On the cosmic ray bound for models of extragalactic neutrino production

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We obtain the maximum diffuse neutrino intensity predicted by hadronic photoproduction models of active galactic nuclei, and other sources such as gamma ray bursts, that is consistent with the observed cosmic ray spectrum and diffuse extragalactic gamma ray background. For this, we compare the contributions to the cosmic ray intensity of extragalactic neutrino sources with the present experimental upper limit on the extragalactic proton contribution to the cosmic ray spectrum, employing a transport calculation of energetic protons traversing cosmic photon backgrounds. We take into account the effects of the photon spectral shape in the sources on the photoproduction process, cosmological source evolution, the optical depth for cosmic ray ejection, and discuss the possible effects of magnetic fields in the vicinity of the sources. We also consider limits set by the observed extragalactic gamma-ray background due to the correlated hadronic gamma-ray emission.

The strongest cosmic ray bound applies to photoproduction sources which are optically thin for the emission of neutrons, and where magnetic field effects can be neglected. We find that our upper bound for that case approximately agrees with the bound obtained by Waxman and Bahcall for neutrino energies in the range of $10^5 - 10^7$ GeV, but allows higher fluxes at both lower and higher energies. We also discuss simple models for neutrino emission from AGN jets, showing that a large fraction of these sources has a high opacity for the emission of ultra-high energy cosmic rays, and that this allows still higher neutrino fluxes. From our models we estimate an upper bound on the neutrino emission from AGN, which implies only little or no corrections to existing predictions for the neutrino event rates correlated to AGN. We also confirm that hadronically emitting active galactic nuclei can produce the extragalactic gamma ray background. Finally, we discuss possible limits that neutrino fluxes, which may be observed in correlation with gamma-ray bursts, can set on the contribution from these sources to the ultra-high energy cosmic rays.

1. INTRODUCTION

The connection between the emission of cosmic rays, gamma rays, and neutrinos from astrophysical accelerators is of considerable interest for the solution of the problem of the origin of cosmic rays [?]. The reason why a fundamental relation between these components must exist can be understood as follows. Particle acceleration mechanisms in cosmic plasmas generally require the presence of a magnetic field which is able to confine the accelerated charged particles, i.e., electrons and protons (or ions). The accelerated electrons lose their energy quickly in synchrotron radiation in the magnetic field. These synchrotron photons provide a target for accelerated protons to undergo photo-hadronic interactions, resulting in the production of mesons, which decay. The particles which eventually emerge from this process are high energy photons, electrons (pairs), neutrons and neutrinos. Neutrinos are directly ejected due to their low interaction cross section. Gamma rays and secondary electrons initiate electromagnetic cascades, shifting the power from ultra-high energies to energies below which the absorption of gamma rays by pair production is unimportant [?]. Finally, the neutrons, which unlike the protons, are not confined in the magnetic field, can escape and convert into cosmic ray protons after $\beta$-decay, but their flux may be diminished by photoinduced $n \rightarrow p$ conversions. The branching ratios which distribute the available energy into the different channels are thereby generally of order unity. This leads to the conclusion that, cosmic proton accelerators produce cosmic rays, gamma-rays and neutrinos with comparable luminosities [?].

The fundamental relation between cosmic ray and gamma ray production has the obvious consequence that AGN, which are known to produce a large fraction of the gamma-rays in the Universe, are a prime candidate for the sources of ultra-high energy cosmic rays (UHECR) [?]. The spectra of the emitted GeV-TeV gamma radiation of AGN also agree with the predictions of a hadronic production of these gamma rays [?]. Other prominent gamma-ray sources, in particular the violent events connected with gamma-ray bursts (GRB), have also been suggested as UHECR source candidates. Moreover, most of the extragalactic gamma-ray energy is found in a diffuse background rather than in point sources, which allows for the possibility that the UHECR sources could be relatively large objects which would have a low gamma-ray
surface brightness, such as radio galaxies [?], galaxy clusters [?], or even larger structures [?]. Whatever the sources are, the fundamental relation between gamma-ray and neutrino fluxes implies that, if in fact the extragalactic gamma-ray emission is due to hadronic processes, a neutrino flux of similar bolometric luminosity must exist. This prediction is the major motivation for high energy neutrino experiments, which are currently operated, under construction, or planned. In fact, most model predictions for extragalactic high-energy neutrino fluxes have been made by using the source model to determine the spectral shape, and then by normalizing the total flux to some fraction of the diffuse extragalactic gamma-ray background (EGRB) [?].

In contrast to the limits set by gamma ray observations, the limits which could arise from the corresponding cosmic ray emission of the neutrino sources have been given little attention, perhaps because of the complications connected with their transport over extragalactic distances (but see Mannheim, 1995 [?]). Recently, it was proposed by Waxman & Bahcall [?] that indeed the measured flux of ultra-high energy cosmic rays provides the most restrictive limit on extragalactic diffuse neutrino fluxes. They claim that this cosmic ray bound is for neutrinos of all energies two orders of magnitude lower than the bound previously used which was based on the EGRB. Besides the obvious restriction of their result to neutrinos from proton accelerators, their claim is mainly based on three assumptions: (i) neutrons produced in photo-hadronic interactions can escape freely from the source, (ii) magnetic fields in the Universe do not affect the propagation of extragalactic cosmic rays, and (iii) the overall injection spectrum of extragalactic cosmic rays is $\propto E^{-2}$. A key role is played by their assumption (iii): By assuming a specific cosmic ray input spectrum, they can normalize their bound at the ultra high energies, where they can show that also (ii) applies. Assumption (i) is justified by showing that some particular sources of specific interest, like the TeV-blazar Mrk 501, or also GRB, are transparent to the emission of neutrons. The authors claim that this new bound set by cosmic ray data essentially rules out the hypothesis that hadronic processes in AGN jets can produce the EGRB, and consequently that their neutrino fluxes are overestimated.

The purpose of the present paper is to revisit the role cosmic ray observations can play to constrain models of neutrino production. The paper is organized as follows. In Section II, we give a brief review on the properties of photo-hadronic interactions, and derive the production spectra for cosmic rays and neutrinos for power-law photon target spectra. In Section III we briefly describe the effect of the propagation of extragalactic cosmic rays. We then follow Waxman & Bahcall in deriving a cosmic ray bound on neutrino fluxes, adopting their assumptions (i) and (ii), but instead of assuming a specific cosmic ray injection spectrum, we assume a specific spectrum for the observable extragalactic cosmic ray flux, which is constructed such that it complies with all existing observational limits on the cosmic ray proton intensity. In Section IV we turn to AGN, and discuss in particular the photo-hadronic opacity of blazar jets as it can be estimated from observations. We shall show that most AGN in fact have large photo-hadronic opacities at ultra-high energies, and we derive an upper bound for the neutrino contribution from AGN considering the effect of neutron opacity. In Section V, we discuss the possible effect of the magnetic fields known to exist in clusters of galaxies, and radio galaxies which are considered the hosts of gamma-ray emitting AGN, and we derive critical energies below which they can affect the bound. We conclude by discussing the combined effect of our results, and to what extent the cosmic ray data can indeed constrain models for expected neutrino fluxes, and vice versa.

II. COSMIC RAY, GAMMA-RAY, AND NEUTRINO EMISSION FROM EXTRAGALACTIC PROTON ACCELERATORS

In this section, we obtain the form of the spectra of cosmic rays, gamma-rays, and neutrinos escaping from cosmic proton accelerators. Here, we shall assume that protons are confined within the acceleration region. This is justified in particular for cosmic ray sources connected to relativistic outflows, like AGN jets or GRB, since the life time of protons is here limited by adiabatic energy losses and generally is much shorter than the diffusive escape time. As a consequence, only protons resulting from the decay of neutrons which have escaped from the acceleration region contribute to the cosmic ray spectrum. Escaping neutrons are produced by accelerated protons which interact with ambient soft photons, together with pions which decay into neutrons. We shall first discuss the properties of photo-hadronic interactions. We shall then obtain the ambient proton spectrum in the emission region resulting from shock acceleration followed by radiative cooling, adiabatic losses, and advection away from the shocked region where the photon density is sufficient for efficient photoproduction of neutrons. From this we obtain the form of the spectra of neutrons and neutrinos on production, and the escaping neutron spectrum which may be modified by neutron absorption in photo-hadronic $n \rightarrow p$ conversions.

A. Photo-hadronic interactions

Photo-hadronic interactions can be divided into two processes: photoproduction of pions (and other mesons), and Bethe-Heitler production of $e^\pm$ pairs. Charged pions decay as $\pi^\pm \rightarrow \mu^\pm\nu\bar{\nu}$, $\mu^\pm \rightarrow e^\pm\nu\bar{\nu}$ (here and in the following we disregard the difference between neutrinos and antineutrinos), neutral pions decay into gamma-rays as $\pi^0 \rightarrow \gamma\gamma$. Electrons and positrons from pion decay and Bethe-Heitler production cascade in the magnetic field and radiation field, and so can be assumed to convert all their energy into synchrotron radiation in the magnetic field required for the acceleration of protons. The production of charged pions allows the production of secondary neutrons through isospin exchange.

The physics of photo-hadronic interactions in ambient photon spectra has been extensively studied in Monte-Carlo simulations [?]. The properties of the production of secondary
particles can be expressed in the fraction $\xi$ of the proton energy given to a specific particle component per interaction. For neutrinos, gamma-rays, and neutrons, the values

\begin{align}
\xi_\nu &\approx \xi_\gamma \approx 0.1 \quad (1) \\
\xi_n &\approx 0.5 \quad (2)
\end{align}

respectively, have been found for power law target spectra typical in AGN jets, while for GRB target spectra the values are $\xi_\nu \approx \xi_\gamma \approx \xi_n \approx 0.2$ [7]. The energy per particle in units of the proton energy have been found for neutrinos and neutrons as $\langle E_\nu \rangle / E_p \approx 0.033$ and $\langle E_n \rangle / E_p \approx 0.83$, respectively, while for GRB they are $\langle E_\nu \rangle / E_p \sim 0.02$ and $\langle E_n \rangle / E_p \sim 0.5$, respectively [7]. From this we can immediately define the relative energy of escaping neutrinos and neutrons as

$$\eta_{\text{en}} = \langle E_\nu \rangle / \langle E_n \rangle \approx 0.04 \quad (3)$$

for both AGN and GRB target spectra. The fractional energy loss of the proton per interaction is $\kappa_\gamma \approx 0.2$ in the AGN case, and $\kappa_\gamma \approx 0.5$ for GRB [7]. Note that the quantities given here as typical for GRB apply only at ultra-high proton energies, at lower energies they approach the values found for AGN [7].

Electromagnetic radiation initiated by the Bethe-Heitler pair production, and by photons and electrons from neutral and charged pion decay, are reprocessed in synchrotron pair cascades. This energy will emerge as a component of the gamma ray background radiation, for AGN mainly in an energy range $10\text{MeV} - 1\text{TeV}$. The contribution of the Bethe-Heitler process to the production of gamma-rays depends on the target energy spectrum index, $\alpha$, since its cross section peaks at energies about two orders of magnitude lower than that of photopion production. Assuming that the power law extends over this range without change, one can find the relation

$$L_\gamma = [1 + \exp(5\alpha - 5)]L_\nu , \quad (4)$$

where $L_\gamma$ and $L_\nu$ are the bolometric photo-hadronic luminosities in gamma rays and neutrinos [7], and we note that for $\alpha = 1$, $L_\gamma = 2L_\nu$. We also note that in general, $L_\gamma \geq L_\nu$ holds as a direct consequence of the isospin-symmetry of charged and neutral pions — hence, for any kind of neutrino production involving pion decay, the bolometric flux in correlated photons sets an robust upper limit on the possible bolometric neutrino flux.

**B. Ambient proton spectrum**

We assume a spectrum of protons on acceleration of the form, $Q(E_p) \propto E_p^{-\alpha} \exp(-E_p/E_{\text{max}})$ (s$^{-1}$ GeV$^{-1}$). In order to calculate the spectra of cosmic ray protons and neutrinos escaping from AGN jets we first need to obtain the ambient spectrum of protons, $N_p(E_p)$ (GeV$^{-1}$), in the shocked regions where the soft photon target density is sufficiently high for pion photoproduction to take place. From the ambient proton spectrum we can then obtain the spectra of neutrinos and neutrons produced, $Q_\nu(E_n)$ and $Q_\nu(E_\nu)$ (s$^{-1}$ GeV$^{-1}$), respectively. Then taking account of the optical depth of the emission region for neutrons escaping photo-induced $n \rightarrow p$ conversions, one can obtain from the neutron production spectrum, the spectrum of cosmic ray protons resulting from the $\beta$-decay of the neutrons escaping from the jet, $Q_p(E_p)$.

The spectrum of protons on acceleration and the ambient proton spectrum are related by the proton loss time scale, $t_p(E_p)$,

$$N_p(E_p) \approx Q_p(E_p)t_p(E_p) . \quad (5)$$

The processes mainly contributing to losses of protons are interactions with radiation, advection away from the shock region of dimension $R$, and adiabatic energy losses if the emission region expands.

The advection time scale is expected to be $t_{\text{adv}} \approx R/(\beta_\text{sh}c)$ where $\beta_\text{sh}$ is the shock velocity and $R$ is the dimension of the jet in the shocked region. In relativistic flows streaming away from a central source, this energy independent time scale is usually in competition with adiabatic energy losses of the protons due to the expansion of the flow [7]. In a relativistic outflow, characterized by a bulk Lorentz factor $\Gamma$ and an opening angle $\Theta$, the expansion velocity in the co-moving frame (in units of $c$) is $\beta_{\text{os}} \approx \Gamma\Theta$ for $\Theta < 1^{-1}$, and $\beta_{\text{os}} \approx 1$ otherwise. This leads to adiabatic cooling on a time scale $t_{\text{ad}} \approx R/(\beta_{\text{os}}c)$. For example, in GRB one can generally assume that $\beta_{\text{os}} \approx 1$, and also observations of superluminal motions in AGN jets are consistent with $\Theta \approx 1^{-1}$ [7], thus $\beta_{\text{os}} \approx 1$. In the following, we shall assume that adiabatic losses are relevant with $0.3 \lesssim \beta_{\text{os}} \lesssim 1$. This has the important consequence that the lifetime of protons in the jet (or outflow) is limited to about one crossing time. The time scale for diffusive escape of protons is usually much longer (except, maybe, near the maximum proton energy), thus protons with $E_p \ll E_{\text{max}}$ can be assumed to be confined in the emitter and do not contribute to the cosmic ray emission.

The photon target spectrum will be assumed to have a power law shape, $\nu(\epsilon) \propto \epsilon^{-\alpha-1}$, extending to energies sufficiently above the threshold for photopion production by protons of energy $E_p$. Then, for $\alpha > 0$ the time scale for energy loss by photo-hadronic interactions is asymptotically of the form

$$t_{\text{pp}}(E_p) \propto E_p^{-\alpha} \quad (6)$$

where $t_{\text{pp}}$ is understood as including Bethe-Heitler and pion production losses, the cooling time for pion production will be called $t_{\text{pp},\pi} > t_{\text{pp}}$. For very flat target spectra, as for example in GRB at ultra-high proton energies, the photoproduction time scale is approximately constant [7], thus, Eq. (6) applies with $\alpha = 0$. We shall confine the discussion to the values $\alpha = 1$, relevant in AGN, and $\alpha = 0$ hereafter. Hence, for $\alpha = 1$ we obtain

$$t_{\text{pp}}(E_p) = (E_1/E_p)^{2}(R/c) , \quad (7)$$

with $t_{\text{pp},\pi}(E_p) \approx 1.5t_{\text{pp}}(E_p)$. $E_1$ is the energy corresponding to unit optical depth for photoproduction losses, $\tau_{\text{pp}}(E_1) = R/[c t_{\text{pp}}(E_1)] = 1$. Setting $t_{\text{pp}}^{-1} = t_{\text{pp,ad}}^{-1} + t_{\text{pp}}^{-1}$ we obtain,
\[ N_p(E_p) = Q_p(E_p)(R/c)[(E_p/E_1) + a]^{-1} \]  
with \( a = \max(\beta_{\text{ex}}, \beta_{\text{sh}}) \), and for \( Q_p(E_p) \propto E^{-2} \) we have
\[
N_p(E_p) \propto \begin{cases} 
    a^{-1}E^{-2} & (E_p < aE_1) \\
    E_1E_p^{-3} & (E_p > aE_1)
\end{cases}.
\]
(9)
(Note that for clarity, here and in the next section we omit the exponential cut-off in the spectrum at \( E_{\text{max}} \).) Obviously, for \( \alpha = 0 \) the optical depth for photoproduction is constant, and \( N_p(E_p) \propto Q_p(E_p) \). One can show that for typical photon densities in GRB fireballs \( \tau_{p\gamma} \ll 1 \), and that Bethe-Heitler losses are unimportant, viz., \( t_{p,\pi} \approx t_{p\gamma} \) [2, 7].

### C. Generic cosmic ray proton and neutrino production spectra

The time scale for photo-hadronic production of neutrons is \( t_{p\gamma-n} \approx t_{p,\pi}k_{p}/\langle N_n \rangle \). For \( \alpha = 1 \), this is \( t_{p\gamma-n} \approx 0.5t_{p\gamma} \propto E_p^{-1} \), while for \( \alpha = 0 \) we have \( t_{p\gamma-n} \approx 2.5t_{p\gamma} \). This immediately gives the production spectrum of neutrons,
\[
Q_n(E_n) \approx N_p(E_p)/t_{p\gamma-n}(E_n) \propto \begin{cases} 
    a^{-1}E_n^{-1} & (E_n < aE_1) \\
    E_1E_n^{-2} & (E_n > aE_1)
\end{cases}.
\]
Neutrons may escape to become cosmic ray protons. However, because neutrons themselves suffer pion photoproduction losses, the cosmic ray production spectrum will differ from \( Q_n(E_n) \) above the energy, \( bE_1 \) at which the optical depth for neutron escape, \( \tau_{n\gamma} \), is one. Neutrons can be considered as “absorbed” after they are converted into a proton, or after they have lost most of their energy in \( n\gamma \) interactions, whichever time scale is shorter. For \( \alpha = 1 \) this means \( \tau_{n\gamma} \approx 2\tau_{p\gamma} \), giving \( b \approx 0.5 \) for AGN jets, while for \( \alpha = 0 \) and typical GRB photon densities, \( \tau_{n\gamma} \approx \tau_{p\gamma} < 1 \), which means that neutron absorption is unimportant in GRB.

We note that in an homogeneous emitter, the escape probability of an interacting particle propagating in straight lines is given as a function of optical depth by
\[
P_{\text{esc}}(\tau) = (1 - e^{-\tau})/\tau \approx \begin{cases} 
    1/\tau & \tau < 1 \\
    1 & \tau > 1
\end{cases},
\]
resulting in the cosmic ray proton production spectrum being steepened above \( bE_1 \), compared with \( Q_n(E_p) \). For \( \alpha = 1 \), the cosmic ray proton production spectrum is therefore
\[
Q_{\text{cr}}(E_p) \propto \begin{cases} 
    a^{-1}E_p^{-1} & (E_p < aE_1) \\
    E_1E_p^{-2} & (aE_1 < E_p < bE_1) \\
    bE_1E_p^{-3} & (E_p > bE_1)
\end{cases}.
\]
(12)
However, because the two break energies, \( aE_1 \) and \( bE_1 \), are probably very close, \( a \sim b \), we shall adopt a single break energy \( E_0 = bE_1 \), and use the following approximations
\[
Q_n(E_n, L_p) \propto L_p \exp\left[\frac{-E_n}{E_{\text{max}}}\right] \begin{cases} 
    E_n^{-1}E_0^{-1} & (E_n < E_b) \\
    E_n^{-2}E_0^{-1} & (E_b < E_n)
\end{cases}.
\]
(13)

### III. PROPAGATION OF NEUTRINOS, PHOTONS, AND PROTONS OVER COSMOLOGICAL DISTANCES

In this section, we discuss propagation of cosmic rays and neutrinos in an expanding Universe filled with the cosmic microwave background radiation. To illustrate the problem, we compare of energy-loss horizons of protons and neutrinos. We shall then briefly discuss the physical problems connected to several approaches to cosmic ray propagation calculations. Using the numerical propagation code described by Protheroe and Johnson [7], we then calculate the observable neutrino and cosmic ray spectra from a cosmological distribution of generic photo-hadronic sources, as described in the last section. Here we assume that the sources are transparent to neutrons, while protons are confined, and that gamma-rays are reprocessed in synchrotron-pair cascades until emitted in the energy range of \( 10 \text{ MeV} - 30 \text{ GeV} \). Using an extrapolated cosmic ray spectrum which is consistent with present observational limits on the light component of cosmic rays (i.e. protons) as an upper limit on the possible extragalactic proton contribution, and the diffuse extragalactic gamma ray background (EGRB) observed by EGRET as an upper limit on the hadronic extragalactic gamma ray flux, we determine
an energy dependent upper bound on the neutrino flux from cosmic ray sources with the assumed properties. Our result is compared with the energy independent bound on extragalactic neutrino fluxes recently proposed by Waxman & Bahcall [7].

A. Comparison of energy-loss horizons

We wish to compare the distances that neutrinos, photons with energies below threshold for cascading in background radiation fields, and protons will travel through the Universe without significant energy losses. We define the energy loss horizon by

$$\lambda = \frac{cE}{|dE/dt|},$$

such that for linear processes the energy is reduced to $1/e$ of its initial value on traversing a distance $\lambda$.

For gamma-rays below $\sim 30$ GeV and neutrinos, the energy-loss process is due to expansion of the Universe [7]. For simplicity, we adopt an Einstein-de Sitter cosmology (i.e., $\Lambda = 0$ and $\Omega = 1$), so that the horizon $\lambda$ can be related to a redshift $z$ by the redshift-distance relation

$$\lambda(z) = \frac{2c}{3H_0}(1 - (1 + z)^{-3/2})$$

where $H_0 = 50h_{50}$ km s$^{-1}$ Mpc$^{-1}$ is the Hubble constant. Since we require the distance for which the energy is reduced by a factor $e$ during propagation, we have $(1 + z) = e$ giving the horizon for redshift losses,

$$\lambda_z = \frac{2c}{3H_0}(1 - e^{-3/2}).$$

This is also the horizon for neutrinos and gamma-rays below $\sim 30$ GeV. Normalizing the neutrino horizon to the radius of the Einstein-de Sitter universe,

$$\hat{\lambda}_\nu = \frac{3H_0}{2e} \lambda_z,$$

we obtain $\hat{\lambda}_\nu = \hat{\lambda}_\nu = \hat{\lambda}_z \approx 0.78$.

In addition to redshift losses, extragalactic cosmic rays suffer energy losses from photo-hadronic interactions with cosmic backgrounds, mainly the microwave background, and this is the reason for the Greisen-Zatsepin-Kuzmin (GZK) cut-off expected for a cosmic ray spectrum originating from a cosmologically homogeneous source distribution [7,7]. Photo-pion production and Bethe-Heitler pair production govern the energy loss in different energy regimes due to their very different threshold energies. The Bethe-Heitler process limits the propagation of protons with energies $E_p > 2m_pm_p c^2/kT_{\text{th}} \approx 4 \times 10^9$ GeV to $\lambda_p \sim 1$ Gpc, while pion production reduces the horizon for protons with $E_p \lesssim m_{\pi} c^2/kT_{\text{th}} \approx 5 \times 10^{11}$ GeV to $\lambda_p \sim 10$ Mpc. Again, in units of the radius of the Einstein-de Sitter Universe, the energy-dependent horizon for protons can be written

$$\lambda_p(E) = \left[1/\lambda_z + 1/\hat{\lambda}_{p,BH}(E) + 1/\hat{\lambda}_{p,\pi}(E)\right]^{-1}$$

where the components expressing redshift, Bethe-Heitler and pion production losses, can be written as

$$\hat{\lambda}_z \approx 0.78,$$

$$\hat{\lambda}_{p,BH}(E) \approx 0.27h_{50} \exp(0.31/E_{10}),$$

$$\hat{\lambda}_{p,\pi}(E) \approx 5 \times 10^{-4}h_{50} \exp(26.7/E_{10}),$$

with $E_{10} = E_p/10^{10}$ GeV. The approximations for $\hat{\lambda}_{p,BH}$ and $\hat{\lambda}_{p,\pi}$ fit the exact functions determined numerically in [7] and the exact interaction kinematics within $\sim 10\%$ up to $E_p \sim 10^{12}$ GeV.

The different energy-loss horizons for gamma rays and neutrinos, and protons strongly affect the relative intensities of their diffuse isotropic background fluxes. This is true in particular for evolving source populations such as quasars, galaxies, or GRBs (if they trace star formation activity), since here most of the energy is released at large redshifts. Cosmic rays above the ankle ($E_{\gamma \text{cr}} \sim 3 \times 10^9$ GeV) originate only from sources with redshifts $z \lesssim 0.27$, while neutrinos and gamma rays originate from sources within $z_\nu = z_\gamma \sim 1.7$. This will give rise to the neutrino intensity being enhanced relative to the protons because of their larger horizon, and because of the evolution of the sources (e.g. quasars) with cosmic time (redshift).

We may illustrate the problem as follows. The basic method of calculating the approximate present-day diffuse fluxes of neutrinos, gamma rays below $\sim 30$ GeV, and cosmic rays of photoproduction origin, would be to integrate the contributions from sources at redshifts up to those corresponding to the respective energy-loss horizons. Assuming a constant source number per co-moving volume element for simplicity, the resulting fluxes are proportional to $V_c(\hat{\lambda})/d_L(\hat{\lambda}) \sim \hat{\lambda}$, where $V_c(\hat{\lambda})$ and $d_L(\hat{\lambda})$ are the cosmological co-moving volume and the luminosity distance, respectively, corresponding to the horizon $\hat{\lambda}$. Assuming photo-hadronic production of neutrinos and cosmic rays in, e.g. a cosmological (non-evolving) distribution of AGN, the relative flux of neutrons (assuming no absorption) with energy $E_n$, and corresponding neutrinos with energy $E_\nu = 0.04E_n$, is at the source given by $[E_n^2N_n(E_\nu)]/[E_\nu^2N_\nu(E_n)] = \xi_\nu/\xi_n \approx 0.2$. The same ratio will be observed in the integrated fluxes, as long $E_n < 4 \times 10^9$ GeV. For higher cosmic ray energies, the flux ratio must be multiplied by a factor $\hat{\lambda}_\nu/\hat{\lambda}_p$, which yields a flux ratio of $\sim 0.6$ for $E_n \sim 10^{10}$ GeV, and $\sim 30$ for $E_n \sim 10^{11}$ GeV. Obviously, the differences would be much larger if we had assumed strong source evolution which enhances the contribution from large distances. If we want to determine a neutrino spectrum from an observed, correlated cosmic ray spectrum from the same sources, the result must therefore approximately reflect the changes in the ratio of the energy loss horizons.
B. Exact calculation of present-day neutrino and cosmic ray spectra

The example above, of simply integrating the cosmic ray and neutrino contribution of cosmologically distributed sources up to the energy-loss horizon, disregards several important aspects of particle propagation. Firstly, the particle number is not conserved in this method. Secondly, the energy evolution of the particles is neglected, which removes the dependence on spectral properties of the source. Both caveats are removed in a method known as the continuous-loss approximation, which follows the particle energy along fixed trajectories as a function of cosmological distance (or redshift) [?, ?]. For example, the trajectories for neutrino energies would be simply \( E(z) = E_0(1+z) \). This method is exact for adiabatic losses due to the expansion of the Universe (i.e., particle redshift), and is still a very good approximation for Bethe-Heitler losses, but it gives only poor results for photo-pion losses. The reason for the latter is the large mean free path, and the large inelasticity of this process, which results in strong fluctuations of the particle energy around its mean trajectory [?, ?]. Cosmic ray transport in the regime where photo-pion losses are relevant is therefore best described by numerical approaches, either solving the exact transport equation [?], or by Monte-Carlo simulations [?]. Another important aspect concerning relative fluxes of cosmic rays, gamma rays and neutrinos is the fact that the interaction of the cosmic rays with the cosmic background radiation themselves produces secondary contributions, which have to be considered as an additional contribution to the primary neutrinos and gamma rays. The full problem is treated by the cascade propagation code which has been described in detail by Protheroe and Johnson [?], and which we shall use also here. The code is based on the matrix-doubling technique for cascade propagation developed by Protheroe and Stanev [?].

For an input spectrum \( (dP_{gal}/dV_{cr})(Q(E, z)) \) per unit comoving volume per unit energy per unit time, the intensity at Earth at energy \( E \) is given by

\[
I(E) \propto \frac{1}{4\pi} \int_{z_{min}}^{z_{max}} M(E, z)(1+z)^2 \frac{dV_c}{dL} \frac{dP_{gal}}{dV_c} \langle Q(1+z|E, z) \rangle dz
\]

where \( dL \) and \( V_c \) are luminosity distance and co-moving volume, and \( M(E, z) \) are “modification factors” for injection of protons at redshift \( z \) as defined by Rachen and Biermann [?]; for neutrinos, \( M(E, z) = 1 \). The modification factors for protons depend on the input spectra, and are calculated numerically using the matrix method [?].

C. An abstract bound on extragalactic neutrino fluxes from neutron-transparent sources

From the above considerations, it is obvious that one can use the observed cosmic ray spectrum to construct a correlated neutrino spectrum, under the assumption that all observed extragalactic cosmic rays are due to neutrons ejected from the same sources as the neutrinos. A difficulty here is that the contribution of extragalactic cosmic rays to the total observed cosmic ray flux is unknown. Since neutrons convert into cosmic ray protons, we can clearly consider only the proton component at all energies. Above the ankle in the cosmic ray spectrum at \( E_{\text{cr}} \sim 3 \times 10^9 \text{ GeV} \), observations are generally consistent with a “light” chemical composition, i.e., a 100% proton composition is possible. Since protons at these energies cannot be confined in the magnetic field of the Galaxy, they are also likely to be of extragalactic origin. At extremely high energies, however, there is the problem that the event statistics is very low, and different experiments disagree on the mean cosmic ray flux at \( \sim 10^{11} \text{ GeV} \) by one order of magnitude (see Bird et al. [?], for a comparison of the results until 1994 of the four major experiments Akeno, Haverah Park, Fly’s Eye, and Yakutsk). This energy region is very important, since we expect here the existence of the GZK cutoff due to photoproduction losses in the microwave background. Currently, no clear evidence for the existence of this cutoff has been found, and the results of at least two major experiments (Haverah Park [?], and AGASA 1998 [?]) are consistent with the result obtained from a superposition of all experiments using...
a maximum likelihood technique [7,8], that is, a continuation of the cosmic ray spectrum as a power law $\propto E^{2.75}$ up to $\gtrsim 3 \times 10^{11}\text{ GeV}$ (see also Fig. 1).

The situation is even more difficult at lower energies: cosmic rays are here assumed to be mainly of Galactic origin, and there is evidence that a considerable, maybe dominant fraction consists of heavy nuclei rather than protons. Around the knee or the cosmic ray spectrum at $E_{\text{cr}} \approx 10^6$–$10^7\text{ GeV}$, recent results from the KASCADE air shower experiment suggest that the fraction of heavy nuclei in the cosmic ray flux is at least $\sim 30\%$, and further increasing with energy [7]. Also below the knee, in the energy range $10^6$–$10^7\text{ GeV}$, the analysis of air shower data has produced tentative evidence of a composition change from heavy to light (with increasing energy), supporting a dominantly heavy composition of cosmic rays between the knee and the ankle [7]. [Note that this result is under dispute, and it has been shown that it depends on the Monte-Carlo simulation codes used to construct the air-shower properties in dependence of the primary particle mass [7]. These simulation codes involve particle interaction models based on extrapolations many orders of magnitude above the energy range currently accessible with particle accelerators [7,8].]

Using all the available data, we find that an extragalactic cosmic ray spectrum of the form

$$N_{\text{p,obs}}(E) = 0.8 \times (E/1\text{ GeV})^{-2.75} \text{ cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^{-1}$$

$$\times (3 \times 10^6 \text{ GeV} < E < 10^{12}\text{ GeV})$$

(25)

is consistent with all data and limits on the cosmic ray proton flux (Fig. 1). It represents the current experimental upper limit on the extragalactic cosmic ray proton flux, which we shall use to construct an upper limit on the possible, diffuse extragalactic neutrino flux.

To construct the neutrino bound, we assume test spectra of the form $Q_{\text{cr}}(E) = Q_{\nu}(E) \propto E^{-1} \exp(-E/E_{\text{max}})$ with $10^6\text{ GeV} < E_{\text{max}} < 10^{12}\text{ GeV}$. The corresponding neutrino spectra are determined using Eq. (16). We assume a source distribution following the cosmological evolution function found for galaxies and AGN (7, see next section). For a given $E_{\text{max}}$, the total contribution of cosmic rays and neutrinos is calculated using Eq. (24). The resulting spectrum is then normalized so that its maximum reaches the cosmic ray flux given by Eq. (25). By varying $E_{\text{max}}$ between $10^6\text{ GeV}$ and $10^{12}\text{ GeV}$, we then obtain the desired maximum flux of neutrinos consistent with the present cosmic ray data (see Fig. 2). We also consider the correlated gamma ray output, assumed to be twice the neutrino energy flux, and check it does not exceed the observed power-law component of the diffuse gamma-ray background (we estimate the background between $3\text{ MeV}$ and $30\text{ GeV}$ to be $\sim 1.5 \times 10^{-5}\text{ GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$, [7]). Note that the cosmic ray curve for $E_{\text{max}} = 10^6\text{ GeV}$ does not reach the estimated cosmic ray proton spectrum in Fig. 2(a) in order to avoid over-producing diffuse gamma rays.

The upper bound on the neutrino flux according to this model is given by the minimum of the cosmic ray and the gamma ray bound, shown in Figure 3 by the curve “$\tau_{\nu} < 1$“. We see that the cosmic ray limit starts to dominate the bound for $E_{\nu} \gtrsim 100\text{ TeV}$, which then decreases to a minimum at $E_{\nu} \approx 10^9\text{ GeV}$, after which it rises again. This rise is a consequence of the strongly increasing ratio of neutrino and proton horizons above this energy, in conjunction with two assumptions we have used: (a) the observed cosmic ray spectrum continuing as an unmodified power law above $3 \times 10^{10}\text{ GeV}$,
the opposite extreme, i.e., sources with a very high neutron energy. This implies only to sources which are transparent to neutrons. For example, if an extended data set would confirm the existence of the GZK cutoff, we will estimate specific neutrino spectra for AGN models. In this case, our estimate of the local cosmic ray source. In this case, our estimate that the lack of evidence for the GZK cutoff in the cosmic ray data is due to the dominant contribution of AGN jets, using the same method as in the previous section, but for the appropriate generic source spectra, Eqs. (14) and (15), varying the break energy $E_b$ over the range allowed by the models.

(b) the distribution of sources being homogeneous in space. While the first assumption is shown to be consistent with present data (although the fluctuations are large), the latter assumption might be questioned — a more realistic scenario might be that the contribution of AGN jets, which typically span the range allowed by the models. Applying the source evolution functions found for BL Lacs and radio-quasars, respectively, we shall derive model estimates for the diffuse neutrino contribution from these sources which are compatible with cosmic ray limits. We shall also construct an upper bound for the extragalactic cosmic ray spectrum to the observed flux at $E_{\nu} = 10^{10}$ GeV. Consequently, their bound (determined for a source evolution $\propto (1+z)^{3.5}$) agrees with ours at $E_{\nu} \sim 5 \times 10^8$ GeV, where the cosmic ray limit is most restrictive. Clearly, this approach implies a correlated extragalactic cosmic ray flux which stays much below the upper limit allowed by observations (except at $10^{10}$ GeV). Of course, their limit applies to models which are based on the same assumptions as they used to derive their bound, as for example the model for diffuse neutrino fluxes from GRB proposed by the same authors [??], and this is also shown in Fig. 3 (note that this prediction assumes no cosmological evolution for GRBs). Also shown in Fig. 3 is a model prediction for diffuse neutrino fluxes from AGN jets (Model A in Mannheim [??]) in which the cosmic ray limit has already been used with the assumption that the emerging neutrons at $\sim 10^{10}$ GeV contribute to the extragalactic cosmic ray spectrum.

IV. DIFFUSE NEUTRINO SPECTRA FROM AGN JETS

In this section, we shall consider the spectra of cosmic ray protons and neutrinos emerging from jets of two classes of gamma-ray emitting AGN, i.e., BL Lac objects and radio-quasars, which are usually combined as the class of *blazars*. We shall use the estimated optical depths of gamma rays to photon-photon pair production in a typical AGN of each type, to infer the corresponding neutron-photon optical depths in these objects. We shall show that high luminosity AGN (like 3C 279) can be expected to be opaque to neutrons at energies above about $10^8-10^9$ GeV, while low luminosity BL Lacs (like Mrk 501) must be transparent to neutrons at all energies. Then assuming a model for the luminosity dependence of the optical depths, and the local luminosity functions of BL Lacs and quasars, we shall estimate the form of the production spectra of cosmic ray protons and neutrinos per unit volume of the local Universe. Applying the source evolution functions found for BL Lacs and radio-quasars, respectively, we shall derive model estimates for the diffuse neutrino contribution from these sources which are compatible with cosmic ray limits. We shall also construct an upper bound for the contribution of AGN jets, using the same method as in the previous section, but for the appropriate generic source spectra, Eqs. (14) and (15), varying the break energy $E_b$ over the range allowed by the models.

A. Cosmic ray proton and neutrino production spectra from blazars

As our starting point, we assume a target photon spectrum with index $\alpha = 1$ which we have already seen leads to $\tau_{\gamma\gamma}(E_\gamma) \propto E_\gamma$. A similar energy dependence applies to
gamma rays interacting with the same photons by photon-photon pair production ($\gamma\gamma \rightarrow e^+ e^-$), and so for $\alpha = 1$ we have
\[
\tau_{p\gamma}(E_p) = \left(\frac{\kappa_p \sigma_{p\gamma}}{\sigma_{\gamma\gamma}}\right) \tau_{\gamma\gamma} \left(\frac{2m_e^2 p}{m_{e^+}m_p + m_e}\right) 
\approx 5 \times 10^{-11} \tau_{\gamma\gamma} \left(\frac{4 \times 10^{-6} E_p}{10^{-9} E_n}\right).
\]
(26)

Here we have used $\langle \kappa_p \sigma_{p\gamma} / \sigma_{\gamma\gamma} \rangle \approx 300 \mu\text{barn}/\sigma_T$ from averaging over a photon spectrum with $\alpha = 1$, where $\langle \kappa_p \sigma_{p\gamma} \rangle$ includes Bethe-Heitler pair production, $\sigma_{\gamma\gamma}$ is the total cross section for the process $\gamma\gamma \rightarrow e^+ e^-$ [2], and $\sigma_T$ is the Thomson cross section. Using $\tau_{n\gamma}(E_n) \approx 2\tau_{p\gamma}(E_n)$, and assuming that $\tau_{n\gamma}(E) \propto E$ holds for a range $10^6 E_n \lesssim E_n \lesssim E_{\text{max}}$, we obtain the relation
\[
\tau_{n\gamma}(E_n) / \tau_{\gamma\gamma}(E_n) \approx 4 \times 10^{-9} E_n / E_{\gamma}.
\]
(27)

We apply this relation to two reference AGN: the BL Lac object Mrk 501, and the quasar 3C 279. The combination of the observed TeV spectrum and the EGRET flux limits for Mrk 501 gives rise to the assumption of a break energy for at which $\tau_{\gamma\gamma} = 1$ at about $0.3-1\text{ TeV}$ [2,7], while for 3C 279 the EGRET spectrum is consistent with a break at $\sim 3-10\text{ GeV}$ [2]. [Note that the recent observation of the spectrum of Mrk 501 up to 25 TeV by the HEGRA Cherenkov telescopes [2] does not imply a low optical depth at that energy, as for a homogeneous emitter the escape probability of an interacting particle propagating in straight lines is such as to steepen the spectrum by one power of energy at energies above which $\tau_{\gamma\gamma} = 1$, see Eq. (11). This is consistent with the observed HEGRA spectrum.] Applying Eq. (27) we find that the break energy in the cosmic ray production spectra should be $E_b \sim 10^{13}\text{ GeV}$ in Mrk 501, and $E_b \sim 10^{9}\text{ GeV}$ in 3C 279.

We note that the break energies derived above depend on the assumption of an undistorted power law target spectrum of photons. This assumption is valid in good approximation for BL Lacs, which show power law photon spectra extending from the infrared (relevant for $p\gamma$ interactions) into the X-ray regime (relevant for $\gamma\gamma$ absorption of $\sim 10-100\text{ GeV}$ photons, assuming a Doppler boosting of the emission by a factor $\delta \sim 10$). In high luminosity radio-quasars, however, this is not the case (see, e.g., [7]), and relation (27) does not necessarily hold. Moreover, in these sources the dominant target spectrum for $p\gamma$ and $\gamma\gamma$ interactions could be given by the external accretion disk photons, forming roughly an (unboosted) power law spectrum with $\alpha = 1$ up to $\sim 10\text{ GeV}$ [2], where it drops by about one order of magnitude (e.g. [2]). This still implies a relation like Eq. (27), but with $\tau_{\gamma\gamma}$ increased relative to $\tau_{p\gamma}$ by a factor of 10 — consequently, the possible break in the gamma-ray spectrum of 3C 279 would correspond to $E_b \sim 10^{9}\text{ GeV}$. A principal lower limit to the break energy in EGRET sources is set at $E_b \sim 10^7\text{ GeV}$, since otherwise EGRET photons ($\gtrsim 1\text{ GeV}$) could not be emitted.

Using cosmic ray proton production spectra with a break at $\sim 10^8\text{ GeV}$ (see Fig. 4) will, of course, have a strong effect on the neutrino bound implied by cosmic ray data. Since the cosmic ray proton spectrum of the source drops after the break faster than the observed extragalactic cosmic ray spectrum, Eq. (25), the bound at a neutrino energy $E_\nu > E_b/25$ is essentially set by the cosmic ray flux at the break energy $E_b$, rather than by the more restrictive flux at $25E_b/2$. If all sources had the same $E_b$, the bound would be increased roughly by a factor $(25E_b/E_b)^{0.75}$.

B. Blazar luminosity functions and generic models for the neutrino contribution from BL Lacs and radio-quasars

In order to obtain a parametrization of blazar neutrino spectra, we need to express $\tau_{n\gamma}$ as a function of the blazar luminosity $L$, given at some frequency. Here we take into consideration that blazars are assumed to be beamed emitters, with a Doppler factor $\delta \sim 10$. The optical depth intrinsic to the emission region, is proportional to $L_1/R$, where $L_1$ is the intrinsic target photon luminosity and $R$ is the size of the emitter. Since blazars are strongly variable objects [2], we can use the variability time scale, $T_{\text{var}}$, to estimate the intrinsic size by $R \sim T_{\text{var}}\delta$. Using also the relation between intrinsic and observed luminosity, $L = L_1\delta^4$, we obtain $\tau_{n\gamma} \propto L/T_{\text{var}}\delta^{-5}$. Although there is no detailed study of a possible systematic dependence of $T_{\text{var}}$ on $L$, the observations are compatible with no such correlation existing — for example, variability time scales of order 1 day are common in both, moderately bright BL Lacs like Mrk 501 or Mrk 421, and powerful quasars like 3C 279, PKS 0528+134, or PKS 1622-297, whose optical luminosities differ by at least three orders of magnitude [2]. For the Doppler factors, there is no evidence for a systematic dependence on $L$ either [2], although unification models for blazars and radio galaxies (see next section) suggest that BL Lacs are on average slightly less beamed ($\langle \delta \rangle \sim 7$) than radio-quasars ($\langle \delta \rangle \sim 11$) [2]. Therefore, we may assume that for
both BL Lacs and quasars, $\tau_{\gamma\gamma}(E, L) \propto L$ holds on average, and that $\tau_{\gamma\gamma}(E, L, \text{BL Lac}) \sim 10 \tau_{\gamma\gamma}(E, L, \text{quasar})$. It is interesting to note that this relation would imply a relation of the break energies of Mrk 501 ($L_{\text{opt}} \sim 10^{44} \text{erg s}^{-1}$) and 3C 279 ($L_{\text{opt}} \sim 10^{47} \text{erg s}^{-1}$) as $E_b(3C 279) \sim 10^{-2} E_b(\text{Mrk 501})$, consistent with our estimate obtained for intrinsic absorption from the gamma ray spectral break. Of course, this does not rule out the possibility that $E_b$ might be systematically lower in 3C 279 due to external photons, as argued above.

To determine the contribution of all blazars in the Universe, we have to relate the proton luminosity $L_p$ to the blazar luminosity $L$ in some frequency range, and then integrate over the luminosity function, $dN/dL$, determined for the same frequency range. Here we have to distinguish between the luminosities in the energy range where the target photons are, $L_o$, and the luminosity of the gamma-rays, $L_\gamma$, which are here assumed to be produced by hadronic interactions. Obviously, the latter implies $L_p \propto L_\gamma$, but on the other hand $\tau_{\gamma\gamma} \propto L_o$. We also have to distinguish between the two classes of blazars: while for BL Lacs $L_o \propto L_\gamma$, observations rather suggest that for quasars $L_\gamma \propto L_o^{1.9}$ [7]. This leads to the following simple models:

For BL Lacs, we use the X-ray luminosity function given by Wolter et al. [7] for X-ray selected BL Lacs,

$$dN_{\text{BL}}/dL_X \propto L_X^{-1.6}$$

and use the relations $L_p \propto L_X$, and $E_b \propto L_X^{-1}$ with $E_b = 10^{11} \text{GeV}$ for $L_X = 3 \times 10^{44} \text{erg s}^{-1}$ (Mrk 501). Here we have assumed that $L_o \propto L_X$.

For quasars, we use the EGRET luminosity function given by Mukherjee & Chiang [7],

$$dN_{\nu}/dL_\gamma \propto L_\gamma^{-2.2}$$

and the relations $L_p \propto L_\gamma$, and $E_b \propto L_\gamma^{1/2}$ where we consider two possible normalizations for $L_\gamma = 10^{48} \text{erg s}^{-1}$ (3C 279), which are $E_b = 10^9 \text{GeV}$ in case that jet-intrinsic photons dominate the target field, and $E_b = 10^8 \text{GeV}$ for the assumption that external photons are the dominant target. For illustration, we show in Fig. 4 cosmic ray and neutrino spectra on emission for $E_b = 10^8 \text{GeV}$ and $E_{\text{max}} = 10^{11} \text{GeV}$.

Then we obtain the form of the production spectra of cosmic ray protons and neutrinos averaged over the local universe due to these two classes of AGN,

$$\langle Q_{\text{cr,nu}}(E) \rangle = \frac{\int Q_{\text{cr,nu}}(E, L)(dN/dL) dL}{\int (dN/dL) dL},$$

where the input spectra $Q_{\text{cr,nu}}$ are given by Eqs. (14) and (15), with $E_{\text{max}} = 10^{11} \text{GeV}$, and $E_b$, $L_p$ given as functions of $L$ as discussed above.

To integrate properly over redshift, we note that while quasars show strong evolution similar to galaxies (see below), BL Lacs show little or no evolution [7], and we shall take this into account when propagating these spectra through the Universe from large redshifts. For quasars, their luminosity per co-moving volume has a pronounced peak at redshifts of $z \sim 2$, and declines or levels off at higher redshifts [7]. We shall assume that this effect is due to evolution of the number of quasars with $z$, rather than evolution of the luminosity of individual sources, which keeps the production spectra $\langle Q_{\text{cr,nu}}(E) \rangle$ independent of $z$. A particular parametrization of the redshift-dependence of the (co-moving frame) UV luminosity density of AGNs as inferred by Boyle and Terlevich [7], assuming an Einstein-de Sitter cosmology and $h_{50} = 1$, is given by

$$\frac{dP_{\text{gal}}}{dV_c} = P_0 \left\{ \begin{array}{ll}
(1 + z)/2.9)^{3.4} & (z < 1.9) \\
1.0 & (1.9 \leq z < 3) \\
\exp[-(z - 3)/1.099] & (z \geq 3)
\end{array} \right.,$$

(31)

where $P_0 = (3.0 \pm 0.3) \times 10^{44} \text{erg s}^{-1} \text{Mpc}^{-3}$. Clearly, the normalization plays no role here since $L_p$ is adapted to match the cosmic ray flux at earth. So for BL Lacs (no evolution), we simply use $dP_{\text{gal}}/dV_c = 1$.

The result is shown in Figure 5. As expected, the diffuse neutrino fluxes from AGN jet models exceed the bound for optically thin sources, but fall into the allowed region for sources optically thick for neutron emission. The BL Lac contribution falls below the bound, because it was derived for a non-evolving source distribution — we note that it still exceeds the corresponding Waxman-Bahcall bound for the case of no evolution. In addition to the models discussed above, we have also constructed and estimated an upper bound on the diffuse neutrino contribution on AGN jets. Here we used the same method as for the construction of Fig. 3, but implying input spectra of the form of Eqs. (14) and (15), with variable $E_b$ and fixed $E_{\text{max}} = 10^{11} \text{GeV}$. The break energy $E_b$ was then varied in the allowed range $10^7 \text{GeV} < E_b < E_{\text{max}} = 10^{11} \text{GeV}$, and the normalization chosen such that the superfused spectra approximately represented the upper limit on the extragalactic cosmic ray spectrum (Fig. 5, left panel). We note that this upper bound corresponds within a factor of 2 with the prediction of a previously published model by Protheroe [7]. Other published models, for example by Halzen & Zas [7], or Mannheim [7, Model B], exceed our bound by about one order of magnitude at energies $E_\nu \gtrsim 10^8 \text{GeV}$, but their predictions for the important energy range below $\sim 10^7 \text{GeV}$ are compatible with this bound. We return to the discussion of these models later, after we discuss the effect of magnetic fields in the AGN environment in the next section.

V. MAGNETIC FIELDS AND THEIR IMPACT ON COSMIC RAY PROPAGATION

Cosmic ray sources which are compact enough to produce high neutrino fluxes are unlikely to occur isolated in extragalactic space. Normally, they can be expected to reside inside galactic or metagalactic structures. X-ray observations and measurements of extragalactic Faraday rotation suggest that
such structures, i.e., galaxy halos, radio galaxy lobes, clusters of galaxies, or superclusters, carry magnetic fields of the order of $0.1 - 10 \, \mu G$ [7]. These can influence the propagation of cosmic ray protons in essentially two ways: (a) particles may be physically confined in the structure for a time \( t \geq t_{\text{HII}} = 1/H_{\text{HII}} \), or (b) the diffusive escape of the particles can lead to adiabatic energy losses. Obviously, magnetic fields on scales larger than the mean free path of a neutron for lead to adiabatic energy losses. Magnetic fields on scales larger than the mean free path of a neutron for lead to adiabatic energy losses. Magnetic fields on scales larger than the mean free path of a neutron for lead to adiabatic energy losses. Magnetic fields on scales larger than the mean free path of a neutron for lead to adiabatic energy losses. Magnetic fields on scales larger than the mean free path of a neutron for lead to adiabatic energy losses. Magnetic fields on scales larger than the mean free path of a neutron for lead to adiabatic energy losses. Magnetic fields on scales larger than the mean free path of a neutron for lead to adiabatic energy losses.

A. Particle confinement in clusters and superclusters

Clusters of galaxies have been recently discussed in the literature as a possible “storage room” for cosmic rays [7, 11, 14], because of their relatively strong magnetic fields ($B_0 \gtrsim 1 \, \mu G$) extending over large scales, i.e., cluster radii, \( R_{\text{cl}} \gtrsim 1 \, \text{Mpc} \). The time scale for diffusive escape of cosmic rays in a turbulent magnetic field of homogeneous strength \( B_0 \) within a central radius \( R_0 \) is given by [14]

\[
t_{\text{esc}} \approx \frac{R_0^2}{6D} \frac{eB_0 R_0^2}{2 \epsilon cr \lambda (E_{\text{cr}})},
\]

where we have used for the diffusion coefficient \( D = \frac{1}{3} \lambda c_{\text{L}} R_{\text{L}} c \), and \( \lambda (E_{\text{cr}}) \) is the scattering length in units of the Larmor radius, \( r_{\text{L}} = E_{\text{cr}}/eB_0 \) of a cosmic ray proton of energy \( E_{\text{cr}} \).

The function \( \lambda \) depends on the turbulence spectrum of the magnetic field, \( \delta B/k \propto k^{-a} \), where \( k \) is the wavenumber. Cosmic ray scattering is dominated by field fluctuations on the scale of the Larmor radius, i.e., \( k \sim r_{\text{L}}^{-1} \). This implies that

\[
\lambda (E_{\text{cr}}) = \left[ \frac{eB_0 a_0}{E_{\text{cr}}} \right]^y \quad \text{for} \quad E_{\text{cr}} < eB a_0,
\]

where \( a_0 = k_{\text{max}}^{-1} \) is the largest scale of the turbulence, i.e., the “cell-size” (or “reversal-scale”) of the magnetic field. For \( k > a_0^{-1} \), the magnetic turbulence in clusters of galaxies seems to be well described by the Kolmogorov law for fully developed hydrodynamical turbulence, i.e., \( y = \frac{5}{3} \). This can be easily seen from relating the typical turbulent magnetic field and cell size found from Faraday rotation measurements, \( a_0 \sim 20 \, \text{kpc} \) and \( B_0 \sim 1 \, \mu G \) [7, 14], to the diffusion coefficient found for electrons of energy \( \sim 1 \, \text{GeV} \) from the synchrotron radio emission spectrum [7], \( D \sim 2 \times 10^{29} \, \text{cm}^2 \, \text{s}^{-1} \).

For \( E_{\text{cr}} > eB a_0 \), i.e. \( r_{\text{L}} > a_0 \), the particle motion is a random walk with scattering angles \( \sim a_0/r_{\text{L}} \) in each step. This
can also be approximately described by Eqs. (32) and (33) by setting \( y = -1 \).

Confinement of cosmic rays over the cluster radius, \( R_0 = R_{\text{cl}} \sim 3 \text{ Mpc} \), is then obtained for \( t_{\text{acc}} > t_\text{H} \), corresponding to a critical energy

\[
E_{\text{acc}}^* \approx 2 \times 10^8 \text{ GeV} \times \left[ \frac{B_0}{1 \mu \text{G}} \right] \left[ \frac{R_{\text{cl}}}{3 \text{ Mpc}} \right]^{3/2} \left[ \frac{a_0}{20 \text{ kpc}} \right]^{-1/2},
\]

as long as \( a_0 > 1 \text{ kpc}(R_{\text{cl}}/3 \text{ Mpc})^2 \). This assumes that the magnetic field strength is homogeneous over the entire cluster. A more realistic scenario may be that the central field of \( \sim 1 \mu \text{G} \) extends only over a central radius \( R_0 \sim 300 \text{ kpc} \), and decreases for \( R_0 < r < R_{\text{cl}} \) while the cell size increases \([?]?\); for simplicity, we shall assume here that \( B_0 \propto \lambda \) is independent of \( r \). In this case, the confinement energy \( E_{\text{acc}}^\ast \) would decrease by about 2–3 orders of magnitude. On the other hand, some observations suggest also larger magnetic fields on smaller reversal scales \([?,?]?\), which would imply larger confinement energies.

The above scenario assumes that the background plasma filling the cluster is at rest. However, simulations of structure formation rather suggest that clusters of galaxies are accreting extragalactic hot gas \([?,?]?\), forming inflows of typical speeds \( v_{\text{fl}} \gtrsim 300 \text{ km s}^{-1} \) downstream of an accretion shock near the outer radius of the cluster \([?]?\). Since particles diffuse relative to the plasma flow, we then have to require that in order to escape from the cluster, the average “speed” of particle diffusion, \( \sim 6D/R_{\text{cl}} \), exceeds the inflow velocity. If we assume the scaling of magnetic turbulence used above, i.e., \( B \sim 0.1 \mu \text{G} \) and \( a \sim 200 \text{ kpc} \) at \( R_{\text{cl}} \sim 3 \text{ Mpc} \), we obtain a critical energy

\[
E_{\text{acc}}^* \approx 10^7 \text{ GeV} \times \left[ \frac{B(R_{\text{cl}})}{0.1 \mu \text{G}} \right] \left[ \frac{R_{\text{cl}}}{3 \text{ Mpc}} \right]^{3/2} \left[ \frac{v_{\text{fl}}}{300 \text{ km s}^{-1}} \right] \left[ \frac{a(R_{\text{cl}})}{200 \text{ kpc}} \right]^{-1/2},
\]

A similar situation might occur in the outflowing flows observed in some rich clusters harboring powerful radio galaxies; here, magnetic fields \( B_0 \sim 5–30 \mu \text{G} \) turbulent on scales \( a_0 \sim 4 \text{ kpc} \) have been inferred from observations for the central \( R_0 \sim 100 \text{ kpc} \) \([?]?\). At this scale, the inflow speed of the flow is \( v_{\text{fl}} \sim 10 \text{ km s}^{-1} \) \([?]?\), yielding

\[
E_{\text{acc}}^* \approx 3 \times 10^5 \text{ GeV} \times \left[ \frac{B_0}{10 \mu \text{G}} \right] \left[ \frac{R_{\text{cl}}}{100 \text{ kpc}} \right] \left[ \frac{v_{\text{fl}}}{10 \text{ km s}^{-1}} \right] \left[ \frac{a_0}{4 \text{ kpc}} \right]^{-1/2},
\]

where \( R_{\text{cl}} \sim R_0 \) is the radius of the cooling flow.

Instead of considering diffusive transport, Waxman & Bahcall have discussed particle escape due to drift in an inhomogeneous magnetic field, \( v_d \sim c(r_L/B)|dB/dr| \sim c r_L/r \) for \( R_0 < r < R_{\text{cl}} \). For cosmic ray energies \( E_{\text{cr}} = 10^7 \text{ GeV} \) and the scenario discussed above, this leads to velocities \( v_d \sim 10 \text{ km s}^{-1} \ll 6D/R_{\text{cl}} \), thus drift can be neglected with respect to diffusion. If we had considered drift only, much higher confinement energies would have been obtained. Obviously, also the decay length of the neutron plays no role, since \( l_n \sim 0.1 \text{ kpc} \) for \( E_n = 10^7 \text{ GeV} \), much smaller than the typical scales involved here.

We see that for the properties of magnetic fields in clusters of galaxies discussed here, cosmic ray propagation may be considerably distorted for energies \( \lesssim 10^7 \text{ GeV} \). Confinement of cosmic rays in clusters would lead to a decrease of the cosmic ray flux measured at earth relative to the corresponding neutrino flux, causing an increase of the neutrino bound for \( E_\nu \lesssim 500 \text{ TeV} \). It is important to note that cosmic ray confinement may exist even on larger scales, i.e., superclusters. It has been shown that magnetic fields \( \sim 0.1 \mu \text{G} \) in superclusters are consistent with observations, and expected in simulations of structure formation \([?,?]?\), which also predict accretion of gas with speeds \( v_{\text{acc}} \sim 1000 \text{ km s}^{-1} \). The confinement energies for these larger scales \( \sim 10 \text{ Mpc} \) could therefore be even larger \( \gtrsim 10^8 \text{ GeV} \). Since our galaxy itself is located in a supercluster, this scenario would tend to increase the cosmic ray flux relative to the corresponding neutrino flux, which fills the Universe homogeneously — thus it could actually decrease the bound below \( E_n \sim 10^7 \text{ GeV} \).

At present, our knowledge on the structure of magnetic fields on supercluster scales, and on the location of cosmic ray sources with respect to clusters of galaxies, is too sparse to make a solid prediction on the observed extragalactic cosmic ray flux below \( 10^8 \text{ GeV} \). A cosmic ray bound on neutrino fluxes can therefore only be reliable where such effects can be excluded on the basis of current data, i.e., for neutrino energies \( \gtrsim 10^7 \text{ GeV} \), and this is beyond the effective energy range of current experiments.

### B. Adiabatic losses in expanding radio lobes and halos

Although cooling flow clusters generally seem to have powerful radio galaxies at their centers, it is also a fact that most powerful radio galaxies are not found in such environments \([?,?]?\). For a radio galaxy located in the normal extragalactic medium, evidence has been found that a pressure equilibrium with the external medium cannot be obtained within the lifetime of the source \( \lesssim 10^8 \text{ yr} \) \([?]?\). Therefore, it can be concluded that the lobes must expand. For a sample of powerful double-lobe (or FR-II) radio galaxies, lobe propagation velocities of order \( 10^4 \text{ km s}^{-1} \) have been inferred \([?]?\). Since the aspect ratio of the sources is found to be independent of their size, consistent with a propagation of the lobes along a constant opening angle of \( \sim 10^\circ \), expansion velocities of the lobes and the connecting “bridges” of \( \sim 1000 \text{ km s}^{-1} \) can be inferred.

Low energy cosmic rays are advected with the outflowing plasma on a time scale of the galaxy lifetime \( t_{\text{H}} \ll t_{\text{H}} \). Expanding radio lobes can therefore not confine cosmic rays at any energy. However, in plasma outflows we must consider the effect of adiabatic losses which affect all particles which are isotropized in the flow due to scattering with plasma turbulence. In this case, kinetic theory implies that the particle energy at ejection from the flow is related to the injected particle energy by
Here, $R_{\text{max}}$ is the radius of the outer termination shock of the flow, and $R_{\text{min}}$ is in general given by some minimal radius $R_{\text{min}}$ where the outflow starts. For the case of neutron ejection from a central source, as considered here, $R_{\text{min}} = l_0$ if $l_0 > R_{\text{min}}$, and $R_{\text{min}} = R_{\text{min}}$ otherwise. Note that the lobes we are considering here are much more extended than the jets in which the particles are accelerated.

The observed synchrotron spectra from radio lobes imply typical magnetic fields in the range $10^{-5}$ to $50 \mu G$, turbulent with a maximum scale (or cell-size) of $\sim 0.5 \text{ kpc}$ [??]. The observed asymmetric depolarization in double radio galaxies leads to the suggestion that the magnetized plasma around the radio galaxy extends into a halo of radius $\sim 300 \text{ kpc}$, with $B \sim 0.3 \mu G$ and a cell size of $\sim 5 \text{ kpc}$ at a radius $r \sim 100 \text{ kpc}$ [?]. The properties of the magnetic field in the central lobe and the halo can be connected by assuming that magnetic field and cell size scale as $B = B_0(r/R_0)^{-2}$ and $a = a_0(r/R_0)^{-2}$, respectively, with $B_0 \sim 30 \mu G$, $a_0 \sim 0.5 \text{ kpc}$ and $R_0 = 10 \text{ kpc}$. Within radius $R_0$, we assume the properties of the turbulent magnetic field are constant. This corresponds to the assumption of an isotropic magnetic field expanding in a plasma outflow with $R_{\text{min}} = R_0$ and $R_{\text{max}} \sim 300 \text{ kpc}$, which will be used as a working hypothesis in the following. Cosmic ray protons are then isotropized in the plasma if $r_1(r) < a(r)$, or for energies $E_{\gamma} < E_{\text{ad}}(r)$ with

$$E_{\gamma}^* (r) = 10^{10} \text{ GeV} \times \left[ \frac{B_0}{30 \mu G} \right]^{-1} \left[ \frac{a_0}{0.5 \text{ kpc}} \right] \left[ \frac{r}{10 \text{ kpc}} \right]^{-1}$$

for $10 \text{ kpc} < r \lesssim 300 \text{ kpc}$. Since these cosmic rays are advected with the flow, they suffer adiabatic losses following Eq. 37, leading to $E_{\gamma} \propto r^{-1}$. This implies, however, that $E_{\gamma}/E_{\gamma}^*$ is independent of $r$, i.e., cosmic rays confined in the outflow at some radius $r$ remain confined for larger radii. If we consider cosmic rays which are ejected as neutrons, the radius where they couple to the magnetic field is given by the $\beta$-decay mean free path, $l_0 \approx 10 \text{ kpc}(E_n/10^9 \text{ GeV})$. The resulting protons are subject to adiabatic losses if $E_n < E_{\gamma}^*(l_0)$, or

$$E_n < 3 \times 10^9 \text{ GeV} \left[ \frac{B_0}{30 \mu G} \right]^{1/2} \left[ \frac{a_0}{0.5 \text{ kpc}} \right]^{1/2}$$

The reduction of energy is then given by $E_{\gamma} = E_n/30$ for $E_n < 10^9 \text{ GeV}$, and $E_{\gamma} = E_n^2/(3 \times 10^{10} \text{ GeV})$ for $10^9 \text{ GeV} < E_n < 3 \times 10^9 \text{ GeV}$. For $E_n > 3 \times 10^9 \text{ GeV}$, cosmic rays (neutrons and protons) traverse the lobe/halo essentially in (nearly) straight lines and adiabatic losses do not apply.

Energy losses of cosmic rays in the lobes and halos of radio galaxies are of particular relevance for models of neutrino production in AGN jets, which we discussed in Section IV. These models apply to radio loud AGN, which are likely to be the beamed counterparts of radio galaxies [??]. The two classes of AGN discussed in the last section hereby correspond to the two Fanaroff-Riley (FR, [?]) classes of radio galaxies: radio quasars might be associated to the powerful double-lobed FR-II radio galaxies, while BL Lac objects might correspond to the less luminous FR-I radio galaxies which generally have diffuse lobes centered around the AGN. The parameters used above for the lobes/halos of radio galaxies were mainly obtained from observations of FR-II radio galaxies or radio quasars. Therefore, the cosmic ray ejection from radio quasar/FR-II sources can be expected to be diminished by more than an order of magnitude below $\sim 10^9 \text{ GeV}$. For the less luminous FR-I galaxies, it could be that the magnetic fields, turbulence scales and halo sizes used above are overestimated, so that $E_{\gamma}^*$ could be lower by about one or two orders of magnitude for these sources.

VI. GENERAL DISCUSSION AND CONCLUSIONS

The aims of the present paper consisted mainly of (a) investigating the constraints on the extragalactic proton contribution to the cosmic ray spectrum for extragalactic neutrino fluxes set by the upper limit to the extragalactic cosmic ray proton spectrum, and (b) to develop viable models and upper limits for diffuse neutrino fluxes from AGN jets, and to compare these with existing predictions. An important related question is whether or not it is possible that AGN jets (or other sources) can produce hadronically the diffuse extragalactic gamma-ray background.

With regard to point (a), we have shown that for sources which are transparent to the emission of ultra-high energy neutrinos, the cosmic-ray bound is indeed more restrictive than the commonly used bound set by the diffuse extragalactic gamma-ray background (EGRB) for a broad range of neutrino energies, $10^8 \text{ GeV} < E_\nu < 10^{11} \text{ GeV}$. In an energy range $10^7 \text{ GeV} < E_\nu < 10^8 \text{ GeV}$, our bound is approximately in agreement with the bound previously proposed by Waxman & Bahcall [?]. The difference at other energies can be understood by the difference in our approaches. Rather than constructing an upper limit on the observable extragalactic cosmic ray proton contribution, as we do, Waxman & Bahcall assume a model input spectrum, which makes their bound insensitive to the actual shape of the measured cosmic ray spectrum. We have also pointed out, referring to recent Monte-Carlo simulations [?], that the fundamental properties of photo-hadronic interactions can affect the bound by up to a factor of 5.

At energies below $10^7 \text{ GeV}$, our bound rises and matches the EGRB bound at about 100 TeV, below which the EGRB constraint dominates. In the same energy region, we have shown that also effects from extragalactic magnetic fields come into play; they could either increase or reduce the bound, depending on the details of the field strength and structure, and the distribution of cosmic ray/neutrino sources. We therefore do not regard the cosmic ray flux as a reliable limit on neutrino fluxes below $10^5 \text{ GeV}$, which is the primary energy region for underwater/ice Cherenkov experiments, such
as AMANDA [7]. Models which predict neutrino emission mainly below $10^7$ GeV are therefore not restricted by cosmic ray data, as for example the model by Berezhensky et al. [7] predicting neutrino fluxes from cosmic rays stored in clusters of galaxies. Such sources could in principle produce neutrino fluxes almost up to the EGRB level within the AMANDA range. However, the neutrino contribution from clusters of galaxies is expected to be much lower, as it is limited by their expected contribution to the EGRB of $\sim 1\%$ [7].

At energies above $10^{11}$ GeV, our bound rises due to the lack of the observation of the Greisen-Zatsepin-Kuzmin (GZK) cutoff in the cosmic ray spectrum. Here we should note that an increased flux of neutrinos in this energy region would only be expected if the continuation of the ultra-high energy cosmic ray (UHECR) spectrum is due to an increased overall activity of the sources. If it is due to the highly-concentrated contribution of one local source, no increase would be expected, and the neutrino flux should stay at or below the Waxman-Bahcall bound. Measuring the actual neutrino flux in this region, which is a program for large air-shower experiments like the Pierre Auger Observatory [7], would therefore be highly relevant for the understanding of the nature and cosmic distribution of UHECR sources.

Our main result concerning point (b), i.e., possible neutrino fluxes from AGN jets (blazars), was to show that most high luminosity blazars are opaque to the emission of UHECR neutrinos, and this allows neutrino fluxes in excess of the bound derived in the first part of our paper. The strongest fluxes are allowed in scenarios where external photons make up the dominant part of the target photons for hadronic interactions. Moreover, we have shown that magnetic fields in the radio lobes/halos of the host galaxies of luminous AGN (radio-quasars) can affect cosmic ray propagation below about $3 \times 10^9$ GeV. Since, for these sources, neutron opacity effects could set in already at $10^{7} - 8$ GeV, we can conclude that the cosmic ray bound for neutron transparent sources does not apply to them at any energy. For lower luminosity blazars (BL Lacs), the allowed diffuse neutrino fluxes are generally lower since they show no evidence for cosmological evolution, as quasars do.

We have estimated an upper limit to the diffuse neutrino fluxes from AGN, using neutron opacities consistent with gamma-ray observations. This limit is roughly consistent with a previously published blazar-neutrino model based on external photon interactions, but below some other models which have been published in the literature. Here we should note that our bound was derived neglecting the effect of the magnetic field in radio lobes/halos — for example, the model of Halzen & Zas [7], predicting a neutrino flux about one order of magnitude above our limit at $E_\nu \sim 10^6$ GeV, cannot be considered as ruled out, because at this energy the cosmic ray ejection may be diminished by an additional factor $\sim 10$ due to adiabatic losses in the expanding radio halos. Generally, there is no conflict between previously estimated AGN correlated neutrino fluxes in the AMANDA range ($< 10^7$ GeV) and our new bound, and we emphasize that a hadronic production of the entire EGRB by AGN is not in conflict with cosmic ray observations.

In all our derivation, we have assumed that only neutrons can be ejected as cosmic rays from the sources. This is justified from a logical point of view, as long we are interested in stating upper limits to the neutrino fluxes, rather than estimates — an additional contribution from directly ejected protons would only lower the allowed correlated neutrino flux, and the upper limit would remain valid. For the most interesting source classes such as AGN jets and gamma-ray bursts (GRB), however, the direct ejection of protons is indeed expected to be strongly suppressed due to the fast expansion of the emission region (and the implied large adiabatic losses). Interestingly, turning the argument around, in order to remain viable as UHECR sources, such objects must eject neutrinos at the level of their allowed bound. This means, for example, that if GRB correlated neutrinos would “only” be observed at the level predicted by Waxman & Bahcall (1997 [7]), they would fall short of producing the observed UHECR flux by about a factor of $10 - 100$, depending on whether or not they follow the observed evolution of star-formation, and noting that the bound is a factor of 5 higher for GRB than for AGN because of their very hard photon spectrum.

Of course, if any neutrino experiment would ever observe neutrino fluxes to correlate with some class of point source at the level of the bound for the same source class, this would be direct evidence for these objects to be the dominant sources of UHECRs. It might therefore be that the riddle of the origin of cosmic rays would not be solved by cosmic ray observations, but by neutrino observations, maybe involving the same experiments. The exploration of the energy range between $10^7$ GeV and $10^9$ GeV, where the cosmic ray/neutrino connection is most rigid, will thereby play a crucial role.

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