A novel, interesting class of scalar-tensor gravity theories is those with a limit on the field motion, where the scalar field either goes to a constant acceleration or stops accelerating and goes to a constant velocity. We combine these with the ability to dynamically cancel a high energy cosmological constant, e.g. through the well tempered or self tuning approaches. One can successfully have a cosmic expansion history with a matter dominated epoch and late time acceleration despite a large cosmological constant, although the late time de Sitter limit may be unstable. Pole models, such as in a Dirac-Born-Infeld action, are of particular interest for a cosmic speed limit.

I. INTRODUCTION

The original cosmological constant problem – how did the universe survive a high energy (possibly Planck scale) cosmological constant to give a universe that looks like ours – is a fundamental issue in physics [1–5]. This is often glossed over in the quest to understand the late time, low energy cosmic acceleration. However, some approaches bring these together and look to resolve them through a dynamical cancellation by an evolving scalar field.

This dynamical approach has been explored under the names of self tuning [6–15] and well tempered [16–22] theories. These require higher order terms in the action involving the scalar field, and fit in well with the most general scalar-tensor theory leading to second order equations of motion, Horndeski gravity.

Here we investigate a particular aspect of the dynamical approach, asking whether the scalar field evolution is unbounded, in field value, velocity, or acceleration. Speed limit theories have been studied in the context of inflation [23–25], without addressing the original cosmological constant problem, with one example being Dirac-Born-Infeld theory.

In Section II we introduce the equations of motion within shift symmetric Horndeski theory, and the conditions we seek: dynamical cancellation of the high energy cosmological constant, while preserving a matter dominated era, and asymptoting to a (low energy cosmological constant) de Sitter state. We then derive the conditions for a field value limit and a field speed limit in Section III, and a field acceleration limit in Section IV, achieving a standard cosmic expansion history despite a high energy cosmological constant. Section V summarizes and concludes, while Appendix A investigates stability of the effective field theory de Sitter background.

II. GENERAL SHIFT SYMMETRIC THEORY

We begin within the most general scalar-tensor theory leading to second order equations of motion, Horndeski gravity, and examine the conditions that allow for dynamical cancellation of a high energy cosmological constant (i.e. removing the original cosmological constant problem). This self tuning imposes relations between the Horndeski action terms, and will set up the investigation of how the asymptotic future field motion – either a field value limit \( \phi \rightarrow 0 \), a speed limit \( \dot{\phi} \rightarrow \text{const} \), or an acceleration limit \( \ddot{\phi} \rightarrow \text{const} \) – further determines the behavior.

The Horndeski action is

\[
S = \int d^4x \sqrt{-g} \left[ G_4(\phi) R + K(\phi, X) - G_3(\phi, X) \square \phi - \Lambda + \mathcal{L}_m[g_{\mu\nu}] \right],
\]

where \( K, G_3, G_4 \) are the Horndeski functions (with \( G_5 = 0 \) and \( G_4 \) purely a function of the scalar field \( \phi \), not its kinetic term \( X = -(1/2)g^{\mu\nu}\phi_\mu \phi_\nu \), to keep gravitational waves propagating at the speed of light). The other quantities are the usual Ricci scalar \( R \), high energy scale cosmological constant \( \Lambda \), and matter Lagrangian \( \mathcal{L}_m \).

Shift symmetry is a valuable property that protects against quantum corrections, preserving a simple form for the theory. Under shift symmetry, the most general forms for the action functions are

\[
K(\phi, X) = \kappa(X) - X^3 \phi
\]

\[
G_3(\phi, X) = G_3(X)
\]

\[
G_4(\phi) = (M_{pl}^2 + M\phi)/2,
\]

(1)
where $M^2_{\text{pl}} = 1/(8\pi G_N)$ and $G_N$ is Newton’s constant. Note the tadpole term $\lambda^3 \phi$ in $K$, and the tadpole-like term $M \phi$ in $G_4$. There is no tadpole-like $b \phi$ in $G_3$ because in the action a term $\phi \Box \phi$ can be converted into a total derivative and a term linearly proportional to $X$, which can be absorbed into $K$ (cf. [18]).

The equations of motion in a Friedmann-Lemaître-Robertson-Walker (FLRW) universe become

\begin{align}
3H^2(M_{\text{pl}}^2 + M \phi) &= \Lambda + 2XK_X - \kappa + \lambda^3 \phi + 6H\dot{\phi}XG_{3X} - 3HM\dot{\phi} + \rho_m \tag{5} \\
-2\dot{H}(M_{\text{pl}}^2 + M \phi) &= \dot{\phi}(M - 2XG_{3X}) - H\dot{\phi}(M - 6XG_{3X}) + 2XK_X + \rho_m + P_m \tag{6} \\
0 &= \ddot{\phi} \left[K_X + 2XK_{XX} + 6H\dot{\phi}(G_{3X} + XG_{3XX})\right] \\
&+ 3H\dot{\phi}K_X + \lambda^3 + 6XG_{3X}(\dot{H} + 3H^2) - 3M(\dot{H} + 2H^2). \tag{7}
\end{align}

Here a subscript $X$ denotes a derivative with respect to $X$ and a dot is a derivative with respect to coordinate time $t$. The Hubble expansion rate $H = \dot{a}/a$ where $a$ is the scale factor, and $\rho_m$ and $P_m$ are the matter energy density and pressure. We define $g \equiv XG_{3X}$.

We will be interested in cosmological solutions that give a late time period of accelerated expansion, and in particular those that have a future de Sitter asymptote, $H \rightarrow h = \text{const}$. One main focus will be to characterize the asymptotic approach.

### III. FIELD LIMIT AND SPEED LIMIT

The Friedmann equation (5) will play an extremely important role, as we need its terms to cancel the high energy cosmological constant $\Lambda$ and to leave a late time de Sitter state $H \rightarrow h$. Those terms will have to interact with each other – giving a connection between the action functions $K$ and $G_3$ – in order to achieve the cancellation dynamically and without fine tuning.

We begin by considering the $\lambda^3 \phi$ term. The field value can asymptotically go to either a constant $\phi_{dS}$ (possibly zero) or to infinity. Unless both the field velocity $\dot{\phi}$ and acceleration $\ddot{\phi}$ go to zero, then we must have $\phi \rightarrow \infty$.

#### A. Field Value Limit

If $\phi \rightarrow \text{const}$ (possibly zero), then as mentioned $\dot{\phi}$ and hence $X$ must go to zero asymptotically. Thus all the terms involving $K$ and $G_3$ must go to either zero, or infinity, depending on whether they depend on negative or positive powers of $X$. If some go to infinity, they must cancel each other or they will overwhelm the de Sitter term $3M_{\text{pl}}^2 h^2$ on the left hand side of the Friedmann equation. However, since additional diverging factors enter the $\dot{H}$ and $\ddot{\phi}$ equations of motion, which have zero on their left hand sides in the de Sitter limit, such as a cancellation is not generally viable. That is, it appears we cannot achieve all three conditions of dynamical cancellation of $\Lambda$, an asymptotic de Sitter state, and solutions to the equations of motion at the same time.

Thus the $K$ and $G_3$ terms must go to zero asymptotically. This is somewhat undesirable as then the action itself shows no sign of anything to cancel the cosmological constant: it is asymptotically just the Einstein-Hilbert action. Looking more closely at the Friedmann equation we see the remaining term to cancel $\Lambda$ is $(\lambda^3 - 3h^2 M)\phi = \text{const}$. That is, the cosmological constant $\Lambda$ is canceled by a constant of integration $\phi_{dS}$, not really dynamically. This can be regarded as fine tuning. Therefore, field value limit theories to cancel the high energy cosmological constant and leave a low energy cosmological constant are not particularly satisfactory.

#### B. Speed Limit

We therefore proceed to speed limit theories. Here, terms proportional to $\phi$ will be growing with time. One possibility is to take $\lambda^3 = 3h^2 M$, so as to preserve the de Sitter desiderata of $3M_{\text{pl}}^2 h^2$ on the left hand side of the Friedmann equation. However, there is a hidden price: the gravitational coupling to the Ricci scalar, $G_4 = (M_{\text{pl}}^2 + M \phi)/2$, blows up as $\phi$ gets large (since $\phi \rightarrow \text{const}$ asymptotically). This will greatly disrupt the equations at the perturbative level, e.g. the growth of matter density perturbations. One might try to arrange that the blow up occurs in the future, beyond the reach of observations, but this requires fine tuning such as choosing initial conditions such that $M \phi$, is extremely small.
This is a shame, as indeed a speed limit theory as simple as
\[
K = -X - \lambda^3 \phi \quad (8)
\]
\[
g = \frac{\lambda^3}{18h^2} + \frac{\dot{\phi}}{6h} \quad ,
\]
will dynamically cancel a high energy cosmological constant, preserve radiation and matter domination, and lead to a late time cosmic acceleration and de Sitter state – i.e. do everything we ask of it at the background level, while furthermore being ghost free and Laplace stable.

Thus we will consider only theories with \( M = 0 \). This will ensure that the left hand side, \( 3M_p^2 H^2 \), of the Friedmann equation is constant for the asymptotic de Sitter state as we wish, but we then require cancellations among terms on the right hand side. That is, there must be a relation between the Horndeski terms \( K \) and \( G_3 \), and the field \( \phi \). At the same time, these must satisfy the equations of motion, i.e. the \( \dot{H} \) and \( \dot{\phi} \) equations. This relation, or degeneracy, is the essence of the self tuning or well tempered mechanisms for dynamically canceling the high energy cosmological constant \( \Lambda \).

Here we take a novel branch of the self tuning class by restricting the asymptotic field motion to speed limit theories (and acceleration limit theories in Sec. IV). For the approach to the de Sitter state we explore the field evolution by writing
\[
\ddot{\phi} \rightarrow \ddot{\phi}_{dS} + \ddot{F}(t) \quad (10)
\]
\[
\dot{\phi} \rightarrow \dot{\phi}_{dS} t + F(t) + \dot{\phi}_1 \quad (11)
\]
\[
\phi \rightarrow (1/2)\dddot{\phi}_{dS} t^2 + \dot{\phi}_1 t + \int dt \, F(t) \quad ,
\]
where \( \dddot{\phi}_{dS} \) is a constant acceleration, \( \dot{\phi}_1 \) a constant speed, and \( F(t) \) an evolution in speed to be determined by the equations of motion. We then look for solutions with either a field speed limit or acceleration limit.

For the speed limit case \( \dddot{\phi}_{dS} = 0 \), plus \( F(t) \) must be subdominant to the constant term \( \dot{\phi}_1 \) at late times: thus the field velocity reaches a bound, \( \dot{\phi} \rightarrow \dot{\phi}_1 \). In the Friedmann equation the term \( \lambda^3 \dot{\phi} \) will be growing as \( t \). Therefore we have to arrange other terms in the Friedmann equation to negate the specific \( \dot{\phi} \) term growth. From Eq. (5) we see this can be accomplished either by \( \phi \) or \( \kappa(X) \). In fact, it cannot be done by \( \phi \) since if \( \dot{\phi} \sim t^0 \), then \( \phi \sim t \) grows faster. Thus we must choose \( \kappa(X) \), and thence \( g(X) \), to enforce the cancellation.

To match \( \phi \rightarrow \dot{\phi}_1 t \) we need \( \kappa(X) \sim t \). Since \( X = \dot{\phi}_1^2 / 2 \), we can write this as
\[
\dot{\phi} = \dot{\phi}_1 + F(t) \quad , \quad \kappa(X) \sim t \sim F^{-1}(\dot{\phi} - \dot{\phi}_1) \quad ,
\]
where \( F^{-1} \) indicates the inverse function (not \( 1/F \)). Since \( t \) gets large at late times while \( \dot{\phi} - \dot{\phi}_1 \) approaches zero, this indicates that speed limit theories require a pole function for \( K_X \), e.g.
\[
K_X \sim (\dot{\phi} - \dot{\phi}_1)^{-p} \quad .
\]
There is nothing wrong with pole functions – indeed Dirac-Born-Infeld kinetic terms are important in various areas of physics – and they can work well cosmologically. One does have to be careful when solving the equations of motion numerically to write the equations in a form such that apparent divergences cancel.

A simple speed limit model to cancel the cosmological constant and give an asymptotic de Sitter state is
\[
K_X = \frac{c}{3h} - \frac{\lambda^3}{3h(\dot{\phi} - \dot{\phi}_{dS})} \quad (16)
\]
\[
g = -\frac{c\dot{\phi}_{dS}}{18h^2\dot{\phi}} + \frac{\lambda^3 \dot{\phi}}{18h^2(\dot{\phi} - \dot{\phi}_{dS})} \quad ,
\]
where \( c \) is a constant of dimension \( \lambda^3 \) and we have written \( \dot{\phi}_1 \) as \( \dot{\phi}_{dS} \) for clarity. Indeed, one can choose \( c = \lambda^3 \) and then
\[
K = -\frac{\lambda^3 \dot{\phi}_{dS}}{3h} \ln \left( \frac{\dot{\phi}}{\dot{\phi}_{dS}} - 1 \right) - \lambda^3 \dot{\phi} \quad .
\]
In the expressions below, however, we keep $c$ general. While $K$ will involve a log (apparently, but see below), recall that in any theory where $g$ has a constant term then $G_3$ has a log also. Specifically,

$$\begin{align}
K(\phi, \dot{\phi}) &= \frac{c\dot{\phi}}{3h} - \frac{\lambda^3(\ddot{\phi} - \dot{\phi}_{\text{dS}})}{3h} - \frac{\lambda^2\dot{\phi}_{\text{dS}}}{3h} \ln \left( \frac{\dot{\phi}}{\dot{\phi}_{\text{dS}}} - 1 \right) - \lambda^3 \phi \\
G_3(\dot{\phi}) &= \frac{c\phi_{\text{dS}}}{9h^2\dot{\phi}} + \frac{\lambda^3}{9h^2} \ln \left( \frac{\phi - \phi_{\text{dS}}}{\phi} \right)
\end{align}$$

(19)

(20)

The scalar field approaches de Sitter with the behavior

$$\begin{align}
\dot{\phi} &= \dot{\phi}_{\text{dS}} (1 - e^{-3ht})^{-1} \\
\dot{\phi} &= -3h\dot{\phi}_{\text{dS}} e^{-3ht} (1 - e^{-3ht})^{-2} \\
\phi &= -\dot{\phi}_{\text{dS}} \ln \left( \frac{\phi - \dot{\phi}_{\text{dS}}}{\dot{\phi}_{\text{dS}}} \right)
\end{align}$$

(21)

(22)

(23)

We now see that the $-\lambda^3 \phi$ term on the de Sitter approach cancels the log term in $K$, leaving

$$K(\phi, \dot{\phi}) \rightarrow \frac{c\dot{\phi}}{3h} - \frac{\lambda^3(\ddot{\phi} - \dot{\phi}_{\text{dS}})}{3h}.$$  

(24)

Thus, we have a theory with $M = 0$, that has a speed limit $\dot{\phi} \rightarrow \dot{\phi}_{\text{dS}}$, vanishing field acceleration $\ddot{\phi} \rightarrow -3h\dot{\phi}_{\text{dS}} e^{-3ht}$, and in fact fulfills the well tempering degeneracy condition (rather than the trivial scalar field equation of self tuning). Note that terms in the equations of motion do not have divergences, but rather are of the form $\dot{\phi}/(\phi - \dot{\phi}_{\text{dS}}) = -3h/(1 - e^{-3ht}) \rightarrow -3h$, $(H - h)/(\phi - \dot{\phi}_{\text{dS}}) \rightarrow -h/\dot{\phi}_{\text{dS}}$, and $H/(\phi - \dot{\phi}_{\text{dS}}) \rightarrow 3h^2/\dot{\phi}_{\text{dS}}$.

However, we are foiled once again at the perturbative level. The braiding property function [26] is $\alpha_B = 2g\dot{\phi}/(HM^2_p)$ and so as $g$ with its pole blows up (this is general, not dependent on the specific form adopted above), so does $\alpha_B$. This not only gives rise to a Laplace instability, but during the evolution crossing $\alpha_B = 2$ the effective gravitational strength $G_{\text{eff}}$ tends to diverge. Since $g$ nears its pole during the de Sitter approach, i.e. late time cosmic acceleration, it again would require fine tuning to push these disasters into the unobserved future. Property functions, stability, and gravitational strength are discussed further in Appendix A.

IV. ACCELERATION LIMIT

We now turn to the acceleration limit class of theories. To assess the possibilities, recall that an acceleration limit theory has $\ddot{\phi}_{\text{dS}} = \text{const} \neq 0$, with the function $F(t)$ taken to die off at late times, i.e. $F \sim t^{c_0}$, and so the field acceleration reaches a bound, $\ddot{\phi} \rightarrow \ddot{\phi}_{\text{dS}}$. The field speed then grows as $t$ at late times, and then the $\lambda^3 \phi$ term in the Friedmann equation has a $t^2$ term. That must be offset by $2X K_X \sim -\kappa(X)/6h\dot{\phi}$. The leading order term in $\ddot{\phi}$ goes as $t$, so $2X K_X \sim t^2 K_X$, implying that a possible resolution is $K_X \sim \text{const}$ (and hence $\kappa(X) \sim X \sim t^2$). The other possible contribution is $\phi g \sim tg$, requiring that $g \sim \phi$. Let us write

$$\begin{align}
K &= bX + p(X) - \lambda^3 \phi \\
g &= g_0 \phi + g_0(X)
\end{align}$$

(25)

(26)

Given the freedom in how $p(X)$, $g_0(X)$, $F(t)$, and $E(t) \equiv H(t) - h$ behave, there can be many classes of theories satisfying the constraining evolution equations near the de Sitter limit.

To keep simplicity foremost, we take $p(X) = so$ and $g_0(X) = \text{const}$. One rationale is that fractional powers of $\phi$ seem less motivated, and negative powers could give difficulties in the early universe where $\phi$ may be small. The $p$ term also gives a $t^1$ term in the Friedmann equation and so can help cancel the additional $t^1$ term from $\lambda^3 \phi$, while of course a constant in $K$ would just be absorbed in $\Lambda$. Basically, we are including all the terms that could contribute to canceling $\lambda^3 \phi$ in the Friedmann equation. Finally, we are free to perform a field redefinition to normalize $|b| = 1$; in fact we later find (see Appendix A) that we need a negative sign, so $b = -1$, in order to be ghost free.

Thus, our Ansatz is

$$\begin{align}
K &= -X + s\dot{\phi} - \lambda^3 \phi \\
g &= g_0 + g_1 \dot{\phi}
\end{align}$$

(27)

(28)
Table I summarizes the asymptotic behaviors for these cases. It seems remarkable that such simple forms for $K$ and $G_3$ can enable dynamical cancellation of the cosmological constant, and clear asymptotic approaches to de Sitter. Note that a subset of the acceleration limit with $F \sim O(t)$, fixing $s = 0$ and $g_0 = 0$, was treated by [14]; our results hold for general $s$ and $g_0$, which also reveal the two new acceleration limit behaviors.

What about the perturbative regime that caused us difficulties previously? Since $g_0$ grows with time, we will again have $\alpha_H$ diverge, with the problems this causes. However, as this goes more slowly that the exponential behavior of the speed limit case, we have the possibility to sweep it to the future and keep the theory compatible with observations. The question of why the de Sitter background is eventually unstable, and what is the appropriate asymptotic background, becomes an issue that needs to be separately resolved (since it generally occurs when $\phi \gg M_{pl}$, other physics may need to enter).

V. CONCLUSIONS

Self tuning and well tempering can provide a path toward canceling a high energy cosmological constant, while preserving a matter/radiation dominated era, and then entering a late time cosmic acceleration with a de Sitter
asymptote. An intriguing question is whether this dynamics can occur within limits: on either the field value, velocity, or acceleration.

We show that no such “cosmic acceleration with limits” is perfect. At sufficiently late times the effective field theory breaks down with an instability in the de Sitter background. This might happen, e.g. for the acceleration limit class, late enough that it would not impact observations. Or it could be pointing to some indication that de Sitter space is not the correct asymptotic background or that higher order terms must enter the effective field theory.

It is remarkable however that at the background level, quite simple models – in either the speed limit or acceleration limit class – can successfully provide a viable cosmic expansion despite the presence of a high energy cosmological constant. Models can be as simple as a minimal coupling to the Ricci scalar, (negative sign) canonical kinetic term with a tadpole, and $g = XG_{3X} \sim \dot{\phi}+\text{const}$, or a simple pole reminiscent of Dirac-Born-Infeld theory. The models presented are highly predictive, for example with just one free action parameter $\lambda$.

We study in particular the asymptotic approach to a de Sitter state, deriving analytic behaviors for the field velocity $\dot{\phi}(t)$ and cosmic expansion $H(t) - \dot{\phi}$. These results indicate that quite simple and tractable modified gravitational theories can have a rich and generally viable cosmology (at least in the expansion history) while treating the high energy cosmological constant problem. Future work could further explore pole models for viable examples, and consider the origin and fate of late time de Sitter instability.

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Appendix A: Property Functions, Ghost Free, and Stability Conditions

At the background level, i.e. for the cosmic expansion, we have seen that we can successfully obtain a current, low energy epoch of cosmic acceleration – despite the presence of a high energy cosmological constant – and a de Sitter asymptote, i.e. the effective dark energy equation of state approaches $w = -1$. However we need to examine the perturbative level as well, i.e. small inhomogeneities away from the homogeneous FLRW background, to assess the impact on cosmic structure growth, not to mention testing the soundness of the theory through freedom from ghosts and gradient instability.

It is convenient to investigate the perturbative quantities through the property functions [26]. The Planck mass running, braiding (between the scalar kinetic and tensor sectors), and kineticity property functions are given for the shift symmetric class of Eqs. (2)–(4) with $M = 0$ as

\begin{align}
\alpha_M &= 0 \\
\alpha_B &= \frac{2\dot{g}}{HM_{pl}^2} \rightarrow \frac{2\dot{\phi}(g_\theta + g_1\dot{\phi})}{HM_{pl}^2} \\
\alpha_K &= \frac{2X(K_X + 2XK_{XX}) + 12H\dot{\phi}Xg_X}{H^2M_{pl}^2} \rightarrow \frac{\dot{\phi}^2}{H^2M_{pl}^2} (6Hg_1 - 1) = \frac{\dot{\phi}^2}{H^2M_{pl}^2}\left(\frac{H}{\dot{\phi}} - 1\right),
\end{align}

where recall $g \equiv XG_{3X}$. The right arrow specializes to the simple acceleration limit model of Eqs. (27), (28), and we have imposed the requirement $g_1 = 1/(6\dot{\phi})$ in the last equation.

At early times, all property functions are small, going as $1/H$. That is, general relativity is a good approximation in the early universe. At late times, $\alpha_K$ vanishes on shell, while $\alpha_B$ gets large as $\dot{\phi}$ does.

To ensure the theory is ghost free, it must satisfy the no ghost condition $\alpha_K + (3/2)\alpha_B^2 \geq 0$. Since $\alpha_K \geq 0$ for the acceleration limit model used in Sec. IV, this is always satisfied. Indeed, this was our motivation for choosing a negative coefficient $K \sim -X$. If one writes $K \sim \epsilon X$ then the equations of motion give $g_1 = -\epsilon/(6\dot{\phi})$ and $\alpha_K \sim -\epsilon$. Thus we need $\epsilon = -1$ to be ghost free. One can also verify that the speed limit pole model used in Sec. III B is ghost free.

The Laplace stability condition delivering a nonnegative scalar sound speed squared is

\begin{equation}
1 - \frac{\alpha_B}{2} \alpha_B + \frac{(H \alpha_B)^2}{H^2} - \frac{2\dot{H}}{H^2} - \frac{\rho_m + P_m}{H^2} \geq 0.
\end{equation}
Now we see the problem with $\dot{\phi}g$, and hence $\alpha_B$, getting large. The condition then becomes

$$-\frac{\alpha_B^2}{2} \geq 0,$$

which is clearly violated. Thus at a minimum the linear effective field theory behind the property functions breaks down at late times. For the acceleration limit theory, $\dot{\phi}$ gets large, while for the speed limit case, although $\dot{\phi} \to \text{const}$, $g$ gets large due to the pole. At early times, the last two terms in the Laplace condition will dominate, with the main contribution to their difference coming from the $\dot{\phi}$ term in Eq. (6), leaving

$$-2g\ddot{\phi} \approx -2g_0\ddot{\phi} \geq 0,$$

as the stability condition: i.e. $g_0\ddot{\phi} \leq 0$. At intermediate times one must check the condition numerically.

Since $\alpha_M = 0$, these models are in the class of No Run Gravity [27], albeit special ones that cancel a high energy cosmological constant. The effective gravitational coupling strengths are [26, 28]

$$G_{\text{matter}} = G_{\text{light}} = \frac{\alpha_B + \alpha_B'}{\alpha_B(1 - \frac{\alpha_B}{2}) + \alpha_B'} = 1 + \frac{\alpha_B^2}{\alpha_B(2 - \alpha_B) + 2\alpha_B'}.$$

In the matter dominated era $\alpha_B \ll 1$ so $G_{\text{matter}} = G_{\text{light}} \to 1$. In the de Sitter state, as expected from the Laplace condition, the strength of gravity blows up as $\alpha_B$ increases too far, and then eventually approaches zero from below. (A theory with coupling $M \neq 0$, and hence $\alpha_M \neq 0$, could avoid this but still tends to cause the gravitational strength to differ from general relativity too early; also see Appendix C of [18] for some further discussion). As long as initial conditions allow a matter dominated era, the values of $G_{\text{matter}}, G_{\text{light}}$ should not be too extreme where we have data, i.e. the past, since the acceleration limit model grows $\phi$ linearly at late times, but this should be numerically evaluated for any theory.

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