Massive Decaying $\tau$ Neutrino and Big Bang Nucleosynthesis

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Abstract

Comparing Big Bang Nucleosynthesis predictions with the light element abundances inferred from observational data, we can obtain the strong constraints on some neutrino properties, e.g. number of neutrino species, mass, lifetime. Recently the deuterium abundances were measured in high red-shift QSO absorption systems. It is expected that they are close to the primordial values, however, two groups have reported inconsistent values which are different in one order of magnitude. In this paper we show how we can constrain on $\tau$ neutrino mass and its lifetime in each case when we adopt either high or low deuterium data. We find that if $0.01 \mathrm{sec} \lesssim \tau_{\nu_{\tau}} \lesssim 1 \mathrm{sec}$ and $10 \mathrm{MeV} \lesssim m_{\nu_{\tau}} \lesssim 24 \mathrm{MeV}$, the theoretical predictions agree with the low D/H abundances. On the other hand if we adopt the high D/H abundances, we obtain the upper bound of $\tau$ neutrino mass, $m_{\nu_{\tau}} \lesssim 20 \mathrm{MeV}$.

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1 Introduction

It is a very important problem for particle physicists and astrophysicists whether a neutrino has a finite mass or not. Neutrinos are likely massive since it is known that neutrinos should have a mass to solve some astrophysical problems such as solar neutrino problem (e.g., \[1\]). Among three species of neutrinos, it is presumed that $\tau$ neutrino is heaviest. The present experimental upper bound on the mass of $\tau$ neutrino is \[2\]

$$m_{\nu_\tau} < 24 \text{ MeV} \quad (95\% \text{ C.L.}).$$

On the other hand, the mass of a stable $\tau$ neutrino should be less than about 100eV otherwise its cosmic density exceeds the critical density of the universe. Therefore massive $\tau$ neutrino with mass between 100eV and 24MeV must be unstable, i.e., decays into other particles. However the radiative decay of a massive neutrino is stringently constrained by laboratory experiments, astrophysics and cosmology and only non-radiative decay is allowed.

Big Bang Nucleosynthesis can strongly constrain on such neutrino properties, in particular the number of neutrino species, mass, lifetime, chemical potential and so on, when we compare the theoretical prediction of light elements ($^4\text{He}$, $\text{D}$, $^3\text{He}$ and $^7\text{Li}$) with the primordial value inferred from the observational data. For example, since neutrino is one of the dominant components of the energy density in the universe at the BBN epoch, the change of neutrino energy density affects the Hubble expansion rate and alters the freeze out time of neutron to proton ratio. Then the predicted light element abundances, especially $^4\text{He}$ in this case, are sensitively changed. More precisely we measure the primordial component of light element abundances, more strongly we constrain on such neutrino properties.

Recently D abundances were measured in high red-shift QSO absorption systems which would not suffer from the effects of the chemical evolution and would have D abundance which is closer to the primordial value. However, two groups have reported inconsistent D/H values which are different in one order of magnitude. Since we do not have any definite models of the galactic and stellar chemical evolutions, it may be premature for us to judge which measurements are more correct.

If the high D abundances, $\text{D}/H = (1.9\pm0.5) \times 10^{-4}$ \[3\], in high red-shift QSO absorption systems are adopted as the primordial values, since they agree with other light element measurements, we can obtain the rigid constraints for some model parameters. On the other hand, if the low D abundances, $\text{D}/H = (3.39\pm0.25) \times 10^{-5}$ \[4, 5\], are adopted, there
is a discrepancy between the standard BBN theory and the observational measurements. This leads to the similar problem which was pointed out by Hata et al. (1995) though they used the data for D and $^3$He in solar neighborhood and a simple chemical evolution model to infer the primordial abundances. They found that theoretical predictions agree with the observational data if the number of neutrino species $N_\nu$ is less than 2.6. The point is that the prediction of the standard ($N_\nu = 3$) BBN for $^4$He is too large if the photon-baryon ratio $\eta$ is chosen to fit the D data (or the prediction of D is too large if $\eta$ is determined by $^4$He data). In this case, we should look for non-standard scenarios.

There are a couple of ideas proposed to solve the discrepancy. One can reduce the abundance of $^4$He by assuming non-standard properties of neutrinos (e.g. degenerate electron neutrino [7], massive neutrino) or can decrease the D abundance by decays of exotic particles [8]. In particular, it was pointed out [9, 10] that a massive and short lived $\tau$ neutrino with mass $O$(MeV) and lifetime about 0.1 sec. can reduce the effective number of neutrino species and hence the abundance of $^4$He. In this paper we study the effect of a massive and short lived $\tau$ neutrino on the BBN with a statistical analysis. Compared with the recent observational data, especially adopting the low D/H in high z QSO absorption systems as a deuterium value, we find that if $0.01 \text{sec} \lesssim \tau_{\nu_\tau} \lesssim 1 \text{sec}$ and $10 \text{MeV} \lesssim m_{\nu_\tau} \lesssim 24 \text{MeV}$ the theoretical predictions agree with the all observational data. In the case where we choose the high D abundance, we find the upper bound of the $\tau$ neutrino mass, $m_{\nu_\tau} \lesssim 20 \text{MeV}$.

2 Cosmological Evolution of Massive Neutrino

We study the BBN effects of the decaying $\tau$ neutrino by solving a set of Boltzmann equations which describe the evolution of momentum distributions of the $\tau$ neutrino and its decay products. A Dirac $\nu_\tau$ with mass larger than few tens of keV is not allowed since it cools SN1987A too fast. Thus, in this paper we only consider a Majorana $\tau$ neutrino. We take into account the three interactions, (1) decay process, (2) pair annihilation and creation process, and (3) scattering process. Concerning the decay process, we suppose that a massive Majorana $\tau$ neutrino with mass $m_{\nu_\tau}$ and lifetime $\tau_{\nu_\tau}$, decays into a $\mu$
neutrino and a scalar particle:\[ \nu_\tau \rightarrow \nu_\mu + \phi, \] (2)

where we assume that $\phi$ and $\nu_\mu$ are massless. The scalar particle may be a Majoron [11] or a Familon [12] and the decay modes are expected to be detected in the future experiments, CLEO III or B factories [13]. The Boltzmann equation for the distribution $f_i$ of $i$-particle ($i = \nu_\tau, \nu_\mu, \phi$) is written as [14] [15],

\[
\frac{\partial f_i}{\partial t} - \frac{a(t)}{a_i} \frac{\partial f_i}{\partial p_i} = C_i(\text{decay}) + C_i(\text{ann}) + C_i(\text{scatt})
\] (3)

where $a(t)$ is the scale factor, $p_i$ is the momentum, and $C_i$s are collision terms which correspond to the three processes.

The collision term $C_{\nu_\tau}$ for the decay process $\nu_\tau(p_{\nu_\tau}) \rightarrow \nu_\mu(p_{\nu_\mu}) + \phi(p_\phi)$ is given by [9]

\[
C_{\nu_\tau}(\text{decay}) = -\Gamma_D^{\nu_\tau} + \Gamma_ID^{\nu_\tau}
\] (4)

where $\Gamma_D^{\nu_\tau}$ and $\Gamma_ID^{\nu_\tau}$ are the collision terms for the decay and the inverse decay processes, respectively. They are given by,

\[
\Gamma_D^{\nu_\tau} = \frac{1}{\tau_{\nu_\tau}} \frac{m_{\nu_\tau}}{\sqrt{m_{\nu_\tau}^2 + p_{\nu_\tau}^2}} f_{\nu_\tau}(p_{\nu_\tau}) \int_{p_{\min}}^{p_{\max}} dp_\phi \left(1 + f_\phi(p_\phi)\right) \left(1 - f_{\nu_\tau}(\sqrt{m_{\nu_\tau}^2 + p_{\nu_\tau}^2} - p_\phi)\right),
\] (5)

\[
\Gamma_ID^{\nu_\tau} = \frac{1}{\tau_{\nu_\tau}} \frac{m_{\nu_\tau}}{\sqrt{m_{\nu_\tau}^2 + p_{\nu_\tau}^2}} (1 - f_{\nu_\tau}(p_{\nu_\tau})) \int_{p_{\min}}^{p_{\max}} dp_\phi f_\phi(p_\phi) f_{\nu_\mu}(\sqrt{m_{\nu_\tau}^2 + p_{\nu_\tau}^2} - p_\phi).
\] (6)

The integration is performed between $p_{\max} = (\sqrt{m_{\nu_\tau}^2 + p_{\nu_\tau}^2} + p_{\nu_\tau})/2$ and $p_{\min} = (\sqrt{m_{\nu_\tau}^2 + p_{\nu_\tau}^2} - p_{\nu_\tau})/2$. In the same way, the collision terms $C_{\nu_\mu}(\text{decay})$ and $C_{\phi}(\text{decay})$ are given by,

\[
C_{\nu_\mu,\phi}(\text{decay}) = \Gamma_D^{\nu_\mu,\phi} - \Gamma_ID^{\nu_\mu,\phi},
\] (7)

where $\Gamma_D^{\nu_\mu,\phi}, \Gamma_ID^{\nu_\mu,\phi}$ are given by

\[
\Gamma_k^{\nu_\tau} = \frac{g_{\nu_\tau}}{g_k} \frac{m_{\nu_\tau}}{\tau_{\nu_\tau}} \frac{1}{p_k^2} (1 \pm f_k(p_k)) \int_{m_{\nu_\tau}/4p_k}^{\infty} dp_k f_{\nu_\tau}(\sqrt{(p_\phi + p_{\nu_\tau})^2 - m_{\nu_\tau}^2}) (1 \pm f_k(p_k))\]

(8)

\[
\Gamma_k^{ID} = \frac{g_{\nu_\tau}}{g_k} \frac{m_{\nu_\tau}}{\tau_{\nu_\tau}} f_k(p_k) \int_{m_{\nu_\tau}/4p_k}^{\infty} dp_k (1 \pm f_{\nu_\tau}(\sqrt{(p_\phi + p_{\nu_\tau})^2 - m_{\nu_\tau}^2})) f_k(p_k),
\] (9)

$^2$The $\tau$ neutrino might decay into an electron neutrino which changes the neutron-proton ratio directly. However, in the present paper we do not consider this decay channel since the Boltzmann equation for $\nu_\tau$ is too complicated. It is expected the emissions of $\nu_\tau$ and $\bar{\nu}_\tau$ increase the neutron-proton ratio and tend to enhance the discrepancy between the BBN theory and the observation.
where \( k \neq k' = \nu_\mu \) or \( \phi \), the spin factors are \( g_{\nu_\tau} = g_{\nu_\mu} = 2 \), and \( g_\phi = 1 \).

For pair annihilation and creation processes \( \nu(p_\nu) + \nu(p'_\nu) \rightarrow l_i(p_i) + l_i(p'_i) \), \( i = e, \nu_e, \nu_\mu \), the collision terms are written as

\[
C(\text{ann}) = -\frac{1}{2\pi^2} \int p_k^2 dp_k (\sigma v)_k (f_k(p_k) f_k(p'_k) - f_{eq}(p_k) f_{eq}(p'_k)),
\]

where \( k = \nu_\tau, \nu_\mu \), and \( f_{eq} \) is the equilibrium distribution and \( (\sigma v)_k \) is the differential cross sections given by

\[
(\sigma v)_{\nu_\tau} = \frac{G_F^2}{9\pi} \sum_{i=e,\nu_e,\nu_\mu} (C_{V_i}^2 + C_{A_i}^2) \left[ 4EE' - \left( \frac{E}{E'} + \frac{E'}{E} \right) m_{\nu_\tau}^2 - 2m_{\nu_\tau}^2 \right],
\]

\[
(\sigma v)_{\nu_\mu} = \frac{4G_F^2}{9\pi} \sum_{i=e,\nu_e,\nu_\mu} (C_{V_i}^2 + C_{A_i}^2) EE',
\]

where \( G_F \) is Fermi coupling constant, and \( C_A = 1/2, C_V = 1/2 - 2\sin^2 \theta_W \) (\( \theta_W \): Weinberg angle) for \( e^\pm \) and \( C_A = 1/2, C_V = 1/2 \) for neutrinos. We do not include the annihilation and creation process for the scalar particle \( \phi \) assuming that their interaction is very weak.

As for scattering processes, it is not easy to solve the Boltzmann equations including the collision terms due to scattering because they are generally much more complicated than collision terms due to the annihilation and the decay. Without scattering, the spectrum of the \( \tau \) neutrino is distorted because of the momentum dependence of the annihilation rate, which leads to overestimation of the density of \( \nu_\tau \) [9]. This effect is significant for the neutrino whose mass is larger than about 10MeV since such heavy neutrino becomes non-relativistic before decoupling and its density decreases through annihilation processes. On the other hand, for neutrinos which are relativistic at decoupling epoch, the momentum distributions do not change by the annihilation and is the same as the equilibrium one. Thus the scattering process is important only for the heavy \( \tau \) neutrino \( (m_{\nu_\tau} \gtrsim 10\text{MeV}) \). Therefore we consider only the scattering process \( \nu_\tau(p) + l(k) \rightarrow \nu_\tau(p') + l(k') \) in the collision term of the Boltzmann equation for \( \nu_\tau \). Using non-relativistic approximation, the collision term is simplified as

\[
C(\text{scatt}) = -\Gamma_f + \Gamma_b,
\]

where \( \Gamma_f \) and \( \Gamma_b \) are forward and backward reaction rates which are given by

\[
\Gamma_f = \sum_{i=e,\nu_e,\nu_\mu} \frac{G_F^2}{\pi^3} (C_{V_i}^2 + C_{A_i}^2) m_{\nu_\tau}^5 \frac{1}{x^2} \frac{e^{\sqrt{x^2 + y^2}/2}}{x\sqrt{x^2 + y^2}} f_{\nu_\tau}(y)
\]

\[
\times \int_0^\infty \frac{y'dy'}{\sqrt{x^2 + y'^2} e^{-\sqrt{x^2 + y'^2}/2}} (y, y')(1 - f_i(y'))),
\]
\[ \Gamma_b = \sum_{i=\nu_e,\nu_\mu} \frac{G_F^2}{\pi^3} (C_{Vi}^2 + C_{Ai}^2) m_{\nu_i}^5 \frac{e^{-\sqrt{x^2+y'^2}/2}}{y \sqrt{x^2 + y'^2}} (1 - f_{\nu_e}(y)) \]  

\[ \times \int_0^\infty \frac{y' dy'}{\sqrt{x^2 + y'^2}} e^{\sqrt{x^2+y'^2}/2} r(y, y') f_i(y'), \]  

where \( x = m_{\nu_e} / T, y = p / T, \) and \( y' = p' / T. \) The function \( r(y, y') \) is given by

\[ r(y, y') = \left(4 + \frac{3}{2} |y - y'| + \frac{1}{4} |y - y'|^2 \right) e^{-\frac{|y-y'|}{2}} \]

\[- \left(4 + \frac{3}{2} (y + y') + \frac{1}{4} (y + y')^2 \right) e^{-\frac{y + y'}{2}}.\]

At \( T \lesssim 0.5 \text{ MeV}, \) electrons and positrons begin to annihilate and heat photons without heating neutrinos. Then we have to deal with the temperature and cosmic time relation carefully. In this situation, we solve a set of Boltzmann equations coupled with the Hubble expansion rate and the entropy conservation equation. The Hubble expansion rate is obtained by solving the Einstein equation:

\[ \frac{\dot{a}(t)}{a(t)} = \left[ \frac{8\pi G}{3} \left( \rho_e + \rho_\gamma + \rho_{\nu_e} + \rho_{\nu_\mu} + \rho_{\nu_\tau} + \rho_\phi \right) \right]^{\frac{1}{2}}, \]

and the entropy conservation is:

\[ \frac{11}{3} T_{\nu}^3 = \frac{4}{3} T_\gamma^3 \left[ 1 + \frac{3}{4} \left( \frac{\rho_e + P_e}{\rho_\gamma} \right) \right], \]

where \( \rho_i \) denotes the energy density of the particle \( \text{"i"} (i = e, \gamma, \nu_e, \nu_\mu, \nu_\tau, \phi) \) and \( P_e \) is the electron pressure.

As initial conditions we take

\[ f_{\nu_e} = f_{\nu_\mu} = f_{eq} \]

\[ f_\phi = 0, \]

where we assume that the scalar particle has been never thermalized and its cosmological density is negligible. We will discuss the justification of this assumption later.

### 3 Time Evolution of \( N_\nu \)

As we have already mentioned, the abundances of \(^4\text{He}\) is sensitive to the total energy density of the universe during the BBN epoch, \( T = 10 \text{ MeV} - 0.1 \text{ MeV}. \) The lower the
energy density, the more slowly the universe expands, which leads to later decoupling of the weak interactions and less neutron-proton ratio. Then the abundance of $^4$He decreases since almost all neutrons are translated into $^4$He. Since the other light elements are insensitive to the cosmic expansion rate, it is expected that the BBN predictions may agree with observational data if the energy density of the universe becomes lower than the standard one by the effect of heavy $\tau$ neutrino decay.

When we discuss the energy density of massive decaying $\tau$ neutrino, it is useful to introduce the effective number of neutrino species $N_\nu$ defined by

$$N_\nu = \frac{\rho_{\nu_e} + \rho_{\nu_\mu} + \rho_{\nu_\tau} + \rho_\phi}{\rho_\nu(m_\nu = 0)}.$$  \hfill (21)

The denominator is the energy density of a massless neutrino and $N_\nu$ is a constant ($= 3$) in the standard model.

In Fig. 1 we show the time evolutions of $N_\nu$ for ($\tau_{\nu_\tau}$ (sec), $m_{\nu_\tau}$ (MeV)) = $(10^{-3}, 1)$, $(1, 10)$ and $(0.1, 14)$. In order to understand their behaviors, it is convenient to introduce the three temperature scales, (1) the epoch when the decaying particle becomes non-relativistic $T_{NR}$, (2) the decaying particle’s decoupling epoch $T_D$, and (3) the epoch when the decay occurs $T_{\text{decay}}$. We define them by,

$$T_{NR} \equiv \frac{m_{\nu_\tau}}{3},$$  \hfill (22)

$$\Gamma(T_D) \equiv H(T_D),$$  \hfill (23)

$$T_{\text{decay}} \equiv T(t = \tau_{\text{eff}}),$$  \hfill (24)

where $H(t)$ is the Hubble expansion rate, and $\tau_{\text{eff}}$ is the effective lifetime of the $\tau$ neutrino. Notice that the effective lifetime of the relativistic neutrino becomes longer than the proper lifetime because of the time dilation ($\tau_{\text{eff}} \sim \tau \sqrt{m_{\nu_\tau}^2 + p_{\nu_\tau}^2 / m_{\nu_\tau}}$).

In the case of $\tau_{\nu_\tau} = 10^{-3}$ sec and $m_{\nu_\tau} = 1$ MeV, the relation of the three time scales is given by $T_{\text{decay}} > T_D > T_{NR}$. At temperatures between $T_{\text{decay}}$ and $T_D$ the decay, inverse decay, annihilation and pair creation processes tend to establish the thermal equilibrium among three species of neutrinos and the scalar particle and $N_\nu$ becomes close to 3.6. Then, after decoupling, $N_\nu$ increases due to the mass effect until $T_{NR}$ at which the inverse decay becomes suppressed and $\nu_\tau$ begins to decay efficiently. On the other hand, in the case of $\tau_{\nu_\tau} = 1$ sec and $m_{\nu_\tau} = 10$ MeV, $T_{NR} > T_D > T_{\text{decay}}$. Between $T_{NR}$ and $T_D$ the energy density of $\tau$ neutrino decrease due to the Boltzmann suppression. After decoupling, the mass effect increases $N_\nu$, and at $T \sim T_{\text{decay}}$ $\nu_\tau$ begins to decay. Thus, if $T_D \sim T_{\text{decay}}$, the increases of $N_\nu$ due to the mass effect is not significant and $N_\nu$ becomes less than three as shown in Fig. 1 for $\tau_{\nu_\tau} = 10^{-1}$ sec and $m_{\nu_\tau} = 14$ MeV.
4 BBN with Massive Decaying $\tau$ Neutrino

In a previous section, we have discussed the time evolution of the energy density in the universe, and have shown that $\tau$ neutrino with $O(10\,\text{MeV})$ mass and short lifetime ($\sim 0.1\,\text{sec}$) can reduce the effective number of neutrino species $N_{\nu}$. In order to calculate the abundances of $D$, $^4\text{He}$, $^3\text{He}$, and $^7\text{Li}$, we incorporate the change of the effective number of neutrino species $N_{\nu}$ into the big bang nucleosynthesis code [10].

In the case of massive and unstable tau neutrino, the effective number of neutrino species is a time-dependent variable. Since only $^4\text{He}$ abundance is sensitive to $N_{\nu}$, if the predicted $^4\text{He}$ abundance is expressed by the corresponding (constant) number of neutrino species in the BBN calculation without massive neutrinos, it helps us to understand the results of our calculation intuitively. Thus, we introduce the equivalent number of neutrino species $\bar{N}_{\nu}$ here which is defined as

$$Y_p(\bar{N}_{\nu}, m_{\nu_{\tau}} = 0, \tau_{\nu_{\tau}} = \infty) \equiv Y_p(N_{\nu} = 3, m_{\nu_{\tau}}, \tau_{\nu_{\tau}}).$$

(25)

In Fig. 2 we show the contour plots of the equivalent number of neutrino species $\bar{N}_{\nu}$ in the parameter space, $\tau_{\nu_{\tau}} - m_{\nu_{\tau}}$. In this plot we expect that the predicted $Y_p$ decreases if $\tau_{\nu_{\tau}} \lesssim 10\,\text{sec}$ and $m_{\nu_{\tau}} \gtrsim 10\,\text{MeV}$.

In the numerical calculation we make use of the Monte Carlo procedure in order to include the experimental uncertainties of the nuclear reaction rates and the neutron lifetime [17, 18]. Since the theoretical $1\sigma$ errors are almost independent of $N_{\nu}$ for $N_{\nu} = 2 - 4$, we adopt the $1\sigma$ errors estimated by the standard BBN ($N_{\nu} = 3$) calculation.

The calculated abundances are compared with the primordial values inferred form the observational data. Concerning the $^4\text{He}$, we adopt the value in the extra galactic HII region and it is given by [19],

$$Y_p = 0.234 \pm 0.002(\text{stat}) \pm 0.005(\text{syst})$$

(26)

where $Y_p$ is the mass fraction of $^4\text{He}$ and “p” denotes the primordial value. It is believed that abundances of $^7\text{Li}$ observed in the population II metal poor halo stars in our galaxy represent the primordial values. We adopt the recent observational data [20],

$$(^7\text{Li}/\text{H})_{p} = (1.73 \pm 0.05(\text{stat}) \pm 0.20(\text{syst})) \times 10^{-10}.$$ 

(27)

For a deuterium abundance, we have some types of the measurements which are the local ISM or the solar system data and the abundances observed in high red-shift QSO absorption systems. The abundances inferred from the local ISM are $(\text{D}/\text{H})_{ISM} = (1.6 \pm$
In the solar system the observed deuterium abundances are \((D/H)_\odot = (2.57 \pm 0.92) \times 10^{-5}\) \(^{22}\). This pre-solar deuterium abundance is obtained by subtracting \(^3\)He/H value observed in carbonaceous chondorites from \((D + ^3\)He)/H observed in the solar winds. From above abundances, we must extrapolate the true primordial deuterium abundance assuming appropriate chemical evolution models.

On the other hand, the observations in high red-shift QSO absorption systems are expected that they are nearly the primordial values because such Lyman \(\alpha\) clouds would be unprocessed. In the last three years, more papers concerning these Lyman limit system observations are published. However, two groups have measured different D/H values which disagree in a order of magnitude. The higher values measured by some persons are \(^{[3]}\),

\[
(D/H)_{\text{QSO}} = (1.9 \pm 0.5) \times 10^{-4}.
\]

The lower values measured by Burles and Tytler. \(^{[4]}\) are,

\[
(D/H)_{\text{QSO}} = (3.39 \pm 0.25) \times 10^{-5}. 
\]

Some chemical evolution models which predict only modest destruction of deuterium during the galaxy evolution have been suggested \(^{[23, 24]}\). Adopting such a recent model \(^{[24]}\) which reports that the deuterium destruction is limited to a factor 3 or less, the above low deuterium values in high red-shift QSO absorption systems would be consistent with the ISM and solar system data. However, there are also the other models in which primordial deuterium can be significantly destroyed into a order of magnitude lower without over-producing the \(^3\)He \(^{[25]}\). The latter models might agree with the high deuterium abundance in high z QSO absorption systems. Considering the circumstances mentioned above, we so far have no conclusive model of the galactic chemical evolution. On the observational side, it is premature to determine which deuterium values in high red-shift QSO absorption systems are reliable. Therefore in this paper, we adopt both the high and low deuterium data in high z QSO absorption systems and for each deuterium value we constrain on the \(\tau\) neutrino mass and its lifetime.

In order to obtain the confidence level of the model parameters, mass \(m_{\nu_\tau}\), lifetime \(\tau_{\nu_\tau}\), and baryon to photon ratio \(\eta\) in the universe, we utilize the Maximum Likelihood analysis. The likelihood function of theoretical abundances is taken to be gaussian which has the one sigma variance calculated by the Monte Carlo procedure. The likelihood function of observational data is taken to be gaussian for the statistical error, and to be top hat distribution for the systematic error (see Ref. \(^{[7]}\)). Also for the \(\tau\) neutrino mass, the experimental bound Eq. \(^{[1]}\) is incorporated as a gaussian distribution.
The obtained two dimensional contours in the parameter space, $\tau_{\nu_r} - m_{\nu_r}$, are shown in Fig. 3 for using the low (D/H)$_{QSO}$, and in Fig. 4 for using the high (D/H)$_{QSO}$. In Fig. 3 the contour with same likelihood as the standard BBN model ($N_\nu = 3$) is shown by the dotted line. Notice that if $0.01 \text{ sec} \lesssim \tau_{\nu_r} \lesssim 1 \text{ sec}$ and $10 \text{ MeV} \lesssim m_{\nu_r} \lesssim 24 \text{ MeV}$, the theoretical predictions give a much better fit to the observational data than the standard BBN. The best fit values are estimated as

$$m_{\nu_r} = 15^{+11}_{-5} \text{ MeV} \quad (68\% \ C.L.),$$

$$\log_{10} \left( \frac{\tau_{\nu_r}}{\text{sec}} \right) = -1.0^{+1.0}_{-1.2} \quad (68\% \ C.L.),$$

for the low D abundances in the QSO absorption systems. The filled square in Fig. 3 represents the best fit point. From Fig. 4 we can obtain the upper limit for the $\tau$ neutrino mass $m_{\nu_r}$,

$$m_{\nu_r} \lesssim 20 \text{ MeV} \quad (68\% \ C.L.),$$

for high D abundances in the QSO absorption systems.

5 Conclusion and Discussion

In this paper we have shown the light element abundances which are predicted by BBN theory with $\tau$ neutrino whose mass is $\sim 10\text{MeV}$ and lifetime is $\sim 0.1 \text{ sec.}$ agree with the primordial values inferred by observational data even if we adopt the low D/H in high $z$ QSO absorption systems. On the other hand, if we adopt the high D/H, we can obtain the strong constraints on $\tau$ neutrino mass $m_{\nu_r}$.

Finally we discuss the annihilation rate for the scalar particle $\phi$. For this purpose we assume that $\phi$ is a Majoron-like particle which has the interaction to $\nu_r$ as

$$L_{\text{int}} = \frac{1}{v} \bar{\nu}_r \gamma_5 \gamma^\mu \nu_r,$$

where $v$ is the characteristic energy scale. In the relativistic limit the ratio of the annihilation rate for $\phi + \phi \rightarrow \nu_r + \nu_r$ to Hubble expansion rate is given by

$$\frac{\Gamma}{H} = \sigma v \nu_r n_\phi / H \simeq 1 \times 10^{20} \left( \frac{m_{\nu_r}}{v} \right)^4 \left( \frac{T}{\text{MeV}} \right)^{-1} \log \left( \frac{1.21 T}{m_{\nu_r}} \right) \quad (T \gg m_{\nu_r}).$$

When the cosmic temperature $T$ is less than $m_{\nu_r}$, the ratio $\Gamma / H \propto T^2$. Thus $\Gamma / H$ takes the maximum value at $T \simeq m_{\nu_r}$:

$$\left( \frac{\Gamma}{H} \right)_{\text{max}} \simeq 2 \times 10^{19} \left( \frac{\text{MeV}}{v} \right)^4.$$
The condition that the scalar particle $\phi$ is not in thermal equilibrium is given by

$$v \gtrsim 7 \times 10^4 \text{MeV}$$  \hspace{1cm} (36)

On the other hand, the lifetime of the $\tau$ neutrino is related to $v$ by

$$\tau_{\nu_\tau} \simeq \frac{16\pi v^2}{m_{\nu_\tau} |U_{\tau,\mu}|^2},$$  \hspace{1cm} (37)

where $U_{\tau,\mu}$ is the mixing angle. Therefore the condition (36) is written as

$$\tau_{\nu_\tau} \gtrsim 4 \times 10^{-5} \text{sec} |U_{\tau,\mu}|^{-2}.$$  \hspace{1cm} (38)

From Eq. (38) the scalar particles are not in thermal equilibrium if $\tau_{\nu_\tau} \gtrsim 1 \times 10^{-5} \text{sec}$ and $|U_{\tau,\mu}| \gtrsim 10^{-2}$. Thus our assumption that the scalar particle has not been thermalized is justified when the mixing angle is large.

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Figure caption

Figure1: Evolution of the effective number of neutrino species $N_{\nu}$ with the cosmic temperature $T$ for $(\tau_{\nu_r}(\text{sec}), m_{\nu_r}(\text{MeV})) = (10^{-3}, 1), (1, 10)$ and $(0.1, 14)$.

Figure2: Contours of the equivalent number of neutrino species $\bar{N}_{\nu}$ in the parameter space, $\tau_{\nu_r} - m_{\nu_r}$. $\bar{N}_{\nu}$ is defined by Eq. (25). In this plot we expect that the predicted $Y_p$ decreases if $\tau_{\nu_r} \lesssim 10 \text{ sec}$ and $m_{\nu_r} \gtrsim 10 \text{ MeV}$. By the definition, $\bar{N}_{\nu}$ does not depend on the baryon to photon ratio $\eta$ at all.

Figure3: Contours of the confidence levels in the parameter space, $\tau_{\nu_r} - m_{\nu_r}$. In this plot the lower deuterium abundance obtained in high red-shift QSO absorption systems is adopted as the primordial value. The dashed line denotes 90% C.L., and the solid line denotes 68% C.L.. The contour with same likelihood as standard BBN model ($N_{\nu} = 3$) is shown by the dotted line. The filled square represents the best fit point.

Figure4: Contours of the confidence levels in the parameter space, $\tau_{\nu_r} - m_{\nu_r}$. In this plot the higher deuterium abundance obtained in high red-shift QSO absorption systems is adopted as the primordial value. The dashed line denotes 90% C.L., and the solid line denotes 68% C.L.. The contour with same likelihood as standard BBN model ($N_{\nu} = 3$) is shown by the dotted line.
$\tau_{\nu} = 10^{-3} \text{ sec}$
$m^*_{\nu} = 1 \text{ MeV}$

$\tau_{\nu} = 1 \text{ sec}$
$m^*_{\nu} = 10 \text{ MeV}$

$\tau_{\nu} = 10^{-1} \text{ sec}$
$m^*_{\nu} = 14 \text{ MeV}$
(D/H)_{QSO} = (3.39 \pm 0.25) \times 10^{-5}
$(D/H)_{\text{QSO}} = (1.9 \pm 0.5) \times 10^{-4}$