Decision making dynamics in corporate boards

Stefano Battiston (1) Eric Bonabeau (2) and Gérard Weisbuch (1)

(1) Laboratoire de Physique Statistique, ENS, 24 rue Lhomond, 75005 Paris, France
(2) Icosystem Corp., 545 Concord Av. Cambridge, MA 02138, USA
Correspondence to: stefano.battiston@ens.fr, tel +33144323623, fax +33144323433

October 24, 2018

Abstract

Members of boards of directors of large corporations who also serve together on an outside board, form the so called interlock graph of the board and are assumed to have a strong influence on each others’ opinion. We here study how the size and the topology of the interlock graph affect the probability that the board approves a strategy proposed by the Chief Executive Officer. We propose a measure of the impact of the interlock on the decision making, which is found to be a good predictor of the decision dynamics outcome. We present two models of decision making dynamics, and we apply them to the data of the boards of the largest US corporations in 1999.
PACS: 89.75 -k; 89.65 -s
Keywords: social networks, opinion dynamics, directorate interlock, phase transition, Ising model.

1 Introduction

The boards of a set of companies together with their directors form a bipartite network. The director network is the network obtained taking the directors as nodes, and a membership in the same board as a link. It is well known that the director network of the largest companies in the US and in other countries has a high degree of interlock, meaning the fact that some directors serve on several boards at the same time so that many boards are connected by shared directors. Interlock convey information and power. For example a bank that lends money to an industry can use interlocked directors in industries of the same domain to get additional information about the real risk of the loan.

As a consequence of economic power concentration over the last decades, ”a special social type emerges spontaneously, a cohesive group of multiple directors tied together by shared background, friendship networks, and economic interest, who sit on bank boards as representative of capital in general”[1]

Now, while part of the public opinion has been since long ago concerned about
the fact that the corporate elite would represent a sort of “financial oligarchy controlling the business of the country” [2], stockholders are more concerned about the effectiveness of boards in overseeing management. Board’s directors should in fact monitor managers’s strategies and decisions to the interest of stockholders. So, after the recent highly visible cases of bankruptcy in the US, the role of boards in the decision making process is now largely debated and more sophisticated forms of corporate control are advocated.

In this regard there are some works in sociology, investigating whether boards have adequate knowledge and information to make meaningful contributions to strategic decision making. Authors try to assess how multiple boards appointments affect directors’ ability to contribute to strategy [3][4]. This kind of study is usually done by means of surveys and no modelling of the dynamics is involved.

Some authors have studied the topological properties of the corporate elite network: Davis and collaborators have shown [5] that the directors network and the boards network of the largest US corporations has Small World properties [6]. German firms too, turn out to form a Small World [7]. Vedres [8] has analyzed the social network composed by directors of the the largest Hungarian companies, banks and government leaders relating the power of social actors to their properties as nodes of the network.

Finally, some authors have studied the diffusion of governance practices such as the so called ‘poison pill’ and ‘golden parachute’ [9], throughout the board network, with an epidemiological approach.

In our work we combine the study of the topological properties of the interlock with the modelling of the dynamics of decision making. Directors have to vote in order for the board to take a decision. It is clear that the social ties linking a director to the other directors in the board influence the formation of his/her opinion. Two directors in the board who also serve in another outside board are likely to take each other’s opinion into account more seriously than the opinion of another director (see below). If there are several directors that, within a board, share additional ties among them, they form a sort of ”lobby”. The question we address here is whether the lobby can influence significantly the decision making process of the whole board. The problem concerns of course not only the boards of large corporations, but many governance structure in social institutions and it is of general interest in social science modelling. We study the boards of the largest corporations because it is a relatively well defined framework and there are data available about the social connections among agents.

Now, one can think of two kinds of decisions a board is faced to: there are decisions regarding a topics specific to a board, such as the appointment of a candidate member. For such decisions, we might suppose that different boards don’t influence each other. There are also decisions about topics related to general trends in the economy such as whether to fire part of the employees, depending on the forecast of economical recession or whether to adopt some governance practice [9]. In those cases, decisions previously taken in some boards might influence other boards. The present work only considers decision of the first kind when a single board decides on some issue independently of
other boards decisions.

In general, models of social choice assume that agents form their opinion according to the information available to them about the state of the world and to the opinions of other agents [10][11][12]. As we said, the interlock comes in the decision making process because we will assume that two directors serving at the same time on several boards have stronger influence on each other. One of the rationale for this assumption is the fact the recruiting mechanism itself relies on personal familiarity: a candidate member is proposed and supported by members who already know him/her because they serve or have served together in another board [4][5]. As a result, interlocked members are likely to be more influential on each other’s opinion.

There exist a large literature about committees and collective decision making, but little numerical or analytical modelling. We start from the standard assumptions of herd behavior [10], which describe individual decisions as based on the successive surveys of other agent opinions; we call this first model to be later fully described as a survey model. A second model is based on the succession of interventions of speakers during the board session, each speaker influencing other directors during his (or her) intervention; this model is called a broadcast model.

We then investigate, for the two models, the effect of the size and topology of the graph of interlocked directors of a board, on the final board decision.

This paper is organized as follow: we first present some statistics based on empirical results concerning the US Fortune 1000 companies. We then describe the survey model, the relevant quantities to be monitored in simulations and check the simulation results with standard mean field results in the absence of interlock. The next section is devoted to a search for a good predictor of the dynamics in the presence of interlock, and to simulation results obtained with test interlock graphs and with empirical board interlock graphs. Similar tests are done for the broadcast models. In the last section we compare and discuss the results.

2 Interlock graphs

In the literature, the topological properties of interlocking directorate are studied for the director network as a whole [5]. We here focus instead on the interlock inside each single board. We call interlock graph of a Board the graph obtained by representing directors of a board as nodes and drawing an edge between two directors if they serve together on an outside board (Figure 1).

Before investigating how the structure of the interlock graph affects the decision making process, we want to know how a typical interlock graph looks like in real boards. We have analyzed data that have been kindly provided by G.Davis [5], about the boards of the US Fortune 1000 companies (year 1999). We found that 321 boards out of 821 have a non empty interlock graph. 20 per cent of all boards have a 1-link interlock graph, another 20 per cent have a more complex
interlock graph. An example of a board with a complex interlock graph, the board of directors of the Bank of America Corp. is shown in Figure 1. Within the 321 interlock graphs there are chains, cliques (subgraphs in which each node is connected to all the others) and various combinations of these components. In particular, we looked at the largest clique in the interlock graph and we found 25 boards with a clique of three nodes, 9 with a clique of 4 nodes. We also looked at the largest connected component (LCC) in the graph and we found 65 boards with a LCC of 3 nodes, 31 boards with a LCC of 4 nodes, 9 boards with a LCC of 5 nodes, 4 boards with a LCC of 6 nodes and 2 boards with a LCC of 8 nodes.

We present in Figure 2 the histograms of board size (number of directors in the board), lobby size (number of directors involved in the interlock graph) and number of links of the interlock graph. Only the 321 boards with non empty interlock graph are considered in the histograms. The average board size is $12.4 \pm 3.6$, the distribution is unimodal, skewed to the right. The smallest and largest board have size 5 and 35 respectively. Lobby size ranges up to 12 nodes.

Another interesting quantity is the ratio between size of the lobby and size of the board (bottom left) which has a mean of .19. The distribution is obviously non gaussian with a long tail.

From the above analysis we see that the fraction of boards of the 1000 Fortune companies, that exhibit a complex interlock graph, is far from being negligible. It is therefore of great interest to try to model its effect on the decision making dynamics.

## 3 The survey model

We want to model the process of decision making on a single board. We first consider the most standard model in economics used to model herd behavior, which we here call the survey model. The model is basically an iterated voting process.

At each time step, one director randomly chosen polls opinions of other agents and makes his opinion accordingly, most often taking the opinion of the majority.

More precisely, at the board meeting, the CEO proposes a strategy for the company. The board directors discuss the strategy and at the end take a decision by voting. We stylize the situation saying that there are only 2 opinions: opinion +1 corresponds to approving CEO's strategy, and -1 to refusing it. The CEO always sticks to opinion +1. The other directors can have opinion +1 or -1. Directors discuss between each other and get to know the opinions of all their colleagues, which they take into account to formulate a new opinion. Other colleagues' opinions define a field: the field is a weighted sum of colleagues' opinions, where the weights depend on the number of boards on which two directors sit together. The new opinion depends stochastically on the intensity of the field.

In formulas the model reads as follows. The opinion of director $i$ is a binary
variable $s_i = \pm 1$. The field acting on director $i$ is:

$$h_i = \sum_{j=1}^{m} J_{ij}s_j$$

$m$ being the size of the board, $J_{i,j}$ being the number of boards on which directors $i$ and $j$ sit together. Obviously, directors take into account their own opinion, hence $J(i, i)$ in equ. 1 must be non zero. Setting $J(i, i)$ to 1 is not very realistic, since it implies that a director with some interlock ties assigns a larger weight to his colleagues’ opinion than to his/her own opinion. We chose to set $J(i, i)$ as the number of boards where director $i$ serve with at least one other director of the same board.

The probability that director $i$ takes some opinion $\pm 1$ at time $t + 1$ is given by:

$$P\{s_i(t + 1) = \pm 1\} = \frac{\exp(\pm \beta h_i(t))}{\exp(\beta h_i(t)) + \exp(-\beta h_i(t))}$$

Parameter $\beta$ in the opinion update acts as the inverse of a temperature. It measures the degree of independence of a director’s opinion from the field. At $T=0$ the opinion dynamics becomes deterministic, at infinite $T$ the dynamics becomes random. The Boltzmann formalism, often referred to by economists as the logit function, can be justified by several considerations such as errors in opinion propagation and random fluctuations of some external conditions. What is meaningful for us is that a small amount of fluctuation is sufficient to remove the system from spurious attractors.

In the next we will refer to dynamics with CEO and without CEO, meaning respectively, that there is a director with a constant opinion $+1$, or not. Formally the model is analogous to an Ising magnetic system.

### 3.1 Variables characterizing the state of the system

We here define some macroscopic variables describing the state of the system. The value of the opinion averaged over directors of the board is called $M$:

$$M \equiv \frac{1}{m} \sum_{j=1}^{m} s_j$$

$M$, is the analog of the magnetization in the Ising model and is a function of time. We denote the magnetization at time 0 and at large time $T$ as respectively: $M^0 = M(t = 0)$ and $M^* = M(t = T)$. In order to evaluate the impact of the interlock on the decision making process we consider the probability that the board votes $+1$ at large time $T$:

$$P_+ = P\{M^* > 0\}$$

If the board is neutral at the beginning i.e. $M^0 = 0$, then in the absence of interlock and CEO there are equal chances of outcome $M^* > 0$ or $M^* < 0$. One way to measure the impact of the interlock is to consider the probability
3 THE SURVEY MODEL

that the board votes +1, conditional to the initial average opinion \( M^0 \) in the board being zero:

\[
P^0_+ = \{ M^* > 0 \mid M^0 = 0 \}
\]

(5)

The fact that the CEO sticks to opinion +1 can be regarded as a constant external field: \( h_{CEO} = J_{CEO,j} s_j = J_{CEO,j} \)

### 3.2 Dynamics in the case of no interlock

In the absence of interlock (\( J_{ij} = 1 \; \forall i = 1 : N \)), and in the absence of the CEO, the dynamics is equivalent to ferromagnetism in the mean field approximation.

The absolute value of \( M^* \), as a function of beta, shows a clear phase transition around \( \beta = 1 \), as predicted by the mean field theory, even for a small number of directors \( N_d = 10 \).

Another way to visualize the phase transition is with a bifurcation diagram: Figure 3 shows the probability distribution of values of \( M^* \) obtained in 1000 runs as a function of beta. When \( \beta = 0 \) one observes a small magnetization due to the CEO: \( M^* \frac{1}{N_d} > 0 \).

Let us note that for \( \beta > 2 \), in practice the only possible values of \( M^* \) are \( \pm 1 \), so that the probability that \( M^* \) is positive coincides with the probability that \( M^* = 1 \).

This means that, at high beta, the board ends up with deciding at unanimity whether to approve or reject CEO’s proposal (unanimity minus one in case of rejection). For \( \beta = 4 \) the attractor is reached within 25-30 steps. This is a realistic scenario because typical discussions end up with very large majority in less than about 3 intervention per director. All the results shown in the following are obtained with \( \beta = 4 \). In order to compute \( M^* \), we run the dynamics for 50 steps and we average the magnetization over time steps 25-50.

### 3.3 Measuring the impact of interlock graphs

We want to investigate the effect on the dynamics due to the presence of a group of directors which serve together on one or more outside boards. The value of \( J_{ij} \) is the number of boards on which directors \( i \) and \( j \) serve together. Obviously \( J_{ij} \) worths at least 1 for all the directors in the board. The subset of directors of the board for which \( J_{ij} > 1 \) form a graph, the interlock graph of the board as we called it in the previous sections. We will refer to it as the ‘lobby’. This graph consists of one or more Connected Components(CC). As a result of the connection structure, a director belonging to a CC feels a stronger influence from a colleague within the CC than from a colleague outside the CC.

This fact will lead in the following section to a definition of force of the lobby.

We first tested our methodology with the set of all graphs with 10 directors connected with at most 3 links. Figure 4 shows all the possible graphs obtained with 1,2,3 links, with a maximum of 2 links per node. They are referred to as interlock graph 1,...,15. In each box the nodes represent the 10 directors of the board. The edges represent the ties between directors which serve together.
THE SURVEY MODEL

on an outside board. The black node is the CEO. Note that graphs 6 and 8 consist of a fully connected subgraph of 3 directors. In the latter case the CEO belongs to the graph.

For a given initial condition the outcome of the dynamics can be either 0 or 1, because of the stochastic nature of the opinion update. Then, in order to estimate the probability $P_+$ that a given board approves the CEO’s proposal, we repeat the dynamics for a large number of runs. We therefore obtain values of $P_+$ for each type of interlock graph, as a function of the initial magnetization inside and outside the lobby.

Now, in order to compare results for lobbies with different sizes and topologies, we seek for a scalar quantity that predicts the impact of an interlock graph. It is clear that the impact of a lobby must depend on the number of nodes in it and on the number of links. The number of links alone does not predict the probability of approving CEO’s proposal. The number of nodes alone doesn’t do better. The fact is that with a same number of links one can build a clique or a chain, so the topological structure must play an important role, too. Moreover the initial opinions of the directors in the interlock graph count a great deal, so we need a quantity which can take them into account.

The best predictor we found is the quantity:

$$F = \frac{1}{m^2} \sum_{ij \in G} J_{ij} s_j(t=0)$$

which we call the force. This scalar quantity is in fact the intensity of the field exerted at time $t=0$, by all the directors in the interlock graph on themselves. The field is normalized with respect to the size $m$ of the whole board, because we want to estimate the impact of the interlock graph with respect to the whole board. The same interlock graph will affect more strongly a small board than a large board.

To have an intuition of the notion we want to capture, suppose that at time $t=0$ all the directors of the interlock graph have opinion +1, but the board as a whole has magnetization $M=0$. The stronger the field the interlock members exert on themselves as compared to the field exerted by the directors outside the interlock graph, the more chances that the directors of the interlock graph stick to their initial opinion at $t \geq 0$. They would then act as an external field driving the board towards positive values of magnetization (although in principle the directors of the interlock graph can change opinion at any time; only the CEO has a fixed opinion).

The force can take several different values according to the different initial opinions +1 and -1 in the interlock graph. Hence, each graph has a set of possible value of force. For each value of the force the dynamics has a certain probability to reach the attractor $M^* = 1$ (as we said, for $\beta > 2$ whenever $M^*$ is positive, it is equal to +1).

Figure 5 displays $P_0^+$ (the probability of approving CEO’s proposal when the board is neutral at time 0) as a function of the force of all the interlock graphs. The fact the $P_0^+$ is an increasing function of the force was quite expected, by construction. What was not clear a priori was that lobbies with
different number of links and different topology but with similar value of the force do have similar value of $P_0^+$ which means that the force is a good predictor of the influence of the lobby on the result of the board decision making. Moreover, it is a linear predictor.

3.4 Voting dynamics simulations in real boards

We ran simulations of the voting dynamics using the interlock graphs that we found in the real boards. As for the elementary interlock graphs, here for each board and for each initial condition, we repeat the dynamics a large number of times in order to estimate the probability $P_+$ that the board will approve CEO’s proposal, as a function of the initial conditions in the lobby. In Figure 6 we show $P_0^+ (P_+ \text{conditional to having } M = 0 \text{ at time } t = 0)$ vs the force, for the real boards of the US 1000 Fortune companies.

To simplify the graph, we considered only points relative to two initial conditions: apart from the CEO, the directors in the interlock graph either have all opinions $+1$ or all $-1$ at time 0. Hence each board is represented by 2 points: one with a positive value of the force and one with a negative one. Figure 6 displays a strong linear correlation between $P_0^+$ and the force.

We left out boards with too large interlock graph i.e. when the number $c$ of nodes in the interlock graph is larger than half the number $m$ of directors. In fact in this case, when all the directors in the lobby start with a same opinion, then, no matter what is the opinion of the directors outside the lobby, there is no configuration with $M_0$ equal to 0. In this case, we set all directors that are not in the lobby as against the lobby, but since $M_0 > 0$, the corresponding data points are not comparable with the ones of the other boards. Boards with $c > m/2$ (not shown) have $P_0^+$ values close to 1.

A complementary set of data is obtained by taking the histogram of the fraction of boards which would agree with the CEO with a given probability. The set of boards is reduced to boards with an interlock graph, and the initial condition are $M_0 = 0$ with all lobby members voting initially as the CEO. Figure 9 display these histogram for four sets of simulation concerning the two models. The top histograms correspond to the survey model with and without a lobby, for the sake of comparison. One reads the histograms in the following way: with the survey model for example (top right frame), 25 per cent of the boards have 75 per cent chances to approve CEO’s proposal, if the directors in the lobby are initially in favor of it. Moreover one can say that 40 per cent of the boards have at least 75 per cent chances of approving the CEO.

4 The broadcast model

We consider now a different model for the voting dynamics, based on the idea that at each time step one director takes his turn to speak while the other directors listen to him/her and are influenced by his/her opinion. We will refer in the following to this model as the broadcast model.
At the board meeting, the CEO proposes a strategy for the company. Again this is stylized saying that there are only two opinions: opinion +1 corresponds to approving CEO’s strategy, and −1 to refusing it. The CEO always sticks to opinion +1. The other directors can have opinion +1 or −1.

- One director \(j\) at a time is chosen to speak. His own opinion is evaluated, as usual, based on the field he experiences according to the logit equation (equ. 2).

- Only the individual field evaluation is changed. When director \(j\) speaks, all directors \(i\) update their individual field according to:

\[
h_i^{\text{new}} = (1 - \gamma)h_i + \gamma J_{ij}s_j \quad \forall i
\] (7)

\(\gamma\) is a parameter which determines the memory length of the agent. At the beginning, the field of the agent \(i\) is initialized as equal to \(J_{ii}s_i\).

As a result, the field experienced by an agent, only takes into account the discounted opinion of the other agents, at the time when they spoke (which might be different from their actual opinion now). This scheme is closer to a class of models based on the Pólya urn, also used by economists[13]. We might then expect some sensitivity to the ordering of agents’ interventions during the board meeting.

In fact, the broadcast model requires to choose at each time step who is going to speak. As modelers we are tempted to use a random order, but in real boards the order is probably far from being random: more convinced directors will likely try to speak first, and moreover the CEO or the chairman plays a role in deciding the order of the speakers. In order to understand the impact of the way in which directors are chosen to speak, two extreme strategies are investigated here:

1. Strategy 1. The speaker is chosen randomly.

2. Strategy 2. For \(t \leq c\) (\(c\) being the size of the interlock graph), the speaker belongs to the interlock graph. For \(t > c\) the speaker is chosen randomly.

We have run simulations of the broadcast dynamics on the elementary interlock graphs and on the real boards of the US 1000 Fortune companies. Similar results were observed for different \(\gamma\) values (\(\gamma = 0.1, 0.3\)). We performed the same analysis as for the survey model: we estimated the probability \(P_{0+}\) that the board will approve CEO’s proposal, conditional to having \(M = 0\) at time \(t = 0\), as a function of the initial conditions in the lobby. As before only the two extreme cases for which the directors in the lobby are all in favor of the CEO, or all against him/her are taken into account. \(P_{0+}\) versus the force is shown in Figure 7.8 for the two strategies of choosing the speakers. The histograms of the fraction of boards which would agree with the CEO with a given probability are shown in Figure 9.
5 Discussion

We have investigated the impact of different structures of corporate directors interlock on the outcome of the decision making process of boards of directors. We have considered two models of decision making process, and we have studied the probability of board approval of the CEO’s strategy as a function of the topology and the size of the interlock structure. We have applied the models on a set of test interlock graphs or lobbies, in order to find a good predictor of the interlock impact, and then we have applied the models to the boards of the largest US corporation.

Figure 10 summarizes our results: the existence of a lobby does influence the vote as compared to the absence of a lobby. The probability $P_0^+$ that the board approves CEO’s proposal when the board is initially neutral, is plotted against the force of the lobby for the different models (for the purpose of comparison, values of the force are grouped in bins of width 0.05 and the corresponding values of $P_0^+$ for different boards inside the bin are averaged together). What surprised us is that this influence is of comparable magnitude for the survey and random broadcast models, at least for small values of $\gamma$, the time discount factor. In the broadcast model when directors of the lobby speak first, the influence of the lobby is enhanced, and even more so in the neighborhood of zero force. This means that a strategic sequence of interventions may enhance the power of the lobby on the decision making process. The discontinuity at $F = 0$ increases with $\gamma$.

We have focused our attention on the case in which initially the whole board is neutral about the decision, that is $M_0 = 0$, while the directors in the lobby have the same opinion, either all +1 or all -1. In this case the probability of approval is related to the power of the lobby, where “power” is used in accordance to Weber’s definition: “power of an actor in a social network is the probability that this actor will carry on his/her will despite resistance of the other actors”[14].

The interest of this investigation for the social sciences consists in offering a framework in which it is possible to make quantitative predictions about the power of a lobby within a board: given the topology of the social ties, we can compute a quantity, the force, which is a good predictor of the power of the lobby. In principle the board should take decisions on the interest of all investors, based on the available information. From our results, a well connected lobby of a minority of directors can drive the decision of the board, and the chances that the board will finally agree with the lobby can be predicted measuring the force of the lobby.

Having a powerful lobby inside the board simply means that the opinion of some directors has counted more than the opinion of others, which is not necessarily bad if, for example, the directors in the lobby were the most competent about the matters in discussion.

But suppose now the lobby rather represents the interest of some minority. This minority could consist of officers of the company itself, reluctant to a change of management or officers of another company that owns a minority
of stocks and want to attack the company. This could be seen as a dangerous situation for the company and the majority of investors. In this perspective, norms could be introduced to limit the force of the lobby e.g. when a new director is proposed for an appointment in the board.

Of course, the prediction of the outcome of the decision making process assumes some simple hypotheses about the influence of board directors on each other’s opinion. The main hypothesis of our models is that the influence $J_{ij}$ of a director $i$ on another director $j$ is a linear function of the number $n_{ij}$ of boards on which the two directors serve together. Some different functional relationships between $J_{ij}$ and $n_{ij}$ could be assumed, provided that the influence is a monotonic increasing function. There is no a priori justification for our linear choice, other than the fact that it is a simple approximation to start with.

For our models, we do not have an estimate of the real value of $\beta$. We made simulations for different values of $\beta$, then we focussed on the case $\beta = 4$, in order to avoid meta-stable states. But for any value $\beta >> 1$ the dynamics converges very rapidly to unanimity, and for $\beta = 4$ in less than 30 steps, i.e. after about 3 interventions on average of each director. This is a quite realistic scenario: in fact this is the typical number of interventions for a board discussion. Moreover, a typical discussion ends up with a consensus or a large majority.

The two models that we have investigated differ in the mechanism of opinion update. The opinion update mechanism we adopted for the survey model is analogous to what is known as ”herd behavior” in the literature of opinion dynamics, but it is also analogous to what is well-known in statistical physics as magnetic system dynamics at finite temperature. In the broadcast model we propose a more realistic mechanism of opinion update, which takes into account the fact that in a real discussion one is not informed of everybody else’s opinion at each step in time. Instead, participants speak once at a time, so that each agent only knows the opinion that another agent had at a certain time, which may differ from the opinion he has at the current time.

The present study focuses on boards of directors, because of the availability of empirical data. Our conclusions can be also applied to the decision dynamics of any political committee or academic board.

One possible extension of this investigation is the study, now in progress, of the dynamics of the decision making process of boards when the decision taken at one board influences the decision process of other boards. In fact, in the case of discussions about adoption of governance practices [9] or decisions that require prior forecasting of economic trends, directors of a board are likely to take into account decisions made in interlocked boards.
6 Acknowledgements

We would like to thank Gerald Davis of the University of Michigan, Business School, for having kindly provided the data of the US Fortune 1000. We also thank Jacques Lesourne for fruitful discussion.

Eurobios company supported two of us, SB and EB, in the early stage of this study. We acknowledge the support of the FET-IST department of the European Community, Grant IST-2001-33555 COSIN.

7 References

1. B. Mintz, and M. Schwartz, The Power Structure of American Business, (1985), University of Chicago Press.
2. L.D. Brandeis, Other People’s Money: And How the Bankers Use It. New York (1914), Frederick A. Stokes.
3. M.A. Carpenter and J.D. Westphal, The strategic context of social network ties: examining the impact of director appointment on board involvement in strategic decision making, forthcoming in Academy of Management Journal.
4. E.M. Fich, L.J. White, (2000), Why do CEO’s reciprocally sit on each other’s boards?, forthcoming
5. G.F. Davis, M. Yoo, W.E. Baker, The small world of the corporate elite, 2002, forthcoming.
6. J.D. Watts and S. Strogatz, Collective Dynamics of “Small World” Networks, Nature 393 (1998) 440-442.
7. B. Kogut and G. Walker, The Small World of Firm Ownership in Germany: Social Capital and Structural Holes in Large Firm Acquisitions - 1993-1997, (1999)
8. B. Vedres, The constellation of economic power: the position of political actors, banks and large corporations in the network of directorate interlock in Hungary, INSNA (1997) 23(1):44-59 2000
9. J. Davis and H. R. Greve, Corporate elite networks and governance changes in the 1980s, Am. J. of Sociology 103 (1996) 1-37.
10. A. Orleans, Bayesian interactions and collective dynamics of opinions: herd behavior and mimetic contagion, J. of Economic Behavior and Organization 28 (1995) pp.257-274.
11. D. Stauffer, Monte Carlo simulations of Sznajd models, J. of Artificial Societies and Social Simulation vol.5 n.1 (2002), http://www.soc.surrey.ac.uk/JASSS/5/1/4.html
12. S. Galam, J-D. Zucker, From individual choice to group decision making, Physica A 287 (2000) 644-659.
13. D. Blackwell, D. Kendall, The martin boundary of Pólya urn schemes... J. Appl. Probability 1 (1964) 284.
14. M. Weber, Theory of Social and Economic Organization, 1920
Figure 1: Example of an interlock graph: The board of directors of the Bank of America Corporation. White nodes represent directors that are not in the management, black nodes represent directors that are also executive of the company. Two directors are connected by a gray edge when they serve on one same outside board. The edge is black when they serve together on more than one outside board.
Figure 2: Histograms of board and interlock characteristics. Top left: board size (number of directors in the board). Top right: lobby size (number of directors involved in an interlock tie with some other directors of the same board). Bottom left: ratio between lobby size and board size. Bottom right: number of links in the interlock graph.
Figure 3: Bifurcation diagram of the magnetization. Frequency distribution of the final value of the magnetization $M^* = M(t = T)$, obtained in 1000 runs, as a function of $\beta$, the temperature parameter. The board has 10 directors. The asymmetry of the diagram is due to the presence of the CEO who always vote +1. Note that for $\beta > 2.5$ the probability that $M^* = 1$ coincides with the probability that $M^* > 0$. 
Figure 4: The simplest interlock graphs for boards with 10 directors. There are 15 different graphs that can be drawn with a maximum of 3 links and with up to 2 links per node. The black node is the CEO.

Figure 5: Survey model simulation of the 15 elementary interlock graphs. Ordinate: probability $P_+^0$ that the board approves CEO’s proposal, conditional to the board being initially neutral ($M^0 = 0$). Abscissa: force of the interlock graph. Data points are the average of 500 runs. Each data point corresponds to a given initial magnetization value of a single interlock graph.
Figure 6: Survey model simulation on the boards of the Fortune 1000 companies. Ordinate and abscissa as in figure 5, each point is an average over 500 runs. Only 2 initial magnetization values of the lobby are considered (all +1, all -1).

Figure 7: Broadcast model simulation on the boards of the Fortune 1000 companies. The order of the speakers is random. Ordinate and abscissa as in figure 5, each point is an average over 500 runs. Only 2 initial magnetization values of the lobby are considered (all +1, all -1).
Figure 8: Broadcast model simulation on the boards of the Fortune 1000 companies. The order of the speakers is as follows: directors of the lobby speak first, then the speaker is chosen randomly. Ordinate and abscissa as in figure 5, each point is an average over 500 runs. Only 2 initial magnetization values of the lobby are considered (all +1, all -1).
Figure 9: Histograms of the fractions of boards which would approve the CEO with a probability given in abscissa.
Top left: values for the survey model without interlock.
Top right: survey model.
Bottom left: broadcast model, strategy 1 (random order of speakers).
Bottom right: broadcast model, strategy 2 (directors in the lobby speak first).
Figure 10: Comparison of the results of the different models. Ordinate and abscissa as in figure 5. Data points correspond to the 321 boards with non-empty interlock graph. Pentagons: survey model with no interlock for control purposes. Circles: survey model. Diamonds: broadcast model with $\gamma = 0.1$, strategy 1 (random order of speakers). Triangles: broadcast model with $\gamma = 0.1$, strategy 2 (directors in the lobby speak first).