Infinitely divisible distributions over locally compact non-archimedean fields

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Abstract

The article is devoted to stochastic processes with values in finite-dimensional vector spaces over infinite locally compact fields with non-trivial non-archimedean valuations. Infinitely divisible distributions are investigated. Theorems about their characteristic functionals are proved. Particular cases are demonstrated.

1 Introduction

It is well-known that infinitely divisible distributions play very important role in the theory of stochastic processes over fields of real and complex numbers [6, 11, 20]. But over infinite fields with non-archimedean non-trivial norms they were not studied. This article is devoted to infinitely divisible distributions of stochastic processes in vector spaces over locally compact fields \( K \). Such fields have non-archimedean norms and their characteristics may be either zero such as for \( \mathbb{Q}_p \) or for its finite algebraic extension, or positive characteristics \( \text{char}(K) = p > 0 \) such as \( \mathbb{F}_p(\theta) \) of Laurent series over a finite field \( \mathbb{F}_p \) with \( p \) elements and an indeterminate \( \theta \), where \( p > 1 \) is a prime number [26]. Multiplicative norms in such fields \( K \) satisfy stronger inequality, than the triangle inequality, \( |x + y| \leq \max(|x|, |y|) \) for each \( x, y \in K \). Non-archimedean fields are totally disconnected and balls in them are either non-intersecting or one of them is contained in another. In works [2, 7]-[10, 12] stochastic processes on spaces of functions with domains of definition in a non-archimedean linear space and with ranges in the
field of real \( \mathbb{R} \) or complex numbers \( \mathbb{C} \) were considered. Different variants of non-archimedean stochastic processes are possible depending on a domain of definition, a range of values of functions, values of measures in either the real field or a non-archimedean field \([18, 19]\), a time parameter may be real or non-archimedean and so on. That is, depending on considered problems different non-archimedean variants arise.

Stochastic processes with values in non-archimedean spaces appear while their studies for non-archimedean Banach spaces, totally disconnected topological groups and manifolds \([13]-[17]\). Very great importance have also branching processes in graphs \([1, 11, 24]\). For finite or infinite graphs with finite degrees of vertices there is possible to consider their embeddings into \( p \)-adic graphs, which can be embedded into locally compact fields. That is, a consideration of such processes reduces to processes with values in either the field \( \mathbb{Q}_p \) of \( p \)-adic numbers or \( \mathbb{F}_p(\theta) \).

In this article theorems about representations of characteristic functionals of infinitely divisible distributions with values in vector spaces over locally compact infinite fields with non-trivial non-archimedean valuations are formulated and proved. Special features of the non-archimedean case are elucidated. Therefore, a part of definitions, formulations of theorems and their proofs are changed in comparison with the classical case. All main results of this paper are obtained for the first time.

There is also an interesting interpretation of stochastic processes with values in \( \mathbb{Q}_p^n \), for which a time parameter may be either real or \( p \)-adic. A random trajectory in \( \mathbb{Q}_p^n \) may be continuous relative to the norm induced by the non-archimedean valuation in \( \mathbb{Q}_p \), but its trajectory in \( \mathbb{Q}^n \) relative to the usual metric induced by the real metric may be discontinuous. This gives new approach to spasmodic or jump or discontinuous stochastic processes with values in \( \mathbb{Q}^n \), when the latter is considered as embedded into \( \mathbb{R}^n \). On the other hand, stochastic processes with values in \( \mathbb{F}_p(\theta)^n \) can naturally take into account cyclic stochastic processes in definite problems.

1. **Notations and definitions.** Let \((\Omega, \mathcal{A}, P)\) - be a probability space, where \( \Omega \) is a space of elementary events, \( \mathcal{A} \) is a \( \sigma \)-algebra of events in \( \Omega \), \( P : \mathcal{A} \rightarrow [0, 1] \) is a probability. Denote by \( \xi \) a random vector (a random variable for \( n = 1 \)) with values in \( \mathbb{K}^n \) such that it has the probability distribution \( P_\xi(A) = P(\{\omega \in \Omega : \xi(\omega) \in A\}) \) for each \( A \in \mathcal{B}(\mathbb{K}^n) \), where \( \xi : \Omega \rightarrow \mathbb{K}^n \), \( \xi \) is \((\mathcal{A}, \mathcal{B}(\mathbb{K}^n))\)-measurable, that is, \( \xi^{-1}(\mathcal{B}(\mathbb{K}^n)) \subseteq \mathcal{A} \), where \( \mathbb{K} \) is a locally compact infinite field with a non-trivial non-archimedean valuation, \( n \in \mathbb{N} \),
Q_p is the field of p-adic numbers, 1 < p is a prime number. Here K is either a finite algebraic extension of the field Q_p or the field Q_p itself for char(K) = 0, or K = F_p(θ) for char(K) = p > 1, B(K^n) is the σ-algebra of all Borel subsets in K^n. Random vectors ξ and η with values in K^n are called independent, if P(\{ξ ∈ A, η ∈ B\}) = P(\{ξ ∈ A\})P(\{η ∈ B\}) for each A, B ∈ B(K^n).

A random vector (a random variable) ξ is called infinitely divisible, if

1. for each m ∈ N there exist random vectors (random variables) ξ_1, ..., ξ_m such that ξ = ξ_1 + ... + ξ_m and the probability distributions of ξ_1, ..., ξ_m are the same.

If ξ = ξ(t) = ξ(t,ω) is a stochastic process with the real time, t ∈ T, T ⊂ R, then it is called infinitely divisible, if Condition (1) is satisfied for each t ∈ T. Introduce the notation B(X, x, R) := \{y ∈ X : ρ(x, y) ≤ R\} for the ball in a metric space (X, ρ) with a metric ρ, 0 < R < ∞, ξ_j(t) are stochastic processes, j = 1, ..., m.

2. Lemma. If ξ and η are two independent random vectors with values in K^n with probability distributions P_ξ and P_η, then ξ + η has the probability distribution P_{ξ+η}(A) = \int_{K^n} P_ξ(A - dy)P_η(dy) for each A ∈ B(K^n).

Proof. Since ξ and η are independent, then P(\{ω ∈ Ω : ξ(ω) ∈ C, η(ω) ∈ B\}) = P(\{ω ∈ Ω : ξ(ω) ∈ C\})P(\{ω ∈ Ω : η(ω) ∈ B\}) for each C, B ∈ B(K^n). Therefore, P(\{ξ + η ∈ A\}) = P(\{ξ ∈ A - y, η = y, y ∈ K^n\}) for each A ∈ B(K^n), consequently, P_{ξ+η}(A) = \int_{K^n} P_ξ(A - dy)P_η(dy).

This means that P_{ξ+η} = P_ξ * P_η is the convolution of measures P_ξ and P_η.

3. Corollary. If ξ is an infinitely divisible random vector, then P_ξ = P_ξ^∞ for each m ∈ N, where P_ξ^m denotes the m-fold convolution P_ξ with itself.

Proof. In view of Lemma 2 and Definition 1 P_ξ = P_{ξ_1} * P_{ξ_2+...+ξ_m} = ... = P_{ξ_1} * P_{ξ_2} * ... * P_{ξ_m}. On the other hand, ξ_1, ..., ξ_m have the same probability distributions, hence P_{ξ_1} * P_{ξ_2} * ... * P_{ξ_m} = P_{ξ}^m.

4. Notes and definitions. Corollary 3 means, that the equality P_ξ = P_{ξ_1}^m implies the relation: P_ξ(A) = \int_{K^n} P_{ξ_1}(A - dy_2)P_{ξ_2}(dy_2 - dy_3)...P_{ξ_m-1}(dy_{m-1} - dy_m)P_{ξ_m}(dy_m), where A ∈ B(K^n). In the case of char(K) = p > 1 Corollary 3 means, that for m = kp, where k ∈ N, if P_ξ(\{0\}) = 0, then P_{ξ_1}(\{y\}) = 0 for each singleton y ∈ K^n, since P(ξ = 0) ≥ P(ξ_1 = ξ_2 = ... = ξ_m) ≥ P_{ξ_1}(\{y\})^m. It is the restriction on the atomic property of P_ξ and P_{ξ_1}.

For p-adic numbers x = \sum_{k=0}^{∞} x_kp^k, where x_k ∈ \{0, 1, ..., p-1\}, N ∈ Z, N = N(x), x_N ≠ 0, x_j = 0 for each j < N, put as usually ord_{Q_p}(x) = N for the order of x, thus its norm is \|x\|_{Q_p} = p^{-N}. Define the function [x]_{Q_p} :=
\[ \sum_{k=N}^{N-1} x_k p^k \] for \( N < 0, [x]_{Q_p} = 0 \) for \( N \geq 0 \) on \( Q_p \). Therefore, the function \([x]_{Q_p}\) on \( Q_p \) is considered with values in the segment \([0, 1] \subset \mathbb{R}\).

For the field \( F_p(\theta) \) put \([x]_{F_p(\theta)} = p^{-N}\), where \( N = \text{ord}_{F_p(\theta)}(x) \in \mathbb{Z}, x = \sum_{j=N}^{\infty} x_j \theta^j, x_j \in F_p \) for each \( j, x_N \neq 0, x_j = 0 \) for each \( j < N \). Then we define the mapping \([x]_{F_p(\theta)} = x_{-1}/p\), where we consider elements of \( F_p = \{0, 1, ..., p-1\} \) embedded into \( \mathbb{R}\), hence \([x]_{F_p(\theta)}\) takes values in \( \mathbb{R}\), where \( 1/p \in \mathbb{R}\), \( x_{-1} = 0 \) when \( N = N(x) \geq 0\).

Consider a local field \( K \) as the vector space over the field \( Q_p\); then it is isomorphic with \( Q_p^b \) for some \( b \in \mathbb{N}\), since \( K \) is a finite algebraic extension of the field \( Q_p\). In the case of \( K = F_p(\theta) \) we take \( b = 1\). Put

\[
(i) \quad F := Q_p \quad \text{for } \text{char}(K) = 0 \quad \text{with } K \supset Q_p, \quad \text{while } \quad (ii) \quad F := F_p(\theta) \quad \text{for } \text{char}(K) = p > 1 \quad \text{with } K = F_p(\theta).
\]

Let \( (x, y) := (x, y)_F := \sum_{j=1}^{\infty} x_j y_j \) for \( x, y \in F, x = (x_1, ..., x_b), x_j \in F; (x, y)_K := \sum_{j=1}^{n} x_j y_j \) for \( x, y \in K^n, x = (x_1, ..., x_n), x_j \in K \).

Define the mapping \(<q >_F := 2\pi[(e, q)]_F\) for each \( q \in K\), which is considered in \((e, q)\) as the element from \( F^b, <q >_F : K \to \mathbb{R}\), where \( e := (1, ..., 1) \in F^b\), particularly \( e = 1 \) for \( b = 1\), that is, either in \((i) K = Q_p\) or in the case \((ii) K = F_p(\theta)\). For the additive group \( K^n\) then there exists the character \( \chi_s(z) := \exp(i <(s, z)_K >_F)\) with values in the field of complex numbers \( \mathbb{C}\) for each value of the parameter \( s \in K^n, s_j(z) := s_j z_j + s_j v_j \) for each \( s_j, z_j, v_j \in K\) and \((s, z + v)_K = (s, z)_K + (s, v)_K, [x + y]_F - [x]_F - [y]_F \in B(F, 0, 1)\) for every \( x, y \in F\), while \([x]_F = 0\) for each \( x \in B(F, 0, 1)\), where \( i = (-1)^{1/2} \in \mathbb{C}\). In particular, \( \chi_0(z) = 1\) for each \( z \in K^n\) for \( s = 0\). The character is non-trivial for \( s \neq 0\).

At the same time \( \chi_s(z) = \prod_{j=1}^{n} \chi_{s_j}(z_j)\), where \( \chi_{s_j}(z_j)\) are characters of \( K\) as the additive group.

For a \( \sigma\)-additive measure \( \mu : \mathcal{B}(K^n) \to \mathbb{C}\) of a bounded variation the characteristic functional \( \hat{\mu}\) is given by the formula: \( \hat{\mu}(s) := \int_{K^n} \chi_s(z) \mu(dz)\), where \( s \in K^n\) is the corresponding continuous \( K\)-linear functional on \( K^n\) denoted by the same \( s\).

In general the characteristic functional of the measure \( \mu\) is defined in the space \( C^b(K^n, K)\) of continuous functions \( f : K^n \to K\)
\[
\hat{\mu}(f) := \int_{K^n} \chi_1(f(z)) \mu(dz), \quad \text{where } 1 \in K.
\]

Let \( \mu\) be a \( \sigma\)-additive finite non-negative measure on \( \mathcal{B}(K^n), \mu(K^n) < \infty\). Consider the class \( C_1 = C_1(K)\) of continuous functions \( A = A_\mu : K^n \to \mathbb{R}\), satisfying Conditions (\( F1 - F3\)):
\[
(F1) \quad A(y + z) = A(y) + A(z) + 2\pi \int_{K^n} f_1(y, z; x) \mu(dx) \quad \text{for each } y, z \in K^n,
\]
(F2) \( A(\beta y) = [\beta]_F A(y) + 2\pi \int_{K^n} f_2(\beta, (e, (y, x)_K)_F) \mu(dx) \) for each \( y \in K^n \), \( \beta \in F \), where either

(F3) if \( F = Q_p \) for \( \text{char}(K) = 0 \), then \( f_1 : (K^n)^3 \to Z \) and \( f_2 : Q_p^2 \to R \) are locally constant continuous bounded functions, \( f_1(y, z; x) \in Z \) and \( f_2(\alpha, \beta)p^{-N(\alpha, \beta)} \in Z \) for \( N(\alpha, \beta) < 0 \) take only integer values, \( N(\alpha, \beta) := \min(\text{ord}_{Q_p}(\alpha), \text{ord}_{Q_p}(\beta)) \); or

(F4) if \( F = F_p(\theta) \) for \( \text{char}(K) = p > 0 \), then \( f_1 : (K^n)^3 \to R \) and \( f_2 : F^2 \to R \) are locally constant continuous bounded functions, \( pf_1(y, z; x) \in Z \) and \( p^2f_2(\alpha, \beta) \in Z \) for \( N(\alpha, \beta) < 0 \) take only integer values, \( N(\alpha, \beta) := \min(\text{ord}_{F_p(\theta)}(\alpha), \text{ord}_{F_p(\theta)}(\beta)) \). While \( f_1(y, z; x) = 0 \) for \( \max(|yx|_K, |zx|_K) \leq 1 \), and \( f_2(\alpha, \beta) = 0 \) for \( \max(|\alpha|_F, |\beta|_F) \leq 1 \) in (F3, F4).

Denote by \( C_2 = C_2(K) \) the class of continuous functions \( B = B_\mu : (K^n)^2 \to K \), satisfying Conditions (B1 – B3):

(B1) \( B(y, z) = B(z, y) \) for each \( y, z \in K^n \), where \( B(y, y) \) is non-negative,

(B2) \( B(q+y, z) = B(q, z) + B(y, z) + 2\pi \int_{K^n} f_1(q, y; x) < (z, x)_K >_F \mu(dx) \) for each \( q, y, z \in K^n \),

(B3) \( B(\beta y, z) = [\beta]_F B(y, z) + 2\pi \int_{K^n} f_2(\beta, (e, (y, x)_K)_F) < (z, x)_K >_F \mu(dx) \), where \( f_1 \) and \( f_2 \) satisfy Condition either (F3) or (F4) depending on the characteristic \( \text{char}(K) \).

For \( y = z \) we shall also write for short \( B(y) := B(y, y) \).

4.1. Lemma. If \( \chi_s(x) : F^n \to C \) is a character of the additive group of \( F^n \) as in Section 4, \( \mu : B(F^n) \to [0, \infty] \) is the Haar measure such that \( \mu(B(F^n, 0, 1)) = 1 \). Then \( \int_{B(F^n, 0, p^k)} \chi_s(x) \mu(dx) = J(s, k) \), where \( J(s, k) = p^{kn} \) for \( |s| \leq p^{-k} \), while \( J(s, k) = 0 \) for \( |s| \geq p^{-k} \).

Proof. The Haar measure \( \mu \) on \( B(F^n) \) is the product of the Haar measures \( \mu_1 \) on \( B(F) \), \( \mu(dx) = \otimes_{j=1}^n \mu_j(dx_j) \), \( \mu_1 = \mu_1 \). Therefore, \( \int_{B(F^n, 0, p^k)} \chi_s(x) \mu(dx) = \prod_{j=1}^n \chi_{\xi_j}(x_j) \mu_j(dx_j) \), where \( \chi_j = \chi_1, \chi_j(x_j) \) is the character of \( F \).

Consider \( n = 1 \). Then \( K := \int_{B(F, 0, p^k)} \chi_s(x) \mu(dx) = \int_{B(F, 0, p^k)} \chi_s(x - y) \mu(dx) \) for each \( y \in B(F, 0, p^k) \). Thus \( K = \chi_s(-y) \int_{B(F, 0, p^k)} \chi_s(x) \mu(dx) \), since \( B(F, 0, p^k) = B(F, y, p^k) \) for each \( y \in B(F, 0, p^k) \), while \( \mu(A - y) = \mu(A) \) for each \( A \in B(F) \). Take \( |s|_F \geq p^{-k+1} \) and \( |y|_F = p^k \) such that \( |sy|_F \neq 0 \) is nonzero. Hence \( K(1 - \chi_s(-y)) = 0 \), but \( \chi_s(-y) \neq 1 \), consequently, \( K = 0 \).

On the other hand, if \( |sx|_F \leq 1 \), then \( \chi_s(x) = 1 \) and inevitably \( \int_{B(F, 0, p^k)} \chi_s(x) \mu(dx) = p^k \), when \( |s|_F \leq p^{-k} \) (see for comparison the case \( F = Q_p \) in Example 6 on page 62 [5]).

5. Theorem. Let \( \{\psi(v, y) : v \in V\} \) be a family of characteristic func-
We prove a weak compactness of the family of measures \( \{ \nu_i \} \).

That is, we need to prove that 

\[
\nu_{\psi}(1 + |x|^2)^{-1} f_{\mathcal{K}^a}(\exp(i < (y, x)_{\mathcal{K}} > F) - 1 < (y, x)_{\mathcal{K}} > F ) (1 + |x|^2)^{-1} + < (y, x)_{\mathcal{K}} > F (1 + |x|^2)^{-1}/2)(1 + |x|^2)/|x|^2 \nu(dx), \nu \geq 0, \nu(\{0\}) = 0.
\]

Proof. Let \( \mu_\nu \) be a measure corresponding to the characteristic functional \( \psi(v, y) \). Put \( \lambda_\nu(A) := |v| f_A |z|^2/[1 + |z|^2] \mu_\nu(\lambda v) \lambda v \) for each \( A \in \mathcal{B}(\mathcal{K}^a) \), where \( |z| := \max_{1 \leq j \leq n} |z_j|_{\mathcal{K}}, z = (z_1, ..., z_n) \in \mathcal{K}^n \), \( z_j \in \mathcal{K} \) for every \( j = 1, ..., n \).

We prove a weak compactness of the family of measures \( \{ \lambda_\nu : v \in \mathcal{V} \} \).

That is, we need to prove that (i) there exists \( L = \text{const} > 0 \) such that \( \sup_{v \in \mathcal{V}} \lambda_\nu(\mathcal{K}^n) \leq L \); (ii) \( \lim_{R \to \infty} \overline{\lim_{n \to 0}} \lambda_\nu(\mathcal{K}^n \setminus \mathcal{B}(\mathcal{K}^a, 0, R)) = 0 \).

The topologically dual space \( \mathcal{K}^n' \) of all continuous \( \mathcal{K} \)-linear functionals on \( \mathcal{K}^n \) is \( \mathcal{K} \)-linearly and topologically isomorphic with \( \mathcal{K}^n \), since \( n \in \mathcal{N} \). Since \( \mathcal{K} \) is the locally compact field, then it is spherically complete (see Theorems 3.15, 5.36 and 5.39 [22]). Since \( \mathcal{K}^n \) as the linear space over \( \mathcal{F} \) is isomorphic with \( \mathcal{F}^{bn} \), then it is sufficient to verify a weak compactness over the field \( \mathcal{F} \), where either \( \mathcal{F} = Q_\mu \) for \( \text{char}(\mathcal{K}) = 0 \) with \( \mathcal{K} \supset \mathcal{Q}_\mu \) and \( b \in \mathcal{N}, \mathcal{F} = F_\mu(\theta) \) for \( \text{char}(\mathcal{K}) = p > 0 \) with \( \mathcal{K} = F_\mu(\theta) \) and \( b = 1 \).

Indeed, apply the non-archimedean variant of the Minlos-Sazonov theorem, due to which there exists the bijective correspondence between characteristic functionals and measures [13], where characteristic functionals are weakly continuous (see also §IV.1.2 and Theorem IV.2.2 about the Minlos-Sazonov theorem on Hausdorff completely regular (Tychonoff) spaces [4]). They are positive definite on \( (\mathcal{K}^n)' \) or \( C^0(\mathcal{K}^n, \mathcal{K}) \), when \( \mu \) is non-negative; \( \hat{\mu}(0) = 1 \) for \( \mu(\mathcal{K}^n) = 1 \). In the considered case \( \mathcal{K}^n \) is a finite dimensional Banach space over \( \mathcal{K} \). Since the multiplication in \( \mathcal{K} \) is continuous, then over \( \mathcal{Q}_\mu \) this gives the continuous mapping \( f_0 : (Q_\mu^b)^2 \to Q_\mu^b \). The composition of \( f_0 \) with all possible \( \mathcal{K} \)-linear continuous functionals \( s : \mathcal{K}^n \to \mathcal{K} \) separates points in \( \mathcal{K}^n \).

Let \( |x| \leq R_1 \), where \( 0 < R_1 < \infty \) is an arbitrarily given number. Due to conditions of this theorem for each \( \delta > 0 \) there exists \( \nu_0 = \nu_0(R_1, \delta) > 0 \) such that for each \( \epsilon > 0 \) there is satisfied the inequality:
(1) $-\text{Reg}(y) + \delta \geq \int_{B(F^{bn},0,\epsilon)} [1 - \cos <(y,x)_F] |x|^{-2} \lambda_v(dx)$ for each $0 < v \leq v_0$, since $e^{i\alpha} = \cos(\alpha) + i \sin(\alpha)$, $-\text{Re}(e^{i\alpha} - 1) = 1 - \cos(\alpha)$ for each $\alpha \in \mathbb{R}$, while $1 + |x|^2 \geq 1$ and $[1 + |x|^2]|x|^{-2} \geq |x|^{-2}$.

If $\epsilon > 1$ and $x \in F^{bn} \setminus B(F^{bn},0,\epsilon)$, then from $|x|_F > \epsilon$ it follows $[1 + |x|^2]|x|^{-2} = 1 + |x|^{-2} \geq 1$ and then for each $\delta > 0$ there exists $v_0 > 0$ such that for each $\epsilon > 1$ and each $0 < v \leq v_0$ there is satisfied the inequality:

(2) $-\text{Reg}(y) + \delta \geq \int_{B(F^{bn},0,\epsilon)} [1 - \cos <(y,x)_F] \lambda_v(dx)$. Integrating these inequalities by $y \in B(Q_p^{bn},0,r)$ and divide on the volume (measure) $\mu(B(F^{bn},0,r))$, where $\mu$ is the nonnegative Haar measure on $F^{bn}$ such that $\mu(B(F^{bn},0,1)) = 1$, $\mu(B(F^{bn},0,r)) = r^{bn}$ for each $r = p^k$ with $k \in \mathbb{Z}$ [3, 26]. Then from (1) it follows:

(3) $-r^{-bn} \int_{B(F^{bn},0,r)} \text{Reg}(y) \mu(dy) + \delta \geq \int_{B(F^{bn},0,\epsilon)} |x|^2 (1 - \cos <(y,x)_F) \lambda_v(dx) \mu(dy) r^{-bn}$. From (2) we get:

(4) $-r^{-bn} \int_{B(F^{bn},0,r)} \text{Reg}(y) \mu(dy) + \delta \geq \int_{B(F^{bn},0,\epsilon)} \lambda_v(dx) \mu(dy) r^{-bn}$. On the other hand, $\cos(<(y,x)_F) = \cos(\sum_{j=1}^{bn} x_j y_j)_F$, since $(y,x) = \sum_{j=1}^{bn} y_j x_j$, also $a + b > F = a > F + b > F + 2w_\pi$ for each $a, b \in F$, where $w$ is an integer number, $w = w(a, b) \in \mathbb{Z}$. For the characters integrals are known due to Lemma 4.1: $\int_{B(F^{bn},0,p^k)} \chi_s(x) \mu(dx) = \prod_{j=1}^{bn} \int_{B(F^{bn},0,p^k)} \chi_{x_j}(x_j) \mu(dx_j) = J(s,k)$, where $J(s,k) = p^{bn} k$ for $|s|_F \leq p^{-k}$, $J(s,k) = 0$ for $|s|_F \geq p^{-k+1}$. Since $(y,x) = (x,y)$ and $\cos(\alpha) = \text{Re}(e^{i\alpha})$ for each $\alpha \in \mathbb{R}$, then $\int_{B(F^{bn},0,p^k)} \cos <(y,x)_F \mu(dy) = J(x,k)$, since $J(x,k) \in \mathbb{R}$. Take in (3, 4) $r = p^k$, then

(5) $-p^{-bn} \int_{B(F^{bn},0,p^k)} \text{Reg}(y) \mu(dy) + \delta \geq (\int_{B(F^{bn},0,\epsilon)} |x|^2 (1 - J(x,k)) \lambda_v(dx))$.

(6) $-p^{-bn} \int_{B(F^{bn},0,p^k)} \text{Reg}(y) \mu(dy) + \delta \geq \int_{B(F^{bn},0,\epsilon)} \lambda_v(dx) \mu(dy)$. Since $J(x,k)p^{-bn} = 1$ for $|x|_F \leq p^{-k}$, while $J(x,k)p^{-bn} = 0$ for $|x|_F \geq p^{-k+1}$, then for $\epsilon > p^{-k+1}$ with $k \in \mathbb{Z}$, where $p \geq 2$, we get $(1 - J(x,k)p^{-bn}) = 1$ for $p^{-k+1} \leq |x|_F \leq \epsilon$, then

$-p^{-bn} \int_{B(F^{bn},0,p^k)} \text{Reg}(y) \mu(dy) + \delta \geq (\int_{B(F^{bn},0,\epsilon)} \lambda_v(dx)) \geq \epsilon^{-2} |\lambda_v(B(F^{bn},0,\epsilon)) - \lambda_v(B(F^{bn},0,\epsilon, 0, p^{-k}))|$, hence

(7) $|\lambda_v(B(F^{bn},0,\epsilon)) - \lambda_v(B(F^{bn},0,\epsilon, 0, p^{-k}))| \leq \epsilon^2 [\delta - p^{-bn} \int_{B(F^{bn},0,p^k)} \text{Reg}(y) \mu(dy)]$. In particular, for $\epsilon_k = p^{-k+2}$ with $\epsilon_k \leq \epsilon$ and $k \rightarrow \infty$ Inequality (7) is satisfied. The summation of both parts of Inequality (7) by such $k$ gives:

(8) $\lambda_v(B(F^{bn},0,\epsilon)) \leq L_1 |\delta - \sum_{k=0}^{\infty} p^{-k} \int_{B(F^{bn},0,p^k)} \text{Reg}(y) \mu(dy)$(, where $L_1 = p^4 \sum_{k=0}^{\infty} p^{-2k} = p^4 / (1 - p^{-2})$, $k_0 \in \mathbb{Z}$ is fixed. At the same time from (6) it follows:
Due to conditions of this theorem the function $g(y)$ is continuous and $g(0) = 0$, consequently, for each $\delta > 0$ there exists sufficiently small $0 < R_1 = p^{k_1} < \infty$ such that $R_1^{-bn} |\int_{B(\mathbb{F}^{bn}, 0, R_1)} \text{Reg}(y)\mu(dy)| < \delta$. In view of Inequality (9) for each $\epsilon > \max(p^{-k_1+1}, 1)$ there is satisfied the inequality $\lambda_v(\mathbb{F}^{bn} \setminus B(\mathbb{F}^{bn}, 0, \epsilon)) < 2\delta$ for each $v \in (0, v_0]$, consequently, the family of measures $\{\lambda_v : v \in V\}$ is weakly compact.

Choose a sequence $h_n \downarrow 0$ such that $\lambda_{v_n}$ is weakly convergent to some measure $\nu$ on $\mathcal{B} (\mathbb{K}^n)$. Due to conditions of this theorem and using the decomposition of $\exp$ into the series, we get the inequality:

(10) $|w(v, y) - 1|/v = \int_{\mathbb{K}^n} (\chi(y,x) - 1)[1 + |x|_{\mathbb{K}}^2]|x|_{\mathbb{K}}^{-2}\lambda_v(dx) = A_v(y) - B_v(y)/2 + \int_{\mathbb{K}^n} f(y, x)\lambda_v(dx),$

where $A_v(y) = \int_{\mathbb{K}^n} < (y, x)_{\mathbb{K}} >_F |x|_{\mathbb{K}}^{-2}\lambda_v(dx)$, $B_v(y) = \int_{\mathbb{K}^n} < (y, x)_{\mathbb{K}} >_F^2 |x|_{\mathbb{K}}^{-2}\lambda_v(dx)$, $f(y, x) = (\exp(i < (y, x)_{\mathbb{K}} >_F) - 1 - i < (y, x)_{\mathbb{K}} >_F [1 + |x|_{\mathbb{K}}^2]^{-1} + < (y, x)_{\mathbb{K}} >_F^2 [1 + |x|_{\mathbb{K}}^2]^{-1}/2] [1 + |x|_{\mathbb{K}}^2]|x|_{\mathbb{K}}^{-2}.$

The multiplier $[1 + |x|_{\mathbb{K}}^2]|x|_{\mathbb{K}}^{-2}$ is continuous and bounded for $|x| \geq R$, where $0 < R < \infty$, $< (y, x)_{\mathbb{K}} >_F = 0$ for $|y|_{\mathbb{K}}|x|_{\mathbb{K}} \leq 1$, hence the function $f(y, x)$ is continuous, it is bounded, when $y$ varies in a bounded subset in $\mathbb{K}^n$, while $x \in \mathbb{K}^n$. Therefore, there exists $\lim_{k \to \infty} \int_{\mathbb{K}^n} f(y, x)\lambda_v(dx) = \int_{\mathbb{K}^n} f(y, x)\nu(dx)$. The functions $< (y, x)_{\mathbb{K}} >_F |x|_{\mathbb{K}}^{-2}$ and $< (y, x)_{\mathbb{K}} >_F < (z, x)_{\mathbb{K}} >_F |x|_{\mathbb{K}}^{-2}$ are locally constant by $x$ for each given value of the parameters $y$ and $z$. These functions are zero, when $|y|_{\mathbb{K}}|x|_{\mathbb{K}} \leq 1$, that is, they are defined in the continuous manner to be zero at the zero point $x = 0$. Since there exists the limit in the left hand side of Inequality (10), then there exist $\lim_{k \to \infty} A_v(y) = A(y) = \int_{\mathbb{K}^n} < (y, x)_{\mathbb{K}} >_F |x|_{\mathbb{K}}^{-2}\nu(dx)$ and $\lim_{k \to \infty} B_v(y) = B(y) = \int_{\mathbb{K}^n} < (y, x)_{\mathbb{K}} >_F^2 |x|_{\mathbb{K}}^{-2}\nu(dx)$. At the same time $B(y) \geq 0$ for each $y \in \mathbb{K}^n$.

Substitute the measure $\nu(U)$ on $\nu(U \setminus \{0\})$ and denote it by the same symbol, where $U \in \mathcal{B}(\mathbb{K}^n)$. Due to the fact that $f(y, 0) = 0$, $< (y, 0)_{\mathbb{K}} >_F = 0$, then for such substitution of the measure the values of integrals $\int_{\mathbb{K}^n} f(y, x)\nu(dx)$, $A(y) = \int_{\mathbb{K}^n} < (y, x)_{\mathbb{K}} >_F |x|_{\mathbb{K}}^{-2}\nu(dx)$ and $B(y, z) = \int_{\mathbb{K}^n} < (y, x)_{\mathbb{K}} >_F < (y, z)_{\mathbb{K}} >_F |x|_{\mathbb{K}}^{-2}\nu(dx)$ do not change.
It is known that \([\alpha + \beta]_F = [\alpha]_F + [\beta]_F + v(\alpha, \beta)\), where \(v(\alpha, \beta) \in \mathbb{Z}\) for \(F = Q_p\), \(p v(\alpha, \beta) \in \mathbb{Z}\) for \(F = F_p(\theta)\), \(0 \leq [\alpha]_p \leq 1\) for each \(\alpha, \beta \in F\). Also \([\alpha \beta]_F = [\alpha]_F [\beta]_F + u(\alpha, \beta)\), where \(p^{-N(\alpha,\beta)}u(\alpha, \beta) \in \mathbb{Z}\) for \(F = Q_p\), \(p^2 u(\alpha, \beta) \in \mathbb{Z}\) for \(F = F_p(\theta)\), since \([\alpha]_{Q_p}[\beta]_{Q_p} = \sum_{k=N(\alpha)}^{l-1} \sum_{l=N(\beta)} \alpha_k \beta_l p^{k+l}\), and \([\alpha \beta]_{Q_p} = \sum_{N(\alpha) \leq k, N(\beta) \leq l, k+l \leq -1} \alpha_k \beta_l p^{k+l}\), where \(\alpha = \sum_{k=N(\alpha)} \alpha_k p^k \in Q_p\), \(\alpha_k \in \{0,1,...,p-1\}\) for each \(k \in \mathbb{Z}\), \(\alpha_{N(\alpha)} \neq 0\), while \([\alpha]_{F_p(\theta)}[\beta]_{F_p(\theta)} = \alpha_1 \beta_1 p^{-2}\), and \([\alpha \beta]_{F_p(\theta)} = \sum_{N(\alpha) \leq k, N(\beta) \leq l, k+l \leq -1} \alpha_k \beta_l p^{-1}\), where \(\alpha = \sum_{k=N(\alpha)} \alpha_k p^k \in F_p(\theta)\), \(\alpha_k \in F_p\) for each \(k \in \mathbb{Z}\), \(\alpha_{N(\alpha)} \neq 0\) \([5, 26]\). At the same time \([\alpha]_F = 0\), when \(\alpha |_F \leq 1\), hence \(v(\alpha, \beta) = 0\) and \(u(\alpha, \beta) = 0\) for \(\max(|\alpha|_F, |\beta|_F) \leq 1\). Then

\[(11) \quad < (y + z, x)_K \succ_F = < (y, x)_K \succ_F + < (z, x)_K \succ_F + 2\pi f_1(y, z; x), \]

where \(f_1 \in \mathbb{Z}\) for \(F = Q_p\), \(p f_1 \in \mathbb{Z}\) for \(F = F_p(\theta)\). Since \(< (y, x)_K \succ_F\) is locally constant and \(0 \leq [\alpha]_F \leq 1\) for each \(\alpha \in F\), then there is the inequality  

\[ -2 \leq f_1(y, z; x) \leq 1 \]

for each \(x, y, z \in K^n\) in \((11)\). On the other hand, \n
\[(12) \quad < (\beta y, x)_K \succ_F = [\beta]_F < (y, x)_K \succ_F + 2\pi f_2(\beta, (e, (y, x)_K)_F), \]

where \(f_2(\alpha, \beta) = u(\alpha, \beta)\) for each \(\alpha, \beta \in F\), since \(F\) is naturally embedded into \(K\) and \(\beta (e, (y, x)_K)_F = (e, (\beta y, x)_K)_F\). Since \([\alpha]_F \in [0,1]\) for each \(\alpha \in F\), then \(-1 \leq f_2(\alpha, \gamma) \leq 1\) for each \(\alpha \in F\) and \(\gamma = (e, (y, x)_K)_F \in F\) in \((12)\). In view of the continuity and the locally constant behavior of \(< (y, x)_K \succ_F\) from this the continuity and local constantness of \(f_1\) and \(f_2\) follow. Thus, \(f_1\) and \(f_2\) satisfy Conditions \((F3, F4)\) depending on \(char(K)\). Therefore, from \((11, 12)\) we get the properties:

\[(13) \quad A(y) = \int_K < (y, x)_K \succ_F |x|^{-2}_K \nu(dx) \quad \text{and} \quad \]

\[(14) \quad B(y, z) = \int_K < (y, x)_K \succ_F < (z, x)_K \succ_F |x|^{-2}_K \nu(dx) \quad \text{with the measure} \quad |x|^{-2}_K \nu(dx) \quad \text{here instead of the measure} \quad \mu \quad \text{in} \quad (F1 - F4), \quad (B1 - B3). \]

By the construction given above the measures in the definitions of \(A\) and \(B\) are nonnegative and the functions in integrals are nonnegative, then \(A(y)\) and \(B(y, z)\) take nonnegative values.

As the metric space \(K^n\) is complete separable and hence is the Radon space (see Theorem 1.2 \([6]\)), that is, the class of compact subsets approximates from below each \(\sigma\)-additive nonnegative finite measure on the Borel \(\sigma\)-algebra \(B(K^n)\). In view of the finiteness and the \(\sigma\)-additivity of the nonnegative measure \(|x|^{-2}_K \nu(dx)\) on \(K^n \setminus B(K^n, 0, 1/|y|_K)\) for \(|y|_K > 0\), \(< (y, x)_K \succ_F = 0\) for \(|(x, y)_K| \leq 1\) and due to continuity and boundedness of the functions in integrals we have that the mappings \(A(y)\) and \(B(y, z)\) are continuous.
6. Corollary. Let the conditions of Theorem 5 be satisfied and there exists $J := \int_{\mathbb{K}^n} |x|^2 \nu(dx) < \infty$. Then $A(y) = -i(\partial \phi(\beta, y)/\partial \beta)|_{\beta=0}$ and $B(y) = -((\beta^2 \phi(\beta, y)/\partial \beta^2)|_{\beta=0},$ where $\phi(\beta, y) = \int_{\mathbb{K}^n} \exp(i < (y, x)_K > F |x|^2 \nu(dx),$ $-1 < \beta < 1$.

Proof. In view of Theorem 5 there exist $A(y)$ and $B(y)$. At the same time the measure $\nu$ is nonnegative as the weak limit of a weakly converging sequence of nonnegative measures, consequently, the measure $\mu(dx) := |x|^2 \nu(dx)$ is nonnegative. In view of the supposition of this Lemma $0 \leq \mu(\mathbb{K}^n) = J < \infty$. If $J = 0$, then $A(y) = 0, B(y) = 0$ and $\phi(\beta, y) = 0$, then the statement of this Lemma is evident. Therefore, there remains the case $J > 0$. Consider the random variable $\zeta := <(y, \eta)_K > F$ with values in $\mathbb{R}$, where $\eta$ is a random vector in $\mathbb{K}^n$ with the probability distribution $P(dx) := J^{-1}|x|^2 \nu(dx)$, where $y \in \mathbb{K}^n$ is the given vector.

Then $\phi(\beta, y) = JM \exp(i\beta \zeta),$ where $M \nu$ denotes the mean value of the random variable $X$ with values in $C$. That is, $M \exp(i\beta \zeta) = \int_{\mathbb{K}^n} \exp(i\beta <(y, x)_K > F )P(dx).$ For $\zeta$ there exists the second moment, since there exists $B(y)$ for each $y \in \mathbb{K}^n$. In view of Theorem II.12.1 [25] about relations between moments of the random variable and values of derivatives of their characteristic functions at zero, we get the statement of this Corollary.

7. Theorem. Let the conditions of Theorem 5 be satisfied and in addition measures $\mu_v(dx)$ possess finite moments of $|x|^2 \nu$ of the second order: $\int_{\mathbb{K}^n} |x|^2 \mu_v(dx) < \infty \forall v \in V$, then for $g(y)$ there is the representation:

(i) $g(y) = iA(y) - B(y)/2 + \int_{B(\mathbb{K}^n, 0, \epsilon)}(\exp(i <(y, x)_K > F ) - 1 - i <(y, x)_K > F + <(y, x)_K > F /2)\eta(dx) + \int_{B(\mathbb{K}^n, 0, \epsilon)}(\exp(i <(y, x)_K > F ) - 1)\eta(dx),$ where $\eta$ is a nonnegative $\sigma$-additive measure on $B(\mathbb{K}^n), \eta(\{0\}) = 0, A(y) \in C_1, B(y, z) \in C_2.$

Proof. Let $\eta_v(A) := v^{-1} \int_A |x|^2 \mu_v(dx),$ where $\{\mu_v : v\}$ is the family of measures corresponding to the characteristic functions $\psi(v, y)$. At first we prove the weak compactness of the family of measures $\{\Psi_B(x)\eta_v(dx) : v \in V\}$ for $B = B(\mathbb{K}^n, 0, R), 0 < R < \infty$, where $\Psi_B(x) = 1$ for $x \in B, \Psi_B(x) = 0$ for $x \notin B,$ $\Psi_B(x)$ is the characteristic function of the set $B$. Using the non-archimedean analog of the Minlos-Sazonov theorem as in §5 we reduce the proof to the case of measures on $\mathbb{F}^{bn}$. Take $0 < R_1 < \infty$. In view of the conditions of this theorem for each $\delta > 0$ there exists $v_0 = v_0(R_1, \delta) > 0$ such that for each $\epsilon > 0$ and each $0 < v \leq v_0$ there is accomplished the inequality $-\text{Reg}(y) + \delta \geq \int_{\mathbb{F}^{bn}} [1 - \cos(<(y, x)_F > F )]|x|^2 \eta_v(dx)$ due to the existence of
lim_{n \to 0} [\psi(v, y) - 1] / v = g(y) uniformly in the ball of the radius $0 < R_1 < \infty$, $\forall y \in F^b : |y| \leq R_1$. Integrate this inequality by $y \in B(F^b, 0, r)$ and divide on the volume $\mu(B(F^b, 0, r)) = r^b$ for $r \in \Gamma_F := \{|x| : x \neq 0, x \in F\} = \{p^k : k \in \mathbb{Z}\}$, where $\mu$ is the Haar nonnegative nontrivial measure on $F^b$. Then $-r^{-bn} \int_{B(F^b_0, 0, r)} Reg(y)\mu(dy) + \delta \geq r^{-bn} \int_{B(F^b_n, 0, r)} (\int_{B(F^b_0, 0, r)} [1 - \cos (<y, x>_{F}) |x|^{-2} \eta_v(dx)] \mu(dy))$ for $\mu \geq 0$ and $\mu \geq 0$ are nonnegative measures. Since $\int_{B(F^b_n, 0, p_k^b)} \chi_s(x)\mu(dx) = J(s, k)$, where $J(s, k) = p^{k-bn}$ for $|s| \leq p^{-k}$, $J(s, k) = 0$ for $|s| \geq p^{-k+1}$, then $-p^{-bn}k \int_{B(F^b_0, 0, p_k^b)} Reg(y)\mu(dy) + \delta \geq \int_{B(F^b_n, 0, p)} [1 - p^{-bn}k J(x, k)] |x|^{-2} \eta_v(dx)$. For $\epsilon > p^{-k+1}$ we then get $[\eta_v(B(F^b_0, 0, \epsilon) - \eta_v(B(F^b_n, 0, p^{-k}))) \leq \epsilon^2 |\delta - p^{-bn}k \int_{B(F^b_0, 0, p_k^b)} Reg(y)\mu(dy)|]$. Then for $\epsilon = p^{-k_0+2}$ and $k_0 \geq \epsilon$, $k \to \infty$ the summing of these inequalities leads to: $\eta_v(B(F^b_0, 0, \epsilon)) \leq L_1 \delta - \sum_{k=k_0}^{\infty} p^{-kbn-2k_0+4} \int_{B(F^b_n, 0, p_k^b)} Reg(y)\mu(dy)$, where $L_1 = p^{k_0-2k_0}/(1-p^{-2})$, $k_0 \in \mathbb{Z}$ is fixed.

In view of the fact that the function $g(y)$ is continuous and $g(0) = 0$, then for each $\delta > 0$ there exists $0 < R_1 < \infty$ such that $R_1^{-bn} |\int_{B(F^b_n, 0, R_1)} Reg(y)\mu(dy)| < \delta$. Then for $\epsilon = p^{-k_0+2}$ there is accomplished the inequality: $\eta_v(B(F^b_n, 0, \epsilon)) < 2L_1 \delta$ for each $v \in (0, v_0)$. Since $\int_{K_0 \setminus B} \Psi_B(x)\eta(dx) = 0$, then the family of measures $\{\Psi_B\eta_v : v \in V\}$ is weakly compact for each given $0 < R < \infty$, $B = B(K^b_0, 0, R)$.

Let $0 < \epsilon < \infty$, then $|\int_{B(K^b_0, 0, \epsilon)} (y, x)_{K^b} F \nu(dx)| < \infty$ and $|\int_{K_0 \setminus B(K^b_0, 0, \epsilon)} (y, x)_{K^b} F |x|^{-2} \nu(dx)| < \infty$, then $J_\epsilon := \int_{K^b_0} (\exp(i \cdot (y, x)_{K^b} F) - 1 - i \cdot (y, x)_{K^b} F [1 + |x|^2]^{-1} + (y, x)_{K^b} F [1 + |x|^2]^{-1} / 2]) |x|^{-2} \nu(dx) = \int_{K^b_0} (\exp(i \cdot (y, x)_{K^b} F) - 1 - i \cdot (y, x)_{K^b} F [1 + |x|^2]^{-1} + (y, x)_{K^b} F [1 + |x|^2]^{-1} / 2] \eta(dx)$, where $\eta(A) := \int_{A} [1 + |x|^2] |x|^{-2} \nu(dx)$ for each $A \in B(K^b_0)$. The measure $\mu \geq 0$ is nonnegative, since $\nu \geq 0$ is nonnegative. From $\nu(\{0\}) = 0$ it follows that $\eta(\{0\}) = 0$. The measure $\eta(A)$ is finite for each $A \in B(K^b_0) \setminus B(K^b_0, 0, \epsilon)$, when $0 < \epsilon < \infty$, since $\nu(K^b_0) < \infty$ and $|x| > \epsilon$ for $x \in K^b_0 \setminus B(K^b_0, 0, \epsilon)$. Therefore, $\eta_\epsilon := (\int_{B(K^b_0, 0, \epsilon)} \exp(i \cdot (y, x)_{K^b} F) - 1 - i \cdot (y, x)_{K^b} F) |x|^{-2} \nu(dx)$. At the same time $\int_{K^b_0} (\exp(i \cdot (y, x)_{K^b} F) - 1 - i \cdot (y, x)_{K^b} F [1 + |x|^2]^{-1} + (y, x)_{K^b} F [1 + |x|^2]^{-1} / 2]) |x|^{-2} \nu(dx) = \int_{B(K^b_0, 0, \epsilon)} (\exp(i \cdot (y, x)_{K^b} F) - 1 - i \cdot (y, x)_{K^b} F [1 + |x|^2]^{-1} + (y, x)_{K^b} F [1 + |x|^2]^{-1} / 2]) |x|^{-2} \nu(dx)$, hence

(1) $g(y) = iA(y) - B(y)/2 + \int_{B(K^b_0, 0, \epsilon)} (\exp(i \cdot (y, x)_{K^b} F) - 1 - i \cdot (y, x)_{K^b} F [1 + |x|^2]^{-1} + (y, x)_{K^b} F [1 + |x|^2]^{-1} / 2]) |x|^{-2} \nu(dx)$,
\[(y,x)_K > F + \langle (y,x)_K > F \rangle^2 / 2) \eta(dx) + \int_{K^n \setminus B(K^n,0,\epsilon)} (\exp(i < (y,x)_K > F) - 1) \eta(dx), \]

where \( \tilde{A}(y) = A(y) + \int B(K^n,0,\epsilon) < (y,x)_K > F \nu(dx) - \int_{K^n \setminus B(K^n,0,\epsilon)} < (y,x)_K > F \eta(dx), \tilde{B}(y) = B(y) + \int_{B(K^n,0,\epsilon)} < (y,x)_K > F \eta(dx) - \int_{K^n \setminus B(K^n,0,\epsilon)} \langle (y,x)_K > F \rangle^2 |x|^{-2} \nu(dx). \]

Using the expressions for \( \tilde{A}(y) \) and \( \tilde{B}(y,z) \) from the proof of Theorem 5, we get
\[
\begin{align*}
(2) \quad & \tilde{A}(y) = \int_{B(K^n,0,\epsilon)} < (y,x)_K > F \nu(dx) + \int_{B(K^n,0,\epsilon)} < (y,x)_K > F |x|^{-2} \nu(dx), \\
(3) \quad & \tilde{B}(y,z) = \int_{B(K^n,0,\epsilon)} < (y,x)_K > F < (z,x)_K > F \nu(dx) + \int_{B(K^n,0,\epsilon)} < (y,x)_K > F < (z,x)_K > F |x|^{-2} \nu(dx). 
\end{align*}
\]

Due to identities (11, 12) with the measure \( [1+|x|^{-2}] \Psi_B \nu(dx) \) here as the measure \( \mu \) in \( \S 4 \), with \( B = B(K^n,0,\epsilon) \), where \( \Psi_B(x) \) is the characteristic function of the set \( B \), \( \Psi_B(x) = 1 \) for \( x \in B \), \( \Psi_B(x) = 0 \) for \( x \in K^n \setminus B \), we get, that \( \tilde{A} \) and \( \tilde{B} \) satisfy Conditions (F1–F4) and (B1–B3) respectively. Since the measures in the definition of \( \tilde{A} \) and \( \tilde{B} \) are nonnegative and the functions in integrals are nonnegative, then \( \tilde{A}(y) \) and \( \tilde{B}(y,z) \) take nonnegative values.

As the metric space \( K^n \) is complete and separable, hence it is the Radon space (see Theorem 1.2 [6]), that is the class of compact subsets approximates from below each \( \sigma \)-additive nonnegative finite measure on the Borel \( \sigma \)-algebra \( B(K^n) \). In view of the finiteness and \( \sigma \)-additivity of the nonnegative measure \( [1+|x|^{-2}] \Psi_B \nu(dx) \) and the boundedness of the continuous functions in integrals the mappings \( \tilde{A}(y) \) and \( \tilde{B}(y,z) \) are continuous.

8. Theorem. A characteristic function \( \psi(y) \) of an infinitely divisible distribution in \( K^n \) has the form \( \psi(y) = \exp(g(y)) \), where \( g(y) \) is given by Formula 5(i). If in addition distributions \( \mu_v(dx) \) from Theorem 5 posses finite moments \( |x|^2 \mu_v(dx) \) the boundedness of the continuous functions in integrals the mappings \( \tilde{A}(y) \) and \( \tilde{B}(y,z) \) are continuous.

9. Definitions. Let there is a random function \( \xi(t) \) with values in \( K^n, t \in T \), where \( (T, \rho) \) is a metric space with a metric \( \rho \). Then \( \xi(t) \) is called stochastically continuous at a point \( t_0 \), if for each \( \epsilon > 0 \) there exists \( \lim_{\rho(t,t_0) \to 0} P(\xi(t) - \xi(t_0) | > \epsilon) = 0 \). If \( \xi(t) \) is stochastically continuous at each point of a subset \( S \) in \( T \), then it is called stochastically continuous on
S.

If \( \lim_{R \to \infty} \sup_{t \in S} P(|\xi(t)| > R) = 0 \), then a random function \( \xi(t) \) is called stochastically bounded on \( S \).

Let \( T = [0, a] \) or \( T = [0, \infty) \), \( a > 0 \). A random process \( \xi(t) \) with values in \( \mathbb{K}^n \) is called a process with independent increments, if \( \forall n, \ 0 \leq t_1 < \ldots < t_n \); random vectors \( \xi(0), \xi(t_1) - \xi(0), \ldots, \xi(t_n) - \xi(t_{n-1}) \) are mutually independent. At the same time the vector \( \xi(0) \) is called the initial state (value), and its distribution \( P(\xi(0) \in B), B \in \mathcal{B}(\mathbb{K}^n) \), is called the initial distribution. A process with independent increments is called homogeneous, if its distribution \( P(t, s, B) := P(\xi(t + s) - \xi(t) \in B), B \in \mathcal{B}(\mathbb{K}^n) \), of the vector \( \xi(t + s) - \xi(t) \) is independent from \( t \), that is, \( P(t, s, B) = P(s, B) \) for each \( t < s + t \in T \).

10. Theorem. Let \( \psi(t, y) \) be a characteristic function of the vector \( \xi(t + s) - \xi(s), t > 0, s \geq 0 \), where \( \xi(t) \) is the stochastically continuous random process with independent increments with values in \( \mathbb{K}^n \). Then \( \psi(t, y) = \exp(tg(y)) \), where \( g(y) \) is given by Formula 5(i). If in addition \( |\xi(t)|_K \) has the second order finite moments, then the function \( g(y) \) is written by Formula 7(i).

Proof. Let \( \xi(t) \) be a homogeneous stochastically continuous process with independent increments with values in \( \mathbb{K}^n \), where \( t \in T \subset \mathbb{R} \). Let \( t > s \), then

\[
|\psi(t, y) - \psi(s, y)| = |M \exp(i < (y, \xi(t))_K > F) - M \exp(i < (y, \xi(s))_K > F)| = M |\exp(i < (y, \xi(t) - \xi(s))_K > F) - 1| \leq M |\exp(i < (y, \xi(t) - \xi(s))_K > F) - 1|.
\]

Therefore, from the stochastic continuity of \( \xi(t) \), it follows continuity of \( \psi(t, y) \) by \( t \). In view of being homogeneous and independency of increments the equalities are accomplished \( \psi(t_1 + t_2, y) = M \exp(i < (y, \xi(t_1 + t_2) - \xi(t_1)) > F + i < (y, \xi(t_1) - \xi(0)) > F) = M \exp(i < (y, \xi(t_1) - \xi(0)) > F) M \exp(i < (y, \xi(t_2) - \xi(0)) > F) = \psi(t_1, y) \psi(t_2, y) \) for each \( t_1, t_2 \in T \). On the other hand, a unique continuous solution of the equation \( f(v + u) = f(v) f(u) \) for each \( v, u \in \mathbb{R} \) has the form \( f(v) = \exp(av) \), where \( a \in \mathbb{R} \). Thus, \( \psi(t, y) = \exp(tg(y)) \), where \( g(y) = \lim_{t \to 0}(\psi(t, y) - 1)/t \).

Applying Theorems 5 and 7, we get the statement of this theorem.

11. Remark. Consider auxiliary random process \( \eta := [\xi]_p \) with values in \( \mathbb{R}^n \), where \( [(q_1, \ldots, q_n)]_p := ([q_1, \ldots, q_n])_p \) for \( q = (q_1, \ldots, q_n) \in \mathbb{K}^n \). If \( \xi(t) \) is a homogeneous process with independent increments, then such is also \( \eta \). Let \( a(t) := M \eta(t) \) is a mean value, while \( R(t, s) := M[(\eta(t) - a(t))^* (\eta(s) - a(s))] \) is the correlation matrix, where \( \eta = (\eta_1, \ldots, \eta_n) \) is the row-vector, \( A^* \) denotes the transposed matrix \( A \). For the process with independent increments and
finite moments of the second order then \( R(t, s) = B(\min(t, s)) \), where the matrix \( B(t) \) is symmetric and nonnegative definite. If \( \xi(t) \) is the homogeneous process with independent increments, \( \eta \) has the finite second order moments, then as it is known \( a(t) = at, R(t, s) = B\min(t, s) \), where \( a \) is the vector, \( B \) is the symmetric nonnegative definite matrix \([11]\).

12. **Theorem.** Let \( P \) and \( Q \) be two nonnegative finite \( \sigma \)-additive measures on the Borel \( \sigma \)-algebra \( \mathcal{B}(K^n) \), where \( K \) is a locally compact infinite field with a nontrivial non-archimedean valuation, \( n \in \mathbb{N} \). If their characteristic functions are equal \( \hat{P}(y) = \hat{Q}(y) \) for each \( y \in K^n \), then \( P(A) = Q(A) \) for each \( A \in \mathcal{B}(K^n) \).

**Proof.** The metric space \( K^n \) is complete and separable, consequently, it is the Radon space, then \( P \) and \( Q \) are Radon measures (see Theorem 1.2 [6]). Then for each \( \delta > 0 \) there exists the ball \( B(K^n, z, R), 0 < R < \infty, z \in K^n \), such that \( P(K^n \setminus B(K^n, z, R)) < \delta \) and \( Q(K^n \setminus B(K^n, z, R)) < \delta \).

For each ball \( B(K^n, z, R_1), z \in K^n, 0 < R_1 < \infty \), due to the Stone-Weierstrass theorem for each \( \epsilon > 0 \) and each continuous bounded function \( f : K^n \to \mathbb{R} \) there exist \( b_1, \ldots, b_k \in C \) and \( s_1, \ldots, s_k \in K^n \) such that \( \sup_{x \in K^n} |b_1^*\chi_{s_1}(x) + \cdots + b_k^*\chi_{s_k}(x) - f(x)| < \epsilon \), where \( \chi_s(x) \) is the character, \( k \in \mathbb{N} \), since the family of all finite \( C \)-linear combinations of characters forms the algebra which is the subalgebra of the algebra of all continuous functions on \( B(K^n, z, R_1) \), the complex conjugation preserves this subalgebra, this subalgebra contains all complex constants and separates points in \( B(K^n, z, R_1) \) (see Theorem IV.10 [21]).

The characteristic function \( \Psi_{B(K^n, z, R)} \) of the set \( B(K^n, z, R) \) is continuous on \( K^n \), since \( K^n \) is totally disconnected and the ball \( B(K^n, z, R) \) is clopen in \( K^n \) (simultaneously open and closed). Take \( z \in K^n, 0 < \delta_k < 1/k, 0 < \epsilon_k < 1/k, R = R(\delta_k) \leq R(\delta_{k+1}) \) for each \( k \). For an arbitrary vector \( z_1 \in K^n \) with \( |z - z_1|_{K^n} < R(\delta_1) \) take the function \( \Psi^\epsilon(x) = b_1\chi_{s_1}(x) + \cdots + b_k\chi_{s_k}(x) \) such that

\[
\sup_{x \in B(K^n, z_1, R_1)} |\Psi^\epsilon(x) - \Psi_{B(K^n, z_1, R_1)}(x)| < \epsilon_k. \quad \text{Then} \]

\[
\int_{K^n} \Psi^\epsilon(x) P(dx) = \int_{K^n} \Psi^\epsilon(x) Q(dx) \quad \text{and} \]

\[
\int_{K^n} \Psi_{B(K^n, z_1, R_1)}(x) P(dx) = P(B(K^n, z_1, R_1)), \quad \int_{K^n} \Psi_{B(K^n, z_1, R_1)}(x) Q(dx) = Q(B(K^n, z_1, R_1)). \quad \text{On the other hand,} \]

\[
|\int_{K^n} \Psi_{B(K^n, z_1, R_1)}(x) P(dx) - \int_{K^n} \Psi_{B(K^n, z_1, R_1)}(x) Q(dx)| \\
\leq |\int_{K^n} \Psi^\epsilon(x) P(dx) - \int_{K^n} \Psi^\epsilon(x) Q(dx)| \\
+ |\int_{K^n} \Psi^\epsilon(x) Q(dx) - \int_{K^n} \Psi_{B(K^n, z_1, R_1)}(x) Q(dx)| + |\int_{K^n} \Psi^\epsilon(x) P(dx) | \\
|\int_{K^n} \Psi^\epsilon(x) Q(dx) - \int_{K^n} \Psi_{B(K^n, z_1, R_1)}(x) Q(dx)| + |\int_{K^n} \Psi^\epsilon(x) P(dx) | \\
|\int_{K^n} \Psi^\epsilon(x) Q(dx) - \int_{K^n} \Psi_{B(K^n, z_1, R_1)}(x) Q(dx)| + |\int_{K^n} \Psi^\epsilon(x) P(dx) |
\]
- $f_{\mathbf{K}^n} \Psi^e(x) Q(dx) \leq \epsilon_k (P(\mathbf{K}^n) + Q(\mathbf{K}^n))$. The right hand side of the latter inequality tends to zero while $k \to \infty$, consequently, $P(B(\mathbf{K}^n, z_1, R_1)) = Q(B(\mathbf{K}^n, z_1, R_1))$ for each ball $B(\mathbf{K}^n, z_1, R_1)$ in $\mathbf{K}^n$, where $0 < R_1 < \infty$, $z_1 \in \mathbf{K}^n$, since $\lim_{k \to \infty} \delta_k = 0$ and $P(\mathbf{K}^n \setminus B(\mathbf{K}^n, z, R(\delta_k)) < \delta_k$, $Q(\mathbf{K}^n \setminus B(\mathbf{K}^n, z, R(\delta_k)) < \delta_k$. Since balls form the base of the topology in $\mathbf{K}^n$, then $P(A) = Q(A)$ for each $A \in \mathcal{B}(\mathbf{K}^n)$.

13. Theorem. Random vectors $\eta_1, ..., \eta_k$ in $\mathbf{K}^n$ are independent if and only if

(1) $M \exp(i < (y_1, \eta_1)_K + ... + (y_k, \eta_k)_K > F) = M \exp(i < (y_1, \eta_1)_K > F)$...$M \exp(i < (y_k, \eta_k)_K > F)$ for each $y_1, ..., y_k \in \mathbf{K}^n$.

Proof. From the independence of $\eta_1, ..., \eta_k$ it follows the independence of $< (y_1, \eta_1)_K > F, ..., < (y_k, \eta_k)_K > F$, consequently, there is satisfied the equality (1), since $\exp(i < (y_1, \eta_1)_K + ... + (y_k, \eta_k)_K > F) = \exp(i < (y_1, \eta_1)_K > F)$...$\exp(i < (y_k, \eta_k)_K > F)$.

Vice versa let (1) be satisfied. Denote by $P_{\eta_1, ..., \eta_k}$ the mutual probability distribution of random vectors $\eta_1, ..., \eta_k$, by $P_{\eta_j}$ denote the probability distribution of $\eta_j$. Then $f_{\mathbf{K}^n} \exp(i < (y_1, x_1)_K + ... + (y_k, x_k)_K > F)P_{\eta_1, ..., \eta_k}(dx) = M \exp(i < (y_1, \eta_1)_K + ... + (y_k, \eta_k)_K > F) = M \exp(i < (y_1, \eta_1)_K > F)$...$M \exp(i < (y_k, \eta_k)_K > F)$ = $\prod_{j=1}^{k} f_{\mathbf{K}^n} \exp(i < (y_j, x_j)_K > F)P_{\eta_j}(dx_j)$, where $x = (x_1, ..., x_k), y_1, ..., y_k, x_1, ..., x_k \in \mathbf{K}^n$. Therefore, by Theorem 12 $P_{\eta_1, ..., \eta_k}(A_1 \times ... \times A_k) = P_{\eta_1}(A_1)...P_{\eta_k}(A_k)$ for each $A_1, ..., A_k \in \mathcal{B}(\mathbf{K}^n)$, consequently, $\eta_1, ..., \eta_k$ are independent.

14. Definitions. A sequence of random vectors $\xi_m$ in $\mathbf{K}^n$ is called convergent by the distribution to a random vector $\xi$, if for each continuous bounded function $f : \mathbf{K}^n \to \mathbf{R}$ there exists $\lim_{m \to \infty} M f(\xi_m) = M f(\xi)$.

Let a metric space $(X, \rho)$ be given with a metric $\rho$ and a $\sigma$-algebra of Borel subsets $\mathcal{B}(X)$.

The family of probability measures $\mathcal{P} := \{P_\beta : \beta \in \Lambda\}$ on $(X, \mathcal{B}(X))$, where $\Lambda$ is a set, is called relatively compact, if an arbitrary sequence of measures from $\mathcal{P}$ contains a subsequence weakly converging to some probability measure.

A family of probability measures $\mathcal{P} := \{P_\beta : \beta \in \Lambda\}$ on $(X, \mathcal{B}(X))$ is called dense, if for each $\epsilon > 0$ there exists a compact subset $C$ in $X$ such that $\sup_{\beta \in \Lambda} P_\beta(E \setminus C) \leq \epsilon$.

A sequence $\{P_m : m \in \mathbf{N}\}$ of probability measures $P_m$ is called weakly convergent to a measure $P$ when $m \to \infty$, if for each continuous bounded function $f : X \to R$ there exists $\lim_{m \to \infty} \int_X f(x) P_m(dx) = \int_X f(x) P(dx)$.
15. **Theorem.** A random vector $\xi$ in $K^n$ is a limit by a distribution of sums $\xi_m := \sum_{k=1}^m \xi_{m,k}$ of independent random vectors with the same probability distribution $\xi_{m,k}$, $k = 1, \ldots, m$, if and only if $\xi$ is infinitely divisible.

**Proof.** If $\xi$ is infinitely divisible, then for each $m \geq 1$ there exists independent random vectors with the same distribution $\xi_{m,1}, \ldots, \xi_{m,k}$ such that the probability distributions of $\xi$ and of the sum $(\xi_{m,1} + \ldots + \xi_{m,k})$ are the same.

Let now $\tilde{\xi}_m$ be a sequence of arbitrary vectors converging by the distribution to $\xi$ when $m \to \infty$. Take $k \geq 1$ and group the summands writing $\tilde{\xi}_{mk}$ in the form: $\tilde{\xi}_{mk} = \xi_{m,1} + \ldots + \xi_{m,k}$, where $\xi_{m,1} = \xi_{m,k,1} + \ldots + \xi_{m,k,m}$, $\ldots, \xi_{m,k,k} = \xi_{m,k,m(k-1)+1} + \ldots + \xi_{m,k,m(k)}$. Since the sequence $\tilde{\xi}_{mk}$ converges by the distribution to $\xi$ while $m \to \infty$, then the sequence of the probability distributions $P_{\tilde{\xi}_{mk}}$ of random vectors $\tilde{\xi}_{mk}$ is relatively compact, consequently, due to the Prohorov Theorem (see §VI.25 [24] or III.2.1 [25]) it is dense.

On the other hand, if $|\tilde{\xi}_{mk}| > R$, then due to non-archimedeanity of the norm in $K^n$ there exists $j$ such that $|\tilde{\xi}_{mj}| > R$, consequently, $P(\tilde{\xi}_{mj} \in K^n \setminus B(K^n, 0, R)) \leq P(\tilde{\xi}_{mk} \in K^n \setminus B(K^n, 0, R))$, since $\tilde{\xi}_{mj}$ are independent and have the same probability distribution. Therefore, $\{P_{\tilde{\xi}_{mj}} : m \in N\}$ is the dense family of probability distributions. Then there exists the sequence $\{m_j : j \in N\}$ and random vectors $\eta_1, \ldots, \eta_k$ such that $\tilde{\xi}_{mj} \to \eta_j$ for $j \to \infty$. In view of the definition of convergence by the distribution this means in particular, that for each $b_1, \ldots, b_k \in K^n$ there exists $\lim_{j \to \infty} M \exp(i < (b_1, \xi_{mj,1})_K + \ldots + (b_k, \xi_{mj,k})_K > F) = M \exp(i < (b_1, \eta_1)_K + \ldots + (b_k, \eta_k)_K > F)$. In view of independence of random vectors $\xi_{mj,1}, \ldots, \xi_{mj,k}$ there is satisfied the equality $M \exp(i < (b_1, \xi_{mj,1})_K + \ldots + (b_k, \xi_{mj,k})_K > F) = M \exp(i < (b_1, \xi_{mj,1})_K > \ldots M \exp(i < (b_k, \xi_{mj,k})_K > F)$, since $\exp(i < y > F)$ is the character of the additive group of the field $K$. Therefore, $\lim_{j \to \infty} M \exp(i < (b_1, \xi_{mj,1})_K + \ldots + (b_k, \xi_{mj,k})_K > F) = M \exp(i < (b_1, \xi_{mj,1})_K > \ldots M \exp(i < (b_k, \xi_{mj,k})_K > F)$, thus, $M \exp(i < (b_1, \eta_1)_K + \ldots + (b_k, \eta_k)_K > F) = M \exp(i < (b_1, \eta_1)_K > \ldots M \exp(i < (b_k, \eta_k)_K > F)$ for each $b_1, \ldots, b_k \in K^n$. Then from Theorem 13 it follows, that the random vectors $\eta_1, \ldots, \eta_k$ are independent.

Since $\xi_{mj,k} = \xi_{mj,1} + \ldots + \xi_{mj,k}$ converges by the distribution to $\eta_1 + \ldots + \eta_k$ and $\tilde{\xi}_{mj,k}$ converges by the distribution to $\xi$, then $\xi$ is equal to $\eta_1 + \ldots + \eta_k$ by the distribution, since $Mf(\xi) = \lim_{j \to \infty} Mf(\tilde{\xi}_{mj,k}) = \lim_{j \to \infty} Mf(\xi_{mj,1} + \ldots + \xi_{mj,k}) = Mf(\eta_1 + \ldots + \eta_k)$ for each continuous bounded function $f : K^n \to R$.  

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16. Particular cases of Theorem 10. 1. If \( A(y) = q < (a, y)_{K} >_{F}, \) \( B = 0, \nu = 0, \) where \( a \in K^{n} \) is some vector, \( q = \text{const} > 0, \) then \( \psi(t, y) = \exp(itq < (a, y)_{K} >_{F}) \). The the random function \( \eta(t) = < (\xi(t), y)_{K} >_{F} \) has the form \( \eta(t) = \eta(0) + tq, \) where \( \xi \) is the initial random vector with values in \( K^{n} \). That is, \( \eta(t) \) corresponds to the uniform motion of the point in \( R \) with the velocity \( q \).

In the case, when \( A(y) = q(v, < y >_{F}), B = 0, \nu = 0, \) where \( v \in R^{n} \) is a given vector, \( 0 \leq v_{j} \leq 1 \) for each \( j = 1, ..., n, v = (v_{1}, ..., v_{n}), q = \text{const} > 0, \) then \( \psi(t, y) = \exp(itq(v, < y >_{F})) \). Therefore, the random variable \( \eta(t) = < (\xi(t) >_{F}, < y >_{F})_{R} \) has the form \( \eta(t) = \eta(0) + tq \).

2. It is possible to consider in formulas for \( A(y) \) and \( B(y, z) \) in \( \S 5 \) and 7 in particular atomic measures, denoting \( \hat{A} \) by \( A \) and \( \hat{B} \) by \( B \) here for the uniformity, then there are the expressions of the form \( \sum_{j} q_{j} < (x_{j}, y)_{K} >_{F} \) and \( \sum_{j} q_{j} < (x_{j}, y)_{K} >_{F} < (x_{j}, z)_{K} >_{F} \), where \( q_{j} = \nu(\{x_{j}\}) > 0 \) or \( q_{j} = \nu(\{x_{j}\})|x_{j}|^{-2} > 0 \) depending on the considered case, \( x_{j} \neq 0 \). In particular, there may be \( x_{j} = e_{j} = (0, ..., 0, 1, 0, ...) \in K^{n} \) with the unity on the \( j \)-th place. These expressions may be transformed using conditions \( (F1 - F4) \) or \( (B1 - B3) \) (see Formulas 5(i, 10, 13, 14) or 7(i, 1-3)). Then there are possible cases \( A(y) = q < (a, y)_{K} >_{F}, A(y) = (v, < y >_{F})_{R}, B(y, z) = \sum_{j=1}^{n} s_{j}y_{j}z_{j} >_{F}, B(y, z) = \sum_{j=1}^{n} q_{j} < y_{j} >_{F} < z_{j} >_{F} \), where \( < y >_{F} = < y_{1} >_{F}, ..., < y_{n} >_{F} \), \( y = (y_{1}, ..., y_{n}) \in K^{n}, y_{k} \in K \) for each \( k, v \in R^{n}, (\ast, \ast)_{R} \) is the scalar product in \( R^{n}, s_{j} \in K, a \in K^{n} \). The consideration of the transition matrix \( Y \) from one basis in \( K^{n} \) to another or the matrix \( X \) of transition from one basis in \( R^{n} \) into another leads to the more general expressions for \( B(y, z) \) such as \( B(y, z) = (b < y >_{F}, < z >_{F})_{R}, B(y, z) = < (hy, z)_{K} >_{F} \), where \( b \) is the symmetric nonnegative definite \( n \times n \) matrix with elements in the field of real numbers \( R, h \) is the symmetric \( n \times n \) matrix with elements in the locally compact field \( K \).

3. If \( A(y) = q < (a, y)_{K} >_{F}, B(y, z) = < (hy, z)_{K} >_{F}, \) where \( a \in K^{n}, h \) is the symmetric \( n \times n \) matrix with elements in the field \( K, \) if the correlation term \( f_{K^{n}}(y, x)\nu(dx) = 0 \) from \( \S 5 \) or \( f_{B(K^{n},y_{0},\zeta)}(\exp(i < (y, x)_{K} >_{F}) - 1 - i < (y, x)_{K} >_{F} + < (y, x)_{K} >_{F}^{2}/2)\eta(dx) + f_{K^{n}\setminus B(K^{n},y_{0},\zeta)}(\exp(i < (y, x)_{K} >_{F}) - 1)\eta(dx) = 0 \) from \( \S 7 \) is zero, then \( \psi(t, y) = \exp(itq < (a, y)_{K} >_{F} - t < (hy, y)_{K} >_{p} /2) \). Then \( \xi(t) \) is one of the non-archimedean variants of the Gaussian process.

4. In the case, when \( A(y) = (v, < y >_{F})_{R}, B(y, z) = (b < y >_{F}, < (hy, y)_{K} >_{F} \).
where $q > n$ respectively here, $B$ it by $y$ the measure view of $|q > n|$. In the particular case of the uniform distribution $\lambda(dx)$, the Gaussian type in the non-archimedean case, but not all (see also [18]).

5. When $A = 0$, $B = 0$ (taking into account $(F1 - F4)$ and $(B1 - B3)$; see Formulas $5(i, 10, 13, 14)$ or $7(i, 1 - 3)$), where $\nu$ is the purely atomic measure, concentrated at the point $z_0$, $\nu(\{z_0\}) = q > 0$, then $\psi(t, y) = \exp(qt(\exp(i < (y, z_0)_K > F)) - 1)$. Therefore, $\xi(t)$ is the non-archimedean analog of the Poisson process.

6. If $\tilde{A}(y) = q < (a, y)_K > F + \int_{B(K^n, 0, \epsilon)} < (y, x)_K > F \eta(dx)$, $\tilde{B}(y) = -\int_{B(K^n, 0, \epsilon)} < (y, x)_K > F + \int_{K^n} \{\exp(i < (y, x)_K > F) - 1\} \lambda(dx)$, where $B$ is the Poisson process with the same probability distribution $\lambda(dx) = \lambda(dx)$ (see Formulas $7(i, 1 - 3)$ and $(F1 - F4)$, $(B1 - B3)$). Therefore, $\psi(t, y) = \exp(itq < (a, y)_K > F) \sum_{k=0}^\infty \exp(-wt)((wt)^k/k!) [\int_{K^n} \exp(i < (y, x)_K > F) \lambda(dx)]^k$. This expression of the characteristic function of the random process $\xi(t)$ is the non-archimedean analog of the generalized Poisson process.

If $\tilde{A}(y) = (v, < y > F)_R + \int_{B(K^n, 0, \epsilon)} < (y, x)_K > F \eta(dx)$, where $\tilde{B}(y)$ is the same as at the beginning of the given paragraph, then $\psi(t, y) = \exp(itv, < y > F)_R + \int_{K^n} \exp(v, < y > F)_R \lambda(dx)$, where $\rho(t)$ has the characteristic function $\exp(itv, < y > F)_R$.

17. Remark. Let a branching random process is realized with values in the ring $\mathbb{Z}_p$ of integer $p$-adic numbers or in the ring $B(F_p(\theta), 0, 1)$, denote it by $B$. In the particular case of the uniform distribution $|x|^2 \nu(dx)$ in $B$ the measure $\nu$ is proportional to the Haar measure $\mu$, $|x|^2 \nu(dx) = q \mu(dx)$, where $q > 0$, $\mu(B) = 1$, $\nu(F \setminus B) = 0$, $K = F = Q_p$ or $K = F = F_p(\theta)$ respectively here, $n = 1$. Then it is possible to calculate $A(y)$ and $B(y)$. In view of §5 in this particular case $A(y) = q \int_B < yx > F \mu(dx)$ and $B(y) = q \int_B < yx > F \mu(dx)$. If $y = 0$, then $A(0) = 0$ and $B(0) = 0$, therefore, consider the case $y \neq 0$. The function $< yx > F$ takes the zero value when $|yx|_F \leq 1$ and is different from zero when $|x|_F > 1/|y|_F$. 

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In the considered case the support of the measure $\nu$ is contained in $B$, then $A(y)$ and $B(y)$ are equal to zero when $|y|_F \leq 1$. But the Haar measure is invariant relative to shifts $\mu(A+z) = \mu(A)$ for each Borel subset in $F$ with the finite measure $\mu(A) < \infty$ and each $z \in F$. Moreover, $\mu(zdx) = |z|_F \mu(dx)$, where $|z|_F = p^{-ord_p(z)}$ (see [26]). Then $A(y) = q \int_{z \in F, |y|_F \geq |z|_F} < z >_F \mu(dz)/|y|_F$ and $B(y) = q \int_{z \in F, |y|_P \geq |z|_P} < z >_P \mu(dz)/|y|_F$, where $|y|_F > 1$. At the same time $z = \sum_{k=N(z)} \infty z_k \theta^k$ for $F = Q_p$ or $z = \sum_{k=N(z)} \infty z_k \theta^k$ for $F = F_p(\theta)$, where $N(z) = ord_p(z)$, $z_k \in \{0, 1, \ldots, p-1\}$ or $z_k \in F_p$. If $\nu(dx)$, then $A(y) = q|y|_F \int_{z \in F, |y|_F \geq |z|_F} < z >_F \mu(dz)$ and $B(y) = q|y|_F \int_{z \in F, |y|_P \geq |z|_P} < z >_P \mu(dz)$. These integrals are expressible in the form of finite sums, since $\mu(B(F, x, p^k)) = p^k$ for each $k \in Z$ and $z \in F$, where the functions in the integrals are locally constant.

The measure $\nu$ is Borelian, $\nu : B(K^n) \to [0, \infty)$, therefore each its atom may be only a singleton. More generally (see Formulas 5(i, 13, 14)), if $\nu = \nu_1 + \nu_2$, where $\nu_2$ is the atomic measure, while $\nu_1(dx) = f(x) \mu(dx)$, where $f(x) = g(|x|_F, < x >_F)$, $g : R^2 \to [0, \infty)$ is a continuous function, then

$$A(y) = \sum_j < yx_j >_F |j|^{-2} \nu_2(\{x_j\}) + \int_{F} < y >_F f(x)|x|_F^{-2} \mu(dx),$$

$$B(y) = \sum_j < yx_j >_F |j|^{-2} \nu_2(\{x_j\}) + \int_{F} < y >_F f(x)|x|_F^{-2} \mu(dx),$$

where $\{x_j\}$ are atoms of the measure $\nu_2$, $\nu_2(\{x_j\}) > 0$, each $x_j \neq 0$ is nonzero. At the same time by the Haar measure $\mu$ on $F$ with functions $< y >_F f(x)|x|_F^{-2}$ and $< yx >_F^2 f(x)|x|_F^{-2}$, where $f(x) = g(|x|_F, < x >_F)$, are expressible in the form of series, since $|x|_F$ and $< x >_F$ are locally constant, hence $f$ is locally constant.

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