Azimuthal angle for boson-jet production in the back-to-back limit

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We show for the first time that the azimuthal angle between a vector boson and a jet, when using the Winner-Take-All recombination scheme, can be predicted at high precision in the back-to-back limit in the transverse plane. Specifically, we present a factorization theorem, and obtain numerical predictions at next-to-next-to-leading logarithmic (NNLL) accuracy. To allow for improved angular resolution, we provide results for track-based jet reconstruction, which only requires minimal changes in the calculation. We also find that linearly-polarized transverse momentum dependent (TMD) beam and jet functions enter at next-to-leading order (NLO) in the factorization theorem, originating from spin superpositions for one gluon, rather than the known case of spin correlations between gluons. We validate the switch from calorimetry to tracks using PYTHIA, and confirm the presence of linearly-polarized TMD functions using MCFM.

Introduction. – At hadron colliders, the production of an electroweak boson recoiling against a jet constitutes one of the simplest processes beyond the 0-jet case, and one of the overall simplest to involve all gauge sectors of the Standard Model (SM). It is a process relevant for many SM investigations (e.g. to control b-tagging for $t\bar{t}$ measurements), and is an important process to investigate medium effects in heavy-ion collisions. A thorough theoretical understanding of these processes is therefore of significant importance.

While the beam-jet-boson system is planar at leading partonic order, additional (primarily QCD) radiation lifts this planarity (see Fig. 1) and decorrelates the azimuthal orientation of jet and boson, which has been measured at ATLAS and CMS. Near the back-to-back limit, the initial and final state QCD radiation is necessarily soft or collinear to the beams or the jet, and gives rise to large logarithms, which can be resummed using Soft-Collinear Effective Theory (SCET). The purpose of this Letter is to investigate this azimuthal angular decorrelation at NNLL accuracy using the Winner-Take-All (WTA) recombination scheme for the jet.

Using the WTA axis both simplifies the factorization by significantly reducing the dependence on soft recoil effects, and allows us to achieve NNLL accuracy in the first place: non-global logarithms (NGLs) appear as new NLL effects when using the standard jet axis, but they are absent for the WTA axis.

To avoid being hamstrung by the poorer angular resolution of the calorimetry-based jet reconstruction, when compared to the inner detectors’ tracking system, we also make use of track functions to reconstruct jets solely from charged hadrons. We find that the switch to tracks merely requires us to change a constant in the factorization theorem: the matching correction for the jet function.

Finally, we find that the broken rotational symmetry around the beam axis forces us to include linearly polarized transverse-momentum-dependent (TMD) beam and jet functions in the factorization theorem at NLO when producing electroweak vector bosons with non-zero virtuality. This is somewhat unexpected for a leading hard process involving only one external gluon (in contrast to the known case of gluon fusion), and represents to our knowledge the first time a linearly-polarized jet function appears.

Factorization. – The azimuthal angle $\Delta \phi$ between the vector boson ($V$) and the jet ($J$) is directly related to the component $p_{x,V}$ of the vector boson transverse momentum (with respect to the beams) which is perpendicular to the plane defined by the colliding beams (labelled $a$, $b$) and the jet axis, see Fig. 1. Explicitly, $\pi - \Delta \phi \equiv \delta \phi \approx \sin(\delta \phi) = |p_{x,V}|/p_{T,V}$, where we introduce $\delta \phi$ such that the back-to-back limit corresponds to $\delta \phi \to 0$. Momentum conservation along this direction implies

$$p_{x,a} + p_{x,b} + p_{x,J} + p_{x,S} + p_{x,V} = 0,$$

where $p_{x,a}, p_{x,b}, (p_{x,S})$ originate from collinear (soft) initial- (initial- and final-) state radiation, and a non-zero jet contribution $p_{x,J}$ arises because the jet momentum and axis are not aligned for the WTA axis. Writing this momentum conservation in terms of the Fourier conjugate variable $b_z$, we obtain the following factorization

$$f_{V,a}(x) = f_{V,b}(x) = f_{V,S}(x) = f_{V}(x),$$

and

$$f_{V,J}(x) = \frac{\int_{-\infty}^{0} d\phi f_{V}(x, \phi)}{\int_{-\infty}^{0} d\phi}.$$
the momentum fractions are given and the soft function
\( S \) of the TMD beam functions, see Eq. (2), have the same virtuality and are only separated in rapidity. This requires a rapidity regulator, for which we adopt the \( \eta \)-regulator \([30,31]\), leading to rapidity divergences of \( 1/\eta \) and a corresponding evolution in the rapidity renormalization scale \( \nu \) that sums (large) rapidity logarithms. (For other choices of rapidity regulators, see e.g. \([32,33]\).) By evaluating the ingredients in Eq. (2) at their natural scales

\[
\mu_H \sim \nu_B \sim \nu_J \sim p_{T,V} \sim m_V, \\
\mu_B \sim \mu_J \sim \nu_S \sim 1/|b_x|,
\]

and evolving them to a common scale using the (rapidity) renormalization group, the logarithms of \( \delta \phi \) are summed. In this Letter we will present numerical results at NLL accuracy, which requires the ingredients in Eq. (2) at one-loop order, their anomalous dimensions at two-loop order \([39,41]\) and the cusp anomalous dimension at three-loop order \([39,45]\). We note that the anomalous dimensions for the linearly-polarized beam and jet functions are the same as their unpolarized counterparts. Furthermore, most of the ingredients for \( \mathcal{N}^3\text{LL} \) resummation are available.

**Ingredients.** – For our NLL predictions we need the hard function in Eq. (2) at one-loop order \([36,47]\), and a new contribution multiplying linearly-polarized gluon beam \([48]\) and jet functions that we calculate here at leading order:

\[
\mathcal{H}_{ij \rightarrow Vk}^L = \frac{x_a x_b p_{T,V}}{8 \pi s^2} |\mathcal{M}_w^L (ij \rightarrow V k)|^2
\]

with

\[
|\mathcal{M}_w^L (qg \rightarrow Vq)|^2 = \frac{32 \pi^2 c_{em} \alpha_s e_q^2}{N_c} \frac{u m_{V^2}}{s t},
\]

\[
|\mathcal{M}_w^L (qg \rightarrow Vg)|^2 = \frac{32 \pi^2 c_{em} \alpha_s e_q^2 (N_c^2 - 1)}{N_c^2} \frac{\hat{s} m_{V^2}}{\hat{u} t}.
\]

Eq. (5) is for the photon, and for the \( Z \) boson

\[
e_q^2 \rightarrow \frac{1 - 4|e_q| \sin^2 \theta_W + 8 e_q^2 \sin^4 \theta_W}{8 \sin^2 \theta_W \cos^2 \theta_W}.
\]

The Mandelstam variables in the hard function are

\[
\hat{s} = m_V^2 + 2 p_{T,V}^2 + 2 p_{T,V} \sqrt{m_V^2 + p_{T,V}^2} \cosh(\eta_J - y_V),
\]

\[
\hat{t} = -p_{T,V}^2 - p_{T,V} \sqrt{m_V^2 + p_{T,V}^2} \exp(\eta_J - y_V),
\]

\[
\hat{u} = -p_{T,V}^2 - p_{T,V} \sqrt{m_V^2 + p_{T,V}^2} \exp(y_V - \eta_J).
\]

(6)
In Eq. (2) a $\mathcal{H}^\perp$ gets accompanied by one linearly-polarized gluon beam or jet function. Since these start at order $\alpha_s$, we only need the LO of $\mathcal{H}^\perp$. Interestingly, the linearly-polarized contributions enter the cross section already at NLO, instead of NNLO for Higgs production \cite{23,49}. This is the first time a linearly-polarized jet function appears, which we discuss in more detail below.

Up to order $\alpha_s^2$, the soft function $S_{ijk}$ can be determined from the standard TMD soft function $S$ \cite{43,44,60}. For exchanges involving only two Wilson lines, we can perform a boost to make them back-to-back. Similar to \cite{51}, our observable is perpendicular to the boost, so only the regulator is affected (see e.g. \cite{52}), yielding

$$S_{ijk}(b_x,\eta,J,\mu,\nu) = -\sum_{i<j} T_i \cdot T_j S_{ij}^{(1)}(b_x,\mu,\nu \sqrt{n_i \cdot n_j}/2);$$

$$S_{ijk}(b_x,\eta,J,\mu,\nu) = -\sum_{i<j} T_i \cdot T_j S_{ij}^{(2)}(b_x,\mu,\nu \sqrt{n_i \cdot n_j}/2)$$

+ $\frac{1}{2} \left[ S_{ij}^{(1)}(b_x,\eta,J,\mu,\nu) \right]^2$. \hspace{1cm} (8)

The color factors are $T_\perp T_\perp = \frac{1}{3}$ and $T_\parallel T_\perp = T_\perp$. $T_\parallel = -\frac{3}{2}$, and $n_{\perp} \cdot n_{\perp} = 2$, $n_{\parallel} \cdot n_{\parallel} = 1 \mp \tan \eta$. The contribution involving exchanges between three Wilson lines vanishes due to color conservation \cite{53}.

The beam functions describe the transverse momentum of the colliding hard parton with respect to the beam axis (subscript $T$) due to collinear initial-state radiation. They have a perturbative matching onto PDFs

$$\mathcal{B}_i(x,\vec{b}_T,\mu,\nu) = \sum_j \int \frac{dx'}{x'} \mathcal{L}_{ij}(x,\vec{b}_T,\mu,\nu) f_j(x',\mu) \times [1 + \mathcal{O}(\vec{b}_T^2 A_{3CD})],$$

(9)

and the matching coefficients are known at two-loop \cite{44,53,57} and partially at three-loop order \cite{58,59}. In Eq. (2), we take $\vec{b}_T = (b_x,0)$.

We recalculated the TMD jet functions \cite{24,60} using the $\eta$-regulator, taking $\delta \phi \ll R$, which removes all dependence on the jet radius. In this limit the momentum of the initial parton is contained in the jet, which simplifies its expression. Writing $\mathcal{J}_i = 1 + \alpha_s/(4\pi) \mathcal{J}_i^{(1)} + \ldots$, \hspace{1cm} (10)

$$\mathcal{J}_q^{(1)}(b_\perp,\mu,\nu) = C_F \left[ L_b \left( 3 + 4 \ln \frac{\nu}{\omega} \right) + 7 - 2 \frac{\pi^2}{3} - 6 \ln 2 \right],$$

$$\mathcal{J}_g^{(1)}(b_\perp,\mu,\nu) = C_A \left[ \frac{11}{3} + 4 \ln \frac{\nu}{\omega} \right] + \frac{131}{18} - 2 \frac{\pi^2}{3} - \frac{22}{3} \ln 2$$

+ $T_{\perp} n_f \left[ -\frac{4}{3} L_b \frac{17}{9} + \frac{8}{3} \ln 2 \right]$, \hspace{1cm} (11)

where $L_b = \ln(\vec{b}_T^2 \mu^2 e^{2\gamma_E}/4)$ and $\omega = 2p_T e^{-2\gamma_E}$. Here $b_\perp$ is transverse to the jet axis, and in Eq. (2) we take it also perpendicular to the beams with $|b_\perp| = |b_x|$.

Linearly-polarized gluon jet function. – The linearly-polarized jet function is defined as

$$\mathcal{J}_g^{L}(b_\perp,\mu,\nu) = \frac{1}{d-3} \left( \frac{g_\perp^{\mu\nu}}{\frac{b_\perp^{\mu\nu}}{b_\perp^{\mu\nu}}} \right)^{T_{\perp} n_f} \frac{1}{N_c^2 - 1} \times \left( \delta(\omega - \vec{n} \cdot P) \delta^{d-2} (P_\perp B_{n_{\perp} \mu}(0) e^{i\vec{b}_\perp \cdot \vec{p}} B_{n_{\perp} \nu}(0) 0) \right).$$

(12)

It differs from the standard jet function by the factor in square brackets (which is otherwise $-\delta^{d-2}(P_\perp B_{a}(0)+\delta(\omega - \vec{n} \cdot P)\delta^{d-2} (P_\perp B^a_{\perp}(0) e^{i\vec{b}_\perp \cdot \vec{p}} B^a_{\perp}(0) 0)$).

Fig. 2 validates our factorization theorem at NLO, highlighting the need to include linearly-polarized gluon beam and jet functions. It shows the difference between the cross section obtained using our factorization in Eq. (2) and MCFM at NLO, with a cut $\delta \phi < \delta \phi^\text{cut}$. This difference should vanish in the limit $\delta \phi^\text{cut} \to 0$, but only does so when including the linearly-polarized gluon beam and jet function. (Note that the linearly-polarized contributions are not visible in the cross section differential in $\delta \phi$. ) The left panel shows the contribution involving $qq$ PDFs, which only involves linearly-polarized beam functions, and in the right panel we focus on the $n_f$ dependent contribution from $qq$ PDFs, to provide evidence for the linearly-polarized jet functions.

Track-based measurement. – The angular resolution of jet measurements is about 0.1 radians, due the size of the calorimeter cells, limiting the access to the resummation region. This can be overcome by measuring the jet using only charged particles, exploiting the superior angular resolution of the tracking systems at the LHC. Here we identify another advantage of the WTA axis: since the effect of soft radiation on the jet algorithm is power suppressed, switching to a track-based measurement only modifies the jet function. (Note that $p_T$, $\eta$ and $\phi$ do not require a fine angular resolution and are therefore measured on the full jet.) Consistency of the factorization theorem in Eq. (2) then implies that this track-based jet function $\mathcal{J}$ has the same anomalous dimension. We reach the same conclusion by a direct calculation using
FIG. 2. Difference between the singular cross section in Eq. (2) and the cross section from MCFM at NLO with a cut $\delta \phi < \delta \phi_{\text{cut}}$. Shown are the contribution from $qq$ (left) and $q\bar{q}$ (right) PDF flavors. Jets are identified by anti-$k_T$ algorithm with $R = 1$ and the leading jet fulfills $p_{T,j} > 60$ GeV and $|\eta_j| < 2$. In the right plot we only consider NLO corrections proportional to $n_f$.

FIG. 3. Predictions from Pythia for the azimuthal angle between the vector boson and jet, using all particles (green) or only charged particles (blue dotted).

track functions [21, 22]. Explicitly, the difference in the one-loop constant for the quark jet function is

$$\hat{f}_q^{(1)} = f_q^{(1)} + 4\alpha_s \int dx \frac{1 + x^2}{1 - x} \ln \frac{x}{1 - x} \int dz_1 T_q(z_1, \mu) \times \int dz_2 T_q(z_2, \mu) \{\theta(z_1 x - z_2(1 - x)) - \theta(x - \frac{1}{2})\},$$

in terms of the track functions $T_q(z, \mu)$. The change reflects the possibility of a hadronization mismatch in the WTA recombination: The losing (in WTA sense) parton may hadronize into the winning tracks. The expression for the gluon jet function involves the appropriate replacement of the splitting functions, and there is no modification to the linearly-polarized gluon jet function at order $\alpha_s$. We have verified using Pythia 8.2 [61] that using tracks only has a minimal effect on this measurement, see Fig. 3. For the standard jet axis, this difference is larger [62]. The conclusions reached here also apply to other angular measurements, such as in [24, 51, 60]. Recently, the ease of track functions for purely collinear measurements was demonstrated [63].

Resummed predictions. – We obtain predictions in Fig. 4 for the LHC with $\sqrt{s} = 13$ TeV, using the factorization formula in Eq. 4. Jets are identified by the anti-$k_T$ clustering algorithm with $R = 0.5$ and the WTA recombination scheme, and we require that the leading jet fulfills $p_{T,j} > 60$ GeV and $|\eta_j| < 2$. The electroweak parameters are $\alpha_{\text{em}} = 1/132.34$, $\cos \theta_W = 0.88168$ and $m_Z = 91.1876$ GeV, and we use the CT14nlo parton distribution functions [64] with $\alpha_s(m_Z) = 0.118$.

We show our resummed predictions in Fig. 4 at NLL+NLO and NNLL+NLO order, and compare to the NLO cross section obtained from MCFM. For our central curve we take $\mu_H = \sqrt{p_{T,V}^2 + m_V^2}$, $\nu_H = \mu_B = 2e^{-\gamma_E}/|b_0|$, $\nu_{B_{c,b}} = x_{a,b} \sqrt{s}$ and $\nu_f = \omega$. We estimate the perturbative uncertainty by varying $\mu_B$ and $\mu_H$ by a factor two around their central values, taking the envelope of the scale variations. The uncertainty bands of the NLL and NNLL predictions overlap, and are substantially reduced for the NNLL in the resummation region $\Delta \phi \gtrsim 170^\circ$. While the resummed predictions go to a constant in the back-to-back limit, the NLO becomes unreliable due to unresummed logarithms. At very low values of $p_{x,V}$, the scale $\mu_H$ hits the Landau pole. To avoid this unphysical behaviour, we apply $b^*$-prescription $b \to b^* = b/\sqrt{1 + b^2}$ [65]. On the other hand, for $\Delta \phi \lesssim 160^\circ$ the fixed-order corrections are important. These are included by matching to the NLO using a transition function, as in e.g. [60]. We also compare to Pythia, including the NLO $K$-factor of 1.6. The difference in shape for $\Delta \gtrsim 170^\circ$ is not significant, given the size of the NLL uncertainty band (a reasonable proxy for the Pythia uncertainty). We have verified that this is not due multiparton interactions or hadronization effects, which have a minimal effect on this observable.
**Conclusions.** – In this Letter we present the first prediction of the azimuthal angular distribution in boson-jet production at NNLL accuracy. Such high theoretical precision is achieved by the use of a recoil-free jet axis in the azimuthal angle definition, ensuring that non-global logarithms are absent. We demonstrate, using simulations at truth particle level, that measuring this angle using charged tracks, to exploit the finest angular resolution, yields almost exactly the same distribution as when using all the jet particles. Our theoretical predictions are based on the factorized expression, derived in SCET, involving all the jet particles. This factorization is checked at NLO by comparing to MCFM, verifying the necessity of including linearly-polarized TMD distributions.

Our work thus establishes a promising channel for precision studies of transverse momentum distributions in initial and final states of high energy collisions. This is intimately related to TMD PDFs and fragmentation functions in the literature, describing the nonperturbative regime $b_T \sim 1/\Lambda_{\text{QCD}}$, where the perturbative matching in Eq. (9) fails. However, the soft function is different compared to e.g. Drell-Yan, and so one cannot simply absorb it into the TMD parton distribution, as is customary [35, 37]. It is interesting to consider polarizaton effects from initial and final states. Besides, our work also serves as a baseline for pinning down the inner-working of the QCD medium produced in heavy-ion collisions [9], where the use of a recoil-free axis will be even more important to suppress effects from the huge underlying event background. All these intriguing questions are left for future research.

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