Light-Cone Quantisation of Matrix Models at $c > 1^*$

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ABSTRACT

The technique of (discretised) light-cone quantisation, as applied to matrix models of relativistic strings, is reviewed. The case of the $c = 2$ non-critical bosonic string is discussed in some detail to clarify the nature of the continuum limit. Further applications for the technique are then outlined.

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1. Introduction

Random surface problems appear in many branches of theoretical and mathematical physics. A number of them may be reformulated as, or arise from, matrix field theories [1, 2, 3, 4, 5, 6], planar diagrams modelling the fluctuating surfaces. The possible physical applications, together with relations to the mathematics of systems of integrable differential equations [7] and moduli space of Riemann surfaces [8, 9], make this a fascinating and novel (ab)use of quantum field theory. While condensed matter problems involve the thermal fluctuations of observable surfaces in 3D Euclidean space, for applications to high-energy physics the surfaces are worldsheets which enter indirectly through the weak coupling expansions of relativistic string theories purporting to describe quantum gravity and confining gauge theories. Matrix models of relativistic strings should be defined in Minkowski space, their spectrum, or a self-consistent truncation of it, comprising string excitations. While knowledge of the fractal geometry of perturbative diagrams is sometimes useful, numerical computation of critical random surface properties in Euclidean space is no substitute for direct calculation of the relativistic string observables; for example, the almost uninvestigated issues of strongly coupled string theories would be otherwise neglected.

I.Klebanov and the author have suggested [10] that discretised light-cone quantisation (DLCQ), introduced with a view to calculating the bound-state spectrum of gauge theories directly [11], may be very appropriately applied to matrix models also. This method, although primarily a numerical one, paints a clear physical picture of otherwise complex dynamics. In the case of string theories, this means the size, shape, and energy of strings. The unfortunate circumstance that the simplest bosonic string theory, to which one would add more structure for a physically realistic object, is (expected to be) tachyonic poses no particular problem to DLCQ of matrix models. On the contrary, one may analyse this pathology with some rigour and, as a result, set about curing it.

The following section contains an elementary review of matrix models, light-cone quantisation for a $c = 2$ model being carried out in section 3. Section 4 describes the critical behaviour of the discretised version, while future avenues of development are discussed in section 5.
2. Matrix Models of Random Surfaces

Consider an $N \times N$ hermitian matrix field $\phi_{ab}(x)$ in $c$ dimensions subject to the following (Euclidean) action

$$S_E = \int d^c x \; \text{Tr} \left( \frac{1}{2} (\partial_\alpha \phi)^2 + \frac{1}{2} \mu \phi^2 - \frac{1}{3\sqrt{N}} \lambda \phi^3 \right).$$

(1)

The Feynman rules are those of ordinary $\phi^3$ field theory except that worldlines are double lines, each line carrying a "colour" index $a$. Colour is conserved along the propagator and at vertices on account of the global $U(N)$ symmetry, $\phi \rightarrow \Omega^\dagger \phi \Omega$, of the action;

$$< \phi_{ab}(x) \phi_{cd}(y) > = \delta_{ad} \delta_{bc} \int \frac{d^c p}{(2\pi)^c} \frac{e^{ip \cdot (x-y)}}{p^2 + \mu},$$

(2)

$$< \phi_{ab}(x) \phi_{cd}(x) \phi_{ef}(x) > = \delta_{bc} \delta_{de} \delta_{fa} N.$$  

(3)

The $1/N$ expansion of the theory described by (1) is a topological expansion in surfaces [1], the Feynman diagrams being understood as fishnet approximations drawn on continuous 2-dimensional surfaces embedded in $R^c$;

$$\int D\phi e^{-S_E} \sim \sum \lambda^v \left( \frac{1}{N} \right)^{-\chi} \int \text{embeddings}.$$

(4)

$v$ is the number of vertices, $\chi$ the Euler number, and one integrates over all embeddings of the graph with rules (2) (3). The discretised surface picture is formalised by considering the dual graphs [3, 4] that join centres of neighbouring loops, which for a $\phi^3$ theory specifies a simplicial triangulation. Each triangle carries unit intrinsic area and its centre coincides with the Feynman vertex; in particular this leaves the angle between neighbouring triangles unspecified in this model. The idea is then to tune the coupling $\lambda$ to a critical value $\lambda_c$ at which surfaces of large intrinsic area $v$ are favourable. At this point one may be able to take a continuum limit for surfaces.*

* There may be no scaling of the intrinsic geometry (curvatures, etc) even though the area scales – the $c \leq 1$ models are examples of this – while at $c > 1$ there is even a danger, commonly attributed to tachyons, of non-scaling area.
The observables are given by the Green’s functions of the matrix field theory. A closed string state is a hole cut into the surfaces at some fixed time, \( t_0 \) say. Such states are given by the singlet operators in the matrix model;

\[
\text{Tr}[\phi(x_1) \ldots \phi(x_B)]_{t=t_0}
\]  

(5)

is a \( B \)-bit string, the bits being embedded at \( x_1, \ldots, x_B \) respectively and forming a closed chain dual to the external legs of the Feynman diagrams. Also present are direct products of (5) (multi-string states) and non-singlets; the latter do not have an interpretation as closed strings and should be eliminated, either by performing a self-consistent truncation or effecting a dynamical decoupling (by gauging the \( U(N) \) for example [12]). The critical point \( \lambda_c \) of large surfaces should manifest itself as singularities of Green’s functions, such as the string propagator \( \sim < \text{Tr}[\phi \cdot \cdot \cdot \phi]\text{Tr}[\phi \cdot \cdot \cdot \phi] > \), and thus in particular the spectrum of string excitations (5) should exhibit some sort of critical behaviour. Applying DLCQ to the matrix field theory in Minkowski space, one derives the spectrum as a function of \( \lambda \) and can search for such behaviour.

The simplest non-trivial example to consider is \( c = 2 \), for which (1) models, in its \( 1/N \) expansion, a discrete version of the worldsheet action for the \( c = 2 \) non-critical bosonic string [13],

\[
A = \int d^2 \sigma \sqrt{-\det g(\Lambda + \nu R(\sigma)) + T \sum_{\mu, \nu = 1}^{2} \eta_{\mu \nu} g^{\alpha \beta} \partial_\alpha x^\mu \partial_\beta x^\nu + O[(\partial x)^4]}. 
\]  

(6)

\( \Lambda, \nu, \) and \( T \) are renormalised parameters associated with \( \lambda, 1/N, \) and \( \mu \) respectively. Stretching energy of the worldsheet is governed by the propagator (2) , which specifies the probability amplitude for the separation of centres of neighbouring triangles. This exponential fall-off gives the Gaussian term in \( A \) plus non-renormalisable higher derivative terms. Naively the latter are irrelevant, but such reasoning assumes scaling of the worldsheet in some sense. The \( c = 2 \) theory (1) is rendered perturbatively finite by normal ordering, this being necessary in any case to eliminate graphs dual to pathological triangulations where two or more sides of the same triangle are identified. According to general arguments [14], the sum of graphs at a given order in \( 1/N \) in a UV finite theory is also finite for sufficiently small coupling constant; the \( 1/N \) expansion itself is only asymptotic. As \( \lambda \to \lambda_c \) one then approaches the
edge of the domain of convergence. The phenomenon is similar to, but certainly different from, the crossover to non-borel summability in the non-matrix field theory ($N = 1$).

3. Light-Cone Quantisation

Let us now rotate $x^0 \rightarrow ix^0$ and find the relativistic string spectrum by light-cone quantisation. Defining light-cone variables $x^\pm = (x^0 \pm x^1)/\sqrt{2}$, with $x^+ = x_- $, the light-cone energy and momentum $P^\pm = \int dx^- T^{\pm \pm}$ are

\[ P^+(x^+) = \int dx^- \text{Tr}(\partial_- \phi)^2, \]
\[ P^-(x^+) = \int dx^- \text{Tr}\left(\frac{1}{2} \mu \phi^2 - \frac{\lambda}{3\sqrt{N}} \phi^3\right). \]  
(7)

The light-cone Hamiltonian $P^-$ propagates a field configuration from one $x^+$ slice to another. Choosing a free field representation at $x^+ = 0$ say $^\ast$, 

\[ \phi_{ij} = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dk^+}{\sqrt{2k^+}} (a_{ij}(k^+)e^{-ik^+x^-} + a_{ij}^\dagger(k^+)e^{ik^+x^-}), \]  
(8)

and imposing the canonical commutation relations at equal $x^+$ lines,

\[ [\phi_{ij}(x^-), \partial_- \phi_{kl}(\tilde{x}^-)] = \frac{i}{2} \delta(x^- - \tilde{x}^-)\delta_{ik}\delta_{jl}, \]  
(9)

the modes $a_{ij}$ satisfy standard commutators;

\[ [a_{ij}(k^+), a_{lk}^\dagger(\tilde{k}^+)] = \delta(k^+ - \tilde{k}^+)\delta_{il}\delta_{jk}. \]  
(10)

An important feature is that longitudinal momentum $k^+$ is positive semi-definite for positive energy quanta $^\dagger$ In the Fock space constructed using (10), the orthonormal single closed-string states are the singlets

\[ N^{-B/2} \text{Tr}[a_{i_1}^\dagger(k^+_{i_1}) \cdots a_{i_B}^\dagger(k^+_{i_B})] |0 >, \sum_{i=1}^B k^+ = P^+. \]  
(11)

If we let $N \rightarrow \infty$ the multi-string states can be neglected, since $1/N$ is the string coupling constant, and $P^-$ propagates without splitting or joining strings. Thus we solve for the free

$^\ast$ The symbol $^\dagger$ is always understood to have purely quantum meaning and never acts on indices.

$^\dagger$ This positivity constraint results in the extra 1/2 in (9), ensuring for example that $[P^+, \phi] = \partial_- \phi$. 

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string spectrum. (Non-singlets states are discarded by hand.) \( P^+ \) and \( P^- \) can be simultaneously diagonalised and so eigenfunctions of \( P^- \) with given \( P^+ \) will be some superposition

\[
\Psi = \sum_{B} \int_{0}^{P^+} dk_1 \cdots dk_B \delta(P^+-\sum k_i) f_B(k_1, \ldots, k_B) N^{-B/2} \text{Tr}[a_1^\dagger(k_1) \cdots a_B^\dagger(k_B)] |0> \tag{12}
\]

Explicitly one finds

\[
P^- := \frac{1}{2} \mu \int_{0}^{\infty} \frac{dk^+}{k^+} a_{ij}^\dagger(k^+) a_{ij}(k^+) - \frac{\lambda}{4\sqrt{N\pi}} \times \int_{0}^{\infty} \frac{dk_1^- dk_2^-}{\sqrt{k_1^- k_2^- (k_1^- + k_2^-)}} \left\{ a_{ij}^\dagger(k_1^+ + k_2^+) a_{ik}(k_2^+) a_{kj}(k_1^+) + a_{ik}^\dagger(k_1^+) a_{kj}^\dagger(k_2^+) a_{ij}(k_1^+ + k_2^+) \right\} \tag{13}
\]

where repeated indices are summed over. For the free string theory each of the terms in (13) has a simple local action on the string. The mass term acts as a tensional energy \( \sim \mu \sum_{i=1}^{B} 1/k_i \) and does not change the state. The cubic term can coalesce two neighbouring bits in the trace, or perform the inverse process, and changes both the length and momentum distribution (structure function) of the string. Since \( c = 2 \), there are no transverse oscillations in the target space. Unlike \( c \leq 1 \) however, we are dealing with truely stringy degrees of freedom – an infinite number of particle fields – which supplement the centre of mass motion.

Free string states will satisfy a relativistic dispersion relation \( 2P^+P^- = M^2 \) and by diagonalising \( P^- \) in the basis of states of a fixed \( P^+ \) we can find the spectrum of masses \( M^2 \). This diagonalisation cannot be performed analytically for the full Hamiltonian in general, so we must introduce a cutoff \([11, 15]\), rendering the number of states of momentum \( P^+ \) finite, and compute numerically. It is therefore important that we can neglect zero momentum modes \( a^\dagger(0) \) since we could in principle include arbitrary numbers of them without changing \( P^+ \). Such modes have infinite energy for \( \mu \neq 0 \), according to (13); however, they can be infinite in number and so a more careful analysis is required to lift this ambiguity. Indeed, it is believed by some that a proper constrained quantisation of these zero modes is necessary to describe spontaneous symmetry breaking and other non-perturbative effects in light-cone formalism. In those cases it is argued that the true vacuum, if one exists at all, is not the
Fock one, \( P^- : |0> = 0 \), but a condensate of \( a^\dagger(0) \) modes. But one should recall that we are interested in (1) only insofar as it generates random surfaces through the \( 1/N \) expansion and perturbation theory. Therefore we should always take the \( \phi = 0 \) vacuum, for which there is no condensate of zero modes, since this is the one with respect to which the planar diagrams are defined. For the \( c = 2 \) model, at each order in \( 1/N \) we work with a convergent perturbation expansion, the non-perturbative effects presumably manifesting themselves through the \( e^{-N} \) corrections to the asymptotic expansion in \( 1/N \). The latter are non-perturbative effects of string theory and specifying their details is equivalent to stating how one is going to stabilise the unbounded \( \phi^3 \) theory (without disturbing the \( 1/N \) expansion), a question that will not be considered here.

Another useful way of viewing these and other issues is to consider triangulations in light-cone perturbation theory. Indeed, for relativistic strings one could have set up the random surface expansion from this vantage point from the very beginning. Using Feynman’s causality prescription on \( x^+ \) rather than \( x^0 \), the single-particle propagator becomes

\[
\lim_{\epsilon \to 0} \left( \int_{-\infty}^{\infty} \frac{dp^+}{4\pi ip^+} \theta(x^+) e^{-i(x^-p^+ + x^+\mu/2p^+)} + \int_{-\infty}^{-\epsilon} \frac{dp^+}{4\pi ip^+} \theta(-x^+) e^{-i(x^-p^+ - x^+\mu/2p^+)} \right) + \frac{\delta(x^+)}{2\pi \mu}.
\]

(14)

The first two terms can be given the usual particle and anti-particle interpretation by viewing the negative energy \( (p^-) \) states as propagating backwards in time \( (x^+) \). The third term is a special contribution from \( p^+ = 0 \) and corresponds in the dual diagram to the propagation of a zero momentum string-bit; neglecting this case, all particles and anti-particles move forward in \( x^+ \) carrying positive \( p^+ \). In early work [16], the quantisation procedure used was the one adopted here, where it was proved not only that \( x^+ \)-ordered and \( x^0 \)-ordered perturbation theory are equivalent, but also that the third term eventually does not contribute in non-vacuum diagrams. This seems to indicate once again that, provided we restrict to perturbation theory of (1), the zero modes can be ignored in computing the string propagator.

4. DISCRETISATION AND CRITICAL BEHAVIOUR

The desired cut-off will be introduced by compactifying \( x^- \) and imposing periodic boundary conditions, \( M_{ij}(x^-) = M_{ij}(x^- + L) \) [11]. Then the allowed momenta are labelled by
positive integers $n_i$;

$$k_i^+ = \frac{2\pi n_i}{L}, \quad P^+ = \frac{2\pi K}{L}, \quad \sum_{i=1}^{B} n_i = K. \quad (15)$$

For fixed $P^+$, removing the cut-off $L \to \infty$ corresponds to sending $K \to \infty$. The “harmonic resolution” $K$ represents the total number of momentum units available to the string. The longest string has $K$ bits of one unit $\star$, the shortest one bit of $K$ units, and in general the states can be labelled by the ordered partitions of $K$ modulo cyclic permutations. Light-cone quantisation (8) - (13) may now be repeated for discrete variables $k^+ \to n$ and one finds the mass relation

$$\frac{2P^+P^-}{\mu} = K(V - xT) ; \quad x = \frac{\lambda}{2\mu \sqrt{\pi}}. \quad (16)$$

$V$ is the discrete version of the mass term in (13) while $T$ is the cubic term. For finite $K$ the r.h.s. is a finite-dimensional symmetric matrix with real dimensionless elements $- \mu$ is the quantity of dimension mass$^2$ which plays the role of string tension $1/\alpha' -$ which may be diagonalised as a function of the dimensionless parameter $x$; $V$ is diagonal while $T$ is off-diagonal.

The following picture of the critical behaviour, supported by the numerical results [10, 17], begins to emerge. In the light cone formalism of critical string theory the longitudinal momentum supported between two points on the string is proportional to the amount of $\sigma$-coordinate space between these points. We can adopt a similar co-ordinate system for the non-critical strings (5). Indeed, fixing a particular bit as origin, we can define a positive scalar field on this $\sigma$-space by $X = \Delta b/\Delta \sigma$, where $b$ is the distance, measured in number of bits, from the origin. For example, the zero mode $\int Xd\sigma$ is the intrinsic length of the string. As we remove the cutoff on the longitudinal momentum allowed for bits ($K \to \infty$), and hence on discreteness of $\sigma$-space, the scalar field will generically take constant values almost everywhere in $\sigma$-space. We would like to be able to tune the theory to a critical point where the scalar field is in a long wavelength regime. In this case it would be somewhat similar to the Liouville mode of Poyakov's string [13]. What would this long wavelength regime mean for the spectrum? Firstly we would expect long string dominance; the expectation value of length for low-lying eigenstates would typically diverge as $x \to x_c$ (and we have been

\* This sector alone represents what one might call “critical string theory” [24]
assuming that this is not distinct from $\lambda_c$ discussed earlier). Since, roughly speaking, each string-bit carries finite energy, we expect that $|M^2| \to \infty$ as a result. $M^2 \to \infty$ at $x = 0$ for infinitely long strings, but as $x \to x_c$, if only for consistency, we must see $M^2 \to -\infty$ for the low-lying eigenstates if they are long. Indeed this will tend to happen to the lowest eigenvalues of any real symmetric matrix as one increases its dimension for sufficiently large off-diagonal elements. Only if the Hamiltonian is an explicitly bounded operator combination ($H \sim O^\dagger O + \text{const}$) can this instability be avoided in general. We might also expect to see a continuous spectrum at $x = x_c$ if we compare with the Liouville theory results [18]

$$2P^+P^-\alpha' = \rho^2 - \frac{1}{6}.$$  \hspace{1cm} (17)

but it has been difficult to confirm this numerically. Moreover the groundstate has finite negative mass squared in (17), while the matrix model’s is infinitely negative at $x = x_c$. It has been suggested [17] that perhaps $\mu$ should be renormalised to zero as a result, but a derivation of this requirement is still lacking. In any case we have the first direct demonstration that the non-critical bosonic string with area action is tachyonic above $c = 1$. The use of an unbounded $\phi^3$ matrix potential does not a priori spell tachyons at any order in $1/N$ in the expansion about $\phi = 0$, but the string theory described at the critical coupling is nevertheless tachyonic.†

5. Future Directions

In order to identify the critical behaviour at $c = 2$ more precisely it is useful to investigate the effects of adding an explicit polymerisation term [19]

$$S = S_E + \int d^2x \frac{g}{N^2}(\text{Tr}[\phi^2])^2.$$  \hspace{1cm} (18)

For sufficiently large $g$ this worldsheet contact interaction seems to favour short strings. This is quite unlike the critical point at $g = 0$ and, assuming that there is a phase transition somewhere in between, casts doubt on branched polymer behaviour of the $c = 2$ matrix

† The use of an unbounded potential is rather a symptom of the divergence of the $1/N$ string perturbation expansion, as commented earlier.
model in Minkowski space; recall that this behaviour was identified in $c > 1$ Euclidean dimensions from numerical simulations [20] and combinatorial estimates [21] of dynamical triangulations at the critical point. Moreover simple polymerisation is not the only possibility in the Euclidean game. The phase diagram needs to be investigated in more detail before a clearer picture can be gained.

The tachyon we found is expected to persist at $c > 2$. To regulate the transverse dimensions we can use a transverse lattice. To eliminate the zero modes associated with $x_\perp$, which would otherwise obscure the stringy part of the spectrum, we can perform the Eguchi-Kawai (EK) compactification [22] to single links in each direction. The resulting field theories are much the same as the $c = 2$ one; namely, we deal with UV finite two-dimensional field theories with convergent perturbation expansions at given order in $1/N$. If we use Hermitian matrix models (1) the EK reduction induces more general interactions $V_{\text{eff}}(\phi)$ in the effective $c = 2$ potential [23]. Unfortunately $V_{\text{eff}}(\phi)$ is not known explicitly and can only be calculated as an expansion in powers of the inverse transverse lattice spacing $1/a$. Truncating the expansion arbitrarily at some order, the resulting model will exhibit polymeric behaviour for sufficiently small $a$ since the leading term is a contact interaction similar to that appended in (18), the induced $g$ being $\sim 1/a^2$. At sufficiently large $a$ the (unreduced) transverse lattice sites are obviously uncoupled and we should recover copies of the $c = 2$ model at each site.

In order to deal with an exact EK reduced system, one can employ complex matrix models similar to the old Weingarten model [2]. Complex matrices $M$ live on the links of a $c$-dimensional hypercubic lattice. For non-critical string theory we must use an action given by traces around all (oriented) loops of length 4 on the lattice [6], which comprises the standard plaquette action plus zero area loops. Expansion of this theory in the coupling constant and $1/N$ reproduces the random surfaces of a dynamical quadrangulation in $c$ dimensions, the the probability distribution for neighbouring vertices $\{x, y\}$ of the quadrangulation in this case being

$$\sum_{\hat{\mu}} \delta(x - y - a\hat{\mu}) + \delta(x - y + a\hat{\mu}) ,$$

for orthonormal vectors $\hat{\mu}$. For $c = 1$ it has been proven that this gives the same answers at the critical point as using Feynman propagators [6]. To perform DLCQ we must take the naive continuum limit for two of the dimensions, which seems to produce an intractable
two-dimensional kinetic term unfortunately. Instead we could use the Feynman propagator for these two continuous dimensions and study actions like [24, 25]

$$\int d^2 x \text{Tr}[\partial_\alpha M \partial^\alpha M^\dagger + \mu M M^\dagger + \lambda_1 M M M^\dagger M^\dagger + \lambda_2 M M^\dagger M M^\dagger].$$ (20)

This is a $c = 3$ model with EK reduction of the transverse direction – a single complex matrix $M$ lies on the periodic link – particularly simple since it has no standard plaquette term. One may now study the DLCQ as a function of $a$. At sufficiently small $a$ there should be a roughening transition from the $c = 2$ to the $c = 3$ phase. It should be noted however that the computational accuracy diminishes significantly for $c > 2$ due to the increase in degrees of freedom, each string bit now carrying at least one more $Z_2$ variable – e.g. the real and imaginary parts of $M$.

Even if reliable data on models such as (20) can be collected, we still expect tachyons. We must try other possibilities to eliminate them, such as introducing some dependence on extrinsic geometry, which is also of interest in condensed matter problems. This is certainly possible for the complex matrix models, at least on the lattice target space, since the orientation of simplices is rather manifest. At a more fundamental level, we should look for tachyon-free matrix models with long string dominance if we wish to describe interacting continuum strings. As indicated earlier, such would be a delicate theory; using a positive definite Hamiltonian tends to counteract precisely the requirement of the critical point – crossover to divergence of perturbation theory. Also the obvious fact should be stressed that, contrary to the popular trends of research, such a theory need not possess an attendant continuum worldsheet expansion to be a sensible string theory.

For other applications of light-cone matrix models, such as to confining gauge theories, continuity of strings may not be so important. For example the interesting Regge-like trajectories found in two-dimensional gauged matrix models [12] are most probably the result of restriction to sectors of fixed (discrete) string length*. According to the suggestion made earlier, this freezes the Liouville-like zero mode and exposes the quasi-harmonic energy levels. While it is unrealistic to expect solution of the gauged models in higher dimensions, since they are more complicated than pure large-$N$ QCD, the two-dimensional models may help

* In ref.[12] low-lying mass eigenstates tended to consist of strings of some given length for mysterious dynamical reasons.
us to understand more clearly the relationship between gluonic and fundamental strings. They describe a limit of higher dimensional gauge theory in which all transverse directions are very compact; $x_\perp$-independent transverse potentials $A_\perp(x^+, x^-)$ play the role of matter $\phi$ in the gauged $c = 2$ matrix model.

Clearly there are many interesting questions in applications of string theory which may be addressed by the light-cone matrix models through analytic and numerical techniques.

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REFERENCES

1. G.’t Hooft, *Nucl. Phys.* B72 461 (1974).
2. D. Weingarten, *Phys. Lett.* 90 (1980) 280.
3. F. David, *Nucl. Phys.* B257 45 (1985).
4. V.A. Kazakov, I.K. Kostov, and A.A. Migdal, *Phys. Lett.* B157 (1985) 295.
5. V.A. Kazakov and A.A. Migdal, *Nucl. Phys.* B311 (1989) 171.
6. S. Dalley, *Mod. Phys. Lett.* A7, 1651 (1992).
7. M.R. Douglas, *Phys. Lett.* B238 (1990) 176.
8. R.C. Penner, J. Diff. Geom. 27 (1988) 35.
9. M. Kontsevich, Commun. Math. Phys. 147 (1992) 1.
10. S. Dalley and I.R. Klebanov, *Phys. Lett.* B298 79 (1993).
11. H.-C. Pauli and S. Brodsky, *Phys. Rev.* D32 (1985) 1993 and 2001.
12. S. Dalley and I.R. Klebanov, *Phys. Rev.* D47 2517 (1993).
13. A.M. Polyakov, *Phys. Lett.* B103 (1981) 207.
14. J. Koplik, A. Neveu, and S. Nussinov, *Nucl. Phys.* B123 (1977) 109.
15. C.B. Thorn, *Phys. Lett.* 70B (1977) 85; *Phys. Rev.* D32 (1978) 1073.
16. Chang and Ma, *Phys. Rev.* 180 (1969) 1506; J. Kogut and Soper, *Phys. Rev.* D1 (1970) 2901.
17. K. Demeterfi and I.R. Klebanov, PUPT–1370, [hep-th/9301006](https://arxiv.org/abs/hep-th/9301006), presented at the 7th Nishinomiya-Yukawa Memorial Symposium “Quantum Gravity”, November 1992.
18. T. L. Curtright and C. B. Thorn, *Phys. Rev. Lett.* 48, 1309 (1982).
19. S. Das, A. Dhar, A. Sengupta, and S. Wadia, *Mod. Phys. Lett.* A5 (1990) 1041.
20. D.V. Boulatov, V.A. Kazakov, I.K. Kostov, and A.A. Migdal, *Nucl. Phys.* B257 (1985) 641.
21. J. Ambjorn, B. Durhuus, and J. Frohlich, *Nucl. Phys.* B257 (1985) 433.
22. T. Eguchi and H. Kawai, *Phys. Rev. Lett.* 48 (1982) 1063.
23. L. Alvarez-Gaumé, C. Crnkovic, and J. F. L. Barbòn, *Nucl. Phys.* **B394** (1993) 383.

24. I. R. Klebanov and L. Susskind, *Nucl. Phys.* **B309**, 175 (1988).

25. S. Dalley and T. R. Morris, *Int. Jour. Mod. Phys.* **A5**, 3929 (1990).