Dynamics of long-lasting relativistic winds in Gamma Ray Bursters

Maxim V. Barkov\textsuperscript{1,2,3} and Maxim Lyutikov\textsuperscript{1}

\textsuperscript{1}Department of Physics and Astronomy, Purdue University, 525 Northwestern Avenue, West Lafayette, IN 47907-2036, USA

\textsuperscript{2}Astrophysical Big Bang Laboratory, RIKEN, 351-0198 Saitama, Japan

\textsuperscript{3}Deutsches Elektronen-Synchrotron (DESY), Platanenallee 6, D-15738 Zeuthen, Germany

Abstract

We perform numerical simulations of the dynamics of magnetized and highly relativistic pulsar-like winds produced by long-lasting central engines in Gamma Ray Bursters (GRB). The very fast wind interacts with the initial relativistically expanding GRB outflow, creating a multiple-shock structure. Depending on the parameters of the model (the energy of the initial explosion, the outside density, the wind power, the delay time for the switch-on of the wind, and the magnetization of the wind) a variety of temporal behaviors can be produced. In certain regimes the dynamics of such secondary engine-driven shocks is self-similar.

\textsuperscript{a} mbarkov@purdue.edu
I. INTRODUCTION

Various observations of early afterglows in long Gamma Ray Bursts (GRBs), at times ≤ 1 day, require a presence of long-lasting active central engine. These observations include the presence of unexpected features like flares and light curves plateaus [1-5], abrupt endings of the plateau phases [6], fast optical variability (e.g., GRB021004 and most notoriously GRB080916C), missing [7] and chromatic [8, 9] jet breaks, missing reverse shocks [10]). These phenomena are hard to explain within the standard fireball model that postulates that the early X-ray are produced in the forward shock, as argued in Refs. [11-13].

As an alternative, in Ref. [13] a model of early GRB afterglows was developed with the dominant X-ray contribution from the reverse shock (RS) propagating in highly relativistic (Lorentz factor $\gamma_w \sim 10^4 - 10^6$) magnetized wind of a long-lasting central engine. The model reproduces, in a fairly natural way, the overall trends and yet allows for variations in the temporal and spectral evolution of early optical and X-ray afterglows. The high energy and the optical synchrotron emission from the RS particles occurs in the fast cooling regime; the resulting synchrotron power $L_s$ is a large fraction of the wind luminosity, $L_s \approx L_w / \sqrt{1 + \sigma_w}$ ($L_w$ and $\sigma_w$ are wind power and magnetization). Thus, plateaus - parts of afterglow light curves that show slowly decreasing spectral power - are a natural consequence of the RS emission. Contribution from the forward shock (FS) is negligible in the X-rays, but in the optical both FS and RS contribute similarly [but see, e.g., 14-16]: the FS optical emission is in the slow cooling regime, producing smooth components, while the RS optical emission is in the fast cooling regime, and thus can both produce optical plateaus and account for fast optical variability correlated with the X-rays, e.g., due to changes in the wind properties. The later phases of pulsar wind interaction with super nova remnant discussed in Ref. [17].

The model developed in Ref. [13] explains, or offers a natural way to explain, many properties of early GRB afterglows: X-ray plateaus (as synchrotron emission from highly magnetized long-lasting ultra-relativistic wind), flares, abrupt endings of the plateau phases and fast optical variations (as response to the reverse shock in the fast cooling regime to the changing wind parameters), fast optical variations as emission from the naked GRBs, missing/chromatic jet breaks, missing orphan afterglows.

In the present paper we perform a number of numerical simulations for the propagation of a highly relativistic magnetized wind that follows a relativistic shock wave. Previously, this problem
was considered analytically in Ref. [18]. In §II we summarize the expectations from the analytical models (and describe their limitations), numerical set-up is described in §III, numerical results are discussed in §IV, finally, in §V we discuss the results.

II. RELATIVISTIC DOUBLE EXPLOSION

A. Triple shock structure

Consider relativistic point explosion of energy $E_1$ in a medium with constant density $\rho_{ex} = m_p n_{ex}$, followed by a wind with constant luminosity $L_w$ [18]. The initial explosion generates a Blandford-McKee forward shock wave (BMFS) [19].

$$\Gamma_1 = \sqrt{\frac{17}{8\pi}} \sqrt{\frac{E_1}{\rho_{ex}c^3}} t^{-3/2}$$

$$p_1 = \frac{2}{3} \rho_{ex} c^2 \Gamma_1^2 f_1(\chi)$$

$$\gamma_1^2 = \frac{1}{2} \Gamma_1^2 g_1(\chi)$$

$$n_1 = 2 n_{ex} \Gamma_1 n_1(\chi)$$

$$f_1(\chi) = \chi^{-17/12}$$

$$g_1(\chi) = 1/\chi$$

$$n_1(\chi) = \chi^{-5/4}$$

$$\chi = \left[1 + 2(m + 1)\Gamma^2\right] \left(1 - r/t\right)$$

(1)

Subscript $ex$ indicates the properties in the surrounding medium; subscript 1 indicates that quantities are measured behind the leading BMFS, hence between the two forward shocks; The Lorentz factor $\Gamma$ depends on time as $\Gamma^2 \propto t^{-m}$, $m = 3$.

We assume that the initial GRB explosion leaves behind an active remnant - a black hole or (fast rotating) neutron star. The remnant produces a long-lasting pulsar-like wind, either using the rotational energy of the newly born neutron star [20], accretion of the pre-explosion envelope onto the BH [21], or if the black hole can keep its magnetic flux for sufficiently long time [22, 23].

One expects that the central engine produces very fast and light wind that will start interacting with the slower, but still relativistically expanding, ejecta. As the highly relativistic wind from the long-lasting engine interacts with the initial explosion, it launches a second forward shock in the
medium already shocked by the primary blast wave. At the same time the reverse shock forms in the wind; the two shocks are separated by the contact discontinuity (CD), Fig. 1.

![Diagram of the model](image)

**FIG. 1.** Cartoon of the model. The long-lasting magnetized wind (region 1) after passing through a reverse shock forms a shocked wind flow (region 2), and launches a second shock (region 3) in the external medium pre-shocked by the primary wind; region 5 is the unshocked external medium. It is assumed that the primary shock is already in self-similar stage.

We assume that external density is constant, while the wind is magnetized with constant luminosity

\[
L_w = 4\pi \gamma_w^2 \left( \rho_w c^2 + \frac{b_w^2}{4\pi} \right) r^2 c
\]

(2)

where \(\rho_w\) and \(b_w\) are density and magnetic field measured in the wind rest frame. Thus

\[
b_w = \sqrt{\frac{\sigma_w}{1+\sigma_w}} \sqrt{\frac{L_w}{c} \frac{1}{r \gamma_w}}
\]

(3)
where
\[ \sigma_w = \frac{b_w^2}{4\pi \rho_w c^2}, \]
is the wind magnetization parameter [24]. The wind is assumed to be very fast, with \( \gamma_w \gg \Gamma_{FS}, \Gamma_{CD} \).

### B. Self-similar stages of relativistic double explosion

Generally, the structure of the flows in double explosions are non-self-similar [18]. First, with time the second forward shock approaches the initial forward shock (FS); for sufficiently powerful winds the second FS may catch up with the primary FS. The presence of this special time violates the assumption of self-similarity.

We can estimate the catch-up time by noticing that the power deposited by the wind in the shocked medium scales as \( L_w/\Gamma_{CD}^2 \). Thus, in coordinate time the wind deposits energy similar to the initial explosion at time when \( \Gamma_{CD} \sim \Gamma_{FS} \),

\[ t_{eq} = \Gamma_{FS}^2 \frac{E_1}{L_w} \approx \left( \frac{E_1^2}{c^5 \rho L_w} \right)^{1/4} \]

At times \( t \leq t_{eq} \) the second shock is approximately self-similar, the CD is located far downstream of the first shock; and is moving with time in the self-similar coordinate \( \chi \), associated with the primary shock, towards the first shock. The motion of the first shock is unaffected by the wind at this stage. At times \( t \geq t_{eq} \) the two shocks merge - the system then relaxes to a Blandford-McKee self-similar solution with energy supply.

Secondly, the self-similarity may be violated at early times if there is an effective delay time \( t_d \) between the initial explosion and the start of the second wind. (This issues is also important in our implementation scheme, §III - since we start simulation with energy injection at some finite distance from the primary shock this is equivalent to some effective time delay for the wind turn-on.)

Suppose that the secondary wind turns on at time \( t_d \) after the initial one and the second shock/CD is moving with the Lorentz factor

\[ \Gamma_{CD}^2 \propto (t-t_d)^{-m} \]

Then, the location of the second shock at time \( t \) is

\[ R_{CD} = (t-t_d) \left( 1 - \frac{1}{2\Gamma_{CD}^2 (m+1)} \right) \]
FIG. 2. Velocity structure of the triple-shock configuration. Leading is the FS that generates a self-similar post-shock velocity and pressure profiles. A fast wind with Lorentz factor $\gamma_w$ is terminated at the reverse shock (RS); the post-RS flow connects through the contact discontinuity (CD, dotted line) to the second shock driven in the already shock media. The CD is located at $r_{CD}$, corresponding to $\chi_{CD}$. The RS and the second forward shock (2nd FS) are located close to $\chi_{CD}$ \[13\].

The corresponding self-similar coordinate of the second shock in terms of the primary shock self-similar parameter $\chi$ is

$$\chi_{CD} = \left( 1 + 8 \gamma_1^2 \right) \left( 1 - \frac{R}{t} \right) \approx \left( \frac{8t_d}{t} + \frac{4}{(m+1)\Gamma_{CD}^2} \right) \gamma_1^2$$ \hspace{1cm} (8)

The effective time delay $t_d$ introduces additional (beside the catch-up time \[5\]) time scales in the problem. Thus, even within the limits of expected self-similar motion, $t \ll t_{eq}$ the effective delay time $t_d$ violates the self-similarity assumption. Still, depending on whether the ratio $t_d/(t\Gamma_{CD}^2)$ is much larger or smaller than unity, we expect approximately self-similar behavior \[13, 18\].

For $t_d \geq t/(2(m+1)\Gamma_{CD}^2)$, the location of the CD in the self-similar coordinate associated with
the first shock is
\[ \chi_{CD} \approx \frac{8 \gamma_2^2 t_d}{t} \propto t^{-4} \quad (9) \]
\[ \Gamma_{CD} = 0.52 \frac{E_1^{5/48} t_d^{5/48} L_w^{1/4}}{c^{85/48} \rho^{17/48} t^{11/12}} \quad (10) \]

Alternatively, for \( t_d \leq t/(2(m+1)\Gamma_{CD}^2) \),
\[ \chi_{CD} = 2.68 \left( \frac{E_1}{c^{5/2} \sqrt{\rho_d} \sqrt{L_w}} \right)^{24/29} \quad (11) \]
\[ \Gamma_{CD} = 0.50 \frac{E_1^{5/58} L_6^{0/29}}{c^{85/58} \rho^{17/58} t^{39/58}} \quad (12) \]

Finally, if the second explosion is point-like with energy \( E_2 \) and no time delay \( \gamma_2 = \sqrt{\frac{71}{2} \left( \frac{17}{\pi} \right)^{5/24} \left( \frac{E_1^{5/48} t_d^{5/48}}{c^{85/48} (m_p n_{ex})^{17}} \right)^{1/24} \sqrt{E_2 t^{-7/3}}} \) \quad (13)

Relation (10-13) indicate that depending on the particularities of the set-up, we expect somewhat different scalings for the propagation of the second shock (we are also often limited in integration time to see a switch between different self-similar regimes).

### III. SIMULATION SETUP

The simulations were performed using a one dimensional (1D) geometry in spherical coordinates using the PLUTO code. Spatial parabolic interpolation, a 3rd order Runge-Kutta approximation in time, and an HLLD Riemann solver were used. PLUTO is a modular Godunov-type code entirely written in C and intended mainly for astrophysical applications and high Mach number flows in multiple spatial dimensions. The simulations were run through the MPI library in the DESY (Germany) cluster. The flow has been approximated as an ideal, relativistic adiabatic gas with and without the toroidal magnetic field, one particle species, and polytropic index of 4/3. The adopted resolution is 192000 cells. The size of the domain is \( r \in [0.95, 4] r_s \) or \( r \in [0.98, 4] r_s \), here \( r_s \) is initial position of shock wave front.

As initial condition we set solution of B&Mc with shock radius 1, Eq (1), the Lorentz factor of the shock was 15. The external matter was assumed uniform with density \( \rho = 1 \) and pressure \( p = 10^{-4} \) (in units \( c = 1 \)). The pressure and density just after shock was determined by B&Mc solution (\( \rho_{BM} = 42.43 \) and \( p_{BM} = 150 \)) with total energy \( E_{BM} = 2.13 \times 10^5 \). From the left boundary (from a center) at radius \( r_w = 0.95 \) or \( r_w = 0.98 \) (models marked by letter ’s’ at the end of its name)
was injected wind with initial Lorentz factor $\gamma_w = 50$. The parameters of the models are listed in Table I.

In the unmagnetized models labeled pXX, we vary wind density. The wind density vary from $10^{-4}$ for pm4 model to $10^2$ for pp2. Magnetized models marked as mXXp1 have constant wind density, where XX indicates magnetization of the flow. Magnetized models marked as mXXep1 have constant wind luminosity, where XX indicates magnetization of the flow.

### TABLE I. Parameters of the models

| Model | $\rho_w$ | $p_w$ | $r_w$ | $\sigma_w$ | $L_w$ |
|-------|---------|------|-------|-----------|-------|
| pm4   | $10^{-4}$ | $10^{-7}$ | 0.95  | 0         | $2.85$ |
| pm3   | $10^{-3}$ | $10^{-6}$ | 0.95  | 0         | $2.85 \times 10$ |
| pm2   | $10^{-2}$ | $10^{-5}$ | 0.95  | 0         | $2.85 \times 10^2$ |
| pm2s  | $10^{-2}$ | $10^{-5}$ | 0.98  | 0         | $2.85 \times 10^2$ |
| pm1   | $10^{-1}$ | $10^{-4}$ | 0.95  | 0         | $2.85 \times 10^3$ |
| pm0   | 1       | $10^{-3}$ | 0.95  | 0         | $2.85 \times 10^4$ |
| pp1   | $10^{1}$ | $10^{-2}$ | 0.95  | 0         | $2.85 \times 10^5$ |
| pp2   | $10^{2}$ | $10^{-1}$ | 0.95  | 0         | $2.85 \times 10^6$ |
| pp2s  | $10^{2}$ | $10^{-1}$ | 0.98  | 0         | $2.85 \times 10^6$ |
| mm1p1 | $10^{1}$ | $10^{-2}$ | 0.95  | 0.1      | $3.13 \times 10^5$ |
| m0p1  | $10^{1}$ | $10^{-2}$ | 0.95  | 1.0      | $5.69 \times 10^5$ |
| m05p1 | $10^{1}$ | $10^{-2}$ | 0.95  | 3.0      | $1.14 \times 10^6$ |
| m1p1  | $10^{1}$ | $10^{-2}$ | 0.95  | 10       | $3.13 \times 10^6$ |
| mm1ep1| 9.09    | $9.1 \times 10^{-3}$ | 0.95  | 0.1      | $2.85 \times 10^5$ |
| m0ep1 | 5.00    | $5.0 \times 10^{-3}$ | 0.95  | 1.0      | $2.85 \times 10^5$ |
| m05ep1| 2.50    | $2.5 \times 10^{-3}$ | 0.95  | 3.0      | $2.85 \times 10^5$ |
| m1ep1 | 0.91    | $9.1 \times 10^{-4}$ | 0.95  | 10       | $2.85 \times 10^5$ |

### TABLE II. Parameters of the models
IV. RESULTS

We performed nine runs without magnetic field and eight runs with different magnetizations. Our numerical model for the primary shock is consistent with analytical solution of BM with an accuracy $\sim 10\%$ (pressure, density and maximal Lorentz factor). On the top of each panel of Figures (3)–(13) we indicate name of the model with parameters presented in the Table I.

A. Unmagnetized secondary wind

In Figure (3) we plot the results of pXX models there we vary power of hydrodynamical wind. At small radius one can clearly identify the location of the reverse shock (RS), where the Lorentz factor suddenly drops. At larger radius the contact discontinuity (CD) is identified by the the position of the tracer drop. Further out is the secondary forward shock, and the initial BM shock.

As we can see on the Figure (4) from pm2 to pp2 model, with increasing wind power, the Lorentz factor of FS and RS are also increase while the distance between these shocks becomes smaller, where positions of the shocks are indicated by jumps of pressure; jump of density at constant pressure identifies the CD. Shift of the wind injection radius (compare models $pm_2$ and $pm_2s$ or $pp_2$ and $pp_2s$) do not change structure of the solution significantly. Change of injection radius shift position of shocked wind structure as a whole. High resolution of our setup allows to resolve structures of density distribution on the radial scale $\sim 10^{-4} r_s$ (see Figure 5).

In Figure 6 three curves are shown for pXX models: (i) theoretical curve based on the expectation from the initial conditions $t_d = (r_s - r_w)/c$; (ii) Inverse square of Lorentz factor; (iii) actual time of delay calculated from position of CD and its Lorentz factor using eq (8). As we can see in the models $pm0$, $pp1$ and $pp2$ (power of the wind comparable to initial explosion) theoretical and actual curves are close. More powerful wind ($L_w r_s/c \geq 0.1 E_{BM}$) can push CD much faster that allows to satisfy conditions (6). Large value of $\gamma_{CD}$ also relax applicability condition of (10). So similar picture we can see on Figure 7 here models $pp2$, $pp1$ and $pm0$ follow theoretically predicted time dependence (see eq (10)) $\gamma_{CD} \propto t^{-11/12}$. Deviations from theoretical curves on Figures (6) and (7) at the late time are due to fact that the wind-triggered FS reach the radius of BMFS, affecting the motion of the initial shock: in this case transition to wind-driven BM solution occurs. The Lorentz factor is fitted by power law $\gamma_{CD} \propto t^{-0.45}$. 
FIG. 3. Hydrodynamic simulations of the double explosion. Potted are Lorentz factor and tracer distribution as a function of radius at the moment $t = 1.9 \frac{r_s}{c}$. The tracer distinguishes the wind from the shocked external medium. The parameters for each panel are encoded in the titles, Table I.

B. Magnetized secondary wind

As a basis for the magnetized wind models, we choose the model $pp1$, which have $L_w r_s/c \approx E_{BM}$, so that the total wind power injected during simulation is compatible to the energy of the initial explosion. Figures (8) and (9) demonstrate the structure of the solution. The main difference from the unmagnetized models is that the thickness of a layer between FS and RS increases with magnetization. This is related to a decrease of compressibility of the magnetized matter. Also note, that in models with similar total power of the wind, the position of FS almost independent of magnetization, while the position of RS strongly depends on the wind magnetization, RS moves slower in highly magnetized models.

Figure [10] demonstrates weak dependence of density profile of double shocked matter if the
FIG. 4. Gas pressure (thick solid lines), density (dotted line) and tracer (dashed line) as functions of radius at the moment $t = 1.9 \ [r_s/c]$.

FIG. 5. Zoom-in to the region close to the CD: Density (solid line) and tracer (dashed line) as functions of radius at the moment $t = 1.9 \ [r_s/c]$. 
FIG. 6. Self-similar coordinate of the second shock $\chi$, eq (8), as function of time for different models. Plotted are values of $8t_d/t$ from simulation (triangles), analytical curve (crosses) [18]. Also plotted square of inverse Lorentz factor (diamonds). Models with high wind power $pm_0$ and $pp_2$ closely follow the theoretical curve.

The total energy of the wind is preserved. On the other hand, if we are preserving hydrodynamic energy flux in the wind and increases its magnetization, due to increasing of the total power of wind double shocked matter suffer stronger compression and layer double shocked matter became thinner. On other hand increase of magnetization decrease compression ratio of the shocked wind.

All magnetized wind models show good agreement between theoretical expectation $I_d$ and actual ones, see Figure [12]. The Lorentz factor of CD, Figure [13] is also nicely fitted by theoretical curve eq. (10).

Figure [14] shows time dependence of Lorentz factor at CD and its $\chi_{CD}$. For high relative wind power the slope of Lorentz factor coincide with theoretical one. Moreover, dependence of the theoretical Lorentz factor on wind power (see eq (10)) $\gamma_{CD} \propto L_w^{1/4} = L_w^{0.25}$ and simulated one (Fig-
FIG. 7. Lorentz factor of the CD as function on time – triangles and analytical expectations [18]. The jumps in the Lorentz factor at later times occurs when the wind driven FS catches with the leading BMFS.

\[ \gamma_{\text{CD}} \propto L_0^{1.18} \]

The jumps in the Lorentz factor are in a good agreement.

Time behavior of theoretically predicted \( \chi_{\text{CD}} \) (\( \chi_{\text{CD}} \propto t^{\alpha_\chi}, \alpha_\chi = -4 \)) is in a good agreement with models with high relative wind power, see Figures (14) and (15) which shows tendency of power slop to \( \alpha_\chi = -3.8 \) at large wind powers. After the moment than wind driven FS reach BMFS, the slope is changed and tends to \( \alpha_\chi = -2.7 \).

The power of the slope of Lorentz factor of CD is in good agreement with theoretical one for wind independent on its magnetization see Figure 16. Moreover, Lorentz factor of CD very weakly depends on magnetization. If power of the wind is conserved \( \gamma_{\text{CD}} \propto \sigma_w^{0.023} \), if we preserve hydrodynamical part of the flow and increase magnetization trough increasing magnetic flux, we get \( \gamma_{\text{CD}} \propto \sigma_w^{0.18} \) that is similar to response of \( \gamma_{\text{CD}} \) on increase of wind power.

The power slope of time dependents of \( \chi_{\text{CD}} \), \( \alpha_{\text{CD}} \) almost do not depends on wind magnetization, Figure 18 and its value close to theoretically predicted slope of \(-4\).
FIG. 8. Lorentz factor and tracer distribution as functions of radius at the moment $t = 1.9$ for models with different magnetization.

V. DISCUSSION

In this paper we consider numerically the dynamics of relativistic double explosions, with applications to Gamma Ray Bursts. Our numerical results are in agreement with theoretical prediction [18]. We find that even for the case of constant external density and constant wind power the dynamics of the wind termination shock shows a large variety - both in temporal slopes of the scaling of the Lorentz factor of the shock, and producing non-monotonic behavior. Non-self-similar evolution of the wind termination shock occurs for two different reasons: (i) at early times due to a delay in the activation of the long-lasting fast wind; (ii) at late times when the energy injected by the wind becomes comparable to the energy of the initial explosion.

The results of fluid simulations confirm the theoretical expectations [13, 18]. For example, for sufficiently high wind power we have $\gamma_{CD} \propto t^{-11/12}$, while after $t_{eq}$ the shocks merge and move as
FIG. 9. Gas pressure (thick solid line), density (dotted line) and tracer (dashed line) as functions of radius at the moment $t = 1.9$ for models with different magnetization.

a single self-similar shock with $\gamma_{CD} \propto t^{-1/2}$.

As a major simplification, we did not include the evolution of the primary reverse shock, produced due to the interaction of the initial GRB explosion with the surrounding medium. We assumed that it already propagated though the ejecta, which reached a self-similar Blandford-McKee configuration. Naturally, taking into account the dynamics of the primary reverse shock will further complicate the evolution of the wind termination shock. We leave this considerations to further studies.

As discussed in Ref. [13], emission from the long-lasting relativistic wind can resolve a number of contradicting GRB observations, like afterglow plateaus and flares, abrupt endings of the plateau phases, fast optical variations, missing/chromatic jet breaks and others. Importantly, the wind termination shock produces emission in the fast cooling regime, converting large fraction of the wind power into radiation. Thus, in order to power afterglows a mildly luminous central source is
FIG. 10. Zoom-in to the regions near the CD. Density (solid line) and tracer (dashed line) as functions of radius at the moment $t = 1.9$ for cases with different magnetization.

needed. In addition, the termination shock luminosity depends on the wind power, and not total injected energy as is the case for the forward shock. As result, its emission can be highly variable.

ACKNOWLEDGMENTS

We thank the PLUTO team for the possibility to use the PLUTO code and for technical support. The visualization of the results performed in the VisIt package [28]. This work had been supported by NASA grants 80NSSC17K0757 and 80NSSC20K0910, and NSF grants 10001562 and 10001521.

The data that support the findings of this study are available from the corresponding author.
FIG. 11. Same as fig. 10 for different set of models.

FIG. 12. Effects of magnetization on flow dynamics. Same as on the Figure 6 for cases with magnetization $\sigma_w = 0$ (left) and $\sigma_w = 10$ (right).
FIG. 13. Effects of magnetization on flow dynamics. Lorentz factor as a function of time – triangles and analytical expectations [18] for cases with magnetization $\sigma_w = 0$ (left) and $\sigma_w = 10$ (right). The jumps in the Lorentz factor at later times occurs when the wind driven FS catches with the leading BMFS.

upon reasonable request.

[1] J. A. Nousek, C. Kouveliotou, D. Grupe, K. L. Page, J. Granot, E. Ramirez-Ruiz, S. K. Patel, D. N. Burrows, V. Mangano, S. Barthelmy, et al., ApJ 642, 389 (2006), astro-ph/0508332.
[2] N. Gehrels and S. Razzaque, Frontiers of Physics 8, 661 (2013), 1301.0840.
[3] A. Lien, T. Sakamoto, S. D. Barthelmy, W. H. Baumgartner, J. K. Cannizzo, K. Chen, N. R. Collins, J. R. Cummings, N. Gehrels, H. A. Krimm, et al., ApJ 829, 7 (2016), 1606.01956.
[4] M. de Pasquale, S. R. Oates, M. J. Page, D. N. Burrows, A. J. Blustin, S. Zane, K. O. Mason, P. W. A. Roming, D. Palmer, N. Gehrels, et al., MNRAS 377, 1638 (2007), astro-ph/0703447.
[5] E. Mazaeva, A. Pozanenko, and P. Minaev, ArXiv e-prints (2018), 1804.10441.
[6] E. Troja, G. Cusumano, P. T. O’Brien, B. Zhang, B. Sbarufatti, V. Mangano, R. Willingale, G. Chincarini, J. P. Osborne, F. E. Marshall, et al., ApJ 665, 599 (2007), arXiv:astro-ph/0702220.
[7] M. De Pasquale, M. J. Page, D. A. Kann, S. R. Oates, S. Schulze, B. Zhang, Z. Cano, B. Gendre, D. Malesani, A. Rossi, et al., MNRAS 462, 1111 (2016), 1602.04158.
[8] A. Panaitescu, MNRAS 380, 374 (2007), 0705.1015.
[9] J. L. Racusin, E. W. Liang, D. N. Burrows, A. Falcone, T. Sakamoto, B. B. Zhang, B. Zhang, P. Evans, and J. Osborne, ApJ 698, 43 (2009), 0812.4780.
FIG. 14. Lorentz factor of contact discontinuity (top panel) and location of the CD $\chi_{CD}$ (bottom panel) as functions of time. The analytical estimations (see Eq. 10) $\gamma_{CD} \propto t^{-0.92}$ and for wind driven shock $\gamma_{CD} \propto t^{-0.5}$ (see thin lines with crosses, stars and circles).
FIG. 15. Dependence of the Lorentz factor of the contact discontinuity (left panel) and power slop $\alpha_x$ (right panel) at $t = 2[R_s/c]$. In the high wind power regime the scaling is close to the expected $\Gamma_{CD} \propto L_w^{1/4}$, Eq. (10).

FIG. 16. Lorentz factor of the contact discontinuity as function of time, left panel, (cf. Eq. (10) $\gamma_{CD} \propto t^{-0.92}$). Dependence of the Lorentz factor of the contact discontinuity on wind magnetization at $t = 1.4[R_s/c]$, right panel. Red curve is constant total power, blue dashed curve is constant matter power. As expected, in for fixed total power the Lorentz factor of the CD is approximately independent of the the wind magnetization.

[10] A. Gomboc, S. Kobayashi, C. G. Mundell, C. Guidorzi, A. Melandri, I. A. Steele, R. J. Smith, D. Bersier, D. Carter, and M. F. Bode, in *American Institute of Physics Conference Series*, edited by C. Meegan, C. Kouveliotou, & N. Gehrels (2009), vol. 1133 of *American Institute of Physics Conference Series*, pp. 145–150.
FIG. 17. Time dependence of $\chi_{CD}$ (cf. Eq. [9]) and $\chi_{RS}$ (location of the CD and RS in self-similar coordinate). In a fully self-similar regime the dynamics of the RS follows that of the CD. The low $\sigma$ models do show this property. As we discussed above in the case of the CD, for smaller wind powers the effective time delay $t_d$ starts to become important, resulting in smaller temporal indecies. We attribute flatter dependence of $\chi_{CD}$ on time (see also Fig. 18) to a somewhat similar effect: for larger $\sigma$ the RS Lorentz factor is smaller, $\propto \Gamma_{CD}/\sqrt{\sigma}$. Thus, beyond some value of $\sigma$ the Lorentz factor of the RS and the corresponding $\chi_{CD}$ are demonstrate flatter temporal profiles.

FIG. 18. Dependence of $\alpha_{\chi}$ on magnetization of the wind ($\chi_{CD} \propto t^{\alpha_{\chi}}$). Diamonds and crosses correspond to contact discontinuity and reverse shock in the case of a preserved energy flux of hydrodynamical flux in the wind. Right triangle and inverted triangle correspond to the case of preserved total energy flux in the wind. (See caption for Fig. [17])

21
[11] M. Lyutikov, ArXiv e-prints 0911.0349 (2009), 0911.0349.
[12] D. A. Kann, S. Klose, B. Zhang, D. Malesani, E. Nakar, A. Pozanenko, A. C. Wilson, N. R. Butler, P. Jakobsson, S. Schulze, et al., ApJ 720, 1513 (2010), 0712.2186.
[13] M. Lyutikov and J. Camilo Jaramillo, ApJ 835, 206 (2017), 1612.01162.
[14] D. C. Warren, D. C. Ellison, M. V. Barkov, and S. Nagataki, ApJ 835, 248 (2017), 1701.04170.
[15] D. C. Warren, M. V. Barkov, H. Ito, S. Nagataki, and T. Laskar, MNRAS 480, 4060 (2018), 1804.06030.
[16] H. Ito, J. Matsumoto, S. Nagataki, D. C. Warren, M. V. Barkov, and D. Yonetoku, Nature Communications 10, 1504 (2019), 1806.00590.
[17] D. Khangulyan, A. V. Koldoba, G. V. Ustyugova, S. V. Bogovalov, and F. Aharonian, ApJ 860, 59 (2018), 1712.10161.
[18] M. Lyutikov, Physics of Fluids 29, 047101 (2017), 1701.06604.
[19] R. D. Blandford and C. F. McKee, Physics of Fluids 19, 1130 (1976).
[20] V. V. Usov, Nature 357, 472 (1992).
[21] J. K. Cannizzo and N. Gehrels, ApJ 700, 1047 (2009), 0901.3564.
[22] M. Lyutikov, Phys. Rev. D 83, 124035 (2011), 1104.1091.
[23] M. Lyutikov and J. C. McKinney, Phys. Rev. D 84, 084019 (2011), 1109.0584.
[24] C. F. Kennel and F. V. Coroniti, ApJ 283, 694 (1984).
[25] Note1, link http://plutocode.ph.unito.it/index.html.
[26] A. Mignone, G. Bodo, S. Massaglia, T. Matsakos, O. Tesileanu, C. Zanni, and A. Ferrari, ApJS 170, 228 (2007), astro-ph/0701854.
[27] A. Mignone, M. Ugliano, and G. Bodo, MNRAS 393, 1141 (2009), 0811.1483.
[28] H. Hank Childs, E. Brugger, B. Whitlock, and et al., in High Performance Visualization–Enabling Extreme-Scale Scientific Insight (2012), pp. 357–372.