Heavy ion collisions and AdS/CFT

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Abstract.
We review some recent applications of the AdS/CFT correspondence to heavy ion collisions including a calculation of the jet quenching parameter in $\mathcal{N} = 4$ super-Yang-Mills theory and quarkonium suppression from velocity scaling of the screening length for a heavy quark-antiquark pair. We also briefly discuss differences and similarities between QCD and $\mathcal{N} = 4$ Super-Yang-Mills theory.

1. Introduction
Understanding the implications of data from the Relativistic Heavy Ion Collider (RHIC) poses qualitatively new challenges [1]. Given its large and anisotropic collective flow and its strong interaction with hard probes, the hot matter produced in RHIC collisions must be described by QCD in a regime of strong, and hence nonperturbative, interactions. In this regime, lattice QCD has to date been the prime calculational tool. However, understanding collective flow, jet quenching and other hard probes requires real-time dynamics, on which information from lattice QCD is at present both scarce and indirect. Complementary methods for real-time strong coupling calculations at finite temperature are therefore desirable.

For a class of non-abelian thermal gauge field theories, the AdS/CFT conjecture provides such an alternative [2]. It maps nonperturbative problems at strong coupling onto calculable problems of classical gravity in a five-dimensional anti-de Sitter (AdS$_5$) black hole spacetime. Although the AdS/CFT correspondence is not directly applicable to QCD, one expects results obtained from closely related non-abelian gauge theories should shed qualitative (or even quantitative) insights into analogous questions in QCD. A beautiful example is the universality of shear viscosity in various gauge theories with a gravity dual [3] and its numerical coincidence with estimates from comparing RHIC data with hydrodynamical model analyses [4].

Here we give a short overview of two recent AdS/CFT calculations of relevance to heavy ion collisions: (i) the jet quenching parameter which controls the description of medium-induced energy loss for relativistic partons in QCD [5, 6]; (ii) velocity-induced quarkonium suppression [7, 6].
2. Jet quenching and AdS/CFT

A high energy parton moving in a QCD quark-gluon plasma will lose energy from interaction with the medium. Medium-induced gluon radiation has been argued to be the dominant mechanism behind jet quenching at RHIC (for reviews see [9]), where the high energy partons whose energy loss is observed in the data have transverse momenta of at most about 20 GeV [1]. In the Baier-Dokshitzer-Mueller-Peigne-Schiff [10] calculation of the medium-modified splitting processes \( q \rightarrow q g \), the quark-gluon radiation vertex is treated perturbatively. However, rescatterings of the radiated gluon and the initial quark with the medium are controlled by \( \alpha_s(T) \) and cannot be treated perturbatively in a strongly interacting quark-gluon plasma. In the high energy limit, at order \( O(1/E) \) with \( E \) the energy of the initial quark, these non-perturbative effects are captured by a single parameter \( q \), which can heuristically be understood as the transverse momentum square transfer from the medium to the light-like initial quark (or radiated gluon) per unit distance. In a heavy ion collision, \( q \) decreases as the hot fluid expands and cools. The time-averaged \( q \) determined in comparison with RHIC data is found to be around 5-15 GeV\(^2/fm\) [11, 12].

A weak-coupling calculation of \( q \) in a static medium yields (up to a logarithm) [10, 13, 14]

\[
\hat{q}_{\text{weak-coupling}} = \frac{8\zeta(3)}{\pi} \alpha_s^2 N^2 T^3
\]

if \( N \), the number of colors, is large. However, given \( \alpha_s(T) \) at RHIC temperature is not small, a weak coupling calculation is not under control. Taking \( \alpha_s = 1/2 \) for temperatures not far above the QCD phase transition and \( N = 3 \), one finds from (1) that \( \hat{q}_{\text{weak-coupling}} \approx 0.94 \text{ GeV}^2/\text{fm} \) for \( T = 300\text{MeV} \) (which is roughly the temperature of RHIC at \( t = 1\text{fm} \)), smaller than the experimental estimate by at least a factor of 5. There is thus strong motivation to calculate \( \hat{q} \) without assuming weak coupling.

In [5, 8] (see also [6] and references therein), a non-perturbative definition of \( \hat{q} \) was given in terms of the short distance limit of the thermal expectation value of a light-like Wilson loop in the adjoint representation

\[
\langle W^A(C_{\text{light-like}}) \rangle \approx \exp \left[ -\frac{1}{4\sqrt{2}} \hat{q} L^- L^2 \right], \quad L^- \gg \frac{1}{T} \gg L
\]

where the contour \( C_{\text{light-like}} \) is a rectangle with large extension \( L^- \) in the \( x^- \)-direction and small extension \( L \) in a transverse direction (see fig 1). At a heuristic level, the two long sides of the Wilson loop can be understood to arise from the eikonal phase of the radiated gluon moving in the medium (which is a lightlike Wilson line along the gluon trajectory) and its complex conjugate. The transverse separation \( L \) is conjugate to the transverse momentum \( k_\perp \) of the emitted gluon. In (2) we again consider a static medium and \( \hat{q} \) is constant.

While it is currently not known how to directly compute (2) for QCD in a strong coupling regime, it is of interest to compute (2) in other non-Abelian gauge theories to extract qualitative information such as how \( \hat{q} \) depends on the number of degrees of
freedom, its coupling constant dependence and so on. At a quantitative level, it would be interesting to know whether other theories exist which can give rise to a \( \hat{q} \) as large as the experimental estimate.

For \( \mathcal{N} = 4 \) Super-Yang-Mills (SYM) theory with gauge group \( SU(N) \) in the limit of large \( N \) and large ’t Hooft coupling \( \lambda = g_{\text{SYM}}^2 N \), the Wilson loop (2) can be calculated using the AdS/CFT correspondence. \( \mathcal{N} = 4 \) SYM is a supersymmetric gauge theory with one gauge field \( A_\mu \), six massless scalar fields \( X^I, I = 1, 2, \ldots, 6 \) and four massless Weyl fermionic fields \( \chi_\xi \), all transforming in the adjoint representation of the gauge group. The theory is conformally invariant and is specified by two parameters: the rank of the gauge group \( N \) and the ’t Hooft coupling \( \lambda = g_{\text{SYM}}^2 N \). (The coupling constant does not run.) In the large \( N \) and large \( \lambda \) limit, the thermal expectation value of a Wilson loop operator \( \langle W(C) \rangle \) in \( \mathcal{N} = 4 \) SYM theory is given by [15, 16]

\[
\langle W(C) \rangle = \exp \left[ i S(C) \right],
\]

where \( S(C) \) is the classical extremal action of a string in a five dimensional anti-de Sitter (AdS\(_5\)) black hole geometry, with the boundary of the string world sheet ending on the curve \( C \) lying in the boundary of the black hole spacetime. The string worldsheet can be considered as the spacetime trajectory of an open string connecting the quark and antiquark which are running along the loop \( C \). The open string “lives” in a \( (4 + 1) \)-d AdS\(_5\) black hole spacetime with our \( (3 + 1) \)-d Minkowski spacetime as its boundary.

For a light-like Wilson loop (2), the extremal string worldsheet is spacelike and \( S(C) \) is pure imaginary. Thus the exponent on the right hand of (3) is real as in that of (2) and one finds that \( \hat{q} \) is given by [5]

\[
\hat{q}_{\text{SYM}} = \frac{\pi^{3/2} \Gamma \left( \frac{3}{4} \right)}{\Gamma \left( \frac{5}{4} \right)} \sqrt{\lambda} T^3 \approx 26.69 \sqrt{\alpha_{\text{SYM}}} N T^3. \tag{4}
\]

Note that \( T^3 \) behavior in (4) can be determined by dimensional analysis since \( \mathcal{N} = 4 \) SYM is conformal. The \( \sqrt{\lambda} \) dependence is non-trivial and is a consequence of strong coupling. Taking \( N = 3 \) and \( \alpha_{\text{SYM}} = \frac{1}{2} \), thinking \( \alpha_{\text{QCD}} = \frac{1}{2} \) for temperatures not far above the QCD phase transition, one finds that

\[
\hat{q}_{\text{SYM}} = 4.5 \text{ GeV}^2/\text{fm} \quad \text{for} \quad T = 300 \text{ MeV}. \tag{5}
\]

It is both surprising and amusing that the \( \mathcal{N} = 4 \) SYM answer is rather close to the experimental estimate mentioned earlier.

One can also use AdS/CFT to evaluate \( \hat{q} \) for other non-abelian gauge theories with a supergravity dual [17]. The value of \( \hat{q} \) is not universal among different theories. Nevertheless, it appears that \( \hat{q} \) can be considered as a measure of the number of degrees of freedom of a theory at an energy scale \( T \) [6]. For example, for any conformal field theory which is dual to a type IIB string theory on AdS\(_5\) \( \times M_5 \) where \( M_5 \) is a 5-d Einstein manifold, one finds that

\[
\frac{\hat{q}_{\text{CFT}}}{\hat{q}_{\mathcal{N} = 4}} = \sqrt{\frac{s_{\text{CFT}}}{s_{\mathcal{N} = 4}}}, \tag{6}
\]
in the limit of large $N$ and large 't Hooft coupling $\lambda$. Given that QCD at a temperature of a few $T_c$ appears to be rather close to being conformal, it is tempting to conjecture that \[ \hat{q}_{\text{QCD}} \approx \sqrt{\frac{47.5}{120}} \approx 0.63 \] as an estimate of the effect of the difference between the number of degrees of freedom in the two theories on $\hat{q}$.

In a relativistic heavy ion collision, the medium itself develops strong collective flow, meaning that the hard parton is traversing a moving medium — it feels a wind. Thus to compare with the experimental estimate we should include the effects of the wind on $\hat{q}$. The behavior of $\hat{q}$ as defined by (2) in a medium which is moving itself can be found by using simple arguments based on Lorentz transformations [6, 18]

\[ \hat{q} = \gamma_f (1 - v_f \cos \theta) \hat{q}_0 , \]

where $v_f$ is the velocity of the wind, $\gamma_f = 1/\sqrt{1 - v_f^2}$, and $\theta$ is the angle between the direction of motion of the hard parton and the direction of the wind. $\hat{q}_0$ is the value of $\hat{q}$ in the absence of a wind. The result (8) for the dependence of $\hat{q}$ on collective flow is valid in QCD and in $\mathcal{N} = 4$ SYM and in the quark-gluon plasma of any other gauge theory, since its derivation relies only on properties of Lorentz transformations. If we crudely guess that head winds are as likely as tail winds, and that the typical transverse wind velocity seen by a high energy parton is about half the speed of light, $\hat{q}$ is increased relative to that in (4) by a factor of 1.16. A credible evaluation of the consequences of (8) for the time-averaged $\hat{q}$ extracted from data will, however, require careful modelling of the geometry of the collision and the time-development of the collective flow velocity.

It is important to emphasize that the motivation behind calculating $\hat{q}$ as defined in (2) for $\mathcal{N} = 4$ SYM and other non-abelian gauge theories is not to understand the full process of energy loss of a high energy parton in those theories. Rather, one seeks insights about $\hat{q}$ in QCD by calculating the analogous quantity in other theories.

The energy loss of an external heavy quark moving in an $\mathcal{N} = 4$ SYM plasma has also been explored recently [19, 20, 21]. On general grounds, one expects the momentum of the quark to satisfy a Langevin equation

\[ \frac{dp_L}{dt} = -\mu(p_L)p_L + \xi_L(t) , \quad \frac{dp_T}{dt} = \xi_T(t) , \]

with

\[ \langle \xi_L(t)\xi_L(t') \rangle = \kappa_L(p)\delta(t - t') \quad \langle \xi_T(t)\xi_T(t') \rangle = \kappa_T(p)\delta(t - t') . \]

$p_L$ and $p_T$ are the longitudinal and transverse momentum of the quark and $\kappa_L(p), \kappa_T(p)$ describe longitudinal and transverse momentum squared transferred to the quark per unit time. It was found in [19, 20] that the drag $\mu(p_L)$ is

\[ \mu(p) = \frac{\pi \sqrt{\lambda}}{2m} T^2 . \]
The momentum-independence of the drag in Eq. (11) highlights that in the high energy limit the energy loss mechanism in strongly coupled $\mathcal{N} = 4$ SYM theory is very different from that in QCD, in which as a result of asymptotic freedom, the dominant energy loss mechanism is perturbative gluon radiation‡. $\kappa_L(p), \kappa_T(p)$ have also been computed [20, 21]

$$
\kappa_T = \frac{\pi \sqrt{\lambda}}{(1 - v^2)^{\frac{3}{4}}} T^3, \quad \kappa_L = \frac{\pi \sqrt{\lambda}}{(1 - v^2)^{\frac{3}{4}}} T^3.
$$

(12)

The divergence at $v = 1$ for these quantities precludes the use of $\kappa_T$ as $\hat{q}$ which is defined based on the BDMPS energy loss formalism, where the initial quark or (radiated gluon) moves strictly along the light-cone. There is no inconsistency, however, because for a fixed quark mass $M$, (12) applies only to velocities $v$ satisfying [21]

$$
\frac{\sqrt{\lambda}}{M} < \frac{(1 - v^2)^{\frac{1}{4}}}{T}
$$

(13)

and $v = 1$ singularity in (12) is never reached. We will elaborate more on the physical meaning of (13) in section 4.2.

3. Quarkonium suppression from velocity scaling of screening length

The dissociation of charmonium and bottomonium bound states has been proposed as a signal for the formation of a hot and deconfined quark-gluon plasma [22]. Recent analyses of this phenomenon are based on the study of the quark-antiquark static potential extracted from lattice QCD (see e.g. [23]). In these calculations, the $q\bar{q}$-dipole is taken to be at rest in the thermal medium. In heavy ion collisions, however,

‡ $\mathcal{N} = 4$ SYM is strongly coupled at all scales, unlike QCD, which is strongly coupled at scales $\sim T$, but, in the high (initial quark) energy limit, is weakly coupled at scales $\sim k_\perp$, the momentum of typical radiated gluons.
quarkonium bound states are produced moving with some velocity $v$ with respect to the medium. In the limit of large quark mass, the velocity-dependent dissociation of such a moving $q\bar{q}$-pair in a medium can be found by evaluating $\langle WF(C_{\text{static}}^{\text{boosted}}) \rangle$ with $C_{\text{static}}^{\text{boosted}}$ depicted in Fig. 1. The orientation of the loop in the $(x_3, t)$-plane changes as a function of $v$. As a working definition, we may take

$$\langle WF(C_{\text{static}}^{\text{boosted}}) \rangle = \exp \left[ -i \mathcal{T} E(L) \right].$$

(14)

where $E(L)$ is (renormalized) free energy of the quark-antiquark system with self-energy of each quark subtracted. Due to screening in the medium, one expects $E(L)$ to become flat for large $L$. The evaluation of (14) with $v \neq 0$ in QCD for a strongly coupled medium is not known at the moment.

From AdS/CFT, one can again evaluate (14) for $\mathcal{N} = 4$ SYM plasma in the limit of large $N$ and large $\lambda = g_{\text{SYM}}^2 N$ using (3). One finds that the worldsheet is time-like and thus $S(C_{\text{static}})$ is real, giving rise to a real $E(L)$. Furthermore, there exists an $L_{\text{max}}$ beyond which $E(L)$ becomes identically zero and thus $L_{\text{max}}$ can be interpreted as the screening length [16]. In [7, 6] (see also [24]), it was found that $L_{\text{max}}$ changes with $v$ as

$$L_{\text{max}} \sim \frac{1}{T}(1 - v^2)^{1/4}.$$  

(15)

If the velocity-scaling of $L_{\text{max}}$ (15) holds for QCD, it will have qualitative consequences for quarkonium suppression in heavy ion collisions [7]. It implies that the temperature $T_{\text{diss}}$ needed to dissociate the $J/\Psi$ decreases as

$$T_{\text{diss}}(v) \sim T_{\text{diss}}(v = 0)(1 - v^2)^{1/4}.$$  

(16)

This indicates that $J/\Psi$ suppression at RHIC may increase markedly for $J/\Psi$'s with transverse momentum $p_T$ above some threshold, on the assumption that RHIC temperature does not reach the dissociation temperature $2.1 T_c$ of $J/\Psi$ at zero velocity [25, 23]. The kinematical range in which this novel quarkonium suppression mechanism is operational lies within experimental reach of future high-luminosity runs at RHIC and will be studied thoroughly at the LHC in both the $J/\Psi$ and Upsilon channels. We should also caution that in modelling quarkonium production and suppression versus $p_T$ in heavy ion collisions, various other effects like secondary production or formation of $J/\Psi$ mesons outside the hot medium at high $p_T$ [26] remain to be quantified. The quantitative importance of these and other effects may vary significantly, depending on details of their model implementation. In contrast, Eq. (16) was obtained directly from a field-theoretic calculation and its implementation will not introduce additional model-dependent uncertainties.

4. Discussions

4.1. $\mathcal{N} = 4$ SYM versus QCD

Given that $\hat{q}$ calculated in $\mathcal{N} = 4$ SYM theory is close to the value extracted from RHIC data, is this agreement meaningful or accidental? More generally, in what respects can
the strongly interacting plasma of \( \mathcal{N} = 4 \) SYM theory give a reasonable description of the quark-gluon plasma in QCD? After all, at a microscopic level \( \mathcal{N} = 4 \) SYM is very different from QCD:

- The theory is conformal, supersymmetric and contains additional global symmetry.
- The coupling does not run and there is no confinement.
- No chiral symmetry and no chiral symmetry breaking.
- No fundamental quarks, additional scalar and fermionic fields in the adjoint representation.

These features imply that physics near the vacuum or at high energy are very different between two theories. However, for QCD at RHIC temperature (about \( 2 T_c \)) almost all differences mentioned above become murky and may be irrelevant:

- There are a variety of indications from lattice QCD calculations (see e.g. [27] for a review) that QCD thermodynamics is reasonably well approximated as conformal in a range of temperatures from about \( 2 T_c \) up to some higher temperature not currently determined.
- Supersymmetry of \( \mathcal{N} = 4 \) SYM is badly broken at finite temperature for physics of the scale of the temperature.
- Above \( T_c \) in QCD, there is no confinement and no chiral condensate. Also if the quark-gluon plasma in QCD is strongly interacting, as indicated by data from RHIC, the asymptotic freedom is not important for those physical quantities probing intrinsic properties of the medium.
- In a strongly interacting liquid there are, by definition, no well-defined, long-lived quasiparticles anyway, making it plausible that observables or ratios of observables can be found which are insensitive to the differences between microscopic degrees of freedom and interactions.

Thus, it does not seems to be too far-fetched to imagine that the quark-gluon plasma of QCD, as explored at RHIC and in lattice QCD calculations, and that of \( \mathcal{N} = 4 \) SYM share certain common properties. Indeed the list of similarities between two theories is growing fast. Examples include the values of thermodynamic quantities like \( \epsilon/T^4 \), \( P/T^4 \), and the velocity of sound, and the static screening length between a quark and antiquark at rest, which can all be compared to lattice QCD calculations, and dynamical quantities like the shear viscosity, \( \hat{\eta} \), \( \kappa T \) which can be compared to inferences drawn from RHIC data. Perhaps (16) will be added to this list, once experiments are done. See sec.6.4 of [6] for more details and also [28].

We are used to the idea that all metals, or all liquids, or all ferromagnets have common, defining, characteristics even though they may differ very significantly at a microscopic level. It is clearly of great interest to understand what are the defining commonalities of quark-gluon plasmas in different theories, and in what instances do these commonalities allow qualitative or semi-quantitative lessons learned about the quark-gluon plasma of one theory to be applied to that of another.
4.2. Lightlike versus time-like Wilson loops

Comparing Eq. (14) and Eq. (2) we see that while the exponent in (14) is pure imaginary, that in (2) is real. So, if we boost a static Wilson loop to the speed of light, the qualitative behavior of the Wilson loop should change as $v \to 1$. How can this happen?

To answer this question, let us first consider a physical set-up in which a boosted single Wilson line with $0 < v < 1$ is realized. The Wilson line can be considered as the non-abelian phase accumulated along the worldline of an external heavy quark moving with a constant velocity $v$. In order for a quark to move at a constant velocity in the medium, some external force has to be supplied, e.g. by applying an external electric field. However, as discussed in the second reference in [21], for a quark with mass $M$, such a set-up can only be realized for velocity $v$ not too close to 1. Otherwise the required electric field is so large that it will create pairs of quark and anti-quark. In the context of $\mathcal{N} = 4$ SYM, such a consideration leads to the inequality (13). In the limit $M \to \infty$, the boundary of (13) is pushed to $v \to 1_-$, but never exactly $v = 1$.

Similarly, one can visualize a boosted Wilson loop with $v < 1$ as the phase associated with the trajectories of a pair of external heavy quark and antiquark, with external forces applied to each of them to keep them moving at a constant velocity $v$. Again, for finite quark mass $M$, such a set-up can only be realized for $v$ satisfying (13). In this regime, $E(L)$ in (14) is real, from which a quark-antiquark potential can be extracted. When $v$ is so close to 1 that (13) is not satisfied, instead, a physical way to realize the Wilson loop is to imagine the quark-antiquark pair as Fock states of a high energy virtual photon in a deep inelastic scattering experiment. As discussed in [5, 6], unitarity then requires that the Wilson loop to have a real exponent as in (2). In the $M \to \infty$ limit, the boundary between two regimes lies at $v \to 1_-$. In [6], an alternative interpretation of (13) was given: when $v$ does not satisfies (13), the Compton wavelength of a quark becomes greater than the screening length of the medium. From this perspective, one also sees that the notion of a static quark potential is not meaningful.

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