Large-eddy simulations of wind farm production and long distance wakes

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Abstract. The future development of offshore wind power will include many wind farms built in the same areas. It is known that wind farms produce long distance wakes, which means that we will see more occasions of farm to farm interaction, namely one wind farm operating in the wake of another wind farm. This study investigates how to perform accurate power predictions on large wind farms and how to assess the long distance wakes generated by these farms. The focus of this paper is the production’s and wake’s sensitivity to the extension of the grid as well as the turbulence when using Large-eddy simulations (LES) with pregenerated Mann turbulence. The aim is to determine an optimal grid which minimizes blockage effects and ensures constant resolution in the entire wake region at the lowest computational cost.

The simulations are first performed in the absence of wind turbines in order to assess how the atmospheric turbulence and wind profile are evolving downstream (up to 12,000 m behind the position where the turbulence is imposed).

In the second step, 10 turbines are added in the domain (using an actuator disc method) and their production is analyzed alongside the mean velocities in the domain. The blockage effects are tested using grids with different vertical extents. An equidistant region is used in order to ensure high resolution in the wake region. The importance of covering the entire wake structure inside the equidistant region is analyzed by decreasing the size of this region. In this step, the importance of the lateral size of the Mann turbulence box is also analyzed.

In the results it can be seen that the flow is acceptably preserved through the empty domain if a larger turbulence box is used. The relative production is increased (due to blockage effects) for the last turbines using a smaller vertical domain, increased for a lower or narrower equidistant region (due to the smearing of the wake in the stretched area) and decreased when using a smaller turbulence box (due to decreased inmixing). The long distance wake behind the row is most impacted by the use of a smaller turbulence box, while the other simulation setups have less influence on these results.

In summary, the results show the importance of having relatively large extensions of the domain, large extensions of the equidistant region and especially large extensions of the turbulence box.

1. Introduction
The future development of wind power offshore will include many wind farms built in the same areas. It is known that these wind farms will produce long distance wakes [1]. This means that we will see more instances of farm to farm interaction where one wind farm is in the wake of another wind farm. A step towards a better understanding of this interaction is to study the long distance wake behind single wind farms using numerical models.
The scale of the long distance wakes is in the order of 10 km [2]. Different types of wake models and meso scale models can be used to study the long distance wakes [1][3]. Large eddy simulations are, although more computational demanding, another alternative to these models considering the continuously increasing computational resources available.

Large eddy simulations (LES) have been used for simulations of wakes inside wind farms in a wide range of studies, see e.g. [4][5][6][7][8][9][10][11]. LES in combination with the actuator disc (ACD) methodology was recently used to study the relative production of the Lillgrund wind farm with good comparison to measurements [11].

Modeling of larger wind farms with long extension and the long distance wakes downstream is, however, associated with increased uncertainties. Large eddy simulations in combination with ACD have been used by Eriksson et al. [12] to study the wake behind Horns Rev. The results showed an increased production at the end of the farm and a faster velocity recovery in the long distance wake compared to the measurements. A parameter study on a row of ten turbines has been performed to determine the impact of grid resolution, Reynolds number, turbulence intensity and internal spacing on production as well as on the velocity recovery in the long distance wake [13]. The results were most sensitive to the turbulence level. The increase in the production in the end of the row could not be explained.

The present study uses LES with ACD on a row of ten turbines and investigates the impact of the grid dimensions, the dimensions of the equidistant region and the extension of the imposed turbulence planes. The study is presented in Section 3 and the simulation setups are presented further in Section 4. First the numerical model is presented in Section 2. The simulations are analysed with respect to production and the development of the wind shear, the velocity as well as the turbulence throughout the domain, see Section 5.

The aim of the present study is to acquire additional knowledge on how to perform accurate power predictions of long wind farms and how to assess the long distance wakes generated by these farms. The focus is on the sensitivity to the extension of the grid and the turbulence when using LES with pregenerated Mann turbulence.

2. Numerical model
The computations are carried out as large-eddy simulations using the EllipSys3D code. The wind turbines are parameterized using an actuator disc model [14]. A controller is used to maintain the optimal rotational speed for each turbine[15]. The sub-grid-scale model from Ta-Phouc[16] is used for the smallest (less than 2 times the grid resolution) non-resolved eddies in the LES.

The atmospheric turbulence is pregenerated using the Mann model [17]. The turbulence and the wind shear are imposed in the computational domain using body forces[10]. The simulations are performed assuming a neutral boundary layer.

The domain is stretched in the inlet, the outlet, towards the sides and towards the top. In the region of interest the domain is equidistant in all directions (uniform). The boundary conditions are fixed values for the inlet (according to the shear), periodic for the sides, convective for the outlet, far field for the top and for the ground. The far field correspond for the ground to a noslip condition and for the top to a fixed velocity (according to the shear).

3. Study of one row of turbines
The study is performed on a row of 10 turbines and includes the long distance wake up to over 6000 m (150 R) downstream from the last turbine, see Figure 1. The simulations are first performed in the absence of wind turbines in order to study the preservation of the flow characteristics throughout the domain (up to 12,000 m behind the position where the turbulence is imposed).

As a second step, 10 turbines are added in the domain and their production is analyzed alongside the mean velocities in the domain. The aim of this step is to determine an optimal grid
which minimizes blockage effects and ensures a constant resolution in the entire wake region at the lowest computational cost. The blockage effects (acceleration dependent of the cross section of the domain versus rotor size) are investigated using grids with different vertical extents. In order to have a high resolution in the wake region an equidistant region is used. The importance of covering the entire wake structure inside the equidistant region is analyzed by decreasing this region. In this step, the importance of the size of the Mann turbulence box is also analyzed.

4. Simulation setup
The 10 turbines used in the simulation have a radius of 80 m, their hub height is 70 m (1.75 R). This turbine is based on a NREL 5 MW [18] and scaled down, to correspond to a Vestas V80 regarding power and thrust coefficient [11].

![Figure 1. Row of 10 turbines.](image)

These are put into a domain, see Figure 1, that has an equidistant region with the length of 302 R (in the streamwise direction z). The extension in width (x-direction) and height (y-direction) varies. The turbulence is inserted at 13 R, which corresponds to 3 R into the equidistant region, and the first turbine is inserted at 17 R. The internal spacing in the row is 14 R. The resolution in the equidistant region is chosen to 0.1 R (in accordance with earlier studies [11]) and a non dimensionalized (with $U_0$ and R) timestep of 0.025 is used for all cases. Full convergence could however not be shown for the downstream turbines with this resolution in an earlier parameter study [13]. The chosen grid resolution also has an impact on to what extent the sub grid model is used. The flow passes through the domain for 26,000 time steps (50 min) before the averaging is started. The averaging is performed over a physical time period of 50 min.

![Figure 2. The grid (in the x-y plane) and extension of the turbulence box (marked with dashed white lines) for (a) Reference case, (b) Lower domain, (c) Turbulence box -small, (d) Equidistant region: High/narrow, (e) Equidistant region: Wide/low, (f) Equidistant region: Small.](image)

The boundary layer in the simulations is described with respect to wind shear and turbulence level. The wind shear follows the logarithmic profile in Equation 1, with a roughness length value ($z_0$) of 0.0001 and constant giving a wind speed of 8 m/s at the hub height 70 m.

$$U(y) = \frac{u_*}{\kappa} \ln\left(\frac{y}{z_0}\right) = 0.5944 \ln\left(\frac{y}{0.0001}\right)$$ (1)
The Mann turbulence is created using the same roughness length \( (z_0) \) of 0.0001 and the same wind speed of 8 m/s at the hub height 70 m. This gives \( \alpha \epsilon^{2/3} \left[ m^{4/3} s^{-2} \right] = 0.009040 \), the eddy lifetime constant of \( \gamma = 3.886 \) and the length scale \( L = 41.3327 \, [m] \). For a detailed description the reader is referred to Mann [19]. The length of the Mann box corresponds to a physical time of 10 min (using the Taylor frozen hypothesis). The resolution is slightly larger in the Mann box compared to the grid with \( dz = 0.117 \, R \) and \( dx = dy = 0.125-0.195 \, R \) (depending on case).

4.1. Studied cases

The simulations are performed for different setups of the domain and sizes of the turbulence box. All modifications are compared to a reference case. The reference case, see Figure 2a, is chosen with rather large extensions and can be seen as the “best affordable”. The other setups study the impact of decreasing the extension to save computational power. The setup for the different cases along with used grid size can be seen in Table 1. The grid and the turbulence box (marked with dashed white lines) are also visually presented in Figures 2.

Table 1: The setup used for the reference and modified cases along with the grid size. The bold text indicate the difference to the reference case.

| Case description            | Short name | Domain \((x*y)\) \([R*R]\) | Equidistant \((x*y)\) \([R*R]\) | Turbulence \((x*y)\) \([R*R]\) | Grid size \([M \text{ cells}]\) |
|-----------------------------|------------|-----------------------------|-------------------------------|-------------------------------|-----------------------------|
| Reference case              | Ref        | \(20*60\)                   | \(10.75*10.75\)               | \(12.5*12.5\)             | \(50.3\)                   |
| Lower domain                | Low        | \(20*20\)                   | \(10.75*10.75\)               | \(12.5*12.5\)             | \(\leq 50.3\)              |
| Turbulence box -Small       | Turb s     | \(20*60\)                   | \(10.75*10.75\)               | \(4*4\)                    | \(50.3\)                   |
| Equidistant region -High    | Equ h      | \(20*60\)                   | \(4*10.75\)                   | \(12.5*12.5\)             | \(25.2\)                   |
| (and narrow)                |            |                             |                               |                               |                             |
| Equidistant region -Wide    | Equ w      | \(20*60\)                   | \(10.75*4\)                   | \(12.5*12.5\)             | \(25.2\)                   |
| (and low)                   |            |                             |                               |                               |                             |
| Equidistant region -Small   | Equ s      | \(20*60\)                   | \(4*4\)                       | \(12.5*12.5\)             | \(12.6\)                   |
| Small turbulence,           | All s      | \(20*20\)                   | \(4*4\)                       | \(4*4\)                    | \(12.6\)                   |
| Small equidistant,          |            |                             |                               |                               |                             |
| Lower domain                |            |                             |                               |                               |                             |

5. Results and discussion

The study is performed with and without turbines. The simulations performed in the absence of wind turbines shows the flow adjusting to the domain and evolving downstream. In the simulations where 10 turbines are added in the domain their production is analyzed alongside the flow in the domain.

The studied parameters are the streamwise flow and a streamwise turbulence measure, (here the streamwise turbulence is defined as the standard deviation of axial wind divided by the mean free stream velocity). The results include the vertical profiles and the values at hub height (both taken at positions along the centerline of the row of turbines).

The legends in Figures 3-24 refer to the short name for the different cases given in Table 1. Note that all results are presented non dimensionalized with the rotor radius \( (R) \) and the incoming wind speed at hub height \( (U) \).
5.1. Downstream development of velocity, turbulence and wind shear - without turbines

The downstream development of the shear for the different cases can be seen in Figure 3 (for the domain without turbines). The profiles follow the initial profile to a relatively high extent also further downstream in the domain. An acceleration of the flow can be seen for the case with a smaller turbulence box. In Figures 4-11 below the development of wind speed and turbulence at hub height can be seen for the different cases.

Figure 3. The downstream wind shear (without turbines) compared to the inlet profile (marked with x’s). For each case, 6 profiles between 168 R - 293 R (every 25 R / 1 km behind the row) are plotted with increasing line width further downstream. The hub height at 1.75 R is marked in the grid.

The wind speed at hub height for both the reference case and the case with lower domain stays close to 1 throughout the empty domain, as can be seen in Figure 4. However a slight increase of the wind speed can be seen, especially for the lower domain. The turbulence shows in both cases the same trend, namely an increase in the beginning followed by a decrease towards a stable value further downstream, see Figure 5.

Figure 4. Impact of height of domain on velocity at hub height, no turbines.

Figure 5. Impact of height of domain on turbulence at hub height, no turbines.

The wind speed stays around 1 and follows the same trend for all the different sizes of equidistant region throughout the empty domain, see Figure 6. A slightly higher wind speed can, however, be seen downstream in the case with the higher equidistant region. The development of the turbulence shows the same trend for all cases, see Figure 7. Only the reference case and the wide case give slightly lower values (at the end of the domain)
The clearest impact on the downstream development is caused by the size of the turbulence box. In the empty domain the smaller turbulence box gives a downstream increase of the wind speed, see Figure 8. The turbulence peaks in the beginning of the domain and decreases towards a clearly lower value in case of a smaller turbulence box, see Figure 9.

In the extreme cases (that using a small equidistant region, a small turbulence box and a low domain) the trends mostly follow the case with the small turbulence box, see Figures 10, 11. Some smaller impact can, however, also be seen downstream from the other parameters with a slightly higher wind speed from the small equidistant region and a slightly higher turbulence from the small equidistant region.

5.2. Production
The relative production shows a clearer dependency on the simulation setup in the different cases. In Figure 12, it can be seen that the relative production for the reference case and the case with the lower domain are similar for the first turbines. For the downstream portion of this long row of turbines a higher production can be seen when using a lower domain.
The size of the equidistant region, see Figure 13, has less impact. The production follows the same trend for all cases. However, the Eqw case shows a slightly higher relative production from turbine 2 and the Eqw h case shows a slightly higher relative production further downstream. This could be explained by the smearing of the wake then reaching outside the equidistant region and that the turbines are closer to the stretched area upwards.

Figure 12. Impact of height of domain on production.

Figure 13. Impact of size of equidistant region on production.

The size of the turbulence box, see Figure 14, shows a high impact. The production decreases gradually when using a smaller box to a level clearly lower than the reference case. This could be explained by the decreased mixing of the flow from above and the sides.

The results of the extreme case (a small equidistant region, a small turbulence box and a low domain) compared to the reference case can be seen in Figure 15. The gradually lower relative production downstream follows the same trend as in Figure 14 with the small turbulence box and the increase in relative production for the most downstream turbines follows the same trend as in Figure 12 with the lower domain.

Figure 14. Impact of size of turbulence box on production.

Figure 15. Impact of the ”all small case” on production.

5.3. Downstream development of velocity, turbulence and wind shear - with turbines

The downstream development of the shear behind the wind farm (each km / 21.5 R) for the different cases is shown in Figure 16. It is noticeable that the profile for the lower domain shows a faster velocity recovery for all heights indicating some blockage effect. For the smaller turbulence box a slower velocity recovery can be seen as well as a higher vertical extension of the wake.
Figure 16. The downstream wind shear (with turbines) compared to the inlet profile (marked with x’s). The cases are separated 0.1 in the x-axis. For each case, 6 profiles between 168 R - 293 R (every 25 R / 1 km behind the row) are plotted with increasing line width further downstream. The hub height at 1.75 R is marked in the grid.

Figures 17-24 show more details of the downstream trends for wind speed and turbulence at hub height for the different cases.

The long distance wake behind the row shows relatively low dependency on the domain height, see Figures 17, 18. The lower domain shows a slightly faster velocity recovery and a higher turbulence level.

Figure 17. Impact of height of domain on velocity at hub height.

Figure 18. Impact of height of domain on turbulence at hub heights.

The size of the equidistant region is of less importance for the long distance wake, see Figures 19, 20.

Figure 19. Impact of size of equidistant region on velocity at hub height.

Figure 20. Impact of size of equidistant region on turbulence at hub height.

The size of the turbulence box is the most important parameter for the long distance wake,
see Figures 21, 22. The wake recovery follows the same trend in both cases. However the wake recovers faster in the reference case. The turbulence level is slightly higher for the reference case in the beginning of the farm wake but levels out to the same value further downstream.

**Figure 21.** Impact of size of turbulence box on velocity at hub height.

**Figure 22.** Impact of size of turbulence box on turbulence at hub height.

In the extreme case (using a small equidistant region, a small turbulence box and a low domain) the accumulated impact results in very small differences for the farm wake, see Figures 23, 24. As in the low domain case the turbulence level is slightly higher compared to the reference case at longer distances behind the row of turbines.

**Figure 23.** Impact of the "all small case" on velocity at hub height.

**Figure 24.** Impact of the "all small case" on turbulence at hub height.

6. Conclusion

Large eddy simulations using the actuator disc method have been performed, both with and without turbines, in order to study the production of a row of turbines and how the atmospheric turbulence and wind profile are evolving downstream in the domain (up to 12,000 m). The sensitivity of the results to varying sizes of domains, different extensions of the imposed turbulence and different extensions of the equidistant region has been studied.

The results show that in the empty domain the conservation of the wind shear and wind speed at hub height are acceptable throughout the domain. The turbulence increases slightly in the beginning of the domain and decreases to a stable but slightly lower value compared to the imposed value. The downstream preservation of both wind speed and turbulence is however impacted negatively by the usage of a smaller turbulence box.

The relative production shows that for a long row of turbines the downstream part is impacted by blockage effects when using a too low domain. The extensions of the equidistant region has some (but less) impact on the relative production. The equidistant wide (Equ w) case shows an increased relative production from turbine 2 and the equidistant high (Equ h) case shows an increased relative production further downstream. The parameter with the greatest impact on the relative production was the size of the turbulence box. The relative production is clearly lower for the small turbulence box.

The size of the turbulence box also caused the largest differences in the long distance wake. Although the farm production was higher in the reference case the velocity recovery in the farm’s wake was faster for the reference case compared to the case with the smaller turbulence box.

Concluding from the results the domain size impacts the relative production downstream, due to blockage effects, and is it therefore important for studies of large and long wind farms.
to simulate large enough domains. The equidistant region needs to be extended to cover the entire wake to avoid a smearing of the wake in the stretched region. The turbulence box needs to cover an area bigger than the crosssection of the wake to allow the mixing in of energy from the surrounding flow. For all cases without turbines a downstream adjustment of the inlet flow and turbulence to the domain can be seen. The characteristic of this adjustment and impact of this on the production needs to be studied in further detail [20].

It can be summarized that it is important to have relatively large extensions of the domain, large extensions of the equidistant region and especially large extension of the turbulence box.

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