A highly predictive and testable $A_4$ flavor model within type-I+II seesaw framework and associated phenomenology

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We investigate neutrino mass model based on $A_4$ discrete flavor symmetry in type-I+II seesaw framework. The model has imperative predictions for neutrino masses, mixing and $CP$ violation testable in the current and upcoming neutrino oscillation experiments. The important predictions of the model are: normal hierarchy for neutrino masses, a higher octant for atmospheric angle ($\theta_{13} > 45^o$) and near-maximal Dirac-type $CP$ phase ($\delta \approx \pi/2$ or $3\pi/2$) at $3\sigma$ C. L.. These predictions are in consonance with the latest global-fit and results from Super-Kamiokande(SK), NO$\nu$A and T2K. Also, one of the important feature of the model is the existence of a lower bound on effective Majorana mass, $|M_{ee}| \geq 0.047$eV(at $3\sigma$) which corresponds to the lower part of the degenerate spectrum and is within the sensitivity reach of the neutrinoless double beta decay($0\nu\beta\beta$) experiments.

PACS numbers: 14.60.Pq, 11.30.Hv, 12.15.Ff, 14.80.Cp
Keywords: Discrete symmetry; seesaw mechanism; neutrino mass model, neutrinoless double-beta decay

I. INTRODUCTION

The neutrino oscillation experiments have conclusively demonstrated that neutrinos have tiny mass and they do mix. Especially, with the observation of non-zero $\theta_{13}$ the $CP$ conserving part of the neutrino mixing matrix is known to high precision: $\theta_{12} = 34.5^\circ_{-1.0}^{+1.2}$, $\theta_{23} = 47.7^\circ_{-1.7}^{+1.2}$, $\theta_{14} = 8.45^\circ_{-0.15}^{+0.16}$. Although, the two neutrino mass squared differences $\Delta m_{12}^2$ and $|\Delta m_{23}^2|$ have, also, been measured but there still exist two possibilities for neutrino masses to be either normal hierarchical(NH) or inverted hierarchical(IH).

Understanding this emerged picture of neutrino masses and mixing, which is at odds with that characterizing the quark sector, is one of the biggest challenge in elementary particle physics. The Yukawa couplings are undetermined in the gauge theories. To understand the origin of neutrino mass and mixing one way is to employ phenomenological approaches such as texture zeros[8–23], hybrid textures[24–28], scaling[29,30] vanishing minor[31,32] etc. irrespective of details of the underlying theory. These different ansätze are quite predictive as they decrease the number of free parameters in neutrino mass matrix. The second way, which is more theoretically motivated, is to apply yet-to-be-determined non-Abelian flavor symmetry. In this approach a flavor symmetry group is employed in addition to the gauge group to restrict the Yukawa structure culminating in definitive predictions for values and/or correlations amongst low energy neutrino mixing parameters.

Discrete symmetry groups have been successfully employed to explain non-zero tiny neutrino masses and large mixing angle in lepton sector[30,48]. There exist plethora of choices for flavor groups having similar predictions for neutrino masses and mixing patterns. In general, a flavor model results in proliferation of the Higgs sector making it sometime discouragingly complex. The group $A_4$[49,52] being the smallest group having 3-dimensional representation is widely employed as the possible underlying symmetry to understand neutrino masses and mixing with in the paradigm of seesaw mechanism[53–61]. It has been successfully employed to have texture zero(s) in the neutrino mass matrix which is found to be very predictive[13,21,22].

Another predictive ansatz is hybrid texture structure with one equality amongst elements and one texture zero in neutrino mass matrix. The hybrid texture of the neutrino mass matrix has been realized under $S_3 \otimes Z_3$ symmetry with in type-II seesaw framework assuming five scalar triplets with different charge assignments under $S_3$ and $Z_3$[27,]. Also, some of these hybrid textures have been realized under Quaternion family symmetry $Q_8$[62]. In this work, we present a simple minimal model based on group $A_4$ with two right-handed neutrinos in type-I+II seesaw mechanism leading to hybrid texture structure for neutrino mass matrix. The same Higgs doublet is responsible for the masses of charged leptons and neutrinos[13]. In addition, one scalar singlet Higgs field $\chi$ and two scalar triplets $\Delta_i (i = 1, 2)$ are required to write $A_4$ invariant Lagrangian.

In Sec. II, we systematically discuss the model based on group $A_4$ and resulting effective Majorana neutrino mass matrix. Sec. III is devoted to study phenomenological consequences of the model. In this section we, also, study the implication to neutrinoless double beta decay ($0\nu\beta\beta$) process. Finally, in Sec. IV, we summarize the predictions of the model and their testability in current and upcoming neutrino oscillation/$0\nu\beta\beta$ experiments.
TABLE I: Field content of the model and charge assignments under SU(2)\(_L\) and A\(_4\).

| Symmetry | \(D_L\) | \(e_R\) | \(\mu_R\) | \(\tau_R\) | \(\nu_3\) | \(\nu_2\) | \(\chi\) | \(\Phi\) | \(\Delta_1\) | \(\Delta_2\) |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| SU(2)\(_L\) | 2      | 1      | 1      | 1      | 1      | 1      | 1      | 1      | 2      | 3      |
| A\(_4\)  | 3      | 1      | 1'     | 1''    | 1''    | 1'     | 1'     | 1      | 3      | 1      |

II. THE A\(_4\) MODEL

The group A\(_4\) is a non-Abelian discrete group of even permutations of four objects. It has four conjugacy classes, thus, have four irreducible representations(\(IRs\)), viz.: 1, 1', 1'' and 3. The multiplication rules of the IRs are: \(1' \otimes 1' = 1'', 1'' \otimes 1'' = 1', 1' \otimes 1'' = 1, 3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3\_a \oplus 3\_b\) where,

\[
(3 \otimes 3)_1 = a_1 b_1 + a_2 b_2 + a_3 b_3, \\
(3 \otimes 3)_1' = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \\
(3 \otimes 3)_1'' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \\
(3 \otimes 3)_a = (a_2 b_3 - b_2 a_3 + a_1 b_3 + a_1 b_2 + a_2 b_1), \\
(3 \otimes 3)_b = (a_2 b_3 - b_2 a_3 + a_1 b_3 - a_1 b_2 + a_2 b_1),
\]

\(\omega \equiv e^{2\pi i/3}\) and \((a_1, a_2, a_3), \,(b_1, b_2, b_3)\) are basis vectors of the two triplets. Here, we present an A\(_4\) model within type-I+II seesaw framework of neutrino mass generation. In this model, we employed one SU(2)\(_L\) Higgs doublet \(\Phi\), one SU(2)\(_L\) singlet Higgs \(\chi\) and two SU(2)\(_L\) triplet Higgs fields\((\Delta_1, \Delta_2)\). The transformation properties of different fields under SU(2)\(_L\) and A\(_4\) are given in Table I. These field assignments under SU(2)\(_L\) and A\(_4\) leads to the following Yukawa Lagrangian

\[
-L \equiv \sum_{i} \frac{1}{2} \left( y_{iL} \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i \right) e_{R} \tau_{R} \\
+ \sum_{i} \left( y_{iL} \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i \right) \mu \tau \mu \tau \\
+ \sum_{i} \left( y_{iL} \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i \right) \nu \nu \nu \\
+ \sum_{i} \left( y_{iL} \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i \right) \nu \nu \nu \\
- \sum_{i} \left( \mu \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i \right) \mu \tau \mu \tau \\
- \sum_{i} \left( \mu \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i \right) \nu \nu \nu \\
- \sum_{i} \left( \mu \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i + \tilde{D}_{iL} \phi i \right) \nu \nu \nu
\]

where, \(\tilde{\phi} = i \tau_2 \phi^*\) and \(y_i(i = e, \mu, \tau, 1, 2, \Delta_1, \Delta_2)\) are Yukawa coupling constants.

The above Lagrangian leads to charged lepton mass matrix \(m_l\), right handed Majorana mass matrix \(m_R\) and Dirac mass matrix \(m_D\) given by

\[
m_l = U_L \text{diag}(y_e, y_\mu, y_\tau) \theta,
\]

\[
m_R = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix},
\]

\[
m_D = \begin{pmatrix} x & y \\ x & \omega y \\ x & \omega^2 y \end{pmatrix},
\]

after spontaneous symmetry breaking with vacuum expectation values(VEVs) as \(\langle \Phi \rangle = \frac{1}{\sqrt{3}}(1, 1, 1)^T\) and \(\langle \chi \rangle = e\) for Higgs doublet and scalar singlet, respectively. Here \(U_L\) is

\[
\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\
\end{pmatrix}
\]

which diagonalizes \(m_l\), \(N = h \chi e\), \(x = \vartheta y_1\) and \(y = \vartheta y_2\). The type-I seesaw contribution to effective Majorana neutrino mass matrix is

\[
m_{\nu_1} = m_D m^{-1}_R m_D^T.
\]

Using Eqns.(3) and (4) we get

\[
m_{\nu_1} = \begin{pmatrix} \frac{x^2}{3M} + \frac{\omega^2 y^2}{M} & \frac{x^2}{3M} + \frac{\omega y^2}{M} & \frac{x^2}{3M} + \frac{y^2}{M} \\ \frac{x^2}{3M} + \frac{\omega^2 y^2}{M} & \frac{x^2}{3M} + \frac{\omega y^2}{M} & \frac{x^2}{3M} + \frac{y^2}{M} \\ \frac{x^2}{3M} + \frac{\omega^2 y^2}{M} & \frac{x^2}{3M} + \frac{\omega y^2}{M} & \frac{x^2}{3M} + \frac{y^2}{M} \end{pmatrix}.
\]

Assuming VEVs \(v_j(j = 1, 2)\) for scalar triplets \(\Delta_1, \Delta_2\), respectively, the type-II seesaw contribution to effective Majorana mass matrix is

\[
m_{\nu_2} = \begin{pmatrix} c + d & 0 & 0 \\ 0 & c + \omega d & 0 \\ 0 & 0 & c + \omega^2 d \end{pmatrix},
\]

where \(c = y_\Delta v_1\) and \(d = y_\Delta v_2\). So, effective Majorana mass matrix is given as

\[
m_{\nu} = m_{\nu_1} + m_{\nu_2}.
\]

The charge lepton mass matrix \(m_l\) can be diagonalized by the transformation

\[
M_l = U_L^\dagger m_l U_R,
\]

where \(U_R\) is unit matrix corresponding to right handed charged lepton singlet fields. In charged lepton basis the effective Majorana mass matrix is given by

\[
M_\nu = \begin{pmatrix} c + \frac{3x^2}{M} & d & 0 \\ d & \frac{3y^2}{N} & c \\ 0 & c & d \end{pmatrix},
\]

which symbolically can be written as

\[
M_\nu = \begin{pmatrix} X & \Delta & 0 \\ \Delta & X & X \\ 0 & X & \Delta \end{pmatrix},
\]
where $\Delta$ denotes the equality between elements and $X$ denotes arbitrary non-zero elements. In literature, such type of neutrino mass matrix structure is referred as hybrid textures. On changing the assignments of the fields we can have two more hybrid textures. For example, if we assign $\nu_2 \sim 1', \chi \sim 1'$ and $\Delta_1 \sim 1'$ we end up with

$$\begin{pmatrix}
X & X & \Delta \\
X & \Delta & 0 \\
\Delta & 0 & X
\end{pmatrix}.$$  \hspace{1cm} (11)

Similarly, the field assignments $\nu_2 \sim 1'$, $\chi \sim 1'$ and $\Delta_2 \sim 1'$ result in effective neutrino mass matrix

$$\begin{pmatrix}
X & 0 & \Delta \\
0 & \Delta & X \\
\Delta & X & X
\end{pmatrix}.$$  \hspace{1cm} (12)

In the next section, we study the phenomenological consequences of these neutrino mass matrices.

### III. PHENOMENOLOGICAL CONSEQUENCES OF THE MODEL

In charged lepton basis, the effective Majorana neutrino mass matrix, $M_\nu$ is given by

$$M_\nu = VM_\nu^{\text{diag}}V^T,$$  \hspace{1cm} (13)

where $V = U, P$ and

$$M_\nu^{\text{diag}} = \begin{pmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{pmatrix}.$$  \hspace{1cm} (14)

$U$ is Pontecorvo-Maki-Nakagawa-Sakata(PMNS) matrix and in standard PDG representation is given by

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The phase matrix, $P$ is

$$P = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{2i\alpha} & 0 \\
0 & 0 & e^{2i(\beta+\delta)}
\end{pmatrix},$$

where $\alpha, \beta$ are Majorana phases and $\delta$ is Dirac-type CP violating phase.

The neutrino mass model described by Eqn.(10) imposes two conditions on the neutrino mass matrix $M_\nu$, viz.:

$$(M_\nu)_{ab} = 0,$$

$$(M_\nu)_{\nu\nu} = (M_\nu)_{mn},$$  \hspace{1cm} (15)

\begin{align*}
R_{13} & \equiv \frac{m_1 e^{-2i(\beta+\delta)}}{m_3}, \\
R_{23} & \equiv \frac{m_2 e^{2i(\alpha-\beta-\delta)}}{m_3}
\end{align*}

where the ratios $R_{13} \equiv \frac{m_1}{m_3}$ and $R_{23} \equiv \frac{m_2}{m_3}$. The ratio $R_{23}$ can be obtained from $R_{13}$ using the transformation

$$m_1 U_{a1}^2 + m_2 U_{a2} U_{b2} e^{2i\alpha} + m_3 U_{a3} U_{b3} e^{2i(\beta+\delta)} = 0,$$  \hspace{1cm} (16)

and

$$m_1 (U_{a1} U_{v1} - U_{m1} U_{n1}) + m_2 (U_{a2} U_{v2} - U_{m2} U_{n2}) e^{2i\alpha} + m_3 (U_{a3} U_{v3} - U_{m3} U_{n3}) e^{2i(\beta+\delta)} = 0.$$  \hspace{1cm} (17)

We solve Eqn. (16) and (17) for mass ratios $\frac{m_1}{m_3}$ and $\frac{m_2}{m_3}$.
\( \theta_{12} \to \frac{\pi}{2} - \theta_{12} \). The mass ratios \( R_{13} \) and \( R_{23} \) along with measured neutrino mass-squared differences provide two values of \( m_3 \), viz.: \( m_3^a \) and \( m_3^b \), respectively and is given by

\[
m_3^a = \sqrt{\Delta m_{12}^2 + \Delta m_{23}^2 \over 1 - R_{13}^2},
\]

\[
m_3^b = \sqrt{\Delta m_{12}^2 \over 1 - R_{23}^2}.
\]

These two values of \( m_3 \) must be consistent with each other, which results in

\[
R_\nu = \frac{R_{23}^2 - R_{13}^2}{|1 - R_{23}^2|} \equiv \frac{\Delta m_{23}^2}{|\Delta m_{21}^2|}.
\]

The ratios \( \frac{m_3^a e^{-2i(\beta + \delta)}}{m_3^a} \) and \( \frac{m_3^b e^{2i(\alpha - \beta - \delta)}}{m_3^b} \), to first order in \( s_{13} \), is given by

\[
\frac{m_1}{m_3} e^{-2i(\beta + \delta)} \approx -\frac{c_{23}^4}{s_{23}} + \frac{e^{-i\delta}}{s_{13}} (s_{23}^2 + c_{23}^2 e^{-2i\delta}) \left( s_{12} - c_{12} c_{23} s_{23} \right),
\]

\[
\frac{m_2}{m_3} e^{2i(\alpha - \beta - \delta)} \approx -\frac{c_{23}^4}{s_{23}} + \frac{e^{-i\delta}}{s_{13}} (s_{23}^2 + c_{23}^2 e^{-2i\delta}) \left( c_{12} + s_{12} c_{23} s_{23} \right).
\]

Using these approximated mass ratios we find

\[
R_{13}^2 \approx \frac{c_{23}^4}{s_{23}^4} - \frac{s_{13}}{s_{12} s_{23}^2} \left( \frac{2 c_{23}^2 \cos \delta - \frac{s_{13}}{s_{12} s_{23}^2} (s_{12} - c_{12} c_{23} s_{23}^2)}{c_{12} s_{23}^2} \right) \left( c_{23}^4 + s_{23}^4 + 2 c_{23}^2 s_{23}^2 \cos 2\delta \right),
\]

\[
R_{23}^2 \approx \frac{c_{23}^4}{s_{23}^4} - \frac{s_{13}}{s_{12} s_{23}^2} \left( 2 c_{23}^2 \cos \delta - \frac{s_{13}}{s_{12} s_{23}^2} (c_{12} + s_{12} c_{23} s_{23}^2) \right) \left( c_{23}^4 + s_{23}^4 + 2 c_{23}^2 s_{23}^2 \cos 2\delta \right),
\]

and

\[
R_{23}^2 - R_{13}^2 \approx s_{13} \left( 4 \sin 2\theta_{12} - \sin 2\theta_{23} \right) \left( c_{23}^4 + s_{23}^4 + 2 c_{23}^2 s_{23}^2 \cos 2\delta \right) s_{13} - 4 \sin 2\theta_{12} \sin^3 2\theta_{23} \cos \delta.
\]

\[
m_2 \text{ must be greater than } m_1, \text{ or equivalently, } R_{23}^2 - R_{13}^2 > 0 \text{ which is possible if}
\]

\[
(4 \sin 2\theta_{12} - \sin 2\theta_{23}) \left( c_{23}^4 + s_{23}^4 + 2 c_{23}^2 s_{23}^2 \cos 2\delta \right) s_{13} > 4 \sin 2\theta_{12} \sin^3 2\theta_{23} \cos \delta,
\]

which translates to constraint on \( \delta \) by

\[
\left( \frac{4 \sin 2\theta_{12} \sin^3 2\theta_{23}}{(4 \sin 2\theta_{12} - \sin 2\theta_{23}) \left( c_{23}^4 + s_{23}^4 \right) s_{13}} \right) \cos \delta - \left( \frac{\sin^2 2\theta_{23}}{2 \left( c_{23}^4 + s_{23}^4 \right)} \right) \cos 2\delta < 1.
\]

Using the experimental data shown in Table II, we find that \( \delta \) can take values only near \( \delta \approx 90^\circ \) or \( \delta \approx 270^\circ \) for the model to be consistent with solar mass hierarchy. However, it will, further, get constrained by the requirement of \( R_\nu \) to be within its experimental range. For normal hierarchy (NH), \( R_{23}^2 - R_{13}^2 > 0 \) and \( 1 - R_{23}^2 > 0 \); for inverted hierarchy (IH), the condition \( R_{23}^2 - R_{13}^2 > 0 \) and \( 1 - R_{13}^2 < 0 \) (or \( R_{23}^2 < 1 \) and \( R_{13} > 1 \) predicts \( \theta_{23} \) below maximality (\( \theta_{23} > 45^\circ \)). Similarly, for inverted hierarchy (IH), the condition \( R_{23}^2 - R_{13}^2 > 0 \) and \( 1 - R_{13}^2 < 0 \) (or \( R_{23}^2 < 1 \) and \( R_{13} > 1 \)) predicts \( \theta_{23} \) below maximality (\( \theta_{23} > 45^\circ \)).
TABLE II: The latest global-fit results of neutrino mixing angles and neutrino mass-squared differences used in this analysis [7].

| Parameters | Best-fit±1σ | 3σ range |
|------------|-------------|----------|
| Δm^2_{21}[10^{-3} eV^2] | 7.55±0.20 | 7.05 - 8.14 |
| Δm^2_{31}[10^{-2} eV^2] (NH) | 2.50±0.03 | 2.41 - 2.60 |
| Δm^2_{31}[10^{-3} eV^2] (IH) | 2.42±0.04 | 2.31 - 2.51 |
| Sin^2θ_{12}/10^{-1} | 3.20±0.16 | 2.73 - 3.79 |
| Sin^2θ_{23}/10^{-1} (NH) | 5.47±0.30 | 4.45 - 5.99 |
| Sin^2θ_{23}/10^{-1} (IH) | 5.51±0.18 | 4.53 - 5.98 |
| Sin^2θ_{13}/10^{-2} (NH) | 2.160±0.083 | 1.96 - 2.41 |
| Sin^2θ_{13}/10^{-2} (IH) | 2.220±0.074 | 1.99 - 2.44 |

analytical discussion. Hence, inverted hierarchy (IH) is ruled out at more than 3σ.

![Graph](image)

FIG. 1: $R_\nu$ as a function of $m_1/m_3$ for normal hierarchy (NH) and inverted hierarchy (IH).

In Fig. 2(a), we have depicted correlation between $\delta$ and $\theta_{23}$ at 3σ. $\theta_{23} = 45^\circ$ is not allowed because $1 - R_{23}^2$ must be less than 1. Also, the point ($\theta_{23} = 45^\circ$, $\delta = 90^\circ$ or $270^\circ$) is not allowed otherwise $R_\nu < 0$. $\theta_{23}$ is found to be above maximality and Dirac-type $CP$ violating phase $\delta$ is constrained to a very narrow region in I$^{st}$ and IV$^{th}$ quadrant. In Fig. 2(b), 2(c) and 2(d), we have shown the normalized probability distributions of $\theta_{23}$ and $\delta$. The 3σ ranges of these parameters are given in Table III.

One of the desirable feature of a neutrino mass model is its prediction of the observable(s) which can be probed outside the neutrino sector. One such process is 0$\nu\beta\beta$ decay, the amplitude of which is proportional to effective Majorana neutrino mass $|M_{ee}|$ given by

$$|M_{ee}| = |m_1c_{12}^2c_{13}^2 + m_2s_{12}^2c_{13}^2c_{2i} + m_3s_{13}^2c_{2i}|.$$  \hspace{1cm} (29)

In Fig. 3, we have shown $\sin^2\theta_{23} - |M_{ee}|$ correlation plot at 3σ. The important feature of the present model TABLE III: Prediction of the model for $\theta_{23}$, $\delta$ and $|M_{ee}|$ (3σ lower bound).

| $\theta_{23}$ | $\delta$ | $|M_{ee}|$ |
|--------------|----------|----------|
| (bfp, 3σ range) | (bfp(s), 3σ range(s)) | (3σ lower bound) |
| 48.68° | 88.58° | ≥0.047 eV |
| 45.17° - 50.70 | 87.50° - 89.70° | 271.42° |
| 270.20° - 272.50° |

is the existence of lower bound $|M_{ee}| > 0.047 eV$ (at 3σ) which is within the sensitivity reach of 0$\nu\beta\beta$ decay experiments like SuperNEMO [63], KamLAND-Zen [64], NEXT [65, 66], nEXO [67].

A similar analysis of neutrino mass matrices shown in Eqns.(11) and (12) reveals that these textures are not compatible with present global-fit data on neutrino masses and mixings including latest hints of normal hierarchical neutrino masses, higher octant of $\theta_{23}$ and near maximal Dirac-type $CP$ violating phase $\delta$ [63, 65, 68, 69]. Furthermore, being a minimal model, it will have interesting implications for leptogenesis which will be discussed elsewhere.
IV. CONCLUSIONS

In conclusion, we have presented a neutrino mass model based on $A_4$ flavor symmetry for leptons within type-I+II seesaw framework. The model is economical in terms of extended scalar sector and is highly predictive. The field content assumed in this work predicts three textures for $M_\nu$, based on the charge assignments under $SU(2)_L$ and $A_4$. However, only one (Eqn.(10)) is found to be compatible with experimental data on neutrino masses and mixing angles. We have studied the phenomenological implications of this texture in detail. The solar mass hierarchy i.e. $R_{23}^2-R_{13}^2>0$ constrains Dirac-type $CP$ violating $\delta$ to narrow ranges $87.50^\circ-89.70^\circ$ and $270.20^\circ-272.50^\circ$ at $3\sigma$. The sharp correlation between Dirac-type $CP$ violating phase $\delta$ and atmospheric mixing angle $\theta_{23}$ demonstrates the true predictive power of the model (Fig. 2(a)). The predictions for these less precisely known oscillation parameters ($\delta$ and $\theta_{23}$) are remarkable which can be tested in neutrino oscillation experiments like T2K, NO$\nu$A, SK and DUNE to name a few. We have, also, calculated effective Majorana neutrino mass $|M_{ee}|$. The important feature of the model is existence of lower bound on $|M_{ee}|$ which can be probed in $0\nu\beta\beta$ decay experiments like SuperNEMO, KamLAND-Zen, NEXT and nEXO. The main predictions of the model are:

i. normal hierarchical neutrino masses.

ii. $\theta_{23}$ above maximality ($\theta_{23}>45^\circ$).

iii. near-maximal Dirac-type $CP$ violating phase $\delta=88.58^\circ$ or $271.42^\circ$.

iv. $3\sigma$ lower bound on effective Majorana neutrino mass $|M_{ee}| \geq 0.047eV$.

A precise measurements of Dirac-type $CP$ violating phase $\delta$, neutrino mass hierarchy and $\theta_{23}$ is important.
to confirm the viability of the model presented in this work.

ACKNOWLEDGMENTS

The authors thank R. R. Gautam for useful discussions. S. V. acknowledges the financial support provided by University Grants Commission (UGC)-Basic Science Research(UGC)-Basic Science Research(BSR), Government of India vide Grant No. F.20-2(03)/2013(BSR). M. K. and S. B. acknowledge the financial support provided by the Central University of Himachal Pradesh. The authors, also, acknowledge Department of Physics and Astronomical Science for providing necessary facility to carry out this work.

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