Unidirectional reflectivity in lossy thin films

Alejandro Padrón-Godínez\textsuperscript{1,2}, Carlos G. Treviño-Palacios\textsuperscript{2}

\textsuperscript{1}Instituto Nacional de Astrofísica, Óptica y Electrónica, Luis Enrique Erro No. 1, Santa María Tonantzintla, C.P. 72840, Puebla - México
\textsuperscript{2}Instituto de Ciencias Aplicadas y Tecnología, Universidad Nacional Autónoma de México, Circuito Exterior S/N, Coyoacán, A. P. 04510, Cd. Universitaria - México

apadron@inaoep.mx, alpago00@unam.mx, carlost@inaoep.mx

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Inspired by non-Hermitian systems, we study reflection and transmission in a stack of thin films composed by the repetition of a bipartite unit cell. We aim for controlled reflection and transmission using lossless and lossy materials in order to develop an optical diode to generate entanglement photons source. Particularly, we show unidirectional reflection using transfer matrix methods and confirm our results by finite element simulation.

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INTRODUCTION

Thin films are a standard option to design optical devices with controlled reflection and transmission. Optics has a long tradition of studying how the basic properties of materials can be used to engineer thin film structures with an overall different behavior \cite{1}, which might be used for industrial applications such as optical camouflage and optical rectifiers, isolators or switches\cite{2,3} to mention a few. Recently, the quantum idea of PT-symmetry has been used to create composite structures with interesting optical properties. PT-Symmetry in quantum mechanics refers to invariance to spatial and temporal reflection. This is provided by complex potentials that obey the property $V^*(x)=-V(x)$ \cite{4,5}. An ideal optical equivalence is to introduce linear media with equal real part of the refractive index and imaginary parts that are the complex conjugate of each other \cite{6}. Such media is practically inexistent in nature and hard to engineer but experimental realizations have shown its feasibility\cite{2}. Furthermore, it has been shown that unidirectional reflection arises from PT-symmetric structures due to the gain-loss balance in optical structures that bring to mind a stack of thin film \cite{7}.

Here, we are interested in the effect of using real-world materials to design unidirectional reflectionless stacks of thin films, Fig.\textsuperscript{1}. First, we are going to model electromagnetic field propagation through dielectric thin films considering left and right normal incidences. We will start with the treatment of a simple layer using multiple beam interference techniques.

Then we will provide their transfer matrix and use them to describe a unit cell composed of just two thin layers \cite{8}. Next, we will optimize the film thicknesses numerically to find the extremal values for reflectivity/transmitivity at the desired wavelength. Then, we will find the transfer matrix results for these optimized parameter values and compare them with finite element simulation. We will use the ideal gain-loss bilayer as benchmark for more realistic passive-loss values of doped silicon dioxide ($SiO_2$) with metal nanoparticles for loss and erbium ions for gain. Finally, we will show results for unidirectional reflectance for a stack composed by three unit cells.

FIG. 1: A unit cell showing unidirectional reflectivity using two thin-films with conjugate.

FIG. 2: Successive reflections and transmissions in a thin film with an incidence angle.
and

\[ t_{i3} = \frac{t_{23}t_{12}e^{i\phi}}{1 - r_{23}t_{21}e^{i\phi}}, \]
\[ r_{i3} = r_{12} + \frac{t_{21}r_{23}t_{12}e^{i\phi}}{1 - r_{23}t_{21}e^{i\phi}}, \]

\[ (1) \]

where \( t_{ij} \) and \( r_{ij} \) are the transmission and reflection coefficients at each boundary, with \( i \neq j \). We will show results for normal incidence for the sake of space. Here the effective transfer matrix \[ M_{neq} = (M_{23},M_{22},M_{12}) = \left( \begin{array}{cc} m_{11} & m_{12} \\ m_{21} & m_{22} \end{array} \right) \]
\[ (2) \]

and

\[ S_{c} = \frac{1}{m_{22}} \left( \begin{array}{cc} Det(M_{nct}) & m_{12} \\ -m_{21} & 1 \end{array} \right) \]
\[ \left( \begin{array}{cc} t_{13} & r_{31} \\ r_{13} & t_{31} \end{array} \right) \]
\[ (3) \]

where \( k_2 \) is the wavenumber in the material, \( d_2 \) is the width of the thin layer, \( n_2 \) is the refraction index, \( t_{13} \) and \( r_{13} \) are the transmission and reflection coefficients for left side incidence then \( r_{31} \) and \( t_{31} \) are the reflection and transmission coefficients considering right side incidence. When we have a layer with complex refractive index, \( n_{ci} = n_i \pm i \cdot \kappa_i \), a negative imaginary part, \( \kappa_i < 0 \), provides gain and loss is obtained with \( \kappa_i > 0 \). In addition, transmission and reflection for a field impinging on the right side are given by \( t_{13} \) and \( r_{13} \) and for a left impinging field \( r_{31} \) and \( t_{31} \).

**OPTIMIZATION**

We are looking for unidirectional reflectionless \[ 10 \ 11 \], that is a reflection that is null in one direction but not in the other. For fixed refractive indexes and impinging wavelength, we can optimize the dimer thickness for reflection or transmission using the first and second numerical derivatives of their transfer matrices. Figure 4 shows numerical derivatives for the reflection and transmission coefficients of a double-layer cell with balanced gain-loss and one double-layer cell with passive-loss materials at \( \lambda = 1550 \text{[nm]} \) and normal incidence, \( \theta_i = 0 \).

The parameters used for the ideal cell were \( n_1 = n_2 = 1.548 \), of silicon dioxide (SiO2) doped with either metal nanoparticles for loss (\( \kappa_2 \)) or erbium ions for gain (\( \kappa_1 \)) with \( \kappa_1 = -\kappa_2 = -0.548 \). For the passive-loss cell \( n_1 = n_2 = 1.548, \kappa_1 = 0, \kappa_2 = 0.548 \), in both cases \( n_0 = n_3 = 1 \), as we showed Fig. 3, 9. The thickness of the layers are equal, \( d_1 = d_2 = d \). We take the zeros of the first derivative that yield positive values of second derivative to find the layer thickness that yields reflectivity minima. When the electromagnetic waves propagate within a dielectric medium, the phases \( \phi \) are cumulative and depend on the refractive index \( n_{ci} \), the wavenumber \( k_0 \) and the width of the layer that in turn it depends on the wavelength \( d_1(\lambda) \). For example, the optimized thickness for reflectivity minima for the balanced bilayer are \( d=220[\text{nm}] \) and \( d=660[\text{nm}] \) at the desired wavelength of 1550[nm]. The optimized thickness for transmission minima for passive-losy cell are the \( d=146[\text{nm}] \) and \( d=200[\text{nm}] \) at the same desired wavelength. In the following we will simulate the propagation of linearly polarized electromagnetic field using these optimized thickness and compare our numerical results with finite element simulation to good agreement.

**RESULTS**

Figure 5 shows the reflectivity and transitivity coefficients for ideal gain-loss structures. Figure 5(a) shows these values for an optimized bilayer thickness of...
$d = 220 [nm]$ where we can see that reflectivity from the left-side, $R_L$, dominates over that from the right-side, $R_R$, at $\lambda = 1550 [nm]$, which is showed as a vertical line. If we wanted to use this bilayer and repeat it as unit cell in a stack, say $N = 3$, the wavelength showing a maximum difference in reflection will shift, Fig. (5b).

Thus, we have to optimize for each and every stack size to recover similar results to the $N = 1$ case, Fig. (5c) with $d = 248 [nm]$. Similarly, Fig. (5d) shows reflectivity and transmissivity values for an optimized bilayer thickness of $d = 660 [nm]$ where we can again see that reflectivity from the left-side dominates over that from the right-side. Again, a wavelength shift occurs if we increase the number of unit cells without further optimization, Fig. (5e) and when we optimized for $N=3$ with $d = 630 [nm]$ we recover results as case $N=1$, Fig. (5f).

Now, we move into a more realistic passive-loss structure. Figure (6a) shows the reflectivity and transmissivity values for an optimized bilayer thickness of $d = 146 [nm]$ where we can see now that, reflectivity from the right dominates over that from the left at $\lambda = 1550 [nm]$. An equivalent wavelength shift is induced when we stack the unit cell $N=3$ without further optimization, Fig. (6b), but this can be easily corrected with optimization for the new stack, Fig. (6c) with $d = 218 [nm]$. Furthermore, Fig. (6d) shows the reflectivity and transmissivity coefficients for passive-loss structures. There we show these values for an optimized bilayer thickness of $d = 220 [nm]$ where we can see that the reflectivity $R_R$, dominates over $R_L$ at $\lambda = 1550 [nm]$ (vertical line). If we wanted to use this bilayer and repeat it as unit cell in a stack, say $N = 3$, the wavelength showing a difference in reflection will shift, Fig. (6b). Thus, we have to optimize for each and every stack size to recover similar results to the $N = 1$ case, Fig. (6f) when $d = 230 [nm]$.

CONCLUSIONS

We have shown that optimization of thin film structures, using transfer matrix analysis, can yield optimal parameters for structures with bidirectional transmission but unidirectional reflection used as a wave rectifier (mirror of one face). That we have to expect in order to develop an optical diode. We presented two cases for two different optimal values of thin film thickness. One being the bipartite unit cell with balanced gain-loss as a one dimer with PT-symmetric and the other a passive-loss structure. Finally, we want to stress that the width of the unit cell must be optimized for the desired stack size, otherwise unidirectional reflectivity will occur at a different wavelength. We desired the resonance at $\lambda = 1550 [nm]$ for optical communications. Of course, in the more realistic passive-loss unit cell, the transmittance will decrease with the size of the stack as a result of the losses. In addition for both circumstances of unit cell, we noted the differences between unidirectional reflectivity; one case by the left side (dimer: balanced) and other by the right side (dimer: passive-loss).
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