A global and non-entropic arrow of time: the double role of the energy-momentum tensor

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In this paper we adopt a global and non-entropic approach to the problem of the arrow of time, according to which the arrow of time is an intrinsic geometrical property of spacetime. Our main aim is to show the double role played by the energy-momentum tensor in the context of our approach. On the one hand, the energy-momentum tensor is the intermediate step that permits to turn the geometrical time-asymmetry of the universe into a local arrow of time manifested as a time-asymmetric energy flow. On the other hand, the energy-momentum tensor supplies the basis for deducing the time-asymmetry of quantum field theory, posed as an axiom in this theory.

I. INTRODUCTION

Since the nineteenth century, the problem of the arrow of time has been one of the most controversial questions in the foundations of physics and one of the main concerns of many physicists. In the last years we were very interested on this problem and have devoted several papers to the cosmological origin of the arrow of time ( [1]). This research has reached its culmination in papers [2] and [3], where we have presented a comprehensive formulation of our view on the subject based on a global and non-entropic approach, according to which the arrow of time is an intrinsic geometrical property of spacetime.

The main aim of this paper is to show the double role played by the energy-momentum tensor in the context of our approach. On the one hand, the energy-momentum tensor is the intermediate step that allows us to turn the geometrical time-asymmetry of the universe into a local arrow of time manifested as a time-asymmetric energy flow. On the other hand, the energy-momentum tensor supplies the basis for deducing the time-asymmetry of quantum field theory, posed as an axiom in this theory.

When the problem of the arrow of time is addressed, the main obstacle to be faced is conceptual confusion: the lack of consensus is primarily due to the fact that different concepts are identified and different problems are subsumed under the same label. Thus, it is not possible to seek an acceptable solution if the terms used are not adequately defined in physical or in mathematical precise terms. For this reason we are forced to devote the initial sections of the paper to clarify several concepts and to disentangle the problems involved in the discussion in order to reach our goal avoiding misunderstandings and misinterpretations. On this basis, the paper is organized as follows. In Section 2 the problems of irreversibility and of the arrow of time are precisely stated and distinguished. In Section 3 the global and non-entropic character of our approach is justified by contrast with other traditional approaches. In Section 4 the necessary conditions for defining a global and non-entropic arrow of time are explained and applied to the case of FLRW models. In Section 5 it is shown the role played by the energy-momentum tensor in translating the global geometrical arrow into the local level. Finally, in Section 6 it is shown how the energy-momentum tensor can be used to justify the time-asymmetry postulate of quantum field theory on global grounds.

II. DISENTANGLING PROBLEMS: IRREVERSIBILITY AND ARROW OF TIME

Traditionally, the problem of irreversibility and the problem of the arrow of time have been identified, as if irreversibility were the clue for understanding the origin and the nature of the arrow of time. In the present section we will show that, when the concepts involved in the debate are precisely clarified, both problems become evidently different. In particular, we will see that the question of irreversibility presupposes the previous answer of the problem of the arrow of time.
A. The problem of irreversibility

In general, the concepts of irreversibility and of time-reversal invariance are invoked in the discussions about the arrow of time, but usually with no elucidation of their precise meanings; this results in confusions that contaminate many interesting considerations. For this reason, we will start from providing some necessary definitions.

**Definition 1:** A dynamical equation is *time-reversal invariant* if it is invariant under the transformation $t \rightarrow -t$; as a result, for each solution $f(t)$, $f(-t)$ is also a solution.

**Definition 2:** A solution of a dynamical equation is *reversible* if it corresponds to a closed curve in phase space.

It is quite clear that both concepts are different to the extent that they apply to different mathematical entities: time-reversal invariance is a property of dynamical equations (laws) and, *a fortiori*, of the set of its solutions (evolutions); reversibility is a property of a single solution of a dynamical equation. Furthermore, they are not even correlated. In fact, time-reversal invariant equations can have irreversible solutions.

When both concepts are elucidated in this way, the problem of irreversibility can be clearly stated: how to explain irreversible evolutions in terms of time-reversal invariant laws. But once it is recognized that irreversibility and time-reversal invariance apply to different mathematical entities, it is easy to find a conceptual answer to the problem of irreversibility: nothing prevents a time-reversal invariant equation from having irreversible solutions.

Of course, even though the conceptual answer is simple, a great deal of theoretical work is needed for obtaining irreversible evolutions from an underlying time-reversal invariant dynamics. This was the problem faced by the founding fathers of statistical mechanics when they sought to describe the irreversible evolutions of thermodynamics by means of the time-reversal invariant laws of classical mechanics. At present, many efforts are directed to explain the irreversible behavior of quantum systems in terms of the time-reversal invariant quantum theory (see, for instance, [4]). Here we only mean that, in order to face the problem of irreversibility, the question about the arrow of time does not need to be addressed: the distinction between the two directions of time is *usually presupposed* when the irreversible evolutions are conceived as processes going from non-equilibrium to equilibrium or from preparation to measurement towards the future.

B. The problem of the arrow of time

In our everyday life we perceive an asymmetry between past and future and experience the time order of the world as "directed". The problem of the arrow of time arises when we seek a physical correlate of this intuitive asymmetry. The main difficulty to be encountered for solving this problem is our anthropocentric perspective, which prevents us from shaking off our temporally asymmetric assumptions. In fact, traditional discussions around the problem of the arrow of time are usually subsumed under the label "the problem of the direction of time", as if we could find an exclusively physical criterion for singling out the direction of time, identified with what we call "the future". But there is nothing in physical laws that distinguishes, in a non-arbitrary way, between past and future as we conceive them in everyday life. It might be objected that physics implicitly assumes this distinction with the use of temporally asymmetric expressions, like "future light cone", "initial conditions", "increasing time", and so on. However this is not the case, and the reason relies on the distinction between "conventional" and "substantial".

**Definition 3:** Two objects are *formally identical* when there is a permutation that interchanges the objects but does not change the properties of the system to which they belong.

In physics it is usual to work with formally identical objects: the two semicones of a light cone, the two spin senses, etc. Now we can define two notions that will be central in the further discussion:

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1For instance, the equation of motion of the pendulum with Hamiltonian:

$$H = \frac{1}{2} p_\theta^2 + \frac{K^2}{2} \cos \theta$$

is *time-reversal invariant*, namely, it is invariant under the transformation $\theta \rightarrow \theta$, $p_\theta \rightarrow -p_\theta$; however, whereas the trajectories within the separatrices are reversible since they are closed curves, the trajectories above (below) the separatrices are *irreversible* since, in the infinite time-limit, $\theta \rightarrow -\infty$ ($\theta \rightarrow +\infty$).
Definition 4: We will say that we establish a *conventional* difference when we call two formally identical objects with two different names, e.g., when we assign different signs to the two spin senses.

Definition 5: We will say that the difference between two objects is *substantial* when we assign different names to two objects which are not formally identical (see [5], [6]). In this case, even though the names are conventional, the difference is substantial.

E.g., the difference between the two poles of the theoretical model of a magnet is conventional since both poles are formally identical; on the contrary, the difference between the two poles of the Earth is substantial because in the north pole there is an ocean and in the south pole there is a continent (and the difference between ocean and continent remains substantial even if we conventionally change the names of the poles).

Once this point is accepted, it turns to be clear that physics uses the labels ”past” and ”future” in a conventional way. Therefore, the problem cannot yet be posed in terms of singling out the future direction of time: the problem of the arrow of time becomes the problem of finding a *substantial difference* between the two temporal directions. But if this is our central question, we cannot project our independent intuitions about past and future for solving it without begging the question. When we want to address the problem of the arrow of time from a perspective purged of our temporal intuitions, we must avoid the conclusions derived from subtly presupposing time-asymmetric notions. As Huw Price [7] claims, it is necessary to stand at a point outside of time, and thence to regard reality in atemporal terms: this is *"the view from nowhen"*. This atemporal standpoint prevents us from using temporally asymmetric expressions in a non-conventional way: the assumption about the difference between past and future is not yet legitimate in the context of the problem of the arrow of time.

But then, what does ”the arrow of time” mean when we accept this constraint? Of course, the traditional expression coined by Eddington has only a metaphorical sense: its meaning must be understood by analogy. We recognize the difference between the head and the tail of an arrow on the basis of its geometrical properties; therefore, we can substantially distinguish between both directions, head-to-tail and tail-to-head, independently of our particular perspective. Analogously, we will conceive the *problem of the arrow of time* in terms of the *possibility of establishing a substantial distinction between the two directions of time on the basis of exclusively physical arguments*.

III. A GLOBAL AND NON-ENTROPIC APPROACH

A. Why global? The traditional local approach

The traditional local approach owes its origin to the attempts to reduce thermodynamics to statistical mechanics: in this context, the usual answer to the problem of the arrow of time consists in defining the future as the direction of time in which entropy increases. However, already in 1912 Paul and Tatiana Ehrenfest [8] noted that, if the entropy of a closed system increases towards the future, such increase is matched by a similar one in the past of the system. Gibbs’ answer to the Ehrenfests’ challenge was based on the assumption that probabilities are determined from prior events to subsequent events. But this answer clearly violates the ”nowhen” standpoint since probabilities are blind to temporal direction. Therefore, any appeal to the distinction between prior and subsequent events commits a *petitio principii* by presupposing the arrow of time from the very beginning.

The point can also be posed in different terms. Let us assume that we have solved the irreversibility problem; so we have the description of all the irreversible evolutions, say, decaying processes, of the universe. However, since we have not yet established a substantial difference between both directions of time, we have no way to decide towards which temporal direction each decay proceeds. Of course, we would obtain the arrow of time if we could coordinate the processes in such a way that all of them parallelly decay towards the same temporal direction. But this is precisely what local physics cannot offer: only by means of global considerations all the decaying processes can be coordinated. This means that the global arrow of time plays the role of the background scenario where we can meaningfully speak of the temporal direction of irreversible processes, and this scenario cannot be built up by means of local theories that only describe phenomena confined in small regions of spacetime.

B. Why non-entropic? The traditional entropic approach

When, in the late nineteenth century, Boltzmann developed the probabilistic version of his theory in response to the objections raised by Loschmidt and Zermelo, he had to face a new challenge: how to explain the highly improbable current state of our world. In order to answer this question, Boltzmann offered the first cosmological approach to the problem: *"The universe, or at least a big part of it around us, considered as a mechanical system, began in a very
improbable state and it is now also in a very improbable state. Then if we take a smaller system of bodies, and we isolate it instantaneously from the rest of the world, in principle this system will be in an improbable state and, during the period of isolation, it will evolve towards more probable states” [9]. Since this seminal work, many authors have related the temporal direction past-to-future to the gradient of the entropy function of the universe. For instance, Feynman asserts: ”For some reason, the universe at one time had a very low entropy for its energy content, and since then entropy has increased. So that is the way towards future. That is the origin of all irreversibility” [10]. In a similar sense, Davies claims that ”There exists an arrow of time only because the universe originates in a less-than-maximum entropy state” [11]. Even if these authors admit the need of global arguments for solving the problem of the arrow of time, they coincide in considering that it must be addressed in terms of entropy.

The global entropic approach rests on two assumptions: that it is possible to define entropy for a complete cross-section of the universe, and that there is an only time for the universe as a whole. However, both assumptions involve difficulties. In the first place, the definition of entropy in cosmology is still a very controversial issue: there is not a consensus regarding how to define a global entropy for the universe. In fact, it is usual to work only with the entropy associated with matter and radiation because there is not yet a clear idea about how to define the entropy due to the gravitational field. In the second place, when general relativity comes into play, time cannot be conceived as a background parameter which, as in pre-relativistic physics, is used to mark the evolution of the system. Therefore, the problem of the arrow of time cannot legitimately be posed, from the beginning, in terms of entropy gradient of the universe computed on a background parameter of evolution.

Nevertheless, these points are not the main difficulty: there is a conceptual argument for abandoning the traditional entropic approach. As it is well known, a given value of entropy is compatible with many configurations of a system: entropy is a phenomenological property. The question is whether there is a more fundamental property of the universe which allows us to distinguish between both temporal directions. On the other hand, if the arrow of time reflects a substantial difference between both directions of time, it is reasonable to consider it as an intrinsic property of time, or better, of spacetime, and not as a secondary feature depending on a phenomenological property. For these reasons we will follow Earman’s ”Time Direction Heresy” [12], according to which the arrow of time is an intrinsic, geometrical property of spacetime embodied in $g_{\mu\nu}(x)$, which does not need to be reduced to non-temporal features. In other words, the geometrical approach to the problem of the arrow of time has conceptual priority over the entropic approach, since the geometrical properties of the universe are more basic than its thermodynamic properties.

IV. CONDITIONS FOR A GLOBAL AND NON-ENTROPIC ARROW OF TIME

A. Time-orientability

In a Minkowski spacetime, it is always possible to define the class of all the future light semicones and the class of all the past light semicones (where the labels ”future” and ”past” are conventional). In general relativity the metric can always be approximated, in small regions of spacetime, to the Minkowski form. However, on the large scale, we do not expect the manifold to be flat because gravity can no longer be neglected. Many different topologies are consistent with Einstein’s field equations. In particular, the possibility arises of spacetime being curved along the spatial dimension in such a way that, e.g., the spacelike sections of the universe become the three-dimensional analogues of a Moebius band; in this case it is said that the spacetime is non-time-orientable.

Definition 6: A spacetime is time-orientable if there exists a continuous non-vanishing vector field $\gamma^\mu(x)$ on it which is everywhere non-spacelike.

By means of this field, the set of all semicones of the manifold can be split into two equivalence classes, $C_+$ and $C_-$. the semicones of $C_+$ contain the vectors of the field and the semicones of $C_-$ do not contain them. On the other hand, in a non time-orientable spacetime it is possible to transform a future pointing timelike vector into a past pointing timelike vector by means of a continuous transport that always keeps non-vanishing timelike vectors timelike; therefore, the distinction between future semicones and past semicones cannot be univocally definable on a global level. This means that the time-orientability of spacetime is a precondition for defining a global arrow of time, since if spacetime is not time-orientable, it is not possible to distinguish between the two temporal directions for the universe as a whole.

Nevertheless, not all accept this conclusion. For instance, Matthews [13] claims that a spacetime may have a regional but not a global arrow of time if the arrow is defined by means of local considerations. However, even from this local approach (which we have rejected in the previous section), time-orientability cannot be avoided. Let us suppose that there were a local non time-reversal invariant law $L$, which defines regional arrows of time that disagree when compared by means of continuous timelike transport. The trajectory of the transport will pass through a
frontier point between both regions: in a region around this point the arrow of time will be not univocally defined, and this amounts to a breakdown of the validity of $L$ in such a point. But this fact contradicts the methodological principle of universality, unquestioningly accepted in contemporary cosmology, according to which the laws of physics are valid in all points of the spacetime. The strategy to escape this conclusion would consist in refusing to assign any meaning to the timelike continuous transport. This strategy would only be acceptable if the two regions with different arrows were physically isolated: this amounts to the disconnectedness of the spacetime. But this fact would contradict another methodological principle of cosmology, that is, the principle of uniqueness, according to which there is only one universe and completely disconnected spacetimes are not allowed\(^2\). These arguments show that the possibility of arrows of time pointing to opposite directions in different regions of the spacetime is not an alternative seriously considered in contemporary cosmology.

B. Cosmic time

As it is well known, general relativity replaces the older conception of space-through-time by the concept of spacetime, where time becomes a dimension of a four-dimensional manifold. But when the time measured by a physical clock is considered, each particle of the universe has its own proper time, that is, the time registered by a clock carried by the particle. Since the curved spacetime of general relativity can be considered locally flat, it is possible to synchronize the clocks fixed to particles whose parallel trajectories are confined in a small region of spacetime. But, in general, the synchronization of the clocks fixed to all the particles of the universe is not possible. Only in certain particular cases all the clocks can be coordinated by means of a cosmic time, which has the features necessary to play the role of the temporal parameter in the evolution of the universe.

The issue can also be posed in geometrical terms. A spacetime may be such that it is not possible to partition the set of all events into equivalent classes such that: (i) each one of them is a spacelike hypersurface, and (ii) the hypersurfaces can be ordered in time. There is a hierarchy of conditions which, applied to a time-orientable spacetime, avoid ”anomalous” temporal features (see [15]). The strongest condition is the existence of a global time.

**Definition 7:** A global time function on the Riemannian manifold $M$ is a function $t : M \to \mathbb{R}$ whose gradient is everywhere timelike.

The value of the global time function increases along every future directed non-spacelike curve. The existence of such a function guarantees that the spacetime is globally splittable into hypersurfaces of simultaneity which define a foliation of the spacetime (see [16]).

Nevertheless, the fact that the spacetime admits a global time function does not yet permit to define this notion of simultaneity in an univocal manner and with physical meaning. In order to avoid ambiguities in the notion of simultaneity, we must choose a particular foliation. The foliation $\tau$ such that there exists a continuous set of worldline curves which are orthogonal to all the hypersurfaces $\tau = const.$ is the proper choice, because orthogonality recovers the notion of simultaneity of special relativity for small regions (tangent hyperplanes) of the hypersurfaces $\tau = const.$ (for the necessary conditions, see [17]). However, even if this condition selects a particular foliation, it permits that the proper time interval between two hypersurfaces of simultaneity be relative to the particular worldline considered for computing it. If we want to avoid this situation, we must impose an additional constraint: the proper time interval between two hypersurfaces $\tau = \tau_1$ and $\tau = \tau_2$ must be the same when measured on any orthogonal worldline curve of the continuous set mentioned above. In this case, the metric results:

$$ds^2 = dt^2 - h_{ij} dx^i dx^j \quad (1)$$

**Definition 8:** When the metric of the spacetime can be expressed as $ds^2 = dt^2 - h_{ij} dx^i dx^j$, $t$ is the cosmic time and $h_{ij} = h_{ij}(t, x^1, x^2, x^3)$ is the three-dimensional metric of each hypersurface of simultaneity.

\(^2\)Although there are quantum cosmologies exhibiting disconnected space-times, such models only play an explanatory role since they are not testably in principle. Anyway, even if disconnected space-times were allowed, each connected region could be considered as a universe by itself, where timelike continuous transport must be valid. This fact is relevant since we are interested in explaining the time direction of our own connected universe. For a different opinion, see [14] and our criticism in [2].
Of course, the existence of cosmic time imposes a significant topological and metric limitation on the spacetime. But with no cosmic time, there is not a single time which can be considered as the parameter of the evolution of the universe and, therefore, it is nonsensical to speak of the two directions of time for the universe as a whole. Therefore, the possibility of defining a cosmic time is a precondition for meaningfully speaking of a global arrow of time. This fact supplies an additional argument against the entropic approach, which takes for granted the possibility of defining the entropy function of the universe. But this amounts to the assumption that: (i) the spacetime can be partitioned in spacelike hypersurfaces on which the entropy of the universe can be defined, and (ii) the spacetime possesses a cosmic time on which the entropy gradient can be computed. When the possibility of spacetimes with no cosmic time is recognized, it is difficult to deny the conceptual priority of the geometrical structure of spacetime over entropic features in the context of our problem.

C. Time-asymmetry

It is quite clear that time-orientability is merely a necessary condition for defining the global arrow of time, but it does not provide a physical, substantial criterion for distinguishing between the two directions of time. As we will see, such a distinction requires the time-asymmetry of the universe.

It is usually accepted that the obstacle to the definition of the arrow of time lies in the fact that the fundamental laws of physics are time-reversal invariant. Nevertheless, this common position can be objected when the concept of time-symmetry is clearly elucidated and compared with the concept of time-reversal invariance: whereas time-reversal invariance is a property of dynamical equations (laws), time-symmetry is a property of a single solution (evolution) of an dynamical equation.

**Definition 9:** A solution \( f(t) \) of a dynamical equation is time-symmetric if there is a time \( t_S \) such that \( f(t + t_S) = f(t - t_S) \).

It is quite clear that the time-reversal invariance of an equation does not imply the time-symmetry of its solutions: a time-reversal invariant law may be such that all or most of the possible evolutions relative to it are individually time-asymmetric. Huw Price [7] illustrates this point with the familiar analogy of a factory which produces equal numbers of left-handed and right-handed corkscrews: the production as a whole is completely unbiased, but each individual corkscrew is asymmetric.

Although these considerations are not applicable to the field equations as originally stated, the existence of a cosmic time allows to present the issue in familiar terms: under this condition, Einstein’s field equations are time-reversal invariant in the sense that if the \( h_{ij}(t, x^1, x^2, x^3) \) of eq.(1) is a solution, \( h_{ij}(-t, x^1, x^2, x^3) \) is also a solution. But the time-reversal invariance of these equations does not prevent us from describing a time-asymmetric universe whose spacetime is asymmetric regarding its geometrical properties along the cosmic time. This idea can also be formulated in terms of the concept of time-isotropy.

**Definition 10:** A time-orientable spacetime \((M, g)\) (where \(M\) is a four-dimensional Riemannian manifold and \(g\) is a Lorentzian metric for \(M\)) is time-isotropic if there is a diffeomorphism \(d\) of \(M\) onto itself which reverses the temporal orientations but preserves the metric \(g\).

However, when we want to express the time-symmetry of a spacetime having a cosmic time, it is necessary to strengthen the definition.

**Definition 11:** A time-orientable spacetime which admits a cosmic time \(t\) is time-symmetric with respect to some spacelike hypersurface \(t = t_S\), where \(t_S\) is a constant, if it is time-isotropic and the diffeomorphism \(d\) leaves fixed the hypersurface \(t = t_S\).

Intuitively this means that, from the hypersurface \(t = t_S\), the spacetime looks the same in both temporal directions. Therefore, if a time-orientable spacetime having a cosmic time is time-asymmetric, we will not find a spacelike hypersurface \(t = t_S\) which splits the spacetime into two "halves", one the temporal mirror image of the other with respect to their intrinsic geometrical properties.

But, how this time-asymmetry allows us to choose a time-orientation of the spacetime? As we have seen, in a time-orientable spacetime a continuous non-vanishing non-spacelike vector field \(\gamma^\mu(x)\) can be defined all over the

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3The exception is the law that rules weak interactions. We will return on this point in the last section.
the simple Robertson-Walker form: of time. But, what can we say about our universe? Cosmology offers a simple answer by means of the cosmological locally can be used $\gamma$ direction of $\gamma$ the time-orientation of the spacetime has been established. Since the field $\gamma$ by making the global continuations of $\gamma$ in any case we have established a substantial difference between $\gamma(x_0)$ and $-\gamma(x_0)$. We can conventionally call the direction of $\gamma(x_0)$ "future" and the direction of $-\gamma(x_0)$ "past" or vice versa, but in any case past is substantially different than future. Now we can extend this difference to the whole continuous fields $\gamma(x)$ and $-\gamma(x)$ obtained by making the global continuations of $\gamma(x_0)$ and $-\gamma(x_0)$ allowed by the definition of time-orientability: in this way, the time-orientation of the spacetime has been established. Since the field $\gamma(x)$ is defined all over the manifold, it can be used locally at each point $x$ to define the future and the past semicones: for instance, if we have called the direction of $\gamma(x)$ "future", $C_+(x)$ contains $\gamma(x)$ and $C_-(x)$ contains $-\gamma(x)$.

D. Application to FLRW models

In the previous subsections we have considered the general conditions necessary for the existence of a global arrow of time. But, what can we say about our universe? Cosmology offers a simple answer by means of the cosmological principle and the assumption of expansion, on the basis of which the metric of the spacetime can be represented in the simple Robertson-Walker form:

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$ (2)

With the Robertson-Walker metric, the Einstein’s field equations can be solved: the corresponding solutions describe the isotropic and homogeneous Friedmann-Lemaître-Robertson-Walker models (FLRW for short), which are the standard models of present day cosmology.

FLRW models provide a simple answer to the question about the first condition for the existence of the arrow of time since they are time-orientable. Moreover, astronomical observations provide empirical evidence that makes implausible the non time-orientability of our spacetime. In particular, there is no astronomical observation of temporally inverted behavior in some (eventually very distant) region of the universe. In fact:

1.- The evolutions of several generations of supernovae always follow the same pattern (let’s say, from birth to death), and there is no trace of a time-inverted pattern in the visible universe. At the decoupling time (400,000 years after the Big-Bang) the universe was essentially composed of light elements like H and He (with traces of isotopes of these elements and of Li and Be) and was virtually free of ions. This matter condensed in stars, where heavy elements were formed. The explosion of these stars as supernovae scattered the heavy elements producing clouds which, in turn, condensed giving rise to a second generation of stars. Analogously, a third generation arose, and so on. We have an indirect evidence of the first generation (known as population III), appeared 200 millions years after the Big-Bang, by the observed re-ionization of the universe which can be explained only by the presence of stars at that period [18]. Then, two new generations (populations II and I) followed. This is the history as manifest in all the universe, and this is a relevant fact in the context of our problem since the supernovae are the markers or standard candles used to measure the more distant galaxies ($z = 1$ and even more) [19].

2.- If we consider that the peak in the quasars formation rate took place when the universe was 3,000 millions years old ($z \sim 2$), and that this rate has always decreased since then, we see that the evolution of the universe is time-asymmetric, at least up to the corresponding distances.

3.- Finally, the process of decoupling between matter an radiation occurred 400,000 years after the Big-Bang ($z = 1000$) and never happened again. This fact proves the time-asymmetry of the visible universe (since 400,000 years after the Big-Bang up to the present).

With regard to the existence of a cosmic time, since FLRW models are spatially homogeneous and isotropic on the large scale, it is possible to find a family of spacelike hypersurfaces which can be labeled by the proper time of the worldlines that orthogonally thread through them: these labels define the cosmic time, which is represented by the variable $t$, and the scale factor $a$ is a scalar only function of $t$.

With regard to time-symmetry, in FLRW models the time-symmetry of spacetime may manifest itself in two different ways according to whether the universe has singular points in one or in both temporal extremities\(^4\). Big Bang-Big

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\(^4\)This depends on the values of the factor $k$ and of the cosmological constant $\Lambda$. 

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Chill universes are manifestly time-asymmetric: since the scale factor $a(t)$ increases with the cosmic time $t$, there is no hypersurface $t = t_S$ from which the spacetime looks the same in both temporal directions. In Big Bang-Big Crunch universes, on the contrary, $a(t)$ has a maximum value: therefore, the spacetime might be time-symmetric about the time of maximum expansion. These are the cases of some FLRW models with dust and radiation. In more general cases (e.g. inflationary models) different fields can represent the matter-energy of the universe. Many interesting results have been obtained, for instance, by modeling matter-energy as a set of scalar fields $\phi_k(t)$: homogeneity is retained and the time-reversal invariance of the field equations is given by the fact that, if $[a(t), \phi_k(t)]$ is a solution, $[a(-t), \phi_k(-t)]$ is also a solution. In these cases, if there is a time $t_{ME}$ of maximum expansion, the scale factor $a(t)$ may be such that $a(t_{ME} + t) \neq a(t_{ME} - t)$ (see, for instance, the models in [20]). This means that a Big Bang-Big Crunch universe may be a time-asymmetric object with respect to the metric of the spacetime, and this asymmetry, essentially grounded on geometrical considerations, allows us to distinguish between the two directions of the cosmic time, independently of entropic considerations.

As Savitt [21] correctly points out, there are two different questions involved in the problem of the arrow of time:

- The "how possible" question: how is it possible to formulate a time-asymmetric model by means of time-reversal laws?
- The "how probable" question: what reason is there to suppose that time-asymmetry is probable?

In the previous subsection we have shown that the time-reversal invariance of laws is not an obstacle to the construction of time-asymmetric models. However, the second question remains. The answer to this question can be given by a simple argument that proves the vanishing probability of perfect time-symmetry (see [2]): such an argument demonstrates that time-symmetric solutions of the universe equations have measure zero in the corresponding phase space. This means that geometrical time-asymmetry is a very specific feature which requires an overwhelmingly improbable fine-tuning of all the state variables of the universe.

V. FROM GEOMETRY TO ENERGY FLOW: THE FIRST ROLE OF THE ENERGY-MOMENTUM TENSOR

A. The arrow of time as energy flow

Up to this point, the arrow of time was defined in terms of the substantial difference between the vector fields $\gamma^\mu(x)$ and $-\gamma^\mu(x)$, grounded on the time-asymmetry of the spacetime. But $\gamma^\mu(x)$ was characterized merely as the vector field that must exist for the time-orientability of spacetime. The question now is whether the arrow of time can be defined in a physical way, that is, by means of a mathematical object that can be interpreted not only geometrically but also in terms of the more familiar magnitudes of physics.

As it is well known, the energy-momentum tensor $T_{\mu\nu}$ represents the density and the flow of energy and momentum at each point of the spacetime. Then, it would be desirable to define the vector field $\gamma^\mu(x)$ in terms of $T_{\mu\nu}$ in order to endow it with a physical meaning. Although this task cannot be accomplished in a completely general case, it is possible to define the arrow of time in terms of $T_{\mu\nu}$ when the energy-momentum tensor satisfies the dominant energy condition everywhere (see [15], [22]).

**Definition 12:** The energy-momentum tensor satisfies the dominant energy condition if, in any orthonormal basis, the energy component dominates the other components of $T_{\alpha\beta}$:

$$T^{00} \geq |T^{\alpha\beta}| \quad \text{for each } \alpha, \beta$$

This means that to any observer the local matter-energy density appears non-negative and the energy flow is non-spacelike. The dominant energy condition does not impose a very strong constraint, since it holds for almost all known forms of matter\(^5\).

---

\(^5\) There are, of course, strange cosmological "objects" whose existence would lead to universes where the dominant energy condition is not satisfied in certain regions of spacetime. For instance, in wormhole spacetimes, the dominant energy condition is violated in the vicinity of the wormhole throat since the wormhole is threaded by negative "exotic" matter (see [22]). Nevertheless, it is plausible to suppose that universes containing such kind of objects will surely not satisfy the stronger conditions necessary for defining the arrow of time, that is, time-orientability and existence of cosmic time.
Let us consider a continuous orthonormal basis \( \{ V^\alpha_\mu (x) \} \) (a tetrad or \textit{vierbein}) consisting of four unitary vectors \( \{ V^\alpha_0 (x), V^\alpha_1 (x), V^\alpha_2 (x), V^\alpha_3 (x) \} \). In this basis, \( g_{\mu \nu} V^\alpha_\mu V^\beta_\nu = \eta_{\alpha \beta} \) are the coordinates of the local Minkowski metric tensor, and \( T^\mu_\nu V^\alpha_\mu V^\beta_\nu = T^\alpha_\beta \) are the coordinates of the energy-momentum tensor. Then, \( T^{0\alpha} V^\mu_\alpha \) can be conceived as a vector representing the energy flow, whose coordinates in that basis are the \( T^{0\alpha} \). Now, if \( T^{00} \geq |T^{0\alpha}| \), then \( T^{00} \geq |T^{00}| \). In turn, \( T^{00} \geq |T^{0\alpha}| \) implies that \( T^{0\alpha} V^\mu_\alpha \) is non-spacelike. On the other hand, if the manifold and the basis field are continuous, \( g_{\mu \nu} \) is continuously defined over it and, provided that the derivatives of \( g_{\mu \nu} \) are also continuous, \( T^{\mu \nu} (T^{0\beta}) \) and, then, \( T^{0\mu} (T^{0\beta}) \) are also continuously defined all over the manifold. Therefore, it seems that, if the density of matter-energy is non-zero everywhere, we have found a physical correlate of the continuous non-vanishing non-spacelike vector field \( \gamma^\mu (x) = T^{0\alpha} (x) V^\mu_\alpha (x) \). The trouble with this conclusion is that \( T^{0\alpha} V^\mu_\alpha \) is not strictly a vector, since it is not transformed as a vector by the Lorentz transformations. Strictly speaking, at each point \( x \) of the spacetime \( T^{0\alpha} (x) V^\mu_\alpha (x) \) is a tetra-magnitude which represents the energy flow only in the basis \( \{ V^\mu_\alpha (x) \} \); thus, it seems that it cannot directly play the role of \( \gamma^\mu (x) \) as initially desired.

Nevertheless, the fact that the energy flow cannot be represented by a vector is not an obstacle to define the arrow of time in terms of such a flow. The dominant energy condition poses a \textit{covariant} condition: if the energy flow is not-spacelike in a reference frame, it is non-spacelike in all reference frames. This means that, no matter which orthonormal basis \( \{ V^\alpha_\mu (x) \} \) is chosen, the energy flow in that basis, represented by \( T^{0\alpha} (x) V^\mu_\alpha (x) \), can be used to define the arrow of time. The usual convention in physics consists in calling the temporal direction of the positive energy flow "future". In this case, at any point \( x \) of the spacetime \( T^{0\alpha} (x) V^\mu_\alpha (x) \) belongs to the future light semicone \( C_+ (x) \): the energy flows towards the future for any observer. But the relevant point is that this sentence acquire a non-conventional meaning only when the substantial difference between past and future has been previously established by the time-asymmetry of the spacetime.

\textbf{B. From the global arrow to the local arrow}

As we have seen, the future light semicone \( C_+ (x) \) at each point \( x \) of the spacetime is defined by the positive energy flow \( T^{0\alpha} (x) V^\mu_\alpha (x) \) at this point. But, is \( T^{0\alpha} \) really the energy flow as conceived by local physics? Let us remember that \( T^{\mu \nu} \) satisfy the "conservation" equation:

\[ \nabla_\mu T^{\mu \nu} = 0 \]

However, this is not a true conservation equation since \( \nabla_\mu \) is a covariant derivative. The usual conservation equation with ordinary derivative reads:

\[ \partial_\mu \tau^{\mu \nu} = 0 \]

where \( \tau^{\mu \nu} \), which is not a tensor, is defined as [17]:

\[ \tau^{\mu \nu} = T^{\mu \nu}_{\text{eff}} = T^{\mu \nu} + t^{\mu \nu} \]

where \( t^{\mu \nu} \) reads:

\[ t^{\mu \nu} = \frac{1}{16\pi} \left[ L g_{\mu \nu} - \frac{\partial L}{\partial g_{\mu \nu}, \lambda} g_{\mu \nu}, \lambda \right] \]

where \( L \) is the system’s Lagrangian of the following equation:

\[ T^{\mu \nu} (x) = \frac{\delta S}{\delta g_{\mu \nu} (x)} = \frac{\delta}{\delta g_{\mu \nu} (x)} \int L \sqrt{-g} dx^4 \]

\( t^{\mu \nu} \) is also an homogeneous and quadratic function of the connection \( \Gamma^\lambda_{\mu \nu} \) [23], precisely:

\[ t^{\mu \nu} = \frac{1}{16\pi k} \left[ (2\Gamma^\chi_{\alpha \beta} \Gamma^\delta_{\chi \delta} - \Gamma^\chi_{\alpha \delta} \Gamma^\delta_{\beta \chi} - \Gamma^\chi_{\beta \chi} \Gamma^\delta_{\alpha \delta} \right) (g^{\mu \alpha} g^{\nu \beta} - g^{\mu \nu} g^{\alpha \beta}) + g^{\mu \alpha} g^{\beta \chi} \left( \Gamma^\nu_{\alpha \delta} \Gamma^\delta_{\chi \beta} + \Gamma^\nu_{\beta \chi} \Gamma^\delta_{\alpha \delta} - \Gamma^\nu_{\chi \delta} \Gamma^\delta_{\alpha \beta} \right) + g^{\mu \nu} g^{\beta \chi} \left( \Gamma^\alpha_{\delta \beta} \Gamma^\delta_{\chi \chi} + \Gamma^\alpha_{\beta \chi} \Gamma^\delta_{\delta \chi} - \Gamma^\alpha_{\chi \delta} \Gamma^\delta_{\beta \alpha} \right) + \]

9
\[
g^{\alpha\beta} \chi^\delta \left( \Gamma^\mu_{\alpha\chi} \Gamma^\nu_{\beta\delta} - \Gamma^\mu_{\alpha\beta} \Gamma^\nu_{\chi\delta} \right)
\]  \hspace{1cm} (4)

Now we can consider the coordinates \( \tau^0 \), which satisfy a usual conservation equation:

\[
\partial_\mu \tau^0 \mu = \partial_0 \tau^{00} + \partial_\tau \tau^0 = 0
\]

where \( \tau^{00} \) represents the energy density and the \( \tau^0 \) are the three components of the spatial energy flow (the Poynting vector).

Analogously to the case of the energy-momentum tensor, in any orthonormal basis \( \{ V^\mu_{(\alpha)} \} \), \( \tau^{00} V^\nu_{(\alpha)} \) is not a vector since it is not transformed as a vector by the Lorentz transformations. But in contrast to that case, the dominant energy condition cannot be expressed in terms of \( \tau^{\mu\nu} \). However, this can be done in the particular case of local inertial frames. In fact, in a local inertial frame, \( \Gamma^\lambda_{\mu\nu} = 0 \); thus, \( t_{\mu\nu} = 0 \) (see eq.(4)) and \( \tau^{\mu\nu} = \Gamma^\lambda_{\mu\nu} \) (see eq.(3)). Therefore, in this case the dominant energy condition implies that:

\[
\tau^{00} \geq |\tau^{\alpha\beta}| \quad \text{for each } \alpha, \beta
\]

As a consequence, in the basis \( \{ W^\mu_{(\alpha)} \} = \{ W^\mu_{(0)}, W^\mu_{(1)}, W^\mu_{(2)}, W^\mu_{(3)} \} \) of the local inertial frame, the energy flow, represented by \( \tau^{00} W^\mu_{(\alpha)} \) and satisfying the usual conservation equation, is non-spacelike.

Although this result cannot be generalized to all the reference frames of the whole spacetime, it is relevant in local contexts since, in small regions of the spacetime, the metric tends to the Minkowski form\(^6\). In fact, any local region of the spacetime can be approximated to the tangent Minkowski space (with orthonormal basis \( \{ V^\mu_{(\alpha)} \} \)) at any point of that region. If \( \{ W^\mu_{(\alpha)} \} \) is the basis of an inertial frame on this flat tangent space, \( \tau^{00} W^\mu_{(\alpha)} \) represents the non-spacelike local energy flow satisfying the usual conservation equation. Moreover, at each point \( x \) of the local region, the local energy flow \( \tau^{00}(x)W^\mu_{(\alpha)}(x) \) belongs to the same light semicone than the one to which \( T^{00}(x)V^\mu_{(\alpha)}(x) \) belongs. Therefore, if we have adopted the usual physical convention in the global level, we can also meaningfully say that future is the temporal direction of the positive local energy flow: the local flow emitted at \( x \) belongs to the light semicone \( C_+(x) \).

This result is particularly relevant because local inertial frames are the reference frames in which the non-relativistic local theories of physics are valid. In turn, ordinary quantum field theory in flat spacetime must be considered as locally formulated in a local inertial frame. This means that the local energy flow directed towards the future is the flow of energy as conceived by this kind of theories, where energy satisfies the usual conservation law expressed by means of ordinary derivatives. Summing up, \( \tau^{00} \) inherits the time-orientation defined at the global level and, to the extent that it has a local physical meaning, it not only transfers the global arrow of time to local contexts, but also translates the global arrow into a usual magnitude of local physical theories.

C. The absolute nature of the arrow of time

As we have seen, the vector \( \tau^{00} \) is always non-spacelike and, therefore, its direction defines the arrow of time. However, we know that, given the time-reversal invariance of Einstein’s field equations, in a time-orientable spacetime where \( t \) is the cosmic time, if \( h_{ij}(t, x^1, x^2, x^3) \) is a solution, \( h_{ij}(-t, x^1, x^2, x^3) \) is also a solution; the first case corresponds to \( \tau^{00} \) and the second case corresponds to \( -\tau^{00} \). At this point, the ghost of symmetry threatens again: it seems that we are committed to supplying a non-conventional criterion for picking out one of both nomologically admissible

---

\(^6\)Near each point \( x_0 \), the metric can be approximated with the metric of the free inertial frame as:

\[
d s^2 = (1 - R_{000m} x^l x^m) dt^2 - \left( \frac{4}{3} R_{0ljm} x^l x^m \right) dt dx^i + \left( \delta_{ij} - \frac{1}{3} R_{ijlm} x^l x^m \right) dx^i dx^j + 0(|x|^3) dx^a d\theta^a
\]

(see [17] eq. (13.73)). Therefore, locally in the inertial frame at \( x_0 \), we can be sure that \( \tau^{00} \geq |\tau^{00}| \).
solutions, one the temporal mirror image of the other. Nevertheless, as we will see, the threat is not so serious as it seems.

To replace \( t \) by \(-t\) is to apply a symmetry transformation, in particular, time-reversal. The point is to understand the meaning of such a transformation. Under the active interpretation, a symmetry transformation corresponds to a change from one system to another; under the passive interpretation, a symmetry transformation consists in a change of the point of view from which the system is described. The traditional position about symmetries assumes that, in the case of discrete transformations, only the active interpretation makes sense: an idealized observer can rotate himself in space in correspondence with the given spatial rotation, but it is impossible to “rotate in time” (see [12], [24]). Of course, this is true when the idealized observer is immersed in the same spacetime as the observed system. But when the system is the universe as a whole, we cannot change our spatial perspective with respect to the universe: it is so impossible to rotate in space as to rotate in time. However, this does not mean that the active interpretation is the correct one: the idea of two identical universes, one translated in space or in time regarding the other, has no meaning. This shows that both interpretations, when applied to the universe as a whole, collapse into conceptual nonsense.

In fact, in cosmology symmetry transformations are neither given an active nor a passive interpretation: two mathematical models for the universe, defined by \((M, g)\) and \((M', g')\), are taken to be equivalent if they are isometric, that is, if there is a diffeomorphism \( \theta : M \to M' \) which carries the metric \( g \) into the metric \( g' \) (see [15]). Since symmetry transformations are isometries, two models related by a symmetry transformation (in particular, time-reversal) are considered equivalent descriptions of one of the same universe. Therefore, it is not necessary to find a non-conventional criterion for selecting one of two nomologically admissible solutions to the extent that both are descriptions of a single possible universe.

These considerations point to the absolute character of the arrow of time embodied in \( \tau^{0\mu} \). This vector (in particular, its direction) is rigidly linked to the spacetime on which it is defined. To change \( \tau^{0\mu} \) by \(-\tau^{0\mu}\) amounts to change the model by its temporal mirror image; but, as we have shown, this move has no physical meaning since both models are merely different descriptions of the same universe.

Let us note that we have not used the term "future" in the present argument. If we adopt the usual terminology, we will call the time direction of the energy flow "future": in this case, we can say that the vector \( \tau^{0\mu} \) points to future. Nevertheless, it is worth remembering that "past" and "future" are words coming from our everyday language but they do not belong to physical theories. Then, the choice of saying that \( \tau^{0\mu} \) points to future is conventional: we can replace "future" by "past" and nothing changes. What remains is the absolute and substantial nature of the arrow of time defined by the unchangeable direction of \( \tau^{0\mu} \).

**D. Breaking the symmetry in time-reversal invariant theories**

As we have seen in subsection 3.A, the Ehrenfest’s objected Gibbs’ approach by pointing out that the increase of the entropy towards the future is always matched by a similar one in the past of the system. It is interesting to note that this old discussion can be generalized to the case of any kind of evolution arising from local time-reversal invariant laws. In fact, time-reversal invariant equations always give rise to what we will call “time-symmetric twins” (see [2]), that is, two mathematical structures symmetrically related by a time-reversal transformation: each “twin”, which in some cases represents an irreversible evolution, is the temporal mirror image of the other “twin”. For instance, electromagnetism provides a pair of advanced and retarded solutions, that are usually related with incoming and outgoing states in scattering situations as described, e.g., by Lax-Phillips scattering theory [25]. In irreversible quantum mechanics, the analytical extension of the energy spectrum of the quantum system’s Hamiltonian into the complex plane leads to poles in the lower half-plane (usually related with decaying unstable states), and symmetric poles in the upper half-plane (usually related with growing unstable states) (see [26]), etc. However, at this level the twins are only conventionally different: we cannot distinguish between advanced and retarded solutions or between lower and upper poles without assuming temporally asymmetric notions, as the asymmetry between past and future or between preparation and measurement. Then, the challenge consists in supplying a non-conventional criterion for choosing one of the twins as the physically relevant: such a criterion must establish a substantial difference between the two members of the pair.

The arrow of time defined by \( \tau^{0\mu} \) is precisely what provides us the criterion for establishing the desired substantial difference. In fact, \( \tau^{0\mu} \) says that at each point of the spacetime the semicones \( C_-(x) \) receive an incoming flow of energy while the semicones \( C_+(x) \) emit an outgoing flow of energy. Therefore, in each case the twin that must be retained as physically relevant is the one describing this kind of energy flow, from \( C_-(x) \) to \( C_+(x) \). For instance, in electromagnetism only retarded solutions fulfill this condition since they describe waves propagating into the semicone \( C_+(x) \). In irreversible quantum mechanics, only decaying unstable states with poles in the lower half-plane have the
VI. THE NON TIME-REVERSAL INVARIANCE OF QUANTUM FIELD THEORY: THE SECOND ROLE OF THE ENERGY-MOMENTUM TENSOR

A. The non time-reversal invariance of axiomatic QFT

The Postulate A.3 of the axiomatic quantum field theory (see [27], p.56, eq.II.1.15) states that the spectrum of the energy-momentum operator $P^\mu$ is confined to a future light semicone, that is, its eigenvalues $p^\mu$ satisfy:

$$p^2 \geq 0 \quad p^0 \geq 0$$

This postulate says that, when we measure the observable $P^\mu$, we obtain a non-spacelike classical $p^\mu$ contained in a future semicone, that is, a semicone belonging to $C_+$. It is clear that condition $p^0 \geq 0$ selects one of the elements of the time-symmetric twins, $p^0 \geq 0$ and $p^0 \leq 0$ that would arise from the theory: by means of Postulate A.3, QFT becomes a non time-reversal invariant theory. In turn, since QFT, being both quantum and relativistic, can be considered one of the most basic theories of physics, the choice introduced by condition $p^0 \geq 0$ is transferred to the rest of physical theories. But such a choice is established from the very beginning, as a postulate of the theory. The challenge is, then, to justify Postulate A.3 by means of independent theoretical arguments.

As it is well known, the components of $T^{\mu\nu}(x)$ can be interpreted as follows:

- $T^{00}(x)$ is the matter-energy density
- $T^{0i}(x)$ is the matter-energy flow
- $T^{ij}(x)$ is the linear momentum density
- $T^{ij}(x)$ is the linear momentum flow

where $(i,j = 1,2,3)$. Since $T^{\mu\nu}$ is a symmetric tensor, $T^{\mu\nu}(x) = T^{\nu\mu}(x)$ and, therefore, $T^{0i}(x) = T^{i0}(x)$; in other words, the matter-energy flow is equal to the linear momentum density. This means that, if the matter energy flow $T^{00}$ can be used to define the arrow of time under the dominant energy condition, this is also the case for the linear momentum density $T^{00}$. On the other hand, we have seen in Subsection 5.B that the local matter-energy flow $T^{0\mu}$ can be conceived as a conservative version of $T^{\mu0}$ in the orthonormal coordinates of a local inertial frame; in this case, the dominant energy condition has the consequence that $T^{0\mu}$ is non-spacelike. Now we know that exactly the same conclusion can be drawn for the local linear momentum density $T^{\mu0}$. But it is precisely the local linear momentum density $T^{\mu0}$ the local magnitude corresponding to the classical $p^\mu$ of QFT; thus, at each point $x$, $p^\mu \sim T^{\mu0} \in C_+(x)$.

In conclusion, the fact that $p^\mu$ at each point $x$ of the local context and, therefore, for every classical particle, must be contained in the future light semicone $C_+(x)$ turns out to be a consequence of the global time-asymmetry of the spacetime when the dominant energy condition holds everywhere. In other words, Postulate A.3 can be justified on cosmological grounds instead of being imposed as a departing point of the axiomatic version of QFT.

B. The non time-reversal invariance of ordinary QFT

In this section we will analyze how time-reversal invariance is introduced in the ordinary version of QFT. In order to do this let us remember again that, in the tangent plane at each point $x$ of our manifold, we can define an orthonormal tetrad $V^\mu_\alpha = \{V^\mu_{(0)}, V^\mu_{(i)}\}$, where $V^\mu_{(0)}$ is a timelike vector and the $V^\mu_{(i)}$ are spacelike vectors.

Definitions 13: The complete Lorentz group $L$ has four components. $L_0$ is the identity component, also known as the proper Lorentz group. The space inversion $P$, defined as $(V^\mu_{(0)} \to V^\mu_{(0)}, V^\mu_{(i)} \to -V^\mu_{(i)})$, is the typical element of the component $L_1$. The combination of $L_0$ and $L_1$ is known as the orthochronous Lorentz group. Analogously, the time inversion $T$, defined as $(V^\mu_{(0)} \to -V^\mu_{(0)}, V^\mu_{(i)} \to V^\mu_{(i)})$, is the typical elements of the component $L_2$. The combination of $L_0$ and $L_2$ is known as the orthospacial Lorentz group.
Finally, the total inversion \((V^\mu_0 \rightarrow -V^\mu_0, V^\mu_0 \rightarrow -V^\mu_0)\) is the typical element of the component \(L_3\). The combination of \(L_0\) and \(L_3\) is known as the unimodular Lorentz group.

The classification of one-particle states according to their transformation under the Lorentz group leads to six classes of four-momenta (see [28]). Of these classes, it is considered that only three have physical meaning: these are precisely the cases which agree with Postulate A.3 of the axiomatic version of QFT. In other words, the symmetry group of QFT is the orthochronous group, where \(\mathcal{P}\) but not \(\mathcal{T}\) are included. This is another way of expressing the non time-reversal invariance of the QFT. In this case, the non time-reversal invariance is introduced not by means of a postulate but on the basis of empirical arguments that make physically meaningless certain classes of four-momenta. However, to the extent that special relativity and standard quantum mechanics are time-reversal invariant theories, they give no theoretically grounded justification for such a break in time-reversal invariance. Nevertheless, as we have seen in the previous subsection, this justification can be given on cosmological grounds.

Let us make the point in different terms. The quantum field correlates of \(\mathcal{P}\) and \(\mathcal{T}\), \(\mathcal{P}\) and \(\mathcal{T}\), are defined as:

\[
\mathcal{P}_i \rho^\mu \rho^{-1} = i \rho^\mu \rho^{-1} \quad \quad \mathcal{T}_i \rho^\mu \rho^{-1} = i \rho^\mu \rho^{-1}
\]

where \(\mathcal{P}\) is a linear and unitary operator and \(\mathcal{T}\) is an antilinear and antiunitary operator. In fact, if \(\mathcal{T}\) were linear and unitary, we could simply cancel the \(i\)'s and, then, \(\mathcal{T} \rho^\mu \rho^{-1} = -\rho^\mu\): the action of the operator \(\mathcal{T}\) on the operator would invert the sign of \(\rho^\mu\), with the consequence that the spectrum of the inverted energy-momentum operator would be contained in a past light semicone. In particular, for \(\nu = 0\), \(\rho^\mu = H\), where \(H\) is the energy operator; then, if \(\mathcal{T}\) were linear and unitary, \(\mathcal{T} \rho^\mu \rho^{-1} = -H\) with the consequence that, for any state of energy \(E\) there would be another state of energy \(-E\). The antilinearity and antiunitarity of \(\mathcal{T}\) avoid these "anomalous" situations in agreement with the conditions imposed by Postulate A.3 and, at the same time, make QFT non time-reversal invariant. Once again, there are good empirical reasons for making \(\mathcal{T}\) antilinear and antiunitary, but not theoretical justification for such a move.

Summing up, in ordinary QFT it is always necessary to take a decision about the time direction of the spectrum of the energy-momentum operator \(P^\mu\). The point that we want to stress here is that, as in the case of Postulate A.3 of the axiomatic version of QFT, the decision can be justified on cosmological grounds, as a consequence of the global time-asymmetry of the universe and the dominant energy condition.

Finally, it is worth reflecting on the role of weak interactions in the problem of the arrow of time. The CPT theorem states that CPT is the only combination of charge conjugation \(C\), parity reflection \(\mathcal{P}\) and time-reversal \(\mathcal{T}\) which is a symmetry of QFT. In fact, it is well known that weak interactions break the \(\mathcal{T}\) of the CPT theorem. According to a common opinion, it is precisely this empirical fact the clue for the solution of the problem of the arrow of time: since the \(\mathcal{T}\) symmetry is violated by weak interactions, they introduce a non-conventional distinction between the two directions of time (see [22]). The question is: is the breaking of \(\mathcal{T}\) what distinguishes both directions of time in QFT? As we have seen, the operator \(\mathcal{T}\) was designed precisely to avoid that certain tetra-magnitudes, such as the linear momentum \(p^\mu\), have the "anomalous" feature of being contained in a past light semicone: the action of the operator \(\mathcal{T}\) onto the energy-momentum operator \(P^\mu\) conserves the time direction of \(P^\mu\) and, therefore, of its eigenvalues. It is this fundamental fact what makes QFT non time-reversal invariant, and not the incidental violation of \(\mathcal{T}\) by weak interactions\(^7\). This non time-reversal invariance of QFT, based on the peculiar features of \(\mathcal{T}\), distinguishes by itself between the two directions of time, with no need of weak interactions. In other words, even if weak interactions did not exist, QFT would be a non time-reversal invariant theory which would define the arrow of time. The real problem is, then, to justify the non time-reversal invariance of a theory which is presented as a synthesis of two time-reversal invariant theories such as special relativity and quantum mechanics. But this problem is completely independent of the existence of weak interactions and the breaking of \(\mathcal{T}\) introduced by them. Summing up, weak interactions do not play a role as relevant in the problem of the arrow of time as it is usually supposed.

\[\text{VII. CONCLUSION}\]

In this paper we have defined a physical object \(\tau^\mu_0 \sim p^\mu\) that can play the role of the arrow of time. Then, the mysterious and phantomlike arrow of Eddington is at last materialized. In particular, we have shown the double role played by the energy-momentum tensor in the context of our approach to the problem of the arrow of time. When

\(^7\)Of course, this leaves open a different problem: to explain why, among all the elementary interactions, only weak interactions break \(\mathcal{T}\).
the matter-energy flow is considered, the energy-momentum tensor translates the geometrical time-asymmetry of the universe in terms of energy flow. When the linear momentum density is considered, the energy-momentum tensor provides the means of justifying the time-asymmetry postulate of axiomatic quantum field theory.

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