Time-frequency analysis of nonlinear and non-stationary weak signals of corona discharge

Lei Wang*, Shanghe Liu, Ming Wei, XiaoFeng Hu
Electrostatic and Electromagnetic Protection Research Institute, Ordnance Engineering College, Shijiazhuang, China, 050003.

Email: wl810603@sina.com

Abstract. It is very useful to study the signals radiated from corona discharges for the purposes of high-voltage line monitoring. Time-frequency analysis can clearly reveal the time-varying spectrum characteristics of such signals, which is very useful for analyzing and processing the non-linear and non-stationary weak signals, such as the signals radiated from corona discharges. Several time-frequency analysis methods, such as the Short-Time Fourier Transform (STFT), Wigner-Ville distribution and the Hilbert-Huang Transform (HHT) and so on, are used in this paper. The simulation data with the same and different amplitudes are comparatively analyzed by these time-frequency distribution methods. It can be concluded that the time-frequency analysis method based on HHT is more efficient to identify and suitable for the non-linear and non-stationary weak signals.

1. Introduction
In the field of power systems, the ultra-UHV transmission has become the development trend of the power transmission. With the improvement of the transmission voltage level, the corona discharge is inevitable, and has become an important factor to affect the safe and stable operation of the high-pressure and UHV transmission lines. It is a new topic for the fault detection of high-voltage transmission line by remotely detecting the signal radiated from corona discharge. However a lot of noise appears in the testing process. So we need to process these signals by some method and detect the weak signals. The time-frequency analysis can clearly reveal the time-varying spectrum characteristics of signals, which is very useful for analyzing and processing the non-linear and non-stationary weak signals. At present, the time-frequency analysis method mainly contains the linear time-frequency analysis method and the bilinear time-frequency distribution based on Fourier transform, and the other method based on Hilbert-Huang transform [1]. Among them, the linear time-frequency analysis method contains the linear time-frequency analysis method and the bilinear time-frequency distribution based on Fourier transform [2], the wavelet transform time-frequency analysis [3] and time-frequency distribution of Gabor [4]; the typical representation of the bilinear time-frequency distribution is the Wigner-Ville time-frequency distribution [5]. The Hilbert-Huang Transform is different from the above two time-frequency analysis methods, which does the frequency decomposition based on

* To whom any correspondence should be addressed.
the local time-varying characteristics of the signal. This paper selects three kinds of time-frequency analysis methods: the short-time Fourier transform time-frequency analysis method, the Wigner-Ville time-frequency distribution and the Hilbert-Huang transform time-frequency analysis method. By analyzing the simulation data and comparing the advantages and disadvantages of this time-frequency analysis methods, the method which is more suitable for time-frequency analysis of weak signals radiated from corona discharge can be obtained.

2. The time-frequency analysis

2.1 The short-time Fourier transform [2, 6]
This is a basic time-frequency analysis. The short-time Fourier transform is the Fourier spectral analysis with a time window. The time-frequency analysis can be obtained by continuous sliding window on the timeline. The defined formula is

\[ STFT(t, \omega) = \int_{-\infty}^{\infty} X(\tau)w(\tau-t)e^{-j\omega \tau} d\tau \]  

(1)

Where, \( X(\tau) \) is the time series signals to be analyzed, and \( w(\tau-t) \) is the sliding time window.

2.2 Wigner-Ville distribution [5, 6]
The time and bandwidth product of the Wigner-ville distribution reached the lower limit of the Heisenbergde uncertainty principle. So the Wigner-ville distribution has good time-frequency resolution rate. It is defined as the instantaneous autocorrelation function of the Fourier transform, which is bilinear distribution proposed by Ville in the study of quantum mechanics. The expression is as follows:

\[ WVD(t, \omega) = \int_{-\infty}^{\infty} X(t + \frac{\tau}{2})X^*(t - \frac{\tau}{2})e^{-j\omega \tau} d\tau \]  

(2)

In the formula, the signal \( X \) appears twice, and without any window function, which avoids the mutual restraint of the time resolution and frequency resolution in the time-frequency linear representation.

2.3 Hilbert-Huang Transform [1]
For any time series, \( X(t) \), in order to determine the phase and eliminate the negative frequency effect in the signal analysis, the Hilbert transform should be done

\[ H(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{X(\tau)}{t-\tau} d\tau \]  

(3)

The analytic signal \( Z(t) \) corresponding to the original signal \( X(t) \) is:

\[ Z(t) = X(t) + jH(t) = A(t)e^{j\theta(t)} \]  

(4)

Where, \( A(t) \) is the amplitude function, \( \theta(t) \) is the phase function. Their expressions are

\[ A(t) = \sqrt{X(t)^2 + H(t)^2} \]  

(5)

\[ \theta(t) = \arctan \frac{H(t)}{X(t)} \]  

(6)

So the instantaneous frequency \( \omega \) can be defined as follow:
\[ \omega = \frac{d\theta(t)}{dt} \quad (7) \]

From the definition of Hilbert transform, it can be seen that the Hilbert transform is the convolution of the signal on \( \frac{1}{t} \). So it stressed the local characteristics of the signal, the instantaneous frequency can be given in some extent. However, many signals may contain multiple oscillation modes, which makes the simple Hilbert transform unable to give the full frequency content of these signals.

For this phenomenon, Huang [1] proposed a new time-frequency analysis method. By using it to analyze the data, two basic steps are required: First, the signal should be decomposed into the Intrinsic Mode Function (IMF) by using some decomposition method. Then, the energy distribution spectra in the time-frequency plane can be obtained by doing the Hilbert transform to the IMF.

The IMF is a function that satisfies two conditions: (1) in the whole data set, the number of extremum and the number of zero crossings must either equal or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

The signal is decomposed into several IMF, and a residue, \( r_n \), which can be either the mean trend or a constant. We finally obtain,

\[ X(t) = \sum_{i=1}^{n} C_i + r_n \quad (8) \]

In the formula, \( C_i \) is the IMF component. The IMF reflects the inherent internal volatility of signals, at any time it only has a single frequency component. So it can be carried out the Hilbert transform to calculate the instantaneous frequency.

3. The analysis of simulation data
The simulation data should be analyzed by different time-frequency analysis methods. So the advantages and disadvantages of these time-frequency analysis methods can be compared.

3.1 The analysis of simulation data with the same amplitude
The simulation signal can be obtained by the equation (9), which contains the frequency modulation signal and the sine signal.

\[ x(t) = \cos(2\pi30t + 0.8\sin(2\pi15t)) + \sin(2\pi150t) \quad (9) \]

The frequency modulation (FM) signal and the sine signal have the same amplitude. The frequency of the sine signal is 150Hz and the frequency of the FM signal is:

\[ f(t) = 30 + 12\cos(2\pi15t) \quad (10) \]

From the above equation, the frequency of the FM signal is from 18 Hz to 42 Hz. The time domain waveform of the simulation signal is shown as figure 1.
The above simulated signal should be analyzed respectively by short-time Fourier transform time-frequency analysis method, Wigner-Ville time-frequency distribution, and Hilbert-Huang transform time-frequency analysis method. The results are illustrated in figure 2.

It can be seen from figure 2, these three time-frequency analysis methods can reflect intact for the sine signal in frequency of 150Hz, and basically reflect the frequency of this signal.

As for the FM signal in frequency of $30 + 12\cos(2\pi15t)$, the results are different. The short-time Fourier transform time-frequency analysis methods can only reflect the carrier wave in frequency of 30Hz, the upper frequency 42Hz and the lower frequency 18Hz; the sine characteristics in this frequency can not be reflected. The Wigner-Ville time-frequency analysis can basically reflect the sine characteristics in the frequency of the FM signal, but it has a serious interference near the frequency of 90 Hz. The HHT time-frequency analysis integrated shows the sine characteristics in the frequency of the FM signal.

Figure 1. The simulation signal with the same amplitude.

Figure 2. Time-frequency analysis of signal with the same amplitude.
As the short-time Fourier transform depending on the traditional Fourier spectrum analysis, it must assume the data to be piecewise smooth, that is, the data is stationary at least in the analysis time window. By the constraint of the Heisenberg uncertainty principle, it can not be minimized simultaneously in time and frequency resolution, which is only suitable for approximating describing to the data that is stationary in subsection. Essentially, short-time Fourier transform is a kind of fixed bandwidth band-pass filter. For the short-time Fourier transform is divided into Grid in time-frequency plane, and its reliability relies on people’s subjective choices, so it is difficult to analyze the complex non-stationary signals. The Wigner-ville distribution is not linear, which contains serious cross-term. If the characteristics of complex signals are not known before analysis, the Wigner-ville distribution will be difficult to distinguish the true nature of the signal. The Hilbert-Huang transform time-frequency analysis does the frequency decomposition based on the local time-varying characteristics of the signal. The definition of the frequency is based on the local features and transient features of the signal waveform, which can be given, from point to point variation between the instantaneous frequency values at each time point of the signal data. It does not need several oscillation periods of the waveform to give a frequency value. The shortcomings of expressing the nonlinear and the non-stationary signals by using the meaningless harmonic components in traditional methods are overcome. By using this method, we can obtain the high frequency resolution and good time-frequency concentration, and achieve the complete, accurate analysis of the nonlinear and non-stationary signals.

Therefore, the Hilbert-Huang transform time-frequency analysis method is more suitable for the nonlinear and non-stationary signals.

3.2 The analysis of simulation data with the different amplitudes

![Figure 3. The simulation signal with the different amplitude.](image)

The amplitude of the sine signal in formula (9) is reduced to 10%, which is given by formula (11). The frequency of this simulation signal doesn’t change, but the intensity the sine signal becomes one-tenth of the original. The time domain waveform of the simulation signal is shown in figure 3.

\[ x(t) = \cos(2\pi 30t + 0.8\sin(2\pi 150t)) + 0.1\sin(2\pi 150t) \]  

(11)

It can be seen from the figure, the sine signal completely submerges in the FM signal. The time-frequency analysis is shown in figure 4.

In figure 4, the sine signal of 150 Hz can not be identified by the short-time Fourier transform time-frequency analysis and Wigner-Ville time-frequency analysis, while it can be identified by the Hilbert-Huang transform time-frequency analysis. Compared with other methods, the Hilbert-Huang transform method has the higher resolution, which is more effective to detect and analyze the weak signal.
Figure 4. Time-frequency analysis of signal with the different amplitude.

4. Conclusions
From the analysis of the simulation data, it can be concluded: compared with other time-frequency analysis methods, the Hilbert-Huang transform method can be more comprehensive and accurate to analyze the time-frequency characteristics of the nonlinear and non-stationary signals; and the Hilbert-Huang transform method has the higher resolution, which is able to detect the weak signal component from the strong background, so this method is more significant in the detection of weak signals radiated from corona discharge.

Acknowledgment
The paper is supported by the Key Programs of the National Natural Science Foundation of China, No. 61172035.

References
[1] Huang N E, Shen Z and Long S R 1998 Proc. R. Soc. Lond., p 903
[2] Kramer M L and Jones D L 1994 Proc. the IEEE-SP Int. Symp. on Time-Freq. and Time-Scale Anal. p 264
[3] Kawada M and Isaka K 2008 Time-Freq. Conf. Rec. of the 2008 IEEE Int. Symp. on Electr. Insul. p 381
[4] Subbanna N K and Eldar Y C 2004 Proc. the 11th IEEE Int. Conf. on Electron., Circuits and Syst., p 650
[5] Tan J L and Sha'ameri bin Z 2008 6th National Conf. on Telecommun. Technol. and 2008 2nd Malaysia Conf. on Photonics, p 260
[6] Shie Q and Chen D P 1996 Prentice Hall PTR.