Ω₀ from the Apparent Evolution of Gas Fractions in Rich Clusters of Galaxies
Rebecca Danos
Whitin Observatory, Wellesley College, Wellesley MA 02181, USA
and Ue-Li Pen
Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge, MA 02138, USA

ABSTRACT

We apply a unique gas fraction estimator to published X-ray cluster properties and compare the derived gas fractions of observed clusters to simulated ones. The observations are consistent with a universal gas fraction of $0.15 \pm 0.01 h^{-3/2} \Omega_0$ for the low redshift clusters that meet our selection criteria. The fair sampling hypothesis states that all clusters should have a universal constant gas fraction for all times. Consequently, any apparent evolution would most likely be explained by an incorrect assumption for the angular-diameter distance relation. We show that the high redshift cluster data is consistent with this hypothesis for $\Omega_0 < 0.63$ (95% formal confidence, flat $\Lambda$ model) or $\Omega_0 < 0.60$ (95% formal confidence, hyperbolic open model). The maximum likelihood occurs at $\Omega_0 = 0.2$ for a spatially flat cosmological constant model.

1. Introduction

It has been proposed that clusters of galaxies should be a fair sample of baryonic matter (White et al. 1993). Rich clusters form through the gravitational collapse of the matter within a 15-30 Mpc diameter volume, where the force of gravity acts equally on all non-relativistic forms of matter. The richest clusters have temperatures above 10 keV, which corresponds to velocity dispersions in excess of 1000 km/sec. Most non-gravitational processes do not appear to affect the bulk of matter at comparable energies, and so we would expect the gas and dark matter to collapse into objects where they are fairly represented. This is confirmed in simulations using a large variety of techniques (Frenk et al. 1998), where it is found that within the virial radius the gas and dark matter are indeed equally represented with deviations of only 10%. Since clusters of galaxies are observed at cosmological distances and are spatially resolved objects, this opens the possibility of directly measuring angular diameter distances if the gas to dark matter ratio were known in advance (Pen 1997). In this paper we compare observed and simulated cluster properties and we estimate the errors in our methods.
2. Method

Observable parameters can be converted to physical quantities by using many simplifying assumptions. Our models most strongly depend on the assumptions of hydrostatic equilibrium and spherical symmetry. These assumptions give sufficient information to solve the equations of hydrostatic equilibrium and thus to measure the ratio of the gas mass to the total mass within the virial radius, a value also corresponding to its global ratio (White et al. 1993). Currently, most astrophysical observations either measure spatially resolved X-ray surface brightness or broad aperture X-ray spectra, but usually not simultaneously. As a result, we can not directly measure total masses. As a first approximation, we shall therefore assume that the gas traces the total mass at all radii, which reduces the free parameters to allow a unique solution of the gas fraction in terms of the three observable parameters: temperature, angular size and X-ray flux (Pen 1997). These quantities are usually translated into physical size and luminosity by assuming some luminosity and angular-diameter distance relation $d_L(z)$, $d_A(z)$. We can estimate the error induced by these assumptions by applying this scheme to the simulated clusters.

The Jones and Forman (1998, hereafter JF98, see also David et al. 1993) catalog published the X-ray properties parameterized by the isothermal $\beta$ model where the gas distribution is modeled as $\rho_g = \rho_0 (1 + r^2 / r_c^2)^{-3\beta/2}$. In our model, the observables will be the total X-ray luminosity $L_X$ derived from the observed flux $F$

$$L_X = 4\pi d_L^2 F = 4\pi \epsilon_{\text{ff}} T^{1/2} n_0^2 r_c^3 \int_0^\infty (1 + u^2)^{-3\beta} u^2 du$$

in cgs units. $\epsilon_{\text{ff}} \sim 1.94 \times 10^{-27}$ is the effective free-free emissivity coefficient, which includes an average Gaunt factor of 1.25 and a hydrogen-helium mixture with 24% Helium by mass (Spitzer 1978). The central electron density, $n_0$ from here on in units $10^{-3}$ cm$^{-3}$, can be solved by

$$n_0^2 = \frac{L_{44} \Gamma[3\beta]}{11.4 r_c^2 \sqrt{T \Gamma[3\beta - 3/2]}}$$

in terms of $L_{44}$, the X-ray luminosity in units of $10^{44}$ erg/sec, the core radius $r_c$ in Mpc, $T$ in keV, and $\Gamma$ functions. The gas fraction $f_g$ is then given by

$$f_g = \frac{9.37 H(\beta) n_0 r_c^2}{T}$$

where $H(\beta) \sim 0.057 (\beta - 4/7)^{-0.787}$ (Pen 1997). If the absolute distance is unknown up to a linear parameter, for example $h_{50}$, the gas fraction in scales as $h_{50}^{-3/2}$. 
3. Simulations

We apply equation (3) to simulated clusters, in which we know the global gas-to-mass ratio, to estimate the systematic and statistical errors induced by the assumptions of spherical symmetry, hydrostatic equilibrium, and gas-traces-mass. We ran several simulations with different volumes, resolutions, and cosmological parameters, and we applied the observational procedure to each set of simulations. We list the simulation parameters in Table 1. Several of the simulations were previously published (Pen 1998b), and a new higher resolution $256^3$ CDM run was added for this analysis. All simulations are performed with the Moving-Mesh-Hydro code (Pen 1998a) on an Origin 2000 at the National Center for Supercomputing applications. The following physical assumptions are imposed: 1. the universe is assumed to consist of two dynamical components, dark matter and baryons in the form of an ideal gas; 2. the dark matter is initially cold and follows the collisionless Boltzmann equation, while the gas is described by the Euler equations in an expanding universe. Only simulation PREHEAT incorporates non-gravitational processes, which simulate the worst case scenario of energy injection. 1 keV of energy per nucleon is instantaneously injected at a redshift of $z = 1$, and the subsequent evolution again follows Euler's equations. This should model the net effect of heat injection from supernova heating, which would also be one of the consequences of cooling.

We first approximate the soft X-ray observations by computing the projected density-square weighted surface brightness $\Sigma = \int \rho^2 dz$. In the soft X-rays, the temperature dependence is very weak, making this a good approximation. We then find the local maxima in the projected 2-D X-ray image. The surface brightness is binned radially centered on the local maxima, and we fit a 1-D surface brightness profiles with two free non-linear parameters $r_c$, and $\beta$ (Press et al. 1992). The results of the radial binning for the highest

| model  | $\Omega_0$ | $\Omega_\Lambda$ | $\sigma_8$ | $h$ | $L(\text{h}^{-1}\text{Mpc})$ | $\Delta x(\text{h}^{-1}\text{kpc})$ |
|--------|-----------|-----------------|-----------|-----|------------------|------------------|
| CDM    | 1         | 0               | 0.6       | 0.5 | 128              | 50               |
| PREHEAT| 1         | 0               | 0.5       | 0.5 | 80               | 62               |
| OCDM   | 0.37      | 0               | 1         | 0.7 | 120              | 94               |
| LCDM   | 0.37      | 0.63            | 1         | 0.7 | 120              | 94               |

Table 1: Simulations used for this study. All simulations except CDM used a $128^3$ grid with $256^3$ particles, while the previous used $256^3$ grid points. All models have a baryonic fraction $\Omega_bh^2 = 0.0125$ and compression limiter $c_{\text{max}} = 10$. The effective grid spacing in virialized objects is $\Delta x$ (Pen 1998a). We used a periodic box of length $L$. 

resolution model are shown in figure 1, where the $\beta$-model is seen to be a good parametric fit.

We can repeat this exercise for each of the three possible projections, and compile a database of inferred gas fractions. The result is shown in figure 2. The assumptions of spherical symmetry, gas-tracing-mass, and hydrostatic equilibrium actually work surprisingly well. The reconstructed gas fraction has a scatter of only 25%, and the mean ratio is almost unbiased, differing from the true mean by less than 8%.

4. Observations

We base our local observational sample on the data set of JF98. Their catalog provides the X-ray luminosities, $\beta$, $r_c$ and $T$, just like the simulated sample. From these observables, we again construct the cluster gas fractions. The dominant error is generally the error in temperature, which we took as the observational error on the gas fraction. In quadrature, we add the expected intrinsic 25% error on the gas fraction seen in the simulations in the absence of measurement error. This scatter arises from the imperfection in the simplifying assumptions. We use this total error to weight each observation. In addition, we impose certain cuts in the model. We require $T_X > 4$ keV and $\beta > 0.6$. The first restriction is to eliminate potential problems with non-gravitational heat sources, such as supernova heating (Loewenstein and Mushotzky 1997). Photoheating is always negligible. The cut in $\beta$ is necessary for two reasons. In the JF98, clusters where $\beta$ could not be directly measured were assigned $\beta = 0.6$, and we felt it important to discard clusters where parameters were based on guesses instead of actual measurements. An additional reason is to obtain a fair estimator of the total cluster temperature (Pen 1997). For small values of $\beta$, much of the X-ray emission arises from the outer regions, and the temperature errors are dominated by radii where significant temperature gradients are expected. Formally, the errors are infinite for $\beta \leq 4/7$. Of the 57 clusters in the JF98 sample with measured temperatures, 25 pass these two criteria. We then apply equation (3) to the clusters and take a weighted average.

The high redshift sample in the literature is still very meager. A search for clusters with redshift $z > 0.5$ satisfying our requirements turned up 3 clusters, MS0451, MS1054 and CL0016 (Donahue et al. 1997, Mushotsky and Scharf 1997, Neumann and Böhringer 1996). Their properties and inferred gas fractions depend on their geometrical distances. For $q_0 = 1/2$, $\Omega = 1$, $H_0 = 50$ km/sec/Mpc, the results are listed in table 2. To convert to other cosmological models, we note that the constrained observables are flux $F$, angular core size $\theta_c$, and temperature. The inferred gas fraction in equation (3) scales as $f_g \propto F^{1/2}d_L^{1/2}T_X^{5/4}$. Recalling that the angular diameter distance $d_A$ is related to
the luminosity distance \( d_L \) by \( d_L = (1 + z)^2 d_A \), we can incorporate the only cosmological dependence into \( f_g \propto d_A^{3/2} \). We will consider the consequences in the next section.

5. Results

For the local cluster sample, the mean gas fraction is very well determined. Applying equation (3) to all qualifying clusters, we find the raw gas fraction \( f_g = 0.14 \pm 0.007 h_{50}^{-3/2} \). If we correct for the likely 8% bias in the estimator as seen in the simulations, and add an equal error for uncertainties in systematics due to Poisson limitations in the simulations, we find

\[
f_g = 0.15 \pm 0.01 h_{50}^{-3/2}
\]

at 1-\( \sigma \) error bars, based on the 25 clusters JF98. The result is consistent with Evrard (1997) \( f_g = 0.17 \pm 0.01 h_{50}^{-3/2} \) which uses an entirely different approach. Our \( \chi^2 \) for the assumed errors is 33.2 for 24 degrees of freedom. This is consistent with the null hypothesis that all clusters have the same intrinsic gas fraction, where all errors are solely due to measurement errors and the 25\% scatter in the gas fraction estimator. We have no need to model any further sources of error.

The high redshift sample allows us to apply the fair sampling hypothesis to constrain geometrical distances. We define \( \chi^2 = \sum (f_g^i - \bar{f}_g)^2 / \sigma_i^2 \), where \( \bar{f}_g = 0.14 \) and each gas fraction and error is taken for the three high redshift clusters from table 2. The individual gas fractions are a function of cosmology \( f_g^i = f_g^i(\Omega = 1) \times [d_A(\Omega_0, z)/d_A(\Omega_0 = 1, z)]^{3/2} \). At a 95\% \( \chi^2 \) limit, we obtain \(-1 < q_0 < 0.2\). This interestingly does not include a flat universe with \( \Omega = 1 \). We can quantify the \( \chi^2 \) distribution for two specific cosmologies: a spatially flat universe with cosmological constant \( \Lambda \) and a spatially curved hyperbolic universe. In

| Cluster | \( z \) | \( L_X \) (10^{44} \text{erg/s}) | \( T_X \) (keV) | \( r_c \) kpc | \( \beta \) | \( f_g \) |
|---------|-------|-------------------|--------------|------------|------|-----|
| MS0451  | 0.55  | 41.7              | 10.4_{-1.3}^{+1.6} | 280        | 1.01 | 0.0843 \pm 0.023 |
| MS1054  | 0.83  | 42.0              | 14.7_{-3.8}^{+4.6} | 500        | 0.80 | 0.0819 \pm 0.020 |
| CL0016  | 0.54  | 30.3              | 8.0_{-1}^{+1}    | 372        | 0.85 | 0.136 \pm 0.036 |

Table 2: High redshift clusters used for this study. The listed quantities are reconstructed from observables by luminosity \( L_X = 4\pi F d_L^2 \) in terms of flux \( F \) and luminosity diameter distance \( d_L \), while core radius \( r_c = \theta_c d_A \) in terms of angular diameter distance \( d_A \) for \( \Omega = 1, H_0 = 50 \).
the first case, the angular diameter distance relation is given by a simple fitting function (Pen 1998c):

\[
d_A = \frac{c}{H_0(1+z)} \left[ \eta(1, \Omega_0) - \eta\left(\frac{1}{1+z}, \Omega_0\right) \right]
\]

\[
\eta(a, \Omega_0) = 2\sqrt{s^3 + 1} \left[ \frac{1}{a^4} - 0.1540 \frac{s}{a^2} + 0.4304 \frac{s^2}{a} + 0.19097 \frac{s^3}{a} + 0.066941 s^4 \right]^{-\frac{1}{2}}
\]

\[
s^3 = \frac{1 - \Omega_0}{\Omega_0}.
\]

(5)

In the open case, Mattig’s relation gives \(d_A = 2[2 - \Omega_0 + \Omega_0 z - (2 - \Omega_0) \sqrt{1 - \Omega_0 z}] / \Omega_0^2\) (Peebles 1993). We then obtain the distribution of \(\chi^2\) shown in figure 4. For a spatially flat \(\Lambda\) model, the minimum \(\chi^2 = 3.04\), which is good fit for 2 degrees of freedom. We see the generic fact that high \(\Omega_0\) universes are strongly disfavored by these clusters. In order to put statistically meaningful limits, we need to understand the full distribution of errors, which is clearly impractical. But if we make the assumption that errors are Gaussian, we obtain formal error estimates. Varying only \(\Omega_0\), we obtain the 95% confidence interval by requiring \(\Delta \chi^2 = 3.841\). Formally, we find that \(\Omega_0 < 0.63\) for a spatially flat cosmological constant dominated universe, while \(\Omega_0 < 0.60\) for a spatially hyperbolic universe with no cosmological constant, both at 95% formal confidence. This result is consistent with the recent program of high-redshift supernova observations (Perlmutter et al. 1998, Glanz 1998). The hyperbolic model is a questionable fit, since we limited the range of \(\Omega_0\) to be positive for a meaningful geometry.

A short analysis of the difference to the Pen (1997) result is in order. That analysis obtained \(q_0 = 0.85 \pm 0.29\), which would argue in favor of high \(\Omega_0\). CL0016 appears in both analyses, albeit with slightly different gas fractions. In the previous analysis, the published central electron density \(n_0\) was used, which is unfortunately interpreted differently by various authors. This analysis uses solely robust observables, namely the flux, angular size, and temperature, which are less subject to interpretational systematic differences.

Several systematic errors can still change the conclusions. The cluster sample is not homogeneous, and may have selection effects. The intrinsic scatter in the observable is 25%, which is comparable to the expected effect at \(z \sim 0.5\). The signal comes partially from the reduction in the error due to the large statistical sample, which assumes independence and randomness of each error source. We quote the formal confidence intervals from the \(\chi^2\) value by assuming Gaussianity, which may not be a good assumption.
6. Conclusions

Comparing a catalog of local cluster properties with simulations, we find that the data is consistent with a universally constant gas fraction of \( f_g = 0.15 \pm 0.01 h_{50}^{-3/2} \). With the present day uncertainty in \( 0.025 < \Omega_b h_{50}^2 < 0.1 \) (Schramm and Turner 1997) and some errors in the Hubble constant, we obtain no useful constant on \( \Omega_0 \) using the low redshift clusters. Independent conservation of baryons and dark matter, however, allows us to constrain \( \Omega_0 \) from the apparent evolution of the cluster gas fraction.

We have shown that the 3 clusters with measured gas fractions at \( z > 0.5 \) are inconsistent with an \( \Omega = 1 \) universe and the fair sampling hypothesis. For a spatially flat cosmological constant dominated universe, we obtain a bound of \( \Omega_0 < 0.63 \) (95% formal confidence) with a best fit value of \( \Omega_0 = 0.2 \). For a spatially hyperbolic universe with only matter, we find \( \Omega_0 < 0.60 \) (95% formal confidence) with the maximum likelihood at \( \Omega_0 = 0 \). The errors are dominated by the temperature uncertainties in the high redshift clusters, and future observations could reduce the errors by a factor of two.

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Fig. 1.— The radial profiles fitted for simulated $\Omega = 1$, $h = 0.5$ clusters. The $\beta$-model provides a very good fit for the simulated projected X-ray surface brightness. $r_c$ is in kpc, and is marked by the vertical line in each panel. Each row contains the three projections of a single cluster.
Fig. 2.— The gas fractions inferred from simulations. Each cluster is projected, centered, and reduced in analogy to the real observed clusters. The solid line is the mean, and the dotted line the standard deviation in the estimates. No measurement errors are included, implying that the scatter and mean reflect the intrinsic scatter and bias of the fitting formula used in this paper. Open boxes are from model CDM. Crosses are LCDM, asterixes are OCDM, circles PREHEAT.
Fig. 3.— The gas fractions of clusters with $T_X > 4$ keV. The error bars are at 90% confidence interval based on temperature errors alone, and do not include a systematic 25% scatter expected from simulations. The round points are the high redshift clusters for $q_0 = 0$, while the crosses are the same clusters for $q_0 = 1/2$. The solid horizontal and dashed lines are the best fit mean and error in the mean respectively.
Fig. 4.— $\chi^2$ as a function of the density parameter for hyperbolic (dashed) and flat $\Lambda$ cosmologies (solid). The horizontal lines indicate the 5% likelihood cutoff for each model defined as $\Delta\chi^2 = 3.841$. 

high redshift cluster constraints