Demand Shaping in Cellular Networks

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Abstract—Demand shaping is a promising way to mitigate the wireless cellular capacity shortfall in the presence of ever-increasing wireless data demand. In this paper, we formulate demand shaping as an optimization problem that minimizes the variation in aggregate traffic. We design a distributed and randomized offline demand shaping algorithm under complete traffic information and prove its almost surely convergence. We further consider a more realistic setting where the traffic information is incomplete but the future traffic can be predicted to a certain degree of accuracy. We design an online demand shaping algorithm that updates the schedules of deferrable applications (DAs) each time when new information is available, based on solving at each timeslot an optimization problem over a shrinking horizon from the current time to the end of the day. We compare the performance of the online algorithm against the optimal offline algorithm, and provide numerical examples to complement the theoretical analysis.

Index Terms—Demand shaping, offline algorithm, online algorithm, steepest descent algorithm, supermartingale, deferrable applications, cellular networks.

I. INTRODUCTION

We have witnessed in recent years rapid increase in demand for wireless data, driven by the proliferation of smart mobile devices. The global mobile traffic in 2016 has nearly reached 84 exabytes, more than 80 times greater than the entire global Internet traffic in 2000; yet, this number is expected to be increasing at a compound annual growth rate (CAGR) of 47% in the coming five years, i.e., a seven-fold growth from 2016 to 2021 [20]. However, despite frequent upgrades of cellular networks technology from 2G to 4G LTE and beyond, wireless service providers fall short of keeping up with this increasing wireless data demand, leading to congestion in the network, especially in areas of dense population. As a result, users’ data rates have to be throttled to ease congestions [2], [6], [9], at the cost of the degraded quality of service (QoS).

Admittedly, the capacity shortfall of cellular networks can be mitigated by allocating more wireless spectrum and deploying more wireless infrastructures including more and smaller cells and WiFi networks offloading, etc. However, spectrum allocation and infrastructure upgrading are not only costly but also time-consuming, while WiFi networks may not always be available and secure. A promising alternative, inspired by the similar problem of demand response in power networks, is to improve spectrum and infrastructure efficiency through managing wireless data traffic (i.e., demand). Notice that wireless traffic or demand usually fluctuates with a large peak-to-valley ratio throughout a day; see Fig. 1 for a trace of smartphone web browsing activity over a day. However, wireless capacity needs to be provisioned to meet the peak demand rather than the average. This means that the cellular network is usually stressed in peak hours while largely underutilized at other times. If the demand profile can be shaped to reduce the peak and smooth the time variation, not only can more traffic be accommodated under limited existing capacity constraints, but also additional spectrum allocation and infrastructure upgrades can be slowed down, which together greatly improve wireless network efficiency and QoS, and yield huge savings for service providers.

In this paper, we focus on designing demand shaping algorithms for cellular networks. We divide wireless traffic into two categories: non-deferrable traffic and deferrable traffic. Non-deferrable traffic refers to the traffic of those applications such as online gaming that have no or low delay tolerance, and constitutes the base traffic whose profile cannot be shaped. Deferrable traffic refers to the traffic of those applications such as file uploading/downloading that are flexible in time and only require being served by a designated deadline, e.g., finishing photo backup on cellphone by 12 am. Deferrable applications (DAs) are further divided into two major types: (1) continuous-rate interruptible applications such as photos backup and applications update that allow any data rates—e.g., the delayed offloading in [27], [30], and (2) discrete-rate non-interruptible applications such as online movie streaming and video conference that usually require certain constant data rate [3], [4] and should not be interrupted once started, e.g., one can schedule movie watching or video conference to the “valley” time to enjoy better graphic quality and incur less data cost if he/she has the time flexibility. See Table I for a summary of traffic types and examples. We seek to schedule the deferrable traffic to flatten the aggregate traffic profile over a day.

Specifically, we formulate the cellular traffic demand shaping as an optimization problem that minimizes the (time)
TABLE I: Traffic/Application types and examples.

| Traffic/Application Type                | Examples                               |
|----------------------------------------|----------------------------------------|
| Non-deferrable application             | Online gaming, web browsing            |
| Discrete-rate non-interruptible DA     | Movie streaming, video conference      |
| Continuous-rate interruptible DA       | Applications update, photos backup     |

variation in the aggregate traffic profile subject to the time and rate specification on each DA. We first assume complete traffic information and design an offline demand shaping algorithm. There are two challenging issues in the offline algorithm design. First, the optimization problem is non-convex because of discrete-rate non-interruptible applications. We instead solve its convex relaxation and design a randomized scheme based on the solution to the relaxed problem. Second, demand shaping involves potentially a huge number of applications and users. A centralized algorithm is not scalable. We instead design an iterative and distributed algorithm based on the descent method. We establish the almost surely convergence for the algorithm based on supermartingale theory.

We then consider a more realistic setting with incomplete information where we can only predict future traffic to a certain degree of accuracy, and design an online and distributed demand shaping algorithm that updates the schedules of DAs each timeslot when new information and updated prediction are available, based on the offline algorithm for an optimization problem over a shrinking horizon from the current time to the end of the day. We compare the performance of the online algorithm against the optimal offline algorithm, and provide numerical examples to complement the theoretical analysis.

The rest of the paper is organized as follows. Section II briefly reviews some related work and discuss some related issues. Section III describes the system model and problem formulation. Section IV presents an offline distributed algorithm for demand shaping under the assumption of complete traffic information and characterizes its performance. Section V considers a realistic setting of incomplete traffic information, and presents an online algorithm for demand shaping. Section VI provides numerical examples to complement theoretical analysis, and Section VII concludes the paper.

II. RELATED WORK AND ISSUES

Demand shaping in cellular networks is similar to demand response in power networks, in terms of design objectives, problem formulation, and the associated algorithmic challenges. Indeed, we borrow insights from demand response in power networks; see, e.g., [12], [14], [15], [29]. In particular, our online demand shaping algorithm is motivated by the solution approach for online control of continuous load in reference [15], and mathematically can be seen as its extension to incorporate discrete decision variables considered in reference [14]. However, our model captures realistic cellular traffic setting, as it includes both continuous and discrete decision variables. Moreover, the integration of discrete decision variables into the online algorithm makes the performance analysis of the algorithm more challenging, compared to that in [15]. Related work also includes Zhao et al [38] that designs a centralized online EV charging algorithm to minimize the peak procurement from the grid under uncertain prediction of future demand and renewable energy supply, and Parise et al [31] that proposes a decentralized charging control for EVs to flatten the aggregate power demand profile. They all consider only continuous decision variables.

To ease the stress from high demand in cellular networks, various demand-shaping-based methodologies as well as traffic offloading strategies have been studied in existing literatures. Tadrous et al in [36] propose a paradigm to proactively serve peak-hour requests during the off-peak time based on prediction to smoothen the traffic demand over time without changing customers’ activity pattern. However, such strategy is limited to routine behaviors only. In [19] Hajiesmaili et al introduce an online procurement auction framework to incentivize mobile devices to participate in device-to-device load balancing to offload traffic from one heavy-loaded base station to adjacent idle ones. Besides, WiFi and femtocell offloading of cellular data is another major approaches to easing the congestion of cellular networks; see [10], [13], [22], [26], [27], [30] for related works.

In this paper we have focused on designing demand shaping algorithms based on a general and simplified system model. We do not investigate the important practical issues such as the timescale and granularity at which we schedule and reschedule the DAs. We plan to develop a platform to enable automatic demand shaping in the future, and will investigate various practical issues then. Also, demand shaping involves not only the design of control algorithms but also the design of right mechanisms to incentivize the users to move out of their “comfortable zone” in wireless applications and data usage. Incentive design for demand shaping is currently an active research area; see, e.g., the smart data pricing in wireless networks [18], [35], [37]; pricing design in general network service to remove congestions [23], [32], pricing/reward signals in power distribution system [28], [40], and the references therein.

Some discussion on the practicality of demand shaping is also in place. People tend to use mobile data services whenever they want, regardless of whether it is at peak time or valley time for the cellular network. However, a survey [17] conducted in India and USA in 2012 shows that, given proper monetary incentive, many people are willing to postpone their mobile data usage, with acceptable postponement varying from minutes to hours, depending on different types of services and different individual preferences [18]. For example, wireless service providers can motivate the users to shift their demand by implementing the time-dependent pricing (TDP) strategy. TDP is now applied as a simple two-period plan by many wireless service providers around the world, in voice services and data services; e.g., Verizon [8] and Sprint [5] in the US have “happy hours” in the night and weekend for voice service, TelCom [7] in South Africa has “Night Surfer” plans giving free data from 11pm to 5am, and Airtel [1] in India provides unlimited data in the night. More refined TDP strategies can be applied to maximize benefits for both wireless service providers and users, by dynamically adjusting prices according to the data usage of the current time and predicted future. For instance, Ha et al [18] have worked on a TDP-based application named TUBE. Trials in cooperation with a local
TABLE II: Main notation.

| Symbol | Description |
|--------|-------------|
| $t$    | time index, $t \in T := \{1, \ldots , T\}$ |
| $n$    | DA index, $n \in N := \{1, \ldots , N\}$ |
| $N'$   | set of $N'$ continuous DAs |
| $N''$  | set of $N'' = N - N'$ discrete DAs |
| $N_0''$ | set of discrete DAs started earlier |
| $N_1$  | set of DAs adjustable at time $t$ |
| $b$    | base traffic profile, $b_0 = \{b(t); t \in T\}$ |
| $p_n$  | data rate profile of DA $n$, $p_{n_0} = \{p_n(t); t \in T\}$ |
| $p_n(t)$ | upper bounds of DA $n$ on the data rate at time $t$ |
| $r_n$  | constant bit rate for DA $n \in N''$ |
| $l_n$  | number of timeslots to finish transmission for DA $n \in N''$ |
| $q$    | virtual deferrable traffic profile |
| $d$    | average traffic profile |
| $\bar{d}$ | average traffic profile of online ODS |
| $\tilde{d}$ | average traffic profile of offline ODS |
| $P_a$  | total traffic required for DA $n$, $P_n = \sum_{t \in T} p_n(t)$ |
| $\bar{p}_n(t)$ | remaining traffic to be served for DA $n \in N'$ |
| $x_n(t)$ | change in traffic profile of DA $n$, $x_n(t) = p_n(t) - \bar{p}_n(t)$ |
| $t_n$  | arrival time of DA $n$ |
| $\tau_n$ | deadline of DA $n$ |
| $A_n$  | number of feasible profiles of DA $n \in N''$ |
| $f_{\alpha,a}$ | $\alpha$-th feasible profile of DA $n \in N''$ |
| $u_{\alpha,a}$ | probability corresponding to $f_{\alpha,a}$ |
| $F_n$  | set of all feasible traffic profiles for discrete DAs, $F_n = \{f_{\alpha,a}; 1 \leq \alpha \leq A_n\}$ |
| $V(d)$ | objective value: (time) variance of $d$ |

We assume that $\delta b(t)$ has a mean of 0 and variance of $\delta^2(t)$, and may be temporally correlated. We further assume that we can make better prediction for the timeslots that are closer to current time, modeled by a time-dependent deviation from the mean, i.e., the base traffic at some future time $\tau \in T$ is predicted at current time $t$ by

$$\hat{b}_n(\tau) = \bar{b}_n + \delta b_n(\tau),$$

(1)

where the subscript $t$ represents the timeslot when the prediction is made, and $\delta b_n(\tau)$ has a decreasing variance $\delta^2(\tau)$ as $\tau$ approaches $\tau$. More concrete model for prediction will be introduced in Section VI. The parameters $\bar{b}$ and $\delta$ will be specified exogenously, and can be estimated from the historical traffic records.

B. Deferrable Applications

Assume that there are $N$ DAs in the network, indexed by $n \in N = \{1, \ldots , N\}$. Each DA $n$ is characterized by an arrival time $t_n$ when it is requested or after which it can be started, a deadline $t_n$ by which its transmission must be done, and certain requirement or constraint on data rate $p_n = \{p_n(t); t \in T\}$. Let $P_n$ denote the total traffic required by DA $n$, i.e., $\sum_{t \in T} p_n(t) = P_n$. We can classify DAs into two main categories: continuous-rate interruptible DAs (or continuous DAs for simplicity) that allow any data rates between certain upper and lower bounds and can be interrupted and resumed at any time before the deadline, and discrete-rate non-interruptible DAs (or discrete DAs for simplicity) that require certain (roughly) constant data rate and cannot be interrupted once they are started. For example, system backup is usually interruptible and allows any continuous data rates, while video conference is usually preferred to be non-interruptible and runs at a constant (thus discrete) data rate once it is started.

Among the total $N$ DAs, we assume there are $N'$ continuous DAs, indexed by $n \in N' = \{1, \ldots , N'\}$. For each continuous DA, denote by $p_{n_0}(t)$ and $\bar{p}_n(t)$ the lower and upper bounds on its data rate at time $t \in T$, i.e.,

$$p_n(t) \leq \bar{p}_n(t) \leq p_n(t), \quad t \in T.$$  

(2)

Naturally, $0 \leq p_n(t) \leq \bar{p}_n(t)$. The lower bounds $p_n(t)$ are usually zero, and the upper bounds $\bar{p}_n(t)$ can be set according to, e.g., the available bandwidth. The arrival time $t_n$ and the deadline $t_n$ can be integrated into the rate constraints (2) by setting $\bar{p}_n(t) = 0$ for $t < t_n$ and $t > t_n$, i.e., no traffic is transmitted before arrival time or after deadline.

Index the rest $N'' = N - N'$ discrete DAs by $n \in N'' = \{N' + 1, \ldots , N\}$. For a discrete DA such as a streaming application, a constant bit rate $r_n$ corresponds to a certain graphic quality, e.g., $r_n = 3$ Mbps for a SD quality movie on Netflix [4], and $r_n = 1.2$ Mbps for a HD video call on Skype [3]. As the graphic quality usually (preferably) does not change during those applications, this seemingly over-simplified assumption of a single discrete rate is reasonable.

For each DA $n \in N''$ with its total traffic $P_n$ and the rate $r_n$, it takes $l_n = P_n/r_n$ consecutive timeslots (or equivalently the other way around, i.e., we calculate $P_n = l_n \ast r_n$ based on $l_n$ and $r_n$). Therefore, the number of its feasible traffic...
profiles is \( A_n = t^d_n - t^n_n - l_n + 1 \), wherein the \( a \)-th feasible profile is denoted as
\[
    f_{n,a} = \{ p_n | p_n(t) = \begin{cases} r_n, & \text{if } t^n_n + a - 1 \leq t \leq t^d_n + a + l_n, \\ 0, & \text{otherwise} \end{cases} \}.
\]
We denote the set of all feasible traffic profiles of DA \( n \in \mathcal{N}'' \) by \( \mathcal{F}_n = \{ f_{n,a} : 1 \leq a \leq A_n \} \), i.e., \( p_n \in \mathcal{F}_n \), \( \forall n \in \mathcal{N}'' \).

Remark 1: All the modeled traffic parameters can be reasonably accessed or estimated in practice. For example, information regarding total required traffic \( P_n \) and video streaming rate \( r_n \) is available from metadata of traffic to be transmitted, parameters like \( t^n_n \) and \( t^d_n \) are specified by the users in advance (and \( \mathcal{F}_n \) can then be calculated accordingly), whereas data rate bounds \( p_n(t) \) and \( p_n(t) \) can be either determined by available bandwidth or designated by the users. See, e.g., [18] for an example system involving similar information requirement and implemented with real users and service provider.

C. Problem Formulation

We aim to schedule the traffic of DAs, so as to flatten the aggregate traffic profile as much as possible. Denote the “average” traffic profile by \( d = \{ d(t); \ t \in \mathcal{T} \} = \frac{1}{\mathcal{T}} (b + \sum_{n \in \mathcal{N}} P_n) \). Traffic flattening can be achieved by minimizing the time variance of \( d \), formulated as the following optimal demand shaping (ODS) problem:

**ODS:**

\[
\begin{align*}
    \min_{p,d} & \quad V(d) = \frac{1}{\mathcal{T}} \sum_{t \in \mathcal{T}} (d(t) - \frac{1}{\mathcal{T}} \sum_{\tau \in \mathcal{T}} d(\tau))^2, \\
    \text{s.t.} & \quad d(t) = \frac{1}{\mathcal{N}} \sum_{n \in \mathcal{N}} p_n(t), \quad t \in \mathcal{T}, \\
    & \quad \sum_{t \in \mathcal{T}} p_n(t) = P_n, \quad n \in \mathcal{N}^\prime, \\
    & \quad p_n(t) \leq p_n(t) \leq p_n(t), \quad t \in \mathcal{T}, \quad n \in \mathcal{N}''. \\& \quad p_n(t) \in \mathcal{F}_n, \quad n \in \mathcal{N}''.
\end{align*}
\]

Notice that the constraints (3e) for discrete DAs are non-convex. In next section, we will investigate an offline algorithm together with a randomized solution for solving the ODS problem under the assumption of complete information on the base traffic and DAs. Then in Section V, we will study an online algorithm for demand shaping under a more realistic setting of incomplete information where we can only predict the future traffic to a certain degree of accuracy. The offline ODS problem and algorithm will later serve as a benchmark to characterize the performance of the online algorithm.

IV. OFFLINE DEMAND SHAPING ALGORITHM

In this section, we assume complete traffic information, i.e., the base traffic and arrival of DAs are accurately known, and study how to solve the resulting offline ODS problem. The offline problem and algorithm will provide insights into the online algorithm design for realistic setting of incomplete information that will be considered in Section V.

A. Convex Relaxation and Randomized Scheme

The offline ODS problem is non-convex, as each discrete DA has to pick a traffic profile from a discrete set; see constraint (3e). Consider the convex hull of \( \mathcal{F}_n \), defined as
\[
\begin{align*}
    \text{conv} & \quad (\mathcal{F}_n) := \{ p_n | p_n = \sum_{a=1}^{A_n} u_{n,a} f_{n,a}, \ u_{n,a} \geq 0 \}
    \quad \text{and} \quad \sum_{a=1}^{A_n} u_{n,a} = 1 \}.
\end{align*}
\]

where \( u_n := \{ u_{n,1}, \ldots, u_{n,A_n} \} \) is the convex combination coefficients, and will be interpreted as probability distribution in the randomized algorithm to be introduced soon. We will instead solve the convex relaxation of the ODS problem by replacing (3e) with the following constraint:
\[
p_n \in \text{conv}(\mathcal{F}_n), \quad n \in \mathcal{N}''.
\]

We call the relaxed problem (3a)-(3d)-(5) the R-ODS problem. However, a solution \( p^*_n \in \text{conv}(\mathcal{F}_n), \ n \in \mathcal{N}'' \) to the R-ODS problem might not be feasible, i.e., \( p^*_n \notin \mathcal{F}_n \). But since by definition (4), a solution \( p^*_n \) can always be written as the convex combination \( \sum_{a=1}^{A_n} u_{n,a} f_{n,a} \) we will randomly pick a traffic profile \( p_n = f_{n,a} \in \mathcal{F}_n \) with corresponding probability \( u_{n,a} \).

That said, we will design a randomized algorithm for the offline ODS problem, based on the solution to the R-ODS problem. We will integrate it into a distributed algorithm next.

B. Distributed Algorithm

Solving the R-ODS problem (and the offline ODS problem) directly in a centralized way requires collecting information on all DAs, which may incur too much communication overhead and is impractical in the real network. Moreover, the users may not be willing to reveal information on DAs due to privacy concern. Therefore, we seek to solve it in a distributed way. Noticing that R-ODS problem has decoupled constraints, we attempt to design an iterative and distributed algorithm based on the decent method [11].

Before deriving the algorithm, we establish the following useful results. At \( k \)-th iteration, let \( p^k = \{ p^k_n; \ n \in \mathcal{N} \} \) be the traffic profiles of all DAs, \( d^k = \frac{1}{\mathcal{N}} (b + \sum_{n \in \mathcal{N}} p^k_n) \) the average traffic profile, and \( x^k_n = p^k_n - p^k_n \) the change in traffic profile of DA \( n \) from iteration \( k \) to \( k+1 \). We have:
\[
\begin{align*}
    E\{ \| \sum_{n \in \mathcal{N}} x^k_n \|_2^2 \} &= \sum_{n \in \mathcal{N}} \text{Var}(x^k_n) + \| \sum_{n \in \mathcal{N}} E[x^k_n] \|_2^2, \\
\end{align*}
\]

where the variance \( \text{Var}(x^k_n) := E\{ \| x^k_n \|_2^2 \} - \| E[x^k_n] \|_2^2 \), and \( E[\cdot] \) denotes the average. By Jensen’s inequality,
\[
\| \sum_{n \in \mathcal{N}} E[x^k_n] \|_2^2 \leq N \sum_{n \in \mathcal{N}} \| E[x^k_n] \|_2^2.
\]

Therefore, one has
\[
\sum_{n \in \mathcal{N}} \text{Var}(x^k_n) \leq \sum_{n \in \mathcal{N}} \text{Var}(x^k_n) + N \sum_{n \in \mathcal{N}} \| E[x^k_n] \|_2^2.
\]

And it follows that
\[
TN^2 (E[V(d^{k+1})] - V(d^k)) = E\{ \| \sum_{n \in \mathcal{N}} x^k_n \|_2^2 + 2(Nd^k, x^k_n) \}
\leq \sum_{n \in \mathcal{N}} \text{Var}(x^k_n) + N \sum_{n \in \mathcal{N}} \| E[x^k_n] \|_2^2 + 2E[(N d^k, x^k_n)]
\]

Notice that we consider a randomized scheme only for discrete DAs. That said, for continuous DAs there is no randomness and their variance is zero.
Denote by $\Pi$ the first term in (9) and $W_2$ the second. For $n \in \mathcal{N}$, we choose $p_{n,k}^{k+1}$ so as to minimize $W_1$, i.e., to solve

$$
\min_{p_n} \ 2(d^k, p_n - p_n^k) + \|p_n - p_n^k\|_2^2
$$

s.t. (3c) - (3d).

On the other hand, after some mathematical manipulations, we have

$$
W_2 = \sum_{n \in \mathcal{N}} \left(2(Nd^k - p_n^k, E[p_n^{k+1}]) + (N - 1)\|E[p_n^{k+1}]\|_2^2\right) + \Pi_k,
$$

where $\Pi_k$ is a constant given $p_{n,k}^k$. For $n \in \mathcal{N}''$, we choose $p_{n,k}^{k+1}$ so as to minimize $W_2$, i.e., to solve

$$
\min_{p_{n,k} \in \text{conv}(F_n)} 2(d^k, p_{n,k}^k - p_n) + \frac{N - 1}{N}\|p_n\|_2^2.
$$

In essence, what we have done is to maximize the expected incremental decrease in the objective value $V(d)$ at each iteration (i.e., steepest descent). This motivates a distributed demand shaping algorithm with the collaboration of a coordinator; see Algorithm 1. The wireless service provider can implement a logical coordinator at the base station.

**Algorithm 1 Offline Demand Shaping (Off-DS) Algorithm**

At $k$-th iteration:

1) Upon gathering traffic profiles $p_n^k$ from DAs, the coordinator calculates the average traffic profile $d^k = \frac{1}{N}(b + \sum_{n \in \mathcal{N}} p_n^k)$ and announces it to DAs (or the end users) over a signaling or control channel.

2) Upon receiving the average traffic profile $d^k$,

- DA $n \in \mathcal{N}$ updates its traffic profile by

$$
p_{n,k}^{k+1} = \arg\min_{p_n} \left\|p_n - p_n^k + d^k\right\|_2^2
$$

s.t. (3c) - (3d),

and submits it to the coordinator.

- DA $n \in \mathcal{N}''$ calculates the average traffic profile by

$$
p_{n,k}^{k+1} = \arg\min_{p_n \in \text{conv}(F_n)} \left\|p_n - \frac{N}{N-1}(p_n^k - d^k)\right\|_2^2,
$$

which is $p_{n,k}^{k+1} = \sum u_{n,a}^{k+1} f_{n,a}$, and then randomly chooses a traffic profile $p_n^{k+1} = f_{n,a}$ with probability $u_{n,a}^{k+1}$ and submits it to the coordinator.

The Off-DS algorithm is a distributed algorithm wherein each DA solves its own simple optimization problem based on its previous decision, the average traffic profile $d^k$, and local constraints, while the coordinator collects the proposed traffic profiles and updates the average traffic profile. Therefore, this algorithm is not only preserving privacy of the users, but also scalable and thus capable of quick response, which is crucial especially in real-time implementation in Section V.

The computational complexity of the Off-DS algorithm is estimated as follows for completeness. Given certain accuracy requirement $\epsilon > 0$ in the objective function value, the descent method requires $O(\log(1/\epsilon))$ iterations [11]. At each iteration, DAs solves an easy quadratic programming with a polynomial complexity of $O(T^{O(1)})$ [33]. On the other hand, the coordinator calculates the average traffic profile which requires $O(N)$ complexity each iteration. As a result, the Off-DS algorithm requires overall computational complexity of $O((N + T^{O(1)})\log(1/\epsilon))$.

Remark 2: For simpler expression, we use $p_n$ as the decision variable for DA $n \in \mathcal{N}''$ in algorithm design and analysis, while in real implementation, it is more convenient to use probability distribution $u_{n,a}$ as the equivalent decision variable. Also notice that, if there is no continuous DA, Algorithm 1 reduces to the stochastic algorithm in [14]. We expect that the solution approach—randomized algorithm based on the “steepest” descent method for the convex relaxed problem—that we lay out in Sections IV-A and IV-B will find broad application in designing efficient algorithms for optimization problems that involve both continuous and discrete decision variables.

**C. Convergence**

Before showing the convergence of the Off-DS algorithm, we first establish two useful relations. For each DA $n \in \mathcal{N}$, since $p_{n,k}^{k+1}$ solves the problem (10), we have the first-order optimality condition

$$
\langle p_{n,k}^{k+1} - p_n, p_n - p_{n,k}^k \rangle \geq 0
$$

for any feasible $p_n$. Set $p_n = p_n^k$ to obtain

$$
\langle d^k, p_{n,k}^{k+1} - p_n \rangle \leq -\|p_{n,k}^{k+1} - p_n^k\|_2^2.
$$

For each DA $n \in \mathcal{N}''$, recalling that $p_{n,k}^{k+1} = E[p_n^{k+1}]$, by the first-order optimality condition, we have

$$
\left\langle \frac{N}{N-1}(d^k - p_n^k) + p_{n,k}^{k+1}, p_n - p_{n,k}^{k+1} \right\rangle \geq 0
$$

for any feasible $p_n$. Set $p_n = p_n^k$ to get

$$
\langle N(d^k - p_{n,k}^{k+1} - p_n^k) \rangle \leq -((N - 1)\|p_{n,k}^{k+1} - p_n^k\|_2^2 + \langle p_{n,k}^{k+1} - p_n^k, p_{n,k}^{k+1} - p_n^k \rangle).
$$

Now, construct a filtration $\Sigma$ of the probability space $\{\Omega, \Sigma, P\}$, where the sample space $\Omega$ is the feasible set specified by the constraints (3c) - (3e), the $\sigma$-algebra $\Sigma_k = \Omega, k \geq 0$, and $P(\Sigma_k) = \{b(p_n^k - p_n^k), n \in \mathcal{N}''; u_{n,a}, 1 \leq a \leq A_n, n \in \mathcal{N}''\}$, i.e., determined by the $k$-th iteration of the Off-DS algorithm.

**Theorem 1:** The pair $(V(d), \Sigma^*)$ is a supermartingale.

**Proof:** First, notice that $V(d)$ is bounded from below. So, $E[\min\{0, V(d)\}] < \infty$. Second, applying relations (13) - (15) to equation (9), we obtain

$$
TN^2\left(\frac{E[V(d^{k+1})]}{p_k} - V(d^k)\right)
\leq \sum_{n \in \mathcal{N}} -N\|x_n^k\|_2^2 + \sum_{n \in \mathcal{N}''} (Var(x_n^k))
+ (-N + 2)\|E[x_n^k]\|_2^2 + 2(p_k, p_{n,k}^{k+1} - p_n^k))
\leq \sum_{n \in \mathcal{N}} -N\|x_n^k\|_2^2 + \sum_{n \in \mathcal{N}''} (-N + 1)\|E[x_n^k]\|_2^2
\leq 0,
$$

i.e., $E[V(d^{k+1})]/p_k \leq V(d^k)$. By definition, $(V(d), \Sigma^*)$ is a supermartingale [16].

Notice that $(V(d), \Sigma^*)$ is a nonnegative supermartingale. By the martingale convergence theorem [16], the following

$$
\sum_{n \in \mathcal{N}} (2(Nd^k, x_n^k) + N\|x_n^k\|_2^2) + \sum_{n \in \mathcal{N}''} (2(Nd^k, E[x_n^k])
+ N\|E[x_n^k]\|_2^2 + Var(x_n^k)) = W_1.
$$

...
result is immediate.

**Corollary 1:** $V(d^\infty) = \lim_{k \to \infty} V(d^k)$ exists almost surely, where $V(d^\infty)$ is some random variable. □

**Theorem 2:** Denote by $P^\infty$ an “equilibrium” distribution over traffic profiles that $(V(d), \Sigma^*)$ converges to. The support of $P^\infty$ is a singleton.

**Proof:** When $(V(d), \Sigma^*)$ converges, $E[V(d^{k+1})|p^k] = V(d^k)$. This requires $E[p^k] = \mathbb{E}[p^k]$, $n, n' \in \mathcal{N}$, $p^k_n = p^k_{n'} = p^k_n$, $n \in \mathcal{N}$, and $p^{k+1}_n = p^k_n$, $n \in \mathcal{N}$ for $(8)$, $(13)$, and $(15)$ to hold with equality. Notice that $p^{k+1}_n = p^k_n$, as different feasible traffic profiles of DA $n \in \mathcal{N}''$ are linearly independent. Thus, $p^{k+1}_n = p^k_n$, $n \in \mathcal{N}$. So, the support of $P^\infty$ contains only one point. □

Denote by $p^\infty$ an “equilibrium” traffic profile of the Off-DS algorithm, i.e., if $p^k = p^\infty$, then $p^{k+1} = p^\infty$. Obviously the set of equilibrium profiles is not empty, as an optimum of the offline ODS problem is an equilibrium. The following result follows immediately from Theorem 2 and Corollary 1.

**Theorem 3:** The Off-DS algorithm converges almost surely to an equilibrium traffic profile. □

By equations $(12)-(14)$, we have the following optimality conditions at equilibrium $p^\infty$: for any feasible $p_n$,

\begin{align}
&\langle b + \sum_{m \in \mathcal{N}} p^\infty_m, p_n - p^\infty_n \rangle \geq 0, \; n \in \mathcal{N}', \\
&\langle b + \sum_{m \neq n} p^\infty_m, p_n - p^\infty_n \rangle \geq 0, \; n \in \mathcal{N}''.
\end{align}

\begin{equation}
(16a, 16b)
\end{equation}

### D. Performance Analysis of the Offline Algorithm

We now characterize the performance of the Off-DS algorithm with respect to the relaxed problem R-ODS that has a better and usually unachievable performance than the ODS problem. Specifically, denote by $p^*$ the solution of the R-ODS, we bound the gap between the equilibrium of the Off-DS algorithm and the solution of the R-ODS problem: $G_{r}^\text{off} := V(d^\infty) - V(d^*)$, where $d^\infty = (b + \sum_{n \in \mathcal{N}} p^\infty_n)/N$ and $d^* = (b + \sum_{n \in \mathcal{N}} p^*_n)/N$. Denote by $G_{r}^\text{off} := (V(d^\infty) - V(d^*))/V(d^*)$ the relative gap achieved by the Off-DS algorithm.

**Theorem 4:** The gap $G_{r}^\text{off}$ is bounded as follows:

\begin{equation}
G_{r}^\text{off} \leq \frac{2}{TN^2} \sum_{n \in \mathcal{N}''} \|p^\infty_n\|^2_2.
\end{equation}

Moreover, the relative gap diminishes as the number $N''$ of discrete DAs increases, i.e.,

\begin{equation}
\lim_{N'' \to \infty} G_{r}^\text{off} = 0.
\end{equation}

**Proof:** For notational simplicity, let $c_d := \sum_{t \in T} d(t)/T$, which is a constant given the total amount of traffic. The objective value can be written as

\begin{align}
V(d) &= \frac{1}{T} \langle d - c_d \cdot 1 \rangle \|1\|^2_2 = \frac{1}{T} (\|b\|^2_2 + c_d^2 \|1\|^2_2 - 2 \langle d, 1 \rangle) \\
&= \frac{1}{T} (\|b\|^2_2 + Tc_d^2 - 2Tc_d),
\end{align}

where only the part $\|d\|^2_2$ contains decision variables. We can thus write the gap $G_{r}^\text{off}$ as

\begin{align}
G_{r}^\text{off} &= V(d^\infty) - V(d^*) = \frac{1}{T} (\|b\|^2_2 - \|d^*\|^2_2) \\
&\leq \frac{1}{T} (2\|d^\infty - d^*\|^2_2 + \|2d^\infty - d^*\|^2_2) \\
&\leq \frac{1}{T} (2\|d^\infty - d^*\|^2_2 + \|2d^\infty - d^*\|^2_2) \\
&\leq \frac{2}{TN^2} \sum_{n \in \mathcal{N}''} \|p^\infty_n\|^2_2,
\end{align}

where the second inequality follows from $(16)$. Note that $\|p^\infty_n\|^2_2$ is a constant for $n \in \mathcal{N}'$. Then the relative gap $G_{r}^\text{off}$ can be bounded as

\begin{equation}
G_{r}^\text{off} \leq \frac{2}{TN^2} \sum_{n \in \mathcal{N}''} \|p^\infty_n\|^2_2/V(d^*) = \frac{\sum_{n \in \mathcal{N}''} \|p^\infty_n\|^2_2}{\|b + \sum_{n \in \mathcal{N}'} p^\infty_n\|^2_2 + N^2(Tc_d^2 - 2Tc_d)},
\end{equation}

whose numerator increases linearly with $N''$ and denominator increases linearly with the square of $N''$. Equation $(18)$ follows.

**Remark 3:** It is worth noting that equation $(18)$ does not necessarily imply a monotone decreasing of $G_{r}^\text{off}$ with respect to $N''$. This can be seen from Fig. 6 in Section VI that does not show a decreasing $G_{r}^\text{off}$ when $N''$ increases.

### V. ONLINE DEMAND SHAPING ALGORITHM

In this section, we consider a realistic setting with incomplete information where we can only predict future traffic to a certain degree of accuracy, and study online demand shaping that makes decision based on the prediction of future traffic and updates the decision as new information becomes available.

A typical algorithm used in this setting is the receding horizon control; see, e.g., (25). However, as the objective function $(3a)$ does not have a nice additive structure, receding horizon control algorithm does not admit an easy analysis. We will instead extend a shrinking horizon control algorithm, which is used in (15) that studies mathematically the same problem with only continuous DAs, to include discrete DAs, and apply it to our online demand shaping (Online DS) problem.

**A. Online Algorithm**

We assume that the number $m_t$ of DAs arriving at time $t$ is randomly distributed with a mean $\lambda_t$ and variance $(\delta \lambda_t)^2$, and the total amount of traffic of each DA is randomly distributed with a mean $P$ and variance $(\delta P)^2$. Denote by $\mathcal{N}''_t = \{1, \cdots, N_t''\}$ the set of continuous DAs and $\mathcal{N}''_t = \{N_t' + 1, \cdots, N_t''\}$ the set of discrete DAs that have arrived by time $t \in T$, and let $\mathcal{N}_t' = \mathcal{N}''_t \cup \mathcal{N}''_t$ and $\mathcal{N}_t'' = \mathcal{N}_t - \mathcal{N}_t'$. Notice that we cannot reschedule the remaining traffic of a discrete DA that has already started. Denote by $\mathcal{N}_t' \subseteq \mathcal{N}_t''$
the set of discrete DAs that have not been started by time \( t \). For DA \( n \in \mathcal{N}_n^p \), denote by \( \mathcal{F}_n(t) = \{ f_{n,a} : 1 \leq a \leq A_n(t) \} \) the set of feasible traffic profiles at time \( t \). Let \( \mathcal{N}_i = \mathcal{N}_i^c \cup \mathcal{N}_i^p \) be the set of DAs whose profiles are still adjustable at time \( t \) (i.e., all the continuous DAs and the discrete DAs that have not started by time \( t \)).

At time \( t \), we make a prediction \( b_t(t : T) \) of base traffic for the rest timeslots of the day, and we also have the information on DA \( n \in \mathcal{N}_i \) and the expected total future deferrable traffic \( \sum_{\tau=t+1}^T P\lambda_\tau \). Following [15], we introduce a virtual deferrable traffic profile \( q(t : T) = \{ q(\tau); t \leq \tau \leq T \} \) with \( q(t) = 0 \) and \( \sum_{\tau=t}^T q(\tau) = \sum_{\tau=t+1}^T P\lambda_\tau \), to emulate the impact of the future deferrable traffic upon the current demand shaping decision. With the above setup, we aim to schedule and reschedule the DAs, so as to solve the following problem at each timeslot \( t \in \mathcal{T} \).

\[
\text{ODS}_i : \quad \min \ V(d) = \frac{1}{T} \sum_{t=1}^T \left( d(\tau) - \frac{\sum_{s=t+1}^T d(s)}{T-t+1} \right)^2 \tag{20a}
\]

over \( p(t : T), d(t : T), q(t : T) \)

s.t. \( d(\tau) = b_t(\tau) + q(\tau) + \sum_{n \in \mathcal{N}_i} P_n(\tau), \quad \tau \geq t, \quad (20b) \)

\( p_n(t), \quad \tau \geq t, \quad n \in \mathcal{N}_i \), \quad (20c)

\( \sum_{n = 1}^{\mathcal{N}_i} P_n(t) = P_n(t), \quad n \in \mathcal{N}_i \), \quad (20d)

\( q(t) = \sum_{\tau=t}^T P\lambda_\tau, \quad (20e) \)

where \( p(t : T) = \{ p_n(\tau) ; t \leq \tau \leq T, \quad n \in \mathcal{N}_i \} \), \( d(t : T) = \{ d(\tau); t \leq \tau \leq T \} \), and \( P_n(t) = P_n - \sum_{\tau=t+1}^T p_n(\tau), n \in \mathcal{N}_i \) is the amount of traffic to be served at or after time \( t \).

We can solve the ODS_1 problem at each timeslot the same way as we solve the offline ODS problem \( \mathcal{D}_i \), constituting an online demand shaping algorithm; see Algorithm \( \mathcal{D}_2 \) wherein the convergence (and computational complexity) of Step 2) can be established (and analyzed) in the same way as Algorithm \( \mathcal{D}_1 \).

### B. Performance Analysis of the Online Algorithm

We now characterize the performance of On-DS algorithm with respect to the result of Off-DS algorithm (i.e., the offline ODS problem) which serves as a benchmark. We will make the following assumptions to simplify the analysis and obtain insights into how uncertainties affect the performance of On-DS algorithm.

**Assumption 1:** The amount of deferrable traffic is large and flexible enough so that a valley-filling schedule exists at every time \( t = 1, \ldots, T \), i.e., there exists some constant \( C(t) \geq b_t(\tau), \forall \tau = t, \ldots, T \) such that

\[
Nd(t) = C(t) = \frac{1}{T-t+1} \left( \sum_{\tau=t+1}^T b(\tau) + \sum_{\tau=t+1}^T P\lambda_\tau + \sum_{n=1}^{\mathcal{N}_i} P_n(t) \right) \tag{21}
\]

**Remark 4:** Assumption \( \mathcal{D}_1 \) looks a strong assumption, and we do not have empirical evidence to support it as demand shaping has not being widely adopted in current cellular networks. However, with increasing penetration of deferrable traffic and users, this assumption expects to hold. One purpose of algorithm design as in this paper and incentive design as in [18] is to facilitate and incentivize wide adoption of demand shaping. On the other hand, valley-filling represents the scenario where demand shaping is most useful and presents a benchmark for the potential of demand shaping. Mathematically, it is very difficult to analyze the performance of the online algorithm under more general assumption than Assumption \( \mathcal{D}_1 \). However, notice that in numerical examples in Section \( \mathcal{V} \), we do not impose Assumption \( \mathcal{D}_1 \) while the results still fall into the bound specified in Theorem \( \mathcal{V} \).

**Assumption 2:** The base traffic prediction at \( t \) is modeled as the following causal filter

\[
b_t(\tau) = \hat{b}(\tau) + \sum_{s=1}^T e(s)f(\tau - s), \quad \tau = 1, \ldots, T, \quad (22)
\]

where \( e = \{ e(s) \}_{s=1}^\infty \) is an uncorrelated sequence of independent and identically distributed random variables with mean 0 and variance \( \sigma^2 \), and \( f = \{ f(\tau) \}_{\tau=-\infty}^{\infty} \) is the impulse

---

**Algorithm 2 Online Demand Shaping (On-DS) Algorithm**

At each timeslot \( t \in \mathcal{T} \):

1. Denote by \( p_n(t-1), n \in \mathcal{N}_i \) the schedules determined by time \( t - 1 \), and by \( \mathcal{N}_n^p \subseteq \mathcal{N}_n^p \) the set of discrete DAs that has been started before time \( t \). For each DA \( n \in \mathcal{N}_n^p \), set its schedule \( p_n(t : T) = \{ p_n(\tau); t \leq \tau \leq T \} \) as \( p_n(\tau) = p_n^{(t-1)}(\tau), t \leq \tau \leq T \).

2. Solve the ODS_t problem iteratively: at \( k \)-th iteration,

   a) Upon gathering traffic profiles \( p_n^{(k)}(t : T) = \{ p_n^{(k)}(\tau); t \leq \tau \leq T \} \) from DAs \( n \in \mathcal{N}_i \), the coordinator solves

\[
\min_{q(t+1: T) \in \mathcal{T}} \frac{T}{t+1} \left( b_t(\tau) + q(\tau) + \sum_{n \in \mathcal{N}_i} P_n(\tau) + \sum_{n \in \mathcal{N}_i} p_n^{(k)}(n, a) \right)^2
\]

s.t. \( (20f) \),

   to obtain a virtual deferrable traffic \( q^{(k)}(\tau); t \leq \tau \leq T \), and then calculates the average traffic \( d^{(k)}(\tau) = \frac{1}{\mathcal{N}_i} \sum_{n \in \mathcal{N}_i} (b_t(\tau) + q^{(k)}(\tau) + \sum_{n \in \mathcal{N}_i} P_n(\tau) + \sum_{n \in \mathcal{N}_i} p_n^{(k)}(n, a)) \) for \( t \leq t \) and announces it to DA \( n \in \mathcal{N}_i \) over a signaling or control channel.

   b) Upon receiving the average profile \( d^{(k)} \),

   - DA \( n \in \mathcal{N}_n^p \) obtains \( p_n^{(k+1)}(t : T) \) by

\[
\min_{p_n(t : T)} \left\| p_n(t : T) - \hat{p}_n(t : T) + d^{(k)}(t : T) \right\|_2
\]

s.t. \( (20e) - (20d) \),

   and submits the updated profile to the coordinator.

   - DA \( n \in \mathcal{N}_n^p \) calculates \( p_n^{(k+1)}(t : T) \) by

\[
\min_{p_n(t : T)} \left\| p_n(t : T) - \frac{\mathcal{N}_i}{\mathcal{N}_i} \hat{p}_n(t : T) - d^{(k)}(t : T) \right\|_2
\]

s.t. \( p_n(t : T) \in \text{conv}(\mathcal{F}_n(t)), n \in \mathcal{N}_n^p \),

   represents it as a convex combination \( p_n^{(k+1)} = \sum_{a=1}^{\mathcal{N}_n^p} u_n^{(k+1)} f_{n,a} \), and randomly chooses a traffic profile \( p_n^{(k+1)} = f_{n,a} \), with probability \( u_n^{(k+1)} \) and submits it to the coordinator.
response with \( f(0) = 1 \). Let \( F(t) := \sum_{s=0}^{t} f(s) \).

We characterize the sub-optimality \( G_{on} \) of the On-DS algorithm with respect to the offline ODS problem using the following metric: \( G_{on} = E[V(\hat{d}) - V(d^\infty)] \), where \( E \) denotes the expectation, and \( \hat{d} \) and \( d^\infty \) denote the average traffic profiles achieved by the On-DS algorithm and the offline ODS problem, respectively.

It turns out that direct calculation of this sub-optimality is difficult. We therefore utilize two intermediate variables: \( d^* \), the average traffic profile achieved by the R-ODS problem, and \( \hat{d}^* \), the average traffic profile achieved by the relaxed online DS, i.e., the counterpart of R-ODS problem.

Accordingly we can improve the result of On-DS algorithm by implementing better prediction mechanism, e.g., On-DS algorithm which updates its prediction to keep the value of prediction error small. Also, if the impulse response \( f \) is chosen to fade quickly enough, then as we have finer time granularity, we have \( T \to \infty \), and \( G_{on} \to 0 \), which intuitively indicates that, with infinitely small timeslot, we can update our decisions frequently enough to mitigate prediction errors, and therefore have negligible sub-optimality.

Lastly, similar to Theorem 4, define a relative sub-optimality \( G_{r_{on}} := G_{on}/V(d^\infty) \). The following result is immediate.

**Theorem 6:** The relative sub-optimality \( G_{r_{on}} \) diminishes as the number of discrete DAs \( N'' \) increases, i.e.,

\[
\lim_{N'' \to \infty} G_{r_{on}} = 0.
\]

VI. NUMERICAL EXAMPLES

In this section, we provide numerical examples to evaluate the performance of the On-DS algorithm. We use certain composite traffic traces to drive simulations to show the impact of base traffic prediction errors, deferrable traffic prediction errors, and deferrable traffic penetration levels. We expect the conclusions obtained to be hold for real traffic.

A. Experimental Setup

Consider a 48-hour period of time starting from 4:00 pm to 3:59 pm two days later. We divide the 48 hours equally into 96 timeslots, each 30 minutes long. We consider scheduling traffic that arrives within the first 24 hours only, which may be allocated to the second 24 hours.

1) Non-deferrable traffic: The “real” trace we use for non-deferrable traffic, or base traffic, is shown in Fig. 3 (red line). It is constructed by random fluctuation around the average base traffic trace (blue line) composed based on North American mobile web browsing activity by time of day in 2013 [21], shown in Fig. 3. As modeled in Section III-A, the prediction of base traffic follows (1), consisting of average base traffic \( \hat{b}(\tau) \) and random deviation \( \delta b(t) \) from the average value. Following [15], at time \( t \), \( \delta b(t) \) is modeled as

\[
\delta b(t) = \sum_{s=t+1}^{T} \omega_s(t), \quad t < \tau \leq T,
\]

where \( \omega_s(t) \) are random variables of Gaussian distribution with 0 mean and variances

\[
E[\omega_s^2(t)] = \frac{\sigma^2}{\tau - s + 1}, \quad 1 \leq s \leq t \leq T.
\]

In this way, \( \delta b(t) \) has decreasing variance as \( t \) approaches \( \tau \), simulating a gradually improving prediction for some future timeslot \( \tau \) as one gets closer to it. In simulation, we take the values of \( \sigma \) in (30) from 0 to 100 with increment of 10, corresponding to a root-mean-square prediction error (RMSE) ranging from 0% to 32%, looking 48 timeslots (24 hours) ahead.

2) Deferrable traffic: We assume that the number of DAs arriving at each timeslot follows a “shifted” Poisson process \( m + \text{poiss}(\lambda_p) \), with \( m \geq 0 \) and \( \text{poiss}(\lambda_p) \) denoting a Poisson process with rate \( \lambda_p \). Here, we set \( \lambda_p = 4 \), while each DA has a 50-50 chance to require continuous- or discrete-rate
traffic. The total traffic $P_n$ of each DA is uniformly distributed in $[P, \overline{P}]$ where we set $P = 12$ and $\overline{P} = 24$. The deadline for DA $n$ is uniformly distributed in $[t_n^a + l_n + D, t_n^a + l_n + \overline{D}]$, where $l_n = \lceil P_n/\overline{P}_n \rceil$ is the minimum number of timeslots required by the DA calculated by ceiling function $\lceil \cdot \rceil$. We set $D = 6$, $\overline{D} = 14$, and a universal bit rate upper bound $\overline{P}_n = 3$.

3) **Benchmarks for comparison:** We compare the performance of the On-DS algorithm with a few typical benchmarks to evaluate the impact of base traffic prediction error, the benefit of updating the prediction in real time, and the impact of deferrable traffic’s penetration level. We thus consider the followings five cases in our experiments:

0) **Offline demand shaping w/ Off-DS algorithm.** We use “real” trace for future base traffic and use arrival information recorded from case (1) below for DAs. Applied with Off-DS algorithm, this case gives the optimal performance used as benchmark to characterize the sub-optimality of other cases.

1) **Online demand shaping w/ On-DS algorithm.** We make prediction for both DAs’ arrival and base traffic in the future. Prediction is updated at each timeslot. We run On-DS algorithm to schedule traffic.

2) **Online demand shaping w/ exact information for base traffic and w/o exact information for DAs.** We use “real” trace for base traffic and prediction for DAs. We apply On-DS algorithm. Comparison of case (2) with case (1) shows the impact of uncertainty in base traffic.

3) **Demand shaping w/ updating prediction of base traffic and w/ exact information for DAs.** We use DAs arrival information recorded from case (1). Instead of applying virtual deferrable traffic, we schedule traffic profiles for all the future deferrable traffic. Since exact base traffic information is not available, we updated base traffic prediction at each timeslot. Comparison of case (3) and (1) shows the impact of uncertainty in DAs arrival prediction.

4) **Demand shaping w/o updating prediction of base traffic and w/ exact information for DAs.** We use prediction of the base traffic at the beginning ($t = 1$) without further updating, and use arrival information recorded from case (1) for DAs. This case shows how the online algorithm benefits from updating prediction at each timeslot.

We use the metric of relative sub-optimality $G_{r} = (V(d) - V(d^*)) / V(d^*)$ to evaluate the performance.

**B. Experiment Results**

Considering randomness in DAs’ arrivals, base traffic prediction, and deciding traffic profiles for discrete DAs, we run simulation for 10 times, and take the average as the final result to present.

1) **Convergence speed:** We first run a case of randomly generated 143 continuous DAs and 150 discrete DAs by Off-DS algorithm with different numbers of iterations ranging from 1 to 40 for 10 times. Because of the random process in choosing traffic profiles for discrete DAs, we observe oscillation in objective function values for each individual run. However, the oscillation has a trend of diminishing as the more iterations are implemented, with satisfying enough results generated from running 20 to 30 iterations. See Fig. 4 for the results. We will implement a number of 30 iterations for each decision to be made in the rest of simulation.

2) **Impact of base traffic prediction error:** As described in Section VI-A we can tune the variance $\sigma$ to emulate situations with different prediction errors in base traffic. As Fig. 5 shows, with updated prediction, case (1)’s performance is barely affected by the increasing prediction error, keeping its sub-optimality under 5%. This is almost as good as that of case (2) with perfect base traffic information. We can also see from the performance of case (3) the pure impact from prediction errors, while case (4) gives an example showing what happens if there is no updated prediction.

3) **Impact of penetration level of discrete DAs:** In this case, we fix the prediction error in base traffic at $\sigma = 40$ and the average number of DAs’ arrival at each timeslot at $\lambda = 4$. We then tune the penetration level of discrete DAs from 25% to 75% with granularity of 5%. As shown in Fig. 6 the sub-optimality maintain relatively unaffected by the changes of
We have formulated demand shaping in cellular networks as an optimization problem that minimizes the time variation in aggregate traffic subject to the rate and time requirements of the applications. We design a distributed and randomized offline demand shaping algorithm under complete traffic information and prove its almost surely convergence. We then consider a realistic setting with incomplete information where we are investigating to integrate the incentive mechanisms such as the smart data pricing into the demand shaping algorithm design. We also plan to develop a platform to enable automatic demand shaping in cellular networks and investigate the related practical issues.

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