Nuclear Spin-Isospin Correlations, Parity Violation, and the $f_\pi$ Problem

Gerald A. Miller
Department of Physics
University of Washington
Seattle, WA 98195-1560

The strong interaction effects of isospin- and spin-dependent nucleon-nucleon correlations observed in many-body calculations are interpreted in terms of a one-pion exchange mechanism. Including such effects in computations of nuclear parity violating effects leads to enhancements of about 10%. A larger effect arises from the one-boson exchange nature of the parity non-conserving nucleon-nucleon interaction, which depends on both weak and strong meson-nucleon coupling constants. Using values of the latter that are constrained by nucleon-nucleon phase shifts leads to enhancements of parity violation by factors close to two. Thus much of previously noticed discrepancies between weak coupling constants extracted from different experiments can be removed.
The problem of determining the parity-violating interaction between nucleons has drawn recent attention with measurements of the $^{133}$Cs anapole moment [1], the TRIUMF measurement of proton-proton scattering [2] and the interactions of epithermal neutrons with heavy nuclei [3]. The parity non-conserving NN potential, $V_{\text{PNC}}$, of Desplanques, Donoghue and Holstein (DDH) [4], constructed so that the only unknown quantities are the weak meson-nucleon coupling constants, which govern the strength of the weak meson-nucleon vertex function, has been the standard tool to analyze these data because the parity-violating observables have been expressed as linear combinations of these, presumably fundamental, weak coupling constants. As explained in various reviews [5–9], a problem occurs because different values of the weak coupling constants are needed to describe different experiments. In particular, the $^{133}$Cs anapole moment requires a pion-nucleon weak coupling constant $f_\pi$ that is larger than that predicted in Ref. [4], but the observation of circularly polarized $\gamma$ rays in the decay of $^{18}$F data require a very small value for $f_\pi$. Furthermore, a recent analysis of heavy compound nuclei [10] finds parity violating effects that are larger (by factors $\sim 1.7 - 3$) than those predicted using DDH potential.

We next explain why the results of nuclear structure calculations [11,12] cause us to examine the effects of spin-isospin, nucleon-nucleon, correlations on parity violation. Nuclear parity violating effects have been typically analyzed in terms of a parity-violating single nucleon shell model potential, constructed from the DDH potential using a Hartree-Fock approximation and RPA correlations (e.g. [5,10,13]). However, two-particle-two-hole correlations are known to be as vital elements of nuclear structure. The spin-independent effects of the short range correlations are often incorporated [5] using the Miller-Spencer correlation function [14], but this does not take into account all of the correlation effects [15,7].

In particular, recent variational studies of nuclear matter [11] and $^{16}$O [12] have demonstrated that spin-isospin correlations are very important. To be specific, consider two-nucleon pair correlation functions defined by the expectation values

$$\rho_\Gamma(r) = \frac{1}{4\pi r^2 A} \langle \Psi | \sum_{i<j} \delta(r - r_{ij}) \Gamma_{i,j} | \Psi \rangle,$$

with $r_{ij} = |r_i - r_j|$, and $\Gamma_{i,j}$ are various two-nucleon operators. For the central term $\Gamma_c = 1$. The work of Refs. [11,12] found that with $\Gamma_{i,j} = \tau_i \cdot \tau_j \sigma_i \cdot \sigma_j$, ($\Gamma = \sigma \tau$) or $\Gamma_{i,j} = \tau_i \cdot \tau_j S_{ij}$ ($\Gamma = \text{ten} \tau$), the $\rho_\Gamma(r)$ can be relatively large, even larger than the well-known effects of short-distance repulsion. Other operators $\Gamma$ cause much smaller effects and are ignored here.

What is the impact of spin-isospin correlations on calculations of parity-violating observables? Consider the construction of the single-particle PNC potential $\hat{U}_{\text{PNC}}$ which governs the interaction between a valence nucleon ($i$) with a spin $0^+$, $N = Z$ core. In using the Hartree-Fock approximation, one folds the operator $V_{\text{PNC}}$ with the density matrix of the core. The pion exchange term contains the operator $i(\tau_i \times \tau_j)_z$, where $j$ represents the core nucleons. The expectation value of this operator vanishes.
But if one includes spin-isospin correlations, the relevant matrix elements include a factor \( \tau_i \cdot \tau_j \), and the product \( i(\tau_i \times \tau_j)_z \tau_i \cdot \tau_j = -2(\tau_i - \tau_j)_z - i(\tau_i \times \tau_j)_z \) contains a non-vanishing term, \(-2\tau_i \). Thus a new non-vanishing contribution will appear.

The starting point for our evaluation of the PNC single-particle potential is the effective parity-violating nucleon-nucleon potential [4] between two nucleons (i,j):

\[
M^{\text{PNC}}(i,j) = i \frac{f_{\pi} g_{\pi NN}}{\sqrt{2}} \left( \frac{\tau_i \times \tau_j}{2} \right)_z (\sigma_i + \sigma_j) \cdot \mathbf{u}_\pi (\mathbf{r})
- g_\rho \left( h_\rho^0 \tau_i \cdot \tau_j + h_\rho^1 \left( \frac{\tau_i + \tau_j}{2} \right)_z + h_\rho^2 \left( \frac{3\tau_i^z \tau_j^z - \tau_i \cdot \tau_j}{2\sqrt{6}} \right) \right)
\times \left( (\sigma_i - \sigma_j) \cdot \mathbf{v}_\rho (\mathbf{r}) + i(1 + \chi_V) (\sigma_i \times \sigma_j) \cdot \mathbf{u}_\rho (\mathbf{r}) \right) - g_\omega \left( h_\omega^0 + h_\omega^1 \left( \frac{\tau_i + \tau_j}{2} \right)_z \right)
\times \left( (\sigma_i - \sigma_j) \cdot \mathbf{v}_\omega (\mathbf{r}) + i(1 + \chi_S) (\sigma_i \times \sigma_j) \cdot \mathbf{u}_\omega (\mathbf{r}) \right) - (g_\omega h_\omega^0 - g_\rho h_\rho^1) \left( \frac{\tau_i - \tau_j}{2} \right)_z
\times (\sigma_i + \sigma_j) \cdot \mathbf{v}_\rho (\mathbf{r}) - g_\rho h_\rho^1 \left( \frac{\tau_i \times \tau_j}{2} \right)_z (\sigma_i + \sigma_j) \cdot \mathbf{u}_\rho (\mathbf{r})
\right)
\]  
(2)

where \( M \) is the nucleon mass, \( \mathbf{v}_m (\mathbf{r}) \equiv \{ \mathbf{p}, f_m (\mathbf{r}) \}, \mathbf{u}_m (\mathbf{r}) \equiv [ \mathbf{p}, f_m (\mathbf{r}) ], f_m (\mathbf{r}) = \exp(-m_m r)/4\pi r \) (with \( m_m = m_{x,\rho,\omega} \)). The strong interaction parameters used by DDH are \( g_{\pi NN} = 13.45, g_{\rho NN} = 2.79, g_{\omega NN} = 8.37, \chi_V = 3.7, \chi_S = -0.12 \). The formula (2) is still used, [10,17]-[19] with these original strong interaction parameters.

The effect we wish to incorporate is that Eq. (2) is not the complete PNC interaction between two-nucleons. This is because the PNC potential acting once, occurs in the midst of all orders of the strong potential, \( V \). The resulting PNC reaction matrix \( G^{\text{PNC}}(E) \) is a generalization of the Bruckner reaction matrix and is given by:

\[
G^{\text{PNC}}(E) = \Omega^\dagger(E) V^{\text{PNC}} \Omega(E),
\]
(3)

\[
\Omega(E) = 1 + \frac{Q}{E - H_0} V \Omega(E) = 1 + \frac{Q}{E - H_0} G(E),
\]
(4)

where \( E \) is an energy to be discussed below, and \( Q \) is an operator which projects onto unoccupied states.

Our focus is on a first estimate of the effects of spin-isospin correlations. Thus we consider a nucleon of momentum \( \mathbf{P} \) outside a core which is approximated as infinite nuclear matter. In this case, the PV single particle potential \( \tilde{U}^{\text{PNC}} \) is given by:

\[
\langle s, t | \tilde{U}^{\text{PNC}}(\mathbf{P}) | s', t' \rangle = \sum_{\mathbf{k}, (k \leq k_F) m, m_t} \langle P, s, t; k, m, t | G^{\text{PNC}}(E) | P, s', t'; k, m, m_t \rangle_A.
\]
(5)

The goal of this work is to assess the influence of tensor and spin dependent correlation effects on calculations of PNC observables. Since these are usually ignored, performing a schematic calculation seems appropriate. It might seem easiest to parameterize the different contributions to \( \Omega(E) \) as function of \( r \), the procedure of Miller & Spencer. Such a strategy will not work here, with the emphasis on finding hitherto
neglected contributions to the direct term. To see this consider the terms of the DDH potential of the form $[p, f_m] \sim \sigma \cdot r$. The Miller-Spencer procedure would be to treat $(r|\Omega(E)|\kappa, m, m_t)$ as a function of $r$, so that the direct matrix element of Eq. (5) would involve a vanishing volume integral of $\sigma \cdot r$ times a function of $r$. Indeed, the PV effective potential arises from the dependence of $\Omega(E)$ on the relative momentum

$$\kappa = (P - k)/2. \quad (6)$$

The importance of the spin-isospin correlations arises from the exchange of pions \cite{11,12}, so we separate the potential $V$ into a one pion exchange term, $V_{OPE}$ and a remainder approximated as being a central potential, $V_c$. We include the effects of the central potential (which include the short-distance repulsion) to all orders and keeps terms of first-order in $V_{OPE}$. The application of the two-potential theorem to Eq. (4) gives

$$\Omega(E) \approx \Omega_c(E) + \Omega_c^\dagger(E) \frac{Q}{E - H_0} V_{OPE} \Omega_c(E), \quad (7)$$

with $\Omega_c(E) \equiv 1 + \frac{Q}{E - H_0} V_c \Omega_c(E)$, and $Q$ is the usual operator which projects onto two-particle-two-hole states. It is natural to model $\Omega_c(E)$ as

$$(r|\Omega_c(E)|\kappa) = (1 + f_c(r)) e^{i\kappa \cdot r}, \quad (8)$$

so that Eq. (7) can be expressed as

$$(r|\Omega(E)|\kappa) = (1 + f_c(r)) e^{i\kappa \cdot r} + \hat{\psi}_\kappa(r), \quad (9)$$

with

$$\hat{\psi}_\kappa(r) = (1 + f_c(r)) \int \frac{d^3p}{(2\pi)^3} \frac{M}{p^2 + \omega^2} e^{ip \cdot r} (p| V_{OPE}(1 + f_c)|\kappa). \quad (10)$$

The notation $|$ denotes a spatial overlap, so that $\hat{\psi}_\kappa(r)$ are operators in spin-isospin space. The parameter $\omega \ (\omega^2/M = -E)$, will be chosen to reproduce the results of Ref. \cite{11}. Using a negative value of $E$ causes the two-particle-two-hole fluctuations to have a short range of order $\sim 1/\omega$. This allows us to neglect the effects of the operator $Q$ because terms involving $1 - Q$ can be regarded as correction terms, of higher order in the density than the terms we examine here.

The main result of this analysis is the simple model (9). It is necessary to compute the matrix elements of the pair-correlation operators of Eq. (1) to see if the results of Ref. \cite{11} can be reproduced with such a simple formula. Evaluating Eq. (1) keeping only two-nucleon correlations leads to the result:

$$\rho_T(r) = \sum_{S, M_S, T, M_T} \int \frac{d^3\kappa}{(2\pi)^3} \theta(k_f - \kappa) f(\kappa) \langle \kappa, S, M_S, T, M_T | \hat{\rho}_T(r) | \kappa, S, M_S, T, M_T \rangle_A, \quad (11)$$

$$\hat{\rho}_T(r) \equiv \frac{2}{4\pi r^2} \Omega^\dagger(E) \sum_{i<j} \delta(r - r_{ij}) \Gamma_{ij} \Omega(E), \quad (12)$$

\[3\]
where $f(\kappa) = 1 - \frac{3}{2}(\kappa/k_F^2) + \frac{1}{2}(\kappa/k_F^3)$ and $|\kappa, S_M, T, M_T\rangle \equiv |\kappa, S, M_S, T, M_T\rangle - (-1)^{S+T}|\kappa, S, M_S, T, M_T\rangle$. We take $k_F = 1.36$ fm$^{-1}$ and choose the previously determined [14] function $f_\epsilon(r) = -e^{-\alpha r^2}(1 - \beta r^2)$ with $\alpha = 1.1$ fm$^{-2}$, $\beta = 0.68$ fm$^{-2}$. The value of $\omega$ that leads to reproducing the results of Ref. [11] is $\omega = 2$ fm$^{-1}$, which is of the expected order $\sim k_F$. As shown in Fig. 1, and the present results reproduce the qualitative features of the pair-correlation functions of Ref. [11]. Furthermore, the integrals of our $\rho_T$ are in quantitative agreement with those of Ref. [11], so the present model should provide an adequate first assessment of the impact of such correlations on calculations of PNC observables.

![FIG. 1. Pair-correlation functions. Solid curves are from Akmal & Pandharipande [11], dashed curves are from this work.](image-url)

Given our model of Eq. (9), we can now evaluate the operator $\hat{U}_\text{PNC}$ of Eq. (5). Using Eq. (9) in Eq. (3), and keeping terms of first order in $V_{OPE}$ leads to

$$G_{\text{PNC}}(E) \approx \tilde{V}_{\text{PNC}} + \Omega^\dagger(E) V_{\text{OPE}} \frac{Q}{E - H_0} \tilde{V}_{\text{PNC}} + \tilde{V}_{\text{PNC}} \frac{Q}{E - H_0} V_{\text{OPE}} \Omega(E),$$

$$= \tilde{V}_{\text{PNC}} + \Delta G_{\text{PNC}}(E),$$

(13)
where $\tilde{V}_{PNC} \equiv (1 + f_c)V_{PNC}(1 + f_c)$. The use of Eq. (13) in Eq. (5) specifies our model [16]. The numerical evaluations are straightforward, so we present the results.

The operator $\tilde{U}_{PNC}(P)$ of Eq. (5) must be a pseudo-scalar spin-isospin operator. For our nuclear matter problem it is of the form

$$\tilde{U}_{PNC}(P) = \sigma \cdot P \rho \frac{\rho}{M^3}(g_0 + g_1 \tau_z).$$

For $^{133}$Cs and $^{205}$Th, $(N - Z)/(N + Z) \approx 0.17$, and we find that keeping the term $g_1$ amounts to keeping a correction to a term that is not large. So we take $N = Z$ causing $g_1$ to vanish. To assess the importance of the spin-isospin correlation, we compare the influence of the two terms of Eq. (14) in Eq. (5). Denoting the results of using the first ($V_{PNC}$) term as $g_0^{(0)}$ and those of using the second term as $\Delta g_0$, with $g_0 = g_0^{(0)} + \Delta g_0$, and evaluating the matrix element numerically leads to the results:

$$g_0^{(0)} = 24 f_\pi - 6.9 h^0_\rho - 3.6 h^1_\rho - 4.1 h^0_\omega - 4.1 h^1_\omega$$

$$\Delta g_0 = 2.43 f_\pi - 1.2 h^0_\rho - 0.66 h^1_\rho - 0.351 h^0_\omega - 0.15 h^1_\omega$$

We see that the coefficient of each term is enhanced by about 10% if $\Delta g_0$ is included. This is in contrast with many other nuclear structure effects which reduce the effects of PNC [8,9]. The present results constitute an argument that the effects of spin-isospin correlations need to be included in quantitative calculations of PNC effects, but are not large enough to not have a major impact [20]. Indeed, uncertainties due to nuclear structure effects might not have a large impact on PNC observables.

But there is a more obvious source of change to present evaluations. Examination of the DDH potential, Eq. (2) immediately reveals the dependence on the product of strong and weak coupling constants. In a one-boson-exchange model one uses one strong and one weak meson-nucleon vertex function to construct the potential. But the strong coupling constants can be separately determined by computing the potential $V$ and choosing the parameters to reproduce experimentally measured scattering observables. This is the procedure of e. g. the Bonn potential [23]. As shown in Table I, the strong coupling constants required to fit phase shifts are much larger than those used originally by DDH (and used presently in Refs. [17–19]).

|       | DDH  | Bonn (OBEPR) |
|-------|------|--------------|
| $g_{\pi NN}$ | 13.45 | 13.68        |
| $g_\rho$      | 2.79  | 3.46         |
| $\chi V$      | 3.7   | 6.1          |
| $g_\omega$    | 8.37  | 20           |
| $\chi S$      | -0.12 | 0.0          |

**TABLE I.** Comparison of strong coupling constants.
The coordinate-space potential (Table 14) of Ref. [23] is used for this comparison because it is a non-relativistic, local potential that is technically compatible with Eq. (2). The large values of $\chi_V$ and $g_\omega$ are essential requirements to reproduce data, if a one-boson-exchange model is used [24].

Now consider the impact of using the Bonn parameters of Table I, in the DDH potential. If one considers a nucleon outside an N=Z core, the parameter combinations $h^0_\rho g_\rho(1 + \chi_V)$ and $h^0_\omega g_\omega$ determine the vector meson contributions to $U^{PNC}$ [5]. Thus using the Bonn parameters is equivalent to increasing the coefficient of the terms proportional to $h^0_{\omega,\rho}$ by a factor of 1.9. The effects of using values such as those of Table I have been noticed previously [25,26].

![FIG. 2. Constraints on the PNC coupling constants ($\times 10^7$) that follow from using the Bonn coupling constants of Table I.](image)

To make a first assessment of the impact of this finding, we revise the results of Table VII and Fig. 9 of Ref. [17] by multiplying the coefficient of the terms proportional to $h^0_{\omega,\rho}$ by a factor of 1.9 [21]. This leads to Fig. 2. The allowed regions are between the dashed lines (pp), each set of solid lines ($^{18,19}F$), and each set of dot-dashed lines $^{133}Cs$. The effects of using values such as those of Table I have been noticed previously [25,26].
The results of the $pp$, and $^{18,19}\text{F}$ experiments can be explained with one set of weak-coupling parameters. The $^{205}\text{Tl}$ result with its large error band is almost consistent with that set of parameters, but the $^{133}\text{Cs}$, and $^{205}\text{Tl}$ experimental results are not compatible, as pointed out earlier [17]. The results from compound nuclei are typically not shown in plots such as Fig. 2, but using the Bonn parameters of Table I, would reduce the discrepancies between theory and experiment.

The constraints arising from $(p,\alpha)$ scattering experiments are not shown in Fig. 2, because currently interpreted provides constraints very similar to that of the $^{19}\text{F}$ experiment, and the calculations are not not complete. For example, $\hat{U}^{\text{PNC}}$, which should be complex, is treated as real. But it is worth remarking that an $(n,\alpha)$ experiment would provide very different constraints. The parity violation predicted using the original DDH potential is very small due to a cancellation between the pionic and vector meson exchange terms [22]. The use of the Bonn coupling constants (Table I) leads to a value of $g_n = 1.9$ (see Eq. (18) of [13]) instead of $g_n = 0.2$. This would be increased further by using a smaller value of $f_\pi$ (indicated in Fig. 2) than the DDH “best guess value” of Ref. [13]. If $f_\pi = 1$, then $g_n = 3.2$.

The ranges of weak-coupling constants covered by Fig. 2 are roughly consistent with the DDH “reasonable ranges” and the same coupling constants account for much of the data, except for that of $^{133}\text{Cs}$. This constitutes some success in explaining PNC phenomena, but the main improvement obtained here by using larger strong coupling constants could itself be true only within one-boson exchange models. All such models have large values of $\chi_V$ and $g_{\omega NN}$, but two-boson exchange models have smaller values of $g_{\omega NN}$ [23]. Furthermore, the one-boson-exchange approach is not consistent with the present state of the art treatments of nucleon-nucleon scattering, see Refs. [27,28], and [29]. An appropriate treatment of PNC effects is one which involves incorporating PNC effects within an updated treatment of the nucleon-nucleon interaction. For example, Holstein has recently advocated an effective field theory treatment [30].

Thus our summary is that the present results demonstrate the need for an improved, updated, consistent incorporation of strong and weak interaction effects, and also indicate that the nuclear structure uncertainties might not be very severe. Thus the improvement of calculations of nuclear PNC effects seems to be an interesting and feasible task, even though much remains to be done.

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