Developing Piaget's Theory in Mistakes Construction of Knowledge when Problem Solving through Analogical Reasoning

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Abstract. This study aims to see the errors of knowledge construction when students solve algebra problems. To get the subject of research conducted exploratory to class VII students with given algebra task sheet in the form of analogy. Through Think out loud method is enhanced by interview obtained verbal data. The results showed that the characteristics of algebraic construction errors in students include: no analogical reasoning, mistake in assimilation and accommodation, and happened pseudo thinking. In addition, mistakes were found in assimilation and accommodation of relationships and strategies.

1. Introduction
In Indonesia, the concept of algebra began to be introduced to students at elementary school level. For example \(5 + \Box = 7\), then results from \(\Box\) can be found by students directly. Likewise, at the junior high school level, algebra in the odd semester in class VII begins by explaining; the coefficients, variables, constants, and tribes in algebraic form, addition and subtraction of algebraic forms, multiplication and algebraic divisions, after which simplification of algebraic form. Judging from the diversity of mathematics subject matter; that one concept deals with the other one concept or more then of concept. This means that one student's difficulty in learning one concept of mathematics can have an impact on students' difficulties in learning other concept of mathematics.

The results of interviews conducted with junior high school teachers in district Ciamis have known that for grades VII and VIII use the curriculum in 2013. While in grad IX varies. There is using curriculum in 2013 and there are schools using an educational unit level curriculum. Reality in learning, both concept given teachers or in books students have been using analogies. Although not directly expressed the basis of analogies and targets of analogy as found in English [1] and Polya [2] but in learning has applied analogical reasoning. One example in the book of class VII [3]: About 6,000 copies of magazines sold in this week. Estimate many magazines to be sold within in the year. This contains can be translated according to the classical analogy as follows: \(1:6000::48\) \(\cdots\) \(\Box\) mean one week as A, 6000 mean magazine sold as B, 48 mean number of weeks in 1 year as C, and \(\Box\) mean as D, classical analogy form is \(A: B :: C: D\). This shows that in the learning has been applied analogical reasoning. Although there are still many teachers who have not used the pedagogical analogy reasoning in learning [4] even the teacher has not understood the logical reasoning as found by Supratman et al. [5].

The meaning of analogical reasoning in learning is how the teacher utilizes the students' knowledge (as the basis of Analogy) to solve new problems (as the analogy target), resulting in the construction of
new knowledge in students' cognition as according to Anthony's opinion [6] in learning. Students learn math means students are directed towards constructing mathematical concepts and building knowledge through the attribution of one concept to another. This goes on and on so that knowledge is awakened. It can be to build up the next knowledge.

In the construction of knowledge cannot be separated from the occurrence of process of assimilation and accommodation. That begins with the occurrence of disequilibrium until the occurrence of the final level equilibrium as Piaget theory [7]. In accordance with Gentner's opinion [8], the process of assimilation and accommodation in Piaget's theory is illustrated by Subaji and Supratman [9] as follows.

![Diagram of Assimilation and Accommodation Process]

**Figure 1.** The occurrence of the Assimilation and Accommodation Process

Students in the construction of knowledge or solve problems, allowing difficulties occur. The difficulty is seen from the error in solving the problem. The incident of Student Error results in the formation of imperfect cognitive structures. This is shown the structure of problem integrated to the cognitive structure, while the problem structure is not the same as the cognitive structure. Illustration of assimilation and accommodation occur when students solve the following task instruments:

"Meli and Joni are brothers, now the age of both is 75 years old, if Meli is 5 years older than Joni. How old is Meli's age?"

In solving this problem there are students who answered Meli's age is 35 years. The possibility of thinking process occurs in imperfections of assimilation and accommodation. In the beginning of the problem-solving in general: students are considering the age of Meli as $x$ and the age of Joni as $y$, but there are also those that let Meli as $x$ and Joni as $y$. So the problem can be explained as shown in Figures 2a, 2b, 2c, 2d, and 2e.

![Diagram of Problem Solution]

**Figure 2a.** Conformity Problem structure with the structure of thinking of type 1
Figure 2b. Conformity Problem structure with the structure of thinking of type 2

Figure 2c. Conformity Problem structure with the structure of thinking of type 3

Figure 2d. The mismatch of problem structure with the structure of thinking of type 1

Figure 2e. The mismatch of problem structures with the structure of thinking type 2

The task instrument contains only the analogy targets that are the number of Meli and Joni's age is 75 years and Meli is 5 years older than Joni as C. How age of Meli? it as D. The teacher has explained earlier as follows: the basic analogous is $x + y = 15$ and $x - y = 5$ as A. is how much x? x is 10 as B.

According to Supratman [10] in the construction of knowledge/problem solving will experience the process of assimilating the problem, assimilation of strategy, assimilation of relationships, accommodation of problem, accommodation of strategy and accommodation of relationships. Thus, the occurrence of the errors of one of the categories of assimilation and accommodation will be results in both the mismatch of problem solving and the construction of new knowledge. Supratman goes so far as to explain that for the analogical reasoning process it will be seen how far the students do correspondence between the base of analogy and the targets of analogy.

Failure in correspondence from the basic of analogy into the target of analogy will be error a construction of concept. The location of the failure of correspondence may occur when assimilating of problem, assimilation of strategy, assimilation of relationships, accommodation of problem, accommodation of strategy and accommodation of relationships.
2. Research Methods
This research is a qualitative-exploratory research as according of Yin [11] and Matthew B. & Huberman, Michael A [12]. Researchers conduct exploration to junior high school students in district of Ciamis, Province of West Java. Of the 72 students involved in the study; As many as 37 students were able to solve the problem correctly, 25 students experienced errors in problem-solving, and 6 students did not give an answer. Research data using verbal data in the form of thinking process of students. To get the data: A student is given a work sheet instrument interchangeably while telling the one she thought. If the student's answer is correct then it is not taken as the subject of research. Next take a student again given the same task and treatment. If the student answers wrong, then the student is taken as the subject of research. Of the 25 students who answered incorrectly can be grouped 2 different categories. For that revealed 2 subjects who have different categories and able to provide information consistently. Furthermore student answers are analyzed based on Piaget's theory. According of Olson G M, Mack R L & Duffy S A [13], Van Someren M W, Barnard Y F and Sandberg J A [14] and Ericsson K A and Simon H A [15] the data retrieval method is said to think out loud or think aloud.

3. Findings
The findings in research on errors in knowledge construction while solving problems through analogical reasoning with Piaget's theoretical framework, include errors; (1) assimilation of problems, (2) assimilation of relationships, (3) assimilation of strategy, (4) accommodation of problem, (5) accommodation of strategy, and (6) accommodation of relationships. To see the analogical reasoning process verbal data needs to be categorized. Categorization is made very simple to facilitate the reading process done when students solve problems through analogical reasoning. As for the encoding/categorization as presented in Table 1

| Code | Meanings |
|------|----------|
| 's   | Problem on the basis of analogy |
| 'a   | Describes the sum of two numbers of (x + y = 15) |
| 'b   | Describes the reduction of two numbers of (x - 5 = y) |
| 'c   | Substituting of (y = x - 5) into (x + y = 15) |
| 'd   | The result of substitution of (y = x - 5) at (x + y = 75) is (x + x - 5 = 15) |
| 'e   | Summing of (x + x - 5 = 15) so as to obtain of (2x = 15 + 5) |
| 'f   | Sums the number of (15 + 5) to get (2x = 20) |
| 'g   | Divide 2 on each segment to get (x = 20/2) |
| 'h   | The calculation result (x = 10) |
| 'i   | For example the age of Meli is 5 years older than Joni with (x = y + 5) |
| 'j   | Substitution of (x = y + 5) into (x + y = 15) is (y + 5 + y = 15) |
| 'k   | Added (y + 5 + y = 15) to get (2y = 15 - 5) |
| 'l   | Reduce the number of (15 - 5) to obtain (2y = 10) |
| 'm   | Divide each segment by 2 to obtain (y = 5) |
| 'n   | Substituting (y = 5) at (x = y + 5) to obtain (x = 5 + 5) |
| 'o   | Describe the age of Meli is 5 years older than Joni with (x - y = 5) |
| 'p   | Added (x + y = 15) with (x - y = 5) to get (2x = 20) |
| s    | About the target analogy |
| a    | For example the ages of Meli and Joni with (x + y = 75) |
| b    | Suppose that Meli's age is 5 years older than Joni with (x-5 = y) |
| c    | Substituting (y = x-5) at (x + y = 75) |
The substitution result \((y = x-5)\) at \((x + y = 75)\) becomes \((x + x-5 = 75)\)

Summing at \((x + x-5 = 75)\) to obtain \((2x = 75 + 5)\)

Add two numbers at \((75 + 5)\) to obtain \((2x = 80)\)

Divide 2 in each space so that it is obtained \((x = 80/2)\)

The result of the calculation is \((x = 40)\)

Suppose that Meli's age is 5 years older than Joni with \((x = y + 5)\)

The substitution result \((x = y + 5)\) at \((x + y = 75)\) is \((y + 5 + y = 75)\)

Summing at \((y + 5 + y = 75)\) to get \((2y = 75 - 5)\)

Reduce the two numbers at \((75 - 5)\) to obtain \((2y = 70)\)

Divide each space by 2 so as to obtain \((y = 35)\)

Substituting \((y = 35)\) at \((x = y + 5)\) is obtained \((x = 35 + 5)\)

Add two numbers at \((35 + 5)\) to get \((x = 40)\)

Suppose that Meli's age is 5 years older than Joni with \((x-y = 5)\)

The thinking process of Character of type student one (S1) is investigated based on Piaget's theory

The process of thinking of group Subject one (S1) examined based on Piaget's theoretical framework, when S1 completing the task sheet instrument can be seen in Diagram 1.
Assimilation problems on target analogy
Assimilation strategy on target analogy
Assimilation relation to the target analogy
Accommodation problems on target analogy
Accommodation relation to the target analogy
Accommodation strategy on target analogy
Not integrated
misrepresented
Impact misrepresents the consequences of producing incorrect calculations
correspondence occurs
Integration of problem structures to cognitive structures

Diagram 1 the Process of Adaptation S1 under the Piaget Framework through Analogical Reasoning

Based on Diagram 1, it shows that structural of problem is not integrated into cognitive structure. The thinking process S1 is dominated by assimilation of relationships, although initially there are accommodation of problem and accommodation of strategy. At the beginning of the problem-solving, S1 describes the number of ages Meli and Joni is 75 year. It as the sum of two numbers x and y. But when S1 describing Meli's age is older five-year than Joni's age, S1 make a mistake. This shows S1 doing pseudo-thinking. S1 does not perform partition on problem structure. In addition, S1 does not use analogical reasoning to solve problems as target analogies.

Pseudo thinking from S1 is not doing really think that is logical and rational. The definition of the partition of the problem structure is to divide some parts of the problem structure on the analogy targets into simple problem structures according to the partitions of the basic of analogy. Understanding S1 does not use analogical reasoning. S1 does not think to do the correspondence between elements that exist of the basic of analogies with the elements required of the target of analogies.

As revealed in the interview researcher to S1 as follows.
Researcher (R): are you sure of your answer?
S1 : yes I'm sure Sir
R : what do you mean by x and y?
S1 : I mean by x is the age of Meli and y is Joni's age
R : What is your goal to x is the age of Meli and y is Joni's age?
S1 : to facilitate the sum of two numbers between x and y. So the ages of Meli and Joni become 
\[x + y = 75\]
R : Why do you describe Meli 5 years older than Joni with \((x + 5 = y)\)? (While pointing to the S1 job)
S1: because the age of Meli is 5 years older than Joni, so I mean \((x + 5 = y)\) Sir.

Answers from S1 are as follows.
Translated version

| S1 suggests that Meli’s age is \( x \) and Joni’s age is \( y \)  |
| then the age of Meli and Joni is 75 years translated \( x + y = 75 \)  |
| then predicted S1 for Meli age 5 years older than Joni is \( x + 5 = y \)  |
| so that \( y \) is substituted at \( x + y = 75 \) by \( x + 5 \)  |
| \( x + x + 5 = 75 \)  |
| \( 2x + 5 - 5 = 75 - 5 \)  |
| \( 2x = 70 \)  |
| \( x = 70/2 \)  |
| \( x = 35 \)  |
| So that Meli’s age is 35 years  |

**Figure 3** S1 fails in accommodation strategy and accommodation relationships

This indicates that S1 failed in doing strategic accommodation. That is, when S1 describes the age of Meli 5 years older than the age of Joni as \( x - 5 = y \) or \( x - y = 5 \) or \( x = y + 5 \). This has an impact on: when S1 conducts relationship accommodation. That is, S1 substitutes \( y \) at \( x + y = 75 \) by \( x + 5 = y \). S1 does not realize that she has made a mistake describing that Meli’s age is 5 years older than Joni’s age pad. The results of the descriptions are \( x + 5 = y \). In addition, S1 does not reflect what he has done, so the possibility of misperception can be avoided. In addition, S1 did not double check the answer. Next S1 finds the value of \( y \), from the calculation \( y = x + 5 \). Then S1 can prove that must \( x > y \). So S1 has confidence that what he did wrong, so he can fix the error.

The thinking process of Character of type students two (S2) is investigated based on Piaget’s theory

The case of the thinking process S1 is almost the same as the case experienced by S2 as for the thinking process S2 can be seen in Diagram 2 as follows.

**Diagram 2** the Process of Adaptation S2 under the Piaget Framework through Analogical Reasoning

Based on Diagram 2, the S2 shows no ability to integrate the problem structure into its cognitive structure. S2 is only able to integrate the substructure of the problem into its cognitive, but S2 makes a mistake in describing of Meli’s age 5 years older than Joni’s age as \( x - 5 = y \) or \( x - y = 5 \) or \( x = y + 5 \). Instead, the S2 describes of Meli’s age 5 years older than Joni’s age as \( x=y-5 \). A very sophisticated S2 fault is Meli as \( x \) and Joni as \( y \). This would give a misleading notion since \( x \) and \( y \) represents not a number but represent an object.
As revealed in the interview researcher to S2 as follows.

**R:** Are you sure of your answer?
**S2:** Sure sir

**R:** Why did you let Meli as x and Joni as y?
**S2:** To be easy and concise

**R:** Are x and y the numbers?
**S2:** No Sir but the variables representing Meli and Joni

**R:** Suppose I borrow the x so I use it for \(x^2\). Approximately what do you think it means?
**S2:** no answer

Furthermore, S2 misrepresents Meli's age is 5 years older than Joni's age. So S2 does pseudo-thinking. S2 does not partition the problem structure. In addition, S2 does not use analogical reasoning to solve problems as a target analogy.

S2 is unaware to assume that Meli's age is 5 years older than the age of Joni as \((x = y - 5)\). It is error accommodation strategy. This implies a wrong in substituting \(x\) at \((x + y = 75)\) by \((x = y - 5)\). So it results in a relationship accommodation error. In addition, S2 does not reflect on what he has done, so the possibility of misperception can be avoided. In addition, S2 did not test the results of its answers. That his answer must be \(x > y\). Suppose S2 compares the values obtained \((y = 40)\) and \((x = 35)\) turns \(y > x\). Realizing it thus means the S2 belief that he perceives wrongly. Answers from S2 are as follows.

Translated version

| \(x\) | \(y\) |
|------|------|
| 35   | 40   |

**Figure 4** S2 fails in accommodation strategy and accommodation relationships

### 4. Discussion

S1 and S2 begin with a new problem, because previously S1 and S2 have not got the problem. This is according to Piaget's theory [9]; S1 and S2 get accommodation problems in the form of looking for the age of Meli. Furthermore, S1 and S2 are able to describe the age of Meli and Joni age is 75 in the algebra model \((x + y = 75)\). This activity is a strategic accommodation. However, S1 and S2 fail to describe the age of Meli 5 years older than Joni age into algebraic models such as \((x - 5 = y)\) or \((x - y = 5)\) or \((x = y + 5)\).
Mistake S1 describes the age of Meli 5 years older than Joni age into an algebraic model like this \((x + 5 = y)\). Similarly, S2 describes Meli's age 5 years older than Joni's age into an algebraic model like this \((x = y - 5)\). This means that S1 and S2 experience a strategy accommodation error. S1 and S2 errors result in errors in relationship accommodation and assimilation of relationships.

The subsequent errors of S1 and S2 do not use the analogy of reasoning to solve the target analogy. Subsequent errors, S1 and S2 do not partition the problem structure into simple structures, according to their cognitive.

5. Conclusion

The occurrence of knowledge construction errors due to:

1) Students do not use analogical reasoning that utilizes knowledge that has been mastered as the basis of analogy to solve the problem as a target analogy
2) Students do not partition the problem structure into a very simple problem according to the concept they have,
3) There is a false thinking because it does not use logic.
4) Students do not reflect on what he has done
5) in the integration of the structure of the problem: when initiated accommodation strategy is wrong, resulting in accommodation relationship is wrong as well. Consequently there is no construction of new knowledge to students.

Acknowledgments:
The Ministry of Research and Technology of the Republic of Indonesia

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