Polarized double dressing control of biphotons interference and quantum tomography

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Abstract

We research the polarized dressing control of biphotons interference and quantum tomography. Different biphotons counting rates are obtained by different polarization dressings of incident beams of lights. We calculated the counting rates of all polarized dressing schemes. The counting rate of circularly polarized dressing has been observed with shorter period of oscillation and a longer coherence time when compared with linear polarization. Since, the different polarization dressings result in different modules of third-order nonlinear susceptibilities and different linewidths. Thus, we can adjust the dressing field by changing the polarization of the incident beam of light to obtain controlled pair of photons with desired longer coherence time. Based on that, we study the impact of the two dressing fields which cause the third-order nonlinear susceptibility to be different. In the absence and presence of external dressing field, the biphoton coincidence rate is completely different. By performing quantum tomography, we get various Werner-states and the polarization states of incident lights, this also helps in comprehensive characterization of output state. Further, we define the interference visibility for different polarized dressed states. The visibility of the circularly polarized dressing field is found to be larger when compared with linearly polarized dressing field. Thus, we can obtain photons with controllable coherence time and desired visibility with different polarization schemes to uncover the indistinguishability of photon interference and promote the study of quantum optics.

1. Introduction

Generating entangled states of nonclassical light is a key task in the application of quantum information processing \cite{1,2}. Quantum correlations and entanglements among multiparticle have great significance for efficient quantum information. There has been a lot of researches about the correlated and polarized entangled multiple photon resources \cite{3,4} for secure quantum communication \cite{5–7} as polarization degree of freedom largely reduced the redundancy \cite{8–10}. Spontaneous four-wave mixing (SFWM) in cold atoms have become an efficient method for producing biphotons which are entangled in energy and time\cite{3,4} with long coherence time. The polarization-entangled shaped photon pairs from SFWM in a laser-cooled \textsuperscript{85}Rb atomic ensemble \cite{11,12} and nondegenerate narrow-bandwidth paired photons with polarization-time-frequency-coupled entanglements \cite{13}, suggests higher information-carrying capacity so a lot of research tends to generate these photons with long coherence time \cite{14–16}. Besides, the polarized properties of photons, have been researched in several metal vapors gases \cite{17,18}. 

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The polarization characteristics and polarization degree of freedom in biphoton bear importance. Recently, our group has modified biphoton correlation time by an unpolarized dressing laser to tune the SFWM nonlinear parametric interaction and linear interaction [19]. While Lan [20] worked on triphoton entangled system by considering the differences of three-photon states and their counting rate in a damped Rabi oscillation regime using six-wave mixing. Lan [20] employed a single polarized laser to control the waveform. Double polarized dressing fields are more complex and can significantly influence the counting rate coherence time by changing the structure of the third-order susceptibility. In this regard, by employing polarization dependency (linear and circular) of double dressing lasers, we theoretically control biphotons coincidence count, biphoton’s waveform oscillation and their coherence time in Rb atomic vapor in the four-level energy system using SFWM. By comparing the biphotons waveform obtained under different laser polarization configurations, we can predict and control the physics behind the period of oscillation and coherence time. We also compare, the differences in counting rate between polarized dressing and double dressing along with quantum tomography to comprehensively characterize biphoton polarized entanglement. The interference visibility of circularly polarized light is observed with larger visibility in comparison with linearly polarized light which means we can modulate the visibility and the degree of entanglement by using such polarization dressing fields.

The paper is organized as follows. In section 2, we present the theoretical formulae of the biphoton wave function. In section 3, we obtain various nonlinear susceptibilities for different polarization states through nonlinear polarization and perturbation chains. In section 4, we calculate the biphoton wave function and influence on wave function subjected to change in the polarization configurations of the incident laser. In section 5, we apply the polarized dressing in quantum state tomography to obtain complete characterization. In section 6, we calculate and define the interference visibilities with different polarization configurations. In section 7, we make a conclusion about this paper and look forward to the future.

2. The schematic of biphoton state function

The schematic of biphoton generation via a four-level \( ^{3}P_{1/2} \) atomic system is shown in figure 1(a). The relevant energy levels are \( 5S_{1/2}, F = 2 (|0\rangle), 5S_{1/2}, F = 3 (|1\rangle), 5P_{1/2}, F = 2 (|2\rangle) \) and \( 5P_{3/2}, F = 3 (|3\rangle) \). Figure 1(a) is a simplified setup for the biphoton-generation process. A half-wave plate (HWP), a polarized beam splitter (PBS) and a quarter-wave plate (QWP) or HWP are used to introduce the different polarization states in incident light (P polarization or S polarization). Similarly, QWP, an HWP, and a PBS are also placed in the propagation path of Stokes (\( E_a \)) and anti-Stokes (\( E_{ac} \)) photons prior to the detector. The angles of both pairs of wave plates can be set independently in order to set the projection state. The idealized atoms or molecules are confined within a long and thin cylindrical volume of length L having atomic density N. In figure 1(b), a pump beam \( E_p \) (frequency \( \omega_p \)) wave vector \( k_p \), Rabi frequency \( \Omega_p \) is applied to connect the between \( |0\rangle \) and \( |2\rangle \) with a detuning \( \Delta_2 \).

\[ \Delta_2 = \omega_20 - \omega_p \]

is defined as the difference between the resonant transition frequency \( \omega_20 \) and laser frequency \( \omega_p \) of the pump filed (\( E_p \)). Another coupling beam \( E_c (\omega_c, k_c, \Omega_c) \) is applied to connect the transition between \( |1\rangle \) and \( |3\rangle \) with a detuning \( \Delta_1 \).

\[ \Delta_1 = \omega_31 - \omega_c \]

is defined as the difference between the resonant transition frequency \( \omega_31 \) and laser frequency \( \omega_c \) of the coupling beam (\( E_c \)). Another polarized beam \( E_s (\omega_s, k_s, \Omega_s) \) has the same transmission direction as the coupling beam and modulate the atomic energy level. \( \Delta_3 = \omega_31 - \omega_s \) is defined as the difference between the resonant transition frequency \( \omega_31 \) and laser frequency \( \omega_s \) of the external dressing field (\( E_s \)). All beams are coupled and focused into the center of medium by optical lenses. Then with the phase-matching condition \( k_p + k_c = k_s + k_{ac} \) and low-gain limit, the SFWM process will occur spontaneously.
which can generate correlated biphoton $E_S$ and $E_a$ satisfying the energy conservation defined as $\omega_p + \omega_a = \omega_S + \omega_a (\sigma_1 + \delta_1 + \tau_{\text{m}} + \delta_2)$.

For simplicity, in this paper, we are not taking into account the Doppler broadening. The field annihilation operators in the time domain on the surfaces are expressed as

$$H_t = \varepsilon_0 \int \, d\tau \chi^{(3)}(\omega) E_{p}^{(+)} E_{a}^{(+)} E_{s}^{(-)} + H.c,$$

(1)

where $H.c$ means the Hermitian conjugate. $\chi^{(3)}$ is the third-order nonlinear susceptibility to the Stokes (or anti-Stokes) field and is defined by the nonlinear polarization. $E_{p}^{(+)}$ and $E_{a}^{(+)}$ are the Hermitian conjugates of quantum-field operators $E_{p}^{(+)}$ and $E_{a}^{(+)}$, respectively. $E_{s}^{(+)}$ and $E_{a}^{(-)}$ are the positive-frequency parts of the coupling and pump field and are considered as classical plane waves, and can be written as

$$E_{p}^{(+)} = E_p e^{i(k_p z - \omega_p \tau)}, \quad E_{a}^{(+)} = E_a e^{i(k_a z - \omega_a \tau)},$$

(2)

where $k_p$ and $k_a$ are field wavenumbers. The $z$-direction is assumed to be parallel to the pump longitudinal propagation. In the Hamiltonian (1), we ignore the reflections from the system surfaces and use the rotating-wave approximation. The generated Stokes and anti-Stokes photons are given by the quantized fields

$$E_{s}^{(+)} (z, t) = \int d\omega \sqrt{\frac{\hbar \omega s}{2\varepsilon_0 n_s V_Q}} \tilde{\alpha}_s(\omega) e^{i[k_s z - \omega_s \tau]},$$

$$E_{a}^{(+)} (z, t) = \int d\omega \sqrt{\frac{\hbar \omega a}{2\varepsilon_0 n_s V_Q}} \tilde{\alpha}_a(\omega) e^{i[k_a z - \omega_a \tau]},$$

(3)

where $V_Q$ is the quantization volume; $\tilde{\alpha}_s$ and $\tilde{\alpha}_a$ are the photon annihilation operators of the output models Stokes and anti-Stokes, respectively, $\varepsilon_0$ is the permittivity of a vacuum. $\omega_s$ and $\omega_a$ are the central frequency of the generated photons. By substituting equations (2) and (3) into equation (1) followed by integration, the equation (1) can be written as

$$\tilde{H}_t = \frac{i\hbar}{2} \int d\omega \omega_{s a} \kappa \Phi(\Delta k) H (\tilde{\alpha}_s + \tilde{\alpha}_a, \rho) \tilde{\alpha}_s^{+} \tilde{\alpha}_a^{+} e^{-i(\omega_s + \omega_a - \omega_m)^2},$$

(4)

In equation (4), $\kappa = -i(\sqrt{\omega_s \omega_a} / 2c) \chi^{(3)}(\omega_s, \omega_a) E_p E_c$ is the nonlinear parametric coupling coefficient. $\Delta k = k_s + k_a - k_c - k_p$ represents the phase mismatching along the $z$-axis for the forward (+) and backward (−) configurations. When $\Delta k = 0$ the phase-matching condition holds perfectly. And $\Phi(\Delta k L) = \sin c(\Delta k L / 2) e^{i(k_s + k_a)/2}$ is the longitudinal detuning function which decides the natural spectral width. We can define $\omega_p + \omega_a - \omega_s - \omega_m = \Delta \omega$, which indicates the frequency difference. $\omega_s$ and $\omega_m$ are the central frequencies of the Stokes and the anti-Stokes fields, respectively.

According to the perturbation theory, the photon state can be regarded as a linear superposition of $|0\rangle$ and $|\Psi\rangle$, where $|0\rangle$ is the vacuum state. Due to the undetectability of the vacuum, we ignore it and can only consider the biphoton part. The biphoton state can be written as

$$|\Psi\rangle_x = \frac{-i}{\hbar} \int_{-\infty}^{+\infty} dt H_t |0\rangle.$$ 

(5)

Replacing $H_t |0\rangle$ from equation (4), the time integral will make $e^{-i(\omega_p + \omega_a - \omega_s - \omega_m)^2}$ a $\delta$ function, $2\pi \delta(\Delta \omega)$, which express the energy conservation and leads to the frequency entanglement of biphoton state. As a consequence, the biphoton state is as follows

$$|\Psi\rangle_x = K \int dt \int d\omega_s d\omega_a \chi^{(3)}(\omega) E_p^{*} E_c \sin c \left(\frac{\Delta k L}{2}\right) \times \left(\frac{\omega_s}{n_s}\right) \left(\frac{\omega_a}{n_a}\right) a_s^{+} a_a^{+} e^{-i\Delta \omega t} H (\tilde{\alpha}_s + \tilde{\alpha}_a, \rho) |0\rangle,$$

(6)

where $\Delta \omega = \omega_p + \omega_a - \omega_s - \omega_m$, $K = i / (4(Cos \theta \times Sin \theta), \theta$ is the rotation angle between the axis of HWP and $x$-axis. As we can obtain from equation (6), the biphoton state is entangled in frequency and wavenumber. Frequency entanglement is the result of energy conservation. $k_{m}$, $k_i$ are included in $\sin c(\Delta k L / 2)$ and the biphoton state is also entangled in wavenumber since the function cannot be factorized as a function of $k_{m}$ times function of $k_i$. To study the optical properties of paired Stokes and anti-Stokes photons generated from such a four-level system, we need to consider the averaged biphoton coincidence counting rate $R_{\text{m}}$, which is the square of the biphoton state. From the above, we can know the third-order nonlinear susceptibility plays an important role. Different polarization involves different nonlinear susceptibilities, so we need to analyze susceptibilities.
3. Nonlinear susceptibilities for different polarization dressing

The pattern of the biphoton amplitude is determined by both third-order nonlinear susceptibility and the longitudinal detuning function. The wave function of biphoton is a mathematical convolution of the linear and nonlinear optical response. In order to obtain the specific form of wave function, we make the longitudinal detuning function $\Phi$ approaching unity representing the linear optical response of the generated fields. Under this circumstance, the nonlinear susceptibility plays an important role in determining the spectral width.

The polarized dependence of the FWM outputs can be modeled by quantum mechanical description [21]. For phase conjugated FWM generation in the atomic system at frequency $\omega_i + \omega_p = \omega_c + \omega_s$, the nonlinear polarization along $i(i = x, y)$ direction is given by

$$\hat{P}_{i,as}^{(3)(\pm)} = \varepsilon_0 \chi_{ijkl}^{(3)(\pm)} E_{i}^{(+)} E_{k}^{(\pm)} E_{l}^{(-)},$$

(7)

where $\chi_{ijkl}^{(3)}$ is the tensor component of the third-order nonlinear susceptibility. By considering all the incident beams and outputs as transverse waves, only four nonzero tensor elements are involved in this system, which are denoted as $\chi_{xxxy}, \chi_{xyyx}, \chi_{yzyx}, \chi_{xzzz}$ [22]. From [22], we know that $\chi_{xxxy} = \chi_{xyyx} + \chi_{yzyx}$, we can also predict the states of Stokes and anti-Stokes subjected to states of incident fields being explicit.

If a HWP is used to change the polarization of the coupling beam $E_c$ while the other beam $E_p$ is polarized in the horizontal direction, the coupling field will have two components: $E_{cx} = E_c \cos \theta$ and $E_{cy} = E_c \sin \theta$ where $\theta$ is the wave plate optics axis. Consequently, the polarization has horizontal components $\hat{P}_{s,as}^{(3)(\pm)} = \varepsilon_0 \chi_{ijkl}^{(3)(\pm)} E_{i}^{(+)} E_{k}^{(\pm)} E_{l}^{(-)} \cos \theta E_{as}^{(+)}$ and perpendicular components $\hat{P}_{s,as}^{(3)(\pm)} = \varepsilon_0 \chi_{ijkl}^{(3)(\pm)} E_{i}^{(+)} E_{k}^{(\pm)} E_{l}^{(-)} \sin \theta E_{as}^{(-)}$. If we use a QWP to adjust the incident beams, the nonlinear susceptibilities will be different, while the excited susceptibilities are the same [13].

Different transition paths are consisting of various transitions between Zeeman levels. Different polarization schemes can excite different transition paths. Here, linearly polarized light can be decomposed into right and left circularly polarized lights. Figure 2 shows the transition paths between Zeeman levels. According to the transition paths and perturbation chains, we can obtain density matrices. Also, the nonlinear polarization along with $i(i = x, y)$ can be written as

$$\hat{P}_{i,as}^{(3)(\pm)} = N \mu P_{i}^{(3)}.$$

(8)

When we don’t consider the external dressing field($E_3$), the third-order nonlinear susceptibility of Stokes and anti-Stokes can be expressed as
where \( N_i = \frac{2|\mu_{ij}|^2\epsilon_0\hbar}{c \Omega_{M1}^2} \) are the electric dipole matrix elements, and \( N \) is atomic density. \( \varepsilon_0 \) is the permittivity of the vacuum. \( \delta \) is resonance linewidth. \( \Delta_1 = \omega_i - \omega_{11} \) is the pump detuning from the atomic transition \( |1\rangle \rightarrow |3\rangle \). \( \Delta_2 = \omega_p - \omega_{20} \) is the pump detuning from the atomic transition \( |0\rangle \rightarrow |2\rangle \).

By making the denominator of nonlinear susceptibility to zero, we can get

\[
\delta_{AF} = -\Delta_1 \pm \sqrt{\Delta_1^2 + 4\Omega_{10}^2 \Gamma_0 + 4(\alpha_0 \Omega_{M1}^2 + \Omega_{M1}^2)^2(\cos^4 \theta + \sin^4 \theta) / 2},
\]

which indicates that two four types of SFWMs appear in the interaction process. Correspondingly, four two types of biphotons can be generated from these SFWMs processes in the spontaneous emission region. Every type of SFWM process should meet the conservation of energy. Thus, the energy conservation equation \( \omega_i + \omega_p = \omega_i + \delta_i + \omega_{11} + \delta_{AF} \) should be satisfied. Similarly, we can get it \( \delta_{AF} \). In such a case, two spontaneous FWM channels occur. From figure 3, we can see the prominent differences between the outputs of linearly (figures 3(a), (b)) and circularly polarized dressing fields (figures 3(c), (d)). As shown in figures 3(a), (b), the two peaks in the linear polarization of the dressing field are closer than the circularly polarized output, which leads to a longer period. Based on this, we can change dispersion relation by choosing selective polarization of incident lights, which can cause a difference in periods of Rabi oscillations. Since line widths of each peak in both polarizations of the dressing field are different, therefore, we can obtain paired photons with different coherence times. This is how the coherence time and the periods of output can be controlled and modulated using a specific type of polarized dressing of incident filed.

When we take the external dressing field \( E_3 \) into count, there are several different combinations of these two fields. \( E_1, E_3 \).

1. The QWP prior to the external dressing field is set at 0 degrees while \( E_3 \) dressing filed is linearly polarized.

Then the third-order nonlinear susceptibility of Stokes and anti-Stokes can be expressed as

\[
\chi^{(3)}_{\text{abM}} = -\frac{N_i}{(\Gamma_{12} + i\Delta_1)} \left( \Gamma_{12} + i\delta \right) \left( \Gamma_{12} + \frac{2\Omega_0 \Omega_{M1}^2 (\cos^2 \theta + \sin^2 \theta)}{\Gamma_{12} + i\Delta_1} \right) \left( \Gamma_{12} + i\delta + i\Delta_1 \right),
\]

\[
\chi^{(3)}_{\text{AF}} = -\frac{N_i}{(\Gamma_{12} + i\Delta_1)} \left( \Gamma_{12} + i\delta + \frac{2\Omega_0 \Omega_{M1}^2 (\cos^2 \theta + \sin^2 \theta)}{\Gamma_{12} + i\Delta_1} \right) \left( \Gamma_{12} + i\delta + i\Delta_1 \right),
\]
\[ \chi^{(3)}_{\text{aM}} = - \frac{iN \mu_{13} \mu_{14} \mu_{23} \mu_{24}}{\varepsilon_0 h^3} \times \frac{1}{(\Gamma_{200} + i\Delta_2)} \times \frac{1}{(\Gamma_{300} + i\delta + i\Delta_3)} \times \left( \Gamma_{01M} - i\delta + \frac{\Omega_{21}^2}{\Gamma_{21M} - i\delta - i\Delta_2} \right) \times \frac{1}{(\Gamma_{21M} - i\delta + i\Delta_2)} \] 

(10)

(2) The QWP prior to the external dressing field is set at 45 degrees while \( E_3 \) is circularly polarized. Then the third-order nonlinear susceptibility of Stokes and anti-Stokes can be expressed as

\[ \chi^{(3)}_{\text{aM}} = - \frac{iN \mu_{13} \mu_{14} \mu_{23} \mu_{24}}{\varepsilon_0 h^3} \times \frac{1}{(\Gamma_{200} + i\Delta_2)} \times \left( \Gamma_{301} + i\Delta_3 \right) \times \frac{1}{(\Gamma_{31M} - i\delta + i\Delta_2)} \times \left( \frac{\Omega_{21}^2}{\Gamma_{21M} - i\delta - i\Delta_2} \right) \times \left( \Gamma_{21M} - i\delta + i\Delta_2 \right) \] 

(11)

(1) The QWP before the external dressing field and coupling field are both set at 0 degrees while \( E_3 \) and \( E_c \) are linearly polarized. Then the third-order nonlinear susceptibility of Stokes and anti-Stokes can be expressed as

\[ \chi^{(3)}_{\text{aM}} = - \frac{iN \mu_{13} \mu_{14} \mu_{23} \mu_{24}}{\varepsilon_0 h^3} \times \frac{1}{(\Gamma_{200} + i\Delta_2)} \times \left( \Gamma_{300} + i\delta + \frac{\Omega_{21}^2}{\Gamma_{21M} - i\delta - i\Delta_2} \right) \times \frac{1}{(\Gamma_{300} + i\delta + i\Delta_3)} \times \left( \frac{\Omega_{21}^2}{\Gamma_{21M} - i\delta - i\Delta_2} \right) \times \left( \Gamma_{21M} - i\delta + i\Delta_2 \right) \] 

(12)

(2) The QWP prior to the external dressing field and coupling field are both set at 45 degrees while \( E_3 \) and \( E_c \) are circularly polarized. Then the third-order nonlinear susceptibility of Stokes and anti-Stokes can be expressed as

\[ \chi^{(3)}_{\text{aM}} = - \frac{iN \mu_{13} \mu_{14} \mu_{23} \mu_{24}}{\varepsilon_0 h^3} \times \frac{1}{(\Gamma_{200} + i\Delta_2)} \times \left( \Gamma_{301} + i\Delta_3 \right) \times \frac{1}{(\Gamma_{31M} - i\delta + i\Delta_2)} \times \left( \frac{\Omega_{21}^2}{\Gamma_{21M} - i\delta - i\Delta_2} \right) \times \left( \Gamma_{21M} - i\delta + i\Delta_2 \right) \] 

(13)
As we can see from figures 4 and 5, the resonances have been divided into three peaks because of three roots when we solve the equation, which indicates a triplet of resonances and three spontaneous FWM channels occur suggesting hyperentangled and would cause biphoton counting to be different.

4. Biphoton counting with different polarization of double dressing fields

Following the third-order nonlinear susceptibility, we can obtain biphoton coincidence count. In this condition, the effective coupling Rabi frequency will cause multi-mode SFW channels occurrence, which can generate multi-mode biphoton along with beating and interference effects. With the multi-mode biphoton beating or destructive interference with each other, the wave function of biphoton with effects of beating and interferences will appear as damped Rabi oscillation.

To study the optical properties of photons generated from such a four-level system, we need to consider the averaged biphoton coincidence counting rate. As shown in simplified setup, anti-Stokes photons and Stokes photons are detected by different detectors with their respective detection time. According to \( \Phi = 1 \), after performing mathematical calculations under the assumption of perfect detection efficiency, the averaged biphoton coincidence counting rate can be written as

\[
R_{cc} = N_{c} E_{1} E_{2} \frac{\omega_{1} \omega_{2}}{2\pi} e^{-\frac{i}{\hbar} \omega_{1} \tau_{1} + \omega_{2} \tau_{2}} \left( -\Delta_{p} + i\Delta_{r} \right) \left( 2e^{-2i\omega_{r}\tau_{12}}(1 - \cos[\Omega_{\text{el}}\tau_{12}])\Theta(\tau_{12}) \right). \tag{14}
\]

In this Eq, \( \Omega_{\text{el}} \) and \( \Gamma_{\text{el}} \) can be written as

\[
\Omega_{\text{el}} = (\Delta_{1}^{2} + 4((CG_{\text{lin}} \times \Omega_{\text{IM}}^{0})^{2})(\cos^{4} \theta + \sin^{4} \theta) + \Gamma_{10}\Gamma_{30})^{1/2},
\]

\[
\Gamma_{\text{el}} = \frac{-\left( \Delta_{1} + \sqrt{\Delta_{1}^{2} + 4((CG_{\text{lin}} \times \Omega_{\text{IM}}^{0})^{2})(\cos^{4} \theta + \sin^{4} \theta) + \Gamma_{10}\Gamma_{30}) \right)}{2\Delta_{1}} + \Gamma_{10}, \tag{15}
\]

where \( \tau_{12} = \tau_{1} - \tau_{2} \) is the time difference between detection of Stokes and anti-Stokes photon while \( \tau_{1} \) is detection time of Stokes photon and \( \tau_{2} \) is detection time of anti-Stokes photon. Obviously, the averaged biphoton coincidence counting rate only has one oscillation period, which results from \( 2\pi / \Omega_{\text{el}} \) in the direction \( \tau_{12} \). The Heaviside step function here is defined as \( \Theta(\tau_{12}) = 1 \) for \( \tau_{12} \geq 0 \), and \( \Theta(\tau_{12}) = 0 \) for \( \tau_{12} < 0 \).
As for linear polarization \( q = 0^\circ \):

\[
R_{\ell} = \left| N_1 E_1 E_2 \frac{\sqrt{\omega_1 \omega_2}}{2\pi} e^{-i(\omega_1 + \omega_2) t}\right|^2 2e^{-2\Omega_{\ell}|\theta=0, q_\ell|\frac{\omega_{\ell}}{2\Delta}\left(1 - \cos\{\Omega_{\ell}|\theta=0, q_\ell|\frac{\omega_{\ell}}{2\Delta}\right)\}} \Theta(\tau_{\ell}).
\] (16)

In this Eq, \( \Omega_{\ell}|\theta=0, q_\ell| \) and \( \Gamma_{\ell}|\theta=0, q_\ell| \) can be written as

\[
\Omega_{\ell}|\theta=0, q_\ell| = \left( \Delta_i^2 + 4\left(\text{CG}_{\text{lin}} \times \Omega_{\ell|M}^0 \right)^2 + \Gamma_{10}\Gamma_{30}\right)^{1/2},
\]

\[
\Gamma_{\ell}|\theta=0, q_\ell| = \frac{-\left(\Delta_i^2 + 4\left(\text{CG}_{\text{lin}} \times \Omega_{\ell|M}^0 \right)^2 + \Gamma_{10}\Gamma_{30}\right)}{2\Delta_i} + \Gamma_{10}.
\] (17)

For circular polarization \( q = 45^\circ \):

\[
R_{c\ell} = \left| N_1 E_1 E_2 \frac{\sqrt{\omega_1 \omega_2}}{2\pi} e^{-i(\omega_1 + \omega_2) t}\right|^2 2e^{-2\Omega_{\ell}|\theta=45, q_\ell|\frac{\omega_{\ell}}{2\Delta}\left(1 - \cos\{\Omega_{\ell}|\theta=45, q_\ell|\frac{\omega_{\ell}}{2\Delta}\right)\}} \Theta(\tau_{\ell}).
\] (18)

In this Eq, \( \Omega_{\ell}|\theta=45, q_\ell| \) and \( \Gamma_{\ell}|\theta=45, q_\ell| \) can be written as

\[
\Omega_{\ell|M}|\theta=45, q_\ell| = \left( \Delta_i^2 + 4\left(\text{CG}_{\text{lin}} \times \Omega_{\ell|M}^0 \right)^2 + \Gamma_{10}\Gamma_{30}\right)^{1/2},
\]

\[
\Gamma_{\ell}|\theta=45, q_\ell| = \frac{-\left(\Delta_i^2 + 4\left(\text{CG}_{\text{lin}} \times \Omega_{\ell|M}^0 \right)^2 + \Gamma_{10}\Gamma_{30}\right)}{2\Delta_i} + \Gamma_{10}.
\] (19)

It can be easily seen from the demonstrated in figure 6 that coincidence counting rate, time period and coherence time of biphoton are obtained to be different with change in the polarization of the incident light. Apparently, the are averaged biphoton coincidence counting rate of circular polarization (figure 6(b)) is appeared to be a higher in amplitude, shorter in the time period and a longer with coherence time when compared with coincidence counting rate obtained with linear polarization (figure 6(a)). The results of both cases are consistent with equations (13) and (15). These differences between the outputs of linear and circular polarization can be also seen in figure 3. The period and attenuation of the averaged biphoton coincidence counting rate depend on \( W_{\text{eM}} \) and \( G_{\text{eM}} \) as \( q \) is included in \( W_{\text{eM}} \) and \( G_{\text{eM}} \). Based on these observations, we can alter the period of damped oscillation amplitude, and attenuation of the averaged biphoton coincidence counting rate. Hence, we can adjust the dressing field by changing the polarization of the incident light to obtain paired photons with the desired coherence time for the application of quantum information processing.

Compared with figure 6, the period of oscillation has been changed a lot when we consider the external dressing field. As we can see from figure 7, the coincidence counts present multi-oscillation periods, and different polarization configurations will change the coincidence rate because the nonlinear susceptibilities are changed when we use various polarized states of the external dressing field.

5. Biphoton quantum tomography with polarized dressing

To comprehensively characterize the polarization of the incident lights, we can measure the output state. So we perform quantum-state tomography using the maximum likelihood estimation method [23]. Here, we obtain various output quantum states by applying HWP with different angles and get these theoretical results to further study the influence on these output quantum states under different susceptibilities. For two qubits, 16 double-coincidence measurements are necessary and here we can access all 16 independent projection states by

![Figure 6. Biphoton coincidence counting rate when don’t consider dressing field with (a) linearly polarized field (b) circularly polarized field.](image-url)
adjusting the angle of QWP and HWP. It should be noted that there is no difference between our method and Daniel F. V. James’s except the direction of projection measurement [23]. The QWP and HWP fast axes are at angles of \( q \) and \( h \) with respect to the horizontal axis, respectively. One of the projection states for the measurement is

\[
|\psi_{\text{proj}}^{(1)}(h, q)\rangle = \hat{U}_{\text{QWP}}(q) \cdot \hat{U}_{\text{HWP}}(h) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

(20)

So the projection state for the two beams can be written as

\[
|\psi_{\text{proj}}^{(2)}(h_1, q_1, h_2, q_2)\rangle = |\psi_{\text{proj}}^{(1)}(h_1, q_1)\rangle \otimes |\psi_{\text{proj}}^{(1)}(h_2, q_2)\rangle
\]

\[= a(h_1, q_1)a(h_2, q_2)|HH\rangle + a(h_1, q_1)b(h_2, q_2)|HV\rangle + b(h_1, q_1)a(h_2, q_2)|VH\rangle + b(h_1, q_1)b(h_2, q_2)|VV\rangle.
\]

(21)

Obviously, the projection state is related to the particular set of wave plate angles \( \{ h_1, h_2, q_1, q_2 \} \). As a result, the average number of coincidence counts is given as

\[n_h = N \langle \psi_h | \hat{\rho} | \psi_h \rangle,
\]

(22)

where \( \hat{\rho} \) is the density matrix and \( N \) is a constant and depends on the photon flux and detector efficiencies. For linear polarization, the average number of coincidence counts is

\[n_{h,\text{lin}} = N_{\text{lin}} \langle \psi_h | \hat{\rho} | \psi_h \rangle \propto |\chi^{(3)}_{\text{lin}}|_{\theta=0^\circ}
\]

(23)

Similarly, for circular polarization

\[n_{h,\text{cir}} = N_{\text{cir}} \langle \psi_h | \hat{\rho} | \psi_h \rangle \propto |\chi^{(3)}_{\text{cir}}|_{\theta=45^\circ}
\]

(24)

Then we can obtain a density matrix in different polarization configurations. Figures 8(a) and (e) are the most common Bell states. As shown in figure 8(a), when the incident beams are \( HH \), the output state can be written as

\[\Phi = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle).
\]

(25)

When the incident beams are \( HV \), the output state can be written as

\[\Psi = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle).
\]

(26)

Thus, different output states can be realized by adjusting the angle between HWP and x-axis (figure 8). According to [24], \( \hat{\rho} = (1/d) \hat{\rho} + ((d - 1)/d) \sigma_x \otimes \rho_y \) can describe the output states. We can achieve different Werner-states by adjusting the angle of the HWP behind the PBS. Different angles of the HWP will change the third-order nonlinear susceptibility. Thus the proportion of \( HH, HV, VH \) and \( VV \) will be different in each Werner-state and the heights are controllable which is being reflected in figure 8. When the angle is zero, the inputs are \( HH \) and the third-order nonlinear susceptibility can be described as \( \chi_{xxxx} \). And when the angle is 90 degree, the inputs are \( HV \) and the third-order nonlinear susceptibility can be described as \( \chi_{xxyy} \).

6. Interference with polarization dressing

Theoretically, we get different visibilities under different polarization configuration by theoretical calculation, while the effects of polarized light on interference visibility are studied.
With linearly polarized dressing field \((\theta = 0^\circ)\)

\[
C_{\text{lin}} = R_{\text{cir}} \max \left( \frac{1}{2} + \frac{1}{2} \cos(\varphi_1 + \varphi_2) \right) + A.
\]  

With circularly polarized dressing field \((\theta = 45^\circ)\)

\[
C_{\text{cir}} = R_{\text{cir}} \max \left( \frac{1}{2} + \frac{1}{2} \cos(\varphi_1 + \varphi_2) \right) + A,
\]

where \(A\) represents accidental counts \(\varphi_1\) and \(\varphi_2\) are polarization angles for Stokes photons and anti-Stokes photons, respectively.

Figure 9 compares the variable visibility dressed by linear polarized laser and by circularly polarized laser. The output state dressed by circularly polarized laser have a higher visibility than dressed by linear polarized laser. Here we fix the Stokes photon polarization angle at \(0^\circ\) and \(180^\circ\). With the maximum coincidence counts \(C_{\text{max}}\) and minimum coincidence counts \(C_{\text{min}}\) in figure 9, we can determine the visibility

\[
V = \frac{(C_{\text{max}} - C_{\text{min}})}{(C_{\text{max}} + C_{\text{min}})}\]

of the biphoton interference. We obtain the interference visibility for linear \((V_{\text{lin}} = 81.7\%)\) and circular \((V_{\text{cir}} = 85.2\%)\), which is larger than \(1/\sqrt{2}\) and violates the Bell–Clauser–Horne–Shimony–Holt inequality and indicates the expected polarization entanglement \([3]\). Since the existence
of accidental counts, the minimum coincidence is not zero experimentally, and the interference visibility cannot reach to 100%. In our case, circularly polarized light has larger interference visibility compared with linearly polarized light. This indicates the variable visibility with different polarization configurations, which can be changed by adjusting the polarization dressing field because of different third-order nonlinear susceptibilities, which is included in Rcc.

7. Conclusion

In conclusion, we obtained different waveforms of coincidence counting rates under different polarization configurations of the double dressing fields. Specifically, the waveform of circular polarization was observed with a shorter period of oscillation and with longer coherence time when compared with linear polarization. The double dressing fields, when compared with the single dressing field, changed third-order nonlinear susceptibility, biphoton response. We also performed quantum tomography to characterize the biphoton polarization entanglement. Theoretically, we obtained different output entangled states by adjusting the angle between the wave plate in the path of incident light and x-axis. At last, we achieved different interference visibilities with employed polarization configurations to explain experimental results. In the future, we would like to analyze the effect on the biphoton coincidence rate when we consider the coupling field and external dressing field both are linearly polarized or circularly polarized.

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