Chiral Supersymmetric Standard Model Spectra from Orientifolds of Gepner Models

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Abstract

We construct $d = 4, N = 1$ orientifolds of Gepner models with just the chiral spectrum of the standard model. We consider all simple current modular invariants of $c = 9$ tensor products of $N = 2$ minimal models. For some very specific tensor combinations, and very specific modular invariants and orientifold projections, we find a large number of such spectra. We allow for standard model singlet (dark) matter and non-chiral exotics. The Chan-Paton gauge group is either $U(3) \times Sp(2) \times U(1) \times U(1)$ or $U(3) \times U(2) \times U(1) \times U(1)$. In many cases the standard model hypercharge $U(1)$ has no coupling to RR 2-forms and hence remains massless; in some of those models the $B-L$ gauge boson does acquire a mass.

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1 Introduction

There are many possible ways in which the Standard Model might emerge from String Theory. One of them is a standard gauge unification scenario using the Heterotic string as a starting point. Another broad class, with particular advantages described extensively in many papers [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14], is through intersecting stacks of branes. In both cases an important issue is to find various kinds of (semi)-realistic examples. Here we will focus on a class of examples that was rather difficult to obtain so far, namely supersymmetric spectra that satisfy all the tadpole cancellation conditions.

There are some results [5, 19, 20] in this area using orientifolds of toric orbifolds [21, 22], but we want to consider here internal CFTs that are non-trivial [23, 24].

We consider all $d = 4, N = 1$ simple current orientifolds of Gepner models (see [25, 26, 27, 28, 29, 30] for some specific cases including chiral spectra.) At this point in moduli space, the underlying $d = 2$ conformal field theory is a $c = 9$ tensor combination of $N = 2$ minimal models. These CFTs are rational, so the results of [31] apply to construct and classify $\alpha'$-exact orientifold vacua.

In this set of M-theory vacua, we are performing a systematic search for finite, perturbatively stable points with just the chiral spectrum of the standard model. We allow for non-chiral matter in standard model gauge group representations, and for any matter in hidden gauge group representations. The total Chan-Paton group includes $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$ or $U(3)_a \times Sp(2)_b \times U(1)_c \times U(1)_d$. The standard model particles are basically realized as in $\mathbb{3}$.

In section 2 we review the material presented in [31] and formulate our precise search criteria. In section 3 we report our results. For some tensor combination of minimal models, like $(6, 6, 6, 6)$, we have found thousands of inequivalent spectra satisfying our conditions. In order to avoid endless tables, we will briefly discuss a few spectra that are chosen from this set.

2 Orientifolds of Simple Current Gepner Models

A fundamental property of D-branes and O-planes is that they are defined in terms of boundary conditions for strings [32]. This means that these objects can be studied by CFT methods, in which the relevant objects are called boundary and crosscap states. These encode the brane/plane tension and RR charges, as well as the perturbative spectrum of string vibrations in a orientifold vacuum.

Based on earlier work of [33, 34, 35, 36, 37], a general formula was presented in [31] for the boundary and crosscap states for a large class of rational CFTs. This class is non-trivial in any RCFT that has simple currents. Such currents are present in abundance in

\[\text{For reviews consult [15, 16, 17, 18].}\]
many cases of interest, and in particular in (tensor products of) $N = 2$ minimal models. In [38, 39] a classification of such invariants was obtained. In [31] a description was given of the simple current based orientifolds of these invariants. Here we consider all these simple current invariants and all orientifolds described, applied to the $c = 9$ tensor products of $N = 2$ minimal models [40].

The “internal” CFT built out of $N = 2$ minimal models is tensored with the space-time NSR sector. We view all factors in the tensor product as non-supersymmetric CFTs. In order to obtain a CFT with global world sheet supersymmetry, the chiral algebra must be extended; a second extension is needed to obtain $N = 1$ supersymmetry in target space. In order to describe these extensions it is convenient to replace the NSR sector by a bosonic CFT, namely a $SO(10)$ level one affine Lie algebra, using the “bosonic string map” (see [41] and references therein, and [42, 43] for recent applications of this method). Hence we consider

$$\mathcal{A}_{\text{tensor}} = D_{5,1} \otimes \bigotimes_{i=1}^{r} A_{k_i}$$

where $A_k$ is the $N = 2$ minimal model at level $k$ with conformal anomaly

$$c_k = \frac{3k}{k+2}.$$  

The constraint $\sum_{i} c_{k_i} = 9$ leads to 168 inequivalent tensor products $\mathcal{A}_{\text{tensor}}$. In each factor we denote the supercurrent as “$v$” (in the NSR sector this is actually a vector that acquires conformal weight $3/2$ by multiplication with $\partial X^\mu$); in each factor this is a simple current. Furthermore each factor contains two Ramond simple currents, which we denote as “$s$” and “$c$”. In order for $\mathcal{A}_{\text{tensor}}$ to be $N = 2$ world-sheet supersymmetric, we need to extend the algebra by the fermion allignment simple currents $(v, v, 0, 0, ...), (v, 0, v, 0, ...), ...$. The resulting algebra is called $\mathcal{A}_{\text{ws}}$. As a result of this extension, all primaries of $\mathcal{A}_{\text{ws}}$ are either in the Ramond (R) or Neveu-Schwarz (NS) sector. Space-time supersymmetry is obtained when we extend by the spectral flow simple current $(s, s, s, s, ...)$ that relates the R and NS sector. This extension is called $\mathcal{A}_{\text{st}}$ and is our starting point. Typically it has several thousands primaries, a few tens or hundreds of which are simple currents. We consider all symmetric simple current invariants generated by the formula of [39]. These invariants may be of automorphism type, or of extension type, or any combination thereof. The heterotic string spectra of these CFTs and their modular invariant partition functions (MIPFs) have been intensively scanned in the past [44, 45, 46, 47, 48, 49, 50] resulting in a large number of spectra with three chiral families only for the combination $(1,16,16,16)$ [51] and some of its modular invariants [46].

In all cases we consider the complete set of boundaries. In the case of extension modular invariants this includes boundaries that do not respect the extended symmetries, as required to fulfill the completeness condition of [32]. Note however, that all boundaries and crosscaps respect the symmetries of $\mathcal{A}_{\text{st}}$, and in particular the same copy of $N = 1$ target space supersymmetry. The formalism could equally well be applied to subalgebras of $\mathcal{A}_{\text{st}}$, such as $\mathcal{A}_{\text{ws}}$, in which case we would be able to consider models with brane supersymmetry breaking [53]. However, due to the huge number of primaries and boundaries this is computationally more challenging, and will not be considered here.
An important ingredient in these computations is the resolution of simple current fixed points that occur for all even values of \( k \). This enters the computation at two points, namely for obtaining the modular \( S \)-matrix of \( A_{st} \), and in the computation of the boundary coefficients for non-trivial MIPFs. A general formula for \( S \) for simple current extended tensor products and coset CFTs was derived in [54]. In addition to this we need a formula for the simple current fixed point resolution matrices \( S^J \) for all simple currents \( J \) of \( A_{st} \). This formula was derived in [55]. In the case we consider here, all these fixed point resolution matrices are of course related to those of \( SU(2) \) level \( k \), which is just a number, but there are several non-trivial phases to keep track of, that originate from field identification in the minimal models, and the extensions that lead to \( A_{st} \). Once these matrices are available, all cases are equally easy to deal with as the “Cardy case” (the charge conjugation modular invariant).

We start by reviewing the construction of Simple Current Gepner Models. Then we present a canonical class of boundary and crosscap states for these theories and write down the tadpole conditions. The spectrum can then be calculated as reviewed in [33, 56, 57]. At the end of this section we formulate our search criteria.

The following notation is understood. Chiral primaries are denoted by \( i, j \) and their characters by \( \chi_i \) and conformal weight by \( h_i \). The superscript in \( i^c \) denotes \( d = 2 \) charge conjugation. Simple currents are denoted by \( J, K, L \) and their order by \( N_J, \ldots \). The monodromy charge of \( i \) with respect to \( J \) is \( Q_J(i) \). Simple current groups are denoted like \( H \) and the number of elements by \( |H| \). The rest of our notation will be explained in the text.

### 2.1 Simple Current Gepner Models

A single \( N = 2 \) minimal model has simple current group \( G_k = \mathbb{Z}_{4k} \) when \( k \) odd and \( \mathbb{Z}_{2k} \times \mathbb{Z}_2 \) when \( k \) even. The extended algebra \( A_{st} \) has a remaining simple current group \( G_{st} \), whose structure depends on the details of the model. For every subgroup \( ^6H \in G_{st} \), and a matrix \( X \), defined modulo integers, that obeys

\[
X(J, K) + X(K, J) = Q_J(K) \mod 1, J \neq K
\]

\[
X(J, J) = -h_J \mod 1
\]

plus the constraints \( N_J X(J, K) \in \mathbb{Z}, X(J, K)N_K \in \mathbb{Z}, \) we can define string vacua with modular invariant torus partition function

\[
Z(H, X) = \sum_{i,j} \chi_i \chi_j c Z_{ij},
\]

where \( Z_{ij} \) is the number of currents \( J \in H \) such that

\[
j = Ji
\]

\[
Q_K(i) + X(K, J) = 0 \mod 1
\]

\(^6\)In addition, all elements of \( H \) must satisfy the condition that spin times order is integral.
for all \( K \in \mathcal{H} \). In this language, the ordinary “Gepner model” corresponds to the choice \( \mathcal{H} = \{0\}, X = 0 \), i.e. the charge conjugation invariant of \( A_{st} \). The number of invariants obtained in this way grows rapidly with the number of cyclic factors in \( G_{st} \).

2.2 Boundary and Crosscap States

We now present the results of [31] in a slightly modified form that is more suitable for our purposes. This only involves some reshuffling of phase factors in the coefficients; the (open and closed) string partition functions are identical to those in [31]. Like in [31], we label the Ishibashi states of (5) by pairs \((m,J)\) that obey

\[
\begin{align*}
m &= Jm , \\
Q_K(m) + X(K,J) &= 0 \mod 1
\end{align*}
\]

for all \( K \in \mathcal{H} \). The boundary labels \([a,\psi_a]\) are \( \mathcal{H} \)-orbits \([a]\) of a chiral sector \( a \). We also need a boundary degeneracy label \( \psi_a \). It is a discrete group character of the central stabilizer \( C_a \) (see below). The boundary states are determined by boundary coefficients. In the simple current case, these are

\[
R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|C_a||S_a|}} \psi_a^*(J) S^J_{am}.
\]

The fixed point resolution matrix \( S^J \), whose rows and columns are labelled by fixed points \( a,m \) of \( J \), implements a modular \( S \)-transformation on the torus with \( J \) inserted. It is unitary and obeys

\[
S^J_{Ki,j} = F_i(K,J)e^{2\pi iQ_K(j)}S^J_{ij}.
\]

The phase \( F \) is called the simple current twist. We can now define the central stabilizer as

\[
C_a = \{J \in S_a | F_a(K,J)e^{2\pi iX(K,J)} = 1 \text{ for all } K \in S_a\}.
\]

For more details we refer to [31]. In contrast to [31] these boundary coefficients are the same for all orientifold choices, and are in fact also valid for oriented strings and non-symmetric modular invariants. They can be used to compute oriented annulus coefficients, defined as

\[
A^I_{[b,\psi_b]} = \sum_{m,J} S^I_{m} R_{[a,\psi_a](m,J)}(R_{[b,\psi_b](m,J)})^*.
\]

Now we have to introduce orientifold choices, and to do so we restrict ourselves to symmetric symbolic invariants. This implies that we only consider symmetric matrices \( X \). The orientifold choice enters into the formalism in two ways, namely through the crosscap coefficients and the definition of the unoriented Annulus. The allowed choices are as follows. One must select

1. A Klein bottle current \( K \). This can be any simple current of \( A \) that is local with all order two currents in \( \mathcal{H} \). Only odd currents outside \( \mathcal{H} \) can give spectra that are inequivalent to those with \( K = 0 \). See [57] for details.
2. A set of phases $\beta(J)$ for all $J \in \mathcal{H}$ that satisfy

$$\beta(J)\beta(J') = \beta(JJ')e^{2\pi i X(J,J')} \quad J, J' \in \mathcal{H}$$

with $\beta(0) = e^{\pi i h_K}$.

In the latter case the freedom is due to that fact that for every even cyclic factor in $\mathcal{H}$ a sign remains undetermined by this condition. The crosscap coefficient of the orientifold $(A_{st}, \mathcal{H}, X, K, \beta)$ is

$$U_{\Omega(m, J)} = \frac{1}{\sqrt{|\mathcal{H}|}} \sum_{L \in \mathcal{R}} \sigma(L)P_{LK,m}\delta_{J,0} \quad \sigma(L) := \beta(L)e^{\pi i [h_{LK} - h_K]} .$$

Here $\Omega$ is a generic notation for the possible orientifold choices. One can show that the $\sigma(L)$ are signs. The matrix $P = \sqrt{TST^2S}\sqrt{T}$ [34]. The unoriented annulus is given by

$$A_{[a, \psi_a][b, \psi_b]}^{\Omega,i} = \sum_{m, J, J'} S^i_m R_{[a, \psi_a](m, J)} g_{J, J'}^{\Omega,m} R_{[b, \psi_b](m, J')} S_{0m}$$

The Ishibashi metric $g^{\Omega,m}$ is defined as

$$g_{J, J'}^{\Omega,m} = \frac{S_{m0}}{S_{mK}} \beta(J)\delta_{J', J^c}$$

The Moebius and Klein bottle amplitude follow from

$$M^i_{[a, \psi_a]} = \sum_{m, J, J'} S^i_m R_{[a, \psi_a](m, J)} g_{J, J'}^{\Omega,m} U_{(m, J')} S_{0m}$$

$$K^i = \sum_{m, J, J'} S^i_m U_{(m, J)} g_{J, J'}^{\Omega,m} U_{(m, J')} S_{0m}$$

The unoriented annuli for the various choices of $\Omega$ can all be derived from the unique oriented annulus [13] by matrix multiplication with the boundary conjugation matrix $A^{\Omega,0}$ that maps a brane $[a, \psi_a]$ to its orientifold image $[a, \psi_a]^c$.

In [57] it is shown that the spectrum is positive and integral. Note that this only determines the boundary and crosscap coefficients up to a common $(m, J)$ dependent sign. Presumably these signs can be determined by solving the sewing constraints, but fortunately they are not relevant for our purposes. In addition integrality is unaffected by the overall sign of the crosscap coefficients. Recently in [58] it was demonstrated that the resulting boundary CFTs are consistent on all orientable surfaces (work on non-orientable surfaces is in progress).
2.3 Tadpole Cancellation

It is our goal to construct stable, finite, supersymmetric four-dimensional string theories. In other words, we will insist on the cancellation of all tadpoles due to both NS-NS and R-R massless scalars. In chiral models, this implies in particular the cancellation of the cubic part of the gauge anomalies [52]. The tadpole cancellation conditions are equations for the Chan-Paton multiplicities $N_{[a,\psi_a]}$, and take the form

$$\sum_{[a,\psi_a]} N_{[a,\psi_a]} R_{[a,\psi_a]}(m,J) = 4\epsilon \eta_m U(m,J)$$  \hspace{1cm} (20)$$

for all Ishibashi labels $(m, J)$ that correspond to massless closed strings in (5). Here $\eta_m = 1$ for $m = 0$, the vacuum, and $\eta_m = -1$ otherwise, and $\epsilon$ is the overall crosscap sign. It is fixed by the dilaton tadpole condition. Note that the aforementioned $(m, J)$ dependent signs cancel in the tadpole equations as well.

In principle one could proceed by solving these equations by computer. This is indeed possible in the six-dimensional case, where we have obtained the complete solution for all orientifolds of all simple current invariants of all $c = 6$ tensor products of $N = 2$ minimal models (see [60] for some special cases.) This is a very useful test of the entire formalism, since anomaly cancellation in six dimensions is a far more powerful constraint than it is in four dimensions.

This method is not feasible in four dimensions, because the number of variables vastly outnumbers the number of conditions. In one of the 168 cases we have been able to do this (for all simple current invariants and orientifolds), namely $(1, 3, 3, 4, 8)$ (which has only 260 primaries, and a chiral spectrum in the Cardy case).

In all other cases we proceed as follows. First we determine a subset of boundaries that produces a desired spectrum, for example the standard model or some of its extensions. Unless one is extremely lucky this set of boundaries and CP multiplicities will not satisfy the tadpole conditions by itself. Therefore we allow additional “hidden” branes. Of course there might be open strings stretching between the standard model and the hidden branes. We will allow such states provided that they are not chiral. The details will be discussed now.

2.4 Chiral Spectrum

There are many conceivable intersecting brane realizations of the Standard Model, but we will aim here for the simplest kind, and in particular the smallest number of branes. We will require that all standard model particles come from strings between different branes (bi-fundamentals), and that baryon and lepton number are conserved perturbatively. This leads almost inevitably to models with four stacks of branes. Following [3] we will label them $a, b, c$ and $d$. The color gauge group $SU(3)$ is associated with brane $a$ and its orientifold image $a^c$, which must produce a Chan-Paton group $U(3)_a$. The weak gauge group $SU(2)$ is associated with brane $b$; this group can either be $U(2)$ or $Sp(2)$; in the latter case $b = b^c$. Branes $c$ and $c^c$ have a $U(1)$ CP-group, as do $d$ and $d^c$. Baryon number
is related to $U(1)_a$, and lepton number to $U(1)_d$. The standard model $Y$-charge is given by $\frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d$.

In table I we summarize all the massless particles with standard model gauge representations that can in principle occur with this brane configuration. We treat all particles as left-handed. The brane representations are denoted as $V$ for vectors, “Adj” for adjoint, “A” for antisymmetric tensor and “S” for symmetric tensor, and a * denotes complex conjugation. The sections of the table denote respectively standard model particles, Higgses, exotics which respect standard model charge quantization, exotics that do not, and hidden matter. In the last column we indicate the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the particles. For representations that do not occur in the standard model we specify, as a subscript, three times baryon number, and lepton number.

To every row $i = 1,...,29$ we associate two non-negative integer multiplicities, $M_i$ and $\bar{M}_i$, where the former is the multiplicity of the particle as shown, and the latter is that of its complex conjugate (of the full representation, including the hidden sector). Of course $M_{16} = \bar{M}_{16}, M_{19} = \bar{M}_{19}$ and $M_{22} = \bar{M}_{22}$. There is a redundancy in the table if the weak group is $Sp(2)$, and hence in that case we can set $M_2 = \bar{M}_2 = M_6 = \bar{M}_6 = M_{16} = 0$ without loss of generality. In the Higgs sector, standard model anomaly cancellation requires that there be an equal number of representations $H_1 = (1,2,-\frac{1}{2})$ and $H_2 = (1,2,\frac{1}{2})$. Hence $M_9 + \bar{M}_{10} = M_{10} + \bar{M}_9 \equiv M_H$. In the case of a weak group $Sp(2)$ the parameters $M_9$ and $\bar{M}_{10}$ are redundant, and hence in that case $M_9 = \bar{M}_{10} = M_H$.

Our requirements on the spectrum are as follows. Define $\Delta_i = M_i - \bar{M}_i$. Then we require that $\Delta_1 + \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 + \Delta_6 = \Delta_7 = \Delta_8 = 3$. We impose no restriction on $\Delta_9$ and $\Delta_{10}$, except that they should be equal as explained above. Note that for $Sp(2)$ $\Delta_9 = \Delta_{10} = 0$. All $\Delta_i$ of the exotic representations, $i = 11,\ldots,28$ are required to vanish. This implies that mass terms are allowed for all exotic representations without breaking the original Chan-Paton group. We are implicitly assuming that such mass terms are indeed generated, so that only the chiral part of the spectrum is accessible with current experiments. Such mass terms may indeed be generated if one moves away from the rational point in the moduli space. We do allow $\Delta_{29} \neq 0$, i.e. the hidden sector may be chiral.

Exotics not respecting charge quantization (nrs. 25 to 28) may occur due to strings stretching between the standard model and the hidden branes. Indeed, the corresponding color singlets always have half-integer electric charge, and hence at least one of them would be stable. There is a variety of ways around this. First of all, these particles may be sufficiently massive and rare to have escaped attention so far; secondly, they may be confined to integer charge hadron like particles by a gauge group from the hidden sector; thirdly, they may simply be absent from the spectrum, and finally the entire hidden gauge sector may be absent, so that they cannot occur at all. Indeed, we found examples of the latter two possibilities, as well as cases where all these strings end on a $U(2)$ or $U(4)$ hidden brane, which are plausible candidates for the second option. On the other hand, one could regard half-integer charge particles as a fairly generic (though not fully general) prediction of this class of models.

In addition to the particles listed in the table there may exist particles that belong
entirely to the hidden sector. We impose no further conditions on this matter. It may in fact even be chiral, although in most cases we have found it is not.

| nr. | $U(3)_a$ | Weak | $U(1)_c$ | $U(1)_d$ | massless particle |
|-----|-----------|-------|-----------|-----------|------------------|
| 1   | $V$       | $V$   | 0         | 0         | $(u, d)$         |
| 2   | $V$       | $V^*$ | 0         | 0         | $(u, d)$         |
| 3   | $V^*$     | 0     | $V$       | 0         | $u^c$           |
| 4   | $V^*$     | 0     | $V^*$     | 0         | $d^c$           |
| 5   | 0         | $V$   | 0         | $V$       | $(\nu, e^-)$    |
| 6   | 0         | $V^*$ | 0         | $V$       | $(\nu, e^-)$    |
| 7   | 0         | 0     | $V$       | $V^*$     | $\nu^c$         |
| 8   | 0         | 0     | $V^*$     | $V^*$     | $e^+$           |
| 9   | 0         | $V$   | $V^*$     | 0         | $H_1$           |
| 10  | 0         | $V$   | $V^*$     | 0         | $H_2$           |
| 11  | $V$       | 0     | 0         | $V$       | $(3,1,-\frac{4}{3})_{1,1}$ |
| 12  | $V$       | 0     | 0         | $V^*$     | $(3,1,\frac{2}{3})_{1,-1}$ |
| 13  | Adj       | 0     | 0         | 0         | $(1,3,0)_{0,0} + (1,1,0)_{0,0}$ |
| 14  | A         | 0     | 0         | 0         | $(3^*,1,\frac{1}{3})_{2,0}$ |
| 15  | S         | 0     | 0         | 0         | $(6,1,\frac{1}{3})_{2,0}$ |
| 16  | 0         | Adj   | 0         | 0         | $(1,3,0)_{0,0} + (1,1,0)_{0,0}$ |
| 17  | 0         | A     | 0         | 0         | $(1,1,0)_{0,0}$ |
| 18  | 0         | S     | 0         | 0         | $(1,3,0)_{0,0}$ |
| 19  | 0         | 0     | Adj       | 0         | $(1,1,0)_{0,0}$ |
| 20  | 0         | 0     | A         | 0         | —               |
| 21  | 0         | 0     | S         | 0         | $(1,1,-1)_{0,0}$ |
| 22  | 0         | 0     | 0         | Adj       | $(1,1,0)_{0,0}$ |
| 23  | 0         | 0     | 0         | A         | —               |
| 24  | 0         | 0     | 0         | S         | $(1,1,-1)_{0,2}$ |
| 25  | $V$       | 0     | 0         | 0         | $(3,1,\frac{1}{3})_{1,0}$ |
| 26  | 0         | $V$   | 0         | 0         | $(1,2,0)_{0,0}$ |
| 27  | 0         | 0     | $V$       | 0         | $(1,1,-\frac{1}{2})_{0,0}$ |
| 28  | 0         | 0     | 0         | $V$       | $(1,1,-\frac{1}{2})_{0,1}$ |
| 29  | 0         | 0     | 0         | 0         | $(1,1,0)_{0,0}$ |

Table 1: List of standard model representations that can appear, and their labelling.

3 Results

In practice, most of the 168 $c = 9$ tensor combinations are accessible by our methods. Indeed, we have been able to analyse 160 combinations under more restrictive conditions than those formulated above. The type of spectra described in the previous section have
so far been searched for in 66 tensor products. In 48 of them the required standard model
brane configuration did not occur for any modular invariant and orientifold choice. In 13
other cases they did occur, but it was possible to show that the tadpole equations have
no solution.

In five cases we did find spectra that satisfy our criteria. At this moment, we have
found such examples for the models listed in table 2. The first column gives the levels
of the minimal models. In order to identify the MIPF, in the second column we list the
hodge numbers and the number of gauge singlets in the corresponding heterotic string
spectrum, for comparison with the tables of [46, 48]. In the third column we give the
number of Ishibashi labels, or equivalently, the number of boundaries. In column four
we specify for how many orientifold choices we have found solutions to all the tadpole
equations satisfying our criteria. For the benefit of the reader we give also the result for
the Cardy case, although no solutions were found. It is amusing to note that in all cases
with solutions \( h_{21} < h_{11} \).

We emphasize that this table is by no means complete. In fact, we expect there to
exist a large, possibly astronomical number of additional solutions. We have only partly
explored the remaining 102 tensor products. In some of them the tadpole conditions are
unsolvable because the number of candidate hidden branes is simply too large.

For all of the cases listed in the table we find many standard model brane configura-
tions, and for many of them a large number of ways of adding hidden sector branes that
saturate the tadpole conditions. For example, for tensor \((6, 6, 6, 6)\), invariant \((3, 59, 223)\)
we have so far identified more than 6000 distinct solutions, without even carefully distin-
guishing all features of the hidden sector, and by only considering the minimal number
of hidden branes. These spectra differ in at least one of the integers \(M_i\) defined above,
and/or in the hidden sector gauge group.

We are still analyzing this enormous set of solutions, and just give here some fairly
randomly chosen examples. We only present the open sector; in addition there are of
course massless particles from the closed sector. All examples we discuss below have a
non-chiral hidden sector.

To specify a model without extra branes it is sufficient to give the integers \(M_i\). A
typical example with an Sp(2) weak gauge group has the following spectrum: quarks
and leptons: \((M_1, M_3, M_4, M_5, M_7, M_8) = (3, 3, 3, 4, 6, 6)\), Adj: \((2, 0, 5, 18)\), A: \((1, 4, 2, 9)\),
S: \((1, 1, 4, 9)\) (these are the number of these representations in each of the four factors)
lepto-quarks: \((M_{11}, M_{12}) = (6, 0)\), Higgs: \(M_9 = M_{10} = 3\). Note that, for example,
\(M_5 = 4\) means that there are four left-handed lepton doublets, and one right-handed one,
since the chiral part of the spectrum is always fixed in the way discussed before. For
the same reason the (anti)-symmetric tensors and lepto-quarks are always non-chiral, \(i.e.
\bar{M}_i = \bar{M}_i\), for \(i = 11 \ldots 24\). This model has three copies of Higgs bosons in the standard
MSSM representation \((1, 2, 1/2) + (1, 2, -1/2)\). If there are extra hidden branes there is
usually a large number of options. A simple example with just a single extra \(U(1)\)-brane
and no SM-hidden bifundamentals of types \(25 \ldots 28\) has the following characteristics:
\((M_1, M_3, M_4, M_5, M_7, M_8) = (5, 5, 5, 7, 5, 5)\), two times \(H_1 + H_2\), two lepto-quarks of each
charge \((M_{11}, M_{12}) = (2, 2)\), and the following multiplicities for the rank-2 fields in the five
groups: Adj: (2,0,2,8,1); S: (1,1,5,2); A: (2,1,2,4,0).

Another example has no mirror quarks and leptons at all, but two extra CP groups $U(9) \times U(1)$ coupling to non-chiral matter of types 25...29. We will not present here the rather large numbers of non-chiral matter in (anti)-symmetric tensors and adjoints, which appears to be a generic feature of these spectra.

Among the models with a weak symmetry group $U(2)$ there are cases with $(M_1,\ldots,M_8) = (1,2,3,3,3,3,3,3)$, but unfortunately no Higgs at all. In this case there are no mirror quarks and leptons, so that the quark/lepton spectrum is exactly that described in [3]. There are additional gauge groups and non-chiral rank-2 fields. There are other $U(2)$ cases with a few mirror quarks or leptons and some Higgs bosons with vanishing brane chirality ($\Delta_9 = \Delta_{10} = 0$). We will not give more examples here; at the moment an important challenge is how to select the most attractive ones from the huge list.

| tensor      | $(h_{21}, h_{11}, S)$ | Boundaries | Orientifolds |
|-------------|-----------------------|------------|--------------|
| (6,6,6,6)   | (149,1,503)           | 9632       | —            |
|             | (5,69,267)            | 400        | 2            |
|             | (9,41,211)            | 800        | 2            |
|             | (3,59,223)            | 368        | 4            |
|             | (5,37,203)            | 368        | 4            |
|             | (3,43,207)            | 400        | 1            |
|             | (17,25,203)           | 1136       | 1            |
| (3,8,8,8)   | (145,1,495)           | 9200       | —            |
|             | (11,47,283)           | 880        | 1            |
| (4,6,6,10)  | (66,6,281)            | 1540       | —            |
|             | (14,38,229)           | 416        | 1            |
| (4,4,10,10) | (128,2,443)           | 7200       | —            |
|             | (10,64,229)           | 406        | 1            |
| (2,5,12,26) | (116,8,453)           | 6006       | —            |
|             | (23,59,327)           | 780        | 1            |
|             | (23,59,327)           | 858        | 1            |

Table 2: Modular invariants for which chiral SSM were found so far. The last column gives the number of distinct orientifold choices which have solutions.

4 Conclusion

The main goal of this paper is to point out that large numbers of vacua with just the chiral standard model spectra can be obtained from orientifolds of non-toric Calabi-Yau compactifications. Since these CFTs correspond to special points in a multi-dimensional moduli space, we focused here on features that are most robust under changes of the moduli: the chiral spectrum. Even with the very limited search we have done so far the
number of solutions is enormous, and more detailed phenomenological input would be needed to reduce this to a more manageable set.

Most examples found so far have an $Sp(2)$ weak gauge group and a large quantity of non-chiral additional matter. We did find examples with a $U(2)$ weak gauge group, but so far they all had Higgses with zero brane chirality. There are examples without hidden branes, without any mirror quarks and leptons, without any half-integer charge exotic matter (even though there is a hidden sector) and examples with hidden sectors that are capable of confining the half-integer charges to integer charges.

We have limited ourselves here to two simple and attractive brane realizations of the standard model. Still more solutions would undoubtedly be found if we allow realizations of the standard model gauge group in larger Chan-Paton groups. On the other hand, with better $a$ priori constraints one could do a dedicated search for models with such desirable features.

The intrinsic limitation of RCFT methods is that one is working on a given point in moduli space. There are many phenomenological issues that could be discussed, but for many of them this restriction is important. For example, three point couplings (in particular fermion-Higgs couplings) are computable in principle in RCFT, but are also modulus dependent. Therefore we see these results primarily as a guide to interesting regions in CY-moduli space.

One issue that can be discussed in RCFT and may remain valid beyond it is the mass of $U(1)$ gauge bosons. In general, $U(1)$ mixed anomalies are of the form $\text{Tr}F_a\text{Tr}(F_b)^2$. They are cancelled by a Green-Schwarz mechanism involving couplings of RR two-form fields to $\text{Tr}(F_b)^2$ and $\text{Tr}F_a$. The latter kind of couplings give masses to $U(1)$ gauge bosons. They must be present for anomalous $U(1)$’s, but may also be present for anomaly-free ones. In our case Baryon and Lepton number are anomalous, but $B - L$ and of course $Y$ are not. We have worked out the coupling of these gauge bosons to the two-form fields and found that in a surprisingly large number of cases all such couplings vanish, for both $B - L$ and $Y$. This includes spectra without hidden branes. This implies that both gauge bosons have zero mass, and that a mass for the $B - L$ gauge boson has to be generated by some other mechanism in order for these spectra to be acceptable. There are also cases where both the $B - L$ and the $Y$ gauge boson have non-zero mass, or only one of the two. In particular we have examples with vanishing $Y$-mass, and non-vanishing $B - L$ gauge boson mass. Some of these examples have no mirror quarks and leptons, but they do have additional gauge groups and non-chiral exotic matter.

We have not yet done a complete analysis of all abelian gauge boson masses in all models we have found so far, and in addition we expect a large number of additional cases to appear when we explore the remaining tensor products. The results of a more complete survey will be presented in a forthcoming publication.

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