Reconstructing Einstein-Aether Gravity from Ordinary and Entropy-Corrected Holographic and New Agegraphic Dark Energy Models

Ujjal Debnath*

Department of Mathematics,
Bengal Engineering and Science University,
Shibpur, Howrah-711 103, India.

(Dated: November 5, 2013)

Here we briefly discuss the Einstein-Aether gravity theory by modification of Einstein-Hilbert action. We find the modified Friedmann equations and then from the equations we find the effective density and pressure for Einstein-Aether gravity sector. These can be treated as dark energy provided some restrictions on the free function $F(K)$, where $K$ is proportional to $H^2$. Assuming two types of the power law solutions of the scale factor, we have reconstructed the unknown function $F(K)$ from HDE and NADE and their entropy-corrected versions (ECHDE and ECNADE). We also obtain the EoS parameter for Einstein-Aether gravity dark energy. For HDE and NADE, we have shown that the type I scale factor generates the quintessence scenario only and type II scale factor generates phantom scenario. But for ECHDE and ECNADE, the both types of scale factors can accommodate the transition from quintessence to phantom stages i.e., phantom crossing is possible for entropy corrected terms of HDE and NADE models. Finally, we show that the models are classically unstable.

PACS numbers: 04.50.Kd, 95.36.+x, 98.80.Cq, 98.80.-k

I. INTRODUCTION

The type Ia Supernovae and Cosmic Microwave Background (CMB) observations indicate that our universe is presently accelerating. This acceleration is caused by some unknown matter which has the property that positive energy density and negative pressure is dubbed as “dark energy” (DE). Observations indicate that dark energy occupies about 70% of the total energy of the universe, and the contribution of dark matter is $\sim 26\%$. Recent WMAP data analysis also give us the confirmation of this acceleration. Although a long-time debate has been done on this well-reputed and

* ujjaldebnath@yahoo.com, ujjal@iucaa.ernet.in
interesting issue of modern cosmology, we still have little knowledge about DE. The most appealing and simplest candidate for DE is the cosmological constant \( \Lambda \). Over the past decade there have been many theoretical models for mimicking the dark energy behaviors, such as \( \Lambda \text{CDM} \), containing a mixture of cosmological constant \( \Lambda \) and cold dark matter (CDM). However, two problems arise from this scenario, namely “fine-tuning” and the “cosmic coincidence” problems. In order to solve these two problems, many dynamical DE theoretical models have been proposed \[5–9\]. The scalar field or quintessence \[10, 11\] is one of the most favored candidate of DE which produce sufficient negative pressure to drive acceleration.

In recent times, considerable interest has been stimulated in explaining the observed dark energy by the \textit{holographic dark energy} (HDE) model \[12–14\]. An approach to the problem of this dark energy arises from the holographic principle \[15\]. For an effective field theory in a box size \( L \) with UV cutoff \( \Lambda_c \), the entropy \( L^3 \Lambda_c^3 \). Taking the whole universe into account the largest IR cut-off \( L \) is chosen by saturating the inequality so that we get the holographic dark energy density as \[13, 16\] in the form \( \rho_\Lambda = 3c^2M_p^2L^{-2} \) where \( c \) is a numerical constant and \( M_p \equiv 1/\sqrt{8\pi G} \) is the reduced Planck mass. The most natural choice of \( L \) is the Hubble horizon \( H^{-1} \). However, Hsu \[17\] and Li \[16\] pointed out that in this case the EoS of holographic dark energy is zero and the expansion of the universe cannot be accelerated. The next choice of \( L \) is the particle horizon. Unfortunately, in this case, the EoS of HDE is always larger than \(-1/3\) and the expansion of the universe also cannot be accelerated. Finally, Li \[16\] found out that \( L \) might be the future event horizon of the universe. On the basis of the holographic principle, several others have studied holographic model for dark energy \[18–20\]. Obviously, in the derivation of HDE, the black hole (BH) entropy \( S_{BH} \) plays an important role. Usually, we know that \( S_{BH} = \frac{A}{4G} \), where \( A (\sim L^2) \) is the area of BH horizon. Due to thermal equilibrium fluctuation, quantum fluctuation, or mass and charge fluctuations, the BH entropy-area relation has been modified in loop quantum gravity (LQG) in the form \[21, 23\] \( S_{BH} = \frac{A}{4G} + \xi \ln \frac{A}{4G} + \zeta \), where \( \xi \) and \( \zeta \) are dimensionless constants of order unity. Recently, motivated by this corrected entropy-area relation in the setup of LQG, the energy density of the entropy-corrected HDE (ECHDE) was proposed by Wei \[24\] and also details discussed in \[23, 25, 26\].

Based on principle of quantum gravity, Cai \[27\] proposed a new dark energy model based on the energy density. The energy density of metric fluctuations of Minkowski-spacetime is given by \( \rho_\Lambda \sim M_p^2/\bar{t}^2 \). As the most natural choice, the time scale \( \bar{t} \) is chosen to be the age of our universe, \( T = \int_0^a \frac{da}{aH} \), where \( a \) is the scale factor of our universe and \( H \) is the Hubble parameter.
Therefore, the dark energy is called *agegraphic dark energy* (ADE) \[27\]. The energy density of agegraphic dark energy is given by \( \rho_\Lambda = 3\alpha^2 M_p^2 T^{-2} \), where the numerical factor \( 3\alpha^2 \) is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe. Since the original ADE model suffers from the difficulty to describe the matter-dominated epoch, so Wei and Cai \[30\] have chosen the time scale \( \tilde{t} \) to be the conformal time \( \eta \) instead of \( T \), which is defined by \( dt = a d\eta \) (where \( t \) is the cosmic time), the energy density is obtained as \( \rho_\Lambda = 3\alpha^2 M_p^2 \eta^{-2} \), which is called *new agegraphic dark energy* (NADE) model \[30\]. It was found that the coincidence problem could be solved naturally in the NADE model. The ADE models have given rise to a lot of interest recently and have been examined and studied in details in \[30, 31\]. Recently, very similar to the ECHDE model, the energy density of the entropy-corrected NADE (ECNADE) was proposed by Wei \[24\] and investigated in details for acceleration of the universe \[26, 32–35\].

Another approach to explore the accelerated expansion of the universe is the modified theories of gravity. In this case cosmic acceleration would arise not from dark energy as a substance but rather from the dynamics of modified gravity \[36\]. Modified gravity constitutes an interesting dynamical alternative to \( \Lambda \)CDM cosmology. The simplest modified gravity is DGP brane-world model \[37\]. The other alternative is \( f(R) \) gravity \[38\] where the Einstein-Hilbert action has been modified. Other modified gravity includes \( f(T) \) gravity, \( f(G) \) gravity, Gauss-Bonnet gravity, Horava-Lifshitz gravity, Brans-Dicke gravity, etc \[39–45\].

Here we have assumed other type of modified gravity developed by Jacobson et al \[46, 47\], known as *Einstein-Aether* theory. Zlosnik et al \[48, 49\] has proposed the generalization of Einstein-Aether theory. These years a lot of work has been done in generalized Einstein-aether theories \[50–56\]. In the generalized Einstein-Aether theories by taking a special form of the Lagrangian density of Aether field, the possibility of Einstein-Aether theory as an alternative to dark energy model is discussed in detail, that is, taking a special Aether field as a dark energy candidate and it has been found the constraints from observational data \[57, 58\]. Meng et al \[57, 58\] have shown that only Einstein-Aether gravity may be generated dark energy, which caused the acceleration of the universe.

Recently the reconstruction procedure or correspondences between various dark energy models became very challenging subject in cosmological phenomena. Correspondence between different DE models, reconstruction of DE/gravity and their cosmological implications have been discussed by several authors \[26, 59–68\]. We shall reconstruct of Einstein-Aether gravity model with HDE and
NADE separately. For this purpose, we first briefly discuss the Einstein-Aether gravity theory by modification of Einstein-Hilbert action in section II. We find the modified Friedmann equations and then from the equations we find the effective density and pressure for Einstein-Aether gravity sector. These can be treated as dark energy provided some restrictions on the free function $F(K)$. Assuming two types of the power law solutions of the scale factor, we can reconstruct the unknown function $F(K)$ from HDE and NADE and their entropy-corrected versions (ECHDE and ECNADE) in section III. Finally, we give some cosmological implications of the reconstructed models in section IV.

II. EINSTEIN-AETHER GRAVITY THEORY AND MODIFIED FRIEDMANN EQUATIONS

Einstein-Aether theory is the extension of general relativity (GR) that incorporates a dynamical unit timelike vector field (i.e., Aether) coupling with the metric. The action of the Einstein-Aether gravity theory with the normal Einstein-Hilbert part action can be written in the form $[48, 57]$

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \mathcal{L}_{EA} + \mathcal{L}_m \right]$$

where $\mathcal{L}_{EA}$ is the Lagrangian density for vector field and $\mathcal{L}_m$ denotes the Lagrangian density for matter field. The Lagrangian density for the vector field $[48, 57]$ is given by

$$\mathcal{L}_{EA} = \frac{M^2}{16\pi G} F(K) + \frac{1}{16\pi G} \lambda (A^a A_a + 1) ,$$

$$K = M^{-2} K^{ab} \nabla_a A^c \nabla_b A^d ,$$

$$K^{ab}_{\quad cd} = c_1 g^{ab}_{\quad cd} + c_2 \delta^a_c \delta^b_d + c_3 \delta^a_d \delta^b_c$$

where $c_i$ are dimensionless constants, $M$ is the coupling constant, $\lambda$ is a Lagrangian multiplier, $A^a$ is a contravariant vector and $F(K)$ is an arbitrary function of $K$. From (1), we get the Einstein’s field equations

$$G_{ab} = T_{ab}^{EA} + 8\pi G T_{ab}^m ,$$

$$\nabla_a \left( F' J^a_b \right) = 2\lambda A_b$$

where

$$F' = \frac{dF}{dK} \text{ and } J^a_b = 2 K^{ad} \nabla_d A^c$$
Here $T_{ab}^m$ is the energy momentum tensor for matter and $T_{ab}^{EA}$ is the energy momentum tensor for the vector field given as follows:

$$T_{ab}^m = (\rho + p)u_au_b + pg_{ab}$$

(8)

where $\rho$ and $p$ are respectively the energy density and pressure of matter and $u_a = (1, 0, 0, 0)$ is the fluid 4-velocity vector and

$$T_{ab}^{EA} = \frac{1}{2} \nabla_d \left[ (J_{(a}^d A_{b)} - J^d_{(a} A_{b)} - J_{(ab)} A^d) F' \right] - Y_{(ab)} F' + \frac{1}{2} g_{ab} M^2 F + \lambda A_a A_b$$

(9)

with

$$Y_{ab} = -c_1 \left[ (\nabla_d A_a)(\nabla^d A_b) - (\nabla_a A_d)(\nabla^b A^d) \right]$$

(10)

where the subscript $(ab)$ means symmetric with respect to the indices and $A^a = (1, 0, 0, 0)$ is time-like unit vector.

We consider the Friedmann-Robertson-Walker (FRW) metric of the universe as

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right]$$

(11)

where $k$ ($= 0, \pm 1$) is the curvature scalar and $a(t)$ is the scale factor. From equations (3) and (4), we get

$$K = \frac{3\beta H^2}{M^2}$$

(12)

where $\beta$ is constant. From eq. (5), we get the modified Friedmann equation for Einstein-Aether gravity as in the following:

$$\beta \left( -F' + \frac{F}{2K} \right) H^2 + \left( H^2 + \frac{k}{a^2} \right) = \frac{8\pi G}{3} \rho$$

(13)

and

$$\beta \frac{d}{dt} (HF') + \left( -2H + \frac{2k}{a^2} \right) = 8\pi G(\rho + p)$$

(14)

where $H (= \frac{\dot{a}}{a})$ is Hubble parameter. Also the conservation equation is given by

$$\dot{\rho} + 3 \frac{\dot{a}}{a}(\rho + p) = 0$$

(15)

Let $\rho_{EA}$ and $p_{EA}$ be the effective energy density and pressure governed by the Einstein-Aether gravity, then we may write the equations (13) and (14) in the following form:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho + \frac{1}{3} \rho_{EA}$$

(16)
\[
\left(-2\dot{H} + \frac{2k}{a^2}\right) = 8\pi G(\rho + p) + (\rho_{EA} + p_{EA})
\]  \hspace{1cm} (17)

and hence we obtain

\[
\rho_{EA} = 3\beta H^2 \left(F' - \frac{F}{2K}\right)
\]  \hspace{1cm} (18)

and

\[
p_{EA} = -3\beta H^2 \left(F' - \frac{F}{2K}\right) - \beta (\dot{HF}' + H\dot{F}')
\]  \hspace{1cm} (19)

The equation of state (EoS) parameter due to the Einstein-Aether contribution is given by

\[
w_{EA} = \frac{p_{EA}}{\rho_{EA}} = -1 - \frac{(\dot{HF}' + H\dot{F}')}{{3H^2} \left(F' - \frac{F}{2K}\right)}
\]  \hspace{1cm} (20)

Since density is always positive, so \(\rho_{EA} > 0\) implies \(F' > \frac{F}{2K}\), where we assume \(\beta > 0\). The effective density and pressure governed by Einstein-Aether gravity generate dark energy if \(\rho_{EA} + 3p_{EA} < 0\) (i.e., strong energy condition violates), which provides the condition \(2H^2 \left(F' - \frac{F}{2K}\right) > -(\dot{HF}' + H\dot{F}')\).

III. RECONSTRUCTION OF EINSTEIN-AETHER GRAVITY MODEL

Since Einstein-Aether theory is the modified gravity theory and this may generate dark energy. So this gravity can be corresponds with other well-known dark energy models. For this purpose, we consider the dark energy models which are holographic dark energy (HDE) and new agegraphic dark energy (NADE) and their entropy-corrected versions (ECHDE and ECNADE). The Einstein-Aether gravity sector contains a function \(F(K)\), so equating the density with the dark energy models, we can find \(F(K)\). So \(F(K)\) can be constructed from the dark energy models. For reconstruction of \(F(K)\) in terms of \(K\), we need to know the form of scale factor \(a(t)\). For this purpose, we assume two types of power-law forms of \(a(t)\) \[26\] for providing the acceleration of the universe and we reconstruct the Einstein-Aether gravity according to the HDE, ECHDE, NADE and ECNADE models:

(i) **Type I**: \(a(t) = a_0 t^m, m > 0\), where the constant \(a_0\) represents the present-day value of the scale factor \[69\]. With this choice of scale factor we obtain \(H, \dot{H}\) and \(K\) in the form:

\[
H = \frac{m}{t}, \dot{H} = -\frac{m}{t^2}, K = \frac{3\beta m^2}{M^2 t^2}
\]  \hspace{1cm} (21)
We see that ${\dot{H}} < 0$, so this model corresponds to only quintessence dominated universe (not phantom) and hence the scale factor may be called the quintessence scale factor.

(ii) **Type II**: 

$$a(t) = a_0(t_s - t)^{-n}, \quad t < t_s, \quad n > 0$$

where the constant $a_0$ represents the present-day value of the scale factor, $t_s$ is the probable future singularity finite time \(^[69, 70]\).

$$H = \frac{n}{t_s - t}, \quad \dot{H} = \frac{n}{(t_s - t)^2}, \quad K = \frac{3\beta n^2}{M^2(t_s - t)^2}$$

We see that $\dot{H} > 0$, so this model corresponds to only phantom dominated universe (not quintessence) and hence the scale factor may be called the phantom scale factor.

**A. Reconstruction from Holographic Dark Energy (HDE) Model**

We now suggest the correspondence between the holographic dark energy scenario and Einstein-Aether dark energy model. The holographic dark energy (HDE) density can be written as \(^[71–73]\)

$$\rho_\Lambda = 3c^2 L^{-2}, \quad L = R_h$$

where $c$ is a constant and $R_h$ represents the future event horizon. From observation, the best fit value of $c$ is $0.818^{+0.113}_{-0.097}$. For type I scale factor, $R_h$ is defined as

$$R_h = a \int_{t}^{t_s} dt \frac{a}{a} = \frac{t}{m - 1}, \quad m > 1$$

(24)

For type II scale factor, $R_h$ is defined as

$$R_h = a \int_{t}^{t_s} dt \frac{a}{a} = \frac{t_s - t}{n + 1}$$

(25)

For type I scale factor, equating the energy densities (i.e., $\rho_{EA} = \rho_\Lambda$), we get reconstructed equation

$$\frac{dF}{dK} - \frac{F}{2K} = \frac{c^2(m - 1)^2}{\beta m^2}$$

(26)

which immediately gives the solution

$$F(K) = \frac{2c^2(m - 1)^2}{\beta m^2} K + A_1 \sqrt{K}$$

(27)

For type II scale factor, we also get similar reconstructed equation as well as (26) and we find the similar solution

$$F(K) = \frac{2c^2(n + 1)^2}{\beta n^2} K + A_2 \sqrt{K}$$

(28)
Figs. 1 and 2 show the variations of $F(K)$ against $K$ for HDE and ECHDE models. Blue line represents for type I model and red line represent for type II model.

Here $A_1$ and $A_2$ are integration constants. For these solutions, the EoS for Einstein-Aether gravity can be obtained as $w_{EA} = -1 + \frac{2}{3m}$ for type I model and $w_{EA} = -1 - \frac{2}{3m}$ for type II model. We see that $-1 < w_{EA} < -\frac{1}{3}$ (i.e., quintessence) if $m > 1$ for type I model and for $n > 0$, $w_{EA} < -1$ (phantom) for type II model. The graphs of $F(K)$ w.r.t $K$ has been drawn in figure 1 for both type I and II models. From figure, we see that the reconstruction function $F(K)$ increases as $K$ increases for HDE model.

**B. Reconstruction from Entropy-Corrected Holographic Dark Energy (ECHDE) Model**

Using the corrected entropy-area relation, the energy density of the ECHDE can be written as

$$\rho_A = \frac{3c^2}{R_h^2} + \frac{\xi}{R_h^4} \ln(R_h^2) + \frac{\zeta}{R_h^4}$$

(29)

where $\xi$ and $\zeta$ are constants. For type I scale factor, equating the energy densities (i.e., $\rho_{EA} = \rho_A$), we get reconstructed equation

$$\frac{dF}{dK} - \frac{F}{2K} = \frac{c^2(m-1)^2}{\beta m^2} + \frac{(m-1)^4 M^2}{(3\beta m^2)^2} K \left[ \zeta + \xi \ln \left( \frac{3\beta m^2}{M^2(m-1)K} \right) \right]$$

(30)

which immediately give the solution

$$F(K) = \frac{2c^2(m-1)^2}{\beta m^2} K + \frac{(m-1)^4 M^2}{9(3\beta m^2)} K^2 \left[ 3\zeta + 2\xi + 2\xi \ln \left( \frac{3\beta m^2}{M^2(m-1)K} \right) \right] + B_1 \sqrt{K}$$

(31)
Here $B_1$ is integration constant. In this case, using eq. (20), the EoS parameter for Einstein-Aether gravity can be written as

$$w_{EA} = -1 + \frac{4}{3m} \left[ \frac{27c^2 t^2 + (m - 1)^2 \left( 9\zeta - \xi + 12\xi \ln \left( \frac{t}{m-1} \right) \right)}{54c^2 t^2 + (m - 1)^2 \left( 9\zeta + 2\xi + 12\xi \ln \left( \frac{t}{m-1} \right) \right)} \right], \quad m > 1 \quad (32)$$

For type II scale factor, we also get the similar solution

$$F(K) = \frac{2c^2(n + 1)^2}{\beta n^2} K + \frac{(n + 1)^4 M^2}{9(3\beta n^2)^2} K^2 \left[ \frac{3\zeta + 2\xi + 12\xi \ln \left( \frac{3\beta n^2 M^2 (n+1)^2 K}{n+1} \right)}{3\beta n^2 M^2 (n+1)^2 K} \right] + B_2 \sqrt{K} \quad (33)$$

Here $B_2$ is integration constant. In this case, using eq. (20), the EoS parameter for Einstein-Aether gravity can be written as

$$w_{EA} = -1 - \frac{4}{3n} \left[ \frac{27c^2 (t_s - t)^2 + (n + 1)^2 \left( 9\zeta - \xi + 12\xi \ln \left( \frac{(t_s-t)}{n+1} \right) \right)}{54c^2 (t_s - t)^2 + (n + 1)^2 \left( 9\zeta + 2\xi + 12\xi \ln \left( \frac{(t_s-t)}{n+1} \right) \right)} \right] \quad (34)$$

The graphs of $F(K)$ w.r.t $K$ has been drawn in figure 2 for both type I and II models. From figure, we see that the reconstruction function $F(K)$ increases as $K$ increases for ECHDE model. The EoS parameter $w_{EA}$ against time $t$ has been drawn in figure 5 for both type I and II models. We see that Einstein-Aether EoS parameter $w_{EA}$ gives the transition from $w_{EA} > -1$ to $w_{EA} < -1$ stages for both type I and II models. So for type I and II models, when we assume ECHDE, the Einstein-Aether DE interpolates from quintessence era to phantom stage. So phantom crossing is possible for these models. Thus we conclude that for both type I and type II models, entropy corrected terms ($\xi \neq 0, \zeta \neq 0$) generate the phantom crossing.

### C. Reconstruction from New Agegraphic Dark Energy (NADE) Model

The energy density of the new agegraphic dark energy (NADE) model is given by

$$\rho_\Lambda = \frac{3\alpha^2}{\eta^2} \quad (35)$$

where, $\eta = \int \frac{dt}{a(t)}$ is the conformal time and the numerical factor $3\alpha^2$ serves to parameterize some uncertainties, which include the effect of curved spacetime, some species of quantum fields in the universe, etc. From observation, the best fit value of $\alpha$ is $2.716^{+0.111}_{-0.109}$.

For type I model, the conformal time is

$$\eta = \int_0^t \frac{dt}{a} = \frac{t^{1-m}}{a_0(1-m)}, \quad m < 1 \quad (36)$$
Figs. 3 and 4 show the variations of $F(K)$ against $K$ for NADE and ECNADE models. Blue line represents for type I model and red line represent for type II model.

For type II model, the conformal time is

$$\eta = \int_t^{t_s} \frac{dt}{a} = \frac{(t_s - t)^{n+1}}{a_0(n + 1)} \quad (37)$$

For type I model, equating the energy densities (i.e., $\rho_{EA} = \rho_{\Lambda}$), we get the reconstructed equation

$$\frac{dF}{dK} - \frac{F^2}{2K} = \frac{a_0^2 \alpha^2 (1 - m)^2}{\beta m^2} \left( \frac{3 \beta m^2}{M^2} \right)^m \frac{1}{K^m} \quad (38)$$

which immediately give the solution

$$F(K) = \frac{2a_0^2 \alpha^2 (1 - m)^2}{\beta m^2 (1 - 2m)} \left( \frac{3 \beta m^2}{M^2} \right)^m K^{1-m} + C_1 \sqrt{K} \quad (39)$$

For type II model, we get the similar reconstructed equation, which gives the similar solution

$$F(K) = \frac{2a_0^2 \alpha^2 (n + 1)^2}{\beta n^2 (2n + 1)} \left( \frac{3 \beta n^2}{M^2} \right)^{-n} K^{n+1} + C_2 \sqrt{K} \quad (40)$$

where $C_1$ and $C_2$ are integration constants. For these solutions, the EoS for Einstein-Aether gravity can be obtained as $w_{EA} = -\frac{5}{3} + \frac{2}{3m}$ for type I model and $w_{EA} = -\frac{5}{3} - \frac{2}{3m}$ for type II model. We see that for $\frac{1}{2} < m < 1$, we have $-1 < w_{EA} < -\frac{1}{3}$ i.e., quintessence model for type I model and $w_{EA} < -\frac{5}{3} < -1$ (phantom divide) for all $n > 0$ for type II model. The graphs of $F(K)$ w.r.t $K$ has been drawn in figure 3 for both type I and II models. From figure, we see that the reconstruction function $F(K)$ increases as $K$ increases for NADE model.
D. Reconstruction from Entropy-Corrected New Agegraphic Dark Energy (ECNADE) Model

Using the corrected entropy-area relation, the energy density of the ECNADE can be written as

\[
\rho_A = \frac{3a^2}{\eta^2} + \frac{\xi}{\eta^4} \ln(\eta^2) + \frac{\zeta}{\eta^4}
\]

(41)

where \(\xi\) and \(\zeta\) are constants. Here we have replaced \(R_h\) by conformal time \(\eta\) in equation (29). For type I model, equating the energy densities (i.e., \(\rho_{EA} = \rho_A\)), we get the reconstructed equation

\[
\frac{dF}{dK} - \frac{F}{2K} = \frac{a_0^2 \alpha^2 (1 - m)^2}{\beta m^2} \left( \frac{3 \beta m^2}{M^2} \right)^m K^{-m} + \frac{a_0^2 (1 - m)^4}{3 \beta m^2} \left( \frac{3 \beta m^2}{M^2} \right)^{2m-1} K^{1-2m} \left[ \zeta + \xi \ln \left( \frac{\left( \frac{3 \beta m^2}{M^2} \right)^{1-m}}{a_0^2 (1 - m)^2} \right) \right]
\]

(42)

which immediately give the solution

\[
F(K) = \frac{2a_0^2 \alpha^2 (1 - m)^2}{\beta m^2 (1 - 2m)} \left( \frac{3 \beta m^2}{M^2} \right)^m K^{1-m} + D_1 \sqrt{K} \frac{2a_0^4 (1 - m)^4}{3 \beta m^2 (3 - 4m)^2} \left( \frac{3 \beta m^2}{M^2} \right)^{2m-1} K^{2-2m} \times \left[ (3 - 4m) \zeta + 2(1 - m) \xi + (3 - 4m) \xi \ln \left( \frac{\left( \frac{3 \beta m^2}{M^2} \right)^{1-m}}{a_0^2 (1 - m)^2} \right) \right]
\]

(43)

Here \(D_1\) is integration constant. In this case, using eq. (20), the EoS parameter for Einstein-Aether gravity can be written as

\[
w_{EA} = -1 + \frac{2(1 - m)}{3m} \left[ \frac{a_0^2 \beta m^2 (2 \zeta - \xi) + \alpha^2 M^2 \left( \frac{3 \beta m^2}{M^2} \right)^m t^{2-2m} + 4a_0^2 \beta m^2 \xi \ln \left( \frac{t^{1-m}}{a_0(1 - m)} \right)}{a_0^2 (1 - m)^2 \left( \frac{3 \beta m^2}{M^2} \right)^m t^{2-2m} + 2a_0^2 \beta m^2 \xi \ln \left( \frac{t^{1-m}}{a_0(1 - m)} \right)} \right], \quad m < 1
\]

(44)

For type II model, we get the similar reconstructed equation, which gives the similar solution

\[
F(K) = \frac{2a_0^2 \alpha^2 (1 + n)^2}{\beta n^2 (1 + 2n)} \left( \frac{3 \beta n^2}{M^2} \right)^{-n} K^{1+n} + D_2 \sqrt{K} \frac{2a_0^4 (1 + n)^4}{3 \beta n^2 (3 + 4n)^2} \left( \frac{3 \beta n^2}{M^2} \right)^{2n-1} K^{2+2n} \times \left[ (3 + 4n) \zeta + 2(1 + n) \xi + (3 + 4n) \xi \ln \left( \frac{\left( \frac{3 \beta n^2}{M^2} \right)^{1+n}}{a_0^2 (1 + n)^2} \right) \right]
\]

(45)

Here \(D_2\) is integration constant. In this case, using eq. (20), the EoS parameter for Einstein-Aether gravity can be written as

\[
w_{EA} = -1 - \frac{2(1 + n)}{3n} \left[ \frac{a_0^2 \beta n^2 (2 \zeta - \xi) + \alpha^2 M^2 \left( \frac{3 \beta n^2}{M^2} \right)^{-n} t^{2+2n} + 4a_0^2 \beta n^2 \xi \ln \left( \frac{t^{1+n}}{a_0(1 + n)} \right)}{a_0^2 \beta n^2 \zeta + \alpha^2 M^2 \left( \frac{3 \beta n^2}{M^2} \right)^{-n} t^{2+2n} + 2a_0^2 \beta n^2 \xi \ln \left( \frac{t^{1+n}}{a_0(1 + n)} \right)} \right]
\]

(46)
Figs. 5 and 6 show the variations of $w_{EA}$ against $t$ for ECHDE and ECNADE models. Blue line represents for type I model and red line represent for type II model.

The graphs of $F(K)$ w.r.t $K$ has been drawn in figure 4 for both type I and II models. From figure, we see that the reconstruction function $F(K)$ increases as $K$ increases for type I model for ECNADE. But for type II model, $F(K)$ first increases with positive value upto a certain value of $K$ and then it sharply decreases from positive value to negative value for increasing $K$. The EoS parameter $w_{EA}$ against time $t$ has been drawn in figure 6 for both type I and II models. We see that Einstein-Aether EoS parameter $w_{EA}$ gives the transition from $w_{EA} > -1$ to $w_{EA} < -1$ stages for both type I and II models. So for type I and II models, when we assume ECNADE, the Einstein-Aether DE interpolates from quintessence era to phantom stage. So phantom crossing is possible for these models. Thus we conclude that for both type I and type II models, entropy corrected terms ($\xi \neq 0$, $\zeta \neq 0$) generate the phantom crossing.

IV. DISCUSSIONS AND CONCLUDING REMARKS

In this work, we have assumed the Einstein-Aether gravity theory by modification of Einstein-Hilbert action in FRW universe. We find the modified Friedmann equations and then from the equations we find the effective density and pressure for Einstein-Aether gravity sector. These can be treated as dark energy provided some restrictions on the free function $F(K)$, where $K$ is proportional to $H^2$. Assuming two types of the power law forms of the scale factor, we have reconstructed the unknown function $F(K)$ from HDE and NADE and their entropy-corrected versions (ECHDE and ECNADE). From figure 1-4, we observed that the function $F(K)$ increases with positive sign
for increasing $K$ for type I and II models when we assumed HDE, ECHDE and NADE. But for type II model in ECNADE, $F(K)$ first increases with positive value upto a certain value of $K$ and then it sharply decreases from positive value to negative value for increasing $K$. We have noticed that from the reconstructions, $F(K) \to 0$ as $K \to 0$ for all our models. We also obtain the EoS parameter for Einstein-Aether gravity dark energy. For HDE and NADE, we have shown that the type I scale factor generates the quintessence scenario ($m > 1$ for HDE and $\frac{1}{2} < m < 1$ for NADE) only and type II scale factor generates phantom scenario ($n > 0$). So these models cannot generate phantom crossing. But for ECHDE and ECNADE, the EoS parameter $w_{EA}$ in terms of time $t$ have been drawn in figures 5 and 6. For ECHDE and ECNADE, the both types of scale factors can accommodate the transition from quintessence to phantom stages i.e., phantom crossing is possible for these models. Hence we conclude that phantom crossing happens for entropy corrected terms ($\xi \neq 0, \zeta \neq 0$) of HDE and NADE models. Also we have observed that the forms of the constructed function $F(K)$ and the EoS parameter $w_{EA}$ are similar for type I and type II models of the scale factor.

We may also assume the scale factor in de Sitter space time as in the form $a(t) = a_0 e^{Ht}$, where $H$ is constant [26], which can describe the early-time inflation of the universe. In this case we shall get $K = \frac{3H^2}{M^2}$, which is constant and hence $F(K)$ must be a constant function. So the reconstruction is not possible for de Sitter space. For this choice of scale factor, we must have $\dot{H} = 0$ and from equation (20), we get $w_{EA} = -1$ which behaves like the cosmological constant. From this point of view, we have noticed that Karami et al [26] have considered reconstruction of $f(R)$ gravity in de Sitter space. For de Sitter space $H = \text{constant}$ implies the Ricci scalar $R = \text{constant}$ which also implies $f(R)$ must be a constant function. But they have obtained $f(R)$ in terms of $R$, so the r.h.s of $f(R)$ must be constant. So for HDE, NADE, ECHDE and ECNADE models, the $f(R)$ cannot be reconstructed in terms of $R$. So their reconstruction analysis in de Sitter space is not correct.

Next we want to examine the stability of the Einstein-Aether gravity model. For this purpose we need to verify the sign of the square speed of sound which is defined by $v_s^2 = \frac{\partial p_{EA}}{\partial \rho_{EA}} = \frac{\dot{p}_{EA}}{\rho_{EA}}$. If $v_s^2 > 0$, the model is stable and $v_s^2 < 0$ implies the model is classically unstable. Some authors [67, 68, 74–78] have shown that HDE, ADE, NADE, Chaplygin gas, holographic Chaplygin, holographic $f(T)$, holographic $f(G)$, new agegraphic $f(T)$, new agegraphic $f(G)$ models are classically unstable because square speed of sound is negative throughout the evolution of the universe. In our reconstructing Einstein-Aether gravity model from HDE, NADE, ECHDE, ECNADE, we have to examine the signs of $v_s^2$. For HDE model, we find $v_s^2 = -1 + \frac{2}{3m}$ for type I model and $v_s^2 = -1 - \frac{2}{3m}$ for type II model.
Since $m > 1$ and $n > 0$, so we obtain $v_s^2 < 0$ for type I and II both models. Also for NADE model, we find $v_s^2 = -\frac{5}{3} + \frac{2}{3m}$ for type I model and $v_s^2 = -\frac{5}{3} - \frac{2}{3n}$ for type II model. Since $\frac{1}{2} < m < 1$ and $n > 0$, so we get $v_s^2 < 0$ for type I and II both models. Hence we conclude that the Einstein-Aether gravity for HDE and NADE models are classically unstable. For ECHDE and ECNADE models, the expressions of $p_{EA}$ and $\rho_{EA}$ are complicated, so we draw the graphs of $v_s^2$ for these models in figures 7 and 8 respectively for type I and II models. From figures, we observe that $v_s^2 < 0$ for Einstein-Aether gravity in ECHDE and ECNADE models. So our all these models are classically unstable at present and future stages of the FRW universe.

Acknowledgements

The author is thankful to Institute of Theoretical Physics, Chinese Academy of Science, Beijing, China for providing TWAS Associateship Programme under which part of the work was carried out. Also UD is thankful to CSIR, Govt. of India for providing research project grant (No. 03(1206)/12/EMR-II).
[1] Perlmutter, S. J. et al, 1998, Nature 391, 51.

[2] Riess, A. G. et al.[Supernova Search Team Collaboration], 1998, Astron. J. 116, 1009.

[3] Briddle, S. et al, 2003, Science 299, 1532.

[4] Spergel, D. N. et al, 2003, Astrophys. J. Suppl. 148, 175.

[5] C. Armendariz - Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. 85 4438 (2000).

[6] A. Sen, JHEP 0207 065 (2002).

[7] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607 35 (2005).

[8] A. Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B 511 265 (2001).

[9] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15 1753 (2006).

[10] P. J. E. Peebles and B. Ratra, Astrophys. J. 325 L17 (1988).

[11] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80 1582 (1998).

[12] K. Enqvist, S. Hannested and M. S. Sloth, JCAP 2 004 (2005).

[13] X. Zhang, Int. J. Mod. Phys. D 14 1597 (2005).

[14] D. Pavon and W. Zimdahl, hep-th/0511053.

[15] W. Fischler and L. Susskind, hep-th/9806039.

[16] M. Li, Phys. Lett. B 603 1 (2004).

[17] S.D.H. Hsu, Phys. Lett. B 594, 13 (2004).
[18] Y. Gong, Phys. Rev. D 70 064029 (2004).

[19] A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Rev. Lett. 82, 4971 (1999).

[20] H. Wei, S.N. Zhang, Phys. Rev. D 76, 063003 (2007).

[21] R. Banerjee and B.R. Majhi, Phys. Lett. B 662, 62 (2008).

[22] S.K. Modak, Phys. Lett. B 671, 167 (2009).

[23] H.M. Sadjadi and M. Jamil, Europhys. Lett. 92, 69001 (2010).

[24] H. Wei, Commun. Theor. Phys. 52, 743 (2009).

[25] M. Jamil and M.U. Farooq, JCAP 03, 001 (2010).

[26] K. Karami and M.S. Khaledian, JHEP 03, 086 (2011).

[27] R. G. Cai, Phys. Lett. B 657, 228 (2007).

[28] M. Maziashvili, Int. J. Mod. Phys. D 16, 1531 (2007).

[29] M. Maziashvili, Phys. Lett. B 652, 165 (2007).

[30] H. Wei and R. G. Cai, Phys. Lett. B 660, 113 (2008).

[31] Y.S. Myung, M.G. Seo, Phys. Lett. B 671, 435 (2009).

[32] K. Karami and A. Sorouri, Phys. Scr. 82, 025901 (2010).

[33] K. Karami et al, Gen. Rel. Grav. 43, 27 (2011).

[34] M.U. Farooq, M. Jamil and M.A. Rashid, Int. J. Theor. Phys. 49, 2278 (2010).

[35] M. Malekjani and A. Khodam-Mohammadi, arXiv:1004.1017 [SPIRES].

[36] S. Tsujikawa, Lect. Notes Phys. 800 99 (2010).
[37] G. R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B 484, 112 (2000).

[38] A. De Felice, T. Tsujikawa, arXiv: 1002.4928 [gr-qc].

[39] M.C.B. Abdalla, S. Nojiri, S.D. Odintsov, Class. Quantum Grav. 22, L35 (2005).

[40] E.V. Linder, Phys. Rev. D 81, 127301 (2010).

[41] K. K. Yerzhanov et al (2010), arXiv:1006.3879v1 [gr-qc].

[42] S. Nojiri and S. D. Odintsov, Phys. Lett. B631, 1 (2005).

[43] I. Antoniadis, J. Rizos, K. Tamvakis, Nucl. Phys. B 415, 497 (1994).

[44] P. Horava, JHEP 0903 020 (2009).

[45] C. Brans and H. Dicke, Phys. Rev. 124, 925 (1961).

[46] T. Jacobson, D. Mattingly, Phys. Rev. D 64, 024028 (2001).

[47] T. Jacobson, D. Mattingly, Phys. Rev. D 70, 024003 (2004).

[48] T.G. Zlosnik, P.G. Ferreira, G.D. Starkman, Phys. Rev. D 75, 044017 (2007).

[49] T.G. Zlosnik, P.G. Ferreira, G.D. Starkman, Phys. Rev. D 77, 084010 (2008).

[50] D. Garfinkle and T. Jacobson, Phys. Rev. Lett. 107, 191102 (2011).

[51] E. V. Linder and R. J. Scherrer, Phys. Rev. D 80, 023008 (2009).

[52] J. D. Barrow, Phys. Rev. D 85, 047503 (2012).

[53] J. Zuntz, T.G. Zlosnik, F. Bourliot, P.G. Ferreira, G.D. Starkman, Phys. Rev. D 81, 104015 (2010).

[54] B. Li, D. Fonseca Mota, J.D. Barrow, Phys. Rev. D 77, 024032 (2008).
[55] M. Gasperini, Class. Quant. Grav. 4, 485 (1987).

[56] M. Gasperini, Gen. Rel. Grav. 30, 1703 (1998).

[57] X. Meng and X. Du, Phys. Lett. B 710, 493 (2012).

[58] X. Meng and X. Du, Comm. Theor. Phys. 57, 227 (2012).

[59] A. K. Mohammadi, P. Majari and M. Malekjani, Astrophys. Space Sci. 331, 673 (2011).

[60] M. H. Daouda, M. E. Rodrigues and M.J.S. Houndjo, Eur. Phys. J. C 72, 1893 (2012).

[61] M. J. S. Houndjo and O. F. Piattella, Int. J. Mod. Phys. D 21, 1250024 (2012).

[62] M. R. Setare, Int. J. Mod. Phys. D 12 2219 (2008).

[63] M.R. Setare, E.N. Saridakis, Phys. Lett. B 670, 1 (2008).

[64] M.R. Setare, M. Jamil, EPL 92, 49003 (2010).

[65] U. Debnath and M. Jamil, Astrophys. Space Sci. 335, 545 (2011).

[66] S. Chattopadhyay and U. Debnath, Int. J. Theor. Phys. 50, 315 (2011).

[67] A. Jawad, S. Chattopadhyay and A. Pasqua, Eur. Phys. J. Plus, 128, 88 (2013).

[68] A. Jawad, A. Pasqua and S. Chattopadhyay, arXiv:1211.7253 [physics.gen-ph].

[69] S. Nojiri and S.D. Odintsov,Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007).

[70] H. M. Sadjadi, Phys. Rev. D 73, 063525 (2006).

[71] X. Wu, Z.-H. Zhu, Phys. Lett. B 660, 293 (2008).

[72] M.J.S. Houndjo, O.F. Piattella, arXiv:1111.4275 [gr-qc].

[73] D. Pavon, W. Zimdahl, Phys. Lett. B 628, 206 (2005).
[74] K.Y. Kim, H.W. Lee, Y.S. Myung, Phys. Lett. B 660, 118 (2008).

[75] Y. S. Myung, Phys. Lett. B, 652 223 (2007).

[76] E. Ebrahimi and A. Sheykhi, Int. J. Mod. Phys. D 20 2369 (2011).

[77] M. Sharif and A. Jawad, Eur. Phys. C 72 2097 (2012).

[78] M.R. Setare, Phys. Lett. B 654, 1 (2007).