User Scheduling and Grouping in Massive MIMO Broadcast Channels with Heterogeneous Users

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Abstract: This paper considers a frequency division duplexing massive multi-input multi-output system with two-stage beamforming. In two-stage beamforming, user scheduling and grouping are applied to reduce inter-user and inter-group interferences, respectively. Conventional scheduling utilizes the channel norm as a user selection metric. However, the metric degrades the multiuser diversity gain due to inter-group interference. To increase multiuser diversity gain from scheduling, an approximate lower bound on expected signal-to-interference-plus-noise ratio (SINR) value is derived in this paper. Its asymptotic sum rate is also derived in a realistic channel model where each user is randomly located in the cell. Analysis results show that the sum-rate growth increases with the pathloss exponent, but is upper-bounded due to inter-group interference. In addition, user grouping for randomly-distributed users is proposed. The approximate lower bound in user selection is used to obtain a new grouping metric by averaging over an instantaneous channel. In simulations, the proposed metric provided higher sum rate than did conventional user grouping methods.

Index Terms: Clustering, joint spatial division and multiplexing, two-stage beamforming, user selection.

I. INTRODUCTION

MULTI-INPUT multi-output (MIMO) is a promising technology for high speed wireless communications [1]–[3]. The use of multiple antennas increases the capacity and reliability of wireless channels. Recently, based on large-scale antenna arrays, massive MIMO systems have been introduced to further improve the spectral efficiency and energy efficiency [4]–[7]. However, large-scale antenna arrays cannot be deployed at the user equipment (UE) due to its small size and limited complexity. Thus, to achieve a massive MIMO gain, multiuser MIMO (MU-MIMO) is considered in massive MIMO systems. Massive MIMO systems can provide an asymptotic orthogonality of channel for different users as the number of transmit antennas increases without complicated signal processing.

The advantages of massive MIMO can be achieved only when channel state information (CSI) is available at the base station (BS). In time-division duplexing (TDD) systems, the BS obtains CSI from uplink training, and exploits the channel reciprocity. The accuracy of CSI in TDD systems is limited by non-orthogonal pilot allocated to the user in the adjacent cell; this effect is called the pilot-contamination problem [6]. In frequency-division duplexing (FDD) systems, the UE estimates CSI, and then feeds it back to the BS [9]. However, as the number of transmit antennas increases, the accuracy of CSI decreases and feedback overhead to the BS increases. To overcome these difficulties, joint spatial division and multiplexing (JSDM) has been introduced for FDD systems [8]. JSDM exploits channel statistic information between BS and UE, and uses the channel statistic information to sort multiple users into several groups. This grouping process can reduce the required amount of feedback by reducing the dimensionality of the effective channel. Then successive outer and inner beamformings are applied to each group; these processes are designed to minimize inter-group interference (IGI) and inter-user interference (IUI), respectively.

To enable JSDM practically, zero-forcing beamforming (ZFBF) was used for outer and inner beamformings in [8], [10]–[13] because of its simple structure and comparable sum rate to the optimal transmit beamforming. However, ZFBF is typically power-inefficient and is inferior to optimal transmit beamforming [14]. Combining ZFBF with a scheduling process can achieve asymptotically optimal sum rate by exploiting the multiuser diversity gain. ZFBF with semi-orthogonal user selection (ZFBF-SUS) in [14] can achieve asymptotically optimal sum rate in multi-input single-output (MISO) broadcast channels, but its sum rate in JSDM has not been reported. Random beamforming (RBF) in [10] was based on the opportunistic user selection. Its sum-rate growth was shown to be \( \beta \log \log (K/G) \) by feeding a signal-to-interference-plus-noise ratio (SINR) back to the BS where \( \beta = \min \{ M, \sum_{g=1}^{G} r_g^2 \} \), \( M \) is the number of transmit antennas, \( K \) is the number of users, \( G \) is the number of groups, and \( r_g^2 \) is effective channel rank. The user selection metric of RBF does not properly estimate the expected SINR of each user. Recently, a new user selection that combines the benefits of RBF and ZFBF-SUS [12] was shown to achieve sum-rate growth of \( \beta \log \log (K/G) \) by constructing a double cone around orthogonal reference beams to select semi-orthogonal users. The studies in [8], [10], [12] analyzed the sum-rate growth under the assumption that all users had the same average SINR, but this assumption is not valid in real cellular systems. Cellular users are typically distributed heterogeneously across the geography of a cell, and therefore have a different average SINRs.

The random geometry of users also provides various channel covariance matrices to the users. Thus, outer beamforming does not completely eliminate the IGI due to the mismatch between true and representative channel covariance matrix of user. User grouping in JSDM reduces this mismatch, and increases the efficiency of outer beamforming. In practice, the user grouping is based on the clustering algorithm. Simple grouping was first introduced in [10]. Based on the similarity of channel statistic
information, $K$-means clustering was applied for user grouping. Instead of chordal distance in $K$-means clustering, weighted likelihood similarity (WLS) was investigated in [15]. By using the weights of different eigenmodes, the WLS had higher sum rate than did $K$-means clustering. However, both groupings in [10] and [15] had a weak sum rate because it was based on the similarity of the channel covariance matrix. Recently, a correlation matrix distance (CMD)-based grouping was introduced in [16] to improve the spatial orthogonality among the groups.

This paper considers user scheduling and grouping for JSDM systems. To effectively exploit multiuser diversity gain, user selection metric should estimate the SINR of each user. However, a user cannot use the inner beamformings before scheduling, so the exact SINR value that corresponds to instantaneous CSI cannot be obtained. To this end, an approximate lower bound for the expected SINR is used. Based on the SINR, a ZFBF-SUS is developed and its asymptotic sum rate is analyzed over realistic channels. In addition, user grouping for practical implementation is proposed. By calculating an expectation for the SINR over instantaneous channel, a grouping metric is obtained, which can increase the sum rate. The proposed grouping has a higher sum rate than do conventional groupings. Our key contributions in this paper are summarized as follows.

- An approximate lower bound of the expected SINR is derived. This lower bound value is the same as the quasi-SINR value in [12], but it is proven that the quasi-SINR metric in ZFBF-SUS can improve the sum rate as an approximate lower bound. In simulations, the approximate lower bound metric in ZFBF-SUS gave a higher sum rate than did other user selection methods.
- The asymptotic sum rate of the ZFBF-SUS is analyzed over heterogeneous channel. The sum-rate growth in MISO broadcast channel with spatial heterogeneity is given by $(M \alpha/2) \log K$ in [3], [17]. Similarly, it may be expected that the sum-rate growth in a JSDM system is $(\sum_{g=1}^{G} r_g^\ast) \log(K/G)$, but it is shown that the sum-rate growth as $(\sum_{g=1}^{G} r_g^\ast)/(2(\alpha) + 1) \cdot \log(K/G)$. The sum-rate growth is upper-bounded by $(\sum_{g=1}^{G} r_g^\ast) \log(K/G)$; this means that the randomness from spatial heterogeneity can not be efficiently exploited because of residual IGI.
- Conventional groupings in [10], [15], [16] exploit the similarity and orthogonality of channel covariance matrix of users. Their groupings do not consider the IGI; thus have a low sum-rate. To improve the sum-rate, the approximate lower bound of the expected SINR is further averaged over the instantaneous CSIs. Unlike conventional groupings, the proposed metric exploits the similarity of outer beamforming and considers the IGI.

The rest of this paper is organized as follows. A system model and user selection metric for JSDM are described in Section II. The sum rate is derived over heterogeneous channel in Section III. A user grouping algorithm is proposed in Section IV. Simulation results and conclusions are presented in Section V and Section VI, respectively. Notations are listed in Table 1.

**Table 1. Notation.**

| Notation | Description |
|----------|-------------|
| $(\cdot)^H$ | Hermitian |
| $[\cdot]_{i,j}$ | entry at the $i$th row and $j$th column |
| $|\cdot|$ | cardinality of the set |
| $\|\cdot\|$ | squared norm |
| $\|\cdot\|_F$ | Frobenius norm |
| $E(\cdot)$ | expectation operation |
| $(\cdot)^{-1}$ | inverse operation |
| tr$(\cdot)$ | trace operation |
| $\Gamma(\cdot)$ | Gamma function |
| $G(x; k, \theta)$ | cumulative distribution function (CDF) of the Gamma distribution with shape $k$ and scale $\theta$ |
| $\Xi$ | unitary matrix after eigen decomposition |
| $f(x) = o(g(x))$ | $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$ |

## II. SYSTEM MODEL

A downlink JSDM system is considered in which the BS has $M$ transmit antennas and serves $K$ single-antenna users. The received signal of users such a system is given as

$$y = H^H s + n = H^H V W x + n = B^H W x + n,$$

where $H \in C^{M \times K}$ is the overall channel matrix, $s \in C^{M \times 1}$ is the transmitted symbol vector with the expected power $P$, i.e., $E[|s|^2] = P$. The information symbol $x$ is precoded by outer matrix $V \in C^{M \times b}$ and inner matrix $W \in C^{b \times S}$. $S$ is the number of scheduled users, $B^H = H^H V$ is the effective channel matrix after outer beamforming, and $n$ is complex Gaussian noise with $CN(0, 1_I)$.

### A. Channel Correlation Model

A Rayleigh correlated channel is considered by using the one-ring scattering model in [8] (Fig. 1) to reflect the reality of the description of a general channel phenomenon. The channel vector $g_k \in C^{M \times 1}$ at the $k$th user is the complex Gaussian vector with $CN(0, R_k)$. The distance model in [18] is used to model the random location of users in the cell. The Karhunen-Loeve representation expresses the channel vector as

$$h_k = \sqrt{d_k^\alpha} g_k = \sqrt{d_k^\alpha} U_k \Lambda_k^{\frac{1}{2}} \eta_k.$$

The pathloss effect in the cell is described as $\sqrt{d_k^\alpha}$ where $d_k \in (0, D)$ is a random distance and $\alpha$ is the pathloss exponent. Each entry of $\eta_k$ has independent complex Gaussian distribution with $CN(0, 1)$. $R_k$ is decomposed as $U_k \Lambda_k U_k^H$ where $U_k \in C^{M \times r_k}$ is a matrix of the eigenvectors that corresponds to the $r_k$ non-zero eigenvalues of $R_k$ and $\Lambda_k \in C^{r_k \times r_k}$ is a diagonal matrix composed of the non-zero eigenvalues of $R_k$. 

The channel covariance matrix is modelled according to the one-ring local scattering model, in which the channel covariance is dependent on the angle spread $\Delta$, angle of arrival $\theta$, and antenna geometry. For the case of a uniform linear array at the BS with the antenna spacing $H$, the channel covariance matrix is expressed as

$$[R_g]_{m,n} = \frac{1}{2\Delta} \int_{-\Delta}^{\Delta} e^{-2j\pi(m-n)\sin(\theta)} d\theta. \quad (3)$$

This paper assumes that the channel covariance matrix varies slower than the effective channel to fully exploit the spatial correlation. In addition, perfect channel estimation of $R_k$ and $\eta_k$ is assumed to focus on user scheduling and grouping. With this assumption, the proposed algorithm and its sum-rate will be derived in Sections II.C and III, respectively.

### B. Joint Spatial Division and Multiplexing

JSDM exploits the channel covariance matrix of users. Users in the cell are partitioned into $G$ groups such that $\sum_{g=1}^{G} K_g = K$ where $K_g$ represents the number of users, and $\sum_{g=1}^{G} S_g = S$ where $S_g$ represents the number of scheduled users in group $g$ (Fig. 1). When users in a group have the same channel covariance matrix, the channel vector of user $k$ in group $g$ is represented by

$$h_{gk} = \sqrt{d_{gk}} U_g \Lambda_g^{\frac{1}{2}} \eta_{gk}, \quad (4)$$

where $U_g$ and $\Lambda_g$ are the representative eigenvector and eigenvalue matrices of channel covariance matrix for group $g$, respectively. From (4), the channel matrix of users in group $g$ can be aggregated as $H_g = [h_{g1}, h_{g2}, \ldots, h_{gK_g}]$. Then, the overall channel matrix can be expressed as $H = [H_1, H_2, \ldots, H_G]$.

By using (1) and (4), the received signal for the users in group $g$ is rewritten as

$$y_g = H_g^H s_g + \sum_{g' \neq g} H_{g'}^H s_{g'} + n_g = H_g^H V_g W_g x_g + \sum_{g' \neq g} H_{g'}^H V_{g'} W_{g'} x_{g'} + n_g$$

$$= B_g^H W_g x_g + \sum_{g' \neq g} H_{g'}^H V_{g'} W_{g'} x_{g'} + n_g, \quad (5)$$

where $x_g$ is the transmitted symbol vector for group $g$. This vector is precoded by the beamforming matrices $V_g \in C^{M \times b_g}$ and $W_g \in C^{b_g \times S_g}$. $W_g$ is composed of $[w_{g1}, w_{g2}, \ldots, w_{gS_g}]$ and depends on the effective channel $B_g^H = H_g^H V_g$ for group $g$. The received signal for the $k$th user in group $g$ is given by

$$y_{gk} = h_{gk}^H V_g w_{gk} x_{gk} + \sum_{k' \neq k} h_{gk'}^H V_g w_{gk'} x_{gk'}$$

$$+ \sum_{g' \neq g} \sum_{k' \neq k} h_{gk'}^H V_{g'} w_{gk'} x_{gk'}, \quad (6)$$

where $x_{gk}$ is the information symbol, and $n_{gk}$ is noise.

Outer beamforming matrix $V_g$ mitigates the IGI based on the representative channel covariance matrices of groups. Block diagonalization (BD) can effectively mitigate IGI [19]. The design criterion of BD can be expressed as

$$U_g V_{g'} \neq 0, \forall g' \neq g. \quad (7)$$

IGI can be perfectly eliminated when

$$\dim \left( \text{Span}(U_g) \cap \text{Span}^\perp (\{U_{g'} : g' \neq g\}) \right) \geq S_g. \quad (8)$$

However, in practice, (8) is difficult to satisfy for finite $M$. Solving this problem requires an alternative design to reduce the IGI to near zero. Approximate BD [8] selects the $r_g^*$ dominant eigenvectors ($= U_g^*$) of the channel covariance matrix such that

$$\dim \left( \text{Span}(U_g^*) \cap \text{Span}^\perp (\{U_{g'}^* : g' \neq g\}) \right) \geq S_g, \quad (9)$$

where $r_g^*$ ($\leq r_g$) is the design parameter, which controls the residual IGI. The residual IGI is generated because the weakest $r_g - r_g^*$ eigenvectors of the channel covariance matrix are not included in $U_g^*$.

### C. User Selection Metric

When the number of users is large, an appropriate user selection can significantly increase the sum rate by exploiting multiuser diversity. Sum rate can be maximized when the scheduler chooses the users that have the largest SINR. To achieve this choice, the scheduler should estimate an accurate SINR based on the CSI. However, the inner beamformings $W_{g'}$ have not been determined, so an accurate SINR cannot be obtained. To solve this difficulty, the expected SINR of user $k$ in group $g$ is used.

With the perfect CSI, ZFBF completely eliminates IUI, so the second term in (6) disappears. Then the received SINR of (6) of
user $k$ in group $g$ can be expressed as

$$\text{SINR}_{g_k} = \frac{|h_{g_k}^H V_g w_{g_k}|^2}{1 + \sum_{g' \neq g} |h_{g_k}^H V_g w_{g'}|^2}. \quad (10)$$

By taking an expectation on (10) over inner beamforming, the expectation of the SINR of user $k$ in group $g$ can be expressed as

$$E\{\text{SINR}_{g_k}\} \approx \frac{E\{|h_{g_k}^H V_g w_{g_k}|^2\}}{E\{1 + \sum_{g' \neq g} |h_{g_k}^H V_g w_{g'}|^2\}} \rho |h_{g_k}^H V_g|^2 \frac{1}{1 + \sum_{g' \neq g} E\{|h_{g_k}^H V_g w_{g'}|^2\}} \geq \frac{\rho |h_{g_k}^H V_g|^2}{1 + \rho r_g^* \sum_{g' \neq g} |h_{g_k}^H V_g w_{g'}|^2}. \quad (11)$$

where $\rho = P/(\sum_{i=1}^{G} r_g^*)$. Equation (a) follows from the fact that the expected value of the ratio $E(x/y)$ for correlated random variables can be approximated as $E(x)/E(y)$ [20]. This relation is not a tight approximation but provides an analytically tractable form. From the further SINR approximation, its sum-rate growth can be obtained as explained in the next section. With appropriate user selection, channel gain reduction of ZFBF can be negligible. Thus, in (b), $E\{|h_{g_k}^H V_g w_{g_k}|^2\} \approx \rho$ where $b_{g_k} = b_{g_k}/|b_{g_k}|$ [14]. Inequality (c) is $|h_{g_k}^H V_g w_{g'}|^2 = |V_g^H h_{g_k}^H W_{g'}^H| \leq \text{tr}\{W_{g'} W_{g'}^H\} \text{tr}\{V_g h_{g_k}^H V_g^H\}$ and $E\{|h_{g_k}^H V_g w_{g'}|^2\} = \rho r_g^*$. This approximate lower bound expression is used for selection as

$$\gamma_{g_k} = \frac{1}{\rho + r_g^* \sum_{g' \neq g} |h_{g_k}^H V_g w_{g'}|^2} |h_{g_k}^H V_g|^2. \quad (12)$$

This value is equivalent to quasi-SINR in [12]. This equivalence means that the approximate SINR lower bound provides an accurate SINR estimate in JSDM systems. The scheduler in [12] with this user selection metric provides the multiuser diversity gain as $\beta \log \log(K/G)$. In this paper, it will prove that this user selection metric can exploit the multiuser diversity gain in the SUS scheduler. The SUS scheduler is used because it is a simple algorithm and can achieve asymptotically optimal sum rate for MISO broadcast channels. The detailed procedure is summarized in Algorithm 1 where $\kappa < 1$ is a positive constant. After this user selection, the channel vectors of the selected users, $\mathcal{A}_g = \{\pi(1), \pi(2), \cdots, \pi(\mathcal{A}_g)\}$ where $\mathcal{A}_g \leq r_g^*$, are used to construct ZFBF vectors as

$$\tilde{W}_g(\mathcal{A}_g) = B_g (B_g^H B_g)^{-1}. \quad (13)$$

Each beamforming vector $w_{g_k}$ is obtained by normalizing each column of $\tilde{W}_g(\mathcal{A}_g)$.

### III. SUM-RATE ANALYSIS

In this section, the sum rate of the ZFBF-SUS is evaluated in JSDM system. First, the CDF of $\gamma_{g_k}$ is derived, and then the sum-rate growth of the ZFBF-SUS is examined. Finally, the multiuser diversity gain is investigated in a JSDM system with spatial heterogeneity. To evaluate the sum rate of the JSDM system, $b_g = S_g = r_g^*$ and $V_g = U_g^{\ast}$ for each $g$ [10] are assumed. In addition, it is assumed that all groups have the same number of users $K' = K_g = K/G$ and that the effective channel $b_{g_k}$ of all users is available at the BS.

The expected sum rate $R_g$ of group $g$ can be approximated by

$$R_g = E\{\sum_{k \in \mathcal{A}_g} \log_2(1 + |h_{g_k}^H V_g w_{g_k}|^2)\} \geq E\{\sum_{k \in \mathcal{A}_g} \log_2(1 + \gamma_{g_k})\} \geq E\{\sum_{i=1}^{r_g^*} \log_2(1 + \max_{k \in \mathcal{T}_i} \gamma_{g_k})\},$$

where (a) follows the expected SINR expression in (12). To describe the limiting behavior of $\gamma_{g_k}$, the CDF of the received SINR is required.

**Lemma 1:** Let $\lambda_{g,j}$ be the $j$th largest eigenvalue of the channel covariance matrix $R_g$. Then, the CDF of $\gamma_{g_k}$ is obtained as

$$\Pr(\gamma_{g_k} \leq x) = 1 - \frac{2\Gamma\left(\frac{\lambda_{g,j}}{\alpha} \frac{\xi}{x}\right)^2}{\alpha D^{\alpha}(1 + \epsilon x)} \sum_{j=1}^{r_g^*} \frac{\lambda_{g,j}^2}{\rho_{g,j}} G\left(D^{\alpha} \left(\frac{\lambda_{g,j}^2}{\rho_{g,j}} \right)^{\alpha/\alpha} \right),$$

where $\xi_j = \Pi_{j=1}^{r_g^*} (1 - (\lambda_{g,j}/\lambda_{g,i}))$ and $\epsilon$ is a small positive constant.

**Proof:** See Appendix A. □

Monte-Carlo simulation was used to verify Lemma 1 (Fig. 2). The analysis results are consistent with the simulation results. As $r_g^*$ increased, the value of the CDF decreased; this inverse relationship occurs because the increased number of dimensions can be constructively exploited. In contrast, as $\alpha$ increased, the value of CDF increased; this relationship indicates that the overall received SINR decreases as $\alpha$ increases. However, the complementary CDF (CCDF) of received SINR in the

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**Algorithm 1** ZFBF-SUS with the approximate SINR lower bound.

1. Initialization: $\mathcal{T}_i = \{1, 2, \cdots, K_g\}$, $\mathcal{A}_g = \emptyset$, $i = 1$.
2. While $\mathcal{T}_i$ is not empty and $i < r_g^*$ do
3. Step 1: For each $k \in \mathcal{T}_i$, calculate $p_k$:
4. If $i = 1$ then
5. $p_k = b_{g_k} - \sum_{j=1}^{i-1} b_{g_k} p_j/\|p_j\| p_j$.
6. Else
7. $p_k = b_{g_k} - \sum_{j=i}^{r_g^*} b_{g_k} p_j/\|p_j\| p_j$.
8. End if
9. Step 2: Find the $i$th selected user ($\pi(i)$):
10. $\pi(i) = \arg\max_{k \in \mathcal{T}_i} \gamma_{g_k}$.
11. Step 3: Update the selected user set $\mathcal{A}_g$:
12. $\mathcal{A}_g = \mathcal{A}_g \cup \pi(i)$.
13. Step 4: Calculate
14. $\mathcal{T}_{i+1} = \{k \in \mathcal{T}_i, k \neq \pi(i) \mid \|b_{g_k} p_{\pi(i)}\|/\|b_{g_k} p_{\pi(i)}\| < \kappa\}$.
15. $i \leftarrow i + 1$
16. End while
proof of Lemma 1 grows at a rate on the order of $x^{-1/(2\alpha + 3)}$. Thus, as $\alpha$ increases, the decrease in CCDF slows; this reduction in the decrease results in an increase in the probability of having a high received SINR. The following theorem shows how ZFBF-SUS exploits this property. Using Lemma 1, the following theorem is derived to describe the limiting behavior of the selected users.

**Theorem 1**: Let $\gamma_{g_k, \alpha}$ be the $j$th largest value of $\{\gamma_{g_k, 1 \geq k \geq K}\}$. Then it is obtained as

$$\Pr\left((\frac{2}{\alpha} + 1) \log(\gamma_{g_k, \alpha}) - \log\left(\frac{2\Gamma\left(\frac{2}{\alpha}\right)\rho^2 \sum_{i=1}^{r_k^*} \frac{\lambda_i}{\xi_i^2}}{\epsilon \alpha D^2} \cdot K'\right)\right)$$

\[
\leq \log \log K' \geq 1 - O\left(\frac{1}{\log K'}\right). \tag{16}
\]

**Proof**: See Appendix B.

From Theorem 1,

$$\log(\gamma_{g_k, \alpha}) = \log\left(\frac{2\Gamma\left(\frac{2}{\alpha}\right)\rho^2 \sum_{i=1}^{r_k^*} \frac{\lambda_i}{\xi_i^2}}{\epsilon \alpha D^2} \cdot K'\right) + O(\log \log K'). \tag{17}
$$

Then, for sufficiently large $K'$, the expected sum rate can be rewritten as

$$R_g \approx \mathbb{E}\left\{\log_2(1 + \gamma_{g_{\alpha_i}, \tau_i})\right\} \approx \mathbb{E}\left\{\log_2(\gamma_{g_{\alpha_i}, \tau_i})\right\}$$

$$\approx \sum_{i=1}^{r_k^*} \log_2\left(\frac{2\Gamma\left(\frac{2}{\alpha}\right)\rho^2 \sum_{i=1}^{r_k^*} \frac{\lambda_i}{\xi_i^2}}{\epsilon \alpha D^2} \cdot \mu_i \cdot K'\right)^{\frac{1}{\alpha + 1}}. \tag{18}
$$

To approximate $|T_n|$, $\mu_i = \lim_{K' \to \infty} |T_n|/K'$ is used where the limit actually exists and satisfies $\mu_i > I_x(i - 1, r_g^* - (i - 1))$ for $1 \leq i \leq r_g^*$ with $I_x(a, b)$ the regularized incomplete beta function [14]. From (18), the sum-rate growth for group $g$ is given by $r_g^*/((2/\alpha) + 1) \log K'$. The sum-rate growth accelerates as the pathloss exponent increases; this relationship occurs because the probability of having a high received SINR increases as the value of the pathloss exponent increases (Lemma 1). ZFBF-SUS selects the users with the highest received SINR, and therefore can provide higher sum-rate growth with increase in $\alpha$

The total expected sum rate can be approximated by

$$R \approx \sum_{g=1}^{G} R_g \approx \sum_{g=1}^{G} \sum_{i=1}^{r_g^*} \log_2\left(\frac{2\Gamma\left(\frac{2}{\alpha}\right)\rho^2 \sum_{i=1}^{r_g^*} \frac{\lambda_i}{\xi_i^2}}{\epsilon \alpha D^2} \cdot \mu_i \cdot K'\right)^{\frac{1}{\alpha + 1}}, \tag{19}
$$

which shows that the sum-rate growth is given by $(\sum_{g=1}^{G} r_g^*)/(\alpha + 1) \log K'$. Homogeneous users that have the same average SINR also have the same asymptotic sum rate $(\sum_{g=1}^{G} r_g^*) \log (K/G)$ [10]. In contrast, heterogeneous users have asymptotic sum rate of $(\sum_{g=1}^{G} r_g^*)/(\alpha + 1) \log (K/G)$ from (19). The sum-rate growth for heterogeneous users increases as $\alpha$ increases. Thus, the multiuser diversity gain increases as $\alpha$ increases. However, the sum-rate growth is upper-bounded by $(\sum_{g=1}^{G} r_g^*) \log (K/G)$ for $\alpha \leq 2$, because the residual IGI limits the benefit that can be extracted from the randomness of spatial heterogeneity.

**IV. USER GROUPING**

In the previous section, it was assumed that users in the same group have the same channel covariance matrix. However, heterogeneous users are not located in the same position, and thus have different channel covariance matrices. Due to this difference, an severe IGI is generated at the UE. Thus, the overall sum rate of JSDM system is degraded. This motivates the use of sophisticated grouping algorithm according to channel covariance matrix for users. To effectively accomplish this grouping, conventional algorithms exploit the similarity and orthogonality of channel covariance matrices. These algorithms are somehow effective, but they do not consider the signal power and IGI power related to sum rate. To this end, a new criterion is presented in subsection IV-C. It is assumed that the BS has perfect knowledge of the channel covariance matrix, which can be obtained from the users through an error-free control channel and be updated intermittently [10].

**A. Related Work**

**A.1 K-means Algorithm [10]**

K-means is a basic clustering algorithm. It partitions $K$ users into $G$ groups by using an iterative procedure. The $K$-means algorithm uses a chordal distance metric between the eigenvectors $U_k$ of the channel covariance matrix $R_k$ and group center $U_g$ as

$$\mathcal{L}(U_k, U_g) = \|U_k U_k^H - U_g U_g^H\|. \tag{20}$$

At each iteration, each user is assigned to the group to which the chordal distance is smallest. After the assignment, the group

$$R \approx \sum_{g=1}^{G} R_g \approx \sum_{g=1}^{G} \sum_{i=1}^{r_g^*} \log_2\left(\frac{2\Gamma\left(\frac{2}{\alpha}\right)\rho^2 \sum_{i=1}^{r_g^*} \frac{\lambda_i}{\xi_i^2}}{\epsilon \alpha D^2} \cdot \mu_i \cdot K'\right)^{\frac{1}{\alpha + 1}}, \tag{19}
$$

which shows that the sum-rate growth is given by $(\sum_{g=1}^{G} r_g^*)/(\alpha + 1) \log K'$. Homogenous users that have the same average SINR also have the same asymptotic sum rate $(\sum_{g=1}^{G} r_g^*) \log (K/G)$ [10]. In contrast, heterogeneous users have asymptotic sum rate of $(\sum_{g=1}^{G} r_g^*)/(\alpha + 1) \log (K/G)$ from (19). The sum-rate growth for heterogeneous users increases as $\alpha$ increases. Thus, the multiuser diversity gain increases as $\alpha$ increases. However, the sum-rate growth is upper-bounded by $(\sum_{g=1}^{G} r_g^*) \log (K/G)$ for $\alpha \leq 2$, because the residual IGI limits the benefit that can be extracted from the randomness of spatial heterogeneity.
center is updated with the dominant eigenvectors of the average eigenspaces of users in that group as

$$U_g = \mathbb{E}\left\{ \frac{1}{|G_g|} \sum_{k \in G_g} U_k U_k^H \right\},$$

(21)

where $G_g$ is the user set of group $g$.

A.2 Weighted Likelihood Similarity (WLS) [15]

The WLS algorithm is also an iterative, but it uses a different distance metric from $K$-means. Thus, the grouping metric and group center should be considered. The WLS algorithm uses the projection of the eigenspaces of the users to that of the group center; its metric can be expressed as

$$\mathcal{L}(R_k, U_g) = \| (U_k A_k H) U_g \|_F^2.$$  

(22)

The WLS algorithm exploits the weight $A_k H$ of different eigenvalues, the similarity measure mostly determines the eigenvectors of dominant eigenvalues. As in the $K$-means algorithm, each user is assigned to the group that has the maximum likelihood measure. Then, the group center is updated as

$$U_g = \mathbb{E}\left\{ \frac{1}{|G_g|} \sum_{k \in G_g} R_k \right\}.$$  

(23)

Unlike the $K$-means algorithm, the group center depends on the weights of dominant eigenspaces.

A.3 Correlation Matrix Distance (CMD) [16]

Weighted likelihood metric in the WLS algorithm does not measure the orthogonality of the groups. To increase the spatial orthogonality between the groups, the correlation matrix distance was introduced. The grouping metric of the CMD algorithm is the same as the WLS algorithm. The difference is that the CMD algorithm sequentially updates the group center of groups by measuring the CMD as

$$n_g = \underset{g'}{\arg \max} \sum_{g'=1}^{g-1} \left( 1 - \frac{\text{tr}(R_k R_{g'})}{\| R_k \|_F \| R_{g'} \|_F} \right),$$

$$U_g = U_{n_g},$$  

(24)

where $g = 1, 2, \cdots, G$. From the sequential updates, the group centers can be selected by maximizing the spatial orthogonality.

B. Proposed User Grouping

In this subsection, user grouping is considered by exploiting the approximate lower bound of the SINR value in (12). The approximate lower bound is a function of the CSI of user and outer beamforming. Because only the channel covariance matrix is available in grouping, the approximate lower bound can be further averaged over the instantaneous CSI. By taking an expectation, the approximate lower bound of the SINR value can be expressed as

$$E\{\gamma_{g_k}\} = E\left\{ \frac{\| h_k^H V_{g} \|^2}{\rho + r_g \sum_{g' \neq g} \| h_{g'}^H V_{g'} \|^2} \right\} \approx \frac{E\{\| h_k^H V_{g} \|^2\}}{\rho + r_g \sum_{g' \neq g} E\{\| h_{g'}^H V_{g'} \|^2\}},$$

(25)

where the tightness of $(a)$ holds for sufficiently large $M$ [20]. It is assumed that the expectation of the ratio of correlated random variables is bounded and its Taylor expansion converges. The interference term can be expressed as

$$E\{\| h_k^H V_{g'} \|^2\} = E\{ h_k^H V_{g} V_{g'}^H h_k \} = d_k^{-\alpha} E\{\eta_k^2 A_k^H U_k V_{g} V_{g'}^H U_k A_k^H \eta_k \}$$

$$= d_k^{-\alpha} \text{tr} \left( A_k^H U_k V_{g} V_{g'}^H U_k A_k^H \right)$$

$$= d_k^{-\alpha} \text{tr} \left( V_{g'}^H R_k V_{g'} \right)$$

$$= d_k^{-\alpha} \| (U_k A_k^H)^2 V_{g'} \|_F^2,$$  

(26)

where $(a)$ follows the Gaussian distribution of $\eta_k$. Similarly, the signal term can be obtained as $d_k^{-\alpha} \| (U_k A_k^H)^2 V_{g} \|_F^2$. By using this fact, the proposed metric can be expressed as

$$\mathcal{L}(R_k, V_g) = \frac{\| (U_k A_k^H)^2 V_{g} \|_F^2}{\rho + r_g \sum_{g' \neq g} \| (U_k A_k^H)^2 V_{g'} \|_F^2}.$$  

(27)

After grouping, the dominant eigenvectors of the average eigenspaces of users is obtained as

$$U_g = \mathbb{E}\left\{ \frac{1}{|G_g|} \sum_{k \in G_g} R_k \right\}.$$  

(28)

By using the updated dominant eigenvectors, the outer beamforming $V_g$ can be updated as in (9). Then, the iterative procedure is performed until it converges. The detailed procedure is described in Algorithm 2 where $\tau$ is a small positive constant related to stopping criterion.

Conventional algorithms exploit the similarity and orthogonality of the channel covariance matrix. In contrast, the proposed user grouping exploits the similarity of the outer beamforming as in (27). Each user selects a group that can maximize

\begin{algorithm}
\caption{Proposed user grouping algorithm}
1: Initialization: $G'_g = \emptyset$, $V_g$, $g = 1, \cdots, G$, $L_{tot}^0 = 0$, $n = 1$.
2: while $|L_{tot}^n - L_{tot}^{n-1}| > \tau L_{tot}^{n-1}$ do
3: \hspace{1em} Step 1: Compute $\mathcal{L}(R_k, V_g^0)$ for each $g$ and $k$.
4: \hspace{1em} Step 2: Assign each user $k$ to the specific group: $k \in G_g^0$ such that $\hat{g} = \max_g \mathcal{L}(R_k, V_g^0)$.
5: \hspace{1em} Step 3: Update group center: $U_g$ and $V_g$ in (9).
6: \hspace{1em} Step 4: Update group center: $U_g$ and $V_g$ in (9).
7: \hspace{1em} Calculate $L_{tot}^n = \sum_{g=1}^{G} \sum_{k \in G_g} \mathcal{L}(R_k, V_g^0)$.
8: \hspace{1em} Calculate $L_{tot}^n = \sum_{g=1}^{G} \sum_{k \in G_g} \mathcal{L}(R_k, V_g)$.
9: \hspace{1em} $n \leftarrow n + 1$
10: \hspace{1em} end while
\end{algorithm}
its own expected signal power by projecting the eigenspace of the outer beamforming. The user also calculates the IGI power by projecting the eigenspaces of other outer beamformings. Thus, by calculating (27), each user can be assigned to a group that increases the expected SINR.

V. SIMULATION RESULTS

In this section, simulation and analysis results for JSDM systems are investigated. From the results, the efficiency of the proposed user scheduling and grouping was verified. The efficiency was investigated by varying the number of users $K$ and transmit antennas $M$. For simulations, the maximum distance from the BS and pathloss exponents are $D = 1000$ m and $\alpha = 3, 4, 5$, respectively. Unless the pathloss exponent is specified, $\alpha = 4$. For the one-ring model, $\theta$ and $\Delta$ for the users were generated randomly between $-60^\circ$ to $60^\circ$ and $5^\circ$ to $15^\circ$, respectively. Perfect CSI and an error/delay-free control channel are assumed. The average received SINR at the cell edge is defined as

$$P_r = \frac{P}{D \sum_{g=1}^{M} r_g^\alpha}.$$  (29)

Then, the BS has $M = 16$, $r_g^\alpha = 4$, $G = 4$, and $P_r = 10$ dB. The diagonal matrix $\Lambda_k$ is generated as $\Lambda_k = \text{diag}(1, r, r^2, r^3, r^4)$ with $r = 0.7$ for $k = 1, 2, 3$, and $\Lambda_k = \text{diag}(1, r, r^2, r^3)$ with $r = 0.7$ for $k = 4$. For approximate BD condition, the eigenvectors that correspond to the eigenvalues and outer beamformings are given as

$$U_1 = F^{16}[:, 1 : 5], V_1 = U_1^* = F^{16}[:, 1 : 4]$$
$$U_2 = F^{16}[:, 5 : 9], V_2 = U_2^* = F^{16}[:, 5 : 8]$$
$$U_3 = F^{16}[:, 9 : 13], V_3 = U_3^* = F^{16}[:, 9 : 12]$$
$$U_4 = F^{16}[:, 13 : 16], V_4 = U_4^* = F^{16}[:, 13 : 16],$$  (30)

where $F^{16}$ is the 16-point fast-Fourier transformation matrix.

Fig. 3 shows the sum rate for increases with $\alpha$ in both simulations and analyses. For comparison, the result in [17] was plotted as $(\sum_{g=1}^{M} r_g^\alpha)(\alpha/2) \log K)$. The analysis results in [17] do not match the simulation results because of residual IGI. However, the proposed analysis considers the residual IGI and provides sum-rate growth that is similar to the simulation result. Note that the approximate SINR lower bound in (12) is not a tight bound due to a rough approximation but its sum-rate growth provides the similar slope with the simulation result. In addition, the sum rate can be increased as the pathloss exponent increases. This relationship occurs because a large pathloss exponent increases the probability of high received SINR. ZFBF-SUS can efficiently exploit this property.

In Fig. 4, the sum rate was compared according to user scheduling. The result of the scheduler in [12] was plotted as a reference-based distributed semi-orthogonal user selection (ReDOS). The sum rate of ZFBF-SUS with approximate SINR lower bound is higher than that of ZFBF-SUS with channel norm in [14] (Fig. 4). This is because the channel norm does not consider the IGI, and is therefore not an appropriate estimate in JSDM. RBF also utilizes the SINR value for JSDM, but obtains the SINR value under the assumption that inner beamforming is an identity matrix. For fewer users, RBF has a higher sum-rate than other algorithms, because ZFBF algorithms have less orthogonality between selected users. However, when the number of users increases, RBF has the lowest sum-rate because of its inaccurate SINR estimate. ZFBF-SUS with approximate SINR lower bound and ReDOS have similar sum-rate growth. However, ZFBF-SUS with approximate SINR lower bound has higher sum rate than ReDOS because orthogonality of users in ReDOS is not guaranteed at small number of transmit antennas. Fig. 5 shows the sum rate of user grouping algorithms over heterogeneous users. WLS algorithm achieved a higher sum rate than did the $K$-means algorithm (Fig. 5(a)), because the WLS
Fig. 5. Sum rate according to user groupings: (a) Different number of users and (b) different number of transmit antennas ($P_r = 10$ dB, $D = 1000$ m, $M = 16$, $r_g^* = 4$, $G = 4$.)

Fig. 6. Sum rate according to user scheduling over heterogeneous users: (a) Different number of users and (b) different number of transmit antennas ($P_r = 10$ dB, $D = 1000$ m, $M = 16$, $r_g^* = 4$, $G = 4$.)

algorithm considers the weights of different eigenmodes, which is beneficial to measure the similarity of channel covariance matrix. The sum rate of CMD algorithm is slightly higher than that of WLS algorithm. This is because its gain maximizes when the outer beamforming in [16] is used, and is limited in heterogeneous users. The proposed algorithm considers the expected SINR by exploiting the outer beamforming. Thus, the proposed algorithm achieved sum rate than did the WLS algorithm. The number of transmit antennas was increased at $K = 100$ and the effective channel rank was set to $r_g^* = M/G$; under these conditions, the proposed algorithm has higher sum rate than other algorithms (Fig. 5(b)). The proposed algorithm has a slight improvement compared with other algorithms. This is because the expected SINR in (25) is derived based on the assumption of perfect grouping. Thus, the proposed algorithm still exhibits a mismatch between the expected SINR and its real value. The difference between the proposed algorithm and WLS algorithm increased as the number of transmit antennas increased. This trend occurs because the accuracy of the approximation in (25) improves as the number of transmit antennas increases.

Fig. 6 shows the sum rate of user scheduling over heterogeneous users. In practical systems, heterogeneous users have an arbitrary channel covariance. Thus, the proposed user grouping was applied to cluster the heterogeneous users. Because the analysis result in (19) is derived based on perfect user grouping, the analysis result is not compared with the simulation result in Fig. 6. ZFBF-SUS with channel norm achieved the lowest sum rate, and achieved little multiuser diversity gain because the IGI was severe (Fig. 6(a)). The slope of ZFBF-SUS with approximate SINR lower bound was similar to those of ReDoS and RBF. The sum rate difference between ReDoS and RBF was lower than in Fig. 4. Orthogonality of the selected users in ReDoS is guaranteed when the true and representative channel covariance matrices are similar. When users are heterogeneous, mismatch between true and representative channel covariance matrices may cause loss of orthogonality, so the efficiency of
ZFBF is degraded. ZFBF-SUS method with approximate SINR lower bound achieved the highest sum rate of the scheduling methods tested, because the proposed algorithm can increase the SINR of selected users by exploiting the expected SINR metrics during user selection and during user grouping. For Fig. 6(b), the effective channel rank was increased by $r_g = M/G$. RBF achieved higher sum rate than ReDOS because the mismatch between true and representative channel covariance matrix increase as the effective channel rank increases. However, ZFBF-SUS with approximate SINR lower bound achieved a higher sum rate than RBF did. The proposed SINR metrics gave a good prediction of the expected SINR value of each user despite the increased number of transmit antennas.

VI. CONCLUSIONS

This paper considered user scheduling and grouping in a JSDM system. To exploit multiuser diversity gain, the instantaneous SINR was considered as a user selection metric. However, inner beamformings are determined after user scheduling, so current SINR cannot be known. Thus, an approximate lower bound of the user selection metric was analyzed over heterogeneous users. To increase sum rate, the SINR was further investigated for practical implementation. To increase the user selection metric was analyzed over heterogeneous users that have different average SINR. The user grouping problem was also investigated for practical implementation. To increase sum rate, the SINR was further averaged over instantaneous CSI and a new grouping metric was derived. By exploiting the outer beamforming, the proposed grouping provided a higher sum rate than did conventional algorithms.

APPENDIX I

PROOF OF LEMMA 1

It is assumed that users are uniformly distributed in the cell. If $Z = d_{y_g}^{-\alpha}$, the CDF of $Z$ is derived as

$$F_Z(z) = Pr(d_{y_g}^{-\alpha} \leq x) = Pr(d_{y_g} \geq x^{-\frac{1}{\alpha}}) = 1 - F_{d_{y_g}}(x^{-\frac{1}{\alpha}}) = 1 - \frac{1}{D^\alpha} z^{-\frac{1}{\alpha}}, z \in [D^{-\alpha}, \infty),$$

(31)

where $F_{d_{y_g}}(x) = d_{y_g}^2/D^2$ in [18]. Differentiating (31) yields the PDF of $Z$ as

$$f_Z(z) = \left\{ \begin{array}{ll} \frac{2}{\alpha D^\alpha} x^{-\frac{1}{\alpha} - 1}, & \text{if } x \in [D^{-\alpha}, \infty), \\ 0, & \text{elsewhere}. \end{array} \right.$$  

(32)

To obtain CDF of $\gamma_{y_g}$, the CCDF of $\phi_{y_g} = \frac{\|g_{y_g}^H V_g\|^2}{\sum_{g'=1}^G \|g_{y_g}^H V_g\|^2}$ is first obtained. If $X = \|g_{y_g}^H V_g\|^2$, then $X = \|g_{y_g}^H U_g^*\|^2 = \sum_{i=1}^{r_g} \lambda_{g,i} |\eta_{g,i}|^2$, which follows the generalized chi-square distribution with order $r_g$ and parameter $\lambda_{g,1}, \ldots, \lambda_{g,r_g}$ as

$$f_X(x) = \sum_{i=1}^{r_g} e^{-\frac{x}{\lambda_{g,i}}} \lambda_{g,i} \xi_{g,i},$$

(33)

where $\xi_{g,i} = \Pi_{j \neq i}(1 - \frac{\lambda_{g,i}}{\lambda_{g,j}})$ and $\sum_{i=1}^{r_g} \xi_{g,i} = 1$.

Similarly, $\|g_{y_g}^H V_g\|^2 = \sum_{r_g=1}^{r_g} \lambda_{g,j} |\eta_{g,j}|^2$. From the approximate BD condition, $U_g V_g' = U_g US^r_g = [0_{G \times r_g}; E_{g,g'}]$, $e_{g,g'}$ is the $j$th row vector of $E_{g,g'}$. The residual IGI from $e_{g,g'}$ is difficult to analyze because of its unknown density function. Despite its unknown property, the residual IGI can be considerably reduced when the design parameter $r_g$ is assigned to be sufficiently large value such that $\sum_{g=1}^G r_g \approx M$.

Then, it may be assumed that $\|x_{g,g'}\|^2$ is upper-bounded by a small positive constant $\epsilon$. The remaining task is to derive a pdf of random variable $Y$ such that $\sum_{r_g=r_g+1}^{r_g} \lambda_{g,j} |\eta_{g,j}|^2$, which also follows the generalized chi-square distribution with order $r_g - r_g$ and parameter $\lambda_{g,r_g+1}, \ldots, \lambda_{g,r_g}$ as

$$f_Y(y) = \sum_{j=r_g+1}^{r_g} e^{-\frac{y}{\lambda_{g,j}}} \lambda_{g,j} \xi_{g,j},$$

(34)

where $\xi_{g,j} = \Pi_{j \neq j}(1 - \frac{\lambda_{g,j}}{\lambda_{g,j}})$ and $\sum_{i=1}^{r_g} \xi_{g,j} = 1$. The random variables of $X$ and $Y$ are independent because $X$ depends on $\lambda_{g,1}, \ldots, \lambda_{g,r_g}$ whereas $Y$ depends on $\lambda_{g,r_g+1}, \ldots, \lambda_{g,r_g}$.

From $X$ and $Y$,

$$Pr(\phi_{y_g} > x) = \int_0^\infty Pr \left( X \geq \frac{x}{\rho (G - 1)} \right) f_X(x) dx$$

$$= \int_0^\infty \int_0^{\frac{x}{\rho (G - 1)}} f_Y(y) dy dx$$

$$= \int_0^{\frac{x}{\rho (G - 1)}} \int_0^\infty f_Y(y) dx dy$$

$$= \int_0^{\frac{x}{\rho (G - 1)}} \int_0^\infty f_Y(y) dy dx$$

$$= \int_0^{\frac{x}{\rho (G - 1)}} \int_0^\infty f_Y(y) dx dy$$

(35)

From (35), the CCDF of $\gamma_{y_g}$ can be derived as

$$Pr(\gamma_{y_g} > x)$$

$$= \int_0^x \frac{\rho^2 \|g_{y_g}^H V_g\|^2}{1 + \rho^2 \sum_{g'=1}^G \|g_{y_g}^H V_g\|^2} dz$$

$$= \int_0^x \frac{\rho^2 \|g_{y_g}^H V_g\|^2}{1 + \rho^2 \sum_{g'=1}^G \|g_{y_g}^H V_g\|^2} dz$$

$$= \int_0^x \frac{\rho^2 \|g_{y_g}^H V_g\|^2}{1 + \rho^2 \sum_{g'=1}^G \|g_{y_g}^H V_g\|^2} dz$$

$$= \int_0^x \frac{\rho^2 \|g_{y_g}^H V_g\|^2}{1 + \rho^2 \sum_{g'=1}^G \|g_{y_g}^H V_g\|^2} dz$$

$$= \int_0^x \frac{\rho^2 \|g_{y_g}^H V_g\|^2}{1 + \rho^2 \sum_{g'=1}^G \|g_{y_g}^H V_g\|^2} dz$$

(36)
Approximation (a) is valid because the CCDF of $\phi_{g_b}$ is independent of the particular value of $\rho$. Thus, $\rho$ is replaced with $\rho_\ast$. In (b), $t = 1/z$ is used. (c) follows the definition of Gamma function $G(y; k, \theta) = \int_0^\infty e^{-\frac{x}{\theta}} x^{k-1} t \, dt$. In (d), $G(y; k, \theta)$ is equivalent to $G(y; \theta; k, 1)$. The term $(\lambda_{g,j}/\lambda_{g,i})$ makes the analysis difficult to handle. However, the value of $(\lambda_{g,j}/\lambda_{g,i}) \cdot (G - 1)e_\ast$ is negligible compared to 1 for small $x$. In addition, the term $(\lambda_{g,j}/\lambda_{g,i}) \cdot (G - 1)$ dominantly depends on $\epsilon$ because it is assumed that $\epsilon \ll 1$. Thus, to simplify the problem, $(\lambda_{g,j}/\lambda_{g,i}) \cdot (G - 1)$ is approximated as $\epsilon$. Monte-Carlo simulations demonstrated this approximation does not change the CDF of $\gamma_{g_b}$.

APPENDIX II

PROOF OF THEOREM 1

The asymptotic behavior of the maximum of $\gamma_{g_j, K}$ is investigated in this proof. Three Theorems 2–4 in [3] are useful to investigate the asymptotic behavior. As in [3], the limiting distributions for $F_{\gamma_{g_b}}$, which denotes a CDF of $\gamma_{g_j, K}$, is first obtained. Then, the behavior of $\gamma_{g_j, K}$ will be investigated as $K$ increases.

Let $a_K = (K \frac{2\Gamma(\frac{2}{\alpha} \frac{d_0}{\lambda_{g,j}})}{\alpha d_0^2}) \sum_{i=1}^{r_i \ast} \left( \frac{d_i}{\lambda_{g,i}} \right)^{\alpha_1} + b_K = 0$ in Theorem 3 [3]. For each $z > 0$, using Lemma 1,

$$\lim_{K \to \infty} K(1 - F(a_K + b_K)) \equiv \lim_{K \to \infty} \frac{K \frac{2\Gamma(\frac{2}{\alpha} \frac{d_0}{\lambda_{g,j}})}{\alpha d_0^2} \sum_{i=1}^{r_i \ast} (\frac{d_i}{\lambda_{g,i}})^{\alpha_1}}{(a_K z)^{\alpha} + (\epsilon (a_K z)^{\alpha} + 1)} = z^{-\frac{2}{\alpha}},$$

where (a) follows $G(\frac{d_0 \alpha_1}{\lambda_{g,j}} \frac{2}{\alpha}, 1) = 1$ as $K \to \infty$. Thus, the limiting distribution of $F_{\gamma_{g_b}}$ belongs to type 1 in Theorem 2 [3] with $(2/\alpha) + 1$.

The convergence rate of the limiting distributions $F_{\gamma_{g_b}}$ can be evaluated by increasing $K$. The evaluation is performed at two points $z_1 = (\log K)^{\frac{2}{\alpha} + 1}$ and $z_2 = \left( \frac{1}{\log \sqrt{K}} \right)^{\frac{2}{2(\alpha) + 1}}$ in Theorem 4, [3]. Then, similarly in Theorem 4 [3], it can be directly obtained as

$$|F_{\gamma_{g_j, K}}(a_K z_1 + b_K) - F_{\gamma_{g_j, K}}(a_K z_2 + b_K)| \geq 1 - O\left( \frac{1}{\log K} \right).$$

Substituting the corresponding values of $a_K, b_K, z_1$, and $z_2$ into (38) completes the proof of Theorem 1.

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