A two-channel, spectrally degenerate polarization entangled source on chip

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Integrated optics provides the platform for the experimental implementation of highly complex and compact circuits for quantum information applications. In this context integrated waveguide sources represent a powerful resource for the generation of quantum states of light. However, the confinement of light in a single spatial mode limits the realization of multi-channel sources. Due to this challenge one of the most adopted sources in quantum information processes, i.e. a source which generates spectrally indistinguishable polarization entangled photons in two different spatial modes, has not yet been realized in a fully integrated platform. Here we overcome this limitation by suitably engineering two periodically poled waveguides and an integrated polarization splitter in lithium niobate. This source produces polarization entangled states with fidelity of \( F = 0.973 \pm 0.003 \) and a test of Bell's inequality results in a violation larger than 14 standard deviations. It can work both in pulsed and continuous wave regime. This device represents a new step toward the implementation of fully integrated circuits for quantum information applications.

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INTRODUCTION

In the last decade quantum photonics has played a crucial role in the development of quantum information processes. In particular the transition from bulk to integrated optics has laid the foundation for the implementation of schemes of high complexity, unachievable with previous setups. In the framework of integrated photonic devices, a major development has been carried out in the realization of linear circuits, ranging from the implementation of single building blocks on chip\(^1\) to the simulation of quantum transport via quantum walks.\(^7\)–\(^10\) The integration of up to a hundred optical components on the same chip has made it possible to experimentally implement the recently proposed boson sampling problem\(^1\)–\(^4\),\(^7\)–\(^17\) opening the way to the experimental investigation of classically hard to solve problems. Although the realization of integrated linear circuits has improved significantly, the conversion toward a fully integrated platform for quantum information requires the development and the combination of sources of quantum light and eventually detectors on chip. Concerning sources on chip, parametric down conversion (PDC) in periodically poled nonlinear waveguides is a promising approach\(^18\)–\(^21\) due to high brightness and intrinsic stability. However the confinement of light in a single channel makes it challenging to emit single photons into different spatial modes without the need of postselection. This is the case of one of the most adopted sources in quantum information experiments as quantum teleportation\(^22\)–\(^23\) quantum key distribution\(^24\)–\(^25\) and quantum secure direct communication\(^26\),\(^27\) i.e. a source of spectrally degenerate polarization entangled states.\(^27\)–\(^28\) While the emission into two different spatial modes is intrinsic in bulk crystals, in integrated waveguides this still remains a challenge and the main reason why a fully on chip source of polarization entangled states was still missing. Indeed nonlinear waveguides have been adopted to realize sources of polarization entangled states, but the entanglement was either generated outside the chip with a probabilistic splitting on different spatial modes,\(^18\)–\(^21\) or the pairs were split deterministically by frequency, leading to spectrally non-degenerate photons.\(^19\) While for some applications this strategy can be useful, degeneracy over all degrees of freedom is a fundamental requirement when dealing with quantum tasks where completely indistinguishable particles are needed. Recently it has been demonstrated that parametric down conversion in a coupled structure allows for the generation of path entangled states.\(^29\)\(^,\)\(^30\) However the adoption of this concept to deterministically address different spatial modes and create entanglement in other degrees of freedom is not straightforward.

Here we overcome these limitations by exploiting the features of titanium indiffused waveguides on lithium niobate (LN) substrates and realize a fully on chip source of polarization entangled states at degenerate wavelengths in the telecom regime. These waveguides can indeed guide both polarizations and present high \(\chi^{(2)}\) nonlinearity.

RESULTS

Working principle and device design

In order to achieve polarization entanglement, we implemented a scheme as shown in Fig. 1. Here, two periodically poled nonlinear waveguides designed to generate orthogonally polarized photons via type II parametric down conversion are connected to a zero-gap directional coupler acting as a polarizing beam splitter (PBS). When a pair is generated in waveguide 1 (wg1) the output state is \(|H, V\rangle\), where \(H, V\) represents a horizontal (vertical) polarized photon in the output mode \(i = A, B\); while when the pair is generated in waveguide 2 (wg2) the resulting state is \(|V, H\rangle\). By enabling PDC in both waveguides the superposition of these two contributions yields to the desired entangled states.
then the output state will read \( \frac{1}{\sqrt{2}} \left[ |H, V \rangle + e^{i\phi} |V, H \rangle \right] \). The phase \( \phi \) is now defined by the phase between the two pump beams.

We can describe the action of our device through the transformation from input to output modes of the integrated PBS as a function of transmissivity and reflectivity:

\[
\begin{align*}
a_{1,H}^1 &= \sqrt{R_H} a_{H,1}^1 - \sqrt{R_V} a_{B,1}^1 \\
 a_{1,V}^1 &= \sqrt{R_V} a_{H,1}^1 + \sqrt{R_H} a_{B,1}^1 \\
 a_{2,H}^1 &= \sqrt{R_H} a_{H,2}^1 - \sqrt{R_V} a_{B,2}^1 \\
 a_{2,V}^1 &= \sqrt{R_V} a_{H,2}^1 + \sqrt{R_H} a_{B,2}^1
\end{align*}
\]

(1)

Here \( a_{k,j}^1 \) is a creation operator acting on a photon in mode \( k \) with polarization \( J \) and \( T_j = 1 - R_j \) are the transmissivity and reflectivity of a photon with polarization \( J \) in either modes. The state generated in the two periodically poled waveguides when both are pumped is \( \langle \psi_m \rangle = \frac{1}{\sqrt{2}} \left[ |a_{H,1}^1 a_{V,1}^1 + e^{i\phi} a_{B,1}^1 a_{V,1}^1 \rangle 0 \right] \). The creation operators evolve according to transformations (1) and then the output state will read

\[
\langle \psi_{out} \rangle = \frac{1}{\sqrt{2}} \left| \left( \sqrt{V_T} T_H + e^{i\phi} \sqrt{R_H} R_V \right) a_{H,1}^1 a_{V,1}^1 + \left( \sqrt{R_H} R_V - e^{i\phi} \sqrt{V_T} T_H \right) a_{B,1}^1 a_{V,1}^1 + \left( \sqrt{V_T} T_H - e^{i\phi} \sqrt{R_V} R_H \right) a_{H,1}^1 a_{B,1}^1 + \left( \sqrt{R_H} R_V + e^{i\phi} \sqrt{V_T} T_H \right) a_{B,1}^1 a_{B,1}^1 \right| 0 \right)
\]

(2)

In the ideal case \( R_V = T_H = 1 \) and the device produces the expected entangled state \( \langle \psi_{int} \rangle = \frac{1}{\sqrt{2}} \left[ |a_{H,1}^1 a_{B,1}^1 - e^{i\phi} a_{B,1}^1 a_{V,1}^1 | 0 \right] \). Deviations from these ideal values lead to a non vanishing probability of having two photons in the same output (for a detailed description see Supplementary Information).

This design has two main challenges: the fabrication of two identical periodically poled waveguides such that the PDC light produced in both of them has the same spectral properties and the realization of an integrated PBS with polarization dependent splitting ratios as close as possible to the ideal ones.

Periodically poled titanium indiffused waveguides on lithium niobate substrates are the optimal candidate to realize this design with the required quality: they guide both polarizations, present very low losses and high \( \chi^{(2)} \) nonlinearity. Moreover, we have achieved a control over the fabrication technique such that our source has excellent performances and higher brightness compared to bulk sources due to the combination of high \( \chi^{(2)} \) nonlinearity of lithium niobate and waveguide confinement.

A fundamental requirement is that the photons have no temporal distinguishability, otherwise it will reduce the entanglement. Lithium niobate is a birefringent material, thus horizontal and vertical polarized photons experience different group velocities when traveling through the waveguides. We can consider that the photons are generated at the center of the poled region of length \( L_p \), this yields a delay between H and V photons which is proportional to an effective crystal length

\[ L_{\text{eff}} = L - L_p/2 \]

\[ \Delta T = \frac{(n_{\phi} - n_{\phi}) L_{\text{eff}}}{c} \]

(3)

where \( n_{\phi} \) is the group refractive index for \( J = H, V \) polarization in LN and \( c \) is the speed of light in vacuum. In order to compensate for this temporal walk off we adopt polarization maintaining fibers (PMFs) and let the fast polarized photon travel through the slow axis of the fiber and vice versa. The length of the fibers \( L_f \) is chosen such that the delay between \( H \) and \( V \) in the fiber is

\[ \Delta T_f = \frac{B L_f}{c} = \Delta T \]

(4)

\( B \) being the birefringence of the PMF. In our experiment the length of our chip is \( L = 49 \text{ mm} \) and the periodic poled waveguides occupy \( L_p = 24 \text{ mm} \) of it (see Methods for detailed description of the circuit design). The effective length is then \( L_{\text{eff}} = L - L_p/2 = 37 \text{ mm} \), which introduces a delay \( \Delta T = 9.31 \text{ ps} \). The length of polarization maintaining fibers is chosen accordingly to be \( L_f = 6.95 \text{ m} \). Let us note that the PMFs can be directly glued to our chip and be considered part of the integrated source itself. However, for some applications, such a temporal compensation can be applied after the total evolution of the two-photon states through a linear network; in this case the PMFs can be considered part of the detection scheme, implying that the presence of the PMFs does not prevent the possibility of directly interfacing this LN chip with other integrated circuits.

A second source of distinguishability is that we should prevent or compensate is a possible non-perfect overlap between the PDC spectra of \( H \) and \( V \) photons. This is directly related to symmetry of the joint spectral amplitude (JSA) of the PDC light produced in each waveguide (see Supplementary Material for more details). For a type II process, where orthogonally polarized pairs are emitted, this condition is true only when the pump bandwidth is small enough to ensure a JSA oriented at \(-45^\circ\) in the plane of signal and idler wavelengths. This case correspond to a process driven by continuous wave (CW) pump light. On the contrary the use of pulses results in an asymmetry of the JSA and a consequent difference in the bandwidths of the spectra for \( H \) and \( V \) photons.

In this case spectral filters have to be introduced. We pumped our source with both CW and picosecond pulses. In the case of pulsed light we adopted fiber Bragg grating (FBG) filters which, as in the case of PMFs, can be considered part of the source or of the detection scheme.

Nonlinear characterization

We first tested the nonlinear properties of the two periodically poled waveguides via second harmonic generation (SHG) in order to find the correct wavelength for a spectrally degenerate process. A CW bright field at telecom wavelength has been injected in each waveguide singularly and photons at half of the wavelength have been detected with a PIN-diode. By varying the wavelength of the bright field the curves in Fig. 2 are recorded. The full width at half maximum (FWHM) of the two curves are \( \Delta \lambda_{wg1} = (0.306 \pm 0.009)\text{nm} \) and \( \Delta \lambda_{wg2} = (0.359 \pm 0.008)\text{nm} \) for waveguide 1 and 2 respectively.

\[ L_{\text{eff}} = L - L_p/2 \]

\[ \Delta T = \frac{(n_{\phi} - n_{\phi}) L_{\text{eff}}}{c} \]

(3)

\[ \Delta T_f = \frac{B L_f}{c} = \Delta T \]

(4)
Quantum characterization

The second step is to characterize the quantum performances of our device. The setup for quantum measurements is described in the Materials and Methods section. We first connect the output of our source directly to superconducting nanowire detectors (SNSPD). By measuring single counts and coincidences between the two SNSPDs we evaluated the high brightness of our source to be $B = (4.8 ± 0.2) \times 10^6 \text{ pas}$. The Klyshko efficiency for signal and idler calculated as ratio between measured coincidences and singles are respectively $\eta_s = 0.120 ± 0.001$ and $\eta_i = 0.095 ± 0.001$ which are mainly due to detection efficiency, coupling efficiencies into single mode fibers and losses from optical components. In order to fully characterize our quantum state, we performed a quantum state tomography$^{31}$ to reconstruct the density matrix describing the produced light. We set the pump wavelength at 777.22 nm to ensure spectral degeneracy and measured all the possible polarization states projections. Real and imaginary parts of the experimental density matrix are reported in Figs. 3a,b for pulsed and in Figs. 3c,d for CW pump light. For pulsed pump light the fidelity (overlap) between the measured state and the singlet state of the Bell basis is $F_\text{Bell} = 0.973 ± 0.003$ and the concurrence$^{32}$ is $C_\text{Bell} = 0.965 ± 0.003$, confirming a high degree of entanglement. These values are mainly affected by the imperfect splitting ratio of the PBS on chip, residual temporal mismatch and possible slight mismatch of the central peak of the adopted FBGs.

We then performed a Bell test according to the scheme of Clause, Horne, Shimony and Holt (CHSH)$^{33}$ and obtained a value of the $S$-parameter of $S_{\text{CHSH}} = 2.694 ± 0.046$ corresponding to a violation of more than 14 standard deviations. We performed the same measurements with a CW pump and no filters and obtained a fidelity of $F_{\text{CW}} = 0.941 ± 0.002$, a concurrence of $C_{\text{CW}} = 0.823 ± 0.003$ and $S$-parameter $S_{\text{CW}} = 2.597 ± 0.027$. This fidelity is lower compared to the pulsed pump case because of slightly different properties between the two poled waveguides (shown already in the SHG measurements). However this can be improved in the fabrication process.

DISCUSSION

We have demonstrated the realization of a completely integrated source of spectrally degenerate polarization entangled photons emitted into different spatial modes. These results show that our fully integrated source of polarization entangled states has high quality performances. In this device the high brightness, low losses and the ability of guiding both polarizations, in combination with a smart waveguide design allow us to achieve a considerable step forward in the realization of integrated multi-channel sources. On one hand the high brightness and stability make this device the optimal candidate for replacing bulk sources commonly adopted in quantum information setups. On the other hand, its integrated geometry will have great potential when combined with integrated linear photonic devices as waveguide arrays for quantum walks$^{34}$, on-chip manipulation of quantum states$^{35}$, boson sampling$^{12-15}$, making it possible to build up a fully integrated platform for generation and manipulation of quantum states.

MATERIALS AND METHODS

A detailed scheme of our device is shown in Fig. 4. The waveguide circuit is realized on lithium niobate through lithographic techniques. Titanium diffusion is carried out for about 9 h at 1060 °C in oxygen environment. The waveguide width is $W = 7 \mu\text{m}$ and the height $85 \text{ nm}$. The input waveguide separation is $80 \mu\text{m}$ while the output one is $127 \mu\text{m}$ which matches the separation of fibers in standard waveguide arrays. The input waveguides present periodic poling with period $\Lambda = 9.08 \mu\text{m}$ for a length of $L_p = 24 \text{ mm}$. These waveguides merge in the structure of a zero-gap directional coupler used as polarization splitter. Its basic design is shown in the inset of Fig. 4. It consists of a central section with length $L_c$ in which the two waveguide branches merge into a single waveguide with width $W_i = 2W$ which is twice the width of the single waveguides. A linear branching is performed to separate the two waveguides, i.e. the single waveguides are inclined by an angle $\Theta$. To avoid problems with the lithography in the range in which the waveguide separation is smaller than $2 \mu\text{m}$ a single tapered waveguide with increasing width is chosen ($L_i$ region in the drawing). In our design $W = 7 \mu\text{m}$, $L_i = 480 \mu\text{m}$ and $\Theta = 0.6^\circ$. Spectrally degenerate pairs at 1554.44 nm are generated when the sample is at a temperature of $T = 50 ^\circ\text{C}$ via type II PDC. We optimized the design of the polarization dependent coupler to achieve optimal splitting ratios for horizontal and vertical polarized light and achieved transmission for H and reflectivity for V of respectively $T_H = 0.996 ± 0.002$ and $R_V = 0.968 ± 0.003$. This corresponds to a probability of generating the singlet state of $P_{\text{singlet}} = 0.942$ (See Supplementary for detailed calculations). The total length of our chip is $L = 1728 \text{ mm}$. The output facet is coated to highly reflect the pump wavelength around 775 nm and to highly transmit the PDC pairs at telecom wavelengths. We achieve a suppression of $\approx 22 \text{ dB}$ for the pump and a transmissivity $\geq 98\%$ for PDC light. Our device presents very low losses, as expected for LN, the average value being $0.03 \text{ dB/cm}$.

The setup for quantum measurements is shown in Fig. 5: it can be divided in three sections: the pump preparation setup, the source itself and the analysis setup. The pump preparation setup consists of a loop: here a diagonally polarized beam is split on a bulk PBS and fed into a Sagnac loop. The two beams are recombined on the PBS, however, no interference occurs because a displacement between them is introduced by tilting one mirror ($M$) of the loop. This displacement is imaged to the separation between the two input waveguides of the chip through a set of lenses (not shown in the picture). Such a configuration ensures a common path geometry for the two pump beams such that their relative phase remains constant in time. The desired phase $\phi$ between them is then set via a liquid crystal device (LCD). After the loop a half wave plate and another PBS are used to set the polarization state of both beams (horizontal). Let us note that by properly rotating the waveplates before and after the Sagnac loop we are able to pump only waveguide 1 or waveguide 2 and generate separable states, or both at the same time and generate polarization entanglement. The second part of the setup is the source itself: the LN chip which generates the polarization entangled state. The last part is the analysis setup: PMFs are used for temporal delay.
In our scheme we chose to use a free space coupling but it is also possible to directly interface a fiber array of PMFs to the output of the chip, as the waveguide separation at the output matches the standard separation between fibers in fiber arrays ($s = 127 \mu m$). In the case of pulsed pump beams FBG filters with bandwidth $\Delta \lambda = 0.25 \text{ nm}$ are inserted to apply spectral filtering. Standard polarization analysis components and superconducting nanowires single photon detectors (SNSPD, Quantum Opus) are used to characterize the state. Coincidence counts between the two outputs are recorded.
Fig. 5  Experimental setup for quantum measurements: the two pump beams are created within a Sagnac loop and displaced by rotating one mirror of the loop (M). The phase between the two beams is set via a liquid crystal device (LCD). After the chip the entangled state is measured with a standard polarization analysis setup: half waveplate (HWP), quarter waveplate (QWP) and a polarizing beam splitter (PBS). The photons are detected with superconducting nanowire single photon detector (SNSPD). Temporal delay is compensated with polarization maintaining fibers (PMF) and fiber Bragg grating filters (FBG) can be possibly inserted to filter the spectrum of the emitted photons

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AUTHOR CONTRIBUTIONS
L.S. and Ch.S. conceived the idea, L.S., K.H.L. and H.H. designed the scheme and calculated the parameters for device fabrication. C.E., R.R. and V.Q. fabricated the device, L.S. carried out the experiment, analyzed the data and wrote the manuscript. All the Authors contributed to the final revision of the manuscript.

COMPETING INTERESTS
The authors declare that they have no competing interests.

REFERENCES
1 Politi, A., Cryan, M. J., Rarity, J. G., Yu, S. & O’Brien, J. L. Silica-on-silicon waveguide quantum circuits. Science 320, 646–649 (2008).
2 Smith, B. J., Kundys, D., Thomas-Peter, N., Smith, P. G. R. & Walmsley, I. A. Phase-controlled integrated photonic quantum circuits. Opt. Express. 17, 13516–13525 (2009).
3 Sansoni, L. et al. Polarization entangled state measurement on a chip. Phys. Rev. Lett. 105, 200503 (2010).
4 Crespi, A. et al. Integrated photonic quantum gates for polarization qubits. Nat. Commu. 2, 566 (2011).
5 Heilmann, R., Gräfe, M., Nolte, S. & Szameit, A. Arbitrary photonic wave plate operations on chip: realizing Hadamard, Pauli-X, and rotation gates for polarisation qubits. Sci. Rep. 4, 4118 (2013).
6 Corrielli, G. & Rotated waveplates in integrated waveguide optics. Nat. Commu. 5, 4249 (2014).
7 Peruzzo, A. et al. Quantum walks of correlated photons. Science 329, 1500–1503 (2010).
8 Sansoni, L. et al. Two-particle bosonic-fermionic quantum walk via integrated photonics. Phys. Rev. Lett. 108, 010502 (2012).
9 Crespi, A. et al. Anderson localization of entangled photons in an integrated quantum walk. Nature Photonics 7, 322–328 (2013).
10 Poulios, K. et al. Quantum walks of correlated photon pairs in two-dimensional waveguide arrays. Phys. Rev. Lett. 112, 143604 (2014).
11 Aaronson, S. & Arkhipov, A. The computational complexity of linear optics. Theory of Computing 9, 143–252, http://www.theoryofcomputing.org/articles/v009a004 (2013).
12 Tillmann, M. et al. Experimental boson sampling. Nature Photonics 7, 540 (2013).
13 Crespi, A. et al. Integrated multimode interferometers with arbitrary designs for photonic boson sampling. Nature Photonics 7, 545–549 (2013).
14 Spring, J. B. et al. Boson sampling on a photonic chip. Science 339, 798–801 (2013).
15 Broome, M. A. et al. Photonic boson sampling in a tunable circuit. Science 339, 794 (2013).
16 Spagnolo, N. et al. Experimental validation of photonic boson sampling. Nat. Photon. 8, 615–620, doi:10.1038/nphoton.2014.135 (2014).
17 Bentivegna, M. et al. Experimental scattershot boson sampling. Sci. Adv. 1, e1400255 (2015).
18 Martin, A. et al. A polarization entangled photon-pair source based on a type-II PPLN waveguide emitting at a telecom wavelength. New. J. Phys. 12, 103005 (2010).
19 Herrmann, H. et al. Post-selection free, integrated optical source of non-degenerate, polarization entangled photon pairs. Opt. Express. 21, 27981–27991, http://www.opticsexpress.org/abstract.cfm?URI=oe-21-23-27981 (2013).
20 Krapick, S. et al. An efficient integrated two-color source for heralded single photons. New. J. Phys. 15, 033010 (2013).
21 Kaiser, F. et al. A versatile source of polarization entangled photons for quantum network applications. Laser Physics Letters 10, 045202 (2013).
22 Bennett, C. H. et al. Teleporting an unknown quantum state via dual classical and einstein-podolsky-rosen channels. Phys. Rev. Lett. 70, 1895–1899, http://link.aps.org/doi/10.1103/PhysRevLett.70.1895 (1993).
23 Boschi, D. et al. Experimental realization of teleporting an unknown pure quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. 80, 1121 (1998).
24 Ekert, A. K. Quantum cryptography based on bell’s theorem. Phys. Rev. Lett. 67, 661–663, http://link.aps.org/doi/10.1103/PhysRevLett.67.661 (1991).
25 Jennewein, T., Simon, C., Weihs, G., Weinfurter, H. & Zeilinger, A. Quantum cryptography with entangled photons. Phys. Rev. Lett. 84, 4729–4732, http://link.aps.org/doi/10.1103/PhysRevLett.84.4729 (2000).
26 Long, G. L. & Liu, X. S. Theoretically efficient high-capacity quantum-key-distribution scheme. Phys. Rev. A. 65, 032302. http://link.aps.org/doi/10.1103/PhysRevA.65.032302 (2002).
27 Kwiat, P., Mattle, K., Weinfurter, H. & Zeilinger, A. New high-intensity source of polarization entangled photon pairs. Phys. Rev. Lett. 75, 4337 (1995).
28 Fedrizzi, A., Herbst, T., Poppe, A., Jennewein, T. & Zeilinger, A. A wavelength-tunable fiber-coupled source of narrowband entangled photons. Opt. Express. 15, 15377 (2007).
29 Kruse, R. et al. Dual-path source engineering in integrated quantum optics. Phys. Rev. A. 92, 053841, http://link.aps.org/doi/10.1103/PhysRevA.92.053841 (2015).
30 Setzpfandt, F. et al. Tunable generation of entangled photons in a nonlinear directional coupler. Laser & Photonics Reviews 10, 131–136, doi:101002/lpor.201500216 (2016).
31 James, D. F. V., Kwiat, P. G., Munro, W. J. & White, A. G. Measurement of qubits. Phys. Rev. A. 64, 052312 (2001).
32 Hill, S. & Wootters, W. K. Entanglement of a pair of quantum bits. Phys. Rev. Lett. 78, 5022–5025, http://link.aps.org/doi/10.1103/PhysRevLett.78.5022 (1997).
33 Clausner, J., Horne, M., Shimony, A. & Holt, R. Proposed experiment to test local hidden-variable theories. Phys. Rev. Lett. 23, 880 (1969).
34 Politi, A., Matthews, J. C. F. & O’Brien, J. L. Shor’s quantum factoring algorithm on a photonic chip. Science 325, 1221 (2009).
35 Shadbolt, P. J. et al. Generating, manipulating and measuring entanglement and mixture with a reconfigurable photonic circuit. Nature Photonics 6, 45–49 (2011).

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