Local $D3/D7$ $\mu$-Split SUSY, 125 GeV Higgs and Large Volume Ricci-Flat Swiss-Cheese Metrics: A Brief Review

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Abstract

In this article, we review briefly recent progress made in realizing local(ized around a mobile space-time filling $D3$-brane in) $D3/D7$ $\mu$-Split Supersymmetry in (the large volume limit of Type IIB) String Theory (compactified on Swiss-Cheese Calabi-Yau orientifolds) as well as obtaining a 125 GeV (light) Higgs in the same set up. We also discuss obtaining the geometric Kähler potential (and hence the Ricci-Flat metric) for the Swiss-Cheese Calabi-Yau in the large volume limit using the Donaldson’s algorithm and intuition from GLSM-based calculations - we present new results for Swiss-Cheese Calabi-Yau (used in the set up) metrics at points finitely away from the “big” divisor.

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1 Introduction and Review of Setup

Despite the success of Standard Model in High Energy Physics, failure of naturalness and fine tuning requirements in the Higgs Sector remain basic motivations for constructing theories beyond the Standard Model. The supersymmetric extension(s) of the Standard Model can solve the fine tuning problem in the Higgs/scalar sector, however for this one requires supersymmetric particles at TeV scale. Though it is possible to achieve gauge coupling unification and obtain a dark matter candidate, yet the existence of naturally large supersymmetric contribution to flavor changing neutral current, experimental value of electron dipole moment (EDM) for natural CP violating phase and dimension-five proton decays are serious issues that can not be solved elegantly in supersymmetric Standard Model. Also, lack of existence of light Higgs boson is one of the major tensions in MSSM. An alternative approach to SUSY was adopted by Arkani-Hamed and Dimopoulos[1] in which they argued given that fine tuning anyway seems to be required to obtain a small and positive cosmological constant (which is one of the most serious issues), one is hence also allowed to assume fine tuning in other sectors of the theory (Higgs Sector) which is a less serious issue in the string theory landscape. Their model based on high scale ($m_s \sim 10^{10}$ GeV) SUSY breaking is named as split SUSY Model. In this scenario all scalar particles acquire heavy masses except one Higgs doublet which is finely tuned to be light while fermions (possibly also gaugino and Higgsino) are light. This interesting class of model has attracted considerable attention though it abandons the primary reason for introducing supersymmetry. This scenario removes all unrealistic features of MSSM while preserves all good features (possibly gauge coupling unification and dark matter candidate). In [2] it is shown that the lightest neutralino can still be taken as a good dark matter candidate in split SUSY. Also gauge coupling unification remains inherent in split SUSY see [3]. One of the other striking feature of this model based on heavy squark masses is the issue of gluino decay discussed in [4]. Kinematically favored three body gluino decays $\tilde{g} \rightarrow \chi_i^0 \bar{q}_I q_J$ or $\tilde{g} \rightarrow \chi_i^{\pm} \bar{q}_I q_J$ (where $\chi_i^0, \chi_i^{\pm}$ correspond to neutralinos and charginos, $q_{I,J}, \bar{q}_{I,J}$ correspond to quarks and antiquarks) occurring via virtual squarks get considerably suppressed due to heavy squark masses and hence gluino remains long lived. Therefore measuring life time of gluino can be adopted as indirect way to measure heavy squark mass i.e limit of SUSY breaking scale in split SUSY scenario.

Despite explaining many unresolved issues of phenomenology in the context of split SUSY, the notorious $\mu$ problem still remains unsolved according to which the stable vacuum that spontaneously breaks electroweak symmetry requires $\mu$ to be of the order of supersymmetry breaking scale. However in case of split SUSY scenario one is assuming $\mu$ to be light while supersymmetry breaking scale to be very high. The other alternative to solve the $\mu$ problem has been discussed by authors in [5] in which they introduce a further split in the split SUSY scenario by raising the $\mu$ parameter to a large value which could be about the same as the sfermion mass or the SUSY breaking scale; this scenario is dubbed as $\mu$-split SUSY scenario. In addition to solving the $\mu$ problem, all the nice features of split supersymmetric model like gauge coupling unification, dark matter candidate remain protected in this scenario.

With the promising approach of string theory to phenomenology as well as cosmology, it is quite interesting to realize the split SUSY scenario within a string theoretic framework. The signatures of the same in the context of type I and type IIA string theory were obtained respectively in [6] and [7]. Recently, in the context of type IIB (“big divisor”) LVS $D3/D7$ Swiss cheese phenomenology, the authors of [8] explicitly showed the possibility of generating light fermion masses as well as heavy squark/sleptons masses including a space-time filling mobile $D3$ brane and stack(s) of (fluxed) $D7$- branes wrapping the “Big” divisor. Matter fields (quarks, leptons and their superpartners) are identified with the (fermionic superpartners of) Wilson line moduli whereas Higgses are identified with space-time filling mobile $D3$-brane position moduli.

In the remainder of this section, we next briefly describe our setup: type IIB compactification on the orientifold of a “Swiss-Cheese Calabi-Yau” in the large volume limit including perturbative $\alpha'$ and world
sheet instanton corrections to the Kähler potential, and the instanton-generated superpotential written out respecting the (subgroup, under orientifolding, of) $SL(2, \mathbb{Z})$ symmetry of the underlying parent type IIB theory, localized around a mobile space-time filling $D3$--brane “restricted” to stacks of $D7$-branes wrapping the “big” divisor along with magnetic fluxes. This is followed by a summary of evaluation of soft supersymmetry breaking parameters, showing that one obtains large open string moduli masses; based on the Yukawas calculated and summarized in Table 1, one conjectures that the $D3$-brane position moduli could be identified with the Higgses and the Wilson line moduli with the first two generations’ squarks/sleptons.

In [9, 10], we addressed some cosmological issues like $dS$ realization, embedding inflationary scenarios and realizing non-trivial non-Gaussianities in the context of type IIB Swiss-Cheese Calabi Yau orientifold in LVS. This has been done with the inclusion of (non-)perturbative $\alpha'$-corrections to the Kähler potential and non-perturbative instanton contribution to the superpotential. The Swiss-Cheese Calabi Yau we are using, is a projective variety in $\text{WCP}^4[1,1,1,6,9]$ given as

$$x_1^{18} + x_2^{18} + x_3^{18} + x_4^2 + x_5^2 - 18\psi\prod_{i=1}^5 x_i - 3\phi x_1^6 x_2^6 x_3^6 = 0,$$

(1)

which has two (big and small) divisors $\Sigma_B(x_5 = 0)$ and $\Sigma_L(x_4 = 0)$[11]. From Sen’s orientifold-limit-of-F-theory point of view corresponding to type IIB compactified on a Calabi-Yau three fold $Z$-orientifold with $O3/O7$ planes, one requires a Calabi-Yau four-fold $X_4$ elliptically fibered (with projection $\pi$) over a 3-fold $B_3 (\equiv CY_3$--orientifold) where $B_3$ is taken be an $n$-twisted $\text{CP}^1$-fibration over $\text{CP}^2$ such that pull-back of the divisors in $CY_3$ automatically satisfy Witten’s unit-arithmetic genus condition. For $n = 6$ [12], the $CY_4$ will be the resolution of a Weierstrass model with $D_4$ singularity along the first section and an $E_6/7/8$ singularity along the second section. The Calabi-Yau three-fold $Z$ then turns out to be a unique Swiss-Cheese Calabi Yau in $\text{WCP}^4[1,1,1,6,9]$ given by (1). We would be assuming an $E_8$-singularity as this corresponds to $h^{1,1}(CY_4) = h^{2,1}(CY_4) \neq 0$[11] which is what we will be needing and using. The required Calabi-Yau has $h^{1,1} = 2, h^{2,1} = 272$. The same has a large discrete symmetry group given by $\Gamma = \mathbb{Z}_6 \times \mathbb{Z}_{18}$ (as mentioned in [13]) relevant to construction of the mirror a la Greene-Plesser prescription. However, as is common in such calculations, one assumes that one is working with a subset of periods of $\Gamma$-invariant cycles - the six periods corresponding to the two complex structure deformations in (1) will coincide with the six periods of the mirror - the complex structure moduli absent in (1) will appear only at a higher order in the superpotential because of $\Gamma$-invariance and can be consistently set to zero.

As shown in [13], in order to support MSSM (-like) models and for resolving the tension between LVS cosmology and LVS phenomenology within a string theoretic setup, a mobile space-time filling $D3$--brane and stacks of $D7$-branes wrapping the “big” divisor $\Sigma_B$ along with magnetic fluxes, are included. The appropriate $\mathcal{N} = 1$ coordinates in the presence of a single $D3$-brane and a single $D7$-brane wrapping the big divisor $\Sigma_B$ along with $D7$-brane fluxes were obtained in [14]; the same along with the details of the holomorphic isometric involution involved in orientifolding, as well as expansion of the complete Kähler potential (including the geometric Kähler potential) and the (non-perturbative) superpotential as a power series in fluctuations about Higgses’ vevs and the corresponding extremum values of the Wilson line moduli, have been summarized in [8].

In Large Volume Scenarios, one considers four stacks of different numbers of multiple $D7$-branes wrapping $\Sigma_B$ but with different choices of magnetic $U(1)$ fluxes turned on, on the two-cycles which are non-trivial in the Homology of $\Sigma_B$ and not the ambient Swiss Cheese Calabi-Yau. By turning on different $U(1)$ fluxes on, e.g., the $3_{QCD} + 2_{EW}$ $D7$-brane stacks in the LVS setup, $U(3_{QCD} + 2_{EW})$ is broken down to $U(3_{QCD}) \times U(2_{EW})$ and the four-dimensional Wilson line moduli $a_I (=1,...,h^{0,1}(\Sigma_B))$ and their fermionic superpartners $\chi^i$ that are valued, e.g., in the $adj (U(3_{QCD} + 2_{EW}))$ to begin with, decompose into the bifundamentals $(3_{QCD}, 2_{EW})$. 

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and its complex conjugate, corresponding to the bifundamental left-handed quarks of the Standard Model (See [15]). The inverse gauge coupling constant squared for the \( j \)-th gauge group \( (j : SU(3), SU(2), U(1)) \), up to open string one-loop level, using [16, 17, 18], will be given by

\[
\frac{1}{g_j^2} = Re(T_{S/B}) + \ln \left( P(\Sigma S) + D_3|_{S_B} \right) + \ln \left( P(\Sigma S) + D_3|_{S_B} \right) + O \left( U(1) - \text{Flux}_j \right),
\]

(2)

where \( U(1) - \text{Flux}_j \) are abelian magnetic fluxes for the \( j \)-th stack. Also, \( P(\Sigma S) + D_3|_{S_B} \) implies the defining hypersurface for the small divisor \( \Sigma S \) written out in terms of the position moduli of the mobile \( D3 \)-brane, restricted to the big divisor \( \Sigma B \). Further, the main idea then behind realizing \( O(1) \) gauge coupling is the competing contribution as compared to the volume of the big divisor \( \Sigma B \) to the gauge kinetic function (and hence to the gauge coupling) coming from the \( D7 \)-brane Wilson line moduli contribution \( c_I j a_I a_I^I \) where the intersection matrix \( c_{IJ} = \int_{\Sigma_B} i^* \omega_B \wedge A_i \wedge A_J \) (the immersion map \( i \) being defined as \( i : \Sigma^B \rightarrow CY_3 \)) and \( \omega_B \in H^{1,1}_+ \) - the Poincare-dual of \( \Sigma_B \), i.e., \( \omega_B = \delta(P_{\Sigma_B})dP_{\Sigma_B} \wedge \delta(P_{\Sigma_B})dP_{\Sigma_B} \) (See [11]) - noting that \( i^* \delta dz_3 \sim \frac{\phi z^6_1 z^2_2(z_2 d_3 + z_1 d_2) - (z^i_1 d_3 + z^i_2 d_2)}{(\phi z^6_1 z^2_2 - z_i^i d_3 - z_i^i d_2)^2} \) (implying \( dz_3 \sim V_{\mathbb{H}} \wedge (d_3 z_2) \), is given by: \( \omega_B \sim \frac{V_{\mathbb{H}}}{2\pi i}(dz_3 \wedge (d_3 z_2)) \). After constructing the following local (i.e. localized around the location of the mobile \( D3 \)-brane in the Calabi-Yau) appropriate involutively-odd harmonic distribution one-forms on the big divisor that lie in \( \text{coker} \left( H^{(0,1)}_{\partial, -} (CY_3) \rightarrow H^{(0,1)}_{\partial, -} (\Sigma_B) \right) \):

\[
A_I \sim \delta \left( |z_3| - V^{\frac{1}{2}} \right) \delta \left( |z_1| - V^{\frac{1}{2}} \right) \delta \left( |z_2| - V^{\frac{1}{2}} \right) \left\{ \omega_I(z_1, z_2) dz_1 + \tilde{\omega}_I(z_1, z_2) dz_2 \right\},
\]

(3)

where \( \omega(z_1, z_2, \tilde{\omega}(z_1, z_2) = -\tilde{\omega}(z_1, z_2) \) and \( \partial_1 \omega = -\partial_2 \tilde{\omega} \) (for the large volume holomorphic isometric involution \( \sigma : z_1 \rightarrow -z_1, z_2, 3 \rightarrow z_2, 3 \)); one obtains (See [13, 8]):

\[
A_1(z_1, z_2, z_3 \sim V^{\frac{1}{2}}) \sim -z_1^{18} z_2^{19} d_1 + z_1^{19} z_2^{18} d_2,
\]

\[
A_2(z_1, z_2, z_3 \sim V^{\frac{1}{2}}) \sim -\left( z_1^{19} + z_2^{18} \right) d_1 + \left( z_1^{19} + z_2^{18} z_1 \right) d_2.
\]

(4)

This also involves stabilization of the Wilson line moduli at around \( V^{-\frac{2}{3}} \) and the \( D3 \)-brane position moduli, the Higgses in our setup, at around \( V^{\frac{1}{3}} \); extremization of the \( N = 1 \) potential, as shown in [13] and mentioned earlier on, shows that this is indeed true. This way the gauge couplings corresponding to the gauge theories living on stacks of \( D7 \) branes wrapping the “big” divisor \( \Sigma B \) (with different \( U(1) \) fluxes on the two-cycles inherited from \( \Sigma_B \)) will be given by: \( g_{YM}^{-2} \sim V^{\frac{1}{3}}, T_B \) being the appropriate \( N = 1 \) Kähler coordinate and the relevant text below the same) and \( \mu_3 \) related to the \( D3 \)-brane tension, implying a finite \( (\mathcal{O}(1)) \) \( g_{YM} \) for \( V \sim 10^6 \). In the dilute flux approximation, the “\( \ln \)” terms in the right hand side of (2) are \( \mathcal{O}(\ln V) \), which for \( V \sim 10^6 \) is taken to be of the same order as \( \sigma^B \) (Big divisor’s volume complexified by four-form axion) \( + \sigma^B - C_I j a_I a_I^J \sim V^{\frac{1}{3} + \frac{1}{2}} \) appearing in \( Re(T_B) \). In the dilute flux approximation, \( \alpha_i(M_s)/\alpha_i(M_{EW}), i = SU(3), SU(2), U(1) \gamma_i \), are hence unified.

As discussed in [19], for the type IIB Swiss-Cheese orientifold considered in our work, guided, e.g.,
by the vanishingly small Yukawa couplings $\hat{Y}_{\hat{A}} Z_i \equiv \frac{e^{\frac{K}{4}} Y_{\hat{A}} Z_i}{\sqrt{(K_{\hat{A}, \hat{A}})^2 K_{Z_i Z_i}}}^2$ obtained from an $ED3$-instanton-generated superpotential (See Table 1 for the single Wilson line modulus case), the spacetime filling mobile $D3$-brane position moduli $z_i$’s and the Wilson line moduli $a_I$’s could be respectively identified with Higgses and the either of the first two generations of sparticles (squarks/sleptons) of some (MS)SM-like model. With a (partial) cancelation between the volume of the “big” divisor and the Wilson line contribution (required for realizing $\sim O(1)g_{YM}$ in our setup), in [13], we calculated in the large volume limit, several soft supersymmetry breaking parameters. The same relevant to this review are summarized in table 1.

| Gravitino mass | $m_3 \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |
|---------------|-------------------------------------|
| Gaugino mass   | $M_2 \sim m_3$                      |
| $D3$-brane position moduli (Higgs mass) | $m_{Z, i} \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |
| Wilson line moduli mass | $m_{\hat{A}} \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |
| A-terms        | $A_{pqr} \sim n^a \mathcal{V}^{-\frac{8}{3}} m_3$ |
|                | $\{ p, q, r \} \in \{ \hat{A}_1, \hat{Z}_1 \}$ |
| Physical $\mu$-terms | $\hat{\mu}_{Z_i Z_j} \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |
|                | $\hat{\mu}_{\hat{A}_1 Z_i} \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |
| Physical Yukawa couplings | $\hat{Y}_{Z_i Z_j \hat{A}_1} \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |
|                | $\hat{Y}_{\hat{A}_1 Z_i} \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |
|                | $\hat{Y}_{Z_i \hat{A}_1 \hat{A}_1} \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |
| Physical $\hat{\mu} B$-terms | $(\hat{\mu} B)_{Z_i Z_j} \sim \mathcal{V}^{-\frac{8}{3}} m_3$ |

Table 1: Results on Soft SUSY Parameters Summarized

2 Obtaining Big Divisor $D3/D7$ $\mu$-Split SUSY as well as 125 GeV Higgs

In split supersymmetry scenario, SUSY breaking scale is high. However, in order to get one light Higgs doublet at EW scale in this scenario, one needs these soft terms to be of $TeV$ order. Since fine tuning is allowed one can assume $\hat{\mu} B \sim m_{\hat{A}_1}^2$ (where $m_{\hat{A}_1}$’s correspond to squark/slepton masses scale which is of the order of high supersymmetry breaking scale as in case of split SUSY, and $\hat{\mu}_{Z_i Z_j}$ is the Higgsino mass parameter). As Higgsino mass contribution ($\hat{\mu}_{Z_i Z_j}$ parameter) is small in most of split SUSY models, one needs $B >> \hat{\mu}_{Z_i Z_j}$ in order to have $\hat{\mu} B \sim m_{\hat{A}_1}^2$. In an alternate approach to split SUSY scenario called “$\mu$-split SUSY scenario” [5], according to which one can assume even $\hat{\mu} \sim m_{\hat{A}_1} \sim B$ i.e large $\mu$ parameter to get $\hat{\mu} B \sim m_{\hat{A}_1}^2$, this choice appears more natural and also helps to alleviate the “$\mu$ problem”; see also [20]. In the single-Wilson-line modulus Large Volume Scenarios set up discussed earlier, values of $\hat{\mu}$ and

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2The Yukawa couplings, $K_{C_i C_j}$, etc. are defined by taking derivatives of the superpotential and Kähler potential with respect to fluctuations $C_{i, 1} = \delta Z_i, \delta \hat{A}_1$ in the open string moduli (about non-zero vevs) written in the basis: $\delta Z_i = \delta z_i + O(1) e^{-\frac{\phi}{2}} \delta \hat{A}_1, \delta \hat{A}_1 = \delta \hat{A}_1 + V^{-\frac{\phi}{2}} (O(1) \delta z_1 + O(1) \delta z_2)$, which diagonalizes $K_{C_i C_j}$ [13].
B terms pertaining to SUSY breaking parameters has been summarized in results in[13], which are of the order $\hat{\mu}^2 \sim \tilde{\mu} B \sim m_{A_1}^2$ (scalar masses) i.e $\hat{\mu} \sim B \sim m_{A_1}$ as in case of $\mu$ split SUSY.

In this section, we demonstrate the possibility to realize $\mu$-Split SUSY in the framework summarized in section 1. We do so by first summarizing our results of [13, 8, 21] wherein we had shown that we obtain very heavy squarks/sleptons, heavy Higgsino mass parameter, light fermions and one light Higgs whose mass could be fine tuned to the desirable 125GeV.

2.1 Generation of $\mu$-Split SUSY Mass Scales for Scalars and Fermions

Fermion (Quark/Lepton) masses are generated by giving some VEVs to Higgses in $\int d^4x \ e^{\hat{K}/2} Y_{ijk} \bar{z}^i \psi^j \psi^k$. The (canonically normalized) fermionic mass matrix is generated by $\hat{Y}_{ijk} < z_i$. For the single Wilson line modulus case, the mass of the fermionic superpartner of $\tilde{A}_1$, as shown in [8] (which based on the near-vanishing value of the Yukawa coupling $\hat{Y}_{ij} \tilde{Z}_i$ in Table 1, is conjectured to be a first/second generation quark/lepton), turns out to be given by: $\nu \sim \frac{10^2}{v_{X}} e^{-n_{\pi}}$ in units of $\mathcal{M}_\nu$, which implies a range of fermion mass $m_{\text{ferm}} \sim \mathcal{O}(\text{MeV} - \text{GeV})$ for Calabi Yau volume $\mathcal{V} \sim \mathcal{O}(6 \times 10^5 - 10^7)$. Interestingly, the mass-scale of 0.5 MeV- the electronic mass scale- could be realized with $\mathcal{V} \sim 6.2 \times 10^5$, $m^\beta = 2$. In MSSM/2HDM models, up to one loop, the leptonical (quark) masses do not change (appreciably) under an RG flow from the intermediate string scale down to the EW scale (See [22]).

The non-zero neutrino masses are generated through the Weinberg(-type) dimension-five operators written out schematically as: $\int d^4x \int d^2\theta e^{\hat{K}/2} \times \left( \tilde{Z}_i \mathcal{A}_1^2 \in \frac{\partial^2 \tilde{Z}_i}{\partial z_i^2} \mathcal{A}_1^2 \right)$, and is given as: $m_{\nu} = v^2 \sin^2 \beta \hat{O}_{Z_i z_i} z_i z_i / 2 M_p$

where $\hat{O}_{Z_i z_i} z_i z_i \equiv \text{coefficient of the physical/normalized quartic in } Z_i$ in the superpotential, and is given as $\hat{O}_{Z_i z_i} z_i z_i = \frac{\sqrt{K_{Z_i Z_i} K_{Z_j Z_j} K_{Z_k Z_k}}}{\sqrt{K_{Z_i Z_i} K_{Z_j Z_j} K_{Z_k Z_k}}} [23]$, $v \sin \beta \equiv \langle H_u \rangle$ and $\sin \beta$ is defined via $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$.

in our setup (See [8]): $O_{Z_i z_i} z_i z_i \sim \frac{2^q}{\pi^2} \times \frac{2^q}{\pi^2} \mu_3 n^s (2 \pi \alpha')^2 \nu \frac{\sigma}{\mathcal{V}} \frac{1}{\sqrt{\mathcal{V}}} \epsilon^{(a+i\beta)}$. Now, $z_i \sim \alpha_1 \nu \frac{\sigma}{\mathcal{V}}$, $i = 1, 2; \beta \sim \alpha_1 \alpha_2$ and $\text{vol}(\Sigma_S) = \gamma_3 \text{ln} \mathcal{V}$ such that $\gamma_3 \text{ln} \mathcal{V} + \mu_3 \beta \mathcal{V} \frac{1}{\sqrt{\mathcal{V}}} = \text{ln} \mathcal{V}$, along with $K_{Z_i Z_i} \sim \frac{\mathcal{V}^{1/2}}{\sqrt{\Sigma_{\beta} n_{\beta}}}$ and the assumption that the holomorphic isometric involution $\sigma$ as part of the Swiss-Cheese orientifolding action $(-)^{F_L} \Omega \cdot \sigma$ is such that $\sum_{i=1}^{2} \nu_i \mathcal{V}^{1/2}$. By analyzmg the RG running of coefficient $\kappa_{ij}$ of dimension-five operator $\kappa_{ij} L_i H L_j H$ and $\langle H_u \rangle$, it was shown in [8] that one can generate a neutrino mass of $\lesssim 1 \text{eV}$ in our setup.

We now summarize our calculations in [21] wherein we had shown that the eigenvalues of the Higgs mass matrix at the EW scale obtained from the solutions to the one-loop RG flow equations assuming non-universality in the open string moduli masses, results in an eigenvalue corresponding to the mass-squared of one of the Higgs doublet to be negative and small and the other to be large and positive with a heavy Higgsino (in addition to heavy squarks/sleptons and light quarks/leptons already demonstrated in [13, 19, 8]) implying the existence of $D3/D7 \ \mu$-Split LVS.

Due to lack of universality in moduli masses but universality in trilinear $A_{ijk}$ couplings, we need to use solution of RG flow equation for moduli masses as given in [24]. From [13], it was shown in [21] that:

$$m_{Z_1}^2 (M_{EW}) \sim m_{Z_1}^2 (M_*) + (0.39) m_{3/2}^2 + \frac{1}{22} \times \frac{19\pi}{100} \times S_0,$$

where $S_0 = Tr(Y m^2) = m_{Z_2}^2 - m_{Z_1}^2 + \sum_{i=1}^{n_{g}} (m_{qL}^2 - 2 m_{uR}^2 + m_{dR}^2 - m_{lL}^2 + m_{eR}^2)$ in which all the masses are
at the string scale and $n_g$ is the number of generations;

$$m^2_{Z_2}(M_{EW}) \sim m_0^2 \delta_2 + (0.32)m^2_{3/2} + (-0.03)n^8 \hat{\mu}_{Z_1 Z_2} m^3_{3/2} + (0.96)m^2_0 - (0.01)(n^8)^2 \hat{\mu}_{Z_1 Z_2} - \frac{19\pi}{2200} \times S_0, \quad (6)$$

where we used $A_{Z,2,3} \sim n^8 \hat{\mu}_{Z_1 Z_2}$ (See [13]). The solution for RG flow equation for $\hat{\mu}^2$ to one loop order is given by [24]:

$$\hat{\mu}^2_{Z_1 Z_2} = - \left[ m_0^2 C_1 + A_0^2 C_2 + m^2_{2} C_3 + m_A^2 C_4 - \frac{1}{2} M^2_{Z_2} + \frac{19\pi}{2200} \left( \frac{\tan^2 \beta + 1}{\tan^2 \beta - 1} \right) S_0 \right], \quad (7)$$

where $C_{1,2,3,4}$ are as given in [24]. The overall minus sign on the right hand side of (7) indicates that our $\hat{\mu}^2_{Z_1 Z_2}$ is negative of $\mu^2$ [24]. In the large $\tan \beta$ (but less than 50)-limit one sees that:

$$\hat{\mu}^2_{Z_1 Z_2} \sim - \left[ \left( \frac{1}{2} + \frac{O(10^3)}{2} \right) m_0^2 - (0.01)(n^8)^2 \hat{\mu}_{Z_1 Z_2} + (0.32)m^2_{3/2} - 1/2 M^2_{EW} + (0.03)n^8 \hat{\mu}_{Z_1 Z_2} m^3_{3/2} + \frac{19\pi}{2200} S_0 \right]. \quad (8)$$

From (6) and (8) one therefore sees that the mass-squared of one of the two Higgs doublets, $m^2_{H_2}$, at the $EW$ scale is given by:

$$m^2_{H_2} = m^2_{Z_2} + \hat{\mu}^2_{Z_1 Z_2} = \left( \frac{1}{2} - \frac{O(10^3)}{2} \right) m_0^2 - (0.06)n^8 \hat{\mu}_{Z_1 Z_2} m^3_{3/2} + \frac{1}{2} M^2_{EW} - \frac{19\pi}{1100} S_0. \quad (9)$$

From [13], we notice: $\hat{\mu}_{Z_1 Z_2} m^3_{3/2} \sim m^2_{Z_1}$, using which in (9), one sees that for an $O(1)$ $n^8$,

$$m^2_{H_2}(M_{EW}) \sim \frac{1}{2} M^2_{EW} - \frac{19\pi}{1100} S_0 - \frac{O(10^3)}{2} \gamma m^3_{3/2}. \quad (10)$$

We have assumed at $m_Z (M_s) = m_{Z_2}(M_s)$. So, $S_0 \approx m^2_{s\text{squark/slepton}}$, which in our setup could be of $O(\hat{\mu}^2)$. Further,

$$m^2_{H_1}(M_{EW}) = (m^2_{Z_1} + \hat{\mu}^2_{Z_1 Z_2})(M_{EW}) \sim m^2_{Z_1}(M_s) + \frac{1}{2} M^2_{EW} + (0.01)(n^8)^2 \gamma^2 m^3_{3/2}. \quad (11)$$

In the results on Soft SUSY Parameters summarized in Table 1, one finds that $\hat{\mu} B \sim \hat{\mu}^2$ at the string scale. By assuming the same to be valid at the string and $EW$ scales, the Higgs mass matrix at the $EW$-scale can thus be expressed as:

$$\left( \begin{array}{cc} m^2_{H_1} & \hat{\mu} B \\ \hat{\mu} B & m^2_{H_2} \end{array} \right) \sim \left( \begin{array}{cc} m^2_{H_1} & \xi \hat{\mu}^2 \\ \xi \hat{\mu}^2 & m^2_{H_2} \end{array} \right). \quad (12)$$

The eigenvalues are given by:

$$\frac{1}{2} \left( m^2_{H_1} + m^2_{H_2} \pm \sqrt{\left( m^2_{H_1} - m^2_{H_2} \right)^2 + 4 \xi^2 \hat{\mu}^4} \right). \quad (13)$$

As (for $O(1)$ $n^8$)

$$m^2_{H_1} + m^2_{H_2} \sim 0.01 \gamma^2 m^3_{3/2} - 0.06 S_0 + ..., \quad (14)$$

$$m^2_{H_1} - m^2_{H_2} \sim 0.01 \gamma^2 m^3_{3/2} + 0.06 S_0 + ..., \quad (14)$$

$$\hat{\mu}^2_{Z_1 Z_2} \sim 0.01 \gamma^2 m^3_{3/2} - 0.03 S_0 + ..., \quad (14)$$
one sees that the eigenvalues are:

\[
0.01V^2m_{3/2}^2 - 0.06S_0 + \ldots \pm \sqrt{\left(0.01V^2m_{3/2}^2 + 0.06S_0 + \ldots\right)^2 + \xi^2 \left(0.02V^2m_{3/2}^2 - 0.06S_0\right)^2}.
\]

(15)

Hence, assuming a universality w.r.t. to the D3-brane position moduli masses \(m_{Z_1, Z_2}\) and lack of the same for the squark/slepton masses, if \(S_0\) and \(\xi\) are fine tuned as follows:

\[
0.01V^2m_{3/2}^2 \sim -0.06S_0 \quad \text{and} \quad \xi \sim \frac{2}{3} + \mathcal{O}(10) \left(\frac{m_{EW}^2}{m_{3/2}^2}\right),
\]

(16)

one sees that one obtains one light Higgs doublet (corresponding to the negative sign of the square root) with a mass of about 125 GeV and one heavy Higgs doublet (corresponding to the positive sign of the square root). Note, however, the Higgsino mass parameter \(\hat{\mu}\) then turns out to be heavy with a value, at the EW scale of around 0.01V\(m_{3/2}\) i.e to the order of squark/slepton mass squared scale which is possible in case of \(\mu\) split SUSY scenario discussed above. This shows the possibility of realizing \(\mu\) split SUSY scenario in the context of LVS phenomenology named as large volume “\(\mu\)-split SUSY” scenario.

2.2 Obtaining Long-Lived Gluinos

The most distinctive feature of split SUSY, decisively differentiating it from the usual Supersymmetric Standard Model, is based on longevity of the gluinos. Since the squarks which mediate its decay are extremely heavy, one expects life time of Gluinos to be high. The decay amplitudes for the three-body tree-level and two-body one-loop diagrams of Fig.1 were evaluated in [21] by considering the contribution of relevant terms in gauged supergravity action of Wess and Bagger [25] given below:

\[
\mathcal{L} = g_{YM}g_{\sigma_B}J X^{\sigma_B} \tilde{\chi}^I \tilde{\lambda}^i + i \sqrt{g} g_{1j} \tilde{\chi}^I \tilde{\sigma}^\mu \nabla_\mu \chi^I
\]

\[
+ \frac{e}{2} (D_i D_j W) \chi^i \chi^j + g_{YM} g_{1j} \tilde{\chi}^I \tilde{\sigma}^\mu A \text{Im} (X^{\sigma_B} K + i D^{\sigma_B}) \chi^I;
\]

(17)

\(W\) is the superpotential as defined in [13, 8, 21], \(\sigma_B\) is the complexified (by four-form axions) big divisor volume, \(\chi/\tilde{\chi}, \lambda/\tilde{\lambda}\) correspond to quarks/antiquarks and gaugino’s and \(X^{\sigma_B} = -6i\kappa^2_3 \mu_7 Q_B\), where \(Q_B = 2\pi\alpha' \int_{\Sigma_B} i^* \omega_b \wedge P_- \tilde{f}\) where \(P_-\) is a harmonic zero-form on \(\Sigma_B\) taking value +1 on \(\Sigma_B\) and -1 on \(\sigma(\Sigma_B)\) - \(\sigma\) being a holomorphic isometric involution as part of the Calabi-Yau orientifold - and \(\tilde{f} \in \tilde{H}^2(\Sigma_B) \equiv \text{coker} \left( H^2(CY_3) \rightarrow H^2(\Sigma_B) \right) ;\) \(D^{\sigma_B} = \frac{4\pi\alpha' \kappa^2_3 \mu_7 Q_B}{\sqrt{v}}\). It is understood that the D3-brane position moduli are indexed by \(i\) and the Wilson line moduli are indexed by \(I = 1, \ldots, h_0^1(\Sigma_B)\).
In principle, due to the presence of a mobile 3-brane, one must also include the geometric Kähler potential $K_{\text{geom}}$ of the Swiss-Cheese Calabi-Yau in the moduli space Kähler potential. In [13], given that we had restricted the mobile 3-brane to $\Sigma_B$, one had estimated (in the large volume limit) $K_{\text{geom}} \sim \frac{\mu}{\sqrt{m\phi}}$ summarized as follows. Using GLSM techniques and the toric data for the given Swiss-Cheese Calabi-Yau, the geometric Kähler potential for the divisor $\Sigma_B$ (and $\Sigma_S$) in the LVS limit was evaluated in [13] in terms of derivatives of genus-two Siegel theta functions as well as two Fayet-Iliopoulos parameters corresponding to the two $C^*$ actions in the two-dimensional $\mathcal{N} = 2$ supersymmetric gauge theory whose target space is our toric variety Calabi-Yau, and a parameter $\zeta$ encoding the information about the $D3$–brane position.
moduli-independent (in the LVS limit) period matrix of the hyperelliptic curve \( w^2 = P(z) \), \( P(z) \) being the sextic in the exponential of the vector superfields eliminated as auxiliary fields, corresponding to \( \Sigma_B \). To be a bit more specific, one can show that upon elimination of the vector superfield (in the IR limit of the GLSM), one obtains an octic in \( e^{2V_2} \), \( V_2 \) being one of the two real gauge superfields. Using Umemura’s result [27] on expressing the roots of an algebraic polynomial of degree \( n \) in terms of Siegel theta functions of genus \( g (> 1) = \left( \frac{n+2}{2} \right) \) :

\[
\theta \left[ \begin{array}{c}
\mu \\
\nu
\end{array} \right] (z, \Omega) = \sum_{n \in \mathbb{Z}^g} e^{i\pi(n+\mu)^T \Omega(n+\mu)+2i\pi(n+\mu)^T(z+\nu)}.
\]

Hence for an octic, one needs to use Siegel theta functions of genus five. The period matrix \( \Omega \) will be defined as follows:

\[
\Omega_{ij} = (\sigma_{ik})^{-1} \rho_{kj}
\]

where

\[
\sigma_{ij} \equiv \oint_{A_j} dz \frac{z^{i-1}}{\sqrt{z(z-1)(z-2)P(z)}}
\]

and

\[
\rho_{ij} \equiv \oint_{B_j} \frac{z^{i-1}}{\sqrt{z(z-1)(z-2)P(z)}}.
\]

\( \{A_i\} \) and \( \{B_i\} \) being a canonical basis of cycles satisfying: \( A_i \cdot A_j = B_i \cdot B_j = 0 \) and \( A_i \cdot B_j = \delta_{ij} \). Umemura’s result then is that a root:

\[
\frac{1}{2} \left( \theta \left[ \begin{array}{c}
\frac{1}{2} \\
0
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4 \left( \theta \left[ \begin{array}{c}
\frac{1}{2} \\
0
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4 \left( \theta \left[ \begin{array}{c}
\frac{1}{2} \\
\frac{1}{2}
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4 \left( \theta \left[ \begin{array}{c}
\frac{1}{2} \\
\frac{1}{2}
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4
\]

\[
\times \left( \theta \left[ \begin{array}{c}
0 \\
0
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4 \left( \theta \left[ \begin{array}{c}
0 \\
0
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4
\]

\[
+ \left( \theta \left[ \begin{array}{c}
\frac{1}{2} \\
\frac{1}{2}
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4 \left( \theta \left[ \begin{array}{c}
\frac{1}{2} \\
\frac{1}{2}
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4
\]

\[
- \left( \theta \left[ \begin{array}{c}
0 \\
\frac{1}{2}
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4 \left( \theta \left[ \begin{array}{c}
0 \\
\frac{1}{2}
\end{array} \right] 0 0 0 0 0 0 0 (0, \Omega) \right)^4
\]

In the LVS limit, the octic reduces to a sextic. Umemura’s result would require the use of genus-four Siegel theta functions. However, using the results of [28], one can express the roots of a sextic in terms of derivatives of genus-two Siegel theta functions as follows:

\[
\begin{bmatrix}
\sigma_{22} d_{dz_1} \theta \left[ \begin{array}{c}
\frac{1}{2} \\
0
\end{array} \right] ((z_1, z_2), \Omega) - \sigma_{21} d_{dz_2} \theta \left[ \begin{array}{c}
\frac{1}{2} \\
0
\end{array} \right] ((z_1, z_2), \Omega) \\
\sigma_{12} d_{dz_1} \theta \left[ \begin{array}{c}
\frac{1}{2} \\
0
\end{array} \right] ((z_1, z_2), \Omega) - \sigma_{12} d_{dz_2} \theta \left[ \begin{array}{c}
\frac{1}{2} \\
0
\end{array} \right] ((z_1, z_2), \Omega)
\end{bmatrix}_{z_1 = z_2 = 0}
\]

9
etc. The symmetric period matrix corresponding to the hyperelliptic curve \( w^2 = P(z) \) is given by:

\[
\begin{pmatrix}
Ω_{11} & Ω_{12} \\
Ω_{12} & Ω_{22}
\end{pmatrix} = \frac{1}{σ_{11}σ_{22} - σ_{12}σ_{21}} \begin{pmatrix}
σ_{22} & -σ_{12} \\
-σ_{21} & σ_{11}
\end{pmatrix} \begin{pmatrix}
ρ_{11} & ρ_{12} \\
ρ_{21} & ρ_{22}
\end{pmatrix},
\]

where \( σ_{ij} = \int_{A_i} \frac{dz}{\sqrt{P(z)}} \) and \( ρ_{ij} = \int_{B_j} \frac{dz}{\sqrt{P(z)}} \) where \( z \) maps the \( A_i \) and \( B_j \) cycles to the \( z \)-plane. The geometric Kähler potential for the divisor \( Σ_B \) in the LVS limit, as shown in [13], then turns out to be given by:

\[
K|_{Σ_B} \sim r_2 - \left[ r_2 - (1 + |z_1|^2 + |z_2|^2) \left( \frac{ζ}{r_1|z_3|^2} \right) \right] \frac{4}{3} \sqrt{ζ/3}
\]

\[
+ |z_3|^2 \left[ \left( r_2 - (1 + |z_1|^2 + |z_2|^2) \left( \frac{ζ}{r_1|z_3|^2} \right) \right) \sqrt{ζ/3} \frac{1}{r_1|z_3|^2} \right]^2
\]

\[
- r_1 ln \left[ \left( r_2 - (1 + |z_1|^2 + |z_2|^2) \left( \frac{ζ}{r_1|z_3|^2} \right) \right) \sqrt{ζ/3} \frac{1}{r_1|z_3|^2} \right] - r_2 ln \left[ \left( ζ/r_1|z_3|^2 \right) \frac{1}{r_1} \right]
\]

\[
\sim ζ^2 / \sqrt{ln V}.
\]

As mentioned earlier, if the space-time filling mobile D3-brane is free to explore the full Calabi-Yau, one would require the knowledge of the geometric Kähler potential of the full Calabi-Yau. We will now estimate \( K_{geom} \) using the Donaldson’s algorithm [29] and obtain a metric for the Swiss-Cheese Calabi-Yau at a generic point finitely separated from \( Σ \), that is Ricci-flat in the large volume limit; we should note that GLSM-based metrics are not expected to yield Ricci-flat metrics.

The crux of the Donaldson’s algorithm is that the sequence \[ \frac{1}{kπ} \partial_i \partial_j \left( ln \sum_{α,β} h^{αβ} s_α s_β \right) \] on \( P(\{z_i\}) \), in the \( k \to \infty \)-limit - which in practice implies \( k \sim 10 \) - converges to a unique Calabi-Yau metric for the given Kähler class and complex structure; \( h_{αβ} \) is a balanced metric on the line bundle \( O_{P(\{z_i\})}(k) \) (with sections \( s_α \)) for any \( k \geq 1 \), i.e.,

\[
T(h)_{αβ} = \frac{N_k}{\sum_{j=1}^{N_k} w_j} \sum_i s_α(p_i) s_β(p_i) w_i = h_{αβ},
\]

where the weight at point \( p_i \), \( w_i \sim \frac{i^\ast(\overline{\partial} \partial_c s_α)}{\overline{\partial} \partial_c s_α} \) with the embedding map \( i : P(\{z_i\}) \to WCP^4 \) and the number of sections is denoted by \( N_k \). The defining hypersurface of the Swiss-Cheese Calabi-Yau in the \( x_2 = 1 \)-coordinate patch in \( WCP^4[1,1,1,6,9] \) is given by: \( 1 + z_1^{18} + z_2^{18} + z_3^3 + z_4^2 - ψ z_1 z_2 z_3 z_4 - 3φ z_1^6 z_2^6 = 0 \). In the large volume limit, the above can be satisfied if, e.g., \( 1 + z_1^{18} + z_2^{18} \sim -z_3^3, z_4^2 \sim ψ z_1 z_2 z_3 z_4 + 3φ z_1^6 z_2^6 \).

For \( z_{1,2} \sim ψ \), one sees the same are satisfied for \( z_{3,4} \sim ψ \), provided \( ψ \sim 3φ \). Therefore:

\[
\begin{align*}
h_{1z_1} & \sim \frac{Ψ^{\frac{1}{6}}}{h^{\frac{3}{2}} z_1^2 Ψ^{\frac{1}{3}}} \\
h_{1z_3} & \sim \frac{Ψ^{\frac{1}{6}}}{h^{\frac{3}{2}} z_1^2 Ψ^{\frac{1}{3}}} \\
h_{1z_4} & \sim \frac{Ψ^{\frac{1}{6}}}{h^{\frac{3}{2}} z_1^2 Ψ^{\frac{1}{3}}} \\
h_{1z_2} & \sim \frac{Ψ^{\frac{1}{6}}}{h^{\frac{3}{2}} z_1^2 Ψ^{\frac{1}{3}}}, etc.,
\end{align*}
\]

(20)
which on being inverted gives:

\[
\begin{pmatrix}
  \frac{\hbar^{2/3}}{4} V_{2/3} & \hbar^{2/3} \sqrt{V} & \hbar^{2/3} V_{11/18} & \hbar^{2/3} \sqrt{V} & \hbar^{2/3} V_{17/36} \\
  \hbar^{2/3} V_{23/36} & \hbar^{2/3} V_{11/18} & \hbar^{2/3} V_{11/18} & \hbar^{2/3} V_{7/12} & \hbar^{2/3} V_{11/36} \\
  \hbar^{2/3} \sqrt{V} & \hbar^{2/3} V_{17/36} & \hbar^{2/3} V_{17/36} & \hbar^{2/3} V_{4/9} & \hbar^{2/3} V_{11/36} \\
  \hbar^{2/3} V_{11/18} & \hbar^{2/3} V_{7/12} & \hbar^{2/3} V_{4/9} & \hbar^{2/3} V_{5/9} & \hbar^{2/3} V_{5/18} \\
  \hbar^{2/3} V_{17/36} & \hbar^{2/3} V_{11/36} & \hbar^{2/3} V_{5/18} & \hbar^{2/3} V_{5/18} & \hbar^{2/3} V_{5/36}
\end{pmatrix}
\]

(21)

Using (21), one hence obtains the following Kähler potential ansatz:

\[
K = \ln \left[ \frac{h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} + h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} V^{23/36} (z_{1} + \bar{z}_{1} + z_{2} + \bar{z}_{2}) + h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} V^{11/18} (z_{1} + \bar{z}_{1} + z_{2} + \bar{z}_{2})}{(z_{1} + \bar{z}_{1} + z_{2} + \bar{z}_{2})} + h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} V^{7/12} (z_{2} z_{1} + z_{2} z_{1} + \bar{z}_{2} \bar{z}_{1} + \bar{z}_{2} \bar{z}_{1} + \bar{z}_{1} \bar{z}_{2} + \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{2}) + h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} V^{5/9} (z_{1} z_{2} + z_{1} z_{2} + z_{1} \bar{z}_{2} + \bar{z}_{1} \bar{z}_{2} + \bar{z}_{1} \bar{z}_{2} + \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{2}) + h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} V^{5/12} (z_{2} z_{1} z_{2} z_{1} + z_{2} z_{1} z_{2} z_{1} + \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} + \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} + \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} + \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{2} \bar{z}_{2} \bar{z}_{2}) + h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} V^{5/18} (z_{1} z_{2} z_{3} \bar{z}_{1} + z_{1} \bar{z}_{2} \bar{z}_{1} + \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{1} + \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} + \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{2} \bar{z}_{2} \bar{z}_{2}) + h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} V^{5/36} (z_{1} z_{2} z_{3} \bar{z}_{1} + z_{1} \bar{z}_{2} \bar{z}_{1} + \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{1} + \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} + \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{2} \bar{z}_{2} \bar{z}_{2}) + h_{z\bar{z}} z_{4} \bar{z}_{4} z_{4} \bar{z}_{4} V^{4/9} (z_{1} z_{2} z_{3} \bar{z}_{1} + z_{1} \bar{z}_{2} \bar{z}_{1} + \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{1} + \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} + \bar{z}_{1} \bar{z}_{2} \bar{z}_{1} \bar{z}_{2} + \bar{z}_{2} \bar{z}_{2} \bar{z}_{2} \bar{z}_{2}) + \sqrt{V} \right]
\]

(22)

From GLSM-based analysis, we had seen in (18) that on \( \Sigma_{B}(z_{4} = 0) \), the argument of the logarithm received the most dominant contribution from the FI-parameter \( r_{2} \sim V^{1/4} \). From (21), one sees that \( h_{11} \sim (z_{4} \bar{z}_{4})^{1/3} V^{1/3} \). For consistency, we should therefore obtain \( h_{z\bar{z}} z_{4} \bar{z}_{4} \sim V^{-1} \). This has been assumed in (22) and will be verified below to correspond to one allowed value of \( h_{z\bar{z}} z_{4} \bar{z}_{4} \) that would yield an approximately Ricci-flat metric (up to within 10%). Using (22), one can show that:

\[
R_{z\bar{z}} \sim \frac{\sum_{n=0}^{n=8} a_{n} \left( h_{z\bar{z}} z_{4} \bar{z}_{4} \right)^{n} V^{n/4}}{(1 + O(1) h_{z\bar{z}} z_{4} \bar{z}_{4} V^{1/4})^{2} V^{1/18} \left( \sum_{n=0}^{n=8} b_{n} \left( h_{z\bar{z}} z_{4} \bar{z}_{4} \right)^{n} V^{n/4} \right)^{2}}.
\]

(23)

Solving numerically: \( \sum_{n=0}^{n=8} a_{n} \left( h_{z\bar{z}} z_{4} \bar{z}_{4} \right)^{n} V^{n/4} = 0 \), as was assumed, one (of the eight values of) \( h_{z\bar{z}} z_{4} \bar{z}_{4} \), up to a trivial Kähler transformation, turns out to be \( V^{-1}, V \sim 10^{9} \). Using this value of \( h_{z\bar{z}} z_{4} \bar{z}_{4} \), one obtains: \( R_{z\bar{z}} \sim 10^{-1} \). Further, as has been assumed that the metric components \( g_{z\bar{z}} z_{4} \bar{z}_{4} \) are negligible as compared to \( g_{z\bar{z}} \) - this was used in showing the completeness of the basis spanning \( H^{1,1} \) in the large volume limit - is born out explicitly, wherein the latter turn out to about 10% of the former.

4 Conclusion and Discussion

We have reviewed recent progress in realizing \( \mu \)-split SUSY scenario localized around a mobile space-time filling \( D3 \)-brane in the context of type IIB Swiss-Cheese orientifold (involving isometric holomorphic involution) compactifications in the L(arge) V(olume) S(cenarios). Generation of very heavy scalars and
light(superpartner) fermions that had already been obtained in the context of L(arge) V(olume) S(cenario) in [13, 8] and reviewed in 1 and 2.1, was adopted as one of the signatures of split supersymmetric behavior. To see it more clearly, in [21] and reviewed in 2.1, we showed how to generate one light Higgs boson with the assumption that fine tuning is allowed in case of split SUSY models. For this, using solution of RG flow equation for the mobile $D3$-brane position moduli masses and Higgsino mass term and further assuming gauge coupling up to one loop order and non-universality in squark/slepton masses (in addition to the non-universality between the Higgs’ and squark/slepton masses), by diagonalizing the mass matrix for the Higgs doublet, we showed how one could obtain one light Higgs (about 125 GeV) and one heavy Higgs, about a tenth of the squark masses. The Higgsino also turns out to about a tenth of the squark mass. Since in our setup, $\mu$ value comes out to be of the order of squark/slepton mass scale i.e high scale, therefore we see the possibility of explicitly realizing $\mu$-split SUSY scenario which we could refer to as “Local $D3/D7$ $\mu$-split SUSY Scenario”.

The most distinctive feature of split SUSY is based on longevity of gluino. Therefore, in order to seek striking evidence of split SUSY in the context of LVS, in [21] and reviewed in 2.2, we estimated the decay width for tree-level three-body gluino decay into a quark, squark and neutralino. By constructing the neutralino mass matrix and diagonalizing the same, we had identified the neutralino with a mass less than that of the gluino (this neutralino in the dilute flux approximation is roughly half the mass of the gluino). This neutralino turns out to be largely a neutral gaugino with a small admixture of the Higgsinos. Using one-loop RG analysis of coefficients of the effective dimension-six gluino decay operators as given in [26], we had showed in [21] that these coefficients at the EW scale are of the same order as that at the squark mass scale; we assume that these coefficients at the EW scale will be of the same order as that at the string scale.

The lower bound on the gluino lifetime via this three-body decay channel was estimated to lie in the range: $10^{-9} - 10^{6}$ seconds depending on which two Wilson line moduli are used to model the two (anti-)quarks produced in gluino decay. We had also calculated the decay width of one-loop two-body gluino decay into gluon and neutralino in [21], results of which, similar to the tree level gluino decay, yield large life time(s) of gluino for this case. The high squark mass, helps to suppress the tree-level as well as one loop gluino decay width. The fact that we have obtained suppressed Gluino decay width for squark masses of the order of $10^{12}$ GeV, is in agreement with the previous theoretical studies based on gluino decays in split SUSY in literature ([26, 30]) and results based on collider phenomenology for stable gluino.

We are currently looking into exploring the neutralino to be a dark matter candidate in a four-Wilson-line moduli (corresponding to the $SU(2)_L$ first-generation quark doublet, $SU(2)_L$ first-generation quark singlet, $SU(2)_L$ first-generation lepton doublet and the first generation $SU(2)_L$ first-generation lepton singlet) local $D3/D7$ $\mu$-Split SUSY framework [31].

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