Mathematical description of automobile headlight contamination when driving on a wet road

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Abstract. The article describes the process of contamination of automobile headlights in order to calculate contamination speed using a mathematical model. The authors published results of the study aimed at solving the problem of negative application of the CAIM (chemical anti-icing materials) on winter roads. In particular, it was found that when driving vehicles on winter roads covered with CAIM, the headlights become contaminated. The headlight contamination with dirty droplets from under the wheels of a vehicle ahead was analyzed. The detachment of droplets from the moving wheel and the path of droplets from the wheel to the headlights of the vehicle behind were analyzed and described. Process is studied of contaminant film formation which decreases light transmission was carried out. Road experiments were carried out. Along with the physicomathematical description of the process under consideration, they give new knowledge about headlight contamination depending on the vehicle speed. The mathematical model supplemented by the experimental study allows us to predict the rate of headlight contamination by the wheels of vehicles ahead when moving along roads covered with chemical anti-icing reagents depending on the speed and the distance between the vehicles.

1. Introduction
Driving on a wet road is more dangerous than driving on a dry road. This is due to the deterioration of traction properties of the wheels [1,2] and poor view of the road situation. If in the daytime the view can be improved using good wipers, in the dark the illumination with headlights is significantly impaired due to their contamination. The headlights are contaminated due to a liquid dirty film formed by dirty droplets from under the wheels of the vehicle ahead [3].

2. Materials and methods
The article aims to describe the process of headlight contamination using a mathematical model developed to predict headlight contamination.

The headlight contamination process consists of three stages:
- detachment of droplets from the moving wheel;
- movement of droplets from the wheel to the headlights of the vehicle ahead;
- formation of a contaminant film on headlights.
Detachment of droplets from a moving wheel occurs if their centrifugal inertia exceeds the centripetal force of attraction to the wheel due to molecular wetting forces [4]. Wetting occurs if attraction forces between the molecules of liquid and solid bodies are greater than between the molecules of liquid bodies. At low speeds, the centripetal force is superior to the centrifugal force (there is no detachment of droplets), and the wetting film is relatively thick. With an increasing speed, the centrifugal force increases, droplets detach from the “thick” film, overcome the weaker attractive force between the liquid molecules, and the film thickness decreases. The latter means that with an increasing wheel speed, the centrifugal centripetal forces increase, since attractive forces between the molecules of a liquid body and a solid body (tire) play a crucial role. Thus, when droplets detach from the wheel, there is no trigger.

The above discussion allows us to draw an important conclusion: the number of droplets detaching from the wheel moving on the wet road increases with its speed.

Let us describe the movement of droplets from the wheel to the headlights of the vehicle ahead. The movement equation is a system of three differential equations

\[ m \frac{d^2 \vec{S}}{dt^2} = \vec{F}_g + \vec{F}_{fr} + \vec{F}_{res}, \]

with the initial condition

\[ \vec{S}(0) = \vec{0}, \quad \frac{d\vec{S}}{dt}(0) = \vec{V}_0. \]

In equations (1-2) \( \vec{V}_0 \) - speed of the droplet, \( m \) - mass of the droplet, \( \vec{S} = (x, y, z) \) - coordinates of the droplet along its trajectory, its movement is determined by the forces acting on it, the first force is \( \vec{F}_g = -m(0,0,g) \) gravity.

In (1), the second term is the viscous friction force described by the Stokes’s formula

\[ \vec{F}_{fr} = -6 \pi \rho \eta r \cdot \left( \frac{d\vec{S}}{dt} - \vec{V}_a \right), \]

where \( \eta = 1.8 \cdot 10^{-5} \text{kgm}^{-1}\text{c}^{-1} \) is the dynamic air viscosity coefficient at 20°C, \( r \) is the radius of the droplet, \( \vec{V}_a \) is the air flow speed behind the car.

The third term is the force of air resistance described by the Newton’s formula

\[ \vec{F}_{res} = -\frac{1}{4} \pi \rho r^2 \cdot \left( \frac{d\vec{S}}{dt} - \vec{V}_a \right)^2, \]

where \( \rho = 1.2 \text{kgm}^{-3} \) is air density at 20°C.

Let us analyze the system of differential equations (1-2).

A) The system (1-2) has an exact solution at \( \rho = \eta = 0 \)

\[ z(t) = \frac{V_0^2 \sin^2 \alpha}{2g} - \frac{g}{2g} \left( 1 - \frac{V_0 \sin \alpha}{g} \right)^2, \quad x(t) = V_0 \cos \alpha \cdot t, \quad t \in \left[ 0, \frac{2V_0 \sin \alpha}{g} \right] \]

\[ x(t) \text{ and } z(t) \text{ are coordinates of the droplet along and perpendicular to the surface, } \alpha \text{ is the angle of speed direction relative to the horizon when the droplet detaches from the wheel, } V_0 \text{ is the modulus of speed of the droplet equal to the modulus of speed of the automobile.} \]

Solution (5) means that the path of the droplet is a parabola in the XOZ plane.

Let us calculate the parameters of the droplet’s path neglecting the forces of friction and air resistance. Let the vehicle speed be 72 km / h (20 m / s) and the droplet detachment angle be 30° to the horizon. The droplet will fly away in the horizontal direction by 35 meters, the maximum lifting height will be 5.1 meters, the time will be 2 seconds. Since for 2 seconds the automobile will drive away 40
meters from the place of detachment of the droplet, at a distance of 75 meters, the headlights could become dirty from the vehicle ahead if there was no influence of the air environment.

B) When analyzing the movement of the droplet in the air, the important criterion is the Reynolds number

$$Re = \frac{2 r \rho V}{\eta},$$

(6)

At large values of the Reynolds number ($Re \geq 500$), the force of viscous friction can be neglected. Let the radius of the droplet be $r = 10^{-3} m$, $V_0 = 20 m s^{-1}$, then $Re = 2600$, i.e. the force of viscous friction is small.

The system (1-2) neglecting the forces of viscous friction and gravity at $V_a = 0$ has a simple analytical solution:

$$\frac{dS}{dt} = \frac{V_0}{1 + aV_d t}, \quad S(t) = \frac{1}{a} \ln(1 + aV_d t),$$

(7)

where $a = \frac{\pi \rho r^2}{4m} = 0.225 m^{-1}$ for a spherical droplet with a density of $10^3 kg m^{-3}$.

Accordingly, the droplet speed decreases from 20 m/s to 2 m/s, while its distance is 10 meters, and the distance between the droplet and the wheel is 50 meters. Thus, if the distance between the vehicles is 50 meters, the headlights of the second vehicle may become contaminated with droplets flying out from under the wheels of the first vehicle. If the radius of the droplet is 2 mm and its density is 2 times higher than density of water, for 2 seconds the droplet will be at a distance of 21 meters moving at an initial speed of 20 m/s, and the distance between the droplet and the wheel will be 61 meters.

It should be noted that accurate calculation of the path of the droplet requires knowledge of speed of air flows $V_a(x, y, z, t)$. It is impossible to calculate them. The speed of air flows is determined by the size, geometry and speed of the vehicle, atmospheric wind, air flows from moving vehicles, and the road landscape.

The processes of detachment and movement of droplets have been described above. An experimental study of these processes is expensive.

Let us describe the process of liquid contaminant film formation on the headlights. The liquid contaminant film decreases light transmission which can be measured. Thus, an experiment is required. It would be possible to install a measuring device next to the headlight. It would measure light transmission of the contaminant film.

However, it is difficult to install this device. A simpler way is to install screens on the headlights, whose light transmission is easy to measure in laboratory conditions. Previously, the authors of the article experiments on automobile lights contamination in wintertime were conducted under real road conditions. Variation in lights intensity due to contamination by deicing chemicals was studied [5,6]. This experiment was conducted when two cars moved at the same speed for five kilometers keeping the distance of 50 meters. Then the headlight screens were replaced with clean ones. The results of this experiment are presented as dots in Fig. 1.

In order to determine the dependence of headlight contamination on speed, it is required to analyze physics of this process. The value of headlight contamination is directly proportional to thickness of the contaminant liquid film $g$, i.e.

$$Y = N(1 - K_{trans}) g,$$

(8)

$K_{trans}$ is the transparency coefficient of the liquid film, depending on its chemical composition, for pure water, its value is close to 1, $N$ is the normalization factor with a value which is the inverse length.

As has already been shown, the number of droplets detaching from the wheel should increase with an increasing speed, which means that at a higher speed, the thickness of the contaminant film is
greater. The contaminant film is formed due to wetting forces determined by the attraction of liquid molecules to the headlight glass molecules. With an increasing film thickness, the outer layer of film molecules is no longer attracted to the glass. Forming droplets, they will slide off the headlight under the influence of gravity and air pressure. The above process can be described by the following differential equation

\[
\frac{dg}{dV} = \frac{Q_{\text{cont}}}{V_{\text{cont}}} - \frac{2}{V_{\text{cont}}} g .
\]  

This equation describes the dependence of thickness of the contaminant film per one kilometer on the vehicle speed.

\[
g(V) = 0.5Q_{\text{cont}}V_{\text{cont}}(1 - e^{-\frac{2V}{V_{\text{cont}}}}),
\]  

where \(V_{\text{cont}}\) is speed at which thickness of the contaminant film is 87% of the limit equal to \(0.5 \cdot Q_{\text{cont}}V_{\text{cont}}\), \(Q_{\text{cont}}\) is conditional time of film formation.

Combining equations (8-10), we have a theoretical formula for the value of headlight contamination at a five-kilometer distance depending on the speed

\[
Y_{hi} = Y_{hi}(1 - e^{-\frac{2V}{V_{\text{cont}}}}), \quad V_{\text{cont}} = 80 \text{ km/h},
\]

The parameters included in (11) were obtained by comparing the experimental data (blue dots in Fig. 1) with the contamination function curve. The limiting calculated degree of contamination turned out to be equal, i.e. under the experimental conditions, the transparency coefficient of liquid forming the film was very high \(K_{\text{trans}} = 0.772\).

![Figure 1. Contamination \(Y_{hl}\), % of Nissan Qashqai headlights depending on its speed at a distance of 50 meters from the vehicle ahead on a road covered with chemical anti-icing reagents](image)

Why is the value of headlight contamination so low (pink dots in Fig. 1) at 10 km/h and 20 km/h? To answer this question, let us use formula (5), according to which the distance between the wheel and the droplet is determined by formula
The maximum distance is reached by a droplet detaching at an angle of 60 degrees equal to

$$X = \frac{2V_0^2 \sin \alpha (1 + \cos \alpha)}{g} \tag{12}$$

Thus, if the vehicles move at 60 km/h, the maximum distance at which the vehicle ahead contaminates the vehicle behind is 72 meters. With a decreasing speed, the distance between the vehicles at decreases; at 20 km/h, it becomes equal to 8 meters.

In the experiment described, the distance between the vehicles was 50 meters, and the headlights should be kept clean at 50 km/h and less. At 10 km/h and 20 km/h, slight contamination might be due to the droplets detaching from the wheels of overtaking vehicles.

At 40 km/h, the contamination value turned out to be unexpectedly high, which might be due to the fact that the distance between the vehicles was sometimes 32 meters.

3. Conclusion
The mathematical model supplemented by the experimental study allows us to predict the rate of headlight contamination by the wheels of vehicles ahead when moving along roads covered with chemical anti-icing reagents depending on the speed and the distance between the vehicles.

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