Fermion masses and mixing, both in the quark and leptonic sector, are discussed within the approach to the Yukawa puzzle proposed by Arkani-Hamed and Schmaltz. In the quark sector we have shown that at least two extra dimensions are necessary in order to obtain sufficient CP violation, while reproducing the correct quark mass spectrum and mixing angles. We have also studied the consequences of suppressing lepton number violating charged lepton decays within this scenario for lepton masses and mixing angles.

1 Introduction

This work summarizes the results obtained in Branco et al. and Barenboim et al. in the context of a new approach to the flavour puzzle proposed by Arkani-Hamed and Schmaltz (AS) in the framework of large extra dimensions.

AS suggested that we live in a thick four dimensional subspace (thick brane) which is infinite in the usual four spacetime dimensions and possesses a finite volume in the extra orthogonal dimensions. Standard Model (SM) fields are constrained to live on this thick brane whilst gravity and possibly other gauge singlets propagate in the whole extradimensional spacetime; furthermore, in this scenario, the Higgs boson and the gauge fields are free to propagate in the entire thick brane, fermions, on the other hand, have higher dimensional wave functions which are localized in specific points in the extra dimensions. In this framework the effective four dimensional Yukawa couplings as well as all effective couplings involving, for instance, four fermions are suppressed by exponential factors that depend on the distance among the different fermion fields localized in the brane. This is an important result since in these models there is no high energy scale above a few Tev responsible for the suppression of unobserved phenomena such as proton decay and lepton number violating charged lepton decays. Direct coupling between fermions are exponentially suppressed by the small overlap of their wavefunctions which are given by narrow Gaussians.

The philosophy behind the AS scenario differs from the traditional explanation of fermion masses and mixing based on flavour symmetries relying instead on the assumption that the necessary suppression and hierarchy of couplings comes from the localization of these fields in the brane. Symmetries are thus replaced by geometry with all higher dimensional couplings assumed to be of order one. It should be noted that the AS scenario leads to startling consequences which may be observed at next generation collider experiments.
The effective four-dimensional Yukawa coupling among \( Q \), denoting a quark \( SU(2) \) doublet and \( U \) an up-type antiquark \( SU(2) \) singlet is given in this framework by \( \lambda = \kappa e^{-\mu^2(l_q-l_u)^2/2} \) with \( 1/(\sqrt{2}\mu) \) the Gaussian width of the fermionic fields \( Q \) and \( U \) whose Gaussian wave functions are centered at \( l_q \) and \( l_u \), respectively and \( \kappa \) the higher-dimensional Yukawa coupling. Likewise for the coupling among \( Q \) and \( D \), with \( D \) denoting the down-type antiquark \( SU(2) \) singlet with \( l_u \) replaced by \( l_d \).

Since in the AS framework quark fields are localized in different places, families are distinguishable, at least in principle, even in the limit where all Yukawa couplings vanish. However, one can still refer to different choices of weak basis (WB) which should be understood as corresponding to different assumptions about the underlying physics.

In the AS approach it is quite natural to obtain effective zeros in the Yukawa matrices, since they correspond to elements which connect fermions “far” apart. On the other hand, equalities or specific relations among elements of the Yukawa matrices usually require fine-tuning.

The question of whether there is any geometrical configuration of quark fields which fits all quark masses and mixing angles with all \( \kappa'_{ij} \)'s of order one was addressed by Mirabelli and Schmaltz \(^7\), and the answer is positive. Therefore, without assuming any flavour symmetry it is possible to accommodate the observed pattern of fermion masses and mixing angles simply by appropriately placing each quark field in a different position. In Ref\(^1\) Branco, Gouvêa and the author addressed the issue of CP violation which was not discussed in Ref\(^7\) and showed that, under these assumptions, it is not possible, with only one extra dimension, to obtain sufficient CP violation. The analysis was done with the help of the WB invariant condition for CP conservation \( Tr[H_u,H_d]^3 = 0 \), derived in Ref\(^8\) where \( H_u \equiv M_u M_u^\dagger \) and \( H_d \equiv M_d M_d^\dagger \); this condition is very useful since it allows to determine whether or not there is CP violation without the need of performing the diagonalization of the mass matrices. It was also shown by Chang and Ng \(^9\) that one can have sufficient CP violation with only one extra dimension at the price of introducing two different Yukawa coupling strengths for the up-type and down-type quarks. The search for a solution with two extra dimensions was done in the nearest neighbour interaction (NNI) basis (exemplified by the pattern of the mass matrices in Eq. \(^2\)). Notice that there is no loss of generality in choosing both \( M_u \) and \( M_d \) of the NNI form \(^10\). An interesting set of locations for the quark fields leading to the correct spectrum of quark masses and pattern of mixing angles was found, allowing for the right strength of CP violation. The locations of the quark fields in the two extra dimensions are depicted in Fig. 1 and correspond to:

\[
q_i = \frac{1}{\mu} \begin{pmatrix} 5.941; 0 \\ -4.008; 0 \\ 0; 0 \end{pmatrix}, \quad u_i = \frac{1}{\mu} \begin{pmatrix} -8.347; 0 \\ 1.815; 0 \\ -0.941; 0 \end{pmatrix}, \quad d_i = \frac{1}{\mu} \begin{pmatrix} -8.421; 0 \\ 2.219; 2.332 \\ -1.253; 2.767 \end{pmatrix},
\]  \( i = 1 \)
assuming $\kappa^i_j v = 1.5 m_t$, for all $i$ and $j$ and $m_t = 166000$ Mev (see Ref[7] for details regarding this choice) it leads to the mass matrices:

$$M_d = \begin{pmatrix}
0 & 16.112 & 0 \\
14.690 & 0 & 121.77 \\
0 & 1400 & 2467.8
\end{pmatrix} \text{MeV}, \quad M_u = \begin{pmatrix}
0 & 50.0 & 0 \\
20.3 & 0 & 2258 \\
0 & 48000 & 160000
\end{pmatrix} \text{MeV}, \quad (2)$$

where the zeros correspond to strongly suppressed matrix elements.

CP violation requires complex entries in the mass terms. In the NNI basis it is possible to eliminate all complex phases from $H_u$, while the off diagonal elements of $H_d$ still have arbitrary phases, by making a transformation of the type

$$H_u \rightarrow K^\dagger H_u K; \quad H_d \rightarrow K^\dagger H_d K,$$

where $K$ is an unitary diagonal matrix. After this transformation $H_d$ is left with two phases which can be factored out into another unitary diagonal matrix $K'$ ($H_d = K'^\dagger H_d K$ with $H_d'$ real). One may choose, without loss of generality $K' = \text{diag}(1, e^{-i\phi}, e^{-i\sigma})$. It was verified that, in this example, the values $\phi = 85^\circ$ and $\sigma = 0^\circ$ lead to the correct amount of CP violation together with values for masses and mixing within the experimental range. This solution and its rationale is analysed in detail in the original reference. It is not possible in such a short contribution to go into further details.

## 3 The leptonic sector

In Ref[2] Barenboim, Branco, Gouvêa and the author studied the consequences of suppressing lepton number violating charged lepton decays within this scenario for lepton masses and mixing angles. Due to limitations of space we just present here the main conclusions obtained in the reference given above.

In the first part of the paper the analysis was done in the framework of the SM with three generations and the addition of three righthanded singlet neutrino fields which were assumed to be localized at different points in the extra dimensions. Furthermore lepton number was also imposed as a conserved symmetry, in order to forbid Majorana masses for both the right-handed neutrinos and the active neutrinos. It was found that the branching ratios for flavour violating muon and tau decays can be very easily suppressed to levels below the current experimental bounds, and that very small neutrino masses can be obtained for separations of order $10^{\mu^{-1}}$. Only configurations which yield hierarchical neutrino masses were found. This is an interesting property of these scenarios. Almost degenerate neutrino masses have extremely interesting consequences in terms of mixing and CP violation[13] as well as in the prediction of a bound for neutrinoless double beta decay[13]. In the AS scenario, the large neutrino mixing which has been observed in the atmospheric neutrino data[14] requires fine-tuning of distances which in principle would be unrelated. This is a peculiar feature of the AS scenario for fermion masses: it naturally accommodates large mass hierarchies and small mixing angles, while it seems to require additional structure (since different pairs of fields have to be separated by very similar distances) in order to explain large mixing angles. The need for fine-tuning in these solutions should be interpreted as an additional challenge in the search for localizing mechanisms. It should be stressed that the AS scenario is particularly suited for explaining the absence of lepton flavour violating muon and tau decays, and can also explain why neutrinos are more than ten orders of magnitude lighter than the top quark, if right-handed neutrinos are introduced in the brane, as opposed to scenarios with bulk neutrinos.

At the end of the paper the effect of explaining the absence of flavour changing tau and muon decays and charged lepton masses in the AS scenario on theories where small Majorana neutrino
masses are generated by breaking lepton number in a far away brane was discussed. In this case, in order to obtain large mixing in the atmospheric sector, we were forced to place different lepton doublet fields so close that the current experimental upper bounds on the branching ratios of some rare tau and muon decays were almost saturated. Therefore, these rare decays are expected to be observed in the next round of experiments, if such a scenario were indeed realised in nature. Furthermore, very hierarchical neutrino mass-squared differences were not attainable, meaning that the solar neutrino puzzle would have to be solved either by the SMA or the LMA solutions.

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