Novel Ameliorated Least Square Method for Beam Forming

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Abstract. The least square method and its variants have been broadly studied and applied in many areas, including antenna beam forming and pattern synthesis. In this paper, based on these progresses, a new ameliorated least square method is presented. The novel approach reforms the least square formula into a new form with adjustable parameters. Through these parameters, the synthesized beampatterns are able to be controlled more flexibly. The results of simulations show that the pattern synthesized by the new approach is better in the key performance indicators, including the peak level of side-lobe and the null beam level, than the pattern formed by the traditional least square method.

1. Introduction

Beam forming and pattern synthesis technologies have been widely investigated and put into use in the fields of multiple-input multiple-output (MIMO) radar and smart antenna in the past decades. Many approaches of beam forming and pattern synthesis have been developed by researchers [1-2]. Many studies focused on beam forming and null beam forming and developed fast calculating approaches to adapt to the need of real time processing [3-7].

The least square (LS) method has been widely applied in many fields. Iterative least square method was applied for antenna beampattern synthesis in [8-11]. However, while non-iterative least square method was used for one dimension (1D) antenna pattern synthesis, it was not excellent. Actually, in many occasions, the pattern synthesis outcomes of the non-iterative least square method were not satisfactory [12].

The least square method and its variants have been broadly studied [12]. In this paper, based on these progresses, a new ameliorated least square method is presented. The novel approach reforms the least square formula into a new form with adjustable parameters. Through these parameters, the synthesized beam and pattern are able to be controlled more flexibly. The results of simulations show that the pattern synthesized by the new approach is better in the key performance indicators, including the peak level of side-lobe and the null beam level, than the pattern formed by the traditional least square method.

2. The New Method

A $d$-spaced $N$ isotropic elements uniform linear antenna array is studied first. A narrow band beam with the central wave length $\lambda$ is supposed to be transmitted by each array element. We study the scenario in far field. The antenna elements from 1 to $N$ are shown in figure 1. In figure 1 $\theta$ is the signal transmitting angle, which is the angle between the signal and the axis of array.
Let \( a(\theta) = [1, e^{j2\pi \cos \theta}, ..., e^{j2\pi (N-1) \cos \theta}]^T \) with superscript \( T \) denoting transpose operation. The antenna element excitation vector is denoted as \( w=[w_1,w_2,...,w_i,...,w_N]^T \) with \( w_i \) being the current excitation of \( i^{th} \) antenna element.

Set \( \theta \) in the range of \([0^\circ, 180^\circ]\). Denote its discrete values as \( \theta_1, \theta_2, ..., \theta_k, ..., \theta_K \). Let the targeted pattern vector be

\[
p = [p_1, p_2, ..., p_k, ..., p_K]^T, \tag{1}
\]

where \( p_k \) is the expected pattern value when \( \theta = \theta_k \).

Let \( \mathbf{G} = [a(\theta_1), a(\theta_2), ..., a(\theta_k), ..., a(\theta_K)]^T \). \( \mathbf{G} \) is called the array manifold matrix. In far field the pattern synthesis issue is to obtain the weight vector by solving equation

\[
\text{abs}(\mathbf{Gw}) = p, \tag{2}
\]

where \( \text{abs} \) denotes absolute value operation. Given the fact matrix \( \mathbf{G} \) has more rows than columns \((K>N)\) in this paper, therefore, application of direct least square method is very important. To use LS method to solve equation (2), in the traditional practice, equation (2) was often changed into

\[
\mathbf{Gw} = p. \tag{3}
\]

The least square solution of equation (3) is

\[
w_{LS} = (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H p, \tag{4}
\]

where superscript \( H \) denotes conjugate transpose operation. Let

\[
\mathbf{D} = \text{diag}(p) = \text{diag} [p_1, p_2, ..., p_K] \tag{5}
\]

with \( \text{diag} \) denoting diagonal matrix. Let

\[
[p_1^{-l-m+1}, p_2^{-l-m+1}, ..., p_k^{-l-m+1}]^T \text{ define } p^{-l-m+1}, \tag{6}
\]

where both \( l \) and \( m \) are integer variables. We consider

\[
\mathbf{Gw} = p^{-l-m+1}. \tag{7}
\]

If \( w \) satisfies equation (7), the pattern formed by \( w \) will have better performance in null beam attenuation than \( p \).

From equation (7) we can get

\[
(\mathbf{D}^H)^T \mathbf{Gw} = (\mathbf{D}^H)^T p^{-l-m+1} = (\mathbf{D}^{-1})^m p. \tag{8}
\]

So

\[
\mathbf{G}^H (\mathbf{D}^H)^T \mathbf{Gw} = \mathbf{G}^H (\mathbf{D}^{-1})^m p. \tag{9}
\]

Then, we can get
\[ w = (G^H(D)^{-1}G^H(D^{-1})^m)p. \] (10)

This is the ameliorated least square method.

3. Simulations and Results

As shown in figure 1, let \( d = \lambda/2 \). Let the signal transmitting angle \( \theta \)'s discrete value \( \theta_1, \theta_2, ..., \theta_k, ..., \theta_K \) equal to \( 0^\circ, 1^\circ, ..., 179^\circ, 180^\circ \) in sequence.

The targeted pattern \( p_1 \) generated by the following steps: firstly, a pattern \( p_0 \) is formed by uniform amplitude excitation; then a pattern \( p_{01} \) with a null beam of -92 dB in \([27^\circ, 33^\circ]\) and another null beam of -88 dB in \([127^\circ, 133^\circ]\) is generated by program based on \( p_0 \); finally, the target pattern \( p_1 \) is the inverse of the pattern \( p_{01} \). The element excitation vector \( w \) is gotten from equation (10). Then the formed pattern can be obtained from equation (3).

In the first example, let \( N = 27 \), the simulation outcomes are shown in figure 2.

In the column of the legend of figure 2, target indicates the expected pattern; LS marks the pattern formed by the traditional least square solution of equation (4), \( l=1 m=1, l=2 m=2, l=3 m=3, l=4 m=4 \) denote the patterns formed by the element current excitation vector of equation (10) while \( l \) and \( m \) have different values.

Figure 2. The simulation results with \( N = 27 \).

In figure 2 the main-lobes of the targeted pattern are in the range \([27^\circ, 33^\circ]\) and \([127^\circ, 133^\circ]\) with a notch at \( \theta = 129^\circ \). Its side-lobe peak level is -38 dB, and there is a null beam in the scope of \([86^\circ, 94^\circ]\) with the lowest level of -89 dB. The main-lobes of the least square method are in the range \([16^\circ, 36^\circ]\) and \([121^\circ, 138^\circ]\) with a notch at \( \theta = 128^\circ \). Its first side-lobe peak level is -11.5 dB. There is no apparent null beam on the pattern of the least square approach. The main-lobes of the new approach, while \( l \) and \( m \) have different values, almost all in the range \([19^\circ, 38^\circ]\) and \([124^\circ, 135^\circ]\). Their first side-lobe peak levels almost all are -14 dB. Their null beams almost all are in the scope \([86^\circ, 94^\circ]\), and its lowest level is -50 dB when \( l=1 m=1, -62 \) dB when \( l=2 m=2, -82 \) dB when \( l=3 m=3, -100 \) dB when \( l=4 m=4 \). In addition, all patterns formed by the new approach have no notch in contrast to that in the targeted one there is an indentation at \( \theta = 129^\circ \).

In the second example, let \( N = 31 \), the spacing of the antenna elements from 9 to 18 is \( d = 2\lambda/5 \) and other elements spacing is \( d = \lambda/2 \). Aside from these, all other conditions are the same as those in the first example. The simulation outcomes are shown in figure 3.
In the column of the legend of figure 3, target indicates the expected pattern; LS marks the pattern formed by the traditional least square solution of equation (4); \( l=1 \) \( m=1 \), \( l=2 \) \( m=2 \), \( l=3 \) \( m=3 \), \( l=4 \) \( m=4 \) denote the patterns formed by the element current excitation of equation (10) while \( l \) and \( m \) have different values.

In figure 3 the outlines of all the patterns nearly are similar with the curves in figure 2. There is still no apparent null beam on the pattern of the least square approach. It is obvious that the new approaches have better performance in the null beam attenuation. And while \( l \) and \( m \) increase from 1 to 4, the lowest level of the null beams decreases from -56 dB to -98 dB. The peak level of side-lobe of the least square method is still higher than those of the new approaches.

From the outcomes of the simulations, we can conclude that the new approach has better performance than the least square method, particularly in null beam forming, the new method is much better.

4. Conclusions
A new ameliorated least square method is presented. The novel approach reforms the least square method into a new form with adjustable parameters. Through these parameters, the synthesized beam and pattern are able to be controlled more flexibly. The results of simulations show that the pattern synthesized by the new approach is better in the key performance indicators, including the peak level of side-lobe and the null beam level, than the pattern formed by the traditional least square method.

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