Three-body model calculation of the $2^+$ state in $^{26}$O

K. Hagino$^{1,2}$ and H. Sagawa$^{3,4}$

1 Department of Physics, Tohoku University, Sendai 980-8578, Japan
2 Research Center for Electron Photon Science, Tohoku University, 1-2-1 Mikamine, Sendai 982-0826, Japan
3 RIKEN Nishina Center, Wako 351-0198, Japan
4 Center for Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8560, Japan

We discuss the energy of the excited $2^+$ state in the unbound nucleus $^{26}$O using the three-body model of $^{24}$O+$n+n$. This model fully takes into account the continuum effects as well as the dineutron correlation of the valence neutrons. The present calculation yields the energy of 1.354 MeV, which is slightly smaller than the unperturbed energy, 1.54 MeV. The energy shifts for the ground and the $2^+$ states with respect to the unperturbed energy suggest that $0^+$ and the $2^+$ states in $^{26}$O show a typical spectrum well described by a short-range pairing interaction acting on two valence nucleons in the same orbit.

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One of the most important problems in the physics of unstable nuclei is to identify the location of the neutron drip line. So far, the drip line has been known experimentally up to oxygen isotopes, for which the last bound nucleus is $^{24}$O. An interesting phenomenon has been found experimentally in the oxygen isotopes. That is, the oxygen ($Z=8$) isotopes are abruptly end at $N=16$, while the fluorine ($Z=9$) isotopes extend significantly more, at least up to $^{31}$F ($N=22$). Recently, this phenomenon has been successfully explained in terms of a three-body interaction.

In order to shed more light on the anomalous behavior of the neutron drip line in the oxygen region, it is useful to investigate the properties of unbound oxygen nuclei beyond the drip line. The neutron decay energy spectrum for the unbound nucleus $^{26}$O was studied for the first time by Hoffman et al., with which the decay energy and the width were determined to be 770$^{+20}_{-10}$ keV and 172(30) keV, respectively. These values have subsequently been confirmed also by Caesar et al., although the decay width was measured to be somewhat smaller, that is, 20$^{+20}_{-10}$ keV. For the $^{26}$O nucleus, its two neutron emission decay from the ground state (that is, $^{26}$O $\rightarrow$ $^{24}$O + $n$ + $n$) was identified for the first time by Lunderberg et al., which was followed by a measurement of Caesar et al.. The decay energy determined in Ref. was 150$^{+150}_{-50}$ keV. Later on, Kohley et al. extracted the decay half-life of $^{26}$O to be 4.5$^{+1.1}_{-1.5}$ $\pm$ 3 ps. These experimental data on the ground state decay of $^{26}$O have been theoretically analyzed by Grigorenko et al. as well as by the present authors. In particular, Grigorenko et al. have concluded based on their three-body model calculation that the decay half life of 4.5$^{+1.1}_{-1.5}$ $\pm$ 3 ps corresponds to the decay energy which is smaller than about 1 keV.

In this paper, we discuss the excited $2^+$ state of $^{26}$O using the same three-body model as the one employed in Ref. A signal of the second peak in the decay spectrum, which is likely due to the excited $2^+$ state based on shell model calculations, was not strong in the experimental data of Lunderberg et al. and Caesar et al., mainly because the statistics were not sufficient. Nevertheless, it is worthwhile investigating the $2^+$ state in $^{26}$O in front of the forthcoming experiment with the SAMURAI facility at RIKEN.

A striking fact is that most of theoretical calculations performed so far have yielded a larger value of the energy of the $2^+$ state, $E_{2^+}$, compared with the unperturbed energy, 1.54 MeV, that is twice the energy of the single-particle $d_{3/2}$ resonance state in $^{25}$O. An ab-initio calculation with chiral NN and 3N interactions predicts $E_{2^+}$ to be 1.6 MeV above the ground state (see also Ref. ). Shell model calculations with the USDA and USDB interactions yield $E_{2^+}$ to be 1.9 and 2.1 MeV, respectively. These theoretical results may imply that the continuum effects, which play an essential role in unbound nuclei, are not taken into account sufficiently well in these calculations. The continuum effects are supposed to be included in a consistent manner in the continuum shell model calculation of Ref. , but the calculation still overestimates the $2^+$ energy to be 1.8 MeV, probably because the ground state energy of $^{25}$O is also overestimated by about 230 keV. Given this puzzling situation, it is intriguing to investigate the $2^+$ state in $^{26}$O using the three-body model, which takes fully into account the continuum effects as well as the dineutron correlation between the valence neutrons.

In order to describe the decay process of $^{26}$O nucleus, we assume the three-body structure of $^{24}$O+$n+n$. We employ the same Hamiltonian as in Ref., that is,

$$H = \hat{h}_{nC}(1) + \hat{h}_{nC}(2) + v(1,2),$$

(1)

in which $\hat{h}_{nC}$ is the single-particle Hamiltonian for a valence neutron interacting with the core, and $v(1,2)$ is the pairing interaction between the valence neutrons. For simplicity, we have neglected the two-body part of the recoil kinetic energy of the core nucleus, while the one-body part is included in the single-particle Hamiltonian $\hat{h}_{nC}$ through the reduced mass. The single-particle potential in $\hat{h}_{nC}$ has a Woods-Saxon form, whose parameters are chosen in order to reproduce the energy of the $d_{3/2}$ reso-


\[ v(r_1, r_2) = \delta(r_1 - r_2) \left( v_0 + \frac{v_p}{1 + \exp(\beta/\lambda)} \right), \]

in which the first term describes the interaction in the vacuum whereas the second term simulates the medium effect through the density dependence of the interaction. See Ref. [7] for the values of the parameters for the single-particle potential and for the pairing interaction.

With the three-body model, we compute the decay energy spectrum for a given angular momentum \( I \),

\[ \frac{dP_I}{dE} = \sum_k |\langle \Psi_k^{(I)} | \Phi_{\text{ref}}^{(I)} \rangle|^2 \delta(E - E_k), \]

where \( \Psi_k^{(I)} \) is a solution of the three-body model Hamiltonian with the angular momentum \( I \) and the energy \( E_k \), and \( \Phi_{\text{ref}}^{(I)} \) is the wave function for a reference state with the same angular momentum. The reference state can be somewhat arbitrary. In Ref. [2], we have used the ground state of the \( ^{27}\text{F} \) nucleus obtained by solving a three-body model Hamiltonian for \( ^{25}\text{F} + n + n \). In this paper, for simplicity, we instead use the uncorrelated state of \( ^{27}\text{F} \) with the neutron \([1d_{3/2} \otimes 1d_{3/2}]^{(1M)}\) configuration, which is dominant in the ground state of \( ^{27}\text{F} \). We have checked that both the reference states lead to similar decay energy spectra for the ground state decay.

With a contact interaction, the continuum effects on the decay energy spectrum can be easily taken into account in terms of the Green’s function. In order to demonstrate this, first notice that Eq. (3) can be expressed as

\[ \frac{dP_I}{dE} = -\frac{1}{\pi} \Im \sum_k |\langle \Phi_{\text{ref}}^{(I)} | \Psi_k^{(I)} \rangle|^2 \frac{1}{E_k - E - i\eta} \langle \Psi_k^{(I)} | \Phi_{\text{ref}}^{(I)} \rangle, \]

\[ = -\frac{1}{\pi} \Im \langle \Phi_{\text{ref}}^{(I)} | G^{(I)}(E) | \Phi_{\text{ref}}^{(I)} \rangle, \]

where \( \Im \) denotes the imaginary part and \( \eta \) is an infinitesimal number. In this equation, the correlated Green’s function, \( G^{(I)}(E) \), is given by

\[ G^{(I)}(E) = \sum_k \langle \Psi_k^{(I)} | \frac{1}{E_k - E - i\eta} \langle \Psi_k^{(I)} | \Phi_{\text{ref}}^{(I)} \rangle. \]

Notice that the correlated Green’s function can be constructed as [20],

\[ G^{(I)}(E) = G_0^{(I)}(E) - G_0^{(I)}(E)v(1 + G_0^{(I)}(E)v)^{-1}G_0^{(I)}(E), \]

where \( G_0^{(I)}(E) \) is the unperturbed Green’s function given by

\[ G_0^{(I)}(E) = \sum_{1,2} \frac{|\langle j_1J_2 |(1M) \rangle |^2}{\epsilon_1 + \epsilon_2 - E - i\eta}. \]

Here, the sum includes all independent two-particle states coupled to the total angular momentum of \( I \). See Ref. [20] for a practical method to perform a summation over a continuum spectrum for the single-particle states.

The upper panel of Fig. 1 shows the decay energy spectrum of \( ^{26}\text{O} \) for the \( I = 0 \) (the dashed line) and \( I = 2 \) (the solid line). For a presentation purpose, we set \( \eta \) in Eq. (3) to be a finite value, that is, \( \eta = 0.21 \text{ MeV} \) [7]. For comparison, we also show the spectrum for the uncorrelated case by the dotted line, which gives the same spectrum both for \( I = 0 \) and \( I = 2 \). For the uncorrelated case, the spectrum has a peak at \( E = 1.54 \text{ MeV} \), that is, twice the single-particle resonance energy, 0.77 MeV. With the pairing interaction between the valence neutrons, the peak energy is shifted towards lower energies. The energy shift is larger in \( I = 0 \) than in \( I = 2 \). That is, the peak in the spectrum appears at \( E = 0.148 \text{ MeV} \) (\( \Delta E = -1.392 \text{ MeV} \) for \( I = 0 \) and at \( E = 1.354 \text{ MeV} \) (\( \Delta E = -0.186 \text{ MeV} \)) for \( I = 2 \).

As a matter of fact, it is quite natural that the \( 2^+ \) state appears at an energy slightly smaller than the unperturbed energy if the three-body picture is reason-
able. In standard textbooks of nuclear physics (see e.g., Refs. [21, 22]), it is shown that the energy shift due to a pairing residual interaction, \( v_{\text{res}}(r_1, r_2) = -g \delta(r_1 - r_2) \), is evaluated for a single-\( j \) orbit as,

\[
\Delta E_I = \frac{(jj)(1M)}{gF_r} = -gF_r \frac{(2j+1)^2}{8\pi} \left( \begin{array}{ccc} j & j & I \\ 1/2 & -1/2 & 0 \end{array} \right)^2,
\]

where \( F_r \) is the radial integral of the four single-particle wave functions. If one applies this formula to the \( 26^O \) nucleus and sets \( j = d_{3/2} \), one obtains

\[
\Delta E_{I=0} = -\frac{16}{8\pi} gF_r \cdot \frac{1}{4},
\]

and

\[
\Delta E_{I=2} = -\frac{16}{8\pi} gF_r \cdot \frac{1}{20}.
\]

That is, \( \Delta E_{I=0}/\Delta E_{I=2} = 5 \), which is compared to the value of \( \Delta E_{I=0}/\Delta E_{I=2} = 7.48 \) obtained in the present three-body model calculation. Eqs. (11) and (12) predict \( \Delta E_{I=0}/\Delta E_{I=2} = -0.278 \text{ MeV} \) for the value \( \Delta E_{I=0} = -1.392 \text{ MeV} \) of the three-body model prediction. Even though \( \Delta E_{I=0}/\Delta E_{I=2} \) in the three-body model somewhat deviates from the simple estimates of Eqs. (11) and (12) due to the many-body continuum effects, the small energy shift for the \( 2^+ \) state can be well understood by these formulas derived for the single-\( j \) model with the residual pairing interaction. The single-\( j \) model indicates that the \( 2^+ \) state in \( 26^O \) shows a typical spectrum governed by a pairing residual interaction and that the \( 2^+ \) energy never exceeds the unperturbed energy. Thus it must be smaller than 1.54 MeV for the \( 26^O \) nucleus since the single-particle resonance energy of \( d_{3/2} \) state is 0.77 MeV in \( 25^O \).

The lower panel of Fig. 1 shows the decay energy spectrum obtained by taking the linear superposition of the \( I = 0 \) and \( I = 2 \) contributions, that is,

\[
\frac{dP}{dE} = (1 - \alpha) \frac{dP_{I=0}}{dE} + \alpha \frac{dP_{I=2}}{dE}.
\]

The actual value of \( \alpha \) would depend on the details of the wave function of \( ^{27}F \) as well as the reaction dynamics of the proton knock-out reaction of \( ^{27}F \) with the initial state of \( 26^O \) was prepared in the experiment of Ref. [3]. Since it is beyond the scope of this paper to evaluate the value of \( \alpha \) in the actual experimental conditions, here we arbitrarily take \( \alpha = 0.1 \). For comparison, the figure also shows the pure \( I = 0 \) component, which is the same as that shown in the upper panel. One can see that the experimental data are reproduced slightly better by mixing the \( I = 2 \) component, although the error bars are large and one may not draw a definite conclusion.

The ground state energy may be slightly overestimated in this calculation in relation to the recent life time measurement of the \( 26^O \) nucleus [6]. We therefore repeat the same calculation by adjusting the \( v_{\text{res}} \) parameter in the pairing interaction, Eq. (2), so that the ground state energy becomes 5 keV, instead of 148 keV. If one uses this interaction, the \( 2^+ \) state in \( 26^O \) appears at \( E = 1.338 \text{ MeV} \). This value is similar to the previous result, \( E = 1.354 \text{ MeV} \), and we thus conclude that the energy of the \( 2^+ \) state is much less sensitive to the \( nn \) interaction as compared to the \( 0^+ \) state. This implies that the \( 2^+ \) state of \( 26^O \) should definitely appear around \( E = 1.3 \text{ MeV} \) as long as the three-body picture is correct.

In summary, we have discussed the \( 2^+ \) state of \( 26^O \) using the three-body model of \( ^{24}O+n+n+n \). We have shown that the \( 2^+ \) state appears at around \( E = 1.35 \text{ MeV} \) and this value does not change much even if we change the \( nn \) interaction to vary the ground state energy from 150 keV to 5 keV. This \( 2^+ \) energy is close to, but slightly smaller than, the unperturbed energy, \( E = 1.54 \text{ MeV} \), and thus the energy shift from the unperturbed energy is much smaller than the energy shift for the \( 0^+ \) state. We have argued that this is a typical spectrum well understood by the single-\( j \) model with the pairing residual interaction. Many shell model calculations have predicted the excitation energy of the \( 2^+ \) state in \( 26^O \) in the opposite trend, that is, they have predicted a higher energy than the unperturbed energy. It is desperately desired to confirm the energy of \( 2^+ \) state experimentally in order to clarify the validity of nuclear models and effective interactions in nuclei on and beyond the neutron drip-line.

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