Josephson junction with magnetic-field tunable current-phase relation

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(Dated: August 21, 2012)

Abstract

We consider a 0-π Josephson junction consisting of asymmetric 0 and π regions of different lengths $L_0$ and $L_{\pi}$ having different critical current densities $j_{c,0}$ and $j_{c,\pi}$. If both segments are rather short, the whole junction can be described by an effective current-phase relation for the spatially averaged phase $\psi$, which includes the usual term $\propto \sin(\psi)$, a negative second harmonic term $\propto \sin(2\psi)$ as well as the unusual term $\propto H \cos \psi$ tunable by magnetic field $H$. Thus one obtains an electronically tunable current-phase relation. At $H = 0$ this corresponds to the $\varphi$ Josephson junction.

PACS numbers: 74.50.+r, 85.25.Cp

Keywords: $\varphi$ Josephson junction
I. INTRODUCTION

Recently we proposed\cite{1} to implement a ϕ Josephson junction (JJ)\cite{2} with magnetic-field tunable current-phase relation (CPR) based on an 0–π JJ with the 0 and π segments of different length $L_0 \neq L_\pi$. This proposal was made keeping in mind YBa$_2$Cu$_3$O$_{7-\delta}$-Nb ramp zigzag JJ technology\cite{3,4} (or a similar one\cite{5} with Nd$_{2-x}$Ce$_x$CuO$_4$-Nb) established recently in our group also\cite{6}. However, in experiment we were more successful\cite{7} in employing superconductor-insulator-ferromagnet-superconductor (SIFS) 0-π JJ\cite{8-10}, where the lengths of 0 and π segments are equal, but critical current densities $j_{c,0}$ and $j_{c,\pi}$ in the 0 and π parts are different.

Therefore, in this paper we present a more general theory, which describes an effective ϕ JJ made of asymmetric 0 and π regions of different lengths $L_0$ and $L_\pi$ having different critical current densities $j_{c,0}$ and $j_{c,\pi}$.

II. MODEL

The static sine-Gordon equation that describes the behavior of the Josephson phase $\phi$ in a 0–π JJ is

$$\frac{\Phi_0}{2\pi\mu_0 d_J} \phi'' - j_c(x) \sin \phi = -j. \quad (1)$$

Here $\mu_0$ is the magnetic flux quantum, $\mu_0 d_J$ is the specific inductance (per square) of the superconducting electrodes forming the JJ and $j$ is the bias current density. The prime denotes the partial derivatives with respect to coordinate $x$. We assume that the critical current density $j_c(x)$ has the form of a step-function

$$j_c = j_{c,0} > 0, \quad 0 \leq x \leq L_0, \quad (2)$$
$$j_c = j_{c,\pi} < 0, \quad -L_\pi \leq x < 0. \quad (3)$$

We write the critical current density $j_c(x)$ as

$$j_c(x) = \langle j_c(x) \rangle [1 + g(x)], \quad (4)$$

where

$$\langle j_c \rangle = \frac{1}{L} \int_{-L_\pi}^{L_0} j_c(x) \, dx = \frac{1}{L} (j_{c,0} L_0 + j_{c,\pi} L_\pi) \quad (5)$$
is the average critical current density, \(L = L_0 + L_π\) is the total length of the junction, and \(\langle g(x) \rangle = 0\). The function \(g(x)\) is defined as

\[
g(x) = \frac{j_c(x)}{\langle j_c(x) \rangle} - 1
\]

that results in

\[
g(x) = \begin{cases} 
  g_0, & 0 < x < L_0, \\
  g_π, & -L_π < x < 0.
\end{cases}
\]

where

\[
g_0 = \frac{(j_{c,0} - j_{c,π})L_π}{j_{c,0}L_0 + j_{c,π}L_π}, \quad g_π = -\frac{(j_{c,0} - j_{c,π})L_0}{j_{c,0}L_0 + j_{c,π}L_π}.
\]

Then we divide Eq. (1) by \(\langle j_c \rangle\) and normalize the coordinate \(x\) to the Josephson length calculated using \(\langle j_c \rangle\), i.e.,

\[
\lambda_J = \sqrt{\frac{\Phi_0}{2\pi\mu_0d_J\langle j_c \rangle}}.
\]

Thus, we obtain a normalized sine-Gordon equation for the phase difference \(φ(x)\)

\[
φ'' - \text{sgn}(\langle j_c \rangle) [1 + g(x)] \sin φ = -γ,
\]

where \(γ = j/\langle j_c \rangle\) is the normalized bias current density. It is worth mentioning that \(\langle j_c \rangle\) can be positive as well as negative. Below, for the same of simplicity, we assume \(\langle j_c \rangle > 0\). Thus, Eq. (10) becomes

\[
φ'' - [1 + g(x)] \sin φ = -γ.
\]

In the case \(\langle j_c \rangle < 0\) the substitution \(φ \to π - φ\) converts Eq. (10) to the same Eq. (11).

We look for a solution of Eq. (11) in the form

\[
φ(x) = ψ + ξ(x) \sin ψ,
\]

where

\[
ψ = \langle φ(x) \rangle
\]

is a constant average phase, while \(ξ(x) \sin ψ\) describes the deviation of the phase from the average value, i.e., \(ξ(x) = 0\). Further we assume that the deviation is small, i.e., \(|ξ(x) \sin ψ| \ll 1\). Then we plug the relation (12) into Eq. (11), expand it in series in \(ξ(x) \sin ψ\), and keep the terms of zero and first order. We get

\[
ξ'' \sin ψ - [1 + g(x)](1 + ξ(x) \cos ψ) \sin ψ = -γ.
\]
The constant terms (zero order of $\xi$ in Eq. (14)) are

$$\gamma = \sin \psi + \langle \xi(x)g(x) \rangle \cos \psi \sin \psi. \quad (15)$$

The terms of first order of $\xi(x)$ in Eq. (14) are

$$\xi'' - g(x) = \{\xi + \xi(x)g(x) - \langle \xi(x)g(x) \rangle \} \cos \psi. \quad (16)$$

Numerical calculations show that the two terms $\propto \cos \psi$ have an extremely weak effect on solutions of Eq. (16). We neglect these terms and obtain for $\xi(x)$

$$\xi'' - g(x) = 0. \quad (17)$$

We treat solutions of Eq. (17) by using the matching continuity (at $x = 0$) and boundary (at $x = -l_\pi \equiv -L_\pi/\lambda_J$, $x = l_0 \equiv L_0/\lambda_J$) conditions

$$\xi_\pi(0) = \xi_0(0), \quad \xi'_\pi(0) = \xi'_0(0), \quad (18)$$

$$\xi'_\pi(-l_\pi) \sin \psi = h, \quad \xi'_0(l_0) \sin \psi = h. \quad (19)$$

The applied field $H$ is normalized by $H_{c1}/2$, i.e.,

$$h = \frac{2H}{H_{c1}}, \quad H_{c1} = \frac{\Phi_0}{\pi \Lambda \lambda_J}, \quad (20)$$

where $\Lambda$ is the effective magnetic thickness of the JJ. We integrate Eq. (17) once and obtain

$$\xi'_0(x) = g_0(x - l_0) + \frac{h}{\sin \psi}, \quad 0 < x < l_0, \quad (21)$$

$$\xi'_\pi(x) = g_\pi(x + l_\pi) + \frac{h}{\sin \psi}, \quad -l_\pi < x < 0. \quad (22)$$

The second integration results in

$$\xi_0(x) = g_0 \left( \frac{x^2}{2} - l_0x \right) + \frac{hx}{\sin \psi} + C, \quad \text{for} \ 0 < x < l_0, \quad (23)$$

$$\xi_\pi(x) = g_\pi \left( \frac{x^2}{2} + l_\pi x \right) + \frac{hx}{\sin \psi} + C, \quad \text{for} \ -l_\pi < x < 0. \quad (24)$$

The integration constant $C$ can be obtained using the condition $\langle \xi(x) \rangle = 0$

$$C = \frac{l_0 - l_\pi}{2} \left( \frac{g_0 l_0 + g_\pi l_\pi}{3} - \frac{h}{\sin \psi} \right). \quad (25)$$
We use Eqs. (7), (23), and (24) and obtain the average $\langle \xi(x)g(x) \rangle$ in the form

$$\langle \xi(x)g(x) \rangle = \Gamma_0 + \Gamma_h \frac{h}{\sin \psi},$$

where the coefficients $\Gamma_0$ and $\Gamma_h$ are given by

$$\Gamma_0 = -\frac{l_0^2 l_{\pi}^2}{3} \left( j_{c,0} - j_{c,\pi} \right)^2,$$

$$\Gamma_h = \frac{l_0 l_{\pi}}{2} \frac{j_{c,0} - j_{c,\pi}}{j_{c,0} l_0 + j_{c,\pi} l_{\pi}}.$$  

(27)

(28)

Using Eqs. (15) and (26) we find the current-phase relation in the form

$$j = \langle j_c \rangle \left( \sin \psi + \Gamma_0 \sin \psi \cos \psi + h \Gamma_h \cos \psi \right).$$

(29)

It is worth noting that there is a simple relation between the coefficients $\Gamma_0$ and $\Gamma_h$. Indeed, it follows from Eqs. (27) and (28) that

$$\Gamma_0 = -\frac{4}{3} \Gamma_h^2.$$  

(30)

In the case of equal lengths of 0 and $\pi$ parts ($l_0 = l_{\pi} = l/2$) we find

$$\Gamma_0 = -\frac{l^2}{12} \left( \frac{j_{c,0} - j_{c,\pi}}{j_{c,0} + j_{c,\pi}} \right)^2, \quad \Gamma_h = \frac{l}{4} \frac{j_{c,0} - j_{c,\pi}}{j_{c,0} + j_{c,\pi}}.$$  

(31)

The energy $U(\psi)$ corresponding to the current-phase relation (29) is given by

$$U(\psi) = \langle j_c \rangle \left( 1 - \cos \psi + h \Gamma_h \sin \psi + \frac{\Gamma_0}{2} \sin^2 \psi \right).$$  

(32)

III. CONCLUSIONS

We have extended our previous results\textsuperscript{1} to the case of arbitrary critical current densities $j_{c,0} \neq j_{c,\pi}$ more relevant for experiment\textsuperscript{2}. The dependence (29) of the CPR on the phase and applied field is the same as in our previous study\textsuperscript{1}. The difference is in the formulas (28) for $\Gamma_0$ and $\Gamma_h$.

\footnotesize

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