Abstract. Dwarf elliptical galaxies are frequently excluded from bright galaxy samples because they do not follow the same linear relations in diagrams involving effective half light radii, $R_e$, or mean effective surface brightnesses, $\langle \mu \rangle_e$. However, using two linear relations which unite dwarf and bright elliptical galaxies we explain how these lead to curved relations when one introduces either $R_e$ or $\langle \mu \rangle_e$. In particular, the curved $\langle \mu \rangle_e - R_e$ relation is derived here. This and other previously misunderstood curved relations, once heralded as evidence for a discontinuity between faint and bright elliptical galaxies at $M_B \approx -18$ mag, actually support the unification of such galaxies as a single population whose structure (i.e. stellar concentration) varies continuously with stellar luminosity and mass.

Elliptical galaxies, and the bulges of disc galaxies, do not have structural homology (e.g. Davies et al. 1988; Caon et al. 1993; D’Onofrio et al. 1994; Young & Currie 1994, 1995; Andredakis et al. 1995). Instead, they have a continuous range of stellar concentrations – quantified by the S´ersic (1968) index $n$ (see Fig. 1a) – that varies linearly with both stellar luminosity and central surface brightness (after correcting for central stellar deficits or excess light). An unappreciated consequence of these two linear relations which unite faint and bright elliptical galaxies across the alleged divide at $M_B \approx -18$ mag is that relations involving either their effective half-light radius ($R_e$) or their effective surface brightness ($\langle \mu \rangle_e$), or the mean surface brightness within $R_e$ ($\langle \mu \rangle_e$), will be non-linear. Such curved relations have often been heralded as evidence that different physical processes must be operating on faint and bright elliptical galaxies because these relations have a different slope at the faint and bright end. To further complicate matters, sample selection which includes faint and bright elliptical galaxies, but excludes the intermediate luminosity population, can effectively break such continuously curved relations into two apparently disconnected relations.
**Fig. 1.** Panel a) Sérsic $R^{1/n}$ surface brightness profiles with effective surface brightness $\mu_e = 10$, and $n=1, 2, 3, 4, 6$ and 10. Panels b) and c) show the difference between the central surface brightness at $R = 0$, denoted by $\mu_0$, and (i) the effective surface brightness $\mu_e$ and (ii) the mean effective surface brightness $<\mu>_e$ within the effective radius $R_e$.

**Fig. 2.** Due to the observed linear relation of the B-band central surface brightness $\mu_{0,B}$ with a) the absolute magnitude $M_B$ (Eq. (0.3)) and b) the logarithm of the Sérsic exponent $n$ (Eq. (0.1)), the relation between the effective radius $R_e$ and the mean surface brightness within this radius $<\mu>_e$ (Eq. (0.5)) is highly curved for elliptical galaxies. The somewhat orthogonal distribution in panel c) is not evidence for two different physical processes operating at the faint and bright end of the elliptical galaxy sequence. Instead it is a consequence of these two linear relations which unify the faint and bright end, and bridge the alleged divide between dwarf and normal elliptical galaxies at $M_B \approx -18$ mag. The “core galaxies” (large filled circles) with partially depleted cores can be seen to have lower central surface brightnesses than the relation in panel a). However, the inward extrapolation of their outer profile yields $\mu_0$ values which follow the linear relation, as first noted by Jerjen & Binggeli (1997). The data are from the compilation by Graham & Guzmán (2003, their Fig. 9).

Figure 2 shows three diagrams for elliptical galaxies, two with linear relations that naturally explain the third panel’s curved relation. The data have been taken from the compilation by Graham & Guzmán (2003), while the two linear relations from that paper have been slightly tweaked here. From the first relation between
central surface brightness $\mu_0$ and Sérsic index $n$, given by

$$\mu_0 = 23 - 15.5 \log(n)$$ (0.1)

and shown in Figure 2b, one can convert $\mu_0$ into $\langle \mu \rangle_e$ using the Sérsic formula

$$\langle \mu \rangle_e = \mu_0 + 2.5b/\ln(10) - 2.5 \log(ne^{bT(2n)}/b^{2n}),$$ (0.2)

where $b \approx 1.9992n - 0.3271$ (Fig. 1c, see Capaccioli 1989 and Graham & Driver 2005, their Eqs. (7)–(9)). The associated effective radius $R_e$ is acquired by matching the second relation between absolute magnitude $M$ and central surface brightness, given by

$$M = 0.6\mu_0 - 28.2$$ (0.3)

and shown in Figure 2a, with the magnitude formula

$$M = \langle \mu \rangle_e - 2.5 \log(2\pi R_e^{2} \text{kpc}) - 36.57$$ (0.4)

(e.g. Graham & Driver 2005, their Eq. (12)). Doing this yields the expression

$$\log R_e = \frac{1}{5} \left\{ 0.4\langle \mu \rangle_e + 1.5 \left[ \frac{b}{\ln(10)} - \log \left( \frac{ne^{bT(2n)}}{b^{2n}} \right) - 6.91 \right] \right\},$$ (0.5)

in which one knows the value of $n$ associated with each value of $\langle \mu \rangle_e$ from the expressions above. Equation (0.5), obtained from two empirical linear relations (Eqs. (0.1)–(0.3)), is a curved relation that is shown in Figure 2c.

The implications of this should not be glossed over. Without any understanding of the $\langle \mu \rangle_e - R_e$ diagram (Fig. 2c), it has in the past been used to claim that faint and bright elliptical galaxies must have obtained their structure from different physical processes – because the faint and bright arms of the galaxy distribution are nearby perpendicular to each other. If there was instead one linear relation in this diagram, it would have been claimed that a single unifying mechanism was operating. As seen in Figures 2a and 2b, linear relations do exist across the faint and bright end of the galaxy distribution in $M - \mu_0$ and $n - \mu_0$ space. In passing we note that because of the linear relation between $M (= \log L)$ and $\log n$ (e.g. Caon et al. 1993; Young & Currie 1994; Jerjen & Binggeli 1997; Graham et al. 2001; Ferrarese et al. 2006), and the associated non-linear behaviour between $\mu_0$ and $\mu_e$ (Fig. 1b), the relation between $M$ and $\mu_e$ is not linear. Similarly, as detailed above, the relation between $\langle \mu \rangle_e$ and $R_e$ is not linear but curved. This result, however, has just been explained from linear relations which unify faint and bright elliptical galaxies.

The departure from the $B$-band $M_B - \mu_{0,B}$ diagram by elliptical galaxies brighter than $M_B \approx -20.5$ mag (Mass $> 0.5 - 1 \times 10^{11} M_{\odot}$), seen in Figure 2a, was explained by Graham & Guzmán (2003) in terms of partially depleted cores relative to their outer Sérsic profile (see also Graham 2004; Trujillo et al. 2004; Merritt & Milosavljević 2005). Such cores are thought to have formed from dry galaxy merger events (Begelman et al. 1980; Ebisuzaki et al. 1991) and resulted
in Graham et al. (2003) and Trujillo et al. (2004) advocating a “new elliptical galaxy paradigm” based on the presence of this central stellar deficit versus either none or an excess of light (see also Gavazzi et al. 2005; Ferrarese et al. 2006; Côté et al. 2007; and later Kormendy et al. 2009). As discussed in Graham & Guzmán (2003), this distinction at $M_B \approx -20.5$ mag between elliptical galaxies with partially depleted cores and those without, is a separate issue from the alleged division between elliptical galaxies and dwarf elliptical galaxies at $M_B \approx -18$ mag (e.g., Kormendy 1985; Kormendy et al. 2009; Tolstoy et al. 2009).

Further evidence for this division at $M_B \approx -20.5$ mag arises from the tendency for the brighter elliptical galaxies to be anisotropically pressure supported systems with boxy isophotes, while the less luminous early-type galaxies are reported to have discy isophotes and often contain a rotating disc (e.g. Carter 1978, 1987; Bender 1988; Peletier et al. 1990; Emsellem et al. 2007; Krajnović et al. 2008). Additional support for the above mentioned dry merging scenario at the high-mass end is the flattening of the colour-magnitude relation above $0.5 - 1 \times 10^{11} M_\odot$. As discussed by Graham (2008, his Sect. 6) and reiterated by Bernardi et al. (2010), this flattening is evident in Baldry et al. (2004, their Fig. 9) and Ferrarese et al. (2006, their Fig. 123), and even Metcalfe et al. (1994). This flattening has since been shown in other data sets (e.g., Skelton et al. 2009, although they reported the transition at $M_R = -21$ mag, i.e. $\approx 1$ mag fainter). Finally, the change in slope of the luminosity-(velocity dispersion) relation is also supportive of a transition at around $M_B \approx -20.5$ mag (e.g. Davies et al. 1983; Held et al. 1992; De Rijcke et al. 2005; Matković & Guzmán 2005).

Together, Figures 1 and 2 reveal that the apparent deviant nature of the dwarf elliptical galaxies from the approximately linear section of the bright-end of the $\langle \mu_e \rangle - R_e$ distribution, known as the Kormendy (1977) relation, does not imply that two different physical processes are operating. Similarly, the location of disc galaxy bulges at the faint end of this distribution does not imply that they must be “pseudobulges”. That is, “pseudobulges”, as opposed to “classical bulges”, can not be identified simply because they are outliers from the Kormendy (1977) relation (Gadotti 2009), which is the bright arm of a longer, continuous and unifying curved relation. While such apparent outliers are associated with bulges having low luminosities, low Sérsic indices, and faint central surface brightnesses, this is not by itself evidence that they experienced a different formation process. For similar reasons, galaxies which do not follow the bright arm of the curved $L-R_e$ relation (derived/explained in Graham & Worley 2008, their Fig. 11) need not be pseudobulges, nor are galaxies which do not follow the bright arm of the curved Mass–$R_e$ relation (presented by Graham et al. 2006, their Fig. 1b). Galaxies which do not follow the bright arm of the continuous, but curved, $L-\langle \mu_e \rangle$ and $L-\mu_e$ relation (e.g., Graham & Guzmán 2003, their Fig. 12) also need not necessarily be pseudobulges (Greene et al. 2008; Fisher & Drory 2010).

While luminous bulges and elliptical galaxies follow the same Fundamental Plane (Djorgovski & Davis 1987; Falcón-Barroso et al. 2002), fainter elliptical galaxies and bulges smoothly depart from the FP (when sample selection biases do not chop out a gulf between the faint and bright systems). Collectively, these
systems appear to follow a continuous trend along what is a curved manifold, of which the FP is the flat portion of this curved hypersurface (Graham & Guzmán 2004; Graham 2005; La Barbera et al. 2005; Zaritsky et al. 2006; Gargiulo et al. 2009). The curved nature of this manifold can be derived from a number of linear relations which span and unify faint and bright elliptical galaxies across the alleged divide at $M_B = -18$ mag, and it implies that some over-riding physical process dictates their structure. Galaxies which appear to branch off from the faint end of the Fundamental need not have formed from different physical mechanisms.

In summary, using curved relations, that can be constructed from unifying linear relations, as a means to identify an allegedly different class of galaxy (i.e. dwarf elliptical galaxies or pseudobulges) is not appropriate. The curved relations involving either $R_e$ or $\langle \mu \rangle_e$, and also $\mu_e$ (see Fig. 1), do not signal a different formation mechanism for low- and high-luminosity elliptical galaxies. Instead, these curved relations can be understood in terms of, and indeed predicted from, linear relations known to unify faint and bright elliptical galaxies. Understanding the implications of structural non-homology (i.e. the range of stellar concentrations) among elliptical galaxies (and bulges in disc galaxies) is key to better understanding galaxies and the connections they share.

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