Power-Sharing Control in Bearingless Multi-Sector and Multi-Three-Phase Permanent Magnet Machines

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Abstract—This article deals with the power-sharing control of bearingless multisector and multithree-phase permanent magnet machines. The proposed control strategy allows to distribute the power flows among the three-phase inverters supplying the machine during bearingless operation of the drive. The control technique is based on the extension of the vector space decomposition modeling approach. The components producing the electromagnetic torque, i.e., the q-axis currents, are controlled independently from the d-axis ones, also with the aim of managing the power flows among the three-phase systems. Conversely, the d-axis currents are exploited for the generation of the radial forces needed to levitate the rotor, while considering the compensation of the forces caused by the q-axis currents in case of unbalanced power sharing strategy. The validity of the proposed method is confirmed by simulations and experimental tests on a prototyped bearingless multisector permanent magnet synchronous machine. The proposed approach is a contribution to the development of advanced control systems employing multiphase drives in the field of bearingless and multiport applications.

Index Terms—Bearingless machines, force control, machine vector control, magnetic levitation, multiphase drives, permanent magnet machines, power-sharing, variable speed drives.

NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| p | Subscript used to identify the sub-windings under the same Pth sector (P = 1, 2, ..., p). |
| N_T | Number of three-phase star connected windings located under each pole pair (sector). |
| T | Subscript used to distinguish the sub-windings under the same Pth sector (T = 1, 2, ..., N_T). |
| i_T,P,M | Current space vector of each Pth three-phase subwinding under the Mth sector. |
| M | Subscripts used to identify the three phases of each subwinding (M = U, V, W). |
| ρ | Order of the space vector, and order of the related harmonic of the stator magnetomotive force distribution. |
| ϑ | Current space vector of space \( ϑ \)th.
| j | Fundamental electrical (mechanical) frequency and angular speed. |
| T | Rotor angular position. |
| k_T | Imaginary unit. |
| F_ρ | Conjugate operator. |
| k_F,ρ | Electromagnetic torque. |
| \( k_{F,ρ} \) | Torque constant. |
| \( F_ρ \) | Radial force contribution related to the \( ρ \)th space (in its x-y components: \( F_ρ = F_{ρ,x} + j F_{ρ,y} \)). |
| \( k_{F,ρ} \) | Radial force constant associated with the contribution related to the \( ρ \)th current space vector. |
| \( c_{m,ρ}, c_{n,ρ} \) | Constants introduced to link the modular three-phase quantities with the multiphase ones. |
| d, q | Subscripts used to identify the variables defined in the synchronous reference frame. |
| \( K_{T,P,d(q)} \) | \( d \) (or \( q \)) axis sharing coefficient of the \( T \)th three-phase sub-winding under the \( P \)th sector. |
| \( K_{ρ,d(q)} \) | \( d \) (or \( q \)) axis current-sharing complex control variable related to the \( ρ \)th vector space, with + and – subscripts used to identify the sharing...
complex variables that are multiplied by $e^{j\rho d_m}$ or $e^{-j\rho d_m}$ in (12), respectively.

$F_{sh}$ Radial force generated by a conventional power-sharing control algorithm.

$R_s$ Phase resistance.

$A, B, C$ Subscripts used to identify the subwindings under the three sectors of the considered triple three-phase machine.

I. INTRODUCTION

MULTIPHASE drive technologies are rapidly reaching different industrial markets, ranging from high-power, and high-voltage generators [1]–[4] to low-voltage hybrid powertrains for automotive applications [5], [6].

The ever increasing interest for multiphase drives, i.e., with more than three phases, is related to the power split (current and/or voltage) among more phases, increased power density, and enhanced reliability [7]–[9]. Therefore, in high-current and high-voltage applications the multilevel converter and the multiphase winding are becoming common solutions, allowing to enhance the drives performance while dealing with the technological limitations of the available high-power electronic components [10], [11]. In other application fields, such as electrical transportation and safety critical systems, the multiphase architecture has been introduced for its internal redundancy [12]. In addition, it presents the opportunity of connecting different inverters, each supplying independent sets of phases, to separated power lines, as illustrated in Fig. 1 for a triple three-phase drive [13]. This control possibility, also known under the name of power-sharing capability of multiphase drives, has been analyzed in-depth in recent research studies [14]–[16].

In the field of bearingless electrical machines, the use of multiphase windings was firstly proposed by Hermann in 1973 [17]. Notable developments and industrial applications of bearingless motors (BMs) are well summarized in [18]–[21]. A variety of bearingless designs has been proposed from the literature, all able to simultaneously generate a flux distribution with periodicity $p$ and $p \pm 1$ needed to produce torque and radial force, respectively. To differentiate among them, it is possible to separate the research studies into two main categories based on the winding arrangement. The first one makes use of two sets of windings, one to generate the torque and the other to produce the force needed for the bearingless operation [22], [23]. The second relies on a combined winding, typically a multiphase one, where all the phases simultaneously contribute to both spin and levitate the rotor [24], [25]. Among the combined winding it is worth mentioning also bridge and parallel path layouts. These configurations have the potential advantage of being “no-voltage” designs, since the suspension terminals do not see an induced voltage by the rotor flux (motional electromotive force) [26], [27]. The multiphase solution is chosen in this article, instead of a separate winding arrangement, because the stator slot area hosting the suspension winding can cover about 25%–40% of the available space. This leads to an oversize of the winding to ensure a reliable force control in all working conditions [20]. Also, multiphase BMs exhibit better fault tolerant capabilities than dual winding ones, because the torque and force controls can still be performed in case of some fault conditions. In fact, contact bearings are one of the most critical parts of an electric motor subject to failure, and the enhancement of the fault tolerance capability of BMs is a promising research topic for the development of safety critical architectures.

Further investigations on bearingless machines have been carried out in terms of machine design [20], [21], [28], [29], control performance [30], [31] and fault tolerant operation [32], [33]. Bearingless systems have been proposed for commercial applications, such as compressors, spindles, drills, flywheels, and turbine generators, where high and ultrahigh rotation speed is a requirement [34]. Disk drives with speed up to 150 kr/min have been reached by employing sliced rotors for high purity and special chemical demand [35], whereas separate winding motors for laser scanning or reaction wheels (proposed for satellite attitude control systems) are tested in the range of 300–500 kr/min in [22] and [36]. To the best of the authors’ knowledge, the highest speed reached by an electrical motor is mentioned in [37], where the authors present a magnetically levitated spinning ball motor able to reach up to 40 260 000 r/min. Also miniature medical pumps [38], high-purity mixers [39] and machines with rotors operating in contact with aggressive fluids [40] are further important fields of application.

Although the industrial application of BMs is still limited to low power applications, they are considered promising substitutes of active magnetic bearings (AMB) in high-power and high-speed applications, where the extra volume and overlength of AMBs limits the speed, due to rotodynamic challenges [41], and the power density of the overall system. In fact, the lower efficiency and power rating of existing BM prototypes make this technology still not ready for industrialization in conventional applications [20]. To enhance the power of BMs, also considering the inverter limitations and fault tolerant requirements, the multithree-phase winding solution seems suitable for high power-rating bearingless applications. Also, the permanent magnet topology of BM has been preferred in this article for its high efficiency and torque density [28].

This article focuses on the power-sharing control of bearingless multisection and multithree-phase machines. The control is developed considering the machine as a whole system, following the multiphase approach known as vector space decomposition (VSD). The equations are written in terms of independent current vectors, directly related to the control of specific space harmonics of the airgap magnetic field. After introducing the equations
describing the electromagnetic behavior of the machine (see Section II), the power-sharing and the control algorithm of the drive are presented (see Sections III and IV) and validated via FE simulations. Section V reports the numerical simulations and experimental tests performed on a prototyped bearingless machine.

II. TORQUE AND RADIAL FORCE EQUATIONS

This section presents the methodology used for the analysis of torque and radial force production in multiphase machines. A model is developed by means of a VSD approach, widely used for the representation of polyphase systems. The presented theory focuses on multisector permanent magnet (MSPM) machines, assuming as negligible the saturation of the magnetic materials and the slotting effect, and a prevalence of the fundamental component of the airgap flux density produced by of the rotor permanent magnets.

The VSD approach allows describing the torque and radial force of a multiphase electric drive as a function of specific current space vectors according to the machine topology [42]–[47]. The currents of an MSPM machine, made of \( N_T \) three-phase star connected windings located under each \( P \)th sector (or pole pair) of the machine, can be described by the conventional three-phase space vector representation of each three-phase subwinding (with phases \( T,P,U, T,P,V, \) and \( T,P,W \)) defined as follows:

\[
\vec{i}_{T,P} = \frac{2}{3} \left( i_{T,P,U} + i_{T,P,V} e^{j \frac{2\pi}{6}} + i_{T,P,W} e^{j \frac{4\pi}{6}} \right) \tag{1}
\]

with \( T = 1, 2, \ldots, N_T \) and \( P = 1, 2, \ldots, p \).

A reasonable assumption is to consider each three-phase subwinding symmetrically distributed under its pole pair. Also, in case of more three-phase subwindings under the same pole pair, the shift between each three-phase subwinding and the following one is assumed to be the same. For example, in case of three three-phase subsystems under each pole pair, the second and the third would be placed shifted by 20 and 40 electrical degrees with respect to the first one, respectively. This choice can be adopted to enhance the torque performance of the multiphase machine keeping the multisector layout.

In case of different winding arrangements, the methodology presented in this article can still be used by adapting the space vector transformations to the considered configuration.

Pursuing the VSD approach, the current space vectors of the considered multi-sector machine can be defined as follows:

\[
\vec{i}_\rho = \frac{2}{3 p N_T} \sum_{T=1}^{N_T} \sum_{P=1}^{p} \left( i_{T,P,U} - i_{T,P,V} e^{-j \frac{2\pi}{6}} - i_{T,P,W} e^{-j \frac{4\pi}{6}} \right) \times e^{j \rho \theta} e^{j \frac{2\pi (T-1)}{6 N_T}} e^{j \frac{2\pi (P-1)}{p}}. \tag{2}
\]

Transformation (2) describes the general \( \rho \)th current space vector of the machine in a way that the \( \rho \)th spatial harmonic of the stator magnetomotive force distribution can be directly evaluated from the current vector of the same order (\( \rho \)). This approach, commonly employed in modeling multiphase machines, allows us to simplify the analysis to a reduced number of field harmonics (i.e., current space vectors). Each current space vector, determined by (2), is built as a sum of the phase currents multiplied by a unit vector that is rotated according to the order of the space (\( \rho \)) and the angular position of the magnetic axis of each phase.

As an example, Fig. 2(a) illustrates the machine geometry of a triple-three-phase MSPM machine, highlighting the phases of the different sectors with different colors, and the coils of the three phases of the same sector with different shades of the same color. The positive and negative terminals of each phase are identified with a “+” (or a “−”) in addition to the name of the phase displayed into each slot.

Fig. 2(b) depicts the magnetic axes of the same machine. As it can be noticed, the magnetic axes are represented in terms of their angular position in mechanical radians. It is worth to note that the magnetic axes of all the phases \( V \) and \( W \) are shown, in Fig. 2(b), in the opposite direction with respect of the convention given by the direction of the phases (as highlighted by the “+” preceding the name of each axis).

The main harmonics of the stator magnetomotive force, responsible for the torque and force production, must be identified to properly select the related current space vectors in (2). For an isotropic permanent magnet rotor with \( p \) pole pairs, it is well known that the torque is mainly generated by the \( p \)th harmonic of the stator magnetomotive force, and more precisely to the quadrature component of the \( p \)th current space vector in the rotor reference frame (\( i_{p,d} e^{-j p \theta} = i_{p,d} + j i_{p,q} \)). As far as the radial force production is concerned, the stator magnetomotive force harmonics of order \( p+1 \) and \( p-1 \) yield constant contributions to the force [18].

Consequently, the simultaneous control of both torque and radial force can be implemented as a function of the three current space vectors of order \( p \), \( p-1 \) and \( p+1 \) as follows [48]:

\[
T = k_T i_{p,q} \tag{3}
\]

\[
F_{p-1} = k_F i_{p-1} e^{j p \theta} \tag{4}
\]

\[
F_{p+1} = k_F i_{p+1} e^{-j p \theta}. \tag{5}
\]

Merging the two contributions in (4) and (5), the fundamental equation for the radial force control results as follows:

\[
F = k_F i_{p-1} e^{j p \theta} + k_F i_{p+1} e^{-j p \theta}. \tag{6}
\]
Under the presented assumptions, (3) and (6), together with the equations of motion, are the main relationships describing the electromechanical behavior of the machine.

The peculiarity of a multisector multithree-phase drive, as the one shown in Fig. 2, is the galvanic insulation of the three-phase subwindings. This insulation permits to independently supply each subwinding with a different inverter, allowing to define fault-tolerant and power-sharing strategies for the control of the overall drive. Therefore, the machine model must be extended accordingly by defining a link between the modular three-phase and the multiphase quantities, i.e., between the transformations (1) and (2). The link can be determined by introducing the new constants

\[
c_{m,\rho} = 1 - e^{-j\left(\frac{2\pi}{3} + \rho \frac{\pi}{6}\right)} - e^{j\left(\frac{2\pi}{3} + \rho \frac{\pi}{6}\right)}
\]

\[
c_{n,\rho} = 1 - e^{-j\left(\frac{2\pi}{3} - \rho \frac{\pi}{6}\right)} - e^{j\left(\frac{2\pi}{3} - \rho \frac{\pi}{6}\right)}.
\]

Thence, (2) can be rewritten as a function of the three-phase current space vectors in (1) as follows:

\[
\tilde{i}_p = \frac{1}{3p\sqrt{T}} \sum_{T=1}^{N_T} \sum_{P=1}^{p} \left(\tilde{i}_{T,P,c_{m,\rho}} + \tilde{i}_{T,P,c_{n,\rho}}\right) \times e^{jp2\pi\left(\frac{T-1}{NP} + \frac{P-1}{p}\right)}.
\]

Equation (9), together with (3) and (6), intends to represent the effect that specific constraints on the currents of the various three-phase inverters have on the torque and radial force controls. For example, \(i_{T,P}\) is zero in case of a three-phase subwinding open-phase fault and it assumes defined values in case of a specific power-sharing request in the multiport-system architecture.

### III. Power-Sharing Algorithm for Multisector Permanent Magnet Machines

The implementation of a power-sharing algorithm for a synchronous machine with surface mounted permanent magnets can be simplified to a torque-sharing algorithm, or, in other terms, to the implementation of different set points for the q-axis components of the current vectors of each three-phase subsystem

\[
\tilde{i}_{T,P} e^{-jpd_{q,m}} e^{j2\pi(T-1)} = \tilde{i}_{T,P,d} + j\tilde{i}_{T,P,q}.
\]

#### A. Radial Force Control in Multisector Drives Under Power-Sharing Operation

Power-sharing algorithms and faults in multi-sector machines introduce unbalances in the machine symmetry which generate undesired radial forces [49]. Therefore, the implementation of a power-sharing algorithm for this machine topology requires additional considerations. In particular, recent works proposed the d-q axes decoupling approach for the development of current sharing controls [13]–[15]. Following a similar approach, the three-phase current space vectors of the MSPM drive can be written in the d-q axes reference frame, by introducing suitable sharing coefficients \(K_{T,P,d}\) and \(K_{T,P,q}\), as follows:

\[
\tilde{i}_{T,P} = N_T p (K_{T,P,d} i_d + j K_{T,P,q} i_q)
\]

assuming the following constraints:

\[
\sum_{T=1}^{N_T} \sum_{P=1}^{p} K_{T,P,d} = 1, \quad \sum_{T=1}^{N_T} \sum_{P=1}^{p} K_{T,P,q} = 1.
\]

Substituting (10) into (9), the space vector equations can be rewritten as follows:

\[
\tilde{i}_p = \left(\tilde{K}_{p,d} c_{m,\rho} e^{jpd_{q,m}} + \tilde{K}_{p,d} c_{n,\rho} e^{-jpd_{q,m}}\right) i_d + j \left(\tilde{K}_{p,q} c_{m,\rho} e^{jpd_{q,m}} - \tilde{K}_{p,q} c_{n,\rho} e^{-jpd_{q,m}}\right) i_q
\]

where the current-sharing complex control variables \(\tilde{K}_{p,d,q}\) are defined as follows:

\[
\tilde{K}_{p,d(q)} = \sum_{T=1}^{N_T} \sum_{P=1}^{p} K_{T,P,d(q)} e^{j2\pi\left(\frac{T-1}{NP} + \frac{P-1}{p}\right)} e^{jpd_{q,m}}.
\]

Once the current-sharing variables in (13) are defined, the space vectors (12) can be evaluated. Consequently, it is possible to determine the torque and force, by (3) and (6), and predict the electromechanical behavior of the machine.

Assuming a set of q-axis sharing coefficients and a reference torque (i.e., the q-axis currents), the radial force \(F_{sh}\) generated by the power-sharing control can be predicted. In fact, substituting (12) into (6), the radial force \(F_{sh}\) resulting from both the q-axis sharing coefficients and the value of \(i_q\) can be expressed as follows:

\[
F_{sh} = j \left[\sum_{T=1}^{N_T} \sum_{P=1}^{p} \left(\tilde{K}_{p-1,d} c_{m,\rho-1} - \tilde{K}_{p-1,q} c_{n,\rho-1}\right) e^{-jpd_{q,m}} + \sum_{T=1}^{N_T} \sum_{P=1}^{p} \left(\tilde{K}_{p+1,d} c_{m,\rho+1} + \tilde{K}_{p+1,q} c_{n,\rho+1}\right) e^{jpd_{q,m}}\right] i_d
\]

The d-axis current \(i_d\) and sharing coefficients \(\tilde{K}_{p,d}\) can be determined according to the desired value of the radial force \(\bar{F}\) acting on the rotor by means of (6) and (14), i.e., as a function of the q-axis current \(i_q\) and sharing coefficients \(\tilde{K}_{p,q}\), as follows:

\[
\left[\sum_{T=1}^{N_T} \sum_{P=1}^{p} \left(\tilde{K}_{p-1,d} c_{m,\rho-1} + \tilde{K}_{p-1,q} c_{n,\rho-1}\right) e^{-jpd_{q,m}} + \sum_{T=1}^{N_T} \sum_{P=1}^{p} \left(\tilde{K}_{p+1,d} c_{m,\rho+1} + \tilde{K}_{p+1,q} c_{n,\rho+1}\right) e^{jpd_{q,m}}\right] i_d = \bar{F} - F_{sh}.
\]

The stator copper losses of the machine can be defined as follows:

\[
P_J = \frac{3p^2 N_T^2}{2} R_s \left(\sum_{T=1}^{N_T} \sum_{P=1}^{p} K_{T,P,d(\rho)}^2 + \sum_{T=1}^{N_T} \sum_{P=1}^{p} K_{T,P,q(\rho)}^2\right).
\]

Therefore, if the q-axis sharing coefficients are fixed by the power sharing algorithm the optimum repartition of the currents, to generate the desired force \(\bar{F}\), can be found (analytically or numerically) from the minimization of the stator copper losses with respect to the d-axis sharing coefficients \(K_{T,P,d}\).

The model presented up to this point is written in a generic form and can be applied to any MSPM machines with \(N_T\) three-phase windings under each pole pair and \(p\) pole pairs. In the next section (see Section III-B) the equations of the behaviour of the radial force during power-sharing operation are rewritten for the
considered triple-three-phase machine and are finally used for the definition of a bearingless control algorithm, in Section IV.

B. Radial Force Control in a Triple-Three-Phase Multisector Drive under Power-Sharing Operation

Limiting the study to a multisector machine as the one depicted in Fig. 2, with three pole pairs \( (p = 3) \) and one three-phase subwinding under each pole pair \( (N_T = 1) \), the equations can be reduced to the space vectors of order 2, 3, and 4. For the sake of clarity, the three-phase windings are named with letters \( A, B, \) and \( C \) rather than numbers 1, 2, and 3, resulting in a current space vector \( i_T \) for each sub-winding \( (T=A, B, C) \), defined as follows:

\[
\vec{i}_T = 3 \left( K_{T,d}i_d + jK_{T,q}i_q \right). \tag{17}
\]

The current space vectors of the considered triple-three-phase machine can be defined from the more general (2) as follows:

\[
\vec{i}_p = \frac{2}{9} \left[ i_{A,U} - i_{A,V} e^{-j\frac{2\pi}{6}} - i_{A,W} e^{j\frac{2\pi}{6}} + \left( i_{B,U} - i_{B,V} e^{-j\frac{2\pi}{6}} - i_{B,W} e^{j\frac{2\pi}{6}} \right) e^{j\frac{2\pi}{3}} + \left( i_{C,U} - i_{C,V} e^{-j\frac{2\pi}{6}} - i_{C,W} e^{j\frac{2\pi}{6}} \right) e^{j\frac{2\pi}{3}} \right]. \tag{18}
\]

The complex current-sharing control variables of the considered system are evaluated from (13) as follows:

\[
\vec{K}_{A,d(q)} = K_{A,d(q)} + K_{B,d(q)} + K_{C,d(q)} \vec{K}_{2,d(q)} = \vec{K}_{d(q)} \vec{K}_{4,d(q)} e^{j\frac{2\pi}{3}}. \tag{19}
\]

with the \( \vec{K}_{d(q)} \) complex variable defined as

\[
\vec{K}_{d(q)} = K_{A,d(q)} + K_{B,d(q)} e^{j\frac{2\pi}{3}} + K_{C,d(q)} e^{-j\frac{2\pi}{3}}. \tag{20}
\]

Also, the current space vectors responsible for the torque and radial force production are evaluated combining (19) with (12) as follows:

\[
\vec{i}_2 = \vec{K}_d \left( c_{m,2} e^{j3\varphi_m} + c_{n,2} e^{-j3\varphi_m} \right) i_d + j\vec{K}_q \left( c_{m,2} e^{j3\varphi_m} + c_{n,2} e^{-j3\varphi_m} \right) i_q \tag{21}
\]

\[
\vec{i}_3 = \left( K_{A,d} + K_{B,d} + K_{C,d} \right) e^{j3\varphi_m} i_d + j \left( K_{A,q} + K_{B,q} + K_{C,q} \right) e^{j3\varphi_m} i_q \tag{22}
\]

\[
\vec{i}_4 = \vec{K}_d \left( c_{m,4} e^{j3\varphi_m} + c_{n,4} e^{-j3\varphi_m} \right) i_d + j\vec{K}_q \left( c_{m,4} e^{j3\varphi_m} + c_{n,4} e^{-j3\varphi_m} \right) i_q. \tag{23}
\]

Equations (19)–(23) fully describe the machine currents under an independent choice of the \( d \)-axis and \( q \)-axis current sharing coefficients introduced in (10). If (19)–(23) are substituted in the expressions of the torque and radial force, (3) and (6), the electromechanical behavior of the considered drive can be summarized by the following relationships:

\[
T = k_T \left( K_{A,q} + K_{B,q} + K_{C,q} \right) i_q \tag{24}
\]

\[
F = \left[ k_{F,2} \left( c_{m,2} e^{j6\varphi_m} + c_{n,2} e^{-j6\varphi_m} \right) + k_{F,4} \left( c_{m,4} e^{j6\varphi_m} + c_{n,4} e^{-j6\varphi_m} \right) \right] \vec{K}_d. \tag{25}
\]

C. Case Study: Radial Forces in a Triple-Three-Phase Multisector Drive Under Power Sharing Operation

In order to validate the model used to develop the control architecture, and highlight its approximations, this paragraph proposes the analysis of a case study of multi-sectored drive operating under unbalanced operation.

Observing (25), and assuming that the considered permanent magnet machine with isotropic rotor is operated with a zero \( d \)-axis current \( (i_d = 0) \), it is possible to highlight the behavior of the resulting radial force. The latter appears, for example, in case of an open phase fault of the drive.

More in detail, there is a first contribution of the radial force in the second term of (25) which has a constant value \( F_{dc} \), and a second one that is composed by the sum of a rotating term and a contrarotating one at twice the electrical frequency of the drive \( (6f_m = 2f_e) \). Also, due to the properties of rotating vectors, the second contribution results in the sum of a pulsating force \( F_{p} \) and a rotating one \( F_{rot} \), both at twice the electrical frequency.

Consequently, during the power-sharing operation of a multisector multithree-phase drive (without controlling the radial forces), the trajectory of the radial force caused by the unbalance of the power flows would be qualitatively represented as follows:

\[
\vec{F}_{sh}(t) = \vec{F}_{dc} + \vec{F}_{p} \cos(2\omega_c t + \varphi) + \vec{F}_{rot} e^{j2\omega_2 t}. \tag{26}
\]

This behavior has been validated by means of finite element simulations in MagNet 2-D (Mentor). Fig. 3 shows the finite-element trajectories of the radial force produced when controlling the machine with 11.5 \( Apk \) \( q \)-axis current in one sector only \((A, B, \) or \( C)\).

The finite-element results are compared with the ones predicted by (26). The mismatch between finite-element and analytical results is mainly caused by the reluctance force, not included in the presented model.

In fact, supplying only sector \( A \) the reluctance force generates an additional attraction force of the rotor in the direction of sector \( A \), i.e., in the \( x \)-direction. This contribution is displayed in Fig. 3 (the central circle highlighted in orange color). The reluctance force contribution has been obtained by means of additional finite element simulations where the magnets have been substituted with air.

Finally, the reluctance contribution has been subtracted from the total radial force, to generate the red dashed trajectory that considers only for the radial force generated by the interaction of the stator magnetomotive force with the rotor magnets. The comparison shows that the main contributions are well predicted.

The small difference between the analytical and finite-element results can be attributed to the higher order harmonics of the magnets and stator magnetomotive force distributions.

It can be concluded that (25) and (26) provide a valid tool to evaluate the main radial force trajectories during the power-sharing operation in a permanent magnet machine with a multi-sector multithree-phase winding arrangement.
multithree-phase permanent magnet synchronous machine is implemented.

On the one hand, as commented in the Section III, the power-sharing operation of such electrical machine topology leads to the generation of an unbalanced radial force. On the other hand, the bearingless operation of a multisector machine does not guarantee that the overall managed power is equally shared among the different three-phase winding systems (the $q$-axis currents are not necessarily balanced). Consequently, the proposed control strategy can be also employed to reduce the radial force produced during the power-sharing operation or to minimize the unbalance of the power flows during the bearingless operation.

During the power-sharing operation the $q$-axis components of the inverters’ currents are generally controlled to different set points. The radial force that results from this unbalanced operation is represented by the second contribution in (25), highlighted in (26), and it cannot be altered. Therefore, the control of the radial force must be implemented by exploiting the $d$-axis components of the inverters’ currents in the most efficient manner. In particular, the optimization strategy developed in this article leads to the minimization of the stator Joule losses in the whole multiphase winding.

Focusing on the force (25), the $d$-axis components of the inverters’ currents are function of the $d$-axis current sharing coefficients and must be controlled according to both the desired reference force $\bar{F}_{ref}$ and the uncontrolled force resulting from the power sharing. Assuming a power-sharing contribution to the force $\bar{F}_{sh}$, the control of the $d$-axis current components must agree to the following equality:

$$
\bar{K}_{d,i} = \frac{\bar{F}_{ref} - \bar{F}_{sh}}{k_{F,d}^{2} (c_{m,2} + e_{n,2} e^{-j\theta_m}) + k_{F,d}^{2} (c_{m,4} + e_{n,4} e^{-j2\theta_m})}.
$$

To ensure that the stator Joule losses needed for the radial force control are minimized, it is possible to explicit the additional losses associated with the $d$-axis currents in (16) as follows:

$$
P_{J,add} = \frac{27}{2} R_s (K_{A,d}^2 + K_{B,d}^2 + K_{C,d}^2) |\bar{i}_{d}|^2.
$$

Therefore, for a given reference value of $\bar{K}_{d,i}$ in (27), the minimum for (28), i.e., the optimum values for the $d$-axis sharing coefficients, can be found analytically. However, owing to the particular representation of the complex sharing coefficients in (20), it follows that an approach similar to the one used to define the inverse Clarke transformation of a polyphase system can be applied to the complex variable $\bar{K}_{d}$. The optimum is simply given by the analytical solution with zero common mode component ($K_{A,d} + K_{B,d} + K_{C,d} = 0$), as follows:

$$
K_{A,d} = \Re \{ \bar{K}_{d} \}; \quad K_{B,d} = \Re \{ \bar{K}_{d} e^{-j\frac{2\pi}{3}} \};
$$

$$
K_{C,d} = \Re \{ \bar{K}_{d} e^{j\frac{2\pi}{3}} \}.
$$

Therefore, by solving (27) and implementing (29), the machine force control under power-sharing operation is fully defined. In fact, the values of the $d$-axis sharing coefficients in (29)
with the value of \( \tilde{K}_{d,i} \) in (27) can be replaced in (21)–(23), together with the \( q \)-axis sharing coefficients and \( i_q \) value resulting from the power-sharing algorithm. By doing so, the set points for the three current space vectors \( \tilde{i}_2 \), \( \tilde{i}_3 \) and \( \tilde{i}_4 \) are uniquely determined as a function of: \( \tilde{F}_{\text{ref}}, i_{q,\text{ref}} \) and the power-sharing coefficients \( K_{A,q}, K_{B,q}, \) and \( K_{C,q} \).

Fig. 4 shows the control diagram of the multisector drive. The \( x-y \) components of the radial force request are determined by the respective PID controllers, acting to eliminate the \( x-y \) position errors. At the same time, the \( i_{q,\text{ref}} \) reference current is shared among the three inverters according to the respective coefficients \( K_{A,q}, K_{B,q}, \) and \( K_{C,q} \). The “VSD sharing” block implements (26), (27) and (29) to compute the reference current space vectors (21)–(23). Overall, there are six current PI regulators which identify the voltages required to track the reference values of the three independent current space vectors of the machine. Finally, the reference voltage space vectors are converted in their respective three-phase voltage vectors, used to define the modulating signals for the PWM implementation in the inverters A, B, and C.

V. SIMULATION AND EXPERIMENTAL RESULTS

Numerical simulations and experimental tests have been performed to validate the model of the drive and the developed control strategy. Table I summarizes the main machine parameters for the prototyped sectored triple-three-phase permanent magnet machine.

The simulations are based on a MATLAB-Simulink model which makes use of look-up tables to represent the electromechanical behavior of the machine. In particular, the relationships between the electromagnetic inputs (phase currents) and outputs (linked fluxes, torque, and force components) are estimated through nonlinear FE 2-D simulations of the machine. For each static simulation, the rotor is rotated by a small angle and each sector is fed with a different current value. The values calculated for the linked fluxes, \( x-y \) force components and torque are stored in the form of lookup table. Finally, this lookup table is used to represent the numerical model of the electrical machine implemented in Simulink.

| Symbol | Quantity | Value |
|--------|----------|-------|
| \( P_{\text{rated}} \) | Rated power | 1.5 kW |
| \( n_m \) | Rated speed \( n_m = \frac{60n_m}{2\pi} \) | 3000 rpm |
| \( p \) | Pole pair number | 3 |
| \( m \) | Number of phases | 9 |
| \( N_T \) | Number of three-phase systems of each pole pair | 1 |
| \( N_c \) | Number of turns per coil | 22 |
| \( i_{pk} \) | Maximum peak current | 20 A |
| \( \sigma \) | Airgap radius | 24.25 mm |
| \( W_R \) | Axial active length | 90 mm |
| \( PM \) | Mass of the rotor | 2 kg |
| \( L_d, L_q \) |\( d-q \) axis inductances of each sector | 0.52 mH |
| \( k_U \) | Unstable position stiffness | 655 N/mm |
| \( k_{F,2} \) | Radial force constant space 2 | 9.60 N/A |
| \( k_{F,4} \) | Radial force constant space 4 | 17.85 N/A |

Fig. 5. Prototype of bearingless multi-sector permanent magnet machine (left) and \( x-y \) displacement sensors (right).

The experimental tests have been carried out on a bearingless drive with two mechanical degrees of freedom, limited by a back-up bearing with 150 \( \mu \)m of clearance on the front side of the machine. Fig. 5 shows the machine prototype. A self-alignment bearing allows avoiding the axial and \( x-y \) displacement of the rotor on the rear side of the shaft, hence avoiding the tilting movement of the rotor during the two degrees of freedom bearingless control. The position of the shaft is measured by means of two eddy current displacement sensors on the \( x \) and \( y \) directions, orthogonal to the rotation axis of the rotor. The
control algorithm is implemented on a custom control platform based on the Microzed board, equipped with the Xilinx Zynq System-on-Chip [50].

The currents and power flows obtained by means of simulation and experimental tests are shown in Figs. 6 and 7, respectively. For better comparison, both simulation and experimental results assume the same operating condition, i.e., 3000 r/min and 1 N·m load torque, 100 V dc-link voltage.

During the test, four meaningful sharing scenarios have been performed. These scenarios are listed in terms of sharing coefficients \( \left( K_{A,q}, K_{B,q}, K_{C,q} \right) \) and providing the related time frames (in square brackets), as follows.

1) Equal distribution of the power (\( K_{A,q} = K_{B,q} = K_{C,q} = 1/3 \)), \([0\-0.05 \text{s} \text{ and } 0.125-0.15 \text{ s}]\).

2) Distribution of the power between inverters A and B, and zero power set point for inverter C (\( K_{A,q} = K_{B,q} = 0.5; K_{C,q} = 0 \)), \([0.05-0.075 \text{ s}]\).

3) Equal distribution of the power between inverters A and B while C has an opposite set point (\( K_{A,q} = K_{B,q} = 1; K_{C,q} = -1 \)), as in Fig. 1, \([0.075-0.1 \text{ s}]\).

4) Inverter A keeps the same set point, whereas B and C double their sharing coefficients, recirculating a higher power (\( K_{A,q} = 1; K_{B,q} = 2; K_{C,q} = -2 \)), \([0.1-0.125 \text{ s}]\).

According to (17) the \( q \)-axis currents of the three inverters (A, B, C) in Figs. 6 and 7 follow the values assumed by their sharing coefficients \( \left( K_{A,q}, K_{B,q}, K_{C,q} \right) \). As expected, also the respective power flows are distributed according to
these set points, validating the assumption of the control algorithm.

The distortion of the currents in the simulation results of Fig. 6 is caused by all the electromagnetic phenomena that are not included in the model and control algorithm. Among these, the authors mainly attribute the distortion to cogging torque, slotting effect, and higher order field harmonics.

The experimental results (see Fig. 7) are in good agreement with the simulation ones (see Fig. 6). The small differences can be ascribed to additional phenomena such as converter nonlinearities and noise in the measurements of the currents and the rotor position.

Finally, Fig. 8 shows the values of the $x$ and $y$ displacements of the shaft from its center. The trajectory is obtained from the measurements acquired during the same experimental test of Fig. 7 and for the overall time frame $[0-0.15 \text{s}]$. The external circumference of Fig. 8 (dashed) represents the maximum displacement allowed by the backup bearing. Also, an internal circumference with a diameter of $60 \mu \text{m}$ has been introduced to highlight the good performance of the bearingless operation during all the considered power-sharing scenarios.

Fig. 9 shows the experimental results carried out in similar test conditions. In this test, the machine is controlled during a speed transient. The reference torque is limited at 5 and 3 N·m before and after the setting of the unbalance of the power flows, respectively. The sharing coefficients are set equal ($K_{A,q} = K_{B,q} = K_{C,q} = 1/3$), $[0.9-1.0 \text{s}]$, and with a zero power set point for inverter $C$ ($K_{A,q} = K_{B,q} = 0.5$; $K_{C,q} = 0$), $[1.0-1.1 \text{s}]$. During the time step $1.0-1.1 \text{s}$ the torque limit is reduced to avoid an overload of the subwindings and to keep the phase currents around their rated values.

Comparing the results of Figs. 7 and 9 it is possible to appreciate that there is less distortion in the phase currents when the load is higher and the unbalance in the power flows is lower. The trajectory of the $x$–$y$ rotor displacement during this test is not illustrated, because it does not significantly differ from the one shown in Fig. 8.
defined. The performance of the proposed control strategy was verified via dynamic simulation and experimentally validated.

Experimental tests were carried out on a prototyped sectored triple-three-phase machine, assembled on an instrumented test rig which allows the bearingless operation with two degrees of freedom. The obtained results confirmed the validity of the proposed theory, showing stable control of the bearingless drive under different power-sharing scenarios.

This article aims to contribute to the development of advanced control systems employing multiphase drives in the field of bearingless and multiport applications. In fact, the proposed control strategy was employed with three different objectives.

1) Simultaneously implement a bearingless and a controlled power-sharing operation.
2) Reduce the radial force produced during the power-sharing control.
3) Minimize the unbalance of the power flows during the bearingless operation.

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