NUMERICAL SIMULATION OF MAGNETIC RECONNECTION AROUND A BLACK HOLE

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ABSTRACT

We performed numerical simulations of general relativistic magnetohydrodynamics with uniform resistivity to investigate the occurrence of magnetic reconnection in a split-monopole magnetic field around a Schwarzschild black hole. We found that magnetic reconnection happens near the black hole at its equatorial plane. The magnetic reconnection has a point-like reconnection region and slow shock waves, as in the Petschek reconnection model. The magnetic reconnection rate decreases as the resistivity becomes smaller. When the global magnetic Reynolds number is $10^4$ or larger, the magnetic reconnection rate increases linearly with time from $2\tau_S$ to $\sim 10\tau_S$ ($\tau_S = r_S/c$, $r_S$ is the Schwarzschild radius and $c$ is the speed of light). The linear increase of the reconnection rate agrees with the magnetic reconnection in the Rutherford regime of the tearing mode instability.

*Keywords:* black hole physics — magnetic fields — plasmas — methods: numerical — Galaxy: nucleus — stars: black holes
1. INTRODUCTION

Galaxies classified as a class of active galactic nuclei (AGNs) are believed to harbor supermassive black holes in their centers. Stellar mass black holes are also thought to reside in long-duration $\gamma$-ray bursts (LGRBs) and microquasars in our Galaxy. From some of these black holes, plasma ejections, like moving radio knots or jets, are observed. As an example of radio knot ejections from AGNs, Acciari et al. (2009) presented simultaneous radio and $\gamma$-ray observations of the nearby active galaxy M87, and showed that radio knots were ejected from the core of the galaxy when the TeV $\gamma$-ray flare occurred. As for relativistic jets from AGNs, Biretta et al. (1999) reported observations of superluminal motion in the M87 jet by the Hubble Space Telescope. Kulkarni et al. (1999) presented optical and near-infrared observations of the afterglow of GRB990123, and argued that the detected $\gamma$-ray is relativistically beamed. Mirabel & Rodriguez (1994) reported superluminal motion of the radio-emitting ejecta from the center of microquasar GRS 1915+105. These plasma boosts are believed to be caused by violent phenomena around black holes.

Such plasma ejections are also observed in solar flares (e.g., Ohyama & Shibata 1998). Solar flares and associated coronal mass ejections are phenomena related to solar magnetic field: they exhibit a sudden energy release of magnetic energy, which is transformed to electromagnetic radiation energy (flares) or mechanical kinetic energy (coronal mass ejections). An idea called magnetic reconnection was proposed to explain solar flares in the 1940s (Giovanelli 1946), and in the 1950–1960s the magnetohydrodynamics (MHD) theory of magnetic reconnection was constructed by Sweet (1958), Parker (1957), and Petschek (1964), known as the Sweet–Parker and Petschek magnetic reconnection models. An important physical quantity of magnetic reconnection is the magnetic reconnection rate, $R_{\text{mr}}$. $R_{\text{mr}}$ expresses the time change rate of magnetic flux at the reconnection point. From the Faraday's law, the parallel component of the electric field to the reconnection line (X-line) at the reconnection point, $E_{\parallel}$, represents this rate. By normalizing $E_{\parallel}$ with the physical quantities of the plasma, $R_{\text{mr}}$ is obtained. From a theoretical point of view, $R_{\text{mr}}$ depends on the reconnection type and stage. The Sweet–Parker and Petschek models describe the stationary (steady) state of magnetic reconnection, so $R_{\text{mr}}$ is time-independent in these models. As for the resistivity-dependence of
$R_{mr}, R_{mr} = 1/\sqrt{S_e}$ for the Sweet–Parker model, and $R_{mr}(\text{max}) = \pi/(8\ln S_e)$ for the Petschek model. Here, $S_e$ is the global magnetic Reynolds number and $R_{mr}(\text{max})$ is the maximum magnetic reconnection rate estimated by Petschek. We give exact definitions of $R_{mr}, S_e$, and the magnetic Reynolds number $S$ in Section 3. Both $S$ and $S_e$ are inversely proportional to resistivity (Parker 1957; Sweet 1958; Petschek 1964; Priest & Forbes 2000; Kulsrud 2005). Considering the typical time scale of a flare, $10^2 - 10^4$ s, the Petschek model better explains the rapid energy release in a flare than the Sweet–Parker model (e.g., Kulsrud 2005; Shibata & Magara 2011). Here we mention that for the occurrence of the Petschek-type reconnection, the local enhancement of resistivity seems to be an essential process (but also see Baty et al. 2009 for a different consideration; we describe it in section 3) (Shibata & Magara 2011).

Before magnetic reconnection, or a current sheet, settles down to the stationary stage as the Sweet–Parker or Petschek-type reconnections, the sheet is subject to the tearing mode instability (Furth et al. 1963), which is a nonstatic state. This state starts from the linear growth stage (Furth et al. 1963) and then shifts to the nonlinear growth stage called the Rutherford regime (Rutherford 1973; Murphy et al. 2008). We illustrate the time development of magnetic reconnection ($R_{mr}$ as a function of time) in plasma with uniform resistivity and an initially uniform current sheet in Figure 1. During the tearing mode instability, $R_{mr}$ varies with time. From the theories, $R_{mr}$ is proportional to $e^{\gamma t}$ where $\gamma \propto 1/\sqrt{S}$ in the linear growth stage (Furth et al. 1963; Biskamp 1993) and $R_{mr}$ is proportional to $t$ in the Rutherford regime (Rutherford 1973). The tearing mode instability was first proposed as the theory to disruptions in laboratory fusion devices such as tokamaks. This instability eventually introduces nonuniformity into the current sheet where a series of magnetic islands or plasmoids is formed. Nonsteady reconnection associated with multiple magnetic islands often causes impulsive bursty reconnection (Priest 1985; Shibata & Magara 2011). The tearing mode instability, especially in the nonlinear Rutherford regime, is a complicated process, but it is important to understand time evolution of magnetic reconnection, and also the intermittent energy release (nonsteady reconnection) commonly observed in the Sun, terrestrial magnetosphere, and laboratory plasmas.
The magnetic reconnection can also be a key process to cause ejection from black holes. To create an occurrence of magnetic reconnection around a black hole, an anti-parallel magnetic field with a current sheet needs to exist there. We here discuss the possibility of spontaneous formation of an antiparallel magnetic field around a black hole. First, let us assume that the initial magnetic field has the uniform magnetic component, aligned with the rotational axis of a rapidly spinning black hole. In this magnetic configuration, one may think that an antiparallel magnetic field is scarcely formed. However, even in the initially uniform magnetic field, the simulation of the general relativistic MHD with zero electric resistivity (ideal GRMHD) showed that the magnetic flux tubes become stationarily antiparallel in the ergosphere around the equatorial plane of the black hole (Komissarov 2005). Next, as the initial condition, we consider magnetic flux tubes connecting an accretion-disk around a rotating black hole and a black hole ergosphere. With this initial condition, ideal GRMHD simulations also indicated that the antiparallel magnetic field is formed spontaneously after a sufficiently long term (Koide et al. 2006; McKinney 2006). As shown in the above two cases of magnetic field, an antiparallel magnetic field (the magnetic configuration where magnetic reconnection occurs) is
relatively easily formed around a spinning black hole; thus, magnetic reconnection is expected to happen frequently around a black hole. This may explain observed plasma boosts from black holes.

Koide & Arai (2008) investigated energy extraction from a rotating black hole by magnetic reconnection in the ergosphere. As they discussed, the phenomena of magnetic reconnection around a black hole should be investigated by numerical simulations of the full GRMHD with nonzero electric resistivity (resistive GRMHD). Two groups, to our knowledge, have developed numerical codes to solve the equations of resistive GRMHD (Bucciantini & Del Zanna 2013; Dionysopoulou et al. 2013). Dionysopoulou et al. (2013) simulated the gravitational collapse of a magnetized, nonrotating neutron star to a black hole, and Dionysopoulou et al. (2015) studied the dynamics of binary neutron stars. In the work of Dionysopoulou et al. (2015), when a black hole had been formed after the merger of two neutron stars, a magnetic-jet structure was formed in the low-density funnel produced by the black hole–torus system; though, a relativistic outflow was not produced in their results. Regarding accretion disks of AGNs, Bugli et al. (2014), with the code of Bucciantini & Del Zanna (2013), considered dynamo action in thick disks around Kerr black holes. However, no simulation work with resistive GRMHD has been released that shows clear magnetic reconnection feature in plasma in the universe.

In this paper, we report the results of numerical simulations using the resistive GRMHD code that we developed to investigate the occurrence of magnetic reconnection in plasma (possibly an accretion disk) just outside a black hole. We numerically explored a time evolution of physical quantities in the field. In our results, magnetic reconnection is for the first time clearly visible in plasma near a black hole. To investigate the basic physics of magnetic reconnection around a black hole, we assumed a Schwarzschild black hole, and split-monopole magnetic field around it as the initial condition of the magnetic field. We found that relatively fast magnetic reconnection happens near the black hole at its equatorial plane. The magnetic reconnection has the point-like reconnection region and the slow shock waves, as in the Petschek reconnection model. We also observed formation of magnetic islands by the magnetic reconnection process.
In Section 2, we show basic equations of our resistive GRMHD code and present assumptions of our simulations. Our numerical simulation results are given in Section 3. We discuss and summarize our work in Section 4.

2. METHOD OF RESISTIVE GRMHD SIMULATIONS

2.1. Covariant form of resistive GRMHD equations

To investigate the basic process of magnetic reconnection around a black hole, we performed numerical simulations with the resistive GRMHD equations of plasmas and electromagnetic fields around a Schwarzschild black hole. In the space-time, \( x^\mu = (t, x^1, x^2, x^3) = (t, r, \theta, \phi) \) around a Schwarzschild black hole with mass \( M \), the line element \( ds^2 \) is given by

\[
 ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \sum_i h_i^2 (dx^i)^2 = -\alpha^2 dt^2 + \frac{1}{\alpha^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,
\]

where \( \alpha = \sqrt{1 - r_S/r} \) is the lapse function and \( r_S = 2M \) is the Schwarzschild radius. Here, Greek subscripts such as \( \mu \) and \( \nu \) run from 0 to 3, whereas Roman subscripts such as \( i \) and \( j \) run from 1 to 3. We use the natural unit system where the speed of light \( c \), the gravitational constant \( G \), the magnetic permeability and electric permittivity in a vacuum \( \mu_0, \varepsilon_0 \), are unity.

The covariant form of standard resistive GRMHD equations consists of the conservation law of particle number, the conservation law of energy and momentum, the Maxwell equations, and the relativistic Ohm’s law with resistivity. Here, we ignore radiation-cooling effects, plasma viscosity, and self-gravity. The equations are written as

\[
 \nabla_\mu (\rho U^\mu) = 0,
\]

\[
 \nabla_\mu T^{\mu\nu} = 0,
\]

\[
 \nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} F^{\mu\nu}) = -J^\nu,
\]

\[
 \nabla_\mu F^{\nu\mu} = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} F^{\nu\mu}) = 0,
\]

\[
 F_{\mu\nu} U^\nu = \eta [J_\mu + (U_\nu J^\nu) U_\mu],
\]

where \( \nabla_\mu \) is the covariant derivative, \( \rho \) is the proper mass density, \( U^\mu \) is the four-velocity, \( T^{\mu\nu} = hU^\mu U^\nu + pg^{\mu\nu} + F^{\mu\sigma} F_{\nu\sigma} - (F^{\rho\sigma} F_{\rho\sigma}) g^{\mu\nu}/4 \) is the energy-momentum tensor of the plasma and the
electromagnetic field \((h)\) is the proper enthalpy density and \(p\) is the pressure of the plasma, \(F_{\mu\nu}\) is the electromagnetic field tensor, \(\ast F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}/2\) is the dual tensor of \(F_{\mu\nu}\), \(J^\mu = (\rho_e, J^1, J^2, J^3)\) is the four-current density \((\rho_e\) is the electric charge density), and \(\eta\) is the resistivity of the plasma. \(\epsilon_{\mu\nu\rho\sigma}\) is the Levi-Civita tensor, which is defined as 
\[
\epsilon_{\mu\nu\rho\sigma} = \frac{\eta_{\mu\nu\rho\sigma}}{\sqrt{-g}},
\]
where \(\eta_{\mu\nu\rho\sigma}\) is the totally asymmetric symbol defined as 
\[
\eta_{\mu\nu\rho\sigma} = \begin{cases} 1 & \text{if the order } [\mu\nu\rho\sigma] \text{ is an even permutation of } [0123], \\ -1 & \text{if the order } [\mu\nu\rho\sigma] \text{ is an odd permutation of } [0123], \\ 0 & \text{otherwise}, \end{cases}
\]
\(\eta_{\mu\nu\rho\sigma}\) unless \(\mu, \nu, \rho, \sigma\) are all different.

### 2.2. 3+1 Formalism of Resistive GRMHD Equations

In order to derive the 3+1 formalism of the resistive GRMHD equations, we introduce the local coordinate frame called the “zero angular momentum observer (ZAMO) frame,” \(\hat{x}^\mu = (\hat{t}, \hat{x}^i)\), which is defined by
\[
ds^2 = \eta_{\mu\nu} \hat{x}^\mu \hat{x}^\nu = -d\hat{t}^2 + \sum_{i=1}^{3} (d\hat{x}^i)^2,
\] (7)
where \(\eta_{\mu\nu}\) is the metric of Minkowski spacetime. Comparing Equations (1) and (7), we have the relation \(d\hat{t} = \alpha dt, d\hat{x}^i = h^i dx^i\). We use the quantities observed by the ZAMO frame because they can be treated intuitively, because the relations between the variables in the ZAMO frame are the same as those in the special theory of relativity. Hereafter we denote the variables observed in the ZAMO frame with the hat.

Using the quantities of the electromagnetic field in the ZAMO frame, the resistive GRMHD equations are written in the following 3+1 formalism,

\[
\frac{\partial D}{\partial t} = -\frac{1}{h_1 h_2 h_3} \sum_i \frac{\partial}{\partial x^i} \left( \frac{\alpha h_1 h_2 h_3}{h_i} D \frac{\partial}{\partial x^i} \right),
\] (8)

\[
\frac{\partial \hat{P}^i}{\partial t} = -\frac{1}{h_1 h_2 h_3} \sum_j \frac{\partial}{\partial x^j} \left( \frac{\alpha h_1 h_2 h_3}{h_j} \hat{\nabla}^j \right) - (\epsilon + D) \frac{1}{h_i} \frac{\partial \alpha}{\partial x^i} + \alpha f^i_{\text{curv}},
\] (9)

\[
\frac{\partial \epsilon}{\partial t} = -\frac{1}{h_1 h_2 h_3} \sum_i \frac{\partial}{\partial x^i} \left[ \alpha h_1 h_2 h_3 (\hat{P}^i - D \hat{v}^i) \right] - \sum_i \hat{P}^i \frac{1}{h_i} \frac{\partial \alpha}{\partial x^i},
\] (10)

\[
\hat{E}_i + \epsilon_{ijk} \hat{v}^j \hat{B}^k = \frac{\eta}{\gamma} \left[ \hat{J}^i - \gamma^2 (\rho_e - \hat{v}_j \hat{J}^j) \hat{v}^i \right],
\] (11)

\[
\frac{\partial \hat{B}_i}{\partial t} = -\frac{h_i}{h_1 h_2 h_3} \sum_{j,k} \epsilon^{ijk} \frac{\partial}{\partial x^j} (\alpha h_k \hat{E}_k),
\] (12)
\[
\sum_i \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial x^i} \left( \frac{h_1 h_2 h_3}{h_i} \hat{B}_i \right) = 0, \quad (13)
\]
\[
\rho_e = \sum_i \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial x^i} \left( \frac{h_1 h_2 h_3}{h_i} \hat{E}_i \right), \quad (14)
\]
\[
\alpha \hat{J}^i + \frac{\partial \hat{E}_i}{\partial t} = \sum_{j,k} \frac{h_i}{h_1 h_2 h_3} \hat{\epsilon}^{ijk} \frac{\partial}{\partial x^j} (\alpha h_k \hat{B}_k), \quad (15)
\]

where \( D = \gamma \rho \) is special relativistic mass density, \( \gamma \) is the Lorentz factor, \( \hat{v}^i = \hat{u}^i / \gamma \) is the three-velocity, \( \hat{P}^i = h \gamma^{2} \hat{v}^i + \hat{\epsilon}^{ijk} \hat{E}_j \hat{B}_k \) is the special relativistic total momentum density, \( \epsilon = \hat{T}^{00} - D = h \gamma^{2} - p - D + \hat{B}^{2} / 2 + \hat{E}^{2} / 2 \) is the special relativistic total energy density, \( \hat{T}^{ij} = h \gamma^{2} \hat{v}^i \hat{v}^j + (p + \hat{B}^{2} / 2 + \hat{E}^{2} / 2) \delta^{ij} - \hat{B}^i \hat{B}^j - \hat{E}^i \hat{E}^j \) is the total stress tensor, \( \hat{E}_i = \hat{F}_{i0} \) is the electric field, \( \hat{B}^i = \ast \hat{F}^0_i \) is the magnetic field, and

\[
f^{i}_{\text{curv}} \equiv - \sum_{j} \left( \frac{1}{h_i h_j h_3} \frac{\partial h_i}{\partial x^j} \hat{T}^{ij} - \frac{1}{h_j^2} \frac{\partial h_j}{\partial x^i} \hat{T}^{jj} \right)
\]

is the term containing the centrifugal force. Here, we used \( \hat{\epsilon}^{ijk} \equiv \eta^{0ijk} \).

We solve these 3+1 form of equations numerically. We extended the numerical method of the resistive special relativistic MHD (resistive RMHD) developed by Watanabe & Yokoyama (2006) to the general relativistic version. Note that Watanabe & Yokoyama (2006) developed the resistive RMHD code for the first time, and they carried out numerical simulations of two-dimensional magnetic reconnection. Resistive RMHD simulations of magnetic reconnection were also presented by Zenitani et al. (2010), who discovered the post-plasmoid vertical shocks and the diamond-chain structure.

We employ the HLL flux solver and the MUSCL interpolation for the numerical simulation (Koide & Morino 2011; Morino 2011). We assume the plasma and field are axisymmetric with respect to the axis of the black hole.

### 2.3. Setup of numerical resistive GRMHD simulations

As the initial condition of the magnetic field, we have the split monopole magnetic field around the Schwarzschild black hole:

\[
\hat{B}^r = \frac{B_0}{r^2} \tanh \left( \frac{\theta - \pi / 2}{\Delta \theta_{cw}} \right), \quad \hat{B}^\theta = \hat{B}^\phi = 0,
\]

(16)
Numerical simulation of magnetic reconnection around black hole

where $B_0$ is a constant and $\Delta \theta_{cw}$ gives the current sheet width (thickness of the current sheet) at the equatorial plane. To introduce this split monopole magnetic field, we refer to the antiparallel magnetic field given by Harris (1962). We set the plasma and the magnetic field around the current sheet at the equatorial plane and they are vertically in equilibrium initially. The initial conditions of the plasma are given as

$$\rho = \frac{\rho_0}{\sqrt{2Mr^3}},$$  \hspace{0.5cm} (17)

$$p = \frac{B_0^2}{2r^4 \cosh^2[(\theta - \pi/2)/\Delta \theta_{cw}]} + p_b, \quad p_b = \frac{\beta_p B_0^2}{8\pi} \left( \frac{\rho}{\rho_0} \right)^{\Gamma},$$  \hspace{0.5cm} (18)

$$\dot{v}^r = -0.8 \sqrt{\frac{2M}{r}}, \quad \dot{v}^\theta = \dot{v}^\phi = 0,$$  \hspace{0.5cm} (19)

where $\beta_p \equiv p/(\dot{B}^2/2)$ is the plasma beta value. We set the resistivity of the plasma $\eta$ uniform in space and constant in time in this paper.

With respect to the radial coordinate $r$, we actually employ the modified tortoise coordinate, $x = \ln[(r - r_{\text{min}})/a_0 + 1]$. Here, $r_{\text{min}}$ is the radial coordinate of the inner boundary near the horizon and $a_0$ is a constant. With a uniform mesh in the $x$-coordinate, the radial mesh width of the $r$-coordinate is proportional to $r - r_{\text{min}} + a_0$. Since the eigenspeed of the MHD waves near the black hole is very small because of the lapse function $\alpha$, the CFL numerical stability condition is the most severe near $r = 1.5r_S$, while it is not severe near the black hole where the mesh width is the smallest. This indicates that these modified tortoise coordinates $(x, \theta, \phi)$ are appropriate for the calculation both near and far from the black hole (Koide et al. 1999). We set the calculation region as $r_{\text{min}} \leq r \leq r_{\text{max}} = r_{\text{min}} + a_0[(1 + \Delta \theta)^I - 1], \Delta \theta/2 \leq \theta \leq \pi - \Delta \theta/2$, where $r_{\text{min}} = 1.001r_S, a_0 = 0.4, \Delta \theta = \pi(1 - 1/J)/J$ is the mesh width of $\theta$, and $I, J$ are grid numbers for $r$ and $\theta$, respectively. We have the radial mesh width $\Delta r = (r - r_{\text{min}} + a_0)\Delta \theta$. Then, the minimum radial mesh width is given at $r = r_{\text{min}}$ as $\Delta r_{\text{min}} = a_0\Delta \theta$. For the numerical calculations, we choose $I, J, \Delta \theta, r_{\text{max}}(\Delta \theta$ is calculated from $J$ and $r_{\text{max}}$ is calculated from $\Delta \theta$ and $I$), and the time interval $\Delta t$, for different $\eta$ values as shown in Table 1. The numerical stability conditions are given by (i) $\Delta t \leq (\Delta r/\alpha)_{\text{min}},$ 

\footnote{1 \hspace{0.5cm} $1 \leq (\Delta r/\alpha)_{\text{min}}/\Delta t \leq \Delta r/(\alpha \Delta t) = r\Delta \theta/(\alpha \Delta t)$.}
and (ii) $\Delta t < 2\eta$. The $\eta$, $\Delta \theta$, and $\Delta t$ combinations in Table 1 satisfy these conditions (these $\eta$ and $\Delta t$ satisfy $\Delta t/\eta < 1/2$).

### Table 1. Numerical Conditions of Resistive GRMHD Simulations

| $\eta/r_S$ | $I$ | $J$ | $r_{\text{min}}$ | $r_{\text{max}}$ | $\Delta \theta$ | $\Delta t/\tau_S$ |
|------------|-----|-----|------------------|------------------|-----------------|------------------|
| $5 \times 10^{-3}$ | 450 | 216 | 1.001$r_S$ | 87.8 | $1.20 \times 10^{-2}$ | $5 \times 10^{-4}$ |
| $3 \times 10^{-3}$ | 600 | 288 | 263 | $1.09 \times 10^{-2}$ | |
| $2 \times 10^{-3}$ | 1200 | 576 | 271 | $5.44 \times 10^{-3}$ | $1 \times 10^{-4}$ |
| $1 \times 10^{-3}$ | 1800 | 864 | 274 | $3.63 \times 10^{-3}$ | $1 \times 10^{-5}$ |
| $5 \times 10^{-4}$ | 5 \times 10^{-5} | 1800 | 864 | 274 | $3.63 \times 10^{-3}$ | $1 \times 10^{-5}$ |

**Note.** The calculation region is $r_{\text{min}} \leq r \leq r_{\text{max}} = r_{\text{min}} + a_0[(1 + \Delta \theta)^I - 1], \Delta \theta/2 < \theta < \pi - \Delta \theta/2$, where $r_{\text{min}} = 1.001r_S, a_0 = 0.4, \Delta \theta = \pi(1 - 1/J)/J$ is the mesh width of $\theta$, and $I, J$ are grid numbers for $r$ and $\theta$. The radial mesh width is $\Delta r = (r - r_{\text{min}} + a_0)\Delta \theta$. Then, the minimum radial mesh width is given at $r = r_{\text{min}}$ as $\Delta r_{\text{min}} = a_0\Delta \theta$. We choose $I, J, \Delta \theta, r_{\text{max}} (\Delta \theta$ is calculated from $J$ and $r_{\text{max}}$ is calculated from $\Delta \theta$ and $I$), and the time interval $\Delta t$ for different $\eta$ values as shown in this table in the simulations. The combinations of $\eta, \Delta \theta, \Delta t$ in this table satisfy the numerical stability conditions mentioned in the text.

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2 From the Ampere’s law, $\frac{\partial}{\partial t} \vec{E} = -\alpha \vec{J}$, and from the Ohm’s law, $\vec{E} = \eta \vec{J}$, we have $\frac{\partial}{\partial t} \vec{E} = -\frac{\alpha \vec{E}}{\eta}, \frac{1}{\Delta t} (\vec{E}^{n+1} - \vec{E}^n) = -\frac{\alpha \vec{E}^n}{\eta}$, and $\vec{E}^{n+1} = (1 - \alpha \Delta t/\eta)\vec{E}^n$. The numerical stability condition is $-1 < (1 - \alpha \Delta t/\eta)$, i.e., $2\eta > \Delta t > \alpha \Delta t$. 

3. NUMERICAL SIMULATION RESULTS

We present the simulation results with the initial conditions of $\rho_0 = 1, B_0 = 10, \Delta \theta_{CW} = 0.1$, and $\beta_p = 0.025$.

Resistivity $\eta$ values are set from $1 \times 10^{-5} r_S$ to $0.005 r_S$. Here we convert these $\eta$ values to the magnetic Reynolds numbers. The magnetic Reynolds number $S$ is defined as $S = L v / \eta$ in MHD, where $L, v$ are typical length-scale and velocity of plasma. In the Sweet–Parker and Petschek mechanisms, we identify $L$ (the length of the reconnection sheet) with the global external length-scale $L_e$ and $S$ therefore with the global magnetic Reynolds number $S_e = L_e v_A / \eta$, where $v_A$ is the Alfvén velocity of the plasma (Parker 1957; Sweet 1958; Petschek 1964; Priest & Forbes 2000). The relativistic Alfvén velocity is calculated as $v_A = \sqrt{B^2 / (\bar{h} + \bar{B}^2)}$, where $\bar{B}$ and $\bar{h}$ are typical values of the magnetic flux density in the ZAMO frame and the proper enthalpy density, respectively, outside of the current sheet. Substituting $\bar{B} \sim 10$ and $\bar{h} \sim \rho_0 = 1$ from the initial conditions, we obtain $v_A \sim 1$. If we regard $L_e = r_S$, then $S_e = r_S / \eta$ with our initial conditions. Thus $\eta = 1 \times 10^{-5} r_S$ corresponds to $S_e = 10^5$ and $\eta = 0.005 r_S$ corresponds to $S_e = 200$ in our simulations.

Figure 2 shows the time evolution of pressure (color), magnetic field (white lines), and velocity (arrows) in the case of $\eta = 0.001 r_S$. $\eta = 0.001 r_S$ corresponds to the global magnetic Reynolds number $S_e = 10^3$. Black regions at the left in Figure 2 show the horizon of the black hole. Initially the plasma and the magnetic field around the current sheet at the equatorial plane are vertically in equilibrium (Figure 2, top panel). The initial magnetic field is split monopole type; above the equatorial plane of the black hole, the magnetic field lines are directed toward the black hole, and below the equatorial plane the field lines are directed away from the black hole. At $t = 9 r_S$, the magnetic reconnection occurs around the current sheet near the horizon at $r \sim 1.2 r_S, \theta = \pi/2$ (Figure 2, middle panel). The reconnection region seems point-like and narrow, and the slow shocks are found along the current sheet outside of the reconnection point, which is similar to the Petschek reconnection model. At $t = 14 r_S$, multiple magnetic reconnections are caused and the plasmoid is formed around $r \sim 1.4 r_S, \theta = \pi/2$ (Figure 2, bottom panel). Because the resistivity is set to be uniform, this relatively fast magnetic reconnection, as in the Petschek model, was not expected until
Figure 2. Time evolution of pressure (color), magnetic field (white lines), and velocity (black arrows) by the resistive GRMHD simulations with the resistivity $\eta = 0.001r_S$ or the global magnetic Reynolds number $S_e = 10^3$. Black regions on the left in the panels show the horizon of the black hole. Top panel: plasma and magnetic field at the initial state. The initial magnetic field is the split monopole type; above the equatorial plane of the black hole, the magnetic field lines are directed toward the black hole, and below the equatorial plane the field lines are directed away from the black hole. Middle panel: at $t = 9\tau_S$, single magnetic reconnection occurs around the current sheet near the horizon at $r \sim 1.2r_S, \theta = \pi/2$. The reconnection region is point-like and narrow, and the slow shock waves are seen, as the Petschek reconnection model. Bottom panel: at $t = 14\tau_S$, multiple reconnections happen and the plasmoid is formed around $r \sim 1.4r_S, \theta = \pi/2$. 
we saw this simulation result. There are arguments, as we mentioned in section 1, that even though
the plasma resistivity is spatially uniform, Petschek-type fast stationary magnetic reconnection is
achieved through the use of a nonuniform viscosity profile (Baty et al. 2009). In the present case,
however, viscosity is assumed to be zero, while the thickness of the current sheet and the lapse of
time are nonuniform (see also section 4). The time lapse described by $\alpha$ is the general relativistic
effect. Thus the general relativistic effect plays an important role for the magnetic reconnection near
the black hole.

We observed the magnetic reconnection rate at the reconnection point for a given $\eta$ at a given time
of the resistive GRMHD simulations. We define the diffusive slip-through rate of magnetic field lines
across plasma at any point as

\[ R_{ms} = \frac{\alpha E'_{\phi}}{v_{A} B} = \frac{\alpha \eta \hat{J}_{\phi}}{v_{A} B}, \tag{20} \]

where $E'_{\phi} \equiv \eta \hat{J}_{\phi}$ is the electric field measured by the plasma rest frame. $\vec{B}$ is the magnetic field
strength just above the reconnection point, and outside of the current sheet. Without $\alpha$, this equation
is the definition of the standard (nonrelativistic) magnetic reconnection rate. To find out the location
of the reconnection point, we checked the profile of $-\hat{B}_\theta$ along the equatorial plane (Figure 3, top).
Because $\hat{B}_\theta$ vanishes at the reconnection point, we identify the position of reconnection point $r = r_X$
by the position of $\hat{B}_\theta = 0$ at the equatorial plane. The magnetic reconnection rate, $R_{mr}$, is given by
$R_{ms}$ at the reconnection point (Figure 3, middle). The radial component of the velocity $\hat{v}_r$ is negative
at the reconnection point (Figure 3, bottom), which means plasma is falling into the black hole at
the reconnection point.

In Figures 4 and 5 we present time developments of magnetic reconnection rate $R_{mr}$ for various
resistivity $\eta$ values in the range from $1 \times 10^{-5}r_S$ to 0.005$r_S$. As shown in Figure 4, $R_{mr}$ decreases as $\eta$
becomes smaller. Regarding the time-development, within the $\eta$ range of $3 \times 10^{-4}r_S$ to 0.005$r_S$, $R_{mr}$
is a function of powers of time, $R_{mr} \propto t^b$ ($b$ is a constant) from $t = 2\tau_S$ to $\sim 10\tau_S$, and it tends to
settle down to a constant value afterwards for large resistivity. For $\eta = 1 \times 10^{-5}r_S, 5 \times 10^{-5}r_S,$ and
$1 \times 10^{-4}r_S$, we fit $R_{mr}$ by a linear function of $t$, $R_{mr} = A(t/\tau_S) + C$, where $A$ and $C$ are constants.
The results of these three low $\eta$ cases are highlighted in Figure 5. $R_{mr}$ values of these low $\eta$ cases are
**Figure 3.** Azimuthal component of the magnetic field ($-\hat{B}_\theta$), diffusive slip-through rate of magnetic field lines across plasma ($R_{\text{ms}}$), and radial component of velocity ($\hat{v}_r$) as functions of $r/r_S$ along the equatorial plane, at $t = 9\tau_S$ in the case of $\eta = 0.001r_S$. $r_X$ is the position of the reconnection point. This figure explains how to determine $r_X$ (the position of $\hat{B}_\theta = 0$) and the reconnection rate $R_{\text{mr}} (R_{\text{ms}}$ value at $r_X$).

well fit by the linear functions of $t$. Regrettably, low-$\eta$ simulations ($\eta = 5 \times 10^{-5}r_S$ and especially $1 \times 10^{-5}r_S$) run only for short times as shown in Figures 4 and 5, because of numerical difficulty in the cases of low-$\eta$. It takes a lot of time for low-$\eta$ runs to complete the job because we should select very large mesh numbers ($I, J$), a very small time interval $\Delta t$, and an appropriate combination of them for low-$\eta$ calculations. The runs often stop due to poor convergence for low-$\eta$ cases. We will improve this problem by establishing a more stable scheme of the resistive GRMHD code in the near future.

The relationship between the resistivity $\eta$ and the power index $b$ is offered by Figure 6. Power index $b$ approaches to unity as $\eta$ becomes smaller or $S_e$ gets larger, and when $\eta \leq 10^{-4}r_S$ or $S_e \geq 10^4$, $R_{\text{mr}}$ is a linear function of $t$, which means $b = 1$ (see Figures 4 and 5). As we described in section 1 and
Numerical simulation of magnetic reconnection around black hole

Figure 4. Time evolutions of magnetic reconnection rate $R_{mr}$ for various resistivity $\eta$ values in the range from $1 \times 10^{-5} r_S$ to $0.005 r_S$. From $\eta = 3 \times 10^{-4}$ to $0.005 r_S$, we fit the data with $R_{mr} = a(t/\tau_S)^b$, where $a$ and $b$ are constants, from $t = 2\tau_S$ to $\sim 10\tau_S$. $\eta$ and the best-fit $b$ values are shown in the figure, together with the best-fit lines. For $\eta = 1 \times 10^{-5} r_S, 5 \times 10^{-5} r_S, 1 \times 10^{-4} r_S$, $R_{mr}$ can be represented by $R_{mr} = A(t/\tau_S) + C$, where $A$ and $C$ are constants. $\eta$, the best-fit $A$ values, and the best-fit lines are in the figure. Orange circle and blue St. Andrew’s cross: $\eta = 5 \times 10^{-3} r_S$, red rectangle and blue Greek cross: $\eta = 3 \times 10^{-3} r_S$, purple diamond shape and yellow-green rectangular: $\eta = 2 \times 10^{-3} r_S$, red square and pink square: $\eta = 1 \times 10^{-3} r_S$, purple St. Andrew’s cross and yellow-green triangle: $\eta = 5 \times 10^{-4} r_S$, orange St. Andrew’s cross and blue St Andrew’s cross: $\eta = 3 \times 10^{-4} r_S$, orange triangle and blue square: $\eta = 1 \times 10^{-4} r_S$, purple Greek cross and yellow-green circle: $\eta = 5 \times 10^{-5} r_S$, orange rectangle and blue rectangle: $\eta = 1 \times 10^{-5} r_S$.

will mention in section 4, among models of magnetic reconnection, only the Rutherford regime of the tearing mode instability explains the results of $R_{mr} \propto t$ for small resistivity (Rutherford 1973).

The theory of the Rutherford regime of the tearing mode instability also predicts $R_{mr} \propto \eta$ (Rutherford 1973; Park et al. 1984; Priest & Forbes 2000), which leads to $R_{mr} \propto t\eta$. We plot the relationship between $\eta$ and $A$ for $\eta = 1 \times 10^{-4} r_S, 5 \times 10^{-5} r_S$, and $1 \times 10^{-5} r_S$ in Figure 7. Figure 7 indicates $R_{mr} \propto \eta$, which gives the evidence that the magnetic reconnection we see with these $\eta$ is the Rutherford regime of the tearing mode instability.
Figure 5. Same as Figure 4 but with scales of $0 \leq t/\tau_s \leq 13$ and $0 \leq R_{mr} \leq 0.004$ to highlight the three low $\eta$ cases: $\eta = 1 \times 10^{-5} \tau_s$, $5 \times 10^{-5} \tau_s$, and $1 \times 10^{-4} \tau_s$.

Figure 6. Dependence of the power index $b$ of $R_{mr}$ time-development (i.e., $R_{mr} \propto t^b$) on the resistivity $\eta$ in the range from $1 \times 10^{-5} \tau_s$ to $0.005 \tau_s$. $b$ approaches to unity as $\eta$ becomes smaller, or $S_e$ gets larger. When $\eta \leq 1 \times 10^{-4} \tau_s$ or $S_e \geq 10^4$, $R_{mr}$ increases linearly as $t$ (see Figures 4 and 5) so we set $b = 1$ for $\eta \leq 1 \times 10^{-4} \tau_s$. From the initial conditions, $\eta = 10^{-5} \tau_s, 10^{-4} \tau_s, 10^{-3} \tau_s,$ and $10^{-2} \tau_s$ correspond to the global magnetic Reynolds number $S_e = 10^5, 10^4, 10^3$, and 100, respectively. These $S_e$ values are also shown below the horizontal axis.
Figure 7. Dependence of time change rate of the magnetic reconnection rate $R_{mr}$ on the resistivity $\eta$ for $\eta = 1 \times 10^{-5} r_S, 5 \times 10^{-5} r_S, \text{and} 1 \times 10^{-4} r_S$. Here $R_{mr}$ is fit by $R_{mr} = A(t/\tau_S) + C$, where $A$ and $C$ are constants (see also Figures 4 and 5). This plot shows $R_{mr} \propto \eta$ (although the results are somewhat skewed), which gives the evidence that the magnetic reconnection is in the Rutherford regime of the tearing mode instability.

Whether the reconnection point $r_X$ shifts with time or not is an interesting topic. The time evolutions of $r_X$ for four different resistivity values can be seen in Figure 8. For all the resistivity values searched, $r_X$ stays almost at the same position at $r \sim 1.2r_S$ during $t \sim 5–12\tau_S$. In the earlier phase ($t < 5\tau_S$) and in the later phase ($t > 12\tau_S$), $r_X$ approaches toward the black hole horizon. The reason of this $r_X$ motion can be understood as follows. We assumed the initial condition of the plasma velocity to be $\dot{v}^r < 0$ (Equation (19)), so in the earlier phase, the plasma initially falls into the black hole, which moves $r_X$ toward the black hole. During $t \sim 5–12\tau_S$, the single magnetic reconnection happens, and the plasma ejection from the reconnection point stops the infall of the plasma, thus $r_X$ stays at the same position at $r \sim 1.2r_S$. In the later phase, multiple magnetic reconnections occur, making a magnetic island, and the magnetic reconnection nearest the black hole is isolated from the outer magnetic field lines thus $r_X$ moves again toward the black hole.
Figure 8. Time evolutions of reconnection point $r_X$ for four different resistivity $\eta$ values. Red square: $\eta = 5 \times 10^{-4}r_S$, blue St. Andrew’s cross: $\eta = 1 \times 10^{-3}r_S$, yellow-green triangle: $\eta = 3 \times 10^{-3}r_S$, purple St. Andrew’s cross: $\eta = 5 \times 10^{-3}r_S$.

4. DISCUSSION

To investigate basic physical process of magnetic reconnection around a black hole, we have developed a resistive GRMHD code, and performed numerical simulations of plasmas and electromagnetic field around a Schwarzschild black hole. We assumed split-monopole magnetic field around the black hole as the initial condition, and electric resistivity $\eta$ to be uniform in space and constant in time. We obtained the following results.

- We found that relatively fast magnetic reconnection happens near the black hole at its equatorial plane. This is the first resistive GRMHD simulation result that shows clear magnetic reconnection feature in plasma around a black hole, to our knowledge. The structure of the reconnection is like the Petschek reconnection model, which has the point-like reconnection region and the slow shock waves, while $\eta$ is assumed to be uniform. We also observed formation of magnetic islands by the magnetic reconnection process.

- The magnetic reconnection rate $R_{mr}$ decreases as $\eta$ becomes smaller. For $\eta > 1 \times 10^{-4}r_S$, $R_{mr}$ is a function of powers of time $t$, $R_{mr} \propto t^b$ ($b$ is the power index), from $t = 2\tau_S$ to $\sim 10\tau_S$, and $R_{mr}$
tends to settle down to a constant value afterwards for large \( \eta \). For \( \eta \leq 1 \times 10^{-4} r_S \) or the global magnetic Reynolds number \( S_e \geq 10^4 \), \( R_{mr} \) increases linearly as time. For this range of \( \eta \) or \( S_e \), \( R_{mr} \) is also proportional to \( \eta \), which means \( R_{mr} \propto \eta t \). These results are in good agreement with the magnetic reconnection in the Rutherford regime of the tearing mode instability.

We discuss here the relationship between present magnetic reconnection models and our simulation results. Note that in astrophysical situations, where plasma is very thin, the magnetic Reynolds number, which is proportional to the inverse of resistivity, is supposed to be sufficiently large (\( S \gg 1 \)). Recent nonrelativistic MHD theories of magnetic reconnection with sufficiently large \( S \) and initially uniform current sheet suggest the form of time development of magnetic reconnection rate \( R_{mr} \) as shown in Figure 1 (e.g., Murphy et al. 2008). The models of magnetic reconnection can be classified into three periods in the time dependence of \( R_{mr} \): the linear growth stage, the nonlinear growth stage, and the stationary stage. The former two stages come from the tearing mode instability. In the linear growth stage, the tearing mode instability starts to appear, and the time dependence of \( R_{mr} \) is \( R_{mr} \propto e^{\gamma t} \), where \( \gamma \propto 1/\sqrt{S} \) (Furth et al. 1963; Biskamp 1993). This exponential growth in time is considered as the beginning phase of magnetic reconnection. After the linear growth stage, magnetic reconnection enters to the nonlinear growth stage. The phenomena in this stage is rather complex. The Rutherford regime is known as the nonlinear growth stage of the tearing mode instability, and according to this theory, \( R_{mr} \propto t \) (Rutherford 1973). This can be regarded as the transition stage of magnetic reconnection. Our results are in good agreement with this model during \( t \sim 2 - 10 \tau_S \). If the magnetic reconnection is in the Rutherford regime of the tearing mode instability, \( R_{mr} \) is also proportional to \( \eta \) (Rutherford 1973; Park et al. 1984; Priest & Forbes 2000). Our results also show \( R_{mr} \propto \eta \) for \( \eta \leq 1 \times 10^{-4} r_S \), which agrees with this theoretical prediction. This confirms that the magnetic reconnection we see in the simulation results for \( \eta \leq 1 \times 10^{-4} r_S \) is the phenomena in this regime. Finally, the steady-state reconnection is achieved as the Sweet–Parker model or the Petschek model. The stationary models predict \( R_{mr} \) to be constant in time (time-independent), and this must be the final stage of magnetic reconnection. The Sweet–Parker model leads \( R_{mr} = 1/\sqrt{S_e} \), whereas the Petschek model tells \( R_{mr}(\text{max}) = \pi/(8\ln S_e) \). Here \( S_e \) is the global magnetic Reynolds number.
and $R_{\text{mr}}(\text{max})$ is the maximum magnetic reconnection rate estimated by Petschek, as explained in Section 3 (Parker 1957; Sweet 1958; Petschek 1964; Priest & Forbes 2000; Kulsrud 2005; Shibata & Magara 2011).

We mentioned above that magnetic reconnection starts from the linear growth stage (Figure 1), while our resistive GRMHD simulations showed that magnetic reconnection starts from the Rutherford regime, which is the second stage of the time-evolution of magnetic reconnection. This is explained by the break-down of the uniformity of the current sheet around the equatorial plane of the black hole. The current sheet is thinner as $r$ is smaller in the initial condition with the split-monopole magnetic field. Then the current density becomes larger as $r$ gets smaller. However, at the horizon, $\alpha$ vanishes, then the diffusive slip-through rate of magnetic field lines across plasma $R_{\text{ms}} = \alpha \eta \hat{j}^\phi / (v_A \hat{B})$ has its maximum value outside of the horizon. The maximum $R_{\text{ms}}$ point is expected to become the reconnection point $r_X$. We examined the maximum $R_{\text{ms}}$ position and $r_X$ at the very early epoch of the numerical simulations. Figure 9 shows $R_{\text{ms}}$ as a function of $r/r_S$ along the equatorial plane at $t = 1 \times 10^{-4} r_S$ in the case of $\eta = 0.001 r_S$. We can see that $R_{\text{ms}}$ has its maximum value at $r \sim 1.7 r_S$. The position of $r_X$ at this time is calculated to be $\sim 1.51 r_S$ from the same analysis as shown in Figure 3. Thus the maximum $R_{\text{ms}}$ position and $r_X$ actually locate at almost the same position, which supports the above discussion.

As the stationary models give the magnetic reconnection rate in the stationary state, let us calculate this rate and compare our simulation results. Our simulation results cover resistivity $\eta$ values in the range from $1 \times 10^{-5} r_S$ to $0.005 r_S$, which corresponds to the global magnetic Reynolds number $S_e \sim 10^5$ to 200. In the Sweet–Parker mechanism, $S_e = 10^5$ yields $R_{\text{mr}} = 1/\sqrt{S_e} \sim 3.2 \times 10^{-3}$. In the Petschek model, $S_e = 10^5$ yields $R_{\text{mr}}(\text{max}) = \pi/(8 \ln S_e) \sim 3.4 \times 10^{-2}$. As shown in Figures 4 and 5, $R_{\text{mr}}$ is smaller than 0.005 for $\eta \leq 1 \times 10^{-4}$ or $S_e \geq 10^4$. Thus these magnetic reconnections can be regarded as the nonlinear growth stage before it settles down to the stationary stage.

With the resistive GRMHD numerical calculations, drastic phenomena around rapidly spinning black holes related with magnetic reconnection will be revealed. One of such phenomena is energy extraction from a black hole through magnetic reconnection (Koide & Arai 2008). Magnetic re-
connection is expected to occur frequently around a rapidly rotating black hole. In this paper, we assumed a Schwarzschild black hole, so we will extend our simulation for the case of a Kerr black hole in the near future. We will also perform the longer-term simulations of magnetic reconnection for larger $S$ and see if the magnetic reconnection settles down to the stationary stage from the non-linear transition stage. Magnetic islands which appear at the later stage are interesting phenomena. Further studies of them are necessary.

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REFERENCES

Acciari, V. A., Aliu, E., Arlen, T., et al. 2009, Sci, 325, 444

Baty, H., Priest, E. R., & Forbes, T. G. 2009, PhPl, 16, 060701

Biretta, J. A., Sparks, W. B., & Macchetto, F. 1999, ApJ, 520, 621

Biskamp, J. A. 1993, Nonlinear Magnetohydrodynamics (Cambridge: Cambridge Univ. Press)

Bucciantini, N., & Del Zanna, L. 2013, MNRAS, 428, 71

Bugli, M., Del Zanna, L., & Bucciantini, N. 2014, MNRASL, 440, L41

Dionysopoulos, K., Alic, D., & Rezzolla, L. 2015, PhRvD, 92, 084064

Dionysopoulos, K., Alic, D., Palenzuela, C., Rezzolla, L., & Giacomazzo, B. 2013, PhRvD, 88, 044020

Furth, H. P., Killeen, J., & Rosenbluth, M. N. 1963, PhFl, 6, 459

Giovannelli, R. G. 1946, Natur, 158, 81

Harris, E. G. 1962, NCim, 23, 115

Koide, S., & Arai, K. 2008, ApJ, 682, 1124

Koide, S., Kudoh, T., & Shibata, K. 2006, PhRvD, 74, 044005

Koide, S., & Morino, R. 2011, PhRvD, 84, 083009

Koide, S., Shibata, K., & Kudoh, T. 1999, ApJ, 522, 727

Komissarov, S. S. 2005, MNRAS, 359, 801

Kulkarni, S. R., Djorgovski, S. G., Odewahn, S. C., et al. 1999, Natur, 398, 389

Kulsrud, R. M. 2005, Plasma Physics for Astrophysics (Princeton, NJ: Princeton Univ. Press)

McKinney, J. C. 2006, MNRAS, 368, 1561

Mirabel, I. F., & Rodriguez, L. F. 1994, Natur, 371, 46

Morino, R. 2011, Master thesis, Kumamoto Univ.

Murphy, G. C., Ouyed, R., & Pelletier, G. 2008, IJMPD, 17, 1715

Ohya, M., & Shibata, K. 1998, ApJ, 499, 934

Park, W., Moutiallo, D. A., & White, R. B. 1984, PhFl, 27, 137

Parker, E. N. 1957, JGR, 62, 509

Petschek, H. P. 1964, NASSP, 50, 425

Priest, E. & Forbes, T. 2000, Magnetic Reconnection: MHD Theory and Applications (Cambridge: Cambridge Univ. Press)

Priest, E. R. 1985, RPPh, 48, 955

Rutherford, P. H. 1973, PhFl, 11, 1903

Shibata, K., & Magara, T. 2011, LRSP, 8, 6

Sweet, P. A. 1958, in IAU Symp. 6, Electromagnetic Phenomena in Cosmical Physics, ed. B. Lehnert (London: Cambridge Univ. Press), 123

Watanabe, N., & Yokoyama, T. 2006, ApJL, 642, L123

Zenitani, S., Hesse, M., & Klimas, A. 2010, ApJL, 716, L214