Application of the least square method for calculating the optimal well profile

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Abstract. The article offers a new approach to calculating a wellbore profile of a directional well. The calculation of the well trajectory based on inclinometric studies using correlation analysis is considered. The coordinates of measurement points are calculated automatically by building a comparison table in Microsoft Excel. Based on the results of calculations, the functional relation the most fully characterizing the well path is selected. We propose the least square method for calculating the optimal well profile. According to revealed functional relation, the length of the well is determined by integration based on the calculated values of displacements. The estimation of wellbore deviation from the well profile design represented as a parabolic dependence at a certain drilling interval was performed.

1. Introduction

In the course of well design, an important task is to determine financially advantageous conditions for the drilling process, in particular, to build an optimal well profile. The exact trajectory of drilling tool penetration in the rock depends on its control and correction, on-line calculation of the spatial coordinates of the well. The wellbore deviation from planned position may lead to reduction in flow rate, oil recovery index, need of drilling additional wells to extract the remaining oil [1 - 5]. There may be a possible leakage of the casing, an increased probability of poor cementing of wells, complicated oil extraction, an extension of the wellbore, the difficult elimination of accidents, etc. [6]. In many cases, controlling the wellbore position is important not only for achieving optimal coordinates, but also for completing the well with the preferred trajectory of entry into the productive formation [7].

At present, there are quite a large number of mathematical calculation methods, that assist in modeling the position of the wellbore in space. These methods include tangential, balanced tangential, the method of average angels, the method of radius of curvatures, the method of minimal curvature, and their combinations [8]. According to [9] the world practice recognises the method of minimal curvature as the most accurate method, since using it the trajectory of the well is the closest to the objective form. The well sections represented in this method as a series of spherical arcs and straight lines are quite simple and do not carry a large number of restrictions associated with data output from the definition area [9]. All existing methods are based on simple geometric interpretations that can be mathematically processed. Calculations are widely used in three-dimensional visualization of the drilling process, so it can be optimized while maintaining safety. However, the literature sources are not systematized, are incomplete, and require additions (mathematical justification). These features make programming controversial [9]. The development of equipment and technology, especially
computing one, assists in searching for more accurate and universal methods for calculating and predicting the optimal well profile. It is obvious that a reliable calculating algorithm is more complex, analyzing, and providing full consideration of the well profile as a function. Correlation analysis is successfully used in various fields, especially for obtaining predictive data.

The article aims to verify the possibility of applying correlation analysis for the design and prediction of the optimal well profile. To achieve this aim, the coordinates and parameters of the well profile are calculated employing the least square method. It is necessary to solve a number of tasks: to determine displacement and length of the wellbore; to determine the accuracy of implementation of the well profile design; to evaluate the deviation of the wellbore from the well profile design represented as a functional relation at a certain drilling interval.

2. Research methodology
Technological processes having a wide variety, are characterized by a set of parameters that reflect certain properties, modify over time and in space. Changes in these parameters are often interdependent and mutually dependent. Sometimes the relation between parameters is very close, and in some cases this relation is not detected at all. The closer the relation is, the more accurate decisions are made and the easier it is to manage the processes. In mathematical statistics, they distinguish two types of dependence functional (strictly deterministic) and statistical. In our study, we consider functional dependence which is always expressed by formulas. It is complete and accurate since the list of factors and mechanisms of their influence on the variable in the form of an equation is usually known. We used the least square method which is the basis of correlation analysis [10 - 12] to determine this type of equation.

Inclinometry technology provides measurements of azimuth a, the zenith angle b, and length of the wellbore from the wellhead to each measurement point at each point of the wellbore. The calculation of the drilling trajectory means to find coordinates of measurement points in the Cartesian coordinate system in relation with the wellhead and the drilling point of the side hole. Otherwise, calculate the vertical depth Z of the measurement point, its horizontal displacements X and Y in the North-South and in the East-West directions [13].

Projections $dx$, $dy$, $dz$ of the elementary interval $dl$ of the wellbore on the coordinate axes have the form:

$$dx = dl \cdot \sin a \cdot \sin b; \quad dy = dl \cdot \cos a \cdot \sin b; \quad dz = dl \cdot \cos b.$$  \hspace{1cm} (1)

By integrating these elementary increments along the length of the interval, you can determine the increments of the coordinates $\Delta x$, $\Delta y$, $\Delta z$ of the wellbore for the interval $\Delta l$ between the measurement points:

$$\Delta x = \int_0^{\Delta l} \sin a \cdot \sin b \, dl; \quad \Delta y = \int_0^{\Delta l} \cos a \cdot \sin b \, dl; \quad \Delta z = \int_0^{\Delta l} \cos b \, dl.$$ \hspace{1cm} (2)

![Figure 1. Projections of the elementary interval of the wellbore on the coordinate axes](image)
In the simplest methods, the results of measuring the zenith angle and azimuth at two adjacent points are used to calculate the increments of the wellbore coordinates. In more accurate methods, the results of measuring at three or more points are used. This assists to reduce errors in constructing an actual wellbore profile using the same measuring tools [13].

Let us consider using the minimum curvature method for calculating the coordinates and trajectory parameters of the actual directional well profile. The zenith angle and azimuth were measured using an inclinometer with a magnetic azimuth detector [13]. The initial data for calculating the well profile are described in table 1.

### Table 1. Initial data for calculating the well profile

| Coordinates of the well profile | Zenith angle, deg. | Azimuth, deg. |
|--------------------------------|--------------------|---------------|
| $X, m$ | $Y, m$ | $Z, m$ | $N$ | $N$ |
| 246.0 | 60.0 | 1473.0 | 14.50 | 102 |
| 240.1 | 84.8 | 1569.7 | 15.00 | 105 |
| 233.2 | 110.4 | 1666.1 | 15.75 | 105 |
| 225.3 | 137.0 | 1762.2 | 16.50 | 108 |
| 216.4 | 164.4 | 1858.0 | 17.00 | 108 |
| 206.6 | 192.8 | 1953.3 | 18.00 | 110 |
| 195.9 | 222.2 | 2048.3 | 18.50 | 110 |
| 184.9 | 252.4 | 2143.0 | 19.00 | 110 |

### 3. Results

Based on the obtained coordinates, horizontal displacements $A$ for each profile point from the vertical line of the wellhead are calculated according to the formula:

$$A = \sqrt{x^2 + y^2}$$

The coordinates of the profile points in the horizontal plane are given in table 2.

### Table 2. Coordinates of the profile points in the horizontal plane

| Coordinates of the well profile | Displacement $A, m$ |
|--------------------------------|---------------------|
| $X, m$ | $Y, m$ | $Z, m$ | $A, m$ |
| 246.0 | 60.0 | 1473.0 | 253.2 |
| 240.1 | 84.8 | 1569.7 | 254.6 |
| 233.2 | 110.4 | 1666.1 | 258.0 |
| 225.3 | 137.0 | 1762.2 | 263.7 |
| 216.4 | 164.4 | 1858.0 | 271.7 |
| 206.6 | 192.8 | 1953.3 | 282.6 |
| 195.9 | 222.2 | 2048.3 | 292.6 |
| 184.9 | 252.4 | 2143.0 | 321.9 |

Well profiles are curves that are similar with a part of a parabola, so we can assume that the correlation $Z = z(A)$ has the form:

$$z(A) = aA^2 + bA + c$$

In this case, the parameters $a$, $b$, $c$ are easiest to determine from the so-called system of normal equations that meet the requirements of the least squares method (LSM). It is possible to write this requirement as $\sum(z - z_i)^2 \to \text{min}$ and determine at what values of parameters $a$, $b$, $c$ the sum of squares of deviation $z$ from $z(A)$ will be minimum. After finding private derivatives of the specified sum for $a$, $b$ and $c$ separately and equating them to zero, it is easy to write down a system of equations, whose solution will give parameters of the required function. The system of equations for determining the second order function is as follows:
Microsoft Excel can help to search for unknown \( a, b, c \) and plot the resulting functional dependence. In the solution process, apart from the parabolic dependency graph, there were also found graphs of other dependencies that approximate the data in Table 2. Figures 2–4 shows the graphs.

\[
\begin{align*}
nc + b \sum A + a \sum A^2 &= \sum z \\
c \sum A + b \sum dA^2 + a \sum A^3 &= \sum A z \\
c \sum dA^2 + b \sum dA^3 + a \sum dA^4 &= \sum dA^2 z.
\end{align*}
\]
The comparative analysis of the presented charts has shown that the data are most clearly approximated by the parabolic function (Fig. 4). Since the vertical axis is directed downwards when designing the well profile (Fig. 1), the searched function \( Z = Z(A) \) is in the area of negative values. Therefore, this function is a functional dependence:

\[
Z(A) = 0.1536 A^2 - 96.941A + 13160. \tag{6}
\]

The mathematical course gives a formula for calculating the length of a curve arc from the point \( M(a); c) \) to the point \( N(b); d) \):

\[
L = \int_{a}^{b} \sqrt{1 + (y'(x))^2} \, dx \tag{7}
\]

Using (6), (7) and integration methods, we will find the wellbore length with displacement in the interval from 253 to 322 m:

\[
L = \int_{253}^{322} \sqrt{1 + (0.3072Ph - 96.941)^2} \, dA \approx 748 \, (m). \]

4. Conclusion
The estimation of wellbore deviation from the well profile design represented as a parabolic dependence at a certain drilling interval from 1500 to 2200 m, was not more than 30 m. This conclusion requires detailed consideration. When conducting further research of more coordinates, it is possible to select the most accurate function with an error tending to zero. It is necessary to consider technical capabilities of drilling tools, drill string bottom layout, geological section of the well, etc.

The research has shown that correlation analysis allows using different mathematical methods in modeling the optimal well profile. It is applicable when other methods do not provide more complete information on the well profile and provide timely data for borehole corrections.

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