A new method of determining decision-making unit congestion under inter-temporal dependence

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Accepted: 11 November 2021 / Published online: 1 February 2022
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Abstract
Congestion is an essential concept in data envelopment analysis (DEA). It occurs when the decrease in at least one decision-making unit (DMU) input increases at least one output without affecting the other inputs or outputs. Although various methods have discussed congestion determination in static DEA, congestion calculation in dynamic DEA under inter-temporal dependence has not been considered in the literature. This paper deals with the congested input levels in multi-period production systems under inter-temporal dependence and the inter-temporal dependence of production periods due to the stock capital. Due to the importance of evaluating DMUs in the presence of time factors, this paper expands the determination of congestion of DMUs under the time factor and dependence of input and output levels in periods. The proposed models determine the congested unit in the assessment window and characterize the extra-input and lack-output amounts in all of the congested unit components in each period of the assessment window. This paper aims to specify production system performance in different evaluation periods by determining the dynamic congestion of DMUs. This paper proposes a method to identify the congested paths. The main advantage of this method is to prevent inappropriate allocation of the resources during evaluation periods of the assessment window. The validity of the proposed method is investigated through an example in the banking sector. Moreover, this method can be utilized in other sectors to homogeneous units with the same structure.

Keywords Data envelopment analysis (DEA) · Congestion · Inter-temporal dependence

1 Introduction
Operational research (OR) is a powerful scientific technique for decision making. This technique can be employed in various scientific applications like management and economics. There are different practical and analytical frameworks for this scientific technique, such as multi-criteria decision making (MCDM) (Ehrgott 2005) and performance evaluation Emrouznejad and Tavana (2014). MCDM is an interesting and notable topic. Many studies have been performed in this field. Decision making under uncertain situations using a neutrosophic multi-criteria decision-making model was presented by Das et al. (2020). Also, Das et al. (2021) adopted tangent similarity measure-based multi-attribute decision making for optimization. One of the branches of operation research is data envelopment analysis (DEA).

In operations research and management science, DEA is a nonparametric method based on mathematical programming, which evaluates the performance of decision-making units. The first DEA model was proposed by Charnes et al. (1978). Since then, various other models have been introduced by other researchers. The time factor is not considered in the static DEA model. By applying the time factor to the DEA models, dynamic DEA models can be established. During the recent decade, researchers have assessed the efficiency of DMUs in dynamic production systems with inter-temporal dependence and have further reviewed the input and output levels. The inter-temporal dependence
factors of assessment periods include stock capital, lagged output, capital output, and quasi-fixed inputs. This survey utilized stock capital to determine the inter-temporal dependence between input and output levels in multi-period production systems. However, overall dynamic efficiency can be obtained by considering quasi-fixed inputs (Nemoto and Goto 2003). Also, Sueyoshi and Sekitani (2005) determined the return to scale in dynamic DEA under the quasi-fixed inputs. Emrouznejad and Thanassoulis (2005) considered the stock capital as an inter-temporal dependence factor of evaluation periods and established a model to evaluate the dynamic-technical efficiency of DMUs. This model was enhanced by Jahanshahloo et al. (2006) to solve the problem of identifying dynamically efficient paths. This inter-temporal dependence case was studied from theoretical and practical aspects in the inverse DEA problems (Ghobadi et al. 2014, 2019; Jahanshahloo 2015). Moreover, estimating units’ cost efficiency (Soleimani-Chamkhorami and Ghobadi 2021), the problem of merging units (Zeinodin and Gholibadi 2020), and ranking of units (Moonesian et al. 2020) have been considered in the literature. Moreover, Kao (2009) studied the inter-temporal dependence of assessment periods caused by the intermediate product or quasi-fixed inputs and derived a mathematical relation between the overall and period efficiencies. Tone et al. (2010) introduced a dynamic non-radial model to divide the carry-over activities between two consecutive terms into four categories: desirable, undesirable, discretionary, and non-discretionary. Ghobadi et al. (2018) established a linear programming model to estimate the technical efficiency of units. The dynamic DEA model was first employed in the industry by Fallah et al. (2014). Li et al. (2017) employed meta-frontier dynamic DEA to evaluate the efficiency of the regional high-tech industry in China. Mariz et al. (2018) reviewed the articles on dynamic data envelopment analysis models from 1996 to 2016. An application of dynamic DEA to achieve the multi-period R&D efficiency of regional systems was introduced by Chen et al. (2018). Simultaneous estimation of input–output levels was studied by Ghobadi (2020) using the ERM model under inter-temporal dependence. Lin et al. (2021) established a multiplier dynamic DEA model to determine the efficiency score of each period based on the directional distance function. Zhou et al. (2021) employed uncertain dynamic DEA to obtain supplier’s goal setting under sustainable conditions. Although numerous studies on dynamic DEA have also been performed in the literature, no research has been reported on dynamic congestion under the inter-temporal dependence of the input and output levels.

Congestion is a significant issue in the economy and a futile step in the production systems to reduce the output. The first survey on congestion was published by Fare and Svensson (1980). Then, Fare and Grosskopf (1983) described the congestion model using the data envelopment analysis model. Fare et al. (1985) examined the existence of congestion based on a DEA radial model. However, their proposed model cannot determine the amount of input congestion and cannot be applied to systems with one input. Congestion was then utilized by Brock et al. (1978) in the Chinese industrial sector. Cooper et al. (1996) introduced a model based on the slack variable for determining the amount of input congestion. Cooper et al. (2002) established a one-model method to calculate the congestion. A method was proposed by Jahanshahloo and Khodabakhshi (2004) to calculate the input congestion based on input relaxation for improving outputs. Tone et al. (2004) provided a non-radial model for identifying the weak congestion and discerned the strong congestion using the multiplier model and the upper bound of the scale elasticity. Sueyoshi et al. solved the problem associated with the multiple optimal solutions in 2009 and derived the theoretical relation between congestion and negative return to scale. Other studies on congestion have been performed, such as Wei and Yan (2004), who identified congestion and variant types of the returns to scale. In 2009, an output-oriented additive model was employed to resolve the same weak congestion. Kao (2010) reduced the congestion by merging DMUs. Asgharian et al. (2010) measured the congestion by stochastic data envelopment analysis. The congestion of DMUs in fuzzy data envelopment analysis was determined by Khirrollahi et al. (2017). Khoveyni et al. (2017) and Mehdiloozad et al. (2018) presented the negative data to identify the DMU congestion. Mitheon et al. (2013) studied congestion in a two-stage supply chain. Shadab et al. (2021) also considered congestion in terms of the role of intermediate products in the sustainable supply chain and identified four scenarios in which congestion can occur in intermediate products. On the other hand, Fang (2015) calculated congestion using the directional distance function in the presence of desirable and undesirable outputs. Ebrahimzadeh Adimi et al. (2019) identified the boundary between the congested and non-congested units by introducing a congestion hyperplane. Shabanpour et al. (2019) evaluated the sustainability of suppliers using the data envelopment analysis congestion method. The definition of directional congestion in specific input and output directions based on DEA was provided by Yang et al. (2020). Also, Khezri et al. (2021) obtained directional congestion when at least one input or output direction is unknown. Mehdiloozad et al. (2020) specified the congestion using a maximal element of a nonnegative polyhedral set and solving a linear programming model. Congestion identification and determination methods have been widely utilized in industrial systems. For example,
Yang et al. (2017) determined the congestion of China’s pure e-commerce firms and evaluated the effects of the congestion and productivity on e-commerce firms’ profitability. Zhang et al. (2020) studied carbon congestion effects in China’s industry using a multiunit model of DEA and considered the balance between CO2 emissions reduction and industrial growth. Wang et al. (2021) assessed the management efficiency of shipping companies using congestion. However, dynamic DEA congestion with inter-temporal dependence between input and output levels has not been studied yet.

According to the DEA literature, various theoretical and practical aspects of the congested unit have been studied by some researchers (Khoveyini et al. 2017; Mehdiloozad et al. 2018; Yang et al. 2017; Zhang et al. 2020; Wang et al. 2021). However, the proposed models in these studies could not identify the congested units under inter-temporal dependence of input and output levels addressed in Emrouznejad and Thanassoulis (2005). Therefore, this study proposes a novel DEA model based on linear programming to identify the congested units under the inter-temporal dependence by influencing the stock capital on output levels in various production periods. The main contribution of this paper is to provide a theoretical and practical framework to identify congested paths in the assessment window. The proposed models determine the congested unit in the assessment window and characterize the extra-input and lack-output amounts in all the congested unit components in each period of the assessment window. The proposed models help the congested unit estimate the excess inputs and the shortage values of outputs in each period of the assessment window. The managers can employ these models to detect congestion paths from non-congestion ones and design useful approaches to improve the performance of the congested paths. These approaches can be considered new approaches in resource allocation management and handling investment analysis problems under the inter-temporal dependence of data. The performance of the proposed method is investigated through an example in the banking sector. Nevertheless, this approach can be adopted in other sectors to homogeneous units with a similar structure.

The remainder of the study is organized as follows. Section 2 presents Cooper’s method for congestion identification using a non-radial model and reviews the dynamic model from Emrouznejad and Thanassoulis (2005). In Sect. 3, a novel method is proposed to calculate the congestion in dynamic DEA under inter-temporal dependence between assessment periods. A practical example of the proposed model is presented in Sect. 4. Finally, Sect. 5 concludes the paper.

2 Preliminaries

In this section, a congestion identification model similar to the Cooper et al. (2002) model is presented. In addition, the Emrouznejad and Thanassoulis (2005) model is also mentioned in this section.

2.1 Identification of the congestion

It is assumed that there are n DMUs, i.e., \(\{DMU_j|j = 1, \ldots, n\}\), and each DMU has m inputs, i.e., \(\{x_{ij}|i = 1, \ldots, m\}\), and s outputs, i.e., \(\{y_{rj}|r = 1, \ldots, s\}\). The congestion production possibility set under the inputs weak disposability is defined as follows:

\[
PPS_{congestion} = \left\{ \left( \begin{array}{c} x \\ y \end{array} \right) \mid R^{m+x} \sum_{j=1}^{n} \lambda_j x_{ij} = x, \sum_{j=1}^{n} \lambda_j y_{rj} \geq y, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \right\}
\]

where \(\lambda_j (j = 1, \ldots, n)\) is the intensity variable. \(\sum_{j=1}^{n} \lambda_j x_{ij} = x\) and \(\sum_{j=1}^{n} \lambda_j y_{rj} \geq y\) are the inputs weak disposability and the outputs strong disposability, respectively. Additionally, \(\sum_{j=1}^{n} \lambda_j = 1\) is the convexity condition.

Cooper’s non-radial one-model method for calculating the amount of the input congestion is as follows:

\[
\begin{align*}
\text{Max} & \quad \sum_{r=1}^{s} s^+_{rk} - \epsilon \sum_{i=1}^{m} s^i_{ik} \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j x_{ij} = x_{ik} \quad i = 1, \ldots, m, \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} - s^+_{rk} = y_{rk} \quad r = 1, \ldots, s, \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad s^+_{rk} \geq 0 \quad i = 1, \ldots, m, \\
& \quad s^+_{rk} \geq 0 \quad r = 1, \ldots, s, \\
& \quad \lambda_j \geq 0 \quad j = 1, \ldots, n.
\end{align*}
\]

In the model (1), \(s^+_{rk}\) and \(s^i_{ik}\) indicate the \(r^{th}\) output slack variable of DMU\(_k\), and the \(i^{th}\) congestion variable of DMU\(_k\), respectively. Also, \(\epsilon\) is a non-Archimedean small positive number. Model (1) is a two-phase process: (a) The maximum output shortage of DMU\(_k\), i.e., \(s^+_{rk}\), is specified by output-oriented non-radial BCC model, and (b) while keeping \(y_{rk} = y_{rk} + s^+_{rk}\), the minimum of the input surpluses of DMU\(_k\) is specified such that DMU\(_k\) is located in
the congested production possibility set. If the optimal solution of phase 1 and phase 2 is positive, it can be concluded that an increase in input reduces output. In other words, there is congestion in DMU. The amount of the input’s congestion is revealed with the following theorem, quite similar to the theorem of Cooper et al. (2002).

**Theorem 2.1.** DMU is congested if and only if in an optimal solution, i.e., \((s^+, s^-)\) of model (1), the following condition is satisfied:

There is at least one \(r (r = 1, \ldots, s)\) so that \(s^+ > 0\) and at least one \(i (i = 1, \ldots, m)\) so that \(s^- > 0\).

In theorem (1), \(s^+\) is the amount of the congestion of \(r\)th input of the DMU and \(s^-\) is the amount of the shortfall in \(r\)th output due to congestion and inefficiency of the DMU.

\[
\text{PPS}_D = \left\{ (x^1, \ldots, x^T, z^1, \ldots, z^T, y^1, \ldots, y^T, Z^I, Z^{I+T}) \mid \begin{align*}
\sum_{j=1}^n \lambda_j x^j_i & \leq x^i, \sum_{j=1}^n \lambda_j y^j_i \leq y^i, \\
\sum_{j=1}^n \lambda_j y^{j^T}_i & \geq y^{j^T}, \sum_{j=1}^n \lambda_j Z^{j^T-1}_i & \leq Z^{j^T-1}, \\
\sum_{j=1}^n \lambda_j Z^{j^T+T}_i & \geq Z^{j^T+T}, \\
\sum_{j=1}^n \lambda_j &= 1, \lambda_j \geq 0, \forall t \in W, \end{align*} \right\}
\]

**2.2 Dynamic DEA under inter-temporal dependence**

In this section, a special model of dynamic DEA under inter-temporal dependence of Emrouznejad and Thanassoulis (2005) has been reviewed. According to Emrouznejad and Thanassoulis, the stock capital change is a specific case of the inter-temporal dependence between input and output levels in a dynamic DEA framework. They measured the performance of n DMU set within assessment of window time period \(W = \{t | t = \tau, \ldots, \tau + T\}\). The initial and terminal time period in assessment window is indicated by \(\tau, \tau + T\), respectively. Each DMU is assumed to have three types of input for every time: period-specific input, stock capital inputs, and initial-stock inputs. Moreover, each DMU produces two types of outputs: period-specific outputs and terminal-stock outputs. The set of inputs and outputs of DMU is defined as follows:

- Period-specific input paths: \(x^i_j \forall i \in I_1, \forall t \in W, \)
- Stock capital input paths: \(z^{j^T}_i \forall i \in I_2, \forall t \in W, \)
- Initial-stock inputs: \(Z^{i^T-1}_j \forall i \in I_2, \)
- Period-specific output paths: \(y^j \forall r \in O, \forall t \in W, \)
- Terminal-stock inputs as output: \(Z^{j^T+T}_i \forall i \in I_2, \) where the set of outputs, \(O = \{1, \ldots, s\}\) and the set of inputs, \(I = \{1, \ldots, m\}\) are divided into two subsets: \(I_1\) (period inputs) and \(I_2\) (capital inputs), such that \(I_1, I_2 \subset I, I_1 \cup I_2 = I, I_1 \cap I_2 = \emptyset\).

Capital input \((z^i)\) and terminal-stock input \((Z^{j^T+T})\) as output are supplied by initial-stock input \((Z^{j^T-1})\). In addition, Jahanshahloo et al. (2015) have been modified the model of Emrouznejad and Thanassoulis (2005) by adding the following equation

\[
Z^{j^T-1} = \sum_{t=\tau}^{\tau+T} z^t + Z^{j^T+T}
\]

Emrouznejad and Thanassoulis (2005) model of production flow is shown in Fig. 1.

They have introduced a dynamic production possibility set (PPS) as follows:

\[
\text{PPS}_D \text{ includes the principles of observations, convexity, strong disposability of period and capital inputs and period and capital outputs, and minimum extrapolation.}
\]

A linear programming problem for estimating the dynamic relative efficiency of DMU was presented by Emrouznejad and Thanassoulis (2005). Again, this model was modified by Jahanshahloo et al. (2006). (For further details, refer to Emrouznejad and Thanassoulis (2005) and Jahanshahloo et al. (2006) hypotheses.)

**3 Proposed method to calculate the congestion in dynamic DEA under inter-temporal dependence**

In this section, first the dynamic congested production possibility set under inputs weak disposability is specified; next, the amount of DMU congestion in dynamic DEA is determined, i.e., when the stock capital has affected inputs and outputs levels in multi-period production systems. In other words, the proposed models not only determine the congested unit in the assessment window, but also characterize the extra-input and lack-output amounts in each of the components of the congested unit in each period of the assessment window.
3.1 The dynamic congested production possibility set (Dynamic congested PPS)

The dynamic congested production possibility set under weak disposability of inputs is defined by:

\[
P_{DC} = \left\{ (x^t, \ldots, x^{t+T}, z^t, \ldots, z^{t+T}, y^t, \ldots, y^{t+T}, Z^{-1}, Z^{t+T}) \middle| \begin{align*}
\sum_{j=1}^{n} \lambda_j x_{ij}^t &= x_i^t, \\
\sum_{j=1}^{n} \lambda_j z_{ij}^t &= z_i^t, \\
\sum_{j=1}^{n} \lambda_j y_{ij}^t &\geq y_i^t, \\
\sum_{j=1}^{n} \lambda_j Z_{ij}^{t-1} &= Z_i^{t-1}, \\
\sum_{j=1}^{n} \lambda_j Z_{ij}^{t+T} &= Z_i^{t+T}, \\
\sum_{j=1}^{n} \lambda_j &\geq 1, \lambda_j \geq 0, \forall t \in W,
\end{align*}\right\}
\]

where \(s_{r}^{t+i}\) indicates the amount of \(r\)th output slack in the time period \(t\) and also \(\lambda_j\) displays the amount of \(j\)th intensity variable. The objective function of Model 2 is the maximum sum of the amounts added to the outputs of the unit under evaluation in dynamic congested production possibility set in all periods.

Here, efficient and congested units from static DEA to dynamic DEA can be defined in details.

**Definition 3.1.** (Dynamically efficient) Suppose \((s_{r}^{t+i}, \lambda_j)\) is an optimal solution of model (2), then the assessment path of \(DMU_k\) is dynamically efficient if and only if for each \(r (r = 1, \ldots, s)\) and for each \(t (t = \tau, \ldots, t + T)\), \(s_{r}^{t+i} = 0\), otherwise, projection of the \(DMU_k\), i.e., \((x_{k}^{t}, z_{k}^{t}, y_{k}^{t}, Z_{k}^{-1}, Z_{k}^{t+T})\) is dynamically efficient.

**Definition 3.2.** (Congested path) The efficient assessment path \((x_{k}^{t}, z_{k}^{t}, y_{k}^{t}, Z_{k}^{-1}, Z_{k}^{t+T})\) of \(DMU_k\) in model (2) is called congested path when an increase (decrease) in at least one input in least one time period is associated with a decrease (increase) in at least one output in least one time period, without negative impact on other inputs and outputs.

3.2 Determination of the amount of DMU congestion in dynamic DEA

In this section, the amount of DMU congestion and the congested paths in dynamic DEA is determined by application of Emrouznejad and Thanassoulis (2005) model where the inter-temporal dependence factor of evaluation periods is stock capital. In model (3), however, in order to calculate the congested path, first the maximum increase in period outputs of \(DMU_k\) is specified. Next, the minimum decrease in period and capital inputs is obtained such that \(DMU_k\) is located in the dynamic congested production possibility set. In addition, the terminal-stock inputs as outputs are considered fixed.
Model (3) has two stages. In step 1, the maximum sum of the amounts added to the period outputs of the evaluated unit is obtained. Then, in step 2, the minimum sum of the reduced amount of period and capital inputs of the unit under assessment is determined. In addition, the unit under evaluation is dynamically efficient. Nonetheless, in the model (3) $s_{i}^{\tau}$, $\delta_{i}^{\tau}$, and $s_{i}^{\tau + T}$ are the period-input, capital-input, and period-output slack variables in the period $\tau$, respectively. Also, $\gamma_{i}^{\tau - 1}$ is the slack variable of initial-stock input. In order to, assessment path of the projection of the $DMU_{k}$ satisfies in the equation $Z^{\tau - 1} = \sum_{i=\tau}^{\tau + T} z_{i} + Z^{\tau + T}$, the constraint $\sum_{i=\tau}^{\tau + T} \delta_{i}^{\tau} = \gamma_{i}^{\tau - 1}$, is used in model (3).

Consequently, on the basis of the following theorem, the congested paths and the amount of DMU congestion at each evaluation period are identified.

\[
\begin{align*}
    \text{Max} & \quad \sum_{t=\tau}^{\tau + T} \sum_{r=1}^{s} s_{i}^{\tau t} - c \left( \sum_{t=\tau}^{\tau + T} \sum_{r=1}^{s} s_{i}^{\tau t} + \sum_{t=\tau}^{\tau + T} \sum_{i=I_{1}}^{s} \delta_{i}^{\tau t} + \sum_{i=I_{2}}^{s} \gamma_{i}^{\tau - 1} \right) \\
    \text{s.t.} & \quad \sum_{j=1}^{n} \lambda_{j} x_{ij}^{\tau t} + s_{i}^{\tau t} = x_{ik}^{\tau t} \\
    & \quad \sum_{j=1}^{n} \lambda_{j} x_{ij}^{\tau t} + \delta_{i}^{\tau t} = z_{ik}^{\tau t} \\
    & \quad \sum_{j=1}^{n} \lambda_{j} Z_{ij}^{\tau t - 1} + \gamma_{i}^{\tau - 1} = Z_{ik}^{\tau - 1} \\
    & \quad \sum_{j=1}^{n} \lambda_{j} y_{ij}^{\tau t} - s_{i}^{\tau t} = y_{ik}^{\tau t} \\
    & \quad \sum_{j=1}^{n} \lambda_{j} Z_{ij}^{\tau + T} = Z_{ik}^{\tau + T} \\
    & \quad \sum_{t=\tau}^{\tau + T} \delta_{i}^{\tau t} = \gamma_{i}^{\tau - 1} \\
    & \quad \sum_{j=1}^{n} \lambda_{j} = 1, \\
    & \quad \lambda_{j} \geq 0 \\
    & \quad s_{i}^{\tau t} \geq 0 \\
    & \quad \delta_{i}^{\tau t} \geq 0 \\
    & \quad \gamma_{i}^{\tau - 1} \geq 0 \\
    & \quad s_{i}^{\tau + T} \geq 0 \\
    & \quad \forall i \in I_{1}, \forall t \in W; \\
    & \quad \forall i \in I_{2}, \forall t \in W; \\
    & \quad \forall i \in I_{1}, \forall r \in O, \forall t \in W; \\
    & \quad \forall i \in I_{2}, \forall t \in W; \\
    & \quad j = 1, \ldots, n, \forall i \in I_{1}, \forall t \in W; \\
    & \quad \forall i \in I_{2}, \forall t \in W; \\
    & \quad \forall i \in I_{1}, \forall r \in O, \forall t \in W; \\
    & \quad \forall i \in I_{2}, \forall t \in W.
\end{align*}
\]

Fig. 1 Production flow in the assessment window
Theorem 3.1. The assessment path 
\( (x_k^{i,\ldots,\tau+T}, x_k^{t,\ldots,\tau+T}, y_k^{i,\ldots,\tau+T}, Z_k^{-1}, Z_k^{+T}) \) of DMU_k is congested if and only if for an optimal solution, i.e., 
\( (s_{i,t}^{r}, \delta_{i,t}^{r}, y_i^{(t-1)}, s_r^{+T}, \lambda_t^{+}) \) in model (3), the following condition exists: 

There is at least one \( r \) (\( r = 1, \ldots, s \)) and at least one \( t \) (\( t = \tau, \ldots, \tau + T \)) such that \( s_{i,t}^{r} > 0 \), and at least one of the following two cases is true:

Case 1. There is at least one \( i \in I_1 \), and at least one \( t \) so that \( s_{i,t}^{r} > 0 \).
Case 2. There is at least one \( i \in I_2 \), and at least one \( t \) so that \( \delta_{i,t}^{r} > 0 \).

Proof. Model (3) is a two-phase process. In the first phase, the maximum increase in DMU_k period output is obtained by dynamic output-oriented non-radial model. Now, assume that \( s_{i,t}^{r} \) is an optimal solution in the first phase. While keeping \( y_k^{i,t} = y_k^{i,t} + s_{i,t}^{r} \) in the second phase, minimum decrease in period and capital inputs is determined such that DMU_k is located in the congested production possibility set. Still, if the optimal solution in phases 1 and 2 is positive, i.e., there is at least one \( r \) and at least one \( t \) and that \( s_{i,t}^{r} > 0 \), and also that at least one of the two cases in 1 and 2 holds true (\( s_{i,t}^{r} > 0 \) or \( \delta_{i,t}^{r} > 0 \)), it can be concluded that an assessment path 
\( \left( x_k^{i,\ldots,\tau+T}, s_k^{i,\ldots,\tau+T}, \delta_k^{i,\ldots,\tau+T}, y_k^{i,\ldots,\tau+T}, Z_k^{-1}, Z_k^{+T} \right) \) is found such that a decrease in at least one of the inputs in at least one of the time periods is associated with an increase in at least one of outputs in at least one of the time periods of DMU_k. Certainly, this is to the significance that DMU_k has congestion.

Remark 1. In optimal solution of model (3), \( s_{i,t}^{r} \) and \( \delta_{i,t}^{r} \) are the amount of the period \( i^{th} \) input congestion and the amount of the capital \( i^{th} \) input congestion in the time period \( t \), respectively.

4 Empirical sample

In this section, the applicability of the proposed method was studied via banking centers empirically. The data set is based on Ghobadi et al. (2019). Twenty Iran commercial bank branches were studied for a period of three months. Each branch was provided two period inputs, a capital input, and three period outputs; period inputs are employee score and deferred claims. Moreover, if any branch appealed for grants, financial assistance from the central bank (capital stock) was provided. According to the bank rating, the maximum amount of central bank grants is fixed during assessment window called initial capital (\( Z_{-1} \)). The initial bank capital is divided between: a) time periods in assessment window and b) remainder of initial capital called terminal stock (\( Z_{+T} \)), used as output for final period of assessment window. Also, for each bank, correlation is always created in terms of:

Initial capital = (sum of period capitals) + terminal capital.

The period outputs are loans, deposit, and profit. Data set is shown in Table 1.

By application of proposed model (3), the dynamic congestion of twenty branches in the assessment window was studied. Results of model (3) demonstrate \( s_{i,t}^{r}, \delta_{i,t}^{r}, \) and \( s_{i,t}^{r} \) for all banks except the first as being zero. Therefore, it can be concluded that bank 1 has a dynamic congested path. Hence, to remove the congested path from bank 1 and to attain an efficient path during the evaluation period, the following results are obtained from Table 2:

1. Amounts of 5.76, 0.97, and 0.97 of bank 1 employees’ scores should be reduced in the first, second, and third periods, respectively.
2. The excess amounts of deferred claims of bank 1 were 3,293,267.40 and 3,159,264.23 in the first and third periods, respectively. There were no extra deferred claims in the second period.
3. Bank 1 needs less capital stock. Therefore, the amounts of 8,905,429.16, 10,993,792.12, and 10,571,478.49 of the capital stock of bank 1 should be reduced in the first, second, and third periods.
4. The shortfall of output (loans) due to congestion and inefficiency was 14,226,309.81, 16,965,193.49, and 20,954,224.32 in the first, second, and third periods, respectively.
5. For elimination of output shortage (deposit) caused by congestion and inefficiency, the amounts of 37,894,332.85, 41,986,953.64, and 45,701,903.51 were added to deposits in the first, second, and third periods.
6. The shortage of outputs (profits) due to congestion and inefficiency was 1,799,012.76, 1,751,931.59, and 1,565,536.16 in the first, second, and third periods, respectively.

Consequently, determining the amount of congestion on a congested path in the assessment window indicates that reducing employee score, deferred claims, and capital stock leads to increased loan, deposit, and profit. In other words, if the first bank hires employees with a lower score and reduces deferred claims and also uses less capital stock, the outputs of bank 1 shall increase in all periods.
### Table 1: The data set of 20 bank branches in the 3-month period

#### The data set in period t = 1

| Bank | Employee score \((x_1)\) | Deferred claims \((x_2)\) | Capital input \((z_1)\) | Loans \((y_1)\) | Deposit \((y_2)\) | Profit \((y_3)\) |
|------|---------------------------|--------------------------|-----------------------|----------------|----------------|----------------|
| 1    | 19.83                     | 4,603,910                | 25,415,944            | 75,097,467     | 80,023,776     | 2,211,465      |
| 2    | 7.08                      | 9547                     | 631,223               | 25,258,238     | 40,413,775     | 767,801        |
| 3    | 4.01                      | 136,115                  | 39,950,793            | 60,530,507     | 48,420,589     | 1,770,276      |
| 4    | 14.92                     | 1,035,215                | 2,823,106             | 65,413,851     | 101,707,340    | 3,898,218      |
| 5    | 5.33                      | 1,030,194                | 3,536,597             | 45,667,593     | 67,411,796     | 1,291,753      |
| 6    | 12.84                     | 2,664,633                | 56,976,524            | 163,070,472    | 4,334,957      |                |
| 7    | 15.72                     | 1,086,083                | 260,385,226           | 462,186,659    | 397,280,289    | 168,494,852    |
| 8    | 10.94                     | 225,665                  | 22,036,925            | 105,618,280    | 130,611,124    | 4,072,227      |
| 9    | 13.08                     | 7,480,348                | 310,092               | 54,863,663     | 90,586,108     | 2,335,849      |
| 10   | 15.97                     | 3,486,536                | 164,983,122           | 175,915,348    | 567,562        |                |
| 11   | 4.53                      | 1,531,195                | 1,978,878             | 48,484,615     | 557,744        |                |
| 12   | 3.88                      | 106,162                  | 2,484,718             | 32,767,317     | 1,044,161      |                |
| 13   | 14.6                      | 430,201                  | 55,740,403            | 164,983,122    | 567,562        |                |
| 14   | 12.19                     | 288,733                  | 42,532,282            | 63,386,263     | 1,779,213      |                |
| 15   | 9.93                      | 79,572                   | 48,940,586            | 72,499,127     | 2,145,795      |                |
| 16   | 3.2                       | 23,274                   | 30,547,469            | 32,161,835     | 923,161        |                |
| 17   | 19.25                     | 655,170                  | 178,762,561           | 209,467,807    | 9,683,107      |                |
| 18   | 6.18                      | 70,840                   | 63,930,945            | 88,958,994     | 4,552,123      |                |
| 19   | 10.44                     | 5,771,009                | 49,562,263            | 96,511,315     | 695,511        |                |
| 20   | 7.86                      | 604,842                  | 5,532,201             | 54,972,684     | 1,539,726      |                |

#### The data set in period t = 2

| Bank | Employee score \((x_1)\) | Deferred claims \((x_2)\) | Capital input \((z_1)\) | Loans \((y_1)\) | Deposit \((y_2)\) | Profit \((y_3)\) |
|------|---------------------------|--------------------------|-----------------------|----------------|----------------|----------------|
| 1    | 9.38                      | 4,258,676                | 33,423,425            | 77,540,480     | 75,205,729     | 2,372,091      |
| 2    | 5.66                      | 9547                     | 2,635,237             | 28,099,162     | 40,501,342     | 767,801        |
| 3    | 4.76                      | 136,115                  | 42,323,282            | 63,866,263     | 49,871,113     | 1,779,213      |
| 4    | 7.69                      | 1,035,215                | 6,186,940             | 69,347,301     | 100,617,551    | 3,917,817      |
| 5    | 3.67                      | 837,568                  | 7,640,404             | 51,678,971     | 69,103,984     | 137,823        |
| 6    | 11                       | 1,217,053                | 73,818,634            | 165,665,373    | 158,442,184    | 4,786,804      |
| 7    | 11.04                     | 406,113                  | 457,271,922           | 408,482,255    | 17,214,726     |                |
| 8    | 7.7                       | 22,915                   | 21,522,051            | 141,070,488    | 4,089,487      |                |
| 9    | 10.44                     | 7,479,093                | 63,460,385            | 96,511,315     | 2,467,889      |                |

#### The data set in period t = 3

| Bank | Employee score \((x_1)\) | Deferred claims \((x_2)\) | Capital input \((z_1)\) | Loans \((y_1)\) | Deposit \((y_2)\) | Profit \((y_3)\) |
|------|---------------------------|--------------------------|-----------------------|----------------|----------------|----------------|
| 1    | 9.39                      | 4,164,067                | 34,302,366            | 79,100,108     | 76,524,475     | 2,608,693      |
### Table 1 (continued)

The data set in period \( t = 3 \)

| Bank | Employee score \((x_1)\) | Deferred claims \((x_2)\) | Capital input \((z_1)\) | Loans \((y_1)\) | Deposit \((y_2)\) | Profit \((y_3)\) |
|------|---------------------------|---------------------------|--------------------------|-----------------|-----------------|-----------------|
| 2    | 5.67                      | 9547                      | 6,014,493                | 31,026,240      | 39,083,163      | 767,801         |
| 3    | 4.76                      | 136,115                   | 39,988,455               | 64,986,239      | 53,471,773      | 1,936,626       |
| 4    | 7.7                       | 1,033,215                 | 9,851,458                | 73,820,219      | 100,647,643     | 3,924,818       |
| 5    | 3.68                      | 786,287                   | 13,105,122               | 58,194,638      | 70,522,315      | 1,400,253       |
| 6    | 11.01                     | 1,267,053                 | 70,323,601               | 174,946,921     | 174,602,767     | 4,898,347       |
| 7    | 11.06                     | 395,522                   | 229,207,401              | 465,979,022     | 426,766,238     | 17,976,626      |
| 8    | 7.71                      | 225,915                   | 17,014,036               | 116,557,642     | 156,249,562     | 4,304,301       |
| 9    | 10.46                     | 7,477,837                 | 10,980,081               | 79,367,959      | 107,494,554     | 2,469,807       |
| 10   | 12.25                     | 3,657,016                 | 213,492,432              | 308,891,608     | 239,236,387     | 7,726,739       |
| 11   | 3.22                      | 1,512,675                 | 8,979,634                | 31,425,356      | 35,479,015      | 579,594         |
| 12   | 4.07                      | 106,162                   | 5,806,457                | 40,643,483      | 54,711,936      | 1,069,796       |
| 13   | 11.2                      | 430,201                   | 60,083,361               | 175,858,151     | 186,753,718     | 6,685,865       |
| 14   | 9.1                       | 277,182                   | 91,013,826               | 153,347,359     | 128,483,906     | 5,174,718       |
| 15   | 5.15                      | 79,572                    | 6,205,102                | 60,613,709      | 86,298,673      | 2,764,479       |
| 16   | 3.37                      | 23,274                    | 14,326,858               | 33,842,858      | 33,071,151      | 966,104         |
| 17   | 12.04                     | 654,770                   | 39,962,610               | 215,702,229     | 274,686,310     | 10,576,311      |
| 18   | 6.33                      | 68,211                    | 61,511,021               | 93,642,456      | 74,504,425      | 5,066,401       |
| 19   | 5.65                      | 148,329                   | 31,628,113               | 54,119,292      | 45,692,855      | 698,963         |
| 20   | 4.1                       | 604,842                   | 6,803,359                | 47,543,624      | 63,980,358      | 1,571,893       |

Initial and terminal capitals

| Bank | Initial capital \((Z^0)\) | Terminal stock \((Z^3)\) |
|------|---------------------------|--------------------------|
| 1    | 117,244,609               | 24,102,874               |
| 2    | 26,566,357                | 17,285,404               |
| 3    | 158,665,116               | 36,793,586               |
| 4    | 29,209,796                | 10,348,292               |
| 5    | 39,252,881                | 14,970,758               |
| 6    | 267,062,245               | 65,943,486               |
| 7    | 1,037,209,839             | 310,091,274              |
| 8    | 87,622,951                | 27,049,939               |
| 9    | 30,043,956                | 15,102,052               |
| 10   | 729,364,697               | 159,059,304              |
| 11   | 34,213,206                | 18,424,628               |
| 12   | 34,626,288                | 22,481,574               |
| 13   | 218,101,708               | 51,819,731               |
| 14   | 282,355,290               | 82,930,801               |
| 15   | 27,056,255                | 14,515,075               |
| 16   | 51,708,898                | 10,444,809               |
| 17   | 234,220,827               | 112,641,044              |
| 18   | 254,370,473               | 67,142,719               |
| 19   | 134,860,848               | 47,752,117               |
| 20   | 25,822,016                | 11,170,386               |
Accordingly, the following items can be considered as the stock capital on output levels in various production periods.

The input congestion and the output shortage due to congestion and inefficiency in the assessment window

| Bank branches | The input congestion ($x_1$) | The input congestion ($x_2$) | The input congestion ($z_1$) | The output shortage ($y_1$) | The output shortage ($y_2$) | The output shortage ($y_3$) |
|---------------|-------------------------------|-------------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1             | 0.97                          | 3,159,264.23                 | 10,571,478.49               | 20,954,224.32               | 45,701,903.51               | 1,565,536.16                |
| The results of the model (3) in the period $t = 1$ |                               |                               |                             |                             |                             |                             |
| 1             | 0.97                          | 3,293,267.40                 | 8,905,429.16                | 14,226,309.81               | 37,894,332.85               | 1,799,012.76                |
| The results of the model (3) in the period $t = 2$ |                               |                               |                             |                             |                             |                             |
| 1             | 0.97                          | 0                             | 10,993,792.12              | 16,965,193.49               | 41,986,953.64               | 1,751,931.59                |
| The results of the model (3) in the period $t = 3$ |                               |                               |                             |                             |                             |                             |

5 Conclusion

So far, no method has been proposed to identify the congestion under inter-temporal dependence.

This research introduces a method for identifying congestion in dynamic data envelopment analysis under inter-temporal dependence, whereby the stock capital is an inter-temporal dependence factor on the input and output levels. The proposed models enable the congested unit to estimate the excess values of inputs and the shortage values of outputs in each period of the assessment window. In fact, this model determines the congested unit in the assessment window and characterizes the extra-input and lack-output amounts in all of the congested unit components in each period of the assessment window. This method helps to identify a unit with a congested path. Besides, the excess input and the output shortage amount of the congested assessment path should eliminate the congested assessment path and determine an efficient assessment path for the period. Determining the amount of the excess input and the output shortage amount of the congested unit in each period of the assessment window has a high informative value for the decision-maker to evaluate the resource allocation and investment analysis of this unit compared with other DMUs. This information clarifies the highest congestion level for this unit in each period compared with other units. Therefore, determining the input surplus caused by the dynamic congestion of DMUs in each evaluation period prevented the inappropriate allocation of resources in different evaluation periods. This means that the managers can employ these models to detect congestion paths from non-congestion ones and design practical approaches to improve the performance of the congested paths. This paper presents new approaches in resource allocation management and handling investment analysis problems under the inter-temporal dependence of data.

This paper studied the problem of identifying congested units under the inter-temporal dependence influencing the stock capital on output levels in various production periods. Accordingly, the following items can be considered as the research aspects:

(i) Identification of the congested units when some outputs produced in a period are considered as inputs in the next period (Ghobadi et al. (2018)).

(ii) Identification of the congested units under the imprecise or vague data, including fuzzy, interval, and stochastic.

Acknowledgements The authors would like to thank anonymous reviewers for their insightful and constructive comments and suggestions, as results the paper has been improved.

Funding This study was not funded by any organization.

Declarations

Conflict of interest Author Tahereh Shahsavan declares that he has no conflict of interest. Author Masoud Sanei declares that he has no conflict of interest. Author Ghasem Tohidi declares that he has no conflict of interest. Author Saeid Ghobadi declares that he has no conflict of interest. Author Farhad Hosseinzadeh Lotfi declares that he has no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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