General scan in flavor parameter space in the models with vector
quark doublets and an enhancement in $B \to X_s \gamma$ process

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Abstract

In the models with vector like quark doublets, the mass matrices of up and down type quarks are related. Precise diagonalization for the mass matrices becomes an obstacle in the numerical studies. In this work we propose a diagonalization method at first. As its application, in the standard model with one vector like quark doublet we present quark mass spectrum, Feynman rules for the calculation of $B \to X_s \gamma$. We find that i) under the constraints of the CKM matrix measurements, the mass parameters in the bilinear term are constrained to a small value by the small deviation from unitarity; ii) compared with the fourth generation extension of the standard model, there is an enhancement to $B \to X_s \gamma$ process in the contribution of vector like quark, resulting a non-decoupling effect in such models.

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I. INTRODUCTION

Though the standard model (SM) has been verified to be correct times by times, many new physics beyond standard model are proposed to solve both experimental and aesthetical problems, such as neutrino masses, $\mu$ anomalous magnetic movement problem or hierarchy problem, etc. Many new models introduce vector like particles (VLP) whose right handed and left handed components transform in the same way under the weak SU(2)×U(1) gauge group. The extension is acceptable because the anomalies generated by the VLPs cancel automatically, and vector quarks can be heavy naturally. VLPs also arise in some grand unification theories. For example, in order to explain the little hierarchy problem between the traditional GUT scale and string scale, a testable flipped SU(5) × U(1)$_X$ model are proposed in Ref. 2 in which the TeV-scale VLPs were introduced 3. Such kind of models can be constructed from the free fermionic string constructions at the Kac-Moody level one 4, 5 and from the local F-theory model 2, 6.

However when we do the flavor physics with doublet VLPs in these models 7, 8, a problem always appears when we are dealing with the mass spectrum of quarks and leptons. Let us start with the SM in which all fermion masses come from the Yukawa couplings. After the spontaneously gauge symmetry breaking, we can get two separate mass matrices $M_U$, $M_D$ for the up and down type fermions. The mass eigen states are obtained after the diagonalization

$$Z^\dagger U M_U U = M^D_U, \quad Z^\dagger D M_D U = M^D_D,$$

where $M^D_U = \text{diag.}[m_u, m_c, m_t]$, $M^D_D = \text{diag.}[m_d, m_s, m_b]$. The physical measurable parameters are $m_i$ and the so called CKM matrix

$$V_{\text{CKM}} = U^\dagger D U.$$

Since $M_U$, $M_D$ come from separate Yukawa couplings, we can always set one of the matrices diagonal, for example $M_U$, and use the CKM matrix to get the Yukawa couplings

$$Z_D \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} V_{\text{CKM}}^\dagger = \begin{pmatrix} Y_{11}^D v & Y_{12}^D v & Y_{13}^D v \\ Y_{21}^D v & Y_{22}^D v & Y_{23}^D v \\ Y_{31}^D v & Y_{32}^D v & Y_{33}^D v \end{pmatrix}$$

for the calculation in flavor physics. Note that $v$ is the vacuum expectation value (VEV) of the Higgs, and $Z_D$ is a random unitary matrix.
Such a trick can not be used in case of the participation of a vector doublet, namely $Q$ with gauge charge $3, 2, \frac{1}{6}$ and $\bar{Q}$ with gauge charge $\bar{3}, 2, -\frac{1}{6}$, resulting bilinear term in the lagrangian

$$M^V Q \cdot \bar{Q}.$$ 

It is clear that in the model, there are the same input parameters in the matrices $M_U, M_D$:

$$M_U = \begin{pmatrix}
Y_{11}^U & Y_{12}^U & Y_{13}^U & \cdots \\
Y_{21}^U & Y_{22}^U & Y_{23}^U & \cdots \\
Y_{31}^U & Y_{32}^U & Y_{33}^U & \cdots \\
M_{41}^V & M_{42}^V & M_{43}^V & \cdots 
\end{pmatrix}, \quad M_D = \begin{pmatrix}
Y_{11}^D & Y_{12}^D & Y_{13}^D & \cdots \\
Y_{21}^D & Y_{22}^D & Y_{23}^D & \cdots \\
Y_{31}^D & Y_{32}^D & Y_{33}^D & \cdots \\
-M_{41}^V & -M_{42}^V & -M_{43}^V & \cdots 
\end{pmatrix}. \quad (4)$$

The mass matrices for up and down type quarks are related to each other. Therefore, we can not set one of the matrices diagonal and the CKM matrix can not be got easily. The shooting method is always used to treat such an obstacle. Random $M_U$ and $M_D$ are generated to meet the requirements after diagonalization: the mass of eigen state and the measurements of elements of CKM matrix. However this is too much time consuming, and precise solution for diagonalization is almost unavailable. Although this is just a numerical problem, when one treats the VLP contributions to the flavor physics seriously, diagonalization of quark matrices will be the first and important step.

In this paper, we will first propose a general method to solve the obstacle in models with vector like quark doublets. As its application, we will study rare B decay $B \to X_s \gamma$ in the SM with one vector like quark doublet. The paper is organized as follows. We show the detail of the trick in Section 2. The simple application to $B \to X_s \gamma$ process, including quark mass spectrum, Feynman rules and the Wilson coefficients, as well as the numerical analysis for calculation of $B \to X_s \gamma$ is shown in Section 3. A summary is given in Section 4.

II. THE TRICK OF DIAGONALIZATION OF VECTOR QUARK DOUBLET

Firstly, we address the problem clearly on how to deal with the diagonalization of $N \times N$ matrix $M_U$ and $M_D$:

$$Z_U^\dagger M_U U_U = M_D^U, \quad Z_D^\dagger M_D U_D = M_D^D \quad (5)$$

in which $M_U^D, M_D^D$ are the diagonal mass matrices for up and down type quark, respectively. Note that $N$ should be greater than 3 and the first three elements in the matrices should be
the three generations of quark multiplates in the SM, other elements with \( N > 3 \) are the new multiplates introduced in new physics beyond the SM. Then we have

\[
M_U = \begin{pmatrix}
Y^U_{11} & Y^U_{12} & Y^U_{13} & \cdots & M_{U1N} \\
Y^U_{21} & Y^U_{22} & Y^U_{23} & \cdots & M_{U2N} \\
Y^U_{31} & Y^U_{32} & Y^U_{33} & \cdots & M_{U3N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
M^U_{N1} & M^U_{N2} & M^U_{N3} & \cdots & M_{UNN}
\end{pmatrix}, \quad
M_D = \begin{pmatrix}
Y^D_{11} & Y^D_{12} & Y^D_{13} & \cdots & M_{D1N} \\
Y^D_{21} & Y^D_{22} & Y^D_{23} & \cdots & M_{D2N} \\
Y^D_{31} & Y^D_{32} & Y^D_{33} & \cdots & M_{D3N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
M^D_{N1} & M^D_{N2} & M^D_{N3} & \cdots & M_{DNN}
\end{pmatrix}.
\]

(6)

The last line of the two matrices has the same parameters except the last elements.

Considering that there are some same parameters in \( M_U \) and \( M_D \), we find that a very simple way is to add two matrices in Eq. (6)

\[
M_U + M_D = (Z_U M^D_U U_{CKMN} + Z_D M^D_D) U_D^\dagger.
\]

(7)

The left side of the equation is

\[
M_U + M_D = \begin{pmatrix}
Y^U_{11} + Y^D_{11} & Y^U_{12} + Y^D_{12} & Y^U_{13} + Y^D_{13} & \cdots & M_{U1N} + M_{D1N} \\
Y^U_{21} + Y^D_{21} & Y^U_{22} + Y^D_{22} & Y^U_{23} + Y^D_{23} & \cdots & M_{U2N} + M_{D2N} \\
Y^U_{31} + Y^D_{31} & Y^U_{32} + Y^D_{32} & Y^U_{33} + Y^D_{33} & \cdots & M_{U3N} + M_{D3N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & M_{UNN} + M_{DNN}
\end{pmatrix}.
\]

(8)

Obviously, the mass inputs from bilinear terms vanish. We can denote the matrix in the form as

\[
M_U + M_D = M_{UD} = \begin{pmatrix}
M_A & M_B \\
M_0 & M_C
\end{pmatrix},
\]

(9)

in which \( M_\text{A}, M_\text{B}, M_0 \) are \((N - 1) \times (N - 1), (N - 1) \times 1 \) and \( 1 \times (N - 1) \) matrices correspondingly.

To prepare for the diagonalization, we chose the diagonal mass matrix elements of quarks \((m_u, m_c, m_t, \cdots m_X)\), \((m_d, m_s, m_b, \cdots m_Y)\) and a matrix \(U_{CKMN}\), which are determined partly by experimental measurements as input parameters

\[
U_{CKMN} = U^\dagger_U U_D = \begin{pmatrix}
(U_{CKM})_{3 \times 3} & \cdots & \cdots \\
\vdots & \ddots & \ddots \\
U_{NN}
\end{pmatrix} = \begin{pmatrix}
U_{ud} & U_{us} & U_{ub} \\
U_{cd} & U_{cs} & U_{cb} \\
U_{td} & U_{ts} & U_{tb} \\
\vdots & \ddots & \ddots \\
U_{NN}
\end{pmatrix}.
\]

(10)
Note that above \( Z_U, Z_D, U_U, U_D \) are unitary matrices, but \((U_{CKM})_{3\times3} \) is not an ordinary CKM matrix \( V_{CKM} \) which is non-unitary in this case. Detailed discussion will be shown in the following section.

What we need to do for the next is to generate a unitary matrix \( U_D \). In the similar way we denote \( U_D \) as

\[
U_D = \begin{pmatrix} U_{DA} & U_{DB} \\ U_{D0} & U_{DNN} \end{pmatrix}.
\]

Both sides of Eq. (7) times the matrix \( U_D \), we can get

\[
M_{UDU_D} = \begin{pmatrix} M_A U_{DA} + M_B U_{D0} & M_A U_{DB} + M_B U_{DNN} \\ M_C U_{D0} & M_C U_{DNN} \end{pmatrix} = (Z_U M_U^P U_{CKMN} + Z_D M_D^P).
\]

From above equation, we can get the last line of \( U_D \) simply by inputting \( M_U^P, M_D^P, U_{CKMN} \) and random \( Z_U, Z_D \):

\[
(Z_U M_U^P U_{CKMN} + Z_D M_D^P)_{\text{last line}} = \begin{pmatrix} M_C U_{D0} \\ M_C U_{DNN} \end{pmatrix} = M_C U_{DN},
\]

where

\[
U_{DN} = \begin{pmatrix} U_{DN1} & U_{DN2} & \cdots & U_{DNN} \end{pmatrix}
\]

is a unit vector in \( N \) dimension.

Next we use the unit vector to generate total \( U_D \). Since \( M_A \) and \( M_B \) are random matrix, \( U_D \) can be random too. The unit vector \( U_{DN-1} \) of \( U_D \) can be determined as

\[
U_{DN-1} = \begin{pmatrix} -\frac{U_{DN2}}{\sqrt{U_{DN1}^2 + U_{DN2}^2}} \\ \frac{U_{DN1}}{\sqrt{U_{DN1}^2 + U_{DN2}^2}} \\ 0 \cdots 0 \end{pmatrix}.
\]

It is clear that the vector is orthogonal to \( U_{DN} \) and normalized to 1. Then we use the first three elements of \( U_{DN} \) and \( U_{DN-1} \) to generate \( U_{DN-2} \): Normalize the algebraic complements of first line of the \( 3 \times 3 \) matrix. Step by step, we can finally get \((U_{D1}, U_{D2}, \cdots, \)
\( U_{DN-1} \) and form a special \( U_D^S \)

\[
U_D^S = \begin{pmatrix}
U_{D1} \\
\vdots \\
U_{DN-2} \\
U_{DN-1} \\
U_D
\end{pmatrix} = \begin{pmatrix}
U_{D11} & U_{D12} & U_{D13} & \cdots & U_{D1N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
U_{D(N-2)1} & U_{D(N-2)2} & U_{D(N-2)3} & \cdots & 0 \\
U_{D(N-1)1} & U_{D(N-1)2} & 0 & \cdots & 0 \\
U_{DN1} & U_{DN2} & U_{DN3} & \cdots & U_{DNN}
\end{pmatrix}.
\] (16)

From above steps, we can see that \((U_{D1}, U_{D2}, \cdots, U_{DN-1})\) can be rotated into any other orthogonal \(N-1\) vectors to construct random matrix \(M_A\) and \(M_B\), only \(U_D\) must be kept unchanged. Therefore, a general unitary matrix can be realized by timesing a unitary \(N \times N\) matrix \(U_R\),

\[
U_D = U_R U_D^S = \begin{pmatrix}
U_{RN-1} & 0 \\
0 & 1
\end{pmatrix} U_D^S
\] (17)

in which \(U_{RN-1}\) is a \((N-1) \times (N-1)\) unitary matrix. We finish the work by

\[
U_U^\dagger = U_{CKMN} U_D^\dagger
\] (18)

\[
M_U = Z_U M_U^D U_U^\dagger
\] (19)

\[
M_D = Z_D M_D^D U_D^\dagger
\] (20)

At this stage, we would like to summarize our method here

- Step 1: Choose \((m_u, m_c, m_t, \cdots, m_X, m_d, m_s, m_b, \cdots, m_y)\) and \(U_{CKMN}\) and generate random unitary matrices \(Z_U\) and \(Z_D\) as the inputs for the model;

- Step 2: Determine the last line of matrix \(Z_U M_U^D U_{CKMN} + Z_D M_D^D\) as

\[
M_C \begin{pmatrix}
U_{DN1} & U_{DN2} & \cdots & U_{DNN}
\end{pmatrix}
\] (21)

and normalize it into a unit vector \(U_D\).

- Step 3: Use the unit vector \(U_D\) to generate other \(N-1\) unitary vectors \((U_{D1}, U_{D2}, \cdots, U_{DN-1})\), and form a special \(U_D^S\)

\[
U_D^S = \begin{pmatrix}
U_{D1} & \cdots & U_{DN-2} & U_{DN-1} & U_D
\end{pmatrix}^T.
\] (22)
- Step 4: Generate a $N - 1$ unitary matrix $U_{R,N-1}$ to form a unitary matrix $U_R$ which is

$$U_R = \begin{pmatrix} U_{R,N-1} & 0 \\ 0 & 1 \end{pmatrix},$$

then, a general $U_D$ is obtained by

$$U_D = U_R U_D^S.$$  \hspace{1cm} (24)

- Step 5: Use these equations

$$U_U^\dagger = U_{CKMN} U_D^\dagger,$$

$$M_U = Z_U M_U^D U_U^\dagger,$$

$$M_D = Z_D M_D^D U_D^\dagger,$$  \hspace{1cm} (25)

to get the inputs for the flavor physics.

We can see that by this trick we can skip the inputs of the bilinear mass terms $M_{N_i}^V$. In physical analysis, the mass of eigen states $m_X, \gamma$ in the VLP models are inputs freely. $Z_U$ and $Z_D$ can be generated randomly, $U_U$ and $U_D$ can also be scanned the most generally if we vary $U_R$ randomly. Thus the method can do the most general scan in the parameter space of mass matrices in the models with VLPs for the numerical studies, which will be shown in the following section.

### III. $B \rightarrow X_s \gamma$ PROCESS IN EXTENSION OF THE SM WITH ONE VECTOR LIKE QUARK DOUBLET

#### A. The standard model with vector like quarks

As an application of the method, in this section we study the VLP contribution to $B \rightarrow X_s \gamma$ in a very simple VLP extension of SM for the demonstration. In the Tab. II we list the gauge symmetry of the matter multiplates in which the first two queues show the quarks in the SM and the last two queues show the VLPs with the anti-gauge symmetry. Note that we ignore partners of the last two queues whose gauge symmetry are exactly the same as the first two queues of the SM. As talked in the introduction, these VLPs can
be heavy naturally. Since gauge symmetry of Higgs $H = (h^+, h^0)^T$ is $(1, 2, 1/2)$, the lagrangian for two quarks of the model is written as:

$$\mathcal{L} = Y_d \bar{Q} H d_R + Y_u \bar{Q} H u_R + Y_{d,u} \bar{V} Q H v L + Y_{d,u} \bar{V} Q H v L + M_d \bar{V} d_L d_R + h.c.,$$

in which $A \cdot B = e^{ij} A_i B_j$. The first line of the lagrangian is Yukawa terms, the second line is the bilinear terms. Note that $Y_u, Y_d$ are $3 \times 3$ matrix, without the bilinear terms, the model will be almost the same as the fourth generation standard model (SM4).

After the electro-weak symmetry breaking, we can get the mass matrices of up and down quarks in the basis of $(u, c, t, V_u)$ and $(d, s, b, V_d)$:

$$M_U = \begin{pmatrix} Y_u^{11} v & Y_u^{12} v & Y_u^{13} v & M_u^1 \\ Y_u^{21} v & Y_u^{22} v & Y_u^{23} v & M_u^2 \\ Y_u^{31} v & Y_u^{32} v & Y_u^{33} v & M_u^3 \\ -M_Q^1 & -M_Q^2 & -M_Q^3 & Y_v v \end{pmatrix}, \quad M_D = \begin{pmatrix} Y_d^{11} v & Y_d^{12} v & Y_d^{13} v & M_d^1 \\ Y_d^{21} v & Y_d^{22} v & Y_d^{23} v & M_d^2 \\ Y_d^{31} v & Y_d^{32} v & Y_d^{33} v & M_d^3 \\ M_Q^1 & M_Q^2 & M_Q^3 & Y_v v \end{pmatrix},$$

where $v$ is the VEV for $H$. The first three elements of last line of the matrices have the same parameter, making the scan of the parameter space very difficult. These two matrices can be diagonalized by unitary matrices $U$ and $Z$,

$$Z_u^\dagger M_u U_u = \text{diag.}[m_u, m_c, m_t, m_X],$$

$$Z_d^\dagger M_d U_d = \text{diag.}[m_d, m_s, m_b, m_Y].$$

Product of the two matrices is denoted as

$$U_{\text{CKM4}} = U_u^\dagger U_d,$$

which is unitary $4 \times 4$ matrix. We stress that the trick we introduced in the above section seems to just give us a numerical tool for quark masses and some quark mixing matrices, but it is important in studying the flavor physics in such models.
For studying VLP contributions to $B \to X_s \gamma$, we now present the Feynman rules for the interaction of $\bar{u}_i d_j \chi^+$, $\chi = W, G$ and $\bar{d}_i d_j Z$ in the Feynman gauge which read:

$$i \frac{g}{\sqrt{2}} \gamma^\mu \left[ g_L^W(i, j) P_L + g_R^W(i, j) P_R \right], \quad (\chi = W, Z),$$

$$i \frac{g}{\sqrt{2} m_W} \left[ g_L^\chi(i, j) P_L + g_R^\chi(i, j) P_R \right], \quad (\chi = G)$$

where

$$g_L^W(i, j) = \sum_{m=1}^3 U_{u}^{*m i} U_{d}^{m, j}, \quad g_R^W(i, j) = Z_{u}^{*i j},$$

$$g_L^G(i, j) = \sum_{k, m=1}^3 Y_u^{km} v Z_{u}^{ski} U_{d}^{m, j} + Y_{d} v Z_{u}^{*i j} U_{d}^{4, j},$$

$$g_R^G(i, j) = -\sum_{k, m=1}^3 Y_d^{km} v Z_{d}^{*kj} U_{d}^{mi} - Y_{V}^{*} v Z_{d}^{*j i} U_{d}^{4, j},$$

$$g_L^Z(i, j) = -\frac{1}{\sqrt{2} \cos \theta_W} \left[ \left( 1 - \frac{2}{3} \sin^2 \theta_W \right) \delta^{ij} - U_{d}^{*i j} U_{d}^{4, j} \right],$$

$$g_R^Z(i, j) = -\frac{1}{\sqrt{2} \cos \theta_W} \left[ -\frac{2}{3} \sin^2 \theta_W \delta^{ij} + Z_{d}^{*i j} Z_{d}^{4, j} \right].$$

Note that $U(1)_{1EM}$ interaction is not changed by the VLPs, thus the vertices of photon and quarks are still the same as those in the SM. From above mass matrices and Feynman rules, we can see that the model has two points to be explored:

- The CKM matrix is got from the $W^+ \bar{u}_i d_j$ vertex in Eq. (32)

$$V_{CKM4}^{ij} = \sum_{m=1}^3 U_{u}^{*m i} U_{d}^{m, j} = U_{CKM4}^{ij} - U_{u}^{*i j} U_{d}^{4, j}.$$ 

which is non-unitary for that the indexes $i, j$ range form 1 to 4, but the summation of index $m$ is from 1 to 3. $V_{CKM4}^{ij}$ is also a $4 \times 4$ matrix of which the upper left elements $(i, j \nu_e 4)$ are physical measurable value of CKM matrix $V$ as in the SM. This is the key difference between VLP models and the SM4. Nevertheless, the loop-level flavor change neutral current (FCNC) will be changed by the Yukawa interactions, then the prediction of process $B \to X_s \gamma$ may be changed significantly.

- The last terms in Eqs. (32)-(36), which we call the “tail terms”, violate the gauge universality of fermions and cause tree-level FCNC processes induced by the processes such as $b \to s \ell^+ \ell^-$, then the constraints on the parameter space need to be explored.
B. Enhancement in $b \to s$ transition

In this subsection we focus attention on VLP contributions to rare B decay $B \to X_s \gamma$. The starting point for rare B decays is the determination of the low-energy effective Hamiltonian obtained by integrating out the heavy degrees of freedom in the theory. For $b \to s$ transition, this can be written as

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10}[C_i(\mu)O_i(\mu) + C'_i(\mu)O'_i(\mu)],$$

where the effective operators $O_i$ are same as those in the SM defined in Ref. [9]. The chirality-flipped operators $O'_i$ are obtained from $O_i$ by the replacement $\gamma_5 \to -\gamma_5$ in quark current[7]. We calculate the Wilson coefficient $C_7$ at matching scale $m_W$. The leading order Feynman diagrams are shown in Fig. 1 and $C_7$ reads

$$C_7(m_W) = \frac{1}{V_{tb}V_{ts}^*} \sum_{i=1}^{4} \left[ g_L^{W*}(i,2)g_L^W(i,3)A(x_i) + \frac{g_L^{G*}(i,2)g_L^G(i,3)}{m_{u_i}^2} x_i B(x_i) ight. \\
+ \frac{g_L^{G*}(i,2)g_R^G(i,3)}{m_{u_i}m_b} x_i C(x_i) + \frac{g_L^{W*}(i,2)g_R^G(i,3)}{m_b} D(x_i) \\
\left. + \frac{m_{u_i}^2}{m_b} g_L^{W*}(i,2)g_L^W(i,3)E(x_i) + \frac{g_L^{G*}(i,2)g_R^W(i,3)}{m_b} D(x_i) \right],$$

where $x_i = m_{u_i}^2/m_W^2$ and the loop function $A(x), B(x), C(x), D(x), E(x)$ are listed in the appendix. The first two lines are the similar contribution as in the SM, while the last lines are the terms come form the tail terms. Note that the contribution of right diagram in the second line of Fig. 1 is zero in the SM. The terms with $1/m_b$ in above equation is extracted to compose the operator $O_7$. There are two differences in the calculation of $B \to X_s \gamma$ processes compared with the SM. One is the tail terms of gauge or Yukawa interactions,
another one is the new type of Yukawa interactions listed in Eqs. (33, 34) which can not be written into the simple form in the SM such as

\[ g_{L}^{G,SM}(i, 2) = m_u V_{us}, \quad g_{R}^{G,SM}(i, 3) = -m_b V_{tb}. \] (40)

| TABLE II: The CKM matrix elements constrained by the tree-level B decays. |

| V_{ud} | 0.97425 \pm 0.00022 | direct measurement from nuclear beta decay |
|--------|-------------------|----------------------------------------|
| V_{us} | 0.2252 \pm 0.0009 | semi-leptonic K-decay |
| V_{ub} | 0.00415 \pm 0.00049 | semi-leptonic B-decay |
| V_{cd} | 0.230 \pm 0.011 | semi-leptonic D-decay |
| V_{cs} | 1.006 \pm 0.023 | (semi-)leptonic D-decay |
| V_{cb} | 0.0409 \pm 0.0011 | semi-leptonic B-decay |
| V_{tb} | 0.89 \pm 0.07 | (single) top-production |

In the model with three generation quarks, the CKM matrix unitarity is already used in the calculations of the loop-level FCNC induced rare B decays. For consistency, in numerical analysis the constraints on CKM matrix element are not from processes occurred at loop level, such as rare B decays, but from tree-level processes shown in Table II [10, 11]. Since there are no tree-level measurements of V_{td}, V_{ts} now, we use above inputs and the unitarity to get 3 \times 3 unitary matrix at first. The method is that we scan (V_{ud}, V_{us}, V_{ub}) randomly (keeping |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1) in range listed in Table II then we define two parameters \( \alpha, \beta \) and solve them by the equations

\[ V_{ud}^* (V_{cd} + \alpha) + V_{us}^* (V_{cs} + \beta) + V_{ub}^* V_{cb} = 0, \]

\[ |V_{cd} + \alpha|^2 + |V_{cs} + \beta|^2 + |V_{cb}|^2 = 1. \] (41)

(V_{td}, V_{ts}, V_{tb}) are got by the unitarity relation with (V_{ud}, V_{us}, V_{ub}) and (V_{cd}, V_{cs}, V_{cb}). After that we times the 3 \times 3 unitary matrix with three matrices

\[
\begin{pmatrix}
1 & \cdots & \cdots & \cdots \\
\cdots & \cos \theta_{4i} & \cdots & \sin \theta_{4i} \\
\cdots & \cdots & \cdots & 1 \\
\cdots & -\sin \theta_{4i} & \cdots & \cos \theta_{4i}
\end{pmatrix}
\] (42)
in which $i = 1, 2, 3$ and $\max(|\theta_{41}|, |\theta_{42}|, |\theta_{43}|) < 0.01\pi$, to generate a $4 \times 4$ unitary matrix $U_{\text{CKM}4}$. $V_{\text{CKM}4}$ are got by the Eq. (37). All the corresponding elements should satisfy the experiment bound list in Table II and $V_{td}, V_{ts}$ ($|V_{ts}| \simeq 0.04$ which is consistent with the fitting results in Ref. [10]) can be got too. With this inputs in hand, the first task is to check the scale of the mass parameter of model, such as $M_Q, m_X, m_Y$. From the $Z\bar{b}b$ vertexes in Eq. (35) and Eq. (36), we can see that in order to keep gauge universality of quarks, the tail terms in the Feynman rules must be much smaller than the SM like terms, namely $|Z_{u,d}^i|^2_{i=1,2,3}, |U_{u,d}^i|^2_{i=1,2,3} \ll \sin^2 \theta_W$. Thus in the numerical studies we require

$$|Z_{u,d}^i|^2_{i=1,2,3}, |U_{u,d}^i|^2_{i=1,2,3} < 10^{-4}. \tag{43}$$

Note that though these elements are greater than $\lambda^3$ (parameter in the Wolfenstein parameterization [12]), they are much smaller than the product of $V_{\text{CKM}3}^T V_{\text{CKM}3}$ (almost equals 1), thus the requirements are suitable for indicating the contraints from the deviation from unitarity.

Since the scanning in the parameter space is freely, we set $m_X = 1172\text{GeV}$ (mass of top quark plus 1000 GeV) and scan $m_Y$ in the range of $(4.2, 1004)\text{GeV}$ (mass of bottom quark plus 1000 GeV), and $Z_{u,d}, U_{u,d}$ randomly (ignoring the CP phases). $M_V$ is defined by

$$M_V = \max(|M_Q^2|, |M_Q^0|, |M_Q^3|). \tag{44}$$
The result for $M_V$ versus $M_Y$ is shown in the FIG. 2 which checks the mass input of vector doublet. We can see that $M_V$ increases as $m_Y$ growing up. However $M_V$ is much smaller than $m_X$ and $m_Y$. Small mixings lead to parameter $M_Q$ which determine the mixing between SM quarks and vector like quarks are also suppressed. This is in agreement with that the deviation from unitarity is suppressed by the ratio $m/m_{X,Y}$ where $m$ denotes generically the standard quark masses, which is a typical result of VLP models. 

![Graph showing $K_1$ and $K_2$ versus $M_V$](image)

**FIG. 3:** $K_1$, (red △) $K_2$ (green □) versus $M_V$ and enhancement of $|C_7(m_W)|$ case of $|Z_{u,d}^{4i}|^2_{i=1,2,3}$, $|U_{u,d}^{4i}|^2_{i=1,2,3} < 10^{-4}$ (color online).

The second task is to check the VLP contribution to $B \to X_s\gamma$. We find the Wilson coefficient of FCNC operator $O_7$ is not so suppressed as the mixing. The new contributions, from the terms of the last line in Eq. (39) are suppressed by the mixing, whereas the terms of first line are almost the same as the SM. The enhancement comes mainly from the $g_G^{(4,3)}/m_b$ terms which are got from the Goldstone loop in $b \to s$ transition. (The right diagram in the first line and diagrams in the second line of the Fig. 1) In order to show the enhancement clearly, we define two factors

\[
K_1 = \frac{g_G^{(4,3)}g_G^{(4,2)}*}{m_X m_b V_{tb} V_{ts}} = \frac{g_{G}^{GVb} g_{G}^{GVs*}}{m_X m_b V_{tb} V_{ts}}, \tag{45}
\]

\[
K_2 = \frac{U_{43}^{*} U_{42}^{*}}{V_{tb} V_{ts}} = \frac{U_{tb} V_{ts}}{V_{tb} V_{ts}} \tag{46},
\]

in which $K_2$ denotes the deviation from the unitarity of $3 \times 3$ CKM matrix, while $K_1$ shows
the enhancement of the contribution from vector like particles. $K_1$ is in fact got from the coefficient of first term in the second line of analytical expression of $C_7(m_W)$ in Eq. (39) when $i = 4$. It will be changed into exactly $K_2$ in case of the SM4. Note that other terms with $g_{G_R}^G(4,3)/m_b$ can give enhancement too, we chose factor $K_1$ for a typical demonstration since it seems that it will be suppressed by $m_X$. Results are shown in the FIG 3 in which the left panel shows $K_1$ and $K_2$ versus $M_V$ while the right panel shows $|C_7(m_W)|$ versus $K_1$. From the left panel, we can see that though $K_2$ increase as $M_V$ increases, it is still much smaller than $V_{tb}V_{ts}^*$, implying that deviation of unitarity are negligible. However the factor $K_1$ can be enhanced up to order $O(1)$ by the increase of $M_V$. From the right panel, we can see that $K_1$ enhances $C_7$ up to a value much larger that the result of the SM. The reason for the enhancement mainly comes from the new type of Yukawa couplings. Combining of Eqs. (28, 33, 34), one can get similar form compared with the SM4

\[
\begin{align*}
\frac{g_{G_L}^G(4,2)}{m_X} &= U_{CKM4}^{42} + \frac{1}{m_X} \left[ \sum_{m=1}^{3} \left( M_Q^m U_d^{m2} Z_u^{44} - M^m_u U_d^{42} Z_u^{m4} \right) + (Y_{Vd} - Y_{Vu}) U_d^{42} Z_u^{44} \right] \\
\frac{g_{G_R}^G(4,3)}{m_b} &= -U_{CKM4}^{43} - \frac{1}{m_b} \left[ \sum_{m=1}^{3} \left( M_Q^m U_u^{m1} Z_d^{43} + M^m_d U_u^{44} Z_d^{m3} \right) + (Y_{Vd}^* - Y_{Vu}^*) U_u^{44} Z_d^{43} \right]
\end{align*}
\]

Since $m_X \simeq Y_{Vu} v, Z_u^{44}, U_u^{44} \simeq 1$, one can easily obtain that

\[
\frac{g_{G_L}^G(4,2)}{m_X} \sim V_{CKM4}^{42},
\]

the suppression of $Z_d^{43}$ (order of $m/m_{X,Y}$) in Eq. (48) are enhanced by terms with factor such as $\frac{Y_{Vu}}{m_b}$, etc., resulting

\[
\frac{g_{G_R}^G(4,3)}{m_b} \gg V_{CKM4}^{43}.
\]

Thus the term $V_{4b}V_{4s}^*$ satisfying the unitary constraint

\[
V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* + V_{4b}V_{4s}^* = 0
\]

is enhanced greatly by heavy VLPs, then the factor leads the enhancement to $C_7$. This is different from those in the SM4 in which the contribution from the fourth generation can be neglected.

In the numerical scan, we vary $Z_{u,d}$ and $U_{u,d}$ randomly, keeping the constraints of $|V_{4i}^4|_{i=1,2,3}^2, |U_{4i}^{4i}|_{i=1,2,3}^2$, scan $m_X$ and $m_Y$ in the range of $(1, 2000)$GeV. Apart from the CKM limits, we use the $B \to X_s \gamma$ process to constrain parameter space. The branching
FIG. 4: $B \to X_s \gamma$ prediction in random scan.

The branching ratio as a function of $m_V$ is shown in FIG. 4, from which we can see that $\text{Br}(B \to X_s \gamma)$ can be enhanced much greater than the experiment bound. Then the measurements of FCNC process can give a stringent constraint on the vector like quark model, especially when the masses of vector quark are much greater than the electro-weak scale. A few remarks should be addressed:

- There is one point of view on the unitarity of the CKM matrix which is that the $3 \times 3$ ordinary quark mixing matrix is regarded as nearly unitary, deviation from unitarity ratio of $B \to X_s \gamma$ is normalized by the process $B \to X_e e \bar{\nu}_e$:

$$\text{Br}(B \to X_s \gamma) = \text{Br}^{\text{ex}}(B \to X_e e \bar{\nu}_e) \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} \left[ |C_{7}^{\text{eff}}(\mu_b)|^2 + |C_{7}^{\prime,\text{eff}}(\mu_b)|^2 \right].$$

Here $z = \frac{m_c}{m_b}$ and $f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z$ is the phase-space factor in the semi-leptonic B decay. The method of running of the operators from $m_W$ scale to $\mu_b$ scale can be found in Ref. [7]. We use the following bounds on the calculation [10]

$$\text{Br}^{\text{ex}}(b \to c e \bar{\nu}_e) = (10.72 \pm 0.13) \times 10^{-2},$$

$$\text{Br}^{\text{ex}}(B \to X_s \gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}.$$
is suppressed by heavy particle in the new physics beyond the SM. In other word, one admits that the extended CKM matrix elements exist, they approach to zero while mass scale of the new physics approaches to infinity. All the new physical effects should decouple from the flavor sector and what should be checked is that if $3 \times 3$ unitarity is consistent in all kinds of flavor processes.

- Another point of view is that, as in the SM case, the $3 \times 3$ ordinary quark mixing matrix elements are only extracted by experiments in the measurements of tree and loop level processes. The unitarity should be checked, experiment measurements on the elements of matrix can be used as the constraints to the new physics beyond the SM. In the numerical analysis, the elements of CKM matrix are regarded as inputs. Thus what should be done is to scan the parameter space generally under these constraints, no prejudice should be imposed. Then the enhancement effect in $B \to X_s \gamma$ will be more clear.

![Graphs showing $K_1$ and $K_2$ versus $m_X$](image)

FIG. 5: Enhancement factor and deviation from unitarity versus $m_X$, red $\triangle$ are excluded by the bound of $B \to X_s \gamma$ measurement which the green $\Box$ are the survived points (color online).

Large parameter space is excluded by the measured branching fraction of $B \to X_s \gamma$ as shown in FIG. 4. The enhancement effect of the VLPs can be seen in FIG. 5 in which the left panel shows the enhancement factor $K_1$ versus $m_X$ while the right panel shows $K_2$. 

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versus $m_X$. From the right panel we can see that deviation from unitarity are very small and almost irrelevant with $m_X$ since we are doing a general scan of $Z_{u,d}$ and $U_{u,d}$. However as we see from the left panel, as $m_X$ increases up, $\text{Br}(B \to X_s\gamma)$ measurement will constrain the enhancement factor and then constrain the input parameter of $m_X$. In all, the enhancement can be summarized as that when mass of vector like particle increases up, it will increase the mass parameter $m_V$ thus give an enhancement factor under very small deviation from unitarity. This should be a special point when we do the study on the vector like quark models.

IV. SUMMARY

In the model with vector doublets, there exist bilinear terms in the lagrangian, making the general scan of the Yukawa coupling very difficult. In this paper, we show a trick to deal with the scan. Our scan method are exactly and the more efficient. We use the trick to study a very simple extension of the SM with vector like quarks. We studied one of the most important rare B decay $B \to X_s\gamma$ process in which we found that even the deviations from the unitarity of quark mixing matrix are small, the enhancement to rare B decay from VLPs are still significant. The enhanced effect is an important feature in the vector like particle model. In this work we just show the scan method, the key point of the enhancement and how stringent constraints on the parameter space from $B \to X_s\gamma$ measurements. What should be done includes models like extension of the SM with VLPs, two higgs doublets models [18] or supersymmetry models [19]. Such effect should be checked in all kinds of rare decays such as inclusive process $b \to s\ell^+\ell^-$ and exclusive processes $B_s \to \mu^+\mu^-, B_s \to \ell^+\ell^-\gamma$ and $B\bar{B}$ mixing et. al. The detailed studies on the parameter space including other rare B decays and new models will appear in our future work.

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Appendix

- The loop functions for calculating the Wilson coefficients at the matching scale are the following

\[
A(x) = \frac{55 - 170x + 127x^2}{36(1-x)^3} + \frac{4x - 17x^2 + 15x^3}{6(1-x)^4} \ln x,
\]

\[
B(x) = \frac{-7 + 5x + 8x^2}{36(1-x)^3} + \frac{-2x + 3x^2}{6(1-x)^4} \ln x,
\]

\[
C(x) = \frac{3 - 5x}{6(1-x)^2} + \frac{2 - 3x}{3(1-x)} \ln x,
\]

\[
D(x) = \frac{3x - 1}{4(1-x)^2} + \frac{x^2}{2(1-x)^3} \ln x,
\]

\[
E(x) = \frac{-17 + 19x}{6(1-x)^2} + \frac{-8x + 9x^2}{3(1-x)^3} \ln x.
\]

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