A method of designing equipment for heat processing of polymer workpieces

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Abstract. The paper describes a method of designing heating equipment that maintains a predetermined temperature field. The method consists in sequential solution of two problems. At the first stage, the heat generation field was calculated using the stationary heat conduction equation. At the second stage, parametric optimization of the temperature field was performed with reference to the power and configuration limits of the heaters. To test this method, the problem of maintaining a predetermined non-uniform temperature field was solved. A practical example of the application of the method for designing a uniform heating plate used in vulcanizing presses was given. To assess the efficiency of the plate, the results of modeling the heat processing of a workpiece from a rubber mixture were presented.

1. Introduction
Heat processing of polymer workpieces (vulcanization, polymerization) requires a high accuracy of maintaining a predetermined temperature [1-4]. As a rule, the design requirements specification for such equipment specifies the permissible deviation from the operating temperature at each point of the working surfaces. For example, during vulcanization with hydraulic presses, such a surface is the upper or lower plane of the heating plate [5,6]. In some cases, zone heating of workpieces is required, which makes it possible to obtain a combination of different mechanical properties within one product [7].

Maintaining the desirable temperature on the working surface is challenging due to a number of factors, such as heat losses from the end surfaces, power and configuration limits of the heaters, the use of molds, as well as the amount of time spent on computations and field experiments.

The problem of designing such equipment is often solved using heuristic approaches [8]. There are also studies on optimization of temperature fields using the design of experiments theory, which makes it possible to significantly reduce the number of iterations [9,10]. However, such parametric optimization is applicable in the presence of the object structure; for example, if the number and shape of the heaters are known, it is required to determine their location and power.

The purpose of the study is to develop a methodology for the design of heating devices that maintain a predetermined temperature field on the working surfaces of the heating parts of equipment for heat processing of polymer workpieces.

2. Problem statement
We consider the heating plate of a vulcanizing press. The problem statement is formulated as follows: it is necessary to determine the number of heaters, their type, shape and location, at which the maximum temperature deviation from the set value at each point of the working surface reaches a minimum:
\[ \max \left\{ T(x, y, h_w) - T^* T(x, y) \right\} \to \min \; ; \; 0 \leq x \leq a \; ; \; 0 \leq y \leq b , \]

where \( T \) is the plate temperature, \( K \); \( h_w \) is \( z \) coordinate of the working surface, \( m \); \( T^* \) is a set temperature on the working surface, \( K \); \( a \) is the plate length, \( m \); \( b \) is the plate width, \( m \).

3. Problem solving

To solve this problem, we propose a method, the idea of which is to solve consistently two subproblems: 1) calculation of the heat generation field; 2) optimization of the temperature field, given the configuration and power limits of the heaters. Since the steady-state mode of operation of heat processing equipment is of practical interest, the specific heat generation field is calculated by the steady-state equation of heat conductivity:

\[ Q = -\lambda \text{div} \text{grad} T , \]

where \( \lambda \) is the heat conductivity coefficient, \( W \cdot m^{-1} K^{-1} \); \( Q \) is the specific heat generation, \( W/m^3 \).

In order to take into account the stage of heating the device, the heat generation field found by equation (2) is sufficient to increase by the value \( Q_0 \), based on the heating rate:

\[ Q_0 = cp \frac{dT}{dt} , \]

where \( c \) is heat capacity, \( J \cdot kg^{-1} K^{-1} \); \( p \) is density, \( kg/m^3 \); \( t \) is time, \( s \).

With a uniform heating rate throughout the entire volume \( (dT/dt = \text{const}) \), \( Q_0 \) is a constant.

As can be seen, it is required to solve the inverse problem at the first stage, i.e. to calculate the heat generation field from the known temperature field. It is noteworthy that the value of the operating temperature must be specified at each point of the design domain. With the help of numerical differentiation, the heat generation field can be found for any predetermined temperature distribution. Since the temperature must be maintained with a given accuracy only on one working surface, this problem can be solved in a two-dimensional plane-parallel formulation.

The second stage involves the determination of the characteristics of the heaters, which approximately reproduce the found heat generation field. To solve this subproblem, heuristic methods can be used, as well as the theory of experiment planning [10].

4. Examples of the method application

4.1. The problem of maintaining a continuous temperature field

Let the temperature field be determined by a function of the form

\[ T(x, y) = 100xy(a - x)(b - y) . \]

The heat generation field is found analytically:

\[ Q(x, y) = -\lambda \text{div} \text{grad} [100xy(a - x)(b - y)] = 200\lambda [x(a - x) + y(b - y)] . \]

Using the level lines, figures 1 and 2 show the temperature field by equation (4) for \( \lambda = 1 \) \( W \cdot m^{-1} K^{-1} \), and the heat generation field by equation (5) for \( a = b = 1 \) \( m \). To verify the solution, the temperature field was calculated in the ANSYS 2019 R2 finite element analysis system [11] with heat generation by equation (5) and boundary conditions of the 1st order:

\[ T(0, y) = T(a, y) = T(x, 0) = T(x, b) = 0 . \]
Figure 1. Temperature field by equation (4).

Figure 2. Heat generation field by equation (5).

The calculated temperature field shown in figure 3 is identical to the original temperature field shown in figure 1.

Figure 3. Verification calculation of the temperature field.

4.2. The problem of maintaining a discontinuous temperature field

Let the temperature field be determined by a discontinuous function of the form
\[ T(x, y) = \begin{cases} \begin{align*} &100, \text{if } (x \geq 0.2) \cup (x \leq 0.8) \cup (y \geq 0.2) \cup (y \leq 0.8); \\ &0, \text{otherwise.} \end{align*} \end{cases} \] (7)

Analytical differentiation of such functions is impossible. Nevertheless, the heat generation field can be calculated using numerical differentiation. Figures 4 and 5 show the temperature field by equation (7) and the heat generation field calculated numerically using equation (2) with a differentiation step of 1/40 m, respectively.

![Figure 4. Temperature field by equation (7).](image)

![Figure 5. Heat generation field, maintaining temperature field by equation (7).](image)

The analysis of the results obtained shows that for a jump-like temperature change, it is necessary to maintain heat generation in the region of high temperature and ensure heat absorption of the same intensity in the region of low temperature. The width of these regions in the case of a discontinuous temperature function depends only on the differentiation step, i.e. the regions of heat generation and heat absorption at the extreme must be infinitely thin. There is no heat generation in regions of uniform temperature. Since the qualitative distribution of heat generation is of the greatest interest in figure 5, the numerical values are not given.

Thus, for any temperature field, the corresponding heat generation field can be found. However, the solutions obtained do not always meet the requirements of process-related feasibility or even physical feasibility. For example, the formation of a heat generation field in the form of a paraboloid, shown in figure 2, is not possible due to process-related limitations. However, this solution can be reached using a combination of ring heaters. In this regard, the heat generation field calculated by equation (2) must be considered as an ideal distribution, to which one should strive for subsequent parametric optimization. Any deviations from this distribution will worsen the temperature field.

4.3. Designing the heating plate

We consider a real example of developing a 600×600×60 mm steel heating plate for a vulcanizing press. As a rule, according to the terms of reference, it is required to maintain a uniform temperature on the working surfaces of such plates. As stated above, in the steady-state mode, heat generation is not required to ensure a uniform temperature. The end surfaces of the plates are usually in direct contact with the surrounding air, heat transfer from them is carried out according to the boundary condition of the 3rd order:
where $\alpha$ is the heat convection coefficient, $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$; $T_{\text{amb}}$ is the ambient temperature, $\text{K}$; $\Omega$ is the heat convection area; $n$ is the direction of the normal to the heat-convection surface.

Consequently, on heat-convection surfaces, it is necessary to compensate for heat losses, the specific value of which is equal to the product $\alpha(T - T_{\text{amb}})$. In addition, in order to heat the plate, it is required to distribute heat generation as uniformly as possible throughout its volume, the intensity of which is determined based on the heating rate according to equation 3.

Figure 6. Design of a combined heating plate.

Figure 7. Temperature field of 1/8 of the working surface of the plate.

Considering the above, the proposed design of the combined heating plate (figure 6) has the following distinctive features:
compensation for heat losses is carried out by a band resistive heater (heaters of this type can be placed as close as possible to the heat-convection end surfaces);

- heating is carried out by three induction heaters, which makes it possible to distribute the heat generation uniformly at the heating stage;

- power control of the compensating heater and the main heaters is performed using two independent proportional-integral-differential (PID) controllers;

- thermal insulation of end surfaces is applied using a composite material with low heat conductivity to reduce heat losses;

- the plate is made from steel with high heat conductivity (67 W·m⁻¹K⁻¹ at 200 °C), thus ensuring the smoothing of the temperature field.

The proposed measures made it possible to maintain the temperature on the entire working surface of the plate at 200±1 °C. Figure 7 shows the temperature field of 1/8 of the working surface of this plate in a steady mode. The temperature field was calculated by a simplified procedure, which made it possible to find a solution of only the transient equation of heat conductivity assuming that there was uniform heat generation in the region of inductors [12].

4.4. Simulation of heat processing of a polymer workpiece

To assess the effectiveness of the proposed design of the combined heating plate, we simulated the heat processing of the “membrane” workpiece from a rubber mixture. In [12], the processing of the same product on a hydraulic press equipped with commercially available induction heating plates was described. The task was to calculate the transient temperature field of a system consisting of two combined heating plates, between which a mold with a workpiece made of a rubber mixture was placed (figure 8). Since the product was thin-walled, the heat generation in the vulcanization process can be neglected.

![Figure 7](image_url)

Figure 7. Temperature field of 1/8 of the working surface of the plate in a steady mode.

According to the process regulations, first the plates and the mold were heated up to a predetermined temperature, and then the rubber mixture was added. The processes of opening and closing the plates, removing and reinstalling the mold were not simulated. It is assumed that adding of the rubber mixture was instantaneous. According to the conclusions in [12], at the end of the heating stage, it took some time to stabilize the temperature fields. The duration of the stabilization stage depends on the plate design and can be determined from the graph of the maximum temperature difference on the working surface.

Since the mold introduced asymmetry in the temperature field, separate power control of the bottom and top plates was required. Thus, when calculating the temperature field, we used the models of four independent PID controllers.
Figure 9 shows graphs of the maximum temperature differences on the working surfaces of the bottom and top plates, as well as in the whole volume of the workpiece being processed. It took 40 minutes to heat the plates to the predetermined temperature. Then there was a stage of stabilization of temperature fields, which was characterized by a sharp decrease in temperature drops. As you can see from the graphs presented, this stage lasted about 15 minutes. At the end of stabilization (at a time of 55 minutes), the rubber mixture was added, which was characterized by a sharp increase in temperature drops and power consumption of the bottom plate as shown in the graphs (figure 10). The power of the top plate changed more smoothly due to the greater distance between the workpiece and the thermocouple of the PID controller, which controlled the power of the induction heaters.

Due to heat loss from the cylindrical wall of the mold, the temperature of the outer part of the workpiece was 2.0 °C below the maximum temperature (figure 11). According to this indicator, combined heating plates gained only 0.3 °C [12]. The problem of uneven heating of the workpiece can be partially eliminated by selecting the operating parameters of the PID controllers. Obviously, to compensate for heat losses and equalize the temperature field in the volume of the workpiece, the temperature of the top plate must be higher. However, even these measures cannot guarantee uniform heating of the workpiece due to the sub-optimal geometry of the mold [13].

**Figure 9.** Maximum temperature difference graphs. 1 - on the surface of the bottom plate; 2 - on the surface of the top plate; 3 - in the volume of the workpiece.

**Figure 10.** Power consumption. 1 - bottom plate; 2 - top plate.

**Figure 11.** Workpiece temperature field.
5. Conclusions
Based on the proposed technique, a combined heating plate with a working surface temperature of 200±1 °C was designed. Simulation of heat processing of a polymer workpiece confirmed that the use of such plates significantly reduces the stabilization time of the temperature field before the vulcanization process (from 30 to 15 minutes in relation to commercially available induction heaters).

Regarding the possibility of maintaining a uniform temperature field in the volume of the workpiece being processed, the combined uniform heating plate did not show high efficiency. This was due to the sub-optimal geometry of the mold. In this regard, the most promising direction is the development of mold designs based on topology optimization methods according to the criteria of the uniformity of the temperature field in the volume of the workpiece and the rigidity of the structure [14,15]. The use of such molds will eliminate the need for lengthy manual adjustment of the heating system for pressing equipment.

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