Constraining properties of neutron stars with heavy-ion reactions in terrestrial laboratories

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Abstract. Heavy-ion reactions provide a unique means to investigate the equation of state (EOS) of neutron-rich nuclear matter, especially the density dependence of the nuclear symmetry energy \(E_{\text{sym}}(\rho)\). The latter plays an important role in understanding many key issues in both nuclear physics and astrophysics. Recent analyses of heavy-ion reactions have already put a stringent constraint on the \(E_{\text{sym}}(\rho)\) around the saturation density. This subsequently allowed us to constrain significantly the radii and cooling mechanisms of neutron stars as well as the possible changing rate of the gravitational constant \(G\).

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1. Introduction

Within the parabolic approximation which has been verified by all many-body theories to date, the EOS of isospin asymmetric nuclear matter can be written as 

\[ E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \]

where \(\delta \equiv (\rho_n - \rho_p)/(\rho_p + \rho_n)\) is the isospin asymmetry and \(E_{\text{sym}}(\rho)\) is the density-dependent nuclear symmetry energy. The latter is very important for understanding many interesting astrophysical problems [1, 2], the structure of radioactive nuclei [3, 4], and the dynamics of heavy-ion reactions [5, 6, 7, 8].

2. Constraining the density dependence of the nuclear symmetry energy in terrestrial nuclear laboratories

Considerable progress has been made recently in determining the density dependence of the nuclear symmetry energy at sub-normal densities in terrestrial nuclear laboratories. Shown in Fig. 1 are several typical theoretical model predictions [9, 10, 11] compared with the constraints obtained from model analyses of experimental data (those labeled...
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Figure 1. Left panel: Density dependence of the nuclear symmetry energy using the MDI interaction with $x = 0$ and $x = -1$ and other many-body theories predictions (taken from [22]). Right panel: symmetry energies obtained from 21 sets of Skyrme interactions and the MDI interaction with $x = -1$ and $x = 0$ [19].

$x = 0$, $x = -1$ and FSU-Gold). The constraints labeled $x = 0$ and $x = -1$ were extracted recently from studying isospin diffusion in the reaction of $^{124}$Sn + $^{112}$Sn at $E_{\text{beam}}/A = 50$ MeV within a transport model [12, 13, 14, 15] using the MDI interaction [16]. For this particular reaction the maximum density reached is about $1.2 \rho_0$. The constraints are thus only valid below this density. Moreover, it was shown that the neutron-skin thickness in $^{208}$Pb calculated within the Hartree-Fock approach using the same underlying Skyrme interactions as the ones labeled $x = 0$ and $x = -1$ is consistent with the available experimental data [17, 18, 19]. The symmetry energy labeled as FSU-Gold was calculated within a Relativistic Mean Field Model (RMF) using a parameter set such that it reproduces both the giant monopole resonance in $^{90}$Zr and $^{208}$Pb, and the isovector giant dipole resonance of $^{208}$Pb [20]. These results all together represent the best phenomenological constraints available on the symmetry energy at sub-normal densities. The various predictions at supra-normal densities still diverge widely. Hopefully, nuclear reactions with high energy radioactive beams will allow us to pin down the high density behavior soon using several probes predicted recently [21].

The available constraints on the symmetry energy limit the nuclear effective interactions in nuclear matter. This can be seen by comparing them with the symmetry energies obtained from Skyrme effective interactions [19]. The right panel of Fig. 1 displays the density dependence of $E_{\text{sym}}(\rho)$ for 21 sets of Skyrme interaction parameters that are currently used widely in nuclear structure studies. Surprisingly, most of these effective interactions lead to symmetry energies rather inconsistent with the constraints discussed above.
3. Constraining the radii and cooling mechanisms of neutron stars

While the maximum mass of neutron stars is mainly determined by the incompressibility of symmetric nuclear matter, their radii are primarily determined by the isospin asymmetric pressure that is proportional to the slope of the symmetry energy \( E'_\text{sym}(\rho) \).

For the simplest case of a neutron-proton-electron (npe) matter in neutron stars at \( \beta \) equilibrium, the pressure is given by

\[
P(\rho, \delta) = P_0(\rho) + P_{\text{asy}}(\rho, \delta) = \rho^2 \left( \frac{\partial E}{\partial \rho} \right)_\delta + \frac{1}{4} \rho_e \mu_e
\]

\[
= \rho^2 \left[ E'(\rho, \delta = 0) + E'_\text{sym}(\rho) \delta^2 \right] + \frac{1}{2} \delta (1 - \delta) \rho E_{\text{sym}}(\rho),
\]

where the electron density is \( \rho_e = \frac{1}{2} (1 - \delta) \rho \) and the chemical potential is \( \mu_e = \mu_n - \mu_p = 4 \delta E_{\text{sym}}(\rho) \). The equilibrium value of \( \delta \) is determined by the chemical equilibrium and charge neutrality conditions, i.e., \( \delta = 1 - 2x_p \) with \( x_p \approx 0.048 \left[ E_{\text{sym}}(\rho)/E_{\text{sym}}(\rho_0) \right]^3 (\rho/\rho_0) (1 - 2x_p)^3 \). Because of the large \( \delta \) value in neutron stars, the electron degenerate pressure is small. Moreover, the isospin symmetric contribution to the pressure is also very small around normal nuclear matter density as \( E'(\rho_0, \delta = 0) = 0 \). The pressure is thus dominated by the term proportional to the slope of the symmetry energy. Since neutron star radii are determined by the pressure at moderate densities where the proton content of matter is small, they are very sensitive to the slope of the symmetry energy near and just above \( \rho_0 \). In particular, a stiffer symmetry energy is expected to lead to a larger neutron star radius.

![Figure 2](image-url)

**Figure 2.** Left panel: mass and proton fraction of neutron star and the symmetry energy as functions of density. Right panel: correlations in neutron star masses and radii [18]. All results are for spherically-symmetric, non-rotating, non-magnetized neutron stars consisting of npe\( \mu \) matter at zero temperature.

The dependence of some basic properties of neutron stars on the nuclear EOS is shown in the left panel of Fig. 3. The top and middle panels give, respectively, the mass of neutron star and its proton fraction \( x_p \), calculated from the MDI interaction with \( x = 0, -1, \) and \(-2\), as functions of its central density. The symmetry energies obtained from these interactions are shown in the lower panel together with that from the AV18+\( \delta V+\)UIX\(^*\) interaction of Akmal et al. (APR) [23]. It is interesting to see that up to about \( 5\rho_0 \) the symmetry energy predicted by APR agrees very well with that
from the MDI interaction with $x = 0$. For $x_p$ below 0.14, the direct URCA process for fast cooling of proto-neutron stars does not proceed because energy and momentum conservation cannot be simultaneously satisfied. For EOSs from the MDI interaction with $x = -1$ and $x = -2$, the condition for direct URCA process is fulfilled for nearly all neutron stars above 1 M$_{\odot}$. For the EOS from $x = 0$, the minimum density for the direct URCA process is indicated by the vertical dotted line, and the corresponding minimum neutron star mass is indicated by the horizontal dotted line. While this indicates that neutron stars with masses above 1.39 M$_{\odot}$ have a central density above the threshold for the direct Urca process, this conclusion may be modified by the presence of terms in the symmetry energy which are quartic in the isospin asymmetry at high density.

The relations between neutron star masses and radii for above EOSs are given in the right panel of Fig. 3. Also given are the constraints due to causality as well as the mass-radius relations from estimates of the crustal fraction of the moment of inertia ($\Delta I/I = 0.014$) in the Vela pulsar and from the redshift measurement from Ref. 26. Allowed equations of state should lie to the right of the causality line and also cross the other two lines. The hatched regions are inferred limits on the radius and the radiation radius (the value of the radius which observed by an observer at infinity) defined as $R_{\infty} = R/\sqrt{1 - 2GM/Rc^2}$ for a 1.4 M$_{\odot}$ neutron star. It is seen that the symmetry energy affects strongly the radius of a neutron star but only slightly its maximum mass. These analyses have led to the conclusion that only radii between 11.5 and 13.6 km (or radiation radii between 14.4 and 16.3 km) are consistent with the EOSs from the MDI interaction with $x = 0$ and $x = -1$ and thus with the terrestrial nuclear laboratory data. The observational determination of the neutron star radius from the measured spectral fluxes relies on a numerical model of the neutron star atmosphere and uses as inputs the composition of the atmosphere, a measurement of the distance, the column density of x-ray absorbing material, and the surface gravitational redshift. Since many of these quantities are difficult to measure, only a paucity of radius measurements are available. Nevertheless, it is interesting to note that the calculations shown in Fig. 3 are consistent with recent observations including the very rapidly rotating pulsar discovered recently.

4. Constraining the changing rate of the gravitational constant G

Testing the constancy of the gravitational constant G has been a longstanding fundamental question in natural science. As first suggested by Jofré, Reisenegger and Fernández, Dirac’s hypothesis of a decreasing gravitational constant $G$ with time due to the expansion of the Universe would induce changes in the composition of neutron stars, causing dissipation and internal heating. Eventually, neutron stars reach their quasi-stationary states where cooling due to neutrino and photon emissions balances the internal heating. As shown in ref. 30, the stationary surface temperature is directly related to the relative changing rate of $G$ via $T_s^\infty = \tilde{D} \left| \frac{\Delta G}{G} \right|^{2/7}$, where the function $\tilde{D}$ is a quantity depending only on the stellar model and the equation of state.
The correlation of surface temperatures and radii of some old neutron stars may thus carry useful information about the changing rate of $G$. Using the constrained symmetry energy with $x = 0$ and $x = -1$ shown in Fig.1 within the gravitochemical heating formalism, as shown in Fig.3 we obtained an upper limit of the relative changing rate of $|\dot{G}/G| \leq 4 \times 10^{-12} yr^{-1}$. This is the best available estimate in the literature\[32]. For a comparison, results with the EOS from the recent Dirac-Brueckner-Hartree-Fock (DBHF)\[35] calculations using the Bonn B potential are also shown. The Bonn B potential gives roughly the same result on the stationary surface temperature but slightly larger radius compared to the $x = 0$ case.

5. Summary

In summary, the symmetry energy of neutron-rich matter is fundamentally important for both nuclear physics and astrophysics. Available data from heavy-ion reactions in terrestrial laboratories allowed us to constrain the symmetry energy at sub-saturation densities. This has led to stringent constraints on not only the nuclear effective interactions but also the cooling mechanisms, radii of neutron stars and the changing rate of the gravitational constant $G$.

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