Prediction of fatigue crack growth life based on a new fusion model

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Abstract. A novel method based on filtering algorithm is proposed to predict fatigue crack growth life of metal structures under repeated loading. Three parts, including improvement of the two-parameter exponential formula, the improved two-parameter exponential formula parameter estimation and residual life prediction, are involved in the proposed method. The improved two-parameter exponential formula is discretized to establish a parameter estimation physical model of state space. Then, the uncertainty parameters α, β and crack length a in the state space physical model are determined according to the crack propagation data obtained by monitoring/detection and the filtering algorithm, to reduce the influence of uncertain factors such as environment and operation conditions on the prediction of fatigue crack growth life of in-service metal structures. Finally, the discretized improved two-parameter exponential formula are employed to predict the growth life of fatigue crack. An example is presented to test the validity of the proposed method. The result demonstrates that the proposed method is available to well deal with the uncertainty of fatigue parameters in the process of fatigue crack growth with the improved two-parameter exponential formula, and the particle filter algorithm can accurately predict the growth residual life of fatigue short crack.

Key words: Remaining life, Filtering algorithm, Fatigue crack, Double parameter exponential formula

1. Introduction

Various methods have been presented to predict the fatigue crack propagation life, among which the two-parameter exponential formula is currently widely used in the prediction of fatigue crack life of metal structures due to its simple structure. The two important material parameters α and β in the two-parameter exponential formula have great influence on the trend of fatigue crack growth life curve. Whereas, the different structure forms, the slight changes in materials and the manufacturing process of the structure all have influence on these two parameters. The material parameters α and β in the two-parameter exponential formula are usually obtained through experiments. Generally speaking, the form of metal structure is complex and the crack propagation parts are scattered. Therefore, the error of describing the crack propagation life with a single fatigue parameter α and β is relatively
large. Although the safety factor can be introduced to ensure the safety of the metal structure, a certain gap still exists between the prediction mode of the crack propagation life and the real state of the structure crack propagation when dealing with one or more components. Consequently, it is of great significance to predict the fatigue crack growth life of metal structures considering the distribution of material parameters $\alpha$ and $\beta$ in the two-parameter exponential formula.

The idea of particle filtering is derived from the Monte Carlo method. It is a method for expressing its distribution form by extracting random state particles from the posterior probability. It is widely used in parameter estimation of nonlinear systems. Zhang [1] combines particle filter algorithm with dynamic background model to realize the detection of target motion and improve the accuracy of foreground detection. Zhang et al [2] propose an improved particle filter algorithm for infrared video pedestrian tracking, which can accurately and effectively track pedestrians in complex scenes and improve the tracking robustness. Sun Q et al [3] present a method based on Trackless particle filter to realize the residual life prediction of power converter. Wang [4] provides a prediction method based on deep learning and hybrid trend particle filter to predict the remaining life of cutting tools.

In this paper, a state space assessment model of fatigue crack growth life is established based on the improved two parameter exponential formula for the uncertainty of metal structure material parameters. The state space parameters are estimated by particle filter algorithm, and the crack growth life is then predicted by discrete improved two parameter exponential formula.

2. Spatial state evaluation model based on improved two-parameter exponential formula

The two-parameter exponential model of fatigue crack growth is proposed by Adib and Baptista [5] to define the fatigue crack growth rate, expressed as follows:

$$\frac{da}{dN} = e^{\alpha} e^{\beta \Delta k}$$

where $\alpha$ and $\beta$ are material parameters; $\Delta k$ is the stress intensity factor amplitude, which can be expressed as:

$$\Delta k = \gamma \Delta \sigma \sqrt{\pi a}$$

in which $\gamma$ is the coefficient related to the loading mode, crack shape and position, $\Delta \sigma$ is the equivalent stress amplitude, and $a$ is the crack length. Thus, the formula of fatigue crack growth rate can be rewritten based on Eq. (2.1) and (2.2):

$$\frac{da}{dN} = e^{\alpha} e^{\beta \gamma \Delta \sigma \sqrt{\pi a}}$$

According to Eq. (2.3), when $\alpha$ and $\beta$ are fixed parameters and $\beta<0$, the crack growth rate increases continuously with the growth of the crack length, but the increase is less than $e^\alpha$. When $\beta>0$, the crack growth rate decreases with the growth of crack length, which is not suitable for life prediction. Therefore, the formula can be used to predict fatigue crack growth when the value of $\beta$ is less than zero, but it is constrained by the $\alpha$ parameter, furthermore, when the crack growth rate is large, the $\alpha$ parameter also needs to produce large changes to achieve the prediction effect, which is not conducive to practical application. Thus, an improved two-parameter exponential model is presented as below:

$$\frac{da}{dN} = e^{\alpha} e^{\beta \gamma \Delta \sigma \sqrt{\pi a}}$$

where $\beta>0$ is required for the improved two-parameter exponential model, to ensure the crack growth rate increases with the length of the crack, and escape the constraint by the parameter $\alpha$.

Assuming $dN=1$, i.e., the crack growth length under a single cycle load. Then, the fatigue crack growth process is discretized into a single cycle crack length accumulation process. Hence, the discretized improved two-parameter exponential model is expressed as
where $k$ is the fatigue cycle. Because of the problems in the process of crack growth, such as nonlinear growth path, crack bifurcation, environment and working condition, there is a certain dispersion in the crack length, so the measurement noise $w_{a,k}$ is introduced in the process of crack growth. The measurement noise follows the distribution $w_{a,k} \sim N(0,Q_{a,k})$, and the crack length expression is described as follows:

$$a_{k+1} = a_k + e^{\alpha} e^{\beta} + w_{a,k}$$  (2.6)

The diversity of working conditions, environment and structural forms will not only cause the dispersion of crack length, but also affect the material parameters $\alpha$ and $\beta$, resulting in a certain degree of dispersion. Therefore, taking the dispersion of $\alpha$, $\beta$ and $a$ into account, the above three parameters can be taken as the estimators, and the spatial state evaluation model is established as follows:

$$X_{k+1} = \begin{bmatrix} a_{k+1} \\ a_k \\ \beta_{k+1} \\ \beta_k \end{bmatrix} = \begin{bmatrix} a_k \\ a_k \\ \beta_k \\ \beta_k \end{bmatrix} + \begin{bmatrix} w_{a,k} \\ w_{a,k} \\ w_{\beta,k} \\ w_{\beta,k} \end{bmatrix} = f_k(X_k) + W_{k+1}$$  (2.7)

where $w_{a,k} \sim N(0,Q_{a,k})$ and $w_{\beta,k} \sim N(0,Q_{\beta,k})$ are Gaussian white noise of $\alpha$ and $\beta$ respectively; $Q_{a,k}$ and $Q_{\beta,k}$ can be determined by material fatigue test respectively.

The observation equation of the system is the fatigue crack length obtained by monitoring/detection. Considering the error in the measurement process, the observation equation is:

$$z_{k+1} = H_{k+1} x_{k+1} + v_{k+1}$$  (2.8)

in which $H_{k+1}$ is the measurement matrix, i.e., the identity matrix, $x_{k+1}$ is the estimated value of the state parameter, here is equivalent to the estimated crack length $a_{k+1}$; $v_{k+1}$ is the measurement error and subjected to $v(k) \sim N(0,a_v^2)$. With the estimated parameter $\alpha, \beta, a$ and $a$ in the equation of state by measuring data, the equation of state and the observation equation in the filter system enable to achieve the purpose of predicting the crack propagation life.

3. State parameter estimation and life prediction based on particle filter algorithm

3.1 Bayesian principle

Bayesian theory aims to solve the problem of state estimation. It is a method to recursively calculate the credibility of the current state estimation value based on a series of data before state estimation, that is, to obtain the posterior probability density through the prior probability density. Bayesian theory consists of two parts: prediction and update.

**Prediction process:** The previous state $X_k$ is used to predict the probability of the future state $X_{k+1}$, denoted as $p(X_{k+1} | Z_k)$, and the observation $Z_k$ is employed to prior verified the future state $X_{k+1}$ of occurrence probability, expressed as $p(X_{k+1} | Z_k)$.

$$p(X_{k+1} | Z_k) = \int p(X_{k+1} | X_k) p(X_k | Z_k) dX_k$$  (3.1)

$$= \int p(X_{k+1} | X_k, Z_k) p(X_k | Z_k) dX_k$$  (3.2)

$$= \int p(X_{k+1} | X_k) p(X_k | Z_k) dX_k$$  (3.3)
**Update process**: Correct the prior probability density \( p(X_{k+1}|Z_k) \) by observing \( Z_{k+1} \) to obtain the posterior probability density \( p(X_{k+1}|Z_{k+1}) \) and the formula is as follows:

\[
p(X_{k+1}|Z_{k+1}) = \frac{p(Z_{k+1}|X_{k+1}) p(X_{k+1}|Z_k)}{p(Z_{k+1}|Z_k)}
\]

(3.2)

where \( p(Z_{k+1}|Z_k) \) can be form into a normalization constant:

\[
p(Z_{k+1}|Z_k) = \int p(Z_{k+1}|X_{k+1}) p(X_{k+1}|Z_k) dX_{k+1}
\]

(3.3)

in which \( p(Z_{k+1}|X_{k+1}) \) is the likelihood function of \( Z_{k+1} \), and \( p(X_{k+1}|Z_k) \) is the probability of measuring the noise distribution.

It can be seen that Bayesian theory requires integral operation. Except for some special systems (linear Gaussian systems), for nonlinear and non-Gaussian systems, it is difficult to obtain analytical solutions of posterior probabilities. Therefore, the particle filter can be considered as an effective tool for the integration problem.

### 3.2 State parameter estimation of fusion model based on particle filter

In order to solve the integration problem in the posterior probability density function, Monte Carlo sampling can be used in the particle filter algorithm. The integration problem can be transformed into the expectation problem in the finite sample, that is, drawing \( N \) samples \( \{X_{k+1}^i, i = 1, \cdots, N\} \) from the posterior probability density functions \( p(X_{k+1}|Z_{k+1}) \). Then, the posterior probability density function can be obtained by the probability estimation method [6]:

\[
p(X_{k+1}|Z_{k+1}) = \frac{1}{N} \sum_{i=1}^{N} \delta(X_{k+1} - X_{k+1}^i)
\]

(3.4)

where \( X_{k+1} \) is a continuous variable, \( \delta(X_{k+1} - X_{k+1}^i) \) is the Dirac function, \( N \) is the total number of particles, and \( X_{k+1}^i \) is the \( i \)th particle samples.

Particle filter uses posterior probability density function to obtain the expected value of the required state function. Supposing the state function is denoted as \( f_k(X_k) \), the filtering formula for the state function is:

\[
E\left[ f_k(X_{k+1}) \right] = \int f_k(X_k)p(X_{k+1}|Z_{k+1})dX_k
\]

\[
\approx \frac{1}{N} \sum_{i=1}^{N} \int f_k(X_k)\delta(X_{k+1} - X_{k+1}^i)dX_k
\]

(3.5)

\[
\approx \frac{1}{N} \sum_{i=1}^{N} f_k(X_{k+1}^i)
\]

A known and sampleable distribution \( q(X_{k+1}|Z_{k+1}) \) needs to be introduced when the posterior probability density function \( p(X_{k+1}|Z_{k+1}) \) is unknown. The principle of selection is to minimize the variance of importance weights, i.e., sampling \( N \) samples \( \{X_{k+1}^i, i = 1, \cdots, N\} \) in the distribution \( q(X_{k+1}|Z_{k+1}) \) and replacing the expected value by sample means:

\[
E\left[ f_k(X_{k+1}) \right] = \sum_{i=1}^{N} W_{k+1}^{i} f_k(X_{k+1}^i)
\]

(3.6)

\[
W_{k+1}^{i} = \frac{W_{k+1}^{i} f_k(X_{k+1}^i)}{\sum_{i=1}^{N} W_{k+1}^{i} f_k(X_{k+1}^i)}
\]

where
\[ W_{k+1}(X_{k+1}) = \frac{p(Z_{k+1}|X_{k+1}) p(X_{k+1})}{q(X_{k+1}|Z_{k+1})} \]  
(3.7)

in which \( p(Z_k|X_k)p(X_k) \) is equivalent to \( p(X_k|Z_k)p(Z_k) \), is the probability that the observed value is equal to the state value in the posterior probability of the previous step. The product of the weight value of particle filter algorithm and the state function obtains the parameter estimation value of the next state. However, the particle degradation problem occurs in the iterative process of particle filter algorithm, that is, the logarithmic particle weight decreases with the increase of iteration, and a few particle weights increase. In order to solve this problem, this paper uses the method of random resampling to index the particles with significant weight many times, and directly omit the particles with small weight.

The procedure for the proposed algorithm based on particle filter and improved two-parameter exponential formula fusion are presented as below:

1. **Initialization of state parameters;**
   - Input state parameter initial value \( X_0 = [a_0, \alpha_0, \beta_0] \) and variance \( Q = [\sigma_a, \sigma_\alpha, \sigma_\beta] \);

2. **Particle collection sampling:**
   - Collect the particle set \( x_i, i = 1, 2, 3, \cdots, N \), and use the state observation equation to generate the state function sample set \( X_{k+1}^i, i = 1, 2, 3, \cdots, N \); \( k+1 \) is the state function step number, \( X \) is the normal function matrix.

3. **Calculation of importance weight:**
   - Calculate the weight value according to the observation equation:
     \[ W_k^i = W_k^i \frac{p(Z_{k+1}|X_{k+1}^i) p(X_{k+1}^i)}{q(X_{k+1}^i|X_k^i, Z_{k+1})} \]
     \( (3.8) \)
   - and normalized weight as \( \tilde{W}_{k+1}^i = \frac{w_{k+1}^i}{\sum_{i=1}^N w_{k+1}^i} \).

4. **Random resampling:**
   - Resample the N particles according to the importance weight \( \tilde{W}_{k+1}^i \). If \( \tilde{W}_{k+1}^i \) is large, the number of allocated particles is large, else if \( \tilde{W}_{k+1}^i \) is direct deleted. Then, the weight value \( W_{k+1}^i = 1/N \) for each particle is redistributed.

5. **Obtain the estimated value of the state parameter:**
   \[ X_{k+1} = \sum_{i=1}^N W_{k+1}^i X_{k+1}^i \]  
(3.9)

### 3.3 Prediction of fatigue crack growth life based on estimated parameters

The parameter \( X_k = [a_k, \alpha_k, \beta_k] \) of the \( k \)th step is obtained using the above three methods to estimate the state parameters. Therefore, in order to predict the residual fatigue life of cracks, it is only necessary to substitute the parameters into Paris formula or improved two-parameter exponential formula discrete equation for recursive calculation to obtain the \( (k+n) \)th step crack length. Because the state parameters change at any time, the smaller \( N \) corresponding to the higher accuracy. The prediction formula of Paris formula or improved two parameter index formula is shown as follows.

\[
\begin{align*}
    a_{k+1} &= a_k + e^{a_k} e^{\alpha_k \sqrt{|a_k|}/\beta_k} \\
    &\vdots \\
    a_{k+n} &= a_{k+n-1} + e^{a_{k+n-1}} e^{\alpha_{k+n-1} \sqrt{|a_{k+n-1}|}/\beta_{k+n}}
\end{align*}
\]
(3.10)
4. Example

4.1 Simulation analysis of fatigue crack growth

A three-dimensional solid Q345 steel plate unit with a length \( L = 115 \text{mm} \), a width \( b = 30 \text{mm} \), and a thickness \( h = 3 \text{mm} \) is established, of which elastic modulus \( E = 2.06 \times 10^{11} \text{pa} \), Poisson’s ratio \( \mu = 0.3 \), the failure criterion is the maximum principal stress failure criterion, its value is 84MPa, and the damage evolution is selected based on energy, linear softening, maximum degradation mode, power law mixed mode, power exponent is 1, related parameters are \( G_1 \equiv G_2 = G_3 \equiv \frac{42200 \text{N}}{\text{mm}} \), \( a = 1 \).

In the analysis step, a direct cycle is adopted, with a fixed constraint at one end in the length direction, a stress of 100 MPa is applied at one end, and the middle edge of the plate is prefabricated with a penetration length of 5 mm. The model with a grid density of 1 mm is shown in figure 4.1 below.

In the plane stress, the relationship between the amplitude of strain energy release rate \( \Delta G \) and the amplitude of stress intensity factor \( \Delta K \) is expressed as:

\[
\Delta G = \frac{\left( \Delta K \right)^2}{E} \tag{4.1}
\]

According to the theory of fracture mechanics, when a single penetration crack is subjected to infinite tensile stress, the formula for solving the stress intensity factor \( K \) is as follows:

\[
K = f \left( \frac{a}{b} \right) \sigma \sqrt{\pi a} \tag{4.2}
\]

Since \( a/b = 0.17 \leq 0.6 \), then

\[
f \left( \frac{a}{b} \right) = 1.12 - 0.23 \left( \frac{a}{b} \right) + 10.6 \left( \frac{a}{b} \right)^2 - 21.71 \left( \frac{a}{b} \right)^3 + 30.38 \left( \frac{a}{b} \right)^4 \tag{4.3}
\]

Then, the simulation value is calculated by Eq. (4.1), and the true value is obtained using Eq. (4.2), as shown in figure 4.2.

![Figure 4.1 Simulation model diagram](image-url)
As seen from figure 4.2, when the number of cycles ranges from 0 to $8 \times 10^5$, the amplitude of the stress intensity factor calculated from the simulated crack length is basically the same as the theoretical value, and the crack propagation trend shown in figure 4.3 is basically the same.

4.2 Error analysis of parameter estimation and prediction

In this section, the crack propagation life of the simulation results in section 4.1 is used to verify the accuracy of the fusion model algorithm. The first $5 \times 10^5$ cycle life is employed to predict the future $1 \times 10^5$ cycle life results. In order to better measure the accuracy of the particle filter algorithm in life prediction, this paper separately conducts 10 simulation predictions on the crack propagation life, and compares the absolute error between the mean value and the true value of 10 simulation results, $\bar{a}_{k+1}$ is the true crack length at the $k+1$th cycle.

Then, the variance of the results and the true value and the mean of the simulation in 10 simulations are calculated as Eq. (4.6) and (4.7) to compare and analyze the accuracy and convergence of the three algorithms.

$$V_{k+1} = \frac{1}{10} \sum_{n=1}^{10} (a_{k+1,n} - \bar{a}_{k+1,real})^2$$  

(4.6)

$$\bar{V}_{k+1} = \frac{1}{10} \sum_{n=1}^{10} (a_{k+1,n} - \bar{a}_{k+1})^2$$  

(4.7)

4.3 Prediction of crack growth life based on improved two-parameter exponential formula fusion model

In the improved two-parameter exponential formula, the initial crack length $a_0 = 16$ mm, and the geometric shape coefficient $\gamma = 1.125$. The assumption of the initial distribution is shown in table 4.1.
Table 4.1. Simulation parameter setting based on improved two-parameter exponential formula fusion model

| Parameter | $\Delta \sigma$/Mpa | $\alpha$ | $\beta$ |
|-----------|---------------------|---------|---------|
| Distributed | $N(100, 0.05^2)$ | $N(-3, 1^2)$ | $N(750, 6^2)$ |

Note: $\Delta \sigma$ is the equivalent stress amplitude; $\alpha$ is the fatigue parameter; $\beta$ is the fatigue parameter; $N$ is the normal distribution function.

The simulation crack life prediction is shown in figure 4.4, which indicates that the fusion model of particle filter algorithm and improved two-parameter exponential formula is more accurate for fatigue crack growth parameter estimation and crack growth life prediction. Then, the number of cycles (510000, 520000, 530000, 540000, 550000, 560000, 570000, 580000, 590000) in the simulation results is selected to obtain the evaluation parameters $\hat{a}_{k+1}$, $\hat{V}_{k+1}$, $\hat{V}_{k+1}$ to compare the advantages and disadvantages of the three methods in the prediction of crack life, as shown in figure 4.5 to figure 4.7.

![Figure 4.4 Fatigue crack life extension life prediction](image)

![Figure 4.5 Relative absolute error of estimation and prediction process](image)

![Figure 4.6 Estimated, predicted and true value variance](image)

![Figure 4.7 Estimated, predicted and mean variance](image)

It can be seen from figure 4.5 that in the parameter estimation stage, the relative absolute error interval between the estimated crack length and the real value is $[0, 0.03]$; in the prediction stage, the relative absolute error interval between the predicted crack length and the real value is $[0, 0.06]$, and the maximum value is 0.06. According to figure 4.6, in the parameter estimation stage, the variance between the estimated value and the real value is small and can be ignored, in the prediction stage,
the variance between the ten predicted values and the real value increases gradually, i.e., with the increase of the number of cycles, the prediction accuracy will continue to decrease, and the maximum value is 0.35. The figure 4.7 indicates that in the parameter estimation stage, the variance between the ten times estimated value and the mean value is almost zero, in the prediction stage, the prediction results are more divergent with the increase of the number of cycles, but the maximum value is only 0.008. Therefore, the particle filter algorithm has good convergence. It summary, the fusion model based on the particle filter algorithm and the improved two parameter exponential formula enable to predict the fatigue crack growth life effectively.

5. Conclusions

In this paper, a discretized improved two parameter exponential formula is proposed. Then, the novel formula is combined with observation equation to establish state space evaluation model. The state space evaluation model is estimated by particle filter, and the fatigue crack growth life is predicted by the estimated value. Finally, the simulation results are used to verify the practicability of the fusion model.

The research indicates that in the parameter estimation stage, the relative absolute error, variance, and variance of the estimated value and the true value are small; the maximum value of the relative absolute error is only 0.03; in the prediction stage, the relative absolute error of the predicted value and the true value, The variance and the variance of the mean are increasing with the number of cycles, but the maximum absolute error relative to the true value is only 0.06, which meets the requirements of life prediction. The variance between the ten-time prediction and the mean is small, which proves the good convergence of particle filter algorithm for life prediction.

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