Presentation of distortion power in an alternating current network and analysis of factors of its occurrence

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Abstract. The article examines the conditions for the appearance of such components of inactive power in electric alternating current networks, which are known as reactive power and distortion power. Through simulation modeling, the validity of the expressions that were obtained analytically and determine the components of inactive power was verified. The validation of expressions based on theoretical concepts. The results are useful in optimizing modes in AC networks and creating effective technical means of compensating latent capacities.

1. Introduction

Optimization of modes in alternating current (AC) networks leads to the need to compensate for all power components that are not able to perform useful work, i.e., which are not active power by definition.

Compensation issues have been studied to the greatest extent for idealized conditions when electric networks have symmetry and sinusoidal phase voltages and balanced loads, while the loads themselves have linear current-voltage characteristics. Under such conditions, the appearance of a component of inactive power, which is known as reactive power (Q), explains the excess of apparent power (S) overactive power (P).

In real conditions, AC electric networks do not have a sinusoidal phase voltage, and most modern loads have sharply non-linear current-voltage characteristics. In these cases, non-sinusoidal currents begin to flow along with circuit elements with non-sinusoidal voltages. This process causes an excess of power S to overpower P, which can no longer be explained by the appearance of reactive power Q. In the scientific literature, this circumstance due to the appearance of another component of inactive power. These components are known as the distortion power, and T [1] or D (Distortions) [2] indicate them. Under certain conditions in an AC network, power D can form as the only component of inactive power.

Optimization of modes in AC networks leads to extremely important when working with various equipment [17, 18], stabilization of compensation of power components during operation of heavily loaded equipment is especially important [19, 20].
2. Statement of the problem
The task is to analyze the most characteristic static modes, which lead to the separate appearance of the power $Q$ and $D$ in electrical AC networks. The results are useful in studying the features of the operating modes of electric networks using simulation tools and creating further effective technical means for compensating latent capacities.

3. Theory
In the theory of electrical engineering, the causes of the appearance of power $Q$ and its negative effects on the operation modes of electric networks with sinusoidal currents and voltages have a fairly good description. Currents and voltages in a significant part of electrical installations, thanks to especially taken measures, can indeed be considered sinusoidal. However, in some cases, currents and voltages are more or less different from sinusoidal. The reason for the appearance of non-sinusoidal voltages and currents can be both generators and energy receivers. Therefore, in practice, such technical means are successfully used, the compensating power of which allows reducing the value $Q_k$ to the minimum value $Q \rightarrow 0$, and thereby bring the value of $S$ closer to the numerical value $P$ of the consumer.

In this case, one of the obvious criteria that form the objective function has the following form of notation

$$S = \sqrt{P^2 + (Q - Q_k)^2} \rightarrow \min.$$  

(1)

Power compensation is a more difficult task if non-sinusoidal voltages or currents operate in the electrical network. In this case, for calculations, they are traditionally represented by the Fourier series [3]

$$u = U_0 + U_{im} \sin(\omega t + \varphi_i) + U_{2m} \sin(2\omega t + \alpha_2) + \ldots + U_{Nm} \sin(N\omega t + \alpha_N),$$

$$i = I_0 + I_{im} \sin(\omega t + \varphi_i) + I_{2m} \sin(2\omega t + \alpha_2) + \ldots + I_{Nm} \sin(N\omega t + \alpha_N + \varphi_N).$$  

(2)

We consider the constant component of series (2) as zero harmonic. Then the total power is calculated by the products of harmonics of voltages and currents with identical numbers

$$S = U \cdot I = \sqrt{\sum_{i=0}^{N} U_i^2 \cdot \sum_{i=0}^{N} I_i^2},$$  

(3)

$U_i = U_{im}/\sqrt{2}$, $I_i = I_{im}/\sqrt{2}$ – the effective values of voltage and current harmonics, $i$ – the harmonic sequence number.

The active component of power is calculated as the average value of the product $u \cdot i$ of all harmonics (2) for the period $T$

$$P = \frac{1}{T} \int_0^T (u \cdot i) dt = U_0 I_0 + U_1 I_1 \cos \varphi_1 + U_2 I_2 \cos \varphi_2 + \ldots + U_N I_N \cos \varphi_N.$$  

(4)

The above equation proves that all harmonics can do useful work in a non-sinusoidal AC network. It also shows that the active power is characterized by the sum of the active powers of the individual harmonics

$$P = P_0 + P_1 + P_2 + \ldots + P_N = \sum_{i=0}^{N} P_i.$$  

(5)

By analogy with power $P$, in the theory of electrical engineering, there is a perception [4] that power $Q$ in AC networks is defined as the sum of the reactive powers of individual harmonics

$$Q = U_1 I_1 \sin \varphi_1 + U_2 I_2 \sin \varphi_2 + \ldots + U_N I_N \sin \varphi_N,$$

$$Q = Q_1 + Q_2 + \ldots + Q_N = \sum_{i=1}^{N} Q_i.$$  

(6)
The component of the inactive power $D$, which is found [5] in networks with non-sinusoidal currents and voltages, is usually determined through the unbalance of the previously found powers $P$ and $Q$ in the inequality $S > \sqrt{P^2 + Q^2}$, using the property of their orthogonality

$$D = \sqrt{S^2 - P^2 - Q^2}.$$  \hspace{1cm} (7)

It should be noted that in the system of state standards for terms and definitions of the basic concepts of electrical engineering (GOST 19880-74), units of physical quantities (GOST 8.417-81), letter designations of the main quantities in electrical engineering (GOST 1494-77), the term "distortion power", universally used in conversion technology, generally absent. There is no generally accepted designation for distortion power. In the theory of non-sinusoidal currents, it is denoted by "T." In this work, for the distortion power, the designation is made with the letter $D$ (from the English distortion) and measurement in volt-amperes [15].

Moreover, among individual scientific schools there is no clear understanding of the physical meaning of the total reactive power, determined by expression (6) for electric networks in which non-sinusoidal voltages and currents operate. This circumstance is reflected in the adopted IEEE 1459-2010 standard [6], which introduced the concept of inactive power, however, the definition of power $Q$ is limited only by the first harmonic.

Based on this, it is advisable to conduct a series of experiments. These experiments devote to modeling the most common static modes of electrical circuits. The results of these experiments determine the most consistent theory and answer the question of the task.

4. The choice of modeling environment and the peculiarity of creating measuring subsystems.

Modes that lead to the separate appearance of $Q$ and $D$ powers in electrical AC networks are the object of analysis. It is advisable to analyze the modes according to the results of model experiments for a series of individual cases, each of which differs in the harmonic composition of the supply voltages, currents, and current-voltage characteristics of the loads. To solve this problem, we used the simulation environment Simulink of the MATLAB package [12] and the experience described in [16].

The functional unit of measurements Power, available in the Simulink environment, confirms the current paradigm of power representation [6]. Because of this, it is not convenient as a tool for measuring the values of $P$ and $Q$. Its algorithm based on the calculation of powers relative to the fundamental harmonic (fundamental frequency), and this requires preliminary settings before each experiment.

Integral methods characterize instantaneous power as $P = u \cdot i$, the integral of which determines the transmitted electric energy $W$ over a time interval. This interval limited by the integration limits $t_1$ and $t_2$ and does not require preliminary consideration of the harmonic composition. For model experiments, it is advisable to calculate the power $P$ for the load in the AC network as follows:

$$P = \frac{W_{t_2-t_1}}{t_2-t_1} = \frac{1}{t_2-t_1} \int_{t_1}^{t_2} (u \cdot i) dt.$$ \hspace{1cm} (8)

Using the resulting expression in the Simulink environment, calculating the power $P$ implements a simple universal model as a measuring subsystem.

For the convenience of constructing the corresponding measuring subsystem, the power $S$ is calculated similarly in the following model, taking into account expression (3):

$$S = \sqrt{\frac{1}{t_2-t_1} \int_{t_1}^{t_2} u^2(t) dt} \cdot \sqrt{\frac{1}{t_2-t_1} \int_{t_1}^{t_2} i^2(t) dt}.$$ \hspace{1cm} (9)

To determine the value of $S$, the resulting subsystem (9) corresponds to the measuring subsystem.
This subsystem also requires preliminary settings in the simulation model.

5. Modeling the conditions of inactive power $Q$ and $D$

Two previously known harmonics models the non-sinusoidally of the voltage in the AC network to simulate the analysis of the results obtained. These are such harmonics as the primary (50Hz) and highest. An example of modeling such harmonics is the tenth (500Hz) is modeled by changing the ratios of their amplitudes.

The ratio of the values of the elements $R$ and $L$ connected in parallel determines the load nature in the model of AC network. Giving the non-linear character of the current-voltage characteristic is carried out by including a semiconductor diode in the load circuit of the load. Then a simple half-wave rectifier circuit is implemented.

The Fourier measuring units determine the monitored values in the model:

- $u_{50}^{\max}$, $u_{500}^{\max}$ – amplitudes of voltage harmonics with a frequency of 50 Hz and 500 Hz;
- $\alpha_{50}^{u}$, $\alpha_{500}^{u}$ – the initial phase angle angles of voltage harmonics with a frequency of 50 Hz and 500 Hz;
- $i_{50}^{\max}$, $i_{500}^{\max}$ – amplitudes of current harmonics with a frequency of 50 Hz and 500 Hz;
- $\alpha_{50}^{i}$, $\alpha_{500}^{i}$ – the initial angle of the phase shift of the harmonics of the current frequency of 50 Hz and 500 Hz;
- $\varphi_{50} = \alpha_{50}^{u} - \alpha_{50}^{i}$, $\varphi_{500} = \alpha_{500}^{u} - \alpha_{500}^{i}$ – phase angle between harmonics of voltages and currents.

The results of a series of model experiments are in Table 1.

| № | $R$ (Ω) | $L$ (H) | $u_{50}^{\max}$ | $u_{500}^{\max}$ | $i_{50}^{\max}$ | $i_{500}^{\max}$ | $\varphi_{50}$ | $\varphi_{500}$ | $P$ (W) | $Q_{1}$ (VA) | $Q_{10}$ (VA) | $D$ (VA) | $S$ (VA) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 100 | - | 100 | 0 | 1.000 | 0 | 0 | - | 50.0 | 0 | 0 | 0 | 50.0 |
| 2 | 100 | - | 100 | 100 | 1.000 | 1.000 | 0 | 0 | 100.0 | 0 | 0 | 0 | 100.0 |
| 3 | - | 0.1 | 100 | 0 | 0 | 3.183 | 0 | -90.0 | - | 0 | 159.2 | 0 | 0 | 159.2 |
| 4 | - | 0.1 | 100 | 0 | 0 | 3.183 | 0.318 | -90.0 | -90.0 | 0 | 159.2 | 15.92 | 0 | 143.2 | 226.2 |
| 5 | 100 | 0.1 | 100 | 0 | 0 | 3.336 | 0 | -72.5 | - | 50.0 | 159.2 | 0 | 0 | 159.2 |
| 6 | 100 | 0.1 | 100 | 0 | 0 | 3.336 | 1.049 | -72.5 | -17.6 | 100.0 | 159.2 | 15.92 | 143.2 | 247.3 |
| 7 | (+VD) | - | 100 | 0 | 0 | 0.500 | - | 0.4 | - | 25.2 | 0.2 | 0 | 25.0 | 35.7 |

An analysis of the results indicates the achievement of a balance of orthogonal capacities for all modes. The analysis confirms the validity of expression (7). The total reactive power of the load is determined by the sum of the reactive powers of the individual harmonics, as follows from expression (6). The power of distortion occurs when the shape of the load current curve is not identical to the shape of the supply voltage curve. Accordingly, combinations of the existing harmonics of voltage ($h$) and current ($k$), the product $u \cdot i$, due to the general properties of orthogonality over a period, give zero average power values (N):

$$N = \frac{1}{T} \int_{0}^{T} (U_{hm} \sin(h \cdot \omega t + \alpha_{h}) \cdot I_{km} \sin(k \cdot \omega t + \alpha_{k})) \, dt = 0,$$

And therefore, the value of $N$ does not apply to power $P$, and is inactive, as it is not able to perform useful work. By the definition $h \neq k$ of expression (6), this power does not form power $Q$. Under such conditions, the inactive power $N$ should be considered as the power of distortions $D$.

6. Analysis of the results

At the end of the work, the most characteristic modes presented in Table 1 are analyzed.
Mode № 1
The supply network is strictly sinusoidal $U_{10} = 0$; the nature of the load is purely active. In this case $I_{10} = 0$, $\cos \varphi_i = 1$, $\sin \varphi_i = 0$. Therefore, it obtained that $Q_i = U_i I_i \sin(\varphi_i) = 0$ and $D = 0$.

Mode № 2
The supply network is non-sinusoidal $U_{10} \neq 0$; the nature of the load is purely active. In this case, we obtain $\cos \varphi_i = 1$, $\cos \varphi_{10} = 1$, $\sin \varphi_i = 0$, $\sin \varphi_{10} = 0$. It turns out that $\frac{U_i}{U_{10}} = \frac{I_i}{I_{10}}$.

Therefore, for this mode $U_1 I_{10} - U_{10} I_1 = 0$.

Thus, in an AC network, even with a non-sinusoidal voltage, but with an active load, the power $D$.

Mode № 6
The supply network is non-sinusoidal; the nature of the load is active-inductive. In this case $0 < \cos \varphi_i < 1$, $0 < \cos \varphi_{10} < 1$, $1 > \sin \varphi_i > 0$, $1 > \sin \varphi_{10} > 0$. For such a regime, having made a series of transformations of expression (11), next formula is relevant

$$D = \sqrt{\left(U_i I_{10} - U_{10} I_i\right)^2} = U_i I_{10} - U_{10} I_i$$

However, in an AC network with a purely active load, the identity between the harmonics of voltages and harmonics of currents is satisfied

$$\frac{U_i}{U_{10}} = \frac{I_i}{I_{10}}$$

(11)

Therefore, for this mode $U_1 I_{10} - U_{10} I_1 = 0$.

Mode № 7
The supply network is sinusoidal; the nature of the load is active but non-linear. In this case $U_i \neq 0$, $U_i = 0$, where $i > 1$, and the alternating current network, only the load current turn out to be non-
sinusoidal, decomposing into a series of harmonics that not useful work by expression (10).

If, as a first approximation, we neglect switching processes, then the main harmonic of the current not have a phase shift relative to the first and only harmonic of the supply voltage, i.e. \( \cos \phi_1 = 1 \), and \( \sin \phi_1 = 0 \). Under these conditions, the value \( Q = 0 \), and the value \( D \) from expressions (3), (4), (5) and (7) expressed as

\[
D = \sqrt{S^2 - P^2} = \sqrt{\sum_{i=0}^{\infty} U_i^2 I_i^2 - \sum_{i=0}^{\infty} U_i^2 I_i^2} = U_j \sum_{i=0}^{\infty} I_i^2
\]

(13)

However, if the current-voltage load characteristics create a primary harmonic of the current for which \( \sin \phi_1 \neq 0 \), then a power \( Q \) is generated in the network with a sinusoidal voltage. For example, this is typical for the operation mode of controlled rectifiers, which form both power \( D \) and power \( Q \), even without specific reactive elements in their circuits. Thus, the mode in which power \( D = 0 \) according to the results of the analysis of expressions (12) and (13) achieves under the following conditions:

- equality of phase shifts of all harmonics of currents and voltages
  \( \phi_i = \phi_j \) (14)
- identity of the relations of the practical values for all harmonics of voltages and currents
  \( \frac{U_i}{I_i} = \frac{U_j}{I_j} \) (15)

If in the model we increase the number of higher harmonics of currents and voltages, then, by analogy with the derivation of expression (11), a general expression obtained for directly finding the quantity \( D \)

\[
D = \sqrt{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} U_i I_j \left( U_i I_j - 2 U_j I_i \cos \phi_i \cdot \cos \phi_j \right) + U_j I_j \left( U_i I_i - 2 U_i I_j \sin \phi_i \cdot \sin \phi_j \right)}
\]

(16)

In the scientific literature, variants of searching for similar expressions in a general form for finding the value of \( D \) also present. So, in [13] it is proposed to determine the value of \( D \) by the formula

\[
D^2 = \sum_{k,l=1}^{N} U_k I_l \left( U_k I_l - 2 U_l I_k \cos(\varphi_k - \varphi_l) \right) + U_j I_j \left( U_j I_j - 2 U_j I_j \cos(\varphi_j - \varphi_j) \right)
\]

and the recording format closest to expression (17) is found in [14].

Thus, the components of power \( N \) can be analyzed and calculated and, at least for simple alternating current circuits in static mode, can be unambiguously divided into components of power \( Q \) according to different harmonics and power \( D \). Closest to the stated results obtained by analytical and confirmed Simulation tools include representations of power \( Q \) and power \( D \) described in [7].

7. Conclusion

The paper analyzes the main modes leading to the separate appearance of the \( Q \) and \( D \) capacities in electrical AC networks. This analysis allows us to conclude that the compensation of power \( Q \) and power \( D \) are most effective if the technical device provides such a correction of the load current, the shape of which as a result is close in shape to the voltage curve of the supply network, which helps to fulfill the condition \( P/S \rightarrow 1.0 \).
The technique of direct derivation of analytical expressions of power $D$ in AC networks is developed. The use of simulation tools made it possible to confirm two simple essential criteria for achieving distortion power compensation, namely, the equality of the phase shifts of all harmonics of the current and load voltage and the equality of the ratios of the valid values of the harmonics of the voltage and load currents.

The obtained patterns and conclusions can be used to model the optimal operating modes of various equipment used in various industries. This opportunity is especially important for the mining industry and heavy-duty pumps [21-24].

References
[1] Zeveke G V, Ionkin P A, Netushil A V 1963 Fundamentals of circuit theory. (M-L: Gosenergoizdat)
[2] Akagi H, Kim H 1999 IEEE International Conference on Power Electronics and Drive Systems, PEDS'99 (Hong Kong)
[3] Ango A 1967 Mathematics for electrical and radio engineers. (M.: Science)
[4] Sharon D 1996 Power factor definitions and power transfer quality in nonsinusoidal situations. IEEE Transactions on Instrumentation and Measurement 45 728-733
[5] Krogeris A F 1993 AC power (Riga, Phys.-Energetic Institute of Latvia. AN)
[6] 2010 IEEE Standard Definition for the measurement of Electric Power Quantities under sinusoidal, non sinusoidal, balanced or unbalanced conditions (IEEE std. 1459TM – 2010). (IEEE Power and Energy Society, New York)
[7] Budenau C 1927 Puissance reactives et fictives. Inst. Romain de l’Energie. (Buharest, Rumania)
[8] Czarnecki L S 1988 Orthogonal decomposition of the currents in a 3-phase nonlinear asymmetrical circuit with a nonsinusoidal voltage source. IEEE Transactions on Instrumentation and Measurement 37 30-34
[9] Fryze S 1931 Active and Apparent power in non-sinusoidal systems. Przeglad Elektrot 7 193-203.
[10] Mayevsky O A 1978 Energy performance of valve converters / OA Mayevsky. (M.: Energy)
[11] Akagi H, Nabae A 1993 The p-q Theory in Three-Phase System under Non-Sinusoidal Conditions. (ETEP)
[12] German-Galkin S G 2013 Virtual laboratories of semiconductor systems in the environment of Matlab - Simulink. (St. Petersburg: Doe)
[13] Bezikovich A A, Shapiro E Z 1980 Measurement of electrical power in the sound frequency range. (Leningrad: Energy)
[14] Shklyarsky Ya E, Bragin A A, Dobush V S 2012 Impact of harmonic components of current and voltage on distortion power. Oil and gas business 4 26-32
[15] Zeveke G V, Ionkin P A, Netushil A V, Strakhov S V 1963 Fundamentals of circuit theory. (M-L: Gosenergoizdat)
[16] Korolev N A, Solovev S V 2017 AC motor diagnostics system based on complex parametric analysis. Materials Science and Engineering 177
[17] Ershov D Y, Zlotnikov E G, Koboyank E L E 2017 Analysis of causes and mathematical description of process of manufacturing errors and local defects in mechanical system nodes. IOP Conference Series: Earth and Environmental Science 87(8) 082015
[18] Avksentiev S Y, Avksentieva E Y 2018 Determining the Parameters of the Hydraulic Transport of Tailings for Processing Iron Ore. IOP Conference Series: Earth and Environmental Science 194 (3) 032003
[19] Serzhan S L, Trufanova I S, Malevannyi D V 2019 Substantiation of the draghead application as a mining unit in conditions of solid minerals deep-sea mining. IOP Conference Series: Earth and Environmental Science 272 1 - 6
[20] Maksarov V V, Keksin A I 2018 Technology of magnetic-abrasive finishing of geometrically-complex products. IOP Conference Series: Materials Science and Engineering 4(327) 42 – 45
[21] Nasonov M Y, Lykov Y V 2017 Setting up excavators with growing cracks in their metal structures for repairs IOP Conference Series: Earth and Environmental Science 87 022015

[22] Lykov Y V, Gorelikov V G, Gantulga B 2017 Analytical research and classification of mechanism of diamond drilling-bits contact with rocks during well sinking. IOP Conference Series: Earth and Environmental Science 87 022012

[23] Krasnyy V A 2018 Application of seals made of directed reinforced polymeric composite materials to improve wear resistance of friction units of oil well pumps. IOP Conf. Series: Earth and Environmental Science 194 042008 doi:10.1088/1755-1315/194/4/042008

[24] Aleksandrov V I, Kibirev V 2018 The Kachkanarsky MCC iron ore processing tailings slurry hydraulic transport parameters determination. Obogashchenie Rud. 56-63 10.17580/or.2018.01.10