Photoproduction of Pseudoscalar Mesons off Nuclei at Forward Angles

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Abstract

With the advent of new photon tagging facilities and novel experimental technologies it has become possible to perform photoproduction cross section measurements of pseudoscalar mesons on nuclei with

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a percent level accuracy. The extraction of the radiative decay widths from these measurements at forward angles is done by the Primakoff method, which requires theoretical treatment of all processes participating in these reactions at the same percent level. In this work we review the theoretical approach to meson photoproduction amplitudes in the electromagnetic and strong fields of nuclei at forward direction. The most updated description of these processes are presented based on the Glauber theory of multiple scattering. In particular, the effects of final state interactions, corrections for light nuclei, and photon shadowing in nuclei are discussed.

1 Introduction

The properties of QCD at low energies are manifested in their most unambiguous form in the sector of light pseudoscalar mesons $\pi^0$, $\eta$ and $\eta'$. The two-photon decays of these mesons are primarily caused by the chiral anomaly [1, 2], the explicit breaking of a classical symmetry by the quantum fluctuations of the quark fields when they couple to the electromagnetic field. This anomalous symmetry breaking is of a pure quantum mechanical origin and can be calculated exactly to all orders in the chiral limit. Particularly, in the case of the $\pi^0$, which has the smallest mass in the hadron spectrum, higher order corrections to chiral anomaly is predicted to be small and can be calculated with a sub-percent accuracy [3, 4, 5]. As a result, the precision measurement of $\pi^0 \to \gamma\gamma$ decay width is widely recognized as an important test of QCD. On the other hand, the system of $\pi^0$, $\eta$ and $\eta'$ contains the fundamental information on the effects of SU(3) and isospin symmetry breaking by the light quark masses. The two-photon decay widths of $\eta$ and $\eta'$ have a significant impact on the knowledge of the quark-mass ratio $(m_d - m_u)/m_s$ [6] and the $\eta-\eta'$ mixing angle. Precision measurements of the two-photon decay widths of these pseudoscalar mesons will provide a rich experimental data set to understand QCD at low energies.

With the recent availabilities of high energy and high precision intense photon tagging facilities [7] together with the novel developments in electromagnetic calorimetry it is feasible to perform percent level differential cross section measurements of light pseudoscalar mesons $\pi^0$, $\eta$ and $\eta'$ on nuclei [8-11]. The two-photon decay widths of these mesons can be extracted from these experiments, performed at forward angles using the Primakoff method [12], which assumes production of mesons in the Coulomb
field of nuclei. However, the production process may also be manifested by exchange of vector mesons having the same quantum numbers as the photon. The angular distributions of these two production mechanisms are realized differently in the cross sections. The Primakoff production is very sharply peaked at forward angles (0.02 degree at $E_\gamma \sim 5$ GeV) with practically vanishing strength at few degrees. The strong production is small under the Primakoff peak, but it begins to dominate at few degrees. The interference of these two coherent amplitudes with its significant magnitude under the Primakoff peak, makes the extraction of the decay widths quite difficult. The full theoretical description of this process, in addition to the above two mechanisms, requires a correct treatment of the final state interactions (FSI) of the produced mesons in nuclear matter, as well as accounting for incoherent processes. This is necessary to provide a percent level extraction of the decay widths from the experimental data set. In this paper we present more general, and perhaps more complete, theoretical descriptions of these photoproduction mechanisms in nuclei. We focus on the example of $\pi^0$ production keeping in mind that the presented results can be extended to $\eta$ and $\eta'$ production with appropriate replacements of the parameters and cross sections.

In 1951 H. Primakoff [12] first proposed to measure the $\pi^0$ meson lifetime from their photoproduction in the Coulomb field of a heavy nucleus (Fig. 1). With that we are considering coherent photoproduction of pseudoscalar mesons on a nucleus at high energies:

$$\gamma + A \rightarrow Ps + A; \quad Ps = \pi^0, \eta, \eta'$$

As mentioned above, the main challenge in the determination of pion lifetime in this way is related to the presence of the strong amplitude in the photoproduction process (Fig. 2). The full amplitude of this coherent process can be described as a sum of the Coulomb, $T_C$, and the strong, $T_S$, parts:

$$T = T_C + e^{i\varphi}T_S$$

where $\varphi$ is the relative phase between the Coulomb and the strong amplitudes. Taking into the account the incoherent processes, the differential cross section can be expressed as:

$$\frac{d\sigma}{d\Omega} = \frac{k^2}{\pi} \frac{d\sigma}{dt} = \left| T_C + e^{i\varphi}T_S \right|^2 + \frac{d\sigma_{inc}}{d\Omega}$$
Figure 1: The pion photoproduction in the nuclear Coulomb field.

Figure 2: The pion photoproduction in the strong field of a nucleus.

where $\frac{d\sigma_{inc}}{d\Omega}$ is the incoherent cross section, i.e., processes involving excitation or breakup of the target nucleus. Each of these amplitudes factorizes into a photoproduction amplitude on a nucleon multiplied by a corresponding form factor. In addition, these form factors must be modified for the final state interactions of the outgoing mesons with nucleons. Interaction of incident high energy photons with nuclear matter gives rise to a shadowing effect, which has to be considered as well.
2 Photoproduction in the Coulomb field of a nucleus

The effect of the pion Final State Interactions (FSI) in nuclei was first discussed in detail by G. Morpurgo [13], who considered absorption of the produced pions in the strong nuclear field using the Distorted Wave Approximation (DWA). Calculations of the electromagnetic and strong form factors in this work have been done with the uniform nuclear density distribution $\rho(r)$. Based on these assumptions, part of the correction to the strong and electromagnetic form factors, which takes into account pion absorption in nuclei, is correctly obtained. However, the effect of the pion re-scattering to forward angles was not taken into account in this work. This effect is important for the precision extraction of the decay widths since pions initially produced at modest angles can re-scatter to small angles. This effect was first considered by G. Fäldt [14] in non-diffractive production processes on nuclei in the framework of the Glauber theory of multiple scattering [15]. A general expression (Eq. 3.2 in [14]) for the electromagnetic amplitude was obtained in this paper, but an analytically integrable formula was derived for the case of equal absorption in a nucleus of the incident and produced particles only (Eq. 3.4 in [14]), which is not the case for the photoproduction processes.

2.1 The electromagnetic form factor

In a more general case, based on Glauber multiple scattering theory and using the independent particle model for nucleons, the Coulomb amplitude of the photoproduction of mesons on nuclei can be expressed as:

$$T_C = Z \sqrt{8\alpha \Gamma} \left( \frac{\beta}{m_\pi} \right)^{3/2} \frac{k^2 \sin \theta}{q^2 + \Delta^2} F_{em}(q, \Delta)$$

(4)

where the electromagnetic form factor is given by [16]:

$$F_{em}(q, \Delta) = \frac{q^2 + \Delta^2}{q} \int J_1(qb) \frac{bd^2dz}{(b^2 + z^2)^{3/2}} e^{i\Delta z} \times \exp \left( -\sigma' \int \rho(b, z')dz' \right) \int_{\sqrt{b^2+z^2}} \rho(x)dx$$

(5)
with the following notation:

\[
\sigma' = \sigma \left( 1 - i \frac{\Re f(0)}{\Im f(0)} \right) = \frac{4\pi}{ik} f(0)
\]

Here \(\sigma\) is the \(\pi N\) total cross section, \(f(0)\) is the \(\pi^0N \rightarrow \pi^0N\) forward amplitude. The invariant momentum transfer is expressed through the two-dimensional transverse momentum \(q = |\vec{q}|\) and longitudinal momentum \(\Delta\):

\[
t = -q^2 - \Delta^2 = -4kp \sin^2 \left( \frac{\theta}{2} \right) - \left( \frac{m^2}{2k} \right)^2
\]

where \(k = |\vec{k}|\) and \(p = |\vec{p}|\) are the photon and pion momenta.

In Eq. 5 \(\rho(r)\) is the nuclear density, \(\text{J}_1(x)\) is the first order Bessel function. The integration in Eq. 5 goes over impact parameter \(b\), which is the two-dimensional vector in the plane perpendicular to the incident photon direction and the longitudinal coordinate \(z\).

The Coulomb form factor, \(F_{em}(q, \Delta)\), acquire an imaginary part due to the longitudinal momentum transfer \(\Delta\) and the presence of the real part of the pion-nucleon elastic forward amplitude in the absorption process.

The main properties of the Coulomb production are: it rises from zero at zero degree angle; reaching its maximum at \(q = \Delta = \frac{m^2}{2k}\) with the specific energy dependence at the peak of \(\frac{d\sigma}{dt} \sim k^2\). These properties allow one to separate the Coulomb production (Primakoff process) from the competing nuclear production in strong field of nuclei, which peaks at relatively large production angles.

For illustration, Fig. 3 shows the angular dependence of the form factors calculated by Eq. 5 for carbon and lead nuclei at 5.2 GeV incident photon energy. In this calculation the nuclear density \(\rho(r)\) was parametrized by Fourier-Bessel analysis from the recent electron scattering data [18, 19]. As evident, the final state interaction effects are more emphasized for the heavier nucleus, though these effects are still very small in the forward Primakoff region.

### 2.2 Effect of light nuclei

The photoproduction of mesons in the electromagnetic field of light nuclei requires a special consideration. Equation 5 was obtained in the optical limit, which is valid for extended nuclear matter (medium and heavy nuclei).
Figure 3: The square of the electromagnetic form factor for carbon (solid line) and lead (dotted line).

Using multiple scattering theory \[15\] we obtain the expression for the electromagnetic form factor for light nuclei suitable for numerical calculations \[20\]:
Here \( I_0(x) \) is the zero order Bessel function of imaginary argument \[17\]. In deriving this expression, the commonly used parametrization \[21\] for the elastic \( \pi N \rightarrow \pi N \) amplitude \( f_s(q) = f_s(0) \exp(-a_s q^2/2) \) has been adopted.

For light nuclei, such as carbon, the nuclear density, \( \rho(r) \), corresponding to the harmonic oscillator potential well is widely accepted in literature \[22\]:

\[
\rho(r) = \frac{4}{\pi^{3/2} a_0^3} \left( 1 + \frac{A - 4 r^2}{6 a_0^2} \right) e^{-\left(\frac{r^2}{a_0^2}\right)}
\]

The nuclear charge distribution, \( \rho_{ch}(r) \), is obtained by convolution of the nuclear density (Eq. 7) with the proton’s charge distribution. For the latter we adopt the simple Gaussian parametrization: \( \rho_p(r) = \frac{1}{\pi^{3/2} r_p^3} e^{-\left(\frac{r^2}{r_p^2}\right)} \). With that, the nuclear charge density can be expressed by\[1\]

\[
\rho_{ch}(r) = \int d^3 r' \rho(r') \rho_p(|r - r'|)
\]

\[
= \frac{2}{\pi^{3/2}(a_0^2 + r_p^2)^{3/2}} \left[ 1 + \frac{(Z - 2)}{3} \left( \frac{3r_p^2}{2(a_0^2 + r_p^2)} + \frac{a_0^2 r_p^2}{(a_0^2 + r_p^2)^2} \right) \right] e^{-\left(\frac{r^2}{r_p^2 + a_0^2}\right)}
\]

Figure 4 shows the difference between these two approaches on the electromagnetic form factor calculations for carbon nucleus. The following parameters are used in these calculations: the proton radius \( r_p = 0.8 \) fm; oscillator parameter in Eq. 7 \( a_0 = 1.65 \) fm. It is seen that the form factor calculated

\[1\]The nuclear density is normalized to the atomic number \( \int \rho(r) d^3r = A \), whereas the charge density is normalized to the number of protons \( Z \).
by the “light nuclei” approach is falling faster at relatively large angles than that based on the “optical limit”. Again, the difference between these two methods in the Primakoff region is practically negligible.

Figure 4: The square of electromagnetic form factor for carbon calculated by expression (6) (dotted line) and (5) (solid line).

2.3 Nuclear excitation by Coulomb exchange

In addition to the coherent photoproduction in the Coulomb field, production of pions with a nuclear collective excitation (for instance, the giant dipole resonance) is also possible. Such processes were first discussed in [23] where
for the inelastic form factor the following approximation was obtained:

\[ |F_n(q)|^2 \approx \frac{1.4N}{2m_pZAE_{av}}|t| \]  \hspace{1cm} (9)

where \( m_p \) and \( E_{av} \) are the proton mass and the average excitation energy of the nucleus having \( Z \) protons and \( N \) neutrons. The longitudinal momentum transfer in the case of \( \pi^0 \) photoproduction with nuclear collective excitation is much larger than in the coherent photoproduction: \( \Delta_m = \Delta + E_{av} \gg \Delta \). Using the fact that the ratio of the “elastic” to the “inelastic” cross sections of the \( \pi^0 \) photoproduction in the Coulomb field can be estimated as:

\[ R = \frac{\frac{d\sigma_{in}}{dt}}{\frac{d\sigma_{el}}{dt}} \approx \frac{(q^2 + \Delta^2)^2 |F_n(q)|^2}{(q^2 + \Delta_m^2)^2 |F(q)|^2} \approx \frac{1.4N}{2m_pZAE_{av}} (q^2 + \Delta^2) \] \hspace{1cm} (10)

For the carbon nucleus the average collective excitation energy is on the level of \( E_{av} \sim 20 - 25 \) MeV which, using Eq. 10 leads to \( R \sim 10^{-7} \) in the Coulomb peak region \( q = \Delta = \frac{m_p^2}{2k} \). Thus, the contribution from the nuclear collective excitations can be safely neglected for the GeV and higher incident photon energies.

### 3 Coherent photoproduction in strong field of nuclei

#### 3.1 The strong amplitude \( T_S \)

The main impact on the lifetime extraction from the measured differential cross sections comes from our knowledge of the strong amplitude \( T_S \) in the coherent process of the reaction:

\[ \gamma + A \rightarrow \pi^0 + A \]  \hspace{1cm} (11)

In the Glauber theory of multiple scattering this coherent photoproduction amplitude is given by [14]:

\[ T_S(q, \Delta) = \frac{ik}{2\pi} \int e^{i(q\vec{b} + \Delta z)} \Gamma_{\rho}(\vec{b} - \vec{s})\rho(\vec{s}, z) \]

\[ \times \left[ 1 - \int \Gamma_{s}(\vec{b} - \vec{s'})\rho(\vec{s'}, z')d^2s'dz' \right] A^{-1} d^2b d^2sdz \]  \hspace{1cm} (12)
The two dimensional vectors $\vec{b}$ and $\vec{s}$ are the impact parameter and the nucleon coordinate in the plane transverse to the incident photon momentum; $z$ is the longitudinal coordinate of the nucleon in the nucleus. The profile functions $\Gamma_{p,s}(\vec{b} - \vec{s})$ are the two dimensional Fourier transforms of the non-spinflip elementary amplitudes for the pion photoproduction off the nucleon $f_p = f(\gamma + N \rightarrow \pi^0 + N)$ and elastic pion-nucleon scattering $f_s = f(\pi + N \rightarrow \pi + N)$:

$$\Gamma_{p,s}(\vec{b} - \vec{s}) = \frac{1}{2\pi i k} \int e^{i\vec{q} \cdot (\vec{b} - \vec{s})} f_{p,s}(q) d^2q$$  \hspace{1cm} (13)

Since the slope of elementary amplitude, $f_s(q)$, is typically much less than the square of the nuclear radii for medium and heavy nuclei, the nuclear density $\rho(r)$ is varying slower than the elastic profile functions $\Gamma_s(\vec{b} - \vec{s})$. With this approximation it is safe to take the $\rho(r)$ off from under the integral sign, which leads to:

$$\int \Gamma_s(\vec{b} - \vec{s}) \rho(s, z) d^2s dz = \sigma' \frac{1}{2} \int \rho(\vec{b}, z) dz$$ \hspace{1cm} (14)

With special care the same procedure can also be applied for the production amplitudes [14].

Isolating near zero angle part of the production amplitude $f_p(q) = (\vec{h} \cdot \vec{q}) \phi(q)$ ($\vec{h} = \frac{[k \times \vec{e}]}{k}$, where $\vec{e}$ is the photon polarization vector, and $\phi(0) \neq 0$) we get:

$$\int \Gamma_p(\vec{b} - \vec{s}) \rho(s, z) d^2s dz = 2\pi k \phi(0) \vec{h} \cdot \frac{1}{k} \int \frac{\partial \rho(\vec{b}, z)}{\partial b} bdb dz$$ \hspace{1cm} (15)

As a result, Eq. [12] can be written in the factorized form:

$$T_S(q) = (\vec{h} \cdot \vec{q}) \phi(0) F_{st}(q, \Delta)$$

$$F_{st}(q, \Delta) = -\frac{2\pi}{q} \int J_1(qb) \frac{\partial \rho(b, z)}{\partial b} bdb dz e^{i\Delta z} \exp \left(-\frac{\sigma'}{2} \int_z^\infty \rho(b, z') dz' \right)$$ \hspace{1cm} (16)

The strong form factor, $F_{st}(q, \Delta)$, can be expressed as a sum of two terms,
as done in [14]:

\[
F_{st}(q, \Delta) = \int e^{i\vec{q} \cdot \vec{b}} \rho(b, z)d^2bdze^{i\Delta z} \exp \left( -\frac{\sigma'}{2} \int_z^\infty \rho(b, z')dz' \right) - \frac{\pi\sigma'}{q} \int J_1(qb)\rho(b, z_1)\frac{\partial \rho(b, z_2)}{\partial b}bdbdz_1dz_2e^{i\Delta z_1} \exp \left( -\frac{\sigma'}{2} \int_{z_1}^\infty \rho(b, z')dz' \right)
\]

(17)

The first term is the usual nuclear form factor and the second one is a correction first introduced by Fäldt [14]. The contribution from the second term is positive (since \( \frac{\partial \rho(b, z)}{\partial b} < 0 \)) and can be interpreted as a result of the final state interaction of the pions produced in the coherent process at nonzero angles which after multiple scattering come to forward direction. Note that this correction was obtained [14] as in the case of electromagnetic part under the assumption that the cross sections for the incident and produced particles are equal. We have considered a more general case for the photoproduction reactions and the results are discussed in the next sections.

### 3.2 Photon shadowing in nuclei

Real photons at high energies are shadowed in nuclei [21]. Photon shadowing in pion photoproduction is a result of the two-step process [24]: the initial photon produces a vector meson in the nucleus, which consequently produces the pseudoscalar meson on another nucleon of the same nucleus. The main contribution to this process comes from the \( \rho \) mesons, as the cross section for \( \rho \) photoproduction from a nucleon is almost one order of magnitude greater than that for \( \omega \) production. In addition, the reaction \( \omega + N \rightarrow \pi^0 + N \) is mainly caused by isospin one exchange (\( \rho \)-exchange) and the production amplitudes on protons and neutrons have different signs. Since the nuclear coherent amplitude is a sum over the elementary photoproduction amplitudes the omega contribution gets an additional suppression.

Using multiple scattering techniques, one can obtain the contribution from intermediate \( \rho \) channel to the strong form factor:

\[
F_1(q) = -\frac{\pi\sigma'}{q} \int J_1(qb)\rho(b, z_1)\frac{\partial \rho(b, z_2)}{\partial b}\theta(z_2 - z_1)bdbz_1dz_2 \times e^{i\Delta \rho(z_1-z_2)+i\Delta z_2} \exp \left( -\frac{\sigma'}{2} \int_{z_1}^\infty \rho(b, z')dz' \right)
\]

(18)
where $\Delta_\rho = \frac{m_\rho^2}{2E}$ is the longitudinal momentum transfer in the $\rho$ meson photoproduction off the nucleon. The strong amplitude accounting for the photon shadowing reads:

$$T_S(q) = \langle \vec{h} \cdot \vec{q} \rangle \phi(0)(F_{st} - wF_I)$$

$$w = \frac{f(\gamma N \rightarrow \rho N)f(\rho N \rightarrow \pi N)}{f(\rho N \rightarrow \rho N)f(\gamma N \rightarrow \pi N)}$$ (19)

where the range of the shadowing parameter $w$ is between zero (no shadowing) and one (Vector Dominance Model).

Equation 18 and Fäldt’s correction (second term in Eq. 17) is very similar and only difference is in the energy dependence through the longitudinal momentum transfer. It is interesting to mention here that at high energies where $\Delta = \Delta_\rho = 0$, assuming a validity of the naive Vector Dominance Model ($w = 1$), the above mentioned two terms cancel each other. Thus, the strong form factor is determined only by the first term in Eq. 17. In this limit the integration over $z$ in Eq. 17 can be done analytically:

$$F_{st}(q) = \frac{2}{\sigma'} \int d^2b e^{i\vec{q}\cdot\vec{b}} \left[ 1 - \exp \left( -\frac{\sigma'}{2} \int \rho(b, z')dz' \right) \right]$$ (20)

This expression looks very similar to the one for coherent production of pions by $\rho$ mesons, a fact well known for the diffractive processes [21].

The effect of photon shadowing on the strong form factor in Eq. 19 is demonstrated in Figs. 5 and 6 for carbon and lead nuclei, calculated from Eqs. 17, 19 for two extreme values of the shadowing parameter: $w = 0$ (no shadowing) and $w = 1$ (naive VDM). As it is seen, the results for both nuclei are strongly dependent on the value of $w$.

### 3.3 The two-step contribution in light nuclei

The optical limit approximations, usually used in the literature for these calculations, can potentially lead to a correction for light nuclei, which can be critical for the precision extraction of decay width from the experimental cross sections.

For light nuclei we parametrize the elementary production amplitudes in
Figure 5: The square of the strong form factor for carbon without shadowing ($w = 0$, solid line) and with maximal photon shadowing ($w = 1$, dotted line).

The following way:

\[
\begin{align*}
    f_p &= \phi(0)(\vec{h} \cdot \vec{q})e^{-\frac{a_p q^2}{2}} \\
    f_s &= f_s(0)e^{-\frac{a_s q^2}{2}}
\end{align*}
\]

(21)

Here $\phi(0)$, $f_s(0)$ and $a_p$, $a_s$ are the forward elementary amplitudes and their
relevant slopes. With these parametrizations one gets:

\[ \Gamma_p(\vec{b} - \vec{s}) = \frac{\vec{h} \cdot (\vec{b} - \vec{s})}{k a_p^2} \phi(0) e^{-\frac{(\vec{b} - \vec{s})^2}{2a_p}} \]

\[ \Gamma_s(\vec{b} - \vec{s}) = \frac{f_s(0)}{i k a_s} e^{-\frac{(\vec{b} - \vec{s})^2}{2a_s}} \]  \hspace{1cm} (22)
Substituting Eq. 22 into Eq. 12 we obtain:

\[ T_S(q, \Delta) = (\vec{n} \cdot \vec{q}) \phi(0) F_{st}(q, \Delta) \]

\[ F_{st}(q, \Delta) = \frac{2\pi}{qa_s^2} \int J_1(qb) \left[ bI_0 \left( \frac{bs}{a_p} \right) - sI_1 \left( \frac{bs}{a_p} \right) \right] \times e^{i\Delta_z} e^{-\frac{|b|^2 + |s|^2}{2a_p^2}} \rho(s, z) [1 - G(b, z)]^{A-1} bdbdsdz \]

\[ G(b, z) = \frac{f_s(0)}{ik\sigma_s} \int e^{-\frac{(b - s')^2}{2a_s^2}} \rho(s', z') d^2s' dz' \]

\[ = \frac{\sigma'}{2a_s} \int e^{-\frac{|b|^2 + |s'|^2}{2a_s^2}} I_0 \left( \frac{bs'}{a_s} \right) \rho(s', z') ds' dz' \quad (23) \]

The amplitude relevant to the two-step process \( \gamma \to \rho \to \pi^0 \) in multiple scattering theory is given by:

\[ T_I(q) = \frac{ik}{2\pi} \frac{A - 1}{A} \int e^{i\vec{q} \cdot \vec{b}} d^2b \Gamma_{\gamma\rho}(\vec{b} - \vec{s}_1) \Gamma_{\rho\pi}(\vec{b} - \vec{s}_2) \rho(s_1, z_1) \rho(s_2, z_2) \times \theta(z_2 - z_1) e^{i\Delta \rho(z_1 - z_2) + i\Delta \rho} [1 - G(b, z_1)]^{A-2} d^2s_1 d^2s_2 dz_1 dz_2 \quad (24) \]

where \( \Delta \rho = \frac{m_p^2}{2E} \) is the longitudinal momentum transfer in the elementary reaction \( \rho + N \to \pi^0 + N \). Using Eq. 22 the relevant form factor can be expressed in a form convenient for numerical integration:

\[ F_I(q, \Delta_\rho, \Delta) = \frac{A - 1}{A} \frac{\pi\sigma'}{qa_s a_p^2} \int J_1(qb) I_0 \left( \frac{bs_2}{a_s} \right) \left[ bI_0 \left( \frac{bs_1}{a_p} \right) - s_1I_1 \left( \frac{bs_1}{a_p} \right) \right] \times \theta(z_2 - z_1) \rho(s_1, z_1) \rho(s_2, z_2) e^{-\frac{(a_p + a_s) b^2}{2a_p a_s}} e^{-\frac{s_1^2}{2a_p^2} - \frac{s_2^2}{2a_s^2}} e^{i\Delta \rho(z_2 - z_1) + i\Delta \rho} \times [1 - G(b, z_1)]^{A-2} bdbds_1 ds_2 dz_1 dz_2 \quad (25) \]

In Fig. 7, the square of the strong form factor for carbon nucleus including the intermediate channel corrections are plotted for two different cases: (a) the solid line is calculated by Eqs. 17-19 with optical model approximations; (b) the dotted line is calculated by multiple scattering theory for light nuclei without optical model approximations (using Eqs. 23-25). The oscillator type of nuclear density (Eq. 7) and the shadowing parameter \( w = 0.25 \) (23, 26) are used in both calculations. As it is seen, the fall of the form factor calculated without the optical model approximations (dotted line) is sizeably faster, and therefore important for the precision extraction of the decay widths. A similar behavior was observed for the electromagnetic form factor shown in Fig. 4.
Figure 7: The square of the strong form factor for carbon nucleus calculated with (solid line) and without (dotted line) optical model approximations.

4 Incoherent photoproduction

Incoherent meson photoproduction is a production with the excitation or a breakup of the target nucleus:

\[ \gamma + A \rightarrow \pi^0 + A' \]  

(26)

The general expression for the incoherent cross section established in the literature \[27, 28\] is given by:

\[ \frac{d\sigma_{inc}}{d\Omega} = \frac{d\sigma_0}{d\Omega} (q) N(0,\sigma) [1 - G(t)] \]  

(27)
where \( \frac{d\sigma_0}{d\Omega} \) is the elementary cross section on the nucleon \( \gamma + N \rightarrow \pi^0 + N \), and
\[
N(0,\sigma) = \int \frac{1 - e^{-\sigma T(b)}}{\sigma} d^2b 
\]

Here \( T(b) = \int \rho(b,z)dz \). The factor \([1 - G(q)]\) takes into account the suppression of pions produced at small angles due to Pauli exclusion principle \([27]\) and goes to zero at small angles. But as was shown for the case of the proton scattering on nuclei \([29]\), the multiple scattering leads to the incoherent cross section, which is different from zero at zero angles. The same effects should also be in the pseudoscalar mesons photoproduction. Assuming that the meson photoproduction cross section on the nucleon is completely determined by the spin-nonflip amplitude, one can parametrize the differential cross section on the nucleon as
\[
\frac{d\sigma_0}{d\Omega} = c_p q^2 e^{-\alpha_p q^2} 
\]

Using this parametrization it can be shown\(^2\) that the incoherent cross section of the process (Eq. 26) can be represented as
\[
\frac{d\sigma_{inc}}{d\Omega} = \frac{d\sigma_0}{d\Omega}(q) \left[ N(0,\sigma) - \frac{|F_{st}(q)|^2}{A} \right] + c_p \xi^2 e^{-\alpha_p q^2} 
\]
\[
\xi^2 = \left| \frac{\sigma'}{2} \right|^2 \int \rho(b,z) \left| \frac{\partial T(b,z)}{\partial b} \right|^2 \exp (-\sigma T(b,z)) d^2bdz 
\]

Here \( F_{st}(q) \) is the strong form factor of the nucleus (Eq. 17). One can easily see that only in the case when absorption is absent \((\sigma = 0)\) the factorization similar to Eq. 27 takes place.

In Fig. 8 we plot the differential cross section for the \( \pi^0 \) photoproduction off carbon nucleus vs laboratory angle. The different curves gives the contribution of relevant mechanism discussed above. All calculations have been

\(^2\) Numerically this parametrization at small momentum transfer coincides with the predictions for \( \pi^0 \) photoproduction cross section on proton obtained \([30]\) in the framework of improved Regge theory

\(^3\) The derivation and detailed discussion of the incoherent production at small angles will be given elsewhere.
done using the expressions obtained in the present work. The spin-nonflip \( \pi^0 \) photoproduction amplitude on the nucleon was parametrized as \([31, 32]\):

\[
f(q) = 10 \left( \frac{k}{k_0} \right)^{1.2} \sin \theta e^{i\varphi}
\]

where \( \varphi \sim 1 \) radian and \( k_0 = 1 \) GeV. The differential cross section of the \( \pi^0 \) photoproduction on the proton was taken in accordance with existing experimental data \([33]\) at forward angles.

Figure 8: Differential cross section for \( \pi^0 \) photoproduction off carbon nuclei as a function of the production angle in the lab system. The long-dashed line shows the electromagnetic (Primakoff) contribution; the dotted line is the strong part; the dash-dotted line is the incoherent cross section; the long-dashed line is the interference between Primakoff and strong amplitudes; the solid line is the full cross section.
5 Summary

We have extended the theoretical treatment of forward production of pseudoscalar mesons off nuclei described in the literature in connection with the extraction of their radiative decay widths. Based on the Glauber theory of multiple scattering we have derived complete analytical formulas for the electromagnetic and strong form factors specific for the photoproduction of pseudoscalar mesons off both light and heavy nuclei, valid for realistic charge and matter distributions. The electromagnetic form factor for the non-diffractive photoproduction processes is derived for the first time. Special attention is paid to light nuclei, since with the increasing photon beam energies the precision decay width extractions is becoming more feasible from the light targets. We have included in our results the difference between the charge and matter distributions for the light nuclei. The photon shadowing effect in the reactions under consideration is correctly treated for the first time. The impact of this effect on the $\pi^0$ meson production in the strong field of both light and heavy nuclei are calculated. An expression for the incoherent photoproduction cross sections of the pseudoscalar mesons at forward angles is derived, which correctly takes into account the mesons’ final state interactions and the exclusion principle. Using the obtained expressions we calculate the differential cross section of the $\pi^0$ meson photoproduction off carbon nucleus and the contributions of different mechanisms considered above. All these give a good challenge to extract the lifetime of pseudoscalar mesons from forthcoming experimental data with high precision.

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