All-electrical detection of the relative strength of Rashba and Dresselhaus spin-orbit interaction in quantum wires

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We propose a method to determine the relative strength of Rashba and Dresselhaus spin-orbit interaction from transport measurements without the need of fitting parameters. To this end, we make use of the conductance anisotropy in narrow quantum wires with respect to the directions of an in-plane magnetic field, the quantum wire and the crystal orientation. We support our proposal by numerical calculations of the conductance of quantum wires based on the Landauer formalism which show the applicability of the method to a wide range of parameters.

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With conventional electronics expected to reach critical boundaries for its performance soon, a new field of research utilizing the spin of the electron has evolved in recent years. Within this field called spintronics much attention has been focussed on spin-orbit interaction (SOI) because it provides a way of controlling the spin degree of freedom electrically in (non-magnetic) semiconductor-based systems without the need of external magnetic fields. However, SOI in two-dimensional electron gases (2DEG) is a double-edged sword, since spin relaxation in disordered 2DEGs, which is typically dominated by the D'yakonov-Perel' (DP) mechanism, is enhanced for disordered quantum wires defined in a 2DEG subject to an in-plane magnetic field. The method is based on the fact, that only for a field parallel to the effective magnetic field due to SOI the weak localization (WL) correction to the conductance survives, while it is suppressed for all other directions. No fit parameters are required, and α/β is straightforwardly related to this specific field direction, where the conductance is minimal.

We propose a method to determine the relative strength, α/β, of Rashba and Dresselhaus SOI from measuring the conductance of narrow quantum wires defined in a 2DEG subject to an in-plane magnetic field. The method is based on the fact, that only for a field parallel to the effective magnetic field due to SOI the weak localization (WL) correction to the conductance survives, while it is suppressed for all other directions. No fit parameters are required, and α/β is straightforwardly related to this specific field direction, where the conductance is minimal.

We numerically calculate the conductance \( G \) of a disordered quantum wire realized in a 2DEG with SOI linear in momentum. The single-particle Hamiltonian of the quantum wire in x-direction reads

\[
\mathcal{H} = \frac{\pi^2 x^2 + \pi^2 y^2}{2m^*} + U(x, y) + \frac{\muBg^*}{2} (\vec{B}_|| + \vec{B}_{so}(\vec{\sigma})) \cdot \vec{\sigma},
\]

with the effective spin-orbit field

\[
\vec{B}_{so}(\vec{\sigma}) = \frac{2}{\muBg^*} \left[ \hat{\sigma}_x (\alpha \pi_y + \beta (\pi_x \cos 2\phi - \pi_y \sin 2\phi)) + \hat{\sigma}_y (-\alpha \pi_x - \beta (\pi_x \sin 2\phi + \pi_y \cos 2\phi)) \right]
\]

and the external in-plane magnetic field

\[
\vec{B}_|| = B_|| (\cos(\theta - \phi) \hat{e}_x + \sin(\theta - \phi) \hat{e}_y).
\]

The vector potential components \( A_i \) in \( \pi_i = (p_i + eA_i) \) arise due to the perpendicular magnetic field \( B_z \) whose contribution to the Zeeman effect we neglect. In Eq. (2) \( \alpha \)
and $\beta$ is the Rashba and Dresselhaus SOI strength respectively and $\phi/\theta$ is the angle between the quantum wire/in-plane magnetic field and the [100] direction of the crystal for a zinc-blende heterostructure grown in the [001] direction. The electrostatic potential $U(x,y)$ includes the confining potential for the quantum wire and the disorder potential from static non-magnetic impurities in a region of length $L$. For the calculations we use a discretized version of the Hamiltonian (1) that allows us to evaluate the transport properties of the wire by computing lattice Green functions. For details see, e.g., Ref. [14]. The dimensionless numerical parameters used in this letter (denoted by a bar) are related to real physical quantities as follows (for square lattice spacing $a$): Energy $\tilde{E} = (2m^*a^2/\hbar^2)E$, SOI strengths $\tilde{\alpha} = (m^*a^2/\hbar^2)\alpha$ and $\tilde{\beta} = (m^*a^2/\hbar^2)\beta$. As a typical lengthscale for the simulations we introduce $\tilde{W}_0 = 20a$. In the calculations, the disorder potential is modeled by Anderson disorder with strength $\tilde{U}_0$. The mean free path is given by $l = 2.4W_0\sqrt{\tilde{E}_F/\tilde{U}_0^2}$, where $\tilde{E}_F$ is the scaled Fermi energy. The conductance of the wire is obtained by averaging over $N_d$ disorder configurations and unless stated otherwise the following parameters are fixed: $\tilde{E}_F = 0.5$ (corresponding to 4 propagating modes for a wire of width $W_0$), $L = 7.5W_0$, $\tilde{U}_0 = 1.4$ (i.e. $l \approx 0.87W_0$) and $N_d = 10000$.

To understand the mechanism for the detection of $\alpha/\beta$, which requires finite $B_{||}$, we first study the conductance of quantum wires at $B_{||} = 0$. Specifically we present the MC for two cases, where WAL is suppressed: (a) Rashba and Dresselhaus spin precession lengths larger than the width of the wire $W$, i.e., $L_\alpha^* = (\pi\hbar^2/m^*\alpha) \gg W$, $L_\beta^* = (\pi\hbar^2/m^*\beta) \gg W$ and (b) $\alpha = \beta$. In Fig. 1a, we plot $G(\Phi_s) - G(0)$ for wires with fixed $\alpha \neq 0, \beta = 0$ and different widths $W$, showing that for smaller $W$ WAL is suppressed, which is in line with earlier experimental results [15] and confirms analytical [16] and numerical treatments [13]. Since spin relaxation is essential for WAL, the mechanism for the suppression of WAL can be attributed to an enhancement of the spin-scattering length in narrow wires [17,18], and more generally, in confined geometries [19,20].

In the case (b), $\alpha = \beta$, over points uniformly into the [110]-direction for all k-vectors and a so-called persistent spin helix forms [21]. There the spin state of an electron is determined only by its initial and final position independent of the exact path in-between. Therefore, charge carriers do not acquire an additional phase due to SOI upon return to their initial positions, resulting in constructive interference of the wavefunctions connected by time reversal, hence WL [22]. This behavior is shown for fixed $W$ and $\alpha$ but variable $\beta$ in Fig. 1b where we observe that WAL is suppressed for $\alpha = \beta$.

In both cases shown in Figs. 1a,b the absence of WAL is caused by the suppression of spin relaxation with the spin relaxation length exceeding the length of the wire $L$, where $L$ in the numerical simulation takes the role of the phase coherence length in the experiment.

We now investigate the influence of an additional in-plane magnetic field on the conductance of a quantum wire where WAL is suppressed. For convenience, we introduce the ratio $\lambda = B_{||}/|\tilde{B}_{so}(k_x)|$ which is the relative strength of the in-plane magnetic field and the effective magnetic field due to SOI for a k-vector along the quantum wire, see Eqs. 2,3. In Fig. 1b: we show the MC for the case $\alpha = \beta$ for several values of $\lambda$. The conductance at $\Phi_s = 0$ is enhanced by a finite $B_{||}$. The form of the MC curves in Fig. 1b can be understood from the expression for the WL/WAL conductance correction from diagrammatic perturbation theory [23]. It is of the form $\Delta G \propto (\lambda^00\sum_{m=-1}^1 C_{1|m})$, where the first (singlet) term $C_{00}$ contributes positively to the conductance and is responsible for the typical WAL peak in systems with SOI. It is unaffected by DP spin relaxation but suppressed by an in-plane magnetic field [24]. The second (triplet) term gives a negative conductance contribution and is suppressed for short spin relaxation times [22]. For the parameters used in Fig. 1b, $C_{00}$ is suppressed for $\lambda \geq 0.15$, thus in the respective curves shown in Fig. 1b: only the triplet term is present in $\Delta G$ resulting in positive MC ($\partial G/\partial \Phi_s > 0$). While for $\lambda = 0$ we observe WL due to $\alpha = \beta$, increasing $\lambda$ gives rise to a transition to $\partial G/\partial \Phi_s \approx 0$ at $\lambda \approx 1$ and back to WL for $\lambda \gg 1$. This can be understood by the change of the spin relaxation in the system: For finite $B_{||}$ in a direction different from [110] ($\theta = 3\pi/4$), the resulting magnetic field $\tilde{B}_{sat}(\vec{\pi}) = \tilde{B}_{||} + \tilde{B}_{so}(\vec{\pi})$ will not be uniform in the [110] direction anymore, but cause spin relaxation, which is strongest for comparable strengths of $\tilde{B}_{||}$ and $\tilde{B}_{so}$ and yields a reduction of the triplet term (green diamonds in Fig. 1b). For in-plane magnetic fields which distinctly exceed the effective magnetic field ($\lambda \gg 1$), on the other hand, WL is restored to some degree (blue triangles in...
The enhancement of \( \vec{B} \) field is anisotropic with respect to the direction of \( \vec{B}_G \) since the resulting \( \vec{B}_G \) is strongly aligned along \( \vec{B}_G \) and spin relaxation is reduced again. The enhancement of \( G(\Phi_s = 0) \) in an in-plane magnetic field is anisotropic with respect to the direction of \( \vec{B}_G \). For \( \theta = (3/4)\pi \), spin remains a good quantum number due to the reduction of the singlet term caused by \( \vec{B}_G \).

In view of the results of Fig. 2, we conjecture that also for a quasi-one-dimensional quantum wire with \( W \ll l_{SO} \) the angle at which the minimum in the conductance appears is given by the direction of the effective magnetic field \( \vec{B}_\alpha(k_x) \) for a \( k \)-vector along the wire direction \( \hat{x} \).

In Fig. 3 we plot Eq. (4) for three different wire orientations \( \phi \) (solid lines), whose validity is nicely confirmed by extracting \( \theta_{min} \) from the numerical \( G(\theta) \) dependence (such as in Fig. 2) for different ratios of \( \alpha/\beta \) (symbols) with fixed \( \alpha + \beta \). In order to use this feature for the determination of the ratio \( \alpha/\beta \) we suggest to measure \( G(\theta) \) for quantum wires oriented either along the [100] or the [010]-direction. Then the angle of the minimum conductance directly provides the unambiguous value for the relative strength and signs of \( \alpha \) and \( \beta \). Choosing, e.g., \( \phi = \pi/2 \) this ratio is given by \( \alpha/\beta = -\cot(\theta_{min}) \), which is representative for the whole sample, since the influence of the lateral confinement on the strength of the SOI is negligible [27]. Considering quantum wires realized in an InAlAs/InGaAs heterostructure (typical values \( m^* = 0.05m_0, g^* = 3 \) and fixing the width \( W_0 = 350 \mu m \), we see that the parameters used in Fig. 3 (\( l \approx 412 nm, B_{||} \approx 0.17 T \) and \( \alpha + \beta \approx 3.5 \times 10^{-12} \psi/\mu m \)) are well in reach of present day experiments [23, 24].

We have neglected effects due to the cubic Dresselhaus SOI term, which becomes increasingly important for wide quantum wells. In general, it induces additional randomization of the spin state, which for the case of a very strong cubic Dresselhaus contribution can result in the absence of the suppression of WAL [22]. Nevertheless, since cubic Dresselhaus coupling is smallest for \( k \)-vectors along [100] or [010] directions, we have neglected it for the determination of \( \alpha/\beta \), since in our proposal the quantum wire is assumed to be oriented in one of those directions. However, in contrast to a 1DQW, it might have an effect.
Fig. 4: (Color online) Geometric determined $\theta_{\text{min}}$ for $W_0$, $\phi = \pi/2$, $(\mu B_g m^* a^2/\hbar^2)B_1 = 0.01$, $N_d = 8000$ and $\alpha/\beta = 3$. Either the mean free path $l$ for fixed $\alpha + \beta = 0.02$ (blue circles) or $\alpha + \beta$ for fixed $l \approx 0.87W_0$ (red squares) was varied. The black line shows the expected value of $\theta_{\text{min}}$ from Eq. (4).

on $\theta_{\text{min}}$, if it is comparable in strength to the linear term.

In order to assess possible limitations of this method, we performed calculations varying several parameters, while keeping the ratio $\alpha/\beta = 3$ constant. In Fig. 4 we show that Eq. (4), $\theta_{\text{min}} = \arctan(-1/3) \approx 0.9\pi$, is fulfilled for a wide range of both SOI strengths (squares) and mean free paths (circles). Further numerical calculations, upon increasing the number of transverse orbital modes in the wire up to 13, showed that Eq. (4) still holds true (not presented here).

In conclusion we have shown, that Eq. (4), derived for a 1DQW, provides a valuable tool to determine the ratio $\alpha/\beta$ also for a quantum wire with several transversal modes, only requiring $W \ll L^{\alpha/\beta}$; i.e. a suppression of WAL due to the confinement [15]. For increasing width, $G(\theta)$ evolves into a behavior typical of a 2DEG [22], [23], where $G(\theta)$ is only anisotropic, if both $\alpha, \beta \neq 0$. Opposed to the narrow quantum wires considered where $\theta_{\text{min}}$, Eq. (4), is a function of $\phi, \alpha, \beta$, in a 2DEG minimum of the conductivity appears either at $\theta = \pi/4$ or $3\pi/4$, depending on the sign of the product $\alpha/\beta$, but independent of the ratio $\alpha/\beta$.

Apart from the condition $W \ll L^{\alpha/\beta}$, the method should be applied at sufficiently small $B_y$ ($\lambda \ll 1$). As can be seen from Fig. 2b,c, when $\lambda \gtrsim 1$, $G$ is increased for any $\theta$, potentially changing the position of $\theta_{\text{min}}$ (see, e.g., blue triangles in Fig. 2e). Only for the case of $\alpha = \beta$ shown in Fig. 2d, $G(\theta_{\text{min}})$ does not increase, since $\vec{B}_n(k) \parallel \vec{B}_y$ for any $k$-vector. In this special case the validity of Eq. (4) is not limited to narrow wires and small magnetic fields.

To summarize, in narrow quantum wires which exhibit weak localization even in the presence of spin-orbit coupling, an in-plane magnetic field can suppress the weak localization effect. We employed the unique angular dependence of this effect to suggest a method for the direct and experimental determination of the ratio between Rashba- and Dresselhaus spin-orbit strengths from transport measurements. Its straightforward applicability may help to facilitate the design of semiconductor-based building blocks for spintronics.

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