Abstract The heterotic and type II superstring actions are identified in different anomaly-free decompositions of a single topological sigma-model action depending on bosonic and fermionic coordinates, $X^\mu$ and $\rho^A$ respectively, and of their topological ghosts. This model results from gauge-fixing the topological gauge symmetry $\delta X^\mu = e^\mu(z,\bar{z})$ ($\mu = 1, 2, \ldots, 10$) and $\delta \rho^\alpha = e^\alpha(z,\bar{z})$. ($\alpha = 1, 2, \ldots, 16$). From another viewpoint the heterotic and type II superstring actions emerge as two different gauge-fixings of the same closed two-form. Comments are also made concerning the possibility of relating $\rho^\alpha$ to a Majorana-Weyl space-time spinor superpartner of $X^\mu$. 

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1 Introduction

There have been various suggestions of an underlying topological basis for string and superstring theory [1]-[5]. In particular, the relevance of twisted nonlinear sigma models to mirror symmetry of Calabi–Yau spaces and of the complexification of space-time has been pointed out in [1] and [6]. In this paper we will pursue the idea of writing superstring actions in the ‘Neveu–Schwarz–Ramond’ (NSR) formalism as sums of exact and $s$-exact terms where $s$ denotes the BRST transformation associated with local redefinitions of world-sheet fields, $X^\mu(z, \bar{z})$ (where $\mu = 1, \ldots, 10$) together with anticommuting coordinates $\rho^\alpha$ ($\alpha = 1, 2, \cdots, 16$). Thus, we will consider theories based on the very large symmetry,

$$\delta X^\mu(z, \bar{z}) = \epsilon^X_\mu(z, \bar{z})$$

$$\delta \rho^\alpha(z, \bar{z}) = \epsilon^\rho_\alpha(z, \bar{z}).$$  \hspace{1cm} (1.1)

A classical action with such a huge symmetry is guaranteed to be purely topological; any lagrangian density invariant under (1.1) must be locally a pure derivative. The rest of the gauge-fixed quantum action is then ghost-dependent and exact under the BRST transformation, $s$, associated with the symmetry defined in (1.1). Once a suitable gauge is chosen the fields $X^\mu$ will describe the target-space coordinates and $\rho^\alpha$ will become the internal symmetry coordinates of the heterotic string. The world-sheet fermionic coordinates associated with $(0, 1)$ supersymmetry for the heterotic theory or $(1, 1)$ supersymmetry for the type II theories arise as combinations of ghosts and antighosts for this symmetry, as will be seen in section 2. The symmetry (1.1) refers to a given world-sheet. In order to define the superstring theory one must also perform the usual sum over all super world-sheets that reduces to integration over the moduli space of two-dimensional metrics and gravitini. These contribute terms in the string quantum action that are also guaranteed to be $s$-exact.

Gauge-fixing will be discussed in section 3. There we show that the choice of superconformal gauge, together with suitable choices of gauge functions for $\rho^\alpha$ and $X^\mu$ leads to an action, $I_B$, that is the sum of a term that is $s$-exact and a topological term. This anomaly-free action may be interpreted as the action for the heterotic string, $I_{het}$, by writing it as $I_B = I_{het} + I'$, where
I" involves fields that may be integrated out. Alternatively, it may be interpreted as the type II action, \( I_{II} \), by writing \( I_B = I_{II} + I'' \), where \( I'' \) also involves fields that may be integrated out. These decompositions are consistent because the terms \( I_{het}, I_{II}, I' \) and \( I'' \) are separately free of anomalies. It is non-trivial that such decompositions exist. The heterotic fermions are identified with elements of the topological BRST quartet of fields in the \( \rho \) sector.

The fact that both the type II and heterotic superstring actions can be viewed as anomaly-free truncations of the same topological \( \sigma \) model with bosonic and fermionic coordinates in the superconformal gauge means that the full theory contains purely topological observables. This topological system also determines matter-dependent observables characterized by the cohomology of the BRST symmetry of the superconformal symmetry, which now appears as a small part of the topological \( \sigma \)-model BRST symmetry. This unification of the type II and heterotic superstring actions requires the introduction of new field variables which decouple from physical quantities but which play a rôle in the computation of the purely topological observables.

The intertwining of world-sheet and space-time symmetries of superstring theory is an interesting and subtle issue. It is tempting to believe that the NSR and the ‘Green–Schwarz’ (GS) formalism (which has manifest space-time supersymmetry) may be viewed as different gauge-fixed versions of a theory that somehow encompasses both. In that case there could be two anticommuting coordinates like \( \rho^\alpha \), one of which has been chosen to be zero in (1.1). After a suitable gauge choice one or both of these coordinates may then be identified with Majorana–Weyl space-time spinor superpartners of \( X^\mu \) in the heterotic or type II theories. A version of the BRST algebra that manifests this space-time supersymmetry will be presented in section 4 although we have not succeeded in obtaining the GS action by gauge fixing in this manner.

2 The BRST algebra for world-sheet matter and supergravity

The BRST symmetry corresponding to the gauge symmetry (1.1) is obtained by changing the parameters \( \epsilon_X \) and \( \epsilon_\rho \) into topological ghosts \( F_X(z, \bar{z}) \) and \( F_\rho(z, \bar{z}) \) and by introducing the antighosts \( \overline{F}_X(z, \bar{z}) \) and \( \overline{F}_\rho(z, \bar{z}) \) with their Lagrange multipliers \( \lambda_X(z, \bar{z}) \) and \( \lambda_\rho(z, \bar{z}) \). The fields \( F_X, \overline{F}_X \) and \( \lambda_\rho \) are fermionic while \( F_\rho, \overline{F}_\rho \) and \( \lambda_X \) are bosonic fields. The graded
differential BRST operator $s$ which encodes the topological gauge symmetry (1.1) is defined as
\[ sX^\mu = iF_X^\mu, \]
\[ sF_X^\mu = 0, \]
\[ sF_X^\mu = \lambda_X^\mu, \quad s\lambda_X^\mu = 0, \] (2.1)
and
\[ s\rho^\alpha = iF_\rho^\alpha, \]
\[ sF_\rho^\alpha = 0, \]
\[ sF_\rho^\alpha = \lambda_\rho^\alpha, \quad s\lambda_\rho^\alpha = 0. \] (2.2)

This symmetry is needed to define a BRST invariant action associated with the symmetry (1.1) on a given worldsheet. The possibility of relating $\rho^\alpha$ to a space-time spinor coordinate will be described in section 4.

The integration over the world-sheet metric and gravitino may be carried out in a superconformal gauge that fixes the super-Weyl and super-reparametrization symmetries. The result is a theory that possesses the BRST symmetry associated with an $N = 1$ superconformal theory. In the following we shall consider the situation in which there is no conformal anomaly. In that case the conformal factors can be gauged away so that the variables that enter the gravitational part of the action are the Beltrami differential $\mu_x^z$ and its anticommuting reparametrization ghost $c^z$, the conformally invariant part of the gravitino $\alpha_{-\frac{1}{2},0}$ and its commuting supersymmetry ghost $\gamma_{-\frac{1}{2},0}$, together with the reparametrization and supersymmetry antighosts $b_{zz}$ and $\beta_{\frac{3}{2},0}$ for the holomorphic sector and the complex conjugates for the other sector. These fields collectively constitute the (super) Beltrami variables. The use of these variables allows for a complete separation between the left-moving and right-moving sectors. The holomorphic sector possesses the factorized BRST symmetry algebra,
\[ s\mu_x^z = \partial_x c^z + c^z \partial_x \mu_x^z - \mu_x^z \partial_x c^z + 2i\alpha_{x,0} \frac{1}{2} \gamma_{x,0}, \]
\[ s\alpha_{x,0} = \partial_x \gamma_{x,0} + \frac{1}{2} \gamma_{x,0} \partial_x \mu_x^z - \mu_x^z \partial_x \gamma_{x,0} + c^z \partial_x \alpha_{x,0} - \frac{1}{2} \alpha_{x,0} \partial_x c^z, \]
\[ sc^z = c^z \partial_x c^z + i \gamma_{x,0}, \]
\[ s\gamma_{x,0} = c^z \partial_x \gamma_{x,0} + \frac{1}{2} \gamma_{x,0} \partial_x c^z, \] (2.3)
with analogous equations in the anti-holomorphic sector. The superconformal gauge conditions are \( \mu \frac{z}{z} = \alpha \frac{1}{2} \frac{0}{0} = 0 \) in the holomorphic sector and \( \mu \frac{z}{z} = \alpha \frac{0}{0} \frac{1}{2} = 0 \) in the antiholomorphic sector. It is well-known that in this gauge the ghost action is given by the \( s \)-exact term [7],

\[
s \left( b_{zz} \mu_{zz} + \beta \frac{1}{2} \frac{0}{0} + b_{zz} \mu_{zz} + \beta \frac{0}{0} \frac{1}{2} \frac{0}{0} \right) = -b_{zz} \partial \bar{z} \bar{c} + \beta \frac{1}{2} \frac{0}{0} \partial \bar{z} \bar{c} + \beta \frac{0}{0} \frac{1}{2} \partial \bar{z} \bar{c}.
\]

From now on we will work in this gauge.

The issue of the consistency of this construction on world-sheets of higher genus is not addressed here.

3 The \( s \)-exact and \( d \)-exact action.

We begin by considering the type II superstring action in a flat target-space metric expressed in the superconformal gauge,

\[
I_{II} = \int d^2 z \sum_{\mu=1}^{10} \left( \partial_z X^\mu \partial \bar{z} X^\mu - i \Psi^{1,1} \partial_z \Psi^{1,1} - i \Psi^{1,2} \partial_z \Psi^{1,2} + i \Psi^{2,1} \partial_z \Psi^{2,1} - i \Psi^{2,2} \partial_z \Psi^{2,2} \right) + I_{II}^{\text{ghost}},
\]

where \( I_{II}^{\text{ghost}} \) is the standard \((1,1)\) superconformal ghost action [2.4] for the type II theories.

The field \( X^\mu \) has conformal weight \((0,0)\) while the conformal weight of the components of the two-dimensional world-sheet Majorana spinor, \( \Psi^{\mu i} \), are \((\frac{1}{2},0)\) for \( i = 1 \) and \((0,\frac{1}{2})\) for \( i = 2 \). In this and subsequent formulae the signature of space-time will be arbitrary.

The next step is to define new fermionic fields, \( F_X \) and \( \bar{F}_X \), that are linear combinations of the NSR fields by

\[
2 \Psi^{a,1} = F_X^{11-a} + F_X^a - i F_X^{11-a} \]
\[
2 i \Psi^{11-a,1} = F_X^{11-a} - F_X^a + i F_X^{11-a} \]
\[
2 \Psi^{a,2} = \bar{F}_X^{a} + F_X^a + i F_X^{11-a} \]
\[
2 i \Psi^{11-a,2} = \bar{F}_X^{a} - F_X^a - i F_X^{11-a} \]

for \( 1 \leq a \leq 5 \). The notation indicates that the fields \( F_X \) and \( \bar{F}_X \) will be identified with the topological ghosts and antighosts introduced in the last section. These field redefinitions can
be considered to be twists of the original fields \[8, 1\]. The inverse relations expressing the
topological ghosts and antighosts in terms of the NSR fields are

\[
2F_X^a = \Psi^{a,2} + \Psi^{a,1} - i\Psi^{11-a,1} - i\Psi^{11-a,2}
\]

\[
2iF_X^{11-a} = \Psi^{a,2} - \Psi^{a,1} + i\Psi^{11-a,1} - i\Psi^{11-a,2},
\]

(3.4)

\[
\overline{F}_X^a = \Psi^{a,2} + i\Psi^{11-a,2}
\]

\[
\overline{F}_X^{11-a} = \Psi^{a,1} + i\Psi^{11-a,1},
\]

(3.5)

In the twisted version of the theory the fields defined in (3.4) have zero world-sheet spin while
the conjugate fields defined in (3.5) have spin one.\footnote{The apparent mismatch in the conformal weights on the left-hand and right-hand sides of these equations can be compensated by field redefinitions involving factors of $\gamma^{\frac{1}{2},0}$ or $\gamma^{0,\frac{1}{2}}$ \cite{[10]}, as will be reviewed later.}

The identification of $F_X$ and $\overline{F}_X$ with the topological ghosts and antighosts follows from
the expression for $I_{11}$ in terms of the new fields,

\[
I_{11} = \int d^2z \sum_{a=1}^{5} \left( -\lambda_X^a \lambda_X^{11-a} + \lambda_X^a (\partial_z X^a + i\partial_\bar{z} X^{11-a}) + \lambda_X^{11-a} (\partial_\bar{z} X^a - i\partial_z X^{11-a}) + i\overline{F}_X^a (\partial_z F^a + i\partial_\bar{z} F^{11-a}) - i\overline{F}_X^{11-a} (\partial_\bar{z} F^a - i\partial_z F^{11-a}) \right) + I_{top} + I_{11}^{\text{ghost}},
\]

(3.6)

where

\[
I_{top} = i \int \sum_{a=1}^{5} dX^a \wedge dX^{11-a} = i \int \sum_{a=1}^{5} d( X^a \wedge dX^{11-a}).
\]

(3.7)

This has a form that is manifestly the sum of a topological term, $I_{top}$, and a $s$-exact term,

\[
I_{11} = \int d \left( i \sum_{a=1}^{5} X^a \wedge dX^{11-a} \right) + \int d^2 z \sum_{a=1}^{5} \left[ b_{zz} \frac{1}{2} \lambda_X^a + b_{zz} \frac{1}{2} \lambda_X^{11-a} + \partial_\bar{z} X^a + i\partial_\bar{z} X^{11-a} + \overline{F}_X^a \left( \frac{1}{2} \lambda_X^a + \partial_\bar{z} X^a - i\partial_z X^{11-a} \right) \right]
\]

(3.8)

The identification of the NSR fermions as combinations of the topological ghosts and antighosts
of the gauge symmetry (1.1) is quite striking.\footnote{The apparent mismatch in the conformal weights on the left-hand and right-hand sides of these equations can be compensated by field redefinitions involving factors of $\gamma^{\frac{1}{2},0}$ or $\gamma^{0,\frac{1}{2}}$ \cite{[10]}, as will be reviewed later.}

Although ghost number is not conserved in these definitions, it is conserved modulo 2, which is all that is required in the quantum theory. The fields $F_X$ and $\overline{F}_X$ are assumed to have boundary values such that $\int d^2 z \partial_\bar{z} (\overline{F} F)$ and $\int d^2 z \partial_\bar{z} (\overline{F} F)$ vanish (for instance with periodic or anti-periodic conditions).
We now turn to consider how the heterotic string action can be obtained by gauge fixing the same topological symmetry. In this case we will find that the heterotic action can be identified with an anomaly-free part of a larger action, $I_B$, that contains other fields that decouple from the fields in the heterotic theory. The starting point is the action for the heterotic string in the superconformal gauge,

$$I_{het} = \int d^2z \left( \sum_{\mu=1}^{10} \partial_z X^\mu \partial_{\bar{z}} X^\mu - i \sum_{\mu=1}^{10} \Psi^{\mu1} \partial_{\bar{z}} \Psi^{\mu1} - b_{zz} \partial_{\bar{z}} e^z + \beta_{\bar{z},0} \partial_{\bar{z}} \gamma^{\bar{1},0} - i \sum_{i=1}^{32} f^i \partial_z f^i - b_{zz} \partial_{\bar{z}} e^{\bar{z}} \right),$$

(3.9)

where the $(0,1)$ superconformal ghost action has been explicitly included. The holomorphic sector is the same as in the type II case while the anti-holomorphic sector contains the 32 anticommuting Majorana-Weyl world-sheet spinors, $f^i$, in addition to the bosonic coordinates and the anti-holomorphic $(b, c)$ ghost system.

In order to obtain this action from an expression that is a sum of $d$-exact and $s$-exact pieces we next observe that the term involving the fermionic fields, $f^i$, can be rewritten as

$$\int d^2z \sum_{i=1}^{32} f^i \partial_z f^i = \int d^2z \sum_{\alpha=1}^{16} \lambda_\rho^\alpha \partial_z \rho_\alpha,$$

(3.10)

where

$$f^\alpha = \rho^\alpha + \lambda^\alpha 2$$

and

$$f^{33-\alpha} = \rho^\alpha - \lambda^\alpha 2i$$

(3.11)

for $1 \leq \alpha \leq 16$. If periodic or antiperiodic boundary spin structures are chosen for all $f^i$ (which defines the $SO(32)$ heterotic string) the term $\int d^2z \partial_z (f^i f^{33-i})$ vanishes. Otherwise (for example, in the $E_8 \times E_8$ case) there is an additional $d$-exact term on the right-hand side of (3.11).

The expression (3.10) can be identified with part of the $s$-exact action,

$$I'' = i \int d^2z \sum_{\alpha=1}^{16} s(\bar{F}_\rho^\alpha \partial_z \rho_\alpha)$$

$$= \int d^2z \sum_{\alpha=1}^{16} (i \lambda_\rho^\alpha \partial_z \rho_\alpha - \bar{F}_\rho^\alpha \partial_z F_\rho^\alpha).$$

(3.12)
This expression has no net propagating fields – the bosonic ghosts and antighosts \( (F^\alpha_\rho \text{ and } F^\alpha_\bar{\rho}) \) balance the heterotic fermions \( (\rho^\alpha \text{ and } \lambda^\alpha) \).

In fact, since \( I'' \) is \( s \)-exact, the action

\[
I_B = I_{II} + I'', \tag{3.13}
\]

would have been an equally good action for the type II theories. The fields in \( I'' \) (the \( \rho \) sector) simply decouple in the functional integral for any correlation function of type II fields (which are in the \( X \) sector). For this to be consistent is essential that the terms \( I_{II} \) and \( I'' \) are separately free of anomalies, which is manifestly the case (taking into account our choice of equal conformal weights for \( \rho \) and \( F_\rho \)). The construction is reminiscent of the definition of topological Yang-Mills theory as the BRST invariant gauge-fixing of the second Chern class \([9]\).

Less obvious is the fact that the action \( I_B \) can also be broken up in another anomaly-free manner,

\[
I_B = I_{het} + I', \tag{3.14}
\]

where \( I_{het} \) was defined in (3.9) and

\[
I' = \int d^2 z \left( -i \sum_{\mu=1}^{10} \Psi^\mu \partial_z \Psi^\mu + \beta_{0,\frac{3}{2}} \partial_z \gamma^{0,\frac{1}{2}} - \sum_{\alpha=1}^{16} F^\alpha_\rho \partial_z F^\alpha_\rho \right). \tag{3.15}
\]

The action \( I' \) only involves anti-holomorphic fields that are absent from the usual heterotic action.

\( I_{het} \) and \( I' \) are not separately BRST exact – only their sum is. The separation of \( I_B \) into these two actions is however consistent and provides two independent theories because each of them is free of gravitational and conformal anomalies. For \( I_{het} \) this follows by the usual arguments. That \( I' \) is independently anomaly-free follows if we attribute conformal weight \( (0, \frac{1}{2}) \) to all the fields of the \( \rho \) sector. In that case the system \( (F^\alpha_\rho, F^\alpha_\bar{\rho}) \) contributes \(-16\) to the conformal anomaly, the NSR fields \( \Psi^\mu \) contribute 5 and the \( (\beta_{0,\frac{3}{2}}, \gamma^{0,\frac{1}{2}}) \) system contributes 11, giving a total conformal anomaly of \( 5 + 11 - 16 = 0 \)\(^2\).

The fields of the usual heterotic theory may be denoted by \( \phi_1 = \{ X, \Psi^1, \rho, \lambda_\rho, b_{zz}, c^z, b_{\bar{z}\bar{z}}, c^\bar{z}, \beta^{0,\frac{3}{2}}, \gamma^{0,\frac{1}{2}} \} \) and the fields in \( I' \) by \( \phi_2 = \{ \Psi^2, F_\rho, \bar{F}_\rho, \beta_{0,\frac{3}{2}}, \gamma^{0,\frac{1}{2}} \} \). A heterotic theory

\(^2\)Recall that a system of conformal fields \( (A, B) \) with Lagrangian \( A \partial B \) has a conformal anomaly equal to \( \pm 2(6n^2 - 6n + 1) \) where \( n \) is the conformal weight of the field \( A \) and the sign \( (+) \) occurs if \( A \) and \( B \) commute (anticommute) \([10]\).
observable \( A(\phi_1) \) is defined by the functional integral,

\[
\langle A(\phi_1) \rangle = \int [d\phi_1][d\phi_2] A(\phi_1) \exp i(I_{\text{het}}[\phi_1] + I_2[\phi_2]) = N \int [d\phi_1] A(\phi_1) \exp iI_{\text{het}}[\phi_1], \tag{3.16}
\]

where \( N \) is the partition function for the theory generated by \( I' \). It is an irrelevant normalisation factor in the context of the usual heterotic theory. Equation (3.16) relies on the fact that both the theories defined by \( I_{\text{het}} \) and \( I' \) are separately anomaly-free and are therefore truly decoupled. It is interesting that there is a mapping between the two theories due to the existence of the topological BRST symmetry which mixes their fields.

We have thus shown that, up to irrelevant terms, the heterotic theory can be obtained from a \( s \)-exact action with the same topological BRST symmetry as in the NSR formulation of the type II theory. The difference between these physically different theories arises from different anomaly-free eliminations of fields. This might be of relevance in the context of the apparently rich set of interrelationships between type II and heterotic theories.

The observables of the type II or heterotic models are defined by the residual symmetries that survive our choices of gauge functions, acting on the remaining (twisted) fields rather than the full topological BRST symmetry. However, there are observables of the theory defined by the large action, \( I_B \), that possess the full topological BRST symmetry of the type encountered in topological \( \sigma \) models.

It is worth noting that the BRST algebra of the beltrami variables (2.3) can be interpreted as a BRST algebra of topological 2-D gravity by the explicit change of variables,

\[
\Psi^z = 2i\alpha^0_\gamma \frac{1}{2}, \quad \Phi^z = \frac{i}{2} \gamma^0_\gamma \frac{1}{2}, \tag{3.17}
\]

in the holomorphic sector and corresponding definitions in the anti-holomorphic sector. The field \( \Psi^z \) is interpreted as the topological ghost of the Beltrami differential, while \( \Phi^z \) is the ghost of this ghost. The explicitly topological BRST algebra on the Beltrami fields is then,

\[
s\mu^z = \Psi^z + 2 \partial_z c^z + c^z \partial_z \mu^z - \mu^z \partial_z c^z,
\]

\[
s\Psi^z = \partial_z \Phi^z + \Phi^z \partial_z \mu^z - \mu^z \partial_z \Phi^z + c^z \partial_z \Psi^z - \Psi^z \partial_z c^z,
\]

\[
s c^z = \Phi^z + c^z \partial_z c^z,
\]

\[
s \Phi^z = c^z \partial_z \Phi^z - \Phi^z \partial_z c^z,
\]
with corresponding equations in the anti-holomorphic sector. One can thus view the ghost system as originating from gauge-fixing topological 2-D gravity in the gauge $\mu\tilde{z} = 0$ and $\Psi\tilde{z} = 0$.

Making use of the fact that $s(\phi_{zz} \Psi\tilde{z}) = \phi_{zz} \partial_z \Phi = 2\gamma^{\tilde{z},0}\phi_{zz} \phi_{\tilde{z}}(\gamma^{\tilde{z},0})$, the holomorphic part of the gravity lagrangian (2.4) can be identified with the $s$-exact expression $s(b_{zz}\mu\tilde{z} + \phi_{zz}\Psi\tilde{z})$ provided the superconformal antighost is identified as

$$\beta_{\tilde{z},0} = 2\phi_{zz}\gamma^{\tilde{z},0}. \quad (3.19)$$

The field definitions (3.17) and (3.19) involve multiplication of fields by $\gamma^{\tilde{z},0}$ in a manner that implements the twists needed to express the gravitational part of the action in a fully topological form (as in [10]). If the twists on the fermionic matter fields are implemented by an analogous change of field variables the complete action $I_B$ assumes the standard form of a topological $\sigma$ model coupled to topological two-dimensional gravity.

Before gauge fixing, the topological theory does not contain specific information about the target-space metric associated with any particular string theory vacuum. Of course, our gauge-fixed derivation of the action $I_B$ is background dependent – we chose the background to be flat, but it presumably could have been more general. In a curved space-time endowed with a closed 2-form $\omega = \omega_{\mu\nu} dX^\mu \wedge dX^\nu$ the invariant $I_{\text{top}}$ can be written as $I_{\text{top}} = \int \omega$. The complexification of the coordinates involved in (3.1) as well as the relation between the NSR fields and the topological ghosts and antighosts can be obtained by using $\omega_{\mu\nu}$ to define the polarizations. Such a generalisation of the construction to curved backgrounds should give a BRST-exact term that depends covariantly on the background metrics along the lines defined in [1] [2]. In these more general backgrounds the distinction between the rôles of type A and type B twistings in the type IIA and type IIB superstring theories should become important.

The action (3.12) is asymmetric with respect to the holomorphic and anti-holomorphic sectors. To obtain a completely symmetrical formulation a further fermionic field, $\rho'$, could be introduced as a holomorphic partner to $\rho$. Together with its topological ghost, antighost and Lagrange multiplier fields it would have the decoupled action $I''' = \int d^2z \sum_{\alpha=1}^{16} s(F^\alpha_{\rho'} \partial_{\tilde{z}} \rho'^\alpha)$ analogous to (3.12). In the type II theory the fields of the $\rho$ and $\rho'$ sectors decouple so that a different gauge choice could have been made where $\rho = \rho' = 0$. This is achieved in a BRST invariant way by replacing $I''$ by $\int d^2z \sum_{\alpha=1}^{16} s(F_{\rho'}^\alpha \rho'^\alpha) = \int d^2z \sum_{\alpha=1}^{16} (\lambda_{\tilde{0}}^\alpha \rho^\alpha - F_{\rho'}^\alpha F_{\rho'}^\alpha)$ with a similar expression replacing $I'''$. 

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4 Relationship to space-time supersymmetry

The GS superstring action for the type II theories involves the fields $X^\mu$ and two space-time Weyl-Majorana fields $\theta^Aa$ ($A = 1, 2, a = 1, \ldots, 16$) that are world-sheet scalars \[12\]. These two fields correspond to the two space-time supersymmetry which have the same space-time chirality in the type IIB theory and opposite chiralities in the type IIA theory. In the heterotic case there is one $\theta^a$ field as well as the usual heterotic fermions, $\rho^\alpha$. In its original derivation, the GS action has no dependence on the NSR fields. It can be interpreted as a nonlinear sigma model with a Wess–Zumino term associated with super-Poincare invariance in the target space \[13\].

In view of the results of last section, where both the type II and heterotic models were related to a topological $\sigma$-model, it is tempting to identify the fermionic variables $\rho^\alpha$ as the 16 independent components of the Majorana-Weyl spinor field $\theta^a$, up to a twist to accommodate the change of the worldsheet conformal weights from zero to one half. The doubling of the $\rho$ sector fields to accommodate the existence of a pair of GS fields is an obvious possibility. In this way, one can imagine promoting the following infinitesimal topological transformations to the rank of a fundamental gauge symmetry,

$$
\delta X^\mu(z, \bar{z}) = \epsilon^\mu_X(z, \bar{z}) - i\tilde{\theta}^A\gamma^\mu\epsilon^A_\theta \\
\delta \theta^Aa(z, \bar{z}) = \epsilon^A_a(z, \bar{z}).
$$

These transformations extend \[(1.1)\] by taking into account local space-time supersymmetry transformations of $X^\mu$ in an equivariant way,

The associated BRST symmetry is

$$
sX^\mu = i F^\mu_X - \tilde{\theta}^A\gamma^\mu F^A_\theta \\
sF^\mu_X = \tilde{F}^A\gamma^\mu F^A_\theta \\
s\tilde{F}^\mu_X = \lambda^\mu_X - \tilde{F}^A\gamma^\mu F^A_\theta \\
s\lambda^\mu_X = i\tilde{\lambda}^A_\theta\gamma^\mu F^A_\theta
$$

and

$$
s\theta^{A\alpha} = i F^{A\alpha}_\theta
$$
\[ sF^A_\theta = 0 \]
\[ sF^a_\theta = \lambda^A_\theta \quad s\lambda^A_\theta = 0 \]  
(4.3)

(we have defined the Dirac conjugation \( \tilde{\rho} = \rho^\dagger \gamma^0 \)).

Although it is not central to this paper it is noteworthy that if one defines the zero curvature Cartan one-form,

\[ (dX^\mu - i\tilde{\rho} \gamma^\mu d\rho) P_\mu + d\rho^a Q_a, \]  
(4.4)

where \( P_\mu \) and \( Q_a \) are the generators of the N=1 super-Poincare symmetry of the target space, this BRST symmetry can be expressed as

\[ ((d + s)X^\mu - i\tilde{\rho} \gamma^\mu (d + s)\rho) P_\mu + (d + s)\rho^a Q_a = iF^\mu_X P_\mu + iF^a_\rho Q_a. \]  
(4.5)

This equation (and its Bianchi identity which determines the way \( F^\mu_X \) and \( F^a_\rho \) transform) may be important in giving a geometrical interpretation of the topological ghosts, and understanding the meaning of the topological term \( \int \omega_{\mu\nu} dX^\mu dX^\nu \) in 10 dimensional space-time.

The form of the BRST transformations (4.2), suggests that the fundamental symmetry of the theory encodes general covariance and local supersymmetry in the target space. Heuristically, one can think of target-space as separated into all possible 2-D surfaces. Thus, if one builds a theory based on the gauge symmetry (4.1) on each of these surfaces, and then sums over them by integrating over all classes of conformally invariant parts of the two-dimensional metrics and gravitini, one formally reconstructs the symmetry of N=2 supergravity, \( \delta X^\mu = \epsilon^\mu(X) - i\tilde{\theta}^A \gamma^\mu \epsilon^A_\theta(X), \delta \theta^A = \epsilon^A_\theta \).

However, the problem of expressing the GS action as a combination of BRST-exact and \( d \)-exact terms remains open. It should be possible to formulate the theory as a topological model based on the above BRST symmetry in such a way that the usual fermionic \( \kappa \) symmetry emerges as a residual local symmetry after gauge-fixing. The \( \kappa \) symmetry allows half of the components of \( \theta^A_a \) to be eliminated in passing to the light-cone gauge by setting \( n_\mu \gamma^\mu \theta^A = 0 \), where \( n^\mu \) is a null vector. In this respect it is intriguing that a null vector naturally arises in the BRST system since the variation, \( sF^\mu_X = \tilde{F}^A_\theta \gamma^\mu F^A_\theta \), in (4.2) is a null vector (due to the well-known properties of gamma matrices in 3, 4, 6 and 10 dimensions).

The distinction between the type II and heterotic models in the GS formulation should reside in different gauge choices for the \( N = 2 \) supercoordinates \( \theta^1 \) and \( \theta^2 \). Thus, in the heterotic
case the fermions for the internal symmetry would be identified with one set of fields, \( \theta^1, \lambda^1 \) with the same gauge function as in the last section, while \( \theta^2 \) would be associated with \( N = 1 \) space-time supersymmetry.

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