Effects of pulse collisions in a multilayer system with noninstantaneous cubic nonlinearity

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Abstract
Numerical simulations of an ultrashort pulse propagation in a one-dimensional nonlinear photonic crystal are carried out. It is known that the relaxation of cubic nonlinearity is the reason for the effect of pulse self-trapping in such a multilayer system. In this paper we study further implications of this effect. It is shown that the trapped light absorbs additional low-intensity pulses which cannot be self-trapped per se. On the other hand, such low-intensity pulses are subject to the so-called induced trapping when light becomes trapped due to a collision of two such pulses. We consider the conditions for this effect in cases of both co- and counter-propagating pulses.

Keywords: photonic crystals, relaxing nonlinearity, self-trapping, ultrashort pulses

1. Introduction
It is known that the nonlinear response of optical media is not generally instantaneous and is described by a certain settling time. This relaxation of nonlinearity can be neglected if the characteristic time of the electromagnetic field (such as a pulse duration) is much greater than the relaxation time. However, in the modern era of ultrashort pulses, there is a growing number of situations when this neglect cannot be justified. The standard means to calculate the relaxation of non-resonant cubic (Kerr) nonlinearity is the so-called Debye model [1],

\[ \frac{d\delta n}{dt} + \delta n = n_2 I, \]  

(1)

where \( \delta n \) is the nonlinear part of the refractive index, \( n_2 \) the Kerr nonlinear coefficient, and \( I_{nl} \) the relaxation time. The latter depends on the specific mechanism of nonlinearity. In this paper it is assumed to be of the order of several femtoseconds.

The influence of the noninstantaneousness of the nonlinearity on the optical response has been studied in a number of works during the last several decades. Not aiming to name all of them, we can mention research on laser beam self-focusing [2–4], filament formation [5], parametric amplification [6], pulse compression [7], modulation instability [8–11], pulse train generation [12], soliton-array generation [13], instability of speckle patterns [14], solitary pulse dynamics [15, 16], optical switching [17, 18], etc.

In this paper we deal with ultrashort (femtosecond) pulse propagation in a one-dimensional nonlinear photonic crystal which is a set of periodically arranged dielectric layers. Such a multilayer structure can be symbolically designated as \((AB)^N\) (\(N\) is the number of pairs of layers of A and B type). One of the first studies of the role of nonlinearity relaxation in such a system is the paper by Vlasov and Smirnov [19] where the problem of pulse compression was under investigation. In recent works [20–23] the effect of pulse self-trapping in the photonic crystal due to relaxing nonlinearity was discovered and discussed. The present paper is a continuation of this research, so it is worth recalling briefly the main results obtained there.
It was shown [20] that a pulse of high enough intensity can be trapped inside a one-dimensional nonlinear photonic crystal due to formation of a dynamical nonlinear ‘cavity’, or ‘trap’. This trap appears only if both linear refractive index modulation and Kerr nonlinearity relaxation are present. The range of pulse durations and relaxation times for such a self-trapping effect to be observed in a multilayer system of several hundred layers was studied as well: \( t_{nl} \) varies from a fraction of a femtosecond to more than 100 fs, and \( t_p \) from about 10 fs to several hundred femtoseconds. This corresponds to the fast electronic mechanism of Kerr nonlinearity. Although we do not refer to any specific materials, it is believed that such nonlinear structures can be composed, for example, from doped glasses with rapidly relaxing nonlinearities. This leads to the requirement of comparatively high intensities of the pulses because of the approximate proportionality \( n_2 \sim n_{nl} \) [1] well known from experiments. Nevertheless, the necessary peak intensities, of the order of 100 GW cm\(^{-2}\), seem to be not excessively high for femtosecond pulses from the viewpoint of optical damage of the materials. Polymeric materials are worth mentioning due to both a high nonlinearity coefficient and fast relaxation [24], that make them prospective candidates for applications in the discussed situations. In [22] we studied in detail the conditions for self-trapping in different configurations of the structure (linear and nonlinear layers, focusing and defocusing nonlinearities, etc) taking into account the correlation between the nonlinearity coefficient and the relaxation time mentioned above. Another problem studied is the spectral transformations of light pulses interacting with a nonlinear photonic crystal in the regime of self-trapping [21]. In particular, under properly chosen conditions, it is possible to generate quasi-monochromatic radiation or a quasi-continuum covering the whole band gap. Finally, the possibility of asymmetric light transmission due to pulse self-trapping was analyzed recently [23].

In this paper we consider the situation not of a single pulse, but of many pulses inside the photonic crystal with relaxing nonlinearity. We are especially interested in investigation of the interaction of probe pulses with the previously trapped radiation. This problem is studied in section 2. Another question is connected with the possibility of light trapping due to a collision of two low-intensity pulses which are not trapped when they propagate separately. This situation, which we call induced trapping, is considered in section 3.

2. Trapping of probe pulses

First, let us state the main equations used in this paper. Propagation of an ultrashort pulse in a one-dimensional nonlinear photonic crystal is described by the Maxwell wave equation

\[
\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 (n^2 E)}{\partial t^2} = 0, \tag{2}
\]

where \( E \) is the electric field strength, \( c \) is the speed of light, and \( n \) is the refractive index, which depends on light intensity \( I = |E|^2 \) as

\[
n(z, t) = n_0(z) + \delta n(l, t). \tag{3}
\]

Here \( n_0(z) \) is the linear part of the refractive index varying periodically along the \( z \) axis, and the nonlinear addition \( \delta n(l, t) \) behaves according to equation (1). Further, we consider femtosecond light pulses of Gaussian shape with the amplitude \( A = A_0 \exp(-t^2/2\tau_p^2) \), where \( \tau_p \) is the pulse duration. To analyze the interaction of such a pulse with a nonlinear photonic crystal, i.e. to solve self-consistently the system (1)–(3), we use the finite-difference time-domain method of numerical simulations, which was described in detail in one of our previous papers [20]. The stability of the algorithm used is governed by the well known von Neumann condition \( \Delta t/\Delta z \leq 1/\nu = n/c \), where \( \Delta t \) and \( \Delta z \) are the time and space steps, respectively. If the ratio of the steps is small enough (taking into account a possible nonlinear change of \( n \)), one provides the necessary stability of calculation. The level of discretization (size of the steps) is chosen to be optimal with respect to both accuracy and calculation time and allows us to obtain the reliable general dependences discussed further.

The parameters of the photonic crystal (the structure of \((AB)^N\) type) used in our calculations are as follows: the linear parts of refractive indices of layers A and B are \( n_2 = 2 \) and \( n_0 = 1.5 \), respectively; their thicknesses \( a = 0.4 \) and \( b = 0.24 \) \( \mu \)m; the number of layers \( N = 200 \). The pulse duration is \( \tau_p = 30 \) fs, and the central wavelength of the initial pulse spectrum is \( \lambda_c = 1.064 \) \( \mu \)m if not stated otherwise. The nonlinear coefficient of the material is defined through the nonlinear term of the refractive index, so \( n_2\delta n_0 = 0.005 \); this means that the pulse amplitude is normalized by the value \( A_0 = \sqrt{n_0} \). The relaxation time of the nonlinearity of both layers is \( t_{nl} = 10 \) fs. We adopt these parameters in this paper, though similar effects can be obtained even in the half-linear structure [22]. It is also important to note that the parameters (in particular, the wavelength) satisfy the requirements on the sign of group velocity dispersion [21, 22].

Figure 1(a) shows the dependence of the output energy leaving the photonic crystal during a certain time interval after pulse incidence on the pulse amplitude. The output energy is calculated by integration of the intensity of light leaving the photonic crystal over time. The relative output energy normalized by the input one is

\[
W = Q_{\text{out}} / Q_{\text{inc}} = \frac{\int f_{\text{out}}(t) \, dt}{\int f_{\text{inc}}(t) \, dt}, \tag{4}
\]

where \( Q_{\text{inc}} \) and \( Q_{\text{out}} \) are the absolute values of energy of the incident and output light, respectively; \( f_{\text{inc}} \) and \( f_{\text{out}} \) are the corresponding intensity profiles. The value calculated by expression (4) at the input edge (incidence plane) of the structure gives the normalized energy of the reflected light, while the energy calculated at the exit edge corresponds to the transmitted light. The sum of these two values is the total output energy. The relative output energy in figure 1(a) is integrated over the time 300\( \tau_p \), which is approximately ten times larger than the pulse transmittance time in the linear regime (about 30\( \tau_p \)). The dip seen in the curve for the total output energy is the feature of self-trapping of the pulse. This
means that the pulses with amplitudes in a certain range are trapped inside the photonic crystal. Outside this range, the low-intensity pulses transmit through the system, while the high-intensity ones are mostly reflected.

The result of self-trapping is the formation of localized light intensity distribution, which is the indication of trap creation. This trap stores most of the pulse energy for times of the order of several thousand \( t_p \) [20]. It is worth noting that self-trapping does not mean light absorption, because the modulated refractive indices remain real. Let us study what happens when the second (probe) pulse interacts with this excited state of the photonic crystal containing trapped light. In this case the intensity of the light in equation (1) is governed by the sum of the field present at a certain space point and time instant. The second pulse starts at the instant \( 100 t_p \) after the first (trapped) one. The results of calculations of the output energy (integrated over the time \( 300 t_p \)) as a function of amplitude of the probe pulse are presented in figure 1(b). It is seen that the trap formed due to a pulse with the amplitude \( A_m = 3A_0 \) blocks propagation of a probe pulse with low enough intensity. This means that the energy of the probe pulse gets stored inside the photonic crystal, so that the trap collects more and more light. Our estimate shows that a probe pulse with \( A_m = A_0 \) loses about 60% of its energy due to interaction with the trap. The region of optimal self-trapping of single pulses (\( 3A_0 < A_m < 4A_0 \)) is naturally the range where most of both pulses is trapped. Finally, a high-intensity probe pulse (with \( A_m \geq 5A_0 \)) cannot be trapped. This, however, is connected not with the breakdown of the trap but with reflection of the high-intensity pulses seen in figure 1(a).

We prove these conclusions in figure 2, where the distributions of light intensity inside the photonic crystal at different time instants are shown. It is seen that, by the time \( t = 100 t_p \) (launch time of the probe pulse), the trap (bell-shaped stable light distribution) formed by the first pulse with the amplitude \( A_m = 3A_0 \) exists inside the multilayer system (the total length of the system is 128 \( \mu m \)). If the probe pulse has low peak amplitude (\( A_p = A_0 \), see figure 2(a)), it is just absorbed by the trap, the peak intensity of the distribution increasing from about 12\( I_0 \) to 14\( I_0 \). This distribution is slowly spreading with time. The situation is totally different for the probe pulse with the amplitude 7\( A_0 \) (figure 2(b)). It is seen that this high-intensity pulse is stopped near the very entrance of the photonic crystal and then effectively reflected. However, some of its light penetrates in the vicinity of the trap and perturbs it. As a result of this perturbation, the peak intensity of the distribution decreases (about 8\( I_0 \) at 300\( t_p \)) and the trap spreads and shifts towards the entrance of the structure. At time \( t = 1000 t_p \) the distribution is strongly widened and has a maximum of the order of 0.5\( I_0 \). In other words, the trap loses light energy faster than in the case of a low-intensity probe pulse and is less stable.

Nevertheless, we can state that, generally, the trap persists and cannot be overcome by a single probe pulse. To study the stability of the trap further, we launch more probe pulses inside the photonic crystal. The first probe pulse starts at \( t = 50 t_p \) after the initial one (which forms the trap); the interval between all subsequent probe pulses is 10\( t_p \). The integral output energy as a function of the number of probe pulses is plotted in figure 3(a). It is seen that at first the fraction of the energy leaving the multilayer structure is growing quite rapidly, but after the sixth probe pulse the trapped energy (in relative units) stays approximately the same. This means that the absolute value of light energy trapped inside the system is growing. The evidence for this conclusion is presented in figure 3(c), where the intensity distribution is shown for the case of 11 probe pulses. The peak value of intensity is about 50\( I_0 \) (at 300\( t_p \)), which is much greater than 14\( I_0 \) for the single probe pulse (see figure 2(a)). The profile of the transmitted radiation shown in figure 3(b) allows us to identify the intensity peaks corresponding to particular probe pulses. Notice that the first strongly pronounced peak is connected with the fourth probe pulse; i.e., the first, second, and third probe pulses are almost entirely absorbed. Every subsequent pulse appears more or less sharply in the output of the photonic crystal.

Returning to figure 3(c), one can note not only the increase in peak intensity of light in the trap but also the

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**Figure 1.** Dependence of the output light energy (normalized to the input energy) (a) on the peak amplitude of the incident pulse and (b) on the peak amplitude of the probe pulse interacting with the photonic crystal in the excited state (after trapping of the initial pulse with the amplitude \( A_m = 3A_0 \)). The energy is integrated over the time 300\( t_p \).
Figure 2. Distribution of light intensity inside the photonic crystal at different time points. The probe pulse amplitude is (a) $A_m = A_0$, (b) $A_m = 7A_0$. The peak amplitude of the first pulse is $A_m = 3A_0$.

shift of the trap towards the input of the system. Obviously, this process continues as we launch more and more pulses in the structure. Such a situation is shown in figure 4. We also see substantial spreading of the distribution for larger number of input pulses. Finally, for 100 probe pulses (lower three panels in figure 4), the bell-shaped distribution tends to collapse sooner than for a lesser number: the trap is absent at $t = 1500t_p$, which is about 500$t_p$ after the last pulse enters the system, while for 25 pulses the trap is still stable after 700$t_p$ (see left upper panel). Nevertheless, even at $t = 2000t_p$ approximately 40% of the energy of 100 pulses remains inside the photonic crystal due to multiple reflections on the numerous boundaries.

3. Induced trapping

In figure 1(a) we see that the transition between the regimes of propagation and trapping is an abrupt one. Therefore, the amplitude of this transition which is approximately equal to $2.2A_0$, can be called the critical amplitude. The pulses with subcritical intensity freely propagate through the photonic crystal (most of the light is simply reflected and transmitted), while the supercritical pulses are self-trapped. In this section we study the possibility for light trapping as a result of collision of two pulses with subcritical intensities. We refer to such a situation as induced trapping.

The first possible scheme is the collision of two co-propagating pulses. This scheme implies that the second
pulses should move faster than the first one and overtake it at a certain point in space. As a result, the light intensity will increase in the vicinity of this point and formation of the trap is expected due to summation of the fields of two subcritical pulses. To demonstrate the possibility of such a course of events, we should select the appropriate values of the parameters of the pulses so that they could meet inside the photonic crystal. It is known that the group velocity $v_{gr}$ of the pulse in the periodic structure depends on the wavelength: it is large in the transmission band and decreases towards the band edge. The length of the system is important at small differences between speeds of the pulses when one of them needs a lot of time to overtake the other; at best, they collide when they move close enough to one to another, i.e. the interval between pulses is not very large.

At $\lambda_{c1} = 1.075 \, \mu m$ we have nearly optimal induced trapping when there is a single low-intensity pulse at the exit. This transmitted pulse corresponds to the first incident one, while the second one is almost entirely absorbed. The trap forms close to the exit and has comparatively high peak intensity (about $9I_0$). As we move further towards the band gap shown in figure 6(a) by the dotted vertical line, the first pulse is reflected more and more, so the transmittance starts to increase as well. However, even near the very band edge (for $\lambda_{c1} = 1.11 \, \mu m$), some of the light energy (about 40%, to be exact) is trapped. The corresponding distribution shows that the trap forms near the entrance of the structure. This trap is perhaps due to multiple reflections from the interfaces between layers as well as the reflection from the output boundary of the photonic crystal. This is corroborated by the position of the trap (near the very output) and the structure of the transmitted radiation (two peaks of lowered intensity, in contrast to the single peak seen in the case of optimal induced trapping at $\lambda_{c1} = 1.075 \, \mu m$). It is worth noting that the interaction between the pulses can be observed when they move close enough to one to another, i.e. the interval between pulses is not very large.

The results of calculations of the output energy as a function of $\lambda_{c1}$ are presented in figure 6(a). The time interval between the peaks of co-propagating pulses is $\Delta t = 5t_p$, their amplitudes are equal and subcritical ($A_{m1} = A_{m2} = 2A_0$). As expected, effective trapping occurs for $\lambda_{c1} > \lambda_{c2}$ (i.e. when $v_{gr1} < v_{gr2}$), so it can be unambiguously interpreted as a result of pulse collision. The details of pulse interaction are clarified in figures 6(b) and (c), where the resulting intensity profiles and distributions (at $300t_p$) are plotted. It is seen that, when the first pulse moves faster ($\lambda_{c1} = 1.05 \, \mu m$) than the second one, there are two sharp peaks at the output. For the pulses with equal wavelengths ($\lambda_{c1} = 1.064 \, \mu m$), these peaks are much less pronounced and about half of the energy of the pulses stays inside the system. There is a bell-shaped light distribution near the very exit of the structure (see figure 6(c), upper right panel), which indicates the trap formed in this case. This trap is perhaps due to multiple reflections from the interfaces between layers as well as the reflection from the output boundary of the photonic crystal. This is corroborated by the position of the trap (near the very output) and the structure of the transmitted radiation (two peaks of lowered intensity, in contrast to the single peak seen in the case of optimal induced trapping at $\lambda_{c1} = 1.075 \, \mu m$). It is worth noting that the interaction between the pulses can be observed when they move close enough to one to another, i.e. the interval between pulses is not very large.

Thus, the effect of induced trapping can be observed in the rather wide range of wavelengths of the pulse near the band edge of the photonic crystal. This range is limited, at a given interval between the pulses, by the length of the structure and, on the other hand, by the reflectance near the band edge. The length of the system is important at small differences between speeds of the pulses when one of them needs a lot of time to overtake the other; at best, they collide near the exit of the system. The importance of the band edge in the spectral range of our interest, namely the near infrared region [25, 26]. In addition, we take only a narrow section of this range from about 1.05 to 1.12 $\mu m$. Thus, the only source of dispersion in the further study is the structural dispersion caused by the order of the photonic crystal layers.
appears in the opposite case of large velocity difference, when, though the pulses collide near the entrance of the system, most of the ‘slow’ pulse is reflected.

In figure 7 we study the dependence of the induced trapping of two identical co-propagating pulses (with amplitudes $2A_0$) on the interval $\Delta t$ between them. The central wavelength of the first pulse ($\lambda_{c1} = 1.075 \ \mu m$) is nearly optimal for trapping. $\Delta t = 0$ means that, in fact, we have a single pulse with the amplitude $4A_0$, i.e. the supercritical one. Trapping with approximately the same efficiency is observed for $\Delta t \leq 5t_p$, but then the output energy starts to increase in a stepwise manner. This is accompanied by a rise of transmitted light as the trap forms nearer to the exit of the system (see the panels corresponding to $\Delta t = 7t_p$ and $9t_p$). Finally, at $\Delta t = 11t_p$, the collisions do not happen: the interval is too large for the pulses to have time to collide. This is the same limit due to the finiteness of the length of the photonic crystal.

The next question is the dependence of the induced trapping on the amplitudes of the pulses at the fixed values of other parameters (we adopt $\Delta t = 5t_p$ and $\lambda_{c1} = 1.075 \ \mu m$). Without loss in generality, we can vary only the amplitude of the second (‘fast’) pulse $A_{m2}$, leaving the amplitude of the first one the same as previously ($A_{m1} = 2A_0$). The results of calculations at different $A_{m2}$ are shown in figure 8. It is seen in figure 8(a) that the output energy starts to decrease at $A_{m2} > A_0$ and reaches the minimum at approximately $2A_0$. At $A_{m2} > 2.5A_0$ there are local peaks in the curves of total and transmitted energy. These peaks can be interpreted as a result of effective trapping of the second, high-intensity pulse preventing the collision. This conclusion is proved in figure 8(c): the transmitted pulse profiles have approximately the same peak intensity in the cases $A_{m2} = 0$ (the single first pulse) and $A_{m2} = 3A_0$ (the trapping of the second pulse), while in the intermediate variant ($A_{m2} = 2A_0$) the transmitted pulse is strongly suppressed due to the induced trapping. The intensity distributions in figure 8(d) demonstrate the difference between the trap formation due to the collision (at $A_{m2} = 2A_0$) and single pulse absorption (at $A_{m2} = 3A_0$). Note that, in the last case, the trap forms at larger distance than in the case of the single $3A_0$ pulse (see figure 2 at $t = 75t_p$). This means that the dynamics of the second pulse in the two-pulse scheme still depend on the first one.

Figure 8(a) does not allow us to demarcate unambiguously the regions of induced trapping and single pulse trapping. To carry out this demarcation, we calculate the excess in output energy, i.e. the difference between output energies in the cases of two pulse and single pulse trapping. The parameters of the structure are given in the text.
Figure 6. (a) Dependence of the output light energy (normalized to the input energy) on the wavelength of the first pulse $\lambda_{c1}$. The amplitude of both pulses is $A_{m1} = A_{m2} = 2A_0$; the wavelength $\lambda_{c2} = 1.064$ $\mu$m; the time interval between the pulses $\Delta t = 5t_p$; the energy is integrated over the time $300t_p$. (b) Profiles of transmitted and reflected light for different $\lambda_{c1}$. (c) Corresponding distributions of light intensity inside the photonic crystal at $t = 300t_p$.

propagation. This value can be written as

$$\Delta W = \frac{W_{12}Q_{12}^{inc} - W_1Q_{1}^{inc} - W_2Q_{2}^{inc}}{Q_{12}^{inc}},$$

where the relative output energies $W_{12}, W_1$ and $W_2$ are defined by equation (4) in the instances of both pulses, single first and single second pulse, respectively. $Q_{12}^{inc}, Q_{1}^{inc}$, and $Q_{2}^{inc}$ are the corresponding absolute energies of incident light ($Q_{12}^{inc} = Q_{1}^{inc} + Q_{2}^{inc}$). The value $\Delta W$ shows how much extra energy leaves the system when both pulses are launched in comparison with the single pulse cases; if it is negative, $\Delta W < 0$, then one can say that the additional energy is trapped inside the multilayer structure due to the interaction between the pulses. The excess value $\Delta W$ extracted from the data in figure 8(a) is shown in (b). This figure is the evidence that, in the region $A_0 < A_{m2} < 2.7A_0$, effective trapping occurs due to the collision of the pulses (up to 80% of the total energy can be additionally trapped). At higher values of the amplitude, $A_{m2} > 2.7A_0$, the excess value is small and approximately constant, $\Delta W \geq -0.1$, which means that only a minor part of light is trapped due to the interpulse interaction. This proves our previous conclusion about practically independent trapping of the second pulse in the high-intensity regime.

Finally, we should consider the case of counter-propagating pulses colliding inside the photonic crystal. Obviously, it is enough to consider the pulses with identical subcritical intensities and central wavelengths. We take the latter to be $\lambda_{c} = 1.064$ $\mu$m, while the former is varied. The results of calculation of the energy excess for this case are presented in figure 9. It is seen that, at low intensities ($A_{m} < 2A_0$), the excess is negligible, which means that the pulses propagate independently from one another. At higher intensities, a small negative excess appears; this means that only a few per cent of the energy of the pulses is trapped due to their interaction. If we increase the pulse amplitude further ($A_{m} > 4A_0$), $\Delta W$ becomes positive, i.e. an additional part of the energy (less than 20%) is released due to the simultaneous presence of both pulses inside the structure.

Thus, there is no evidence of induced trapping in the situation of counter-propagating pulses. One can suppose that the reason is the short interaction time between the pulses. Indeed, in the case of co-propagating pulses, they travel one after another exchanging energy for a long time. In contrast, the intersection of two subcritical counter-propagating pulses is too short lived to lead to any substantial result. The calculations at different wavelength show that slowing down of pulses does not help to improve this situation. Only at
Figure 7. (a) Dependence of the output light energy (normalized to the input energy) on the time interval between the pulses $\Delta t$. The amplitude of both pulses is $A_{m1} = A_{m2} = 2A_0$; the wavelengths $\lambda_{c1} = 1.075 \, \mu m$, $\lambda_{c2} = 1.064 \, \mu m$; the energy is integrated over the time $300t_p$. (b) Profiles of transmitted and incident light for different $\Delta t$. (c) Corresponding distributions of light intensity inside the photonic crystal at $t = 300t_p$.

high intensities, when the pulses are trapped very soon after launching, do the traps (which are unstable in this case) seem to be sensitive to the presence of the second pulse.

4. Conclusion

To sum up, in this paper we have considered the interaction of many ultrashort pulses with a photonic crystal possessing relaxing cubic nonlinearity. As known from our previous investigations, there is a possibility of pulse self-trapping in such a structure. First, we analyzed the influence of trapped light on the behavior of additional (probe) pulses. It is shown that the trap formed by the high-intensity (supercritical) pulse can absorb probe pulses with low intensity, i.e. subcritical pulses which do not demonstrate self-trapping on their own account. This effect can serve as a peculiar absorber for low-intensity pulses. We have also studied the changes produced in the trap by the incidence of many probe pulses.

The second topic of this paper concerns induced trapping, i.e. the effect of light trapping as a result of collision of two subcritical pulses. We have shown that this phenomenon occurs in the case of two co-propagating pulses with different velocities: when one of them overtakes the other, their interaction leads to effective trapping of their energy. In contrast, the collision of two counter-propagating pulses does not allow us to observe induced trapping.

The results presented in this paper are of general character and therefore leave the question of their particular realization open. Here we briefly discuss some important issues. We have used the simple and well known Debye model of relaxation, though the particular nonlinear media can relax according to different laws. This raises an interesting question of the dependence of the results on the relaxation model, though it is likely that the particular model is not as important for the effects calculated as the presence of relaxation per se. The choice of model is perhaps closely associated with the choice of relaxation time. The relaxation times used in our research (up to 10 fs) make another demand of appropriate materials. Such short $t_{nl}$ values are characteristic, for example, of the electronic mechanism and its combinations with other contributions to the Kerr nonlinearity. However, our analysis is purely phenomenological and does not include these important details.

Another important issue is connected with the value of light intensities necessary to obtain the effects reported, in particular self-trapping of the pulse. Obviously, intensities should be very large, putting a question of optimization of
Figure 8. (a) Dependence of the output light energy (normalized to the input energy) on the amplitude of the second pulse $A_{m2}$. The amplitude of the first pulse is $A_{m1} = 2A_0$; the wavelengths $\lambda_{c1} = 1.075 \, \mu\text{m}$, $\lambda_{c2} = 1.064 \, \mu\text{m}$; the time interval between the pulses $\Delta t = 5t_p$; the energy is integrated over the time $300t_p$. (b) The excess in output energy in comparison with the case of single $A_{m1}$ and $A_{m2}$ pulses. (c) Profiles of transmitted light for different $A_{m2}$. (d) Corresponding distributions of light intensity inside the photonic crystal at $t = 300t_p$.

Figure 9. The excess in total output energy in the case of two counter-propagating pulses in comparison with the case of a single $A_{m}$ pulse. The wavelength $\lambda_{c} = 1.064 \, \mu\text{m}$; the energy is integrated over the time $300t_p$; both pulses start simultaneously.

the system in order to reduce the field. According to this paper, one way to reach this is to use induced trapping of relatively low-intensity pulses instead of one high-intensity pulse. Another possible approach is the adjustment of the photonic structure aimed at lowering the requirements on the materials and pulse parameters. This work is still to be done as well as analysis of more general situations. Such situations include taking into account absorption, spatial finiteness of the laser beam (we have considered only the plane wave approximation as yet), and other nonlinear contributions (for example, light frequency conversion).

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