Two-Stage Algorithm for the Open Vehicle Routing Problem

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Abstract. Open Vehicle Routing Problem (OVRP) is a problem that is often found in transportation/distribution problems. On this issue it is known that some consumers are served by several homogeneous vehicles that depart from the depot where after visiting the last consumer the vehicle does not go back to the depot as in the OVRP the vehicle is not possessed by the company. The distribution cost is calculated from the depot until delivery to the last consumer. OVRP will be solved using the 2-stage method. The first stage is the initial construction phase of the solution while the second stage is the quality improvement phase of the solution. The Nearest Neighbor and 2-Opt algorithm are applied to generate the initial solution. The second stage uses Multi-Level Algorithm. The algorithm is equipped with four local searches. The data set from the literature is used to test the algorithm.

1. Introduction

This paper is an initial investigation (work in progress) to solve the Open Vehicle Routing Problem (OVRP). The OVRP is a problem that is often found in transportation/distribution problems. The OVRP is a variant of the well-known Capacitated Vehicle Routing Problem (CVRP). In the CVRP, the objective is to find the shortest distance in serving the customers. Here, we need to consider that consumers are served by several homogeneous vehicles that start and return to the depot. The difference with the OVRP is in the OVRP the vehicle does not go back to the depot after visiting the last consumer as they are not owned by the company. The distribution cost is calculated from the depot until the delivery to the last consumer is done. The aim of the OVRP is to minimize the number of vehicles used and produce the shortest distance in serving the consumer.

There are several methods/algorithms that have been developed to tackle the OVRP. The OVRP was first addressed by Sariklis and Powell [20] when they are solving the distribution management issues. They proposed the Cluster First, Route Second (CFRS) based algorithm. Brandao [3] and Fu et al. [6] proposed Tabu Search algorithms. Tarantilis et al. [23] applied a Threshold Accepting based algorithm called the List Based Threshold Accepting (LBTA) to tackle the OVRP data set and the actual problem (distributing goods from a warehouse to customers throughout the greater Athens Metropolitan area). Pisinger and Ropke [15] used the Adaptive Large Neighborhood Search (ALNS). They also applied their algorithm to solve several variants of the VRP and it produced excellent results. Li et al. [12] developed a Record to Record (RTR) Travel based algorithm to address the OVRP. They also created and solved large data set for the OVRP. Letchford et al. [10] proposed an exact method to solve the OVRP. They used the Branch and Cut algorithm. A variable neighborhood search based approach was used by Fleszar et al. [5]. An Integer Linear Programming (ILP) improvement procedure was proposed by Salari et al. [17]. MirHassani and Abolghasemi [13] proposed a Particle Swarm Optimization based algorithm to tackle the OVRP. Pichpibul and
Kawtummachai [14] developed Clark and Wright algorithm based method. They obtained good results. Sedighpur et al. [21] applied a hybrid ant colony optimization based algorithm. The results produced by Pichpipul and Kawtummachai [14] are discussed in Sorensen et al. [22]. Sorensen et al. [22] found that the results are questionable.

The applications of the OVRP in the real problem can be seen in the distribution problem of lubricants in Greece (Repoussis et al. [16]), the problem for determining school bus routes in Hong Kong (Li and Fu [11]), the train service problem through the Channel Tunnel (Fu and Wright [7]) and in determining the delivery routes for FedEx aircraft (Bodin et al. [2]).

The paper is written as follows, the introduction is given in section 1. Section 2 explain of the proposed method. In section 3 the results obtained by the proposed method are presented and discussed. In the final section, the conclusions and suggestions for improving the results are given.

2. The proposed algorithm
In this study, the OVRP will be solved using the 2-stage method. The first stage is the initial construction phase of the solution, while the second stage is the quality improvement phase of the solution. In the first stage, the nearest neighbor algorithm followed by applying 2-Opt intra-route local search are used to generate the initial solution. In the second stage, the solution generated by the first stage is used as an input Multi-Level heuristic based algorithm of Salhi and Sari [19] are implemented.

Four local searches such as 2 types 1-0 insertions and 2 types swap (1-1 insertion) are applied in the framework of Multi-Level Algorithm to enhance the solution quality. The first type is a local search for improving the route by considering one route (intra-route) while the second type is an improvement procedure that consider two routes (inter-route) when conducting 1-0 and 1-1 move.

In the case for solving the vehicle routing problem and its variants, the application of the Multi-Level algorithm can be found in Imran et al. [9] for Heterogeneous Fleet Vehicle Routing Problem (HFVRP), Salhi et al. [18] for the Multi-Depot Heterogeneous Vehicle Routing Problem (MDHFVRP), Salhi and Sari [19] for MDHFVRP and Multi-Depot Vehicle Routing Problem (MDVRP) and in Imran et al. [8] for the Heterogeneous Fixed Fleet Vehicle Routing Problem (HFFVRP).

1-0 insertion consider removing a customer to a new feasible position while 1-1 insertion (swap) swap position of a customer with another customer. A move is taken when it can produce better solution. The illustration of 1-0 and 1-1 moves for inter-route case is given by Figure 1, Figure 2 and Figure 3. In Figure 1 we have two routes R1 and R2. R1 have 5 customers (c1, c2, c3, c4, c5) and R2 also have 5 customers (c6, c7, c8, c9, c10).

![Figure 1. Route R1 and R2](image1.png)

![Figure 2. 1-0 insertion inter route.](image2.png)
In Fig 2 customer c3 of R1 is inserted between customer c8 and c9 of R2.

In Figure 3, customer c3 of R1 is swapped with c9 of R2.

The local searches are used in Multi-Level algorithm of Salhi and Sari [19]. The search begins by generating the initial solution by using the Nearest Neighbor Algorithm. The algorithm picks the closest customer to the depot as the next customer. It continues by selecting the nearest customer to the last customer picked in the route. The process is done by considering vehicle capacity. The algorithm stops after all customers are allocated. The routes obtained is improved by applying 2-opt algorithm. The Multi-Level algorithm starts by using the first local search \( L_1 \) to obtain the best solution \( s'' \). If it is found that \( s'' \) is smaller than \( s \), then \( s = s'' \) and the search returns to \( L_1 \), otherwise the second local search \( (L_2) \) is used to find \( s'' \). If \( s'' \) is smaller than \( s \), then \( s = s'' \) and the search goes back to \( L_1 \), otherwise the third local search search \( (L_3) \) is applied to find \( s'' \). The search is repeated until \( L_4 \) cannot obtain a better solution. The best solution is recorded. The algorithm stops when all neighborhoods are used and there is no better solution is found.

The steps of the proposed algorithm are given in Figure 4.

Define a set of local searches \( L_j \), for \( j = 1, ..., j_{\text{max}} \)

Generate an initial solution \( s \),
set \( s_{\text{best}} = s \).

\( j = 1 \)

(a) Apply local search \#1 \( (L_1) \).
   If the local optimum \( s'' \) is smaller than the incumbent \( s \)
   Set \( s \leftarrow s'' \) and \( j \leftarrow j + 1 \)

(b) Apply local search \#2 \( (L_2) \).
   If the local optimum \( s'' \) is smaller than the incumbent \( s \)
   Set \( s \leftarrow s'' \) and \( j \leftarrow 1 \)
   Else
   Set \( j \leftarrow j + 1 \)
   If \( (j < j_{\text{max}}) \)
      go to (a)
   Else
   Stop

Where, \( j_{\text{max}} \) is the maximum number of local search used in search process.
The number of vehicles used \((nv)\) is determined by calculating the minimum number of vehicles by using Equation 1.  

\[
 nv = \sum_{i=1}^{n} \frac{Q_i}{C}
\]  

(1)

The minimum number of vehicles \((nv)\) is the ratio between the sum of the demand \((Q_i)\) and the vehicle capacity \((C)\). After that, the route is formed with respect to the limits of vehicle capacity and other restrictions.

3. Results

The proposed algorithm was implemented in C++ and executed by using the computer with Intel core i3 processor and 1 GB of RAM. Five data from a data set of Augerat et al. [1] are solved to testing the algorithm. The solutions obtained and the computation time used are then compared with the results obtained by Letchford et al. [10] and Pichpibul and Kawtummachai [14] to see the quality of the proposed algorithm.

The results of the test are presented in Table 1 and Table 2. Table 1 contains solutions obtained by the proposed algorithms and solutions obtained by Letchford et al. [10] and Pichpibul and Kawtummachai [14]. Table 2 contains the computational time used by the two stage algorithm and by the similar paper with Table 1. From Table 1 it can be seen that the proposed method produced one solution similar to the best known solution \(#1\) and solutions \(#2\ #3\ #4\ and \#5\) are not as good as the best known solutions or the solutions obtained by Letchford et al. [10] and Pichpibul and Kawtummachai [14].

In terms of the computation time, the proposed algorithm can be said has used a reasonable computation time when compared with the computation time of other algorithm. However, we need to solve large data set in order to observe the computation time.

| Table 1. Solutions (Augerat et al. data set [1]) |
|---|---|---|---|---|
| No | #Customer | Best Solution | Letchford et al. [10] | Pitchibul and Kawtummachai [14] | Proposed Algorithm |
| 1 | 16 | 235.06 | 235.06 | 235.06 | 235.06 |
| 2 | 19 | 168.57 | 168.57 | 168.57 | 169.92 |
| 3 | 20 | 170.28 | 170.28 | 170.28 | 174.20 |
| 4 | 21 | 163.88 | 163.88 | 163.88 | 164.36 |
| 5 | 22 | 167.19 | 167.19 | 167.19 | 185.05 |

| Table 2. Computation time (in second) |
|---|---|---|---|
| No | #Customer | Letchford et al. [10] | Pitchibul and Kawtummachai [14] | Proposed Algorithm |
| 1 | 16 | 0.09 | 1.17 | 1.35 |
| 2 | 19 | 0.2 | 1.57 | 0.79 |
| 3 | 20 | 0.2 | 1.71 | 1.47 |
| 4 | 21 | 0.02 | 1.73 | 0.94 |
| 5 | 22 | 0.03 | 1.89 | 0.83 |
4. Conclusion
In this study, a two stage algorithm is developed to address the OVRP. The proposed algorithm is tested to solve the data set from the literature. However, the quality of the solutions of the proposed algorithm needs to be improved as they are not as good as the solutions obtained by the two methods that are used for comparison. For further research, the use of more neighborhood and other local search types such as 2-Opt local search (intra and inter-route), Or-Opt local search (intra-and inter-route) and perturbation local search can be applied in order to solve larger data set and to improve the solution quality. Adding a procedure that provides the possibility to obtain new solutions as inputs if the proposed algorithm is repeated from the beginning of the algorithm can also be introduced.

5. References
[1] Augerat P, Belenguer J, Benavent E, Corbern A, Naddef D and Rinaldi G 1995 Computational results with a branch and cut code for the capacitated vehicle routing problem Research Report 949-M Univesite Joseph Fourier Grenoble France.
[2] Bodin L, Golden B, Assad A and Ball M O 1983 Routing and scheduling of vehicles and crews; the state of the art Comp. & Op. Res. 10 63-211.
[3] Brandão J 2004 A tabu search algorithm for the open vehicle routing problem European J. of Op. Res. 157 552–64.
[4] Clarke G and Wright J W 1964 Scheduling of vehicles from a central depot to a number of delivery points Op. Res. 12 568–81.
[5] Fleszar K, Osman I H and Hindi K S 2009 A variable neighbourhood search algorithm for the open vehicle routing problem European J. of Op. Res 195 803–9.
[6] Fu Z, Eglese R W and Li L Y O 2005 A new tabu search heuristic for the open vehicle routing problem J. of the Op. Res. Soc 56 267–74.
[7] Fu Z, Wright M 1994 Train plan model for British Rail freight services through the Channel Tunnel J. of the Op. Res. Soc. 45 384–91.
[8] Imran A, Luis M and Okdinauati L 2016 A variable neighborhood search for the heterogeneous fixed fleet vehicle routing problem J. Teknologi 78 53-8.
[9] Imran A, Salhi S and Wassan N A 2009 A variable neighborhood-based heuristic for the heterogeneous fleet vehicle routing problem European J. of Op. Res 197 509-18.
[10] Letchford A, Lysgaard J and Eglese R W 2007 A branch and cut algorithm for the capacitated open vehicle routing J. of the Op. Res. Soc. 58 1624-51.
[11] Li L Y O and Fu Z 2002 The school bus routing problem: a case study J. of the Op. Res. Soc. 53 552–8.
[12] Li F, Golden B and Wasil E 2007 The open vehicle routing problem: algorithms, large-scale test problems, and computational results Comp. & Op. Res. 34 2918–30.
[13] MirHassani S A and Abolghasemi N 2011 A particle swarm optimization for the open vehicle routing problem Expert Syst. with Appl. 38 11547–51.
[14] Pichpibul T and Kawtummachai R 2013 A heuristic approach based on Clarke-Wright algorithm for open vehicle routing problem The Scientific World J.
[15] Pisinger D and Ropke S 2007 A general heuristic for vehicle routing problems Comp. & Op. Res. 34 2403–35.
[16] Repoussis P P, Paraskevopoulos D C, Zobolas G, Tarantilis C D and Ioannou G 2009 A web-based decision support system for waste lube oils collection and recycling European J. of Op. Res. 195 676–700.
[17] Salari M, Toth P and Tramontani A 2010 An ILP improvement procedure for the open vehicle routing problem Comp. & Op. Res. 12 2106-02.
[18] Salhi S, Imran A and Wassan N A 2014 The multi-depot vehicle routing problem with heterogeneous vehicle fleet: Formulation and a variable neighborhood search implementation Comp. & Op. Res. 52 315-25.
[19] Salhi S and Sari M 1997 A multi-level composite heuristic for the multi-depot vehicle fleet mix problem European J. of Op. Res. 103 95-112.
[20] Sariklis D and Powell S 2000 A heuristic method for the open vehicle routing problem The J. of the Op. Res. Soc. 51 564–73.
[21] Sedighpour M, Ahmadi V, Yousefikhoshbakht M, Didehvar F and Rahmati F 2014 Solving the open vehicle routing problem by a hybrid ant colony optimization Kuwait J. of Science 41 139-62.
[22] Sorensen K, Arnold F and Cuervo D P 2017 A critical analysis of the improved Clarke and Wright savings algorithm Int. Trans. on Op. Res.DOI: 10.1111/itor.12443.
[23] Tarantilis C D, Ioannou G, Kiranoudis C T and Prastacos G P 2005 Solving the open vehicle routing problem via a single parameter metaheuristic algorithm J. of the Op. Res. Soc. 56 588-96.