Features of collimation and focusing of few-cycle optical beams

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Abstract. The paper reports theoretical features of collimation and focusing of few-cycle optical beams in transparent isotropic dielectric medium. It was showed, that a spatial collimation of such short waves results in a peculiar shaping of spatio-temporal structure with the central part moving faster than its outlying areas and collimated beam loses with distance half cycle of temporal electromagnetic field oscillation. In focus of focusing mirror, number of oscillations in temporal field structure of beams depends on only temporal profile of radiation source, when focusing collimated wave packet.

1. Introduction
Development of the methods of generation terahertz radiation made it possible to obtain few-cycle (less than 10 field oscillations) electromagnetic waves [1]. Such radiation is widely applicable in spectroscopy, in detection systems of drugs and explosive materials and for medical diagnostics [2]. The optimization of optical systems of emission, collimation and focusing initially few-cycle waves is important problem in this area [3].

Laws of paraxial difraction and collimation of electric field spatio-temporal structure, which is formed in far field region of diffraction of initially Gaussian one-cycle wave are well studied in the present time [3-4]. In this paper, we study collimation and focusing of initially Gaussian few-cycle waves propagation in homogeneous isotropic medium.

2. Paraxial radiation dynamics of spatio-temporal spectrum
Paraxial radiation dynamics of spatio-temporal spectrum

\[ g(\omega, k_x, k_y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(t, x, y, z) \exp(-i(\omega t + k_x x + k_y y)) d\omega dk_x dk_y \]  

(1)

in an isotropic homogenous optical medium can be described by the expression [4]:

\[ g(\omega, k_x, k_y, z) = g_{x,y}(\omega, k_x, k_y) \exp \left( -i k(\omega) z \left( 1 - \frac{k_x^2 + k_y^2}{2k(\omega)^2} \right) \right) \],  

(2)

where \( x, y, z \) are Cartesian axes coordinates (\( z \) axis coincides with wave propagation direction), \( C_x \) and \( C_y \) are assumed to be known spectral components of radiation at \( z = 0 \).
\( \omega \) is the temporal frequency of radiation, \( k_x \) and \( k_y \) are the spatial frequencies, \( k = \frac{\omega n(\omega)}{c} \) is the wave number, \( E(t, x, y, z) \) is the spatio-temporal distribution of electric field strength in optical beam, \( n(\omega) \) is the refractive index of the medium, \( c \) is the speed of light in a vacuum.

Diffraction of the initial Gaussian beams at the distance \( z_0 \) can be presented in the form, which is equivalent to expression in [4]

\[
G(\omega, x, y, z_0) = z_R(\omega) \frac{z_R(\omega) + iz_0}{z_R^2(\omega) + z_0} \exp \left( -\frac{x^2 + y^2}{\rho^2} \frac{z_R(\omega) + iz_0}{z_R^2(\omega) + z_0} - ik(\omega) z_0 \right) G_0(\omega) \quad (3)
\]

Where \( z_R(\omega) = \frac{\rho^2 n(\omega) \omega}{2c} \) is the Rayleigh range, \( \rho \) is the transverse beam width at \( z = 0 \).

We set collimating mirror with reflection function \( F_{col}(\omega, x, y, z) = \exp \left( ik(\omega) (x^2 + y^2) / 2f_{col} \right) \) at distance \( z = 0 \) and focal length \( f_{col} = z_0 \). Hence, spatial distribution of temporal spectrum immediately after reflection can be described by

\[
G_{col}(\omega, x, y, z_0) = G(\omega, x, y, z_0) \times F_{col}(\omega, x, y), \quad (4)
\]

where \( G(\omega, x, y, z_0) \) is defined by expression (3), in contrast to paper [3], where collimation mirror has been placed in far field diffraction region,

\[
G_{col}(\omega, x, y, z_0) = iz_R(\omega) \frac{iz_R(\omega) - z_0}{z_R(\omega)^2 + z_0^2} \times \exp \left( -\frac{(x^2 + y^2)}{\rho^2} \frac{iz_R(\omega) - z_0}{z_R(\omega)^2 + z_0^2} - ik(\omega) z_0 \right) G_0(\omega). \quad (5)
\]

Spatio-temporal spectrum is determined by Fourier transform

\[
g(\omega, k_x, k_y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} G(\omega, x, y, z) \exp(-i(k_x x + k_y y)) \, dk_x \, dk_y. \quad (6)
\]

Inserting of expression (5) in formula (6) gives us spatio-temporal spectrum

\[
g_{col}(\omega, k_x, k_y, z_0) = i\pi \rho^2 \frac{z_0}{z_R(\omega)} \exp \left( -\frac{k_x^2 + k_y^2}{4} \rho^2 \frac{z_0 + iz_R(\omega)}{z_R(\omega)^2} - ik(\omega) z_0 \right) G_0(\omega). \quad (7)
\]

For boundary condition (7) expression (2) takes form

\[
g_{col}(\omega, k_x, k_y, z) = i\pi \rho^2 \frac{z_0}{z_R(\omega)} \exp \left( -\frac{k_x^2 + k_y^2}{4} \rho^2 \frac{z_0 - iz_R(\omega)(z-z_0)}{z_R(\omega)^2} - ik(\omega)(z_0 + z) \right) G_0(\omega). \quad (8)
\]

Spatial distribution of temporal spectrum is described by Fourier transform formula

\[
G(\omega, x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g_{col}(\omega, k_x, k_y, z) \exp(i(k_x x + k_y y)) \, dk_x \, dk_y, \text{ therefore the dynamics of } \text{electro-magnetic field after collimation mirror, are described by expression}
\]

\[
G_{col}(\omega, x, y, z) = iz_0 \frac{z_{col}(\omega) + i(z - z_0)}{(z_{col}(\omega))^2 + (z - z_0)^2} \exp \left( -\frac{x^2 + y^2}{\rho^2} \frac{z_R(\omega)(z_{col}(\omega) + i(z - z_0))}{(z_{col}(\omega))^2 + (z - z_0)^2} - ik(\omega)(z_0 + z) \right) G_0(\omega), \quad (9)
\]

where \( z_{col}(\omega) = \frac{z_0^2}{z_R(\omega)}. \)
For far field region $z - z_0 \gg z_{col}(\omega)$ relation (9) is simplified to form

$$G_{col}^{Far}(\omega, x, y, z) = -\frac{z_0}{(z - z_0)} \exp \left( -\frac{x^2 + y^2}{\rho^2 \left( (z - z_0) \right)^2} \right) G_0(\omega).$$  (10)

Let us consider spatio-temporal structure of collimated beam. Such structure may occur in, for example, initially single-cycle waves on condition that refractive index is present in the form $n(\omega) = N = \text{const}$. At distances given in this article atmospheric air for THz radiation may be an example of such media [5].

Spectrum of single-cycle wave on the source can be described by the expression

$$G_0(\omega) = -\frac{\sqrt{\pi}}{2} E_0 \tau_p^2 \omega \exp \left( -\frac{\tau_p \omega^2}{2} \right),$$  (11)

where $\tau_p$ is the duration of radiation at $z = 0$.

Using Fourier transform formula

$$E(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} G(\omega, x, y, z) \exp(i\omega t) d\omega$$  (12)

we can take expression that describes spatio-temporal structure of collimated beam at the focus of collimation mirror ($z = z_0$)

$$E_{col}(t', x, y, z) = E_0 \frac{2 \nu^2 \tau_p}{\nu^2 \tau_p} \left( 1 - 2 \left( \frac{t'}{\tau_\theta} \right)^2 \right) \exp \left( -\left( \frac{t'}{\tau_\theta} \right)^2 \right),$$  (13)

where $t' = t - 2\frac{z_0 N}{c}$, which is called ”delay time” [4] and duration $\tau_\theta = \tau_p \sqrt{1 + \frac{z^2 + y^2}{\rho^2 \left( \frac{\nu^2}{\tau_p^2} \right)^2}},$ which is directly proportional to the distance from beam axis.

Radiation in far field region can be described by the Fourier transform (12) of function (10) with the radiation spectrum of the emitter (11):

$$E_{col}^{Far}(t'', x, y, z) = -E_0 \frac{z_0}{(z - z_0)} \frac{t''}{\tau_p} \exp \left( -\frac{x^2 + y^2}{\rho^2 \left( (z - z_0) \right)^2} \right) \exp \left( -\left( \frac{t''}{\tau_p} \right)^2 \right),$$  (14)

where delay time is $t'' = t - \frac{N}{c} \left( z_0 + z + \frac{z_0^2 + y^2}{\rho^2 (z - z_0)} \right),$ while duration has become the constant in cross section of wave packet.

Figure 1 illustrates expressions for collimation of initial one-cycle wave packet. Red and blue areas correspond to maximum positive and minimum negative field strength in two-dimensional representation. Here we take $\rho/\lambda_{max} = 10$, where $\rho$ is the initial transverse beam width, $\lambda_{max}$ is the wavelength corresponding to the maximum of input wave packet spectrum. For example, for terahertz single-cycle waves this value is usually $\lambda_{max} = 300 \mu m$.

Figure 1 and analysis of equation (9) show that in the focus of collimation mirror the central part of spatio-temporal structure of collimated beam moves faster than its outlying areas and that radiation have extra half cycle oscillation of temporal electromagnetic field (figure 1 (b)), which fades with distance in far field diffraction region (figure 1 (c)).
Figure 1. Spatio-temporal structure of electric field at the source of radiation (a); after collimation at the distance \(z_0\) (b) build by expression (13); after collimation at the distance \(80z_0\) (c) build by expression (14); where \(z_0\) is the focusing distance of collimation mirror.

3. Focusing of collimated radiation

Focusing collimated radiation (9) by using the mirror with reflection function \(F_{\text{foc}}(\omega, x, y) = \exp(ik(\omega)(x^2 + y^2)/2f_{\text{foc}})\) can be described by expression

\[
G_{\text{foc}}(\omega, x, y, z, f_{\text{foc}}) = \frac{iz_0}{z_{\text{col}}(\omega)^2} + \frac{i(z_f-z_0)}{(z_f-z_0)^2} \times \exp\left(-\frac{x^2 + y^2}{\rho^2}zR(\omega) - \frac{z_0}{\rho^2}z\right) A(z, z_f, z_0, f_{\text{foc}}) G_0(\omega). \tag{15}\]

where \(z_f\) is the distance from collimation mirror to focusing, \(f_{\text{foc}}\) is focal length.

By analogy with the collimated radiation we have derive the expression that describes spatial distribution dynamics of the temporal spectrum

\[
G_{\text{foc}}(\omega, x, y, z, z_f, f_{\text{foc}}) = \frac{iz_0}{z_{\text{col}}(\omega)^2} + \frac{i(z_f-z_0)}{(z_f-z_0)^2} \times \exp\left(-\frac{x^2 + y^2}{\rho^2}zR(\omega) - \frac{z_0}{\rho^2}z\right) A(z, z_f, z_0, f_{\text{foc}}) G_0(\omega). \tag{16}\]

Where

\[
A(z, z_f, z_0, f_{\text{foc}}) = \frac{z_{\text{col}}(\omega)^2}{z_{\text{col}}(\omega)^2 + \frac{z_0}{f_{\text{foc}}}} \left(\frac{(z_f-z_0)-z}{f_{\text{foc}}-z_0} + \frac{z_0}{f_{\text{foc}}-z} \right)^2 + \frac{i(z_f-f_{\text{foc}}-z_0)}{(f_{\text{foc}}-z_0)^2} \left(\frac{z_{\text{col}}(\omega)^2}{z_{\text{col}}(\omega)^2 + \frac{z_0}{f_{\text{foc}}}} \left(\frac{(z_f-z_0)-z}{f_{\text{foc}}-z_0} + \frac{z_0}{f_{\text{foc}}-z} \right)^2 \right) \tag{17}\]

is the complex factor, which determine duration and wavefront curvature of focusing radiation.

In focus of focusing mirror expression (16) can be simplified and we can find analytical solution of Fourier integral (12), which can be represented in the form

\[
E_{\text{foc}}^{z=z_0}(\omega, x, y, z, f_{\text{foc}}) = -E_0 \frac{z_0}{f_{\text{foc}}} t' \exp\left(-\frac{t'}{\tau_p}\right) - \frac{x^2 + y^2}{\rho^2} \frac{z_0}{f_{\text{foc}}} \tag{18}\]

where delay time is \(t' = t + \frac{x^2 + y^2}{2c} N(z_f-f_{\text{foc}}) - \frac{N}{c}(z_0+z_f+z)\).
Figure 2. Spatio-temporal structure of electric field at focus of focusing mirror:
(a) distance between mirrors $z_f = f_{col}$ and their focal length correlated as $f_{foc} = 0.5f_{col}$,
(b) distance between mirrors $z_f = 0.5f_{col}$ and their focal length correlated as $f_{foc} = f_{col}$,
(c) distance between mirrors $z_f = 5f_{col}$ and their focal length correlated as $f_{foc} = 0.5f_{col}$.

Analysis of equation (16) shows that number of electric field oscillations is determined only by temporal profile of radiation source and that electric field has phase shift relative to initial wave at the emitter at focus of focusing mirror.

4. Conclusion
We obtained analytical expressions, which describe paraxial features of spatio-temporal electric field structure that is forming due to collimation and focusing of initially few-cycle waves. We have demonstrated that duration of wave packet is directly proportional to the distance from beam axis and that the spatio-temporal structure of electric field has extra half cycle of oscillation at the focus of collimation mirror. In far field region, collimated beam loses that field oscillation and wavelength becomes constant in cross section of wave packet. In focus of focusing mirror electric field has phase shift relative to initial wave at the emitter. Curvature of wavefront depends on distance, which the radiation passed, and focal length of collimation and focusing mirror. These features can be used for optimization of optical systems using THz radiation.

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