On nature of the vector resonance $Y(2175)$

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Spectroscopic parameters and decay channels of the vector resonance $Y(2175)$ are studied by considering it as a diquark-antidiquark state with the quark content $su\bar{s}$. The mass and coupling of the tetraquark $Y(2175)$ are calculated using the QCD two-point sum rules by taking into account various quark, gluon and mixed condensates up to dimension 15. Partial widths of its strong decays to $\phi f_0(980)$, $\eta f_0$, and $\phi f_0'$ are computed as well. To this end, we explore the vertices $Y\phi f_0(980)$, $Y\phi\eta$, and $Y\phi f_0'$, and calculate the corresponding strong couplings by means of the QCD light-cone sum rule method. The coupling $g_{\gamma}\phi f_0(980)$ is found using the full version of this method, and by treating the scalar meson $f_0(980)$ as a diquark-antidiquark tetraquark state. The couplings $g_{\gamma}\phi f_0$ and $g_{f_0}\phi f_0'$, however, are calculated by applying the soft-meson approximation to the light-cone sum rule method. Prediction for the mass of the resonance $m_Y = (2173 \pm 85)$ MeV is in excellent agreement with the data of the BaBar Collaboration [1], and within errors of calculations is compatible with the result reported by BESIII [2]. The full width $I_{\text{full}} = (91.1 \pm 20.5)$ MeV of the $Y(2175)$ saturated by its three strong decay channels is in a reasonable agreement with existing experimental data.

I. INTRODUCTION

The resonances $\{Y\}$ with the quantum numbers $J^{PC} = 1^{--}$ constitute two families of particles, interpretation of which is one of interesting and yet unsettled problems of the high energy physics. Members of the first family populate the mass region $m = 4.2 - 4.7$ GeV, and were observed by different collaborations. These resonances reside very close to each other, and are more numerous than vector charmonia $\Upsilon$ from this mass range. Hence, at least some of these resonances have different quark-gluon structure, and are presumably states built of four valence quarks. Besides a suggestion about the tetraquark nature of heavy $\{Y\}$ states, there are various alternative models to account for their parameters and decay channels.

Another family of the $\{Y\}$ resonances occupies the light segment of meson spectroscopy and incorporates the famous “old” state $Y(2175)$, and new ones $X(2239)$ and $X(2100)$ seen recently. The structure $Y(2175)$ was discovered by the BaBar Collaboration in the initial-state radiation process $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi f_0(980)$ as a resonance in the $\phi f_0(980)$ invariant mass spectrum [3]. The mass and width of this resonance measured by BaBar amount to $m = (2175 \pm 10 \pm 15)$ MeV and $\Gamma = (58 \pm 16 \pm 20)$ MeV, respectively. The same result was also seen by the BE- SIII Collaboration in the exclusive decay $J/\psi \rightarrow \eta \phi \pi^+ \pi^-$. The spectroscopic parameters of the $Y(2175)$ extracted in this experiment differ from original results and are $m = (2200 \pm 6 \pm 5)$ MeV and $\Gamma = (104 \pm 15 \pm 15)$ MeV. Because $Y(2175)$ was observed by BaBar and confirmed by the BESII, BESIII, and Belle collaborations [2,4,], its existence is not in doubt, but an uncertain situation with the mass and full width of this resonance requires further experimental and theoretical studies.

Other resonances that may be considered as candidates to light exotic vector mesons were discovered by the BE- SIII Collaboration. The first of them, i.e., $X(2239)$ was fixed in the process $e^+e^- \rightarrow K^+K^-$ as a resonance structure in the cross section shape line [5]. The second resonance $X(2100)$ was seen in the $\phi\eta$ mass spectrum in the process $J/\psi \rightarrow \phi\eta f_0(980)$. The quantum numbers of $X(2239)$ were determined unambiguously, whereas a situation with $X(2100)$ remains unclear. Indeed, because of a scarcity of experimental information the collaboration could not clearly distinguish two $1^+$ and $1^-$ assumptions for the spin-parity $J^P$ of the resonance $X(2100)$. Hence, BESIII extracted its mass and full width using both of these options. Obtained results differ from each other and depend on assumption about the parity of the state $X(2100)$.

Theoretical interpretations of the light vector resonances comprise all available models and approaches of the high energy physics. Thus, the $Y(2175)$ was considered as $2^1D_1$ excitation of the ordinary $\Upsilon$ meson [6,8]. It was explained also as a dynamically generated state in the $\phi K\bar{K}$ system [9], or as a resonance appeared due to self-interaction between $\phi$ and $f_0(980)$ mesons [10]. A hybrid meson with structure $\Upsilon_g$ and a baryon-antibaryon $qq\bar{q}qq\bar{q}$ state which couples strongly to the $\Lambda\bar{\Lambda}$ channel are among alternative models of the $Y(2175)$ resonance.

There were attempts to interpret $Y(2175)$ as a vector tetraquark with $ss\bar{s}s$ or $ss\bar{s}s$ contents [11,12] (see Ref. [3] for other models). The resonance $X(2100)$ was examined in the framework of the QCD sum rule method in Refs. [14,15].

Recently, we explored the light resonances $X(2100)$ and $X(2239)$ as the axial-vector and vector $ss\bar{s}s$ tetraquarks [16], respectively. Besides spectroscopic parameters we also investigated the strong decays
X(2100) → φf' and X(2100) → φη, and calculated their partial widths. Predictions obtained for the mass and width of the axial-vector state allowed us to identify it with the resonance X(2100), because our theoretical predictions are very close to its parameters measured by the BESIII Collaboration. We classified X(2239) as the vector tetraquark ss̅s̅ and found a reasonable agreement between theoretical and experimental results.

In the present work, we continue our investigations of the light vector resonances and concentrate on features of the state Y(2175) (hereafter, Y). Our treatment of this state differs from existing analyses. Thus, we consider it as a vector tetraquark with content su̅su̅ rather than as a state ss̅s̅s̅. The traditional assumption about the quark content of the Y is inspired by the fact, that it was discovered in φf(980) invariant mass distribution. Because in the standard model of mesons one treats the φ and f0(980) as vector and scalar particles with the same ηs structure, then it is natural to assume that Y is built of four valence s quarks.

But the conventional quark-antiquark model of mesons in the case of light scalar nonets meets with evident difficulties. In fact, the nonet of scalar mesons in the SU(3) model may be realized as 1P0 states. In accordance with various computations, masses of the scalars 1P0 are higher than 1 GeV. They were identified with the isoscalar mesons f0(1370) and f0(1710), the isovector a0(1450) or isospinor K0(1430) states, i.e., with scalars from the second light nonet. But masses of the mesons from the first nonet are lower than 1 GeV, and they cannot be included into this scheme. Therefore, to explain experimental information on their masses, and an unusual mass hierarchy inside of the nonet, Jaffe made suggestion on a four-quark nature of these particles 17.

An updated model of the light scalar nonets is based on assumption about a diquark-antidiquark structure of these particles, which appear as mixtures of spin-0 diquarks from (3, 3̅f) representation with spin-1 diquarks from (6c, 3̅f) representation of the color-flavor group 18. In Refs. 19, 20 we investigated the scalar mesons f0(500) and f0(980) as admixtures of the SU(3) basic light L = [ud]|σ| and heavy Η = ([su]|σ| + [ds]|σ|)/√2 tetraquark states, and calculated their spectroscopic parameters and full widths. Obtained predictions agree with existing experimental data, therefore we consider the f0(980) as the exotic four-quark meson. Once we accept this model, a treatment of the Y as a vector tetraquark Y = [su]|σ| becomes quite reasonable.

We calculate the spectroscopic parameters of the vector tetraquark Y = [su]|σ| and explore some of its decay channels. The mass and coupling of the Y are evaluated using the QCD two-point sum rule method 21, 22. We investigate the strong decays Y → φf(980), Y → φη, and Y → φf', and find their partial widths. To this end, we use the QCD light-cone sum rule (LCSR) method 23, and calculate the couplings GYφf, gYφη, and gYφf' corresponding to the strong vertices Yφf(980), Yφη, and Yφf', respectively. The coupling GYφf is computed by employing the full version of the LCSR method, whereas in the case of gYφη and gYφf' this method is supplemented by a technique of the soft-meson approximation 24, 25. Because the light component of f0(980) is irrelevant for analysis of the decay Y → φf(980), we treat f0(980) as a pure H state.

This article is organized as the following way: In Sec. II we calculate the mass and coupling of the tetraquark Y. The strong decays of this state are considered in sections III and IV. In Sec. III we analyze the process Y → φf(980) using the LCSR method and find partial decay width of this channel. The partial widths of the decay modes Y → φη, and Y → φf' are calculated in the next section IV. In Sec. V we analyze the obtained results, and give our conclusions.

II. SPECTROSCOPIC PARAMETERS OF THE TETRAQUARK Y: THE MASS mY AND CURRENT COUPLING fY

To evaluate the mass mY and coupling fY of the vector tetraquark Y, we use the QCD two-point sum rule method and start our calculations from analysis of the correlation function

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0| T \{ J^Y_\mu(x) J^{Y\dagger}_\nu(0) \} | 0 \rangle,$$  

(1)

where

$$J^Y_\mu(x) = [u^T_a(x) C \gamma_\nu s_b(x)] [\bar{u}_a(x) \gamma_\mu \gamma_5 \sigma^{\mu\nu} b(x)]$$

-$$[u^T_a(x) C \gamma_\nu s_b(x)] [\bar{u}_a(x) \gamma_\mu \gamma_5 C \sigma^{\mu\nu} b(x)],$$

(2)

is the interpolating current for the Y. Here, a and b are color indices, and C is the charge-conjugation matrix.

To get the sum rules for the mass and coupling of the tetraquark Y, we should express the correlation function in terms of these spectral parameters, and also calculate \Pi_{\mu\nu}(p) using quark-gluon degrees of freedom. The first expression forms the physical side of the sum rules \Pi^{\text{phys}}_{\mu\nu}(p), whereas the second one constitutes their QCD side \Pi^{\text{QCD}}_{\mu\nu}(p). In terms of the tetraquark’s parameters the correlation function has the following form

$$\Pi^{\text{phys}}_{\mu\nu}(p) = \frac{\langle 0| J^Y_\mu | Y(p) \rangle \langle Y(p) | J^{Y\dagger}_\nu | 0 \rangle}{m^2_Y - p^2} + \ldots$$

(3)

Equation (3) is derived by saturating the correlation function with a complete set of P = 1−− states and carrying out integration in Eq. (1) over x. As usual, contributions arising from higher resonances and continuum states are denoted above by dots.

The correlator \Pi^{\text{phys}}_{\mu\nu}(p) can be further simplified if one introduces the matrix element

$$\langle 0| J^Y_\mu | Y(p) \rangle = f_Y m_Y \epsilon_\mu,$$  

(4)
In the formula above get into Eq. (1), and contracting the quark fields, we get
\[ \Pi_{\mu \nu}^{\text{OPE}}(p) = \int d^4x e^{ipx} \left\{ \text{Tr} \left[ \gamma_5 \bar{S}_q^b(x)\gamma_5\gamma_\mu S_q^a(x) \right] \right. \\
\times \left[ \bar{S}_q^b(x)\gamma_\mu S_q^a(x) \right] + \left[ \gamma_5 \bar{S}_q^b(x)\gamma_\mu S_q^a(x) \right] \gamma_5 \\
\times \left[ \bar{S}_q^b(x)\gamma_\mu S_q^a(x) \right] + \left[ \gamma_5 \bar{S}_q^b(x)\gamma_\mu S_q^a(x) \right] \gamma_5 \\
\left. + \left[ \gamma_5 \bar{S}_q^b(x)\gamma_\mu S_q^a(x) \right] \gamma_5 \right\}, \tag{6} \]
where
\[ \bar{S}_q(x) = CS_q^T(x)C. \tag{7} \]
In the formula above \( S_q(x) \) is the light quark propagator, for which we employ the expression
\[ S_q^{\mu \nu}(x) = i \frac{f_{\pi}}{2 F_{\pi}^2} \delta_{\mu \nu} - \frac{m_q}{4 F_{\pi}^2} \delta_{\mu \nu} - \frac{\left< \bar{q}q \right>}{12} \left( 1 - i \frac{m_q f_{\pi}}{4 F_{\pi}^2} \right) \delta_{\mu \nu}, \tag{8} \]
where \( \gamma_E \approx 0.577 \) is the Euler constant and \( \Lambda \) is the QCD scale parameter.

To find the required sum rules, we extract the invariant amplitude \( \Pi_{\mu \nu}^{\text{OPE}}(p^2) \) corresponding to the structure \( g_{\mu \nu} \), and equate it to \( \Pi_{\mu \nu}^{\text{phys}}(p^2) \). We apply the Borel transformation to both sides of the obtained equality, which is necessary to suppress contributions of the higher resonances and continuum states. At the next stage, using an assumption on quark-hadron duality, we carry out the continuum subtraction. After these standard manipulations the sum rule depends on new parameters \( M^2 \) and \( s_0 \), the first of them is \( M^2 \) the Borel parameter generated by the Borel transformation, whereas \( s_0 \) is the continuum threshold parameter that dissects contributions of the ground-state and higher resonances from each other. Remaining operations to find the sum rules for \( m_V \) and \( f_Y \) are similar to ones presented numerously in the literature, and therefore, we skip further details. It is worth noting that calculation of \( \Pi_{\mu \nu}^{\text{OPE}}(p^2) \) in the present article is performed by taking into account nonperturbative terms up to dimension 15.

The obtained sum rules contain various vacuum condensates, and depend on the quark’s mass and on two auxiliary parameters \( M^2 \) and \( s_0 \). Values of the vacuum condensates and the mass of a quark used in numerical computations are collected in Table I. Here, we also write down the parameters of the \( \phi, f_0(980), \eta, \) and \( \eta' \) mesons which are necessary to calculate partial widths of the decay processes.

The condensates characterize nonperturbative features of the vacuum and do not depend on a problem under consideration. On the contrary, the Borel and continuum threshold parameters \( M^2 \) and \( s_0 \) should be chosen for each sum rule computations individually and must meet restrictions imposed on them by the QCD sum rule method. The main constraints on \( M^2 \) and \( s_0 \) are connected with convergence of the operator product expansion (OPE) which we fix by means of the ratio
\[ R(M^2) = \frac{\Pi_{\text{DimN}}^{\text{OPE}}(M^2, s_0)}{\Pi(M^2, s_0)}, \tag{9} \]
and with the restriction on the pole contribution (PC)
\[ \text{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}. \tag{10} \]

In Eqs. (9) and (10) \( \Pi(M^2, s_0) \) is the invariant amplitude \( \Pi_{\mu \nu}^{\text{OPE}}(p^2) \) obtained after the Borel transformation and subtraction procedures, and \( \Pi_{\text{DimN}}^{\text{OPE}}(M^2, s_0) \) denotes a last term (or a sum of last few terms) in OPE. At the minimum of the working window for the Borel parameter, we require \( R(M^2) \approx 0.01 \) and use a sum of three terms.

### Table I: Vacuum condensates and spectroscopic parameters of the mesons used in numerical computations

| Quantity       | Value                           |
|----------------|---------------------------------|
| \( \left< qq \right> \) | \((0.24 \pm 0.01) \) GeV²        |
| \( \left< \bar{q}q \right> \) | \((0.8 \pm 0.1) \) GeV²          |
| \( m_0^2 \)     | \((701.07 \pm 0.01) \) GeV²      |
| \( m_{\pi^0} \)  | \((0.16 \pm 0.01) \) GeV²        |
| \( m_{\eta'} \)  | \((0.68 \pm 0.01) \) GeV²        |
| \( m_{\phi} \)   | \((1.01 \pm 0.01) \) GeV²        |
| \( f_{\pi} \)   | \((1.95 \pm 0.01) \) GeV²        |
| \( f_{\eta} \)   | \((0.95 \pm 0.01) \) GeV²        |
| \( f_{\eta'} \)  | \((0.95 \pm 0.01) \) GeV²        |
| \( f_{\phi} \)   | \((0.95 \pm 0.01) \) GeV²        |

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Dim$N = \dim(13 + 14 + 15)$ to estimate $R(M^2_{\text{min}})$. At maximum allowed value of $M^2$, we demand fulfillment of the condition $PC > 0.2$.

In general, quantities extracted from the sum rules should not depend on $M^2$ and $s_0$. But actual computations demonstrate a residual dependence of results on these parameters. Let us note that uncertainties in the choice of the $M^2$ and $s_0$ are main sources of theoretical errors in the sum rule calculations, which however can be systematically kept under control. So, a minimum sensitivity of the extracted quantities on errors in the sum rule calculations, which however can be.

Numerical analysis allows us to fix the regions

$$M^2 \in [1.2, 1.7] \text{ GeV}^2, \quad s_0 \in [6, 6.5] \text{ GeV}^2$$

as ones which obey the constraints imposed on $M^2$ and $s_0$. Thus, at $M^2 = 1.2 \text{ GeV}^2$ the convergence of the OPE is fulfilled, because a contribution of the last three terms to the Borel transformed and subtracted invariant amplitude $\Pi(M^2, s_0)$ does not exceed 0.3% of its value. At $M^2 = 1.2 \text{ GeV}^2$ the pole contribution forms 60% of $\Pi(M^2, s_0)$, whereas at $M^2 = 1.7 \text{ GeV}^2$ it amounts to approximately 30% of the whole result.

Our results for the spectroscopic parameters of the tetraquark $Y$ are

$$m_Y = (2173 \pm 85) \text{ MeV},$$

$$f_Y = (2.8 \pm 0.5) \times 10^{-3} \text{ GeV}^4. \quad (12)$$

As is seen, the $m_Y$ is in excellent agreement with the datum of the BaBar Collaboration $(2175 \pm 10 \pm 15) \text{ MeV}$. It is lower than the new result $(2200 \pm 6\pm 5) \text{ MeV}$ reported by BESIII, but within errors of calculations is compatible with this measurement as well. In this situation decays of the tetraquark $Y$ become of special interest, because predictions for partial widths of the different channels and for the full width of the $Y$ are important to verify our assumption on its structure.

III. THE DECAY $Y \to \phi f_0(980)$

The process $Y \to \phi f_0(980)$ is one of dominant strong decays of the tetraquark $Y$. To calculate partial width of this channel, we extract the strong coupling $G_{Y\phi f}$ of the vertex $Y\phi f_0(980)$ in the context of the LCSR method and express it in terms of various vacuum condensates and distribution amplitudes (DAs) of the $\phi$ meson.

To derive the light-cone sum rule for the coupling $G_{Y\phi f}$, we start from analysis of the correlation function

$$\Pi_\mu(p, q) = i \int d^4xe^{ipx} \langle \phi(q) | T \{ J^Y_\mu(x) J^Y_\mu(0) \} | 0 \rangle, \quad (13)$$

where $J^Y_\mu(x)$ is the interpolating current of $Y$ introduced in Eq. (2).

As it has been emphasized above, we consider the scalar meson $f_0(980)$ [in formulas we use $f = f_0(980)$] as a pure $H$ state. Interpolating current for such state is given by expression

$$J^Y_\mu(x) = \frac{\epsilon \epsilon'}{\sqrt{2}} \left\{ [u^T_\mu(x) C \gamma_5 s_6(x)] [\bar{\pi}_c(x) \gamma_5 C \sigma^T_c(x)]ight. + \left. [d^T_\mu(x) C \gamma_5 s_6(x)] [\bar{d}_c(x) \gamma_5 C \sigma^T_c(x)] \right\}, \quad (14)$$

where $\epsilon \epsilon' = \epsilon d abc d' c' e'$. Then, the phenomenological side of the sum rule is determined by the formula

$$\Pi^\text{Phys}_\mu(p, q) = \frac{\langle 0 | J^Y_\mu(p) | f(q) \rangle}{p^2 - m^2_Y} \langle f(p) \phi(q) | Y(p') \rangle \times \langle Y(p') | J^Y_\mu(0) | 0 \rangle\frac{1}{p'^2 - m^2_Y} + \cdots, \quad (15)$$
where \( p', \) and \( p, \ q \) are 4-momenta of the initial and final particles, respectively. To simplify \( \Pi_{\mu}^{\text{Phys}}(p,q) \) we express the matrix elements in terms of physical parameters of the particles involved into the decay process. The matrix element \( \langle Y(p')|J_{\mu}|0 \rangle \) is given by Eq. (14), whereas for \( \langle 0|J_{\mu}|f(p) \rangle \) we use

\[
\langle 0|J_{\mu}|f(p) \rangle = F^{\mu}m_{f}.
\]  

(16)

We parameterize the vertex \( \langle f(p)\phi(q)|Y(p') \rangle \) by means of the expression

\[
\langle f(p)\phi(q)|Y(p') \rangle = G_{Y}\Phi \left( (p' \cdot q)(\varepsilon^{*} \cdot \varepsilon') \right) \\
-(q \cdot \varepsilon')(p' \cdot \varepsilon').
\]

(17)

where \( G_{Y}\Phi \) is the strong coupling which should be determined using the sum rule, and \( \varepsilon^{*}_{\mu} \) is the polarization vector of the \( \Phi \) meson. This information on the matrix elements is enough to get the phenomenological side of sum rule which reads

\[
\Pi_{\mu}^{\text{Phys}}(p,q) = G_{Y}\Phi \frac{m_{Y}f_{Y}m_{J}F_{J}}{2(p^{2} - m_{Y}^{2})(p^{2} - m_{J}^{2})} \\
\times \left[ m_{J}^{2} - m_{Y}^{2} - m_{Q}^{2} \right] \varepsilon^{*}_{\mu} + \left( m_{X}^{2} + m_{Y}^{2} - m_{Q}^{2} \right) p \cdot \varepsilon q_{\mu}.
\]

(18)

It is seen that the function \( \Pi_{\mu}^{\text{Phys}}(p,q) \) contains two Lorentz structures which can be employed to derive the required sum rule. In the present study we choose the structure proportional to the polarization vector \( \varepsilon^{*}_{\mu} \).

The second component of the sum rule is obtained by substituting the interpolating currents into the correlation function \( \Pi_{\mu}^{\text{OPE}}(p,q) \) contracting the relevant quark fields, and expressing \( \Pi_{\mu}^{\text{OPE}}(p,q) \) in terms of quarks’ light-cone propagators \( S_{\alpha}(x) \), and distribution amplitudes of the \( \Phi \) meson. After contracting the quark fields the matrix element in Eq. (13) contains numerous terms of the forms

\[
[A(x)]^{ab}_{\alpha\beta} \langle \phi(q)|S_{\alpha}(x)s_{b}(0)|0 \rangle, \\
[B(x)]^{ab}_{\alpha\beta} \langle \phi(q)|S_{\alpha}(0)s_{b}(x)|0 \rangle,
\]

(19)

where \( \alpha, \beta \) are the spinor indices. Here \( A(x) \) and \( B(x) \) are some combinations of the propagators \( S_{\pm}(\pm x), \ S_{\alpha}(\pm x), = C \bar{S}_{\alpha}(\pm x)C, \text{ and } \gamma_{\sigma}(\sigma) \) matrices. In calculations we use the light-cone propagator of the \( u, \ d \), and \( s \) quarks, which is determined by the expression

\[
\begin{align*}
S_{q}^{ab}(x) &= \frac{i}{2\pi^{2}x^{2}} \delta_{ab} - \frac{m_{q}}{4\pi^{2}x^{2}} \delta_{ab} - \frac{\langle q\bar{q} \rangle}{12} \left( 1 - i \frac{m_{q}}{4} \frac{\not{x}}{\not{y}} \right) \delta_{ab} \\
&- \frac{x^{2}}{192} m_{q}^{2} \langle q\bar{q} \rangle \left( 1 - i \frac{m_{q}}{6} \frac{\not{x}}{\not{y}} \right) \delta_{ab} \\
&- \frac{x^{2}}{16} m_{q}^{2} \langle q\bar{q} \rangle \left( 1 - i \frac{m_{q}}{6} \frac{\not{x}}{\not{y}} \right) \delta_{ab} \\
&- ig_{s} \int_{0}^{1} du \left\{ \frac{i}{16\pi^{2}x^{2}} \frac{G_{\mu\nu}(ux)\sigma_{\mu\nu}}{G_{\mu\nu}(ux)\gamma_{\nu}} \\
- \frac{i}{16\pi^{2}x^{2}} \frac{G_{\mu\nu}(ux)\sigma_{\mu\nu}}{G_{\mu\nu}(ux)\gamma_{\nu}} \left[ \ln \left( -\frac{x^{2}A^{2}}{4} \right) + 2\gamma_{E} \right] \right\}.
\end{align*}
\]

(20)

Apart from propagators the function \( \Pi_{\mu}^{\text{OPE}}(p,q) \) depends also on non-local matrix elements of the quark operator \( \bar{s}s \) sandwiched between the vacuum and \( \Phi \) state. To express these matrix elements using the \( \phi \) meson’s distribution amplitudes, we expand \( \bar{s}(0)s(x) \) as well over the full set of Dirac matrices \( \Gamma^{J} \) and project them onto the color-singlet states

\[
\bar{s}_{\alpha}(x)s_{b}(0) \rightarrow \frac{1}{12}\delta_{ab}\Gamma_{\beta\alpha}^{J} \left[ \bar{s}(0)\Gamma^{J}s(0) \right],
\]

(21)

where \( \Gamma^{J} \)

\[
\Gamma^{J} = 1, \gamma_{5}, \gamma_{\mu}, i\gamma_{5}\gamma_{\mu}, \sigma_{\mu\nu}/\sqrt{2}.
\]

(22)

The matrix element of the operators \( \bar{s}(0)\Gamma^{J}s(0) \) can be expanded over \( x^{2} \) and written down in terms of the \( \phi \) meson’s two- and three-particle DAs of different twist. In the case \( \Gamma^{J} = 1 \) and \( i\gamma_{5}\gamma_{\mu} \) we use the definitions

\[
\langle 0|\bar{s}(x)s(0)|\phi(q) \rangle = -if_{\phi}^{\parallel} \varepsilon \cdot x m_{\phi}^{2} \int_{0}^{1} du e^{i\varepsilon x} \psi_{\phi}^{\parallel}(u),
\]

(23)

and

\[
\langle 0|\bar{s}(x)\gamma_{\mu}s(0)|\phi(q) \rangle = f_{\phi}^{\parallel} m_{\phi} \epsilon_{\mu\alpha\beta} \varepsilon^{\nu} q^{\nu} x^{\beta} \\
\times \int_{0}^{1} du e^{i\varepsilon x} \psi_{\phi}^{\parallel}(u).
\]

(24)

For the structures \( \Gamma^{J} = \gamma_{\mu} \) and \( \sigma_{\mu\nu} \) we have

\[
\langle 0|\bar{s}(x)\gamma_{\mu}s(0)|\phi(q) \rangle = f_{\phi}^{\parallel} m_{\phi} \epsilon_{\mu}\frac{\varepsilon \cdot x}{q_{\cdot x}} q_{\mu} \\
\times \int_{0}^{1} du e^{i\varepsilon x} \left[ \phi_{\parallel}^{\parallel}(u) + \frac{m_{\phi}^{2}x^{2}}{4}\phi_{\parallel}^{\perp}(u) \right] \\
+ (\epsilon_{\mu} - q_{\mu}\frac{\varepsilon \cdot x}{q_{\cdot x}}) \int_{0}^{1} du e^{i\varepsilon x} \phi_{\parallel}^{\perp}(u) \\
- \frac{1}{2} x^{2} \frac{\varepsilon \cdot x}{(q_{\cdot x})^{2}} m_{\phi}^{2} \int_{0}^{1} du e^{i\varepsilon x} C(u) + ... \right\},
\]

(25)

and

\[
\langle 0|\bar{s}(x)\sigma_{\mu\nu}s(0)|\phi(q) \rangle = if_{\phi}^{\parallel} \left\{ (\epsilon_{\mu} q_{\nu} - \epsilon_{\nu} q_{\mu}) \right. \\
\times \int_{0}^{1} du e^{i\varepsilon x} \left[ \phi_{\parallel}^{\parallel}(u) + \frac{m_{\phi}^{2}x^{2}}{4}\phi_{\parallel}^{\perp}(u) \right] \\
+ \left( \frac{1}{2} (\epsilon_{\mu} q_{\nu} - \epsilon_{\nu} q_{\mu}) \right) \int_{0}^{1} du e^{i\varepsilon x} \left[ \psi_{\phi}^{\parallel}(u) - \phi_{\parallel}^{\perp}(u) \right] \\
+ (q_{\mu} q_{\nu} - q_{\nu} q_{\mu}) \frac{\varepsilon \cdot x}{(q_{\cdot x})^{2}} m_{\phi}^{2} \int_{0}^{1} du e^{i\varepsilon x} D(u) + ... \right\},
\]

(26)

respectively. Here \( \parallel = 1 - u, \) and \( m_{\phi} \) and \( \varepsilon \) are the mass and polarization vector of the \( \phi \) meson, respectively. In
the equations above the functions $C(u)$ and $D(u)$ denote the combinations of the two-particle DAs
\begin{align*}
C(u) &= \psi_4^\parallel(u) + \phi_4^\parallel(u) - 2\phi_4^\perp(u), \\
D(u) &= \phi_3^\parallel(u) - \frac{1}{2}\phi_3^\parallel(u) - \frac{1}{2}\psi_4^\parallel(u).
\end{align*}
(27)

The twists of the distribution amplitudes are shown as subscripts in the relevant functions. It is seen that the $C(u)$ and $D(u)$ include the two-particle leading twist DAs $\phi_4^\parallel(u)$, the twist-3 distribution amplitudes $\phi_3^\parallel(u)$ and $\psi_4^\parallel(u)$, as well as twist-4 distributions $\phi_4^\perp(u)$ and $\psi_4^\perp(u)$. Expressions of the matrix elements $\langle 0|\Phi(x)\Gamma^\mu G_{\mu\nu}(vx)s(0)|\phi(q)\rangle$ in terms of the higher twist DAs of the $\phi$ meson, as well as detailed information on their properties were reported in Refs. [27, 31].

The explicit expression of the $\Pi_{\mu}^{\text{OPE}}(p, q)$ as a function of the $\phi$ meson’s DAs is cumbersome, therefore we do not provide it here. In our analysis we employ the invariant amplitude $\Pi_{\mu}^{\text{OPE}}(p^2, p^2)$ proportional to $\epsilon_\mu^\nu$ and match it to corresponding function from $\Pi_{\mu}^{\text{phys}}(p, q)$. These amplitudes depend on $p^2$ and $p^2$, therefore one should perform the double Borel transformation over the variables $p^2$ and $p'^2$

$$
\Pi_{\text{OPE}}(M_1^2, M_2^2) = B_{p^2} M_1^2 B_{p'^2} M_2^2 \Pi_{\text{OPE}}(p^2, p'^2).
$$
(28)

The Borel transformed amplitude $\Pi_{\text{OPE}}(M_1^2, M_2^2)$ can be calculated in accordance with a scheme explained in Ref. [32], and expressed as a double dispersion integral. To simplify manipulations following after the Borel transformation, we can relate the parameters $M_1^2$ and $M_2^2$ to each other using $M_1^2 M_2^2 = \frac{m_1^2 m_2^2}{m_1^2}$ and introduce a common parameter $M^2$ through the relation

$$
\frac{1}{M^2} = \frac{1}{M_1^2} + \frac{1}{M_2^2}.
$$
(29)

This implies replacements

$$
M_1^2 = \frac{m_1^2 + m_2^2}{m_1^2} M^2,
$$
$$
M_2^2 = \frac{m_1^2 + m_2^2}{m_1^2} M^2,
$$
(30)

in the sum rules, and allows us to perform integration over one of variables in the double dispersion integral. The obtained expressions in this step depend also on the parameter $u_0$ with

$$
u_0 = \frac{M_1^2}{M_1^2 + M_2^2} = \frac{m_1^2}{m_1^2 + m_2^2}.
$$
(31)

As a result of the above procedure we get a single integral representation for $\Pi_{\text{OPE}}(M^2)$ which considerably simplifies the continuum subtraction: formulas necessary to carry out this procedure can be found in Appendix B of Ref. [32].

The DAs of the $\phi$ meson depend on numerous parameters. For example, the leading twist DAs of the longitudinally and transversely polarized $\phi$ meson are given by the expression

$$
\phi_2^\parallel(\perp)(u) = 6u\pi \left[ \sum_{n=2}^{\infty} a_n^\parallel(\perp) C_n^{3/2}(2u - 1) \right],
$$
(32)

where $C_n^{m}(2u - 1)$ are the Gegenbauer polynomials. Equation (32) is the general expression for $\phi_2^\parallel(\perp)(u)$. In our calculations we employ twist-2 DAs with a non-asymptotic term $a_2^\parallel(\perp) \neq 0$. The models for the higher twist DAs, and values of the relevant parameters at the normalization scale $\mu_0 = 1$ GeV are taken from Refs. [30, 31] (see Tables 1 and 2 in Ref. [31]).

The sum rule for the coupling $G_{Y \phi f}$ contains the quark, gluon and mixed condensates and the $s$-quark mass which are moved to Table I. The spectroscopic parameters of the particles involved into the decay $Y \to \phi f_0(980)$ are also input information of computations. The mass and coupling of the tetraquark $Y$ have been evaluated in the present work. For the mass of the $\phi$ and $f_0(980)$ mesons we use their experimental values (see Table I). The coupling $F_f$ of the meson $f_0(980)$ is borrowed from Ref. [19], where it was treated as the four-quark system

$$
F_f \equiv F_H = (1.35 \pm 0.34) \times 10^{-3} \text{ GeV}^4.
$$
(33)

Finally, the sum rule depends on the Borel and continuum threshold parameters: $M^2$ and $s_0$ are auxiliary parameters of computations, and the result should be insensitive to their choices. But in real analysis we can only minimize these effects and fix convenient working windows for the $M^2$ and $s_0$

$$
M^2 \in [2.4, 3.4] \text{ GeV}^2, \ s_0 \in [6, 6.5] \text{ GeV}^2.
$$
(34)

In accordance with our studies the strong coupling $G_{Y \phi f}$ is equal to

$$
G_{Y \phi f} = (1.62 \pm 0.41) \text{ GeV}^{-1}.
$$
(35)

The width of the decay $Y \to \phi f_0(980)$ is determined by the expression

$$
\Gamma(Y \to \phi f) = \frac{G_{Y \phi f} m_\phi^2}{24\pi} \lambda \left( 3 + \frac{2\lambda^2}{m_\phi^2} \right),
$$
(36)

where

\begin{align*}
\lambda &\equiv \lambda(m_Y, m_\phi, m_f) = \frac{1}{2m_Y} \left[ m_Y^4 + m_\phi^4 + m_f^4 \\
&\quad - 2(m_\phi^2 m_Y^2 + m_Y^2 m_f^2 + m_\phi^2 m_f^2) \right]^{1/2}.
\end{align*}
(37)

Then computations yield

$$
\Gamma(Y \to \phi f) = (49.2 \pm 17.6) \text{ MeV}.
$$
(38)

The prediction for $\Gamma(Y \to \phi f)$ is the main result of this section which will be used to estimate the full width of the tetraquark $Y$. 


IV. THE DECAYS $Y \to \phi \eta$ AND $Y \to \phi \eta'$

The next two strong decays of the tetraquark $Y$ are the channels $Y \to \phi \eta$ and $Y \to \phi \eta'$. Here, we consider in a detailed form the dominant process $Y \to \phi \eta$, and write down final results for the second mode $Y \to \phi \eta'$. In the framework of the LCSR method the correlation function necessary to study the vertex $Y \phi \eta$ is given by the expression

$$\Pi_{\mu\nu}(p, q) = i \int d^4x e^{ipx} \langle \eta(q) | T \{ J^\phi_\mu(x) J^Y_\nu(0) \} | 0 \rangle,$$  

where $J^\phi_\mu(x)$ is the interpolating current for the vector meson

$$J^\phi_\mu(x) = \pi(x) \gamma_\mu s(x).$$  

(40)

The phenomenological side of the sum rule can be written down in the form

$$\Pi^{\text{phys}}_{\mu\nu}(p, q) = \frac{\langle 0 | J^\phi_\mu | \phi(p) \rangle}{p^2 - m^2_\phi} \langle \phi(p) \eta(q) | Y(p') \rangle \times \langle Y(p') | J^Y_\nu(0) \rangle \frac{1}{p^2 - m^2_Y} + \cdots,$$  

(41)

and simplified further using the matrix elements:

$$\langle 0 | J^\phi_\mu | \phi(p) \rangle = f_\phi m_\phi \epsilon^\mu_\phi,$$  

(42)

and

$$\langle \phi(p) \eta(q) | Y(p') \rangle = g_{Y \phi \eta} \epsilon_{\mu \nu \alpha \beta} p^\mu q^\nu \epsilon^* \epsilon^\alpha, \epsilon^\beta,$$  

(43)

where $\epsilon^\alpha$ is the polarization vector of the tetraquark $Y$, and $g_{Y \phi \eta}$ is the strong coupling corresponding to the vertex $Y \phi \eta$.

Simple manipulations allow us to recast $\Pi^{\text{phys}}_{\mu\nu}(p, q)$ into the form

$$\Pi^{\text{phys}}_{\mu\nu}(p, q) = g_{Y \phi \eta} \frac{f_\phi m_\phi f_Y m_Y}{(p^2 - m^2_\phi)(p'^2 - m^2_Y)} \epsilon_{\mu \nu \alpha \beta} P^\alpha q^\beta + \cdots,$$  

(44)

where the only term is the contribution arising from the ground-state particles: effects of the higher resonances and continuum states are denoted by dots. The correlation function $\Pi^{\text{phys}}_{\mu\nu}(p, q)$ has a simple Lorentz structure. The invariant amplitude $\Pi^{\text{phys}}_{\mu\nu}(p^2, p'^2)$, which will be used to derive the sum rule for the coupling $g_{Y \phi \eta}$, can be obtained from Eq. (44) by factoring out the structure $\epsilon_{\mu \nu \alpha \beta} P^\alpha q^\beta$.

We extract the second component of the sum rule, i.e., the invariant amplitude $\Pi^{\text{OPE}}_{\mu\nu}(p^2, p'^2)$ from the correlation function $\Pi^{\text{OPE}}_{\mu\nu}(p, q)$. In the case under analysis it is given by the following expression

$$\Pi^{\text{OPE}}_{\mu\nu}(p, q) = -i \int d^4x e^{ipx} \left[ \gamma_5 \bar{S}_b^\nu(x) \gamma_\mu \bar{S}_a^\mu(-x) \gamma_5 \gamma_\nu \right] \langle \eta(q) | \pi_\alpha(0) u^*_\beta(0) | 0 \rangle + \gamma_5 \gamma_5 \bar{S}_b^\nu(x) \gamma_\mu \bar{S}_a^\mu(-x) \gamma_5 \right]_{\alpha \beta} \langle \eta(q) | \pi_\alpha(0) u^*_\beta(0) | 0 \rangle.$$  

(45)

As is seen, the correlation function is written down in terms of the quark propagators and local matrix elements of the $\eta$ meson. Dependence of $\Pi^{\text{OPE}}_{\mu\nu}(p, q)$ on the local matrix elements of a final meson is the distinctive feature of the LCSR method when applied to tetraquark-meson-meson vertices. Treatment of such vertices requires some additional manipulations, which we are going to explain below. But before that we have to find $\Pi^{\text{OPE}}_{\mu\nu}(p, q)$ by rewriting the matrix elements $\langle \eta(q) | \pi_\alpha(0) u^*_\beta(0) | 0 \rangle$ in terms of the $\eta$ meson’s parameters. To this end, we expand $\pi_\alpha(0) u^*_\beta(0)$ and determine the standard matrix elements of the $\eta$ meson that contribute to the correlation function. These operations have been discussed in the previous section, therefore here we omit further details.

Performed analysis shows that the matrix element $\langle \eta(q) | \pi_\alpha(0) \gamma_5 u(0) \rangle$ contributes to the correlation function $\Pi^{\text{OPE}}_{\mu\nu}(p, q)$. It is defined by the formula

$$\langle \eta(q) | \pi_\alpha \gamma_5 u(0) \rangle = -i \frac{f_\eta}{\sqrt{2} \eta},$$  

(46)

where $f_\eta$ is the decay constant of the $\eta$ meson’s “$q$” component. The matrix element (46) differs from similar expressions of other pseudoscalar mesons. This is connected with the mixing in the $\eta - \eta'$ system which can be described using either the octet-singlet or quark-flavor basis of the flavor $SU_f(3)$ group. The latter is more convenient and simple for applications, and was used in Refs. 33, 34 to explore different exclusive processes. This scheme is utilized in the present work as well.

In the quark-flavor basis the decay constants of the mesons $\eta$ and $\eta'$ can be extracted from the equality

$$\begin{pmatrix} f_{\eta} & f_{\eta'} \\ f_{\eta'} & f_{\eta} \end{pmatrix} = U(\varphi) \begin{pmatrix} f_s & 0 \\ 0 & f_s \end{pmatrix},$$  

(47)

where $U(\varphi)$ is the mixing matrix

$$U(\varphi) = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix}.$$  

(48)

In Eqs. (47) and (48) the constants $f_s$ and $f_s$ are given by the formulas

$$f_\eta = (1.07 \pm 0.02) f_\pi,$$  

$$f_s = (1.34 \pm 0.06) f_\pi,$$  

(49)

where $f_\pi = 131$ MeV is the pion decay constant, and $\varphi = 39^\circ.3 \pm 1^\circ.0$ is the mixing angle in the quark-flavor basis.

Using Eqs. (45) and (46), we can obtain the invariant amplitude $\Pi^{\text{OPE}}_{\mu\nu}(p^2, p'^2)$ which should be equated to $\Pi^{\text{phys}}_{\mu\nu}(p^2, p'^2)$ in order to derive the sum rule for the strong coupling $g_{Y \phi \eta}$. But, as we have been emphasized above, a treatment of tetraquark-meson-meson vertices in the context of the LCSR method differs from standard analysis of the previous section [28]. In fact, the LCSR for
vertices of conventional mesons depends on distribution amplitudes of one of final mesons, which contain all information about nonperturbative dynamical features of the meson. The same arguments are valid for the tetraquark-tetraquark-meson vertices as well [32]. But in the case of the tetraquark-meson-meson vertices, after contracting relevant quark fields, due to a four-quark structure of the tetraquark, the correlation function contains only local matrix elements one of final mesons [33]. Then the momentum of this meson should be set \( q = 0 \) which is necessary to satisfy the four-momentum conservation at the vertex. This leads to essential modifications in the calculational scheme, because now we have to complete the LCSR method with technical methods of the soft-meson approximation [24, 26].

In the soft limit \( q \to 0 \), we get \( p' = p \), as a result we have to perform one-variable Borel transformation of the invariant amplitudes [24]. For the physical (phenomenological) side this leads to the formula

\[
\mathcal{B}_\text{P}(p^2) = g_{Y\phi\eta} f_\phi m_\phi f_Y m_Y \frac{e^{-m^2/M^2}}{M^2} + \cdots,
\]

where \( m^2 = (m_\phi^2 + m_Y^2)/2 \).

In the soft-meson approximation the phenomenological side of the sum rule has a more complicated organization than in the case of full LCSR method. The reason is that in the soft limit contributions connected with higher resonances and continuum states demonstrate complicated behavior. Indeed, some of these terms even after the Borel transformation remain unsuppressed and appear as contaminations in the phenomenological side [24]. Therefore, before carrying out the continuum subtraction they should be excluded from \( \mathcal{B}_\text{P}(p^2) \) by means of some manipulations. This problem is solved by acting on the phenomenological side of the sum rule by the operator [24, 25]

\[
\mathcal{P}(M^2, m^2) = \left( 1 - M^2 \frac{d}{dM^2} \right) M^2 e^{-m^2/M^2},
\]

that eliminates contaminating terms. Then contributions of higher resonances with regular behavior can be subtracted from the QCD side by benefitting from the quark-hadron duality assumption.

The operator \( \mathcal{P}(M^2, m^2) \) should also be applied to the QCD side of the sum rule. Then the strong coupling \( g_{Y\phi\eta} \) can be determined from the sum rule

\[
g_{Y\phi\eta} = \frac{1}{f_\phi m_\phi f_Y m_Y} \mathcal{P}(M^2, m^2) \Pi^\text{OPE}(M^2, s_0),
\]

where \( \Pi^\text{OPE}(M^2, s_0) \) is the invariant amplitude \( \Pi^\text{OPE}(p^2) \) after the Borel transformation and continuum subtraction procedures. Our calculations carried out by taking into account nonperturbative terms up to dimension 5 yield

\[
\Pi^\text{OPE}(M^2, s_0) = \frac{f_\phi^2 m_s}{8\sqrt{2}\pi^2} \int_{4m_s^2}^{s_0} ds e^{-s/M^2} + \frac{f_\phi^2 m_s^2}{6\sqrt{2}M^2} (s) + \frac{f_\phi^2 m_s^3}{12\sqrt{2}M^2} (s) (s_g G). \tag{53}
\]

The width of the decay \( Y \to \phi\eta \) is given by the following expression

\[
\Gamma(Y \to \phi\eta) = \frac{g_{Y\phi\eta}^2}{12\pi} (m_Y, m_\phi, m_\eta), \tag{54}
\]

Numerical analysis leads to the results

\[
g_{Y\phi\eta} = (1.85 \pm 0.38) \text{ GeV}^{-1}, \quad \Gamma(Y \to \phi\eta) = (35.8 \pm 10.4) \text{ MeV}. \tag{55}
\]

It is worth noting that in computations of \( g_{Y\phi\eta} \), we have used the following working regions for \( M^2 \) and \( s_0 \)

\[
M^2 \in [1.3, 1.8] \text{ GeV}^2, \quad s_0 \in [6, 6.5] \text{ GeV}^2. \tag{56}
\]

The partial width of the second process \( Y \to \phi\eta' \) can be computed by utilizing the expressions obtained for the first decay. The corrections are connected with mass of the \( \eta' \) meson and coupling \( f_{\eta'}^0 \), and required replacements

\[
f_{\eta'}^0 = f_\phi \sin \varphi, \quad \lambda \to \lambda(m_Y, m_\phi, m_\eta'), \tag{57}
\]

can be easily implemented into analysis. For the parameters of the second process we obtain

\[
g_{Y\phi\eta'} = (1.59 \pm 0.31) \text{ GeV}^{-1}, \quad \Gamma(Y \to \phi\eta') = (6.1 \pm 1.7) \text{ MeV}. \tag{58}
\]

Saturating the full width of the \( Y \) resonance by three decay channels considered in the present work, we get

\[
\Gamma_{\text{full}} = (91.1 \pm 20.5) \text{ MeV}. \tag{59}
\]

This estimate coincides neither with the BaBar data nor with measurements of the BESIII Collaboration, but is close to the latter.

V. ANALYSIS AND CONCLUDING NOTES

We have explored the resonance \( Y \) by modeling it as a light vector tetraquark with the content \( [ss][SU] \). In the tetraquark model it was considered till now as a vector \( [ss][SS] \) or \( (S)(SS) \) particles. Our treatment is motivated by the dominant decay channel \( Y \to \phi f_0(980) \) of the \( Y \), where it was observed as a resonant structure in the \( \phi f_0(980) \) invariant mass distribution. A suggestion on the quark content of the \( Y \) depend on the structures of the final-state particles: one can consider the \( f_0(980) \) either as a scalar meson \( SS \) or as a particle composed of the four valence quarks. In the second picture the
vector compound $Y = [su][su]$ emerges as a quite natural assignment for this resonance. Calculations carried out in the present work lead to the following predictions for $m_Y$ and $\Gamma_{\text{full}}$ of such state

$$m_Y = (2173 \pm 85) \text{ MeV}, \quad \Gamma_{\text{full}} = (91.1 \pm 20.5) \text{ MeV},$$

(60)

The result for the mass $m_Y$ is in accord with the BaBar data, but is compatible with BESIII measurements as well. The full width $\Gamma_{\text{full}}$ has the small overlapping region with $\Gamma = (58 \pm 16 \pm 20) \text{ MeV}$ extracted in Ref. [1], but agreement with data of the BESIII Collaboration is considerably better. In calculations of the $\Gamma_{\text{full}}$, we have taken into account only three strong decays of the resonance $Y$. But decay modes $Y \to \phi\pi\pi$, $K^+K^-\pi^+\pi^-$, $K^*(892)^0\overline{K}(892)^0$ of $Y$ (seen experimentally and/or theoretically possible) and other channels have not been included into analysis. Partial width of these decays may significantly improve the present estimate for $\Gamma_{\text{full}}$.

Encouraging is our estimate for the ratio

$$\frac{\Gamma(Y \to \phi\eta)}{\Gamma(Y \to \phi f)} \approx 0.73,$$

which almost coincides with its experimental value $\approx 0.74$. The latter has been extracted from an available information on the ratios [36] $[Y$ is denoted there $\phi(2170)]$

$$\frac{\Gamma(Y \to \phi\eta) \times \Gamma(Y \to e^+e^-)}{\Gamma_{\text{total}}} = 1.7 \pm 0.7 \pm 1.3, \quad (61)$$

and

$$\frac{\Gamma(Y \to \phi f) \times \Gamma(Y \to e^+e^-)}{\Gamma_{\text{total}}} = 2.3 \pm 0.3 \pm 0.3. \quad (62)$$

Unfortunately, precision of the experimental data and uncertainties of the theoretical results do not allow us to make more strong statements about decay modes of the tetraquark $Y$.

As is seen, our suggestion on a nature of the resonance $Y(2175)$ as the vector tetraquark with the content $[su][su]$ has led to reasonable agreements with existing experimental data. Theoretical analyses of decay channels left beyond the scope of the present work, as well as their detailed experimental studies will be of great help to answer open questions about the structure of the resonance $Y(2175)$. 

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