Melting curves and other phase transitions in a two dimensional electron assembly in a transverse magnetic field as a function of the Landau level filling factor

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Besides Laughlin/composite Fermion liquid, and Wigner solid, there have been proposals for Hall crystals and condensed phases of skyrmions for inclusion in the QHE systems phase diagram. Some results on Hall crystals are reported, which suggest that at $\nu = 1/3$, such crystals can be the ground state of the 2DEG in a magnetic field. At $\nu = 1$ and 2, Chen et al experimentally established that the Wigner solid is the ground state. An explanation is also given here for the coexistence in these experiments of an integral QHE plateau and the presence of the Wigner crystal phase near $\nu = 1$. Further, Mandal et al showed that at $\nu = 1/7$, and most probably at $\nu = 1/9$, an incompressible liquid state should be the ground state. Thus, at least three non-entrant solid phases appear to exist: $0 < \nu \leq 1/9$, $1/9 < \nu < 1/7$ and, less certainly, $1/7 < \nu < 1/5$. In addition, points of solid nature are present at $\nu = 1$ and $2$ (Wigner solid) and possibly at $\nu = 1/3$ (Hall crystal).

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The boundaries between different phases of the 2DEG at high magnetic fields has been the subject of much interest recently. A schematic diagram of the melting curve characterizing the equilibrium between a Wigner solid and a Laughlin liquid was proposed more than a decade ago by Buhmann et al. This was followed by a thermodynamical discussion based on a first order phase transition by Lea, March and Sung. These authors wrote for the slope of the melting curve $T_m(\nu)$, with $\nu$ the Landau level filling factor,

$$\frac{\partial}{\partial \nu} T_m(\nu) = \frac{B \Delta M}{\nu \Delta S}. \quad (1)$$

Here $B$ is the strength of the transverse magnetic field, while $\Delta M$ and $\Delta S$ are the changes in the magnetization $M$ and entropy $S$ across the melting line. To be precise, with "s" "solid, and "l" "liquid:

$$\Delta M = M_s - M_l.$$

and

$$\Delta S = S_s - S_l.$$

Lea, March and Sung subsequently applied microscopic theory, for both anyon and composite Fermion models, to demonstrate that the main features of the Buhmann et al diagram could be explained by invoking the magnetization, of the de Haas-van Alphen character, of the Laughlin/CF liquid phase.

Key additions to this field are the very recent studies of Mandal, Peterson and Jain, who presented Monte Carlo results for $\nu = 1/7, 1/9$ and 1/11. They then write that ‘the principal conclusion of this work is that, for the model considered, the ground state is a liquid at $\nu = 1/7$. This is in agreement with the Buhmann et al schematic diagram. The second addition, also very recent, is the experimental study of Chen et al. on microwave resonance measurements which establishes that a pinned crystal is the ground-state at both $\nu = 1$ and 2. Some explanations about their experiment are offered below. These two new contributions will here be embodied in a proposal to refine and extend somewhat the phase diagram of Buhmann et al. In the course of such refinement, we shall want to consider further phases besides the Wigner solid: a ‘Hall’crystal (HC) and a ‘Skyrme’ crystal (SC). Beginning with the HC state, two of us (A.C and F.C.) have focussed on the filling factor $\nu = 1/3$. Their starting point was to build on the charge density wave states (CDW) found in early work by F.C using the Hartree-Fock (HF) approximation. As $\nu$ is varied, the energy per particle of such states presents cusps at all fillings with odd denominator revealing a gap, while they behave as metals at all even denominators. Cabo and Claro argued that in addition the HC shows strong cohesive energy determined by cooperative rings of exchange effects. The second order correlation correction turned out sufficiently strong or the energy of such filling factors to go below that of the Yoshioka-Lee HF approximation for the Wigner solid (WS). The source of the intensity of these cohesive correlations is here illustrated in Fig. 1.
where the form of the electron density of the HC state is shown in comparison with the one corresponding to the Yoshioka-Lee state.

As can be observed, the channel like regions connecting the density maxima imply that the Wannier like localized states of the problem should have appreciable overlap. This is in contrast with the structure of the density in the Yoshioka-Lee CDW that shows rather isolated gaussian-shaped peaks with no channels between them. Therefore, the contributions of the cooperative rings of exchange effects can be expected to produce substantial cohesive effects. To be fully operative they require long paths and will not be properly accounted for in numerical calculations involving small samples and very few electrons. Although a precise evaluation of the total energy is needed for a definitive conclusion to be extracted, at the present state of understanding the above characteristics of these HC states lead us to propose them here as feasible candidates for the ground state at \( \nu = \frac{1}{3} \). The investigation of this question will be considered in detail elsewhere.

Besides the above implication that the HC is a strong candidate for the ground state at \( \nu = \frac{1}{3} \), we have attempted in Fig. 2 to refine and extend the proposed phase diagram of Buhmann et al.\(^1\) in the light of these findings. We show \( 0 < \nu \leq \frac{1}{3} \) in the main panel and \( \nu \) from \( \frac{1}{3} \) to 2 in the inset. Although still very schematic, we think the HC phase may come into its own in the three ‘solid’ reentrant phases shown, and have then sketched further the phase boundaries between the HC and WS states. In the inset, it is relevant to add the ground states: both WS near \( \nu = 1 \) and 2. It should be remarked that an earlier proposal was made for n SC around \( \nu = \frac{11}{12}, \frac{13}{12} \). While we in no way rule out the possible appearance of an SC phase elsewhere in the schematic phase diagram of Fig. 2, Chen et al.\(^8\) argue conclusively that, since Skyrmions do not exist at \( \nu = 2 \) and their microwave resonances show major similarities around \( \nu = 1 \) and \( \nu = 2 \), the Skyrmion crystal is not the explanation for their data near \( \nu = 1 \). They present conclusive evidence that the microwave resonance is caused by the pinning mode of a crystalline phase.

In connection with \( \nu = 1 \) and 2 we would like to also include here some comments related with the detection of WS like states in the work of Chen et al.\(^8\) They find that the magnetic field region in which the microwave absorption shows the peaks are well inside the plateaus\(^11\). Then, the question about the compatibility of the crystalline nature of the ground state and the quantized value of the Hall conductivity emerges. We just want to point out in this note that the occurrence of such an effect is coherent with the results of the work of Cabo and Martinez.\(^18\) It is then argued that the linear response of a 2DEG in the integral quantum Hall regime can be roughly described as ‘forcing’ the impurities to act as effective ‘charge reservoirs’, receiving or releasing the exact amount of electrons to ensure the local integral filling condition in large sample areas. Therefore, the following picture seems a plausible way to explain the findings of Chen et al.\(^8\) First, when the filling factor is near the values \( \nu = 1, 2 \) for a very clean sample, it can be expected that the ‘charge reservoirs’ accumulating the excess or defect charges are the small number of imperfections present in the clean sample. However, when \( B \) deviates more, i.e., at \( \nu = \frac{8}{9}, \frac{10}{9} \), it could happen that the limited quantity of localization centers in the high quality sample employed are saturated to their capacity. Therefore, a next imaginable step for the system to continue localizing defect (excess) charges is to start situating them in pinned crystalline regions which could be viewed as a system of dynamically generated localization centers.

Returning to the proposed phase diagram in Fig. 2, we notice that near \( \nu = 1/q \) the form shown suggests melting of the HC, but to a solid-solid phase transition further away. If our proposal proves to be useful, the WS is always the high temperature phase of those solid-solid transitions since correlations in favor the HC state. Due to the thermodynamic result \(1\), which of course applies everywhere to the phase boundaries shown in the figure, assuming that they are all representing first-order phase transitions the magnetization \( M \) and the entropy \( S \) of the HC are subjects well worth further investigation. For the CF liquid, \( M_l \) and \( S_l \) were treated at some length more than a decade ago by Lea, March and Sung.\(^4\)
FIG. 2: Phase boundaries in the $T$ vs $\nu$ plane. WS is the Wigner Solid phase, HC the Hall crystal, and CF indicates the composite fermion liquid. The inset refers to the segment $[1/3,2]$ of the filling factor.

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