Some Probability Distributions and L-Moment in Hydrological Engineering

Khurshid Ahmad, Bashir Ahmad, Akhtar Alam

Abstract: Extreme hydrological situations constantly disturb the earth activities and life, to envisage such extreme activities we need a system that alarms well on time and recognized the expected danger; to prepare such systems one must have knowledge of the significant factors that are actively responsible for such extreme situations and we should have a reliable statistical technique that helps to prepare a useful model for such systems. In this paper we investigate the historical data of peak flood from several gauging stations of river Jhelum in Kashmir, India. A reliable estimation technique (L-moment) is applied for parametric estimation of the probability distributions and a reliable testing techniques are used to check the accuracy of fitting of the distribution, in additional to that L-moment ratio diagram (LMRD) is used to impart information about fitting of distribution. Log Pearson-III distribution shows better results and satisfies tests of distribution fitting, same probability distribution is globally accepted for flood forecasting.

Key Words: L-Moments, Jhelum, P-P plot, L-moment ratio diagram, Return period.

I. INTRODUCTION

Extreme hydrologic situations cause miseries to human beings and earth activities. To identify the trend and the movements of such situations we should have good knowledge of the significant factors involving in system and the reliable statistical techniques, in order to save the human and economic losses, we should build such a system that not only provides the warning before the extreme situation but also support and remain effective in those conditions. As far as the hydrologic forecast and allied constructional/managerial scrutiny is considered one must have the adequate knowledge about the concerning factors that plays vital role and latest statistical techniques that is used to frame better models. Some methodologies are used to make decisions about flood warning response, flood warning zones [8,10], optimal decisions are taken for hydrological forecasting based on the adequate information and efficient models [2,9,12].

Different researchers suggest that a good number of probability distributions are used for flood frequency analysis [5,7]. Mostly the probability distributions that are used in flood frequency distributions are extreme value Type –I, Generalized Pareto distribution, Log Pearson-III, Generalized extreme value distribution and normal distribution [3].

Parametric estimation in these distributions are mostly done by using method of moments, in this method assessing to higher moments becomes very difficult and this method is considered as less accurate comparative to other methods, to overcome such shortcomings L-moment method (LM) provides better results [4], Kumar, Saf and Yue used the L-moment methods for estimations of parameters of probability distributions [11,13,16].

In the valley of Kashmir river Jhelum is the main source of hydrological extremes, flows through the region from south to north and entered into Pakistan (figure-I), we have investigate the instrumental data of peak flood of 58 years of this region from the different gauging stations, Sangum in south Kashmir, Ram-Munshibagh in central Kashmir and Asham in north Kashmir.
II. METHOD

L-moments (LM) are linear functions of ordered statistics having less biasing, less affected by sampling variability and are more robust than conventional moments [4,15]. The L-moments for the order statistics is:

\[ \lambda_r = \frac{1}{r} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} E(X_{r-k:r}) \]

Three probability distributions are used, two of them are three parametric (GEV and LP3) and one is two parametric (Gumbel). The aptness of the distributions is investigated by various tests like Kolmogorov-Smirnov (KS), Anderson-Darling (AD), and the Chi-Squared.

2.1 Generalized Extreme Values (GEV)

Generalized extreme value distribution is the combination of Weibull, Gumbel and Frechet probability distributions, it has three parameters: location parameter \( z \), scale parameter \( \alpha \) and shape parameter \( \kappa \). Probability distributions having more number of parameters will be considered as more efficient specially in hydrological engineering [6], even two parametric distributions sometimes have good results but in case of small sample size [1].

2.2 Log Pearson Type-III Distribution (LP3)

Similar to GEV, log pearson Tpe-III also uses three parameters: location parameter \( \mu \), scale parameter \( \sigma \) and shape parameter \( \gamma \), it belongs to the family of pearson type-III.

2.3 Gumbel Distribution (EV1)

Gumbel distribution is a two parametric distribution, location parameter \( z \) and scale parameter \( \alpha \). Standard error of two parametric distributions like (EV1) are smaller but biasing is large as compared to 3 or 4 parametric distributions [1].
III. GOODNESS OF FIT TESTS

Various tests (parametric and non-parametric) are used to check the efficient fitting of probability distribution to the given data, in these tests, test statistics are calculated and analyzed. Here we use non-parametric tests like Kolmogorov-Smirnov, Chi-Square and Anderson-Darling [14].

Table I

| Station          | Distributions               | Sample Size | Kolmogorov-Smirnov | Chi-Square | Anderson-Darling |
|------------------|-----------------------------|-------------|--------------------|------------|------------------|
| I Sangum         | Generalized Extreme Value   | 59          | 0.06857            | 0.90131    | 2.9735           | 0.70213           | 0.2228           |
|                  | Log-Pearson III             | 59          | 0.0716             | 0.9010     | 2.376            | 0.795             | 0.22142          |
|                  | Gumbel                      | 59          | 0.1239             | 0.1239     | 4.884            | 0.299             | 0.42365          |
| II Ram MunshiBagh| Generalized Extreme Value   | 59          | 0.0548             | 0.99023    | 2.3259           | 0.80245           | 0.1625           |
|                  | Log-Pearson III             | 59          | 0.05429            | 0.99122    | 1.2423           | 0.94076           | 0.1531           |
|                  | Gumbel                      | 59          | 0.05516            | 0.98948    | 2.3142           | 0.80419           | 0.16677          |
| III Asham        | Generalized Extreme Value   | 59          | 0.05892            | 0.97913    | 2.6497           | 0.75381           | 0.19416          |
|                  | Log-Pearson III             | 59          | 0.06357            | 0.95869    | 0.903            | 0.97              | 0.18036          |
|                  | Gumbel                      | 59          | 0.07675            | 0.85148    | 1.143            | 0.95022           | 0.3523           |

IV. L-MOMENT RATIO DIAGRAM (LMRD)

An additional mode to check the fitting of distributions is the (LMRD), it the diagram which represents the relationship between L-Skewness and L-Kurtosis (τ3,τ4) of data plotted against points and constant lines. Actually LMR Diagram is the visual check of the necessary fitting of the distribution.

![L-Moment Ratio Diagram](image)

Fig. III. L-Moment Ratio Diagram

V. RETURN PERIOD

The return period (year) of three gauging stations using Log Pearson’s probability distributions (LP3) is given in Table II,
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Table II

| Return Period (Year) | Station I (Sangum) | Station II (Ram-MunshiBagh) | Station III (Asham) |
|----------------------|--------------------|-----------------------------|---------------------|
| 2                    | 549.54             | 602.55                      | 732.34              |
| 5                    | 1000.00            | 977.24                      | 1114.36             |
| 10                   | 1348.96            | 1174.89                     | 1344.61             |
| 25                   | 1819.70            | 1479.11                     | 1606.69             |
| 50                   | 2137.96            | 1698.24                     | 1781.77             |
| 100                  | 2511.88            | 1862.08                     | 1940.90             |
| 200                  | 2884.03            | 2041.74                     | 2086.54             |

VI. CONCLUSION

L-moment method provides better estimation with less biasing of unknown parameters of important probability distributions using in hydrological system, throughout the world it is a main requirement. How to judge the trends of extreme hydrological situation, so that we can reduce economical and human losses. Knowing the key factors involving the system and using the proper statistical techniques we may be able to establish a reliable system that predicts and monitor the situations. Here we see that the Log Pearson-3 distribution shows better results with support of non-parametric tests, L-Moment Ratio Diagrams (LMRD) which is an additional way to measure goodness of fit to supports the claims. Log Pearson-3 distribution are used globally for flood forecasting.

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AUTHORS PROFILE

Dr. Khurshid Ahmad Bhat, Ph.D in Mathematical statistics, working as Senior Lecturer (vice Principal) in State Department of Education J&K, published a good number of papers in international journals and attended many conferences and workshops in reputed institutes like IIT Rorkee, University of Delhi, IISI Kolkata.

Dr. Bashir Ahmad, Ph.D in Geology, working as lecturer in State Department of Education J&K, published a good number of papers in international journals and attended many conferences of national and international repute.

Dr. Akhter Alam, Ph.D in Geography, working as assistant professor Department of Geography in university of Kashmir, published a good number of papers in international journals of repute.