The Higgs boson mass in a natural minimal supersymmetric standard model with nonuniversal gaugino masses at the grand unification theory scale

Hiroyuki Abe*, Junichiro Kawamura*, and Hajime Otsuka*

Department of Physics, Waseda University, Tokyo 169-8555, Japan
*E-mail: abe@waseda.jp; junichiro-k@ruri.waseda.jp; hajime.13.gologo@akane.waseda.jp

Published January 1, 2013

We identify a parameter region where the mass of the lightest CP-even Higgs boson resides at 124.4–126.8 GeV, and at the same time the degree of tuning a Higgsino-mass parameter (the so-called $\mu$-parameter) is relaxed above 10% in the minimal supersymmetric standard model (MSSM) with soft supersymmetry breaking terms, by solving the full set of one-loop renormalization group equations numerically. It is found that certain nonuniversal values of gaugino-mass parameters at the so-called grand unification theory (GUT) scale $\sim 10^{16}$ GeV are important ingredients for the MSSM to predict, without severe fine-tuning, the Higgs boson mass $\sim 125$ GeV, indicated by recent observations at the Large Hadron Collider. We also show a typical superparticle spectrum in this parameter region.

Subject Index B13

1. Introduction

Low-energy supersymmetry is one of the most promising candidates for new physics beyond the standard model (SM) of elementary particles, due to the absence of quadratic divergences. The supersymmetric partners of SM particles cancel the radiative corrections to the mass of Higgs bosons, then protect the electroweak (EW) scale $M_{\text{EW}} \sim 10^2$ GeV against huge corrections. The lightest supersymmetric particle (LSP) can be a candidate for the dark matter required from cosmological observations. Moreover, the minimal supersymmetric standard model (MSSM) predicts that three gauge coupling constants are unified at a high-energy scale around $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, and the electroweak symmetry is broken in a wide range of the parameter space of the MSSM with soft supersymmetry breaking terms, due to logarithmic radiative corrections that cause sizable running of parameters from $M_{\text{GUT}}$ to $M_{\text{EW}}$. (For a review, see Ref. [1].)

On a stable vacuum where the EW symmetry is broken successfully, the mass of the Z boson is determined by

$$m_Z^2 = \frac{|m_{H_d}^2(M_{\text{EW}}) - m_{H_u}^2(M_{\text{EW}})|}{\sqrt{1 - \sin^2(2\beta)}} - m_{H_u}^2(M_{\text{EW}}) - m_{H_d}^2(M_{\text{EW}}) - 2|\mu(M_{\text{EW}})|^2 \simeq -2|\mu(M_{\text{EW}})|^2 - 2m_{H_u}^2(M_{\text{EW}}),$$

(1)
where $\mu(M_{\text{EW}})$ and $m_{H_u}(M_{\text{EW}})$ are supersymmetric and soft supersymmetry breaking masses, respectively, for the Higgs field evaluated at $M_{\text{EW}}$. The Higgsino mass is also given by $\mu$ and we refer to this parameter as the $\mu$-parameter. The above-mentioned radiative EW breaking means roughly $m_{H_u}^2(M_{\text{EW}}) < -|\mu(M_{\text{EW}})|^2 < 0$ even with $m_{H_u}^2(M_{\text{GUT}}) > 0$ at $M_{\text{GUT}}$. The observed value $m_Z = 91.2$ GeV indicates $|\mu(M_{\text{EW}})| \sim |m_{H_u}(M_{\text{EW}})| \sim M_{\text{EW}}$, otherwise fine-tuning is required between parameters $\mu$ and $m_{H_u}$. One of the guiding principles toward a more fundamental theory beyond the MSSM can be provided by an argument of naturalness. Because the mass parameters $\mu$ and $m_{H_u}$ should have essentially different origins from the viewpoint of supersymmetry, there is no reason that these parameters are closely related.

However, from the recent results in the search for Higgs and supersymmetric particles at the Large Hadron Collider (LHC), the mass of scalar quarks (squarks) in the first and second generations is indicated above about 1.3 TeV [2], and the allowed region of the mass of the lightest CP-even Higgs boson has been reported in between 124.4 and 126.8 GeV [3,4]. The latter implies a large radiative correction for the mass of the Higgs boson, which is lighter than the $Z$ boson at the tree level [5]. In this case the mass of scalar top quarks must be much heavier than $M_{\text{EW}}$, which dominantly contribute to such a correction due to a large top Yukawa coupling. These observations indicate that the mass scale of soft supersymmetry breaking parameters tends to be much larger than $m_Z$, which in general cause the fine-tuning problem mentioned above.

It was pointed out in Ref. [6] that certain nonuniversal gaugino masses at $M_{\text{GUT}}$ relax the degree of the above-mentioned fine-tuning in the MSSM. In this paper, we update the analysis based on the recent experimental data. Because the latest Higgs mass bound shown above indicates a larger value of $\tan \beta$ than the value adopted in Ref. [6], in the following analysis, we solve the full set of one-loop renormalization group equations (RGEs), in contrast to the previous analysis in Ref. [6] where all the Yukawa (and scalar trilinear) couplings are neglected except for those involving only (scalar) top quarks.

The following sections are organized as follows. In Sect. 2, we roughly estimate the effects of certain nonuniversal gaugino masses on the mass of the lightest CP-even Higgs boson and on the degree of tuning the $\mu$-parameter. Based on these implications, in Sect. 3, we perform a full numerical analysis at the one-loop level and identify a parameter region where the Higgs mass resides at 124.4–126.8 GeV and the degree of tuning is relaxed above 10% at the same time. Finally, Sect. 4 is devoted to conclusions and discussions. In Appendix A, we show the boundary conditions for RGEs at the GUT scale adopted in this paper. The relevant RGEs for analyzing the Higgs mass are given in Appendix B.

2. Implications of nonuniversal gaugino masses

The superpotential and soft supersymmetry breaking terms in the MSSM are shown in Eqs. (A1) and (A2) in Appendix A, respectively. We use the notations and conventions adopted in Appendix A throughout this paper.

The radiative corrections to the Higgs mass are dominated by loops of scalar top quarks. With an approximation that the mass eigenstates of top squarks are nearly degenerate, the mass of the lightest

\footnote{A similar analysis was performed recently in Ref. [7].}
CP-even Higgs boson is evaluated as [8]
\[
m_h^2 \simeq m_Z^2 \cos^2(2\beta) \left( 1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} \right) + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[ \frac{1}{2} X_t + t + \frac{1}{16\pi^2} \left( \frac{3}{2} \frac{m_t^2}{v^2} - 32\pi^3 \right) (X_t + t^2) \right],
\]
with
\[
X_t = \frac{2\tilde{A}_t^2}{M_{st}^2} \left( 1 - \frac{\tilde{A}_t^2}{12M_{st}^2} \right), \quad \begin{align*}
\tilde{A}_t &\equiv A_t(m_Z) - \mu(m_Z) \cot \beta, \\
M_{st}^2 &\equiv \frac{1}{\sqrt{2}} \frac{m_{Q_3}^2(m_Z)}{m_{Q_3}^2} \frac{m_{Q_3}^2(m_Z)}{m_{Q_3}^2} m_{Q_3}^2(m_Z),
\end{align*}
\]
where \( m_t \) is the on-shell running mass computed from the physical top mass \( m_t = 173.5 \text{ GeV} \) [9], i.e. \( m_t = 165.2 \text{ GeV}. \) \( y_t \equiv y_{33}^u \) is the top Yukawa coupling, \( a_{33}^u \equiv y_t A_{33}^u \equiv y_t A_t \) is the scalar trilinear coupling involving only top squarks, \( m_{Q_3}^2(m_Z) \equiv (m_{Q_3}^2(m_Z)) \) is the left-(right-)handed top squark mass square, and \( t = \ln(M_{st}^2/m_t^2) \). Here \( v = 174 \text{ GeV} \) is related to the VEVs of up- and down-type Higgs fields \( \mu \) and \( \mu \) as \( v_u = v \sin \beta \) and \( v_d = v \cos \beta \), respectively. As is well known, the value of \( X_t \) is maximized with \( |\tilde{A}_t| \sim \sqrt{6}M_{st} \), and then the loop corrections are enhanced in Eq. (2).

If all the Yukawa (and scalar trilinear) couplings are neglected except for those involving only (scalar) top quarks, the one-loop RGEs show that the soft supersymmetry breaking parameters evaluated at the \( Z \) boson mass scale \( m_Z \) are related to those at \( M_{GUT} \) as
\[
m_{Q_3}^2(m_Z) \simeq -0.02 M_1^2 + 0.38 M_2^2 - 0.02 M_1 M_3 - 0.07 M_2 M_3 + 5.63 M_3^2
+ (0.02 M_2 + 0.09 M_3 - 0.02 A_t) A_t - 0.14 m_H^2 + 0.86 m_{Q_3}^2 - 0.14 m_{U_3}^2,
\]
\[
m_{U_3}^2(m_Z) \simeq 0.07 M_1^2 - 0.01 M_1 M_2 - 0.21 M_2^2 - 0.03 M_1 M_3 - 0.14 M_2 M_3 + 4.61 M_3^2
+ (0.01 M_1 + 0.04 M_2 + 0.18 M_3 - 0.05 A_t) A_t - 0.27 m_H^2 - 0.27 m_{Q_3}^2 + 0.73 m_{U_3}^2,
\]
\[
A_t(m_Z) \simeq -0.04 M_1 - 0.21 M_2 - 1.90 M_3 + 0.18 A_t,
\]
where the soft parameters without any arguments on the right-hand sides represent those evaluated at \( M_{GUT} \). Here the numerical values of the MSSM gauge couplings and the top Yukawa coupling are chosen in such a way that these values become the observed ones at low energies, and we take \( \tan \beta = 15 \) as declared at the end of Appendix A. The numerical values of the coefficients in Eq. (3) suggest \( |\tilde{A}_t|/M_{st} < 1 \) for soft parameters of the same orders of magnitude at \( M_{GUT} \), without any cancellation between terms on the right-hand sides of Eq. (3).

This insists that a certain cancellation is required in Eq. (3) in order to enhance radiative corrections in Eq. (2) for obtaining a heavier Higgs mass. We find that the gluino mass-square term with the largest coefficient can be cancelled by the wino mass-square term in \( m_U^2(m_Z) \) with the ratio \( M_2/M_3 \sim \sqrt{4.6/0.21} \sim 4.8 \), and then \( 0 < m_U(m_Z) < |A_t(m_Z)| \) is realized. This phenomenon can be understood as follows. The gaugino masses and soft scalar masses act as positive and negative driving forces, respectively, in the renormalization group evolution of the soft scalar mass square from the GUT to the EW scale. The relevant RGEs are shown in Appendix B. Therefore, the mass square of the left-handed scalar quark \( m_Q^2 \) tends to increase for a larger wino mass \( M_2/M_3 > 1 \), and then the mass square of the right-handed squark \( m_U^2 \) tends to decrease with increasing \( m_Q^2 \). The averaged top squark mass \( M_3 \) decreases because the increasing contribution from \( m_Q^2 \) is dominated by the decreasing one from \( m_U^2 \). As we can estimate in Eq. (3), the contribution from the wino mass...
satisfying $2 \lesssim M_2/M_3 \lesssim 5$ to reduce $m_{U}^2$ dominates the one to increase $m_{Q}^2$, and then $|\tilde{A}_t|/M_{\text{st}} > 1$ is achieved.

The above rough estimation can be verified numerically. Figure 1 shows contours of $M_{\text{st}}/m_{Q_3}$ (left) and $r_a \equiv |\tilde{A}_t|/M_{\text{st}}$ (right) in the $(r_1, r_2)$ plane, evaluated by a numerical solution for the full set of one-loop RGEs with the boundary conditions at the GUT scale shown in Appendix A, where the bino–gluino and the wino–gluino mass ratios at the GUT scale are denoted respectively by

$$r_1 \equiv M_1/M_3, \quad r_2 \equiv M_2/M_3.$$ 

The ratios $r_1$ and $r_2$ are varied in the following analysis. From Fig. 1 we find that $M_{\text{st}}$ decreases and then $|\tilde{A}_t|/M_{\text{st}}$ increases as the wino mass parameter $M_2$ increases, confirming the above rough estimation.

On the other hand, concerning the fine-tuning, a smaller absolute value of the soft scalar mass square for the up-type Higgs $m_{H_u}^2$ is favored. A similar relation to the ones in Eq. (3) is obtained for
\[ m^2_{H_u} \] as
\[ m^2_{H_u}(m_Z) \simeq -0.01 M_1 M_2 + 0.17 M_2^2 - 0.05 M_1 M_3 - 0.20 M_2 M_3 - 3.09 M_3^2 \]
\[ + (0.02 M_1 + 0.06 M_2 + 0.27 M_3 - 0.07 A_t) A_t \]
\[ + 0.59 m^2_{H_u} - 0.41 m^2_{Q_3} - 0.41 m^2_{\tilde{t}_3} . \] (4)

The numerical values of the coefficients suggest that \( m^2_{H_u}(m_Z) \) is of the order of soft parameters for those of the same orders of magnitude at \( M_{GUT} \), without any cancellation between terms in the right-hand side of Eq. (4).

This insists again that a certain cancellation is required in Eq. (4) in order to realize \( -m^2_{H_u}(m_Z) \sim m^2_Z \) to reduce the fine-tuning required by Eq. (1). We find that the gluino mass-square term with the largest coefficient can be cancelled by the wino mass-square term in \( m^2_{H_u}(m_Z) \) with the ratio \( M_2/M_3 \sim \sqrt{3.1/0.17} \sim 4.3 \), which is within the range \( 2 \lesssim M_2/M_3 \lesssim 5 \) required for \( |\tilde{A}_t|/M_{st} > 1 \).

3. The Higgs mass and naturalness

Motivated by the above rough estimations based on Eqs. (2), (3), and (4), we search a parameter space of the MSSM with soft terms, in particular a region of gaugino mass ratios, where the mass of the lightest CP-even Higgs boson \( m_h \) resides at 124.4–126.8 GeV and at the same time the degree of tuning the \( \mu \)-parameter is relaxed above 10%. Because we take \( \tan \beta = 15 \) as declared at the end of Appendix A, the effect of Yukawa couplings other than the top one, especially the bottom Yukawa coupling, can induce sizable corrections; therefore we numerically solve the full set \(^2\) of one-loop RGEs including all the Yukawa couplings. (The complete set of MSSM RGEs can be found in, e.g., Ref. [11] at the two-loop level.)

By utilizing the numerical solutions of RGEs, we evaluate the Higgs mass \( m_h \) based on the mass matrix with one- and two-loop contributions from top and bottom squarks, respectively, derived in Ref. [8]. Here we do not adopt the approximate expression (2) because left- and right-handed top squarks are not degenerate in the favored region \( 2 \lesssim M_2/M_3 \lesssim 5 \) with the maximal-mixing of top squarks. Although the above rough estimation is based on Eq. (2), it turns out to be valid and consequently a heavier Higgs mass can be obtained in the full numerical analysis, as we will see later.

We assume a certain mechanism of supersymmetry breaking, which determines soft supersymmetry breaking parameters at the GUT scale, and then the ratios between these parameters are fixed with some accuracy. The \( \mu \)-parameter in the MSSM superpotential (6) is in general independent of the mechanism of supersymmetry breaking. As a measure of the degree of tuning of the \( \mu \)-parameter at the GUT scale, we adopt a parameter \( \Delta_{\mu} \) defined by
\[ \Delta_{\mu} = \frac{|\mu|}{2m^2_Z} \frac{\partial m^2_Z}{\partial |\mu|} , \] (5)
which represents the sensitivity [12] of the Z boson mass \( m_Z \) at the EW scale to the \( \mu \)-parameter at the GUT scale. The degree of tuning to obtain \( m_Z = 91.2 \) GeV is then estimated as 100 \( \times |\Delta_{\mu}^{-1}| \) %.

\(^2\) Although the effects of light generations are negligible, for concreteness, we input numerical values for all the Yukawa couplings determined by the Froggatt–Nielsen mechanism [10] at the GUT scale, shown in Appendix A. Also, our calculation covers the situation in which the scalar charm quark masses are about \( 10^2 \) times higher than scalar top quark masses and the correction to \( m^2_{H_u} \) running is sizable, as can be seen in Appendix B.
It seems that, with the current experimental status, \(|\Delta_\mu| > 100\) is inevitable with universal values of soft parameters at the GUT scale, which requires more severe fine-tuning than the degree of 1%.

The results of the direct search at the LHC set the lower bounds of the gluino mass at about 1 TeV [13] and those of the squark masses in the first and second generations at 1320 GeV [2]. We adopt the severest bound, although it may be lowered in models with some nonuniversal gaugino mass ratios [14]. Taking these stringent bounds and the other experimental bounds for top squarks, neutralinos, and charginos [9,15] into account, we fix the numerical values of the soft parameters other than the wino and bino masses at the GUT scale as shown in Appendix A, and vary the gaugino mass ratios \(r_1, r_2\) at the GUT scale in the evaluation of the Higgs mass \(m_h\) and the parameter \(|\Delta_\mu|\).

Figure 2 shows contours of \(m_h\) (GeV) and \(100 \times |\Delta_\mu^{-1}|\) (%) in the parameter space \((r_1, r_2)\).

From Fig. 2, we find that the mass of the Higgs boson resides at 124.4–126.8 GeV in the region \(3.0 \lesssim r_2 \lesssim 5.5\) for the wide range of \(r_1 \gtrsim -5\). Moreover, we emphasize that the region overlaps with \(5.2 \lesssim r_2 \lesssim 5.5\), where the degree of tuning of the \(\mu\)-parameter is relaxed above 10%. Therefore we find a parameter region where the Higgs boson mass \(\sim 125\) GeV indicated by recent LHC observations is predicted in a natural MSSM with certain nonuniversal gaugino masses at the GUT scale. Compared with Ref. [6], here a Higgs boson mass above 115 GeV is achieved by a larger value of \(\tan \beta = 15\); this is one of the reasons we employed the full set of MSSM RGEs including all the

\[
\begin{align*}
\text{Fig. 2. Contours of } m_h \text{ (GeV) and } 100 \times |\Delta_\mu^{-1}| \text{ (%) in the parameter space } (r_1, r_2). 
\end{align*}
\]
Bosons are shown in Tables 1 and 2, by setting the gaugino mass ratios breaking and mediation mechanisms. By naturalness. For example, concrete fixed values for the ratios are shown in Ref. [16] for various and mediation mechanisms that fix the gaugino mass ratios to the above numerical values favored ratios of the gaugino masses is not so stringent. There could exist some supersymmetry breaking Yukawa couplings. Although the overlapping region is not so wide, the required accuracy for the ratios of the gaugino masses is not so stringent. There could exist some supersymmetry breaking and mediation mechanisms that fix the gaugino mass ratios to the above numerical values favored by naturalness. For example, concrete fixed values for the ratios are shown in Ref. [16] for various breaking and mediation mechanisms.

A typical superparticle (sparticle) spectrum at the EW scale and the masses of neutral and charged Higgs bosons as well as the gaugino masses at the EW scale for \((r_1, r_2) = (13, 5.4)\). The degree of tuning the \(\mu\)-parameter, \(100 \times |\Delta^{-1}_\mu|\) (%), is also shown.

| sparticle | mass (GeV) | sparticle | mass (GeV) |
|-----------|------------|-----------|------------|
| \(\tilde{u}_1\) | 2209 | \(\tilde{e}_1\) | 2302 |
| \(\tilde{u}_2\) | 2324 | \(\tilde{e}_2\) | 2456 |
| \(\tilde{c}_1\) | 2204 | \(\tilde{\mu}_1\) | 2301 |
| \(\tilde{c}_2\) | 2320 | \(\tilde{\mu}_2\) | 2439 |
| \(\tilde{t}_1\) | 752.1 | \(\tilde{\tau}_1\) | 1735 |
| \(\tilde{t}_2\) | 1457 | \(\tilde{\tau}_2\) | 1936 |
| \(\tilde{d}_1\) | 1903 | \(\tilde{\chi}^0_1\) | 2064 |
| \(\tilde{d}_2\) | 2326 | \(\tilde{\chi}^0_2\) | 1713 |
| \(\tilde{s}_1\) | 1874 | \(\tilde{\chi}^0_3\) | 187.4 |
| \(\tilde{s}_2\) | 2322 | \(\tilde{\chi}^0_4\) | 192.2 |
| \(\tilde{b}_1\) | 1071 | \(\tilde{\chi}^{\pm}_1\) | 189.4 |
| \(\tilde{b}_2\) | 1436 | \(\tilde{\chi}^{\pm}_2\) | 1713 |

| parameter | value |
|-----------|-------|
| \(m_H\) (GeV) | 126.0 |
| \(m_H\) (GeV) | 1639 |
| \(m_H\) (GeV) | 1641 |
| \(100 \times |\Delta^{-1}_\mu|\) (%) | 11.48 |
| \(M_1(m_Z)\) (GeV) | 2063 |
| \(M_2(m_Z)\) (GeV) | 1709 |
| \(M_3(m_Z)\) (GeV) | 1104 |

Yukawa couplings. Although the overlapping region is not so wide, the required accuracy for the ratios of the gaugino masses is not so stringent. There could exist some supersymmetry breaking and mediation mechanisms that fix the gaugino mass ratios to the above numerical values favored by naturalness. For example, concrete fixed values for the ratios are shown in Ref. [16] for various breaking and mediation mechanisms.

A typical superparticle spectrum at the EW scale and the masses of neutral and charged Higgs bosons are shown in Tables 1 and 2, by setting the gaugino mass ratios \((r_1, r_2) = (13, 5.4)\) inside the favored region found above. From these tables, we find that all the experimental lower bounds on the masses of gluinos, neutralinos, charginos, squarks, and sleptons as well as neutral and charged Higgs bosons can be satisfied with 11.48% tuning of the \(\mu\)-parameter. With this parameter choice, Fig. 3 shows the running of gaugino and soft scalar masses from the GUT to the EW scale. We find that gaugino masses tend to degenerate at low energies; this is a typical signal of this parameter region, as discussed in Ref. [6].

Here we comment on charge and color breaking minima [17,18]. If we take \(A_t\) larger than \(M_\text{st}\), there is a possibility that such minima appear along the direction satisfying \(|\tilde{q}_3| = |\tilde{u}_3| = |h_u|\) in the field space. The minima exist unless the following condition,

\[
|A_t|^2 \leq 3(m_Q^2 + m_U^2 + m^2_H + |\mu|^2),
\]

is satisfied. It has been noted that the parameters yielding \(r_u \simeq \sqrt{6}\) are dangerous with the parameters satisfying \(m_Q = m_U^3\) due to the inequality (6) [19]. On the other hand, in our analysis, the important region \(\sqrt{3} \leq r_u \leq \sqrt{7}\) shown in Fig. 1 appears with the parameters satisfying \(|A_t(m_Z)| \sim m_Q^3(m_Z)\) and \(0 < m_U^3(m_Z) < m_Q^3(m_Z)\), and then the inequality (6) is guaranteed.
Finally, we mention the next to minimal supersymmetric standard model (NMSSM). When a singlet chiral multiplet is added, the MSSM superpotential can be modified to have a positive correction to the Higgs mass square at the tree level. There is, however, another negative contribution to the Higgs mass square induced by mixing with the singlet field. The latter negative contribution also becomes larger in general when the parameters are chosen to make the former positive contribution larger. This fact makes the analysis of the NMSSM complicated (for a review, see Ref. [20]). At the current stage, we cannot make a definitive statement that the NMSSM is better than the MSSM from the viewpoint of both the Higgs mass and naturalness, even though there have been interesting studies concerning these issues in the NMSSM [21–23].

4. Conclusions and discussions

By solving the full set of one-loop RGEs numerically for all the parameters in the MSSM with soft supersymmetry breaking terms, we identified a parameter region where the mass of the lightest CP-even Higgs boson resides at 124.4–126.8 GeV, indicated by the recent LHC results, and at the same time the degree of tuning of the μ-parameter is relaxed above 10%. The region is characterized by certain nonuniversal values of gaugino mass parameters at the GUT scale, as indicated in Ref. [6] before the LHC observations. We have confirmed that, even after the LHC results, the main suggestions in Ref. [6] are still valid with a larger value of tan β based on more accurate analyses than those in Ref. [6].

We also derived a superparticle spectrum for a typical parameter choice inside the identified region. We find that all the experimental lower bounds on the masses of gluinos, neutralinos, charginos, squarks, and sleptons as well as neutral and charged Higgs bosons can be satisfied. One of the outstanding features of the spectrum is the degenerate gaugino masses at low energies as a consequence of a particular choice of gaugino mass ratios at the GUT scale favored by naturalness, as mentioned in Ref. [6]. It would be interesting to study cosmological features in detail in this parameter region of the MSSM.\(^3\)\(^4\).

\(^3\) A possibility for the neutralino dark matter was studied in Ref. [24] based on Ref. [6] before the LHC results.

\(^4\) Candidates for dark matter in this region are the gravitino, the right-handed sneutrino, and the Higgsino-like neutralino. In the third case, Higgsino-like neutralino dark matter would be possible due to an enhancement
One of the important guiding principles for the physics beyond the MSSM is provided by the argument of its naturalness. The EW scale is unstable under a tiny numerical deviation of the $\mu$-parameter when severe tuning is required to realize the observed mass of $W$ and $Z$ bosons. Therefore, an underlying theory free from such fine-tuning is desirable; this becomes a strong guiding principle when we study particle physics models at a more fundamental level. We adopted Eq. (5) as the measure of the degree of tuning. The measure can be extended to include all the other soft supersymmetry breaking parameters like, e.g., in Refs. [28,29]. However, we have assumed that the ratio between the breaking parameters is fixed with enough accuracy by concrete supersymmetry breaking and the mediation mechanism. Even in this situation, the supersymmetric parameter $\mu$ would be independent to the other parameters, and the issues of the fine-tuning should be relevant to the $\mu$-parameter.

There could exist some supersymmetry breaking and mediation mechanisms that fix the gaugino mass ratios [16] to the numerical values favored by naturalness. One such candidate is the mirage mediation model [30,31], where the gaugino mass ratio at the GUT scale is determined by the ratio of contributions to gaugino masses from the modulus and the anomaly mediated supersymmetry breaking [32,33]. Issues of fine-tuning in this model are studied in Refs. [34,35]. Another and more general origin of nonuniversal gaugino masses is moduli-mixing gauge kinetic functions, which appear even at the tree level in a certain effective supergravity action. For example, in some superstring models, such moduli-mixings appear from nontrivial D-brane configurations, where the gaugino mass ratios are determined by, e.g., numbers of windings, intersections, and magnetic fluxes of D-branes [36] (for a review, see Ref. [37]). Even without mentioning superstrings, in a minimal extension of the MSSM with a single extra dimension, namely, in 5D supergravity models, the situation is similar and the gaugino mass ratios can be determined by data for the very special manifold governing the structure of $\mathcal{N} = 2$ vector multiplets [38]. In all cases with moduli-mixing gauge kinetic functions, the mechanism of moduli stabilization is important to determine the gaugino mass ratios [39,40].

The LHC is now exploring the parameter space of the MSSM and the other supersymmetric models. It is possible that the recent and near-future observations could guide us in a direction toward a more fundamental theory of nature through the results obtained here.

**Acknowledgement**

The authors would like to thank T. Higaki for stimulating discussions. The work of H.A. was supported by the Waseda University Grant for Special Research Projects No. 2012B-151.

**Appendix A: Boundary conditions at the GUT scale**

In this appendix we show numerical values of parameters in the MSSM with soft supersymmetry breaking terms selected as a boundary condition at the GUT scale, $M_{\text{GUT}} = 2 \times 10^{16}$ GeV, for our numerical analysis of the full set of MSSM one-loop RGEs.

For the chiral multiplets $Q_i$, $U_i$, $D_i$, $L_i$, and $E_i$ carrying the $i$th generation of the left-handed quark doublet, the right-handed up quark, the right-handed down quark, the left-handed lepton doublet, and the right-handed electron, and those $H_u$ and $H_d$ containing the up- and down-type Higgs doublet, respectively, the MSSM superpotential is given by

$$W_{\text{MSSM}} = \mu H_u H_d + y_{ij}^u H_u Q_i U_j + y_{ij}^d H_d Q_i D_j + y_{ij}^e H_d L_i E_j,$$

(A1)

from non-thermal decay (gravitino [25], Axino [26], or moduli decay [27]), although the thermal abundance of the Higgsino-like neutralino is sub-dominant to explain the current dark matter abundance.
where \( y_{ij}^{u,d,e} \) are Yukawa coupling constants, and \( \mu \) is the Higgsino mass parameter referred to as the \( \mu \)-parameter. The summations over the generation indices \( i, j = 1, 2, 3 \) are implicit. The soft supersymmetry breaking terms are defined by

\[
\mathcal{L}_{\text{soft}} = -\frac{1}{2} \left( \sum_{a=1}^{3} M_a \text{tr} \lambda^a \lambda^a + \text{h.c.} \right) - \sum_{\Phi} (m_{\Phi}^2)_{ij} \tilde{\varphi}_i \tilde{\varphi}_j - m_{H_u}^2 |h_u|^2 - m_{H_d}^2 |h_d|^2 - (a_{ij}^u h_u \tilde{q}_i \tilde{u}_j + a_{ij}^d h_d \tilde{d}_i \tilde{d}_j + a_{ij}^e h_d \tilde{l}_i \tilde{e}_j + B \mu h_u h_d + \text{h.c.}),
\]

which can be realized by the Froggatt–Nielsen mechanism \[10\] or quasi-localized matter fields in 5D spacetime \[41,42\] yielding observed masses and mixings of quarks and charged leptons at the EW scale. Here \( \epsilon = 0.225 \) represents the magnitude of mixing by the Cabibbo angle. Note that these numerical values for the elements involving light generations are just for concreteness of the numerical evaluation, and these concrete values for the light generations are not essential for our results. The basic results are valid for the other ansatz of Yukawa matrices that generate observed quark and lepton masses and mixings, as long as it coincides with a value of \( \tan \beta \gtrsim 10 \), as will be selected in Eq. \( \text{(A3)} \).
The parameters in soft supersymmetry breaking terms are selected as follows. The gluino mass is chosen as

\[ M_3 = 385 \text{ GeV}, \]

which is almost the lowest value satisfying the experimental lower bound at the EW scale. Note that the gaugino mass ratios \( r_1 \) and \( r_2 \), namely the bino mass \( M_1 \) and the wino masses \( M_2 \), are varied at the GUT scale with the above fixed gluino mass \( M_3 \) in our numerical analysis. The soft scalar masses and the scalar trilinear couplings at the GUT scale are fixed as

\[
\sqrt{(m^2_{\Phi})_{ij}} = \begin{cases} 
1500 \text{ GeV} & (i = j = 1, 2) \\
200 \text{ GeV} & (i = j = 3) \\
0 \text{ GeV} & (i \neq j)
\end{cases}, \quad \Phi = Q, U, D, L, E, \\
m_{H_u} = m_{H_d} = 200 \text{ GeV},
\]

and

\[
a_{ij}^{u,d,e} = \gamma^u_{ij} A_{ij}^{u,d,e}, \quad A_{ij}^{u,d,e} = -400 \text{ GeV}, \quad i, j = 1, 2, 3.
\]

Although we adopt vanishing off-diagonal elements of the soft scalar mass matrices for simplicity, these essentially do not affect the main results of this paper.

We choose the following ratio:

\[ \tan \beta \equiv \frac{v_u}{v_d} = 15, \quad (A3) \]

where \( v_u = v \sin \beta \) and \( v_d = v \cos \beta \) are the VEVs of the up- and down-type Higgs fields, \( h_u \) and \( h_d \), respectively, and \( v = 174 \) GeV. The smaller value of \( \tan \beta \) makes the Higgs mass tend to be below 120 GeV in the MSSM. The larger value makes the hierarchy between \( v_u \) and \( v_d \) severer; this is unfavorable from the viewpoint of naturalness. The \( \mu \)-parameter in the MSSM superpotential (A1) as well as the \( B_\mu \)-parameter in the soft supersymmetry breaking terms (A2) is determined at the GUT scale in such a way that the correct \( Z \) boson mass is realized at the EW scale with the above set of the other parameters. Note that the numerical values of these parameters change as \( r_1 \) and \( r_2 \) vary.

**Appendix B: RGEs relevant to the Higgs mass**

In this appendix, we show the one-loop RGEs relevant to our discussion for the mass of the lightest CP-even Higgs boson:

\[
\frac{d m_Q^2}{dt} = \frac{1}{4 \pi^2} \left( \frac{8}{3} g_3^2 |M_3|^2 + \frac{3}{2} g_2^2 |M_2|^2 + \frac{1}{30} g_1^2 |M_1|^2 - \frac{1}{20} g_1^2 S \right) \hat{1} \\
+ \frac{1}{8 \pi^2} \left( \frac{1}{2} y^u(y^u) m_Q^2 + \frac{1}{2} m_Q^2 y^u(y^u) + y^u m_Q^2 (y^u) + (m_{H_u}^2) y^u(y^u) + A^u (A^u)^\dagger \right) \\
+ \frac{1}{8 \pi^2} \left( \frac{1}{2} y^d(y^d) m_Q^2 + \frac{1}{2} m_Q^2 y^d(y^d) + y^d m_Q^2 (y^d) + (m_{H_d}^2) y^d(y^d) + A^d (A^d)^\dagger \right),
\]

11/13
\[
\frac{d m_U^2}{d t} = -\frac{1}{4\pi^2} \left( \frac{8}{3} \frac{g_3^2 |M_3|^2}{3} + \frac{8}{15} \frac{g_1^2 |M_1|^2}{5} + \frac{1}{5} \frac{g_1^2 S}{5} \right) \hat{I} \\
+ \frac{1}{4\pi^2} \left( \frac{1}{2} (y_u^\dagger y_u)^2 m_U^2 + \frac{1}{2} m_y^2 (y_u^\dagger y_u + (y_u^\dagger m_Q^{-1} y_u + (m_H^2 + m_L^2 + m_D^2 + m_E^2)),
\frac{d m_H^2}{d t} = -\frac{1}{4\pi^2} \left( \frac{3}{2} \frac{g_2^2 |M_2|^2}{2} + \frac{3}{10} \frac{g_1^2 |M_1|^2}{5} - \frac{3}{20} \frac{g_1^2 S}{5} \right) \right) \\
+ \frac{3}{8\pi^2} \text{tr} \{ y_u^2 m_Q^{-1} y_u + (y_u^\dagger m_Q^{-1} y_u + (m_H^2 y_u^\dagger y_u + A_u^\dagger A_u) \},
S = m_H^2 - m_M^2 + \text{tr} (m_Q^2 - m_L^2 - m_U^2 - m_D^2 + m_E^2),
\]
where \( t = \ln(E/M_{GUT}) \) is a logarithmic energy scale measured by \( M_{GUT} = 2 \times 10^{16} \) GeV, and \( \hat{I} \) is a \( 3 \times 3 \) unit matrix. The complete set of MSSM RGEs can be found in, e.g., Ref. [11] at the two-loop level.

References

[1] S. P. Martin, in Perspectives on Supersymmetry II, ed. G. L. Kane, pp. 1–153 (1997).
[2] G. Aad et al. [ATLAS Collaboration], arXiv:1208.0949 [hep-ex].
[3] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012).
[4] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012).
[5] I. Niessen, A. Strubig, J. High Energy Phys. 1206, 013002 (2012).
[6] J. Cao, Z.-X. Heng, J. M. Yang, Y.-M. Zhang, and J.-Y. Zhu, J. High Energy Phys. 1207, 083502 (2005).
[7] K. Choi, A. Falkowski, H. P. Nilles, and M. Olechowski, Nucl. Phys. B 718, 113 (2005).
[8] K. Choi, K. S. Jeong, T. Kobayashi, and K.-i. Okumura, Phys. Lett. B 633, 355 (2006).
[9] R. Kitano and Y. Nomura, Phys. Lett. B 631, 58 (2005).
[10] D. Lust, P. Mayr, R. Richter, and S. Stieberger, Nucl. Phys. B 696, 205 (2004).
[37] R. Blumenhagen, B. Kors, D. Lust, and S. Stieberger, Phys. Rep. 445, 1 (2007).

[38] A. Ceresole and G. Dall’Agata, Nucl. Phys. B 585, 143 (2000).

[39] H. Abe, T. Higaki, and T. Kobayashi, Phys. Rev. D 73, 046005 (2006).

[40] H. Abe, T. Higaki, and T. Kobayashi, Nucl. Phys. B 742, 187 (2006).

[41] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D 61, 033005 (2000).

[42] D. E. Kaplan and T. M. P. Tait, J. High Energy Phys. 0006, 020 (2000).