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A Method of Constructing Non-Homogeneous Model With Weibull Distribution

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Abstract: When the slope is numerically simulated, the more realistic it is, the more authentic the simulation results are. How to correctly deal with the uncertainty and randomness of rock slope materials is always a difficult problem in modeling of slopes. Taking an open pit slope as an example, in order to get a more realistic slope model, through the analysis of the slope point load data of the open pit mine, it is known that the mechanical properties of the slope are in accordance with weibull distribution. Based on the weibull distribution, the volume modulus, shear modulus, cohesion and tensile strength of the slope model are randomly assigned to the slope by monte-carlo method, and the non-homogenization modeling of the slope is carried out. The modeling analysis process and method of non-homogenization modeling based on flac3d have certain reference significance to the construction of similar models.

1. Introduction

With the development of computer technology, computer numerical simulation has become an important means to study the stability of slope[1]. Due to the complex structure of rock composition and long-term geological action, one of the basic characteristics of rock is non-homogeneity[2]. Scholars at home and abroad have conducted a lot of studies on the heterogeneity of rock. Li Guoying[3] studied and analyzed the meso heterogeneity of rock materials by establishing the non-homogeneous rock model based on digital image technology, the non-homogeneous rock model with random defects, and the non-homogeneous rock model satisfying the statistical distribution rule. Lei Jinsheng et al.[4] established a porosity model conforming to weibull distribution by summarizing the distribution law of soil parameters, and assigned values to the unit according to the relationship formula between basic physical parameters such as permeability coefficient and elastic modulus, so as to build an inhomogeneous two-dimensional geological profile and three-dimensional digital formation model. Lo Wing[5] in the content of different kinds of materials in the rocks, on the basis of the different defines the category rule of material component unit, and according to the law, in turn, the numerical model with Monte - Carlo method to determine all the units in the category and the assignment, and on the basis of the composition for geotechnical material itself describe the heterogeneity of material properties. Established by Flac5d in this paper a slope model as an example, using the Monte - Carlo method produces obedience (0, 1] interval the uniform distribution of pseudo - random sequence, the sequence into the inverse function of Weibull distribution, obtained by mechanical property parameters of Weibull distribution on, using Fish language to traverse all unit, mechanical property parameters of Weibull distribution on separately to each unit, which completed
the heterogeneity of Weibull distribution on the slope model is set up. Based on this model, it is more practical to study the influence of precipitation seepage, earthquake, blasting and other effects on the slope and to find the dangerous sliding crack surface.

2. Basic principle

2.1 The Weibull distribution

Weibull distribution was proposed by Wallodi Weibull, a Swedish physicist in 1939 and widely used in reliability engineering. It is worth mentioning that Weibull distribution is related to Rayleigh distribution, exponential distribution and even distribution, and can be converted to each other \([6]\). The probability density formula of two-parameter Weibull distribution is as follows:

\[
f(x) = \begin{cases} \frac{\beta}{\alpha} \left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^\beta}, & x \geq 0 \\ 0, & x < 0 \end{cases}
\]

The cumulative distribution formula is as follows:

\[
F(x) = \int_{-\infty}^{x} f(t)dt = \begin{cases} 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta}, & x \geq 0 \\ 0, & x < 0 \end{cases}
\]

In the formulas, first is the scale parameter, generally is the average value of mechanical properties, and its influence on probability density curve and cumulative distribution curve is shown in FIG. 1 and FIG. 2. The shape parameter represents the uniformity of the rock mass in the rock mass model. The larger the value is, the more uniform the rock mass is, and it plays a decisive role in determining the shape of the probability curve, as shown in FIG. 3 and FIG. 4. When the probability curve is larger than 3.5, the probability density curve is similar to the normal distribution. The peak value appears at the value of \(x\) equal to the initial value, and the larger the slope, the more concentrated the distribution.
2.2 Methods of heterogeneous model construction

According to a statistical distribution, a sample space consisting of units subject to the distribution can be generated. The distribution state of the sample space depends on two important parameters: average value and homogeneity. For the same sample space, even if the mean remains the same, the integral space will be different, because the expression can change. Under the constraint of the average value, the microscopic average value of the unit set is roughly equal to the overall average value, but the random arrangement of the unit makes the space obviously disordered, which characterizes the discretization characteristics inside the rock.

Unit distribution of the disorder, can through the Monte Carlo method to produce a set with
Weibull cumulative distribution function value one-to-one pseudo-random sequence \( R \), the pseudo random sequence into a Weibull cumulative distribution formula of inverse function can be based on this set of sequence of the independent variables, the corresponding function was obtained by Weibull cumulative distribution formula can get its inverse function formula for:

\[
\alpha_i = F^{-1}(R_i) = \alpha_0 \left[ \ln \left( \frac{1}{1 - R_i} \right) \right]^{\frac{1}{\beta}}, \quad 0 < R_i \leq 1
\]

When \( \alpha, \beta \) are obtained from the analysis of material data and taken into account, the mechanical parameter \( \{\alpha_i|i=1,2,3,\ldots\} \) that corresponds to the sequence \( \{R_i\leq1|i=1,2,3,\ldots\} \) and is subject to weibull distribution can be obtained. As shown in figure 5, monte-carlo method is used to generate pseudo-random number sequences uniformly distributed on the interval \((0,1)\), so that the cumulative distribution \( F^{-1}(R_i) = \alpha_i \). In other words, \( R_i \) is substituted into the inverse function formula, and the coordinate is obtained in FIG. 5, forming \( \{R_i\} \sim \{\alpha_i\} \). Thus, a series of Numbers subject to Weibull distribution is formed. which is assigned to different units of the model separately to complete the construction of the model subject to Weibull distribution.

3. Engineering examples

3.1 Weibull distribution test of rock strength

At different positions of an open pit slope, equivalent blocks of rock were taken for point load test, and a total of 80 groups of tests were conducted. According to 《the standards for testing methods of engineering rock mass (GBT50266-2013)》, the strength test of point load is to place the rock specimen between a pair of spherical cones and apply concentrated load until failure. During the test, the oil pressure gauge reading can be read, measuring the distance between the two loading points on the failure surface of the test sample and the average width perpendicular to the loading point, which can be converted into the load \( P \) of the failure of the test sample and the equivalent core diameter \( D_e \). The uncorrected load strength of the rock point can be calculated according to the following formula:

\[
I_s = \frac{P}{D_e^2}
\]

Where:

- \( I_s \) -- uncorrected strength of rock point load;
- \( P \) -- damage load;
- \( D_e \) -- equivalent core diameter.
The equivalent core diameter of the irregular rock sample shall be converted according to the following formula:

$$D_e^2 = \frac{4WD}{\pi}$$

Where:

- $W$ -- the width or average width of the minimum cross section through two loading points;
- $D$ -- distance between two loading points.

The test data are shown in table 1. In order to verify whether the rock properties can be described by weibull distribution, the point load strength of the uncorrected irregular test block [7] was taken in the table.

| NO. | $W$ /cm | $D$ /cm | $D_e^2$ /cm$^2$ | $P$ /kN | $I_s$ /MPa |
|-----|---------|---------|-----------------|--------|-----------|
| 1   | 7.8     | 4.5     | 44.69           | 32.34  | 72.36     |
| 2   | 7       | 5.5     | 49.02           | 23.72  | 48.39     |
| 3   | 11      | 5.8     | 81.23           | 27.26  | 63.56     |
| 4   | 7.1     | 5.5     | 49.72           | 29.56  | 59.45     |
| 5   | 7.6     | 5.9     | 7.09            | 47.16  | 82.60     |
| 6   | 7.8     | 5.2     | 51.64           | 20.36  | 39.42     |
| 7   | 6.4     | 6.2     | 50.52           | 60.72  | 120.18    |
| 8   | 4.9     | 5.3     | 33.07           | 6.82   | 20.63     |
| 9   | 5.5     | 5.5     | 38.52           | 16.68  | 43.31     |
| 10  | 4.6     | 4.4     | 25.77           | 40.26  | 156.23    |
| 11  | 9.2     | 4.7     | 55.05           | 29.13  | 52.91     |
| 12  | 9.4     | 4.9     | 58.65           | 51.82  | 88.36     |
| 13  | 6.9     | 4.5     | 39.53           | 6.71   | 16.97     |
| 14  | 5.4     | 4.3     | 29.56           | 14.28  | 48.30     |
| 15  | 13.4    | 4.4     | 75.07           | 23.42  | 31.20     |
| 16  | 6.6     | 6       | 50.42           | 24.96  | 49.50     |
| 17  | 8.4     | 4.3     | 45.99           | 18.23  | 39.64     |
| 18  | 5.9     | 3.4     | 25.54           | 10.18  | 39.86     |
| 19  | 10.8    | 5.1     | 70.13           | 46.68  | 66.56     |
| 20  | 6.8     | 2.8     | 24.24           | 9.87   | 40.71     |
| 21  | 6.9     | 4.3     | 37.78           | 6.34   | 16.78     |
| 22  | 9.6     | 3.7     | 45.23           | 26.49  | 58.57     |
| 23  | 8.3     | 3.5     | 34.87           | 15.9   | 45.59     |
| 24  | 6.1     | 4.7     | 36.50           | 29.23  | 80.07     |
| 25  | 6.8     | 4.9     | 42.42           | 23.56  | 55.53     |
| 26  | 9.2     | 2.4     | 28.11           | 13.52  | 48.09     |
| 27  | 9.4     | 5.3     | 63.43           | 8.86   | 13.97     |
| 28  | 8.6     | 5.7     | 62.41           | 60.74  | 97.32     |
| 29  | 5.7     | 4.2     | 30.48           | 20.62  | 67.65     |
| 30  | 6.4     | 5.1     | 41.56           | 13.98  | 33.64     |
| 31  | 8.2     | 4.7     | 49.07           | 20.03  | 40.82     |
| 32  | 8.5     | 3.4     | 36.80           | 15.68  | 42.61     |
| 33  | 9.8     | 3.7     | 46.17           | 15.88  | 34.40     |
| 34  | 5.3     | 5.3     | 35.77           | 8.34   | 23.32     |
| 35  | 7.4     | 3.7     | 34.86           | 46.03  | 132.04    |
| 36  | 7.1     | 4.6     | 41.58           | 12.67  | 30.47     |
| 37  | 6.2     | 4.3     | 33.94           | 22.32  | 65.75     |
| 38  | 6       | 3.9     | 29.79           | 4.74   | 15.91     |
| 39  | 9.7     | 4.2     | 51.87           | 17.63  | 33.99     |
| 40  | 5.1     | 4.3     | 27.92           | 18.37  | 65.79     |

The uncorrected strength value of rock point load obtained in table 1 above was imported into MATLAB to test the distribution of strength value. Since the form of normal distribution is the most common form of distribution, it is respectively assumed that the rock strength value conforms to the
normal distribution and weibull distribution, and the fitting goodness test in MATLAB is used to analyze which statistical distribution of rock strength value is more consistent.

In MATLAB, a variety of tools can be embedded to test whether the distribution of data satisfies a certain statistical distribution. Such functions include chi2gof, jarque-bera test (jbtest), kolmogorov-smirnov (k-s) test (kstest) of single sample, kolmogorov-smirnov (k-s) test of double sample (kstest2) and lillietest. Chi-square goodness of fit test (chi2gof) was used to quantitatively examine the distribution of rock strength values and analyze whether the distribution was more normal or Weibull statistical distribution.

To test whether the distribution of rock strength value conforms to the Normal distribution, the parameters for fitting rock strength data X with the function Q = fitdist (X, 'Normal') were first solved in MATLAB. Then, [h, p, stats] = chi2gof (X, 'CDF', Q) is used to test whether the rock strength data X obeys the normal distribution when the parameter is Q, and the CDF in the function represents the cumulative distribution function when the parameter is Q. The function test of the null hypothesis of rock strength data under the significance level of 0.05 X is normal distribution, the output function parameter h value is 0 or 1, when the output of h value is 0, p value is greater than the significance level of 0.05, says under the significance level of 0.05 to accept the null hypothesis, that rock strength data X is obey the normal distribution; When the output h value is equal to 1 and the p value is not greater than the significance level of 0.05, the original hypothesis is rejected at the significance level of 0.05, that is, the rock strength data X is not subject to normal distribution. When the distribution of rock strength value is verified to conform to Weibull statistical distribution, only the 'Normal' in the fitdist function can be modified to be 'Weibull', and the running results are shown in FIG.6 and FIG.7.

It can be seen from FIG.6 that when performing the fitting test of normal distribution of rock strength data, the output results h value is 1 and p value is 0.064 less than 0.05. Therefore, the assumption that rock strength conforms to normal distribution is rejected when the significance level is 0.05. As can be seen from figure 7, when fitting the rock strength data to weibull statistical distribution, the output results h value is 0 and p value is 0.1114 greater than 0.05. Therefore, the original hypothesis that rock strength conforms to weibull statistical distribution is accepted when the significance level is 0.05. Therefore, it is relatively reasonable to assume that the rock parameters of the slope conform to weibull statistical distribution.

3.2 Establishment of slope model
When modeling with Flac\textsuperscript{3D} slope, the size of the numerical model of the slope is expanded according to literature \[8\]. That is, the slope height H and the base rock height at the bottom of the slope are equal to H, the top width of slope is 2.5H and the bottom width of slope is 2H. The specific size of the model is \(x = 180\) m, \(y = 30\) m, \(z = 60\) m. The slope height H= 30m, and the slope gradient is 1:1.5, as shown in figure 8. The top of the slope is forward along the x axis to enlarge the cell in the proportion of 1:1.1, and in the height range below the slope foot (z=0), the cell is amplified in the negative direction of the z axis according to the proportion of 1:1.1. A total of 8450 units and 9,966 nodes are generated. According to relevant data\[9\], mechanical property parameters of rock slope are taken as shown in table 2.

![Open-pit mine slope model](image)

### Table 2: Mechanical properties of rock slope

| Name of rock | Density (Kg \(\cdot\) m\(^{-3}\)) | Cohesive force (MPa) | Angle of internal friction (\(^\circ\)) | Tensile strength (MPa) | Elastic modulus (MPa) | Poisson's ratio |
|--------------|---------------------|---------------------|---------------------------------|---------------------|-----------------------|----------------|
| Mixed rock   | 2600                | 0.375               | 31                              | 0.2                 | 1180                  | 0.25           |

In Flac\textsuperscript{3D}, the deformation parameters used are volume modulus K and shear modulus G, while non-elastic modulus (young's modulus) E and poisson's ratio is ideal. Therefore, it is necessary to perform the transformation and then perform the value.

\[
K = \frac{E}{3*(1-2*\nu)} \quad G = \frac{E}{2*(1+\nu)}
\]

According to the values of elastic modulus and poisson's ratio in table 2, the volume modulus is 787 MPa and the shear modulus is 472 MPa.

### 3.3 Selection of Weibull distribution parameters

In Weibull distribution with two parameters, the maximum likelihood estimation system of parameters \[10\] is:
There is no analytic solution to this system, and numerical methods are needed to solve it. Newton iteration algorithm has good convergence for solving nonlinear equations. Newton iteration formula \[11\] is:

\[
F(x^{(k+1)}) = x^k - \left[ F'(x^{(k)}) \right]^{-1} F(x^{(k)}),
\]

\(k = 0, 1, 2, \ldots\)

Where \(x(0)\) is the initial value, \(x(k)\) is the \(K\)th iteration value, \(F\) is the derivative of the vector function, called the jacobian matrix. In the calculation, the linear equation is solved first, the vector is obtained, and then the iteration is made, the resulting vector sequence \(\{x(k)\}\) is satisfied, then \(x^*\) is a solution of the system. When the Newton iteration formula is used to solve equation (1), the jacobian matrix is expressed as:

\[
F'(\alpha, \beta) = \begin{pmatrix}
\frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial \beta} \\
\frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial \beta} \\
\frac{\partial F_3}{\partial \alpha} & \frac{\partial F_3}{\partial \beta}
\end{pmatrix}
\]

In which that:

\[
\frac{\partial F_1}{\partial \alpha} = \beta \alpha^{\beta-1} \sum_{i=1}^{n} x_i^\beta;
\]

\[
\frac{\partial F_1}{\partial \beta} = \alpha^\beta \log \alpha \sum_{i=1}^{n} x_i^\beta + \alpha^\beta \sum_{i=1}^{n} x_i^\beta \log x_i;
\]

\[
\frac{\partial F_2}{\partial \alpha} = -\beta \alpha^{\beta-1} \sum_{i=1}^{n} x_i^\beta \log x_i;
\]

\[
\frac{\partial F_2}{\partial \beta} = -\frac{n}{\beta^2} - \alpha^\beta \log \alpha \sum_{i=1}^{n} x_i^\beta \log x_i - \alpha^\beta \sum_{i=1}^{n} x_i^\beta \log x_i^2.
\]

Taking the rock strength data in table 1 as the sample, select (60,1) as the initial value, carry out iterative calculation in Matlab, and finally obtain the slope value of 1.9262. It can be seen that the rock property is relatively dispersed, and it is unreasonable to simulate the slope with the homogeneous model. Therefore, when Weibull distribution is used to describe the properties of rocks, the mean value is obtained, that is, the reference value shown in table 2, and the outlier value is 1.9262.

3.4 Random assignment of mechanical parameters of slope model

Fish is an embedded programming language of Flac3d and can write custom functions and variables as needed. Using Fish language of \(p_z = \text{zone} \_ \text{head} \) traverse on the slope model of each unit, the material assignment can use built-in \(z \_ \text{prop} \) fish program function, it can be carried out in accordance with the parameters for the name of the unit assignment, unit attribute parameter values, according to the formula to calculate the inverse weibull function \((0, 1)\) in the type range evenly distributed pseudo-random number \(R\) in Fish language by urand command implementation. Since the system will automatically determine the value of young's modulus and poisson's ratio according to the value of the previous volume modulus and shear modulus, when the volume modulus and shear modulus are modified, the young's modulus can be directly modified, and the volume modulus and shear modulus will change accordingly.

Generally, poisson's ratio and the Angle of internal friction of the rock do not change much, so only the volume modulus, shear modulus, cohesive force and tensile strength are randomly assigned. The model after the non-homogenization of the mechanical parameters of the rock is realized by the above
method, as shown in FIG. 9.

Fig.9  Non-homogeneous slope model with volume modulus satisfying weibull distribution

4. Conclusion
The geotechnical properties of rock and soil present heterogeneous distribution state under long-term geological action. Based on the analysis of the probability distribution of the load strength at different positions of an open pit slope, the results show that the distribution of rock mechanical properties can be described by Weibull distribution.

The intensity value of point load is calculated iteratively in Matlab, and the shape parameter of Weibull distribution is 1.9262, and the scale parameter is taken as the average value of rock mechanical property, that is, the data reference value. By using the monte-carlo method, the uniformly distributed pseudo-random sequence of the interval subject to (0,1) was generated, and the sequence was introduced into the inverse function of Weibull distribution, and the mechanical property parameters subject to Weibull distribution were obtained. All the elements were traversed using the Fish language, and the mechanical property parameters subject to Weibull distribution were assigned to each unit respectively. The method of establishing a nonhomogeneous slope model whose rock mechanical parameters are subject to Weibull distribution can be realized by using the mathematical function of Fish language in Flac3d, as well as the function of unit traversal and unit assignment.

The non-homogeneous slope model plays a decisive role in further correctly analyzing the actual influence of precipitation seepage, earthquake, blasting and other factors on the slope. The analysis process and modeling method of establishing the non-homogeneous slope model have certain reference value for the numerical simulation modeling of similar projects.

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