Evidence for Non-Exponential Elastic Proton-Proton Differential Cross-Section at Low $|t|$ and $\sqrt{s} = 8$ TeV by TOTEM

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Abstract

The TOTEM experiment has made a precise measurement of the elastic proton-proton differential cross-section at the centre-of-mass energy $\sqrt{s} = 8$ TeV based on a high-statistics data sample obtained with the $\beta^*=90$ m optics. Both the statistical and systematic uncertainties remain below 1%, except for the $t$-independent contribution from the overall normalisation. This unprecedented precision allows to exclude a purely exponential differential cross-section in the range of four-momentum transfer squared $0.027 < |t| < 0.2$ GeV$^2$ with a significance greater than 7 $\sigma$. Two extended parametrisations, with quadratic and cubic polynomials in the exponent, are shown to be well compatible with the data. Using them for the differential cross-section extrapolation to $t = 0$, and further applying the optical theorem, yields total cross-section estimates of $(100.8 \pm 2.1)$ mb and $(101.2 \pm 2.1)$ mb, respectively, in agreement with previous TOTEM measurements.

Keywords: proton-proton interactions, elastic scattering, total cross-section, LHC, TOTEM

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1. Introduction

The differential cross-section $d\sigma/dt$ of hadronic proton-(anti)proton scattering at low $|t|$ has traditionally been parametrised with a simple exponential function, $e^{-B|t|}$, giving a satisfactory description of all past experimental data. Nonetheless, a few experiments have already reported about hints of slight deviations from this behaviour. At the ISR, for $\sqrt{s}$ between 21.5 GeV and 52.8 GeV, elastic pp and partly $\bar{p}p$ data have shown a change of slope [1, 2] or have been better parametrised with quadratic exponential functions, $e^{-B|t| - Ct^2}$ [3, 4]. At the S$pS$, for $\sqrt{s} = 546$ GeV, a change of slope at $|t| \approx 0.14$ GeV$^2$ has been observed, while the inclusion of a quadratic term in the exponent did not improve the fit significantly [5]. At the Tevatron [6, 7, 8, 9] no deviations from pure exponential functions were observed, except at larger $|t|$ where the influence of the shoulder ($\sim 0.8$ GeV$^2$ at $\sqrt{s} = 0.546$ TeV and $\sim 0.6$ GeV$^2$ at 1.8 and 1.96 TeV) becomes visible. At the LHC, at 7 TeV as well as at 8 TeV, all data published so far [10, 11, 12, 13] have been compatible with a pure exponential shape.

This report presents a new data sample of elastic scattering at the energy of $\sqrt{s} = 8$ TeV. Thanks to its high statistics, an unprecedented precision has been reached in the region $0.027 \lesssim |t| \lesssim 0.2$ GeV$^2$. Both the statistical and systematic components of the differential cross-section uncertainty are controlled at a level below 1%, except for the overall normalisation (Section 5.2.6). Consequently, the functional form of the cross-section can be strongly constrained, thus having more impact on theoretical model building and, in particular, on the extrapolation to $t = 0$ used for total cross-section determination. Neglecting the influence of Coulomb scattering in the observed range, the often used purely exponential extrapolation has been found inadequate, and extended parametrisations are provided, still yielding total cross-section values compatible with the previous TOTEM results [12] at the same energy.

This article is organised as follows. Section 2 outlines the detector apparatus used for this
measurement. Section 3 summarises the data-taking conditions; details on the LHC beam optics are given in Section 4. Section 5 describes the data analysis and reconstruction of the differential cross-section. In Section 6 three parametrisations of the differential cross-section are tested, and from those compatible with data the total cross-section is derived. The results are summarised in Section 7.

2. Experimental Apparatus

The TOTEM experiment is located at the LHC interaction point (IP) 5 together with the CMS experiment. In this article only the Roman Pot (RP) system, the sub-detector relevant for elastic scattering measurement, is outlined, whereas TOTEM’s full experimental apparatus is described elsewhere [14]. RPs are movable beam-pipe insertions that approach the LHC beam very closely in order to detect particles scattered to very small angles. They are organised in two stations placed symmetrically around the IP: one on the left side (in LHC sector 45), one on the right (sector 56). Each station is formed by two units: near (214 m from the IP) and far (220 m). Each unit includes three RPs: one approaching the beam from the top, one from the bottom and one horizontally. Each RP hosts a stack of 10 silicon strip sensors (pitch 66 \( \mu \text{m} \)) with strongly reduced insensitive region at the edge facing the beam (few tens of micrometres). The sensors are equipped with trigger-capable electronics. Since elastic scattering events consist of two anti-parallel protons, the detected events can have two topologies, called diagonals: 45 bottom – 56 top and 45 top – 56 bottom.

This report will use a reference frame where \( x \) denotes the horizontal axis (pointing out of the LHC ring), \( y \) the vertical axis (pointing against gravity) and \( z \) the beam axis (in the clockwise direction).

3. Data Taking

The measurement presented here is based on data taken in July 2012, during the LHC fill number 2836 providing protons colliding at the centre-of-mass energy \( \sqrt{s} = 8 \text{ TeV} \). The vertical RPs were inserted at a distance of 9.5 times the transverse beam size, \( \sigma_{\text{beam}} \). Initially two, later three colliding bunch-pairs were used, each with a typical population of \( 8 \cdot 10^{10} \) protons, yielding an instantaneous luminosity of about \( 10^{28} \text{ cm}^{-2} \text{s}^{-1} \) per bunch. The main trigger required a coincidence between the RPs in both arms, combining the near and far units of a station in OR to ensure maximal efficiency. During the about 11 h long data-taking, a luminosity of 735 \( \mu \text{b}^{-1} \) was accumulated, giving \( 7.2 \cdot 10^6 \) tagged elastic events.

4. Beam Optics

The beam optics relates the proton state at the IP to its state at the RP location. At the IP, the direction of a proton can be described by the scattering angle \( \theta^\ast \) (with respect to the \( z \) axis) and azimuthal angle \( \phi^\ast \) (about the \( z \) axis). Alternatively, the horizontal \( (x) \) and vertical \( (y) \) projections of the scattering angle can be used:

\[
\theta_x^\ast = \theta^\ast \cos \phi^\ast, \quad \theta_y^\ast = \theta^\ast \sin \phi^\ast.
\]
Table 1: Optical functions for elastic proton transport. The values refer to the right arm, for the left one they are very similar.

| unit | \( L_x \) | \( v_x \) | \( L_y \) | \( v_y \) |
|------|----------|----------|----------|----------|
| near | 2.45 m  | -2.17 m  | 239 m    | 0.040    |
| far  | -0.37 m | -1.87 m  | 264 m    | 0.021    |

A proton with such scattering angles, emerging from the vertex \((x^*, y^*)\) with relative momentum loss \(\xi \equiv \Delta p/p\) arrives at the RPs with transverse position

\[
x = L_x \theta_x^* + v_x x^* + D_x \xi, \quad y = L_y \theta_y^* + v_y y^* + D_y \xi, \tag{2}
\]
given by the optical functions: effective length \(L\), magnification \(v\) and dispersion \(D\). Since protons lose no momentum in elastic collisions (i.e. \(\xi = 0\)), the dispersion terms can be ignored for this process.

For the reported measurement, a special \(\beta^* = 90\) m optics was used, with essentially the same characteristics as at \(\sqrt{s} = 7\) TeV \[10\], see Table 1 for details. In the vertical plane, it features parallel-to-point focussing (\(v_y \approx 0\)) and large effective length \(L_y\). In the horizontal plane, the almost vanishing effective length \(L_x\) simplifies the separation of elastic and diffractive events: any sizeable horizontal displacement must be due to a momentum loss \(\xi\).

5. Analysis

The analysis method is similar to the ones used in the previous publications \[11, 12\]. However, a different normalisation approach is used (Section 5.2.6) that makes all \(t\)-independent scaling factors irrelevant.

The analysis is presented in two main blocks. Section 5.1 covers all aspects related to the reconstruction of a single event. Section 5.2 describes the steps of transforming a raw \(t\)-distribution into the differential cross-section. The \(t\)-distributions for the two diagonals are analysed separately. After comparison (Section 5.3) they are finally merged (Section 5.4).

5.1. Event Analysis

Event kinematics are determined from the coordinates of track hits in the RPs after proper alignment (see Section 5.1.2), using the LHC optics (see Section 5.1.3).

5.1.1. Kinematics Reconstruction

The scattering angles and vertex position are first determined for each proton (i.e. from each arm) separately by inverting the proton transport, Eq. (2), assuming \(\xi = 0\). The following formulae optimise the robustness against optics imperfections:

\[
\theta_{x, y}^{LR} = \frac{v_x^L x^F - v_x^F x^N}{v_x^L L_x^F - v_x^F L_x^N}, \quad \theta_{y}^{LR} = \frac{1}{2} \left( \frac{y_N^N}{L_y^N} + \frac{y_F^F}{L_y^F} \right), \quad x^{LR} = \frac{L_x^F x^N - L_x^N x^F}{v_x^L L_x^F - v_x^F L_x^N}, \tag{3}
\]
where the N and F superscripts refer to the near and far units, L and R to the left and right arm, respectively. This one-arm reconstruction is used for tagging elastic events, where the left and right arm protons are compared.

Once an event is selected, the information from both arms is merged yielding better angular resolution:

\[
\theta^* = \frac{\theta_x^L + \theta_x^R}{2}, \quad \phi^* = \frac{\theta_y^L + \theta_y^R}{2}.
\]  

Eventually, the full scattering angle and four-momentum transfer squared are calculated as

\[
\theta^* = \sqrt{\theta^2 + \phi^2}, \quad t = -p^2 \left( \theta^2 + \phi^2 \right),
\]

where \( p \) denotes the beam momentum.

5.1.2. Alignment

The standard three-step procedure [15] has been applied: beam-based alignment prior to the run (as for LHC collimators) followed by two off-line methods. First, track-based alignment for relative positions among RPs, and second, alignment with elastic events for absolute position with respect to the beam. The final uncertainties per unit (common for top and bottom RPs) are: 2\( \mu \)m (horizontal shift), 100\( \mu \)m (vertical shift) and 0.2 mrad (rotation about the beam axis). Propagated through Eqs. (3) and (4) to the scattering angles reconstructed from both arms, the shifts lead to uncertainties of 0.8\( \mu \)rad (horizontal) and 0.2\( \mu \)rad (vertical). The relatively large impact of horizontal misalignment is due to the almost vanishing effective length \( L_x \) (cf. Eq. (3)). RP rotations induce a bias in the reconstructed horizontal scattering angle:

\[
\theta_x^* \to \theta_x^* + c\theta_y^*,
\]

where the proportionality constant \( c \) has a mean of 0 and a standard deviation of 0.02.

5.1.3. Optics

In order to reduce the impact of imperfect optics knowledge, the LHC optics calibration [16] has been applied. This method uses various RP observables to determine fine corrections to the optical functions presented in Eq. (2).

The residual errors induce a bias in the reconstructed scattering angles:

\[
\theta_x^* \to (1 + d_x) \theta_x^*, \quad \theta_y^* \to (1 + d_y) \theta_y^*.
\]

For the two-arm reconstruction, Eq. (4), the biases \( d_x \) and \( d_y \) have uncertainties of 0.21\% and 0.25\%, respectively, and a correlation factor of \(-0.70\). These estimates include the effects of magnet field harmonics. For evaluating the impact on the \( t \)-distribution, it is convenient to decompose the correlated biases \( d_x \) and \( d_y \) into eigenvectors of the covariance matrix:

\[
\begin{pmatrix}
  d_x \\
  d_y
\end{pmatrix} = \eta_1 \begin{pmatrix}
  -0.182 \% \\
  +0.235 \%
\end{pmatrix} + \eta_2 \begin{pmatrix}
  -0.096 \% \\
  -0.074 \%
\end{pmatrix}
\]

normalised such that the factors \( \eta_{1,2} \) have unit variance.
\[ \frac{d\sigma}{dt}(\text{bin}) = \mathcal{N} \mathcal{U}(\text{bin}) \mathcal{B} \frac{\sum_{\text{bin}} \mathcal{A}(\theta^*, \theta^*_y) \mathcal{E}(\theta^*_y)}{\Delta t}, \]

5.1.4. Resolution

Statistical fluctuations in the reconstructed scattering angles are caused by the beam divergence and, in the horizontal projection (due to the small \( L_x \)), also by the sensor resolution. They are studied by comparing the scattering angles reconstructed from the two arms, in particular through differences \( \theta_{x,y}^R - \theta_{x,y}^L \) as illustrated in Figure 1. The distributions exhibit small deviations from a Gaussian shape which decrease with time.

Since in good approximation the fluctuations are independent in each arm, the angular resolution for the two-arm reconstruction, Eq. (4), is given by half of the standard deviation of the \( \theta_{x,y}^R - \theta_{x,y}^L \) distributions. As shown in Figure 2, the resolution deteriorates slightly with time, which can be expected mainly due to the emittance growth. The small difference in \( \theta^*_y \) resolution between the diagonals can be attributed to different RPs, each with slightly different spatial resolution, being involved in the two diagonals.

Measurements of beam emittances [17] show that the vertical beam divergences of the two beams are equal within a tolerance of about 15%. Exploiting this equality, one can deconvolute the distribution of \( \theta_y^R - \theta_y^L \) in order to obtain the beam-divergence distribution, used e.g. for acceptance corrections discussed in Section 5.2.3 (only required in the vertical plane).

5.2. Differential Cross-Section

For a given \( t \) bin, the differential cross-section is evaluated by selecting and counting elastic events:
where $\Delta t$ is the width of the bin, $N$ is a normalisation factor, and the other symbols stand for various correction factors: $U$ for unfolding of resolution effects, $B$ for background subtraction, $A$ for acceptance correction and $E$ for detection and reconstruction efficiency.

5.2.1. Event Tagging

The cuts used to select elastic events are summarised in Table 2. Cuts 1 and 2 require the reconstructed-track collinearity between the left and right arm. Cuts 3 and 4 control the elasticity – if a proton loses momentum, the vertical position-angle correlation at the RPs is lost. Cut 5 ensures that the two protons come from the same vertex (horizontally). The correlation plots corresponding to these cuts are shown in Figure 3.

Monte-Carlo simulation suggests that applying all the five cuts at $3 \sigma$ level would lead to a loss of about $2\%$ of elastic events. Setting the thresholds to $4 \sigma$ yields a tolerable loss of about $0.07\%$ and therefore the cuts are applied at the $4 \sigma$ level.

The tagging efficiency is studied experimentally by applying the cuts also at the $5 \sigma$ level. This selection yields about $0.5\%$ more events in every $t$ bin – thus the inefficiency is irrelevant for this analysis since the overall normalisation is determined from another dataset, see Section 5.2.6.

Table 2: The elastic selection cuts. The superscripts $R$ and $L$ refer to the right and left arm, $N$ and $F$ correspond to the near and far units, respectively. The constant $\alpha = L_y^R/L_y^N - 1 \approx 0.11$. The right-most column gives a typical RMS of the cut distribution.

| discriminator | cut quantity | RMS $(\equiv 1\sigma)$ |
|---------------|--------------|------------------------|
| 1             | $\theta_x^R - \theta_x^L$ | $9.5 \mu$rad          |
|               | $\theta_y^R - \theta_y^L$ | $3.3 \mu$rad          |
| 3             | $\alpha y_{yNL} - (y_{yFL} - y_{yNR})$ | $18 \mu$m           |
| 4             | $\alpha y_{yLN} - (y_{yFL} - y_{yNL})$ | $18 \mu$m           |
| 5             | $x^R - x^L$ | $8.5 \mu$m            |

Figure 2: Angular resolution for the two-arm reconstruction, Eq. (4), as a function of time (from the beginning of the run). The step in $\theta_x^*$ resolution around 7h is due to inclusion of another colliding bunch-pair with a larger vertical emittance.
Figure 3: Correlation plots for the event selection cuts summarised in Table 2, using all events with diagonal topology 45 top – 56 bottom. The black solid lines delimit the signal (±4 σ) region.
5.2.2. Background

Expectable background (i.e. non-elastic events passing the tagging cuts) may come from central diffraction as well as pile-up of single diffraction and/or beam-halo protons. The background rate is studied by plotting the discriminators from Table 2 under various cut combinations, see an example in Figure 4. While the central part (signal) remains essentially constant, the tails (background) are strongly suppressed with increasing number of cuts applied. This interpretation is further supported by the discriminator distributions from non-diagonal RP configurations, see the dotted curves in the figure. While these top – top or bottom – bottom configurations cannot contain any elastic signal, they are likely to have a similar share of events causing background to the presented analysis. And indeed, the figure shows a good agreement at the distribution tails. Integrating the non-diagonal curve over the signal region (see the dashed lines in the figure) yields a background estimate of $1 - B < 10^{-4}$.

5.2.3. Acceptance Correction

Two proton detection limitations have been identified: detector coverage (mostly at the edge facing the beam, i.e. relevant for small $|\theta_y|$) and LHC apertures ($|\theta_y| \approx 100 \mu$rad). The correction accounting for these limitations includes two contributions – a geometrical correction $A_{\text{geom}}$ reflecting the fraction of the phase space within the acceptance and a component $A_{\text{fluct}}$ correcting for fluctuations around the acceptance limits (cuts in $\theta_y$):

$$ A(\theta^*, \theta_y^c) = A_{\text{geom}}(\theta^*) A_{\text{fluct}}(\theta_y^c). $$

Figure 4: Distributions of discriminator 1, i.e. the difference between the horizontal scattering angle reconstructed from the right and the left arm. Solid curves: data from diagonal 45 top – 56 bottom, the different colours correspond to various combinations of the selection cuts (see numbering in Table 2). Dotted curves: data from non-diagonal RP configurations, obtained by inverting track coordinates in the left arm. The vertical dashed lines represent the boundaries of the signal region ($\pm 4 \sigma$).
The calculation of the geometrical correction $A_{\text{geom}}$ is based on the azimuthal symmetry of elastic scattering, experimentally verified for the data within acceptance. As shown in Figure 5, for a given value of $\theta^*$ the correction is given by:

$$A_{\text{geom}}(\theta^*) = \frac{\text{full arc length}}{\text{arc length within acceptance}}.$$  \hfill (11)

The correction $A_{\text{fluct}}$ is calculated analytically from the probability that any of the two elastic protons leaves the region of acceptance due to the beam divergence. The beam divergence distribution is modelled as a Gaussian with the spread determined by the method described in Section 5.1.4. This correction contribution is sizeable only close to the acceptance limits but is kept below 1.5 by discarding data with larger corrections. The uncertainties are related to the resolution parameters (vertical beam divergence, left-right asymmetry and non-Gaussian shape), and all stay below 0.1 %.

Figure 6 shows an example for the $t$-dependence of the acceptance correction for one diagonal. Since a single diagonal cannot cover more than half of the phase space, the minimum value of the correction is 2. As indicated in the figure, data points with too large correction ($A \gtrsim 5$) are discarded.

5.2.4. Inefficiency Corrections

Since the overall normalisation is determined from another dataset (see Section 5.2.6), any inefficiency correction that does not alter the $t$-distribution shape does not need to be considered in this analysis (trigger, data acquisition and pile-up inefficiency discussed in [11, 12]). The inefficiencies left are related to inability of a RP to resolve the elastic proton track.
One such case is when a single RP cannot detect and/or reconstruct a proton track, with no correlation to other RPs. This type of inefficiency, $I_{3/4}$, is evaluated by removing the RP from the tagging cuts (Table 2), repeating the event selection and calculating the fraction of recovered events. A typical example is given in Figure 7, showing that the efficiency decreases gently with the vertical scattering angle. This dependence stems from the fact that protons with larger $|\theta^*_y|$ hit the RPs further from their edge and therefore the potentially created secondary particles have more chance to induce additional signal in the sensors and thus prevent from resolving the elastic proton track.

Another source of inefficiency are proton interactions in a near RP affecting simultaneously the far RP downstream. The contribution from these near-far correlated inefficiencies, $I_{2/4}$, is determined by evaluating the rate of events with high track multiplicity ($\gtrsim 5$) in both near and far RPs. Events with high track multiplicity simultaneously in a near top and near bottom RP are not counted as such a shower is likely to have started upstream from the RP station and thus unrelated to the elastic proton interacting with detectors. The outcome, $I_{2/4} \approx 1.5\%$, is compatible between left/right arms and top/bottom RP pairs, in addition it compares well to Monte-Carlo simulations (e.g. section 7.5 in [18]).

The full correction is calculated as

$$E(\theta^*_y) = \frac{1}{1 - \left( \sum_{i \in \text{RPs}} I_{3/4}^i(\theta^*_y) + 2I_{2/4} \right)}.$$  \hspace{1cm} (12)

The first term in the parentheses sums the contributions from the four RPs of a diagonal and grows from about 7 to 10\% from the lowest to the highest $|\theta^*_y|$. The second term amounts to about 3\%. 

Figure 6: Full acceptance correction, $\mathcal{A}$, for diagonal 45 bottom – 56 top. The points give the mean value per bin, the error bars indicate the standard deviation. The sharp shape change at $|t| \approx 0.16\text{GeV}^2$ is caused by the LHC aperture cuts. The data left of the dashed vertical line are discarded due to excessively large acceptance correction.
5.2.5. Unfolding of Resolution Effects

The correction for resolution effects has been determined by the following iterative procedure. The differential cross-section data are fitted by a smooth curve which serves as an input to a Monte-Carlo simulation using the resolution parameters determined in Section 5.1.4. Making a ratio between simulated histograms with and without smearing effects gives a set of per-bin correction factors. Applying them to the yet uncorrected differential cross-section yields a better estimate of the true $t$-distribution which can be used as input to the next iteration. The iterations stop when the difference between the input and output $t$-distributions becomes negligible, which is typically achieved after the second iteration. Thanks to the good angular resolution (see Section 5.1.4), the final correction is not large, as shown in Figure 8.

For the uncertainty estimate, the uncertainties of $\theta^{x}_{c}$ and $\theta^{y}_{c}$ resolutions (accommodating the full time variation) as well as fit-model dependence have been considered, the first contribution being dominant.

5.2.6. Normalisation

The normalisation $N$ is determined by requiring the same cross-section integral between $|t| = 0.027$ and $0.083$ GeV$^2$ as for dataset 1 from [12], where the luminosity-independent calibration was applied. The leading uncertainty of the scaling factor 4.2% comes from the luminosity-independent method.

5.2.7. Binning

Two binnings have been considered. The “optimised” option sets the bin size to $1 \sigma$ of the resolution in $t$. The “per-mille” binning is built such that each bin collects about one per-mille of the events.
5.2.8. Systematic Uncertainties

Besides the systematic uncertainties mentioned at the above analysis steps, the uncertainty of the beam momentum, 0.1 %, needs to be considered when the scattering angles are translated into $t$, see Eq. (5).

The systematic effects are propagated to the $t$-distribution with help of a Monte-Carlo simulation. A fit of the final differential cross-section data is used to generate the true $t$-distribution. Simultaneously, another $t$-distribution is built, having introduced one of the above mentioned systematic effects at 1 $\sigma$ level. The difference between the $t$-distributions gives the systematic effect on the differential cross-section. Formally, this procedure is equivalent to evaluating

$$\delta s_q(t) \equiv \frac{\partial (d\sigma/dt)}{\partial q} \delta q,$$

where $\delta q$ corresponds to 1 $\sigma$ bias in the quantity $q$ responsible for a given systematic effect.

The Monte-Carlo simulations show that the combined effect of several systematic errors is well approximated by linear combination of the individual contributions from Eq. (13).

5.3. Systematic Cross-Checks

Compatible results have been obtained from data originating from different bunches, different diagonals and different time periods.

In addition, the complete analysis chain has been applied in two independent analysis implementations, yielding compatible results.

5.4. Final Data Merging

Finally, the differential cross-section histograms from both diagonals are merged. This is accomplished by a per-bin weighted average, with the weight given by inverse squared statistical
Figure 9: Impact of $t$-dependent systematic effects on the differential cross-section. Each curve corresponds to a systematic error at 1 $\sigma$, cf. Eq. (13). The two contributions due to optics correspond to the two vectors in Eq. (8). The envelope is determined by summing all shown contributions in quadrature for each $|t|$ value. The right plot provides a zoom of the left one.
uncertainty. The statistical and systematic uncertainties are propagated accordingly. For the sys-
tematic ones, the correlation between the diagonals is taken into account. For example, the vertical
(mis-)alignment of the RPs within one unit is almost fully correlated, thus the effect on the dif-
terential cross-section is opposite in the two diagonals, and consequently its impact is strongly
reduced once the diagonals are merged.

The final systematic uncertainties, except the 4.2% coming from the normalisation, are sum-
marised in Figure 9 where their impact on the differential cross-section is shown. The leading un-
certainties include normalisation, optics imperfections and beam momentum offset. Their effects
are quantified in Table 3, which can be used to approximate the covariance matrix of systematic
uncertainties:

\[ V_{ij} = \sum_q \delta s_q(i) \delta s_q(j), \] (14)

where \( i \) and \( j \) are bin indices (row numbers in Table 3) and the sum goes over the leading error
contributions \( q \) (four right-most columns in the table).

5.5. Statistical Uncertainty Adjustment

The statistical fluctuations in the differential cross-section using the “optimised” binning have
been slightly overestimated, whereas the “per-mille” binning does not suffer from this problem.
One way to demonstrate this is to split the data into groups of consecutive points small enough
for a linear function to approximate well the differential cross-section within each group. Then,
performing straight-line fits through each group yields on average \( \chi^2 \) values slightly too low. Al-
ternatively, the issue can be demonstrated as follows. The data sample is divided into several
sub-samples corresponding to the same luminosity, and the analysis method described in the ear-
lier sections is repeated for each of these sub-samples. Then, fluctuations of each bin content
are determined from the several sub-samples, giving values slightly lower than the uncertainty
estimates.

As a remedy, the statistical uncertainties in the “optimised” binning have been divided by a
factor of 1.176. This value has been determined by requiring both binnings to give the same value
of \( \chi^2/\text{ndf} \) for fits of \( d\sigma/dt \) to the fit function in Eq. (15) with \( N_b = 3 \) which has enough flexibility
to describe the data.

6. Results

The final differential cross-section in the “optimised” binning is presented in Table 3 and Figure
10. In order to visualise small deviations from the leading pure-exponential behaviour, Figure 11
shows the relative difference of the cross-section from a reference exponential. This plot imme-
diately suggests a non-exponentiality of the data: pure exponentials would look like (almost) linear
functions in this kind of representation.

To study the detailed behaviour of the differential cross-section, a series of fits has been made
using the parametrisation:

\[ \frac{d\sigma}{dt}(t) = \frac{d\sigma}{dt}\bigg|_{t=0} \exp \left( \sum_{i=1}^{N_b} b_i t^i \right), \] (15)
Table 3: The elastic differential cross-section as determined in this analysis using the “optimised” binning. The three left-most columns describe the bins in $t$. The representative point gives the $t$ value suitable for fitting [19]. The other columns are related to the differential cross-section. The four right-most columns give the leading systematic biases in $\mathrm{d}x/\mathrm{d}t$ for 1$\sigma$-shifts in the respective quantities, $\delta x_j$, see Eqs. (13) and (14). The two contributions due to optics correspond to the two vectors in Eq. (8).

| $|t|$ bin [GeV$^2$] | $\mathrm{d}x/\mathrm{d}t$ [mb/GeV$^2$] |
|------------------|--------------------------|
|                  | value | statistical uncertainty | systematic uncertainty | normalisation $N$ | optics mode 1 | optics mode 2 | optics beam momentum |
| 0.02697 0.03005 0.02850 | 300.94 | 0.520 | 12.68 | +12.66 | -0.473 | -0.259 | +0.254 |
| 0.03005 0.03325 0.03164 | 284.04 | 0.471 | 11.91 | +11.90 | -0.495 | -0.214 | +0.204 |
| 0.03325 0.03658 0.03491 | 265.59 | 0.430 | 11.17 | +11.15 | -0.484 | -0.172 | +0.157 |
| 0.03658 0.04005 0.03831 | 247.90 | 0.395 | 10.44 | +10.43 | -0.472 | -0.133 | +0.114 |
| 0.04005 0.04365 0.04184 | 231.95 | 0.366 | 9.740 | +9.727 | -0.459 | -0.0968 | +0.0740 |
| 0.04365 0.04740 0.04551 | 215.35 | 0.338 | 9.061 | +9.048 | -0.445 | -0.0638 | +0.0378 |
| 0.04740 0.05129 0.04933 | 199.89 | 0.314 | 8.405 | +8.393 | -0.431 | -0.0338 | +0.0051 |
| 0.05129 0.05534 0.05330 | 184.55 | 0.291 | 7.775 | +7.763 | -0.415 | -0.0069 | -0.0240 |
| 0.05534 0.05956 0.05743 | 170.71 | 0.271 | 7.171 | +7.159 | -0.399 | +0.0170 | -0.0498 |
| 0.05956 0.06394 0.06173 | 156.62 | 0.251 | 6.594 | +6.581 | -0.382 | +0.0380 | -0.0721 |
| 0.06394 0.06850 0.06620 | 142.96 | 0.233 | 6.044 | +6.031 | -0.365 | +0.0561 | -0.0913 |
| 0.06850 0.07324 0.07085 | 131.31 | 0.216 | 5.521 | +5.508 | -0.348 | +0.0715 | -0.107 |
| 0.07324 0.07817 0.07568 | 119.59 | 0.201 | 5.027 | +5.013 | -0.330 | +0.0842 | -0.120 |
| 0.07817 0.08329 0.08071 | 108.28 | 0.186 | 4.560 | +4.546 | -0.312 | +0.0944 | -0.130 |
| 0.08329 0.08862 0.08593 | 97.732 | 0.172 | 4.122 | +4.107 | -0.293 | +0.102 | -0.138 |
| 0.08862 0.09417 0.09137 | 87.916 | 0.158 | 3.711 | +3.696 | -0.275 | +0.108 | -0.143 |
| 0.09417 0.09994 0.09702 | 78.866 | 0.146 | 3.329 | +3.313 | -0.257 | +0.112 | -0.145 |
| 0.09994 0.10593 0.10290 | 70.641 | 0.135 | 2.973 | +2.957 | -0.239 | +0.113 | -0.146 |
| 0.10593 0.11217 0.10902 | 62.480 | 0.124 | 2.644 | +2.628 | -0.221 | +0.113 | -0.145 |
| 0.11217 0.11866 0.11538 | 55.454 | 0.113 | 2.341 | +2.325 | -0.204 | +0.112 | -0.142 |
| 0.11866 0.12540 0.12199 | 48.733 | 0.104 | 2.063 | +2.047 | -0.187 | +0.109 | -0.138 |
| 0.12540 0.13242 0.12887 | 42.712 | 0.0948 | 1.810 | +1.793 | -0.170 | +0.106 | -0.132 |
| 0.13242 0.13972 0.13602 | 37.277 | 0.0864 | 1.580 | +1.563 | -0.155 | +0.101 | -0.126 |
| 0.13972 0.14730 0.14346 | 32.207 | 0.0784 | 1.372 | +1.356 | -0.140 | +0.0961 | -0.118 |
| 0.14730 0.15520 0.15120 | 27.731 | 0.0710 | 1.185 | +1.169 | -0.125 | +0.0911 | -0.110 |
| 0.15520 0.16340 0.15925 | 23.827 | 0.0650 | 1.016 | +1.002 | -0.0942 | +0.0855 | -0.102 |
| 0.16340 0.17194 0.16761 | 20.364 | 0.0608 | 0.865 | +0.854 | -0.0582 | +0.0793 | -0.0938 |
| 0.17194 0.18082 0.17632 | 17.249 | 0.0566 | 0.733 | +0.723 | -0.0298 | +0.0729 | -0.0853 |
| 0.18082 0.19005 0.18537 | 14.480 | 0.0536 | 0.618 | +0.609 | -0.0080 | +0.0664 | -0.0769 |
| 0.19005 0.19965 0.19478 | 12.124 | 0.0498 | 0.517 | +0.508 | +0.0052 | +0.0598 | -0.0687 |
which includes the pure exponential ($N_b = 1$) and its straight-forward extensions ($N_b = 2, 3$).

The fits have been performed by the standard least-squares method, in particular minimising:

$$\chi^2 = \Delta^T V^{-1} \Delta, \quad \Delta = (\text{data} - \text{fit}), \quad V = V_{\text{stat}} + V_{\text{syst}},$$

where $\Delta$ is a vector of differences between the differential cross-section data and a fit function evaluated at the bin representative points (see Table 3). The covariance matrix $V$ is given by the sum of the statistical component $V_{\text{stat}}$ (statistical uncertainty squared from Table 3 on the diagonal) and the systematic component $V_{\text{syst}}$ (see Eq. (14)).

The quality of fits is judged on the basis of several measures. The first is the value of $\chi^2$ after minimisation divided by the number of degrees of freedom (ndf). Secondly, the p-value stands for the probability that a $\chi^2$ value greater than the observed one would be drawn from the $\chi^2$ distribution with the given number of degrees of freedom. Finally, significance means the half-width of a central region that needs to be excluded from a normal distribution to get the same integrated probability as the p-value. The significance is expressed in multiples of sigma, the standard deviation of the normal distribution.

Figure 11 shows several fits of the differential cross-section with the parametrisation in Eq. (15) and different numbers of parameters in the exponent, $N_b$. The corresponding fit quality is given in Table 4, indicating that the purely exponential fit ($N_b = 1$) is excluded at 7.2$\sigma$ significance. The other two fits ($N_b = 2, 3$) are of reasonable quality and can, therefore, be used for a total cross-section estimation with the optical theorem in the form

$$\sigma_{\text{tot}}^2 = \frac{16\pi (\hbar c)^2}{1 + \rho^2} \left. \frac{d\sigma_{el}}{dt} \right|_{t=0},$$

which neglects the effects due to the Coulomb interaction. Using the COMPETE [20] preferred-
Table 4: Fit quality measures for fits in Figure 11.

| $N_b$ | $\chi^2$/ndf | p-value | significance |
|-------|-------------|--------|--------------|
| 1     | 117.5/28 = 4.20 | 6.1 $\cdot 10^{-13}$ | 7.2 $\sigma$ |
| 2     | 29.3/27 = 1.09  | 0.35   | 0.94 $\sigma$ |
| 3     | 25.5/26 = 0.98  | 0.49   | 0.69 $\sigma$ |

Figure 11: Differential cross-section using the “optimised” binning and plotted as relative difference from a reference exponential (see vertical axis). The black dots represent data points with statistical uncertainty bars. The coloured lines correspond to fits with parametrisation Eq. (15) and different numbers of parameters in the exponent. The red line lies seemingly too high with respect to the data points, which is a consequence of the systematic degrees of freedom included in the fit: some of the effects in Figure 9 may flatten the distribution which at the same time changes the overall normalisation. The yellow band corresponds to the full systematic uncertainty, the brown-hatched one includes all systematic contributions except the normalisation. Both bands are centred around the fit curve with $N_b = 3$.

model extrapolation of $\rho = 0.140 \pm 0.007$ yields

\[
N_b = 2 : \quad \sigma_{\text{tot}} = (100.8 \pm 2.1) \text{ mb},
N_b = 3 : \quad \sigma_{\text{tot}} = (101.2 \pm 2.1) \text{ mb},
\]

which are well compatible with the previous measurement using the luminosity-independent method [12].

The incompatibility between a pure-exponential behaviour and the data with the “per-mille” binning can be shown equally well. However, since the number of points is drastically increased, the straight-forward $\chi^2$ test does not have sufficient sensitivity, and a different test is used. Assuming that the data can be described by a pure exponential, the fit parameters should have compatible values for fits over different ranges. Figure 12 shows a fit (minimisation of $\chi^2$ from Eq. (16)) with the parametrisation

\[
\frac{d\sigma}{dt}(t) = \begin{cases} 
    a_1 e^{b_1|t|} & |t| < 0.07 \text{ GeV}^2 \\
    a_2 e^{b_2|t|} & |t| > 0.07 \text{ GeV}^2
\end{cases}
\]

which is given by

\[
\frac{d\sigma}{dt}(t) = \begin{cases} 
    a_1 e^{b_1|t|} & |t| < 0.07 \text{ GeV}^2 \\
    a_2 e^{b_2|t|} & |t| > 0.07 \text{ GeV}^2
\end{cases}
\]
Figure 12: Differential cross-section using the “per-mille” binning and plotted as relative difference from the reference exponential (see vertical axis). The black dots represent data points with statistical uncertainty bars. The red line shows pure exponential fits in regions below and above $|t| = 0.07$ GeV$^2$, see Eq. (19). The yellow band corresponds to the full systematic uncertainty, the brown-hatched one includes all systematic contributions except the normalisation. Both bands are centred around the fit curve.

giving a reasonable fit quality (p-value of 0.57). The compatibility of the parameters in the two $|t|$ regions can be verified by evaluating

$$
\chi^2_p = \Delta_p^T V_p^{-1} \Delta_p , \quad \Delta_p = \begin{pmatrix} a_1 - a_2 \\ b_1 - b_2 \end{pmatrix},
$$

where $V_p$ is the covariance matrix for the difference vector $\Delta_p$. It yields $\chi^2_p = 65.4$ which with 2 degrees of freedom corresponds to a p-value of $6.2 \cdot 10^{-15}$ and a significance of 7.8$\sigma$. This, in turn, rules out the hypothesis of a purely exponential behaviour of the data over the entire observed range.

Since parameters estimated with the least squares method are unbiased, the test in Eq. (20) is asymptotically binning independent. Indeed, applying it to the data in the “optimised” binning (prior to the statistical uncertainty rescaling, Section 5.5) yields $\chi^2_p = 65.7$ which corresponds to a significance of 7.8$\sigma$. After the uncertainty rescaling, the exclusion significance increases to 9.0$\sigma$.

7. Conclusions and Outlook

Thanks to a very-high statistics data set TOTEM has excluded a purely exponential differential cross-section for elastic proton-proton scattering with significance greater than 7$\sigma$ in the $|t|$ range from 0.027 to 0.2 GeV$^2$ at $\sqrt{s} = 8$ TeV. The data are described satisfactorily with an exponent quadratic or cubic in $t$. Using this refined parametrisation for the extrapolation to the optical point, $t = 0$, yields total cross-section values compatible with the previous measurement, in all cases neglecting the effects due to the Coulomb interaction.
In an upcoming analysis, this proof of non-exponentiality in a $t$-domain strongly dominated by hadronic interactions will be combined with a measurement of elastic scattering in the Coulomb-nuclear interference region, aiming at constraints on the phenomenological description of the hadronic phase and of the interference mechanism.

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