Equation of State of Symmetric And Asymmetric Nuclear Matter At Various Densities And Temperatures

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Abstract. The equation of state (EOS) of nuclear matter (NM) is an important ingredient in the study of properties of nuclei at and far from stability, of structures and evolution of compact astrophysical objects, such as neutron stars and core-collapse supernovae, and of heavy-ion collisions (HIC). We will review the current status of the EOS of NM, as deduced from theoretical and experimental studies of structure and reactions of nuclei.

1. Introduction
The saturation point of the equation of state (EOS), i.e., the energy as a function of matter density, for the symmetric nuclear matter (SNM) at zero temperature (T = 0) is well determined from ground state properties of nuclei, such as binding energies and central matter densities, by extrapolation to infinite nuclear matter (NM). To extend our knowledge of the EOS beyond the saturation point of the SNM, an accurate value of the NM incompressibility coefficient K, which is directly related to the curvature of the EOS, is needed. An accurate value of the symmetry energy coefficient, J, is needed for the EOS of asymmetric NM. It is well known that the energies of compression modes in nuclei, such as the isoscalar giant monopole resonance (ISGMR) and the isoscalar giant dipole resonance (ISGDR), are sensitive to the value of K and the energies of the isovector giant dipole resonance (IVGDR) are sensitive to J. We consider, in particular, the issue of determining the parameters of the effective nucleon-nucleon interaction from properties of finite nuclei within the Hartree-Fock (HF) approach. We will present results of fully self-consistent HF-based random-phase-approximation (RPA) for the ISGMR and compare with experimental data.

A proper description of nuclei away from stability and the structure and evolution of compact astrophysical objects requires knowledge of the EOS at extreme conditions of density (low and high), isospin asymmetry and temperature. Due to the important effect of correlations the mean-field approach is inadequate. Studies of multifragmentation of hot nuclear systems, created in high energy proton induced reactions or in intermediate energy heavy-ion collisions (HIC), have been employed to determine the EOS of nuclear matter at extreme conditions. It is common to adopt the freeze-out assumption and/or the coalescence model to reconstruct the properties of the dense excited matter using the fragment yields. We will discuss important effects that should be accounted for in the analysis of experimental data and comment on the values of T, matter density and the symmetry energy density at low density determined from HIC.

2. EOS at zero temperature
The development of a modern and more realistic nuclear energy density functional (EDF) [1] for accurate predictions of properties of nuclei is the subject of enhanced activity, since it is very
important for the study of properties of rare nuclei with unusual neutron-to-proton ratios that are difficult to produce experimentally and likely to exhibit interesting new phenomena associated with isospin, clusterization and the continuum. Since the pioneering work of Brink and Vautherin [2], continuous efforts have been made to readjust the parameters of the Skyrme-type effective nucleon-nucleon (NN) interaction to better reproduce experimental data [3]. We have recently considered the Skyrme EDF [4] associated with the standard parameterization of Skyrme type interactions [5]:

\[ V_{12} = t_0 (1 + x_0 P_{12}^\sigma) \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2} t_1 (1 + x_1 P_{12}^\sigma) [\vec{k}_{12}^2 \delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2) \vec{k}_{12}^2] \\
+ t_2 (1 + x_2 P_{12}^\sigma) \vec{k}_{12} \delta(\vec{r}_1 - \vec{r}_2) \vec{k}_{12} + \frac{1}{6} t_3 (1 + x_3 P_{12}^\sigma) \rho \rho \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \delta(\vec{r}_1 - \vec{r}_2) + \right] \\
i W_0 \vec{k}_{12} \delta(\vec{r}_1 - \vec{r}_2) (\vec{\sigma}_1 + \vec{\sigma}_2) \times \vec{k}_{12}, \tag{1} \]

where \( t_i, x_i, \alpha \), and \( W_0 \) are the parameters of the interaction, \( P_{12}^\sigma \) is the spin exchange operator, \( \vec{\sigma}_i \) is the Pauli spin operator, \( \vec{k}_{12} = -i(\vec{\nabla}_1 - \vec{\nabla}_2)/2 \), and \( \vec{\nabla}_1 = -i(\vec{\nabla}_1 - \vec{\nabla}_2)/2 \). Here, the right and left arrows indicate that the momentum operators act on the right and on the left, respectively. We determined within the HF approximation a new and more realistic Skyrme interaction (named KDE0) by fitting [4] a set of extensive data on binding energies, "bare" single particle energies, charge root mean square (rms) radii, and rms radii of valence nucleon density distribution of nuclei. We have included in the fit, for the first time, the data on the constraint energies of the isoscalar giant monopole resonances (ISGMR) of nuclei and imposed additional constraints, such as a non-negative value for the slope of the symmetry energy density at high nuclear matter (NM) density (at three times the saturation density of NM) and the Landau stability constraints on nuclear matter. We have implemented, for the first time, the simulated annealing method (SAM) together with an advanced least square method to search for the global minima. In Table 1 we present the results of fully self-consistent HF-based RPA for the centroid energies of the ISGMR (see Refs. [6,7]) for several nuclei, using the KDE0 [4], SK255 [6], and SGII [8] Skyrme interactions and the results obtained within the relativistic mean-field (RMF)-based RPA obtained for the NL3 interaction [9] and compare with the experimental data of Refs. [10,11]. Considering the results for the SK255 and NL3, we conclude that it is possible to build \textit{bona fide} Skyrme forces with \( K \) close to that obtained in relativistic models. We have also found that (see the review in Ref. [12]) \( K_{nm} = 240 \pm 20 \text{ MeV} \). The uncertainty of 20 MeV is mainly due to the uncertainty in the value of \( J \).

Table 1. Fully self-consistent RPA results for the centroid energies of the ISGMR for interactions with various values of \( K \) and \( J \) (in MeV), compared with experimental data.

| Nucleus | \( \omega_1 - \omega_2 \) | Expt. | NL3 | SK255 | SGII | KDE0 |
|---------|----------------|-------|-----|-------|------|------|
| \(^{100}\text{Zr}\) | 0-60 | 17.81±0.30 | 18.7 | 18.85 | 17.87 | 17.98 |
| | 10-35 | 16.1 | 17.1 | 17.31 | 16.36 | 16.58 |
| \(^{116}\text{Sn}\) | 0-60 | 15.85±0.20 | 16.1 | 16.3 | 16.21 | 15.26 | 15.46 |
| | 10-35 | 15.40±0.40 | 16.3 | 15.19 | 15.22 | 15.44 |
| \(^{144}\text{Sm}\) | 0-60 | 13.96±0.20 | 14.2 | 14.34 | 13.57 | 13.79 |
| | 10-35 | 13.48 | 14.03 | 13.58 | 13.84 |
| \(^{208}\text{Pb}\) | 0-60 | 272 | 255 | 215 | 229 |
| | 10-35 | 37.4 | 37.4 | 26.8 | 33.0 |

\( K \) (MeV) | 229
\( J \) (MeV) | 33.0
3. Temperature and density dependence of EOS from heavy ion collisions

We point out that the decay of highly excited nuclear matter produced in intermediate energy heavy-ion collisions is a complex dynamic process. During the collision a dense and hot nuclear matter is created which then expands and fragments. A simple approach for describing the process is the freeze-out concept, in which one assumes that the expanding nuclear matter reaches thermal equilibrium. With decreasing density, the reaction rates decrease and the equilibration process is suppressed. At this point the thermal and chemical equilibriums are frozen out and the reaction product distribution is identical to the cluster distribution at the freeze-out point. In a macrocanonical ensemble, the statistical properties of the system can be evaluated using the grand partition sum

$$\mathcal{Z} = \sum_{\ell} e^{(\mu_n N_f + \mu_p Z_f)/T} Q_f, \quad Q_f = \exp(-F_f/T).$$

(2)

In Eq. (1), $N_f$, $Z_f$, $Q_f$, and $F_f$ are the total neutron number, the total charge, the partition function, and the free energy for a given event $f$, respectively. The Lagrange multipliers $T$, $\mu_n$, and $\mu_p$ are determined by the corresponding conservation laws. Using (2) one finds [13] for the relative yield of fragments $s$ the expression

$$\frac{\bar{n}_s}{\bar{n}_s^{(a)}} = \frac{1}{2} \left( \frac{1 + \kappa}{2\chi} \right)^{A_s-1} \left( \frac{\lambda T}{\chi} \right)^{A_s/2} Q_s^{1/2} e^{-(B_s + E_{C}(s))/T}.$$

(3)

Here, $\bar{n}_s$ is the average density of clusters $s$ with $A_s=N_s+Z_s$ nucleons, $\chi = V/V_0$ is the hindrance factor, $\lambda T$ is the thermal nucleon wave-length, $Q_s$ is the internal partition function, and $B_s$ and $E_{C}(s)$ are the binding energy and the Wigner-Seitz Coulomb energy corrections, respectively.

A key element in the investigation of the nuclear multifragmentation phenomena is the derivation of the temperature $T$ at the freeze-out point, which can be simply obtained from (3) by considering ratios of the yields of fragments [14]. In particular, one finds that the dependence of the excitation energy of the disassembling system on $T$, i.e., the caloric curve, shows irregularities which may be interpreted as a possible liquid-gas phase transition.

![Figure 1](image.png)

**Figure 1.** The ratio between the baryon density determined without ($n_B^{(a)}$) and with ($n_B$) medium effect.
In the analysis of the experimental data of fragment yields we have considered the effects of:

(i) The long range Coulomb interactions among fragments in the freeze-out volume [15]. Here we employ the Wigner-Seitz approximation;

(ii) The radial collective flow [16]. An expanding system, in a strict thermodynamic sense, is not in equilibrium. However, if the time scale involved in the expansion is much larger compared to the equilibration times in the expanding complex, i.e., the flow velocity is quite small compared to the average nucleonic velocity, the assumption of thermodynamic equilibrium may not be inappropriate;

(iii) The post emission decay (secondary decay) processes of the fragments emitted from the freeze-out surface [17];

(iv) The effect of the medium on the binding energies of clusters [18]; and,

(v) The effect of clusterization on the symmetry energy density [19];

We have demonstrated that: (i) Coulomb effects are important when considering ratios of yields of isotones; (ii) The flow affects the freeze-out volume; (iii) Taking into account the post emission decay one extracts the same freeze-out temperature using different thermometers (double yields ratios); (iv) Although, at low density, the temperature calculated from given yields changes only modestly if medium effects are taken into account, larger discrepancies are observed when the nucleon densities are determined from measured yields, see Figure 1 (taken from Ref. [18]); (v). Due to clusterization, at low density nuclear matter, the symmetry energy is much larger than that predicted by mean-field models (see Ref. [19]).

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