Massive MIMO As Extreme Learning Machine

Dawei Gao and Qinghua Guo

Abstract—This work shows that massive multiple-input multiple-output (MIMO) with low-resolution analog-to-digital converters (ADCs) forms a natural extreme learning machine (ELM), where the massive number of receive antennas act as hidden nodes of the ELM, and the low-resolution ADCs serve as the activation function of the ELM. It is demonstrated that by adding biases to received signals and optimizing the ELM output weights, the system can effectively tackle hardware impairments, e.g., the power amplifier nonlinearity at transmitter side. It is interesting that the low-resolution ADCs can bring benefit to the receiver in handling nonlinear impairments, and the most computation-intensive part of the ELM is naturally accomplished by signal transmission and reception.

Index Terms—Massive MIMO, extreme learning machine (ELM), signal detection, nonlinearity, low-resolution ADC, hardware impairments.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO), where the base station is equipped with a large number of antennas, is a promising technology for 5G and future generation wireless communications [1]. However, numerous radio frequency chains in massive MIMO lead to high power consumption. To address this challenge, low-resolution analog-to-digital converters (ADCs) can be used [2], [3]. Besides the hardware imperfection at base station, there are also hardware impairments at the user side. For example, the use of cheap power amplifier may lead to nonlinear distortions to the transmitted signals. The hardware impairments have to be properly handled to avoid severe performance degradation. Many investigations have been carried out, e.g., the works in [4], [5], which either address the impairments at transmitter side or receiver side. There are few works addressing the impairments at both user and base station sides. In this work, we bring up a brand-new method to address the challenges in massive MIMO by treating massive MIMO as a natural extreme learning machine (ELM).

ELM is a single-hidden layer feedforward neural network, where the input weights and biases are randomly initialized and fixed [6]. The parameters to be learned in ELM are the output weights, which boils down to solving a linear system, making ELM fast in learning. ELM has been investigated for light emitting diode (LED) communications in our previous works [7], [8] to tackle LED nonlinearity and/or cross-LED interference. We have designed ELM based non-iterative and iterative receivers [8], and our investigations demonstrate that ELM is very effective to handle nonlinearity, delivering much better performance than polynomial based techniques [9], [10]. ELM has also been used for channel estimation and detection for OFDM systems [11], [12].

In this work, we consider the uplink of a massive MIMO system where transmitted signals of users suffer from nonlinear distortions, and the base station is equipped with a massive number of antennas with low-resolution ADCs. It is interesting that the massive MIMO itself can be treated as (part of) an ELM. In particular, the transmit antennas of users can be regarded as the input nodes of the ELM, the massive number of antennas at base station acts as the hidden nodes of the ELM, so the massive MIMO channel matrix functions as the input weight matrix of the ELM. Furthermore, the low-resolution ADCs serve as the activation function of the ELM. Then we add biases to the received signals before analog-to-digital conversion and obtain the output weights of the ELM with training signals. We show that the ELM can effectively handle the nonlinear impairments, and particularly, the low-resolution ADCs are helpful to handle the nonlinear distortion at transmitter side.

The rest of the paper is organized as follows. In Section II, the signal model for massive MIMO with hardware impairments is presented. ELM is briefly introduced in Section III. In Section IV, an ELM receiver is borrowed from [7] for massive MIMO detection. In Section V, the new ELM based receiver is proposed, where the massive MIMO itself is treated as part of the ELM. Simulation results are provided in Section VI, followed by conclusions in Section VII.

II. MASSIVE MIMO WITH HARDWARE IMPAIRMENTS

We consider the uplink transmission in a massive MIMO system with K active users. Assume that each user has a single antenna and the base station is equipped with N antennas, where N can be much larger than K. In this work, we particularly consider the nonlinear distortion of power amplifiers at transmitter (user) side and low-resolution ADCs at receiver (base station) side.

The nonlinear distortion of the power amplifier can be characterized by the nonlinear amplitude to amplitude conversion (AM/AM) and amplitude to phase conversion (AM/PM) [13]

\[
A(a) = \frac{a \alpha(a)}{1 + \beta a^2}, \quad \Phi(a) = \frac{\alpha \phi a^2}{1 + \beta \phi a^2},
\]

where \( a \) is the amplitude of the signal input to the power amplifier, and \( A(a) \) and \( \Phi(a) \) represent the amplitude distortion and phase distortion of the power amplifier, respectively.

The transmitted baseband signal vector at sampling time instant \( m \) can be represented as

\[
y[m] = H f(x[m]) + n[m],
\]

where \( H \) is a \( K \times N \) channel matrix, \( x[m] = [x_1[m], x_2[m], ..., x_K[m]]^T \) is the transmitted signals of all users.
active users, \( (\Gamma)^T \) denotes the transpose operation, \( n[m] \) denotes an additive white Gaussian noise vector, and \( f(x) \) is an element-wise function that accounts for the distortions of the power amplifier to the amplitude and phase of the transmitted signal, i.e.,

\[
    f(x) = A(|x|)e^{i\text{angle}(x)+\Phi(|x|)}.
\]

After low resolution ADC, the signal can be represented as

\[
    r[m] = Q(y[m]),
\]

where \( Q(\cdot) \) denotes the quantization operation. The aim of the receiver is to recover \( x[m] \) based on the quantized signal \( r[m] \).

III. EXTREME LEARNING MACHINE

The structure of ELM is shown in Fig. 1. ELM is a single-hidden layer feedforward neural network, where the input weights \( \{\omega_l\} \) and biases \( \{b_l\} \) are randomly initialized and fixed without tuning \([6]\). The parameters to be learned in ELM are the output weights, and hence ELM can be formulated as a linear model with respect to the parameters, which boils down to solving a linear system, making ELM efficient in learning. Suppose there are \( M \) distinct training samples \( \{(s[m], t[m])\in \mathbb{R}^U \times \mathbb{R}^V \}_{m=1}^M \), where \( s[m] = [s_1[m], s_2[m], \ldots, s_L[m]]^T \) and \( t[m] = [t_1[m], t_2[m], \ldots, t_L[m]]^T \), the \( \psi \)th output of the ELM shown in Fig. 1 can be expressed as

\[
    
    \psi_v[m] = \sum_{l=1}^L \beta_{vl}(\omega_l^T s[m] + b_l), \quad m = 1, \ldots, M,\tag{5}
\]

where \( L \) is the number of hidden nodes, \( \omega_l = [\omega_{l1}, \omega_{l2}, \ldots, \omega_{lU}]^T \) is the input weight vector that connects all input nodes to the \( l \)th hidden node, \( b_l \) is the bias of the \( l \)th hidden node, \( g(\cdot) \) is the activation function of the hidden layer, and \( \beta_{vl} \) denotes the output weight that connects the \( l \)th hidden node and the \( v \)th output node.

We can express the \( M \) equations in (5) in a matrix form as

\[
    \psi_v = Z\beta_v,\tag{6}
\]

with \( \beta_v = [\beta_{v1}, \beta_{v2}, \ldots, \beta_{vL}]^T, \quad \psi_v = [\psi_v[1], \psi_v[2], \ldots, \psi_v[M]]^T \) and the hidden layer output matrix

\[
    Z = [z[1], z[2], \ldots, z[M]]^T,\tag{7}
\]

The structure of ELM is shown in Fig. 1. ELM is a single-hidden layer feedforward neural network, where the input weights \( \{\omega_{il}\} \) and biases \( \{b_l\} \) are randomly initialized and fixed without tuning \([6]\). The parameters to be learned in ELM are the output weights, and hence ELM can be formulated as a linear model with respect to the parameters, which boils down to solving a linear system, making ELM efficient in learning. Suppose there are \( M \) distinct training samples \( \{(s[m], t[m])\in \mathbb{R}^U \times \mathbb{R}^V \}_{m=1}^M \), where \( s[m] = [s_1[m], s_2[m], \ldots, s_L[m]]^T \) and \( t[m] = [t_1[m], t_2[m], \ldots, t_L[m]]^T \), the \( \psi \)th output of the ELM shown in Fig. 1 can be expressed as

\[
    
    \psi_v[m] = \sum_{l=1}^L \beta_{vl}(\omega_l^T s[m] + b_l), \quad m = 1, \ldots, M,\tag{5}
\]

where \( L \) is the number of hidden nodes, \( \omega_l = [\omega_{l1}, \omega_{l2}, \ldots, \omega_{lU}]^T \) is the input weight vector that connects all input nodes to the \( l \)th hidden node, \( b_l \) is the bias of the \( l \)th hidden node, \( g(\cdot) \) is the activation function of the hidden layer, and \( \beta_{vl} \) denotes the output weight that connects the \( l \)th hidden node and the \( v \)th output node.

We can express the \( M \) equations in (5) in a matrix form as

\[
    \psi_v = Z\beta_v,\tag{6}
\]

with \( \beta_v = [\beta_{v1}, \beta_{v2}, \ldots, \beta_{vL}]^T, \quad \psi_v = [\psi_v[1], \psi_v[2], \ldots, \psi_v[M]]^T \) and the hidden layer output matrix

\[
    Z = [z[1], z[2], \ldots, z[M]]^T,\tag{7}
\]

where \( z[m] = g(\Omega s[m] + b) \)

is the hidden layer output vector at time constant \( m \), \( \Omega = [\omega_1, \omega_2, \ldots, \omega_L]^T \)

is the input weight matrix, and \( b = [b_1, b_2, \ldots, b_L]^T \).

ELM randomly selects input weights and biases, and output weights \( \beta_v \) are obtained by minimizing the cost function \( \sum_{m=1}^M \|\psi[m] - t[m]\|^2 \). The regularized smallest norm least-squares solution is given by \([14]\)

\[
    \beta_v = (Z^T Z + \gamma I)^{-1} Z^T t_v, \quad v = 1, \ldots, V,\tag{9}
\]

where \( I \) is an identity matrix, \( \gamma \) is a regularization parameter and \( t_v = [t_v[1], t_v[2], \ldots, t_v[M]]^T \).

IV. ELM RECEIVER BORROWED FROM [7]

In [7], we proposed an ELM based receiver to handle both the LED nonlinearity and cross-LED interference in MIMO LED communications. Here, we borrow the ELM receiver in [7]. As shown in Fig. 2, the input to the ELM is the quantized signal \( r[m] \) in (4). With the training signals \( \{t[m], m = 1, \ldots, M\} \) and the corresponding received signals \( \{r[m], m = 1, 2, \ldots, M\}, \) the ELM can be trained.

Here, as the signals are complexed valued, we arrange the input vector as \( r'[m] = [\Re(t[m])^T, \Im(t[m])^T]^T \), and the expected output vector as \( t'[m] = [\Re(t[m])^T, \Im(t[m])^T]^T \).

Then, the output weight vectors \( \beta_k^{Re}, \beta_k^{Im}, k = 1, 2, \ldots, K \) can be obtained using (9), and each pair of weight vectors correspond to a user. Then, the trained ELM can be used to detect the transmitted data of each user, i.e., the estimator for \( x_k[m] \) can be represented as

\[
    \hat{x}_k[m] = (\beta_k^{Re})^T g(\Omega r'[m] + b) + i(\beta_k^{Im})^T g(\Omega r'[m] + b),\tag{10}
\]

where \( \Omega \in \mathbb{R}^{L \times 2N} \) and \( b \in \mathbb{R}^L \). Then, the decision based on \( \hat{x}_k[m] \) can be expressed as

\[
    \hat{x}_k[m] = \arg\min_c \|\hat{x}_k[m] - c\|^2,\tag{11}
\]

where \( c \) belongs to the symbol alphabet.

It can be seen in (10) that, intensive calculations are involved in the product of the input weight matrix \( \Omega \) and the input data vector \( r'[m] \), leading to a quadratic complexity \( O(LN) \). As we proposed in [7], we can put a constrain on the structure of \( \Omega \), i.e., it is a (partial) circulant input weight matrix, enabling an implementation using the fast Fourier transform (FFT) with significantly reduced complexity. Refer to [7] for details.
V. MASSIVE MIMO AS ELM

A. New ELM Based Massive MIMO

We treat massive MIMO with low-resolution ADCs as a natural ELM, based on which a new receiver is designed. It is noted that the idea and receiver here are completely different from those in Section IV.

Figure 3 illustrates the ELM based massive MIMO system where the transmit antennas, massive MIMO channel and receive antennas serve as part of the ELM. As a common assumption in massive MIMO, we assume that the number of active users $K$ is less than the number of receive antennas $N$ at base station. By comparing Fig. 3 with the ELM in Fig. 1, the $K$ transmit antennas are analogous to the input nodes of the ELM, and the signals are transmitted over the air, which are picked up by the receive antennas. We treat the receive antennas as the hidden nodes of the ELM, and the channel matrix $H$ is analogous to the input weight matrix $\Omega$ of the ELM. To mimic the ELM, we add a bias $b_n$ to the received signal $y_n$ at each receive antenna. Then the biased signals are the input to the low-resolution ADCs. Hence the signal vector after ADC can be represented as

$$r[m] = Q(Hs[m] + b + n[m]),$$

where $s[m] = f(x[m])$ is the distorted signal vector. We treat $Q(\cdot)$ as the activation function, and the only difference between (12) and (8) is the extra noise term $n[m]$. If we ignore the noise term, $r[m]$ represents the hidden layer output vector. As shown in Fig. 3, we only need to determine the ELM output weight $\{\beta_k, k = 1, 2, \ldots, K\}$, by using training sequences.

We still assume real ELM, and separate the real and imaginary parts of $r[m]$ as $r^T[m] = [\text{Re}(r[m])^T, \text{Im}(r[m])]^T$. Then, the output matrix from ADCs can be represented as,

$$R' = [r'[1], r'[2], \ldots, r'[M]]^T.$$  \hspace{1cm} (13)

So, $\beta^\text{Re}_k$ and $\beta^\text{Im}_k$ can be obtained by solving two regularized LS problems, i.e.,

$$\beta^\text{Re}_k = (R'^T R' + \gamma I)^{-1} R'^T t^\text{Re}_k, \hspace{1cm} (14)$$

$$\beta^\text{Im}_k = (R'^T R' + \gamma I)^{-1} R'^T t^\text{Im}_k, \hspace{1cm} (15)$$

where $t^\text{Re}_k = \text{Re}(t_k)$, $t^\text{Im}_k = \text{Im}(t_k)$ and $t_k = [t_k[1], t_k[2], \ldots, t_k[M]]^T$ is the training sequence of the $k$th user. The trained output weights can be applied to received signals to estimate the transmitted data of each user, i.e.,

$$\hat{s}_k[m] = (\beta_k^\text{Re})^T r'[m] + i(\beta_k^\text{Im})^T r'[m], \hspace{1cm} (16)$$

Then, the decision based on $\hat{s}_k[m]$ can be expressed as

$$\hat{s}_k[m] = \arg\min_c \|\hat{s}_k[m] - c\|^2,$$  \hspace{1cm} (17)

where $c$ belongs to the symbol alphabet.

B. Comparisons with Receivers and Remarks

1) Conventional ZF receiver with perfect channel state information: With the perfect knowledge of the channel matrix $H$, the weight of the ZF detector can be represented as

$$w_k^\text{ZF} = (HH^T)^{-1}h_k^*, \hspace{1cm} (18)$$

where $h_k$ is the channel vector of the $k$th user, $(\cdot)^H$ denotes the conjugate transpose, and $(\cdot)^*$ denotes the conjugate operation. The detector simply ignores the nonlinear distortion to the transmitted signal and the impact of the low-resolution ADCs at the receiver side, which leads to very poor performance as shown in Section VI.

2) Conventional ZF receiver with training: The detector is directly trained using training signals. In this case the weight of the detector can be expressed as

$$w_k^\text{ZF} = (R^H R)^{-1}R^H t_k, \hspace{1cm} (19)$$

where $R = [r[1], r[2], \ldots, r[M]]^T$. It is interesting that the directly trained detector performs slightly better than the detector with perfect $H$, as shown in the Section VI. This is because the training considers the impact of nonlinearity, although it is still a linear one.

3) ELM receiver in Section IV: As shown in Fig. 2 the ELM receiver in Section IV treats the quantized received signals as the input, and it needs a large number of hidden nodes. The new ELM based receiver shown in Fig. 3 is very different. In the new ELM based receiver, the multiplication of the input weight matrix with the input vector is naturally accomplished by signal transmission over the air, and the output of the ADCs is the output of the activation function. Clearly, compared to the ELM receiver in Section IV, the new ELM based receiver has lower complexity of training and significantly lower complexity of detection. Once trained, the new ELM based receiver only needs to carry out (16) and (17) for detection. However, the ELM receiver in Section IV needs to carry out (10) and (11) for detection, which involves matrix-vector multiplication. In addition, as shown in Section VI, the new ELM based receiver can even achieve considerably better performance.

VI. SIMULATION RESULTS

We consider a massive MIMO system with $K = 10$ transmit antennas and $N = 256$ receive antennas, and 16-QAM is used. According to [13], the parameter setting for the power amplifier nonlinearity is as follows: $\alpha_0 = 1.96$, $\beta_0 = 0.99$, $\alpha_{\phi} = 2.53$ and $\beta_{\phi} = 2.82$. ADCs with 6-bit quantization are used. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = P_s/P_n$, where $P_s$ is the power of the signal.
have poor performance due to their weak capability to mitigate nonlinearity. In comparison, the ELM receiver in Section IV can effectively handle the hardware impairments. It also can be seen that the new ELM based receiver delivers the best performance, with significantly lower complexity compared to the ELM receiver in Section IV.

To examine the impact of received signal biasing and low resolution ADCs, we carry out an interesting experiment. We assume a trained ZF receiver without quantization (i.e., infinite number of bits for ADC) is used. One ZF detector is trained without adding biases to the received signal, and the other one is trained with biased received signal. The results are shown in Fig. 5. It is interesting that the ZF with biased received signal performs better than the ZF without biasing. This demonstrates that, even for a linear detector, adding biases to the received signal is helpful in dealing with the nonlinear distortion at the transmitter side. It is also interesting that, from Fig. 5 the ZF detector with biasing delivers performance much worse than that of the new ELM based receiver. This indicates that the low resolution ADCs are even helpful to deal with the nonlinear distortion when they are exploited as activation function of the ELM.

As a final remark, we note that, as ELM allows fast learning (only output weights need to be updated), adaptive ELM receiver can be developed to handle time varying massive MIMO channels, which requires shorter training sequences once the output weights are initialized.

VII. CONCLUSION

In this letter we have shown that massive MIMO with low resolution ADCs can be treated as a natural ELM where the massive number of antennas act as the hidden nodes and the ADCs act as the activation function of the ELM. By adding biases to the received signals and optimizing the output weights, the ELM can effectively handle hardware impairments in massive MIMO. The effectiveness of the receiver has been demonstrated.

REFERENCES

[1] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, “What will 5g be?” IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1065–1082, June 2014.
[2] S. Jacobsson, G. Durisi, M. Coldrey, U. Gustavsson, and C. Studer, “Throughput analysis of massive mimo uplink with low-resolution adcs,” IEEE Trans. Wireless Commun., vol. 16, no. 6, pp. 4038–4051, April 2017.
[3] J. Singh, O. Dabeer, and U. Madhow, “On the limits of communication with low-precision analog-to-digital conversion at the receiver,” IEEE Trans. Commun., vol. 57, no. 12, pp. 3629–3639, December 2009.
[4] E. Björnson, J. Hoydis, M. Kountouris, and M. Debbah, “Massive mimo systems with non-ideal hardware: Energy efficiency, estimation, and capacity limits,” IEEE Trans. Inf. Theory, vol. 60, no. 11, pp. 7112–7139, Nov 2014.
[5] L. Ding, G. T. Zhou, D. R. Morgan, Zhengxiang Ma, J. S. Kenney, Jaehyeong Kim, and C. R. Giardina, “A robust digital baseband predistorter constructed using memory polynomials,” IEEE Trans. Commun., vol. 52, no. 1, pp. 159–165, January 2004.
[6] G.-B. Huang, Q.-Y. Zhu, and C.-K. Siew, “Extreme learning machine: theory and applications,” Neurocomputing, vol. 70, no. 1, pp. 489–501, Dec. 2006.
[7] D. Gao and Q. Guo, “Extreme Learning Machine-Based Receiver for MIMO LED Communications,” arXiv e-prints, Feb. 2019, [Published in Digit. Signal Process. in Dec. 2019].
[8] D. Gao, Q. Guo, J. Tong, N. Wu, J. Xi, and Y. Yu, “Extreme-learning-machine-based noniterative and iterative nonlinearity mitigation for led communication systems,” IEEE Syst. J., pp. 1–10, March 2020.

[9] H. Qian, S. Yao, S. Cai, and T. Zhou, “Adaptive postdistortion for nonlinear LEDs in visible light communications,” IEEE Photon. J., vol. 6, no. 4, pp. 1–8, Aug. 2014.

[10] K. Ying, Z. Yu, R. J. Baxley, H. Qian, G. Chang, and G. T. Zhou, “Nonlinear distortion mitigation in visible light communications,” IEEE Wireless Commun., vol. 22, no. 2, pp. 36–45, April 2015.

[11] J. Liu, K. Mei, X. Zhang, D. Ma, and J. Wei, “Online extreme learning machine-based channel estimation and equalization for ofdm systems,” IEEE Commun. Lett., vol. 23, no. 7, pp. 1276–1279, 2019.

[12] L. Yang, Q. Zhao, and Y. Jing, “Channel equalization and detection with elm-based regressors for ofdm systems,” IEEE Commun. Lett., vol. 24, no. 1, pp. 86–89, Nov. 2020.

[13] A. A. M. Saleh, “Frequency-independent and frequency-dependent non-linear models of twt amplifiers,” IEEE Trans. Commun., vol. 29, no. 11, pp. 1715–1720, Nov. 1981.

[14] W. Deng, Q. Zheng, and L. Chen, “Regularized extreme learning machine,” in 2009 IEEE Symp. CIDM, March 2009, pp. 389–395.