Electric Dipole Moments of Light Nuclei from Chiral Effective Field Theory

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Abstract Recent calculations of EDMs of light nuclei in the framework of chiral effective field theory are presented. We argue that they can be written in terms of the leading six low-energy constants encoding \( CP \)-violating physics. EDMs of the deuteron, triton, and helium are explicitly given in order to corroborate our claim. An eventual non-zero measurement of these EDMs can be used to disentangle the different sources and strengths of \( CP \)-violation.

Keywords \( CP \)-violation · Electric Dipole Moments · Chiral Symmetry

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1 Introduction

A search for new sources of \( CP \)-violation can be done by looking at electric dipole moments (EDMs) of quantum objects like fundamental particles, nuclei, atoms and some molecules. A permanent EDM of a particle violates time-reversal symmetry (\( T \)) which, according to the \( CPT \) theorem, is equivalent to \( CP \) violation. \( CP \)-violating phases in the CKM matrix provides an EDM to the neutron of the order of \( |d_n| \sim 10^{-32} \text{ cm} \) \cite{1}. On the other hand, the existing upper limits on the neutron EDM \cite{2}, \( |d_n| < 2.9 \times 10^{-26} \text{ cm} \), and the proton EDM (\( |d_p| < 7.9 \times 10^{-25} \text{ cm} \), from \( ^{199}\text{Hg} \) EDM \cite{3}) constrain the vacuum angle to be \( \theta < 10^{-9} \). With new prospects of improving the sensitivity of current EDM measurements down up to two orders of magnitude \cite{2}, together with plans to measure EDMs of charged light nuclei in storage rings \cite{4}, any non-zero EDM signal will either indicate new physics or set the magnitude of the QCD \( \theta \) parameter. A method to disentangle the different \( T \)-violating (\( T \)) sources that can be
2 Chiral constraints

Chiral effective field theory ($\chi$EFT) is nowadays a well-established method to deal with the non-perturbative QCD dynamics in terms of low-energy hadronic degrees of freedom —see, for instance, Refs. [5]. SM electroweak operators can easily be incorporated via their chiral behavior, and here we use the same idea to deal with the leading $T$ interactions [17].

2.1 $T$ operators at the QCD scale

The $T$ interactions can be written in terms of quark, gluon, and photon degrees of freedom at the QCD energies $M_{QCD} \sim 1$ GeV, well below its characteristic $M_T$ scale. The $\theta$-term and other (up to dimension-6) $T$ operators from possible extensions of the SM are effectively represented at the QCD scale by the following operators [17]

$$L^{(T)} = m_s \theta \bar{q} i \gamma_5 q - \frac{1}{2} \bar{q}(d_0 + d_3) \sigma^{\mu \nu} i \gamma_5 \lambda^a q G_{\mu \nu}^a - \frac{1}{2} \bar{q}(d_0 + d_3) \sigma^{\mu \nu} i \gamma_5 q F_{\mu \nu}$$

$$+ \frac{d_W}{6} \epsilon_{\mu \nu \rho \sigma} \Gamma^{abc} G_{\mu \rho} G_{\nu \sigma} + \frac{1}{4} \text{Im} \Sigma_1(\bar{q} q i \gamma_5 q - \bar{q} \tau q \cdot \bar{q} \tau i \gamma_5 q)$$

$$+ \text{Im} \Sigma_8(\bar{q} \lambda^a q \bar{q} \lambda^a i \gamma_5 q - \bar{q} \lambda^5 \tau q \cdot \bar{q} \lambda^5 \tau i \gamma_5 q),$$

where $m_s = m_u m_d / (m_u + m_d) \sim m^2 / M_{QCD}$, with $m_s$ the pion mass, $G_{\mu \nu}$, and $F_{\mu \nu}$ are the usual gluon and photon field strengths. See [17] for details. The $\theta$-term operator is the $4\mathbf{8}$ component of a chiral vector, Lorentz pseudoscalar $P = (\bar{q} \tau q, \bar{q} i \gamma_5 q)$, the $3\mathbf{3}$ component of it being an isospin-breaking term. Therefore, isospin-breaking and $T$ from the $\theta$-term have a close relation via chiral symmetry that can be further explored [10].

The isoscalar and isovector terms of the quark chromo-EDM (qCEDM) operator, proportional to $\bar{q}d_0, \bar{q}d_3$, transform as the $4\mathbf{8}$ and $3\mathbf{3}$ component of the chiral vectors $\mathbf{V} = \frac{1}{2}(\bar{q} \sigma^{\mu \nu} \tau^a \lambda^5 q, q \bar{q} \sigma^{\mu \nu} \bar{\gamma}_5 \bar{\lambda}^a q) G_{\mu \nu}^a$ and $W = \frac{1}{2}(-i q \sigma^{\mu \nu} \tau^a \lambda^a q, q \bar{q} \sigma^{\mu \nu} \bar{\lambda}^a q) G_{\mu \nu}^a$, with no useful relation to $T$-conserving operators. Analogous remarks also apply to the $d_0$ and $d_3$ terms in the quark EDM (qEDM) operator. On the other hand, the gluon chromo-EDM (gCEDM) and the four-quark operators $(4q_1, 4q_8)$ behave as singlets under the chiral group [17] —chiral symmetry cannot disentangle these three interactions— therefore they are grouped together in a chiral invariant $(\chi_1)$ operator. In terms of chiral objects, the leading $T$ Lagrangian reads

$$L^{(T)} = m_s \theta \bar{q} P_1 - d_0 V_4 + d_3 W_3 - d_0 \bar{V}_4 + d_3 \bar{W}_3 + d_w \chi I.$$

In terms of $M_T$, $\bar{m} = \frac{1}{4}(m_u + m_d)$, electric charge $e$, and dimensionless couplings $d_{0,3}$, $\delta_{0,3}$, and $\bar{u}$, the coefficients $d_{0,3}$, $\bar{d}_{0,3}$, and $d_w$ are expected to scale as [2]

$$d_{0,3} \sim O\left(\frac{\bar{m}}{M_T^2}\right), \quad \bar{d}_{0,3} \sim O\left(\frac{4\pi \bar{d}_{0,3} \bar{m}}{M_T^2}\right), \quad d_w \sim O\left(\frac{4\pi w}{M_T^2}\right).$$

assessed by EDMs is therefore called for. As we are going to see, the chiral symmetry of QCD can be explored to accomplish such task.
2.2 $T$ operators at the hadronic scale

$\chi$EFT provides a systematic way of building up the chiral operators in Eq. (2) in terms of hadronic degrees of freedom. At LO the most relevant terms to EDMs are [9]

$$\mathcal{L}_{\text{eff}}^{(T)} = -2\bar{N}(d_0 + \tilde{d}_1 \tau_3) S^\mu V^\nu F_{\mu\nu} - \frac{1}{f_\pi} \bar{N}((\tilde{g}_0 \tau \cdot \pi + \tilde{g}_1 \tau_3)N$$

$$+ \tilde{c}_1 \bar{N}N \partial_\mu (\bar{N} S^\mu N) + \tilde{c}_2 \bar{N} \tau N \cdot \partial_\mu (\bar{N} S^\mu \tau N)$$  \hspace{1cm} (4)$$

where the above six low-energy constants (LECs) receive contributions from dimensions 4 and 6 $T$ operators with distinct weights. Given the isospin-breaking parameter $\varepsilon = (m_d - m_u)/(m_d + m_u)$ and using naive dimensional analysis (NDA) one arrives at the following estimates [9]:

- $\theta$-term: $\tilde{d}_{0,1} \sim \theta \varepsilon m^2_{QCD}$, $\tilde{g}_0 \sim \theta \varepsilon m^2_{QCD}$, $\tilde{g}_1 \sim \theta \varepsilon m^4_{QCD}$,
- nucleon EDM: $\tilde{d}_{0,1} \sim \delta_{0,3} \varepsilon m^2_{QCD} M^2_F$,
- $\chi$: $\tilde{d}_{0,1} \sim w \varepsilon m^2_{QCD} M^2_F$,
- $\tilde{g}_0 \sim (\tilde{g}_0^0 + \tilde{g}_0^3) m^2_{QCD} M^2_F$,
- $\tilde{g}_1 \sim \delta_{0,3} m^2_{QCD} M^2_F$,

and $\tilde{C}_{1,2} \sim w \varepsilon M^2_{QCD} M^2_F$.

2.3 EDM of a light nucleus

The EDM of an $A \geq 2$ nucleus receives in general two distinct contributions: a $T$ dipole operator $D_T$ (derived from $T$ electromagnetic current $J_T^\mu$) and evaluated between the nucleus bra-ket states $|\Psi_A\rangle$ and $|\bar{\Psi}_A\rangle$, and a $T$-conserving dipole operator $D_T$ (derived from $J_T^\mu$) evaluated between $|\bar{\Psi}_A\rangle$ and the corresponding ket-state with a $T$ admixture $|\Psi_A\rangle$. The expression reads [9]

$$d_A = \frac{1}{\sqrt{6}} \left[ \langle \Psi_A| D_T |\bar{\Psi}_A\rangle + 2 \langle \bar{\Psi}_A| D_T |\Psi_A\rangle \right],$$  \hspace{1cm} (6)$$

where $|\Psi_A\rangle$ and $|\bar{\Psi}_A\rangle$ satisfy $(E - H_0 - V_T)|\Psi_A\rangle = 0$ and $(E - H_0 - V_T)|\bar{\Psi}_A\rangle = V_T|\Psi_A\rangle$. The $T$ and $T$ electromagnetic currents, as well as the $T$ nucleon-nucleon potential $V_T$ are derived from the $T$ and $T$ chiral effective Lagrangians. However, for $^2$H, $^3$H, and $^4$He considered in this work we use the $T$ wave functions from realistic phenomenological potentials. This hybrid approach is justified whenever the short-distance details are not relevant, which is partially confirmed when comparing our results with previous studies [11]. Our results are summarized in the Tables 1. The LECs $\tilde{d}_0$ and $\tilde{d}_1$ were renormalized in a way to produce the neutron and proton EDMs, $d_n = \tilde{d}_0 - \tilde{d}_1$ and $d_p = \tilde{d}_0 + \tilde{d}_1$. In general one expresses the EDMs of light nuclei in terms of the six LECs in different combinations, as one sees for helium and triton. For $N = Z$ nuclei, however, isospin selection rules make this dependence go down to three, as one can verify for the deuteron case [9].

Although one needs six independent EDM measurements to pin down the six LECs, there are still some predictions that can be tested only with the nuclei considered here, depending on the dominance of the $T$ source. From the scalings of Eq. (4) one gets
- qEDM: $d_{3H} \simeq d_n + d_p$, $d_{3He} + d_{3H} \simeq 0.84(d_n + d_p)$, and $d_{3He} - d_{3H} \simeq 0.94(d_n - d_p)$;
- qCEDM: $d_{3He} + d_{3H} \simeq 3d_{3H}$;
- $\theta$-term: $d_{3He} + d_{3H} \simeq 0.84(d_n + d_p)$ and $d_{3He} - d_{3H} \neq -1.88\bar{d}_1 \simeq -0.94(d_n - d_p)$.

Disentangling $\chi$ sources is much harder to achieve since all LECs are involved and may appear with comparable strength. If, for some unknown mechanism, all but $\bar{d}_{0,1}$ are much more suppressed than NDA estimates, then it would be very difficult to distinguish $\chi$ sources from qEDM. The hope is to look at other light nuclei to check if additional relations among LECs can be retrieved.

To summarize, we calculated the contribution of different leading sources of $CP$-violating interactions to the EDMs of deuteron, helion, and triton using $\chi$EFT. We argue that EDMs of light nuclei can be expressed, in general, in terms of six $\mathcal{T}$ LECs. For $N = Z$ nuclei, isospin selection rules probably reduce this number, as in the deuteron case. From our expressions, exploring the distinct chiral properties and using naive dimensional analysis, one is able to derive relations among EDMs and, in principle, disentangle the different $\mathcal{T}$ sources.

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