Light neutrino and heavy particle exchange in $0\nu\beta\beta$-decay

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A simple and precise method is presented to compare contributions to neutrinoless double-beta decay ($0\nu\beta\beta$-decay) from heavy particle exchange and light Majorana neutrino exchange. This procedure makes no assumptions about the momentum transfer between the two nucleons involved in the $0\nu\beta\beta$-decay process. It is shown that for a general particle physics model, the characteristic $0\nu\beta\beta$-decay scale > 4.4 TeV when all the coupling constants are assumed to be natural and of $O(1)$.

With the discovery of neutrino oscillations a few years ago [1, 2, 3], the fundamental question of whether at least some neutrinos have mass has been answered in the positive. The parallel questions of a) the magnitudes of the neutrino masses and b) the nature of the neutrino mass matrix remain to be answered. If the neutrinos have a Dirac mass matrix, then lepton number is not violated by neutrino interactions while the right-handed neutrinos and left-handed anti-neutrinos are electroweak singlets. Alternatively, for a Majorana neutrino mass matrix, lepton number is violated by two units, and processes like neutrinoless double-beta decay ($0\nu\beta\beta$-decay) are permitted as demonstrated by the Feynman diagram of Fig. 1a.

The observation of $0\nu\beta\beta$-decay would shed light on the neutrino mass magnitude and whether it is Dirac or Majorana, but additional input would be required. Indeed, a number of particle physics models beyond the standard model (SM) have lepton-number violating (LNV) operators that allow $0\nu\beta\beta$-decay through the exchange of heavy particles such as the neutralino [4, 5, 6] or a heavy right-handed neutrino [7, 8, 9]. Thus, the observation of $0\nu\beta\beta$-decay would provide a unique window on physics beyond the SM with broad implications for LNV.

Although it has been known for some time that $0\nu\beta\beta$-decay mediated by light neutrino exchange is suppressed by the ratio of the neutrino mass to charged weak boson mass, $m_{\nu}/m_W$, the fundamental question of whether at least one neutrino has mass has been answered in the positive [10]. The SM low-energy effective Lagrangian with Majorana neutrinos

$$L_{SM} = \frac{G_F}{\sqrt{2}} \bar{\nu}_L \gamma_\mu d_L \bar{\nu}_L \gamma_\mu \nu_L + h.c., \quad (1)$$

gives rise to the $0\nu\beta\beta$-decay operator of Fig. 1b, where a light Majorana neutrino is exchanged. In Eq. (1), $G_F = \sqrt{2}g^2/(8M_W^2)$ is the Fermi constant, $M_W$ the charged weak boson mass, $g = 0.65265$, $u$ and $d$ are the

This might seem counter-intuitive since $0\nu\beta\beta$-decay mediated by light neutrino exchange is suppressed by the ratio of the neutrino mass to $Q^2$ (the momentum squared flowing through the neutrino propagator $|Q| \sim 100$ MeV). In contrast, $0\nu\beta\beta$-decay is suppressed by $\Lambda_{\beta\beta}$ when it occurs through heavy particle exchange, where $\Lambda_{\beta\beta}$ is the heavy scale (typically of the order of 1 TeV or larger) that characterizes the strength of the $0\nu\beta\beta$-decay operator.

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FIG. 1: a) $0\nu\beta\beta$-decay through the exchange of a light Majorana neutrino. b) $0\nu\beta\beta$-decay through the exchange of heavy particles, in this case two squarks and a gluino in RPV SUSY.
up and down quark fields respectively, $e$ the electron field and $\nu$ the neutrino field. From the Feynman rules, the amplitude for this diagram is simply

$$8G_F^2\bar{\nu}_L\gamma^\mu d_L \frac{m_\nu}{Q^2} \bar{u}_L\gamma_\mu d_L \bar{e}_L e_L.$$  \hfill (2)$$

where $m_\nu$ is the neutrino mass.

The current-current interaction of Eq. (1) gives rise to lepton-hadron vertices ($\pi\nu e$, $NN\nu e$) that contribute to $0\nu\beta\beta$-decay through the operators shown in Fig. 2. \hfill (12)

The vertex is assumed to be

\begin{figure}
\begin{center}
\includegraphics[width=0.8\textwidth]{fig2.png}
\end{center}
\caption{0$\nu\beta\beta$-decay operators with light Majorana neutrino exchange. There exists another diagram like (b) where the pion and neutrino lines are exchanged.}
\end{figure}

Operators that contribute to $0\nu\beta\beta$-decay are either suppressed or enhanced by powers of $(p/\Lambda_H)^n$ (where $n$ is the chiral power of the operator) with $p \sim 0.1$ GeV and where $\Lambda_H$ is a hadronic scale $\sim 1$ GeV. The chiral power of a $0\nu\beta\beta$-decay operator can be calculated with the following rules:

- a derivative in a vertex counts as one power of $p$;
- pion and light neutrino propagators count as $p^{-2}$.

Considering that the parity-conserving pion-nucleon vertex is $(g_A/f_\pi)N\gamma^\mu\gamma^5N\partial_\mu\pi$ and noting the derivative in the pion-lepton operator in Eq. (3), one finds that the $0\nu\beta\beta$-decay operators of Fig. 2 are all of $O(p^{-2})$. From Ref. 12, it is seen that these operators have the same chiral power as the LO $0\nu\beta\beta$-decay heavy particle exchange operators. This observation suggests a more precise method to compare heavy and light particle exchange contributions to $0\nu\beta\beta$-decay.

Consider the amplitude for the Feynman graph of Fig. 2b:

\begin{align*}
\text{Fig. 2b} = & \nonumber \\
8G_F^2g_A^2M^2m_\nu \frac{q_1 \cdot q_2}{Q^2 - m_\nu^2} \frac{\bar{p}_1\gamma^\mu n_3}{q_1^2 - m^2_n} \frac{\bar{p}_2\gamma^\mu n_4}{q_2^2 - m^2_n} \times \bar{e}_L e_L c, \\
& \approx 8G_F^2g_A^2M^2m_\nu \frac{\bar{p}_1\gamma^\mu n_3}{q_1^2 - m^2_n} \frac{\bar{p}_2\gamma^\mu n_4}{q_2^2 - m^2_n} \times \bar{e}_L e_L c, \quad (4)
\end{align*}

where the error stemming from writing $q_1 \cdot q_2/Q^2 \cong 1$ is of $O(Q \cdot (k_1 - k_2)/Q^2)$ with $|k_1 + k_2| \cong 2.5$ MeV being the energy carried off by the electrons. The approximation in Eq. (4) is therefore very good. Eq. (4) has the same form as the LO $0\nu\beta\beta$-decay hadronic operators stemming from the exchange of a heavy particle 12. It follows that to a high degree of precision, one can introduce a new “short-distance” $0\nu\beta\beta$-decay operator that stems from light neutrino exchange:

$$L_{\pi\nu ee} \equiv 2m_\nu G_F^2f_\pi^2\bar{\pi}^\nu - \bar{\pi}^\nu \bar{e}_L e_L^c + \text{h.c.} \quad (5)$$

This operator combined with the parity-conserving pion-nucleon vertex yields Eq. (4). In this form, comparing heavy and light particle exchange contributions to $0\nu\beta\beta$-decay is relatively easy. The only caveat is the existence of a possible suppression of Fig. 2a relatively to Fig. 2b, due to the fact that the pion has a finite range. In coordinate space, the suppression of the nuclear matrix elements will occur through exponentials of the form $e^{-m_\nu r}$. In momentum space, the suppression is due to a factors of the form $Q^2/m_\nu^2$. The way to handle this is discussed further below.

From Ref. 12, a LO operator has the form

$$\frac{\lambda^2}{\Lambda_{\beta\beta}^2} \bar{u}_d \bar{u}_d \bar{e}_L e_L^c \quad (6)$$

where the general $0\nu\beta\beta$-decay vertex is assumed to be suppressed by five powers of the $0\nu\beta\beta$-decay scale, $\Lambda_{\beta\beta}$ as occurs in many popular particle physics models. 2

2 Note that the authors of Ref 11 insert an extra factor of $G_F^2M_N^4$ in Eq. (4).
example, this suppression by five powers occurs in R-parity violating supersymmetry (RPV SUSY) and the left-right symmetric model (LRSM).\textsuperscript{3}

This LO operator leads to the $\pi\pi e e$ operator

$$
\frac{\lambda^2}{\Lambda_{\beta\beta}} \bar{u}d \bar{u} c_L e^c_L \rightarrow \beta \frac{\lambda^2}{\Lambda_{\beta\beta}} (\frac{f^2}{2} \pi^- \pi^- e^c_L e^c_L),
$$

where $\beta$ is a parameter of $O(1)$ generated by the hadronization of the quark currents as discussed in Ref.\textsuperscript{12}.

Comparing Eq. (7) with Eq. (5), it is seen that the contributions to $0\nu\beta\beta$-decay from light neutrino exchange and heavy particle exchange processes are equal when

$$\frac{m_\nu}{1 \text{ eV}} = 3.8 \times 10^3 \frac{\lambda^2}{\alpha_M \Lambda_{\beta\beta}} \text{ TeV}^5,$$

where $\alpha_M$ is a number that takes into account the fact that the matrix elements of the pion exchange diagram of Fig. 2 is suppressed with respect to the matrix element of the operator generated by the graph of Fig. 2.

In Ref.\textsuperscript{14}, the nuclear matrix elements from pseudoscalar couplings (corresponding to Fig. 2a) and axialvector couplings (corresponding to Fig. 2c) were computed for nine nuclei and tabulated in their Table II. On average, for light neutrino exchange, the matrix elements stemming from pseudoscalar coupling and denoted $M_{Pp}^{\text{light}}$ in Ref.\textsuperscript{14} are ten times smaller than the matrix elements stemming from axialvector coupling and denoted $M_{AA}^{\text{light}}$. The value $\alpha_M = 10$ will therefore be used. Limits on $\Lambda_{\beta\beta}$ derived below are not very sensitive to the exact value of $\alpha_M$ since $\Lambda_{\beta\beta}$ appears to the fifth power in Eq. (8). factors of two or three in $\alpha_M$ can change the results appearing below by at most 1.2.

Eq. (5) is plotted in Fig. 3. Above the LHE line, light neutrino exchange is larger than the heavy particle contribution to $0\nu\beta\beta$-decay; the reverse is true below the LHE line.

The dashed line in Fig. 3 is the upper-limit on $m_\nu < 0.23$ eV from the WMAP\textsuperscript{13}. The upper-limit on $m_\nu$ implies that the $0\nu\beta\beta$-decay heavy particle exchange operator is larger than the light neutrino exchange contribution for

$$\Lambda_{\beta\beta}\lambda^{-2/5} < 4.4 \text{ TeV},$$

indicated by the first arrow in Fig. 3. This is essentially a model independent limit. If one uses the neutrino mass limit $m_\nu < 0.04$ eV that could be reached by the future Planck mission\textsuperscript{14}, and represented by the dotted line in Fig. 3 then

$$\Lambda_{\beta\beta}\lambda^{-2/5} < 6.2 \text{ TeV},$$

as indicated by the second arrow in the plot.

We can evaluate in specific models the point at which heavy particle exchange becomes larger than light neutrino exchange. Consider the LRSM first, we have\textsuperscript{8,12}

$$\frac{\lambda^2}{\Lambda_{\beta\beta}^5} = \zeta \frac{g^4 M_R^2}{32 M_W^2} \frac{1}{M_R^4 N_R},$$

$$\lambda^2 \equiv \zeta \frac{g^4 M_R^2}{32 M_W^2} < 5 \times 10^{-4},$$

where the limits $\zeta < 10^{-3}$, $M_R > 800$ GeV on the weak gauge boson mixing angle and the right-handed weak boson mass were used in evaluating Eq. (12). Thus, the upper-limit on the right-handed particle masses below which the heavy particle exchange contribution dominates is $M_R \sim N_R \sim \Lambda_R < 0.9$ TeV. Using the lower-limit on the half-life of $0\nu\beta\beta$-decay of $T_{1/2}^{0\nu\beta\beta} > 10^{25}$ years, one obtains a lower-bound on the mass of the heavy right-handed particles of $\sim 1$ TeV\textsuperscript{12}; this implies that the region in the LRSM parameter space where the heavy particles dominate over the light neutrino exchange contributions is essentially ruled out for the current limits on $m_\nu$.

Similarly in RPV-SUSY, the diagram that provides the strongest constraints on $\lambda_{111}^\prime$ is the one with gluino ex-
change shown in Fig. 1b. From Ref. [17], we have
\[ \frac{\lambda^2}{\Lambda_{\beta\beta}^5} \equiv \alpha_S \frac{8\pi \lambda_{\nu\beta\beta}^2}{9 m_q^2 m_S}. \] (13)

Taking \( m_q = m_S = \Lambda_S = 1 \) TeV, \( \alpha_S = 1 \), and substituting in Eq. (9), one obtains that the heavy particle exchange contribution is largest when \( \lambda_{\nu\beta\beta}^2 > 1.5 \times 10^{-2} \). The lower-limit on \( \lambda_{\nu\beta\beta}^2 \) extracted from 0νββ-decay and \( T^{0\nu\beta\beta}_{1/2} > 10^{29} \) years is \( \lambda_{\nu\beta\beta}^2 < 6.3 \times 10^{-2} \) derived from 0νββ-decay in Ref. [17] and Ref. [18] respectively. Hence, the region in RPV-SUSY where the heavy particle exchange contribution dominates over the light neutrino exchange is also ruled out for current limits on \( m_\nu \).

Using the limit in Eq. (10) instead, one obtains
\[ \Lambda_R < 1.4 \text{ TeV}, \quad \lambda_{\nu\beta\beta}^2 > 6.3 \times 10^{-3} \] (14)
instead.

Note that with current limits on 0νββ-decay, one requires the coupling constant \( \lambda \) in Eq. (6) to be \( \ll 1 \) if one demands that \( \Lambda_{\beta\beta} \approx 1 \) TeV. This observation is here verified in the LRSM and RPV SUSY. Although such a small value of \( \lambda \) is clearly allowed, it should be explained since naturalness suggests \( \lambda \sim 1 \) instead. In this case, one would expect \( \Lambda_{\beta\beta} > 4.4 \) TeV.

In this note, a simple and precise method of comparing contributions to 0νββ-decay was presented. It was shown that LO ππe−e− operators can be written down for both light neutrino exchange and heavy particle exchange contributions to 0νββ-decay. This observation facilitated their comparison and allowed us to plot a graph in the neutrino mass and heavy particle scale \( \Lambda_{\beta\beta} \) parameter space to discern the regions where one contribution may be larger than the other. Using current limits on 0νββ-decay, it was also shown that \( \Lambda_{\beta\beta} \gtrsim 4.4 \) TeV in a general particle physics model where the 0νββ-decay operator coupling constant is assumed to be of \( \mathcal{O}(1) \).

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