The relationship between hydraulic resistance and heat transfer in the crossflow-line of staggered tube bundles

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Abstract. The paper develops the idea that the semi-empirical formula for calculating heat transfer in turbulent flow in pipes, expressed in terms of ratio of the dynamic speed to the average flow velocity in the pipe, are universal and can be used for the calculation of heat transfer in channels with complex geometry, where separated flows take place. A relationship between flow resistance and heat transfer at cross turbulent flow in line and staggered bundles of rods was found. The resulting formulas generalize experimental data available in the literature better than the empirical formulas.

1. Introduction
One of the urgent tasks of heat transfer theory and hydrodynamics is to establish correlations between hydraulic resistance and heat transfer for specific cooling surface. Within the framework of the semiempirical Prandtl theory of turbulence this problem is almost solved for smooth direct channels, and in the case of complex shapes with loose-channel eddy currents (crossflow tube bundles, ball filling, the porous material, etc.), where the friction loss of pressure constitute a small part of the total pressure loss for pumping coolant, this question requires serious research. Different approaches to solve this problem are available in literature publications [1-5]. The universal relationship between hydraulic resistance and heat and mass transfer in porous media proposed and substantiated in [1-4]. This universal relationship summarizes a large body of experimental data obtained by different authors for many structures with a deviation of no more than 30%. So, the heat transfer and hydrodynamics in the pebble bed and granular layers crossflow tube bundles, grid inserts, perforated plates packets are bind by the same formula. This suggests that advanced in [1-4] hypothesis of the universal nature of the heat transfer laws, derived from semi-empirical theory, is correct. However, the universal formula is not sufficiently accurate for reliable thermal-hydraulic calculations. That’s why, further studies are required to clarify the heat transfer model for each type of porous structures. In particular, proposed in [1-4] universal formula is suitable for cross flow staggered bundles, but it does not describe satisfactorily correlation between hydraulic resistance and heat transfer at crossflow line bundles. This work is aimed to further study of the interrelations of hydraulic resistance and heat transfer in tube bundles in crossflow within the framework of the proposed in [1-4] approach.
2. The interrelation between flow resistance and heat transfer in turbulent flow in smooth straight pipes

In [6-9] showed that in turbulent flow in pipes in the area of stabilized heat transfer for constant heat load heat transfer is well described obtained by theoretical calculation formula

\[ Nu = \left( \frac{\zeta}{\sqrt{8}} \right) \frac{Re \cdot Pr}{k_1 \sqrt{Pr + \frac{\zeta}{8} (Pr^{\frac{2}{3}} - 1)}} \]  

(1)

where \( Nu = \frac{\alpha d}{\lambda} \), \( d \) - tube diameter; \( \alpha \) – heat transfer coefficient; \( \lambda \) – thermal conductivity coefficient of the coolant; \( \zeta = (1.81 \log_{10} Re - 1.64)^2 \) - hydraulic resistance coefficient of the pipe, \( k_1 = 1.07 \), \( k_2 = 12.7 \). Equation (1) is valid for Re numbers from \( 10^4 \) to \( 5 \times 10^5 \) and Pr numbers from 0.5 to 2000. An important characteristic of the coolant velocity distribution near the pipe wall is a dynamic rate \( \nu_* \), which is related to the average velocity and drag coefficient \( \nu_*/\nu = \sqrt{\zeta}/8 \). Let’s mark the ratio of the dynamic speed to the average speed through \( \phi = \nu_*/\nu \), and assume that \( k_1 = 1 \) and \( k = k_2 \). Then the formula (1) can be reduced to the form:

\[ St = \frac{\phi^2}{1 + k \phi (Pr^{\frac{2}{3}} - 1)} \]  

(2)

where \( St = \frac{Nu}{(Re \cdot Pr)} \) - the Stanton number. In [1-4], is assumed that the equation (2) is universal and applicable to the calculation of the average heat transfer in complex configuration channels with separated flows. Thus the main objective is to look at the relationship between the dynamic speed and the average speed of the flow in the channel, i.e. expression relating \( \phi \) to the characteristics of averaged flow. In [1-4] proposed to determine the speed of a dynamic power through the coolant pumping costs. Dynamic speed associated with power costs from dimensional considerations for pumping expression

\[ \nu_* = c \left( \frac{\nu \nabla P}{\rho} \right)^{0.25} \]  

(3)

where \( \nu \) - kinematic viscosity coefficient, \( c \) – proportionality factor, which depends in general on the geometry of the channel and the number of Re. For any channel we have the relation:

\[ \phi = \frac{\nu_*}{\nu} = c \left( \frac{zeta}{2Re \Pi} \right)^{0.25} \]  

(4)

In [1-4] on the basis of experimental data analysis it was found that for many complex heat exchange surfaces (granular layers, crossflow staggered bundles, of mesh inserts, packets perforated plates) factor "c" can be assumed to be constant and equal to about 3. The expression (2) becomes a universal for all considered in [1-4] surfaces, and the whole array of studied experimental data is summarized with 30% accuracy.

Set out above model describes a limiting case of separated flows in the winding channels, when the coolant is constantly changing its direction of motion and intensively mixed in the communicating channel. For example, in crossflow line bundles, where portions of rectilinear motion of the coolant can be identified in the annulus, the assumption that \( c = 3 \) is false. Calculation of heat transfer by the
formula (2) with reasonable accuracy requires knowledge of «c» channel coefficient dependence on the geometry and coolant flow rate for each type of heat-exchange surfaces.

3. Line and staggered tube bundles.

In [10-12] presented experimental data on heat transfer and hydraulic resistance in the tube bundles in crossflow and empirical formulas are offered for engineering calculations. Note that all investigated range of Reynolds numbers for line bundles and staggered bundles is divided into three sections. And each site has its own formula for calculations. Wherein calculations on recommended formulas are in good agreement with the experimental data only for the bundles with a large value of porosity \( \Pi \geq 0.3 \). And for the dense packing of tube bundles \( \Pi < 0.3 \) the results of calculations and the experimental data disagree strongly (up to 70 \%).

For heat transfer calculations by means of equation (2) it is necessary to know the parameter \( \varphi \), which in turn depends on the constant "c". Processing of the experimental data from [11] gives the following dependences:

for line bundles: \( k = 11.13 \)

\[
c = \begin{cases} 
0.76 + 1.93\Pi, & \Pi \geq 0.30 \\
(0.97\Pi - 0.03)Re^{0.157}, & \Pi < 0.30 
\end{cases}
\]  

(5)

for staggered bundles: \( k = 8.3 \)

\[
c = \begin{cases} 
1.6 + 0.32\Pi, & \Pi \geq 0.30 \\
3.5 + 2.17Re \cdot 10^{-6}, & \Pi < 0.30 
\end{cases}
\]  

(6)

From (5) and (6) it follows that for crossflow in line and staggered bundles with greater porosity \( \Pi \geq 0.3 \) coefficient "c" is a function of the geometric parameters of the bundle and does not depend on the Reynolds number. In dense bundles of tubes with \( \Pi = 0.14 - 0.3 \) dependence "c" of the Reynolds number appears. Calculation formula using a set of dependencies for the "c" in good agreement with the experimental data [11] (Figure 1-2).

![Figure 1](image)

**Figure 1.** The dependence of heat transfer from the complex number Re for a corridor beam with high porosity (a) and low porosity (b): 1- calculation guidelines [4], 2- formula calculation (1), 3- experimental values.
Figure 2. The dependence of heat transfer from the complex number Re for staggered with high porosity (a) and low porosity (b): 1- calculation guidelines [4], 2- formula calculation (1), 3- experimental values.

4. Conclusion
The results obtained demonstrate the possibility of applying the given approach to the calculations for average heat transfer in channels of different shapes according to the flow resistance of these channels. It also demonstrates the universality of the laws of heat transfer in a turbulent flow.

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