Spacetime rotation-induced Landau quantization

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We investigate non-inertial and gravitational effects on quantum states in electromagnetic fields and present the analytic solution for energy eigenstates for the Schrödinger equation including non-inertial, gravitational and electromagnetic effects. We find that in addition to the Landau quantization the rotation of spacetime itself leads to the additional quantization, and that the energy levels for an electron are different from those for a proton at the level of gravitational corrections.

Introduction.— In many quantum systems, gravitational interaction is usually neglected because of the weakness of the interaction. Hence, gravitational effects on quantum systems remain to be fully elucidated. At present, such circumstance may be an obstacle to understand the interplay between the quantum theory and the gravitational theory. As for verifications of gravitational effects on quantum systems, several experiments have been conducted. Colella, Overhauser and Werner\textsuperscript{[1]} for the first time experimentally showed an physical phenomena involving both the Planck constant $\hbar$ and the gravitational constant $G$ by using a neutron interferometer elegantly. Since then, ingenious experiments using neutrons\textsuperscript{[2,3]} or atoms\textsuperscript{[4]} have been conducted to reveal gravitational or non-inertial effects on quantum systems.

On the other hand, electromagnetic fields are ubiquitous in the universe. Around magnetized compact objects such as magnetized neutron stars and magnetars, the couplings between gravitational effects, quantum effects and electromagnetic effects will come into play. Actually, signatures of Landau quantization in X-ray cyclotron absorption lines were observed on a neutron star surface\textsuperscript{[5]} where the gravitational effect are much stronger than that on the Earth. While non-inertial and gravitational effects on quantum systems in unmagnetized circumstances have been well studied theoretically so far\textsuperscript{[6,12]}, there are only a few reports\textsuperscript{[13–17]} about those effects in magnetized circumstances in the literature.

In this paper, we investigate non-inertial and gravitational effects on quantum states in electromagnetic fields by solving the Schrödinger equation seriously for non-relativistic magnetized matter in slowly rotating Kerr spacetime, and find the analytic solution for the quantum states of a charged particle including non-inertial, gravitational, and electromagnetic effects for the first time, in which we neglect the effect of the intrinsic spin of a particle\textsuperscript{[10,12,17]}.

Spacetime metric.— First of all, we discuss the metric around a rotating star. We assume that the rotational axis is aligned with the z-axis. In this paper, we explicitly use the gravitational constant $G$ and the speed of light $c$ for later conveniences. The spacetime metric is approximated by the slow rotation limit of the Kerr metric

\begin{equation}
 ds^2 = \left(1 - \frac{2M_\ast}{r}\right) c^2 dt^2 + \frac{4M_\ast a}{r} \sin^2 \theta \ dt d\phi - \frac{dr^2}{1 - 2M_\ast / r} - r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right),
 \end{equation}

where $M_\ast = GM/c^2$, $M$ is the mass of the star and $a$ is the Kerr parameter and is considered to be small throughout this paper. After the coordinate transformation $(x, y, z) = (\rho \sin \theta \cos \phi, \rho \sin \theta \sin \phi, \rho \cos \theta)$, where $\rho = (r - M_\ast + \sqrt{r^2 - 2Mr})/2$, we derive

\begin{equation}
 ds^2 = F^2 G^{-2} \gamma^2 dt^2 + \frac{4M_\ast a}{\rho^3} G^{-2} (xdy - ydx) c \ dt
 - G^4 (dx^2 + dy^2 + dz^2),
 \end{equation}

where $F \equiv 1 - M_\ast/(2\rho)$ and $G \equiv 1 + M_\ast/(2\rho)$. Furthermore we consider the coordinate transformation to the rotating frame on the stellar surface, i.e., $(x, y, z) = (x' \cos \Omega t - y' \sin \Omega t, x' \sin \Omega t + y' \cos \Omega t, z')$, where $\Omega$ is the angular velocity of the rotating star. Dropping the prime after the transformation, we obtain

\begin{equation}
 ds^2 = \left[F^2 G^{-2} \gamma^2 + \frac{4M_\ast a}{\rho^3} G^{-2} \Omega (x'^2 + y'^2) - G^4 \Omega^2\right]
 \times (x'^2 + y'^2) dt^2 + 2 \left(\frac{2M_\ast a}{\rho^3} G^{-2} - G^4 \Omega\right)
 \times (xdy - ydx) dt - G^4 (dx^2 + dy^2 + dz^2).
 \end{equation}

Equation\textsuperscript{[3]} provides the spacetime metric which is described by an observer on the surface of a rotating star.

The Schrödinger equation with general relativistic corrections.—Let us obtain the Schrödinger equation with general relativistic corrections from the Klein-Gordon equation\textsuperscript{[3]}. Our approach is validated only when we neglect the intrinsic spin. The Klein-Gordon equation for a massive scalar field $\phi$ in the presence of an electromagnetic field is given by\textsuperscript{[13,14,16]}

\begin{equation}
 g^{\mu\nu} \left(\nabla_\mu - \frac{iq}{\hbar} A_\mu\right) \left(\nabla_\nu - \frac{iq}{\hbar} A_\nu\right) \phi + \frac{m^2 c^2}{\hbar^2} \phi = 0,
 \end{equation}

where $q = e$ is the charge of the particle and $A_\mu$ is the electromagnetic potential.

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where \( m \) is the mass of the field \( \phi \), \( q \) is the electric charge, \( g^{\mu \nu} \) is the metric, and \( \nabla_{\mu} \) denotes the covariant derivative. Now we focus on the polar region of the rotating star (see FIG. 1). The magnetic field is approximated by a uniform magnetic field in the polar region. Here we assume that the magnetic field lines are aligned with the rotational axis. This special configuration enables us to obtain the analytic solution for the field. Thus we take the electromagnetic 4-potential as \( A_{\mu} = (0, -By/2, Bx/2, 0) \). To derive the Schrödinger equation, we expand the field \( \phi \) as \( \phi(t, x, y, z) = \psi(t, x, y, z) \exp[-i(m c^2 / \hbar) t] \). From Eq. (4), up to \( O(c^{-1}) \), we obtain

\[
\frac{i \hbar}{\partial t} \psi = \left[ -\frac{\hbar^2}{2m} \nabla^2 - \frac{GMm}{\rho} - \left( \frac{qB}{2m} + \varpi(\rho) \right) L_z \right. \\
+ \left. \left( \frac{qB^2}{8m} + \frac{qB}{2} \varpi(\rho) \right) (x^2 + y^2) \right] \psi, \tag{5}
\]

where \( \nabla^2 \equiv \partial_x^2 + \partial_y^2 + \partial_z^2 \), \( L_z \equiv -i \hbar (x \partial_y - y \partial_x) \), and \( \varpi(\rho) \equiv \Omega - 2GMa/(c \rho^3) \). To take the origin of the \( z \) axis at the surface, we transform \( z \) as \( z \to R + z \), where \( R \) is the radius of the star. Here we have \( x, y, z < R \) for the polar region and derive \( \rho \approx R \left( 1 + z/R^2 \right) \). For energy eigenstates, we obtain

\[
E \psi = \left[ -\frac{\hbar^2}{2m} \nabla^2 + \frac{mu}{m} + mgz - \left( \frac{qB}{2m} + \varpi(R) \right) L_z \right. \\
+ \left. \left( \frac{qB^2}{8m} + \frac{qB}{2} \varpi(R) \right) (x^2 + y^2) \right] \psi. \tag{6}
\]

where \( E \) is the energy, \( U \equiv -GM/R \) is the gravitational potential, and \( q \equiv GM/R^2 \) is the gravitational acceleration. Here we have neglected the term proportional to \( az/R^4 \) in \( \varpi \). Equation (3) governs quantum states on the surface of a rotating star.

Quantum states on the surface of a rotating star;— We discuss quantum states on the stellar surface. To solve Eq. (3) for \( \psi \), we assume the separation of variables as \( \psi(x, y, z) = F(x, y) G(z) \), where functions \( F \) and \( G \) are introduced. From Eq. (3), we can derive the differential equation for \( F \) and that for \( G \) in the cylindrical coordinates \((r, \theta, z)\)

\[
(E - mU - K) F(r, \theta) = \left[ -\frac{\hbar^2}{2m} \left( \partial_r^2 + \frac{1}{r} \partial_r \right) + \frac{\rho_0^2}{2m r^2} - \left( \frac{qB}{2m} + \varpi(R) \right) \rho_\theta \right. \\
+ \left. \left( \frac{q^2 B^2}{8m} + \frac{qB}{2} \varpi(R) \right) r^2 \right] F(r, \theta), \tag{7}
\]

\[
KG(z) = -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + mgz \right] G(z), \tag{8}
\]

where \( \rho_\theta \equiv -i \hbar \partial_\theta \), and \( K \) is a constant introduced by the method of separation of variables. When we consider a neutron star, we should recall that the electron capture process is dominant inside a neutron star. When we take account of the potential inside a neutron star, we should replace the latter equation (8) with the equation

\[
KG(z) = \left[ -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + V_{\text{eff}}(z) \right] G(z), \tag{9}
\]

where the effective potential \( V_{\text{eff}}(z) \) is assumed to be

\[
V_{\text{eff}}(z) = \left\{ \begin{array}{ll}
mgz & (z > 0) \\
\infty & (z \leq 0)
\end{array} \right.. \tag{10}
\]

Here the form of \( V_{\text{eff}} \) for \( z \leq 0 \) may be somewhat ideal. (See Discussion & Conclusion for anticipation of more realistic cases.) Thus, we can find quantum states on the neutron star by solving Eqs. (7) and (9).

First we discuss Eq. (10) for the wave function in the \( z \)-direction. This equation was already investigated in [8, 18]. The solution of Eq. (10) is given by the Airy function

\[
G(z) = \text{Ai} \left( \left( \frac{2m^2 q}{\hbar^2} \right)^{1/3} \left( z - \frac{K}{mg} \right) \right). \tag{11}
\]

Here \( K \) is quantized due to the boundary condition at \( z = 0 \), i.e., \( G(0) = 0 \), as \( K_n = \hbar \omega_{z}(m) \lambda_n \), where \( n = 0, 1, 2, \cdots, \omega_z(m) \equiv (mg^2/(2\hbar))^1/3 \) and \( \lambda_n \) denotes the zero points of the Airy function, i.e., \( \text{Ai}(\lambda_n) = 0 \). Therefore, the wave function in the \( z \)-direction is given by Eq. (11) with quantized energy \( K = K_n \).

Next, we discuss Eq. (7) in the \( xy \)-plane. Let us take eigenstates for \( \rho_\theta \), i.e., \( F(r, \theta) = \exp[i (p_\theta / \hbar) \theta] f(r) \), where \( f \) is a function of \( r \) only. From Eq. (7), we derive

\[
\left[ \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \beta^2 r^2 - \frac{\rho_0^2}{2 \hbar^2 r^2} + \mathcal{E} \right] f = 0, \tag{12}
\]

where

\[
\beta = \left( \frac{q^2 B^2}{4\hbar^2} + \frac{mgB}{\hbar^2} \varpi(R) \right)^{1/3}, \tag{13}
\]

\[
\mathcal{E} = \frac{2m}{\hbar^2} (E - mU - K_n) + \left( \frac{qB}{\hbar^2} + \frac{2m}{\hbar^2} \varpi(R) \right) p_\theta. \tag{14}
\]
Furthermore, we assume $f(r) = r^e e^{-\frac{\beta}{r}} \tilde{f}(r)$, where $\ell$ is defined as
\[
\ell = \pm \frac{p_\theta}{\hbar} \quad \text{for} \quad q = \pm e,
\] (15)
where $e > 0$ is the elementary charge. When we adopt the variable $x = \beta r^2$, we derive
\[
\left[\frac{d^2}{dx^2} + \{(\ell + 1) - x\} \frac{d}{dx} + \left(\frac{E}{4\beta} - \frac{\ell + 1}{2}\right)\right] \tilde{f}(x) = 0.
\] (16)
This equation is equivalent to the confluent hypergeometric equation $x y'' + (\gamma - x)y' - \alpha y = 0$. Hence the solutions of Eq. (16) are given by the confluent hypergeometric functions in the form
\[
y = i F_I(\alpha; \gamma; x) = 1 + \frac{\alpha x}{\gamma 1!} + \frac{\alpha (\alpha + 1) x^2}{\gamma (\gamma + 1) 2!} + \cdots.
\] (17)
We now discuss the integrability condition of the wave function $F$. The integral of $F^* F$ is calculated as
\[
\int_0^{2\pi} \int_0^\infty r dr F^* F = \int_0^\infty dx x e^{-x} \left| \tilde{f}(x) \right|^2
\] (18)
where $F^*$ denotes the complex conjugate of $F$. Thus, when the series in Eq. (17) ends at a finite order, the integral of Eq. (15) becomes finite. Therefore, to make the wave function integrable, the constant $\alpha$ in Eq. (17) must be zero or negative integers, i.e., $\alpha = -n'$, where $n' = 0, 1, 2, \cdots$. In the same way, from Eq. (16), we obtain the condition
\[
\frac{E}{4\beta} - \frac{\ell + 1}{2} = n'.
\] (19)
In this case, we can find integrable wave functions. When the condition Eq. (19) is satisfied, the solution of Eq. (16) is given by the associated Laguerre polynomials $\tilde{f}(x) = L_{n'}^\ell(x)$. Thus, we obtain
\[
F(r, \theta) = e^{i\frac{\beta m}{\hbar} r^2} e^{-\frac{\beta}{r}} L_n^\ell(\beta r^2).
\] (20)
For $q = \pm e$, Eq. (19) is approximately calculated as
\[
E - mU - K_n \approx \hbar \left(\frac{eB}{m} \pm 2\omega(R)\right) \left(n' + \frac{1}{2}\right).
\] (21)
Equations (20) and (21) describe the Landau quantization with general relativistic corrections in the $xy$-plane. Consequently, we obtain the wave function on the polar region
\[
\psi = A r^e e^{-\frac{\beta}{r}} L_n^\ell(\beta r^2) \text{Ai}\left(\frac{2m^2 q}{\hbar^2} z - \lambda_n\right)
\] (22)
where $A$ is a normalization factor, $\ell$ and $\ell$ is given by Eq. (15), and must be zero or positive. The energy eigenvalues for $q = \pm e$ are given by
\[
E_{nn'} \approx mU + \hbar \omega_\perp(m) n + \hbar \left(\frac{eB}{m} \pm 2\omega(R)\right) \left(n' + \frac{1}{2}\right).
\] (23)
where the positive sign corresponds to the case for a proton, and the negative sign corresponds to the case for an electron. Equations (22) and (23) provide the quantum states of the field $\psi$ on the surface of a rotating star.

**Discussion & Conclusion.**—We discuss physical consequences of the quantized states with the general relativistic corrections. The energy states in Eq. (19) are characterized by two integers $n$ and $n'$. In Eq. (23), the first term is the gravitational potential and merely shifts the zero-point energy of the system. The second term denotes the energy levels in the vertical direction. The vertical energy levels depend on the mass $m$ only, not on the magnetic field $B$. Thus the energy levels for a proton are different from those for an electron (see also TABLE I). The third term denotes the Landau quantization with the general relativistic correction. The sign in front of the correction $2\omega$ depends on the charge of a particle. Thus the energy step for a proton is different from that for an electron. Therefore, in principle, we could determine which particle comes into play on the surface from the fine structure of the eigenstates due to the gravitational corrections.

It is worth noting that the Landau quantization caused by $\omega$ survive in the limit of $B \rightarrow 0$. Therefore, the rotation of the spacetime cause the Landau quantization without magnetic fields. This effect might be called *spacetime rotation-induced (or geometric) Landau quantization* (see also 19, 20).

Next, we discuss observability of the quantum states discussed above for neutron stars. Here it should be noted that we actually observe the energy that is subject to the gravitational redshift, i.e., $E_{\text{obs}} = \gamma_{\text{red}} E$, where $\gamma_{\text{red}}$ is the factor for gravitational redshift. For the vertical quantum levels, we can obtain the order estimates $\gamma_{\text{red}} \omega_\perp(m_e) \sim 10^{16}[\text{Hz}]$, $\gamma_{\text{red}} \omega_\perp(m_p) \sim 10^{18}[\text{Hz}]$, where $m_e$ is the mass for an electron, and $m_p$ is the mass for a proton. The first few eigen frequencies for the vertical energy levels are shown in TABLE I. In practice, the potential well $V_{\text{eff}}$ would be broadened out in the direction of $z < 0$, and the intervals of the energy levels would become narrower. When the magnetic field strength varies from $10^8[\text{G}]$ to $10^{15}[\text{G}]$, we derive the order estimates for cyclotron frequencies, $\gamma_{\text{red}} eB/m_e \sim 10^{15.10^{26}}[\text{Hz}]$, $\gamma_{\text{red}} eB/m_p \sim 10^{12.10^{19}}[\text{Hz}]$. For millisecond pulsars,

**TABLE I.** Vertical energy eigenstates for an electron are compared with those for a proton. $m_e$ and $m_p$ denote the mass for an electron and that for a proton, respectively. We adopt $M = M_\odot$ and $R = 10[\text{km}]$ for the estimates.

| $n$ | $\lambda_n$ | $K_n(m_e)/[\text{Hz}]$ | $K_n(m_p)/[\text{Hz}]$ |
|-----|-------------|------------------------|------------------------|
| 1   | 2.33811     | 4.61512 x 10^{10}      | 5.65135 x 10^{10}      |
| 2   | 4.08795     | 8.6908 x 10^{10}       | 9.88083 x 10^{10}      |
| 3   | 5.52056     | 1.08969 x 10^{10}      | 1.33435 x 10^{11}      |
| 4   | 6.78671     | 1.33961 x 10^{10}      | 1.64039 x 10^{11}      |
| ... | ...         | ...                    | ...                    |
we derive the order estimates of the rotational terms in $\omega$ as $2\gamma\omega\Omega \sim 10^{13}[\text{Hz}]$ and $\gamma\omega^4GMa/(cR^3) = \gamma\omega^4GM/(c^2R) \cdot J/(MR^2) \sim 10^{20}[\text{Hz}]$. Thus the cyclotron frequency is the most energetic for neutron stars. The vertical energy step is the second most energetic, and the general relativistic correction to the Landau energy is the lowest. Hence, we can determine the quantity $B/m$ from the most energetic absorption lines that are almost given by the cyclotron frequency. While we can determine the mass $m$, in principle, from the gravitational corrections. Once we could detect the gravitational corrections from observations, we could determine the magnetic field strength itself. However, it would be difficult to detect the gravitational corrections at present. In general, the absorption lines are broadened by thermal, quantum and environmental effects \cite{21}. The probability of absorption in the vicinity of an absorption line would be proportional to the factor $\exp(-|\Delta E|/(kB_T))$, where $\Delta E$ is the energy difference from the absorption line, $kB_T$ is the Boltzmann constant, and $T$ is the temperature of the environment. Since the surface of a neutron star typically has a temperature $T \sim 10^6[K]$ \cite{22}, the absorption line is broadened by the frequency width $\Delta\omega_T \sim kB_T/h \sim 10^{17}[\text{Hz}]$. Thus the absorption lines below $\Delta\omega_T$ would be blurred. Although we could easily detect the cyclotron frequency above $\Delta\omega_T$, it would be difficult to detect the gravitational corrections owing to the thermal turbulence. Nonetheless, the information of the gravitational corrections is certainly hidden in the features of broaden absorption curves in spectra. This would be investigated in the future work.

Although we have focused our attention on a neutron star, the spacetime rotation-induced Landau quantization is universal phenomena. Thus the effect may be detectable with physical systems of ultra-low temperature such as superconductor in laboratories on the Earth \cite{23} rather than on a neutron star. Such an experimental verification would be awaited.

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