A novel error equivalence model on the kinematic error of the linear axis of high-end machine tool

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Abstract
The kinematic errors of the linear axis play a key role in machining precision of high-end CNC (computer numerical control) machine tool. The quantification of error relationship is still an urgent problem to be solved in the assembly process of the linear axis, especially considering the effect of the elastic deformation of rollers. In order to obtain the kinematic errors of the linear axis of machine tool, a systematic error equivalence model of slider is proposed. The linear axis contains the base, the linear guide rail, and carriage. Firstly, the geometric errors of assembly surface of linear guide rail are represented by small displacement torsor. Then, according to the theory of different motion of robots, the error equivalence model of a single slider is established, namely the geometric errors of assembly surface of linear guide rail and the pose error of slider are equivalent to the elastic deformation of roller. Based on the principle of vector summation, the kinematic error of a single slider is mapped to the carriage, and the kinematic error of the linear axis is obtained. At the same time, experiments validation of kinematic error model of the linear axis is carried out. It is indicated that the proposed model is accurate and feasible. The analysis of key design parameters shows that the proposed model can provide an accurate guidance for the manufacturing and operation performance of the linear axis in quantification, and a more effective reference for the engineers at the design and assembly stage.

Keywords Linear axis · Kinematic error · Error equivalence · Small displacement torsor

1 Introduction

With the rapid development of precision manufacturing and unceasing enhancement demand, high-precision machining centers, namely multi-axis CNC machine tools, are widely used in all kinds of advanced manufacturing fields, such as automotive engines, aero-engines and gearboxes, valve bodies, and other pivotal components. Multi-axis precision CNC machine tools are composed of the linear axis, rotation axis, and other components. As an important translational component and with bearing capacity, the linear axis plays a crucial role in the machining accuracy of workpieces. Generally, the horizontal CNC machine tools with four axes consist of three linear axes (x, y, and z axis) and a rotation axis. The linear axis is composed of a base, two linear guide rails, four sliders, and a carriage. The kinematic error of the linear axis is mainly affected by the geometric error of the base and the kinematic error of the slider, meanwhile it also directly affects the machining accuracy of workpieces. However, the geometric tolerances of the installing surface of the linear axis are often given according to the designers’ experience; moreover, the influence factors of roller deformation inside the slider are ignored in the actual working situations. Therefore, in the design stage and manufacturing process, an accurate kinematic error prediction model of the linear axis is extremely essential. In addition, the kinematic error and machining accuracy of the linear axis can be improved from the aspects of design, manufacture, and assembly techniques.
At present, in the aspect of three-dimensional geometric error characterization analysis of parts, the mainstream methods include T-Map [1, 2], Jacobian Torsor model [3], and small displacement Torsor [4]. T-Map method is difficult to describe the interaction relationship and distribution of errors between parts. Jacobian Torsor model adopts the hypothesis of small angle and uses the constrained rotation vector to represent the small variation. The algorithm is simple, but the scale of transformation matrix is different, and the portability is weak. Small displacement Torsor employs translational vector and rotation vector to represent the error between the actual feature and the ideal feature; moreover, any part geometric feature can be represented. It can be widely applied. Asante [5] expressed the accumulated errors with small displacement torsor parameters, and obtained the machining characteristic boundary of the parts by solving and calculating the tolerance model. Jin et al. [6] applied the small displacement torsor method to the tolerance solution of the cone model, and proposed a representation method of the cone surface and its tolerance model, finally obtained the statistical analysis results through an example.

On one hand, in order to obtain the kinematic errors of the linear axis, the kinematic errors of a single slider moving along the guide rail need to be acquired first. Over the past few decades, a large number of studies have been carried out on the kinematic errors of a single slider. There exist three categories of mainstream research. The first one is that only the geometric errors of the linear guide rail are considered; the contact analysis between the guide rail and slider is ignored. With the measuring form errors of guideway tier, He et al. [7] proposed a systematic method of motion error estimation of a linear motion bearing table. Zha et al. [8] investigated the relationship between the profile error of guide rail and vertical motion straightness of y axis slider, and provide theoretic guidance for the precision design of open hydrostatic guideways. Xue et al. [9] presented a new method to analyze the motion error of a closed hydrostatic guideway and found that the wavelength of each profile error component is the main affecting factor. Considering the geometric profile error of guideway, Fan et al. [10] introduced a systematic approach to predict the kinematic error of machine tool’s guideway based on the guideway tolerance. In view of the installing base bending shape, Zhong et al. [11] established the error transfer model to obtain the accurate kinematic errors of a translational axis of machine tool. Zhang et al. [12] proposed a new approximate model to study the effect of geometric errors of guide rails and table on the motion errors of hydrostatic guideways; the result showed that the geometric error of guide rail is the main influence factor. The second one is that only the contact analysis between the guide rails and slides is performed to study the kinematic errors of linear guide rail; the geometric errors of guide rail and base are neglected. Using the finite element analysis, Chlebus et al. [13] presented a method to obtain the static properties of guideway joints with contact layers. Pawel Majda [14] introduced an analytical examination approach, aiming to study the influence of static characteristics of linear guideway on the joint kinematic errors of machine tool table. Based on Hertz contact theory, the elastic deformation of guideway and roller was considered to construct the static model to study the kinematic errors of linear guideway [15, 16]. Jeong et al. [17] established the eight-spring equivalent model to link the motion components and guideways, and adopted the finite element method to analyze the static and dynamic properties of linear guideway. Zou and Wang [18] investigate the contact stiffness variation of linear rolling guides due to the effect of friction and wear, and indirectly analyzed the precision condition of linear rolling guides by the mode shape. The third one is that the geometric error and elastic deformation of guide rail are both considered to study the kinematic errors of linear guideway, namely error equivalent model construction considering deformation factors; however, the research object and the key point are different. Different from the line contact form of the linear guide rail with rollers, the linear guide rail with rolling ball is the point contact form. Ma et al. [19] proposed a method of combining the geometric error of rolling guide and the elastic deformation of rolling ball, aiming to study the motion error rule of slider under different working situations. Khim et al. [20] established the transfer function between the geometric error of the guide rail and the restoring force, and also the balance equation in the motion process to analyze the motion error of the linear guide rail. In view of the difficulty in obtaining the geometric error of guide rail, Khim et al. [21] further substituted the geometric error of guide rail with the straightness error of base, and analyzed the motion error of linear guide rail. Taking into account both the influence of the form error of guide rails and the reaction force of a bearing pad, Khim et al. [20] presented a simple and systematic estimation method of motion errors of 5-DOF aerostatic linear motion stage. Kim et al. [22] proposed a motion error estimation method for a slide table, considering the effect of the reaction moment resulting from a pad pitch error; this method constructed a new reaction force-moment model with a double spring system.

On the other hand, the model of how to obtain the kinematic error of the linear axis by the kinematic errors of a single slider has been studied by many scholars. Zhong et al. [11] presented the error transfer model of a translational axis based on the screw theory, aiming to figure out how the based geometric errors or slider kinematic errors are transferred to the carriage kinematic errors. Based on the hierarchical ways, He et al. [7] proposed an approach to estimate the motion errors of a linear motion bearing table and developed a mapping model from the form errors of guideway Tier to the motion errors of Slider Tier, and to Table Tier. As for the aerostatic bearings, Ekinci et al. [23, 24] proposed an equation system, aiming at
investigating the relationship from the geometric errors of guideway to the motion errors of axis. Khim et al. [25] introduced a prediction model of motion accuracy of a multi-sliders motion table and constructed the transfer function between the form errors of rail and the bearing force of a bearing block. Pawel Majda [14] used the finite element method to equivalently obtain the geometric errors of guideways, and established an analysis model of the influence of guideway errors on the kinematic errors of linear guideway of machine tools.

As can be seen from the above researches, the scholars gave many insights about the geometric errors modeling and measurement of linear guideway, also the relationship between the geometric error of guideway, and the kinematic error of the linear axis. However, majority of them are mainly focused on the geometric errors modeling of a single linear guide rail or a single slider or the base; the kinematic errors of carriage and the linear axis are not concerned. Moreover, in some studies, the effect of the rolling body deformation on the kinematic errors of the linear axis is omitted. In addition, in the design process, the information of geometric error of guide rail and base is far from enough. Generally, the processing position change of workpiece is mainly guaranteed by the linear axis precision, so it is very essential to obtain the kinematic error of linear axis. Furthermore, the kinematic error of the linear axis is influenced by the assembly process, including the hierarchical error transfer of base, guide rail, and carriage. More importantly, the kinematic error prediction model of the linear axis of machine tool can provide an accurate guidance for precision design of new machine tools. Obviously, little attention has been devoted to the kinematic errors of the linear axis based on the geometric errors of guide rail and the elastic deformation of rollers in the literatures. In other words, little research has been devoted to an accurate and quantitative theoretical guidance in the relationship between the geometric errors of guide rail and the elastic deformation value of rollers and the kinematic errors of the linear axis. Hence, an accurate model of predicting the kinematic errors of the linear axis based on the error equivalence model of slider is extremely significant in the actual engineering design, manufacturing, and assembly techniques.

In view of the above-mentioned limits, in this paper, in order to obtain the kinematic error of linear axis, a systematic error equivalence model is proposed. Firstly, the geometric errors of assembly surface of linear guide rail are represented by small displacement torsor. Then, according to the theory of different motion of robots, the error equivalence model of a single slider is established. Little work focused on the error equivalence method, the contribution point of the proposed model is that the geometric error of assembly surface of linear guide rail and the pose error of slider are equivalent to the elastic deformation of roller. Afterwards, based on the principle of vector summation, the kinematic error of a single slider is mapped to the carriage, and the kinematic error of the linear axis is obtained. Finally, a measuring experiment of the kinematic errors of the linear axis is performed, aiming to verify the validity of the proposed model by comparing the theoretical results and the experimental values. The rest of this paper is organized as follows: Section 2 gives a geometric error representation of linear guide rail based on MCS(Monte Carlo Simulation)method. Section 3 proposes an error equivalence model of guide rail. The contribution point is that the geometric error of guide rail and the pose error of slider are equivalent to the elastic deformation of rollers. Subsequently, the mapping model of the kinematic errors from a single slider to the linear axis is established. Finally, the predict model of kinematic errors of linear axis is obtained. Section 4 performs a measuring experiment to verify the validity of the proposed model by comparing the theoretical results and experimental values. Section 5 discusses the influence factors on the kinematic errors of linear axis. Section 6 draws conclusions and plans the future work.

2 Geometric error representation based on MCS method

The linear axis of machine tool is mainly composed of base, guide rail, slider, roller, carriage, and other parts, the carriage reciprocates along the direction of the guide rail. The kinematic error of linear axis depends on the geometric error and deformation of the parts. The geometric error value of base and linear guide rail is unknown in the design stage, while its tolerance value is the interval value that restricts its change. The geometric tolerance is easier to obtain, but it is difficult for the precise distribution function of the geometric error. The sampling statistical analysis method is often used to analyze its uncertain parameters under the unknown distribution function. Therefore, the interval parameters and small displacement torsor method are adopted to describe the uncertain geometric error. Besides, MCS method is employed for simulation error; the geometric error distribution of linear guide rail is obtained under meeting the geometric tolerance constraints.

2.1 Geometric error constraints of guide rail

Torsor theory is an important mathematical tool for studying space motion. Torsor is composed of two three-dimensional vectors, which respectively represent the direction and position of vectors [26]. Furthermore, small displacement torsor is commonly used, which is a vector composed of weeny displacement generated by a rigid body with six motion components. In 1996, Bourdet et al. [27] first introduced small displacement torsor into the tolerance modeling field, and proposed to replace the variation of the actual tolerance surface in tolerance zone with the variation of small displacement torsor
parameters. In three-dimensional space, the geometric elements of the geometric error can be expressed by their respective small displacement torsors [28]. Ref. [29] listed several common torsor expressions corresponding to form and position errors.

The kinematic error of the linear axis of a machine tool is mainly manifested as the deviation between the actual and ideal position of rolling guide rail, which can be expressed by small displacement torsor parameters \((u,v,0,\alpha,\beta,\gamma)\). According to the design requirements, neglecting the dimension error, the pose error of rolling guide rail is determined by the planeness error of the assembly main datum surface, the perpendicularity error, and the parallelism error of the assembly subordinate datum surface. In this research, they are called the geometric error of linear guide rail, as shown in Fig. 1. For the purpose of simplifying analysis, the coordinate system is set at the center of the main assembly datum surface. In Fig. 2a, the width and length of the assembly main datum system is set at the center of the main assembly datum surface.

(1) Flatness error

The assembly datum surface of the guide rail usually has the main datum surface \(A\) and two subordinate datum surfaces \(B\) and \(C\). The geometric errors (flatness, perpendicularity, and parallelism) of the assembly surface of guide rail are shown in Fig. 1. For the purpose of simplifying analysis, the coordinate system is set at the center of the main assembly datum surface. In Fig. 2a, the width and length of the assembly main datum surface \(A\) are \(a\) and \(d\), and surface \(C\) and \(B\) are the subordinate datum surface for the left and right guide rail respectively; the height of the assembly main datum surface \(A\) are \(H\); the distance of surface \(B\) and \(C\) along \(x\) axis is \(b\). According to the design requirements, the flatness error of assembly main datum surface is \(T_a\); the perpendicularity error of right datum surface \(B\) and \(A\) is \(T_b\), and the parallelism error of left datum surface \(C\) and \(B\) is \(T_c\). Assuming that the dimension tolerance is neglected, the flatness error of the assembly main datum surface \(A\) is \(T_a\), so the torsor model is expressed as \((0, v_T, 0, 0, \alpha_T, \beta_T, \gamma_T)\), and the variation range constraints [6] of the torsor parameters are as follows:

\[
\begin{bmatrix}
-T_a/2 \\
-T_a/d \\
-T_a/a
\end{bmatrix}
\leq
\begin{bmatrix}
v_T \\
\alpha_T \\
\beta_T
\end{bmatrix}
\leq
\begin{bmatrix}
T_a/2 \\
T_a/d \\
T_a/a
\end{bmatrix}
\quad (1)
\]

Fig. 1 Geometric errors of assembly surface of linear guide rail

The variation range formulas of main assembly datum surface \(A\) are as follows:

\[
\begin{bmatrix}
y_T \\
\Delta y_T
\end{bmatrix}
=\begin{bmatrix}
v_T \\
\delta_T \\
\alpha_T
\end{bmatrix}
\begin{bmatrix}
x \\
\gamma \\
z
\end{bmatrix}
\quad (2)
\]

where \(\Delta y_T\) represents the distance between any two points on the surface. The length and width of rectangular surface are limited within the flatness error \(T_a\); the extreme value occurs at the vertex of the rectangular surface. Therefore, the torsor parameter constraint formulas of flatness error of assembly main datum surface are expressed as follows:

\[
\begin{bmatrix}
-T_a/2 \\
-T_a/2
\end{bmatrix}
\leq
\begin{bmatrix}
y_T \\
\Delta y_T
\end{bmatrix}
\leq
\begin{bmatrix}
T_a/2 \\
T_a/2
\end{bmatrix}
\quad (3)
\]

where, \(x,z\) value is the four limit positions of surface \(A\), that is four corners on the assembly main datum surface \(A\) in the figure, namely the setting range of \(x\) value is \((-a/2, a/2)\), and the setting range of \(z\) value is \((-d/2, d/2)\).

(2) Parallelism error

According to the design requirements, the assembly subordinate datum surface \(C\) of the left guide rail is required to have a parallelism error \(T_c\) with the assembly main datum surface \(A\); furthermore, the corresponding small displacement torsor model is \((u_c, 0, 0, 0, \beta_c, \delta_c)\), and the variation range constraints [6] are as follows:

\[
\begin{bmatrix}
-T_c/2 \\
-T_c/d \\
-T_c/H
\end{bmatrix}
\leq
\begin{bmatrix}
u_c \\
\beta_c \\
\delta_c
\end{bmatrix}
\leq
\begin{bmatrix}
T_c/2 \\
T_c/d \\
T_c/H
\end{bmatrix}
\quad (4)
\]

Similar to the constraints form of flatness error, the torsor parameters constraints of parallelism error are as follows:

\[
\begin{bmatrix}
-T_c/2 \\
-T_c/2
\end{bmatrix}
\leq
\begin{bmatrix}
\delta_c \\
\beta_c \\
\gamma
\end{bmatrix}
\leq
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\quad (5)
\]

where, \(y,z\) value is the four limit positions of surface \(C\), that is four vertices on the assembly main datum surface \(C\) in the figure, namely the setting range of \(y\)
value is \( (0, H) \) and the setting range of \( z \) value is \( (-d/2, d/2) \), as shown in Fig. 2b.

(3) Perpendicularity error

Similar to the subordinate datum surface \( C \) of left guide rail, according to the design requirements, the assembly subordinate datum surface \( B \) of the right guide rail is required to have a perpendicularity error \( T_c \) with the assembly main datum surface \( A \); moreover, the corresponding small displacement torsor model is \( (u_Y, 0, 0, \beta_Y, \delta_Y) \), and the variation range constraints [6] are as follows:

\[
\begin{bmatrix}
-T_b/2 \\
-T_b/d \\
-T_b/H
\end{bmatrix} \leq \begin{bmatrix}
u_Y \\
\beta_Y \\
\delta_Y
\end{bmatrix} \leq \begin{bmatrix}
T_b/2 \\
T_b/d \\
T_b/H
\end{bmatrix}
\]

(6)

Similar to the constraints form of parallelism error, the torsor parameters constraints of perpendicularity error are as follows:

\[
\begin{bmatrix}
-T_b/2 \\
-T_b/d \\
-T_b/H
\end{bmatrix} \leq \begin{bmatrix}
\delta_Y \\
\beta_Y \\
\delta_Y
\end{bmatrix} \begin{bmatrix}
y \\
z
\end{bmatrix} + \begin{bmatrix}
0 \\
u_Y
\end{bmatrix} \leq \begin{bmatrix}
T_b/2 \\
T_b/d \\
T_b/H
\end{bmatrix}
\]

(7)

where \( y,z \) value is the four limit positions of surface \( B \), that is four corners on the assembly subordinate datum surface \( B \) in the figure, namely the setting range \( y \) value is \( (-d/2, d/2) \) and the setting range of \( z \) value is \( (0, H) \), as shown in Fig. 2c.

2.2 Geometric error simulation of guide rail

MCS is a method to solve approximate solutions of problems through statistical tests and stochastic simulation of random variables [30]. It has many advantages such as strong adaptability, simple calculation method, and solving errors independent of problem dimensions. With the MCS method, small displacement torsor parameters of geometric error are sampled; the sampling flow chart of MCS simulation is shown in Fig. 3. The generated corresponding random parameters are utilized to simulate flatness and parallelism and perpendicularity error. Taking the flatness error of linear guide rail’s main assembly datum surface \( A \) as an example, the steps to simulate the actual variation range of flatness error by MCS are as follows:

(1) Determine an ideal probability distribution model of geometric error torsor parameters. In general, the geometric error distribution of parts manufactured by mechanical processing conforms to the law of normal distribution [31], and its probability density function is assumed to be as follows:

\[
\phi(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left(-\frac{(x-u)^2}{2\sigma^2}\right) \hspace{1cm} (-\infty < x < +\infty, \sigma > 0)
\]

(8)

(2) Determine the mean value and variance of ideal probability distribution for torsor parameters of geometric error. From the perspective of engineering practice, it is generally considered that the distribution range of the normal distribution is \( \pm 3\sigma \). According to the torsor variation in Eq. (1), the mean and variance of the ideal distribution of torsor parameters \( v_y, \alpha_y, \delta_y \) are \( 0, T_d/6, 0, T_d/3d, 0, T_d/3a \), respectively. Similarly, the mean and variance of the torsor parameters \( u_y, \beta_y, \delta_y \) of parallelism error are \( 0, T_c/6, 0, T_c/3d, 0, T_c/3H \) and the torsor

\[\text{ideal surface}\]

\[\text{datum}\]

\[\text{ideal surface}\]

\[\text{datum}\]

Fig. 2 Variation range. a Flatness. b Parallelism. c Perpendicularity
parameters \( u_I, \beta_Y, \delta_T \) of perpendicularity error are \((0, T_y/6, 0, T_y/3d, 0, T_y/3H)\), respectively.

(3) According to the constraint requirements, the torsor parameters of geometric errors are sampled, and the total sample size in the current study was set at 10,000.

Similarly, both torsor parameter \((u_T, 0, 0, 0, \beta_z, \delta_T)\) samples of parallelism error and torsor parameter \((u_T, 0, 0, \beta_Y, 0, \delta_T)\) samples of perpendicularity error are also obtained.

The structure of the linear axis contains the base, linear guide rail, and carriage in Fig. 4. The kinematic error of the linear axis is determined by the combined pose error, composed by the assembly main datum surface and subordinate datum surface of left and right guide rails. According to the master-slave relation of the assembly guide rails, the right guide rail is taken as the datum one; the torsor parameters of pose error for left and right guide rail are \((u_T, v_T, 0, \alpha_T, \beta_z, \delta_T)\) and \((u_T, v_T, 0, \alpha_T, \beta_Y, \delta_T)\), respectively.

According to mentioned contents in above sections, torsor parameter of flatness error is \((0, v_T, 0, \alpha_T, 0, \delta_T)\); torsor parameter of parallelism error is \((u_T, 0, 0, 0, \beta_z, \delta_T)\), and torsor parameter of comprehensive pose error is \((u_T, v_T, 0, \alpha_T, \beta_z, \delta_T)\). The simulation results by MCS method is shown in Fig. 5.

Similarly, torsor parameter of flatness error is also \((0, v_T, 0, \alpha_T, 0, \delta_T)\); torsor parameter of perpendicularity error is \((u_T, v_T, 0, 0, 0, \beta_Y, \delta_T)\), and the torsor parameter of comprehensive pose error is \((u_T, v_T, 0, 0, \beta_Y, \delta_T)\). The simulation results by MCS method is shown in Fig. 6.

3 Error equivalence model of linear axis

Section 3 consists of two parts: kinematic error model of a single slider and kinematic error model of the linear axis. Firstly, the kinematic error of a single slider is calculated, and then kinematic error of the linear axis can be obtained through the principle of vector summation. Finally, as for the kinematic error solution process of linear axis, the flow chart of technical route is shown in Fig. 7, as follows.

3.1 Error equivalence model of a single slider

In order to obtain the kinematic error of the linear axis of machine tool, the kinematic error of a single slider is firstly calculated. We assume that the linear guide rail keep balance under the load namely its gravity and the elastic restoring force of rollers. Moreover, the geometric error of guide rail and the position error of slider will influence the contact condition among the slider, guide rail, and rollers in Fig. 8. At the same time, the elastic deformation of rollers can be considered. The error equivalence model is proposed, namely the geometric error of guide rail and the position error of slider are equivalent to the elastic deformation of rollers. Besides, Hertz’s contact theory is adopted to construct the static balance equations of linear guide rail. Consequently, the kinematic error of linear guide rail based on the error equivalence model is obtained.
The positioning accuracy of linear axis of the machine tool is mainly determined by the driving unit and the control unit. When the motion system of the linear axis moves along the $z$ axis, there occurs six kinematic errors; the positioning accuracy is neglected, including two translational errors ($\Delta z$ and $\Delta x$) along the $y$ and $x$ axis, and three rotational errors ($\Delta u$, $\Delta v$, $\Delta w$) around $x$, $y$, and $z$ axis, as shown in Fig. 9. Furthermore, the kinematic errors of these five degrees of freedom of the machine tool mainly derive from the straightness of the guide rail. Hence, in this paper, five items of kinematic errors, namely the

![Fig. 4 Structure sketch of the linear axis](image1)

![Fig. 5 MCS results of comprehensive pose error for left guide rail](image2)
straightness along \( x \) and \( y \) axis (\( \Delta y \) and \( \Delta x \)), and angular errors of pitch, yaw, and roll (\( \Delta u \), \( \Delta v \), \( \Delta w \)); the geometric representation of each error is listed in Table 1.

According to the practical load condition of linear guide rail in milling machining, it can be decomposed into the normal load \( F_{vi} \) on the upper surface of slider, the lateral load \( F_{Hi} \) perpendicular to the side surface of slider, and the rotational torque \( M_z \) around \( Z \) axis of translational direction. Assuming that four columns of rollers are denoted by number 1, 2, 3, and 4 in Fig. 10, the coordinate system \( o-xyz \) is established at the symmetrical center point \( O \) of rollers. As for each roller in \( i \)th column, the normal load from the slider contact surface is \( Q_i \) ('\( i = 1, 2, 3, 4 \)' ); moreover, the preload [32] of guide rail plays a key role in the kinematic error of a single slider; the initial deformation of each column of roller under the preload is \( \delta_0 \); the elastic deformation of the \( i \)th column of rollers is \( \delta_i \) ('\( i = 1, 2, 3, 4 \)' ). The center distance between two column of rollers are \( l_x \) and \( l_y \) along \( x \) and \( y \) axis, respectively. The angle between the rollers and raceway is \( \alpha \). Therefore, the displacement of slider under load in \( x \) and \( y \) axis is denoted as \( \delta_x \) and \( \delta_y \); the angle around \( z \) axis under rotational torque is \( \theta \). As a result, considering the structure characteristics of linear guide

Fig. 6  MCS simulation results of comprehensive pose error for left guide rail
rail, the elastic deformation of the $i$th column of rollers is expressed as in Eq. (9).

$$
\begin{align*}
\delta_1 &= \delta_0 + \delta_v \cos \alpha + \delta_H \sin \alpha - \frac{\theta (l_x \cos \alpha + l_y \sin \alpha)}{2} \\
\delta_2 &= \delta_0 + \delta_v \cos \alpha - \delta_H \sin \alpha + \frac{\theta (l_x \cos \alpha + l_y \sin \alpha)}{2} \\
\delta_3 &= \delta_0 - \delta_v \cos \alpha - \delta_H \sin \alpha - \frac{\theta (l_x \cos \alpha + l_y \sin \alpha)}{2} \\
\delta_4 &= \delta_0 - \delta_v \cos \alpha + \delta_H \sin \alpha + \frac{\theta (l_x \cos \alpha + l_y \sin \alpha)}{2}
\end{align*}
$$

(9)

When the slider is under the loads, the elastic deformation occurs between rollers and raceway. Since the width of the contact region between roller and raceway is far less than the curvature radius of contact point, the interaction between roller and raceway can be equivalent to the contact problem between an elastic cylindrical and a rigid plane [33]. Therefore, as for the motion of a finite length roller in the raceway, the relationship between the elastic deformation and load can be obtained by the Palmgren empirical formula [34], as shown in Eq. (10).
δ_i = 1.36 \times \left( \frac{(1-\nu_1)^2}{E_1} + \frac{(1-\nu_2)^2}{E_2} \right)^{0.9} \times \frac{Q_i}{l_e^{0.8}} \quad (10)

where the constant \( E_1, E_2, \nu_1, \) and \( \nu_2 \) are the elastic modulus and Poisson’s ratio of two contact body, respectively; \( l_e \) represents the effective length of roller. According to Eq. (10), for the elastic deformation \( \delta_i \) of the \( i \)th column of rollers, when \( \delta_i \leq 0 \), the initial deformation of the \( i \)th column of rollers disappears under the preload.

When \( \delta_i > 0 \), there occurs the roller deformation, so the normal load of contact surface on each column of roller is written as Eq. (11).

\[
Q_i = \begin{cases} \left( \frac{\delta_i l_e^{0.8}}{3.84 \times 10^5} \right)^{n_e}, & \delta_i > 0 \\ 0, & \delta_i \leq 0 \end{cases}
\]

(11)

For single linear guide rail and slider, force and moment keep balance at the coordinate origin O of guide rail, so the interaction between single guide rail and slider is obtained in Eq. (12). In addition, there are three assumptions in the proposed model: (1) the friction between roller and raceway is ignored; (2) the normal contact forces of all rollers in each raceway of a single slider are equal; (3) The center line between the raceway of guide rail and slider is parallel before and after rollers deformation. The load analysis of single guide rail and slider is shown in Fig. 10.

\[
\begin{align*}
(Q_1 \cdot \cos \alpha + Q_2 \cdot \cos \alpha - Q_3 \cdot \cos \alpha - Q_4 \cdot \cos \alpha) \times n_e &= F_w \\
(Q_1 \cdot \sin \alpha - Q_2 \cdot \sin \alpha - Q_3 \cdot \sin \alpha - Q_4 \cdot \sin \alpha) \times n_e &= F_H \\
\left(\frac{1}{2} + \frac{Q_1 \cdot (l_e \cdot \cos \alpha + l_y \cdot \sin \alpha)}{Q_2 \cdot (l_e \cdot \cos \alpha + l_y \cdot \sin \alpha)}ight)n_e &= M_z \\
\end{align*}
\]

(12)

where \( n_e \) denotes the number of effective rollers subject to load in each column. The normal force of contact surface calculated by Eq. (11) is substituted into Eq. (12), so the...
relationships between the normal load $F_{Vi}$, the side load $F_{Hi}$, rotational moment $M_z$, and the displacement $\delta_v$ along $x$ axis, the displacement along $y$ axis, and the angle $\theta$ around $z$ axis, is written as follows:

$$
\delta_v = f_1(F_{Vi}, F_{Hi}, M_z),
\delta_H = f_2(F_{Vi}, F_{Hi}, M_z),
\theta = f_3(F_{Vi}, F_{Hi}, M_z)
$$

(13)

where $f_1$, $f_2$, $f_3$ are the three different functions, respectively.

After the deformation and rotation angle of a single slider are obtained, the kinematic error of a single slider is calculated by the error equivalent model of linear guide rail. The geometric error of guide rail and the pose error of slider are equivalent to the elastic deformation of roller. The geometric error of guide rail is reflected as the offset of guide rail raceway center, and the pose error of slider is reflected as the offset of slider raceway center. Hence, the offset of the center distance of guide rail raceway and slider raceway is equivalent to the elastic deformation of roller. Ideally, the contact model of rollers before and after elastic deformation is shown in Fig. 11.

In Fig. 11a, $o$ is the geometric center of roller, $\alpha$ is the angle between the center line of roller and raceway and horizontal direction before deformation. $o_g$ and $o_s$ are the curvature center of guide rail and slider before deformation; $L_o$ is the distance of raceway center between guide rail and slider before deformation; $L_{xo}$ and $L_{yo}$ are the components in $x$ and $y$ direction in Eq. (14), respectively. In Fig. 11b, $\alpha'$ is the angle between the center line of roller and raceway and horizontal direction after deformation. $o'_g$ and $o'_s$ are the curvature center of guide rail and slider after deformation; $\delta_{ijkx}$ and $\delta_{ijky}$ are the horizontal and vertical offset of the actual curvature center of the guide rail raceway after deformation; $u_{ijkx}$ and $u_{ijky}$ are the horizontal and vertical offset of the actual curvature center of the slider after deformation, $L_{ijk}$ is the distance of raceway center between guide rail and slider after deformation; $L_{ijkx}$ and $L_{ijky}$ are the components in $x$ and $y$ direction in Eq. (16), respectively.

Fig. 10 Load analysis of single guide rail

Fig. 11 a Contact model of linear guide rail before deformation. b Error equivalence model after deformation
\[
\begin{align*}
L_{x0} &= D_a \times \cos \alpha \\
L_{y0} &= D_a \times \sin \alpha 
\end{align*}
\]  
(14)

Where \(D_a\) is the diameter of slider before deformation. On the basis of the deformation of slider in Eq. (13), the distance of raceway center between guide rail and slider after deformation is calculated by Eq. (15).

\[
\Delta = \sqrt{\left(\delta_v^2 + \delta_H^2\right)}
\]  
(15)

Then Eq. (15) is submitted into Eq. (16), so the center distance \(L_{ijk}\) between guide rail and slider raceway is deduced in Eq. (17). \(\lambda\) denotes the preload, in fact, the linear guide rail can be preloaded directly by increasing the roller diameter. Therefore, if the roller diameter is increased by \(\lambda\), the actual deformation is expressed as Eq. (16).

\[
\begin{align*}
\Delta &= \left(L_{ijk}^2 + L_{ijky}^2\right)^{1/2} - D_a + \lambda \\
L_{ijk} &= L_{ijk}^2 + L_{ijky}^2
\end{align*}
\]  
(16)

(17)

The center distance component of raceway after deformation in \(x\) and \(y\) direction is written as Eq. (18). Besides, the angle between the center line of raceway and horizontal direction after deformation is also as follows.

\[
\begin{align*}
L_{ijkx} &= L_{ijk} \times \cos \alpha' \\
L_{ijky} &= L_{ijk} \times \sin \alpha' \\
\alpha' &= \alpha + \theta
\end{align*}
\]  
(18)

The kinematic error of linear guide rail is expressed as the pose error of slider and guide rail; the pose error of slider and guide rail at a certain position is resulted from the comprehensive factors, namely the geometric error and the elastic deformation under external load. Relative to ideal position, there occurs five groups of errors for slider and guide rail, namely the horizontal displacement error \(\delta_x, u_x\) along \(x\) direction, the vertical displacement error \(\delta_y, V_T\) along \(y\) direction, the angular error pitch \(\varepsilon_x, \alpha_T\) around \(x\) axis, the angular error yaw \(\varepsilon_y, \beta_z\) around \(y\) axis, and the angular error roll \(\varepsilon_z, \delta_T\) around \(z\) axis. Thus, take the left guide rail as an example. Due to the smaller error value, according to the theory of different motions of robots [35], the error equivalence model of a single slider is established, namely the geometric error of assembly surface of linear guide rail and the pose error of slider is equivalent to the elastic deformation of roller. The pose error transformation matrix of the slider and guide rail can be expressed as Eq. (19) and (20), respectively.

\[
\Delta T_g = \begin{bmatrix}
1 & -\varepsilon_x & \varepsilon_y & \delta_x \\
\varepsilon_z & 1 & -\varepsilon_x & \delta_y \\
-\varepsilon_y & \varepsilon_x & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(19)

Taking a single-row four-track guide rail as an example, assuming that the vector of the curvature center of the raceway is Eq. (21), corresponding to the \(j\)th roller in the \(i\)th raceway. Therefore, the deviations of the pose errors of slider and guide rail in the horizontal and vertical directions are respectively written as Eq. (22) and (23).

\[
\begin{bmatrix}
x_{ijk} & y_{ijk} & z_{ijk} & 1
\end{bmatrix}^T
\]  
(20)

\[
\begin{align*}
\mu_{ijk} &= \delta_x + \varepsilon_x \cdot z_{ijk} - \varepsilon_y \cdot y_{ijk} \\
\mu_{ijky} &= \delta_y - \varepsilon_y \cdot z_{ijk} + \varepsilon_x \cdot y_{ijk}
\end{align*}
\]  
(22)

\[
\begin{align*}
\delta_{ijks} &= u_x - \delta_T \cdot y_{ijk} + \beta_z \cdot z_{ijk} \\
\delta_{ijky} &= V_T + \delta_T \cdot x_{ijk} - \varepsilon_x \cdot z_{ijk}
\end{align*}
\]  
(23)

Then, caused by the geometric error and the elastic deformation, the horizontal displacement \(L_{ijks}\), and vertical displacement \(L_{ijky}\) of the center distance of raceway are written as follows:

\[
\begin{align*}
L_{ijks} &= L_{x0} - \mu_{ijk} + \delta_{ijks} \\
L_{ijky} &= L_{y0} - \mu_{ijky} + \delta_{ijky}
\end{align*}
\]  
(24)

Finally, according to Eqs. (9)–(24), the kinematic error of a single slider can be obtained, and written as follows:

\[
\begin{align*}
\delta_x &= L_{x0} - L_{ijks} + u_x \\
\delta_y &= L_{y0} - L_{ijky} + V_T \\
\varepsilon_x &= \alpha_T \\
\varepsilon_y &= \beta_z \\
\varepsilon_z &= \delta_T
\end{align*}
\]  
(25)

### 3.2 Kinematic error model of the linear axis

The geometric error of base and the kinematic error of a single slider are studied in above sections. Next, we will establish the kinematic error of the linear axis. In addition, the kinematic error of the linear axis is the same as that of a single slider, and there are five categories of errors: straightness along \(x\) and \(y\) axis, pitch, yaw, and roll angular error. In this section, the kinematic error of a single slider is transformed to derive and obtain the kinematic error of the linear axis. As shown in Fig. 12, the linear axis of machine tool is composed of base, linear guide rail, and carriage.

As for force analysis of each slider of the linear guide rail, when milling a workpiece in a machine tool, in addition to the weight of the table and the workpiece, the most important load is the milling force [36]. Moreover, the milling force has a crucial influence on the kinematic error of linear axis [37].
The total load of slider keeps balanced in the horizontal and vertical direction; besides, the gravity and milling force are decomposed into horizontal and vertical components. Therefore, the horizontal load and vertical load of each slider are calculated as in Eq. (26)–(27).

\[
\begin{align*}
F_{V1} &= P_1 + P'_1 \\
F_{V2} &= P_2 + P'_2 \\
F_{V3} &= P_3 + P'_3 \\
F_{V4} &= P_4 + P'_4 \\
F_{H1} &= P_{1T} \\
F_{H2} &= P_{2T} \\
F_{H3} &= P_{3T} \\
F_{H4} &= P_{4T}
\end{align*}
\] (26)

\[
\begin{align*}
P_1 &= G - \frac{G \cdot l_1 - G \cdot l_2}{2d_s} \\
P_2 &= G - \frac{G \cdot l_1 - G \cdot l_2}{2d_s} \\
P_3 &= G - \frac{G \cdot l_1 + G \cdot l_2}{2d_s} \\
P_4 &= \frac{G}{4} + \frac{G \cdot l_1 + G \cdot l_2}{2d_s} \\
P'_{1T} &= P'_{2T} = -\frac{F_m l_4}{2b_g} \\
P'_{3T} &= P'_{4T} = \frac{2b_g}{2d_s} \\
P_{1T} &= P_{4T} = -\frac{F_m l_3}{4d_s} + \frac{F_m l_4}{2d_s} \\
P_{2T} &= P_{3T} = -\frac{F_m l_3}{4d_s} + \frac{F_m l_4}{2d_s}
\end{align*}
\] (28)

where \( L_i (i = 1, 2, 3, 4) \) represents the vertical load of each slider under the workpiece gravity \( G \) in Fig. 13, similarly, \( P'_i (i = 1, 2, 3, 4) \) is the vertical load of each slider under the milling force \( F_m \), \( L_i (i = 1, 2, 3, 4) \) denotes the horizontal load of each slider under the milling force \( F_m \) in Fig. 14, \( d_s \) is the distance between two sliders along \( z \) axis; \( b_g \) is the distance between two guide rails along \( x \) axis; \( l_1 \) is the distance from the gravity center to the carriage center along \( x \) axis; \( l_1 \) is the distance from the gravity center to the carriage center along \( x \) axis; \( l_3 \) is the distance from the milling force point to the carriage center along \( z \) axis, and \( l_4 \) is the distance from the milling force point to the slider center along \( y \) axis.

By substituting Eqs. (28)–(29) into (26)–(27), the horizontal and vertical load of each slider can be calculated. Afterwards, Eqs. (26)–(27) are substituted into Eq. (12) in Section 3.1. Finally, the kinematic error of each slider under the workpiece gravity and milling force can be obtained.

As for the straightness solution of carriage in horizontal and vertical direction, based on the principle of vector summation, the straightness of each slider in horizontal and vertical direction; besides, the gravity and milling force are decomposed into horizontal and vertical components. Therefore, the horizontal load and vertical load of each slider are calculated as in Eq. (26)–(27).

\[
\begin{align*}
F_{V1} &= P_1 + P'_1 \\
F_{V2} &= P_2 + P'_2 \\
F_{V3} &= P_3 + P'_3 \\
F_{V4} &= P_4 + P'_4 \\
F_{H1} &= P_{1T} \\
F_{H2} &= P_{2T} \\
F_{H3} &= P_{3T} \\
F_{H4} &= P_{4T}
\end{align*}
\] (26)

\[
\begin{align*}
P_1 &= G - \frac{G \cdot l_1 - G \cdot l_2}{2d_s} \\
P_2 &= G - \frac{G \cdot l_1 - G \cdot l_2}{2d_s} \\
P_3 &= G - \frac{G \cdot l_1 + G \cdot l_2}{2d_s} \\
P_4 &= \frac{G}{4} + \frac{G \cdot l_1 + G \cdot l_2}{2d_s} \\
P'_{1T} &= P'_{2T} = -\frac{F_m l_4}{2b_g} \\
P'_{3T} &= P'_{4T} = \frac{2b_g}{2d_s} \\
P_{1T} &= P_{4T} = -\frac{F_m l_3}{4d_s} + \frac{F_m l_4}{2d_s} \\
P_{2T} &= P_{3T} = -\frac{F_m l_3}{4d_s} + \frac{F_m l_4}{2d_s}
\end{align*}
\] (28)

\[
\begin{align*}
P'_1 &= P'_{2T} = -\frac{F_m l_4}{2b_g} \\
P'_3 &= P'_{4T} = \frac{2b_g}{2d_s} \\
P'_{1T} &= P'_{4T} = -\frac{F_m l_3}{4d_s} + \frac{F_m l_4}{2d_s} \\
P'_{2T} &= P'_{3T} = -\frac{F_m l_3}{4d_s} + \frac{F_m l_4}{2d_s}
\end{align*}
\] (29)

By substituting Eqs. (28)–(29) into (26)–(27), the horizontal and vertical load of each slider can be calculated. Afterwards, Eqs. (26)–(27) are substituted into Eq. (12) in Section 3.1. Finally, the kinematic error of each slider under the workpiece gravity and milling force can be obtained.

As for the straightness solution of carriage in horizontal and vertical direction, based on the principle of vector summation, the straightness of each slider in horizontal and vertical direction; besides, the gravity and milling force are decomposed into horizontal and vertical components. Therefore, the horizontal load and vertical load of each slider are calculated as in Eq. (26)–(27).
vertical directions is mapped into the carriage [38], aiming to calculate the kinematic error of the linear axis. So the carriage error parameters are deduced in Eqs. (30)–(33); numbers 1–4 of the following Eqs. (30)–(33) stands for the carriage position corresponding to slider position in Fig. 15.

\[
\begin{align*}
\Delta x_i &= \delta_{1x} - \frac{d_i}{2} \sin \varepsilon_{1y} + h \sin \varepsilon_{1z} \\
\Delta y_i &= \delta_{1y} - \frac{d_i}{2} \sin \varepsilon_{1x} - b_i \sin \varepsilon_{1z} \\
\Delta z_i &= \delta_{1z} + \frac{d_i}{2} \sin \varepsilon_{1y} + h \sin \varepsilon_{1x} \\
\Delta \phi_i &= \delta_{1x} + \frac{d_i}{2} \sin \varepsilon_{1y} + h \sin \varepsilon_{1x} \\
\end{align*}
\]  

(30)

Through a series of derivations and calculations, the straightness of the linear axis in horizontal and vertical directions is expressed as follows:

\[
\begin{align*}
\delta_{cx} &= -\Delta_{1x} + \Delta_{2x} + \Delta_{3x} + \Delta_{4x} \\
\delta_{cy} &= \Delta_{1y} + \Delta_{2y} + \Delta_{3y} + \Delta_{4y} \\
\end{align*}
\]  

(34)

As for the angular errors solution (pitch, yaw, and roll) of carriage, the angular error of carriage is determined by itself straightness and manufacturing error. Therefore, the angular error calculation (pitch, yaw, and roll) of four positions of carriage corresponding to slider position is shown in Eqs. (35)–(38) below.

\[
\begin{align*}
E_{3x} &= \delta_{3y} + \varepsilon_{3x} (S_h / 2) \\
E_{3y} &= \delta_{3x} + \varepsilon_{3y} (S_d / 2) \\
E_{3z} &= \delta_{3y} + \varepsilon_{3z} (S_h / 2) \\
E_{2x} &= \delta_{2y} + \varepsilon_{2x} (S_d / 2) \\
E_{2y} &= \delta_{2x} + \varepsilon_{2y} (S_d / 2) \\
E_{2z} &= \delta_{2y} + \varepsilon_{2z} (S_h / 2) \\
E_{4x} &= \delta_{4y} + \varepsilon_{4x} (S_d / 2) \\
E_{4y} &= \delta_{4x} + \varepsilon_{4y} (S_d / 2) \\
E_{4z} &= \delta_{4y} + \varepsilon_{4z} (S_h / 2) \\
E_{1x} &= \delta_{1y} + \varepsilon_{1x} (S_d / 2) \\
E_{1y} &= \delta_{1x} + \varepsilon_{1y} (S_d / 2) \\
E_{1z} &= \delta_{1y} + \varepsilon_{1z} (S_h / 2) \\
\end{align*}
\]  

(35)

(36)

(37)

(38)

In Fig. 15, \( h \) denotes the distance from the top surface of carriage to a slider center along \( y \) axis.
where \( E_{\alpha}(i = 1, 2, 3, 4) \) is the pitch error of carriage position corresponding to slider position around \( x \) axis, the same to the \( E_{\beta}(i = 1, 2, 3, 4) \) and \( E_{\gamma}(i = 1, 2, 3, 4) \) around \( y \) and \( z \) axis. In Fig. 15, \( S_a \) is the length of slider, \( S_b \) is the width of slider, and \( S_c \) is the height of slider.

Through a series of derivations and calculations, the angular error (pitch, yaw, and roll) of the linear axis around \( x \), \( y \) and \( z \) are expressed as follows:

\[
\begin{aligned}
\epsilon_{cx} &= \arctan\left(\frac{(E_{ax}-E_{1y})/2-(E_{2x}-E_{3x})/2}{d_s}\right) \\
\epsilon_{cy} &= \arctan\left(\frac{(E_{ay}-E_{1y})/2-(E_{2y}-E_{3y})/2}{d_s}\right) \\
\epsilon_{cz} &= \arctan\left(\frac{(E_{ax}+E_{3x})/2-(E_{1x}+E_{2x})/2}{d_b}\right)
\end{aligned}
\tag{39}
\]

Hence, by adopting the error equivalent model, the kinematic error of the linear axis is obtained, as shown in Eq. (40).

\[
\begin{aligned}
\delta_{cx} &= -\Delta_{1x}+\Delta_{2x}+\Delta_{3x}+\Delta_{4x} \\
\delta_{cy} &= \Delta_{1y}+\Delta_{2y}+\Delta_{3y}+\Delta_{4y} \\
\epsilon_{cx} &= \arctan\left(\frac{(E_{ax}-E_{1y})/2-(E_{2x}-E_{3x})/2}{d_s}\right) \\
\epsilon_{cy} &= \arctan\left(\frac{(E_{ay}-E_{1y})/2-(E_{2y}-E_{3y})/2}{d_s}\right) \\
\epsilon_{cz} &= \arctan\left(\frac{(E_{ax}+E_{3x})/2-(E_{1x}+E_{2x})/2}{d_b}\right)
\end{aligned}
\tag{40}
\]

4 Experimental validation of kinematic error model

In order to verify the accuracy of the proposed model in this paper, a 4-axis horizontal machining center of a company is taken as a case. The \( z \)-axis geometric error of the machine tool is measured, including the straightness in horizontal and vertical directions and angular deviation of pitch, yaw, and roll. Five measurement results of geometric error are compared with the corresponding calculation results of the proposed theoretical model, aiming to verify the proposed model.

4.1 Experiment setup

Some approaches have been utilized to measure the kinematic error of the linear axis [39]; in the current study, the horizontal machine tool of a certain company is regarded as a case to carry out the verification experiments. The Renishaw multi-laser interferometer XL-80 is employed to measure angular errors (pitch and yaw) of \( z \)-axis of machine tool; the instrument resolution is 0.01°; the angle measurement range is ±10°; the angle accuracy is ±0.2%±0.5±0.1M)um/m; \( M \) is the measurement distance, and the unit is m. The dial indicator and marble square are adopted to measure the straightness along the plane \( xz \) and \( yz \); the electronic level meter is used to measure the angular error (roll), as shown in Fig. 16. Besides, the type NSK RA55 of the linear guide rail is used in the machine tool; some parameters are listed in Table 2. All guide rail, roller, and slider of linear guide rail use the material GCr15; the elasticity modulus is 206GPa, and the Poisson’s ratio is 0.3. Therefore, all five kinematic errors of \( z \)-axis of horizontal machine tool have been measured [40], as shown in Fig. 17.

After the dial indicator and marble square are adjusted, and the linear axis of machine tool is moved along \( z \) axis, then the straightness is measured. The top and side surfaces of the marble square are measured with the dial indicator, respectively, namely the straightness of surface \( yz \) and \( xz \) along \( z \) axis in Fig. 17a, b. Secondly, after the laser interferometer is adjusted and the linear axis of machine tool is also moved along \( z \) axis, the angular error of pitch and yaw is measured in Fig. 17c, d. Finally, after the electronic level is adjusted and the linear axis of machine tool is also moved along \( z \) axis, the angular error of roll is measured in Fig. 17e. In addition, the \( z \) axis moving range of the machine tool is 610mm. The above moving along the \( z \) axis of machine tool is set up to 10 measuring points each stroke and repeated for 10 times. A total of 100 groups of measurement data are obtained.

| Items Type                | RA55       |
|--------------------------|------------|
| Slider dimension /mm     | 128×98×70  |
| Guide rail dimension /mm | 1316×53×43.5 |
| Roller diameter /mm      | 6          |
| Roller contact length /mm| 5          |
| Static bearing capacity /N| 330000    |
| Dynamic bearing capacity /N| 129000   |
| Moderate preload /N      | 4361       |
| Initial contact angle /° | 45         |
| Number of grooves        | 4          |
| Number of each row of contact rollers | 18 |
4.2 Verification

With measured straightness and angular error of z axis of machine tool, the corresponding kinematic errors of the linear axis are all calculated by the proposed model in this paper, as shown in Fig. 18. In order to verify the validity of the proposed model, the theoretical model results are compared with the experiment data, as shown in Fig. 19.

Five groups of measured kinematic error parameters were plotted as the frequency histogram, and then Gaussian fitting was performed to obtain the Gaussian curve. Afterwards, five kinds of kinematic error parameters calculated by the proposed theoretical model were plotted as the frequency to error curves; moreover, the peak lines of the frequency curves of the experiment data were compared and analyzed with the theoretical model. The comparison results show that for five kinds of kinematic errors, the frequency value of the theoretical model curve is much smaller than that measured by the experiment in Table 3. It is mainly due to the larger sample size of the theoretical model than the experimental model. Furthermore, the comparison mainly focuses on the frequency peak value, namely the average value of kinematic error.

When machine tool moves along z axis, on one hand, as for the straightness of two surfaces, and surface xz along z axis, as shown in Table 3, the error of the maximum frequency value between the theoretical model and experiment results is 0.18% in Fig. 18a and 19a. Similarly, surface yz along z axis, the error of the maximum frequency value between the theoretical model and experiment results is 0.26% in Fig. 18b and 19b. On the other hand, as for the angular error around x, y and z axis, the pitch error around x axis between the theoretical model and experiment results is 6.46% in Fig. 18c and 19c, and the yaw error around y axis between the theoretical model and experiment results is 8.24% in Fig. 18d and 19d, and the roll error around z axis between the theoretical model and experiment results is 8.54% in Fig. 18e and 19e.

It can be seen from the above results that the errors of five kinematic error values calculated by the theoretical model are all within 10%; thus, it belongs to the acceptable accuracy range. Therefore, the results show that the theoretical model proposed in this paper is accurate and feasible. In addition, the theoretical model proposed in this paper can also provide a scientific and reasonable guidance for the design, manufacture, and assembly of the linear axis of the machine tools.

5 Discussions

In the manufacturing and assembly process of the linear axis, the geometric error of parts and the external load play a key role in making a precision linear axis of the machine tool. According to above verification results in Section 4.2, some effect factors on the kinematic error of the linear axis is researched in the proposed model, including the preload of guide rail, the geometric error of the assembly surface of linear guide rail, and the external load, and moment.

5.1 Effect of preload

According to literatures [19, 41], as for a single slider, the preload played an important role in the straightness of the assembly main datum surface A and the
Subordinate datum surface B or C; however, it can be seen from Fig. 20 that the preload has almost no influence on all five kinematic errors of the linear axis. It can be inferred that the kinematic errors of four sliders of the linear axis produce the elimination each other. Although the preload has no obvious effect on the kinematic error of the linear axis, the selection of linear guide rail must comply with the machine tool design rules in the other aspects of bearing capacity and accuracy requirements.

5.2 Effect of flatness

As shown in Fig. 21, the flatness error of the assembly main datum surface A has a significant on the kinematic error of the linear axis, especially the straightness error of surface xz moving along z axis, and more on the angular error of pitch and roll. With the improvement of precision grade, five kinematic errors of the linear axis show an obvious tendency to decrease; moreover, the distribution range shrinks. When the flatness...
tolerance decreases from 0.04 to 0.015mm, the straightness error of surface $x_z$ of the linear axis also decreases from 0.05 to 0.02mm, and the angular error of pitch decreases from 9 to 3”。 A notable change has occurred. Meanwhile, it is found that the angular error of roll has the similarly obvious change, and decreases from 3.5 to 1.5”。 In addition, the proposed model takes the elastic deformation of rollers into account; hence, the mean center line of the kinematic error curve has a deviation and is not at the position of zero point.

5.3 Effect of parallelism

It can be seen from Fig. 22 that the parallelism error of the subordinate datum surface $C$ plays a key role in the kinematic error of the linear axis, especially the straightness error of

![Graphs showing calculated kinematic errors of the linear axis.](image)

Fig. 18 Calculated kinematic errors of the linear axis. a Straightness $\delta_{cx}$. b Straightness $\delta_{cy}$. c Pitch $\varepsilon_{cy}$. d Yaw $\varepsilon_{cy}$. e Roll $\varepsilon_{cz}$. 

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Fig. 19 Measured kinematic errors of the linear axis. a Straightness $\delta_{cx}$. b Straightness $\delta_{cy}$. c Pitch $\varepsilon_{cx}$. d Yaw $\varepsilon_{cy}$. e Roll $\varepsilon_{cz}$.

| Items | Straightness $\delta_{cx}(\times10^{-3} \text{mm})$ | Straightness $\delta_{cy}(\times10^{-3} \text{mm})$ | Pitch $\varepsilon_{cx}(\arcmin)$ | Yaw $\varepsilon_{cy}(\arcmin)$ | Roll $\varepsilon_{cz}(\arcmin)$ |
|-------|-----------------|-----------------|----------------|----------------|----------------|
| Experiment result | 7.68018 | -5.34184 | 2.11241 | 1.75225 | 0.92568 |
| Proposed model | 7.66611 | -5.32795 | 1.97593 | 1.60787 | 0.84652 |
| Errors | 0.1832% | 0.260% | 6.461% | 8.240% | 8.543% |
surface $yz$ moving along $z$ axis, and the angular error of yaw. With the improvement of precision grade, five kinematic errors of the linear axis have a tendency to decrease, and the distribution range shrinks. When the parallelism tolerance decreases from 0.033 to 0.013mm, the straightness error of surface $yz$ also decreases from 0.03 to 0.01mm, and the angular error of roll decreases from 8 to 3″, a remarkable change has been done. However, the other three kinematic errors do not change obviously.

5.4 Effect of perpendicularity

As shown in Fig. 23, the influence trend of perpendicularity on the kinematic error of the linear axis is similar to that of parallelism; both have a great influence on the straightness error of surface $yz$ moving along $z$ axis and the angular error of yaw. With the improvement of precision grade, five kinematic errors of the linear axis have a tendency to decrease, and the distribution range shrinks. Especially, the straightness
error of surface $yz$ moving along $z$ axis, and the angular error of yaw show obvious changes. When the perpendicularity tolerance decreases from 0.03 to 0.01 mm, the straightness error of surface $yz$ moving along $z$ axis decreases from 0.035 to 0.018 mm, and the angular error of yaw decreases from 7.5” to 3.5”, showing a great change. However, similar to the parallelism error, the other three kinematic errors do not have such an obvious change.

5.5 Effect of load

(1) Milling force

It can be seen that from Fig. 24 the milling force has little effect on the kinematic error of the linear axis. When the milling force increases to 1200N, both the straightness error of surface $yz$ moving along $z$ axis and the angular error of yaw are larger than the other three kinematic errors, while the
overall distribution range is not highly sensitive to the milling force.

(2) Gravity

As shown in Fig. 25, the workpiece gravity has almost no influence on the kinematic error of the linear axis. When the workpiece gravity increases from $1.60 \times 10^4N$ to $1.72 \times 10^4N$, the overall distribution range is not sensitive to the gravity.

In conclusion, according to the analysis of the effect factors in all the above figures, the effect degree comparison of effect factors on the kinematic error of the linear axis is shown in Table 4. As can be seen from the following Table, flatness is the most crucial effect factor, especially on the straightness $\delta_{cy}$ along $yz$ plane, Pitch $\varepsilon_{cx}$, and Roll $\varepsilon_{cz}$. Then, parallelism and perpendicularity are also the key effect factors on the
straightness $\delta_{cx}$ along $xz$ plane and Yaw $\varepsilon_{cy}$. In addition, the effect of gravity and milling force on the kinematic error of the linear axis is relatively smaller. Therefore, based on the analysis results, some guidelines can be provided for the design and machining of the linear axis of high-end machine tool.

6 Conclusions

A systematic error equivalence model of slider is presented in this paper, aiming to predict the kinematic errors of the linear axis of machine tool. The proposed methodology contains (i) the error equivalence method for the kinematic error of slider...
and (ii) the vector summation method for the kinematic error of the linear axis. The verification experiment shows a great consistence. It indicates that the proposed model is accurate and effective. Through the effect analysis of the kinematic error of the linear axis, it is obvious that the influence of flatness is relatively greater among all factors. In summary, the proposed model can be used to obtain the kinematic error of the linear axis at the design stage. Furthermore, the quantitative guidance can be provided in the design and manufacturing of the linear axis of machine tool. It improves the design and manufacturing efficiency significantly.

In future study, we will add the study of the surface topography effect of linear guide rail on the kinematic error of the linear axis, and further obtain the accuracy retention rules of the linear axis, aiming to predict the operation life of machine tool.

Fig. 24 Kinematic error of the linear axis under different milling force. a Straightness $\delta_{cx}$. b Straightness $\delta_{cy}$. c Pitch $\varepsilon_{cx}$. d Yaw $\varepsilon_{cy}$. e Roll $\varepsilon_{cz}$.
Fig. 25  Kinematic error of the linear axis under different gravity. a Straightness $\delta_{cx}$. b Straightness $\delta_{cy}$. c Pitch $\varepsilon_{cx}$. d Yaw $\varepsilon_{cy}$. e Roll $\varepsilon_{cz}$

Table 4  Effect analysis on the kinematic error of linear axis

| Kinematic error | Effect degree comparison |
|-----------------|--------------------------|
| Straightness $\delta_{cx}$ (along $x_z$ plane) | Parallelism=Perpendicularity=Preload=Flatness=Gravity=Milling force |
| Straightness $\delta_{cy}$ (along $y_z$ plane) | Flatness=Preload=Parallelism=Perpendicularity=Milling force=Gravity |
| Angular error | Flatness=Preload=Parallelism=Perpendicularity=Milling force=Gravity=Flatness=Preload=Parallelism=Perpendicularity=Milling force=Gravity |

Table 4  Effect analysis on the kinematic error of linear axis

| Kinematic error | Effect degree comparison |
|-----------------|--------------------------|
| Straightness $\delta_{cx}$ (along $x_z$ plane) | Parallelism=Perpendicularity=Preload=Flatness=Gravity=Milling force |
| Straightness $\delta_{cy}$ (along $y_z$ plane) | Flatness=Preload=Parallelism=Perpendicularity=Milling force=Gravity |
| Angular error | Flatness=Preload=Parallelism=Perpendicularity=Milling force=Gravity=Flatness=Preload=Parallelism=Perpendicularity=Milling force=Gravity |
Author contribution  Xinxin LI: Conceptualization, Methodology, Software, Validation, Visualization, Writing—original draft, Writing—review and editing
Zhimin LI: Conceptualization, Methodology, Validation, Visualization, Supervision, Writing—review and editing
Sun JIN: Conceptualization, Supervision
Jichang ZHANG: Conceptualization, Supervision
Zhihua NIU: Conceptualization, Visualization
Siyi DING: Conceptualization, Software

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