THE DISPERSION OF HORIZONTAL TECTONIC STRESSES IN THE EARTH’S CRUST IN THE BALTIC REGION

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Abstract. GPS measurements recorded within the period from 1992 to 2003 were employed to investigate horizontal tectonic stresses in the Earth’s crust in the Baltic region. To avoid the impact of discrepancies in the systems of coordinates upon the parameters of deformations, the method of tensor analysis was applied thus estimating parameters employing the method of finite elements. Computations were performed using the created algorithms and applying ANSYS code.

The values of tectonic stresses in the Earth’s crust in the territory of the Baltic Sea region were calculated considering changes in maximum and minimum principal stresses. The value of change in maximum principal stress in the territory of the Baltic Sea region varies between –0.0013 MPa and +0.0032 MPa; the value of change in minimum principal stress varies between –0.0084 MPa and +0.0009 MPa. Positive values are dominating in directions of changes in maximum principal stresses (extension), whereas negative values – in directions of changes in minimum principal stresses (compression).

Keywords: finite element modelling, horizontal strains, tensor analysis, GPS, tectonic stresses.

1. Introduction

Horizontal deformations of the Earth’s crust can be identified from changes in geodetic coordinates and other elements of the points of geodetic networks performing repeated geodetic measurements (Barba et al. 2010; Dwivedi and Hayashi 2010; Ponraj et al. 2010; Romtognanni 2010; Stanionis 2008; Zakarevičius 2003; Zakarevičius et al. 2009, 2010a, 2010b; Zakarevičius and Stanionis 2007; Zhu and Shi 2011). The carried out measurements appear as continuous and / or differential regimes.

Among the latest technologies for measuring a geodetic network, GPS is the most widely used approach. The repeated measurements of GPS networks enable a definition of horizontal stresses affected by the Earth’s crust.

The objective of the present study is to evaluate the applicability of tensor analysis and finite element modelling approach dealing with horizontal stresses using geodetic measurements. Data on the geodetic network of the Baltic region were employed.

2. Data

Data on GPS campaigns organized within the period from 1992 and 2003 GPS were used in the performed analysis. The network consists of 354 triangles (Fig. 1) comprising 19 geodetic sites (Fig. 2).

Fig. 1. Finite element meshing of the model
The EUREF-BAL’92 campaign was carried out from August 29 to September 4, 1992 (Ehrensperger 1995; Madsen, F., Madsen, B. 1993). Each day, the morning and afternoon sessions of approximately 5 hours duration took place. The observations were made by Norwegian, Swedish, Finish and Danish geodesists using Ashtech dual frequency receivers. 24 geodetic sites were measured using 20 GPS receivers. The sites in Landskrone (401), Vaivara (402), Tartu (403), Ohtja (404) and Saarde (405) were measured in Estonia, the sites in Riga (201), Kaugari (406), Indra (407) and Arajas (410) – in Latvia and those in Akmeniskiai (311), Meskonys (312), Saseliai (408) and Dainavele (409) – in Lithuania. The EUREF network was tied to geodetic stations in Poland (Borowiec (216), Barowabora (217), Lamkowko (302), Masze (303), Germany (Wettzell (013), Klinta (015), Visby (411) and Denmark (København (412). Stations 011, 013, 015, 035, 313, 412 were fixed with reference to EUREF-89 geodetic coordinates. Processing was performed as a traditional network densification of the original EUREF-89 campaign. TOPAS software was used for reducing observations and FILLNET – for vector adjustment.

The EUREF-POL’2001 GPS campaign was carried out in September 2001 (Jaworski et al. 2002). Five 24 hour-duration sessions were performed for the quality assurance of the Polish part of the EUREF-POL’1992 campaign (Zielinski et al. 1994). The solution was computed in ITRF 2000 epoch 2001.74 and then transformed to ETRS89. Data on sites 302 and 303 were used for analyzing strains on the geodetic network.

The 2003 GPS campaign under the framework of the Nordic Geodetic Commission (NKG) was carried out in GPS-week 1238 (28 September to 4 October 2003) (Jival et al. 2005a, 2005b, 2007). The campaign mainly covered permanent GPS stations in the Nordic and Baltic areas as well as Iceland, Greenland and Svalbard. The geodetic points of ETRS 89 were also included in Latvia, Lithuania and Denmark. Processing the NKG GPS 2003 campaign was performed in four centres of analysis using three different software packages (Bernese version 4.2, version 5.0, Gipsy/Globk, Gipsy/Oasis II). The final solution to ITRF 2000 epoch 2003.75 is the average of four solutions after aligning them all to the average of two global solutions (Gipsy and Gamit). The estimated accuracy at a level of 95% is 0.5–1 cm for horizontal components and 1–2 cm for the vertical ones. New ETRS 89 coordinates based on the NKG 2003 campaign have been calculated.

Finally, all coordinates were converted to the plane coordinates of Transverse Mercator projection (Table 1).

### Table 1. Plane rectangular coordinates of GPS sites and their changes

| GPS sites | \( x_{1992} \) (m) | \( y_{1992} \) (m) | \( x_{2003} \) (m) | \( y_{2003} \) (m) | \( \Delta x \) (m) | \( \Delta y \) (m) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 11        | 6677031.9509    | 521909.1789    | 521909.1805    | -0.0167         | 0.0016          |
| 201       | 6312913.3231    | 503565.2750    | 503565.2747    | -0.0179         | -0.0003         |
| 216       | 5815531.6150    | 27711.9353     | 27711.9271     | -0.0104         | -0.0082         |
| 217       | 5819167.2339    | 298609.6680    | 298609.6586    | -0.0114         | -0.0094         |
| 302       | 5977885.3762    | 281147.2077    | 281147.1945    | -0.0075         | -0.0132         |
| 303       | 6133606.0460    | 414581.4432    | 414581.4523    | -0.0117         | 0.0091          |
| 311       | 6089118.3447    | 584389.8085    | 584389.7969    | -0.0127         | -0.0116         |
| 401       | 6590192.0115    | 541864.6581    | 541864.6568    | -0.0074         | -0.0013         |
| 402       | 6589613.4649    | 718703.8336    | 718703.8348    | -0.0127         | 0.0012          |
| 403       | 6473252.8898    | 658701.1614    | 658701.1584    | -0.0166         | -0.0030         |
| 404       | 6478886.3818    | 404611.4777    | 404611.4691    | -0.0146         | -0.0086         |
| 405       | 6444302.5676    | 559477.0009    | 559476.9926    | -0.0076         | -0.0083         |
| 406       | 6334917.5992    | 717738.2104    | 717738.2073    | -0.0221         | -0.0031         |
| 407       | 6199760.1464    | 725910.0411    | 725910.9935    | 0.0144          | -0.0476         |
| 408       | 6156799.5186    | 481318.6422    | 481318.6496    | -0.0228         | 0.0074          |
| 409       | 6015165.7893    | 460745.2649    | 460745.2578    | -0.0220         | 0.0071          |
| 410       | 6264461.9294    | 363483.8445    | 363483.8345    | -0.0203         | -0.0100         |
| 411       | 6405424.0359    | 164014.3807    | 164014.3760    | -0.0206         | -0.0047         |
3. Strain and Stress Field Determination

Strains $\varepsilon_{xx}$, $\varepsilon_{yy}$, $\varepsilon_{xy}$ are linked to shifts $u$ and $v$ and are calculated by three geometric (Koshi) equations in a horizontal plane at a point of the deformed body (Atkočiūnas and Nagevičius 2004; Zakarevičius and Stanionis 2004b):

$$
\begin{align*}
\varepsilon_{xx} &= \frac{\partial u}{\partial x}, \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y}, \\
\varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}.
\end{align*}
$$

(1)

In an operational matrix form, Koshi geometric equations are (Atkočiūnas and Nagevičius 2004; Zakarevičius and Stanionis 2004b):

$$
\varepsilon = \nabla^T \cdot u,
$$

(2)

$$
\varepsilon = [\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{xy}]^T,
$$

(3)

$$
u = [u \quad v]^T,
$$

(4)

where $\varepsilon$ is the vector of horizontal strains, $\nu$ is the vector of shifts, $\nabla^T$ is transposed Hamilton operator.

Strains on plane stress state $\varepsilon_{xx} = 0$, $\varepsilon_{yy} = 0$, $\varepsilon_{xy} \neq 0$ (Zakarevičius et al. 2005):

$$
\varepsilon_{zz} = -\frac{\nu}{(1-v)}(\varepsilon_{xx} + \varepsilon_{yy}),
$$

(5)

where $v$ – Poisson’s ratio (0,25), $\varepsilon_{xx}$ and $\varepsilon_{yy}$ – relative shear strains, $\varepsilon_{xy}$ – relative linear strain.

When horizontal relative linear and shear strains are calculated, it is possible to evaluate change in tectonic stress (for certain time span).

The inverse Hook’s Law may be applied to model tectonic stresses in the horizontal plane ($\sigma_{zz} = 0$, $\sigma_{xy} = 0$, $\sigma_{xx} = \sigma_{yy} = 0$) (Atkočiūnas and Nagevičius 2004; Zakarevičius et al. 2005; Zakarevičius and Stanionis 2004a):

$$
\begin{align*}
\sigma_{xx} &= \frac{E}{1-v^2}(\varepsilon_{xx} + \nu \varepsilon_{yy}), \\
\sigma_{yy} &= \frac{E}{1-v^2}(\varepsilon_{yy} + \nu \varepsilon_{xx}), \\
\sigma_{xy} &= G \varepsilon_{xy} = \frac{E}{2(1+v)} \varepsilon_{xy},
\end{align*}
$$

(6)

where $G$ is shear modulus, $E$ is Young’s modulus $\left(7 \cdot 10^{10}\ \text{N/m}^2\right)$, $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{zz}$ – normal stresses, $\sigma_{xy}$, $\sigma_{xz}$, $\sigma_{yz}$ – shear stresses.

Physical relationships (6) can be written in a matrix form (Zakarevičius et al. 2005; Zakarevičius and Stanionis 2004a):

$$
\sigma = K \cdot \varepsilon,
$$

(7)

$$
\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}^T,
$$

(8)

$$
K = \begin{bmatrix} \frac{E}{1-v^2} & \nu \varepsilon_{xy} \\ \nu \varepsilon_{xy} & 1 \varepsilon_{yy} \end{bmatrix},
$$

(9)

$$
\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{xy} \end{bmatrix}^T,
$$

(10)

where $\sigma$ is the vector of tectonic stress, $\varepsilon$ – the vector of horizontal strains, $K$ is - stiffness matrix.

Following the law of shear stress duality, $\sigma_{xy} = \sigma_{yx}$.

Accordingly, tectonic stress state in a horizontal plane is defined by the symmetric stress tensor (Atkočiūnas and Nagevičius 2004):

$$
\tilde{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix},
$$

(11)

The second rank stress tensor $\tilde{\sigma}$ does not depend on the selected coordinate system.

Principal tectonic stresses are calculated as a quadratic equation (Atkočiūnas and Nagevičius 2004; Zakarevičius et al. 2005):

$$
\sigma_I^2 - I_1 \cdot \sigma + I_2 = 0,
$$

(12)

that is obtained by extending the determinant:

$$
\begin{vmatrix} \sigma_{xx} - \sigma & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} - \sigma \end{vmatrix} = 0,
$$

(13)

$$
I_1 = \sigma_{xx} + \sigma_{yy},
$$

(14)

$$
I_2 = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{vmatrix},
$$

(15)

where $\sigma$ is – principal stresses, $I_1$, $I_2$ – stress tensor invariants.

By solving the quadratic equation (12), two actual roots $\sigma_1$ and $\sigma_2$ ($\sigma_1 \geq \sigma_2$) are obtained, i.e. $\sigma_1$ is maximum principal stress and $\sigma_2$ is minimum principal stress.
4. 2-D Finite Element Modelling of Tectonic Stresses

GPS sites are rather regularly spaced (Fig. 2). Data on 19 GPS marks were used in the conducted analysis. Measurements in the Baltic Sea region were carried out in 1992 and 2003. The coordinates of GPS marks of two measurement cycles are presented in Table 1. The relative errors of the network chords (zero class) do not exceed \( \approx 0.1 \times 10^{-6} \).

Following the above described approach for calculating horizontal stresses, the two-dimensional (2-D) thin-shell body was modelled to define horizontal stresses affecting the Baltic region. The finite element approach was applied assuming that the geometric elements (triangles) of the limited size deform isotropically. Zero movements of a model contour define boundary conditions. The model incorporates four fixed points and 19 mobile GPS sites. The area of the model is larger than that of the Baltic region and consists of 354 finite elements (triangles), 68 of which cover the Baltic Region area (Figs 1, 2). An increase in the model area is required to avoid artifacts (if any) at the edges of the model.

The finite element is described by six nodes: I, J, K, L, M and N (Fig. 3). Each node of the triangle has two degrees of freedom (north and east shifts).

A deformation of the finite element is described by (Ansys Theory Reference 1998):

\[
\begin{align*}
    u_i &= u_{I} (2L_3 - 1)L_1 + u_{J} (2L_2 - 1)L_2 + \\
    &+ u_{K} (2L_3 - 1)L_3 + u_{M} (4L_2 L_3) + \\
    &+ u_{N} (4L_2 L_1), \\

    v_i &= v_{I} (2L_4 - 1)L_1 + v_{J} (2L_4 - 1)L_2 + \\
    &+ v_{K} (2L_2 - 1)L_3 + v_{M} (4L_2 L_3) + \\
    &+ v_{N} (4L_2 L_1),
\end{align*}
\]

where \( u_{I}, u_{J}, u_{K}, u_{M}, u_{N} \), \( v_{I}, v_{J}, v_{K}, v_{M}, v_{N} \) are shifts of node coordinates; \( L_1, L_2, L_3 \) are normalized coordinates (range from 0 to 1 in the finite element).

The modelled stress field of the Baltic region defined from the GPS network

Changes in principal stresses were estimated for the finite element nodes (Fig. 4). The value of change in maximum principal stress in the territory of the Baltic Sea region varies between \(-0.0013\) MPa and \(+0.0032\) MPa, whereas the value of minimum principal stress change is between \(-0.0084\) MPa and \(+0.0009\) MPa (Table 2). The calculated principal stress directions are presented in Fig. 5.
5. Geodynamic Interpretation

Modeling a GPS network reveals significant changes in horizontal tectonic stress affecting the Baltic region subject to horizontal extension the direction of which rotates from NNE-SSW in the south and west to NW-SE in the north and east (Fig. 5). The eastern part of Lithuania and Latvia and the northern part of central Poland are subject to predominating horizontal compression of respectively N-S and NW-E direction.

The identified low-rate strain rates are compatible to those obtained from other cratonic areas (e.g. Fennoscandian Shield, North America, and India). Furthermore, the domination of extensional deformations in the western and northern parts of the Baltic region correlate with GPS data obtained from Fennoscandia that is accounted to the post-glacial up-doming of the lithosphere induced by glacial isostasy. It may explain higher seismic activity of the extension-dominated area of the Baltic region (Latvia and Estonia).

The inferred pattern of distributing parameters of horizontal deformation from geodetic networks is important for understanding seismic processes in the Baltic region.

6. Conclusions

1. Tectonic stresses in the Earth’s crust in the Baltic region were calculated. Positive values of stresses are prevailing in the direction of maximum principal stress; the values of minimum principal stress are negative within the whole territory (except three nodes).

2. Three different provinces were identified, which shows different stress regimes. It implies different geodynamic mechanisms involved in the Baltic area. The obtained stresses are compatible to other cratonic regions.

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