The Evolution of Universe with the B-I Type Phantom Scalar Field

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Abstract

We considered the phantom cosmology with a lagrangian $L = \frac{1}{\eta}[1 - \sqrt{1 + \eta g^{\mu\nu} \phi, \mu, \nu}] - u(\phi)$, which is original from the nonlinear Born-Infeld type scalar field with the lagrangian $L = \frac{1}{\eta}[1 - \sqrt{1 - \eta g^{\mu\nu} \phi, \mu, \nu}] - u(\phi)$. This cosmological model can explain the accelerated expansion of the universe with the equation of state parameter $w \leq -1$. We get a sufficient condition for a arbitrary potential to admit a late time attractor solution: the value of potential $u(X_c)$ at the critical point $(X_c, 0)$ should be maximum and large than zero. We study a specific potential with the form of $u(\phi) = V_0(1 + \frac{\phi}{\phi_0})e^{(-\frac{\phi}{\phi_0})}$ via phase plane analysis and compute the cosmological evolution by numerical analysis in detail. The result shows that the phantom field survive till today (to account for the observed late time accelerated expansion) without interfering with the nucleosynthesis of the standard model (the density parameter $\Omega_{\phi} \simeq 10^{-12}$ at the equipartition epoch), and also avoid the future collapse of the universe.

Keywords: Dark energy; Born-Infeld type scalar field; Attractor solution; Phantom cosmology.

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1 Introduction

A recent great finding in modern cosmology is that the universe is undergoing a
accelerating expansion at present epoch and have a decelerated expansion phase in the recent
past. This dynamics property is not only inferred in high redshift surveys of type Ia supernovae[1],
but also independently implied from seven cosmic microwave background experiments(including the latest WMAP results)[2]. People have proposed many candidates for
dark energy to fit the current observations, several important candidates may be the cos-
mological constant (or vacuum density), quintessence[3](a time varying scalar field evolving
in a specific potential) and the "k-essence"(see literatures[4,5] and References). The major
difference among these models is that they predict different equations of state parameter of
dark energy and different accelerating style of present universe. Very recently, joint analysis
of CMB+SN-Ia+HST+LSS data hold that the equation of state parameter can lie in the
rang of \( -1.32 < w < -0.82 \) with a best-fit value of \( w \sim -1.04 \) at the \( 2 - \sigma \) confidence levels,
slightly preferring "phantom" models[6]. Though phantom models[7] poses a challenge to the
wildly accepted energy condition[8,9] and may lead to rapid vacuum decay[10], it is still very
meaningful to study these models in the sense that it fits the observation very well, and also
is phenomenologically interesting.

Nonlinear Born-Infeld theory has been considered widely in string theory and cosmology.
In 1934[11], Born and Infeld put forward a theory of nonlinear electromagnetic field to resolve
the singularity in classical electromagnetic dynamics. The lagrangian density is

\[
L_{BI} = b^2 \left[ 1 - \sqrt{1 + \left( \frac{1}{2b^2} \right) F_{\mu\nu} F^{\mu\nu}} \right]
\] (1)

In order to describe the process of meson multiple production connected with strong field
regime[12], Heisenberg proposed the following nonlinear scalar field lagrangian firstly:

\[
L = \frac{1}{\eta} \left[ 1 - \sqrt{1 - \eta g^{\mu\nu} \phi,_{\mu} \phi,_{\nu}} \right]
\] (2)

This lagrangian density(2) possesses some interesting characteristics: (i)it is exceptional in
the sense that shock waves do not develop under smooth or continuous initial conditions[13],
(ii)because nonlinearities have been introduced, nonsingular scalar field solutions can be gen-
erated, (iii) if \( g^{\mu\nu} \phi,_{\mu} \phi,_{\nu} << \frac{1}{\eta} \), by Taylor expansion, Eq.(2) approximates to the lagrangian
of linear scalar field,

\[
\lim_{\eta \to 0} L = \frac{1}{2} g^{\mu\nu} \phi,_{\mu} \phi,_{\nu}
\] (3)
the linear theory is recovered. H.P.de Oliveira has investigated qualitatively the static and spherically symmetric solutions of this nonlinear scalar field\cite{14}. One of our authors has investigated the quantum cosmology based on the same nonlinear B-I type scalar field\cite{15}. Especially, if the potential $u(\phi)$ equals to $\frac{1}{\eta}$, we find that $p\rho = -\frac{1}{\eta^2}$, this describes a Chaplygin gas. The Chaplygin gas has raised a renewed interest recently because of its many remarkable and intriguing unique features\cite{16}.

We have proposed a dark energy model based on the lagrangian Eq(2) in literature\cite{8}, and found the universe with this B-I type scalar field with a potential can undergo a phase of accelerating expansion, the corresponding equation of state parameter lies in the range of $-1 < w < -\frac{1}{3}$, for the nonlinear B-I lagrangian with a negative kinetic energy term, when $u(\phi)$ is constant, the corresponding equation of state parameter can lie in the range $w < -1$.

In this paper, we consider the phantom cosmology of this nonlinear B-I lagrangian, i.e, with a negative kinetic energy term, $L = \frac{1}{\eta} \left[ 1 - \sqrt{1 + \eta^2 \dot{\phi}^2} \right] - u(\phi)$, where $u(\phi)$ is taken the form of $u(\phi) = V_0 (1 + \frac{\phi}{\phi_0}) e^{-\frac{\phi}{\phi_0}}$, investigate the global structure of the dynamical system via phase plane analysis, and calculate the cosmological evolution by numerical analysis.

The paper is organized as follows: In section 2, we start the phantom model with the B-I lagrangian, in section 3, the dynamical evolution of the phantom field without the presence of matter and radiation is considered, we discuss the evolution of the phantom field at the presence of matter and radiation in section 4, section 5 is summary.

## 2 The Phantom Model with B-I Lagrangian

We consider the kinetic energy term is negative, namely the phantom cosmology. For the spatially homogeneous phantom scalar field, we have the following lagrangian:

$$L = \frac{1}{\eta} \left[ 1 - \sqrt{1 + \eta^2 \dot{\phi}^2} \right] - u(\phi)$$  \hspace{1cm} (4)

where $u(\phi)$ is the positive potential, $\eta$ is a constant. First we consider the simple case where the phantom field $\phi$ is dominant only. In the spatially flat Robertson-Walker metric $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$, the Einstein equation $G_{\mu\nu} = \xi T_{\mu\nu}$, can be written as

$$\begin{cases} H^2 = \frac{\xi}{3} \rho_{\phi} \\ \frac{\ddot{a}}{a} = -\frac{2}{3} (\rho_{\phi} + 3P_{\phi}) \end{cases}$$  \hspace{1cm} (5)
For a spatially homogenous phantom field $\phi$, we have the equation of motion

$$\ddot{\phi} + 3H\dot{\phi}(1 + \eta\dot{\phi}^2) - u'(\phi)(1 + \eta\dot{\phi}^2)^{\frac{3}{2}} = 0 \quad (6)$$

where the overdot represents the differentiation with respect to $t$ and the prime denotes the differentiation with respect to $\phi$, the constant $\xi = 8\pi G$, where $G$ is a Newtonian gravitation constant. The Energy-moment tensor is

$$T^\mu_\nu = -\frac{g^{\rho\sigma}\phi_{,\rho}\phi_{,\sigma}}{\sqrt{1 + \eta\phi^2}} - \delta^\mu_\nu \mathcal{L} \quad (7)$$

From Eq.(7), we have

$$\rho_\phi = T^0_0 = \frac{1}{\eta\sqrt{1 + \eta\phi^2}} - \frac{1}{\eta} + u \quad (8)$$

$$P_\phi = -T^i_i = \frac{1}{\eta} - \frac{\sqrt{1 + \eta\phi^2}}{\eta} - u \quad (9)$$

From Eqs.(8) and (9), we obtain

$$\rho_\phi + p_\phi = -\frac{\dot{\phi}^2}{\sqrt{1 + \eta\phi^2}} \quad (10)$$

It is clear that the equation of state parameter $w_\phi < -1$ is completely confirmed by Eq.(10) which agrees with the recent analysis of observation data. We also can get

$$\rho_\phi + 3p_\phi = \frac{2}{\eta} - \frac{2}{\eta}\sqrt{1 + \eta\phi^2} - \frac{\dot{\phi}^2}{\sqrt{1 + \eta\phi^2}} - 2u \quad (11)$$

It is obvious that $\rho_\phi + 3p_\phi < 0$. Eq.(11)shows that the universe is undergoing a phase of accelerating expansion. The model of phantom B-I scalar field without potential $u(\phi)$ is hard to understand. We can always find $\rho_\phi = \frac{1}{\eta\sqrt{1 + \eta\phi^2}} - \frac{1}{\eta} < 0$ in this case, it is unreasonable apparently. However, in the model of phantom B-I scalar field with potential $u(\phi)$, when

$$u(\phi) > \frac{1}{\eta} - \frac{1}{\eta\sqrt{1 + \eta\phi^2}} \quad (12)$$

$\rho_\phi$ is always greater than zero. In phantom B-I scalar model with a potential $u(\phi)$, we also find the strong and weak energy condition always failed from Eqs.(10)and (11). The equation of state is

$$w_\phi = \frac{P_\phi}{\rho_\phi} = -1 - \frac{\eta\dot{\phi}^2}{1 + |\eta u(\phi) - 1|\sqrt{1 + \eta\phi^2}} \quad (13)$$

It is always less than $-1$ if Eq(12) is satisfied.
We also can write the speed of sound in these phantom model:

\[ c_s^2 = \frac{P, X}{\rho, X} = \frac{L_{\phi, X}}{L_{\phi, X} + 2XL_{\phi, X}} \]  

(14)

where \( P = L_{\phi}(\phi, X) \) and \( \rho = 2XL_{\phi, X} - L_{\phi}(\phi, X) \) with \( X = \frac{1}{2}(\partial_{\mu}\phi)^2 \). In our model \( X = \frac{1}{2}\dot{\phi}^2 \), so the speed of sound of the phantom with BI lagrangian is

\[ c_s^2 = \frac{P, X}{\rho, X} = 1 + \eta\dot{\phi}^2 \]  

(15)

It is clear from Eq.(15) that the \( c_s \) is always greater than 1 unless \( \dot{\phi}^2 = 0 \), this means that the perturbation of the background field can travel faster than light as measured in the preferred frame where the background field is homogeneous. For a time dependent background field, this is not a Lorentz invariant state. However, it does not violate causality because the underlying theory is manifestly Lorentz invariant and it is not possible to transmit information faster than light along arbitrary spacelike directions or create closed timelike curves[17]. In Ref[18], they also shows that a fluid with \(|w| > 1\) does not contradict causality if \( w \) is not constant. In our model, \( w \) is clear a time-dependent parameter.

### 3 Dynamical Evolution Of The Phantom Field

As we consider the phantom field becomes dominant, we can neglect the nonrelativititic and relativistic components (matter and radiation) in the universe, then from Eq.(5,8,6), we have

\[ \ddot{\phi} + \dot{\phi}(1 + \eta\dot{\phi}^2) \sqrt{3\xi\left[\frac{1}{\eta\sqrt{1 + \eta\dot{\phi}^2}} - \frac{1}{\eta} + u(\phi)\right]} = 0 \]  

(16)

to study an numerical computation, it is convenient to introduce two independent variables

\[
\begin{cases}
    X = \phi \\
    Y = \dot{\phi}
\end{cases}
\]

(17)

then Eq.(16) can be written

\[
\begin{cases}
    \frac{dX}{dt} = Y \\
    \frac{dY}{dt} = u'(X)(1 + \eta Y^2)^{\frac{3}{2}} - Y(1 + \eta Y^2) \sqrt{3\xi\left[\frac{1}{\eta\sqrt{1 + \eta Y^2}} - \frac{1}{\eta} + u(X)\right]}
\end{cases}
\]

(18)

we can obtain this system’s critical point from

\[
\begin{cases}
    \frac{dX}{dt} = 0 \\
    \frac{dY}{dt} = 0
\end{cases}
\]

(19)
then its critical point is \((X_c,0)\), where the critical value \(X_c\) is determined by \(u'(X_c) = 0\). Linearizing Eq.(18) around the critical point, we have

\[
\begin{aligned}
\frac{dX}{dt} &= Y \\
\frac{dY}{dt} &= u''(X_c)(X - X_c) - \sqrt{3\xi u(X_c)}Y
\end{aligned}
\]  

(20)

the types of the critical point are determined by the eigenvalue of system

\[
\lambda^2 + \alpha \lambda + \beta = 0
\]  

(21)

where \(\lambda = \sqrt{3\xi u(X_c)}, \beta = -u''(X_c)\), the two eigenvalues are \(\lambda_1 = -\sqrt{3\xi u(X_c)+2\xi u(X_c)+4u''(X_c)}, \lambda_2 = -\sqrt{3\xi u(X_c)-2\xi u(X_c)+4u''(X_c)}\). For a positive potentials, if \(u''(X_c) < 0\), then the critical point \((X_c,0)\) is a stable node, which implies that the dynamical system admits attractor solutions. We can also conclude that if a potential possesses the general properties: \(u(X_c) > 0, u'(X_c) = 0\) and \(u''(X_c) < 0\), then our phantom model with this potential must have a attractor solution and will predict a late time de-sitter like behavior \((w_\phi = -1)\). Some authors have studied the cosmological dynamics of phantom scalar field with special potentials which possessed above properties, such as \(u(\phi) = V_0 e^{-\phi^2/\sigma^2}[19], u(\phi) = V_0 [\cosh(2\phi/M_\phi)]^{-1}[20], u(\phi) = V_0 [1 + \alpha \phi^2/M_\phi]^{-1}\).

Next we specify the potential with a special form. We choose a widely studied potential[21] as

\[
u(\phi) = V_0(1 + \frac{\phi}{\phi_0})e^{-\frac{\phi}{\phi_0}}
\]  

(22)

where \(V_0 > 0\). It is easy to find that the critical \(X_c = 0\) and \(u''(X_c) = -\frac{V_0}{\phi_0^2} < 0\) in such a potential. Therefore this model has an attractor solution which corresponds to its attractor regime, the equation of state \(w \leq -1\). Substitute Eq.(22) into Eq.(18), we obtain

\[
\begin{aligned}
\frac{dX}{dt} &= Y \\
\frac{dY}{dt} &= -\frac{V_0}{\phi_0^2} X e^{-\frac{\phi}{\phi_0}} (1 + \eta Y^2)^{\frac{\gamma}{2}} - Y (1 + \eta Y^2) \sqrt{3\xi[\frac{1}{\eta(1 + \eta Y^2)} - \frac{1}{\eta} + \frac{1}{\phi_0^2}]} + V_0 (1 + \frac{X}{\phi_0}) e^{-\frac{\phi}{\phi_0}}
\end{aligned}
\]  

(23)

We will solve this equations system via the numerical approach, to do this, we rescale the quantities as \(x = \frac{X}{\phi_0}, s = (\phi_0^2 \eta)^{-\frac{1}{\gamma}} t, y = \sqrt{\eta} Y\), Then Eq.(22) becomes

\[
\begin{aligned}
\frac{dx}{ds} &= y \\
\frac{dy}{ds} &= -\gamma x e^{-x}(1 + y^2)^{\frac{\gamma}{2}} - \phi_0 y (1 + y^2) \sqrt{3\xi[\frac{1}{\sqrt{1 + y^2}} - 1 + \gamma(1 + x)e^{-x}]} + V_0 (1 + \frac{x}{\phi_0}) e^{-\frac{\phi}{\phi_0}}
\end{aligned}
\]  

(24)

where \(\gamma = V_0 \eta\) are parameters, we take \(\xi = 1\) for convenience. The numerical results with different initial condition are plotted in Figs.1 – 3 and the parameters \(\phi_0 = \sqrt{0.1}, \gamma = 3\).
Fig1. This plot shows the evolution of the scalar field in different initial conditions, solid line is for $\phi_{in} = 0.7\phi_0$, dotted line is for $\phi_{in} = \phi_0$, dashed line and dot-dashed line for $\phi_{in} = 1.3\phi_0, 1.7\phi_0$ respectively, they are all plotted for a fixed value of $y_{in} = 0.1$.

Fig2. The evolution of $w$ with respect to $s$, the initial condition is the same as fig1.
Fig3. The attractor property of the system in the phase plane, the initial condition is the same as fig1.

As we know, when $\eta \to 0$, our model come back quintessence case. In order to see the nonlinear effect, we plot the quintessence model with our model in fig4 and fig5.

Fig4. the evolution of scalar field with respect to $s$, solid line is quintessence model, dotted line and dashed line are nonlinear scalar field, dotted line is for $\eta = 2/3$, dashed line is for $\eta = 1/3$. 
Fig5. the evolution of $w$ with respect to $s$, solid line is quintessence model, dotted line and dashed line are nonlinear scalar field, dotted line is for $\eta = 2/3$, dashed line is for $\eta = 1/3$.

From fig1, fig2, and fig3, we can easily find the system admit a attractor solution, the equation of state parameter $w$ start from nearly -1, then quickly evolve to the regime of smaller than -1, turn back to execute the damped oscillation, and reach to -1 eventually for ever. Due to the unusual physical properties in phantom model, the phantom field, released at a distance from the origin with a small kinetic energy, moves towards the top of the potential and crosses over to the other side and then turns back to execute the damped oscillation around the critical point, after a certain time the motion ceases and the phantom field settles on the top of the potential permanently to mimic the de Sitter-like behavior ($w_\phi = -1$). Fig4 and fig5 indeed shows that the nonlinear scalar field will come back to quintessence case when $\eta$ decreases to zero. the nonlinear effect does not affect the global attractor behavior but change the evolution of $w$ and scalar field $\phi$ in details.

4 Evolution of the phantom at the presence of matter and radiation

In the previous section, we studied the case when phantom dominates over all other energy density. Now we consider the more general case that the effect of matter and radiation can
not be neglected, thus the Eq.(5) becomes

$$H^2 = \frac{\xi}{3} (\rho_\phi + \rho_M + \rho_r)$$ (25)

where $\rho_M$ and $\rho_r$ denote the energy density of nonrelativistic and relativistic components (namely, matter and radiation), respectively. We know that

$$\begin{cases} 
\rho_M a^3 = \rho_M a_i^3 = \text{constant} \\
\rho_r a^4 = \rho_r a_i^4 = \text{constant} 
\end{cases}$$ (26)

So we can rewrite Eq.(25) as

$$H^2 = H_i^2 \left[ \frac{\rho_\phi}{\rho_{ci}} + \Omega_M \left( \frac{a_i}{a} \right)^3 + \Omega_r \left( \frac{a_i}{a} \right)^4 \right]$$ (27)

where $H_i^2 = \frac{\xi \sqrt{\rho_{ci}}}{4}$, $\rho_{ci}$ is the critical energy density of the universe at the initial $t_i$, $\rho_{mi}, \rho_{ri}$ are the matter and the radiation energy densities at $t_i$, $\Omega_{mi}, \Omega_{ri}$ are the cosmic density parameters for matter and radiation at $t_i$, $a_i$ is the initial scale factor at $t_i$, we will specify $a_i = 1$ for convenience. We can finally give the form of Eq.(27) as

$$H^2 = H_i^2 \left[ \frac{\rho_\phi}{\rho_{ci}} + \Omega_M a^{-3} + \Omega_r a^{-4} \right]$$ (28)

we introduce the new variables $x = \frac{\phi}{\phi_0}$, $y = \sqrt{\frac{3 \rho_\phi}{\rho_{ci}}} N = \ln a$, we can express Eq.(16) as

$$\begin{cases} 
\frac{dx}{dN} = \frac{y}{\Pi_i \phi_0 \eta^\psi} \\
\frac{dy}{dN} = -3y(1 + y^2) + \sqrt{\frac{3 \rho_\phi}{\rho_{ci}}} \frac{\eta V_0}{H_i} (1 + y^2)^{\frac{3}{2}} \psi 
\end{cases}$$ (29)

where $\psi(x, y, N) = \left[ \frac{1}{\rho_{ci}} \left( \frac{1}{\eta \sqrt{1 + \eta^2}} - \frac{1}{\eta} + u \right) + \Omega_M e^{-3N} + \Omega_r e^{-4N} \right]^{-\frac{1}{2}}$. Substitute the potential Eq.(22) into Eq.(29), we get

$$\begin{cases} 
\frac{dx}{dN} = \frac{y}{\Pi_i \phi_0 \sqrt{\eta} \psi'} \\
\frac{dy}{dN} = -3y(1 + y^2) - \frac{\sqrt{3 \rho_\phi \eta}}{\phi_0 H_i} \left( 1 + y^2 \right)^{\frac{3}{2}} \psi' 
\end{cases}$$ (30)

where $\psi'(x, y, N) = \left[ \frac{V_0 \xi}{3H_i^2} \left( \frac{1}{\eta \sqrt{1 + \eta^2}} - \frac{1}{\eta \sqrt{\eta V_0}} + (1 + x) e^{(-x)} \right) + \Omega_M e^{-3N} + \Omega_M e^{-4N} \right]^{-\frac{1}{2}}$. The numerical results are plotted in Figs.6–10. We specify our starting point as the equipartition epoch, at which $\Omega_M = \Omega_r = 0.5$. We plot the results by choosing the parameter $\frac{\sqrt{3 \rho_\phi V_0}}{H_i} = 10^{-8}$.
Fig6. The evolution of the scalar field with respect to N in the presence of matter and radiation. They are plotted in different initial conditions, solid line is for $\phi_{in} = 10.6\phi_0$, dotted line, dashed line and dot-dashed line for $\phi_{in} = 8\phi_0, 6\phi_0, 4\phi_0$ respectively, $y_{in}$ is set zero all the time.

Fig7. The evolution of $w$ with respect to N in the presence of matter and radiation, we chose the same initial condition as fig6.
Fig8. The attractor property of the scalar field in the presence of matter and radiation, the initial condition is the same as in fig6.

Fig9. The evolution of density parameter $\Omega$ with respect to $N$, solid line is for scalar field, dotted line for matter and dashed line for radiation, where the initial value of $\phi_{in} = 10.6\phi_0$, $y_{in} = 0$. 
Fig10. The evolution of decelerating factor $q$ with respect to $N$, the initial condition is the same as fig9.

From these results (fig6,7,8) we can conclude that there is still a late time attractor solution even in the presence of matter and radiation. The phantom field energy density will dominant over the matter and radiation composition ($\Omega_\phi \to 1$) with its state parameter $w_\phi \to -1$, mimicking the cosmological constant. Such evolution behaviors will avoid the cosmic doomsday [9]. Fig10 quantitatively shows the universe is speeding up at present but undergoes a decelerated regime at the recent past. Since $T_{eq} \simeq 5.64(\Omega_0 h^2)eV \simeq 2.843 \times 10^4 K$ (where we set $h = 0.71$), $T_0 \simeq 2.7K$, $a_i = a_{eq} = 1$, the scale factor at the present epoch $a_0$ would nearly be $1.053 \times 10^4$, then we know at present epoch $N = lna_0 \simeq 9.262$, we calculate the current value of $\Omega_{\phi 0} \simeq 0.705$, decelerating factor $q_0 \simeq -0.603$ and the transition shift $Z_T \simeq 0.672$ in our model, this is consistent with current observation data.

5 Summary

We have shown that it is possible to construct a viable cosmological model based on nonlinear Born-Infeld scalar field. We get the condition to admit a late-time attractor solution: $u(X_c) > 0, u'(X_c) = 0$ and $u''(X_c) < 0$ at critical point, this condition is also true for quintessence model. However, we should emphasized that this condition is a sufficient condition. The attractive behavior of the solution may resolve the "coincidence problem" in dark energy models in principle. We investigate the global structure of the dynamical system via phase plane analysis and calculate the cosmological evolution by numerical analysis with
and without the presence of radiation and matter, the result shows the universe will evolve to a de Sitter like attractor regime in the future and the phantom field energy density will dominate the whole universe and behave as a cosmological constant. An obvious excellent character of our model is that the phantom field survive till today (to account for the observed late time accelerated expansion) without interfering with the nucleosynthesis of the standard model (the density parameter $\Omega_\phi \approx 10^{-12}$ at the equipartition epoch), and also avoid the future collapse of the universe, it was indicated by Carroll, Hoffman, and Trodden[19].

Theoretical cosmology with phantom models has become an active area of theoretical research. However, recently, some authors showed in [22] that quantum effects (without a negative energy kinetic term) can also render $\omega < -1$ on cosmological scales. Since the idea of phantom cosmology is pretty new, there are many questions remaining open yet.

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