Heavy-light decay constants from clover heavy quark action in QCD with two flavors of dynamical quarks

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We present results on an analysis of the decay constants \( f_B \) and \( \xi = f_B / f_{B_s} \), in conjunction with the (future) experimental data on \( \Delta m_d \) (and \( \Delta m_s \)). Their calculation in the quenched approximation, using a number of different formulations for heavy quarks and the results are converging. A major uncertainty in these results, however, is that they may be susceptible to large corrections due to the effect of sea quarks. The penultimate step in eliminating this uncertainty is to consider the effect of two flavours of sea quark (\( N_f = 2 \)). Here we present the status of such a study, using the clover action for heavy quark, which we started last year.

As the lattice spacings available to us are relatively coarse, we deal with the large mass of b quark \( m_b a > 1 \) in the formalism of Ref. which has been previously applied in the quenched calculation of B meson decay constants.

In order to reduce discretisation effects, we have employed an RG-improved action in the gluon sector. Since this action was not considered in previous studies of \( f_B \), we repeat the calculation in quenched QCD to compare with full QCD results.

A comparison is also made with preliminary NRQCD results for the decay constants obtained on the same full QCD configurations, presented elsewhere in these proceedings.

1. Introduction

An accurate determination of the parameters \( f_{B_d} \sqrt{B_{B_d}} \) and \( \xi = f_{B_s} \sqrt{B_{B_s}} / f_{B_d} \sqrt{B_{B_d}} \), in conjunction with the (future) experimental data on \( \Delta m_d \) (and \( \Delta m_s \)) will provide excellent constraints on \( |V_{td}| \) and \( |V_{ts}| / |V_{td}| \). Their calculation in the quenched approximation, using the plaquette gluon action, has been carried out using a number of different formulations for heavy quarks and the results are converging.

A comparison is also made with preliminary NRQCD results for the decay constants obtained on the same full QCD configurations, presented elsewhere in these proceedings.

### 2. Computational Details

Gauge configurations are computed using the RG-improved gauge action and a tadpole-improved SW clover fermion action to create the background for two flavours of sea quark. As listed in Table, three bare gauge couplings were used. At each bare gauge coupling (\( \beta \)) runs were made for four sea quark masses (\( m_{\text{sea}}^a \)) in the

### Table 1

| \( N_s^3 \times N_t \) | \( \beta \) | \( a_{\text{chiral}}^{-1} \) | \( a_{\text{adj}}^{-1} \) |
|----------------------|---------|-----------------|-----------------|
| 12\(^3\) \times 24   | 1.8     | 2.8 fm          | 0.835 GeV       |
| 16\(^3\) \times 32   | 1.95    | 2.5 fm          | 1.263 GeV       |
| 24\(^3\) \times 48   | 2.1     | 2.6 fm          | 1.807 GeV       |
range $m_\pi/m_\rho \approx 0.8 - 0.6$. More details of these gauge configurations are presented in Ref. [11].

At each $(\beta, m_{\text{sea}})$, propagators for 8 heavy and 2 light valence quark masses were computed. The same tadpole-improved clover action is employed for both heavy and light quarks. Details of the propagators computation and the choice of smearing can be found in Ref. [6].

The heavy quark masses ranged roughly from charm to bottom. One of the light valence quark masses was chosen so that $m_{PS}/m_V = 0.688$, which approximately corresponds to the strange quark mass. The other light valence quark mass was chosen equal to the bare sea quark mass parameter. For each $\beta$, both types of matrix element were fitted as a function of $(m_{\text{sea}})^2$ to compute $f_{B_s}$ and $f_{B_d}$ at zero sea quark mass and finite lattice spacing.

We include the correction to the axial vector current obtained by the following substitution to the heavy quark field [7],

$$h(x) \rightarrow (1 - d_1 a \gamma \cdot \bar{D})h(x).$$

Using tadpole-improved tree-level value for $d_1$, we find that this correction has a contribution of up to 8% of the original current.

As a definition for the ground state mass of the heavy-light meson we used the “HQET” mass definition, as discussed in Ref. [13]. The physical scale of lattice spacing is fixed by the string tension measured for each sea quark mass using $\sqrt{\sigma} = 427$ MeV.

We made the quenched calculation with the same action at 5 different gauge couplings each on $24^3 \times 48$ and $16^3 \times 32$ lattices. The couplings were tuned so that they match the string tensions of full QCD for the four values of finite sea quark mass and the extrapolated zero sea quark mass. For more details see Ref. [1].

3. Results

In Fig. 1 we plot results for $f_{B_s}$ for $N_f = 2$ and in the quenched approximation ($N_f = 0$); sea quark mass is extrapolated to the chiral limit for $N_f = 2$. A clear increase of 10-20% is seen from two flavours of sea quark.

We also include the preliminary results from NRQCD [10] in Fig. 2, for two values of finite sea quark mass at $\beta = 1.95$ in full QCD (filled squares), and one value in quenched QCD (open squares). In Fig. 2 we present a similar comparison of the Fermilab and NRQCD approaches for $f_{B_d}$. In both figures we find good consistency of results between the two approaches.

Finally, we plot the ratio $f_{B_s}/f_{B_d}$ in Fig. 3 using the $K$ meson mass to set the strange quark mass. We observe only mild variation of the ratio with respect to $a$ for even the coarsest of our lattice spacings.

As a preliminary result we quote

$$f_{B_s}^{n_f=2} = 251 \pm 3 \pm 4(m_s) \pm 15_{27}^{27} \text{ (fit)} \text{ MeV},$$

Figure 1. A comparison of $f_{B_s}$ for full and quenched QCD. NRQCD results are also shown (squares). The scale is set by string tension $\sqrt{\sigma} = 427$ MeV.

Figure 2. A comparison of preliminary NRQCD and clover results for $f_{B_d}$. NRQCD [10] in Fig. 1, for two values of finite sea quark mass at $\beta = 1.95$ in full QCD (filled squares), and one value in quenched QCD (open squares). In Fig. 2 we present a similar comparison of the Fermilab and NRQCD approaches for $f_{B_d}$. In both figures we find good consistency of results between the two approaches.

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(2)
The first errors are statistical. The error labelled $(m_s)$ is due to the ambiguity of using the mass of the $\phi$ or $K$ to set the strange quark mass. The central values are determined by assuming the results independent of $a$ for the two finer lattice spacings. We should also add an uncertainty due to the choice of scale; our preliminary estimate is 15–20 MeV from comparison of results using the scale determined from $m_\rho$.

4. Conclusions

At this point it seems clear that there exists a systematic difference between the $N_f = 0$ and 2 data. Encouragingly enough, the preliminary NRQCD results are also in agreement with the relativistic results in both cases as well. One worrying point is that the quenched results for $f_{B_s}$ is approximately 10% larger than the quenched results using the Wilson action\cite{2,3}. Clearly this effect needs further examination. The ratio $f_{B_s}/f_{B_d}$ appears to be less affected by discretisation effects and is not substantially different from previous quenched calculations. It should be noted that even a systematic error of 10% for this ratio would be of substantial use for phenomenologists. It seems plausible then that a calculation of $f_{B_s}/f_{B_d}$ could be carried out for three flavors of dynamical quarks on a comparatively coarse lattice.

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