Data from $e^+e^-$ annihilation into hadrons collected by the JADE experiment at centre-of-mass energies between 14 and 44 GeV were used to study the 4-jet rate using the Durham algorithm as well as the first five moments of event shape observables. The data were compared with NLO QCD predictions, augmented by resummed NLLA calculations for the 4-jet rate, in order to extract values of the strong coupling constant $\alpha_S$. The preliminary results are $\alpha_S(M_Z) = 0.1169 \pm 0.0026$ (4-jet rate) and $\alpha_S(M_Z) = 0.1286 \pm 0.0072$ (moments) consistent with the world average value. For some of the higher moments systematic deficiencies of the QCD predictions are observed.

1 Introduction

The production of hadrons in $e^+e^-$ annihilation allows precise tests of the gauge theory of strong interactions, Quantum Chromodynamics (QCD). In this paper recent and preliminary analyses of JADE data using the 4-jet rate based on the Durham algorithm [1] and using the first five moments of event shape observables are presented [2, 3].

The data used in our analyses were collected at centre-of-mass (cms) energies $\sqrt{s} = 14.0, 22.0, 34.6, 35.0, 38.3$ and 43.8 GeV between 1981 and 1986 with the JADE detector [4]. The data samples consist of $\mathcal{O}(1000)$ events at $\sqrt{s} = 14.0, 22.0, 38.3$ and 43.8 GeV while at $\sqrt{s} = 34.6$ (35.0) GeV about 14000 (21000) events are used.

The software employed to perform the analyses includes the original JADE detector simulation, event reconstruction and event display programs, see [2, 3] for details. It is possible to use recent Monte Carlo event generators such as PYTHIA, HERWIG or ARIADNE to generate simulated events, to pass these through the JADE detector simulation and to reconstruct them in essentially the same way as the data. The event generators were used with parameter settings obtained by OPAL after adjusting to LEP 1 data. The original data are only available after a further step of data reduction has been performed, resulting in information about 4-vectors of reconstructed particles and some quantities for event selection.

The selection of well reconstructed tracks from the tracking detectors and clusters from the electromagnetic calorimeter as well as the selection of hadronic events follows previous analyses. Most importantly requirements on visible energy and momentum, balance of momentum along the beam direction and track multiplicity suppress events from two-photon interactions, $\tau$ production and other backgrounds [4–7].

The contribution of $e^+e^- \rightarrow b\bar{b}$ events is subtracted from the data using simulated events. Then the data are corrected for the effects of detector resolution and acceptance of selection.
cuts by correction factors determined from simulated events before and after the JADE detector simulation.

Experimental systematic uncertainties of the analyses include variation of the event selection cuts, comparison of data sets resulting from different versions of the JADE event reconstruction program and comparing experimental corrections derived from different event generators.

Additional systematic uncertainties are studied for the extraction of values of the strong coupling constant $\alpha_S$. Hadronisation uncertainties are evaluated by using different Monte Carlo generators to compute the hadronisation corrections. Theoretical systematic uncertainties are found by changing the renormalisation scale $\mu$ of the QCD predictions from $\mu = \sqrt{s}$ to $\mu = \sqrt{s}/2$ and $\mu = 2\sqrt{s}$.

## 4-Jet Rate

Jets are reconstructed in the hadronic events using the Durham jet clustering algorithm [1].

Figure 1 (left) shows the data for the 4-jet rate corrected for experimental effects as a function of $y_{\text{cut}}$ measured at $\sqrt{s} = 35$ GeV. The data are compared to predictions from the event generators PYTHIA, HERWIG and ARIADNE. We find good agreement between the data and the predictions within the statistical and experimental errors and conclude that the models can be used to correct for hadronisation effects.

The QCD predictions are next-to-leading-order (NLO), i.e. $\mathcal{O}(\alpha_S^2)$ in leading-order (LO) with $\mathcal{O}(\alpha_S^3)$ radiative corrections, combined with resummed next-to-leading-logarithm (NLLA) calculations [8]. We find good agreement between data and theory for large values of $y_{\text{cut}} = \mathcal{O}(10^{-2})$ where mostly 3- and 4-jet events are found and the theory considering at most five partons is expected to be reliable. The resulting values of $\alpha_S$ are shown in figure 1 (right). The values for $\alpha_S(M_{Z^0})$ from the data at $\sqrt{s} = 22.0$, 34.6, 35.0, 38.3 and 43.8 GeV are combined taking into account correlations between experimental, hadronisation and theoretical systematic uncertainties. The fits at $\sqrt{s} = 14.0$ GeV have large experimental and hadronisation
uncertainties and are therefore excluded from the average. The result is $\alpha_S(M_{Z0}) = 0.1169 \pm 0.0004({\text{stat.}}) \pm 0.0012({\text{exp.}}) \pm 0.0021({\text{had.}}) \pm 0.0007({\text{theo.}})$, $\alpha_S(M_{Z0}) = 0.1169 \pm 0.0026$ (total error), consistent with the current world average $\alpha_S(M_{Z0}) = 0.1182 \pm 0.0027$ [9]. As shown in [8] the theoretical uncertainty rises with small $y_{\text{cut}}$ but is small within the fit ranges.

3 Moments of Event Shape Observables

The first five moments of the distributions of the event shape observables $1-T, C, B_T, B_W, y_{23}$ and $M_H$ are calculated according to $\langle y^n \rangle = \int_{y_{\text{min}}}^{y_{\text{max}}} y^n \frac{1}{\sigma} d\sigma dy'$, where $y$ denotes one of the observables, $y_{\text{max}}$ is the kinematically allowed upper limit of the observable and $n = 1, \ldots, 5$.

The calculation of perturbative QCD predictions in NLO ($\mathcal{O}(\alpha_S)$) in LO with $\mathcal{O}(\alpha_S^2)$ radiative corrections) involves a full integration over phase space. This analysis is thus complementary to tests of the theory using the differential distributions which are commonly only compared with data in restricted regions, where the theory is able to describe the data well, see e.g. [5].

Figure 2: (left) The corrected data for the first five moments of the observables $1-T, M_H$ and $C$ are presented with error bars showing statistical and experimental error added in quadrature. The lines indicate the predictions of Monte Carlo models. The lower panels show the differences between data and models at $\sqrt{s} = 14$ and 35 GeV, divided by the errors [3]. (right) Measurements of $\alpha_S(M_{Z0})$ using fits to moments of six event shape observables are shown. The inner error bars represent statistical errors, the middle error bars include experimental errors and the outer error bars show the total errors. The dotted line indicates the weighted average described in the text; only the measurements indicated by solid symbols were used for this purpose [3].

Figure 2 (left) presents the data for the first five moments of $1-T, M_H$ and $C$ corrected for experimental effects compared with predictions by the same event generators as in section 2. There is generally good agreement between data and model predictions; HERWIG is seen to describe the data somewhat less well than PYHTIA or ARIADNE. We will use the models to derive hadronisation corrections in order to compare the data with predictions from perturbative QCD.

We fitted the QCD predictions corrected for hadronisation to the data for a given observable and moment $n = 1, \ldots, 5$ individually with $\alpha_S(M_{Z0})$ as the only free parameter. The results for $\alpha_S(M_{Z0})$ are summarised in figure 2 (right). The fit to $\langle M_H \rangle$ did not converge and therefore
no result is shown. We observe that the values of $\alpha_S(M_{Z^0})$ increase with $n$ for the observables $\langle (1 - T)^n \rangle$, $\langle C^n \rangle$ and $\langle B^n_T \rangle$, while for the other observables $\langle B^n_W \rangle$, $\langle (y_{23})^n \rangle$ and $\langle M^n_H \rangle$, $n = 2, \ldots, 5$, the results are fairly stable.

We evaluated the ratio $K$ of NLO and LO coefficients for the six observables used in our fits and found a clear correlation between the steeply increasing values of $\alpha_S(M_{Z^0})$ and increasing values of $K$ with $n$ for $\langle (1 - T)^n \rangle$, $\langle C^n \rangle$ and $\langle B^n_T \rangle$. The other observables $\langle B^n_W \rangle$, $\langle (y_{23})^n \rangle$ and $\langle M^n_H \rangle$, $n = 2, \ldots, 5$, have fairly constant values of $K$ and correspondingly stable results for $\alpha_S(M_{Z^0})$. We also noted that $\langle M^1_H \rangle$ has a large and negative value of $K$ which is the cause that the fit did not converge.

In order to find a combined value of $\alpha_S(M_{Z^0})$ we considered only those results for which the NLO term is less than half the LO term (i.e. $|K\alpha_S/2\pi| < 0.5$), namely $\langle 1 - T \rangle$, $\langle C \rangle$, $\langle B_T \rangle$, $\langle B^0_W \rangle$ and $\langle (y_{23})^n \rangle$, $n = 1, \ldots, 5$ and $\langle M^n_H \rangle$, $n = 2, \ldots, 5$; i.e. results from 17 observables in total. The purpose of this requirement was to select observables with an apparently converging perturbative prediction. Correlations between statistical, experimental, hadronisation and theoretical uncertainties were considered when forming the average. The result is $\alpha_S(M_{Z^0}) = 0.1286 \pm 0.0007$ (stat.) $\pm 0.0011$ (exp.) $\pm 0.0022$ (had.) $\pm 0.0068$ (theo.), $\alpha_S(M_{Z^0}) = 0.1286 \pm 0.0072$ (total error), above but still consistent with the world average value. It has been observed previously in comparisons of distributions of event shape observables with NLO QCD predictions with renormalisation scale $\mu = \sqrt{s}$ that fitted values of $\alpha_S(M_{Z^0})$ tend to be large, see e.g. [10].

4 Summary

We have presented preliminary results of measurements of the 4-jet rate based on the Durham algorithm and the first five moments of event shape observables using JADE data at $\sqrt{s} = 14.0$ to 43.8 GeV. The predictions of the Monte Carlo models PYTHIA, HERWIG and ARIADNE tuned by OPAL to LEP 1 data were found to be in reasonable agreement with the data. The data have also been used to extract measurements of the strong coupling constant $\alpha_S(M_{Z^0})$ with the results $\alpha_S(M_{Z^0}) = 0.1169 \pm 0.0026$ (4-jet rate) and $\alpha_S(M_{Z^0}) = 0.1286 \pm 0.0072$ (moments) in agreement with the world average value. The higher moments of $1 - T$, $C$ and $B_T$ are observed to yield systematically larger values of $\alpha_S(M_{Z^0})$.

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