The Maximum Detectable Momentum for cosmic ray muons in the MINOS far detector

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Abstract: A magnetic detector such as MINOS which is measuring the sign of muons has to deal with issues of bending, which depend on the magnetic field configuration, and multiple scattering, which depends on the amount of material which is traversed. Above some momentum which depends on these factors, the momentum cannot be resolved. Issues related to measurement of the muon charge ratio in MINOS are discussed.

Introduction

The MINOS detector was designed to measure neutrinos produced in a beamline at Fermilab starting with protons at the 120 GeV Main Injector (NuMI). The far detector is located 735 km from the NuMI target and in the Soudan Underground Laboratory 710 m underground. The detector consists of 486 2.5 cm octagonal iron plates perpendicular to the NuMI beam, arranged in two “supermodules”. There are 484 1 cm thick scintillator planes with corresponding 2.5 cm wide air gaps. The scintillator strips are 4.1 cm wide and 3.4 - 8.0 m long. The strips are oriented at ± 45° to the (vertical) y axis, in orthogonal “u” and “v” planes. The steel has been magnetized in a toroidal configuration to an average value of 1.3 T. The field is saturated at 1.8 T near the central coil hole and falls to 1 T near the octagon edges.

Since MINOS studies a beam with many more neutrinos than antineutrinos, the magnet polarity is chosen to focus $\mu^-$ coming from the Fermilab direction. A coordinate system along this beamline and detector is shown in Figure 1. The z axis is the central axis of the detector, which is also the approximate direction of the Fermilab neutrino beam. MINOS has measured the cosmic ray muon charge ratio underground [1]. MINOS can only measure the charge of muons that have less than the maximum detectable momentum (MDM) at the detector. Above that momentum, muon tracks are too straight to measure charge. For tracks which do not exit the detector in z, the MDM only depends on the impact parameter b, which is the distance of closest approach to the axis (z axis) of the magnet, and $\theta_z$, the angle with respect to the z-axis. As discussed in Reference [1], the full magnetic field map, reconstruction software and detector geometry are used to track muons. In this note, certain simplifying assumptions have been made to qualitatively illustrate certain features of the detector and the MDM.

The calculation of the Maximum Detectable Momentum

We define the Maximum Detectable Momentum (MDM) as that momentum for which a nearly straight real track will have a measured curvature (determined from a fit to points along the track) which is one standard deviation from zero. The MDM is simply the reciprocal of the error (s.d.) of the curvature measurement, when the curvature is expressed in $(GeV/c)^{-1})$. The essential features of the magnetic response to cosmic rays for the MINOS detector can be characterized by considering a detector which is a right circular cylinder of radius 4m and length 29 m divided into two supermodules. The radiation length, for multiple coulomb scattering, is 3.83 cm. The magnetic
field is taken to be uniform (in the steel and air), azimuthal and 0.6 tesla. All throughgoing tracks which do not enter or leave the front or back of the detector curve one way for half of their trajectory in MINOS and the other way for the other half. This will be referred to as an S shaped track. More than 95% of cosmic ray muons in MINOS are S shaped tracks. This contrasts with charged tracks along the beam direction, which are C shaped, as long as they don’t cross the center. The relevant curvature is thus measured twice along half of the track length. The approximate length of a half-track is:

$$L = \sqrt{(R^2 - b^2)/\sin(\theta_z)}$$  \(1\)

where \(R = 4\ m\) is the detector Radius, \(b\) is the impact parameter and varies from -4 m to 4 m, and \(\theta_z\) is the angle along the z axis or beam axis (it is not the cosmic ray zenith angle).

To calculate the precision of momentum measurement, we need the component of the magnetic field which is perpendicular to the track direction. We approximate this by finding \(B\) at the midpoint of the half-track and then resolving it into components parallel and perpendicular to the track. It is

$$B_{\text{perp}} = B \times \sqrt{1 - \sin^2(\theta_z) \times \cos^2(\arctan(\sqrt{(R^2 - b^2)/2b})]}$$

Note that both \(L\) and \(B_{\text{perp}}\) depend only on \(b\) and \(\theta_z\).

A fairly typical cosmic ray muon in the MINOS detector is shown in Figures 2 and 3. Figure 2 shows the 3 sides of the detector, uz, vz and xy with units in meters. The individual plane crossings are too small to distinguish in Fig. 2. To see the curvature we compare the hits to a straight line fit. \(\Delta u\) and \(\Delta v\) are plotted versus \(z\) in Figure 3. Shown are the centers of the 4.1 cm strips, the strips themselves, and the best fit. Note that the largest deviations are only about 2 cm, which is comparable to the strip width. This muon deviates from straight by almost 5 sigma, while most cosmic muons have a substantially smaller deviation from straightness. The largest deviation from the straight line is termed the sagitta of the half track, and for this track is about 2 cm.

The MDM calculated here is tabulated in Table 2 as a function of \(\theta_z\) and impact parameter, for cosmic ray muon tracks in a single supermodule. For the track in Figs 2 & 3, the MDM is about 200 GeV/c.

**Implications**

The MDM has been useful in reducing the two different kinds of charge misidentification which provide systematic errors in the measurement of the atmospheric muon charge ratios. We call these errors bias errors and randomization. Bias errors can have any effect on the measured charge ratio. They could be due, for example, to the acceptance, poor modeling of the magnetic field or alignment errors. Acceptance effects can be accurately studied in a Monte Carlo, but modeling errors cannot. Bias errors do cancel, however, by taking the geometric mean of the charge ratio from forward and reverse running of the magnetic field. Randomization errors are those in which the charge of the muon is assigned at random. These always cause the measured \(\mu\) charge ratio to be closer to unity than the true charge ratio, and since the charge ratio \(r > 1\), they make the measured \(r\) lower. They cannot be canceled by using reverse field data.

The curvature resolution is the inverse of the MDM. The MDM does not account for bias which must be dealt with separately. Before forward and reverse field data were combined, there was evidence for several unexplained bias effects in the MINOS data. One such effect is a bump in the charge ratio versus momentum. We have calculated that a bias in the sagitta of 2 mm, corresponding to a bias in the curvature calculation of 1/(2000 GeV/c) together with a curvature resolution of 1/(200 GeV/c) leads to a bump in the charge ratio versus momentum distribution similar to that seen in our data. When forward and reverse field were combined, this bias was completely removed.

We expected randomization effects at large muon momentum above the MDM. These were minimized by cutting on the charge confidence parameter \(\frac{1}{\sigma} / \sigma(\frac{1}{\sigma})\). The charge confidence parameter is approximately equal to the MDM divided by the measured momentum.

We also encountered another randomization effect at low measured momentum. This came from an unexpected source and can be explained as follows. In Table 1, the ratio of the mean angular deflec-
tion from Multiple Coulomb Scattering (MCS) to the angular deflection from bending is calculated as $0.232/[B_{\text{perp}} \times \sqrt{L}]$ with $B$ in tesla and $L$ in meters. When this fraction approaches unity, a straight track will give a good fit to any momentum. Then a few hits which should not be fit to the track may be incorrectly included in the fit, and will usually give a low momentum with a random sign of charge. This happened most frequently for directions corresponding to large values in Table 1. These events were eliminated by requiring track directions inconsistent with this possibility.

Figure 1: The MINOS detector coordinate system. $z$ is along the beam axis.

Figure 2: Three views of a cosmic ray muon data event in MINOS. Even though this muon curves more than most cosmic rays in MINOS, the curvature is not apparent in these views. Hits which are not in the track fit are not shown. The track has a fit momentum of $50.4 \pm 10.2$ GeV/c, and a $\chi^2$/ndof = 119/97. The charge confidence is about 5σ.

Table 1: Relative angle change from Multiple Coulomb scattering versus bending as a function of angle from the $z$ axis (in degrees) and the absolute value of the impact parameter (in meters).

| $\theta$ | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|
| 10      | .14 | .14 | .14 | .14 | .15 | .15 | .15 | .15 |
| 20      | .14 | .15 | .15 | .15 | .15 | .15 | .15 | .17 |
| 30      | .14 | .15 | .15 | .16 | .17 | .19 | .22 | .22 |
| 40      | .15 | .16 | .18 | .19 | .21 | .24 | .28 | .28 |
| 50      | .17 | .18 | .20 | .22 | .25 | .29 | .36 | .36 |
| 60      | .18 | .20 | .22 | .26 | .30 | .36 | .47 | .47 |
| 70      | .19 | .21 | .24 | .29 | .35 | .45 | .64 | .64 |
| 80      | .19 | .20 | .22 | .25 | .31 | .39 | .54 | .88 |

Acknowledgments

This work is supported by US DOE and NSF, the UK Particle Physics and Astronomy Research Council, and the University of Minnesota. We wish to thank the Minnesota Department of Natural Resources for use of the facilities of the Soudan Underground State Park, and also the large crews of workers who helped construct the detector and its components, and the mine crew for help in operating the detector.

References

[1] S. Mufson for the MINOS collaboration, these proceedings.
Table 2: Maximum detectable momentum (in GeV/c) in MINOS as a function of angle from the z axis (in degrees) and absolute value of the impact parameter (in meters).

| $b$ | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\theta_z$ | 471 | 470 | 469 | 468 | 467 | 466 | 465 | 464 |
| 10   | 471 | 469 | 465 | 460 | 455 | 451 | 447 | 246 |
| 20   | 471 | 467 | 458 | 447 | 391 | 298 | 197 | 92  |
| 30   | 444 | 333 | 304 | 262 | 213 | 159 | 103 | 47  |
| 40   | 235 | 226 | 203 | 171 | 135 | 98  | 62  | 28  |
| 50   | 232 | 223 | 197 | 162 | 124 | 87  | 52  | 22  |
| 60   | 231 | 221 | 193 | 156 | 115 | 77  | 44  | 17  |
| 70   | 270 | 257 | 223 | 177 | 128 | 82  | 43  | 15  |
| 80   | 232 | 221 | 193 | 156 | 115 | 77  | 44  | 17  |

Figure 3: This shows the deviations from straightness in $\Delta u$ and $\Delta v$ of the cosmic ray muon in Fig. 2. Deviations from a straight line fit were everywhere less than 2 cm. The green squares represent the centers of the 4 cm wide scintillator strips that had hits on the track fit. The vertical lines represent the width of the strip in u or v. The solid blue symbols represent the 3D position of the track fit. The “S” shape in both views is indicative of bending in the MINOS toroid and a good track fit.