A Quantum Bit Commitment Protocol Based on EPR States

M. Ardehali

Abstract

A protocol for quantum bit commitment is proposed. The protocol is feasible with present technology and is secure against cheaters with unlimited computing power as long as the sender does not have the technology to store an EPR particle for an arbitrarily long period of time. The protocol is very efficient, requiring only tens of particles.

1email:ardehali@mel.cl.nec.co.jp
2Atago, Tama-shi, Tokyo 206 Japan; permanent address: Microelectronics Research Laboratories, NEC Corporation, Sagamihara, Kanagawa 229 Japan Correspondence should only be sent to permanent address.
Quantum cryptography was initiated by the pioneering work of Wiesner in late Sixties (his paper, however, was not published until 1983 [1]). Over the past two decades, many applications of quantum cryptography have been discovered [2], [3]. One of the most outstanding applications of quantum cryptography is quantum bit commitment.

The interest in bit commitment is motivated by a recent trend in cryptographic research to reduce or more preferably to eliminate the complexity assumptions from the protocols. Many protocol problems that were previously solved subject to complexity assumptions are now being solved without these assumptions. These breakthroughs demonstrate the weaknesses of the unproven complexity assumptions. One way of eliminating the complexity assumptions from a protocol is to build the protocol using a small set of relatively simple primitives. The security of the protocol then entirely depends on the security of its primitives. One of the most fundamental primitives is the bit commitment. The extreme generality and usefulness of bit commitment primitive has been demonstrated by several authors [4].

Let us briefly review the goal of the bit commitment protocol:

1. Alice has a bit $\lambda$ in mind to which she would like to be committed toward Bob.
2. Bob should not learn any information about $\lambda$ before Alice opens up the commitment.
3. Alice should not be able to change the value of $\lambda$ after the commitment.

In the past, several bit commitment protocols based on complexity assumptions have been proposed. However, none of these protocols are safe against cheaters with unlimited computing power. Bit commitment protocols based on uncertainty principle have also been proposed [5], [6], [7]. However,
all these protocols are insecure against EPR attack [8-11]. In this paper, we
describe a new and efficient quantum bit protocol which requires only tens of
EPR particles and is feasible with present technology. However, the present
protocol, similar to previous schemes, is not secure against a cheating Al-
ice who has the technology to store an EPR particle for an arbitrarily long
period of time.

Before proceeding, it is useful to review some elementary features of quan-
tum mechanics. We consider a pair of particles in the EPR entangled state
\[ | \Phi \rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \]. After particles are spatially separated, the spin of
the first (second) particle \( m_1^a \) \((m_2^b)\) is measured along an arbitrary axis \( \vec{a} \) \((\vec{b})\),
with the z axis being along the direction of flight of particles. If the spin of
the first (second) particle is up, then \( m_1^a = 1 \) \((m_2^b = 1)\), and if the spin of
the first (second) particle is down, then \( m_1^a = -1 \) \((m_2^b = -1)\). The expected
value of the product of the spins of the particles is

\[
E_{\Phi}(\vec{a}, \vec{b}) = \langle \Phi | \sigma_1^a \sigma_2^b | \Phi \rangle 
= \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\phi_1 + \phi_2),
\]

where \( \theta_1(\theta_2) \) is the polar angle between \( \vec{a} \) \((\vec{b})\) and the z axis, and \( \phi_1(\phi_2) \) is
the azimuthal angle between \( \vec{a} \) \((\vec{b})\) and the x axis. Similarly for a pair of
particles in the EPR entangled state \( | \Phi' \rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle) \),

\[
E_{\Phi'}(\vec{a}, \vec{b}) = \langle \Phi' | \sigma_1^a \sigma_2^b | \Phi' \rangle 
= \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \cos (\phi_1 + \phi_2).
\]

For a pair of particles in the entangled state \( | \Psi \rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle + i |\downarrow\downarrow\rangle) \),

\[
E_{\Psi}(\vec{a}, \vec{b}) = \langle \Psi | \sigma_1^a \sigma_2^b | \Psi \rangle 
= \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \sin (\phi_1 + \phi_2).
\]
Finally for a pair of particles in the entangled state $|\Psi'\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - i |\downarrow\downarrow\rangle)$, 

$$E_{\Psi'}(\vec{a}, \vec{b}) = \langle \Psi' | \sigma_1^a \sigma_2^b | \Psi' \rangle$$ 

$$= \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \sin (\phi_1 + \phi_2). \quad (4)$$

For states $|\Phi\rangle$, $|\Phi'\rangle$, $|\Psi\rangle$, and $|\Psi'\rangle$, the probability that the product of the spins of the two particles $m_1^a m_2^b$ is +1 or −1 is

$$p(m_1^a m_1^b = 1) = \frac{1 + E(\vec{a}, \vec{b})}{2}, \quad (5)$$

$$p(m_1^a m_1^b = -1) = \frac{1 - E(\vec{a}, \vec{b})}{2}.$$

With the above in mind, we now proceed to describe a quantum bit commitment protocol based on EPR states. Alice and Bob initiate the following steps:

1. Alice and Bob agree on a security parameter $n$. They also agree that if Alice wants to be committed to bit $\lambda = 1$, then she prepares a sequence of $n$ states, randomly chosen from $|\Phi\rangle$ or $|\Phi'\rangle$, and if she wants to be committed to bit $\lambda = 0$, then she prepares a sequence of $n$ states, randomly chosen from $|\Psi\rangle$ or $|\Psi'\rangle$. They also agree on a security parameter $n$.

2. Bob chooses a vector $B = (\theta_1, \theta'_1, \phi_1, \phi'_1, \ldots, \theta_n, \theta'_n, \phi_n, \phi'_n)$ such that $\theta_i, \theta'_i, \phi_i,$ and $\phi'_i$ satisfy one of the following relations:

   1. $\theta_i = \theta'_i = 90^\circ$ and $\phi_i + \phi'_i = 0^\circ$,
   2. $\theta_i = \theta'_i$ and $\phi_i + \phi'_i = 0^\circ$,
   3. $\theta_i + \theta'_i = 180^\circ$ and $\phi_i + \phi'_i = 0^\circ$,
   4. $\theta_i = \theta'_i = 90^\circ$ and $\phi_i + \phi'_i = 90^\circ$,
   5. $\theta_i = \theta'_i$ and $\phi_i + \phi'_i = 90^\circ$,
   6. $\theta_i + \theta'_i = 180^\circ$ and $\phi_i + \phi'_i = 90^\circ$. 

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Bob will measure the spin of the first (second) particle at polar angle \( \theta_i (\theta'_i) \) and azimuthal angle \( \phi_i (\phi'_i) \). He keeps the vector \( B \) secret.

(4) \[ \text{Alice sends the } i \text{th EPR pair to Bob. Bob measures the spin of the first particle, } m^a_1, \text{ along axis } \vec{a} \text{ at polar (azimuthal) angle } \theta_i (\phi_i) \text{ and spin of the second particle, } m^b_1, \text{ along axis } \vec{b} \text{ at polar (azimuthal) angle } \theta'_i (\phi'_i). \text{ Bob keeps the results of his measurements secret.} \]

Note that Bob does not learn any information about the bit \( \lambda \) since if Alice selects states \( |\Phi\rangle \) or \( |\Phi'\rangle \), then

\[
p(m^a_1 m^b_1 = 1) = \frac{1}{2} \left( \frac{1 + E_{\Phi}(\vec{a}, \vec{b})}{2} + \frac{1 + E_{\Phi'}(\vec{a}, \vec{b})}{2} \right),
\]

\[
p(m^a_1 m^b_1 = -1) = \frac{1 - \cos \theta_1 \cos \theta_2}{2},
\]

and if she selects states \( |\Psi\rangle \) or \( |\Psi'\rangle \), then

\[
p(m^a_1 m^b_1 = 1) = \frac{1 + \cos \theta_1 \cos \theta_2}{2},
\]

\[
p(m^a_1 m^b_1 = -1) = \frac{1 - \cos \theta_1 \cos \theta_2}{2}.
\]

Thus according to the standard rules of quantum mechanics, Bob does not learn any information about Alice’s bit no matter along which axis he performs his measurement.

We now consider the opening of the commitment. Alice and Bob initiate the following steps:

(1) Alice reveals the bit \( \lambda \) to Bob [As previously stated, \( \lambda = 1 \) indicates that Alice has chosen state \( |\Phi\rangle \), or \( |\Phi'\rangle \), and \( \lambda = 0 \) indicates that Alice has chosen the state \( |\Psi\rangle \) or \( |\Psi'\rangle \)].

(2) If \( \lambda = 1 \), then \[ \text{Bob checks that if the } i \text{th EPR state is } |\Phi\rangle, \text{ then} \]

\[
p(m^a_1 m^b_1 = 1) = \frac{1 + \cos \theta_1 \cos \theta_2}{2},
\]

\[
p(m^a_1 m^b_1 = -1) = \frac{1 - \cos \theta_1 \cos \theta_2}{2}.
\]
\[ m_a^1 m_b^2 = 1 \quad \text{whenever} \quad \theta_i = \theta'_i = 90^\circ \quad \text{and} \quad \phi_i + \phi'_i = 0^\circ, \quad \text{and whenever} \quad \theta_i = \theta'_i \quad \text{and} \quad \phi_i + \phi'_i = 0^\circ. \]

If the \( i \)th EPR state is is \(| \Phi' \rangle\), then Bob checks that \( m_a^1 m_b^2 = -1 \)
whenever \( \theta_i = \theta'_i = 90^\circ \) and \( \phi_i + \phi'_i = 0^\circ \), and whenever \( \theta_i + \theta'_i = 180^\circ \) and \( \phi_i + \phi'_i = 0^\circ \).

If \( \lambda = 0 \), \( \sum_{i=1}^{n} \) Bob checks that if the \( i \)th EPR state is \(| \Psi \rangle\), then \( m_a^1 m_b^2 = 1 \)
whenever \( \theta_i = \theta'_i = 90^\circ \) and \( \phi_i + \phi'_i = 90^\circ \), and whenever \( \theta_i = \theta'_i \) and \( \phi_i + \phi'_i = 90^\circ \). If the \( i \)th EPR state is is \(| \Psi' \rangle\) then Bob checks that \( m_a^1 m_b^2 = -1 \)
whenever \( \theta_i = \theta'_i = 90^\circ \) and \( \phi_i + \phi'_i = 90^\circ \), and whenever \( \theta_i + \theta'_i = 180^\circ \) and \( \phi_i + \phi'_i = 90^\circ \).

(3) If these conditions are satisfied, then Bob accepts that Alice had indeed committed to the bit \( \lambda \).

We now show that the above bit commitment protocol is perfectly secure against cheaters with unlimited computing power provided Alice does not have the technology to store an EPR particle for an arbitrarily long period of time. First we note that if Alice is honest, and if no transmission errors occur [12], then condition (2) is always satisfied. Now suppose that a cheating Alice tries to commit in a way that will enable her to change \( \lambda \) at a later time. In order to achieve this, she must tell Bob that she had selected (for example) state \(| \Phi \rangle\), when in reality she had committed to the state \(| \Psi \rangle\).

Consider an instance when Bob measures the spin of the first particle along axis \( \vec{a} \) and the spin of the second particle along axis \( \vec{b} \), and obtains \( m_a^1 m_b^2 = 1 \).

If she cheats and tells Bob that she had used \(| \Phi \rangle\), then the probability that her guess is correct is \( \frac{1}{2} \). Therefore, in the long run, the probability that Alice cheats and succeeds is \( \left( \frac{1}{2} \right)^2 \) [note that Bob uses approximately \( \frac{n}{2} \) particles to reach a decision: When Alice tells Bob that she has committed to state \(| \Phi \rangle \) or \(| \Phi' \rangle\), then Bob considers only instances when \( \phi_i + \phi'_i = 0^\circ \) which
happens for approximately $\frac{n}{2}$ particles. Similarly when Alice tells Bob that she has committed to state $|\Psi\rangle$ or $|\Psi'\rangle$, then Bob considers only instances when $\phi_i + \phi'_i = 90^\circ$ which again happens for approximately $\frac{n}{2}$ particles]. For sufficiently large security parameter $n$, the probability of success of a cheating Alice can be made arbitrarily small.

We now show that the proposed protocol is not secure against EPR attack (see also [8-11]), that is, we show that the protocol is not secure against a cheating Alice who has the technology to store the third particle of the following EPR state $|A\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)$ for an arbitrarily long period of time. We consider a spin variable at polar angle $\theta$ and at azimuthal angle $\phi$ along direction $\vec{n}$. The states of spin-up $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$ can be expanded in terms of the states of spin-up $|+\rangle$ and spin-down $|-\rangle$ along direction $\vec{n}$ so that we have the expressions

\[|\uparrow\rangle = e^{(i\phi/2)} \left[ \cos (\theta/2) |+\rangle - \sin (\theta/2) |-\rangle \right],\]
\[|\downarrow\rangle = e^{(-i\phi/2)} \left[ \sin (\theta/2) |+\rangle + \cos (\theta/2) |-\rangle \right].\]  \hspace{1cm} (8)

By expanding $|\uparrow\rangle$ and $|\downarrow\rangle$ in terms of $|x\rangle$ and $|-x\rangle$, we obtain

\[|\uparrow\rangle = \frac{1}{\sqrt{2}} (|x\rangle - |-x\rangle),\]
\[|\downarrow\rangle = \frac{1}{\sqrt{2}} (|x\rangle + |-x\rangle).\]  \hspace{1cm} (9)

Similarly by expanding $|\uparrow\rangle$ and $|\downarrow\rangle$ in terms of $|y\rangle$ and $|-y\rangle$, we obtain

\[|\uparrow\rangle = \frac{1 + i}{2} (|y\rangle - |-y\rangle),\]
\[|\downarrow\rangle = \frac{1 - i}{2} (|y\rangle + |-y\rangle).\]  \hspace{1cm} (10)

Using Eqs. (9) and expanding the third particle of state $|A\rangle$ in terms of
\[ | x \rangle \text{ and } | -x \rangle, \text{ we obtain} \]
\[
| A \rangle = \frac{1}{2} [ | \uparrow \uparrow \rangle (| x \rangle - | -x \rangle)] + \frac{1}{2} [ | \downarrow \downarrow \rangle (| x \rangle + | -x \rangle)].
\]

(11)

Rearranging the terms,
\[
| A \rangle = \frac{1}{\sqrt{2}} [ | \phi \rangle | x \rangle - | \phi' \rangle | -x \rangle].
\]

(12)

Similarly by expanding the third particle of state \(| A \rangle\) in terms the \(| y \rangle\) and \(| -y \rangle\), we obtain
\[
| A \rangle = \frac{1 + i}{2} [ | \psi' \rangle | y \rangle - | \psi \rangle | -y \rangle].
\]

(13)

Now if Alice wants to pretend that she had committed to bit 0, that is, if she wants to pretend she had chosen states \(| \Phi \rangle\) or \(| \Phi' \rangle\), then she measures the spin of the third particle along the \(x\) axis. If the result of her measurement is 1 (−1), then she tells Bob that she had chosen state \(| \Phi \rangle\) (\(| \Phi' \rangle\)). Similarly if Alice wants to pretend that she had committed to bit 1, that is, if she wants to pretend that she had chosen states \(| \Psi \rangle\) or \(| \Psi' \rangle\), then she measures the spin of the third particle along the \(y\) axis. If the result of her measurement is 1 (−1), then she tells Bob that she had chosen state \(| \Psi' \rangle\) (\(| \Psi \rangle\)).

In summary, we have shown that the present protocol is secure even against cheaters with unlimited computing power, However, the proposed scheme is not secure against a cheating Alice who has the technology to store an EPR particle for an arbitrarily long period of time.

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[12] There is a practical problem that is very easy to deal with. In practical detectors and polarizers, transmission errors occur due to misalignment of polarizers, dark counts, etc. If the error rate is $\eta$% then Bob should check that condition (2) in opening the commitment is satisfied for at least $(100 - \eta)$% of the time.