PDC Synthesis for T-S Fuzzy Large-Scale Systems

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Abstract - This paper deals with the decentralized stabilization problem for a T-S fuzzy large-scale system. The interconnection between any two subsystems may be nonlinear and satisfies some matching condition. The decentralized parallel distributed compensation (PDC) fuzzy control for each subsystem is synthesized in which the control gain depends on the strength of interconnections, maximal number of the fired rule in each subsystem and the common positive matrix P.

Based on Lyapunov criterion and Riccati-inequality, some sufficient conditions are derived and the common P is solved by linear matrix inequalities (LMI) toolbox of Matlab so that the whole closed loop large-scale fuzzy system with the synthesized fuzzy control is asymptotically stable. Furthermore, we also discuss the robustness of the closed loop system with perturbations. Finally, a numerical example is given to illustrate the control synthesis and its effectiveness.

Keywords: Large-scale system, T-S fuzzy model, Fuzzy control.

1 Introduction

It is known that, in today's-technology, many real-life problems such as electric power systems, nuclear reactors, aerospace systems, economic systems, process control systems, etc. have become increasingly large in scope and complex in structure. Such systems, called large-scale systems, may consist of a number of independent subsystems which serve particular functions, share resources, and are governed by a set of interrelated goals and constraints [10]. Over the past decade and before, many researchers [3], [7]-[9], [17]-[19], [21], [22] have paid a great deal of attention to various control problems of large-scale systems. As we know, hierarchical control and decentralized control are two main control methods for large-scale systems. Although the studies mentioned above have gained great harvest, the model of a large-scale system should be established before the control design. However, the modeling for a large-scale system may be complicated or difficult to be described in practice.

Recently, the fuzzy system with IF-THEN rules has become one of the most useful modeling approaches for complex systems. It has the capability of modeling complex nonlinear processes to arbitrary degrees of accuracy [2], [16]. [11] proposes a kind of fuzzy inference system so-called Takagi-Sugeno (T-S) fuzzy model which combines the flexibility of fuzzy logic theory and the rigorous mathematical analysis tools in linear system theory into a unified framework. [12] proposes a control concept, that is, parallel distributed compensation (PDC) for fuzzy control design of T-S fuzzy system. Under some conditions, PDC can stabilize the closed loop fuzzy system asymptotically. Linear matrix inequality (LMI) methods to find the common positive matrix P always plays the key role work in PDC design [15].

Now let us consider the control problem of large-scale systems under the base of T-S fuzzy model. The large-scale system is first decomposed into a set of fuzzy regions and in each region a subsystem's nonlinear behavior is described by a T-S fuzzy model. The global model for the large-scale systems can be then achieved by smoothly connecting the local linear model in each fuzzy subspace together via the membership functions. The main difficulty in these schemes is how to ensure the global stability of the large-scale fuzzy systems. Recently, the task of effectively controlling large-scale T-S fuzzy system has been studied by some literature [1], [4]-[6], [14] and [20]. In [1], the fuzzy system is viewed as an interconnected system and the stability conditions are derived for the system via the concepts of Lyapunov functions and M-matrices. [4] combines an adaptive control and a fuzzy sliding mode control for large-scale systems to achieve the output tracking objective. [5] and [6] provide the stability criterion in terms of Lyapunov's direct method with the aids of LMI tools. Their stability criterion includes firing-strength of fired rules. [14] deals with the model reference tracking control problem using $H_{\infty}$ decentralized fuzzy control. In [20], the robust fuzzy
decentralized controller is derived for nonlinear large-scale descriptor system. In this paper, we consider the stabilization problem of a large-scale system. The system is composed of a number of T-S fuzzy subsystems. Based on decentralized control concept, we like to synthesize a PDC controller for each subsystem such that the whole system can be stabilized asymptotically. Furthermore, the robust stabilization is also investigated.

The article is organized as follows. The considered system and problem are described in Section 2. In Section 3, a synthesis strategy of the decentralized PDC fuzzy control to guarantee the stabilization of the whole system is proposed. The allowable perturbation bound of the closed loop system is obtained in Section 4. Section 5 provides an example to illustrate the proposed design. Finally, a conclusion is given in Section 6.

2 System description and problem formulation

Suppose there is a large-scale fuzzy system composed of \( J \) subsystems. Each subsystem \( S_i \) can be represented by a T-S fuzzy model as follows

\[
S_i^j : \begin{cases} 
\text{If } z_n \text{ is } F^j_n \text{ and... and } z_q \text{ is } F^j_q, \\
\text{Then } \dot{x}_i(t) = A'_i x_i(t) + B'_i u_i(t) + \sum_{j=1}^{J} f_q(x_j(t)) + g'_i(x_i(t)), 
\end{cases}
\]

(1)

where \( A'_i \) and \( B'_i \) with appropriate dimensions are the system matrices of rule-1 in subsystem \( S_i \), and by hypothesis, all pairs \((A'_i, B'_i)\), \( i = 1, 2, ..., \), and \( l = 1, 2, ..., \), are controllable; \( r_i \), \( f_q(x_j(t)) \), and \( F^j_q(i = 1, 2, ..., q) \) represent the number of the fuzzy rules in subsystem \( S_i \), the interconnection between subsystem \( S_i \) and subsystem \( S_j \), and the linguist fuzzy sets of the rule \( l \), respectively; \( z_i(t) = [z_{i1}, z_{i2}, ..., z_{iq}] \) are some measurable premise variables for subsystem \( S_i \). \( g'_i(x_i(t)) \) is a bounded nonlinear perturbation vector. Moreover, the interconnection \( f_q(x_j(t)) \) satisfies the matching condition as follows.

\[
f_q(x_j(t)) = B'_i f_q(x_j(t)) \text{ and } \| f_q(x_j(t)) \| \leq \mu_{f_q} \| x_j(t) \|. 
\]

(2)

where \( f_q = 0 \), \( \mu_{f_q} \) is a constant, and \( \| f_q(x_j(t)) \| \) is the Euclidean norm of the vector \( f_q(x_j(t)) \).

If we utilize the standard fuzzy inference method, i.e., a singleton fuzzifier, minimum fuzzy inference, and central-average defuzzifier, (1) can be inferred as

\[
\dot{x}_i(t) = \sum_{j=1}^{J} \mu_i^j(z_q(t)) \left[ A'_i x_i(t) + B'_i u_i(t) + \sum_{j=1}^{J} f_q(x_j(t)) + g'_i(x_i(t)) \right], 
\]

(3)

where

\[
\mu_i^j(z_q(t)) = \frac{\delta^j_i}{\sum_{k=1}^{J} \delta^k_i}, 
\]

\[
w_i^j(z_q(t)) = \min(F^j_q(z_q(t))), \quad \mu_i^j(z_q(t)) = \frac{w_i^j(z_q(t))}{\sum_{k=1}^{J} w_k^j(z_q(t))}.
\]

(4)

\( F^j_q(z_q(t)) \) is the grade of membership of \( z_q(t) \) in \( F^j_q \). We have two tasks to be dealt with in this paper. One is to synthesize the decentralized PDC \( u_i(t) \) for each subsystem such that the closed loop large-scale T-S fuzzy system (3) without perturbation is asymptotically stable. The other is to estimate the tolerable upper bound of the perturbation \( g'_i(x_i(t)) \) in the closed-loop system.

3 Stabilization and PDC synthesis

In this section, we consider the system (1) but without perturbation. The decentralization concept and PDC approach are employed to synthesize the desired controller. The design strategy is that a local feedback control is synthesized for each local subsystem. Let the \( i \)-th rule of the \( i \)-th fuzzy controller be as the PDC form

\[
C^j_i : \begin{cases} 
\text{If } z_n \text{ is } F^j_n \text{ and... and } z_q \text{ is } F^j_q, \\
\text{Then } u_i(t) = -K^j_i x_i(t), 
\end{cases}
\]

(5)

\( i = 1, 2, ..., J; \ l = 1, 2, ..., r_i \). For convenience, we use the same weight notation \( \mu_i^j(z_q(t)) \) as in subsystem \( S_i \). Analogous to (3), the final output of the fuzzy controller \( C_i \) for the corresponding subsystem \( S_i \) is

\[
u_i(t) = -\sum_{j=1}^{J} \mu_i^j K^j_i x_i(t).
\]

(6)

Thus, combining (3) with \( g_i(x_i(t)) = 0 \) and (5), the closed-loop fuzzy subsystem becomes

\[
\dot{x}_i(t) = \sum_{j=1}^{J} \mu_i^j [A'_i - B'_i K^j_i] x_i(t) + \sum_{j=1}^{J} \mu_i^j f_q(x_j(t)).
\]

(7)

Now, our work is to synthesize \( K^j_i \) such that the whole large-scale fuzzy system (6) is stabilized asymptotically.

Referring to the results of [17], let us set the local state feedback gain of rule-1 of subsystem \( S_i \)

\[
K^j_i = \frac{H^j_i}{2} B'_i P^j_i, \ i = 1, 2, ..., J; \ l = 1, 2, ..., r_i.
\]

(8)

where \( H^j_i \) is a constant and satisfying
and \( P_i \) is a symmetric positive definite matrix satisfying the following Riccati-inequality,
\[
\begin{aligned}
A_i^T P_i + P_iA_i' - P_iB_i'B_i^T P_i &\leq -Q_i',
\end{aligned}
\]
for \( i = 1, 2, \ldots, J \); \( l = 1, 2, \ldots, r_i \) (9)

where \( Q_i' \) is a positive definite matrix, and satisfying
\[
\min_l (\lambda_{\text{min}}(Q_i')) > \tilde{\rho}_J,
\]
where \( \tilde{\rho}_J \) is the maximal number of the fired rules by any instant input \( x_i(t) \) in subsystem \( S_i \); \( \lambda_{\text{min}}(Q_i') \) is the minimal eigenvalue of matrix \( Q_i' \). Thus we have the following result.

Theorem 1: For the large-scale fuzzy system (1) with PDC (5) but without perturbation, if the local state feedback gain is as (7), and \( P_i \) is a symmetric positive definite matrix satisfying (9). Moreover the following inequalities hold
\[
\begin{aligned}
\text{for } i = 1, 2, \ldots, J; \text{ and } l = 1, 2, \ldots, r_i \),\( (11)
\end{aligned}
\]
where \( G_i' = A_i' - B_i'K_i' \). Then the overall closed loop large-scale fuzzy system is asymptotically stable.

4 Robustness of the system

In this section, the perturbation \( g_i'(x_i(t)) \) in the system (1) is considered. Suppose \( g_i'(x_i(t)) \) is a bounded nonlinear perturbation vector satisfying
\[
\|g_i'(x_i(t))\| \leq \tilde{g}_i\|x_i(t)\|, \quad i = 1, 2, \ldots, r_i,
\]
with \( \tilde{g}_i > 0 \) [17]. Then, with the PDC (5), we can rewrite (3) as follows
\[
\begin{aligned}
\dot{x}_i(t) &= \sum_{m=1}^{3} \nu_i^m \left( A_i' x_i(t) + B_i' [\sum_{m=1}^{3} \nu_i^m K_i^m x_i(t)] + \sum_{j=1}^{3} f_j(x_i(t)) + g_i'(x_i(t)) \right) \\
&= \sum_{m=1}^{3} \sum_{j=1}^{3} \nu_i^m \nu_j^m \left( A_i' - B_i'K_i^m \right) x_j(t) + \sum_{j=1}^{3} f_j(x_i(t)).
\end{aligned}
\]
(13)

We like to know the tolerable bound of \( g_i'(x_i(t)) \) under which the closed-loop system (13) with PDC (5) remains asymptotically stable.

Theorem 2: The large-scale fuzzy system (1) with the bounded nonlinear perturbation (12) can be stabilized by PDC (5) asymptotically, if the local state feedback gain is as (7), and \( P_i \) is a common positive definite matrix satisfying (9) and (11) for all rules, where the tolerable perturbation bound of each rule will be
\[
\tilde{g}_i \leq \frac{g_i - \tilde{\rho}_J}{2\lambda_{\text{max}}(P_i)},
\]
where \( \tilde{\rho}_J = \max_i \tilde{\rho}_i \), and \( \lambda_{\text{max}}(P_i) \) denotes the maximal eigenvalue of the matrix \( P_i \).

5 An illustrative example

Here, we give an example to demonstrate the decentralized stabilization for a large-scale fuzzy system. Consider a large-scale system \( S \) composed of three fuzzy subsystems \( S_i, i = 1, 2, 3 \), as follows, in which each state of each subsystem has two dimensions.

Subsystem \( S_1 \):

Rule 1: If \( x_{11}(t) \) is small and \( x_{12}(t) \) is big,
\[
\dot{x}_1(t) = A_1' x_1(t) + B_1' u_1 + \sum_{j=1}^{3} f_j(x_j(t)).
\]

Rule 2: If \( x_{11}(t) \) is small and \( x_{12}(t) \) is small,
\[
\dot{x}_1(t) = A_2' x_1(t) + B_2' u_1 + \sum_{j=1}^{3} f_j(x_j(t)).
\]

Rule 3: If \( x_{11}(t) \) is big and \( x_{12}(t) \) is small,
\[
\dot{x}_1(t) = A_3' x_1(t) + B_3' u_1 + \sum_{j=1}^{3} f_j(x_j(t)).
\]

Subsystem \( S_2 \):

Rule 1: If \( x_{21}(t) \) is small and \( x_{22}(t) \) is small,
\[
\dot{x}_2(t) = A_4' x_2(t) + B_4' u_2 + \sum_{j=1}^{3} f_j(x_j(t)).
\]

Rule 2: If \( x_{21}(t) \) is big and \( x_{22}(t) \) is small,
\[
\dot{x}_2(t) = A_5' x_2(t) + B_5' u_2 + \sum_{j=1}^{3} f_j(x_j(t)).
\]

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\]

Rule 3: If \( x_{11}(t) \) is big and \( x_{12}(t) \) is small,
\[
\dot{x}_1(t) = A_3' x_1(t) + B_3' u_1 + \sum_{j=1}^{3} f_j(x_j(t)).
\]

where \( A_1' = \begin{bmatrix} -2 & 3 \\ 1.5 & -2.2 \end{bmatrix} \), \( A_2' = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} \), \( A_3' = \begin{bmatrix} -2 & 3 \\ -6 & -11 \end{bmatrix} \), \( B_1' = \begin{bmatrix} 0.15 \\ 0.6 \end{bmatrix} \), \( B_2' = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} \), \( B_3' = \begin{bmatrix} 0.2 \end{bmatrix} \), \( f_{1j} = 0.5\|x_j(t)\| \), \( f_{2j} = 0.125\|x_j(t)\| \), \( f_{3j} = 0.2\|x_j(t)\| \), \( f_{4j} = 0.6\|x_j(t)\| \), \( f_{5j} = 1.5\|x_j(t)\| \), and \( f_{6j} = 0.3\|x_j(t)\| \).

Subsystem \( S_2 \):

Rule 1: If \( x_{21}(t) \) is small and \( x_{22}(t) \) is small,
\[
\dot{x}_2(t) = A_4' x_2(t) + B_4' u_2 + \sum_{j=1}^{3} f_j(x_j(t)).
\]

Rule 2: If \( x_{21}(t) \) is big and \( x_{22}(t) \) is small,
\[
\dot{x}_2(t) = A_5' x_2(t) + B_5' u_2 + \sum_{j=1}^{3} f_j(x_j(t)).
\]
where $A_2 = \begin{bmatrix} -3 & 1 \\ 5 & -3 \end{bmatrix}$ and $A_2 = \begin{bmatrix} -2 & 1 \\ 3 & -0.3 \end{bmatrix}$; $B_1 = \begin{bmatrix} 0.1 \\ 0.6 \end{bmatrix}$ and $B_2 = \begin{bmatrix} 0.2 \\ 1.2 \end{bmatrix}$; $f_1 = 0.2\|x(t)\|$; $f_2 = 0.1\|x(t)\|$, and $f_3 = 0.3\|x(t)\|$.  

\( \text{Subsystem } S_1: \)

Rule 1: If $x_{31}(t)$ is small and $x_{32}(t)$ is big, 
Then $u_3(t) = -K_1^2 x_1(t)$.

Rule 2: If $x_{31}(t)$ is small and $x_{32}(t)$ is small, 
Then $u_3(t) = -K_1^2 x_1(t)$.

Fuzzy controller $C_1$: 

Rule 1: If $x_{11}(t)$ is small and $x_{12}(t)$ is big, 
Then $u_1(t) = -K_1^1 x_1(t)$.

Rule 2: If $x_{11}(t)$ is small and $x_{12}(t)$ is small, 
Then $u_1(t) = -K_1^1 x_1(t)$.

Fuzzy controller $C_2$: 

Rule 1: If $x_{21}(t)$ is small and $x_{22}(t)$ is small, 
Then $u_2(t) = -K_1^1 x_1(t)$.

Rule 2: If $x_{21}(t)$ is big and $x_{22}(t)$ is small, 
Then $u_2(t) = -K_1^1 x_1(t)$.

Fuzzy controller $C_3$: 

Rule 1: If $x_{31}(t)$ is big and $x_{32}(t)$ is small, 
Then $u_3(t) = -K_1^2 x_1(t)$.

Rule 2: If $x_{31}(t)$ is big and $x_{32}(t)$ is big, 
Then $u_3(t) = -K_1^2 x_1(t)$.

Assume that all $f_q$ satisfy the matching condition (2).

The membership functions of each state are shown in Fig. 1. The above systems (15) with $u(t) = 0$ has unstable response as shown in Fig. 2, in which the initial conditions are $x_1(0) = [1.5 -1]^T$, $x_2(0) = [-0.5 \ 0.5]^T$, and $x_3(0) = [0.7 \ -0.3]^T$, respectively. In order to stabilize the system, three decentralized PDC fuzzy controllers will be synthesized in the following.

Fuzzy controller $C_1$: 

Rule 1: If $x_{11}(t)$ is small and $x_{12}(t)$ is big, 
Then $u_1(t) = -K_1^1 x_1(t)$.

Rule 2: If $x_{11}(t)$ is small and $x_{12}(t)$ is small, 
Then $u_1(t) = -K_1^1 x_1(t)$.

Fuzzy controller $C_2$: 

Rule 1: If $x_{21}(t)$ is small and $x_{22}(t)$ is small, 
Then $u_2(t) = -K_1^1 x_1(t)$.

Rule 2: If $x_{21}(t)$ is big and $x_{22}(t)$ is small, 
Then $u_2(t) = -K_1^1 x_1(t)$.

Hereby, the closed-loop global fuzzy system with the above decentralized PDC is as the following form

\[
\dot{x}_1 = \sum_{q=1}^{3} \sum_{i=1}^{n} \mu_i^1 \left[ A_1 x_1 + B_1^T u_1 + \sum_{j=1}^{n} f_1(x_j(t)) \right], \quad x_1(0) = [0.7 \ 0.3]^T
\]

Based on the LMI toolbox of Matlab, we can obtain the common $P_i$, $i = 1, 2, 3$, as

\[
P_1 = \begin{bmatrix} 26.1971 & 27.3258 \\ 27.3258 & 48.9412 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 60.0928 \ -16.7848 \\ -16.7848 & 12.2827 \end{bmatrix},
\]

and $P_3 = \begin{bmatrix} 45.0909 & -10.725 \\ -10.725 & 10.3799 \end{bmatrix}$, where $Q_i = 10 I$, $Q_i = 18 I$, $Q_i = 14 I$, $Q_i = 23 I$, $Q_i = 7 I$, $Q_i = 28.5 I$, and $Q_i = 11 I$ are chosen. It is seen that $P_i$ satisfies (9) for all rules and all subsystems and (10) holds too.

Let us choose $h_1 = 6.49$, $h_2 = 1.34$, $h_1 = 2.37$ to satisfy (8) with $f_1 = 0.5$, $f_2 = 0.125$, $f_3 = 0.25$, $f_4 = 0.6$, $f_5 = 0.15$, and $f_6 = 0.3$. From (7), we have $K_i = [21.6186 \ 29.1822]$, $K_i = [17.8545 \ 24.1012]$, and $K_i = [15.7893 \ 21.3134]$ for subsystem-1. Set $h_1 = 2.6$, $h_2 = 1.4$ to satisfy (8) with $f_1 = 0.2$, $f_2 = 0.1$, $f_3 = 0.6$, and $f_4 = 0.3$. $K_i = [-5.2801 \ 7.3985]$, and $K_i = [-5.6863 \ 7.9676]$ are obtained for subsystem-2. Let us choose $h_1 = 4.2$, $h_2 = 13.8$ to satisfy (8) with
\[ j_{1} = 0.8, j_{2} = 1.6, j_{3} = 0.4, \text{ and } j_{4} = 0.8. \] We obtain for subsystem-3. Meanwhile, it is checked that (11) is satisfied. Hence, according to Theorem 1, the overall closed-loop large-scale fuzzy systems composed of three subsystems \( S_1 \), \( S_2 \), and \( S_3 \) but without perturbation should be stabilized asymptotically. Assume there is a perturbation \( g_i(x_i(t)) \) presenting on the \( I \)-th rule of subsystem \( S_i \), by Theorem 2, the tolerable bounds are

\[ \hat{g}_1 \leq 0.0074, \hat{g}_2 \leq 0.0076, \text{ and } \hat{g}_3 \leq 0.0519. \] (16)

The complete simulation results with the initial condition \( x_1(0) = [1.5 \ -1]^T \), \( x_2(0) = [-0.5 \ 0.5]^T \), and \( x_3(0) = [0.7 \ -0.3]^T \) and with the following perturbations are shown in Fig. 3. The perturbation sets in the simulation are as the following form:

\[
\begin{align*}
g_1(x_1(t)) &= \hat{g}_1 \begin{bmatrix} x_1(t) \sin(3x_1(t)) \\ x_2(t) \sin(5x_2(t)) \end{bmatrix}, \\
g_2(x_2(t)) &= \hat{g}_2 \begin{bmatrix} x_3(t) \sin(3x_3(t)) \\ x_4(t) \sin(5x_4(t)) \end{bmatrix}, \text{ and} \\
g_3(x_3(t)) &= \hat{g}_3 \begin{bmatrix} x_5(t) \sin(0.5x_5(t)) \\ x_6(t) \sin(0.5x_6(t)) \end{bmatrix}, \text{ for all } I. \end{align*}
\] (17)

6 Conclusion

This paper has studied the stability, stabilization and robustness problems of the large-scale T-S fuzzy systems. The decentralized PDC has been synthesized under some conditions such that the whole closed-loop system is asymptotically stable. LMI tool still plays the role to find the common matrix \( P \). It has been shown that the gain of each PDC depends on the strength of interconnections, maximal number of the fired rules and the common \( P \).

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Fig. 1. Membership functions

Fig. 2. The responses of the system (26) with $u(t) = 0$

Fig. 3. The state responses with control and perturbations.