Affleck-Dine-Seiberg from Seiberg-Witten

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Abstract

Perturbing the Seiberg-Witten curves for $\mathcal{N} = 2 U(N_c)$ and $SU(N_c)$ super Yang-Mills theory with $N_f < N_c$ flavours with a mass term for the adjoint field completely lifts the quantum vacuum degeneracy. The generated $\mathcal{N} = 1$ effective superpotential can be obtained from the factorization formulae of Seiberg-Witten curves with matter. We show that the Affleck-Dine-Seiberg superpotential emerges. Moreover it appears additive with respect to the classical superpotential for the meson superfields, as expected from the Intriligator-Leigh-Seiberg linearity principle.
1 Introduction

Since the Dijkgraaf-Vafa conjecture [1] there has been an increased activity in the field of $\mathcal{N} = 1$ gauge theories. In this note we want to connect recent results concerning factorization of Seiberg-Witten curves obtained by using Dijkgraaf-Vafa with the Affleck-Dine-Seiberg superpotential, derived from symmetries, anomalies, holomorphicity and an instanton calculation [2] [3].

We focus on $\mathcal{N} = 1$ $U(N_c)$ and $SU(N_c)$ super Yang-Mills theory (SYM) with a chiral superfield $\Phi$ in the adjoint and $N_f < N_c$ massive flavours, consisting of chiral superfields $Q_i$ and $\tilde{Q}_i$, $i = 1..N_f$, transforming respectively in the fundamental and antifundamental of the gauge group. We consider the tree level superpotential:

$$W_{\text{tree}} = \frac{M}{2} Tr \Phi^2 + \sum_{i=1}^{N_f} \tilde{Q}_i \Phi Q_i + \sum_{i=1}^{N_f} \tilde{Q}_i m_i Q_i,$$ (1)

As the superpotential is quadratic in $\Phi$, one can integrate it out exactly at the classical level. This results in a $\mathcal{N} = 1$ SYM theory with $N_f$ flavours and a classical superpotential:

$$W_{\text{class}}(X_{ij} = \tilde{Q}_i Q_j; M, m_i)$$ (2)

whose exact form depends on whether one considers $U(N_c)$ or $SU(N_c)$.

The non-renormalization theorems state that there are no perturbative quantum corrections to this superpotential. Though there are non-perturbative corrections generated by the dynamics of the SYM theory with flavours. These effects are captured by adding to the classical superpotential [2] the Affleck-Dine-Seiberg superpotential (ADS):

$$W_{\text{ADS}} = (N_c - N_f) \left( \hat{\Lambda}^{3N_c-N_f} \right) \left( \frac{1}{\det X_{ij}} \right)^{\frac{N_c-N_f}{N_c}}$$ (3)

where $\hat{\Lambda}$ is the dynamically generated scale of the 'downstairs' SYM theory with $N_f$ flavours. It is related to the scale $\Lambda$ of the 'upstairs' theory before integrating out the adjoint by:

$$\hat{\Lambda}^{3N_c-N_f} = M^{N_c} \Lambda^{2N_c-N_f}$$ (4)

The ADS potential is independent of the parameters appearing in the classical tree level potential [2].
The purpose of this note is to obtain this non-trivial structure of the quantum effective superpotential and in particular the Affleck-Dine-Seiberg superpotential from the factorization of Seiberg-Witten curves with matter [5].

In [4] agreement between the matrix model calculation and a field theory calculation using the ADS potential has been shown for $U(2)$ super Yang-Mills with 1 flavour. Afterwards [6] showed that the ADS potential in terms of the meson superfield can be obtained using Random Matrix Model techniques à la Dijkgraaf-Vafa. It appeared from the large $N$ limit of the Jacobian arising from changing integration variables from $Q_i$ and $\tilde{Q}_i$ to the gauge invariant integration variable $X_{ij}$. The Jacobian was calculated by means of constrained matrix integrals. Recently in [7] there was some progress towards a diagrammatic derivation of the Veneziano-Yankielowicz-Taylor superpotential.

The plan of this note is as follows: in section 2 we briefly sketch the original derivation of ADS superpotential. The computation of the effective superpotential by integrating out $\Phi$ and adding ADS can be found in section 3. Factorization formulae of Seiberg-Witten curves are collected in section 4. From these formulae we compute in section 5 the effective superpotential and show how the Affleck-Dine-Seiberg superpotential emerges. We close the note with a discussion.

## 2 Affleck-Dine-Seiberg potential

In this section we will briefly sketch the derivation of the Affleck-Dine-Seiberg potential in the case of $\mathcal{N} = 1$ SYM with $N_f < N_c$ massless flavours. An excellent review is [3].

Using the global $U(N_f)_L \times U(N_f)_R \times U(1)_R$ symmetry of the theory, anomaly considerations and holomorphicity one can show that the unique form of the non-perturbative superpotential, consistent with all symmetries is given by:

$$W_{\text{eff}} = C(N_c, N_f) \left( \frac{\hat{\Lambda}^{3N_c-N_f}}{\det X_{ij}} \right)^{\frac{1}{N_c-N_f}},$$

where $\hat{\Lambda}$ is the dynamical scale of the theory.

By giving a large expectation value to one of the flavours $< Q_k > = < \tilde{Q}_k > = a_k$ one obtains a relation between $C(N_c, N_f)$ and $C(N_c-1, N_f-1)$. 


By adding a mass term for one of the massless flavours one obtains a relation between $C(N_c, N_f)$ and $C(N_c, N_f - 1)$. Consistency requirements amongst these relations reduces $C(N_c, N_f)$ to a constant $C$:

$$C(N_c, N_f) = (N_c - N_f)C^{\frac{1}{N_c - N_f}} \tag{6}$$

The constant $C$ can then be computed via an one instanton calculation for $N_f = N_c - 1$. The non-abelian gauge group is then completely higgsed and the instanton computation is reliable. Detailed analysis for $N_c = 2$ and $N_f = 1$ reveals that $C = 1$ in the DR scheme, which concludes the derivation.

It is a very non-trivial statement that the non-perturbative effects in the presence of a tree level superpotential for the meson superfields $W_{tree}(X_{ij})$ are captured by merely adding the same ADS superpotential to $W_{tree}(X_{ij}) \ [9]$. The form of the quantum generated superpotential, consistent with the symmetries is then not anymore unique. There are other possible terms that one can write down involving the parameters of the classical potential. Though one can show, using some appropriate limiting procedures that the only consistent quantum effective superpotential is given by:

$$W_{eff}(X_{ij}, \hat{\Lambda}) = W_{class}(X_{ij}) + (N_c - N_f) \left( \frac{\hat{\Lambda}^{\hat{\Lambda} N_c - N_f}}{\det X_{ij}} \right)^{\frac{1}{N_c - N_f}} \tag{7}$$

### 3 Superpotential from integrating out $\Phi$

Starting point is the tree level superpotential (11):

$$W_{tree} = \frac{M}{2} Tr\Phi^2 + \sum_{i=1}^{N_f} \tilde{Q}_i \Phi Q_i + \sum_i \tilde{Q}_i m_i Q_i, \tag{8}$$

As the results for integrating out the superfield in the adjoint of $U(N_c)$ and $SU(N_c)$ are slightly different, we discuss them seperately. Note that the ADS non-perturbative potential has the same form for both cases.

**$U(N_c)$ with $N_f$ flavours**

Integrating out $\Phi_{ab}$ in the adjoint of $U(N_c)$ leads straightforward to:

$$W_{class} = -\frac{1}{2M} Tr X^2 + Tr m_X, \tag{9}$$
where \( m \) is the diagonal \( N_f \times N_f \) mass matrix diag\((m_1, \ldots, m_{N_f})\) and \( X_{ij} = \tilde{Q}_i Q_j \). The non-perturbative dynamics of \( U(N_c) \) SYM theory with \( N_f \) flavours is captured by the addition of the ADS potential:

\[
W_{\text{eff}}(X, M, m_i, \Lambda) = -\frac{1}{2M} \text{Tr} X^2 + Tr m X + (N_c - N_f) \left( \frac{\Lambda^{3N_c-N_f}}{\text{det} X_{ij}} \right)^{\frac{1}{2N_c-N_f}}
\]

(10)

Note that \( \Lambda \) is the dynamical scale of the 'downstairs' theory after integrating out \( \Phi \). Invoking the matching of the scales (4) and the field redefinition \( X = M\tilde{X} \) one can rewrite the superpotential as:

\[
W_{\text{eff}}(\tilde{X}, M, m_i, \Lambda) = M \left( -\frac{1}{2} \text{Tr} \tilde{X}^2 + Tr m \tilde{X} + (N_c - N_f) \left( \frac{\Lambda^{2N_c-N_f}}{\text{det} \tilde{X}_{ij}} \right)^{\frac{1}{2N_c-N_f}} \right)
\]

(11)

Note that upon using the scale \( \Lambda \) of the theory with adjoint, the superpotential is linear in its mass \( M \). This is a consequence of the Intriligator-Leigh-Seiberg linearity principle [9].

The final step consists of integrating out the modified meson superfield \( \tilde{X}_{ij} \) from the superpotential (11). All field equations are solved by a diagonal meson matrix \( \tilde{X} = \text{diag}(\tilde{X}_{11}, \ldots, \tilde{X}_{N_fN_f}) \). The superpotential and the equations of motion of the diagonal elements reduce to:

\[
W_{\text{eff}} = M \left( -\frac{1}{2} \sum_{i=1}^{N_f} \tilde{X}_{ii}^2 + \sum_{i=1}^{N_f} m_i \tilde{X}_{ii} + (N_c - N_f) \left( \frac{\Lambda^{2N_c-N_f}}{\prod_{i=1}^{N_f} \tilde{X}_{ii}} \right)^{\frac{1}{2N_c-N_f}} \right) \\
- \tilde{X}_{kk}^2 + m_k \tilde{X}_{kk} - \left( \frac{\Lambda^{2N_c-N_f}}{\prod_{i=1}^{N_f} \tilde{X}_{ii}} \right)^{\frac{1}{2N_c-N_f}} = 0, \quad k = 1..N_f
\]

(12)

These equations define implicitly the quantum effective superpotential in terms of the flavour masses and the scale \( \Lambda \). It is linear in the mass of the adjoint superfield.

**SU\((N_c)\) with \( N_f \) flavours**

Integrating out \( \Phi_{ab} \) in the adjoint of \( SU(N_c) \) is a little more subtle:

\[
W_{\text{class}} = -\frac{1}{2M} \text{Tr} X^2 + \left( \frac{\text{Tr} X}{2N_cM} \right)^{2} + Tr m X
\]

(13)
where $m$ is again the diagonal $N_f \times N_f$ mass matrix. Addition of the ADS superpotential encodes all non-perturbative effects:

$$ W_{\text{eff}}(X, M, m_i, \hat{\Lambda}) = -\frac{1}{2M} Tr X^2 + \frac{(Tr X)^2}{2N_c M} + TrmX + (N_c - N_f) \left( \frac{\hat{\Lambda}^{3N_c-N_f}}{\det X_{ij}} \right)^{\frac{1}{N_c-N_f}} $$

This superpotential is linear in $M$, when written in terms of the scale of the upstairs theory:

$$ W_{\text{eff}} = M \left( -\frac{1}{2} Tr \tilde{X}^2 + \frac{(Tr \tilde{X})^2}{2N_c} + Trm\tilde{X} + (N_c - N_f) \left( \frac{\Lambda^{2N_c-N_f}}{\det \tilde{X}_{ij}} \right)^\frac{1}{N_c-N_f} \right) $$

Integrating out the modified meson superfields $\tilde{X}_{ij} = X_{ij}/M$ gives rise to an implicit expression for the superpotential $W_{\text{eff}}(M, m_i, \Lambda)$:

$$ W_{\text{eff}} = M \left( \sum_{i=1}^{N_f} -\tilde{X}_{ii}^2 + m_i \tilde{X}_{ii} + \left( \sum_{i=1}^{N_f} \tilde{X}_{ii} \right)^2 + (N_c - N_f) \left( \frac{\Lambda^{2N_c-N_f}}{\prod_{i=1}^{N_f} \tilde{X}_{ii}} \right)^\frac{1}{N_c-N_f} \right) $$

$$ -\tilde{X}_{kk} + \frac{1}{N_c} \sum_{i=1}^{N_f} \tilde{X}_{ii} + m_k - \frac{1}{\tilde{X}_{kk}} \left( \frac{\Lambda^{2N_c-N_f}}{\prod_{i=1}^{N_f} \tilde{X}_{ii}} \right)^\frac{1}{N_c-N_f} = 0, \quad k = 1..N_f $$

4 Factorization of Seiberg-Witten curves with matter

In the remaining sections we derive the effective superpotential from the factorization formulae of Seiberg-Witten curves with matter. We show that the Affleck-Dine-Seiberg superpotential emerges with all correct coefficients in addition to the classical superpotential for the meson superfields.

$\mathcal{N} = 2 U(N_c)$ SYM theory with $N_f < N_c$ flavours has a quantum Coulomb branch parametrized by $u_p = Tr \frac{\phi_p}{p}$. At each point of this $N_c$ dimensional vacuum manifold the low energy effective field theory is an abelian $U(1)^{N_c}$ theory. All the relevant quantum dynamics can be recast at each point of the moduli space in terms of the Seiberg-Witten curve [10][11]:

$$ y^2 = P_{N_c}(x, u_k)^2 - 4\Lambda^{2N_c-N_f} \prod_{i=1}^{N_f} \left( x + m_i \right) $$

(17)
It is a family of genus $N_c - 1$ hyperelliptic curves parametrized by $u_p$.

Perturbing this $\mathcal{N} = 2$ theory with a tree level mass term $M$ for $\Phi$ completely lifts the vacuum degeneracy. An effective potential given by [10][12][3]:

$$W_{\text{eff}}(R, T) = M u_p^{\text{fact.}}(u_1, \Lambda, m_i)$$

is generated. Note that this effective superpotential is manifest linear in $M$. All non-trivial information is encoded in $u_p^{\text{fact.}}$. It represents the tuning of the $u_2$ parameter in the Seiberg-Witten curve such that it completely factorizes:

$$y^2 = P_{N_c}(x, u_k)^2 - 4 \Lambda^{2N_c - N_f} \prod_{i=1}^{N_f}(x + m_i) = F_2(x)H_{N_c - 1}^2(x),$$

where the subscript on $P, F$ and $H$ denote the degree of the polynomial. This tuning corresponds to a restriction of the original Coulomb branch to a 1 dimensional submanifold where all $N_c - 1$ mutually local monopoles become massless.

These factorization formulae have been found recently using Random Matrices techniques in the spirit of Dijkgraaf-Vafa [5] and give an expression for the moduli $u_p$ in terms of one parameter $u_1$, the scale of the theory $\Lambda$ and the flavour masses $m_i$:

$$u_p^{\text{fact.}} = N_c U_p^{\text{pure}}(R, T) + \sum_{i=1}^{N_f} U_p^{\text{matter}}(R, T, m_i)$$

where

$$U_p^{\text{pure}}(R, T) = \frac{1}{2} \sum_{q=0}^{[p/2]} \binom{p}{2q} \binom{2q}{q} R^q T^{p-2q-1}$$

$$U_{p \geq 2}^{\text{matter}}(R, T, m) = \sum_{n=0}^{p-2} c_{p,n} R^{f_n}(z) - v_p \frac{1}{2} \left( m + T - \sqrt{(m + T)^2 - 4R} \right)$$

In the above formula the coefficients $c_{p,n}$ and $v_p$ are given by:

$$c_{p,n} = 2^n R^{\frac{p-n-2}{2}} \sum_{k=0}^{[p-n-2]/2} \binom{2k}{k} \binom{p-1}{2k+n+1} R^k T^{p-n-2-2k}$$

$$v_p = \sum_{q=0}^{[p/2]} \frac{p-2q}{p} \binom{p}{2q} \binom{2q}{q} R^q T^{p-2q-1}.$$
Finally \( f_0(z) \) are functions of 
\[ z = \frac{m+T}{2R} \left( m + T + \sqrt{(m + T)^2 - 4R} \right). \]
For the purpose of this note \( f_0(z) = (2z - 2)^{-1} \) will be sufficient. We refer to the original paper [5] for the explicit expressions for some of the other functions.

All formulae are in terms of two parameters \( R \) and \( T \), expressing the \( \Lambda \) and \( u_1 \) dependence:

\[
\begin{align*}
\Lambda^{2N_c-N_f} &= R^{N_c-N_f} \frac{1}{N_c} \left( m_i + T - \sqrt{(m_i + T)^2 - 4R} \right) \quad (26) \\
u_1(R, T, m) &= N_cT - \sum_{i=1}^{N_f} \frac{1}{2} \left( m_i + T - \sqrt{(m_i + T)^2 - 4R} \right) \quad (25)
\end{align*}
\]

5 Affleck-Dine-Seiberg from factorized curves

Using the above formulae one can easily write down the effective potential \( u_{\text{fact.}}^2 \) where \( u_{\text{fact.}}^2 \) is given by:

\[
u_{\text{fact.}}^2 = N_c \left( \frac{1}{2} T^2 - R \right) + \frac{1}{4} \sum_{i=1}^{N_f} \left( (m_i - T)(m_i + T - \sqrt{(m_i + T)^2 - 4R}) - 2R \right) \quad (27)
\]

where \( R \) and \( T \) are related to \( u_1 \) and \( \Lambda \) by (25) and (26). As all non-trivial information is encoded in \( u_{\text{fact.}}^2 \), we will denote it as the superpotential, keeping in mind that one still has to multiply everything with \( M \).

Defining \( x_i = \frac{1}{2} \sqrt{(m_i + T)^2 - 4R} \) for \( i = 1...N_f \), one can easily eliminate \( R \) and \( T \) and obtain an implicit expression for the superpotential:

\[
u_{\text{fact.}}^2 (u_1, x_i, \Lambda) = \frac{u_1^2}{2N_c} + \frac{u_1}{2N_c} \sum_{i=1}^{N_f} x_i - \frac{1}{2} \sum_{i=1}^{N_f} m_i x_i + \left( N_c - \frac{1}{2} N_f \right) \left( \frac{\Lambda^{2N_c-N_f}}{\prod_{i=1}^{N_f} x_i^i} \right)^{\frac{1}{N_c-N_f}} \quad (28)
\]

where the \( x_i \) are functions of \( u_1 \) and \( \Lambda \) obtained by their definition:

\[
x_k \left( N_c u_1 \right) = x_k^2 - \frac{1}{N_c} \left( \sum_{i=1}^{N_f} x_i \right) x_k - m_k x_k + \left( \frac{\Lambda^{2N_c-N_f}}{\prod_{i=1}^{N_f} x_i^i} \right)^{\frac{1}{N_c-N_f}}, \quad k = 1...N_f \quad (29)
\]

Summing these constraints over \( k \) gives:

\[
\frac{u_1}{N_c} \sum_{i=1}^{N_f} x_i = \sum_{i=1}^{N_f} x_i^2 - \frac{1}{N_c} \left( \sum_{i=1}^{N_f} x_i \right)^2 - \sum_{i=1}^{N_f} m_i x_i + N_f \left( \frac{\Lambda^{2N_c-N_f}}{\prod_{i=1}^{N_f} x_i^i} \right)^{\frac{1}{N_c-N_f}} \quad (30)
\]
Note that this form of the superpotential arises merely from a rewriting of the factorization formulae.

**SU($N_c$) with $N_f$ flavours**

The results for $SU(N_c)$ super Yang-Mills theories are easily obtained by setting explicitly $u_1 = 0$. The superpotential reduces to:

$$W_{\text{eff}} = M \left( \frac{1}{2} \sum_{i=1}^{N_f} m_i x_i + \left( N_c - \frac{1}{2} N_f \right) \left( \frac{\Lambda^{2N_c-N_f}}{\Pi_{i=1}^{N_f} x_i} \right) \right)$$

$$x_k^2 - \frac{1}{N_c} \left( \sum_{i=1}^{N_f} x_i \right) x_k - m_k x_k + \left( \frac{\Lambda^{2N_c-N_f}}{\Pi_{i=1}^{N_f} x_i} \right) = 0, \quad k = 1..N_f \quad (31)$$

This result is identical as the previous one obtained from the ADS superpotential (16). To make the agreement manifest one can combine the superpotential with the constraint (30):

$$W_{\text{eff}} = M \left( \sum_{i=1}^{N_f} - \frac{x_i^2}{2} + m_i x_i + \left( \frac{\sum_{i=1}^{N_f} x_i}{2N_c} \right)^2 + \left( N_c - N_f \right) \left( \frac{\Lambda^{2N_c-N_f}}{\Pi_{i=1}^{N_f} x_i} \right) \right)$$

$$x_k^2 - \frac{1}{N_c} \left( \sum_{i=1}^{N_f} x_i \right) x_k - m_k x_k + \left( \frac{\Lambda^{2N_c-N_f}}{\Pi_{i=1}^{N_f} x_i} \right) = 0, \quad k = 1..N_f \quad (32)$$

This complete form, including the Affleck-Dine-Seiberg superpotential arises from the factorization formulae of Seiberg-Witten curves. Notice that we not only get the same endresult, but that the complete structure of integrating out the meson superfields is present upon choosing the 'good' variables to rewrite the factorization formulae. The classical superpotential for the meson superfield, as well as the ADS superpotential is manifest with all the correct coefficients.

**U($N$) with $N_f$ flavours**

In this case the superpotential is an implicit function of $u_1$ and one need to integrate it out. Minimizing the superpotential with respect to $u_1$ while using the constraints gives:

$$\frac{d}{du_1} W_{\text{eff}}(M, m_i, \Lambda, u_1, x_k) = u_1 \frac{N_f}{N_c} + \sum_{i=1}^{N_f} \left( \frac{\partial W_{\text{eff}}(M, m_i, \Lambda, u_1, x_k)}{\partial x_i} \right) \frac{\partial x_i}{\partial u_1} = 0 \quad (33)$$
The last part can be calculated by using:

$$\frac{\partial x_i}{\partial u_1} = \left(\frac{\partial u_1}{\partial x_i}\right)^{-1} \left(1 - \sum_{j \neq i} \frac{\partial u_1}{\partial x_j} \frac{\partial x_j}{\partial u_1}\right)$$  \hspace{1cm} (34)$$

By manipulating carefully the equations we obtain:

$$\frac{\partial W_{eff}}{\partial x_i} \left(\frac{\partial u_1}{\partial x_i}\right)^{-1} = \frac{1}{N_c} \sum_{k=1}^{N_f} x_k$$  \hspace{1cm} (35)$$

As this expression is independent of the label $i$ the total result simplifies to:

$$\frac{u_1}{N_c} + \frac{\sum_{k=1}^{N_f} x_k}{N_c} \sum_{i=1}^{N_f} \left(1 - \sum_{j \neq i} \frac{\partial u_1}{\partial x_j} \frac{\partial x_j}{\partial u_1}\right) = 0$$  \hspace{1cm} (36)$$

The remaining summation is trivial and the value for $u_1$ minimizing the potential is given by

$$u_1 = - \sum_{i=1}^{N_f} x_i$$  \hspace{1cm} (37)$$

The final step consists of plugging this value for $u_1$ into the superpotential:

$$W_{eff}(M, m_i, \Lambda, x_i) = M \left(\frac{1}{2} m_i x_i + (N_c - \frac{1}{2} N_f) \left(\frac{\Lambda^{2N_c-N_f}/\prod_{i=1}^{N_f} x_i}{N_c-N_f}\right)^{\frac{1}{N_c-N_f}}\right)$$

$$x_k^2 - m_k x_k + \left(\frac{\Lambda^{2N_c-N_f}/\prod_{i=1}^{N_f} x_i}{N_c-N_f}\right)^{\frac{1}{N_c-N_f}} = 0 \hspace{1cm} k = 1..N_f$$  \hspace{1cm} (38)$$

Proceeding as above and using the constraint, one can make the agreement with the result using the explicit form of ADS manifest:

$$W_{eff} = M \left(-\frac{1}{2} \sum_{i=1}^{N_f} x_i^2 + \sum_{i=1}^{N_f} m_i x_i + (N_c - N_f) \left(\frac{\Lambda^{2N_c-N_f}/\prod_{i=1}^{N_f} x_i}{N_c-N_f}\right)^{\frac{1}{N_c-N_f}}\right)$$

$$x_k^2 - m_k x_k + \left(\frac{\Lambda^{2N_c-N_f}/\prod_{i=1}^{N_f} x_i}{N_c-N_f}\right)^{\frac{1}{N_c-N_f}} = 0 \hspace{1cm} k = 1..N_f$$  \hspace{1cm} (39)$$

Again the ADS superpotential arises additive with respect to the classical superpotential for the meson superfields as expected. Moreover the form of the effective superpotential mimics the integrating out of the meson superfield from the method involving the ADS superpotential.
Discussion

Perturbing the $\mathcal{N} = 2$ Seiberg-Witten curve with $N_f$ flavours with a mass term for the adjoint breaks the supersymmetry to $\mathcal{N} = 1$. We computed the effective superpotential from the factorization formulae of the Seiberg-Witten curves and showed that the Affleck-Dine-Seiberg superpotential emerges. It appears additive with respect to the classical superpotential for the meson superfields as expected from the Intriligator-Leigh-Seiberg linearity principle. Moreover the complete structure of integrating out the meson fields can be retrieved from the factorization formulae.

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