Structure of fracture of ice cover, finger rafting

N M Osipenko
Ishlinsky Institute for Problems in Mechanics, Moscow, Russia
E-mail: osipnm@mail.ru

Abstract. The mechanism of fracture in the vicinity of the region of mutual cutting of ice cover plates is considered on the basis of the analysis of sea ice cover compression structures development. A model of the cutting process based on the representation of this region as the longitudinal shear is proposed. The obtained structures of fracture at shear in the model materials containing capillary pores were analyzed. In particular, it was shown that during the formation of the cracks the distance between cracks along the shear front in the primary echelon structure of brittle fracture is linearly related to the incision tip radius. An estimate is obtained for the effective fracture toughness of ice at longitudinal shear.

1. Introduction
The mutual cutting of the contacting ice floes in the sea ice cover takes place at the front of the finger rafting. The mechanism of fracture is associated with a tendency to develop a shift along the grain axis and a system of capillary pores [1]. The presence of pores creates anisotropy of the shear strength [2, 3]. The least shear resistance is observed when the shear is oriented across the plane of the ice cover. At finger rafting main part of the plates on each side of the slot lie on each other. The concentration of longitudinal shear stresses is created on the interaction front in the mutual cutting region. The present study is devoted to simulation of the internal problem of longitudinal shear initiation in a capillary porous medium on model materials. As a result, a model is proposed for the process of finger rafting of ice, based on the concept of the shear crack front propagation.

2. Model experiments
To simulate this form of failure, a number of experiments were performed on samples with a system of holes simulating capillaries loaded with shear loads. Figure 1a shows the scheme of experiments. When loading by shear in the vicinity of the notch, whose apex has the shape of a capillary, a system of feathering cracks develops (figure 1b). It is analogous to fracture systems in the vicinity of the longitudinal shear crack [4]. On the surface of failure from two concentrators under shear, the structure of opposing fracture systems are observed (figure 1c). The following stages of the process can be identified (figure 2):

- initiation of the cracks on the contours of the nearest holes on the extension of the incision line (figure 2a) (stage I);
- fusion of feathering cracks between the holes on the extension of the incision (figure 2b) (stage II);
- the formation of a system of particles between the surfaces of the growing main shear (figure 2c) (stage III).
The particle system forms a moving layer (third body) between the contacting surfaces under shear. Figure 3 shows the test scheme and the shape of the sample when modeling the echelon of cracks. The location of the crack system (the primary echelon of cracks) formed in the shear crack tip region is given in figure 3, on the right.

The cracks formed on the continuation of the shear have the character of fractures of rupture. In particular, in the absence of normal stresses on the shear plane (stress intensity factor $K_I = 0$), the angle between the planes of cracks and the plane of shear is $\alpha \approx \pi / 4$. The interrelation of the distance between the cracks in the primary echelon ($h$) and the transverse dimension (radius) of the end region of the incision ($R$) for the materials used in experiments is shown in figure 4. We obtain the ratio: $h \approx 4R$. With a brittle mechanism of fracture, the structure in the fracture region does not depend on the deformation properties of the material.

![Figure 1. Experiment on the fracture of model samples under shear.](image1)

![Figure 2. Development structure of fracture top of longitudinal shear.](image2)

![Figure 3. Initiation of fracture on the contour of the hole at the top of the shear.](image3)

3. **Model representation of the end zone of the longitudinal shear**

The initiation of the fracture occurs on the continuation of the initial incision on the internal surface of the hole which is the end region of the shear, where the stress concentration is created (stage I). Then the crack propagates in an unstable regime in a plane inclined to the shear axis at an angle of $\sim \pi / 4$ (stage II). Further crack growth together with adjacent cracks formation occurs in a stable regime (stage III). It should be noted that the main events in the initiation of the crack system are tied to certain planes of the vicinities of the end region of the shear. These are the plane of the crack, the plane of the macro shear and the plane touching the end region, transverse to it.
Figure 4. Interrelation of the distance between cracks in the primary echelon \( (h) \) and the radius of the end region of the incision \( (R) \) (* - cheese, o - gypsum).

We give some estimates. Consider a set of parallel cracks in the vicinity of the longitudinal shear that is cut with a finite radius \( R \). We use the solution for the stress intensity factor in a system of closely spaced parallel rectilinear cracks, in a plane loaded uniformly by a uniform load at infinity [4]. For a plane stress state

\[
K_I \approx \sigma_y \sqrt{t/2} ,
\]

where \( t \) - is the distance between the cracks, and \( \sigma_y \) - the stresses along the normal to the crack line.

The relation (1) coincides with the estimate from the numerical solution [5] for a plane with an infinite periodic system of parallel cracks of the same length, loaded with a uniform stress along the normal to the lines of cracks at \( t \leq \ell \)

\[
K_I \approx 0.71 \sigma_y \sqrt{t} .
\]

We also mention the coincidence with the solution given in [6] for a system of parallel cuts loaded with homogeneous internal pressure.

The stress \( \sigma_y = \sigma_1 \) in the vicinity of the end region of the shear is estimated from the asymptotic behavior of the stresses in the external problem of fracture mechanics for the end region crack of the longitudinal shear [6]. The principal stresses on the selected plane of the section at the end region of the shear are

\[
\sigma_{1,2} = \pm \frac{K_{\text{III}}}{\sqrt{2\pi r}} \cos \theta \frac{\theta}{2} \sqrt{\cos \theta} ,
\]

where \( K_{\text{III}} \) is the stress intensity factor of longitudinal shear stresses for a cut as a crack of longitudinal shear. \( \theta \) is the angle between the stress vector and the axis of the crack.
Assuming that the level of $K_{III}$ and the level of stresses provided by it at the unstable initial phase of cracks formation change insignificantly at this phase, and also that the process is concentrated near the contour of the notch on its continuation (i.e. $r \rightarrow R, \theta \rightarrow 0$), we obtain from (1)

$$K_{III} \approx \sigma_1 \sqrt{2 \pi R} \rightarrow \sigma_1 \approx K_{III} / \sqrt{2 \pi R}.$$ 

We will assume that in the state of limiting equilibrium on the contour of a single crack of normal rupture in the scale of the external problem of cracks we have $K_1 = K_{IC}$, where $K_{IC}$ is the fracture toughness of the material. From here

$$t \approx \left( \frac{K_{IC}}{\sigma_1} \right)^2 = 4\pi R \left( \frac{K_{IC}}{K_{III}} \right)^2.$$  

According to the accepted fracture scenario, the initial stage of cracks development (as isolated cracks) that takes place in the stress field (2) is unstable. Then the cracks are inhibited, as the stress intensity decreases when the front of the crack leaves the stress concentrator region [4]. The level $K_1$ corresponding to the moment of initiation of the next crack is determined by the relation

$$K_1 \approx \sigma_1 \sqrt{\pi \ell} \approx K_{IC},$$

where $\ell$ is the size of the primary equilibrium crack in the plane under consideration. The size of the primary equilibrium crack $\ell$ upon initiation of the next crack on the contour of the notch is controlled by the size (radius) of the tip of the cut, i.e. $\ell \sim fR, (f < 1)$. The condition relating the limiting parameters for the cut and cracks has the form

$$\frac{K_{IC}}{\sqrt{\pi fR}} \approx \frac{K_{III}}{\sqrt{2 \pi R}}; K_{IC} \approx K_{III} \sqrt{\frac{f}{2}}.$$ 

From (3) and (4), we obtain a relation showing the linear relationship between the radius of the cut tip and the distance between the cracks in the primary echelon of cracks:

$$t \approx 2\pi fR.$$ 

The distance between the centers of the cracks $h$, taking into account the orientation of the cracks relative to the axis of the echelon in this approximation is determined by the relation

$$h = t / \cos \alpha \approx 2\pi fR / \cos \alpha,$$

where $\alpha$ is the angle between the shear axis and the plane of the crack. The result is correlated with the experimental data for $f = 0.45$, $\alpha = \pi / 4$.

The location of the foci of next crack initiation is determined by the maximum of the tensile stresses in the vicinity of the primary crack. The stress field in the vicinity of the initial crack is composed of a field created by the longitudinal shear and a local perturbation, the source of which is the initial crack. We give some asymptotic estimates in the framework of the plane problem, for a single initial crack. Let us write down the principal stresses at the site perpendicular to the front of the longitudinal shear along its continuation in the coordinates associated with the initial crack

$$\sigma_y = \frac{K_{III}}{2\sqrt{\pi r}} \sqrt{\left(1 + N^{-1/2}\right)N^{-1/2}}, \quad N = \left(\frac{x}{r}\right)^2 \sin^2 \alpha + 1, \quad (5)$$

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where $K_{III}$ is the stress intensity factor at the longitudinal shear front, $r$ - is the distance from the axis of the shear front to the site, and $\alpha$ - the orientation of the normal to the trajectory of the principal stresses (here, the orientation of the plane of the crack).

We assume that $r \sim R$, where $R$ is the characteristic radius of curvature of the top of the shear. We also assume that, in the asymptotic approximation, the perturbation of the stress state in the vicinity of the macro-shear, associated with the presence of the initial crack, is provided by a singularity of the stress field in the region of the crack tip. In the end zones of the equilibrium initial crack of a normal discontinuity, the Irvine condition ($K_1=K_{IC}$) is fulfilled. For initiation of the next crack, the most interesting is the situation in which initial crack having a size close to the size of radius of curvature of the top of the shear.

Stress $\sigma_y$, coinciding with the direction $\sigma_1$ of the external stress field at the site without cracks at the coordinates associated with the fracture plane is [6]

$$\sigma_y = \frac{K_1}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right].$$

(6)

Passing further to dimensionless coordinates $x = x/\ell; y = y/\ell$, we obtain from (5) and (6) the asymptotic formulas for estimating the normal stress $\sigma_y$ at an arbitrary point of the site in the variant of $K_1=K_{IC}$ and $\alpha = \pi/4$.

$$\sigma_y = \frac{K_1}{2\sqrt{\pi} \ell} \left(\frac{K_{III}}{K_{IC}}\sqrt{1+N^{*-1/2}}\right)^{N^{*-1/2}} + \frac{\sqrt{y^2 + (1-x)^2} - 1 + x \left(1.5 - 3x + 1.5x^2 + 0.5y^2\right)}{\sqrt{y^2 + (1-x)^2} \left(y^2 + (1-x)^2\right)}$$

$$N^* \approx 1 + 0.5(x - y)^2$$

Figure 5 shows the isolines of the parameter characterizing the stresses $\beta = \sigma_y \sqrt{2\pi \ell / K_{IC}}$ in the vicinity of the crack for different values of the ratio $k = K_{III}/K_{IC}$. The figure gives an asymptotic estimate of the location of a maximum of tensile stresses in a distant neighborhood of the initial crack. It can be seen that if the shear is predominant ($k = 10$), the stress maximum $\sigma_y$ (denoted by a cross) is observed on the shear axis outside the unloading zone at a distance of about $4R$ from the center of the primary crack along the shear axis. This estimate coincides with the data of the model experiment which coincides with the model experiment data $h \approx 4R$.

This region can serve as a point for initiating the next crack in the developing echelon of cracks. Thus, the characteristic scale of the echelon of cracks in the vicinity of the top of the shear is given. As $k$ decreases, the distance $h$ decreases, and in the region $k \sim 2$ the local maximum of the parameter is practically absorbed by the stress concentration region near the crack tip. With a small fraction of the shear in the field of external stresses, a change in the fracture scenario takes place - instead of a crack echelon, a single zigzag crack is formed at the front of the cut.
Figure 5. The isolines of the parameter \( \beta = \sigma_2 K \) in the neighborhood of the top of the shear.

4. Conclusion, effective crack resistance
As shown earlier, cracks surrounding the capillaries on the continuation of the main shear form an echelon of cracks between the capillaries. The contour of the individual crack at the top of the shear expands at an angle \( \alpha = \pm \pi/2 \) [4, 7]. With the growth and fusion of cracks between the two capillaries, a crack is formed in a gap inscribed in a square with a diagonal close to the distance between the centers of the capillaries (\( L \)) (Figure 6). When the dimensions of the cracks in the echelon reach the boundaries of the square mentioned above, the limit equilibrium state of the system is attained. The subsequent integration of cracks and the formation of moving fragments (elements of the third body, Figure 2) are not accompanied by the increase in the applied load, according to our experimental data. These estimates do not take into account the interaction with the third body in the implementation of the shear.

Figure 6. Scheme of development of a single crack between capillaries at shear along the axis of capillaries.

It is convenient to consider the model situation in a plane section normal to the plane of shear on its continuation. The principal stresses on the plane of the section in a small neighborhood of the top of the shear, as before, are estimated from (2). Suppose, as usual [8], that the fractures of the normal...
rupture formed on the continuation of the shear are oriented along the normal to the maximum principal stress. In the situation of shear dominance under the orientation of feathering cracks at an angle \((\alpha \approx \pi/4)\) to the axis of the echelon

\[
\frac{K_{IC}}{\sqrt{R/2}} \approx \frac{K_{III}}{2\sqrt{(\pi L)}}.
\]

From this we obtain the condition of limiting equilibrium in a system of cracks in a neighborhood of the shear along the capillaries

\[
K_{IIIc} \approx 2K_{IC} \sqrt{\frac{\pi L}{2R}}.
\]

The capillary porosity \((n)\) is related to the geometry of the porous space in the plane by the relation

\[
R/L \approx \sqrt{n}.
\]

From this we obtain an estimate of the level of effective crack resistance at longitudinal shear of the capillary porous medium

\[
K_{IIICc} \approx m\pi^{0.5}K_{IC}n^{-0.25}; m \sim 1.7.
\]

The described scenario of the finger rafting mainly refers to the case high porosity and to the situation of active ice movement, since when it stops the friction surfaces of ice floes may freeze and change the fracture mechanism.

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**References**

[1] Osipenko N M and Chencow A B 2017 *Preprint IPMech RAS* 1151 p 28
[2] Tyshko K P, Cherepanov N V and Fedotov V I 2000 *Crystalline Structure of Sea Ice Cover* (St. Petersburg: Gidrometeoizdat) p 66
[3] Cherepanov N V 1976 *Proc. of AARI* 331 77-9
[4] Goldstein R V and Osipenko N M 2012 *Mechanics of Solids* 47 505-16
[5] Murakami Y 1987 *Stress Intensity Factors Handbook* (New York: Pergamon Press) p 1464
[6] Cherepanov G P 1978 *Mechanics of Brittle Fracture* (New York: McGrawHill) p 950
[7] Goldstein R V and Osipenko N M 2014 *Fatigue Fract. Eng. Mater. Struct.* 37 1292–305
[8] Barenblatt G I 1961 *J. Appl. Mech. Tech. Phys.* 4 3-56