Balitskiy, Alexey; Berdnikov, Aleksandr
Local-to-global Urysohn width estimates. (English) Zbl 07436421
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Summary: The notion of the Urysohn $d$-width measures to what extent a metric space can be approximated by a $d$-dimensional simplicial complex. We investigate how local Urysohn width bounds on a Riemannian manifold affect its global width. We bound the 1-width of a Riemannian manifold in terms of its first homology and the supremal width of its unit balls. Answering a question of Larry Guth, we give examples of $n$-manifolds of considerable $(n-1)$-width in which all unit balls have arbitrarily small 1-width. We also give examples of topologically simple manifolds that are locally nearly low-dimensional.

MSC:
53C23 Global geometric and topological methods (à la Gromov); differential geometric analysis on metric spaces
53C22 Geodesics in global differential geometry
54E35 Metric spaces, metrizability

Full Text: DOI arXiv

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