Research on Turbofan Engine Model above Idle State Based on NARX Modeling Approach

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Abstract. The nonlinear model for turbofan engine above idle state based on NARX is studied. Above all, the data sets for the JT9D engine from existing model are obtained via simulation. Then, a nonlinear modeling scheme based on NARX is proposed and several models with different parameters are built according to the former data sets. Finally, the simulations have been taken to verify the precise and dynamic performance the models, the results show that the NARX model can well reflect the dynamics characteristic of the turbofan engine with high accuracy.

1. Introduction
The modern aircraft engine is a complicated thermomechanical system; therefore, it is a challenging work to modeling it with high accuracy and real time characteristic. In general, there are two basic methods for modeling the system: the theory modeling and the identification modeling \cite{1-2}. Among the existing modeling methods, the nonlinear autoregressive exogenous (NARX) modeling approach can well capture the dynamics of many nonlinear dynamical systems \cite{3-4}. The NARX is a kind of identification technology based on time series \cite{5}, this modeling approach has been employed to the modeling work of gas turbine engine by several researchers. For instance, a nonlinear autoregressive exogenous model of the PGTA10B1 General Electric-Nuovo Pignone gas turbine operating in isolated and non-isolated conditions was reported by M.Basso et al. \cite{6}. The NARX models of a heavy-duty single-shaft gas turbine(GT) were developed and validated by Hamid Asgari et al. \cite{7}. In this article, the modeling of the JT9D engine is chosen as the plant, and the data for modeling are based on the component level turbofan model of JT9D \cite{8}. By refining the identification parameters, several NARX models are built, and the mean errors of the best one are lower than 0.01, which can well reflect the dynamics characteristic of the turbofan engine with high accuracy.

2. The specification of the turbofan engine
The identification modeling of nonlinear system should be based on actual flight or test data sets in practical environment. Due to the restriction of our research work, it hard to get the real experimental data, thus the component level model of a turbofan engine introduced by the NASA is employed as the plant and the data sets are obtained from its simulation. Then, the data sets are applied to develop the
NARX model of the turbofan engine. The main specifications of the turbofan engine employed in this article are illustrated in table 1. The component level model consists eight parts, which are respectively the inlet, fan, low pressure compressor, high pressure compressor, burner, high pressure turbine, low pressure turbine and nozzle. And its input signals are the flight height $H$, Mach number $M_a$ and fuel-air ratio $FAR$, meanwhile, the output signals are low pressure rotation speed $n_L$ and high pressure rotation speed $n_H$.

Table 1. The main specifications of the turbofan engine.

| Take-off thrust (kN) | Air flow rate (kg/s) | Bypass ratio | Total compression ratio | Thrust-weight ratio | High pressure rotation speed (rpm) |
|---------------------|----------------------|--------------|-------------------------|--------------------|----------------------------------|
| 222.4               | 721.2                | 5.1          | 23.5                    | 5.63               | 7800                             |

3. NARX model

The NARX model is based on the extension of the autoregressive exogenous (ARX) model [9-11], and its mathematical structure could be described as equation (1):

$$y(t) = f[y(t - 1), ..., y(t - m_a), u(t - m_k), ..., u(t - m_k - m_b - 1)]$$

(1)

Where $y(t)$ is the output of the NARX model, $u(t)$ is the input of the NARX model, and $f$ is a nonlinear function based on a finite number of input and output data; $m_a$, $m_b$ and $m_k$ are number of past output terms, number of past input terms and delay from input to the output in terms of the number of samples, respectively. The NARX model structure is shown as figure 1:

As presented in figure 1, the NARX model structure is consisted of two stages which are the regressors block and the nonlinearity estimator block. In this article, the wavelet network is used as the nonlinear function in the nonlinearity estimator [12-13].

4. NARX model identification

4.1. The NARX model identification process

Figure 2 presents the NARX model identification process. The input and output data sets obtained through running the turbofan engine component level model are divided into two groups, which are respectively employed to develop and validate the NARX mode.
4.2. The NARX model structure and parameters identification

As shown in figure 1, both the input values and past output values are applied in the regressors block of the NARX model. Therefore, according to the equation (1), the following expressions can be obtained:

\[
n_H(t) = f[n_H(t-1), n_H(t-2), \ldots, n_H(t-n_{a11}), n_L(t-1), n_L(t-2), \ldots, n_L(t-n_{a22}), \text{FAR}(t-n_{k11}), \ldots, \text{FAR}(t-n_{k11}-n_{b11}+1)]
\]

(2)

\[
n_L(t) = f[n_H(t-1), n_H(t-2), \ldots, n_H(t-n_{a21}), n_L(t-1), n_L(t-2), \ldots, n_L(t-n_{a22}), \text{FAR}(t-n_{k21}), \ldots, \text{FAR}(t-n_{k21}-n_{b21}+1)]
\]

(3)

Where \( n_L \) and \( n_H \) are low and high pressure rotation speed of the NARX model, respectively; \( \text{FAR} \) is the fuel-air ratio of the NARX model; \( n_a, n_b \) and \( n_k \) can be expressed as follows:

\[
\begin{bmatrix}
n_{a11} & n_{a12} \\
n_{a21} & n_{a22}
\end{bmatrix}, \quad
\begin{bmatrix}
n_{b11} \\
n_{b21}
\end{bmatrix}, \quad
\begin{bmatrix}
n_{k11} \\
n_{k21}
\end{bmatrix}
\]

(4)

Based on the NARX model structure of the turbofan engine and the limitation of computing ability, \( n_a, n_b \) and \( n_k \) are limited within the range of [1, 5].

4.3. The input and output data sets acquisition

In view of the turbofan engine component level model mentioned above, the input data change scope is set to make the engine work in idling and above in this article. The range of the \( \text{FAR} \) are from 0.008 to 0.023, and the flight height \( H \) is 0m and Mach number \( M_a \) is 0. In the simulation, the integral method based on the limited white noise is applied to simulate the input data sets (that is \( \text{FAR} \)), and the limits on the integral are \( \text{FAR} \) ranges; Then, when \( \text{FAR} \) is imported into the turbofan engine component level model, the output data sets (that are \( n_L \) and \( n_H \)) can be obtained. The acquisition time is 200 seconds, and the step time is 0.04 seconds. The data sets in the first 80 seconds are utilized to develop the NARX model, and the rest of data sets are applied to validate the model.

4.4. The NARX model verification

The NARX model is employed to identify the nonlinear system, which can select different input and output terms and the delay, and then a series of NARX model can be determined. Therefore, the Best Fit as evaluation standard is employed to obtain the best NARX model from the determined model.

The Degree of fitting between the output of the turbofan engine component level model and the output of the determined NARX model is calculated, and the numerical most one has the highest accuracy.
\[ f_{FIT} = \left(1 - \frac{|y - \hat{y}|}{|y - \bar{y}|} \right) \times 100\% \] (5)

Where \( f_{FIT} \) is the degree of fitting, \( y \) is the output value of the turbofan engine component level model, that are low and high pressure rotation speed; \( \hat{y} \) is the output value of the NARX model; \( \bar{y} \) is the mean value of \( y \).

The best NARX model named Best-name is required to meet two factors: (1) Both of the Degree of fitting of high pressure rotation speed \( n_L \) and the Degree of fitting of high pressure rotation speed \( n_H \) are more than 95%; (2) On the basis of meeting (1), the sum of the Degree of fitting of \( n_L \) and \( n_H \) is highest. With the value of \( n_a \), \( n_b \) and \( n_k \) changed and the Degree of fitting of the determined NARX model calculated, the best-model can be obtained. Table 2 shows results of the best-model and three other NARX models.

Table 2. The degree of fitting for the best-model and three other NARX models.

| MODEL    | \( n_a \)     | \( n_b \)     | \( n_k \)     | \( n_H \)FIT\% | \( n_L \)FIT\% |
|----------|---------------|---------------|---------------|----------------|----------------|
| Best-model | [2 1; 2 1]   | [1; 1]        | [1; 1]        | 96.43          | 96.95          |
| M1       | [3 1; 1 1]   | [2; 3]        | [2; 3]        | 80.83          | 89.49          |
| M2       | [4 2; 5 4]   | [5; 4]        | [2; 4]        | 59.52          | 68.50          |
| M3       | [2 2; 2 2]   | [2; 4]        | [3; 4]        | 36.46          | 46.03          |

Figure 3 reveals variations of the rotation speed for the NARX model and component level model when the input data are same. NARX model output curves are consistent with component level model output curves, and the degree of fitting for \( n_L \) and \( n_H \) are 96.43% and 96.95%. The results show that the NARX model can be successfully applied to modeling of the turbofan engine component level model, and it meets the requirements for further research.

For further comparison the variations, the error \( \Delta n \) is defined:

\[ \Delta n = \sqrt{\left(\frac{n_L - n'_L}{n_L}\right)^2 + \left(\frac{n_H - n'_H}{n_H}\right)^2} \] (6)

Where \( n \) and \( n' \) are the output value of component level model and the NARX model at the same time, respectively. And figure 4 shows the errors tendency. And the errors of the Best-model, the mean values of \( \Delta n \) are 0.008, are smallest and being within the acceptable range.
Figure 3. Comparison between component level model and the NARX model.

Figure 4. The errors of the output data between component level and the NARX model.

5. Conclusion
The NARX model structure and its identification process are introduced in the article. The nonlinear modeling scheme based on NARX is proposed and several models with different parameters are built according to the former data sets. Finally, the simulations have been taken to verify the precise and dynamic performance the models, the results show that the NARX model can well reflect the dynamics characteristic of the turbofan engine with high accuracy.

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