Effect of conjugate heat transfer on the side and horizontal walls on the structure of convective flow in the Rayleigh-Benard convection mode

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Abstract. Rayleigh-Benard convection in rectangular cavities with different relative size with walls of finite thermal conductivity is studied numerically in a two-dimensional conjugate formulation. The influence of relative cavity sizes on the spatial shape and evolution of convective flows with increasing Rayleigh numbers is investigated. Velocity and temperature fields in the liquid, temperature fields, temperature gradients and heat fluxes in solid walls are calculated. The effect of conjugate heat transfer with walls of finite thermal conductivity on the spatial shape and local characteristics of convective heat transfer is studied.

1. Introduction

Buoyant-driven convection in a horizontal liquid layer uniformly heated from below is one of the canonical objects of physical hydrodynamics. This type of flows is intensively studied due to the large number of theoretical and practical applications [1, 2]. Many technological and technical processes, such as the crystal growth and epitaxial films growth, solar collectors are largely dependent on this type of free convection. Multi-scale natural phenomena such as heat transfer at the interface of ocean and atmosphere, in the boundary layer and the cloud atmosphere, in the Earth's mantle and in the photosphere of the Sun have a physical nature and spatial form, similar to the observed in horizontal layers, heated from below.

In theoretical studies, as a rule, an infinite layer in the horizontal plane with rotary and translational symmetry is considered. However, in real experiments and technological processes the layer is always limited and has side walls of finite thermal conductivity. The presence of sidewalls was discussed as a layer limitation factor affecting the selection of wavelengths of increasing perturbations and the critical values of the Rayleigh number. Similarly, the influence of the finite thermal conductivity of the horizontal boundaries of liquid layers with different Prandtl numbers was considered [1-7]. Taking into account the finite thermal conductivity of the walls limiting the liquid layers complicates the formulation of the problem of the stability of the mechanical equilibrium and the establishment of a finite-amplitude convective flow. This problem is the subject of this work. Practically important are the studies of the effect of conjugate convective heat transfer with walls on the processes of establishing the spatial form of finite-amplitude convection and on local and integral heat transfer. The influence of the layer limitation on the convective flow structure and its evolution with increasing characteristic temperature drops is poorly studied, especially taking into account the conjugate convective heat transfer at the rigid boundaries of the region occupied by the liquid. This work is a
continuation of a series of studies conducted in its SB RAS [3-7]. Data on the spatial shape of convective flows and velocity fields in the liquid were obtained experimentally. The results of numerical simulation presented below are verified by the available experimental data and significantly supplement them with information about the temperature fields in solid walls and local features of heat transfer.

2. Model
Numerical simulation was carried out in dimensionless form in two-dimensional conjugate formulation in Cartesian coordinates. Figure 1 shows the computational domain scheme.

![Figure 1. The computational domain scheme.](image)

The computational domain consists of the following regions: 1 – fluid (ethanol); 2 – glass plate with relative thickness l1; 3 – plexiglass walls with relative thickness l2; H – highly conductive massive wall maintained at a constant high temperature. The calculated area contains the boundaries: Γ1 – isothermal, hot surface; Γ2 – isothermal, cold surface; Γ3 – thermally insulated external surfaces of vertical walls; Γ4, Γ5, Γ6 – the boundaries between different materials with a given condition of ideal thermal contact. Convective heat transfer in a fluid is described by a dimensionless system of Navier-Stokes equations in the Boussinesq approximation, written in terms of temperature, velocity vorticity, and stream function:

\[
\begin{align*}
&\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{1}{Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \\
&\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \\
&\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} = \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + Gr \frac{\partial T}{\partial x} 
\end{align*}
\]

where \(T\) is the temperature, \(\psi\) is the stream function, \(\omega\) is the velocity vorticity. The system of equations is reduced to a dimensionless form using the following scales of dimensional quantities: length scale – \(H\), temperature – \(\Delta T = T_1 - T_2\) (\(T_1\) – temperature at the lower hot border, \(T_2\) – temperature on the outer side of the upper cold border), velocity – \(v/H\), stream function – \(v\), velocity vorticity – \(v/H^2\). In the system of equations appear two dimensionless complexes – two similarity criteria: \(Pr = \nu/\alpha\) – Prandtl number, \(Ra = \beta \cdot g \cdot \Delta T \cdot H^3\) – Rayleigh number. Here \(\nu\) is the coefficient of kinematic viscosity, \(\alpha\) is the coefficient of thermal conductivity of the liquid; \(g\) – acceleration of
gravity, \( a_t = \lambda_f/\rho \cdot c_p \) - coefficient of thermal diffusivity of the liquid, \( \beta = \frac{1}{\rho \cdot c_p} \left( \frac{\partial \rho}{\partial T} \right)_\rho \) - coefficient of volumetric expansion, geometric parameters of the region \( L \) and \( H \) – width and height of the liquid layer respectively.

Conductive heat transfer in solid walls is described by the heat equation:

\[
\frac{\partial T}{\partial t} + \alpha_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0
\]

here \( \alpha_s \) is the coefficient of thermal conductivity of the solid wall material.

At the border of \( \Gamma_1 \) maintained a constant high temperature \( T|_{\Gamma_1} = 1 \), at the border of \( \Gamma_2 \) constant low temperature \( T|_{\Gamma_2} = 0 \). The boundary of \( \Gamma_3 \) is insulated \( \frac{\partial T}{\partial n}|_{\Gamma_3} = 0 \). At the boundaries of \( \Gamma_4, \Gamma_5, \Gamma_6 \) given the condition of ideal thermal contact \( T|_s = T|_m \), \( \lambda_f \frac{\partial T}{\partial n}|_s = \lambda_s \frac{\partial T}{\partial n}|_m \).

The calculations were performed in the range of relative cavity sizes \( 2 \leq L/H \leq 10 \), the range of solid wall thickness \( 0.1 \leq (l_1, l_2)/H \leq 1 \), with Prandtl number \( Pr=16 \) in the ranges of Rayleigh numbers \( 1000 \leq Ra \leq 90000 \). The following thermophysical parameters were used in the calculations: specific heat capacity of liquid (ethanol) \( c_p = 2.4 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} \); liquid density \( \rho = 790 \text{ kg} \cdot \text{m}^{-3} \); thermal conductivity of liquid \( \lambda_f = 0.179 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1} \); thermal conductivity of plexiglass \( \lambda_{s1} = 0.187 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1} \); thermal conductivity of mirror glass \( \lambda_{s2} = 0.814 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1} \); volume expansion coefficient \( \beta = 1.08 \cdot 10^{-3} \text{ K}^{-1} \); the kinematic viscosity of the fluid \( \nu = 1.462 \cdot 10^{-6} \text{ m}^{2} \cdot \text{s}^{-1} \) [8].

The calculations were performed using the own software package based on the finite element method. Triangular finite elements with linear basis functions given on them are used. The number of nodes in the grid: \( N_Y = H \cdot 50; N_X = L/H \cdot 50 \). The initial distribution of the temperature field is entered into the system using a pseudorandom number generator. This creates the initial perturbation is random noise, from which, by analogy with the experimental conditions, a cellular flow arises. Further, depending on the given value of the Rayleigh number, there is either a damping of disturbances (at Rayleigh numbers below the critical ones) or a stared convection mode.

Velocity fields in the fluid are calculated, profiles of the horizontal and vertical velocity components, local heat flux distributions, and the dependence of the Nusselt integral numbers on the Rayleigh numbers are obtained.

3. Results and discussion

The results of experimental studies of the evolution of the flow spatial form and hydrodynamics are partially presented in figure 2.

**Figure 2.** The structure of the flow in the vertical section of the cavity: a – Ra=21000, L/H=150; b – Ra=47200, L/H =150; c – Ra=95000, L/H =120; d – Ra=200000, L/H =120.
Figure 3. Isotherm fields (above) and contour fields of the stream function in fluid in the cavity of size $L/H = 4$ at: a – $l_1=0.1$, $l_2=0.1$, $Ra=32000$; b – $l_1=0.1$, $l_2=0.1$, $Ra=80000$; c – $l_1=0.2$, $l_2=0.2$, $Ra=32000$; d – $l_1=0.2$, $l_2=0.2$, $Ra=80000$; e – $l_1=0.4$, $l_2=0.4$, $Ra=32000$; f – $l_1=0.4$, $l_2=0.4$, $Ra=80000$; g – $l_1=0.6$, $l_2=0.6$, $Ra=32000$; h – $l_1=0.6$, $l_2=0.6$, $Ra = 80000$; i – $l_1=1$, $l_2=1$, $Ra = 32000$; j – $l_1=1$, $l_2=1$, $Ra = 80000$. 
Figure 4. Isotherm fields (top) and contour fields in fluid of the stream function at wall thickness $l_1=0.5$, $l_2=0.2$ and at: a – $L/H=4$, $Ra=2000$; b – $L/H=8$, $Ra=2000$; c – $L/H=4$, $Ra=8000$; d – $L/H=8$, $Ra=8000$; e – $L/H=4$, $Ra=16000$; f – $L/H=8$, $Ra=16000$; g – $L/H=4$, $Ra=32000$; h – $L/H=8$, $Ra=32000$; i – $L/H=4$, $Ra=80000$; j – $L/H=8$, $Ra=80000$. 

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Track patterns of flow in the vertical section for narrow cavities with a rigid upper boundary are presented. The images are obtained by folding 200 frames of the captured video (equivalent to an exposure time of 8 seconds). Video films of visualized fluid flow in the process of structure formation and under stationary boundary conditions for temperature.

In experimental studies of the spatial form of Rayleigh-Benard convection, the upper boundary of the layer is usually made of mirror glass with a relatively low thermal conductivity. It was necessary to study the effect of conjugate heat transfer at the liquid-glass interface on the characteristics of the convective flow. For this purpose, numerical simulation was carried out, the results of which were verified by the available experimental data and significantly supplemented the experimental data. Figure 3 shows the results of calculations for two values of Rayleigh numbers and the same relative sizes of liquid layers. The variables were the thickness of the upper horizontal wall and the thickness of the side walls. The results are presented as l1 and l2 grow. Isotherm fields in this case are shown only in the liquid layer. An increase in the Rayleigh number leads to an increase in the horizontal size of the shafts. This effect, which coincides with the experimental data [3-5], is observed clearly from the comparison of columns in figure 3. When Ra=32000 with increasing wall thickness, a similar pattern is seen, which is due to a decrease in the temperature drop in the liquid layer as the thickness of the upper wall increases. At Ra=80000, a generally similar, but less pronounced, process occurs. The average temperature at the liquid-glass interface changes with increasing glass wall thickness as follows: 0.067, 0.123, 0.211, 0.275, 0.376.

The final thermal conductivity of the glass wall, even of a relatively small thickness, makes noticeable changes in the characteristics of the fluid flow, in the distribution of the fields of isotherms and isolines of the stream function. The evolution of temperature fields in the liquid layer and in the glass upper boundary with the growth of the Rayleigh number is shown in figure 4 at two values of the relative size of the cavity. In the presence of a glass upper boundary, there is a spatial temperature modulation at the lower boundary of the glass wall. The amplitude of deviations of the local temperature from its mean value on the glass plate increases with the Rayleigh number. When L/H = 4 with increasing Rayleigh number from 16000 to 32000 in the central part of the cavity observed fragmentation of cells. Significant restructuring of the isotherm fields in the side walls is not observed. The flow directions at the side walls do not change at Ra ≤ 32000. Thus, the observed cell fragmentation is a property of cellular flow regimes in a horizontally bounded cavity, which is accompanied by a sharp increase in wavelength when moving to higher Rayleigh numbers. A similar rearrangement of the spatial form is observed at L/H = 8. This can be seen in figures 4i, j. Thus, in the investigated range of Ra numbers, there are preconditions for the development of the instability Echousa opposite types: first, the crushing of cells, and then merge them.

![Figure 5](image)

**Figure 5.** Dependence of Nusselt number on Rayleigh number and area sizes: 1 – L/H=4, l1=0.5, l2=0.2; 2 – L/H=4, l1=0.5, l2=0.4; 3 – L/H=8, l1=1, l2=0.4; 4 – L/H=8, l1=0.5, l2=0.2.
Local characteristics were obtained: profiles of the vertical velocity component, temperature distribution over the layer height in ascending and descending flows, local heat fluxes. Integral heat fluxes are obtained from the distributions of local heat fluxes, which determine the integral heat transfer coefficients (Nusselt numbers) from the lower wall (figure 5).

![Graph](image)

**Figure 6.** Dependence of Reynolds number on Rayleigh number at L/H=8.

The amplitude of the vertical velocity component $V_y$ is not significantly affected by the thickness of the upper wall at a given temperature drop. As $Ra$ increases, the amplitude of the vertical velocity component increases. This can be seen in figure 6, which shows the dependence of the Reynolds number $Re = V_y \cdot H/\nu$ on $Ra$. The results of numerical simulation of the restructuring of the flow structure with an increase in the number $Ra$ at a fixed horizontal size of the cavity $L/H$ show that there is a clear tendency in the system to increase the wavelength with an increase in $Ra$. But the fixed size of the cavity leads to the fact that the wavelength increase can only occur in a discrete way at the secondary critical values of the Rayleigh numbers $Ra_2$. With the growth of $Ra$, Eckhaus instability leads to an increase in the number of cells in the cavity at first, and then to a sharp decrease. To study the effect of relative size on the development of this type of instability, calculations are performed with a discrete increase in the length of the cavity. Such rearrangements of the spatial flow form lead to a nonmonotonic dependence of the Nusselt number on the Rayleigh number in figure 5.

**Conclusion**

The effect of the relative size of the cavities uniformly heated from below on the structure and intensity of the convective fluid flow with the number $Pr = 16$ in the Rayleigh-Benard convection regime was experimentally investigated. The dependence of the spatial shape of the convective flow on the Rayleigh number for narrow cavities with relatively large horizontal dimensions is studied. Rayleigh-Benard convection in bounded cavities uniformly heated from below is numerically investigated. The aim of the work was to study the influence of the relative dimensions of the cavity on the spatial flow form, velocity and temperature fields, local and integral heat fluxes. The equations of Buoyant-driven convection in Boussinesq approximation on triangular grids are solved by the finite element method. The finite thermal conductivity of the upper horizontal boundary and the side walls of the cavity is taken into account. The thermal conductivity of the boundaries was chosen in accordance with the experimental conditions. The obtained results are necessary for an adequate analysis of the results of the physical experiment, which uses a relatively low-thermal upper and lateral boundaries. In experimental studies of the spatial form of Rayleigh-Benard convection, the upper boundary of the layer is usually made of mirror glass with a relatively low thermal conductivity. It was necessary to study the effect of conjugate heat transfer at the liquid-glass interface on the
characteristics of the convective flow. The information obtained shows that the spatial form of the flow does not change dramatically due to the non-isothermy of the glass-liquid interface. Local heat fluxes change. The results obtained in this work significantly complement the physical experiment in terms of detailed information about the temperature fields and local heat fluxes.

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