Nanofluids modeling and energy transformation under the influence of mixed convection: Role of aluminum and \(\gamma\)-aluminum nanoparticles

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Abstract

The analysis of nanofluids under various physical scenarios convinced the researchers and scientists because of their broad range of applications in potential area of the current time like chemical engineering, biomedical engineering and applied thermal engineering etc. To give the final shape of many industrial and engineering processes, enhanced heat transfer desired, therefore, the study of \(\text{Al}_2\text{O}_3\)-\(\text{H}_2\text{O}\), \(\gamma\text{Al}_2\text{O}_3\)-\(\text{H}_2\text{O}\), \(\text{Al}_2\text{O}_3\)-\(\text{C}_2\text{H}_6\text{O}_2\), and \(\gamma\text{Al}_2\text{O}_3\)-\(\text{C}_2\text{H}_6\text{O}_2\) nanofluids is reported. The model successfully achieved after mathematical operations and by appealing similarity transforms. To examine the behavior of heat transfer, numerical tools utilized and performed the results. It is observed that enhanced heat transfer in \(\text{Al}_2\text{O}_3\)-\(\text{H}_2\text{O}\), \(\gamma\text{Al}_2\text{O}_3\)-\(\text{H}_2\text{O}\), \(\text{Al}_2\text{O}_3\)-\(\text{C}_2\text{H}_6\text{O}_2\), and \(\gamma\text{Al}_2\text{O}_3\)-\(\text{C}_2\text{H}_6\text{O}_2\) could be attained by setting nanoparticles concentration up to 20\%. For \(\text{Al}_2\text{O}_3\)-\(\text{H}_2\text{O}\), \(\gamma\text{Al}_2\text{O}_3\)-\(\text{H}_2\text{O}\), optimum heat transfer trends noticed due to their prominent thermophysical values. Also, fewer effects of combined convection on \(\theta(\eta)\) examined.

Keywords

\(\text{Al}_2\text{O}_3\) and \(\gamma\text{Al}_2\text{O}_3\) nanoparticles, unsteady flow, stretching sheet, different base fluids, numerical technique

Introduction

A lot of researchers studied temperature behavior in the base fluids such as ethylene glycol, kerosene oil, and \(\text{H}_2\text{O}\). These fluids have limited applications due to their low thermal performance, as for many industrial uses high thermal transport is required to finish the processes of different products. Therefore, researchers focused on the development of a new class of fluids with high thermal performance than the regular liquids. These fluids gained much intentions of the researches and industrialists and strengthen their roots in the modern world.

Choi\(^1\) potentially contributed in the development of nanofluids. Finally, his untiring efforts opened a new lock in the modern world. They explored that thermal performance in the regular liquids could be intensify by adding nano additives of various metals and oxides. These particles are saturated in the liquid uniformly and thermally in equilibrium. The addition of nanoparticles in the regular liquid boosts thermal performance of the fluid. These became very popular among the researcher’s community and resolved many problems of the industrialist which they faced from many decades in the context of heat transport. Therefore, the investigation of thermal transport in nanofluids prepared by various metals oxides by dispersing in...
various regular liquids are reported by Ashraf et al.\textsuperscript{2,3} and Chu et al.\textsuperscript{4} 

The study related to nanofluids containing $\text{Al}_2\text{O}_3$ and $\gamma$-$\text{Al}_2\text{O}_3$ nanomaterials is described by Maciver et al.\textsuperscript{5} The effective thermal conductivity of nanofluids is imperative and playing significant role to boost the internal energy of the nanofluids which ultimately results in rise in the fluid temperature. Therefore, numerous thermal conductance correlations were introduced and to analyze their thermal capability, researchers and engineers utilized those correlations in the modeling of nanofluids and examined their influences in the fluid thermal transport ability. The fluid dynamists made their efforts day by day and reported the nanofluid temperature behavior prepared by various metals and regular liquids. The significant studies in this regard under the impacts of various physical quantities like viscus dissipation, thermal radiations, combined convection effects, nonlinear thermal radiations, chemical reaction, and internal heat generation and absorption are reported in Hamad,\textsuperscript{6} Vajravelu et al.,\textsuperscript{7} and Ashraf et al.\textsuperscript{8} Synthetization process of nanofluids given in Figure 1(a).

Thermal conductivity and thickness reproductions are expenditure for $\text{Al}_2\text{O}_3$. Individually investigative in addition mathematical investigation are supported available. The flow above a stretching surface is an important problem in lots of industrial procedures with uses in manufacturing such as melt-spinning, extrusion, the boiling developing, wire illustration, glass–fiber manufacture, rubber sheets and refrigeration of a big metal plate in immersion, which may be an electrolyte, etc. The production, polymer pieces, production of plastic, and fibers are industrial in unbroken extrusion of polymer commencing die to a mechanical wave, which is placed at a limited coldness. Considered the fixed 2D incompressible flow of a Newtonian fluid instigated using the widening of an elastic smooth sheet which transfers in its specific horizontal velocity changing linearly as the distance from a steady idea due to the use of a constant pressure. Problematic is mostly stimulating since a particular result of the 2D Navier–Stokes equations has been developed by Tiwari and Das.\textsuperscript{9}

Later on, Crane\textsuperscript{10} considered an effective substance to study the heat transfer mechanism and unsteady flow of nanofluid past a stretching sheet. They modeled the problem for nanofluid and examined the behavior of thermal transport under the impacts of various flow governing quantities. The important investigations of nanofluids upshot temperature under physical phenomena like thermal radiations and convective conditions etc. reports in Ashraf et al.\textsuperscript{11,12} The increase of the unusual kind of flow due to reduction was first explored by Wang\textsuperscript{13} and considered the performance of fluid over an unsteady stretching sheet. They studied the mixed convection heat transfer on a non-linear stretching geometry with constant fluid done by Prasad et al.\textsuperscript{14} Yazdi et al.\textsuperscript{15} conversed the slip flow and heat transfer over a non-linear porous stretching surface. The investigation of the nanofluid dynamics saturated by aluminum alloys past a stretching surface was described in Zaidi et al.\textsuperscript{16} Recently, Rana and Bhargava\textsuperscript{17} studied the boundary layer flow of nanofluid flow on a non-linearly stretching sheet. Just, Das\textsuperscript{18} considered heat transfer convective of nanofluids over a stretching sheet in the existence of partial slip-up and thermal radioactivity. First idea on show in their investigated that the stretching of the sheet may not automatically linear by Gupta and Gupta.\textsuperscript{19}

Thermal enhancement and entropy\textsuperscript{20–23} study in nanofluids convinced by thermal radiations behavior made place in the scientist’s community. The forced scientists from various applied areas of research to investigate and explore fascinating insights in thermal and enhancement
studies. Carbon nanotubes which are found to be very useful for enhanced heat transport due to their unique structure compelled the researchers to examine the dynamics of nanofluids. The Fluid Dynamists made several attempts to study the role of CNTs in the heat as well as entropy of the fluid. The studies conducted by adding variety of body forces and under various conditions specified at the boundary domain, such studies available in the literature. Among the distinct heat transport mode, convection with addition of CCHFM (Cattaneo Christov Heat Flux Model) is one of the prominent sources to upshot the heat transfer inside the working nanofluid. Recently, many studies reported in this regard by taking various sorts of working nanofluids synthesized by different metals and host solvents. The literature revealed that inside temperature of the nanofluids could be enriched by inducing different physical phenomena in the modeling of the heat transfer problem. Further, many fascinating and useful results presented in the studies and found excellent heat transport which would be helpful to the industrialist and engineers to cope their issues in the current time.

From the above literature, it is found that the novel comparative analysis of heat transfer in Al2O3-H2O, γAl2O3-H2O, Al2O3-C2H6O2, and γAl2O3-C2H6O2 nanofluids is absent which is significant research gap. The γAl2O3 nanoparticles are very famous due to their progressive applications in the world specifically in biomedical engineering. Therefore, in this study the behavior of Al2O3-H2O, γAl2O3-H2O, Al2O3-C2H6O2, and γAl2O3-C2H6O2 heat transfer with outstanding thermophysical attributes of two base solvents and updated nanofluid models for γAl2O3 is presented. Further, the model will be upgraded with new insights of combined convection which is an important factor in the analysis of nanofluids. This study will fulfill the above gap and help the researchers in the selection of parameters for better heat transfer.

The study will subject to the following limitations:

- The nanoparticles uniformly dispersed in the base solvents (H2O, C2H6O2) and no slip exist between them.
- The supporting nanofluids effective thermophysical models will be used during the analysis.
- The flow is unsteady, laminar and no slip effects imposed over the surface.
- Both the nanomaterial and base solvents are thermally in equilibrium.
- Mixed convection effects incorporated during the model development.

**Formulation of the problem**

Considered an unsteady incompressible flow of nanofluids composed by Al2O3, γ-Al2O3 nanomaterials and the regular liquids namely C2H6O2 and H2O above a stretching sheet. The flow is configurated due to a stretching surface in two-dimensions. The velocity of the sheet is $u_w = ax$ along the x-axis and temperature at the stretchable surface is $T_s = T_w + bx^2$. Where a, b are constants. The configuration of nanofluid flow over a stretchable surface is depicted in Figure 1.

The basic physical laws that govern the flow of Al2O3-H2O, γAl2O3-H2O, Al2O3-C2H6O2 and γAl2O3-C2H6O2 over unsteady stretching surface are described as below in vector form:  

\[
\nabla \mathbf{V} = 0
\]

\[
\rho_{nf} \left( \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right) = \mu_{nf} \nabla^2 \mathbf{V} + S_l
\]

\[
\rho_{cp} \left( \frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T \right) = k_{nf} \nabla^2 T
\]

Where; $S_l$ incorporates the effects of combined convection.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho_{nf} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu_{nf} \frac{\partial^2 u}{\partial y^2} + g \left( \beta p \right) (T - T_w). \quad (2)
\]

\[
\left( \rho C_p \right)_{nf} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K_{nf} \frac{\partial^2 T}{\partial y^2} \quad (3)
\]

The flow is subject to the following flow conditions:

at $y = 0$:

\[
U = U_w, \quad v = 0, \quad T_w = T_0 + bx^2,
\]

as $y \to \infty$:

\[
U \to 0, \quad T \to T_0.
\]

**Thermal physical values**

The effective correlations which work particularly for γAl2O3 nanoparticles and by considering the base solvents are listed in equations (7)–(12). These properties of the nanofluids will be utilized in the development of Al2O3-H2O, γAl2O3-H2O, Al2O3-C2H6O2 and γAl2O3-C2H6O2 thermal transport models after incorporating in the basic governing laws.
\[ \rho_{nf} = (1-\phi)\rho_f + \phi \rho_s \]
\[ (\rho C_p)_{nf} = (1-\phi)((\rho C_p)_f + \phi ((\rho C_p)_s) \]
\[ (\rho \beta)_nf = (1-\phi)((\rho \beta)_f + \phi ((\rho \beta)_s) \]

Wherever \( \phi \) represent the nanoparticles volumetric fraction.

\[ \mu_{nf} = \frac{1}{(1-\phi)^{2.5}} \]
(for \( Al_2O_3-H_2O \), and \( Al_2O_3-C_2H_6O_2 \)).

\[ \mu_{nf} = 306\phi^2 - 0.19\phi + 1. \text{ (for } \gamma = Al_2O_3-C_2H_6O_2 \) (8)

\[ \mu_{nf} = 123\phi^2 - 7.3\phi + 1. \text{ (for } \gamma = Al_2O_3-H_2O \) (9)

Thermal conductivity of the nanofluid is given by the relations:

\[ K_{nf} = K_f \left[ \frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + \phi(K_f - K_s)} \right] \]
(for \( Al_2O_3-C_2H_6O_2 \), and \( Al_2O_3-C_2H_6O_2 \)).

\[ K_{nf} = K_f \left[ 28.905\phi^2 + 2.8273\phi + 1 \right] \]
(for \( Al_2O_3-C_2H_6O_2 \)).

\[ K_{nf} = K_f \left[ 4.97\phi^2 + 2.72\phi + 1 \right] \]
(for \( Al_2O_3-H_2O \)).

Now, we introduce the following dimensionless functions \( F \) and \( \theta \), and the similarity variable \( \eta \) as:

\[ \eta = \sqrt[2]{\frac{a}{y t^{1-c}}} \]
\[ \psi = \sqrt[2]{\frac{a \psi_f}{t^{1-c}}} \]
\[ \theta(\eta) = \frac{T - T_w}{T_m - T_w} \]

Here, \( \psi \) is the stream function, \( u \) and \( v \) are the velocity components and defined as:

\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \]

The transformed momentum and energy equations combine with the boundary conditions are given by equations (2)–(4), can be written as:

\[ F''''(\eta) + \left[ 1 - \phi + \frac{\rho_s}{\rho_f} \right] (1-\phi)^{2.5} \]
\[ F(\eta) F''''(\eta) - (F''(\eta))^2 \]
\[ -A \left( F''(\eta) + \frac{1}{2} \eta F''''(\eta) \right) \]
\[ + (1-\phi)^{2.5} \left( 1 - \phi + \phi \left( \frac{\rho \beta}{\rho \beta}_f \right) \right) \gamma(\eta) = 0, \]
(for \( Al_2O_3-H_2O \) and \( Al_2O_3-C_2H_6O_2 \)).

\[ F''''(\eta) + \left[ 1 - \phi + \frac{\rho_s}{\rho_f} \right] \left( 306\phi^2 - 0.19\phi + 1 \right) \]
\[ -A \left( F''(\eta) + \frac{1}{2} \eta F''''(\eta) \right) \]
\[ + \left[ 1 - \phi + \phi \left( \frac{\rho \beta}{\rho \beta}_f \right) \right] (1-\phi)^{2.5} \gamma(\eta) = 0 \]
(for \( Al_2O_3-C_2H_6O_2 \)).

\[ F''''(\eta) + \left[ 1 - \phi + \frac{\rho_s}{\rho_f} \right] \left( 123\phi^2 - 7.3\phi + 1 \right) \]
\[ -A \left( F''(\eta) + \frac{1}{2} \eta F''''(\eta) \right) \]
\[ + \left[ 1 - \phi + \phi \left( \frac{\rho \beta}{\rho \beta}_f \right) \right] (123\phi^2 - 7.3\phi + 1) \gamma(\eta) = 0 \]
(for \( Al_2O_3-C_2H_6O_2 \)).

The boundary conditions are:

\[ F(0) = 0, F'(0) = 1, \text{ and } F''(\infty) = 0, \theta(0) = 1, \theta'(\infty) = 0. \]

So, the energy equation becomes:

\[ K_s - 2K_f + 2\phi(K_f - K_s) \]
\[ K_s - 2K_f + \phi(K_f - K_s) \]
\[ Pr \left( 1 - \phi + \frac{\rho \beta}{\rho \beta}_f \right) \theta''(\eta) - \]
\[ \frac{1}{2} \eta \theta'(\eta) + \theta'(\eta) F(\eta) = 0, \]
(for \( Al_2O_3-H_2O \) and \( Al_2O_3-C_2H_6O_2 \)).
The dimensionless parameters appeared in the final model due to the induction of combined convection and unsteady effects are $A = \frac{c}{a}$ (unsteady parameter), $Pr = \frac{(\nu_f)(\rho_{c_f})_f}{g(\rho \beta)}$ (Prandtl number) and $\gamma = \frac{\rho_f}{a^2}$ (buoyancy parameter).

**Procedure for solution**

The numerical technique based on shooting algorithm is adopted to tackle the model. This technique usually based on a set of first order ordinary differential equations which are obtained from the nonlinear flow model by using some sort of substitutions. For under consideration model, these substitutions for the velocity and temperature equations are made in the following way and the method implementation is also given in the chart:

Further,

$$
\left( y_1, y_2, y_3, y_4, y_5, y_6 \right) = (F, F', F'', F', \theta', \theta^*)
$$

$$
y_2' = \frac{1}{Pr \left(1 - \phi + \phi \frac{\rho_f}{\rho_{c_f}} \right)} \left[ \left(1 - \phi + \phi \frac{\rho_f}{\rho_{c_f}} \right) A \left( y_1 + \frac{1}{2} \eta y_3 \right) + y_5 - y_1 y_3 \right],
$$

for $Al_2O_3 - H_2O$ and $Al_2O_3 - C_2H_6O_2$

$$
y_3' = \frac{1}{306\phi^2 - 0.19\phi + 1} \left[ \left(1 - \phi + \phi \frac{\rho_f}{\rho_{c_f}} \right) A \left( y_1 + \frac{1}{2} \eta y_3 \right) + y_5 - y_1 y_3 \right],
$$

for $\gamma Al_2O_3 - C_2H_6O_2$

$$
y_5' = \frac{1}{123\phi^2 - 7.3\phi + 1} \left[ \left(1 - \phi + \phi \frac{\rho_f}{\rho_{c_f}} \right) A \left( y_1 + \frac{1}{2} \eta y_3 \right) + y_5 - y_1 y_3 \right],
$$

for $\gamma Al_2O_3 - H_2O$

$$
y_6' = \frac{1 - \phi + \phi \frac{\rho_f}{\rho_{c_f}}}{K_s - 2K_f + 2\phi (K_f - K_s)} \left( y_1 y_6 - \frac{1}{2} \eta y_6 \right),
$$

for $Al_2O_3 - H_2O$ and $Al_2O_3 - C_2H_6O_2$.

(20)
Quantities of engineering concern

The analysis of skin friction coefficient and local heat transport mechanism is significant from industrial and technological point of view. Further, thermophysical values described in Table 1. For under consideration model, the dimensional mathematical relations define as:

\[ C_f = \frac{-2 \mu_f \left( \frac{\partial u}{\partial y} \right)_{y=0}}{\rho_f u_x^2} \quad \text{and} \quad \text{Nu}_x = \frac{q_w}{k_f (T_w - T_x)} \]

(28)

Where \( q_w \) is the heat transfer from sheet and can be written as:

\[ q_w = -K_{sf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \]

(29)

Applying the dimensional transformation (12) we obtain:

\[ \frac{1}{2} Re_x C_f = -\left(1 - \phi \right)^{-2} F^* (0) \quad \text{for } Al_2O_3 - H_2O \]

(30)

\[ \frac{1}{2} Re_x C_f = -\left(123 \phi^2 + 7.3 \phi + 1 \right) F^* (0) \quad \text{for } Al_2O_3 - C_2H_6O_2 \]

(31)

\[ \frac{1}{2} Re_x C_f = -\left(123 \phi^2 + 7.3 \phi + 1 \right) F^* (0) \quad \text{for } Al_2O_3 - C_2H_6O_2 \]

(32)

\[ \frac{1}{2} Re_x \frac{1}{2} Nu_x = -\left(123 \phi^2 + 7.3 \phi + 1 \right) \theta' (0) \quad \text{for } Al_2O_3 - H_2O \]

(33)

\[ \frac{1}{2} Re_x \frac{1}{2} Nu_x = -\left(123 \phi^2 + 7.3 \phi + 1 \right) \theta' (0) \quad \text{for } Al_2O_3 - C_2H_6O_2 \]

(34)

\[ \frac{1}{2} Re_x \frac{1}{2} Nu_x = -\left(123 \phi^2 + 7.3 \phi + 1 \right) \theta' (0) \quad \text{for } Al_2O_3 - C_2H_6O_2 \]

(35)

\[ \frac{1}{2} Re_x \frac{1}{2} Nu_x = -\left(123 \phi^2 + 7.3 \phi + 1 \right) \theta' (0) \quad \text{for } Al_2O_3 - C_2H_6O_2 \]

(36)

Results and discussion

In the study of nanofluids, the behavior of working fluid regarding the momentum and energy transport is of huge significance which make them prominent from the common fluids. In this research, variety of nanoliquids (Al₂O₃, H₂O, γAl₂O₃-H₂O, Al₂O₃-C₂H₆O₂, and γAl₂O₃-C₂H₆O₂) which prepared by two distinct base solvents (H₂O, C₂H₆O₂) and important nanomaterial (Al₂O₃, γAl₂O₃) are taken in the existence of combined convection and unsteady effects. These nanomaterials became much popular and can be utilized significantly in industrial applications due to their superior chemical properties. The comparative alterations in the velocity (\( F'(\eta) \)) and temperature (\( \theta(\eta) \)) profiles under varying flow quantities are demonstrated in this section.
The velocity $F'(\eta)$ variations for $\text{Al}_2\text{O}_3$-$\text{H}_2\text{O}$, $\gamma\text{Al}_2\text{O}_3$-$\text{H}_2\text{O}$, due to unsteady effects $A$ are captured in Figure 2(a) and (b). Keen observations of the plotted results reveal that the motion of the both nanoliquids decays under stronger unsteady effects. The movement of nanoliquids based on simple $\text{Al}_2\text{O}_3$ promptly decays than $\gamma\text{Al}_2\text{O}_3$. Physically, the nanoliquids $\text{Al}_2\text{O}_3$-$\text{H}_2\text{O}$ and $\text{Al}_2\text{O}_3$-$\text{C}_2\text{H}_6\text{O}_2$ are more-denser and in the view of their thermophysical characteristics which opposes the motion. The viscosity inside the nanoliquids intensifies which resists the particles motion. Figure 2(c) to (f) demonstrating thermal distribution ($\theta(\eta)$) for the nanofluids against increasing unsteady effects $A$. These results reveal that nanoliquids are better for thermal transport in which thermal conductivity of $\text{Al}_2\text{O}_3$ and $\gamma\text{Al}_2\text{O}_3$ are have important role.

The velocity trends against increasing $\phi$ are depicted in Figure 3(a) and (b) for $\text{Al}_2\text{O}_3$-$\text{H}_2\text{O}$, $\gamma\text{Al}_2\text{O}_3$-$\text{H}_2\text{O}$, $\text{Al}_2\text{O}_3$-$\text{C}_2\text{H}_6\text{O}_2$, and $\gamma\text{Al}_2\text{O}_3$-$\text{C}_2\text{H}_6\text{O}_2$ nanoliquids, respectively.
Physically, increment in the velocity occurs because for higher fraction $\phi$, viscosity and density are decreases and the fluid particles move freely within the domain. Figure 3(c) to (f) furnished to investigate the temperature behavior by enhancing volume concentration. Thermal conductivity of nanoparticles playing significant role in the temperature enhancing of the nanoliquids. Physically, additional thermal conductivity of $\gamma\text{Al}_2\text{O}_3$ is a key tool for thermal enhancement because these are better conductor and intensify the heat transport inside the fluid.

Figure 4(a) and (b) furnished for the analysis of $F(\eta)$ due to increasing buoyancy effects. It is noticed that the velocity grows almost inconsequentially by strengthen $\gamma$. Further, momentum boundary layer portion rises for $\gamma\text{Al}_2\text{O}_3$ and it decays for $\text{Al}_2\text{O}_3$ and $\text{Al}_2\text{O}_3$. Figure 4(c) to (f) demonstrating impacts of imposed mixed convection effects ($\gamma$) on the temperature distribution $\theta(\eta)$. These results showing that the temperature drops due to upsurges in $\gamma$. Physically, the motion declines due to stronger $\gamma$ as a result the particles collide slowly and friction between the successive layer of the fluid reduces. This reduction in collision affects the temperature and hence it increases very slowly.
Conclusions

The flow of nanofluids comprising $\gamma\text{Al}_2\text{O}_3$, $\text{Al}_2\text{O}_3$ nanomaterials is conducted under unsteady effects. The model is handled numerically and demonstrated the results against the significant physical quantities. From the graphical representation of the results, it is examined that the nanofluid motion declines abruptly for the nanofluid composed by $\text{Al}_2\text{O}_3$ than $\gamma\text{Al}_2\text{O}_3$. The volumetric fraction of the metallic particles is a key ingredient for thermal enhancement in the nanofluid and showing dominant behavior for $\gamma$-nanomaterials based nanofluid. Dual behavior of the temperature is investigated against combined convection effects. Finally, it is concluded that the nanofluids are reliable for thermal enhancement and beneficial for many industrial and technological purposes as compared to that of base liquids which have low thermal performance characteristics.

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### Appendix

#### Notation

- [u, v] velocity components [m/s]
- T temperature [K]
- Uw surface velocity [m/s]
- Tw surface temperature [K]
- T∞ ambient temperature [K]
- nf stands for nanofluids
- ρnf density of nf [kgm⁻³]
- ρf base solvent density [kgm⁻³]
- ρs nanoparticles density [kgm⁻³]
- µnf dynamic viscosity [kgm⁻¹s⁻¹]
- µf base solvent density [kgm⁻¹s⁻¹]
- knf nanofluids thermal conductivity [Wm⁻¹K⁻¹]
- ks nanoparticles thermal conductivity [Wm⁻¹K⁻¹]
- kf base solvent thermal conductivity [Wm⁻¹K⁻¹]
- cpnf heat capacity of nanofluids [JK⁻¹]
- cf base fluid heat capacity [JK⁻¹]
- φ concentration factor
- η similarity variable
- ψ stream function
- F′(η) dimensionless velocity
- θ(η) dimensionless temperature
- γ mixed convection number
- Pr Prandtl number