Can EDGES observation favour any dark matter model?

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ABSTRACT

The recent detection of the 21-cm absorption signal by the EDGES collaboration has been widely used to constrain the basic properties of dark matter particles. However, extracting the parameters of the 21-cm absorption signal relies on a chosen parametrisation of the foreground radio emission. Recently, the new parametrisations of the foreground and systematics have been proposed, showing significant deviations of the 21-cm signal parameters from those assumed by the original EDGES paper. In this paper, we consider this new uncertainty, comparing the observed signal with the predictions of several dark matter models, including the widely-used cold dark matter (CDM) model, 1–3 keV warm dark matter models (WDM), and 7 keV sterile neutrino (SN7) model, capable of producing the reported 3.5 keV line. We show that all these dark matter models cannot be statistically distinguished using the available EDGES data.

Key words: dark matter – cosmic background radiation – cosmology: observations – dark ages, reionization, first stars

1 INTRODUCTION

The possibility of observation of the cosmological 21-cm hydrogen signal was of interest to cosmologists even before the first observational constrains became real (see, e.g., Hogan & Rees 1979; Scott & Rees 1990; Madau et al. 1997). There are two possible ways in which this observation may shed light on the structure formation processes in the early Universe. The first one is the 21-cm tomography, which is based on the study of the spatial distribution of fluctuations of the 21-cm signal generated by HI clouds at the Dark Ages and reionization epochs (see, e.g., Madau et al. 1997; Ciardì & Ferrara 2005; McQuinn et al. 2006; Mao et al. 2008; Morales & Wyithe 2010). The second one is detection of the sky-averaged signal from the Dark Ages epoch (see, e.g., Mirocha et al. 2013; Mirocha et al. 2015; Cohen et al. 2017) produced by absorption of the cosmic microwave background radiation (CMB) by the neutral hydrogen. This absorption is caused by the Wouthuysen–Field coupling between the spin temperature of hydrogen and Ly-α radiation of the first galaxies. For a detailed description of the theoretical and observational challenges of 21-cm signal detection, see, e.g., Furlanetto et al. (2006); Pritchard & Loeb (2008, 2012) and references therein.

Detection of the global absorption signal claimed by the EDGES collaboration (Bowman et al. 2018) caused great excitement among physicists. While the central frequency of the absorption peak (which is linked to the redshift at which it was generated) is in a good agreement with the predictions of the Lambda-CDM model (Cohen et al. 2017), the amplitude (∼ 0.5 K) of the observed signal appears to be at least twice that of the most extreme absorption predicted in the Lambda-CDM model (see, e.g., Cohen et al. 2017).

Observation of the global 21-cm absorption by EDGES motivated a wide search of possible mechanisms that would explain the observed depth and position of the absorption signal. This opened up a possibility to study and put constrains on the early star formation rate (see, e.g., Madau 2018; Schauer et al. 2019), structure formation in dark matter models (Safarzadeh et al. 2018; Schneider 2018; Chatterjee et al. 2019; Boyarsky et al. 2019a; Leo et al. 2019) and non-standard X-ray sources that can heat the IGM such as the first black holes (Clark et al. 2018) and decaying or annihilating dark matter (Mitridate & Podo 2018; Cheung et al. 2019; Liu & Slatyer 2018; Yang 2018; D’Amico et al. 2018; Fraser et al. 2018; Hektor et al. 2018; Clark et al. 2018; Chatterjee et al. 2019). Additional mechanisms beyond the standard cosmological scenario were proposed to explain the depth of the absorption signal, such as interaction between baryonic matter and dark matter (Barkana 2018; Fialkov et al. 2018) and extreme radio background during the Dark Ages (Feng & Holder 2018; Ewall-Wice et al. 2018; Fialkov & Barkana 2019).

At the same time, a well-known problem of detecting the 21-cm signal is that the galactic synchrotron emission and ionospheric emission and absorption are dominant over...
the signal by four orders of magnitude at frequencies below 100 MHz (Bernardi et al. 2015). Therefore, it is not surprising that soon after the EDGES publication several works appeared addressing the technical details of extracting the 21-cm signal from the observation and questioning the reliability and validity of the observed profile. Hills et al. (2018) reanalysed the data reported by EDGES. They showed that the original analysis assumed controversial structure in the spectrum of the foreground (galactic) emission in the studied spectral band and led to non-physical properties of the ionosphere. Bradley et al. (2019) reported a possible systematic artefact in the observations that can affect the determination of the absorption signal.

This work has the following plan. In the next section we describe how we use the foreground modelling proposed in Hills et al. (2018) and take into account the ground plane artefact discussed in Bradley et al. (2019) to best fit the EDGES data via the non-linear least-squares procedure. Next, we model the 21-cm global signal with the ARES code for different models of dark matter (CDM, thermal relic WDM and 7 keV sterile neutrino decaying dark matter) motivated by the recently detected 3.5 keV line Boyarsky et al. (2014); Bulbul et al. (2014); Boyarsky et al. (2018)\(^1\). Then, in the Results section we fit the EDGES data subtracting the different absorption signals and compare the fitting scores in order to select the preferred dark matter model. In the last, Conclusions & Discussion section we briefly summarise our results and discuss the possibility to constrain the parameters of structure formation during the Dark Ages and, in particular, dark matter properties by the EDGES data and by the planned observations of 21-cm absorption signal.

2 METHODS

2.1 Fitting the EDGES data

We represent the EDGES data with a sum of three components,

\[
T(v) = T_{\text{sky}}(v) + T_{\text{res}}(v) + T_{21}(v).
\]

Here, the first term is the sky foreground brightness temperature. In the original EDGES paper (Bowman et al. 2018) this foreground was modelled in the linearised form. However, Hills et al. (2018) argued that the parameters of such representation have nonphysical values in the best-fitting model. In our fits we used the nonlinear model with the ionospheric absorption and emission terms being connected through the electronic temperature \(T_e\), obtained by expanding the foreground temperature around some central frequency \(v_c\) as described in Hills et al. (2018):

\[
T^\text{H18\,sky}(v) = b_0 \left( \frac{v}{v_c} \right)^{-2.5+3b_1+b_2} e^{-b_2(v/v_c)^2} + T_e \left( 1 - e^{-b_2(v/v_c)^2} \right).
\]

Here, the electronic temperature \(T_e\) is allowed to vary in the range [200; 2000] K, according to the EDGES measurements, see fig. 2 of Rogers et al. (2015). The parameter \(b_1\) is a correction to the overall power-law index, which varies by \(-0.1\) across the sky according to Bowman et al. (2018). In our fits we allow this parameter to vary in a broader range of \([-0.2; 0.2]\). The parameter \(b_2\) was left unconstrained, although its best-fit value should be controlled to be \(-0.1\) according to Bernardi et al. (2015). The ionospheric opacity \(b_0\) is allowed to vary between 0 and 0.03 according to Hills et al. (2018).

The second term in Eq. (1) is the sum of three resonant absorption components described in Bradley et al. (2019):

\[
T_{\text{res}}(v) = -\sum_{i=1}^{3} A_i v^3 v_{0i}^2 + Q_i (v^2 - v_{0i}^2)^2.
\]

This term is characterised by three parameters: the central frequencies \(v_{0i}\), the depths of the profiles \(A_i \neq A(v_{0i})\), and the quality factors \(Q_i\) which are the ratios of \(v_{0i}\) to the spectral widths of absorption. The phenomenological origin of this term is the fact that soil itself is a resonator, of which one cannot get rid. EDGES uses the ground antennae, and possible discontinuities on the edges of the ground plate produce resonant features and distort the signal.

The last term in Eq. (1), \(T_{21}(v)\), is the global 21-cm absorption signal. While Bowman et al. (2018) modelled it in the form of flattened Gaussian, we do not represent it in any analytical form. Instead, from the initial EDGES data we subtract the absorption profile obtained in the ARES simulation, taking into account the specific structure formation model, including the influence from the particular type of the halo mass function and DM decays, see Sec. 2.2.

To fit our model to the data we use the non-linear least-squares procedure implemented in the lmfit python package (Newville et al. 2018). Following Bradley et al. (2019), we choose the objective function in the form of the minimal log-likelihood \(-\ln L = \frac{1}{2} \sum_{kl} (y_k - \hat{y}_k) C_{kl} (y_l - \hat{y}_l)\) with the noise covariance matrix \(C_{kl} \propto \delta_{kl}\), where \(y_i\) are the data points and \(\hat{y}_i\) are the model values. Because the underlying Levenberg–Marquardt algorithm depends on the initial guess for the parameters, we repeat fits many times, randomly varying these initial values. We use the well-known RandomizedSearchCV cross-validation procedure of the scikit-learn package (Pedregosa et al. 2011) with 100 runs and the RepeatedKFold validation scheme to compare between different runs and to choose the best-fitting model. Namely, on each run the data points are randomly split into five groups, and the fits are performed five times, using only four of them as data to model, and leaving one group to be the validation data. The root mean square (r.m.s.) score is calculated over the validation data. Finally, we take the mean value for the score. The model with the best validation score is chosen. This procedure allows us to select the best initial parameter values, simultaneously preventing the selection of the model which overfits the data, i.e., fits the noise. Finally, the r.m.s. of the chosen best-fitting model over the entire data is calculated. These values are used to compare the models.

To quantify the difference between models we use the Bayesian information criterion (BIC), which could be written as

\[
\text{BIC} = N \ln \left( \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \right) + k \ln N
\]

\(^1\) Such dark matter is in a good agreement with the Ly-\(\alpha\) forest analysis (Baur et al. 2017) and reionization history data (Rudakovskiy & Iakubovskyi 2019).
in case of Gaussian errors (Schwarz 1978; Priestley 1983). Here \( y_i \) denotes the data points and \( \hat{y}_i \) are the model values, \( N \) is the number of the data points (\( N = 123 \) for the EDGES data) and \( k \) is the number of free model parameters (\( k = 14 \) for our model). Notice that the extraction of the simulated signal from the raw spectra does not change the values of \( N \) and \( k \). The model with the lower BIC could be treated as strongly preferred if the difference between BICs is higher than 6, and one cannot talk of any preference for \( \Delta \text{BIC} < 2 \) (Kass & Raftery 1995).

2.2 Modelling the global 21-cm absorption profile

We use the open-sourced Accelerated Reionization Era Simulations (ARES) code (Mirocha 2014) to compute the global 21-cm neutral hydrogen signal in different DM models. This code produces the profile of the global 21-cm signal for the given star formation rate, halo mass function and cosmology. The PopII stars in the galaxies are assumed to be sources of the Lyman-\( \alpha \) and ionising UV-radiation, and black holes produce X-rays. Also, there is a possibility to add new sources of radiation, like, for example, decaying DM.

For each of the DM models under consideration, we generate the corresponding halo mass functions for redshifts \( z \) between 0 and 50:

\[
\frac{dn}{d\ln M} = f(x) \frac{\rho_m}{M} \frac{d\sigma_{\text{red}}^{-1}}{d\ln M},
\]

where \( x = \left( \frac{\delta_c(z)}{\sigma} \right), \delta_c(z) = 1.686 \frac{D(z)}{\Omega_m}, D(z) \) is the growth factor (Heath 1977), \( \sigma(M) \) is the variance of the density fluctuations on a mass scale \( M \) and \( \rho_m \) is the mean matter density of the Universe. For all DM models we use the Sheth-Tormen approximation (Sheth & Tormen 2002):

\[
f(x) = A_{ST} \sqrt{\frac{2\delta_c}{\pi}} \left(1 + (\alpha x)^p\right) e^{-x^2/2}.
\]

We consider a sharp-\( k \) filter for \( \sigma(M) \) calculation, which provides a good fit for the CDM and WDM halo mass functions, obtained during N-body simulations at different redshifts (Benson et al. 2013; Schneider 2015, 2018): \( \sigma^2 = \int_0^k P(k) \frac{dk}{(2\pi)^3}, \) where the mass of the halo and \( k_c \) are related as \( M = \frac{4\pi}{3} \rho_c \left( \frac{k_c}{2\pi} \right)^3 \). For the CDM and thermal relic WDM we use the \( \text{hmf} \) public code (Murray et al. 2013).

We assume the parameters of the Sheth-Tormen halo mass function to be \( A_{ST} = 0.322, q = 0.93, \) and \( p = 0.3 \). In this case the calculated halo mass functions are in a good agreement with the CDM and WDM simulations, shown on fig.1 of Schneider (2018).\(^{2}\)

The star-formation rate density (\( \text{SFRD} \)) is modelled by the standard so-called "fcoll" model used in ARES:

\[
\dot{\rho}_* = f_\text{SF} \rho_\text{b} \frac{d}{dt} f_{\text{coll}}(z).
\]

The star-formation efficiency \( f_\text{SF} \) in our analysis is considered to be a constant, which cannot exceed the unity. The collapsed fraction \( f_{\text{coll}} \) is defined as

\[
f_{\text{coll}}(z) = \frac{1}{\rho_m} \int_{M_{\text{min}}}^\infty \frac{dn}{d\ln M} dM,
\]

where \( M_{\text{min}} \) is the minimal mass of the source, determined by the virial temperature \( T_{\text{vir}} \), similarly to Barkana & Loeb (2001). In this work we assume that \( T_{\text{vir}} = 10^4K \). The specific emissivity in a particular spectral band between \( E_{\text{min}} \) and \( E_{\text{max}} \) is proportional to \( \text{SFRD} \) (see, e.g., Mirocha 2014):

\[
e(E, z) = c \dot{\rho}_*(z) I(E),
\]

where \( c \) is the conversion factor between emissivity and intensity \( I(E) \) is the spectral density normalised as \( \int_{E_{\text{min}}}^{E_{\text{max}}} I(E) dE = 1 \).

Throughout this work we assume the default ARES parameters (except \( f_\text{SF} \)) for the Ly\( \alpha \) and LyC photons produced by PopII stars in the first galaxies and the X-ray generated by the accretion of baryonic matter onto the first black holes.

Another possible source of the X-ray photons are decaying DM particles. The recently detected narrow 3.5 keV line in the spectra of the DM dominated objects (Boyarsky et al. 2014, 2015; Bulbul et al. 2014) may be emitted during the DM decay. One of the best motivated DM candidates, which might explain the 3.5 keV line, is ~ 7 keV sterile neutrino (see, e.g., Drewes et al. 2017; Boyarsky et al. 2019b, and references therein). We focus on the resonantly produced 7 keV sterile neutrino DM with the lepton asymmetry \( \mathcal{L}_q = 12 \), corresponding to the mixing angle \( \sin^2 2\theta = 1.6 \cdot 10^{-11} \) according to (Boyarsky et al. 2018), and lifetime \( \tau_{\text{DM}} = 2.677 \cdot 10^{28} \) s consistent with (Boyarsky et al. 2014). For decaying DM models (e.g., the model of 7 keV sterile neutrino), the luminosity density of photons with energy \( E_\gamma \) generated via decays is

\[
L_{\text{decays}} = E_\gamma \frac{\rho_{\text{DM}}}{m_{\text{DM}}} \frac{1}{\tau_{\text{DM}}}. \tag{10}
\]

3 RESULTS

Firstly, we basically reproduce the modelling by Bowman et al. (2018) and Bradley et al. (2019) to justify our fitting method (see Appendices A, B).

Then we generate the absorption signals for the different dark matter models (CDM, 1–3 keV thermal relic WDM, 7 keV decaying sterile neutrinos), each with different values of \( f_\text{SF} \) in the range from 0.01 to 1. We subtract these signals from the data and perform the fits as described in 2.1. All the best-fitting models give the comparable r.m.s. of the order of 20.9 mK. The differences in the values of BIC are all below 2, so we conclude that all the models to be indistinguishable. Examples of specially interesting cases are provided in Figs. 1, 2.

4 CONCLUSIONS & DISCUSSION

We modelled the EDGES data with the alternative to Bowman et al. (2018) physically motivated foreground model proposed by Hills et al. (2018), taking into account the ground plane artefact absorption by Bradley et al. (2019), and subtracting different simulated 21-cm absorption profiles in order to constrain the underlying dark matter model.
We explicitly showed that the fit quality of such model does not depend on the assumed dark matter particle model, concluding that the EDGES observation cannot be used as a good tool for quantitative constraining of dark matter particle models.

Unlike the papers in which the form and position of the signal reported by Bowman et al. (2018) are used to constrain the dark matter models (see, e.g., Safarzadeh et al. 2018; Schneider 2018; Clark et al. 2018; Hektor et al. 2018; Chatterjee et al. 2019), our paper uses the “raw” signal $T(\nu)$ to perform the modelling.

Bradley et al. (2019) proposed a physically motivated instrumental feature; however, one can try to check other forms of systematics. For example, Singh & Subrahmanyan (2019) showed that the EDGES spectrum is consistent with the standard cosmology if the maximally smooth polynomial foreground and sinusoidal systematics are assumed. Their fit gives $\text{BIC} = -894.2$, formally strongly preferred over our best value of $-884.9$. It is a matter of choosing the form of the foreground and of the term describing the systematics. This brings in uncertainty and model dependence into the exploration of the global absorption signal. Moreover, the global 21-cm signal is averaged over the sky and contains contributions from many different sources; thus no simple physical model may be appropriate to fit it.

The developed radiometer experiments such as BIGHORNS (Sokolowski et al. 2015), SARAS 2 (Singh et al. 2018) and LEDA (Price et al. 2018), which will be focused on the global 21-cm signal, may shed new light on the 21-cm absorption feature. However, there are large uncertainties in the star formation in galaxies during the reionization and Dark Ages epochs, which makes it difficult to constrain a dark matter scenario by using the global 21-cm absorption signal (Boyarsky et al. 2019a). Nevertheless, the future studies of the statistics of the spatial distribution of the 21-cm signal by radio interferometers such as MWA, HERA and SKA may help to break the degeneracy between the baryonic and dark matter effects during the reionization and Dark Ages epochs (see, e.g., Mesinger et al. 2014; Sitwell et al. 2014; Bull et al. 2018).

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APPENDIX A: REPRODUCING THE ORIGINAL EDGES RESULTS

In the original EDGES paper (Bowman et al. 2018) the foreground was modelled as

\[ T_{\text{sky}}^{(B18)}(v) = a_0 \left( \frac{v}{v_c} \right)^{-2.5} + a_1 \left( \frac{v}{v_c} \right)^{-2.5} \log \left( \frac{v}{v_c} \right) + a_2 \left( \frac{v}{v_c} \right)^{-2.5} \left[ \log \left( \frac{v}{v_c} \right) \right]^2 + a_3 \left( \frac{v}{v_c} \right)^{-4.5} + a_4 \left( \frac{v}{v_c} \right)^{-2}. \] (A1)

The absorption signal was assumed in the form of a flattened Gaussian:

\[ T_{\text{21}}^{(B18)}(v) = \frac{4(v - \nu_0)^2}{w^2} \log \left[ \frac{1}{\tau} \log \left( \frac{1 + e^{-\tau}}{2} \right) \right]. \] (A2)

where

\[ B = \frac{4(v - \nu_0)^2}{w^2} \log \left[ \frac{1}{\tau} \log \left( \frac{1 + e^{-\tau}}{2} \right) \right]. \] (A3)

We fit the data with the sum of the sky foreground brightness temperature in the form of Eq. A1, and the 21-cm absorption term in the form of the flattened gaussian, Eq. A2.

Our best-fitting model closely reproduces the results reported by the EDGES collaboration. The best-fitting values for the 21-cm model parameters are \( A = 0.056 \text{K}, \) \( w = 18.8 \text{MHz} \) and \( \tau = 5.8. \) The resulting r.m.s. is 24.5 mK (close to 0.025 K reported by Bowman et al. (2018)). The summary of our results is plotted in Fig. A1.

APPENDIX B: REPRODUCING THE ORIGINAL BRADLEY’S RESULTS

We also reproduce the findings by Bradley et al. (2019) so that the EDGES spectrum can be modelled with a sum of cosmic foreground continuum in the linearised form of Eq. A1, taking into account only the first two terms, and three resonant features of the ground plane patch absorber (Eq. 3). The obtained best root mean square residual is 20.8 mK, the same as reported by Bradley et al. (2019). The resonant absorption profile together with the model residuals are plotted in Fig. B1.
Figure A1. Best-fitting results, reproducing Bowman et al. (2018). Top: Residuals after fitting and removing only the foreground model, Eq. A1 (see their fig. 1b). Middle: The best-fitting model of the 21-cm absorption feature, Eq. A2 (see their fig. 1d). Bottom: Residuals after removing both the foreground and the 21-cm absorption model (see their fig. 1c).

Figure B1. Best-fitting results, reproducing Bradley et al. (2019). Top: Residuals after fitting and removing only the foreground model (first two terms of Eq. A1). Middle: The best-fitting model of the instrumental resonant absorption feature, Eq. 3. Bottom: Residuals after removing both the foreground and the resonant absorption model.

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