Evidence of non-local impact ionization in CNT and HgCdTe

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Abstract. Two aspects of the non-local nature of impact ionization, dead space and resonance, are investigated. The very small excess noise factor measured for mercury cadmium telluride photodiodes can only be explained if the hole to electron ionization coefficient ratio, \( k \), is very small and the impact ionization dead space is also considered. A maximum value of \( k \) for HgCdTe is estimated in this paper. In addition, recent measurements of the reverse photocurrent in single wall carbon nanotubes show a well defined flat region at a multiplication of 1.6. This is argued to be evidence for resonant behaviour in impact ionization for carbon nanotubes.

1. Introduction

The classic 1966 theory of McIntyre [1] gives the following relationship for the excess noise factor, \( F(M) \), as a function of the multiplication, \( M \), and the ionization coefficient ratio, \( k \).

\[
F(M) = kM + \left(2 - \frac{3}{k}\right)(1 - k)
\]  

(1)

For diodes where the injected carriers are electrons, \( k \) is the ratio of the hole ionization coefficient, \( \beta \), to the electron ionization coefficient, \( \alpha \). McIntyre [1] assumed that the ionization coefficients are a function of the local electric field only. However, the probability of impact ionization depends upon kinetic energy rather than field, so that, although ionization coefficients will tend to an asymptotic value for a given field, there is a region where carriers can not impact ionize because they have insufficient energy. This ‘dead space’ region is a non-local effect and was first investigated by van Vliet et al [2] in 1979, then independently in 1990 by Marsland [3] and Saleh et al [4] and by numerous others since. These papers all demonstrated that the dead space acts to reduce the excess noise factor so that equation (1) is an upper limit which applies in cases where the dead space effect can be neglected. HgCdTe photodiodes provide the clearest experimental evidence [5,6] of the dead space effect. The very small excess noise factor at high multiplications requires the combination of a significant dead space effect and \( k \) to be very small. However an upper limit for \( k \) in HgCdTe has not been established and that is addressed in this paper.

Marsland [7-9] has predicted that there is a resonance effect in addition to the dead space effect. Although resonance effects have been demonstrated by Monte Carlo simulation for HgCdTe [10], there has been no experimental evidence due to the very high electric field required. However recently Gabor et al [11] have reported reverse photocurrent measurements in single wall carbon nanotubes...
which show a distinct flat region at a multiplication of 1.6. It is argued in section 3 that this is the first experimental evidence for the resonance effect.

2. Non-local impact ionization in mercury cadmium telluride

Multiplication of up to 5,300 has been measured in HgCdTe photodiodes with excess noise factors very close to unity [5,6]. McIntyre’s noise equation predicts an excess noise factor \( F(M) = 2 + 5300k \) for \( M = 5300 \). An excess noise factor significantly less than 2 can only be explained if the non-local nature of impact ionization is considered as has been demonstrated by [2-4]. The Monte Carlo study reported by Ma et al [10] showed that a resonant behaviour was observed in the excess noise factor as shown by the solid line in figure 1. Very similar resonant behaviour has been predicted by Marsland [8,9] for \( k = 0 \) and that is shown by a dashed line on figure 1. There is excellent agreement between the two results for \( M \leq 2 \). It should be noted that if each injected electron results in a multiplication of either 1 or 2, and there is no possibility of a multiplication greater than 2, then the excess noise factor is given by equation (2) for a multiplication (average over all carriers) in the interval 1 to 2.

\[
F(M) = \frac{(3M - 2)}{M^2} \quad 1 \leq M \leq 2
\]

\[F(M)\] versus multiplication; solid line Monte Carlo [10], dashed line [8,9] \( \sigma=1 \), \( \lambda=0.00333 \), \( l=0.0415 \)

The results diverge for \( M > 2 \) although they both show the same resonant minima at 4, 8 and 16. Exact agreement is not expected because the results from [8,9] assume a parameter set \((\sigma=1, \lambda=0.00333, l=0.0415)\) appropriate for an extremely high electric field. Also there may be some sampling uncertainty in the Monte Carlo simulation and the curve shown in figure 1 from [8,9] assumes \( k = 0 \). More recent Monte Carlo simulations [12] do not reproduce the resonance effect when more realistic alloy scattering and impact ionization rates are used and this may be why the resonance effect has not been observed experimentally.

2.1. Excess noise factor for \( M = 1000 \) and \( k \neq 0 \).

Following the methodology described in [8] the excess noise factor is calculated for \( M \) up to 1000 and non zero values of \( k \). Whereas \( k \) can be unambiguously defined when \( \alpha \) and \( \beta \) are history independent, in the nonlocal case this is not as straightforward because \( \alpha \) and \( \beta \) are dependent on the distance, \( z \), travelled from a point \((z = 0)\) where the carriers have no kinetic energy. In this paper \( k \) is taken to be the ratio of the asymptotic values of the ionization coefficients given by the following equation.

\[
k = \frac{\beta(z \to \infty)}{\alpha(z \to \infty)}
\]

The ionization coefficients are parameterized using three coefficients; \( l \) the dead space length and \( \sigma \) and \( \lambda \) which determine the slope of the rise and fall of the ionization pathlength p.d.f. as described in [9]. The asymptotic ionization coefficients are given by equation (4).

\[
\alpha, \beta(z \to \infty) = (\lambda(\sigma + \sigma^{-1}) + l)^{-1}
\]
In the results shown in figure 2, two sets of parameters have been used for \( \alpha \); for the dotted line and the squares the parameters are \( \sigma = \lambda = 0.3162 \) and \( l = 0.15 \) representing a high field with an overshoot factor of 0.97 and for the dashed line and circles, \( \sigma = 1, \lambda = 0.00333, l = 0.0415 \), representing an extremely high field with an overshoot factor of 5.36. The parameters for \( \beta \) are selected to achieve either \( k = 0.01 \) or 0.001 with \( l \) constant for both carriers and either \( \sigma \) constant (open symbols) or the \( \sigma \lambda \) product constant (closed symbols). The dashed and dotted lines show the \( k = 0 \) case and the solid line shows McIntyre’s equation (1) for comparison.

![Figure 2. Excess noise factor versus multiplication. Solid line McIntyre’s equation (1), dotted line \( k = 0 \) overshoot = 0.97, dashed line \( k = 0 \) overshoot = 5.36, squares overshoot = 0.97, circles overshoot = 5.36, closed symbols \( \sigma \lambda \) constant, open symbols \( \sigma \) constant.](image)

Considering the graph for \( k = 0.01 \), all four sets of results give \( F(M) \) greater than 2 at \( M \) less than 1000 so it can be concluded that \( k \) is less than 0.01 for HgCdTe. The open symbols represent the best case (i.e. least noise) but they are probably unrealistic because the constant \( \sigma \) assumption results in a nonlocal ionization coefficient for holes that has a very ‘soft’ turn on; \( \beta \) only achieves 53% (squares) and 28% (circles) of its asymptotic value within the multiplication region.

As expected \( F(M) \) is less for \( k = 0.001 \). Indeed using the constant \( \sigma \) approach (open symbols) there appears to be very little deviation from the \( k = 0 \) case. However this is misleading because the hole ionization has an extremely soft turn on so that the nonlocal ionization coefficient for a hole that traverses the complete multiplication region is only 9% (squares) and 4% (circles) of \( \beta(z \rightarrow \infty) \). So \( k \) is meaningless in these cases and the closed symbols are more realistic for \( k = 0.001 \). In conclusion the experimental results for \( F(M) \) in HgCdTe implies that \( k \) is less than 0.001 in that material.

3. Evidence for the resonance effect in single wall carbon nanotubes
Avalanche multiplication and breakdown has been observed in semiconducting single wall carbon nanotubes (CNT) [13,14]. Recently CNT pin diodes, fabricated and measured at Cornell University and University of Alberta [11], have shown a stepped response in the photocurrent as shown in figure 3(a). These measurements were made at 4K using CNT with a bandgap of 0.28eV. The reverse bias data from the three curves in figure 3(a) with the greatest photon flux is plotted in figure 3(b) normalized to a multiplication, \( M \), of 1 at zero applied bias.

The flattening in the multiplication curve at \( M \sim 1.6 \) and possible further flat regions at higher multiplications may be the first experimental observation of the resonance effect predicted in [8]. However the first predicted level region occurs at \( M = 2 \). The lower multiplication value of 1.6 can be explained by considering phonon interactions in CNT. Unlike conventional semiconductors, the optical phonon m.f.p. and energy is a significant fraction of the m.f.p. and threshold energy for impact
ionization. Under these conditions, the ionization pathlength p.d.f. can split into two (or more) peaks; a peak at shorter pathlengths, of population $N$, for quasi-ballistic carriers which do not emit an optical phonon and further peak (or peaks), of population $(1 - N)$, for carriers that have one or more optical phonon interactions. The first flat region in the multiplication curve would correspond to $M = (1 + N)$. An excess noise factor equal to 1 has been predicted [8] for level regions in the multiplication curve at $M = 2, 4, 8, \text{etc.}$ However the same cannot be predicted for a level region at $M = 1.6$ where $F(M) \approx 1.1$ using equation (2).

Figure 3. (a) Photocurrent versus applied voltage from [11] and (b) multiplication versus reverse bias.

4. Conclusion
Calculations of the excess noise factor for small, non-zero $k$ values using non-local ionization coefficients indicate that $k$ is less than 0.001 in HgCdTe. In photocurrent measurements made on carbon nanotubes, a distinct flat region at a multiplication of 1.6 is argued to be the first experimental evidence for the resonance effect.

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