What is the best RNN-cell structure for forecasting each time series behavior?

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Abstract

It is unquestionable that time series forecasting is of paramount importance in many fields. The most used machine learning models to address time series forecasting tasks are Recurrent Neural Networks (RNNs). Typically, those models are built using one of the three most popular cells, ELMAN, Long-Short Term Memory (LSTM), or Gated Recurrent Unit (GRU) cells, each cell has a different structure and implies a different computational cost. However, it is not clear why and when to use each RNN-cell structure. Actually, there is no comprehensive characterization of all the possible time series behaviors and no guidance on what RNN cell structure is the most suitable for each behavior. The objective of this study is two-fold: it presents a comprehensive taxonomy of all-time series behaviors (deterministic, random-walk, nonlinear, long-memory, and chaotic), and provides insights into the best RNN cell structure for each time series behavior. We conducted two experiments: (1) The first experiment evaluates and analyzes the role of each component in the LSTM-Vanilla cell by creating 11 variants based on one alteration in its basic architecture (removing, adding, or substituting one cell component). (2) The second experiment evaluates and analyzes the performance of 20 possible RNN-cell structures. Our results showed that the MGU-SLIM3 cell is the most recommended for deterministic and nonlinear behaviors, the MGU-SLIM2 cell is the most suitable for random-walk behavior, FB1 cell is advocated for long-memory behavior, and LSTM-SLIM1 for chaotic behavior.

Keywords: Forecasting; Time series; Times series behavior; RNN models; LSTM cells

1. Introduction

Many real-world prediction problems involve a temporal dimension and typically require the estimation of numerical sequential data referred to as time series forecasting. Time series forecasting is one of the major stones in data science playing a pivotal role in almost all domains, including meteorology \cite{Murat2018}, natural disasters control \cite{Erdelj2017}, energy \cite{Bourdeau2019}, manufacturing \cite{Wang2018}, finance \cite{Liu2019}, econometrics \cite{Siami2018}, telecommunication \cite{Maeng2019}.
healthcare (Khaldi et al. 2019b) to name a few. Accurate time series forecasting requires robust forecasting models.

Currently, Recurrent Neural Network (RNN) models are one of the most popular machine learning models in sequential data modeling, including natural language, image/video captioning, and forecasting (Sutskever et al. 2014, Vinyals et al. 2015, Chimmula and Zhang 2020). Such RNN models are built as a sequence of the same cell structure, for example, ELMAN cell, Long-Short Term Memory (LSTM) cell or Gated Recurrent Unit (GRU) cell. The simplest RNN cell is ELMAN, it includes one layer of hidden neurons. While, LSTM and GRU cells incorporate a gating mechanism, three gates in LSTM and two gates in GRU, where each gate is a layer of hidden neurons. Many other cell structures have been introduced in the literature (Zhou et al. 2016, Lu and Salem 2017, Mikolov et al. 2014, Pulver and Lyu 2017). However, to solve time series forecasting tasks, the building of RNN models is typically limited to the three aforementioned cell structures (Sezer et al. 2020, Runge and Zmeureanu 2021, Liu et al. 2021, Rajagukguk et al. 2020, Alkhayat and Mehmood 2021), as they provide very good accuracy (Runge and Zmeureanu 2021, Sezer et al. 2020).

Nevertheless, building robust RNN models for time series forecasting is still a challenging task as there does not exist yet a clear understanding of times series data itself and hence there exist very little knowledge about what cell structure is the most appropriate for each data type. In general, when facing a new problem, practitioners select one of the most popular cells, usually LSTM, and use it as a building block for the RNN model without any guarantee on the appropriateness of this cell to the current data. The objective of this work is two-fold. It presents a comprehensive characterization of time series behaviors, and provides guidelines on the best RNN cell structure for each behavior. As far as we know, this is the first work in providing such insights. The main contributions of this study can be summarized as follows:

- Provide a better understanding of times series data by presenting a comprehensive characterization of their behaviors.
- Determine the most appropriate cell structure for each time series behavior (i.e., whether a specific cell structure should be avoided for certain behaviors).
- Identify differences in predictability between behaviors (i.e., whether certain behaviors are easier or harder to predict across all cell models).
- Provide useful guidelines that can assist decision-makers and scholars in the process of selecting the most suitable RNN-cell structure from both, a computational and performance point of views.

The remainder of this study is organized as follows: Section 2 states the related works. Section 3 presents a taxonomy of time series behaviors. Section 4 presents a taxonomy of RNN cells. Section 5 describes the experiment. Section 6 exhibits and discuss the obtained results. Finally, the last section concludes the findings and spots light on future research directions.
2. Related works

The last decades have known an explosion of time series data acquired by automated data collection devices such as monitors, IoT devices, and sensors (Murat et al. 2018, Erdelj et al. 2017, Bourdeau et al. 2019). The collected time series describe different quantitative values: stock price, amount of sales, electricity load demand, weather temperature, etc. In parallel, a large number of comparative studies have been carried out in the forecasting area (Parmezan et al. 2019, Godahewa et al. 2021, Athiyarath et al. 2020, Divina et al. 2019, Choubin et al. 2018, Sagheer and Kotb 2019, Bianchi et al. 2017). These studies can be divided into two categories: (1) the first category tries to find a universal forecasting model (Parmezan et al. 2019, Godahewa et al. 2021, Athiyarath et al. 2020). They compare multiple models on a set of datasets from different fields to conclude which model is the universal predictor. The selection of the used datasets is not based on any clear criteria. Whereas, (2) the second category focuses on selecting the most performing forecasting model (Divina et al. 2019, Choubin et al. 2018, Sagheer and Kotb 2019, Bianchi et al. 2017). They compare a set of models on one or multiple datasets coming from a the same field. Nevertheless, the best predictor does not ensure stable performance over different datasets even if they come from the same field. Actually, after an extensive analysis on highly diverse datasets, Keogh and Kasetty (2003) demonstrated that there is a need for more comprehensive time series benchmarks and more careful evaluations in the data mining community. In addition, datasets should have a large size to train and test the models, and incorporate specific behaviors that challenge their modeling. Such knowledge of the dataset properties is required to facilitate a better interpretation of the modeling results.

The natural approach to create such benchmarking time series datasets is to collect data from real applications. For instance, the NN5 dataset (Crone 2008), the CIF 2016 dataset (Godahewa et al. 2020), the M4 dataset (Makridakis et al. 2018), and the Monash archive that gather 20 publicly available time series datasets (Godahewa et al. 2021). Although, real time series are always business-oriented which make them either proprietorial or expensive to obtain (Dau et al. 2019), they can take decades to become mature and ready to be used for machine learning purposes, their diversity testing is tedious (Dau et al. 2019, Spiliotis et al. 2020), and most importantly, their Data Generation Processes (DGPs) are unknown which make the interpretation of the models and the explanation of their decisions challenging.

An alternative solution is to generate synthetic time series datasets with known embedded patterns (Olson et al. 2017). For instance, Zhang and Qi (2005) investigated the issue of how to effectively model artificial time series with deterministic behavior due to the existence of trend and seasonality using Artificial Neural Networks (ANNs). López-Caraballo et al. (2016) examined ANNs on time series with noiseless and noisy chaotic behavior generated by Mackey-Glass series. Li and Lin (2016) applied the Self-Constructing Fuzzy Neural Network (SCFNN) on chaotic time series including Logistic and Henon data. Yeo (2017) evaluated the performance of LSTM on three different time series with chaotic behavior (delay-time chaotic dynamical systems, Mackey-Glass and Ikeda equations). Fischer et al. (2018) presented an experimental evaluation of seven machine learning models applied on a set of eight DGPs reflecting linear and nonlinear
behaviors. Parmezan et al. (2019) used 40 synthetic datasets of deterministic, stochastic and chaotic time series to compare eleven parametric and non-parametric models. Kang et al. (2020) used mixture auto-regressive (MAR) models to create GRATIS dataset based on which they compared different statistical models. However, all the aforementioned studies remain non comprehensive of the main time series behaviors that can be faced in real datasets.

Another categorization of these comparative studies can be made based on the types of the evaluated models. Here, three categories can be set: (1) Studies based on parametric models where scientists try to evaluate the forecasting performance of different statistical models (Godahewa et al. 2021, Yu et al. 2020, Kim and Jung 2018). (2) Studies based on non-parametric models where they assess the performance of machine learning models (Granata 2019, Dudek 2016, Sagheer and Kotb 2019). (3) Studies based on parametric and non-parametric models where both types of models are compared (Parmezan et al. 2019, Khaldi et al. 2019a, Yamak et al. 2019, Siami-Namini and Namin 2018). RNNs are one of the most used machine learning models in time series forecasting (Sezer et al. 2020). Nevertheless, their usage is limited to three RNN variants (ELMAN, LSTM, and GRU). In the financial field, Sezer et al. (2020) reported that from 2005 to 2019, 52.5% of publications used RNN models to perform time series forecasting, where LSTM model represents 60.4%, ELMAN (vanilla RNN) represents 29.7%, and GRU represents 9.89%. In the energy field, Runge and Zmeureanu (2021) stated in their review that ELMAN, GRU, and LSTM are the main applied deep learning models to building energy forecasting. In the environmental field, Liu et al. (2021) asserted in their review, from 2015 to 2020, that LSTM and GRU are the most practical RNN models in the air quality forecasting. In the renewable energy field, Rajagukguk et al. (2020) reviewed, from 2005 to 2020, that in the photovoltaic power forecasting, 60% of publications used LSTM, 20% used ELMAN, and 13% used GRU. While, in the solar irradiance forecasting, they reported that LSTM represent 44% of the used deep learning models, ELMAN represents 25%, and GRU 19%. Similarly, Alkhayat and Mehmood (2021) reported in their review, from 2016 to 2020, that ELMAN, LSTM, and GRU are the only RNN models used in wind and solar energy forecasting. They outlined that the usage of RNNs within this field increased from 2% in 2016 to 25% in 2020.

Therefore, there is a strong need for a comprehensive analysis of different types of RNN models, including the above three variants, in forecasting the main time series behaviors, and a strong need for an RNN-based models guide to assist practitioners in their process of selection and structure of the best RNN cell for each time series behavior.

3. Taxonomy of times series behaviors

As far as we know, this is the first work in introducing a complete formal characterization of real-world time series. Time series emerging from real-world applications can either follow a stochastic mechanism or a chaotic mechanism, and are usually contaminated by white noise (Wales 1991, Cencini et al. 2000, Zumino et al. 2012, Boaretto et al. 2015).
3.1. Stochastic behavior

In the stochastic behavior, real time series are generated by a random stable system, and can exhibit the following behaviors:

1. Deterministic behavior. These time series are characterized by the existence of deterministic patterns. They usually incorporate at least one of the following patterns: increasing or decreasing deterministic trend, simple deterministic seasonality, and complex deterministic seasonality (Figure 1). The trend pattern is a long-term evolution in the data, it can be increasing or decreasing, and it can have different forms (linear, exponential and damped) (Montgomery et al. 2015). An increasing trend can appear in the demand for technologies in the social fields, while a decreasing trend is related to epidemics, mortality rates, and unemployment (Parmezan et al. 2019). The seasonality pattern can be described as the occurrence of short-term regular variation that repeats at known and relatively constant time intervals. This type of patterns can occur in different types of data including time series of sales, e.g., the increase in sales of warm clothing in winter and air conditioners in summer (Parmezan et al. 2019).

2. Random-Walk behavior. The time series of this behavior are characterized by the existence of unit-root patterns. This behavior appears when the time series have a stochastic increasing or decreasing trend and/or stochastic seasonality (Figure 2). Here the current observation is equal to the previous observation plus a random step. The lag between these two observations is equal to one in the case of trend, and equal to the seasonality period in the case of seasonality.

The presence of deterministic or random-walk behavior induce the non-stationarity in time series. The stationarity is a relevant feature in time series which basically implies that the mean, the variance, and the covariance do not depend on time (Montgomery et al. 2015). These two types of behaviors are the most important features in time series, and usually characterize business and macro-economic data (Salles et al. 2019, Liu et al. 2019).

3. Nonlinear behavior. Time series observations can often exhibit correlations with different degrees of non-linearity (Figure 3). This type of behavior is present in almost all real-world time series data, such as in stream-flow forecasting (Wang et al. 2020), and in financial markets forecasting (Bukhari et al. 2020).

4. Long-memory behavior. Some time series may present properties of long-range dependencies in time which implies strong coupling effect between the values of observations at different time steps, i.e. the correlations between observations has a slower exponential decay compared to short range dependencies (Figure 4). This type of behavior can occur in hydrology forecasting (Papacharalampous and Tyralis 2020), network traffic forecasting (Ramakrishnan and Soni 2018), financial market forecasting (Bukhari et al. 2020), etc.
Figure 1: Time series with deterministic behavior: (a) increasing trend, (b) simple seasonality, (c) complex seasonality, (d) increasing trend and simple seasonality and (e) increasing trend and complex seasonality.

Figure 2: Time series with random-walk behavior: (a) trend random-walk, (b) seasonal random-walk, (c) trend and seasonal random-walk.

Figure 3: Time series with nonlinear behavior generated by: (a) Nonlinear Auto-Regressive (NAR) process, (b) Smooth Transition Auto-Regressive (STAR) process, (c) Threshold Auto-Regressive (TAR) process.
3.2. Chaotic behavior

The chaotic mechanism can be expressed as a nonlinear deterministic dynamical system that is often unknown or incompletely understood (Li and Lin 2016), real time series can exhibit a noisy chaotic behavior (Figure 5). These time series are sensitive to initial conditions (butterfly effect) where a small smooth perturbations in the system or measurement errors generate an abrupt change in the behavior of the time series (bifurcation). This type of behaviors is unstable since it tends to be deterministic at short term but random at long term. Such kind of time series are usually present in many science and engineering fields such as weather forecasting (Tian 2019), financial markets forecasting (Bukhari et al. 2020), energy forecasting (Bourdeau et al. 2019), intelligent transport and trajectory forecasting (Giuliari et al. 2021), etc.
Based on the literature (Chandra and Zhang 2012, Montgomery et al. 2015, Liu et al. 2017, Fischer et al. 2018), the five aforementioned behaviors (deterministic, random-walk, nonlinear, long-memory, and chaotic) are the main behaviors encountered in real applications. Real-world time series can either express an individual behavior or an aggregation of more than one behavior. To identify which type of behavior a real-world time series can include, different statistical preprocessing tools and tests can be applied (Grau-Carles 2005, Inglada-Perez 2020). In Table 1, we present a set of tools that can be used to identify the existence of the five aforementioned behaviors in the time series data.

| Tool | Time series behavior | Reference |
|------|----------------------|-----------|
| Data visualization | Deterministic behavior | Chatfield (2013) |
| Correlogram | Deterministic behavior | Box et al. (2015) |
| Time series decomposition | Deterministic behavior | Chatfield (2013) |
| Smoothing | Deterministic behavior | Montgomery et al. (2015) |
| Data visualization | Random-Walk behavior | Montgomery et al. (2015) |
| Augmented Dickey Fuller (ADF) test | Random-Walk behavior | Dickey and Fuller (1979) |
| Phillips–Perron (PP) test | Random-Walk behavior | Phillips and Perron (1988) |
| Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test | Random-Walk behavior | Kwiatkowski et al. (1992) |
| Kaplan test | Nonlinear behavior | Kaplan (1994) |
| Keenan test | Nonlinear behavior | Keenan (1985) |
| Tsay test | Nonlinear behavior | Tsay (1986) |
| Teräsvirta test | Nonlinear behavior | Teräsvirta et al. (1993) |
| White test | Nonlinear behavior | White (1989) |
| Correlogram | Long-memory behavior | Palma (2007) |
| Qu test | Long-memory behavior | Qu (2011) |
| R/S analysis | Long-memory behavior | Mandelbrot and Wallis (1968) |
| Modified R/S | Long-memory behavior | Lo (1991) |
| Geweke and Porter-Hudak (GPH) test | Long-memory behavior | Geweke and Porter-Hudak (1983) |
| Detrended Fluctuation Analysis (DFA) | Long-memory behavior | Peng et al. (1994) |
| Correlation Dimension | Chaotic behavior | Grassberger and Procaccia (1984) |
| Lyapunov Exponent | Chaotic behavior | Bensáïda and Litimi (2013) |
| MGRM test | Chaotic behavior | Matilla-García and Marín (2010) |
| Recurrence Plots | Chaotic behavior | Eckmann and Ruelle (1985) |
| 0/1 test | Chaotic behavior | Gottwald and Melbourne (2004) |
4. Taxonomy of RNN cells

Humans do not start their thinking from zero every second, our thoughts have persistence in the memory of our brains. For example, as the reader reads this paper, he/she understands each word based on his/her understanding of the words before. The absence of memory is the major shortcoming in traditional machine learning models, particularly in feed-forward neural networks (FNNs). To overcome this limitation, RNNs integrate the concept of feedback connections in their structure (Figure 6, where \( x_t \) and \( h_t \) are the input state and the hidden state at time step \( t \), respectively). This mechanism enables RNNs to have a certain memory capable of capturing the dynamics in sequential data by conveying information through time.

![Figure 6: The folded (left) and unfolded (right) architecture of RNN model.](image)

Table 2: A collection of RNN cell structures historically sorted.

| RNN cell structure | Year | Reference |
|--------------------|------|-----------|
| JORDAN             | 1989 | Jordan (1989) |
| ELMAN              | 1990 | Elman (1990) |
| LSTM-NFG           | 1997 | Hochreiter and Schmidhuber (1997) |
| LSTM-Vanilla       | 2000 | Gers et al. (2000) |
| LSTM-PC            | 2000 | Gers and Schmidhuber (2000) |
| SCRN               | 2014 | Mikolov et al. (2014) |
| GRU                | 2014 | Cho et al. (2014) |
| IRNN               | 2015 | Le et al. (2015) |
| LSTM-FB1           | 2015 | Jozefowicz et al. (2015) |
| MUT                | 2015 | Jozefowicz et al. (2015) |
| LSTM-CIFG          | 2015 | Nina and Rodriguez (2015) |
| Differential LSTM  | 2015 | Veeriah et al. (2015) |
| MRNN               | 2016 | Abdulkarim (2016) |
| MGU                | 2016 | Zhou et al. (2016) |
| Phased LSTM        | 2016 | Neil et al. (2016) |
| Highway Connections | 2016 | Irie et al. (2016) |
| LSTM with Working Memory | 2017 | Pulver and Lyu (2017) |
| SLIM               | 2017 | Lu and Salem (2017), Dey and Salem (2017), Heck and Salem (2017) |
| GORO               | 2019 | Jing et al. (2019) |
RNN models are built based on one specific cell structure which is the core of all computations that occur in the network. Multiple cell structures have been created since 1989 (Table 2). The early cell structure is named JORDAN (Jordan 1989), where at each time step the previous output state is fed into the cell (Figure 7a). Later, ELMAN cell was proposed by Elman (1990). Unlike the JORDAN cell, each time step in the ELMAN cell calls the previous hidden state (Figure 7b). In 2016, a combination of both JORDAN and ELMAN cells in one cell named multi-recurrent neural network (MRNN) was evaluated by Abdulkarim (2016). In this cell structure, at each time step, both previous output and hidden states are presented to the cell (Figure 7c).

Figure 7: (a) JORDAN cell structure. (b) ELMAN cell structure. (c) MRNN cell structure.

It was proved that the ELMAN cell suffers from the vanishing and exploding gradient problems which impedes the capturing of long term dependencies (Pascanu et al. 2013). To overcome the memory limitation of this cell, novel cell structures have been proposed. In 2014, the Structurally Constrained Recurrent Network (SCRN) was proposed by Mikolov et al. (2014). They integrated a slight structural modification in ELMAN cell that consists in adding a new slowly changing state at each time step called context state $s_{t-1}$ (Figure 8a). In 2015, Le et al. (2015) created a new cell called Identity Recurrent Neural Network (IRNN) as a modification of ELMAN by setting the ReLu as activation function, the identity matrix as an initialization of the hidden states weight matrix, and zero as an initialization of the bias (Figure 8b).

Figure 8: (a) SCRN cell structure. (b) IRNN cell structure.
A different way to handle the vanishing and exploding gradient problems resulted in creating different cell structures characterized by the gating mechanism that regulate the flowing of the information flux. The gates can be seen as filters that only hold useful information and selectively remove any irrelevant information from passing through. To perform this control of information (i.e., which information to pass and which information to discard), the gates are equipped with parameters that need to be trained through the model learning process using the back-propagation through time algorithm (Werbos 1990). Thus, this mechanism provides the RNN cell with an internal permanent memory able to store information for long time periods (Weston et al. 2014, Graves et al. 2014).

In 1997, the first version of this type of cells named Long-Short Term Memory with No Forget Gate (LSTM-NFG) was created by Hochreiter and Schmidhuber (1997). This cell contains two gates: the input gate \( \Gamma_i \) and the output gate \( \Gamma_o \). Later in 2000, the concept of the forget gate \( \Gamma_f \) was introduced by Gers et al. (2000) creating LSTM-Vanilla that has been widely used in most applications (Figure 9a). In the same year, LSTM cell with peephole connections (LSTM-PC) was proposed by Gers and Schmidhuber (2000). The peephole connections connect the previous cell state \( c_{t-1} \) with the input, forget, and the output gates (Figure 9b). These connections enable the LSTM cell to inspect its current cell states (Gers and Schmidhuber 2001), and to learn precise and stable timing without teacher forcing (Gers et al. 2002).

In 2014, Gated Recurrent Unit (GRU) was proposed by Cho et al. (2014) as a simpler variant of LSTM that shares many of the same properties. The idea behind GRU cell was to reduce the gating mechanism of LSTM cell from three gates to two gates (relevance gate \( \Gamma_r \) and update gate \( \Gamma_u \)) in order to decrease the number of parameters and to improve the learning velocity (Figure 10a). In 2015, ten thousand RNN cell structures were evaluated by Jozefowicz et al. (2015) using a mutation-based search process. They identified a cell architecture that outperforms both LSTM and GRU on some tasks. This cell consists in adding a bias of 1 to the LSTM forget gate creating LSTM-FB1. Further, they discovered three optimal cell architectures named MUT1, MUT2, and MUT3 that are similar to GRU but have some modifications in their gating mechanism and in their candidate hidden state \( \hat{h}_t \) (Figure 10b and 10c). During that year, coupling both the input and the forget gates into one gate was proposed by Nina and Rodriguez (2015) creating LSTM-CIFG cell (Figure 11a). Further, the differential LSTM was proposed by Veeriah et al. (2015) to solve the impact

Figure 9: (a) LSTM-Vanilla cell structure. (b) LSTM-PC cell structure.

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Figure 9: (a) LSTM-Vanilla cell structure. (b) LSTM-PC cell structure.
of spatial-temporal dynamics by introducing the differential gating scheme in LSTM cell.

![Figure 10: (a) GRU cell structure. (b) MUT1 cell structure. (c) MUT3 cell structure.](image)

In 2016, the Minimal Gate Unit (MGU) cell was created by Zhou et al. (2016) to further reduce the number of parameters by decreasing the gating mechanism to one forget gate. This variant has simpler structure and fewer parameters compared to LSTM cell and GRU cell (Figure 11b). In the same year, eight variants of LSTM-PC (based on modifying, adding, or removing one cell component at each time) were evaluated by Greff et al. (2016) on three different types of task: speech recognition, polyphonic music modeling, and handwritten recognition. They demonstrated that the forget and the output gates are the most critical components in the LSTM cell. In addition, their results show that none of the evaluated variants can overcome the LSTM-PC cell. During that year, phased LSTM cell was introduced by Neil et al. (2016), where they added a time gate which updates the cell sparsely, and makes it converge faster than the basic LSTM. Further, highway connections were added to GRU and LSTM cells by Irie et al. (2016). In 2017, LSTM with working memory was created by Pulver and Lyu (2017), where they substituted the forget gate with a functional layer whose input depends on the previous cell state. In 2019, a Gated Orthogonal Recurrent Unit (GORO) was introduced by Jing et al. (2019), where they added to the GRU cell an orthogonal matrix that replaced the hidden state loop matrix.

![Figure 11: (a) LSTM-CIFG cell structure. (b) MGU cell structure.](image)
While very powerful in long term dependencies, the basic cells (LSTM, GRU, and MGU) have complex structure with relatively large number of parameters. In 2017, the concept of parameter reduction was differently tackled through the creation of new cells called SLIM \cite{Lu2017,Dey2017,Heck2017}. These variants aim to reduce aggressively the parameters in order to achieve memory and time savings while necessarily retaining a performance comparable to the basic cells. The new parameter-reduced variants of these cells eliminate the combinations of the input state, the hidden state, and the bias from the individual gating signals, creating SLIM1, SLIM2, and SLIM3, respectively. The SLIM1 cell consists in removing from the mechanism of all the gates the input state and its associated parameter matrix (Figure 12(a)). SLIM2 cell consists in maintaining only the hidden state and its associated parameter matrix (Figure 12(b)). Whereas, SLIM3 cell consists in removing the input state, the hidden state, and their associated parameters matrices (Figure 12(c)). The cellular calculations within the displayed RNN cells along with the evaluated ones are provided in Appendix.

5. Experimental structure

Two experiments have been carried out in this study. The first experiment analyzes the utility of each LSTM-Vanilla cell component in forecasting time series behaviors. The second experiment evaluates different variants of RNN cell structures in forecasting time series behaviors. In this section, we first describe the process we followed to generate the dataset for each time series behavior (Section 5.1). Then, we present the selected models for the first and second experiment (Section 5.2). Finally, we provide the setup of the used models (Section 5.3).

5.1. Synthetic data generation

To simulate the five aforementioned time series behaviors, we used 21 different DGPs with white Gaussian noise $\epsilon_t \sim \mathcal{N}(\mu = 0, \sigma = 0.2)$. From each DGP we created time series of length 3000 observations replicated 30 times through a Monte Carlo simulation experiment using different initial random seeds for the white noise term $\epsilon_t$ (Table 3).

To generate time series with deterministic behavior, 5 DGPs were used (Table 4): Trend process (T), Simple Seasonality process (SS), Complex Seasonality process (CS), Trend and Simple Seasonality process (TS), Trend and Simple Seasonality process...
(TSS), and Trend and Complex Seasonality process (TCS). To simulate the random-walk behavior, 3 DGPs were used (Table 5): Trend Random-Walk process (TRW), Seasonal Random-Walk process (SRW), and Trend and Seasonal Random-Walk process (TSRW). To simulate time series with nonlinear behavior, we used 6 most popular nonlinear models commonly used in the forecasting literature having an increasing levels of non-linearity (Zhang et al. 2001) (Table 6): Sign Auto-Regressive process (SAR), Nonlinear Moving Average process (NMA), Nonlinear Auto-Regressive process (NAR), Bilinear process (BL), Smooth Transition Auto-Regressive process (STAR), and Threshold Auto-Regressive process (TAR). These models are motivated by many nonlinear characteristics commonly observed in practice. To artificially generate time series with long memory behavior, we used the Auto-Regressive Fractionally Integrated Moving Average process ARFIMA(p,d,q) since it is one of the best-known long memory processes (Liu et al. 2017). In order to evaluate the performance of RNN cell structures with respect to DGPs with an increasing memory structure, 3 DGPs were created based on the variation of the fractional order of ARFIMA process \( d = \{0, 0.2, 0.4\} \). A higher fractional order \( d \) implies longer dependency structure (Table 7). To ensure the stationarity of the generated time series, we set the values of the fractional order strictly less than 0.5. Finally, to simulate the noisy chaotic behavior, 4 most known chaotic DGPs were used (Table 8): Mackey-Glass process, Lorenz process, Rössler process, and Hénon-Map process. Then, we added the white Gaussian noise \( \epsilon_t \) to the deterministic signals to create noisy chaotic time series (Sangiorgio et al. 2021).

Table 3: Number of data generation processes and time series in each behavior.

| Behavior                  | # DGPs | # Time series |
|---------------------------|--------|---------------|
| Deterministic behavior    | 5      | 150           |
| Random-walk behavior      | 3      | 90            |
| Nonlinear behavior        | 6      | 180           |
| Long-memory behavior      | 2      | 60            |
| Chaotic behavior          | 4      | 120           |
| Total                     | 21     | 600           |

Table 4: The DGPs used to simulate time series with deterministic behavior.

| Process name                      | Process mathematical model               |
|-----------------------------------|------------------------------------------|
| Trend process                     | \((T) : z_t = 10 + 0.02t + \epsilon_t\) |
| Simple Seasonality process        | \((SS) : z_t = 2 \sin(2\pi t/5) + \epsilon_t\) |
| Complex Seasonality process       | \((CS) : z_t = \sin(2\pi t/100) + 0.5 \sin(2\pi t/5) + \epsilon_t\) |
| Trend and Simple Seasonality process | \((TSS) : z_t = 10 + 0.02t + 5 \sin(2\pi t/5) + \epsilon_t\) |
| Trend and Complex Seasonality process | \((TCS) : z_t = 10 + 0.02t + \sin(2\pi t/100) + 0.5 \sin(2\pi t/5) + \epsilon_t\) |
Table 5: The DGPs used to simulate time series with random-walk behavior.

| Process name                              | Process mathematical model                                      |
|-------------------------------------------|-----------------------------------------------------------------|
| Trend Random-Walk process                 | (TRW) : \( z_t = z_{t-1} + \epsilon_t \)                      |
| Seasonal Random-Walk process              | (SRW) : \( z_t = z_{t-4} + \epsilon_t \)                      |
| Trend and Seasonal Random-Walk process    | (TSRW) : \( z_t = z_{t-1} + z_{t-4} - z_{t-5} + \epsilon_t \) |

Table 6: The DGPs used to simulate time series with nonlinear behavior.

| Process name                                | Process mathematical model                                      |
|---------------------------------------------|-----------------------------------------------------------------|
| Sign Auto-Regressive process               | \( SAR(2) : z_t = \text{sign}(z_{t-1} + z_{t-2}) + \epsilon_t \) |
| Nonlinear Moving Average process           | \( NMA(2) : z_t = \epsilon_t - 0.3\epsilon_{t-1} + 0.2\epsilon_{t-2} + 0.4\epsilon_{t-1}\epsilon_{t-2} - 0.25\epsilon_{t-2}^2 \) |
| Nonlinear Auto-Regressive process          | \( NAR(2) : z_t = \frac{0.7z_{t-1}}{|z_{t-1}| + 2} + \frac{0.35|z_{t-2}|}{|z_{t-2}| + 2} + \epsilon_t \) |
| Bilinear process                           | \( BL(2) : z_t = 0.4z_{t-1} - 0.3z_{t-2} + 0.5z_{t-1}\epsilon_{t-1} + \epsilon_t \) |
| Smooth Transition Auto-Regressive process  | \( STAR(2) : z_t = 0.3z_{t-1} + 0.6z_{t-2} + \frac{0.1-0.9z_{t-1}+0.8z_{t-2}}{1+e^{-10z_{t-1}}} + \epsilon_t \) |
| Threshold Auto-Regressive process          | \( TAR(2) : z_t = \begin{cases} 0.9z_{t-1} + 0.05z_{t-2} + \epsilon_t & \text{for } |z_{t-1}| \leq 1 \\ -0.3z_{t-1} + 0.65z_{t-2} - \epsilon_t & \text{for } |z_{t-1}| > 1 \end{cases} \) |

Table 7: The DGPs used to simulate time series with long-memory behavior.

| Process name      | Process mathematical model                                      |
|-------------------|-----------------------------------------------------------------|
| \( ARFIMA(p = 2, d = 0.0, q = 2) \) | \( z_t^{(0.0)} = 0.7z_{t-1}^{(0.0)} - 0.1z_{t-2}^{(0.0)} - 0.5\epsilon_{t-1} + 0.4\epsilon_{t-2} + \epsilon_t \) |
| \( ARFIMA(p = 2, d = 0.2, q = 2) \) | \( z_t^{(0.2)} = 0.7z_{t-1}^{(0.2)} - 0.1z_{t-2}^{(0.2)} - 0.5\epsilon_{t-1} + 0.4\epsilon_{t-2} + \epsilon_t \) |
| \( ARFIMA(p = 2, d = 0.4, q = 2) \) | \( z_t^{(0.4)} = 0.7z_{t-1}^{(0.4)} - 0.1z_{t-2}^{(0.4)} - 0.5\epsilon_{t-1} + 0.4\epsilon_{t-2} + \epsilon_t \) |

Table 8: The DGPs used to simulate time series with chaotic behavior.

| Process name          | Process mathematical model                                      |
|-----------------------|-----------------------------------------------------------------|
| Mackey-Glass process  | \( \frac{dx}{dt} = a - \frac{x(t-\tau)}{1+x^2(t-\tau)} - bx(t) \) such that \( \tau = 17, a = 0.2, b = 0.1, c = 10 \) \( \text{(Ma et al. 2007)} \) |
| Hénon-Map process     | \( x_{t+1} = 1 + y_t - ax_t^2 \); \( y_{t+1} = bx_t \) such that \( a = 1.4, b = 0.3 \) \( \text{(Li and Lin 2016)} \) |
| Rössler process       | \( \dot{x} = -y - z \); \( \dot{y} = x + ay \); \( \dot{z} = b + z(x - c) \) \( \text{(Lim and Puthusserypady 2007)} \) |
| Lorenz process        | \( \dot{x} = \sigma(y - x) \); \( \dot{y} = -xz + rx - y \); \( \dot{z} = xy - bz \) \( \text{(Lim and Puthusserypady 2007)} \) |
5.2. RNN-cells used for experiment 1 and 2

To evaluate each cell structure with respect to each times series behavior, we conducted two experiments as summarized in Table 9. The first experiment evaluates LSTM-Vanilla and 11 of its variants created based on one alteration in the basic Vanilla architecture that consists of (1) removing, (2) adding, or (3) substituting one cell component (Table A.16): (1) The first three variants NIG (No Input Gate), NFG (No Forget Gate), and NOG (No Output Gate) were created through the deactivation of the input gate, the forget gate, and the output gate, respectively. The four subsequent variants NIAF (No Input Activation Function), NFAF (No Forget Activation Function), NOAF (No Output Activation Function), and NCAF (No Candidate Activation Function) were constructed through the elimination of the input, forget, output, and candidate activation function, respectively. (2) The two subsequent variants PC (Peephole Connections), and FGR (Full Gate Recurrence) were designed through the creation of new connections between the cell state and the gates, and between the current states and the previous states of the gates, respectively. (3) Eventually, FB1 (Forget Gate Bias 1) and CIFG (Coupled Input Forget Gate) were conceived by setting the forget gate bias to one, and by coupling the input and the forget gate into one gate, respectively.

The second experiment evaluates and analyzes the performance of 20 possible RNN-cell structures: JORDAN, ELMAN, MRNN, SCRN, IRNN, LSTM-Vanilla, GRU, MGU, MUT1, MUT2, MUT3, and 9 SLIM variants mapping LSTM, GRU, and MGU. A summary of the evaluated cells related to each experiment is presented in Table 9, and the cellular calculations inside each cell is presented in Table A.17.

The studied RNN cells have different degrees of complexity, which is referred to as theoretic complexity. This complexity is defined by the number of parameters inside each cell which depends on the number of inputs, the number of hidden nodes, the number of context nodes (in the case of SCRN model), and the number of outputs. During the hyper-parameter tuning the complexity of the cell may increase or decrease depending on the optimal number of hidden nodes found. Thus, the complexity of the cell defined after the hyper-parameter tuning is called empirical complexity.

5.3. Experimental setup

Before starting the modeling process, each time series data was partitioned into three subsets: the first 2000 observations were used to train the models in order to find the best parameters, the next 500 observations were used to select the best configuration of each model, and the last 500 observations were used to test the out-of-sample performance of these models. Each partition was normalized, then reshaped using the estimation window size (number of lags) and the forecasting window size (number of horizons) to convert them into supervised learning data (input-target). To find the best configuration of the studied models, we fixed some hyper-parameters (Table 13) and varied other ones. The process of hyper-parameter optimization consists in finding the best number of hidden neurons and the best estimation window size using the Grid Search algorithm. The ranges of values used to find the best estimation window size for each DGP are presented in Table 11. Each model’s configuration (combination of number of hidden nodes and estimation window size) was run 10 times to test its performance stability.
Table 9: List of evaluated RNN models with regard to each experiment with their theoretic complexity, number of weight matrices, and number of bias vectors. \( n_I \) is the number of inputs, \( n_H \) is the number of hidden nodes, \( n_S \) is the number of context nodes, and \( n_O \) is the number of outputs.

| RNN models                     | Short name | Full name                                      | Theoretic complexity | \# Weight matrices | \# Bias vectors |
|--------------------------------|------------|------------------------------------------------|----------------------|---------------------|----------------|
| NIG                            | LSTM with No Input Gate | \( 3n_In_H + 3n_I^2 + 3n_H \) | 6                    | 3                   |
| NFG                            | LSTM with No Forget Gate | \( 3n_In_H + 3n_I^2 + 3n_H \) | 6                    | 3                   |
| NOG                            | LSTM with No Output Gate | \( 3n_In_H + 3n_I^2 + 3n_H \) | 6                    | 3                   |
| CIFG                           | LSTM with Coupled Input Forget Gate | \( 3n_In_H + 3n_I^2 + 3n_H \) | 6                    | 3                   |
| FBI                            | LSTM with Forget Gate Bias 1 | \( 4n_In_H + 4n_I^2 + 3n_H \) | 6                    | 3                   |
| NIAF                           | LSTM with No Input Activation Function | \( 4n_In_H + 4n_I^2 + 4n_H \) | 8                    | 4                   |
| NFAF                           | LSTM with No Forget Activation Function | \( 4n_In_H + 4n_I^2 + 4n_H \) | 8                    | 4                   |
| NOAF                           | LSTM with No Output Activation Function | \( 4n_In_H + 4n_I^2 + 4n_H \) | 8                    | 4                   |
| NCAF                           | LSTM with No Candidate Activation Function | \( 4n_In_H + 4n_I^2 + 4n_H \) | 8                    | 4                   |
| Vanilla                        | LSTM Vanilla | \( 4n_In_H + 4n_I^2 + 4n_H \) | 8                    | 4                   |
| PC                             | LSTM with Peephole Connections | \( 4n_In_H + 7n_I^2 + 4n_H \) | 11                   | 4                   |
| FGR                            | LSTM with Full Gate Recurrence | \( 4n_In_H + 13n_I^2 + 4n_H \) | 17                   | 4                   |
| ELMAN                          | ELMAN      | \( n_In_H + n_I^2 + n_H \) | 2                    | 1                   |
| RNN                            | Identity Recurrent Neural Network | \( n_In_H + n_I^2 + n_H \) | 2                    | 1                   |
| JORDAN                         | JORDAN     | \( n_In_H + n_OIn_H + n_H \) | 2                    | 1                   |
| MRNN                           | Multi-Recurent Neural Network | \( n_In_H + n_I^2 + n_OIn_H + n_H \) | 3                    | 1                   |
| SCRN                           | Structurally Constrained Recurrent Network | \( n_In_H + n_I^2 + n_OIn_H + n_H \) | 4                    | 1                   |
| MGU-SLIM3                      | Minimal Gate Unit SLIM3 | \( n_In_H + n_I^2 + 2n_H \) | 2                    | 2                   |
| MGU-SLIM2                      | Minimal Gate Unit SLIM2 | \( n_In_H + 2n_I^2 + n_H \) | 3                    | 1                   |
| MGU-SLIM1                      | Minimal Gate Unit SLIM1 | \( n_In_H + 2n_I^2 + 2n_H \) | 3                    | 2                   |
| MGU                            | Minimal Gate Unit | \( 2n_In_H + 2n_I^2 + 2n_H \) | 4                    | 2                   |
| GRU-SLIM3                      | Gated Recurrent Unit SLIM3 | \( n_In_H + n_I^2 + 3n_H \) | 2                    | 3                   |
| GRU-SLIM2                      | Gated Recurrent Unit SLIM2 | \( n_In_H + 3n_I^2 + n_H \) | 4                    | 1                   |
| GRU-SLIM1                      | Gated Recurrent Unit SLIM1 | \( n_In_H + 3n_I^2 + 3n_H \) | 4                    | 3                   |
| MUT1                           | Gated Recurrent Unit Mutation 1 | \( 2n_In_H + 2n_I^2 + 3n_H \) | 4                    | 3                   |
| MUT2                           | Gated Recurrent Unit Mutation 2 | \( 2n_In_H + 3n_I^2 + 3n_H \) | 5                    | 3                   |
| MUT3                           | Gated Recurrent Unit Mutation 3 | \( 3n_In_H + 3n_I^2 + 3n_H \) | 6                    | 3                   |
| GRU                            | Gated Recurrent Unit | \( 3n_In_H + 3n_I^2 + 3n_H \) | 6                    | 3                   |
| LSTM-SLIM3                     | LSTM SLIM3 | \( n_In_H + n_I^2 + 4n_H \) | 2                    | 4                   |
| LSTM-SLIM2                     | LSTM SLIM2 | \( n_In_H + 4n_I^2 + n_H \) | 5                    | 1                   |
| LSTM-SLIM1                     | LSTM SLIM1 | \( n_In_H + 4n_I^2 + 4n_H \) | 5                    | 4                   |
| LSTM-Vanilla                   | LSTM Vanilla | \( 4n_In_H + 4n_I^2 + 4n_H \) | 8                    | 4                   |

Table 10: The values of the hyper-parameters used to train the studied RNN models.

| Hyper-parameter                  | Value |
|----------------------------------|-------|
| Mini-batch size                  | 100   |
| Maximum number of epochs         | 500   |
| Initial learning rate            | 0.01  |
| Learning algorithm               | Adam  |
| Forecasting window size (horizon)| 1     |
| Number of hidden neurons         | (1:1:10) |
| (initial: step: final)           |       |
Table 11: The range of values used as inputs for the studied RNN models with respect to the DGPs of each time series behavior.

| Behavior          | DGP             | Estimation window size |
|-------------------|-----------------|------------------------|
|                    | T               | 1:1:10                 |
|                    | SS              | 1:1:5                  |
| Deterministic behavior | CS             | 1:1:5                  |
|                    | TSS             | 1:1:5                  |
|                    | TCS             | 1:1:5                  |
|                    | TRW             | 1:1:10                 |
| Random-walk behavior | SRW            | 1:1:4                  |
|                    | TSRW            | 1:1:5                  |
|                    | SAR(2)          | 1:1:5                  |
| Nonlinear behavior | NMA(2)          | 1:1:5                  |
|                    | NAR(2)          | 1:1:5                  |
|                    | BL(2)           | 1:1:5                  |
|                    | STAR(2)         | 1:1:5                  |
|                    | TAR(2)          | 1:1:5                  |
| Long-memory behavior | ARFIMA(2, 0, 2) | 1:1:5                  |
|                    | ARFIMA(2, 0.2, 2) | 1:1:20                |
|                    | ARFIMA(2, 0.4, 2) | 1:1:40                |
| Chaotic behavior   | Mackey-Glass    | 1:1:7                  |
|                    | Hénon map       | 1:1:3                  |
|                    | Rössler         | 1:1:14                 |
|                    | Lorenz          | 1:1:25                 |

Once finding the best configuration of each model, the training and the validations sets were blended to form a new training set on which these models were retrained on for 500 epochs. To evaluate the forecasting performance of the models the Root Mean Square Error (RMSE) was used:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^{n} (y_t - \hat{y}_t)^2}
\]

Where \(y_t\) and \(\hat{y}_t\) are the real value and the predicted value at time step \(t\), respectively. And \(n\) is the number of observations in the time series. The predicted values are computed as:

\[
\hat{y}_t = h_t.W_{hy} + b_y
\]
$W_{hy}$ is the weight matrix between the hidden and the output layer, and $b_y$ is the output layer bias.

6. Results and discussion

In this section, we present the results of the two conducted experiments: (1) The first experiment consists of evaluating and analyzing the role of each component in LSTM-Vanilla cell with respect to the five time series behaviors. The evaluated architectures were generated by removing (NIG, NFG, NOG, NIAF, NFAF, NOAF, and NCAF), adding (PC and FGR), or substituting (FB1 and CIFG) one cell component. (2) The second experiment aims at evaluating and analyzing the performance of all possible RNN cell structures (JORDAN, ELMAN, MRNN, SCRN, IRNN, LSTM-Vanilla, GRU, MGU, MUT1, MUT2, MUT3, and 9 SLIM variants) in forecasting the five behaviors.

The RNN cells presented in the following figures are ordered based on their theoretic complexity in order to maintain the same order in all the figures of the five behaviors so as to facilitate their interpretation. To grab the reader’s attention to the best RNN cell within each DGP and within each behavior, we used the star symbol (i.e. ★ and ⋆). Two colors were used so as to differentiate between models having the same forecasting errors but different empirical complexities. The symbol ★ is used to highlight the model having the smallest forecasting error and the smallest empirical complexity. However, the symbol ⋆ is used to highlight the model(s) having similar forecasting error(s) as the ★ model but with a relatively higher empirical complexity compared to this one. In the case where more than one cell structure provide similar forecasting errors, the simplest architecture (i.e., the ★ model) is recommended based on the principle of Occam’s Razor [Blumer et al. 1987].

6.1. Experiment 1: Utility analysis of LSTM cell components in forecasting time series behaviors.

The impact of each component (i.e., input gate, forget gate, output gate, coupled input-forget gate, input activation function, forget activation function, output activation function, candidate activation function, fixing the forget bias to 1, peephole connections, and full gate recurrence) on the performance of LSTM-Vanilla model for predicting the deterministic behavior, random-walk behavior, nonlinear behavior, long-memory behavior, and chaotic behavior is shown, respectively, in Figures 13, 14, 15, 16 and 17.

As it can be observed from Figure 13 for modeling time series with deterministic behavior, LSTM cell structures behave differently to each deterministic time series patterns (T, SS, CS, TSS, and TCS) (Figure 13). The forget and the candidate activation functions are the most relevant cell components in forecasting time series with TSS pattern. Substituting the forget gate bias with the value one deteriorates the cell forecasting ability towards time series with TSS pattern. The existence of the output gate is necessary in forecasting time series with SS pattern. In almost all models, forecasting the CS pattern is very challenging compared to the other patterns, while the T pattern is easily captured. To model time series with random-walk behavior, the input activation function is of paramount importance to capture the TSRW pattern. The
FGR variant outperforms in capturing the SRW pattern, however, its performance widely degrades with respect to TSRW pattern. Adding the peephole connections make the LSTM cell behave similarly to both patterns TRW and SRW. Whereas, coupling the input and the forget gates make the LSTM cell perform equally to SRW and TSRW patterns (Figure 14).

Figure 13: Forecasting performance of LSTM-Vanilla cell structures with respect to time series with deterministic behavior generated by 5 types of DGPs: T, SS, CS, TSS, and TCS.

![Deterministic behavior](image)

Figure 14: Forecasting performance of LSTM-Vanilla cell structures with respect to time series with random-walk behavior generated by 3 types of DGPs: TRW, SRW, and TSRW.

![Random-walk behavior](image)

Regarding the nonlinear behavior, the LSTM structures behave similarly to all the 6 nonlinear time series patterns (Figure 15). Their errors decrease with the increase in the non-linearity order of the time series. This proves that all the components in the LSTM cell are able to capture the strong non-linearity in time series more than the soft non-linearity pattern. With regard to the long-memory behavior, the LSTM structures display approximately similar ability in forecasting the 3 long-memory degrees (Figure 16). Their errors decrease with the increase in the long memory order. Nevertheless, the NOAF cell exhibits a different
behavior. It presents a peak in the RMSE with respect to the highest long-memory degree. Therefore, the output activation function plays a major role in capturing the long range dependencies in time series data.

Figure 15: Forecasting performance of LSTM-Vanilla cell structures with respect to time series with nonlinear behavior generated by 6 types of DGPs: SAR(2), NMA(2), NAR(2), BL(2), STAR(2), and TAR(2).

Figure 16: Forecasting performance of LSTM-Vanilla cell structures with respect to time series with long-memory behavior generated by 3 types of DGPs using ARFIMA with \( d = \{0, 0.2, 0.4\} \).

Within the chaotic behavior, almost all the LSTM variants behave similarly to all the 4 chaotic behaviors (Figure 17). The Mackey-Glass process followed by the Hénon process are more challenging compared to the Rössler process followed by the Lorenz process. Although, with the NIAF, the FB1, and the PC variants, the forecasting error of the Hénon process exceeds the one of the Mackey-Glass process. The input and the forget activation functions are compulsory in modeling the Mackey-Glass process. The reasons that can degrade the forecasting performance of LSTM-vanilla cell with regard to the Hénon process are: removing the input activation function, coupling the input and the forget gates, substituting the forget gate bias by one, and adding the peephole connections.
To identify the general forecasting ability of all the LSTM-vanilla variants for each time series behavior, we computed the mean RMSE over all the DGPs of each behavior (Figure 18). To capture the deterministic behavior, the CIFG model is the most recommended. The forget and the candidate activation functions are the most mandatory components in the LSTM cell. In addition, the bias of the forget gate should be set as a learnable parameter and not equal to one as in FB1. To model the random-walk behavior, the CIFG is the best model. The input activation function is very necessary, although, the Vanilla version of LSTM should be avoided. To model the nonlinear behavior, almost all the cell structures present similar performance, such that the NOG structure is the most recommended. To capture the long range dependencies in time series with long-memory behavior, the existence of the output activation function is very critical (i.e., when it is removed, the forecasting error gets multiplied by 2). Almost all the remaining LSTM structures present
similar results, however, the FB1 variant is the most performing. To model the chaotic behavior, all the LSTM structures have approximately the same performance, such that the NOAF structure is the most recommended. In Table 12, we provide guidelines about the best LSTM cell structure to be used in each process and in each time series behavior.

Table 12: Guidelines on the best LSTM cell structure for each time series behavior. The Mean refers to the best cell structure over all the DGP's of the same behavior. The two colored stars are used to differentiate between models with the same forecasting error but with different empirical complexity. (⋆) refers to model with the smallest forecasting error. (★★) refers to model with the smallest forecasting error and empirical complexity.

| Behaviors  | DGP's | NIG | NFG | NOG | CIFG | FB1 | NIAF | NFAF | NOAF | NCAF | Vanilla | PC | FGR |
|------------|-------|-----|-----|-----|------|-----|------|------|------|------|--------|----|-----|
| Deterministic | T     |     |     |     |      |     |      |      |      |      |        |    |     |
|             | SS    | ★   | ★   | ★   | ★    | ★   | ★    | ★    | ★    | ★    |        |    |     |
|             | CS    |     |     |     |      |     |      |      |      |      |        |    |     |
|             | TSS   |     |     |     |      |     |      |      |      |      |        |    |     |
|             | TCS   |     |     |     |      |     |      |      |      |      |        |    |     |
|             | Mean  |     |     |     |      |     |      |      |      |      |        |    |     |
| Unit-root   | TRW   |     |     |     |      |     |      |      |      |      |        |    |     |
|             | SRW   |     |     |     |      |     |      |      |      |      |        |    |     |
|             | TSRW  |     |     |     |      |     |      |      |      |      |        |    |     |
|             | Mean  |     |     |     |      |     |      |      |      |      |        |    |     |
| Nonlinear   | SAR(2)| ★   |     | ★   |      | ★   |      |      |      |      |        |    |     |
|             | NMA(2)|     | ★   |     |      |     |      |      |      |      |        |    |     |
|             | NAR(2)| ★   | ★   | ★   |      | ★   | ★    |      |      |      |        |    |     |
|             | BL(2)| ★   | ★   | ★   | ★    | ★   |      |      |      |      |        |    |     |
|             | STAR(2)| ★  | ★   | ★   |      | ★   |      |      |      |      |        |    |     |
|             | TAR(2)| ★   |     | ★   |      |     |      |      |      |      |        |    |     |
|             | Mean  | ★   |     | ★   |      |     |      |      |      |      |        |    |     |
| Long-memory | ARFIMA(2,0,2)|     |      |      |      |      |      |      |      |      |        |    |     |
|             | ARFIMA(2,0.2,2)| ★| ★   | ★   | ★    | ★    |      |      |      |      |        |    |     |
|             | ARFIMA(2,0.4,2)|     |      |      |      |      |      |      |      |      |        |    |     |
|             | Mean  | ★   |     | ★   |      |     |      |      |      |      |        |    |     |
| Chaotic     | Mackey| ★   | ★   |     |      | ★   |      |      |      |      |        |    |     |
|             | Hénon |     |     | ★   |      | ★   |      |      |      |      |        |    |     |
|             | Rössler| ★ |     |     |      | ★   |      |      |      |      |        |    |     |
|             | Lorenz| ★   |     |     |      | ★   |      |      |      |      |        |    |     |
|             | Mean  | ★   |     | ★   |      |     |      |      |      |      |        |    |     |
6.2. Experiment 2: Performance analysis of different RNN cell structures in forecasting time series behaviors.

The performance of each RNN cell structure (JORDAN, ELMAN, MRNN, SCRN, IRNN, LSTM-Vanilla, GRU, MGU, MUT1, MUT2, MUT3, and 9 SLIM variants) for predicting the deterministic behavior, random-walk behavior, nonlinear behavior, long-memory behavior, and chaotic behavior is shown, respectively, in Figures 19, 20, 21, 22, and 23.

The RNN models behave similarly to the different deterministic time series patterns (Figure 19). Typically, for the majority of RNN cell structures, the CS pattern is the most challenging, while the T pattern is the less challenging. The simple recurrent cells (ELMAN, IRNN, JORDAN, MRNN, and SCRN) and the MGU-SLIM1 cell are unable to model time series with TSS pattern because they exhibit a strong rise in the RMSE compared to the other models.

Regarding the random-walk behavior, the RNN models perform differently with respect to the different patterns (Figure 20). The models ELMAN, IRNN, JORDAN, and MRNN are unable to capture the TRW pattern since they depict an intense increase in the forecasting error. For the same pattern, MUT1 followed by SCRN also depict a higher error compared to the remaining MGU, GRU, and LSTM variants. In addition, to model the SRW pattern, the MRNN model shows the highest error compared to all the other models. With respect to the three patterns, the SLIM1 and SLIM2 variants of MGU are more stable than those of GRU and LSTM. While, the SLIM3 variant of LSTM is more stable than the SLIM3 variant of MGU and GRU.

![Deterministic behavior](image)

Figure 19: Forecasting performance of RNN cell structures with respect to time series with deterministic behavior generated by 5 types of DGPs: T, SS, CS, TSS, and TCS.

With regard to the nonlinear behavior, all the models express similar abilities in forecasting the 6 nonlinear time series patterns (Figure 21). Their forecasting errors decrease with the increase in the nonlinear order of the time series. However, the JORDAN and SCRN models find some difficulties in learning the STAR process compared to the other models.
Similar findings can be noticed with the long-memory time series patterns where all the models express similar behavior (Figure 22). Their forecasting errors decrease with the increase in the long-range dependency order of the ARFIMA process. Although, the MRNN model shows an opposite behavior due to the occurrence of the vanishing gradient problem. Eventually, for the chaotic behavior, the RNN models exhibit approximately the same performance with regard to the 4 chaotic processes (Figure 23). In almost all RNN models, the Mackey-Glass process followed by the Hénon process are more challenging compared to the Rössler process followed by the Lorenz process. Nevertheless, the forecasting error of the Hénon process exceeds the one of the Mackey-Glass process with the following models: IRNN, MGU-SLIM1, GRU-SLIM3, GRU-SLIM1, and MUT2. In addition, the ELMAN, MRNN, and SCRN models demonstrate poor performance in forecasting the Rössler process compared to the remaining models.
Figure 22: Forecasting performance of RNN cell structures with respect to time series with long-memory behavior generated by 3 types of DGPs using ARFIMA with $d = \{0, 0.2, 0.4\}$.

Figure 23: Forecasting performance of RNN cell structures with respect to time series with noisy chaotic behavior generated by 4 types of DGPs: Mackey-Glass, Lorenz, Rössler, and Hénon map.

To examine the general ability of RNN models with respect to each behavior, we present the overall mean RMSE over all the DGPs of each behavior in Figure 24. To model the deterministic behavior, MGU-SLIM3 is the most advocated, nevertheless, the simple RNN models (ELMAN, IRNN, JORDAN, MRNN, and SCRN) and the MGU-SLIM1 model should be avoided. To predict the random-walk behavior, the MGU-SLIM2 model is the most recommended, and the following models should be avoided: ELMAN, IRNN, JORDAN, and MRNN. To capture the nonlinear behavior, all the RNN models present similar performance, while MGU-SLIM3 is the most performing. To forecast the long memory behavior, almost all the RNN models exhibit similar results, such that MRNN is the less performing. However, the most recommended model is LSTM-SLIM3. To model the Chaotic behavior in time series, LSTM-SLIM1 is the most suitable. In Table 13, we provide guidelines about the best RNN model to be used with each process and with each behavior.
Table 13: Guidelines on the best RNN cell structure for each time series behavior. The Mean refers to the best cell structure over all the DGPs of the same behavior. The two colored stars are used to differentiate between models with the same forecasting error but with different empirical complexity. (⋆) refers to model with the smallest forecasting error. (★) refers to model with the smallest forecasting error and empirical complexity.

| Behaviors     | Deterministic | Random-walk | Nonlinear | Long-memory | Chaotic |
|---------------|---------------|-------------|-----------|-------------|---------|
|               | RNN variants  |             |           |             |         |
|               | ELMAN | IRNN | JORDAN | MRNN | SCRN | MGU SLIM1 | MGU SLIM2 | MGU SLIM3 | MGU SLIM4 | MGU SLIM5 | GRU SLIM1 | GRU SLIM2 | GRU SLIM3 | MUT1 | MUT2 | MUT3 | GRU | LSTM SLIM1 | LSTM SLIM2 | LSTM SLIM3 | LSTM SLIM4 | LSTM SLIM5 | LSTM SLIM6 | LSTM SLIM7 | LSTM SLIM8 | LSTM SLIM9 | LSTM SLIM10 | LSTM SLIM11 | LSTM SLIM12 | LSTM SLIM13 | LSTM SLIM14 | LSTM SLIM15 | LSTM SLIM16 |
| Deterministic | T     |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |
|               | SS    |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | CS    |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | TSS   |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | TCS   |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | Mean  |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
| Random-walk   | TRW   |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | SRW   |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | TSRW  |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | Mean  |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
| Nonlinear     | SAR(2)|      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | NMA(2)|      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | NAR(2)|      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | BL(2) |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | STAR(2)|      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | TAR(2)|      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | Mean  |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
| Long-memory   | ARFIMA(2.0,0.2)|             |             |             |             |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | ARFIMA(2.0,0.2)|             |             |             |             |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | ARFIMA(2.0,2.2)|             |             |             |             |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | Mean  |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
| Chaotic       | Mackey|      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | Hénon |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | Rössler|      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | Lorenz|      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
|               | Mean  |      |      |      |      |             |             |             |             |             |             |             |             |             |     |     |     |     |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |
The results of the two experiments are summarized in Table 14. Over all the DGPs of the deterministic and the nonlinear behaviors, the most recommended cell is the SLIM3 version of MGU cell. For time series with random-walk behavior, both CIFG and MGU-SLIM2 cells exhibited similar forecasting abilities, however, we suggest to use MGU-SLIM2 cell since it is simpler than CIFG. Eventually, FB1 and LSTM-SLIM1 are the most recommended for long-memory and chaotic behaviors, respectively.

Based on the outcomes of the two experiments, we can also derive insights about the predictability of each behavior. In Figure 25, we present the average forecasting errors over all the 31 RNN structures used in this study for each of the five time series behaviors. We can notice that all the behaviors have different degrees of predictability. The most straightforward is the random-walk followed by the deterministic behavior. Then, the chaotic behavior is ranked in the third place followed by the long-memory behavior. Finally, the most challenging one is the nonlinear behavior.

| Behaviors      | Experiment 1 | Experiment 2 | Recommended cell |
|----------------|--------------|--------------|------------------|
|                | Best cell   | RMSE         | Best cell       | RMSE         |
| Deterministic  | CIFG        | 0.048        | MGU-SLIM3       | 0.0468       | MGU-SLIM3    |
| Random-walk    | CIFG        | 0.0286       | MGU-SLIM2       | 0.0286       | MGU-SLIM2    |
| Nonlinear      | NOG         | 0.1418       | MGU-SLIM3       | 0.1333       | MGU-SLIM3    |
| Long-memory    | FB1         | 0.1163       | MGU-SLIM3       | 0.1166       | FB1          |
| Chaotic        | NOAF        | 0.0667       | LSTM-SLIM1      | 0.0662       | LSTM-SLIM1   |

Figure 24: Average forecasting performance of RNN cell structures with respect to five time series behaviors.
7. Conclusions

In this paper, we proposed a comprehensive taxonomy of all possible time series behaviors, which are: deterministic, random-walk, nonlinear, long-memory, and chaotic. Then, we conducted two experiments to show the best RNN cell structure for each behavior. In the first experiment, we evaluated LSTM-Vanilla model and 11 of its variants created based on one alteration in its basic architecture that consists in (1) removing (NIG, NFG, NOG, NIAF, NFAF, NOAF, and NCAF), (2) adding (PC and FGR), or (3) substituting (FB1 and CIFG) one cell component. While, in the second experiment, we evaluated LSTM-Vanilla along with a set of 19 RNN models based on other recurrent cell structures (JORDAN, ELMAN, MRNN, SCRN, IRNN, GRU, MGU, MUT1, MUT2, MUT3, and 9 SLIM variants).

In the first experiment, We showed that CIFG cell is the most suitable for non-stationary time series due to the existence of deterministic behavior or random-walk behavior. We also, experimentally, proved that all the LSTM-Vanilla cells have approximately comparable forecasting ability in the case of nonlinear behavior, such that NOG is the most stable. The obtained results showed that the output activation function is a critical component that help LSTM cell capture long term dependencies. In the same context, we demonstrated that FB1 cell is the most recommended in forecasting time series with long-term behavior. Eventually, we demonstrated that to forecast time series with chaotic behavior, the output activation function should be removed since the NOAF cell displayed the best results.

In the second experiment, over the 20 evaluated RNN models, the best forecasting results were achieved by the new parameter-reduced variants of MGU and LSTM. With time series of deterministic, nonlinear, and long memory behaviors, the best cell was MGU-SLIM3. For the random-walk behavior, the most performing model was based on MGU-SLIM2 cell. Finally, for chaotic behavior, RNN model-based LSTM-SLIM1 cell outperformed all the other models.

Based on the outcomes of both experiments, we arrived to demonstrate that the SLIM3 version of MGU cell has the highest ability to increase the performance of RNN model in forecasting deterministic and nonlinear behaviors. While, its SLIM2 version is recommended in the case of time series with random-walk behavior. Finally, for the long-memory and the chaotic behaviors, FB1 and LSTM-SLIM1 cells are strongly advocated, respectively. The outcomes of our study are limited to the time series with a single behavior.
However, in real-world problems, combined behaviors (i.e., more than one behavior in the same time series) can also occur. As future work, evaluating the best RNN cell with respect to such types of time series can complement the guidelines provided by this study.

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Appendix A. Architectures of the studied RNN cells

In this section, we provide the cell structures of the different RNN models along with their cellular calculations. To understand the calculations, Table A.15 presents deception of the mathematical notations. To better understand the components inside LSTM-Vanilla cell, we present below the role of the main elements:

- **Input state** $x_t$: it contains the data features at time step $t$.
- **Output state** $y_t$: it contains the output of the model at time step $t$.
- **Hidden state** $h_t$: it represents the short term memory of the cell at time step $t$.
- **Cell state** $c_t$: it represents the long-term memory of the cell at time step $t$.
- **Candidate cell state** $\tilde{c}_t$: it contains the new information we can use to update the cell state at time step $t$.
- **Input gate** $\Gamma_i$: it filters from the current candidate cell state the information that should be used to update the current cell state.
- **Forget gate** $\Gamma_f$: it filters from the previous cell state the information that should be used to update the current cell state.
- **Output gate** $\Gamma_o$: it filters from the current cell state the information that should be exposed to the external network (the next time step and the next hidden and/or output layer).

| Symbol | Significance |
|--------|--------------|
| $x_t$  | the input state at time step $t$. |
| $h_t$  | the hidden state at time step $t$. |
| $y_t$  | the output state at time step $t$. |
| $\Gamma_i$ | the input gate at time step $t$. |
| $\Gamma_f$ | the forget gate at time step $t$. |
| $\Gamma_o$ | the output gate at time step $t$. |
| $\Gamma_u$ | the update gate at time step $t$. |
| $\Gamma_r$ | the relevance gate at time step $t$. |
| $c_t$  | the cell state at time step $t$. |
| $\tilde{c}_t$ | the candidate cell state at time step $t$. |
| $W_{xh}$ | the weight matrix between the input and the hidden states. |
| $W_{hh}$ | the weight matrix between the previous and the current hidden states. |
| Symbol | Significance |
|--------|--------------|
| $W_{hy}$ | the weight matrix between the hidden and the output states. |
| $W_{yh}$ | the weight matrix between the output and the hidden states. |
| $W_{xs}$ | the weight matrix between the input and the context states. |
| $W_{sh}$ | the weight matrix between the context and the hidden states. |
| $W_{sy}$ | the weight matrix between the context and the output states. |
| $W_{xi}$ | the weight matrix between the input state and the input gate. |
| $W_{hi}$ | the weight matrix between the hidden state and the input gate. |
| $W_{xo}$ | the weight matrix between the input state and the output gate. |
| $W_{ho}$ | the weight matrix between the hidden state and the output gate. |
| $W_{xf}$ | the weight matrix between the input state and the forget gate. |
| $W_{hf}$ | the weight matrix between the hidden state and the forget gate. |
| $W_{xz}$ | the weight matrix between the input state and the candidate cell state. |
| $W_{hx}$ | the weight matrix between the hidden state and the candidate cell state. |
| $W_{sx}$ | the weight matrix between the input state and the candidate hidden state. |
| $W_{sh}$ | the weight matrix between the hidden state and the candidate hidden state. |
| $W_{ci}$ | the weight matrix between the cell state and the input gate. |
| $W_{cf}$ | the weight matrix between the cell state and the forget gate. |
| $W_{co}$ | the weight matrix between the cell state and the output gate. |
| $W_{xu}$ | the weight matrix between the input state and the update gate. |
| $W_{hu}$ | the weight matrix between the hidden state and the update gate. |
| $W_{xr}$ | the weight matrix between the input state and the relevance gate. |
| $W_{hr}$ | the weight matrix between the hidden state and the relevance gate. |
| $W_{ii}$ | the weight matrix between the previous and the current input gates. |
| $W_{ff}$ | the weight matrix between the previous and the current forget gates. |
| $W_{oo}$ | the weight matrix between the previous and the current output gates. |
| $W_{if}$ | the weight matrix between the previous input gate and the current forget gate. |
| $W_{io}$ | the weight matrix between the previous input gate and the current output gate. |
| $W_{fi}$ | the weight matrix between the previous forget gate and the current input gate. |
| $W_{fo}$ | the weight matrix between the previous forget gate and the current output gate. |
| $W_{oi}$ | the weight matrix between the previous output gate and the current input gate. |
| $W_{of}$ | the weight matrix between the previous output gate and the current forget gate. |
| $b_{h}$ | the bias related to the hidden state. |
| $b_{y}$ | the bias related to the output state. |
| $b_{c}$ | the bias related to the candidate cell state. |
Table A.15: Nomenclature

| Symbol | Significance |
|--------|--------------|
| $b_h$  | the bias related to the candidate hidden state. |
| $b_i$  | the bias related to the input gate. |
| $b_o$  | the bias related to the output gate. |
| $b_f$  | the bias related to the forget gate. |
| $b_u$  | the bias related to the update gate. |
| $b_r$  | the bias related to the relevance gate. |
| $g$    | the activation function of the output state (the identity function). |
| $\otimes$ | the point-wise multiplication (Hadamard product) presented in the figures as $\odot$. |
| $\sigma$ | the sigmoid activation function. |

Table A.16: Description of different LSTM-Vanilla cell structures created based on one change in its architecture.

| Cell name    | Cell architecture | Cell computations |
|--------------|-------------------|-------------------|
| LSTM-Vanilla | ![LSTM-Vanilla Diagram](image) | $c_t = \tanh(x_t.W_{\tilde{c}} + h_{t-1}.W_{h\tilde{c}} + b_{\tilde{c}})$  
$\Gamma_{i_t} = \sigma(x_t.W_{x_i} + h_{t-1}.W_{hi} + b_i)$  
$\Gamma_{f_t} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + b_f)$  
$\Gamma_{o_t} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + b_o)$  
$c_t = \Gamma_{f_t} \otimes c_{t-1} + \Gamma_{i_t} \otimes \tilde{c}_t$  
$h_t = \Gamma_{o_t} \otimes \tanh(c_t)$ |
| LSTM-NIG    | ![LSTM-NIG Diagram](image) | $c_t = \tanh(x_t.W_{\tilde{c}} + h_{t-1}.W_{h\tilde{c}} + b_{\tilde{c}})$  
$\Gamma_{f_t} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + b_f)$  
$\Gamma_{o_t} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + b_o)$  
$c_t = \Gamma_{f_t} \otimes c_{t-1} + \tilde{c}_t$  
$h_t = \Gamma_{o_t} \otimes \tanh(c_t)$ |
| LSTM-NFG    | ![LSTM-NFG Diagram](image) | $c_t = \tanh(x_t.W_{\tilde{c}} + h_{t-1}.W_{h\tilde{c}} + b_{\tilde{c}})$  
$\Gamma_{i_t} = \sigma(x_t.W_{x_i} + h_{t-1}.W_{hi} + b_i)$  
$\Gamma_{o_t} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + b_o)$  
$c_t = c_{t-1} + \Gamma_{i_t} \otimes \tilde{c}_t$  
$h_t = \Gamma_{o_t} \otimes \tanh(c_t)$ |
| Cell name   | Cell architecture | Cell computations |
|------------|-------------------|-------------------|
| LSTM-NOG  | ![LSTM-NOG Diagram] | $\hat{c}_t = \tanh(x_t.W_{\hat{c}} + h_{t-1}.W_h + b)$  
|           |                   | $\Gamma_{ii} = \sigma(x_t.W_{x_i} + h_{t-1}.W_{hi} + b_i)$  
|           |                   | $\Gamma_{if} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + b_f)$  
|           |                   | $c_t = \Gamma_{if} \odot c_{t-1} + \Gamma_{ii} \odot \hat{c}_t$  
|           |                   | $h_t = \tanh(c_t)$ |
| LSTM-NIAF | ![LSTM-NIAF Diagram] | $\hat{c}_t = \tanh(x_t.W_{\hat{c}} + h_{t-1}.W_h + b)$  
|           |                   | $\Gamma_{ii} = x_t.W_{x_i} + h_{t-1}.W_{hi} + b_i$  
|           |                   | $\Gamma_{if} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + b_f)$  
|           |                   | $\Gamma_{io} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + b_o)$  
|           |                   | $c_t = \Gamma_{if} \odot c_{t-1} + \Gamma_{ii} \odot \hat{c}_t$  
|           |                   | $h_t = \Gamma_{io} \odot \tanh(c_t)$ |
| LSTM-NFAF | ![LSTM-NFAF Diagram] | $\hat{c}_t = \tanh(x_t.W_{\hat{c}} + h_{t-1}.W_h + b)$  
|           |                   | $\Gamma_{ii} = \sigma(x_t.W_{x_i} + h_{t-1}.W_{hi} + b_i)$  
|           |                   | $\Gamma_{if} = x_t.W_{xf} + h_{t-1}.W_{hf} + b_f$  
|           |                   | $\Gamma_{io} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + b_o)$  
|           |                   | $c_t = \Gamma_{if} \odot c_{t-1} + \Gamma_{ii} \odot \hat{c}_t$  
|           |                   | $h_t = \Gamma_{io} \odot \tanh(c_t)$ |
| LSTM-NOAF | ![LSTM-NOAF Diagram] | $\hat{c}_t = \tanh(x_t.W_{\hat{c}} + h_{t-1}.W_h + b)$  
|           |                   | $\Gamma_{ii} = x_t.W_{x_i} + h_{t-1}.W_{hi} + b_i$  
|           |                   | $\Gamma_{if} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + b_f)$  
|           |                   | $\Gamma_{io} = x_t.W_{xo} + h_{t-1}.W_{ho} + b_o$  
|           |                   | $c_t = \Gamma_{if} \odot c_{t-1} + \Gamma_{ii} \odot \hat{c}_t$  
|           |                   | $h_t = \Gamma_{io} \odot \tanh(c_t)$ |
| LSTM-NCAF | ![LSTM-NCAF Diagram] | $\hat{c}_t = x_t.W_{\hat{c}} + h_{t-1}.W_h + b$  
|           |                   | $\Gamma_{ii} = \sigma(x_t.W_{x_i} + h_{t-1}.W_{hi} + b_i)$  
|           |                   | $\Gamma_{if} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + b_f)$  
|           |                   | $\Gamma_{io} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + b_o)$  
|           |                   | $c_t = \Gamma_{if} \odot c_{t-1} + \Gamma_{ii} \odot \hat{c}_t$  
|           |                   | $h_t = \Gamma_{io} \odot \tanh(c_t)$ |
| Cell name   | Cell architecture | Cell computations |
|------------|-------------------|-------------------|
| LSTM-FB1   | ![LSTM-FB1 Diagram](image1) | $\tilde{c}_t = \tanh(x_t.W_{2\tilde{c}} + h_{t-1}.W_{h\tilde{c}} + b_\tilde{c})$
|            |                   | $\Gamma_{i_t} = \sigma(x_t.W_{xi} + h_{t-1}.W_{hi} + b_i)$
|            |                   | $b_f = 1$
|            |                   | $\Gamma_{f_t} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + b_f)$
|            |                   | $\Gamma_{o_t} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + b_o)$
|            |                   | $c_t = \Gamma_{f_t} \odot \overline{c}_{t-1} + \Gamma_{i_t} \odot \tilde{c}_t$
|            |                   | $h_t = \Gamma_{o_t} \odot \tanh(c_t)$ |
| LSTM-CIFG  | ![LSTM-CIFG Diagram](image2) | $\tilde{c}_t = \tanh(x_t.W_{2\tilde{c}} + h_{t-1}.W_{h\tilde{c}} + b_\tilde{c})$
|            |                   | $\Gamma_{i_t} = \sigma(x_t.W_{xi} + h_{t-1}.W_{hi} + c_t.W_{ci} + b_i)$
|            |                   | $\Gamma_{f_t} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + \overline{c}_{t-1}.W_{cf} + b_f)$
|            |                   | $\Gamma_{o_t} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + c_t.W_{co} + b_o)$
|            |                   | $c_t = \Gamma_{f_t} \odot \overline{c}_{t-1} + \Gamma_{i_t} \odot \tilde{c}_t$
|            |                   | $h_t = \Gamma_{o_t} \odot \tanh(c_t)$ |
| LSTM-PC    | ![LSTM-PC Diagram](image3) | $\tilde{c}_t = \tanh(x_t.W_{2\tilde{c}} + h_{t-1}.W_{h\tilde{c}} + b_\tilde{c})$
|            |                   | $\Gamma_{i_t} = \sigma(x_t.W_{xi} + h_{t-1}.W_{hi} + c_t.W_{ci} + b_i)$
|            |                   | $\Gamma_{f_t} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + \overline{c}_{t-1}.W_{cf} + b_f)$
|            |                   | $\Gamma_{o_t} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + \overline{c}_{t-1}.W_{co} + b_o)$
|            |                   | $c_t = \Gamma_{f_t} \odot \overline{c}_{t-1} + \Gamma_{i_t} \odot \tilde{c}_t$
|            |                   | $h_t = \Gamma_{o_t} \odot \tanh(c_t)$ |
| LSTM-FGR   | ![LSTM-FGR Diagram](image4) | $\tilde{c}_t = \tanh(x_t.W_{2\tilde{c}} + h_{t-1}.W_{h\tilde{c}} + b_\tilde{c})$
|            |                   | $\Gamma_{i_t} = \sigma(x_t.W_{xi} + h_{t-1}.W_{hi} + \Gamma_{i(t-1)}.W_{ti} + \Gamma_{f(t-1)}.W_{ft} + \Gamma_{o(t-1)}.W_{ot} + b_i)$
|            |                   | $\Gamma_{f_t} = \sigma(x_t.W_{xf} + h_{t-1}.W_{hf} + \Gamma_{i(t-1)}.W_{if} + \Gamma_{f(t-1)}.W_{ff} + \Gamma_{o(t-1)}.W_{of} + b_f)$
|            |                   | $\Gamma_{o_t} = \sigma(x_t.W_{xo} + h_{t-1}.W_{ho} + \Gamma_{i(t-1)}.W_{io} + \Gamma_{f(t-1)}.W_{fo} + \Gamma_{o(t-1)}.W_{oo} + b_o)$
|            |                   | $c_t = \Gamma_{f_t} \odot \overline{c}_{t-1} + \Gamma_{i_t} \odot \tilde{c}_t$
|            |                   | $h_t = \Gamma_{o_t} \odot \tanh(c_t)$ |
Table A.17: Description of different architectures of RNN cells.

| Cell name  | Cell architecture | Cell computations |
|------------|-------------------|-------------------|
| **LSTM-SLIM1** | ![Diagram](image) | \( \tilde{c}_t = \tanh(x_t.W_{x\tilde{c}} + h_{t-1}.W_{\tilde{c}h} + b_{\tilde{c}}) \) |
|           |                   | \( \Gamma_i = \sigma(h_{t-1}.W_{hi} + b_i) \) |
|           |                   | \( \Gamma_f = \sigma(h_{t-1}.W_{hf} + b_f) \) |
|           |                   | \( \Gamma_o = \sigma(h_{t-1}.W_{ho} + b_o) \) |
|           |                   | \( c_t = \Gamma_f \odot c_{t-1} + \Gamma_i \odot \tilde{c}_t \) |
|           |                   | \( h_t = \Gamma_o \odot \tanh(c_t) \) |
| **LSTM-SLIM2** | ![Diagram](image) | \( \tilde{c}_t = \tanh(x_t.W_{x\tilde{c}} + h_{t-1}.W_{\tilde{c}h} + b_{\tilde{c}}) \) |
|           |                   | \( \Gamma_i = \sigma(h_{t-1}.W_{hi}) \) |
|           |                   | \( \Gamma_f = \sigma(h_{t-1}.W_{hf}) \) |
|           |                   | \( \Gamma_o = \sigma(h_{t-1}.W_{ho}) \) |
|           |                   | \( c_t = \Gamma_f \odot c_{t-1} + \Gamma_i \odot \tilde{c}_t \) |
|           |                   | \( h_t = \Gamma_o \odot \tanh(c_t) \) |
| **LSTM-SLIM3** | ![Diagram](image) | \( \tilde{c}_t = \tanh(x_t.W_{x\tilde{c}} + h_{t-1}.W_{\tilde{c}h} + b_{\tilde{c}}) \) |
|           |                   | \( \Gamma_i = \sigma(b_i) \) |
|           |                   | \( \Gamma_f = \sigma(b_f) \) |
|           |                   | \( \Gamma_o = \sigma(b_o) \) |
|           |                   | \( c_t = \Gamma_f \odot c_{t-1} + \Gamma_i \odot \tilde{c}_t \) |
|           |                   | \( h_t = \Gamma_o \odot \tanh(c_t) \) |
| **GRU**   | ![Diagram](image) | \( \Gamma_u = \sigma(x_t.W_{xu} + h_{t-1}.W_{hu} + b_u) \) |
|           |                   | \( \Gamma_r = \sigma(x_t.W_{xr} + h_{t-1}.W_{hr} + b_r) \) |
|           |                   | \( \tilde{h}_t = \tanh(x_t.W_{x\tilde{h}} + (\Gamma_r \odot \tilde{h}_{t-1}).W_{h\tilde{h}} + b_{\tilde{h}}) \) |
|           |                   | \( h_t = \Gamma_u \odot \tilde{h}_t + (1 - \Gamma_u) \odot h_{t-1} \) |
| **MUT1**  | ![Diagram](image) | \( \Gamma_u = \sigma(x_t.W_{xu} + b_u) \) |
|           |                   | \( \Gamma_r = \sigma(x_t.W_{xr} + h_{t-1}.W_{hr} + b_r) \) |
|           |                   | \( \tilde{h}_t = \tanh(\tanh(x_t) + (\Gamma_r \odot \tilde{h}_{t-1}).W_{h\tilde{h}} + b_{\tilde{h}}) \) |
|           |                   | \( h_t = \Gamma_u \odot \tilde{h}_t + (1 - \Gamma_u) \odot h_{t-1} \) |
| Cell name      | Cell architecture | Cell computations |
|---------------|-------------------|-------------------|
| MUT2          | ![Diagram of MUT2](image) | $\Gamma_{u_t} = \sigma(x_t.W_{xu} + h_{t-1}.W_{hu} + b_u)$  
$\Gamma_{r_t} = \sigma(x_t + h_{t-1}.W_{hr} + b_r)$  
$\tilde{h}_t = \tanh(x_t.W_{x\tilde{h}} + (\Gamma_{r_t} \odot h_{t-1}).W_{h\tilde{h}} + b_{\tilde{h}})$  
$h_t = \Gamma_{u_t} \odot \tilde{h}_t + (1 - \Gamma_{u_t}) \odot h_{t-1}$ |
| MUT3          | ![Diagram of MUT3](image) | $\Gamma_{u_t} = \sigma(x_t.W_{xu} + \tanh(h_{t-1}).W_{hu} + b_u)$  
$\Gamma_{r_t} = \sigma(x_t.W_{xr} + h_{t-1}.W_{hr} + b_r)$  
$\tilde{h}_t = \tanh(x_t.W_{x\tilde{h}} + (\Gamma_{r_t} \odot h_{t-1}).W_{h\tilde{h}} + b_{\tilde{h}})$  
$h_t = \Gamma_{u_t} \odot \tilde{h}_t + (1 - \Gamma_{u_t}) \odot h_{t-1}$ |
| GRU-SLIM1     | ![Diagram of GRU-SLIM1](image) | $\Gamma_{u_t} = \sigma(h_{t-1}.W_{hu} + b_u)$  
$\Gamma_{r_t} = \sigma(h_{t-1}.W_{hr} + b_r)$  
$\tilde{h}_t = \tanh(x_t.W_{x\tilde{h}} + (\Gamma_{r_t} \odot h_{t-1}).W_{h\tilde{h}} + b_{\tilde{h}})$  
$h_t = \Gamma_{u_t} \odot \tilde{h}_t + (1 - \Gamma_{u_t}) \odot h_{t-1}$ |
| GRU-SLIM2     | ![Diagram of GRU-SLIM2](image) | $\Gamma_{u_t} = \sigma(h_{t-1}.W_{hu})$  
$\Gamma_{r_t} = \sigma(h_{t-1}.W_{hr})$  
$\tilde{h}_t = \tanh(x_t.W_{x\tilde{h}} + (\Gamma_{r_t} \odot h_{t-1}).W_{h\tilde{h}} + b_{\tilde{h}})$  
$h_t = \Gamma_{u_t} \odot \tilde{h}_t + (1 - \Gamma_{u_t}) \odot h_{t-1}$ |
| GRU-SLIM3     | ![Diagram of GRU-SLIM3](image) | $\Gamma_{u_t} = \sigma(b_u)$  
$\Gamma_{r_t} = \sigma(b_r)$  
$\tilde{h}_t = \tanh(x_t.W_{x\tilde{h}} + (\Gamma_{r_t} \odot h_{t-1}).W_{h\tilde{h}} + b_{\tilde{h}})$  
$h_t = \Gamma_{u_t} \odot \tilde{h}_t + (1 - \Gamma_{u_t}) \odot h_{t-1}$ |
Table A.17: – Continued from previous page

| Cell name | Cell architecture | Cell computations |
|-----------|-------------------|-------------------|
| MGU       | ![MGU Diagram]    | \[\Gamma_{f_t} = \sigma(x_tW_{xf} + h_{t-1}W_{hf} + b_f)\]  
            |                   | \[\tilde{h}_t = \tanh(x_tW_{x\tilde{h}} + (\Gamma_{f_t} \otimes h_{t-1})W_{h\tilde{h}} + b_{\tilde{h}})\]  
            |                   | \[h_t = \Gamma_{f_t} \otimes \tilde{h}_t + (1 - \Gamma_{f_t}) \otimes h_{t-1}\] |
| MGU-SLIM1 | ![MGU-SLIM1 Diagram] | \[\Gamma_{f_t} = \sigma(h_{t-1}W_{hf} + b_f)\]  
           |                   | \[\tilde{h}_t = \tanh(x_tW_{x\tilde{h}} + (\Gamma_{f_t} \otimes h_{t-1})W_{h\tilde{h}} + b_{\tilde{h}})\]  
           |                   | \[h_t = \Gamma_{f_t} \otimes \tilde{h}_t + (1 - \Gamma_{f_t}) \otimes h_{t-1}\] |
| MGU-SLIM2 | ![MGU-SLIM2 Diagram] | \[\Gamma_{f_t} = \sigma(h_{t-1}W_{hf})\]  
           |                   | \[\tilde{h}_t = \tanh(x_tW_{x\tilde{h}} + (\Gamma_{f_t} \otimes h_{t-1})W_{h\tilde{h}} + b_{\tilde{h}})\]  
           |                   | \[h_t = \Gamma_{f_t} \otimes \tilde{h}_t + (1 - \Gamma_{f_t}) \otimes h_{t-1}\] |
| MGU-SLIM3 | ![MGU-SLIM3 Diagram] | \[\Gamma_{f_t} = \sigma(b_f)\]  
           |                   | \[\tilde{h}_t = \tanh(x_tW_{x\tilde{h}} + (\Gamma_{f_t} \otimes h_{t-1})W_{h\tilde{h}} + b_{\tilde{h}})\]  
           |                   | \[h_t = \Gamma_{f_t} \otimes \tilde{h}_t + (1 - \Gamma_{f_t}) \otimes h_{t-1}\] |
| SCRN      | ![SCRN Diagram]   | \[s_t = (1 - \alpha)x_tW_{zs} + \alpha.s_{t-1}\]  
            |                   | \[\alpha \in [0, 1]\]  
            |                   | \[h_t = \tanh(x_tW_{zh} + h_{t-1}W_{hh} + s_{t-1}W_{sh} + b_h)\] |
Table A.17:  – Continued from previous page

| Cell name | Cell architecture | Cell computations |
|-----------|-------------------|-------------------|
| MRNN      | ![MRNN Diagram](image) | $h_t = \tanh(x_t.W_{xh} + h_{t-1}.W_{hh} + \hat{y}_{t-1}.W_{yh} + b_h)$ |
| JORDAN    | ![JORDAN Diagram](image) | $h_t = \tanh(x_t.W_{xh} + \hat{y}_{t-1}.W_{yh} + b_h)$ |
| IRNN      | ![IRNN Diagram](image) | $h_t = \text{ReLU}(x_t.W_{xh} + h_{t-1}.W_{hh} + b_h)$ |
| ELMAN     | ![ELMAN Diagram](image) | $h_t = \tanh(x_t.W_{xh} + h_{t-1}.W_{hh} + b_h)$ |