1. Introduction

1.1. Neutrino Flare and a "cocktail" of Flavours

The recent peculiar solar flares on October-November 2003 were source of high energetic charged particles with energies: \( 15\text{GeV} \geq E_p \geq 100\text{MeV} \). A large fraction of these primary particles, i.e. solar flare cosmic rays, became a source of both neutrons and secondary kaons, \( K^\pm \), pions, \( \pi^\pm \) by their particle-particle spallation on the Sun surface. Consequently, \( \mu^\pm \), muonic and electronic neutrinos and anti-neutrinos, \( \nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e \), \( \gamma \) rays, are released by the chain reactions

\[
\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu), \quad \pi^0 \rightarrow 2\gamma, \quad \mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \nu_\mu(\bar{\nu}_\mu).
\]

There are two different sites for these decays.
to occur, and two corresponding neutrino emissions (see 1, 8): A brief and sharp solar flare (a solar neutrino burst), originated within the solar corona and a diluted and delayed terrestrial neutrino flux, produced by flare particles hitting the Earth’s atmosphere. The first is a prompt neutrino burst (few seconds/minutes onset) due to charged particles scattering onto the solar shock waves, associated with prompt gamma, X, neutron events. For instance the largest event which occurred at 19:50 UT on November 4 2003 was recorded as an maximal X28, most intense X ray event. The consequent solar flare neutrinos reached the Earth with a well defined directionality and within a narrow time range. The corresponding average energies $< E_{\nu_e} >$, $< E_{\nu_\mu} >$ are in principle larger compared to an event in the Earth’s atmosphere since the associated primary particles ($\pi^\pm, \mu^\pm$) decay in flight at low solar densities, where they suffer negligible energy loss: $< E_{\nu_e} > \simeq 50$ MeV, $< E_{\nu_\mu} > \simeq 100 \div 200$ MeV; (however the proton solar flare spectra is generally softer than atmospheric one leading to a kind of compensation). The delayed neutrino flux originated in the Earth’s atmosphere is due to the arrival of prompt solar charged particles nearly ten minutes later than onset of the radio-X emission. Such nearly relativistic cosmic rays are charged and bent by inter-planetary particles and fields. Therefore their arrival and the corresponding neutrino production in the Earth’s atmosphere occurs few tens of minutes or even few hours later than the solar X-radio sharp event. They will hit preferentially terrestrial magnetic poles and the South Atlantic Anomaly (SAA) area. As a result, their signal is widely spread and diluted in time and difficult to observe. Therefore we shall focus only on the prompt solar neutrino burst both outward the sun (where density is low and decreasing) or inward the solar surface (where density is growing) while pointing at the same time to the (Earth) observer. Because of the very different consequent target solar atmosphere, the $\pi^\pm, \mu^\pm$, and $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ production is different. Our estimate of the solar flare neutrino burst 1 is scaled by an integrated flare energy $E_{FL}$, which is assumed to be of the order of $E_{FL} \simeq 10^{31} \div 10^{32}$ erg, by comparison with the known largest solar flare events as those in 1956 and 1989 2. The exact solar flare spectra on recent 2003 are yet unknown but their energies are well extending above the few GeV threshold necessary to the pion production 4. In the final figures we describe their spectra emergence over the more abundant (but lower energetic) thermal solar neutrinos 5 as well as in respect with cosmic back ground Super-Novae relic and atmospheric neutrinos 6.
1.2. Upward and downward solar neutrino flare burst

An energetic proton ($E_p \approx 2$ GeV) may scatter inelastically with a target proton at rest in solar atmosphere, whose density behaves as (reference 3, 8). $n_\odot = N_0 e^{-\frac{r}{\Delta}}$; $N_0 = 2.26 \cdot 10^{17} \text{ cm}^{-3}$, $h_0 = 1.16 \cdot 10^7 \text{ cm}$ where $h_0$ is the photosphere height where flare occurs. The inelastic proton-proton cross section for energetic particles ($E_p > 2$ GeV) is nearly constant (reaching a maxima at the $\Delta$ resonant peak): $\sigma_{pp}(E > 2 \text{ GeV}) \simeq 4 \cdot 10^{-26} \text{ cm}^2$.

Therefore the scattering probability $P_{\text{up}}$ for an upward and downward (respect to the solar surface) energetic proton $p_E$, to produce pions (or kaons) via nuclear reactions is: $P_{\text{up}} = 1 - e^{-\int_{h_0}^{\odot} \sigma_{pp} n_\odot dh} \simeq 0.1$ and $P_{\text{down}} \simeq 1$ because for the downward hitting the density is growing and the probability reach and overcome unity. Moreover, because of the kinematics, only a fraction smaller than 1/2 of the energetic proton will be released to pions (or kaons) formation. In the simplest approach, the main source of pion production is $p + p \rightarrow \Delta^{++} n \rightarrow p \pi^+ n$; $p + p \rightarrow \Delta^+ p \pi^0 n$ at the center of mass of the resonance $\Delta$ (whose mass value is $m_\Delta = 1232$ MeV).

The ratio $R_{\pi p}$ between the pion to the proton energy is: $R_{\pi p} = \frac{E_\pi}{E_p} = \frac{m_\Delta^2 + m_p^2 - m_{\pi}^2}{m_\Delta^2 + m_p^2 - m_{\pi}^2} = 0.276$ Therefore the total pion flare energy due to upward proton is: $E_{\pi F L} = P R_{\pi p} E_{F L} = 2.76 \cdot 10^{-2} E_{F L}$. Because of the isotopic spin, the probability to form a charged pion over a neutral one in the reactions above: $p + p \rightarrow p + n + \pi^+$, $p + p \rightarrow p + p + \pi^0$, is given by the square of the Clebsh Gordon coefficients: $C_{2,0}^2 = \frac{3}{5}$.

This ratio imply a larger (nearly an order of magnitude) neutrino fluence over the gamma one (as the 1991 and recent gamma 2002 flare 13). The ratio of the neutrino and muon energy in pion decay is also a small dimensional fraction $R_{\nu, \mu}$.

$R_{\nu, \mu} = \frac{E_{\nu \mu}}{E_\mu} = \frac{m_{\pi}^2 - m_{\mu}^2}{m_{\pi}^2 + m_{\mu}^2} = 0.271$. Therefore, at the rest frame of the charged pion one finds the mono-cromatic energies for the neutrino and muon: $E_{\nu \mu} = 29.8 \text{ MeV}$; $E_\mu = 109.8 \text{ MeV}$. One might notice that the correct averaged energy (by Michell parameters) for neutrino decay $\mu^+$ at rest are: $E_{\bar{\nu}_\mu} = E_{\nu_e} = \frac{3}{4} m_\mu \simeq \frac{1}{4} m_\pi$. However as a first approximation and as an useful simplification after the needed boost of the secondaries energies one may assume that the total pion $\pi^+$ energy is equally distributed, in average, in all its final remnants: ($\bar{\nu}_\mu, e^+, \nu_e, \nu_\mu$): $E_{\bar{\nu}_\mu} \simeq E_{\nu_e} \simeq \frac{1}{4} E_\pi$.

Similar reactions (at lower probability) may also occur by proton-alpha scattering leading to: $p + n \rightarrow \Delta^+ n \rightarrow n \pi^+ n$; $p + n \rightarrow \Delta^0 p \pi^+$.
we neglect the $\pi^-$ additional role due to the flavor mixing and the dominance of previous reactions $\pi^+$ production at soft flare spectra. To a first approximation the oscillation will lead to a decrease in the muon component and it will make the electron neutrino component harder. Indeed the oscillation length at the energy considered is small respect Earth-Sun distance: 

$$L_{\nu_{\mu} - \nu_e} = 2.48 \cdot 10^9 \text{cm} \left( \frac{E_{\nu}}{10^9 \text{eV}} \right) \left( \frac{\Delta m^2_{31}}{10^{-2} \text{eV}^2} \right)^{-1} \ll D_{\odot} = 1.5 \cdot 10^{13} \text{cm}. \tag{1}$$

We take into account this flavor mixing by a conversion term re-scaling the final muon neutrino signal and increasing the electron spectra component. While at the birth place the neutrino fluxes by positive charged pions $\pi^+$ are $\Phi_{\nu_e} : \Phi_{\nu_\mu} : \Phi_{\nu_\tau} = 1 : 1 : 0$, after the mixing assuming a democratic number redistribution we expect $\Phi_{\nu_e} : \Phi_{\nu_\mu} : \Phi_{\nu_\tau} = (\frac{2}{3}) : (\frac{2}{3}) : (\frac{2}{3})$. Naturally in a more detailed balance the role of the most subtle and hidden parameter (the neutrino mixing $\Theta_{13}$) may be deforming the present averaged flavour balance. On the other side for the anti-neutrino fluxes we expect at the birth place: $\Phi_{\bar{\nu}_e} : \Phi_{\bar{\nu}_\mu} : \Phi_{\bar{\nu}_\tau} = 0 : 1 : 0$ while at their arrival (within a similar democratic redistribution) :$\Phi_{\bar{\nu}_e} : \Phi_{\bar{\nu}_\mu} : \Phi_{\bar{\nu}_\tau} = (\frac{1}{3}) : (\frac{1}{3}) : (\frac{1}{3})$. Because in $\pi$-$\mu$ decay the $\mu$ neutrinos secondary are twice the electron ones, the anti-electron neutrino flare energy is, from the birth place on Sun up to the flavour mixed states on Earth:

$$E_{\bar{\nu}_e} F L \simeq E_{\nu_\mu} F L \simeq 2.6 \cdot 10^{28} \left( \frac{E_{FL}}{10^{31} \text{erg}} \right) \text{ erg}. \tag{2}$$

The corresponding neutrino flare energy and number fluxes at sea level (for the poor up-going flare) are: $\Phi_{\bar{\nu}_e} F L \simeq 9.15 \left( \frac{E_{FL}}{10^{31} \text{erg}} \right) \text{ cm}^{-2}$; $N_{\bar{\nu}_e} \simeq \frac{N_{\nu_e}}{2} \simeq 5.7 \cdot 10^4 \left( \frac{E_{FL}}{10^{31} \text{erg}} \right) \left( \frac{<E_{\bar{\nu}_e}>}{100 \text{ MeV}} \right)^{-1} \text{cm}^{-2}.$

This neutrino number flux in 100 s. time duration is larger by two order of magnitude over the atmospheric one. Therefore in this rough approximation we may expect in S.K. a signal at the threshold level (below one event). Let us consider horizontal flares similar to horizontal and upward neutrino induced air-showers inside the Earth Crust (see \cite{9, 10}). The solar neutrino flare production is enhanced by a higher solar gas density where the flare beam occurs. Moreover a beamed X-flare (due to relativistic electron bremsstrahlung) may suggest a primary beamed pion shower whose thin jet naturally increases the neutrino signal. High energetic protons flying downward (or horizontally) to the Sun center are crossing larger (and deeper) solar densities and their interaction probability $P_d$ is larger than the previous one ($P_{up}$). The proton energy losses due to ionization, at the atmospheric solar densities are low.

We foresee, in general, that the flare energy relations are: $E_{\pi F L} \equiv \eta E_{FL} \leq E_{FL} ; E_{\bar{\nu}_e F L} \simeq \left( \frac{E_{\nu e F L}}{2} \right) \simeq 9.4 \cdot 10^{30} \eta \left( \frac{E_{FL}}{10^{31} \text{erg}} \right) \text{ erg}$. Let us estimate
the Solar neutrino flare events in SK-II. We expect a solar flare spectrum with an exponent equal or larger (i.e. softer) than the cosmic ray proton spectrum one. Estimated averaged neutrino energy \( < E_\nu > \) below GeV for down-going rich flare is \(^1\):

\[
< N_\nu_e > \simeq 1.72 \cdot 10^6 \eta \left( \frac{< E_\nu_e >}{100 \text{ MeV}} \right)^{-1} \left( \frac{E_{FL}}{10^{31} \text{ erg}} \right) \text{ cm}^{-2}; \quad < N_\nu_\mu > \simeq 4.12 \cdot 10^6 \eta \left( \frac{< E_\nu_\mu >}{100 \text{ MeV}} \right)^{-1} \left( \frac{E_{FL}}{10^{31} \text{ erg}} \right) \text{ cm}^{-2}.
\]

We now consider the solar flare neutrino events due to these number fluxes following known \( \nu \)-nucleons cross-sections at these energies \(^{11,12,14}\) at Super-Kamiokande II: \( N_{ev} \simeq 7.5 \cdot \eta \left( \frac{E_{FL}}{10^{31} \text{ erg}} \right) \) (for more details and explanations see \(^1\)).

The events in the terrestrial flux should almost double the common integral daily atmospheric neutrino flux background (5.8 event a day) but as we mentioned with little statistical meaning. Finally we summarized the expectation event numbers at SK-II for solar neutrino burst assuming, before mixing, a more pessimistic detector thresholds calibrated with the observed Supernovae 1987A event fluxes \(^1\): \( N_{ev_\nu_e} \simeq 0.63 \eta \left( \frac{E_{\nu_e}}{50 \text{ MeV}} \right) \left( \frac{E_{FL}}{10^{31} \text{ erg}} \right); \quad \bar{E}_{\nu_e} \leq 100 \text{ MeV}; \quad N_{ev_\nu_\mu} \simeq 1.58 \eta \left( \frac{E_{\nu_\mu}}{100 \text{ MeV}} \right) \left( \frac{E_{FL}}{10^{31} \text{ erg}} \right); \quad \bar{E}_{\nu_\mu} \geq 100 - 1000 \text{ MeV}; \quad N_{ev_{\bar{\nu}_e}} \simeq 3.58 \eta \left( \frac{E_{\bar{\nu}_e}}{100 \text{ MeV}} \right) \left( \frac{E_{FL}}{10^{31} \text{ erg}} \right); \quad \bar{E}_{\bar{\nu}_e} \geq 200 - 1000 \text{ MeV}; \) where the efficiency factor \( \eta \leq 1 \).

The neutrino events in Super-Kamiokande may be also recorded as stimulated beta decay on oxygen nuclei \(^7\). Let us underline the the surprising role of Solar Neutrino Flavor mixing: the \( \mu \) and \( \tau \) appearance; indeed the oscillation and mixing guarantee the consequent tau flavor rise and the anti neutrino electron component hardening respect to the one at its birth. This will also increase the neutrino electron component while it will reduce the corresponding muon component leading to: \( \frac{N_{ev_{\nu_e}}}{N_{ev_{\nu_\mu}}} \simeq \frac{1}{7} \) and to \( N_{ev_{\nu_\mu}} \simeq 4 \cdot \frac{< E_{\nu_\mu} >}{200 \text{ MeV}} \left( \frac{< E_{\nu_{e\mu}} >}{10^{31} \text{ erg}} \right); \quad N_{ev_{\bar{\nu}_e}} \simeq N_{ev_{\bar{\nu}_\mu}} \simeq 4 \left( \frac{< E_{\bar{\nu}_e} >}{200 \text{ MeV}} \right) \left( \frac{< E_{\bar{\nu}_{e\mu}} >}{10^{31} \text{ erg}} \right), \) as well as a comparable, \( \nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu \) energy fluence and spectra. At energies above the \( \tau \) threshold energy \( E_{\nu_{\tau}} \geq 3.46 \text{ GeV} \) a surprising \( \tau \) appearance may occur: this requires a hard \( (E_{\nu_\mu} \rightarrow E_{\nu_\tau} \simeq 4 \text{GeV}) \) flare spectra.

Any positive evidence for such events will mark a new road to Neutrino Astrophysics, to be complementary to lower neutrino energy from Sun and Supernovae. New larger generations of neutrino detectors (with the capability to reveal \( 10 - 1000 \text{MeV} \) lepton tracks) will be more sensitive to such less powerful, but more frequent and energetic solar flares, than to the rarest galactic and even extragalactic supernovae events (as the one from Andromeda) \(^9\). The background due to energetic atmospheric neutrinos at the Japanese detector is nearly 5.8 event a day corresponding to a rate \( \Gamma \simeq 6.71 \times 10^{-5} \text{s}^{-1} \). The lowest and highest predicted
event numbers \((1 \div 5) \eta, (\eta \leq 1)\) within the narrow time range defined by the sharp X burst (100s), are well above the background. Indeed the probability to find by chance one neutrino event within a 1 – 2 minute \(\Delta t \simeq 10^2\)s in that interval is \(P \simeq \Gamma \cdot \Delta T \simeq 6.7 \cdot 10^{-3}\). For a Poisson distribution the probability to find \(n = 1, 2, 3, 4, 5\) events in a narrow time window might reach extremely small values: \(P_n \simeq e^{-P} \cdot \frac{P^n}{n!} \approx \frac{P^n}{n!} = (6.7 \cdot 10^{-3}, 2.25 \cdot 10^{-5}, 5 \cdot 10^{-8}, 8.39 \cdot 10^{-11}, 1.1 \cdot 10^{-13})\). Therefore the possible presence of one or more high energetic (tens-hundred MeVs) positrons (or better positive muons) as well as negative electrons or muons (twice as much because privileged \(\pi^+\) primary), in Super-Kamiokande at X-flare onset time, may be a well defined signature of the solar neutrino flare. A surprising discover of the complete mixing from the \(\tau\) appearance may occur for hard \((E_{\nu_{\mu}} \rightarrow E_{\nu_{\tau}} > 4GeV)\) flare spectra. The expected matter-antimatter asymmetry differ from the observed atmospheric one and will help to untie the two signals. Our considerations are preliminary and they must be taken cautiously (given the delicate chain of assumptions and simplification). We suggest to control the Super-Kamiokande data records on October – November solar flare X-radio peak activity, namely on 26 – 28 – 30th October and 2nd, at 19:48 U.T. on 4th November 2003 and 13 November X-ray onset. A more accurate control neutrino correlation with largest solar flares may be also done in old past data records (during SK past life activity). Finally we notice that the new larger neutrino detectors such as UNO might be at the same time ideal laboratories for solar neutrino flare and flavour mixing, as well as the most rapid alert system monitoring the huge coronal mass ejection, dangerous for orbiting satellites.

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Figure 1. Summary of Solar neutrino Flare fluxes over known neutrino background: solar neutrinos by known nuclear activities as well as the expected Supernova Relic fluxes and the atmospheric noises. The Solar flare are estimated on the Earth at peak activity of the flare supposed to held 100 s. The two primordial flavour $\nu_e, \bar{\nu}_e$ and their correspondent $\nu_\mu, \bar{\nu}_\mu$ are shown before their oscillation while flying and mixing toward the Earth. Their final flavour are nearly equal in all states $\nu_e, \nu_\mu, \nu_\tau$ (twice as large as $\nu_e, \bar{\nu}_e, \bar{\nu}_\mu$, not shown for sake of simplicity); however each final $\nu_e, \nu_\mu, \nu_\tau$ fluxes are almost corresponding to the lower curve for the electron neutrino spectra. Both highest and lowest activities are described following an up-going (poor flux) or down-going (richer flux) solar "burst" scenario. The primary solar flare spectra is considered like the atmosphere one at least within the energy windows $E_{\nu_\mu} \simeq 10^{-3} \text{GeV}$ up to 10 GeV.

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Figure 2. Solar Neutrino $\nu_e$ Flare Before the mixing; note the thin $\nu_e$ peak due to prompt neutronization flash.

Figure 3. Solar Neutrino $\nu_e$ Flare After the mixing; note the marginal 0.33 loss of intensity due to the mixing.
Figure 4. Solar Neutrino $\nu_\mu$ Flare Before the mixing; note the absence of the thermal solar lines and spectra.

Figure 5. Solar Neutrino $\nu_\mu$ Flare After the mixing; note the presence of two different fluxes (upward and downward $\nu_\mu$) due to the well known muon oscillation and disappearance into $\tau$ flavour.
Figure 6. Solar Neutrino $\nu_\tau$ Flare Before the mixing: note the remarkable absence of any primary $\nu_\tau$ noises due to the paucity of their production on the sun or in atmospheric showers.

Figure 7. Solar Neutrino $\nu_\tau$ Flare After the mixing; note the presence of a different fluxes (upward) due to the well known muon oscillation and disappearance into $\tau$ flavour.