Triple-Product Correlations in $B \rightarrow V_1V_2$ Decays and New Physics

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Abstract

In this paper we examine T-violating triple-product correlations (TP’s) in $B \rightarrow V_1V_2$ decays. TP’s are excellent probes of physics beyond the standard model (SM) for two reasons: (i) within the SM, most TP’s are expected to be tiny, and (ii) unlike direct CP asymmetries, TP’s are not suppressed by the small strong phases which are expected in $B$ decays. TP’s are obtained via the angular analysis of $B \rightarrow V_1V_2$. In a general analysis based on factorization, we demonstrate that the most promising decays for measuring TP’s in the SM involve excited final-state vector mesons, and we provide estimates of such TP’s. We find that there are only a handful of decays in which large TP’s are possible, and the size of these TP’s depends strongly on the size of nonfactorizable effects. We show that TP’s which vanish in the SM can be very large in models with new physics. The measurement of a nonzero TP asymmetry in a decay where none is expected would specifically point to new physics involving large couplings to the right-handed $b$-quark.

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1 Introduction

There is a great deal of interest these days in the study of CP violation in the $B$ system. By examining CP-violating effects in $B$ decays, we hope to get some clues as to the origin of CP violation in the quark sector. If we are lucky, the standard model (SM) explanation of CP violation — a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix — will be shown to be insufficient to explain the data, and we will therefore have found indirect evidence for the presence of physics beyond the SM.

Most of the theoretical work on this subject has concentrated on mixing-induced CP-violating asymmetries in neutral $B$ decays, while a smaller fraction has focussed on direct CP asymmetries \[1\]. However, there is another class of CP-violating effects which has received considerably less attention, and which can also reveal the presence of new physics: triple-product correlations. These take the form $\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$, where each $v_i$ is a spin or momentum. These triple products (TP’s) are odd under time reversal (T) and hence, by the CPT theorem, also constitute potential signals of CP violation. One can establish the presence of a nonzero TP by measuring a nonzero value of the asymmetry

$$A_T \equiv \frac{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) - \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) + \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)} ,$$

where $\Gamma$ is the decay rate for the process in question.

Of course, there is a well-known technical complication for such effects: strong phases can produce a nonzero value of $A_T$, even if the weak phases are zero (i.e. there is no CP violation). Thus the TP asymmetry $A_T$ is not a true T-violating effect (we refer to it as $T$-odd). However, one can still obtain a true T-violating (and hence CP-violating) signal by comparing $A_T$ with $\bar{A}_T$, where $\bar{A}_T$ is the T-odd asymmetry measured in the CP-conjugate decay process \[2\].

TP asymmetries are similar to direct CP asymmetries in two ways: (i) they are both obtained by comparing a signal in a given decay with the corresponding signal in the CP-transformed process, and (ii) both are nonzero only if there are two interfering decay amplitudes. However, there is an important difference between the two. Denoting $\phi$ and $\delta$ as the relative weak and strong phases, respectively, between the two interfering amplitudes, the signal for direct CP violation can be written

$$A_{CP}^{dir} \propto \sin \phi \sin \delta ,$$

while, as we shall see, that for the (true T-violating) TP asymmetry is given by

$$A_T \propto \sin \phi \cos \delta .$$

The key point here is that one can produce a direct CP asymmetry only if there is a nonzero strong-phase difference between the two decay amplitudes. However, it
has been argued that, due to the fact that the $b$-quark is heavy, all strong phases in $B$ decays should be rather small. In this case, all direct CP-violation signals will be tiny as well. On the other hand, TP asymmetries are maximal when the strong-phase difference vanishes. Thus, it may well be more promising to search for triple-product asymmetries than direct CP asymmetries in $B$ decays.

One class of $B$ processes in which triple products are generally expected to appear are the decays of a $B$-meson (charged or neutral) into two final-state vector mesons: $B \rightarrow V_1 V_2$. In the rest frame of the $B$, the TP takes the form $\vec{q} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$, where $\vec{q}$ is the momentum of one of the final vector mesons, and $\vec{\varepsilon}_1$ and $\vec{\varepsilon}_2$ are the polarizations of $V_1$ and $V_2$. Since $B^0$ and $B^{\pm}$ mesons are already being produced copiously at the $B$-factories BaBar and Belle, the study of such TP signals can be performed now.

Some triple-product signals in the $B$ system have been studied within the SM in past analyses – they were first examined many years ago by Valencia [2], and several general studies of $B \rightarrow V_1 V_2$ decays were subsequently performed [3, 4, 5, 6]. In these papers, it is found that the TP’s with ground state vector mesons are (almost) all small. As we show in the present paper, this result can be understood, in a general analysis based on factorization, in terms of mass and flavour suppressions. On the other hand, these suppressions are small or absent for decays involving excited vector mesons. We therefore note that the most promising decays for measuring TP’s in the SM involve radially-excited mesons, and we provide estimates of the TP’s in such decays, as well as in several other modes not considered previously. However, as we show, most of these TP asymmetries are also expected to be small in the SM. The fact that most TP’s are expected to be small in the SM makes their measurements a very promising method for searching for new physics.

We begin in Sec. 2 with a general review of triple-product correlations in $B \rightarrow V_1 V_2$ decays. Using factorization, we describe the conditions which must be present in order to produce a TP in a given decay. We then present a detailed list of exclusive decays which are expected to yield TP’s in the SM. We also discuss the possibility of generating TP’s via mixing. In Sec. 3, we turn to the question of the experimental prospects for measuring TP’s. It is well known that one can disentangle the helicities of the $V_1 V_2$ final state via an angular analysis. We briefly review this analysis, stressing that this is precisely how TP’s are measured. As we will show, TP’s are typically suppressed by a factor of at least $m_{\nu}/m_{b}$, and are further suppressed if $V_1$ and $V_2$ are related by a symmetry. Consequently, the TP’s in $B$ decays to ground-state vector mesons are all expected to be very small, and this has been found by previous analyses. On the other hand, decays in which the final-state vector mesons are unrelated, and as heavy as possible, are less affected by these suppressions. The most promising decays for the detection of TP’s are therefore those which involve final-state radially-excited mesons. In this section, we estimate the size of the TP’s, as well as the branching ratios, for such decays. Although there are some TP’s which may be large, the great majority of $B$ decays exhibit very small TP’s. This makes them an ideal place to look for physics beyond the SM – should
any large TP be found, this would be a clear signal of the presence of new physics. We also address the issue of nonfactorizable effects in this section, as well as how TP’s may help in the resolution of discrete ambiguities in the measurement of the angles of the unitarity triangle. Finally, in Sec. 4, we examine the properties of new physics which can modify the SM predictions for TP’s in various $B \to V_1 V_2$ decays. Specifically, we show that if the new physics involves significant couplings to the right-handed $b$-quark, large TP asymmetries can be produced. We illustrate this in the context of a specific new-physics model, supersymmetry with broken R-parity. This demonstrates quite clearly that the measurement of TP’s is an excellent way to search for new physics. We summarize our results in Sec. 5.

2 Triple Products in $B \to VV$ decays

2.1 General Considerations

In this subsection, we follow the analysis of Ref. [2], and use the following notation at the meson level: $B(p) \to V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)$. The decay amplitude can then be expressed as follows:

$$M = a \, \varepsilon_1^* \cdot \varepsilon_2^* + \frac{b}{m_B^2} (p \cdot \varepsilon_1^*) (p \cdot \varepsilon_2^*) + i \frac{c}{m_B^2} q^\mu q'^\nu \varepsilon_1^{*\mu} \varepsilon_2^{*\nu},$$

where $q \equiv k_1 - k_2$. (Note that we have normalized terms with a factor $m_B^2$, rather than $m_1 m_2$ as in Ref. [2]. With the above normalization, each of $a$, $b$ and $c$ is expected to be the same order of magnitude.) The $a$ and $b$ terms correspond to combinations of $s$- and $d$-wave amplitudes while the $c$ term corresponds to the $p$-wave amplitude for the final state. The quantities $a$, $b$ and $c$ are complex and will in general contain both CP-conserving strong phases and CP-violating weak phases.

In $|M|^2$, a triple-product correlation arises from interference terms involving the $c$ amplitude, and will be present if $\text{Im}(ac^*)$ or $\text{Im}(bc^*)$ is nonzero. In the rest frame of the $B$ meson, this TP takes the form $\vec{q} \cdot (\vec{\varepsilon}_1^* \times \vec{\varepsilon}_2^*)$.

However, as discussed above, due to the presence of strong phases, such TP’s are not necessarily true T-violating effects. To obtain a true measure of T violation, one has to compare the triple product measured in $B \to V_1 V_2$ with that obtained in the CP-conjugate process. Using CPT, the amplitude for the CP-conjugate process $\bar{B}(p) \to \bar{V}_1(k_1, \bar{\varepsilon}_1) + \bar{V}_2(k_2, \bar{\varepsilon}_2)$ can be expressed as follows:

$$\bar{M} = \bar{a} \, \bar{\varepsilon}_1^* \cdot \bar{\varepsilon}_2^* + \frac{\bar{b}}{m_B^2} (p \cdot \bar{\varepsilon}_1^*) (p \cdot \bar{\varepsilon}_2^*) - i \frac{\bar{c}}{m_B^2} q^\mu q'^\nu \bar{\varepsilon}_1^{*\mu} \bar{\varepsilon}_2^{*\nu},$$

where $\bar{a}$, $\bar{b}$ and $\bar{c}$ can be obtained from $a$, $b$ and $c$ by changing the sign of the weak phases. If CP is conserved, one has $\bar{a} = a$, $\bar{b} = b$ and $\bar{c} = c$. 
Note that CPT leaves invariant each of the three Lorentz scalars in Eq. (4). Thus, because the $p$-wave amplitude in $\bar{M}$ changes sign relative to that of $M$, the sign of the T-odd asymmetry in $|\bar{M}|^2$ is opposite that in $|M|^2$. The true T-violating asymmetry is therefore found by adding the T-odd asymmetries in $|\bar{M}|^2$ and $|M|^2$:

$$A_T \equiv \frac{1}{2}(A_T + \bar{A}_T).$$

Writing

$$a = \sum_i a_i e^{i\phi_i^a} e^{i\delta_i^a}, \quad \bar{a} = \sum_i a_i e^{-i\phi_i^a} e^{i\delta_i^a},$$

$$b = \sum_i b_i e^{i\phi_i^b} e^{i\delta_i^b}, \quad \bar{b} = \sum_i b_i e^{-i\phi_i^b} e^{i\delta_i^b},$$

$$c = \sum_i c_i e^{i\phi_i^c} e^{i\delta_i^c}, \quad \bar{c} = \sum_i c_i e^{-i\phi_i^c} e^{i\delta_i^c},$$

where the $\phi_i^{a,b,c}$ ($\delta_i^{a,b,c}$) are weak (strong) phases, we see that

$$\frac{1}{2} \left[ \text{Im}(ac^*) - \text{Im}(\bar{c}\bar{a}^*) \right] = \sum_{i,j} a_i c_j \sin \left( \phi_i^a - \phi_j^c \right) \cos \left( \delta_i^a - \delta_j^c \right),$$

$$\frac{1}{2} \left[ \text{Im}(bc^*) - \text{Im}(\bar{c}\bar{b}^*) \right] = \sum_{i,j} b_i c_j \sin \left( \phi_i^b - \phi_j^c \right) \cos \left( \delta_i^b - \delta_j^c \right),$$

which explains the form of Eq. (3).

### 2.2 Factorization

Not all $B \to V_1 V_2$ decays will necessarily yield triple products. In this subsection, we use the framework of naive factorization to examine the conditions which are required in order to produce a TP in a given decay. It should be noted that there have been recent developments in the study of nonleptonic decays, such as QCD factorization [7] and PQCD [8], in which corrections to naive factorization proportional to $\alpha_s$ have been calculated in the heavy $m_B$ limit. QCD factorization has been applied to some $B \to V_1 V_2$ decays [9]. However, some of the corrections to naive factorization turn out to be divergent, so that predictive power is lost.

Previous analyses, using naive factorization, have found that most TP asymmetries with ground state vector mesons are expected to be small in the SM [2, 3, 4, 5, 6]. As will be shown, we agree with this result. Note that this conclusion will necessarily hold even if one employs QCD factorization or PQCD, since the dominant contribution comes from naive factorization in these approaches. It is possible that nonfactorizable effects are significant in certain $B \to V_1 V_2$ decays, particularly those dominated by colour-suppressed amplitudes. We discuss these effects in some detail later (Sec. 3.3), and attempt to take them into account in our analysis. For
B decays to radially-excited vector mesons, which have not been considered previously, we also use naive factorization to estimate the TP asymmetries. (Note that the methods of QCD factorization or PQCD have not been developed or used with radially-excited states.)

The starting point for factorization is the SM effective Hamiltonian for B decays:

$$H_{\text{eff}}^q = \frac{G_F}{\sqrt{2}} [V_{tb} V_{tq}^* (c_1 O_1^q + c_2 O_2^q) - \sum_{i=3}^{10} (V_{ub} V_{uq}^* c_i^u + V_{cd} V_{cq}^* c_i^c + V_{tb} V_{tq}^* c_i^t) O_i^q] + h.c.,$$  \hspace{1cm} (12)$$

where the superscript $u$, $c$, $t$ indicates the internal quark, $f$ can be the $u$ or $c$ quark, and $q$ can be either a $d$ or $s$ quark. The operators $O_i^q$ are defined as

$$O_{1,7}^q = \bar{q}_a \gamma_\mu L f_\beta \bar{f}_\beta \gamma^\mu L b_\alpha, \quad O_{2,6}^q = \bar{q}_a \gamma_\mu L f_\beta \bar{f}_\beta \gamma^\mu L b_\alpha,$$

$$O_{3,5}^q = \bar{q}_a \gamma_\mu L b_\beta \bar{q}_\beta \gamma^\mu (R(L)) q', \quad O_{4,6}^q = \bar{q}_a \gamma_\mu L b_\beta \bar{q}_\beta \gamma^\mu L (R) q'_\alpha,$$

$$O_{7,9}^q = \frac{3}{2} \bar{q}_a \gamma_\mu L b_{e\gamma'} \bar{q}_\gamma' \gamma^\mu R (L) q', \quad O_{8,10}^q = \frac{3}{2} \bar{q}_a \gamma_\mu L b_\beta e_{\gamma'} \bar{q}_\gamma' \gamma^\mu R (L) q'_\alpha,$$

where $R(L) = 1 \pm \gamma_5$, and $q'$ is summed over $u$, $d$, $s$, $c$. $O_2$ and $O_1$ are the tree-level and QCD-corrected operators, respectively. $O_{3-6}$ are the strong gluon-induced penguin operators, and operators $O_{7-10}$ are due to $\gamma$ and $Z$ exchange (electroweak penguins), and "box" diagrams at loop level. In what follows, the important point is that all SM operators involve a left-handed $b$-quark.

Within factorization, the amplitude for $B \rightarrow V_1 V_2$ can be written as

$$\mathcal{A}(B \rightarrow V_1 V_2) = \sum_{\mathcal{O}, \mathcal{O}'} \langle V_1 | \mathcal{O} | 0 \rangle \langle V_2 | \mathcal{O}' | B \rangle + \langle V_2 | \mathcal{O} | 0 \rangle \langle V_1 | \mathcal{O}' | B \rangle,$$

where $\mathcal{O}$ and $\mathcal{O}'$ are pieces of the $O_i^q$ operators above. The specific quark content of these operators depends on the final state $V_1 V_2$. (As mentioned above, we return to the question of nonfactorizable effects in Sec. 3.)

In the following, we write the quark-level decay as $b \rightarrow q\bar{q}' q'$, and call the spectator quark $\bar{q}_s$. As noted above, there are two categories of operators which contribute to this decay: (i) tree contributions, which have the form $\bar{q}' \gamma^\mu (1 - \gamma_5)b \bar{q}_s \gamma_\mu (1 - \gamma_5)q'$, and (ii) penguin operators, which have the form $\bar{q}_s \gamma^\mu (1 - \gamma_5)b \bar{q}' \gamma_\mu (1 - \gamma_5)q'$. (In all operators, both colour assignments are understood.)

Consider now the first term in the above expression, $\sum_{\mathcal{O}, \mathcal{O}'} \langle V_1 | \mathcal{O} | 0 \rangle \langle V_2 | \mathcal{O}' | B \rangle$. Let us first suppose that $V_1 = q\bar{q}'$ (so-called colour-allowed decays). Then

$$\langle V_1 | \bar{q}' \gamma^\mu q' | 0 \rangle = m_1 g_{V_1} \varepsilon^\mu_{V_1}.$$

The tree operator involves the factor $\bar{q}_s \gamma_\mu (1 - \gamma_5)q'$, and therefore has the right form. On the other hand, one must perform a Fierz transformation on the penguin operators to obtain the correct form. Those operators of the form $\bar{q}_s \gamma^\mu (1 - \gamma_5)b \bar{q}' \gamma_\mu (1 - \gamma_5)q'$
Fierz-transform into \( \bar{q}'\gamma^\mu(1-\gamma_5)b\bar{q}\gamma_\mu(1-\gamma_5)q' \), just like the tree contributions (modulo colour factors). However, penguin operators of the form \( \bar{q}'\gamma^\mu(1-\gamma_5)b\bar{q}\gamma_\mu(1+\gamma_5)q' \) Fierz-transform into \( -2\bar{q}'(1-\gamma_5)b\bar{q}(1+\gamma_5)q' \), and these will not contribute to the decay, since \( \langle V_1 | \bar{q}(1+\gamma_5)q' | 0 \rangle = 0 \). We therefore find that

\[
\sum_{\mathcal{O},\mathcal{O}'} \langle V_1 | \mathcal{O} | 0 \rangle \langle V_2 | \mathcal{O}' | B \rangle = X\varepsilon_1^\mu \langle V_2 | \bar{q}'\gamma_\mu(1-\gamma_5)b | B \rangle ,
\]

where \( X \) is a factor which includes a combination of Wilson coefficients and weak CKM phases (e.g. \( V_{cb}V_{cq}^* \) and \( V_{tb}V_{tq}^* \)). The upshot is that, within the SM, there is only one decay amplitude (i.e. operator) for this term, and the fundamental reason for this is that the SM involves only left-handed \( b \)-quarks.

One obtains a similar expression for the case where \( V_1 = q'\bar{q}' \) (so-called colour-suppressed (or electroweak penguin) decays). The only difference is that, in this case, the penguin operators have the correct form, but the tree operator must be Fierz-transformed. However, one still ends up with an expression for the amplitude similar to that above.

Now, in order to cast Eq. (16) in the same form as Eq. (4), one must express the remaining matrix element above in terms of form factors. This can be done as follows:

\[
\langle V_2(k_2) | \bar{q}'\gamma_\mu b | B(p) \rangle = i \frac{2V^{(2)}(r^2)}{(m_B + m_2)} \epsilon_{\mu\nu\rho\sigma} p^\nu k_2^\rho \varepsilon_2^\sigma ,
\]

\[
\langle V_2(k_2) | \bar{q}'\gamma_\mu(1-\gamma_5)b | B(p) \rangle = (m_B + m_2)A_1^{(2)}(r^2) \left[ \varepsilon_{2\nu} - \frac{\varepsilon_2^\rho r_\rho}{r^2} r_\mu \right]
- A_2^{(2)}(r^2) \frac{\varepsilon_2^\rho r_\rho}{m_B + m_2} \left( p_\mu + k_2\mu \right) - \frac{m_B^2 - m_2^2}{r^2} r_\mu \right]
+ 2im_2 \frac{\varepsilon_2^\rho r_\rho}{r^2} r_\mu A_0^{(2)}(r^2) ,
\]

where \( r = p - k_2 \), and \( V^{(2)}, A_1^{(2)}, A_2^{(2)} \) and \( A_0^{(2)} \) are form factors. Thus, the first term of Eq. (17) is given by

\[
\sum_{\mathcal{O},\mathcal{O}'} \langle V_1 | \mathcal{O} | 0 \rangle \langle V_2 | \mathcal{O}' | B \rangle = -(m_B + m_2)m_1 g_V X A_1^{(2)}(m_1^2) \varepsilon_1^* \varepsilon_2^*
+ 2 \frac{m_1}{(m_B + m_2)} g_V X A_2^{(2)}(m_1^2) \varepsilon_2^* \cdot p \varepsilon_1^* \cdot p
- i \frac{m_1}{(m_B + m_2)} g_V X V^{(2)}(m_1^2) \epsilon_{\mu\nu\rho\sigma} q' p^\rho \varepsilon_1^* \varepsilon_2^* \varepsilon_2^* (18)
\]

where we have used \( k_2 = (p-q)/2 \). The key point here is that all phase information is contained within the factor \( X \), which is common to all three independent amplitudes. Thus, these quantities all have the same phase.
A similar analysis holds for the second term in Eq. (14):

$$
\sum_{\mathcal{O}, \mathcal{O}'} \langle V_2 | \mathcal{O} | 0 \rangle \langle V_1 | \mathcal{O}' | B \rangle = -(m_B + m_1)m_2 g_{V_2} Y A_1^{(1)}(m_2^2) \varepsilon_1^* \cdot \varepsilon_2^* + 2 \frac{m_2}{m_B + m_1} g_{V_2} Y A_2^{(1)}(m_2^2) \varepsilon_2^* \cdot p \varepsilon_1^* \cdot p - i \frac{m_2}{(m_B + m_1)} g_{V_2} Y V^{(1)}(m_2^2) \epsilon_{\mu \nu \rho \sigma} p^\mu q^\nu \varepsilon_1^* \varepsilon_2^* \varepsilon_{\rho \sigma} (19)
$$

As before, all three independent amplitudes have the same phase, $Y$ (though this is not necessarily equal to that of the first term, $X$).

We can now express the quantities $a$, $b$ and $c$ of Eq. (14) as follows:

$$
a = -m_1 g_{V_1} (m_B + m_2) A_1^{(2)}(m_1^2) X - m_2 g_{V_2}(m_B + m_1) A_1^{(1)}(m_2^2) Y
$$

$$
b = 2m_1 g_{V_1} \frac{m_B}{(m_B + m_2)} m_B A_2^{(2)}(m_1^2) X + 2m_2 g_{V_2} \frac{m_B}{(m_B + m_1)} m_B A_2^{(1)}(m_2^2) Y
$$

$$
c = -m_1 g_{V_1} \frac{m_B}{(m_B + m_2)} m_B V^{(2)}(m_1^2) X - m_2 g_{V_2} \frac{m_B}{(m_B + m_1)} m_B V^{(1)}(m_2^2) Y (20)
$$

At this point we can make an important general observation. As noted previously, TP’s will be produced in $B \to V_1 V_2$ decays as long as $\text{Im}(ac^*)$ or $\text{Im}(bc^*)$ is nonzero. However, from the above equation, we see that if either $X$ or $Y$ is zero, then $a$, $b$ and $c$ will all have the same phase, so that $\text{Im}(ac^*) = \text{Im}(bc^*) = 0$. Therefore, in order to have a triple-product correlation in a given decay, both of the amplitudes in Eq. (14) must be present.

This is perhaps a surprising result. Naively, one would think that if a particular decay receives both tree and penguin contributions, with different weak phases, T-violating TP’s would automatically arise. However, as we have shown above, this is not necessarily so. The reason is that TP’s are a kinematical CP-violating effect. It is therefore not enough to have two decay amplitudes with a relative weak phase. What one really needs is two different kinematical amplitudes with a relative weak phase.

There is a second important point: if we replace the index ‘2’ by ‘1’ in Eq. (20) above, $a$, $b$ and $c$ will once again have the same phase. We therefore see explicitly that if $V_1 = V_2$, no TP’s can be produced. (This is to be expected since, from Eq. (14), there is only a single amplitude in this case.) However, it also indicates that if $V_1$ and $V_2$ are similar, i.e. related by a symmetry, the phases of $a$, $b$ and $c$ will also be similar, and the TP correspondingly suppressed. This will be important when we estimate the sizes of TP’s for specific exclusive decays.

### 2.3 Triple products in specific exclusive decays

We now turn to establishing which specific exclusive $B \to V_1 V_2$ decays are expected to have triple-product correlations in the SM. As we have noted above, two kinematical amplitudes are necessary in order for a TP to be produced.
To a first approximation, there is a simple rule for determining which processes have two such amplitudes: in the quark-level decay $b \to q \bar{q}' q''$, if the spectator quark $\bar{q}_s$ is the same as $\bar{q}'$, then the two amplitudes of Eq. (14) will be present. However, this is not sufficient to generate a true T-violating TP. It is also necessary that the two kinematical amplitudes have different weak phases. Thus, if the quark-level decay is dominated by a single decay amplitude, a TP can never be generated. This is the case for the quark-level decays $b \to c \bar{c} s$, whose tree and penguin contributions have approximately the same weak phase (an example of such a decay at the meson level is $B \to J/\psi K^*$). It also holds for pure $b \to s$ penguin decays, which are dominated by internal $t$-quark exchange (e.g. $B \to \phi K^*$).

This rule must be modified slightly when the flavour wavefunction of one of the final-state vector mesons contains more than one piece (e.g. the $\rho^0$ is composed of both $uu$ and $dd$ pairs). In this case, several different quark-level $b \to q \bar{q}' q''$ decays can contribute to the final state, and $\bar{q}_s$ must be the same as one of the $\bar{q}'$ quarks. Furthermore, it is necessary that $V_1$ and $V_2$ have different flavour wavefunctions. For example, suppose that $V_1 = \rho^0$ and $V_2 = \rho^{0'}$, where $\rho^{0'}$ is an excited state. In this case, even though $V_1 \neq V_2$, there will still be no TP since the two kinematical amplitudes will have the same phase, i.e. one will have $X = Y$ in Eq. (20).

With these constraints, we find that only a small number of $B$ decays can yield TP's in the SM. These are listed below. In the discussion of each decay mode, we use the following notation to denote the main decay amplitudes: $T$ (colour-allowed tree amplitude), $C$ (colour-suppressed tree amplitude), $P$ (gluon-mediated penguin amplitude), and $P_{EW}$ (electroweak penguin amplitude). We ignore the smaller decay amplitudes such as the OZI-suppressed gluonic penguin and the colour-suppressed electroweak penguin amplitudes (although they will be included in our numerical calculations in the next section). We also note which CKM matrix elements govern each of the decay amplitudes. For $T$, $C$, $P_{EW}$ and $b \to s$ $P$ amplitudes these CKM elements are always well-defined. On the other hand, $b \to d$ penguin amplitudes receive contributions from internal $u$, $c$ and $t$ quarks, which involve different combinations of CKM matrix elements. Using the unitarity of the CKM matrix, this amplitude can always be written in terms of a piece proportional to $V_{tb}V_{td}^*$ and another piece proportional to either $V_{ub}V_{ud}^*$ or $V_{cb}V_{cd}^*$. However, for most decays of interest, this second piece can always be absorbed into a $T$ or $C$ amplitude. Thus, the $b \to d$ penguin amplitudes can usually be thought of as effectively governed by $V_{tb}V_{td}^*$ (there is one exception, noted below). Note also that the final-state mesons in the list below can be in the ground state or in an excited state.

- $B_c^- \to J/\psi D^{*-} (b \to c \bar{c} d)$. There are four contributing decay amplitudes: $T$ and $C$ ($V_{cb}V_{cd}^*$), and $P$ and $P_{EW}$ ($V_{tb}V_{td}^*$). $T$ and $P$ are kinematically similar, as are $C$ and $P_{EW}$. Thus, TP's arise from the interference of $T$ and $P_{EW}$ or $C$ and $P$.

- $B^- \to \rho^0 K^{*-}, \omega K^{*-} (b \to u \bar{u} s)$. There are 4 contributing decay amplitudes:
to this decay. In this case, the interference of $T$ and $P_{EW}$ or $C$ and $P$.

- $\bar{B}^0_d \to K^{*0}\rho^0, K^{*0}\omega$. This is more complicated. There are 3 contributing amplitudes: $P$ and $P_{EW}$ ($V_{tb}V_{ts}^*$) arise from the quark-level decay $b \to d\bar{d}s$, while $C$ ($V_{ub}V_{us}^*$) comes from $b \to u\bar{u}s$. The TP comes from the interference of $C$ and $P$.

- $B^- \to \rho^-\rho^0, \rho^-\omega$. This is the most complicated decay. Here there are 4 contributing amplitudes. $T$ and $C$ ($V_{ub}V_{us}^*$) correspond to the quark-level decay $b \to u\bar{u}d$, while $P$ and $P_{EW}$ ($V_{tb}V_{td}^*$) come from both $b \to u\bar{u}d$ and $b \to d\bar{d}d$. However, the penguin decays $b \to u\bar{u}d$ and $b \to d\bar{d}d$ are kinematically different. The TP arises mainly from the interference of $T$ and the $b \to d\bar{d}d$ $P$, but other interferences can also contribute.

- $\bar{B}^0_d \to \rho^0\omega$. There can be a TP in this decay due to the fact that $\rho^0$ and $\omega$ have different flavour wavefunctions: $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}, \omega = (u\bar{u} + d\bar{d})/\sqrt{2}$. There are several contributions: $C$ ($b \to u\bar{u}d: V_{ub}V_{ud}^*$), $P$ ($b \to d\bar{d}d: V_{tb}V_{td}^*$) and $P_{EW}$ ($b \to u\bar{u}d$ and $b \to d\bar{d}d$: $V_{tb}V_{td}^*$). The TP is due mainly to $C$–$P$ interference.

- $\bar{B}^0_\phi \to \phi K^{*0} (b \to s\bar{s}d)$. This is a pure penguin decay, and there are 2 contributing amplitudes: $P$ and $P_{EW}$. In this case, the TP arises from the interference of the $V_{tb}V_{td}^*$ and $V_{ub}V_{ud}^*$ pieces of these two $b \to d$ penguin amplitudes.

Some of these decays have been studied previously: except for the $B^-_c$ decays, the other decays in the SM, with the final-state vector mesons in the ground state, have been examined in Ref. [3].

In addition, there is another class of $B \to V_1V_2$ decays, not considered in earlier calculations, which can potentially yield triple-product correlations:

- $B^- \to D^{*0}K^{*-} (b \to c\bar{s}f)$ receives contributions from $T$ and $C$, while $B^- \to \bar{D}^{*0}K^{*-} (b \to u\bar{c}s)$ is due to $C$ alone.

- $B^- \to D^{*0}\rho^- (b \to c\bar{u}d)$ receives contributions from $T$ and $C$, while $B^- \to \bar{D}^{*0}\rho^- (b \to uc\bar{d})$ is due to $C$ alone.

- $B^-_c \to \bar{D}^{*0}D_s^{*-} (b \to u\bar{c}s)$ receives contributions from $T$ and $C$, while $B^-_c \to D^{*0}D_s^{*-} (b \to c\bar{u}s)$ is due to $C$ alone.

- $B^-_c \to \bar{D}^{*0}D^{*-} (b \to u\bar{c}d)$ receives contributions from $T$ and $C$, while $B^-_c \to D^{*0}D^{*-} (b \to c\bar{u}d)$ is due to $C$ alone.

The main decay modes of the $D^{*0}$ are $D^{0}\pi^0$ (60%) and $D^{0}\gamma$ (40%). Similarly, the $\bar{D}^{*0}$ decays to $\bar{D}^{0}\pi^0$ and $\bar{D}^{0}\gamma$. Therefore, if we consider a final state to which both $D^{0}$ and $\bar{D}^{0}$ can decay, the two amplitudes in each of the items above can contribute to this decay. In this case, the interference of $T (b \to c\bar{u}f, f = d, s)$ and $C (b \to u\bar{c}f)$
will lead to a TP. (These types of interferences are similar to those proposed for the extraction of the CP phase \( \gamma \).)

In Sec. 3, we will provide estimates of the expected size of the TP’s within the SM for decays which have not been studied previously, namely charmless \( \bar{B}^0_d \), \( B^- \) and \( \bar{B}^0_s \) decays to final-state excited vector mesons, \( B^- \) decays, and \( B^- \) and \( B^- \) decays to final states which include \( D^0 \) and \( \bar{D}^0 \) mesons. Estimates of TP’s for charmless \( \bar{B}^0_d \) and \( B^- \) decays to ground-state mesons have already been given in Ref. [3] and will not be repeated here. We will, however, provide general arguments of why TP’s with ground-state mesons in the SM are small.

### 2.4 Mixing-induced triple products

Finally, there is one more possibility which must be examined. A decay such as \( \bar{B}^0_d \rightarrow D^{*+}D^{*-} \) is not expected to yield a triple-product correlation because, while the transition \( \bar{B}^0_d \rightarrow D^{*+} \) is allowed, \( \bar{B}^0_d \rightarrow D^{*+} \) is not. That is, not both of the amplitudes in Eq. (14) are present. However, the missing amplitude can be generated via \( \bar{B}^0_d - B^0_d \) mixing: \( \bar{B}^0_d \rightarrow B^0_d \rightarrow D^{*-} \). Thus, one might wonder whether this can lead to a TP. In addition, even if a TP is expected in \( B^0 \rightarrow V_1 V_2 \), if \( \bar{B}^0 \) can also decay to \( V_1 V_2 \), the TP may be modified in time due to \( B^0 - \bar{B}^0 \) mixing. In this subsection, we investigate these possibilities, which were first examined by Valencia in Ref. [2].

In the presence of \( B^0 - \bar{B}^0 \) mixing, the states \( B^0 \) and \( \bar{B}^0 \) can be written as a function of time as follows:

\[
B^0(t) = e^{-i(M - i\beta)^t} \left[ \cos \left( \frac{\Delta m t}{2} \right) B^0 - i e^{2i\phi_M} \sin \left( \frac{\Delta m t}{2} \right) \bar{B}^0 \right],
\]

\[
\bar{B}^0(t) = e^{-i(M - i\beta)^t} \left[ -i e^{2i\phi_M} \sin \left( \frac{\Delta m t}{2} \right) B^0 + \cos \left( \frac{\Delta m t}{2} \right) \bar{B}^0 \right],
\]

where \( \phi_M \) is the weak phase in \( B^0 - \bar{B}^0 \) mixing [\( \phi_M = \beta \) (0) for \( B^0 = B^0_d \) (\( B^0_s \))]. Following Eq. (14), we define [2]

\[
A(B^0 \rightarrow V_1 V_2) = a_1 s + b_1 d + ic_1 p
\]

\[
A(\bar{B}^0 \rightarrow V_1 V_2) = a_2 s + b_2 d + ic_2 p
\]

\[
A(\bar{B}^0 \rightarrow \bar{V}_1 \bar{V}_2) = \bar{a}_1 s + \bar{b}_1 d - i\bar{c}_1 p
\]

\[
A(\bar{B}^0 \rightarrow V_1 \bar{V}_2) = \bar{a}_2 s + \bar{b}_2 d - i\bar{c}_2 p,
\]

where \( s, d \) and \( p \) are defined as in Eq. (14): \( s \equiv \varepsilon_1^* \cdot \varepsilon_2^*; \ d \equiv (p \cdot \varepsilon_1^*) (p \cdot \varepsilon_2^* )/m_B^2; \)

\[
p \equiv \epsilon_{\mu
u\rho\sigma} p^\mu q^\nu \varepsilon_1^{*\mu} \varepsilon_2^{*\nu}/m_B^2.
\]

In the above, the barred amplitudes are obtained from the corresponding unbarred ones by changing the sign of the weak phases. We can then write

\[
M \equiv A(B^0(t) \rightarrow V_1 V_2) = e^{-i(M - i\beta)^t} [as + bd + icp],
\]

(23)
with
\[ a = a_1 \cos \left( \frac{\Delta m t}{2} \right) - i e^{-2i\phi_M} \sin \left( \frac{\Delta m t}{2} \right) a_2 , \]
\[ b = b_1 \cos \left( \frac{\Delta m t}{2} \right) - i e^{-2i\phi_M} \sin \left( \frac{\Delta m t}{2} \right) b_2 , \]
\[ c = c_1 \cos \left( \frac{\Delta m t}{2} \right) - i e^{-2i\phi_M} \sin \left( \frac{\Delta m t}{2} \right) c_2 . \] (24)

Similarly,
\[ \bar{M} \equiv A(B^0(t) \rightarrow \bar{V}_1\bar{V}_2) = e^{-i(M-\frac{i1}{2})t} \left[ \bar{a}s + \bar{b}d - i\bar{c}p \right] , \] (25)

with
\[ \bar{a} = \bar{a}_1 \cos \left( \frac{\Delta m t}{2} \right) - i e^{2i\phi_M} \sin \left( \frac{\Delta m t}{2} \right) \bar{a}_2 , \]
\[ \bar{b} = \bar{b}_1 \cos \left( \frac{\Delta m t}{2} \right) - i e^{2i\phi_M} \sin \left( \frac{\Delta m t}{2} \right) \bar{b}_2 , \]
\[ \bar{c} = \bar{c}_1 \cos \left( \frac{\Delta m t}{2} \right) - i e^{2i\phi_M} \sin \left( \frac{\Delta m t}{2} \right) \bar{c}_2 . \] (26)

Now, since we are interested in TP’s, we will consider only the \( a-c \) interference terms in \(|M|^2\) and \(|\bar{M}|^2\) (the conclusions will be identical for \( b-c \) interference) \[6\].

The T-violating term which interests us is found in the sum of \(|M|^2\) and \(|\bar{M}|^2\) \[2\]:
\[ |M|^2_{ac} + |\bar{M}|^2_{ac} \sim \text{Im}(a\,c^*) - \text{Im}(\bar{a}\,\bar{c}^*) \]
\[ = \cos^2 \left( \frac{\Delta m t}{2} \right) \text{Im}(a_1 c_1^* - \bar{a}_1 \bar{c}_1^*) + \sin^2 \left( \frac{\Delta m t}{2} \right) \text{Im}(a_2 c_2^* - \bar{a}_2 \bar{c}_2^*) \]
\[ + \sin \left( \frac{\Delta m t}{2} \right) \cos \left( \frac{\Delta m t}{2} \right) \text{Re} \left[ e^{-2i\phi_M} a_2 c_1^* - e^{2i\phi_M} \bar{a}_2 \bar{c}_1^* \right. \]
\[ \left. - e^{2i\phi_M} a_1 c_2^* + e^{-2i\phi_M} \bar{a}_1 \bar{c}_2^* \right] . \] (27)

The first term above is nonzero only if there is a TP in \( B^0 \rightarrow V_1 V_2 \), and describes how this TP evolves in time (note that it is the only term which does not vanish at \( t = 0 \)). Similarly, the second term, which is generated due to \( B^0-\bar{B}^0 \) mixing, describes the time evolution of the TP in \( \bar{B}^0 \rightarrow V_1 V_2 \). Note that if the final state is self-conjugate, \( \bar{V}_1\bar{V}_2 = V_1 V_2 \), we have [see Eq. (22)]
\[ \bar{a}_2 = a_1 \ , \quad \bar{a}_1 = a_2 \ , \quad \bar{b}_2 = b_1 \ , \quad \bar{b}_1 = b_2 \ , \quad \bar{c}_2 = -c_1 \ , \quad \bar{c}_1 = -c_2 . \] (28)

In this case, the first two terms of Eq. (27) add, and the third term vanishes, so that the TP in \( B^0 \rightarrow V_1 V_2 \) is independent of time.

Now consider the third term in Eq. (27). This is the term which can potentially generate a TP due to \( B^0-\bar{B}^0 \) mixing even if the TP in \( B^0 \rightarrow V_1 V_2 \) is absent. Perhaps
the easiest way to see what is happening here is to explicitly write the amplitudes $a_1, a_2, \text{etc.}$ as in Eq. (29):

$$a_1 = \sum_i a_{1i} e^{i\phi_{i1}} e^{i\delta_{i1}}, \quad \bar{a}_1 = \sum_i a_{1i} e^{-i\phi_{i1}} e^{i\delta_{i1}},$$

$$a_2 = \sum_i a_{2i} e^{i\phi_{i2}} e^{i\delta_{i2}}, \quad \bar{a}_2 = \sum_i a_{2i} e^{-i\phi_{i2}} e^{i\delta_{i2}},$$

$$c_1 = \sum_i c_{1i} e^{i\phi_{i1}} e^{i\delta_{i1}}, \quad \bar{c}_1 = \sum_i c_{1i} e^{-i\phi_{i1}} e^{i\delta_{i1}},$$

$$c_2 = \sum_i c_{2i} e^{i\phi_{i2}} e^{i\delta_{i2}}, \quad \bar{c}_2 = \sum_i c_{2i} e^{-i\phi_{i2}} e^{i\delta_{i2}}. \quad (29)$$

Then the third term can be written as

$$(\sin \Delta mt) \sum_{i,j} \left[ a_{2i} c_{1j} \sin(\phi_{i2}^a - \phi_{j1}^c - 2\phi_M) \sin(\delta_{i1}^a - \delta_{j2}^c) - a_{1i} c_{2j} \sin(\phi_{i1}^a - \phi_{j2}^c + 2\phi_M) \sin(\delta_{i1}^a - \delta_{j2}^c) \right]. \quad (30)$$

There are several points to be discussed here. It is indeed possible to generate a T-violating triple product via $B^0 - \bar{B}^0$ mixing even if the TP in $B^0 \to V_1 V_2$ is absent (note that we disagree with Ref. [2] on this point). However, unlike TP’s generated directly [e.g. Eq. (11)], these mixing-induced TP’s are similar to direct CP asymmetries in that they vanish when the strong-phase differences vanish. Mathematically, the reason for this can be traced to the factor of $i$ in the expression for the time-dependent $B^0$ and $\bar{B}^0$ states [Eq. (21)]. But this can also be understood physically. As mentioned above, if the transition $B^0 \to V_1$ is allowed, but $B^0 \to V_2$ is not, there will be no TP. From Eq. (30), it appears that one can generate a TP through $B^0 - \bar{B}^0$ mixing if $B^0 \to V_2$ is allowed. However, as we have stressed several times, TP’s are kinematical CP-violating effects. That is, we do not expect to generate any TP’s when the kinematics of the two amplitudes are the same. Thus, the TP will still vanish if $B^0 \to V_2$ is kinematically identical to $B^0 \to V_1$. Since the kinematics are related in part to the strong phases, it is not surprising that mixing-induced TP’s vanish when the strong-phase differences vanish.

In fact, this point can be quantified. Suppose the final state $V_1 V_2$ is self-conjugate, in which case the amplitudes satisfy the relations in Eq. (28). It is then straightforward to show that the TP asymmetry described by Eq. (30) vanishes! Thus, for example, even when $B^0_d - \bar{B}^0_d$ mixing is taken into account, one can never generate a TP in the decay $B^0_d \to D^{*+} D^{-}$, mentioned at the beginning of this subsection, because the final state is self-conjugate. (That is, the transition $B^0_d \to D^{*-}$ is kinematically identical to $\bar{B}^0_d \to D^{*+}$, so that $B^0_d - \bar{B}^0_d$ mixing cannot lead to a TP.)

In light of this, we can now elaborate the conditions for generating a TP via $B^0 - \bar{B}^0$ mixing: (i) the final state $V_1 V_2$ must be one to which both $B^0$ and $\bar{B}^0$ can decay, and (ii) it must not be self-conjugate. Decays for which mixing can generate
a TP include $B_d^0 \rightarrow D^{*-}D'$, $B_s^0 \rightarrow K^*+K'$, $B_d^0 \rightarrow D^{*+}\rho^-$ \[15\], etc., where $D'$ and $K^*$ are excited states.

Still, as noted above, this class of TP's is very similar to direct CP asymmetries in that both involve the quantity $\sin \phi \sin \delta$ [see Eq. (2)]. Thus, compared to direct CP asymmetries, we do not get additional information from these TP's. For this reason we will not consider them further.

3 Experimental Prospects

In the previous section, we found several $B$ decays which are predicted to exhibit triple-product correlations in the SM. The relevant question now is: what are the prospects for detecting such TP's experimentally? There are several issues here. What are the experimental signals for TP's? For a given decay, what is the branching ratio, and what is the expected size of the TP? In this section, we provide answers to these questions.

3.1 Experimental Signals

In order to obtain experimental information from $B \rightarrow V_1V_2$, it is necessary to perform an angular analysis. For this purpose, it is useful to use the linear polarization basis. In this basis, one decomposes the decay amplitude into components in which the polarizations of the final-state vector mesons are either longitudinal ($A_0$), or transverse to their directions of motion and parallel ($A_\parallel$) or perpendicular ($A_\perp$) to one another. One writes \[4, 5\]

$$M = A_0 \varepsilon_1^L \cdot \varepsilon_2^L - \frac{1}{\sqrt{2}} A_\parallel \varepsilon_1^T \cdot \varepsilon_2^T - i \frac{1}{\sqrt{2}} A_\perp \varepsilon_1^T \times \varepsilon_2^T \cdot \hat{p},$$

(31)

where $\hat{p}$ is the unit vector along the direction of motion of $V_2$ in the rest frame of $V_1$, $\varepsilon_i^L = \varepsilon_i^T \cdot \hat{p}$, and $\varepsilon_i^T = \varepsilon_i^L - \varepsilon_i^L \hat{p}$. $A_0, A_\parallel, A_\perp$ are related to $a, b$ and $c$ of Eq. (4) via

$$A_\parallel = \sqrt{2}a, \quad A_0 = -ax - \frac{m_1m_2}{m_B^2}b(x^2 - 1), \quad A_\perp = 2\sqrt{2} \frac{m_1m_2}{m_B^2}c\sqrt{x^2 - 1},$$

(32)

where $x = k_1 \cdot k_2/(m_1m_2)$. (A popular alternative basis is to express the decay amplitude in terms of helicity amplitudes $A_\lambda$, where $\lambda = 1, 0, -1$ \[3, 4\]. The helicity amplitudes can be written in terms of the linear polarization amplitudes via $A_{\pm 1} = (A_\parallel \pm A_\perp)/\sqrt{2}$, with $A_0$ the same in both bases.)

The angular distribution of the decay depends on the decay products of $V_1$ and $V_2$. For the case where both vector mesons decay into pseudoscalars, i.e. $V_1 \rightarrow P_1P_1'$, $V_2 \rightarrow P_2P_2'$, one has \[11, 12\]

$$\frac{d\Gamma}{d\cos \theta_1 d\cos \theta_2 d\phi} = N \left( |A_0|^2 \cos^2 \theta_1 \cos^2 \theta_2 + \frac{|A_\perp|^2}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \phi \right.$$

$$\left. + \frac{|A_\parallel|^2}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \phi \right)$$

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\[ + \frac{|A_\perp|^2}{2} \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \phi + \frac{\text{Re}(A_0 A_0^\ast)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \phi \]
\[ - \frac{\text{Im}(A_\perp A_0^\ast)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \phi - \frac{\text{Im}(A_\perp A_\parallel^\ast)}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi \], \tag{33}

where \( \theta_1 \) (\( \theta_2 \)) is the angle between the directions of motion of the \( P_1 \) \( (P_2) \) in the \( V_1 \) \( (V_2) \) rest frame and the \( V_1 \) \( (V_2) \) in the \( B \) rest frame, and \( \phi \) is the angle between the normals to the planes defined by \( P_1 P'_1 \) and \( P_2 P'_2 \) in the \( B \) rest frame. (For other decays of the \( V_1 \) and \( V_2 \) (e.g. into \( e^+ e^- \), \( P \gamma \) or three pseudoscalars), one will obtain a different angular distribution, see Refs. \[3, 4, 5\].)

Now, the above angular distribution already appears in most of the papers in Refs. \[3, 4, 5\]. We repeat it here to emphasize the following point. The terms which are of interest to us are those proportional to \( \text{Im}(A_\perp A_0^\ast) \) and \( \text{Im}(A_\perp A_\parallel^\ast) \). From Eq. (32) above, these are related to \( \text{Im}(ac^\ast) \) and \( \text{Im}(bc^\ast) \). In other words, these two terms in Eq. (33) are precisely the triple-product correlations. Thus, by performing a full angular analysis, one can in fact obtain the TP’s.

Note that these terms are often referred to as CP-violating in Refs. \[3, 4, 5\]. However, as we have already noted, this is not accurate – they are really T-odd terms, and it is only by adding the TP’s in \( |M|^2 \) and \( |\overline{M}|^2 \) that one can obtain a truly T-violating effect.

### 3.2 Sizes of Triple Products – Factorization

In this subsection we estimate the sizes of the triple products within factorization. We concentrate on those TP’s which are generated directly (i.e. not via mixing) because they do not vanish when the strong phases vanish. Note: from the point of view of searching for new physics, the precise predicted value of a given TP is not particularly important. What is relevant is the question of whether that TP is measurable (\( > 5\% \)) or not. If it is expected to be small within the SM, then the measurement of a large value for that TP would point clearly towards the presence of physics beyond the SM. As we will see, most TP’s are expected to be very small in the SM.

As mentioned in the previous subsection, the presence of the terms \( \text{Im}(A_\perp A_0^\ast) \) or \( \text{Im}(A_\perp A_\parallel^\ast) \) in the angular distribution will indicate a nonzero TP asymmetry. In order to estimate the size of T violation in a given decay, we define the following T-odd quantities:

\[ A_T^{(1)} \equiv \frac{\text{Im}(A_\perp A_0^\ast)}{A_0^2 + A_\parallel^2 + A_\perp^2}, \quad A_T^{(2)} \equiv \frac{\text{Im}(A_\perp A_\parallel^\ast)}{A_0^2 + A_\parallel^2 + A_\perp^2}. \tag{34} \]

The corresponding quantities for the charge-conjugate process, \( \overline{A}_T^{(1)} \) and \( \overline{A}_T^{(2)} \), are defined similarly. The comparison of the TP asymmetries in a decay and in its
corresponding CP-conjugate process will give a measure of the true T-violating asymmetry for that decay.

In order to calculate the TP quantities defined above, we first need the values of \(a\), \(b\) and \(c\) in Eq. (20). These are obtained using estimates of form factors, along with the latest Wilson coefficients (including strong phases), decay constants and CKM matrix elements. From these, we then calculate the linear polarization amplitudes of Eq. (32) to obtain the branching ratios (BR’s) and T-odd TP’s. As indicated above, in order to get true T-violating TP’s, we need to calculate \(\bar{a}\), \(\bar{b}\) and \(\bar{c}\) as well.

3.3 SM Triple Products in \(B\) Decays to Ground-State Vector Mesons

As mentioned earlier, the sizes of TP’s for \(B\) decays to ground-state vector mesons in the SM have been already estimated for many modes \(^3\), and have been found to be small. Without performing any actual calculations, we can understand this result by making some general observations. First, as noted at the end of Sec. 2.3, if \(V_1\) and \(V_2\) are the same particle, the TP vanishes. Similarly, if \(V_1 = V_2\) in some symmetry limit, then there is again no TP in this limit since \(a\), \(b\) and \(c\) of Eq. (20) are all proportional to \(X + Y\), and there is no relative phase. Thus, in this case the size of the TP is related to the size of the symmetry breaking. We call this *flavour suppression*. For example, we expect the TP in \(B^- \rightarrow \rho^- \rho^0\) to be tiny because the \(\rho^-\) and \(\rho^0\) are related by isospin. Similarly, the TP in \(B \rightarrow K^* \rho\) is expected to be small since the \(K^*\) and \(\rho\) are related by flavour \(SU(3)\) symmetry.

Second, consider \(B \rightarrow V_1V_2\) decays in which the final vector mesons are light: \(m_{1,2} \ll m_B\). Neglecting terms of \(O(m_{1,2}^2/m_B^2)\), we can then approximate \(E_1 \sim E_2 \sim |\vec{k}| = E = m_B/2\). Then, using Eq. (32), we have for the various linear polarization amplitudes

\[
A_0 \approx -(2a + b) \frac{E^2}{m_1 m_2}, \\
A_\parallel \approx \sqrt{2}a, \\
A_\perp \approx \sqrt{2}c. 
\]

Naively, since \(a\), \(b\) and \(c\) are expected to be of the same order, this then implies that

\[
\frac{A_{\parallel,\perp}}{A_0} \sim \frac{m_1 m_2}{E^2}, 
\]

We therefore expect the TP effects in \(A_T^{(1)}\) to be suppressed by \(m_1 m_2/E^2\), while those in \(A_T^{(2)}\) are even smaller: \(A_T^{(2)} \sim (m_1 m_2/E^2)^2\). This behavior can be understood rather simply. The form of the TP term in Eq. (1) requires that both \(V_1\) and \(V_2\) be transversely polarized. However, the polarization vector for transverse polarization...
is suppressed relative to that for longitudinal polarization by $m/E$. This leads to the above suppression factors for TP’s in $B \to V_1 V_2$. (This to be contrasted with T violation in $\Lambda_b \to F_1 V$ decays, where $F_1$ is a spin-1/2 baryon and $V$ a vector meson. Here the final-state $V$ can be longitudinally polarized, so that a T-violating asymmetry can be produced without any suppression by powers of $m_V/m_{\Lambda_b}$.

Assuming factorization and using the expressions for $a$, $b$ and $c$ given in Eq. (20), from Eq. (35) we obtain

\[ A_0 = A_{0X} + A_{0Y} , \]

\[ A_{0X} \approx 2 m_B m_1 g_V X \left[ (A_1^{(2)} - A_2^{(2)}) + \frac{m_2}{m_B} (A_1^{(2)} + A_2^{(2)}) \right] \frac{E^2}{m_1 m_2} , \]

\[ A_{0Y} \approx 2 m_B m_2 g_V Y \left[ (A_1^{(1)} - A_2^{(1)}) + \frac{m_1}{m_B} (A_1^{(1)} + A_2^{(1)}) \right] \frac{E^2}{m_1 m_2} , \]

\[ A_\parallel \approx -\sqrt{2} m_B \left[ m_1 g_{V_1} \left( 1 + \frac{m_2}{m_B} \right) A_1^{(2)} (m_1^2) X + m_2 g_{V_2} \left( 1 + \frac{m_1}{m_B} \right) A_1^{(1)} (m_2^2) Y \right] , \]

\[ A_\perp \approx -\sqrt{2} m_B \left[ m_1 g_{V_1} \left( 1 - \frac{m_2}{m_B} \right) V^{(2)} (m_1^2) X + m_2 g_{V_2} \left( 1 - \frac{m_1}{m_B} \right) V^{(1)} (m_2^2) Y \right] . \]  

(37)

The above equations exhibit the same suppression of the $A_{\parallel,\perp}$ amplitudes relative to $A_0$ as that given in Eq. (36). However, from the expression for $A_0$, one sees that if we have $A_1 \approx A_2$ then the suppression of $A_{\parallel,\perp}$ relative to $A_0$ will be diluted from $m_1 m_2 / E^2$ to simply $m/E$ (where $m = m_1$ or $m_2$). In fact, this may well be the case: if the dominant contribution to the form factors comes from soft gluon interactions between the quarks inside the mesons then one has the following relations between the vector form factors:

\[ A_1 = A_2 + O(m/E) , \quad V = A_1 + O(m/E) . \]  

(38)

On the other hand, in the presence of hard gluon interactions, the relations in the above equation no longer hold. Still, even for this scenario, the form factors $A_1$ and $A_2$ have been found numerically to be very similar.

The main point here is that all triple-product correlations in charmless $B \to V_1 V_2$ decays are suppressed by some power of $m/E$. We call this mass suppression.

Thus, all TP’s in $B \to V_1 V_2$ decays suffer from a combination of flavour and mass suppression. These suppressions are most severe for the $B$ decays to ground-state vector mesons which have been studied previously. For example, even for interfering amplitudes of similar size, the flavour suppression from isospin symmetry will produce a negligible TP. In fact, the earlier calculations find that most SM TP’s in $B$ decays with final-state ground state mesons are small, less than 5%, and we largely agree with these results (we have checked these estimates using updated values for the CKM parameters $\rho$ and $\eta$, the Wilson coefficients, and various form factors).
There is one point on which we disagree with previous analyses. Some of the papers in Ref. [3] find large $O(10\%)$ TP’s for $B$ decays involving ground-state vector mesons. In general, these large TP’s correspond to decays with final-state $\omega$ mesons. In principle, such decays might avoid flavour suppression since the $\omega$ is an $SU(3)$ singlet. However, since the mass, form factors and decay constants for the $\omega$ are very similar to those of the $\rho$, flavour suppression is expected to be present even for decays involving $\omega$’s, and this is what we find through explicit calculation. We therefore conclude that the TP’s for $B$ decays to ground-state vector mesons are all small in the SM.

3.4 SM Triple Products: Radially-Excited Vector Mesons and other New Decays

Based on the discussion above, it is clear that the measurement of TP asymmetries will be facilitated if one uses the heaviest final-state vector mesons possible. This will minimize the mass suppression of the TP’s. For example, one could consider decays of the $B_c$ mesons using $b \to c$ transitions. For charmless $B$ decays it might be more useful to consider decays to radially-excited states of the vector mesons $\rho$, $K^*$ or $\phi$. As shown in Ref. [19], such transitions can have branching ratios which may be larger than, or of the same size as, decays to the ground state configurations. This is easily understood in the context of factorization. Consider the decay $B \to V_1'V_2$ where $V_1'$ is the radially-excited meson and $V_{1,2}$ are the ground-state mesons. We assume that both $V_1'$ and $V_2$ are light mesons. The amplitude for the process is then

$$\langle V_2(\bar{p}) V_1'(-\bar{p}) | T | B \rangle = \langle V_1'(-\bar{p}) | J_{\mu} | B \rangle \langle V_2(\bar{p}) | J_{\mu}^B | 0 \rangle ,$$

(39)

where $J_{1,2}$ are currents that occur in the effective Hamiltonian [Eq. (12)]. The transition matrix element for the hadronic decay can then be written in terms of $B \to V_1'$ form factors and the $V_2$ meson decay constant. The form factors can be expressed as overlap integrals of the $B$ and the $V_1'$ meson wavefunctions. When $V_1'$ is a light meson, with a mass much smaller than that of the $B$ meson, the main contributions to the overlap integrals come from the high-momentum components, or the tail, of the meson wavefunctions. For a radially-excited meson $V_1'$, which has more high-momentum components (i.e. a longer tail), the overlap integrals will be enhanced compared to those of the ground-state meson $V_1$. As a consequence, the $B \to V_1'$ form factors are likely to be increased compared to those of $B \to V_1$. This would then translate into a larger branching ratio for $B \to V_1'V_2$ than $B \to V_1V_2$. In Ref. [19], this effect was demonstrated explicitly with various confining potentials for the mesons.

Another advantage of using radially-excited mesons is that the TP asymmetries will not be suppressed by flavour symmetries. For instance, although the TP in $B^- \to \rho^- \rho^0$ is tiny due to isospin symmetry, the TP asymmetry in $B^- \to \rho^- \rho^0$,
where $\rho^{0'}$ is a radially-excited state, does not suffer a corresponding flavour suppression.

In this section we provide estimates of the sizes of TP’s involving radially-excited vector mesons, as well as several other new modes not considered before.

Like any CP-violating signal, TP’s will be largest when the two interfering amplitudes are of comparable size. Also, TP asymmetries will be maximized when the largest decay amplitude is involved. (If not, then the denominator will be larger than the numerator, thereby decreasing the asymmetry.)

With these general ideas in hand, we use the framework of factorization to estimate numerically the size of the TP’s within the SM. In particular, we use factorization to estimate the matrix elements of the various operators that appear in the effective Hamiltonian [Eq. (12)]. The values of the various Wilson coefficients are given by

$$
c_{1f} = -0.185, \quad c_{2f} = 1.082,
$$

$$
c_{3} = 0.014, \quad c_{4} = -0.035, \quad c_{5} = 0.010, \quad c_{6} = -0.041,
$$

$$
c_{7} = -1.24 \times 10^{-5}, \quad c_{8} = 3.77 \times 10^{-4}, \quad c_{9} = -0.010, \quad c_{10} = 2.06 \times 10^{-3}
$$

Note that the tree operators in Eq. (12) can generate via rescattering the $u$- and $c$-quark penguin pieces, proportional to $V_{ub}V_{uq}^*$ and $V_{cb}V_{cq}^*$ ($q = d, s$) respectively.

The coefficients associated with the short-distance rescattering effects are given by

$$
c_{3,5} = -c_{4,6}/N_c = P_s^i/N_c, \quad c_{7,9} = P_e^i, \quad c_{8,10} = 0, \quad i = u, c,
$$

where $N_c$ is the number of colours. The leading contributions to $P_{s,e}^i$ are given by

$$
P_s^i = (\frac{8}{\pi^2})c_2(\frac{4}{3} + G(m_i, \mu, q^2)) \quad \text{and} \quad P_e^i = (\frac{8}{\pi^2})(N_c c_1 + c_2)(\frac{4}{3} + G(m_i, \mu, q^2))
$$

in which the function $G(m, \mu, q^2)$ takes the form

$$
G(m, \mu, q^2) = 4 \int_0^1 x(1-x)\ln \frac{m^2 - x(1-x)q^2}{\mu^2} \, dx,
$$

where $q$ is the momentum carried by the virtual gluon in the penguin diagram. In our calculations, we use a value of $q^2 = m_b^2/2$.

In Sec. 2.3, we presented a list of decays which can yield triple-product asymmetries in the SM. Below we provide estimates of the expected size of these TP’s for decays which have not been previously examined. Because we expect that $A_T^{(2)} < A_T^{(1)}$, the estimates are given for $A_T^{(1)}$ only. We also give the expected branching ratios for both a given decay and its CP-conjugate decay. From these numbers one can easily obtain the direct CP asymmetry expected in the decay.

The amplitudes for the various decays depend on combinations of Wilson coefficients, $a_i$, where $a_i = c_i + c_{i+1}/N_c$ for $i$ odd and $a_i = c_i + c_{i-1}/N_c$ for $i$ even. The terms described by the various $a_i$’s can be associated with the different decay topologies introduced earlier. The terms proportional to $a_2$ and $a_1$ are, respectively,
the colour-allowed and colour-suppressed tree amplitudes $T$ and $C$. The term proportional to $a_4$ is the colour-allowed penguin amplitude, $P$, while the terms $a_3$ and $a_5$ represent the OZI-suppressed amplitudes. Finally, the dominant electroweak penguin $P_{\text{EW}}$ is represented by term proportional to $a_9$, while $a_7$ and $a_{10}$ are additional small electroweak-penguin amplitudes.

In addition, as mentioned earlier, certain amplitudes can potentially receive large nonfactorizable corrections. In the past it has been customary to take into account such nonfactorizable corrections by treating $N_c$ as a free parameter. In our calculations we adopt the same prescription, and provide estimates of TP’s with two standard choices: $N_c = 3$ (pure factorization) and $N_c = \infty$ (large nonfactorizable effects included). Now, it is known that $N_c \to \infty$ is inconsistent with data on $B \to PP$ and $B \to PV$ decays [20]. In fact, for charmless $B$ decays the effective $N_c$ may be different for operators with different chiral structure in the effective Hamiltonian [20]. In this paper we are dealing with $VV$ final states, and the effective value of $N_c$ which is applicable here will only be known when there are enough experimental data to carry out a detailed analysis. Our choice of $N_c = \infty$ can be considered as an extreme case of nonfactorizable effects. Although it will probably turn out to be inconsistent with data on nonleptonic $B \to VV$ decays, our purpose here is simply to be most conservative (perhaps excessively so) in our estimation of nonfactorizable effects. Realistically, we expect the true value of the TP to lie somewhere between its values for $N_c = 3$ and $N_c = \infty$. In most cases, this allows us to clearly establish that the TP in question is expected to be small in the SM, even in the presence of unrealistically large nonfactorizable effects. (A more complete discussion of nonfactorizable effects can be found in the next subsection.)

Note that since $c_1$ and $c_2$ have opposite signs, the colour-suppressed tree amplitude, described by $a_1 = c_1 + c_2/N_c$, is further suppressed because of an accidental cancellation between its Wilson coefficients. The effect of this suppression depends strongly on the value taken for $N_c$. For $N_c = 3$, one obtains $a_1 = 0.176$, while for $N_c = \infty$ we have $a_1 = -0.185$, so that even the sign of the colour-suppressed amplitude is different in the two cases. For the OZI-suppressed terms $a_{3,5}$ the difference can also be quite dramatic: for $N_c = 3$ we have $a_3 = 0.002$ and $a_5 = -0.0036$, while for $N_c = \infty$ we have $a_3 = 0.014$ and $a_5 = 0.01$. In this latter case the OZI terms can be of the same order as the colour-allowed penguins. This fact will be important in understanding the numbers for the T-violating asymmetries given below. Whether or not the OZI terms are important in $B$ decays is a matter of debate, and several tests to find evidence for their presence in $B$ decays have been discussed recently [21].
3.4.1 $B_c^- \to J/\psi D^*$

The amplitude for this process is given by

$$ A[B_c^- \to J/\psi D^*] = \frac{G_F}{\sqrt{2}} [X P_{D^*} + Y P_{J/\psi}], $$

(43)

with

$$ X = V_{cb} V_{cs}^* a_2 - \sum_{q=u,c,t} V_{q} V_{qd}^* (a_4^q + a_5^q), $$

$$ Y = V_{cb} V_{cs}^* a_1 - \sum_{q=u,c,t} V_{q} V_{qd}^* (a_3^q + a_4^q + a_5^q), $$

$$ P_{D^*} = m_{D^*} g_{D^*} e_D^{*\mu} \langle J/\psi | \bar{c} \gamma_\mu (1 - \gamma_5) b | B_c^- \rangle, $$

$$ P_{J/\psi} = m_{J/\psi} g_{J/\psi} e_J^{*\mu} \langle D^* | \bar{d} \gamma_\mu (1 - \gamma_5) b | B_c^- \rangle. $$

(44)

Note that the main difference between the two amplitudes $X$ and $Y$ is simply the fact that some Wilson coefficients are multiplied by $1/N_c$ in one amplitude, while they are multiplied by 1 in the other. This is the case for most of the decays we consider.

We can now calculate $a$, $b$ and $c$ from Eq. (20) with the identification $V_1 = D^*$ and $V_2 = J/\psi$. For numerical results we will use the following inputs: the CKM parameters are $\rho = 0.17$ and $\eta = 0.39$; the decay constants are $g_{J/\psi} = 0.405$ GeV [22] and $g_{D^*} = 0.237$ GeV [23]; the form factors for $B_c^- \to J/\psi$ transitions are given by $A_1^{(J/\psi)}(m_{D^*}^2) = 0.73$, $A_2^{(J/\psi)}(m_{D^*}^2) = 0.75$ and $V^{(J/\psi)}(m_{D^*}^2) = 1.1$, while for $B_c^- \to D^*$ they are $A_1^{(D^*)}(m_{J/\psi}^2) = 0.70$, $A_2^{(D^*)}(m_{J/\psi}^2) = 1.2$ and $V^{(D^*)}(m_{J/\psi}^2) = 2.1$ [24].

| Process       | BR     | $A_T^{(1)}$ (%) | $N_c$ |
|---------------|--------|----------------|-------|
| $B_c^- \to J/\psi D^*$− | 3.48 (3.45) $\times 10^{-3}$ | 0.011 (−0.03) | 3    |
| $B_c^- \to J/\psi D^*$− | 3.02 (3.0) $\times 10^{-3}$ | 0.11 (−0.06) | $\infty$ |

Table 1: Branching ratios (BR) and triple-product asymmetries ($A_T^{(1)}$) for $B_c^- \to J/\psi D^*$−, for $N_c = 3$ (pure factorization) and $N_c = \infty$ (large nonfactorizable effects). The results for the CP-conjugate process are given in parentheses.

We present our results in Table 1 including the branching ratio and the T-odd triple product $A_T^{(1)}$ for both process and CP-conjugate process. Regardless of the value taken for $N_c$, the T-violating asymmetries are expected to be tiny. This is understandable because, while T-violation comes from $C–P$ or $T–P_{EW}(P_{OZI})$ interference, there is a large colour-allowed tree contribution to the amplitude. Thus, the denominator of $A_T^{(1)}$ [Eq. (34)] is always much larger than the numerator, resulting
in a small TP asymmetry. As expected, the T-violation is larger for \( N_c = \infty \) because of enhanced OZI terms, but the asymmetries are still too small to be measurable.

### 3.4.2 \( B^- \to \rho^0 K^{*-}, \rho^0 K^{*-}, \omega' K^{*-}, \omega K^{*-} \)

The amplitude for \( B \) decays to radially-excited vector mesons has a similar form to that for \( B \) decays to ground-state vector mesons. We therefore start with the decay amplitude for ground-state mesons. For \( V = \rho^0 \) or \( \omega \), this amplitude can be written as

\[
A[B^- \to K^{*-}V] = \frac{G_F}{\sqrt{2}} [X_V P_K^V + Y_V P_K^{V*}],
\tag{45}
\]

with

\[
X_\rho = X_\omega = V_{ub} V_{us}^* q_2 - \sum_{q=u,c,t} V_{qb} V_{qs}^* (a_4^q + a_{10}^q),
\]

\[
Y_\rho = V_{ub} V_{us}^* q_1 - \sum_{q=u,c,t} V_{qb} V_{qs}^* \left( \frac{3}{2} a_4^q + \frac{3}{2} a_5^q \right),
\]

\[
Y_\omega = V_{ub} V_{us}^* q_1 - \sum_{q=u,c,t} V_{qb} V_{qs}^* \left( 2a_3^q + 2a_5^q + \frac{1}{2} a_7^q + \frac{1}{2} a_9^q \right),
\]

\[
P_{K^*}^{\rho^0} = m_{K^*} g_{K^*} \varepsilon_{K^*}^{*\mu} \langle \rho^0 | \bar{u} \gamma_\mu (1 - \gamma_5)b | B^- \rangle,
\]

\[
P_{K^*}^{\omega} = m_{\omega} g_{\omega} \varepsilon_{K^*}^{*\mu} \langle K^{*-} | \bar{s} \gamma_\mu (1 - \gamma_5)b | B^- \rangle,
\]

\[
P_{K^*}^{\rho_0} = m_{\rho_0} g_{\rho_0} \varepsilon_{K^*}^{*\mu} \langle K^{*-} | \bar{s} \gamma_\mu (1 - \gamma_5)b | B^- \rangle,
\]

\[
P_{K^*}^{\omega} = m_{\omega} g_{\omega} \varepsilon_{K^*}^{*\mu} \langle K^{*-} | \bar{s} \gamma_\mu (1 - \gamma_5)b | B^- \rangle.
\tag{46}
\]

The decays which interest us involve \( \rho(1450), \omega(1420) \) and/or \( K^*(1410) \) in the final state. Quark-model predictions classify these states as radially-excited states of the \( \rho, \omega \) and the \( K^* \). Henceforth we will label these states as \( \rho^0r, \omega' \) and \( K^{*-}r \). The amplitude for a decay involving an excited final state can be obtained simply from Eqs. \(45\) and \(46\) by replacing \( \rho^0, \omega \) and/or \( K^* \) by \( \rho^0r, \omega' \) and/or \( K^{*-}r \).

To calculate the TP asymmetries for these decays, we need the form factors for transitions of a \( B \)-meson to such radially-excited states. These are obtained by assuming a linear confining potential for the light mesons, and the wavefunction \( N e^{-p^2/2m_F^2} \) with the fermi momentum \( p_F = 0.3 \) (0.5) \( \text{GeV} \) for \( B_d^0 (B_s^0) \) mesons \[19\]. The results for the form factors are

\[
\frac{A_{\rho^0}^\prime (q^2 = m_{\rho^0}^2)}{A_{\rho^0} (q^2 = m_{\rho^0}^2)} = 1.38,
\]

\[
\frac{A_{\omega}^\prime (q^2 = m_{\omega}^2)}{A_{\omega} (q^2 = m_{\omega}^2)} = 1.2,
\]

\[21\]
\[
\frac{V^{\rho^\prime}(q^2 = m_{\rho^\prime}^2)}{V^{\rho}(q^2 = m_{\rho}^2)} = 1.66. \tag{47}
\]

We assume the same values for the ratios of form factors for \( B \to \omega^\prime \) and \( B \to K^{*\prime} \) transitions. This is reasonable since any SU(3) breaking effects should cancel in the ratios of form factors. Note that Eq. (47) is given in terms of the form factors for ground-state mesons. Identifying \( V_1 = K^* \) and \( V_2 = \rho^\prime \) for \( B^- \to \rho^0 \) these are \( A_1^{(\rho)}(m_{K^*}^2) = 0.26, A_2^{(\rho)}(m_{K^*}^2) = 0.24 \) and \( V_1^{(\rho)}(m_{K^*}^2) = 0.31 \), while for \( B^- \to K^{*-} \) transitions the form factors are given by \( A_1^{(K^*)}(m_{\rho}^2) = 0.36, A_2^{(K^*)}(m_{\rho}^2) = 0.32 \) and \( V^{(K^*)}(m_{\rho}^2) = 0.44 \). We assume that the form factors for \( B \to \omega \) are the same as for \( B \to \rho^0 \).

An estimate of the triple products for these decays also involves the decay constants of the radially-excited vector mesons. These are found to be similar to the decay constants for the ground state mesons. We take \( f_{\rho} = f_{\rho^\prime} = f_{\omega} = f_{\omega^\prime} = 0.190 \) GeV and \( f_{K^*} = f_{K^{*\prime}} = 0.214 \) GeV.

There are two advantages to using a final state with one radially-excited vector meson: the \( m_V/m_B \) suppression is reduced, and there is no flavour symmetry relating the final-state vector mesons which would result in a further suppression. One could also consider final states containing two radially-excited states. In this case, however, the suppression due to flavour symmetry would apply again and for this reason we do not examine these decays here. One could also consider final states with radially-excited and orbitally-excited vector mesons. This would require the calculation of form factors for transitions of \( B \) mesons to orbitally-excited states. This interesting possibility is beyond the scope of this work and will be investigated elsewhere.

As we will see, for final states with a single radially-excited state measurable TP asymmetries may be possible, and this should encourage a more thorough study of TP’s in \( B \) decays to radially and orbitally-excited vector mesons.

Based on Eq. (47) we can make the following observations. For the decays \( B^- \to \rho^0K^{*-} \) and \( B^- \to \rho^0K^{*-\prime} \), the interfering amplitudes are of unequal size, which further suppresses the TP asymmetry. For \( B^- \to \omega^*K^*- \) and \( B^- \to \omega^*K^{*-\prime} \), there is a possibility of an enhanced OZI contribution for \( N_c = \infty \). This could interfere with the colour-allowed tree which is suppressed by CKM factors.

The results of Table 2 are consistent with these observations. All TP’s are expected to be very small with two exceptions: for \( N_c = \infty \) a measurable TP asymmetry is predicted for \( B^- \to \omega^*K^{*-} \) (8%) and possibly \( B^- \to \omega^*K^{*-\prime} \) (4%). These TP asymmetries are generated mainly from \( T-P_{OZI} \) interference (the \( C-P \) interference is small). Thus, it is possible that nonfactorizable effects can generate large TP asymmetries for these decays to radially-excited states. We should stress, however, that this is far from guaranteed – the \( N_c = \infty \) prescription is just an estimate. The true nonfactorizable effects could be much smaller than this. Conversely, for other decays, it appears unlikely that such effects can lead to measurable TP’s – these decays are therefore excellent places to search for new physics.
| Process          | BR              | \(A_T^{(1)}\) % | \(N_c\) |
|------------------|-----------------|-----------------|--------|
| \(B^- \to \rho^0 K^*^-\) | 11.8 (7.9) \times 10^{-6} | 0.53 (-0.13) | 3      |
| \(B^- \to \rho^0 K^*^-\) | 13.1 (10.0) \times 10^{-6} | 1.2 (0.45) | ∞      |
| \(B^- \to \rho^0 K^*^-\) | 10 (7.0) \times 10^{-6} | -0.87 (0.21) | 3      |
| \(B^- \to \rho^0 K^*^-\) | 11.1 (9.1) \times 10^{-6} | -1.9 (-0.68) | ∞      |
| \(B^- \to \omega' K^*^-\) | 10 (6) \times 10^{-6} | 0.16 (-0.29) | 3      |
| \(B^- \to \omega K^*^-\) | 2.3 (3.6) \times 10^{-6} | -10.6 (-5.2) | ∞      |
| \(B^- \to \omega K^*^-\) | 7.7 (4.7) \times 10^{-6} | -0.29 (0.51) | 3      |
| \(B^- \to \omega K^*^-\) | 6.5 (8.8) \times 10^{-6} | 5.4 (3.0) | ∞      |

Table 2: Branching ratios (BR) and triple-product asymmetries (\(A_T^{(1)}\)) for \(B^- \to \rho^0 K^*^-, \rho^0 K^*^-, \omega' K^-*\) and \(\omega K^*^-\), for \(N_c = 3\) (pure factorization) and \(N_c = \infty\) (large nonfactorizable effects). The results for the CP-conjugate process are given in parentheses.

\[\begin{align*}
3.4.3 \quad \bar{B}_d^0 & \to K^{*0} \rho^0, K^{*0} \rho^0, K^{*0} \omega, K^{*0} \omega \\
\end{align*}\]

As before, we first present the amplitude for the ground-state mesons. For the decays \(\bar{B}_d^0 \to K^{*0}V\) with \(V = \rho^0\) or \(\omega\), the amplitude is given by

\[
A[\bar{B}_d^0 \to K^{*0}V] = \frac{G_F}{\sqrt{2}}[X_V P_{K^*} V + Y_V P_{K^*} V],
\]

with

\[
X_{\rho} = -X_{\omega} = \sum_{q=u,c,t} V_{qb} V_{qs}^* (a_4^q + a_{10}^q),
\]

\[
Y_\rho = V_{ub} V_{us}^* a_1 - \sum_{q=u,c,t} V_{qb} V_{qs}^* \left( \frac{3}{2} a_7^q + \frac{3}{2} a_9^q \right),
\]

\[
Y_\omega = V_{ub} V_{us}^* a_1 - \sum_{q=u,c,t} V_{qb} V_{qs}^* \left( 2 a_3^q + 2 a_5^q + \frac{1}{2} a_2^q + \frac{1}{2} a_6^q \right),
\]

\[
P_{K^*}^\rho = m_{K^*} g_{K^*} \langle K^* | \bar{d} \gamma_\mu (1 - \gamma_5) b | \bar{B}_d^0 \rangle,
\]

\[
P_{K^*}^\omega = m_{K^*} g_{K^*} \langle K^* | \bar{d} \gamma_\mu (1 - \gamma_5) b | \bar{B}_d^0 \rangle,
\]

\[
P_{K^*}^\rho = m_{K^*} g_{K^*} \langle \omega | \bar{d} \gamma_\mu (1 - \gamma_5) b | \bar{B}_d^0 \rangle,
\]

\[
P_{K^*}^\omega = m_{K^*} g_{K^*} \langle \omega | \bar{d} \gamma_\mu (1 - \gamma_5) b | \bar{B}_d^0 \rangle.
\]

Again, to obtain the amplitude for a decay involving an excited final state, one simply replaces \(\rho^0, \omega\) and/or \(K^*\) by \(\rho^{0'}, \omega'\) and/or \(K^{*'}\) in the above equation.

Above, we have used the flavour wavefunction \(\rho^0 = (\bar{u}u - \bar{d}d)/\sqrt{2}\) and \(\omega = (\bar{u}u + \bar{d}d)/\sqrt{2}\), and similarly for the excited states. The effect of the relative sign in
the flavour wavefunctions of the \( \rho^0 \) and \( \omega \) has been included in the definition of the phases \( X_{\rho,\omega} \) and \( Y_{\rho,\omega} \), and so does not have to be included in the \( B \to \rho^0(\omega) \) form factors. We shall follow this convention in subsequent decays involving \( \rho^0 \) and \( \omega \).

The triple-product correlations for these decays are presented in Table 3. The TP’s are from \( C-P \) interference, which is small, so that we do not find large TP’s in this case. It is only in the decay \( \overline{B}_d^0 \to K^{*0} \omega \) that a marginally measurable TP \((\sim 3-4\%)\) might be found. However this again relies on large nonfactorizable effects, which may or may not be present.

| Process | BR                  | \( A_{T}^{(1)} \) % | \( N_c \) |
|---------|---------------------|---------------------|-----------|
| \( B_d^0 \to K^{*0}\rho^0 \) | 11.3 \((10.9) \times 10^{-6}\) | 0.05 \((-0.61)\) | 3         |
| \( B_d^0 \to K^{*0}\rho^0 \) | 13.5 \((14.6) \times 10^{-6}\) | 0.64 \((-0.14)\) | \( \infty \) |
| \( B_d^0 \to K^{*0}\rho^0 \) | 9.7 \((9.3) \times 10^{-6}\) | -0.08 \((1.0)\) | 3         |
| \( B_d^0 \to K^{*0}\rho^0 \) | 12.0 \((12.9) \times 10^{-6}\) | -1.0 \((0.23)\) | \( \infty \) |
| \( B_d^0 \to K^{*0} \omega' \) | 9.1 \((8.6) \times 10^{-6}\) | -0.14 \((-0.52)\) | 3         |
| \( B_d^0 \to K^{*0} \omega' \) | 1.0 \((0.7) \times 10^{-6}\) | 3.8 \((3.13)\) | \( \infty \) |
| \( B_d^0 \to K^{*0} \omega \) | 7.0 \((10) \times 10^{-6}\) | 0.26 \((0.65)\) | 3         |
| \( B_d^0 \to K^{*0} \omega' \) | 5.5 \((4.9) \times 10^{-6}\) | -1.0 \((-0.73)\) | \( \infty \) |

Table 3: Branching ratios (BR) and triple-product asymmetries \( A_{T}^{(1)} \) for \( \overline{B}_d^0 \to K^{*0} \rho^0, K^{*0} \rho^0, K^{*0} \omega' \) and \( K^{*0} \omega \), for \( N_c = 3 \) (pure factorization) and \( N_c = \infty \) (large nonfactorizable effects). The results for the CP-conjugate process are given in parentheses.

### 3.4.4 \( B^- \to \rho^- \rho^0, \rho^- \rho^0, \rho^- \omega', \rho^- \omega \)

The amplitude for the ground-state decay \( B^- \to \rho^- V \) with \( V = \rho^0 \) or \( \omega \) is given by

\[
A[B^- \to \rho^- V] = \frac{G_F}{\sqrt{2}} \left[ X_V P_{\rho^-}^V + Y_V P_{\rho^-}^\omega \right],
\]

with

\[
X_\rho = X_\omega = V_{ub} V_{ud}^* a_2 - \sum_{q=u,c,t} V_{qb} V_{qd}^* (a_4^q + a_{10}^q),
\]

\[
Y_\rho = V_{ub} V_{ud}^* a_1 - \sum_{q=u,c,t} V_{qb} V_{qd}^* \left( -a_4^q + \frac{3}{2} a_7^q + \frac{3}{2} a_9^q + \frac{1}{2} a_{10}^q \right),
\]

\[
Y_\omega = V_{ub} V_{ud}^* a_1 - \sum_{q=u,c,t} V_{qb} V_{qd}^* \left( a_4^q + 2 a_3^q + 2 a_9^q + \frac{1}{2} a_7^q + \frac{1}{2} a_{10}^q - \frac{1}{2} a_{10}^q \right),
\]

\[
P_{\rho^-}^{\rho^0} = m_\rho g_{\rho^-} \varepsilon_{\rho^-}^{\rho^0} \langle \rho^0 | \bar{u} \gamma_\mu (1 - \gamma_5) b | B^- \rangle,
\]

\[
P_{\rho^-}^{\omega} = m_\rho g_{\rho^-} \varepsilon_{\rho^-}^{\omega} \langle \rho^0 | \bar{u} \gamma_\mu (1 - \gamma_5) b | B^- \rangle.
\]

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\[
P_{\rho^-} = \frac{1}{\sqrt{2}} m_{\rho} g_{\rho} \varepsilon_{\rho}^{\mu} \left\langle \rho^- \right| \bar{d} \gamma_{\mu} (1 - \gamma_5) b \left| B^- \right\rangle ,
\]
\[
P_{\omega^-} = m_{\omega} g_{\omega} \varepsilon_{\omega}^{\mu} \left\langle \omega \right| \bar{u} \gamma_{\mu} (1 - \gamma_5) b \left| B^- \right\rangle ,
\]
\[
P_{\rho^-} = \frac{1}{\sqrt{2}} m_{\omega} g_{\omega} \varepsilon_{\omega}^{\mu} \left\langle \rho^- \right| \bar{d} \gamma_{\mu} (1 - \gamma_5) b \left| B^- \right\rangle .
\] (51)

The amplitude for a decay involving an excited final state is obtained by replacing \(\rho^0\) and/or \(\omega\) by \(\rho^0'\) and/or \(\omega'\).

In this case, the TP’s arise mostly from \(T - P\) interference, neither of which is CKM-suppressed. However, the penguin amplitude \(P\) is only about 4% of the tree amplitude \(T\), so that the maximum TP asymmetry turns out to be small, \(\sim 1\%\).

The results of our calculations are presented in Table 4. Note that, if one assumes isospin conservation, flavour suppression leads to an identically vanishing TP for the ground-state decay \(B^- \rightarrow \rho^- \rho^0\).

| Process | BR $|10^{-6}|$ | \(A_T^{(1)}\)% | \(N_c\) |
|---------|-------------|----------------|-------|
| \(B^- \rightarrow \rho^- \rho^0\) | 29.3 (29.7) | 0.51 (0.33) | 3 |
| \(B^- \rightarrow \rho^- \rho^0\) | 18.3 (18.7) | 0.66 (0.41) | \(\infty\) |
| \(B^- \rightarrow \rho^- \rho^0\) | 23.8 (23.5) | -0.63 (-0.42) | 3 |
| \(B^- \rightarrow \rho^- \rho^0\) | 12 (11.7) | -1.0 (-0.65) | \(\infty\) |
| \(B^- \rightarrow \rho^- \omega\) | 27.4 (32.7) | -0.58 (-0.38) | 3 |
| \(B^- \rightarrow \rho^- \omega\) | 17.9 (19.8) | 0.1 (0.04) | \(\infty\) |
| \(B^- \rightarrow \rho^- \omega\) | 15.8 (20.0) | -1.16 (-0.72) | 3 |
| \(B^- \rightarrow \rho^- \omega\) | 4.0 (4.2) | 0.49 (0.22) | \(\infty\) |

Table 4: Branching ratios (BR) and triple-product asymmetries \(A_T^{(1)}\) for \(B^- \rightarrow \rho^- \rho^0\), \(\rho^- \rho^0\), \(\rho^- \omega\) and \(\rho^- \omega\), for \(N_c = 3\) (pure factorization) and \(N_c = \infty\) (large non-factorizable effects). The results for the CP-conjugate process are given in parentheses.

### 3.4.5 \(\overline{B}_{d}^0 \rightarrow \rho^0 \omega'\), \(\rho^0 \omega\)

The amplitude for the ground-state decay \(\overline{B}_{d}^0 \rightarrow \rho^0 \omega\) is given by

\[
A[\overline{B}_{d}^0 \rightarrow \rho^0 \omega] = \frac{G_F}{\sqrt{2}} [XP_{\rho^0} + YP_{\omega}] ,
\] (52)

with

\[
X = V_{ub} \bar{V}_{ud} a_1 + \sum_{q=u,c,t} V_{qb} V_{qd}^* \left( a_4^q + \frac{3}{2} a_7^q + \frac{3}{2} a_9^q \right) ,
\]

\[
y = V_{ub} \bar{V}_{ud} a_1 + \sum_{q=u,c,t} V_{qb} V_{qd}^* \left( a_4^q + \frac{3}{2} a_7^q + \frac{3}{2} a_9^q \right) .
\]
\[ Y = -V_{ub} V_{ud}^{*} a_1 + \sum_{q=u,c,t} V_{qb} V_{qd}^{*} \left( a_4^q + 2a_3^q + 2a_2^q + \frac{1}{2}a_1^q + \frac{1}{2}a_0^q + \frac{1}{2}a_{10}^q \right), \]

\[ P_{\rho} = \frac{1}{\sqrt{\rho}} m_{\rho} g_{\rho} \langle \omega \mid \bar{d} \gamma_\mu (1 - \gamma_5) b \mid B_d^0 \rangle, \]

\[ P_{\omega} = \frac{1}{\sqrt{\omega}} m_{\omega} g_{\omega} \langle \rho^0 \mid \bar{d} \gamma_\mu (1 - \gamma_5) b \mid B_d^0 \rangle. \] (53)

The TP’s in this case are due principally to $C-P$ interference. Neither of these amplitudes is CKM-suppressed, and they are of similar size. As a consequence, while the TP’s for the ground-state decay are small, due to flavour and mass suppressions, we find measurable asymmetries for decays with radially-excited vector mesons in the final state (see Table 5). Unfortunately, the branching ratios for all these decays are expected to be in the $10^{-7}$ range. Furthermore, the TP asymmetry changes sign as $N_c$ is varied from 3 to $\infty$. Thus, there is again no guarantee of a large TP – it is possible that nonfactorizable effects are such that the actual TP is small.

| Process | BR | $A_T^{(1)}$ | $N_c$ |
|---------|----|-------------|------|
| $B_d^0 \to \rho^0 \omega'$ | $4.5 (1.8) \times 10^{-7}$ | 6.2 (10.2) | 3 |
| $B_d^0 \to \rho^0 \omega'$ | $0.5 (0.52) \times 10^{-7}$ | $-17.2 (-11.1)$ | $\infty$ |
| $B_d^0 \to \rho^0 \omega$ | $6 (3.3) \times 10^{-7}$ | 6.0 (6.3) | 3 |
| $B_d^0 \to \rho^0 \omega$ | $2.45 (2.08) \times 10^{-7}$ | $-4.0 (-3.2)$ | $\infty$ |

Table 5: Branching ratios (BR) and triple-product asymmetries ($A_T^{(1)}$) for $B_d^0 \to \rho^0 \omega'$ and $\rho^0 \omega$, for $N_c = 3$ (pure factorization) and $N_c = \infty$ (large nonfactorizable effects). The results for the CP-conjugate process are given in parentheses.

### 3.4.6 $B_d^0 \to \phi' K^*$, $\phi K^*$

We are also interested in the pure $b \to d$ penguin decay $B_d^0 \to \phi^{(*)} K^{(*)}$, where $\phi'$ corresponds to the radially-excited state $\phi(1680)$. For the form factors for these decays, we obtain

\[ \frac{A_{1}^{\phi'}(q^2 = m_{K^{*}}^2)}{A_{1}^{\phi}(q^2 = m_{K^{*}}^2)} = 1.5, \]

\[ \frac{A_{2}^{\phi'}(q^2 = m_{K^{*}}^2)}{A_{2}^{\phi}(q^2 = m_{K^{*}}^2)} = 1.35, \]

\[ \frac{V_{1}^{\phi'}(q^2 = m_{K^{*}}^2)}{V_{1}^{\phi}(q^2 = m_{K^{*}}^2)} = 1.8. \] (54)
The form factors for the ground-state transitions can be found in Ref. [25]. For the
decay constants we use $f_\phi = f_\phi' = 0.237$ GeV.

The amplitude for the ground-state decay $B_0^s \to \phi K^{*0}$ is given by

$$A[B_0^s \to \phi K^{*0}] = \frac{G_F}{\sqrt{2}} [XP_{K^*} + YP_\phi] ,$$

with

$$X = - \sum_{q=u,c,t} V_{qb}V_{qd}^* \left( a_4^q - \frac{1}{2}a_{10}^q \right) ,$$

$$Y = - \sum_{q=u,c,t} V_{qb}V_{qd}^* \left( a_3^q + a_5^q - \frac{1}{2}a_7^q - \frac{1}{2}a_9^q \right) ,$$

$$P_{K^*} = m_{K^*}g_{K^*} \langle \phi | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_0^s \rangle ,$$

$$P_\phi = m_\phi g_\phi \langle \phi | \bar{d} \gamma_\mu (1 - \gamma_5) b | B_0^s \rangle .$$

In this case, the TP's arise mainly from $P - P_{EW}(P_{OZI}$ interference. We find a
marginally measurable TP asymmetry only for $B_0^s \to \phi K^{*'}$ with $N_c = \infty$, i.e. with
enhanced OZI terms, and the branching ratio for this decay is tiny, $O(10^{-8})$. Our
results are presented in Table 6.

| Process          | BR         | $A_T^{(1)} \%$ | $N_c$ |
|------------------|------------|----------------|------|
| $B_0^s \to \phi' K^*$ | $11 (5.5) \times 10^{-7}$ | $-0.17 (0.21)$ | 3    |
| $B_0^s \to \phi' K^*$ | $2.8 (1.3) \times 10^{-7}$ | $-1.14 (1.51)$ | $\infty$ |
| $B_0^s \to \phi K^{*'}$ | $6.3 (3.1) \times 10^{-7}$ | $0.23 (-0.31)$ | 3    |
| $B_0^s \to \phi K^{*'}$ | $0.15 (0.06) \times 10^{-7}$ | $16.8 (-22.9)$ | $\infty$ |

Table 6: Branching ratios (BR) and triple-product asymmetries ($A_T^{(1)}$) for $B_0^s \to \phi K^*$ and $\phi K^{*'}$, for $N_c = 3$ (pure factorization) and $N_c = \infty$ (large nonfactorizable
effects). The results for the CP-conjugate process are given in parentheses.

### 3.4.7 $B^- \to D^{*0} K^{*-}, \bar{D}^{*0} K^{*+}$; $B^- \to D^{*0} \rho^-, \bar{D}^{*0} \rho^-$

We now examine $B^-$ decays in which the final-state $D^{*0}$ or $\bar{D}^{*0}$ mesons subsequently
decay to the same state. We assume that $D^{0*} \to D^{0} \pi^0$ and $D^{0*} \to D^{0} \pi^0$, with
$D^{0}, \bar{D}^{0} \to f$, where $f = K^+ \pi^-$ or $f = \pi^+ \pi^-$.  

Consider first $B^- \to D^{*0} K^{*-}$ and $B^- \to \bar{D}^{*0} K^{*+}$. The decay amplitude is given by

$$A[B^- \to K^{*-}f] = \frac{G_F}{\sqrt{2}} [XP_{K^*} + YP_{D^*}] ,$$

where

- $X = - \sum_{q=u,c,t} V_{qb}V_{qd}^* \left( a_4^q - \frac{1}{2}a_{10}^q \right) ,$
- $Y = - \sum_{q=u,c,t} V_{qb}V_{qd}^* \left( a_3^q + a_5^q - \frac{1}{2}a_7^q - \frac{1}{2}a_9^q \right) ,$
- $P_{K^*} = m_{K^*}g_{K^*} \langle \phi | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_0^s \rangle ,$
- $P_\phi = m_\phi g_\phi \langle \phi | \bar{d} \gamma_\mu (1 - \gamma_5) b | B_0^s \rangle .$$
with

\[
X = V_{cb}V_{us}^* a_2 \sqrt{B_1 \sqrt{B_2}},
\]

\[
Y = V_{cb}V_{us}^* a_1 \sqrt{B_1 \sqrt{B_2}} + V_{ub}V_{cs}^* a_1 \sqrt{B_1 \sqrt{B_2}},
\]

\[
P_{K^*} = m_K g_K \varepsilon_{K^*}^* \left< D^0 \big| \bar{c} \gamma_\mu (1 - \gamma_5) b \big| B^- \right>,
\]

\[
P_{D^*} = m_D g_D \varepsilon_{D^*}^* \left< K^{*-} \big| \bar{s} \gamma_\mu (1 - \gamma_5) b \big| B^- \right>,
\]

(58)

where \(B_1\) is the branching ratio for \(D^{0*} \to D^0\pi^0\) and \(B_2(B^*_2)\) are the branching ratios for \(D^0(D^0) \to f\). The values for the form factors for \(B \to D^*\) transitions are \(A_1(m_{K^*}^2) = V(m_{K^*}^2) = 0.783\) and \(A_2(m_{K^*}^2) = 0.772\) \([20]\). (Due to heavy quark symmetry, the form factors have very similar values.) Now, the relative strong phase between the amplitudes \(D^0 \to f\) and \(D^0 \to f\) is unknown. In our estimates, we choose this phase to be zero. This assumption is not unreasonable since these transitions go through colour-allowed tree decays, so that any strong phases generated by nonfactorizable effects are likely to be small.

| Process | BR | \(A_T^{(1)}\)% | \(N_c\) |
|---------|----|---------------|--------|
| \(B^- \to K^{*-} (f = K^+\pi^-)\) | \(3.5 \times 10^{-6}\) | 0.03 (0.1) | \(3\) |
| \(B^- \to K^{*-} (f = K^+\pi^-)\) | \(5.8 \times 10^{-4}\) | 4.1 (4.1) | \(3\) |
| \(B^- \to K^{*-} (f = \pi^+\pi^-)\) | \(5.8 \times 10^{-7}\) | 0.52 (0.52) | \(3\) |
| \(B^- \to \rho^- (f = K^+\pi^-)\) | \(5.8 \times 10^{-7}\) | 0.56 (0.56) | \(3\) |
| \(B^- \to \rho^- (f = K^+\pi^-)\) | \(5.8 \times 10^{-7}\) | 1.03 (1.03) | \(3\) |
| \(B^- \to \rho^- (f = K^+\pi^-)\) | \(5.8 \times 10^{-7}\) | 0.03 (0.03) | \(3\) |
| \(B^- \to \rho^- (f = \pi^+\pi^-)\) | \(5.8 \times 10^{-7}\) | 0.04 (0.04) | \(3\) |

Table 7: Branching ratios (BR) and triple-product asymmetries \((A_T^{(1)})\) for \(B^- \to D^{*0}K^{*-}\) and \(B^- \to \bar{D}^{*0}K^{*-}\), as well as \(B^- \to D^{*0}\rho^-\) and \(B^- \to \bar{D}^{*0}\rho^-\), for \(N_c = 3\) (pure factorization) and \(N_c = \infty\) (large nonfactorizable effects). It is assumed that \(D^0, \bar{D}^0 \to f\), with \(f = K^+\pi^-\) or \(f = \pi^+\pi^-\). The results for the CP-conjugate process are given in parentheses.

We present our results in Table 4. We find that the T-violating asymmetries may be measurable for the decays with \(f = K^+\pi^-\), but are small for \(f = \pi^+\pi^-\). These results can be understood as follows. The decay \(B^- \to D^{*0}K^{*-}\) is dominated by a colour-allowed tree diagram \((T)\) and involves the CKM matrix elements \(V_{cb}V_{cd}^*\), while \(B^- \to \bar{D}^{*0}K^{*-}\) is colour-suppressed \((C)\) and involves \(V_{ub}V_{cs}^*\). Thus, these two amplitudes are of very different size – the latter is roughly 5% of the former. However, in order to obtain a sizeable TP, it is necessary to have two decay amplitudes of similar magnitudes. This can occur if the decays \(D^0 \to f\) and \(\bar{D}^0 \to f\) are,
respectively, doubly-Cabibbo-suppressed and Cabibbo-allowed, which is the case for $f = K^+\pi^-$. (This is similar to the method for obtaining $\gamma$ proposed in Ref. [14].) Unfortunately, the net branching ratio is small $O(10^{-7})$. On the other hand, for $f = \pi^+\pi^-$, both the $D^0$ and $\bar{D}^0$ decays are singly-Cabibbo-suppressed, so the TP is small.

We now turn to the decays $B^- \to D^{*0}\rho^-$ and $B^- \to \bar{D}^{*0}\rho^-$. The amplitude in this case given by

$$A[B^- \to \rho^- f] = \frac{G_F}{\sqrt{2}}[XP_{\rho^-} + YP_{D^*}],$$

and

$$X = V_{cb}V^*_{ud}a_2\sqrt{B_1\sqrt{B_2}},$$
$$Y = V_{cb}V^*_{ud}a_1\sqrt{B_1\sqrt{B_2}} + V_{ub}V^*_{cd}a_1\sqrt{B_1\sqrt{B_2}},$$
$$P_{\rho^-} = m_{\rho^-}g_{\rho^-}\varepsilon^*_{\rho^-} \left\langle \bar{D}^{*0}\right| \bar{c}\gamma_\mu(1 - \gamma_5)b \left| B^- \right\rangle,$$
$$P_{D^*} = m_{D^*}g_{D^*}\varepsilon^*_{D^*} \left\langle \rho^- \right| \bar{d}\gamma_\mu(1 - \gamma_5)b \left| B^- \right\rangle.$$

In this case, the second decay ($B^- \to \bar{D}^{*0}\rho^-$) is also suppressed relative to the first ($B^- \to D^{*0}\rho^-$). However, here the suppression is much larger than in $B^- \to D^{*0}K^{*-}$, $\bar{D}^{*0}K^{*-}$ decays—in addition to the ratio $C/T$, there is also a suppression due to the ratio of CKM matrix elements, $|V_{ub}V^*_{cd}|/|V_{ub}V^*_{cd}|$. Thus, regardless of the final state $f$ in $D^0$, $\bar{D}^0 \to f$, the two amplitudes remain very different in size, leading to small TP’s. This expectation is borne out in Table 7.

### 3.4.8 $B^- \to \bar{D}^{*0}D_s^{*-}$, $D^{*0}D_s^{*-}$; $B^- \to \bar{D}^{*0}D_s^-$, $D^{*0}D_s^-$

Finally, we consider pairs of $B^-_c$ decays to final states including $D^{*0}$ or $\bar{D}^{*0}$ mesons. Unfortunately, there are no calculations yet of the form factors for $B^-_c \to \bar{D}^{*0}$, $D_s^-$ and $D^{*0}$ transitions. As a result, we can only present “back-of-the-envelope” estimates of the triple products for these decays. (Still, based on our analyses of the previous decays, these estimates are probably reasonably accurate.)

Consider first $B^-_c \to \bar{D}^{*0}D_s^-$, $D^{*0}D_s^-$. The decay $B^-_c \to \bar{D}^{*0}D_s^-$ is dominated by $T$ and involves $V_{ub}V^*_{cs}$, while $B^-_c \to D^{*0}D_s^-$ is governed by $C$ and $V_{ub}V^*_{us}$. The two amplitudes are therefore comparable in size, which naively suggests that one can obtain a measurable TP by using decays such as $D^0, \bar{D}^0 \to \pi^+\pi^-$, which are both singly-Cabibbo-suppressed. However, note that, within factorization, the two $B^-_c$ decay amplitudes are proportional to $f_{B^-_c \to \bar{D}^{*0}}f_{D_s^-}$ and $f_{B^-_c \to D^{*0}}f_{D_s^0}$, which are related by flavour $SU(3)$ symmetry. We therefore expect the TP asymmetries to be small for these decays. However, the TP’s could be measurable if one uses final states involving excited mesons.

The situation is similar for $B^-_c \to \bar{D}^{*0}D_s^-$, $D^{*0}D_s^-$. In this case, the amplitude for the second $B^-_c$ decay is actually larger than the first (by about a factor of 10).
Thus, in order to obtain roughly equal overall amplitudes, one has to use doubly-Cabibbo-suppressed decays such as \( D^0, D^0 \rightarrow K^+\pi^- \). However, even in this case one expects tiny TP asymmetries: the two \( B_{c}^- \) decay amplitudes are proportional to \( f_{B_{c}^- \rightarrow D^{*-0}} f_{D^{*-}} \) and \( f_{B_{c}^- \rightarrow D^{*-0}} f_{D^{*-0}} \), which are related by isospin. The only way to obtain measurable TP’s is if the final states involve excited mesons.

### 3.5 Nonfactorizable effects

In our analysis, we have used factorization to calculate the expected size of triple-product asymmetries in certain \( B \rightarrow V_1 V_2 \) decays. We have included potential nonfactorizable contributions by considering also the case \( N_c = \infty \) in the \( a_i \) (which are combinations of the Wilson coefficients and \( N_c \)). In this subsection, we examine in more detail nonfactorizable effects. In particular, we are interested in establishing which decays are likely to be most (and least) affected by such effects. We also explore the properties of those nonfactorizable effects which can modify the TP predictions. The determination of which TP predictions are the most reliable in turn indicates which decay modes are best to use in the search for new physics.

The most interesting decays are those for which the TP asymmetries are predicted to be very small (or zero) in the framework of factorization within the SM. If it can be established that nonfactorizable effects do not significantly affect these predictions, the measurement of a sizeable nonzero TP asymmetry would clearly signal the presence of new physics. In such decays, within factorization, we can express the various linear polarization amplitudes in the following form:

\[
A_i = R_i [P_1 + P_2 e^{i\phi} e^{i\Delta}] = R_i X ,
\]

where \( i = 0, \parallel \) and \( \perp \). The weak and strong phases are denoted by \( \phi \) and \( \Delta \), respectively, while the \( R_i \) are real numbers that depend on form factors and decay constants. The quantities \( P_{1,2} \) depend on combinations of the Wilson coefficients and the magnitude of the CKM elements, and are therefore real. With the above parametrization, it is clear that there are no TP asymmetries, since all amplitudes have the same phase, i.e. \( \text{Im}[A_0 A^*_1] = \text{Im}[A_\parallel A^*_\perp] \sim \text{Im}[X X^*] = 0 \).

TP asymmetries can potentially be generated in such decays if nonfactorizable effects are present. One possibility is that there are additional contributions, such as annihilation diagrams, which contribute to the decay. In this case, the new amplitude can interfere with the amplitude in Eq. (61) to generate a TP asymmetry. The full decay amplitude then has the general form

\[
A_i = R_i X + R'_i Y ,
\]

where \( X \) and \( Y \) depend differently on the weak and the strong phases. In practice, however, such annihilation contributions are suppressed in the heavy-quark limit. The annihilation terms can be estimated in the framework of QCD factorization [7].
For VV final states, the annihilation terms are not chirally enhanced, unlike PP and PV states. Thus, these contributions are purely power suppressed ($\sim O(1/m_b)$) in the heavy-quark expansion, and are small.

Another class of nonfactorizable effects are those which modify the individual $P_{1,2}$ amplitudes in Eq. (61). The general form of the amplitudes is then

$$A_i = R_i [P_1 (1 + a_i e^{i\alpha_i}) + P_2 (1 + b_i e^{i\beta_i}) e^{i\phi} e^{i\Delta}],$$

(63)

where $\alpha_i$ and $\beta_i$ are strong phases generated by the nonfactorizable effects. Note that if the quantities $a_i$, $b_i$, $\alpha_i$ and $\beta_i$ are the same for all three linear polarization amplitudes, then the TP asymmetries will still vanish, even in the presence of nonfactorizable effects. Thus, it is only nonfactorizable contributions that affect the amplitudes $A_0$, $A_{\parallel}$ and $A_{\perp}$ differently which can generate a TP asymmetry.

One can see explicitly how a TP is generated by nonfactorizable effects by rewriting the $A_i$ in Eq. (63):

$$A_i = R_i X_i + R_i Y_i,$$

(64)

with

$$X_i = (P_1 + P_2 e^{i\phi} e^{i\Delta})(1 + a_i e^{i\alpha_i}),$$

$$Y_i = P_2 (b_i e^{i\beta_i} - a_i e^{i\alpha_i}) e^{i\phi} e^{i\Delta}.$$  

(65)

The interference of $X_i$ with $Y_i$ (specifically, $P_1 - Y_i$ interference) leads to a TP. (Note that the interference of two different $X_i$’s does not lead to a true T-violating effect since $X$, the term containing the weak-phase information, is the same for all three amplitudes.) Thus, not only must the nonfactorizable effects be different for the three $A_i$, but the nonfactorizable corrections to $P_1$ and $P_2$ should also be different. If this were not the case, then the $Y_i$ would vanish.

We have therefore seen that TP’s can be generated by the interference of factorizable and nonfactorizable contributions. This then begs two questions: (i) which amplitudes are most likely to be affected by nonfactorizable effects, and (ii) how big are such effects? The first question is easy to answer: contributions which are suppressed in the factorization framework, such as the colour-suppressed tree amplitude $C$, are likely to receive large nonfactorizable contributions. This was already seen in the previous subsection in which we parametrized nonfactorizable effects by varying $N_c$. The value and the sign of $a_1 = c_1 + c_2/N_c$, which describes $C$, depend strongly on the value chosen for $N_c$. Thus, TP asymmetries that arise from the interference of colour-allowed and colour-suppressed transitions, as happens for several of the decay modes, can be significantly modified by nonfactorizable effects.

The second question is more difficult to answer. Various methods to calculate nonfactorizable effects have been considered recently in the literature, but there is no compelling evidence for the validity of any one approach. For example, in QCD factorization, nonfactorizable effects can be different for the different helicity
states (linear polarization states) \[9\]. However, some of these corrections, such as the hard spectator corrections, are dominated by soft configurations and turn out to be divergent. Hence nothing quantitative can be said about the size of TP asymmetries generated by these nonfactorizable corrections. Still, it should be noted that qualitatively these corrections are quite significant for colour-suppressed amplitudes while the fractional change in colour-allowed amplitudes is proportional to \(\alpha_s(m_b) \sim 0.2\) and is small. It is very likely that the measurement of triple-product asymmetries in \(B \to V_1V_2\) decays will provide useful information about the dynamics of nonleptonic decays.

The conclusion here is that the most reliable predictions for TP’s are for those decays where both \(P_1\) and \(P_2\) are colour allowed. There are many examples of these. For example, the decays \(\bar{B}_d^0 \to D^{*+} D^{*-}\), \(B^- \to D^{*0} D^{*-}\), \(\bar{B}_s^0 \to D_s^{*+} D_s^{*-}\), \(\bar{B}_d^0 \to \rho^+ \rho^-\), \(\bar{B}_s^0 \to K^{*+} \rho^-\), \(B_c^- \to \bar{D}^{*0} \rho^-\) have both colour-allowed tree and colour-allowed weak penguin contributions; \(\bar{B}_d^0 \to K^{*-} \rho^+\), \(\bar{B}_s^0 \to K^{*+} K^{*-}\), \(B_c^- \to D^{*0} K^{*-}\) have both colour-allowed tree and colour-allowed weak penguin contributions; \(\bar{B}_d^0 \to K^{-} \bar{K}^{*0}\), \(B^- \to K^{*-} K^{*-}\), \(B_c^- \to K^{*0} D^{*-}\) are pure colour-allowed weak penguin decays. Within factorization, the TP asymmetries in all of these decays are expected to vanish, even though there are two decay amplitudes \((P_{1,2})\) with a relative weak phase. Since the nonfactorizable effects in these decays are expected to be small, any measurement of large T-violating triple products in these decays will be a clear signal of new physics. (Of course, one can add to this list processes which are dominated by a single amplitude, such as \(B \to J/\psi K^*\) \((b \to c\bar{c}s)\) or \(B \to \phi K^*\) (pure \(b \to s\) penguin), since no TP asymmetries are expected in these decays.)

### 3.6 Discrete Ambiguities

Although most of the triple-product asymmetries are predicted to be small in the decays we have studied, a handful of TP’s may be measurable. What can we learn from them? The answer is that, apart from testing our knowledge of hadronic \(B\) decays, these TP’s can potentially be used to remove an important discrete ambiguity in the measurements of the CP angles of the unitarity triangle.

Within the SM, CP violation is signalled by nonzero values of \(\alpha, \beta\) and \(\gamma\), the three internal angles of the so-called unitarity triangle (UT) \[22\]. By measuring CP-violating rate asymmetries in the \(B\) system, one can obtain the three CP angles \(\alpha, \beta\) and \(\gamma\). Any inconsistency among the angles and sides of the UT indicates the presence of new physics. The standard decays used for obtaining these CP angles are \(B_d^0(t) \to J/\psi K_s\) \((\beta)\), \(B_d^0(t) \to \pi^+ \pi^-\) \((\alpha)\), and \(B^\pm \to DK^\pm\) \((\gamma)\) \[1\]. Unfortunately, these decays only allow the extraction of \(\sin 2\alpha, \sin 2\beta\) and \(\sin^2 \gamma\), which leads to a fourfold ambiguity for each of the angles.

If one assumes that the three angles add up to \(180^\circ\), which holds even in the presence of new physics in \(B_d^0 - \bar{B}_d^0\) mixing \[27\], then most of the values for the angle sets \((\alpha, \beta, \gamma)\) are forbidden. However, one still has a twofold ambiguity in the
construction of the UT [28]. Given that \( \sin 2\beta \) has been measured to be positive, there are two scenarios: (i) if \( \sin(2\alpha) > 0 \), then both UT’s point up, while (ii) if \( \sin(2\alpha) < 0 \), then one UT points up, while the other points down. In either scenario, if one of the solutions is consistent with the SM, while the other is not, then it is necessary to resolve this discrete ambiguity in order to be certain that new physics is present.

Consider now a decay for which the TP is predicted to be large. As noted in the introduction, the TP is proportional to \( \sin \phi \cos \delta \), where \( \phi \) and \( \delta \) are weak and strong phases, respectively. In fact, it is straightforward to show that all TP’s are proportional to the CKM parameter \( \eta \). \( \eta \) measures the height of the UT, so that if \( \eta > 0 \) (< 0), the UT points up (down). Therefore, the sign of the TP can tell us whether the UT points up or down, thus resolving the discrete ambiguity in the second scenario above.

Unfortunately, things are not quite so easy. Once again, one needs to understand well the nonfactorizable effects. For example, consider the decay \( B_0^+ \rightarrow \rho^0 \omega' \) (Table 5). If \( N_c = 3 \) (pure factorization), then the TP asymmetry is predicted to be +8%, while if \( N_c = \infty \) (large nonfactorizable effects included), the asymmetry is −14%. Suppose, then, that an asymmetry of −8% is measured. This could imply one of two things: either (i) there are no nonfactorizable effects, but \( \eta < 0 \), or (ii) nonfactorizable effects are important, and \( \eta > 0 \). Unless one can distinguish theoretically between these two possibilities, the measurement of the TP asymmetry will not tell us whether the UT points up or down. Thus, TP’s can potentially resolve the above discrete ambiguity, but significant theoretical input will probably be required.

4 New Physics

In almost all of the decays we have studied, the triple-product asymmetries are predicted to be very small, so that these are good places to search for physics beyond the SM. In this section, we examine in more detail the kinds of new physics which can generate such TP’s.

Consider \( B \rightarrow V_1V_2 \) decays which have only one kinematical amplitude in the standard model (or for which one such amplitude dominates). Because there is only a single amplitude, no T-violating TP asymmetry can be produced. Now, as we saw earlier, the effective SM Hamiltonian involves only a left-handed \( b \)-quark, and so contains only \((V-A) \times (V-A)\) and \((V-A) \times (V+A)\) operators. However, some types of new physics can couple to the right-handed \( b \)-quark, producing \((V+A) \times (V-A)\) and/or \((V+A) \times (V+A)\) operators. These new-physics operators will produce different kinematical amplitudes, leading to different phases for \( a, b \) and \( c \), and giving rise to a TP asymmetry.

This can be seen explicitly as follows. Suppose that only \( B \rightarrow V_2 \) transitions occur in the decay \( B \rightarrow V_1V_2 \). The SM contribution to such a decay, \( A_{SM} \), is given
in Eq. (16), repeated here for convenience:

\[ A_{SM} \sim \sum \langle V_1 | \mathcal{O} | 0 \rangle \langle V_2 | \mathcal{O}' | B \rangle = X \varepsilon_1^{*\mu} \langle V_2 | q' \gamma_\mu (1 - \gamma_5) b | B \rangle . \] (66)

Recall that all weak-phase information is contained in the factor \( X \). Now assume that there is a new-physics contribution with a \((V + A) \times (V - A)\) or \((V + A) \times (V + A)\) structure. The new amplitude is then

\[ A_{NP} \sim \sum \langle V_1 | \mathcal{O}_{NP} | 0 \rangle \langle V_2 | \mathcal{O}_{NP}' | B \rangle = Y \varepsilon_1^{*\mu} \langle V_2 | q' \gamma_\mu (1 + \gamma_5) b | B \rangle , \] (67)

where \( Y \) contains the new-physics weak phase information. In the presence of the new-physics contribution the amplitudes \( a \), \( b \) and \( c \) of Eq. (4) can now be written as

\[
\begin{align*}
    a &= -m_B m_1 g_{V_1} A_1^{(2)}(m_1^2) \left( 1 + \frac{m_2}{m_B} \right) \left[ X - Y \right], \\
    b &= 2m_B m_1 g_{V_1} A_2^{(2)}(m_1^2) \left( 1 + \frac{m_2}{m_B} \right)^{-1} \left[ X - Y \right], \\
    c &= -m_B m_1 g_{V_1} V^{(2)}(m_1^2) \left( 1 + \frac{m_2}{m_B} \right)^{-1} \left[ X + Y \right].
\end{align*}
\] (68)

Thus, when the new-physics contributions are included, \( \text{Im}(ac^*) \) and \( \text{Im}(bc^*) \) are nonzero. That is, a TP asymmetry will arise due to the interference of \( X \) and \( Y \). Furthermore, since the SM and new-physics operators have different structures, there is no flavour symmetry relating the two contributions, i.e. the phases of \( a \), \( b \) and \( c \) are different even if \( V_1 = V_2 \). That is, in the presence of new physics there is no suppression of the TP asymmetry due to flavour symmetries. Note also that these TP asymmetries can be generated by the interference of two colour-allowed amplitudes (most TP’s in the SM are due to the interference of a colour-allowed and a colour-suppressed amplitude).

We therefore see that the measurement of a nonzero TP asymmetry in this class of decays would be a smoking-gun signal for the presence of nonstandard operators, specifically those involving a right-handed \( b \)-quark \[29\]. In fact, as was shown in Ref. \[30\], by studying TP’s in several such modes, one can test various models of new physics. As an example of this, we concentrate on the decay \( B \to \phi K^* \).

Within the SM, the CP asymmetries in both \( B_0(t) \to J/\psi K_s \) and \( B_0(t) \to \phi K_s \) are expected to measure \( \sin 2\beta \). Any differences between these two measurements should be at most at the level of \( \mathcal{O}(\lambda^2) \), where \( \lambda \sim 0.2 \). However, at present there appears to be an inconsistency. The world averages for these measurements are \[31\] 32:

\[
\begin{align*}
    \sin(2\beta(J/\psi K_s)) &= 0.734 \pm 0.054 \\
    \sin(2\beta(\phi K_s)) &= -0.39 \pm 0.41 
\end{align*}
\] (69)
Now, decays that have significant penguins contributions are most likely to be affected by physics beyond the SM. In particular, it was pointed out some time ago that $B_d^0 \rightarrow \phi K_s$ is sensitive to new physics because it is a pure $b \rightarrow s$ penguin decay. For this reason, there have been several recent papers discussing possible new-physics scenarios which can account for the above discrepancy. (Some of these have sought a simultaneous explanation of the CP asymmetry measurements in $B_d^0(t) \rightarrow \phi K_s$ and the $B \rightarrow \eta' K$ branching ratios. However, it should be pointed out that the SM explanation of these branching ratios is far from being ruled out.)

Assuming that there is physics beyond the SM in $B_d^0 \rightarrow \phi K_s$, the question then is: what is the nature of this new physics? More concretely, what is the structure of the new-physics operators that contribute to the effective Hamiltonian for $B$ decays?

A partial answer to this question can be found in the measurement of $T$-violating triple products in the sister decay $B \rightarrow \phi K^*$. As we have argued above, if $T$-products vanish in certain decays in the SM, they can be generated in models of new physics which involve couplings to the right-handed $b$-quark. However, not all of the models proposed to explain the CP asymmetry in $B_d^0(t) \rightarrow \phi K_s$ contain such couplings. One can therefore partially distinguish among these models by examining $T$-products in $B \rightarrow \phi K^*$. (Note that one can also look at $T$-products in $\Lambda_b \rightarrow \Lambda \phi$ as the underlying $b \rightarrow s \bar{s}s$ transition in this decay is the same as in $B \rightarrow \phi K_s$.)

We do not present here a comprehensive analysis, but rather focus on one particular new-physics model, that of supersymmetry with $R$-parity violation. (Note that the analysis here can easily be extended to a more general approach, in which one examines new-physics operators without reference to a particular model. Such an approach was presented in Ref. [30].) Assuming that $R$-parity-violating SUSY is the explanation for the CP measurements in $B_d^0(t) \rightarrow \phi K_s$, we estimate here the expected TP asymmetries in $B \rightarrow \phi K^*$.

For the $b \rightarrow s \bar{s}s$ transition, the relevant terms in the $R$-parity-violating SUSY Lagrangian are

$$L_{\text{eff}} = \frac{\lambda'_{32} \lambda'_{22}^*}{4m^2_{\nu_i}} \bar{s}(1 + \gamma_5)s \bar{s}(1 - \gamma_5)b + \frac{\lambda'_{22} \lambda'_{23}^*}{4m^2_{\nu_i}} \bar{s}(1 - \gamma_5)s \bar{s}(1 + \gamma_5)b .$$

(We refer to Ref. [36] for a full explanation of SUSY with R-parity violation.) The amplitude for $B \rightarrow \phi K^*$, including the new-physics contributions, can then be written as

$$A[B \rightarrow \phi K^*] = \frac{G_F}{\sqrt{2}} [(X + X_1)P_\phi + X_2 Q_\phi] ,$$

with

$$X = - \sum_{q=u,c,t} V_{qb} V_{qs}^* \left[ a_3^q + a_4^q + a_5^q - \frac{1}{2} (a_7^q + a_9^q + a_{10}^q) \right] ,$$

$$X_1 = \frac{\sqrt{2} \lambda'_{32} \lambda'_{22}^*}{G_F 24 m^2_{\nu_i}} .$$
\[
X_2 = -\frac{\sqrt{2} \lambda'_{22} \lambda'_{23}}{G_F 24 m^2_{\tilde{\nu}_i}},
\]
\[
P_\phi = m_\phi g_\phi \bar{s}_\mu (1 - \gamma_5) b B, \\
Q_\phi = m_\phi g_\phi \bar{s}_\mu (1 + \gamma_5) b B. 
\] (72)

For \(B_d^0 \to \phi K_S\) it is the combination \(X_1 + X_2\) which contributes [36], and we can define the quantity \(X_R\) via
\[
X_1 + X_2 = \frac{\sqrt{2}}{G_F 12 M^2} X_R e^{i\phi}, 
\] (73)
where \(\phi\) is the weak phase in the R-parity-violating couplings, and \(M\) is a mass scale with \(M \sim m_{\tilde{\nu}_i}\). In order to reproduce the CP-violating \(B_d^0(t) \to \phi K_S\) measurement in Eq. (69), one requires \(|X_R| \sim 1.5 \times 10^{-3}\) for \(M = 100\) GeV, along with a value for the phase \(\phi\) near \(\frac{\pi}{2}\). In our calculations of TP’s in \(B \to \phi K^*\) we make the simplifying assumption that \(X_1 = X_2\), and choose \(\phi = \frac{\pi}{2}\).

We present our results in Table. 8. Note that these hold for both neutral and charged \(B\) decays. The branching ratio for \(B \to \phi K^*\) is slightly larger than the measured branching ratios \(BR(B^+ \to \phi K^{*+}) = 10^{+7}_{-4} \times 10^{-6}\) and \(BR(B_d^0 \to \phi K^{*0}) = 9.5^{+2.4}_{-2.0} \times 10^{-6}\) [22], but it is well within the theoretical uncertainties of the calculation. The important observation is that we expect very large (15–20%) TP asymmetries for these decays, as well as for those with radially-excited final states.

In fact, these results are not unique to supersymmetry with R-parity violation. One expects to find large TP asymmetries in many other models of physics beyond the SM. The measurement of such TP asymmetries would not only reveal the presence of new physics, but more specifically it would point to new physics which includes large couplings to the right-handed \(b\)-quark.

There is one final point which must be stressed here. The standard method of searching for new physics in such decays is to try to measure direct CP asymmetries. However, here such asymmetries are small, at most 4%. The reason is simply that direct CP asymmetries are proportional to \(\sin \delta\), where \(\delta\) is the strong phase difference between the two decay amplitudes [Eq. (2)], and for this set of decays the strong phase difference is very small. Indeed, this is the case for many \(B\) decays. This emphasizes the importance of measuring triple-product asymmetries in order to search for physics beyond the SM. If one relies only on direct CP asymmetries, it is easy to miss the new physics.

5 Summary

A great deal of work, both theoretical and experimental, has been devoted to the study of CP violation in the \(B\) system. As always, the hope is that we will discover physics beyond the standard model. Most of this work has concentrated on
Table 8: Branching ratios (BR) and triple-product asymmetries ($A_T^{(1)}$) for $B \to \phi K^*$ and excited states, for $N_c = 3$ (pure factorization). The results for the CP-conjugate process are given in parentheses.

| Process          | BR          | $A_T^{(1)}$ | $N_c$ |
|------------------|-------------|-------------|-------|
| $B \to \phi K^*$ | $16.7 (17.4) \times 10^{-6}$ | $-16.3 (-15.6)$ | 3     |
| $B \to \phi' K^*$ | $19.1 (20.7) \times 10^{-6}$ | $-21.0 (-19.3)$ | 3     |
| $B \to \phi K^{*'}$ | $28.0 (28.9) \times 10^{-6}$ | $-15.4 (-14.8)$ | 3     |

indirect CP-violating asymmetries, while a smaller fraction has focussed on direct CP violation. However, one subject which has been largely neglected is T-violating triple-product correlations (TP’s) which take the form $\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$, where each $v_i$ is a spin or momentum. One point we have attempted to emphasize in this paper is that TP’s are an excellent way to look for new physics.

The idea is straightforward. If one measures a nonzero value for a quantity which is expected to vanish in the SM, one will clearly have found new physics. Now, direct CP asymmetries are proportional to $\sin \phi \sin \delta$, where $\phi$ and $\delta$ are, respectively, weak and strong phase differences. In $B$ decays, the strong phases are expected to be small in general, so that the direct CP asymmetries will be unmeasurable. Note that weak-annihilation contributions induced by $(S-P)(S+P)$ penguin operators can lead to large strong phases in certain $PP$ and $PV$ decays [8], leading to measurable direct CP asymmetries. However, the annihilation amplitude in the $VV$ case does not gain a chiral enhancement of order $m_B^2/(m_q m_b)$ – it is truly power suppressed in the heavy-quark limit [9]. Hence, in $B \to VV$ decays, the strong phases are expected to be small, so that direct CP asymmetries will be tiny.

These strong phases will also be small in the presence of new physics. This is because the new-physics amplitude is typically expected to be of the same size as loop amplitudes in the SM, and so any rescattering effects from these new operators will be small, resulting in small strong phases. (Note that in the SM the strong phases for $B \to VV$ decays arise dominantly from rescattering of tree-level amplitudes.) Furthermore, even though the new-physics contribution may contain different short-distance physics than that of the SM, the process of hadronization to the final-state mesons is a QCD phenomena, and so is expected to be same with or without new physics. Hence, if the SM strong phases are small in $B \to VV$ decays, they are likely to be small even with new physics. Thus, even if new physics is present, it will probably be undetectable in $B \to VV$ decays using direct CP asymmetries.

On the other hand, triple-product asymmetries are proportional to $\sin \phi \cos \delta$, which are maximized if $\delta \approx 0$. Thus, if a TP is predicted to vanish in the SM, this is an excellent place to look for new physics because there is no suppression from the strong phases. In particular, if new physics is present, it will be detected in TP’s but not in direct CP asymmetries.
In this paper we have examined in detail triple-product asymmetries in $B \to V_1V_2$ decays $[2, 3]$. It is well known that one can perform an angular analysis on such decays (usually to separate the final state into CP-even and CP-odd pieces). However, it is rarely emphasized that the TP’s are in fact the coefficients of some of the terms in the angular analysis. Thus, the TP asymmetries can be obtained from such an analysis.

Within factorization, there are relatively few $B \to V_1V_2$ decays which are expected to have TP’s. The point is that it is not enough to have two decay amplitudes with a relative weak phase (e.g. a tree and penguin amplitude) – what one really needs are two kinematical amplitudes with a relative phase. In particular, because the SM interactions are purely left-handed, both $B \to V_1$ and $B \to V_2$ transitions must be allowed. This strongly limits the number of decays in which TP’s are expected, which helps in the search for new physics.

Like previous analyses $[2, 3]$, we have found several $B$ decays which satisfy these criteria. However, there are two factors which can suppress the TP’s in such decays. First, if $V_1$ and $V_2$ are related by a symmetry such as isospin or $SU(3)$ flavour, the TP asymmetry is suppressed by the size of symmetry breaking. It is therefore best to use decays in which the two final-state vector mesons are unrelated by such a symmetry. Second, all TP’s are suppressed by at least one power of $m_{V}/m_{B}$, so that it is best to use heavy final-state mesons. The upshot is that it is advantageous to consider decays which involve excited mesons in the final state. In such decays, the above suppressions are minimized (and the branching ratios are expected to be of the same size as those involving ground-state mesons). In this paper, we have therefore concentrated mainly on decays which involve excited mesons in the same final state. In such decays, the above suppressions are minimized (and the branching ratios are expected to be of the same size as those involving ground-state mesons). In this paper, we have therefore concentrated mainly on decays with radially-excited vector mesons. We have also considered new modes involving $B_{c}^{-}$ decays, as well as $B$ decays to $D^{*0}$ and $D^{0}$ which then decay to the same final state.

For those decays which can have nonzero triple products in the SM, we have calculated the expected size of these TP’s. We have found that most TP’s are very small. The only processes where large TP’s (>$5\%$) can occur are in $B$ decays to excited final-state vector mesons, specifically $B^{-} \to K^{*-}\omega'$, $\bar{B}_{d}^{0} \to \rho^{0}\omega'$ and $\bar{B}_{d}^{0} \to \rho^{0}\omega$. Decays with TP’s of several percent (i.e. only marginally measurable) include $B^{-} \to K^{*-}\omega'$, $\bar{B}_{d}^{0} \to K^{*0}\omega'$ and $\bar{B}_{d}^{0} \to K^{*0}\omega'$. We have also considered $B$ decays to final states which include $D^{*0}$ or $\bar{D}^{*0}$ mesons, in which these mesons decay to the same final state. Large TP’s are possible only for $B^{-} \to K^{*-}D^{*0}$ and $B^{-} \to K^{*-}\bar{D}^{*0}$, with $D^{*0} \to D^{0}\pi^{0}$ and $\bar{D}^{*0} \to D^{0}\pi^{0}$, and $D^{0}, \bar{D}^{0} \to K^{+}\pi^{-}$.

Note that the sizes of these TP’s all depend on the size of the nonfactorizable effects. In particular, if large TP’s are not found in these decays, it does not necessarily indicate new physics – it could simply be that the nonfactorizable effects are such that the TP’s are small.

The most reliable estimates of TP’s are for those decays in which nonfactorizable effects are expected to be small. These are decays which are dominated by colour-allowed transitions. As it turns out, most TP’s in such decays are expected to
vanish, so that these are excellent processes in which to search for physics beyond the standard model. As an example of how new physics can affect triple products, we considered a supersymmetric model with R-parity violation, and calculated the size of TP’s in $B \to \phi K^*$ decays. In the SM, the TP for this decay vanishes, but when the new-physics contribution is added, very large TP’s are obtained, in the range 15%-20%. Indeed, this type of result is expected in many models of new physics. The measurement of a nonzero TP asymmetry where none is expected would not only reveal the presence of new physics, but more specifically it would point to new physics which includes large couplings to the right-handed $b$-quark. This illustrates quite clearly the usefulness of triple-product correlations in $B$ decays for finding new physics.

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