Collective Thomson scattering from high-temperature high-density plasmas revisited

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Abstract

The theory of collective Thomson scattering from high-temperature high-density plasmas is revisited from the viewpoint of plasma fluctuation theory. Three subtle effects are addressed. The first is the correction of the first order of $v/c$, where $v$ is the particle velocity and $c$ is the light speed, the second is the plasma dielectric effect and the third is the finite scattering volume effect. When the plasma density is high, the first effect is very significant in inferring plasma parameters from the scattering spectra of electron plasma waves. Without the inclusion of the correction, the electron temperature may be significantly underestimated. The second is also notable but much less significant. When the size of the scattering volume is much larger than the probe wavelength, the third is negligible.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Collective Thomson scattering has been widely used as a key diagnostic tool to characterize laser-produced plasmas in relevance to inertial confinement fusion (ICF) in recent years [1–9]. In comparison with the plasmas in the field of magnetic confinement fusion, laser-produced plasmas have some unique properties, such as much higher electron density, much greater parameter gradients, more frequent Coulomb collisions and super-Gaussian electron velocity distribution due to strong inverse bremsstrahlung heating. The theory of collective Thomson scattering is developed to include various effects that may be important for laser-produced plasmas, such as super-Gaussian electron velocity distribution [10, 11], Coulomb
collisions [12, 13] and plasma inhomogeneity [14, 15]. However, there are still some subtle effects usually neglected in inferring plasma parameters from Thomson scattering spectra of laser-produced plasmas. For example, the correction in the first order of $v/c$ to the scattering spectrum (where $v$ and $c$ are the electron and light speeds, respectively) [16], the finite scattering volume effect [17] and the plasma dielectric effect [18]. These effects have been studied mainly for incoherent Thomson scattering. These may also be important for collective Thomson scattering. In this paper, we revisit the theory of collective Thomson scattering from high-temperature high-density plasmas because Thomson scattering is usually operated in the collective regime in the field of ICF, and address these effects from the viewpoint of fluctuation theory. The correction term of the first order of $v/c$ that we obtain in this paper is the same as that given by Sheffield in his well-known monograph [19]. However, the so-called finite scattering volume effect first pointed out by Pechacek and Trivelpiece [17] and lately adopted by Sheffield [19] is essentially incorrect, which was first revealed by Kukushkin [20] and lately by Chen and Marshall [21]. We can see that the correction due to the finite $v/c$ is significant in analyzing the scattering spectra from thermal electron plasma waves. The plasma dielectric effect is also notable in the theory. When the size of the scattering volume is much larger than the probe wavelength, the finite scattering volume effect is negligible.

The paper is organized as follows. In section 2, we derive the basic equations of Thomson scattering. We then discuss the spectral power of Thomson scattering with the correction of the first order of $v/c$ in section 3. The plasma dielectric effect is addressed in section 4, and the finite scattering volume effect is discussed in section 5. Finally, we give a summary in section 6.

2. Basic equations of Thomson scattering

We start our calculations from the electric and magnetic fields emitted from an accelerated electron [22]:

$$E(R, r, t) = -\frac{e}{c^2|R - r(\tau)|[1 - n_s \cdot v(\tau)/c]^3}n_s \times [(n_s - v(\tau)/c) \times \dot{v}(\tau)],$$

$$B(R, r, t) = n_s \times E,$$

where $-e$ is the electron charge, $\dot{v}(\tau) = dv(\tau)/d\tau$ is the charge acceleration, $r$ is the charge coordinate, $R$ is the observation coordinate, $n_s = [R - r(\tau)]/[R - r(\tau)]$ is the unit vector from the electron to the observer and $\tau$ is the retarded time defined as

$$t = \tau + \frac{1}{c} |R - r(\tau)|.$$

As usual, we choose the origin of our coordinate system in the interior of the charge system. In the wave zone, i.e. $R \gg r$, we can make the following approximations:

$$n_s = \frac{R}{R},$$

$$t = \tau + \frac{R}{c} = \frac{1}{c} n_s \cdot r(\tau).$$

The emitted magnetic field in the wave zone can now be approximately written as

$$B(R, r, t) = -\frac{e}{c^2[R[1 - n_s \cdot v(\tau)/c]]^3}d\tau \left[ \frac{v(\tau) \times n_s}{1 - n_s \cdot v(\tau)/c} \right].$$

The spectral energy emitted into the solid angle $d\Omega$ is then given by

$$\frac{d^2E_{n_s\omega_s}}{d\Omega d\omega_s} = \frac{e^2}{8\pi^2c^3} \left[ \int_{-\infty}^{\infty} d\tau \left[ \frac{v(\tau) \times n_s}{1 - n_s \cdot v(\tau)/c} \right] e^{i\omega_s\tau - i(\omega_s/c)n_s \cdot r(\tau)} \right]^2.$$
where \( \omega_0 \) denotes the frequency of the emitted electromagnetic waves. In the case when the radiation process involves many electrons, equation (3) should be rewritten as

\[
\frac{d^2 \mathcal{E}_{\text{th}}}{d\Omega \, d\omega_0} = \frac{e^2}{8\pi^2 c^3} \sum_{i=1}^{N} \int_{-\infty}^{\infty} d\tau \left[ \frac{v_i(\tau) \times n_s}{1 - n_s \cdot v_i(\tau)/c} \right] e^{i\omega_0\tau - i\omega_0/c n_s \cdot r_i(\tau)} d\tau \frac{2}{c},
\]

(4)

where \( N \) is the total electron number in the system of interest.

In the process of Thomson scattering, an electron is accelerated under the action of an incident electromagnetic wave and then emits scattering waves. The intensity of the incident wave is assumed so weak as not to change the velocity of the electron. For the sake of simplicity, we further assume that the incident wave is plane, monochromatic and linearly polarized, i.e. the electric and magnetic fields of the incident wave are described as

\[
E_0 = e_0 E_0 \cos(k_0 \cdot r - \omega_0 t),
\]

(5a)

\[
B_0 = (n_0 \times e_0) E_0 \cos(k_0 \cdot r - \omega_0 t),
\]

(5b)

where \( \omega_0 \) and \( k_0 \) are the frequency and wave vector of the incident wave, \( e_0 \) is the polarization vector of the wave and \( n_0 \) is the propagation direction of the incident wave. In this paper, we are only interested in Thomson scattering within the correction to the first order of \( v/c \). Therefore, we neglect those terms which are proportional to the higher order of \( v/c \) in the following calculations. The acceleration of an electron in the wave fields of equations (5a) and (5b) is given by

\[
\dot{v}(\tau) = -\frac{eE_0}{m_e} \left[ e_0 + \frac{1}{c} v \times (n_0 \times e_0) \right] \cos[k_0 \cdot r(\tau) - \omega_0 \tau] + O[(v/c)^2].
\]

(6)

With the aid of equation (6), we have

\[
\frac{d}{d\tau} \left[ \frac{v(\tau) \times n_s}{1 - n_s \cdot v(\tau)/c} \right] = -\frac{eE_0}{m_e} \left\{ (e_0 \times n_s) + \frac{1}{c} (e_0 \times n_s) [(n_s - n_0) \cdot v] + \frac{1}{c} (n_0 \times n_s) (e_0 \cdot v) + \frac{1}{c} (e_0 \cdot n_s) (v \times n_s) \right\} \\
\times \cos[k_0 \cdot r(\tau) - \omega_0 \tau] + O[(v/c)^2].
\]

(7)

Substituting equation (7) into equation (4), we obtain the spectral energy of Thomson scattering:

\[
\frac{d^2 \mathcal{E}_{\text{th}}}{d\Omega \, d\omega_0} = \frac{1}{\pi} I_0 r_e^2 \sum_{i=1}^{N} \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \left[ (e_0 \times n_s)^2 + \frac{1}{c} \mathbf{a} \cdot [v_i(\tau) + v_j(\tau')] \right] \\
\times \cos[k_0 \cdot r_i(\tau) - \omega_0 \tau] \cos[k_0 \cdot r_j(\tau') - \omega_0 \tau'] \\
\times e^{i\omega_0\tau - i\omega_0/c n_s \cdot r_i(\tau)} e^{-i\omega_0\tau' + i\omega_0/c n_s \cdot r_j(\tau')} \frac{2}{c},
\]

(8)

where \( I_0 = cE_0^2/8\pi \) is the intensity of the incident wave, \( r_e = e^2/m_e c^2 \) is the classical electron radius, \( r_i(\tau) \) and \( v_i(\tau) \) are the coordinate and velocity of the \( i \)th electron, respectively, and the vector \( \mathbf{a} \) is defined as

\[
\mathbf{a} = (e_0 \times n_s)^2 (n_s - n_0) - (e_0 \cdot n_s)(n_s \cdot n_0)e_0 - (e_0 \cdot n_s)^2 n_s + (e_0 \cdot n_s)e_0.
\]

(9)

In the case when \( e_0 \) is perpendicular to the scattering direction \( n_s \), the vector \( \mathbf{a} \) is greatly simplified,

\[
\mathbf{a} = (e_0 \times n_s)^2 (n_s - n_0).
\]

(10)
The electrons in the plasmas can be described with the Klimontovich distribution function [23]

\[ F_e(r, v, \tau) = \sum_{i=1}^{N_e} \delta[r - r_i(\tau)] \delta[v - v_i(\tau)]. \]  

(11)

Here it is not necessary to assume that \( N \) is the number of electrons located within the scattering volume. We just make the assumption that \( N \) is the total electron number of the plasma within a volume of \( V \) which is much larger than the scattering volume. In the thermodynamic limit, \( N \to \infty \), \( V \to \infty \), and the electron number density \( N/V = n_e \) is finite. In order to take into account the effect of finite scattering volume, we introduce a geometric factor \( G(r) \) which is defined as

\[ G(r) = \begin{cases} 
1, & \text{when } r \text{ is in } V_s, \\
0, & \text{when } r \text{ is not in } V_s, 
\end{cases} \]  

(12)

where \( V_s \) is the scattering volume which is a small part of the plasma volume \( V \). With the geometric factor, only those electrons located within the scattering volume \( V_s \) now contribute to the scattering power that can be observed. With the Klimontovich distribution function (11) and the geometric factor (12), the spectral energy (8) can be rewritten as

\[
\frac{d^2 \mathcal{E}_{e_{\text{obs}}}}{d\Omega \, d\omega_s} = \frac{i \nu_e^2}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\alpha \times n_e)^2 + \frac{1}{c} \alpha \cdot (v + v') G(r) G(r') \\
\times \mathcal{F}_e(r, v, \tau) \mathcal{F}_e(r', v', \tau') \cos(k_0 \cdot r - \omega_0 t) \\
\times \cos(k_0 \cdot r' - \omega_0 t') e^{i \omega_0 (t - t')} - i(\alpha/c)n_e(r - r').
\]

Introducing the new variables \( \Delta \tau = \tau - \tau' \) and \( t = \tau' \), we rewrite the above equation in the following form:

\[
\frac{d^2 \mathcal{E}_{e_{\text{obs}}}}{d\Omega \, d\omega_s} = \frac{i \nu_e^2}{\pi} \int_{-\infty}^{\infty} d(\Delta \tau) \int d^3 r \, d^3 r' \, d^3 v \, d^3 v' \left[ (\alpha \times n_e)^2 + \frac{1}{c} \alpha \cdot (v + v') \right] \\
\times G(r) G(r') \times \mathcal{F}_e(r, v, \tau + \Delta \tau) \mathcal{F}_e(r', v', \tau) e^{i \omega_0 (\tau - \tau')} \\
\times \cos(k_0 \cdot r - \omega_0 (t + \Delta \tau)) \cos(k_0 \cdot r' - \omega_0 t).
\]  

(13)

In many experiments, it is power but not energy that is actually measured. At the observation point, the relation between energy and power is given by the following relation:

\[
\frac{d^2 \mathcal{E}_{e_{\text{obs}}}}{d\Omega \, d\omega_s} = \int_{-\infty}^{\infty} \frac{d^2 P_{e_{\text{obs}}}}{d\Omega \, d\omega_s} \, dt.
\]  

(14)

Comparing equation (14) with equation (13), we obtain the observed spectral power:

\[
\frac{d^2 P_{e_{\text{obs}}}}{d\Omega \, d\omega_s} = \frac{i \nu_e^2}{\pi} \int_{-\infty}^{\infty} d(\Delta \tau) \int d^3 r \, d^3 r' \, d^3 v \, d^3 v' \left[ (\alpha \times n_e)^2 + \frac{1}{c} \alpha \cdot (v + v') \right] \\
\times G(r) G(r') \times \mathcal{F}_e(r, v, \tau + \Delta \tau) \mathcal{F}_e(r', v', \tau) e^{i \omega_0 (\tau - \tau')} \\
\times \cos(k_0 \cdot r - \omega_0 (t + \Delta \tau)) \cos(k_0 \cdot r' - \omega_0 t') e^{i \omega_0 (t - t')} - i(\alpha/c)n_e(r - r').
\]  

(15)

The scattering power given by equation (15) is a fluctuating quantity. In a real experiment, any detector owns some temporal resolution \( \Delta T \). In this sense, what is observed in the experiment is in fact the time-averaged spectral power:

\[
\frac{d^2 P_{\text{obs}}}{d\Omega \, d\omega_s} = \frac{1}{\Delta T} \int_{-\Delta T/2}^{\Delta T/2} d^2 P_{e_{\text{obs}}} \, dt.
\]  

(16)
When the plasma is in the equilibrium or quasi-equilibrium state and $\Delta T$ is much larger than the timescales of fluctuations in the plasma, the time average on the right hand side of equation (16) could be replaced with a proper ensemble average [24], i.e.,

$$\frac{1}{\Delta T} \int_{-\Delta T/2}^{\Delta T/2} d^{2}P_{n,\omega} \, dt = \left\langle \frac{d^{2}P_{n,\omega}}{d \Omega \, d \omega} \right\rangle,$$

(17)

where $\langle \cdots \rangle$ denotes the ensemble average. The observed scattering spectral power is now given by

$$\frac{d^{2}P_{\text{obs}}}{d \Omega \, d \omega} = \left\langle \frac{d^{2}P_{n,\omega}}{d \Omega \, d \omega} \right\rangle.$$

(18)

It should be pointed out that the radiation power emitted by a moving radiator is not equal to the radiation power observed at a fixed point. This is because the time interval at the observation point is not equal to the time interval of the moving radiator due to the retardation effect of equation (1b). In Thomson scattering, this effect was first explained as the finite transition time effect by Pechacek and Trivelpiece [17], which is assumed to exist when the scattering volume is finite. Lately, this explanation was adopted by Sheffield in his monograph [19]. We point out here that this so-called finite transition effect cannot be interpreted as a retardation effect. As to the retardation effect, it is absent in the systems where the space-velocity distribution function of radiation emitters or scatterers in the observation volume is a steady-state one. This has already been well explained by Ginzburg et al in the calculation of synchrotron radiation from an electron system [25].

We split the Klimontovich function $F_{e}$ into an ensemble-averaged part $f_{e}$ and a fluctuating part $\delta f_{e}$:

$$F_{e}(r, v, t) = f_{e}(v, r) + \delta f_{e}(r, v, t),$$

where the fluctuation part $\delta f_{e}$ satisfies the condition

$$\langle \delta f_{e} \rangle = 0.$$

When the plasma is in the equilibrium or quasi-equilibrium state, the ensemble-averaged part $f_{e}$ is a Maxwellian distribution function:

$$f_{e} = \frac{n_{e}}{(2\pi)^{3/2}v_{e}^{3}} \exp \left( -\frac{v^{2}}{2v_{e}^{2}} \right),$$

where $v_{e} = \sqrt{T_{e}/m_{e}}$ is the thermal velocity of the electrons. After ensemble-averaging, the spectral power is given by

$$\frac{d^{2}P_{\text{obs}}}{d \Omega \, d \omega} = \frac{I_{0}v_{e}^{2}}{\pi} \int_{-\infty}^{\infty} d(\Delta t) \int d^{3}r \, d^{3}r' \, d^{3}v \, d^{3}v' \left[ \left( e_{0} \times n_{s} \right)^{2} + \frac{1}{c} a \cdot (v + v') \right]$$

$$\times G(r)G(r') \langle \delta f_{e}(r, v, t + \Delta t) \delta f_{e}(r', v', t) \rangle e^{ika \Delta t - i(\omega_{0}/c)na_{0} \cdot (r-r')}$$

$$\times \cos[k_{0} \cdot r - \omega_{0}(t + \Delta t)] \cos[k_{0} \cdot r' - \omega_{0}t].$$

(19)

If the fluctuation properties of the plasma are known, the spectral power of Thomson scattering is fully determined.

Fluctuations are usually described by the spectral densities of various correlation functions. With the Fourier component of the fluctuations,

$$\delta f_{e}(\omega, k, v) = \int e^{i\omega t - ik \cdot r} \delta f_{e}(r, v, t) \, dt \, d^{3}r,$$

(20)
the spectral density of the correlation function of a homogeneous stationary plasma is defined by [23]

$$\langle \delta f_c(\omega, \mathbf{k}, \mathbf{v})\delta f_c(\omega', \mathbf{k}', \mathbf{v}') \rangle = (2\pi)^3 \delta(\omega + \omega') \delta(\mathbf{k} + \mathbf{k}') \langle \delta f_c^2 \rangle_{\omega, \mathbf{k}, \mathbf{v}, \mathbf{v}'},$$

(21)

With the aid of equations (20) and (21), the spectral power of Thomson scattering from a homogeneous stationary plasma is given by

$$\frac{d^2 p_{\text{obs}}}{d \Omega d \omega} = \frac{1}{2\pi} I_{0f} c^2 \int \frac{d^3 q}{(2\pi)^3} d^3 v d^3 v' |G(q)|^2 (\delta f_c^2)_{k-q, \omega, v, v'} \times \left[ (e_0 \times n_s)^2 + \frac{1}{c} (v + v') \cdot a \right].$$

(22)

where $\omega$ and $k$ are the differential frequency and wave vector defined as

$$\omega = \omega_e - \omega_0,$$

(23a)

$$k = \frac{\omega}{c} n_s - k_0,$$

(23b)

$G(q)$ is the spatial Fourier component of the geometric factor $G(r)$:

$$G(q) = \int G(r) e^{-iq \cdot r} d^3 r.$$

(24)

The second term in brackets on the right hand side of equation (22) is the corrections of the first order of $v/c$. And the convolution of $|G(q)|^2$ and $(\delta f_c^2)_{k-q, \omega, v, v'}$ is the consequence of finite scattering volume.

3. First order corrections

We first discuss the corrections to the first order of $v/c$. For the sake of simplicity, we assume that the scattering volume is infinite. With this assumption, we have

$$G(q) = (2\pi)^3 \delta(q),$$

$$|G(q)|^2 = \lim_{V \to \infty} (2\pi)^3 \delta(q) V_c.$$

Equation (22) now becomes

$$\frac{d^2 p_{\text{obs}}}{d \Omega d \omega} = \frac{V_s}{2\pi} I_{0f} c^2 \int d^3 v d^3 v' (\delta f_c^2)_{k, \omega, v, v'} \left[ (e_0 \times n_s)^2 + \frac{1}{c} (v + v') \cdot a \right].$$

(25)

For a collisionless quasi-equilibrium plasma, the spectral density $(\delta f_c^2)_{k, \omega, v, v'}$ is already obtained [23]:

$$\langle \delta f_c^2 \rangle_{k, \omega, v, v'} = 2\pi \delta(v - v') f_c(v) \delta(\omega - k \cdot v)
+ \frac{e^2}{m_e^2} \frac{(k \cdot \partial f_c / \partial v)(k \cdot \partial f_c / \partial v')}{32\pi^3} \frac{32\pi^3}{k^4 (\omega - k \cdot v + i0) (\omega - k \cdot v' + i0) \delta^4(\omega, k)}
\times \sum_{a,e,i} \int f_a(v) \delta(\omega - k \cdot v) d^3 v
\frac{8\pi^2 e^2}{k^2} \left[ \frac{k \cdot \partial f_c(v) / \partial v}{m_e e_i(\omega, k)(\omega - k \cdot v + i0)} \delta(\omega - k \cdot v')
+ \frac{f_c(v) [k \cdot \partial f_c(v') / \partial v']}{m_e e_i(\omega, k)(\omega - k \cdot v' - i0)} \delta(\omega - k \cdot v) \right],$$

(26)
where $\epsilon_l(\omega, k)$ is the longitudinal permittivity of the plasma. After integrating over the velocities $v$ and $v'$, we obtain (see the appendix)

$$\frac{d^2 P_{\text{obs}}^{n_s,\omega_s}}{d\Omega d\omega_s} \approx r_z^2 I_0 N_i S(k, \omega) \left[ (e_0 \times n_s)^2 + \frac{2\omega}{k c} k \cdot a \right],$$

(27)

where $N_i = n_e V_s$ is the average electron number in the scattering volume $V_s$. Here $S(k, \omega)$ is the normal dynamic form factor of a collisionless plasma [24]:

$$S(k, \omega) = \frac{1}{\sqrt{2\pi k v_c}} \left| 1 - \frac{\chi_e(\omega, k)}{\epsilon_l(\omega, k)} \right|^2 \exp \left( -\frac{\omega^2}{2k^2 v_c^2} \right) + \frac{Z}{\sqrt{2\pi k v_i}} \left| \frac{\chi_e(\omega/k)}{\epsilon_l(\omega/k)} \right|^2 \exp \left( -\frac{\omega^2}{2k^2 v_i^2} \right),$$

(28)

where $\chi_e$ is the electron susceptibility and $Z$ is the ion charge state. Here the wave number $k$ is given by

$$k = \frac{(n_s - n_0)}{c} \sqrt{\frac{2}{1 - n_s \cdot n_0}}.$$  

(29)

The second term on the right hand side of equation (27) can be further simplified in the case of $\sqrt{2(1 - n_s \cdot n_0)} \gg \omega/\omega_0$:

$$k \approx \frac{(n_s - n_0)}{\sqrt{2(1 - n_s \cdot n_0)}},$$  

(30)

$$k \approx \frac{\omega_0}{c} \sqrt{2(1 - n_s \cdot n_0)}.$$  

(31)

Then we have

$$\frac{2\omega k}{k c} \cdot a \approx (e_0 \times n_s)^2 \frac{2\omega}{\omega_0}.$$  

Equation (27) is then simplified as

$$\frac{d^2 P_{\text{obs}}^{n_s,\omega_s}}{d\Omega d\omega_s} = r_z^2 I_0 N_i (e_0 \times n_s)^2 S(k, \omega) \left( 1 + \frac{2\omega}{\omega_0} \right).$$  

(32)

This result is essentially the same as equation (9.3.8) in Sheffield’s monograph [19]. Here it should be pointed out that the differential wave number $k$ in the dynamic form factor $S(k, \omega)$ is given by the exact formula (29). In the zeroth order approximation, however, the differential wave number $k$ is given by equation (31). We also point out that equation (32) is not suitable in the case when the condition $\sqrt{2(1 - n_s \cdot n_0)} \gg \omega/\omega_0$ is not satisfied.

Thomson scattering is usually operated in the collective regime in the measurement of laser-produced plasmas. In most such experiments, scattering spectra of thermal ion-acoustic waves are detected [1–3, 5–8]. The typical frequency shift of the ion-acoustic features of Thomson scattering spectra is in the order of

$$\frac{\omega}{\omega_0} \sim -\frac{Z T_e}{A m_p},$$  

where $A$ is the ion mass number and $m_p$ is the proton mass. For a laser-produced plasma in relevance to ICF, the electron temperature is usually lower than 5 keV [26]. We can see that the correction in the first order of $v/c$ is indeed negligible. However, scattering spectra from thermal electron plasma waves are also successfully measured in laser-produced plasmas. In the experiment performed by Glenzer et al [4], the frequency shift $\omega$ of the electron plasma
Figure 1. The profile of the spectral intensities of collective Thomson scattering, where the solid line is the uncorrected result and the dashed line is the corrected one. Here $T_e = 2$ keV, $n_e = 2.1 \times 10^{20}$ cm$^{-3}$, the probe wavelength is 0.5266 µm and the scattering angle is 104°.

Wave feature becomes significant. With a 0.5266 µm probe, the spectral maximum locates at 0.735 µm. The relative frequency shift is

$$\left| \frac{\omega}{\omega_0} \right| = 0.28.$$  

The correction factor is given by

$$2 \frac{\omega}{\omega_0} = -0.57.$$  

This correction factor is rather large and can no longer be neglected. However, the authors did not include the correction term when fitting the experimental spectra although many other effects were taken into account [14]. Hence, some errors may be introduced in the process of inferring plasma parameters from the experimental data. In figure 1 we have plotted the profile of the scattering power spectrum with the correction (32), and compared it with that without correction. As seen in figure 1, the curve without the correction overestimated the intensity of the peak corresponding to thermal electron plasma waves in the plasma. The parameters that we take in the calculations are the same as those used by Glenzer et al [3]: $T_e = 2$ keV, $n_e = 2.1 \times 10^{20}$ cm$^{-3}$, the probe wavelength is 0.5266 µm and the scattering angle is 104°.

As clearly seen in figure 1, in comparison with the curve without correction, apart from that the intensity becomes smaller, the peak position of the scattering spectrum is a little blue-shifted and the width of the spectrum becomes a little narrower. The smaller intensity is due to the factor $(1 + 2\omega/\omega_0)$ in equation (32). Other differences between the corrected and uncorrected results can be largely ascribed to the use of the exact differential wave number $k$ in the corrected theory. When the wavelength of the scattering wave is $\lambda_s = 0.735$ µm, with the parameters used in figure 1, the exact differential wave number is $1.36 \omega_0/c$, which is 0.86 times the approximate differential wave number given by equation (31) that is usually used to calculate the dynamic form factor. From the kinetic theory of electron waves, we know that when the other parameters are fixed a smaller wave number leads to a lighter damping rate and a lower frequency of the electron plasma waves. Without the inclusion of the correction,
we may therefore underestimate the electron density and temperature. In order to illustrate this, we use the uncorrected theory to generate an artificial scattering spectrum, which is fitted with the corrected theory, see figure 2. The plasma parameters used to generate the solid line are $T_e = 2$ keV and $n_e = 2.1 \times 10^{20}$ cm$^{-3}$ while those for the dashed line are $T_e = 2.7$ keV and $n_e = 2.15 \times 10^{20}$ cm$^{-3}$. The probe wavelength is $0.5266 \mu$m and the scattering angle is $104^\circ$.

In analyzing experimental data, it is the profile but not the absolute intensity of the collective scattering spectrum that is usually used in inferring plasma parameters. This makes the factor $(1 + 2\omega/\omega_0)$ relatively unimportant in the process of inferring plasma parameters. Since the differential wave number has a much stronger influence on the profile of the collective scattering spectrum, the exact differential wave number of equation (29) should be adopted in analyzing scattering data from electron plasma waves. Therefore, we can conclude that in the Thomson scattering measurement of electron plasma waves in plasmas with a density of a few percent of the critical density of the probe beam the correction due to the finite $v/c$ could not be neglected. An easy way to include the correction is to use equation (29) instead of equation (31). However, it may be rather difficult to quantitatively evaluate this correction because of large gradients in laser-produced plasmas.

4. Plasma dielectric effect

When plasma density is high, the plasma dielectric effect may also be significant. In the case when the plasma dielectric effect cannot be neglected, the intensity of an electromagnetic wave in the plasma is given by

$$I = \frac{c}{8\pi}\sqrt{\varepsilon(\omega_0)E^2},$$
Figure 3. The profile of the spectral power of Thomson scattering with (dashed curve) and without (solid curve) the plasma dielectric effect, where the parameters are the same as those used in figure 1.

where \( \epsilon(\omega) = 1 - \omega_p^2/\omega^2 \) is the plasma dielectric constant. And the differential wave vector also changes a little:

\[
k = \frac{\omega_s}{c} \sqrt{\epsilon(\omega_s) n_s} - \frac{\omega_0}{c} \sqrt{\epsilon(\omega_0) n_0}, \tag{33}
\]

The scattering power is now given by

\[
\frac{d^2 P_{n_s,\omega obss}}{d\omega d\Omega} = \frac{r_e^2 I_0 N_s (\epsilon_0 \times n_s)^2}{\epsilon(\omega_s) \epsilon(\omega_0) S(k, \omega) \left( 1 + \frac{2\omega}{\omega_0} \right)}. \tag{34}
\]

Here, the plasma dielectric effect on the correction term is neglected.

In figure 3 we have plotted the profiles of the features of the scattering spectrum of a thermal electron plasma wave with and without the dielectric effect. As seen in figure 3, there are some differences when the dielectric effect is taken into account: the wavelength shift becomes smaller and the intensity becomes a little higher. The reason is quite understandable. When the dielectric effect is included, the differential wave vector becomes a little smaller, leading to a lighter damping and a lower frequency of the waves. The plasma dielectric effect would become more notable when the plasma density is higher. In the case when \( n_e/n_c \) is about several percent, the dielectric effect should be included in inferring the plasma parameters from the features of electron plasma waves.

The dielectric effect on the ion-acoustic features of Thomson scattering is much less significant. Since \( \omega_s \) is very close to the probe frequency \( \omega_0 \) in this case, the dielectric effect just leads to a minor frequency shift \( \delta \omega/\omega_a \sim 0.5(n_e/n_c) \), which is usually \( \lesssim 10^{-2} \) in experiment. Therefore, the dielectric effect can be neglected in fitting the ion-acoustic feature of a Thomson spectrum.

5. Effect of finite scattering volume

When the scattering volume is small, the geometry factor may become important. When discussing the finite scattering volume effect, we neglect the correction terms in the first order
of \( v/c \). We then have

\[
\frac{d^2 P_{n,obs}^m}{d\omega d\Omega} = n e^2 I_0 (e_0 \times n_s)^2 \int \frac{d^3 q}{(2\pi)^3} |G(q)|^2 S(|k - q|, \omega). \tag{35}
\]

In the case of \( k \gg q \), we can approximately have

\[
\frac{d^2 P_{n,obs}^m}{d\omega d\Omega} \approx n e^2 I_0 (e_0 \times n_s)^2 \int \frac{d^3 q}{(2\pi)^3} |G(q)|^2 \left[ S(k, \omega) - q \cdot \frac{\partial}{\partial k} S(k, \omega) \right] \sim N_s e^2 I_0 \left[ 1 + \frac{\Delta q}{k} \right].
\]

Hence \( \Delta q \) is the range in which \( G(q) \) is notably different from zero. With the assumption that the scattering volume is cubic with a length of \( L \), \( \Delta q \sim 2\pi / L \). Then the correction due to the finite scattering volume effect is about

\[
\frac{\Delta q}{k} \sim \frac{2\pi}{k L} \sim \frac{\lambda_0}{2 L \sin(\theta_s/2)},
\]

where \( \theta_s \) is the scattering angle and \( \lambda_0 \) is the probe wavelength. For a scattering experiment with a laser wavelength of 0.53 \( \mu \)m, a scattering volume size of 50 \( \times \) 50 \( \times \) 50 \( \mu \)m\(^3\) and a scattering angle of 90\(^\circ\), this correction is about \( 10^{-2} \) and can be thereby neglected.

6. Summary

We revisit the theory of collective Thomson scattering from high-temperature high-density plasmas. Three effects are discussed: the correction due to finite \( v/c \), the plasma dielectric effect and the finite scattering volume effect. Among these three effects, the correction due to finite \( v/c \) is the most significant for analyzing the Thomson scattering spectrum of electron plasma waves. Without the correction, electron temperature may be significantly underestimated. The plasma dielectric effect is less important but still notable. The finite scattering volume effect can be neglected if the size of the scattering volume is about \( 10^2 \) of the probe wavelength and the scattering angle is not very small.

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Appendix

We first demonstrate that the normal dynamic form factor of a collisionless plasma can be obtained from equation (28) with the spectral density (26).

The electron susceptibility of a collisionless plasma is given by

\[
\chi_e(k, \omega) = \frac{4\pi e^2}{m_e k^2} \int \frac{k \cdot \partial f_e(v) / \partial v}{\omega - k \cdot v + i0} d^3 v.
\]

When the distribution function of the electrons is Maxwellian,

\[
f_e = \frac{n_e}{(2\pi)^{3/2} v_e^3} \exp \left( -\frac{v^2}{2v_e^2} \right),
\]

and
the electron susceptibility can be written as
\[ \chi_e = \frac{1}{k^2 \lambda_B^2} \left[ \frac{\omega}{\sqrt{2} k v_c} W \left( \frac{\omega}{\sqrt{2} k v_c} \right) \right], \]

where \( W(\nu) \) is the plasma dispersion function. We further introduce the functions \( F_\alpha \):
\[ F_\alpha(\omega/k) = \int f_\alpha(v) \delta(\omega - k \cdot v) \, d^3v = \frac{n_\alpha}{\sqrt{2 \pi k v_\alpha}} \exp \left( - \frac{\omega^2}{2 k^2 v_\alpha^2} \right). \]

Then we have
\[ \int d^3v \, d^3v' 2\pi \delta(\nu - \nu') f_e(v) \delta(\omega - k \cdot v) = 2\pi F_e(\omega/k), \]
\[ \int d^3v \, d^3v' m_e^2 \omega(\omega - k \cdot v + i0)(\omega - k \cdot v' - i0) k^2 \epsilon_i (\omega, k)^2 \sum_{a=e,i} e_a^2 \int f_a(v) \delta(\omega - k \cdot v) \, d^3v \]
\[ = 2\pi \left( \frac{\chi_e(\omega/k)}{\epsilon_i (\omega/k)} \right)^2 \left[ F_e(\omega/k) + Z^2 F_i(\omega/k) \right], \]
\[ - \frac{8\pi^2 e^2}{k^2} \int d^3v \, d^3v' \left\{ \frac{[k \cdot \partial f_e(v)/\partial v](k \cdot \partial f_e(v'))}{m_e \epsilon_i (\omega, k)(\omega - k \cdot v + i0)} \right\} = -2\pi \left( \frac{\chi_e + \chi_\alpha}{\epsilon_i} \right) F_e(\omega/k). \]

Combining the above results, we have
\[ \int d^3v \, d^3v' \langle \delta f_e^2 \rangle_{\omega, \nu, \nu'} = 2\pi \left\{ \left[ 1 + \frac{\chi_e(\omega, k)}{\epsilon_i (\omega, k)} \right]^2 F_e(\omega/k) + Z^2 \left[ \frac{\chi_e(\omega/k)}{\epsilon_i (\omega/k)} \right]^2 F_i(\omega/k) \right\}. \]

Finally, we obtain the normal dynamic form factor,
\[ S(k, \omega) = \frac{1}{\sqrt{2 \pi k v_c}} \left[ 1 + \frac{\chi_e(\omega, k)}{\epsilon_i (\omega, k)} \right]^2 \exp \left( - \frac{\omega^2}{2 k^2 v_c^2} \right) \]
\[ + \frac{Z}{\sqrt{2 \pi k v_c}} \left[ \frac{\chi_e(\omega/k)}{\epsilon_i (\omega/k)} \right]^2 \exp \left( - \frac{\omega^2}{2 k^2 v_c^2} \right). \]

Equation (27) is verified as follows. It is easy to demonstrate the following two equations:
\[ \int v f_e(v) \delta(\omega - k \cdot v) \, d^3v = \frac{k \omega}{k} F_e(\omega/k), \]
\[ \int v(k \cdot \partial f_e/\partial v) \omega - k \cdot v + i0 \, d^3v = \frac{k \omega}{k} \int \frac{(k \cdot \partial f_e/\partial v)}{\omega - k \cdot v + i0} \, d^3v. \]

With the aid of these two equations, we have
\[ \frac{1}{2\pi n_e} \int v(\delta f_e^2)_{\omega, \nu, \nu'} \, d^3v \, d^3v' = \frac{k \omega}{k} S(k, \omega). \]

Then we obtain equation (27).
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