Monopoles, confinement and deconfinement in lattice compact QED in (2+1)D with external fields

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Finite temperature compact electrodynamics in (2+1) dimensions is studied in the presence of external electromagnetic fields. The deconfinement temperature is found to be insensitive to the external fields. This result corroborates our observation that external fields create additional small–size magnetic dipoles from the vacuum which do not spoil the confining properties of the model at low temperature. However, the Polyakov loop is not an order parameter of confinement. It can vanish in deconfinement in the presence of external field. This does not mean the restoration of confinement for certain external field fluxes. As a next step in the study of (2+1)D QED, the influence of monopoles on the photon propagator is studied. First results are presented showing this connection in the confining phase (without external field).

1. INTRODUCTION AND MODEL

In 3D compact QED confinement is proven and understood. The confinement is due to presence of Abelian monopoles – topological defects which appear due to the compactness of the gauge group. The confining property is lost at sufficiently high temperature. In \cite{1} we have demonstrated that the monopoles are sensitive to the confinement-deconfinement transition. In the confinement phase the monopoles are in the plasma state while in the deconfinement phase the monopoles appear in the form of a dilute gas of magnetic dipoles. Here we present results on the influence of an external electromagnetic field on the confining and monopole properties of the compact Abelian gauge model in (2+1)D \cite{2}.

The action with an external field is given by \cite{1}\cite{2}

\begin{equation}
S[\theta, \theta^\text{ext}] = -\beta \sum_p \cos (\theta_p - \theta^\text{ext}_p)
\end{equation}

with $\beta = 1/(a g_3^2)$, $T/g_3^2 = \beta/L_t$. The non-zero quantized external electric ($E$) or magnetic field ($B$) is directly coupled to plaquettes in the 31 or 12 plane, respectively ($n_{E/M} \in \mathbb{Z}$):

\begin{equation}
E = \theta^\text{ext}_{31} = 2\pi n_E/(L_3 L_1) \quad B = \theta^\text{ext}_{12} = 2\pi n_M/(L_2 L_1).
\end{equation}

Due to the form of action the maximal number of external flux quanta is restricted. We used a Monte Carlo algorithm, which combines a local Monte Carlo step (Metropolis and microcanonical sweep) with a global update step changing the internal field by a number of flux units, in order to improve ergodicity.

2. POLYAKOV LOOP AND STRING TENSION

Usually, the Polyakov loop $L(x)$ is used to probe confinement. Due to its Abelian nature the correlator of two Polyakov loops in the presence of a external field $F^\text{ext}$ (we use $F^\text{ext} = (0, E, 0)$ or $(0, 0, B)$) can be written as:

\begin{equation}
\langle L(0)L^*(\mathbf{r}) \rangle_{F^\text{ext}} \propto e^{i\Phi_C(F^\text{ext}) - L_v V(\mathbf{r}; F^\text{ext})},
\end{equation}

where $V$ is the potential and $\Phi_C$ is the non-vanishing flux of the internal field $F$ which pene-
trates the surface spanned on the contour $C$ given by the test particle trajectories.

Also the internal fluxes through the corresponding planes are quantized. Only the internal electric field contributes to the flux $\Phi_C(F_{\text{int}})$.

An external magnetic field is directed along the Polyakov loop, therefore, the induced internal field does not contribute to $\Phi_C$. Consider the correlator in an external electric field

$$\langle L(0, 0) L^*(x, y) \rangle_E \propto e^{2\pi i n_{\text{int}} x / L_s} e^{V(x, y; E)}$$

with $\Phi_C = 2\pi n_{\text{int}} x / L_s$. Our numerical results suggest that the internal fluxes can adequately be described by taking into account only the most probable flux state $n_{\text{int}} = n_{\text{int}}(n_{\text{ext}}, \beta, L_i)$. The oscillating part of the correlator is defined by the electric component of the internal field. The correlator (Fig. 1) simultaneously characterizes both

$$\langle L_\parallel(0) L_\parallel^*(y) \rangle_{n_E} = \text{const} \cdot \cosh \left[ \sigma_L \left( y - \frac{L_s}{2} \right) \right],$$

$$\sigma_{\text{eff}} = \frac{1}{L_t} \arccosh \left[ \cosh(\sigma L_t) - \cos \frac{2\pi n_{\text{int}}}{L_s} + 1 \right].$$

The essential lesson is that the effective string tension $\sigma_{\text{eff}}$ does not tell anything about confinement properties described by $\sigma$. The fitted string tension (Fig. 2), the confinement property and

![Figure 1. Real part of the Polyakov loop correlator in the x-y plane for $n_E = 4$ in deconfinement phase ($\beta = 2.6$). Half of a lattice $32^2 \times 8$ is shown.](image)

the screening of the external field (phase factor) and the potential of the test electric charges (by its modulus).

To evaluate the string tension $\sigma$, we use two Polyakov “plane” operators which are defined as a sum of Polyakov loops along and perpendicular to the external field. It can be shown that in the presence of an electric field

$$\langle L_\parallel(0) L_\parallel^*(x) \rangle_{n_E} = \text{const} \cdot e^{2\pi i n_{\text{int}} x / L_s} \cosh \left[ \sigma L_t \left( x - \frac{L_s}{2} \right) \right],$$

therefore, the plane–plane Polyakov loop correlator parallel to the electric field oscillates with a decreasing amplitude.

The plane–plane correlator perpendicular to the field decreases exponentially (without oscillations) as function of the distance between the planes:

$$\langle L_\perp(0) L_\perp^*(y) \rangle_{n_E} = \text{const} \cdot \cosh \left[ \sigma_{\text{eff}}(\sigma, n_{\text{int}}) L_t \left( y - \frac{L_s}{2} \right) \right],$$

$$\sigma_{\text{eff}} = \frac{1}{L_t} \arccosh \left[ \cosh(\sigma L_t) - \cos \frac{2\pi n_{\text{int}}}{L_s} + 1 \right].$$

The phase structure are not changed due to the presence of the external fields.

On the other side, in the case of a non-vanishing $E$, the Polyakov loop expectation value may vanish due to its tree level contribution regardless of the value of the actual string tension (see Fig. 3).

![Figure 2. Fitted string tension for various $\beta$-values as function $n_E$.](image)

3. MONOPOLE PROPERTIES AND THE PHOTON PROPAGATOR

Now we present a few results based on a cluster analysis of the monopole configurations [1]. We observe that the plasma component of the single monopole ensemble does not feel the external field. This is in agreement with the observation made before that the confinement property does not depend on the external field.
Figure 3. The absolute value of the bulk Polyakov loop \( |L| \) vs. the external electric flux \( n_E \) and its fit in the deconfinement phase.

On the contrary, the dipole density changes drastically as the external field increases (Fig. 4): the field creates additional monopoles popping up as dipoles from the vacuum. The larger the temperature (or \( \beta \)), the larger the increase of the dipole density. With a non–zero electric field, the system is anisotropic in all directions. As the external field increases, the dipoles become elongated, increasingly with the external field, along the direction of the applied field while in an external magnetic field dipoles become more polarized along the (temporal) \( z \)-direction similar to increasing temperature.

At zero temperature the model is confining. It is interesting how this is reflected in the photon propagator and how monopoles are showing up there. In Fig. 5 we show in lattice momentum space how singular (monopole-like) and regular (photon-like) lattice gauge field modes contribute to the photon propagator at zero temperature. The propagator has been measured in Landau gauge taking into account Gribov copies and removing zero momentum modes. Details will be presented elsewhere [5]. Similar investigation of the influence of center vortices on the gluon propagator was done in Ref. [1].

Figure 4. Clusters of two monopoles (dipoles) vs. \( n_E/M \) \((r)\) for some \( \beta \) values.

Figure 5. Different contributions to \( T = 0 \) photon propagator \( G(p^2) \) in Landau gauge vs. lattice momentum squared, \( p^2_L \) (lattice \( 16^3 \), \( \beta = 2.0 \)).

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