Performance Analysis of Intelligent Reflecting Surface in Multi-user MIMO Systems

Shaobo Liu\textsuperscript{1}, Limin Xiao\textsuperscript{1}, Ming Zhao\textsuperscript{1}, Xibin Xu\textsuperscript{1} and Yunzhou Li\textsuperscript{1}

Beijing National Research Center for Information Science and Technology, Dept. of Electronic Engineering, Tsinghua University, Beijing 100084, China
Email: lsb17@mails.tsinghua.edu.cn, xiaolm@tsinghua.edu.cn, zhaoming@tsinghua.edu.cn, xuxb@tsinghua.edu.cn, liyunzhou@tsinghua.edu.cn

Abstract. Intelligent Reflecting Surface (IRS) is a revolutionary technology which is able to improve the performance of wireless data transmission system. Specially, large numbers of small reflecting unit are jointly adjusted to reconfigure the wireless signal transmitting environment. In this paper, we study the use of IRS in a multi-user MIMO system where IRS is deployed to assist the data transmission in both uplink and downlink phases. We first present an overview of the technology of intelligent surfaces, including its fundamental principles and practical applications. Then we derive the signal model of IRS aided multi-user MIMO system and provide data transmission schemes. Finally, numerical results are demonstrated to show the superb signal enhancement with the use of intelligent surfaces in wireless system.

1. Introduction

Various technology advances such as ultra-dense network (UDN), massive multiple-input multiple-output (MIMO), millimetre wave (mmWave) communication and so on have witnessed the increasing data rate and channel capacity demands for fifth Generation (5G) wireless communication [1]. These requirements can be met via deployment of multi-antenna base stations and access points [2]. However, the high complexity of hardware implementations and increasing energy consumptions are still practical challenges need to be solved. For example, massive MIMO technology requires elaborated design and configuration of large numbers of antennas at both sides of the wireless channel, as well as complex signal processing and costly energy consuming devices. Millimetre wave (mmWave) communication has severe degradation under adverse weather condition, which may consume more energy while not meet transmission requirements. Therefore, researches on finding both low-complexity and energy efficient techniques for the green and sustainable fifth Generation (5G) is imperative.

Intelligent reflecting surface (IRS) is an entirely new concept in wireless communication [3], which can be seen as an extension of massive MIMO [4]. Recently, it has been proposed to be a promising technology with significant potential in reducing energy consumption of wireless networks, while being capable of achieving unprecedented massive MIMO gains in theory. IRS is a large surface attached on a wall or a flat facade, consisting of large numbers of low-cost reflecting electronic elements with reconfigurable parameters, such as conventional reflect-arrays [5] and varactor diodes or micro-electrical-mechanical systems, whose resonant frequency can be electronically controlled [6]. These tiny electronic elements are capable of reflecting electromagnetic waves cast on them. Thus the amplitude and phase of the reflecting waves are affected, as well as the electromagnetic behavior of the wireless propagation channel. Each of these element unit can contribute a phase shift, hence the combined configuration is able to meet certain communication demands. IRS performs as a signal...
scatterer and does not require any dedicated energy supply for channel estimation or signal decoding, which is fundamentally different from a multi-antenna relay [7]. Despite its many benefits, IRS aided wireless communication system consists of both active and passive components, which is more complex than traditional ones with only active devices. This article provides an overview of passive intelligent surface, including its signal model, channel estimation, deployment parameters and so on.

The rest of the paper is organized as follows. In Section 2, we describe the system model of the IRS aided multi-user MIMO data signal transmission. We also demonstrate the data rate and channel capacity in downlink scenario. In particular, through numerical calculations, methods of achieving desired data rate are presented. In Section 3, we derive some of the precoding schemes such as channel inversion and block-diagonalization, analyze the signal enhancement that IRS is capable of achieving. Numerical results are presented in Section 4, and Section 5 summarizes the paper.

2. System Model

In this section, we first derive our IRS aided multi-user MIMO channel model system, as well as our analysis of channel capacity.

2.1. Signal Model for IRS aided Multi-user MIMO System

In a single user MIMO system, spatial multiplexing technique can support the point-to-point high data rate while provide spatial diversity gain. However, under most of the circumstances, communication systems deal with multiple users who are sharing the same radio resources. Suppose that the base station and each terminal are equipped with $N_B$ and $N_M$ antennas respectively. As K independent users from a virtual set of $K \cdot N_M$ antennas communicate with a single BS with $N_B$ antennas, the BS wishes to convey information bearing signals simultaneously to K users. In this multi-user communication system, multiple antennas allow the independent users to transmit their own data stream in the uplink stage at the same time or the base station to transmit the multiple user data streams to be decoded by each user in the downlink stage.

This communication is assisted by an IRS attached to the facade of a wall or a flat surface existing in the vicinity of the multi-user MIMO system. The IRS is comprising of many passive elements, or reflect-arrays, is capable of control the electromagnetic behavior of the wireless propagation channel. It can be dynamically reconfigured via a controller to program the scattering of incident signals from both BS and terminals, which effectively act as low resolution phase shifters impacting the impinging information bearing electromagnetic field.

Let $x_u \in \mathbb{C}^{N_M \times 1}$ and $y_{UL,u} \in \mathbb{C}^{N_B \times 1}$ denote the transmit signal from the $u$th user, $u = 1, 2, 3, \ldots K$, and the received signal at the BS respectively. The direct channel gain between BS and the $u$th user is denoted by $H_{B,u}^{DL} \in \mathbb{C}^{N_B \times N_M}$, $u = 1, 2, 3, \ldots K$. Similarly, $H_{B,S}^{UL} \in \mathbb{C}^{M \times N_M}$ represents the direct channel between IRS and the $u$th user, and we use the notation $H_{B,S}^{UL} \in \mathbb{C}^{N_B \times M}$ for the channel between BS and IRS where $M$ is the number of passive elements of IRS.

We assume there is no dependence or correlations between any of the above mentioned matrices. The received signal at BS is expressed as

$$y_{MAC} = \sum_{u=1}^{K} (H_{B,u}^{UL} + \beta_H^{UL} \phi H_{B,S}^{UL}) x_u + z$$

where $\phi = \text{diag}[^{\beta_1, \phi_1, \beta_2, \phi_2, \ldots, \beta_M, \phi_M]}$ is a diagonal matrix, $\beta_n \in (0, 1)$ and $\phi_n \in [0, 2\pi]$ specify the reflection coefficient of the reflected signal’s attenuation and the effective phase shifting value respectively, and $z \in \mathbb{C}^{N_B \times 1}$ is the additive noise in the receiver and it is modeled as a zero-mean circular symmetric complex Gaussian (ZMCSGC) random vector.

On the other hand, the downlink channel, known as the broadcast channel (BC) is also considered. $x \in \mathbb{C}^{N_M \times 1}$ is the transmit signal from the BS and $y_u \in \mathbb{C}^{N_B \times 1}$ is the received signal at the $u$th user, $u = 1, 2, 3, \ldots K$. Similar to the uplink channel, the direct channel gain between BS and the $u$th user is denoted by $H_{B,u}^{DL} \in \mathbb{C}^{N_M \times N_B}$, $u = 1, 2, 3, \ldots K$. $H_{B,S}^{UL} \in \mathbb{C}^{N_M \times M}$ represents the direct channel between IRS and BS, and we use the notation $H_{S,u}^{DL} \in \mathbb{C}^{M \times N_B}$ for the channel between IRS and the $u$th user.
user. In the multiple access channel, the received signal at the \( u \)th user is given as

\[
y_u = (H_{B,u}^{DL} + H_{B,S}^{DL} \phi H_{S,u}^{DL}) x_u + z_u
\]

where \( z_u \in \mathbb{C}^{N_M \times 1} \) is the additive ZMCGNC noise at the \( u \)th user.

### 2.2. Analysis of Channel Capacity

#### 2.2.1 Capacity of Multiple Access Channel

Capacity region of multiple access channel (MAC) was first introduced in [8]. Let \( P_u \) and \( R_u \) denote the power and data rate of the \( u \)th user in our K-user IRS-aided MIMO system, \( u = 1, 2, 3, \ldots, K \). The multiple access channel capacity region for \( K = 2, N_M = 1 \) is given as

\[
R_1 \leq \log_2 \left( 1 + \left\| H_{B,1}^{UL} + H_{B,S}^{UL} \phi H_{S,1}^{UL} \right\|^2 \frac{P_1}{\sigma_z^2} \right)
\]

\[
R_2 \leq \log_2 \left( 1 + \left\| H_{B,2}^{UL} + H_{B,S}^{UL} \phi H_{S,2}^{UL} \right\|^2 \frac{P_2}{\sigma_z^2} \right)
\]

\[
R_1 + R_2 \leq \log_2 \left( 1 + \left\| H_{B,1}^{UL} + H_{B,S}^{UL} \phi H_{S,1}^{UL} \right\|^2 \frac{P_1}{\sigma_z^2} + \left\| H_{B,2}^{UL} + H_{B,S}^{UL} \phi H_{S,2}^{UL} \right\|^2 \frac{P_2}{\sigma_z^2} \right)
\]

where \( \sigma_z^2 \) is the noise power of \( z \in \mathbb{C}^{N_B \times 1} \) at base station. In this situation, the received signal is expressed as

\[
y_{MAC} = (H_{B,1}^{UL} + H_{B,S}^{UL} \phi H_{S,1}^{UL}) x_1 + (H_{B,2}^{UL} + H_{B,S}^{UL} \phi H_{S,2}^{UL}) x_2 + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}
\]

\[
= \begin{bmatrix} H_{B,1}^{UL} + H_{B,S}^{UL} \phi H_{S,1}^{UL} \\ H_{B,2}^{UL} + H_{B,S}^{UL} \phi H_{S,2}^{UL} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}
\]

In order to achieve the maximum data transmission rate of user 2, signal \( x_1 \) is detected by assuming interfered with the signal from user 2. Once the signal \( x_1 \) is detected correctly, which is possible only if the transmission rate of user 1 is less than the channel capacity

\[
R_1 = \log_2(1 + \| H_{B,1}^{UL} + H_{B,S}^{UL} \phi H_{S,1}^{UL} \|^2 P_1 / (\sigma_z^2 + \| H_{B,2}^{UL} + H_{B,S}^{UL} \phi H_{S,2}^{UL} \|^2 P_2))
\]

\( R \) can be canceled from the joint received signal as

\[
\hat{y}_{MAC} = y_{MAC} - (H_{B,1}^{UL} + H_{B,S}^{UL} \phi H_{S,1}^{UL}) x_1 = (H_{B,2}^{UL} + H_{B,S}^{UL} \phi H_{S,2}^{UL}) x_2 + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}
\]

From the interference-free signal above mentioned, \( x_2 \) is detected to achieve rate

\[
R_2 = \log_2(1 + \| H_{B,2}^{UL} + H_{B,S}^{UL} \phi H_{S,2}^{UL} \|^2 P_2 / \sigma_z^2)
\]

Meanwhile, the maximum data transmission rate of user 1 can be attained in the other way around. In [9], it was shown that the sum rate capacity of multiple access channel is proportional to \( \min(N_B, K \cdot N_M) \).

#### 2.2.2 Capacity of Broadcast Channel

For the single antenna broadcast channel, sum rate channel capacity is achieved by transmitting to the user with the largest channel norm, since it falls into the class of degraded broadcast channel, for which it is known that the sum rate channel capacity is equal to the largest single-user capacity in the system. However, this is not generally suitable for a multiple transmit antenna broadcast channel. For
the latter situation, sum rate channel capacity is achieved by using multi-user precoding techniques to transmit signals to several users simultaneously. At the transmitter, the transmit power is distributed on each transmit antenna, and different transmitting data is loaded on each antenna. The receiver of each user estimates transmit symbols destined for the user using a linear minimum mean square error (MMSE) detector [10]. In MIMO systems with these configurations, each transmit antenna creates a spatial channel [11], resulting in total spatial channels for each time slot.

We consider the downlink scenario of an IRS aided multi user MIMO system with \( N_B \) transmit antennas and \( N_M \) receive antennas for each of the \( K \) users. As mentioned in the previous section, the received signal is mathematically represented as equation (2). In this subsection, we discuss an achievable downlink channel capacity for a special case of \( N_B = 2 \), \( N_M = 1 \) and \( K = 2 \). Using LQ-decomposition, we can derive the received signal as

\[
y_{BC} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_{DL,1}^{DL} + H_{DL,5}^{DL} \phi H_{S,1}^{DL} \\ H_{DL,2}^{DL} + H_{DL,5}^{DL} \phi H_{S,2}^{DL} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} L_{11} \\ L_{21} \\ L_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}
\] (8)

where

\[
L_{11} = \|H_{DL,1}^{DL} + H_{DL,5}^{DL} \phi H_{S,1}^{DL}\|, q_1 = \frac{1}{L_{11}} (H_{DL,1}^{DL} + H_{DL,5}^{DL} \phi H_{S,1}^{DL})^H,
\]

\[
L_{21} = q_1 (H_{DL,2}^{DL} + H_{DL,5}^{DL} \phi H_{S,2}^{DL})^H, 
\]

\[
L_{22} = \|H_{DL,2}^{DL} + H_{DL,5}^{DL} \phi H_{S,2}^{DL} - L_{21} q_1\|, 
\]

\[
q_2 = \frac{1}{L_{22}} (H_{DL,2}^{DL} + H_{DL,5}^{DL} \phi H_{S,2}^{DL} - L_{21} q_1). 
\]

It is obvious that \( \{q_i\}_{i=1,2} \) in equation (8) are orthonormal row vectors. Obtaining \( \{q_i\}_{i=1,2} \) from the channel information, we are capable of precoding the transmitted signal as

\[
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} q_1^H \\ q_2^H \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 - \frac{L_{21}}{L_{22}} \hat{x}_1 \end{bmatrix}
\] (9)

Transmitting the above precoded data, the received signal is expressed as

\[
y_{BC} = \begin{bmatrix} L_{11} \\ L_{21} \\ L_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \begin{bmatrix} q_1^H \\ q_2^H \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 - \frac{L_{21}}{L_{22}} \hat{x}_1 \end{bmatrix} + z
\]

\[
= \begin{bmatrix} L_{11} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + z = \begin{bmatrix} \|H_{DL,1}^{DL} + H_{DL,5}^{DL} \phi H_{S,1}^{DL}\| \\ 0 \\ \|H_{DL,2}^{DL} + H_{DL,5}^{DL} \phi H_{S,2}^{DL} - L_{21} q_1\| \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + z
\] (10)

From the above equation, we can see that the correlations between two signals \( \hat{x}_1 \) and \( \hat{x}_2 \) are canceled. Meanwhile two virtual interference-free channels have been established. We assume that the total transmitting power \( P \) is distributed to \( \alpha P \) and \((1 - \alpha)P\) for the first and the second users respectively. Then the capacities for the two users are given as

\[
R_1 = \log_2 \left( 1 + \|H_{DL,1}^{DL} + H_{DL,5}^{DL} \phi H_{S,1}^{DL}\|^2 \frac{\alpha P}{\sigma^2} \right)
\] (11)
\[ R_2 = \log_2 \left( 1 + \left\| H_{B,2}^{DL} + H_{S,2}^{DL} \phi H_{S,2}^{DL} - L_{21}q_1 \right\| \frac{(1-\alpha)P}{\sigma_z^2} \right) \]

(12)

If the second user is selected such that \( L_{21} = 0 \), we have

\[ R_2 = \log_2 \left( 1 + \left\| H_{B,2}^{DL} + H_{B,S}^{DL} \phi H_{S,2}^{DL} \right\|^2 \frac{(1-\alpha)P}{\sigma_z^2} \right) \]

(13)

We can see from the result that the channel capacity of BC is similar to that of the multiple access channel. In [12], the duality of the uplink and downlink channel capacities was used to derive the capacity of broadcast channel. It was also shown that the downlink channel capacity using Dirty paper coding (DPC) is the same as that of using the multiple access channel [13].

3. Pre-coding Transmission Schemes for Broadcast Channel

In this section we discuss the pre-coding transmission schemes, which is a specific means of implementing the multi user MIMO system for the downlink. It is not quite straightforward to detect coordinated signals at the receiver antennas, which is the main challenge in broadcast channel data transmission. As a consequence, interference cancellation methods at BS are inevitably required. In this section, we consider some of the different transmission methods, and apply them in our IRS aided multi user MIMO system.

3.1. Channel Inversion

Consider a similar broadcast channel situation to the one mentioned above, where \( N_M = 1 \) for all users and \( K = N_B \). Let \( \hat{x}_u \) denote the \( u \)th user, \( u = 1, 2, 3, \ldots K \). The received signal of the \( u \)th user is given as

\[ y_u = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_K \end{bmatrix} + z_u, u = 1, 2, \ldots K. \]

(14)

The received signal of all users can be represented as

\[
\begin{bmatrix}
  y_1 \\
y_2 \\
\vdots \\
y_K
\end{bmatrix} = 
\begin{bmatrix}
  H_{B,1}^{DL} + H_{B,S}^{DL} \phi H_{S,1}^{DL} \\
  H_{B,2}^{DL} + H_{B,S}^{DL} \phi H_{S,2}^{DL} \\
  \vdots \\
  H_{B,K}^{DL} + H_{B,S}^{DL} \phi H_{S,K}^{DL}
\end{bmatrix} 
\begin{bmatrix}
  \hat{x}_1 \\
  \hat{x}_2 \\
  \vdots \\
  \hat{x}_K
\end{bmatrix} + 
\begin{bmatrix}
  z_1 \\
  z_2 \\
  \vdots \\
  z_K
\end{bmatrix}
\]

(15)

The receiving signal at each user terminal in equation (15) is a scalar while each user’s received signal in equation (2) is a vector. Due to the limitation that each user is only equipped with single antenna, interferences created by other signals cannot be canceled. Instead pre-coding techniques such as channel inversion and regularized channel inversion can be considered [14]. In the multi user MIMO scenario, channel inversion is the same processing as the zero forcing (ZF) pre-equalization [15]. As in the single user MIMO case, noise enhancement can be mitigated using the MMSE criterion [16]. It can be derived that in regularized case, the pre-coding matrix is expressed as

\[ W = \sqrt{\frac{N_B}{\text{tr}\left(MM^H\right)}} M \]

(16)
where $\mathbf{M} = \mathbf{H}' \cdot (\mathbf{HH}' + \sigma_n^2 \mathbf{I})^{-1} \cdot \mathbf{H}$ =
\[
\begin{bmatrix}
\mathbf{H}_{B,1}^{DL} + \mathbf{H} \mathbf{D}_{B,S} \mathbf{H}_{S,1}^{DL} \\
\mathbf{H}_{B,2}^{DL} + \mathbf{H} \mathbf{D}_{B,S} \mathbf{H}_{S,2}^{DL} \\
\vdots \\
\mathbf{H}_{B,K}^{DL} + \mathbf{H} \mathbf{D}_{B,S} \mathbf{H}_{S,K}^{DL}
\end{bmatrix}
\] and $\text{tr} (A)$ represents the trace of matrix $A$.

A. Values of IRS matrix $\phi$ will be discussed in the next section.

### 3.2 Block Diagonalization

The above mentioned channel inversion methods are to deal with multiple users, each with one single antenna. All signal $x_u (u \neq u_T)$ is considered as interference, except for the target signal where $u = u_T$. Therefore the interference signals are canceled from $y_{u_T}$ via precoding. For the multiple users with multiple antennas scenario, a similar scheme can be applicable.

Since the inter-antenna interference of the same user in the specific signal as well as other user interference are canceled or mitigated in the channel inversion processes, the noise enhancement becomes more severe from the perspective of target users. In this case, the method called block diagonalization (BD) is more suitable [17-18]. Unlike in the channel inversion processes, in the block diagonalization method, only interferences from other users are canceled in the processes of precoding. Then, the inter-antenna interferences for each user can be canceled or mitigated by various signal detection methods such as minimum mean-square error (MMSE), maximum likelihood (ML), etc.

Let $N_u$ denote the number of antennas for the $u$th user, $u = 1, 2, 3, \ldots, K$. For the $u$th user signal $\tilde{x}_u \in \mathbb{C}^{N_u \times 1}$, using the denotations in the previous sections, the received signal is $y_u \in \mathbb{C}^{N_M \times 1}$ is expressed as

$$y_u = (\mathbf{H}_{B,u}^{DL} + \mathbf{H}_{B,S}^{DL} \phi \mathbf{H}_{S,u}^{DL}) \sum_{k=1}^{K} \mathbf{W}_k x_k + z_u$$

(17)

The signal consists of target signal, interference signal and noise. \{$(\mathbf{H}_{B,u}^{DL} + \mathbf{H}_{B,S}^{DL} \phi \mathbf{H}_{S,u}^{DL}) \mathbf{W}_k$\} form an effective channel matrix between the $u$th user receiver and the $k$th user transmit signal. When $u \neq k$, $u$ incur interferences to the $u$th user receiver unless $(\mathbf{H}_{B,u}^{DL} + \mathbf{H}_{B,S}^{DL} \phi \mathbf{H}_{S,u}^{DL}) \mathbf{W}_{u \neq k} = 0_{N_M \times N_u}$. In other words, the interference-free transmission will be guaranteed as long as the effective channel matrix can be block-diagonalized, that is $\forall u \neq k$, the above mentioned equation holds true.

In order to meet the transmit power constraint, the precoders $\mathbf{W}_u \in \mathbb{C}^{N_B \times N_u}$ is supposed to be unitary. We now discuss how to obtain the desired $\phi$ and $\mathbf{W}_u$ that satisfy the block-diagonalized condition. Let us first establish the following channel matrix that contains the channel gains of all users except for the $u$th user.

$$\hat{\mathbf{H}}_u^{ex} = \begin{bmatrix}
(H_{u,1}^{ex} + H_{u,2}^{ex} \phi H_{S,1}^{ex})^\gamma \\
(H_{u,2}^{ex} + H_{u,2}^{ex} \phi H_{S,2}^{ex})^\gamma \\
\vdots \\
(H_{u,K}^{ex} + H_{u,K}^{ex} \phi H_{S,K}^{ex})^\gamma
\end{bmatrix} = \begin{bmatrix}
H_{u,1}^{ex} \\
H_{u,2}^{ex} \\
\vdots \\
H_{u,K}^{ex}
\end{bmatrix}^\gamma$$

(18)

When $N_M = \sum_{u=1}^{K} N_u = N_B$, i.e., the total number of antennas used by all active users is the same as the number of all BS antennas, the block-diagonalized condition is equivalent to $\hat{\mathbf{H}}_u^{DL} : \mathbf{W}_u = 0_{(N_M - N_u) \times N_u}$. This implies that the precoder matrix $\mathbf{W}_u \in \mathbb{C}^{N_B \times N_u}$ must be designed to lie in the null space of $\hat{\mathbf{H}}_u^{DL}$. We note that the dimension of matrix $\hat{\mathbf{H}}_u^{DL} \in \mathbb{C}^{(N_M - N_u) \times N_u}$ is less than $\min(N_M - N_u, N_B)$. Assuming that $N_M = N_B$ and $\min(N_M - N_u, N_B) = N_B - N_u$, the singular value decomposition (SVD) of $\hat{\mathbf{H}}_u^{DL}$ is $\hat{\mathbf{H}}_u^{DL} = \hat{\mathbf{U}}_u \hat{\mathbf{\Sigma}}_u \hat{\mathbf{V}}_u^{\text{non-zero}}$, where $\hat{\mathbf{V}}_u^{\text{non-zero}} \in \mathbb{C}^{(N_M - N_u) \times N_B}$ and $\hat{\mathbf{V}}_u^{\text{zero}} \in \mathbb{C}^{N_u \times N_B}$ are composed of right singular vectors that correspond to non-zero singular values and zero singular values respectively. As a consequence, we have $\hat{\mathbf{H}}_u^{DL} : \hat{\mathbf{V}}_u^{\text{zero}} = 0_{(N_M - N_u) \times N_B}$, where $\hat{\mathbf{V}}_u^{\text{zero}}$ is exactly in the null space of $\hat{\mathbf{H}}_u^{DL}$. In another word, when a signal is transmitted in the direction of $\hat{\mathbf{V}}_u^{\text{zero}}$, all but the $u$th user receives no signal at all. Thus, $\mathbf{W}_u = \hat{\mathbf{V}}_u$ can be the precoding matrix for the $u$th user.

Now we discuss the appropriate value for IRS matrix $\phi$. Using the block-diagonalization methods mentioned above, we are capable of cancel the interferences between multiple users. With the aid of
IRS, we are able to enhance the target signal. Target signal is given as $y_u = H_{B,u}^D L + H_{S,u}^D L \phi H_{S,u}^D L + z_u$. We note that $H_{B,u}^D L$ and $H_{B,S}^D L \phi H_{S,u}^D L$ are the main channel gains contributing to the received signal. If the two channels are somehow focus on similar directions, lie in the same row space, for example, the target signal’s energy is aggregated instead of diverged. In a more specific way, our goal is to maximum the norm of channel gain $|H_{B,u}^D L + H_{B,S}^D L \phi H_{S,u}^D L|$. Using the assumptions above mentioned, $r(H_{B,u}^D L) \leq N_B = N_M, r(H_{B,S}^D L) \leq \min(N_M, M)$ and $r(H_{S,u}^D L) \leq \min(N_B, M)$. Thus, if $M \geq \max(N_B, N_M)$, we can carefully set $\phi$ to meet the requirements. The optimal target is given as

$$\phi = \arg\max_\phi |H_{5,u}^D L \phi H_{5,u}^D L + 2H_{B,u}^D L H_{B,S}^D L \phi H_{S,u}^D L|$$ (19)

Similar problems have been proposed and discussed in [19]. One challenge of the IRS design is that each element lies in the discrete amplitude and phase-shift levels. Instead of using exhaustive search, a more practical solution is to first relax the constraints and solve the problem with continuous phase-shift values, then quantize the solutions to the nearest feasible discrete value [20]. Once we construct the interference-free signal, various signal detection methods mentioned before can be applied to estimate $\hat{x}_u$.

4. Numerical Results
In this section we simulate the data transmission in broadcast channel (BC) with multiple users. Some precoding techniques are applied to demonstrate the energy focusing performance of IRS.

Figure 1 demonstrates the theoretical ergodic capacity of the proposed MIMO system. We set the number of users $K = 1$. As is indicated in the figure, the capacity grows almost linearly as the SNR increases. In the region where SNR is smaller than 20dB, the signal enhancement of IRS is able to raise the capacity about 20%-50%.

![Figure 1. Ergodic capacity of MIMO system with different $N_M$ and $N_B$](image)

In Figure 2, we compare the bit error rate (BER) of different transmission schemes in single user antenna scenario. We set the number of reflecting elements of the IRS $N = 20$, and $N_M = N_B = 4$. In order to control the signal power, all the transmitting signal are the same and the channel information state (CSI) is normalized. First, it is observed that as the SNR increases, BER has a logarithmic descent tendency. Second, due to the consideration of noise signal power, regularized channel
inversion has better performance than ordinary channel inversion, which decreases BER around 60% in most of the experiment cases. With the aid of IRS, the BER’s decreasing ratio is even larger, up to 70% compared to the cases without IRS. In the high SNR situations, the BER has a slight tendency to saturate, which results from the negligible influence from noise to the target signals.

![Figure 2. Received signal BER versus SNR using channel inversion](image2)

As shown in Figure 3, under multiple user antennas circumstances, block-diagonalization technique is applied in the precoding process. The settings are similar to that in Figure 1, but each user is equipped with 2 receiving antennas. The curves have a similar trend as the ones in Figure 2. In such cases, IRS performs almost the same as in the single receiving antenna scenario, reducing the BER around 70% under most of the experiment SNRs. In high SNR region (larger than 25dB for example), the BER in IRS aided scenario is smaller than $10^{-3}$.

![Figure 3. Received signal BER versus SNR using block diagonalization](image3)
Figure 4 depicts the received signal power versus different numbers of IRS elements unit for $N_M = N_B = 4$ and $N_u = 2$. Different curves indicate the different phase resolution. The curves imply that as the resolution of phases increases, the power of received signal increases as well. When the number of element is small, $N = 20$ for example, the performance of different resolution (1-bit, 2-bit and 3-bit) is almost the same. As the number of elements increases to above 150, the power gap between different resolutions has a tendency to keep in unaltered condition (around 0.6dB between 1-bit and 2-bit resolution). It is obvious that the theoretical limit where IRS has infinite phase resolution is not achievable in practice.

![Figure 4. Signal power versus number of IRS elements with different phase resolution](image)

5. Conclusions
In this paper, we have considered the multi-user MIMO system aided by an IRS. The capacity analysis of both uplink and downlink channel are provided, as well as some precoding schemes in broadcast channel data transmission scenario. In particular, we applied a new approach to enhance the receiving signal power and reduce bit error rate (BER) with the deployment of IRS. As the IRS aided transmission system is a new concept and remains largely uninvestigated, we hope that this paper would provide an effective guide for the future researches on Intelligent Surfaces.

6. Acknowledgements
This work is supported by the National Natural Science Foundation of China (No.61631013), Beijing National Research Center for Information Science and Technology, National Major Project (NO. 2018ZX03001006-003).

7. References
[1] F. Boccardi et al., “Five Disruptive Technology Directions for 5G,” IEEE Commun. Mag., vol. 52, no. 2, Feb. 2014, pp. 74–80
[2] T. S. Rappaport et al., May 2013, “Millimeter wave mobile communications for 5G cellular: It will work!” IEEE access, vol. 1, pp. 335–349
[3] S. Hu, F. Rusek, and O. Edfors, Dec. 2012, “The potential of using large antenna arrays on intelligent surfaces,” in Proc. IEEE 85th Veh. Technol. Conf., Sydney, Jun. 2017, pp. 1–6
[4] F. Rusek et al., “Scaling up MIMO: Opportunities and challenges with very large arrays,” IEEE Signal Process. Mag., vol. 30, no. 1, pp. 40-60
[5] S. V. Hum and J. Perruisseau-Carrier, Jan. 2014 “Reconfigurable reflect arrays and array lenses for dynamic antenna beam control: A review,” IEEE Trans. Ant. Prop., vol. 62, no. 1, pp. 183–198

[6] T. J. Cui, M. Q. Qi, X. Wan, J. Zhao, and Q. Cheng, Oct. 2014, “Coding meta-materials, digital meta-materials and programmable meta-materials,” Light: Science & Applications, vol. 3, no. 10, p. e218

[7] B. Sainath and N. B. Mehta, Nov. 2012, “Generalizing the amplify-and-forward relay gain model: An optimal SEP perspective,” IEEE Trans. Wireless Commun., vol. 11, no. 11, pp. 4118–4127

[8] Shin, O.S. and Lee, K.B. (2003) Antenna-assisted round robin scheduling for MIMO cellular systems. IEEE Commun. Letters, 7(3), 109–111

[9] Jindal, N. and Goldsmith, A. (2005) Dirty paper coding vs. TDMA for MIMO broadcast channel. IEEE Trans. Info. Theory, 51(5), 1783–1794

[10] R. W. Heath, S. Sandhu, and A. J. Paulraj, Apr. 2001, “Antenna selection for spatial multiplexing systems with linear receivers,” IEEE Commun. Lett., vol. 5, pp. 142–144.

[11] R. W. Heath, M. Airy, and A. J. Paulraj, Nov. 2001, “Multiuser diversity for MIMO wireless systems with linear receivers,” in Proc. Asilomar Conf. Signals, Systems, and Computers, Pacific Grove, CA, pp. 1194–1199

[12] Weingarten, H., Steinberg, Y., and Shamai, S. (2006) The capacity region of the Gaussian MIMO broadcast channel. IEEE Trans. Info. Theory, 52(9), 3936–3964

[13] Boppana, S., and J. M. Shea. 2006, "Downlink User Capacity of Cellular Systems: TDMA vs Dirty Paper Coding." IEEE International Symposium on Information Theory

[14] Choi, R. and Murch, R. (2003) A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach. IEEE Trans. Wireless Commun., 2(2), 20–24

[15] Sakamoto, Kazumitsu , et al. "A Novel ZF-Based Fast Beamforming Method for Short-Range MIMO Transmission." IEEE Wireless Communications Letters 4.5(2015):557-560

[16] Yan, Liu. "Linear Mmse Interference Cancellation Detection for MIMO-OFDM System." 2017 9th International Conference on Measuring Technology and Mechatronics Automation (ICMTMA) IEE

[17] Spencer, Q.H., Swindlehurst, A.L., and Haardt, M. (2004) Zero-forcing methods for downlink spatial multiplexing in multi-user MIMO channels. IEEE Trans. Signal Processing, 52(2) 461–471

[18] Pan, Z., Wong, K.K., and Ng, T. (May2003) MIMO antenna system for multi-user multi-stream orthogonal space division multiplexing. IEEE ICC’03, vol. 5, pp. 3220–3224

[19] Chongwen Huang, George C. Alexandropoulos, Alessio Zappone, Merouane Debbah and Chau Yuen. 2018, “Energy Efficient Multi-User MISO Communication using Low Resolution Large Intelligent Surfaces.”

[20] Wu, Qingqing, and R. Zhang. 2018, "Intelligent Reflecting Surface Enhanced Wireless Network: Joint Active and Passive Beamforming Design."