Strong Phases in the Decays $B$ to $D\pi$

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Abstract

The observed strong phase difference of $30^\circ$ between $I = \frac{3}{2}$ and $I = \frac{1}{2}$ final states for the decay $B \to D\pi$ is analyzed in terms of rescattering like $D^*\pi \to D\pi$, etc. It is concluded that for the decay $B^o \to D^+\pi^-$ the strong phase is only about $10^\circ$. Implications for the determination of $\sin(2\beta + \gamma)$ are discussed.

The weak decay amplitude to a specific final state can be written as $Ae^{i\delta}$ where $A$ is the decay amplitude, in general complex, and $\delta$ is the ”strong phase”. For a final eigenstate $\delta$ is simply the elastic scattering phase in accordance with the Watson theorem. For the case of $B$ decays the final scattering is primarily inelastic; $\delta$ arises from the absorptive part of the decay amplitude corresponding to a weak decay to intermediate states followed by a strong scattering to the final state. Many papers have discussed the expected size of $\delta$ [1].

Recently data has suggested a significant non-zero phase for the decays $\overline{B}$ to $D\pi$ [2]. These decays can be analyzed in terms of two amplitudes $A_3 e^{i\delta_3}$ and $A_1 e^{i\delta_1}$ corresponding to the final isospin states $\frac{3}{2}$ and $\frac{1}{2}$. The amplitudes for the three decays of interest are the following:

\begin{align*}
A_{o-} &= A(D^o \pi^-) = A_3 e^{i\delta_3} \\
A_{+ -} &= A(D^+ \pi^-) = \frac{1}{3} A_3 e^{i\delta_3} + \frac{2}{3} A_1 e^{i\delta_1} \\
A_{oo} &= A(D^o \pi^o) = \frac{\sqrt{2}}{3} (A_3 e^{i\delta_3} - A_1 e^{i\delta_1})
\end{align*}

(1a) (1b) (1c)

The experimental result gives the ratio of decay probabilities
\[ D^0\pi^- : D^+\pi^- : D^0\pi^o = 46:27:2.9 \]

From this one deduces

\[ \frac{A_1}{A_3} = 0.69, \quad \cos(\delta_3 - \delta_1) = 0.86 \quad (2) \]

Thus we find approximately a 30 degree phase difference, significantly different from zero. We discuss here the implications of such a phase difference; there remain, of course, sizable errors on this value.

We now make the assumption that the major rescattering comes from states of the form \( D^+_i \pi^-_j \) such as \( D^{*+}\pi^- \), \( D^+\rho^- \), etc.\(^3\) \(^4\) Such states are expected in factorization and about 10% of the \( b \) to \( u\bar{c} \) transitions have been identified to be of this type. It is much less likely that complicated many-particle states should rescatter to \( D\pi \).

We consider first the simple factorization (large \( N_c \)) limit:

\[ A_{+ -} = A_{o -}, \quad A_{oo} = 0 \]

Here the \( \pi^- \) is assumed to come directly from the \( \bar{u} \bar{d} \) current, the amplitudes are real and there is no \( D^0\pi^o \) decay. This corresponds to \( A_3 = A_1 = A \). We now add rescattering from states of the form \( D^+_i \pi^-_j \). We label these amplitudes \( X_3 \) and \( X_1 \) and again \( X_3 = X_1 \). The most obvious rescattering occurs via the exchange of an isospin 1 particle, either \( \pi \) or \( \rho \). As a result the rescattering amplitude is proportional to \( \tau_i \cdot T_j \) with the values \((\frac{1}{2}, -1)\) for \( I = (\frac{3}{2}, \frac{1}{2}) \).

The resultant imaginary amplitudes

\[ \frac{\text{Im } A_3}{\text{Im } A_1} = \frac{X_3 (\frac{1}{2})}{X_1 (-1)} = -\frac{1}{2} \quad (3) \]

Considering the \( \text{Im } A_i \) as fairly small, this means

\[ \delta_1 = -2\delta_3 \quad (4a) \]

If we use the empirical value \((\delta_3 - \delta_1) = 30^o \)

\[ \delta_3 = 10^o, \quad \delta_1 = -20^o \quad (4b) \]

and to lowest order in \( \delta_1 \) and \( \delta_3 \)

\[ A_{o -} = A e^{i\delta_3}, \quad A_{+ -} = A e^{-i\delta_3} \]
\[ A_{oo} = \sqrt{2} \quad iA \delta_3 \]

Thus the phase difference of 30° corresponds to a fairly small phase of magnitude 10° for both of the favored decays. Of course in this approximation the \( D^0\pi^o \) decay is purely imaginary entirely due to rescattering.
If we now use the empirical value $A_1 = 0.7 A_3 = 0.7 A$ but still assume $X_3 = X_1$, we have using Eq. (1) to lowest order in $\delta_1$ and $\delta_3$.

$$
A_3 = A + i \delta_3 A \\
A_1 = 0.7 A - 2 i \delta_3 A \\
\delta_1 = \frac{-2 \delta_3}{0.7} \\
A_{+-} = 0.8 A - i A \delta_3
$$

and with $\delta_3 - \delta_1 = 30^o$ we get $\delta_1 = -22^o$, $\delta_3 = 8^o$ and the phase for $B \to D^+ \pi^-$ is again $10^o$.

Finally if we also assume $X_1 = 0.7 X_3$ we get Eqs. (4) again and

$$
A_{+-} = 0.8 A - i 0.6 A \delta_3
$$

and the phase for $B \to D^+ \pi^-$ is $7.5^o$.

Thus we conclude that $(\delta_3 - \delta_1) = 30^o$ corresponds to a small phase of order $10^o$ for $B \to D^+ \pi^-$. In contrast the phase for the unfavored decay $\bar{B} \to D^o \pi^o$ is greater than $45^o$. Similar results are implied by more detailed analysis [3].

One reason for the interest in the strong phase for $\bar{B} \to D^+ \pi^-$ is the possible use of this decay or the related decay $B \to D^+ \pi^-$ in the determination of the phase $\gamma$ in the CKM matrix. One can look at the time-dependence of the decay due to interference with the double-Cabibbo suppressed decay $B \to D^o \pi^o$ which corresponds to $\bar{b} \to \bar{u} + c + \bar{d}$. The time-dependent term can be used [5] to find $\sin (2\beta + \gamma)$. The detailed analysis [6] [7] involves the strong phase $\Delta$, which is the difference between the strong phase for $B \to D^+ \pi^-$ and that for $B \to D^+ \pi^-$. There is an ambiguity in the result unless one can assume $\Delta$ is small.

The same isospin analysis given for $\bar{B} \to D^+ \pi^-$ can be applied to $B \to D^+ \pi$ and one expects again that the final state phase is due to the same rescattering from status like $D^* \pi$, $D \rho$, etc. The relative importance will be different for the case of $B$ as compared to $\bar{B}$, but theoretical estimates [8] indicate the difference is not large. Thus $\Delta$ is expected to involve a cancellation between the two strong phases and thus be smaller than either one. Given our conclusion that the phase for $\bar{B} \to D^+ \pi^-$ is of order $10^o$ we conclude that $\Delta$ is very small.

All the analysis here holds equally well for the decays to $D^* \pi$. In fact the experimental results [8] for the decays to $D^* \pi$ are the same within errors as for $D \pi$ and give essentially the same strong phase shift Eq. (2).

Decays in which the final $\pi^-$ is replaced by a $\rho^-$ are found to have a branching ratio 2 to 3 times as large as those with a $\pi^-$. Thus it may be expected that rescattering from states $D_i \rho$ to $D_i \pi$ may have a larger effect than $D_i \pi$ to $D_i \rho$. Thus while our general analysis might be applicable to $D_i \rho$ we expect the
magnitude of the strong phase shifts would be smaller. This seems to be true from the first data on the $D^0\rho^0$ decay \cite{9}.

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