Double charm production $e^+e^-\to J/\psi + c\bar{c}$ at B factories with next-to-leading order QCD corrections

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The inclusive $J/\psi$ production in $e^+e^-\to J/\psi c\bar{c}$ at B factories is one of the most challenging open problems in heavy quarkonium physics. The observed cross section of this double charm production process is larger than existing leading order (LO) QCD predictions by a factor of 5. In the nonrelativistic QCD (NRQCD) factorization formalism, we calculate the next-to-leading order (NLO) QCD virtual and real corrections to this process, and find that these corrections can substantially enhance the cross section with a $K$ factor of about 1.8. We further take into account the feeddown contributions from higher charmonium states (mainly the $\psi(2S)$ as well as $\chi_{cJ}$) and the two-photon contributions, and find that the discrepancy between theory and experiment can be largely removed.

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The double charm production in $e^+e^-$ annihilation at B factories is one of the most challenging open problems in heavy quarkonium physics and nonrelativistic QCD (NRQCD) (for a review of related problems, see[1]). The exclusive production cross section of double charmonium in $e^+e^-\to J/\psi \eta_c$ at $\sqrt{s}=10.6$ GeV measured by Belle[2] and BaBar[3] are larger than the leading order (LO) calculations $3.8 \sim 5.5fb$[3] in NRQCD by possibly almost an order of magnitude (see also[2]).

Moreover, the inclusive $J/\psi$ production cross section via double $c\bar{c}$ in $e^+e^-\to J/\psi c\bar{c}$ at $\sqrt{s}=10.6$ GeV measured by Belle[2]

$$\sigma[e^+e^-\to J/\psi + c\bar{c} + X] = (0.87^{+0.21}_{-0.19} \pm 0.17) \text{ pb}, \quad (1)$$

is about a factor of 5 higher than theoretical predictions including both the color-singles[8,9] and color-octet[9] $c\bar{c}$ contributions at leading order (LO) of $\alpha_s$ in the NRQCD factorization formalism[11]. This is another intriguing challenge in the double charm production problem, aside from the exclusive $J/\psi \eta_c$ production in $e^+e^-$ annihilation.

Some theoretical studies were attempted in order to resolve the large discrepancy in $e^+e^-\to J/\psi + c\bar{c}$. Liu, He, and Chao calculated the color-octet contribution[9] and the two-photon contribution to $J/\psi + c\bar{c}$ production[11] in NRQCD. But those contributions are small and can not make up such a large discrepancy. Hagihara et al. assumed a large renormalization $K$ factor ($K \sim 4$) for the $J/\psi c\bar{c}$ cross section[12]. Kaidalov introduced a nonperturbative quark-gluon-string model[13]. Kang et al. got $\sigma(e^+e^-\to J/\psi + c\bar{c} + X)/\sigma(e^+e^-\to J/\psi + X) = 0.049$ in the color-evaporation-model[14]. Other suggestions to resolve this problem may be found in Ref.[1].

In order to further clarify this problem, in this paper we present a result for the next-to-leading order (NLO) QCD correction to the inclusive $J/\psi$ production process of $e^+ + e^- \to J/\psi + c\bar{c}$. And we have already found that the NLO QCD correction to the exclusive production process of $e^+ + e^- \to J/\psi + \eta_c$ is crucial, for which the $K$ factor (the ratio of NLO to LO) may reach to a value of about 2, and hence essentially reduces the large discrepancy between theory and experiment of $e^+ + e^- \to J/\psi + \eta_c$[15](with significant relativistic corrections further considered this discrepancy is probably resolved[16,17]; enhancement effects due to use of light-cone formalism and relativistic corrections are also proposed in[18]).

At LO in $\alpha_s$, $J/\psi + c\bar{c}$ can be produced at order $\alpha^2\alpha_s^2$ (see, e.g., Refs.[6,17]). There are four Feynman diagrams, two of which are shown in Fig. 1. and the other two diagrams can be found through reversing the arrows on the quark lines. Momenta for the involved particles are assigned as $e^-(k_1)e^+(k_2) \to \gamma^*(Q) \to J/\psi(p_1) + e(p_\gamma) + \bar{c}(p_c)$. In the calculation, we use FeynArts[19] to generate Feynman diagrams and amplitudes; FeynCalc[20] for the tensor reduction; and LoopTools[21] for the numerical evaluation of the infrared (IR)-safe one-loop integrals. Finally we use Mathematica to integrate over phase space and get the numerical results.

To NLO calculation, the cross section is

$$\sigma = \sigma_{\text{Born}} + \sigma_{\text{virtual}} + \sigma_{\text{real}} + O(\alpha^2\alpha_s^4), \quad (2)$$

FIG. 1: Two of the four Born diagrams for $e^-e^+\to J/\psi c\bar{c}$.
The quark lines that are connected with can be obtained through reversing the arrows on the of them are shown in Fig. 3. The other fifteen diagrams external charm quark and anti-charm quark in the and Real N15, in which the gluon is connected with the tors of Real N4, Real N6, Real N8, Real N11, Real N13, respectively. We find that Real N1, Real N7, Real N12, and Real N14 are canceled by themselves. The other five diagrams BoxN9 + BoxN2 + PentagonN11 are IR finite respectively. IR terms of BoxN9 + BoxN2 + PentagonN11 are canceled by Vertex diagrams. The IR term in counter terms and BoxN5 + BoxN8 + PentagonN10 should be canceled by the real corrections. And the Coulomb singularity terms in BoxN8 + PentagonN10 should be mapped into the wave functions of \( J/\psi \). Similar to Ref. [13], we use \( D = 4 - 2\epsilon \) space-time dimension and the relative velocity \( v \) to regularize the IR and Coulomb singularities. The IR and Coulomb singularity terms in the virtual corrections are

\[
d\sigma^{1R, Coulomb}_{virtual} = d\sigma^{Born}_{virtual} \frac{4\alpha_s}{3\pi} \left( \frac{\pi^2}{v} \frac{1}{\epsilon} - \frac{p_c \cdot p_{\bar{c}} \ln(-s_{\bar{c}c})}{\sqrt{(p_c \cdot p_{\bar{c}})^2 - m^2}} \right),
\]

where \( s_{\bar{c}c} = \frac{\sqrt{1-4m^2/(p_c + p_{\bar{c}})^2} - 1}{\sqrt{1-4m^2/(p_c + p_{\bar{c}})^2} + 1} \).

There are thirty diagrams for real corrections, and half of them are shown in Fig. 3. The other fifteen diagrams can be obtained through reversing the arrows on the quark lines that are connected with \( J/\psi \). The calculation of the real corrections is similar to the leading order calculation, but there should appear the IR singularity [22]. We find that Real N1, Real N7, Real N12, and Real N14 are associated with IR singularity. And the eikonal factors of Real N4, Real N6, Real N8, Real N11, Real N13, and Real N15, in which the gluon is connected with the external charm quark and anti-charm quark in the \( J/\psi \), are canceled by themselves. The other five diagrams Real N2, Real N3, Real N5, Real N9, and Real N10 are independent of IR singularity. Using the eikonal approximation, we get the IR singularity terms in real corrections

\[
d\sigma^{1R, Coulomb}_{real} = d\sigma^{Born}_{real} \frac{4\alpha_s}{3\pi} \left( \frac{\ln(-s_{\bar{c}c})}{\sqrt{(p_c \cdot p_{\bar{c}})^2 - m^2}} \right).
\]

They just cancel the IR singular terms of \( d\sigma^{1R, Coulomb}_{virtual} \). The Coulomb singular terms in \( d\sigma^{1R, Coulomb}_{virtual} \) can be mapped into the \( J/\psi \) wave function. We now turn into numerical calculations for the cross section of \( e^+ + e^- \rightarrow J/\psi + c\bar{c} + X \). To be consistent with the above NLO calculation, the value of the \( J/\psi \) wave function squared at the origin should be extracted from the leptonic width at NLO of \( \alpha_s \) (see e.g. [10]).

\[
|R_S(0)|^2 = \frac{9m^2_{J/\psi}}{16\alpha^2(1 - 4C_F\alpha_s/\pi)} \left( J/\psi \rightarrow e^+e^- \right).
\]

Using the experimental value \( 5.55 \pm 0.14 \pm 0.02 \) KeV [23], we obtain \( |R_S(0)|^2 = 1.01 \text{GeV}^3 \), which is a factor of 1.25 larger than 0.810\text{GeV}^3 that was used in Ref. [2] from potential model calculations. Taking \( m_{J/\psi} = 2m \) (in the nonrelativistic limit), \( m = 1.5 \) GeV, \( \Lambda_{MS} = 338 \text{MeV} \), \( \alpha_s(\mu) = 0.259 \) for \( \mu = 2m \) (these are the same as in Ref. [9] except here a larger \( |R_S(0)|^2 \) is used), the cross section for \( e^+ + e^- \rightarrow J/\psi + c\bar{c} + X \) at NLO of \( \alpha_s \) is

\[
\sigma(e^+ + e^- \rightarrow J/\psi + c\bar{c} + X) = 0.33 \text{ pb}.
\]

It is a factor of 1.8 larger than the LO result 0.18 pb obtained with the same parameters. For \( \mu = m \) and \( \sqrt{s}/2 \), we have \( \alpha_s = 0.369 \) and 0.211, and get the cross section 0.53 pb and 0.24 pb respectively. If we set \( m = 1.4 \text{GeV} \) and \( \mu = 2m \), the cross section at NLO of \( \alpha_s \) is

\[
\sigma(e^+ + e^- \rightarrow J/\psi + c\bar{c} + X) = 0.47 \text{ pb}.
\]

It is about a factor of 1.7 larger than leading order cross section 0.27 pb. The dependence of the cross section on the renormalization scale \( \mu \) is shown in Fig. 3. When \( \mu \) changes from \( m_c = 1.5 \) GeV to \( \sqrt{s}/2 = 5.3 \) GeV, the ratio \( \sigma(\mu)/\sigma(\sqrt{s}/2) \) is found to vary from 3.05 to 1 in LO result. For NLO, with \( m_c = 1.4(1.6) \) GeV, the ratio \( \sigma(\mu)/\sigma(\sqrt{s}/2) \) varies from 2.22 (2.26) to 1. We see that, as expected, the scale-dependence in NLO is considerably reduced compared with that in the LO. A detailed discussion will be given elsewhere.

We should also include the QED contribution as well as the two-photon contribution of \( e^+e^- \rightarrow 2\gamma \rightarrow J/\psi + c\bar{c} \). Furthermore, since the experimental data are for the prompt \( J/\psi + c\bar{c} + X \) production, we should consider the feeddown contributions from higher charmonium states such as \( e^+e^- \rightarrow \psi(2S) + c\bar{c} + X \rightarrow J/\psi + c\bar{c} + X \), and \( e^+e^- \rightarrow \chi_{cJ} + c\bar{c} + X \rightarrow J/\psi + c\bar{c} + X \).

Two of the six QED diagrams of \( e^+e^- \rightarrow \gamma \rightarrow J/\psi + c\bar{c} \) are shown in Fig. 5. The other four diagrams can be obtained by replacing the gluon with the photon shown in Fig. 1. Contributions from QED diagrams can interfere with that from QCD Born diagrams, and resulted in a cross section of 8 fb at order \( O(\alpha_s^3) \).

\[
e^+e^- \rightarrow 2\gamma \rightarrow J/\psi + c\bar{c} \text{ has been calculated by Liu et al. [11]. Using their result, this cross section is } 23 \times \frac{1.01}{0.810} = 29 \text{ fb, where the factor } \frac{1.01}{0.810} \text{ is due to using the new value of the } J/\psi \text{ wave function at the origin.}
At leading order in $v$ (the relative velocity of quark and antiquark in the charmonium rest frame), the difference between $e^+e^- \rightarrow \psi(2S) + c\bar{c}$ and $e^+e^- \rightarrow J/\psi + c\bar{c}$ is in the wave functions at the origin. Using Eq. (1), the contribution from the transition $\psi(2S) \rightarrow J/\psi$ is to enlarge the direct production of $J/\psi + c\bar{c} + X$ by a factor of $\frac{|R_{\psi(2S)}(0)|^2}{|R_{\psi(0)}(0)|^2} B(\psi(2S) \rightarrow J/\psi X)$. By using $\Gamma(\psi(2S) \rightarrow e^+e^-) = 2.48 \pm 0.06\text{KeV}$ and the branching ratio for the $\psi(2S) \rightarrow J/\psi X$ transition fraction $B = 56.1 \pm 0.9\%$ [23], we find the enlarging factor to be 0.355.

The cross sections of $e^+e^- \rightarrow \chi_{cJ} + c\bar{c} + X$ with both color singlet and octet contributions were calculated in Ref.[9]. Moreover, the color octet contribution to $J/\psi c\bar{c}$ production was also estimated to be about 11 fb in Ref.[9]. Using their results and the observed branching ratios for $\chi_{cJ} \rightarrow J/\psi \gamma$ transitions $B = 1.31\%, 35.6\%, 20.2\%$ for $J = 0, 1, 2$ respectively [23], we find the sum of feeddown from $\chi_{cJ}$ and color-octet for $J/\psi$ contributions to be 21 fb.

Combining all these contributions, the prompt cross section of $e^+e^- \rightarrow J/\psi + c\bar{c} + X$ at NLO of $\alpha_s$ is

$$\sigma_{\text{prompt}}(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.51 \text{ pb}.$$ (9)

It is 59% of the data value in Eq. (11). If we set $m = 1.4\text{GeV}$ and $\mu = 2m$, and ignore the differences of other contributions due to the change of mass, then the prompt cross section of $e^+e^- \rightarrow J/\psi + c\bar{c} + X$ at NLO of $\alpha_s$ is

$$\sigma_{\text{prompt}}(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.71 \text{ pb}.$$ (10)
FIG. 4: Direct production cross sections of $e^+ + e^- \rightarrow J/\psi + c\bar{c} + X$ as functions of the renormalization scale $\mu$. Here $|R_{e}(0)|^2 = 1.01\text{GeV}^3$, $\Lambda = 0.338\text{GeV}$. NLO results are represented by solid lines and LO one by dashed lines; the upper line is for $m = 1.4\text{GeV}$ and the corresponding lower line is for $m = 1.5\text{GeV}$.

FIG. 5: Two of the six QED diagrams for $e^+e^- \rightarrow J/\psi c\bar{c}$. It is 82% of the experimental value in Eq. (1).

In order to see the uncertainties due to parameters $m_{c}$, $\mu$, $\alpha_s(\mu)$, we set $\mu = 2.8(5.3)\text{ GeV}$, $m = 1.4\pm 0.2\text{ GeV}$, and then get the cross section in the range $0.71^{+0.94}_{-0.31}(0.53^{+0.59}_{-0.23})\text{ pb}$. The NLO QCD correction to $e^+e^- \rightarrow J/\psi + c\bar{c}$ is large, despite of existing parametric uncertainties.

In conclusion, we find that by taking into consideration all NLO virtual corrections with self energy, triangle, box, and pentagon diagrams, and the real corrections, and factoring the Coulomb singular term into the $c\bar{c}$ bound state wave function, we get an ultraviolet (UV) and infrared (IR) finite correction to the direct production cross section of $e^+e^- \rightarrow J/\psi + c\bar{c}$ at $\sqrt{s} = 10.6\text{ GeV}$, and find that the NLO QCD correction can substantially enhance the cross section with a $K$ factor of about 1.8. With $m = 1.4\text{GeV}$ and $\mu = 2m$, the cross section of direct $J/\psi c\bar{c}$ production through one-photon is estimated to be 0.47 pb. Adding the feeddown contributions from higher charmonium states (mainly the $\psi(2S)$ as well as $\chi_{cJ}$) and contributions from two-photon process and color-octet channels, the prompt production cross section of $e^+e^- \rightarrow J/\psi + c\bar{c}$ at NLO in $\alpha_s$ is found to be 0.71 pb, which is 82% of the experimental value 0.87 pb. Hence the discrepancy between theory and experiment is largely removed, despite of certain theoretical uncertainties.

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