The Scale on Chiral Symmetry Breaking

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We study the relation between the scale of chiral symmetry spontaneously breaking and constituent quark mass. We argue that this relation partly reveals strong interaction origination of chiral symmetry breaking. We show that the relation can be obtained via checking unitarity region of low-energy effective field theory of QCD. This effective field theory must manifestly include consistent quark mass as parameter. Thus we derive this effective field theory from naive chiral constituent quark model. The phenomenological value obtained by this method agree with usual one determined by pion decay constant.

PACS numbers: 12.38.Aw,12.38.Lg,11.30.Rd,12.39.Fe

The typical difficulty on studies on QCD is from its dramatic properties when dynamics of QCD lies in non-perturbative region. The analysis of renormalization group shows that QCD is asymptotic freedom at high energy scale, but should lie in confinement phase at very low energy. Consequently, a phase transition must occur when energy scale varies from higher to lower one. The phase transition are dynamically characterized by well-known fermion (quark) condensation phenomena. A dynamical scale (it is usually referred as $\Lambda_{QCD}$ in QCD) is consequently generated by the quark condensation. It is just order parameter associating the phase transition. Sometimes this scale is also transferred to another effective parameter: so-called constituent quark mass $m$ and treated it as order parameter. Focusing on dynamics of QCD with light flavor quarks only, however, the story is more complicated: The quark condensation also breaks the (approximate) global chiral symmetry of QCD. Accompanying with the chiral symmetry spontaneously breaking (CSSB), another scale $\Lambda_{CSSB}$ must be dynamically generated and Goldstone bosons appear as dynamical degrees of freedom. CSSB is one of the most important features for the hadron physics. It together with color confinement governs full low-energy dynamics of QCD. A typical example is success of chiral perturbation theory (ChPT).

An interesting issue is that global chiral symmetry is broken due to pure strong interaction. To localize the global chiral symmetry one has to introduce electroweak interactions. Because the quark condensation breaks both of local and global chiral symmetry, CSSB actually involves both of strong and electroweak interactions (in contrast to color confinement caused by pure strong interaction). The fact of electroweak relevance of CSSB has been shown in determination on CSSB scale via weak decay constant of pion, i.e., $\Lambda_{CSSB} \sim 2\pi F_\pi \sim 1.2\text{GeV}$. The studies on role of strong interaction in CSSB, however, seems to be more difficult, since complete understanding on this issue requires underlying knowledge on dynamical mechanics of color confinement. During the past decades, CSSB has been extensively studied along this way, i.e., so called the formalism of the gap equations (or Schwinger-Dyson equations, see refs. and the references within, and ). This method is rigorous and achieves some successes, but still far from our final expectation so far. Alternately, it should be also possible to explore CSSB by starting within confinement phase. The key point is to find relation between $\Lambda_{CSSB}$ and constituent quark mass $m$ (or $\Lambda_{QCD}$). It will partly reflect the role of strong interaction in CSSB. This is just purpose of this letter. In this phase, the dynamical description is replaced by effective one with constituent quarks and Goldstone bosons. In such effective description a critic-like energy scale must exist. Above this energy scale, this effective description on the system collapses and below it, the description works. This critic-like energy scale should be just the scale of CSSB, $\Lambda_{CSSB}$.

The natural criterion on whether a quantum effective field theory (QED) description collapses or not is to check unitarity of the QED. This claim bases on the fact that QED does not describe full degrees of freedom of fundamental theory. When energy is higher than characteristic scale of the QED, some new degrees of freedoms will
be excited consequently the unitarity of the QEFT is lost. In this letter, we will derive a low-energy QEFT of pure meson interaction from naive chiral constituent quark model, and to obtain \( \Lambda_{CSSB} \) via checking unitarity region of that QEFT.

It is well-known that a low energy effective meson theory should be a well-defined perturbative theory in \( N_c^{-1} \) expansion. Therefore, unitarity condition of \( S \)-matrix, or optical theorem, has to satisfied order by order in powers of \( N_c^{-1} \) expansion,

\[
\text{Im}(T_{\beta,\alpha})_n = \frac{1}{2} \sum_{\gamma} \sum_{m \leq n} (T_{\gamma,\alpha})_m (T^{*}_{\gamma,\beta})_{n-m}.
\]

(1)

where the \( T_{\beta,\alpha} \) is transition amplitude from state initial \( \alpha \) to final state \( \beta \), and \( \gamma \) denotes all possible intermediate states on mass shells, and \( T_n \sim O((1/\sqrt{N_c})^n) \). According to standard power counting law on large \( N_c \) expansion in meson interaction, any transition amplitudes with \( n_V \) vertices, \( n_e \) external meson lines, \( n_i \) internal meson lines and \( n_l \) loops of mesons are of order

\[
N_c^{n_V-n_i-n_e/2} = (N_c^{-\frac{1}{2}})^{2n_i+n_e-2},
\]

(2)

where topological relation \( n_l = n_i - n_e + 1 \) has been used. We focus on transition amplitude from single meson initial state \( \alpha \) to \( k \) mesons final state \( \beta = \{\beta_1, \beta_2, \ldots, \beta_k\} \). Assuming intermediate state \( \gamma \) includes \( s \) mesons \( \{\gamma_1, \gamma_2, \ldots, \gamma_s\} \), then using the power counting rule, eq. (2) can be written as

\[
\text{Im}(T_{\beta,\alpha})_{(2n_l+k-1)} = \frac{1}{2} \sum_{\gamma(s)} \sum_{n''} (T_{\gamma,\alpha})_{(2n_l+s-1)} (T^{*}_{\gamma,\beta})_{(2n''+s+k-2)},
\]

(3)

where \( n_l, n'_l \) and \( n''_l \) are meson loop numbers in transition amplitude \( T_{\beta,\alpha}, T_{\gamma,\alpha} \) and \( T_{\gamma,\beta} \) respectively. Both side of eq. (3) should be of the same order, thus

\[
n'_l + n''_l + s = 1 + n_l.
\]

(4)

For the case of leading order of transition amplitude \( T_{\beta,\alpha} \), i.e., \( n_i = 0 \), we have \( n'_l = n''_l = 0 \) and \( s = 1 \) according to eq. (4). Consequently only \( \gamma = \alpha \) is allowed at the leading order. Since meson fields are free point-particle at limit \( N_c \to \infty \), we have \( (T_{\alpha,\alpha})_0 = 0 \). Therefore, it can be claimed that, if any effective meson theory is unitary below its characteristic scale, the on-shell transition amplitude from any meson state to any multi-meson state must be real at leading order of \( N_c^{-1} \) expansion,

\[
\text{Im}(T_{\beta,\alpha})_{k-1} = 0.
\]

(5)

where the superscript \( (0) \) denotes the leading order of \( N_c^{-1} \) expansion. This claim will serve as equivalent description of unitarity for any QEFTs on meson interaction.

A convenient effective description on the low energy QCD is naive chiral constituent quark model (ChQM) proposed in ref. [2]. The constituent quark mass as order parameter associating to phase transition is manifestly appear in this model. Thus this model provides a possible framework to explore relation between \( \Lambda_{CSSB} \) and order parameter. The simplest ChQM is parameterized by the following SU(3) invariant Lagrangian

\[
\mathcal{L}_{\text{ChQM}} = i\bar{q}(\partial + \vec{p} + g_A \Delta_5 - i\gamma)q - m\bar{q}q - \bar{q}Sq - \kappa\bar{q}P\gamma_5q + \frac{F^2}{16} < \nabla_{\mu}U\nabla^{\mu}U^\dagger > + \frac{1}{4} m_3^2 < V_{\mu}V^{\mu} > .
\]

(6)

Here \( V_{\mu} \) are vector meson octet, \( < \cdots > \) denotes trace in SU(3) flavor space, \( \bar{q} = (\bar{u}, \bar{d}, \bar{s}) \) are constituent quark fields, \( g_A = 0.75 \) is fitted by beta decay of neutron, and

\[
\begin{align*}
\Delta_{\mu} &= \frac{1}{2} [\xi^\dagger (\partial_{\mu} - ir_{\mu})\xi - \xi (\partial_{\mu} - il_{\mu})\xi^\dagger], \\
\Gamma_{\mu} &= \frac{1}{2} [\xi (\partial_{\mu} - ir_{\mu})\xi^\dagger + \xi^\dagger (\partial_{\mu} - il_{\mu})\xi], \\
\nabla_{\mu}U &= \partial_{\mu}U - ir_{\mu}U + iUr_{\mu} = 2\xi \Delta_{\mu}\xi, \\
\nabla_{\mu}U^\dagger &= \partial_{\mu}U^\dagger - il_{\mu}U^\dagger + iU^\dagger r_{\mu} = -2\xi^\dagger \Delta_{\mu}\xi^\dagger, \\
S &= \frac{1}{2} (\xi^\dagger \tilde{\chi} \xi^\dagger + \xi \tilde{\chi} \xi), \\
P &= \frac{1}{2} (\xi^\dagger \tilde{\chi} \xi^\dagger - \xi \tilde{\chi} \xi).
\end{align*}
\]

(7)
where \( l_\mu = v_\mu + a_\mu \) and \( r_\mu = v_\mu - a_\mu \), \( \bar{\chi} = s_{\text{ext}} + M + ip \) with external fields \( v_\mu \) (vector), \( a_\mu \) (axial-vector), \( s_{\text{ext}} \) (scalar), \( p \) (pseudoscalar), and current quark mass matrix \( M = \text{diag}\{m_u,m_d,m_s\} \) respectively. \( \xi \) associates with non-linear realization of spontaneously broken global chiral symmetry \( G = SU(3)_L \times SU(3)_R \) introduced by Weinberg [10],

\[
\xi(\Phi) \rightarrow g_R \xi(\Phi) h^\dagger(\Phi) = h(\Phi) \xi(\Phi) g_L^\dagger,
\]

where \( \lambda^1, \ldots, \lambda^8 \) are SU(3) Gell-Mann matrices in flavor space, and the Goldstone bosons \( \Phi^a \) are identified to pseudoscalar meson octet.

The transformation law under SU(3)_V for any quantities defined in eqs. (6) and (8) are

\[
q \rightarrow h(\Phi) q,
\]

\[
\Delta_\mu \rightarrow h(\Phi) \Delta_\mu h^\dagger(\Phi),
\]

\[
\Gamma_\mu \rightarrow h(\Phi) \Gamma_\mu h^\dagger(\Phi) + h(\Phi) \partial_\mu h^\dagger(\Phi).
\]

The homogenous transformation law on vector meson field is usually referred as WCCWZ realization on vector meson [11].

Because there is no kinetic term for vector fields in \( \mathcal{L}_{\text{ChQM}} \), they serve as auxiliary fields in this formalism. From the equation of motion \( \delta \mathcal{L}_{\text{ChQM}}/\delta V = 0 \), we can see the vector fields in \( \mathcal{L}_{\text{ChQM}} \) are the composite fields of constituent quarks. Therefore, WCCWZ method is actually a way to catch the effects of constituent quark bound states in the ChQM. \( F, g_A, m, \kappa \) and \( m_0 \) in eq. (9) are free parameters of the model.

The effective action on meson interaction, \( S_{\text{eff}}[U,V] \), can be obtained via integrating out quark fields,

\[
S_{\text{eff}}[U,V] = \ln \det(D) + \int d^4x \left\{ \frac{F^2}{16} < \nabla_\mu U \nabla^\mu U^\dagger > + \frac{1}{4} m_0^2 < V_\mu V^\mu > \right\},
\]

where \( D = \tilde{\phi} + \gamma + g_A \tilde{\Delta}_5 - i \gamma \cdot m - \gamma \cdot P \gamma_5 \), \( \phi \) and \( m_0 \) will receive quark loop effects and then are renormalized into \( F = 186 \text{MeV} \) and the physical masses \( m_V \) of vector mesons respectively. Then \( S_{\text{eff}}[U,V] \) parameterizes a QFT on pure meson interaction.

Now let us consider unitarity of this QFT. In particular, we focus on \( V \rightarrow \Phi \Phi \) decay amplitude and impose eq. (9) to find unitarity region of the QFT. To separate relevant effective action from \( S_{\text{eff}}[U,V] \) and rewrite it into appropriate form

\[
S_{\text{eff}}^{\Phi \Phi} = \sum_{abc} \int \frac{d^4q_1 d^4q_2}{(2\pi)^4} \delta(p + q_1 + q_2) V_{abc}(p) \Phi_{bc}(q_1) \Phi_{ca}(q_2) f_{abc}(p^2,q_1^2,q_2^2),
\]

we have

\[
T_{\Phi \Phi}^{(0)} = \sum_{abc} |\Phi_{bc}(q_1) \Phi_{ca}(q_2)| T^{(0)}(V_{abc}(p,\lambda)) = (2\pi)^4 \delta(p - q_1 - q_2) \epsilon^\lambda \epsilon_{abc}(p^2,q_1^2,q_2^2),
\]

where \( \epsilon^\lambda \) is the polarization vector of the vector meson \( V_{abc}(p,\lambda) \). Consequently

\[
\text{Im} T_{\Phi \Phi}^{(0)} \propto \text{Im} f_{abc}(p^2,q_1^2,q_2^2).
\]

The form factor \( f_{abc}(p^2,q_1^2,q_2^2) \) can be rewritten as \( f_{abc}(p^2,q_1^2,q_2^2) = f_2(p^2) + f_3(p^2,q_1^2,q_2^2) \), where \( f_2 \) and \( f_3 \), with subscript \( abc \) suppressed, are two-point Green function (Fig. 1-a) and three-point Green function (Fig.1-b) of constituent quark fields respectively, and are linearly independent. Explicitly, the calculations on form factors \( f_2(p^2) \) and \( f_3(p^2,q_1^2,q_2^2) \) are straightforward,
where \( g, h \) and \( \tilde{g}, \tilde{h} \) are definite real and polynomial functions of \( u = k_E^2 \) \((k_E^\mu = -ik_0, k_x, k_y, k_z)\), \( M_a = m + m_a \) \((a = u, d, s)\) and

\[
D_2 = M_a^2(1 - x) + M_d^2x - p^2x(1 - x), \\
D_3 = M_a^2x(1 - y) + M_d^2(1 - x) + M_s^2xy - p^2x(1 - x)(1 - y) - q_1^2xy(1 - x) - q_2^2x^2y(1 - y). 
\]

Using principle value formula

\[
\frac{1}{z \pm i\epsilon} = \frac{P}{z} \mp i\pi\delta(z),
\]

where \( \frac{P}{z} \) is the principle value and is real, we can express \( f_2 \) and \( f_3 \) as

\[
f_2(p^2) = -\int_0^1 dx \int_0^\infty du \frac{\partial}{\partial u} \left( \frac{P}{u + D_1} + i\pi\delta(u + D_1) \right), \\
f_3(p^2, q_1^2, q_2^2) = \frac{1}{2} \int_0^1 dx \int_0^1 dy \int_0^\infty du \frac{\partial^2}{\partial u^2} \left( \frac{P}{u + D_2} + i\pi\delta(u + D_2) \right).
\]

Then we obtain

\[
\text{Im} f_2(p^2) \propto \int_0^1 dx \int_0^\infty du \frac{\partial \tilde{g}(x, u, p^2)}{\partial u} \delta(u + D_2), \\
\text{Im} f_3(p^2, q_1^2, q_2^2) \propto \int_0^1 dx \int_0^1 dy \int_0^\infty du \frac{\partial^2 \tilde{h}(x, y, u, p^2, q_1^2, q_2^2)}{\partial u^2} \delta(u + D_3).
\]

Finally we have

\[
\text{Im} f_i(p^2) = 0 \iff u + D_i \neq 0 \iff D_i > 0 \quad i = 2, 3, 
\]

where \( u > 0, 0 < x, y < 1 \) have been considered. More precisely,

\[
\text{Im} T^{(0)}_{\Phi\Phi, V} = 0 \iff \begin{cases} 
D_1 > 0 & (0 < x < 1) \\
D_2 > 0 & (0 < x, y < 1). 
\end{cases}
\]

This will lead to a restriction on the range of \( p^2 \). The former inequality will hold in domain \( 0 \leq x \leq 1 \) if and only if

\[
M_{V^a} = \sqrt{p^2} \leq M_a + M_b.
\]

As to the latter, the right side of it has no stationary point in \( x - y \) plane, therefore this inequality holding in the square domain is equivalent to it holding at boundary of the square, which gives

\[
\begin{cases} 
M_{V^a} = \sqrt{p^2} \leq M_a + M_b, \\
M_{\Phi^a} = \sqrt{q^2} \leq M_a + M_b.
\end{cases}
\]

Because \( M_{\Phi^a} < M_{V^a} \), we see that the second condition is satisfied if the first one does. Therefore we conclude that the necessary condition for the effective theory to be unitary is

\[
M_{V^a} = \sqrt{p^2} \leq \Lambda^a = 2m + m_a + m_b.
\]
In the beginning of this letter, we have actually argued an important fact that $\Lambda_{ab} \equiv 2m + m_a + m_b$ is a critical energy scale in the meson QFT parameterized by $S_{\text{eff}}[U, V]$. As $\sqrt{p^2}$ is below $\Lambda_{ab}$, the $S$-matrices yielded from the Feynman rules of that QFT are unitary, while as $\sqrt{p^2}$ is above this scale, the unitarity of that QFT will be violated. This fact indicates that the well-defined QFT describing the meson physics in the framework of ChQM exists only as the characteristic energy is below $\Lambda_{ab}$. When energy is above $\Lambda_{ab}$, the effective meson Lagrangian description of the dynamics is illegal in principle because the unitarity fails. This is precisely a critical phenomenon, or quantum phase transition in quantum field theory, which is caused by quantum fluctuations in the system[12]. Recalling the meaning of the scale $\Lambda_{\text{CSSB}}$ of chiral symmetry spontaneously breaking in QCD, we can see that $\Lambda_{ab}$ play the same role as $\Lambda_{\text{CSSB}}$. Then, in the framework of ChQM, we identify

$$\Lambda_{\text{CSSB}} = \Lambda_{ab} \equiv 2m + m_a + m_b.$$  \hspace{1cm} (23)

The above equation just explores a simple relation between $\Lambda_{\text{CSSB}}$ and constituent quark mass, and is the main result of this letter. It should be a simple fact that $\Lambda_{\text{CSSB}}$ is flavor-dependent as we expect. This reflects the fact that although the vacuum (quark condensation) is $SU(3)_V$ invariant at chiral limit, it is explicitly broken to Abelian subgroup of $SU(3)_V$ when current quark masses are turned on. As discussed at the beginning of this letter, $\Lambda_{\text{QCD}}$ should be unique scale of QCD at low energy. In other words, other dimensional quantities, even including $\Lambda_{\text{CSSB}}$, should be related to $\Lambda_{\text{QCD}}$. Thus more fundamental task is to find relation between $\Lambda_{\text{CSSB}}$ and $\Lambda_{\text{QCD}}$ from eq. (23). Roughly we can expect $m \simeq \Lambda_{\text{QCD}}$, at some definite low energy limits at least. However, the precise coefficient is no longer 1. A direct evidence is that if interaction between gluons and constituent quarks are turned on (this coupling is usually expected to be weak, but should not vanish exactly), the self-energy diagram of constituent quarks will contribute to mass term of constituent quarks. To explore relation between $\Lambda_{\text{CSSB}}$ and $\Lambda_{\text{QCD}}$ means that we should explore exactly relation between $\Lambda_{\text{QCD}}$ and constituent quark mass. It actually requires that we should know underlying dynamical mechanism of low energy QCD and thus will be great challenge.

Phenomenologically, it is also interesting to fix numerical value of $\Lambda_{\text{CSSB}}$. The low-energy limit of the QFT can obtained via integrating out vector meson fields[13, 14]. It means that, at very low energy, the dynamics of vector mesons are replaced by pseudoscalar meson fields. Expanding the resulted Lagrangian up to $O(p^4)$ in terms of Schwinger’s proper time method[15, 16], we get $O(p^4)$ ChPT-coefficients as follows

$$L_1 = \frac{1}{2}, \quad L_2 = \frac{1}{128\pi^2}, \quad L_3 = \frac{3}{64\pi^2} + \frac{1}{64\pi^2}g_A^4,$$

$$L_4 = L_6 = 0, \quad L_5 = \frac{3m}{32\pi^2B_0}g_A^2,$$

$$L_8 = \frac{F_\pi^2}{128B_0m}(3 - \kappa^2) + \frac{3m}{64\pi^2B_0}\left(\frac{m}{B_0} - \kappa g_A - \frac{g_A^2}{2} - \frac{B_0}{6m}g_A^4\right) + \frac{L_5}{2},$$

$$L_9 = \frac{1}{16\pi^2}, \quad L_{10} = -\frac{1}{16\pi^2} + \frac{1}{32\pi^2}g_A^2.$$  \hspace{1cm} (24)

The above expressions of $L_i$ have been obtained in some previous refs[13, 18, 19] (except $L_8$). Then inputting experimental values of $L_5$ and $L_8$ and taking $g_A = 0.75$ (fitted by $n \to p e^-\bar{\nu}_e$ decay[2]) and $m_u + m_d \simeq 11\text{MeV}$, we can fix phenomenological values of other free parameters as $B_0 \simeq 1.8\text{GeV}$, $m \simeq 460\text{MeV}$ and $\kappa \simeq 0.5$. The numerical results for those low energy constants are listed in table II.

| $L_i$ | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ | $L_8$ | $L_9$ | $L_{10}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| ChPT  | $0.7 \pm 0.3$ | $1.3 \pm 0.7$ | $-4.1 \pm 2.5$ | $-0.3 \pm 0.5$ | $1.4 \pm 0.5$ | $-0.2 \pm 0.3$ | $0.9 \pm 0.3$ | $6.9 \pm 0.7$ | $-5.2 \pm 0.3$ |
| ChQM  | $0.79$ | $1.58$ | $-4.25$ | $0$ | $1.4^{a}$ | $0$ | $0.9^{a}$ | $6.33$ | $-4.55$ |

**TABLE I:** $L_i$ in units of $10^{-3}$, $\mu = m_{\nu}$. a)input. b)contribution from gluon anomaly.

Numerically, for $ud$-flavor system (e.g., $\pi - \rho - \omega$ physics),

$$\Lambda_{\text{CSSB}}(ud) \simeq 2m = 920\text{MeV}.$$  \hspace{1cm} (25)

For $u(d)s$-flavor system (e.g., $K - K^*$ physics),

$$\Lambda_{\text{CSSB}}(u(d)s) \simeq 2m + m_s = 1090\text{MeV}.$$  \hspace{1cm} (26)

For $ss$ case (e.g., $\phi$-physics),

$$\Lambda_{\text{CSSB}}(ss) \simeq 2(m + m_s) = 1260\text{MeV}.$$  \hspace{1cm} (27)
Since $m_{\rho} < \Lambda_{\text{CSSB}}(ud)$, $m_{K^*} < \Lambda_{\text{CSSB}}(u(d)s)$ and $m_{\phi} < \Lambda_{\text{CSSB}}(ss)$, the effective meson field theory derived by resummation derivation in ChQM in this paper is unitary. And the low energy expansions in powers of $p$ are legitimate and convergent due to $p^2/\Lambda_{\text{CSSB}}^2 < 1$. It means that all light flavor vector meson resonances can be included in ChQM consistently. It is remarkable that the quantum phase transitions in ChQM can be explored successfully in resummation derivation method, and the corresponding critical scales are determined analytically.

To conclude, it is shown that the scale of CSSB is not independent of the scale of color confinement. The relation between two scales reveals strong interaction origination of CSSB phenomena. However, to explore this relation precisely is very difficult due to lack of underlying knowledge on color confinement. Instead we argued that this relation can be partly replaced by one between $\Lambda_{\text{CSSB}}$ and constituent quark mass $m$. We used naive chiral constituent quark model to find this relation via checking unitarity region of induced QFT of meson interaction. Phenomenologically, we determined numerical value of $\Lambda_{\text{CSSB}}$ in terms of consistent fit on values of free parameters of ChQM. The result agree with usual value of $\Lambda_{\text{CSSB}}$ determined by pion decay constant. Our evaluation also shows that lowest order vector meson resonances can be consistently included in naive ChQM.

ACKNOWLEDGMENTS

This work is partially supported by NSF of China 90103002 and the Grant of the Chinese Academy of Sciences. The authors wish to thank Yong-Shi Wu (Utah U) for his stimulating discussions on quantum phase transitions.

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