We report on the extraction of $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ and indirect measurement of the mass of the W boson from the forward-backward asymmetry of $\mu^+\mu^-$ events in the Z boson mass region. The data sample collected by the CDF detector corresponds to the full 9 fb$^{-1}$ run II sample. We measure $\sin^2 \theta_{\text{lept}}^{\text{eff}} = 0.2315 \pm 0.0010$, $\sin^2 \theta_W = 0.2233 \pm 0.0009$ and $M_W$(indirect) = 80.365 ± 0.047 GeV/c$^2$, where each uncertainty includes both statistical and systematic contributions.

1 Introduction

Now that the Higgs mass is known, the Standard Model is over constrained. Therefore, any inconsistency between precise measurements of SM parameters would be indicative of new physics. The parameter that needs to be measured more precisely is $M_W$ (with errors <15 MeV), or equivalently $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$ (with errors <0.0003). Similarly, in order to help resolve the long standing 3$\sigma$ difference in $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ between SLD and LEP (shown in bottom left part of Figure 1), new measurements of $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ should have errors similar to SLD or LEP (±0.0003).

Precise extractions of $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ and $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$ using the forward-backward asymmetry ($A_R$) of dilepton events produced in pp and pp collisions are now possible for the first time because of three new innovations:

- The use a new technique$^1$ for calibrating the muon energy scale as a function of detector $\eta$ and $\phi$ (and sign), thus greatly reducing systematic errors from the energy scale. A similar method can also used for electrons.

- The implementation$^2$ of Z fitter electroweak radiative corrections into the theory predictions of POWHEG and RESBOS which allows for a measurement of both $\sin^2 \theta_{\text{lept}}^{\text{eff}}(M_Z)$ and $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$.

- Use of a new event weighting technique$^3$. With this technique all experimental uncertainties in acceptance and efficiencies cancel (by measuring the cos $\theta$ coefficient $A_4$ and using the
Figure 1 – **Top-left**: CDF raw $A_{FB}$ measurement in bins of $\mu^+\mu^-$ invariant mass. Only statistical uncertainties are shown. The Monte Carlo simulation (PYTHIA) includes the effect of resolution smearing and FSR. The PYTHIA $|y| < 1$ asymmetry curve does not.

**Top-right**: $A_{FB}$ unfolded for resolution and QED-FSR. The PYTHIA calculation uses $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.232$. The EBA-based RESBOS calculation uses $\sin^2 \theta_{\text{eff}}^{W} = 0.2233$ ($\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.2315$).

**Bottom-left**: Measurements of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$. **Bottom-right**: Direct and indirect measurement of $M_W$. Also shown are the errors from 9 fb$^{-1}$ $e^+e^-$ sample in CDF which are expected to be smaller by a factor of 2.

Relation $A_{FB} = 8 A_4/3$. Similarly, additional weights are included for antiquark dilution, which makes the analysis independent of the acceptance in dilepton rapidity.

## 2 Analysis of CDF $\mu^+\mu^-$ full 9 fb$^{-1}$ run II sample

We report on the published analysis of the full 9 fb$^{-1}$ run II $\mu^+\mu^-$ data sample$^4$ collected by the CDF detector.$^3$

After applying the calibrations and muon scale corrections to the experimental and simulated data, $A_{FB}$ is measured in bins of $\mu^+\mu^-$ invariant mass using the event-weighting method. This measurement is denoted as the raw $A_{FB}$ measurement because the event-weighting method provides a first-order acceptance correction, but does not include resolution unfolding and final-state (FSR) QED radiation. The raw $A_{FB}$ measurement in bins of the muon-pair invariant mass is shown on the top left part of Fig. 1. Only statistical uncertainties are shown. The Monte Carlo simulation (PYTHIA) includes the effect of resolution smearing and FSR. The PYTHIA $|y| < 1$ asymmetry curve does not include the effect of resolution smearing or FSR.

With the event weighting technique, the events near $\cos \theta = 0$ are assigned zero weight. Therefore, the migration of events between positive and negative $\cos \theta$ is negligible. Resolution smearing and FSR primarily transfer events between bins in invariant mass.
The raw $A_{fb}$ in bins of invariant is unfolded for resolution smearing and FSR using a transfer matrix which is obtained from the Monte Carlo simulation. The unfolded $A_{fb}$ is shown in the top-right side of Fig. 1.

The electroweak (EWK) mixing parameters $\sin^2 \theta_{\text{lept}}^\text{eff}$ and $\sin^2 \theta_W$ are extracted from the fully unfolded $A_{fb}$ measurements using $A_{fb}$ templates calculated with different values of $\sin^2 \theta_W$. Three QCD calculations are used: LO (tree), resbos NLO, and powheg-box NLO. The calculations were modified to include EWK radiative correction using the effective Born approximation (EBA). For the EBA electroweak form-factor calculations, the EW parameter is $\sin^2 \theta_W$.

The $A_{fb}$ measurement is directly sensitive to the effective-mixing parameters $\sin^2 \theta_{\text{lept}}^\text{eff}$ which are combinations of the form-factors and $\sin^2 \theta_W$. Most of the sensitivity to $\sin^2 \theta_{\text{lept}}^\text{eff}$ comes from the Drell-Yan $A_{fb}$ near the $Z$ pole, where $A_{fb}$ is small. In contrast, $A_{fb}$ at higher mass values where $A_{fb}$ is large, is mostly sensitive to the axial coupling, which is known.

The measurement and templates are compared using the $\chi^2$ statistic evaluated with the $A_{fb}$ measurement error matrix. Each template provides a scan point for the $\chi^2$ function ($\sin^2 \theta_W$, $\chi^2(\sin^2 \theta_W)$). The scan points are fit to a parabolic $\chi^2$ functional form. The $\chi^2$ distribution of the scan over templates from the resbos NLO calculation (with CT10 PDFs) is shown in the right side of Fig. 2. The EBA-based resbos calculations of $A_{fb}$ are used to extract the central value of $\sin^2 \theta_W$. The other calculations are used to estimate the systematic error from the electroweak radiative corrections and QCD NLO radiation.

### 3 Systematic errors in the extraction of $\sin^2 \theta_{\text{lept}}^\text{eff}$ from the full 9 fb$^{-1}$ run II sample

In all QCD calculations, the mass-factorization and renormalization scales are set to the muon-pair invariant mass. To evaluate the effects of different scales, the running scales are varied independently by a factor ranging from 0.5 to 2 in the calculations. The largest observed deviation of the best-fit value of $\sin^2 \theta_W$ from the default value is considered to be the QCD-scale uncertainty. This uncertainty is $\Delta \sin^2 \theta_W$ (QCD scale) = $\pm 0.00003$.

The CT10 PDFs are derived from a global analysis of experimental data that utilizes 26 fit parameters and the associated error matrix. In addition to the best global-fit PDFs, PDFs representing the uncertainty along the eigenvectors of the error matrix are also derived. For each eigenvector $i$, a pair of PDFs are derived using 90% C.L. excursions from the best-fit parameters along its positive and negative directions. The difference between the best-fit $\sin^2 \theta_W$ values obtained from the positive (negative) direction excursion PDF and the global best-fit PDF is denoted as $\delta^+_i$ ($\delta^-_i$). The 90% C.L. uncertainty for $\sin^2 \theta_W$ is given by the expression $\frac{1}{2} \sqrt{\sum_i (|\delta^+_i|^2 + |\delta^-_i|^2)}$. 

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Figure 2 – **Left:** Extracted values of $\sin^2 \theta_{\text{lept}}^\text{eff}$, $\sin^2 \theta_W$, and $M_W$ from the CDF measurement of $A_{fb}$ in the 9 fb$^{-1}$ $\mu^+\mu^-$ sample (with sources of systematic errors). **Right:** $\chi^2$ comparison of the CDF $A_{fb}$ $\mu^+\mu^-$ measurement with resbos-EBA NLO templates.
where the sum \(i\) runs over the 26 eigenvectors. This value is scaled down by a factor of 1.645 for the 68.3% C.L. (one standard-deviation) uncertainty yielding \(\Delta \sin^2 \theta_W (\text{PDF}) = \pm 0.00036\). The PDF error is expected to be a factor of 2 smaller with more modern PDFs.

The RESBOS \(A_\text{eff}\) templates are the default templates for the extraction of \(\sin^2 \theta^\text{lep}_\text{eff}\). The scan with the POWHEG-BOX or the tree templates yields slightly different values for \(\sin^2 \theta_W\). The difference, denoted as the EBA uncertainty, is \(\Delta \sin^2 \theta_W (\text{EBA}) = \pm 0.00012\). Although the RESBOS and POWHEG-BOX predictions are fixed-order NLO QCD calculations at large boson \(P_T\), they are all-orders resummation calculations in the low-to-moderate \(P_T\) region, which provides most of the total cross section. The EBA uncertainty is a combination of differences between the resummation calculations and the derived value of \(\sin^2 \theta_W\) with and without QCD radiation.

In summary, the total systematic uncertainties on \(\sin^2 \theta_W\) from the QCD mass-factorization and renormalization scales, and from the CT10 PDFs is \(\pm 0.00036\). All component uncertainties (shown in the left side of Fig. 2) are combined in quadrature. With the inclusion of the EBA uncertainty, the total systematic uncertainty is \(\pm 0.00038\).

4 Summary of results from the 9 fb\(^{-1}\) \(\mu^+\mu^-\) sample

The left side of Fig. 2 shows the best fit extracted values of \(\sin^2 \theta^\text{lep}_\text{eff}\), \(\sin^2 \theta_W\), and \(M_W\) from the CDF measurement of \(A_\text{eff}\) in the 9 fb\(^{-1}\) \(\mu^+\mu^-\) sample with statistical errors, and the various sources of systematic errors. With statistical and systematic errors added in quadrature, the best-fit values are

\[
\sin^2 \theta^\text{lep}_\text{eff} = 0.2315 \pm 0.0010 \\
\sin^2 \theta_W = 0.2233 \pm 0.0009 \\
M_W (\text{indirect}) = 80.365 \pm 0.047 \text{ GeV}/c^2.
\]

The results for \(\sin^2 \theta^\text{lep}_\text{eff}\) are consistent with other measurements at the Z-boson pole, as shown on the bottom left part of Fig. 1. The results for \(M_W\) are consistent with other direct and indirect measurements of \(M_W\) as shown on the bottom-right side of Fig. 1.

Also shown is the most recent (Aug. 2014) value\(^5\) of \(\sin^2 \theta^\text{lep}_\text{eff}\) extracted from the full 9.7 fb\(^{-1}\) run II \(e^+e^-\) sample in D0\(^5\) (0.2315 \(\pm 0.0005\). The results from the CDF full 9 fb\(^{-1}\) run II \(e^+e^-\) sample are expected by end of 2014. Because of the larger angular acceptance for electrons, the expected error in \(\sin^2 \theta^\text{lep}_\text{eff}\) are smaller (\(\pm 0.0005\). The expected error in the CDF extracted value of \(M_W^{\text{indirect}}\) (\(\pm 24\) MeV) will be competitive with the direct measurements of \(M_W\). The uncertainty in the average of the CDF and D0 legacy 9 fb\(^{-1}\) measurement of \(\sin^2 \theta^\text{lep}_\text{eff}\) will be competitive with LEP and SLC.

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