EFFECTIVE LAGRANGIANS
AND TRIPLE BOSON COUPLINGS

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ABSTRACT

I review the use of effective lagrangians in describing the physics beyond the standard model, several theoretical and practical aspects are discussed. It is argued that the only situations where new physics can be observed corresponds to cases where the standard model contributions are extremely suppressed and where high quality data is available.
Effective lagrangians have been used often in the past with great success as, for example, the four-fermi approach to low energy weak interactions and the chiral lagrangian approach to the strong interactions demonstrate. In this talk I will apply this formalism to describe the low energy effects of physics beyond the standard model [1,2,3,4,5]. This approach is self-consistent and model independent but cannot (by its very nature) determine unambiguously the kind of physics present at high energies. A well known fact (though often forgotten) is that the effective lagrangian is associated with a cutoff $\Lambda$ which is a measure of the scale of new physics; it then follows that this approach has a limited energy range and cannot be applied for energies above $\Lambda$.

The effective lagrangian is defined as the most general (local) object which obeys certain symmetries and contains a given set of fields (the light excitations). For the case at hand I will choose the standard model fields (including the Higgs) as the light excitations; in this case the decoupling theorem [6] insures that all observable low effects produced by the new interactions can be described as a series in $1/\Lambda$ [1]. When the Higgs is not included in the light sector a chiral lagrangian classification of the induced operators is required and has been studied extensively elsewhere [4]. In this talk I will consider only first possibility due to space limitations.

An example of an effective lagrangian extensively studied in the literature [7] is the triple gauge bosons vertex for two $W$’s and one neutral ($Z, \gamma$) vector boson,

$$\mathcal{L}_{WWV} = g_{WVV} \left[ ig_1^Y \left( W^\dagger_{\mu\nu} W^{\mu\nu} - \text{h. c.} \right) + i\kappa V W^\dagger_{\mu} W_{\nu} V^{\mu\nu} ight)$$

$$+ i \frac{\lambda V}{m^2_W} W^\dagger_{\lambda\mu} W^{\mu\nu} V^{\lambda\nu} - g_4 W^\dagger_{\mu} W_{\nu} \left( \partial^{\mu} V^{\nu} + \partial^{\nu} V^{\mu} \right)$$

$$+ \text{CP violating terms} \right] \right.$$  \hspace{1cm} (1)

where the imposed symmetries are $U(1)^{EM}$ (for the case $V = \gamma$) and Lorentz invariance. This expression predicts the following static moments for the $W$: $\mu_W = e(1+\kappa_\gamma+\lambda_\gamma)/(2m_W)$ for the magnetic dipole moment and $Q_W = -e(\kappa_\gamma-\lambda_\gamma)/m^2_W$.  

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for the electric quadrupole moment; for the standard model $g_1^\gamma = 1$, $\lambda_{Z,\gamma} = 0$, $\kappa_{Z,\gamma} = 1$.

Going back to the general formalism I need now to choose the symmetries to be obeyed by the effective interactions. In this respect there has been some controversy as to whether gauge invariance should be imposed; the point [8] is that, by an extension of the Stuckelberg trick [9], any lagrangian can be thought as being the the unitary gauge version of a gauge invariant lagrangian. The idea is simple: given a set of vector fields $A_\mu$, introduce an auxiliary unitary field $U$ and construct the object $A_\mu = U^\dagger(\partial_\mu + iA_\mu)U$; if we now assume that the $A_\mu$ are in fact gauge fields, then a gauge transformation for $U$ can be chosen so that $A$ is gauge covariant. Then if the original lagrangian is $\mathcal{L}(A)$ the object $\mathcal{L}(A)$ is gauge invariant and coincides with $\mathcal{L}(A)$ in the unitary limit $U \rightarrow 1$.

Based on this it would seem that gauge invariance is indeed a red herring (since anything can be thought of as being gauge invariant) but I don’t believe that this is the case. The important point often missed in this discussion is that the gauge group is not fixed. For example, $\mathcal{L}_{WWV}$ in (1) can be written as an $SU(2) \times U(1)$ or as a $U(1)^3$ invariant lagrangian, and the imposition of a given gauge group does have non-trivial content. Since the standard model gauge symmetry is in agreement with all experimental observations [10], I will also require the effective interactions to obey this symmetry.

From a theoretical standpoint gauge invariance is also very important for the naturalness of the theory: if absent, strong arguments indicate that radiative corrections will drive the mass of the vector bosons to the cutoff [11].

Based on the above discussion the effective lagrangian can be written in the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum_\mathcal{O} \alpha_\mathcal{O} \mathcal{O} + O\left(\frac{1}{\Lambda^3}\right)$$

where catalogues of the operators $\mathcal{O}$ can be found in Refs. 1, 12; some examples
\[ O_W = \epsilon_{abc} W_{\mu\nu}^a W^{b\lambda} W^c_{\lambda\mu}; \quad O_{WB} = \left( \phi^{\dagger} \gamma^a \phi \right) W_{\mu\nu}^a B^{\mu\nu} \]  

where \( W_{\mu\nu}^a \) and \( B_{\mu\nu} \) denote respectively the \( SU(2) \) and \( U(1) \) field curvatures and \( \phi \) the scalar doublet. These operators contribute to (1), for example: \( \kappa - 1 = (4m_W^2/gg'\Lambda^2)\alpha_{WB} \) and \( \lambda = (6m_W^2/g\Lambda^2)\alpha_W \).

The requirement that the approach be self consistent fixes the order of magnitude of the coefficients, for example, if the underlying physics is weakly coupled it can be shown that, since \( O_W \) and \( O_{WB} \) can be generated only via loops [13],

\[ \alpha_W \sim \frac{g^3}{16\pi^2}; \quad \alpha_{WB} \sim \frac{gg'}{16\pi^2} \]  

where I used the fact that each vector bosons is always accompanied with its corresponding gauge coupling. Therefore the natural size for the anomalous moments is

\[ |\kappa - 1|, |\lambda| \sim 10^{-3} \quad (\Lambda \sim 250\text{GeV}) \]  

This is bad news for precision measurements at colliders. To see this note that effective lagrangians are useful only if the underlying physics is not apparent, thus if, for example, we hope to use this approach in a 1TeV electron collider, we should study the predictions with \( \Lambda > 1\text{TeV} \) only. Applying this to LEP2, I find that the expected magnitude of \( |\kappa - 1| \) and \( \lambda \) is \( 10^{-3} \), while the expected sensitivity is only \( \sim 0.1 \). The sensitivity is expected to improve by an order of magnitude at the NLC (\( \sqrt{s} = 0.5\text{TeV} \)), but then the natural scale of the above parameters becomes \( \sim 4 \times 10^{-4} \). It is, however, possible for the underlying theory to have relatively light resonances, this in some instances can produce an improvement of \( \sim 10 \) in the estimates for the \( \alpha_O \); if this is actually realized several collider proposals to measure \( \kappa \) and \( \lambda \) will be marginally sensitive to new physics, some examples are the SSC (sensitivity \( \sim 10^{-2} \)) [14] and a 1.5TeV \( e^+e^- \) collider (sensitivity \( \sim 10^{-3} \)) [15].
A conclusion which can be immediately drawn is that there are *never* large effects from $\mathcal{L}_{\text{eff}}$. This results from the suppression by powers of $1/\Lambda$ multiplying the $\mathcal{O}$ and from the limited energy range of applicability. The existing claims to the contrary correspond to situations where the energies are larger than the cutoff $[16]^\star$, or from unnaturally large coefficients which violate (4) by several orders of magnitude. Therefore the best places to find effects from the operators $\mathcal{O}$ is either in high precision experiments (such as the muon’s anomalous magnetic moment) or in cases where the standard model contributions are accidently suppressed (such as the $\rho$ parameter).

I shall present two examples corresponding to each of these possibilities. First I present the results for the anomalous moment of the muon [18]. I will consider two contributions, first from the operators $\mathcal{O}_W$ and $\mathcal{O}_{WB}$ at the one loop level, and the tree level ones generated by the “direct” operators

$$
\mathcal{O}_{\mu B} = (\bar{\nu}_\mu, \bar{\mu})_L \sigma^{\mu\nu} B_{\mu\nu} \mu_R \phi \quad \mathcal{O}_{\mu W} = (\bar{\nu}_\mu, \bar{\mu})_L \sigma^{\mu\nu} \sigma^{a} W^a_{\mu\nu} \mu_R \phi
$$

The loop contributions give

$$
\delta a_\mu = 10^{-10} \frac{\alpha W}{\Lambda_{\text{TeV}}^2} + 2 \times 10^{-9} \left(1 + \frac{\ln \Lambda_{\text{TeV}}^2}{7}\right) \frac{\alpha}{\Lambda_{\text{TeV}}^2} \approx 3 \times 10^{-12} \left(1 + \frac{\ln \Lambda_{\text{TeV}}^2}{7}\right) \frac{1}{\Lambda_{\text{TeV}}^2}
$$

where $\Lambda_{\text{TeV}}^2$ is the cutoff in TeV units and I used (4). It is clear from this expression that the Brookhaven experiment AGS 821 [19] will be insensitive to $\mathcal{O}_W$, $\mathcal{O}_{WB}$, the corresponding contributions are just too small. On the other hand, the effects of $\mathcal{O}_{\mu W, B}$ are much larger and in fact a sensitivity to $\Lambda$ of $\sim 700$ GeV can be inferred [18] for the above experiment.

As the second example I consider a modified standard model in which the low energy fields are the standard model ones with the addition of an extra scalar doublet $^\dagger$. Within this model there is a scalar excitation which is CP odd and

\[ \star \] This is also true for the “delayed unitarity” scenario [17].

\[ \dagger \] I will impose the usual discrete symmetry [20].
which I’ll denote by $a_o$; its decay into two photons is strongly suppressed in the limit where the ration of the $\langle s \rangle$, denoted by $\tan \beta$, is large [21]. Therefore this is a promising decay to study when considering the effects of the effective operators $\mathcal{O}$ (which describe now the physics beyond this two doublet model).

There is no dimension six operator contributing to $a_o \to \gamma \gamma$ at tree level, so that the $\mathcal{O}$-induced loop contributions must be finite. This is indeed verified by explicit computation [22]. For simplicity I will consider here only

$$\mathcal{O} = (\phi_1^\dagger \phi_2)(\bar{q}_R t \phi_1),$$

where $q$ denotes the top-bottom left-handed fermion doublet and $t_R$ the right-handed top quark field. The full calculation will appear in Ref. 22.

Evaluating the relevant graphs and taking $m_{\text{top}} = 170 \text{GeV}$ and $\alpha_\mathcal{O} v^2 = \Lambda^2$ I find that the contribution from this operator dominates the usual one provided $\tan \beta > 6$, the general order of magnitude is $\Gamma(a_o \to \gamma \gamma) \sim 10^{-9} \text{GeV}$ when $m_{a_o} \sim m_{\text{top}}$ and it peaks at $m_{a_o} \simeq 2 m_{\text{top}}$. It is worth pointing out that the effective operators will also generate angular distributions for the two photon final state which differs from the standard one. The branching ratio is very small, however, and this will probably be unobservable. This reaction is a good place to look for physics beyond, for example, the minimal supersymmetric extension of the standard model [20] if the large $\tan \beta$ scenario is realized.

I now discuss some technical aspects of the effective lagrangian formalism.

1) Equations of motion. It has been shown that the number of operators $\mathcal{O}$ can be reduced when the classical equations of motion are imposed [1,5,23]. However, even if some operators are related via the classical equations of motion, they can have very different origins. Consider for example

$$\left\{(D_\mu \phi)^\dagger (D_\mu \phi) + \phi^\dagger [D_\mu, D_\nu] \phi\right\} B^{\mu\nu}$$

which can be generated only via loops [13]. On the other hand the use of the equations of motion show that it is equivalent to $(\phi^\dagger D_\nu \phi) j^\nu$ where $j^\nu$ is the source
current for $B$; this last operator can be generated at tree level. Therefore even if the S-matrix elements cannot distinguish between the first and second operators, there is a very large quantitative difference whether the underlying physics generates the second one or not.

\[ \text{ii)} \text{ Blind directions} \] The above comments should be kept in mind when studying effects from operators to which we are not currently sensitive (blind directions \[3\]). In the final analysis, the statement that blind operators have coefficients similar to the ones we are sensitive to is an additional assumption. To illustrate this point consider a model with a light scalar field $\phi$ interacting with two heavy fermions $\psi_a (a = 1, 2)$. The lagrangian is

\[
\mathcal{L} = \frac{1}{2}(\partial \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{6}\sigma\phi^3 - \frac{1}{24}\lambda\phi^4 + \sum_{a=1}^{2} \bar{\psi}_a (i\partial - M + (-)^ag\phi) \psi
\]

(10)

When the fermions are integrated out they produce an effective action even in $\phi$.

If odd powers of $\phi$ are blind in this toy world then the claim that, say, the coefficient of $\phi^5$ will be of the same order as that of $\phi^6$ (times $M$) is wrong. On the other hand if even powers of $\phi$ are blind, then very precise measurements on the coefficient of $\phi^5$ will produce very misleading conclusions about the scale $M$.

\[ \text{[24]} \text{ iii)} \text{ Anomalies} \] Though there are new fermionic couplings no new anomalies are generated

\[ \text{iv)} \text{ Gauge invariance} \] It might seem puzzling to assume, from a calculational point of view, that the effective operators are gauge invariant. After all these are generated in a large mass expansion from an underlying (presumably gauge) theory. In the process one must fix the (underlying) gauge, add Fadeev-Popov ghost, etc. etc. and in this process explicit gauge invariance is generally lost. To solve this puzzle it is just necessary to recall that one can use a background gauge fixing method \[25\] which guarantees a gauge invariant effective action.
v) Renormalizability. Contrary to the standard lore, effective lagrangians are renormalizable: all infinities can be absorbed in redefining the coefficients of some effective operator already present (by definition) in the theory, moreover this can be done order-by-order in $1/\Lambda$. The only remnant of the (logarithmic) divergences is the renormalization group running of the coefficients $\alpha_O$. All power divergences are unobservable. Many loop calculations have already appeared in the literature [26].

I would like to conclude by stating that the effective lagrangian formalism is a self-consistent model independent way of talking about what we don’t know. The approach is fully renormalizable and the divergences are unobservable, loops can be calculated as usual.

Most effects are predicted to be too small for observation and the expected sensitivity to be derived from near-future and existing experiments is modest (in the few hundred GeV range). It is possible, however, for the coefficients $\alpha_O$ to be anomalously small [27], in this case it may very well be that no precision measurement will suggest the presence of new resonances which, in fact, are just around the corner. There is also the possibility of some modest ($\lesssim 10$) enhancements of the $\alpha_O$ due, for example, to low lying resonances, but even in this case the sensitivity of future colliders will be at best marginal to the effects of the operators $\mathcal{O}$. 


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