Study of spatial and energy characteristics of relativistic electron bursts in magnetosphere with robust methods

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Abstract. Electron bursts are well-known phenomena of fast increase in particle fluxes in near-Earth space. Powerful local geophysical events like earthquakes or thunderstorms can induce precipitation of electrons with defined energy spectrum from the radiation belt, which would be registered as fast increase in particle count rate on board the low orbit satellite. Using particle burst energy spectrum evolution in time one can detect the area of particles precipitation. Background particles are registered by instruments too and can’t be separated from burst particles. High level of background particles can have large impact on detection of the area of particles precipitation. A robust regression method to solve problem of background particles is introduced and compared with standard method of linear regression. Results of comparison between various data analysis methods in application to study of spatial and energy characteristics of relativistic electron bursts in the Earth magnetosphere are presented in this work. Robust method proved to be optimal for data analysis of energy spectrum evolution in time for search of zones of local radiation belt disturbances.

1. Introduction
The shape of Earth’s magnetic field forms a trap for charged particles. It is called the Earth radiation belt. Some geophysical phenomena (earthquakes, thunderstorms) are known to be the cause of particle precipitations below the Earth radiation belt to heights of low orbital satellites. Precipitated particles take part in three types of motion: rotation around geomagnetic field line, bouncing between mirror points and longitudinal drift around the Earth. Satellite experiment below radiation belt with particle detection instrument crossing disturbed L-shell would detect precipitated particles as short time increase in count rate of particles. This well-known phenomenon is called particle burst and has been studied for long time in many satellite experiments [1, 2, 3, and references therein].

ARINA experiment is conducted on board the low-orbit russian sattelite Resurs-DK1 with altitude 350-600 km and inclination 70° since 2006 and VSPLESK experiment were carried out from 2008 to 2013 on board the International Space Station on altitudes 350-400 km
with inclination 51°[3]. ARINA and VSPLEKS instruments based on multilayer scintillation telescope-spectrometer detector with 10 layers of plastic scintillator plates, instruments can detect 3-30 MeV electrons and protons in energy range 30-100 MeV.

If one assumes that particle precipitation occurs instantly in single point-like region(locally), one can expect that precipitated particles would drifts along disturbed L-shell as a cloud of particles and the shape of this cloud will be mainly defined by energy spectrum of precipitated particles. Study of energy spectrum and time characteristics of particles of cloud in principle will determine parameters of precipitation area.

This article discusses features of various data analysis methods in application to study of electron bursts in the Earth magnetosphere using numerical models of local particles precipitation and background particle fluxes.

2. Particle longitudinal drift process simulation
As it was mentioned above, longitudinal drift velocity of particles depends on their energy. The time\( T_{dr} \) for particle to perform a complete rotation around the Earth defined by formula (1) (based on article [4]):

\[
T_{dr} = K_1 \frac{1 + \varepsilon}{\varepsilon(1 + \varepsilon)} \frac{K_2}{L}
\]

where \( \varepsilon \) is the ratio of the kinetic energy of the particle to its rest energy, \( L \) - drift shell coordinate.

\[
K_1 = \frac{4}{3} \pi Z |e| M \frac{M}{m_0 c^3}
\]

\( M \) - geomagnetic field momentum, \( m_0 \) - electron mass. The \( K_2 \) depends on latitude of particle’s mirror point and varies from 1.0 to 1.5, where \( K_2 = 1.0 \) on geomagnetic equator.

\[
t_{dr}(\Delta \lambda, \varepsilon) = T_{dr}(\varepsilon) \frac{\Delta \lambda}{360}
\]

where \( \Delta \lambda \) - longitudinal distance between points of precipitation and registration and \( \varepsilon = E/E_0 \), where \( E \) is particle energy.

Using this model, one can assume that registered particles of cloud would group along the curve \( t_{dr}(\Delta \lambda, \varepsilon) \) (3), where arrival time to detector from precipitation region depends on value of \( \Delta \lambda \) and \( \varepsilon \).

In this work, using Monte-Carlo method two sets of electrons were generated and followed all the way to their registration. The first set are electrons precipitated locally and instantly from radiation belt (see figure 1), and the second one consists of background albedo electrons (poisson process, random in time) (see figure 2). It is natural to search time of registration on particle energy dependence in form of plot where on x-axis particle energy is plotted, and on y-axis its registration time. Main objective of this research is to find burst electrons from merged sets of background and burst electrons, and determine \( \Delta \lambda \) parameter.

3. Data analysis methods
In this work two regression analysis methods were used: Linear regression method (Ordinary Least Squares) and Robust regression method. General formula for this type of analysis is shown on (4)[5, 6]:

\[
\text{General formula for regression analysis}
\]
In order to reconstruct curve and find longitudinal distance between zones of precipitation and registration, a global minimum of (4) have to be found, where $t_i$ - time of particle registration, $E_i$ - particle energy, $\sigma_i$ - standard deviation of measured particle energy.

In case of ordinary least squares method parameter $w_i$ is equal for all registered particles. In case of robust regression, using some assumptions or additional information about dataset, one need to create some weight function to provide $w_i$.

Using this weight function, one assumes that some of particles are part of burst and others are part of background.

4. Minimum linear distance

As it can be seen from figure 2, albedo electrons lays chaotically in relative to the curve, withal burst electrons lay close to the curve. Using this knowledge we assume that the particle lying far from the curve is has less probability to be part of the burst.

$$d_i = \sqrt{\frac{(E_i - E_{min})^2}{A*E_{lim}} + \frac{(t_i - t_{dr}(E_{min}, \Delta \lambda))^2}{B*t_{lim}}}$$  \hspace{1cm} (5)

$$w_i = \frac{1}{1 + e^{-\beta_0 + \beta_1 d_i}}$$  \hspace{1cm} (6)

In this work we used minimum linear distance from curve to each point (5) to measure burst electron likeness using logistic function (6) with defined parameters $\beta_0$ and $\beta_1$, where value $w_i = 1$, means that particle $i$ - is burst electron, and for value $w_i = 0$ particle $i$ - is background albedo electron. In this work first approximation curve is calculated using linear regression method, then we calculate $w_i$ of particles using their minimum linear distance to first approximation curve. Using robust regression of parameter, one can minimize impact of
background albedo electrons on $\Delta \lambda$ calculation. As one can see from figure 3, linear method failed to converge to original curve, in the same time, robust curve fits almost perfect to it. The results of that exact simulation round are the following: $\Delta \lambda_{\text{robust}} = 175 \pm 11$ with real $\Delta \lambda_{\text{sim}} = 180$.

**Figure 3.** Results of $t_{dr}$ curve reconstruction. Burst particles are shown as • and background particles as ■. First (linear) [ ] and second (robust) [—] curve approximations, with original (see figure 1) [—] curve.

5. **Linear and robust regression method comparison**

Regression methods were compared using a Monte-Carlo simulation of particle precipitation, followed by their longitudinal drift and registration by instrument on board satellite. Some number of background particles were injected and registered with burst particles. Instruments uncertainties of energy detection also took into account.

Methods were compared by varying parameters such as count of precipitated particles and background particles, their energy spectrum, level of instrument uncertainties.

As can be seen from figure 4, linear regression very fast deviates from original curve, nevertheless robust regression fits to original curve within the errors until the level of background particles less than 80% of precipitated (burst) particles.

Robust regression method show no deviation when background to burst ration is lower than 50%, but linear regression show deviation greater than 10% even in conditions where background to burst ratio is 20%.

Both methods have tendency to higher deviation, when background to burst ratio increases, but when ratio reaches 170% linear method show same deviation, because the method converges to straight line.

On figure 5 fit parameter deviation from original parameter dependency on total count of registered particles is shown. As one can see, errors become smaller when number of particles become bigger, but because with growing number of burst particles, number of background particles grows too. The whole image become less clearer and minimal linear distance heuristic performs poorer, than on small particles count.

Fit deviation dependency of robust method have a minimum at 20 particles, because in situation with low particle count, each background particle misinterpreted as burst particle have higher impact on fit, as particle count rises, effect from misinterpreted particles drops and the method shows the best results, after that as number of particles rises, cumulative effect of misinterpreted particles grows shifting fit results from original.
6. Conclusion

Using Monte-Carlo simulation the comparison between linear and robust regression methods have been carried out. Using robust regression method we managed to improve quality of fit and minimize the impact of background particles on analysis.

Robust regression method based on minimal linear distance heuristic with current algorithm settings in average make correct results (deviation less than 10%) in application to longitudinal drift distance determination before following conditions are met: background to burst ration is less than 80%, total number of registered burst and background particles less than 100.

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