Application of density dependent parametrization models to asymmetric nuclear matter

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Density dependent parametrization models of the nucleon-meson effective couplings, including the isovector scalar $\delta$-field, are applied to asymmetric nuclear matter. The nuclear equation of state and the neutron star properties are studied in an effective Lagrangian density approach, using the relativistic mean field hadron theory. It is known that the introduction of a $\delta$-meson in the constant coupling scheme leads to an increase of the symmetry energy at high density and so to larger neutron star masses, in a pure nucleon-lepton scheme. We use here a more microscopic density dependent model of the nucleon-meson couplings to study the properties of neutron star matter and to re-examine the $\delta$-field effects in asymmetric nuclear matter. Our calculations show that, due to the increase of the effective $\delta$ coupling at high density, with density dependent couplings the neutron star masses in fact can be even reduced.

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1. Introduction

The understanding of the properties of nuclear matter, at both normal and high density regions, is of crucial importance in explaining the formation and structure of neutron stars after the supernova explosion. The experiments with unstable nuclear beams and relativistic heavy ions are potential tools in determining which are the best equations of state (EOS) that are able to describe hot and dense matter. The properties of neutron stars (NS) are characterized by masses and radii, which are obtained from an appropriate EOS at high densities. The EOS can be derived either from relativistic or potential models.

The nonlinear Walecka model (NLWM)\cite{1,2} and derivative scalar couplings\cite{3,4,5,6,7}, based on the relativistic mean-field (RMF) approach, have been extensively used to study the properties of nuclear and neutron matter, $\beta$-stable nuclei and then extended to the drip-line regions. In the last years some authors\cite{4,5,6,7,8} have stressed the importance of including the isovector scalar virtual $\delta(a_0(980))$ field in hadronic effective field theories for asymmetric nuclear matter. The role of the $\delta$ meson in isospin channels appears relevant at high densities\cite{4,5,6,7,8} and so of great interest in nuclear astrophysics.

In order to describe the medium dependence of nuclear interactions, a density dependent relativistic hadron field (DDRH) theory has been recently suggested\cite{8,9,10,11}. The density dependent meson-nucleon couplings are based on microscopic Dirac-Brueckner (DB) calculations\cite{11,12,13} and adjusted to reproduce some nuclear matter and finite nuclei properties\cite{9,10,11}. The main intention of this work is to apply different parametrizations of the density dependent meson-nucleon couplings, including the $\delta$ meson, to asymmetric nuclear matter. In particular we will see the predictions of the density dependent coupling models when applied to the neutron stars (NS). In fact it is known that the introduction of $\delta$-meson in the constant coupling model\cite{8} leads to heavier neutron stars in a nucleon-lepton picture. This is not obvious for density dependent models.

The paper is arranged as follows. In Sect.2 the model formalism is shortly derived. The meson-nucleon coupling parametrizations are presented in Sect.3. Results and discussions are given in Sects.4,5.

2. The model formalism

The Lagrangian density, with $\delta$ mesons, used in this work reads

\[
\mathcal{L} = \bar{\psi} i\gamma_\mu \partial^\mu - (M - g_\sigma \sigma - g_\delta \vec{\tau} \cdot \vec{\delta}) - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \rho^\mu \cdot \vec{b}_\mu |\psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 + \frac{1}{2} \omega_\mu \omega^\mu + \frac{1}{2} \rho_\mu \rho^\mu \cdot \vec{b}^\mu
\]
with the isoscalar (scalar,vector) \( \sigma, \omega, \rho \) and isovector (scalar,vector) \( \delta, \rho, \omega \), named \( \delta, \tilde{b}, \delta, \tilde{b} \), effective fields. \( F_{\mu \nu} \equiv \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu} \) and \( \dot{G}_{\mu \nu} \equiv \partial_{\mu} \delta_{\nu} - \partial_{\nu} \delta_{\mu} \).

The most important difference to conventional RMF theory is the contribution from the rearrangement self-energies to the DDRH baryon field equation. The meson-nucleon couplings \( g_{\sigma}, g_{\omega}, g_{\rho} \) and \( g_{\delta} \) are assumed to be vertex functions of Lorentz-scalar bilinear forms of the nucleon field operators. In most applications of DDRH theory, these couplings are chosen as functions of the vector density \( \rho^2 = j_\mu j^\mu \) with \( j_\mu = \bar{\psi} \gamma_\mu \psi \).

The equation of state (EOS) for nuclear matter at T=0 is obtained from the energy-momentum tensor. In a mean field approximation the energy density has the form \[ \epsilon = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} E_i^*(k) + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{2} m_\delta^2 \delta^2, \] and the pressure is

\[
p = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} E_i^*(k) - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{2} m_\delta^2 \delta^2 - \Sigma^R \rho, \]

where \( E_i^* = \sqrt{k^2 + M_i^*} \), \( i = p, n \). The nucleon effective masses are \( M^*_{p,n} = M - g_\sigma \sigma - g_\delta \delta_3 \) and \( M^* = M - g_\sigma \sigma + g_\delta \delta_3 \), where the scalar fields, \( \sigma \) (isoscalar) and \( \delta_3 \) (isovector) are expressed in terms of the corresponding local scalar densities. In the pressure a rearrangement term appears, in the density dependent cases, as

\[
\Sigma^R \rho = \left( \frac{\partial g_\sigma}{\partial \rho} \right) \frac{g_\sigma}{m_\sigma^2} \rho^2 + \left( \frac{\partial g_\delta}{\partial \rho} \right) \frac{g_\delta}{m_\delta^2} \rho^2 - \left( \frac{\partial g_\omega}{\partial \rho} \right) \frac{g_\omega}{m_\omega^2} \rho^2.
\]

The charge neutrality condition of its chemical potential, the charge neutrality condition

\[
\rho_c = \frac{1}{3\pi^2} \rho^3 = \rho_p = X_p \rho.
\]

Then, for a given \( \rho \), the \( X_p \) is related to the nuclear symmetry energy by

\[
3\pi^2 \rho X_p - [4E_{sym}(\rho)(1 - 2X_p)]^3 = 0.
\]

In the case of the \( (n, p, e^-, \mu^-) \) system, the constituents of neutron stars are neutrons, protons, electrons and muons. The threshold density for the appearance of muons is when the electron chemical potential is larger than the muon rest mass : \( \mu_e > m_\mu = 106.55 \text{ MeV} \). The chemical equilibrium for the \( (n, p, e^-, \mu^-) \) system reads

\[
\mu_\mu = \mu_e = \mu_n - \mu_p.
\]

The charge neutrality condition is

\[
\rho_p = \rho_e + \rho_\mu.
\]
with the muon density $\rho_\mu$ expressed as a function of its chemical potential

$$\rho_\mu = \frac{1}{3\pi^2} (\mu_\mu^2 - m_\mu^2)^{3/2}(\mu_e - m_\mu). \quad (13)$$

The proton fraction $X_p$ for $(npe)$ and $(npe\mu)$ systems can be obtained by solving Eq.10 and Eqs.11\,12\,13, respectively. The EOS for the $\beta$-stable $(npe)$ and $(npe\mu)$ matter can be estimated by using the obtained values of $X_p$. The equilibrium properties of the neutron stars can be finally studied by solving Tolman-Oppenheimer-Volkov (TOV) equations [16, 17] inserting the derived nuclear EOS as an input. We note that the presence of muons slightly increases the proton fraction for a fixed density, making the matter softer. We will see this effect in the final equilibrium properties.

### Table 1. Parameters of the model .

| Meson  | $m_i$ (MeV) | $\sigma$ | $\omega$ | $\rho$ | $\rho$ | $\delta$ |
|--------|-------------|---------|---------|-------|-------|-------|
|        | $g_i(\rho_0)$ | $a_i$  | $b_i$  | $c_i$  | $d_i$  |
|        | 550         | 1.36   | 0.23   | 0.41   | 0.90   |
| $m_\pi$| 783         | 1.40   | 0.17   | 0.34   | 0.98   |
| $m_\omega$| 770       | 0.095  | 2.17   | 0.05   | 17.84  |
| $m_\rho$| 770         | 0.095  | 2.17   | 0.05   | 17.84  |
| $m_\delta$| 980       | 0.02   | 3.47   | -0.09  | -9.81  |

$\rho_0 = 0.153 \text{ fm}^{-3}$

### FIG. 1: Density dependence of the meson-nucleon couplings.

### 3. Parametrizations of the meson-nucleon couplings

The parameters of the model include nucleon, ($M = 939\text{ MeV}$), and meson ($m_\sigma$, $m_\omega$, $m_\rho$, $m_\delta$, see Table 1) masses and the density dependent meson-nucleon couplings. The density dependence parametrization used here, inspired by DB calculations [11, 12, 13], was proposed [10] as:

$$g_i(\rho) = g_i(\rho_0) f_i(x), \quad \text{for } i = \sigma, \omega, \rho, \delta, \quad (14)$$

with

$$f_i(x) = \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}, \quad i = \sigma, \omega,$$

$$f_i(x) = a_i \exp[-b_i(x - 1)] - c_i(x - d_i), \quad i = \rho, \delta(15)$$

where $x = \rho/\rho_0$ and $\rho_0$ is the saturation density.

Parametrization form and parameters are taken from ref.[10] for $\sigma, \omega$ mesons and from ref.[7, 18] for $\rho, \delta$ mesons, respectively. All parameters are listed in Table 1. The density dependent couplings as a function of baryon density are displayed in Fig. 1.

### FIG. 2: Density dependence of symmetry energy. Insert: proton fraction of the corresponding $\beta$-equilibrium $(npe)$ system.

For symmetric matter at saturation ($\rho_0 = 0.153 \text{ fm}^{-3}$) we get a binding energy $E/A = \epsilon/\rho - M = -16.25\text{ MeV}$ and a compressibility modulus $K = 240\text{ MeV}$. In order to remark the effects of the coupling density dependence we will compare the results with a non-linear (NL) relativistic mean field model with constant couplings which presents very similar saturation properties (Set A of ref.[8]), including a symmetry energy $E_{sym} = 31.3\text{ MeV}$. Both effective models, NL and DDRH are rather soft for symmetric matter at high density, in agreement with relativistic collision data, [19, 20], and Dirac-Brueckner expectations [21, 22].

As shown in refs.[4, 8] when we include the $\delta$ coupling we have to increase the $\rho$ coupling in order to keep the same symmetry term at saturation (see Table 1). Since at higher densities the $\delta$ coupling is increasing while the $\rho$ one is decreasing (see Fig.1), as a result in the DDRH choice the symmetry term will be less repulsive than in the NL case. This can be clearly seen in Fig. 2 at densities above $2.5\rho_0$. In the insert we present the corresponding proton fraction in a $\beta$-equilibrated $npe$ system.
4. Neutron star results

The $\beta$-equilibrium nuclear matter is relevant for the composition of the neutron stars, as discussed in the previous Section. The EOS, pressure vs. density, for (npe) matter in the density dependent DDRH vs. NL - RMF models is reported in Fig. 3. We see that, at variance with the NL results, in the DDRH cases the EOS without $\delta$-meson is stiffer than that with the $\delta$-meson. This is partially due to the softening of the symmetry term in the DDRH$\rho\delta$ choice joined to a larger negative contribution to the pressure from the rearrangement term, see Eq. (3), as shown inside Fig. 3. We note that both effects are related to the density increase of the effective $g_\delta$ coupling (see Fig. 4), as expected from Dirac-Brueckner calculations [11, 12, 13].

We use the two effective nucleon-meson lagrangians, with and without density dependent couplings, to calculate neutron star (NS) properties, with particular attention to the $\delta$-field effects. The correlation between neutron star mass and radius for the $\beta$-equilibrium (npe) and (npe$\mu$) matter obtained by the DDRH (density dependent) and $NL-\text{RMF}(\text{constant couplings})$ parametrizations are shown in Fig. 4. The obtained maximum mass, corresponding radius and central density for the (npe) and (npe$\mu$) neutron star matter are reported in Table 2.

We first note that the NL$\rho$ and DDRH$\rho$ results are rather similar, with the DDRH$\rho$ interaction leading to a little softer matter, slightly smaller NS mass $M_{S}$ and radius $R$ and larger central density (see Table 2). When we include the $\delta$ coupling we observe a clear effect in opposite directions: the DDRH case becomes much softer while the NL - RMF choice shows a much stiffer behavior. This can be seen from Table 2, for the variations in $M_{S}/R$ and central densities, but in fact it is quite impressive as it appears in Fig 4, with reference to the close DDRH$\rho$/NL$\rho$ curves we see a clear shift to the “left” of the DDRH$\rho\delta$ predictions and just the opposite to the “right” for the NL$\rho\delta$ expectations.

In general we also see, in particular from Table 2, that the (npe) star matter, for all models, has slightly larger masses and radii, and lower central densities, than the (npe$\mu$) star matter. This is due to the fact that the (npe$\mu$) star matter has some larger proton fraction in the regions above a critical baryon density where the muon appears, as already noted at the end of Sect. 2.

5. Conclusion and outlook

All microscopic approaches of Dirac-Brueckner type to an effective meson-nucleon Lagrangian picture of the nuclear matter are predicting a density dependence of the couplings. We have studied the relative effects on the nuclear EOS at high baryon and isospin density, with application to nucleon-lepton neutron star properties. In particular we have focussed our attention on the contribution of the isovector-scalar $\delta$-meson. In fact in the “constant coupling” (NL - RMF) scheme the $\delta$ leads to very repulsive symmetry energy at high density. At variance in the “density dependence” case (DDRH) we can have a “softer” dense asymmetric matter due to combined mechanism of a decrease of the isovector-vector $g_\rho$ coupling and an increase of the $g_\delta$ (isovector scalar), which even leads to a larger pressure reduction from the rearrangement terms. The effect is clearly seen on equilibration properties of (npe) and/or (npe$\mu$) neutron stars, with an interesting decrease of the NS mass in the DDRH case when the $\delta$ contribution is included. We note that pure nucleon-lepton models cannot easily predict NS masses below two solar units. Our results seem to indicate that the large uncertainty of nucleon matter predictions, see the recent review [22], of relevance even for hybrid quark models, can be associated to the density dependence of the effective meson-nucleon couplings, in particular of the
Table 2. Maximum mass, corresponding radius and central density of the star by the different models.

| Model | Dens.Dip | RMF |
|-------|----------|-----|
|       | DDRHρ   | DDRHρδ | NLρ | NLρδ |
| neutron star properties | M$_S$/M$_⨀$ | 2.108 | 2.01 | 2.14 | 2.21 |
| (npe) matter | R(km) | 11.00 | 10.29 | 11.02 | 11.55 |
| (npe) matter | R$_c$/ρ$_0$ | 6.99 | 7.41 | 6.78 | 6.44 |
| (npeµ) matter | M$_S$/M$_⨀$ | 2.106 | 1.98 | 2.12 | 2.18 |
| (npeµ) matter | R(km) | 10.91 | 10.27 | 10.91 | 11.30 |
| (npeµ) matter | R$_c$/ρ$_0$ | 7.14 | 7.44 | 6.93 | 6.71 |

$g_\delta$.

In conclusion we remark the interest of future work on two main directions:

i) The importance of further DB confirmations of the high density behavior of the meson-nucleon effective couplings, in particular of some fundamental ground for the expected increase of the $g_\delta$;

ii) The study of dynamical effects of the isovector meson fields at the high baryon and isospin densities that can be reached in relativistic heavy ion collisions with exotic beams. Differential flows and particle productions appear to be rather promising observables, see the recent refs. [24, 25, 26, 27, 28].

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