Supersymmetric Aether

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Abstract

It has been suggested by Groot Nibbelink and Pospelov that Lorentz invariance can be an emergent symmetry of low-energy physics provided the theory enjoys a non-relativistic version of supersymmetry. We construct a model that realizes the latter symmetry dynamically: it breaks Lorentz invariance but leaves the supersymmetry generators intact. The model is a supersymmetric extension of the dynamical aether theory of Jacobson and Mattingly. It exhibits rich dynamics and possesses a family of inequivalent vacua realizing different symmetry breaking patterns. In particular, we find stable vacua that break spontaneously spatial isotropy. Supersymmetry breaking terms give masses to fermionic and bosonic partners of the aether field. We comment on the coupling of the model to supergravity and on the implications for Hořava gravity.

1 Introduction

Recently the idea that Lorentz invariance (LI) may not be a fundamental symmetry of nature has gained considerable attention. Indeed, violation of LI is present in one form or another in many theories of gravity devised to solve the problems of Einstein’s general relativity (GR). This is the case of the infrared modifications of gravity, which attempt to address unresolved problems of cosmology such as the nature of dark matter and dark energy. Examples are the ghost condensate [1], Lorentz violating massive gravity [2], the Galileons [3] and kinetic gravity braiding [4]. It has been shown [5] that violation of LI allows a simple and technically
natural explanation of the dark energy. Also breaking of LI can be a consequence of quantum
gravity. In particular, in the approach to quantum gravity recently proposed by P. Hořava
[6] (see also [7, 8, 9, 10]) one manages to construct a potentially renormalizable theory of
quantum gravity at the price of abandoning LI at very high energies.

Of course, LI is one of the best tested symmetries experimentally [11, 12, 13] and one
may wonder how this can be reconciled with the ideas mentioned above. In other words, in
order to be viable, any Lorentz breaking model must incorporate a mechanism that ensures
the emergence of LI to a very high accuracy, at least within the Standard Model (SM) of
particle physics at relatively low energies.

In [14, 15] it was suggested that such mechanism can be provided by supersymmetry
(SUSY). Namely, consider the standard SUSY algebra and remove the boosts from it [16, 17]
1. The remaining generators still form a closed algebra that we will call non-relativistic SUSY
(or, where it will not lead to confusion, just SUSY for short). As will be discussed below, this
is actually the most general minimal superalgebra containing spatial rotations and space-time
translations. A theory invariant under the non-relativistic SUSY is in general not Lorentz
invariant: it is possible to explicitly construct supersymmetric terms in the Lagrangian that
violate LI. However, it turns out that with the field content of the Minimal Supersymmetric
Extension of the SM (MSSM) all these Lorentz violating operators are of dimensions five or
higher2. In other words, LI emerges as an accidental symmetry at the renormalizable level.
The eventual breaking of SUSY introduces Lorentz violation also at this level, but the effect
is within the existing bounds, provided the masses of the MSSM superpartners lie sufficiently
below the Lorentz violation scale [14, 15].

The analysis of [14, 15] is performed in the flat space-time where violation of LI cor-
responds to the existence of a globally defined preferred frame. However, when gravity is
taken into account the space-time becomes dynamical and it is clear that the preferred frame
must become dynamical as well. Thus to implement the mechanism of [14, 15] in gravita-
tional models with Lorentz violation it is necessary to construct SUSY extensions of theories
describing the preferred frame dynamics.

In the present paper we consider a representative theory of this class – the Einstein-aether
model [18, 19]. In this model the preferred frame is defined by a vector field $u^m$ (aether)

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1Refs. [14, 15, 16, 17] further restrict the SUSY algebra stripping off from it the spatial rotations as well. Here, we prefer to keep the invariance under rotations explicit and thus retain the corresponding generators in the symmetry algebra.

2Dimension five operators can be forbidden by imposing additionally CPT invariance, leaving dimension six terms as the lowest Lorentz violating contributions.
which is constrained to be time-like and have unit norm. We present a supersymmetric extension of the model and analyze its consequences. We restrict to the case of global SUSY deferring the coupling of the model to supergravity (SUGRA) for the future. This is a good approximation when the energy scale characterizing Lorentz violation is small compared to the Planck mass. Interestingly enough, we will find that the fluctuations of the aether field around its vacuum expectation value (VEV) also exhibit LI at the level of low-dimension operators. This extends the mechanism of [14, 15] to the supersymmetric aether sector.

The construction we present in this paper allows to realize the non-relativistic SUSY as the residual symmetry left over by the aether VEV in an otherwise super-Poincaré invariant setup. In this language the mechanism of [14, 15] can be formulated as follows: so long as the aether VEV preserves SUSY, the Lorentz breaking does not propagate into the MSSM sector at the renormalizable level. The eventual breaking of SUSY, that must be incorporated in any realistic model, is unrelated to the dynamics of the aether. It is assumed to come from a different source characterized by a lower energy scale. It is worth mentioning two interesting recent papers [20, 21] where a general recipe was given to construct supersymmetric versions of a wide class of theories including Lorentz breaking models such as ghost condensation. However, in the models constructed in this way, the Lorentz violating backgrounds inevitably break SUSY at the same time. This results in the absence of a hierarchy of scales between SUSY and Lorentz breaking and so the LI of MSSM is not protected in these models. Our approach differs in spirit. Our main goal is to construct a Lorentz violating theory with unbroken SUSY and study what kind of restriction this requirement imposes on the structure of the theory.

The paper is organized as follows. In Sec. 2 we describe the Einstein-aether model. In Sec. 3 we introduce the non-relativistic SUSY algebra and briefly review the arguments of [14, 15]. The SUSY extension of the aether model is constructed in Sec. 4. In Sec. 5 we analyze the effects of SUSY breaking on the super-aether. We conclude in Sec. 6 by discussing our results and future directions.

Throughout the paper we follow the conventions and notations of [22]. In particular, the metric signature is (−, +, +, +); Lorentz indices are denoted by the letters from the middle of the latin alphabet, \( m, n, \cdots = 0, 1, 2, 3 \); letters from the beginning of the alphabet, \( a, b, \ldots \), are used for purely spatial indices. We work with two-component spinors, the spinor indices are denoted by Greek letters with or without an overdot depending on the chirality. The spinor indices are raised and lowered using the two-index antisymmetric tensor \( \varepsilon^{\alpha \beta} \):
\[ \psi^\alpha = \varepsilon^{\alpha\beta} \psi_\beta, \quad \bar{\psi}_\alpha = \varepsilon_{\alpha\beta} \bar{\psi}^\beta, \quad \text{where} \quad \varepsilon^{12} = \varepsilon_{21} = 1. \] We will often omit the spinor indices adopting the following convention for the multiplication of spinors: \( \psi \eta \equiv \psi^{\alpha} \eta_\alpha, \quad \bar{\psi} \bar{\eta} \equiv \bar{\psi}_\alpha \bar{\eta}^\alpha. \)

## 2 Review of Einstein-aether

The action of the Einstein-aether theory has the form [18, 19]

\[ S = S_{GR} + S_\alpha, \quad (1) \]

where

\[ S_{GR} = -\frac{M_p^2}{2} \int \sqrt{-g} \, R \, d^4x \quad (2) \]

is the standard Einstein-Hilbert action for gravity and the aether action is given by

\[
S_\alpha = -\frac{M_\alpha^2}{2} \int \sqrt{-g} \left( c_1 \nabla_m u^n \nabla^m u^n + c_2 (\nabla_m u^m)^2 + c_3 \nabla_n u_m \nabla^m u^n - c_4 u^s \nabla_r u_m \nabla^s u^m + \lambda (u_m u^m + 1) \right) d^4x. \quad (3)
\]

Here \( u^m \) is a dynamical vector field and \( \lambda \) is a Lagrange multiplier that enforces \( u^m \) to be time-like with unit norm,

\[ u_m u^m = -1. \quad (4) \]

The dimensionful parameter \( M_\alpha \) sets the scale of violation of LI; the dimensionless constants \( c_1, \ldots, c_4 \) are the remaining free parameters of the theory. Eq. (3) is the most general action for a vector with fixed norm containing up to two derivatives.

Two comments are in order. The third term in (3) can be cast after integration by parts into the same form as the second term plus a non-minimal coupling of the vector to gravity,

\[
c_3 \nabla_n u_m \nabla^m u^n \sim c_3 (\nabla_m u^m)^2 + c_3 R_{mn} u^m u^n, \quad (5)
\]

where \( R_{mn} \) is the Ricci tensor. Next, the energy-momentum tensor of the aether is proportional to \( M_\alpha \). Thus if the scale of Lorentz breaking is much lower than the Planck scale, \( M_\alpha \ll M_{pl} \), the effect of the aether on gravity is small. In this regime it is self-consistent to

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\(^3\text{This parameterization is redundant: the change of } M_\alpha \text{ can be absorbed into redefinition of } c_i \text{'s. One could fix the ambiguity by choosing } M_\alpha = M_{pl} \text{ as in [19]. Then the scale of Lorentz violation would be set by the product } M_{pl} c_i. \text{ However, we find convenient to keep the scale of Lorentz breaking as an explicit parameter while assuming the constants } c_i \text{ to be of order one.}\)
consider the aether in external metric, neglecting its back-reaction on the geometry. In this paper we concentrate on the aether theory in flat space-time.

The ground state corresponds to a homogeneous configuration of the aether which by a Lorentz transformation can always be cast into the form

\[ u^m_{\text{vac}} = (1, 0, 0, 0) \.
\]

This configuration breaks LI down to the \( SO(3) \) subgroup of spatial rotations. The model describes three propagating degrees of freedom: one helicity-0 and two helicity-1 modes. These have linear dispersion relations

\[ \omega = s_{(I)}|k| \]

with the velocities\(^4\) [19]

\[ s^2_{(0)} = \frac{c_1 + c_2 + c_3}{c_1 + c_4} , \quad s^2_{(1)} = \frac{c_1}{c_1 + c_4} . \]

The difference in the propagation speeds of the modes with different helicities clearly manifests the breakdown of LI.

Generally we can consider coupling the aether to other sectors of the theory including SM. This will lead to a strong violation of LI in these sectors. Let us illustrate this point on an example of a scalar field \( \phi \), that we take as a toy model for the SM sector. Possible interactions of \( \phi \) with the aether include a dimension-four operator

\[ \frac{\kappa}{2} u^m u^n \partial_m \phi \partial_n \phi . \]

In the background (6) this will modify the dispersion relations of the \( \phi \)-particles,

\[ \omega^2 = s^2 k^2 + m^2 , \]

where now the maximum propagation velocity \( s \) is not equal to one,

\[ s^2 = \frac{1}{1 + \kappa} . \]

For a single field this modification can be absorbed by redefinition of the units of time or space. However, the real problem arises when we consider several particle species. In general, their interactions with the aether will involve different coupling constants leading to different maximal propagation velocities. Non-observation of such differences within SM leads to very tight constraints on the couplings of the form (9) [13]. As we now discuss, the unwanted couplings can be forbidden by SUSY.

\(^4\)To obtain (8) from the expressions of [19] one should take the limit \( c_i \ll 1 \) corresponding to the case when back-reaction of the aether on gravity can be neglected.
3 Lorentz invariance from non-relativistic SUSY

Consider the non-relativistic symmetry algebra consisting of the generators of 3-dimensional translations $P_a$ and rotations $J_a$, $a = 1, 2, 3$, supplemented by the time translation $P_0$. Its minimal supersymmetric extension contains two supercharges $Q_\alpha$, $\alpha = 1, 2$, transforming in the spinor representation of the $SO(3)$ group of rotations, and their Hermitian conjugates $\bar{Q}^\alpha$. The most general commutation relations compatible with the $SO(3)$ symmetry and Jacobi identities are\(^5\),

\[
\begin{align*}
\{Q_\alpha, Q_\beta\} &= 2A\sigma^a_{\alpha\beta}P_a, \\
\{\bar{Q}^\alpha, \bar{Q}^\beta\} &= -2A^* (\sigma^a)^{\alpha\beta}P_a, \\
\{Q_\alpha, \bar{Q}^\beta\} &= 2B(\sigma^a)^\beta_\alpha P_a - 2C\delta^\beta_\alpha P_0, \\
\{P_a, Q_\alpha\} = [P_a, \bar{Q}^\alpha] = [P_0, Q_\alpha] = [P_0, \bar{Q}^\alpha] &= 0.
\end{align*}
\]

Here $A$ is a complex constant, $B$ and $C$ are real, and $(\sigma^a)^\beta_\alpha$ are the Pauli matrices\(^6\). The commutators involving the angular momenta $J_a$ have the standard form dictated by the $SO(3)$ symmetry and we do not write them explicitly. Note that by choosing the dimension of the supercharges to be (length)$^{-1/2}$ the constants $A$ and $B$ are made dimensionless. Then $C$ has the dimension of velocity. Existence of this universal velocity encoded in the SUSY algebra allows to connect units of space and time and, as we are going to see, eventually leads to the emergence of LI at low energies. We will refer to the algebra (12) as “non-relativistic SUSY”.

We now show that it is equivalent to the standard 4-dimensional super-Poincaré algebra with the boosts stripped off. Indeed, it is straightforward to check that by an appropriate linear transformation of the supercharges

\[Q_\alpha \mapsto \tilde{Q}_\alpha = a_1 Q_\alpha + a_2 \varepsilon_{\alpha\beta} \bar{Q}^\beta\]

the coefficient $A$ can be set to zero. Then, assuming the generic case when both $B$ and $C$ are non-zero, we can set $B = C = 1$ by rescaling the generators $P_a$, $P_0$ (i.e., choosing

\(^5\)We consider the case of non-zero $C$ in (12c). If $C = 0$ there is an extra solution with

\[[P_0, Q_\alpha] = C_1 Q_\alpha + C_2 \varepsilon_{\alpha\beta} \bar{Q}^\beta,
\]

where $C_1$ is real and $C_2 = -C_1 A/B$.

\(^6\)We limit our consideration to proper algebras, in the sense that the r.h.s. of the commutation relations involves the generators only linearly. As was pointed out in [23, 24], one could consider more general constructions allowing the coefficients in (12) to depend on the Casimir operators such as $P_a P_a$ and $P_0$. We do not pursue this route in the present paper.
appropriately the units of length and time). Finally, redefining
\[ \bar{Q}^\alpha \mapsto \bar{Q}_\alpha \]  
(14)
one obtains

\[
\{ Q_\alpha, \bar{Q}_\beta \} = 2 \sigma^m_{\alpha\beta} P_m , \\
\{ Q_\alpha, Q_\beta \} = \{ \bar{Q}_\dot{\alpha}, \bar{Q}_\dot{\beta} \} = [P_m, Q_\alpha] = [P_m, \bar{Q}_\dot{\alpha}] = 0 ,
\]
(15a)

where \( \sigma^0_{\alpha\beta} \equiv -\delta^\beta_\alpha \). This is nothing but the commutation relations of 4d SUSY.

We now briefly review the argument of \cite{14, 15} showing that the symmetry (15) leads
to LI at the renormalizable level of MSSM. As in the relativistic case, the representations
of the algebra (15) are classified in terms of the superfields: indeed, the boost generators
never appear in the superspace construction. The MSSM Lagrangian is constructed out of
a number of chiral matter \( \Phi(I) \) and real gauge \( V(J) \) superfields. It involves integration over
the Grassmann variables \( \theta_\alpha \): \( \int d^2\theta \) in the superpotential term and \( \int d^2\theta d^2\bar{\theta} \) in the kinetic
(Kähler) part. Table 1 summarizes the mass dimensions of the objects that can appear in
the Lagrangian.

| Object                        | Dimension |
|-------------------------------|-----------|
| matter field \( \Phi(I) \)    | 1         |
| gauge field \( V(J) \)        | 0         |
| gauge field strength \( W(J) \alpha \) | 3/2 |
| space-time derivative \( \partial_m \) | 1       |
| supercovariant derivatives \( D_\alpha, \bar{D}_{\dot{\alpha}} \) | 1/2 |
| chiral measure \( \int d^2\theta \) | 1       |
| super-space measure \( \int d^2\theta d^2\bar{\theta} \) | 2       |
| Lorentz breaking VEV \( u^m \) | 0         |

Table 1: The mass dimensions of various objects entering the MSSM Lagrangian.

Let us try to construct from these ingredients a Lorentz violating contribution to the
Lagrangian. This contribution must be invariant under the remaining symmetries of the
theory such as gauge invariance. It is convenient to adopt a formally Lorentz covariant
description using the vector \( u^m \) with a Lorentz breaking VEV (6) as a compensator\(^7\). Then a

\(^7\)In Sec 4.2 we will see that the SUSY aether can provide richer patterns of Lorentz breaking. For the
sake of the argument, we concentrate here on the simplest pattern corresponding to the VEV (6).
Lorentz violating term in the Lagrangian will contain a number of Lorentz indices contracted with $u^m$. The superfields $\Phi(I)$, $V(J)$ are Lorentz scalars. Thus the Lorentz violating term must involve at least one derivative or the gauge field strength $W(J)\alpha$. By inspection, with the account of Table 1, one finds that there are no Lorentz violating terms of dimensions 2 or 3. The only allowed term of dimension 4, that is not a total derivative, has the form

$$\mathcal{L}_{LV}^{(4)} = \kappa_{IJ} u^m \int d^2\theta \Phi(I) \partial_m \Phi(J) + \text{h.c.},$$

(16)

where $I \neq J$. However, in MSSM this term is forbidden by the gauge invariance. Indeed, all fields of MSSM are charged under the gauge group and the plain derivative in (16) is not gauge invariant$^8$. One could try to remedy this by introducing the gauge-covariant derivative

$$\mathcal{D}_m \Phi = -\frac{i}{4} \bar{\sigma}^a_m \bar{D}^a e^{-V} D_\alpha e^V \Phi.$$  

(17)

However, this expression is no longer chiral,

$$\bar{D}_\beta \mathcal{D}_m \Phi \neq 0,$$

and cannot be used in the superpotential term. Thus one concludes that in MSSM it is impossible to write any Lorentz violating operator of dimension 4 or less.

To avoid confusion let us stress that the above argument does not imply complete restoration of LI. One can easily construct higher dimension operators that do violate LI. An example is the dimension 6 contribution

$$\mathcal{L}_{LV}^{(6)} = \frac{u^m u^n}{M_{LV}^2} \int d^2\theta d^2\bar{\theta} \mathcal{D}_m \bar{\Phi} \mathcal{D}_n \Phi$$  

(18)

in the kinetic part. Here $M_{LV}$ is the scale of Lorentz violation. Due to the eventual SUSY breaking the Lorentz violation from higher-order operators feeds into the lower-dimensional ones [14, 15]. In particular, the operator (18) gives rise to the dimension 4 terms of the form (9). However, the coefficients in front of these terms are suppressed by the ratio $m_{SUSY}^2 / M_{LV}^2$, where $m_{SUSY}$ is the scale of SUSY breaking in MSSM (the scale of the superpartner masses). This ratio can be small enough to satisfy the experimental bounds on Lorentz violation.

$^8$Note that terms of the form (16) do appear if we include into consideration sterile neutrinos that are gauge singlets.
4 Super-aether

4.1 The Lagrangian

We now turn to the construction of the supersymmetric aether theory. We impose the requirement that this theory must be compatible with the mechanism for emergence of LI reviewed in the previous section. This determines the choice of the SUSY multiplet to embed the aether vector \( u^m \). Indeed, for the argument of the previous section to hold it is necessary that a constant VEV of \( u^m \) preserves SUSY. This implies that \( u^m \) must be the lowest component of the multiplet. The simplest multiplet with these properties corresponds to a chiral superfield

\[
\bar{D}_\alpha U^m = 0 .
\] (19)

This superfield is a Lorentz vector. In components we write

\[
U^m = u^\mu(y) + \sqrt{2}\theta^\alpha \eta^m_\alpha(y) + \theta^2 G^m(y) ,
\] (20)

where \( \theta^2 \equiv \theta^\alpha \theta_\alpha \) and \( y^m = x^m + i\theta \sigma^m \bar{\theta} \). Note that the aether vector \( u^m \) is now allowed to be complex,

\[
u^m = u^m_R + iu^m_I ,
\] (21)

with \( u^m_R, u^m_I \) real. The vector-spinor field \( \eta^m_\alpha \) in (20) is the aether superpartner and \( G^m \) is the auxiliary component. By analogy with Eq. (4) we impose the constraint

\[
U^m U_m = -1 .
\] (22)

Note that \( U^m \) has vanishing mass dimension.

Following the ordinary aether theory as the guideline we look for the super aether Lagrangian in the form

\[
\mathcal{L} = M^2 a \tilde{\mathcal{L}} .
\] (23)

In other words, we factor out the scale of Lorentz breaking from the Lagrangian. As long as we are interested in the low-energy physics we can restrict to the operators of dimensions up to 2 in \( \tilde{\mathcal{L}} \). We also assume that the aether field is the only source of Lorentz violation, and thus the Lagrangian must be a Lorentz scalar. With these restrictions the only possible form of the kinetic term is a function of the dimensionless combination \( U^m \bar{U}_m \) because the superspace integration already contributes dimension 2, see Table 1. Turning to the superpotential part,

\footnote{The alternative choice of the constraint \( U^m \bar{U}_m = -1 \) does not lead to a consistent theory.}
all terms without derivatives are trivial due to the constraint (22). Moreover, terms with one space-time derivative vanish due to the identity

\[ U^m \partial_n U_m = 0 \]

that is a consequence of (22). Finally, the terms with more derivatives are of higher dimensions. One concludes that at the level of operators of dimensions less or equal 2 the superpotential vanishes. In this way we arrive at the most general super-aether Lagrangian

\[ \mathcal{L}^{(2)} = M_2^2 \left[ \int d^2 \theta d^2 \bar{\theta} \ f(U^m \bar{U}_m) + \left( \int d^2 \theta \ \Lambda(U^m U_m + 1) + \text{h.c.} \right) \right], \quad (24) \]

where \( f \) is an arbitrary function and we have implemented the constraint (22) by a super-potential term with a Lagrange multiplier chiral superfield \( \Lambda \).

One makes an important observation. Besides the usual LI that acts on the coordinates and the indices of the field \( U^m \), the Lagrangian (24) possesses an extra \( SO(3,1) \) invariance that acts only on the \( U^m \) indices. In fact, if one treats the index ‘\( m \)’ as internal, one immediately recognizes in (24) the Lagrangian of the standard supersymmetric non-linear sigma-model with internal Lorentz group. Thus one concludes that at the level of the lower-dimension operators the super-aether is equivalent to a Lorentzian sigma-model with Kähler potential \( f(U^m \bar{U}_m) \). The symmetries of (24) contain the product

\[ \text{Lorentz invariance} \times \text{internal} \ SO(3,1). \]

Even when the vector \( u^m \) acquires non-zero VEV the diagonal subgroup of this product remains unbroken. This means that the theory for perturbations around the ground state effectively continues to be Lorentz invariant. In particular, as we will see explicitly below, all perturbations have the same dispersion relation. Thus LI emerges as an accidental symmetry due to SUSY even in the Lorentz violating sector! Let us emphasize that this is happening despite that \( U_m \) is neutral with respect to all (internal) gauge symmetries. Rather, the emergence of LI in this sector appears as a consequence of the unit-norm condition.

Let us stress again that this does not mean that LI is completely restored. Once we go beyond the low-energy theory it is easy to write higher-dimensional contributions into the super-aether Lagrangian where \( U^m \) is contracted with space-time derivatives, such as the operators

\[ \int d^2 \theta \ (\partial_m U^m)^2 + \text{h.c} , \quad \int d^2 \theta d^2 \bar{\theta} \ \bar{U}^m U^m \partial_m \bar{U}_l \partial_n U^l \]

multiplied by the appropriate dimensionful coupling constants. This removes the accidental internal \( SO(3,1) \) symmetry and thus leads to a genuine breaking of LI by the VEV of \( u^m \).
Alternatively we can consider coupling of the super-aether to another sector of the theory. If this sector contains (at least two) chiral gauge singlets, it is possible to construct a dimension 4 coupling

\[ L_{LV}^{(4)} = \kappa_{(IJ)} \int d^2 \theta \ U'^m \Phi_{(I)} \partial_m \Phi_{(J)} + \text{h.c.} \]  

(25)

that reduces to the Lorentz violating operator (16) in the constant aether background. If gauge singlets are absent, as in the case of MSSM, there are always higher-dimensional couplings such as, e.g.,

\[ L_{LV}^{(6)} = M_{LV}^{-2} \int d^2 \theta d^2 \bar{\theta} \ U'^m U'^m \bar{D}_m \bar{D}_n \Phi . \]  

(26)

This is the generalization of (18) to the case of dynamical aether. Finally, the internal \( SO(3,1) \) will be broken by gravity that mixes the space-time and vector indices through covariant derivatives.

It is instructive to write the Lagrangian (24) in components. Using (20) and a similar decomposition

\[ \Lambda = l(y) + \sqrt{2} \theta \xi(y) + \theta^2 H(y) , \]  

(27)

after integration over the anticommuting variables we obtain,

\[ \mathcal{L} = \mathcal{L}_{bos} + \mathcal{L}_{ferm} , \]  

(28)

where the bosonic and fermionic parts are

\[ \mathcal{L}_{bos} = M_\infty^2 \left[ -f'^m \partial_r \bar{u}_m \partial^r u_n + f^m \tilde{G}_m G_n + \left[ H(u_m u^m + 1) + 2l u_m G^m + \text{h.c.} \right] \right] , \]  

(29)

\[ \mathcal{L}_{ferm} = \frac{M_\infty^2}{2} \left[ i f'^{mr} \partial_m \bar{\eta}_n \bar{\sigma}^m \eta_r 
\right. 
\left. + f'' \left( -i \bar{u}_m \partial_r u_n \bar{\eta}^m \bar{\sigma}^r \eta^n - i \bar{u}_m \partial_r u^r \bar{\eta}_n \bar{\sigma}^m \eta^n - u_m G_n \bar{\eta}^m \eta^n + \frac{1}{2} (\bar{\eta}_m \eta_n) (\eta^m \eta^n) \right) 
\right. 
\left. + f''' \left( -i u_m \bar{u}_n \bar{u}_r \partial_s u^r \bar{\eta}^m \bar{\sigma}^s \eta^n - u_m u_n \bar{u}_s G^s \bar{\eta}^m \eta^n + u_m \bar{u}_n (\bar{\eta}^m \eta_r) (\eta^n \eta^r) \right) 
\right. 
\left. + \frac{1}{4} f'''' u_m u_n \bar{u}_s \bar{u}_r (\bar{\eta}^m \eta^n)(\eta^s \eta^r) - l \bar{\eta}_m \eta^m - 2 \xi \eta_m u^m + \text{h.c.} \right] . \]  

(30)

Here the “effective metric” is

\[ f^{mn} = f'(w) \eta^{mn} + f''(w) u^m \bar{u}^n , \]  

(31)

and primes denote derivatives of the function \( f \) with respect to its argument

\[ w \equiv u^m \bar{u}_m . \]  

(32)
Let us set the fermionic components to zero and concentrate on the bosonic part (29). The equations of motion for the fields $G^m$, $l$ imply that these fields vanish, i.e. they are non-dynamical. The field $H$ is a Lagrange multiplier that enforces the unit-norm constraint on the aether $u^m$. For purely real aether Eq. (29) reduces to the standard aether Lagrangian (3) with
\[ c_1 = 2f'(-1), \quad c_2 + c_3 = c_4 = 0. \] (33)
Due to the relation (5) considerations in flat space-time do not allow to determine the coefficients $c_2$, $c_3$ separately, but only their sum.

4.2 Vacua and fluctuations

We now classify the ground states of the model in the general case of complex aether. The unit-norm constraint implies two equations for the real and imaginary parts of $u^m$,
\[ u_R^m u_{Rm} - u_I^m u_{Im} = -1, \] (34a)
\[ u_R^m u_{1m} = 0. \] (34b)
Consider first the case when the vector $u_R^m$ is time-like. By a suitable Lorentz rotation it can be aligned with the time axis. Then due to (34b) $u_I^m$ has only spatial components and by a 3d rotation we direct it along the third axis. Taking into account (34a) we obtain a family of ground states,
\[ u^m_{vac} = (\cos \alpha, 0, 0, i \sin \alpha), \] (35)
parameterized by $\alpha \in [0, \pi/2]$. Note that for $\alpha \neq 0$ the ground state, besides breaking boosts, breaks also the spatial isotropy. Indeed, for $\alpha \neq 0, \pi/2$ the group of invariance of (35) reduces to the $SO(2)$ rotations in the $(1,2)$-plane. For $\alpha = \pi/2$ the unbroken group is enhanced up to $SO(2,1)$. Only for $\alpha = 0$ one recovers the 3-dimensional $SO(3)$ rotations. Thus we conclude that SUSY has introduced a qualitatively new feature into the aether model: existence of a family of inequivalent vacua, generally breaking spatial isotropy together with LI. Let us stress, however, that breaking of rotational invariance has a very different physical meaning from the breaking of boosts. In a sense, the boost invariance is broken *explicitly*. Indeed, there is no state in the theory where it would be recovered\(^{10}\). On the other hand, breaking

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\(^{10}\)One may think that the boost invariance can be at least partially restored, as it appears to happen in the vacuum (35) at $\alpha = \pi/2$ and in the vacua (36). However, we are going to see that these vacua contain a ghost in the spectrum of perturbations and thus are dynamically inaccessible.
of the rotation group is truly *spontaneous*: the symmetry breaking configurations smoothly connect to the vacuum with $\alpha = 0$ where the full $SO(3)$ group is restored.

Apart from (35) the constraints (34) possess a branch of solutions when both $u^m_R$ and $u^m_1$ are space-like. Due to the orthogonality condition (34b) we can direct these vectors along the second and third axes. Then Eq. (34a) yields

$$u^m_{\text{vac}} = (0, 0, \text{sh} \beta, i \text{ch} \beta),$$  

(36)

where $\beta > 0$. These vacua leave unbroken the subgroup $SO(1,1)$ of the Lorentz group corresponding to the boosts in the $(0,1)$-plane.

To get more insight into the dynamics let us study small perturbations around the vacua. We start with the background (35). Writing

$$u^m = u^m_{\text{vac}} + v^m,$$  

(37)

one finds that the unit-norm constraint relates the components $v_0$ and $v_3$. It is convenient to express them in terms of a single field $v_\parallel$,

$$v_0 = -iv_\parallel \sin \alpha, \quad v_3 = v_\parallel \cos \alpha.$$  

Substituting this into (29) we obtain the quadratic Lagrangian,

$$\mathcal{L}_{\text{bos}} = -M^2 \left[ ( -wf' + (1 - w^2)f'') \partial_m v_\parallel \partial^m \bar{v}_\parallel + f' \sum_{a=1,2} \partial_m v_a \partial^m \bar{v}_a \right],$$  

(38)

where

$$w = -\cos 2\alpha.$$  

(39)

Note that this Lagrangian is effectively Lorentz invariant if we treat the perturbation components $v_1, v_2, v_\parallel$ as scalars. In particular, all modes have the same linear dispersion relation with unit velocity. This is the manifestation of the emergent LI discussed above. On the other hand, the symmetry between $v_\parallel$ and $v_1, v_2$ is clearly broken in (38). The Lagrangian is free of ghosts or any other pathologies as long as

$$f' > 0, \quad -wf' + (1 - w^2)f'' > 0.$$  

Therefore, the model allows breaking of spatial isotropy (in the vacuum with $\alpha \neq 0$) and still possesses completely stable spectrum of perturbations. Note that this situation cannot be realized in the real-aether model of Sec. 2. In that case considering space-like aether
corresponds to switching the sign in the constraint (4). Then it is straightforward to check that the spectrum of perturbations contains a ghost. Instead, for the super-aether model changing the sign in the constraint (22) does not affect the physics: it simply corresponds to multiplying \( U^m \) by \( i \). We have seen that the model is stable so long as one of the components of the vector – its real or imaginary part – is time-like.

In the purely spatial background (36) the situation is different and, actually, is analogous to the case of spatial real aether. Here the unit-norm constraint relates \( v_2 \) and \( v_3 \), and the quadratic Lagrangian reads

\[
L_{bos} = -M_\pi^2 \left[ -f' \partial_m v_0 \partial^m \bar{v}_0 + f' \partial_m v_1 \partial^m \bar{v}_1 + (wf' + (w^2 - 1)f'') \partial_m v_\parallel \partial^m \bar{v}_\parallel \right],
\]

where now \( w = \text{ch} \ 2\beta \) and \( v_\parallel \) is introduced as

\[
v_2 = -iv_\parallel \text{ch} \beta, \quad v_3 = v_\parallel \text{sh} \beta.
\]

Clearly, one of the modes is always a ghost. Thus the purely space-like aether background is pathological and we do not consider it in what follows.

Finally we analyze the fermionic content of the model. The spinor field \( \xi \) in the last line of (30) is a Lagrange multiplier that enforces the constraint

\[
u^m \eta_{m\gamma} = 0.
\]

Solving this constraint in the background (35),

\[
\eta_0\gamma = -i\eta_\parallel\gamma \sin \alpha, \quad \eta_3\gamma = \eta_\parallel\gamma \cos \alpha,
\]

we obtain

\[
L_{\text{ferm}} = \frac{iM_\pi^2}{2} \left[ -wf' + (1 - w^2)f'' \right] \partial_m \bar{\eta}_\parallel \sigma^m \eta_\parallel + f' \sum_{a=1,2} \partial_m \bar{\eta}_a \sigma^m \eta_a + \text{h.c.} \right] + \ldots,
\]

where \( w \) is given by (39) and dots stand for the terms with higher powers of the fermionic fields. This Lagrangian describes three fermions in the Weyl representation with respect to the emergent LI that match the three complex scalars of (38).

5 Breaking of SUSY

In this section we discuss the effects of SUSY breaking on the super-aether model. The breaking of SUSY is conveniently described using spurion superfields whose higher components have non-vanishing VEVs. We will assume the spurions and their non-zero components
to be Lorentz scalars, so that they do not source additional violation of LI. Then the SUSY breaking Lagrangian of the lowest dimension is,

\[ \mathcal{L}_{SB} = -M_{\infty}^2 \int d^2 \theta d^2 \bar{\theta} \left[ S_{(1)} g_{(1)}(U^m \bar{U}_m) + S_{(2)} g_{(2)}(U^m \bar{U}_m) \right], \tag{43} \]

where \( g_{(1)}, g_{(2)} \) are arbitrary functions and the spurions have the form

\[ S_{(1)} = m_{(1)}^2 \theta^2 \bar{\theta}^2, \tag{44a} \]
\[ S_{(2)} = m_{(2)}(\theta^2 + \bar{\theta}^2). \tag{44b} \]

The parameters \( m_{(1)}, m_{(2)} \) have the dimension of mass; they set the scale of SUSY breaking in the aether sector. The Lagrangian (43) reads in components

\[ \mathcal{L}_{SB} = -M_{\infty}^2 \left[ m_{(1)}^2 g_{(1)}(w) + m_{(2)}^2 \left( g'_{(2)}(w) \bar{G}_m u^m - \frac{1}{2} g''_{(2)}(w) u^m u_n (\bar{\eta}^m \eta^n) + \text{h.c.} \right) \right], \tag{45} \]

where \( w \) is defined in (32). This must be combined with the supersymmetric part (29), (30).

It is straightforward to work out the effect of the first term in (45). Consider the aether vacuum (35). One observes that the modulus \( \alpha \) acquires a potential proportional to \( g_{(1)}(-\cos 2\alpha) \). If \( g'_{(1)}(-1) > 0 \) the modulus is stabilized at \( \alpha = 0 \). Thus we conclude that SUSY breaking lifts the degeneracy of the aether vacua and, under broad assumptions, singles out the purely time-like aether configuration. Note that this configuration is real and preserves the spatial isotropy. The imaginary part of the aether perturbations acquires a mass of order \( m_{(1)} \) in this vacuum.

The effect of the contribution proportional to \( m_{(2)} \) in (45) is subtler. At first sight, the second term in the round brackets seems to be a candidate for the fermionic mass term. However, due to the constraint (41), this term actually vanishes to quadratic order in the purely real aether background. Nevertheless, the fermions do get masses, though in an indirect way. We observe that the \( m_{(2)} \)-contribution modifies the equations for the non-dynamical fields \( G^m, l \), so that the field \( l \) no longer vanishes. Instead we obtain in the vacuum

\[ l_{\text{vac}} = \frac{m_{(2)} g'_{(2)}}{2}. \tag{46} \]

Substituted into the last line of the fermionic Lagrangian (30) this endows \( \eta^m \) with the mass of order \( m_{(2)} \).

To sum up, we have found that the general result of SUSY breaking is to generate masses for the fermions and the imaginary part of the aether. Below the SUSY breaking
scale one is left with the ordinary real aether described in Sec. 2. However, the memory of the SUSY origin of the theory is not totally lost: it is encoded in the relations (33) between the coefficients of the aether action\(^{11}\).

6 Discussion and outlook

In this paper we have considered a prototypical model of Lorentz violation with a dynamical preferred frame – the Einstein-aether theory – and have constructed the supersymmetric extension of the aether sector. Our model allows to implement the SUSY-based mechanism of [14, 15] that ensures emergence of LI at low energies within the SM sector. SUSY turned out to be more powerful in this respect than one could a priori expect: it leads to emergent low-energy LI even within the aether sector itself. We have found that the dynamics of the super-aether is richer than of its non-SUSY counterpart. In particular, the model possesses a family of inequivalent vacua exhibiting different symmetry breaking patterns. Remarkably, the model tolerates a certain breaking of spatial isotropy while remaining stable and ghost free. Finally, we have analyzed the effects of SUSY breaking.

In this paper we have restricted to the case of global SUSY. An important next step is the coupling of the super-aether model to supergravity. This task appears straightforward but presents a non-trivial technical exercise. Leaving it for the future, let us mention the following subtlety. It is impossible to embed the model of the present paper in the framework of the minimal SUGRA [25, 26]. The reason is that in this framework, due to non-vanishing supercurvature, the covariant spinor derivatives do not anticommute when acting on a vector superfield,

\[ \{ \bar{\nabla}_\alpha, \bar{\nabla}_\beta \} U^m \neq 0. \]

This prevents the definition of the chiral aether vector [27]. To overcome this obstruction one has to resort to the non-minimal formulation of SUGRA [28, 29] where it is possible to construct improved spinor derivatives, that do anticommute [30].

It would be interesting to generalize the construction of this paper to other theories with dynamical preferred frame. To illustrate possible problems let us consider the khrono-metric model of [9]. This model arises as the low-energy limit of the healthy Hořava gravity [6, 7] and is closely related to the Einstein-aether theory [31]. Violation of LI in this model is

\(^{11}\)Strictly speaking, the SUSY breaking will also affect the relations (33) through higher-dimensional operators. However, these corrections are suppressed by powers of the ratio of the SUSY breaking scale to \(M_\text{pl}\). They are small provided there is a hierarchy between these scales.
described by a real scalar field $\varphi$ – the khronon – that has non-vanishing time-like gradient. One imposes invariance under reparameterizations of $\varphi$,

$$\varphi \mapsto \tilde{\varphi}(\varphi) \ ,$$

which implies that $\varphi$ enters the Lagrangian only through the unit time-like vector

$$u_m = \frac{\partial_m \varphi}{\sqrt{-\partial_n \varphi \partial^n \varphi}} \ .$$

In terms of this vector the action has the same form (3) as the aether action\textsuperscript{12}. Note, however, that in terms of the fundamental field $\varphi$ the action contains higher derivatives leading to fourth-order equations of motion. As discussed in [9], this does not lead to inconsistencies. Due to the special structure of the fourth-order operator it is always possible to choose a coordinate frame in such a way that the equations contain only two time-derivatives, the extra two derivatives being purely space-like. The preferred frame is determined by the khronon field itself. Namely, the time coordinate must be chosen to coincide with the background value of the field,

$$t = \varphi_{\text{background}} \ .$$

We stress that the possibility to make this choice is crucial for the consistency of the model. As we now discuss, this property is lost when we attempt to construct a SUSY extension of the model, and higher time derivatives persist in the equations of motion.

By the analogy with the aether case, a natural choice of the khronon superfield is a chiral scalar,

$$\Phi = \varphi(y) + \sqrt{2} \theta^\alpha \psi_\alpha(y) + \theta^2 F(y) \ .$$

Importantly, this requires that the khronon field $\varphi$ be complexified. From this scalar one constructs a vector superfield

$$U_m = \frac{\partial_m \Phi}{\sqrt{-\partial_n \Phi \partial^n \Phi}} \ .$$

The lowest components of $U_m$ and $\Phi$ are related by Eq. (49). Note that $U_m$ is automatically chiral and satisfies the unit-norm constraint (22). The Lagrangian of the theory has the form (24) with the only difference that now there is no need for a Lagrange multiplier.

The khronon vacuum corresponding to (35) has the form

$$\varphi_{\text{vac}} = t \cos \alpha + iz \sin \alpha \ .$$

\textsuperscript{12}For the hypersurface-orthogonal vector (49) the four terms in the aether action are not independent. One of them can be eliminated in favor of the rest.
Note that this vacuum preserves SUSY despite of the fact that the SUSY variation of the fermionic component is non-zero,

$$\delta \psi = i \sqrt{2} \sigma^m \zeta \partial_m \varphi_{\text{vac}} = \sqrt{2} (i \sigma^0 \cos \alpha - \sigma^3 \sin \alpha) \zeta ,$$  \hspace{1cm} (54)

where \( \zeta \) is the parameter of the SUSY transformation. The reason is the large symmetry group of the theory. The expression (52) is invariant not only under arbitrary reparameterizations of \( \Phi \), but also under all transformations of the form

$$\Phi \mapsto \tilde{\Phi}(\Phi, \theta^\alpha)$$  \hspace{1cm} (55)

that involve arbitrary dependence on the holomorphic Grassmann coordinates. An appropriate transformation of this form compensates the variation (54).

Consider now small perturbations on top of the vacuum (53). For our argument it is sufficient to concentrate on the perturbations of the khronon, leaving aside the fermionic and auxiliary fields. We write

$$\varphi = \varphi_{\text{vac}} + \chi ,$$  \hspace{1cm} (56)

and expand the action to quadratic order in \( \chi \). We can use the result (38) for the aether: it remains to express the aether perturbations \( v_{1,2}, v_{\parallel} \) in terms of \( \chi \). A straightforward calculation yields

$$v_a = \partial_a \chi , \hspace{1cm} a = 1, 2 , \hspace{1cm} (57a)$$

$$v_{\parallel} = \cos \alpha \partial_3 \chi - i \sin \alpha \dot{\chi} . \hspace{1cm} (57b)$$

If \( \alpha = 0 \) we recover the quadratic Lagrangian of [9] for the (complexified) khronon perturbations containing only first time derivatives of \( \chi \). However, in the general case \( \alpha \neq 0 \) the action involves second time derivatives \( \ddot{\chi} \) and results in the fourth-order equations of motion. This leads to very fast instabilities that are clearly unacceptable. Note that the problem cannot be eliminated even by breaking SUSY or simply by choosing the background with \( \alpha = 0 \), since the higher time derivatives will inevitably appear in interaction terms. The origin of this behavior can be traced back to the fact that the khronon is complex and therefore there is no real coordinate system in which it coincides with the would-be preferred time. Thus we conclude that new ideas are required to supersymmetrize the khronon model and, eventually, its UV completion – Hořava gravity.

Another issue related to the problem of constructing a SUSY Hořava gravity, but also interesting on its own right, is the extension of the notion of SUSY to theories invariant
under anisotropic scaling of space and time coordinates. Recently, it has been demonstrated
[24] that the requirements of the anisotropic scaling and gauge invariance are incompatible
with the non-relativistic SUSY algebra (12). Namely, it is impossible to construct any
gauge theory\textsuperscript{13} with anisotropic scaling invariant under (12). As a way out one could try
to deform the SUSY algebra by allowing non-linear functions of the spatial momentum $P_a$
appear on the r.h.s. of the commutation relations [23]. This would make SUSY compatible
with the anisotropic scaling. However, as pointed out in [24], construction of interacting
models invariant under the deformed algebra may present a non-trivial task because the
SUSY transformations now contain higher spatial derivatives. In particular, one cannot
use the superfield formalism to build invariant Lagrangians. Indeed, any attempt to realize
the supercharges on the superspace will involve higher-order differential operators. These
supercharges will not satisfy the Leibniz rule, implying that the product of two superfields
does not transform as a superfield and thus the superfield formalism becomes useless. At
present only free theories invariant under the deformed SUSY have been constructed [24].

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