The Radiative Corrections to the Mass of the Kink Using an Alternative Renormalization Program

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ABSTRACT: In this paper we compute the radiative correction to the mass of the kink in $\phi^4$ theory in 1+1 dimensions, using an alternative renormalization program. In this newly proposed renormalization program the breaking of the translational invariance and the topological nature of the problem, due to the presence of the kink, is automatically taken into account. This will naturally lead to uniquely defined position dependent counterterms. We use the mode number cutoff in conjunction with the above program to compute the mass of the kink up to and including the next to the leading order quantum correction. We discuss the differences between the results of this procedure and the previously reported ones.

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1. Introduction

The quantum radiative corrections to the mass of the solitons have been of great interest since the 1970’s, and has had a long, complicated, and at times controversial history. In 1974, Dashen et al. [1] computed the one-loop correction to the mass of the bosonic kink in \( \phi^4 \) field theory for the first time. In that article they used the mode number cutoff method with a continuous form for the phase shifts of the scattering states. After Dashen, several other authors have used similar methods to compute similar corrections for analogous problems [2, 3]. Ever since Dashen’s work, several other methods have been invented or used for analogous problems, the most important five of which are the following. First, the energy momentum cutoff using discontinuous form of the scattering states phase shifts [3, 4, 5]. The above two approaches have sparked remarkable controversies [6, 7, 8]. Second, the derivative expansion of the effective action using summation of the series for the exactly solvable cases, which embeds an analytic continuation, or Padé approximation or Borel summation formula for the approximately solvable cases [9]. Third, the scattering phase shift method in which the change in the density of states due to the presence of the disturbance is represented by the scattering phase shifts [10]. Fourth, the dimensional regularization technique in which the zero point energy of the free vacuum is subtracted and dimensional regularization is used [11, 12]. Fifth, the zeta function regularization technique which completely bypasses the explicit subtraction of free vacuum energy [13]. Obviously all these methods have eventually confirmed the DHN result. As a side note we should mention that analogous corrections to the mass of the bosonic kink have been computed in supersymmetric models (see for example [14, 15, 16, 17]).

The presence of either non-trivial boundary conditions or non-perturbative backgrounds, e.g. solitons, have important manifestations in the physical properties of the systems. In particular, an alternative renormalization program has been proposed [18] which is fully consistent with the boundary conditions and its use has led to new results for the Next to Leading Order (NLO) Casimir effect within \( \phi^4 \) theory [19, 20]. The main purpose of this paper is to explore another manifestation of this newly proposed renormalization program by presenting an analogous study for systems with non-trivial backgrounds. In particular, we calculate the quantum correction to the mass of the \( \phi^4 \) kink, which is analogous to the Casimir problem with the kink as its static background.

In this paper we explain briefly the alternative renormalization program as tailored to our problem. The starting part of our computation parallels closely Dashen’s work. That is, we use the mode number cutoff with continuous phase shifts. However, the counterterms that we derive differ form the free counterterms, by which we mean the ones derived specifically for the free case, i.e. cases with no non-trivial boundary conditions or spatial backgrounds. The main issue of the alternative renormalization program is that the presence of non-trivial boundary conditions or strong backgrounds such as solitons, which could also affect the boundary conditions, are in principle non-perturbative effects. Therefore, they define the overall structure and the properties of the theory and obviously cannot be ignored or even taken into account perturbatively. The alternative renormalization program is founded on the principle that the solution to the problem
should be self-contained and the renormalization procedure be done self-consistently with the nature of the problem.

An additional justification supporting this method is the fact that the presence of either non-trivial boundary conditions or non-trivial backgrounds or both break the translational symmetry of the system. In our case this occurs when we fix the position of the soliton. Obviously the breaking of the translational symmetry has many manifestations. Most importantly all the \( n \)-point functions of the theory will have in general non-trivial position dependence in the coordinate representation. The procedure to deduce the counterterms from the \( n \)-point functions in a renormalized perturbation theory is standard and has been available for over half a century. This, as we shall show, could lead to uniquely defined position dependent counterterms. Then, the radiative corrections to all the input parameters of the theory, will be in general position dependent. In that case, the information about the non-trivial boundary conditions or position dependent background is carried by the full set of \( n \)-point functions, the resulting counterterms, and the renormalized parameters of the theory. When we compute the mass counterterm systematically by setting the tadpole diagrams equal to zero, it turns out to be proportional to Green’s function, as usual, which obviously has non-trivial position dependence in this problem. This counterterm turns out to be different from the trivial sector only by some finite localized contributions which are proportional to the bound state distributions.

We have organized the paper in four sections as follows. In Section 2 we set up the usual problem of \( \phi^4 \) theory for a real scalar field in 1+1 dimensions, in the spontaneously broken phase. We find the static background solutions which include the trivial and the kink sectors. We also exhibit the quantum fluctuations in both sectors, the latter of which includes two bound states. Our main calculational tools in Section 3 are the renormalized perturbation theory and the expansion of the Lagrangian about the two different static sectors. We then calculate NLO correction to the kink mass by subtracting the vacuum energies of the two sectors. The part of this energy which does not depend on the counterterms is calculated using the mode number cutoff. We then calculate the contribution from the mass counterterms, which are fixed by the no tadpole renormalization condition. When we add up all the contributions we find an extra term which is due to our non-trivial counterterm in the kink sector. Finally, in Section 4 we compare our methods and results to some earlier work.

2. Kink solutions and their quantum fluctuations

In this section we shall very briefly state the standard results for the static background solutions and their quantum fluctuations for the bosonic \( \phi^4 \) theory. For a comprehensive review of the standard materials, see for example [3]. We start with the Lagrangian density for a neutral massive scalar field, within \( \phi^4 \) theory, appropriate for the spontaneously broken symmetry phase in 1+1 dimensions,

\[
\mathcal{L} = \frac{1}{2} \left( \frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 - U[\phi(x)],
\]

(2.1)

where \( U[\phi] = \frac{\lambda}{4} \left( \phi^2 - \frac{\lambda_0^2}{\phi} \right)^2 \). The Euler-Lagrange equation can be easily obtained and is a second-order non-linear PDE with the following solutions: Two non-topological static solutions \( \phi_{\text{vac}}(x) = \pm \frac{\lambda_0}{\sqrt{\lambda}} \), and two topological static ones \( \phi_{\text{kink}}(x) = \pm \frac{\lambda_0}{\sqrt{\lambda}} \tanh\left[ \frac{\mu_0(x-x_0)}{\sqrt{2}} \right] \) which are called kink and antikink, respectively. The presence of \( x_0 \) indicates the translational invariance, and this will lead to a zero mode. The total kink energy, sometimes called the classical kink mass can be easily calculated and is given by \( M_{cl} = \frac{2\sqrt{2}\lambda_0^2}{3} \). In order to find the quantum corrections to this mass, we have to make a functional Taylor expansion of the potential about the static solutions which yields the stability equation

\[
\left[ -\nabla^2 + \frac{d^2U}{d\phi^2} \bigg|_{\phi_{\text{static}}(x)} \right] \eta(x) = \omega^2 \eta(x),
\]

(2.2)

where we have defined \( \phi = \phi_{\text{static}} + \eta \). The results in the trivial sector are the following continuum states \( \psi(x) = \exp(ikx) \) with \( k^2 = \lambda^2 + 2\lambda \). In the kink sector we have the following two bound states:

\[
\begin{align*}
\psi_{\text{trivial}}(x) &= e^{ikx} \left( \frac{\lambda}{\sqrt{\lambda}} \right) \tanh\left[ \frac{\mu_0(x-x_0)}{\sqrt{2}} \right], \\
\psi_{\text{kink}}(x) &= e^{i(kx + \frac{\pi}{2})} \left( \frac{\lambda}{\sqrt{\lambda}} \right) \tanh\left[ \frac{\mu_0(x-x_0)}{\sqrt{2}} \right],
\end{align*}
\]

as the solutions to the eigenvalue equation.

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\[
\begin{align*}
\psi_{\text{trivial}}(x) &= e^{i(kx + \frac{\pi}{2})} \left( \frac{\lambda}{\sqrt{\lambda}} \right) \tanh\left[ \frac{\mu_0(x-x_0)}{\sqrt{2}} \right], \\
\psi_{\text{kink}}(x) &= e^{ikx} \left( \frac{\lambda}{\sqrt{\lambda}} \right) \tanh\left[ \frac{\mu_0(x-x_0)}{\sqrt{2}} \right],
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\end{align*}
\]

as the solutions to the eigenvalue equation.
states and continuum states:

\[ \eta_0(z) = \sqrt{\frac{3m_0}{8}} \frac{1}{\cosh^2 z}, \]
\[ \eta_B(z) = \sqrt{\frac{3m_0}{4}} \frac{\sinh z}{\cosh^2 z}, \]
\[ \eta_q(z) = e^{iqz} N_q \left[ -3 \tanh^2 z + 1 + q^2 + 3iq \tanh z \right], \]

where \( m_0 = \mu_0 \sqrt{2} \), \( \omega_0^2 = 0 \), \( \omega_B^2 = \frac{3}{4}m_0^2 \) and \( \omega_q^2 = m_0^2(q^2 + 1) \). Here \( N_q^2 = \frac{16\omega_q^2}{m_0^2}(\omega_q^2 - \omega_B^2) \), and \( z = \frac{m_0 x^2}{2} \). The continuum states \( \eta_q(z) \) have the following asymptotic behavior for \( x \to \pm \infty \),

\[ \eta_q(z) \to \exp[iqz \pm \frac{i}{2} \delta(q)], \]

where \( \delta(q) = -2 \arctan\left[ \frac{3q}{2 - q^2} \right] \) is the phase shift for the scattering states. We believe the phase shifts should be in principle defined to be continuous functions of their arguments. This is particularly apparent in their use in the strong and the usual forms of the Levinson theorem (see for example [21]). For this particular case the phase shift is illustrated in figure 1.

![Figure 1](image)

**Figure 1**: A direct calculation of the phase shift yields the graph on the top. However, in all physical applications that we are aware of, the proper form to use is a continuous as the one illustrated on the bottom.

### 3. First order radiative correction to the kink mass

In this section we calculate the first order quantum correction to the kink mass. As is well known, this is analogous to the Casimir problem for this case. That is, the exact kink mass is the difference between the vacuum energies in the presence and absence of the kink. To calculate this effect we set up renormalized perturbation theory. We should mention that in these (1+1)-dimensional problems, one usually chooses a minimal renormalization scheme defined at all loops by \[ Z_\lambda = 1, \quad Z_\eta = 1 \] and \( m_0^2 = m^2 - \delta_m \).

The sufficiency of these conditions is supported by the fact that for any theory of a scalar field in two dimensions with non-derivative interactions, all divergences that occur in any order of perturbation theory can be removed by normal-ordering the Hamiltonian [22]. However, relaxing the first condition might lead to some extra finite contributions, and this deserves a further investigation. Here, for mere comparison reasons, we stay focused on the renormalization program with the above stated conditions but with non-trivial mass counterterm \( \delta m_{\text{kink}} \).

Now we can split the expression for mass of the kink into the following two parts,

\[ M = (E_{\text{kink}} - E_{\text{vac}}) + (\Delta E_{\text{kink}} - \Delta E_{\text{vac}}), \]

where the first part is in the vacuum sector of each, and the second part is due to the counterterms. We put our solutions in a box of length \( L \) and impose periodic boundary conditions. The continuum limit is reached by taking \( L \) to infinity and the sum turns into an integral. Now we use the usual
mode number cutoff as advocated by R.F. Dashen [1] to calculate the first part of Eq. (3.2). In this method one subtracts the energies of the bound states in the presence of solitons from the same number of lowest lying quasi-continuum states in the vacuum of the trivial sector. Then one subtracts the remaining quasi-continuum states from each other in ascending order. In this case we have two bound states which are to be subtracted from $\omega_{+1}$. Then we subtract the quasi-continuum $q_n$ from the remaining $k_{n+1}$ one by one. The periodic boundary condition implies,

$$k_{n+1}L - 2\pi = 2n\pi = q_n \frac{mL}{2} + \delta(q_n).$$

(3.3)

The first part of Eq. (3.2) can be easily calculated as follows

$$E_{\text{kink}} - E_{\text{vac.}} = M_{\text{cl.}} + \frac{1}{2} \left( \sum \omega - \sum \omega' \right)$$

$$= \frac{m^3}{3\lambda} + \frac{1}{2} \left[ \omega_0 + \omega_B - (\omega_1 + \omega'_{-1}) + 2 \sum_{n=1}^{N} (\omega_n - \omega'_{n+1}) \right]$$

$$= \frac{m^3}{3\lambda} + \frac{\sqrt{3m}}{4} - m + \sum_{n=1}^{N} \left[ \frac{m(2n^2/4 + 1)^2}{k_n^2 + m^2/2} \right]$$

$$\rightarrow \frac{m^3}{3\lambda} + \frac{\sqrt{3m}}{4} - \frac{3m}{2\pi} \int_{-\infty}^{\infty} \frac{dk}{\sqrt{k^2 + m^2(k^2 + \frac{m^2}{4})}},$$

(3.4)

where in the last step we have taken the continuum limit, performed an integration by parts taking the appropriate boundary values of the phase shift into account, as explained earlier. Note that the last term is logarithmically divergent.

Now we calculate the second part of Eq. (3.2):

$$\Delta E_{\text{kink}} - \Delta E_{\text{vac.}} = -\frac{1}{2} \int dx \left[ \delta_{m_{\text{kink}}} \phi_{\text{kink}}^2(x) - \delta_{m_{\text{vac.}}} \phi_{\text{vac.}}^2(x) \right],$$

(3.5)

where $\delta_{m_{\text{kink}}}$ and $\delta_{m_{\text{vac.}}}$ are the mass counterterms in the kink and vacuum backgrounds, respectively, and are calculated below. We first start with the mass counterterms in vacuum background. The procedure for obtaining this quantity is well known, e.g. by setting the tadpole equal to zero [23].

The result is

$$\delta_{m_{\text{vac.}}} = \frac{3\lambda}{4\pi} \int_{-\infty}^{\infty} \frac{dk}{\sqrt{k^2 + m^2}},$$

(3.6)

which is logarithmically divergent.

Next we calculate the mass counterterm in the kink sector by expanding the Lagrangian, which includes the mass counterterm, around the kink background as follows

$$\phi(x, t) \rightarrow \phi_{\text{cl}}(x) + \eta(x, t) = \frac{m}{\sqrt{\lambda}} \tanh \left( \frac{m}{\sqrt{\lambda}} x \right) + \eta(x, t),$$

(3.7)

where $\phi_{\text{cl}}(x)$ can be any of the static solutions, for example the kink solution as indicated above. Then the Lagrangian which incudes the mass counterterm becomes,

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^2 + \frac{1}{2} (m^2 - \delta_m) \phi^2 - \frac{\lambda}{4} \phi^4 - \frac{(m^2 - \delta_m)^2}{4\lambda}$$

$$= \frac{1}{2} (\partial_{\mu} \eta)^2 + \frac{1}{2} (m^2 - 3\lambda \phi_{\text{cl}}^2) \eta^2 - \lambda \phi_{\text{cl}} \eta^4 - \frac{\lambda}{4} \eta^4 - \delta_m \phi_{\text{cl}} \eta - \frac{1}{2} \delta_m \eta^2$$

$$- \frac{1}{2} (\partial_{\mu} \phi_{\text{cl}})^2 + \frac{1}{2} (m^2 - \delta_m) \phi_{\text{cl}}^2 - \frac{1}{4} \lambda \phi_{\text{cl}}^4 - \frac{(m^2 - \delta_m)^2}{4\lambda} + (m^2 \phi_{\text{cl}} - \lambda \phi_{\text{cl}}^3 + \partial_{\mu} \phi_{\text{cl}} \partial^\mu \eta).$$

(3.8)

Note that the last term in the above equation which is proportional to $\eta$ vanishes exactly after an integration by parts and using the equation of motion. Therefore, the condition of setting the tadpole equal to zero simply becomes

$$\partial_x = \mathcal{O}_x + \mathcal{O} \mathcal{O} + \ldots = 0.$$
Accordingly, up to first order in $\lambda$ we obtain,

$$i\delta_m(x, t) \phi_{\text{cl}}(x) = \frac{1}{2} \left[ -6i\lambda \right] \phi_{\text{cl}}(x) G(x, t; x, t),$$

(3.10)

where $G(x, t; x', t')$ is the propagator for the particular problem under investigation. We finally obtain the following general result, which is also obtained in [3, 19, 20] using analogous general arguments but a slightly different method,

$$\delta_m(x, t) = -3\lambda G(x, t; x, t).$$

(3.11)

Note that Eq. (3.11) is a special case of this equation. More importantly, note that the counterterms in general naturally turn out to be position dependent. Since $G(x, t; x', t')$ is uniquely determined by the nature of the problem, so is $\delta_m(x, t)$ via Eq. (3.11). The Green function for this problem in the presence of a kink is,

$$G(x, t; x', t') = i \int \frac{d\omega}{2\pi} e^{i\omega(t-t')} \left( \sum_{n \neq 0} \frac{\eta_n^2(x) \eta_n(x')}{\omega_n^2 - \omega^2} + \int dk \frac{\eta_k^2(x) \eta_k(x')}{\omega_k^2 - \omega^2} \right),$$

(3.12)

where the sum indicates the contributions of the bound states and the integral the continuum states. Note that the zero mode is neglected since it is only the manifestation of the translational invariance of the system and is to be treated as a collective coordinate [3, 16]. The above equation, when the two space-time points are set to be equal and the $\omega$ integration is performed, becomes

$$G(x, t; x, t) = -\frac{\eta_B^2(x)}{2\omega_B} - \int \frac{dk}{2\pi} \frac{\eta_k(x)}{2\omega_k}. $$

(3.13)

Calculating this integral is very cumbersome, but we can use an interesting relation which is the local version of the completeness relation [24, 23, 22]:

$$|\phi(k, x)|^2 = 1 - \frac{m}{\omega^2} \eta_B^2(x) - \frac{2m}{\omega^2} \eta_0^2(x).$$

(3.14)

Using the above equation, Green’s function is easily computable by performing simple integrals. Putting Eq. (3.14) into Eq. (3.13) and using Eq. (3.11) the mass counterterm in the kink background becomes,

$$\delta_{m_{\text{kink}}} = \frac{\lambda}{\sqrt{3m}} \eta_B^2(x) - \frac{3\lambda}{4\pi m} \eta_0^2(x) + \frac{3\lambda m}{4\pi} \int_{-\infty}^{\infty} \frac{1}{\sqrt{k^2 + m^2}} dk.$$

(3.15)

which as expected earlier is different from mass counterterm in the trivial sector, i.e. the last term in Eq. (3.13). In fact it has extra localized finite $x$-dependent terms due to the presence of the bound states, and obviously this difference tends to zero as $x \rightarrow \pm \infty$. An alternative reasoning is that the kink solution also tends to either of the trivial vacuum states as $x \rightarrow \pm \infty$. To complete the calculation we need to calculate Eq. (3.14) by inserting the expressions for $\delta_{m_{\text{kink}}}$ and $\delta_{m_{\text{vac}}}$. into it. The result is

$$\Delta E_{\text{kink}} - \Delta E_{\text{vac}} = \frac{1}{2} \int_{-\infty}^{\infty} \text{dx} \left\{ \delta_{m_{\text{vac}}} \left[ \phi_{\text{kink}}^2(x) - \phi_{\text{vac}}^2(x) \right] + \left[ \frac{\lambda}{\sqrt{3m}} \eta_B^2(x) - \frac{3\lambda}{4\pi m} \eta_0^2(x) \right] \phi_{\text{kink}}^2(x) \right\}$$

$$= \frac{m}{\lambda} \delta_{m_{\text{vac}}} - \frac{\sqrt{3\pi} - 3}{20\pi} m.$$

(3.16)

Inserting the expressions obtained in Eqs. (3.14, 3.16) into Eq. (3.12) we obtain the following expression for $M$:

$$M = \frac{m^3}{3\lambda} + \frac{\sqrt{3m}}{4} - \frac{3m^2}{2\pi} - \frac{\sqrt{3\pi} - 3}{20\pi} m$$

$$- \frac{3m}{4\pi} \int_0^{\infty} \frac{2k^2 + m^2}{\sqrt{k^2 + m^2}} dk + \frac{3m}{2\pi} \int_0^{\infty} \frac{dk}{\sqrt{k^2 + m^2}}.$$

(3.17)

The logarithmic divergences cancel and the final result is:

$$M = \frac{m^3}{3\lambda} + \frac{m}{4\sqrt{3}} - \frac{3m^2}{2\pi} - \frac{\sqrt{3\pi} - 3}{20\pi} m.$$

(3.18)

Our result differs slightly from the previously reported result [1], by the last term in Eq. (3.18).
4. Conclusion

In this paper we have calculated the NLO correction to the mass of the kink using the newly proposed alternative renormalization program. The use of this renormalization program is justified by the fact that the presence of non-trivial boundary conditions or strong non-trivial backgrounds, such as solitons, which could also affect the boundary conditions are in principle non-perturbative effects. Therefore, they define the overall structure and the properties of the theory and obviously cannot be ignored or even taken into account perturbatively. We believe that the solution to the problem should be self-contained and the renormalization procedure be done self-consistently with the nature of the problem. Moreover, the presence of a kink with a fixed position breaks the translational symmetry of the system and this has profound consequences. In particular all of the $n$-point functions of the theory, the counterterms and the renormalized parameters of the theory will in general become position dependent. We have shown this explicitly for the mass counterterm in this problem. This will affect the quantum corrections to the kink mass. In particular in Eq. (3.15) we have shown explicitly the difference between $\delta m_{\text{kink}}$ and $\delta m_{\text{vac}}$. This has led to a small correction to the result obtained by using free counterterms.

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