Comment on “Loophole-free Bell inequality”

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Abstract

A recent experiment yielding results in agreement with quantum theory and violating Bell inequalities was interpreted [Nature 526 (29 Octobert 2015) p. 682 and p. 649] as ruling out any local realistic theory of nature. But quantum theory itself is both local and realistic when properly interpreted using a quantum Hilbert space rather than the classical hidden variables used to derive Bell inequalities. There is no spooky action at a distance in the real world we live in if it is governed by the laws of quantum mechanics.

Comment

One can admire the technical skill that went into planning and carrying out the experiment reported in [1] without necessarily agreeing with the some of the conclusions drawn by the authors or found in the accompanying commentary [2]. At the beginning of the abstract of [1] one finds the assertion:

More than 50 years ago, John Bell proved that no theory of nature that obeys locality and realism can reproduce all the predictions of quantum theory: in any local-realist theory, the correlations between outcomes of measurements on distant particles satisfy an inequality that can be violated if the particles are entangled.

On the contrary, there is a theory of nature that is both local and realistic, and reproduces all the predictions of quantum theory. It is known as quantum mechanics or, to be more precise, let us call it Hilbert-space quantum mechanics. What John Bell actually proved was that a theory based upon hidden variables rather than the quantum Hilbert space, and satisfying an additional assumption of locality, makes predictions (Bell inequalities) that disagree with quantum theory, and with a series of experiments of increasing precision, accuracy and sophistication, of which those reported in [1] are among the most recent. These experimental results rule out hidden variables, while being perfectly compatible with the locality and realism of Hilbert space quantum mechanics.

In his first course in quantum physics the student learns that $PX$ is not the same thing as $XP$: quantum operators associated with physical quantities, such as momentum and position, in general do not commute. Such noncommutation is the most fundamental way in which quantum theory differs from classical mechanics, and the connection of this with Bell (and CHSH) inequalities was pointed out by Fine in 1982 [3]:

...I believe that [the material presented earlier in the article] shows what hidden variables and the Bell inequalities are all about; namely, imposing requirements to make well defined precisely those probability distributions for noncommuting observables whose rejection is the very essence of quantum mechanics.

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In other words, Bell inequalities based on hidden variables employ probabilities in a manner inconsistent with the principles of quantum mechanics. Let us look at this in more detail.

A spin-half particle is the simplest example of a system whose correct description requires the use of a Hilbert space[1] in this case a two-dimensional complex vector space with an inner product. The \( x \) and \( z \) components of angular momentum of such a particle, \( S_x \) and \( S_z \), are operators (matrices) whose eigenvalues are \(+1/2\) and \(-1/2\) in units of \( \hbar \). Probabilities can be assigned to the two possible values of \( S_z \) because they are mutually exclusive and exhaust all possibilities: \( S_z \) takes no values apart from \(+1/2\) and \(-1/2\). In standard (Kolmogorov) probability theory an exhaustive set of mutually-exclusive possibilities is called a sample space, and probabilities are assigned to elements of the sample space, or sets of elements from the sample space. From the physics point of view it was the Stern-Gerlach experiment that first identified this sample space.

Similarly, \( S_x \) can take on only the values \(+1/2\) and \(-1/2\), and these again constitute a sample space. Were one concerned with classical and not quantum physics, it would be sensible to talk about a joint probability distribution based on a sample space of the four mutually exclusive possibilities in which both \( S_z \) and \( S_x \) have well-defined values: \( S_z = +1/2 \) AND \( S_x = +1/2 \), \( S_z = +1/2 \) AND \( S_x = -1/2 \), etc. This is the assumption made, explicitly or implicitly, in various hidden-variables models. But in Hilbert-space quantum mechanics as developed by von Neumann (who, incidentally, invented the term ‘Hilbert space’) there is no such thing as \( S_z = +1/2 \) AND \( S_x = +1/2 \). The reason is that quantum properties are associated with subspaces of the Hilbert space; see Sec. III.5 of [4]. Within the two-dimensional Hilbert space of a spin-half particle there is a one-dimensional subspace associated with \( S_z = +1/2 \), another with \( S_z = -1/2 \), another with \( S_x = +1/2 \), and so forth; a mathematically-distinct subspace is associated with every direction in physical space. But this Hilbert space contains no one-dimensional subspace that can be associated with \( S_z = +1/2 \) AND \( S_x = +1/2 \), or with any of the three other possibilities needed if one is to assign a joint probability distribution to \( S_x \) and \( S_z \); this reflects the fact that they are incompatible observables, the operators do not commute[2].

The preceding remarks agree with what students are taught in an introductory quantum course: it is impossible to simultaneously measure \( S_x \) and \( S_z \) for a spin-half particle. Alas, they are rarely told the reason behind this: there is no property represented by a subspace in the quantum Hilbert space that corresponds to \( S_x \) and \( S_z \) simultaneously having specific values. And even skilled experimentalists cannot measure what does not exist. However the derivation of the CHSH version of a Bell inequality, which is Eq. (1) in [1], has as one of its assumptions that \( S_x \), \( S_z \), and other components of spin can be replaced by classical, which is to say commuting, quantities, which have a joint probability distribution, directly contrary, as Fine pointed out, to the principles of quantum mechanics. That an inequality based on such assumptions is violated by experimental results is not surprising if the real world is quantum mechanical. And if the experimental values agree, as seems to be the case, with quantum mechanics, what this would seem to tell us is that our world is quantum mechanical and correctly described using properties associated with a Hilbert space, rather than by classical hidden variables.

If we understand realism to mean, in line with the commentary in [2], that measurements reveal pre-existing physical properties of the world, Hilbert space quantum mechanics is realistic, as long as one does not make the unreasonable demand that measurements reveal things which do not exist in the real (quantum) world. Many physicists believe that the macroscopic outcomes

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[1] The term “Hilbert space” was first used in the study of infinite-dimensional spaces of functions, but is nowadays also employed for a finite-dimensional space.

[2] In the quantum logic of Birkhoff and von Neumann [5], \( S_z = +1/2 \) AND \( S_x = +1/2 \) is assigned the zero-dimensional subspace consisting of the zero vector, a property which is always false. Alas, quantum logic has not turned out to provide an approach to quantum mechanics useful for giving it a physical interpretation. The fundamental difficulty as seen from a physicist’s perspective is discussed for the case of a spin-half particle in Sec. 4.6 of [6].
of their experiments indicate previous microscopic states-of-affairs, and Hilbert-space quantum mechanics supports this conviction, as long as those microscopic situations can be described using Hilbert subspaces. It is the attempt to describe these pre-existing physical properties using classical hidden variables, while ignoring noncommutativity, that disagrees with experiment. The reasonable conclusion would seem to be that the real world is quantum mechanical, not classical. Thus the term “local realism” in [1,2] would be less misleading were it replaced by “local classical realism.”

But is Hilbert-space quantum mechanics, the kind that agrees with experiment, a local theory? In answering this question we must be careful about what we mean by “local.” Sometimes events that occur at widely separated places can be correlated with each other in a way that indicates they have a common cause in the past. This kind of nonlocality, common in classical physics, is not surprising. What is of interest for the present discussion is whether quantum theory allows for, or implies the existence of, nonlocal dynamical influences whereby a cause at one location can produce an effect at a different location when the two are at spacelike separation, i.e., the events are sufficiently far apart that no signals can pass from one to the other without exceeding the speed of light. Such influences are what Einstein referred to as “spooky action at a distance.” Even those who believe, on the basis of violations of Bell inequalities, that such superluminal influences exist will concede that they are “non-signaling”: they cannot be used to convey information from one location to another. This precludes any direct experimental test for their existence. Instead the evidence is indirect: there are correlations that cannot be explained by one’s local theory, and from this one infers that the world is nonlocal. But perhaps the problem lies with the theory.

Bell argued that the correlations predicted by quantum mechanics, and confirmed by experiment, cannot be explained by a local hidden variables theory. Alas, he did not possess a consistent formulation of Hilbert-space quantum mechanics that would have allowed him to compare its predictions with those provided by local hidden variables. Standing in his way was the infamous measurement problem of quantum foundations, which is to find a consistent and fully quantum mechanical description of the physical processes that go on in a measurement, starting with the microscopic property that is to be measured, and ending with the macroscopic property (often called a “pointer position”) correlated with, and thus indicating, the earlier microscopic property. Bell never solved this problem, as is evident from one of his last papers [7]. Taken together with his other publications it shows that he was unaware of, or at least had not given serious thought to, an approach developed in the 1980s by Gell-Mann, Hartle, Omnès, and the undersigned, and which did not reach a fully consistent form until the mid 1990s after Bell’s death. It is known as the consistent or decoherent histories formulation of quantum mechanics, and it assigns probabilities in a way consistent with the use of Hilbert subspaces, taking proper account of the noncommutativity of quantum operators. An analysis based on this approach, with details given in [10], shows that quantum dynamics is consistent with the following principle of Einstein locality:

Objective properties of isolated individual systems do not change when something is done to another non-interacting system.

This agrees with the characterization of “local” found in [2], and shows that Hilbert-space quantum mechanics is local as well as being in good agreement with experiment.

In summary, the impressive experimental results reported in [11] are in complete accord with, and indeed confirm, the validity of Hilbert-space quantum mechanics, which is both realistic and local when proper account is taken of the fact that the quantum world, the real world we live in, differs in important respects from the world of classical physics. It is the use of classical hidden variables in derivations of Bell inequalities which leads to results in disagreement with quantum
theory. And there is a very simple explanation of why spooky action at a distance is unable to convey information: it does not exist.

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