Magnetic field generation and amplification in an expanding plasma

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Particle-in-cell simulations are used to investigate the formation of magnetic fields, B, in plasmas with perpendicular electron density and temperature gradients. For system sizes, L, comparable to the ion skin depth, d_i, it is shown that $B \sim d_i/L$, consistent with the Biermann battery effect. However, for large $L/d_i$, it is found that the Weibel instability (due to electron temperature anisotropy) supersedes the Biermann battery as the main producer of B. The Weibel-produced fields saturate at a finite amplitude (plasma $\beta \approx 100$), independent of L. The magnetic energy spectra below the electron Larmor radius scale are well fitted by power law with slope $-16/3$, as predicted in Schekochihin et al., Astrophys. J. Suppl. Ser. 182, 310 (2009).

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Introduction. The origin and amplification of magnetic fields is a central problem in astrophysics [1]. The turbulent dynamo [2, 3] is generally thought to be the basic process behind the amplification of a magnetic seed field; however, some other process is required to originate the seed itself. Amongst the few mechanisms able to do so is the Biermann battery effect, due to perpendicular electron density and temperature gradients [4]. It is often conjectured that the observed magnetic fields in the universe may be of Biermann origin, subsequently amplified via dynamo action [1]. However, simple theoretical estimates suggest that Biermann-generated magnetic fields, B, should be such that $5–7$

$$\beta \equiv 8\pi P/B^2 \sim (d_i/L)^{-2},$$ (1)

where $P$ is the plasma pressure, $d_i = c/\omega_{pi}$ is the ion inertial length (with c the speed of light and $\omega_{pi}$ the ion plasma frequency) and L is the characteristic length scale of the system. Given the extremely small values of $d_i/L$ typical of astrophysical systems, it is an open question whether such seeds are sufficiently large to account for the microgauss fields observed today.

Megagauss magnetic fields are observed to form in intense laser-solid interaction laboratory experiments [8–12]. In these experiments, the laser generates an expanding bubble of plasma by ionizing a foil of metal or plastic. The plasma is denser closer to the plane of the target foil, and hotter closer to the laser beam axis. Perpendicular density and temperature gradients are thus generated, giving rise to magnetic fields via the Biermann effect. Besides their intrinsic interest, these experiments offer a unique opportunity to illuminate a fascinating, and poorly understood, astrophysical process.

In this Letter we perform ab initio numerical investigations of the generation and growth of magnetic fields in a configuration akin to that of laser-generated plasma systems. For small to moderate values of the parameter $L/d_i$ our simulations confirm the theoretical predictions of Haines [7]; in particular, for $L/d_i \gtrsim 1$ the magnetic fields obey the scaling of Equation (1). However, when $L/d_i \gg 1$, we find that the plasma is unstable to the Weibel instability [13], which amplifies the magnetic fields such that $\beta \approx 100$, independent of L. These results have strong implications for the interpretation of laser-solid interaction experiments; they also shed new light on the currently accepted view of the origin of the observed cosmic magnetic fields.

Computational Model. We perform a set of particle-in-cell (PIC) simulations using the OSIRIS framework [14, 15]. The initial fluid velocity, electric field, and magnetic field are all uniformly zero. We start with a spheroid distribution of density, that has a shorter length scale in one direction: $n = (n_0 - n_b) \cos(\pi R_1/2L_T) + n_b$, if $R_1 < L_T, n_b$, otherwise, where $R_1 = \sqrt{x^2 + (L_T/L_n)^2 + z^2}$ and $n_b = 0.1 n_0$ is the uniform background density. The characteristic lengths of the temperature and density gradients generated by the laser beam are denoted by $L_T$ and $L_n$, respectively, to represent the recently ionized foil, which is flatter in the direction of the laser, y, we set $L_T/L_n = 2$. (This is a generic choice that appears to be qualitatively consistent with experiments, e.g., [9–12]; note, however, that the specific value of $L_T/L_n$ depends on target and laser properties.) Ion thermal velocity $v_T\theta$. The spatial profile for the electron thermal velocity is cylindrically symmetric along the y direction, where it is hottest in the center: $v_{Te} = (v_{Te0} - v_{Teb}) \cos(\pi R_2/2L_T) + v_{Teb}$, if $R_2 < L_T, v_{Teb}$, otherwise, where $R_2 = \sqrt{x^2 + z^2}$, resulting in a maximum initial electron pressure $P_e = m_e n_0 v_{Te0}^2/2$. The numerical values of the thermal velocities are $v_{Te0} = 0.2c$ and $v_{Teb} = 0.01c$. Note that in our setup the pressure is dominated by the electrons, and thus $\beta \approx \beta_e \equiv 8\pi P_e/B^2$. For simplicity the boundaries are periodic, but the box is large enough that they do not interfere with the dynamics [1]. In order to investigate a larger range of $L_T/d_i,$
the simulations are run with a reduced mass ratio of 25. The spatial resolution is 16 gridpoints/\(d_e\), or 2.26 gridpoints/\(\lambda_d\), where \(d_e = c/\omega_{pe}\) is the electron inertial length (\(\omega_{pe}\) is the electron plasma frequency) and \(\lambda_d\) is the electron Debye length. The time resolution is \(\Delta t\omega_{pe} = 0.07\). The 2D simulations have 196 or 64 particles per cell (ppc); the 3D simulation has 27 ppc.

**Biermann regime.** Figure 1 shows contours of constant magnetic energy density and magnetic field lines from a 3D simulation with \(L_T/d_e = 50\) taken at \(t\omega_{pe} = 235.2\), after the magnetic field strength saturates (see Fig. 3). As expected based on the initial conditions, we observe the formation of large-scale azimuthal Biermann magnetic fields which are nearly axisymmetric. Although Biermann generation of magnetic fields has been investigated before [16], this is the first fully self-consistent kinetic 3D simulation.

The axisymmetry in the 3D simulation suggests that a scaling study in system size can be performed using a more computationally efficient 2D setup. To this end, we take a cut of the 3D system at \(z = 0\), where the azimuthal (out-of-plane) magnetic fields are in the \(z\) direction, and perform a set of 2D simulations with \(L_T/d_e = (4, 8, 16, 25, 32, 50, 64, 128, 200, 400)\). For \(4 \leq L_T/d_e \leq 128\) we use 196 ppc. For \(L_T/d_e = 200, 400\) we use 64 ppc instead due to computing time limitations; convergence studies at lower values of \(L_T/d_e\) do not show significant differences between 196 and 64 ppc. A snapshot taken at the same time of a 2D version of the simulation presented in Fig. 1 is shown in Fig. 2(a) for comparison. The same large-scale magnetic field structure is manifest, with very similar levels of \(B_z\).

The time trace of the maximum magnetic field strength for a selection of cases can be seen in Fig. 3(a). For small systems, \(L_T/d_e < 50\), the magnetic field reaches a maximum and then decays away. On the other hand, we observe that for \(L_T/d_e > 50\) the magnetic field saturates at around its peak value.

Figure 3(b) shows the scaling with system size of the maximum and the average magnitude of the magnetic field (the square root of \(B^2\) averaged in a box \(2L_T \times 2L_n\) surrounding the expanding bubble) at the time when the field saturates (or peaks for \(L_T/d_e < 50\)). There are three distinct regions in this plot. For \(L_T/d_e < 25\) (i.e., \(L_T/d_i \leq 5\)), the magnetic field increases with system size. This stage is followed by a region where the saturated amplitude of the field decreases as \(d_i/L_T\), which lasts while \(L_T/d_e < 100\). These two stages confirm the theoretical prediction of Haines [7]: in very small systems, there is a competition between the Biermann battery effect and microinstabilities (the ion acoustic and the lower hybrid drift instabilities), triggered by an electron drift velocity in excess of the ion acoustic speed, which suppress the Biermann fields. As the system becomes larger, the electron drift velocity decreases. (Larger systems have larger-scale magnetic fields, and therefore lower currents.) The microinstabilities thus become progressively less viru-
The maximum $B_z$ vs. time for a selection of system sizes ($L_T/d_e$). The inset shows the $L_T/d_e = 400$ case (black). The magenta line is the maximum Weibel growth rate, $\gamma_{max}$, at $y = 0$. Dashed lines identify the times at which the spectra of Fig. 5 are calculated. (b) Maximum (asterisks), and average magnitude (diamonds) of $B_z$ vs. $L_T/d_e$. The triangle represents the maximum $B_z$ for the 3D run. The solid curve is $max(B_z)/\sqrt{8\pi T_e} = \sqrt{2d_e/L_T}$; the dotted line indicates $L_T/d_i = 5$. The inset shows the time to maximum magnetic field, $t_{max}$, vs. $L_T/d_e$. The solid line indicates $t_{max} = L_T/v_{Te}0$.

**FIG. 3.**

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**FIG. 4.** Electric field in the $x$-direction, $E_x$, for $L_T/d_e = 25$ at $\omega_{pe} = 142.8$.

lent until their complete suppression, whereupon we encounter a “pure” Biermann regime, as described in Equation (1). Inspection of the simulations for $L_T/d_e < 50$ at times after the magnetic field reaches its peak value shows clear electric field perturbations along $y = 0$, consistent with the ion acoustic instability. These are exemplified for $L_T/d_e = 25$ in Fig. 4. Note that the density gradient going to zero at $y = 0$, ruling out the lower hybrid drift instability as the cause of the decay of the magnetic field.

**Weibel regime.** An unexpected third regime is encountered for $L_T/d_e > 100$. In that region of Fig. 3(b), the magnetic field produced in our simulations no longer follows the predicted $d_e/L_T$ Biermann scaling, but rather increases with the system size and appears to tend to a constant, finite value, $\beta_e \approx 100$.

In this new regime the magnetic fields are produced by the Weibel instability [13]. The initial cloud of plasma expands due to the imposed density gradient, generating both outward ion and hot electron flows. The velocity of the electron flows vary along the temperature gradient. The higher temperature flows originating in the center, stream past lower temperature inward flows originating further outward, which maintain quasineutrality. This generates a larger velocity spread (larger temperature) in the direction of the flow, while the perpendicular temperature remains unaffected. It is this temperature anisotropy that drives the Weibel instability [13]. Note that along $x = 0$, where the temperature gradient is zero no anisotropy is generated, and thus the Weibel instability is not observed [see Fig. 2(b)].

As exemplified in Fig. 2(b) for our largest simulation ($L_T/d_e = 400$), the large-scale coherent Biermann magnetic fields characteristic of the smaller systems are replaced by non-propagating magnetic structures with very large wavenumbers ($kd_e \sim 0.2$), and with a transverse wave vector, $k$, perpendicular to the direction with a larger temperature. These features are consistent with the Weibel instability [13, 17, 18]. In addition we have compared our results with the analytic growth rate predicted by Weibel [13]. In our simulations we observe an enhanced temperature in the direction of the density gradient (parallel) as high as $A \equiv T[e]/T[ee] - 1 \approx 2.0$. In a cut at $y = 0$ we calculate the Weibel growth rate, $\gamma$, for the fastest growing $k$, $k_{max}$, using the locally measured values of $n$, $T[e]$, and $A$. The maximum $\gamma$ of this cut, $\gamma_{max}$, is plotted vs. time in Fig. 3(a), showing a peak when the magnetic field strength rises exponentially, and a subsequent drop corresponding to the loss of anisotropy after saturation. The magnitude of the growth rate thus calculated is also consistent within a factor of 2, with $k_{max}/d_e \approx 0.2$, analogous to the structures in Fig. 2(b).

The transition between the Biermann and Weibel regimes is also visible in the inset plot in Fig. 3(b), where we show the time to reach the maximum magnetic field, $t_{max}$, as a function of system size. For $L_T/d_e < 50$, we find that $t_{max} \sim L_T/v_{Te}0$. A linear in time scaling is indeed to be expected for Biermann generated fields;
We note that both of these regimes can in principle be probed by existing experiments. For example, the L_T/d_e ≈ 1 regime (Biermann) is accessible to the Vul-can laser [9], whereas L_T/d_i ≈ 100 (Weibel) is reachable by an OMEGA laser [10]. In practice, however, collision frequencies large compared to the electron transit time prohibit electron temperature anisotropies, thereby inhibiting the Weibel instability. If less collisional regimes can be attained in the experiments, it may be possible to experimentally investigate the transition from Biermann to Weibel produced magnetic fields that we have uncovered here.

In the context of (largely collisionless) astrophysical plasmas, our results may significantly impact the canonical picture of cosmic magnetic field generation [1], by suggesting that Biermann seed fields may be pre-amplified exponentially fast via the Weibel instability up to rea-
sonably large values (i.e., independent of the system size) previous to turbulent dynamo action.

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[1] R. M. Kulsrud and E. G. Zweibel, Rep. Prog. Phys. 71, 046901 (2008).
[2] R. M. Kulsrud and S. W. Anderson, Astrophys. J. 396, 606 (1992).
[3] A. Brandenburg, D. Sokoloff, and K. Subramanian, Space Sci. Rev. 169, 123 (2012).
[4] L. Biermann, Z. Naturforsch. 5a, 65 (1950).
[5] C. E. Max, W. M. Manheimer, and J. J. Thomson, Phys. Fluids 21, 128 (1978).
[6] R. S. Craxton and M. G. Haines, Plasma Phys. 20, 487 (1978).
[7] M. G. Haines, Phys. Rev. Lett. 78, 254 (1997).
[8] J. A. Stamper, K. Papadopoulos, R. N. Sudan, S. O. Dean, and E. A. McLean, Phys. Rev. Lett. 26, 1012 (1971).
[9] P. M. Nilson, L. Willingale, M. C. Kaluza, C. Kamperidis, S. Minardi, M. S. Wei, P. Fernandes, M. Notley, S. Bandyopadhyay, M. Sherlock, R. J. Kingham, M. Tatarakis, Z. Najmudin, W. Rozmus, et al., Phys. Rev. Lett. 97, 255001 (2006).
[10] C. K. Li, F. H. Séguin, J. A. Frenje, J. R. Rygg, R. D. Petrasco, R. P. J. Town, P. A. Amendt, S. P. Hatchett, O. L. Landen, A. J. Mackinnon, P. K. Patel, V. A. Smalyuk, T. C. Sangster, and J. P. Knauer, Phys. Rev. Lett. 97, 135003 (2006).
[11] C. K. Li, F. H. Seguin, J. A. Frenje, J. R. Rygg, R. D. Petrasco, R. P. J. Town, O. L. Landen, J. P. Knauer, and V. A. Smalyuk, Phys. Rev. Lett. 99, 055001 (2007).
[12] N. L. Kugland, D. D. Ryutov, P.-Y. Chang, R. P. Drake, G. Fiksel, D. H. Froula, S. H. Glenzer, G. Gregori, M. Grosskopf, M. Koenig, Y. Kuramitsu, C. Kuranz, M. C. Levy, E. Liang, et al., Nature Phys. 8, 809 (2012).
[13] E. S. Weibel, Phys. Rev. 114, 18 (1959).
[14] R. A. Fonseca, L. O. Silva, F. S. Tsung, V. K. Decyk, W. Lu, C. Ren, W. B. Mori, S. Deng, S. Lee, T. Katsouleas, and J. C. Adam, Lect. Notes Comp. Sci. 2331, 342 (2002).
[15] R. A. Fonseca, S. F. Martins, L. O. Silva, J. W. Tonge, F. S. Tsung, and W. B. Mori, Plasma Phys. Control. Fusion 50, 124034 (2008).
[16] A. Thomas, M. Tzoufras, A. Robinson, R. Kingham, C. Ridgers, M. Sherlock, and A. Bell, J. Comput. Phys. 231, 1051 (2012).
[17] F. Califano, F. Pegoraro, S. V. Bulanov, and A. Mangeney, Phys. Rev. E 57, 7048 (1998).
[18] R. A. Fonseca, L. O. Silva, J. W. Tonge, W. B. Mori, and J. M. Dawson, Plasma Phys. and Controlled Fusion 50, 055001 (2008).
[19] K. Molvig, Phys. Rev. Lett. 35, 1504 (1975).
[20] D. V. Romanov, V. Y. Bychenkov, W. Rozmus, C. E. Capjack, and R. Fedosejevs, Phys. Rev. Lett. 93, 215004 (2004).
[21] A. A. Shchekochihin, S. C. Cowley, W. Dörland, G. W. Hammett, G. G. Howes, E. Quataert, and T. Tatsuno, Astrophys. J. Suppl. Ser. 182, 310 (2009).
[22] E. Camporeale and D. Burgess, Astrophys. J. 730, 8 (2011).