Physical vacuum as the source of the standard model particle masses

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Abstract

We present an effective model for mass generation of the standard model particles in which the fermions acquire their masses from their interactions with the physical vacuum and the gauge bosons from the charge fluctuations of the vacuum. A remarkable aspect of this model is that the left-handed neutrinos are massive because they have weak charge. We obtain consistently the masses of the electroweak gauge bosons in terms of the masses of the fermions and the running coupling constants of the strong, electromagnetic and weak interactions. In this paper we focus our interest in finding some phenomenological consequences of this effective model as for instance a restriction on the possible number of families, a prediction of the top quark mass, and an upper limit for the sum of the the square of the neutrino masses.

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1 Introduction

The Higgs mechanism is based on the fact that the potential must be such that one of the neutral components of the Higgs field doublet spontaneously acquires a non-vanishing vacuum expectation value. Since the vacuum expectation value of the Higgs field is different from zero, the Higgs field vacuum can be interpreted as a medium with a net weak charge. In this way the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry is spontaneously broken into the $SU(3)_C \times U(1)_{em}$ symmetry [1]. In the current picture of the Higgs mechanism [2] the masses of the particles are generated through the interactions among the electrically charged fermions and the electroweak gauge bosons with the weakly charged Higgs field vacuum.

We present an effective model for particle mass generation [3] in which the role of the Higgs field vacuum is played by the physical vacuum. This physical vacuum can be described as a virtual medium at zero temperature which is formed by massless fermions and antifermions that interact among themselves exchanging massless gauge bosons. The fundamental model describing the dynamics of the physical vacuum is the Standard Model without the Higgs Sector (SMWHS), which is based in the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. We assume that each fermion flavor in the physical vacuum has associated a chemical potential $\mu_f$ which describes an excess of virtual antifermions over virtual fermions.

In this effective model, the masses of the fermions are obtained starting from their self-energies, which represent the fundamental interactions of a massless fermion with the physical vacuum [4]. On the other hand, gauge bosons masses are obtained from the charge fluctuations of the physical vacuum, which are described by the vacuum polarization tensors [4]. A remarkable fact of our model is that the left-handed neutrinos are massive because they have weak charge. The weak interaction among the massless left-handed neutrinos and the physical vacuum is the source of their masses.

We find that the masses of the fermions and gauge bosons are functions of the vacuum fermionic chemical potentials $\mu_f$ which are unknown input parameters in this effective model. In this way we can obtain the masses of the electroweak gauge bosons in terms of the masses of the fermions and the running coupling constants of the strong, electromagnetic and weak interactions.

Before considering the real physical vacuum which is described by the SMWHS, in section 2 we first present a more simple case in which the dynamics of the vacuum is described by a non-abelian gauge theory, and in this context we introduce the masses of a fermion and a gauge boson. In section 3 we consider the SMWHS as the model which describes the dynamics of the physical vacuum and we obtain the fermion masses (quarks and leptons) and those of the electroweak gauge bosons ($W^\pm$ and $Z^0$). We obtain consistently the masses of the electroweak gauge bosons in terms of the masses of the fermions and the running coupling constants of the three fundamental interactions.
In section 4 we focus our interest in finding a restriction about the possible number of families, a prediction of the top quark mass, and an upper limit for the sum of the neutrino squared masses. Our conclusions are summarized in section 5.

2 Mass generation in a non-abelian gauge theory

We initially study the case in which the dynamics of the vacuum is described by mean of a gauge theory invariant under the non-abelian gauge group SU(N). Consequently the physical vacuum is thought to be a quantum medium at zero temperature constituted by virtual massless fermions and antifermions interacting among them through the N-1 massless gauge bosons. We assume that there is an excess of virtual antifermions over virtual fermions in the vacuum. This antimatter-matter asymmetry of the vacuum is described by non-vanishing fermionic chemical potentials \( \mu_f \), where \( f_i \) represents the different fermion species. For simplicity, we take \( \mu_{f_1} = \mu_{f_2} = \ldots = \mu_f \). The fermion mass is generated by the SU(N) gauge interaction among the massless fermions with the vacuum. The charge fluctuations of the vacuum is the source of the mass of the gauge bosons.

We follow the next general procedure to calculate the particle masses: (i) Initially we write the one-loop self-energies and the one-loop polarization tensors at finite density and finite temperature. (ii) Next we calculate the dispersion relations from the poles of the fermion and gauge boson propagators. (iii) Starting from these dispersion relations we obtain the fermion and gauge boson effective masses at finite density and finite temperature. (iv) Finally we identify these particle effective masses at zero temperature with the physical particle masses. This identification can be performed because the virtual medium at zero temperature is representing the physical vacuum.

It is well known in the context of the quantum field theory at finite temperature and density, that as a consequence of the statistical interactions among massless fermions with a medium at temperature \( T \) and fermionic chemical potential \( \mu_f \), the fermions acquire an effective mass \( M_F \) given by [5]:

\[
M_F^2(T, \mu_f) = \frac{g^2 C(R)}{8} \left( T^2 + \frac{\mu_f^2}{\pi^2} \right),
\]

being \( g \) the interaction coupling constant and \( C(R) \) the quadratic Casimir invariant of the representation of the gauge group SU(N). For the fundamental representation, \( C(R) = (N^2 - 1)/2N \) [6]. The expression for \( M_F^2 \) given by [1] is in agreement with [7]-[10]. For the case in which the interaction among the massless fermions with the medium is mediated by U(1) abelian gauge bosons, the effective mass of the fermions is also given by the expression [1] but now \( g^2 C(R) \to e^2 \), being \( e \) the interaction coupling constant associated with the U(1) gauge group. The effective mass of the fermions is
gauge invariant because it was obtained at leading order in temperature and chemical potential [10]. We are interested in the effective mass at $T = 0$, which corresponds precisely to the case in which the vacuum is described by a virtual medium at zero temperature. For this case, the effective mass of the fermion is:

$$M_F^2(0, \mu_F) = M_F^2 = \frac{g^2C(R)\mu_f^2}{8\pi^2}. \quad (2)$$

In the limit $k \ll M_F$, the fermion dispersion relation can be written as [3]:

$$\omega^2(k) = M_F^2 \left[1 + \frac{2}{3} \frac{k}{M_F} + \frac{5}{9} \frac{k^2}{M_F^2} + \ldots\right] \quad (3)$$

It is very well known that the relativistic energy in the vacuum for a massive fermion at rest is $\omega^2(0) = m_f^2$. It is clear from (3) that if $k = 0$ then $\omega^2(0) = M_F^2$ and thereby we can identify the fermion effective mass at zero temperature as the rest mass of the fermion, i.e. $m_f = M_F$. By this reason, we can conclude that the gauge invariant fermion mass, which is generated from the SU(N) gauge interaction of the massless fermion with the vacuum, is:

$$m_f^2 = \frac{g^2C(R)\mu_f^2}{8\pi^2}. \quad (4)$$

On the other hand, as a consequence of the charge fluctuations of the medium, the non-abelian gauge boson acquires an effective mass $M_{B(na)}$ given by [5]:

$$M_{B(na)}^2(T, \mu_f) = \frac{1}{6}Ng^2T^2 + \frac{1}{2}g^2C(R) \left[\frac{T^2}{6} + \frac{\mu_f^2}{2\pi^2}\right], \quad (5)$$

being $N$ the gauge group dimension. The non-abelian effective mass [5] it was also calculated in [3]. If the dynamics of the medium were described by mean of a U(1) gauge invariant theory, the abelian gauge boson would acquire an effective mass $M_{B(a)}$ given by [5]

$$M_{B(a)}^2(T, \mu_f) = e^2 \left[\frac{T^2}{6} + \frac{\mu_f^2}{2\pi^2}\right]. \quad (6)$$

The abelian effective mass [6] is in agreement with [11]. Because the vacuum is described by a virtual medium at $T = 0$, then the non-abelian gauge boson effective mass generated by the quantum fluctuations of the vacuum is:

$$M_{B(na)}^2(0, \mu_f) = M_{B(na)}^2 = g^2C(R)\frac{\mu_f^2}{4\pi^2}, \quad (7)$$

and the abelian gauge boson effective mass is:

$$M_{B(a)}^2(0, \mu_f) = M_{B(a)}^2 = e^2\frac{\mu_f^2}{2\pi^2}. \quad (8)$$
in agreement with the result obtained at finite density and zero temperature \[12\]. For the limit \( k \ll M_B \), it is possible to obtain the dispersion relations for the transverse and longitudinal propagation modes \[13\]:

\[
\omega^2_L = M^2_B + \frac{3}{5} k_L^2 + \ldots
\]

\[
\omega^2_T = M^2_B + \frac{6}{5} k_T^2 + \ldots
\]

It is clear from (9) and (10) that for \( k = 0 \) then \( \omega^2(0) = M^2_B \) and it is possible to recognize the gauge boson effective mass as the true gauge boson mass. The non-abelian gauge boson mass is:

\[
m^2_{b(\text{na})} = M^2_{B(\text{na})} = g^2 C(R) \frac{\mu^2_f}{4\pi^2},
\]

and the abelian gauge boson mass is:

\[
m^2_{b(\text{a})} = M^2_{B(\text{a})} = e^2 \frac{\mu^2_f}{2\pi^2}.
\]

We observe that the gauge boson mass is a function on the chemical potential that is a free parameter in this effective model. It is important to note that if the fermionic chemical potential has an imaginary value, then the gauge boson effective mass, given by (11) or (12), would be negative \[14\].

### 3 Fermions and electroweak gauge bosons masses

In this section we present the fermion and gauge boson masses for the case in which the dynamics of the physical vacuum is described by mean of the SMWHS. The dynamics of the vacuum associated with the strong interaction is described by Quantum Chromodynamics (QCD), while the electroweak dynamics of the physical vacuum is described by the \( SU(2)_L \times U(1)_Y \) electroweak standard model without a Higgs sector. The physical vacuum is assumed to be a virtual medium at zero temperature constituted by virtual massless quarks, antiquarks, leptons and antileptons interacting among them through massless \( G \) gluons (for the case of the quarks and antiquarks), massless \( W^\pm \) electroweak gauge bosons, massless \( W^3 \) gauge bosons and massless \( B \) bosons. In this quantum medium there is an excess of virtual antifermions over virtual fermions. This fact is described by non-vanishing chemical potentials associated with the different fermion flavors. The chemical potentials are represented for the six quarks by the six symbols \( \mu_u, \mu_d, \mu_c, \mu_s, \mu_t, \mu_b \). For the chemical potentials of the charged leptons we use \( \mu_e, \mu_\mu, \mu_\tau \) and for neutrinos \( \mu_{\nu_e}, \mu_{\nu_\mu}, \mu_{\nu_\tau} \). These non-vanishing chemical potentials are free parameters in the effective model of particle mass generation.
Considering the Feynman rules of the SMWHS we can calculate the self-energies for each of the six quark flavors. The one-loop Feynman diagrams which contribute to the self-energies of the left-handed quarks \( i (i = u_L, c_L, t_L) \) are shown in Figure 1. Using the general expression for the fermion mass given by (4), we obtain that the left-handed quarks masses are [3]:

\[
m_i^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right] \frac{\mu_i^2}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu_{i_L}^2}{8\pi^2},
\]

\[
m_i^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right] \frac{\mu_i^2}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu_{i_L}^2}{8\pi^2},
\]

being \( g_s, g_w \) and \( g_e \) the running coupling constants of the strong, weak and electromagnetic interactions, respectively. In the expressions (13) and (14) the couple of indexes \((i, I)\) run over left-handed quarks \((u_L, d_L), (c_L, s_L)\) and \((t_L, b_L)\). We can identify in (13) and (14) the contributions to the masses of the left-handed quarks from the \( G, W^3, B \) and \( W^\pm \) interactions among the massless left-handed quarks and the physical vacuum.

\[
\begin{align*}
\text{Figure 1: Feynman diagrams contributing to the self-energy of the left-handed quark } i.
\end{align*}
\]

If we call

\[
a_q = \frac{1}{8\pi^2} \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right],
\]

\[
b_q = \frac{1}{8\pi^2} \left[ \frac{1}{2} g_w^2 \right],
\]

it is easy to see that the quark masses (13) and (14) lead to

\[
\mu_{u_L}^2 = \frac{a_q m_u^2 - b_q m_d^2}{a_q^2 - b_q^2},
\]

\[
\mu_{d_L}^2 = \frac{-b_q m_u^2 + a_q m_d^2}{a_q^2 - b_q^2},
\]
and similar expressions for the other two quark doublets \((c_L, s_L)\) and \((t_L, b_L)\).

The masses of the left-handed leptons are obtained considering the contributions to the lepton self-energies. We obtain that these masses are given by [3]:

\[
m^2_i = \left(\frac{1}{4} g_w^2 \frac{\mu^2_{iL}}{8\pi^2} + \frac{\mu^2_{iL}}{8\pi^2}\right),
\]

(19)

\[
m^2_I = \left(\frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right) \frac{\mu^2_{I_L}}{8\pi^2} + \left[\frac{1}{2} g_w^2 \frac{\mu^2_{I_L}}{8\pi^2}\right],
\]

(20)

where the couple of indexes \((i, I)\) run over leptons \((\nu_{eL}, e_L), (\nu_{\mu L}, \mu_L)\) and \((\nu_{\tau L}, \tau_L)\). A remarkable fact of our model is that the left-handed neutrinos are massive because they have weak charge. The \(W^3\) and \(W^\pm\) interactions among the massless neutrinos with the physical vacuum are the origin of the left-handed neutrinos masses, as we can observe from (19).

If we introduce the following definitions

\[
a_l = \frac{1}{8\pi^2} \left[\frac{1}{4} g_w^2 \right],
\]

(21)

\[
b_l = \frac{1}{8\pi^2} \left[\frac{1}{2} g_w^2 \right],
\]

(22)

\[
c_l = \frac{1}{8\pi^2} \left[\frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right],
\]

(23)

then the lepton masses (19) and (20) lead to

\[
\mu_{\nu_L}^2 = \frac{c_l m^2_{\nu} - b_l m^2_{e}}{a_l c_l - b_l^2},
\]

(24)

\[
\mu_{\nu_{eL}}^2 = \frac{-b_l m^2_{\nu} + a_l m^2_{e}}{a_l c_l - b_l^2},
\]

(25)

and similar expressions for the other two lepton doublets \((\nu_{\mu L}, \mu_L)\) and \((\nu_{\tau L}, \tau_L)\).

It is possible to observe that for five of the six fermion doublets the square of the left-handed chemical potential associated to the down fermion of the doublet has a negative value. This behavior is observed if there is a large difference between the masses of the two fermions of the doublet. In other words, the mentioned behavior is not observed for the left-handed quark doublet which is formed by the up and down quarks, because the two quark mass values are very close. In this case, the chemical potentials associated to these two left-handed quarks are positive.

On the other hand, applying the expressions (11) and (12) in the SMWHS, we
obtain that the masses of the gauge bosons are [3]:

\[ M_{W^\pm}^2 = \frac{g_w^2}{2} \left( \mu_{u_L}^2 + \mu_{d_L}^2 + \mu_{e_L}^2 - \mu_{s_L}^2 + \mu_{c_L}^2 - \mu_{b_L}^2 + \sum_{i=1}^{3} (\mu_{\nu_{iL}}^2 - \mu_{\nu_{eL}}^2) \right), \quad (26) \]

\[ M_{W^3}^2 = \frac{g_w^2}{4} \frac{\mu_{u_L}^2 + \mu_{d_L}^2 + \mu_{e_L}^2 - \mu_{s_L}^2 + \mu_{c_L}^2 - \mu_{b_L}^2 + \sum_{i=1}^{3} (\mu_{\nu_{iL}}^2 - \mu_{\nu_{eL}}^2)}{2\pi^2}, \quad (27) \]

\[ M_B^2 = \frac{g_e^2}{4} \frac{\mu_{u_L}^2 + \mu_{d_L}^2 + \mu_{e_L}^2 - \mu_{s_L}^2 + \mu_{c_L}^2 - \mu_{b_L}^2 + \sum_{i=1}^{3} (\mu_{\nu_{iL}}^2 - \mu_{\nu_{eL}}^2)}{2\pi^2}, \quad (28) \]

where the sum running over the three leptons families. It is important to remember that if the fermionic chemical potential has an imaginary value, then its contribution to the gauge boson effective mass, as in the case (11) or (12), would be negative. This fact means that finally the contribution from each fermionic left-handed chemical potential to the gauge boson masses is always positive.

Due to well known physical reasons, the \( W^3 \) and \( B \) gauge bosons are mixed. After diagonalizing the mass matrix, we get the physical fields \( A_\mu \) and \( Z_\mu \) corresponding to the massless photon and the neutral \( Z^0 \) boson of mass \( M_Z \) respectively, with the relations [15, 16]:

\[ M_Z^2 = M_W^2 + M_B^2, \quad (29) \]
\[ \cos \theta_w = \frac{M_W}{M_Z}, \quad \sin \theta_w = \frac{M_B}{M_Z}, \quad (30) \]

where \( \theta_w \) is the weak mixing angle:

\[ Z^0 = B_\mu \sin \theta_w - W^3_\mu \cos \theta_w, \quad (31) \]
\[ A_\mu = B_\mu \cos \theta_w + W^3_\mu \sin \theta_w. \quad (32) \]

Substituting the expressions for the fermionic left-handed chemical potentials given by (17), (18), (24), (25) into the expressions (26), (27), (28), we obtain the masses of the electroweak gauge bosons \( W \) and \( Z \) in terms of the masses of the fermions and the running coupling constants of the strong, weak and electromagnetic interactions. The masses of the electroweak gauge bosons are [3]

\[ M_W^2 = g_w^2 (A_1 + A_2 + A_3 - A_4), \quad (33) \]
\[ M_Z^2 = (g_e^2 + g_w^2) (A_1 + A_2 + A_3 - A_4), \quad (34) \]
where the parameters $A_1$, $A_2$, $A_3$ and $A_4$ are

\[
A_1 = \frac{m_u^2 + m_d^2}{B_1},
\]

(35)  

\[
A_2 = \frac{m_c^2 - m_s^2 + m_t^2 - m_b^2}{B_2},
\]

(36)  

\[
A_3 = \frac{3(m_e^2 + m_e^2 + m_\mu^2)}{B_3},
\]

(37)  

\[
A_4 = \frac{(3 + g_e^2/g_w^2)(m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau})}{B_3},
\]

(38)  

being

\[
B_1 = \frac{4}{3}g_s^2 + \frac{3}{4}g_w^2 + \frac{1}{4}g_e^2,
\]

(39)  

\[
B_2 = \frac{4}{3}g_s^2 - \frac{1}{4}g_w^2 + \frac{1}{4}g_e^2,
\]

(40)  

\[
B_3 = \frac{3}{4}g_w^2 - \frac{1}{4}g_e^2.
\]

(41)  

It is clear that if we take the experimental central values for the strong coupling constant at the $M_Z$ scale as $\alpha_s(M_Z) = 0.1176$, the fine-structure constant as $\alpha_e = 7.2973525376 \times 10^{-3}$ and the cosine of the electroweak mixing angle as $\cos \theta_w = M_W/M_Z = 80.403/91.1876 = 0.881732$ \cite{17}, then $g_s = 1.21565$, $g_w = 0.641911$ and $g_e = 0.34344$. Putting these values for $g_s$, $g_w$ and $g_e$ and the central values for the masses of the electrically charged fermions, given by \cite{17}, $m_u = 0.00225$ GeV, $m_c = 1.25$ GeV, $m_s = 0.095$ GeV, $m_t = 172.371$ GeV, $m_b = 4.20$ GeV, $m_e = 0.51099892 \times 10^{-3}$ GeV, $m_\mu = 0.105658369$ GeV, $m_\tau = 1.77699$ GeV, into the expressions (33) and (34), and assuming the neutrinos to be massless, $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$, we obtain that the theoretical masses of the $W$ and $Z$ electroweak gauge bosons are

\[
M_{W^\pm} = 80.403 \text{ GeV}
\]

(42)  

\[
M_Z = 91.1876 \text{ GeV}.
\]

(43)  

These theoretical masses are in agreement with theirs experimental values given by $M_W^{\exp} = 80.403 \pm 0.029$ GeV and $M_Z^{\exp} = 91.1876 \pm 0.0021$ GeV \cite{17}. The parameters $A_1$, $A_2$, $A_3$ and $A_4$ in the expressions (33) and (34) have the following values $A_1 = 1.30201 \times 10^{-5}$, $A_2 = 15655$, $A_3 = 34.0068$ and $A_4 = 0$. We observe that $A_2$ is very large respect to $A_3$ and $A_1$. Through the definition of the parameter $A_2$, given by (36), it is possible to conclude that the masses of the electroweak gauge bosons coming specially from the top quark mass $m_t$ and the strong running coupling constant $g_s$. 


Neutrino masses are not known but direct experimental results shown that neutrino masses are of order 1 eV \[17\], and cosmological interpretations from five-year WMAP observations find a limit on the total mass of massive neutrinos of $\Sigma m_\nu < 0.6$ eV (95% CL) \[18\]. These results ensure that the values of the left-handed lepton chemical potentials obtained taking neutrinos to be massless will not change much if we take the true small values for the neutrino masses.

4 Some consequences of our model

We note that expressions (33) and (34) establish a relation very close among the twelve fermion masses and the three interaction running coupling constants with the masses of the $W$ and $Z$ electroweak gauge bosons. This fact allows to obtain some phenomenological consequences that we present below.

We conclude that the expressions (33) and (34), due to the experimental uncertainties for the electroweak gauge bosons masses, restrict the existence of a new fourth family of fermions in the SMWHS. We can arrive to this conclusion if we represent the two new leptons as $\nu_n$ and $l_n$, the two new quarks as $u_n$ and $d_n$, and the masses of these fermions by $m_{\nu_n}$, $m_{l_n}$, $m_{u_n}$ and $m_{d_n}$. We hope that the masses of these fermions should be more heavy than the ones of the third family. These masses can satisfy: (i) The non-hierarchy condition given by $m_{\nu_n} \sim m_{l_n}$ and $m_{u_n} \sim m_{d_n}$; (ii) the hierarchy condition expressed as $m_{\nu_n} \ll m_{l_n}$ and $m_{u_n} \gg m_{d_n}$. For the first condition, the expressions (33) and (34) are modified by the inclusion of terms which are proportional to $m_{\nu_n}^2 + m_{l_n}^2$ and $m_{u_n}^2 + m_{d_n}^2$. For the second condition, these expressions are modified by terms which are proportional to $m_{l_n}^2 - m_{\nu_n}^2$ and $m_{u_n}^2 - m_{d_n}^2$. Both cases are strongly suppressed by the experimental uncertainties for the electroweak gauge bosons masses. In this form, our model establishes a strong restriction to the existence of a new fourth fermion family in the SMWHS.

We also obtain a prediction for the top quark mass starting from the expression (34). Using the experimental values and their uncertainties for the $Z$ and $W$ electroweak gauge boson, for the coupling constants of the strong and electromagnetic interactions and for the masses of the electrically charged fermions, and assuming the neutrinos to be massless, we predict from (34) that the top quark mass is $m_{t}^{th} = 172.371 \pm 1.679$ GeV. This theoretical value is in agreement with the experimental value for the top quark mass given by \[17\] $m_{t}^{ex} = 172.5 \pm 2.7$ GeV.

If we write (38) as

$$A_4 = \frac{(3 + g_2^2/g_w^2)(\Sigma m_\nu^2)}{B_3},$$

from (34) we obtain that the sum of the squares of the neutrino masses $\Sigma m_\nu^2$ can be
written as
\[ \Sigma m^2_\nu = \left[ A_1 + A_2 + A_3 - \frac{M^2_{Z_{\text{min}}}}{g^2_\nu + g^2_{w}} \right] \frac{B_3}{3 + g^2_c/g^2_w}, \]  
(45)

being \( M_{Z_{\text{min}}} \) the smallest experimental value for the Z mass given by \( M_{Z_{\text{min}}} = 91.1855 \) GeV. Using the central experimental values for the masses of the fermions and for the running coupling constants that we have used in section 3, we obtain that \( \Sigma m^2_\nu = 0.06117 \) GeV\(^2\). The effective model for mass generation that we have presented here predicts that the left-handed neutrinos are massive, but it can not predict the values of their masses because the fermionic chemical potentials of the vacuum are free parameters. However we find an upper limit for the sum of the squares of the neutrino masses given by \( \Sigma m^2_\nu < 0.06117 \) GeV\(^2\).

5 Conclusions

We presented an effective model for particle mass generation in which we extracted some generic features of the Higgs mechanism that do not depend on its interpretation in terms of a Higgs field. The physical vacuum has been assumed to be a medium at zero temperature which is formed by virtual massless fermions and antifermions interacting among themselves by means of massless gauge bosons. The fundamental effective model describing the dynamics of this physical vacuum is the SMWHS. We have assumed that each fermion flavor in the physical vacuum has associated with it a chemical potential \( \mu_f \) in such a way that there is an excess of virtual antifermions over virtual fermions. This fact implies that the vacuum is thought to be a virtual medium having a net antimatter finite density.

Fermion masses are calculated starting from the fermion self-energy, which represents the fundamental interactions of a massless fermion with the physical vacuum. The gauge boson masses are calculated from the charge fluctuations of the physical vacuum, which are described by the vacuum polarization tensor. Using this effective model for particle mass generation, we obtain the masses of the electroweak gauge bosons in agreement with their experimental values.

A further result of this effective model is that the left-handed neutrinos are massive because they have weak charge. Additionally our model establishes a strong restriction to the existence of a new fourth fermion family in the SMWHS. As a consequence of this model we predict that the top quark mass is \( m^{th}_t = 172.371 \pm 1.679 \) GeV. Finally, we obtain an upper limit for the sum of the squares of the neutrino masses given by \( \Sigma m^2_\nu < 0.06117 \) GeV\(^2\).
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