Some Properties of Kenmotsu Manifolds Admitting a Semi-symmetric Non-metric Connection

S. K. Chaubey
Section of Mathematics, Department of IT, Shinas college of technology, Shinas, P.O. Box 77, Postal Code 324, Sultanate of Oman. Email: sk22_math@yahoo.co.in

A. C. Pandey
Department of Mathematics, Bramanand P. G. College, Kanpur – 208004, U. P., India. Email: acpbnd73@gmail.com

N. V. C. Shukla
Department of Mathematics and Astronomy, Lucknow University - 226007, U.P., India. Email: nvcshukla72@gmail.com

Abstract
The aim of this paper is to study generalized recurrent, generalized Ricci-recurrent, weakly symmetric and weakly Ricci-symmetric Kenmotsu manifolds with respect to the semi-symmetric non-metric connection.

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1 Introduction
Let \((M_n, g)\) be a Riemannian manifold of dimension \(n\). A linear connection \(\nabla\) in \((M_n, g)\), whose torsion tensor \(T\) of type \((1, 2)\) is defined as

\[
T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y],
\]

for arbitrary vector fields \(X\) and \(Y\), is said to be torsion free or symmetric if \(T\) vanishes, otherwise it is non-symmetric. If the connection \(\nabla\) satisfy \(\nabla g = 0\) in \((M_n, g)\), then it is called metric connection otherwise it is non-metric. Friedmann and Schouten [1] introduced the notion of semi-symmetric linear connection on a differentiable manifold. Hayden [2] introduced the idea of semi-symmetric linear connection with non-zero torsion tensor on a Riemannian manifold. The idea of semi-symmetric metric connection on Riemannian manifold was introduced by Yano [3]. He proved that a Riemannian
manifold with respect to the semi-symmetric metric connection has vanishing curvature tensor if and only if it is conformally flat. This result was generalize for vanishing Ricci tensor of the semi-symmetric metric connection by T. Imai ([4], [6]). Various properties of such connection have studied in ([11], [22]) and by many other geometers. Agashe and Chaflie [13] defined and studied a semi-symmetric non-metric connection in a Riemannian manifold. This was further developed by Agashe and Chaflie [14], De and Kamilya [17], Pandey and Ojha [15], Chaturvedi and Pandey [19] and others. Sengupta, De and Binh [22], De and Sengupta [16] defined new type of semi-symmetric non-metric connections on a Riemannian manifold and studied some geometrical properties with respect to such connections. In this connection, the properties of non-metric connections have studied in ([37], [24], [23], [38], [39]) and many others. In 2008, Tripathi introduced the generalized form of a new connection in Riemannian manifold [36]. Chaubey [20, 21] defined semi-symmetric non-metric connections on an almost contact metric manifold and studied its different geometrical properties. Some properties of such connections have been noticed in ([25], [26], [27], [28], [43]) and others.

In 1972, K. Kenmotsu [18] introduced a class of contact Riemann manifold known as Kenmotsu Manifold. He studied that if a Kenmotsu manifold satisfies the condition $R(X,Y)Z = 0$, then the manifold is of negative curvature -1, where $R$ is the Riemannian curvature tensor of type $(1,3)$ and $R(X,Y)Z$ is derivative of tensor algebra at each point of the tangent space. Several properties of Kenmotsu Manifold have been studied by Sinha and Srivastav [7], De [8], De and Pathak [9], Chaubey et al. ([10], [11], [12]) and many others. Ozgur [32] studied generalised recurrent Kenmotsu manifold and proved that if $M$ be a generalised recurrent Kenmotsu manifold and generalised Ricci Recurrent Kenmotsu manifold then $\beta = \alpha$ holds on $M$. Sular studies the generalised recurrent and generalised Ricci recurrent Kenmotsu manifolds with respect to semi symmetric metric connection and proved that $\beta = 2\alpha$ where $\alpha$ and $\beta$ are smooth functions and $M$ is generalised recurrent and generalised Ricci recurrent Kenmotsu manifold admitting a semi-symmetric connection [35]. In the present paper, we studied the properties of semi-symmetric non-metric connection in Kenmotsu manifolds.

The present paper is organized as follows. Section 2 is preliminaries in which basic concepts of Kenmotsu manifolds are given. Section 3 deals with the brief account of semi-symmetric non-metric connection. In section 4, we define a generalized recurrent Kenmotsu manifolds with respect to the semi-symmetric non-metric connection and studied its some properties. Section 5 is concerned with the weakly symmetric Kenmotsu manifolds with respect to the semi-symmetric non-metric connection.

## 2 Preliminaries

An $n$-dimensional Riemannian manifold $(M_n, g)$ of class $C^\infty$ with a 1-form $\eta$, the associated vector field $\xi$ and a $(1,1)$ tensor field $\phi$ satisfying

$$\phi^2 X + X = \eta(X)\xi,$$  \hspace{1cm} (2)
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for arbitrary vector field $X$, is called an almost contact manifold. This system $(\phi, \xi, \eta)$ is called an almost contact structure to $M_n$ [5]. If the associated Riemannian metric $g$ in $M_n$ satisfy

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$

for arbitrary vector fields $X$, $Y$ in $M_n$, then $(M_n, g)$ is said to be an almost contact metric manifold. Putting $\xi$ for $X$ in (4) and using (3), we obtain

$$g(\xi, Y) = \eta(Y).$$

Also,

$$\phi(X, Y) = g(\phi X, Y)$$

with $\phi = d\eta$ is 2-form.

If moreover

$$D_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X,$$

$$D_X \xi = X - \eta(X)\xi,$$

hold in $(M_n, g)$, where $D$ being the Levi-Civita connection of the Riemannian metric $g$, then $(M_n, g)$ is called a Kenmotsu manifold [18]. Also the following relations hold in a Kenmotsu manifold

$$K(X, Y)\xi = \eta(X)Y - \eta(Y)X,$$

$$K(\xi, X)Y = \eta(Y)X - g(X, Y)\xi,$$

$$S(X, \xi) = -(n - 1)\eta(X),$$

$$D_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y)$$

for arbitrary vector fields $X$ and $Y$, where $K$ and $S$ denote the Riemannian curvature and Ricci tensors of the connection $D$ respectively.

3 Semi-symmetric non-metric connection

A linear connection $\nabla$ on $(M_n, g)$ is said to be a semi-symmetric non-metric connection if the torsion tensor $T$ of the connection $\nabla$ and the Riemannian metric $g$ satisfy the following conditions:

$$T(X, Y) = 2\phi(X, Y)\xi,$$

$$(\nabla_X g)(Y, Z) = -\eta(Y)\phi(X, Z) - \eta(Z)\phi(X, Y),$$
for arbitrary vector fields $X$, $Y$ and $Z$, where $\eta$ is $1$–form on $(M_n, g)$ with $\xi$ as associated vector field. If $D$ denotes the Levi-Civita connection, then the semi-symmetric non-metric connection \[ \nabla \] on $(M_n, g)$ is defined as

\[ \nabla_X Y = D_X Y + g(\phi X, Y)\xi, \]  

for arbitrary vector fields $X$ and $Y$.

The curvature tensor $R$ of the semi-symmetric non-metric connection \[ \nabla \] is defined as

\[ R(X, Y)Z = K(X, Y)Z + g(\phi Y, Z)D_X \xi - g(\phi X, Z)D_Y \xi + g((D_X \phi)(Y) - (D_Y \phi)(X), Z)\xi. \]  

From (3), (5), (8) and (9), it follows that

\[ R(X, Y)Z = K(X, Y)Z + g(\phi Y, Z)X - g(\phi X, Z)Y + 2\eta(Z)g(\phi X, Y)\xi \]  

which give

\[ \tilde{S}(Y, Z) = S(Y, Z) + (n - 1)g(\phi Y, Z) \]  

and

\[ \tilde{r} = r. \]

Here $\tilde{S}$ and $\tilde{r}$ denote the Ricci tensor and scalar curvature with respect to the semi-symmetric non-metric connection $\nabla$ and $r$ is the scalar curvature with respect to the Levi-Civita connection $D$. From (20) we lead the following corollary:

**Corollary 1** Let $M_n$ be an $n$–dimensional Kenmotsu manifold equipped with a semi-symmetric non-metric connection $\nabla$, then the scalar curvature with respect to semi-symmetric non-metric connection is equal to scalar curvature with respect to Levi-Civita connection.

Replacing $Z$ by $\xi$ in (18) and (19) and then using (3) and (5), we get

\[ R(X, Y)\xi = K(X, Y)\xi + 2g(\phi X, Y)\xi \]  

and

\[ \tilde{S}(Y, \xi) = S(Y, \xi). \]

### 4 Generalized Recurrent Kenmotsu Manifolds

**Definition 1** A non-flat $n$–dimensional differentiable manifold $M_n$, $(n > 3)$, is called generalized recurrent manifold \[ 22 \] if its curvature tensor $K$ satisfies the condition

\[ (D_X K)(Y, Z)W = A(X)K(Y, Z)W + B(X)[g(Z, W)Y - g(Y, W)Z], \]  

where $A(X)$ and $B(X)$ are functions on $M_n$. This condition is called the recurrence relation for the curvature tensor.
where $A$ and $B \neq 0$ are 1–forms defined as

$$A(X) = g(X, \rho_1), \quad B(X) = g(X, \rho_2),$$

for arbitrary vector fields $X$, $Y$, $Z$ and $W$. Here $\rho_1$ and $\rho_2$ are the vector fields associated with the 1–forms $A$ and $B$ respectively.

**Definition 2** A non-flat $n$–dimensional differentiable manifold $M_n$, ($n > 3$), is called generalized Ricci-recurrent [29] if its Ricci tensor $S$ satisfies the condition

$$(D_X S)(Y, Z) = A(X) S(Y, Z) + (n - 1) B(X) g(Y, Z),$$

for arbitrary vector fields $X$, $Y$ and $Z$, where $A$ and $B$ are defined as in (24).

In the similar fashion, we defined the following definitions:

**Definition 3** A non-flat $n$–dimensional differentiable manifold $M_n$, ($n > 3$), is called generalized recurrent with respect to the semi-symmetric non-metric connection $\nabla$ if its curvature tensor $R$ satisfies the condition

$$(\nabla_X R)(Y, Z)W = A(X) R(Y, Z)W + B(X) [g(Z, W)Y - g(Y, W)Z],$$

for arbitrary vector fields $X$, $Y$, $Z$ and $W$.

**Definition 4** A non-flat $n$–dimensional differentiable manifold $M_n$, ($n > 3$), is called generalized Ricci-recurrent with respect to the semi-symmetric non-metric connection $\tilde{\nabla}$ if its Ricci tensor $\tilde{S}$ satisfies the condition

$$(\nabla_X \tilde{S})(Y, Z) = A(X) \tilde{S}(Y, Z) + (n - 1) B(X) g(Y, Z),$$

for arbitrary vector fields $X$, $Y$, $Z$, where $A$ and $B$ are defined as in (24). Now we consider the generalized recurrent and generalized Ricci-recurrent Kenmotsu manifolds admitting the semi-symmetric non-metric connection $\nabla$ and prove the following theorems:

**Theorem 4.1** Let $M_n$ be an $n$–dimensional generalized recurrent Kenmotsu manifold equipped with a semi-symmetric non-metric connection $\nabla$. Then $B = A$ holds on $M_n$.

**Proof 1** Replacing $Y$ and $W$ by $\xi$ in (26) and using (3) and (5), we obtain

$$(\nabla_X R)(\xi, Z)\xi = A(X) R(\xi, Z)\xi + B(X) [\eta(Z)\xi - Z].$$

In consequence of (10) and (21), (28) becomes

$$(\nabla_X R)(\xi, Z)\xi = (B(X) - A(X)) [\eta(Z)\xi - Z].$$

It can be easily seen that

$$(\nabla_X R)(\xi, Z)\xi = \nabla_X R(\xi, Z)\xi - R(\nabla_X \xi, Z)\xi - R(\xi, \nabla_X Z)\xi - R(\xi, Z)\nabla_X \xi.$$
From (3), (10), (21) and (30), it follows that
\[(\nabla_X R)(\xi, Z)\xi = 0.\] (31)

In view of (29) and (31), we get
\[(B(X) - A(X)) [\eta(Z)\xi - Z] = 0.\] (32)

Since $Z \neq \eta(Z)\xi$ in general, therefore $B = A$.

**Theorem 4.2** If an $n$–dimensional generalized Ricci-recurrent Kenmotsu manifold $M_n$ admitting a semi-symmetric non-metric connection $\nabla$, then $B = A$ holds on $M_n$.

**Proof 2** Replacing $Z$ by $\xi$ in (27) and then using (5), (12) and (22), we obtain
\[(\nabla_X \tilde{S})(Y, \xi) = (n - 1)\eta(Y) [B(X) - A(X)].\] (33)

It is obvious that
\[(\nabla_X \tilde{S})(Y, \xi) = \nabla_X \tilde{S}(Y, \xi) - \tilde{S}(\nabla_X Y, \xi) - \tilde{S}(Y, \nabla_X \xi).\] (34)

In consequence of (3), (5), (12), (13), (16) and (22), (34) becomes
\[(\nabla_X \tilde{S})(Y, \xi) = -(n - 1)g(X, Y) + 2(n - 1)g(\phi X, Y) - S(X, Y).\] (35)

From (33) and (35), it follows that
\[(n - 1)\eta(Y) [B(X) - A(X)] = -(n - 1)g(X, Y) + 2(n - 1)g(\phi X, Y) - S(X, Y).\] (36)

Putting $Y = \xi$ in (36) and using (3), (5) and (12), we obtain $B = A$.

5 Weakly symmetric Kenmotsu manifolds

**Definition 5** A non-flat $n$–dimensional differentiable manifold $M_n$, ($n > 3$), is called pseudo symmetric [30] if there is a 1–form $A$ on $M_n$ such that
\[
(D_X K)(Y, Z)W = 2A(X)K(Y, Z)W + A(Y)K(X, Z)W + A(Z)K(Y, X)W
+ A(W)K(Y, Z)X + g(K(Y, Z)W, X)\rho_1,
\] (37)

where $D$ is the Levi-Civita connection and $X$, $Y$, $Z$ and $W$ are arbitrary vector fields on $M_n$. The vector field $\rho_1$ associated with the 1–form $A$ is defined by $A(X) = g(X, \rho_1)$.
Definition 6 A non-flat \( n \)-dimensional differentiable manifold \( M_n \), \( (n > 3) \), is called weakly symmetric \([33, 34]\) if there are 1-forms \( A, B, C \) and \( D \) on \( M_n \) such that

\[
(D_X K)(Y, Z)W = A(X)K(Y, Z)W + B(Y)K(X, Z)W + C(Z)K(Y, X)W \\
+ D(W)K(Y, Z)X + g(K(Y, Z)W, X)\sigma,
\]

where \( X, Y, Z, W \) are arbitrary vector fields on \( M_n \). The vector field \( \sigma \) associated with the 1-form \( p \) is defined as \( p(X) = g(\sigma, X) \). A weakly symmetric manifold \( M_n \) is said to be pseudo symmetric if \( B = C = D = A = \sigma = \rho_1 \) and \( A \) is replaced by \( 2A \), locally symmetric if \( A = B = C = D = 0 \) and \( \sigma = 0 \). A weakly symmetric manifold is said to be proper if at least one of the 1-forms \( A, B, C \) and \( D \) is non zero or \( \sigma \neq 0 \).

Definition 7 A non-flat \( n \)-dimensional differentiable manifold \( M_n \), \( (n > 3) \), is called weakly Ricci-symmetric \([33, 34]\) if there are 1-forms \( \alpha, \beta \) and \( \gamma \) on \( M_n \) such that

\[
(D_X S)(Y, Z) = \alpha(X)S(Y, Z) + \beta(Y)S(X, Z) + \gamma(Z)S(X, Y),
\]

where \( X, Y \) and \( Z \) are arbitrary vector fields on \( M_n \). A weakly Ricci-symmetric manifold \( M_n \) is called pseudo Ricci-symmetric if \( \alpha = \beta = \gamma \). Contracting \((38)\) with respect to \( Y \), we get

\[
(D_X S)(Z, W) = A(X)S(Z, W) + B(K(X, Z)W + C(Z)S(W, X) \\
+ D(W)S(X, Z) + p(K(X, W)Z),
\]

where \( p \) is defined as \( p(X) = g(\sigma, X) \) for arbitrary vector field \( X \). The author \([40]\) studied the properties of weakly and weakly Ricci symmetric manifolds with examples.

Similarly we define the following definitions:

Definition 8 A non-flat \( n \)-dimensional differentiable manifold \( M_n \), \( (n > 3) \), is called weakly symmetric with respect to the semi-symmetric non-metric connection \( \nabla \) if there are 1-forms \( A, B, C \) and \( D \) on \( M_n \) such that

\[
(\nabla_X R)(Y, Z)W = A(X)R(Y, Z)W + B(Y)R(X, Z)W + C(Z)R(Y, X)W \\
+ D(W)R(Y, Z)X + g(R(Y, Z)W, X)\sigma,
\]

where \( X, Y, Z, W \) are arbitrary vector fields on \( M_n \) and the 1-forms \( A, B, C, D \) and the vector field \( \sigma \) are defined previously.

Definition 9 A non-flat \( n \)-dimensional differentiable manifold \( M_n \), \( (n > 3) \), is called weakly Ricci-symmetric with respect to the semi-symmetric non-metric connection \( \nabla \) if there are 1-forms \( \alpha, \beta \) and \( \gamma \) on \( M_n \) such that

\[
(\nabla_X S)(Y, Z) = \alpha(X)S(Y, Z) + \beta(Y)S(X, Z) + \gamma(Z)S(X, Y),
\]

where \( X, Y, Z \) are arbitrary vector fields on \( M_n \).
Contracting (41) with Y, we get
\[
(\nabla_X \tilde{S})(Z, W) = A(X)\tilde{S}(Z, W) + B(R(X, Z)W) + C(Z)\tilde{S}(W, X) + D(W)\tilde{S}(X, Z) + p(R(X, W)Z),
\] (43)
where \(p\) is defined as \(p(X) = g(X, \sigma)\) for arbitrary vector field \(X\).

Özgür [31] considered weakly symmetric and weakly Ricci-symmetric Kenmotsu manifolds and proved the following theorems:

**Theorem 5.1** There is no weakly symmetric Kenmotsu manifold \(M\), \((n > 3)\), unless \(A + C + D\) is everywhere zero.

**Theorem 5.2** There is no weakly Ricci-symmetric Kenmotsu manifold \(M\), \((n > 3)\), unless \(\alpha + \beta + \gamma\) is everywhere zero.

Sular [33] considered weakly symmetric and weakly Ricci-symmetric Kenmotsu manifolds with respect to the semi-symmetric metric connection and proved the following results:

**Theorem 5.3** There is no weakly symmetric Kenmotsu manifold \(M\) admitting a semi-symmetric metric connection, \((n > 3)\), unless \(A + C + D\) is everywhere zero.

**Theorem 5.4** There is no weakly Ricci-symmetric Kenmotsu manifold \(M\) admitting a semi-symmetric metric connection, \((n > 3)\), unless \(\alpha + \beta + \gamma\) is everywhere zero.

Now we consider the weakly symmetric and weakly Ricci-symmetric Kenmotsu manifolds admitting the semi-symmetric non-metric connection \(\nabla\) and prove the following theorems:

**Theorem 5.5** Let \(M_n\), \((n > 3)\) be an \(n\)−dimensional weakly symmetric Kenmotsu manifold admitting a semi-symmetric non-metric connection \(\nabla\) then there is no \(M_n\), unless \(A + C + D\) is everywhere zero.

**Proof 3** Replacing \(W\) by \(\xi\) in (43) and using (3), (10), (11), (12), (18), (21) and (22) we obtain
\[
(\nabla_X \tilde{S})(Z, \xi) = -(n-1)A(X)\eta(Z) + \eta(X)B(Z) - \eta(Z)B(X) - (n-1)C(Z)\eta(X) + D(\xi)S(X, Z) - \eta(Z)p(X) + p(\xi)g(X, Z) + (n-1)D(\xi)g(\phi X, Z) - p(\xi)g(\phi X, Z). \] (44)

From (35) and (44), it follows that
\[
-(n-1)g(X, Z) + 2(n-1)g(\phi X, Z) - S(X, Z)
= -(n-1)A(X)\eta(Z) + \eta(X)B(Z) - \eta(Z)B(X) - (n-1)C(Z)\eta(X) + D(\xi)S(X, Z) - \eta(Z)p(X) + p(\xi)g(X, Z) + (n-1)D(\xi)g(\phi X, Z) - p(\xi)g(\phi X, Z). \] (45)
Replacing $X$ and $Z$ by $\xi$ in (45) and using (3), (5) and (12), we get
\[ A(\xi) + C(\xi) + D(\xi) = 0. \] (46)

Putting $Z = \xi$ in (43) and using (3), (5), (10), (11), (12), (18), (21), we obtain
\[ (\nabla_X \bar{S})(\xi, W) = -(n - 1)A(X)\eta(W) + g(X, W)B(\xi) - \eta(W)B(X) + \eta(X)p(W) \]
\[ -g(\phi X, W)B(\xi) + C(\xi)S(W, X) + (n - 1)C(\xi)g(\phi W, X) \]
\[ -(n - 1)D(W)\eta(X) - \eta(W)p(X) + 2g(\phi X, W)p(\xi). \] (47)

In consequence of (33) and (47), we have
\[ -(n - 1)g(\phi X, W) - S(X, W) \]
\[ -(n - 1)A(X)\eta(W) + g(X, W)B(\xi) - \eta(W)B(X) + \eta(X)p(W) \]
\[ -g(\phi X, W)B(\xi) + C(\xi)S(W, X) + (n - 1)C(\xi)g(\phi W, X) \]
\[ -(n - 1)D(W)\eta(X) - \eta(W)p(X) + 2g(\phi X, W)p(\xi). \] (48)

Putting $W = \xi$ in (48) and using (3), (5) and (12), we get
\[ \eta(X)B(\xi) - p(X) - B(X) - (n - 1)C(\xi)\eta(X) \]
\[ -(n - 1)D(\xi)\eta(X) + \eta(X)p(\xi) - (n - 1)A(X) = 0. \] (49)

Replacing $X$ with $\xi$ in (48) and then using (3), (5) and (12), we find
\[ -(n - 1)A(\xi)\eta(W) - (n - 1)C(\xi)\eta(W) - (n - 1)D(W) + p(W) - \eta(W)p(\xi) = 0. \] (50)

Replacing $W$ by $X$ in (50), we get
\[ -(n - 1)A(\xi)\eta(X) - (n - 1)C(\xi)\eta(X) - (n - 1)D(X) + p(X) - \eta(X)p(\xi) = 0. \] (51)

Adding (49) and (51) and using (46), we obtain
\[ \eta(X)B(\xi) - (n - 1)A(X) - (n - 1)D(X) - B(X) - (n - 1)C(\xi)\eta(X) = 0. \] (52)

Taking $X = \xi$ in (43) and then using (3), (5) and (12), we find
\[ B(Z) - (n - 1)A(\xi)\eta(Z) - \eta(Z)B(\xi) - (n - 1)C(Z) - (n - 1)D(\xi)\eta(Z) = 0. \] (53)

Replacing $Z$ by $X$ in (53), we get
\[ B(X) - (n - 1)A(\xi)\eta(X) - \eta(X)B(\xi) - (n - 1)C(X) - (n - 1)D(\xi)\eta(X) = 0. \] (54)

Adding (52) and (54) and using (46), we have
\[ A(X) + C(X) + D(X) = 0. \] (55)

Hence the statement of the theorem.
Theorem 5.6 Let $M_n$, $(n > 3)$, be an $n$–dimensional weakly Ricci-symmetric Kenmotsu manifold admitting a semi-symmetric non-metric connection $\nabla$ then there is no $M_n$, unless $\alpha + \beta + \gamma$ is everywhere zero.

Proof 4 Putting $Z = \xi$ in (42) and then using (12), (19) and (22), we find

$$
(\nabla_X \tilde{S})(Y, \xi) = -(n-1)\alpha(X)\eta(Y) - (n-1)\beta(Y)\eta(X) + \gamma(\xi)S(X, Y) + (n-1)\gamma(\xi)g(\phi X, Y).
$$

(56)

In consequence of (33) and (56), we have

$$
-(n-1)g(X, Y) + 2(n-1)g(\phi X, Y) - S(X, Y) = -(n-1)\alpha(X)\eta(Y) - (n-1)\beta(Y)\eta(X) + \gamma(\xi)S(X, Y) + (n-1)\gamma(\xi)g(\phi X, Y).
$$

(57)

Taking $X = Y = \xi$ in (57) and using (3), (5) and (12), we find

$$
\alpha(\xi) + \beta(\xi) + \gamma(\xi) = 0.
$$

(58)

Replacing $X$ by $\xi$ in (57) and using (3), (5), (12) and (58), we get

$$
\beta(Y) = \beta(\xi)\eta(Y).
$$

(59)

Again replacing $Y$ by $\xi$ in (57) and using (3), (5), (12) and (58), we obtain

$$
\alpha(X) = \alpha(\xi)\eta(X).
$$

(60)

From (3), (9), (12), (13), (16) and (22), it follows that

$$
(\nabla_\xi \tilde{S})(\xi, X) = 0.
$$

(61)

In view of (42), above equation becomes

$$
\alpha(\xi)\tilde{S}(\xi, X) + \beta(\xi)\tilde{S}(\xi, X) + \gamma(\xi)\tilde{S}(\xi, \xi) = 0.
$$

(62)

With the help of (3), (5), (12) and (22), equation (62) gives

$$
\gamma(X) = \gamma(\xi)\eta(X).
$$

(63)

Adding (57), (60) and (63) and using (58), we get the statement of the theorem.
References

[1] A. Friedmann and J. A. Schouten, Über die Geometric der halbsymmetrischen Übertragung, Math., Zeitschr., 21 (1924), 211-223.

[2] H. A. Hayden, Subspace of a space with torsion, Proceedings of London Mathematical Society II Series 34 (1932), 27-50.

[3] K. Yano, On semi-symmetric metric connections, Rev. Roumaine Math. Pures Appl., 15 (1970), 1579-1586.

[4] T. Imai, Notes on semi-symmetric metric connections, Tensor N. S., 24 (1972), 293-296.

[5] D. E. Blair, Contact manifold in Riemannian Geometry, Lecture notes in Mathematics, 509, Springer Verlag Berlin, 1976.

[6] T. Imai, Hypersurfaces of a Riemannian manifold with semi-symmetric metric connection, Tensor N. S., 23 (1972), 300-306.

[7] B. B. Sinha and A. K. Srivastava, Curvature on Kenmotsu manifold, Indian J. Pure and Appl. Math. 22,(1) (1991),23-28.

[8] U. C. De, On φ symmetric Kenmotsu manifolds, International electronic Jour. of geometry.1 (2008),33-38.

[9] U. C. De and G. Pathak, On three dimensional Kenmotsu manifolds, Indian Jour. Pure Appl. Math ,35(2004) 159-165.

[10] S. K. Chaubey and R. H. Ojha : On the m-projective curvature tensor of a Kenmotsu manifold, Differential Geometry-Dynamical Systems, 12, 2010, 52-60.

[11] S. K. Chaubey and C. S. Prasad, ON generalized φ recurrent Kenmotsu manifolds, TWMS J. App. Eng. Math. V.5, N.1, 2015, pp. 1-9.

[12] S. K. Chaubey, S. Prakash and R. Nivas, Some properties of m—projective curvature tensor in Kenmotsu manifolds, Bulletin of Mathematical Analysis and Applications, 4 (3), (2012), 48-56.

[13] N. S. Agashe and M. R. Chafle, A semi-symmetric non-metric connection in a Riemannian manifold, Indian J. Pure Appl. Math., 23 (1992), 399–409.

[14] N. S. Agashe and M. R. Chafle, On submanifolds of a Riemannian manifold with semi-symmetric non-metric connection, Tensor N. S., 55, no. 2 (1994), pp. 120–130.

[15] L. K. Pandey, and R. H. Ojha, Semi-symmetric metric and non-metric connections in Lorentzian Paracontact manifold, Bull. Cal. Math. Soc., 93, no. 6 (2001), pp. 497–504.
[16] U. C. De and J. Sengupta, On a type of semi-symmetric non-metric connection, Bull. Cal. Math. Soc., 92, no. 5 (2000), pp. 375–384.

[17] U. C. De and D. Kamila, Hypersurfaces of a Riemannian manifold with semi-symmetric non-metric connection, J. Indian Inst. Sci., 75, (1995), pp. 707–710.

[18] K. Kenmotsu, A class of almost contact Riemannian manifolds, Tohoku Math. J., 24 (1972), 93-103.

[19] B. B. Chaturvedi and P. N. Pandey, Semi-symmetric non-metric connection on a Kähler manifold, Differential Geometry- Dynamical Systems, 10, (2008), pp. 86–90.

[20] S. K. Chaubey and R. H. Ojha, On semi-symmetric non-metric and quarter-symmetric metric connections, Tensor N. S., 70, 2. (2008) 202–213.

[21] S. K. Chaubey, On a semi-symmetric non-metric connection, https://arxiv.org/abs/1711.01035.

[22] J. Sengupta, U. C. De and T. Q. Binh, On a type of semi-symmetric non-metric connection on a Riemannian manifold, Indian J. Pure Appl. Math., 31, no. (1-2) (2000), pp. 1659–1670.

[23] A. K. Dubey, S. K. Chaubey and R. H. Ojha, On semi-symmetric non-metric connection, International Mathematical Forum, 5, no. 15 (2010), pp. 731–737.

[24] A. Kumar and S. K. Chaubey, A semi-symmetric non-metric connection in a generalised cosymplectic manifold, Int. Journal of Math. Analysis, 4, no. 17 (2010), pp. 809–817.

[25] J. P. Jaiswal and R. H. Ojha, Some properties of K-contact Riemannian manifolds admitting a semi-symmetric non-metric connection, Filomat 24:4, (2010), pp. 9–16.

[26] S. K. Chaubey and R. H. Ojha, On a semi-symmetric non-metric connection, Filomat, 26:2, (2012), pp. 63-69.

[27] S. K. Chaubey, Almost contact metric manifolds admitting semi-symmetric non-metric connection, Bulletin of Mathematical Analysis and Applications, 3 (2), (2011), 252-260.

[28] S. K. Chaubey and A. C. Pandey, Some properties of a semi-symmetric non-metric connection on a Sasakian manifold, Int. J. Contemp. Math. Sciences, Vol. 8, 2013, no. 16, 789 - 799.

[29] U. C. De and N. Guha, On generalized recurrent manifolds, Proc. Math. Soc., 7 (1991), 7-11.
Some Properties of Kenmotsu Manifold admitting ... connection

[30] M. C. Chaki, On pseudo symmetric manifolds. Analele Stiintifice ale Universitatii Al. I. Cuza din Iasi, 33: 5358, 1987.

[31] C. Özgür, On weakly symmetric Kenmotsu manifolds, Differential Geometry- Dynamical Systems, 8 (2006), 204-209.

[32] C. Özgür, On generalized recurrent Kenmotsu manifolds, World Applied Sciences J., 2 (2007), no. 1, 29-33.

[33] L. Tamássy and T. Q. Binh, On weakly symmetric and weakly projective symmetric Riemannian manifolds, Colloq. Math. Soc. J. Bolyai, 56 (1992), 663-670.

[34] L. Tamássy and T. Q. Binh, On weak symmetries of Einstein and Sasakian manifolds, Tensor N. S., 53 (1993), no. 1, 140-148.

[35] S. Sular, Some properties of a Kenmotsu manifold with a semi-symmetric metric connection, International Electronic Journal of Geometry, 3 (2010), no. 1, 24-34.

[36] M. M. Tripathi, A new connection in a Riemannian manifold, International Electronic Journal of Geometry, 1 No. 1 (2008), pp. 15-24.

[37] C. Özgür, On submanifolds of a Riemannian manifold with a semi-symmetric non-metric connection, Kuwait J. Sci. Engg., (Accepted).

[38] A. K. Dubey, R. H. Ojha and S. K. Chaubey, Some properties of quarter-symmetric non-metric connection in a Kähler manifold, Int. J. Contemp. Math. Sciences, Vol. 5, 2010, no. 20, 1001 - 1007.

[39] S. K. Chaubey and R. H. Ojha, On quarter-symmetric non-metric connection on almost Hermitian manifold, Bulletin of Mathematical Analysis and Applications, Volume 2 (2) (2010), 77-83.

[40] S. K. Chaubey, On weakly m projectively symmetric manifolds, Novi Sad J. Math, 42 (1) (2012), 67-79.

[41] S. K. Chaubey and Ashok Kumar, Semi-symmetric metric $T$-connection in an almost contact metric manifold, International Mathematical Forum, 5(23) (2010), 1121-1129.

[42] S. K. Yadav, Pankaj and S. K. Chaubey, Riemannian manifolds admitting a projective semi-symmetric connection, https://arxiv.org/pdf/1710.00622.pdf.

[43] Pankaj, S. K. Chaubey and R. Prasad, Trans-Sasakian manifolds with respect to a non-symmetric non-metric connection, Global Journal of Advanced Research on Classical and Modern Geometries, Vol.7, (2018), Issue 1, pp.1-10.