Synchronization Control of Complex Network Based on Extended Observer and Sliding Mode Control

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ABSTRACT The synchronization of complex networks is one of the key points to study the dynamic law of network. Due to the influence of external environment or physical conditions, nonlinear interference is inevitable. As a result, system parameters will not be obtained completely, which is not conducive to realize synchronization of network. In order to get system state variables and realize synchronization of complex networks with external interference. A synchronization control method is proposed based on the extended observer theory and sliding mode control theory in this paper. Firstly, aiming at the problem that the system state variables can’t be obtained accurately. A method of designing extended observer is proposed to observe the unknown state variables of system. Secondly, on the basis of getting the system state variables, in order to realize system synchronization control, the synchronization controller is designed based on the sliding mode control theory. And the correctness of this method is proved by Lyapunov stability theory. Finally, the validity of the proposed method is verified by simulation of Lorenz chaotic system of random 30-node and IEEE39-bus system.

INDEX TERMS Complex network, synchronization, extended observer, sliding mode control.

I. INTRODUCTION
As a basic tool to understand real dynamic behavior systems, complex networks have attracted wide attention in many aspects. In human social activities, complex networks exist everywhere, such as the Internet, biological networks, power networks, social networks and scientific citation networks, and affect all aspects of our lives [1]–[3]. Complex network is a graph that is composed of a large number of information-intensive systems or nodes of a specific basic unit of dynamics, and the edges can be regarded as the relationship between systems or units [4], [5]. The complexity of complex networks is primarily manifested in the complexity of structural links, the large number of nodes and the complexity of the space-time evolution process [6]. The research of complex networks focuses on the interaction between local node dynamics and global topology [7]–[10]. Synchronization is not only an important branch of complex network research, but also a typical collective behavior, which has been extensively discussed and applied in many projects [11].

Synchronization refers to the behaviors of nodes that tend to the same state with the increase of time, these behaviors can explain many complex phenomena in nature and have great significance to human society [12]. In general, complex network can’t achieve complete synchronization by only depending on the mutual coupling between the nodes. Thus, controller needs to be designed to realize synchronization. Existing typical control methods include adaptive synchronization control [13], pinning synchronization control [14]–[16] and periodic intermittent synchronization control [17], these mainly apply different controllers to all or part of the network nodes to realize synchronization. At the moment, however, most studies investigate the complex dynamic network in ideal conditions; few studies involve the complex network with external interference. Even if there are synchronization methods of complex network with external interference, they also has some limitations apply. Such as reference [18], although the $H_\infty$ synchronization control of
uncertain complex network systems can be realized based on the non-fragile state feedback control scheme, the unknown interference must be bounded. However, in an actual network, there are various interference factors such as random forces in the physical system or noise generated by uncertain factors in the experimental environment [19], their boundaries are usually unknown. Thus, this method has some limitations. In [20], the synchronization method of complex dynamic network under one-sided Lipschitz nonlinear condition is proposed. But, the nonlinear part of system state equation also some limitations apply.

In the field of control, if we want to control the system with external interference, we usually adopt the strategy of observation before control [21]. The extended observer technique has become one of the most frequently used method to solve this problem, and has become a research hotspot [22]–[24]. Thus, in order to realize the synchronization of complex networks with unknown interference and be not restricted to the application condition, a synchronous control method is proposed in this paper. Firstly, in order to get the state variables of the system, a design method of extended observer is proposed based on dynamic coupling state equation of network. Secondly, on the basis of getting the system state variables, a synchronization control method of complex network is proposed based on the sliding mode control. Finally, the feasibility of this paper method is verified by simulation.

II. DESIGNING EXTENDED OBSERVER

Assume that a continuous dynamic network with $N$ nodes, the state equation of the continuous-time dissipative coupled dynamic network is expressed as:

$$\dot{X}_i(t) = f(X_i(t)) + d \sum_{j=1}^{N} l_{ij} HX_j(t) + B_i U_i(t) + W_i(t)$$

(1)

where $X_i = [x_{i1}, x_{i2}, x_{i3}, \cdots, x_{im}]^T$ represent the state variables of node $i$, $m$ is the number of state variables of node $i$, $d$ is the network coupling intensity, $d > 0$, $H$ is the external coupling function between the state variables of each node, $L = [l_{ij}]_{N \times N}$ represents the topological structure of the network, which is referred to as the externally coupled matrix and generally satisfies $\sum_{j=1}^{N} l_{ij} = 0$, $\sum_{j=1}^{N} l_{ij} = N$ for $j \neq i$ number of system nodes, $f(\cdot) = [f_1(\cdot), \cdots, f_m(\cdot)]^T$ is continuously differentiable function which is used to describe the dynamics of each isolated (uncoupled) node, $W_i(t) = [w_{i1}(t), w_{i2}(t), \cdots, w_{im}(t)]^T$ is the unknown external disturbance quantity, $U_i = [u_{i1}, \cdots, u_{ik}, \cdots, u_{im}]^T$ is the controller designed to synchronize the network as soon as possible, and $B_i = \text{diag}(b_{i1}, \cdots, b_{ik}, \cdots, b_{im})$ is the coefficient vector of the input variables.

**Definition 1:** For a state variable $x_{i\xi}$ of a node, there is the variable $\gamma$ making $\dot{\gamma} = x_{i\xi}$, where $x_{i\xi}$ is the $\xi$th state variable of node $i$. Thus, $x_{i\xi}$ can be written as:

$$\begin{align*}
\dot{\gamma} &= x_{i\xi} \\
\dot{x}_{i\xi} &= f_i(x_{i1}(t), x_{i2}(t), \cdots, x_{im}(t)) + b_{i\xi} u_{i\xi}(t) \\
&\quad + d \sum_{j=1}^{N} l_{ij} HX_j(t) + w_{i\xi}(t)
\end{align*}$$

(2)

where $b_{i\xi} \in B_i$, and $u_{i\xi}(t) \in U_i$.

Formula (2) can be rewritten as follows:

$$\begin{align*}
\dot{\gamma} &= x_{i\xi} \\
\dot{x}_{i\xi} &= b_{i\xi} u_{i\xi}(t) + F
\end{align*}$$

(3)

where $F = f_i(x_{i1}(t), \cdots, x_{im}(t)) + d \sum_{j=1}^{N} l_{ij} HX_j(t) + w_{i\xi}(t)$.

**Theorem 1:** According to reference [25], the extended observer of complex networks (2) can be achieved with the following as:

$$\begin{align*}
\dot{\hat{\gamma}} &= \hat{x}_{i\xi} + \frac{\alpha_1}{\epsilon} (\gamma - \hat{\gamma}) \\
\dot{\hat{x}}_{i\xi} &= b_{i\xi} u_{i\xi}(t) + \hat{\sigma} + \frac{\alpha_2}{\epsilon^2} (\gamma - \hat{\gamma}) \\
\dot{\hat{\sigma}} &= \frac{\alpha_3}{\epsilon^2} (\gamma - \hat{\gamma})
\end{align*}$$

(4)

where $\gamma$, $\hat{x}_{i\xi}$ and $\hat{\sigma}$ are observer states, $\alpha_1$, $\alpha_2$ and $\alpha_3$ are positive real numbers.

**Proof:** Defining $\eta = [\eta_1, \eta_2, \eta_3]^T$, where

$$\begin{align*}
\epsilon \dot{\eta}_1 &= \frac{\dot{\gamma} - \hat{\gamma}}{\epsilon} = \frac{1}{\epsilon} (x_{i\xi}(t) - (\hat{x}_{i\xi} + \frac{\alpha_1}{\epsilon}(\gamma - \hat{\gamma}))) \\
&= \frac{1}{\epsilon} (x_{i\xi}(t) - \hat{x}_{i\xi} - \frac{\alpha_1}{\epsilon}(\gamma - \hat{\gamma})) \\
&= -\frac{\alpha_1}{\epsilon^2} (\gamma - \hat{\gamma}) + \frac{1}{\epsilon} (x_{i\xi}(t) - \hat{x}_{i\xi}) \\
&= -\alpha_1 \eta_1 + \eta_2 \\
\epsilon \dot{\eta}_2 &= \frac{\dot{\hat{x}}_{i\xi} - \hat{x}_{i\xi}}{\epsilon} \\
&= (b_{i\xi} u_{i\xi}(t) + F - (b_{i\xi} u_{i\xi}(t) + \hat{\sigma} + \frac{\alpha_2}{\epsilon^2} (\gamma - \hat{\gamma}))) \\
&= -\frac{\alpha_2}{\epsilon^2} (\gamma - \hat{\gamma}) + (F - \hat{\sigma}) \\
&= -\alpha_2 \eta_1 + \eta_3 \\
\epsilon \dot{\eta}_3 &= \epsilon (\hat{F} - \hat{\sigma}) \\
&= \epsilon \hat{F} - \frac{\alpha_3}{\epsilon^2} (\gamma - \hat{\gamma}) \\
&= -\alpha_3 \eta_1 + \epsilon \hat{F}
\end{align*}$$

(5)

The observer error state equation can be written as:

$$\epsilon \dot{\hat{\eta}} = \epsilon \begin{bmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \\ \hat{\eta}_3 \end{bmatrix} = \epsilon \begin{bmatrix} -\alpha_1 \eta_1 + \eta_2 \\ -\alpha_2 \eta_1 + \eta_3 \\ -\alpha_3 \eta_1 + \hat{F} \end{bmatrix}$$

(6)

$$\epsilon \begin{bmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \\ \hat{\eta}_3 \end{bmatrix} = \epsilon \begin{bmatrix} -\alpha_1 & 1 & 0 \\ -\alpha_2 & 0 & 1 \\ -\alpha_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} + \epsilon \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \hat{F}$$

(7)

$$\epsilon \dot{\hat{\eta}} = \epsilon A \eta + \epsilon B \hat{F}$$

(8)
where
\[
A = \begin{bmatrix} -\alpha_1 & 1 & 0 \\ -\alpha_2 & 0 & 1 \\ -\alpha_3 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.
\]

The characteristic equation of matrix $A$ is:
\[
|\lambda I - A| = \begin{bmatrix} \lambda + \alpha_1 & -1 & 0 \\ \alpha_2 & \lambda & -1 \\ \alpha_3 & 0 & \lambda \end{bmatrix} = 0
\]
(9)
Thus,
\[
(\lambda + \alpha_1)\lambda^2 + \alpha_3 + \alpha_2\lambda = \lambda^3 + \alpha_1\lambda^2 + \alpha_2\lambda + \alpha_3 = 0
\]
(10)

The quality of the extended observer is only related to the pole assignment of matrix $A$, so long as the pole assignment of matrix $A$ is reasonable, the good quality of the extended observer can be guaranteed. Thus, choosing the values of $\alpha_1$, $\alpha_2$ and $\alpha_3$ to make sure that the matrix $A$ is Hurwitz matrix, which can make the extended observer have good quality.

**Assumption 1:** For any given symmetric positive definite matrix $Q$, the symmetric positive definite matrix $P$ satisfies the following Lyapunov equation:
\[
A^T P + PA + Q = 0
\]
(11)
The Lyapunov function of observer is defined as follows:
\[
V_1 = \varepsilon \eta^T P \eta
\]
(12)
Thus,
\[
\dot{V}_1 = \varepsilon \eta^T P \dot{\eta} + \varepsilon \dot{\eta}^T P \eta = (A\eta + \varepsilon B \hat{F})^T P \eta + \eta^T P (A\eta + \varepsilon B \hat{F}) = \eta^T (A^T P + PA) \eta + 2\varepsilon \eta^T PB \hat{F}
\]
\[
\leq -\varepsilon \eta^T \eta + 2\varepsilon \| PB \| \cdot \| \eta \| \cdot | \hat{F} | \]
(13)
And,
\[
\dot{V}_1 \leq -\lambda_{\min}(Q) \| \eta \|^2 + 2\varepsilon \| PB \| \cdot \| \eta \|
\]
(14)
where $\lambda_{\min}(Q)$ is the minimum eigenvalue of $Q$, and $| \hat{F} | \leq \beta$.

The convergence condition of observer is expressed as follows:
\[
\| \eta \| \leq \frac{2\varepsilon \| PB \|}{\lambda_{\min}(Q)}
\]
(15)
In (15), the convergence rate of observation error $\eta$ is related to $\varepsilon$. When the parameter $\varepsilon$ is very small, according to the singularly perturbed system theory [26], the error dynamic equation (3) is a fast-varying subsystem. The smaller $\varepsilon$ is, the faster convergence of $\eta$ is. With the decrease of $\varepsilon$, the observation error $\| \eta \|$ tends to zero.

**Remark 1:** It can be seen that the Theorem 1 can be applied in solving observation problem of state variables. But, the extended observer belongs to the high gain observer. When the initial value differs from the initial value of the object, a peak phenomenon will exist for a very small $\varepsilon$, which causes poor convergence effect of the observer. To prevent the peak phenomena, the design is expressed as:
\[
\frac{1}{\varepsilon} = \left\{ \begin{array}{ll}
\mu \frac{1 - e^{-\frac{t}{\varepsilon}}}{1 + e^{-\delta t}} & 0 \leq t \leq t_{\max} \\
\mu & t > t_{\max}
\end{array} \right.
\]
(16)
where $\mu > 0$, $\zeta > 0$ and $\delta > 0$.

### III. Designing Sliding Mode Controller based on Expansion Observer

Defining the sliding mode function is:
\[
s = ce + \dot{e}
\]
(17)
where $c > 0$, $\dot{e} = y - z_d$, $z_d = \int y_{i\xi} dt$, and $y_{i\xi}$ is the synchronous state of $x_{i\xi}$.

**Theorem 2:** The synchronization of complex networks (2) can be achieved with the following controllers
\[
u_{i\xi}(t) = \frac{1}{b_{i\xi}}(-k s - \dot{v} - \hat{F})
\]
(18)
where $\dot{v} = \dot{\hat{e}} - \dot{z}_d$, $\hat{s} = \hat{c}e + \hat{\dot{e}}$, $\hat{e} = \hat{\gamma} - z_d$, $\hat{\dot{e}} = \hat{s}_{i\xi} - y_{i\xi}$, and $\hat{F}$ is observation value of $F$.

**Proof:** Considering that the Lyapunov function is $V_2 = \frac{1}{2} \hat{s}^2$, then,
\[
\dot{V}_2 = \frac{1}{2} \ddot{s}
\]
\[
= s(c\dot{e} + \ddot{e})
\]
\[
= s(c\dot{e} + b_{i\xi} u_{i\xi}(t) + F - \dot{\hat{F}})
\]
\[
= s(c\dot{e} - k\hat{s} - \dot{v} - \hat{F} + F - \dot{\hat{F}})
\]
\[
= -k\hat{s}\ddot{s} + s(\ddot{v} + \hat{F})
\]
\[
= -k\hat{s}\ddot{s} + s(\ddot{v} + \hat{F} + k\hat{s})
\]
(19)
where $\hat{F} = F - \dot{\hat{F}}$, $\ddot{v} = \ddot{v} - \dot{\hat{s}} = c(\ddot{\gamma} - \dot{s}_{i\xi}) = c\ddot{x}_{i\xi}$, and $\hat{s} = s - \hat{s} = c\ddot{x}_{i\xi} + \ddot{x}_{i\xi}$. $\ddot{v} + \hat{F} + k\hat{s}$ depends on the observational errors of the states of the extended observer. Considering that $\Delta_{\max} \geq |\ddot{v} + \hat{F} + k\hat{s}|$, thus:
\[
\dot{V}_2 \leq -k\hat{s}\ddot{s} + \frac{1}{2}(s^2 + \Delta_{\max}^2)
\]
\[
= -(k - \frac{1}{2})s^2 + \frac{1}{2}\Delta_{\max}^2
\]
\[
= -(2k - 1)V_2 + \frac{1}{2}\Delta_{\max}^2
\]
(20)
According to lemma in reference [27], and considering $\theta = 2k - 1$ and $F = \frac{1}{2}\Delta_{\max}^2$, the solution of (20) is expressed as follows:
\[
\dot{V}_2 \leq e^{-\theta(t-t_0)} V_2(t_0) + \frac{1}{2}\Delta_{\max}^2 \int_{t_0}^{t} e^{-\theta(t-\tau)} d\tau
\]
\[
= e^{-\theta(t-t_0)} V_2(t_0) - \frac{1}{2\theta}\Delta_{\max}^2 \int_{t_0}^{t} e^{-\theta(t-\tau)} d(-\theta(t-t_0))
\]
\[
= e^{-(2k-1)(t-t_0)} V_2(t_0) - \frac{1}{2(2k-1)}\Delta_{\max}^2 (1 - e^{-\theta(t-t_0)}
\]
(21)
Thus \( V \) as follows observer and controller, the Lyapunov function is expressed shown in FIGURE 1. The simulation process is as follows:

Considering \( k > \frac{1}{2} \), then

\[
\lim_{t \to \infty} V \leq \frac{1}{2(2k-1)} \Delta_{\text{max}}^2
\]  

(22)

Because \( V_2 \geq 0 \), when \( t \to \infty \), \( V_2 = \frac{1}{2(2k-1)} \Delta_{\text{max}}^2 \) and the convergence rate depends on the control gain \( k \) and the observer parameter \( \epsilon \).

Considering the closed-loop system, which consists of the observer and controller, the Lyapunov function is expressed as follows:

\[
V = V_2 + V_1
\]  

(23)

Thus:

\[
\dot{V} = \dot{V}_1 + \dot{V}_2 \\
\leq -\lambda_{\text{min}}(Q) \eta^2 + 2\epsilon \beta \|PB\| \|\eta\| \\
- (2k - 1)V_2 + \frac{1}{2} \Delta_{\text{max}}^2
\]  

(24)

Because of (15) and (22), (24) may be simplified to

\[
\dot{V} \leq -\lambda_{\text{min}}(Q) \cdot \left( \frac{2\epsilon \beta \|PB\|}{\lambda_{\text{min}}(Q)} \right)^2 + 2\epsilon \beta \|PB\| \cdot \frac{2\epsilon \beta \|PB\|}{\lambda_{\text{min}}(Q)} \\
- (2k - 1) \cdot \frac{1}{2(2k-1)} \Delta_{\text{max}}^2 + \frac{1}{2} \Delta_{\text{max}}^2 \\
\leq -\frac{(2\epsilon \beta \|PB\|)^2}{\lambda_{\text{min}}(Q)} + \frac{(2\epsilon \beta \|PB\|)^2}{\lambda_{\text{min}}(Q)} - \frac{1}{2} \Delta_{\text{max}}^2 + \frac{1}{2} \Delta_{\text{max}}^2 \\
\leq 0
\]  

(25)

To summarize, as long as the right \( \epsilon \) and \( k \) are selected, \( \dot{V} \leq 0 \) can be satisfied.

Remark 2: It can be seen that the Theorem 2 can be applied in solving synchronization problem of uncertain network. This means the derived Theorem 2 is universal.

IV. SIMULATION

The classical Lorenz chaotic system is selected to construct a complex network of 30-node for simulation. The topology is shown in FIGURE 1. The simulation process is as follows:

(1) The state equation of the Lorenz chaotic system is:

\[
\begin{align*}
\dot{x}_1 &= 10(x_2 - x_1) + u_1 + w_{11}(t) \\
\dot{x}_2 &= 28x_1 - x_2 + x_1x_3 + u_2 + w_{12}(t) \\
\dot{x}_3 &= -\frac{8}{3}x_3 + x_1x_2 + u_3 + w_{13}(t)
\end{align*}
\]

where \( w_{11}(t), w_{12}(t) \) and \( w_{13}(t) \) are external unknown disturbances.

(2) Observation results of single-node variables.

(i) When \( u_{11}(t) = 2 \sin(t), u_{12}(t) = 2 \cos(t), u_{13} = \sin(2t) \) and \( w_{11}(t) = w_{12}(t) = w_{13}(t) = \sin(t) \), the observation results are as shown in FIGURE 2.

(ii) When \( u_{11} = 4 \sin(3t), u_{12} = 2 \cos(5t), u_{13} = 3 \sin(4t) \) and \( w_{11}(t) = w_{12}(t) = w_{13}(t) = 20 \sin(2t) \), the observation results are as shown in FIGURE 3.

As shown in FIGURE 2 and FIGURE 3, the observer designed in this paper can observe the system state variables. Thus, this paper method is correct.

(3) The 30 nodes are selected as control nodes to simulate in two different situations. In addition, \( H = \text{diag}[1, 1, 1] \) and \( d = 1 \).

(i) When the external interference is \( w_{11}(t) = w_{12}(t) = w_{13}(t) = 3 \sin(t) \) and the synchronization states are \( y_{11} = 2 \cos(2t), y_{12} = 3 \cos(3t) \) and \( y_{13} = -2 \sin(2t) \),
the simulation results of the synchronous state curves, observation curves, observation error curves and synchronous error curves are as shown in FIGURE 4, FIGURE 5, FIGURE 6 and FIGURE 7.

(ii) When the external interference is \( w_{11}(t) = w_{12}(t) = w_{13}(t) = 10 \cos(2t) \) and the synchronization states are \( y_{11} = 4 \cos(4t) \), \( y_{12} = -6 \sin(6t) \) and \( y_{13} = -5 \sin(5t) \), the simulation results of the synchronous state curves, observation curves, observation error curves and synchronous error curves are as shown in FIGURE 8, FIGURE 9, FIGURE 10 and FIGURE 11.

As shown in FIGURE 4 to FIGURE 11, the proposed method can achieve synchronization control of the system. To prevent the peak value in the entire control process, the low
gain is adopted when the extended observer is designed, which reduces the initial dynamics of the extended state observer and the convergence speed of control error. However, the requirement of synchronous control is satisfied. Thus, this paper method is feasible.

(4) In order to verify the correctness of this paper method, the 10 machine 39-bus New England system is simulated. The simplified topology of the system is shown in FIGURE 12. The synchronous error curves of state variable are shown in FIGURE 13, FIGURE 14 and FIGURE 15.

The state equation of generator is as follows:

\[
\begin{align*}
\dot{\delta} &= (\omega - 1)\omega_0 \\
\dot{\omega} &= \frac{1}{H'}[P_m - P_e - D(\omega - 1)] \\
\dot{\dot{P}}_m &= \frac{2}{T_w}(-P_m + \alpha - T_w\dot{\alpha}) \\
\dot{\alpha} &= \frac{1}{T_s}(-\alpha + \alpha_0 + u)
\end{align*}
\]

where \(\omega\) is the rotor speed, \(D\) is the mechanical damping coefficient, \(H'\) is the inertia time constant of the rotor, \(\omega_0 = 2\pi f_0, f_0 = 50\text{Hz}\), and \(P_e\) is the output power of the generator.

The simulation parameters are defined as follows: \(w_{i1}(t) = w_{i2}(t) = w_{i3}(t) = w_{i4}(t) = 4\sin(t)\), \(D = 0.5\), \(H' = 4\), \(T_w = 4\), \(T_s = 4\), \(d = 1\) and \(H = \text{diag}[1, 1, 1, 1]\). According to the state equation of generator, the synchronous error equation of node is as follows:

\[
\begin{align*}
\dot{e}_{i1} &= 314e_{i2} + d \sum_{j=1}^{10} l_{ij}HE_j + u_{i1} + w_{i1}(t) \\
\dot{e}_{i2} &= 0.25e_{i3} - 0.125e_{i2} + d \sum_{j=1}^{10} l_{ij}HE_j + u_{i2} + w_{i2}(t) \\
\dot{e}_{i3} &= -0.5e_{i3} + e_{i4} + d \sum_{j=1}^{10} l_{ij}HE_j + u_{i3} + w_{i3}(t) \\
\dot{e}_{i4} &= -0.25e_{i4} + d \sum_{j=1}^{10} l_{ij}HE_j + 0.25u_{i4} + w_{i4}(t)
\end{align*}
\]

In FIGURE 14 and FIGURE 15, taking control time as evaluation standard, the simulation of IEEE39 node system with this method is better than that in reference [28], which
network, there is also time-delay in control process. In the future, the synchronization of time-delay system with external interference will be studied to make the synchronization method more close to the actual situation. And, whether other control methods can be applied to realize the synchronization of complex networks is also a research area.

V. CONCLUSION

Research on synchronization of the dynamic behaviors of complex network systems has become an indispensable and important research topic in the field of control. In practical engineering, external interference inevitably exists. If the impact of external interference on network synchronization is not considered, some adverse consequences will occur. In this paper, firstly, the observer is designed to observe the system variables. Secondly, after the system variables are observed, the synchronous control of the system is realized by designing a sliding mode controller. Finally, the feasibility of this paper method is verified by the simulation of random 30-node system and IEEE39-bus system. In this paper, the observer theory and the sliding mode theory are used to realize the synchronization of system with uncertain interference, which is an extension of the existing methods and is conducive to the study of the dynamic laws of complex networks. This paper is only a preliminary study and mainly analyzes synchronization of complex networks with interference. However, in an actual network, there is also time-delay in control process. In the future, the synchronization of time-delay system with external interference will be studied to make the synchronization method more close to the actual situation. And, whether other control methods can be applied to realize the synchronization of complex networks is also a research area.

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