The Large Scale Structures in the Solar System:
I. Cometary Belts With Resonant Features
Near the Orbits of Four Giant Planets

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ABSTRACT

We employ an efficient numerical approach to simulate a stationary distribution of test objects, which results from their gravitational scattering on the four giant planets, with accounting for effects of mean motion resonances. Using the observed distribution of the Kuiper belt objects, we reconstruct, in the space of orbital coordinates, the distribution function $n(a, e, i)$ for the population of minor bodies beyond Jupiter. We confirm that thousands of large yet cold comets and Centaurs might be located between the orbits of Jupiter and Neptune. Moreover, we find as an important result that they are concentrated into four circumsolar belts, with a highly non-uniform and well structured distribution of the objects. This huge yet unrevealed population, with only a few of its representatives presently known, is expected to have, like our simulations demonstrate, a rich resonant structure containing both density maxima and gaps. The resonant structure is formed due to gravitational perturbations, i.e. in a non-dissipative way. If plotted in the $(a, e, i)$-space of orbital coordinates, the belts contain gaps (including those between resonant groups), quite similar to the Kirkwood gaps in the main asteroid belt. An appreciable fraction of the test bodies reveals, for some time, an accumulation near (rather than in) the resonances, both interior and exterior, with the giant planets.

An accompanying paper considers the population simulated in this work as the major source of dust in the outer Solar system. The simple but fast and efficient numerical approach employed in this work would allow the reader for applying it to many other problems of his/her interest.
1. Introduction

Not much is known about the size of the population of minor bodies of the Solar System between the orbits of Jupiter and Neptune, nor how these bodies are distributed there. The known representatives of that population, the Centaurs, are the objects with \( 6 \lesssim a \lesssim 26 \) AU intermediate between comets and asteroids, having presumably short lifetimes compared to the Solar system’s age (Asher & Steel 1993). These objects, as well as the other known populations, such as the Jupiter-family comets, could have a common origin in the Kuiper belt (Luu & Jewitt 1996). For Jupiter-family comets, an alternative evolutionary path from the Oort cloud has been found unlikely (see e.g. review by Kresák 1994 and refs. therein). If the Kuiper belt objects (called ‘kuiperoids’ hereinafter) are indeed responsible for progressive replenishment of the observable populations, gravitational scattering on all four giant planets would be responsible for the transport of these objects from the trans-Neptunian region all the way inward, down to Jupiter (Levison & Duncan, 1997). The present paper aims at a related, but rather different subject, viz. computing the distributions of minor bodies between Neptune and Jupiter, including analysis of those distributions in the space of orbital coordinates, \( a, e, i \). An accompanying paper deals with the distribution of dust expected to be produced by these bodies.

2. Processes Accounted for: Gravitational Scattering and Influence of Resonances

Gravitational scattering of test bodies on the planets (Carusi, Valsecchi, & Greenberg, 1990) and influence of resonances (Jackson & Zook 1989, Roques et al. 1994) have been studied previously by a number of investigators. Here, we undertake a combined numerical study of these effects to demonstrate that, when put together, they mutually enhance each other by creating qualitatively new phenomena. While using a different numerical approach and employing different tools to analyze the computational results, we also make emphasis on a different subject – the stationary distributions of comets near each of the giant planets.
and its structure in the space of orbital coordinates. [A systematic use of distributions in these coordinates is a part of the kinetic approach employed by us in studying the structure, evolution, and origin of the interplanetary dust and its sources (Gor’kavyi et al. 1997a,b, 1998)]. Furthermore, as opposed to the usual consideration of a dissipative mechanism of captures into mean motion resonances (e.g., due to the tidal dissipation or Poynting-Robertson drag), we deal here with an entirely different type of non-dissipative captures, when the Tisserand constant is conserved. Indeed, gravitational scattering as the only accompanying process in our consideration here is an elastic process. A dissipationless capture of a minor body into a mean motion resonance with the planet becomes possible at a large eccentricity of a minor body, when an exchange of angular momentum between the planet and the minor body becomes especially effective due to their very close approach.

In our numerical study, we adopt the approximations of a restricted 3-body problem (the Sun, the planet on a circular orbit, and a massless minor body). It is convenient to define the planet’s zone of gravitational influence in the \((a, e)\)-plane of orbital coordinates:

\[
\begin{align*}
a(1 - e) & \leq a_p \quad \text{if} \quad a > a_p, \\
 a(1 + e) & \geq a_p \quad \text{if} \quad a < a_p,
\end{align*}
\]

where \(a\) is the semi-major axis of a test body, \(a_p\) is the semi-major axis of the planet, and \(e\) is eccentricity of the test body. This zone looks like a triangle and we call it hereinafter ‘the triangle zone’.

Our study examines the distribution of minor bodies within each planet’s zone of influence. We also evaluate qualitatively how the bodies are scattered from one zone to another.

Obviously, a restricted 3-body problem is a highly simplified approach to examine the distribution of minor bodies in the gravitational field produced by the four giant planets. However, as the reader will see below, even this approximation reveals a rich and sophisticated structure in the distribution of test bodies. Our follow-up research will study how the revealed distribution is modified when the influence of three other planets, with
accounting for non-circular orbits as well, is taken into consideration.

3. Computational Method: Simulation of a Quasi-stationary Distribution of Test Bodies in the Planet’s Zone of Influence

A quasi-stationary distribution of minor bodies has been established in the outer parts of the Solar system due to multiple gravitational scatterings on the four giant planets. Each giant planet maintains this quasi-stationarity in its zone of gravitational influence by ejecting the minor bodies in the amounts comparable to what flows into this zone from outside. As a convenient approach to simulate such a quasi-stationary distribution of massless minor bodies around a giant planet, we applied the following computational procedure: a record of coordinates and velocities of a test body was taken after certain number of revolutions (usually each 10 revolutions) of the planet around the Sun and these data were then used to characterize the positions of many test bodies over the entire time span, beginning at an initial instant, and ending at the instant of ejection of the test body from the planet’s zone of influence.

We computed 44 stationary distributions of test bodies totalling $0.5 \times 10^6$ ($a, e, i$)-orbital elements. Details of computational runs are given in Table 1.

%% PUT TABLE 1 HERE%%

Meanwhile a small fraction of minor bodies is scattered from the zone of influence of each outer giant planet into the next innermost giant neighbor’s zone. As a result, there is a flow of these objects from the trans-Neptune region inward the Solar system.

4. The Results: Four Cometary-Asteroidal Belts and Their Resonant Structure

Our results of orbit integrations are shown, in the orbital coordinates $a, e$ and $a, i$, for Neptune in Fig. 1a,b; for Uranus in Fig. 3a,b; for Saturn in Fig. 5a,b; and for Jupiter in
4.1. Four Cometary-Asteroidal Belts

The minor bodies (kuiperoids and Centaurs) from the trans-Neptune regions can be captured into mean motion resonances with Neptune (Morbidelli 1997, Levison & Duncan 1997, Malhotra 1998). A minor body being in a resonance with Neptune increases its eccentricity, approaches closely the planet and undergoes a strong gravitational scattering on it. The scattering results in filling a quasi-triangle zone with numerous resonances and gaps (see Fig. 1a).

The minor bodies populating Neptune’s zone of gravitational influence should form a broad belt along Neptune’s orbit – the ‘Neptune belt’. Furthermore, a fraction of minor bodies from Neptune’s quasi-triangle zone is intercepted by Uranus and fills in the quasi-triangle zone of the latter in the same fashion as it happens for Neptune’s zone of influence. A similar process of ‘leakage’ of a part of minor bodies occurs from the belt of each outer giant planet into the next innermost giant neighbor’s sphere of influence. Some of them (e.g. Uranus) are able to eject, by gravitational scattering, a minor body with such a high velocity in a particular direction that the body can reach Jupiter.

As a net result, there is a flow of these objects from the trans-Neptune region downward to Jupiter. The minor bodies form a rather wide belt around each giant planet’s orbit. Figs. 1a, 3a, 5a, and 7a indicate that the bulk of test bodies is located within each planet’s triangle zone of influence, although a substantial number of bodies form a ‘beard-like’ distribution with lower eccentricities.

4.2. Resonant Structure of the Four Belts

Two types of structures in the zone of gravitational influence of each giant planet are clearly seen in Figs. 1 to 8: (i) resonant gaps and (ii) resonant groups of test bodies.
There is a substantial difference between them: gaps represent a robust feature, whereas probability to be captured into a resonant group is rather sensitive to the number of computational runs used.

We would like to emphasize an interesting phenomenon of a near-resonance accumulation: the test bodies tend to avoid location directly within the resonances, both interior and exterior ones. Instead, they prefer to be positioned slightly aside, mostly at both sides of the resonant gaps. Those gaps seem to be similar to the well-known Kirkwood gaps in the main asteroid belt.

The locations of gaps are clearly seen from the number of test bodies as a function of semi-major axis, which is shown, for each of the giant planets, in Figs. 2, 4, 6, and 8. Occurrence of locations is a measure of a probability for a test body, \( p(a) \), to be located between \( a \) and \( a + da \) (with \( a \) measured in \( a_{\text{planet}} \), where \( da = 0.05 \) and the total probability to be in the planet’s triangle zone is \( \int p(a) \, da = 1 \).

There are two possible mechanisms for gap formation. The first one is associated directly with gravitational scattering of a test body on the planet. The scattering can be stronger when the minor body’s period of revolutions is commensurable with the planet’s period, and this is not accompanied by the resonant capture thereby producing a gap. The second mechanism is associated with the resonance influence: by getting an excessive angular momentum from the planet, the minor body leaves the resonance.

When the minor body leaves the resonance, the balance of the angular momentum forces the minor body to librate, thereby spending much of the time at the edge of the resonant region. This results in a near-resonant accumulation. We observe numerous examples of such accumulations (see e.g. Figs. 3 and 5). Without going into a detailed classification, we will not discriminate here between usual resonance and near-resonant accumulation.

Test bodies captured into resonances in the triangle zone, undergo a characteristic pattern of evolution in \( e \) and \( i \) within the resonances: As long as the eccentricity of the
minor body captured into a particular resonance increases (decreases), its inclination (shown in Figs. 1b, 3b, 5b, and 7b) decreases (increases) so that the Tisserand constant, $T$, is kept invariable. In our computations, $T$ is only insignificantly (at the typical level of a fraction of 1%) fluctuates.

A detailed consideration of the dynamics leading to the capture into, as well as strength of, resonances when gravitational scattering is the only accompanying process is considered elsewhere (Gor’kavyi & Ozernoy 1999).

4.3. The Expected General Picture of the Belts

For illustrative purposes, in Fig. 9 we show in $(a,e)$-coordinates the positions of all four cometary belts we expect to exist near the orbits of the four giant planets. The known minor bodies in the main asteroid belts and the known Centaurs are also shown Fig. 9. Although this picture neglects eccentricities of the giant planets as well as gravitational perturbations from three other giant planets in each planetary zone of influence (thereby secular resonances are out of consideration here), the resonant patterns in each zone are expected to be preserved. It is remarkable that, as is seen in Fig. 9, at least 4 of 6 known Centaurs are located close to the resonances.

Finally, Figs. 10 and 11 illustrate, in the above approximations, the large-scale structure of the Solar system formed by the cometary-asteroidal belts envisaged in this paper. The maxima in minor body concentrations clearly delineate the positions of these belts, although the actual density contrasts should be determined in further, more detailed computations.

5. Conclusions

1. As our simulations indicate, the distribution of cometary and asteroidal bodies between the orbits of Jupiter and Neptune is expected to be structured into four belts.
These belts are well separated in the \((a, e)\)-plane of the orbital coordinates. Moreover, they are expected to be distinguishable in space as well.

2. The cometary-asteroidal belts near all four giant planets contain a rich resonant structure. Each belt plotted in the orbital coordinate \((a, e, i)\)-space, is characterized by sharp edges and an internal structure that includes density maxima and gaps around resonances. The latter are similar to the Kirkwood gaps in the main asteroid belt.

3. The minor bodies near the boundary of (or inside) the planet’s triangle zone described by Eqs.\((1)-(2)\) can be captured into non-dissipative resonances. More precisely, the bodies tend to be located not directly in the resonances but, instead, they reveal a near-resonance accumulation on the both sides of the resonance. Dynamics of such bodies can be explained by a balance between resonant interactions with the planet and close approaches to it.

4. We expect that an appreciable part of newly discovered Centaurs between Jupiter and Neptune be in resonances with the appropriate planet and located outside the ‘triangle zones’ of the giant planets.

The resonant structure of the cometary-asteroidal belts near the outer planets simulated in this work has some general features which could be revealed both in the distribution of AAA-asteroids near Earth and near exo-planets.

The envisaged cometary-asteroidal belts, which represent the largest structures in the Solar system, are expected to serve as the major sources of dust in the outer parts of our planetary system. The dust distribution produced by these sources is the subject of our accompanying paper.

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TABLE 1

Details of Computational Runs

|                | Neptune | Uranus | Saturn | Jupiter |
|----------------|---------|--------|--------|---------|
| Number of runs | 3       | 5      | 16     | 20      |
| Number of planet’s revolutions during one run\(^{(1)}\) | from $0.500 \times 10^6$ to $0.985 \times 10^6$ | from $0.76 \times 10^6$ to $3.0 \times 10^6$ | from $1.69 \times 10^4$ to $3.92 \times 10^5$ | from $1.50 \times 10^3$ to $0.59 \times 10^5$ |
| Number of computed positions | 198,500 \(^{(2)}\) | 103,495 \(^{(3)}\) | 144,226 \(^{(2)}\) | 36,421 \(^{(2)}\) |
| Number of positions plotted in Figs. 1 to 4 | 29,885 \(^{(4)}\) | 25,024 \(^{(5)}\) | 28,848 \(^{(6)}\) | 23,602 \(^{(7)}\) |

\(^{(1)}\) until the test body was ejected (otherwise the run was stopped).

\(^{(2)}\) taken with time step = 10 revolutions of the planet.

\(^{(3)}\) taken with time step = 100 revolutions of the planet.

\(^{(4)}\) taken with time step = 60 revolutions of the planet, with removing the positions at $a > 10 a_p$.

\(^{(5)}\) taken with time step = 400 revolutions of the planet, with removing the positions at $a > 10 a_p$.

\(^{(6)}\) taken with time step = 50 revolutions of the planet, with removing the positions at $a > 10 a_p$.

\(^{(7)}\) taken with time step = 10 revolutions of the planet, with removing the positions at $a > 10 a_p$. 
Figure Captions

Figure 1.

a. Eccentricity vs. the semi-major axis of minor bodies in and near Neptune’s zone of influence from a simulation of 29,885 test body positions. The sides of the ‘triangle zone’ given by Eqs. (1)-(2) are shown by dashed lines, and its base (the upper boundary of eccentricities of test bodies) is determined by the Tisserand constant. Test bodies on resonant and chaotic orbits below the triangle zone’s sides delineate a ‘beard’. Both the triangle zone and its ‘beard’ contain numerous gaps and resonances, including those of a high order (e.g. 7:1, 6:1, etc.)

b. Inclination vs. the semi-major axis of minor bodies in and near Neptune’s zone from the same simulation. Note an interesting two-side accumulation of test bodies around 4:3 resonance as well as one-side accumulation of test bodies around 1:1 resonance.

Figure 2. Occurence of various resonances and gaps in and near Neptune’s zone from the same simulation as Fig. 1.

Figure 3.

a. Eccentricity vs. the semi-major axis of minor bodies in and near Uranus zone of influence from a simulation of 25,024 test body positions.

b. Inclination vs. the semi-major axis of minor bodies in and near Uranus zone from the same simulation. Note several ‘muffs’ (i.e. symmetric accumulations localized within a short range of inclinations) near the resonances 2:1, 3:1, 6:1, and 7:1.

Figure 4. Occurence of various resonances and gaps in and near Uranus zone from the same simulation as Fig. 3.
Figure 5.

a. Eccentricity vs. the semi-major axis of minor bodies in and near Saturn’s zone of influence from a simulation of 28,848 test body positions. Numerous gaps and are clearly seen in the resonances 7:1, 13:2, 6:1, etc.

b. Inclination vs. the semi-major axis of minor bodies in and near Saturn’s zone from the same simulation. Note several ‘muffs’ similar to those seen in Fig. 2b. They are formed at \((1 - e) \approx 1\), i.e. near the edge of the triangle zone.

Figure 6. Occurrence of various resonances and gaps in and near Saturn’s zone from the same simulation as Fig. 5.

Figure 7. a. Eccentricity vs. the semi-major axis of minor bodies in and near Jupiter’s zone of influence from a simulation of 23,602 test body positions. One can see that the Jovian gaps have wider and less sharp edges compared to those of less massive giant planets.

b. Inclination vs. the semi-major axis of minor bodies in and near Jupiter’s zone from the same simulation. Note numerous ‘muffs’ similar to those seen in Figs. 1b, 3b, and 5b.

Figure 8. Occurrence of various resonances and gaps in and near Jupiter’s zone from the same simulation as Fig. 5. The structure of the resonant region between resonances 2:1 and 4:1 looks very similar to the Kirkwood gap in the main asteroid belt between 1:4 and 1:2 resonances.

Figure 9. The large-scale structure of the outer part of the Solar system shown in the orbital \((a, e)\)-coordinates. The simulated cometary-asteroidal belts are shown along with the currently known objects. Crosses stand for asteroids of the main belt (100 objects), triangles stand for Jupiter-family comets (112 objects), squares stand for Centaurs (6 objects), and diamonds stand for kuiperoids (50 objects). The simulated population fills
the densest parts of the triangle zones of all the giant planets. The known Centaurs and those Kuiperoids whose orbits cross the Neptune’s orbit are in this densest part as well. One can see that bulk of the known Jupiter-family comets are located far from the simulated, yet unrevealed, cometary population in the densest part of Jupiter’s triangle zone. We explain this by observational selection: the major part of the known Jupiter-family comets has perihelia $p < 2$ AU (above the dashed line). The sides of the ‘triangle zone’ given by Eqs. (1)-(2) are shown by heavy lines.

**Figure 10.** The large-scale structure of the outer part of the Solar system shown face-on. For convenience, positions of Jupiter, Saturn, Uranus, and Neptune are indicated. One can see (especially clearly for Neptune and Jupiter) the simulated cometary-asteroidal belts associated with the giant planet orbits.

**Figure 11.** The large-scale structure of the outer part of the Solar system shown edge-on. For convenience, positions of Jupiter, Saturn, Uranus, and Neptune are indicated. One can see (especially clearly for Neptune and Jupiter) the simulated cometary-asteroidal belts associated with the giant planet orbits.
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