Optimization of maintenance period for the elements of resource-supplying networks with branching structure

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Abstract. A restorable system with linear branching structure and finite reliability is considered. Preventive maintenance of its elements is carried out. Operation and restoration periods are assumed to be random values of general kind. Iteration processes of calculation of stationary reliability and economical characteristics of the network are constructed. Optimal intervals between elements’ maintenance are obtained as the functions of their times-to-failure. The examples of electricity supply network and commutation network equipment at the enterprise are given. The optimal choice of elements’ preventive maintenance is shown to result in economical network indexes improvement.

1. Introduction
Practical necessity has resulted in a considerable widening of studies in the theory of complex technical systems’ reliability, specifically resource-supplying networks. The problems of reliability are quite many-sided. They are connected with technological, constructive, organizational aspects. The necessity of development of fundamental mathematics adapted for specific problems in reliability is evident. But in most cases calculation formulas are obtained under assumption of exponential distribution laws of restoration and operation times of elements. In this case Markov random processes are applicable.

Considering general kinds of the above variables complicates the problem of getting reliability and economical characteristics of systems considerably. And recurrent algorithms of calculating stationary characteristics of resource-supplying networks with branching structure are constructed in this type of problem setting. Optimal period of preventive maintenance is obtained.

2. Problem setting
Let us consider resource-supplying network having a many-layered linear branching structure in which every element of some layer is over several elements of lower layer. Each element is connected with the elements of next layers only [1]. The structure of such a network is given in Fig. 1.

The main element \(a_0\) is connected with \(a_1\) elements of the first layer, each of which, in its turn, is connected with \(a_2\) elements of the second level and so on. Each element of the second to last \((n-1)\)th layer is connected with \(a_n\) layers of the last \(n\)th layer. The elements of the last layer are called output ones.
Let us describe the operation of the above network. The failure of any element of the network is discovered immediately and at once its restoration begins. At the moment of failure, both operation and restoration of an element are stopped. The network is supposed to be homogeneous, i.e. the elements of one layer are the same. Time-to-failure of elements of \(i^{th}\) layer is a random variable \(\alpha_i\) with the distribution function \(F_i(t) = P(\alpha_i \leq t), i = 0, n\); and restoration time is a random variable \(\beta_i\) with the distribution function \(G_i(t) = P(\beta_i \leq t), i = 0, n\). Preventive maintenance is carried out according to the age strategy [2]. Maintenance is carried out after operation for \(\tau_i\). It completely restores the elements. Maintenance duration is a random variable \(\beta_i^p\) with the distribution function \(G_i^p(t) = P(\beta_i^p \leq t), i = 0, n\).

\[
\begin{align*}
K_i(\tau_i) &= \frac{T_i^{(1)}(\tau_i)}{T_i^{(1)}(\tau_i) + T_i^{(0)}(\tau_i) + T_i^{(2)}(\tau_i)}; \\
S_i(\tau_i) &= c_i T_i^{(1)}(\tau_i) - c_i^0 T_i^{(0)}(\tau_i) - c_i^p T_i^{(2)}(\tau_i); \\
C_i(\tau_i) &= c_i^0 T_i^{(0)}(\tau_i) + c_i^p T_i^{(2)}(\tau_i). 
\end{align*}
\]

Figure 1. Structural scheme of resource-supplying network

The criterion of network failure is the absence of at least one operating way connecting the main and output elements.

The economical parameters of elements of \(i^{th}\) layer (\(i = 0, n\)) of the network are: \(c_i\) is the income per unit of operation time, \(c_i^0\) is the cost per unit of restoration time and \(c_i^p\) is the cost per unit of maintenance time.

The purpose of the research is to construct iteration algorithm of calculating stationary availability factor \(K(\tau_0, \tau_1, \ldots, \tau_n)\) of the network, average specific income \(S(\tau_0, \tau_1, \ldots, \tau_n)\) per calendar time unit and average specific cost \(C(\tau_0, \tau_1, \ldots, \tau_n)\) per time unit of network operation and to obtain optimal maintenance period of its elements.

2.1. Stationary characteristics definition
Let us denote by \(K_i(\tau_i)\) the stationary availability factor, by \(S_i(T_i)\) the average specific income, by \(C_i(T_i)\) the average specific cost of the element of the \(i^{th}\) layer. These characteristics are defined by the formulas (1) [2, 3]:

\[
K_i(\tau_i) = \frac{T_i^{(1)}(\tau_i)}{T_i^{(1)}(\tau_i) + T_i^{(0)}(\tau_i) + T_i^{(2)}(\tau_i)}; \\
S_i(\tau_i) = c_i T_i^{(1)}(\tau_i) - c_i^0 T_i^{(0)}(\tau_i) - c_i^p T_i^{(2)}(\tau_i); \\
C_i(\tau_i) = c_i^0 T_i^{(0)}(\tau_i) + c_i^p T_i^{(2)}(\tau_i). 
\]
where \( T_i^{(1)}(\tau_i) = \int_{0}^{\tau_i} F_i(t) dt \) is the average operation time; \( T_i^{(0)}(\tau_i) = F_i(\tau_i)E\beta_i \) is the average restoration time; \( T_i^{(2)}(\tau_i) = F_i(\tau_i)E\beta_i^2 \) the average maintenance time of the element of \( i \)th layer per regeneration period, \( i = 0, n \).

To get stationary characteristics of the network we apply formulas from [3] to the structure considered. The following recurrent formulas are obtained. For the family of \( n \)th layer with \( a_n \) output elements controlled by one element of \((n-1)\)th layer the characteristics are as follows:

\[
K^{(n)}(\tau_n) = 1 - \left[ 1 - K_n(\tau_n) \right]^{a_n} ;
\]

\[
S^{(n)}(\tau_n) = a_n S_n(\tau_n) ;
\]

\[
C^{(n)}(\tau_n) = \frac{a_n C_n(\tau_n) K_n(\tau_n)}{K^{(n)}(\tau_n)} .
\]

For one family of elements of \( m \)th layer having \( a_m \) elements controlled by one element of \((m-1)\)th layer, the formulas are (for \( m = 1, n \)):

\[
K^{(m)}(\tau_m, \ldots, \tau_n) = 1 - \left[ \sum_{i=0}^{a_m-1} \left( K_m(\tau_m)K^{(m+1)}(\tau_{m+1}, \ldots, \tau_n) - K_m(\tau_m)K^{(m+1)}(\tau_{m+1}, \ldots, \tau_n) \right) \right]^{a_m} ;
\]

\[
S^{(m)}(\tau_m, \ldots, \tau_n) = \frac{a_m S_m(\tau_m)K^{(m+1)}(\tau_{m+1}, \ldots, \tau_n) + K_m(\tau_m)S^{(m+1)}(\tau_{m+1}, \ldots, \tau_n)}{K_m(\tau_m) + K^{(m+1)}(\tau_{m+1}, \ldots, \tau_n) - K_m(\tau_m)K^{(m+1)}(\tau_{m+1}, \ldots, \tau_n)} ;
\]

\[
C^{(m)}(\tau_m, \ldots, \tau_n) = \frac{a_m C_m(\tau_m) + C^{(m+1)}(\tau_{m+1}, \ldots, \tau_n) - K_mK^{(m+1)}(\tau_{m+1}, \ldots, \tau_n)}{K_m + K^{(m+1)}(\tau_{m+1}, \ldots, \tau_n) - K_mK^{(m+1)}(\tau_{m+1}, \ldots, \tau_n)} \cdot \sum_{i=0}^{a_m-1} \left( K_m + K^{(m+1)}(\tau_{m+1}, \ldots, \tau_n) - K_mK^{(m+1)}(\tau_{m+1}, \ldots, \tau_n) \right) .
\]

The characteristics of the whole network with branching structure are:

\[
K(\tau_0, \tau_1, \ldots, \tau_n) = \frac{K_0(\tau_0)K^{(1)}(\tau_1, \ldots, \tau_n)}{K_0(\tau_0) + K^{(1)}(\tau_1, \ldots, \tau_n) - K_0(\tau_0)K^{(1)}(\tau_1, \ldots, \tau_n)} ;
\]

\[
S(\tau_0, \tau_1, \ldots, \tau_n) = \frac{S_0(\tau_0)K^{(1)}(\tau_1, \ldots, \tau_n) + K_0(\tau_0)S^{(1)}(\tau_1, \ldots, \tau_n)}{K_0(\tau_0) + K^{(1)}(\tau_1, \ldots, \tau_n) - K_0(\tau_0)K^{(1)}(\tau_1, \ldots, \tau_n)} ;
\]

\[
C(\tau_0, \tau_1, \ldots, \tau_n) = C_0(\tau_0) + \frac{a_1 \left[ C_1 + C^{(2)}(1)K^{(2)}(1) - K_1K^{(2)}(1) \right]^{a_1-1}}{\sum_{i=0}^{a_1-1} \left[ K_1 + K^{(2)}(1) - 2K_1K^{(2)}(1) \right] \left[ K_1 + K^{(2)}(1) - K_1K^{(2)}(1) \right]^{a_1-1}} .
\]

### 2.2. Optimization of maintenance period of network elements

Stationary characteristics of the network are the functions of ages \( \tau_i \) of the elements. That is why the problem of optimal maintenance period for the elements is reduced to the problem of getting points of absolute extrema of the chosen criteria functions (2):

\[
K(\tau_0, \tau_1, \ldots, \tau_n) \rightarrow \max_{\tau_i \in (0, \infty), i = 0, n} ;
\]

\[
S(\tau_0, \tau_1, \ldots, \tau_n) \rightarrow \max_{\tau_i \in (0, \infty), i = 0, n} ;
\]
\[ C \left( \tau_0, \tau_1, \ldots, \tau_n \right) \rightarrow \min_{\tau_i \in (0, \infty); i=0,n} \quad . \]

2.2.1 Example 1: electricity supply network. As a rule, electricity travels big distances from electric stations to the customers. So, to organize electricity supply a branching network including elements changing technical characteristics of electricity are necessary [4, 5].

Let us consider main elements of the electricity supply network: 2 transformers increasing the voltage (elements of the first layer), and 36 transformers gradually decreasing the voltage (6 transformers up to 220 kV and 30 transformers to 6-10 kV). According to the instructions, maintenance of the network elements has to be carried out not less than once \( \tau_0 = 100 \) days, \( \tau_1 = 80 \) days, \( \tau_2 = 60 \) days, \( \tau_3 = 30 \) days to ensure the necessary reliability of the network.

The random variables \( \alpha_i, \beta_i, \beta_i^p \) for the network elements have Erlangian distributions with the distribution functions:

\[
F_i(t) = 1 - e^{-\beta_i t} \sum_{j=0}^{2} \frac{\left( \alpha_i t \right)^j}{j!} , \quad G_i(t) = 1 - e^{-\beta_i^p t} \sum_{j=0}^{2} \frac{\left( \mu_i t \right)^j}{j!} , \quad G_i^p(t) = 1 - e^{-\beta_i^p t} \sum_{j=0}^{2} \frac{\left( \mu_i^p t \right)^j}{j!} .
\]

Input characteristics of the elements are given in the table 1.

| Layer number | Amount of family elements | Average time-to-failure \( M\alpha_i \), days | Average restoration time \( M\beta_i \), days | Average maintenance time \( M\beta_i^p \), hours | Element’s income \( c_i \), m.u./ month | Element’s cost per restoration \( c_i^0 \), m.u./ month | Element’s cost per maintenance \( c_i^p \), m.u./ month |
|--------------|---------------------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|-------------------------------------|----------------------------------------|------------------------------------------|
| 0            | \( a_0 = 1 \)             | 200                                         | 8,6                                         | 16,0                                         | 1500                                | 2000                                    | 500                                      |
| 1            | \( a_1 = 2 \)             | 75                                          | 6,0                                         | 14,5                                         | 1200                                | 1600                                    | 300                                      |
| 2            | \( a_2 = 3 \)             | 54                                          | 4,6                                         | 14,1                                         | 1000                                | 1200                                    | 200                                      |
| 3            | \( a_3 = 5 \)             | 46                                          | 3,8                                         | 12,9                                         | 1000                                | 900                                     | 200                                      |

Table 2. Optimization results for different criteria (example 1)

| Layer number | \( \tau_i \), days | \( \tau_i^* \), days | \( S_{max} \), m.u./ month | \( S^* \), m.u./ month | \( \tau_i^c \), days | \( C_{min} \), m.u./ month | \( C^* \), m.u./ month |
|--------------|-------------------|---------------------|---------------------------|------------------------|-------------------|--------------------------|--------------------------|
| 0            | 100.0             | 79.6                | 34061,5                   | 32882,1                | 44.2              | 623,5                    | 1261,6                   |
| 1            | 80.0              | 33,4                | 32882,1                   | 32882,1                | 8,6               | 623,5                    | 1261,6                   |
| 2            | 60.0              | 26,2                | 32882,1                   | 32882,1                | 9,8               | 623,5                    | 1261,6                   |
| 3            | 30.0              | 20,0                | 32882,1                   | 32882,1                | 12,2              | 623,5                    | 1261,6                   |

In table 2 we denote by \( S^* \), \( C^* \) economical indexes of the network in case of recommended strategy \( \tau_0 = 100 \) days, \( \tau_1 = 80 \) days, \( \tau_2 = 60 \) days, \( \tau_3 = 30 \) days.

Preventive maintenance at the levels of time-to-failure \( \tau_i^*, \tau_i^c, i=0,3 \), improves the indexes for 3.5% and 50.6% correspondingly depending on the criterion.

2.2.2. Example 2: network equipment. Let us consider the example of calculating characteristics and maintenance period optimization for elements of the commutation network equipment at the enterprise.
Let us consider the network consisting of the central commutation and 6 commutations (elements of the first layer). Each commutation serves 15 personal computers. According to the instructions, maintenance of elements should be carried out not less than \( \tau_0 = 215 \text{ days} \), \( \tau_1 = 115 \text{ days} \), \( \tau_2 = 70 \text{ days} \) to ensure necessary reliability of the network. The distribution functions of the random variables \( \alpha_i, \beta_i, \beta_i^p \) describing network elements are:

\[
F_i(t) = 1 - e^{-\lambda_i t}, \quad G_i(t) = 1 - e^{-\mu_i t}, \quad G_i^p(t) = 1 - e^{-\mu_i^p t}.
\]

Input characteristics of the elements are given in the table 3.

### Table 3. Input data for the network from example 2

| Layer number | Amount of family elements | Average time-to-failure \( M\alpha_i \), days | Average restoration time \( M\beta_i \), days | Average maintenance time \( M\beta_i^p \), hours | Element’s income \( c_i \), m.u./month | Element’s cost per restoration \( c_i^0 \), m.u./month | Element’s cost per maintenance \( c_i^f \), m.u./month |
|--------------|---------------------------|--------------------------------------------|---------------------------------------------|---------------------------------------------|--------------------------------------|---------------------------------------------|---------------------------------------------|
| 0            | \( a_0 = 1 \)             | 450                                        | 4.0                                        | 12.3                                        | 1500                                 | 2800                                        | 1300                                        |
| 1            | \( a_1 = 6 \)             | 225                                        | 3.5                                        | 11.6                                        | 1200                                 | 2400                                        | 1200                                        |
| 2            | \( a_2 = 15 \)            | 129                                        | 3.2                                        | 11.4                                        | 1000                                 | 2000                                        | 800                                         |

### Table 4. Optimization results for different criteria (example 2)

| Layer number | \( \tau_i \), days | \( \tau_i^f \), days | \( S_{\max} \), m.u./month | \( S^* \), m.u./month | \( t_i^C \), days | \( C_{\min} \), m.u./month | \( C^* \), m.u./month |
|--------------|--------------------|----------------------|----------------------------|-----------------------|------------------|--------------------------|--------------------------|
| 0            | 300.0              | 214.3                | 94782.56                   | 93574.47              | 159.7            | 1405.2                   | 2320.26                  |
| 1            | 200.0              | 107.8                | 80.6                       | 45.7                 |                  |                          |                          |
| 2            | 100.0              | 53.2                 |                            |                      |                  |                          |                          |

In table 4 we denote by \( S^* \), \( C^* \) economical indexes of the network in case of recommended strategy \( \tau_0 = 215 \text{ days} \), \( \tau_1 = 115 \text{ days} \), \( \tau_2 = 70 \text{ days} \). Preventive maintenance at the levels of time-to-failure \( \tau_i^C \), \( i=0,3 \), improves the indexes for 1,2% and 39,4% correspondingly depending on the criterion.

### 3. Conclusions

Iteration processes for calculating stationary reliability and economical indexes of networks with branching structure is constructed with regard to preventive maintenance of the elements. The examples of specific resource-supplying networks show the optimal choice of maintenance intervals can result in the network characteristics improvement. Stationary characteristics of the network can be improved in comparison with the existing structure: for electric supplying network the average specific income increases for 3,5%, the average specific cost reduces for 50,6%; for the network equipment the average specific income increases for 1,2%, the average specific cost reduces for 39,4%.

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