About the impact value on the track in the «train – bridge» system

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Abstract. The article shows the necessity of taking into account the full vertical acceleration of the pair of drivers during high-speed operation. Dynamic coupling of a rolling load depending, among other things, on the material of an elastic-viscous interlayer between the rail and the bridge deck slab with a bridge is important during the high-speed operation of rolling stock along bridges. Material characteristics can be selected using a mathematical model of interaction, implemented in numerical experiments. The article shows the necessity of taking into account the full vertical acceleration of the pair of drivers during high-speed operation.

Introduction
The bridges dynamics of high-speed railway lines (HSR) remain relevant [1-13]. Calculations on railway load in practice are carried out both when taking into account only the impellents system effect on the bridge replacing the rolling stock [7], as well as using inertial rolling load when driving cars along a discrete track (taking into account the rail grid and sleepers) [11,12]. The latter approach causes large computing costs.

Main part
Number of issues research, in the context of solving a task related to the load movement along the railbridge beam system with an elastic interlayer between them, is of interest. To demonstrate the need for high-speed operation taking into account the full vertical acceleration of the contact points of the train pairs of drivers and the rail. To check the possibility of using a simplified dynamic model of the permanent way in the “train-bridge” during high-speed operation, i.e. a model in the form of an added mass to the wheel and an elastic-viscous element under the wheel, moving together with the wheel on the bridge beam (refer with fig. 2b). It is reasonable to expand the possibilities of using this track model from [2] for bridges on HSR [3, 14, 15].

Figure 1. (a, b)
For the system shown in figure 1a, the differential equations describing its motion are of the form

\[ E_1 \frac{\partial^4 v}{\partial y^4} + \mu_1 E_1 \frac{\partial^2 v}{\partial y^2 \partial t} + m_1 \frac{\partial^2 v}{\partial t^2} + k(v-u) + \gamma \left( \frac{\partial v}{\partial t} - \frac{\partial u}{\partial t} \right) = \delta(y - \tilde{C}_i)(R_1 + P)\theta(t) \]  

(1)

\[ E_2 \frac{\partial^4 u}{\partial y^4} + \mu_2 E_2 \frac{\partial^2 u}{\partial y^2 \partial t} + m_2 \frac{\partial^2 u}{\partial t^2} + k(u-v) + \gamma \left( \frac{\partial u}{\partial t} - \frac{\partial v}{\partial t} \right) = 0, \quad R_1 = -Mw_1, \quad \tilde{C}_i = C_1 + Vt \]  

(2)

where \( E_1, \mu_1 \), and \( E_2, \mu_2 \) - rail and beam bending stiffness; \( m_1, m_2 \) - rail and beam mass of unit length; \( \mu_1, \mu_2 \) and \( \gamma \) - coefficients of viscosity; \( k \) - coefficients of interlayer elasticity; \( \delta(y) \) - delta function; \( \theta(t) \) - the Heaviside function; \( v(y,t), u(y,t) \) - rail and beam vertical displacements; \( C_1 \) - abscissa of the load touch-down point; \( \tilde{C}_i \) - abscissa of the moving load; \( V \) - movement speed of the load (pair of drivers); \( P \) - axiload on the load; \( R_1 \) - dynamic additive to the static load reaction; \( \ell \) - bridge span; \( w_1 \) - the vertical acceleration of a load moving on a rail; \( M \) - load weight.

For the system shown in figure 2b, the differential equation describing its movement is of the form

\[ E_2 \frac{\partial^4 u}{\partial y^4} + \mu_2 E_2 \frac{\partial^2 u}{\partial y^2 \partial t} + m_2 \frac{\partial^2 u}{\partial t^2} = \delta(y - \tilde{C}_i)R_2\theta(t), \quad R_2 = -M_w w_1 + P \]  

(3)

where \( M_w \) - load weight and the added rail mass.

To solve equations (1-3), we apply the Fourier method.

\[ v(y,t) = \sum_{i=1}^{N} W_i(y)q_i(t), \quad u(y,t) = \sum_{i=1}^{N} W_i(y)q_1(t), \quad \text{where} \quad W_i(y) = \sin(\pi i \ell) \quad \pi i = \pi i \]  

(4)

where \( q_i(t), q_1(t) \) - are the generalized coordinates defining the fields of vertical displacements, respectively, of the track and bridge beam; \( N, N_1 \) - the number of members of the series in (4).

To solve the problem, after its spatial discretization, we use the stepping procedure and the accounting method the rolling inertial load, proposed in [9,10]. We perform a problem discretization in time, introducing integer the whole \( t_i \) and half nodes \( t_{j+1/2} \). In the step \( \Delta t_j = t_{j+1} - t_j \) the system (1, 2) has the form

\[ \bar{M}\dot{q}_{j,i} + \bar{C}_i \dot{q}_{j,i} + \bar{K}_i q_{j,i} = \bar{D}_j, \quad i = 1, \ldots, N, \quad \bar{M}_i = E \]  

(5)

and its solution, in accordance with [9], is

\[ \dot{q}_{j,i} = U_{ij} \dot{q}_{i+1} + U_{ij} \dot{q}_{i-1} + G_i \bar{D}_{j,i+1/2}, \quad \ddot{q}_{j,i} = \ddot{q}_{i+1} + \ddot{q}_{i-1} + D_{ij}, \quad \bar{K}_i = \left[k_{m,n,i}\right], \quad \bar{C}_i = \left[c_{m,n,i}\right] \]

\[ n, m = 1, 2 \]

(6)

\[ \ddot{q}_{j,i} = \sum_{j=1}^{N} W_i(t) \left\{ \ddot{q}_{j,i} + \ddot{q}_{j,i} \Delta t_j + \left[U_{i1} \ddot{q}_{j,i} + U_{i2} \ddot{q}_{j,i} + G_i \bar{D}_{j,i+1/2} \frac{\Delta t_j^2}{2} \right] \right\}, \quad \bar{q}_i = [q_{1,i}, q_{2,i}] \]

(7)

where \( G_i = \bar{M}_i + \bar{C}_i \frac{\Delta t_j}{2} + \frac{\bar{K}_i \Delta t_j^2}{4} \), \( G_i^{-1} = \left[g_{n,m,i}\right] \), \( k_{11,i} = \omega_{1,i}^2 + \frac{k}{m_1} \), \( k_{12,i} = \frac{k}{m_2} \), \( k_{22,i} = \omega_{1,i}^2 + \frac{k}{m_2} \), \( k_{21,i} = -\frac{k}{m_1} \), \( k_{12} = \frac{k}{m_2} \)

\[ U_{i1} = [u_{n,m,i}] = -G_i^{-1} \bar{K}_i, \quad U_{i2} = [u_{n,m,i}] = -G_i^{-1} (\bar{C}_i + \bar{K}_i \frac{\Delta t_j}{2}), \quad c_{22,i} = \mu_2 \omega_{2,i}^2 + \frac{\gamma}{m_2} \]  

(8)
To solve the problem we use the expression of the total vertical acceleration of the load [2, 10], moving along a beam with a constant speed $V$, in the form (8) and its variant, often used, its approximation during acceleration in the form (9)

$$\ddot{w}_i = \frac{\partial^2 \ddot{u}(V_1, t)}{\partial t^2} + 2V \frac{\partial^2 \ddot{u}(V_1, t)}{\partial y \partial t} + V^2 \frac{\partial^2 \ddot{u}(V_1, t)}{\partial y^2}$$

(8)

$$\ddot{w}_i = \frac{\ddot{u}(V_1, t)}{\partial t^2}$$

(9)

We will take into account the conditions of continuity of displacements and speeds in the system “load - deck”. As a result, using the system solution (1), (2) and the replacement $R_1 = -M\ddot{w}_1$, at step $[t_{j+1}, t_j]$ we have at the moment $t=t_{j+1/2}$ full load acceleration $\ddot{w}_{i,j+1/2} = \ddot{q}_{i,j+1/2}$ in the form

$$\ddot{q}_{i,j+1/2} = Z_1/Z_2 \text{ where } Z_1 = 1 + \bar{b}_1 M \sum_{i=1}^{N} g_{i}^{(2)} \beta_i r_i sin(\frac{r_i}{\ell} \beta_i)$$

$$Z_2 = \sum_{i=1}^{N} \left( (\beta_i \ddot{u}_{1i}^{(1)} + \ddot{u}_{12i}^{(2)} + \ddot{u}_{22i}^{(2)} + \ddot{u}_{12i}^{(2)} + \ddot{u}_{12i}^{(2)} + g_{i}^{(1)} \bar{b}_1 P_{j+1/2} s_{i,j+1/2} \right)$$

$$s_{i,j+1/2} = \sin(\frac{r_i}{\ell} \beta_i)$$

$$\bar{b}_1 = \frac{2}{m_i \beta_i}$$

$$\beta_i = \sin(\frac{r_i}{\ell} \beta_i)$$

$$\beta_i = 2V(r_i/\ell) \cos(\frac{r_i}{\ell} \beta_i) - V^2(r_i/\ell)^2(\Delta t_i/2) \sin(\frac{r_i}{\ell} \beta_i)$$

Let us follow the progress of solving the entire problem with $j=0, 1, 2, \ldots$ At step $[t_{j+1}, t_j]$, at initial conditions at moment $t_1$, the acceleration $\dot{q}_{1,j+1/2}$ is determined from (10), further using (4), (5), (6), the dynamic reaction $R_{1,j+1/2}$, displacement fields, velocities for the system “load-deck” are calculated at the moment $t_{j+1}$. Further process repeats. When solving problems, we will use the following data: $E_1 = 2.1 \times 10^6$ kN/m$^2$; $f = 0.6 \times 10^{-6}$ m$^{-1}$; $\mu_1 = 0$; $m_1 = 12$ t/m; $k_1 = 50000$ kN/m$^2$; $\gamma = 6$ kNs/m$^2$; $P = 170$ kN; $M = 2.6$ t; $E_2 = 0.33 \times 10^8$ kN/m$^2$; $f_2 = 18.22$ m$^{-1}$; $\mu_2 = 0.167 \times 10^{-3}$ s; $m_2 = 26.8$ t/m; $C_1 = 14$ m; $\ell = 44$ m. The parameters of the deck (load beam used on the HSR) are taken from [13], the other parameters are traditional [3, 14, 15].

Let us estimate the degree of approximation of the problem solution, using (9), depending on the movement speed. Let us carry out the preliminary task testing. The first test task – effect on a rail bearer in the combined system (refer with fig. 1a) of static load $\delta(y - 0.5\ell)P$. Proposed stepping procedure in [9] is used to solve both dynamic and static problems. Under zero initial conditions and $\gamma = 0$ already at the first step of integrating the equations system (1), (2) equal, for example, $\Delta t_j = 10^3 \text{c}$, we have, according to [9],

$$\bar{q}_j = \sum_{i=1}^{N} W_i (0.5) q_{2i} = 2\delta_{SD} = 0.000134 \text{ m}, \text{ where } \delta_{SD} = 0.0005017 \text{ m is the static deflection of the carrier (lower) beam under the specified load affect. In the well-known static problem, only for a carrier beam, with hinged ends and a load of } \delta(y - 0.5\ell)P, \text{ we have } \delta_{CT} = 2P\ell^3 / (\pi^2 E_2J_2) = 0.0004955 \text{ m.}$$
For test purposes, in the second case, we exclude the lower beam from the work in the beam system (refer with figure 1a), in the belief that $k=\gamma=0$ in (1). Replace its parameters with the parameters of the carrier (lower) beam and consider the load movement on it at high speed. Note that to obtain an approximate solution in (10), we should put $\beta_0 = \sin (\pi C_{ij}/l) \cdot \beta_1 = 0$. Stepping procedure (5) - (10), with $k=\gamma=0$ in (1), (2), is implemented for a hinged beam with relations between the input parameters of the problem $g_m P \lambda^2 = \frac{\alpha}{\beta}$, $\lambda = \lambda_m / \Delta t$, where $P = \bar{P}$ is the weight of the load. Figure 2 (a, b) with $\alpha = 0.3$ and $\beta = 3$ presents, depending on $L = Vt / \ell$, changes $R = \bar{R} / \bar{P}$ and $Y = Y_0 / \delta_{id}$, where $\bar{R} = R + \bar{P}$ is load pressure on the beam; $Y_0$ - displacement in the beam center; $L$ - load position on a beam. Note that figure 2 (a, b) shows: the computing results of $R$ and $Y$, where lines 1 correspond to the use of expression (8) (lines 1 completely coincide with the diagrams from [6, p. 30]) and the computing results $R$ and $Y$, where lines 2 correspond to the use of expression (9). Note that with a different mass ratio between the load beam and the load, for example, when the load equal to the pair of drivers mass, the interaction pattern changes. With $\alpha = 0.3$ and $\beta = M / m_2 \ell = 0.002205$, presented in figure 2 (a, b) curves (lines 1 and 2) merge, and approximate solutions for $R$ and $Y$ values practically coincide with solutions when using acceleration in the form (8). Note that with the task parameters selected above, in the cases $\alpha = 0.3$, $\beta = 3$ and $\beta = M / m_2 \ell = 0.002205$ the load moved along the beam with a speed of $V = 101.36$ m/s, and the calculations were performed when solving the problem at $N = 200$ and $\Delta t = 0.0001$ s in (5) - (10). In railway practice, the pair of drivers mass and the reduced mass of the rail in contact with it on an elastic foundation are comparable, therefore the investigation of the moving load (pair of drivers) interaction and the track on the bridge at high speeds in this situation is of interest. After testing procedure, let us return to the structure shown in figure 1a, choosing the weight load equal to the pair of drivers mass, and conduct a series of numerical experiments.
To estimate the approximation effect, in the form (9), on the final result, we will calculate the displacements $\ddot{Y}$ (m) - under the load moving along the beam (refer to fig. 1a), numerically solving the system of equations (1), (2) and changing the load movement speed. In this case, it is advisable to consider the case of the load touch-down on a rail beam at some distance from the slant leg in order to bring the solution closer to the mode of interaction of the load with an endless rail beam on HSR. Figure 3 (a, b) - 5 (a, b) shows the changes $Y=\ddot{Y}$ (m) depending on the time $t$ (s) and the load movement with speeds: $V=180$ km/h (refer to figure 3 (a, b)); $V=360$ km/h (refer to figure 4 (a, b), 5 (a, b)). The results in figure 5 (a, b) correspond to the load movement only along a rail bearer based on Winkler. Note that figure 3a - 5a show the changes $Y=\ddot{Y}$ (m) when constructing a solution using the full acceleration in the form (8), and figure 3b - 5b shows the changes only when the first term is retained in (8). At the same time, the results of a series of numerical experiments with a load speeding up, confirm the fact that, starting from 200 km/h, the error of the approximate solutions presented selectively in figures 3b - 5b, increases significantly, which indicates the need to use the expression for full acceleration at the contact points of wheels and rails, in the form (8), with high-speed movement of rolling stock on HSR.

To solve the second task at hand (on a simplified dynamic model of the track on bridges), with the diagram shown in figure 1b, we use a similar approach with the approach to solve the first task. To solve equation (3) in the case of the load touch-down, after the equation discretization, equation (1) can be applied, setting $k = \gamma = 0$ with replacement, respectively, $m_1$, $E_1$, $J_1$, $\mu_1$, $M \rightarrow m_2$, $E_2$, $J_2$, $\mu_2$, $M_\gamma$: $R_{1j+1/2} \rightarrow R_{2j+1/2}$; $P_{j+1/2} \rightarrow 0$ (used as in (10) changing $R_{2j}=-M_\gamma \ddot{q}_j + P$)

$$R_{2j+1/2} = -D_j L \dddot{q}_{j+1/2} + c_j L (q_{j+2} - \ddot{q}_j) + (\gamma_j + c_j \frac{\Delta t_j}{2}) L (\ddot{q}_{j+1} - \ddot{q}_j) + \frac{D_j L}{M_\gamma} P_{j+1/2}$$

where $D_j = c_j \frac{\Delta t_j^2}{4} + \gamma_j \frac{\Delta t_j}{2}$, $L = \frac{M_\gamma}{M_\gamma + D_j}$
$c_\ast, \gamma_\ast$ - rigidity and viscosity of an elastic-viscous element; $q_z$ - the vertical load displacement; $\ddot{q}_t$ is the vertical full acceleration of the contact point of the beam and the elastic-viscous element.

Using the system of equations (1) - (3), at the stage of solving the problem at hand, the parameters $c_\ast, \mu_\ast$ were selected applying numerical experiments by comparing the diagrams for dynamic pressures and deflections obtained for the two models shown in figure 1 (a, b). As a result, the following parameters were selected from the results of numerical experiments: $c_\ast=67\times10^3$ kN/m; $\gamma_\ast=12$ kNs/m with $\Delta t_j=0.0001s$.

![Figure 6 (a, b)](image)

![Figure 7 (a, b)](image)

As before, the problems of load touch down were solved with $C_1=14m$ and the change in V from 90 to 360 km/h. Note that the results of solving problems at $V=360$ km/h for the first model (refer with figure 1a) are presented in figure 6a - 7a, and for the second (refer with figure 1b) in figure 6b - 7b. Figure 6 (a, b) presents the dynamic reactions $R$(kN) under the load when it touched down on the bridge. Figure 7 (a, b) shows load deflections $Y=\ddot{q}_t$(m) when it touched down on the bridge. The results of the diagrams comparison presented in figure 6 (a, b) - 7 (a, b), they indicate the expediency of using a simplified track model and while a high-speed cars moving on the bridge structure, expanding the capabilities of this model, previously used in [2].

**Summary**

The numerical experiments carried out on the basis of methodology tested in this article and in [3, 10, 14] for studying the interaction of inertial rolling loads and rods, justifiably indicate the need while dynamic calculating of the rolling stock on HSR, to choose the structural design of “the car-rail-deck system”, taking into account the use of mathematical modeling of full vertical and transverse accelerations of the contact points of pairs of drivers and rails when the cars move along bridges.

In the strength calculation of the decks on HSR, it is advisable to use a simplified dynamic model of the track on the bridges.

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