On the sensitivity of the Mohr-Coulomb ductile fracture criterion in the mixed stress-strain space on varying hardening curve approximations

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Abstract. In recent years the use of the Mohr-Coulomb type fracture criteria has become popular in ductile fracture prediction. In ductile fracture prediction, fracture criteria are often transformed into the mixed stress strain space formulation and applied to numerical simulations in this formulation. This paper will investigate the shape of the Mohr Coulomb fracture criterion in the mixed stress strain space as a function of the used hardening curve formulation, in particular the hardening curve exponent n that determines the slope of the hardening curve approximation for large strains. It is found that the shape of the fracture criterion significantly depends on the slope of the hardening curve. This makes a correct extrapolation of the hardening curve for large strain values essential for a correct ductile fracture prediction using the Mohr-Coulomb fracture model in its mixed stress strain space formulation.

1 Introduction

Recent studies have shown good accuracy of Mohr Coulomb type of fracture criteria in ductile fracture prediction [1-4]. In order to account for non-constant stress-states that are dominant in almost every forming process, usually a damage accumulation rule is assumed (equation (1)).

\[ D = \int \frac{n}{\varepsilon_f(\eta, \theta)} D_{n-1} \frac{n-1}{n} d\varepsilon \]  

(1)

In this equation \( \varepsilon_f(\eta, \theta) \) is the current fracture strain predicted by the Mohr-Coulomb fracture criterion. In order to calculate it, the criterion is usually expressed in its mixed stress-strain space formulation.

1.1 The mixed stress strain space

In accordance with Mohr’s nomenclature [2] the space of stress triaxiality \( \eta \), normalized Lode angle parameter \( \bar{\theta} \) and fracture strain \( \varepsilon_f \) will be referred to as the “mixed stress strain space”. Equations (2) and (3) give the definitions of \( \eta \) and \( \bar{\theta} \) used in this paper, where \( \sigma_h \) is the acting
hydrostatic stress, $\bar{\sigma}$ is the equivalent von-Mises stress and $\sigma_i$ are the Cauchy stress tensor’s principal stresses.

\[ \eta = \frac{\sigma_h}{\bar{\sigma}} \]  
\[ \bar{\theta} = 1 - \frac{2}{\pi} \arccos \left( \frac{27}{2} \frac{(\sigma_1 - \sigma_h)(\sigma_2 - \sigma_h)(\sigma_3 - \sigma_h)}{\bar{\sigma}^3} \right) \]  

1.2 The Mohr-Coulomb (MC) fracture criterion

The Mohr Coulomb (MC) [5] fracture criterion assumes that fracture occurs as soon as a critical weighted sum of acting shear stresses and acting normal stresses is exceeded on an arbitrary plane in the material. The MC fracture criterion is therefore given by equation (4).

\[ |\tau| + c_1 \sigma_n = c_2 \]  

It can be proven [2] from considerations on Mohr’s stress circles that this failure criterion is fully equivalent with equation (5).

\[ (\sigma_1 - \sigma_3) + c(\sigma_1 + \sigma_3) = b \]  

Equation (5) can be transformed to an expression of $\eta$ and $\bar{\theta}$ by purely changing the stress state parametrization [4]. Equation (6) gives this formulation, where $\bar{\sigma}_f$ is the equivalent von Mises stress at which fracture occurs.

\[ \bar{\sigma}_f = c_2 \left[ \frac{1 + c_1^2}{3} \cos \left( \frac{\bar{\theta} \pi}{6} \right) + c_1 \left[ \eta + \frac{1}{3} \sin \left( \frac{\bar{\theta} \pi}{6} \right) \right] \right]^{-1} \]  

To transform this equation into its mixed stress strain formulation ($\varepsilon_f, \eta, \bar{\theta}$) the material hardening curve formulation (equation (7)) is used. In particular this hardening curve formulation will be given by equations (10) and (11) with varying parameters.

\[ \bar{\sigma}_f = k(\varepsilon_f) \]  

This paper will focus on how the material hardening curve formulation influences the shape of the Mohr-Coulomb fracture criterion in the mixed stress strain space ($\varepsilon_f, \eta, \bar{\theta}$).
2 Material parametrization

In this section, the material’s hardening and fracture behavior are characterized. The investigated steel is a low-alloyed boron tempered steel.

2.1 Hardening behavior

To investigate the material’s hardening behavior, dilatometer compression tests were carried out on cylindrical probes at a Temperature of 363K (90°C) at a strain rate of 0.01/s. The experimentally measured data were then corrected for the influence of friction by equation (8) [6].

\[
\sigma_y \cong \frac{F(\varepsilon)}{\pi r(\varepsilon)^2} \left[ 1 + \frac{2\mu r(\varepsilon)}{3h(\varepsilon)} \right]
\]

The Coulomb friction coefficient \( \mu \) was found by measuring the probes barreling and comparing it to the barreling of the probe in numerical simulations with varying \( \mu \). The simulation with the probe barreling closest to the experimentally observed one was assumed to have the correct friction coefficient \( \mu \). Both a Hockett Sherby and a Hensel-Spittel hardening curve approach were used for parameter identification. The parameter identification was performed by minimizing square errors while at the same time fulfilling the constraint given in equation (9). This constraint ensures the equality of the experimental and approximated slope of the hardening curve in the last measured data point of \( \varepsilon=40\% \). This ensures a better agreement of the hardening curves in the extrapolated regions for large strains.

\[
\frac{d\sigma_{fit}}{d\varepsilon_{fit}}|_{\varepsilon=40\%} = \frac{d\sigma_{meas}}{d\varepsilon_{meas}}|_{\varepsilon=40\%}
\]

Figure 1 gives the experimental hardening curve, as well as the two approximations and their slopes.

![Figure 1. Experimental hardening curve and mathematical approximations](image)

The resulting flow curve approximations are given in equations (10) (Hensel-Spittel) and (11) (Hocket-Sherby). For simplicity reasons, the flow curve is assumed as independent of temperature and strain rate. Although these effects shouldn’t be neglected in actual forming processes, for this study they do not have to be taken into account.
\[
\sigma_{\text{Hence}} = A \varepsilon^n = 750.5 \varepsilon^{0.0944} \text{[MPa]}
\]

\[
\sigma_{\text{Hocket}} = A - (A - B)e^{-mt_\varepsilon^n} = 741.6 - (741.6 - 195.1)e^{-3.01e^{0.4855}} \text{[MPa]}
\]

2.2 Fracture behaviour

The material’s fracture behavior was investigated in pure torsion tests, Torsion tests with superimposed compression/tension as well as unnotched and notched tensile tests. Figure 2, left gives the specimen geometries used. The results are given in figure 2 right, where varying \(\eta\) and \(\bar{\theta}\) were averaged according to equation (12).

\[
\eta_{\text{av}} = \int_0^{\epsilon_f} \frac{\eta}{\epsilon_f} \, d\epsilon
\]

![Figure 2. Experimental results and specimen geometries used for calibration of parameters of the MC fracture criterion](image)

These experimental results were then used to determine the material’s Mohr-Coulomb-fracture criteria parameters. The found values are given in figure 4 right under letter c (grey background). They were identified by minimizing square errors. The resulting shape of the MC criterion in the mixed stress-strain space is given in figure 4 left, letter c.

3 Varying the hardening curve approximation

3.1 Re-Parametrization of the hardening curve approximation

In this part of the paper, the material’s hardening curve will be re-parametrized. It is assumed that there is a small error in the measurement of the hardening curve. In dilatometer compression tests, this error could be caused by numerous disturbances of the measurement, such as temperature effects, non-compensation of friction effects or a small error in the determined friction coefficient \(\mu\), causing an error in the friction compensation according to equation (8). This error will be introduced into the parametrization procedure by setting the hardening exponent \(n\) to a predefined value. In order to not only investigate a small error’s influence on the MC fracture criterion but also the general shape of the criterion as a function of the hardening curve, \(n\) values are varied in a wide range from \(n=0.04\) to \(n=0.20\). The Hocket-Sherby flow curve can be seen as the limiting case of \(n\to0\) since the slope of the hardening curve will approach zero for \(\epsilon\to\infty\) for this saturating hardening curve approximation. The
parameter $A$ that is found by minimizing square errors in function of parameter $n$ is given in figure 3 right. Figure 3 left shows the different hardening curve approximations and the experimental result.

![Figure 3. Varying hardening curve approximations](image)

| Configuration | $n$ | $A$  |
|---------------|-----|------|
| a             | 0.2 | 891.7|
| b             | 0.114 | 765.8|
| c             | 0.0944 | 750.5|
| d             | 0.0744 | 710.6|
| e             | 0.04 | 665.0|

3.2 Re-Parametrization of the MC-fracture criterion

Under the varying hardening curve approximations, the Mohr-Coulomb fracture criteria’s parameters have been re-identified from the experimental data set given in chapter 2.2 by minimizing square errors. The identified MC-parameters are given figure 4 right in function of the hardening curve formulation. Figure 4 left shows the resulting shapes of the MC fracture criteria in the mixed stress strain space.

![Figure 4. Varying shapes of the MC fracture criterion in function of used hardening curve approximation](image)

| Configuration | $c_1$ | $c_2$ |
|---------------|-------|-------|
| a             | 0.1815 | 573.66|
| b             | 0.1377 | 469.99|
| c             | 0.1286 | 455.71|
| d             | 0.1201 | 462.91|
| e             | 0.1076 | 392.01|
| f             | 0.0976 | 420.76|

The differences in fracture strains making use of varying hardening curve formulations can be significant, depending on the stress state. Using the Hocket-Sherby flow curve approximation there are stress states, such as low stress triaxiality and high norm of the normalized lode angle parameter, under which fracture would not occur at all. These stress states are indicated in yellow...
in figure 5. This is due to the saturating nature of the hardening curve approximation that will never reach equivalent stresses under which fracture will occur for certain stress states.

![Image](image_url)

**Figure 5.** Stress states under which fracture wouldn’t occur using the Hockett-Sherby flow curve

4 **Conclusions**

The following conclusion can be made from the findings in this paper:

- The hardening curve formulation and its slope for high strains changes the shape of the MC fracture criterion in the mixed stress-strain space in a considerable extent.
- When extrapolating the flow curve, special care has to be taken in order to achieve correct results for fracture strains given through the MC criterion in the mixed stress strain space.
- Since there is some uncertainty with respect to the large strain hardening of materials, there is some uncertainty with respect to the shape of the MC-criterion for high strains.
- The uncertainty with respect to large strain hardening should be minimized. One way to do so, could be the approximation of the large strain hardening behavior by hardness measurements.

**References**

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