Selection of third party reverses logistic providers: an approach of BCF-CRITIC-MULTIMOORA using Archimedean power aggregation operators

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Abstract

One of the most powerful tools to operate imprecision is bipolar complex fuzzy sets (BCFSs), which is an enlargement of bipolar fuzzy sets (BFSs) as well as complex fuzzy sets (CFSs). This paper deals with an integrated MULTIMOORA (multi-objective optimization on the basis of ratio analysis plus full multiplicative form) framework as a generalization of fuzzy MULTIMOORA procedure to assess the multi-criteria decision-making (MCDM) problems with BCFSs. We develop BCF-Archimedean power weighted (ordered weighted) arithmetic and geometric aggregation operators (AOs) and discuss their properties from this point of view. The proposed Archimedean power-weighted AOs can eliminate the influence of extreme evaluating criteria values from some biased experts with different preference attitudes under the BCF setting. Afterward, we put forward an integrated MULTIMOORA algorithm based on the proposed AOs, where criteria weights are estimated using the CRITIC (criteria importance through inter-criteria correlation) method, which is a well-known objective weighting method based on aggregated score values of options, intensity contrast of every criteria and conflict among attributes. In the proposed methodology, criteria values are aggregated based on the MULTIMOORA method that involves three sub-methods: the ‘ratio system’, the ‘reference point’ and the ‘full multiplicative form’ and thus takes less computational time, minimum mathematical evaluations and bears good stability. In the following, third-party reverse logistics providers’ (3PRLP) selection problem is brought into consideration to manifest the sufficiency of the developed methodology. At the end of this study, we draw attention to a comparison between the proposed decision-making approach with the corresponding BCF-CRITIC-TOPSIS and BCF-CRITIC-WASPAS methods.

Keywords Bipolar complex fuzzy set · BCF-archimedean power weighted aggregation operators · BCF-CRITIC-MULTIMOORA · Third-party reverse logistics providers selection

Abbreviations

| Abbreviation | Meaning |
|--------------|---------|
| RLS          | Reverse logistics |
| 3PRLPs       | Third party reverse logistics providers |
| FS           | Fuzzy set |
| IFS          | Intuitionistic fuzzy set |
| IVPHFS       | Interval-valued Pythagorean hesitant FS |
| BD           | Belongingness degree |
| ND           | Non-belongingness degree |
| BFS          | Bipolar fuzzy set |
| BF           | Bipolar fuzzy |
| CFS          | Complex fuzzy set |
| BCF          | Bipolar complex fuzzy set |
| BCFN         | Bipolar complex fuzzy number |
| DE           | Decision expert |
| MCDM         | Multi-criteria decision-making |

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To begin with reverse logistics (RLs), it is uniformly essential to distinguish what RL is and how it functions. Council of Logistics Management (CLM) illustrates ‘RLs’ as “to achieve the purpose of recycling value and proper disposal, a process from the point of consumption to the starting point in an efficient and economical way that plans, implements, and controls raw materials, semi-finished inventory, finished goods and related information [70].” In other words, RL is defined as the procedure of planning, executing, and controlling the efficient, cost-effective flow of raw materials, in-process stock, finished merchandise, and associated information from the point of expenditure to origin for the goal of suitable disposal [10]. Using rendering profit with second-hand products reinstatement and protecting the environment through reprocessing and appropriate disposal [68, 99], the RLs might show the avenue of a stable and compatible parity in between environmental and economic affairs.

The inherent behavior of RLs shows its viable effort [90]. The viability based components are covered up by RLs through reduction of prices economically, keeping save the atmosphere with 3R (reduce, reuse, recycle), disposal and other practices as an environmental aspect, and lastly, providing security to resources obtained from nature thinking about future generations socially [50, 69]. Consequently, RLs turn into an inexorable policy for production-based industries in the modern period [1]. Thus far, using this policy requires a skilled support panel, in addition, formation of both design and plan of a robust network together with mending and
charging professionals for keeping the network operational, owing to the complications associated with RLs performance [88, 99]. On account of quantifying difficulty, very often some of the criteria from a considerable large set of criteria, affects fixing the plan and design of a network, particularly reverse logistic in nature resulting it to become complicated and gradually becomes a vital job in case of any organization. As a result, several companies engaged themselves in outsourcing these logistics functions into learned third-party reverse logistics providers (3PRLPs) that in turn effectively reducing price and upraise the efficiency of regaining of second-hand products that are already dispatched for creating competition towards the advantages [34, 74]. In addition, a vital role by the 3PRLPs is being played to assist the organizations for the development and proper execution of the reverse supply chains on account of returns. Thus, the 3PRLP assessment decision can be a strategic critical partnering problem handled by processes and reverse supply chain executives’ sustaining managerial strategic competitive advantage. Accordingly, it is difficult for the companies to choose the most accessible 3PRLP among a set of provider alternatives by considering the desired evaluation criteria. Hence, the evaluation and selection of a desirable 3PRLP is a multi-faceted and complex decision-making task due to multiple qualitative and quantitative attributes [74]. The criteria involved in this process may fluctuate based on the type of considered item and often conflict with each other [59].

In recent times, the assessment of the 3PRLPs selection process has received great attention from the researchers. Numerous scholarly articles on the selection of the best 3PRLP alternative have been presented in the literature. However, more studies are required to manage the preferences of different expertise, different backgrounds, and knowledge levels on reverse logistics with considering social, environmental, and economic aspects simultaneously. Consequently, the present study is concentrated on introducing a novel decision-making method for 3PRLP selection under uncertain contexts. The concept of bipolar complex fuzzy sets (BCFSs) [7] is pioneered as an innovative tool to describe the bipolar nature in the lack of sureness and periodicity semantics by applying the BFSs range in the domain of complex geometry. In BCFSs theory, the amplitude term corresponding to membership (non-membership) degree gives the extent of belongingness (non-belongingness) of an object, and the phase term associated with membership (non-membership) degree gives the additional information, generally related with periodicity. BFS theory deals with only one dimension at a time, which results in information loss in some instances. On the other hand, CFS theory deals with the two-dimensional information of an object. However, in day-to-day life, we come across complex natural phenomena where it becomes essential to consider the two-dimensional and the bipolar information (positive and negative information) of an object. BCFSs [7] can efficiently deal with this situation. To illustrate the significance of BCFS, consider the example (adapted from [7]): "As indicated by Chinese devo-tees, all universe articles can be seen from the perspective of Yin and Yang components [23]. Many factors with the viewpoint of bipolarity have affected simultaneously with types of food Yin and Yang to have a fair body like our everyday exercises sitting actually is Yin, practice is Yang, our current circumstance a chilly climate atmosphere and a sleepy country town is more Yin, a more blazing atmosphere and busy city is Yang, and our level of profound mindfulness and change. Cousens [23] introduced that "the level of otherworldly mindfulness and change influences how much our brain is moved by the yan and yang energy of foods in a fairly unexpected manner in comparison to different elements influencing yin and yang". These kinds of data convey the bipolarity of uncertainty (food types) and the bipolarity of periodicity (day-by-day exercises, climate, or level of otherworldly mindfulness and change). The present circumstance can’t be displayed precisely utilizing CFS and BFS theory as none of them can deal with two factors at the same time used to find the ideal body balance in the Yin and Yang food framework. An ideal approach to speak to this is BCFS theory. Thus BCFS is more general compared to FS [97], BFS [103] and CFS [72]. At present, very few scholars have focused their attention on BCFSs. Based on its unique amenities, in this paper, our discussion encompasses the BCFS environment. It is clear from the literature that there has been no study on developing the integrated MCDM tool associating the CRITIC and MULTIMOORA approaches with BCFS information. Also, there has been no study in the literature regarding the developed hybrid approach, namely BCFS-CRITIC-MULTIMOORA, in assessing the 3PRLP selection process. The novel contributions are as follows:

- BCFS-Archimedean power AOs have been developed and their basic characteristics are surveyed.
- Novel integrated BCFS-CRITIC-MULTIMOORA methodology has been developed to deal with MCDM problems.
- To illustrate the feasibility and usefulness of BCFS-CRITIC-MULTIMOORA approach, an empirical case study of 3PRLP selection has been studied in the BCFSs setting.
- A comparative discussion has been deployed to show the strength of the introduced approach.

We summarize the remaining paper as follows: In “Literature review”, we give a concise literature review. In “Prerequisites”, we recall the definition of a BCFN and some related concepts such as score, accuracy value, ranking rules of the BCFSs, Archimedean operational laws for BCFSs and the definition of the power aggregation (PA) operator. In “BCF-Archimedean power weighted aggre-
gation operators”, we develop some BCF Archimedean power-weighted AOs, such as $BCFAPWAA$, $BCFAPOWA$, $BCFAPWGA$, and $BCFAPOWGA$. Also, we discuss the essential postulates of proposed operators. In “BCF-CRITIC-MULTIMOORA methodology for decision-making”, we develop a novel BCF-CRITIC-MULTIMOORA framework with CRITIC method and the proposed AOs where the criteria values take the form of BCFNs. In “Case study: 3PRLPs selection”, we deploy a case study on 3PRLPs selection to BCFNs. In “Comparative study” deals with the CRITIC method and the proposed AOs where the criteria values take the form of BCFNs. In “Case study: 3PRLPs selection”, we deploy a case study on 3PRLPs selection to BCFNs. Also, we discuss the comparative discussion to affirm the prevalence of the developed technique. In the end, in “Conclusions”, we make some conclusions upon this entire study and give an outline of future prospects.

Literature review

Here, a comprehensive review related to this study is presented.

Bipolar complex fuzzy sets

The doctrine of FSs, pioneered by Zadeh [97], has received huge interest from several authors in handling uncertainty in diverse fields. However, FSs cannot deal with complex problems as they only have a belongingness degree (BD). Next, Atanassov [9] developed the idea of IFSs, which is considered BD and non-belongingness degree (ND). Over the last few decades, various authors have initiated several kinds of algorithms to solve the MCDM problems by using the FSs and IFSs theories, but it has been observed commonly for the data assessment of an element that analogous to each postulate, there exists some counter postulate. To conquer this issue, Zhang [103, 104] pioneered BFSs, which consists of positive BD and negative BD. The positive BD lies in $[0, 1]$ and the negative BD lies in $[-1, 0]$. Zhang and Zhang [106] put forth the notion of bipolar logic and fuzzy logic to represent how quantum fields are merged with neural biology networks, equilibrium combines with bipolar disorder, and gets to know how especial hypothesis get united with brain and behavior. Alghamdi et al. [5] suggested MCDM techniques by reporting the BF concept. Akram and Arshad [2] initiated BF linguistic variables and BF numbers as a generalization of BFSs. The notion of BFSs has widely been applied in medical diagnosis, bipolar disorder, decision making, optimization, and others [3, 4, 40, 77, 78, 105, 15].

Ramot et al. [72] pioneered the concept of CFSs, characterized by a BD, whose limit is expanded to a circle with a unit radius in the complex plane in place of $[0, 1]$. The concept of extending the span of FS to a broader limit of CFS lies in its capacity to collect the semantics comprising the uncertainty and periodicity news altogether. Ramot et al. [71] gave an additional term named the phase term to handle the enigma in transforming some complex-valued functions on physical expressions to human language and vice versa. In Cartesian and polar structures, the membership grade for complex fuzzy may be expressed with two fuzzy components [85]. As a potent trick to establish the notion of BCFNs [7], the phase term of complex numbers (CNs) is taken into consideration.

The new idea of BCFS may be deployed to illustrate the imprecision and difficulty in periodicity of bipolar fuzzy messages in complex geometry in a combined manner. Firstly, Singh [79] suggested the bipolar complex fuzzy lattice ideas by its possible infliction to circumnavigate or decompose the BCFSs and their semantics by utilizing a demonstrative example. Alkouri et al. [7] studied the mathematical structure of BCFS and its applications. Al-Husban et al. [6] presented an overview of BCFS and its basic concepts.

CRITIC methods

In the process of MCDM, determining the criteria weights is a significant concern for DEs. The criteria weight determination approaches are divided into objective and subjective weights [65]. The CRITIC model, propounded by Diakoulaki et al. [25], is one of the weighting tools to determine the objective criteria weights. In this approach, with the help of the contradictory intensity of each criterion, known as standard deviation, criteria’s significance can be judged. In contrast, it is treated the controversy in between the criteria as the correlation coefficient among them. The basis of the CRITIC approach is the intensity of the contrast in the construction of decision-making issues [25]. Recently, few hybrid methods have been developed by combining CRITIC and many other MCDM approaches under uncertain environments. For example, Ghorabae et al. [30] suggested an integrated model with CRITIC and WASPAS approaches to assess the third-party logistics providers. Ghorabae et al. [29] designed a hybrid fuzzy MCDM framework based on the CRITIC, SWARA and EDAS methods. Peng et al. [67] presented an integrated Pythagorean fuzzy CRITIC and CoCoSo based methodology for 5G industry evaluation. Wei et al. [92] studied an integrated method by combining GRA and CRITIC approaches to evaluate and select the desired location for electric vehicle charging stations under probabilistic uncertain linguistic term sets the context. Peng and Huang [66] proposed a combined methodology by integrating CRITIC and CoCoSo approaches for financial risk evaluation. Liang [54] gave an MCDM method with the CRITIC and EDAS methods to gradually compute the attribute weights and the favor ordering of the alternatives.
MULTIMOORA method

An MCDM, a part of decision theory, is an act of selecting an ideal choice from a given set of decision variants. Due to the wide-spread changes and the development of socio-economic environment, real-world decision-making issues are becoming more and more complex. Over the last few decades, many new approaches have been proposed to deal with real-life MCDM problems, where each of them has its own advantages and limitations. The MOORA model, proposed by Brauers and Zavadskas [18], is an efficient and renowned MCDM method consisting of RS and RP models. To increase the robustness of MOORA model, Brauers and Zavadskas [19] pioneered the MULTIMOORA approach, which consists of three aggregation models with different functions: the RS method, the RP model, and FMF procedure. In comparison with AHP, TOPSIS, VIKOR, PROMETHEE, LINMAP, and ELECTRE, the MULTIMOORA approach has more superiority, easy mathematical expressions, less computation time, and strong robustness [17]. Due to its unique advantages over other MCDM methods, the classical MULTIMOORA method has been employed for various MCDM concerns [80, 94].

Further, to tackle uncertain information arises in MCDM problems, several extensions of MULTIMOORA have been introduced under diverse uncertain environments (see Table 1).

The 3PRLPs selection

A variety of criteria are involved in the evaluation of 3PRLPs selection procedure; accordingly, this selection process can be observed as an MCDM problem. Existing studies on the 3PRLP selection problem confirm the emergent interest of scholars and manufacturers. Over the last few years, copious MCDM models have been established in the setting of 3PRLP assessment problem. Realistic reverse logistics outsourcing assessments are commonly prepared under imprecise and

Table 1 Literature on MULTIMOORA approach in different environments

| Author and year | Benchmarks | Application |
|-----------------|------------|-------------|
| Brauers et al. [20] | F-MULTIMOORA approach | Selection of EU member states |
| Balezentis et al. [12] | F-MULTIMOORA approach | Personnel selection |
| Balezentis and Zeng [14] | IVF-MULTIMOORA approach | Personnel selection |
| Datta et al. [24] | Grey MULTIMOORA approach | Robot selection |
| Balezentis et al. [13] | Intuitionistic fuzzy MULTIMOORA approach | Case study of personnel management |
| Li [49] | Hesitant fuzzy MULTIMOORA approach | Software selection |
| Stanujkic et al. [82] | IVF-MULTIMOORA approach | Comminution circuit design selection |
| Zavadskas et al. [100] | IVIF-MULTIMOORA approach | An examples related to civil engineering problems |
| Hafezalkotob and Hafezalkotob [37] | Shannon entropy weighted MULTIMOORA | Materials selection |
| Hafezalkotob et al. [39] | Interval MULTIMOORA | Materials selection of power gears |
| Hafezalkotob and Hafezalkotob [38] | Interval MULTIMOORA | Biomaterials selection |
| Stanujkic et al. [83] | Neutrosophic MULTIMOORA | Comminution circuit design selection |
| Tian et al. [89] | SNL-MULTIMOORA | Enterprise resource planning system selection |
| Gou et al. [32] | DHHFL-MULTIMOORA | Assessment of air pollution control procedures for treating haze |
| Chen et al. [21] | Triangular fuzzy MULTIMOORA | Wastewater treatment evaluation |
| Maghsoodi et al. [58] | MULTIMOORA based on integrated Shannon’s entropy | Evaluation of performance appraisal |
| Liang et al. [52] | SNL-SWARA-MULTIMOORA approach | Case study of optimal mining method selection |
| Geetha et al. [28] | IHF-MULTIMOORA | Evaluation of HCW D technologies |
| Liang et al. [53] | IVPF-MULTIMOORA | Hospital open-source EHR system selection |
| Stanujkic et al. [81] | BF-MULTIMOORA | Location selection |
| Gündo˘gdu [36] | Spherical fuzzy MULTIMOORA | Personnel selection |
| Liao et al. [55] | HF-Choquet integral based MULTIMOORA | Talent selection |
| Dong et al. [26] | Hesitant fuzzy linguistic term set based MULTIMOORA | Assessment of the innovative ability of universities |
| Zhang et al. [102] | IF-MULTIMOORA approach | Evaluation of energy storage methods |
| Xian et al. [95] | I2TPFL-MULTIMOORA | Evaluation of financial management performances in universities |
vague environment due to multiple indicators, like as partial ignorance, imprecise estimation, partial or inaccessible decision information [16, 27]. Consequently, crisp values are usually unsuitable for modelling such types of practical decision conditions.

The FSs theory and their extensions have proven to be suitable tools to handle uncertain and vague information in realistic MCDM settings. Efendigil et al. [27] designed a two-way method by integrating fuzzy logic and artificial neural networks to assess an ideal 3PRLP option. A structured procedure with AHP on FSs was developed by Kannan [43] for evaluating the 3PRLP selection problem. Govindan and Murugesan [33] used the fuzzy extent assessment approach for choosing the desirable 3PRLP for a battery manufacturing industry. Senthil et al. [76] suggested a combined model with AHP and TOPSIS approaches for evaluating an ideal reverse logistics contractor. In a further study by Tajik et al. [84], a hybrid fuzzy decision-making framework was introduced for choosing the most suitable 3PRLP alternative by considering all three aspects of sustainability. Later, Uygun et al. [91] planned and selected an outsourcing provider for the plastics industry and further ranked the sustainable 3PRLPs through MOORA method within FSs context. Tavana et al. [86] suggested a combined method with the integration of ANP and grey superiority and inferiority methods on intuitionistic fuzzy sets to assess the 3PRLPs selection process. Li et al. [51] used a combined cumulative prospect doctrine with hybrid-information MCDM methodology to evaluate 3PRLPs from sustainability perspectives. Zarabkhshnia et al. [98] weighted the assessment criteria through fuzzy-SWARA method and ranked the sustainable 3PRLPs by employing COPRAS method under fuzzy environment. Liu et al. [57] suggested an innovative IVPHF-BWM to research the selection of 3PRLPs. Bai and Sarks [11] pioneered multi-stage, multi-method, and MCDM tool with TOPSIS, VIKOR and neighborhood rough set for the evaluation of 3PRLP selection decision. Zhang and Su [107] introduced a dominance-score dependent heterogeneous linguistic model to assess the best sustainable 3PRLP for a car manufacture industry. Mishra et al. [62] introduced a hybrid approach using the CoCoSo method and discrimination measure on HFSs to deal with the 3PRLP assessment problem. Mishra et al. [63] presented an integrated model with CRITIC and evaluation based on distance from average solution (EDAS) models for Fermatean fuzzy sets (FFSs) to tackle with the S3PRLP assessment. To select the optimal S3PRLP, Mishra and Rani [61] initiated a hybrid approach with combined compromise solution (CoCoSo) and CRITIC approaches on single-valued neutrosophic sets (SVNSs). Chen et al. [22] gave a projection model to analyze, rank, evaluate and select the optimal 3PRLPs on IVIFSs.

**Prerequisites**

In this section, we present the definition, score and accuracy value, ranking rules, distance measure, and Archimedean operations of BCFNs. At the end, we recall the definition of power AO.

**Definition 1** [7] Let $U$ denotes a universe set (finite). Then a BCFN $A$ on $U$ is expressed by

$$\bar{A} = \left\{ \left( u, \left( \mu^+_A(u), \mu^-_A(u) \right) \right) : u \in U \right\},$$

where the terms $\mu^+_A(u)$ and $\mu^-_A(u)$ are known as complex-valued positive BD and complex valued negative BD of the object $u \in U$. The values of $\mu^+_A(u)$ and $\mu^-_A(u)$ lie within the unit disc $D = \{ z \in C : |z| \leq 1 \}$ ($C$ denotes the set of all complex numbers). So, without loss of generality, we may accept that $\mu^+_A(u) = \alpha(u)e^{i\omega\theta(u)}$ and $\mu^-_A(u) = \beta(u)e^{i\omega\theta(u)}$, where $\alpha(u), \delta(u) \in [0, 1]$ and $\beta(u), \vartheta(u) \in [-1, 0]$ for any $u \in U$ and $i = \sqrt{-1}$. $\omega \in (0, 2\pi)$ is called the scaling factor and it is utilized to restrict the elucidation of phases inside the unit disk and the interval $(0, 2\pi]$. $\delta(u)$ and $\vartheta(u)$ are known as positive and negative phase values of the object $u \in U$. Without these phase values, the BCFS $\bar{A}$ is reduced to a traditional BFS. Moreover if we set $\beta(u) = 0 \forall u \in U$, then BCFS reduces to a traditional CFS.

Thus the BCFS $\bar{A}$ can be rewritten as $\bar{A} = \left\{ \left( u, (\alpha(u)e^{i\omega\delta(u)}, \beta(u)e^{i\omega\vartheta(u)}) \right) : u \in U \right\}$. For any $u \in U$, the pair $(\alpha(u)e^{i\omega\delta(u)}, \beta(u)e^{i\omega\vartheta(u)})$ is termed as a bipolar complex fuzzy number (BCFN). For easiness, the symbol $\xi = (\alpha e^{i\omega\delta}, \beta e^{i\omega\vartheta})$ is used to denote a BCFN. The set of all BCFN on $U$ is signified as $BCFN^U$.

**Definition 2** [56] Let $\xi = (\alpha e^{i\omega\delta}, \beta e^{i\omega\vartheta}) \in BCFN^U$. Then the score value of $\xi$ is defined as $S(\xi) = \frac{1}{2} (2 + \alpha + \beta + \delta)$.

Clearly, $0 \leq S(\xi) \leq 1$. It is observed that the score function can’t be effectively used to discriminate various BCFNs in several specific cases. For instance, if $\xi_1 = (0.5 \times e^{i(0.7\pi)}, -0.3 \times e^{i(-0.7\pi)})$ and $\xi_2 = (0.6 \times e^{i(0.5\pi)}, -0.3 \times e^{i(-0.6\pi)})$, then $S(\xi_1) = S(\xi_2)$ (taking $\omega = \pi$). To tackle this scenario, the notion of accuracy value of a BCFN was proposed by Liu et al. [56].
Definition 3 [56] Let \( \xi = (\alpha \times e^{(\text{i} \omega \delta)}, \beta \times e^{(\text{i} \omega \vartheta)}) \in BCFN^U \). Then the accuracy value of \( \xi \) is defined as
\[
AC(\xi) = \frac{1}{4} (\alpha - \beta + \beta - \vartheta).
\]

Clearly, \( 0 \leq AC(\xi) \leq 1 \).

Corresponding to the score and accuracy values of BCFNs, a comparative process of BCFNs is described as.

Definition 4 [56]: Let \( \xi_1, \xi_2 \in BCFN^U \). Then:

(I) If \( S(\xi_1) > S(\xi_2) \), then \( \xi_1 > \xi_2 \) (or \( \xi_2 < \xi_1 \)).

(II) If \( S(\xi_1) = S(\xi_2) \), then:

(i) if \( AC(\xi_1) > AC(\xi_2) \), then \( \xi_1 > \xi_2 \) (or \( \xi_2 < \xi_1 \)).

(ii) if \( AC(\xi_1) = AC(\xi_2) \), then \( \xi_1 = \xi_2 \).

Based on Archimedean operational laws [46], Liu et al. [56] introduced Archimedean operational laws for BCFNs which are presented by.

Definition 5 [56]: Let \( \xi_1 = (\alpha_1 e^{(\text{i} \omega_1 \delta_1)}, \beta_1 e^{(\text{i} \omega_1 \vartheta_1)}), \xi_2 = (\alpha_2 e^{(\text{i} \omega_2 \delta_2)}, \beta_2 e^{(\text{i} \omega_2 \vartheta_2)}) \in BCFN^U \). Then the Archimedean operational laws of BCFNs are:

(i) \( \xi_1 \heartsuit \xi_2 = (\left( g^{-1}(g(\alpha_1) + g(\alpha_2))e^{\omega(\text{a}g(\omega_1) + g(\omega_2)))}, -(h^{-1}(h(\beta_1) + h(\beta_2)))e^{\omega(h^{-1}(h(\delta_1) + h(\delta_2)))}) \right) \)

(ii) \( \xi_1 \clubsuit \xi_2 = (\left( h^{-1}(h(\alpha_1) + h(\alpha_2))e^{\omega(h^{-1}(h(\beta_1) + h(\beta_2)))}, -(g^{-1}(g(\beta_1) + g(\beta_2)))e^{\omega(g^{-1}(g(\delta_1) + g(\delta_2)))}) \right) \)

(iii) \( \lambda \heartsuit \xi_1 = (\left( g^{-1}(\lambda g(\alpha_1))e^{\omega(g^{-1}(\lambda g(\delta_1)))}, -(h^{-1}(\lambda h(\beta_1)))e^{\omega(h^{-1}(\lambda h(\vartheta_1)))}) \right) (\lambda > 0) \)

(iv) \( \lambda \clubsuit \xi_1 = (\left( h^{-1}(\lambda h(\alpha_1))e^{\omega(h^{-1}(\lambda h(\delta_1)))}, -(g^{-1}(\lambda g(\beta_1)))e^{\omega(g^{-1}(\lambda g(\vartheta_1)))}) \right) (\lambda > 0) \)

BCF-Archimedean power weighted aggregation operators

In this current section, we build up some BCF-Archimedean power-weighted AOs with the help of the Archimedean operations of BCFNs.

BCF-Archimedean power weighted arithmetic AOs:

Here, we propose \( BCFAPWAA \) and \( BCFAPOWAA \) operators and study their properties.

Definition 7 Suppose \( \xi_j = (\alpha_j e^{(\text{i} \omega_j \delta_j)}, \beta_j e^{(\text{i} \omega_j \vartheta_j)}) \in BCFN^U \) (\( j \in N_n \)). Then the \( BCFAPWAA \) operator a function \( BCFAPWAA : BCFN^U \rightarrow BCFN^U \) given by:

\[
BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \frac{n}{\sum_{j=1}^{n} w_j (1 + \Delta(\xi_j))} \times \xi_j,
\]

where \( w_j > 0 \) (\( j \in N_n \)) is the weight of \( \xi_j \) with \( \sum_{j=1}^{n} w_j = 1 \).

Here \( \Delta(\xi_j) = \sum_{j=1, j \neq i}^{n} \text{Supp}(\xi_j, \xi_i) \).

Next, the theorem given below follows from Definition 7.

Theorem 1 The aggregated \( BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) \) is also a BCFN and

\[
BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \left( \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \right) e^{\omega \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\beta_j) \right) \right)},
\]

\[
\times \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) e^{-\omega \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right)} \right)
\]

(1)

where \( \theta_j = \frac{(1 + \Delta(\xi_j))w_j}{\sum_{j=1}^{n} w_j (1 + \Delta(\xi_j))} \) (\( j \in N_n \)).
Theorem 2 (Shift invariance) Suppose $\xi_j \in BCFN^U$ ($j \in N_n$) and $\xi_0(\neq \xi_j) \in BCFN^U$. Then
classical operator (BCFAPWAA) on elements $\xi_1, \xi_2, \xi_3, \ldots, \xi_n$.

Proof is given in Appendix.

Theorem 3 (Idempotency) Suppose $\xi_j \in BCFN^U$ ($j \in N_n$) and $\xi_0 \in BCFN^U$ such that $\xi_j = \xi_0 \forall j$. Then we have, BCFAPWAA $(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \xi_0$.

Proof is given in Appendix.

Theorem 4 (Boundedness) Suppose $\xi_j \in BCFN^U$ ($j \in N_n$). Then, $\xi^- < BCFAPWAA(\xi_1, \xi_2,\ldots, \xi_n) < \xi^+$ where $\xi^- = (\alpha^+ e^{(i\sigma)}), \Phi^- e^{(i\varphi)}$ and $\xi^+ = (\alpha^+ e^{(i\sigma)}), \beta^- e^{(i\varphi)}$.

Proof is given in Appendix.

Theorem 5 (Monotonicity) Suppose $\xi_j, \xi'_j \in BCFN^U$ ($j \in N_n$) satisfying $\alpha_j \leq \alpha'_j, \delta_j \leq \delta'_j, \beta_j \geq \beta'_j, \theta_j \geq \theta'_j$ where $\xi'_j = (\alpha'_j e^{(i\sigma)}, \beta'_j e^{(i\varphi)})$. Then we have,

BCFAPWAA $(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) < BCFAPWAA(\xi'_1, \xi'_2, \xi'_3, \ldots, \xi'_n)$.

Proof is given in Appendix.

Next, based on BCFAPWAA operator, we develop the BCFAPWAA operator as follows:

Definition 8 Suppose $\xi_j \in BCFN^U$ ($j \in N_n$). Then the BCFAPWAA is a function $BCFAPWAA : BCFN^U \rightarrow BCFN^U$ which is defined as follows:

$$BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \sum_{j=1}^{n} \left(\theta_j \times \xi_{\sigma(j)}\right),$$

where $\sigma(1), \sigma(2), \sigma(3), \ldots, \sigma(n)$ is an arrangement of $\xi_j$ with $\xi_{\sigma(j)} \neq \xi_{\sigma(j)} \forall j \in N_n$.

The following theorem follows from Definition 8.

Theorem 6 The aggregated value $BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n)$ is also a BCFN and

$$BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \left(\left(\sum_{j=1}^{n} \theta_j g(\alpha_{\sigma(j)})\right) e\left(\sum_{j=1}^{n} \theta_j g(\beta_{\sigma(j)})\right)\right) \times \left(\sum_{j=1}^{n} \theta_j h(\varphi_{\sigma(j)})\right) e\left(\sum_{j=1}^{n} \theta_j h(\psi_{\sigma(j)})\right).$$

(2)

In particular, if $w_j = \frac{1}{n} \forall j \in N_n$, then the BCFAPWAA reduces to the BCFAPWAA.

Theorem 7 (Shift invariance) Suppose $\xi_j \in BCFN^U$ ($j \in N_n$) and $\xi_0(\neq \xi_j) \in BCFN^U$. Then

$$BCFAPWAA(\xi_0\oplus\xi_1, \xi_0\oplus\xi_2, \ldots, \xi_0\oplus\xi_n) = \xi_0 \oplus BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n).$$

Theorem 8 (Idempotency) Suppose $\xi_j \in BCFN^U$ ($j \in N_n$) and $\xi_0 \in BCFN^U$ satisfying $\xi_j = \xi_0 \forall j$. Then we have, $BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \xi_0$.

Theorem 9 (Boundedness) Suppose $\xi_j \in BCFN^U$ ($j \in N_n$). Then,

$$\xi^- < BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) < \xi^+,$$

where $\xi^- = (\alpha^+ e^{(i\sigma)}), \Phi^- e^{(i\varphi)}$ and $\xi^+ = (\alpha^+ e^{(i\sigma)}), \beta^- e^{(i\varphi)}$.

Theorem 10 (Monotonicity) Suppose $\xi_j, \xi'_j \in BCFN^U$ ($j \in N_n$) such that $\alpha_j \leq \alpha'_j, \delta_j \leq \delta'_j, \beta_j \geq \beta'_j, \theta_j \geq \theta'_j$ where $\xi'_j = (\alpha'_j e^{(i\sigma)}, \beta'_j e^{(i\varphi)})$.

Then, we have $BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) < BCFAPWAA(\xi'_1, \xi'_2, \xi'_3, \ldots, \xi'_n)$.

Proofs are similar to above.

**BCF-Archipedeanean power weighted geometric AOs**

In this sub-section, we propose BCF Archimedean power-weighted geometric AO (BCFAPWGA) and BCF Archimedean power ordered weighted geometric AO (BCFAPWGGA).

Definition 9 Suppose $\xi_j \in BCFN^U$ ($j \in N_n$). Then the BCFAPWGGA is a function $BCFAPWGGA : BCFN^U \rightarrow BCFN^U$ which is defined as follows:

$$BCFAPWGGA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \sum_{j=1}^{n} \left(\frac{1 + \Delta(\xi_j)w_j}{\sum_{j=1}^{n} w_j(1 + \Delta(\xi_j))} \circ \xi_j\right),$$

where $w_j > 0$ is the weight of $\xi_j$ with $\sum_{j=1}^{n} w_j = 1$.

Here $\Delta(\xi_j) = \sum_{j=1, j \neq i}^{n} \text{Supp}(\xi_i, \xi_j)$.

The given theorem follows the Definition 9.
Theorem 11 The aggregated value BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) is also a BCFN and

\[
BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) = \left( \left( h^{-1} \left( \sum_{j=1}^{n} θ_j h(ξ_j) \right) \right) \right) e^{\left( \sum_{j=1}^{n} θ_j g(β_j) \right) / \sum_{j=1}^{n} \Delta θ_j}
\]

where \( θ_j = \frac{(1+Δ(ξ_j))}{\sum_{j=1}^{n}(1+Δ(ξ_j))} \) w_j (j ∈ Nₙ).

Theorem 12 (Shift invariance) Suppose ξ_j ∈ BCFN \(^{U}\) (j ∈ Nₙ) and ξ₀ ≠ ξ_j ∈ BCFN \(^{U}\) Then BCFAPWGA(ξ₀ ⊗ ξ₁, ξ₀ ⊗ ξ₂, ..., ξ₀ ⊗ ξₙ) = ξ₀ ⊗ BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ).

Theorem 13 (Idempotency) Suppose ξ_j ∈ BCFN \(^{U}\) (j ∈ Nₙ) and ξ₀ ∈ BCFN \(^{U}\) such that ξ_j = ξ₀ ∀ j. Then we have BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) = ξ₀.

Theorem 14 (Boundedness) Suppose ξ_j ∈ BCFN \(^{U}\) (j ∈ Nₙ). Then,

\[ ξ^- < BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) < ξ^+ \]

where \( ξ^- = (φ^e(iαν^φj), Φ^−e(iαν^ψj)) \) and \( ξ^+ = (α^e(iαν^k), β^−e(iαν^σj)) \), such that \( φ^- = min_j(α_j), η^- = min_j(δ_j), Φ^- = max_j(β_j), ψ^- = max_j(ψ_j), α^+ = max_j(α_j), β^- = min_j(β_j), θ^- = min_j(θ_j) \).

Theorem 15 (Monotonicity) Suppose ξ_j, ξ_j' ∈ BCFN \(^{U}\) (j ∈ Nₙ) such that \( α_j ≤ α_j', δ_j ≤ δ_j', β_j ≥ β_j', θ_j ≥ θ_j' \) where \( ξ_j' = \left( α_j'e(iαν^k), β_j'e(iαν^σj) \right) \).

Then, we have BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) < BCFAPWGA(ξ₁', ξ₂', ξ₃', ..., ξₙ').

Next, based on BCFAWPGA operator, we shall develop the BCFAWPGA as follows:

Definition 10 Consider a collection ξ_j ∈ BCFN \(^{U}\) (j ∈ Nₙ). Then the BCFAPWGA is a function BCFAPWGA : BCFN \(^{U}\) → BCFN \(^{U}\) which is defined as follows:

\[
BCFAOWPGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) = \sum_{j=1}^{n} (θ_j ⊙ ξ_{σ(j)})
\]

where \( (σ(1), σ(2), σ(3), ..., σ(n)) \) is an arrangement of ξ_j satisfying \( ξ_{σ(j-1)} ≥ ξ_{σ(j)} \) ∀ j ∈ Nₙ.

Next, the mentioned theorem follows from Definition 10.

Theorem 16 The aggregated value BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) is also a BCFN and

\[
BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) = \left( \left( h^{-1} \left( \sum_{j=1}^{n} θ_j h(ξ_j) \right) \right) \right) e^{\left( \sum_{j=1}^{n} θ_j g(β_j) \right) / \sum_{j=1}^{n} \Delta θ_j}
\]

If w_j = \( \frac{1}{n} \) ∀ j ∈ Nₙ; then the BCFAPWGA reduces to the BCFAPWG.

Theorem 17 (Shift invariance) Suppose ξ_j ∈ BCFN \(^{U}\) (j ∈ Nₙ) and ξ₀ (≠ ξ_j) ∈ BCFN \(^{U}\) Then BCFAPWGA(ξ₀ ⊗ ξ₁, ξ₀ ⊗ ξ₂, ..., ξ₀ ⊗ ξₙ) = ξ₀ ⊗ BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ).

Theorem 18 (Idempotency) Suppose ξ_j ∈ BCFN \(^{U}\) (j ∈ Nₙ) and ξ₀ ∈ BCFN \(^{U}\) such that ξ_j = ξ₀ ∀ j. Then we have BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) = ξ₀.

Theorem 19 (Boundedness) Suppose ξ_j ∈ BCFN \(^{U}\) (j ∈ Nₙ). Then,

\[ ξ^- < BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) < ξ^+ \]

where \( ξ^- = (φ^e(iαν^φj), Φ^−e(iαν^ψj)) \) and \( ξ^+ = (α^e(iαν^k), β^−e(iαν^σj)) \) such that \( φ^- = min_j(α_j), η^- = min_j(δ_j), Φ^- = max_j(β_j), ψ^- = max_j(ψ_j), α^+ = max_j(α_j), δ^+ = max_j(β_j), η^+ = min_j(θ_j) \).

Theorem 20 (Monotonicity) Suppose ξ_j, ξ_j' ∈ BCFN \(^{U}\) (j ∈ Nₙ) satisfying \( α_j ≤ α_j', δ_j ≤ δ_j', β_j ≥ β_j', θ_j ≥ θ_j' \) where \( ξ_j' = \left( α_j'e(iαν^k), β_j'e(iαν^σj) \right) \).

Then, we have BCFAPWGA(ξ₁, ξ₂, ξ₃, ..., ξₙ) < BCFAPWGA(ξ₁', ξ₂', ξ₃', ..., ξₙ').

BCF-CRITIC-MULTIMOORA methodology for decision-making

In this present section, an integrated CRITIC-MULTIMOORA approach with BCF data in view of the introduced AOs is developed.

To solve a MCGDM problem comprising m different alternatives A₁, A₂, ..., Aₘ in which the alternatives are assessed by DEs D₁, D₂, ..., Dᵢ in BCF environment over a set of n distinct attributes C₁, C₂, ..., Cₙ, we develop an integrated BCF-CRITIC-MULTIMOORA methodology as follows (see Fig. 1):
Step 1: Consider the BCF-matrices representing the initial assessments of DEs. Suppose \( \wp_k = \begin{bmatrix} d_{krj} \end{bmatrix}_{m \times n} \) represents the initial assessment of the DE \( D_k \).

Step 2: Normalize the BCF-matrices \( \tilde{d}_{krj} = \begin{bmatrix} \tilde{\alpha}_{krj}, \tilde{\beta}_{krj} \end{bmatrix} \) where

\[
\Delta_1 = \frac{1}{3} \left( |\tilde{\alpha}_{krj} - \tilde{\alpha}_{srj}| + |\tilde{\delta}_{krj} - \tilde{\delta}_{srj}| + |\tilde{\beta}_{krj} - \tilde{\beta}_{srj}| + |\tilde{\vartheta}_{krj} - \tilde{\vartheta}_{srj}| \right)
\]

Step 3: Find out the supports \( \text{Supp}(\tilde{d}_{krj}, \tilde{d}_{srj}) \) using the below expression

\[
\text{Supp}(\tilde{d}_{krj}, \tilde{d}_{srj}) = 1 - D(\tilde{d}_{krj}, \tilde{d}_{srj}) (r \in N_m; j \in N_n; k \neq l)
\]

Step 4: Obtain the aggregated BCF matrix.

where \( D(\tilde{d}_{krj}, \tilde{d}_{srj}) \) is the distance between BCFNs \( \tilde{d}_{krj} \) and \( \tilde{d}_{srj} \) given by Eq. (7).

Step 5: Obtain the aggregated BCF matrix using BCFPAWAA or BCFPAWGA operator.
We use the proposed BCFAPWAA (or BCFAPWGA) operator to get the aggregated BCF matrix \( \left[ d_{rj} \right]_{m \times n} \) as follows:

\[
d_{rj} = BCFAPWAA(\tilde{a}_{1rj}, \tilde{a}_{2rj}, ..., \tilde{a}_{drj}) \\
= \left\{ \begin{array}{l}
g^{-1} \left( \sum_{k=1}^{l} \theta_{k}^j g(\tilde{a}_{krj}) \right) e^{\omega \left( \sum_{k=1}^{l} \phi_{k}^j h(\tilde{a}_{krj}) \right)} , \\
\times \left( -h^{-1} \left( \sum_{k=1}^{l} \theta_{k}^j h(\tilde{a}_{krj}) \right) \right) e^{\omega \left( -h^{-1} \left( \sum_{k=1}^{l} \phi_{k}^j h(\tilde{a}_{krj}) \right) \right)} \end{array} \right. \\
\text{or}
\]

\[
d_{rj} = BCFAPWGA(\tilde{a}_{1rj}, \tilde{a}_{2rj}, ..., \tilde{a}_{drj}) \\
= \left\{ \begin{array}{l}
h^{-1} \left( \sum_{k=1}^{l} \theta_{k}^j h(\tilde{a}_{krj}) \right) e^{\omega \left( h^{-1} \left( \sum_{k=1}^{l} \phi_{k}^j h(\tilde{a}_{krj}) \right) \right)} , \\
\times \left( -g^{-1} \left( \sum_{k=1}^{l} \theta_{k}^j g(\tilde{a}_{krj}) \right) \right) e^{\omega \left( -g^{-1} \left( \sum_{k=1}^{l} \phi_{k}^j g(\tilde{a}_{krj}) \right) \right)} \end{array} \right. \\
\text{Suppose the aggregated BCF matrix is} \left[ d_{rj} \right]_{m \times n} = \left[ \begin{array}{c} \alpha_{rj}g(\theta_{krj}) \end{array} \right]_{m \times n}. \\
\text{Step 6: Computations of criteria weights.}
\text{Let} \ w = (w_1, w_2, ..., w_n)^T \text{ such that} \sum_{j=1}^{n} w_j = 1, w_j \in [0, 1] \text{ be weight values for the criterion set. The indispensable attribute weights could uncover abundant data connecting in each of them, which is known as “objective weight”. The CRITIC is a methodology for processing the objective weights of the considered criteria. The weights inferred by this methodology associated both intensity contrast of every criteria and conflict among attributes. Intensity contrast of attribute is esteemed to standard deviation (SD) and conflict among them is calculated by the correlation coefficient (CRC). In this step, we implement this methodology into BCFNs.}
\text{Step 6.1: Utilizing the score values of BCFNs} \ d_{rj}, \text{ we construct the score matrix} \ S = \left[ S(d_{rj}) \right]_{m \times n}, \text{ where} \ S(d_{rj}) = \text{score value of the BCFN} \ d_{rj} \text{ where}
\]

\[
S(d_{rj}) = \frac{2+\alpha_{rj}+\beta_{rj}+\delta_{rj}}{2(\alpha_{rj}+\beta_{rj}+\delta_{rj})} \text{ if} \ C_j \in Q_B \\
\frac{2(1-\alpha_{rj})+\beta_{rj}+(1-\delta_{rj})}{4} \text{ if} \ C_j \in Q_C
\]

\text{Step 6.2: Convert the score matrix} \ S \text{ into the standard BCF-matrix} \ \tilde{S} = \left( \tilde{s}_{rj} \right)_{m \times n} \text{ where}

\[
\tilde{s}_{rj} = \begin{cases} 
\frac{S(d_{rj}) - \zeta^-}{\zeta^+ - \zeta^-}, & \text{if} \ C_j \in Q_B \\
\frac{\zeta^+ - S(d_{rj})}{\zeta^+ - \zeta^-}, & \text{if} \ C_j \in Q_C
\end{cases}
\]

\text{Step 6.3: Compute the attribute SDs by Eq. (14):}

\[
\sigma_j = \sqrt{\frac{\sum_{r=1}^{m} (\tilde{s}_{rj} - \overline{s}_{rj})^2}{m}}, \text{ where} \overline{s}_{rj} = \frac{1}{m} \sum_{r=1}^{m} \tilde{s}_{rj}.
\]

\text{Step 6.4: Estimate the correlation coefficient (CRC) utilizing Eq. (4):}

\[
r_{rj} = \frac{\sum_{r=1}^{m} (\tilde{s}_{rj} - \overline{s}_{rj})(\tilde{s}_{ry} - \overline{s}_{ry})}{\sqrt{\sum_{r=1}^{m} (\tilde{s}_{rj} - \overline{s}_{rj})^2 \sum_{r=1}^{m} (\tilde{s}_{ry} - \overline{s}_{ry})^2}}
\]

\text{Step 6.5: Analyze the amount of information of each attribute as}

\[
c_j = \sigma_j \sum_{y=1}^{n} (1 - r_{rj}).
\]

\text{Step 6.6: Obtain the criteria weights using:}

\[
w_{rj} = \frac{c_j}{\sum_{j=1}^{m} c_j}.
\]

\text{Step 7: Obtain the best-suited alternative by the RS approach.}
\text{In the following sub steps it may be explored the choice of the best alternative and the ranking order of the alternatives with this approach in the suggested BCF-CRITIC-MULTIMOORA method.}
\text{Step 7.1: Compute} Y^+_j \text{ and} Y^-_j \text{ by utilizing the BCFAWAA operator [56] as given below:}

\[
Y^+_j = \text{BCFWAWA}(d_{r1}, d_{r2}, ..., d_{rn}) \\
= \left\{ \begin{array}{l}
g^{-1} \left( \sum_{j=Q_B} w_{rj} g(\theta_{krj}) \right) e^{\omega \left( \sum_{j=Q_B} w_{rj} g(\theta_{krj}) \right)} , \\
\times \left( -h^{-1} \left( \sum_{j=Q_B} w_{rj} h(\beta_{krj}) \right) \right) e^{\omega \left( -h^{-1} \left( \sum_{j=Q_B} w_{rj} h(\beta_{krj}) \right) \right)} \end{array} \right.
\]

\text{or}

\[
Y^-_j = \text{BCFWAWA}(d_{r1}, d_{r2}, ..., d_{rn}) \\
= \left\{ \begin{array}{l}
h^{-1} \left( \sum_{j=Q_B} w_{rj} h(\beta_{krj}) \right) e^{\omega \left( h^{-1} \left( \sum_{j=Q_B} w_{rj} h(\beta_{krj}) \right) \right)} , \\
\times \left( g^{-1} \left( \sum_{j=Q_B} w_{rj} g(\theta_{krj}) \right) \right) e^{\omega \left( g^{-1} \left( \sum_{j=Q_B} w_{rj} g(\theta_{krj}) \right) \right)} \end{array} \right.
\]
where \( Y^*_r \) and \( Y^-_r \) represent the alternative’s \((A_r)\) significance that are achieved subject to the respective benefit and cost criteria. Clearly, \( Y^*_r \) and \( Y^-_r \) are BCFNs.

Step 7.2: Compute the score values of the BCFs \( Y^*_r \) and \( Y^-_r \) (\( r \in N_m \)) by using Definition 2.

Step 7.3: Compute the overall significance for each alternative using the formula:

\[
\Omega_r = S(Y^*_r) - S(Y^-_r) \quad (r \in N_m).
\]

Step 7.4: Selection of the best alternative is to be performed after of their ranking. Similar to the RS approach underlying the ordinary MULTIMOORA method, the process of giving the ranking order can be entertained at this step.

Step 8: Obtain the ranking order of alternatives based on the RP approach.

Step 8.1: Compute the RP. Here, each coordinate \( r^*_j (j = 1, 2, \ldots, n) \) of the RP \( r^* = \{r^*_1, r^*_2, \ldots, r^*_n\} \) is a BCFN that are calculated by the following way:

\[
\begin{align*}
 r^*_j = & \left\{ \begin{array}{ll}
 \max_{j \in Q^B} \alpha_{rj} \times e^{(\omega_{\max} \beta_{j})} + \min_{j \in Q^B} \beta_{rj} \times e^{(\omega_{\min} \beta_{j})} & , \quad \text{for } j \in Q^B \smallskip \\
 \min_{j \in Q^B} \alpha_{rj} \times e^{(\omega_{\max} \beta_{j})} + \max_{j \in Q^B} \beta_{rj} \times e^{(\omega_{\min} \beta_{j})} & , \quad \text{for } j \in Q^C.
\end{array} \right.
\end{align*}
\]

Step 8.2: Distance between RPs and each alternative is to be calculated using the condition:

\[
D_{rj} = w_j \times D\left( d_{rj}, r^*_j \right),
\]

in which \( D_{rj} \) represents the alternative’s \((A_r)\) distance which is determined on the basis of evaluation criterion \( C_j \) obtained by Eq. (7).

Step 8.3: Using the following relation, each alternative’s highest distance is to be measured.

\[
d_r = \max_j D_{rj} \quad (r \in N_m)
\]

Step 8.4: Selection of the best alternative is to be performed after their ranking. Similar to the RP approach underlying the ordinary MULTIMOORA method, the process of giving the ranking order can be entertained at this step.

Step 9: Obtain the ranking order of alternatives based on the FMF procedure.

Step 9.1: Utilizing the BCFAWGA operator [56], calculate \( \Gamma_r \) and \( \Phi_r \) as follows:

\[
\begin{align*}
\Gamma_r &= \text{BCFAWG}(d_1, d_2, \ldots, d_n) \\
&= \left\{ \begin{array}{ll}
 \left( g^{-1} \left( \sum_{j \in Q^B} w_j h(\alpha_{ij}) \right) \right) e^{\omega \left( h^{-1} \left( \sum_{j \in Q^C} w_j h(\beta_{ij}) \right) \right)} & , \quad \text{for } j \in Q^B \smallskip \\
 \left( g^{-1} \left( \sum_{j \in Q^C} w_j h(\beta_{ij}) \right) \right) e^{\omega \left( h^{-1} \left( \sum_{j \in Q^B} w_j h(\alpha_{ij}) \right) \right)} & , \quad \text{for } j \in Q^C.
\end{array} \right.
\end{align*}
\]

where \( \Gamma_r \) and \( \Phi_r \) are BCFNs representing the multiplicative forms corresponding to benefit-type and cost-type attributes, respectively.

Step 9.2: Estimate the score values of the BCFNs \( \Gamma_r \) and \( \Phi_r \) using Definition 2.

Step 9.3: The overall effectiveness value for each alternative by FMF method is calculated by:

\[
\eta_r = \frac{\Gamma_r}{\Phi_r} \quad (r \in N_m).
\]

Step 9.4: Select the best alternative after getting the ranking order.

Step 10: Determine the final ranking order of the alternatives.

The overall assessment value of alternative by improved Borda Rule [93] is obtained by

\[
I_{\text{BR}}(A_r) = \hat{\Omega}_r \times m - \rho(\hat{\Omega}_r) + 1 \quad \frac{\tilde{d}_r \times \rho(\tilde{d}_r)}{m(m+1)/2} + \tilde{\eta}_r \times \frac{m - \rho(\tilde{\eta}_r) + 1}{m(m+1)/2} \quad (r \in N_m),
\]

where \( \hat{\Omega}_r, \tilde{d}_r, \tilde{\eta}_r \) are the normalized score values and \( \rho(\hat{\Omega}_r), \rho(\tilde{d}_r), \rho(\tilde{\eta}_r) \), are the final ranks of the alternative \( A_r \) by RS, RP and FMF approaches, respectively. The best alternative has the maximum value of \( I_{\text{BR}}(A_r) \).
Case study: 3PRLPs selection

Problem description

In order to reveal the application of the developed framework, an illustrative case study of Chinese electronics’ company has been presented. The preferred company was established in the early 21st era and placed in the southwestern province of China to rise into an enterprise leader in computer manufacturing. At the moment, the company has an annual manufacturing capacity in excess of 2.5 million computers. However, the end-of-life (EOL) products generated a large volume of waste largely generating plastics and metal waste which had environmental effects and even polluted the water and land. Taking into consideration the increasing public awareness on environmental issues, increase in charge of raw materials, and compulsory green legislation in China, this manufacturer has decided to create a sustainable closed-loop supply chain with recycle the green products in forms of energy conservation. Consequently, the executives had arrived at a contract in implementation of a reverse logistics structure to efficiently organize and evoke the worth of reverse flow by reuse, recycling, reproducing, and eco-friendly disposal. On the other hand, the company considered in this study has a lack of experience and accessible organization capacity for RLs, and therefore decided to outsource RLs execution to 3PRLPs. After the open bidding, ten 3PRLPs displayed their curiosity in offering services. On the basis of preliminary analysis and discussions with experts, the company identified five potential 3PRLPs (A1, A2, A3, A4, A5). A group of experts has been invited to evaluate the present 3PRLPs selection problem over fifteen identified criteria. The details of the criteria are depicted in Table 2.

Problem solution

To solve the problem described above we take \( \omega = \pi \). To reduce the shape and size of each table and for the purpose of simplistic presentation of each entry, in this subsection, the notation \( (\alpha, \delta, \beta, \theta) \) is used to signify a BCFN \( e^{i(\pi \alpha, \beta, \theta)} \).

Step 1: In this step, the decision experts will assess the five options A1, A2, A3, A4, A5 in relation to considered attributes \( C_j \) ( \( j \in N_{15} \)). The initial assessment results are given in the form of the matrices \( [d_{rk}]_{5 \times 15} \) ( \( k \in N_3 \)) is given in the form of Table 3 (taking \( \omega = \pi \)).

Steps 2–3: We normalize the matrices \( [d_{rk}]_{5 \times 15} \) ( \( k \in N_3 \)) by making use of Eq. (5). Then we calculate the supports \( \text{Supp}(d_{rk}, d_{rk}) \) ( \( r \in N_5; j \in N_{15}; k, s \in N_3 \)).
|   | $A_1$       | $A_2$       | $A_3$       | $A_4$       | $A_5$       |
|---|------------|------------|------------|------------|------------|
| $D_1$ | C1  | (0.5, 0.2; −0.6, −0.4) | (0.3, 0.5; −0.7, −0.8) | <0.8, 0.9; −0.5, −0.6) | (0.4, 0.6; −0.1, −0.3) | (0.7, 0.8; −0.2, −0.3) |
|     | C2  | (0.1, 0.3; −0.7, −0.6) | (0.2, 0.4; −0.8, −0.9) | <0.7, 0.8; −0.2, −0.3) | (0.3, 0.5; −0.4, −0.4) | (0.8, 0.9; −0.5, −0.6) |
|     | C3  | (0.7, 0.8; −0.2, −0.3) | (0.1, 0.2; −0.3, −0.4) | <0.5, 0.5; −0.4, −0.5) | (0.1, 0.3; −0.7, −0.6) | (0.5, 0.5; −0.4, −0.5) |
|     | C4  | (0.5, 0.5; −0.4, −0.5) | (0.8, 0.9; −0.5, −0.6) | (0.6, 0.8; −0.3, −0.4) | (0.8, 0.9; −0.5, −0.6) | (0.1, 0.2; −0.3, −0.4) |
|     | C5  | (0.2, 0.4; −0.8, −0.9) | (0.3, 0.5; −0.4, −0.4) | (0.4, 0.6; −0.1, −0.3) | (0.2, 0.4; −0.8, −0.9) | (0.5, 0.2; −0.6, −0.4) |
|     | C6  | (0.4, 0.6; −0.1, −0.3) | (0.5, 0.5; −0.4, −0.5) | (0.1, 0.2; −0.3, −0.4) | (0.4, 0.6; −0.1, −0.3) | (0.3, 0.5; −0.4, −0.4) |
|     | C7  | (0.3, 0.5; −0.7, −0.8) | (0.5, 0.7; −0.2, −0.4) | (0.5, 0.2; −0.6, −0.4) | (0.3, 0.5; −0.7, −0.8) | (0.7, 0.8; −0.2, −0.3) |
|     | C8  | (0.6, 0.8; −0.3, −0.4) | (0.7, 0.8; −0.2, −0.3) | (0.8, 0.9; −0.5, −0.6) | (0.5, 0.7; −0.2, −0.4) | (0.6, 0.8; −0.3, −0.4) |
|     | C9  | (0.5, 0.7; −0.2, −0.4) | (0.5, 0.5; −0.4, −0.5) | (0.3, 0.5; −0.4, −0.4) | (0.8, 0.9; −0.5, −0.6) | (0.2, 0.4; −0.8, −0.9) |
|     | C10 | (0.8, 0.9; −0.5, −0.6) | (0.2, 0.4; −0.8, −0.9) | (0.3, 0.5; −0.7, −0.8) | (0.2, 0.3; −0.7, −0.6) | (0.8, 0.9; −0.5, −0.6) |
|     | C11 | (0.3, 0.5; −0.4, −0.4) | (0.4, 0.6; −0.1, −0.3) | (0.3, 0.5; −0.7, −0.8) | (0.6, 0.8; −0.3, −0.4) | (0.4, 0.6; −0.1, −0.3) |
|     | C12 | (0.7, 0.8; −0.2, −0.3) | (0.7, 0.9; −0.3, −0.4) | (0.5, 0.5; −0.4, −0.5) | (0.3, 0.5; −0.4, −0.4) | (0.3, 0.5; −0.7, −0.8) |
|     | C13 | (0.4, 0.6; −0.1, −0.3) | (0.8, 0.9; −0.5, −0.6) | (0.1, 0.2; −0.3, −0.4) | (0.2, 0.4; −0.8, −0.9) | (0.5, 0.2; −0.6, −0.4) |
|     | C14 | (0.1, 0.2; −0.3, −0.4) | (0.2, 0.4; −0.8, −0.9) | (0.5, 0.7; −0.2, −0.4) | (0.7, 0.8; −0.2, −0.3) | (0.1, 0.2; −0.3, −0.4) |
|     | C15 | (0.5, 0.5; −0.4, −0.5) | (0.1, 0.3; −0.7, −0.6) | (0.6, 0.8; −0.3, −0.4) | (0.8, 0.9; −0.5, −0.6) | (0.5, 0.5; −0.4, −0.5) |

**Table 3** Initial decision matrix
and quantity of information of each factor are computed and Eqs. (14)–(16), the standard deviation, correlation coefficient Table 6).

\[ S \approx (\zeta_{ij})_{m \times n} \]

by utilizing the Eq. (13). Next, by applying Eqs. (14)–(16), the standard deviation, correlation coefficient and quantity of information of each factor are computed and depicted in Table 7. The criteria weights are computed by using Eq. (17) and are depicted in the final column of Table 7.

\[ S^{(12)} = S^{(21)} = \]

\[ S^{(13)} = S^{(31)} = \]

\[ r_1^* = \begin{pmatrix} 0.686673872 \\ -0.647148423 \end{pmatrix} \times e^{(0.810622539i)} \times e^{(-0.750635151i)} \]

\[ r_2^* = \begin{pmatrix} 0.630747981 \\ -0.57688361 \end{pmatrix} \times e^{(0.791276869i)} \times e^{(-0.57688361i)} \]

\[ r_3^* = \begin{pmatrix} 0.295691311 \\ -0.271312913 \end{pmatrix} \times e^{(0.163487059i)} \times e^{(-0.272904272i)} \]

\[ r_4^* = \begin{pmatrix} 0.767096534 \\ -0.665506341 \end{pmatrix} \times e^{(0.870258995i)} \times e^{(-0.7103762i)} \]

\[ r_5^* = \begin{pmatrix} 0.639349863 \\ -0.760180273 \end{pmatrix} \times e^{(0.693125864i)} \times e^{(-0.707049938i)} \]

\[ r_6^* = \begin{pmatrix} 0.691723482 \\ -0.431668642 \end{pmatrix} \times e^{(0.818116623i)} \times e^{(-0.533942955i)} \]

\[ r_7^* = \begin{pmatrix} 0.25965885 \\ -0.202353406i \end{pmatrix} \times e^{(0.249700265i)} \times e^{(-0.202353406i)} \]

\[ r_8^* = \begin{pmatrix} 0.635180188 \\ -0.677697106i \end{pmatrix} \times e^{(0.775736758i)} \times e^{(-0.677697106i)} \]

\[ r_9^* = \begin{pmatrix} 0.734764713 \\ -0.750936934 \end{pmatrix} \times e^{(0.855967720i)} \times e^{(-0.752943434i)} \]

\[ r_{10}^* = \begin{pmatrix} 0.654960339 \\ -0.707049938i \end{pmatrix} \times e^{(0.778172555i)} \times e^{(-0.707049938i)} \]

\[ r_{11}^* = \begin{pmatrix} 0.57326954 \end{pmatrix} \times e^{(0.632781179i)} \times e^{(-0.615991472i)} \]

\[ r_{12}^* = \begin{pmatrix} 0.730943836 \\ -0.46515778 \end{pmatrix} \times e^{(0.831633305i)} \times e^{(-0.6561423i)} \]

\[ r_{13}^* = \begin{pmatrix} 0.705745217 \\ -0.688555132 \end{pmatrix} \times e^{(0.852557965i)} \times e^{(-0.656988502i)} \]

\[ \begin{array}{cccccc}
A_1 & A_2 & A_3 & A_4 & A_5 \\
0.633333 & 0.733333 & 0.533333 & 1 & 0.666667 \\
0.7 & 0.433333 & 0.433333 & 0.76667 & 0.533333 \\
0.9 & 0.7 & 0.7 & 0.96667 & 0.866667 \\
1 & 0.5 & 1 & 1 & 0.7 \\
1 & 1 & 1 & 1 & 1 \\
0.633333 & 0.733333 & 0.4 & 1 & 0.666667 \\
0.833333 & 0.6 & 0.6 & 0.63333 & 0.633333 \\
0.9 & 0.9 & 0.76667 & 0.8 & 0.533333 \\
0.866667 & 0.7 & 0.86667 & 0.433333 & 0.6 \\
0.4 & 1 & 0.86667 & 0.4 & 0.4 \\
0.7 & 0.43333 & 0.76667 & 0.76667 & 0.533333 \\
0.9 & 0.9 & 0.7 & 0.8 & 0.533333 \\
0.866667 & 0.7 & 0.8 & 0.433333 & 0.6 \\
0.4 & 1 & 0.93333 & 0.4 & 0.4 \\
1 & 0.5 & 1 & 1 & 0.7 \\
0.5667 & 0.8667 & 0.8 & 0.53333 & 0.733333 \\
0.6667 & 0.4667 & 0.7 & 0.8 & 0.433333 \\
0.8 & 0.83333 & 0.7 & 0.63333 & 0.666667 \\
0.6667 & 0.6 & 0.6667 & 0.8 & 0.8 \\
0.9 & 0.5 & 0.53333 & 0.76667 & 0.76667 \\
0.5667 & 0.8667 & 0.8 & 0.53333 & 0.733333 \\
0.6667 & 0.43333 & 0.9 & 0.6 & 0.4 \\
0.53333 & 0.6 & 0.6667 & 0.53333 & 0.6 \\
0.9 & 0.53333 & 0.53333 & 0.76667 & 0.6667 \\
0.6667 & 0.7667 & 0.7667 & 0.9667 & 0.6 \\
0.7 & 0.8 & 0.9 & 0.73333 & 0.7 \\
0.8 & 0.63333 & 0.8 & 0.63333 & 0.5667 \\
0.6667 & 0.43333 & 1 & 0.86667 & 0.9 \\
0.83333 & 0.6 & 0.6 & 0.63333 & 0.63333 \\
0.8667 & 0.73333 & 0.9 & 0.6 & 0.8667 \\
0.8667 & 0.43333 & 1 & 0.86667 & 0.9 \\
0.8 & 0.63333 & 0.8 & 0.63333 & 0.5667 \\
0.6667 & 0.43333 & 1 & 0.86667 & 0.9 \\
0.53333 & 0.6 & 0.6667 & 0.53333 & 0.6 \\
0.9 & 0.53333 & 0.53333 & 0.7667 & 0.6667 \\
\end{array} \]
|        | $A_1$       | $A_2$       | $A_3$       | $A_4$       | $A_5$       |
|--------|-------------|-------------|-------------|-------------|-------------|
| $\theta^1_{jr}$ |             |             |             |             |             |
| $[\theta^1_{jr}]_{15 \times 5} = $ |             |             |             |             |             |
| C_1    | 0.357862    | 0.366935    | 0.347025    | 0.377841    | 0.358974    |
| C_2    | 0.330672    | 0.338223    | 0.357925    | 0.343945    | 0.329545    |
| C_3    | 0.364243    | 0.349773    | 0.349773    | 0.350986    | 0.367911    |
| C_4    | 0.366279    | 0.315462    | 0.356816    | 0.364213    | 0.36        |
| C_5    | 0.360294    | 0.370643    | 0.360294    | 0.370643    | 0.375348    |
| C_6    | 0.35171     | 0.37908     | 0.287671    | 0.356635    | 0.332682    |
| C_7    | 0.355224    | 0.36        | 0.36        | 0.362623    | 0.362623    |
| C_8    | 0.352729    | 0.358033    | 0.334643    | 0.364276    | 0.309924    |
| C_9    | 0.354118    | 0.36        | 0.359618    | 0.371636    | 0.36        |
| C_10   | 0.331081    | 0.360294    | 0.343905    | 0.319685    | 0.331081    |
| C_11   | 0.338164    | 0.344094    | 0.353666    | 0.349194    | 0.350539    |
| C_12   | 0.359972    | 0.381427    | 0.323699    | 0.344262    | 0.30867     |
| C_13   | 0.347826    | 0.351619    | 0.337838    | 0.340392    | 0.327373    |
| C_14   | 0.346645    | 0.372951    | 0.364973    | 0.352134    | 0.345291    |
| C_15   | 0.354895    | 0.346249    | 0.377841    | 0.362219    | 0.355525    |
| $\theta^2_{jr}$ |             |             |             |             |             |
| $[\theta^2_{jr}]_{15 \times 5} = $ |             |             |             |             |             |
| C_1    | 0.251743    | 0.235215    | 0.233711    | 0.269886    | 0.242165    |
| C_2    | 0.259481    | 0.25453     | 0.215486    | 0.262976    | 0.267857    |
| C_3    | 0.256838    | 0.240104    | 0.240104    | 0.247887    | 0.255878    |
| C_4    | 0.261628    | 0.268065    | 0.249141    | 0.260152    | 0.228571    |
| C_5    | 0.257353    | 0.264745    | 0.257353    | 0.264745    | 0.268106    |
| C_6    | 0.240754    | 0.241098    | 0.273973    | 0.254739    | 0.250651    |
| C_7    | 0.238806    | 0.22        | 0.22        | 0.22951     | 0.22951     |
| C_8    | 0.24895     | 0.259016    | 0.261952    | 0.253165    | 0.274809    |
| C_9    | 0.241176    | 0.228571    | 0.238949    | 0.203636    | 0.22        |
| C_10   | 0.236486    | 0.257353    | 0.258085    | 0.251969    | 0.236486    |
| C_11   | 0.258799    | 0.227439    | 0.237215    | 0.276285    | 0.242681    |
| C_12   | 0.267547    | 0.268611    | 0.260116    | 0.252207    | 0.272795    |
| C_13   | 0.254658    | 0.241243    | 0.26018     | 0.239216    | 0.25447     |
| C_14   | 0.231629    | 0.266393    | 0.260695    | 0.221037    | 0.224215    |
| C_15   | 0.253497    | 0.251443    | 0.269886    | 0.258728    | 0.243651    |
| $\theta^3_{jr}$ |             |             |             |             |             |
| $[\theta^3_{jr}]_{15 \times 5} = $ |             |             |             |             |             |
| C_1    | 0.390395    | 0.397849    | 0.419263    | 0.352273    | 0.39886     |
| C_2    | 0.409847    | 0.407248    | 0.426589    | 0.39308     | 0.402597    |
| C_3    | 0.378919    | 0.410123    | 0.410123    | 0.401127    | 0.37621     |
| C_4    | 0.372093    | 0.416472    | 0.394044    | 0.375635    | 0.411429    |
| C_5    | 0.382353    | 0.364611    | 0.382353    | 0.364611    | 0.356546    |
| C_6    | 0.407537    | 0.379822    | 0.438356    | 0.388626    | 0.416667    |
| C_7    | 0.40597     | 0.416       | 0.416       | 0.414426    | 0.414426    |
| C_8    | 0.39832     | 0.382951    | 0.403405    | 0.38256     | 0.415267    |
| C_9    | 0.404706    | 0.411429    | 0.401434    | 0.424727    | 0.416       |
| C_10   | 0.432432    | 0.382353    | 0.39801     | 0.428346    | 0.432432    |
| C_11   | 0.403037    | 0.428467    | 0.409119    | 0.37452     | 0.40678     |
| C_12   | 0.372481    | 0.349962    | 0.416185    | 0.403531    | 0.418535    |
| C_13   | 0.397516    | 0.407138    | 0.401544    | 0.420392    | 0.418157    |
| C_14   | 0.421725    | 0.360656    | 0.374332    | 0.426829    | 0.430493    |
| C_15   | 0.391608    | 0.402308    | 0.352273    | 0.379052    | 0.400824    |
|   | $A_1$                                                                 | $A_2$                                                                 | $A_3$                                                                 | $A_4$                                                                 | $A_5$                                                                 |
|---|-----------------------------------------------------------------------|-----------------------------------------------------------------------|-----------------------------------------------------------------------|-----------------------------------------------------------------------|-----------------------------------------------------------------------|
| $C_1$ | $(0.6339423, 0.7016362; - 0.3599141, - 0.435865)$                      | $(0.3179776, 0.4623839; - 0.6471484, 0.7092740)$                      | $(0.5494609, 0.7092740; - 0.6471484, 0.4081181)$                      | $(0.3360066, 0.5128354; - 0.1984746, - 0.3891341)$                    | $(0.6866738, 0.8106225; - 0.3409192, - 0.4204782)$                    |
| $C_2$ | $(0.3373275, 0.441156; - 0.4813110, 0.5011949)$                      | $(0.5772322, 0.3791261; - 0.3891341, - 0.5768833)$                    | $(0.7690574, 0.370783; - 0.5876156, - 0.5743009)$                    | $(0.8244176, 0.6745848; - 0.4685608, - 0.4766906)$                    | $(0.2956913, 0.1634870; - 0.7173783, - 0.418548)$                    |
| $C_3$ | $(0.2956913, 0.1634870; - 0.7173783, - 0.418548)$                    | $(0.4668186, 0.6446797; - 0.655063, - 0.71037)$                      | $(0.5397342, - 0.4016016)$                                            | $(0.563236, 0.765351; - 0.2557017, - 0.4)$                           | $(0.377640, 0.4044453; - 0.3593942, - 0.4601616)$                    |
| $C_4$ | $(0.1631486, 0.3635730; - 0.7601802, - 0.7707499)$                    | $(0.2650757, 0.465632; - 0.5150138, - 0.5376129)$                    | $(0.553380, 0.6469602; - 0.3518180, - 0.4612324)$                    | $(0.4309768, 0.3233441; - 0.721412, - 0.4711775)$                    | $(0.7276862, 0.5259465; - 0.3389222, - 0.2203340)$                    |
| $C_5$ | $(0.2707023, 0.678446; - 0.3207504, 0.3725345)$                      | $(0.5944799, 0.6538969; - 0.4209309, - 0.5212770)$                    | $(0.4624495, 0.6327811; - 0.2345801, - 0.4324969)$                    | $(0.6498645, 0.5538214; - 0.234826, - 0.255508)$                     | $(0.5930344, 0.7649443; - 0.2896811, - 0.3930425)$                    |
| $C_6$ | $(0.5758289, 0.6720562; - 0.4023451, - 0.5038108)$                    | $(0.1631486, 0.3635730; - 0.7601802, - 0.7707499)$                    | $(0.3156918, 0.5173494; - 0.2906437, - 0.4356189)$                    | $(0.4624495, 0.6327811; - 0.2345801, - 0.4324969)$                    | $(0.7276862, 0.5259465; - 0.3389222, - 0.2203340)$                    |
| $C_7$ | $(0.2116530, 0.4129153; - 0.6113418, - 0.5699141)$                    | $(0.596448, 0.7884209; - 0.3606496, 0.6538969)$                      | $(0.7309438, 0.8316333; - 0.461507, - 0.565614)$                     | $(0.4997353, 0.5982905; - 0.1919051, - 0.396351)$                    | $(0.5078292, 0.6720362; - 0.016211, - 0.3731880)$                    |
| $C_8$ | $(0.6080393, 0.6648140; - 0.3356303, - 0.3606496)$                    | $(0.896316, 0.8525579; - 0.565165, 0.8478899)$                      | $(0.7309438, 0.8316333; - 0.461507, - 0.565614)$                     | $(0.5793735, 0.5982905; - 0.1919051, - 0.396351)$                    | $(0.5078292, 0.6720362; - 0.016211, - 0.3731880)$                    |
| $C_9$ | $(0.565165, 0.7177855; - 0.1571534, - 0.3228032)$                    | $(0.7057452, 0.8525579; - 0.3848292, - 0.4866059)$                    | $(0.347259, 0.4139570; - 0.3029354, - 0.4734955)$                    | $(0.4997353, 0.5982905; - 0.1919051, - 0.396351)$                    | $(0.5078292, 0.6720362; - 0.016211, - 0.3731880)$                    |
| $C_{10}$ | $(0.4646014, 0.6310599; - 0.2124681, - 0.3891867)$                | $(0.3247307, 0.4381840; - 0.6230484, - 0.7280765)$                    | $(0.461516, 0.5331840; - 0.1942984, - 0.3710983)$                    | $(0.4997353, 0.5982905; - 0.1919051, - 0.396351)$                    | $(0.5078292, 0.6720362; - 0.016211, - 0.3731880)$                    |
| $C_{11}$ | $(0.4295979, 0.5; - 0.481621)$                                       | $(0.4762138, 0.653767; - 0.294012, - 0.4539880)$                    | $(0.5123854, 0.7238065; - 0.5106280)$                                 | $(0.4419814, 0.5145213)$                                             | $(0.213454, 0.2814880)$                                             |

Table 6 Aggregated BCF-matrix
Table 7 The standard BCF-matrix, SD, amount of information and criteria weights using CRITIC

|    | A1 | A2 | A3 | A4 | A5 | σj | cj | wj |
|----|----|----|----|----|----|----|----|----|
| C1 | 0.860 | 0.000 | 0.920 | 0.655 | 1.000 | 0.362 | 4.564 | 0.0581 |
| C2 | 0.000 | 0.674 | 1.000 | 0.914 | 0.838 | 0.359 | 5.012 | 0.0639 |
| C3 | 1.000 | 0.356 | 0.234 | 0.000 | 0.516 | 0.335 | 5.286 | 0.0673 |
| C4 | 0.209 | 0.000 | 0.864 | 1.000 | 0.016 | 0.428 | 5.391 | 0.0687 |
| C5 | 0.000 | 0.378 | 1.000 | 0.216 | 0.685 | 0.352 | 4.819 | 0.0614 |
| C6 | 0.728 | 0.000 | 0.685 | 1.000 | 0.970 | 0.361 | 4.622 | 0.0589 |
| C7 | 0.013 | 0.770 | 0.202 | 0.000 | 1.000 | 0.411 | 6.871 | 0.0876 |
| C8 | 1.000 | 0.962 | 0.626 | 0.837 | 0.000 | 0.366 | 5.752 | 0.0733 |
| C9 | 1.000 | 0.792 | 0.533 | 0.848 | 0.000 | 0.351 | 5.421 | 0.0691 |
| C10 | 0.797 | 0.000 | 1.000 | 0.884 | 0.797 | 0.356 | 4.310 | 0.0549 |
| C11 | 0.000 | 0.650 | 0.952 | 0.548 | 1.000 | 0.359 | 5.158 | 0.0657 |
| C12 | 1.000 | 0.804 | 0.657 | 0.000 | 0.758 | 0.341 | 5.740 | 0.0731 |
| C13 | 1.000 | 0.918 | 0.393 | 0.416 | 0.000 | 0.370 | 6.344 | 0.0808 |
| C14 | 0.912 | 0.000 | 0.761 | 1.000 | 0.660 | 0.353 | 4.435 | 0.0565 |
| C15 | 0.000 | 0.513 | 0.409 | 1.000 | 0.320 | 0.325 | 4.762 | 0.0607 |

Comparative study

To validate our result, we compare our proposed BFCRITIC-MULTIMOORA method with the corresponding BFCRITIC-TOPSIS and BFCRITIC-WASPAS approaches.

(a) TOPSIS model, introduced by Hwang and Yoon [42] focuses on relative closeness to the optimal solution. In other words, according to the TOPSIS method, the selected alternatives should maintain the minimum and maximum geometric distance from the PIS and the NIS, respectively. Actually, for comparative study, we made original extensions of TOPSIS by combining it with the CRITIC technique in BCF setting. The algorithm for this extended TOPSIS method, i.e., BFCRITIC-TOPSIS method is given below:

Steps 1–6: Same as discussed in Sect. 6

At the end of step 6, we get the aggregated decision matrix

\[
d_{ij}^\alpha = \left[\alpha_j e^{i\alpha_\omega j} \right]_{m \times n}
\]

Table 8 Overall importance and rank of the alternative based on RS technique

|    | \(Y_+\) | \(Y_+^w\) | \(\Omega_r\) |
|----|--------|---------|------------|
| A1 | 0.4225966, 0.5474440; -0.3818371, -0.4974646 | (0.1285016, 0.0744819; -0.8894645, -0.8260330) | 0.4008132 |
| A2 | 0.4425807, 0.5974804; -0.4891042, -0.5943579 | (0.1375916, 0.1199040; -0.9376415, -0.9019055) | 0.3846627 |
| A3 | 0.4717833, 0.6302160; -0.3562212, -0.4891728 | (0.1710891, 0.1728958; -0.8769665, -0.8906689) | 0.4200639 |
| A4 | 0.5028514, 0.6663226; 0.4047916, -0.5334460 | (0.2187768, 0.1407743; -0.8067707, -0.8342712) | 0.3781066 |
| A5 | 0.4274683, 0.5393345; 0.4580571, -0.5409282 | (0.0739845, 0.0583823; -0.9252436, -0.8823078) | 0.4107505 |

Table 9 Distance from each alternative to the RPs

|    | A1 | A2 | A3 | A4 | A5 |
|----|----|----|----|----|----|
| C1 | 0.255907 | 0.238978 | 0.334082 | 0.488264 | 0.210929 |
| C2 | 0.239743 | 0.065948 | 0.133515 | 0.061409 | 0.206372 |
| C3 | 0.197237 | 0.559454 | 0.480286 | 0.422639 | 0.254388 |
| C4 | 0.470491 | 0.175282 | 0.342983 | 0.186342 | 0.592428 |
| C5 | 0.268585 | 0.360024 | 0.400082 | 0.298611 | 0.193585 |
| C6 | 0.238724 | 0.154020 | 0.000000 | 0.308566 | 0.174887 |
| C7 | 0.28279 | 0.327463 | 0.396162 | 0.243167 | 0.279038 |
| C8 | 0.232187 | 0.192375 | 0.151389 | 0.153274 | 0.146336 |
| C9 | 0.465415 | 0.301343 | 0.541312 | 0.100672 | 0.317115 |
| C10 | 0.270029 | 0.302137 | 0.395172 | 0.195029 | 0.270029 |
| C11 | 0.193827 | 0.291339 | 0.226392 | 0.138817 | 0.177858 |
| C12 | 0.208054 | 0.124891 | 0.000000 | 0.350405 | 0.170712 |
| C13 | 0.380313 | 0.157696 | 0.451722 | 0.231219 | 0.272608 |
| C14 | 0.266087 | 0.127177 | 0.350127 | 0.268403 | 0.221066 |
| C15 | 0.249538 | 0.218258 | 0.128248 | 0.000000 | 0.356659 |
Let us take Δ \(_j^+\) = \(\alpha^+_j e^{(i\theta^+_j)}\), β\(_j^+\) e\(^{(i\vartheta^+_j)}\) and Δ\(_j^-\) = \(\alpha^-_j e^{(i\theta^-_j)}\), β\(_j^-\) e\(^{(i\vartheta^-_j)}\).

**Step 9:** Estimate the BCF-distances \(D(d^W_{rj}, Δ^+_j)\) and \(D(d^W_{rj}, Δ^-_j)\) \((r \in N_5; j \in N_{15})\) where the values \(D(d^W_{rj}, Δ^+_j)\) and \(D(d^W_{rj}, Δ^-_j)\) are calculated using Eqs. (29) and (30).

\[
D(d^W_{rj}, Δ^+_j) = \frac{1}{3} \left(|\alpha^+_j - \alpha^-_j| + |\delta^+_j - \delta^-_j| + |\beta^+_j - \beta^-_j| + |\vartheta^+_j - \vartheta^-_j|\right)
\]

**Step 10:** The distances of the alternatives from the PIS and the NIS are calculated as:

\[
\bar{\Omega}^+_r = \sum_{j=1}^{n} D(d^W_{rj}, Δ^+_j) \text{ and } \bar{\Omega}^-_r = \sum_{j=1}^{n} D(d^W_{rj}, Δ^-_j) \text{ for } r \in N_5.
\]

**Step 11:** Obtain the closeness index values of all alternatives by utilizing the formula given below:

\[
\varrho_r = \frac{\bar{\Omega}^+_r}{\bar{\Omega}^+_r + \bar{\Omega}^-_r} \quad (r \in N_5)
\]

**Step 12:** Alternatives are ranked according to their closeness index values \(\varrho_r\) \((r \in N_5)\).

In Table 12, we depict the distances of alternatives from PIS and NIS. The closeness index of all alternatives and their final ranks are also given in Table 12.
suggested by the BCF-CRITIC-MULTIMOORA method is validated and credible.

(b) WASPAS model developed by Zavadskas et al. [101] has the utility to determine the optimal alternative that is very close to the optimal solution. WASPAS, an integration of WSM and WPM, is the emphatic new MCDM procedure. WASPAS model is more accurate compare to WSM and WPM. Moreover, WASPAS technique enables us to meet the highest accuracy of estimation. For the purpose of comparison, we consider BCF-CRITIC-WASPAS method [56] which is an original extension of WASPAS by combining it with CRITIC technique in BCF setting. We apply this in the case study considered earlier and the final score values of $A_1, A_2, A_3, A_4, A_5$ are respectively 0.798277, 0.763399, 0.848130, 0.845793, 0.756968 according to which the ranking order is $A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5$ and the best 3PRLP is $A_3$. This also means that the ranking result suggested by the BCF-CRITIC-MULTIMOORA method is validated and credible.

The above results are summarized in Fig. 2.

Next, to illustrate the strengths of the developed approaches, we also apply the existing methods [3, 7, 16, 30, 35, 59] to the same numerical example discussed earlier. The results are summarized in Table 13. Table 13 clearly demonstrates the superiority of the proposed method over the existing methods [3, 7, 16, 30, 35, 39].

**Conclusions**

In today’s complex environment, selecting an appropriate 3PRLP becomes more significant for most companies to accomplish the objectives of sustainable development and environmental safety. This process involves quantitative and qualitative criteria to choose the most desirable provider. Several methods have already been propounded by numerous researchers to get the best 3PRLP provider. We know that uncertainty is one of the widespread and major problems arising in the procedure of MCDM because of time-bound, a dearth of information, or larger complexity of socio-economic conditions. In this context, the more versatile and flexible BCFSs, as the successive extension of FSs, BFSs and CFSs, can be exploited to tackle the incertitude of real-world decisive problems as BCFSs mainly can negotiate with erratic and periodic bipolar fuzzy data in complex geometry. This present paper deals with an authentic integrated BCF-CRITIC-MULTIMOORA approach developed through the BCF-Archimedean power weighted AOs to achieve aggregated results, CRITIC Method to compute criteria weights and MULTIMOORA method to pick out the optimal option under the BCF environment. Next, we consider a 3PRLP selection problem in the regime of BCF to ensure the effectiveness of the method developed in this study. Afterward, a comparison is studied with the introduced and corresponding related BCF-CRITIC-TOPSIS and BCF-CRITIC-WASPAS methods that validate the outcomes. The outcomes implicate that the proposed BCF-CRITIC-MULTIMOORA approach is serviceable and well-consistent.

The proposed BCF-CRITIC-MULTIMOORA method has the following advantages:

- As BCFSs are extended versions of FSs and CFSs, so they can deal more dubious complex data that exists in practical decision-making problems. Thus, our developed method is more general.
- The proposed Archimedean weighted AOs can eliminate the influence of extreme evaluating criteria values from some biased DEs with different preference attitudes under the BCF setting. In other words, BCF-Archimedean weighted can reduce the impact of extreme assessment criteria values from some biased decision-experts with various inclination perspectives. Thus, the inclusion of these operators in the decision-making process makes the process more reasonable.
| Factors                                      | Methods                                      | Proposed  | [16] Govindan et al. | [35] | [59] | [30] | [7] Akram et al. |
|---------------------------------------------|---------------------------------------------|-----------|----------------------|------|------|------|-----------------|
| Information type                            | Bipolar complex fuzzy                       | Fuzzy     | Fuzzy                | Fuzzy| Interval type-2 fuzzy | Bipolar complex fuzzy | Bipolar fuzzy |
| Whether deals with group decision-making?   | Yes                                         | No        | Yes                  | No   | No   | No   | Yes             |
| Methods for aggregation                     | MULTIMOORA                                  | Fuzzy TOPSIS | Fuzzy TOPSIS      | Fuzzy MOORA | WASPAS | NIL | Fuzzy PROMETHEE |
| Criteria weights calculation                | CRITIC                                      | Direct    | Direct              | Fuzzy SWARA | CRITIC | Direct | Shannon Entropy measure |
| Calculation of DEs weight                   | Power aggregation                           | NA        | NA                  | NA   | NA   | NA   | NA              |
| Generality and flexibility of the Aggregation operators | Very high                                  | NA        | NA                  | NA   | High | NA   | NA              |
| Whether captures two-dimensional information | Yes                                         | No        | No                  | No   | No   | No   | Yes             |
| Whether captures bipolar information        | Yes                                         | No        | No                  | No   | No   | No   | No              |
| Ranking of alternatives                     | $A_3 > A_1 > A_4 > A_5 > A_2$               | Can’t be determined | Can’t be determined | Can’t be determined | Can’t be determined | Can’t be determined | $A_3 > A_1 > A_4 > A_2 > A_5$ |

*Table 13* Comparative study: proposed with existing methods
To develop the BCF-power weighted AOs, we have used Archimedean operations (Archimedean norm and conorm) between BCFNs because Archimedean operations are flexible and decision-makers can adopt the suitable functions depending on the risk preferences. Thus our proposed AOs are much flexible.

Our proposed method determines the criteria weights by using the CRITIC method which is a well-known objective method. This framework is based on aggregated score values of options, intensity contrast of every criterion and conflict among attributes. Intensity contrast of attribute is esteemed to standard deviation (SD) and conflict among them is calculated by the correlation coefficient (CRC). Thus, the inclusion of CRITIC technique makes the decision-making problem more realistic.

Our proposed approach is based on MULTIMOORA approach which is one of the most renowned MCDM tools to enhance the MOORA model. MULTIMOORA framework involves three sub-methods, that is, the RS procedure, the RP procedure, and the FMF procedure.

A characteristic comparison between MULTIMOORA method and other MCDM methods can be found in Table 14 presented as follows:

As mentioned above, the proposed BCF-CRITIC-MULTIMOORA methodology has several advantages. But it has certain drawbacks too as mentioned below:

1. It is based on Archimedean power aggregation operators on the bipolar complex fuzzy environment and thus cannot consider the interrelationships among criteria.

2. It does not consider both the subjective and objective weights of experts.

3. It is not suitable when the number of experts is more than 11 because in that case, the problem becomes a large-scale group decision-making problem.

To overcome the drawbacks, in the future, other AOs namely Bonferroni mean operators, Hamy mean operators, Maclaurin symmetric mean operators, and others can be developed with BCFSs, new decision models with integrated approaches like integrated MARCOS method, integrated TODIM method and others can be developed for providing a practical solution to decision problems, namely-cluster analysis, pattern recognition, charging station’s site selection for electric vehicle, treatment technology selection for medical waste, technological forecasting method selection, cloud vendor selection problem etc. Further, information measures such as divergence measures and uncertain measures for BCFSs can be developed for the determination of criteria weights. Moreover, based on consistency harmonious weight coefficient and similarity between DEs preferences, subjective and objective weights of DEs can be formulated. Lastly, a consensus-based behavioral TOPSIS method can be developed with BCF information if the number of experts exceeds 11. It is pertinent to mention that the proposed methodology can be extended to bipolar complex Pythagorean fuzzy and bipolar complex q-rung orthopair fuzzy environments.

Declarations

Conflict of interest The authors declare that they don’t have any conflict of interest.

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Appendix: Proof of the Theorems

Proof of Theorem 1 The first result holds immediately from definition 7. Now to show the rest part, we use the method of mathematical induction on n which are summarized as follows:

For n = 1, the result is obvious. For n = 2, we have,

$$BCFAPWAA(ξ_1, ξ_2) = (θ_1 * ξ_1) \text{̃⊕} (θ_2 * ξ_2)$$

$$= \left( \left( g^{-1}(θ_1 g(α_1)) e^{α g^{-1}(θ_1 g(δ_1)))} \right) + \left( g^{-1}(θ_2 g(α_2)) e^{α g^{-1}(θ_2 g(δ_2)))} \right) \right)$$

$$= (h^{-1}(θ_1 h(β_1))) e^{α g^{-1}(θ_1 g(δ_1))) + (h^{-1}(θ_2 h(β_2))) e^{α g^{-1}(θ_2 g(δ_2)))$$

Thus Eq. (1) holds good for n = 2. Let us assume that Eq. (1) holds for n = r. Then,

$$BCFAPWAA(ξ_1, ξ_2, ..., ξ_r)$$
Thus, Eq. (1) holds good for \( n = r + 1 \) also. Hence, by the Principle of induction, we conclude that Eq. (1) is true for all natural numbers \( n \).

**Proof of Theorem 2** Suppose \( \xi_0 = \{a_0 e^{(\omega_0)}, b_0 e^{(\omega_0)}\} \).

Then we have by Eq. (1),

\[
\begin{align*}
\text{BCFAPWA} &= (\xi_0 \oplus \xi_1, \xi_0 \oplus \xi_2, \ldots, \xi_0 \oplus \xi_n). \\
\text{Then we have by Eq. (1),}
\end{align*}
\]

Next utilizing the proposed operations and using Eq. (1), we get,

\[
\begin{align*}
\xi_0 \oplus \text{BCFAPWA} &= (\xi_1, \xi_2, \ldots, \xi_n) \\
\text{Then we have by Eq. (1),}
\end{align*}
\]
= \left( g^{-1} \left( g(\alpha_0) + \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \right) e^{\omega \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\delta_j) \right) \right)},
- \left( h^{-1} \left( h(\beta_0) + \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) e^{-\omega \left( h^{-1} \left( h(\beta_0) + \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right)}
\right)\).$$

Hence, $\text{BCFAPWA}(\xi_0, \xi_1, \xi_2, \ldots, \xi_n) = \xi_0 \oplus \text{BCFAPWA}(\xi_1, \xi_2, \xi_3, \ldots, \xi_n)$.

**Proof of Theorem 3** Suppose $\xi_0 = \langle \alpha_0 e^{(\phi_0)}, \beta_0 e^{(\omega_0)} \rangle$.

Since $\xi_j = \xi_0 \forall j$, we have, $\alpha_0 = \alpha_j, \beta_0 = \beta_j, \phi_0 = \phi_j \forall j = 1, 2, 3, \ldots, n$. Then from Eq. (1), we get,

$\text{BCFAPWA}(\xi_1, \xi_2, \xi_3, \ldots, \xi_n)$

$$= \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \right) e^{\omega \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\delta_j) \right) \right)},
- \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) e^{-\omega \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right)}
\right)\).$$

$\text{BCFAPWA}(\xi_1, \xi_2, \ldots, \xi_n)$

$$= \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \right) e^{\omega \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\delta_j) \right) \right)},
- \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) e^{-\omega \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right)}
\right)\).$$

$\text{Proof of Theorem 4}$ For any $j \in \{1, 2, 3, \ldots, n\}$, we have, $\min_j(\alpha_j) \leq \alpha_j \leq \max_j(\alpha_j)$ and $\min_j(\delta_j) \leq \delta_j \leq \min_j(\delta_j)$. Since $g$ and $g^{-1}$ are increasing functions, we get,

$$\sum_{j=1}^{n} \theta_j g(\min_j(\alpha_j)) \leq \sum_{j=1}^{n} \theta_j g(\alpha_j) \leq \sum_{j=1}^{n} \theta_j g(\max_j(\alpha_j))$$

and

$$\sum_{j=1}^{n} \theta_j g(\min_j(\delta_j)) \leq \sum_{j=1}^{n} \theta_j g(\delta_j) \leq \sum_{j=1}^{n} \theta_j g(\max_j(\delta_j))$$

$$\Rightarrow g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\min_j(\alpha_j)) \right) \leq g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \leq g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\max_j(\alpha_j)) \right)$$

$$\Rightarrow g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\min_j(\delta_j)) \right) \leq g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\delta_j) \right) \leq g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\max_j(\delta_j)) \right)$$

$$\Rightarrow \min_j(\alpha_j) \leq g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \leq \max_j(\alpha_j) \text{ and } \min_j(\delta_j) \leq g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\delta_j) \right) \leq \max_j(\delta_j)$$

$$\Rightarrow \phi^+ \leq g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \leq \alpha^+ \text{ and } \eta^+$$

$$\leq g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\delta_j) \right) \leq \delta^+.$$
Similarly, we can show that $\Phi^- \leq -h^{-1}$ and $\psi^- \leq -h^{-1}$.

Now $S(\xi^-) = \frac{1}{4} (2 + \psi^- + \alpha^- + \Phi^- + \psi^-) \\
\leq \frac{1}{4} \left[ 2 + g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) + g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\delta_j) \right) \right] \\
- \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) - h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\vartheta_j) \right) \\
= S(BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n))$.

Again, $S(BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n)) = \frac{1}{4} \left[ 2 + g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) + g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\delta_j) \right) \right] \\
- \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) - h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\vartheta_j) \right) \\
\leq \frac{1}{4} (2 + \alpha^- + \delta^- + \beta^- + \vartheta^-) \\
= S(\xi^-)$.

Combining these two results, we get, $S(\xi^-) \leq S(BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n)) \leq S(\xi^+)$. Hence, by ranking rules of BCFNs, we get, $\xi^- \prec BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) \prec \xi^+$.

**Proof of Theorem 5** We have from Theorem 1,

$BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \left( \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \right) e^{-w(\sum_{j=1}^{n} \theta_j g(\delta_j))} \right) \times \\
- \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) e^{-w(\sum_{j=1}^{n} \theta_j h(\vartheta_j))} \right)$

$BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \left( \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \right) e^{-w(\sum_{j=1}^{n} \theta_j g(\delta_j))} \right) \times \\
- \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) e^{-w(\sum_{j=1}^{n} \theta_j h(\vartheta_j))} \right)$

$BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) = \left( \left( g^{-1} \left( \sum_{j=1}^{n} \theta_j g(\alpha_j) \right) \right) e^{-w(\sum_{j=1}^{n} \theta_j g(\delta_j))} \right) \times \\
- \left( h^{-1} \left( \sum_{j=1}^{n} \theta_j h(\beta_j) \right) \right) e^{-w(\sum_{j=1}^{n} \theta_j h(\vartheta_j))} \right)$

Hence, $\xi^- \prec BCFAPWAA(\xi_1, \xi_2, \xi_3, \ldots, \xi_n) \prec \xi^+$.

**References**

1. Agrawal S, Singh RK, Murtaza Q (2016) Disposition decisions in reverse logistics: graph theory and matrix approach. J Cleaner Prod 137:93–104
2. Akram M, Arshad M (2019) A novel trapezoidal bipolar fuzzy TOPSIS method for group decision-making. Group DecisNeg 28:565–584
3. Akram M, Shumaiza, A.N.Al-Kenani (2020a) Multi-criteria group decision-making for selection of green suppliers under bipolar fuzzy PROMETHEE process. Symmetry 12(1):77
4. Akram M, Shumaiza, A.N.Al-Kenani (2020b) Bipolar fuzzy TOPSIS and bipolar fuzzy ELECTRE-I methods to diagnosis. Comput Appl Maths 39(7):1–21
5. Alghamdi MA, Alshehri NO, Akram M (2018) Multi-criteria decision-making methods in bipolar fuzzy environment. Int J Fuzzy Syst 20:2057–2064
6. Al-Husban A, Amournah A, Jaber JI (2020) Bipolar complex fuzzy sets and their properties. Ital J Pure Appl Maths 43:754–761
7. Alkouri AUMJ, Massa’deh MO, Ali M (2020) On bipolar complex fuzzy sets and its application. J Intell Fuzzy Syst. https://doi.org/10.3233/IFS-191350
8. Andersson D, Normann A (2002) Procurement of logistics services—a minutes work or a multiyear project. Eur J Purch Supply Manag 8(1):3–14
9. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
10. Azadi M, Saen RF (2011) A new chance-constrained data envelopment analysis for selecting third-party reverse logistics providers in the existence of dual-role factors. Expert Syst Appl 38:12231–12236
11. Bai C, Sarkis J (2019) Integrating and extending data and decision tools for sustainable third-party reverse logistics provider selection. Comput Oper Res 110:188–207
12. Baležentis A, Baležentis T, Brauers WKM (2012) MULTIMOORA-FG: a multi-objective decision-making method for linguistic reasoning with an application to personnel selection. Informatica 23(2):173–190
13. Baležentis T, Zeng SZ. Baležentis A (2014) MULTIMOORA-MIFN: a MCDM method based on intuitionistic fuzzy number for performance management. Econ Comput Econ Cybern Stud Res 48(4):85–102
14. Baležentis T, Zeng SZ, Baležentis A (2013) Group multi-criteria decision making based upon interval-valued fuzzy numbers: an extension of the MULTIMOORA method. Exp Syst Appl 40(2):543–550
15. Banaeian N, Bobil H, Neilsen IE, Omid M (2015) Criteria definition and approaches in green supplier selection—a case study for raw material and packaging of food industry. Prod Manuf Res 3(1):149–168
16. Bottani E, Rizzi A (2006) A fuzzy TOPSIS methodology to support outsourcing of logistics services. Supply Chain Manag 11(4):294–308
17. Brauers WKM, Zavadskas EK (2012) Robustness of MULTIMOORA: a method for multi-objective optimization. Informatica 23(1):1–25
18. Brauers WKM, Zavadskas EK (2006) The MOORA method and its application to privatization in a transition economy. Control Cybern 35(2):445–469
19. Brauers WKM, Zavadskas EK (2010) Project management by MULTIMOORA as an instrument for transition economies. Technol Econ Dev Econ 16(1):5–24
20. Brauers WKM, Baležentis A, Baležentis T (2011) Multimora for the EU member states updated with fuzzy number theory. Technol Econ Dev Econ 17(2):259–290
21. Chen X, Zhao L, Liang H (2018) A novel multi-attribute group decision-making method based on the MULTIMOORA with linguistic evaluations. Soft Comput 22:5347–5361
22. Chen L, Duan D, Mishra AR, Alrasheedi M (2021) Sustainable third-party reverse logistics provider selection to promote circular economy using new uncertain interval-valued intuitionistic fuzzy-projection model. J Enterp Inf Manag. https://doi.org/10.1108/JEIM-02-2021-0066
23. Cousens MDG (2000) Consensus eating. North Atlantic Books, Berkeley
24. Datta S, Sahu N, Mahapatra S (2013) Robot selection based on grey-MULTIMOORA approach. Grey Syst Theory Appl 3(2):201–232
25. Diakoulaki D, Mavrotas G, Papayannakis L (1995) Determining objective weights in multiple criteria problems: the CRITIC method. Comput Oper Res 22:763–770
26. Dong L, Gu X, Wu X, Liao H (2019) An improved MULTIMOORA method with combined weights and its application in assessing the innovative ability of universities. Expert Syst Appl. https://doi.org/10.1011/exsy.12362
27. Efendigil T, Onut S, Kongar E (2008) A holistic approach for selecting a third party reverse logistic provider in the presence of vagueness. Comput Ind Eng 54(2):269–287
28. Geetha S, Narayananmooorthy S, Kang D, Kureethara JV (2019) A novel assessment of healthcare waste disposal methods: intuitionistic hesitant fuzzy MULTIMOORA decision-making approach. IEEE Access 7:130283–130299
29. Ghorabae MK, Amiri M, Zavadskas EK, Antuchevicj E (2018) A new hybrid fuzzy MCDM approach for evaluation of construction equipment with sustainability considerations. Arch Civ Mech Eng 18(1):32–49
30. Ghorabae MK, Amiri M, Zavadskas EK, Antuchevicj E (2017) Assessment of third-party logistics providers using a CRITIC–WASPAS approach with interval type-2 fuzzy sets. Transport 32(1):66–78
31. Goebel P, Reuter C, Pibernik R, Sichtmann C (2012) The influence of ethical culture on supplier selection in the context of sustainable sourcing. Int J Prod Econ 140(1):7–17
32. Gou X, Liao H, Xu Z, Herrera F (2017) Double hierarchy hesitant fuzzy linguistic term set and MULTIMOORA method: a case of study to evaluate the implementation status of haze controlling measures. Inf Fusion 38:22–34
33. Govindan K, Murugesan P (2011) Selection of third-party reverse logistics providers using fuzzy extent analysis. Benchmarking 18(1):149–167
34. Govindan K, Palaniappan M, Zhu Q, Kannan D (2012) Analysis of third party reverse logistics provider using interpretive structural modeling. Int J Prod Econ 140(1):204–211
35. Govindan K, Pokharel S, Kumar PS (2009) A hybrid approach using ISM and fuzzy TOPSIS for the selection of reverse logistics provider. Resour Conserv Recycl 54(1):28–36
36. Gündoğdu FK (2019) A spherical fuzzy extension of MULTIMOORA method. J Intell Fuzzy Syst 38(2):1–16
37. Hafezalkotob A, Hafezalkotob A (2016) Extended MULTIMOORA-FG: a multi-objective decision-making approach. Int J Fuzzy Syst 23(6):2358–2370
38. Hafezalkotob A, Hafezalkotob A (2017) MOORA method based on Shannon entropy weight for materials selection. Inf Fusion 28:565–584
39. Hafezalkotob A, Hafezalkotob A, Sayadi MK (2016) Extension of MULTIMOORA with interval type-2 fuzzy sets and its properties. Ital J Pure Appl Maths 43:754–761
40. Han Y, Shi P, Chen S (2015) Bipolar-valued rough fuzzy set and its applications to decision information system. IEEE Trans Fuzzy Syst 23(6):2358–2370
41. Hussain M, Awasthi A, Tiwari MK (2016) Interpretive structural modeling-analytic network process integrated framework for evaluating sustainable supply chain management alternatives. Appl Math Model 40:3671–3687
42. Hwang CL, Yoon KS (1981) Multiple attribute decision-making: methods and applications. Springer, pp 58–191
43. Kannan G (2009) Fuzzy approach for the selection of third party reverse logistics Provider. Asia Pac J Mark Logist 21(3):397–416
44. Kannan G, Murugesan P, Senthil P, Haq AN (2009) Multicriteria group decision making for the third party reverse logistics service provider in the supply chain model using fuzzy TOPSIS for transportation services. Int J Serv Technol Manag 11(2):162–181
45. Kim M, Park M, Jeong D (2004) The effects of customer satisfaction and switching barrier on customer loyalty in Korean mobile telecommunication services. Telecommun Policy 28:145–159
46. Klement EP, Mesiar R (2005) Logical, algebraic, analytic and probabilistic aspects of triangular norms. Elsevier, New York
47. Kwang JK, Jeon JJ, Park JC, Park YJ, Kim CG, Kim TH (2007) The impact of network service performance on customer satisfaction and loyalty: high-speed internet service case in Korea. Exp Syst Appl 32:822–831
48. Langley CJ, Allen OR, Tyndall OR (2002) Third party logistics study 2002: results and findings of the seventh annual study. Council of Logistics Management, Chicago
49. Li ZH (2014) An extension of the Multimoora method for multicriteria group decision making based upon hesitant fuzzy sets. J Appl Math. https://doi.org/10.1155/2014/527836
50. Li W, Wu H, Jin M, Lai M (2017) Two-stage remanufacturing decision makings considering product life cycle and consumer perception. J Clean Prod 161:581–590
51. Li YL, Ying CS, Chin KS, Yang HT, Xu J (2018) Third-party reverse logistics provider selection approach based on hybrid-information MCDM and cumulative prospect theory. J Clean Prod 195:573–584
52. Liang W, Darko AP, Zeng J (2019) Interval-valued Pythagorean fuzzy power average based MULTIMOORA method for multicriteria decision-making. J Exp Theor Artif Intell. https://doi.org/10.1080/0952813X.2019.1694589
53. Liang W, Zhao G, Hong C (2019) Selecting the optimal mining method with extended multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) approach. Neural Comput Appl 31:5871–5886
54. Liang Y (2020) An EDAS method for multiple attribute group decision-making under intuitionistic fuzzy environment and its application for evaluating green building energy-saving design projects. Symmetry. https://doi.org/10.3390/sym12030484
55. Liao Z, Liao H, Gou X, Xu ZS, Zavadskas EK (2019) A hesitant fuzzy linguistic Choquet Integral-based MULTIMOORA method for multi-criteria decision-making. J Exp Theor Artif Intell. https://doi.org/10.1080/0952813X.2019.1694589
56. Liu P, Saha A, Misra AR, Rani P, Dutta D, Baidya J (2020) An extension of the Multimoora method for solving complex decision-making problems based on the use of combined compromise solution framework. Clean Responsible Consum 2:100011
57. Mishra AR, Rani P, Krishankumar R, Zavadskas EK, Cavallaro F, Ravichandran KS (2021) A hesitant fuzzy combined compromise solution framework-based on discrimination measure for ranking sustainable third-party reverse logistic providers. Sustainability 13:2064
58. Mishra AR, Rani P, Pandey K (2021) Fermatean fuzzy CRITIC-EDAS approach for the selection of sustainable third-party reverse logistics providers using improved generalized score function. J Ambient Intell Humaniz Comput. https://doi.org/10.1007/s12652-021-02902-w
59. Mohr J, Spekman R (1994) Characteristics of partnership success: partnership attributes, communication behavior, and conflict resolution techniques. Strat Manag J 15(2):135–152
60. Peng X (2019) New multi-parametric similarity measure and distance measure for interval neutrosophic set with IoT industry evaluation. IEEE Access 7:28258–28280
61. Peng X, Huang H (2020) Fuzzy decision making method based on CoCoSo with CRITIC for financial risk evaluation. Techn Econ Dev Econ. https://doi.org/10.3846/tede.2020.11920
62. Peng X, Zhang X, Luo Z (2020) Pythagorean fuzzy MCDM method based on CoCoSo and CRITIC with score function for 5G industry evaluation. Arti Intell Rev 53:3813–3847
63. Peretti U, Tatham P, Wu Y, Sgarbossa F (2015) Reverse logistics in humanitarian operations: challenges and opportunities. J Human Logist Supply Chain Manag 5(2):253–274
64. Prakash C, Barua MK (2016) An analysis of integrated robust hybrid model for third-party reverse logistics partner selection under fuzzy environment. Resour Conserv Recycl 108:63–81
65. Ramot D, Friedman M, Langholz G, Kandel A (2003) Complex fuzzy logic. IEEE Trans Fuzzy Syst 11:450–461
66. Ramot D, Milo R, Friedman M, Kandel A (2002) Complex fuzzy sets. IEEE Trans Fuzzy Syst 10(2):171–186
67. Razzaque MA, Sheng CC (1998) Outsourcing of logistics functions: a literature survey. Int J Phys Distrib Logist Manag 28(2):89–107
68. Saen RF (2009) A mathematical model for selecting third-party reverse logistics providers. Int J Procure Manag 2(2):180–190
69. Saen RF (2010) A new model for selecting third-party reverse logistics providers in the presence of multiple dual-role factors. Int J Adv Manuf Technol 46(1):405–410
70. Senthil S, Srirangacharyulu B, Ramesh A (2014) A robust hybrid multi-criteria decision-making methodology for contractor evaluation and selection in third-party reverse logistics. Exp Syst Appl 41(1):50–58
71. Shumaiza AM, Al-Kenani AN, Alcantud JCR (2019) Group decision-making based on the VIKOR method with trapezoidal bipolar fuzzy information. Symmetry 11(10):1313
72. Shumaiza AM, Al-Kenani AN (2019) Multiple-attribute decision making ELECTRE II method under bipolar fuzzy model. Algorithms 12(11):1–24
73. Singh PK (2020) Bipolar l-equivalent complex fuzzy concept lattice with its application. Neu Comput Appli 32:2405–2422
74. Stankevičienė J, Maidenos D, Kraujaliene I (2019) MULTIMOORA as the instrument to evaluate the technology transfer process in higher education institutions. Econ Sociol 12(2):345–360
75. Stanujkic D, Karabasevic D, Zavadskas EK, Smarandache F, Brauers WKM (2019) A bipolar fuzzy extension of the MULTIMOORA method. Informatica 30(1):135–152
76. Stanujkic D, Zavadskas EK, Brauers WKM, Karabasevic D (2015) An extension of the multimoora method for solving complex decision-making problems based on the use of

Complex & Intelligent Systems (2021) 7:2503–2530 2529

67. Mishra AR, Rani P (2021) Assessment of sustainable third party reverse logistic provider using the single-valued neutrosophic

combined compromise solution framework. Clean Responsible Consum:100011

68. Mishra AR, Rani P, Krishankumar R, Zavadskas EK, Cavallaro F, Ravichandran KS (2021) A hesitant fuzzy combined compromise solution framework-based on discrimination measure for ranking sustainable third-party reverse logistic providers. Sustainability:13:2064

69. Mishra AR, Rani P, Pandey K (2021) Fermatean fuzzy CRITIC-EDAS approach for the selection of sustainable third-party reverse logistics providers using improved generalized score function. J Ambient Intell Humaniz Comput. https://doi.org/10.1007/s12652-021-02902-w

70. Mohr J, Spekman R (1994) Characteristics of partnership success: partnership attributes, communication behavior, and conflict resolution techniques. Strat Manag J:15(2):135–152

71. Peng X (2019) New multi-parametric similarity measure and distance measure for interval neutrosophic set with IoT industry evaluation. IEEE Access:7:28258–28280

72. Peng X, Huang H (2020) Fuzzy decision making method based on CoCoSo with CRITIC for financial risk evaluation. Techn Econ Dev Econ. https://doi.org/10.3846/tede.2020.11920

73. Peng X, Zhang X, Luo Z (2020) Pythagorean fuzzy MCDM method based on CoCoSo and CRITIC with score function for 5G industry evaluation. Arti Intell Rev:53:3813–3847

74. Peretti U, Tatham P, Wu Y, Sgarbossa F (2015) Reverse logistics in humanitarian operations: challenges and opportunities. J Human Logist Supply Chain Manag:5(2):253–274

75. Ramot D, Friedman M, Langholz G, Kandel A (2003) Complex fuzzy logic. IEEE Trans Fuzzy Syst:11:450–461

76. Ramot D, Milo R, Friedman M, Kandel A (2002) Complex fuzzy sets. IEEE Trans Fuzzy Syst:10(2):171–186

77. Razzaque MA, Sheng CC (1998) Outsourcing of logistics functions: a literature survey. Int J Phys Distrib Logist Manag:28(2):89–107

78. Saen RF (2009) A mathematical model for selecting third-party reverse logistics providers. Int J Procure Manag:2(2):180–190

79. Saen RF (2010) A new model for selecting third-party reverse logistics providers in the presence of multiple dual-role factors. Int J Adv Manuf Technol:46(1):405–410

80. Senthil S, Srirangacharyulu B, Ramesh A (2014) A robust hybrid multi-criteria decision-making methodology for contractor evaluation and selection in third-party reverse logistics. Exp Syst Appl:41(1):50–58

81. Shumaiza AM, Al-Kenani AN, Alcantud JCR (2019) Group decision-making based on the VIKOR method with trapezoidal bipolar fuzzy information. Symmetry:11(10):1313

82. Shumaiza AM, Al-Kenani AN (2019) Multiple-attribute decision making ELECTRE II method under bipolar fuzzy model. Algorithms:12(11):1–24
interval-valued triangular fuzzy numbers. Transform Bus Econ 14(2B):355–375
83. Stanujkic D, Zavadskas EK, Smarandache F, Brauers WK, Karabasevic D (2017) A neutrosophic extension of the MULTIMOORA method. Informatica 28(1):181–192
84. Tajik G, Azadnia AH, Ma’aram A, Hassan S (2013) Hybrid fuzzy MCDM approach for sustainable third-party reverse logistics provider selection. Adv Mater Res 845:521–526
85. Tamir DE, Lin J, Kandel A (2011) A new interpretation of complex membership grade. Int J Intell Syst 26:285–312
86. Tavana M, Zareinejad M, Santos-Arteaga FJ (2018) An intuitionistic fuzzy-grey superiority and inferiority ranking method for third-party reverse logistics provider selection. Int J Syst Sci Oper Logist 5(2):175–194
87. Tavana M, Zareinejad M, Santos-Arteaga FJ, Kaviani MA (2016) A conceptual analytic network model for evaluating and selecting third-party reverse logistics providers. Int J Adv Manuf Technol 86:1705–1721
88. Tian G, Liu X, Zhang M, Yang Y, Zhang H, Lin Y, Ma F, Wang X, Qu T, Li Z (2019) Selection of take-back pattern of vehicle reverse logistics in China via Grey-DEMATEL and Fuzzy-VIKOR combined method. J Clean Prod 220:1088–1100
89. Tian ZP, Wang J, Wang JQ, Zhang HY (2017) An improved MULTIMOORA approach for multi-criteria decision-making based on interdependent inputs of simplified neutrosophic linguistic information. Neural Comput Appl 28(1):585–597
90. Turki S, Sauvey C, Rezg N (2018) Modelling and optimization of a manufacturing/remanufacturing system with storage facility under carbon cap and trade policy. J Clean Prod 193:441–458
91. Uygun Ö, Kaçamak H, Kahraman ÜA (2015) An integrated DEMATEL and fuzzy ANP techniques for evaluation and selection of outsourcing provider for a telecommunication company. Comput Ind Eng 86:137–146
92. Wei G, Lei F, Lin R, Wang R, Wei Y, Wu J, Wei C (2020) Algorithms for probabilistic uncertain linguistic multiple attribute group decision making based on the GRA and CRITIC method: application to location planning of electric vehicle charging stations. Econ Res Ekonomiska Istraživanja 33(1):828–846
93. Wu X, Liao H, Xu Z, Hafezalkotob A, Herrera F (2018) Probabilistic linguistic MULTIMOORA: a multi-criteria decision-making method based on the probabilistic linguistic expectation function and the improved Borda rule. IEEE Trans Fuzzy Syst 26(6):3688–3702
94. Wu SM, You XY, Liu HC, Wang LE (2020) Improving quality function deployment analysis with the cloud MULTIMOORA method. Int Trans Oper Res 27(3):1600–1621
95. Xian S, Liu Z, Gou X, Wan W (2020) Interval 2-tuple Pythagorean fuzzy linguistic MULTIMOORA method with CIA and their application to MCGDMM. Int J Intell Syst 35(4):650–681
96. Yager RR (2001) The power average operator. IEEE Trans Man Cybern 31(6):724–731
97. Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353
98. Zarbakhshnia N, Soleimani H, Ghaderi H (2018) Sustainable third-party reverse logistics provider evaluation and selection using fuzzy SWARA and developed fuzzy COPRAS in the presence of risk criteria. Appl Soft Comput 65:307–319
99. Zarbakhshnia N, Wu Y, Govindan K, Soleimani H (2020) A novel hybrid multiple attribute decision-making approach for outsourcing sustainable reverse logistics. J Clean Prod. https://doi.org/10.1016/j.jclepro.2019.118461
100. Zavadskas EK, Antucheviciene H, Hajjagha SHR, Hashemi SS (2015) The interval-valued intuitionistic fuzzy MULTIMOORA method for group decision-making in Engineering. Math Probl Eng Article ID 560690, 1–13
101. Zavadskas EK, Turkis Z, Antucheviciene J (2012) Optimization of weighted aggregated sum product assessment. Electron Electr Eng 6:122. https://doi.org/10.5755/j01.eee.122.6.1810
102. Zhang C, Chen C, Streimikiene D, Balezentis T (2019) Intuitionistic fuzzy MULTIMOORA approach for multi-criteria assessment of the energy storage technologies. Appl Soft Comput 79:410–423
103. Zhang WR (1994) Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis. In: The workshop on fuzzy information processing society bi conference, pp 305–309.
104. Zhang WR (1998) Bipolar fuzzy sets. In: Proceedings of FUZZY IEEE, pp 835–840
105. Zhang WR, Pandurangi KA, Peace KE, Zhang Y, Zhao Z (2011) Mental squares: A generic bipolar support vector machine for psychiatric disorder classification, diagnostic analysis and neurobiological data mining. Int J Data Min Bioinf 5(5):532–572
106. Zhang WR, Zhang L (2004) Bipolar logic and bipolar fuzzy logic. Inf Sci 165(3–4):265–287
107. Zhang X, Su T (2020) The dominance degree-based heterogeneous linguistic decision-making technique for sustainable 3PRLP selection. Complexity Article ID 6102036:1–18

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