Radio spectra and radio-loudness of low-luminosity AGNs

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Abstract. We investigated the empirical relation between black hole mass ($m_{\text{BH}}$), Eddington ratio ($L_{\text{bol}}/L_{\text{Edd}}$), and radio loudness ($R_{\text{RL}}$: a ratio of radio to optical luminosity) of nearby low-luminosity active galactic nuclei (LLAGNs). A best-fit plane was found in the three-dimensional space using a sample of 48 nearby LLAGNs: $R_{\text{RL}} = m_{\text{BH}}^{0.52 \pm 0.14}(L_{\text{bol}}/L_{\text{Edd}})^{-0.39 \pm 0.08}$. This suggests that spectral energy distributions of LLAGNs are controlled by both the black hole mass and accretion rate.

1. Introduction

The Palomar optical spectroscopic survey has detected nuclear emission lines in nuclear regions of 86 per cent of 486 nearby bright galaxies with $B_T < 12.5$ mag ($B_T < 12.5$ is the total apparent B-band flux), mainly in the northern sky [12]. Active galactic nuclei (AGNs) reside in about one half of these nuclear emission line galaxies [13]. Almost all of them are low-luminosity AGNs (LLAGNs), which are usually formally defined as $L_{\text{H}\alpha} < 10^{40}$ ergs s$^{-1}$. Their low luminosities are thought to be caused by low accretion rates and low radiative efficiencies in the accretion flows. Such accretion has often been explained by an optically-thin accretion flow such as the advection-dominated accretion flow (ADAF, [31]) rather than an optically thick disc (‘standard disc’; [38]), which applies to luminous AGNs. Although the ADAF model successfully explains the broadband spectra of LLAGNs, there is a significant departure from observations in the radio regime (e.g., [25, 35, 10]).

From an observational point of view, the connection between radio properties and accretion phenomena in AGNs has not been clear for a long time. Radio loudness, conventionally defined as the ratio of radio at 5 GHz to B-band flux densities [42, 40, 18], spreads more than four orders of magnitude at the same optical luminosities (e.g., [15], and cf. [41]). The jet powers are not well correlated with the radiation powers from accretion discs. On the other hand, there are many reports about radio–optical continuum (or line) flux correlations (e.g., [43, 15]). The coincidences of the jet producing events and soft X-ray flux variations in 3C 120 [26] are one of the phenomenological evidence for the jet–disc connection in the AGNs. The jet–disc connection seems to be weak but real. However, the principal parameters that control jet activities in accretion discs have so far been hardly identified by observations. Recently, several implications have been reported that jet activities are related to black hole masses [5, 27, 32, 19, 11] and/or accretion rates [20, 11, 19], both of which are thought to be two principal parameters that
describe the accretion phenomena. For example, radio-loud quasars have been found exclusively in very high-mass black hole systems [20]; radio-louder AGNs tend to show lower accretion rates [11]. A ‘fundamental plane’ has been found in the three-dimensional space spanned by the radio and X-ray luminosities of the core component and by the black hole mass that applies to stellar-mass as well as supermassive black hole systems ([28], see also [4]). Recently developed methods for estimating black hole masses allow to make such studies. These results suggest that the black hole mass and accretion rate may be principal parameters for the jet activities.

In the present paper, we suggest a unified interpretation that the radio powers and radio loudness can be described using only the two principal parameters (black hole mass and accretion rate) for nearby AGNs, most of which are LLAGNs.

2. Sample
We compiled two samples for nearby AGNs: sample A and sample B. Sample A consists of 48 radio sources that had been significantly detected (>1 mJy) in systematic surveys with the VLA A-configuration at 15 GHz from [29, 30] and archival data. Sample B consists of the other 22 nearby AGNs whose reliable black hole masses have been compiled by [28]; uniform radio data are not available for these sources. All of the AGNs were selected from 486 nearby bright galaxies of the Palomar optical spectroscopic survey [12]. The optical survey collects galaxies with $B_T < 12.5$ mag mainly in the northern sky from the Revised Shapley-Arnes Catalogue of Bright Galaxies [37].

Properties of observed optical emission lines were taken from [12, 14], and were extinction-corrected using the mean extinction law of [3], assuming the intrinsic $H\alpha/H\beta$ flux ratio = 3.1. Consequently, most of the AGNs in our samples are LLAGNs. The formal definition of LLAGNs, $L_{H\alpha} < 10^{40}$ ergs s$^{-1}$ for the narrow-line component, is not adopted in our selections, so that nine nearby AGNs are included in the samples.

3. Estimates of several parameters
3.1. Bolometric luminosities
The nuclear continuum emission from most LLAGNs is dominated by the stellar light of their host galaxies in almost the entire part of their spectral energy distributions (SEDs). Only if sensitive, high-angular resolution observations with instruments like the Hubble space telescope and the Chandra X-ray observatory are used, the non-stellar continuum emission can be extracted [16, 41]. Since complete data were not available for all objects in our samples, we estimated their bolometric luminosities from Balmer line luminosities, according to [21]. An empirical $H\beta$ line–$B$-band continuum relation in AGNs including LLAGNs [15] and an intrinsic ratio of $H\alpha/H\beta = 3.1$ provide

$$\log L_{bol} = 1.176 \log L_{H\alpha} - 4.91,$$

where $\log L_{bol}$ is the bolometric luminosity in ergs s$^{-1}$. This nonlinearity is consistent with that of the line-continuum correlation in quasars [33]. With this relation, the operational definition of LLAGNs, $L_{H\alpha} < 10^{40}$, corresponds to $L_{bol} < 10^{42.1}$ in ergs s$^{-1}$. The resultant bolometric luminosities, not listed in the present paper, will be used for estimating mass accretion rates.

3.2. Black hole masses
The black hole mass is thought to be one of the fundamental parameters that drive the various activities in all accreting systems; the other one is the accretion rate. We examine in the present paper whether the observed properties of the nearby LLAGNs can be described with the two fundamental parameters.
We compiled black hole masses, mainly provided by the relation between the central stellar velocity dispersion and the black hole mass \[8, 7\]. We used the relation that has been found by \[6\] as \[m_{\text{BH}} = 1.66 \times 10^8 (\sigma_*/200)^{4.58}\], where \(m_{\text{BH}}\) is the black hole mass in \(M_\odot\) and \(\sigma_*\) is the central stellar velocity dispersion in \(\text{km s}^{-1}\). The HYPERLEDA catalogue \[39, 34\] provides the values of velocity dispersion. This method can estimate black hole masses with an uncertainty of \(\sim 20–30\) per cent.

3.3. Accretion rates

The actual dimensionless accretion rate, \(\dot{m} = \dot{M}/\dot{M}_{\text{Edd}}\) where \(\dot{M}\) is the physical accretion rate and \(\dot{M}_{\text{Edd}}\) is the Eddington rate, cannot be measured by observing the radiation from the nucleus because the radiative efficiency in the accretion disc is unknown. Instead of the actual accretion rate, a ratio of bolometric luminosity to Eddington luminosity, \(L_{\text{bol}}/L_{\text{Edd}}\), is generally used. The HYPERLEDA catalogue \[39, 34\] provides the values of velocity dispersion. This method can estimate black hole masses with an uncertainty of \(\sim 20–30\) per cent.

Therefore, we assume a radiative efficiency of the ADAF type for the nearby AGNs of our samples. In the ADAF regime, \(L_{\text{bol}} \propto \dot{m}^2\), while \(L_{\text{bol}} \propto \dot{m}\) in the standard-disc regime; the difference between them is small at \(\dot{m} \sim 10^{-2}\), but significant at \(\dot{m} \ll 10^{-2}\). Although the conversion factor between \(L_{\text{bol}}\) and the actual accretion rate may be different for different objects, we adopt a uniform scaling law for the ADAF case (eq. (58) of \[24\]):

\[
\dot{m}_{\text{ADAF}} = \alpha_{\text{vis}} \left( \frac{L_{\text{bol}}}{0.20 L_{\text{Edd}}} \right)^{1/2},
\]

where \(\dot{m}_{\text{ADAF}}\) is the dimensionless accretion rate in the ADAF. We assume a viscosity parameter of \(\alpha_{\text{vis}} = 0.1\) in the present paper. Note that the ADAF is established only for accretion rates lower than a critical value \(\dot{m} < \dot{m}_{\text{crit}} \sim \alpha_{\text{vis}}^2\), i.e. \(\sim 10^{-2}\) \[36\]. In our samples, five objects with relatively high accretion rates do not fulfill this criterion; these objects may be dominated by the standard disc. Note that \(L_{\text{bol}}/L_{\text{Edd}}\) is a model-independent parameter, while \(\dot{m}_{\text{ADAF}}\) is a model-dependent parameter.

3.4. Radio Loudness

Conventionally, the radio loudness has been defined as the ratio between the 5-GHz radio and the \(\text{B}\)-band optical flux densities \[42, 40, 18\]. In the present paper, we define the radio loudness of the core component as \(R_{\text{RL}} = P_{\text{VLA-A}}/P_{\text{B}}\), where \(P_{\text{VLA-A}}\) is the monochromatic flux density that was measured with the VLA in the A-configuration at 15 GHz with a resolution of \(\sim 0.15\)-arcsec and \(P_{\text{B}}\) is the \(\text{B}\)-band monochromatic flux density, which was estimated from the extinction-corrected \(H\beta\) luminosity as described in section 3.1.

4. Results

4.1. \((P_{15}\text{VLA-A}, m_{\text{BH}}, \dot{m})\) plot

The three-dimensional plot of radio power, black hole mass and accretion rate for the sample A is presented in Fig. 1. We found a best-fit plane in the three-dimensional space,

\[
\log P_{15}\text{VLA-A} = (1.52 \pm 0.14) \log m_{\text{BH}} + (0.61 \pm 0.08) \log (L_{\text{bol}}/L_{\text{Edd}}) + (11.4 \pm 0.97)
\]

\[
= (1.52 \pm 0.14) \log m_{\text{BH}}
\]
This is a ‘fundamental plane of radio power’. The radio powers are well-correlated with a function of both the black hole mass and accretion rate, although the radio powers are little correlated individually with the black hole mass or accretion rate, as seen in the two-dimensionally projected plots of $m_{\text{BH}}$ vs. $P_{15}^{\text{VLA-A}}$ and $\dot{m}_{\text{ADAF}}$ vs. $P_{15}^{\text{VLA-A}}$ in Fig. 1. The correlation coefficient between log $m_{\text{BH}}$ and log $P_{15}^{\text{VLA-A}}$ is 0.61; the standard deviation of log $P_{15}^{\text{VLA-A}}$ from a regression line is 0.67. The correlation coefficient between log $\dot{m}_{\text{ADAF}}$ and log $P_{15}^{\text{VLA-A}}$ is 0.10; the standard deviation of log $P_{15}^{\text{VLA-A}}$ from a regression line is 0.84. In a two-dimensional plot projected from an edge-on view (log $P_{15}^{\text{VLA-A}}$ vs. 1.52 log $m_{\text{BH}} + 1.22 \log \dot{m}_{\text{ADAF}}$, not illustrated in the present paper), the correlation coefficient is 0.85; the standard deviation from the best-fit plane is 0.45.

This best-fit plane suggests that AGNs with higher mass black holes and higher accretion rates tend to have cores with more powerful radio emission. The dependence of the radio power on the black hole mass and its dependence on the accretion rate are nearly the same: $P_{15}^{\text{VLA-A}} \propto m_{\text{BH}}^{1.52 \pm 0.14} \cdot \dot{m}_{\text{ADAF}}^{1.22 \pm 0.16}$. It is very important that the dependence of radio powers is slightly but significantly different from that of the bolometric luminosities, $L_{\text{bol}} \propto m_{\text{BH}}^{0.52} \dot{m}_{\text{ADAF}}^{-0.78}$. Such a relation can be seen in the three-dimensional plot of radio loudness, black hole mass and accretion rate (Fig. 2). The best-fit plane in equation (4) is also expressed as

$$+(1.22 \pm 0.16) \log \dot{m}_{\text{ADAF}}$$
$$+(12.20 \pm 0.94). \quad (4)$$

This is the fundamental plane of radio loudness. The radio loudness is correlated with both the black hole mass and accretion rate. The correlation coefficient between log $m_{\text{BH}}$ and log $R_{\text{RL}}$ is 0.55; the standard deviation of log $R_{\text{RL}}$ from a regression line is 0.71. The correlation coefficient between log $\dot{m}_{\text{ADAF}}$ and log $R_{\text{RL}}$ is −0.75; the standard deviation of log $R_{\text{RL}}$ from a regression line is 0.51. In a two-dimensional plot projected from an edge-on view (log $R_{\text{RL}}$ vs. 0.52 log $m_{\text{BH}} − 0.78 \log \dot{m}_{\text{ADAF}}$ plot, not illustrated in the present paper), the correlation coefficient is 0.82; the standard deviation from the best-fit plane is 0.45. This best-fit plane of radio loudness suggests that AGNs with higher mass black holes and lower accretion rates tend to be radio-louder AGNs.

4.2. ($R_{\text{RL}}, m_{\text{BH}}, \dot{m}$) plot
As mentioned in the previous section, the dependence of radio powers is different from that of the bolometric luminosities on black hole mass and accretion rate: $P_{15}^{\text{VLA-A}} \propto m_{\text{BH}}^{1.52} \cdot \dot{m}_{\text{ADAF}}^{1.22}$. This indicates that also the radio loudness depends on the black hole mass and accretion rate, because of its definition as $R_{\text{RL}} \equiv P_{15}^{\text{VLA-A}}/P_{\text{B}} \propto P_{15}^{\text{VLA-A}}/L_{\text{bol}} \propto m_{\text{BH}}^{0.52} \dot{m}_{\text{ADAF}}^{-0.78}$. Such a relation can be seen in the three-dimensional plot of radio loudness, black hole mass and accretion rate (Fig. 2). The best-fit plane in equation (4) is also expressed as

$$\log R_{\text{RL}} = (0.52 \pm 0.14) \log m_{\text{BH}}$$
$$-(0.39 \pm 0.08) \log (L_{\text{bol}}/L_{\text{Edd}})$$
$$-(3.48 \pm 0.97) \quad (5)$$

$$= (0.52 \pm 0.14) \log m_{\text{BH}}$$
$$-(0.78 \pm 0.16) \log \dot{m}_{\text{ADAF}}$$
$$-(3.98 \pm 0.94). \quad (6)$$

This is the fundamental plane of radio loudness. The radio loudness is correlated with both the black hole mass and accretion rate. The correlation coefficient between log $m_{\text{BH}}$ and log $R_{\text{RL}}$ is 0.55; the standard deviation of log $R_{\text{RL}}$ from a regression line is 0.71. The correlation coefficient between log $\dot{m}_{\text{ADAF}}$ and log $R_{\text{RL}}$ is −0.75; the standard deviation of log $R_{\text{RL}}$ from a regression line is 0.51. In a two-dimensional plot projected from an edge-on view (log $R_{\text{RL}}$ vs. 0.52 log $m_{\text{BH}} − 0.78 \log \dot{m}_{\text{ADAF}}$ plot, not illustrated in the present paper), the correlation coefficient is 0.82; the standard deviation from the best-fit plane is 0.45. This best-fit plane of radio loudness suggests that AGNs with higher mass black holes and lower accretion rates tend to be radio-louder AGNs.

4.3. Two-dimensional plot of ($m_{\text{BH}}, \dot{m}$)
A two-dimensional plot of the black hole mass vs. the accretion rate is shown in Fig. 3. It is the projection of the three-dimensional plot onto the ($m_{\text{BH}}, \dot{m}$) plain, but objects of sample B are also included. Almost all of the nearby AGNs in our samples lie at $\dot{m}_{\text{ADAF}} < 10^{-2}$, where
accretion flows can be advection-dominated. Black hole masses are distributed over a range of $10^6$–$10^{9.5}$.

In this plot, we illustrate three kinds of lines as follows. The dashed lines represent constant values of bolometric luminosities, which were derived from the extinction-corrected H$\beta$ luminosities of the narrow-line components, on the basis of the hypothesis that photoionization is induced by the radiation of the accretion flow. Blue dot-dashed lines represent constant values of radio power on the best-fit plane (see section 4.1). Nearby AGNs are expected from the best-fit plane to have the radio powers of $\sim10^{19}$–$10^{22}$ W Hz$^{-1}$. Red solid lines represent constant values of radio loudness on the best-fit plane of radio loudness (Section 4.2). The nearby AGNs from our samples are expected from the best-fit plane to show the radio loudness of $10^1$–$10^4$, i.e. to be almost always radio-loud.

Type 1s (S1.2–1.9, L1.9) and type 2s (S2, L2, T2) are described with different symbols in Fig. 3. We made statistical tests between the two types in terms of extinction-corrected H$\alpha$ luminosities, black hole masses, accretion rates, radio powers, and radio loudness. When the parameter includes upper- or lower-limit data, we made a Gehan’s generalized Wilcoxon test using the ASURV package for two sample test [17, 22]. When the parameter includes only detected data, we made a Wilcoxon test. We did not found any significant differences between type 1s and 2s for H$\alpha$ luminosities, black hole masses, radio powers, or radio loudness. On the other hand, the statistical test indicates that type 1s and 2s are not from the same parent population of accretion rate with the significance level of 95 per cent; type 2s show systematically significantly lower accretion rates than type 1s.
5. Discussion

The best-fit plane of radio loudness may explain why quasars are found to be both radio-loud and radio-quiet objects, why LLAGNs are exclusively radio-loud, and why narrow-line Seyfert 1s (NLSY1s) are exclusively radio-quiet. The objects with higher luminosities should tend to be radio-quieter as shown in Fig. 3. In the accretion regime of $L_{\text{bol}} / L_{\text{Edd}} > 10^{-1}$, where NLSY1s and quasars should exist, their radio loudness is expected from the best-fit plane of radio loudness to be $10^{-1} - 10^2$. The quasars with low mass black holes ($m_{\text{BH}} \approx 10^6 - 10^8$) are expected to be radio quiet, while those with high mass black holes ($m_{\text{BH}} \approx 10^8 - 10^{10}$) are expected to be radio loud. In fact, radio-loud quasars have been found exclusively in high mass black hole systems in elliptical host galaxies, while the quasars with $m_{\text{BH}} < 3 \times 10^8$ are practically all radio-quiet [20]. NLSY1s, which have been found exclusively as radio-quiet objects [44], are thought to be luminous Seyferts with very high-accretion rates ($L_{\text{bol}} / L_{\text{Edd}} \sim 1$) on to low mass black holes ($m_{\text{BH}} \sim 10^6 - 10^7$; [23, 2]). Thus, the observed properties of quasars, NLSY1s and LLAGNs are consistent with the best-fit plane of radio loudness. We suggest ‘a unified interpretation of radio loudness’: the dependence of bolometric luminosity and radio power on black hole mass and accretion rate provides the best-fit plane of radio loudness, which can phenomenologically explain the radio loudness across the whole luminosity range of AGNs.

However, the best-fit plane of radio loudness was established only with the nearby AGNs, almost all of which are low-accretion rate systems, where the ADAF model is applicable. For quasars and NLSY1s, the standard-disc (or slim-disc) model is more plausible. The radiative efficiency of a standard disc is $L_{\text{bol}} = \eta_{\text{eff}} M c^2$, where $\eta_{\text{eff}}$ is the mass-to-energy conversion factor. The radio power of nonthermal jets from the standard discs are theoretically predicted to be $P_{\text{radio}} \propto m_{\text{BH}}^{0.42} \dot{m}^{1.00}$. As a result, we find that $R_{\text{RL}} \equiv P_{\text{radio}} / P_{\text{B}} \propto P_{\text{radio}} / L_{\text{bol}} \propto m_{\text{BH}}^{0.42} \dot{m}^{-1.00}$ is expected in AGNs with a standard disc. Such a dependence is similar to that of our best-fit plane of radio loudness and the model of radiatively inefficient accretion. In fact, radio powers seem to follow a single best-fit plane among stellar-mass black hole systems, LLAGNs and quasars [28]. Therefore, the best-fit plane for radio loudness could be also valid in the quasars and NLSY1 regimes. It is important to examine the best-fit plane also for high accretion-rate AGNs.

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References
[1] Abramowicz M. A., Czerny B., Lasota J. P., Szuszkiewicz E., 1988, ApJ, 332, 646
[2] Boroson T. A., 2002, ApJ, 565, 78
[3] Cardelli J. A., Clayton G. C., Mathis J. S., 1989, ApJ, 345, 245
[4] Falcke H., Küröd E., Markoff S., 2004, A&A, 414, 895
[5] Franceschini A., Vercellone S., Fabian A. C., 1998, MNRAS, 297, 817
[6] Ferrarese, L. (astro-ph: 0203047)
[7] Ferrarese L., Merritt D., 2000, ApJ, 539, L9
[8] Gebhardt K. et al., 2000, ApJ, 539, L13
[9] Heinz S., Sunyaev R. A., 2003, MNRAS, 343, L59
[10] Ho L. C., 1999, ApJ, 516, 672
[11] Ho L. C., 2002, ApJ, 564, 120
[12] Ho L. C., Filippenko A. V., Sargent W. L. W., 1997a, ApJS, 112, 315
[13] Ho L. C., Filippenko A. V., Sargent W. L. W., 1997b, ApJ, 487, 568
[14] Ho L. C., Filippenko A. V., Sargent W. L. W., 2003, ApJ, 583, 159
[15] Ho L. C., Peng C. Y., 2001, ApJ, 555, 650
[16] Ho L. C. et al., 2001, ApJ, 549, L51
[17] Isobe, T., Feigelson, E. D. 1990, BAAS, 22, 917
[18] Kellermann K. I., Sramek R. A., Schmidt M., Green R. F., Shaffer D. B., 1994, AJ, 108, 1163
[19] Lacy M., Laurent-Muehleisen S. A., Ridgway S. E., Becker R. H., White R. L., 2001, ApJ, 551, L17
[20] Laor A., 2000, ApJ, 543, L111
[21] Laor A., 2003, ApJ, 590, 86
[22] LaValley, M., Isobe, T., Feigelson, E. D., 1992, in ASP Conf. Ser. 25, Astronomical Data Analysis Software and Systems I, ed. D. M. Worrall, C. Biemesderfer, & J. Barnes (San Francisco: ASP), 245
[23] Leighly K. M., 1999, ApJS, 125, 297
[24] Mahadevan R., 1997, ApJ, 477, 585
[25] Mannmoto T., Mineshige S., Kusunose M., 1997, ApJ, 489, 791
[26] Marscher A. P., Jorstad S. G., Gómez J., Aller M. F., Teräsranta H., Lister M. L., Stirling A. M., 2002, Natur, 417, 625
[27] McLure R. J., Kukula M. J., Dunlop J. S., Baum S. A., O’Dea C. P., Hughes D. H., 1999, MNRAS, 308, 377
[28] Merloni A., Heinz S., di Matteo T., 2003, MNRAS, 345, 1057
[29] Nagar N. M., Falcke H., Wilson A. S., Ho L. C., 2000, ApJ, 542, 186
[30] Nagar N. M., Falcke H., Wilson A. S., Ulvestad J. S., 2002a, A&A, 392, 53
[31] Narayan R., Yi I., 1994, ApJ, 428, L13
[32] Nelson C. H., 2000, ApJ, 544, L91
[33] Netzer H., Laor A., Gondhalekar P. M., 1992, MNRAS, 254, 15
[34] Patrauël G., Petit C., Prugniel P., Theureau G., Rousseau J., Brouty M., Dubois P., Cambrésy L., 2003, A&A, 412, 45
[35] Quataert E., di Matteo T., Narayan R., Ho L. C., 1999, ApJ, 525, L89
[36] Rees M. J., Phinney E. S., Begelman M. C., Blandford R. D., 1982, Natur, 295, 17
[37] Sandage, A. R. & Tammann, G. A. 1981, A Revised Shapley-Ames Catalog of Bright Galaxies (Washington: Carnegie Institution of Washington)
[38] Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
[39] Simien F., Prugniel P., 2002, A&A, 384, 371
[40] Stocke J. T., Morris S. L., Weymann R. J., Foltz C. B., 1992, ApJ, 396, 487
[41] Terashima Y., Wilson A. S., 2003, ApJ, 583, 145
[42] Visnovsky K. L., Impey C. D., Foltz C. B., Hewett P. C., Weymann R. J., Morris S. L., 1992, ApJ, 391, 560
[43] Xu C., Livio M., Baum S., 1999, AJ, 118, 1169
[44] Zhou H., Wang T., 2002, ChJAA, 2, 501