ANALYTICAL STUDY ON THERMOHALINE CONVECTIVE INSTABILITY IN A MICROPOLAR FERROFLUID

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Abstract: The present investigation is on linear analytical study of thermohaline convective instability in micropolar ferrofluid using perturbation technique. The fluid layer is heated from below and salted from above. The theory of linear stability is used to reduce the non-linear effects on governing equations and normal mode analysis is taken to study. The critical magnetic thermal Rayleigh number \(N_{SC} \) is obtained for sufficient large value of \(M_1\) and an oscillatory instability is determined. The parameters \(N_1\) and \(N_5\) are analyzed for stabilizing behavior and \(N_3, \tau\) and \(M_3\) give the destabilizing behavior. The results are depicted graphically.

Key words: Thermohaline convection, Micropolar ferrofluid, Salinity Rayleigh number, Perturbation technique, linear stability analysis

1. Introduction

Ferrofluids are colloidal suspension of fine magnetic mono domain nano-particles in non-conducting liquids. Such types of ferrofluids have wonderful applications in science and technology. Generally, the ferrofluids are used for cancer treatment in the biomedicine field. An excellent introduction and reviews of this extremely interesting monograph has been given by Rosensweig [1]. In his monograph, the fascinating information is introduced on magnetization. The convection in ferromagnetic fluid is analyzed in various aspects by Chandrasekhar [2]. Finlayson [3] has been investigated the convection in ferrofluid in single component fluid with uniform magnetic fluid. This investigation is extended to porous medium by Vaidyanathan et al. [4]. In non-presence of buoyancy effects, the thermoconvective instability in ferrofluid is given by Lalas and Carmi [5].

The micropolar fluids respond to spin inertia and micro-rotational motions. It can support couple stress and distributed body couples. Eringen [6] introduced the micropolar fluids theory. This theory has been developed by Eringen [7] on thermal effect. An excellent reviews and applications of this fluids theory can be obtained in by Ariman et al. [8] and Eringen [9]. Later, Ahmadi [10] employed firstly the energy method on convective instability of micropolar fluid with use of stability analysis. Pérez-Garcia and Rubi [11] analyzed the micropolar fluids with the effects of overstable motions. Narasimma Murty [12] examined the instability of the Bénard convection in a micropolar fluid using linear stability analysis.

In the effect of porous media, the double-diffusive convection is of greatest interest in mechanical and chemical engineering. In some special case, sodium chloride and temperature field are involving and this is often called as thermohaline convection. Thermohaline convection in a ferrofluid has been analyzed by Vaidyanathan et al. [13] with two-component fluid. The presence of porous medium on ferrothermohaline convection has been given by Vaidyanathan et al. [14].

The theoretical investigation of a micropolar ferromagnetic two component fluid in non-presence of Darcy porous effect has been undertaken by Sunil et al. [15]. The Soret effect is investigated on two component ferrofluid by Vaidyanathan et al. [16] and this is continued to large and small porous effect by Sekar et al. [17, 18]. Reena and Rana [19] have been analyzed the thermostolutal convective instability of micropolar rotating fluids in a porous effect. They used the Darcy model. Chand [20] studied the porous effect on triple-diffusive convective instability in micropolar ferromagnetic fluid.

In present investigation, our intension is to consider salinity gradient on magnetization and magnetic potential equation and thermal convection problem in micropolar fluid of Eringen extend to the thermohaline convection in micropolar ferrofluid. Also, an effect of salinity gradient and how micropolar parameters affect the stability in micropolar ferromagnetic fluid heated from below and salted from above. The stationary and oscillatory instabilities are studied.

2. Mathematical Formulation Of Problem
Here we consider, an infinite horizontal micropolar ferrofluid layer heated from below and salted from above. The fluid layer is of thickness \( d \) and the fluid is considered as an electrically non-conducting incompressible one. The temperature and salinity at the bottom and top surfaces are \( T_0 \pm (dT / dz) \) and \( S_0 \pm (DS / dz) \), respectively and \( \beta_i ( = |dT / dz| ) \) and \( \beta_s ( = |DS / dz| ) \) are maintained. The governing equations are

\[
\nabla \cdot \mathbf{q} = 0
\]

The momentum and internal angular momentum equations are

\[
\rho_0 \left( \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \right) \mathbf{q} = -\nabla p + \rho \mathbf{g} + \nabla (\mathbf{HB}) + 2 \zeta ( \nabla \times \mathbf{q} ) + ( \zeta + \eta ) \nabla^2 \mathbf{q}
\]

\[
\rho_i \mathbf{L} \left( \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \right) \mathbf{\omega} = 2 \zeta ( \nabla \times \mathbf{q} - 2 \mathbf{\omega} ) + \mu_0 ( \mathbf{M} \times \mathbf{H} ) + ( \lambda + \eta ) \nabla ( \nabla \cdot \mathbf{\omega} ) + \eta \nabla^2 \mathbf{\omega}
\]

The temperature equation is

\[
\rho_0 C_{r,H} \left( \frac{\partial T}{\partial t} + \mathbf{q} \cdot \nabla T \right) + \rho_0 C_{i,H} \left( \frac{\partial T}{\partial t} + \mathbf{\omega} \cdot \nabla T \right) = -\nabla \cdot \mathbf{q} + \mu_0 ( \mathbf{M} \times \mathbf{H} ) + \frac{\partial (\mathbf{H} \cdot \mathbf{M})}{\partial t} + \frac{\partial (\mathbf{M} \cdot \mathbf{H})}{\partial t}
\]

The mass flux equation is

\[
\rho_0 (\mathbf{q} / \partial t + \mathbf{q} \cdot \nabla \mathbf{S}) = K_\alpha \nabla^2 \mathbf{S}
\]

We can assume the magnetization using Maxwell’s equation for non-conducting fluids [16-18] is

\[
\mathbf{M} = \mathbf{H} \mathbf{M} (H, T, S) / H. \text{ The linearized magnetic equation in term of } H_0, T_a \text{ and } S_a \text{ is }
\]

\[
M = M_0 + \chi (H - H_0) - K (T - T_0) + K_\alpha (S - S_a)
\]

The density equation of state is

\[
\rho = \rho_0 [1 - \alpha_t (T - T_0) + \alpha_s (S - S_a)]
\]

where \( \mathbf{q} \) - velocity of fluid, \( \rho_0 \) - mean density of the clean fluid, \( p \) - pressure, \( \rho \) - density of the fluid, \( \mathbf{g} \) - gravitational field, \( \mathbf{H} \) - magnetic field, \( \mathbf{B} \) - magnetic induction, \( \zeta \) - coupling viscosity, \( \omega \) - microrotation, \( \eta \) - shear viscosity coefficient, \( I \) - moment of inertia, \( \mathbf{M} \) - magnetization, \( \lambda \) - bulk spin viscosity, \( \eta \) - shear spin viscosity, \( C_{r,H} \) - effective heat capacity at constant volume, \( C_t \), \( C_s \) - specific heat solid material, \( H_0 \) - viscosity of the fluid when the applied magnetic field is absent, \( K_1 \) - thermal diffusivity, \( T \) - temperature, \( \delta \) - micropolar heat conduction coefficient, \( S \) - solute concentration, \( K_1 \) - concentration diffusivity, \( H_0 \) - uniform magnetic field, \( T_a \) - average temperature, \( S_a \) - average salinity, \( \alpha_t \) - thermal expansion coefficient and \( \alpha_s \) - analogous solvent coefficient.

The basic state quantities are
\[ M_b(z) = \begin{bmatrix} M_0 + \frac{K \beta_1 z - K_2 \beta_5 z}{1 + \chi} \end{bmatrix} k, \quad \rho(z) = \rho_0 [1 + \alpha \beta_1 z - \alpha_5 \beta_5 z], \quad q = q_b = (0, 0, 0). \]

\[ H_b(z) = \begin{bmatrix} H_0 - \frac{K \beta_1 z + K_2 \beta_5 z}{1 + \chi} \end{bmatrix} k, \quad \omega = \omega_b = (0, 0, 0). \]

\[ T_b = T_0 - \beta z, \quad S_b = S_0 - \beta_3 z, \quad p = p_b(z) \quad \text{and} \quad M_0 + H_0 = H_0^{'}. \]

where subscript \( b \) — the basic state and \( \hat{k} \) — unit vector vertical direction.

A small thermal disturbance is made on the system. Let us take the perturbed components of \( M \) and \( H \) be \([M_1^{'}, M_2^{'}, M_0(z) + M_3^{'})\) and \([H_1^{'}, H_2^{'}, H_0(z) + H_3^{'})\) respectively. The perturbed quantities are

\[ H = H_b(z) + H', \quad \omega = \omega_b + \omega', \quad p = p_b(z) + p', \quad S = S_b(z) + S', \]

\[ M = M_b(z) + M', \quad q = q_b + q', \quad \rho = \rho_b + \rho'.T = T_b(z) + \theta, \]

the superscript ‘ denotes perturbed state.

The perturbed density equation can be calculated as

\[ \rho' = \rho_0 (-\alpha \theta + \alpha, S') \]

1. normal mode analysis method

We undertake the perturbation quantities by use of normal modes are

\[ w(x, y, z, t) = w(z, t) \exp[i k_x x + i k_y y] \]

\[ \theta(x, y, z, t) = \theta(z, t) \exp[i k_x x + i k_y y] \]

\[ \phi(x, y, z, t) = \phi(z, t) \exp[i k_x x + i k_y y] \]

\[ S(x, y, z, t) = S(z, t) \exp[i k_x x + i k_y y] \]

In Eq. (2), one can get the \( k^0 \) component is

\[ \left( \rho_0 \frac{\partial}{\partial t} \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \right) w = \mu_0 K \beta_1 k_0 \frac{\partial^2 \phi}{\partial z^2} - \left( \frac{\mu_0 K^2 \beta_1}{1 + \chi} \right) k_0^2 \theta + \left( \frac{\mu_0 K K_2 \beta_1}{1 + \chi} \right) k_0^2 S - \mu_0 K_2 \beta_5 k_0 \frac{\partial^2 \phi}{\partial z^2} + \left( \frac{\mu_0 K K_2 \beta_5}{1 + \chi} \right) k_0^2 \theta \]

\[ \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) k_0^2 S - \rho_0 g \alpha \kappa_0 \theta + \rho_0 g \alpha \kappa_5 k_0^2 S + 2 \zeta \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \Omega_3 + \left( \zeta + \eta \right) \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right)^2 \]

Internal angular Eq. (3) can be manipulated as

\[ \rho_0 J \frac{\partial \Omega_3}{\partial t} = -2 \zeta \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) w + 2 \Omega_3 + \eta \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right)^2 \Omega_3 \]

Eq. (4) can be calculated as

\[ \rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial^2 \phi}{\partial z^2} = K_1 \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \theta + \rho C_2 \beta_1 \left( \frac{\mu_0 K^2 T_0^2 \beta_1}{1 + \chi} \right) + \left( \frac{\mu_0 K K_2 \beta_5}{1 + \chi} \right) w - \delta \beta_1 \Omega_3 \]

The Salinity equation is

\[ (\partial / \partial t) S + \beta S w = K_S \left( \partial^2 / \partial z^2 - k_0^2 \right) S \]

Using Vaidyanathan et al. [14], one gets

\[ (1 + \chi) \frac{\partial^2 \phi}{\partial z^2} - \left( 1 + \frac{M_0}{H_0} \right) k_0^2 \phi - K \frac{\partial \theta}{\partial z} + K_2 \frac{\partial S}{\partial z} = 0 \]

The non-dimensional equations can be derived by use of normal mode method as
\[
\left(\frac{\partial}{\partial t} - D^2 - a^2 \right) w^* = a M_1 R^{1/2} \frac{\partial}{\partial t}D \phi^* - a(1 + M_1) R^{1/2} D \phi^* + a R^{1/2}_S(1 + M_4) S^* - a R^{1/2}_T(1 + M_5) T^* + a R^{1/2}_M(1 + M_5) M_2 S^* + 2 N_1 (D^2 - a^2) \Omega_1^* + (\zeta + \eta)(D^2 - a^2)^2 w^*
\]
}\]
\[
I' \frac{\partial \Omega_2^*}{\partial t} = -2 \left[ (D^2 - a^2) w^* + 2 \Omega_3^* \right] N_1 + (D^2 - a^2) N_3 \Omega_3^* \]
\]
\[
P \frac{\partial T^*}{\partial t} - M_2 P \frac{\partial}{\partial t}(D \phi^*) = (D^2 - a^2) T^* + a (1 - M_2 - M_5 M_4) R^{1/2} w^* - a N_2 R^{1/2} \Omega_2^* \]
\]
\[
P \frac{\partial S^*}{\partial t} = \tau (D^2 - a^2) S^* - a R^{1/2}_{S}\left(\frac{M_5}{M_6}\right) w^*, \quad \tau = \frac{R}{R_S} \frac{1}{R^{1/2}} \]
\]
\[
D^2 \phi^* - M_2 a^2 \phi^* - D T^* + M_5 \left(\frac{R}{R_S}\right)^{1/2} D S^* = 0,
\]
\]
where the dimensionless quantities are
\[
\begin{align*}
w^* &= \frac{w d}{v}, \quad t^* = \frac{vt}{d^2}, \quad T^* = \left(\frac{k_a a R^{1/2}}{\rho C_{v, r} K \beta v d}\right) \theta, \quad \phi^* = \left(\frac{(1 + \chi) K_a a R^{1/2}}{\rho C_{v, r} K \beta v d^2}\right) \phi, \quad z^* = \frac{z}{d}, \quad a = k_p d, \quad D = \frac{\partial}{\partial z^*} \\
S^* &= \left(\frac{K_a a R^{1/2}}{\rho C_{v, r} K \beta v d^2}\right) \frac{S}{v} = \left(\frac{\beta}{\rho_0}\right) \frac{\Omega_1}{v} d^3, \quad M_1 = \left(\frac{\mu_0 K^2 \beta}{(1 + \chi) \rho_0 \beta v d}ight), \quad M_2 = \left(\frac{\mu_0 K^2 T}{(1 + \chi) \rho_0 C_{v, r}}\right), \quad N_1 = \zeta, \quad N_3 = \eta, \quad M_3 = \frac{1 + M_4}{(1 + \chi)}, \quad M_4 = \frac{K_a a R^{1/2}}{\rho C_{v, r} K \beta v d^2}, \quad M_5 = \frac{K_a a R^{1/2}}{\rho C_{v, r} K \beta v d^2}, \quad \tau = \frac{\rho_0 C_{v, r} K}{K_3}, \quad N_5 = \frac{\eta^*}{\eta d^2}, \\
N_4 &= \frac{\delta}{\rho C_{v, r} d^2} \quad I' = \frac{1}{d^2}, \quad P_s = \frac{v}{K_1} \rho C_{v, r}, \quad P_r = \frac{v}{K_1} \rho C_{v, r}, \quad R_s = \frac{\rho C_{v, r} \beta \alpha g d^4}{v K_3}, \quad R = \frac{\rho C_{v, r} \beta \alpha g d^4}{v K_1}
\end{align*}
\]
\]
2. Linear stability analysis
\]
The stationary and oscillatory instabilities have been studied using linear theory. The boundary conditions are
\[
w^* = D^2 w^* = D \phi^* = S^* = \Omega_1^* = T^* = 0 \quad \text{at} \quad z^* = \pm 1/2.
\]
\[
\text{The exact solutions satisfying above Eq. (23) are}
\]
\[
w^* = X_1 e^{\alpha z^*}, \quad T^* = X_2 e^{\alpha z^*} \cos \pi z^*, \quad S^* = X_3 e^{\alpha z^*} \cos \pi z^*, \quad \phi^* = X_4 e^{\alpha z^*} \sin \pi z^*, \quad \Omega_1^* = X_5 e^{\alpha z^*} \cos \pi z^*
\]
\]
where \(X_1, X_2, X_3, X_4\) and \(X_5\) are constants. Eqs. (17)–(21) can be mathematically manipulated using Eq. (23) as
\[
(\pi^2 + a^2) \left[ \sigma + a \frac{(1 + N_1)(\pi^2 + a^2)}{2} \right] X_1 - a R^{1/2} \left[1 + (1 + M_4) M_2\right] X_2 + a \left(1 + M_4 + M_4 M_1^{-1}\right) R^{1/2} X_3 - a(1 + M_5) R^{1/2} M_2 X_4 - 2(\pi^2 + a^2) N_1 X_5 = 0
\]
\]
\[
-2(\pi^2 + a^2) N_1 X_1 + \left[4 N_1 + (\pi^2 + a^2) N_3 + I' \sigma \right] X_5 = 0
\]
\]
\[
a(1 - M_2 - M_5 M_4) R^{1/2} X_1 - (P_1 \sigma + \pi^2 + a^2) X_2 + P_1 \sigma M_4 X_4 - a N_1 R^{1/2} X_5 = 0
\]
\]
\[
a M_4 R^{1/2} X_1 + \left[\pi^2 + a^2\right] \tau + \sigma P_1 X_1 = 0
\]
\]
\[
- R^{1/2}_S \pi^2 X_2 + R^{1/2}_S \pi^2 (M_5 / M_6^{-1}) X_3 + R^{1/2}_S (\pi^2 + a^2 M_4) X_5 = 0
\]
To evaluate the Eigen function, determination of the co-efficient of $X_1$, $X_2$, $X_3$, $X_4$ and $X_5$ in Eqs. (25)–(29) is equal to zero. Using the analyses Vaidyanathan et al. [13, 14], Eqs. (25)–(29) have been adopted to get

$$T_1\sigma^4 + T_2\sigma^3 + T_3\sigma^2 + T_4\sigma + T_5 = 0$$  \hspace{1cm} (30)$$

where

$$T_1 = P_1(I' - x_1), \quad T_2 = P_1'I'x_1^2(1 + P_1') - P_1x_1(P_1' + I')$$

$$T_3 = a^2x_2P_1'I'R - x_8x_2P_1'^2 + x_1^2(1 + P_1')(P_1' + I') + x_8P_1'I(a^2x_2 - x_7^2) - x_1^2x_8x_7\tau P_1'$$

$$+ a^2M_6P_1'I'R(x_4x_6 + x_5x_8)$$

$$T_4 = -a^2\pi^2x_4x_1'I'R - a^2\pi^2P_1'x_4R(x_5 - x_8N_5') - x_8x_6P_1'(a^2x_2N_4'R + x_1x_6) - x_1^2x_8x_6\tau P_1'$$

$$+ (P_1' + I')(a^2x_2x_1R - x_7^2) - x_1^2x_8x_6\tau(1 + P_1') + x_8P_1'I(x_7 - x_1I')$$

$$x_5 = x_1\tau(\pi^2(a^2x_2x_1R - a^2x_2x_8N_4'R) + x_8(a^2x_2R + x_1x_6) - x_7(a^2x_2R - x_7^2))$$

$$- x_1^2x_8x_6(x_4x_6 + x_5x_8)$$

$$x_1 = \pi^2 + a^2, \quad x_2 = 1 + x_4, \quad x_3 = 1 + M_4 + (M_4/M_5), \quad x_4 = M_4(1 + M_5),$$

$$x_5 = 2N_4, \quad x_6 = x_4x_5, \quad x_7 = 4N_1 + x_1N_3', \quad x_8 = \pi^2 + a^2M_3, \quad x_9 = M_5\pi^2/M_6$$

4.1 The case of stationary instability

For steady state, we have $\sigma = 0$ at the marginal stability. Then the Eq. (30) leads to get Eigen value $R_{ec}$ for which solution exists. Using the analyses [14–15], the critical magnetic Rayleigh number $R_{ec}$ has been obtained using

$$R_{ec} = \frac{Nr}{Dr}$$  \hspace{1cm} (31)$$

where

$$Nr = (\pi^2 + a^2)^2\left((4N_1 + (\pi^2 + a^2)N_3')\left(1 + N_1\right) - 4N_2^2\right) - a^2\left(1 + M_4 + M_4/M_5\right)\left(4N_1 + (\pi^2 + a^2)N_3\right)M_6R_s$$

$$Dr = a^2\left(1 + M_4(1 + M_5)\right)\left(4N_1 + (\pi^2 + a^2)\left(N_3' - 2N_1N_5\right)\right)$$

$$- a^2\pi^2M_4\left(1 + M_5\right)\left(4N_1 + (\pi^2 + a^2)N_3\right)\left(1 + M_5\pi^2\right) - 2N_1N_3'(\pi^2 + a^2)$$

When $M_4$ is very large, one can gets $N_{ec} = (M_4R_{ec})$.

$$N_{ec} = \frac{Nr}{Dr}$$  \hspace{1cm} (32)$$

where

$$Dr = a^2\left(1 + M_5\right)\left(4N_1 + (\pi^2 + a^2)\left(N_3' - 2N_1N_5\right)\right)$$

$$- \frac{a^2\left(1 + M_4\right)\pi^2}{\pi^2 + a^2M_3}\left(4N_1 + (\pi^2 + a^2)N_3\right)\left(1 + M_5\pi^2\right) - 2N_1N_3'(\pi^2 + a^2)$$

Here $a$ is denoted as critical wave number $a_c$. Analysis of the classical results is given below:

Assuming $a = 1, \quad N_3 = 1, \quad N_1 = 0, \quad$ and $N_5' = 0$ in Eq. (32), one get

$$N_{ec} = \frac{(\pi^2 + a^2)^2 - a^2\left(1 + M_4 + M_4/M_5\right)M_6R_s\pi^2}{a^2\left(1 + M_5\right)\left(1 - \pi^2\left(1 + M_5\pi^2\right)\right)\left(\pi^2 + a^2M_3\right)}$$  \hspace{1cm} (33)$$

which is an expression for $N_{ec}$ of Vaidyanathan et al. [13].

Moreover, if $M_4$, $M_5$, $\pi^{-1}, \quad R_s = 0$ in Eq. (33), it gives $N_{ec}$ of the Finlayson [3], a single component fluid.

4.2 The case of oscillatory instability
Taking \( \sigma = i\sigma_c \) \((\sigma_c > 0)\) in Eq. (30), it leads to \( R_{nc} \) has been derived using

\[
R_{nc} = \left( T_1 Y_0 \sigma_c^6 + (T_1 Y_5 + T_2 Y_3) \sigma_c^4 + (Y_2 (Y_5 + Y_6) + Y_3 Y_4) \sigma_c^2 + Y_5 (Y_5 + Y_6) \right) / Dr
\]

where

\[
Y_1 = a^2 M_6 P_1' (x_2 x_0 + x_3 x_5) - a^2 x_2 x_6 P_1' I - a^2 x_2 x_5 P_1' I,
Y_2 = x_7 (1 + P_1 (P_1' + I') - x_9 I)^2 - x_7 x_5 P_1' I - x_7 x_6 P_1' I - x_7 x_5 x_6 x_7,
Y_3 = -a^2 \pi^2 I \epsilon x_7 x_4 - a^2 \pi^2 P_1' (x_7 - x_6 N_5') - a^2 x_2 x_6 P_1 x_7 x_5 x_6 x_7 - x_7 x_5 x_6 \epsilon x_7 x_6 x_7 P_1' + a^2 x_2 (P_1' + I')
\]

\[
- a^2 M_6 x_2 x_6 (P_1 x_7 - x_6 I'),
Y_4 = -x_9 x_6 P_1 \epsilon - (P_1 + I') x_5^2 + x_7 x_5 \epsilon (1 + P_1) - (P_1 x_7 - x_6 I) a^2 M_6 x_3 x_6 R_5,
Y_5 = x_7 \pi^2 (a^2 x_4 x_7 - a^2 x_3 x_6 N_5') + a^2 x_2 x_5 x_6 x_7 N_5' + a^2 x_2 x_6 x_7 x_8 + a^2 M_6 x_2 x_4 x_7 x_8,
Y_6 = \epsilon x_7 x_8 x_9 + a^2 M_6 P_1 x_7 x_8 - \epsilon x_7 x_8 x_9,
\]

\[
Dr = (Y_2 \sigma_c^2 + Y_3)^2 - \sigma_c^2 Y_3^2, \quad \sigma_c^2 = (-A_2 \pm \sqrt{A_2^2 - 4A_3 A_4}) / 2 A_4,
A_3 = Y_2 Y_5 - Y_5 (Y_5 + Y_6)
\]

3. Discussion of Results

In this investigation, thermohaline convection in micropolar ferrofluid layer is studied. The fluid layer is heated from below and salted from above and the convective system is subjected to a transverse uniform magnetic field. The thermal perturbations and linear stability analysis are used in the study. Here we consider the free boundary conditions. The magnetic numbers \( M_1 \) and \( M_2 \) are considered the values 1000 and 0, respectively. \( M_3 \) is taken from 5 to 25 (Vaidyanathan et al. [14]) and \( T \) ranges from 0.05 to 0.11 (Vaidyanathan et al. [14]) and \( R_5 \) taken from -500 to 500. The magnetization parameters \( M_4 \) and \( M_5 \) are taken to be 0.1 and 0.5 (Vaidyanathan et al. [16]). Further, \( N_1 \), \( N_3 \), and \( N_5 \) are taken to be non-negative values which is presented by Eringen [21] and he assumed the clausius-Duhem inequality. \( P_i \) is taken as 0.01.

The variation of \( N_{nc} \) with the coupling parameter \( \epsilon \) is depicted in Fig. 2 (a) and (b). It is observed from the Fig. 2 (a) that the convective system gives stabilizing behavior, when increasing values of \( M_3 \) and \( R_S \). Due to the increasing value of \( N_1 \) from 0 to 1 and \( R_S \) from -500 to 500 on the system, \( N_{nc} \) gets the highest values and the system has more stabilizing effect. But, an increasing of \( M_3 \) from 5 to 25, the system shows the stabilizing effect and it is less pronounced. \( M_3 \) analyzed for destabilizing behavior always [13-14, 16-17]. But, introducing of \( N_1 \) on \( M_5 \), the system gets stabilizing effect. Fig. 2 (b) represents the plot of \( N_{nc} \) versus \( N_1 \) for different \( T \). This figure shows that \( N_1 \) has the stabilizing behavior for increasing value of \( T \). This is because, the greater the mass and heat transports and more buoyancy energy, which contributes to thermal instability. Also, it is shown from the Fig. 2 (c) that the increase in \( N_1 \) stabilizes the system for increasing of \( T \) and \( R_5 \). Also, in the presence of \( R_S = 500, a_5 \) is close to zero. In this moment, the system has an equilibrium state.

Figs. 3 (a) and (b) display the variation of \( N_{nc} \) versus spin diffusion parameter \( N_3 \), for increasing of \( R_S \) and \( M_3 \) and \( T \), respectively. In Fig. 3 (a), we observe that \( N_{nc} \) decreases with increasing of \( N_3 \), which leads to destabilize the system. Moreover, when \( R_S = 500, N_{nc} \) gets zero value. Therefore, the system has a null effect. From Fig. 3 (b), it is seen that \( N_3 \) increases from 2 to 8, there is a decrease in \( N_{nc} \) indicating destabilization for different \( T \). Fig. 3 (c) shows the variation of \( a_6 \) versus \( N_3 \), for various \( T \), \( R_S \) and \( M_3 \). When \( T \) increases from 0.05 to 0.1, \( R_S \) increases from -500 to 500 and \( M_3 \) increases from 5 to 25, there is a decrease in \( a_6 \). It clear that there is a destabilization on the system which is not much pronounced and when \( R_S = 500 \), there is an oscillation in \( a_6 \).

Figs. 4 (a) and (b) show the variation of \( N_{nc} \) versus micropolar heat conduction parameter \( N_5 \) for different \( M_3, T \) and. It is clear from the Fig. 4 (a) that \( N_5 \) leads to an increase in \( N_{nc} \). Therefore, \( N_5 \) has a stabilizing effect. It is very clear from the Fig. 4 (b) that increase in \( N_5 \), it is stabilizing behavior for various \( R_S \). Fig. 4 (c) represents the critical wave number \( a_6 \) versus the \( N_5 \) for various physical parameter \( T, R_S \) and \( M_3 \). In this figure \( N_5 \) shows a stabilizing behavior. In such situation also the system no effect when \( R_S = 500 \).
The increase in non-buoyancy magnetization parameter $M_3$ is obtained to cause large destabilization, because both thermal and magnetic mechanisms favor destabilization. This can be studied from Figs. 5 (a) and (b) in which the increase in $M_3$ and $\tau$ decrease in $N_{nc}$ and $\alpha_c$, respectively.

From Fig. 6 (a), it is seen that an increase in $R_S$, decrease in $N_{nc}$. An increase of $R_S$ would means that the system is salted from above. Also, when $R_S = 500$ and $\tau (=0.05, 0.07, 0.09)$, the $N_{nc}$ gets small values. But for the value $\tau = 0.11$, suddenly $N_{sc}$ gets highest value. In this moment, the convective system gets stabilizing effect. From Fig. 6 (b), it is seen that an increase in $R_S$, decrease in $N_{sc}$. An increase of $R_S$ would means that the system is salted from above. Also, when $R_S = 500$ and $\tau (=0.05, 0.07, 0.09)$, the $N_{nc}$ gets small values. But for the value $\tau = 0.11$, suddenly $N_{sc}$ gets highest value. In this moment, the convective system gets stabilizing effect. That is, the system converges to the small values. But, when the highest values of $\tau (=0.11)$, the system has an equilibrium position.

4. Conclusion

In the present analysis, the results of a theoretical study on thermohaline convection in a micropolar ferrofluid are considered with free boundary conditions. We conclude that the effect of non-buoyancy magnetization $M_3$, salinity effect $R_S$, spin diffusion parameter $N_{3}'$ have destabilizing behavior and the effect of coupling parameter $N_1$ and the micropolar heat conduction parameter $N_{5}'$ have a stabilizing effect due to the microrotation on the onset of convection.
Fig. 3 (a). Variation of $N_{sc}$ versus $N_{3}'$ for different $M_3$ and $R_S$. $N_i = 0.2$, $N_S = 0.2$ and $\tau = 0.05$.

Fig. 3 (b). Variation of $N_{sc}$ versus $N_{3}'$ for different $\tau$. $N_i = 0.2$, $N_S = 0.2$, $M_3 = 5$ and $R_S = -500$.

Fig. 3 (c) – Variation of $a_c$ versus $N_{3}'$ for different $R_S$, $M_3$ and $\tau$. $N_i = 0.2$, $N_S = 0.2$ and $M_3 = 5$.

Fig. 4 (a). Variation of $N_{sc}$ versus $N_{5}'$ for different $M_3$ and $\tau$. $N_i = 0.2$, $N_S = 2$ and $R_S = -500$.

Fig. 4 (b). Variation of $N_{sc}$ versus $N_{5}'$ for different $R_S$, $\tau = 0.05$, $N_i = 0.2$, $N_S = 2$, $M_3 = 5$ and $R_S = -500$. 
Fig. 4 (c). Variation of \(a_c\) versus \(N_S^*\) for different \(R_s, M_3\) and \(\tau, N_I = 0.2, N_{S^*} = 0.2\) and \(M_3 = 5\).

Fig. 5 (a). Variation of \(N_{sc}\) versus \(M_3\) for different \(\tau, N_I = 0.2, N_{S^*} = 2, N_{S^*} = 0.2\) and \(R_s = -500\).

Fig. 5 (b). Variation of \(a_c\) versus \(M_3\) for different \(\tau, N_I = 0.2, N_{S^*} = 2, N_{S^*} = 0.2\) and \(R_s = -500\).

Fig. 6 (a). Variation of \(N_{sc}\) versus \(\tau\) for different \(R_s, N_I = 0.2, N_{S^*} = 2, N_{S^*} = 0.2\) and \(M_3 = 5\).

Fig. 6 (b). Variation of \(a_c\) versus \(R_s\) for different \(\tau, N_I = 0.2, N_{S^*} = 2, N_{S^*} = 0.2\) and \(M_3 = 5\).
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