Numerical Simulation of Polydisperse Two-phase
Unidirectional Fibrous Structures Electroelastic Deformation

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Abstract. In this work the problem formulation is stated, and a mathematical model is developed for calculating the electroelastic deformation of polydisperse two-phase unidirectional fibrous structures. The object of the research was a unidirectional transversely isotropic composite with a two-phase polydisperse structure - a piezo actuator cell and fragments of polydisperse fibrous structures. The constitutive relations at the composite macrolevel were obtained for longitudinal and transverse electromagnetic loading with allowance for the phase conductivity. The coefficients of the electromagnetic coupling of the composite were numerically determined depending on the volume fraction of the piezoelectric phase in the fibers and matrix form. The analysis of the obtained numerical solutions was carried out, the coincidence of the analytical solution with the solution of the asymptotic averaging method for an ideal periodic fibrous structure was obtained. Recommendations on the volumetric content of phases in a two-phase polydisperse composite are presented.

1. Introduction
Piezoceramics is now widespread and is applied in piezomechanical devices, such as piezo actuators, designed to actuate mechanisms, systems or control them based on the piezoelectric effect [1-6]. Piezo actuators are used, for example, in transport and aerospace engineering for state monitoring, geometry control [7,8], vibration and noise reduction [9-11], thus creating various types of SMART structures. When designing SMART structures, it becomes necessary to develop new mathematical models of piezo actuators, develop methods for solving related boundary value problems of electromagnetoelasticity, and predict piezoelectroelastic characteristics. In this research, a mathematical model was developed and the calculation of the effective moduli of a unidirectional fiber composite with piezoelectric and piezomagnetic phases was carried out. A comparative analysis of the obtained numerical solutions with the results of analytical solutions for an ideal periodic fibrous structure, a cell of a piezo actuator, was carried out.

2. Numerical model
In Fig. 1 an actuator mathematical model and fragments of polydisperse two-phase fibrous structures unidirectional along z are showed, where the 1st phase is indicated in black, and the 2nd phase is indicated in white. The size distribution of particles (cross-sections of single-phase cylinders in Fig. 1, b and composite two-phase cylinders in Fig. 1, a and Fig. 1, c) is quite wide, including infinitesimal ones, which makes it possible to fill the entire V area of the composite with such polydisperse
particles. The structure model in Fig. 1, a is formed from composite particles - this is a fiber from the 1st phase, surrounded by a layer of the 2nd phase of the composite.

In this research a unidirectional transversely isotropic fiber composite with a polydisperse structure was considered in more detail in Fig. 1, a. The axes of electric and magnetic polarization of both phases: fibers (1st phase) and matrix (2nd phase) coincide with the \( z \) coordinate axis and with the fiber axes direction orientation. The \( z \) axis of the \( r, \theta, z \) cylindrical coordinate system is compatible with the axis of symmetry of the considered undefined composite cylinder. Constitutive relations for both piezoactive phases 1, 2

\[
\begin{align*}
\sigma_{rr} &= C_{f(111)} e_{rr} + C_{f(112)} e_{\theta\theta} + C_{f(133)} e_{zz} - e_{f(131)} \tilde{E}_z - h_{f(331)} \tilde{H}_z, \\
\sigma_{\theta\theta} &= C_{f(112)} e_{rr} + C_{f(111)} e_{\theta\theta} + C_{f(133)} e_{zz} - e_{f(131)} \tilde{E}_z - h_{f(331)} \tilde{H}_z, \\
\sigma_{zz} &= C_{f(113)} \left( e_{rr} + e_{\theta\theta} \right) + C_{f(333)} e_{zz} - e_{f(331)} \tilde{E}_z - h_{f(333)} \tilde{H}_z, \\
\sigma_{\theta r} &= \sigma_{r\theta} = C_{f(133)} \gamma_{\theta r} - e_{f(331)} \tilde{E}_\theta - h_{f(331)} \tilde{H}_\theta; \\
\tilde{D}_r &= e_{f(331)} \gamma_{\theta r} + \lambda_{f(111)} \tilde{E}_r, \\
\tilde{D}_\theta &= e_{f(113)} \gamma_{\theta \theta} + \lambda_{f(111)} \tilde{E}_\theta, \\
\tilde{D}_z &= e_{f(331)} \left( e_{rr} + e_{\theta\theta} \right) + e_{f(333)} e_{zz} + \lambda_{f(333)} \tilde{E}_z; \\
\tilde{B}_r &= h_{f(331)} \gamma_{\theta r} + \mu_{f(111)} \tilde{H}_r, \\
\tilde{B}_\theta &= h_{f(113)} \gamma_{\theta \theta} + \mu_{f(111)} \tilde{H}_\theta, \\
\tilde{B}_z &= h_{f(331)} \left( e_{rr} + e_{\theta\theta} \right) + h_{f(333)} e_{zz} + \mu_{f(333)} \tilde{H}_z
\end{align*}
\]

connect the components of the stress tensor \( \sigma \), the induction vectors of the electric \( \tilde{D} \) and magnetic \( \tilde{B} \) fields with the components of the deformation tensor \( \epsilon \), the vectors of the electric \( \tilde{E} \) and magnetic \( \tilde{H} \) fields through the components of the tensors of elastic properties \( C_f \), piezoelectric \( e_f \) and piezomagnetic \( h_f \) properties, dielectric \( \lambda_f \) and magnetic \( \mu_f \) permeabilities considered known for each \( f \) phase; shear angles \( \gamma_{\theta r} = 2e_{\theta r}, \gamma_{\theta \theta} = 2e_{\theta \theta}, \gamma_{r\theta} = 2e_{r\theta} \).

The solution of the problem was carried out in two statements: axisymmetric and transverse loading of a polydisperse composite, which are reduced to considering the corresponding problems for a single cell of a composite two-phase cylinder.

3. Longitudinal electromagnetic loading (axisymmetric problem)

Let at the macrolevel of the composite \( \tilde{E}^* \neq 0, \tilde{H}^* \neq 0, \epsilon^* \neq 0, \epsilon_{zz}^* \neq 0 \), the remaining components of the vectors of the electric \( \tilde{E}^* \) and magnetic \( \tilde{H}^* \) fields strengths and the strain tensor \( \epsilon^* \) are equal to zero, \( \epsilon^* = \epsilon_{rr}^* + \epsilon_{\theta\theta}^* \), the relative change in volume during plane deformation. For the considered axisymmetric problem, the equilibrium equations

\[
\begin{align*}
\sigma_{rr} &= C_{f(111)} e_{rr} + C_{f(112)} e_{\theta\theta} + C_{f(133)} e_{zz} - e_{f(131)} \tilde{E}_z - h_{f(331)} \tilde{H}_z \\
\sigma_{\theta\theta} &= C_{f(112)} e_{rr} + C_{f(111)} e_{\theta\theta} + C_{f(133)} e_{zz} - e_{f(131)} \tilde{E}_z - h_{f(331)} \tilde{H}_z \\
\sigma_{zz} &= C_{f(113)} \left( e_{rr} + e_{\theta\theta} \right) + C_{f(333)} e_{zz} - e_{f(331)} \tilde{E}_z - h_{f(333)} \tilde{H}_z \\
\sigma_{\theta r} &= \sigma_{r\theta} = C_{f(133)} \gamma_{\theta r} - e_{f(331)} \tilde{E}_\theta - h_{f(331)} \tilde{H}_\theta;
\end{align*}
\]
\[
\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0
\]  

(2)

Cauchy relations

\[
e_{rr} = \frac{du}{dr}, \quad e_{\theta\theta} = \frac{u_r}{r}
\]  

(3)

and nonzero movements in the xy plane will be only radial movements \( u \); electric and magnetic potentials \( \varphi = -E_z^*z \), \( \psi = -H_z^*z \). General solutions for radial movements in fiber when \( r \in (0;a) \)

\[
u_{(1)r} = Cr
\]  

(4)

and in the matrix when \( r \in (a;b) \)

\[
u_{(2)r} = Ar + \frac{B}{r}
\]  

(5)

of the composite cylinder. Integration constants \( A, B, C \) are determined from the conditions of continuity of radial displacements \( u \) and stresses on the boundary when \( r=a \)

\[
u_{(1)r} = u_{(2)r}, \quad \sigma_{(1)rr} = \sigma_{(2)rr}
\]  

(6)

and condition when \( r=b \)

\[
u_{(2)r} = \frac{\varepsilon}{2} b
\]  

(7)

which follows from the condition \( \langle \varepsilon \rangle = \varepsilon^* \), where \( \langle ... \rangle \) is the averaging operator over the volume of the composite cylinder (the composite). Note that from the defining relations for \( \sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \) (2), taking into account the Cauchy relations (3), general solutions for displacements (4), (5) and additional equalities \( E_z = E_z^*, H_z = H_z^* \), \( \varepsilon_z = \varepsilon_z^* \) the expressions for the stresses in the fiber follow

\[
\sigma_{(1)rr} = \sigma_{(1)\theta\theta} = 2k_{(1)}C + C(1)_{133}\varepsilon_{zz}^* - e_{(1)331}E_z^* - h_{(1)311}H_z^*,
\]

\[
\sigma_{(1)zz} = 2C(1)_{133}C + C(1)_{1333}\varepsilon_{zz}^* - e_{(1)333}E_z^* - h_{(1)333}H_z^*
\]  

(8)

and in the matrix

\[
\sigma_{(2)rr} = 2k_{(2)}A - \frac{2G_{(2)}B}{r^2} + C(2)_{133}\varepsilon_{zz}^* - e_{(2)331}E_z^* - h_{(2)311}H_z^*,
\]

\[
\sigma_{(2)\theta\theta} = 2k_{(2)}A + \frac{2G_{(2)}B}{r^2} + C(2)_{133}\varepsilon_{zz}^* - e_{(2)331}E_z^* - h_{(2)311}H_z^*,
\]

\[
\sigma_{(2)zz} = 2C(2)_{133}A + C(2)_{1333}\varepsilon_{zz}^* - e_{(2)333}E_z^* - h_{(2)333}H_z^*
\]  

(9)
where is the bulk modulus of plane deformation \( k_{(f)} = \left( C_{(f)111} + C_{(f)112} \right) / 2 \) and the shear modulus \( G_{(f)} = \left( C_{(f)111} - C_{(f)112} \right) / 2 \) in the plane of \( xy \) isotropy of both phases ( \( f = 1, 2 \)).

For the fiber and matrix, the average stresses \( \sigma_{(f)} = \left( \sigma_{(f)x} + \sigma_{(f)y} \right) / 2 \) in the \( xy \) plane taking into account (8), (9)

\[
\sigma_{(1)} = 2k_{(1)}C + 11133 \varepsilon_{zz}^* - 11313 \tilde{E}_{zz}^* - 11131 \tilde{H}_{zz}^*; \\
\sigma_{(2)} = 2k_{(2)}A + 11133 \varepsilon_{zz}^* - 11313 \tilde{E}_{zz}^* - 11131 \tilde{H}_{zz}^* 
\]

and induction of electric and magnetic fields

\[
\tilde{D}_{(1)} = 2e_{(1)313}C + e_{(1)333} \varepsilon_{zz}^* + 13313 \tilde{E}_{zz}^* + 11331 \tilde{H}_{zz}^*; \\
\tilde{B}_{(1)} = 2h_{(1)313}C + h_{(1)333} \varepsilon_{zz}^* + 13313 \tilde{E}_{zz}^* + 11331 \tilde{H}_{zz}^*; \\
\tilde{D}_{(2)} = 2e_{(2)313}A + e_{(2)333} \varepsilon_{zz}^* + 13313 \tilde{E}_{zz}^* + 11331 \tilde{H}_{zz}^*; \\
\tilde{B}_{(2)} = 2h_{(2)313}A + h_{(2)333} \varepsilon_{zz}^* + 13313 \tilde{E}_{zz}^* + 11331 \tilde{H}_{zz}^* 
\]

After the \( \langle \ldots \rangle \) operator averaging the quantities (10), (11), we obtain the form of the constitutive relations at the macrolevel of the composite

\[
\sigma' = k' \varepsilon' + C_{1133} \varepsilon_{zz}^* + e_{313} \tilde{E}_{zz}^* + h_{313} \tilde{H}_{zz}^*; \\
\sigma_{zz}' = C_{1133} \varepsilon_{zz}^* + C_{3333} \varepsilon_{zz}^* - e_{333} \tilde{E}_{zz}^* - h_{333} \tilde{H}_{zz}^*; \\
\tilde{D}_{x}' = e_{313} \varepsilon_{zz}^* + e_{333} \varepsilon_{zz}^* + \lambda_{333} \tilde{E}_{zz}^* + \lambda_{333} \tilde{H}_{zz}^*; \\
\tilde{B}_{x}' = h_{313} \varepsilon_{zz}^* + h_{333} \varepsilon_{zz}^* + \mu_{333} \tilde{H}_{zz}^* + \kappa_{333} \tilde{E}_{zz}^* 
\]

with the sough for solutions for efficient modules

\[
k' = k_{(2)} + v_1k_{a_{11}}, \\
C_{1133} = \langle C_{1133} \rangle + v_1k_{a_{12}}; \\
e_{313} = \langle e_{313} \rangle - v_1k_{a_{13}}, \\
h_{313} = \langle h_{313} \rangle - v_1k_{a_{14}}; \\
C_{3333} = C_{(2)1133} + v_1 \tilde{C}_{1133}a_{11}, \\
C_{3333} = \langle C_{3333} \rangle + v_1 \tilde{C}_{1133}a_{12}; \\
e_{333} = \langle e_{333} \rangle - v_1 \tilde{C}_{1133}a_{13}, \\
h_{333} = \langle h_{333} \rangle - v_1 \tilde{C}_{1133}a_{14}; \\
e_{333} = \langle e_{333} \rangle + v_1 \tilde{e}_{313}a_{11}, \\
\lambda_{333} = \langle \lambda_{333} \rangle + v_1 \tilde{e}_{313}a_{13}; \\
\chi_{33} = v_1 \tilde{e}_{313}a_{14}; \\
h_{313} = h_{(2)313} + v_1 \tilde{h}_{313}a_{11}, \\
h_{333} = h_{(2)313} + v_1 \tilde{h}_{313}a_{12}; \\
k_{333} = v_1 \tilde{h}_{313}a_{13}, \\
\mu_{333} = \langle \mu_{333} \rangle + v_1 \tilde{h}_{313}a_{14}, 
\]

where the averaged values \( \langle C_{1133} \rangle , \ldots , \langle \mu_{333} \rangle \) are calculated according to the "mixture rule", for example, \( \langle C_{1133} \rangle = v_1 C_{(1)1133} + (1 - v_1)C_{(2)1133} \).
4. Transverse electromagnetic loading

Let at the macrolevel of the composite $E^* \neq 0, \ H^* \neq 0$, the remaining components of the vectors of the electric $E^*$ and magnetic $H^*$ fields and all components of the deformation tensor $\epsilon^*$ are equal to zero. Similarly to the longitudinal loading solution for the problem under consideration, the following form of constitutive relations at the composite macrolevel follows:

$$
\begin{align*}
\vec{D}' &= \lambda_{11}' \vec{E}^* + \chi_{11}' \vec{H}^*, \\
\vec{B}' &= \mu_{11}' \vec{H}^* + \kappa_{11}' \vec{E}^*, \\
\sigma_v &= -\epsilon_{11}' \vec{E}^* - h_{11}' \vec{H}^*,
\end{align*}
$$

where

$$
\begin{align*}
\lambda_{11}' &= \lambda_{12ii} + \nu \left( \bar{\epsilon}_{11} b_{11} - \bar{\lambda}_{11} b_{11} \right), & \chi_{11}' &= \nu \left( \bar{\epsilon}_{11} b_{12} - \bar{\lambda}_{11} b_{12} \right), \\
\kappa_{11}' &= \nu \left( \bar{\mu}_{11} b_{11} - \bar{\mu}_{11} b_{11} \right), & \mu_{11}' &= \mu_{12ii} + \nu \left( \bar{h}_{11} b_{12} - \bar{\mu}_{11} b_{12} \right), \\
\epsilon_{11}' &= \epsilon_{12ii} - \nu \left( \bar{\epsilon}_{11} b_{12} + \bar{\epsilon}_{11} b_{22} \right), \\
h_{11}' &= \nu \left( \bar{h}_{11} b_{11} + \bar{h}_{11} b_{22} + \bar{\epsilon}_{11} b_{22} \right),
\end{align*}
$$

with complex real parts $\lambda_{1jli}', \ \mu_{1jli}'$ and specific conductivities $\gamma_{1jli}'$ of phases: $j = 1$ for the fiber, $j = 2$ for the matrix. The circular frequency of the applied electric field, which leads to complex values of the effective piezoelectromagnetic composite constants and, as a consequence, to the emergence at the macrolevel of the composite of dispersion and losses in alternating electric fields, known as "Maxwell -Wagnerian relaxation". Complex dielectric constants $\lambda_{1jli}'$ were not included in the solution for the longitudinal coefficient $\kappa_{11}'$, but were included in the solution for the transverse electromagnetic coupling coefficient $\kappa_{11}'$ of the composite. Therefore, the coefficients are: $\kappa_{33}'$ - real, and $\kappa_{11}'^*$ complex, which we represent by decomposition into real $\kappa_{11}'''$ and imaginary $\kappa_{11}''''$ components.

$$
\kappa_{11}' = \kappa_{11}''' - i \kappa_{11}''''
$$
5. Numerical simulation results
The independent elastic, dielectric, and piezomechanical constants of the transversely isotropic electroelastic properties of fibers - PVDF polymer piezoelectric with the z axis of symmetry are known: elastic constants

\[
\begin{align*}
C_{(1)111} &= 0.86 \cdot 10^{10} \text{ Pa}, & C_{(1)122} &= 0.56 \cdot 10^{10} \text{ Pa}, \\
C_{(1)133} &= 0.54 \cdot 10^{10} \text{ Pa}, & C_{(1)333} &= 0.71 \cdot 10^{10} \text{ Pa}, & C_{(1)313} &= 0.10 \cdot 10^{10} \text{ Pa}; \\
C_{(2)111} &= 22 \cdot 10^{10} \text{ Pa}, & C_{(2)313} &= 5.5 \cdot 10^{10} \text{ Pa},
\end{align*}
\]  

(18)

relative dielectric constants

\[
\lambda_{(1)311} / \lambda_0 = 14.7, \quad \lambda_{(1)333} / \lambda_0 = 12.4;
\]

(19)

piezoelectric constants

\[
e_{(1)311} = -1.1 \text{ C/m}^2, \quad e_{(1)333} = 2.9 \text{ C/m}^2, \quad e_{(1)111} = 2.3 \text{ C/m}^2,
\]

(20)

dielectric constant of vacuum \( \lambda_0 \approx 8.85 \cdot 10^{-12} \text{ F/m}. \)

Isotropic elastic properties of a piezomagnetic ferrite matrix are specified in terms of the independent components of the elastic properties tensor

\[
C_{(2)111} = 22 \cdot 10^{10} \text{ Pa}, \quad C_{(2)313} = 5.5 \cdot 10^{10} \text{ Pa},
\]

(21)

transversely isotropic magnetic properties with a \( z \) axis of symmetry are given through piezomodules

\[
h_{(2)311} = h_{(2)322} = -400 \text{ T}, \quad h_{(2)333} = 800 \text{ T}, \quad h_{(2)413} = h_{(2)223} = 200 \text{ T}
\]

(22)

and magnetic permeability

\[
\mu_{(2)11} = \mu_{(2)22} = 3.14 \cdot 10^{-5} \text{ Tm/A}, \quad \mu_{(2)33} = 2.51 \cdot 10^{-5} \text{ Tm/A}.
\]

(23)

Nonzero magnetic permeabilities for piezoelectric fibers \( \mu_{(1)11} = \mu_{(1)22} = \mu_{(1)33} \) were assumed to be equal to the magnetic constant of vacuum \( \mu_0 \approx 1.256 \cdot 10^{-6} \text{ T m/A} \), and the dielectric constants of ferrite \( \lambda_{(2)311} = \lambda_{(2)322} = \lambda_{(2)333} = \lambda_0 \).

Fig. 2 shows the results of calculating the effective coefficients of electromagnetic coupling \( \kappa_{11}^*, \quad \kappa_{33}^* \) of a fibrous piezoelectromagnet depending on the content of piezoelectric PVDF ( \( \square \) ) fibers \( v_1 \) in a ferrite matrix (Fig. 1, a). The results ( \( \vartriangle \) ) correspond to the case of inversion of the properties of the 1st and 2nd phases, i.e. when the matrix was a PVDF piezoelectric and the fibers were ferrite; for clarity, the comparison of the graphs here, as before, denotes the relative volumetric content of PVDF in the composite through \( v_1 \). For comparison, the calculation results ( \( \circ \) ) in the singular approximation [20] (when the comparison medium with averaged \( \langle \ldots \rangle \) over the volume \( V \) composite properties) for the structure in Fig. 1b in the absence of the matrix property and with invariance to the inversion of the phase properties.
Figure 2. Coefficients of electromagnetic coupling $\kappa_{11}^*$ (a), $\kappa_{33}^*$ (b) of the composite depending on the volume fraction of the piezoelectric phase $v_1$ in the form of fibers (○, □) or matrix (△, ○).

The (○) solution at "small" degrees of $v_1$ filling is close to the (□) solution for the 2nd phase matrix structure (Fig. 1, b) and at "large" $v_1$ to the (△) solution for the 1st phase matrix structure (Fig. 1 , d). The analytical solution $\kappa_{33}^*$ (Fig. 2) (□) for a composite with a polydisperse structure exactly coincided with the solution of the asymptotic averaging method [19] for an ideal periodic fibrous structure. The calculation results in Fig. 2 allow us to conclude that the inversion of the properties of the fibers and the matrix of the composite can lead to a significant increase in the absolute values of the effective coefficients of the $\kappa_{11}^*$, $\kappa_{33}^*$ electromagnetic coupling of the composite at fixed values of the volumetric content of the piezoelectric $v_1$ and piezomagnetic $1-v_1$ phases. In particular, with the volume fraction $v_1 \approx 0.2$ for PVDF and for ferrite - 0.8, the absolute values of $\kappa_{33}^*$ for a composite with ferrite fibers in a PVDF matrix are more than two times higher than the $\kappa_{33}^*$ for a composite with PVDF fibers in a ferrite matrix, i.e. PVDF is preferred as the matrix of the composite.

6. Conclusion
Thus, based on the results of the research, the problem was formulated and a mathematical model was developed for calculating the electroelastic deformation of polydisperse two-phase unidirectional fibrous structures. The constitutive relations at the composite macrolevel were obtained for longitudinal and transverse electromagnetic loading with allowance for the phase conductivity. The coefficients of the composite electromagnetic coupling were numerically determined depending on the volume fraction of the piezoelectric phase in the form of fibers and matrix. The analysis of the obtained numerical solutions and their comparison with the analytical solution was carried out. Recommendations on the volumetric content of phases in a two-phase polydisperse composite were formulated.

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