Efficient calibration and modelling of charge amplifiers for dynamic measurements

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Abstract. Charge amplifier calibration at low frequencies is a time-consuming service. This paper presents a method to speed up the process by measuring decades of frequencies in a single run. Based on the calibration results gained, a filter design process is introduced that facilitates a multi-rate approach to digital IIR filters in order to circumvent the complications arising from high-pass filtering with low corner frequency at high sampling rates.

1. Motivation
In applications using dynamic measurements, the conditioning amplifiers used have to be judged by their frequency response [1]. While voltage or bridge amplifiers usually have only an upper frequency limit and can be modelled as low-pass filters, charge amplifiers exhibit a clear bandpass behaviour with different characteristics for the high- and the low-pass part. Earlier papers have dealt with the modelling of these bandpass characteristics [2] [3], however, this contribution deals with the problem using a slightly different approach aiming at the use of a small number of parameters and the use of prior knowledge in terms of technical design details.

For a typical dynamic measurement task in which charge amplifiers are involved, both ends of the applicable frequency spectrum are of concern. For high intensity shocks, e.g. Hopkinson bar devices or pressure shock tubes, the low pass may limit the dynamic information gained from an experiment with short transients. For dynamic measurements with medium- or long-term transients, the high-pass characteristics imply a signal drift, which is difficult to correct.

Any systematic correction of influences from the transfer function requires sound knowledge of the amplifier based on a dynamic calibration and a validated model that utilizes such calibration results.

2. Calibration
The typical measurement setup for the traceable dynamic calibration of charge amplifiers has already been described in detail in e.g. [1] and [4]; this should not be repeated here. However, it has to be noted that the classical procedure for a calibration is quite time-consuming for the low frequency range. This is due to the frequency-by-frequency approach, where the excitation signal is a single sine which has to be measured over several periods with some repetitions. The measured data is subjected to a sine approximation algorithm using the applied frequency.

The signal generation and data acquisition for this paper applied an extended version of this procedure. The excitation signal was a multi-sine function $U_{\text{inp}}$ of $N$ frequencies $f_k$ with a
random initial phase $\varphi_k$ of the form

$$U_{\text{inp}}(t_i) = 1 \cdot \sum_{k=1}^{N} \frac{1}{N} \sin(2\pi f_k t_i + \varphi_k). \quad (1)$$

This signal could, of course, only be applied by using an arbitrary waveform generator. The sample rate for the time stamps $t_i$ was taken as 1 MS/s. The initial phase was taken from a rectangular distribution in the interval $[0, 2\pi[$ (half-open interval). In a single run, the signal contained eight frequency points within one order of magnitude in an almost logarithmic partitioning scheme. The distribution was only approximatively logarithmic, because the chosen frequencies were taken to fit as a multiple of the sampling rate and as integer fractions of the inverse of the measurement interval. These conditions were emphasized to prevent uncertainty components from leakage in the subsequent analysis by multi-sine approximation.

Provided that the transfer characteristics of the device under test are sufficiently linear, the sampled output $U_{\text{out}}(t_i)$ includes only frequency components that were already included in the excitation signal. Hence, a similar equation to (1)

$$U_{\text{out}}(t_i) = \sum_{k=1}^{N} [a_{\text{out},k} \sin(2\pi f_k t_i) + b_{\text{out},k} \cos(2\pi f_k t_i)] + c_{\text{out}} \quad (2)$$

can be used for the analysis by linear least squares. And accordingly, the excitation signal is sampled and fitted as

$$U_{\text{inp}}(t_i) = \sum_{k=1}^{N} [a_{\text{inp},k} \sin(2\pi f_k t_i) + b_{\text{inp},k} \cos(2\pi f_k t_i)] + c_{\text{inp}}. \quad (3)$$

With this multi-sine fit function and the resulting coefficients $a_k$ and $b_k$, the complex transfer function $H$ of the amplifier can be evaluated for several frequencies at once. It is

$$H(f_k) = \frac{\sqrt{a_{\text{out},k}^2 + b_{\text{out},k}^2}}{\sqrt{a_{\text{inp},k}^2 + b_{\text{inp},k}^2}} \cdot \exp \left\{ j \cdot \left[ \tan^{-1} \left( \frac{a_{\text{out},k}}{b_{\text{out},k}} \right) - \tan^{-1} \left( \frac{a_{\text{inp},k}}{b_{\text{inp},k}} \right) \right] \right\}, \quad (4)$$

where $j = \sqrt{-1}$ is the imaginary unit. Figure 1 depicts an example of a measured transfer function using different colours for the different frequency intervals measured in a single run.

3. Modelling and fitting

3.1. Continuous time model

The frequency response or transfer function determined in the previous section is a representation considering continuous time. Accordingly, a first step in modelling is the determination of a continuous transfer function. Due to the chosen settings of the device under test the high pass is supposed to have a corner frequency of 0.3 Hz and the low pass one of 100 kHz. A first order filter for each of these given corner frequencies $f_c = \frac{\omega_c}{2\pi}$ has one of the following functional forms,

$$H_{\text{hp}}(j\omega) = \frac{j\omega \frac{1}{\omega_c}}{j\omega \frac{1}{\omega_c} + 1} \quad \text{or} \quad H_{\text{lp}}(j\omega) = \frac{1}{j\omega \frac{1}{\omega_c} + 1}, \quad (5)$$

where the subscript hp represents the high pass, lp represents the low pass and $\omega = 2\pi f$, the angular frequency.
In a first approach, the simple product $H(j\omega) = H_{hp}(j\omega) \cdot H_{lp}(j\omega)$ of a first order high pass and low pass with the given corner frequencies was compared to the measured data. Figure 2 shows the filter as a continuous blue line. Although the magnitude approximation is not too bad for the simple approach, the phase response exhibits strong deviations.

Further investigation and testing showed that

1) for the high pass, a second filter stage with lower cut-off frequency was implemented in the device and is therefore needed in the model,
2) for the low pass, a (much) higher order of the filter was necessary, whereas a second filter stage did not significantly improve the model,
3) for both filters, a numerical optimization of the cut-off improved the consistency between model and data.

The numerical calculations were performed in Python using SciPy, which is “a Python-based ecosystem of open-source software for mathematics, science, and engineering” (citation from...
the SciPy website). In order to optimize the complex-valued functions, the real and imaginary parts of the fit functions were optimized in a least squares sense with respect to the real and imaginary parts of the measured transfer function. The fitting was performed by using the `scipy.optimize.curvefit(...)` function. Only frequencies less or equal to 200 Hz were considered for the high pass, accordingly the low pass was fitted to frequencies beyond 200 Hz. Since the equations (5) provide unity gain filters, the measured data were normalized with reference to the maximum measured magnitude prior to the fitting.

The results of this process were a high-pass filter consisting of the product of two first order stages with corner frequencies of 0.278 Hz and 0.066 Hz and a low pass of sixth order consisting of a first order low pass with a corner frequency of 279.9 kHz raised to the power of 6. The results are included in figure 2 as a green curve. At first glance, the fit is quite good, except for the phase response at high frequencies, where an increasing deviation can be observed. Taking the difference of the phase from the measurement and the phase of the filter response as seen in figure 3, however, the data reveals a linear phase deviation. Hence, the designed filter only suffers from a constant time delay. A straight-line fit to this phase difference gives a slope of $7.5 \cdot 10^{-6}$ rad/Hz, which corresponds to a constant delay of approximately 1.2 $\mu$s.

### 3.2. Discrete time model

For the application of the modelled charge amplifier response to measured or simulated time series data, a transform into the discrete time domain is necessary. This is achieved by applying the bilinear transform or z-transform, which is available in SciPy as `scipy.signal.bilinear(b,a,Ts,...)`. This function takes the coefficients of polynomials of $j\omega$ as arguments, where the first argument refers to the numerator polynomial and the second to the denominator polynomial.

By following this scheme of polynomial coefficients and facilitating `scipy.poly1d` objects, it is easy to construct complex filter structures by multiplying the polynomial objects of the fundamental filter stages. In the case presented here, the high pass was a product of two polynomials, whereas the low-pass coefficients could be calculated by exponentiating the first order polynomial to the power of 6. The third mandatory argument is the sample interval. In [2] it was presented that with increasing sample rate and thus, decreasing sample interval, the precision requirements for the coefficients increase. Specifically for the high-pass filter high sampling rates were found to be detrimental.

### 4. Making use of the model

The discrete time IIR filter is applied to a time series by the well-known equation

$$a_0y_n = \sum_{i=0}^{N_h} b_i x_{n-i} - \sum_{i=1}^{N_l} a_i y_{n-i},$$

(6)
where \( x_i \) is the input time series, \( y_i \) is the filtered (output) time series and \( a_0 \) is usually set to unity. The coefficients \( a \) and \( b \) depend on the analogue filter characteristics and the sampling interval or the sampling rate.

Although high sampling rates are necessary for many applications, the degradation of the high-pass filter response can be avoided by using a multi-rate approach. For this, a time series at a high sample rate \( R_h \) is divided into \( M \) time series of sample rate \( R_h/M \) with the reduction factor \( M \):

\[
[x_1, x_2, x_3, \ldots] \rightarrow \begin{bmatrix}
    x_1 & x_{M+1} & x_{2M+1} & \ldots \\
    x_2 & x_{M+2} & x_{2M+2} & \ldots \\
    x_3 & x_{M+3} & x_{2M+3} & \ldots \\
    \vdots & \vdots & \vdots & \ldots \\
    x_M & x_{2M} & X_{3M} & \ldots
\end{bmatrix} = X_{ji}
\]

(7)

The coefficients for the high-pass filter are calculated for the lower sample rate \( R_h/M \) and the filter is applied subsequently to all time series in the rows of the matrix \( X_{ji} \) in (7). In cases where \( R_h \) is not an integer multiple of \( M \), the last row or time series needs to be zero padded. Finally, the matrix is again reshaped to the original time series format. The high pass can be applied directly to the time series using the original sample rate \( R_h \).

5. Validation

In order to validate the described approach, a sequence of calculated single-sine time series at the same frequency points as the original measurements was subjected to the calculated IIR filters. The sample rate for the time series was 1 MS/s and the duration was 15 seconds, corresponding to 1.5 periods of the lowest frequency of 0.1 Hz. The reduction factor for the application of the high pass was 100. The filtered time series was analysed by single-frequency sine approximation to calculate the amplitude and the initial phase. Combined with the knowledge of the amplitude and the initial phase of the simulated input time series, a transfer function was evaluated at each frequency point of the original calibration measurement. The result is depicted in figure 4 in comparison to the original measurement. The calculated phase response exhibits the increasing deviation already found for the analogue filter. By shifting the filtered time series by a single sample, a time delay correction of 1 \( \mu s \) of the formerly calculated 1.2 \( \mu s \) (c.f. 3.1) can be easily achieved. The corrected phase response is added in the lower graph of figure 4.

**Figure 4.** Comparison of the magnitude (top) and the phase (bottom) response of the IIR filter (continuous lines) with the measured data (crosshair). The filter’s phase response is given as original data (blue) and as data corrected for a time delay of 1 \( \mu s \) (green).
6. Summary

Charge amplifier calibrations can be performed more efficiently by the multi-sine excitation presented, which is a logical extension to the commonly applied single-sine excitation with subsequent sine approximation. The only adaptation necessary for the experimental setup is the use of an arbitrary waveform generator in contrast to a simple generator. The frequency response function gained in the frequency domain is transformed to a continuous time (analogue) IIR filter and subsequently to a sample-rate-dependent discrete time IIR filter. For cases of a high sampling rate, a multi-rate approach is introduced that allows the application of a reduced sample rate for the high-pass filter part at low frequencies.

References
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