Cosmic String Global Superconducting Dirac Born Infeld

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Abstract. Superconducting cosmic string possibly plays an important role in the formation of the universe structure. The physics of this phenomenon has been explored by studying the field theory in the string interior. Numerical solutions of superconducting strings with all relevant fields are presented in this paper. The field is constructed from a generalization of the usual field theory of superconducting global string, but the kinetic term consists of the Dirac Born Infeld (DBI). Some changes in the characteristic of the superconducting string DBI from the usual superconducting string case have been observed. The observation includes physical mechanism of all related fields.

1. INTRODUCTION
Cosmology is the study of the origin of the universe and its evolution of forming various kinds of particles that have been observed at present. There have been a number of theories proposed to describe the condition of the universe at its beginning, but the most relevant one for today is perhaps the Big Bang Theory. This theory explains the formation of the universe started with a huge explosion that took place about 15 billion years ago [1]. According to Kibble [2], as long as the universe expanded and the temperature dropped after the Big Bang, then there would be a condition, when the universe reached some certain critical temperature known as the Curie Temperature or commonly indicated by Tc. At the time, when the temperature was around Tc, the phase transitions occurred, accompanied by the destruction of symmetry in the field. The symmetry destruction could bring events of topological defect. In other words, topological defect would occur during the process of the destruction of symmetry, in the transition phase shortly after the Big Bang [3]. In physics community, topological defect is known as a set of the solutions of nonlinear equations of a field [4]. It is called as topological because the solution of the nonlinear equations is related to the topology of the terrain and is said as defect due to the destruction of the symmetry of the field in formation [5].

One of the type of topological defect was most likely to occur in the past described the immediate aftermath of the Big Bang is a cosmic string. This type of topological defect has very interesting properties and it is possible to play an important role in shaping the structure of the universe. Cosmic strings are one-dimensional object that is very thin and can stretch across the universe. This string consists of a complex scalar field configuration [1]. Some observations about cosmic string yield a fairly typical trail and still be within the range of observation [6, 7]. In his work, Witten argued that under certain conditions the cosmic strings can behave like superconductors strings. This situation will result in a variety of interesting effects which is
possibly observable. This superconducting charge carriers can be either bosons or fermions. These events may be observed as a synchrotron source or as a source of high-energy cosmic rays [8]. Several studies have been carried out to extend the original works of Witten and study this issue [9, 10, 11]. In their paper, Davis and Shellard have discussed in a more detail the field and current profiles in the superconducting cosmic string interior. These efforts have produced a numerical solution for all field-related string and explained the physics behind the current quenching mechanism in the string [12].

In an effort to explore more about the cosmic string, Sarangi introduced a global string solution which is a generalization of the usual field theory global cosmic string when the kinetic term is DBI. From his work, it was found that if the solution of the current global string is replaced by the DBI kinetic term, the same effect as regular global solution string emerges [13]. Given that the DBI only occurs at high energy levels, then these results may indicate the possibility of the existence of cosmic strings at very high energy, namely when the Big Bang had just happened.

There have been a number of studies, which attempt to develop a theory of superconducting cosmic string and DBI cosmic string. So far, there is no study that shows the existence of cosmic strings solution, when these two concepts are combined. It is the purpose of this paper to discuss the numerical solution of superconducting global cosmic string DBI, for charged boson and to show the existence of the global DBI superconducting cosmic string.

1.1. Cosmic String Global Superconductors

Since the occurrence of cosmic strings involves the destruction of symmetry, the Lagrangian describing the condition is the symmetry breaking Lagrangian, which was previously proposed by Goldstone [14]. The Lagrangian for the global case reads

\[ \mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) - V(\phi), \]  

with

\[ V(\phi) = \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2. \]  

The Lagrangian consists of a scalar field \( \phi \) and a potential that depends on the scalar field. The corresponding variables of this potential are \( x \) and \( y \) coordinates. The \( x \)-axis describes the imaginary part of the scalar field, the \( y \)-axis is the real part of the scalar field, and the \( z \)-axis corresponds to the potential \( V(\phi) \). This potential is commonly referred to as the Mexican hat potential. By adding a scalar field, which is destroy the symmetry in the core part of the string, Witten revealed that a cosmic string can behave as superconductor [8]. Therefore, in this case there are two scalar fields equation of motion that are mutually coupled to one another. The global cosmic string superconductor Lagrangian proposed by Witten is

\[ \mathcal{L} = |\partial_\mu \phi|^2 + |\partial_\mu \sigma|^2 - V(\phi, \sigma) \]  

The Lagrangian above contains two complex scalar fields \( \phi \) and \( \sigma \) and a symmetry breaking potential \( V(\phi, \sigma) \) that works on both fields with [15]

\[ V(\phi, \sigma) = \frac{1}{4} \lambda_\phi (|\phi|^2 - \nu_\phi^2)^2 + \frac{1}{4} \lambda_\sigma (|\sigma|^2 - \nu_\sigma^2)^2 + \beta |\phi|^2 |\sigma|^2 \]  

The parameter \( \lambda_\phi \) and \( \lambda_\sigma \) are the coupling constants for scalar fields \( \phi \) and \( \sigma \). The constant \( \beta \) is a coupling constant relating \( \phi \) and \( \sigma \), whereas \( \nu_\phi \) and \( \nu_\sigma \) are the vacuum scales to the fields \( \phi \) and \( \sigma \), respectively. Although the Lagrangian formulation of the kinetic rate is the same, the two fields have opposite characteristics to each other. In this case, \( \phi \) is a field that describes the
model of cosmic strings. The symmetry of this field is unbroken at the string core but it will be broken far away from the core. While the symmetry of $\sigma$ will be broken in the core part and unbroken at far away from the core. Because of these conditions, Witten proposed a specific boundary condition [8]. Heuristically, we can see that there are basically three requirements, which must be satisfied by the parameters. First, we require to have the symmetry of $\phi$ broken and $\sigma$ unbroken as a global minimum to the theory. This means

$$\lambda_{\phi} v_{\phi}^4 > \lambda_{\sigma} v_{\sigma}$$

The second requirement is to ensure that $\sigma$ goes to zero off the string, since otherwise the electromagnetism would be broken everywhere. The effective mass term for $\sigma$ at large distance must therefore be positive, so another condition implies

$$\beta v_{\sigma}^2 - \frac{1}{2} \lambda_{\sigma} v_{\sigma}^2 > 0$$

The final requirement is that the Compton wavelength of $\sigma$ must be small enough to fit inside the string in order the condensate to form. This gives [12]

$$\sqrt{\lambda_{\phi}} v_{\phi} < \sqrt{\lambda_{\sigma}} v_{\sigma}$$

2. Cosmic String Global DBI Superconductor

As is already known, it is possible that cosmic string formed shortly after the Big Bang or at the high energy condition. In string theory (a theory that studies the phenomenon of physics at high energy condition) this is known as the DBI kinetic rate. According to this theory, kinetic term in the Lagrangian equation can be modified into a DBI kinetic rate due to physical factors field conditions, when it is at a high energy level. Departing from the two of such information, several studies have been performed to investigate the relationship between the cosmic strings and the DBI [13, 16]. The DBI Lagrangian equation for cosmic strings solution is formulated as follows

$$\mathcal{L} = \kappa^2 \left( \sqrt{1 + \frac{2}{\kappa^2} |\partial_{\mu} \phi|^2} - 1 \right) - V(\phi).$$

Here $\phi$ is a complex scalar field and $|\phi| = \sqrt{\phi \phi^*}$ is its absolute value. The parameter $\kappa$ is the coupling constant of DBI. The potential $V(\phi)$ is a potential symmetry breaking which has a Mexican hat shape.

As the Lagrangian equation for global cosmic strings ordinary superconductors proposed by Witten [8], the equation of global superconducting cosmic string DBI also consists of two scalar fields, $\phi$ and $\sigma$. In the present study the field forming string $\phi$ is converted into kinetic rate DBI. Witten proposed an additional scalar field $\sigma$, so that a string can behave like a superconductor. The equation for global superconducting cosmic string DBI is modeled as follows

$$\mathcal{L} = \kappa^2 \left( \sqrt{1 + \frac{2}{\kappa^2} |\partial_{\mu} \phi|^2} - 1 \right) + |\partial_{\mu} \sigma|^2 - V(\phi, \sigma),$$

where

$$V(\phi, \sigma) = \frac{1}{4} \lambda_{\phi} (|\phi|^2 - \nu_{\phi}^2)^2 + \frac{1}{4} \lambda_{\sigma} (|\sigma|^2 - \nu_{\sigma}^2)^2 + \beta |\phi|^2 |\sigma|^2.$$
Numerical solution of the superconducting DBI cosmic string with \( \lambda = 0.1 \), \( \nu = 1 \), \( \lambda_\sigma = 1 \), \( \nu_\sigma = 0.4 \), and \( \beta = 0.10 \).

at large distance. The boundary condition for \( \sigma \) is that \( \phi \to 0 \) far away from the string and that \( \sigma \) should be smooth at the center. By assuming \( \alpha \) as a constant, Davis defines the shape of the field \( \sigma \equiv \sigma(r)e^{i\alpha} \) [12]. From these definitions the field \( \sigma \) turns out to depend only on the radius of the string \( r \).

Since the equation of motion consists of two scalar fields, it must be treated separately for each field. The obtained derivative of equation of motion for scalar field \( \phi \), which has been converted into a kinetic rate DBI, reads

\[
\frac{d}{dr} \left( \gamma \frac{d\phi}{dr} \right) + \frac{\gamma}{r} \frac{d\phi}{dr} - \frac{\lambda}{r^2} \phi \left( \phi^2 - \nu^2 \right) - \beta \phi \sigma^2 = 0, \tag{11}
\]

where

\[
\gamma = \frac{1}{\sqrt{1 - \frac{2}{r^2} \left( \left( \frac{d\phi}{dr} \right)^2 + \phi^2 \right)}}. \tag{12}
\]

The equation of motion of the scalar field \( \sigma \) is unchanged and the kinetic rate equation still uses the usual global cosmic string of Witten,

\[
\frac{d}{dr} \left( \frac{d\sigma}{dr} \right) + \frac{1}{r} \frac{d\sigma}{dr} - \frac{\lambda_\sigma}{2} \sigma \left( \sigma^2 - \nu_\sigma^2 \right) - \beta \phi^2 \sigma = 0 \tag{13}
\]

Both of them are nonlinear form and mutually coupled to each other, so that it is difficult to obtain the solution analytically. Therefore, to obtain a solution of the field equations it is necessary to exploit the numerical methods. The numerical result of those equations is shown in Fig. 1. The result shows that \( \phi \) tends to zero at the core of the string and will come a certain value, which is equal to the current value of \( \nu_\phi \) further away from the core. This means that when the field becomes zero, then symmetry in the core of the string is not damaged, because it has a certain energy value, and in fact is quite large. When the value of \( \phi = \phi_\lambda \), the symmetry will be broken because the field is at a vacuum value, which is the current state of minimum energy.

Unlike \( \phi \), the field of \( \sigma \) is in charge of making these strings superconducting. Our result shows that the core part has a certain value, but very close to zero. From Fig. 1 it seems that
the $\sigma$ will disappear close to zero when it is away from the core string. This condition describes the symmetry is broken at the core string but unbroken when further away from the core. The symmetry of $\sigma$ is broken in the core, which will allow the string to be enormous energized and uninhibited. It means that the magnitude of the current inside the string will never reduce, or in other words, the string is likely a superconductor. This condition is in accordance with the proposed idea about the cosmic string superconductors of Witten [8]. In addition, the results using DBI kinetic rate turns out to have the same characteristics as the results of Shellard Davis [12], which displays the condition of cosmic strings for ordinary superconductors.

From Fig. 1 one can also see that the numerical result of global superconducting cosmic strings for both DBI and non DBI exhibits the same characteristic. The condition of accelerated $\phi$ toward the value of $\nu_\phi = 1$ means that the symmetry is broken faster. The gradient of the plot of $\phi$ indicates the amount of energy in the string. Numerical results show that the DBI converted into kinetic energy in the string tends is getting greater. This may explain the possibility of the existence of the cosmic strings superconducting shortly after the Big Bang, given that the DBI only occurred at very high energy levels. Beside that, the variation of DBI coupling constants $\kappa$ can influence the obtained result. The smaller value of this parameter, the steeper is the gradient, which means that more energy in the string and $\phi$ is increasing toward the value of $\nu_\phi = 1$, or it implies a more rapid symmetry breaking. If we use a larger coupling constants, the obtained results are closer to the ordinary superconducting cosmic string without DBI.
The parameter of $\lambda_\phi$ is the coupling constants for $\phi$. When these constants are varied, the obtained result is shown in Fig. 2. From the figure, it appears that the greater value of parameter $\lambda_\phi$ the steeper the plot of $\phi$ will be obtained. This situation illustrates that the larger the selected coupling constant value $\phi$ the larger the impact on the growing energy in the string. Besides that the $\phi$ field is also increasing to the value of $\nu_\phi = 1$ which means that the sooner the destruction of symmetry. Although $\lambda_\phi$ is the coupling constant for the $\phi$ field, it also affects the physical state of the $\sigma$ field. This can be seen in the plot that shows the numerical result of different $\sigma$ fields in the core part of the string.

The parameter of $\lambda_\sigma$ is the coupling constant for $\sigma$. When these constants are varied, the results obtained is shown in Fig. 3. The effect of the parameter variation turns out to be contrary to the variation $\lambda_\phi$. The increase in the value of the parameter $\lambda_\sigma$ is proportional to the magnitude of the field at the core of the string. Although $\lambda_\sigma$ is a coupling constants for $\sigma$, its also influence the plot of $\phi$ field. Variation of the $\lambda_\sigma$ parameter values leads to an increasingly large $\phi$. This situation can be interpreted as the magnitude selection of the coupling constants for the scalar field $\sigma$, which is inversely proportional to the amount of energy of the string. It is also apparent that the magnitude determination of $\lambda_\sigma$ also affects the size of the string. The greater the selection parameter value the longer vacuum value of $\phi$ field, which means the size of the string is getting larger.

Finally, we will discuss the comparison between the results of the numerical solution global superconducting cosmic strings DBI and that of non-DBI with variation of $\beta$. This is shown in Fig. 4. Both results show similar characteristics. Presumably, the $\beta$ value has a greater influence on $\sigma$ rather than the $\phi$. It is apparent that the rise in the value of $\beta$ is inversely proportional to the scalar field $\sigma$ in the core part of the string. Both DBI and non-DBI cases show the same conditions. As for the plot of $\phi$ itself, it does not show a significant variation, which is seen from the position of the plots which are almost coincident with each other.

3. CONCLUSIONS
To understand the mechanism of superconducting cosmic strings, it is necessary to review the numerical solution of the cosmic string Lagrangian. The present study has been able to show that the scalar field $\phi$ and $\sigma$ of the superconducting cosmic string Lagrangian equation DBI exists. Overall, the numerical solutions of the ordinary superconducting cosmic strings and the superconducting cosmic string DBI have a similar characteristic. This situation shows that the solution of DBI superconducting cosmic strings can exist. In the future, a revised version that includes the presence of gauge fields is expected, in which the flow characteristics of quenching...
can be seen. This may strengthen the evidence of the possibility of the superconducting cosmic string DBI.

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