Chiral magnetic effect in the hadronic phase

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We study the chiral magnetic effect (CME) in the hadronic phase. The CME current involves pseudoscalar mesons to modify its functional form. This conclusion is independent of microscopic details. The strength of the CME current in the hadronic phase would decrease for two flavors.

I. INTRODUCTION

Among a rich variety of quantum phenomena driven by a magnetic field [1, 2], the chiral magnetic effect (CME) [3], that is, generation of an electric current along a magnetic field in chirally imbalanced systems, is notable for its salient characteristics. By virtue of its anomalous origin, the strength of the CME current is topologically protected even in the strong coupling limit [4, 5], as substantiated by holographic approaches [6–9]. As the CME persists in the long-wavelength regime, it modifies the hydrodynamic and kinetic descriptions of chiral fluids [10–13]. The nondissipative nature of the CME, which is concisely understood in terms of the time-reversal symmetry [14], is also unusual and enables one to determine the corresponding transport coefficient in the chiral hydrodynamics [10].

Heavy ion collision experiments provide experimental probes to study the CME. (See also the realizations in Weyl semimetals [15–20] and lattice simulations [21–23].) This is because ions passing with each other would generate an intense magnetic field of order $eB \sim m^2_{\pi}$ [24–26] and the fluctuating gauge topology generates the chirality imbalance [27–29]. The STAR Collaboration at Relativistic Heavy Ion Collider (RHIC) and the ALICE Collaboration at Large Hadron Collider (LHC) have reported the charge-dependent azimuthal correlators which are qualitatively consistent with the charge separations caused by the CME [30–33]. The signals of the CME-driven collective excitation, called chiral magnetic wave [34–38], is also expected to be observed as charge-dependent elliptic flows [39, 40]. The beam energy scan program at RHIC is continuing to examine the energy dependence of charge separations toward the low energy regime [41–43]. Further understanding would be achieved by these ongoing experiments as well as firm quantification of the transport properties [44–52] and the real-time dynamics [53–57] of matter under a strong magnetic field.

One complication of heavy ion collisions is that created fireballs would undergo hadronization. However, the CME in the hadronic phase has not been studied much detail. (Exceptions include Ref. [58].) This is one of the paramount issues, especially when scanning the low beam energy regions where quark-gluon plasma would have a short lifetime or even not be generated. In order to examine the CME signals in heavy ion collisions with low energy, it is indispensable to quantify the CME in the hadronic phase.

This work is to present two conclusions. Firstly, the CME current in the hadronic phase is, at the functional level, modified from the chiral phase by involving the pseudoscalar mesons. This conclusion is independent of microscopic details. The involvement of the pseudoscalar mesons to the CME is not quite unnatural. Although the absence of higher-order corrections in the anomalous current is perturbatively proved [5], it is not shown nonperturbatively. Indeed, the anomalous triangle diagram can, in general, involve the pseudoscalar particles as exemplified by the $\pi^0 \rightarrow \gamma\gamma$ decay and the Primakoff effect. Secondly, the involvement of the pseudoscalar mesons would reduce the strength of CME by a few percents. We will analytically demonstrate this conclusion for the two-flavor case.

This paper is organized as follows. We first review the CME in the chiral phase in Sec. II. We compute the effective action to get the renowned CME formula. Then we study the CME in the hadronic phase in Sec. III. The CME is firstly derived by means of a chiral effective model to clarify the physical picture. Afterwards, we show the model independence of the result. It is clarified that the CME in the hadronic phase involves the pseudoscalar mesons to modify its functional form. We then analyze how this modification influences the strength of the CME current in Sec. IV. We carry out an analytical evaluation of the current strength by limiting ourselves to a two-flavor free pion gas. We will find that the CME current would be reduced by a few percents. The last section, Sec. V, is devoted to the summary and outlook.

II. CHIRAL MAGNETIC EFFECT IN THE CHIRAL PHASE

We outline the CME in the chiral phase. The original work [3] provides fourfold ways of derivations, among which we adopt a derivative expansion of the effective action.
We consider a system in the chiral phase with an external magnetic field and imbalance between right-handed and left-handed fermions. One way to derive the current in this system is through the effective action

\[ S_{\text{eff}} = -i \log \text{Det}(i \partial - m) . \]  

(1)

The covariant derivative \( iD_\mu = i\partial_\mu - eQA_\mu - \gamma_5 a_\mu \) incorporates the electromagnetic field \( A_\mu \) associated with the charge matrix \( Q \) and the axial gauge field \( a_\mu = (\mu_5, 0) \) encoding the axial chemical potential. The electromagnetic field strength is denoted by \( F_{\mu\nu} \). The determinant is over the coordinate space as well as the flavor, color, and Dirac indices. The quark masses are set to the equal value \( m \) for brevity. The derivative expansion of the effective action (1) is performed as Ref. [59]. To determine the effective action without the renormalization scheme dependence, it is necessary to impose the physical requirement that the current should generate the canonical anomalous divergence

\[ \partial_\mu j^\mu = \frac{e^2 N_c}{16\pi^2} \left( F^{R\mu\nu} \tilde{F}^{R\mu\nu} - F^{L\mu\nu} \tilde{F}^{L\mu\nu} \right) , \]  

with \( N_c \) being the number of colors. This requirement identifies the effective action as

\[ S_{\text{eff}} = \frac{e^2 N_c}{4\pi^2} \int d^4x a_\mu A_\nu \tilde{F}^{\mu\nu} \text{tr}(Q^2) \]  

(3)

in the leading order of the derivative expansion. We omitted the irrelevant terms to the CME. A functional derivative of Eq. (3) with respective to \( A \) yields the CME current

\[ j = \frac{e^2 N_c}{2\pi^2} \mu_5 B \text{tr}(Q^2) . \]  

(4)

We note that this derivation as such assumes a static and homogeneous magnetic field.

### III. CHIRAL MAGNETIC EFFECT IN THE HADRONIC PHASE

We have observed that the CME in the chiral phase reads as simple as Eq. (4). This section is to derive the CME in the hadronic phase. The derivation is performed in two ways. Firstly, we adopt a chiral effective model to get the effective action. This method clarifies the physical picture of the result. Secondly, we argue that the Wess-Zumino-Witten (WZW) action [60] gives the identical effective action. This method verifies the model independence of the result. The obtained effective action will yield the CME current.

Now, we consider a system in the hadronic phase with an external magnetic field and chirality imbalance. We first adopt the chiral effective model called the non-linear quark-meson model [61, 62]. This model represents the chiral symmetry in terms of the pseudoscalar meson multiplet \( \Sigma \equiv \exp(i\pi^A\lambda^A/f_\pi) \) with \( A \) being the adjoint flavor indices. The pseudoscalar mesons are the external source for the moment while we will treat them as the dynamical fields in Sec. IV. The Lagrangian of the quark sector reads

\[ \mathcal{L} = \bar{q}(i\partial - gM)q , \]

(5)

where \( M = P_R \Sigma + P_L \Sigma^\dagger \) with \( P_R \) and \( P_L \) being chiral projectors while \( g \neq 0 \) denotes the coupling constant. The quarks \( q \) are constituent ones. The covariant derivative incorporates the magnetic field and the axial chemical potential as in Eq. (1). To derive the electric current, we again seek the effective action

\[ S_{\text{eff}} = -i \log \text{Det}(i \partial - gM) . \]

(6)

Note that this effective action formally reduces to Eq. (1) if the pseudoscalar mesons are absent, namely, \( \Sigma = 1 \). The perturbative expansion of the effective action yields

\[ S_{\text{eff}} = -ie^2 \cdot \text{Tr} \left( \frac{\gamma_5 \partial + gM^\dagger}{-\partial^2 - g^2} Q A \frac{i\partial + gM^\dagger}{-\partial^2 - g^2} Q A \frac{i\partial + gM^\dagger}{-\partial^2 - g^2} \right) , \]

(7)

among which the nonvanishing contributions are depicted by the triangle diagrams in Fig. 1. We omitted the irrelevant terms to the CME. In contrast to the chiral phase, the effective action involves the pseudoscalar mesons through \( M \) in Eq. (7). Further computation can be performed by imposing the physical requirement that the effective action should reduce to Eq. (3) for \( \Sigma = 1 \). The result, in the leading order of the derivative expansion, reads

\[ S_{\text{eff}} = \frac{e^2 N_c}{12\pi^2} \int d^4x a_\mu A_\nu \tilde{F}^{\mu\nu} \text{tr}(2Q^2 + Q\Sigma Q^\dagger) , \]  

(8)

which is independent of \( g \). The derivation is given in Appendix. A.

The effective action (8) is also obtained from the WZW action. Although the WZW action in Ref. [63], for example, as such yields the different action from Eq. (8) by a term proportional to \( a_\mu A_\nu \tilde{F}^{\mu\nu} \text{tr}(Q^2) \), this term is regarded as the difference of counterterms. One can tame this renormalization scheme dependence by the same requirement we used in the previous paragraph to obtain Eq. (8). This argument qualifies the effective action (8) to be, in the low energy regime, independent of microscopic details. Besides, the topological nature of the
The effective action (8) yields the CME current

\[ j = \frac{e^2 N_c}{2\pi^2} \mu_5 B \text{tr} \left( Q^2 + \frac{1}{6} [Q, \Sigma]|[Q, \Sigma^*] \right). \tag{9} \]

This is our main result. One can clearly see that the CME current involves the pseudoscalar mesons to modify its functional form from that in the chiral phase (4).

Before leaving this section, we make a remark on the disagreement with the work by Fukushima and Mameda [58]. Their work demonstrates that the CME current in the hadronic phase maintains the same functional form with the chiral phase. One possible root of this disagreement is the difference in the ways to incorporate the axial chemical potential. Ref. [58] mimics the involvement of the pseudoscalar mesons to modify the CME current in the hadronic phase maintains the same functional form with the chiral phase (4). This limitation enables an analytical evaluation of the current strength. We will see that the current strength is decreased by a few percents.

We hereby treat the pions as the dynamical fields. In this case, the flow of the charged pions also carry the electric current, but we ignore this non-anomalous contribution. The pions are assumed to be free with their mass \( m_\pi \) and decay constant \( f_\pi \) being substituted by the thermal effective values given in Ref. [64]. The magnetic-field dependence of \( m_\pi \) and \( f_\pi \) are ignored. The vacuum or thermal expectation value of the CME current (9) reads

\[ \langle j \rangle = \kappa \frac{e^2 N_c}{2\pi^2} \mu_5 B \text{tr}(Q^2), \tag{10} \]

where we defined

\[ \kappa \equiv \left\langle \text{tr} \left( Q^2 + \frac{1}{6} [Q, \Sigma]|[Q, \Sigma^*] \right) \right\rangle / \text{tr}(Q^2). \tag{11} \]

The bracket \( \langle \cdots \rangle \) denotes the vacuum or thermal expectation value. Comparing this with Eq. (4), the coefficient \( \kappa \) is interpreted as the generalized dielectric constant incorporating the medium effects of pseudoscalar mesons [44]. As shown in Appendix. B, the dielectric constant (11) is evaluated in terms of the thermal Green function at the coincidental point [65]

\[ G \equiv f_\pi^{-2} \langle \pi^A(x)\pi^A(x) \rangle \]

\[ = G_0 - \frac{m_\pi^2}{16\pi^2 f_\pi^2} + \frac{m_\pi T}{2\pi^2 f_\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} K_1 \left( \frac{m_\pi n}{T} \right), \tag{12} \]

where \( K_1(z) \) denotes the modified Bessel function of the second kind. The sum over \( A = 1, 2, 3 \) is not taken here. The constant \( G_0 \) is the counterterm to be determined shortly. The result reads

\[ \kappa = \frac{1}{5} (12 + 3e^{-2G} + 9e^{-G} - 18e^{-G} + G). \tag{13} \]

In order to determine \( G_0 \), we impose the requirement that the dielectric constant (13) should be unity at the temperature of chiral symmetry restoration, \( T = 180 \text{ MeV} \). This requirement gives \( G_0 = 1.48 \text{ MeV} \).

The temperature dependence of the dielectric constant \( \kappa \) is shown in Fig. 2. The strength of the CME current is reduced from that in the chiral phase. It is interesting to note that the beam energy scan programs in ALICE and STAR have reported the reduced charge separations for low beam energies [32, 33], for which quark-gluon plasma would have a short lifetime until the system hadronizes.

\[ \text{FIG. 2. Temperature dependence of the dielectric constant.} \]

\[ \text{V. SUMMARY AND OUTLOOK} \]

We studied the CME in the hadronic phase by means of the chiral effective model as well as the WZW action. The CME current reads Eq. (9) and involves the pseudoscalar mesons. This result is independent of microscopic details. The involvement of the pseudoscalar mesons can either increase and decrease the strength of the CME current. Especially, the analysis of the two-flavor case implies that the CME signal would be reduced when the collision energy is so low that the system quickly hadronizes. This
result qualitatively agrees with the observations of the beam energy scan programs. The large multi-pion correlations, which is recently reported by the ALICE collaboration [66, 67], might considerably influence the behavior of the CME signals. A recent theoretical study implies that these large correlations could be the manifestation of the Bose-Einstein condensation of the charged pions [68]. (See also Refs. [69, 70] for related works.) Although we have limited ourselves to a free pion gas in Sec. IV, further analysis including the multi-pion correlations is necessary to inspect these interesting phenomena.

Other chiral transport phenomena, most of which have been examined in the chiral phase, could be enriched by hadronic environments. For instance, it is clear from Eq. (8) that the chiral separation effect (CSE) also involves the pseudoscalar mesons. Accordingly, the chiral magnetic wave, which is derived by combining the CME and the CSE, would change its behavior in the hadronic phase. Besides, one could examine the chiral vortical effect and the chiral torsional effect [71, 72] in the hadronic phase by incorporating the pseudoscalar mesons. These intriguing transport phenomena deserve further investigations.

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Appendix A: Derivation of Eq. (8)

We derive the effective action (8). We neglect the momenta of the pseudoscalar mesons in what follows. The perturbative expansion of the effective action (6) yields

\[ S_{\text{eff}} = -i N_c \sum_{n=1}^{\infty} \frac{1}{n} \operatorname{Tr} \left[ \left( i \partial - g M \right)^n \right]. \]

The trace is over the coordinate space as well as the flavor and Dirac indices. As we are interested in CME, we focus on the term that is linear in \( a_\mu \) and quadratic in \( A_\mu \), which is from \( n = 3 \). This term reads, in the momentum space,

\[ S_{\text{eff}} = -i N_c \int \frac{d^4 p}{(2\pi)^4} \sum_{\mu} a_\mu A_\mu(p) A_\mu(-p) K^{\mu\nu\rho}(p). \]  

The kernel \( K^{\mu\nu\rho}(p) \) is given by the triangle diagrams in Fig. 1, or, an integral

\[ K^{\mu\nu\rho}(p) = e^2 \int \frac{d^4 k}{(2\pi)^4} \Gamma^{\mu\nu\rho\lambda} \left( \frac{k + gM^g}{k^2 - g^2} Q^{\gamma\lambda} \frac{k + p + gM^f}{(k + p)^2 - g^2} Q^{\rho\lambda} \frac{k + gM^f}{k^2 - g^2} \right). \]

By virtue of the trace identity of gamma matrices, only the terms involving even numbers of \( M \) are nonvanishing. The term with two \( M \) is given by a convergent integral and reads

\[ K_1^{\mu\nu\rho}(p) = -\frac{e^2}{12\pi^2} \varepsilon^{\mu\nu\rho\sigma} p_\sigma \left( Q^2 + Q^2 \Sigma^\dagger \right), \]

in the leading order of the derivative expansion. On the other hand, the term without \( M \) is divergent and thus depends on regularization schemes. We now impose a requirement that the CME current with \( \Sigma = 1 \) must reproduce that in the chiral phase (4). This requirement identifies the coefficient with the form

\[ K^{\mu\nu\rho}(p) = -\frac{e^2}{12\pi^2} \varepsilon^{\mu\nu\rho\sigma} p_\sigma \left( 2Q^2 + Q^2 \Sigma^\dagger \right). \]

By plugging this into Eq. (A1), we reach the effective action which reads Eq. (8) in the coordinate space.

Appendix B: Derivation of Eq. (13)

We consider a two-flavor free pion gas to derive the dielectric constant (13). We represent the pions by \( \Sigma = \delta_{\pi^+ \pi^-} + e^{i \Delta^\pi \cdot q} \). We consider a two-flavor free pion gas to derive the dielectric constant (13). We represent the pions by \( \Sigma = \delta_{\pi^+ \pi^-} + e^{i \Delta^\pi \cdot q} \). The Pauli matrices are denoted by \( \tau^A \) \( (A = 1, 2, 3) \). The quantity of our interest reads

\[ \operatorname{tr}(2Q^2 + Q^2 \Sigma^\dagger) = 6q^0 q^0 + q^i q^j (4\delta^{XY} + T^{XY}), \]

where \( T^{XY} \equiv \langle \operatorname{tr}(\tau^X \tau^Y \Sigma^\dagger) \rangle \). This trace is given by

\[ T^{XY} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{i^n(-i)^m}{n! m!} \langle \Pi^1 \cdots \Pi^n \Pi^1 \cdots \Pi^m \rangle \]

\[ \cdot \operatorname{tr}(\tau^X \tau^Y \Sigma^\dagger \Pi^1 \cdots \Pi^n \Pi^1 \cdots \Pi^m) \].

Bosonic nature of pions and the anticommutative nature of Pauli matrices imply that the indices \( A_1, \ldots, A_n \) and \( B_1, \ldots, B_m \) in Eq. (B1) respectively take the same values. This observation simplifies this sum as

\[ T^{XY} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{i^n(-i)^m}{n! m!} \langle (\Pi^1)^n (\Pi^1)^m \rangle \cdot \operatorname{tr}[\tau^X (\tau^A) \cdots (\tau^A) \tau^Y (\tau^B) \cdots (\tau^B)]. \]

We further observe that this trace is nonvanishing only when \( n \) and \( m \) are both odd or both even. For odd \( n \) and \( m \), the vacuum expectation value in Eq. (B2) is nonvanishing only when the indices \( A \) and \( B \) are equal. Then Wick’s theorem gives

\[ T^{XY}|_{\text{odd } n,m} = \sum_{n'=0}^{\infty} \sum_{m'=0}^{\infty} \frac{i^{2n'+1}(-i)^{2m'+1}}{(2n'+1)! (2m'+1)!} \langle (\Pi^1)^{2n'+2m'} \rangle \cdot \operatorname{tr}[\tau^X (\tau^A) \cdots (\tau^A) \tau^Y (\tau^A) \cdots (\tau^A)] = \delta^{XY} (e^{-2G} - 1). \]
The sum with even $n$ and $m$ can also be readily evaluated. After all, we obtain
\[
T^{XY} = \delta^{XY}(10 + 4e^{-2G} + 12e^{-G} - 24e^{-\frac{1}{2}G}).
\]
Thus, for the charge matrix of the $u$- and $d$-quark, $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3})$,
\[
\text{tr}(2Q^2 + QSQ\Sigma^I) = \frac{1}{3}(11 + 3e^{-2G} + 9e^{-G} - 18e^{-\frac{1}{2}G}).
\]

It gives rise to the dielectric constant (13).

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