The quantum effects on neutrino mass and lepton flavor mixing (MNS) matrices

Naoyuki Haba

Faculty of Engineering, Mie University, Tsu Mie 514-8507, Japan

1. Introduction

Previously, people thought the effects of quantum corrections on neutrino mass matrix may be negligible, since the Yukawa couplings of neutrinos are much much smaller than other quarks and leptons. However, is it true?? Besides, are “maximal” mixings in the lepton flavor mixing matrix stable against quantum corrections? My talk is concentrating on these topics in the framework of the minimal supersymmetric standard model with the effective dimension-five operator $\kappa_{ij}$ which gives the Majorana masses of neutrinos.

2. RGE of neutrino sector

The renormalization group equations (RGEs) of neutrino mass and MNS matrices are given by simple formulas thanks to the non-renormalization theorem of supersymmetry. The RGE of the operator $\kappa_{ij}$ is given by

$$\frac{d}{dt}\kappa_{ij} = (\gamma_i + \gamma_j + 2\gamma_H)\kappa_{ij}$$

where, $\gamma_i,H$ is the anomalous dimensions of $i$-th generation lepton and Higgs doublets, respectively. This equation induces the following two important consequences $^1$.

1. None of phases of $\kappa_{ij}$ depend on the energy scale.
2. The energy scale dependence of the MNS matrix is governed only by $n_g - 1$ real parameters.

Here $n_g$ is the generation number. Above two consequences make the RGE analyses of neutrino mass and MNS matrices be simple. Let us show the three-generation case. The neutrino mass matrix at the high energy $m_\nu(M_R)$ is related to that at the low energy $m_\nu(m_Z)$ as $^2^3$

$$m_\nu(M_R) = c \begin{pmatrix} 1 - \epsilon_e & 0 & 0 \\ 0 & 1 - \epsilon_\mu & 0 \\ 0 & 0 & 1 \end{pmatrix} m_\nu(m_Z) \begin{pmatrix} 1 - \epsilon_e & 0 & 0 \\ 0 & 1 - \epsilon_\mu & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

where $\epsilon_e,\mu$ is the quantities determined by the values of $\tan \beta$ and $M_R$ as shown in Fig.1. Since the difference of the magnitudes between $Y_e$ and $Y_\mu$ comparing to $Y_\tau$ is negligible, the quantum effects can be estimated by only one parameter $\epsilon(\approx \epsilon_e,\mu)$ $^2^3$.

Now let us show quantum effects on neutrino mass and MNS matrices in $2 \times 2$ case, $3 \times 3$ case, and democratic-type of mass matrices.

3. $2 \times 2$ case

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$^1$Talk at Post Summer Institute 2000, Yamanashi, Japan, 21 Aug - 24 Aug 2000.
$^2$E-mail: haba@eken.phys.nagoya-u.ac.jp
Two-generation example is the good study for understanding “more complicated” three-generation case. The results of the three-generation case are completely understood in the analogy of two-generation case. There are two cases for the maximal mixings, one is (a) : hierarchical, and the other is (b) : degenerate. For the first order, these cases are classified as $m_\nu^{(a)} = \text{diag.}(0,1)$, $m_\nu^{(b1)} = \text{diag.}(-1,1)$, and $m_\nu^{(b2)} = \text{diag.}(1,1)$ in the diagonal base of neutrino mass matrix. Here (b1) and (b2) are cases with opposite signs. We can easily show that the mixing angle of cases (a) and (b1) are stable, and (b2) is unstable against quantum corrections\[3\]. The case of degenerate and the same sign has a risk of instability of mixing angle\[3\]. In other words, a “maximal” mixing is possibly obtained by the RGE effects in the case of (b2). We had shown a “maximal” mixing can be realized by the quantum corrections in degenerate neutrinos with (the same sign) masses of order 0.1 eV\[4\].

We can also show that the cases of (b1) and (b2) are connected with each other by the physical Majorana $CP$ phase $\phi$. Therefore, this $CP$ phase should be the order parameter connecting the “stable” region with the “unstable” region as shown in Fig.2\[5\].

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3 It depends on the values of Majorana mass and $\tan \beta$\[3\].

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4. 3 × 3 case

Figure 1: $\tan \beta$ dependence of $\epsilon_{e,\mu}[3]$. The solid-line (dashed-line) shows $M_R = 10^{13}\text{GeV}$ (10$^6\text{GeV}$). Each dotted-line shows (a):$4.6 \times 10^{-2}$, (b):$1.9 \times 10^{-3}$, (c):$1.0 \times 10^{-3}$ and (d):$2.3 \times 10^{-4}$.

Figure 2: contour plot of $\sin^2 2\theta_{23}$ for $\phi$ and $m_2^2$[5]. $A : \sin^2 2\theta_{23} < 0.05$, $B : 0.05 \leq \sin^2 2\theta_{23} < 0.1$, $C : 0.1 \leq \sin^2 2\theta_{23} < 0.5$, $D : 0.5 \leq \sin^2 2\theta_{23} < 0.9$, $E : 0.9 \leq \sin^2 2\theta_{23} < 0.99$, $F : 0.99 \leq \sin^2 2\theta_{23}$. 
The stabilities of mixing angles in the $3 \times 3$ case can be understood in the analogy of $2 \times 2$ case. The results are shown in Ref.[3], that is, cases of hierarchical or the opposite signs between the generations guarantee stable mixing angles against quantum corrections. As for the $CP$ phases, there are two physical Majorana phases, which connect the “stable” region with the “unstable” region[6] as in the case of $2 \times 2$ case.

5. Stability of the democratic-type of mass matrix

Democratic type of mass matrix[4] can naturally explain why masses of the third generation fields are much heavier than those of other generation fields. In the democratic-type of mass matrix, the origin of the “maximal” mixing exists in the unitary matrix diagonalizing the charged lepton sector, where the neutrino masses should be degenerate with the same signs and negligibly small flavor mixings. As shown in $2 \times 2$ case of (b2), this case has a risk that the mixing angles become unstable due to quantum corrections. Actually, stabilities of the mixing angles depend on the solar solutions (degrees of degeneracy). When we take the right-handed Majorana mass scale as $10^{13}$ GeV, the vacuum solution is completely destroyed by the quantum corrections, and the large angle and the small angle MSW solutions require $\tan \beta < 10$ in order not to be destroyed by the quantum corrections.

6. Summary

The quantum effects of lepton flavor mixing angles can be easily estimated by using the technique in Ref.[1]. The cases of hierarchical or the opposite signs between the generations guarantee stable mixing angles against quantum corrections. We also show that physical Majorana phases connect the “stable” region with the “unstable” region.

References

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\[4\]In order to realize the democratic-type of mass matrix, there should be, for example, $S_{3L} \times S_{3R}, O(3)_L \times O(3)_R$ flavor symmetries behind.