Constraining vacuum gap models of pulsar radio emission using the intensity modulation index

Janusz Gil\textsuperscript{1} & Fredrick A. Jenet\textsuperscript{2}

\textbf{ABSTRACT}

Recent observations suggest that the level of pulse-to-pulse intensity modulation observed in a given radio pulsar may depend on its period and period derivative. Such a “modulation index relationship” (MIR) may be an important tool for determining the physical processes behind the radio emission. In the context of sparking gap models, the exact functional form of the MIR depends on the physical processes occurring on the surface of the neutron star in the region known as the “vacuum gap.” Several possible vacuum gap models are studied here in order to determine the expected MIR for a given model. Current observations are consistent with two of the four models studied: the curvature radiation driven vacuum gap and the curvature radiation driven near threshold vacuum gap (CR-NTVG). It is shown that the inverse Compton scattering driven vacuum gap models are not supported by the current data. Given that the current data best supports the CR-NTVG model, it possible that all pulsars have strong ($\approx 10^{13}$ G) surface magnetic fields.

\textit{Subject headings:} pulsars:general

1. \textbf{INTRODUCTION}

The physical mechanism of the coherent pulsar radio emission has remained elusive since their discovery over 30 years ago. The observed high brightness temperatures together with enormous amount of phenomenology exhibited makes these sources very difficult to understand. However, it is generally accepted that pulsar radio emission is generated within a dense electron-positron plasma, created near the polar cap and flowing along open magnetic field lines. The observed pulse-to-pulse intensity modulation can arise from the time-dependent

\textsuperscript{1}\textit{Institute of Astronomy, University of Zielona Góra, Lubuska 2, 65-265, Zielona Góra, Poland}

\textsuperscript{2}\textit{California Institute of Technology, Jet Propulsion Laboratory}
4800 Oak Grove Drive, Pasadena, CA 91109
lateral structure of this flow, probed once per pulsar period by the observer’s line of sight. Ruderman & Sutherland (1975, RS75 henceforth) proposed a pulsar model in which the lateral structure was in the form of localized spark discharges in a charge depleted region near the stellar surface know as the “vacuum gap.” Gil & Sendyk (2000, GS00 henceforth) explored the RS75 model in an attempt to relate the observed radio emission properties to a pulsar’s period $P$ and its derivative $\dot{P}$. They postulated that the polar cap is populated as densely as possible with a number of sparks, each having a characteristic size as well as separation from adjacent sparks approximately equal to the height $h$ of the vacuum gap acceleration region. This leads directly to the so-called complexity parameter

$$ a = \frac{r_p}{h} \quad (1) $$

equal to the ratio of the polar cap radius to the characteristic spark dimension. One can show that $a$ is approximately equal to the maximum number of sparks across the polar cap and thus the maximum number of sub-pulses that can appear in the pulse window. Therefore, $a$ describes the complexity of single pulses and/or mean profile (see GS00 for details).

The pulse-to-pulse intensity modulation can be quantified by the phase resolved modulation index defined as

$$ m(\phi) = \sqrt{< I_s(\phi)^2 > - < I_s(\phi) >^2 } < I_s(\phi) >, \quad (2) $$

where $I_s(\phi)$ is the pulsar intensity at pulse phase $\phi$ and the angle brackets represent averaging over the ensemble of single pulses (for details see Jenet, Anderson & Prince 2001). Within the framework of sparking gap model the observed pulse-to-pulse intensity modulation is due to the presence of a number sparks moving over the polar cap either erratically or in an organized manner (drifting). As the number of sparks increase, one expects to see less pulse-to-pulse intensity modulation. Hence, the modulation index (Eq. [1]) should be anticorrelated to the complexity parameter (Eq. [2]), to the extent that at very high values of $a$ there should be no detectable intensity modulation (GS00).

In general, the complexity parameter (Eq. [1]) can be expressed in the form $a \propto P^\beta \dot{P}^\gamma$. Since the modulation index may be directly related to some unknown function of $a$, it is useful to use the Spearman rank ordered correlation (SROC) technique in order to determine if a correlation exists between $a$ and $m$. The SROC technique is independent, up to a sign, of any monotonic function applied to $m$ and/or $a$. In this case, only the ratio $\alpha = \beta / \gamma$ is important. Jenet & Gil (2003a, JG03 henceforth) used the SROC technique to search for the proposed relationship, hereafter referred to as a modulation index relationship (MIR), using a sample of 12 pulsars. This preliminary investigation found evidence for a MIR with $\alpha$ between $[-5, -2]$. The most significant value of $\alpha = -2.7$. In general, a MIR will provide an important constraint on any model of pulsar radio emission. Even its non-existence may be
an important piece of information. For the purposes of this letter, only spark gap models will be considered. These models predict the existence of a MIR together with a value of $\alpha$. The specific value of $\alpha$ depends on the vacuum gap model in question. Thus, an observationally determined range of $\alpha$ values will rule out various vacuum gap models.

In the next section, four models of the vacuum gap acceleration region are studied in order to determine the expected value of $\alpha$ for each model. This letter is summarized in §3.

2. MODELS OF VACUUM GAP ACCELERATION REGIONS

All vacuum gap (VG) models assume that the free outflow of electrons or ions from the polar cap surface is strongly inhibited, which leads to formation of a charge depleted region just above the polar cap. These regions are conventionally called the vacuum gap even if the charge density is not exactly zero within it (for review see Gil, Melikidze & Geppert 2003). The charge depletion with respect to the corotational value (Goldreich & Julian 1969) results in an extremely high potential drop across the VG, which is discharged by photon induced pair creation in the strong and curved surface magnetic field. The details of this discharge depend on the strength and curvature of the surface magnetic field and on the physical process that generate the electron-positron pair plasma.

The analysis presented here will calculate the complexity parameter for each VG model in question. In order to do this, both the vacuum gap height, $h$, and the polar cap radius, $r_p$, need to be estimated. Both of these quantities will depend on the structure of the magnetic field near the surface of the neutron star.

Classically, the magnetic field is assumed to be dipolar. There is a growing evidence that the actual surface magnetic field is highly non-dipolar (for review see Geppert, Rheinhardt, & Gil 2003; Urpin & Gil 2003). The total surface field, $B_s$, may be written as

$$B_s = bB_d,$$

where $b$ is a constant of proportionality which relates the dipolar field component, $B_d$, to the total surface field strength. For VG models that include the effects of non-dipolar fields (i.e. $b \neq 1$), it will be useful to estimate $b$ in terms of $P$ and $\dot{P}$. Two assumptions will be made in order to do this. First, it will be assumed that the observationally inferred surface magnetic field is actually a measure of the surface dipole component of the field. This is a reasonable assumption since the power radiated by the dipolar magnetic field dominates the other multi-pole components. The surface dipolar magnetic field strength, $B_d$, is estimated by

$$B_d = 6.4 \times 10^{19}(P\dot{P})^{0.5} G = 2 \times 10^{12}(P\dot{P}_{-15})^{0.5} G,$$
where $P$ is the pulsar period and $\dot{P}$ is the period derivative with respect to time (Shapiro & Teukolsky 1983). The second assumption will be that the total surface field, $B_s$, will be approximately equal to $10^{13} G$ for all pulsars, independent of $P$ and $\dot{P}$ (Gil & Melikidze 2002, and references therein). With these two assumption, $b$ may be written as

$$b = 5B_{13}(P\dot{P})^{-0.5},$$

(5)

where $B_{13} = B_s/10^{13}$ G is of order unity.

The size of the polar cap is determined by those magnetic field lines which connect to the interstellar medium magnetic field. For all the VG models discussed below, the polar cap radius will be estimated using

$$r_p = b^{-0.5}R_6^{1.5}10^4P^{-0.5} \text{cm},$$

(6)

where $R_6$ is the neutron star radius in units of $10^6$ cm. This is the classical dipolar field polar cap radius modified by a factor of $b^{-0.5}$ due to magnetic flux conservation over the tube of open magnetic field lines.

Four VG models will be discussed in the next four subsections. The models are classified by the structure of the surface magnetic field and the physics of the electron-positron pair plasma production process. The prefix “CR” stands for curvature radiation induced pair production (Erber 1966, RS75) while the prefix “ICS” stands for resonant inverse Compton scattering induced pair production(Zhang et al. 1997, and references therein). The suffix “VG” is used for models that assume $b = 1$ (i.e. a pure dipolar field) while the suffix “NTVG” is for “near threshold vacuum gap” models that assume $b$ is given by Equation 5. The results of the next four subsections are summarized in Table 1 which gives $a$ and $\alpha$ for each model.

| Model     | Complexity Parameter ($a$) | $\alpha$ | References |
|-----------|--------------------------|---------|------------|
| CR-VG     | $2.9R_6^{-2/7}R_6^{3/2}P^{-9/14}\dot{P}^{2/7}$ | -2.25   | 1, 2       |
| CR-NTVG   | $3.0R_6^{-2/7}R_6^{1/2}B_{13}^{1/14}P^{-19/28}\dot{P}^{1/4}$ | -2.71   | 3, 4       |
| ICS-VG    | $1.1R_6^{-0.57}R_6^{3/2}\dot{P}^{0.14}P^{-0.79}$ | 0.18    | 3, 5       |
| ICS-NTVG  | $4.5R_6^{-0.57}R_6^{3/2}B_{13}^{0.5}P^{-0.39}\dot{P}^{0.25}$ | -1.56   | 3, 4       |

References. — (1) Ruderman & Sutherland (1975), (2) Gil & Sendyk (2000), (3) Gil & Mitra (2001), (4) Gil & Melikidze (2002), (5) Zhang et al. (1997)
2.1. CR-VG model

The model of RS75 is the prototype of a curvature radiation induced vacuum gap. The gap height is determined by the condition \( h = l_{ph} \), where \( l_{ph} \) is the mean free path for a photon to collide with the strong magnetic field and form an electron-positron pair. This model is valid for a dipolar magnetic field with \( B_d \leq 4.4 \times 10^{12} \) G. The gap height in this model is \( h = (3.5 \times 10^3) R_6^{2/7} P_1^{1/7} \dot{P}_{-15}^{-2/7} \) cm (RS75, Gil & Melikidze 2002), where \( R_6 \approx 1 \) is the radius of curvature of field lines in units of neutron star radius \( R = 10^6 \) cm. The complexity parameter, \( a \), takes the form \( a = 2.9 R_6^{-2/7} R_6^{3/2} P^{-9/14} \dot{P}_{-15}^{2/7} \) and \( \alpha = -2.25 \). This value of \( \alpha \) is currently in the range allowed by the analysis of JG03.

2.2. CR-NTVG model

The CR-VG model described above has a fundamental problem: the binding energy is much too small to prevent thermionic emission from the polar cap surface (e.g. Abrahams & Shapiro 1991; Usov & Melrose 1995, 1996). Gil & Mitra (2001) argued that this problem can be solved by assuming that pulsars have an extremely strong, non-dipolar surface magnetic field whose magnitude, \( B_s \), is close to about \( 10^{13} \) G, independent of \( P \) and \( \dot{P} \). In such a strong magnetic field \( (B_s \geq 4.4 \times 10^{12} \) G (e.g. Usov & Melrose 1995)), the curvature radiated photons will pair produce near the kinematic threshold (e.g. Daugherty & Harding 1983). Hence, this type of model is called a curvature radiation induced near threshold vacuum gap (CR-NTVG) model. The gap height in this model is \( h = (3 \times 10^3) R_6^{2/7} b^{3/7} P^{3/14} \dot{P}_{-15}^{-3/14} \) cm (Gil & Melikidze 2002, GM02 hereafter). Using Equations 1, 5, and 6 one obtains \( a = 3.0 R_6^{-2/7} R_6^{3/2} B_{13}^{-1/14} P^{-19/28} \dot{P}_{-15}^{1/4} \). This gives \( \alpha = -2.71 \), amazingly close to the maximum significant value obtained by JG03.

2.3. ICS-VG model

In this model, the electron-positron pairs are produced by inverse Compton scattered seed photons in a relatively low strength dipolar magnetic field (Zhang et al. 1997, and references therein). The gap height in this model is \( h = (8.8 \times 10^3) R_6^{0.57} P^{-0.64} \dot{P}^{-0.79} \) cm (Zhang, Harding & Muslimov 2000; Gil & Mitra 2001). Thus, the complexity parameter is \( a = 1.1 R_6^{-0.57} R_6^{1.5} P^{0.14} \dot{P}_{-15}^{0.79} \) and \( \alpha = 0.18 \). This value is well outside the range of \( \alpha \) values supported by the analysis of JG03.
2.4. ICS-NTVG model

This is a near-threshold version of the ICS-VG model, valid for $B_s > 4.4 \times 10^{12}$ G. The gap height in this model is $h = (5 \times 10)^3 R_6^{0.57} b_1 P^{-0.36} \dot{P}^{-0.5} \text{ cm}$ (GM02). Using equations 1, 5, and 6 one obtains $a = 4.5 R_6^{-0.57} R_6^{1.5} B_{13}^{0.5} P^{-0.39} \dot{P}^{0.25}_{-15}$ and $\alpha = -1.56$. This value of $\alpha$ is also outside the current range of supported values.

3. SUMMARY AND DISCUSSION

By analyzing the pulse-to-pulse intensity fluctuations of a set of 12 pulsars, JG03 found a significant correlation between the measured intensity modulation index (Eq. [2]) and $a(\alpha) = P^\alpha \dot{P}$ when $\alpha$ is within the range $[-5,-2]$. The most significant correlation was found when $\alpha = -2.7$. JG03 concluded that this modulation index relationship (MIR) is consistent with the RS75 vacuum gap model which predicts $\alpha = -2.25$. The work presented here calculates the expected values of $\alpha$ for four models of the vacuum gap including the RS75 model. Assuming that future observations confirm the results of JG03, the inverse Compton scattering (ICS) driven gap models can be ruled out since they predict values of $\alpha$ outside the allowed range. Note that JG03 performed their analysis on “core” emission components only, hence the ICS models are only ruled out for such components. The curvature radiation driven gap models are consistent with the JG03 results. Moreover, we demonstrate that the near threshold vacuum gap model of GM02 reproduces the observed value of the most likely exponent $\alpha = -2.7$. Since this model requires large amplitude, non-dipolar, surface magnetic fields, confirmation of a MIR with $\alpha = -2.7$ could be evidence for non-dipolar surface fields of order $10^{13} G$. Such fields may be generated by small-scale turbulent dynamo action just after the formation of the neutron star (Urpin & Gil 2003) or by Hall-effect processes occurring on the stellar surface (Geppert, Rheinhardt, & Gil 2003).

Part of this research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. GJ acknowledges the support of the Polish State Committee for scientific research under Grant 2 P03D 008 19. We thank E. Gil for technical help.

REFERENCES

Abrahams, A.M., & Shapiro, S. L. 1991, ApJ, 374, 652
Daugherty J., & Harding, A.K. 1983, ApJ, 273, 761
Erber, T. 1966, Rev. Mod. Phys., 38, 626

Geppert, V., Rheinhardt, M., & Gil, J. 2003, A&A Letters, accepted (astro-ph/0311121)

Gil, J. & Sendyk, M. 2000, ApJ, 541, 351 (GS00)

Gil, J. & Mitra, D. 2001, ApJ, 550, 383

Gil, J. & Melikidze, G.I. 2002, ApJ, 577, 909 (GM02)

Gil, J., Melikidze, G.I., & Geppert, U. 2003, A&A, 407, 315 (GMG03)

Goldreich, P. & Julian, H. 1969, ApJ, 157, 869

Jenet, F.A., Anderson, S.B., & Price, T.A. 2001, ApJ, 546, 394

Jenet, F.A. & Gil, J. 2003a, ApJ, 596, L215 (JG03)

Jenet, F.A. & Gil, J. 2003b, ApJ, submitted

Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51

Shapiro, S.L., & Teukolsky S.A. 1983, Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects (New York: Wiley)

Urpin, U., & Gil, J. 2003, A&A, in press (astro-ph/0311180)

Usov, V.V., & Melrose, D. B. 1995, Aust. J. Phys. 48, 571

Usov, V.V., & Melrose, D. B. 1996, ApJ, 464, 306

Zhang, B., Qiao, G.J., Lin, W.D., et al. 1997, ApJ, 478, 313

Zhang, B., Harding, A., & Muslimov, A.G. 2000, ApJ, 531, L135

This preprint was prepared with the AAS LaTeX macros v5.0.