Nonperturbative “Lattice Perturbation Theory”*  

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We discuss a program for replacing standard perturbative methods with Monte Carlo simulations in short distance lattice gauge theory calculations.

In principle, perturbation theory is unnecessary to solve QCD with lattice methods. Even the short distance calculations relating the lattice action to continuum actions could in principle be done by $a \to 0, V \to \infty$ brute force Monte Carlo calculations. In practise, perturbation theory has been essential to the progress of lattice methods. It is much easier and more powerful than Monte Carlo for some purposes:

- The $a \to 0, V \to \infty$ limits are much easier to take in perturbative calculations.
- The values of perturbative coefficients can be calculated much more accurately than typical quantities in numerical calculations.
- When short distance quantities can be computed with very different calculational methods, such as perturbation theory and Monte Carlo simulation, confidence is bolstered in both methods.

In this talk, we will reexamine the question of when it may be advantageous and feasible to do “Feynman diagrams” nonperturbatively. An important motivation for reexamining methods for perturbative calculations now is the increasing understanding and use of improved actions. $O(a)$ improved actions for Wilson fermions are known to be crucial for calculating some quantities (such as the hyperfine splitting in quarkonium systems). They may be important for many more quantities. The perturbation theory for $O(a)$ improved actions is much harder than the perturbation theory for unimproved actions (already hard enough), but still tractable. Most of the most important calculations for this action are in the process of being done.

The frontier in the practical application of improved action is $O(a^2)$ improvement. For example, in Lepage’s talk at this conference, he showed that a mean-field improved Weisz action (the pure gauge action constructed from plaquettes and flat six-link loops) plus the improved fermion action of NRQCD is capable of calculating the charmonium spectrum correctly to a few per cent, even at lattice spacings as large as $a^{-1} \sim 400$ MeV. Even if the program to perform spectrum calculations with improved actions is very successful, that is only part of the total program for lattice QCD. Extraction of decay constants, form factors and quark masses requires short distance calculations which have been done with perturbative methods. The action and Feynman rules for this action are much, much more complicated than those for the unimproved theory. Redoing all existing perturbative calculations with this action will be terribly complicated, even with large increases in the amount of work devoted to perturbative calculations. The job would be more difficult than the job of redoing all of the simulation programs for the new action.

An even more extreme example is the “perfect action”. Classical corrections to the action are much more tractable than quantum corrections. They can be done more or less “perfectly”, but at the cost of adding many additional terms to the action. The work required for deriving Feynman rules and performing perturbative calculations with such an action is hard to imagine.

*Talk given by Paul Mackenzie
The purpose of this talk is to ask whether it is possible to perform short distance calculations without deriving Feynman rules and doing Dirac and Lorentz algebra. There are several possible goals of short distance Monte Carlo calculations. One, which is not the subject of this talk but which is potentially more important, is the search for nonperturbative effects at short distances. For example, the expectation value of the plaquette is expected to have, in addition to its expansion in powers of $g^2$, nonperturbative effects which fall off as some power of the lattice spacing. The condensate picture of short distance nonperturbative effects suggests that this power is four, the dimension of the operator $F_{\mu\nu}^2$. Little is known from first principles, however. This subject is 1) hugely important, and 2) not the subject of this talk but one, which is not the subject of this talk but.

The questions addressed by this talk are more modest. What are the coefficients of the powers of $g^2$ in the perturbative expansion? To what extent can we find methods to test perturbation theory where we can use conventional methods, and to replace perturbation theory where we cannot? In cases where the correct perturbative coefficient is known, can we use nonperturbative methods to

- recalculate the first order coefficient correctly?
- bound or estimate the $O(\alpha^2), O(\alpha^3), \ldots$ coefficients correctly?
- redo the calculation with ten times as complicated an action?

Some short distance nonperturbative calculations are easier than others. For example, much may be learned about the extraction of $\alpha_s$ from nonperturbatively calculated short distance quantities with relatively simple calculations. Other quantities, such as operator normalizations and quark mass extractions, are more complicated. As an illustrative example, we will consider one of the more difficult quantities, extracting the quark mass from nonperturbative short distance calculations. The goal will be to determine numerically the coefficients of the first power or two in $\alpha_s$ for a given value of the quark mass in lattice units, with the idea of using the coefficients in the way that normal perturbative results for the same value of $ma$ would be used in phenomenological calculations. (Perturbative coefficients are explicit functions of the quark mass when one is not in the limit $am \to 0$.) We will do this by performing a Monte Carlo simulation with very small lattice spacing, fixing the gauge, creating a quark propagator with wall source and sink, and calculating an effective (pole) mass for the quark in the usual way.

There are several problems which arise in this calculation. They are mostly associated in one way or another with the limiting procedure. The nice limit for perturbative calculations is the limit $V \to \infty$, followed by $g^2 \to 0$. But Monte Carlo calculations (on noninfinite computers) require $V$ finite, while we would still like $g^2 \to 0$. This is a complicated limit for the theory. In particular, as the quark mass in lattice units becomes small, we do not have the desirable property $am >> 1/L$ (where $a$ is the lattice spacing, $m$ is the quark mass, and $L$ is the box size in lattice units).

One problem is the effect of tunneling between the $(81) \ Z3$ vacua of the gauge theory. Quarks have energies of order $1/L$ in nontrivial vacua. Only in the trivial vacuum do these not dominate the $O(g^2)$ correction unless $L \to \infty$. Only the trivial vacuum part of the path integral corresponds to the usual perturbative momentum sum.

Even in the trivial vacuum, zero modes give a contribution to the quark mass which vanishes only in the large volume limit. Dimensional analysis suggests that this effect also goes like $1/L$. These effects must be calculated. They are especially important for light quarks.

Gauge dependence is an additional problem. The pole mass is gauge invariant, but infrared sensitive gauges may have problems. Axial gauges are famously IR sensitive. Landau gauge (and Feynman and other covariant gauges) have gauge dependent infrared logarithms in their wave function renormalizations. They therefore have no isolated poles, but have branchcuts instead. Among the commonly used gauges in Monte Carlo simulations, only Coulomb gauges (where an initial gauge fixing to $A_0 = 0$ gauge fixes the time direction gauge freedom) has no
known infrared problems. An additional infrared healthy gauge (perhaps Yennie gauge) would be desirable in our calculations to test gauge dependence.

Bearing in mind these complications, which are not all understood and which may limit the applicability of the approach, we will now ask if a Monte Carlo determination of the quark mass correction if feasible. We perform a simulation with periodic quark boundary conditions (N.B.), at $\beta = 60.0$, and at large volume, $20^3 \times 32$, to limit tunneling. Fig. 1 shows an effective mass plot for Wilson fermions calculated by Monte Carlo methods in the three gauges mentioned above, and compared with tree-level and one-loop perturbation theory. The gauges with infrared problems (axial and Landau) do not agree well with one-loop perturbation theory, but the Coulomb gauge results do agree. Fig. 2 shows that the same is true for nonrelativistic fermions.

The next practical calculations to be addressed include the calculation of the second order additive mass renormalizations for NRQCD and Wilson fermions, and the first order additive mass renormalizations for $O(\alpha^2)$ improved actions. Questions of principle which remain to be fully addressed include the effects of boundary conditions, gauge dependence, and zero modes.

Summary:

- Conventional perturbative calculations get harder faster than Monte Carlo calculations as the action gets more complicated.
- $\alpha^2$ corrected actions will be terribly complicated for perturbation theory.
- Brute force evaluation of quark-gluon Green’s functions is clearly possible in principle.
- Brute force evaluation of quark-gluon Green’s functions may also be of practical importance if various complications can be understood.

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