Stochastic Flexibility Evaluation for Virtual Power Plant by Aggregating Distributed Energy Resources

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Abstract—To manage huge amount of flexible distributed energy resources (DERs) in the distribution networks, the virtual power plant (VPP) is introduced in industry. The VPP can optimally dispatch these resources in a cluster way and provide flexibility for the power system operation as a whole. Most existing works formulate the equivalent power flexibility of the aggregating DERs as deterministic optimization models without considering their uncertainties. In this paper, we introduce the stochastic power flexibility range (PFR) to describe the power flexibility of VPP, which is formulated as a chance constrained optimization model. In this model, both operational constraints and the randomness of DERs’ output are incorporated, and a combined model and data-driven solution is proposed to obtain the stochastic PFR and cost function of VPP. Finally, numerical tests are conducted to verify the correctness and efficiency of the proposed method.

Index Terms—Virtual power plant, stochastic power flexibility, chance constrained optimization, combined model and data-driven

NOMENCLATURE

A. Parameters

\[K, G, J\]

\[b, c, d\]

\[A_{i,SG}, b_{i,SG}\]

\[A_{i,PV}, b_{i,PV}\]

\[A_{i,ESS}, b_{i,ESS}\]

\[A_{i,wind}, b_{i,wind}\]

\[P_{i,load}, P_{i, PV}, \hat{P}_{i, PV}, \hat{P}_{i, wind}\]

Forecast value of activate load power at phase \(\psi\) of bus \(i\) at time \(t\)

Power factor of load at phase \(\psi\) of bus \(i\) at time \(t\)

Sets of wye- and delta-connection buses

Sets of wye- and delta-connection phases

Maximum and minimum voltage amplitude at phase \(\psi\) of bus \(i\)

Risk probability of voltage amplitude exceeding the upper and lower limits

Maximum current amplitude at phase \(\psi\) of branch \(j\)

Risk probability of positive and negative currents amplitude exceeding the limits of branches

Quadric cost coefficients of synchronous generator at bus \(i\)

Grid electricity price at the PCC bus of the VPP at time \(t\)

Discharge and charge cost parameter of ESS of bus \(i\)

Time interval of one period

The \(k\)-th sample of activate output power of VPP and corresponding minimum cost

\[s_i, s_i^*, s_i^k\]

\[V\]

\[s_0, i_j\]

\[P_{i,SG}, Q_{i,SG}\]

\[P_{i,PV}, Q_{i,PV}\]

\[P_{i,ESS}, Q_{i,ESS}\]

\[P_{i,wind}, Q_{i,wind}\]

\[\hat{P}_{i,PV}, \hat{P}_{i,wind}, \hat{P}_{i,wind}\]

Real-time maximum output power and curtailed power of photovoltaic generator at phase \(\psi\) of bus \(i\) at time \(t\)

The complex injection power of wye- and delta-connection sources at bus \(i\)

All the wye- and delta-connection sources in the VPP

Voltage amplitudes vector of all phases of all the buses

Complex power injection at PCC

Branch current vector of all phases of all branch

Activate and reactivate output power of synchronous generator at phase \(\psi\) of bus \(i\) at time \(t\)

Activate and reactivate output power of photovoltaic generator at phase \(\psi\) of bus \(i\) at time \(t\)

Activate and reactivate output power of energy storage system at phase \(\psi\) of bus \(i\) at time \(t\)

Activate and reactivate output power of wind turbine at phase \(\psi\) of bus \(i\) at time \(t\)

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CDF of standard Gaussian distribution

**A. Motivation**

High penetration of renewable energy in distribution networks bring new challenges in terms of voltage violation, power quality deterioration, protection relay failure and insufficient flexibility. On the one hand, it is very difficult to control thousands or even millions of small DERs directly. On the other hand, these resources cannot participate in the electricity market individually due to their small capacity. The concept of virtual power plant (VPP) provides a promising solution to this problem [1]. VPP is a collection of distributed generators (DGs), distributed energy storage units and controllable loads. It uses advanced regulation and communication technologies to manage these DERs in a cluster way [2], [3]. All the DERs in a VPP can be considered as a whole to dispatch and control.

Since these DERs are aggregated together as a VPP, evaluating its power flexibility is critical to participate in power system operation or electricity market [4]. Some literatures have proposed methods to assess the deterministic power flexibility range [5]–[7]. However, there are many random factors that can affect the flexibility range of VPP, such as the loads’ variations and the fluctuations of renewable generators’ output. The evaluation of VPP’s flexibility should reflect the characteristics of these uncertainties. i.e., it should be formulated as a stochastic model.

In this paper, we propose a model to assess the flexibility of VPP considering the randomness of DERs. In this model, we focus on the activate and reactivate power adjustable capability of the whole VPP at the point of common coupling (PCC). The randomness of the model is described as the confidence that the operational constraints are satisfied. The stochastic flexibility model can help the VPP operators to evaluate voltage regulation and power regulation capabilities. This model can also be incorporated in the stochastic unit commitment [8], risk dispatching [9], bidding in the electricity market [10] and so on.

**B. Related work and contribution**

The aggregation of power flexibility problem has been studied in several previous works. The power capability diagram is used in [6] to describe the power capability of microgrid and the impacts of PHEVs, capacitor banks and storage devices are discussed. Similarly, the power flexibility of DERs in different scenarios and degrees of control is introduced in [11]. Silva et al developed the methodology [12] to find the flexibility area at the TSO/DSO boundary node. Then, they proposed the Interval Constrained Power Flow (ICPF) method to estimate the flexibility and corresponding adjustment costs [13]. In [14], the grid scanning method is used to obtain the maximum flexibility potential of the active distribution grid. The work of [15] estimates the flexibility of an active distribution network by Monte Carlo simulation. Resorting to a large number of sampling, the probability of feasibility can be obtained in the future time interval. The linear models of flexibility ranges of units and linear power flow model are used in [16] to reduce the computation time while maintaining accuracy. Reference [17] proposed the linear OPF-based flexibility aggregation algorithm and use beta distribution to consider the random variables. It can be conveniently used in the large grid models with a big amount of flexibility resources. Besides, the effect of different constraints on the shape of the power capability chart is discussed in [17]–[19].

In this paper, we propose a method to aggregate the power flexibility ranges of the DERs in the VPP. As shown in Fig. 1, each kind of DER has a power flexibility range, which can be expressed as a range on the P-Q panel. These ranges constitute the technical constraints of DERs. Besides, the network...
constraints should be considered, such as the voltage limit constraints of buses and the capacity constraints of transmission lines. The main goal of our method is to aggregate the power flexibility ranges of DERs and assess the power flexibility at the point of common coupling while meeting these constraints. However, the loads and maximum output power of renewable generators are random variables, which can be described by the probability distribution functions. Therefore, the performance of the power flexibility range at the VPP’s point of common coupling is also stochastic. Hereafter, it is named as the stochastic power flexibility range (PFR) of VPP.

II. NETWORK AND DER MODEL

A. Network model

In VPPs, their distribution networks are usually multi-phase unbalanced. The DERs in the network can be wye-connection, delta-connection, or the combination of the two. In this paper, we use a multi-phase unbalanced network model [21], [22] to simulate the real situations. This model considers all the connection types above and linearizes the multi-phase unbalanced power flow. The complex injection power of wye- and delta-connection sources at bus $i$ can be denoted as the vectors $s^i := (s^i_p, s^i_q)^T$ and $s^h := (s^h_p, s^h_q, s^h_{θ})^T$, respectively. And their collections are defined as

$$s^i := (s^1_p, s^1_q, \ldots, s^N_p, s^N_q)^T = p^i + j q^i \quad (1)$$

$$s^h := (s^1_p, s^1_q, \ldots, s^N_p, s^N_q, s^N_{θ})^T = p^h + j q^h \quad (2)$$

Here, $p^i, q^i, p^h, q^h$ represent the active and reactive injection power in different connection forms. Finally, we arrange all the injection power as

$$x := [(p^i)^T, (q^i)^T, (p^h)^T, (q^h)^T]^T \quad (3)$$

Then, the voltage amplitudes $V$, complex power injection $s^h$ at PCC, and branch current $i^h$ can be expressed as the following linear form, respectively.

$$V = Kx + b \quad (4)$$

$$s^h = Gx + c \quad (5)$$

$$i^h = Jx + d^h \quad (6)$$

where, the matrices $K, G, J$, vectors $b, c, d^h$ are all the system parameters, whose detail definitions can be referred in [21], [22]. This linearization method is essentially a linearized interpolation of the two load-flow solutions: the given operation point and a known zero-load operation point. This multiphase linear model also has a good approximation accuracy performance. According to the numerical tests results of [21], the relative errors of voltages amplitude in the IEEE 13 system and a real system with about 2000 nodes are less than 0.2% and 0.6%, respectively.

B. DER model

Fig. 2. The power regulation capability charts of (a) synchronous generator; (b) inverter-interfaced DG; (c) energy storage battery; and (d) doubly-fed induction generator. The grey polygons represent the linearized regulation capability.
The regulation capability charts of DERs’ outputs have different shapes [5]. In this section, we formulate the following different DERs including the synchronous generator (SG), inverter-interfaced DG (such as photovoltaic), energy storage system (ESS) and doubly-fed induction generator (DFIG) [23]. Their regulation capability charts consist of the constraints associated with DERs’ parameters, which usually include nonlinear or even non-convex constraints. Therefore, we linearize their boundaries and convert these charts into polygons as shown in Fig. 2.

The polygonal power regulation capability charts of these DERs can be described as the following polygons:

\[ A_{\psi,SG}^i \cdot \left[ \frac{P_{\psi,SG}^{i,t}}{Q_{\psi,SG}^{i,t}} \right] \leq b_{\psi,SG}^i \]  \hspace{1cm} (7)
\[ A_{\psi,PV}^i \cdot \left[ \frac{P_{\psi,SV}^{i,t}}{Q_{\psi,SV}^{i,t}} \right] \leq b_{\psi,SV}^i \]  \hspace{1cm} (8)
\[ A_{\psi,ESS}^i \cdot \left[ \frac{P_{\psi,ESS}^{i,t}}{Q_{\psi,ESS}^{i,t}} \right] \leq b_{\psi,ESS}^i \]  \hspace{1cm} (9)
\[ A_{\psi,wind}^i \cdot \left[ \frac{P_{\psi,wind}^{i,t}}{Q_{\psi,wind}^{i,t}} \right] \leq b_{\psi,wind}^i \]  \hspace{1cm} (10)

Where, these linear constraints represent the feasible operation region of the DERs, depicted as the grey polygonal areas in Fig. 2. For example, \( P_{\psi,SG}^{i,t} \) denotes the active power output of SG on the phase \( \psi \) of bus \( i \) at time \( t \). The superscript \( \psi \) denotes the connection phases of the DERs. If a SG is delta-connection on the \( ab \) and \( bc \) phase, then \( \psi = \{ ab, bc \} \).

Moreover, the maximum output power of renewable energy, such as photovoltaics and wind turbines, depend on the weather condition. Their forecast output power can be described as the sum of expected forecast value ( \( \bar{P}_{\psi,SV}^{i,t} \), \( \bar{P}_{\psi,wind}^{i,t} \)) and forecast errors ( \( \hat{P}_{\psi,SV}^{i,t} \), \( \hat{P}_{\psi,wind}^{i,t} \)). Curtailed power \( \bar{P}_{\psi,SV}^{i,t} \) and \( \bar{P}_{\psi,wind}^{i,t} \) are used to provide flexibility.

\[ \bar{P}_{\psi,SV}^{i,t,max} = \bar{P}_{\psi,SV}^{i,t,max} + \hat{P}_{\psi,SV}^{i,t} \]  \hspace{1cm} (11)
\[ p_{\psi,SV}^{i,t} = \bar{P}_{\psi,SV}^{i,t,max} - \bar{P}_{\psi,SV}^{i,t} \]  \hspace{1cm} (12)
\[ 0 \leq p_{\psi,SV}^{i,t} \leq \bar{P}_{\psi,SV}^{i,t,max} \]  \hspace{1cm} (13)
\[ \bar{P}_{\psi,wind}^{i,t,max} = \bar{P}_{\psi,wind}^{i,t,max} + \hat{P}_{\psi,wind}^{i,t} \]  \hspace{1cm} (14)
\[ p_{\psi,wind}^{i,t} = \bar{P}_{\psi,wind}^{i,t,max} - \bar{P}_{\psi,wind}^{i,t} \]  \hspace{1cm} (15)
\[ 0 \leq p_{\psi,wind}^{i,t} \leq \bar{P}_{\psi,wind}^{i,t,max} \]  \hspace{1cm} (16)

The load in VPP depends on the behavior of consumers, and they are also random variables. We can assume that their power factor is constant, [16], that is:

\[ P_{\psi,load}^{i,t} = \bar{P}_{\psi,load}^{i,t} + \hat{P}_{\psi,load}^{i,t} \tan (\varphi_{\psi,load}^{i,t}) \]  \hspace{1cm} (17)

By summing the power DERs and loads, we can get the power injection of buses:

\[ P_{\psi,SV}^{i,t} = P_{\psi,SV}^{i,t} + P_{\psi,wind}^{i,t} + p_{\psi,SV}^{i,t} + p_{\psi,wind}^{i,t} \]  \hspace{1cm} (18)
\[ Q_{\psi,SV}^{i,t} = Q_{\psi,SV}^{i,t} + Q_{\psi,wind}^{i,t} + Q_{\psi,SV}^{i,t} + Q_{\psi,wind}^{i,t} - \hat{Q}_{\psi,load}^{i,t} \]  \hspace{1cm} (19)

Accordingly, the power injection in the wye- and delta-connection format at time \( t \) is given by

\[ P_{\psi}^{t} = \left[ P_{\psi,SV}^{t} \right]_{\psi,SV}^{t} \]  \hspace{1cm} (21)
\[ Q_{\psi}^{t} = \left[ Q_{\psi,SV}^{t} \right]_{\psi,SV}^{t} \]  \hspace{1cm} (22)
\[ P_{\psi}^{t} = \left[ P_{\psi,ESS}^{t} \right]_{\psi,ESS}^{t} \]  \hspace{1cm} (23)
\[ Q_{\psi}^{t} = \left[ Q_{\psi,ESS}^{t} \right]_{\psi,ESS}^{t} \]  \hspace{1cm} (24)

Based on the multi-phase unbalanced network model, we can calculate buses’ voltages and the branches’ currents. Considering the randomness of renewable energy generators and loads, the network operational constraints are formulated as chance constraints as follows:

\[ Pr \{ P_{\psi}^{t} \leq V_{\psi}^{t,max} \} \geq 1 - \alpha_{\psi}^{t} \]  \hspace{1cm} (25)
\[ Pr \{ Q_{\psi}^{t} \geq V_{\psi}^{t,max} \} \geq 1 - \alpha_{\psi}^{t} \]  \hspace{1cm} (26)
\[ Pr \{ I_{\psi}^{t} \leq I_{\psi}^{t,max} \} \geq 1 - \alpha_{\psi}^{t} \]  \hspace{1cm} (27)
\[ Pr \{ I_{\psi}^{t} \geq I_{\psi}^{t,min} \} \geq 1 - \alpha_{\psi}^{t} \]  \hspace{1cm} (28)

Due to the volatility of loads and renewable energy generators, the random capability of maximum output active power at PCC, denoted as \( P_{\psi,rand}^{t} \), is also a random variable.

\[ \bar{P}_{\psi,rand}^{t} = \sum_{i} \bar{P}_{\psi,SV}^{i,t} + \bar{P}_{\psi,wind}^{i,t} - \bar{P}_{\psi,load}^{i,t} \]  \hspace{1cm} (29)

Then, the following chance constraint can be used to express the influence of volatility on the output active power capability.

\[ Pr \{ P_{\psi,rand}^{t} \leq \bar{P}_{\psi,rand}^{t} \} \geq 1 - \alpha_{\psi}^{t} \]  \hspace{1cm} (30)

Where, \( P_{\psi,rand}^{t} \) denotes the actual capability of maximum output active power at PCC, as shown in (31):

\[ P_{\psi,rand}^{t} = \sum_{i} \left( \bar{P}_{\psi,SV}^{i,t} + \bar{P}_{\psi,wind}^{i,t} - \bar{P}_{\psi,load}^{i,t} \right) \]  \hspace{1cm} (31)

III. SOLUTION PROCEDURE

A. Decision variables

To simplify the expression of decision variables, we use the vectors to collect all the output power of DERs at time \( t \):

\[ P_{SV}^{t} = \left[ P_{SV}^{t} \right]_{\psi,SV}^{t} \]  \hspace{1cm} (32)
\[ Q_{SV}^{t} = \left[ Q_{SV}^{t} \right]_{\psi,SV}^{t} \]  \hspace{1cm} (32)
\[ S \in \left \{ SG, PV, ESS, wind \right \} \]

Then, we use \( \mathbf{X}^{t} \) to represent the decision variables vector at time \( t \), which is made up of all the output power of controllable DERs:

\[ \mathbf{X}^{t} = \left[ P_{SV}^{t}, Q_{SV}^{t}, P_{ESS}^{t}, Q_{ESS}^{t} \right] \]  \hspace{1cm} (33)

B. Modeling of uncertainties

The forecast errors of renewable energy generators and loads constitute the random variables. We use vector \( \mathbf{\epsilon}^{t} \) to denote the collection of them at time \( t \).

\[ \mathbf{\epsilon}^{t} = \left[ \epsilon_{\psi,SV}^{t}, \epsilon_{\psi,wind}^{t}, \epsilon_{\psi,load}^{t} \right] \]  \hspace{1cm} (34)

The joint probability density function (PDF) of \( \mathbf{\epsilon}^{t} \) can be estimated from the historical data. In this paper, we use
Gaussian Mixture Model (GMM) to characterize uncertainties of forecast errors. GMM can be used to fit any PDF of random variables by adjusting its parameters and keeps affine invariance [8]. With GMM, \( \hat{e} \) can be expressed by an affine combination of multivariate Gaussian distributions as follows:

\[
PDF_{\hat{e}}(\mathbf{x}) \sum_{j=1}^{\infty} a_j N(x; \mu_j, \Sigma_j)
\]

(35)

where, \( a_j \) is the weight coefficient; \( \mu_j \) is the expectation vector, and \( \Sigma_j \) is the covariance matrix of \( j \)-th Gaussian distribution vector.

C. Conversion of chance constraints

In our model, the network operational constraints are formulated as chance constraints. To make them solvable, they should be converted to deterministic ones. We take the chance constraints (25)-(26) related \( \tilde{V}_{\gamma} \) as the example to demonstrate the solution.

Since the standard Gaussian distributions of variables \( \tilde{V}_{\gamma} \) can be transformed into the equivalent deterministic constraints with

\[
V_{\gamma}^{\max} - (b_{\gamma}^e)^T \mathbf{x} - c_{\gamma}^e \geq Quant_{\gamma}(1-\alpha) (a_{\gamma}^e)^T \hat{e}
\]

(43)

\[
V_{\gamma}^{\min} - (b_{\gamma}^e)^T \mathbf{x} - c_{\gamma}^e \leq Quant_{\gamma}(1-\alpha) (a_{\gamma}^e)^T \hat{e}
\]

(44)

Similarly, chance constraints (27)-(28) related to variable \( \tilde{P}_{\gamma} \) and chance constraint (30) can be transformed into

\[
P_{\gamma}^{\max} - (b_{\gamma}^p)^T \mathbf{x} - c_{\gamma}^p \geq Quant_{\gamma}(1-\alpha) (a_{\gamma}^p)^T \hat{e}
\]

(45)

\[
P_{\gamma}^{\min} - (b_{\gamma}^p)^T \mathbf{x} - c_{\gamma}^p \leq Quant_{\gamma}(1-\alpha) (a_{\gamma}^p)^T \hat{e}
\]

(46)

\[
(b_{\gamma})^T \mathbf{x} - c_{\gamma}^{p2} \leq Quant_{\gamma}(\alpha) (a_{\gamma}^p)^T \hat{e}
\]

(47)

where, the quantiles are defined as the inverse of cumulative density function (CDF):

\[
Quant(\gamma | \hat{x}) = CDF_{\hat{e}}^{-1}(\alpha)
\]

(48)

For example, if \( Quant(\gamma | \hat{x}) = q = \alpha = CDF_{\hat{e}}(q) \), the value of \( q \) can be solved iteratively with Newton method [8] as the pseudo-code in Algorithm 1.

Algorithm 1. Calculate Quantile with Newton Method

1: Given the initial value: \( q_0, i = 0 \), maximum error \( \varepsilon \)
2: LOOP UNTIL \( \left| CDF_{\hat{e}}(q_i) - \alpha \right| < \varepsilon \)
3: \( q_{i+1} = q_i - \frac{CDF_{\hat{e}}(q_i) - \alpha}{PDF_{\hat{e}}(q_i)}, i \leftarrow i + 1 \)
4: END LOOP

D. Solution of stochastic power flexibility range

1) OPF model for PFR

Based on the multi-phase linear network model (5), we can calculate the maximal complex power injection of PCC at each time \( t \), denoted by \( P_{\text{PCC}} \) and \( Q_{\text{PCC}} \), which should consider the operational constraints related to networks and DERs. Specifically, for given the confidence \( \gamma = 1 - \alpha \) for chance constraints (25)-(28), (30) and the power factor \( \Phi \) of PCC, an optimal power flow (OPF) is developed to calculate the power flexibility range:

\[
\max_{x} P_{\text{PCC}} \cos(\Phi) + Q_{\text{PCC}} \sin(\Phi)
\]

(49)

The constraints include four parts:

(i) network model (5) and

\[
Q_{\text{PCC}} = P_{\text{PCC}} \tan(\Phi)
\]

(50)

(ii) the constraints of DERs (7)-(10);

(iii) maximum output power of renewable energy constraints (13) and (16);

(iv) the equivalent deterministic constraints of network (43)-(47).

Here, the power factor \( \Phi \) is a parameter. We can solve the OPF problem to find out the maximal regulation power capability under a constant power factor \( \Phi \). By varying \( \Phi \) from 0 to \( 2\pi \) [18], a series of OPFs are conducted to get a group of results, forming the power flexibility range with the confidence level \( \gamma \). As shown in Fig. 3, the power flexibility range varying with \( \Phi \) can be obtained, which represents the maximal regulation power capability.
If the confidence level changes, the values of quantiles in the network constraints (43)-(46) change accordingly. Then a new PFR is obtained under the new confidence level. A function can be constructed, which maps from the PFR to the confidence.

\[
\gamma = \text{Conf}_i \left( P_{\text{PCC}}, Q_{\text{PCC}} \right)
\]

In the 3-dimensional space of \( P_{\text{PCC}}, Q_{\text{PCC}} \) and \( \gamma \), this function is in the form of a surface. Each group of results under the same confidence forms a contour of this surface. By solving the OPFs with different confidence, the points on the corresponding contours can be calculated. Furthermore, all the points on the surface can be obtained, which is the stochastic PFR of the VPP.

2) Analytic reformulation of the stochastic PFR

In the section above, the stochastic PFR of VPP is gained, which is a surface composed of a large number of scattered points. Nevertheless, an analytic expression is preferable to describe the function (51). Here, we use the convex piecewise-linear fitting algorithm [24] to fit this function, whose process is explained in detail in the supplementary file [25]. Therefore, the analytic reformulation of the stochastic PFR can be expressed as the convex piecewise-linear function with \( m \) partitions:

\[
\text{Conf}_i \left( \mathbf{Y} \right) = \text{ReLU} \left( \min_{j=1,...,m} \left\{ a_j^T \mathbf{Y}^j + b_j \right\} \right)
\]

Where, \( \mathbf{Y} \) is the 2-dimensional variables vector composed of \( \left\{ P_{\text{PCC}}, Q_{\text{PCC}} \right\}^T \), \( \text{ReLU}(\cdot) \) denotes the Rectified Linear Unit function. Since \( \gamma \) is confidence level and it cannot be negative, we just need to fit the positive part of the function. The convex piecewise-linear fitting algorithm can be applied in the fitting process, because the positive part of this function is approximately concave.

IV. CALCULATING THE COST FUNCTION OF VPP

To participate in system operation or market bidding, the VPP need calculate its aggregating cost function besides its PFR. This section will introduce the solution for generating the cost function of VPP.

A. Cost model of DERs

Firstly, the cost functions of different types of DERs are explained here. We use \( P_{i,s} \) to represent the output power of DERs on bus \( i \) at time \( t \), that is

\[
P_{i,s} = \sum_{\gamma} P_{i,s,\gamma}, s \in \{ \text{SG, PV, ESS, wind} \}
\]

The cost functions of SGs are quadratic:

\[
C_{i,\text{SG}}(P_{i,\text{SG}}) = a_{i,\text{SG}} \left( P_{i,\text{SG}}^2 \right) + b_{i,\text{SG}} \left( P_{i,\text{DG}} \right) + c_{i,\text{SG}}
\]

The cost functions of PV and wind power take the cost of curtaining power into consideration, where \( P_{\text{grid}} \) is the grid electricity price at the PCC bus of the VPP at time \( t \).

\[
C_{i,\text{wind}}(P_{i,\text{wind}}) = P_{i,\text{grid}} \left( P_{i,\text{wind}} - P_{i,\text{wind}} \right)
\]

\[
C_{i,\text{PV}}(P_{i,\text{PV}}) = P_{i,\text{grid}} \left( P_{i,\text{PV}} - P_{i,\text{PV}} \right)
\]

The operation and maintenance cost of ESS is as follows, with \( K_i^e \) and \( K_i^{dis} \) denote the charge and discharge cost parameters of ESS, respectively.

\[
C_{i,\text{ESS}}(P_{i,\text{ESS}}) = \max \left\{ K_i^e P_{i,\text{ESS}} \Delta t - K_i^{dis} P_{i,\text{ESS}} \Delta t \right\}
\]

Where, \( \Delta t \) denotes the time interval of one period.

B. Piecewise-linear fitting the cost function of VPP

Using the stochastic PFR model presented in Section III, we can gain the minimum and maximum active output power of VPP at time \( t \), denoted by \( P_{\text{PCC}}^{\text{min}} \) and \( P_{\text{PCC}}^{\text{max}} \). Then, we can sample a series of operation points equally in interval \( \left[ P_{\text{PCC}}^{\text{min}}, P_{\text{PCC}}^{\text{max}} \right] \), and solve the flowing OPF to find the minimum operating cost of VPP, corresponding to the given \( P_{i,\text{PCC}} \).

\[
\min_{i=1,...,N} C_{\text{VPP}}^{(k)} = \sum_{i=1,...,N} \left( C_{i,\text{DG}}(P_{i,\text{DG}}) + C_{i,\text{wind}}(P_{i,\text{wind}}) + C_{i,\text{PV}}(P_{i,\text{PV}}) + C_{i,\text{ESS}}(P_{i,\text{ESS}}) \right)
\]

s.t. \( (5), (7)-(10), (13), (16), (43)-(47) \)

\[
P_{\text{PCC}} = P_{\text{PCC}}^{(k)}
\]

Here, \( C_{\text{VPP}}^{(k)} \) denotes the minimum cost of VPP corresponding to the \( k \)-th sample \( P_{\text{PCC}}^{(k)} \). Therefore, every tuple \( \left( P_{\text{PCC}}^{(k)}, C_{\text{VPP}}^{(k)} \right) \) corresponds to a point on the cost function curve of VPP. In this way, we can get \( K \) sample points in total. With the convex piecewise-linear fitting algorithm in the supplementary file [25], these samples can be further used to fit the analytic expression of the cost function into \( m \) linear partitions and can be easily incorporated to any optimization problem as follows:

\[
\min \text{Cost} (P_{\text{PCC}})^=r
\]

s.t. \( a_j^T P_{\text{PCC}} + b_j \leq r, j=1,...,m \)

V. NUMERICAL TESTS

A. Simulation setup

Numerical tests are carried out on the 15-bus modified European medium voltage distribution network benchmark [26]. The topology of network and detail parameters of DER units can refer to the supplementary file [25]. The maximum and minimum limits of buses’ voltage are set to 1.05 p.u. and 0.95 p.u., respectively. The output power data of PVs, wind turbines and loads are cited from [27] and [28]. These historical data are used to generate the probability density function of forecast errors using GMM.

The test case was conducted on a laptop with Intel Core i7-8550U CPU, 1.80 GHz and 16 GB RAM. The MATLAB software with YALMIP toolbox and CPLEX solver were used to solve the optimization problems.
We firstly use the proposed methodology to evaluate the stochastic PFR in section B. Then, the comparison of computational efficiency is presented in Section C. Finally, Section D introduces the aggregated piecewise-linear cost function of VPP.

### B. Stochastic power flexibility range evaluation

Based on our proposed method, we can obtain the stochastic PFR of VPP. We use the operational state of the VPP at 12:00 as an example. The result is a three-dimensional surface composed of many scattered points. The X-axis and Y-axis represents the injection active and reactive power at PCC of VPP, and the Z-axis represents the confidence level corresponding to the injected power. To express the results analytically, the obtained scatters are fitted into a 16 partitions’ piecewise-linear function, as shown in Fig. 4 (a).

Therefore, given a specific confidence level $\gamma$, we can calculate the corresponding PFR based on (52). It can be expressed as a polygon.

$$A'_yY' \leq b'_y \quad (61)$$

Where, $A'_y$ and $b'_y$ are the constant coefficients ($16 \times 2$) matrix and ($16 \times 1$) vector of this polygon, respectively. If the confidence level is set as $\gamma = 0.8$, the numerical values of $A'_y$ and $b'_y$ can refer to the supplementary file [25].

What is more, the stochastic PFR of VPP can also be obtained point by point by the Monte Carlo simulation method. Based on the expectation vectors and covariance matrices of GMM, we can generate the realization scenarios of the loads and maximum output power of renewable generators. Subsequently, at each realization of injection power at PCC ($P_{PCC}^{(t)}$, $Q_{PCC}^{(t)}$), we can construct all the operational constraints and find out the proportion of feasible scenarios as the confidence level of this point $(P_{PCC}^{(t)}$, $Q_{PCC}^{(t)}$). After finishing scanning all the operating points, we can get stochastic PFR of VPP at time $t$. In this test case, we scanned possible injection power points on a 200×200 grid and generated 1,200 scenarios at each point to calculate the confidence level. The results of Monte Carlo simulation is shown in Fig. 4 (b).

Although the proposed method and the Monte Carlo simulation method can obtain similar results, the computational efficiency of the two differs greatly. The calculation time and RMSE of our proposed method and the Monte Carlo simulation are listed in TABLE II. Obviously, the Monte Carlo simulation cannot be used for real application since of its ultra-heavy computational burden.

### C. Comparison of computational efficiency

#### TABLE II

**COMPUTATIONAL EFFICIENCY COMPARISON**

| Item                  | Calculation time | RMSE |
|-----------------------|------------------|------|
| Our proposed method   | 83.19 s          | 0.0173 |
| Monte Carlo simulation| 12,372 s         | 0.0176 |

#### TABLE I

**ERRORS COMPARISON (RMSE: ROOT MEAN SQUARED ERROR, $R^2$: THE COEFFICIENT OF DETERMINATION)**

| Item                  | RMSE     | $R^2$  |
|-----------------------|----------|--------|
| Convex piecewise-linear fitting | 0.0173 | 0.9982 |
| Monte Carlo simulation | 0.0176 | 0.9979 |

D. Aggregated cost function of VPP

With the aggregated cost function generation method, we can solve the piecewise-linear cost function of VPP. The scatters in Fig. 5 shows the original calculation results of the VPP’s cost and the polyline is the piecewise-linear fitting result at 12:00.

![Fig. 5. The cost function of VPP at 12:00.](image)

Although the proposed method and the Monte Carlo simulation method can obtain similar results, the computational efficiency of the two differs greatly. The calculation time and RMSE of our proposed method and the Monte Carlo simulation are listed in TABLE II. Obviously, the Monte Carlo simulation cannot be used for real application since of its ultra-heavy computational burden.

In this numerical test, the cost function is divided into five partitions. The generation cost function of VPP is adaptively divided into five linear partitions based on the forecast value of loads and output power of renewable energy generators. When the active output power of VPP is negative, the generation cost of VPP is also negative, which means the VPP need to purchase some electricity to meet the demand in it.

Fig. 6 shows the cost function with the granularity of 15 minutes. Because of their fluctuations, the output power range of VPP also changes throughout the day and the cost changes.
We can further calculate the maximum, minimum, and average RMSE and $R^2$ of the cost function fitting at different times, as shown in TABLE III.

| Item     | RMSE    | $R^2$   |
|----------|---------|---------|
| Maximum  | 0.0140  | 0.9999  |
| Minimum  | 0.0019  | 0.99974 |
| Average  | 0.0099  | 0.99982 |

Combining the stochastic PFR results with the corresponding generation cost functions, the operators of VPPs can further construct their own bidding strategies. They can send the PFR of VPP in the form of a polygon as (61), the segmented nodes of 5 partitions and the corresponding operational cost of VPP to the power system control center. Therefore, the VPPs can participate in the electricity market as a special power plant.

VI. Conclusion

In this paper, we propose a method to assess the stochastic power flexibility range and operational cost function of virtual power plant. This problem can be formulated as a chance constrained optimization problem and equivalently converted a deterministic convex optimization:

- Using Gaussian mixture model to characterize the uncertainties of forecast errors;
- Transforming the risk constraints into equivalent deterministic constraints; and
- Linearizing the models of network and DERs.

Moreover, we use the convex piecewise-linear fitting algorithm to fit the stochastic PFR and cost function of VPP with the data-driven method and make it easily embedded into the market-clearing model.

The results of numerical test verify the correctness of our model and show the high efficiency of our proposed method.

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