Resilient fuzzy control design of singular stochastic biological economic fishery model

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Abstract
This research paper focuses on designing a resilient fuzzy controller for a kind of singular stochastic biological economic fishery model with a variable economic profit. Initially, the singular stochastic biological economic fishery model is formulated as T-S fuzzy singular systems. By using the ideas of continuous frequency distribution and Lyapunov approach, a fresh collection of linear matrix inequalities is developed which are sufficient to establish the asymptotic stability of the singular biological economic fishery model. Later these conditions are extended to obtain some novel sufficient finite-time stability conditions for the addressed model. The ultimate objective of this research paper is to develop a more generalized version of the resilient controller against nonlinear actuator faults such that it makes the considered system stable within a finite interval of time. Moreover, the developed conditions are based on the solutions of LMIs which provides maximal estimation of region of attraction. Conclusively, simulation results are provided to embellish the usefulness of the obtained control design.

1 | INTRODUCTION

In present situations, mankind face a lot of problem due to the deterioration of natural resources and sinking of the environment conditions, which is mainly due to the fact that human needs have increased exponentially. Biological process models in [1, 2] are designed to affect agro-ecological processes. Certain models as in [3–5] were quite sophisticated, but only a particular sub-component of an ecosystem are considered while modelling. For instance, a three-species discrete hybrid food chain system with the consideration of commercial harvesting is proposed in [6]. Economic profit is an essential factor, which needs to be considered to explore biological economic systems. Further to expedite the results of their research, the authors in [7] have considered a constant economic profit, which may not be realistic. In reality, a constant economic profit can never happen since it will be altered according to some diverse factors, like seasonality, earnings, market interest, harvesting charge and so on. Therefore, it would be more fair to consider a variable economic profit.

Singular systems yield a better description while dealing with real processes when compared to state-space ones [8, 9], which can also be referred to as descriptor systems or generalized state-space systems. By applying equivalent sets approach, the state feedback controller is designed for continuous singular systems in [10]. On the other hand, stochastic systems have been extensively applied in the fields of economics, electric power, manufacture, communication and so on, to handle plants whose structure is mostly subject to erratic sudden changes [11]. In biological models, due to continuous environment fluctuations like changes in sunlight intensity, water level, temperature and so on, plant model parameters are not always an absolute constant instead they keep fluctuating around some fixed mean value. Due to these reasons, the population density usually does not attain a constant scalar value as the time advances but somewhat shows a continuous oscillation around some fixed mean value. On the basis of these factors, population models with stochastic perturbations have earned a great interest [12]. However, a very meagre amount of literature works has been reported for the biological economic systems with stochastic perturbations on a generalized state-space structure.

Moreover, it is well known that fuzzy models are brilliant approximations for nonlinear dynamical systems [13]. Takagi-Sugeno (T-S) fuzzy models are very famous due to its endurance to approximate any fine nonlinear function and it has numerous number of applications in many practical systems. Some
beneficial results on the stability of biological T-S fuzzy models have been obtained in [14], whereas in [13], the admissibility criteria of T-S fuzzy singular systems had been studied. But there exist very few results on the stability for fuzzy based singular stochastic economic fishery model. In this research paper, we study the singular stochastic biological economic fishery model based on the fuzzy model approach. The analysis on designing a controller and thereby stabilizing singular biological economic systems based on fuzzy control has been dealt with in [16]. This model studies the interaction between the density of the population and the economic profit per unit harvesting effort, where the common differential equations which describe the traditional deterministic models are used.

Moreover, in practice, many economic factors are not always deterministic and hence saturation is irresistible in feedback control systems. If it is neglected in the design, a controller may completely fail, possibly turning out into a degraded output or even instability. On the other hand, better performance may be anticipated if a controller is designed by considering the saturation effect a priori. Lately, research societies have diverted their interest on this subject and have made a number of outstanding results [17]. The problem of quantized feedback stabilization for continuous time-delay systems is investigated subject to actuator saturation in [18]. In the case of fisheries, over fishing can be done due to high population densities. Therefore by estimating the saturation region, the harvesting level could be decided, so that ecology could be maintained [19, 20]. Further, failures in control design are irresistible in all biological systems and may cause unexpected changes in the system stability. A suitable fault-tolerant control not only supports to enhance ability of the biological plant model, but also improves the accuracy of the biological plant model and sustains an admissible performance level [21–23].

It should be emphasised that in many works on the fault-tolerant control design, the designed controller is linear. The prospects of the appearance of nonlinear control perturbation have been captivated only by few researchers. A fault-tolerant controller, in which there is a amalgamation of both linear and nonlinear control part, has been reported in [24]. Moreover, in many biological plant models, uncertainties may arise while designing a controller. Therefore, the incorporation of controller which is resilient would be more relevant in the biological plant model to validate the stability [25, 26, 27]. As well known, the research studies based on traditional Lyapunov stability deal with the steady-state behaviour of controlled biological plant dynamics over an infinite interval of time and primarily solves the asymptotic performance of biological plant model. However, in many biological plant models, system state does not cross some limit during some time interval, in particular, large values of the biological plant model states are not desirable in the presence of saturations.

Under these circumstances, we have to examine if the biological plant model states remain inside the specified bound in a fixed finite-interval of time. Therefore, to accommodate to these temporal performance of controlled biological plant models, finite-time stability concept is developed. Due to insufficient workable test conditions for finite-time stability, the theories on stability within a finite-time has been studied again and again in the light of linear matrix inequality theory [28]. Wielding linear matrix inequality theory, many number of results were developed to ensure finite-time stability, finite-time boundedness and finite-time stabilization of various dynamical models including linear system models, nonlinear system models and stochastic system models [29]. Though some studies have been presented for singular stochastic biological economic fishery model still there exist many difficulties in analysis, modeling and designing the control which have not yet been fully exploited. Despite the fact that a large number of research have been carried out on control synthesis and stability analysis of biological economic fishery models. However, there exist a large number of crucial issues during the design of control, namely, nonlinearities, saturations and variations in the control gain. These particulars inspired us and caught our attention leading to the study on resilient fuzzy controller design for singular stochastic input saturated biological economic fishery systems. The core contributions of this research paper are highlighted below.

- A complete model of singular stochastic biological economic fishery system is established in which the controller contains actuator faults and input saturations.
- A fault-tolerant resilient control model which is an amalgamation of both linear and nonlinear term is designed to the addressed model.
- Sufficient conditions are obtained for the finite-time stability of stochastic singular input saturated biological economic fishery model in the presence of nonlinear faults in the controller.
- Simulation results reveal that the suggested controller can combat to the nonlinearities and controller gain fluctuations originating in the controller. Moreover, in spite of the occurrence of actuator faults and input saturation, the designed controller can accomplish good efficiency towards the system stability within a finite-time.

2 | BIOLOGICAL MODEL DESCRIPTION AND PRELIMINARIES

In the present section of this research paper, initially, we consider the singular biological economic fishery model which represents a complete ecosystem model and it can be utilized to investigate the aspects of predation, harvesting and production. Taking into account all these factors, the authors in [16] have modelled a system by further considering the over exploitation of resources, raising resource price and economic profit. If \( \rho(k) \) is the density of the fishes, \( \epsilon(k) \) is the unit price of the fishes and \( e(k) \) the economic profit per unit harvesting effort at time \( k \), then the singular biological economic fishery model is represented by

\[
\begin{align*}
    d\rho(k) &= \left[ \rho(k) \left( 1 - \frac{\rho(k)}{c} \right) - H \rho(k) + u_F(k) \right] dk, \\
    dc(k) &= [\epsilon(m - c(k) - H\rho(k))] dk, \\
    0 &= Hv(k)\rho(k) - hH\rho(k) - \epsilon(k).
\end{align*}
\] (1)
Further, in order to stabilize the economy directed from the market $e(k)$ and the density of biological population of fishes $\rho(k)$, we have utilized a control variable $u_F(k)$ in the above system. The positive constants $r$ and $c$ are the intrinsic growth rate and the carrying capacity of the fishes; $b$ represents the harvesting cost per unit. The parameters $H > 0$ and $m > 0$ represent the harvesting effort and the market capacity. The perfect elasticity of price or in other words, the market competition is denoted by $s$, which reflects the price speed adjustment and the market competition. For the sake of simplicity, let us denote $\varepsilon(k) = e(k) - m$ and rewrite model (1) as below:

$$d\rho(k) = \left[r\rho(k) \left(1 - \frac{\rho(k)}{c}\right) - H\rho(k) + u_F(k)\right]dk,$$

$$d\varepsilon(k) = [-sH\rho(k) - s\varepsilon(k)]dk,$$

$$0 = (m - b)H\rho(k) + H\rho(k)\varepsilon(k) - e(k).$$

(2)

If $b_1(\rho(k)) = \frac{c - \rho(k)}{2c}$ and $b_2(\rho(k)) = \frac{c + \rho(k)}{2c}$ represent the fuzzy weighting function, then the above model can be expressed as a fuzzy model by utilizing the fuzzy inference approach. Obviously, $b_1(\rho(k)) \geq 0$, $b_2(\rho(k)) \geq 0$ and $\sum_{i=1}^{2} b_i(\rho(k)) = 1$. Therefore, the overall fuzzy singular biological economic fishery model can be inferred as follows:

$$\sum_{j=1}^{2} b_j(\rho(k)) [\bar{A}_jX(k) + \bar{B}_ju_F(k)],$$

(3)

where $X(k) = [\rho(k) \; \varepsilon(k)]^T$, $\bar{\Xi} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

$$\bar{A}_1 = \begin{bmatrix} 2r - H & 0 & 0 \\ -sH & -s & 0 \\ (m - b)H & -cH & -1 \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} -H & 0 & 0 \\ -sH & -s & 0 \\ (m - b)H & cH & -1 \end{bmatrix},$$

$$\bar{B}_1 = \bar{B}_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T.$$

It is well known that the white noise always exists and it not easy to neglect the clout of the white noise in biological economic fishery model. Taking this into consideration, in this paper, we have further considered singular stochastic biological economic fishery model. By denoting $\Omega$ as the stochastic perturbation, the above biological economic fishery model (3) can be rewritten as

$$\sum_{j=1}^{2} b_j(\rho(k)) [\bar{A}_jX(k) + \bar{B}_ju_F(k)]dk + \bar{D}_j\varepsilon(k)d\omega(k),$$

(4)

where $\bar{D}_1 = \bar{D}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\omega(k)$ is one-dimensional Brownian motion with its initial value as 0 and $t^2$ is intensity of white noise.

Fishery models are complicated due to the complicated biology of the available fish stocks which are being handled due to the changes in nature. When using harvest strategies, it is necessary to define a harvest control rule for the quantity of fish that can be caught based on market capacity in order to meet the objective. In particular, it is of utmost importance to stabilize the above fuzzy models (3) and (4) by proposing a suitable controller. In biological economic systems, exceptional conditions cannot be predicted as the system is based on environmental conditions. Within this scope, we design a control protocol which can maintain overall system stability and acceptable performance, even under unexpected environmental conditions. In fishery models, nonlinear characteristic of actuators will occur inevitably. Therefore in this research paper we use the actuator nonlinearity $\phi(u(k))$ which will be interpreted in the upcoming description of fault tolerant controller. For this purpose, we consider a generalized controller $u_F(k)$ in the form

$$u_F(k) = \bar{F}u(k) + \phi(u(k)),$$

(5)

where the fault tolerant matrix $\bar{F}$ takes the value such that $0 \leq \bar{F}^T \leq \bar{F}^T \leq \bar{F} \leq 1$, where $\bar{F} = 0$ means that the actuator completely fails, $\bar{F} = 1$ means that the actuator is normal. Define $\bar{F} = \bar{F}^T + \bar{F}^T$ and $\bar{F} = \bar{F}^T - \bar{F}^T$. Then,

$$\bar{F} = \bar{F} + \bar{F}\Omega,$$

(6)

where $-1 \leq \Omega \leq 1$. Moreover, the actuator nonlinearity $\phi(u(k))$ is assumed to satisfy the lipschitz condition $|\phi(u(k))| \leq \sqrt{\mathfrak{F}|u(k)|}$ for a scalar $\mathfrak{F} > 0$ and hence

$$\phi^T(u(k))\phi(u(k)) = u^T(k)\mathfrak{F}u(k).$$

(7)

Due to certain perturbations on controller gains, the controller might be fragile. Therefore, to stabilize the biological population of the fishes and economic profit, we design a control protocol that will be insensitive to some variations in control gains. For this purpose, we consider a resilient fuzzy state feedback controller of the form

$$u(k) = \sum_{j=1}^{2} b_j(\rho(k))K_jX(k),$$

(8)

with $K_j = \bar{K}_j + \Delta \bar{K}_j$, where $\bar{K}_j$ denotes the control gain matrix which is to be determined. $\Delta \bar{K}_j$ is the matrix which is used to denote the possible gain variations in the additive form $\Delta \bar{K}_j = H_j\Delta N_j(k)M_j$, where $\Delta N_j(k)\Delta N_j(k) \leq I$ and $H_j, M_j$ are constant matrices with known values. Using (8) in (5), we can obtain the output of the nonlinear resilient fault-tolerant fuzzy controller as

$$u_F(k) = \bar{F} \sum_{j=1}^{2} b_j(\rho(k))K_jX(k) + \phi(u(k)).$$
\[
\mathcal{F} \sum_{j=1}^{2} b_j(\varphi(k))(\overline{R}_j + \Delta \overline{R}_j(k))X(k) + \phi(u(k))
\]
\[
= \sum_{j=1}^{2} b_j(\varphi(k))\{\overline{F}_jX(k) + \overline{F}_j\Delta \overline{R}_j(k)X(k) + \phi(u(k))\}.
\]

By using the nonlinear resilient fault-tolerant fuzzy controller (9) in (3), the state space description of the closed-loop singular biological economic fishery system can be given as

\[
\dot{\Xi}(k) = (A_F + B_F)X(k) + C_F\phi(u(k)),
\]

where

\[
A_F = \sum_{i,j=1}^{2} b_i(\varphi(k))b_j(\varphi(k))[\overline{A}_i + \overline{B}_i\overline{F}_j],
\]

\[
B_F = \sum_{i,j=1}^{2} b_i(\varphi(k))b_j(\varphi(k))\overline{B}_i\overline{F}_j\overline{F}_j^T\overline{N}_j(k)\overline{M}_j
\]

and

\[
C_F = \sum_{i,j=1}^{2} b_i(\varphi(k))b_j(\varphi(k))\overline{B}_j.
\]

Moreover, for singular stochastic biological economic fishery system (4), the state space description of the addressed model can be obtained in a similar way as

\[
\dot{\Xi}(k) = (A_F + B_F)X(k) + C_F\phi(u(k)) + \omega(k),
\]

where

\[
D_F = \sum_{i,j=1}^{2} b_i(\varphi(k))b_j(\varphi(k))D_i.
\]

Remark 1. It should be mentioned that the biological economic fishery models (3) and (4) contain two fuzzy rules which can be generalized for biological economic fishery control systems with g fuzzy rules for any scalar \( g \) via the proper selection of the fuzzy weighting functions.

Now, before proceeding into the further part of the section, the subsequent lemmas are provided, which will be adapted for the derivation of the next section.

Lemma 1. [30] Let \( l \) and \( m \) be real vectors of equal dimensions. Then, \( 2l^TM \leq \overline{\beta}_1l^TMl + \overline{\beta}_1^{-1}m^TMm \), where \( \overline{\beta}_1 \) is any positive scalar.

Lemma 2. [17] Let \( A = A^T, B \) and \( C \) be real matrices of appropriate dimension with \( D \) satisfying \( DD^T \leq I \), then \( A + BDC + C^TD^TB \) \( < 0 \) if and only if there exists a scalar \( \overline{\beta}_2 > 0 \) such that \( A + \overline{\beta}_2BB^T + \overline{\beta}_2^{-1}CC^T < 0 \), or equivalently

\[
\begin{bmatrix}
A & \overline{\beta}_2B & C^T \\
\overline{\beta}_2^T & 0 & 0 \\
0 & 0 & -\overline{\beta}_2^T
\end{bmatrix} < 0.
\]

3 MAIN RESULTS

This main section is divided into two subsections 3.1 and 3.2. The first subsection 3.1 concentrates in providing sufficient conditions to establish the asymptotic stability of singular biological economic fishery system (3) and the subsequent subsection 3.2 concentrates in providing sufficient conditions to establish the finite-time stability of singular stochastic biological economic fishery system (4).

3.1 Admissibility of singular input saturated biological economic fishery system

In this upcoming subsection, initially, a nonlinear resilient fuzzy fault-tolerant control protocol is designed to guarantee the stabilization of biological economic fishery system (3) without taking into account the saturation effect. Subsequently, by considering the aspect of saturation effect, we establish a set of conditions which are sufficient to prove the existence of invariant ellipsoids. These conditions have been used to outline the admissibility of the system (3) under study.

3.1.1 Fault-tolerant controller design without saturation effect

In this subsection, we deal with the stabilization of singular biological economic fishery model in the presence of nonlinear faults in the control input. More specifically, in the following theorem, we propose a contemporary set of sufficient conditions to establish the asymptotic stability of (10), when the controller gain matrix is resilient to controller gain fluctuations.

Theorem 1. For an unknown actuator failure \( \overline{F} \) of the format (6), if there exist real positive scalars \( \overline{\beta}_1, \overline{\beta}_2, \overline{\beta}_3, \) symmetric matrix \( Q > 0 \) and any matrices \( R_{ij} \) for \( i, j = 1, 2 \) the following LMIs hold:

\[
[\Phi_{ij}^F] < 0, i = j
\]

\[
[\Phi_{ij}^F] + [\Phi_{ji}^F] < 0, i < j
\]

where

\[
\Phi_{ij}^F = \begin{bmatrix}
\Phi_{11}^F & \Phi_{12}^F & R_{ij}^T & \Phi_{14}^F & \Phi_{15}^F & \Phi_{16}^F & \Phi_{17}^F \\
\Phi_{12}^T & \Phi_{22}^F & 0 & \Phi_{15}^T & \Phi_{16}^T & \Phi_{17}^T & 0 \\
R_{ij} & \Phi_{15} & \Phi_{16} & \Phi_{17} & 0 & 0 & 0 \\
\Phi_{14} & \Phi_{15}^T & \Phi_{16}^T & \Phi_{17}^T & \Phi_{17} & 0 & 0 \\
\Phi_{13} & \Phi_{15} & \Phi_{16} & \Phi_{17} & 0 & 0 & 0 \\
\Phi_{12} & \Phi_{15} & \Phi_{16} & \Phi_{17} & 0 & 0 & 0 \\
\Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17}
\end{bmatrix},
\]

where

\[
\Phi_{11}^F = \overline{A}_iQ + \overline{B}_i\overline{F}_j, \quad \Phi_{12}^F = \overline{\beta}_1\overline{B}_j, \quad \Phi_{13}^F = \overline{\beta}_2\overline{\beta}_3\overline{F}_j\overline{H}_j, \quad \Phi_{14}^F = \overline{\beta}_2\overline{\beta}_3\overline{F}_j\overline{H}_j, \\
\Phi_{15}^F = \overline{Q}_M, \quad \Phi_{16}^F = \overline{\beta}_3\overline{H}_j, \quad \Phi_{17}^F = R_{ij}, \quad \Phi_{33}^F = -\overline{\beta}_3I, \quad \Phi_{47}^F = \overline{\beta}_2\overline{H}_j \text{ and } \Phi_{47}^F = \overline{\beta}_2\overline{H}_j
\]

then the considered system (10) is asymptotically stable. Moreover, controller gain matrices are designed by \( \overline{R}_j = R_{ij}Q^{-1}. \)

Proof. According to Lyapunov theorem, if \( V(k) > 0 \) and \( \dot{V}(k) \leq 0 \) then the asymptotic stability of (10) can be guaranteed. On this context, we consider a Lyapunov function for (10) of the form, \( \dot{V}(X(k)) = \dot{X}^T(k)\overline{P}_2X(k) > 0, \) where \( \overline{P} \) is a positive definite matrix. Now, along the trajectories of system...
we compute \( \dot{V}(X(k), k) \) and obtain

\[
\dot{V}(X(k), k) = 2X^T(k)\hat{\Sigma}^T\hat{P}\Sigma X(k)
\]

\[
= 2X^T(k)\hat{\Sigma}^T\hat{P}((A_F + B_F)X(k) + C_F\phi(u(k))].
\]

Now, by utilizing Lemma 1 we can have

\[
2X^T(k)\hat{\Sigma}^T\hat{P}C_F\phi(u(k)) = 2\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\rho(k))h_j(\rho(k))X^T(k)\hat{\Sigma}^T\hat{P}C_F\phi(u(k))
\]

\[
\overline{P}B\phi(u(k)) \leq \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\rho(k))h_j(\rho(k))\beta_iX^T(k)\hat{\Sigma}^T\hat{P}C_F\phi(u(k)).
\]

Further by (7), we can rewrite the above as

\[
2X^T(k)\hat{\Sigma}^T\hat{P}C_F\phi(u(k)) \leq 2\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\rho(k))h_j(\rho(k))X^T(k)\hat{\Sigma}^T\hat{P}C_F\phi(u(k))
\]

\[
|\beta_i\Sigma^T\overline{P}B\beta_i\hat{\Sigma} + \beta_i^{-1}|K_j + H_j\overline{N}_j(k)\overline{M}_j|X(k).
\]

Now by using (15) in (14), we can obtain

\[
\dot{V}(X(k), k) \leq 2\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\rho(k))h_j(\rho(k))X^T(k)\hat{\Sigma}^T\hat{P}(\overline{A}_j + \overline{B}_j)
\]

\[
\overline{F}K_j + \overline{B}_j\overline{F}H_j\overline{N}_j(k)\overline{M}_j|X(k)| + \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\rho(k))
\]

\[
b_i(\rho(k))X^T(k)|\beta_i\Sigma^T\overline{P}B\beta_i\hat{\Sigma} + (\overline{K}_j + \overline{H}_j\overline{N}_j(k)\overline{M}_j)|X(k).
\]

Using Schur complement and Lemma 2 along with the condition \( \overline{F} = \overline{F} + \overline{P}\Omega \), the above inequality deduces to the form

\[
\dot{V}(X(k), k) \leq \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\rho(k))h_j(\rho(k))X^T(k)|\beta_i\Sigma^T\overline{P}B\beta_i\hat{\Sigma} + (\overline{K}_j + \overline{H}_j\overline{N}_j(k)\overline{M}_j)|X(k).
\]

where \( \hat{\phi}_{ij} = \hat{\Sigma}^T\hat{P}\overline{A}_j + \hat{\Sigma}^T\overline{P}B\beta_i\overline{K}_j \), and

\[
\hat{\phi}_{ij} = \Sigma^T\overline{P}\overline{A}_j + \Sigma^T\overline{P}B\beta_i\overline{K}_j,
\]

with \( \hat{\phi}_{i1} = \hat{\Sigma}^T\overline{P}\overline{A}_j + \hat{\Sigma}^T\overline{P}B\beta_i\overline{K}_j \), \( \hat{\phi}_{i2} = \beta_i\Sigma^T\overline{P}B_j \),

\[
\hat{\phi}_{i3} = \hat{\Sigma}^T\overline{P}\overline{A}_j + \hat{\Sigma}^T\overline{P}B\beta_i\overline{K}_j \), \( \hat{\phi}_{i4} = \beta_i\Sigma^T\overline{P}B_j \),

\[
\hat{\phi}_{i5} = \hat{\Sigma}^T\overline{P}\overline{A}_j + \hat{\Sigma}^T\overline{P}B\beta_i\overline{K}_j \), \( \hat{\phi}_{i6} = \beta_i\Sigma^T\overline{P}B_j \),

\[
\hat{\phi}_{i7} = \hat{\Sigma}^T\overline{P}\overline{A}_j + \hat{\Sigma}^T\overline{P}B\beta_i\overline{K}_j \), \( \hat{\phi}_{i8} = \beta_i\Sigma^T\overline{P}B_j \).

Therefore, it can be reasoned out that if LMIs (12) and (13) hold, then \( \dot{V}(X(k), k) < 0 \). Thus, by Lyapunov stability theory, the singular biological economic fishery model (3) is asymptotically stabilized via a resilient fuzzy state feedback controller (8) subject to actuator failures. This concludes the proof. \( \square \)

## 3.1.2 Fault-tolerant controller design with saturation effect

Now, we first derive a set of invariance conditions in Theorem 2 for the controller subject to saturation effect and prove that the system (3) is regular and impulse free. Subsequently, in Theorem 3, we obtain a set of sufficient conditions for the existence of invariant ellipsoids and for designing a nonlinear fault-tolerant resilient fuzzy controller subject to saturation effects such that the biological economic singular fishery model (3) is admissible.

Saturation nonlinearities are omnipresent in control systems. In real practice, every concrete actuator is conditional to saturation due to its maximum and minimum bounds. Due to the effect of saturation, the control system may not be stabilized. Hence it is essential to determine a bounded region of the initial states to estimate the domain. For this purpose, we assume the actuator output to be affected by saturation effect and derive a set of invariance conditions. Under this scenario, system (3) takes the following form

\[
\tilde{X}(k) = \sum_{i=1}^{2} h_i(\rho(k))\tilde{A}_iX(k) + \tilde{B}_i\text{sat}(u_i(k)),
\]

where \( \text{sat}(u_i(k)) \) is the standard vector valued saturation function. Without loss of generality, let us take unit saturation level and the matrices \( \tilde{\Sigma}, \tilde{A}_i \), and \( \tilde{B}_i \), in the following form

\[
\tilde{\Sigma} = \begin{bmatrix} l_2 & 0 \\ 0 & l_2 \end{bmatrix}, \quad \tilde{A}_i = \begin{bmatrix} A_{i1}^{(1)} & A_{i1}^{(2)} \\ A_{i2}^{(1)} & A_{i2}^{(2)} \end{bmatrix} \quad \text{and} \quad \tilde{B}_i = \begin{bmatrix} b_{i1}^{(1)} \\ b_{i2}^{(1)} \end{bmatrix}.
\]
Accordingly, let us partition \( X(k) \) as \( X(k) = \begin{bmatrix} X^{(1)} \\ X^{(2)} \end{bmatrix} \). Now, following the same lines as in [31], the estimate for the domain of attraction can be taken in the form \( \{X(k) : X^{(1)} \in \Theta(P, 1), A^{(21)} X^{(1)} + A^{(22)} X^{(2)} + B^{(2)} \text{ sat}(u_F(k)) = 0 \} \), where \( P \) is a positive definite matrix, \( \Theta(P, 1) = \{X(k) : X^{(1)^T} P X^{(1)} \leq 1 \} \) is a contractively invariant ellipsoid and \( u_F(k) \) is defined as in (9).

Now, to establish conditions for \( \Theta(P, 1) \) to be contractively invariant ellipsoid, let us define \( Y(K_i) = \{X(k) : |k^{(1)}_i X(k)| \leq 1 \} \), where \( k^{(1)}_i \) is the \( m^{th} \) row of matrix \( K_i \). In this paper, we take \( m = 1 \). Let us define a set \( S \) whose elements \( S_j \) are diagonal matrices with diagonal values either 1 or 0. Since \( S_j \subset S \), \( S_j^c = I - S_j \subset S \). It is noted that in the case of non-unit saturation levels, the effect of saturation will be absorbed into the control matrix \( B_i \) and the controller gain matrix \( K_j \). Now, before proceeding further into the full part of the subsection, the following definition and lemma are provided, which will be adapted in the derivation of the set invariance conditions.

**Definition 1.** [33] Consider the singular system \( E \dot{x}(k) = A x(k) \) which is regular if \( \det(\lambda E - A) \) is not identically zero; impulse free if \( \deg(\det(\lambda E - A)) = \text{rank}(E) \); admissible, if it is regular, impulse-free and asymptotically stable.

**Lemma 3.** [33] Let \( K_j, \bar{T}_j \in \mathbb{R} \). Then for any \( X(k) \in Y(K_i) \), we have

\[
\text{sat}(u_F(k)) \in \partial \left\{ S_j \bar{T}_j X(k) + S_j \varphi(u(k)) + S_j^c \bar{T}_j X(k) \right\}
\]

\[
g = \{1, 2, ..., 2^w\}, \text{where } co \text{ stands for the convex hull.}
\]

From Lemma 3, if there exists an auxiliary matrix \( \bar{T}_j \) satisfying the condition \( |k^{(1)}_j X(k)| \leq 1 \) then for \( Y_\psi \geq 0 \) and \( \sum_{j=1}^{2^w} Y_\psi = 1 \), we can have

\[
\text{sat}(u_F(k)) = \sum_{j=1}^{2^w} Y_\psi (S_j \bar{T}_j X(k) + S_j \varphi(u(k)) + S_j^c \bar{T}_j X(k)).
\]

In the following Theorem 2, in order to establish set invariance conditions for the ellipsoid \( \Theta(P, 1) \), we define a nonsingular matrix

\[
P = \begin{bmatrix} P^{(1)} & 0 \\ P^{(3)} & P^{(5)} \end{bmatrix},
\]

where \( P^{(1)} > 0 \) and \( P^{(4)} \) is a nonsingular matrix.

**Theorem 2.** For an unknown fault matrix \( F \) in (6), if there exist a matrix \( P > 0 \) of the form (19) and \( \bar{T}_j \in \mathbb{R} \) such that

\[
\sum_{i=1}^{2} \sum_{j=1}^{2} b_i(\varphi(k)) b_j(\varphi(k)) \left\{ \begin{array}{l}
\left[ A_i + B_i F S_k j + B_i F S_k \varphi(u(k)) \right]
\\
+ B_i S_k^c \bar{T}_j \right)^T P + P^T \left[ A_i + B_i F S_k j + B_i F S_k \varphi(u(k)) \right]
\\
+ B_i S_k^c \bar{T}_j \right\} < 0, g = 1, 2, ..., 2^w
\]

and \( \Theta(\bar{Z}, P, 1) \subset Y(K_j) \), then system (16) is regular and impulse free within \( \Theta(\bar{Z}, P, 1) \). Also \( \Theta(P^{(1)}, 1) \) is a contractively invariant set.

**Proof.** First, let us prove the system (16) is regular and impulse free within \( \Theta(\bar{Z}, P, 1) \). For this purpose, we define two matrices of the form \( \bar{T}_j = \begin{bmatrix} \bar{T}_j^{(1)} & \bar{T}_j^{(2)} \end{bmatrix} \) and \( K_j = \begin{bmatrix} K_j^{(1)} & K_j^{(2)} \end{bmatrix} \), where \( K_j^{(1)} = \bar{K}_j^{(1)} + \Delta \bar{K}_j^{(1)} \) for \( j = 1, 2 \). Also let \( \varphi(u(k)) = \begin{bmatrix} \varphi_1(u(k)) \varphi_2(u(k)) \end{bmatrix} \). Now substituting all these matrices in (20) and by some simple calculations, it is easy to see that

\[
\sum_{i=1}^{2} \sum_{j=1}^{2} b_i(\varphi(k)) b_j(\varphi(k)) \left\{ \begin{array}{l}
\left[ A_i^{(22)} + \bar{B}_i^{(2)} F S_k j^{(2)} + \bar{B}_i^{(2)} F S_k \varphi(u(k)) \right]
\\
+ \bar{B}_i^{(2)} S_k^c \bar{T}_j^{(2)} \right)^T P + P^T \left[ A_i^{(22)} + \bar{B}_i^{(2)} F S_k j^{(2)} + \bar{B}_i^{(2)} F S_k \varphi(u(k)) \right]
\\
+ \bar{B}_i^{(2)} S_k^c \bar{T}_j^{(2)} \right\} < 0,
\]

\( g = 1, 2, ..., 2^w \). This proves that \( \bar{T}_j^{(22)} + \bar{B}_i^{(2)} F S_k j^{(2)} + \bar{B}_i^{(2)} F S_k \varphi(u(k)) \) is nonsingular. Further, we can obtain \( \bar{T}_j^{(22)} = 0 \). Therefore \( \bar{T}_j^{(22)} + \bar{B}_i^{(2)} F S_k j^{(2)} + \bar{B}_i^{(2)} F S_k \varphi(u(k)) \) is nonsingular for \( g = 1, 2, ..., 2^w \). We know that \( \bar{T}_j^{(22)} X^{(1)} + \bar{A}_j^{(22)} X^{(2)} + \bar{B}_j^{(2)} \text{ sat}(u_F(k)) = 0 \) for system (16), hence \( \text{sat}(u_F(k)) \in \{ F S_k j^{(2)} X^{(1)} + \bar{F} S_k \varphi(u(k)) + \bar{F} S_k j^{(2)} X^{(2)} \} \), for \( g, b = 1, 2, ..., 2^w \) and \( \forall X(k) \in \Theta(\bar{Z}, P, 1) \), where \( e_j \in \{-1, 1\} \). Moreover, \( \bar{A}_j^{(22)} + \bar{B}_j^{(22)} F S_k j^{(2)} + \bar{B}_j^{(22)} F S_k \varphi(u(k)))X^{(2)} = -\bar{A}_j^{(21)} + \bar{B}_j^{(22)} F S_k j^{(1)} + \bar{B}_j^{(22)} F S_k \varphi(u(k)))X^{(1)} - \bar{B}_j^{(22)} F S_k^c e_j \) and \( \bar{A}_j^{(22)} + \bar{B}_j^{(22)} F S_k j^{(2)} + \bar{B}_j^{(22)} S_k \varphi(u(k)) \) is nonsingular, hence we can solve for \( X^{(2)} \) and obtain a normal linear equation. This proves that system (16) is regular and impulse free.

Let us now prove that \( \Theta(P^{(1)}, 1) \) is contractively invariant. By virtue of Lemma 3, it follows that

\[
\sum_{j=1}^{2} b_j(\varphi(k)) \left[ \bar{T}_j^{(1)}(k) + \bar{B}_j \text{ sat}(u_F(k)) \right] \in \Theta(\bar{Z}, P, 1)
\]

or

\[
\left( \sum_{j=1}^{2} b_j(\varphi(k)) \right)^2 \left( \sum_{j=1}^{2} b_j(\varphi(k)) \right) \bar{T}_j^{(1)}(k)
\]

\[
\left( \sum_{j=1}^{2} b_j(\varphi(k)) \right)^2 \left( \sum_{j=1}^{2} b_j(\varphi(k)) \right) \bar{T}_j^{(1)}(k) \]
\[ \begin{aligned}
&+ \overline{B}_i (S_j K_j + \overline{B}_i (S_j \phi(u(k)) + S_i T_j) X(k)), \quad (21) \\
\end{aligned} \]

where \( g = 1, 2, \ldots, 2^m \). Choose a matrix \( \overline{P} > 0 \) such that \( \overline{P} = \begin{bmatrix} P^{(1)} & P^{(2)} \\ 0 & P^{(1)} \end{bmatrix} \). Also let us consider the following matrices \( Y_0 = \begin{bmatrix} 0 \\ f \end{bmatrix} \) and \( Z = \begin{bmatrix} 0 \\ P^{(1)} \end{bmatrix} \). Then we can observe that \( \overline{P} \overline{X} + Y_0 Z = P \) and \( \overline{X} Y_0 = 0 \).

Consider the Lyapunov candidate as

\[ V(X(k), k) = X^T(k) P^{(1)} X(k) = X^T(k) \overline{P} X(k) > 0. \]

Taking the derivative along the trajectories of system (16), we obtain

\[ \dot{V}(X(k), k) = 2X^T(k) (\overline{P} \overline{X} + Y_0 Z)^T \overline{X} = 2X^T(k) (\overline{P} \overline{X} + Y_0 Z)^T \overline{X} + 2 \sum_{j=1}^{2^m} b_j(\rho(k))[\overline{A}_j + \overline{B}_i F S_j K_j + \overline{B}_i F S_j \phi(u(k)) + \overline{B}_i S_i T_j)] X(k) \]

\[ \leq \max_{g = 1, 2, \ldots, 2^m} 2X^T(k) P^{(1)} \left( 2 \sum_{j=1}^{2^m} b_j(\rho(k)) b_j(\rho(k)) \right) + \overline{B}_i F S_j K_j + \overline{B}_i F S_j \phi(u(k)) + \overline{B}_i S_i T_j) X(k). \]

Therefore, if (20) holds, then \( \dot{V}(X(k), k) < 0, \forall X(k) \in \tilde{\Omega}(\overline{P}, 1) \). This proves that \( \Theta(\overline{P}, 1) \) is contractively invariant.

The further part of this subsection concentrates on analysing the admissibility of the singular biological economic model when the nonlinear resilient control is subject to faults and saturation within the contractively invariant ellipsoid \( \tilde{\Omega}(\overline{P}, 1) \).

In Theorem 3, valid sufficient conditions for the solvability of the nonlinear fault-tolerant resilient control problem will be acquired, when the input control is affected by saturation effect.

Before proceeding to Theorem 3, by considering (8), (9) and (18), we rewrite (16) in a simplified form as

\[ \tilde{\Omega} \overline{X}(k) = \sum_{g=1}^{2^m} \gamma_g (\overline{A}_j + B_i) X(k) + C_j \phi(u(k))), \quad (23) \]

where

\[ A_j = \sum_{j=1}^{2^m} b_j(\rho(k)) b_j(\rho(k)) \overline{A}_j + \overline{B}_i F S_j K_j + \overline{B}_i S_i T_j), B_i = \sum_{j=1}^{2^m} b_j(\rho(k)) b_j(\rho(k)) \overline{B}_i F S_j \overline{F} \overline{N}_j(k) \overline{M}_j \]

and

\[ C_j = 2 \sum_{i=1}^{2^m} b_i(\rho(k)) b_j(\rho(k)) \overline{B}_i S_i \]

\[ \Box \]

**Theorem 3.** Consider the closed-loop system (23) affected by saturation effect. This system is admissible for an unknown fault matrix \( \overline{F} \) and any matrices \( T_j \) for \( i, j = 1, 2 \) if the following LMIs hold:

\[ \begin{bmatrix} -1 & R_j \\ 0 & -Q \end{bmatrix} \leq 0, \quad (24) \]

\[ [\hat{\phi}_j^1]^T < 0, i = j \]

\[ [\hat{\phi}_j^1] + [\hat{\phi}_j^1] < 0, i < j \]

where

\[ \hat{\phi}_j = \begin{bmatrix} \hat{\phi}_j^1 & \hat{\phi}_j^2 & \hat{\phi}_j^3 & \hat{\phi}_j^4 & \hat{\phi}_j^5 & \hat{\phi}_j^6 & \hat{\phi}_j^7 \\ -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{\phi}_j^{23} & \hat{\phi}_j^{24} & \hat{\phi}_j^{25} & \hat{\phi}_j^{26} & \hat{\phi}_j^{27} & \hat{\phi}_j^{28} & \hat{\phi}_j^{29} \\ -\beta_2 & 0 & 0 & 0 & \hat{\phi}_j^{24} & \hat{\phi}_j^{25} & \hat{\phi}_j^{26} \end{bmatrix}, \]

with \( \hat{\phi}_j^{11} = \beta_1 \hat{\phi}_j^{11} + \beta_2 \hat{\phi}_j^{12} + \beta_3 \hat{\phi}_j^{13} + \beta_4 \hat{\phi}_j^{14} + \beta_5 \hat{\phi}_j^{15} + \beta_6 \hat{\phi}_j^{16} + \beta_7 \hat{\phi}_j^{17} + \beta_8 \hat{\phi}_j^{18} + \beta_9 \hat{\phi}_j^{19} + \beta_{10} \hat{\phi}_j^{110} \). Moreover, the desired control gains are given by \( \hat{K}_j = \overline{K}_j Q_j^{-1}, T_j = T_j Q_j^{-1} \) and the remaining parameters take the form as described in Theorem 1.

**Proof.** Now, according to Theorem 2 we have \( \Theta(\overline{P}, 1) \subset Y(K_j) \) and \( \Theta(\overline{P}, 1) \) is contractively invariant, we obtain \( \overline{K}_j^{(2)} = 0 \).

For a detailed proof, one can refer [31]. Moreover, \( \Theta(\overline{P}, 1) \subset Y(K_j) \) is equivalent to \( k_j^{(1)} (P_j^{(1)})^{-1} k_j^{(1)} T_j \leq 1 \), for \( s = 1, 2, \ldots, m \), where \( k_j^{(1)} \) is the \( s \)-th row of \( k_j^{(1)} \). With the aid of Schur complement, this can be equivalently written as

\[ \begin{bmatrix} -1 & k_j^{(1)} \\ k_j^{(1)} T_j & -P_j^{(1)} \end{bmatrix} \leq 0, \quad j = 1, 2, \ldots, m. \]

For \( (\tilde{\Omega}^T P_j)^{-1} = Q = \begin{bmatrix} Q_j^{(1)} & Q_j^{(3)} \\ P_j^{(1)} & P_j^{(3)} \end{bmatrix} \), one can deduce that \( \tilde{\Omega}^T Q^{(1)} = Q^{(1)} \). Also, let us define the following

\[ R_j = \begin{bmatrix} k_j^{(1)} & 0 \\ 0 & Q_j^{(1)} \end{bmatrix}, \quad R_j = \begin{bmatrix} k_j^{(1)} & 0 \\ 0 & Q_j^{(1)} \end{bmatrix}, \quad R_j = \begin{bmatrix} k_j^{(1)} & 0 \\ 0 & Q_j^{(1)} \end{bmatrix}. \]

**References:**

1. [SUSANA RAMYA AND LEELAMANI](#)
2. [Theorem 3.](#)
Then (27) is pre- and post-multiplied by $\text{diag} [I, Q]$ and $\text{diag} [I, Q]$, respectively. After doing some simple calculations, we can notice that inequality (24) holds. Now, by adhering to the approach as in Theorem 1, by substituting $A_F$, $B_F$ and $C_F$ in the place of $A$, $B$ and $C$, respectively, and by defining $T = T_Q$, we can obtain (25) and (26). Hence if (25) and (26) hold, system (23) is asymptotically stable. Moreover, from Theorem 2, the system (23) is regular and impulse-free, this proves that the system (23) is admissible and that the trajectories of $X(k)$ always lie within $\Theta(\Xi, P, 1)$.

3.2 Finite-time stability of singular stochastic input saturated biological economic fishery system

For this subsection of our research paper, let us retrospect some essential definitions and results.

Definition 2. [32] The closed-loop continuous-time singular system (11) is stochastic finite time bounded with respect to $(f_1, f_2, \tau, \ddot{R})$ with $f_1 < f_2$ and $\ddot{R} > 0$, if the stochastic system (11) is regular and impulse free in time interval $[0, \tau]$ and satisfies the following constraints

$$E[X^T(0)\dot{\Xi} \ddot{R} X(0)] \leq f_1^2 \Rightarrow E[X^T(k)\dot{\Xi} \ddot{R} X(k)] \leq f_2^2,$$

for all $k \in [0, \tau]$.

Lemma 4. [32] The following statements are true.

(i) Let rank($\Xi$) = 8, there exist two orthogonal matrices $X$ and $Y$ such that $\Xi$ can be decomposed of the form

$$\Xi = X(\Delta_8 \oplus 0)Y^T = X(I_8 \oplus 0)Y^T,$$

where $\Delta_8 = \text{diag}[\delta_1, \delta_2, \ldots, \delta_8]$ with $\delta_l > 0, \forall l = 1, 2, \ldots, 8$. Let us partition $X = (X_1, X_2), Y = (Y_1, Y_2)$ and $\Xi = (\Xi_1, \Xi_2)$ with $\Xi_{12} = 0$ and $X^T \Xi_1 = 0$.

(ii) If $P$ is such that $\Xi P^T = P \Xi^T \geq 0$, then $P = X^T P Y$ if and only if $P = \left(\begin{array}{cc} P^{(1)} & 0 \\ 0 & P^{(2)} \end{array}\right)$ with $X$ and $Y$ satisfying (29) and $P^{(1)} \geq 0$.

Further, if $P$ is nonsingular, then $P^{(1)} > 0$ and det$(P^{(3)}) \neq 0$. In addition, $P$ can be parameterized as $P = \Xi U^T U \Xi^T + X_1 X_2^T$, where $U = \text{diag}(P^{(1)}), \varphi), V = \left(\begin{array}{cc} P^{(1)} & 0 \\ 0 & P^{(2)} \end{array}\right)$ and $\varphi$ is an arbitrary parameter real matrix.

(iii) If $P$ is a nonsingular matrix, $\Xi$ and $\varphi$ are two positive definite symmetric matrices, $U$ is a diagonal matrix and the following equality holds;

$$P^{-1} \Xi = \Xi^T R^{1/2} Q R^{1/2} \Xi,$$

then the positive definite symmetric matrix which is a solution of (30) is of the form $Q = R^{1/2} X^T \Xi^T \Xi^T R^{1/2}$.

Also if $P \in \mathcal{R}$ is a symmetric matrix such that $P^{-1} \Xi \geq 0$, then for any positive scalar $\nu$ we can define a set $\zeta(P^{-1} \Xi, \nu)$ as $\zeta(P^{-1} \Xi, \nu) = \{X(k) \in \mathcal{R} : X^T(k) P^{-1} \Xi X(k) \leq \nu\}$. Now, according to Lemma 3, the overall singular stochastic input saturated biological economic fishery model can be described as

$$\dot{\Xi} = \sum_{j=1}^{2^\nu} \gamma(j) (A_j + B_j) X(k) + C, \varphi(u(k))dk + D_j X(k)d\omega(k),$$

where $A_j = \sum_{i=1}^{2} \sum_{i=1}^{2} b_i(\varphi(k)) b_j(\varphi(k)) A_i + B_i S F R_j + B_i S F T_j, B_j = \sum_{i=1}^{2} b_i(\varphi(k)) b_j(\varphi(k)) B_i R_i S_j F_j(k) M_j$, and $D_j = \sum_{i=1}^{2} \sum_{i=1}^{2} b_i(\varphi(k)) b_j(\varphi(k)) D_i$. In the further part of this subsection, we aim to study the stability of singular stochastic input saturated biological economic fishery system (31). In order to ease the analysis, that is for solving the singular stochastic biological economic fishery system (31) directly, it is sufficient to prove (31) is stochastic finite time bounded. More precisely, by using Lyapunov-Krasovskii functional method, the finite time boundedness of (31) will be proved in the following theorem.

3.2.1 Stability analysis of singular stochastic input saturated biological economic fishery system

Theorem 4. For a time constant $\tau > 0$ and for given scalars $\alpha \in [0, 1], f_1 > 0, f_2 > 0$ the closed-loop singular stochastic input saturated biological economic fishery system (31) is stochastic finite-time bounded with respect to $(f_1, f_2, \tau, \ddot{R})$ with initial conditions belonging to $\zeta(P^{-1} \Xi, \nu)$, if there exist real positive scalars $\beta_1, \beta_2$ and $\beta_3$, a nonsingular matrix $\beta$ and a symmetric positive definite matrix $Q_i$ satisfying the following LMI:

$$P \Xi^T = \Xi P^T \geq 0,$$

$$\left(\begin{array}{cc} \Lambda_{ij} & D_j R_{i} \Xi^T \\ \Xi D_j R_{i} & -Q_j \end{array}\right) < 0, i = j,$$

$$\left(\begin{array}{cc} \Lambda_{ij} & \Xi D_j R_{i} \Xi^T \\ \Xi D_j R_{i} & -Q_j \end{array}\right) < 0, i < j.$$
that the system (31) is regular and impulse free in time interval
implies that the singular stochastic biological economic singular
system (31) is regular and impulse free within the time interval
by Lemma 4, we can obtain
\[
\begin{bmatrix}
\Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} & \Lambda_{16} & \Lambda_{17} \\
* & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Lambda_{33} & \Lambda_{34} & 0 & 0 & 0 \\
* & * & * & -\beta_2 & 0 & 0 & 0 \\
* & * & * & * & -\beta_2 & 0 & 0 \\
* & * & * & * & * & \Lambda_{47} \\
* & * & * & * & * & -\beta_3 \\
\end{bmatrix}
\]

Therefore it follows that
\[
\begin{aligned}
\overline{V}(X(k), k) & = \alpha V(X(k), k) = \\
& X^T(k)[2P^{-1}\{\overline{A}_i + \overline{B}_i\} + \overline{D}_i P^{-1}\overline{D}_i]X(k) \\
& + 2X^T(k)P^{-1}\overline{C}, \phi(n(k)).
\end{aligned}
\]

Now by combining the matrix \(\overline{Q}_2 = \overline{R}^{-1/2} \overline{Q}_1 \overline{R}^{-1/2}\) with (35), we further obtain
\[
\begin{aligned}
\overline{V}(X(k), k) & = \alpha V(X(k), k) = \\
& X^T(k)\left[2P^{-1}\{\overline{A}_i + \overline{B}_i\} + \overline{D}_i P^{-1}\overline{D}_i]X(k) \\
& + 2X^T(k)P^{-1}\overline{C}, \phi(n(k))\right].
\end{aligned}
\]

Further, by following the same derivations as in Theorem 1 and applying Schur complement, the above equation becomes
\[
\begin{aligned}
\overline{V}(X(k), k) & = \alpha V(X(k), k) = \\
& X^T(k)\left[2P^{-1}\{\overline{A}_i + \overline{B}_i\} - \alpha P^{-1}\overline{Z} - \alpha P^{-1}\overline{Z} \right]X(k) \\
& + 2X^T(k)P^{-1}\overline{C}, \phi(n(k)).
\end{aligned}
\]

Through the matrix decomposition method and using Lemmas 1 and 2, the above equation takes the form
\[
\begin{aligned}
\overline{V}(X(k), k) & = \alpha V(X(k), k) = \\
& \sum_{i=1}^{2} \sum_{j=1}^{2} b_i(\phi(k))b_j(\phi(k))X^T(k)\left[\Lambda_{ij} \overline{D}_j \overline{Z} \overline{Q}_2 X(k) \right] \\
& \text{which can be equivalently written as}
\end{aligned}
\]
\[
\begin{aligned}
\overline{V}(X(k), k) & = \alpha V(X(k), k) = \\
& \sum_{i=1}^{2} \sum_{j=1}^{2} b_i(\phi(k))b_j(\phi(k))X^T(k)\left[\Lambda_{ij} \overline{D}_j \overline{Z} \overline{Q}_2 X(k) \right] \\
& + X(k) + \sum_{i=1}^{2} b_i^2(\phi(k))X^T(k)\left[\Lambda_{ii} \overline{D}_i \overline{Z} \overline{Q}_2 X(k) \right].
\end{aligned}
\]
where $\Lambda_{ij}$ is defined in the theorem statement. Therefore if (33) and (34) holds, then

$$
\overline{V}(x(k), k) - \alpha \overline{V}(x(k), k) < 0, \forall x(k) \in \zeta(P^{-1} \Xi, \nu), \tag{39}
$$

By integrating both sides of (39) from 0 to k with $k \in [0, \tau]$ and then taking expectation, yields

$$
\mathbb{E}[\overline{V}(x(k), k)] < \overline{V}(x(0), 0) + \alpha \int_0^k \mathbb{E}[\overline{V}(x(\theta), \theta)] d\theta.
$$

Using Gronwall's inequality, we can obtain that

$$
\mathbb{E}[\overline{V}(x(k), k)] < \overline{V}(x(0), 0)e^{\alpha \tau}. \tag{40}
$$

Now by combining (35) and (38) and by using the definition of $\lambda_1$ and $\lambda_2$ given in the theorem statement, we can obtain

$$
\mathbb{E}[\overline{V}(x(k), k)] = \mathbb{E}[V^T(x(k))R^1/2 \overline{Q}_i R^{1/2} \overline{X}(x(k))] \geq \lambda_{\text{min}}(\overline{Q}_i) \mathbb{E}[V^T(x(k))R \overline{X}(x(k))] = \lambda_1 \mathbb{E}[V^T(x(k))R \overline{X}(x(k))]. \tag{41}
$$

$$
V(x(0), 0)^{\alpha \tau} = X^T(0)P^{-1} \Xi X(0)^{\alpha \tau} \geq \lambda_1 \mathbb{E}[V^T(x(k))R \overline{X}(x(k))] \leq X^T(0)P^{-1} \Xi X(0)^{\alpha \tau} \leq \lambda_2 f_1^{2 \alpha \tau}. \tag{42}
$$

By Definition 2, if $X^T(0)P^{-1} \Xi X(0) \leq f_2^2$, then (42) can be condensed as

$$
\overline{V}(x(0), 0)^{\alpha \tau} \leq \lambda_2 f_2^{2 \alpha \tau}. \tag{43}
$$

Now by combining (40), (41) and (43), we can derive that

$$
\lambda_1 \mathbb{E}[V^T(x(k))R \overline{X}(x(k))] \leq \mathbb{E}[\overline{V}(x(k), k)] \leq \overline{V}(x(0), 0)^{\alpha \tau} \leq \lambda_2 f_2^{2 \alpha \tau}.
$$

Hence, $\mathbb{E}[V^T(x(k))R \overline{X}(x(k))] \leq \frac{\lambda_2 f_2^{2 \alpha \tau}}{\lambda_1}$. Therefore, if (36) holds, then $\mathbb{E}[V^T(x(k))R \overline{X}(x(k))] \leq f_2^2, \forall t \in [0, \tau]$. This proves that the system (31) is bounded within a finite interval of time according to Definition 2. Let us denote the $k^{th}$ row of $K_j$ as $b_{kj}$ for $k = [1, 2, ..., m]$. If condition (37) holds, then $|b_{kj}x(k)| \leq 1, \forall x(k) \in \zeta(P^{-1} \Xi, \nu)$ according to [34]. Then clearly $\zeta(P^{-1} \Xi, \nu)$ is a contractively invariant set. Therefore, the system (31) is finite-time bounded inside the contractively invariant set $\zeta(P^{-1} \Xi, \nu)$.

### 3.2.2 Controller design of singular stochastic input saturated biological economic fishery system

**Theorem 5.** For a prescribed time constant $\tau > 0$ and for given scalars $0 \leq \alpha < 1, f_1 > 0, f_2 > 0$ the close-loop singular stochastic input saturated biological economic fishery system (31) is singular stochastic finite-time bounded with respect to $(f_1, f_2, \tau, \overline{R})$, if there exist a nonsingular matrix $P$, a symmetric positive definite matrix $\overline{Q}_i$ and any appropriate dimensioned matrices $\overline{K}_j$ and $\overline{T}_j$ such that (32), (35), (36) and the following conditions are satisfied:

$$
(\hat{\Lambda}_i + \hat{\Lambda}_j + \hat{\Lambda}_k) \overline{D}^T \overline{P} \overline{Q}_i \overline{D} \overline{P}^T < 0, i, j \in \{1, 2, 3\} \tag{44}
$$

and $t_{kj} = b_{kj}(\frac{1}{\nu} \Xi)^{-1} - \frac{1}{\nu} \Xi$ with $\hat{\Lambda}_1 = \hat{\Lambda}_1 P^T + \hat{\Lambda}_2 S_{j} \Sigma_{j} R_j - \alpha \Xi P^T + \overline{B}_j \overline{D} \overline{T}_j \overline{P} T^T, \hat{\Lambda}_2 = \hat{\Lambda}_2 P^T, \hat{\Lambda}_4 = \hat{\Lambda}_4 P^T, \hat{\Lambda}_5 = \hat{\Lambda}_5 P^T,$

$$
\hat{\Lambda}_1 = \hat{\Lambda}_1 P^T + \hat{\Lambda}_2 S_{j} \Sigma_{j} R_j - \alpha \Xi P^T + \overline{B}_j \overline{D} \overline{T}_j \overline{P} T^T, \hat{\Lambda}_2 = \hat{\Lambda}_2 P^T, \hat{\Lambda}_4 = \hat{\Lambda}_4 P^T, \hat{\Lambda}_5 = \hat{\Lambda}_5 P^T,$
$$

and the other parameters used in this theorem statement take the same description as in Theorem 4. Furthermore, if the aforesaid LMIs hold, the controller gain matrices $\overline{K}_j$ and $\overline{T}_j$ can be computed by using the relation $\overline{K}_j = \overline{R}_j \overline{P} \overline{T}$ and $\overline{T}_j = \overline{T} \overline{P} \overline{T}$, respectively.

**Proof.** The proof of this theorem can readily obtained from Theorem 1 when the left hand side of equations (33) and (34) are pre- and post-multiplied by diag($P, L_{\infty}$) and diag($P^T, L_{\infty}$), respectively. Thereby, by applying the change of variables $R_j \overline{P} \overline{T} = \overline{K}_j$ and $T_j \overline{P} \overline{T} = \overline{T}_j$, we obtain the matrix inequality (44) and (45).

From [34], inequality (37) is equivalent to

$$
\nu b_{kj} \overline{P}^{-1} \Xi b_{kj}^T \leq 1 \Rightarrow \left( \begin{array}{cc} 1 & b_{kj}(\frac{1}{\nu} \Xi)^{-1} \\ (\frac{1}{\nu} \Xi)^{-T} b_{kj}^T & (\frac{1}{\nu} \Xi)^{-1} \end{array} \right) \geq 0, \tag{47}
$$
where \( k = [1, 2, \ldots, m] \). Denote \( T_j \left( \frac{p^{-1} \Xi}{\nu} \right)^{-1} = \bar{T}_j \). If the \( k \)-th row of \( T_j \) is denoted as \( t_k \), then \( t_k = h_k \left( \frac{p^{-1} \Xi}{\nu} \right)^{-1} \) as defined in the theorem statement. Hence by using the parameters \( t_k \) in (47), we obtain that (37) is equivalent to (46). The proof is completed. \( \square \)

**Remark 2.** The developed LMIs in Theorem 5 offers set invariance conditions for a wider choice of invariant ellipsoids for optimization. In this subsection, we choose the largest ellipsoid which satisfies the LMIs of Theorem 5. Furthermore, the obtained largest ellipsoid will be used to get an estimate of the domain of attraction with least conservatism.

To prove the following Corollary, we define the following; Let \( \bar{S}_D \) be an arbitrary bounded convex set and let \( S \) be a set such that \( \eta_D(\mathcal{S}) = \sup \eta > 0 : \eta \bar{S}_D \subset \mathcal{A} \). If \( \eta_D(\mathcal{S}) \geq 1 \), then \( \bar{S}_D \subset S \) and the ellipsoids \( \bar{S}_D = \bar{\zeta}(R, 1) = \{ X(k) \in \mathcal{R} : X^T(k)RX(k) \leq 1 \} \).

**Corollary 1.** A maximal estimate of the domain of attraction can be got from the optimization algorithm:

\[
\inf_{\mathcal{D} > 0, T_j} \gamma \\
\text{subject to } \left( -\frac{\gamma R}{I} \right) \leq 0, k = \{1, 2, \ldots, m\} (48) \\
\text{and LMIs } (32), (35), (36), (44), (45), (46),
\]

where \( \bar{Q} = \frac{1}{\eta^2} \bar{\Xi} \) and \( \gamma = \frac{1}{\eta^2} \).

**Proof.** From Theorem 5, we obtain a requisite for an ellipsoid to be inside the domain of attraction. Now we consider all ellipsoids \( \bar{\zeta}(\bar{p}^{-1} \Xi, \nu) \) which satisfy the LMIs of Theorem 5 such that \( \eta_D(\bar{\zeta}(\bar{p}^{-1} \Xi, \nu)) \) is maximized. Following the same derivations as in [34], the domain of attraction can be obtained from the conditions

\[
\sup_{\bar{\Xi} > 0, \nu, T_j} \eta \\
\text{subject to } \eta \bar{S}_D \subset \bar{\zeta}(\bar{p}^{-1} \Xi, \nu) (49) \\
\text{and LMIs } (32), (35), (36), (44), (45), (46).
\]

By the definition of \( \bar{S}_D \), (49) can be written as

\[
\frac{R}{\eta^2} \leq \frac{\bar{p}^{-1} \Xi}{\nu}.
\]

Further with the aid of Schur complement, the above can be equivalently written as

\[
\left( \begin{array}{cc} -\frac{R}{\eta^2} & I \\ I & \frac{\bar{p}^{-1} \Xi}{\nu} \end{array} \right) \leq 0.
\]

Now by using the terms \( \gamma \) and \( \bar{Q} \) defined in the Corollary statement, we can obtain (48). The proof is completed. \( \square \)

**4 NUMERICAL EXAMPLE**

In this section, to exhibit clearly the validity and practicality of the obtained results accomplished in this paper, we consider the ecological parameters as given in Moroccan western coastline area biological sardine data [16]. The ecological parameters of system (1) is taken as \( r = 0.015, c = 1702.8, H = 0.008, m = 800, s = 0.005 \) and \( b = 200 \) in appropriate units. Using these ecological parameters, system (3) takes the form

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\dot{X}(k) = \sum_{i=1}^{2} b_i(\rho(k))[\bar{\mathcal{S}}]X(k) + \begin{bmatrix} 1 & 0 \end{bmatrix} u_f(k),
\]

where \( \mathcal{A}_1 = \begin{bmatrix} 0.0220 & 0.0000 & 0.0000 \\
-0.00001 & -0.0050 & 0.0000 \\
4.8000 & -13.6224 & -1.0000 \end{bmatrix} \)

and the fuzzy weighting functions \( b_1(\rho(k)) \) and \( b_2(\rho(k)) \) as already defined. Our main purpose is to model a suitable controller, through which we can attain a better performance by incorporating possible issues such as variations in controller gain, input nonlinearities and saturations.

**4.1 Validation of Theorem 1**

To achieve the stability of (10) by means of the nonlinear resilient fault-tolerant controller (9), the parameters are considered as \( \bar{\mathcal{L}}_1 = \begin{bmatrix} 0.15 & 0 & 0 \end{bmatrix}^T, \bar{\mathcal{L}}_2 = \begin{bmatrix} 0.2 & 0 & 0 \end{bmatrix}^T, \bar{\mathcal{M}}_1 = \begin{bmatrix} 0.01 & 0 & 0 \end{bmatrix}, \bar{\mathcal{M}}_2 = \begin{bmatrix} 0.02 & 0 & 0 \end{bmatrix} \) and \( \bar{N}_1(k) = \bar{N}_2(k) = sin(k) \). Moreover, we consider \( \phi(u(k)) = sin(6.1u(k)) \), \( \bar{\mathcal{A}} = 7 \) and the range within which the fault lies as \( 0.13 \leq \bar{F} \leq 0.3F \). The main aim of modelling the controller is to find the gain matrices \( \bar{K}_1 \) and \( \bar{K}_2 \) which could stabilize (10). With the above defined parameters, we can arrive at the feasible solution by solving the inequalities (12) and (13) in Theorem 1. The associated control gain matrices are obtained as

\[
\bar{K}_1 = \begin{bmatrix} -0.0010 & -0.1614 & 0.0005 \end{bmatrix} \text{ and } \bar{K}_2 = \begin{bmatrix} 0.0001 & -0.0794 & 0.0002 \end{bmatrix}.
\]

The evolution of the states of (10) with the above obtained controller gain values \( \bar{K}_j, j = 1, 2 \) are displayed in Figure 1 with \( X(0) = \begin{bmatrix} 100 & 150 \end{bmatrix}^T \). From Figure 1, we monitor that there is a random fluctuation in the economic profit. This
fluctuation is due to the fact that the economic profit is always contrived by the population density and unit price of the fishes in the proposed biological economic model (1).

4.2 Validation of Theorem 3

Now, in this subsection, we achieve the stability of (23) with the proposed controller (18) which is affected by saturations. On this behalf, with the same biological economic parameters and the other values as in the above subsection, we solve the inequalities in Theorem 3. The associated control gain matrices are computed as

\[
\begin{align*}
\overline{K}_1 &= \begin{bmatrix} -0.1225 & -1.3292 & 0.0280 \end{bmatrix}, \\
\overline{K}_2 &= \begin{bmatrix} -0.0913 & -0.9315 & 0.0216 \end{bmatrix}, \\
\overline{T}_1 &= \begin{bmatrix} -0.0222 & -6.9061 & 0.0220 \end{bmatrix}, \\
\overline{T}_2 &= \begin{bmatrix} 0.0264 & -4.1461 & 0.0097 \end{bmatrix}.
\end{align*}
\]

The corresponding state trajectories of the system (23) are shown in Figure 2. Figure 2 infers that the system trajectories are convergent which means that the biological economic system (23) can be stabilized. This reveals the fact that the designed controller can adequately tolerate nonlinear faults and saturation in its design.

The trajectory of the nonlinear fault-tolerant fuzzy controller which is also resilient to control gain fluctuations and which asymptotically stabilizes system (23) is shown in Figure 3.

Figure 4 shows the pictorial representation of the contractively invariant ellipsoids under two cases, that is, when the actuator fault is normal and when the actuator fault is defective. The uncoloured contractively invariant ellipsoids shows the region where the actuator is normal. The coloured contractively
invariant ellipsoids shows the region where the actuator is defective. It can be observed from Figure 4 that the coloured contractively invariant ellipsoid lies inside the uncoloured contractively invariant ellipsoid. Hence, it can be inferred that, the area of the contractively invariant ellipsoid increases when the actuator is normal. This proves that the actuator failures in the designed controller highly influences the system’s stability.

To highlight the results in this paper, the two figures, Figures 5 and 6 are presented. Figure 5 shows the trajectories of the density of the fishes $\rho(k)$ for various values of $r$. As $r$ is the intrinsic growth rate of the fishes at time $k$, we observe from Figure 5 that when the value of $r$ is increased, $\rho(k)$ is decreasing. In particular, when $r = 0.015, \rho(k) = 6324$; $r = 0.018, \rho(k) = 6180; r = 0.02, \rho(k) = 6085$ and when $r = 0.022, \rho(k) = 5980$. Specifically, from Figure 5, we come to an understanding that, even though the intrinsic growth rate of the fishes are increased to a certain level, the density of the fishes in the waters is getting decreased which is due to the fact of biological overfishing. In other words, fish are caught from the sea so rapidly that the replenishment of fish stock by multiplication decreases. If the replenishment keeps decreasing for long enough, replenishment will have an adverse effect and the population of fishes will go down.

Bio economic overfishing over and above that considers the cost of fishing thereby defining overfishing as a condition of negative marginal growth of resource rent. Figure 6 shows that economic profit $e(k)$ is negative which is because of the fishes are being caught from the sea so quickly that the growth in the profitability of fishing decreases. If this keeps prolonging for a long time, profitability will decreases. Therefore, suitable measures should be taken to reduce bio economic overfishing. Further, from Figure 6 we can observe that as the price speed adjustment and the market competition $s$ takes various constant values then the the economic profit $e(k)$ is improved.

### 4.3 Validation of Theorem 5

To procure the finite-time stability of biological economic fishery system (4) with white noise $\Delta = 0.01$ by means of the nonlinear resilient fault-tolerant controller (9), the following parameters are further considered $\bar{D}_1 = \bar{D}_2 = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\alpha = 0.001, f_1 = 10, c_2 = 1.6 \times 10^9, \tau = 100$ and $\bar{R} = I$. With the same set of ecological parameters as given in the previous subsections, we solve the LMIs in Theorem 5 and arrive at a feasible solution. The corresponding control gain matrices are computed as

$$\begin{align*}
\bar{K}_1 &= \begin{bmatrix} -0.0760 & -0.8303 & 0.0174 \\ -0.0566 & -0.5819 & 0.0134 \\ -0.0150 & -6.8840 & 0.0203 \end{bmatrix}, \\
\bar{K}_2 &= \begin{bmatrix} 0.0326 & -4.1337 & 0.0082 \end{bmatrix}.
\end{align*}$$
The corresponding state trajectories of the input saturated stochastic system (31) is depicted in Figure 7. Therefore, we can clearly infer from Figure 7 that the system trajectories converge and that the singular stochastic input saturated biological economic system (31) can be stabilized. This reveals the fact that the controller which is designed in this paper can adequately tolerate saturation effect. Further, from Figure 7, it can be clearly observed that the economic profit gravitates towards stable over the finite interval of time $[0, 100]$. Also, from Figure 7, we can observe that the population density of fishes $\rho(k)$ and the unit price $c(k)$ fluctuates randomly due to the presence of stochastic perturbations, thereby affecting the economic profit $e(k)$.

The trajectory of the nonlinear fault-tolerant saturated fuzzy controller which is also resilient to control gain fluctuations and which stabilizes system (31) is shown in Figure 8.

The invariant ellipsoid and the state response of (31) is shown in Figure 9. We notice from Figure 9 that the evolution of trajectories originating from the invariant ellipsoid converges to the origin and stays within the ellipsoid itself.

To highlight the results in this paper, the simulation results of Figure 10 are presented. The economic profit $e(k)$ at time $t = 100$ s for various values of $r$ are obtained and plotted as bar graph. We can infer from Figure 10(a) that the economic profit keeps increasing as the intrinsic growth rate increases. Similarly
from Figure 10(b), we can conclude that the economic profit can be increased by increasing the elasticity in price \( \varepsilon \).

5 | CONCLUSION

Here, in this research paper, we have obtained the stability conditions of biological economic singular fishery model with variable economic profits. A more comprehensive form of nonlinear fuzzy fault-tolerant controller is designed which is subject to gain variations and saturation effects. By employing the Lyapunov approach, we have derived a set of sufficient conditions for the admissibility of the considered biological economic fishery model in terms of LMIs. Also, set invariance conditions for singular stochastic biological economic fishery model has been derived. In short, a resilient fuzzy controller which is also fault-tolerant has been aptly designed, which can accommodate variations in the control gain and saturation. Further the results are broadened to obtain conditions for finite-time stabilization for singular stochastic biological economic fishery model. The invariant ellipsoid for both the considered deterministic and stochastic singular biological economic fishery model has been displayed. Some graphical results have been provided to show the applicability of developed control design.

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