Emission of charged particles from four- and five-dimensional black holes

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Abstract

Recently Das and Mathur found that the leading order Hawking emission rate of neutral scalars by near-extremal $D = 5$ black holes is exactly reproduced by a string theoretic model involving intersecting D-branes. We show that the agreement continues to hold for charged scalar emission. We further show that similar agreement can be obtained for a class of near-extremal $D = 4$ black holes using a model inspired by M-theory. In this model, BPS saturated $D = 4$ black holes with four charges are realized in M-theory as 5-branes triply intersecting over a string. The low-energy excitations are signals traveling on the intersection string.

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1 Introduction

During recent months, impressive progress has been made towards a more fundamental explanation of the semi-classical properties of black holes. Strominger and Vafa found a statistical explanation of the Bekenstein-Hawking entropy \([1, 2]\) for a class of supersymmetric black holes in \(D = 5\) \([3]\). When such black holes carry three different \(U(1)\) charges, their horizon area is non-vanishing even for the extremal solution. In string theory, the Bekenstein-Hawking entropy of macroscopic black holes is exactly reproduced by the counting of states of a configuration of Dirichlet branes \(\text{[4]}\) which carries the same charges \(\text{[2]}\). This result was extended to \(D = 5\) black holes near extremality in \(\text{[5, 6]}\). Furthermore, in \(\text{[5]}\) it was shown how Hawking emission from the near-extremal black holes takes place in the stringy description. The model involves \(n_1\) 1-branes marginally bound to \(n_5\) 5-branes, with some longitudinal momentum along the 1-branes carried by left moving open strings. Near extremality, a small number of right movers is also present, so that a left moving and a right moving open string may collide to produce an outgoing closed string \(\text{[5, 7]}\). In \(\text{[5]}\) it was shown that this mechanism leads to a thermal distribution for the massless outgoing particles, as expected of Hawking radiation. The inverse of this process, which gives the leading order contribution to the absorption of closed strings, was also found to be in agreement with the semi-classical gravity, up to an overall normalization \(\text{[8]}\). More recently, in an impressive paper \(\text{[9]}\) Das and Mathur carefully normalized the leading emission and absorption rates, both in semi-classical gravity and in the D-brane picture, and found perfect agreement! The specific picture used in \(\text{[9]}\) follows that suggested in \(\text{[10, 11]}\): the low-energy dynamics of the D-brane configuration is captured by a single string with winding number \(n_1 n_5\) which is free to vibrate only within the 5-brane hyperplane. Vibrations in the transverse directions are not allowed because the 1-branes are bound to the 5-branes, so only transverse scalars are emitted at low energies, in agreement with semi-classical gravity.

In this paper we extend the result of \(\text{[9]}\) in two directions. First we generalize the calculation to outgoing scalars which are charged and massive (their mass is proportional to charge according to the BPS condition). The \(D = 5\) black hole carries three different charges which in the string context are realized as the number of 1-branes, the number of 5-branes, and the Kaluza-Klein charge (the momentum along the 1-brane direction). Endowing the outgoing scalars with the Kaluza-Klein charge in both the D-brane and the gravity calculations, we once again find perfect agreement.

Our second extension is to supersymmetric four-dimensional black holes with regular horizons. In \(\text{[12, 13]}\) it was shown that for an extremal \(D = 4\) black hole to have a finite horizon area, it must carry four different \(U(1)\) charges. Such black holes can be embedded into string theory, but the necessary configurations involve solitonic 5-branes or Kaluza-Klein monopoles in addition to the D-branes \(\text{[14, 15]}\). In \(\text{[16, 17, 18]}\) it was argued that it is advantageous to view the \(D = 4\) black holes as dimensionally reduced configurations of intersecting branes in M-theory. A specific configuration useful for explaining the Bekenstein-Hawking entropy is the \(5 \perp 5 \perp 5\) intersection \(\text{[17]}\): there are \(n_1\) 5-branes in the \((12345)\) hyperplane, \(n_2\)
5-branes in the (12367) hyperplane, and \( n_3 \) 5-branes in the (14567) hyperplane. One also introduces a left moving momentum along the intersection string (in the \( \hat{1} \) direction). If the length of this direction is \( L_1 \), then the momentum is quantized as \( 2\pi n_K / L_1 \), so that \( n_K \) plays the role of the fourth \( U(1) \) charge. Upon compactification on \( T^7 \) the metric of the \( 5 \perp 5 \perp 5 \) configuration reduces to that of the \( D = 4 \) black hole with four charges. Just like in the D-brane description of the \( D = 5 \) black hole, the low-energy excitations are signals propagating along the intersection string. In M-theory the relevant states are likely to be small 2-branes with three holes glued into the three different hyperplanes \([17]\). As a result, the effective winding number of the intersection string is \( n_1 n_2 n_3 \). This fact, together with the assumption that these modes carry central charge \( c = 6 \), is enough to reproduce the Bekenstein-Hawking entropy, \( S = 2\pi \sqrt{n_1 n_2 n_3 n_K} \) \([17]\). In this paper we go further and show that this “multiply-wound string” model of the four-charge \( D = 4 \) black hole is also capable of accounting for Hawking radiation. It exactly reproduces the emission rate found using the methods of semi-classical gravity for scalars carrying Kaluza-Klein charge.

2 The string theory analysis

Let us start by recapitulating in a streamlined way the string theory treatment of the five-dimensional black hole \([9, 8, 11]\). The essential assumption is that the \( n_1 \) D-strings bound to \( n_5 \) 5-branes act as a single D-string of winding number \( n_1 n_5 \), which is free to move only within the 5-brane hyperplane. This multiply-wound D-string is described by the following six-dimensional effective action:

\[
S = T_D \int d^2 \xi e^{-\phi} \varepsilon^{\alpha\beta} \frac{1}{2} \left( \delta_{ij} + 2\kappa_6 h_{ij} \right) \partial_\alpha X^i \partial_\beta X^j + \int d^6 x \left( \frac{1}{2} \partial_M h_{ij} \right) \left( \partial^M h^{ij} \right) + \ldots .
\]

Here \( \alpha \) and \( \beta \) are coordinates on the D-string worldsheet, \( i \) and \( j \) run over the coordinates \( y_2, \ldots y_5 \) compactified on \( T^4 \), and \( M \) and \( N \) run over the six other coordinates. The spatial coordinate \( y_1 \) is parallel to the D-string, whose tension we denote by \( T_D \). In the first line of (1), the D-string action is of the standard Dirac-Born-Infeld form, while the gravity action is part of the standard type IIB action.

In (1) we have suppressed those parts of the action not directly relevant to the calculations below: in the first line, kinetic terms for the dilaton and the antisymmetric tensor are omitted, as well as the fermion terms both on the D-string and in the bulk of spacetime; in the second line, the only terms listed are the leading order interaction term coupling the bosonic excitations of the D-string with the internal gravitons \( h_{ij} \) (scalars in six dimensions) and the kinetic terms for these fields.

One more subtlety involves the range of the coordinate \( y_1 \). Strictly speaking, the action in (1) describes only the vibrations of a single 1-brane within the world-volume of a single
5-brane. When we compactify down to five dimensions by wrapping the D-string around a circle of circumference \( L_1 \), \( y_1 \) becomes periodic in the obvious manner, \( y_1 \sim y_1 + L_1 \). There is a neat trick [9, 10] for handling the case of multiple 1- and 5-branes: the effective theory in five dimensions is obtained simply by identifying \( y_1 \sim y_1 + L_{\text{eff}} \) where \( L_{\text{eff}} = n_1 n_5 L_1 \). This prescription corresponds to the entropically favored configuration of a single 1-brane and a single 5-brane wound \( n_1 \) and \( n_5 \) times, respectively [11]. Note that the total momentum of the D-string is quantized in units of \( 2\pi/L_1 \), as is the momentum parallel to the D-string of particles in the bulk of spacetime. By contrast, the bosonic excitations of the D-string, which are described by quanta of the fields \( X^i \), have momentum quantized in units of \( 2\pi/L_{\text{eff}} \).

The numbers of left moving (\( E = -p \)) and right moving (\( E = p \)) bosonic excitations are assumed to follow separate thermal distributions

\[
\rho_L(p_0) = \frac{1}{e^{\beta p_0} - 1} \quad \rho_R(q_0) = \frac{1}{e^{\beta q_0} - 1}
\]

with \( T_R \ll T_L \). Equivalently, (2) can be thought of as a single thermal distribution for all the modes subject to a constraint on the total momentum imposed via a chemical potential, as explained for instance in [9].

We have now finished setting the stage for the analysis of the dominant decay processes of the D-string. It is remarkable that the relatively complex string theoretic structure of multiply wound intersecting 1- and 5-branes with a condensate of open strings running along the intersection should boil down to such a simple description as a single long string described by the action on the second line of (1). While this picture undoubtedly needs a more precise justification, it works very well. It correctly predicts the black hole entropy [11], the neutral particle emission [9], and, as we will show, the charged particle emission.

The invariant amplitude for the process where a left moving \( X^i \) excitation with momentum \( p \) and a right moving \( X^j \) excitation with momentum \( q \) collide and turn into a graviton with polarization in the \( ij \) direction and momentum \( k \) is

\[
\mathcal{M} = \sqrt{2k_0} p \cdot q .
\]

Let us fix the graviton’s momentum parallel to the D-string: \( k_1 = -e \) where \( e \) is the Kaluza-Klein charge in five dimensions. (We conventionally take \( e > 0 \) for left movers in order that when \( T_R \ll T_L \) the black hole has an overall positive charge.) The total rate to produce such a graviton with definite transverse momentum \( \vec{k} \) follows directly from (3) and kinematical arguments:

\[
\Gamma(\vec{k}) \frac{d^4k}{(2\pi)^4} = 2 \int_{-\infty}^{0} \frac{L_{\text{eff}} dp_1}{2\pi} \rho_L(-p_1) \int_{0}^{\infty} \frac{L_{\text{eff}} dq_1}{2\pi} \rho_R(q_1) \cdot (2\pi)^2 \delta(p_0 + q_0 - k_0) \delta(p_1 + q_1 - k_1) \frac{1}{8L_1 L_{\text{eff}}} \frac{|\mathcal{M}|^2}{p_0 q_0 k_0} \frac{d^4k}{(2\pi)^4}
\]

\[
= \frac{\kappa^2 L_{\text{eff}}}{4} \frac{k_0^2}{\omega} \rho_L \left( \frac{\omega + e}{2} \right) \rho_R \left( \frac{\omega - e}{2} \right) \frac{d^4k}{(2\pi)^4}
\]

(4)
In this equation and in the following, $k = |\vec{k}|$ is the magnitude of the particle’s momentum in the four noncompact spatial dimensions. The factor of 2 outside the integral in the first line of (4) accounts for the fact that the same graviton can be produced by a left moving $X^i$ quantum colliding with a right moving $X^j$ quantum or by a left moving $X^j$ quantum colliding with a right moving $X^i$ quantum. To obtain a rate which can be directly compared with semi-classical calculations, we make the crucial high temperature expansion

$$\rho_L(E) \approx \frac{1}{\beta L E}$$

and use the results of previous papers [3, 5, 9, 11] to express the right and left moving temperatures in terms of properties of the classical five-dimensional near-extremal geometry:

$$T_L = \frac{S_L}{\pi L_{\text{eff}}} \approx \frac{S_{\text{BH}}}{\pi L_{\text{eff}}} = \frac{A_h}{4 \pi L_{\text{eff}} G_5}, \quad T_R = \frac{1}{2} T_H.$$  

In normalizing the left moving temperature we used the fact that there are four species of massless bosons and four species of massless fermions on the string (i.e. the central charge is $c = 6$). Now the differential rate of emission of a given species of scalar particle of charge $e$ and mass $m = |e|$ is

$$\Gamma(\omega) \frac{d\omega}{2\pi} = \frac{1}{8\pi^2} \frac{A_h k^2 (\omega - e)}{\omega^{3/2} (\omega - e)^2 - 1} d\omega.$$  

Note that the ten possible polarizations of $h_{ij}$ give ten different species of scalars. For a given species, the total power radiated for frequencies between $\omega_1$ and $\omega_2$ is

$$P(\omega_1, \omega_2) = \int_{\omega_1}^{\omega_2} \omega \Gamma(\omega) \frac{d\omega}{2\pi}.$$  

In the next section we carry out a parallel calculation of the charged scalar emission rate by solving the scalar field equation in the classical black hole background.

### 3 The five-dimensional case

The main result of [9] was to show that for $e = 0$, (7) can be reproduced by a semi-classical calculation in the long wavelength limit $Q/k^2 \ll 1$. Our goal in this section is to extend this result to the case of charged particles.

Let us start by reviewing the classical geometry [20, 5, 6]. The 1- and 5-brane configuration in ten dimensions is described by the following string metric and dilaton:

$$ds^2_{(10, \text{str})} = \frac{1}{\sqrt{f_1 f_5}} \left(-dt^2 + dy_1^2 + K(dt + dy_1)^2\right) + \sqrt{f_1/f_5} \left(dy_2^2 + \ldots + dy_5^2\right) +$$

\[\text{We thank J. Maldacena and A. Strominger for informing us of their independent verification of the charged particle case [17].}\]
The charges $Q_1$ and $Q_5$ are proportional to the numbers of 1- and 5-branes. In the following, we will only need the normalization of the product of the two charges [5, 17]:

$$Q_1 Q_5 = \kappa_5^2 L_1 n_1 n_5 / 4 \pi^3 ,$$

where $\kappa_5^2 = \kappa_{10}^2 / \prod_{i=1}^5 L_i$. The Kaluza-Klein charge is also quantized [5, 17]:

$$Q_K = \kappa_5^2 n_K / \pi L_1 .$$

Reducing to six dimensions by compactifying $y_2, \ldots, y_5$ on a four-torus $T^4$ yields the metric which describes a string pointing along the $\hat{1}$-direction:

$$ds^2(6) = \frac{1}{\sqrt{f_1 f_5}} \left( -dt^2 + dy_1^2 + K(dt + dy_1)^2 \right) \sqrt{f_1 f_5} \left( dr^2 + r^2 d\Omega_{S^3}^2 \right) .$$

The six-dimensional dilaton is constant. Reduction from the string in $D = 6$ to the black hole in $D = 5$ is achieved by comparing (13) to the form

$$ds^2(5) = e^{-2D/3} ds^2(5) + e^{2D} \left( dy_1 + A_\mu dx^\mu \right)^2 ,$$

where the $x^\mu$ are coordinates for the five noncompact directions. The factors of $e^D$ in (14) are arranged to put the five-dimensional metric in Einstein frame. The result is

$$ds^2(5) = -f^{-2/3} dt^2 + f^{1/3} \left( dr^2 + r^2 d\Omega_{S^3}^2 \right) ,$$

$$f = \left( 1 + \frac{Q_1}{r^2} \right) \left( 1 + \frac{Q_5}{r^2} \right) \left( 1 + \frac{Q_K}{r^2} \right) ,$$

$$A_0 = K / (1 + K) = Q_K / (Q_K + r) .$$

We have chosen the normalization of $A_0$ so that $m = |e|$ for BPS saturated charged scalars, as in section 2. The horizon area and Bekenstein-Hawking entropy following from (15) are

$$A_h = 2 \pi^2 \prod_{i=1}^3 \sqrt{Q_i} ,$$

$$S_{BH} = \frac{A_h}{4 G_5} = 2 \pi \prod_{i=1}^3 \sqrt{n_i} .$$

$$\left( d\Omega_{S^3}^2 \right) .$$

$$\left( d\Omega_{S^3}^2 \right) .$$
where for convenience we have set $Q_2 = Q_5$, $Q_3 = Q_K$, $n_2 = n_5$, and $n_3 = n_K$.

To describe slight departures from extremality, one introduces a small parameter $\mu$ with the same dimensions ($[\text{length}]^2$) as the $Q_i$ [21, 22]. The near-extremal entropy has the form characteristic of $1+1$ dimensional field theory if we choose the charges in the following way:

$$\mu \ll Q_K \ll Q_1, Q_5 .$$  \hfill (17)

This condition is necessary to insure that the anti-onebranes and anti-fivebranes [23] are suppressed, so that the departure from extremality is due only to the right movers on the intersection string. The changes in the entropy and the Hawking temperature are then given by

$$\Delta S = \sqrt{\pi E_{\text{eff}}} , \quad T_H = 2 \sqrt{\frac{E}{\pi L_{\text{eff}}}} , \hfill (18)$$

where $E = M - M_0$.

When the five-dimensional black hole is raised slightly above extremality, the dominant Hawking radiation processes are those where scalars are emitted in an $s$-wave. The $s$-wave is dominant because the near-extremal black hole emits mostly particles with wavelength much longer than its typical length scales $\sqrt{Q_i}$. Emission rates for higher partial waves and for higher spin particles are expected to be suppressed by powers of $A_h/\lambda^3$. The analysis of section 2 enables us to compare to string theory the emission rate of scalars coming from the $T^4$ polarizations of the ten-dimensional graviton.

The Hawking rate is determined by the classical absorption probability, which for the case at hand may be calculated using the extremal geometry [3]. Intuitively speaking, the reason why the extremal geometry may be used is because its horizon has finite area and finite electrostatic potential which are corrected only at order $\mu$ in the near extremal case: consequently the classical absorption probability also suffers only $O(\mu)$ departures from its value at extremality.

The classical field equation in five dimensions for an $s$-wave scalar follows from plugging the ansatz $\phi(t, y_1, r) = e^{-i \omega t} e^{-i y_1} R(r)$ into the six-dimensional Laplace equation:

$$\Box_{(6)} \phi = \frac{1}{\sqrt{-g^{(6)}}} \partial_M \sqrt{-g^{(6)}} g^{MN(6)} \partial_N \phi = 0 . \hfill (19)$$

The radial equation takes the remarkably simple form

$$\left[ (\omega - e A_0)^2 - \frac{m^2}{(1 + K)^2} + \frac{1}{f r^3} \frac{d}{dr} r^3 \frac{d}{dr} \right] R(r) = 0 . \hfill (20)$$

Following the method developed by Unruh [24] and used in [8, 9], we solve the radial equation to leading order in the small quantities $Q_i k^2$ and extract the classical absorption probability. The solution is achieved by patching together three regions:
I. For \( r \ll \sqrt{Q_i} \), that is to say very close to the horizon, the dominant terms of (20) are

\[
\left( \frac{1}{r^3} \frac{d}{dr} r^3 \frac{d}{dr} + \frac{P}{r^6} \right) R(r) = 0 \tag{21}
\]

where

\[
P = (\omega - ea_0)^2 \prod_{i=1}^{3} Q_i \tag{22}
\]

and \( a_0 = A_0|_{r=0} = 1 \) is the electrostatic potential on the horizon. The solution which determines classical absorption by the black hole must represent purely infalling matter close to the horizon. The infalling solution to (21) is

\[
R(r) = e^{i\sqrt{P}/(2r^2)} \tag{23}
\]

II. The key to matching the near and far regions is to use the long wavelength limit to trivialize (20) for \( r \sim \sqrt{Q_i} \). The result is

\[
\frac{1}{r^3} \frac{d}{dr} r^3 \frac{d}{dr} R(r) = 0 \tag{24}
\]

and the solution is of the form

\[
R(r) = C + D/r^2 \tag{25}
\]

III. For \( r \gg \sqrt{Q_i} \), an expansion in powers of \( 1/r \) up to \( 1/r^2 \) yields

\[
\left[ \frac{1}{r^3} \frac{d}{dr} r^3 \frac{d}{dr} + k^2 \right] 1 + \frac{\sum_{i=1}^{3} Q_i - 2eQ_3/(\omega + e)}{r^2} \right] R(r) = 0 \tag{26}
\]

where \( k^2 = \omega^2 - m^2 \). The general solution is

\[
R(r) = \frac{\alpha J_\nu(kr) + \beta J_{-\nu}(kr)}{kr} \\
\nu = \sqrt{1 - k^2 \left( \sum_{i=1}^{3} Q_i - 2eQ_3/(\omega + e) \right)} \tag{27}
\]

To obtain matching between II and III it is necessary to have \( \nu \) close to 1. For \( e > 0 \), \( \nu = 1 - O(Q_i k^2) \), so matching is insured by the long wavelength limit. For \( e < 0 \), \( 1 - \nu \) is not small unless also \( Q_k m \omega \ll 1 \). The condition \( Q_k m \omega \ll 1 \) is necessary to insure that the particle is perturbatively scattered: if it fails, then resonance with bound states affects
scattering processes significantly, and re-absorption must be taken into account in Hawking emission calculations.

Matching the three regions to leading order in $1 - \nu$, we find

$$\alpha = 2 \quad \beta = \frac{ik^2 \sqrt{P}}{4(1 - \nu)}.$$  

We have allowed the normalization of $R(r)$ to be fixed by the coefficient on the near-horizon solution (23). This normalization is arbitrary, but for the purpose of determining the S-matrix element, only the relative coefficient between outgoing and ingoing waves far from the black hole is relevant. This relative coefficient is determined by the ratio $\beta/\alpha$, as we shall see.

The standard asymptotic form for an $s$-wave that one derives from a partial wave expansion is

$$R(r) \sim S_0 e^{ikr} - i e^{-ikr} \frac{1}{(kr)^{3/2}} \quad \text{as } r \to \infty.$$  

Comparison of (29) with the $r \to \infty$ asymptotics of (27) allows one to read off the S-matrix element:

$$S_0 = e^{i\pi(1-\nu)} \frac{1 - \frac{\beta}{\alpha} e^{-i\pi(1-\nu)}}{1 - \frac{\beta}{\alpha} e^{i\pi(1-\nu)}}.$$  

The classical absorption probability is

$$1 - |S_0|^2 \approx \frac{\pi}{2} k^2 (\omega - ea_0) \prod_{i=1}^{3} \sqrt{Q_i} = \frac{k^2 (\omega - ea_0) A_h}{4\pi}.$$  

The Hawking rate is computed from the classical absorption probability via a standard formula of quantum field theory in curved spacetime [25]:

$$\Gamma(\omega) \frac{d\omega}{2\pi} = \frac{1 - |S_0|^2}{e^{\beta H(\omega-ea_0)} - 1} \frac{d\omega}{2\pi} \approx \frac{k^2 (\omega - ea_0) A_h}{8\pi^2 e^{\beta H(\omega-ea_0)} - 1} d\omega,$$  

in exact agreement with (3) since $a_0 = 1$ at extremality. Note how the electrostatic potential on the horizon, $a_0$, enters as a chemical potential. We have verified that (32) holds far from extremality as well, provided $\mu m^2 \ll 1$. For the near-extremal geometry, $a_0 = 1 - O(\mu)$ where $\mu$ is the parameter measuring deviation from extremality. It would be interesting to see how the D-brane picture reproduces the $O(\mu)$ corrections.

It is worth pointing out two simple physical properties of (32). First, Hawking emission of particles with $e > 0$ is enhanced relative to $e < 0$ particles of the same mass, whereas the classical absorption of the former is suppressed relative to the latter. This is just what one expects from a black hole with positive charge and hence $a_0 > 0$. Second, the phenomenon of super-radiance cannot occur here because $\omega - ea_0 \geq \omega - m \geq 0$: in effect it is forbidden by supersymmetry.
4 The four-dimensional case

The semi-classical computation for the four-dimensional case is another straightforward application of the methods developed in [24]. As in the five-dimensional case, it suffices to compute the S-matrix element for long wavelength s-wave scattering to zeroth order in $\mu$, that is to say for the extremal black hole. The 11-dimensional configuration of three sets of 5-branes intersecting along a common 1-brane is described by the metric [21]

$$\text{ds}_{(11)}^2 = (f_1 f_2 f_3)^{-1/3} \left[ -dt^2 + dy_1^2 + K(dt + dy_1)^2 \right] + (f_1 f_2 f_3)^{-1/3} \left[ f_3 \left( dy_2^2 + dy_3^2 \right) + f_2 \left( dy_4^2 + dy_5^2 \right) + f_1 \left( dy_6^2 + dy_7^2 \right) \right] + (f_1 f_2 f_3)^{2/3} \left( dr^2 + r^2 d\Omega_{S^2}^2 \right)$$

$$f_i = 1 + \frac{Q_i}{r}, \quad K = \frac{Q_K}{r}.$$  \tag{33}

The charges $Q_i$ are related to the numbers of 5-branes [17]:

$$Q_1 = \frac{n_1}{L_6 L_7} \left( \frac{\kappa_{11}}{4\pi} \right)^{2/3}, \quad Q_2 = \frac{n_1}{L_4 L_5} \left( \frac{\kappa_{11}}{4\pi} \right)^{2/3}, \quad Q_3 = \frac{n_1}{L_2 L_3} \left( \frac{\kappa_{11}}{4\pi} \right)^{2/3},$$  \tag{34}

where $L_i$ is the range of the coordinate $y_i$. The Kaluza-Klein charge $Q_K$ is quantized as [17]

$$Q_K = \kappa_4^2 \frac{n_K}{L_1},$$  \tag{35}

where $\kappa_4^2 = \kappa_{11}^2 / \prod_{i=1}^7 L_i$.

To obtain the string in five dimensions one compactifies on a six-torus involving the coordinates $y_2, \ldots, y_7$. This gives the following metric:

$$\text{ds}_{(5)}^2 = (f_1 f_2 f_3)^{-1/3} \left[ -dt^2 + dy^2 + K(dt + dy)^2 \right] + (f_1 f_2 f_3)^{2/3} \left( dr^2 + r^2 d\Omega_{S^2}^2 \right)$$  \tag{36}

while the five-dimensional dilaton turns out to be constant. Reduction from the string in $D = 5$ to the black hole in $D = 4$ is achieved by comparing (36) to the form

$$\text{ds}_{(5)}^2 = e^{-D} \text{ds}_{(4)}^2 + e^{2D} \left( dy_1 + A_\mu dx^\mu \right)^2.$$  \tag{37}

where the factors of $e^D$ are arranged to put the four-dimensional metric in Einstein frame. The results are quite similar to the five-dimensional black hole:

$$\text{ds}_{(4)}^2 = -f^{-1/2} dt^2 + f^{1/2} \left( dr^2 + r^2 d\Omega_{S^2}^2 \right)$$

$$f = \prod_{i=1}^4 \left( 1 + \frac{Q_i}{r} \right),$$

$$A_0 = K/(1 + K) = Q_K/(Q_K + r).$$  \tag{38}
where we have set $Q_4 = Q_K$. The horizon area and Bekenstein-Hawking entropy are given by

\[
A_h = 4\pi \prod_{i=1}^{4} \sqrt{Q_i},
\]

\[
S_{BH} = \frac{A_h}{4G_4} = 2\pi \prod_{i=1}^{4} \sqrt{n_i} .
\]  

(39)

As shown in [21], the near-extremal entropy has the form (18) characteristic of $1+1$ dimensional field theory if the charges satisfy

\[
\mu \ll Q_K \ll Q_1, Q_2, Q_3,
\]

(40)

where again $\mu$ parametrizes small departures from extremality. The Hawking temperature is also given by (18) with $L_{\text{eff}} = L_1 n_1 n_2 n_3$.

As in the case of the five-dimensional black hole and for the same reasons, the dominant Hawking radiation processes for this four-dimensional black hole are those where a four-dimensional scalar is emitted in an $s$-wave. The classical field equation has almost the same form as in the five-dimensional case: plugging the ansatz $\phi(t, y_1, r) = e^{-i\omega t} e^{-i\omega y} R(r)$ into the equation

\[
\Box_{(5)} \phi = \frac{1}{\sqrt{-g^{(5)}}} \partial_M \sqrt{-g^{(5)}} g^{MN} \partial_N \phi = 0 ,
\]

(41)

one obtains

\[
\left[ (\omega - e A_0)^2 - \frac{m^2}{(1 + K)^2} + \frac{1}{fr^2} \frac{d}{d r} \frac{d}{d r} \right] R(r) = 0 .
\]  

(42)

We have again chosen conventions so that $m = |e|$. One can solve (42) to leading order in the small quantities $Q_k r$ (where $k^2 = \omega^2 - m^2$) using simpler approximations than in the five-dimensional case. The three regions can be treated as follows:

I. In the near region, the dominant terms of (42) are

\[
\left( \frac{1}{r^2} \frac{d}{d r} r^2 \frac{d}{d r} + \frac{P}{r^4} \right) R(r) = 0
\]

(43)

where

\[
P = (\omega - e a_0)^2 \prod_{i=1}^{4} Q_i .
\]

(44)

The infalling solution is
\[ R(r) = e^{\sqrt{P}/r} . \]  

II. In the intermediate region, the same long wavelength limit as used in the five-dimensional case makes the equation trivial:

\[ \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} R(r) = 0 . \]  

The solution is of the form

\[ R(r) = C + D/r . \]  

III. Far from the black hole, it turns out to be sufficient to use the free particle equation,

\[ \left( \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} + k^2 \right) R(r) = 0 . \]  

The general solution is

\[ R(r) = \alpha \frac{\sin kr}{kr} - \beta \frac{\cos kr}{kr} . \]  

To insure perturbative scattering it is necessary to assume \( Q_K m \omega / k \ll 1 \) when \( e < 0 \). Matching the three regions yields

\[ \alpha = 1 \quad \beta = -ik\sqrt{P} . \]  

Comparing the solution (49) with the standard asymptotic form

\[ R(r) \sim \frac{S_0 e^{ikr} - e^{-ikr}}{kr} \quad \text{as } r \to \infty \]  

yields for the S-matrix element

\[ S_0 = \frac{1 - i\beta / \alpha}{1 + i\beta / \alpha} = \frac{1 - k\sqrt{P}}{1 + k\sqrt{P}} . \]  

The Hawking rate is computed as before:

\[ \Gamma(\omega) \frac{d\omega}{2\pi} = \frac{1 - |S_0|^2}{e^{\beta_H(\omega - e a)} - 1} \frac{d\omega}{2\pi} \approx \frac{1}{2\pi^2} \frac{k(\omega - e a_0) A_p}{e^{\beta_H(\omega - e a)} - 1} d\omega , \]  

where we have used (39). (53) holds far from extremality, provided \( \mu m \ll 1 \).

Now we would like to argue that, for the scalars arising in four dimensions from 11-dimensional gravitons polarized in the \( y_2, \ldots, y_7 \) directions, this emission rate is exactly reproduced by a simple model analogous to the one used for the \( D = 5 \) black hole. This
model is based on a long string whose left and right moving fluctuations collide to produce outgoing charged scalars. From the M-theory point of view, this long string pointing in the \( \hat{1} \) direction is at the triple intersection of 5-branes. If the length of the \( y_1 \) direction is \( L_1 \), then the effective length of the intersection string is \( L_{\text{eff}} = n_1 n_2 n_3 L_1 \). If we assume that there are four massless bosons and four massless fermion modes on this long string, then the Bekenstein-Hawking entropy \( (39) \) is correctly reproduced \([17, 21]\).  

While the details of the dynamics of intersecting M-branes are unknown, our calculation is largely independent of them. We just assume the geometrical coupling of the long string to the gravitons polarized in the \( y_2, \ldots, y_7 \) directions:

\[
T_D \int d^2 \xi \frac{1}{2} (\delta_{ij} + 2 \kappa_5 h_{ij}) \partial_\alpha X^i \partial^\alpha X^j .
\]

This is identical to the coupling in \((1)\), but with \( \kappa_6 \) replaced by \( \kappa_5 \). In fact, the entire calculation of the emission from a long string, presented in section 2, carries over to the \( D = 4 \) case with minor alterations. The only changes needed in \((1)\) are the replacement of \( \kappa_5 \) by \( \kappa_4 \) and of \( d^4 k/(2\pi)^4 \) by \( d^3 k/(2\pi)^3 \). One then obtains

\[
\Gamma(\vec{k}) \frac{d^3 k}{(2\pi)^3} = \frac{\kappa_4^2 L_{\text{eff}}}{4} \frac{k^2}{\omega} \rho_L \left( \frac{\omega + e}{2} \right) \rho_R \left( \frac{\omega - e}{2} \right) \frac{d^3 k}{(2\pi)^3}
\]

for the differential rate in \( D = 4 \). Making use of \((1)\), and of \((1)\) with \( G_5 \) replaced by \( G_4 \) (which is applicable because we again assume that there are four species of massless bosons and fermions), we bring the differential rate into the form

\[
\Gamma(\omega) \frac{d\omega}{2\pi} = \frac{1}{2\pi^2} A_4 k(\omega - e) \frac{d\omega}{\omega^2} ,
\]

which is identical to \((53)\) derived in the context of semi-classical gravity! While a better derivation of the emission model based on the multiply wound string is clearly necessary, this model incorporates important properties of semi-classical black holes.

5 Discussion

In this paper we have presented new evidence in favor of a microscopic picture behind semi-classical near-extremal black holes. There are two simple classes of black holes whose extremal limits preserve some supersymmetry and are characterized by finite horizon area. These are the \( D = 5 \) black hole with three \( U(1) \) charges and the \( D = 4 \) black hole with four \( U(1) \) charges. Both cases may be represented as branes intersecting along a string: in the former case it is sufficient to use D-branes alone, while in the latter one may use triply intersecting 5-branes of M-theory. We have confirmed that microscopic models based on these brane descriptions predict a charged particle emission rate which agrees exactly, including the normalization, with the semi-classical treatment of Hawking radiation. Once the brane
calculations are reduced to small fluctuations of the intersection string, they become quite simple and almost entirely independent of the details of the higher dimensional theory.

The formulae for charged particle emission reveal some simple physically expected properties. If the black hole is positively charged, then the emission of positively charged particles of mass $m$ is enhanced compared to that of negatively charged particles of the same mass. In fact, the emission rate for negatively charged particles of long wavelengths contains an exponential suppression factor, $e^{-2\beta H m}$, consistent with the semi-classical interpretation of tunneling.

It would be interesting to calculate the net emission rate for charge and compare it with the energy emission rate. Presumably, after radiating some charge and mass, the near-extremal black hole will stabilize at some values of charge and mass which satisfy the extremality relation. We hope to return to this issue in a later publication.

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