Meson–Nucleon Coupling from AdS/QCD

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Plan

1: Introduction
2: Meson sector (review)
3: Meson-Nucleon coupling (review & our results)
4: Summary
Introduction

AdS/CFT correspondence has opened a new avenue to study strongly coupled gauge theories

Application to QCD and hadron physics

Top down: String theory $\rightarrow$ QCD
Sakai & Sugimoto (2005)

Bottom up: QCD $\rightarrow$ 5D holographic model
Erlich, Katz, Son & Stephanov (2005)
Da Lord & Pomarol (2005)
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$\Rightarrow$ 5D model of Meson sector
  (vector, axial-vector meson masses, decay consts, $g_{\rho \pi \pi}$, ...)

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As for baryons, two approaches are known

1: Skyrmion
   Hashimoto, Sakai & Sugimoto (2008)
   and many papers

2: Bulk fermion
   Hong, Inami & Yee (2007)[spin 1/2]
   Ahn, Hong, Park & Siwach (2009)[spin 3/2]
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$\pi$ NN coupling
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Our work

By extending the model of Hong, Inami & Yee, we calculated (Axial-)Vector Meson-Nucleon couplings using the 5D Holographic QCD
Meson sector

“QCD and a Holographic Model of Hadrons”
Erlich, Katz, Son and Stephanov
PRL95 261602 (2005)

“Chiral Symmetry Breaking from Five Dimensional Spaces”
Da Lord and Pomarol
NPB721 79 (2005)
Following this dictionary, we guess a holographic model of QCD.
5D $SU(N_f)_L \times SU(N_f)_R$ gauge theory on a slice of $AdS_5$

$$ds^2 = \frac{1}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right), \quad \varepsilon \leq z \leq z_m \left(= \Lambda_{QCD}^{-1} \right)$$

(Axial-)Vector meson (spin 1) $\leftrightarrow$ (Axial-)Vector gauge field

| 4D: $O(x)$ | 5D: $\phi(x,z)$ | $SU(N_f)_L \times SU(N_f)_R$ | $p$ | $\Delta$ | $M_5^2 = (\Delta - p)(\Delta + p - 4)$ |
|------------|----------------|-------------------------------|-----|-------|---------------------------------|
| $\bar{q}_L \gamma^\mu t^a q_L$ | $L_\mu$ | $(\text{adj},1)$ | 1 | 3 | 0 |
| $\bar{q}_R \gamma^\mu t^a q_R$ | $R_\mu$ | $(1,\text{adj})$ | 1 | 3 | 0 |
| $\bar{q}_R^\alpha q_L^\beta$ | $z^{-1} X^{\alpha\beta}$ | $(\bar{N}_f,N_f)$ | 0 | 3 | $-3$ |

$$S_{\text{meson}} = \int d^5x \sqrt{-g} \text{Tr} \left[ -\frac{1}{2 g_s^2} \left( L_{MN}^2 + R_{MN}^2 \right) + |D_M X|^2 - M_5^2 |X|^2 \right]$$
Bulk scalar field “X”
Source of chiral symmetry breaking

Classical solution

\[ X(z) = Mz + \Sigma z^3 \]

"explicit breaking”
Quark Mass

UV B.C.
\[ \left. \frac{X(z)}{z} \right|_{z=\varepsilon} = M \]

"spontaneous breaking”
Chiral condensate

\[ \langle \bar{q}q \rangle = \left. \frac{\partial}{\partial M} \exp \left[ i \int d^4x M \bar{q}q \right] \right|_{M=0} \]
\[ = \left. \frac{\delta}{\delta M} S_S[X] \right|_{X=X_{cl}, M=0, z=\varepsilon} \sim \Sigma \]
Defining \( V_M = \frac{1}{2} (L_M + R_M) \), \( A_M = \frac{1}{2} (L_M - R_M) \) and adding

\[
\mathcal{L}_{gf} = -\frac{1}{2\xi V g_s^2 z} \left[ \partial_\mu V^\mu - \xi_V z \partial_z \left( \frac{V_z}{z} \right) \right]^2 - \frac{1}{2\xi_A g_s^2 z} \left[ \partial_\mu A^\mu - \xi_A z \partial_z \left( \frac{A_z}{z} \right) + 2\sqrt{2} g_s^2 \xi_A \langle X \rangle^2 P \right]^2
\]

leads to the quadratic Lagrangian in the unitary gauge

\[
\mathcal{L}_{\text{quadratic}} = -\frac{1}{4 g_s^2 z} V_\mu^a \left[ -\eta^{\mu \nu} \partial^2 + \partial_\mu \partial^\nu + \eta^{\mu \nu} z \partial_z \left( \frac{1}{z} \right) \partial_z \right] V_\nu^a \quad X = \langle X \rangle e^{iP}
\]

\[
-\frac{1}{4 g_s^2 z} A_\mu^a \left[ -\eta^{\mu \nu} \partial^2 + \partial_\mu \partial^\nu + \eta^{\mu \nu} z \partial_z \left( \frac{1}{z} \right) \partial_z \right] A_\nu^a + \frac{\langle X \rangle^2}{2z^3} \left( \partial_\mu P^a - A_\mu^a \right)^2
\]

Mode equations and boundary conditions

\[
0 = \left[ m_n^2 + z \partial_z \left( \frac{1}{z} \right) \partial_z \right] f_n^V, \quad f_n^V (\varepsilon) = \partial_z f_n^V (z_m) = 0
\]

\[
0 = \left[ m_n^2 + z \partial_z \left( \frac{1}{z} \right) \partial_z - \frac{2g_s^2 \langle X \rangle^2}{z^2} \right] f_n^A, \quad f_n^A (\varepsilon) = \partial_z f_n^A (z_m) = 0
\]
Meson-Nucleon coupling

“Baryons in AdS/QCD”
Hong, Inami and Yee, PLB646 165 (2007)

“Meson-Nucleon Coupling from AdS/QCD”
N.M. and Motoi Tachibana
arXiv:0904.3816(accepted in EPJC)
Spin $\frac{1}{2}$ Baryon $\Leftrightarrow$ Spin $\frac{1}{2}$ Dirac fermion

Hong, Inami & Yee (2007)

\[
S_{\text{Baryon}} = \int d^5x \sqrt{-g} \left[ i\bar{N}_1 e^M_A \Gamma^A D_M N_1 + i\bar{N}_2 e^M_A \Gamma^A D_M N_2 - \frac{5}{2} \bar{N}_1 N_1 + \frac{5}{2} \bar{N}_2 N_2 \right]
\]

\[
D_\mu = \partial_\mu + \frac{1}{2z} \Gamma_z \Gamma_\mu - iL_\mu, \quad D_z = \partial_\mu - iL_z, \quad m_s^2 = (\Delta - 2)^2 = \left(\frac{9}{2} - 2\right)^2
\]

To incorporate chiral symmetry breaking, the following Yukawa coupling is introduced

\[
S_{\text{Yukawa}} = \int d^5x \sqrt{-g} \left[ -g_Y \bar{N}_1 X N_1 - g_Y \bar{N}_2 X^\dagger N_2 \right]
\]

\[
\begin{pmatrix}
\hat{\partial}_z - \frac{\Delta}{z} \\
-g_Y \langle X \rangle \\
-\frac{4-\Delta}{z}
\end{pmatrix}
\begin{pmatrix}
f^n_{1L} \\
f^n_{2L}
\end{pmatrix} = -m_n \begin{pmatrix}
f^n_{1R} \\
f^n_{2R}
\end{pmatrix},
\begin{pmatrix}
\hat{\partial}_z - \frac{4-\Delta}{z} \\
g_Y \langle X \rangle \\
\hat{\partial}_z - \frac{\Delta}{z}
\end{pmatrix}
\begin{pmatrix}
f^n_{1R} \\
f^n_{2R}
\end{pmatrix} = m_n \begin{pmatrix}
f^n_{1L} \\
f^n_{2L}
\end{pmatrix}
\]

\[
f^n_{1R}(z_m) = f^n_{2L}(z_m) = f^n_{1L}(\varepsilon) = f^n_{2R}(\varepsilon) = 0, \text{ Other B.C. from EOM}
\]
**π NN coupling**

π NN coupling is generated from

1: Gauge coupling (5th component)
2: Yukawa coupling
3: Pauli term (through Goldberger-Treiman relation)

\[ \mathcal{L}_{\pi NN} = \int_{0}^{z_m} dz \sqrt{-g} \left[ \frac{i}{2} \overline{N}_1 \Gamma^z (-iA_z^L) N_1 - \frac{i}{2} (iA_z^L \overline{N}_1) \Gamma^z N_1 + (L \leftrightarrow R, 1 \leftrightarrow 2) \right] \]

\[ + \int_{0}^{z_m} dz \sqrt{-g} \left[ -g_Y \overline{N}_1 X N_1 - g_Y \overline{N}_2 X^\dagger N_2 \right] \]

\[ g_{\pi N^l N^{l'}} = \int_{0}^{z_m} dz \frac{1}{z^4} \left[ f_\pi \left( f_{1L}^l f_{1R}^{l*} - f_{2L}^l f_{2R}^{l*} \right) - \frac{g_Y}{2v(z)zg_5} \partial_z \left( \frac{f_\pi}{z} \right) \left( f_{1L}^l f_{2R}^l - f_{2L}^l f_{1R}^{l*} \right) \right] \]
Vector & Axial-vector Meson-Nucleon coupling

Vector & Axial-vector meson-nucleon couplings come from

1: Gauge coupling
2: Pauli term

\[
\mathcal{L}_{\text{gauge}} = \int_0^{z_m} dz \sqrt{-g} \left[ \frac{i}{2} N_1 e_A^M \Gamma^A \left( -i A_M^L \right) N_1 - \frac{i}{2} \left( i A_M^L \bar{N}_1 \right) e_A^M \Gamma^A N_1 \right. \\
\left. + \frac{i}{2} N_2 e_A^M \Gamma^A \left( -i A_M^R \right) N_2 - \frac{i}{2} \left( i A_M^R \bar{N}_2 \right) e_A^M \Gamma^A N_2 \right] \\
\Rightarrow \int_0^{z_m} dz \frac{1}{z^4} \left[ \bar{N}_1 \gamma^\mu V_\mu N_1 + \bar{N}_2 \gamma^\mu V_\mu N_2 + \bar{N}_1 \gamma^\mu A_\mu N_1 - \bar{N}_2 \gamma^\mu A_\mu N_2 \right]
\]
\[ \mathcal{L}_{\text{Pauli}} = c \int_0^z dz \sqrt{-g} \left[ iN_1 \Gamma^{MN} F_{MN}^L N_1 - iN_2 \Gamma^{MN} F_{MN}^R N_2 \right] \]

\[ = -c \int_0^z dz \frac{1}{Z^3} \left[ \overline{N}_{1L} \gamma^\mu \gamma^5 \left( \partial_\mu V_z - \partial_z V_\mu \right) N_{1L} + \overline{N}_{1R} \gamma^\mu \gamma^5 \left( \partial_\mu V_z - \partial_z V_\mu \right) N_{1R} - (1 \leftrightarrow 2) \right] \]

\[ - c \int_0^z dz \frac{1}{Z^3} \left[ \overline{N}_{1L} \gamma^\mu \gamma^5 \left( \partial_\mu A_z - \partial_z A_\mu \right) N_{1L} + \overline{N}_{1R} \gamma^\mu \gamma^5 \left( \partial_\mu A_z - \partial_z A_\mu \right) N_{1R} + (1 \leftrightarrow 2) \right] \]

\[ g_{V_{N'}^{N''}} \equiv \int_0^z dz \frac{1}{Z^4} \left[ f_n^V + cz \partial_z f_n^V \right] \left[ |f_{1L}^I|^2 + |f_{1R}^I|^2 \right] \]

\[ g_{A_{N'}^{N''}} \equiv \int_0^z dz \frac{1}{Z^4} \left[ f_n^A + cz \partial_z f_n^A \right] \left[ |f_{1L}^I|^2 + |f_{1R}^I|^2 \right] \]

\[ g_{\partial \pi_{N'}^{N''}} \equiv -c \int_0^z dz \frac{1}{Z^3} f_\pi \left[ |f_{1L}^I|^2 + |f_{1R}^I|^2 \right] \Rightarrow g_{\partial \pi NN} = \frac{m_\pi}{2m_N} g_{\pi NN} (GT) \]
### Numerical Results

Data: \( g_\rho_{NN} = 4.2 \sim 6.5, g_\pi_{NN} \sim 13.6 \)

| \( z_m^{-1} \) (GeV) | \( g_Y \) | \( g_\rho_{NN} \) | \( g_{a_1_{NN}} \) | \( g_\pi_{NN} \) |
|---------------------|----------|-----------------|-----------------|-----------------|
| 0.6                 | 26.5 ~ 26.9 | −4.3 ~ −6.2    | −8.2 ~ −10.5   | −20.1 ~ −22.0   |
| 0.7                 | 33.6 ~ 34.0 | −5.1 ~ −6.2    | −10.1 ~ −11.4  | −19.8 ~ −20.7   |
| 0.8                 | 38.6 ~ 40.2 | −4.2 ~ −6.4    | −10.0 ~ −13.1  | −18.8 ~ −20.5   |
| 0.9                 | 42.5 ~ 44.1 | −5.1 ~ −6.5    | −13.0 ~ −15.1  | −19.8 ~ −20.9   |
| 1.0                 | 39.0 ~ 43.8 | −4.2 ~ −6.5    | −13.7 ~ −17.6  | −19.9 ~ −21.7   |

fitted prediction 50% dev.

Parameters: \( z_m, g_Y \) (free parameters), \( m_N=0.94\text{GeV} \)
\( M=2.34\text{MeV}, \Sigma=(311\text{MeV})^3 \rightarrow m_\pi, f_\pi \)
\( g_5=2 \pi \rightarrow \text{Matching to pQCD} \) \( \text{Erlich et al} \ (2005) \)
\( c \rightarrow \text{Nucleon g-2} \) \( \text{Hong, Kim, Siwach & Yee} \ (2007) \)
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Comparison to Skyrmion approach for baryons

Similar results: \( g_{\rho NN} = 5.8, g_{\pi NN} \sim 7.46 \)

Hashimoto, Sakai & Sugimoto (2008)
We have formulated meson-baryon couplings in 5D holographic QCD. Spin $\frac{1}{2}$ baryon $\leftrightarrow$ 5D Dirac fermion. $\pi NN$, $\rho NN$, $a_1NN$ couplings were computed. In particular, $a_1NN$ coupling is a prediction, our model can be tested by measuring this coupling.
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$\pi \text{NN, } \rho \text{NN, } a_1 \text{NN}$ couplings were computed.

In particular, $a_1 \text{NN}$ coupling is a prediction our model can be tested by measuring this coupling.

Ways out to improve the results in our simplified model:

- Deformation of the metric
- Anomalous dimension $\rightarrow$ Bulk mass correction
- Quantum gravity, Stringy effects
Backup Slides
$\rho$ meson mass & its mode function

\[ m_\rho \sim \frac{3}{4} \pi z_m^{-1}, \quad f^\rho (z) \sim \frac{z J_1 (m_\rho z)}{\sqrt{\int_0^{z_m} dz z J_1 (m_\rho z)^2}} \]

$\alpha_1$ meson mass & its mode function

Mode equation cannot be solved analytically due to a $z$-dependent mass

Approximation: brane localized mass@ IR brane

\[ \tan \left( m_{\alpha_1} z_m - \frac{3}{4} \pi \right) \sim \frac{g_5^2 \sigma^2}{2m_{\alpha_1}} z_m^4, \quad f^{\alpha_1} (z) \sim \frac{z J_1 (m_{\alpha_1} z)}{\sqrt{\int_0^{z_m} dz z J_1 (m_{\alpha_1} z)^2}} \]
Pion mode function

Note that in the unitary gauge, if \( z \partial_z \left( \frac{A_z}{z} \right) - 2\sqrt{2} \frac{v^2 g_5^2}{z^2} P = 0 \)

its orthogonal combination of \( A_z \) & \( P \) are massless \( \Rightarrow \) Pion

\[
\mathcal{L}(A_z, P) = \frac{z^4}{2g_5^2} \left( \partial_\mu A_z \right)^2 + \frac{z^8}{8g_5^2 v^2} \left[ \partial_z \left( \frac{\partial_\mu A_z}{z} \right) \right]^2 - \frac{v^2 z^2}{8g_5^4} \left[ \partial_z \left( \frac{z^3}{v^2} \partial_z \left( \frac{A_z}{z} \right) \right) - 4g_5^2 A_z \right]^2
\]

Pion mode function is defined from \( A_z = f_\pi(z) \pi(x) \)
and obtained by

\[
\partial_z \left( \frac{z^3}{v^2} \partial_z \left( \frac{f_\pi}{z} \right) \right) - 4g_5^2 f_\pi = 0, \quad 1 = \int_0^{z_m} dz \left[ \frac{1}{2g_5^2 z} f_\pi^2 + \frac{z^3}{8v^2 g_5^4} \left( \partial_z \left( \frac{f_\pi}{z} \right) \right)^2 \right]
\]