STABILITY OF THE EW VACUUM  
HIGGS BOSON AND NEW PHYSICS 

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based on :  
V. Branchina, E. Messina, Phys.Rev.Lett.111, 241801 (2013)  
work in progress with : E. Messina, A. Platania, M. Sher 

Rencontres de Moriond – QCD and High Energy Interactions  
La Thuile, March 23, 2014
Higgs One-Loop Effective Potential $V^{1l}(\phi)$

\[
V^{1l}(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{24} \phi^4 + \frac{1}{64\pi^2} \left[ \left( m^2 + \frac{\lambda}{2} \phi^2 \right)^2 \left( \ln \left( \frac{m^2 + \frac{\lambda}{2} \phi^2}{\mu^2} \right) - \frac{3}{2} \right) \right.
\]
\[
+ 3 \left( m^2 + \frac{\lambda}{6} \phi^2 \right)^2 \left( \ln \left( \frac{m^2 + \frac{\lambda}{6} \phi^2}{\mu^2} \right) - \frac{3}{2} \right) + 6 \frac{g_1^4}{16} \phi^4 \left( \ln \left( \frac{\frac{1}{4} g_1^2 \phi^2}{\mu^2} \right) - \frac{5}{6} \right)
\]
\[
+ 3 \frac{(g_1^2 + g_2^2)^2}{16} \phi^4 \left( \ln \left( \frac{1}{4} \left( g_1^2 + g_2^2 \right) \phi^2 \right) - \frac{5}{6} \right) - 12 \frac{g^4}{\mu^2} \phi^4 \left( \ln \frac{g^2 \phi^2}{\mu^2} - \frac{3}{2} \right) \right]
\]
Depending on the values of $M_H$ and $M_t$, the second minimum can be:

1. lower than the EW minimum (as in the figure); 
2. at the same level of the EW minimum; 
3. higher than the EW minimum.
Metastability Scenario

Tunnelling between the Metastable EW Vacuum and the True Vacuum.

As long as EW vacuum lifetime larger than the age of the Universe ...

.... we may well live in the Meta-Stable (EW) Vacuum ....
EW vacuum lifetime ( = Tunneling Time $\tau$)

$$\Gamma = \frac{1}{\tau} = T_U^3 \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' [-\partial^2 + V''(\phi_b)]}{\det [-\partial^2 + V''(v)]} \right|^{-1/2} e^{-S[\phi_b]}$$

$\phi_b(r)$ : Bounce Solution

Solution to the Euclidean Equation of Motion with appropriate boundary conditions

S. Coleman, Phys. Rev. D 15 (1977) 2929

C.G. Callan, S. Coleman, Phys. Rev. D 16 (1977) 1762
Instability for large $\phi$:

$$V_{\text{eff}}(\phi) \sim \frac{\lambda_{\text{eff}}(\phi)}{24} \phi^4 \ ; \ \lambda_{\text{eff}} < 0 \ ; \ \lambda_{\text{eff}} \sim \text{const}$$

$$\implies V(\phi) = \frac{\lambda}{24} \phi^4$$

with constant negative $\lambda$ is a good approximation in this range.

Having the potential, we can compute the Bounce Solutions.
Bounces:  \[ \phi_b(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2} \]

\( R = \text{bounce size} \) – Classical degeneracy:  \[ S[\phi_b] = \frac{8\pi^2}{3|\lambda|} \]

Degeneracy removed at the Quantum Level
Phase Diagram in the $M_H - M_t$ plane

Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, JHEP 1312 (2013) 089.
Phase Diagram in the $M_H - M_t$ plane

Stability region: $V_{eff}(v) < V_{eff}(\phi^{(2)}_{\text{min}})$. Meta-stability region: $\tau > T_U$.
Instability region: $\tau < T_U$. Dashed line: $V_{eff}(v) = V_{eff}(\phi^{(2)}_{\text{min}})$. Dashed - dotted line: $M_H$ and $M_t$ such that $\tau = T_U$. 
FOCUS : The scenario that we are discussing

The scenario is: Let’s assume that the SM is a theory valid all the way up to the Planck scale

i.e. No New Physics below the Planck scale

Within this scenario we study the stability of the EW vacuum

... This is a perfectly legitimate scenario to explore ...

..... However .....
$M_H = 126 \text{ GeV} \quad M_t = 173.1 \text{ GeV}$

New minimum at $\phi \sim 10^{31} \text{GeV}$ !!!!

SM Effective Potential extrapolated well above $M_P$ !!!

Does it make any sense ???
In order to make sense of this nonsensical potential, the proponents of this scenario observe that the New Physics Interactions that appear at the Planck scale $M_P$ eventually stabilize the potential at $M_P$ (see: Isidori, Ridolfi Strumia,)

However, they argue that these New Physics Interactions at the Planck scale have no impact on the computation of the EW vacuum lifetime (we shall see that this is not true and that this has profound consequences on the stability problem)
Let us explicitly consider New Physics Interactions at $M_P$

Add, for instance, $\phi^6$ and $\phi^8$ to the SM Higgs potential:

$$V(\phi) = \frac{\lambda}{4} \phi^4 + \frac{\lambda_6}{6} \frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8} \frac{\phi^8}{M_P^4}$$

The Effective Potential $V_{_{\text{eff}}}^{\text{new}}(\phi)$ is modified as :

$$V_{_{\text{eff}}}^{\text{new}}(\phi) = V_{_{\text{eff}}}^{}(\phi) + \frac{\lambda_6(\phi)}{6M_P^2} \xi(\phi)\phi^6 + \frac{\lambda_8(\phi)}{8M_P^4} \xi(\phi)\phi^8$$

For $\phi < M_P$ : $V_{_{\text{eff}}}^{\text{new}}(\phi)$ practically coincides with $V_{_{\text{eff}}}^{}(\phi)$

For $\phi \sim M_P$ : $V_{_{\text{eff}}}^{\text{new}}(\phi)$ depends on $\lambda_6$ and $\lambda_8$

Now, take : $\lambda_6(M_P) = -2 \quad \lambda_8(M_P) = 2.1$
Effective Potential \( M_H = 126 \) \( M_t = 173.1 \) Log-Log Plot

Blue line : \( V_{\text{eff}}(\phi) \) no higher order terms

Red line : \( V_{\text{eff}}^{\text{new}}(\phi) \) with \( \lambda_6(M_P) = -2 \) \( \lambda_8(M_P) = 2.1 \)
Zoom around the Planck scale

Blue line: $V_{\text{eff}}(\phi)$ no higher order terms

Red line: $V_{\text{eff}}^{\text{new}}(\phi)$ with $\lambda_6(M_P) = -2 \quad \lambda_8(M_P) = 2.1$
Bounces for the new potential

$$V(\phi) = \frac{\lambda}{4} \phi^4 + \frac{\lambda_6}{6} \frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8} \frac{\phi^8}{M_P^4}$$

$$(\lambda_6 = -2 \quad , \quad \lambda_8 = 2.1)$$

Eucl. equation of motion : 

$$\frac{d^2 \phi}{dx^2} + \frac{3}{x} \frac{d\phi}{dx} - \lambda \varphi^3 - \lambda_6 \varphi^5 - \lambda_8 \varphi^7 = 0 \quad (1)$$

Remember : for \(\lambda_6 = 0\) and \(\lambda_8 = 0\) , Solution seen before :

$$\phi_b^{(1)}(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2} \quad \text{Action degenerate with} \ R : \ S[\phi_b^{(1)}] = \frac{8\pi^2}{3|\lambda|}$$

Degeneracy lifted by the quantum fluctuations :

With \(M_H = 126 \text{ GeV}\) and \(M_t = 173.1 \text{ GeV}\) : \(R_M \sim 8 \times 10^{-18} \text{ GeV}^{-1}\)

- Important : \(\phi_b^{(1)}(r)\) also approximate solutions of \((1)\) for \(R \gg 1/M_P\)

- And : also numerical exact solution (forward-backward shooting)

$$\phi_b^{(2)}(r) \quad \text{with size} \quad \bar{R} \sim \frac{5.06}{M_P}$$
New Phys. Interactions not included: Only $\phi^{(1)}_b$ (Literature case)

$$\Gamma = \frac{1}{\tau} = \frac{1}{T_U} \left[ \frac{S[\phi^{(1)}_b]^2}{4\pi^2} \frac{T_U^4}{R_M^4} e^{-S[\phi^{(1)}_b]} \right] \times [e^{-\Delta S_1}]$$

New Phys. Interactions included: $\phi^{(2)}_b$ and $\phi^{(1)}_b$ (quasi-sol) (Our case)

$$\Gamma = \Gamma_1 + \Gamma_2 = \frac{1}{\tau_1} + \frac{1}{\tau_2} = \frac{1}{T_U} \left[ \frac{S[\phi^{(1)}_b]^2}{4\pi^2} \frac{T_U^4}{R_M^4} e^{-S[\phi^{(1)}_b]} \right] \times [e^{-\Delta S_1}] + \frac{1}{T_U} \left[ \frac{S[\phi^{(2)}_b]^2}{4\pi^2} \frac{T_U^4}{R^4} e^{-S[\phi^{(2)}_b]} \right] \times [e^{-\Delta S_2}]$$

Neglecting for a moment the $\Delta S$ (quantum) contributions

Literature: $S[\phi^{(1)}_b] \sim 1833$, $R_M \sim 8 \times 10^{-18} GeV^{-1} \Rightarrow \tau \sim 10^{555} T_U$

Our case: $S[\phi^{(1)}_b] \sim 82$, $\overline{R} \sim \frac{5}{M_P} \Rightarrow \tau \sim 10^{-208} T_U$

Contribution from $\phi^{(1)}_b$ exponentially suppressed!

New Physics Interactions at the Planck scale do matter !!!
Quantum fluctuations do not change significantly these “classical” results

### Literature : Loop contributions to $\tau$

| $e^{\Delta S_H}$ | 2.87185 |
| $e^{\Delta S_t}$ | $1.20708 \times 10^{-18}$ |
| $e^{\Delta S_{gg}}$ | $1.26746 \times 10^{50}$ |

$\Rightarrow \quad \tau_{cl} \sim 10^{555} T_U \quad \rightarrow \quad \tau \sim 10^{588} T_U$

### Our case : Loop contributions to $\tau$

| $e^{\Delta S_H}$ | $2.82295 \times 10^{10}$ |
| $e^{\Delta S_t}$ | $8.62404 \times 10^{-5}$ |
| $e^{\Delta S_{gg}}$ | $4.97869 \times 10^9$ |

$\Rightarrow \quad \tau_{cl} \sim 10^{-208} T_U \quad \rightarrow \quad \tau \sim 10^{-189} T_U$
Back to the Phase Diagram in the $M_H - M_t$ plane \textbf{(Literature)}

obtained neglecting New Physics Interactions at $M_P$

\begin{figure}
\centering
\includegraphics[width=\textwidth]{phase_diagram}
\end{figure}

For $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV

- SM inside the Metastability Region
- Extremely long-lived Metastable State
- Very popular nowadays to speculate on the position of the black dot in the Phase Diagram above: SM close to Stability line (criticality)

But remember : extreme sensitivity to New Physics Interactions!
Lesson on “Near Criticality”

Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, Investigating the near-criticality of the Higgs boson, JHEP 1312 (2013) 089

“We believe that near-criticality of the SM vacuum is the most important message we have learnt so far from experimental data on the Higgs boson”
Lesson on “Higgs Inflation”

I. Masina, The Gravitational Wave Background and Higgs False Vacuum Inflation, arXiv:1403.5244 [astro-ph.CO]

“For a narrow band of values of the top quark and Higgs boson masses, the Standard Model Higgs potential develops a shallow local minimum at energies of about $10^{16}$ GeV, where primordial inflation could have started in a cold metastable state”

Lesson on “Precision Measurement of $M_t$”
Conclusions

• Lifetime $\tau$ of the EW vacuum strongly depends on New Physics Interactions. The same is true for the SM Phase Diagram.

• The Metastability Scenario is not the generic outcome of “SM valid all the way up to $M_P$” + “$M_H \sim 126 \text{ GeV}$ and $M_t \sim 173.1 \text{ GeV}$$

• These results provide constraints on New Physics beyond SM.

• A similar analysis can be done also if the new physics scale lies below the Planck scale. Say GUT scale.

• This analysis also relevant for: Higgs potential with two degenerate minima, $\lambda(M_P) \sim 0$, $\beta(\lambda(M_P)) \sim 0$ (criticality), Higgs inflation... : High sensitivity to New Physics Interactions!