Cylindrical cavity design and particle-tracking simulation in cyclotron auto-resonance accelerator

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Abstract. The Cyclotron Auto-Resonance Accelerator (CARA) is a novel concept of accelerating continuous-wave (CW) charged-particle beams. This type of accelerator has applications in environment improvement area and generation of high-power microwaves. In CARA, the CW electron beam follows a gyrating trajectory while undergoing the interaction with a rotating TE₉₁₁-mode RF field and tapered static magnetic field. The cylindrical cavity operating at TE₁₁₆-mode is adapted to accelerate electron beam. The cavity size is optimized to obtain a beam with designed energy, then a design method of the TE₁₁₆-mode acceleration cavity is described here. Moreover, regardless of space charge effect, several particle-tracking simulations of CARAs are showed.

1. Introduction

Researches and experiments have been implemented to study the concept underlying Cyclotron Auto-Resonance Accelerator (CARA) over the past 50-years [1-5], wherein gyrating electron beam is maintained in phase synchronism with a rotating TE₁₁₁-mode waveguide RF field using tapers in magnetic field or waveguide radius. CARA has many unique features, including remarkably high RF-to-beam efficiency (measured at >96%) [3], generation of a continuous self-scanning beam (without bunching and external scanning structures), simple one-stage cylindrical accelerating structure [6] and lower beam loading on output window, as compared with the typical linear accelerator. It has many applications in environment improvement area and generation of high-power microwaves [7]. Two concepts described here based and go beyond prior work, namely (a) design method of a TE₁₁₆-cylindrical cavity for CARA; and (b) several particle-tracking simulations of CARAs under different RF power frequency.

2. Theoretical analysis

2.1. Gyroresonant acceleration theory

In CARA, the electron beam follows a gyrating trajectory under the rotating TE₁₁₁-mode waveguide field and axial static magnetic field [1, 3, 5]. The TE₁₁₁-mode field only has transverse E-field, which can be used to accelerate electron transversely as shown in figure 1. The blue arrow represents the transverse E-field on the cross section; the red point is the electron; and the red circle represents the
cyclootron motion of the electron. When the direction of E-field is parallel (180 degree) to the tangential velocity of the electron, the electron will get accelerated most effective. At resonance, the electron gyrates with the rotating electric field at the same cyclootron frequency, then the electron will be accelerated at any time as displayed in figure 1. The continuously accelerating is achieved, named “RF gyroresonant acceleration”. 

In the presence of the axial magnetic field \( B_0(z) \), the particle executes an additional cyclootron motion. The cyclootron frequency of particles of charge \( e \) and rest mass \( m_0 \) in consideration of relativistic factor \( \gamma \) is

\[
\Omega = eB_0(z)(m_0\gamma)^{-1}
\]

(1)

The frequency of RF wave seen by the particle (in the laboratory frame) is \( \omega - k_z c \beta_z \), where \( \omega \) is the frequency of RF source, \( k_z \) is the wave number in \( z \) direction, \( c \) is the light speed and \( \beta_z \) is the normalized axial velocity of particle. If these two frequencies are made to coincide, one expects some resonance effect to occur in the particle motion. So the resonance condition [7] is

\[
\omega - k_z c \beta_z - \Omega = 0 \quad \text{or} \quad b_0 = \gamma (1 - n \beta_z)
\]

(2)

Where \( n = k_z c / \omega \) is the effective refractive index for the operating mode.

So the synchronous B-field is

\[
\Omega = eB_0(z)(m_0\gamma)^{-1}
\]

(3)

\[
B_0(z) = \frac{m_0 \omega \gamma}{e}(1 - n \beta_z)
\]

(4)

\( B_0(z) \) is related to the energy and velocity of particle, which are varying with time and are affected by the magnetic field at last position. In this paper, adapting multiple iterations to obtain \( B_0(z) \).

The maximum energy can be reached using an up-tapered B-field, until mirror-reflection leads to stalling [6, 7]. The maximum accelerated energy is given approximated by

\[
\gamma_{\text{max}} = \gamma_0 + \left( n_1^2 - 1 \right)^{1/2} \gamma_0 \left( 1 - n_1^2 \right)
\]

(5)

Where \( n_1 \) is the refractive index at the end of CARA, \( \gamma_0 \) is the initial value of the particle energy factor. Practical implications of the limit are illustrated in figure 2: the larger \( n_1 \) and \( \gamma_0 \) of particle, the larger the energy limit \( \gamma_{\text{max}} \); and \( n_1 \) required to close to unity to get large \( \gamma_{\text{max}} \).

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**Figure 1.** TE\(_{11}\) E-field and electron on cross section.

**Figure 2.** Energy limit in CARA plotted as the refractive index \( n \) approaches unity for several particle injection energy.
2.2. Design of TE_{11p}-mode cavity in CARA

The CARA can be achieved both in a cylindrical waveguide or cavity. It is suggested to employ standing-wave cavity comparing with traveling-wave waveguide in order to avoid the size and complexity of the resonant ring. Besides, the standing-wave cavity can offer comparable RF-to-beam efficiencies and wall losses are similar when the extra length of the resonant ring is taken into account [7]. This paper adapts TE_{11p} cylindrical cavity. And the up-tapered B-field is used to achieve the resonance condition.

For TE_{11p}-mode in cylindrical cavity, defining axial field cycle length is \( l_1 \) and the radius is \( r \), so the total length of the cavity is \( l = l_1p \). Then the refractive index of TE_{11p}-mode is

\[
n = \left( \frac{1.841l_1}{r\pi} \right)^2 + 1 \right)^{1/2} \tag{6}
\]

And the resonant frequency is

\[
f = \frac{c}{2\pi} \left( \frac{1.841}{r} \right)^2 + \left( \frac{\pi}{l_1} \right)^2 \right)^{1/2} \tag{7}
\]

Which indicates the refractive index and resonant frequency are both only related to \( l_1 \) and \( r \) for TE_{11p}-mode in cylindrical cavity. Supposing the injection energy of electron beam is 0.1 MeV, the refractive index \( n \), resonant frequency \( f \) and energy limit \( \gamma_{\text{max}} \) at different cavity size \( l_1 \) and \( r \) are shown in figure 3. \( \gamma_{\text{max}} \) is almost linear with \( r \); large \( n \) can be achieved at large \( r \) and small \( l_1 \); and \( r \) has little effect on \( f \) while \( l_1 \) has great influence under the change scale of \( r \) and \( l_1 \). And the smaller the RF power frequency is, the larger \( l_1 \) is. Then the required \( r \) is larger in order to get large \( \gamma_{\text{max}} \). Given the frequency of RF power \( f \), the beam injection energy \( W_0 \) and the required output energy \( W_1 \), then the cavity size \( r \) and \( l_1 \) can be designed:

(a) The range of the radius \( r \) can be calculated from the cut-off frequency of TE_{11p}-mode:

\[
r \geq 1.841c/(2\pi f)^{-1} \tag{8}
\]

(b) \( f \) is almost affected by \( l_1 \), so the approximate range of \( l_1 \) can be determined by \( f \) (the \( r \) value is arbitrarily given 0.2):

\[
l_1 = \pi f \left( \left( 2\pi f / c \right)^2 + \left( 1.841 / r \right)^2 \right)^{1/2} \text{ and } l_1 > c / 2f \tag{9}
\]

(c) The minimum value of required refractive index \( n \) can be derived from the input and output energy of particle by the \( \gamma_{\text{max}} \) formula:

\[
n = \left( 1 - \frac{\gamma_0^2 - 1}{\gamma_{\text{max}}^2 - \gamma_0^2} \right)^{1/2} \tag{10}
\]

(d) The value of \( r \) can be obtained from the formula of \( n \) after \( n \) and \( l_1 \) are determined:

\[
r = 1.841l_1 \left( \left( 1 - \frac{n^2}{\pi^2} \right)^{1/2} \right) \tag{11}
\]

(e) The \( f \) formula is used to correct \( r \) and \( l_1 \): if \( f \) is too large, then correct \( r \) (increase \( r \)); or if \( f \) is too small, then correct \( l_1 \) (decrease \( l_1 \)). Because increasing \( r \) or decreasing \( l_1 \) can both increase \( n \).

\[
l_{1,\text{correct}} = \pi \left( \left( 2\pi f / c \right)^2 - \left( 1.841 / r \right)^2 \right)^{1/2} \tag{12}
\]
After determining the cavity size \( r \) and \( l_1 \), the guided axial B-field, RF field strength and axial cycle number \( p \) are required to optimize to realize the resonance condition between the RF field and electron beam.

\[
 r_{\text{correct}} = 1.841 \left( \frac{2\pi f / c}{\left( \frac{\pi}{l_1} \right)^2} \right)^{-1/2} \tag{13}
\]

Figure 3. Dependence of (a) energy limit; (b) refractive index \( n \); and (c) resonant frequency \( f \) on cavity radius \( r \) for different axial cycle length \( l_1 \).

3. Numerical particle-tracking simulations
The peak E-field in the cavity is fixed at 2 MV/m. The injection and target output energy are 0.1 MeV and ~2 MeV respectively. The cylindrical cavity is designed according to the above steps when the RF power frequency is 1.3, 2.856, and 5.712 GHz, respectively. Supposing the injection electron beam has only axial velocity, and particle-tracking simulation is performed regardless of the space charge effect. Table 1 lists computed parameters for several 1.3, 2.856, 5.712 GHz, ~2 MeV CARAs. The corresponding simulation results of the electron cyclotron frequency, gyro radius, \( \gamma \), axial velocity \( V_z \), transverse velocity \( V_T \) and the optimized guided axial B-field are shown in figure 4. The beam trajectory is shown in figure 5.

Table 1. Parameters for several 1.3, 2.856, 5.712 GHz, ~2 MeV CARAs.

|                  | Case 1 | Case 2 | Case 3 |
|------------------|--------|--------|--------|
| Driven Frequency (GHz) | 1.3    | 2.856  | 5.712  |
| Cavity Radius (cm)    | 39.26  | 19.63  | 8.83   |
| Refractive Index \( n \) | 11.71  | 5.31   | 2.66   |
| \( \gamma_{\text{max}} \) (theoretical) | 5      | 5.3    | 4.96   |
| Axial Cycle Numbers \( p \) | 30     | 65     | 121    |
| Total Length (m)     | 3.51   | 3.45   | 3.22   |
| \( \gamma_{\text{out}} \)      | 5.26   | 5.2    | 5      |
| Wall loss (W)         | 5.09e5 | 6.66e5 | 8.16e5 |

Table 1 indicates: Under the same electron injection and output energy, \( r \) and \( l_1 \) need to be smaller with the higher RF power frequency. In order to get ~2 MeV output beam, the total cavity length \( l \) at three different RF source frequencies are almost the same when the peak E-field is fixed. The axial cycle numbers \( p \) and wall loss \( P_c \) increases with RF source frequency.
Figure 4. Dependence of cyclotron radius, cyclotron frequency, $\gamma$, axial velocity $V_z$ and transverse velocity $V_T$ on time $t$ and the optimized guided axial B-field on $z$.

Figure 5. The electron beam trajectory in CARA.
From the results in figure 4, the law of electron beam parameters during acceleration can be obtained:

(a) The axial synchronous B-field gradually increases along z direction with the slope becomes larger and larger;

(b) The cyclotron frequency of electron gradually decreases while cyclotron radius gradually increases;

(c) The $\gamma$ value increases approximately linearly;

(d) $V_z$ and $V_r$ both increase and $V_r$ increases more than $V_z$.

The axial B-field and cyclotron frequency increase while gyration radius falls with RF power frequency. However, the $\gamma$ and velocity of electron change with time at different RF power frequency are similar. Numerical simulations show a slight energy excess above the theoretical prediction when an optimized B-field is introduced. Since the peak E-field in the cavity is fixed, and the electron beam is accelerated by the tangential force of the transverse RF E-field, as shown in figure 1, the cyclotron motion circumstance of the electron directly affects the energy increment. The circumstance is proportional to the cyclotron radius and frequency. Although the cyclotron frequency increases with the RF power frequency, the cyclotron radius decreases. So there is not much different in total cavity length at different RF power frequency. From the energy limit formula, the large cavity radius $r$ can reach large output electron energy; however, this strategy has limited utility, since for a fixed RF power level, the accelerating gradient drops as $r$ increases, thus lengthening the structure and thereby limiting its practical utility. A standing-wave is the superposition of two counter-propagating travelling waves. In fact, electron interacts only with the traveling wave propagating in the positive $z$ direction while detunes with the other one, which will destroy the electron stability during acceleration. Therefore, it can be seen that there is a certain oscillation in the gamma and velocity curves, and the higher the RF power frequency, the larger the oscillation frequency and the amplitude.

4. Conclusions

The design process of TE$_{11p}$-mode cylindrical cavity in CARA has been established. And several 1.3, 2.856, 5.712 GHz, ~2 MeV CARAs have been designed and simulated, from which the effect of cavity size on CARA performance, the electron beam motion during acceleration are analysed. The CARA competitive edge arises from (a) a simpler and cheaper geometry of the accelerator structure; (b) the continuous accelerating process with high RF-to-beam efficiency; (c) the self-scanning output beam without external device. However, the ungainly-sized structure exposes a possible disadvantage of CARA.

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