Quantum Mechanics and Black Holes in Four-Dimensional String Theory

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Abstract

In previous papers we have shown how strings in a two-dimensional target space reconcile quantum mechanics with general relativity, thanks to an infinite set of conserved quantum numbers, “W-hair”, associated with topological soliton-like states. In this paper we extend these arguments to four dimensions, by considering explicitly the case of string black holes with radial symmetry. The key infinite-dimensional W-symmetry is associated with the $SU(1,1)/U(1)$ coset structure of the dilaton-graviton sector that is a model-independent feature of spherically symmetric four-dimensional strings. Arguments are also given that the enormous number of string discrete (topological) states account for the maintenance of quantum coherence during the (non-thermal) stringy evaporation process, as well as quenching the large Hawking-Bekenstein entropy associated with the black hole. Defining the latter as the measure of the loss of information for an observer at infinity, who ignoring the higher string quantum numbers - keeps track only of the classical mass, angular momentum and charge of the black hole, one recovers the familiar a quadratic dependence on the black-hole mass by simple counting arguments on the asymptotic density of string states in a linear-dilaton background.
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1 Introduction and Summary

String theory offers the possibility of resolving once and for all many of the deepest problems in quantum gravity, such as finiteness, the reconciliation of quantum mechanics with general relativity, the vanishing of the cosmological constant, the origin and flatness of the Universe, and even the satisfactory definition of the gravitational path integral. Varying amounts of progress have been made in presenting the actual solutions to these fundamental problems. For example, it has been shown that all n-loop string amplitudes calculated in a fixed flat background are finite [1], but the enumeration of classical non-perturbative backgrounds and understanding of the string path integral are far from complete. Some of these fundamental problems have been addressed from the point of view of the effective field theory of light string states, such as the existence of hair that might help us understand whether string black holes respect quantum coherence [2], and the discovery of “no-scale” models [3] that ensure the vanishing of the cosmological constant in some approximation. However, the resolutions of all these problems presumably require non-perturbative string theory techniques.

Such techniques have recently been successfully developed and applied to two-dimensional string quantum gravity, leading first to the non-perturbative solution of matrix models [4] and more recently to the construction of two-dimensional string black holes [5, 6, 7]. Earlier constructions exact cosmological solutions of subcritical string theory as conformal Wess-Zumino models had also been known [8]. One could hope that these non-perturbative techniques have advanced sufficiently for the resolutions of some of the above-mentioned fundamental problems to be within reach. Indeed, we have argued in a recent series of papers [9, 10, 11] that string theory reconciles quantum mechanics and two-dimensional gravity. We have identified an infinite set of exactly-conserved gauge quantum numbers, “W-hair”, which are associated with the topological solitons [1] that form the final stages of two-dimensional black hole evaporation [3, 4]. As a result of this W-symmetry, the two-dimensional phase space volume of the matrix model is conserved under time evolution [14, 10], excluding the introduction of modifications of the conventional S-matrix or Hamiltonian evolution of the density matrix [15] in two dimensions. Furthermore, we have demonstrated explicitly that the evaporation of a two-dimensional black hole is a purely quantum-mechanical higher-genus effect that does not introduce mixed states [11].

In this paper we extend these arguments to argue that four-dimensional string black holes do not lead to mixed states, and hence that string reconciles quantum mechanics with general relativity also in four dimensions. The central problem of quantum coherence was seen originally for spherically-symmetric four-dimensional black holes [12]. However their physical significance, especially for black hole physics, seems not to have been recognized by these authors.

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1 These discrete states were discovered in the context of matrix models by Gross and Klebanov [12], and in the continuum Liouville theory by Polyakov [13]. However their physical significance, especially for black hole physics, seems not to have been recognized by these authors.
black holes [16, 17], and the study of rotating four-dimensional black holes has not altered the dilemma, so we emphasize here spherically-symmetric four-dimensional black holes. These are described by dimensional reduction of four-dimensional string theories. The dimensionally-reduced theory is expressible as a $SU(1,1)/U(1)$ coset conformal field theory of the two dimensional string black hole, and thus is an exact solution. The radially-dependent dilaton field enters just as in the two-dimensional model, whose massless “tachyon” represents model-dependent matter fields in four dimensions. There is infinite-dimensional $W$-hair associated with this $SU(1,1)/U(1)$ coset model that conserves the 2-dimensional radial $s$-wave phase space volume element, and thereby prevents the appearance of non-quantum-mechanical terms in the $S$-matrix description of $s$-wave scattering. Just as in two dimensions, the quantum-mechanical evaporation of the four-dimensional black hole is a higher-genus effect that does not involve mixed states. Nevertheless, by simple counting arguments on the multiplicity of string states, one recovers the Hawking-Bekenstein formula [17, 19] for the apparent entropy of a black hole ($S \propto M^2$) if one restricts oneself to the classical conserved charges of a black hole (mass, angular momentum and electric charge) and disregards the infinite string set of conserved quantities.

2 Non-local stringy gauge symmetries

The central obstruction to reconciling quantum mechanics with general relativity, and thereby avoiding the evolution of pure states into mixed states, is the apparent loss of information across the horizon surrounding, e.g., a conventional black hole. This can be expressed mathematically via the formula [17, 19]

$$S = \frac{1}{4} k_B A$$

for the entropy $S$ associated with a horizon of area $A$ ($k_B$ denotes Boltzmann’s constant). Using the usual relation between the mass and horizon area of a spherically-symmetric black hole, we find

$$S = \frac{1}{\hbar} k_B M^2$$

The entropy (1,2) is just one aspect of the black hole thermodynamics induced by quantum effects in local field theories. There is also an apparent temperature

$$T = \frac{\hbar}{8\pi M}$$

The extension of our arguments to rotating black holes is technically more complicated, but presumably does not raise any fundamental issues of principle, since string has infinitely many gauge symmetries. See the discussion of the Kerr solution in section 4.

The one-dimensional axion “hair” found in the effective field theory approach [18] is just a single strand of this infinite-dimensional $W$-hair, and cannot by itself reconcile black hole dynamics with quantum mechanics.
associated with a spherically-symmetric black hole. There is an alternative statistical
definition of the entropy of a black hole:

\[ S = -k_B \ln N_H \]  

(4)

where \( N_H \) denotes the total number of quantum-mechanical distinct ways that a
black hole, of given mass, angular momentum, and charge could have been made. The number \( N_H \) can be viewed as counting the number of possible independent
microscopic states of the black hole atmosphere. The problem is that in any local
field theory there is only a finite number of conserved gauge quantum numbers, so
the entropy (1) or (2) can only be accommodated by a mixed state. Also it is clear
that a thermal state is necessarily mixed. Recently attacks have been made on this
problem using effective field theories derived from the string, which contain a finite
number of additional conserved quantum numbers, associated, e.g., with the axion
[18] or with discrete gauge symmetries [2]. However, these still do not touch the
core of the problem presented by the entropy (1) or (2), which seems to require an
infinite number of exactly conserved quantities, if the black hole state is not to be
mixed.

However, a stringy black hole has an infinite set of hair associated with the infinity
of gauge symmetries that characterise any string theory [20, 21]. In this case the large
entropy (1) defined by Hawking is avoided by the following argument [3]. Classically,
mass, angular momentum and charge are the only type of observable hair that a
black hole can have, and hence the necessity of a mixed state to account for the large
entropy. In string theories the entropy is zero, since quantum mechanics is valid and
pure states never mix, due to the arguments in [9], and the evaporation scenario
of [14] is in vacuo and does not involve mixed states. It is the information carried
by the infinite string hair that makes the difference from the previous calculation
of the entropy (1,2). To get the latter, one considers only the classical charges of
the black hole and treats the (infinity of) string gauge charges (associated with the
the rest of the excited string states—which in the effective two-dimensional case are
topological) as unobservable, using them just to count the number of quantum-
mechanically distinct ways that a black hole of given mass, electric charge, and
angular momentum is made. In this way, eq. (1) accounts for the large Hawking-
Bekenstein entropy, as we shall show explicitly in section 3.

It is instructive to review briefly at this point target-space gauge symmetries in
critical strings. The first approach to such symmetries was that of refs. [20, 21],
who showed that there exists in string theory an infinite set of generalized Ward
identities inter-relating states of different spin and mass. The lowest such identity

\[ \text{Notice that a local field theory has necessarily a finite number of conserved charges, hence}
\text{the thermal evaporation scenario for the black holes seems the only consistent one, with all the}
\text{inevitable consequences on the loss of quantum-mechanical coherence.} \]
is that expressing general coordinate invariance:

\[ q^\mu < V^G_{\mu\nu}(q) \Pi_{i=1}^{N} V^T(k_i) > = \sum_{i=1}^{N} k_{i\nu} < V^T(k_i + q) \Pi_{j \neq i} V^T(k_j) > \] (5)

and another involves states of rank four, three, and two [21]

\[ k^\mu A_{(\mu|\nu|\rho|\sigma)} - iB_{\nu\rho\sigma} = \sum_{i=1}^{N} k_{i\nu} G_{\rho\sigma}(k_i + k; \{k_j; j \neq i\}) + \text{perms}(\nu, \rho, \sigma) \] (6)

where \(\mu|\nu|\rho \equiv \mu\nu\rho + \rho\nu\mu\). In on-shell cases the sums on the rhs of the above equations (5,6) vanish on the basis of the cancelled propagator argument [22]. For our purposes we shall only deal with on-shell modes, in which case one avoids the usual ambiguities of extending these identities off string-shell [21]. Ref. [23] gave a conformal field theory analysis of such gauge symmetries, showing that there was one associated with every (1,0) or (0,1) operator, of which string theories have an infinite number. Contained within this infinite set of gauge symmetries is the particular \(W_{\infty+1}\) symmetry located in studies of two-dimensional string gravity, whose associated “ground ring” algebraic structure of (1,0) and (0,1) operators has been discussed in ref. [14].

A simple counting argument indicates that the number of such (1,0) and (0,1) operators in a four-dimensional string theory is comparable to the entropy (2) of a massive black hole, and hence might be adequate to accommodate this entropy without the necessity of a mixed state. This is based on the fact that the number of gauge symmetries is at least in correspondence with the number of string levels. For example, a subclass of (1,0) operators corresponding to string level \(2N\) assumes the generic form [21]

\[ \int d\sigma \Psi(\partial X)^N (\partial X)^{N-1} \] (7)

where \(\sigma\) is a space-like world sheet parameter, and \(\Psi\) is a \((2N-1)\)-index tensor space-time field that is symmetric on the first \(N\) and last \(N-1\) indices, as well as divergence-free on each index. Clearly this is an infinite set of operators in any string theory. The actual symmetries are bigger [23]. On the basis of general arguments, one could expect that each gauge stringy symmetry would lead to a conserved charge, which could participate in characterising the black hole.

In two dimensions, the existence of an infinite set of conserved quantum numbers was first demonstrated in matrix models [12]. We subsequently pointed out that they should also appear as gauge symmetries of two-dimensional black holes [2], associated with the massive topological discrete discrete states that are known to exist in continuum Liouville models of two-dimensional gravity [13]. Indeed, the gauge nature of these symmetries was subsequently demonstrated in ref. [24]. We argued [7] that this infinite set of conserved charges, “W-hair”, should be sufficient to
maintain quantum coherence for two-dimensional black holes, consistently with the
known existence of an $S$-matrix for two-dimensional matrix models. This quantum-
mechanical behaviour was subsequently given an elegant geometrical interpretation
in terms of an infinite phase-space area-preserving symmetry [10], which originates
from the ground ring of $(1, 0)$ or $(0, 1)$ world-sheet operators mentioned earlier [14].
In confirmation of this point, it was shown subsequently [11] that the quantum
evaporation of a two-dimensional black hole was related to the imaginary part of a
formally divergent higher-genus string amplitude, associated with an integral over
large tori. This evaporation did not have a finite-temperature interpretation, and did
not lead to a mixed state. For the purposes of the later discussion we note that this
evaporation mechanism did not seem specific to two dimensions, and appeared to
be generalizable to four-dimensional black holes.

3 Spherically-Symmetric Four-dimensional Black
Holes

We note first that the original arguments given by Hawking referred to spherically-
symmetric black holes originated by the spherically-symmetric collapse of macro-
scopic matter [17]. Spherically-symmetric solutions to gravity theories in arbitrary
dimensions have been classified in a wide class of theories [25]. In particular, the
so-called second-order formalism has been adopted for a description of gravity the-
ories in arbitrary number of dimensions, involving in general higher powers of the
curvature tensor. The result is that, with the exception of some unphysical cases,
all spherically-symmetric solutions are static [25] and some of them are known to
exhibit singularities hidden by event horizons, and therefore are of black hole type.
Since all such spherically-symmetric singularities can be regarded as in some sense
two-dimensional, the angular variables being inessential, we analyze them using re-
sults from string theory in two-dimensional space-time, which we now review briefly.

Witten [6] showed that it is possible to describe the region of two-dimensional
target space-time around the singularity by an exact conformal field theory, which
is a coset $SU(1,1)/U(1)$ Wess-Zumino $\sigma$-model formulated on an arbitrary Riemann sur-
face $\Sigma$. In [11] we have argued that summation over Riemann surfaces of arbitrary
topology, as required by a consistent string formalism, produces modular infinities
which have to be regularised by analytic continuation, thereby leading to imaginary
mass shifts of the black hole solution and hence instabilities. The latter will cause
the black hole to evaporate, but such an evaporation, although quantum in origin,
is different from the thermal scenario argued by Hawking [17] in conventional local
field theories of gravity. The evaporation is necessitated by the fact that the string
black hole solutions carry an infinite number of conserved quantum numbers, aris-
ing from stringy gauge symmetries [20, 23, 1] mixing the various mass levels. In the
case of two-dimensional black holes these charges are known [24, 10] to form a $W_{\infty+1}$
extended conformal algebra. This symmetry is a subgroup of an area-preserving infinite dimensional algebra generated by world-sheet currents of conformal spin (1, 0) or (0, 1) \[14\]. Due to this fact, we have shown in \[10\] that the symmetry is elevated into a target space one leading to the infinity of conserved charges mentioned before. The reason is that such world-sheet symmetries constitute a canonical deformation of the conformal field theory (stringy σ-model) describing the world-sheet dynamics \[23\]. The latter are represented as induced transformations of the (target space) background fields of the σ-model. The important feature, relevant for issues of quantum coherence, is that this symmetry preserves the phase-space area of the matrix model \[14\], which describes the interaction of \(c = 1\) matter with the black hole. In \[10\] we pointed out that it is precisely this property of the infinite-dimensional string symmetry that ensures preservation of quantum coherence during the black hole evaporation process. This was the feature that was believed to be violated according to the Hawking arguments \[16\] on the non-factorisability of the conventional scattering matrix due to the presence of space-time singularities \[5\].

These arguments were originally formulated in two-dimensional string cases, and one can naively think that they have nothing to do with the real four-dimensional case. However we shall now argue that this is not the case, since spherically-symmetric solutions of four-dimensional gravity theories are effectively two-dimensional theories. We conjecture that, in order to describe the spherically-symmetric singularities one can use the formalism of two-dimensional strings. This conjecture seems also to be in agreement with Witten’s point of view \[27\]. However, as we argued already in \[8, 10\] and we shall repeat below, it seems to us that full consistency of general relativity with quantum mechanics is achieved only upon inclusion of the entire spectrum of topological string states, which in two dimensions constitute the remnants of excited string states in higher-dimensional target spaces.

To be systematic, we start from the observation that in a \(D\)-dimensional target-space string theory there is an infinity of discrete topological states, which are similar in nature to those of the two-dimensional case \[13\]. Indeed these states can be seen in the gauge conditions for a rank \(n\) tensor multiplet,

\[
D_{\mu_1} A_{\mu_2...\mu_n} = 0
\]  

(8)

where \(D_\mu\) is a (curved space) covariant derivative. To illustrate our arguments, consider the simplified case of weak gravitational perturbations around flat space,

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5 According to Hawking \[10\] these constituted an obstruction to the analytic continuation from Euclidean to Minkowskian space, causing the non-factorisability property. Such modifications would invalidate the CPT-theorem of quantum mechanics in its ordinary sense, as the later is incompatible with a non-factorisable \(\$\)-matrix. However if one abandons the concept of a superscattering operator, while keeping the density matrix formalism as fundamental, then CPT-invariance can be preserved \[26\], perhaps at the cost of not having definite mixed states, a situation even less deterministic than that of Hawking \[17\]. In our case, the factorisability of the Hawking matrix \$ is guaranteed due to symmetries of the theory and hence CPT-invariance holds in the strong (ordinary) sense.
with a linear dilaton field of the form $\Phi(X) = Q_\mu X^\mu$. One finds the following Fourier transform of (8),

$$(p + Q)^{\mu_1} \tilde{A}(k)^{\mu_1,\mu_2,...,\mu_n} = 0$$

We then observe that there is a jump in the number of degrees of freedom at discrete momenta $p = -Q$. Due to the complete uncertainty in space, such states are delocalised, and can be considered as quasi-topological and non-propagating soliton-like states. In ordinary string theories, such states presumably carry a small statistical weight, due to the continuous spectrum of the various string modes. However, in strings propagating in spherically-symmetric four-dimensional background space-times, these discrete states become relevant. Such backgrounds are effectively two-dimensional, and therefore all the transverse modes of higher rank tensors can be gauged away using Ward identities of the form (8), except for the topological modes. In a four-dimensional spherically-symmetric background formalism, these are s-wave topological modes. For spherically-symmetric black holes, these modes constitute the final stage of the evaporation [9, 5], and they are responsible for the maintenance of quantum coherence [9, 10]. For clarity we shall recapitulate the arguments of [9, 10, 11], emphasizing that now one is really dealing with four-dimensional space-time spherically-symmetric singularities.

The analysis of [25] implies that in pure gravity all the classical spherically-symmetric solutions to the equations of motion obtained from higher-derivative gravitational actions with an arbitrary number of curvature tensors are static. A similar result occurs in the case of string-theoretic black holes at tree string-level. However, the arguments of [11] imply instabilities in spherically-symmetric black hole solutions, since these are massive string states and as such should be able to decay to lighter states [28, 29]. This mechanism also exists for superstring theories. Consider then a superstring theory, and a spherically-symmetric gravitational background of black hole type. The metric tensor will be given by an Ansatz of the form:

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta + e^{W(r,t)}d\Omega^2$$

where $W(r,t)$ is a non-singular function and $x^\alpha, x^\beta$ denote $r, t$ coordinates. Also, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ denotes the line element on a spherical surface that does not change with time. It can be shown that the standard Schwarzschild solution of the spherically-symmetric four-dimensional black hole [30] can be put in the above form by an appropriate transformation of variables. Consider the Schwarzschild solution in Kruskal-Szekeres coordinates [30]

$$ds^2 = -\frac{32M^3}{r}e^{-\frac{r}{2M}}dudv + r^2d\Omega^2$$

Here $r$ is a function of $u, v$, since it is given by

$$\left(\frac{r}{2M} - 1\right)e^{\frac{r}{2M}} = -uv$$
Notice that despite the static character of the black hole solution, upon changing variables, the two-dimensional metric components depend on both variables \( u, v \).

Changing variables to

\[
e^{-\frac{r}{2M}} u = u' \\
e^{-\frac{r}{2M}} v = v'
\]

and taking into account the Jacobian \( J \) of the transformation in the (positive-definite) area element \( du dv \), we can put the two-dimensional metric in the form

\[
g_{bh}(u', v') = \frac{e^{D(u', v')}}{1 - u'v'}
\]

where the scale factor is given by \( 16M^2 e^{-\frac{r(u', v')}{2M}} J(u', v') \), with \( r' \) the function \( r \) expressed in \( u', v' \) coordinates. This form of the metric is just a conformally-rescaled form of Witten’s two-dimensional black hole solution \(^{6}\). Since the latter is described by an exact conformal field theory, so is the conformally-rescaled metric, which from a \( \sigma \)-model point of view simply expresses a sort of renormalisation scheme change. From now on we shall work directly with the conformally-rescaled metric. The global properties (singularities) remain unchanged from the two-dimensional string case. In particular, according to the interpretation of Witten’s work \(^{6}\) by Eguchi \(^{31}\), a (conformal) Wess-Zumino coset model is suitable for the description of the region of target space-time around the singularity, where the conventional \( \sigma \)-model formalism breaks down.

To understand this point better, let us consider the gravitational sector of a four-dimensional supersymmetric string effective action It has the generic form

\[
S_{\text{eff}} = \int d^4 x \sqrt{G} e^\Phi \left\{ \frac{1}{\kappa^2} R^{(4)} - \frac{1}{2} (\nabla \mu \phi)^2 - e^{-2\sqrt{2}\kappa \phi} H_{\mu \nu \rho}^2 + \ldots \right\}
\]

where \( \phi \) is a four-dimensional dilaton field, and \( H_{\mu \nu \rho} = \partial_{[\mu} B_{\nu \rho]} + \omega_L - \omega_Y \) is the field strength of an antisymmetric tensor field, which by a duality transformation, upon using the equations of motion, is equivalent to a pseudoscalar \( \lambda \). The dots denote higher-derivative terms as well as gauge or other matter fields coming from compactification, in the case that one starts from a string theory in the critical dimension. Their presence does not affect our discussion. Upon dimensionally reducing (15) to discuss the spherically-symmetric gravitational background, one observes that another dilaton \( (W(r, t)) \) is going to be generated by the angular part of the Ansatz (10), as well as a two-dimensional cosmological constant term, even if the four-dimensional theory has zero cosmological constant. The reasons are simple. Since the metric is spherically-symmetric, there will be the radial part \( g_{00} \) in (14) (depending on time in general), which yields a two-dimensional scalar.

\(^{6}\)The function \( D(u, v) \) can be regarded also as a part of the two-dimensional dilaton in the given renormalisation scheme.
curvature term $R^{(2)}$. From the four-dimensional determinant one obtains exponential $W$-dilaton factors accompanying the two-dimensional metric determinant $\sqrt{g}$, and from the derivatives with respect to $r$ and $t$ of the angular part one gets two-dimensional $W$-dilaton kinetic terms. The angular part of the metric (10), with constant two-dimensional curvature, yields a cosmological constant part $\mathcal{V}$. Thus the effective description of the theory is given by the following two-dimensional effective action (to lowest order in derivatives)

$$4\pi \int d^2 x e^W \sqrt{g} (R^{(2)} - (\nabla W)^2 - 2 + \nabla^2 + ...)$$

(16)

We are interested in the extra charges that the black-hole can have in a string effective model. In the case of spherical geometry, this implies a spherically-symmetric Ansatz for the matter fields in (15). This leads to scalar s-mode structures for the antisymmetric tensor (axion) and higher-dimensional dilaton or other matter fields’ s-wave modes which are collectively represented as a two-dimensional string “tachyonic” mode, $T^8$.

Having expressed the theory as a two-dimensional effective string model, one can apply the whole machinery of two-dimensional strings to study the dynamics of the evaporation of the black holes and determine the final stage. It should be stressed that, from the general analysis mentioned in the beginning [25], the static character of the physically-interesting solutions to string-inspired gravitational theories implies probably that the only way that these black holes evaporate is the one suggested in [11], i.e. through string quantum corrections, requiring a formulation in higher world-sheet genera and summation over them. Independent arguments to support this claim will be given below. At present, we note that such decay is non-thermal, and hence maintains quantum coherence, as guaranteed by the $W_\infty$-symmetry associated with the discrete topological s-wave modes. The s-wave matter phase-space, which would be the problematic one from the point of view of quantum coherence, due to modes going into and not coming out or vice versa, is two-dimensional in the spherically-symmetric case and hence the arguments of ref. [14] apply. The symmetry of the effective two-dimensional target space is phase-space volume (area in two dimensions) preserving, and hence Liouville’s theorem for the time evolution of the density matrix remains valid. Thus, there is no modification of the evolution equation of the density matrix in the presence of a spherically symmetric black hole [15], and the factorisation of Hawking’s superscattering operator holds.

7 The $d\Omega^2$ represents the metric of a 2-sphere of unit radius, with scalar curvature 2. Integration over the angular variables yields an extra factor of $4\pi$.

8 We should stress that in our formalism the “dilaton” of the two-dimensional string model occurs necessarily in the spherical Ansatz for four-dimensional gravity, and therefore one does not have to start from a higher-dimensional string model and compactify. All such compactification modes that occur in traditional superstring-inspired models [32] appear in our two-dimensional effective model as matter “tachyon” $T$-fields.
A further comment concerning the thermodynamical relations (1), (2) of the black hole solutions is in order. We noted in [11] that the quantum instabilities associated with modular infinities, which cause the evaporation of two-dimensional black holes (and hence of spherically-symmetric four-dimensional configurations as well), are non-thermal in origin. Arguments have been given [11] for the thermal stability of these objects on the basis of compactified $c = 1$ matrix models, believed to represent a stringy regularisation of two-dimensional strings (and, in view of the picture in this article, of four-dimensional strings propagating in spherically-symmetric backgrounds). Here we would like to give an independent argument in support of the absence of thermal evaporation, at least in the conventional sense, by showing that the available string states account for the quadratic mass dependence of the black hole entropy (2).

The argument is based on the fact that the black hole is a particular string state of mass $M$. The Hawking entropy is viewed as the number of ways $N(M)$ one can construct a state of this mass (ignoring the associated string $W$ quantum numbers, which, in view of our previous arguments, would make the exact string entropy vanish). In this picture $N(M)$ may be considered the same as the multiplicity of string states of mass level $M$. This entropy is measured by an observer at spatial infinity, where the string propagates in a flat background with a linear dilaton field, $Q_{\mu}X^{\mu}$. In the two-dimensional black hole case the black hole mass is determined by a constant shift $2a$ in the dilaton which is non-trivial. The precise relation is [6, 5]

$$\sqrt{\alpha'\pi} = Q e^{2a}$$

(17)

A choice of $a$ selects a particular black-hole configuration (i.e. a vacuum for the string). If we rescale the Regge slope by $e^{-2a}$, then in units of the rescaled slope the black hole mass is given by $Q$. The situation is then analogous to that of ref. [8], where for large mass levels the multiplicities are given asymptotically by

$$N(M) = \lim_{a,M \to \infty} e^{2\pi\sqrt{\alpha'}\sqrt{1+Q^2 e^{2a}}} = e^{2\pi\alpha' M^2}$$

(18)

where $\alpha'$ is the redefined Regge slope, depending on the particular black hole background. There is no loss of generality if we express the black hole mass in units of this. The entropy is determined by taking the logarithm $-k_B ln N(M)$, thereby leading to quadratic mass dependence of the form (2), as argued by Hawking and Bekenstein [17, 19] using classical thermodynamics and information theory. It should be noticed that as the mass decreases there will be a point where formula (18) will be no longer applicable, and thus the situation is analogous to that of Hawking [17] where quantum effects become important for small black hole masses and classical thermodynamics arguments cannot be used.

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9We should stress that the same arguments of [8] apply here, despite the Wick-rotated $Q$ relative to their case.
So far we have dealt with static black holes. Let us now consider the above thermodynamical relations in connection with the evaporation mechanism described in [11]. The latter is based on the fact that any massive string state decays to lighter states in such a way that the stringy gauge symmetries associated with the relevant mass levels remain intact. The formal origin of such an instability appears as a modular divergence of the two-point function of the state in question. Regularisation of the infinity by means of, say, analytic continuation yields an imaginary part \( I \) [28, 29]. The latter, in view of the validity of the \( S \)-matrix formalism due to the \( W \)-hair property of the string black holes, implies due to the optical theorem a decay with a decay rate \( \Gamma = -2I \). The dimensionality of space-time plays a crucial rôle in determining the life-time. To see this, let us consider a simple field theory example, which however captures all the essential features of the string case. Consider [29] two fields \( \Phi \) and \( \phi \) of masses \( M \) and \( m \) respectively, with \( M > 2m \), and a “string inspired” \( \frac{1}{2} \lambda \Phi \phi^2 \) interaction. The imaginary part of the one-loop two-point function of the state \( M \) can be computed in terms of \( M, m \) [29]

\[
\Gamma \equiv \frac{1}{M} \frac{dM}{dt} = \frac{\lambda^2 \pi M^{D-5}}{(16\pi)^{\frac{D}{2}} \Gamma \left( \frac{D-1}{2} \right)} \left( 1 - \frac{4m^2}{M^2} \right)^{\frac{D-3}{2}} \tag{19}
\]

The above computation necessarily goes off-mass shell for the propagating light particle in the loop. In the two-dimensional case one observes that the decay rate \( \frac{dM}{dt} \) is proportional to \( M^{-2} \). This property, when transcribed into the case of black hole decay \textit{in vacuo} yields the entropy relation (4), with coefficient dependent on the details of the process [4]. In string theory one has an infinity of states propagating in the loop. However, in the two-dimensional \textit{effective string theory} the only propagating states are those of the massless “tachyons” [13]. The rest of the string states are topological. As a crude estimate therefore of the corresponding decay rate we take (19) with \( m = 0, d = 2 \) and \( M \) the black hole mass. Although detailed computations must be done to estimate the magnitude of the proportionality coefficients, we believe the heuristic argument we gave above is sufficient to demonstrate the essential physics of black hole evaporation. Although the evaporation is not thermal, one can give a thermodynamic interpretation à la Hawking [17], if one restricts oneself to the entropy defined in the classical black hole case (see the discussion in section 2 and in the previous paragraph). Then, the thermodynamic relation (2) follows from the simple fact that the decay \textit{in vacuo} occurs with a rate proportional to \( M^{-2} \). Upon time-averaging during the decay from an initial black hole mass \( M \) down to mass zero, we observe that the average Hawking entropy still obeys a quadratic-mass law. This is a feature only of the inverse squared-mass behaviour of the decay rate. As a side remark, we would like to point out that in local field theories the relation (2) can be also explained in a thermal way by considering the evaporation of the black hole \textit{in equilibrium} with a heat bath of temperature approaching the Hawking one from below [33]. In such a scenario, the crucial point for getting a large statistical

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10 The proportionality coefficient depends on detailed string computations, to which we hope to return in the near future.
entropy is the zero-point energy of *summed up* excited states which disappear if we allow the black hole to evaporate down to a final mass zero. This is to be compared with the situation in the present case, where however the evaporation/decay takes place *in vacuo*. The topological states, which are responsible for the maintenance of quantum coherence during the evaporation process, also “disappear” as external states in the limiting zero-mass case represented by matrix model. However, their presence is essential in yielding the enormous statistical entropy for the stringy black hole. In the formal sense, their presence guarantees the correct value of the proportionality coefficient in (2) to match in order of magnitude that of Hawking’s original computation, based on classical black body radiation [17].

### 4 Non-Spherically-Symmetric Black Holes

What about spherically non-symmetric singularities? Such objects are known as solutions of Einstein’s equations, the Kerr rotating black holes for example [34]. In the context of the present formalism, rotating objects with event horizons can be constructed by appropriate tensoring of Wess-Zumino models in two dimensions [35]. The original papers on the possibility of the loss of quantum coherence have concentrated on the spherically-symmetric case, which we have argued does not have any such problems in the string case. What happens in spherically-non-symmetric cases is not yet fully understood. However, we now note that even in the case of Kerr black holes, one can argue that in physically-interesting cases the final stage of the evaporation excites *s*-wave topological modes that are similar to the spherically-symmetric case.

Consider the Kerr metric [36]

\[
\begin{align*}
\text{ds}^2_{Kerr} = & \frac{r^2 + A^2 \cos^2 \theta - 2Mr}{r^2 + A^2 \cos^2 \theta} dt^2 - \frac{r^2 + A^2 \cos^2 \theta}{r^2 + A^2 - 2Mr} dr^2 - (r^2 + A^2 \cos^2 \theta) d\theta^2 - \frac{[(r^2 + A^2)^2 - A^2 \sin^2 \theta (r^2 + A^2 - 2Mr)] \sin^2 \theta}{r^2 + A^2 \cos^2 \theta} d\phi^2 - \frac{4MAr \sin^2 \theta}{r^2 + A^2 \cos^2 \theta} dt d\phi \tag{20}
\end{align*}
\]

where \(M\) is the mass of the black hole, and \(A = \alpha M\) is the angular momentum. For physically-interesting cases \([30, 34]\) \(0 < \alpha^2 < M^2\). The region \(M^2 < \alpha^2\) corresponds to very rapid rotation of the body, which probably does not occur for real physical bodies, as they would fly apart before rotating so rapidly. The final stage of the

\[\text{For classical Einstein gravity coupled to Maxwell’s electromagnetism there are uniqueness theorems for rotating black holes [34]. The situation is less clear for quantum theories, especially the ones obtained as a low energy limit of string theories, where higher-derivative modifications of Einstein-Maxwell’s equations occur. For our purposes we shall concentrate on the Kerr solution, which is the only one studied extensively so far. We believe that this is sufficient to demonstrate our arguments.}\]
evaporation, $M \to 0$, therefore, would correspond to $\alpha \to 0$ as well. Expanding in powers of $M$ and keeping only the leading order it is straightforward to see from (23) that the limit is again one with a topological $Q$-graviton as in the two-dimensional string case (upon redefining $r \to e^Q r$, where $Q$ is $2\sqrt{2}$, as required by the conformal field theory interpretation of the spherically-symmetric case [3, 14]). Perturbations around the Kerr solution by the other stringy modes will presumably lead to an excitation of the rest of the topological $s$-wave modes in the final stage of the evaporation, which is similar to that in the spherically-symmetric case.

This oversimplified argument suggests that $W$-symmetry, or rather a generalised form of it, appropriate for spherically-non-symmetric matter, generated by topological (discrete) stringy modes, characterises in general singularities in four-dimensional target space-times. The formal reason is that the latter are described by topological theories which are in some sense characterised by an infinite-dimensional extended conformal symmetry. We expect that the generalisation (to non-symmetric spaces) of the area-preserving $W$-symmetry characterising the two-dimensional symmetric case will be a phase-space volume preserving algebra. In the same way that $W$-symmetry characterises $SU(1,1)$ coset Wess-Zumino models [37], one might find that the appropriate extension of these theories to describe spherically-non-symmetric matter would be phase-space volume-preserving groups. This, however, is still a speculation, but one expects that the quantum coherence problem can be solved in general due to the enormous stringy symmetries, without reference to any particular geometry for the singularity.

5 Conclusions

We have argued that spherically-symmetric four-dimensional objects with physical curvature singularities and event horizons (black holes) can be described by effective two-dimensional string theories. Such models have necessarily a dilaton field, and can be represented as coset $SU(1,1)/U(1)$ Wess-Zumino models, which are known to possess $W$-symmetries that preserve the two-dimensional $s$-wave phase-space under time evolution, and thus maintain quantum coherence during the (non-thermal) stringy evaporation process of a spherically-symmetric four-dimensional black hole. The latter expresses decay of the massive stringy black hole state due to instabilities induced by the string propagation in summed up world-sheet topologies. We have argued that such an evaporation is consistent with a large Hawking entropy associated with the black hole, the latter being viewed as the entropy the black hole appears to have if one does not take into account the infinity of (observable) quantum charges associated with topological excited string states. The quadratic black-hole-mass dependence of this entropy, conjectured by Bekenstein and Hawking [13, 17], can be obtained from simple formulas yielding the asymptotic density of string states in a linear dilaton background, which resembles the asymptotic form of the black-hole space-time.
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