Structure identification and state initialization of spin networks with limited access

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Abstract

For reliable and consistent quantum information processing carried out on a quantum network, the network structure must be fully known and a desired initial state must be accurately prepared on it. In this paper, for a class of spin networks with only its single node accessible, we provide two continuous-measurement-based methods to achieve the above requirements; the first identifies the unknown network structure with a high probability, based on a continuous-time Bayesian update of the graph structure and the second is, with the use of an adaptive measurement technique, able to deterministically drive any mixed state to a spin coherent state for network initialization.

1. Introduction

Quantum information processing is usually performed on a highly networked system composed of many subsystems [1, 2]. In particular, a universal quantum computation is possible if we can ideally control all those subsystems [3–6]. However, this ‘global control’ approach inevitably introduces noise to the controlled subsystems, which accumulate, and as a result can largely degrade the performance of information processing. Also, engineering all the control actuators costs a lot for a large network. A different view is that all the network components are usually not accessible, such as in a solid network system whose surface can only be manipulated or...
measured; hence in this case the global control approach cannot be taken. These facts thus stimulate the development of methodologies dealing with networks that allow access only to a small set of subsystems.

Now let us turn our attention to the requirements imposed on a network for quantum computation [1]. In particular, the following are critical: the dynamics of the whole network must be fully known, a desired initial state of the network must be accurately prepared, and universal gate operation must be possible. Together with the fact mentioned in the first paragraph, we are thus reasonably motivated to develop a scheme for achieving these requirements in a network that allows us access to only a part of the whole system. Actually there has been notable progress along this direction: the authors of [7–12] deal with the problem of estimating the parameters of a limited-access spin network with known structure (topology); also we find a probabilistic system initialization method in [13, 14]; moreover, some approaches to quantum computation have been developed, in terms of controllability analysis, in [15–19].

The purpose of this paper is to complete the procedures necessary for quantum computation on a limited-access network. That is, we aim to develop methods achieving (i) structure identification and (ii) deterministic state initialization, for spin networks with only a single node accessible. Indeed, the accomplishment of these tasks, together with the results referred to above, i.e. parameter estimation [7–12] and universal gate operation [17, 18], enable us to carry out quantum computation on a limited-access network. A schematic illustrating our tasks is given in figure 1, where in case (i) we try to identify the structure of an unknown network hidden in a ‘black box’ and in case (ii) we try to stabilize a spin coherent state of a known network. Both of the schematics are based on continuous measurement [20–22], which is a continuous-time repetition of a Bayesian update of the system state based on the measurement result. Thus, it can be directly applied to the problems of parameter estimation [22–27] and feedback control for state preparation [22, 28–31], implying the suitability of the continuous measurement approach in our case as well.

In what follows, we describe the significance of the goals (i) and (ii) in a more detailed way and also, briefly, the results. First, regarding the goal (i), we are motivated by the fact that the graph structure itself is often unknown; for instance, in the case of solid systems, subsystems are served by atoms produced at different sites possibly randomly, and only some of them appear...
near the surface. Hence, in general, it is important to develop a scheme for identifying the graph structure from only accessible nodes, as illustrated in figure 1(a). In this paper, toward achieving the goal (i), we provide an algorithm to test whether any given pair of nodes of the network are connected or not; then it will be demonstrated numerically that our scheme correctly identifies the graph structure with a high probability of success. It is worth noting that this kind of structure identification problem can be found in the classical regime, e.g. the reconstruction of the graph structure of gene mRNA concentrations [32] and the estimation of relationships in social networks [33].

Once the network structure is correctly identified, which means that the system parameters can be further estimated using the results in [7–12], then the next step is to initialize the network. In this paper, we aim to deterministically stabilize a spin coherent state $|0^\otimes N\rangle = |0, 0, \ldots, 0\rangle$ in a limited-access network originally prepared in any mixed state. A scheme has not been developed satisfying all these requirements, though it is clearly an important subject, particularly in quantum computation. The continuous measurement is again useful to solve the problem, because it continuously reduces the entropy of the system; furthermore, to overcome the issue that any measurement induces a probabilistic (i.e. non-deterministic) behavior of the system, we employ the adaptive measurement technique [34, 35], which is a kind of feedback control that changes the measured observable continuously in time, depending on the past measurement results. It will be actually demonstrated that, in some examples, this method realizes the deterministic stabilization of the target state. At the same time, we clarify a situation where the adaptive scheme does not work; more precisely, it is proven that the permutation symmetry property [19] of the network prohibits such desirable convergence.

Lastly we remark that the presented schemes can be straightforwardly extended to the case where two neighboring nodes of the network are accessible; this is indeed a necessary requirement to perform universal quantum computation on a limited-access network [18].

**Notation.** The spin-up and spin-down states are represented by $|0\rangle = (1, 0)^T$ and $|1\rangle = (0, 1)^T$, respectively. $I_n$ is the $n \times n$ identity matrix. $\sigma^x$, $\sigma^y$ and $\sigma^z$ are the Pauli matrices.

### 2. Spin network under continuous measurement

We first describe the general setup of continuous measurement. This can be physically realized by coupling an optical probe field to the system of interest and measuring the output field continuously in time. In particular, when a homodyne detector is used for measurement, the time evolution of the system state $\rho_t$ conditioned on the measurement results $Y_t = \{Y_s : 0 \leq s \leq t\}$ is given by the following stochastic master equation (SME) [20–22]:

$$
\begin{align*}
\frac{d\rho_t}{dt} &= -i[H, \rho_t] \, dt + \gamma D[c] \rho_t \, dt + \sqrt{\gamma} H[c] \rho_t \, dW_t, \\
\frac{dY_t}{dt} &= \sqrt{\gamma} \, \text{Tr}[(c + c^\dagger) \rho_t] \, dt + dW_t,
\end{align*}
$$

(1)

(2)

where $H$ is the system Hamiltonian. The measurement operator $c$ represents the coupling between the system and the probe field, and $\gamma$ is the measurement strength. $dW_t$ is the standard Wiener increment with mean zero and variance $dt$. Also we have defined

$$
D[c] \rho = c \rho c^\dagger - \frac{1}{2}(c^\dagger c \rho + \rho c^\dagger c), \quad H[c] \rho = c \rho + \rho c^\dagger - \text{Tr}[(c + c^\dagger) \rho] \rho.
$$

Note that the set of equations (1) and (2) is a quantum counterpart to the classical Kushner–Stratonovitch equation describing the time evolution of a conditional probability
density. Hence, as in the classical case, the conditional expectation $\text{Tr}(A\rho_t)$ represents the least mean squared error estimate of an observable $A$ at time $t$.

In this paper, we study an $N$-spins network whose structure is captured by the graph $G$ with the set of nodes (vertices) $V(G)$ and that of edges $E(G)$, i.e. each node represents a single spin and $E(G)$ denotes the set of a pair of spins connected with each other. We make two assumptions on the system as follows. First, the interaction between the nodes is given by the XY coupling Hamiltonian [36]

$$H = \sum_{(j,k) \in E(G)} \lambda_{jk} \left( \sigma_j^x \otimes \sigma_k^x + \sigma_j^y \otimes \sigma_k^y \right),$$

where $\sigma_j^x, \sigma_j^y$ and $\sigma_j^z$ are the Pauli matrices acting on the $j$th spin; thus the notation means, e.g. $\sigma_j^z = I_2 \otimes \cdots \otimes I_2 \otimes \sigma_j \otimes I_2$. The second assumption is that only the first node is accessible; here we continuously measure the $z$ component of the first spin, in which case the measurement operator $c$ in equations (1) and (2) is given by

$$c = \sigma_1^z = \sigma^z \otimes I_2^{\otimes (N-1)}.$$

As a result, the conditional state of the whole spin network with graph $G$, whose first node is continuously measured, is subjected to the SME (1) with Hamiltonian (3) and the measurement operator (4).

3. Structure identification via continuous measurement on a single node

3.1. The structure estimator

We are concerned with the situation where the structure of graph $G$ is unknown; that is, we want to know which nodes of the network are connected with each other and how strong those connections are. But this general setting makes the problem too difficult, thus let us temporarily assume that the coupling constants $\lambda_{jk}$ are known and are uniformly given by $\lambda$. Of course, this assumption does not hold in general, so we will return to the original problem in section 3.4. Also, the measurement strength $\gamma$ is assumed to be known. Consequently, here we concentrate on the problem of identifying the structure of the graph $G$. This is equivalent to correctly choosing the true graph $\tilde{G}^{(i)}$ from all possible nominal graphs $\tilde{G}^{(1)}, \ldots, \tilde{G}^{(m')}$, where $m' = 2^{N^2}$ is the number of all combinations of the edges contained in the $N$-spins network. For a network composed of three spins, for instance, we have in total eight candidates of graph, $\tilde{G}^{(1)}, \ldots, \tilde{G}^{(8)}$, whose edges are respectively given by $E(\tilde{G}^{(1)}) = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$, $E(\tilde{G}^{(2)}) = \{\{1, 2\}, \{1, 3\}\}$, $E(\tilde{G}^{(3)}) = \{\{1, 2\}, \{2, 3\}\}$, and $E(\tilde{G}^{(8)}) = \{\}$. As shown in the left side of figure 2.

Clearly, the above classification is redundant, because the observer who accesses only to the first node cannot distinguish, for instance, $\tilde{G}^{(3)}$ and $\tilde{G}^{(5)}$; hence these two graphs have to be identified as the graph $\tilde{G}^{(3)}$, which is shown in the right side of figure 2. From the same reason, $\tilde{G}^{(4)}$ and $\tilde{G}^{(6)}$ are identified as the two-nodes graph $\tilde{G}^{(4)}$. Also $\tilde{G}^{(7)}$ and $\tilde{G}^{(8)}$ correspond to $\tilde{G}^{(5)}$. Consequently, for the three-nodes spin network, we can reduce the number of possible graph structure from $m' = 8$ to $m = 5$; the true graph $\tilde{G}^{(0)}$ is included in the set $G_3 = \{\tilde{G}^{(1)}, \ldots, \tilde{G}^{(5)}\}$. Note that $G_3$ contains the set of single-node graph $\tilde{G}_1 = \{\tilde{G}^{(5)}\}$ and that of two-nodes graph $\tilde{G}_2 = \{\tilde{G}^{(4)}\}$; therefore, identifying the graph structure by choosing one element from $G_3$ implies that at the same time we are estimating the number of nodes of the network, which has to be less than or equal to $k$. 


To attack the problem, we employ the estimation technique, which is found for instance in [22–27]. The basic idea is that, based on the measurement data $Y_i$, we attempt to estimate the value of both the index $i \in \{1, \ldots, m\}$ and a system observable in a recursive (continuous-time) manner. For this purpose, let us define the classical probability distribution $\{p_i^{(1)}, \ldots, p_i^{(m)}\}$ with $p_i^{(t)} = \mathbb{P}(G = G^{(i)})|Y_t|$ denoting the conditional probability that the true graph of the network is given by $G^{(i)}$. Then the above-mentioned goal can be attained by constructing an update law of $\{p_i^{(t)}\}$, such that it changes in time and will get the maximum value at the index $i = i_0$. At the same time, we need to update the system state conditioned on the measurement results $Y_t$; let us denote $\rho_t^{(i)}$ the whole network state corresponding to the $i$th nominal graph $G^{(i)}$. Now, the system with graph $G^{(i)}$ is driven by the Hamiltonian

$$H^{(i)} = \sum_{(j,k) \in E(G^{(i)})} \lambda \left( \sigma_j^x \otimes \sigma_k^x + \sigma_j^y \otimes \sigma_k^y \right)$$

while the measurement operator (4) is commonly taken for all nominal graphs. By using the same technique for deriving the SME (1) and (2), we have the following update laws of $\rho_t^{(i)}$ and $p_t^{(i)}$ (two methods to derive these equations are given in appendix A):

$$d\rho_t^{(i)} = -i[H^{(i)}, \rho_t^{(i)}]dt + \gamma D[c] \rho_t^{(i)}dt + \sqrt{\gamma} \mathcal{H}[c] \rho_t^{(i)} dt \left( dY_t - 2\sqrt{\gamma} \operatorname{Tr}(c\tilde{\rho}_t) dr \right),$$

$$dp_t^{(i)} = 2\sqrt{\gamma} \left\{ \operatorname{Tr}(c\tilde{\rho}_t^{(i)}) - \operatorname{Tr}(c\tilde{\rho}_t) \right\} p_t^{(i)}(dY_t - 2\sqrt{\gamma} \operatorname{Tr}(c\tilde{\rho}_t) dr),$$

where $\tilde{\rho}_t := \sum_{i=1}^m p_t^{(i)} \rho_t^{(i)}$. Here $Y_t$ is the measurement result generated from the true system having the true Hamiltonian $H = H^{(i_0)}$ and the measurement operator (4), i.e.

$$d\rho_t^{(i_0)} = -i[H^{(i_0)}, \rho_t^{(i_0)}]dt + \gamma D[c] \rho_t^{(i_0)}dt + \sqrt{\gamma} \mathcal{H}[c] \rho_t^{(i_0)} dt dW_t,$$

$$dY_t = 2\sqrt{\gamma} \operatorname{Tr}(c\tilde{\rho}_t^{(i_0)}) dt + dW_t.$$  

We recursively calculate the above equations to update the probability distribution $p_t^{(i)}$ as well as the state $\rho_t^{(i)}$, using the measurement result $Y_t$; what we expect is that, again, $p_t^{(i)}$ will get the maximum value at the index $i = i_0$ after many iterations. Note that in reality equations (8) and (9) cannot be computed since $H^{(i_0)}$ is unknown, but only the experimental data $Y_t$ is obtained; in numerical simulations, however, we do that in order to generate $Y_t$. Figure 3 illustrates the configuration of the estimation scheme.
Figure 3. Configuration of the structure estimator. We perform a continuous-time measurement on the accessible node of the spin network whose graph structure $G(i_0)$ is unknown. The measurement result $Y_t$ is used to update $p_t^{(i)}$, the probability that $G^{(i)}$ is the true graph, as well as $\rho_t^{(i)}$, the quantum state of the system with graph $G^{(i)}$. The update laws are given by equations (6) and (7).

3.2. Example 1: three-spins case

Let us consider the simple network composed of three nodes; in this case, as depicted in figure 2, we have $m = 5$ candidates as the graph structure. The true system is chosen to be the chain-type network $G^{(i_0=3)}$. The initial distribution is set to the uniform one $p_0^{(i)} = 1/5 \forall i$, because the graph structure is assumed to be completely unknown at the initial time $t = 0$. From a similar reason, we should set the initial density matrix to the maximal mixed state $\rho_0^{(i)} = (I_2/2)^{\otimes 3} \forall i$.

In this setting, we run the algorithm (6) and (7) to compute $p_t^{(i)}$. Figure 4(a) shows the averaged time evolution of 50 sample paths of $p_t^{(i)}$, denoted by $\langle p_t^{(i)} \rangle$; from this we clearly see that the correct convergence of $p_t^{(i)}$ to the distribution with $p^{(3)} = 1$ occurs most frequently. Hence, our estimator correctly identifies the true graph $G^{(3)}$.

We now discuss why the identification is possible by measuring only a part of the network. For this purpose let us focus on the estimate (conditional expectation) of the $z$-component of the measured spin. The continuous measurement tends to increase the absolute value of the estimate of $\sigma^z$ [29], while now the value of the $z$-component of the measured spin is distributed over the network due to the XY coupling Hamiltonian [37, 38], i.e. spin diffusion occurs. Hence, intuitively, if the network is ‘small’ in the sense that the path length from the accessible node to every terminal node is relatively short, then the spin wave quickly gets back to the measured spin and consequently the estimate of the $z$-component of the measured spin will change very fast, while in the opposite case the estimate will change slowly. Figure 4(b) plots the trajectories of $Z_t^{(i_0=3)} = \text{Tr}(c\rho_t^{(i_0=3)})$ and $Z_t^{(i)} = \text{Tr}(c\rho_t^{(i)})$. These figures support the validity of the above observation; because the chain is a relatively ‘large’ network, the true estimate $Z_t^{(i_0=3)}$ actually changes slowly. Remarkably, only the nominal estimate $Z_t^{(3)}$ shows a similar trajectory to that of the true one $Z_t^{(i_0=3)}$, while the other nominals do not. This fact means that the measurement, even only on a part of the network, certainly brings useful information for identifying the whole structure. At the same time, figure 4(b) tells us that the time-evolution of $Z_t^{(3)}$ produced from the large network is singularly different from those produced from the small networks, i.e. $Z_t^{(1)}$, $Z_t^{(2)}$ and $Z_t^{(4)}$, which all behave in a similar fashion. In general, if the interaction strengths are uniform and the upper bound of the total spin number of the network is known, then there are a
Figure 4. Numerical simulation for the three-spins network with parameter $\lambda = \gamma$. The true graph structure is set to $G^{(i_0=3)}$. Figure (a) shows the time evolution of the probability distribution $\langle p^{(i)} \rangle$ averaged over 50 sample paths, which takes the highest value at the true index $i_0 = 3$. Figure (b) shows typical sample paths of the estimate of $c = \sigma^z_1$, for the true system with graph $G^{(i_0=3)}$ (upper left) and for the nominal systems.

3.3. Example 2: five-spins case

We next consider the five-nodes network with true graph $G^{(i_0=25)}$, which is depicted in the inset of figure 5(a). For networks composed of up to five nodes, there are in total $m = 74$ graph structures, so the true graph is contained in the set $G_5 = \{G^{(1)}, \ldots, G^{(74)}\}$. Note that, without taking into account the redundancy, the number of possible graph structures is $m' = 1024$; hence the efficiency of the classification method introduced in section 3.1 warrants a special mention.
Figure 5. (a) Time evolution of the probability distribution $\langle p_i(t) \rangle$ averaged over 50 sample paths. The true graph is $G_{(i_0=25)}$, which has a uniform coupling constant. The parameters are set to $\lambda = \gamma$. (b)–(d) There are several examples where the coupling constants of the true network are not uniform. The number along the edge $\{j, k\}$ represents $\lambda_{jk}/\gamma$. The time evolution of $\langle p_i(t) \rangle$ is computed by averaging 50 sample paths. The true graph and the estimated graph are both depicted in the inset.

The time evolution of $\langle p_i(t) \rangle$ is computed by averaging over 50 sample paths of $p_i(t)$ and is shown in figure 5(a). The initial distribution is set to the uniform one $p_{0_i} = 1/74\forall i$, for the same reason as explained before. Also $p_{0_i} = (I_2/2)^{\otimes 5}\forall i$. Note that this system is a relatively small network with a length of up to 2 from the accessible node; hence, as discussed in section 3.2, the system may be less distinguishable compared to the chain-structured one. Nonetheless, the trajectory still converges and takes the maximum value at $i = 25$, thus the estimator correctly identifies the true graph structure.
3.4. Example 3: networks with non-uniform coupling constants

We have observed that our estimator can identify the true graph structure with high probability, under the assumption that the coupling constants of the true network are known and are uniform. Thus here we should return to the original problem, i.e. the structure identification of a network having non-uniform coupling constants. So the true network has a Hamiltonian of the form

\[ H^{(\text{true})} = \sum_{(j,k) \in E(G^{(\text{true})})} \lambda_{jk} \left( \sigma_j^x \otimes \sigma_k^x + \sigma_j^y \otimes \sigma_k^y \right) . \]  

(10)

\( G^{(\text{true})} \) is the true graph with unknown coupling constants \( \lambda_{jk} \). To attack this general problem, in this paper we follow the strategy to estimate only the graph structure; that is, we apply the same estimator as before, which assumes uniform coupling constants for the underlying network. Thus the estimator attempts to choose a most-likely graph from the set \( G_m = \{ G^{(1)}, \ldots, G^{(m)} \} \) composed of the graphs with uniform coupling constants; hence \( G^{(\text{true})} \) is not contained in \( G_m \).

There are two related reasons behind this approach: firstly, as mentioned in section 3.3, this is a relatively small network, hence the distinguishability, or roughly speaking the identifiability, would become worse. Figure 5(c) shows this is indeed the case; although \( \langle p_t^{(i)} \rangle \) takes the maximum value at \( i = 25 \) and thus the estimator correctly identifies the true graph structure, the probability of reaching the point \( i = 25 \) becomes smaller than the case of the chain. Note also that the success probability becomes smaller than the previous case shown in figure 5(a).

Lastly, we consider a star-type network with the weighted graph \( G^{(\text{true})} \) depicted in the inset of figure 5(d). In this case, as shown in the figure, \( \langle p_t^{(b)} \rangle \) has two comparable peaks at the points corresponding to the correct answer \( G^{(11)} \) and the wrong one \( G^{(31)} \); although \( \langle p_t^{(11)} \rangle \) is a bit larger than \( \langle p_t^{(31)} \rangle \), we should take both answers as the identified model. That is, the estimator cannot definitively identify the true graph. This result makes sense, because the true system is the smallest five-node network that would have a number of other types of small networks producing a similar output signal \( Y_t \); in particular, \( G^{(11)} \) and \( G^{(31)} \) generate very similar time evolutions of the estimate of \( c = \sigma_1^z \), hence it is hard to distinguish them.

3.5. Discussion

Our main question is the following: for what kind of network systems does the algorithm work well and generate the correct answer? The key concept enabling us to approach this problem would be identifiability, which has already appeared above without a formal definition, i.e. if two
different systems generate different outputs for a given common input, they can be distinguished from the output data and thus can be called identifiable. This is a fundamental notion in the field of system identification \[39\], whose quantum version has started to be studied only very recently \[40, 41\]. So our conjecture is as follows.

**Conjecture 1.** If the true system is identifiable, then the solution of the SME converges to the correct answer with a high probability.

Actually, it can be proven that the chain-formed network is identifiable \[40, 41\]. On the other hand, the system in figure 5(d) seems to be not identifiable, which can be seen from the fact that the systems \(G(11)\) and \(G(31)\) generate almost the same amount of the probability \(\langle p_i \rangle\). Exploring the connection of the structure identification problem to the general identifiability analysis is very important and should bring useful facts.

4. State initialization via continuous measurement on a single node

If the network structure is correctly identified, the estimation methods developed in \[7–12\] can be applied to determine the coupling constant \(\lambda_{jk}\) in the Hamiltonian \(3\), and then we can further move forward to the stage of system initialization. In this section, under the assumption that the Hamiltonian \(3\) is completely known, we provide a scheme that deterministically stabilizes the spin coherent state \(|0^N\rangle = |0, 0, \ldots, 0\rangle\). The scheme is again based on the continuous measurement performed only on a single node of the network. As mentioned in section 1, in general, measurement can reduce the entropy of the system state, while it must drive the state probabilistically; to achieve the deterministic state preparation, we thus employ the mechanism of adaptive measurement. To make this idea clear, this section is first devoted to present an adaptive measurement method for state preparation of a single-spin system. Then it is applied to quantum networks with only a single node accessible.

Before describing the results, we make two remarks. Firstly, for a solid system, we can effectively prepare a product of ground states \(|g, g, \ldots, g\rangle\) by cooling the system. The situation considered in this paper, however, does not allow the standard cooling method that extracts entropy from all the nodes via the global system–refrigerator coupling. Rather we here need to extract entropy from only a part of the system. Secondly, a completely different type of initialization method was proposed in \[42\]; the idea is to utilize a state transfer architecture from an ancilla system to the network system through accessible nodes. This method is effective if initializing the ancilla can be done easily.

4.1. Adaptive measurement for single spin state preparation

As indicated by equation \(1\), measuring a quantum system always brings a stochastic driving of the state. This means that only a fixed measurement does not deterministically stabilize the state. Combining measurement with feedback control is thus expected to overcome this issue, as is actually demonstrated in several studies \[22, 28–31\]. Adaptive measurement is a kind of feedback control, which does not introduce an additional actuator for control but instead changes the detector configuration based on the past measurement results. Thus a merit of adaptive measurement may appear in a practical situation where it costs less than adding an additional actuator. The applicability of this scheme to single spin stabilization has been
demonstrated in [34, 35]; we here consider the same problem studied in these references and show a new result.

The problem of stabilizing a single spin state via adaptive measurement is described as follows. Recall that the state under continuous measurement evolves in time according to equation (1). We here set $H = 0$ and assume that the measurement operator $c$ can be changed in time as a function of $\rho_t$; let us parameterize $c$ in the following form:

$$c_t = \begin{pmatrix} \cos \theta_t & e^{-i\delta_t} \sin \theta_t \\ e^{i\delta_t} \sin \theta_t & -\cos \theta_t \end{pmatrix}.$$  \hspace{1cm} (11)

Then the problem is to determine the time evolution of the parameters $(\theta_t, \delta_t)$ so that the single spin state governed by the SME (1) with $H = 0$ and equation (11) deterministically converges to a desired target state. In particular, we set the target to be $|0\rangle$.

To solve the problem, let us consider the following cost function:

$$J_t = 1 - \text{Tr}(\sigma^z \rho_t).$$

This is non-negative and takes the minimum value 0 only when $\rho_t = |0\rangle\langle 0|$. Also we parameterize the state as

$$\rho_t = \frac{1}{2} \begin{pmatrix} 1 + r_t \cos \alpha_t & r_t e^{-i\beta_t} \sin \alpha_t \\ r_t e^{i\beta_t} \sin \alpha_t & 1 - r_t \cos \alpha_t \end{pmatrix}.$$  \hspace{1cm} (12)

Then, the derivative of $E[J_t]$ is given by

$$\frac{dE[J_t]}{dt} = -\frac{r_t}{2} (\cos(2\theta_t) - 1) \cos \alpha_t + \sin(2\theta_t) \sin \alpha_t \cos(\delta_t - \beta_t)).$$  \hspace{1cm} (13)

Hence, by choosing the tuning parameters as

$$(\theta_t, \delta_t) = (\alpha_t/2, \beta_t), \quad (-\alpha_t/2, \beta_t + \pi)$$  \hspace{1cm} (14)

we have

$$\frac{dE[J_t]}{dt} = -\frac{r_t}{2} (1 - \cos \alpha_t) \leq 0.$$

Then, from the theory of stochastic stability [43], $dE[J_t]/dt \to 0$ holds, thus equivalently $\alpha_t \to 0$ or $r_t \to 0$ is guaranteed. This means that after a long time limit the state lies on the positive half of the z-axis in the Bloch sphere. But it is well known that an ideal continuous measurement of an observable always increases the purity of the conditional state, i.e. $r_t \to 1$. Combining these two results, we can conclude that the state converges to the target state $|0\rangle$ almost surely. The adaptive measurement law (14) has been found in [35], though without rigorous proof. Hence here we present the result as a new contribution.

**Theorem 1.** The single spin state subjected to the SME (1) with $H = 0$ and the adaptive measurement law (11), (12) and (14) converges to the target state $|0\rangle$ almost surely.

### 4.2. Network initialization via adaptive measurement on a single node

Now we apply the adaptive measurement scheme developed in the previous subsection to an $N$-spins quantum network with only a single node accessible. Let us set the target to be the spin coherent state $|0^{\otimes N}\rangle$. Then, the goal is to stabilize the target $|0^{\otimes N}\rangle$ by applying the continuous adaptive measurement performed on the accessible spin.
We follow the same procedure as before. Now the whole state is governed by the SME (1) with Hamiltonian (3) and the measurement operator

$$e_i' = \left( \begin{array}{cc} \cos \theta_i' & e^{-i\delta_i'} \\ e^{i\delta_i'} \sin \theta_i' & -\cos \theta_i' \end{array} \right) \otimes I_2^{(N-1)}$$

(15)

instead of equation (4). The adaptive measurement law of the parameters \((\theta_i', \delta_i')\) can be determined from the following cost function:

$$J'_i = N - \text{Tr}\left( \sum_{j=1}^{N} \sigma_j \rho_i \right)$$

(16)

which is non-negative and takes the minimum value 0 only when \(\rho_i = |0^\otimes N\rangle \langle 0^\otimes N|\). The time derivative of \(E[J'_i]\) is given by

$$\frac{dE[J'_i]}{dt} = -\frac{r'_i}{2} \left\{ (\cos(2\theta'_i) - 1) \cos \alpha'_i + \sin(2\theta'_i) \sin \alpha'_i \cos(\delta'_i - \beta'_i) \right\}$$

(17)

where \((r'_i, \alpha'_i, \beta'_i)\) are the parameters of the reduced quantum state

$$\rho'_i = \text{Tr}_{(2,3,4,\ldots,N)}[\rho_i] = \frac{1}{2} \left( \begin{array}{cc} 1 + r'_i \cos \alpha'_i & r'_i e^{-i\beta'_i} \sin \alpha'_i \\ r'_i e^{i\beta'_i} \sin \alpha'_i & 1 - r'_i \cos \alpha'_i \end{array} \right).$$

(18)

The point is that equation (17) has the same form as that for the single spin case, equation (13). Thus the adaptive law

$$\left( \theta'_i, \delta'_i \right) = \left( \alpha'_i/2, \beta'_i \right), \quad \left( -\alpha'_i/2, \beta'_i + \pi \right)$$

(19)

gives rise to \(dE[J'_i]/dt \to 0\) as before, which thus concludes \(\rho'_i \to |0\rangle \langle 0|\). Also note that the measurement operator (15) then converges to equation (4).

Although the above result does not necessarily mean the deterministic convergence of the whole network state \(\rho_i\) to the target \(|0^\otimes N\rangle\), the following two facts suggest that it would actually occur. Firstly, we now know that the value of the \(z\)-component of the first spin, which is continuously raised via the adaptive measurement, is distributed over the whole network due to the XY coupling Hamiltonian; hence the \(z\)-components of all spins may also increase. Secondly, it can be proven that the target \(|0^\otimes N\rangle\) is a steady state of the SME with equations (3) and (4) (see appendix B). In view of these two facts, we pose the following conjecture.

**Conjecture 2.** The whole network state \(\rho_i\) will deterministically converge to the target \(|0^\otimes N\rangle\), if it is the unique steady state of the controlled SME, i.e. the SME containing the adaptive measurement schematic.

### 4.3. Example: five-spins network

Here we consider some five-spins networks shown in figure 6, whose first spin is continuously measured with the adaptive law (19). To evaluate the performance, we use the fidelity \(F_i = \langle 0^\otimes 5 | \rho_i | 0^\otimes 5 \rangle\), which takes the maximum value 1 only when \(\rho_i = |0^\otimes 5\rangle \langle 0^\otimes 5|\). The initial state is \(\rho_0 = (I_2/2)^\otimes 5\). In each panel of figure 6, some sample paths of \(F_i\) are displayed.
Figure 6. Sample paths of the fidelity $F_t = \langle 0^{\otimes 5} | \rho_t | 0^{\otimes 5} \rangle$ for several five-spins networks. The graph (e) contains the permutation symmetry structure. The number along the edge $\{j, k\}$ represents $\lambda_{jk}/\gamma$.

First let us focus on the systems shown in figures 6(a)–(c) and (f). The remarkable fact is that, for all these cases, the target $|0^{\otimes 5}\rangle$ is the unique steady state of the controlled SME. Hence, conjecture 2 mentioned above suggests that our scheme realizes the deterministic state stabilization; actually all trajectories converge to 1. Also this result shows that, for a variety of network structures, the goal can be achieved.

On the other hand, the figures show that, in the cases (d) and (e), the adaptive scheme does not work well. Indeed, in these cases, it can be proven that the controlled SME has a steady state other than the target, implying that the contraposition of conjecture 2 is true. In particular, the system shown in figure 6(e) has a special symmetric structure; indeed this structure is what prohibits the desirable convergence and will be examined in detail in the next subsection.
4.4. Permutation symmetry

The above results imply that, as suggested by conjecture 2, the deterministic convergence needs the condition that the target \(|0^{\otimes 5}\rangle\) is the unique steady state of the controlled SME. Hence, it should be useful to characterize a system that does not have such uniqueness property. In particular, systems having permutation symmetry [19] are important; this property means that the Hamiltonian is invariant under the exchange of some specific pairs of spins. The system shown in figure 6(e) has this property; actually the Hamiltonian is

\[
H = \gamma (XXIII + YIII) + 1.2 \gamma (IXXII + IYIII) + 0.9 \gamma (IIIXI + IYYI) \\
+ 1.2 \gamma (IIIIX + IIIIY) + \gamma (XIIIIX + YIIIIY),
\]

where e.g. \(XXIII = \sigma^x \otimes \sigma^x \otimes I_2 \otimes I_2 \otimes I_2\), and it is permutation symmetric with respect to the exchanges between the second and the fifth spins and between the third and the fourth spins. We can then find that the entangled state

\[
|\phi'\rangle = \frac{1}{\sqrt{2(1+a^2)}} \left( |00001\rangle + a|00010\rangle - a|00100\rangle - |01000\rangle \right), \quad a = \frac{-3 \pm \sqrt{73}}{8}
\]

satisfies \(c|\phi'\rangle = |\phi'\rangle\) and \(H|\phi'\rangle = 1.2a|\phi'\rangle\), which thus implies that, in addition to \(|0^{\otimes 5}\rangle\), \(|\phi'\rangle\) is a steady state of the controlled SME because of the fact described in appendix B. Note that \(|\phi'\rangle\) is invariant under the above-mentioned permutation operation. Thus, the state can also move toward \(|\phi'\rangle\), implying that the deterministic convergence to the target \(|0^{\otimes 5}\rangle\) would not be expected. Actually, as seen in figure 6(e), the state does not converge to the target, even with the aid of the adaptive measurement; rather it converges to a mixed state on the subspace spanned by the steady states of the controlled SME.

The above result can be generalized, as shown below. The proof is given in appendix C.

**Theorem 2.** Suppose that the network has the property of permutation symmetry. Then, the target state is not the unique steady state of the controlled SME.

But note again that the system shown in figure 6(f) shows the deterministic convergence. Hence the deterministic stabilization of the state at the target is simply recovered, if the system experiences some perturbation and loses the permutation symmetry.

4.5. Discussion

The numerical simulations support the validity of conjecture 2 posed at the end of section 4.2: if the target is the unique steady state of the controlled SME, then the adaptive scheme achieves the deterministic state initialization. Therefore, to make the presented scheme stronger, we need to prove this conjecture and also characterize the network structure such that the spin coherent state is the unique steady solution of the controlled SME. These problems are both difficult due to the huge variety of the network structure, but we expect that the results [44] and [31] could be applied to solve them; the former shows the uniqueness condition of the steady state of a general master equation, and the latter gives a rigorous proof of the deterministic convergence in the case \(c = \sum_k \sigma_k^z\) and \(H = u(t) \sum_k \sigma_k^z\), with \(u(t)\) the feedback input. Both problems need careful mathematical analysis and should be investigated in the future work.
5. Concluding discussion

In this paper, we have provided continuous-measurement-based methods to achieve the structure identification and deterministic state initialization of spin networks with a single node only accessible. In each case, as numerically demonstrated, the performance of the scheme fully depends on the network structure. So surely it is very important to clarify what kind of graph structure is suitable for achieving both goals. This general question is, of course, not straightforward to answer, but here we try to deduce a conjecture from an intuitive observation.

First, to succeed in the structure identification, we need enough information that can be extracted from the accessible node; in general, when the system is in a highly mixed state, such information leaking occurs. On the other hand, if the system state is in a pure state, or in our case the state initialization has been completed, no meaningful information is available anymore. Hence, it seems that the identification and the initialization are in a trade-off relationship. Is there a system that allows us to achieve both goals? Indeed, we have seen that the presented two schemes work well particularly for chain-formed system, as observed in figures 5(b) and 6(a). This can be understood by looking at the fact that the chain is a relatively large network that needs a longer time until being purified; hence a large network offers more information during the measurement process compared to some other small networks. Indeed figure 6(a) shows that the chain-formed network takes the longest time to be initialized. Based on these observations, we now have the following general conjecture.

Conjecture 3. A system having an infection property [8, 10, 12, 17, 40, 41] is the best suitable for quantum computation on a limited-access network.

Actually, an infective system is essentially equivalent to a chain-formed system. The importance of this class of systems lies in the following three facts: firstly, an infective system can be parameter estimable [8, 10, 12]; secondly, it is possible to perform a universal gate operation on an infective system [17] and thirdly, it was proven in [40, 41] that an infective system is identifiable in the sense discussed in section 3.5. Therefore, here we are interested in proving the following conjecture:

Conjecture 4. The controlled SME of an infective system does not have a steady state other than the spin coherent state.

By proving the above conjecture and further conjectures 1 and 2 stated in sections 3.5 and 4.5, together with the above three facts, we can conclude that conjecture 3 is true, although a more precise meaning of the ‘best’ structure should be clarified.

Lastly, we remark that, in both the identification and stabilization problems, the total computation time for running the algorithm largely increases with the size of the network, so both schemes are inefficient for exponentially large systems. In this sense, quantum communication or metrology formulated within the indirect control framework, for example, would be suitable subjects to which our method should be first applied.

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Appendix A. Derivation of the SME (6) and (7)

Here we derive the SME (6) and (7), using two methods. We refer to [25–27] for a more-detailed description.

The key idea of the first approach is that, by embedding the classical probability distribution \( \{ p_i^{(i)} \} \) into a space of density matrices, we apply the quantum filtering theory to the augmented system composed of the classical (fictitious) system and the quantum system. For this purpose, let \( \{|\psi_i\rangle\} \) be the set of \( m \)-dimensional orthonormal vectors, with the index \( i \) corresponding to the \( r \)th graph. Then, the state of the augmented system is represented by

\[
\rho_i^E = \sum_{i=1}^{m} p_i^{(i)}|\psi_i\rangle \langle \psi_i| \otimes \rho_i^{(i)}.
\]

In the same manner, the Hamiltonian and the measurement operator acting on the whole space are respectively given by

\[
H^E = \sum_{i=1}^{m} |\psi_i\rangle \langle \psi_i| \otimes H^{(i)}, \quad c^E = I_m \otimes c
\]

with \( c \) given by equation (4). Then, the SME for the augmented system is given by

\[
d\rho_i^E = -i [H^E, \rho_i^E] dt + \gamma D[c] \rho_i^E dt + \sqrt{\gamma} \mathcal{H}[c] \rho_i^E dW_i^E. \tag{A.1}
\]

\[
dY_t = 2\sqrt{\gamma} \text{Tr}(c^E \rho_i^E) dt + dW_i^E. \tag{A.2}
\]

In particular, in the basis \( |\psi_i\rangle \), equation (A.1) gives

\[
dp_i^{(i)} = -i [H^{(i)}, \rho_i^{(i)}] dt + \gamma D[c] \rho_i^{(i)} dt + \sqrt{\gamma} \left[ c \rho_i^{(i)} + \rho_i^{(i)} c - 2 \text{Tr}(c^E \rho_i^E) \rho_i^{(i)} \right] dW_i^E. \tag{A.3}
\]

Then, the trace operation on the above equation yields

\[
dp_i^{(i)} = 2\sqrt{\gamma} \left\{ \text{Tr}(c \rho_i^{(i)}) - \text{Tr}(c^E \rho_i^E) \right\} \rho_i^{(i)} dW_i^E \tag{A.4}
\]

which is equation (7), where we have expressed \( \text{Tr}(c^E \rho_i^E) = \text{Tr}(c \tilde{\rho}_i) \) with \( \tilde{\rho}_i = \sum_i \rho_i^{(i)} \). To obtain the equation of \( \rho_i^{(i)} \), we assume that it follows \( d\rho_i^{(i)} = A_i dt + B_i dW_i^E \); then substituting this equation and equation (A.4) for equation (A.3), we have \( B_i = \mathcal{H}[c] \rho_i^{(i)} \) and

\[
A_i = -i [H^{(i)}, \rho_i^{(i)}] dt + \gamma D[c] \rho_i^{(i)} dt + 2\sqrt{\gamma} \left\{ \text{Tr}(c^E \rho_i^E) - \text{Tr}(c \rho_i^{(i)}) \right\} \mathcal{H}[c] \rho_i^{(i)}.
\]

Hence we obtain equation (6).

Next, we give an alternative way to derive equations (6) and (7). The idea is that we explicitly use the classical Bayes rule to obtain the update law of \( p_i^{(i)} \), unlike the above approach where the Bayes rule was implicitly used through the filtering procedure. Let us begin with the assumption that, through the estimation process up to time \( t \), we have estimated the true graph to be \( G^{(t)} \); then, under this condition, the measurement result is given by

\[
dY_t = 2\sqrt{\gamma} \text{Tr}(c \rho_i^{(i)}) dt + dW_t. \]

With this information, the conditional state \( \rho_i^{(i)} \) is updated to \( \rho_{i+dt}^{(i)} \).
through the usual quantum filtering technique, which leads to equation (6). Moreover, it is used for updating \( p_{t}^{(i)} = P((\mathcal{G} = G^{(i)}) | \mathcal{Y}_{t}) \) via the Bayes rule (\( \mathcal{N} \) is the normalization constant)

\[
P_{t+dt}^{(i)} = \frac{P(\mathcal{Y}_{t+dt} | \{G = G^{(i)}\}) p_{t}^{(i)}}{\sum_{i} P(\mathcal{Y}_{t+dt} | \{G = G^{(i)}\}) p_{t}^{(i)}} = \frac{1}{\mathcal{N}} \exp \left[ -\frac{1}{2dt} \left( dY_{t} - 2\sqrt{\gamma} \text{Tr}(c\rho_{t}^{(i)}) dt \right)^{2} \right] p_{t}^{(i)} = \frac{1 + 2\sqrt{\gamma} \sum_{i} \text{Tr}(c\rho_{t}^{(i)}) p_{t}^{(i)} dY_{t}}{1 + 2\sqrt{\gamma} \sum_{i} \text{Tr}(c\rho_{t}^{(i)}) p_{t}^{(i)} dY_{t}}
\]

hence we have equation (7).

Here we remark that in reality the output \( Y_{t} \) is generated from the true system, thus from equation (9) the innovation term is given by

\[
dW'_{t} = dW_{t} + 2\sqrt{\gamma} \text{Tr}(c\rho_{t}^{(i)}) dt - 2\sqrt{\gamma} \text{Tr}(c\rho_{t}^{(i)}) dt
\]

which is not the standard Wiener increment when \( \rho_{t}^{(i)} \neq \rho_{t}^{(i)0} \). As a result, particularly when the graph \( G^{(i)} \) largely differs from the true one \( G^{(i)0} \), the drift term of equation (6) (the term proportional to \( dt \)) can take a big number such that the constraint \( \text{Tr}(\rho) = 1 \) or \( \rho \geq 0 \) is numerically violated; consequently in the simulation, the time evolution of \( \rho_{t}^{(i)} \) becomes unstable and it sometimes diverges. Thus, we have introduced a normalization operation in the simulator (MATLAB) for numerically preserving those constraints.

**Appendix B. Steady state of the SME**

In general, a pure state \(|\psi\rangle\) is a steady state of the SME (1) if and only if \(|\psi\rangle\) is a common eigenvector of \( iH + c^{2}c/2 \) and \( c \), which can be directly proved using the results [44, 45]. Now, \(|0^{\otimes N}\rangle\) is clearly an eigenvector of \( c = \sigma_{z}^{i} \). Also noting the relation \((\sigma_{z}^{i} \otimes \sigma_{z}^{j} + \sigma_{y}^{i} \otimes \sigma_{y}^{j})(00) = 0\), we readily have \( H|0^{\otimes N}\rangle = 0 \). Therefore, \(|0^{\otimes N}\rangle\) is a common eigenvector of \( iH + c^{2}c/2 \) and \( c \), hence it is a steady state of the SME. Note that the above fact does not mean that \(|0^{\otimes N}\rangle\) is a unique steady state of the SME.

**Appendix C. Proof of theorem 2**

The goal is to prove that the controlled SME, having the property of permutation symmetry, has a steady state other than \(|0^{\otimes N}\rangle\). This can be achieved by showing that, based on the fact mentioned in appendix B, there exists a common eigenstate of \( c \) and \( H \) such that \( c|\phi\rangle = |\phi\rangle \) and \(|\phi\rangle \neq |0^{\otimes N}\rangle\). Note that the eigenstate satisfying \( c|\phi\rangle = -|\phi\rangle \) cannot be a steady state due to the adaptive measurement mechanism.

First, let \( P \) be a permutation matrix exchanging the indices 0 and 1 of two specific spins; which, however, does not act on the first node. Then, \( J_{z} = \Sigma_{i}^{1} \sigma_{z}^{i} \) satisfies \( [P, J_{z}] = [H, J_{z}] = [c, J_{z}] = 0 \), since \( P, H \) and \( c \) preserve the total \( z \) component of the network. Thus, \( P, H \) and \( c \) can be block-diagonalized into \( N + 1 \) blocks corresponding to the eigenspaces of \( J_{z} \),
i.e. $P = \text{diag}(P_0, \ldots, P_N)$, $H = \text{diag}(H_0, \ldots, H_N)$ and $c = \text{diag}(c_0, \ldots, c_N)$, where the $j$th component acts on the subspace spanned by the states with $j$ excitations (i.e. the states composed of $j$ spins with $|1\rangle$ and $N - j$ spins with $|0\rangle$). In particular, the subspace corresponding to $j = 1$ is spanned by $|100\ldots0\rangle$, $|010\ldots0\rangle$, $\ldots$, $|000\ldots1\rangle$. Now remove $|100\ldots0\rangle$ from this space and define

$$S_1 = \text{span}\{|010\ldots0\rangle, |001\ldots0\rangle, \ldots, |000\ldots1\rangle\}.$$  

Then, any state $|\phi\rangle \in S_1$ satisfies $c|\phi\rangle = |\phi\rangle$. Also note that $|0^\otimes N\rangle \not\in S_1$.

Let us now define $P' = \text{diag}(P_1, P_1, I_2, \ldots, I_1)$; then $P'$ must have two eigenvalues $\pm 1$, implying that there exists an eigenstate $|\phi\rangle \in S_1$ satisfying $P'|\phi\rangle = -|\phi\rangle$. We here use the assumption that $H$ is permutation symmetric, which leads to $[H, P'] = 0$ and further $[H, P'] = 0$. Then, from $[H, P']|\phi\rangle = 0$ we have

$$P'(H|\phi\rangle) = -(H|\phi\rangle)$$

which implies $H|\phi\rangle \in S_1$. Noting that this relation holds for any state satisfying $P'|\phi\rangle = -|\phi\rangle$, we can conclude that $H$ has an eigenstate $|\phi'\rangle \in S_1$. Together with the fact that $c|\phi'\rangle = |\phi'\rangle$ and $|\phi'\rangle \not= |0^\otimes N\rangle$, we obtain the assertion.

In the case of the five-spins network shown in figure 6(e), the steady state $|\phi'\rangle$ other than the target, the existence of which is shown in the above proof, can be found as follows. First, the operators acting on the space $S_1$, i.e. $P_1$, $H_1$ and $c_1$, have the following matrix representation:

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} 0 & 2.4 & 0 & 0 \\ 2.4 & 0 & 1.8 & 0 \\ 0 & 1.8 & 0 & 2.4 \\ 0 & 0 & 2.4 & 0 \end{pmatrix}, \quad c_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$  

The eigenvector of $P_1$ corresponding to the eigenvalue $-1$ is of the form $x, y, -y, -x)^T$. This further becomes an eigenvector of $H_1$ if it takes the form $(1, a, -a, -1)^T/\sqrt{2(1+a^2)}$ with $a = (-3 \pm \sqrt{73})/8$, which is of course an eigenvector of $c_1$. Then we find that it has the following representation in the whole system space:

$$|\phi'\rangle = \frac{1}{\sqrt{2(1+a^2)}}((00001) + a(00010) - a(00100) - (01000)),$$

References

[1] Divincenzo D P 2000 *Fortschr. Phys.* 48 771
[2] Nielsen M and Chuang I 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[3] Deutsch D 1989 *Proc. R. Soc. Lond. A* 425 73
[4] DiVincenzo D P 1995 *Phys. Rev. A* 50 1015
[5] Lloyd S 1995 *Phys. Rev. Lett.* 75 346
[6] Barenco A et al 1996 *Phys. Rev. A* 52 3457
[7] Burgarth D, Maruyama K and Nori F 2009 *Phys. Rev. A* 79 020305
[8] Burgarth D and Maruyama K 2009 *New J. Phys.* 11 103019
[9] Franco C D, Paternostro M and Kim M S 2009 *Phys. Rev. Lett.* 102 187203
[10] Burgarth D, Maruyama K and Nori F 2011 *New J. Phys.* 13 013019
[11] Lapasar E H, Maruyama K, Burgarth D, Takui T, Kondo Y and Nakahara M 2012 New J. Phys. 14 013043
[12] Maruyama K, Burgarth D, Ishizaki A, Takui T and Whaley K B 2012 Quantum Inform. Comput. 12 763
[13] Nakazato H, Takazawa T and Yuasa K 2003 Phys. Rev. Lett. 90 060401
[14] Nakazato H, Unoki M and Yuasa K 2004 Phys. Rev. A 70 012303
[15] Schirmer S G, Kandasamy G and Devitt S J 2008 Proc. ISCCSP 2008 (Malta, 12–14 March 2008) pp 966–71
[16] Schirmer S, Pullen I and Pemberton-Ross P 2008 Phys. Rev. A 78 062339
[17] Burgarth D, Bose S, Bruder C and Giovannetti V 2009 Phys. Rev. A 79 060305
[18] Burgarth D, Maruyama K, Murphy M, Montangero S, Calarco T, Nori F and Plenio M B 2010 Phys. Rev. A 81 040303
[19] Wang X, Pemberton-Ross P and Schirmer S G 2012 IEEE Trans. Autom. Control 57 1945
[20] Belavkin V P 1992 J. Multivariate Anal. 42 171–201
[21] Bouten L, van Handel R and James M R 2007 SIAM J. Control Optim. 46 2199–241
[22] Wiseman H M and Milburn G J 2009 Quantum Measurement and Control (Cambridge: Cambridge University Press)
[23] Mabuchi H 1996 Quantum Semiclassic. Opt. 8 1103
[24] Gambetta J and Wiseman H M 2001 Phys. Rev. A 64 042105
[25] Chase B A and Geremia J M 2009 Phys. Rev. A 79 022314
[26] Ralph J F, Jacobs K and Hill C D 2011 Phys. Rev. A 84 052119
[27] Gammelmark S and Molmer K 2013 Phys. Rev. A 87 032115
[28] Stockton J K, van Handel R and Mabuchi H 2004 Phys. Rev. A 70 022106
[29] van Handel R, Stockton J K and Mabuchi H 2005 IEEE Trans. Autom. Control 50 768–80
[30] Yamamoto N, Tsumura K and Hara S 2007 Automatica 43 981–92
[31] Mirrahimi M and van Handel R 2007 SIAM J. Control Optim. 46 445–67
[32] Rice J J, Tu Y and Stolovitzky G 2005 Bioinformatics 21 765
[33] Siciliano M D, Yenigunc D and Ertabn G 2012 Soc. Netw. 34 585
[34] Jacobs K 2010 New J. Phys. 12 043005
[35] Tanaka S and Yamamoto N 2012 Phys. Rev. A 86 062331
[36] Lieb E, Schultz T and Mattis D 1961 Ann. Phys. 16 407–66
[37] Bloembergen N 1949 Physica 15 386–426
[38] Negoro M, Tateishi K, Kagawa A and Kitagawa M 2011 Phys. Rev. Lett. 107 050503
[39] Ljung L 1987 System Identification: Theory for the User (Englewood Cliffs, NJ: Prentice-Hall)
[40] Burgarth D and Yuasa K 2012 Phys. Rev. Lett. 108 080502
[41] Guta M and Yamamoto N 2013 Proc. 52nd IEEE CDC (Florence, Italy, 10–13 December 2013) pp 1930–7 (arXiv:1303.3771)
[42] Burgarth D and Giovannetti V 2007 Phys. Rev. Lett. 99 100501
[43] Has’minskii R Z 1980 Stochastic Stability of Differential Equations (Alphen aan den Rijn: Sijthoff & Noordhoff)
[44] Kraus B, Büchler H P, Diehl S, Kantian A, Micheli A and Zoller P 2008 Phys. Rev. A 78 042307
[45] Yamamoto N 2005 Phys. Rev. A 72 024104