Simple method of optical ring cavity design and its applications

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Abstract: When an optical ring cavity is designed, the beam radii at some special positions, especially at the beam waists are very interested in, since the gain mediums, nonlinear crystals and others important optical elements are generally located at the beam waist. In this paper, we firstly presented a simple method for designing optical ring cavities based on the self-consistency theory and the fact that q parameter is uniquely determined by the waist beam radius and its position. This approach is different from ABCD method and it no longer requires cumbersome calculation. The calculations of designing optical ring cavities are simplified because q parameter only has imaginary part at beam waist plane. Moreover, designing the resonant cavity through the calculation of beam waist radii and their position has great practical significance, because it is very easy to adjust the waist radii and the positions at the important optical elements. We employed this method to design an end-pumped six-mirror ring cavity continuous-wave passively mode locked laser. The experiment of a highly stabilized continuous-wave mode locked (CWML) laser was investigated and the results coincided with the theoretical studies very well. The investigation results show that the simple method can be used to design optical ring cavities conveniently, intuitively and efficiently. ©2014 Optical Society of America

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References and links

1. M. V. Okhapkin, M. N. Skvortsov, A. M. Belkin, N. L. Kvashnin, and S. N. Bagayev, “Tunable single-frequency diode-pumped Nd:YAG ring laser at 1064/532 nm for optical frequency standard applications,” Opt. Commun. 203(3-6), 359–362 (2002).
2. R. L. Folk, B. I. Greene, and C. V. Shank, “Generation of optical pulses shorter than 0.1 psec by colliding pulse mode locking,” Appl. Phys. Lett. 38(9), 671–672 (1981).
3. H. Zhao, P. Wang, Z. Y. Wei, J. R. Tian, D. H. Li, Z. H. Wang, and J. Zhang, “Highly efficient and stable ring regenerative amplifier for chirped-pulse amplification at repetition rate 1 kHz,” Chin. Phys. Lett. 24(1), 115–118 (2007).
4. S. Diddams, B. Atherton, and J. C. Diels, “Frequency locking and unlocking in a femtosecond ring laser with application to intracavity phase measurements,” Appl. Phys. B 63(5), 473–480 (1996).
5. S. Schwartz, G. Feugnet, and J. P. Pocholle, “Biasing the beat regime of a solid-state ring laser: from a magnetometer to a multioscillator rotation sensor,” J. Opt. Soc. Am. B 30(8), 2157–2160 (2013).
6. W. W. Chow, J. Gea-Banacloche, L. M. Pedrotti, V. Sanders, W. Schleich, and M. Scully, “The ring laser gyro,” Rev. Mod. Phys. 57(1), 61–104 (1985).
7. W. W. Rigrod, “The optical ring resonator,” Bell Syst. Tech. J. 44(5), 907–916 (1965).
1. Introduction

Compared to linear cavities, laser rings have certain properties that make them useful for a number of applications. The round trip dispersion caused by the active medium in ring laser is half of that in a linear laser since the pulse travels only once per round trip in the active medium in ring laser. The thermal properties in such lasers are also better for the same reason, which reduce the losses associated with thermally induced aberrations. In addition, more stored energy in the active medium could be obtained because of the absence of the effect of spatial hole burning in a travelling-wave ring laser. Consequently, an optical ring cavity which is inherently stable, is especially desirable to be used in solid-state single-frequency lasers [1], colliding pulse mode locking lasers [2], regenerative amplifiers [3] and so on. Furthermore, ring lasers have numerous potential applications in areas such as ultra-sensitive detectors of phase shifts [4], sensors [5], frequency standard [1], and in particular, laser gyroscopes [6].

The performance of a laser largely depends on the cavity design quality, and an optical cavity plays an important role in a laser especially in a ring laser. A significant number of research works [7–17] concentrated on the method of designing optical ring cavities based on $ABCD$ matrix were reported. However, a simple and powerful method for designing optical ring cavities is needed. The conventional method of optical cavity design mainly uses transform circle graphic theory [15–17] and $ABCD$ matrix [7–14]. It is intuitive and reliable to design linear cavity configuration lasers by using transform circle graphic theory, but no one has used it to design ordinary optical ring cavities so far. $ABCD$ matrix is usually used to calculate the spot size and curvature radius of an optical cavity with certain parameters. Though it is suitable for analyzing resonator, it is not convenient to design an optical cavity, especially for an optical ring cavity, owing to the tedious calculations to implement the analysis of optical cavities. Taking into account the complexity of optical ring cavities, it is very difficult and blindfold to use $ABCD$ matrix to design cavities by modifying the cavity parameters. Moreover, when an optical ring cavity is designed, not every beam radius and curvature radius at each position of an optical ring cavity are interested in. Only those at some special positions, especially at the beam waist need to be paid close attention to, since saturable absorber elements, nonlinear crystals and others important optical elements are generally located at the beam waist. If we only consider the beam radius and curvature radius at beam waists in a ring cavity, designing ring cavity no longer requires cumbersome calculation. It becomes very simple and more effective.

In this letters, we present a simple but useful method to design optical ring cavity, based on the self-consistency theory and the fact that $q$ parameter is uniquely determined by the beam waist radius and its position. This method can be used to design optical ring cavities.
efficiently, and easily design suitable beam waist radii in specific positions. We employ this method to design an end-pumped six-mirror ring cavity continuous-wave passively mode locked laser. The experiment of a highly stabilized continuous-wave mode locked (CWML) laser is investigated and the results coincide with the theoretical studies very well.

2. Method for optical cavity design

The Gaussian field envelope is assumed to be the following expression:

\[ U = \frac{A_0}{q} \exp\left(-j \frac{\pi r^2}{4q}\right). \]  

(1)

Here \( A_0 \) is the amplitude, \( \lambda \) is the wavelength, \( r \) denotes the radial distance from the center axis of the beam. The complex \( q \) parameter is related to the beam radius \( w \) and the radius of phase front curvature \( R \) by the following form:

\[ \frac{1}{q} = \frac{1}{R} - j \frac{\lambda}{\pi w^2}. \]  

(2)

The parameters \( R \) and \( w \) can be recovered from the real and imaginary parts of \( 1/q \). We regard \( q_1 \) and \( q_2 \) as the beam parameters in the input and output planes of an optical system described by its ray \((ABCD)\) matrix. We see that

\[ q_2 = \frac{Aq_1 + B}{Cq_1 + D}, \]  

(3)

which is a \( ABCD \) law. If we let \( q_1 \) be a complex beam parameter of the Gaussian beam leaving a plane in a resonator, the complex \( q \) after one round-trip must equal \( q_1 \). This is the \( q \) parameter self-consistency, \( i.e. \) a traditional laser analysis and design method. Self-consistency condition requires a Gaussian beam in the resonator is the one that reproduces itself after one round trip. Using the self-consistency condition, the beam parameter of Gaussian at a plane in a general resonator can be obtained by solving the Eq. (3). Generally speaking, \( ABCD \) matrix of an optical cavity, especially optical ring cavity, is very complicated. The complex \( q \) parameter has real and imaginary parts, therefore using the traditional method to solve equation and design optical cavities need tedious calculations. If we consider that the complex \( q \) parameter only has imaginary part at beam waist positions, designing optical ring cavities will become simple and easy.

As the beam curvature \( R \) approaches infinity at beam waist, the \( q \) parameter is uniquely determined by the beam waist radius \( w_0 \) and its position. The \( q \) parameter at beam waist plane can be simplified as:

\[ \frac{1}{q} = j \frac{\lambda}{\pi w_0^2}. \]  

(4)

Obviously the formula (4) is simpler obviously compared with the formula (2). Furthermore, the beam radius at some special positions, especially at the beam waist are most concerned by researchers, when they design a laser cavity, in on account of the fact that saturable absorber elements, nonlinear crystals and others important optical elements are generally located at the beam waist positions. Therefore, the simplification of \( q \) parameter not only makes the design of optical cavities become simple and easy. Designing the resonant cavity through the calculation of beam waist radii and their position has great practical significance.

In the following, we will present a simple method to design an optical ring cavity based on the self-consistency theory and the fact that \( q \) parameter is uniquely determined by the beam waist radius and its position. In our approach, we design an optical ring cavity through the calculation of beam waist radii and their positions. When the Gaussian beam propagates...
through a thin lens or spherical mirror, the relations between the beam waist radii before and after the lens can be found, yielding:

\[ w_0^2 = \frac{f^2 w_0^2}{(l - f)^2 + \left(\frac{\pi w_0^2}{\lambda}\right)^2}, \tag{5} \]

\[ l' = f + \frac{(l - f) f^2}{(l - f)^2 + \left(\frac{\pi w_0^2}{\lambda}\right)^2}. \tag{6} \]

\( w_0 \) and \( l \) represent for two parameters of the Gaussian beam, i.e. the beam waist radius and the spacing between the beam waist position and the thin lens, respectively. The new Gaussian beam is characterized by the parameters of the new beam waist \( w_0' \) and the distance \( l' \) away from the lens. The sign with superscript ' represents in image space, while the sign without superscript represents in object space. The subscript 0 denotes the parameters of Gaussian beam at waist. The parameter \( f \) is the lens focal length and \( b_0 = \pi w_0^2/\lambda \) is the Rayleigh range.

![Fig. 1. Configuration of an optical ring cavity.](image)

In the following, we will take a typical optical ring cavity consisting of three curved mirrors as an example and describe a simple method of optical ring cavity design based on the self-consistency of Gaussian beam waist. Figure 1 is a schematic drawing that illustrates a triangular laser ring resonator. For the sake of convenience, the thermal lens effect and the astigmatism in the optical ring cavity are not considered. Plane reflection mirrors can be inserted into the three curved mirrors cavity, which constitutes other shapes multiple mirrors ring cavity, such as four-mirror ring cavity, five-mirror or six-mirror ring cavity. The insertion plane mirrors do not affect the distribution of the spot radii and the curvature radii in a ring cavity. \( M_1, M_2 \) and \( M_3 \) represent curved mirrors. \( w_{01}, w_{02}, w_{03} \) are beam waist radii and \( l \) denotes the spacing between the beam waist position and the curved mirrors. It is assumed...
that a Gaussian beam with a beam waist radius $w_{01}$ and a distance $l_{11}$ away from the curved mirror $M_1$ propagate through curved mirrors $M_1$ and $M_2$, respectively. Using formula (5) and (6), we can obtain a new Gaussian beam, which has a beam waist radius $w_{03}$ and a distance $l_{32}$ away from the curved mirror $M_2$ in image space. Properly choosing of a curvature radius and a location of the mirror $M_3$, we can achieve a new Gaussian beam which satisfies the following two conditions. Firstly, the beam waist is located at the original beam waist position where away from the curved mirror $M_1$ a distance $l_{11}$. Secondly, the beam waist radius is equal to the original beam waist radius $w_{01}$. Satisfying these two conditions implies that the Gaussian beam can be self-consistency when it propagates through in a ring cavity one circle. 

In brief, a beam waist radius and its position of a Gaussian beam are firstly selected according to the practical situation of the application and experimental requirements. When the Gaussian beam passes through a curved mirror or a lens, the parameters of beam waist are changed. The beam waist radius and its position can be obtained by using the above formulas. A given Gaussian beam propagates for a cycle in an optical cavity. It returns to the original position, where is exactly the new beam waist position, and the news beam waist radius is equal to the old one. This is the Gaussian beam self-consistency of optical ring cavity. In this case the cavity is a stable cavity. The beam waist radii of each arm and their positions are as described above.

![Image](image_url)

Fig. 2. Beam waist radius $w_{02}$ and the distance $l_{22}$ between the beam waist position and the curved mirrors in image space vary with the distance $l_{12}$ in the object space. The calculation parameters of the radii of curvature of concave mirrors $M_i$ is 200 mm.

According to the actual situation, we choose the beam waist $w_{01} = 60 \mu m$ in which is used to place a semiconductor saturable absorber mirror (SESAM), and select a 200 mm curvature radius common curved mirror as mirror $M_1$. Many variations of the beam waist radius at the different beam waist position can be obtained in the image space of mirror $M_1$ by changing the distance $l_{11}$. Figure 2 shows the beam waist radius $w_{02}$ and the distance $l_{21}$ in image space vary with the distance $l_{11}$ in the object space, respectively. It can be seen from Fig. 2 that beam waist radius $w_{02}$ reaches a maximum value 564 $\mu m$, when $l_{11}$ is equal to the focal length.
100 mm of curved mirror $M_1$. In this circumstance, $l_{21}$ is equal to the focal length 100 mm which represents the beam waist is located on the image focal plane of the curved mirror $M_1$. Except the beam waist radius maximum value, a definite size beam waist radius can be obtained at two different positions corresponding to two different values $l_{21}$ (for a negative $l_{21}$ one has a virtual beam waist). It can also be shown from Fig. 2 that the variations of $l_{21}$ are evident for the case that the beam waist is nearly located at the focal plane of curved mirror $M_1$ but are slight for the case that the beam waist is nearly located at close to or far away from the curved mirror $M_1$. Figure 3 gives the beam waist radius $w_{03}$ and the distance $l_{32}$ as a function of the distance $l_2$ between the curved mirrors $M_1$ and $M_3$, respectively. It can be seen from Fig. 3 that the beam waist radius $w_{03}$ and the distance $l_{32}$ don’t change monotonously as the distance $l_2$ is increased.

Fig. 3. Beam waist radius $w_{03}$ and the distance $l_{32}$ as a function of the distance $l_2$ between the curved mirrors $M_1$ and $M_3$, respectively. The calculation parameters of the radii of curvature of concave mirrors $M_1$ and $M_2$ are 200 mm and 500 mm respectively, $l_{11}$ = 13 cm.

The Gaussian beam waist radius $w_{01}$, $w_{02}$, $w_{03}$ as well as their distances which are away from $M_1$ and $M_2$ are determined, respectively. The following study will focus on how to choose and where to place the curved mirror $M_3$. Appropriately selecting the curved mirror $M_3$ make the Gaussian beam waist $w_{03}$ match the Gaussian beam waist $w_{01}$. By using formula (5) and self-consistency condition, the beam waist radius $w_{01}$ can be described as:

$$w_{01}^2 = \frac{f_1^2 w_{03}^2}{(l_{33} - f_1)^2 + \left(\frac{\pi w_{03}^2}{\lambda}\right)^2}$$

(7)

We solve the Eq. (7), yielding:

$$l_{33} = f_1 \pm \sqrt{f_1^2 \frac{w_{03}^2}{w_{01}^2} - \left(\frac{\pi w_{03}^2}{\lambda}\right)^2}$$

(8)
Obviously, the necessary condition when above Eq. (7) has real solutions is

$$f_3 \geq \frac{\pi w_{03} w_{01}}{\lambda} \quad (9)$$

When the Gaussian beam passes through the curved mirror $M_3$, in order to satisfy the condition that the beam waist $w_{03}$ matches the beam waist $w_{01}$, the distance $l_{13}$ has two possible values. We insert the formula (8) into the following formula:

$$l_{13} = f_3 + \frac{(l_{33} - f_3) f_3^2}{(l_{33} - f_3)^2 + \left(\frac{\pi w_{03}}{\lambda}\right)^2} \quad (10)$$

and find the two different values $l_{13}$. Therefore, each parameters of a ring cavity including curvature radii of mirrors, the distances between each mirrors have been determined. Moreover, beam waist radii and their positions were obtained during the process of calculation. By now, we have completed the optical ring cavity design based on the self-consistency theory and the fact that $q$ parameter is uniquely determined by the waist beam radius and its position. This method is not only efficient and simple but also is very easy to design an optical ring cavity with the suitable beam waist radius in a specific position.

In the following section, we will discuss a special case that the Gaussian beam waist is individually located on the focus of curved mirrors. A Gaussian beam has a beam waist a distance $l_{11}$ away for a curved mirror with a beam waist radius $w_{01}$ and $l_{11}$ is exactly equal to the focal length of the curved mirror. When the Gaussian beam passes through the curved mirror, the curved mirror produces another beam waist with a beam waist radius $w_{02}$ and a distance $l_{21}$ away from the mirror. Using formulas (5) and (6), we obtain:

$$w_{02} = \frac{\lambda f_1}{\pi w_{01}}, \quad l_{21} = f_1 \quad (11)$$

According to the above two formulas, we know that when a beam waist is at one of the focal planes, another beam waist is at the image focal planes. Analogously, when the beam waist $w_{02}$ is in the focal planes of the curved mirror $M_2$, we find

$$w_{03} = \frac{\lambda f_2}{\pi w_{02}}, \quad l_{32} = f_2 \quad (12)$$

It is assumed that the beam waist radius $w_{03}$ is in the object focal planes of the curved mirror $M_3$. The relation of the beam waist radii $w_{01}$ and $w_{03}$ is consistent with mode matching. We can obtain:

$$w_{01} = \frac{\lambda f_3}{\pi w_{03}}, \quad l_{33} = f_3 \quad (13)$$

We combine (10), (11) with (12), and obtain the waist radii of each arm in the resonant:

$$w_{01}^2 = \frac{\lambda f_1 f_3}{\pi f_2}, \quad w_{02}^2 = \frac{\lambda f_1 f_2}{\pi f_3}, \quad w_{03}^2 = \frac{\lambda f_2 f_3}{\pi f_1} \quad (14)$$

All the Gaussian beam waist are individually located on the focus of curved mirrors. The cavity length of the optical ring cavity is $2(f_1 + f_2 + f_3)$. Consequently, it becomes extremely simple to design an optical ring cavity.
3. Experimental study

According to the sketch in Fig. 1, a six-mirror ring cavity as illustrated in Fig. 4 was well designed in order to obtain stable continuous wave mode locked pulse trains. The six-mirror is composed of three curved mirrors, a flat output coupler mirror OC with a partial transmittance of 5%, a dichroic flat mirror DF (HT at 808 nm, HR at 1064 nm) and a SESAM flat mirror. The output coupler mirror, the dichroic mirror and SESAM are flat mirrors, therefore place these flat mirrors do not affect optical ring cavity spot radii distribution. The radii of curvature of concave mirrors \( M_1, M_2 \) and \( M_3 \) are 200 mm, 500 mm and 300 mm, respectively. These concave mirrors separately correspond to the curved mirrors in Fig. 1. The SESAM (provided by Institute of Semiconductors, Chinese Academy of Science) was adhered to a copper heat sink by silicon grease. According the theoretical calculation in the above section, beam waist at SESAM is equal to 60 \( \mu m \). The pump source is an 808 nm fiber-coupled laser diode (LIMO GmbH, Germany), whose maximum output power is 35 W, and the output fiber has a core diameter of only 400 \( \mu m \). The numerical aperture of the fiber is 0.22. An \( a \)-cut 3 \( \times \) 3 \( \times \) 10 mm\(^3\) Nd:YVO\(_4\) crystal with Nd\(^{3+}\) concentration of 0.3 at.% was used as the gain medium in our experiment. The crystal, wrapped with indium film, was bedded in a copper heat sink to transfer heat. The temperature of the laser diode and crystal was controlled at 25 °C by a cool-water temperature controller.

Simultaneous lasing for both clockwise (CW) and counter-clockwise (CCW) light waves was observed behind each folding mirror. Their average output powers were very similar values. The output behaviour of the laser in both sides was investigated. The variation of average output power versus laser diode incident pump power was as depicted in Fig. 5. The threshold for continuous operation was as low as 3 \( W \) of pump power incident on the crystal. When the incident pump power is less than 6.7 \( W \), the ring laser operated in free running regime. At the incident pump power of 10.4 \( W \), the laser operation transformed from the Q-switched mode locked state to the continuous wave mode-locked (CWML) state. We found that the maximum 1064 nm laser total output was 3.1 \( W \) when the incident pump power was 17.5 \( W \) at 880 nm, corresponding to an optical-to-optical efficiency of 17.7%.
Fig. 5. Output power and total light-light efficiency versus input power at 808 nm, respectively. CW: clockwise; CCW: counter-clockwise; QML: Q-switch mode locking; CWML: continuous wave mode locking.

Fig. 6. Oscilloscope traces of both clockwise and counter-clockwise outputs of the ring laser.

The temporal profiles of the passively mode-locked Nd:YVO$_4$ ring laser with bidirectional operation were monitored and analyzed by a 1 GHz analog bandwidth high speed digital oscilloscope (DPO4104B, Tektronix, Inc., USA) and a 1 ns rise time fast photodetector (DET10A/M, Thorlabs, Inc., USA). Stable CW mode locked pulse trains are obtained in the both sides of the ring laser when the ring laser operates at CWML state. Figure 6 (a) and (b) are the short term pulse trains (10 ns/div) and the long term pulse trains (40 μs/div), respectively. The red and green marking pulses represent the pulses of ring laser in clockwise and counter-clockwise side, respectively. The repetition rate of the laser was about 119 MHz. The mode-locked ring laser operated at stabilized CWML state, and this state could continue
for more than several hours without readjustment. The experimental results show that the optical ring cavity designed by us, is very suitable for SESAM mode-locked laser.

4. Conclusion

In conclusion, we proposed a simple method for designing a laser resonator, based on the self-consistency theory and the fact that $q$ parameter is uniquely determined by the waist beam radius and its position. The calculations of designing optical ring cavities are simplified because the $q$ parameter only has imaginary part at beam waist plane. Furthermore, saturable absorber elements, nonlinear crystals and others important optical elements are generally located at the beam waist positions, therefore designing the resonant cavity through the calculation of beam waist radii and their position has great practical significance. We employed this method to design an end-pumped six-mirror ring cavity SESAM continuous-wave passively mode locked (CWML) laser. The experiment of a highly stabilized continuous-wave mode locked (CWML) laser was investigated. The results coincide with the theoretical studies very well. The investigation results show that the simple method can be used to design optical ring cavities very conveniently, intuitively and efficiently.

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