ELECTROMAGNETIC THERMAL INSTABILITY WITH MOMENTUM AND ENERGY EXCHANGE BETWEEN ELECTRONS AND IONS IN GALAXY CLUSTERS

Anatoly K. Nekrasov
Institute of Physics of the Earth, Russian Academy of Sciences, 123995 Moscow, Russia; anatoli.nekrassov@t-online.de, anekrasov@ifz.ru
Received 2011 April 28; accepted 2011 June 22; published 2011 September 14

ABSTRACT

Thermal instability in an electron–ion magnetized plasma, which is relevant in the intragalactic medium of galaxy clusters, solar corona, and other two-component plasma objects, is investigated. We apply the multicomponent plasma approach where the dynamics of all species are considered separately through electric field perturbations. General expressions for the dynamical variables obtained in this paper can be applied over a wide range of astrophysical and laboratory plasmas also containing neutrals and dust grains. We assume that background temperatures of electrons and ions are different and include the energy exchange in thermal equations for electrons and ions along with the collisional momentum exchange in equations of motion. We take into account the dependence of collision frequency on density and temperature perturbations. The cooling–heating functions are taken for both electrons and ions. A condensation mode of thermal instability has been studied in the fast sound speed limit. We derive a new dispersion relation including different electron and ion cooling–heating functions and other effects mentioned above and find its simple solutions for growth rates in limiting cases. We show that the perturbations have an electromagnetic nature and demonstrate the crucial role of the electric field perturbation along the background magnetic field in the fast sound speed limit. We find that at the conditions under consideration, condensation must occur along the magnetic field while the transverse scale sizes can be both larger and smaller than the longitudinal ones. The results obtained can be useful for interpreting observations of dense cold regions in astrophysical objects.

Key words: conduction – galaxies: clusters: general – instabilities – magnetic fields – plasmas – waves

1. INTRODUCTION

If a medium in thermal equilibrium can become cooler due to radiation and fluid contraction, it can get unstable, leading to the formation of density condensations with lower temperatures than in the surrounding medium (Parker 1953; Field 1965). This instability, called thermal or radiation-condensation instability, has been studied for more than five decades in astrophysical objects and plasma physics applications (for reviews see, e.g., Meerson 1996; Vázquez-Semadeni et al. 2003; Elmegreen & Scalo 2004; Cox 2005; Heiles & Crutcher 2005). Many papers in the astrophysical literature considered thermal instability in the neutral (Field 1965; Begelman & McKee 1990; Hennebelle & Péralta 1999; Koyama & Inutsuka 2000, 2002; Burkert & Lin 2000; Kritsuk & Norman 2002; Sánchez-Salcedo et al. 2002; Audit & Hennebelle 2005; Vázquez-Semadeni et al. 2006; Hennebelle & Audit 2007, and references given above) and magnetized interstellar medium (ISM; Field 1965; Hennebelle & Péralta 2000; Stiele et al. 2006; Fukue & Kamaya 2007; Inoue & Inutsuka 2008; Shadmehri et al. 2010), solar corona where prominences are formed (e.g., Field 1965; Nakagawa, 1970; Heyvaerts 1974; Mason & Bessey 1983; Karpen et al. 1988), planetary nebulae (e.g., Field 1965), galaxy clusters, and the intragalactic medium (IGM; Field 1965; Mathews & Bregman 1978; Balbus & Soker 1989; Loewenstein 1990; Balbus 1991; Bogdanović et al. 2009; Parrish et al. 2009; Sharma et al. 2010). This instability was studied for rotating (e.g., Field 1965; Nipoti 2010), expanding (e.g., Field 1965; Gomez-Pelaez & Moreno-Insertis 2002) and dynamical systems with steady or time-dependent flows and nonstationary background parameters (e.g., Mathews & Bregman 1978; Balbus 1986; Balbus & Soker 1989; Burkert & Lin 2000). The nonlinear stage of thermal instability resulting in the formation of nonlinear structures (localized clouds) in the ISM was investigated in, e.g., Trevisan & Ibáñez (2000), Sánchez-Salcedo et al. (2002), and Yatou & Toh (2009), and in solar prominences by Mason & Bessey (1983), Karpen et al. (1989) and Trevisan & Ibáñez (2000).

In majority of papers studying thermal instability of astrophysical objects in the magnetic field, one uses the one-fluid ideal magnetohydrodynamics (MHD) model. In some papers, the two-fluid model with the ideal magnetic induction equation has been used (e.g., Fukue & Kamaya 2007; Inoue & Inutsuka 2008). The nonideal effects in the induction equation have been included in, e.g., Heyvaerts (1974), Stiele et al. (2006), and Shadmehri et al. (2010).

In the plasma physics literature devoted to thermal instability, researchers considered, in addition to the one-fluid (e.g., Bora & Taiwari 1993) and two-fluid (e.g., Birk 2000) ideal MHD models (from the point of view of the magnetic induction equation), a multicomponent medium where the presence of electrons, ions, dust grains, and neutrals is taken into account (Kopp et al. 1997; Birk & Wiechen 2001; Pandey & Krishan 2001; Pandey et al. 2003; Shukla & Sandberg 2003; Kopp & Shchekinov 2007). Such an effect as dust charge variation is also included when studying thermal instability (Pandey & Krishan 2001; Ibáñez & Shchekinov 2002; Pandey et al. 2003). Analytical investigation of thermal instability in multicomponent magnetized media with physical effects such as collisions between different species, ionization and recombination, dust charge dynamics, gravity, self-gravity, rotation, and so on is a very challenging problem. General basic equations have very complex forms (see, e.g., Kopp et al. 1997). Therefore, one usually utilizes simplified models, considering, for example, potential perturbations in nonmagnetized (Kopp et al. 1997; Pandey & Krishan 2001; Ibáñez & Shchekinov 2002; Pandey et al. 2003; Shukla & Sandberg 2003; Kopp & Shchekinov 2007) and magnetized (Kopp et al. 1997; Shukla & Sandberg 2003) plasmas.
Neglecting the energy exchange between species in thermal equations is one of the usual simplifying assumptions. It is justified when a collisional coupling is weak or when the energy exchange frequency is sufficiently large, so that the temperatures of species are equal. In general, one can encounter considerable complications. Nevertheless, some authors have taken the energy exchange into account for thermal instability in the two-fluid MHD framework (e.g., Birk 2000; Birk & Wiechen 2001).

The terms describing the energy exchange contribute to the dispersion relation not only through temperature (or pressure) perturbations (Birk 2000; Birk & Wiechen 2001) but also through the perturbation of collision frequency, which depends on the density and temperature. This effect is important when background temperatures of species are different. Such a situation can occur, for example, in galaxy clusters (see Markevitch et al. 1996; Fox & Loeb 1997; Ettori & Fabian 1998; Takizawa 1998).

In this paper, we investigate the thermal instability in the electron–ion magnetized plasma, which is relevant in the IGM of galaxy clusters, solar corona, and other two-component plasma objects. We apply the multicomponent plasma approach when the dynamics of all species are considered separately through electric field perturbations (the \(E\) approach; see, e.g., Nekrasov 2009a, 2009b, 2009c; Nekrasov & Shadmehri 2010, 2011). General expressions developed in this paper can be applied to a wide range of astrophysical and laboratory plasmas. We assume that the background electron and ion temperatures are different and include the energy exchange in thermal equations for electrons and ions. We take into account the dependence of collision frequency on density and temperature perturbations. Different cooling–heating functions are assumed for electrons and ions. We include neither ionization and recombination effects nor gravity. Some expressions for electron and ion perturbations are used in their general forms, which can be used for other species (dust grains and neutrals). Here, we treat the condensation mode of thermal instability in the fast sound speed limit. We derive the general dispersion relation, taking into account the effects mentioned above, and find its simple solutions for the growth rates in limiting cases.

This paper is organized in the following manner. In Section 2, we give the fundamental equations used in this paper. The equilibrium state is considered in Section 3. Equations for temperature perturbations are presented in Section 4. In Section 5, we give specific conditions for further consideration. In Section 6, equations for components of velocity perturbations are given in the fast sound speed limit. Components of the perturbed current are calculated in Section 7. These components for the simplified collision contribution are given in Section 8. In Section 9, we derive the dispersion relation. Its limiting cases are considered in Section 10. We discuss the results in Section 11. The possible astrophysical implications are considered in Section 12. A summing up of the main points is given in Section 13.

2. BASIC EQUATIONS

The fundamental equations that we consider here are the following:

\[
\frac{\partial \mathbf{v}_j}{\partial t} + \mathbf{v}_j \cdot \nabla \mathbf{v}_j = -\frac{\nabla p_j}{m_j n_j} + \mathbf{F}_j + \frac{q_j}{m_j e} \mathbf{v}_j \times \mathbf{B},
\]

the equation of motion,

\[
\frac{\partial n_j}{\partial t} + \nabla \cdot n_j \mathbf{v}_j = 0,
\]

the continuity equation, and

\[
\frac{\partial T_i}{\partial t} + \mathbf{v}_i \cdot \nabla T_i + \frac{\gamma - 1}{\gamma - 1} T_i \nabla \cdot \mathbf{v}_i = -\left(\gamma - 1\right) \frac{1}{n_i} \mathcal{L}_e(n_i, T_i) + v^2_{re}(n_e, T_e)(T_e - T_i)
\]

and

\[
\frac{\partial T_e}{\partial t} + \mathbf{v}_e \cdot \nabla T_e + \frac{\gamma - 1}{\gamma - 1} T_e \nabla \cdot \mathbf{v}_e = -\left(\gamma - 1\right) \frac{1}{n_e} \mathbf{q}_e - \left(\gamma - 1\right) \frac{1}{n_e} \mathcal{L}_e(n_e, T_e) - v^2_{re}(n_i, T_e)(T_e - T_i)
\]

are the temperature equations for ions and electrons. In Equations (1) and (2), the index \(j = i, e\) denotes the ions and electrons, respectively. The force \(\mathbf{F}_j\) in Equation (1) is given by

\[
\mathbf{F}_i = \frac{q_i}{m_i} \mathbf{E} - v_{ie}(\mathbf{v}_i - \mathbf{v}_e), \quad \mathbf{F}_e = \frac{q_e}{m_e} \mathbf{E} - v_{ie}(\mathbf{v}_e - \mathbf{v}_i).
\]

Other notations in Equations (1)–(5) are the following: \(q_j\) and \(m_j\) are the charge and mass of species, \(j = i, e\), \(\mathbf{v}_j\) is the hydrodynamic velocity, \(n_j\) is the number density, \(p_j = n_j T_j\) is the thermal pressure, \(T_j\) is the temperature, \(v_{ie}\) (\(v_{ei}\)) is the collision frequency of ions (electrons) with electrons (ions), \(v^2_{re}(n_e, T_e) = 2v_{re}(v_{ei}(n_i, T_e))\) is the frequency of the thermal energy exchange between ions (electrons) and electrons (ions) (Braginskii 1965), \(n_i v^2_{ei}(n_i, T_e) = n_e v^2_{re}(n_i, T_e)\), \(\gamma\) is the ratio of specific heats, \(\mathbf{E}\) and \(\mathbf{B}\) are the electric and magnetic fields, respectively, and \(c\) is the speed of light in a vacuum. The value \(\mathbf{q}_e\) in Equation (4) is the electron heat flux (Braginskii 1965). As for the latter, we will consider a weakly collisional plasma when the electron Larmor radius is much smaller than the electron collisional mean free path. In this case, the electron thermal flux is mainly directed along the magnetic field,

\[
\mathbf{q}_e = -\chi_e \mathbf{b} (\mathbf{b} \cdot \nabla) T_e.
\]

where \(\chi_e\) is the electron thermal conductivity coefficient and \(\mathbf{b} = \mathbf{B}/B\) is the unit vector along the magnetic field. In other respects, a relation between cyclotron and collision frequencies of species stays arbitrary in general expressions considered below. We only take into account the electron thermal flux (6) because the corresponding ion thermal conductivity is considerably smaller (Braginskii
1965). We also assume that the thermal flux in the equilibrium is absent. The cooling and heating of plasma species in Equations (3) and (4) are described by the function \( \mathcal{L}_j(n_j, T_j) = n_j^2 \Lambda_j(T_j) - n_j \Gamma_j \), where \( \Lambda_j \) and \( \Gamma_j \) are the cooling and heating functions, respectively. There is some difference between the form of this function and the frequently used cooling–heating function \( \mathcal{F} \) (Field 1965). Both functions are connected to each other via the equality \( \mathcal{L}_j(n_j, T_j) = m_j n_j \mathcal{F}_j \). Our choice is analogous to those of Begelman & Zweibel (1994), Pandey & Krishan (2001), Shukla & Sandberg (2003), Bogdanović et al. (2009), and Parrish et al. (2009). The function \( \Lambda_j(T_j) \) can be found, for example, in Tozzi & Norman (2001).

Electromagnetic equations are Faraday’s law,
\[
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t},
\]
and Ampere’s law,
\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j},
\]
where \( \mathbf{j} = \sum q_j n_j \mathbf{v}_j \). We consider wave processes with typical timescales much larger than the time it takes for the light to cover the wavelength of perturbations. In this case, one can neglect the displacement current in Equation (8) that results in quasi-neutrality for both the electromagnetic and purely electrostatic perturbations. The magnetic field \( \mathbf{B} \) includes the background magnetic field \( \mathbf{B}_0 \), the magnetic field \( \mathbf{B}_{0\text{pert}} \) of the background electric current (when it is present), and the perturbed magnetic field.

For general, we assume in the meanwhile that \( n_i \) and \( n_e \) are different, keeping in mind that some solutions obtained below can be applied to multicomponent plasmas.

### 3. Equilibrium State

First, we will consider an equilibrium state. We assume that the background velocities of species are absent. Here, we do not involve an equilibrium inhomogeneity. Therefore, the thermal equations (3) and (4) in equilibrium take the form
\[
\begin{align*}
(n_i - 1) \frac{1}{n_i} \frac{\partial \mathcal{L}_i(n_i, T_i)}{\partial t} - \nu_{ie}^i(n_i, T_i, T_0)(T_e - T_i) &= 0, \\
(n_e - 1) \frac{1}{n_e} \frac{\partial \mathcal{L}_e(n_e, T_e)}{\partial t} + \nu_{ie}^e(n_e, T_e, T_0)(T_e - T_i) &= 0,
\end{align*}
\]
respectively, where the index 0 denotes the unperturbed values.

### 4. Linear Equations for the Temperature Perturbations

We now consider Equations (3) and (4) in the linear approximation. Using Equations (2) and (9) for ions, we find that
\[
D_{1i} T_{i1} - D_{2i} T_{e1} = C_{1i} \nabla \cdot \mathbf{v}_{i1} - C_{2i} \nabla \cdot \mathbf{v}_{e1},
\]
where the index 1 denotes the perturbed values. The following notations are introduced:
\[
D_{1i} = \left( \frac{\partial}{\partial t} + \Omega_{Fi} + \Omega_{Te} \right) \frac{\partial}{\partial t}, \quad D_{2i} = \left( \Omega_{Fi} + \Omega_{Te} \right) \frac{\partial}{\partial t}, \quad C_{1i} = T_{i0} \left[ -(n_i - 1) \frac{\partial}{\partial t} + \nu_{ni}^i \right], \quad C_{2i} = \Omega_{Te}(T_e - T_{i0}).
\]
Analogously, we obtain for the electrons
\[
D_{1e} T_{e1} - D_{2e} T_{i1} = C_{1e} \nabla \cdot \mathbf{v}_{e1} + C_{2e} \nabla \cdot \mathbf{v}_{e1},
\]
where
\[
D_{1e} = \left( \frac{\partial}{\partial t} + \Omega_{Fe} + \Omega_{Te} + \Omega_{Tei} + \Omega_{Te} \right) \frac{\partial}{\partial t}, \quad D_{2e} = \Omega_{Te} \frac{\partial}{\partial t}, \quad C_{1e} = T_{e0} \left[ -(n_e - 1) \frac{\partial}{\partial t} + \nu_{ne}^e \right], \quad C_{2e} = \Omega_{Te}(T_e - T_{i0}).
\]
In Equations (11) and (13), we introduce the following frequencies:
\[
\begin{align*}
\Omega_{Fi} &= (n_i - 1) \frac{\partial}{\partial T_i}, \quad \Omega_{Fe} = (n_e - 1) \frac{\partial}{\partial T_e}, \\
\Omega_{Te} &= (n_e - 1) \frac{\partial}{\partial T_e} + \frac{\partial \mathcal{L}_e(n_e, T_e, T_0)}{\partial T_e} \\
\Omega_{Tei} &= (n_e - 1) \frac{\partial}{\partial T_e} + \frac{\partial \mathcal{L}_i(n_i, T_i, T_0)}{\partial T_i},
\end{align*}
\]
and
\[
\begin{align*}
\Omega_{ni} &= \frac{\partial \nu_{ni}^i(n_i, T_0)}{\partial T_i}(T_e - T_{i0}), \\
\Omega_{ne} &= \frac{\partial \nu_{ne}^e(n_e, T_0)}{\partial T_e}(T_e - T_{i0}), \\
\Omega_{Tei} &= \frac{\partial \nu_{Te}^e(n_e, T_0)}{\partial T_e}(T_e - T_{i0}).
\end{align*}
\]
We assume that the background magnetic field $B_0$ is directed along the z-axis. In notations (11) and (13), we use an equilibrium state and the fact that $v_{ei}^*(n_{i0}, T_{i0}) \sim n_{i0}$ and $v_{ei}^*(n_{e0}, T_{e0}) \sim n_{e0}$. We see from Equations (10) and (12) that the temperature perturbations are connected to a velocity divergence. Solutions for $T_{e1}$ and $T_{i1}$ are given by

$$DT_{e1} = G_1 \cdot \nabla \cdot v_{e1} + G_2 \cdot \nabla \cdot v_{i1},$$  

and

$$DT_{i1} = G_3 \cdot \nabla \cdot v_{e1} + G_4 \cdot \nabla \cdot v_{i1},$$  

where the following notations are introduced:

$$D = D_{1i} D_{1e} - D_{2i} D_{2e}, \quad G_1 = D_{1i} C_{1e} - D_{2e} C_{2i}, \quad G_2 = D_{1i} C_{2e} + D_{2e} C_{1i},$$

$$G_3 = D_{2i} C_{1e} - D_{1e} C_{2i}, \quad \text{and} \quad G_4 = D_{1i} C_{1i} + D_{2i} C_{2e}.$$  

To find the temperature perturbation $T_{j1}$, we have to calculate expressions for $\nabla \cdot v_{j1}$. General equations for the velocities $v_{j1}$ and $\nabla \cdot v_{j1}$ are shown in the Appendix, where expressions for $D$ and $G_1$, $l = 1, 2, 3$, and 4 are also given. In their general form, the components of $v_{j1}$ are very complex. Therefore, to proceed further analytically, here we restrict ourselves to a limiting case in which the dynamical frequency $\partial / \partial t$ is smaller than the sound frequency. Thus, we consider sufficiently short-wavelength perturbations along the magnetic field (see below). Some additional simplifying conditions that are satisfied in magnetized plasmas are also used.

5. SPECIFIC CASE: FAST SOUND SPEED LIMIT

Equations (A27), (A28), and (A30) are written in their general forms, which allows us to consider different simplified specific cases corresponding to real astrophysical conditions. We also consider the case in which

$$\omega_i^2 \gg \left( \frac{\partial}{\partial z} \right)^2 \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \frac{\partial^2}{\partial t^2}.$$  

(18)

Usually, since $\omega_i^2 \gg \partial^2 / \partial t^2$ in magnetized plasmas, Equation (18) is satisfied for a wide range of the transverse wavelengths of perturbations $(\partial / \partial y)^{-1}$ in comparison with longitudinal wavelengths $(\partial / \partial z)^{-1}$. The other simplifying condition is

$$\omega_i^2 \gg \left[ T_{e0} T_{i0} \frac{\partial}{\partial t} + (T_{e0} + T_{i0}) \Omega_{cie} \right] \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$  

(19)

This inequality means that the effective ion Larmor radius is much smaller than the perturbation wavelengths.

Below, we consider the case in which the perturbation frequency is much smaller than the sound frequency, i.e.,

$$\frac{\partial^2}{\partial t^2} \ll \left[ T_{e0} T_{i0} \frac{\partial}{\partial t} + (T_{e0} + T_{i0}) \Omega_{cie} \right] \frac{\partial^2}{\partial z^2}.$$  

(20)

This condition can be written as $\partial^2 / \partial t^2 \ll c_s^2 \partial^2 / \partial z^2$, where $c_s$ can be considered as the effective sound velocity. Equations (19) and (20) are given in the approximate forms to unite two cases, $\frac{\partial}{\partial t} \gg (\ll) \Omega_{cie}$. We note that Equation (18) follows from Equations (19) and (20).

Under Equations (19) and (20), we find, using Equations (A10), (A30), and (A31), the following equations for $P_{i, e1}$ (see Equations (A27) and (A28)):

$$\frac{\partial^2 P_{i1}}{\partial z^2} = -(1 - \alpha_i) \left( \frac{1}{\omega_i^2} \frac{\partial^3 Q_{i1y}}{\partial y \partial t^2} + \frac{\partial^2 F_{i1z}}{\partial z \partial t} \right) + \beta_i \left( \frac{1}{\omega_i^2} \frac{\partial^3 Q_{i1y}}{\partial y \partial t^2} + \frac{\partial^2 F_{i1z}}{\partial z \partial t} \right),$$  

(21)

and

$$\frac{\partial^2 P_{e1}}{\partial z^2} = -(1 - \alpha_e) \left( \frac{1}{\omega_e^2} \frac{\partial^3 Q_{e1y}}{\partial y \partial t^2} + \frac{\partial^2 F_{e1z}}{\partial z \partial t} \right) + \beta_e \left( \frac{1}{\omega_e^2} \frac{\partial^3 Q_{e1y}}{\partial y \partial t^2} + \frac{\partial^2 F_{e1z}}{\partial z \partial t} \right),$$  

(22)

where

$$\alpha_i = \frac{m_i}{K} \left( DT_{e0} - G_1 \frac{\partial}{\partial t} \frac{\partial^2}{\partial z^2} \right) - \beta_i = \frac{m_e}{K} \left( DT_{i0} - G_1 \frac{\partial}{\partial t} \frac{\partial^2}{\partial z^2} \right),$$

$$\beta_i = \frac{m_i}{K} G_2 \frac{\partial}{\partial z} \frac{\partial^3}{\partial t^3}, \quad \text{and} \quad \beta_e = \frac{m_e}{K} G_3 \frac{\partial}{\partial z} \frac{\partial^3}{\partial t^3}.$$  

(23)

Below, we will need correction terms proportional to $\alpha_{e,i} \ll 1$ and $\beta_{e,i} \ll 1$. We note that the contribution of the correction term proportional to $Q_{e1y}$ in Equation (21) is at least of the order of $m_e / m_i$ in comparison with the term proportional to $Q_{i1y}$. However,
for convenience we keep this small term for the symmetry with equation for $P_{ei}$. The ratio of contributions of the correction terms proportional to $F_{elz}$ and $F_{i1z}$ in Equation (21) is of the order of

$$\frac{(T_0 + T_{i0})\Omega_{ie}}{T_0 \dot{\rho}_{i} + (T_0 + T_{i0})\Omega_{ie}}.$$  

The same for terms proportional to $Q_{iy}$ and $Q_{e1y}$ in Equation (22) is

$$\frac{m_i}{m_e (T_0 + T_{i0})\Omega_{ie}}$$

and for the ratio of terms $\sim F_{i1z}$ and $F_{elz}$ we have

$$\frac{(T_0 + T_{i0})\Omega_{ie}}{T_{i0} \dot{\rho}_{i} + (T_0 + T_{i0})\Omega_{ie}}.$$  

6. EQUATIONS FOR COMPONENTS OF VELOCITIES $v_{i,e1}$

We now obtain equations for components of velocities $v_{i,e1}$, using Equations (21) and (22).

6.1. Equations for $v_{i,e1y}$

From Equations (A3), (21), and (22), we find, using notations (A6),

$$v_{i1y} = -\frac{1}{\omega_{ci}} F_{i1x} + \frac{1}{\omega_{ci}^2} \left[ \frac{\alpha_i}{\omega_{ci}} \left( \frac{\partial}{\partial y} \right)^{-2} \right] \frac{\partial^2 F_{i1x}}{\partial y^2} + \frac{1}{\omega_{ci}} \frac{\beta_{ei}}{\alpha_i} \left( \frac{\partial}{\partial y} \right)^{-2} \frac{\partial^3 F_{i1y}}{\partial y \partial t} - \frac{1}{\omega_{ci}^2} \left[ \frac{1}{\omega_{ci}^2} \left( \frac{\partial}{\partial y} \right)^{-2} \right] \frac{\partial^3 F_{i1y}}{\partial y^2 \partial t}$$

and

$$v_{e1y} = \frac{1}{\omega_{ce}} F_{e1x} + \frac{1}{\omega_{ce}^2} \left[ \frac{\alpha_e}{\omega_{ce}} \left( \frac{\partial}{\partial y} \right)^{-2} \right] \frac{\partial^2 F_{e1x}}{\partial y^2} + \frac{1}{\omega_{ce}} \frac{\beta_{ei}}{\alpha_e} \left( \frac{\partial}{\partial y} \right)^{-2} \frac{\partial^3 F_{e1y}}{\partial y \partial t} - \frac{1}{\omega_{ce}^2} \left[ \frac{1}{\omega_{ce}^2} \left( \frac{\partial}{\partial y} \right)^{-2} \right] \frac{\partial^3 F_{e1y}}{\partial y^2 \partial t}$$

We see that these equations are derived from one another by changing $i \leftrightarrow e$. We note that $\alpha_j, \beta_j \sim m_j$. However, we keep some small terms in these equations for the symmetry of both equations. The terms proportional to $\omega_{ci}^3$ are needed in the equations for $v_{i1x}$. We also note that $\alpha_i \sim (m_i/T_0)(\partial/\partial z)^{-2} \partial^2 / \partial t^2$ and $\beta_i \sim \alpha_i \Omega_{ei} / (\partial/\partial t + \Omega_{ei})$ if $T_0 \sim T_{i0} \sim T_0$. For these estimations, we use expressions (A18), (A25), (A26), and (A29), assuming $W_j \sim \partial/\partial t$.

6.2. Equations for $v_{i,e1x}$

Equations for $v_{i,e1x}$ are derived from Equation (A2) by using Equations (24) and (25).

6.3. Equations for $v_{i,e1z}$

From Equations (A7), (21), and (22), we obtain, using Equation (A6),

$$\frac{\partial^2 v_{i1z}}{\partial z \partial t} = \frac{1}{\omega_{ci}^2} \frac{\partial^2 F_{i1x}}{\partial y \partial t} - \frac{\alpha_i}{\omega_{ci}} \left( \frac{\partial}{\partial y} \right)^{-2} \frac{\partial^3 F_{i1y}}{\partial y^2 \partial t}$$

and

$$\frac{\partial^2 v_{e1z}}{\partial z \partial t} = \frac{1}{\omega_{ce}^2} \frac{\partial^2 F_{e1x}}{\partial y \partial t} - \frac{\alpha_e}{\omega_{ce}} \left( \frac{\partial}{\partial y} \right)^{-2} \frac{\partial^3 F_{e1y}}{\partial y^2 \partial t}$$

We note that in Equation (26) (Equation 27) the terms proportional to $\beta_e F_{e1x} \times (\alpha_e F_{e1x}, \beta_e F_{e1x})$ are small compared with $\alpha_i F_{i1x} \times (\beta_i F_{i1x})$. However, we also keep them for the symmetry of these equations.

7. COMPONENTS OF CURRENT

We now find components of the linear current $j_i = \sum_j q_j n_j \partial \psi_{j1}$, It is convenient to calculate the value $4\pi (\partial/\partial t)^{-1} j_i$. We also consider the electron–ion plasma in which $n_{ei} = n_{i0}, q_e = -q_i$. In our calculations, we use the equality $m_e v_{ei} = m_i v_{ie}$. From
Equations (A2), (24)–(27), and (5) in the linear approximation, we find

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1x} = a_{xx} E_{1x} - a_{xy} E_{1y} + a_{xz} E_{1z} - b_{tx}(v_{1x} - v_{e1x}) + b_{ty}(v_{1y} - v_{e1y}) - b_{tz}(v_{1z} - v_{e1z}), \]  

(28)

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1y} = a_{yx} E_{1x} + a_{yy} E_{1y} - a_{yz} E_{1z} - b_{yx}(v_{1x} - v_{e1x}) - b_{yy}(v_{1y} - v_{e1y}) + b_{yz}(v_{1z} - v_{e1z}), \]  

(29)

and

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1z} = -a_{xz} E_{1x} - a_{zy} E_{1y} + a_{zz} E_{1z} + b_{tx}(v_{1x} - v_{e1x}) + b_{ty}(v_{1y} - v_{e1y}) - b_{tz}(v_{1z} - v_{e1z}). \]  

(30)

Here, the following notations are introduced:

\[ a_{xx} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left[ 1 + \frac{\partial^2}{\partial y^2} \left( \frac{\partial}{\partial \zeta} \right)^{-2} \right], \quad a_{xy} = a_{yx}, \quad a_{yy} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left[ 1 + \frac{\partial^2}{\partial y^2} \left( \frac{\partial}{\partial \zeta} \right)^{-2} \right], \quad a_{yy} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left( \frac{\partial}{\partial y} \frac{\partial}{\partial \zeta} \right)^{-1}, \quad a_{zz} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left( \frac{\partial}{\partial y} \frac{\partial}{\partial \zeta} \right)^{-1}, \quad b_{ij} = a_{ij} \frac{m_i}{m_e}, \]  

(31)

where \( \omega_{pi} = (4\pi n_0 q_i^2 / m_i)^{1/2} \) is the ion plasma frequency. The value \( \delta \) in the expression for \( a_{zz} \) has the form

\[ \delta = \left( \frac{\alpha_s}{m_i} + \frac{\alpha_e - \beta_s}{m_e} \right). \]  

(32)

8. SIMPLIFICATION OF COLLISION CONTRIBUTION

The relationship between \( \omega_{ce} \) and \( v_{ei} \), or \( \omega_{ci} \) and \( v_{ie} \) (that is, the same), can be arbitrary in Equations (28)–(30) (except for that in thermal conduction). We proceed further by taking into account that \( \partial / \partial t \ll \omega_{ci} \). In this case, we can neglect the collisional terms proportional to \( b_{xy} \) and \( b_{yx} \) (see Equation (31)). However, a system of Equations (28)–(30) remains sufficiently complex to find \( j_1 \) through \( E_1 \). Therefore, we further consider the case in which the following condition is satisfied:

\[ 1 \gg \frac{v_{ie} \partial}{\omega_{ci}^2} (\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial \zeta^2}) \left( \frac{\partial}{\partial \zeta} \right)^{-2}. \]  

(33)

It is clear that Equation (33) can be realized easily. In this case, we can neglect collisional terms proportional to \( b_{x,zz} \) and \( b_{y,zy} \). Then, the system of Equations (28)–(30) takes the form

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1x} = \varepsilon_{xx} E_{1x} - \varepsilon_{xy} E_{1y} + \varepsilon_{xz} E_{1z}, \]  

(34)

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1y} = \varepsilon_{yx} E_{1x} + \varepsilon_{yy} E_{1y} - \varepsilon_{yz} E_{1z}, \]  

(35)

and

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1z} = -\varepsilon_{xz} E_{1x} - \varepsilon_{zy} E_{1y} + \varepsilon_{zz} E_{1z}. \]  

(36)

Here,

\[ \varepsilon_{xx} = a_{xx} \varepsilon_{xy} = a_{xy} \frac{v_{ie}}{\omega_{ci}^2} \left( \frac{\partial^2}{\partial y \partial t} \right)^{-1}, \quad \varepsilon_{xz} = a_{xz} \frac{v_{ie}}{\omega_{ci}^2} \left( \frac{\partial^2}{\partial z \partial t} \right)^{-1}, \quad \varepsilon_{xy} = a_{yx} \frac{v_{ie}}{\omega_{ci}^2} \left( \frac{\partial^2}{\partial y \partial t} \right)^{-1}, \quad \varepsilon_{yy} = a_{yy} \frac{v_{ie}}{\omega_{ci}^2} \left( \frac{\partial^2}{\partial z \partial t} \right)^{-1}, \quad \varepsilon_{yz} = a_{yz} \frac{v_{ie}}{\omega_{ci}^2} \left( \frac{\partial^2}{\partial y \partial t} \right)^{-1}, \quad \varepsilon_{zz} = a_{zz} \frac{v_{ie}}{\omega_{ci}^2} \left( \frac{\partial^2}{\partial z \partial t} \right)^{-1} \]  

(37)

where

\[ d = a_{zz} \frac{v_{ie}}{\omega_{pi}^2} \frac{\partial}{\partial t} = v_{ie} m_i \delta. \]  

(38)

Parameter \( d \) defines the collisionless, \( d \ll 1 \), and collisional, \( d \gg 1 \), regimes. We derive the dispersion relation below.
9. DISPERSION RELATION AND ELECTRIC FIELD POLARIZATION

We further consider Equations (34)–(36) in the Fourier representation, assuming that perturbations have the form \( \exp(\text{i}k \mathbf{r} - \text{i} \omega t) \). Then, using Equations (7) and (8), we obtain the following system of equations:

\[
(n^2 - \varepsilon_{xx}) E_{1 \text{zk}} + \varepsilon_{yy} E_{1 \text{yk}} - \varepsilon_{zz} E_{1 \text{zk}} = 0, \\
-\varepsilon_{yx} E_{1 \text{zk}} + (n_z^2 - \varepsilon_{yy}) E_{1 \text{yk}} + (-n_y n_z + \varepsilon_{yz}) E_{1 \text{zk}} = 0, \quad \text{and} \\
\varepsilon_{zx} E_{1 \text{zk}} + (-n_y n_z + \varepsilon_{yz}) E_{1 \text{yk}} + (n_y^2 - \varepsilon_{zz}) E_{1 \text{zk}} = 0,
\]

(39)

where \( E_{1 \text{zk}} \) is the Fourier image of the electric field perturbation, \( \mathbf{n} = k \mathbf{c}/\omega \). The index \( k \) by \( E_{1 \text{zk}} \) is equal to \( k = (k, \omega) \). For the Fourier images of operators \( \varepsilon_{ij} \) and \( d \), we keep the same notations. In general, we see that the longitudinal electric field \( E_{1 \text{zk}} \sim E_{1 \text{yk}} \) inevitably arises when \( k_y \neq 0 \) and \( n_z^2 - \varepsilon_{zz} \neq 0 \). The dispersion relation can be found by setting the determinant of the system, Equation (39), equal to zero. The contribution of terms proportional to \( \varepsilon_{yy} \) and \( \varepsilon_{yz} \) into the dispersion relation is small compared with that of terms \( \sim \varepsilon_{xx}, \varepsilon_{xy} \) according to Equation (18) and \( \alpha_i, \beta_i \ll 1 \). Neglecting these small terms, we obtain

\[
(n^2 - \varepsilon_{xx}) E_{1 \text{zk}} - \varepsilon_{xz} E_{1 \text{zk}} = 0, \quad (n_z^2 - \varepsilon_{yy}) E_{1 \text{yk}} + (-n_y n_z + \varepsilon_{yz}) E_{1 \text{zk}} = 0, \quad \text{and} \\
\varepsilon_{zx} E_{1 \text{zk}} + (-n_y n_z + \varepsilon_{yz}) E_{1 \text{yk}} + (n_y^2 - \varepsilon_{zz}) E_{1 \text{zk}} = 0.
\]

(40)

Below, we consider the dispersion relation for this system in the collisionless \( (d \ll 1) \) and collisional \( (d \gg 1) \) cases.

9.1. Collisionless Case

Consider the case

\( d \ll 1, \)

(41)

where an estimation of \( d \) is \( d \sim v_{le}/k_z^2 c_s^2 \) for \( \delta \sim \alpha_i/m_i \). Then the dispersion relation has two solutions when \( n_z^2 - a_{yy} = 0 \) and \( n_z^2 - a_{yy} \neq 0 \) (see the second equation in the system of Equation (40)).

\( n_z^2 - a_{yy} = 0 \). This case describes the Alfvén waves \( \omega^2 = k_z^2 c_A^2 \), where \( c_A = B_0/(4\pi n_0 m_i)^{1/2} \). Polarization of the electric field has the forms \( E_{1 \text{yk}} \neq 0 \) and \( E_{1 \text{zk}} = E_{1 \text{zk}} = 0 \).

\( n_z^2 - a_{yy} \neq 0 \). In this case, the dispersion relation reduces to \( \varepsilon_{zz} \sim 0 \). The contribution of all other terms can be shown to be small using Equations (31) and (37). The last equation means that

\( \delta \sim 0. \)

(42)

Polarization of these perturbations is the following:

\( E_{1 \text{zk}} \neq 0, \quad E_{1 \text{yk}} = \frac{n_z}{n_z} E_{1 \text{zk}}, \quad \text{and} \quad E_{1 \text{zk}} = \frac{a_{zz}}{(n_z^2 - a_{xx})} E_{1 \text{zk}}. \)

(43)

We see that the electric field in the plane of the wave vector is a potential one. However, the component \( E_{1 \text{zk}} \) can be large, \( E_{1 \text{zk}} \sim \alpha_i (k_z k_x \omega_{ci}/k^2 \omega) E_{1 \text{zk}} \). Thus, in general, perturbations have an electromagnetic nature.

9.2. Collisional Case

In the collisional case,

\( d \gg 1, \)

(44)

the dispersion relation of Equation (40) takes the form

\[
(n_z^2 - a_{yy}) a_{zz} = 0.
\]

(45)

We see from Equation (45) that the same cases, \( n_z^2 - a_{yy} = 0 \) and \( a_{zz} = 0 \), take place analogously to the case of \( d \ll 1 \) considered above. However, polarization in the case of \( n_z^2 - a_{yy} = 0 \) is different: \( E_{1 \text{zk}} = 0, \quad E_{1 \text{yk}} \neq 0, \quad \text{and} \quad E_{1 \text{zk}} = (n_z n_y d/a_{zz}) E_{1 \text{zk}} \sim (v_{le}/\omega_{ci}) E_{1 \text{yk}} \). This is a mixture of the Alfvén and magnetosonic waves. For the dispersion relation \( a_{zz} = 0 \), the electric field polarization is the following:

\[
E_{1 \text{zk}} \neq 0, \quad E_{1 \text{zk}} = \frac{a_{zz}}{(n_z^2 - a_{xx})} E_{1 \text{zk}}, \quad \text{and} \quad E_{1 \text{yk}} = \frac{(n_z n_y d - a_{zz})}{(n_z^2 - a_{yy})} E_{1 \text{zk}}.
\]

(46)

Thus, the electric field perturbation has all three components.

We see that in the case of \( \varepsilon_{zz} = 0 \), the electric field perturbation along the magnetic field plays the crucial role in the fast sound speed regime (Equation (20)).
10. SOLUTION OF DISPERSION RELATION $\varepsilon_{zz} = 0$

We now consider the dispersion relation, Equation (42), which is suitable also for the collisional case. Using Equations (23), (32), and (A16)–(A20), we obtain the following dispersion relation:

$$
T_0 \left( \gamma \frac{\partial}{\partial t} + \Omega_x + \Omega_{Te} - \Omega_{ne} \right) \left( \frac{\partial}{\partial t} + \Omega_{T_i} + 2\Omega_e \right) + T_0 \left( \gamma \frac{\partial}{\partial t} + \Omega_{T_i} - \Omega_m \right) \left( \frac{\partial}{\partial t} + \Omega_x + \Omega_{Te} + 2\Omega_e \right) - \Omega_{ne} (T_0 - T_0) \left( \frac{\partial}{\partial t} + \Omega_e + \Omega_{T_i} \right) \\
+ T_0 \Omega_{T_i e} \left( \frac{\partial}{\partial t} + \Omega_{T_i} \right) + T_1 \Omega_{T_i e} \left( \gamma \frac{\partial}{\partial t} + \Omega_{T_i} - \Omega_m \right) + T_0 \Omega_{T_i e} \left[ \left( \gamma - 1 \right) \frac{\partial}{\partial t} - \Omega_{ne} \right] = 0,
$$

(47)

where $\partial/\partial t = -i\omega$ and $\Omega_x = (\gamma - 1)(\chi_{e0}/n_0)k^2$. All terms $\Omega$ are defined by Equation (14). We see that the first four terms on the left-hand side of Equation (47) are symmetric concerning the contribution of electrons and ions. The third and fourth terms have appeared due to the dependence of collision frequency $v_{e,i}(n_{i,e}, T_i)$ on density perturbation. The last three terms are connected to the perturbation of this frequency because of the electron temperature perturbation. All terms proportional to $\Omega_{e,i,e}$ and $\Omega_{T_i,i,e}$ are connected with the energy exchange in thermal equations. We see that taking into account the collision frequency perturbation in a general case such as $T_0 \neq T_0$ results in considerable modification of the dispersion relation. However, this effect should be involved because the absence of thermal equilibrium between electrons and ions can occur, for example, in galaxy clusters (e.g., Markovitch et al. 1996; Fox & Loeb 1997; Ettori & Fabian 1998; Takizawa 1998). We further use the fact that $n_{i0} = n_{i0}$ in our case and $v_{e,i} \sim T_e^{-3/2}$ (Braginskii 1965). Then we obtain $\Omega_{e,i} = \Omega_e$ and $\Omega_{T_i,e} = \Omega_{T,i} = -3\Omega_e(T_0 - T_0)/2T_0$. In this case, Equation (47) takes the form

$$
2\gamma^2 \omega^2 + \left( [(\gamma + 1)(\Omega_x + \Omega_{Te} + \Omega_{T_i}) - \Omega_{ne} - \Omega_m] \omega - 2(\Omega_x + \Omega_{Te})\Omega_{T_i} + \Omega_m \Omega_{T_i} + \Omega_m(\Omega_x + \Omega_{Te}) \right) = 0.
$$

(48)

Below, we will consider the different limiting cases of Equation (48).

10.1. The Case of $\Omega_{ie} = 0$

If we do not take into account the energy exchange, $\Omega_{ie} = 0$, and set $T_0 = T_0$, we obtain the equation

$$
2\gamma^2 \omega^2 + \left( [(\gamma + 1)(\Omega_x + \Omega_{Te} + \Omega_{T_i}) - \Omega_{ne} - \Omega_m] \omega - 2(\Omega_x + \Omega_{Te})\Omega_{T_i} + \Omega_m \Omega_{T_i} + \Omega_m(\Omega_x + \Omega_{Te}) \right) = 0.
$$

(49)

It follows from Equation (49) that the ion cooling–heating function modifies the growth rate that takes place without this function. Neglecting the contribution of the ion cooling and heating, $\Omega_{T_i} = \Omega_m = 0$, we have

$$
\omega = -\frac{i}{2\gamma} [(\gamma + 1)(\Omega_x + \Omega_{Te}) - \Omega_{ne}].
$$

If we neglect the ion temperature perturbation, i.e., neglect the second term $\sim T_0$ in Equation (48), we obtain the usual isobaric solution

$$
\omega = -\frac{i}{\gamma}(\Omega_x + \Omega_{Te} - \Omega_{ne}).
$$

We also see from Equation (48) that for the short-wavelength perturbations when $\Omega_x \gg \omega, \Omega_{Te}, \Omega_{ne}$, thermal instability can arise due to the ion cooling function

$$
\omega = -\frac{i}{T_0 + \gamma T_0} \left[ (T_0 + T_0)\Omega_{T_i} - T_0 \Omega_m \right].
$$

10.2. The Case of $\Omega_{ie} = \infty$

When the frequency $\Omega_{ie}$ is much larger than other frequencies, $2\Omega_{ie} \gg \partial/\partial t, \Omega_x, \Omega_{T_i,i,e}$, and $T_0 = T_0$, the dispersion relation becomes the following:

$$
\omega = -\frac{i}{2\gamma} (\Omega_x + \Omega_{Te} - \Omega_{ne} + \Omega_{T_i} - \Omega_m).
$$

(50)

This is an isobaric solution with the electron and ion cooling.

For the different temperatures of electrons and ions, $T_{ie} \neq T_{0i}$, we obtain

$$
\frac{i}{2} \left[ T_0 + \left( \frac{3T_0}{T_{ie}} \right) T_{0i} \right] \omega = (3T_{ie} - T_{0i})(\Omega_x + \Omega_{Te}) - \left[ \frac{5}{2} T_{0i} - 3T_{0i} \left( 1 + \frac{T_{0i}}{2T_0} \right) \right] \Omega_{T_i} - \frac{1}{2} (3T_{ie} + T_{0i}) \left( \Omega_{ne} + \frac{T_{0i}}{T_{ie}} \Omega_m \right).
$$

(51)
In the case of $T_e \gg T_i$, this equation takes the form
\[ \omega = -\frac{i}{\gamma} [6(\Omega_{x} + \Omega_{Te}) - \Omega_{ne} - 5\Omega_{T_i}]. \]

In the opposite case, $T_e \ll T_i$, we obtain
\[ \omega = -\frac{i}{\gamma} (\Omega_{T_i} - \Omega_{ni}). \]

10.3. General Case

In a general case, Equation (48) can be written in the form
\[ g_0\omega^2 + ig_1\omega - g_2 = 0, \tag{52} \]
where
\[ g_0 = \gamma(T_e + T_i), \quad g_1 = [(\gamma T_i + T_e)(\Omega_{x} + \Omega_{Te}) + (\gamma T_i + T_e)\Omega_{T_i} - T_e\Omega_{ne} - T_i\Omega_{ni}] \]
\[ + \frac{1}{2} \gamma \left[ T_e + T_i \left( 4 + \frac{3}{T_e} \right) \Omega_{ne}, \quad \text{and} \right] \]
\[ g_2 = T_e(\Omega_{x} + \Omega_{Te} - \Omega_{ne})\Omega_{T_i} + T_i(\Omega_{x} + \Omega_{Te} - \Omega_{ni})\Omega_{T_i} + (3T_e - T_i)\Omega_{ne}(\Omega_{x} + \Omega_{Te}) \]
\[ - \left[ \frac{5}{2} T_e - 3T_i \left( 1 + \frac{T_i}{2T_e} \right) \Omega_{ni}\Omega_{T_i} - \frac{1}{2}(T_e + 3T_i)\Omega_{ne} \right. \]
\[ \left. \left( \Omega_{ne} + \Omega_{ni} \frac{T_e}{T_i} \right) \right). \tag{53} \]

Equation (52) can be solved numerically for different cooling functions.

11. DISCUSSION

From the results obtained above, we can estimate the relative perturbations of the number density and pressure in the fast sound speed regime. Using Equations (24)–(27) and keeping the main terms, we find expressions for $\nabla \cdot \mathbf{v}_j$,
\[ \frac{\partial}{\partial t} \nabla \cdot \mathbf{v}_1 \simeq \frac{\partial}{\partial z}(\alpha_i F_{i1z} + \beta_e F_{e1z}), \quad \frac{\partial}{\partial t} \nabla \cdot \mathbf{v}_e \simeq \frac{\partial}{\partial z}(\alpha_e F_{e1z} + \beta_i F_{i1z}). \tag{54} \]

Thus, the number density perturbation $n_{j1} \sim \nabla \cdot \mathbf{v}_j$ is determined by the longitudinal electric field $E_z$. If we use the condition of quasi-neutrality, $\nabla \cdot \mathbf{v}_1 = \nabla \cdot \mathbf{v}_e$, and the relation $m_i F_{i1} = -m_e F_{e1}$, we obtain the dispersion relation, Equation (42). From Equations (21) and (22), taking into account polarizations (43) and (46), we have
\[ \frac{\partial P_{j1}}{\partial z} \simeq -\frac{\partial F_{j1z}}{\partial t}, \tag{55} \]
where the value $P_{j1}$ is connected with the pressure perturbation through the relation
\[ P_{j1} = -\frac{1}{m_j n_j} \frac{\partial p_{j1}}{\partial t}. \tag{56} \]

From the linear continuity equation and Equations (54)–(56), we find
\[ \frac{\partial^2 n_{i1}}{\partial t^2} = -\left( \frac{\alpha_i}{m_i} - \frac{\beta_e}{m_e} \right) \frac{\partial^2 P_{i1}}{\partial z^2}, \quad \text{and} \quad \frac{\partial^2 n_{e1}}{\partial t^2} = -\left( \frac{\alpha_e}{m_e} - \frac{\beta_i}{m_i} \right) \frac{\partial^2 P_{e1}}{\partial z^2}. \]

It follows from here that the relation between $p_{i1}$ and $p_{e1}$ is
\[ \left( \frac{\alpha_i}{m_i} - \frac{\beta_e}{m_e} \right) p_{i1} = \left( \frac{\alpha_e}{m_e} - \frac{\beta_i}{m_i} \right) p_{e1}. \]

The sum of pressures, $p_{e1} + p_{i1}$, is equal to
\[ p_{e1} + p_{i1} = \delta \left( \frac{\alpha_i}{m_i} - \frac{\beta_e}{m_e} \right)^{-1} p_{e1} \ll p_{e1}. \]

Thus, the total pressure almost does not change. An estimation of the value $\alpha_i/m_i - \beta_e/m_e$ gives (see Equation (23))
\[ \alpha_i/m_i - \beta_e/m_e \sim \frac{1}{T_0} \left( \frac{\partial}{\partial z} \right)^{-2} \left( \frac{\partial^2}{\partial t^2} \right) \]
\[ (T_{e0} \sim T_{i0} \sim T_0). \]

Thus, we obtain $n_{i1}/n_0 \sim p_{i1}/p_0$. 

9
Our dispersion relation, Equation (47), does not depend on the wave vector $\mathbf{k}$ (except $\Omega_\perp$) and magnetic field $B_0$. This independence from $\mathbf{k}$ is connected with the limiting case (Equation (20)). The absence of the magnetic field is a result of the main contribution to $\mathbf{k}$ in the case of a strong coupling. The intermediate case is described by Equation (52), where coefficients in Equation (53) contain different perturbations given in some papers (see Section 1) is not adequate. In particular, Equation (49) is available for a weak thermal coupling and equal temperatures, while Equations (50) and (51) describe different cooling–heating functions and temperatures allows us to consider various cases that can be realized in real situations.

The general form of the dispersion relation, Equation (48), including the thermal exchange between electrons and ions and their different cooling–heating functions and temperatures allows us to consider various cases that can be realized in real situations. In particular, Equation (49) is available for a weak thermal coupling and equal temperatures, while Equations (50) and (51) are appropriate in the case of a strong coupling. The intermediate case is described by Equation (52), where coefficients in Equation (53) contain different perturbations and cooling functions. The large thermal conductivity $\chi$ stabilizes some perturbations. We note that one particular Field length (Field 1965) can be obtained from the condition $\Omega_\perp = -\Omega_T e + \Omega_{ne}$. However, in the limit $\Omega_\perp \gg \omega$, $\Omega_T e, \Omega_{ne}, 2\Omega_{ri}$, Equation (48) describes instability due to ion cooling (see Section 10).

We have shown that unstable perturbations have an electromagnetic nature (see Equations (43) and (46)). Thus, the consideration of potential perturbations given in some papers (see Section 1) is not adequate. From the system, Equation (40), we see that for condensation mode $\epsilon_{zz} = 0$ in the fast sound speed regime (Equation (20)) the longitudinal electric field $E_1$ plays a crucial role. The transverse wavelengths of unstable perturbations can be both larger and smaller than the longitudinal ones.

The contribution of collisions between electrons and ions in the momentum equations depends on the parameter $d$ defined by Equation (38). In both limits in Equations (41) ($d \ll 1$) and (44) ($d \gg 1$), the dispersion relation has the same form, $\epsilon_{zz} = 0$.

12. ASTROPHYSICAL IMPLICATIONS

The fundamental purpose of this paper is to investigate thermal instability, taking into account the real multicomponent nature of an appropriate medium (e.g., for galaxy clusters) in a straightforward manner using the $E$ approach. Therefore, the different cooling–heating functions for electrons and ions, the energy exchange between them, and the perturbation of the energy exchange collision frequency for the non-equilibrium background state need to be included. The last effect considered for the first time for thermal instabilities contributes to the dispersion relation with the same order of magnitude as other effects and considerably modifies it (see, e.g., Equations (50) and (51)). We note that the effects mentioned above are not considered in the astrophysical literature using the MHD approach. The key condition for the results obtained is Equation (20). It denotes that we consider sufficiently short wavelengths along the magnetic field. In the simplest case without the energy exchange and ion temperature perturbations, the growth rate is in the region of the isobaric solution

$$\omega = -\frac{i}{\gamma}(\Omega_\perp + \Omega_T e - \Omega_{ne})$$

(see Section 10.1). We see the same growth rate for the condensation mode, for example, for the neutral and magnetized media when the perturbation wavelength tends to infinity (e.g., Field 1965; Heyvaerts 1974; Loewenstein 1990; Balbus 1991). Thus, from Equation (48), we can recover the classical result.

Conditions on the perturbation scale lengths transverse to the magnetic field are given by Equations (18), (19), and (33), which can be written in the form

$$\frac{k^2}{k_z} \ll \frac{\omega^2}{\omega_i^2}, \frac{\omega^2}{v_i \omega}, k_z^2 \ll \frac{\omega^2}{c_s^2}.$$
short-wavelength perturbations, which must be stable because of the large electron thermal conduction, can be unstable due to
the contribution to the cooling of a medium from ions (see Section 10). In the fast sound regime, when growth rates do not depend
on the magnetic field and the wavelengths of perturbations, both filaments as well as clouds and pancakes can be observed. It
is important to keep in mind that in this regime the electric field and electric currents of species along the magnetic field lines play a
crucial role. This result could be used for the diagnostics of the magnetic field direction in unstable domains.

13. CONCLUSION

We have treated thermal instability in the electron–ion magnetized plasma, which is relevant to galaxy clusters, solar corona, and
other two-component plasma objects. The multicomponent plasma approach has been applied to derive the dispersion relation for
the condensation modes in the case in which the dynamical frequency is much slower than the sound frequency. Our dispersion relation
takes into account the electron and ion cooling–heating functions, collisions in momentum equations, energy exchange in thermal
equations, different background temperatures of electrons and ions, and perturbation of the energy exchange frequency due to density
and temperature perturbations. Different limiting cases of the dispersion relation have been considered and simple expressions for
the growth rates have been obtained. We have shown that perturbations have an electromagnetic nature. The important role of the
electric field perturbation along the background magnetic field has been demonstrated. We have found that at the conditions under
consideration, condensation must occur along the magnetic field lines, while the transverse scale sizes can be both larger and smaller
than the longitudinal ones. General expressions for the dynamical variables obtained in this paper can be applied to a wide range of
astrophysical and laboratory plasmas also containing neutrals and dust grains. The results obtained can be useful for interpreting
observations of dense cold regions in astrophysical objects such as IGM, solar corona, and so on.

I thank Dr. Mohsen Shadmehri for valuable discussions and suggestions and also the anonymous referee, whose useful comments
helped to improve the manuscript.

APPENDIX

A.1. Perturbed Velocities of Species

In the linear approximation, Equation (1) for the perturbed velocity \( v_{j1} \) takes the form

\[
\frac{\partial v_{j1}}{\partial t} = -\nabla p_{j1} + \frac{q_j}{m_j c} v_{j1} \times B_0,
\]

(A1)

where \( p_{j1} = n_j T_{j1} + n_{j1} \rho_{j1} \). From this equation, we can find solutions for the components of \( v_{j1} \). For simplicity, we assume that \( \partial / \partial x = 0 \) because a system is symmetric in the transverse direction relative to the \( z \)-axis. Then the x-component of Equation (A1) gives

\[
\frac{\partial v_{j1x}}{\partial t} = F_{j1x} + \omega_{cj} v_{j1y},
\]

(A2)

where \( \omega_{cj} = q_j B_0 / m_j c \) is the cyclotron frequency of species \( j \). Differentiating Equation (A1) over \( t \) and using Equation (2) in the linear approximation as well as Equations (15), (16), and (A2), we obtain for the y-component of Equation (A1):

\[
\left( \frac{\partial^2}{\partial t^2} + \omega_{cj}^2 \right) v_{j1y} = \frac{\partial P_{j1}}{\partial y} + Q_{j1y},
\]

(A3)

where

\[
P_{e1} = -\frac{G_2}{D_{me}} \frac{\partial}{\partial t} \nabla \cdot v_{i1} + \left( \frac{T_{e0}}{m_e} - \frac{G_1}{D_{me}} \frac{\partial}{\partial t} \right) \nabla \cdot v_{e1},
\]

and

\[
P_{i1} = -\frac{G_3}{D_{mi}} \frac{\partial}{\partial t} \nabla \cdot v_{e1} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{D_{mi}} \frac{\partial}{\partial t} \right) \nabla \cdot v_{i1}.
\]

(A4)

The value \( P_{j1} \) is connected with the pressure perturbation (see Equation (A1)). Using Equations (A2) and (A3), we find

\[
\frac{\partial}{\partial t} \left[ \left( \frac{\partial^2}{\partial t^2} + \omega_{cj}^2 \right) v_{j1x} - Q_{j1x} \right] = \frac{\partial P_{j1}}{\partial y}.
\]

(A5)

In Equations (A3) and (A5), notations

\[
Q_{j1y} = -\omega_{cj} F_{j1y} + \frac{\partial F_{j1y}}{\partial t}, \quad Q_{j1x} = \omega_{cj} F_{j1x} + \frac{\partial F_{j1x}}{\partial t},
\]

(A6)

are introduced. We see from these equations that the thermal pressure effect on the velocity \( v_{j1x} \) is much larger than that on \( v_{j1y} \) when \( \partial / \partial t \ll \omega_{cj} \). The z-component of Equation (A1) can be written in the form

\[
\frac{\partial^2 v_{j1z}}{\partial t^2} = \frac{\partial P_{j1}}{\partial z} + \frac{\partial F_{j1z}}{\partial t}.
\]

(A7)
A.2. Calculation of $\nabla \cdot \mathbf{v}_{j1}$ and $P_{j1}$

We have

\[
\nabla \cdot \mathbf{v}_{j1} = \frac{\partial v_{j1y}}{\partial y} + \frac{\partial v_{j1z}}{\partial z}.
\]

(A8)

Using Equations (A3), (A4), (A7), and (A8), we obtain

\[
L_{1e} \nabla \cdot \mathbf{v}_{e1} + L_{2e} \nabla \cdot \mathbf{v}_{i1} = H_{e1}, \quad \text{and} \quad L_{1i} \nabla \cdot \mathbf{v}_{i1} + L_{2i} \nabla \cdot \mathbf{v}_{e1} = H_{i1}.
\]

(A9)

Here,

\[
H_{j1} = \frac{\partial^2 Q_{j1y}}{\partial y \partial t^2} + \left( \frac{\partial^2 F_{j1z}}{\partial z \partial t} + \omega_c^2 \right)
\]

(A10)

and operators $L_{1j}$ and $L_{2j}$ are the following:

\[
L_{1e} = \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^2}{\partial t^2} - L_{3e} \left( T_{e0} \frac{\partial}{\partial t} - \frac{G_1}{Dm_e} \frac{\partial}{\partial t} \right), \quad L_{1i} = \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^2}{\partial z^2} - L_{3i} \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right).
\]

(A11)

From the system in Equation (A9), we find

\[
L \nabla \cdot \mathbf{v}_{e1} = -L_{2e} H_{i1} + L_{1i} H_{e1}, \quad \text{and} \quad L \nabla \cdot \mathbf{v}_{i1} = -L_{2i} H_{e1} + L_{1e} H_{i1},
\]

(A12)

where

\[
L = L_{1e} L_{1i} - L_{2e} L_{2i}.
\]

(A13)

The values $P_{i1}$ and $P_{e1}$ can be found by substituting solutions in Equation (A12) for expressions in Equation (A4),

\[
LP_{i1} = \left[ G_3 \frac{\partial}{\partial t} L_{2e} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right) L_{1e} \right] H_{i1} - \left[ G_3 \frac{\partial}{\partial t} L_{1i} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right) L_{2i} \right] H_{e1},
\]

(A14)

and

\[
LP_{e1} = \left[ G_2 \frac{\partial}{\partial t} L_{2i} + \left( \frac{T_{e0}}{m_e} - \frac{G_1}{Dm_e} \frac{\partial}{\partial t} \right) L_{1i} \right] H_{e1} - \left[ G_2 \frac{\partial}{\partial t} L_{1e} + \left( \frac{T_{e0}}{m_e} - \frac{G_1}{Dm_e} \frac{\partial}{\partial t} \right) L_{2e} \right] H_{i1}.
\]

(A15)

A.3. Expressions for $D$ and $G_{1,2,3,4}$

We now give expressions for values given by Equation (17):

\[
D = \left( \frac{\partial}{\partial t} + \Omega_x + \Omega_{Te} \right) \left( \frac{\partial}{\partial t} + \Omega_{Ti} + \Omega_{Te} \right) \frac{\partial^2}{\partial t^2} + (\Omega_{ei} + \Omega_{Te}) \left( \frac{\partial}{\partial t} + \Omega_{Ti} \right) \frac{\partial^2}{\partial t^2}.
\]

(A16)

\[
G_1 = T_{e0} \left[ \Omega_{ne} - (\gamma - 1) \frac{\partial}{\partial t} \right] \left( \frac{\partial}{\partial t} + \Omega_{Ti} + \Omega_{Te} \right) \frac{\partial}{\partial t} + \Omega_{ei} (T_{e0} - T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_{Ti} \right) \frac{\partial}{\partial t}.
\]

(A17)

\[
G_2 = \Omega_{ei} T_{i0} \left[ \Omega_{ni} - (\gamma - 1) \frac{\partial}{\partial t} \right] \frac{\partial}{\partial t} + \Omega_{ei} (T_{e0} - T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_{Ti} \right) \frac{\partial}{\partial t}.
\]

(A18)

\[
G_3 = (\Omega_{Te} + \Omega_{Te}) T_{e0} \left[ \Omega_{ne} - (\gamma - 1) \frac{\partial}{\partial t} \right] \frac{\partial}{\partial t} - \Omega_{ei} (T_{e0} - T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_x + \Omega_{Te} \right) \frac{\partial}{\partial t}.
\]

(A19)

and

\[
G_4 = T_{i0} \left( \frac{\partial}{\partial t} + \Omega_x + \Omega_{Te} + \Omega_{Te} + \Omega_{ei} \right) \left[ \Omega_{ni} - (\gamma - 1) \frac{\partial}{\partial t} \right] \frac{\partial}{\partial t} - \Omega_{ei} (T_{e0} - T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_x + \Omega_{Te} \right) \frac{\partial}{\partial t}.
\]

(A20)
A.4. Simplification of Equations (A14) and (A15)

We further calculate coefficients by $H_{i1}$ in Equations (A14) and (A15). Using Equation (A11), we find

$$\frac{G_3}{D m_i} \frac{\partial}{\partial t} L_{i1} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{D m_i} \frac{\partial}{\partial t} \right) L_{i2} = \frac{G_3}{D m_i} \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^3}{\partial t^3}$$

(A21)

and

$$\frac{G_3}{D m_i} \frac{\partial}{\partial t} L_{i2} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{D m_i} \frac{\partial}{\partial t} \right) L_{i1} = \frac{1}{D} \left( \frac{T_{i0}}{m_i} - \frac{G_4}{m_i} \frac{\partial}{\partial t} \right) \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^2}{\partial t^2} + \frac{1}{D m_i m_i L_{3i} K}.$$  \hspace{1cm} (A22)

In Equation (A22), we have introduced the notation

$$K = \frac{1}{D} \left( G_2 G_3 - G_1 G_4 \right) \frac{\partial^2}{\partial t^2} + \left( T_{i0} G_4 + T_{i0} G_1 \right) \frac{\partial}{\partial t} - D T_{i0} T_{i0}.$$ \hspace{1cm} (A23)

Calculations show that the value $(G_2 G_3 - G_1 G_4)$ has a simple form, i.e.,

$$\frac{1}{D} \left( G_2 G_3 - G_1 G_4 \right) = \Omega_{i e} \left( T_{e0} - T_{i0} \right) T_{i0} \left[ \Omega_{n e} - (\gamma - 1) \frac{\partial}{\partial t} \right] + \Omega_{i e} \left( T_{i0} - T_{e0} \right) T_{i0} \left[ \Omega_{n i} - (\gamma - 1) \frac{\partial}{\partial t} \right]$$

$$- T_{i0} T_{i0} \left[ \Omega_{n e} - (\gamma - 1) \frac{\partial}{\partial t} \right] [\Omega_{n i} - (\gamma - 1) \frac{\partial}{\partial t}].$$ \hspace{1cm} (A24)

Using Equations (A16), (A17), (A20), and (A24), we obtain the simple form for the operator $K$ (Equation (A23)):

$$K = -\Omega_{i e} T_{e0} W_e \frac{\partial^2}{\partial t^2} - \left( \Omega_{i e} T_{i0} + \Omega_{T i} T_{i0} \right) T_{i0} W_i \frac{\partial^2}{\partial t^2} - T_{i0} T_{i0} W_i W_i \frac{\partial^2}{\partial t^2},$$ \hspace{1cm} (A25)

where notations

$$W_e = \gamma \frac{\partial}{\partial t} + \Omega_{T e} - \Omega_{n e}, \quad \text{and} \quad W_i = \gamma \frac{\partial}{\partial t} + \Omega_{T i} - \Omega_{n i}$$ \hspace{1cm} (A26)

are introduced. Using Equations (A21) and (A22), Equation (A14) for $P_{t1}$ takes the form

$$D L P_{t1} = \left[ \left( \frac{T_{i0}}{m_i} - \frac{G_4}{m_i} \frac{\partial}{\partial t} \right) \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^3}{\partial t^3} + \frac{1}{m_i} L_{3i} K \right] H_{i1} - \frac{G_3}{m_i} \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^3}{\partial t^3} H_{i1}.$$ \hspace{1cm} (A27)

A similar consideration of Equation (A15) leads to the following equation for $P_{e1}$:

$$D L P_{e1} = \left[ \left( \frac{T_{e0}}{m_e} - \frac{G_1}{m_e} \frac{\partial}{\partial t} \right) \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^3}{\partial t^3} + \frac{1}{m_e} L_{3i} K \right] H_{e1} - \frac{G_2}{m_e} \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^3}{\partial t^3} H_{e1}.$$ \hspace{1cm} (A28)

Operators

$$D \frac{T_{i0}}{m_i} - \frac{G_1}{m_i} \frac{\partial}{\partial t} \quad \text{and} \quad D \frac{T_{e0}}{m_e} - \frac{G_4}{m_e} \frac{\partial}{\partial t}$$

can be found by using Equations (A16), (A17), (A20), and (A26):

$$D \frac{T_{i0}}{m_i} - \frac{G_1}{m_i} \frac{\partial}{\partial t} = \frac{T_{i0}}{m_i} W_i \left( \frac{\partial}{\partial t} + \Omega_{T i} + \Omega_{c i} \right) \frac{\partial^2}{\partial t^2} + \frac{1}{m_i} \left( T_{i0} \Omega_{T i} + \Omega_{c i} \right) \frac{\partial^2}{\partial t^2}, \quad \text{and}$$

$$D \frac{T_{e0}}{m_e} - \frac{G_4}{m_e} \frac{\partial}{\partial t} = \frac{T_{e0}}{m_e} W_i \left( \frac{\partial}{\partial t} + \Omega_{c e} + \Omega_{T e} + \Omega_{e i} + \Omega_{T e} \right) \frac{\partial^2}{\partial t^2} + \frac{T_{e0}}{m_i} \Omega_{e i} \left( \frac{\partial}{\partial t} + \Omega_{c e} + \Omega_{T e} \right) \frac{\partial^2}{\partial t^2}. \hspace{1cm} (A29)$$

A.5. Operator $L$ in a General Form

Using Equation (A11), we find from Equation (A13):

$$L = M - N - \frac{1}{m_i} D L_{3i} K,$$ \hspace{1cm} (A30)

where

$$M = \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^4}{\partial t^4}, \quad \text{and} \quad N = \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^2}{\partial t^2} L_{3i} \left( \frac{T_{i0}}{m_i} - G_1 \frac{\partial}{\partial t} \right) + \left( \frac{\partial^2}{\partial t^2} + \omega_c^2 \right) \frac{\partial^2}{\partial t^2} L_{3i} \left( \frac{T_{i0}}{m_i} - \frac{G_4}{D m_i} \frac{\partial}{\partial t} \right). \hspace{1cm} (A31)$$
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