Dynamics of finite size neutrally buoyant particles in isotropic turbulence

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Abstract. The dynamics of neutrally buoyant particles suspended in a turbulent flow is investigated experimentally, with particles having diameters larger than the Kolmogorov length scale. To that purpose, a turbulence generator have been constructed and the resulting flow characterized. The fluid was then seeded with polystyrene particles of diameter about 1 mm and their velocity measured separately and simultaneously with the surrounding fluid. Comparison of the velocities statistics between the two phases shows no appreciable discrepancy. However, simultaneous velocity measurement shows that particles may move in different direction from the underlying flow.

1. Introduction

The main aim of the present work is to study the dynamics of neutrally buoyant finite size particles and its dependence on the turbulent flow properties. The case of neutrally buoyant particles is for example of interest for applications to the study of zoo-plankton dynamics in the ocean, or the use of tracers for velocimetry in turbulent flows with very high Reynolds numbers. In both cases particles size is of the order or larger than the Kolmogorov length scale, so the non-linear terms in the equation of fluid flow motion at the scale of the cannot be neglected. In several experimental and numerical studies, the effect of particles size on velocity and acceleration statistics has been investigated in both two-dimensional flows and the three-dimensional turbulent flow (see Homann & Bec (2010) ; Xu & Bodenschatz (2008) ; Ouellette & al (2008) ; Qureshi & al (2007)). In the case of isotropic turbulence Homann & Bec (2010) shows that while the probability density function (P.D.F) of the velocity statistics of particles normalized by its variance does not varies with particle size, the variance itself depends on the particles size. A scaling relation has been proposed by Homann & Bec (2010) in order to take into account the Faxen correction due to the finite size of the particles. The relation relates linearly the discrepancy between particle and fluid velocity variances to the square of particle diameter, and is shown to be valid for particles with diameter up-to four times the Kolmogorov length scale of the carrier flow. On the other hand in the case of a chaotic two-dimensional flow, experimental investigation by Ouellette & al (2008) have revealed no noticeable differences between velocity statistics of the two phases. Although a discrepancy between the two phases velocities may occur locally. The particles trajectories separates from the fluid trajectories exponentially at hyperbolic stagnation points where the deformation is dominating, as found
analytically by Babiano & al (2008). One interest of our present study is to extend this analysis to the case of a 3D turbulent flow. The present paper is organized as follows: the experiment set-up and measurements techniques are first described, then the properties of the generated turbulent flow are presented in the second section, the third section is devoted to the results concerning particles velocity statistics.

2. Experimental set-up and measurements techniques

The turbulence is generated using an active grid set-up similar to the one used by Morize & al (2005): One realization of the turbulent flow is produced by moving the grid upward in a water tank at a velocity of \( U = 1 \) m/s along a stroke of 0.5 m (with acceleration and deceleration along 0.1 m), the turbulence then decays freely until the next stroke (500 independent realizations are considered), the time origin is taken at the end of grid stroke. The grid is made of square rods of sides 1 cm with a mesh size \( M = 4.5 \) cm, and a solidity \( \sigma = 0.395 \). The estimation of the parameters of the turbulence at the non-dimensional time \( t.U/M = 66.6 \) based on the measured values of turbulent root mean square velocity (r.m.s) \( u' = 3.10^{-2} \) m/s, and the mesh size as an estimation of the integral length scale are presented in table 1.

Particle Image velocimetry (PIV) measurements are first performed to measure the fluid velocity in an area of 20 cm × 11.8 cm in the vertical \( (x_1, x_2) \) plan, where \( x_2 \) is the vertical direction of the grid motion. Measurements resolution is limited by the size of the interrogation window \( B = 2.26 \) mm, which is not sufficient to resolve the scales of the flow since \( B/\eta_k \approx 20 \) at \( t.U/M = 66.6 \), however we expect to catch a significant portion of the inertial range since \( B = \lambda/1.8.0 \).

The flow is seeded with polystyrene particles of diameter \( d_p = 0.9 - 1.12 \) mm with a small mass loading of \( \Phi_m = 4.10^{-5} \), in order to avoid as much as possible a global modulation of the turbulent carrier flow, and to neglect the effects of inter-particles interactions. To achieve a similar density for the two phases, water density is increased to about \( \rho_f = 1.03 \) g/ml by salt addition. Settling velocity of the particles is measured and found to be \( V_s = 6.76.10^{-4} \) m/s in average for a water density of \( \rho_f = 1.037g/ml \). Settling velocity is negligible compared to fluid velocity r.m.s since \( V_s/u' = 2.24.10^{-2} \) at \( t.U/M = 66.6 \). Using a Particle Tracking Velocimetry (PTV) technique, the particles images are tracked in two successive camera frames. The matched number of particles images for each double frame is 10 in average, and the number of considered flow realizations is 960. Using an optical discrimination technique (Poelma & al, 2005), the two measurements techniques PIV and PTV were combined in a two camera set-up, where each camera is dedicated to one phase, in order to measure particles and fluid velocities simultaneously. The flow was seeded with fluorescent tracers and the dedicated camera equipped with a cut-off filter, thus receiving only the fluorescence signal and avoiding to catch the light reflected by the polystyrene particles.

3. Turbulence properties

The main aim of is to define the stages of the decay where the turbulence is sufficiently intense with satisfying homogeneity and isotropy properties. Each PIV measurement results in a velocity field with one vector per interrogation window, we define the local averaging operator over the realization \( \langle \ldots \rangle_L \), and the spatial averaging operator over the whole measurement field \( \langle \ldots \rangle_S \). The total averaging operator defined as the combination of the two previous operators is written as \( \langle \ldots \rangle_T = \langle \langle \ldots \rangle_L \rangle_S \). The local mean velocity components for each position \( (x_1, x_2) \) (corresponding to one window’s center) are \( U_{m,i}(x_1, x_2) = \langle u_i(x_1, x_2) \rangle_L \) where \( u_i \) is the velocity component along the directions \( i = 1, 2 \). Turbulent velocity is hence defined as \( u_{t,i} = u_i - U_{m,i} \) and the turbulent r.m.s velocity components is defined as \( u_{t,i}^2 = \langle u_{t,i}^2 \rangle_L \).
the turbulent kinetic energy is estimated from the two available velocity components by the relation: $e_t(x_1, x_2) = \frac{1}{2} \left\langle u_I^2(x_1, x_2) + u_I^2(x_1, x_2) \right\rangle_L$.

The large scales isotropy is checked by computing the spatial average of ratio of the turbulent r.m.s velocity $I = \langle \hat{u}_2 / \hat{u}_1 \rangle_S$. It is found that until $t.U/M = 150$, the ratio $I$ remains in the range $[1.1, 1.2]$, which compares well with the values reported for static grid turbulence in a wind tunnel without contraction (Comte-Bellot & Corrsin, 1966). In order to quantify turbulence's homogeneity, the mean spatial dispersion of the velocity r.m.s about the mean for the two measured components is obtained by using the quantity $H_i = 100 \left\langle \left( \langle u'_i \rangle - \langle u' \rangle \right) / \langle u' \rangle \right\rangle_S$. Until $t.U/M = 150$, spatial dispersion is found smaller than 11%, while in the case of wind-tunnel grid turbulence with no upstream contraction, previous studies (Grant & Nisbet, 1957) have reported a maximum dispersion of 30% for the turbulence r.m.s velocity in the direction of the mean flow. The inhomogeneity is often greater for the r.m.s velocity in the direction of the grid motion, suggesting that the initial large scale inhomogeneity induced by the grid motion persists throughout the decay. This inhomogeneity may also originate from a non-negligible mean motion which occurs due to the confinement by the wall of the tank (McKenna & McGillis, 2004). The intensity of the turbulent motion compared to the mean motion is quantified through the ratio of r.m.s to mean velocity components $\langle u'_i / \langle U_{m,i} \rangle \rangle_S$, as shown in figure 1a) the mean flow is stronger along the direction of the grid motion as the ratio remains in the range $[2, 3.5]$. In conclusion, the turbulence shows satisfying homogeneity and isotropy with sufficient intensity of the turbulent motion at the first stages of the decay i.e until $t.U/M = 150$, hence the focus will be made on this time interval in the following. Finally the decay of the component energies is shown in figure 1b), the measured decay exponent is found to be $n = 1.06$, which is comparable with values reported in the literature for wind tunnel grid turbulence (Mohamed & Larue, 1990).

In order to compute the Kolmogorov length and time scales we measure the viscous dissipation throughout the decay. Under the local isotropy hypothesis the viscous dissipation in our experiments may be computed from the available components of the velocity gradients $\epsilon = 3\nu \left( \left\langle s_{11}^2 \right\rangle_L + \left\langle s_{22}^2 \right\rangle_L \right) + 12\nu \left\langle s_{12}^2 \right\rangle_L$, with $s_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ being the local shear stress. The resulting spatial average $\epsilon_S = \langle \epsilon \rangle_S$ is compared to the decay rate of the turbulent kinetic energy spatial average $\epsilon_T = - \frac{\partial}{\partial t} \langle e_t \rangle_S$ (figure 2a)). The two quantities being equal in an isotropic turbulence, any discrepancy comes from the terms related to the mean motion or gradient of other statistical quantities, as shown by the transport equation of the turbulent kinetic energy. As the discrepancy between the two quantities remains small until $t.U/M = 100$ those terms can be neglected in comparison with the viscous dissipation term. The Kolmogorov length and time scales are then computed from $\eta_k = (\nu^3/\epsilon_S)^{1/4}$ and $\tau_k = (\nu/\epsilon_S)^{1/2}$, and then compared respectively to the particle diameter $d_p$ and relaxation time $\tau_p = d_p^2/12\nu$ (figure 2b)). Until $t.U/M = 110$ $d_p/\eta_k \geq 4$ and $St = \tau_p/\tau_k > 1$, hence for the considered stages of the decay the particles are inertial with a size larger than Kolmogorov length scale.

### Table 1. Turbulence parameters at $t.U/M = 66.6$. $Re_L = u'.M/\nu$: Reynolds number, $Re_\lambda = u'.\lambda/\nu$: Taylor-Reynolds number, $\eta_k = L/Re_\lambda^{3/4}$: Kolmogorov length scale, $\lambda = u'\sqrt{15\nu_T/\epsilon}$: Taylor length scale

| $Re_L$ | $Re_\lambda$ | $\eta_k$ | $\lambda$ |
|--------|---------------|----------|-----------|
| 1400   | 180           | 170 μm   | 6 mm      |
In order to compare the velocity statistics of the two phases we define the fluctuating velocities as $v_{p,i} = u_{p,i} - \langle u_{p,i} \rangle_T$ for the particles and $v_{f,i} = u_{f,i} - \langle u_{f,i} \rangle_T$ for the fluid, and the fluctuating velocities r.m.s as $v'_{p,i} = \langle v_{p,i}^2 \rangle_T^{1/2}$ and $v'_{f,i} = \langle v_{f,i}^2 \rangle_T^{1/2}$. The P.D.F of the normalized fluctuating velocities for the two phases $v_{p,i}/v'_{p,i}$ and $v_{f,i}/v'_{f,i}$ are compared at different considered stages of the decay as reported in figure 3a) and b), using separate velocity measurements for each phase. No significant differences between the normalized fluctuating velocity component P.D.F for the two phases is found. Nevertheless a slight difference is seen for the second velocity component, which may be related to a significant difference in mean velocities $\langle u_{p,i} \rangle_T$ and $\langle u_{f,i} \rangle_T$ resulting from the spatial inhomogeneity of the second velocity component statistics. Comparison of the velocity variances between the two phases shows a discrepancy between $-5\%$ and $15\%$ at different stages of the decay. As shown by Homann & Bec (2010), the variance difference scales with $(d_p/\lambda)^2$. As shown in figure 2b) this ratio is of the order of $\mathcal{O}(10^{-2})$ in our case, hence
Figure 3. Comparison of the normalized velocity P.D.F of the two phases at different stages of the decay. Solid lines for the fluid. For the particles $\bigcirc t.U/M = 57.7$ $\square t.U/M = 102.2$ $\Diamond t.U/M = 146.6$. The P.D.Fs have been offset for clarity.

Figure 4. a) Normalized speed difference P.D.F $\delta u = (|u_p| - |u_f|)/\langle u_f^2 \rangle^{1/2}$ at time $t.U/M = 57.7$ $\bigcirc$ at time $t.U/M = 151.1$ b) P.D.F of the angle $\theta$ between the fluid and particle velocity vectors at time $t.U/M = 57.7$ $\bigcirc$ at time $t.U/M = 151.1$

no sensible velocity variance difference is expected to be measured in our experiments. Using simultaneous velocity measurements results, statistics are made on the local speed differences between the two phases: $\delta u = (|u_p| - |u_f|)/\langle u_f^2 \rangle^{1/2}$ and the angle between the velocity vectors of particle and the surrounding fluid $\theta$, for two stages of the decay $t.U/M = 57.7$ and $t.U/M = 151.1$. First results in figure 4 shows a finite probability for particles to move in direction different from that of the fluid and with different speed, as have been reported in the case of a 2D chaotic flow (Ouellette & al, 2008). This probability is shown to decrease with the decay, as suggested by the scaling with $(d_p/\lambda)^2$ since the Taylor scale increases.
5. Conclusion and perspectives

In this study the dynamics of finite size neurally buoyant particles and the turbulent, freely decaying carrier flow have been measured and compared. The turbulent flow was generated through an active grid set-up and showed a sufficient isotropy, homogeneity and intensity for the first stages of the decay. Particles were then added to the flow and their velocity statistics measured separately and compared to that of the fluid, and no significant differences between the statistics of the two phases. Further work will focus on simultaneous velocity measurements of the two phase for different particles diameters, in order to quantify local velocity differences.

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