The influence of design features on the properties of the metal network

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Abstract. In this paper, based on the electrodynamic model, the effect of cell pitch, yarn radius of the filament and the conductivity of the material on the network at the reflection, transmission, and absorption coefficients are studied. On the basis of numerical calculations, the fulfillment of the boundary conditions and the power balance of the incident, absorbed, and transmitted fields are shown. An abnormally high power absorption of a set of weaves was revealed at certain radii of the cross section of the thread.

1. Analysis of the properties of a metal network with a rectangular cell with a normal incidence of a plane wave

Important qualitative properties of mesh structures can be studied in the analysis of the infinite structure of parallel cylindrical filaments. Under the restrictions on the wavelength, grating step and radius of the filament cross section $\lambda >> d >> a$, the exciting field in the cross section of each filament can be considered uniform. With an arbitrary polarization of the incident field, it can be represented by a superposition of fields polarized parallel and perpendicular to the axis of each thread, which induce zero and first wave harmonics, respectively, inside the rod. In this case, the intensity of the zero harmonic is several orders of magnitude higher than the intensity of the first. Hence, it can be expected that for a net structure of two mutually perpendicular systems of parallel threads, their study with a normal incidence of the exciting plane wave can be carried out independently of each other, decomposing the incident field into two mutually perpendicular components, each of which excites its own parallel system. The presence or absence of contacts between two mutually perpendicular filaments of the structure should not greatly affect the reflective properties of the metal network. They can only be affected by the violation of the perpendicularity and uniformity of the material of the filaments in the places of their connection. In the case of fabrication of the mesh structure by weaving technology, the inevitable bends of the threads will also affect the electrophysical properties of the structure. In this paper, we restrict ourselves to the consideration of two mutually perpendicular systems of parallel cylindrical filaments forming a rectangular grid.

$$E_0 = z_0 E_0$$ (1)

field inside each thread
is made up of the field excited by (1), the sum of the fields of adjacent threads and the polarization currents inside the thread itself. When analyzing the mesh structure, the case of a strong surface effect is of the greatest interest, when the radius of the cross section of the thread is much larger than the penetration depth and the internal field of the thread exists only in a thin layer near the border. Under the constraints considered, the basis for constructing a model is a single-thread excitation model.

2. The structure of the field near a single thread

The problem of excitation of a single ideally conducting cylinder by a plane wave was considered in [1], where the results of the numerical calculation of the scattered field pattern are presented. This approach can be extended to the case of a cylinder with finite conductivity. The exciting field can be represented as a plane wave polarized parallel to the axis of the rod (z-axis):

\[ E_0 = z_0 E_0 e^{-i k_0 x} \]  

and extending in the x direction. Turning to the cylindrical coordinates, we can represent the incident field in the form of an expansion in cylindrical wave functions:

\[ E_0 = z_0 E_0 \left( J_0(k_0 \rho) + 2 \sum_{n=1}^{\infty} J_n(k_0 \rho) \cos(n \varphi) \right), \]  

where \( J_n(k_0 \rho) \) – is the nth order Bessel function.

At the considered orders of magnitudes of the wavelength \( \lambda \), the rod radius \( a (\lambda \approx 1m, a = 50 \cdot 10^{-6}m) \) the cross-sectional size is \( \frac{2a}{\lambda} = 10^{-4} \) and in (4) can be limited to one member of the expansion:

\[ E_0 = z_0 E_0 J_0(k_0 \rho) \]  

It corresponds to the magnetic field of the incident wave:

\[ H_0 = \phi_0 \frac{k_0}{i \omega \mu_0} E_0 J_1(k_0 \rho) \]  

The internal field is presented in a similar way:

\[ E_i = z_0 E_i J_0(k_i \rho) \]  

\[ H_i = \phi_0 \frac{k_i}{i \omega \mu_i} E_i J_1(k_i \rho) \]  

The external stray field is expressed in terms of the Hankel functions of the second kind:

\[ E_e = z_0 E_e H_0^{(2)}(k_0 \rho) \]  

\[ H_e = \phi_0 \frac{k_0}{i \omega \mu_0} E_e H_1^{(2)}(k_0 \rho) \]  

The unknown coefficients \( E_i \) and \( E_e \) are to be determined from the conditions of continuity of the tangential components of the electric and magnetic fields when crossing the rod’s border at a given value of the exciting electric field \( E_0 \), which leads to the following relations for the field inside the rod and the external stray field:

\[ \vec{E}_i = \tilde{z}_0 E_0 T J_0(k_i \rho) \]  

\[ \vec{E}_e = \tilde{z}_0 E_0 S H_0^{(2)}(k_0 \rho) \]  

where

\[ T = \frac{2}{\pi i (H_0^{(2)}(k_0 a) J_1(k_i a) \frac{k_i a}{\mu_i} - J_0(k_i a) H_1^{(2)}(k_0 a) k_0 a)}, \]
The coefficients $T$ and $S$ can be considered, respectively, as transmission and scattering coefficients (reflections). For the field inside the rod, the propagation coefficient $\gamma$, associated with $k_i$ by the ratio $\gamma = ik_i$ is a complex value with a sufficiently large module:

$$\gamma = \alpha + i\beta$$

$$\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}}.$$ 

Specific conductivity values for the most common materials used in reticular blades can be found in reference books (see, for example, [3]). In particular, for copper, tungsten and steel, the conductivity $\sigma$ has, respectively, the values: $5.9 \cdot 10^7$, $1.8 \cdot 10^7$, $1.01 \cdot 10^7 \text{ S/m}$. The module of constant distribution:

$$|\gamma| = \sqrt{\omega \mu \sigma}$$

for the frequency of 300 MHz is in these cases significant values: 18.69, 10.74, 7.7.

Below (figure 1–2), the calculation results are given using (11), (12) scattering and transmission coefficients as functions of the radius of the filament section. In the lattice, in addition to the primary plane wave, the field of neighboring filaments acts on the filament, the closest of which are at a distance $d$. For further consideration, it is necessary to investigate the possibility of assessing the influence of neighboring threads. Assuming that each adjacent thread scatters a set of cylindrical harmonics, it is advisable to consider their effect on the thread under consideration.

**Figure 1.** The dispersion coefficient of the copper rod as a function of section radius $t$.

**Figure 2.** Copper bar transmission coefficient as a function of section radius $t$.

To do this, we can use the addition theorem for cylindrical functions [2], which gives a representation of each cylindrical harmonics of the adjacent cylinder in the form of expansion in cylindrical harmonics of the cylinder under consideration. With the help of estimates of the numerical values of these functions, it can be shown that the selected considered thread is in the total uniform field, in turn, responding to this field with the main first harmonic. This greatly simplifies the construction of a model for the excitation of an infinite lattice. To complete the picture, it is necessary to consider the excitation of a filament by orthogonal polarization, when the electric field vector is perpendicular to the axis of the rod. For this case, the necessary calculated ratios are obtained from the above relations (11), (12) on the basis.
of the duality principle and are not explicitly given here, for brevity purpose. The corresponding results of the calculations of the reflection coefficient and transmission are presented in figure 3 - 4. The conclusion important for practical calculations follows from them: the reflection coefficient for the field of orthogonal polarization is 7–8 orders of magnitude smaller than the reflection coefficient for parallel polarization, the field of perpendicular polarization practically does not interact with it at the considered.

![Figure 3](image1.png)

**Figure 3** The scattering coefficient of the copper rod as a function of the radius of section \( t \) for orthogonal polarization.

![Figure 4](image2.png)

**Figure 4** Copper rod transmission coefficient as a function of section radius \( t \) for orthogonal polarization.

3. **Excitation of a metal net by a plane wave at normal incidence**

Since the external scattered field is of primary interest for a net-hollow, a variant of the equation is considered here with respect to the external resultant exciting field of each filament, which is an analogue of the integral equation. The field \( E_0' \), which excites each thread, is made up of the field of the incident wave \( E_0 \) and the fields created by the neighboring threads located at a distance from this thread at the distance \( m d \):

\[
E_0' = E_0 + 2 \sum_{m=1}^{\infty} E_0' S H_0^{(2)}(k_0 m d)
\]

From here we obtain the required effective exciting field of each thread

\[
E_0' = \frac{E_0}{1 - 2S \sum_{m=1}^{\infty} H_0^{(2)}(k_0 m d)}
\]
Since the parameter $k_0d$ can be considered small ($k_0d = 2\pi \cdot 10^{-3}$) the sum in the denominator of this expression can be considered with some approximation as integral and, using well-known expressions for integrals of cylindrical functions ([41], P.303), we can obtain an approximate expression for the sum, taking into account that for an effective exciting field we get:

$$E_0' = \frac{E_0}{1 - 2S(\sum_{m=1}^{M}H_0^{(2)}(k_0 md) + \frac{1}{k_0d}\left\{1 - \int_0^{2M+1}H_0^{(2)}(t)dt\right\})}$$

(17)

Acceptable accuracy is governed by the number of row members $M$ and is achieved already with $M = 2 \div 4$.

This ratio plays an important role in assessing the performance of the net-baling machine. Reflection coefficient $S$ takes into account the diameter and material properties of the threads. It is of interest to evaluate how far the field $E_0'$, excitant separate thread in the system is different from $E_0$, acting on a single thread. We give some examples of the calculation. For a lattice of copper filaments with parameters $\sigma = 5.9 \cdot 10^7$ S/m, $d = 1$ mm, $a = 50$ μm, the effective exciting field is $E_0' = (3.263 \cdot 10^{-3} + i0.016)E_0$. That is, in comparison with a single thread, in a system of conducting threads with a given section radius and grid step, the exciting field for each thread decreases in magnitude by about 6 times and shifts in phase by $\pi/2$.

When calculating the field in the immediate vicinity of the filaments, it is advisable to use the following formula:

$$E = E_0 + E_0' S \left(2H_0^{(2)}(k_0\rho) +ight.$$  

$$+ 2E_0' S(\sum_{m=1}^{M}H_0^{(2)}(k_0 md) + \frac{1}{k_0d}\left\{1 - \int_0^{2M+1}H_0^{(2)}(t)dt\right\}) \right).$$

(18)

At a distance from the lattice ($|x| \gg a$), the more accurate result is given by the formula:

$$E(x) = E_0 e^{-ik_0x} + E_0' S \left(2H_0^{(2)}(k_0x) + 2E_0' S(\sum_{m=1}^{M}H_0^{(2)}(k_0\sqrt{(md)^2 + x^2}) +ight.$$  

$$+ \frac{1}{k_0d}\left\{e^{-ik_0|x|} - \int_0^{2M+1}H_0^{(2)}(\sqrt{t^2 + (k_0x)^2})dt\right\}\right),$$

(19)

with the help of which the field dissipated by an infinite lattice can be represented as:

$$E_s(x) = z_0'(E_0'SH_0^{(2)}(k_0x) + 2E_0' S(\sum_{m=1}^{M}H_0^{(2)}(k_0\sqrt{(md)^2 + x^2}) +$$  

$$+ \frac{1}{k_0d}\left\{e^{-ik_0|x|} - \int_0^{2M+1}H_0^{(2)}(\sqrt{t^2 + (k_0x)^2})dt\right\})$$

(20)

It is easy to see that when it is necessary to calculate the field in the immediate vicinity of the lattice rods, it is necessary to use the exact formula (18). At a distance from the lattice (20) gives the formed reflected wave.
4. Net absorbed power
To calculate the power absorbed by the grating in its filaments, you can calculate the flow of the real part of the Poynting vector flowing through the surface of the filament per unit length:

\[ P_{st} = -|E'_0|^2 |T|^2 \pi a \text{Re} \left( J_0(k_1a) \left( \frac{k_i}{i \omega \mu} \right) \right), \]  
(21)

Since \( 1 / d \) of rods per unit of width, per unit area of the grid we get:

\[ P_{st} = -|E'_0|^2 |T|^2 \frac{\pi a}{d} \text{Re} \left( J_0(k_1a) \left( \frac{k_i}{i \omega \mu} \right) \right), \]  
(22)

The figure 6 shows the results of the calculation of the power density absorbed by the copper metal mesh. An extreme point is detected at values of the radius of the section from the value \( 0.25 \times 10^{-5}m \) to \( 0.3 \times 10^{-5}m \).

Figure 6 Density of power absorbed by copper sheet as a function of yarn radius

For orthogonal polarization, the formula for absorbed power can be obtained, as mentioned above, using the duality principle:

\[ P_{\perp st} = \frac{|E'_0|^2}{|W_0|^2} |T_h|^2 \frac{\pi a}{d} \text{Re} \left( J_0(k_1a) \left( \frac{k_i}{i \omega \mu} \right) \right), \]  
(23)
Comparison of losses by formulas (22) and (23) for a lattice of copper filaments gives the following result: $2.087 \cdot 10^{-7} \frac{W}{m^2}$ и $1.34 \cdot 10^{-12} \frac{W}{m^2}$.

It is seen that the orthogonal polarization practically does not interact with the lattice.

Its contribution to losses in the lattice is more than five orders of magnitude smaller than the contribution of parallel polarization.

It is interesting to compare with the absorption of a solid screen of the same metal:

$$P = \frac{|E_0|^2 Re(W_i)}{|W_i|^2 + 2 Re(W_i) \cdot W_0 + W_0^2} = 4.464 \cdot 10^{-8} \frac{W}{m^2}$$

The solid screen absorbs about five times smaller than the lattice with parallel polarization. Analysis of the energy characteristics of the sheet allows you to check the adequacy of the model, since it can be seen that for metals the field values are close to the values for an ideally conducting conductor. Meanwhile, for him, the power absorption of the web was not taken. Obviously, the objective criterion is to check the balance of power: the power supplied to the web must coincide with the sum of the power absorbed by the web and the power transmitted by the web. For the considered combination of material parameters and geometrical dimensions (radius of the conductor cross section and grid spacing), the relative mismatch is 0.34%, which corresponds to the acceptable quality of the model. The adequacy of the model is confirmed by the calculation of the fulfillment of the boundary conditions. The error does not exceed 0.5%. For an infinite orthogonal grid with square cells of cylindrical conductors for metallic materials used for the manufacture of rebar, the constructed model is analytical, strictly satisfying the boundary conditions and, therefore, not requiring their averaging, which is confirmed by calculations made in the MATHCAD software environment.

In this paper, we consider the case of normal wave incidence on a net-hollow. The general formulas for an arbitrary direction of the incident wave taking into account the interaction arising in this case between two orthogonal systems of filaments are given in [5].

5. Conclusion

The proposed method allows to take into account the effect of the mesh pitch, the radius of the filament section, the mesh material on its performance: the power absorbed by the metal mesh, the transmission coefficient through the mesh. The calculations carried out using the MATHCAD software demonstrate the adequacy of the model in studying the characteristics of the web with a relative error not exceeding 0.5% of the fulfillment of the boundary conditions and energy balance conditions of 0.34%.

References

[1] Potekhin A I 1948 Some problems of diffraction of electromagnetic waves (Moscow: Sovetskoye Radio)

[2] Yanke E, Emde F and Loesh F 1968 Special functions (Moscow: Nauka)

[3] Bakharev S I and Volman V I 1982 Handbook on calculation and design of microwave microstrip devices (Moscow, Radio i Svyaz) p 328

[4] Abramowitz M and Stegun I A 1979 Handbook of Mathematical Functions (Moscow: Nauka) p 832

[5] Dautov O Sh, Romanov A G, Golovenkin E N, Skachkov V A and Shatrov A K 2018 IOP Conf. Series: Materials Science and Engineering 450 022034