INTRODUCTION TO THE THEORY OF GOYAKS
OPERATOR MANIFOLD APPROACH
TO GEOMETRY AND PARTICLE PHYSICS

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The fundamental question that guides our discussion is "how did the geometry and particles come into being?" To explore this query we suggest the theory of goyaks, which reveals the primordial deeper structures underlying fundamental concepts of contemporary physics. It address itself to the question of the prime-cause of origin of geometry and basic concepts of particle physics such as the fundamental fields of quarks and leptons with the spins and various quantum numbers, internal symmetries and so forth; also basic principles of Relativity, Quantum, Gauge and Color Confinement, which are, as it was proven, all derivative and come into being simultaneously. The substance out of which the geometry and particles are made is a set of new physical structures - the goyaks, which are involved into reciprocal linkage establishing processes.

The most promising aspect of our approach so far is the fact that many of the important anticipated properties, basic concepts and principles of particle physics are appeared quite naturally in the framework of suggested theory.

In pursuing the original problem further we have elaborated a new mathematical framework in order to describe the persistent processes of creation and annihilation of goyaks in the definite states. It is, in fact, a still wider generalization of familiar methods of secondary quantization with appropriate expansion over the geometric objects.

One interesting offshoot of this generalization directly leads to the formalism of operator manifold $\hat{G}(2.2.3)$, which will frame our discussion throughout this paper. In broad sense it consequently yields the quantization of geometry, which differs in principle from all earlier studies. It was found out a contingency arisen at the very beginning that all states of goyaks are degenerate with the degree of degeneracy equal 2. That is, the goyaks are turned out to be fermions with half-integral spins, which subsequently give rise to origin of the spins of particles.

The nature of $\hat{G}(2.2.3)$ provides its elements with both the field and geometric aspects.

We have directly passed from $\hat{G}(2.2.3)$ to wave-manifold $\tilde{G}(2.2.3)$, which is practically

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2a goyak in Armenian means an existence (an existing structure). This term has been firstly used in (Ter-Kazarian, 1986)
made of goyak’s eigen-functions and was endowed by a statistical-probabilistic nature.

We have dealt with the principles of operator $\hat{F}$ and wave $\tilde{F}$ groups and their representations, which refer to continuous and discrete symmetries of operator and wave manifolds.

To render the discussion more transparent the generalized causal Green’s functions of regular goyaks in terms of wave manifold have been introduced, by means of which the geometry realization condition has been cleared up. There are two different choices of realization of geometry. First one is a trivial, which leads to geometry without particles and interactions. But the second choice subsequently yields the geometry $G(2.2.3)$ with the particles and interactions.

We employ the wave functions of distorted goyaks to extend the knowledge here gained regarding the quark fields are being introduced in the color space of internal degrees of freedom. The local distortion rotations around each fixed axis yield the quarks or antiquarks. They obey exact color confinement, and the spectrum of hadrons would emerge as the spectrum of the color singlet states.

Finally, it has been shown that the gauge principle holds for any physical system which can be treated as a definite system of distorted goyaks.

The theory of goyaks predicts a class of possible models of internal symmetries, which utilize the idea of gauge symmetry and reproduce the known phenomenology of electromagnetic, weak and strong interactions.

Here we focused our attention mainly on developing the mathematical foundations for our viewpoint. Hence our discussion has been rather general and abstract. Of course, much remains to be done for a larger contribution into the particle physics. However, we believe that the more realistic final theory of particles and interactions can be found within a context of suggested theory of goyaks.

**Key words:** Geometry-Particles-Basic Principles
1 Introduction

The contemporary understanding of physical processes is mainly based on the concepts of local symmetries and gauge fields (Weil, 1918; Yang et al, 1954; Utiyama, 1956; Glashow, 1961; Salam, 1959; Schwinger, 1962; Kibble, 1961).

Non-Abelian gauge theories may have something to do with nature. They are utilized by theoreticians for the description of both electro-weak interactions (Salam et al, 1964; Schwinger, 1957; Glashow, 1961; Weinberg, 1967; Salam, 1968; Abers et al, 1973) and strong interactions (Fritzsh et al, 1972; Weinberg, 1973; Gross et al, 1973; Marciano et al, 1978).

The theoretical picture of the weak interactions has became rather definite. At present a consistent description of the known weak interactions can be given in terms of gauge theory based on the group \( SU^{loc}(2) \otimes U^{loc}(1) \).

Many physicists suspect that the underlying theory of strong interactions is the gauge theory with the local unitary group \( SU^{loc}_C(3) \), consisting of colored quarks of various types coupled to eight colored gauge bosons (gluons). The existence of the internal symmetry group \( SU^{loc}_C(3) \) allows oneself to introduce a gauge theory in color space, with the color charges acting as exactly conserved quantities. The role played by quarks in hadron physics has became rather transparent. In building up the particle spectrum, they act as constituents such that states of the type \( qqq \) describe the baryons, and states of the type \( q\bar{q} \) the mesons. One can postulate the confinement condition such that all physical particles and observables are required to be singlets under the local color transformations implemented on the quarks through the \( SU^{loc}_C(3) \) rotation matrices in the fundamental representation. This leads automatically to the correct spectrum and eliminates quarks \( (q) \), diquarks \( (qq) \) etc. as physical particles. That is, a color would be confined and the spectrum of hadrons would emerg as the spectrum of the color singlet states. This indicates to the hidden realization of a symmetry: only singlets of the symmetry group exist as the physical states. This possibility requires the symmetry to be an exact symmetry.

At present there is a strong tendency among theoreticians to believe in the forceful
arguments brought up in favor of absolute confinement. It remains to be seen if there is the absolute color confinement, which is certainly the simplest and most attractive possibility, but perhaps not the one chosen by nature.

It will be advantageous to unify all interactions within one simple group. Several models of this type have been discussed by various authors (Weinberg, 1972; Pati et al, 1973; Georgi & Glashow, 1974; Georgi et al, 1974, 1979; Gunaydin et al, 1974; Ramond, 1977). As one might expected from a theory, which unifies the color and flavor groups of the quarks and the flavor group of the leptons in one simple group, the conservation of baryon number or the stability of the proton with respect to its decay into leptons is a nontrivial aspect (Gell-Mann et al, 1978; Harari, 1978; Goldman et al, 1980; Ellis et al, 1980; Segre et al, 1980). Which of the various schemes, if any, is realized either exactly or at least approximately in nature, remains to be seen.

The intensive attempts have been made in order to construct the unified field theory of particles and all interactions including also the gravitation, based on the concepts of supersymmetries and superstrings (Wess et al, 1974; Freedman et al, 1976; Deser et al, 1976; Schwarz, 1984; van Nieuwenhuizen, 1981; Schwarz, 1985; Green et al, 1985). There are a number of advantages to this suggestion, but on the theoretical side there are also several serious problems one has to deal with in these schemes. Of course, much remains to be done before one can determine whether this approach can ever make a larger contribution into the physics of particles and interactions.

Although a definite pattern for the theoretical description of particle physics has emerged, which is attractive enough both theoretically and phenomenologically, but it could not be regarded as the final word in particle physics, and many fundamental questions have yet to be answered. It is worth emphasizing that it is yet very vulnerable, especially on the theoretical basis. As a last remark, we note that just as in these earlier studies, the fundamental concepts and right symmetries, also basic principles of particle physics have been put into theory by hand. The difficulties associated with this step are notorious.
Thus, in spite of the considerable progress achieved over the entire subsequent period in the study of the fundamental constituents of the matter and forces, the physical theory is still far from being complete, and not free of many serious difficulties. Moreover, we are only at the very beginning of deeper understanding of the prime-cause of the origin of the basic concepts and principles of the particle physics, which are uncertain by now and there is still a long way to go.

This brings us to the greatest questions of physics: How did the fundamental concepts such as the space-time continuum, quarks and leptons, their masses at rest, various quantum numbers; and also basic principles of physics such as Relativity, Quantum, Gauge and Color Confinement principles come into being? Are they all primordial or derivative?

To explore these queries we are led to consider the big problem, as it was seen at the outset, which is how to find out the substance or basic fundamental structures, something deeper than the geometric continuum of four dimension (representing the arena of space-time, within which the phenomenon of the particles is presumed to appear), that underly both geometry and particles?

It is likely that these questions cannot be answered within the scopes of conventional physics. The absence of the vital physical theory, which is able to reveal these structures and answer to the right questions mentioned above, imperatively stimulates the search for general constructive principles, which becomes of paramount importance for particle physics.

Following Ter-Kazarian (1986, 1989, 1992) we learned that the manifold $G(2.2.3)$ underlies the space-time continuum. The perception of the space and time has been suggested by means of new concepts. While the problems and dynamics of the processes, which are of interest of Relativity and Quantum Field Theories, have been studied from a specific novel point of view. The spacial-time concepts have been properly substituted by the appropriate new ones.

On the examples of simple fields the possibility of introducing a concept of mass at rest of particle as a function of the internal degree of freedom has been shown.
The general theory of distortion of space-time has been suggested. Within its scopes, as it is widely believed, the space-time is not pre-determined background on which physical processes take place, but a dynamical entity itself.

Curvature of space-time continuum is considered as a particular regime of distortion, which enables one to construct a new theory of gravitation.

The other regime of "pure-inner distortion" of the space-time continuum below some small length scale has been discussed. It is shown that the theory predicts new physical phenomena, which have a dominant contribution in high energy region with respect to all other processes.

This theory in practical application in astrophysical problem, for example, enables oneself to provide an alternative approach for understanding of the internal structure of supermassive compact stationary celestial bodies such as active galactic nuclei. There are a number of advantages to this approach, which differs in principle from the standard black hole accretion models (Ter-Kazarian, 1989, 1989, 1990, 1991, 1992) and is in good agreement with the observational data.

The guiding line framing our discussion throughout this paper is a generalization and further expansion of basic ideas of the theory of distortion of space-time continuum, in order to find out the deeper structures underlaying both geometry $G(2.2.3)$ and particles. It will be appropriate to turn to them as the primordial deeper structures. Suggested theory will address itself to the question of the origin of fundamental concepts of particle physics such as the quarks, leptons, their spins, internal symmetries and associated with them different charges. Within its context we will clear up the prime-cause of origin of basic principles, especially of the most challenging ones such as Gauge and Color Confinement principles.

The paper is organized as follows: Being confronted by the problems mentioned above, in first part of treatment our task will be to develop and understand the conceptual foundations for our viewpoint in general. To begin with a description of theory we choose a simple setting and consider new formations designed to endow certain physical properties
and satisfying the rules stated in section 2. It forms the starting point for them to get into the processes of establishing the reciprocal "linkage" between different type of goyaks in given state.

We will elaborate a new mathematical framework in order to describe the persistent processes of creation and annihilation of goyaks. It will be, in fact, a still wider generalization of the familiar methods of secondary quantization or similar processes, with appropriate expansion over the geometric objects. Hence it gives rise to the formalism of "operator manifold" \( \hat{G}(2.2.3) \). We will discuss two aspects of it. First one is a quantum field aspect, but a second- is a differential geometric aspect.

A closer examination of the properties of this formalism practically compulsory leads to the other formalism of "wave manifold" \( \tilde{G}(2.2.3) \). The latter is a manifold of eigen-functions of goyaks and endows a statistical-probabilistic nature.

This one, in its turn, precedes to ordinary geometry \( G(2.2.3) \), within which subsequently the phenomena of fundamental fields with the half-integral spins, different charges and the unitary groups of internal symmetries emerge.

In the second part, which is devoted mainly to the dynamics of distorted goyaks, we will formulate a principle of identity of regular goyaks and extend the knowledge gained in outlined mathematical framework regarding the origin of fundamental fields, Gauge and Quark Confinement principles, also internal symmetries.

In last section we will discuss a class of models of internal symmetries, which reproduce the known phenomenology of electromagnetic, weak and strong interactions. In order to save writing we guess it worthwhile to leave the other concepts such as the flavors and so forth with associated aspects of particle physics for another treatment. It will not concern us here and must be further discussed. Surely this is an important subject for separate research.

Here, as far as we should fix our attention mainly on developing the mathematical foundations for our viewpoint, a discussion would be rather general and abstract. Actually, this kind of mathematical treatment will necessarily be schematic and introductory by
nature, since a complete discussion requires more realistic investigation.

However, we believe we would make good headway by presenting a reasonable framework whereby one will be able to verify the basic ideas and illustrate the main features of the theory of goyaks.

Part I. Regular Goyaks

2 The Goyaks and Link-Establishing Processes Between Them

Before embarking on a description of goyaks, just a very brief recapitulation of the main points of the structure of manifold (Ter-Kazarian, 1989, 1992)

\[
G(2.2.3) = ^*G(2.2) \otimes G(3),
\]

with the set of unit elements \(\{e_{(\lambda,\mu,\alpha)}\}\) \((\lambda, \mu = 1, 2; \alpha = 1, 2, 3)\) providing the basis

\[
e_{(\lambda,\mu,\alpha)} = O_{\lambda,\mu} \otimes \sigma_{\alpha}.
\]

The set of the linear unit bi-pseudo-vectors \(\{O_{\lambda,\mu}\}\)

\[
< O_{\lambda,\mu}, O_{\tau,\nu} > = \, * \delta_{\lambda,\tau} * \delta_{\mu,\nu}, \quad * \delta_{\lambda,\tau} = \begin{cases} 1 & \text{if } \lambda \neq \tau, \\ 0 & \text{otherwise}, \end{cases}
\]

is the basis in \(2 \times 2\) dimensional linear bi-pseudo-space \(^*G(2.2)\).

The \(G(3)\) is the three-dimensional real linear space with the basis consisted of the ordinary unit vectors \(\sigma_{\alpha}\)

\[
< \sigma_{\alpha}, \sigma_{\beta} > = \delta_{\alpha,\beta} = \begin{cases} 1 & \text{if } \alpha = \beta, \\ 0 & \text{otherwise}. \end{cases}
\]

Henceforth we always let the first two subscripts in the parentheses specify the bi-pseudo-vector components, the third refers to the ordinary-vector components.

The metric in \(G(2.2.3)\) is bilinear, local, symmetric and positive defined reflection

\[
\hat{g} : T_p \otimes T_p \rightarrow C^\infty(G(2.2.3)),
\]

where \(T_p\) is the whole set of vector fields on \(G(2.2.3)\).

The metric \(\hat{g}\) reads in component form with respect to the basis \(\{e_{(\lambda,\mu,\alpha)}\}\)

\[
g_{(\lambda,\mu,\alpha)(\tau,\nu,\beta)} = g(e_{(\lambda,\mu,\alpha)}, e_{(\tau,\nu,\beta)}) = g(e_{(\tau,\nu,\beta)}, e_{(\lambda,\mu,\alpha)}).
\]
Bilinear form on vectors $\zeta = e^{(\lambda,\mu,\alpha)} \zeta_{(\lambda,\mu,\alpha)} \in G(2.2.3)$ with respect to holonomic basis is given as follows:

$$\hat{g} = g^{(\lambda,\mu,\alpha)}(\tau,\nu,\beta) d\zeta_{(\lambda,\mu,\alpha)} \otimes d\zeta^{(\tau,\nu,\beta)}. \quad (2.7)$$

Analogical form on co-vectors $\bar{\zeta} = e_{(\lambda,\mu,\alpha)} \zeta_{(\lambda,\mu,\alpha)}$ reads

$$\hat{g} = g_{(\lambda,\mu,\alpha)}(\tau,\nu,\beta) d\zeta^{(\lambda,\mu,\alpha)} \otimes d\zeta_{(\tau,\nu,\beta)}. \quad (2.8)$$

Except where stated otherwise, here as usual, the double occurrence of the dummy indices will be taken to denote a summation extended over their all values.

The manifold $G(2.2.3)$ decomposes into two 6-dimensional manifolds

$$G(2.2.3) = G(2.3) \oplus G(2.3). \quad (2.9)$$

The set of unit elements

$$e^0_{i(\lambda\alpha)} = O^i_{\lambda} \otimes \sigma_\alpha, \quad (\lambda = \pm; \alpha = 1, 2, 3) \quad (2.10)$$

is the basis in manifolds $G(2.3)(i = \eta)$ and $G(2.3)(i = u)$, in which the positive metric forms are defined

$$\eta^2 = \eta_{(\lambda\alpha)} \eta^{(\lambda\alpha)} \in G(2.3), \quad u^2 = u_{(\lambda\alpha)} u^{(\lambda\alpha)} \in G^u(2.3), \quad (2.11)$$

where

$$e^0_{i(+)\alpha} = \frac{1}{\sqrt{2}} (e_{(1,1,\alpha)} + \varepsilon_i e_{(2,1,\alpha)}),$$

$$e^0_{i(-)\alpha} = \frac{1}{\sqrt{2}} (e_{(1,2,\alpha)} + \varepsilon_i e_{(2,2,\alpha)}), \quad (2.12)$$

$$\varepsilon_i = \begin{cases} 1 & \text{if } i = \eta, \\ -1 & \text{if } i = u, \end{cases}$$

and

$$\eta_{(+)\alpha} = \frac{1}{\sqrt{2}} (\zeta_{(1,1,\alpha)} + \zeta_{(2,1,\alpha)}), \quad u_{(+)\alpha} = \frac{1}{\sqrt{2}} (\zeta_{(1,1,\alpha)} - \zeta_{(2,1,\alpha)}),$$

$$\eta_{(-)\alpha} = \frac{1}{\sqrt{2}} (\zeta_{(1,2,\alpha)} + \zeta_{(2,2,\alpha)}), \quad u_{(-)\alpha} = \frac{1}{\sqrt{2}} (\zeta_{(1,2,\alpha)} - \zeta_{(2,2,\alpha)}). \quad (2.13)$$

The following scalar products of vectors (co-vectors) are available

$$< O^i_{\lambda}, O^j_\tau >= \varepsilon_i \delta_{ij}^r \delta_{\lambda\tau} \quad (< O^i_{\lambda}, O^r_\tau >= \varepsilon_i \delta_{ij}^r \delta_{\lambda\tau}),$$

$$< O^i_{\lambda}, O^j_\tau > = \varepsilon_i \delta_{ij}^\lambda \delta^\lambda_r, \quad (2.14)$$
and
\[ < e^0_{i (\lambda \alpha)}, e^0_{j (\tau \beta)} > = \varepsilon_i \delta_{i j} \delta_{\lambda \tau} \delta_{\alpha \beta}, \quad < e^{(\lambda \alpha)}_{i 0}, e^0_{j (\tau \beta)} > = \varepsilon_i \delta_{i j} \delta^\lambda \delta_{\alpha \beta}. \] (2.15)

provided
\[ O^\lambda_{i} = \delta^{\lambda \tau}_{i} O_{\tau}, \quad e^{(\lambda \alpha)}_{i 0} = \delta^{\lambda \tau} \delta_{\alpha \beta} e^0_{i (\tau \beta)}. \] (2.16)

To render our discussion here more transparent next we will develop the foundations for our viewpoint and proceed to general definitions and conjectures of the theory of goyaks directly. At this, for simplest sense, we may consider it briefly hoping to mitigate a shortage of insufficient rigorous treatment by the further exposition of the theory and make them complete and discussed in broad sense in due course.

**Conjecture 1** The 6-dimensional basis vectors \( e^0_{i (\lambda \alpha)} \) (or co-vectors \( e^{(\lambda \alpha)}_{i 0} \)) we explore from a specific novel point of view, as being the main characteristics of the real existing structures called ”goyaks”. Below we distinguish two type of goyaks: \( \eta \)-type \( (i = \eta) \) and \( u \)-type \( (i = u) \), respectively.

**Conjecture 2** The goyaks establish reciprocal ”linkage” between themselves. The links are described by means of ”link-function” vectors \( \Psi_{\eta (\lambda \alpha)} (\eta, p_\eta) \) and \( \Psi_{u (\lambda \alpha)} (u, p_u) \) (or co-vectors \( \Psi^{(\lambda \alpha)}_{\eta} (\eta, p_\eta) \) and \( \Psi^{(\lambda \alpha)}_{u} (u, p_u) \)):
\[ \Psi_{\eta (\pm \alpha)} (\eta, p_\eta) = \eta(\pm \alpha) \Psi_{\eta (\pm \alpha)} (\eta, p_\eta), \] (2.17)