Study of $B_s \rightarrow \pi \rho$ decays in the perturbative QCD approach

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The $B$ meson rare decays provide a good place for testing the Standard Model (SM), studying $CP$ violation and looking for possible new physics beyond the SM. A lot of theoretical studies have been done in recent years, which are strongly supported by the running $B$ factories in KEK and Stanford Linear Accelerator Center (SLAC). Looking forward to the future CERN Large Hadron Collider beauty experiments (LHC-b), a large number of $B_s$ and $B_c$ mesons can also be produced. So the studies of $B_s$ meson rare decays are necessary in the next a few years.

In this paper, we study the rare decays $B_s \rightarrow \pi \rho$ in Perturbative QCD approach (PQCD) [1]. Comparing with QCD factorization approach [2], PQCD approach can make a reliable calculation for pure annihilation diagrams in $k_T$ factorization. The endpoint singularity occurred in QCD factorization approach can be cured here by the Sudakov factor from resummation of double logarithms.

In PQCD approach, the decay amplitude can be written as:

$$\text{Amplitude} \sim \int d^4 k_1 d^4 k_2 d^4 k_3 \text{Tr}[C(t)\Phi_B(k_1)\Phi_\pi(k_2)\Phi_\rho(k_3)H(k_1, k_2, k_3, t)]e^{-S(t)}. \quad (1)$$

In our following calculations, the Wilson coefficient $C(t)$, Sudakov factor $S_i(t)|i = B_s, \pi, \rho|$ and the non-perturbative but universal wave function $\Phi_i$ can be found in the Refs. [3, 4, 5, 6, 7]. The hard part $H$ are channel dependent but fortunately perturbative calculable, which will be shown below.

Like the $B_s \rightarrow \pi^+ \pi^-$ decay [8], the $B_s \rightarrow \pi \rho$ decays are pure annihilation type rare decays, which are difficult to be calculated in method other than PQCD approach. Fig. 1 shows the lowest order Feynman diagrams to be calculated in PQCD approach where the big dots denote the quark currents in four quark operators for $B_s \rightarrow \pi^+ \rho^-$ according to the effective Hamiltonian of $b$ quark decay [9]. First for the usual factorizable diagram (a) and (b), the sum of $(V-A)/(V-A)$ current contributions is given by

$$M_a[C] = 16\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ [x_3 \phi_A^A(x_2)\phi_\rho(x_3) + 2r_\pi r_\rho(1 + x_3)\phi_A^\rho(x_2)\phi_\rho^\rho(x_3)] + 2r_\pi r_\rho(x_3 - 1)\phi_A^\rho(x_2)\phi_\rho^\rho(x_3) \right\} \alpha_s(t_a^2) h_a(x_3, x_2, b_2, b_3) \exp[-S_\pi(t_a^2) - S_\rho(t_a^2)]C(t_a^2)$$

$$- [x_2 \phi_A^A(x_2)\phi_\rho(x_3) + 2r_\pi r_\rho(1 + x_2)\phi_A^\rho(x_2)\phi_\rho^\rho(x_3) + 2r_\pi r_\rho(x_2 - 1)\phi_A^\rho(x_2)\phi_\rho^\rho(x_3)] \alpha_s(t_a^2) h_a(x_3, x_2, b_3, b_2) \exp[-S_\pi(t_a^2) - S_\rho(t_a^2)]C(t_a^2), \quad (2)$$

where $r_\pi = m_{\pi}/m_B = m_\pi^2/[m_B(m_u + m_d)]$, $r_\rho = m_\rho/m_B$. $C_F = 4/3$ is the group factor of the $SU(3)$ gauge group. $\phi_i$ is light cone distribution amplitude, which describes the momentum distribution of the meson wave function. The

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results from the threshold resummation of QCD radiative corrections to hard amplitudes [10].

\[ J_{(V-A)(V-A)} \text{ operator's contribution is:} \]

\[ \text{comes from the Fourier transformation of propagators of virtual quark and gluon in the hard part calculations. } S_i(x) \text{ results from the threshold resummation of QCD radiative corrections to hard amplitudes} \]

\[ H_0(z) = J_0(z) + iY_0(z), \]

\[ J_0 \text{ and } Y_0 \text{ are Bessel functions. The sum of (V-A)(V+A) current contributions of diagrams (a) and (b) is } -M_0[C]. \]

For the non-factorizable annihilation diagrams (c) and (d), all three meson wave functions are involved. The (V-A)(V-A) operator’s contribution is:

\[ M_{\gamma}[C] = \frac{-1}{\sqrt{2N_c}} 64\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_b db_1 b_2 db_2 \phi_B(x_1, b_1) \]

\[ \times \left\{ [-x_2 \phi_\rho(x_3)\phi_\sigma(x_2) + r_\rho r_\pi(x_3 - x_2)\phi_\rho(x_3)\phi_\sigma(x_2) - r_\rho r_\pi(x_2 + x_3)\phi_\rho(x_3)\phi_\sigma(x_2) - r_\pi r_\rho(x_2 + x_3)\phi_\rho(x_3)\phi_\sigma(x_2) - r_\pi r_\rho(x_3 - x_2)\phi_\rho(x_3)\phi_\sigma(x_2) - r_\pi r_\rho(x_2 + x_3)\phi_\rho(x_3)\phi_\sigma(x_2) - r_\pi r_\rho(x_3 - x_2)\phi_\rho(x_3)\phi_\sigma(x_2)] \right\} \]

\[ C(t_c^1)\alpha_s(t_c^1)h_c^{(1)}(x_1, x_2, x_3, b_1, b_2) \exp\left[-S_B(t_c^1) - S_{\pi}(t_c^1) - S_{\rho}(t_c^1)\right] \]

\[ + r_\rho r_\pi(x_3 - x_2)\phi_\rho(x_3)\phi_\sigma(x_2) - r_\rho r_\pi(x_2 + x_3)\phi_\rho(x_3)\phi_\sigma(x_2) + r_\pi r_\rho(x_2 + x_3)\phi_\rho(x_3)\phi_\sigma(x_2) + r_\pi r_\rho(x_3 - x_2)\phi_\rho(x_3)\phi_\sigma(x_2)] \]

\[ C(t_c^2)\alpha_s(t_c^2)h_c^{(2)}(x_1, x_2, x_3, b_1, b_2) \exp\left[-S_B(t_c^2) - S_{\pi}(t_c^2) - S_{\rho}(t_c^2)\right]. \]
For the penguin operators, there are also $(V-A)(V+A)$ type operators, whose contribution is given as

$$M_{ij}^P[C] = \frac{1}{\sqrt{2}N_c} 64\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^x b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \times \bigl\{ [-x_3 \phi_p(x_3) \phi_A(x_2) - r_\rho \phi(x_3) - r_\rho \phi(x_2) + 2x_3 \phi_\rho(x_3)] \phi^P_\rho(x_2) \phi^F_\rho(x_2) - r_\rho \phi(x_3) - r_\rho \phi(x_2) + 2x_3 \phi_\rho(x_3)] \phi^F_\rho(x_2)

+ r_\rho \phi(x_3) - r_\rho \phi(x_2) + 2x_3 \phi_\rho(x_3)] \phi_\rho(x_2) - r_\rho \phi(x_3) - r_\rho \phi(x_2) + 2x_3 \phi_\rho(x_3)] \phi^F_\rho(x_2)

\bigr\} C(t_c^0) \alpha_s(t_c^0) h_c^{(1)}(x_1, x_2, x_3, b_1, b_2) \exp[-S_B(t_c^0) - S_\rho(t_c^0)] - [-x_2 \phi_\rho(x_3)] \phi_\rho(x_2) + 2x_3 \phi_\rho(x_3)] \phi^F_\rho(x_2)

C(t_c^0) \alpha_s(t_c^0) h_c^{(2)}(x_1, x_2, x_3, b_1, b_2) \exp[-S_B(t_c^0) - S_\rho(t_c^0)] \bigr\}, \quad (5)

where

$$h_c^{(j)}(x_1, x_2, x_3, b_1, b_2) = \begin{cases} \theta(b_2 - b_1) \frac{\pi}{2} H_0^{(1)}(M_B \sqrt{x_2 x_3} b_2) J_0(M_B \sqrt{x_2 x_3} b_1) \\
+ (b_1 \leftrightarrow b_2) \end{cases} \times \left( \frac{K_0(M_B F_{(j)} b_1)}{2} H_0^{(1)}(M_B \sqrt{|F_{(j)}^2|} b_1), \quad \text{for } F_{(j)}^2 > 0 \right), \quad \frac{K_0(M_B F_{(j)} b_1)}{2} H_0^{(1)}(M_B \sqrt{|F_{(j)}^2|} b_1), \quad \text{for } F_{(j)}^2 < 0 \right), \quad (6)

K_0 is modified Bessel function and $F_{(j)}$’s are defined by

$$F_{(1)} = x_1 x_2 - x_2 x_3; \quad F_{(2)} = x_1 x_2 + x_3 - x_1 x_2 - x_2 x_3. \quad (7)

The hard scale $t_c$ in Eqs. (2), (3) are chosen as the largest energy scale appearing in each diagram to kill the large logarithmic corrections:

$$t_a^1 = \max(M_B \sqrt{x_3}, 1/b_2, 1/b_3), \quad t_a^2 = \max(M_B \sqrt{x_2}, 1/b_2, 1/b_3), \\
t_c^1 = \max(M_B \sqrt{|F_{(1)}^2|}, M_B \sqrt{x_2 x_3}, 1/b_1, 1/b_2), \quad t_c^2 = \max(M_B \sqrt{|F_{(2)}^2|}, M_B \sqrt{x_2 x_3}, 1/b_1, 1/b_2).$$

The total decay amplitude is then

$$A(B_s \to \pi^+ \rho^-) = f_B M_a \left[ V_{ub} V_{us} (C_1 + \frac{1}{3} C_2) - V_{tb} V_{ts} (2C_3 + \frac{2}{3} C_4 - 2C_5 - \frac{5}{3} C_6 - \frac{1}{2} C_7 - C_8 + \frac{1}{2} C_9 + \frac{1}{6} C_{10}) \right] + M_c \left[ V_{ub} V_{us} C_2 - V_{tb} V_{ts} (2C_4 + \frac{1}{2} C_{10}) - V_{tb} V_{ts} M_c^P \left( 2C_6 + \frac{1}{2} C_8 \right) \right], \quad (8)$$

and the decay width is expressed as

$$\Gamma(B_s \to \pi^+ \rho^-) = \frac{G_F^2 M_B^3}{128 \pi} |A(B_s \to \pi^+ \rho^-)|^2. \quad (9)$$

The most important contribution here is the factorizable penguin diagram, which is CKM enhanced. If we exchange the $\pi$ and $\rho$ in Fig. 1, by the same method, we can compute the $B_s \to \pi^- \rho^+$ decay. The expressions are similar. The decay amplitude for $B_s \to \pi^- \rho^+$ is

$$A(B_s \to \pi^- \rho^+) = A(B_s \to \pi^+ \rho^-) + A(B_s \to \pi^- \rho^+), \quad (10)$$

and the decay width can be written as

$$\Gamma(B_s \to \pi^- \rho^+) = \frac{G_F^2 M_B^3}{512 \pi} |A(B_s \to \pi^+ \rho^-)|^2. \quad (11)$$
In the following, we first give the branching ratios of $B_s \to \pi \rho$. Just as in Ref. [8], we leave the CKM angle $\gamma$ as a free parameter in our numerical calculations. Because there are four decay channels: $B_s/B_s \to \pi^+ \rho^-$, $B_s/B_s \to \pi^- \rho^+$, it is not possible to distinguish the initial state by detecting the final states. We average the sum of $B_s/B_s \to \pi^+ \rho^-$ as one channel, and $B_s/B_s \to \pi^- \rho^+$ as another, which is distinguishable by experiments. Using the same parameters as refs. [4, 5], we get

$$\begin{align*}
\text{Br}(B_s/B_s \to \pi^+ \rho^-) &= (5.1^{+0.8}_{-0.5}) \times 10^{-7}, \\
\text{Br}(B_s/B_s \to \pi^- \rho^+) &= (5.4^{+0.5}_{-0.8}) \times 10^{-7}, \\
\text{Br}(B_s/B_s \to \pi^0 \rho^0) &= (4.2^{+0.6}_{-0.7}) \times 10^{-7},
\end{align*}$$

(12)

where all channels are averaged for $B_s$ and $\bar{B}_s$.

In Ref. [11], Beneke et al. have estimated the branching ratio of $\bar{B}_s \to \pi^+ \rho^-$ in the QCD factorization approach. Weak annihilation diagrams are power suppressed in the heavy quark limit and, in general, not calculable in QCD factorization approach. In order to avoid the end-point singularities, they introduced phenomenological parameters to replace the divergent integral. With those parameters they estimated that the branching ratio of $\bar{B}_s \to \pi^+ \rho^-$ is $(0.03 - 0.14) \times 10^{-7}$. In PQCD approach, the annihilation amplitude is calculable. In the rest frame of the $B_s$ meson, the $d$ or $\bar{d}$ quark included in $\rho$ or $\pi$ has momentum $O(M_B/4)$, and the gluon producing them has momentum $q^2 = O(M_B^2/4)$. This is a hard gluon, the PQCD can be safely used because of asymptotic freedom of QCD [12]. We have tested that the bulk of the result comes from the region with $\alpha_s/\pi < 0.2$, where a figure was show in Ref. [13].

Our predicted result is larger than their estimation, which can be tested by the future experiments.

Using the same definition in Ref. [8], we study the $CP$ violation parameters $A_{CP}^{dir}$ and $a_{\epsilon+\epsilon'}$ in the process of $B_s \to \pi^0 \rho^0$ decay. We find the direct $CP$ violation parameter $A_{CP}^{dir}(B_s \to \pi^0 \rho^0)$ is about $2\%$ when $\gamma$ is near $100^\circ$, the small direct $CP$ asymmetry is also a result of small tree level contribution. The mixing $CP$ violation parameter $a_{\epsilon+\epsilon'}(B_s \to \pi^0 \rho^0)$ is large, whose peak is close to $16\%$ (see Fig. 2).

![FIG. 2: Mixing CP violation parameter of $B_s \to \pi^0 \rho^0$ decay as a function of CKM angle $\gamma$](image)

The $CP$ violations of $B_s/\bar{B}_s \to \pi^\pm \rho^\mp$ are very complicated. There are four decay amplitudes, which can be expressed as:

$$\begin{align*}
g &= \langle \pi^+ \rho^- | H_{eff} | B_s \rangle; & h &= \langle \pi^+ \rho^- | H_{eff} | \bar{B}_s \rangle; \\
g' &= \langle \pi^- \rho^+ | H_{eff} | B_s \rangle; & h' &= \langle \pi^- \rho^+ | H_{eff} | \bar{B}_s \rangle.
\end{align*}$$

(13)

We introduce four parameters to describe the $CP$ asymmetries in the processes, which are given by [14]

$$\begin{align*}
C_{\pi\rho} &= \frac{1}{2}(a_{\epsilon'} + a_{\epsilon'}), & \Delta C_{\pi\rho} &= \frac{1}{2}(a_{\epsilon'} - a_{\epsilon'}), \\
S_{\pi\rho} &= \frac{1}{2}(a_{\epsilon+\epsilon'} + a_{\epsilon+\epsilon'}), & \Delta S_{\pi\rho} &= \frac{1}{2}(a_{\epsilon+\epsilon'} - a_{\epsilon+\epsilon'}),
\end{align*}$$

(14)
where

\[ a_{\epsilon'} = \frac{|g|^2 - |h|^2}{|g|^2 + |h|^2}, \quad a_{\epsilon+\epsilon'}' = -2 \frac{Im(h/g)}{1 + |h/g|^2}, \]

\[ a_{\bar{\epsilon}'} = \frac{|\bar{h}|^2 - |\bar{g}|^2}{|\bar{h}|^2 + |\bar{g}|^2}, \quad a_{\bar{\epsilon}+\bar{\epsilon}'}' = -2 \frac{Im(\bar{g}/\bar{h})}{1 + |\bar{g}/\bar{h}|^2}. \]

(15)

We calculate the above four CP asymmetry parameters and show the CKM angle \( \gamma \) dependence in Fig. 3. We do not plot the \( C_{\pi\rho} \) in this figure, since its value is near zero. The decay branching ratios depends heavily on the shape of wave functions and decay constants etc. But the CP asymmetry should not since the dependence will be cancelled. The direct CP asymmetry can be affected by the power corrections and next-to-leading order contributions easily. We investigate the CP asymmetry parameters’s dependence on the hard scale \( t \) in Eq. (1), which characterize the size of next-to-leading order contribution. The CP asymmetry numbers are shown in Table I with those uncertainties. By changing the hard scale \( t \) from 0.9 to 1.3, we find the CP asymmetries of \( B_s(\bar{B}_s) \to \pi^\pm \rho^\mp \) change little: for \( \gamma = 60^\circ \), the uncertainty is less than 1% for \( C_{\pi\rho} \), 4% for \( \Delta C_{\pi\rho} \), 3% for \( S_{\pi\rho} \) and 7% for \( \Delta S_{\pi\rho} \), respectively. The reason is that mixing induced CP is dominant here, but the direct CP of \( B_s \to \pi^0 \rho^0 \) really changed much, as shown in Table I.

| Scale | \( C_{\pi\rho} \) | \( \Delta C_{\pi\rho} \) | \( S_{\pi\rho} \) | \( \Delta S_{\pi\rho} \) | \( A_{\text{CP}}^{\pi^0}(B_s \to \pi^0 \rho^0) \) | \( a_{\epsilon+\epsilon'}(B_s \to \pi^0 \rho^0) \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.9 t | 0.1%           | 10%            | 21%            | 0.6%           | 11%            |
| t     | 1%             | 11%            | 14%            | 1.6%           | 15%            |
| 1.3 t | 1.8%           | 14%            | 10%            | 2.6%           | 16%            |

FIG. 3: CP violation parameters of \( B_s(\bar{B}_s) \to \pi^\pm \rho^\mp \) decays: \( \Delta C \) (solid line), \( S \) (dashed line) and \( \Delta S \) (dotted line) as a function of CKM angle \( \gamma \). In conclusion, we study the branching ratio and CP asymmetries of \( B_s \to \pi \rho \) decays in PQCD approach. We find the branching ratio of \( B_s \to \pi^+ \rho^- + \pi^- \rho^+ \) is at order \( 10^{-6} \), which is larger than QCD factorization’s estimation. We also predict CP asymmetries in the process, which may be measured in the future LHC-b experiments. We thank M.-Z. Yang for useful discussions. This work is partly supported by National Science Foundation of
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