Effect of stacking sequence on the coefficients of mutual influence of composite laminates

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Abstract. Fiber reinforced polymeric (FRP) composites are nowadays widely used in engineering applications due to their outstanding features, such as high specific strength and specific stiffness as well as good corrosion resistance. A major advantage of fibrous polymeric composites is that their anisotropy can be controlled through suitable choice of the influencing parameters. The unidirectional fiber reinforced composites provide much higher longitudinal mechanical properties compared to the transverse ones. Therefore, composite laminates are formed by stacking two or more laminas, with different fiber orientations, as to respond to complex states of stresses. These laminates experience the effect of axial-shear coupling, which is caused by applying normal or shear stresses, implying shear or normal strains, respectively. The normal-shear coupling is expressed by the coefficients of mutual influence. They are engineering constants of primary interest for composite laminates, since the mismatch of the material properties between adjacent layers can produce interlaminar stresses and/or plies delamination. The paper presents the variation of the in-plane and flexural coefficients of mutual influence for three types of multi-layered composites, with different stacking sequences. The results are obtained using the Classical Lamination Theory (CLT) and are illustrated graphically in terms of fiber orientations, for asymmetric, antisymmetric and symmetric laminates. Conclusions are formulated on the variation of these coefficients, caused by the stacking sequence.

1. Introduction

One of the most important advantages of laminated composites is their potential to orient the laminas in different suitable directions or to choose the convenient stacking sequence configurations, such that the structural response to the complex state of stresses can be improved [1].

The coefficients of mutual influence or the shear-extension coupling coefficients describe the structural response of the multi-layered composites with shear or normal strains, caused by normal or shear stresses, respectively.

Depending on the applied stresses, the different mechanical strains induced at the interface of the adjacent laminas, can be high enough to cause failure [2]. The stacking sequence, which is an important parameter, firstly studied in [3-4], has a major influence on the failure modes at the free edges such as: delamination and transverse cracks through the thickness of the laminate, caused by interlaminar...
stresses. The mismatch of the material properties between adjacent plies, quantified by the Poisson's ratio or by the coefficients of mutual influence, is still an actual concern in the design of composite laminates.

The delamination, which may occur in multi-layered composites due to the mismatch of the material or due to the residual stresses induced by the manufacturing process, is frequently studied through the suitable experimental tests or numerical modelling analysis using FEM [5, 6].

Changing the stacking sequences, while maintaining the same orientation angles in composite laminates, influences the bending response and the coupling effects, which may occur, but it does not influence the in-plane response [7].

The effect of the stacking sequence on the in-plane and flexural coefficients of mutual influence is studied in this paper, through the suitable choice of three lay-up configurations as illustrated in figure 1: a general asymmetric laminate \([\pm \theta_1(\theta)_2]\), an antisymmetric angle-ply laminate \([\pm \theta_2]\) and a symmetric angle-ply laminate \([\pm \theta_S]\).

![Figure 1. Stacking sequences of the analysed laminates: a. \([\pm \theta_1(\theta)_2]\) general asymmetric laminate; b. \([\pm \theta_2]\) antisymmetric angle-ply laminate; c. \([\pm \theta_S]\) symmetric angle-ply laminate.](image)

Special particularities of the variation of the coefficients of mutual influence are discussed, depending on the chosen stacking sequences.

2. Evaluation of the coefficients of mutual influence

The constitutive relations of the multi-layered composites expressed by the force-deformation and moment-deformation equations are:

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} +
\begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix}
\begin{bmatrix}
k_x \\
k_y \\
k_{xy}
\end{bmatrix}
\]

(1.1)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} =
\begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} +
\begin{bmatrix}
D_{11} & D_{12} & D_{16} \\
D_{12} & D_{22} & D_{26} \\
D_{16} & D_{26} & D_{66}
\end{bmatrix}
\begin{bmatrix}
k_x \\
k_y \\
k_{xy}
\end{bmatrix}
\]

(1.2)

where: (\(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\)) are the laminate mid-plane strains; (\(k_x, k_y, k_{xy}\)) are the laminate curvatures; \([A], [B], [D]\) represent the terms of the extensional stiffness matrix \([A]\), bending-stretching coupling matrix \([B]\) and bending stiffness matrix \([D]\), respectively.

The effect of the axial-shear coupling are expressed by the coefficients of mutual influence that have been introduced by Lekhnitskii [8] and defined in [9-13]:

\(\eta_{ij}\) represents the in-plane coefficient of mutual influence of the first kind that describes the stretching in the \(i\) direction, induced by the shear stress in the \(ij\) plane;
\[ \eta_{ij,\bar{i}} \] represents the in-plane coefficient of mutual influence of the second kind that describes the shearing in the \( ij \) plane, induced by normal stress in the \( i \) direction;

\[ \eta_{i,j}^f \] represents the flexural coefficient of mutual influence of the first kind that describes the bending with respect to the \( i \) direction, induced by twisting in the \( ij \) plane;

\[ \eta_{ij,i} \] represents the flexural coefficient of mutual influence of the second kind that describes the twisting in the \( ij \) plane, induced by bending moment with respect to the \( i \) direction.

In the present paper the relations for the evaluation of the in-plane and flexural coefficients of mutual influence, valid for any type of general composite laminate, are developed; however, they are also available for special orthotropic laminates or other particular multi-layered composites. The equations are developed, starting from the reports formulated by Nettles [14], where the in-plane engineering constants of non-symmetric composite laminates are written in terms of the fully-populated \([A], [B], [D]\) stiffness matrices, after solving a Cramer system. Therefore, the expressions for the in-plane and flexural coefficients of mutual influence are presented, both for the general case of composite laminates and for a particular case of symmetric angle-ply laminates, when \([B] = 0\).

2.1. In-plane coefficients of mutual influence

If laminate bending-stretching coupling does not occur, the in-plane coefficients of mutual influence are characterized by the extensional stiffness matrix through the terms \(A_{16}\) and \(A_{26}\). Otherwise, almost all the terms of the assembled stiffness matrix are implied in the evaluation of the in-plane structural response, produced by the axial-shear coupling.

The in-plane coefficients of mutual influence are defined according to equation (2):

\[ \eta_{x,xy} = \varepsilon \gamma_{xy}^0, \eta_{y,xy} = \varepsilon \gamma_{xy}^0, \text{ when } \tau_{xy} \neq 0, \sigma_x = 0, \sigma_y = 0; \]

\[ \eta_{xy,x} = \gamma_{xy}^0 \varepsilon_x^0, \text{ when } \sigma_x \neq 0, \sigma_y = 0, \tau_{xy} = 0; \quad (2) \]

\[ \eta_{xy,y} = \gamma_{xy}^0 \varepsilon_y^0, \text{ when } \sigma_y \neq 0, \sigma_x = 0, \tau_{xy} = 0. \]

2.1.1. General composite laminates

The in-plane coefficients of mutual influence for any general case of composite laminate are determined with equations (3)-(6), as following:

\[ \eta_{x,xy} = \begin{bmatrix} A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{22} & A_{26} & B_{22} & B_{26} & B_{26} \\ B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{22} & B_{26} & D_{22} & D_{26} & D_{26} \\ B_{26} & D_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \]

\[ \eta_{y,xy} = \begin{bmatrix} A_{11} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{26} & B_{12} & B_{22} & B_{26} \\ B_{11} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{26} & D_{12} & D_{26} & D_{26} \\ B_{16} & D_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \]
2.2.1. Flexural coefficients of mutual influence

The flexural coefficients of mutual influence of general composite laminates are determined with equations (12), as following:

\[ \eta_{xy,x} = \begin{bmatrix} A_{12} & A_{22} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & D_{16} & D_{26} & D_{66} \end{bmatrix} (5), \quad \eta_{xy,y} = \begin{bmatrix} A_{11} & A_{12} & B_{11} & B_{12} & B_{16} \\ A_{16} & A_{26} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & D_{16} & D_{26} & D_{66} \end{bmatrix} (6) \]

\[ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} (7.1), \quad \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} (7.2) \]

The flexural coefficients of mutual influence can be expressed through the terms \( D_{16} \) and \( D_{26} \) of the bending stiffness matrix:

\[ \eta_{x,xy} = \frac{A_{16}}{A_{66}} (8), \quad \eta_{y,xy} = \frac{A_{26}}{A_{66}} (9), \quad \eta_{xy,x} = \frac{A_{16}}{A_{11}} (10), \quad \eta_{xy,y} = \frac{A_{26}}{A_{22}} (11) \]

2.2.2. Particular case of symmetric angle-ply laminates

The direct and inverse form of force-deformation constitutive relation for symmetric composite laminates are presented in equations (7.1) and (7.2), as following:

\[ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} (7.1), \quad \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} (7.2) \]

For the particular case of symmetric angle-ply laminates, the in-plane coefficients of mutual influence can be established with equations (8) – (11):

\[ \eta_{x,xy} = \frac{A_{16}}{A_{66}} (8), \quad \eta_{y,xy} = \frac{A_{26}}{A_{66}} (9), \quad \eta_{xy,x} = \frac{A_{16}}{A_{11}} (10), \quad \eta_{xy,y} = \frac{A_{26}}{A_{22}} (11) \]

2.2. Flexural coefficients of mutual influence

The assessment of the bending structural response of the general composite laminates, caused by the normal-shear coupling, can be realized with the implication of all terms of the stiffness matrices.

In case of symmetric laminates, the flexural coefficients of mutual influence are expressed through the terms \( D_{16} \) and \( D_{26} \) of the bending stiffness matrix.

The flexural coefficients of mutual influence are defined according to equation (12), as following:

\[ \eta_{x,xy}^f = k_x / k_{xy}, \quad \eta_{y,xy}^f = k_y / k_{xy}, \quad \text{when} \quad \tau_{xy}^f \neq 0, \quad \sigma_x^f = 0, \quad \sigma_y^f = 0; \]

\[ \eta_{xy,x}^f = k_{xy} / k_x, \quad \text{when} \quad \sigma_x^f \neq 0, \quad \sigma_y^f = 0, \quad \tau_{xy}^f = 0; \]

\[ \eta_{xy,y}^f = k_{xy} / k_y, \quad \text{when} \quad \sigma_x^f \neq 0, \quad \sigma_y^f = 0, \quad \tau_{xy}^f = 0. \]

2.2.1. General composite laminates

The flexural coefficients of mutual influence of general composite laminates are determined with equations (13) – (16), such as:
The coefficients of mutual influence are
\[ \eta_{x,xy} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{22} & D_{26} \end{bmatrix} \]

(13), \[ \eta_{y,xy} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \]

(14)

\[ \eta_{k,xy,x} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} \\ A_{12} & A_{22} & A_{26} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} \\ B_{12} & B_{22} & B_{26} & D_{22} & D_{26} \end{bmatrix} \]

(15), \[ \eta_{k,xy,y} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \]

(16)

2.2.2. Particular case of symmetric angle-ply laminates
The direct and inverse form of moment-deformation constitutive relations are performed by equations
(17.1) and (17.2):
\[ \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \]

(17.1), \[ \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} \]

(17.2)

The evaluation of the flexural coefficients of mutual influence for symmetric angle-ply laminates is
simplified, based on the equations (18) – (21):
\[ \eta_{x,xy} = \frac{D_{16}}{D_{66}} \]

(18), \[ \eta_{y,xy} = \frac{D_{26}}{D_{66}} \]

(19), \[ \eta_{x,y,x} = \frac{D_{16}}{D_{11}} \]

(20), \[ \eta_{x,y,y} = \frac{D_{26}}{D_{22}} \]

(21)

3. Results
The variations of the coefficients of mutual influence are represented graphically with respect to the
fiber orientation angles, for the three analyzed composite laminates. The plies of the multi-layered
composites are realized alternatively of S glass or E glass fibers, embedded in an epoxy resin matrix.
The mechanical characteristics of the composite material constituents are illustrated in table 1.
### Table 1. Mechanical properties of constituent materials [15]

| Constituents of composite plies | Young’s modulus (GPa) | Poisson’s ratio |
|---------------------------------|------------------------|-----------------|
| S glass fibres                 | 85.5                   | 0.22            |
| E glass fibres                 | 72.4                   | 0.22            |
| Epoxy resin matrix             | 4.1                    | 0.40            |

The results are centralized such that three distinct comparative analysis can be observed, depending on the stacking sequence, on the in-plane or bending structural response and function of the fiber volume fractions for the considered lay-ups.

The differences and the particularities of the studied asymmetric, antisymmetric and symmetric composite laminates, in terms of in-plane and flexural coefficients of mutual influence of first kind and second kind are illustrated in figure 2.

Figure 3 and figure 4 show a comparative analysis between the in-plane and flexural coefficients of mutual influence, both for the general composite and for the symmetric angle-ply laminate.

In figures 5-7, the variation of the in-plane and flexural coefficients of mutual influence is presented for every studied configuration with non-zero results, for suggestive fiber volume fractions.

![Figure 2. In-plane and flexural coefficients of mutual influence for the analysed laminates.](image-url)
Figure 3. In-plane and flexural coefficients of mutual influence for general asymmetric laminates.

Figure 4. In-plane and flexural coefficients of mutual influence for symmetric angle-ply laminates.
Figure 5. In-plane coefficients of mutual influence for general asymmetric laminates.

Figure 6. Flexural coefficients of mutual influence for general laminates.
4. Discussion

The general asymmetric $[\pm \theta_1/\theta_2]_2$ composite laminate presents nonzero values for both in-plane and flexural coefficients of mutual influence, having the flexural engineering constants greater than the in-plane ones.

The analyzed antisymmetric $[\pm \theta]_2$ angle-ply laminate has all coefficients of mutual influence equal to zero, since the angle-ply laminate is a special orthotropic laminate. All antisymmetric special orthotropic laminates have zero terms for $A_{16}, A_{26}, D_{16}, D_{26}$, which implies no in-plane and flexural coefficients of mutual influence. However, this is not a general rule for every antisymmetric laminate.

The symmetric $[\pm \theta]_k$ angle-ply laminate has zero in-plane coefficients of mutual influence and nonzero flexural coefficients of mutual influence. The results are confirmed by the relations which express that the terms $A_{16} = A_{26} = 0$, while $D_{16}, D_{26} \neq 0$, valid for symmetric angle-ply laminates.

Comparison between the flexural shear-extension coupling coefficients shows greater values in case of the general laminate than for the symmetric angle-ply laminate. Moreover, the coefficients of influence of first kind are smaller than the ones of second kind. As to clearly show the differences, in case of the studied configuration of laminates and composite materials constituents, the maximum general values for the coefficients of influence of first kind range from 0.05 to -0.40, while in case of the coefficients of mutual influence of second kind the values extend between 0.05 to -1.50.

In the case of the in-plane and flexural coefficients of mutual influence for general laminate or for the flexural coefficients of mutual influence for symmetric angle-ply laminate, the analysis has been run for fiber volume fractions of 20% and 60%. Increases of the coefficients’ values with fiber volume fraction are noticed, until $V_f$ reaches 60%.
5. Conclusions
The stacking sequence of the composite laminate has a major importance on the variation of the flexural coefficients of mutual influence, rather than the in-plane ones.

The special orthotropic laminate configurations are generally the recommended solutions for engineering applications, due to their reductions of the stiffness matrix terms and consequently for the simplification of the laminate analysis. The symmetric special orthotropic laminates have zero in-plane coefficients of mutual influence, but nonzero values for the flexural ones. The antisymmetric angle-ply laminate is a good alternative when both in-plane and flexural coefficients of mutual influence have to be zero, due to the complex loading actions.

Therefore, the configuration lay-up of a multi-layered element can be rationally designed, such as the differences in the coefficients of mutual influence between the adjacent plies to be as small as possible, to avoid failure caused by interlaminar stresses and/or ply delamination at the free edges.

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