Abstract

In this paper, we investigate the effects of an additional trusted relay node on the secrecy of multiple-access wiretap channel (MAC-WT) by considering the model of multiple-access relay wiretap channel (MARC-WT). More specifically, first, we investigate the discrete memoryless MARC-WT. Three inner bounds (with respect to decode-forward (DF), noise-forward (NF) and compress-forward (CF) strategies) on the secrecy capacity region are provided. Second, we investigate the degraded discrete memoryless MARC-WT, and present an outer bound on the secrecy capacity region of this degraded model. Finally, we investigate the Gaussian MARC-WT, and find that the NF and CF strategies help to enhance Tekin-Yener’s achievable secrecy rate region of Gaussian MAC-WT. Moreover, we find that if the channel from the transmitters to the relay is less noisy than the channels from the transmitters to the legitimate receiver and the wiretapper, the DF strategy performs even better than the NF and CF strategies, i.e., the noise-forward strategy is not always the best way to enhance the security.

Index Terms

Multiple-access wiretap channel, relay channel, secrecy capacity region.

I. INTRODUCTION

Equivocation was first introduced into channel coding by Wyner in his study of wiretap channel [2]. It is a kind of discrete memoryless degraded broadcast channels. The object is to transmit messages to the legitimate receiver, while keeping the wiretapper as ignorant of the messages as possible. Based on Wyner’s work, Leung-Yan-Cheong and Hellman studied the Gaussian wiretap channel (GWC) [3], and showed that its secrecy capacity was the difference between the main channel capacity and the overall wiretap channel capacity (the cascade of main channel and wiretap channel).

After the publication of Wyner’s work, Csiszár and Körner [4] investigated a more general situation: the broadcast channels with confidential messages (BCC). In this model, a common message and a confidential message were sent through a general broadcast channel. The common message was assumed to be decoded correctly by the legitimate receiver and the wiretapper, while the confidential message was only allowed to be obtained by the legitimate receiver. This model is also a generalization of [5], where no confidentiality condition is imposed. The capacity-equivocation region and the secrecy capacity region of BCC [4] were totally determined, and the results

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were also a generalization of those in [2]. Furthermore, the capacity-equivocation region of Gaussian BCC was
determined in [21].

By using the approach of [2] and [4], the information-theoretic security for other multi-user communication
systems has been widely studied, see the followings.

- For the broadcast channel, Liu et al. [6] studied the broadcast channel with two confidential messages (no
  common message), and provided an inner bound on the secrecy capacity region. Furthermore, Xu et al. [7]
studied the broadcast channel with two confidential messages and one common message, and provided inner
and outer bounds on the capacity-equivocation region.

- For the multiple-access channel (MAC), the security problems are split into two directions.
  - The first is that two users wish to transmit their corresponding messages to a destination, and meanwhile,
    they also receive the channel output. Each user treats the other user as a wiretapper, and wishes to
    keep its confidential message as secret as possible from the wiretapper. This model is usually called the
    MAC with confidential messages, and it was studied by Liang and Poor [8]. An inner bound on the
    capacity-equivocation region is provided for the model with two confidential messages, and the capacity-
    equivocation region is still not known. Furthermore, for the model of MAC with one confidential message
    [8], both inner and outer bounds on capacity-equivocation region are derived. Moreover, for the degraded
    MAC with one confidential message, the capacity-equivocation region is totally determined.
  - The second is that an additional wiretapper has access to the MAC output via a wiretap channel, and
    therefore, how to keep the confidential messages of the two users as secret as possible from the additional
    wiretapper is the main concern of the system designer. This model is usually called the multiple-access
    wiretap channel (MAC-WT). The Gaussian MAC-WT was investigated in [9], [10]. An inner bound on the
    capacity-equivocation region is provided for the Gaussian MAC-WT. Other related works on MAC-WT
    can be found in [11], [12], [13], [14], [15].

- For the interference channel, Liu et al. [6] studied the interference channel with two confidential messages,
  and provided inner and outer bounds on the secrecy capacity region. In addition, Liang et al. [16] studied
  the cognitive interference channel with one common message and one confidential message, and the capacity-
  equivocation region was totally determined for this model.

- For the relay channel, Lai and Gamal [17] studied the relay-eavesdropper channel, where a source wishes to
  send messages to a destination while leveraging the help of a trusted relay node to hide those messages from
  the eavesdropper. Three inner bounds (with respect to decode-forward, noise-forward and compress-forward
  strategies) and one outer bound on the capacity-equivocation region were provided in [17]. Furthermore,
  Tang et. al. [20] introduced the noise-forward strategy of [17] into the wireless communication networks, and
  found that with the help of an independent interferer, the security of the wireless communication networks
  is enhanced. In addition, Oohama [18] studied the relay channel with confidential messages, where a relay
  helps the transmission of messages from one sender to one receiver. The relay is considered not only as a
sender that helps the message transmission but also as a wiretapper who can obtain some knowledge about the transmitted messages. Measuring the uncertainty of the relay by equivocation, the inner and outer bounds on the capacity-equivocation region were provided in [18].

Recently, Ekrem and Ulukus [19] investigated the effects of user cooperation on the secrecy of broadcast channels by considering a cooperative relay broadcast channel. They showed that user cooperation can increase the achievable secrecy rate region of [6].

In this paper, we study the multiple-access relay wiretap channel (MARC-WT), see Figure 1. This model generalizes the MAC-WT by considering an additional trusted relay node. The motivation of this work is to investigate the effects of the trusted relay node on the secrecy of MAC-WT, and whether the achievable secrecy rate region of [10] can be enhanced by using an additional relay node.

First, we provide three inner bounds on the secrecy capacity region (achievable secrecy rate regions) of the discrete memoryless model of Figure 1. The decode-forward (DF), noise-forward (NF) and compress-forward (CF) relay strategies are used in the construction of the inner bounds. Second, we investigate the degraded discrete memoryless MARC-WT, and present an outer bound on the secrecy capacity region of this degraded case. Finally, the Gaussian model of Figure 1 is investigated, and we find that with the help of this additional trusted relay node, Tekin-Yeners achievable secrecy rate region of the Gaussian MAC-WT [10] is enhanced.

In this paper, random variables, sample values and alphabets are denoted by capital letters, lower case letters and calligraphic letters, respectively. A similar convention is applied to the random vectors and their sample values. For example, $U^N$ denotes a random $N$-vector $(U_1, ..., U_N)$, and $u^N = (u_1, ..., u_N)$ is a specific vector value in $U^N$ that is the $N$th Cartesian power of $U$. $U_i^N$ denotes a random $N - i + 1$-vector $(U_i, ..., U_N)$, and $u_i^N = (u_i, ..., u_N)$ is a specific vector value in $U_i^N$. Let $P_{V}(v)$ denote the probability mass function $Pr\{V = v\}$. Throughout the paper, the logarithmic function is to the base 2.

The organization of this paper is as follows. Section II provides the achievable secrecy rate regions of the discrete memoryless model of Figure 1. The Gaussian model of Figure 1 is investigated in Section III. Final conclusions are provided in Section IV.
II. DISCRETE MEMORYLESS MULTIPLE-ACCESS RELAY WIRETAP CHANNEL

A. Inner bounds on the secrecy capacity region of the discrete memoryless MARC-WT

The discrete memoryless model of Figure [1] is a five-terminal discrete channel consisting of finite sets $X_1$, $X_2$, $X_r$, $Y$, $Y_r$, $Z$ and a transition probability distribution $P_{Y,Y_r,Z|X_1,X_2,X_r}(y,y_r,z|x_1,x_2,x_r)$. $X_1^N$, $X_2^N$ and $X_r^N$ are the channel inputs from the transmitters and the relay respectively, while $Y^N$, $Y_r^N$, $Z^N$ are the channel outputs at the legitimate receiver, the relay and the wiretapper, respectively. The channel is discrete memoryless, i.e., the channel outputs $(y_i, y_{r,i}, z_i)$ at time $i$ only depend on the channel inputs $(x_{1,i}, x_{2,i}, x_{r,i})$ at time $i$.

Definition 1: (Channel encoders) The confidential messages $W_1$ and $W_2$ take values in $\mathcal{W}_1$, $\mathcal{W}_2$, respectively. $W_1$ and $W_2$ are independent and uniformly distributed over their ranges. The channel encoders $f_{E_1}$ and $f_{E_2}$ are stochastic encoders that map the messages $w_1$ and $w_2$ into the codewords $x_1^N \in \mathcal{X}_1^N$ and $x_2^N \in \mathcal{X}_2^N$, respectively. The transmission rates of the confidential messages $W_1$ and $W_2$ are $\frac{\log \| W_1 \|}{N}$ and $\frac{\log \| W_2 \|}{N}$, respectively.

Definition 2: (Relay encoder) The relay encoder $\phi_i$ is also a stochastic encoder that maps the signals $(y_{r,1}, y_{r,2}, \ldots, y_{r,i-1})$ received before time $i$ to the channel input $x_{r,i}$.

Definition 3: (Decoder) The Decoder for the legitimate receiver is a mapping $f_D : \mathcal{Y}^N \rightarrow \mathcal{W}_1 \times \mathcal{W}_2$, with input $Y^N$ and outputs $\hat{W}_1$, $\hat{W}_2$. Let $P_e$ be the error probability of the legitimate receiver, and it is defined as $Pr\{ (W_1, W_2) \neq (\hat{W}_1, \hat{W}_2) \}$.

The equivocation rate at the wiretapper is defined as

$$\Delta = \frac{1}{N} H(W_1, W_2 | Z^N).$$

A secrecy rate pair $(R_1, R_2)$ (where $R_1, R_2 > 0$) is called achievable if, for any $\epsilon > 0$ (where $\epsilon$ is an arbitrary small positive real number and $\epsilon \rightarrow 0$), there exists a channel encoder-decoder $(N, \Delta, P_e)$ such that

$$\lim_{N \rightarrow \infty} \frac{\log \| W_1 \|}{N} = R_1, \quad \lim_{N \rightarrow \infty} \frac{\log \| W_2 \|}{N} = R_2,$$

$$\lim_{N \rightarrow \infty} \Delta \geq R_1 + R_2, \quad P_e \leq \epsilon.$$  \hspace{1cm} (2.2)

The secrecy capacity region $\mathcal{R}^d$ is a set composed of all achievable secrecy rate pairs $(R_1, R_2)$. Three inner bounds (with respect to DF, NF and CF strategies) on $\mathcal{R}^d$ are provided in the following Theorem [1][2][3].

Our first step is to characterize the inner bound on the secrecy capacity region $\mathcal{R}^d$ by using Cover-Gamal’s Decode and Forward (DF) Strategy [22]. In the DF Strategy, the relay node will first decode the confidential messages, and then re-encode them to cooperate with the transmitters. The superposition coding and random binning techniques will be combined with the classical DF strategy [22] to characterize the DF inner bound of Figure [1].

Theorem 1: (Inner bound 1: DF strategy) A single-letter characterization of the region $\mathcal{R}^{d1}$ ($\mathcal{R}^{d1} \subseteq \mathcal{R}^d$) is as
follows,
\[ R^{d1} = \{(R_1, R_2) : \]
\[ R_1 \leq \min\{I(X_1; Y_r | X_r, X_2, V_1, V_2), I(X_1, X_r; Y | X_2, V_2)\} - I(X_1; Z), \]
\[ R_2 \leq \min\{I(X_2; Y_r | X_r, X_1, V_1, V_2), I(X_2, X_r; Y | X_1, V_1)\} - I(X_2; Z), \]
\[ R_1 + R_2 \leq \min\{I(X_1, X_2; Y_r | X_r, V_1, V_2), I(X_1, X_2, X_r; Y)\} - I(X_1, X_2; Z)\}, \]

for some distribution \[ P_{Y,Z,Y_r,X_r,X_2,V_1,V_2}(y, z, y_r, x_r, x_1, x_2, v_1, v_2) = \]
\[ P_{Y,Z,Y_r|X_r,X_2,V_1,V_2}(y, z, y_r|x_r, x_1, x_2)P_{X_r|V_1,V_2}(x_r|v_1,v_2)P_{X_1|V_1}(x_1|v_1)P_{X_2|V_2}(x_2|v_2)P_{V_1}(v_1)P_{V_2}(v_2). \]

Proof:
The achievable coding scheme is a combination of [25], [20] and [10], and the details about the proof are provided in Appendix A.

Remark 1: There are some notes on Theorem 1, see the following.

- If we let \( Z = \text{const} \) (which implies that there is no wiretapper), the region \( R^{d1} \) reduces to the region \( R^{marc} \), where
  \[ R^{marc} = \{(R_1, R_2) : \]
  \[ R_1 \leq \min\{I(X_1; Y_r | X_r, X_2, V_1, V_2), I(X_1, X_r; Y | X_2, V_2)\}, \]
  \[ R_2 \leq \min\{I(X_2; Y_r | X_r, X_1, V_1, V_2), I(X_2, X_r; Y | X_1, V_1)\}, \]
  \[ R_1 + R_2 \leq \min\{I(X_1, X_2; Y_r | X_r, V_1, V_2), I(X_1, X_2, X_r; Y)\}\}. \tag{2.3} \]

Here note that the region \( R^{marc} \) is exactly the same as the DF region of the discrete memoryless multiple-access relay channel [25], [20].

- If we let \( Y_r = Y \) and \( V_1 = V_2 = X_r = \text{const} \) (which implies that there is no relay), the region \( R^{d1} \) reduces to the region \( R^{mac-wt} \), where
  \[ R^{mac-wt} = \{(R_1, R_2) : \]
  \[ R_1 \leq I(X_1; Y | X_2) - I(X_1; Z), \]
  \[ R_2 \leq I(X_2; Y | X_1) - I(X_2; Z), \]
  \[ R_1 + R_2 \leq I(X_1, X_2; Y) - I(X_1, X_2; Z)\}. \tag{2.4} \]

Also note that the region \( R^{mac-wt} \) is exactly the same as the achievable secrecy rate region of discrete memoryless multiple-access wiretap channel [10].

The second step is to characterize the inner bound on the secrecy capacity region \( R^d \) by using the noise and forward (NF) strategy. In the NF Strategy, the relay node does not attempt to decode the messages but sends codewords that are independent of the transmitters’ messages, and these codewords aid in confusing the wiretapper.
More specifically, if the channel from the relay to the legitimate receiver is less noisy than the channel from the relay to the wiretapper, we allow the legitimate receiver to decode the relay codeword, and the wiretapper can not decode it. Therefore, in this case, the relay codeword can be viewed as a noise signal to confuse the wiretapper.

On the other hand, if the channel from the relay to the legitimate receiver is more noisy than the channel from the relay to the wiretapper, we allow both the receivers to decode the relay codeword, and therefore, in this case, the relay codeword can be viewed as a noise signal to confuse the wiretapper.

\[ R_{\text{r}} = \min \{ I(X_1; Y), I(X_1; Z|X_1), I(X_1; Z|X_2) \} \]

\[ P_{Y,Z,Y_r,X_1,X_2}(y, z, y_r, x_r, x_1, x_2) = P_{Y,Z,Y_r,X_1,X_2}(y, z, y_r, x_r, x_1, x_2)P_{X_r}(x_r)P_{X_1}(x_1)P_{X_2}(x_2), \]

and \( R_{\text{r}} \) is denoted by

\[ R_{\text{r}} = \min \{ I(X_1; Y), I(X_1; Z|X_1), I(X_1; Z|X_2) \}. \]

**Proof:**

The achievable coding scheme is a combination of [17, Theorem 3] and [10], and the details about the proof are provided in Appendix B.

**Remark 2:** There are some notes on Theorem 2, see the following.

- The region \( \mathcal{L}^1 \) is characterized under the condition that the channel from the relay to the legitimate receiver is less noisy than the channel from the relay to the wiretapper \( I(X_1; Y) \geq I(X_1; Z) \). Then, in this case, the legitimate receiver is allowed to decode the relay codeword, and the wiretapper is not allowed to decode it. The rate of the relay is defined as \( R_{\text{r}} = \min \{ I(X_1; Y), I(X_1; Z|X_1), I(X_1; Z|X_2) \} \), and the relay codeword is viewed as pure noise for the wiretapper.
• The region $\mathcal{L}^3$ is characterized under the condition that the channel from the relay to the legitimate receiver is more noisy than the channel from the relay to the wiretapper ($I(X_r; Y) \leq I(X_r; Z)$). Then, in this case, both the legitimate receiver and the wiretapper are allowed to decode the relay codeword. The rate of the relay is defined as $R_r = I(X_r; Y)$, and the relay codeword does not make any contribution to the security of the model of Figure 1.

The third step is to characterize the inner bound on the secrecy capacity region $\mathcal{R}^d$ by using a combination of Cover-Gamal's compress and forward (CF) strategy [22] and the NF strategy provided in Theorem 2, i.e., in addition to the independent codewords, the relay also sends a quantized version of its noisy observations to the legitimate receiver. This noisy version of the relay’s observations helps the legitimate receiver in decoding the transmitters’ messages, while the independent codewords help in confusing the wiretapper.

**Theorem 3:** (Inner bound 3: CF strategy) A single-letter characterization of the region $\mathcal{R}^d$ ($\mathcal{R}^d \subseteq \mathcal{R}^3$) is as follows,

$$\mathcal{R}^d = \mathcal{L}^3 \bigcup \mathcal{L}^4,$$

where $\mathcal{L}^3$ is given by

$$\mathcal{L}^3 = \bigcup_{P_{Y,Z,Y_r,Y_r,Y_r,Y_r,X_r,X_r,X_r,X_r,X_r,X_r,X_r,X_r}} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq I(X_1; Y, \hat{Y}_r|X_2, X_2) - I(X_1; X_r; Z) + R^*, \\ R_2 \leq I(X_2; Y, \hat{Y}_r|X_1, X_r) - I(X_2; X_r; Z) + R^*, \\ R_1 + R_2 \leq I(X_1, X_2; Y, \hat{Y}_r|X_r) - I(X_1, X_2, X_r; Z) + R^*. \end{array} \right\},$$

$$R^*_1 = \min \{I(X_r; Z|X_1), I(X_r; Z|X_2), I(X_r; Y) \},$$

and $\mathcal{L}^4$ is given by

$$\mathcal{L}^4 = \bigcup_{P_{Y,Z,Y_r,Y_r,Y_r,Y_r,X_r,X_r,X_r,X_r,X_r,X_r,X_r,X_r}} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq I(X_1; Y, \hat{Y}_r|X_2, X_2) - I(X_1; Z|X_r), \\ R_2 \leq I(X_2; Y, \hat{Y}_r|X_1, X_r) - I(X_2; Z|X_r), \\ R_1 + R_2 \leq I(X_1, X_2; Y, \hat{Y}_r|X_r) - I(X_1, X_2; Z|X_r). \end{array} \right\}.$$

The joint probability $P_{Y,Z,Y_r,Y_r,Y_r,Y_r,X_r,X_r,X_r,X_r,X_r,X_r,X_r,X_r}(y, z, y_r, \hat{y}_r, x_r, x_1, x_2)$ satisfies

$$P_{Y,Z,Y_r,Y_r,Y_r,Y_r,X_r,X_r,X_r,X_r,X_r,X_r,X_r}(y, z, y_r, \hat{y}_r, x_r, x_1, x_2) = \sum_{x_r} P_{X_r}(x_r)P_{X_1}(x_1)P_{X_2}(x_2).$$

**Proof:**

The achievable coding scheme is a combination of [17] Theorem 4] and [10], and the details about the proof are provided in Appendix C.

**Remark 3:** There are some notes on Theorem 3 see the following.

• In $\mathcal{L}^3$, the channel from the relay to the legitimate receiver is less noisy than the channel from the relay to the wiretapper ($I(X_r; Y) \geq I(X_r; Z)$). Then, in this case, the legitimate receiver is allowed to decode the relay codeword, and the wiretapper is not allowed to decode it. Here note that $R^*$ is the rate of pure noise generated
by the relay to confuse the wiretapper, while $R_{r} - R^*$ is the part of the rate allocated to send the compressed signal $\hat{Y}_r$ to help the legitimate receiver. If $R^* = R_{r}$, this scheme is exactly the same as the NF scheme.

- In $\mathcal{L}^4$, the channel from the relay to the legitimate receiver is more noisy than the channel from the relay to the wiretapper ($I(X_r; Y) \leq I(X_r; Z)$). Then, in this case, both the legitimate receiver and the wiretapper are allowed to decode the relay codeword. However, The relay can still help to enhance the security of the model of Figure 1 by sending the compressed signal $\hat{Y}_r$ to the legitimate receiver. Thus, the region $\mathcal{L}^4$ is characterized by combining the $\mathcal{L}^2$ of Theorem 2 with the classical compress and forward (CF) strategy [22].

B. Outer bound on the secrecy capacity region of the degraded discrete memoryless MARC-WT

Compared with the discrete memoryless model of Figure 1, the degraded discrete memoryless MARC-WT implies the existence of a Markov chain $(X_1, X_2, X_r, Y_r) \rightarrow Y \rightarrow Z$. The secrecy capacity region $\mathcal{R}^{dd}$ of the degraded discrete memoryless MARC-WT is a set composed of all achievable secrecy rate pairs $(R_1, R_2)$. An outer bound on $\mathcal{R}^{dd}$ is provided in the following Theorem 4.

**Theorem 4: (Outer bound)** A single-letter characterization of the region $\mathcal{R}^{ddo}$ ($\mathcal{R}^{dd} \subseteq \mathcal{R}^{ddo}$) is as follows,

$$
\mathcal{R}^{ddo} = \{(R_1, R_2) : \\
R_1 \leq I(X_1, X_r; Y|X_2, U) - I(X_1; Z|U) \\
R_2 \leq I(X_2, X_r; Y|X_1, U) - I(X_2; Z|U) \\
R_1 + R_2 \leq I(X_1, X_2, X_r; Y|U) - I(X_1, X_2; Z|U)\}
$$

for some distribution

$$
P_{Z,Y,Y_r,X_r,X_1,X_2,U}(z,y,y_r,x_r,x_1,x_2,u) = \\
P_{Z|Y}(z|y)P_{Y,Y_r|X_1,X_2,X_r}(y,y_r|x_1,x_2,x_r)P_{U,X_1,X_2,X_r}(u,x_1,x_2,x_r).
$$

**Proof:**

The details about the proof are provided in Appendix D.

III. GAUSSIAN MULTIPLE-ACCESS RELAY WIRETAP CHANNEL

In this section, we investigate the Gaussian multiple-access relay wiretap channel (GMARC-WT). The signal received at each node is given by

$$
Y_r = X_1 + X_2 + Z_r, \\
Y = X_1 + X_2 + X_r + Z_1, \\
Z = X_1 + X_2 + X_r + Z_2,
$$

(3.1)
where \( Z_r \sim \mathcal{N}(0, N_r) \), \( Z_1 \sim \mathcal{N}(0, N_1) \), \( Z_2 \sim \mathcal{N}(0, N_2) \), and they are independent, \( E[X_1^2] \leq P_1 \), \( E[X_2^2] \leq P_2 \), \( E[X_3^2] \leq P_r \).

The remainder of this section is organized as follows. Subsection III-A shows the achievable secrecy rate regions of GMARC-WT, and the numerical examples and discussions are given in Subsection III-B.

A. Capacity results on GMARC-WT

**Theorem 5:** The DF inner bound on the secrecy capacity region of the Gaussian case of Figure [1] is given by

\[
\mathcal{R}^1 = \bigcup_{0 \leq \gamma \leq 1} \left\{ (R_1, R_2) : \begin{array}{l}
R_1 \leq \min\left\{ \frac{1}{2} \log(1 + \frac{P_0}{N_1}), \frac{1}{2} \log(1 + \frac{P_2 + \gamma P_r}{N_1}) \right\} - \frac{1}{2} \log \left( \frac{P_i + P_2 + P_r + N_k}{P_i + P_2 + P_r + N_2} \right), \\
R_2 \leq \min\left\{ \frac{1}{2} \log(1 + \frac{P_0}{N_2}), \frac{1}{2} \log(1 + \frac{P_3 + (1-\gamma)P_r}{N_2}) \right\} - \frac{1}{2} \log \left( \frac{P_i + P_3 + P_r + N_k}{P_i + P_3 + P_r + N_2} \right), \\
R_1 + R_2 \leq \min\left\{ \frac{1}{2} \log(1 + \frac{P_0 + N_k}{N_1}), \frac{1}{2} \log(1 + \frac{P_3 + P_r}{N_2}) \right\} - \frac{1}{2} \log \left( \frac{P_i + P_3 + P_r + N_k}{P_i + P_3 + P_r + N_2} \right). \end{array} \right\} \quad (3.2)
\]

**Proof:**
First, let \( X_r = V_2 + V_2 \), where \( V_1 \sim \mathcal{N}(0, \gamma P_r) \) and \( V_2 \sim \mathcal{N}(0, (1-\gamma)P_r) \).

Let \( X_1 = \sqrt{\frac{(1-\alpha)P_1}{2r}} V_1 + X_{10} \), where \( 0 \leq \alpha \leq 1 \) and \( X_{10} \sim \mathcal{N}(0, \alpha P_1) \).

Analogously, let \( X_2 = \sqrt{\frac{(1-\beta)P_2}{2r}} V_2 + X_{20} \), where \( 0 \leq \beta \leq 1 \) and \( X_{20} \sim \mathcal{N}(0, \beta P_2) \).

Here note that \( V_1, V_2, X_{10} \) and \( X_{20} \) are independent random variables.

The region \( \mathcal{R}^1 \) is obtained by substituting the above definitions into Theorem [1] and maximizing \( \alpha \) and \( \beta \) (the maximum of \( \mathcal{R}^1 \) is achieved when \( \alpha = \beta = 1 \)). Thus, the proof of Theorem [5] is completed.

**Theorem 6:** Then, the NF inner bound on the secrecy capacity region of the Gaussian case of Figure [1] is given by

\[
\mathcal{R}^2 = \mathcal{G}^1 \bigcup \mathcal{G}^2,
\]

where \( \mathcal{G}^1 \) is given by

\[
\mathcal{G}^1 = \bigcup_{N_1 \leq N_2} \left\{ (R_1, R_2) : \begin{array}{l}
R_1 \leq \frac{1}{2} \log(1 + \frac{P_i}{N_1}) - \frac{1}{2} \log(1 + \frac{P_0 + P_r}{N_1}) + R_r, \\
R_2 \leq \frac{1}{2} \log(1 + \frac{P_i}{N_2}) - \frac{1}{2} \log(1 + \frac{P_3 + P_r}{N_2}) + R_r, \\
R_1 + R_2 \leq \frac{1}{2} \log(1 + \frac{P_i + P_2}{N_1}) - \frac{1}{2} \log(1 + \frac{P_r + P_2}{N_2}) + R_r. \end{array} \right\}
\]

\[R_r = \min\left\{ \frac{1}{2} \log(1 + \frac{P_0}{P_i + P_2 + N_1}), \frac{1}{2} \log(1 + \frac{P_3}{P_r + N_2}), \frac{1}{2} \log(1 + \frac{P_r}{P_i + N_2}) \right\} \]

and \( \mathcal{G}^2 \) is given by

\[
\mathcal{G}^2 = \bigcup_{N_1 \geq N_2} \left\{ (R_1, R_2) : \begin{array}{l}
R_1 \leq \frac{1}{2} \log(1 + \frac{P_i}{N_1}) - \frac{1}{2} \log(1 + \frac{P_0}{N_1}), \\
R_2 \leq \frac{1}{2} \log(1 + \frac{P_i}{N_2}) - \frac{1}{2} \log(1 + \frac{P_r}{N_2}), \\
R_1 + R_2 \leq \frac{1}{2} \log(1 + \frac{P_i + P_2}{N_1}) - \frac{1}{2} \log(1 + \frac{P_r + P_2}{N_2}). \end{array} \right\}
\]

**Proof:**
Here note that $N_1 \leq N_2$ implies $I(X_r; Y) \geq I(X_r; Z)$. The region $G^1$ is obtained by substituting $X_1 \sim N(0, P_1)$, $X_2 \sim N(0, P_2)$ and $X_r \sim N(0, P_r)$ into the region $L^1$ of Theorem 2 and using the fact that $X_1$, $X_2$ and $X_r$ are independent random variables.

Analogously, $N_1 \geq N_2$ implies $I(X_r; Y) \leq I(X_r; Z)$. The region $G^2$ is obtained by substituting $X_1 \sim N(0, P_1)$, $X_2 \sim N(0, P_2)$ and $X_r \sim N(0, P_r)$ into the region $L^2$ of Theorem 2 and using the fact that $X_1$, $X_2$ and $X_r$ are independent random variables. Thus, the proof of Theorem 6 is completed.

**Theorem 7:** Next, the CF inner bound on the secrecy capacity region of the Gaussian case of Figure 1 is given by

$$R_{\text{CF}} = G^3 \cup G^4,$$

where $G^3$ is given by

$$G^3 = \bigcup_{N_1 \leq N_2} \left\{ (R_1, R_2) : \begin{aligned} R_1 &\leq \frac{1}{2} \log \left(1 + \frac{P_3(Q+P_3)(Q+3N_r)}{N_1(N_r+Q)}\right) \bigg(1 + \frac{P_1}{P_2+N_2}\right) + R^*, \\
R_2 &\leq \frac{1}{2} \log \left(1 + \frac{P_3(Q+P_3)(Q+3N_r)}{N_1(N_r+Q)}\right) - \frac{1}{2} \log \left(1 + \frac{P_1}{P_2+N_2}\right) + R^*, \\
R_1 + R_2 &\leq \frac{1}{2} \log \left(1 + \frac{(P_1+P_2)(Q+3N_r)}{N_1(N_r+Q)}\right) - \frac{1}{2} \log \left(1 + \frac{P_1+P_2}{N_2}\right) + R^*.
\end{aligned} \right\},$$

$R^*$ satisfies $0 \leq R^* = \min \left\{ \frac{1}{2} \log \left(1 + \frac{P_3}{P_1+P_2+N_1}\right), \frac{1}{2} \log \left(1 + \frac{P_3}{P_2+N_2}\right), \frac{1}{2} \log \left(1 + \frac{P_3}{P_1+N_1}\right) \right\} - \frac{1}{2} \log \left(1 + \frac{P_1+P_2}{Q}\right)$, and $G^4$ is given by

$$G^4 = \bigcup_{N_1 \geq N_2} \left\{ (R_1, R_2) : \begin{aligned} R_1 &\leq \frac{1}{2} \log \left(1 + \frac{P_3(Q+P_3)(Q+3N_r)}{N_1(N_r+Q)}\right) - \frac{1}{2} \log \left(1 + \frac{P_1}{P_2+N_2}\right), \\
R_2 &\leq \frac{1}{2} \log \left(1 + \frac{P_3(Q+P_3)(Q+3N_r)}{N_1(N_r+Q)}\right) - \frac{1}{2} \log \left(1 + \frac{P_1+P_2}{N_2}\right), \\
R_1 + R_2 &\leq \frac{1}{2} \log \left(1 + \frac{(P_1+P_2)(Q+3N_r)}{N_1(N_r+Q)}\right) - \frac{1}{2} \log \left(1 + \frac{P_1+P_2}{N_2}\right).
\end{aligned} \right\},$$

here $Q$ satisfies $Q \geq \frac{(P_1+P_2)^2+(P_1+P_3)(N_r+1)+N_r}{P_r}$.

**Proof:**

Here note that $N_1 \leq N_2$ implies $I(X_r; Y) \geq I(X_r; Z)$. The region $G^3$ is obtained by substituting $X_1 \sim N(0, P_1)$, $X_2 \sim N(0, P_2)$, $X_r \sim N(0, P_r)$, $\hat{Y}_r = Y_r + Z_Q$ and $Z_Q \sim N(0, Q)$ into the region $L^3$ of Theorem 3 and using the fact that $X_1$, $X_2$ and $X_r$ are independent random variables.

Analogously, $N_1 \geq N_2$ implies $I(X_r; Y) \leq I(X_r; Z)$. The region $G^4$ is obtained by substituting $X_1 \sim N(0, P_1)$, $X_2 \sim N(0, P_2)$, $X_r \sim N(0, P_r)$, $\hat{Y}_r = Y_r + Z_Q$ and $Z_Q \sim N(0, Q)$ into the region $L^4$ of Theorem 3 and using the fact that $X_1$, $X_2$ and $X_r$ are independent random variables. Thus, the proof of Theorem 7 is completed.

**Theorem 8:** Finally, remember that [10] provides an achievable secrecy rate region $R_{\text{Gi}}$ of the Gaussian multiple-
access wiretap channel (GMAC-WT), and it is given by

\[ \mathcal{R}^{Gi} = \left\{ (R_1, R_2) : \begin{array}{l}
R_1 \leq \frac{1}{2} \log(1 + \frac{P_1}{N_1}) - \frac{1}{2} \log(1 + \frac{P_2}{N_1 + P_2}) \\
R_2 \leq \frac{1}{2} \log(1 + \frac{P_2}{N_1}) - \frac{1}{2} \log(1 + \frac{P_1}{N_1 + P_1}) \\
R_1 + R_2 \leq \frac{1}{2} \log(1 + \frac{P_1 + P_2}{N_1}) - \frac{1}{2} \log(1 + \frac{P_1 + P_2}{N_2})
\end{array} \right\} . \]

**Proof:**

The proof is in [10], and it is omitted here.

---

**B. Numerical Examples and Discussions**

Letting \( P_1 = 5 \), \( P_2 = 6 \), \( P_r = 20 \), \( N_1 = 2 \), \( N_2 = 14 \) and \( Q = 200 \), the following Figure 2, 3 and 4 show the achievable secrecy rate regions of the GMARC-WT and the achievable secrecy rate region of the GMAC-WT for different values of \( N_r \).

Compared with the achievable secrecy rate region \( \mathcal{R}^{Gi} \) of GMAC-WT, it is easy to see that the NF (\( \mathcal{R}^{g2} \)) and CF (\( \mathcal{R}^{g3} \)) strategies help to enhance \( \mathcal{R}^{Gi} \) (no relay). However, for the DF strategy (\( \mathcal{R}^{g1} \)), we find that when \( N_r \) is much larger than \( N_1 \), the DF region \( \mathcal{R}^{g1} \) is even smaller than \( \mathcal{R}^{Gi} \), i.e., the relay makes the things even worse. When \( N_r \) is close to \( N_1 \) (still larger than \( N_1 \)), the DF region \( \mathcal{R}^{g1} \) is larger than \( \mathcal{R}^{Gi} \), but it is still smaller than the NF and CF regions. When \( N_r \) is smaller than \( N_1 \), as we can see in Figure 4, the DF region performs the best!

In addition, when \( Q \to \infty \), the CF region \( \mathcal{R}^{g3} \) is exactly the same as the NF region \( \mathcal{R}^{g2} \).

---

**Fig. 2:** The achievable secrecy rate regions of GMARC-WT and GMAC-WT for \( N_r = 5 \)
In this paper, first, we provide three inner bounds on the secrecy capacity region (achievable secrecy rate regions) of the discrete memoryless model of Figure 1. The decode-forward (DF), noise-forward (NF), and compress-forward (CF) relay strategies are used in the construction of these inner bounds. Second, we investigate the degraded discrete memoryless MARC-WT, and present an outer bound on the secrecy capacity region of this degraded case. Finally,
we study the Gaussian model of Figure [1] and find that the NF and CF strategies help to enhance Tekin-Yener’s achievable secrecy rate region of Gaussian MAC-WT. Moreover, we find that if the channel from the transmitters to the relay is less noisy than the channels from the transmitters to the legitimate receiver and the wiretapper, the DF strategy performs even better than the NF and CF strategies.

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APPENDIX A
PROOF OF THEOREM [1]

We only need to prove that the pair

\[ (R_1 = \min \{I(X_1; Y_r | X_r, X_2, V_1, V_2), I(X_1, X_r; Y | X_2, V_2)\} \}

is achievable, and the achievability proof for \((R_1 = \min \{I(X_1; Y_r | X_r, V_1, V_2), I(X_1, V_1; Y)\} - I(X_1; Z | X_r)\), \(R_2 = \min \{I(X_2; Y_r | X_r, V_1, V_2), I(X_2, V_1; Y)\} - I(X_2; Z | X_1)\)

follows by symmetry.

The coding scheme combines the decode-and-forward (DF) strategy of MARC [20], random binning, superposition coding, and block Markov coding techniques, see the followings.

First, define the messages \(W_1\) and \(W_2\) taken values in the alphabets \(\mathcal{W}_1\) and \(\mathcal{W}_2\), respectively, where

\[ \mathcal{W}_1 = \{1, 2, \ldots, 2^{NR_1}\}, \mathcal{W}_2 = \{1, 2, \ldots, 2^{NR_2}\}. \]

Here note that \(R_1\) and \(R_2\) satisfy

\[ R_1 = \min \{I(X_1; Y_r | X_r, X_2, V_1, V_2), I(X_1, X_r; Y | X_2, V_2)\} - I(X_1; Z), \quad (A1) \]

and

\[ R_2 = \min \{I(X_2; Y_r | X_r, V_1, V_2), I(X_2, V_1; Y)\} - I(X_2; Z | X_1). \quad (A2) \]

**Code Construction:** Fix the joint probability mass function \(P_{Y, Z, Y_r, X_r, x_1, v_1}(y, z, y_r | x_r, v_1) \) with \(P_{X_r, V_1, V_2}(x_r, v_1, v_2) \) and \(P_{X_1, V_1}(x_1 | v_1) P_{X_2, V_2}(x_2 | v_2) P_{V_1}(v_1) P_{V_2}(v_2) \). For arbitrary \(\epsilon > 0\), define

\[ R_1^* = I(X_1; Z) - \epsilon, \quad (A3) \]

\[ R_2^* = I(X_2; Z | X_1) - \epsilon, \quad (A4) \]

\[ R_{v_1} = I(V_1, X_r; Y | X_2, V_2) - \epsilon, \quad (A5) \]

\[ R_{v_2} = I(V_2; Y) - \epsilon. \quad (A6) \]
Relay Code-books Construction:

- Generate at random $2^{NR_1}$ i.i.d. sequences $v_1^N$ according to $P_{V_1^N}(v_1^N) = \prod_{i=1}^N P_{V_1}(v_{1,i})$. Index them as $v_1^N(s_1)$, where $s_1 \in [1, 2^{NR_1}]$.

  Analogously, generate at random $2^{NR_2}$ i.i.d. sequences $v_2^N$ according to $P_{V_2^N}(v_2^N) = \prod_{i=1}^N P_{V_2}(v_{2,i})$. Index them as $v_2^N(s_2)$, where $s_2 \in [1, 2^{NR_2}]$.

- Generate at random $2^{(NR_1+R_1)}$ i.i.d. sequences $x_1^N$ according to $P_{X_1^N|V_1^N,V_2^N}(x_1^N|v_1^N, v_2^N) = \prod_{i=1}^N P_{X_{1,i}}(v_{1,i}, v_{2,i})$.

  Index them as $x_1^N(s_1, s_2)$, where $s_1 \in [1, 2^{NR_1}]$ and $s_2 \in [1, 2^{NR_2}]$.

Transmitters’ Code-books Construction:

- Generate at random $2^{(NR_1+R_1)}$ i.i.d. sequences $x_1^N(w_1, w_1^i|s_1)$ ($w_1 \in [1, 2^{NR_1}], w_1^i \in [1, 2^{NR_1}], s_1 \in [1, 2^{NR_1}]$) according to $\prod_{i=1}^N P_{X_1|V_1}(x_{1,i}|v_{1,i})$. In addition, partition these $2^{(NR_1+R_1)}$ i.i.d. sequences $x_1^N$ into $2^{NR_1}$ bins. These bins are denoted as $\{a_1, a_2, ..., a_{2^{NR_1}}\}$, where $a_i$ ($1 \leq i \leq 2^{NR_1}$) contains $2^{(NR_1+R_1)-R_1}$ sequences about $x_1^N$. Note that here for given $w_1, w_1^i$ and $s_1$, the index of the bin which $x_1^N(w_1, w_1^i|s_1)$ belongs to, is totally determined.

- Analogously, generate at random $2^{(NR_2+R_2)}$ i.i.d. sequences $x_2^N(w_2, w_2^i|s_2)$ ($w_2 \in [1, 2^{NR_2}], w_2^i \in [1, 2^{NR_2}], s_2 \in [1, 2^{NR_2}]$) according to $\prod_{i=1}^N P_{X_2|V_2}(x_{2,i}|v_{2,i})$. In addition, partition these $2^{(NR_2+R_2)}$ i.i.d. sequences $x_2^N$ into $2^{NR_2}$ bins. These bins are denoted as $\{b_1, b_2, ..., b_{2^{NR_2}}\}$, where $b_i$ ($1 \leq i \leq 2^{NR_2}$) contains $2^{(NR_2+R_2)-R_2}$ sequences about $x_2^N$. Note that here for given $w_2, w_2^i$ and $s_2$, the index of the bin which $x_2^N(w_2, w_2^i|s_2)$ belongs to, is totally determined.

Encoding: Encoding involves the mapping of message indices to channel inputs, which are facilitated by the sequences generated above. We exploit the block Markov coding scheme, as argued in [22], the loss induced by this scheme is negligible as the number of blocks $n \rightarrow \infty$. For block $i$ ($1 \leq i \leq n$), encoding proceeds as follows.

First, for convenience, the messages $w_1, w_1^i, w_2, w_2^i, s_1$ and $s_2$ transmitted in the $i$-th block are denoted by $w_{1,i}, w_{1,i}^*, w_{2,i}, w_{2,i}^*, s_{1,i}$ and $s_{2,i}$, respectively.

- **(Channel encoders)**
  
  1) The message $w_{1,i}^*$ ($1 \leq i \leq n$) is randomly chosen from the set $\{1, 2, ..., 2^{NR_1}\}$. The transmitter 1 (encoder 1) sends $x_1^N(w_{1,1}, w_{1,1}^i|1)$ at the first block ($s_{1,1} = 1$), $x_1^N(w_{1,i}, w_{1,i}^*|s_{1,i})$ from block $2 \leq i \leq n-1$, and $x_1^N(1, 1|s_{1,n})$ at block $n$ ($w_{1,n} = w_{1,n}^* = 1$).

  2) The message $w_{2,i}^*$ ($1 \leq i \leq n$) is randomly chosen from the set $\{1, 2, ..., 2^{NR_2}\}$. The transmitter 2 (encoder 2) sends $x_2^N(w_{2,1}, w_{2,1}^i|1)$ at the first block ($s_{2,1} = 1$), $x_2^N(w_{2,i}, w_{2,i}^*|s_{2,i})$ from block $2 \leq i \leq n-1$, and $x_2^N(1, 1|s_{2,n})$ at block $n$ ($w_{2,n} = w_{2,n}^* = 1$).

- **(Relay encoder)**
  
  The relay sends $(v_1^N(1), v_2^N(1), x_1^N(1, 1))$ at the first block, and $(v_1^N(\hat{s}_{1,i}), v_2^N(\hat{s}_{2,i}), x_r^N(\hat{s}_{1,i}, \hat{s}_{2,i}))$ from block $2 \leq i \leq n$.

Decoding: Decoding proceeds as follows.

1) (At the relay) At the end of block $i$ ($1 \leq i \leq n$), the relay already has an estimation of the $s_{1,i}$ and $s_{2,i}$
(denoted by \( \hat{s}_{1,i} \) and \( \hat{s}_{2,i} \), respectively), and will declare that it receives \( \hat{s}_{2,i+1} \) if this is the only triple such that 

\[
(x_2^N(\hat{w}_{2,i}, \hat{w}_{2,i}^*|\hat{s}_{2,i}), x_2^N(\hat{w}_{1,i}, \hat{w}_{1,i}^*|\hat{s}_{1,i}), y^N_r(i))
\]

are jointly typical. Here note that \( y^N_r(i) \) indicates the output sequence \( y^N_r \) in block \( i \). \( \hat{s}_{2,i+1} \) is the index of the bin which \( x_2^N(\hat{w}_{2,i}, \hat{w}_{2,i}^*|\hat{s}_{2,i}) \) belongs to, and it will be used in the \( i+1 \)-th block. Based on the AEP, the probability \( Pr\{\hat{s}_{2,i+1} = s_{2,i+1}\} \) goes to 1 if 

\[
R_2 + R_2^* \leq I(X_2; Y_r | X_r) \overset{(a)}{=} I(X_2; Y_r, X_2, V_1, V_2),
\]

where (a) is from the Markov chains \( (V_1, V_2) \to X_r \to Y_r \) and \( (V_1, V_2) \to (X_2, X_r) \to Y_r \).

After the relay successfully decodes \( \hat{w}_{2,i} \), \( \hat{w}_{2,i}^* \) and the corresponding \( \hat{s}_{2,i+1} \), he tries to find a unique codeword 

\[
x_2^N(\hat{w}_{1,i}, \hat{w}_{1,i}^*|\hat{s}_{1,i}), x_2^N(\hat{w}_{2,i}, \hat{w}_{2,i}^*|\hat{s}_{2,i}), x_2^N(\hat{w}_{1,i}, \hat{w}_{1,i}^*|\hat{s}_{1,i}), y^N_r(i)
\]

are jointly typical. Here \( \hat{s}_{1,i+1} \) is the index of the bin which \( x_2^N(\hat{w}_{1,i}, \hat{w}_{1,i}^*|\hat{s}_{1,i}) \) belongs to, and it will be used in the \( i+1 \)-th block. Based on the AEP, the probability \( Pr\{\hat{s}_{1,i+1} = s_{1,i+1}\} \) goes to 1 if 

\[
R_1 + R_1^* \leq I(X_1; Y_r | X_r, X_2) \overset{(b)}{=} I(X_1; Y_r, X_2, V_1, V_2),
\]

where (b) is from the Markov chains \( (V_1, V_2) \to (X_2, X_r) \to Y_r \) and \( (V_1, V_2) \to (X_1, X_2, X_r) \to Y_r \).

2) (At the legitimate receiver)

- The legitimate receiver decodes from the last block, i.e., block \( n \). Suppose that at the end of block \( n \), the legitimate receiver will declare that \( \hat{s}_{2,n} \) is received if \( (v_2^N(\hat{s}_{2,n}), y^N(n)) \) jointly typical. Based on the AEP, the probability \( Pr\{\hat{s}_{2,n} = s_{2,n}\} \) goes to 1 if 

\[
R_{r2} \leq I(V_2; Y).
\]

After getting \( \hat{s}_{2,n} \), the legitimate receiver can get an estimation of \( s_{2,i} \) (\( 1 \leq i \leq n - 1 \)) in a similar way.

- After decoding \( \hat{s}_{2,i} \) and \( \hat{s}_{2,i+1} \) (\( 1 \leq i \leq n \)), the legitimate receiver tries to find a \( a_2^N(\hat{w}_{2,i}, \hat{w}_{2,i}^*|\hat{s}_{2,i}) \) such that 

\[
(x_2^N(\hat{w}_{2,i}, \hat{w}_{2,i}^*|\hat{s}_{2,i}), v_2^N(\hat{s}_{2,i}, y^N(i)) \)

are jointly typical. Based on the AEP, the probability \( Pr\{a_2^N(\hat{w}_{2,i}, \hat{w}_{2,i}^*|\hat{s}_{2,i}) \to w_{2,i}^*\} \) goes to 1 if 

\[
R_2 + R_2^* - R_{r2} \leq I(X_2; Y | V_2).
\]

- After decoding \( \hat{s}_{2,i} \), \( \hat{w}_{2,i} \) and \( \hat{w}_{2,i}^* \) (\( 1 \leq i \leq n \)), the legitimate receiver tries to find \( v_1^N(\hat{s}_{1,i}, s_{2,i}) \) and \( x_2^N(\hat{s}_{1,i}, \hat{s}_{2,i}) \) such that 

\[
(v_1^N(\hat{s}_{1,i}), x_2^N(\hat{s}_{1,i}, \hat{s}_{2,i}), v_2^N(\hat{w}_{2,i}, \hat{w}_{2,i}^*|\hat{s}_{2,i}), v_2^N(\hat{s}_{2,i}, y^N(i)) \)

are jointly typical. Based on the AEP, the probability \( Pr\{\hat{s}_{1,i} = s_{1,i}\} \) goes to 1 if 

\[
R_{r1} \leq I(V_1, X_r; Y | X_2, V_2) \overset{(c)}{=} I(X_r; Y | X_2, V_2),
\]

where (c) is from the Markov chain \( V_1 \to (X_2, V_2, X_r) \to Y \).

- After decoding \( \hat{s}_{2,i} \), \( \hat{w}_{2,i} \), \( \hat{w}_{2,i}^* \), \( \hat{s}_{1,i} \) and \( \hat{s}_{1,i+1} \) (\( 1 \leq i \leq n \)), the legitimate receiver tries to find a \( a_1^N(\hat{w}_{1,i}, \hat{w}_{1,i}^*|\hat{s}_{1,i}) \) such that 

\[
(x_1^N(\hat{w}_{1,i}, \hat{w}_{1,i}^*|\hat{s}_{1,i}), v_1^N(\hat{s}_{1,i}, \hat{s}_{2,i}), x_2^N(\hat{w}_{2,i}, \hat{w}_{2,i}^*|\hat{s}_{2,i}), v_2^N(\hat{s}_{2,i}, y^N(i)) \)

are jointly typical. Based on the AEP, the probability \( Pr\{a_1^N(\hat{w}_{1,i}, \hat{w}_{1,i}^*|\hat{s}_{1,i}) \to w_{1,i}^*\} \) goes to 1 if 

\[
R_1 + R_1^* - R_{r1} \leq I(X_1; Y | V_1, X_2, V_2, X_r) \overset{(d)}{=} I(X_1; Y | X_2, V_2, X_r),
\]

where (d) is from the Markov chains \( V_1 \to (X_2, V_2, X_r) \to Y \) and \( V_1 \to (X_1, X_2, V_2, X_r) \to Y \).
The following Table I shows the transmitted codewords in the first three blocks.

By using (A1), (A2), (A3), (A4), (A5), (A6), (A7), (A8), (A9), (A10), (A11) and (A12), it is easy to check that $P_e \leq \epsilon$. It remains to show that $\lim_{N \to \infty} \Delta \geq R_1 + R_2$, see the followings.

**Equivocation Analysis:**

\[
\lim_{N \to \infty} \Delta = \lim_{N \to \infty} \frac{1}{N} H(W_1, W_2 | Z^N) = \lim_{N \to \infty} \frac{1}{N} (H(W_1 | Z^N) + H(W_2 | W_1, Z^N)). \tag{A13}
\]

The first term in (A13) is bounded as follows.

\[
\lim_{N \to \infty} \frac{1}{N} H(W_1 | Z^N) = \lim_{N \to \infty} \frac{1}{N} (H(W_1, Z^N) - H(Z^N)) = \lim_{N \to \infty} \frac{1}{N} (H(W_1, Z^N, X_1^N) - H(X_1^N | W_1, Z^N) - H(Z^N)) \\
\overset{(a)}{=} \lim_{N \to \infty} \frac{1}{N} (H(Z^N | X_1^N) + H(X_1^N) - H(X_1^N | W_1, Z^N)) = \lim_{N \to \infty} \frac{1}{N} (H(X_1^N) - I(X_1^N; Z^N) - H(X_1^N | W_1, Z^N)), \tag{A14}
\]

where (a) follows from $W_1 \rightarrow X_1^N \rightarrow Z^N$ and $H(W_1 | X_1^N) = 0$.

Consider the first term in (A14), the code-book generation of $x_1^N$ shows that the total number of $x_1^N$ is $2^{N(R_1 + R_1^*)} = 2^N \min \{ I(X_1; Y_1 | X_r, X_r, V_1, V_2), I(X_1, X_r; Y | X_1, V_2) \} - \epsilon \ (\epsilon \to 0$ as $N \to \infty$). Thus, using the same approach as that in [8, Lemma 3], we have

\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N) \geq \min \{ I(X_1; Y_1 | X_r, X_r, V_1, V_2), I(X_1, X_r; Y | X_1, V_2) \}. \tag{A15}
\]

For the second term in (A14), using the same approach as that in [8, Lemma 3], we get

\[
\lim_{N \to \infty} \frac{1}{N} I(X_1^N; Z^N) \leq I(X_1; Z). \tag{A16}
\]

Now, we consider the last term of (A14). Given $Z^N$ and $W_1$, the total number of possible codewords of $x_1^N$ is
$2^{NR'_1} = 2^{N(I(X_1;Z^\epsilon) - \epsilon)}$ ($\epsilon \to 0$ as $N \to \infty$). By using the Fano’s inequality, we have

$$\lim_{N \to \infty} \frac{1}{N} H(X_1^N | W_1, Z^N) = 0.$$  \hfill (A17)

Substituting (A15), (A16) and (A17) into (A14), we have

$$\lim_{N \to \infty} \frac{1}{N} H(W_1 | Z^N) \geq \min\{I(X_1; Y_r | X_r, X_2, V_1, V_2), I(X_1, X_r; Y | X_2, V_2)\} - I(X_1; Z) = R_1.$$  \hfill (A18)

The second term in (A13) is bounded as follows.

$$\lim_{N \to \infty} \frac{1}{N} H(W_2 | W_1, Z^N) \geq \lim_{N \to \infty} \frac{1}{N} H(W_2 | W_1, Z^N, X_1^N)$$

\[\geq (1) \lim_{N \to \infty} \frac{1}{N} H(W_2 | Z^N, X_1^N)\]

\[= \lim_{N \to \infty} \frac{1}{N} (H(W_2, Z^N, X_1^N) - H(Z^N, X_1^N))\]

\[= \lim_{N \to \infty} \frac{1}{N} (H(W_2, Z^N, X_2^N) - H(X_2^N | W_2, Z^N, X_1^N) - H(Z^N, X_1^N))\]

\[\geq (2) \lim_{N \to \infty} \frac{1}{N} (H(Z^N | X_1^N, X_2^N) + H(X_1^N) + H(X_2^N) - H(X_1^N | W_2, Z^N, X_1^N) - H(Z^N | X_1^N) - H(X_1^N))\]

\[= \lim_{N \to \infty} \frac{1}{N} (H(X_2^N) - I(X_2^N; Z^N | X_1^N) - H(X_2^N | W_2, Z^N, X_1^N)),\]  \hfill (A19)

where (1) is from the Markov chain $W_1 \to (Z^N, X_1^N) \to W_2$, and (2) is from the Markov chain $W_2 \to (X_1^N, X_2^N) \to Z^N$.

Consider the first term in (A19), the code-book generation of $x_2^N$ shows that the total number of $x_2^N$ is $2^{N(R_2 + R'_2)} = 2^{N\min\{I(X_2; Y_r | X_r, V_1, V_2), I(X_2, V_2; Y)\} - \epsilon}$ ($\epsilon \to 0$ as $N \to \infty$). Thus, using the same approach as that in [8] Lemma 3], we have

$$\lim_{N \to \infty} \frac{1}{N} H(X_2^N) \geq \min\{I(X_2; Y_r | X_r, V_1, V_2), I(X_2, V_2; Y)\}.$$  \hfill (A20)

For the second term in (A19), using the same approach as that in [4] Lemma 3], we get

$$\lim_{N \to \infty} \frac{1}{N} I(X_2^N; Z^N | X_1^N) \leq I(X_2; Z | X_1).$$  \hfill (A21)

Now, we consider the last term of (A19). Given $Z^N$, $X_1^N$ and $W_2$, the total number of possible codewords of $x_2^N$ is $2^{NR_2^\epsilon} = 2^{I(X_2; Z | X_1) - \epsilon}$ ($\epsilon \to 0$ as $N \to \infty$). By using the Fano’s inequality, we have

$$\lim_{N \to \infty} \frac{1}{N} H(X_2^N | W_2, Z^N, X_1^N) = 0.$$  \hfill (A22)

Substituting (A20), (A21) and (A22) into (A19), we have

$$\lim_{N \to \infty} \frac{1}{N} H(W_2 | W_1, Z^N) \geq \min\{I(X_2; Y_r | X_r, V_1, V_2), I(X_2, V_2; Y)\} - I(X_2; Z | X_1) = R_2.$$  \hfill (A23)

Substituting (A18) and (A23) into (A13), $\lim_{N \to \infty} \Delta \geq R_1 + R_2$ is proved.

The proof of Theorem 1 is completed.
Theorem 2 is proved by the following two cases.

- **(Case 1)** If the channel from the relay to the legitimate receiver is less noisy than the channel from the relay to the wiretapper ($I(X_r; Y) \geq I(X_r; Z)$), we allow the legitimate receiver to decode $x_r^N$, and the wiretapper cannot decode it.

For case 1, it is sufficient to show that the pair $(R_1, R_2) \in \mathcal{L}^1$ with the condition

$$R_1 = I(X_1; Y|X_2, X_r) - I(X_1; X_r; Z) + R_r, \quad R_2 = I(X_2; Y|X_r) - I(X_2; Z|X_1, X_r)$$

(A24)

is achievable. The achievability proof of $(R_1 = I(X_1; Y|X_r) - I(X_1; Z|X_2, X_r), R_2 = I(X_2; Y|X_1, X_r) - I(X_2; X_r; Z) + R_r)$ follows by symmetry.

- **(Case 2)** If the channel from the relay to the legitimate receiver is more noisy than the channel from the relay to the wiretapper ($I(X_r; Y) \leq I(X_r; Z)$), we allow both the receivers to decode $x_r^N$.

For case 2, it is sufficient to show that the pair $(R_1, R_2) \in \mathcal{L}^2$ with the condition

$$R_1 = I(X_1; Y|X_2, X_r) - I(X_1; Z|X_r), \quad R_2 = I(X_2; Y|X_1, X_r) - I(X_2; Z|X_r)$$

(A25)

is achievable. The achievability proof of $(R_1 = I(X_1; Y|X_r) - I(X_1; Z|X_2, X_r), R_2 = I(X_2; Y|X_1, X_r) - I(X_2; Z|X_r)$) follows by symmetry.

Fix the joint probability mass function $P_{Y,Z,Y_r,X_r,X_1,X_2}(y, z, y_r, x_r, x_1, x_2)$. Define the messages $W_1, W_2$ taken values in the alphabets $\mathcal{W}_1, \mathcal{W}_2$, respectively, where

$$\mathcal{W}_1 = \{1, 2, \ldots, 2^{NR_1}\}, \quad \mathcal{W}_2 = \{1, 2, \ldots, 2^{NR_2}\}.$$

**Code-book Construction for the Two Cases:**

- **Code-book construction for case 1:**
  - First, generate at random $2^{N(R_r-\epsilon)}$ (as $N \to \infty$) i.i.d. sequences at the relay node each drawn according to $P_{X_r^N}(x_r^N) = \prod_{i=1}^{N} P_{X_r}(x_{r,i})$, index them as $x_r^N(a)$, $a \in [1, 2^{N(R_r-\epsilon)}]$, where

$$R_r = \min\{I(X_r; Z|X_1), I(X_r; Z|X_2), I(X_r; Y)\}.$$  

(A26)

Here note that

$$R_r \geq I(X_1; Z).$$  

(A27)

- Second, generate $2^{N(I(X_2; Y|X_r)-\epsilon)}$ i.i.d. codewords $x_2^N(I)$ according to $P_{X_2}(x_2)$, and divide them into $2^{NR_2}$ bins. Each bin contains $2^{N(I(X_2; Y|X_r)-\epsilon-R_2)}$ codewords, where

$$I(X_2; Y|X_r) - \epsilon - R_2 = I(X_2; Z|X_1, X_r) - \epsilon.$$  

(A28)

- Third, generate $2^{N(I(X_1; Y|X_2, X_r)-\epsilon)}$ i.i.d. codewords $x_1^N$ according to $P_{X_1}(x_1)$, and divide them into $2^{NR_1}$ bins. Each bin contains $2^{N(I(X_1; Y|X_2, X_r)-\epsilon-R_1)}$ codewords.
Code-book Construction for case 2:

- Generate at random \(2^{N(R_r-\epsilon)}\) \((\epsilon \to 0 \text{ as } N \to \infty)\) i.i.d. sequences at the relay node each drawn according to \(P_{XY}(x_N^r) = \prod_{i=1}^{N} P_{X_r}(x_{r,i})\), index them as \(x_N^r(a)\), \(a \in [1, 2^{N(R_r-\epsilon)}]\), where

\[
R_r = I(X_r; Y) \leq I(X_1; Z). \tag{A29}
\]

- Second, generate \(2^{N(I(X_2; Y|X_r)-\epsilon)}\) i.i.d. codewords \(x_2^N\) according to \(P_{X_2}(x_2)\), and divide them into \(2^{N R_2}\) bins. Each bin contains \(2^{N(I(X_2; Y|X_r)-\epsilon-R_2)}\) codewords, where

\[
I(X_2; Y|X_r) - \epsilon - R_2 = I(X_2; Z|X_1, X_r) - \epsilon. \tag{A30}
\]

- Third, generate \(2^{N(I(X_1; Y|X_2, X_r)-\epsilon)}\) i.i.d. codewords \(x_1^N\) according to \(P_{X_1}(x_1)\), and divide them into \(2^{N R_1}\) bins. Each bin contains \(2^{N(I(X_1; Y|X_2, X_r)-\epsilon-R_1)}\) codewords, where

\[
I(X_1; Y|X_2, X_r) - \epsilon - R_1 = I(X_1; Z|X_r) - \epsilon. \tag{A31}
\]

Encoding for both cases:

The relay uniformly picks a codeword \(x_N^r(a)\) from \([1, 2^{N(R_r-\epsilon)}]\), and sends \(x_N^r(a)\).

For a given confidential message \(w_2\), randomly choose a codeword \(x_2^N\) in bin \(w_2\) to transmit. Similarly, for a given confidential message \(w_1\), randomly choose a codeword \(x_1^N\) in bin \(w_1\) to transmit.

Decoding for both cases:

For a given \(y^N\), try to find a sequence \(x_N^r(\hat{a})\) such that \((x_N^r(\hat{a}), y^N)\) are jointly typical. If there exists a unique sequence with the index \(\hat{a}\), put out the corresponding \(\hat{a}\), else declare a decoding error. Based on the AEP and \((\ref{A26})\) (or \((\ref{A29})\)), the probability \(Pr\{\hat{a} = a\}\) goes to 1.

After decoding \(\hat{a}\), the legitimate receiver tries to find a sequence \(x_2^N(\hat{w}_2)\) such that \((x_2^N(\hat{w}_2), x_N^r(\hat{a}), y^N)\) are jointly typical. If there exists a unique sequence with the index \(\hat{w}_2\), put out the corresponding \(\hat{w}_2\), else declare a decoding error. Based on the AEP and the construction of \(x_2^N\) for both cases, the probability \(Pr\{\hat{w}_2 = w_2\}\) goes to 1.

Finally, after decoding \(\hat{a}\) and \(\hat{w}_2\), the legitimate receiver tries to find a sequence \(x_1^N(\hat{w}_1)\) such that \((x_1^N(\hat{w}_1), x_2^N(\hat{w}_2), x_N^r(\hat{a}), y^N)\) are jointly typical. If there exists a unique sequence with the index \(\hat{w}_1\), put out the corresponding \(\hat{w}_1\), else declare a decoding error. Based on the AEP and the construction of \(x_1^N\) for both cases, the probability \(Pr\{\hat{w}_1 = w_1\}\) goes to 1.

\(P_{\epsilon} \leq \epsilon\) is easy to be checked by using the above encoding-decoding schemes. Now, it remains to prove \(\lim_{N \to \infty} \Delta \geq R_1 + R_2\) for both cases, see the followings.

Equivocation Analysis:

**Proof of** \(\lim_{N \to \infty} \Delta \geq R_1 + R_2\) **for case 1:**

\[
\lim_{N \to \infty} \Delta = \lim_{N \to \infty} \frac{1}{N} H(W_1, W_2|Z^N) = \lim_{N \to \infty} \frac{1}{N} (H(W_1|Z^N) + H(W_2|W_1, Z^N)). \tag{A32}
\]
The first term in (A32) is bounded as follows.
\[
\lim_{N \to \infty} \frac{1}{N} H(W_1|Z^N) = \lim_{N \to \infty} \frac{1}{N} (H(W_1, Z^N) - H(Z^N)) = \lim_{N \to \infty} \frac{1}{N} (H(W_1, Z^N, X_1^N, X_r^N) - H(X_1^N, X_r^N|W_1, Z^N) - H(Z^N)) \\
\overset{(a)}{=} \lim_{N \to \infty} \frac{1}{N} (H(Z^N|X_1^N, X_r^N) + H(X_1^N) + H(X_r^N) - H(X_1^N, X_r^N|W_1, Z^N) - H(Z^N)) \\
= \lim_{N \to \infty} \frac{1}{N} (H(X_1^N) + H(X_r^N) - I(X_1^N, X_r^N; Z^N) - H(X_1^N, X_r^N|W_1, Z^N)), \quad (A33)
\]
where (a) follows from \( W_1 \to (X_1^N, X_r^N) \to Z^N, H(W_1|X_1^N) = 0 \) and the fact that \( X_1^N \) is independent of \( X_r^N \).

Consider the first term in (A33), the code-book generation of \( x_1^N \) shows that the total number of \( x_1^N \) is \( 2^{N(I(X_1;Y|X_2,X_r) - \epsilon)} \) (\( \epsilon \to 0 \) as \( N \to \infty \)). Thus, using the same approach as that in [8, Lemma 3], we have
\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N) \geq I(X_1; Y|X_2, X_r). \quad (A34)
\]
For the second term in (A33), the code-book generation of \( x_r^N \) guarantees that
\[
\lim_{N \to \infty} \frac{1}{N} H(X_r^N) \geq R_r. \quad (A35)
\]
For the third term in (A33), using the same approach as that in [4, Lemma 3], we get
\[
\lim_{N \to \infty} \frac{1}{N} I(X_1^N, X_r^N; Z^N) \leq I(X_1, X_r; Z). \quad (A36)
\]
Now, we consider the last term of (A33). Given \( w_1 \), the wiretapper can do joint decoding. Specifically, given \( z^N \) and \( w_1 \),
\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N, X_r^N|W_1, Z^N) = 0, \quad (A37)
\]
is guaranteed if \( R_r \leq I(X_r; Z|X_1) \) and \( R_r \geq I(X_r; Z) \), and this is from the properties of AEP (similar argument is used in the proof of [17, Theorem 3]). By using (A26) and (A27), (A37) is obtained.

Substituting (A34), (A35), (A36) and (A37) into (A33), we have
\[
\lim_{N \to \infty} \frac{1}{N} H(W_1|Z^N) \geq I(X_1; Y|X_2, X_r) + R_r - I(X_1, X_r; Z) = R_1. \quad (A38)
\]
The second term in (A32) is bounded as follows.
\[
\lim_{N \to \infty} \frac{1}{N} H(W_2|W_1, Z^N) \geq \lim_{N \to \infty} \frac{1}{N} H(W_2|W_1, Z^N, X_1^N, X_r^N)
\]
\[
\overset{(1)}{=} \lim_{N \to \infty} \frac{1}{N} H(W_2|Z^N, X_1^N, X_r^N)
\]
\[
= \lim_{N \to \infty} \frac{1}{N} \left( H(W_2, Z^N, X_1^N, X_r^N) - H(Z^N, X_1^N, X_r^N) \right)
\]
\[
= \lim_{N \to \infty} \frac{1}{N} \left( H(W_2, Z^N, X_1^N, X_r^N) - H(X_2^N|W_2, Z^N, X_1^N, X_r^N) - H(Z^N, X_1^N, X_r^N) \right)
\]
\[
\overset{(2)}{=} \lim_{N \to \infty} \frac{1}{N} \left( H(Z^N|X_1^N, X_r^N) + H(X_2^N) \right)
\]
\[
- H(X_2^N|W_2, Z^N, X_1^N, X_r^N) - H(Z^N|X_1^N, X_r^N) - H(X_1^N) - H(X_r^N)
\]
\[
= \lim_{N \to \infty} \frac{1}{N} \left( H(X_2^N) - I(X_2^N; Z^N|X_1^N, X_r^N) - H(X_2^N|W_2, Z^N, X_1^N, X_r^N) \right),
\]
(A39)

where (1) is from the Markov chain \(W_1 \to (Z^N, X_1^N, X_r^N) \to W_2\), and (2) is from the Markov chain \(W_2 \to (X_1^N, X_2^N, X_r^N) \to Z^N, H(W_2|X_2^N) = 0\), and the fact that \(X_1^N, X_2^N\) and \(X_r^N\) are independent.

Consider the first term in (A39), the code-book generation of \(x_2^N\) shows that the total number of \(x_2^N\) is \(2^{N(I(X_2^N;Y|X_r^N)-\epsilon)}\) \((\epsilon \to 0\) as \(N \to \infty\)). Thus, using the same approach as that in [8] Lemma 3, we have
\[
\lim_{N \to \infty} \frac{1}{N} H(X_2^N) \geq I(X_2;Y|X_r).
\]
(A40)

For the second term in (A39), using the same approach as that in [4] Lemma 3, we get
\[
\lim_{N \to \infty} \frac{1}{N} I(X_2^N; Z^N|X_1^N, X_r^N) \leq I(X_2;Z|X_1, X_r).
\]
(A41)

Now, we consider the last term of (A39). Given \(Z^N, X_1^N, X_r^N\) and \(W_2\), the total number of possible codewords of \(x_2^N\) is \(2^{N(I(X_2^N;Y|X_r^N)-\epsilon-R_2)}\) \((\epsilon \to 0\) as \(N \to \infty\)). By using the Fano’s inequality and (A28), we have
\[
\lim_{N \to \infty} \frac{1}{N} H(X_2^N|W_2, Z^N, X_1^N, X_r^N) = 0.
\]
(A42)

Substituting (A40), (A41) and (A42) into (A39), we have
\[
\lim_{N \to \infty} \frac{1}{N} H(W_2|W_1, Z^N) \geq I(X_2;Y|X_r) - I(X_2;Z|X_1, X_r) = R_2.
\]
(A43)

Substituting (A38) and (A43) into (A32), \(\lim_{N \to \infty} \Delta \geq R_1 + R_2\) for case 1 is proved.

**Proof of** \(\lim_{N \to \infty} \Delta \geq R_1 + R_2\) for **case 2**:

\[
\lim_{N \to \infty} \Delta = \lim_{N \to \infty} \frac{1}{N} H(W_1, W_2|Z^N)
\]
\[
= \lim_{N \to \infty} \frac{1}{N} \left( H(W_1|Z^N) + H(W_2|W_1, Z^N) \right).
\]
(A44)

The first term in (A44) is bounded as follows.
\[
\lim_{N \to \infty} \frac{1}{N} H(W_1|Z^N) \geq \lim_{N \to \infty} \frac{1}{N} H(W_1|Z^N, X_1^N) \\
= \lim_{N \to \infty} \frac{1}{N} \left( H(W_1, Z^N, X_1^N) - H(Z^N, X_1^N) \right) \\
= \lim_{N \to \infty} \frac{1}{N} \left( H(W_1, Z^N, X_1^N, X_r^N) - H(X_1^N|W_1, Z^N, X_r^N) - H(Z^N, X_r^N) \right) \\
= \lim_{N \to \infty} \frac{1}{N} \left( H(Z^N|X_1^N, X_r^N) + H(X_1^N) - H(X_1^N|W_1, Z^N, X_r^N) \\
- H(Z^N|X_r^N) - H(X_r^N) \right) \\
= \lim_{N \to \infty} \frac{1}{N} \left( H(X_1^N) - I(X_1^N; Z^N|X_r^N) - H(X_1^N|W_1, Z^N, X_r^N) \right),
\]
where (a) follows from \( W_1 \to (X_1^N, X_r^N) \to Z^N, H(W_1|X_1^N) = 0 \) and the fact that \( X_1^N \) is independent of \( X_r^N \).

Consider the first term in (A45), the code-book generation of \( x_1^N \) shows that the total number of \( x_1^N \) is \( 2^{N(I(X_1; Y|X_2, X_r) - \epsilon)} (\epsilon \to 0 \text{ as } N \to \infty) \). Thus, using the same approach as that in [6, Lemma 3], we have

\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N) \geq I(X_1; Y|X_2, X_r).
\]

For the second term in (A45), using the same approach as that in [4, Lemma 3], we get

\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N; Z^N|X_r^N) \leq I(X_1; Z|X_r).
\]

Now, we consider the last term of (A45). Given \( Z^N, X_r^N \) and \( W_1 \), the total number of possible codewords of \( x_1^N \) is \( 2^{N(I(X_1; Y|X_2, X_r) - \epsilon - R_1)} (\epsilon \to 0 \text{ as } N \to \infty) \). By using the Fano’s inequality and (A31), we have

\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N|W_1, Z^N, X_r^N) = 0.
\]

Substituting (A46), (A47) and (A48) into (A45), we have

\[
\lim_{N \to \infty} \frac{1}{N} H(W_1|Z^N) \geq I(X_1; Y|X_2, X_r) - I(X_1; Z|X_r) = R_1.
\]

The second term in (A44) is bounded the same as that for case 1, and thus, we have

\[
\lim_{N \to \infty} \frac{1}{N} H(W_2|W_1, Z^N) \geq I(X_2; Y|X_r) - I(X_2; Z|X_1, X_r) = R_2.
\]

The proof is omitted here.

Substituting (A49) and (A50) into (A44), \( \lim_{N \to \infty} \Delta \geq R_1 + R_2 \) for case 2 is proved.

The proof of Theorem 2 is completed.

**APPENDIX C**

**PROOF OF THEOREM 2**

Theorem 2 is proved by the following two cases.

- **(Case 1)** If the channel from the relay to the legitimate receiver is less noisy than the channel from the relay to the wiretapper \( I(X_r; Y) \geq I(X_r; Z) \), we allow the legitimate receiver to decode \( x_r^N \), and the wiretapper can not decode it.
For case 1, it is sufficient to show that the pair $(R_1, R_2) \in \mathcal{L}^d$ with the condition

$$ R_1 = I(X_1; Y, \hat{Y}_r|X_2, X_r) - I(X_1, X_r; Z) + R^*, \quad R_2 = I(X_2; Y, \hat{Y}_r|X_r) - I(X_2; Z|X_1, X_r) $$  \hspace{1cm} (A51)

is achievable. The achievability proof of $(R_1 = I(X_1; Y, \hat{Y}_r|X_r) - I(X_1; Z|X_2, X_r), R_2 = I(X_2; Y, \hat{Y}_r|X_1, X_r) - I(X_2, X_r; Z) + R^*)$ follows by symmetry. Here note that $R^*$ satisfies

$$ \min\{I(X_r; Z|X_1), I(X_r; Z|X_2), I(X_r; Y)\} - R^* \geq I(Y_r; \hat{Y}_r|X_r). $$  \hspace{1cm} (A52)

- **(Case 2)** If the channel from the relay to the legitimate receiver is more noisy than the channel from the relay to the wiretapper $(I(Y_r; \hat{Y}_r|X_r) \leq I(X_r; Y) \leq I(X_r; Z))$, we allow both the receivers to decode $x_r^N$.

For case 2, it is sufficient to show that the pair $(R_1, R_2) \in \mathcal{L}^d$ with the condition

$$ R_1 = I(X_1; Y, \hat{Y}_r|X_2, X_r) - I(X_1; Z|X_r), \quad R_2 = I(X_2; Y, \hat{Y}_r|X_1, X_r) - I(X_2; Z|X_1, X_r) $$  \hspace{1cm} (A53)

is achievable. The achievability proof of $(R_1 = I(X_1; Y, \hat{Y}_r|X_r) - I(X_1; Z|X_2, X_r), R_2 = I(X_2; Y, \hat{Y}_r|X_1, X_r) - I(X_2; Z|X_1, X_r))$ follows by symmetry.

Fix the joint probability mass function $P_{Y_r|Y_r, X_r}(\hat{y}_r|y_r, x_r)P_{Y_r,Z|X_r}(y, z, y_r, x_1, x_2)P_{X_r}(x_r)P_{X_1}(x_1)P_{X_2}(x_2)$.

Define the messages $W_1, W_2$ taken values in the alphabets $\mathcal{W}_1, \mathcal{W}_2$, respectively, where

$$ \mathcal{W}_1 = \{1, 2, ..., 2^{NR_1}\}, \quad \mathcal{W}_2 = \{1, 2, ..., 2^{NR_2}\}. $$

**Code-book Construction for the Two Cases:**

- **Code-book construction for case 1:**
  - First, generate at random $2^{N(R_1^* - \epsilon)}$ (as $N \rightarrow \infty$) i.i.d. sequences $x_r^N$ at the relay node each drawn according to $P_{X_r^N}(x_r^N) = \prod_{i=1}^{N} P_{X_r}(x_r^N)$, index them as $x_r^N(a)$, $a \in [1, 2^{N(R_1^* - \epsilon)}]$, where

$$ R_{r1}^* = \min\{I(X_r; Z|X_1), I(X_r; Z|X_2), I(X_r; Y)\}. $$  \hspace{1cm} (A54)

Here note that

$$ R_{r1}^* \geq I(X_r; Z). $$  \hspace{1cm} (A55)

For each $x_r^N(a)$ ($a \in [1, 2^{N(R_1^* - \epsilon)}]$), generate at random $2^{N(R_1^* - \epsilon - R^*)}$ i.i.d. $\hat{y}_r^N$ according to $P_{Y_r^N|X_r^N}(\hat{y}_r^N|x_r^N) = \prod_{i=1}^{N} P_{Y_r|X_r}(\hat{y}_r^N|x_r^N)$. Label these $\hat{y}_r^N$ as $\hat{y}_r^N(m, a)$, $m \in [1, 2^{N(R_1^* - \epsilon - R^*)}]$, $a \in [1, 2^{N(R_1^* - \epsilon)}]$. Equally divide $2^{N(R_1^* - \epsilon)}$ sequences of $x_r^N$ into $2^{N(R_1^* - \epsilon - R^*)}$ bins, hence there are $2^{NR^*}$ sequences of $x_r^N$ at each bin.

  - Second, generate $2^{N(I(X_2; Y, \hat{Y}_r|X_r) - \epsilon)}$ i.i.d. codewords $x_2^N$ according to $P_{X_2}(x_2)$, and divide them into $2^{NR_2}$ bins. Each bin contains $2^{N(I(X_2; Y, \hat{Y}_r|X_r) - \epsilon - R_2)}$ codewords, where

$$ I(X_2; Y, \hat{Y}_r|X_r) - \epsilon - R_2 = I(X_2; Z|X_1, X_r) - \epsilon. $$  \hspace{1cm} (A56)

  - Third, generate $2^{N(I(X_2; Y, \hat{Y}_r|X_r, X_2, X_r) - \epsilon + R^* - R_1^*)}$ i.i.d. codewords $x_1^N$ according to $P_{X_1}(x_1)$, and divide them into $2^{NR_1}$ bins. Each bin contains $2^{N(I(X_2; Y, \hat{Y}_r|X_2, X_r; X_r) - \epsilon + R^* - R_1^* - R_2)}$ codewords. Here note that
from (A52) and (A54), we know that \( R^* \leq R_{r1}^{*} \), and thus, we have

\[
I(X_1; Y, \hat{Y}_r | X_2, X_r) - \epsilon + R^* - R_{r1}^* \leq I(X_1; Y, \hat{Y}_r | X_2, X_r) - \epsilon.
\]  (A57)

In addition, by using \( R_1 = I(X_1; Y, \hat{Y}_r | X_2, X_r) - I(X_1, X_r; Z) + R^* \), the codewords \( x_1^n \) in each bin is upper bounded by

\[
I(X_1; Y, \hat{Y}_r | X_2, X_r) - \epsilon + R^* - R_{r1}^* - R_1
\]
\[
= I(X_1; Y, \hat{Y}_r | X_2, X_r) - \epsilon + R^* - R_{r1}^* - (I(X_1; Y, \hat{Y}_r | X_2, X_r) - I(X_1, X_r; Z) + R^*)
\]
\[
= I(X_1, X_r; Z) - R_{r1}^* - \epsilon
\]
\[
\leq I(X_1, X_r; Z) - I(X_r; Z) - \epsilon
\]
\[
= I(X_1; Z | X_r) - \epsilon,
\]  (A58)

where (a) is from (A55).

**Code-book Construction for case 2:**

- First, generate at random \( 2^{N(R_{r2}^* - \epsilon)} \) (\( \epsilon \to 0 \) as \( N \to \infty \)) i.i.d. sequences \( x_r^n \) at the relay node each drawn according to \( P_{X_r}(x_r) = \prod_{i=1}^{N} P_{X_r}(x_{r,i}) \), index them as \( x_r^n(a), a \in [1, 2^{N(R_{r2}^* - \epsilon)}] \), where

\[
R_{r2}^* = I(X_r; Y) \leq I(X_r; Z).
\]  (A59)

For each \( x_r^n(a) (a \in [1, 2^{N(R_{r2}^* - \epsilon)}]) \), generate at random \( 2^{N(R_{r2}^* - \epsilon)} \) i.i.d. \( \hat{y}_r^n \) according to \( P_{\hat{Y}_r^n | X_r^n}(\hat{y}_r^n | x_r^n) = \prod_{i=1}^{N} P_{\hat{Y}_r | X_r}(\hat{y}_{r,i} | x_{r,i}) \). Label these \( \hat{y}_r^n \) as \( \hat{y}_r^n(a), a \in [1, 2^{N(R_{r2}^* - \epsilon)}] \).

- Second, generate \( 2^{N(I(X_2; Y, \hat{Y}_r | X_r) - \epsilon)} \) i.i.d. codewords \( x_2^n \) according to \( P_{X_2}(x_2) \), and divide them into \( 2^{N R_{r2}} \) bins. Each bin contains \( 2^{N(I(X_2; Y, \hat{Y}_r | X_r) - \epsilon - R_{r2})} \) codewords, where

\[
I(X_2; Y, \hat{Y}_r | X_r) - \epsilon - R_{r2} = I(X_2; Z | X_1, X_r) - \epsilon.
\]  (A60)

- Third, generate \( 2^{N(I(X_1; Y, \hat{Y}_r | X_2, X_r) - \epsilon)} \) i.i.d. codewords \( x_1^n \) according to \( P_{X_1}(x_1) \), and divide them into \( 2^{N R_{r1}} \) bins. Each bin contains \( 2^{N(I(X_1; Y, \hat{Y}_r | X_2, X_r) - \epsilon - R_{r1})} \) codewords, where

\[
I(X_1; Y, \hat{Y}_r | X_2, X_r) - \epsilon - R_{r1} = I(X_1; Z | X_r) - \epsilon.
\]  (A61)

**Encoding:**

Encoding involves the mapping of message indices to channel inputs, which are facilitated by the sequences generated above. We exploit the block Markov coding scheme, as argued in [22], the loss induced by this scheme is negligible as the number of blocks \( n \to \infty \). For block \( i \) (\( 1 \leq i \leq n \)), encoding proceeds as follows.

First, for convenience, the messages \( w_1 \) and \( w_2 \) transmitted in the \( i \)-th block are denoted by \( w_{1,i} \) and \( w_{2,i} \), respectively. \( y_r^n(i) \) and \( \hat{y}_r^n(i) \) are the \( y_r^n \) and \( \hat{y}_r^n \) for the \( i \)-th block, respectively.

**Encoding for case 1:**
At the end of block $i$ ($2 \leq i \leq n$), assume that $(x_i^N(a_i), y_i^N(i), \hat{y}_i^N(m_i, a_i))$ are jointly typical, then we choose $a_{i+1}$ uniformly from bin $m_i$, and the relay sends $x_i^N(a_{i+1})$ at block $i + 1$. In the first block, the relay sends $x_1^N(1)$.

For a given confidential message $w_2$, randomly choose a codeword $x_2^N$ in bin $w_2$ to transmit. Similarly, for a given confidential message $w_1$, randomly choose a codeword $x_1^N$ in bin $w_1$ to transmit.

- **Encoding for case 2:**

In block $i$ ($1 \leq i \leq n$), the relay randomly choose an index $a_i$ from $[1, 2^{N(R_r - \epsilon)}]$, and sends $x_i^N(a_i)$ and $\hat{y}_i^N(a_i)$.

For a given confidential message $w_2$, randomly choose a codeword $x_2^N$ in bin $w_2$ to transmit. Similarly, for a given confidential message $w_1$, randomly choose a codeword $x_1^N$ in bin $w_1$ to transmit.

**Decoding:**

- **Decoding for case 1:**

  (At the relay) At the end of block $i$, the relay already has $a_i$, it then decides $m_i$ by choosing $m_i$ such that $(x_i^N(a_i), y_i^N(i), \hat{y}_i^N(m_i, a_i))$ are jointly typical. There exists such $m_i$, if

  $$R^*_{r_1} - R^* \geq I(Y_r; \hat{Y}_r|X_r),$$

  (A62)

  and $N$ is sufficiently large. Choose $a_{i+1}$ uniformly from bin $m_i$.

  (At the legitimate receiver) The legitimate receiver does backward decoding. The decoding process starts at the last block $n$, the legitimate receiver decodes $a_n$ by choosing unique $\hat{a}_n$ such that $(x_n^N(\hat{a}_n), y_n^N(n))$ are jointly typical. Since $R^*_{r_1}$ satisfies (A54), the probability $Pr\{\hat{a}_n = a_n\}$ goes to 1 for sufficiently large $N$.

  Next, the legitimate receiver moves to the block $n-1$. Now it already has $\hat{a}_n$, hence we also have $\hat{m}_{n-1} = f(\hat{a}_n)$ (here $f$ is a deterministic function, which means that $\hat{m}_{n-1}$ can be determined by $\hat{a}_n$). It first declares that $\hat{a}_{n-1}$ is received, if $\hat{a}_{n-1}$ is the unique one such that $(x_{n-1}^N(\hat{a}_{n-1}), y_{n}(n - 1))$ are joint typical. If (A54) is satisfied, $\hat{a}_{n-1} = a_{n-1}$ with high probability. After knowing $\hat{a}_{n-1}$, the destination gets an estimation of $w_{2,n-1}$ by picking the unique $\hat{w}_{2,n-1}$ such that $(x_{2}^N(\hat{w}_{2,n-1}), \hat{y}_{r}^N(\hat{m}_{n-1}, \hat{a}_{n-1}), y_{n}(n - 1), x_r^N(\hat{a}_{n-1}))$ are jointly typical. We will have $\hat{w}_{2,n-1} = w_{2,n-1}$ with high probability, if the codewords of $x_2^N$ is upper bounded by $2^{NI(X_2;Y_{r}X_{1}|X_{2})}$ and $N$ is sufficiently large.

  After decoding $\hat{w}_{2,n-1}$, the legitimate receiver tries to find a quintuple such that $(x_1^N(\hat{w}_{1,n-1}), x_2^N(\hat{w}_{2,n-1}), \hat{y}_{r}^N(\hat{m}_{n-1}, \hat{a}_{n-1}), y_{n}(n - 1), x_r^N(\hat{a}_{n-1}))$ are jointly typical. Based on the AEP, the probability $Pr\{\hat{w}_{1,n-1} = w_{1,n-1}\}$ goes to 1 if the codewords of $x_1^N$ is upper bounded by $2^{NI(X_1;Y_{r}X_{1}|X_{2},X_{r})}$ and $N$ is sufficiently large.

  The decoding scheme of the legitimate receiver in block $i$ ($1 \leq i \leq n - 2$) is similar to that in block $n - 1$, and we omit it here.

- **Decoding for case 2:**

  (At the relay) The relay does not need to decode any codeword.

  (At the legitimate receiver) In block $i$ ($1 \leq i \leq n$), the legitimate receiver decodes $a_i$ by choosing unique $\hat{a}_i$
such that \((x_1^N(\hat{a}_i), y^N(i))\) are jointly typical. Since \(R_{e2}'\) satisfies \(A59\), the probability \(Pr\{\hat{a}_i = a_i\}\) goes to 1 for sufficiently large \(N\).

Now since the legitimate receiver has \(\hat{a}_i\), he also knows \(\hat{y}_r^N(\hat{a}_i)\). Then he gets an estimation of \(w_{2,i}\) by picking the unique \(\hat{w}_{2,i}\) such that \((x_2^N(\hat{w}_{2,i}), \hat{y}_r^N(\hat{a}_i), y^N(i), x_r^N(\hat{a}_i))\) are jointly typical. We will have \(\hat{w}_{2,i} = w_{2,i}\) with high probability, if the codewords of \(x_2^N\) is upper bounded by \(2^{NI(X_2;Y_r|X_r)}\) and \(N\) is sufficiently large.

After decoding \(\hat{w}_{2,i}\), the legitimate receiver tries to find a quintuple such that \((x_1^N(\hat{w}_{1,i}), x_2^N(\hat{w}_{2,i}), \hat{y}_r^N(\hat{a}_i), y^N(i), x_r^N(\hat{a}_i))\) are jointly typical. Based on the AEP, the probability \(Pr\{\hat{w}_{1,i} = w_{1,i}\}\) goes to 1 if the codewords of \(x_1^N\) is upper bounded by \(2^{NI(X_1;Y_r|X_2,X_r)}\) and \(N\) is sufficiently large.

\(P_e \leq \epsilon\) is easy to be checked by using the above encoding-decoding schemes. Now, it remains to prove \(\lim_{N \to \infty} \Delta \geq R_1 + R_2\) for both cases, see the followings.

**Equivocation Analysis:**

**Proof of \(\lim_{N \to \infty} \Delta \geq R_1 + R_2\) for case 1:**

\[
\lim_{N \to \infty} \frac{1}{N} H(W_1|Z^N) = \lim_{N \to \infty} \frac{1}{N} (H(W_1, Z^N) - H(Z^N))
\]
\[
= \lim_{N \to \infty} \frac{1}{N} (H(W_1, Z^N, X_1^N, X_r^N) - H(X_1^N, X_r^N|W_1, Z^N) - H(Z^N))
\]
\[
\overset{(a)}{=} \lim_{N \to \infty} \frac{1}{N} (H(Z^N|X_1^N, X_r^N) + H(X_1^N) + H(X_r^N) - H(X_1^N, X_r^N|W_1, Z^N) - H(Z^N))
\]
\[
= \lim_{N \to \infty} \frac{1}{N} (H(X_1^N) + H(X_r^N) - I(X_1^N, X_r^N; Z^N) - H(X_1^N, X_r^N|W_1, Z^N)),
\]

where (a) follows from \(W_1 \to (X_1^N, X_r^N) \to Z^N, H(W_1|X_1^N) = 0\) and the fact that \(X_1^N\) is independent of \(X_r^N\).

Consider the first term in \(A64\), the code-book generation of \(x_1^N\) shows that the total number of \(x_1^N\) is upper bounded by \(A58\). Thus, using the same approach as that in \([8\) Lemma 3], we have

\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N) \geq I(X_1;Y_r|X_2,X_r) + R^* - R_{r1}^*.
\]

(A65)

For the second term in \(A64\), the code-book generation of \(x_2^N\) and \([8\) Lemma 3] guarantee that

\[
\lim_{N \to \infty} \frac{1}{N} H(X_2^N) \geq R_{r1}^*.
\]

(A66)

For the third term in \(A64\), using the same approach as that in \([4\) Lemma 3], we get

\[
\lim_{N \to \infty} \frac{1}{N} I(X_1^N, X_r^N; Z^N) \leq I(X_1, X_r; Z).
\]

(A67)
Now, we consider the last term of (A64). Given \( w_1 \), the wiretapper can do joint decoding. Specifically, given \( z^N \) and \( w_1 \),

\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N, X_r^N | W_1, Z^N) = 0,
\]

(A68)
is guaranteed if \( R_r \leq I(X_r; Z|X_1) \) and \( I(X_1; Y, \hat{Y}_r|X_2, X_r) - \epsilon + R^* - R_{r1} - R_1 \leq I(X_1; Z|X_r) \), and this is from the properties of AEP (similar argument is used in the proof of [17, Theorem 3]). By using (A54) and (A58), (A68) is obtained.

Substituting (A65), (A66), (A67) and (A68) into (A64), we have

\[
\lim_{N \to \infty} \frac{1}{N} H(W_1|Z^N) \geq I(X_1; Y, \hat{Y}_r|X_2, X_r) + R^* - I(X_1, X_r; Z) = R_1.
\]

(A69)

The second term in (A63) is bounded as follows.

\[
\begin{align*}
\lim_{N \to \infty} \frac{1}{N} H(W_2|W_1, Z^N) & \geq \lim_{N \to \infty} \frac{1}{N} H(W_2|W_1, Z^N, X_1^N, X_r^N) \\
& \overset{(1)}{=} \lim_{N \to \infty} \frac{1}{N} H(W_2|Z^N, X_1^N, X_r^N) \\
& = \lim_{N \to \infty} \frac{1}{N} \left( H(W_2, Z^N, X_1^N, X_r^N) - H(Z^N, X_1^N, X_r^N) \right) \\
& = \lim_{N \to \infty} \frac{1}{N} \left( H(W_2, Z^N, X_1^N, X_r^N) - H(X_2^N|W_2, Z^N, X_1^N, X_r^N) - H(Z^N, X_1^N, X_r^N) \right) \\
& \overset{(2)}{=} \lim_{N \to \infty} \frac{1}{N} \left( H(Z^N|X_1^N, X_r^N) + H(X_1^N) + H(X_r^N) \\
& - H(X_2^N|W_2, Z^N, X_1^N, X_r^N) - H(Z^N|X_1^N, X_r^N) - H(X_1^N) - H(X_r^N) \right) \\
& = \lim_{N \to \infty} \frac{1}{N} \left( H(X_2^N) - I(X_2^N; Z^N|X_1^N, X_r^N) - H(X_2^N|W_2, Z^N, X_1^N, X_r^N) \right),
\end{align*}
\]

(A70)

where (1) is from the Markov chain \( W_1 \rightarrow (Z^N, X_1^N, X_r^N) \rightarrow W_2 \), and (2) is from the Markov chain \( W_2 \rightarrow (X_1^N, X_2^N, X_r^N) \rightarrow Z^N, H(W_2|X_2^N) = 0 \), and the fact that \( X_1^N, X_2^N \) and \( X_r^N \) are independent.

Consider the first term in (A70), the code-book generation of \( x_2^N \) shows that the total number of \( x_2^N \) is \( 2^{N(I(X_2;Y,\hat{Y}_r|X_r)-\epsilon)} (\epsilon \to 0 \text{ as } N \to \infty) \). Thus, using the same approach as that in [3] Lemma 3], we have

\[
\lim_{N \to \infty} \frac{1}{N} H(X_2^N) \geq I(X_2; Y, \hat{Y}_r|X_r).
\]

(A71)

For the second term in (A70), using the same approach as that in [3] Lemma 3], we get

\[
\lim_{N \to \infty} \frac{1}{N} I(X_2^N; Z^N|X_1^N, X_r^N) \leq I(X_2; Z|X_1, X_r).
\]

(A72)

Now, we consider the last term of (A70). Given \( Z^N, X_1^N, X_r^N \) and \( W_2 \), the total number of possible codewords of \( x_2^N \) is \( 2^{N(I(X_2;Y,\hat{Y}_r|X_r)-\epsilon-R_2)} (\epsilon \to 0 \text{ as } N \to \infty) \). By using the Fano’s inequality and (A56), we have

\[
\lim_{N \to \infty} \frac{1}{N} H(X_2^N|W_2, Z^N, X_1^N, X_r^N) = 0.
\]

(A73)

Substituting (A71), (A72) and (A73) into (A70), we have

\[
\lim_{N \to \infty} \frac{1}{N} H(W_2|W_1, Z^N) \geq I(X_2; Y, \hat{Y}_r|X_r) - I(X_2; Z|X_1, X_r) = R_2.
\]

(A74)
Substituting (A69) and (A74) into (A63), \( \lim_{N \to \infty} \Delta \geq R_1 + R_2 \) for case 1 is proved.

**Proof of \( \lim_{N \to \infty} \Delta \geq R_1 + R_2 \) for case 2:**

\[
\lim_{N \to \infty} \Delta = \lim_{N \to \infty} \frac{1}{N} H(W_1, W_2 | Z^N) = \lim_{N \to \infty} \frac{1}{N} (H(W_1 | Z^N) + H(W_2 | W_1, Z^N)). \tag{A75}
\]

The first term in (A75) is bounded as follows.

\[
\lim_{N \to \infty} \frac{1}{N} H(W_1 | Z^N) \geq \lim_{N \to \infty} \frac{1}{N} H(W_1 | Z^N, X_r^N) = \lim_{N \to \infty} \frac{1}{N} (H(W_1, Z^N, X_r^N) - H(Z^N, X_r^N)) = \lim_{N \to \infty} \frac{1}{N} (H(W_1, Z^N, X_r^N) - H(X_r^N | W_1, Z^N, X_r^N)) = \lim_{N \to \infty} \frac{1}{N} (H(Z^N | X_r^N) + H(X_r^N) - H(X_r^N | W_1, Z^N, X_r^N)) \tag{A76}
\]

where (a) follows from \( W_1 \to (X_1^N, X_r^N) \to Z^N, H(W_1 | X_1^N) = 0 \) and the fact that \( X_1^N \) is independent of \( X_r^N \).

Consider the first term in (A76), the code-book generation of \( x_1^N \) shows that the total number of \( x_1^N \) is \( 2^{N(I(X_1^N; Y_r | X_2^N, X_r) - \epsilon)} (\epsilon \to 0 \text{ as } N \to \infty) \). Thus, using the same approach as that in [3] Lemma 3], we have

\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N) \geq I(X_1; Y_r | X_2, X_r). \tag{A77}
\]

For the second term in (A76), using the same approach as that in [4] Lemma 3], we get

\[
\lim_{N \to \infty} \frac{1}{N} I(X_1^N; Z^N | X_r^N) \leq I(X_1; Z | X_r). \tag{A78}
\]

Now, we consider the last term of (A76). Given \( Z^N \), \( X_r^N \) and \( W_1 \), the total number of possible codewords of \( x_1^N \) is \( 2^{N(I(X_1^N; Y_r | X_2^N, X_r) - \epsilon)} (\epsilon \to 0 \text{ as } N \to \infty) \). By using the Fano’s inequality and (A61), we have

\[
\lim_{N \to \infty} \frac{1}{N} H(X_1^N | W_1, Z^N, X_r^N) = 0. \tag{A79}
\]

Substituting (A77), (A78) and (A79) into (A76), we have

\[
\lim_{N \to \infty} \frac{1}{N} H(W_1 | Z^N) \geq I(X_1; Y, Y_r | X_2, X_r) - I(X_1; Z | X_r) = R_1. \tag{A80}
\]

The second term in (A75) is bounded the same as that for case 1, and thus, we have

\[
\lim_{N \to \infty} \frac{1}{N} H(W_2 | W_1, Z^N) \geq I(X_2; Y, Y_r | X_r) - I(X_2; Z | X_1, X_r) = R_2. \tag{A81}
\]

The proof is omitted here.

Substituting (A80) and (A81) into (A75), \( \lim_{N \to \infty} \Delta \geq R_1 + R_2 \) for case 2 is proved.

The proof of Theorem 3 is completed.
APPENDIX D

PROOF OF THEOREM 4

In this section, we prove Theorem 4 all the achievable secrecy pairs $(R_1, R_2)$ of the degraded discrete memoryless MARC-WT are contained in the set $\mathcal{R}^{djo}$. We will prove the inequalities of Theorem 4 in the remainder of this section.

(Proof of) $R_1 \leq I(X_1, X_r; Y|X_2, U) - I(X_1; Z|U)$:

\[
\frac{1}{N} H(W_1) \overset{(1)}{=} \frac{1}{N} H(W_1|Z^N)
\]
\[
= \frac{1}{N} (H(W_1|Z^N) - H(W_1|Z^N, W_2, Y^N) + H(W_1|Z^N, W_2, Y^N))
\]
\[
\overset{(2)}{\leq} \frac{1}{N} (I(W_1; W_2, Y^N|Z^N) + \delta(P_e))
\]
\[
\leq \frac{1}{N} (H(W_1|Z^N) - H(W_1|Z^N, W_2, Y^N, X_2^N) + \delta(P_e))
\]
\[
\overset{(3)}{=} \frac{1}{N} (I(W_1; Y^N, X_2^N|Z^N) + \delta(P_e))
\]
\[
\leq \frac{1}{N} (H(Y^N, X_2^N|Z^N) - H(Y^N, X_2^N|Z^N, W_1, X_1^N) + \delta(P_e))
\]
\[
\overset{(4)}{=} \frac{1}{N} (I(Y^N, X_2^N; X_1^N|Z^N) + \delta(P_e))
\]
\[
\leq \frac{1}{N} (I(Y^N, X_2^N; X_1^N|Z^N) - I(X_1^N; Z^N) + \delta(P_e))
\]
\[
\leq \frac{1}{N} I(X_1^N; Y^N|X_2^N) - I(X_1^N; Z^N) + \delta(P_e))
\]
\[
= \frac{1}{N} \sum_{i=1}^{N} (H(Y_i|Y^{i-1}, X_2^N) - H(Y_i|X_{1,i}, X_{2,i}, X_{r,i}) - H(Z_i|Z^{i-1}) + H(Z_i|Z^{i-1}, X_1^N)) + \frac{\delta(P_e)}{N}
\]
\[
\overset{(6)}{\leq} \frac{1}{N} \sum_{i=1}^{N} (H(Y_i|Y^{i-1}, X_2^N, Z^{i-1}) - H(Y_i|X_{1,i}, X_{2,i}, X_{r,i}, Z^{i-1}) - H(Z_i|Z^{i-1}) + H(Z_i|Z^{i-1}, X_1^N)) + \frac{\delta(P_e)}{N}
\]
\[
\overset{(7)}{=} \frac{1}{N} \sum_{i=1}^{N} (H(Y_i|X_{2,i}, Z^{i-1}, J = i) - H(Y_i|X_{1,i}, X_{2,i}, X_{r,i}, Z^{i-1}, J = i) - H(Z_i|Z^{i-1}, J = i)
\]
\[
+ H(Z_i|Z^{i-1}, X_{1,i}, J = i)) + \frac{\delta(P_e)}{N}
\]
\[
= H(Y_j|X_{2,j}, Z^{j-1}, J = j) - H(Y_j|X_{1,j}, X_{2,j}, X_{r,j}, Z^{j-1}, J = j) - H(Z_j|Z^{j-1}, J = j) + H(Z_j|Z^{j-1}, X_{1,j}, J) + \frac{\delta(P_e)}{N}
\]
\[
\overset{(8)}{=} I(X_1, X_r; Y|X_2, U) - I(X_1; Z|U) + \frac{\delta(P_e)}{N}
\]

(A82)
where (1) is from the definition of the perfect secrecy, (2) is from the Fano inequality, (3) is from $H(W_2 | X_2^N) = 0$, (4) is from $H(W_1 | X_1^N) = 0$, (5) is from the Markov chain $X_1^N \rightarrow (X_2^N, Y^N) \rightarrow Z^N$ and the fact that $X_1^N$ is independent of $X_2^N$, (6) is from the Markov chains $Y_i \rightarrow (Y^{i-1}, X_i^N) \rightarrow Z^{i-1}$ and $Y_i \rightarrow (X_{i,i}, X_{i,i}, X_{r,i}) \rightarrow Z^{i-1}$, (7) is from $J$ is a random variable (uniformly distributed over $\{1, 2, ..., N\}$), and it is independent of $X_1^N$, $X_2^N$, $X_r^N$, $Y^N$ and $Z^N$, (8) is from $J$ is uniformly distributed over $\{1, 2, ..., N\}$, and (9) is from the definitions that $X_1 \triangleq X_{1,i}$, $X_2 \triangleq X_{2,j}$, $X_r \triangleq X_{r,j}$, $Y \triangleq Y_j$, $Z \triangleq Z_j$ and $U \triangleq (Z^{J-1}, J)$.

By using $P_e \leq \epsilon$, $\epsilon \rightarrow 0$ as $N \rightarrow \infty$, $\lim_{N \rightarrow \infty} \frac{H(W_1)}{N} = R_1$ and (A82), it is easy to see that $R_1 \leq I(X_1, X_r; Y | X_2, U) - I(X_1; Z | U)$.

**Proof of $R_2 \leq I(X_2, X_r; Y | X_1, U) - I(X_2; Z | U)$:**

The proof is analogous to the proof of $R_1 \leq I(X_1, X_r; Y | X_2, U) - I(X_1; Z | U)$, and it is omitted here.

**Proof of** $R_1 + R_2 \leq I(X_1, X_2, X_r; Y | U) - I(X_1, X_2; Z | U)$:

\[
\lim_{N \rightarrow \infty} \Delta = \lim_{N \rightarrow \infty} \frac{1}{N} H(W_1, W_2 | Z^N)
\leq \lim_{N \rightarrow \infty} \frac{1}{N} (H(W_1, W_2 | Z^N) + \delta(P_e) - H(W_1, W_2 | Y^N, Z^N))
\leq \lim_{N \rightarrow \infty} \frac{1}{N} (H(Y^N | Z^N) - H(Y^N | W_1, W_2, X_1^N, X_2^N) + \delta(P_e))
\leq \lim_{N \rightarrow \infty} \frac{1}{N} (H(Y^N | Z^N) - H(Y^N | X_1^N, X_2^N) + \delta(P_e))
\leq \lim_{N \rightarrow \infty} \frac{1}{N} (H(Y_i | Y^{i-1}) - H(Y_i | X_{i,i}, X_{i,i}, X_{r,i}, Z^{i-1}) - H(Z_i | Z^{i-1}) + H(Z_i | X_{i,i}, X_{r,i}, Z^{i-1})) + \frac{\delta(P_e)}{N}
\leq \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^{N} (H(Y_i | Y^{i-1}) - H(Y_i | X_{i,i}, X_{i,i}, X_{r,i}, Z^{i-1}) - H(Z_i | Z^{i-1}) + H(Z_i | X_{i,i}, X_{r,i}, Z^{i-1})) + \frac{\delta(P_e)}{N}
\leq \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^{N} (H(Y_i | Z^{i-1}, J = i) - H(Y_i | X_{i,i}, X_{i,i}, X_{r,i}, Z^{i-1}, J = i)) + \frac{\delta(P_e)}{N}
\leq \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^{N} (H(Y_j | Z^{J-1}, J) - H(Y_j | X_{j,i}, X_{i,i}, X_{r,i}, Z^{J-1}, J) + H(Z_j | X_{j,i}, X_{r,i}, Z^{J-1}, J) + \frac{\delta(P_e)}{N})
distributed over \( \{1, 2, \ldots, N\} \), and (7) is from the definitions that \( X_1 \triangleq X_{1,J} \), \( X_2 \triangleq X_{2,J} \), \( X_r \triangleq X_{r,J} \), \( Y \triangleq Y_J \), \( Z \triangleq Z_J \) and \( U \triangleq (Z^J - 1, J) \), and the fact that \( P_e \to 0 \) as \( N \to \infty \).

By using \( \lim_{N \to \infty} \Delta \geq R_1 + R_2 \) and (A83), it is easy to see that \( R_1 + R_2 \leq I(X_1, X_2, X_r; Y|U) - I(X_1, X_2; Z|U) \).

The proof of Theorem 4 is completed.

REFERENCES

[1] C. E. Shannon, “Communication theory of secrecy systems,” The Bell System Technical Journal, vol. 28, pp. 656-714, 1949.

[2] A. D. Wyner, “The wire-tap channel,” The Bell System Technical Journal, vol. 54, no. 8, pp. 1355-1387, 1975.

[3] S. K. Leung-Yan-Cheong, M. E. Hellman, “The Gaussian wire-tap channel,” IEEE Trans Inf Theory, vol. IT-24, no. 4, pp. 451-456, July 1978.

[4] I. Csiszar and J. Korner, “Broadcast channels with confidential messages,” IEEE Trans Inf Theory, vol. IT-24, no. 3, pp. 339-348, May 1978.

[5] J. Korner and K. Marton, “General broadcast channels with degraded message sets,” IEEE Trans Inf Theory, vol. IT-23, no. 1, pp. 60-64, January 1977.

[6] R. Liu, I. Marie, P. Spasojevic and R.D Yates, “Discrete memoryless interference and broadcast channels with confidential messages: secrecy rate regions,” IEEE Trans Inf Theory, vol. IT-54, no. 6, pp. 2493-2507, Jun. 2008.

[7] J. Xu, Y. Cao, and B. Chen, “Capacity bounds for broadcast channels with confidential messages,” IEEE Trans Inf Theory, vol. IT-55, no. 6, pp. 4529-4542. 2009.

[8] Y. Liang and H. V. Poor, “Multiple-access channels with confidential messages,” IEEE Trans Inf Theory, vol. IT-54, no. 3, pp. 976-1002, Mar. 2008.

[9] E. Tekin and A. Yener, “The Gaussian multiple access wire-tap channel,” IEEE Trans Inf Theory, vol. IT-54, no. 12, pp. 5747-5755, Dec. 2008.

[10] E. Tekin and A. Yener, “The general Gaussian multiple access and two-way wire-tap channels: Achievable rates and cooperative jamming,” IEEE Trans Inf Theory, vol. IT-54, no. 6, pp. 2735-2751, June 2008.

[11] E. Ekrem and S. Ulukus, “On the secrecy of multiple access wiretap channel,” in Proc. Annual Allerton Conf. on Communications, Control and Computing, Monticello, IL, Sept. 2008.

[12] Raef Bassily and Sennur Ulukus, “A New Achievable Ergodic Secrecy Rate Region for the Fading Multiple Access Wiretap Channel,” in Proc. Annual Allerton Conf. on Communications, Control and Computing, Monticello, IL, Sept. 2009.

[13] Moritz Wiese and Holger Boche, “An Achievable Region for the Wiretap Multiple-Access Channel with Common Message,” Proceedings of 2012 IEEE International Symposium on Information Theory, 2012.

[14] Xiang He, Ashish Khisti, and Aylin Yener, “MIMO Multiple Access Channel With an Arbitrarily Varying Eavesdropper: Secrecy Degrees of Freedom,” IEEE Trans Inf Theory, vol. IT-59, no. 8, pp. 4733-4745, 2013.

[15] Peng Xu, Zhiguo Ding, and Xuchu Dai, “Rate Regions for Multiple Access Channel With Conference and Secrecy Constraints,” IEEE Trans Inf Forensics and Security, vol. 8, no. 12, pp. 1961-1974, 2013.

[16] Y. Liang, A. Somekh-Baruch, H. V. Poor, S. Shamai, and S. Verdu, “Capacity of cognitive interference channels with and without secrecy,” IEEE Trans Inf Theory, vol. IT-55, pp. 604-619, 2009.

[17] L. Lai and H. El Gamal, “The relay-eavesdropper channel: cooperation for secrecy,” IEEE Trans Inf Theory, vol. IT-54, no. 9, pp. 4005-4019, Sep. 2008.

[18] Y. Oohama, “Coding for relay channels with confidential messages,” in Proceedings of IEEE Information Theory Workshop, Australia, 2001.

[19] E. Ekrem and S. Ulukus, “Secrecy in cooperative relay broadcast channels,” IEEE Trans Inf Theory, vol. IT-57, pp. 137-155, 2011.

[20] G. Kramer, M. Gastpar and P. Gupta, “Cooperative strategies and capacity theorems for relay networks,” IEEE Trans Inf Theory, vol. IT-51, pp. 3037-3063, 2005.

[21] Y. Liang, H. V. Poor and S. Shamai, “Secure communication over fading channels,” IEEE Trans Inf Theory, vol. IT-54, pp. 2470-2492, 2008.
[22] T. M. Cover and A. El Gamal, “Capacity theorems for the relay channel,” IEEE Trans Inf Theory, vol. IT-25, pp. 572-584, 1979.

[23] K. Marton, “A coding theorem for the discrete memoryless broadcast channel,” IEEE Trans Inf Theory, vol. IT-25, pp. 306-311, 1979.

[24] A. A. El Gamal and E. C. van der Meulen, “A proof of Marton’s coding theorems for the discrete memoryless broadcast channel,” IEEE Trans Inf Theory, vol. IT-27, pp. 120-122, 1981.

[25] L. Sankaranarayanan, G. Kramer and N. B. Mandayam, “Capacity theorems for the multiple-access relay channel,” Proceedings of Allerton Conference on Communications, Control and Computing, 2004.

[26] X. Tang, R. Liu, P. P. Spasojevic and H. V. Poor, “Interference assisted secret communication,” IEEE Trans Inf Theory, vol. IT-57, pp. 3153-3167, 2011.