Nonlinear Creep Model and Parameter Determination of Asphalt

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Abstract. Asphalt is a kind of typical viscoelastic material. Adding different materials in asphalt composite materials is an important way in road engineering materials. The creep experiments under different stress of asphalt, we used the multivariate model to fit the experimental data to get Burgers model and the creep law of asphalt correlation is best. The Burgers model be expanded by Taylor series, the polynomial creep model of creep compliance and relaxation characterized the coefficient of asphalt specimen can be obtained. Compared to other models, the creep law of asphalts well characterized by polynomial model, multivariate model has a wider range of adaptability compared to polynomial model, and polynomial model of mathematical calculation more simple than multivariate model.

1. Introduction
Asphalt is a kind of typical viscoelastic material, with good cohesiveness, plasticity, waterproof and moisture resistance, widely used in industry, traffic, water conservancy and civil engineering construction. Multi-class composite materials composed of asphalt, stone and gravel are the most important road materials in road engineering. At present, 80% of the road pavement in the world is asphalt pavement. Asphalt in the low temperature (high viscosity) and under instantaneous load, elastic deformation occupies the main position, while in high temperature (low viscosity) and under long time load, its deformation is close to viscosity, in most practical use, asphalt deformation is in the viscous elasticity range [1, 2].

At present, a wide variety of asphalt of the rheological properties of composite material was studied and many rheological models have been established by researchers [3-6], but the research of single asphalt is relatively small, asphalt as a main component of medium, studying the characteristics of its mechanical related to the performance of different asphalt compound adjustment and evaluation is of great significance.

At first, this paper used the method of combination model to discuss several models on two groups of asphalt creep experimental results of fitting error, and the params of each model are determined by MATLAB. Taking Burgers model with the best fitting effect bogs as an example, it is verified that the polynomial creep model can better represent creep law of asphalt. It is basically consistent with the fitting results of the Burgers model, and the polynomial model can describe the creep law of asphalts simply, efficiently and accurately, which has wide value in practical application.
2. Creep experiment
The asphalt sample is cylindrical with a height-diameter ratio of 1, which is prepared by injection molding. Specimen size and experimental conditions listed in table 1. Asphalt specimen compression displacement is measured with a dial indicator, the experiment device is shown in figure 1.

Table 1. Sample size and experimental conditions.

| Sample | Diameter (mm) | High (mm) | T (℃) | Load (g) | Stress (Pa) |
|--------|---------------|-----------|-------|----------|-------------|
| 1      | 19            | 19        | 18    | 1549     | 49300       |
| 2      | 19            | 19        | 21    | 776      | 24700       |

Figure 1. Pitch creep test device. Figure 2. Creep curves of asphalt under different stress levels.

3. The analysis of creep results and fitting of combined model
The creep curves of asphalt under different stress are shown as Figure 2. It can be seen that different from the sheer creep curve of asphalt, the compress creep curves of asphalt mainly include two stages of the decaying and stable creep. The creep stage of the decaying of Sample 2 is longer than Sample 1 on the creep curves due to the higher temperature. However, after a certain period of time, they reach an almost similar number at the stage of accelerating. So, we can hypothesis: on the premise of ignore other errors, for a certain size of the asphalt sample, within a certain stress range, the strain of asphalt sample when it reaches stability does not change with stress, and the difference stress level will only change the time to reach equilibrium. Higher stress levels, shorter time; lower stress levels, longer time.

Burgess model, Lethersich model and Kelvin model, Maxwell model were used to fit the
experimental curve (Figure 3, Figure 4.) in MATLAB. As shown in the figure, using Burgess model, Lethersich model and Kelvin model to fit curve is more relevant with measured curve for the compress creep characteristics of asphalt. That will produce very big errors to describe the creep curves of asphalt by Maxwell model which is One-dimensional linear model, so the law of nonlinear creep model is suitable for the compress creep of asphalt. Obviously, the Burgess model is more coincident than the above two nonlinear combination models.

4. Comparison of polynomial model and Burgess mode

The Burgess model is composed by a Maxwell model series connection a Kelvin model (Figure 5) [7]. The stress of the two elements is equal and the total strain is the sum of the strain of the two models.

\[
\sigma + \left( \frac{\eta_1}{E_1} + \frac{\eta_2}{E_2} \right) \sigma + \frac{\eta_1}{E_1} \dot{\sigma} + \frac{\eta_2}{E_2} \ddot{\sigma} = \eta_1 \dot{\varepsilon} + \frac{\eta_2}{E_2} \ddot{\varepsilon}
\]

To eliminate \( \varepsilon_1 \) and \( \varepsilon_2 \), so the constitutive equation of the Burgers model is as follows:

\[
\varepsilon(t) = \sigma_0 \left( \frac{1}{E_1} + \frac{t}{\eta_1} + \frac{1}{E_2} \left[ 1 - e^{-\eta_1/E_2 t} \right] \right)
\]

In the right side of the equal sign representing three respectively:

- \( \frac{\sigma_0}{E_1} \) — Instantaneous elastic strain;
- \( \frac{\sigma_0 t}{\eta_1} \) — Viscous strain;
- \( \frac{\sigma_0}{E_2} \left[ 1 - e^{-\eta_1/E_2 t} \right] \) — Retarded elastic strain;

In this paper, the creep equation of Burgers model is expanded in polynomial form by using the series of the Taylor:

\[
\varepsilon(t) = \sigma_0 \left( \frac{1}{E_1} + \frac{t}{\eta_1} + \frac{1}{E_2} \left[ 1 - e^{-\eta_1/E_2 t} \right] \right) = \sigma_0 \left( \frac{1}{E_1} + \frac{t}{\eta_1} + \frac{1}{E_2} \frac{1}{\eta_2} \right) \left[ 1 - e^{-\eta_1/E_2 t} \right] \approx \sigma_0 \left[ \frac{1}{E_1} + \frac{1}{\eta_1 (1 + \frac{\eta_2}{\eta_1})} t - \frac{E_2}{2 \eta_2} t^2 + \frac{E_2^2}{6 \eta_2^2} t^3 - \frac{E_2^3}{24 \eta_2^3} t^4 + \frac{E_2^4}{120 \eta_2^4} t^5 \right]
\]

So

\[
\frac{\varepsilon(t)}{\sigma_0} = \frac{1}{E_1} + \frac{1}{\eta_1} \frac{1}{\eta_2} t - \frac{E_2}{2 \eta_2} t^2 + \frac{E_2^2}{6 \eta_2^2} t^3 - \frac{E_2^3}{24 \eta_2^3} t^4 + \frac{E_2^4}{120 \eta_2^4} t^5
\]

Where \( \frac{\varepsilon(t)}{\sigma_0} \) indicates the creep caused by stress of a unit, which changes with time. In other words, the creep function caused by unit stress can be approximated by a polynomial when the strain is small (less time). This polynomial is \( a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \). The polynomial is compared with the formula (4), we get

\[
a_0 = \frac{1}{E_1}; a_1 = \frac{1}{\eta_1}; a_2 = \frac{E_2}{2 \eta_2}; a_3 = \frac{E_2^2}{6 \eta_2^2}; a_4 = \frac{E_2^3}{24 \eta_2^3}; a_5 = \frac{E_2^4}{120 \eta_2^4}.
\]
It is obvious that the coefficients are related to the inherent properties of asphalt and the temperature of the environment, which are not related to the level of stress. Based on creep compliance of asphalt creep and relaxation time to define the formula (6) [8]:

$$J_1 = \frac{1}{E_1}; J_2 = \frac{1}{E_2}; \lambda_1 = \eta_1; \lambda_2 = \eta_2$$

Take the (6) into the (5) to get the formula (7):

$$a_0 = J_1; a_1 = J_1 \frac{1}{\lambda_1}; a_2 = -\frac{J_2}{2\lambda_2}; a_3 = \frac{J_2^2}{6\lambda_2^2}; a_4 = -\frac{J_2^3}{24\lambda_2^3}; a_5 = -\frac{J_2^4}{120\lambda_2^4}.$$  

Formula (7) expresses the coefficient of creep polynomial expression of asphalt with relaxation time and creep compliance. The macroscopic significance of relaxation time $\lambda$ is the time required to reduce the stress to 0.368(e$^{-1}$) times the initial stress $\sigma(0)$. The relaxation time is related to the structure and temperature of the asphalt. When the temperature rises, the frequency of molecular motion increases and the relaxation time decreases. The relaxation time is the ratio of the viscosity coefficient and the elastic coefficient, which indicates that the relaxation process must be both viscous and elastic. The creep compliance $J$ is the reciprocal of the modulus, that is the strain caused by the unit strain. It can be used to indicate the degree of difficulty of deformation.

From Figure 6 to Figure 9, respectively, for the two sample of the test results of the quartic, quintic polynomial and the experimental data, it can be seen that the quartic and the quintic polynomial model for the experimental data of asphalt creep is excellent.

Table 2, Table 3, respectively, for the sample 1, 2, multi-model fitting rheological parameters, polynomial model fitting coefficients. It can be seen that the coefficients of the polynomial model have a smaller range of variation than the rheological parameters of the multivariate model. Therefore, the polynomial model has a wider adaptability when the polynomial model and the multivariate model can better reflect the creep law of the asphalt material. And the mathematical solution of the polynomial model is simpler than the multivariate model.

Figure 6. 49300Pa stress creep experiments of the quartic polynomial fitting.  
Figure 7. 49300Pa stress creep experiments of the quintic polynomial fitting.
Comparing the polynomial coefficients obtained by substituting the rheological parameters of the multivariate model in Table 2 into equation (5) with the coefficients obtained by fitting the polynomial model, as shown in Table 4, the polynomial fitting coefficient is very similar to the polynomial coefficient calculated by the multivariate model-Burgess model, in the case of small strain (shorter time), the two can be considered the same. That is, fitting the creep curve with polynomials ensures its accuracy.

Table 2. Matlab model of the rheological parameters derived from fitting.

| Multivariate model & Rheological parameters | Sample 1     | Sample 2     | Sample 1/Sample 2 |
|--------------------------------------------|--------------|--------------|------------------|
| Burgess model                              |              |              |                  |
| \(E_1\)                                   | 1.3686e+007  | 7.1862e+013  | 1.904e-007       |
| \(\eta\)                                   | 1.375e+011   | 3.2617e+015  | 4.216e-006       |
| \(E_2\)                                   | 2.6728e+007  | 2.2458e+006  | 1.190e+001       |
| \(\eta^2\)                                | 8.3784e+010  | 1.7769e+010  | 4.715e+000       |

Table 3. Fitting coefficients of polynomial model.

| Polynomial model                   | Sample 1     | Sample 2     | Sample 1/Sample 2 |
|------------------------------------|--------------|--------------|------------------|
| quartic polynomial model           |              |              |                  |
| \(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x + a_5 x^5\) |              |              |                  |
| \(a_0\)                            | 6.085E-11    | 8.1E-11      | 0.751            |
| \(a_1\)                            | -6.09E-15    | -4E-15       | 1.52             |
| \(a_2\)                            | 4.057E-19    | 2.02E-19     | 2.01             |
| \(a_3\)                            | -6.09E-24    | -3.2E-24     | 1.9              |
| \(a_4\)                            | 8.114E-09    | -2E-08       | 0.406            |
| \(a_5\)                            | 6.085E-11    | 4.05E-11     | 1.5              |
| quintic polynomial model           |              |              |                  |
| \(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5\) |              |              |                  |
| \(a_0\)                            | -8.11E-15    | 4.05E-16     | 20               |
| \(a_1\)                            | 8.114E-19    | -4E-19       | 2.03             |
| \(a_2\)                            | -2.03E-23    | 4.05E-23     | 0.501            |
| \(a_3\)                            | 4.057E-28    | -8.1E-28     | 0.501            |

Table 4. Polynomial model fitting coefficients calculated with the coefficients of multiple model.

| Multivariate model & coefficient | Polynomial fitting coefficient \(A_i\) | Multivariate model coefficient \(A_2\) | \(A_1 / A_2\) |
|----------------------------------|----------------------------------------|----------------------------------------|---------------|
| quartic polynomial               | \(a_0\)                                | 1.217E-08                              | 7.3E-08       | 1.67E-01     |
| \(a_0 + a_1 x + a_2 x^2\)       | \(a_1\)                                | 6.085E-11                              | 1.92E-11      | 3.17E+00     |
| \(a_2\)                          |                                        | -6.09E-15                              | -1.9E-15      | 3.21E+00     |
| \(a_3\)                          |                                        | 4.057E-19                              | 2.02E-19      | 2.01E+00     |
| \(a_4\)                          |                                        | -6.09E-24                              | -1.6E-24      | 3.81E+00     |
| quintic polynomial model         | \(a_0\)                                | 8.114E-09                              | 7.3E-08       | 1.11E-01     |
5. Conclusion

(1) As time increases, asphalt creep eventually reaches a steady value. Creep rate is always reduced, the final creep rate is zero, that is, creep tends to be stable.

(2) The more the number of multivariate model components, the closer the fitting curve is to the measured creep curve. The quaternary Burgess model can fit the measured curve well.

(3) The coefficient of the corresponding two test model of the quartic/quintic polynomials have a smaller range of variation than the rheological parameters of the Gaussian model. Therefore, the polynomial model has a wider adaptability when the polynomial model and the multivariate model can better reflect the creep law of the asphalt material. And the mathematical solution of the polynomial model is simpler than the multivariate model.

(4) The polynomial fitting coefficient is very similar to the polynomial coefficient calculated by the multivariate model-Burgess model, in the case of small strain (shorter time), the two can be considered the same. Fitting the creep curve with polynomials ensures its accuracy.

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