Strings versus supersymmetric GUTs: can they be reconciled?

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Abstract

We describe a class of supersymmetric unified models with the following properties: i) the full breaking of the gauge group is achieved by Higgs fields in the fundamental representation; ii) the correct unification of the strong and electroweak coupling constants is obtained without the need of any intermediate scale; iii) the problems of the doublet-triplet splitting and of the proton decay at dimension-5 level may receive a natural solution. The models, other than being interesting unified field theories per se, may constitute examples of string-derivable GUTs.

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In this note we describe a class of supersymmetric models based on the gauge group $G \otimes G$, with $G \supset SU(5)$, with the following properties:

i) the full breaking of the gauge group is achieved by Higgs fields in the fundamental representation;

ii) the correct unification of the strong and electroweak gauge couplings is obtained without the need of any intermediate scale;

iv) the problems of the doublet-triplet splitting and of the proton decay at dimension-5 level may receive a natural solution.

The models, other than being interesting unified field theories per se, may constitute examples of string-derivable GUTs. We have in mind the difficulty to obtain, in a string theory context, gauge models with adjoint or higher representations in their spectra [1]. Our focus, on the other hand, is on the comparison of standard GUTs [2] versus ununified string models [3] in their prediction of the gauge couplings at low energy. In this respect, ununified string models with conventional $k_i$-factors, although in principle more predictive, are not as successful as standard GUTs, unless large string threshold corrections are invoked [4]. Such corrections seem indeed to be there. The question is however: why should these corrections maintain the relation between the couplings characteristic of the Grand Unified symmetry, if such a symmetry is not actually realized? Needless to say, flipped $SU(5) \otimes U(1)$ [6] does not differ in this respect from $SU(3) \otimes SU(2) \otimes U(1)$ or any ununified group.

For concreteness we discuss an $SU(5)^{'} \otimes SU(5)^{''}$ model. The trivial extension to larger groups, in particular to $SO(10) \otimes SO(10)$ or $SU(6) \otimes SU(6)$, will be briefly described later on. The full breaking of the gauge group to the standard $SU(3) \otimes SU(2) \otimes U(1)$ is achieved by the vacuum expectation values (vev) of one, or more, multiplets

$$Z_{\mu i}^a = (5, \bar{5})_i, \quad Z_{\mu i}^{a'} = (\bar{5}, 5)_i, \quad (1)$$

with $SU(5)^{'}$ and $SU(5)^{''}$ indices, $a'$ and $a''$, in the fundamental respective representations. With a generic superpotential $W(Z_i, \bar{Z}_i)$, it is easy to show that the supersymmetric potential in the scalar components of the superfields $Z_i, \bar{Z}_i$ has a supersymmetric minimum for $Z = \bar{Z}$ of the following possible forms

$$Z_1 = V_1 \cdot \text{diag}(1, 1, 1, 1, 1), \quad (2a)$$
$$Z_2 = V_2 \cdot \text{diag}(0, 0, 0, 1, 1), \quad (2b)$$
$$Z_3 = V_3 \cdot \text{diag}(1, 1, 1, 0), \quad (2c)$$
$$Z_4 = V_4 \cdot \text{diag}(1, 1, 1, x, x), \quad x \neq 1. \quad (2d)$$

In general it is sufficient that the superpotential be dependent at least upon two invariants, like $\text{Tr}(Z \bar{Z})$ and $\text{Tr}(Z \bar{Z} Z \bar{Z})$. Various $Z_i, \bar{Z}_i$ fields may be coupled to each other in the superpotential, as needed to avoid unwanted massless particles, and still have different orientations of their vevs, as in eq.s (2).

These vevs lead respectively to the breakings of the gauge group $SU(5)^{'} \otimes SU(5)^{''}$ down to

$$SU(5) \quad (3a)$$
$$SU(3)^{'} \otimes SU(3)^{''} \otimes SU(2) \otimes U(1) \quad (3b)$$
$$SU(3) \otimes SU(2)^{'} \otimes SU(2)^{''} \otimes U(1) \quad (3c)$$
$$SU(3) \otimes SU(2) \otimes U(1) \quad (3d)$$

1 From the point of view of the present paper, $Z, \bar{Z}$ multiplets transforming as $(5, 5)$ and $(\bar{5}, 5)$, rather than $(5, 5)$ and $(\bar{5}, 5)$, with a consequent change in the representation of the matter multiplets, are equally apt to their purpose. The two cases, however, may not be equivalent from the point of view of a more fundamental theory.
where the unprimed factors correspond to obvious diagonal subgroups.

Any pair of the first three vevs, as (2d) alone, give rise to the usual low energy group. Furthermore, irrespective of the values of the SU(5)' and SU(5)'' couplings, $g'$ and $g''$, the standard gauge couplings $g_3, g_2, g_1$ are unified at a common scale $M$, as in standard SU(5), in any of the following cases (among others):

i) $V_1 \geq V_2 \approx V_3 \approx M$ (no $V_4$);

ii) $V_2 \approx V_3 \approx M$ (no $V_1, V_4$);

iii) $V_4 \approx M$ (no $V_1, V_2, V_3$).

In case i) the theory is actually indistinguishable from simple SU(5) up to the scale $V_1$, which may be arbitrarily high. Notice that in all cases the low energy group lives in the diagonal SU(5). We find this essential to achieve the desired unification of couplings. Had we considered either SU(3) or SU(2) embedded in $G'$ or $G''$, with the symmetric group factor broken at high energy, we would have not obtained the correct boundary condition on the gauge couplings even with $g' = g''$. In turn, this is what prevents the consideration of models fully symmetric under interchange of the simple group factors $G'$ and $G''$, since one does not want a doubling of the light matter particles with symmetric Yukawa coupling.

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The matter multiplets are taken to transform as $5 \oplus 10$ representations of either SU(5) factors. It is in fact possible that different families transform under different SU(5)'s, leading to a possibly interesting asymmetry between them. For simplicity we take here

$$f_i = (\bar{5} \oplus 10, 1)_i \equiv (\bar{5}' \oplus 10')_i, \quad i = 1, 2, 3. \quad (4)$$

From the point of view of the diagonal SU(5), the light Higgs doublets must live in $5 \oplus \bar{5}$ representations. Also in this case, therefore, one has several possible choices for the transformation properties of the corresponding multiplets under the full $SU(5)' \otimes SU(5)''$ group. An interesting possibility, also taking into account the anomaly cancellation requirement, would be to take

$$H' = (5 \oplus 10, 1), \quad \bar{H}' = (1, 5 \oplus \bar{10}).$$

In the following, however, we shall stick to the choice

$$H' = (5, 1), \quad \bar{H}' = (\bar{5}, 1), \quad H'' = (1, 5), \quad \bar{H}'' = (1, \bar{5}).$$

with a doubling $H', \bar{H}' \rightarrow H'', \bar{H}''$ that may or may not be necessary.

The presence of Z-multiplets with the vevs (2b), (2c) suggests simple ways to overcome the difficulties of usual GUTs associated with the doublet-triplet splitting and with the possible dimension-5 operators mediating the proton decay. On one side, the heavy mass for the triplet fields may arise from couplings of the form

$$H' Z_3 \bar{H}'', \quad \bar{H}' Z_3 H''.$$

On the other side, the same triplets may even be decoupled from the light generation if the required Higgs coupling is obtained through the non-renormalizable operators

$$\frac{1}{M} 5' 10' \bar{Z}_2 \bar{H}'', \quad \frac{1}{M} 10' 10' Z_2 H''.$$

At the same time, of course, the wanted couplings will have to be forbidden by appropriate symmetries.

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2The possibility to break a group $G \otimes G$ in an interesting way by means of Higgs multiplets in the fundamental representation is pointed out in ref. 2.

3An interesting possible way out is offered by theories based on the gauge group $G \otimes G \otimes G$.

4A mechanism of this type to suppress the proton decay has been suggested in an SO(10) context in ref. 3.
As a specific example, consider a model where three $Z_i \oplus \bar{Z}_i$ fields are present, $i = 1, 2, 3$, acquiring the vevs (2a), (2b), (2c) respectively. Suppose further that the theory has a $Z_3^{(1)} \otimes Z_3^{(2)}$ discrete symmetry, under which

$$Z_3^{(1)} : \{ \tilde{H}', H'', Z_1 \} \rightarrow e^{2\pi i/3} \{ \tilde{H}', H'', Z_1 \}, \quad \tilde{Z}_1 \rightarrow e^{-2\pi i/3} \tilde{Z}_1$$

$$Z_3^{(2)} : \{ \tilde{H}'', H', Z_3 \} \rightarrow e^{2\pi i/3} \{ \tilde{H}'', H', Z_3 \}, \quad \tilde{Z}_3 \rightarrow e^{-2\pi i/3} \tilde{Z}_3$$

These discrete symmetries, without inhibiting the required terms in $W(Z, \bar{Z})$, restrict the renormalizable couplings between the $H$ and the $Z$-fields in the superpotential to be of the form

$$\tilde{H}'Z_1H'' + H'\tilde{Z}_3H''.$$ 

As a result, one obtains a pair of massless Higgs doublets, only contained in $H', \tilde{H}'$. Even non-renormalizable couplings, if present, can be inhibited up to a sufficiently high level not to disturb the lightness of these doublets. On the other hand, it is easy to extend the discrete symmetries to the matter multiplets, in such a way that the allowed Yukawa couplings be of the form

$$10' \ 10'H' + \bar{5}' \ 10'\bar{Z}_2\tilde{H}''.$$ 

Notice in particular the necessary asymmetry between the dimensionality of the operators responsible for the couplings to the light Higgs doublets of the up-type quarks and of the down-type quarks (or of the charged leptons) respectively.

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The model described here can be trivially extended to the SO(10) $\otimes$ SO(10) gauge group. In this last case, other than the Higgs supermultiplets in the fundamental vector representations

$$Z_i = (10, 10)_i, \quad H' = (10, 1), \quad H'' = (1, 10),$$

one must also have the fundamental spinor representation, e. g.

$$\Psi' = (16, 1), \quad \bar{\Psi}'' = (1, 16)$$

to reduce, as usual, the rank of the group.

In particular, it is easy to see how the discussion of the Higgs superpotential can be adapted to the SO(10) case. Experts will recognize a variant of the Dimopoulos-Wilczek \[10\] mechanism, designed in SO(10) to understand the doublet-triplet splitting, as a natural consequence of the present scheme, which can in fact be trivially applied to any SU($n$) $\otimes$ SU($n$) group.

In this last case, a model which looks to us especially elegant and economical is based on the SU(6) $\otimes$ SU(6) group. Suppose that the Higgs supermultiplets only contain the representations

$$Z = (6, \bar{6}), \quad \bar{Z} = (\bar{6}, 6)$$

$$H = (1, \bar{6}), \quad \bar{H} = (1, 6)$$

and that the Higgs superpotential is of the form

$$W = W^{(1)}(Z, \bar{Z}) + W^{(2)}(H, \bar{H})$$

with no $Z - H$ interaction, at least up to some level. Suppose further that $W^{(1)}$ and $W^{(2)}$ be such that, among their minima, one is obtained for

$$Z = \bar{Z} = V_Z \cdot \text{diag}(1, 1, 1, 1, x, x) \quad x \neq 1$$

$$H = \bar{H} = V_H \cdot (1, 0, 0, 0, 0)^T$$
This model represents a simpler, maybe string-derivable, variant of a model previously discussed \[1\]. The larger symmetry of the Higgs superpotential, with an extra global SU(6) factor, leads, after spontaneous symmetry breaking, to the masslessness of the Higgs doublets whereas the Goldstone triplets are eaten by the simultaneous breaking of the gauge group. It has also been noticed elsewhere \[2\] how this asymmetry between the SU(2) doublets and the SU(3) triplets in \(H, \bar{H}\) may inhibit or prevent at all the proton decay operators mediated by the colour triplets. This comes about because of the absence of any \(F\)-term-type mass for the SU(3) triplets.

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In conclusion, the focus on the successful unification of the gauge couplings in standard GUTs and on the constraints on model building from string theory suggests to consider models based on the group \(G \otimes G\), spontaneously broken by Higgs supermultiplets in the fundamental representation. It is surprising to see how this viewpoint may lead, at the same time, to a simple solution of the classic problems of standard GUTs. We presume that it may be interesting to study possible constraints on these kinds of models from the point of view of the string theory construction. From our side, we plan to concentrate on the fermion mass problem, along lines similar to those explored in ref. \[1\].

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