The charged Z(4430) in the diquark–antidiquark picture

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\textbf{Abstract.} We identify a newly found Z(4430) with the first radial excitation of the tetraquark basic supermultiplet to which \(X(3872)\) and \(X(3876)\) belong. Experimental predictions following from this hypothesis are spelled out.

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1. Introduction

A possible interpretation of the new \(X\) and \(Y\) states observed by BELLE and BABAR \cite{1} is that they are made by a color antitriplet diquark, \([cq]\), bound to a color triplet antidiquark, \([\bar{c}\bar{q}]\), with \(q = u, d\), a tetraquark for short \cite{2}. Other tetraquark potential models have already been studied, with more problematic conclusions on the existence of tetraquark-bound states being drawn in Vijande \textit{et al} 2007 \cite{3}. The proposed diquark–antidiquark structure for the \(Z\) extends to high mass the picture proposed in \cite{4}\textsuperscript{3} for the light, sub-giga-electron-volt, scalar mesons \cite{5},

\textsuperscript{3} A more recent discussion is in Jaffe and Wilczek 2003, quoted under \cite{4}.
thereby aiming at a unified description of mesons which do not seem to fit into the $q\bar{q}$ picture. We note that the diquark–antidiquark picture at low energy is supported by the confirmation of a full $SU_3$ nonet made of $\sigma(500)$, $\kappa(800)$, $f_0(980)$ and $a_0(980)$.

At present, other explanations of $X$ and $Y$ particles are still possible, in terms of hadronic molecules for $X(3872)$ [6] and hybrid states [7], molecules [8] or baryonium [9] for $Y(4260)$. It is important to explore the predictions of each model, to allow for an experimental discrimination of the different alternatives.

A firm prediction of the tetraquark scheme is the existence, for given $J^{PC}$, of four states, two neutrals and two charged, which form an $I = 1, 0$ complex. $C$ is the charge conjugation eigenvalue of the neutral states, which may be superpositions of the two $I_3 = 0$ states, possibly with large mixing [2]. Isospin violation in neutral states is evidenced by the simultaneous decay of $X(3872)$ into $\psi \rho$ and $\psi \omega$.

The recent observation by BELLE and BABAR [10] of two different masses for the decays of the $X(3872)$ has been interpreted in [11] as an indication of the two neutral states associated with the $J^{PC} = 1^{++}$ multiplet:

$$X(3872) = X^0_{dd}(1^{++}; 1S) \rightarrow J/\psi \pi^+ \pi^-$$
$$X(3876) = X^0_{uu}(1^{++}; 1S) \rightarrow D^0 \bar{D}^0 \pi^0.$$  \hspace{1cm} (1)

Theoretical bounds to the observation of the charged partners have been derived and shown to be compatible with present limits [11].

The BELLE Collaboration has reported the observation of a new structure, $Z(4430)$, via the decay [12]:

$$Z(4430) \rightarrow \psi (2S) + \pi^\pm.$$  \hspace{1cm} (2)

Assuming the $Z$ structure to be a normal hadron resonance and assuming isospin conservation, the decay (2) indicates $I = 1$ and $G = +1$. Further assuming S-wave decay, we obtain $J^{PC} = 1^{++}$.

In this paper, we propose to interpret $Z(4430)$ as one of the long-sought charged charmonium states, with the composition $[cu][\bar{c}\bar{d}]$, and work out the predictions for masses and decay modes of other particles related to it in the tetraquark picture.

The tetraquark hypothesis implies a proliferation of states, already suggested by the anomalous $p_\perp$ distribution of $\psi$ in $B$ decay [13], that may be at the basis of the rich spectroscopy of $X$ and $Y$ states the experiments are starting to unveil ([14] and references therein).

At the time of our first communication on the subject [15], a different interpretation had been proposed [16], explaining the $Z$ structure in terms of a cusp effect related to the $D^*(2010) – D_1(2420)$ threshold. A cusp interpretation has also been suggested in [17] whereas the tetraquark picture, in a different variant, has been studied in [18]. Other authors underscore the four-quark hypothesis [19], or explore the possibility that the $Z(4430)$ is a molecular [20] or a baryonium state [21]. Independent of its internal structure, the properties of the $SU_3$ flavor partners of $Z^+$ have been investigated in [22].

2. Isospin, charge conjugation and G-parity properties of tetraquarks

To derive the correct selection rules for the decays of tetraquark states, it is useful to discuss primarily the related issues of isospin and G-parity invariance of these states.
Isospin symmetry in QCD is broken by the \( u - d \) quark mass difference and by the second-order photon and weak boson exchange processes. Consider first the limit in which all these effects are neglected and decay amplitudes are exactly isospin invariant. For self-conjugate isospin multiplets, charge conjugation invariance leads to the conservation of \( G \)-parity, where \( G = C(-1)^I \) and \( C \) is the charge conjugation of the neutral member of the isospin multiplet. Next, we introduce isospin breaking by inserting the corresponding effective Lagrangian in the isospin symmetric amplitudes. This gives a small effect, of the order of few per cent, except for the cases where the effective Lagrangian is inserted in the external legs. In the latter case, if there are near degeneracies between states of different isospin, the effect is enhanced by the presence of a small denominator. The situation occurs for the neutral members of the \( I = 0 \) and 1 multiplets considered before, where the mass difference may be of the same order as the \( u - d \) quark mass difference, as it was argued in [23] and references therein. In this case, we have to treat isospin breaking exactly by describing mass eigenstates as superposition of states with different isospin, thereby breaking \( G \)-parity and keeping charge conjugation invariance.

The upshot of this discussion is that when we consider the charged tetraquarks we may assume isospin and \( G \)-parity as good quantum numbers, to a few per cent accuracy, and use them to derive selection rules for production and decay. For neutral states, instead, the only good quantum number is charge conjugation. We shall use the charge conjugation of the neutral members, \( C \), to characterize the different \( I = 0, 1 \) complexes. \( C \) determines the \( G \)-parity of the associated charged states and their properties.

3. \( Z \) as a radially excited tetraquark

We have studied in [2] the lowest lying tetraquark supermultiplet within the constituent quark model. Besides the one associated with \( X(3872) \)/\( X(3876) \), the supermultiplet contains two \( I = 0, 1 \) multiplets with quantum numbers \( J^{PC} = 1^{+-} \). The properties of the two \( 1^{+-} \) states were estimated to be:

\[
X(1^{+-}; 1S)_{1,2}:
M_{1,2} \sim 3880, 3750; \quad X_{ud}^{+}(1^{+-}; 1S)_{1,2} \to \psi(1S) + \pi^+ \text{ or } \eta_c(1S) + \rho^+.
\]

The \( Z \) mass is too high for it to be associated to either of these complexes. It is interesting to observe, however, that the mass difference between \( Z(4430) \) and either one of the expected \( X(1^{+-}; 1S) \) is close to the mass difference:

\[
M_{\psi(2S)} - M_{\psi(1S)} \sim 590 \text{ MeV}
\]

so as to suggest that, in fact, \( \text{BELLE may have observed the first radial excitation} \) of one of the two \( 1^{+-} \) tetraquarks. The decay in \( \psi(2S) \) rather than \( \psi(1S) \) lends credibility to the statement.

In the following, we elaborate on a few testable predictions that would follow from the assignment:

\[
Z(4430) = X_{ud}^{+}(1^{+-}; 2S).
\]

The numerics between (3) and (4) work better for the highest \( 1^{+-} \) state, but either possibility is in fact compatible with our qualitative argument.
4. Neutral partners of Z(4430)

It goes almost without saying that Z(4430) must have neutral partners according to the scheme:

$$X_{u\bar{u},d\bar{d}}^0(1^+; 2S) :$$  
$$M \sim 4430 \pm \text{few MeV}; \quad X_{u\bar{u},d\bar{d}}^0(1^+; 2S) \rightarrow \psi(2S) + \pi^0 \text{ or } \eta \eta_c(2S) + \rho^0 \text{ or } \omega.$$  \hfill (6)

5. The 1^−, 1S and 2S, charged states

A crucial consequence of (5) is that two charged states decaying in $\psi + \pi^\pm$ or $\eta_c + \rho^\pm$ should be found around 3880 and 3750 MeV, according to (3). In addition, another state with similar decay to the Z should be seen at about $M(Z) \pm 130$ MeV, to be identified with the second, $J^{PC} = 1^+\text{ radial excitation}$.

6. Other states of the 2S supermultiplet

Replicating the mass spectrum found in [2] for the lowest excitations and assuming that the Z(4430) is the highest of the two 1^−, 2S states, one concludes that:

- The 1^++ radial excitations of $X(3872)$ should be at:
  $$X_{q\bar{q}}(1^{++}; 2S) :$$  
  $$M \sim 4200 - 4300; \quad X_{q\bar{q}}(1^{++}; 2S) \rightarrow D\bar{D}^{(*)}, \quad \psi' \pi^+\pi^-.$$  \hfill (7)

- The 2^{++}, 2S states should be around 4600 MeV.

In the case that the Z(4430) is the lowest 1^−, 2S state, we have to shift masses upwards by 150 MeV.

There are indications of an X state with decay in $D^*\bar{D}^*$ produced in:

$$e^+e^- \rightarrow \psi D^*\bar{D}^*$$  \hfill (8)

at a mass $M = 4160$ MeV. The charge conjugation of such a state is +1, so that $J^{PC} = 0^{++}$ or 2^{++} if S-wave decay is assumed. This particle could be identified with one of the two $X_{q\bar{q}}(0^{++}; 2S)$ although a conventional classification as $\eta_c(3S)$ is also viable at present.

7. The baryon–antibaryon threshold

The tetraquark mesons have a strong affinity for the decay into baryon–antibaryon channels. The thresholds are:

$$2M_{\Lambda_c} = 4572 \text{ MeV}; \quad M_{\Lambda_c} + M_{\Sigma_c} = 4739 \text{ MeV}.$$  \hfill (9)

for neutral and charged hidden-charm tetraquarks, respectively. X mesons with masses above this limit are expected to be considerably broader than the presently observed ones, which would provide another conclusive test of the present scheme.

Finally, we comment on the search for charged tetraquarks. Predicting the observable quantity: $B(B \rightarrow X + \text{anything}) \times B(X \rightarrow f)$, where $f$ is a given final state, is made difficult by two problems [2, 11].

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Firstly, there are two independent $B$ non-leptonic decay amplitudes which could combine differently in decays containing charged versus neutral tetraquarks.

Secondly, while the decay rates $\Gamma(X^+ \rightarrow f^+)$ and $\Gamma(X^0 \rightarrow f^0)$ are related by isospin, the corresponding branching ratios could be different, because neutral tetraquarks have additional multigluon decay channels. The decays of the charged $X^{\pm}$ associated with $X(3872)/X(3876)$ have the additional problem that they are strongly dependent on the $X^{\pm}$ mass due to the very close thresholds of the most favored channels $D^- D^0 \pi^0$ or $D^+ D^0 \pi^0$.

The two latter problems should not be so important for the radially excited tetraquarks, which have rates dominated by quark rearrangement diagrams where isospin relations should be valid.

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