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Deliverable table

| Deliverable no. | D3.3          |
|-----------------|---------------|
| Deliverable name| OAM-detector by phase unfolding |
| WP no.          | 3             |
| Lead beneficiary no. | 4 (UGLAS) |
| Nature          | P             |
| Dissemination level | PU         |
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| Actual delivery date     | 30 September 2012 |
D3.3) OAM-detector by phase unfolding: Demonstration of a OAM detector based on the phase unfolding concept. Optimized (possibly custom-made) lenses (or model on SLM) will be employed. The achieved channel capacity and detection efficiency will be evaluated, both in a classical and quantum regime. The prototype quality and the extent to which its performances reach our target goals will be assessed and validated by the steering committee. A short report or scientific publications will illustrate the device working principle and performances. [month 24] [Excerpt from the GA-Annex I describing the deliverables of WP3, page 21]

We have developed a demonstrator for our modesorter, comprising two custom built refractive optical elements that allow to transform between the cylindrical LG coordinates characterised by l and p and cartesian coordinates, thus allowing to sort OAM. The hardware elements are diamond machined using a Natotech, 3 axis (X,Z,C) ultra precision lathe in combination with a Nanotech NFTS6000 fast tool servo. The mode sorter prototype is shown in Fig. 1a) and a typical setup in 1.b). The work has been published in Optics Express [1] with the following abstract:

We have developed a mode transformer comprising two custom refractive optical elements which convert orbital angular momentum states into transverse momentum states. This transformation allows for an efficient measurement of the orbital angular momentum content of an input light beam. We characterise the channel capacity of the system for 50 input modes, giving a maximum value of 3.46 bits per photon. Using an electron multiplying CCD (EMCCD) camera with a laser source attenuated such that on average there is less than one photon present within the system per measurement period, we demonstrate that the elements are efficient for the use in single photon experiments.

Several mode-sorters have been installed in various international groups.

The mapping achieved by the mode sorter transforms a set of concentric rings at the input plane into a set of parallel lines in the output plane. This allows for the sorter to differentiate not only between different OAM states but also between radial modes. The sorting of both angular and radial variable was observed for over 50 spatial light modes. This work was presented at , and a paper on this was submitted to New J. Phys. [2] and presented at a SPIE conference [3] with the following abstract:
Previously we have demonstrated that the orbital angular momentum (OAM) of the light beam may be measured by image transformation mapping the azimuthal to linear transverse co-ordinate. For each input OAM state the light is focussed to a different transverse position enabling simultaneous measurement over many states (Lavery et al. (2012) Opt. Express, 20, 3). Here we significantly improve our earlier design, extending the measurement bandwidth to > 50 OAM states and showing simultaneous measurement of the radial coordinate.

PHORBITECH contribution to this deliverable
PHORBITECH in this work has supported a fraction of the cost of the PhD student Martin Lavery and of Miles Padgett and the purchase of some lab materials.

PHORBITECH contributors to this deliverable
UGLAS: Martin Lavery (PhD student), Miles Padgett

PROTOTYPE VALIDATION. The quality of the device prototype for OAM sorting and detection based on the phase unfolding concept has been assessed by the PHORBITECH steering committee in its meeting held in Leiden on September 13, 2012, after a detailed presentation of the device features and performances. The committee unanimously judged that the prototype performances meet fully the target goals set in the proposal (the members of the committee who are directly involved in this work did not participate in this final assessment). These include resolving more than 20 OAM states (50 modes resolution has been demonstrated), high-speed (this is only limited by the speed of the detectors utilized for the final detection), efficiency of at least 50% (85% has been demonstrated for the sole mode sorter and 50% when including the effect of all optical components and of the detection camera). The device is also fairly compact and simple to align. The only drawback is a small degree of cross-talk between adjacent modes, which is a fundamental unavoidable limitation of the proposed approach. The convenience of the demonstrated device is also confirmed by the fact that several prototypes of the device have been sent out to different labs (not only within the PHORBITECH consortium) and are currently being used in many different experiments.

Publications included in this deliverable:

1. M. P. J. Lavery, D. J. Robertson, G. C. G. Berkhout, G. D. Love, M. J. Padgett, and J. Courtial, “Refractive elements for the measurement of the orbital angular momentum of a single photon”, Opt. Express 20, 2110-2115 (2012) doi:10.1364/OE.20.002110

2. M. P. J. Lavery, D. J. Robertson, A. Sponselli, J. Courtial, N. K. Steinhoff, G. A. Tyler, A. Wilner and Miles J. Padgett, “Efficient measurement of orbital angular momentum over 50 different states”, submitted to New J. Phys. (preprint included(717,793),(776,914))

3. M. P. J. Lavery, D. Robertson, M. Malik, B. Rodenburg, J. Courtial, R. W. Boyd, and M. J. Padgett. “The efficient sorting of light’s orbital angular momentum for optical communications”, Proc. SPIE 7950, Complex Light and Optical Forces V, 79500E (February 16, 2011); doi:10.1117/12.876119
Refractive elements for the measurement of the orbital angular momentum of a single photon

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Abstract: We have developed a mode transformer comprising two custom refractive optical elements which convert orbital angular momentum states into transverse momentum states. This transformation allows for an efficient measurement of the orbital angular momentum content of an input light beam. We characterise the channel capacity of the system for 50 input modes, giving a maximum value of 3.46 bits per photon. Using an electron multiplying CCD (EMCCD) camera with a laser source attenuated such that on average there is less than one photon present within the system per measurement period, we demonstrate that the elements are efficient for the use in single photon experiments.

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OCIS codes: (060.4510) Optical communications; (050.4865) Optical vortices; (060.5565) Quantum communications; (080.3630) Lenses.

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The desire to increase the amount of information that can be encoded onto a single photon has driven research into many areas of optics. One such area is optical orbital angular momentum (OAM) [1]. The work by Allen et al. in 1992 showed that beams with an transverse amplitude profile of $A(r) \exp(i\ell \phi)$ carry an orbital angular momentum of $\ell \hbar$ per photon [1,2]. An example are Laguerre-Gaussian (LG) beams which have a helical phase structure, with $r$ and $\phi$ as the radial and angular coordinates respectively. The integer $\ell$ is unbounded, giving a large state space in which to encode information [3–7].

The use of diffractive optical elements (DOEs) containing an $\ell$-fold fork dislocation has become commonplace for the generation of beams carrying OAM [8,9]. The forked diffraction grating, when illuminated with a Gaussian beam, for example from a single-mode fibre, produces the helical mode in the first diffraction order. This grating can also be used in reverse to couple light with a helical phase into a single-mode fibre, measuring the power in that mode [10]. Sequentially changing the dislocation in the fork allows a range of $\ell$ values to be measured, but checking for $N$ states require at least $N$ photons [10]. Similar techniques have been demonstrated using spiral phase plates and q-plate technology in place of the DOE [11,12].

A method to route OAM at the single photon level was demonstrated by Leach et al. It required a $N - 1$ Mach-Zehnder interferometers with a Dove prism in each arm [13] for the routing of $N$ states. In principle, this routing can be achieved with 100% efficiency and with no loss of the input beam’s mode structure. However, simultaneously maintaining the alignment of $N$ interferometers has proved technically challenging.

We recently showed that two diffractive optical elements, implemented on spatial light modulators (SLMs), can be used to transform OAM states into transverse momentum states [14]. This was achieved through the use of mapping of a position $(x, y)$ in the input plane to a position $(u, v)$ in the output plane, where $u = -a \ln(\sqrt{x^2 + y^2} / b)$ and $v = a \arctan(y/x)$ [15–17]. A mapping of this type transforms a set of concentric rings at the input plane into a set of parallel lines in the output plane. The combination of the two diffractive optical elements transforms both the phase and intensity of the beam in the form $\exp(i\ell \phi)$, to give a complex amplitude at the output plane of the form $\exp(i\ell v/a)$. A lens can then separate the resulting transverse momentum states into specified lateral positions, allowing for the efficient measurement of multiple states simultaneously [18,19]. In our previous proof-of-principle demonstration, approximately three
quarters of the input light was lost due to the limited diffraction efficiency of the SLMs [14].

In this paper we replace the previously used diffractive optical elements with refractive elements which carry out the desired optical transformation (Fig. 1). The transmission efficiency of the combination of elements is approximately 85%, which makes them attractive for use with single photons. The number of components was also reduced through the integration of the transform lens previously required between the diffractive optical elements into the transformation elements themselves. The height profiles for the refractive elements (Fig. 2) were derived from the equations defining the phase profile of the diffractive elements [14], along with the addition of a lens term, indicated in Eq. (1) and Eq. (2) shown below.

When light of a particular wavelength, $\lambda$, passes through a material of height $Z$, and with a refractive index $n$, the effective optical path length changes with respect to the same distance of propagation in a vacuum. The change in path length can be expressed as a change in phase of $\Delta \Phi = 2\pi(n - 1)Z/\lambda$, hence the first element requires a height profile of

$$Z_1(x,y) = \frac{a}{f(n-1)} \left[ \arctan \left( \frac{y}{x} \right) - x \ln \left( \frac{\sqrt{x^2 + y^2}}{b} \right) + x - \frac{1}{a} \left( \frac{1}{2} (x^2 + y^2) \right) \right], \quad (1)$$

where $f$ is the focal length of the integrated lens. There are two free parameters, $a$ and $b$, which determine the scaling and position of the transformed beam. The parameter $a$ takes the value $a = d/2\pi$, ensuring that the azimuthal angle range ($0 \rightarrow 2\pi$) is mapped onto the full width of the second element, $d$. The parameter $b$ is optimised for the particular physical dimensions of the sorter. The second of these elements has a height profile

$$Z_2(x,y) = -\frac{ab}{f(n-1)} \left[ \exp \left( -\frac{u}{a} \right) \cos \left( \frac{v}{a} \right) - \frac{1}{ab} \left( \frac{1}{2} (u^2 + v^2) \right) \right], \quad (2)$$

Fig. 1. (a) Conversion of OAM states into transverse momentum states with refractive optical elements. An image of the beam was captured in several transverse planes and overlaid (in red) to give the image shown above. (b) A beam carrying OAM is prepared through the use of a $\ell$-forked hologram, realised using a spatial light modulator (SLM) and then passed through the two elements, represented as the green rectangle, required to perform the transformation of both the phase and intensity of the beam.
where $u$ and $v$ are the coordinates in the output plane. This element is placed a distance $f$ behind the first element. Each surface is wavelength independent, but dispersion effects in the material manifest themselves as a change in the focal length of the integrated lens for different wavelengths. Hence, the system can be tuned to a specific wavelength by changing the distance between the elements.

The elements were diamond machined using a Natotech, 3 axis (X,Z,C) ultra precision lathe (UPL) in combination with a Nanotech NFTS6000 fast tool servo (FTS) system to provide a fast (W) axis superimposed on the machine Z-axis. The machining programme was generated using proprietary code written within commercially available software, DIFFSYS. This programme converts the input data, in the form of an X,Y,Z cloud of points, into the requisite UPL machine and FTS system machining files.

Generally, when machining freeform surfaces it is normal to separate out the symmetrical and non-symmetrical components to realise minimum departure, of the FTS tool and therefore maximise machining performance [20]. However, as the total sag height difference for each part was relatively small (115µm for surface 1 and 144µm for surface 2) and as both surfaces are highly asymmetric resulting in a small component of symmetric departure the elements were machined using FTS tool movement in W only. The surfaces are shown in Fig. 2(b) and 2(d).

In our experiment we generate Laguerre-Gaussian (LG) beams by expanding a HeNe laser onto a $\ell$-forked hologram, realised using a SLM, by programming the SLM with both phase and intensity information. The beam generated in the first order of the hologram was selected with an aperture and the plane of the SLM is imaged onto the plane of the first element. The beam is then passed through the elements transforming it into the form exp(\(i\ell v/a\)), giving a transverse direction state which is then focussed into an elongated spot on a camera. The transverse position of the spot is dependent on $\ell$.

An important consideration in any communication system is the cross talk between the channels in that system. To assess this the camera was portioned into $N$ adjacent regions, where each
Fig. 3. (a) Channel capacity for a \( N \) of LG modes, where \( N = 2, 4, 6, \ldots, 50 \). Detector noise was measured with no light incident on the camera, which was overcome by setting a threshold with a signal to noise ratio of 3000 to 1. (b) The ratio of energy measured in each of the detector regions showing the degree of cross talk.

region is centred on one elongated spot, and the measured intensity of the pixels in the region was summed for each region. For a single input mode, one would expect the majority of the energy to be detected in the bin corresponding to the input mode and any energy readings in other regions represent cross talk between channels. Our transformation from orbital angular momentum states into transverse momentum states gives rise to inherent cross talk due to the diffraction limit. The inherent degree of cross talk can be deduced from Fourier theory, which predicts approximately 80% of the input light will be present in the bin corresponding to that input OAM mode value. A common method of evaluating the degree of cross talk in a communications system is the channel capacity, which is the maximum amount of information that can be reliably transmitted by an information carrier [21]. In a multi-channel system, a photon can be in one of \( N \) input states and the maximum channel capacity value is \( \log_2 N \) bits per photon.

To evaluate the range of modes the system is able to detect efficiently, the system is tested using LG beams over the mode range \( \ell = -25 \) to \( \ell = 25 \). The choice of LG beams allows the beam waist to be controlled, and the experimental result to be very closely matched to numerical modelling of the system. The channel capacity was measured for \( N \) modes, where \( N = 2, 4, 6, \ldots, 50 \). For each measurement the range \( \ell = -N/2 \) to \( \ell = N/2 \) was used while leaving \( \ell = 0 \) free as an alignment channel. The values measured are shown in Fig. 3. The optical transformation we utilise is only perfect for rays which are normally incident on the transformation elements. Helically phased beams are inherently not of this type, and have a skew angle of the rays of \( \theta_s = \ell / kr \), where \( k \) is the wavenumber of the light and \( r \) is the distance from the beam centre [22,23]. A numerical simulation of the experimental setup was carried out using plane wave decomposition [14]. Comparing channel capacity values from the simulated and experimentally obtained results, with that of the maximum possible channel capacity, one sees the difference increase at higher mode ranges. These results are consistent with the larger skew angle at higher \( \ell \) causing errors in the transformation, hence increasing the channel cross talk at these \( \ell \) values. Simulations show that reducing the separation between the components or increasing the aperture size of the system can reduce these skew angle effects at higher \( \ell \) values, hence reducing the cross talk within the system.

The optical efficiency of the transformation elements is very important for the use of such a technique within quantum communications. To test that our transformation elements are adequately efficient for use with single photons, we replace the standard camera with an electron multiplying CCD (EMCCD) camera which is sensitive to single photons. The power of the input beam before the first element was attenuated to a power of approximately \( 2 \times 10^{-17} \) W,
Fig. 4. Using a EMCCD camera in single photon counting mode, images were generated by summing over 16383 frames. Each pixel has a dark count rate, generating noise on every pixel in the camera. The images shown are the raw captured images. The dark count rate was assessed by counting the photons over the same capture period with the camera shutter closed. A threshold was set with a value corresponding to the mean, plus one standard deviation of the dark count rate. The corresponding graph is a sum of each column, in blue, and superimposed with the results when a Wiener Noise reduction filter is applied shown in red. Summing under the red curve gives us an approximation of the number of photons received at the camera plane.

corresponding to approximately 75 photons per second entering the first element. The camera was set to capture 100 frames per second, hence on average we record less than one photon per measurement.

To verify that our measurement corresponds to the expected number of photons, we first measure the unattenuated power before entering the first element and at the camera plane giving a measured efficiency of approximately 75%, 10% lower than the combination of transformation elements. This difference arises from the losses due to scatter of the other optical components. The efficiency could be further improved by adding anti-reflective coating to the elements. A measurement of the efficiency at the level of single photons was made by counting the number of photons detected over a large number of accumulated camera frames when used in single photon counting mode. The images produced are shown in Fig. 4. The quantum efficiency of the entire system (including the effects of all optical components and the efficiency of the EMCCD camera) was measured to be approximately 50%.

In conclusion, we have developed a mode transformer comprising two refractive elements which can separate beams carrying OAM into discrete regions on a detector with an efficiency of 85%. In the case of many photons, the experimental system was characterised to have a channel capacity for 8 modes of 1.85 bits per photon, 16 modes of 2.68 bits per photon and for 32 modes of 3.26 bits per photon. An attenuated laser source, where on average there was less than one photon in the system within any measurement period, demonstrates the elements are capable of separating the OAM states of the input light at the level of single photons. This approach could be used to generate and detect OAM states used within quantum communications or quantum key distribution systems, increasing the amount of information one can encode onto a single photon.

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Efficient measurement of orbital angular momentum over 50 different states

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Abstract. Previously we have demonstrated that the orbital angular momentum (OAM) of the light beam may be measured by image transformation mapping the azimuthal to linear transverse coordinate. For each input OAM state the light is focused to a different transverse position enabling simultaneous measurement over many states (Lavery et al. (2012) Opt. Express, 20, 3). Here we significantly improve our earlier design, extending the measurement bandwidth to > 50 OAM states and showing simultaneous measurement of the radial coordinate.

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The information bandwidth of optical communications systems are constantly being pushed by our requirements to transmit larger amounts of data across our networks. In an attempt to increase the available bandwidth the use of various optical characteristics other then just polarisation, wavelength and intensity, have become an area of interest to many researchers optical communications field. One such additional characteristic is orbital angular momentum (OAM) [1]. Allen et al. showed that beams with an transverse complex amplitude profile of $\psi_x = A(r) \exp(i \ell \theta)$, with $r$ and $\theta$ as the radial and angular coordinates respectively with a real amplitude of $A(r)$, carry an orbital angular momentum of $\ell h$ per photon [1, 2]. The variable $\ell$ is an unbounded integer, potentially leading to access of a larger alphabet in both free space [3, 4, 5, 6] and fibre based communications schemes [7]. In addition to classical considerations, this larger alphabet has been suggested to improve the security of cryptographic keys transmitted with a quantum key distribution system [8].

We recently developed two freeform refractive optical elements, that can be used to transform OAM states into transverse momentum states [9, 10]. Acting together these elements map a position $(x, y)$ in the input plane to a position $(u, v)$ in the output plane, where $u = -a \ln(x^2 + y^2)/b$ and $v = -a \arctan(y/x)$, where $a$ and $b$ are constants relating to the size of the optical components [9]. This mapping transforms a set of concentric rings at the input plane into a set of parallel lines in the output plane and hence transforms azimuthal phase term $\exp(i \ell \theta)$ into a transverse phase gradient. A spherical lens placed after this second element then separates the resulting transverse momentum states into specified lateral positions in its back focal plane $(u', v')$, thus allowing for the efficient measurement of multiple OAM states simultaneously. A specific input value of $r$ maps to a specific value of $u$ and hence the full range of $u'$, meaning that $r$ cannot be measured by this configuration.

Key to the transformation is that $u, v$ should depend only on $x, y$ and not on the local ray direction. Our reformatter works by the first element of radius $r$ introducing a spatially dependent angular deviation to the local ray direction, such that upon propagation over a distance, $L$, to the second element the angular deviation gives the require transverse displacement. The role of the second element is to undo the angular deviation introduced by the first. This approach works for collimated beams, incident at normal incidence to the first component. For a non collimated beam we require that the angles associated with reformating, $\approx r/L$ dominate over the angles associated with the beam divergence, alignment or other properties. Irrespective of their divergence, beams carrying OAM are never collimated since we require an azimuthal component to the Poynting vector to give the angular momentum in the direction of propagation, see Fig.1. For all beams described by an azimuthal phase term $\exp(i \ell \theta)$, the local skew angle of the Poynting vector, and hence ray direction, is given by $\beta = \ell/kr$[11]. Hence for accurate reformating we require $\ell/kr << r/L$, which gives

$$\ell << \frac{2\pi r^2}{L\lambda}$$  \hspace{1cm} (1)

we note that the RHS of this equation is proportional to the Fresnel number of the reformating optics.

In this article we discuss a solution to mitigate the effects arising from the inherent skew angle of beam carrying OAM, allowing us to increase the bandwidth of the sorter. In addition we present a further modification which allows for the separation of beams of diffraction radius in $r$ in the detector plane. Increasing the OAM measurement
bandwidth is simply a matter of increasing the Fresnel number of the transforming optics. This is a technical limitation based upon their method of manufacture, both in terms of the largest possible aperture and the gradient of any surface. The ability to measure \( r \) is more subtle. When using a spherical lens the mapping of \( u \rightarrow u' \) and \( v \rightarrow v' \) is a Fourier transform. If a second cylindrical lens is added to image of the out of the sorter, such that \( v \rightarrow mv \), with some magnification \( m \), while maintaining the mapping \( u \rightarrow u' \), an input value of \( r \) is mapped to a specific value of \( mv \). Hence, it is now possible to measure OAM and radial co-ordinate simultaneously.

To test the connection between the Fresnel number and the detection bandwidth we machined optical elements with a larger optical radius, of 12\( \text{mm} \), separated from each other by \( L = 300\text{mm} \), giving a maximum Fresnel number of approximately \( F = 1500 \). The elements were diamond machined using a Natotech, 3 axis (X,Z,C) ultra precision lathe (UPL) in combination with a Nanotech NFTS6000 fast tool servo (FTS) system to provide a fast (W) axis superimposed on the machine Z-axis. This increase in the overall aperture size results in deeper cuts as overall thickness variation of the surface has increased. A vacuum chuck is generally used in such a lathe, however due to the large cut depth a semi-permanent bonding technique is used. A water soluble wax, produced by Nexgen Optical, was used as the bonding agent as this can easily de-bonded with the use of a standard ultrasonic system. Although the aperture of the components sets an upper limit to the Fresnel number, the effective Fresnel number is set by the radius or the optics, or the input beam, whichever is the smaller. The output plane of the reformatted was transformed to the plane of the detector using \( L_3 = 500\text{mm} \) spherical lens and \( L_4 = 40\text{mm} \) cylindrical lens giving a lateral displacement of 17.6\( \mu \text{m} \) for an increase \( \Delta = 1 \) in OAM value and a radial magnification of 0.096, see Fig. 2.

We generate the input OAM test modes by the use of a simple forked diffraction grating created using a spatial light modulator (SLM) that is illuminated by the expanded Gaussian beam produced by a He Ne laser. Rather than producing a pure
Laguerre-Gaussian mode, this results in a helically phased beam with a Gaussian intensity distribution. To obtain radial control of the intensity distribution we apply an spatially dependent reduction of the contrast allowing us to create in the first order diffracted beam a close approximation of any Laguerre Gaussian mode of choice [12]. This approach allows control over the beam waist $w_0$ independent of the mode index. The SLM is then imaged to a chosen proportion of the diameter input pupil of the OAM mode sorter. Varying $w_0$ allows a set of different effective Fresnel numbers to be tested and the corresponding OAM bandwidth of the reformatter to be measured.

We chose an input test range of $\ell = \pm28$, at a several specific value of $w_0$. A camera was placed at the focal plane of the final lens where, adjacent, equally sized regions were selected, each corresponding to a specific $\ell$-value. The total intensity over all the pixels in each region was summed to give the relative power in each of the OAM modes. As discussed in our earlier work, this approach has some residual crosstalk which arising from the mapping of the periodic angular variable to the transverse variable of finite aperture, resulting in diffraction [9, 10] which is manifest as off-diagonal components Fig. 3. The inherent degree of cross talk can be deduced from simple Fourier theory, which predicts approximately 80% of the input light will be present in the bin corresponding to that input OAM mode value [10]. It can be seen that the there is an increased in crosstalk for higher $\ell$ values, Fig.3 (a) which arises from the mapping errors discussed above. However, when the effective Fresnel number is increased Fig.3 (b),(c) the crosstalk within for these higher $\ell$ values is reduced.

To show that this modified configuration can measure OAM and $r$ simultaneously we generated a superposition of 8 different modes $\ell = -6, -3, +3, +6$, with beam waists $w_0 = 0.4\text{mm}$ and $2\text{mm}$ and $\ell = 0$ with a beam waist of $2\text{mm}$. Both numerically modelled and experimentally measured results as shown in Fig. 4. It should be noted...
Efficient measurement of orbital angular momentum over 50 different states

Figure 3. (a-c) An increase in the Fresnel number of the system was achieved by changing the beam waist of the input mode into the sorter. The increase in measurement bandwidth is denoted by the green arrows. (d) For clarity the fraction of power in the correct measurement bin $\Delta \ell$ and crosstalk into bins $\Delta \ell_{\pm 1}$ and $\Delta \ell_{\pm 2}$. 
Efficient measurement of orbital angular momentum over 50 different states

By imaging along the \( v' \) axis, sorting radial component \( r \) is made possible. Simulation of an input mode, (a), \( \ell = -6, -3, 0, 3, 6 \) with a beam waist of \( w_0 = 2\text{mm} \), and \( \ell = -6, -3, 3, 6 \) with a beam waist of \( w_0 = 0.4\text{mm} \). The contrast in (a) has been altered to show the modes fine structure. Simulated output is where \( \ell \) is sorted left to right and \( r \) sorted up and down, were \( w_0 = 2\text{mm} \) corresponds to the \( r_1 \) line, and \( w_0 = 0.4\text{mm} \) to the \( r_2 \) line (b) along. (c) Experimental measurement of the same modes.

that sorting of different beam waist can increase the available channels, but effective Fresnel number of the system is different for each \( w_0 \) and hence smaller beams will have a more restricted \( \ell \) bandwidth.

In conclusion, we have presented a scalable solution to increasing the available OAM bandwidth of a mode sorter based on reformatting, and the demonstrated the diffraction limited sorting of over 50 modes. A further improvement has been presented where a change in the beam waist of an input mode is mapped onto a specific position in output plane, allowing for the sorting of modes spatially separated in \( r \). These improvement will increase the feasibility of such a sorting technique being implemented as a tool in the fields of optical communications and quantum optics.

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The efficient sorting of light’s orbital angular momentum for optical communications

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ABSTRACT

We have developed a mode transformer comprising two custom refractive optical elements which convert orbital angular momentum states into transverse momentum states. This transformation efficiently measures the orbital angular momentum content of an input light beam, allowing the decoding of OAM states within a free-space communications channel. Turbulence is a key issue within such a channel. Through the use of a phase only spatial light modulator, turbulence is simulated, and the cross talk between detected OAM modes is observed. We study this crosstalk for eleven OAM modes, showing that turbulence equally degrades the purity of all the modes within this range.

Keywords: orbital angular momentum, turbulence, optical communications

1. INTRODUCTION

We are transmitting ever larger amounts of data across optical communications links, and demand is pushing research into many optical properties. One such property is optical orbital angular momentum (OAM).\textsuperscript{1} The work by Allen \textit{et al.} in 1992 showed that beams with a transverse complex amplitude profile of the form $A(r) \exp(i\ell \phi)$ (where $r$ and $\phi$ are the radial and angular coordinates, respectively, and $A(r)$ is the radial part of the amplitude profile) carry an orbital angular momentum of $\ell \hbar$ per photon.\textsuperscript{1,2} An example for such beams are Laguerre-Gaussian (LG) modes, which have a helical phase structure. The integer $\ell$ is unbounded, giving a large state space in which to encode information.\textsuperscript{3–8}

The use of diffractive optical elements (DOEs) containing an $\ell$-fold fork dislocation has become commonplace for the generation of beams carrying OAM.\textsuperscript{9,10} The forked diffraction grating, when illuminated with a Gaussian beam, for example from a single-mode fibre, produces the helical mode in the first diffraction order. This grating can also be used in reverse to couple light with a helical phase into a single-mode fibre, to measure the power in that mode.\textsuperscript{11} Sequentially changing the number of dislocations in the fork allows a range of $\ell$ values to be measured, but checking for $N$ states require at least $N$ photons.\textsuperscript{11} Similar techniques have been demonstrated using spiral phase plates and q-plate technology in place of the DOE.\textsuperscript{12,13} A method to route OAM at the single photon level was demonstrated by Leach \textit{et al.} It required $N – 1$ Mach-Zehnder interferometers with a Dove prism in each arm\textsuperscript{14,15} for the routing of $N$ states. In principle, this routing can be achieved with 100% efficiency and with no loss of the input beam’s mode structure. However, simultaneously maintaining the alignment of $N$ interferometers has proved technically challenging.

We previously showed that two phase-only diffractive elements can be used to transform OAM states into transverse momentum states.\textsuperscript{16} This was achieved through the use of mapping of a position $(x, y)$ in the input

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Figure 1. (a) Conversion of OAM states into transverse momentum states with refractive optical elements. An image of the beam was captured in several transverse planes and overlaid (in red) to give the image shown above. (b) A beam carrying OAM is prepared through the use of a $\ell$-forked hologram, realised using a spatial light modulator (SLM) and then passed through the two elements, represented as the green rectangle, required to perform the transformation of both the phase and intensity of the beam.

plane to a position $(u, v)$ in the output plane, where $u = -a \ln(\sqrt{x^2 + y^2}/b)$ and $v = a \arctan(y/x)$. A set of concentric rings in the input plane is transformed into a set of parallel lines in the output plane, but the combination of the two diffractive optical elements transforms not only the intensity of the beam but also its phase: the azimuthal phase term $\exp(i\ell\phi)$ in the input plane becomes, in the output plane, $\exp(i\ell v/a)$, i.e. the beam now has a transverse momentum $\hbar \ell/a$ in the $v$ direction. A lens can then separate the resulting transverse momentum states into specified lateral positions, allowing for the efficient measurement of multiple states simultaneously.

Separating OAM states in this way presents an opportunity for this larger alphabet to improve the data capacity of a communications link and has potential applications both in the classical and quantum regime. For example, in free-space communications links the use of an alphabet of OAM could present advantages over other encoding techniques. However, the natural random time-dependent variations in temperature and pressure of the atmosphere result in changes in density of the atmosphere, which in turn give a spatially-dependent change of the refractive index, leading to a phase distortion across a transmitted beam. A phase distortion of this type can be approximated to a phase screen, and is commonly referred to as thin phase turbulence. Such a phase distortion is a concern for any free space communications channel where the atmospheric turbulence may affect the cross-talk between channels, an important consideration in any communication system.

We use two bespoke refractive elements which carry out the optical transformation described above (Fig.1). Hence the elements can be used as a decoder for information encoded with OAM states. We then use these elements to experimentally study the effects of atmospheric turbulence on a communications system utilising OAM modes as the information carrier.

The height profiles for the refractive elements were derived from the equations defining the phase profile of the diffractive elements presented in our earlier work, along with the addition of a lens term, indicated in equations (1) and (2) shown below. When light of a particular wavelength, $\lambda$, passes through a material of height $Z$ and with a refractive index $n$, the effective optical path length changes with respect to the same distance of propagation in a vacuum. The change in path length can be expressed as a change in phase of
Figure 2. (a) A beam carrying OAM is prepared through the use of a $\ell$-forked hologram, seen in (b), realised using a spatial light modulator (SLM), illuminated by an expanded He-Ne laser. The first order beam is imaged onto the front aperture of a OAM mode sorter (MS) which converts OAM states into transverse momentum states with the use of two refractive optical elements. These transverse momentum states are then focused to specific spatial locations on a CCD. The power measured in each of these locations gives a measure of the OAM superposition incident on the mode sorter.

Thin phase turbulence is added to the $\ell$-forked hologram changing the OAM superposition measured by the system.

\[ \Delta \Phi = 2\pi(n - 1)Z/\lambda, \]

hence the first element requires a height profile of

\[ Z_1(x, y) = \frac{a}{f(n - 1)} \left[ y \arctan \left( \frac{y}{x} \right) - x \ln \left( \frac{\sqrt{x^2 + y^2}}{b} \right) + x - \frac{1}{a} \left( \frac{1}{2} (x^2 + y^2) \right) \right], \tag{1} \]

where $f$ is the focal length of the integrated lens. There are two free parameters, $a$ and $b$, which determine the scaling and position of the transformed beam. The parameter $a$ takes the value $a = d/2\pi$, ensuring that the azimuthal angle range $(0 \rightarrow 2\pi)$ is mapped onto the full width of the second element, $d = 8$mm. The parameter $b$ is optimised for the particular physical dimensions of the sorter. The second of these elements has a height profile

\[ Z_2(x, y) = -\frac{ab}{f(n - 1)} \left[ \exp \left( -\frac{u}{a} \right) \cos \left( \frac{v}{a} \right) - \frac{1}{ab} \left( \frac{1}{2} (x^2 + y^2) \right) \right], \tag{2} \]

where $u$ and $v$ are the coordinates in the output plane. This element is placed a distance $f$ behind the first element. Each surface is wavelength independent, but dispersion effects in the material manifest themselves as a change in the focal length of the integrated lens for different wavelengths. Hence, the system can be tuned to a specific wavelength by slightly changing the distance between the elements.

In our experiment we generate Laguerre-Gaussian (LG) beams by expanding a HeNe laser onto an $\ell$-forked hologram (Fig. 2(b)), realised using an SLM, by programming the SLM with both phase and intensity information. The beam generated in the first order of the hologram was selected with an aperture and the plane of the SLM is imaged onto the plane of the first element. The beam is then passed through the elements transforming it into the form $\exp(i\ell\nu/a)$, giving a transverse direction state which is then focused into an elongated spot on a camera. The transverse position of the spot is dependent on $\ell$.

Our transformation from orbital angular momentum states into transverse momentum states gives rise to inherent cross talk due to the diffraction limit. The inherent degree of cross talk can be deduced from Fourier theory, which predicts approximately 80% of the input light will be present in the bin corresponding to the input OAM mode value. To experimentally assess the inherent crosstalk of the system, a camera was portioned into $N$ adjacent regions, where each region was centred on one of the elongated spots that correspond to a particular value of $\ell$, and the measured intensity of the pixels in the region was summed for each region. For a single input
mode, one would expect/hope for the majority of the energy to be detected in the bin corresponding to the input mode; any energy readings in other regions represent cross talk between channels. A common method of evaluating the degree of cross talk in a communications system is the channel capacity, which is the maximum amount of information that can be reliably transmitted by an information carrier. In a multi-channel system, a photon can be in one of \( N \) input states and the maximum channel capacity value is \( \log_2 N \) bits per photon.

To evaluate the range of modes the system is able to detect efficiently, the system is tested using LG beams over the mode range \( \ell = -25 \) to \( \ell = +25 \). LG modes were specifically chosen to allow precise control of the beam waist, and for the experimental result to be most closely matched to our numerical modelling of the system. The channel capacity was measured for \( N \) modes, where \( N = 2, 4, 6, ..., 50 \). For each measurement, the range \( \ell = -N/2 \) to \( \ell = N/2 \) was used while leaving \( \ell = 0 \) free as an alignment channel. The values measured are shown in Fig. 3.

The optical transformation we utilise is only perfect for rays which are normally incident on the transformation elements. Light carrying OAM, such as LG modes, is inherently not of this type as the rays have a skew angle of \( \theta_s = \ell / kr \), where \( k \) is the wavenumber of the light and \( r \) is the distance from the mode’s centre. A numerical simulation of the experimental setup was carried out using plane wave decomposition. Comparing channel capacity values from the simulated and experimentally obtained results with that of the maximum possible channel capacity, one sees the difference increase at higher mode ranges. Such results are consistent with the larger skew angles at higher \( \ell \) causing errors in the transformation, hence increasing the channel cross talk at these \( \ell \) values. Our simulations show that a reduction in crosstalk at higher \( \ell \) values can be achieved by an increase in separation between the components or an increase in the aperture size of the system. A study of this effect has recently been submitted.

We introduce precisely simulated amounts of thin phase turbulence through the use of a phase only spatial light modulator. Once the turbulence is introduced, we can measure the resulting cross talk between OAM modes and relate this to atmospheric conditions. The phase screen corresponding to a particular turbulence strength is generated by considering Kolmogorov turbulence theory. The aberrations introduced by atmospheric turbulence can be considered as normal random variables, where the ensemble average can be written as \( \langle [\phi(r_1) - \phi(r_2)]^2 \rangle \) and is known as the phase structure function. \( \phi(r_1) \) and \( \phi(r_2) \) are two randomly generated phase fluctuations. From Kolmogorov statistics it can be shown that this ensemble average must meet the requirement that

\[
\langle [\phi(r_1) - \phi(r_2)]^2 \rangle = 6.88 \left( \frac{r_1 - r_2}{r_0} \right)^{5/3}.
\]

The value \( r_0 \) is the Fried parameter, and is a measure of the transverse distance scale over which the refractive index is correlated. To characterise the effect of turbulence on the optical system, a ratio \( D/r_0 \) is considered,
Figure 4. Power, $s$, in modes with indices $\ell + \Delta$, after an incident mode with index $\ell$ has been propagating through a range of turbulent phase screens with strength characterised by $D/r_0$. (a) $\ell = 0$; (b) other values of $\ell$. For each experimentally measured value of $D/r_0$, denoted as crosses, the measured power in each detected mode was averaged over 100 randomly generated phase screens. These modal powers were then co-plotted against the theoretical predictions given by equation 4, shown as lines.

where $D$ is the aperture of the system. This ratio sets two limiting cases: in the first case, when $D/r_0 < q_l$, the resolution of the system is limited by its aperture; in the second case, when $D/r_0 > q_l$, the atmosphere limits the system’s ability to resolve an object.

In 2005, Paterson predicted the spread in the OAM spectrum resulting from thin phase turbulence. For a single OAM mode with index $\ell$, transmitted through an ensemble average of many turbulent phase screens, the power, $s$, in a particular mode with index $\ell + \Delta$ ($\Delta = 0, \pm 1, \pm 2, \ldots$) is given by

$$s = \frac{1}{\pi} \int_0^1 \rho \, d\rho \int_0^{2\pi} d\theta \, e^{-3.44 \left[ \left( \frac{\rho}{\pi} \right)^2 \left( \rho \sin \frac{\Delta}{2} \right)^{\frac{1}{3}} \right]} \cos \Delta \theta,$$

(4)

where $\rho = 2\pi / D$.

Rather than producing a pure Laguerre-Gaussian mode, we produce helically phased modes where the radius of the mode is specifically controlled and the plane of the SLM then imaged, giving a near Gaussian intensity distribution. This approach maintains the ratio $D/r_0$ independent of the mode index. A particular turbulent phase screen can then be added to this hologram to simulate the presence of atmospheric turbulence. The SLM is then imaged to the 8 mm diameter input pupil of the OAM mode sorter to decompose the resulting beam into its constituent OAM modes.

To measure the effect of turbulence on the channel cross talk, we consider a smaller mode range of $\ell = -5$ to $\ell = +5$. We consider 24 different values of $D/r_0$; for each value, 100 randomly generated phase screens are applied to the input mode and the OAM spectrum was measured (Fig. 4). The power was measured across all the 11 regions and normalised with respect to the power measured for $\ell = 0$ with no turbulence applied. The measured power was then plotted as a function of the turbulence strength, $D/r_0$. As predicted by equation (4), the crosstalk between OAM modes increases with turbulence and in this mid/high turbulence regime we find the agreement between theory and our measurements is good.

The weightings of the known input states can be described by an $N = 11$ element column vector, $(I_0, I_1, \ldots)^T$. These are mapped by an $N \times N$ cross-talk matrix onto the measured $N$-element output vector $(O_0, O_1, \ldots)^T$:

$$
\begin{bmatrix}
O_0 \\
O_1 \\
\vdots \\
O_N
\end{bmatrix} =
\begin{bmatrix}
1 - g & a & \ldots & b \\
1 - h & d & \ldots & \ldots \\
\vdots & \ddots & \ddots & \vdots \\
1 & \ldots & f & \ldots
\end{bmatrix}
\begin{bmatrix}
I_0 \\
I_1 \\
\vdots \\
I_N
\end{bmatrix}.
$$

(5)
For the case of zero residual crosstalk, this matrix would have a leading diagonal of 1’s and 0’s elsewhere. For finite crosstalk, the coefficients of the crosstalk matrix are measured at zero turbulence and then this matrix is used to predict the measured OAM output spectrum for an input OAM state subject to the atmospheric cross-talk from the Paterson model (Eqn (4)).

It is seen in Fig. 4 that, at high turbulence values, where $D/r_0 \gg 1$, the average power is spread between all the possible detections modes. It should be noted that we are only considering the proportion of the power detected within the detector regions and not considering the power incident outside our detectors.

Paterson’s earlier work indicates that the probability of modal cross talk resulting from atmospheric turbulence is independent of the mode propagating through that turbulence. For each of these OAM modes in the range from $\ell = -5$ to $\ell = +5$, the same set of turbulent phase screens was applied, and cross-talk measured (Fig. 4(b)). We note that the observed cross-talk is indeed similar for the range of OAM modes we examine.

In this work we have studied the case where turbulence can be considered as thin phase screen. Such an approach is widely used in astronomy, as when one considers the distance to an astronomical light source, the largest proportion of the turbulence is experienced, relatively, very close to the observer. In the case of long distance point to point communications on earth, turbulence is characterised more accurately by multi-plane turbulence, however, we expect similar principles to apply in the two cases.

In conclusion, we have presented a method utilising bespoke optical elements to sort OAM for a wide mode range. We have studied the effect of turbulence on an optical communications channel that uses an OAM-based alphabet. The effects of turbulence were experimentally shown to be independent of mode number and hence a well known turbulence mitigation systems could be implemented to preserve the integrity of the communications link. Such a optical communications link could be attractive to both classical and quantum optical systems.

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