PQCD approach to parton propagation in matter

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Abstract

We review recent theoretical developments in understanding the many-body perturbative QCD dynamics of strongly-interacting hard probes in dense nuclear matter. The relation between initial- and final-state parton scattering in nuclei and the quark-gluon plasma and experimental measurements of the Cronin effect, nuclear shadowing, jet quenching, modified di-hadron correlations and forward rapidity particle suppression in p+A and A+A reactions is clarified. Our approach emphasizes the process dependence of nuclear effects and outlines the techniques for their dynamical calculation and incorporation in simulations that build upon collinear factorization in QCD.

Key words: multi-parton dynamics, Cronin effect, coherence, nuclear shadowing, energy loss, jet quenching

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1 Introduction

In high energy p+p collisions jet and particle production is well described in the framework of the collinear PQCD factorization approach. However, when a partonic process is embedded in nuclear matter, the cross sections are modified by the soft scatterings that precede and/or follow the hard interaction [1]. One can gain insight in the various aspects of perturbative many-body QCD dynamics by following the history of a parton from early times, \( t \to -\infty \), in the parton distribution function to its asymptotic state, \( t \to \infty \), in the wave function of the observed hadron. In our approach, the strength of jet interactions is controlled by microscopic and non-perturbative parameters characterizing

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| Type of scattering                | $p_T$ dependence of the nuclear effect                                                                 |
|----------------------------------|---------------------------------------------------------------------------------------------------------|
| Elastic (incoherent)             | $I_1P$: Cronin effect; $I_2P$: di-jet acoplanarity                                                     |
| Inelastic (final-state)          | $I_1P$: quenching at all $p_T$; $I_2P$: quenching/enhancement at high/low $p_T$                         |
| Inelastic (initial-state)        | $I_1P$: suppression at all $p_T$; $I_2P$: same as $I_1P$                                               |
| Coherent ($t-$, $u-$ channel)    | $I_1P$: suppression at low $p_T$; $I_2P$: same as $I_1P$                                               |
| Coherent ($s-$ channel)          | $I_1P$: enhancement at low $p_T$; $I_2P$: same as $I_1P$                                               |

Table 1
Effect of elastic, inelastic and coherent multiple scattering on the transverse momentum dependence of single ($I_1P$) and double inclusive ($I_2P$) hadron production in the perturbative regime.

the medium, such as temperature $T$, energy density $\epsilon$, the scale of high twist corrections $\xi^2$, typical momentum transfer squared $\mu^2$, and mean free path $\lambda$. Results are summarized [1] in Table 1.

2 Cronin effect and dynamical shadowing

We first consider partonic $2 \to 2$ processes, which we call elastic as opposed to inelastic $2 \to 2 + n$ processes associated with gluon bremsstrahlung. Multiple interactions of the incoming quarks and gluons lead to transverse momentum diffusion [2]. For initial parton flux $d^2N_i(k) = \delta^2(k)d^2k$ the distribution of partons at the hard collision vertex reads:

$$\frac{d^2N_f(k)}{d^2k} = \sum_{n=0}^{\infty} \frac{\chi^n}{n!} \int \prod_{i=1}^{n} d^2q_i \left[ \frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^2q_i} (e^{-q_i \cdot \nabla_{\bar{k}}} - 1) \right] \frac{d^2N_i(k)}{d^2k} \approx \frac{1}{2\pi} \chi \mu^2 \xi, \quad \chi = \frac{L}{\lambda}, \quad \xi = \xi(k^2). \quad (1)$$

Such broadening can be implemented in the PQCD formalism as follows:

$$\langle k_i^2 \rangle = \langle k_{i \text{vac}}^2 \rangle + 2\chi_i \mu^2 \xi, \quad i = 1, 2, \quad (2)$$

and leads to $\sqrt{s}$-dependent enhancement of the hadron cross sections at intermediate $p_T \sim$ few GeV. The left panel of Fig. 1 shows theoretical calculations at $y = 0$ compared to p+W/p+Be data and predictions for d+Au collisions at RHIC. We used $2\mu^2/\lambda_q \approx 0.12$ GeV$^2$/fm, $2\mu^2/\lambda_g \approx 0.28$ GeV$^2$/fm. For a survey of models of the Cronin effect see [3].
Fig. 1. Left panel: the ratio of $A$-scaled $p+W/p+Be$ data on $\pi^+,\pi^-$ production at $\sqrt{s} = 27.4, 38.8$ GeV. Right panel: all-twist resummed $F_2(A)/F_2(D)$ calculation. The band corresponds to the choice $\xi^2 = 0.09 - 0.12$ GeV$^2$.

A distinctly different effect arises from coherent final-state interactions in large nuclei [4]. When the light cone momentum fraction $x_B < 0.1$ the longitudinal extent of the exchange virtual meson $l_c = 1/2x_Bm_N$ exceeds the nucleon size $r_0$ and the struck parton scatters coherently on several nucleons. The high twist scale per nucleon is defined as:

$$\xi^2 = \frac{3\pi\alpha_s(Q^2)}{8\pi r_0^2} \langle p | \hat{F}^2(\lambda_i) | p \rangle = \frac{3\pi\alpha_s(Q^2)}{8\pi r_0^2} \lim_{x \to 0} \frac{1}{x} xG(x).$$

(3)

Nuclear shadowing can be cleanly studied only in deep inelastic scattering (DIS). Resummation of all nuclear-enhanced power corrections to the inclusive cross sections generates dynamical mass for the final-state parton $m_{dyn}^2 = \xi^2A^{1/3}$. On the example of the transverse structure function we have:

$$F_T^A(x, Q^2) \approx A F_T^{(LT)} \left( x + \frac{x \xi^2(A^{1/3} - 1)}{Q^2}, Q^2 \right) = F_T^{(LT)} \left( x + \frac{xm_{dyn}^2}{Q^2}, Q^2 \right).$$

(4)

The left panel of Fig. 1 shows good description of the world’s data on DIS on nuclei [4] with $\xi^2 = 0.09 - 0.12$ GeV$^2$ and $Q^2 \geq m_N^2$. Similar results were derived for $p+A$ reactions [5] and our calculations have been generalized to include the case of heavy quarks [1]. For a survey of models of nuclear shadowing see [3].
Initial-state multiple scattering in the coherent regime, in contrast to final-state, leads to enhancement of the cross sections and Cronin effect [1]. Such distinctly different behavior points to the process dependence of nuclear effects and the need for their dynamical evaluation.

3 Initial- and final-state energy loss

The acceleration of the incoming and outgoing partons by the soft interactions in the dense nuclear matter results in medium-induced gluon bremsstrahlung [7]. Given its quantitative importance for the description of the large jet attenuation in a quark-gluon plasma (QGP) of initial temperature $T_0 \sim 400$ MeV and energy density $\epsilon_0 \sim 20$ GeV/fm$^3$, recent perturbative calculations have been focused on final-state radiative energy loss. The goal of such studies is to describe the large five-fold suppression of high-$p_T$ hadron production in central Au+Au reactions [8,9].

We here rely on the Gyulassy-Levai-Vitev (GLV) approach for describing the multiple interactions of the propagating jet+gluon system [10,11] since it was specifically developed to describe the QCD dynamics of heavy ion (p+A and A+A) reactions. It is well equipped to account for some of the most important characteristics of nuclear collisions: $L/\lambda_g \leq \text{few} \ll \infty$ and $E_{\text{jet}} \ll \infty$. To first order in the correlation between multiple scattering centers the differential bremsstrahlung spectrum is given by [10]:

$$
\omega \frac{dN_g}{d\omega \ d^2 k} = \frac{C_R \alpha_s}{\pi^2} \left[ \frac{\mu^2(z)}{\pi (q^2 + \mu^2(z))^2} \right] \times \\
\int_{z_0}^{L} d\Delta z \int d^2 q \frac{\mu^2(z)}{\pi (q^2 + \mu^2(z))^2} \times \\
\left[ 1 - \cos \left( \frac{(k - q_{\text{dir}} - q)^2 \Delta z}{2\omega} \right) \right]. \quad (5)
$$

Here $q_{\text{dir}}$ is possible $\neq 0$ directed momentum transfer, for example transverse flow. Clearly, this effect amounts to a change of the reference frame for the radiative spectrum but neither distorts it nor changes the amount of lost energy $\Delta E$.

Integrating over the gluon bremsstrahlung intensity in the limit $2E/\mu^2 L \gg 1$ we find:

$$
\Delta E \approx \frac{C_R \alpha_s \mu^2}{4 \lambda_g} L^2 \ln \frac{2E}{\mu^2 L}, \quad \Delta E_T \approx \frac{9\pi \alpha_s^3}{4} \frac{1}{A_\perp} \frac{dN_g}{dy} L_T \ln \frac{2E_T}{\mu^2 L_T} \quad (6)
$$

for static and (1+1)D Bjorken-expanding plasmas, respectively [10,11]. Tests
of parton energy loss at RHIC and the LHC can be carried out via the centrality dependence of jet quenching. With $dN^g/dy \propto N_{\text{part}}$, $L \propto N_{\text{part}}^{2/3}$ and $A_\perp \propto N_{\text{part}}^{2/3}$, the GLV approach gives a definite prediction for the system size dependence of energy loss and jet quenching [12]:

$$\epsilon = \Delta E/E \propto N_{\text{part}}^{2/3}, \quad \ln R_{AA} \approx -k N_{\text{part}}^{2/3}. \quad (7)$$

Our analytic result for the system size dependence of $R_{AA}(p_T)$ is shown in the left panel of Fig. 2. It is fixed by the magnitude of the suppression established in central Au+Au collisions. From this analysis we expect a factor $\sim 2$ suppression in central Cu+Cu collisions. Comparison to the PHENIX $p_T > 7$ GeV data [8] and STAR $p_T > 6$ GeV data [9] is also shown.

To verify the universal suppression pattern, Eq. (7), we carry out numerical simulations of $R_{AA}$ versus $p_T$ and centrality with QGP properties constrained by the entropy density, or equivalently $dN^g/dy$ and $\tau_0$ at RHIC. The modification of inclusive hadron production from final state radiative energy loss can be represented as [12]:

$$D_{h/c}(z) \Rightarrow \int_0^{1-z} d\epsilon P(\epsilon) \frac{1}{1-\epsilon} D_{h/c} \left( \frac{z}{1-\epsilon} \right) + \int_z^1 d\epsilon \frac{dN^g}{d\epsilon}(\epsilon) \frac{1}{\epsilon} D_{h/g} \left( \frac{z}{\epsilon} \right). \quad (8)$$
In Eq. (8) the first term is associated with the quenching of leading hadrons and $P(\epsilon)$ is the probabilistic distribution for the fractional energy loss $\epsilon = \Delta E/E = \sum_{i=1}^{n} \epsilon_i$, $\epsilon_i = \omega_i/E$, due to multiple gluon emission. The right panel of Fig. 2 shows the jet quenching results versus $p_T$ for three different centralities in Au+Au and Cu+Cu collisions at RHIC [12]. Data is from PHENIX [8].

With fractional energy loss as large as $\sim 25\%$ for quarks and $\sim 50\%$ for gluons [12], the quenching of hard jets should be correlated with measurable enhanced production of low-$p_T$ hadrons [13]. The second term in Eq. (8) reflects the gluon feedback contribution, which for single inclusive measurements becomes important only at the LHC. For away-side di-hadron correlations this effect is already large at RHIC [14]. It follows from Eq. (5) that the angular behavior of the medium-induced gluon bremsstrahlung is significantly broader when compared to vacuum radiation. Its most important feature is the cancellation for the collinear $k \to q_{dir}$ radiation. The left panel of Fig. 3 shows the angular distribution of $\omega = 3$ GeV radiative gluon off a quark jet for $L = 5$ fm, $\mu = 1$ GeV, $\lambda_g = 1$ fm [14]. The remnant of the leading parton is not shown.

Depletion of hadrons from the quenching of the parent parton alone leads to a large suppression of the double inclusive cross section with weak $p_{T2}$ dependence, shown in the right panel of Fig. 3. Hadronic feedback from the medium-induced gluon radiation, however, completely changes the nuclear di-hadron modification factor $R^{(2)}_{AA}$. It now shows a clear transition from a quenching of the away-side jet at high transverse momenta to enhancement at $p_{T2} \leq 2$ GeV [14] compatible with STAR measurements [13].

The physics process that has so far not been addressed is initial state energy loss in cold nuclear matter [1,15]. To first order in opacity:
Fig. 4. Left panel: calculated suppression of $\pi^0$ production at $y = 4$ for d+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at RHIC. Right panel: nuclear modification of single inclusive $\pi^0$ (left) and $D^0 + D^+$ mesons (right) in d+Au collisions at RHIC versus rapidity and centrality.

$$\frac{\omega dN_g}{d\omega d^2k} = \frac{C_R\alpha_s}{\pi^2} \int_{0}^{L} d\Delta z \frac{\mu^2(z)}{\lambda_g(z)} \int d^2q \frac{\mu^2(z)}{\pi(q^2 + \mu^2(z))^2} \times \left[ \frac{q^2}{k^2(k - q)^2} - 2 \frac{q^2 - k \cdot q}{k^2(k - q)^2} \cos \left( \frac{k^2\Delta z}{2} \right) \right].$$  \hfill (9)

From Eq. (9) with the requirement $\omega dN_g/d\omega d^2k \geq 0$ we can evaluate the energy loss and implement it in the PQCD hadron production formalism:

$$\epsilon = \frac{\Delta E}{E} \propto \frac{L}{\lambda} = \kappa A^{1/3}, \quad \phi_{q,g/N}(x, Q^2) \rightarrow \phi_{q,g/N} \left( \frac{x}{1 - \epsilon} , Q^2 \right).$$  \hfill (10)

As shown in the left panel of Fig. 4, shadowing calculations can only account for $\sim 50\%$ of the hadron suppression at forward rapidities p+A reactions at RHIC [1,5]. Taking into account initial-state energy loss, we find good description of the $y = 4$ $R_{AA}(p_T)$ for $\pi^0$s by STAR [16]. The right panel of Fig. 4 shows the same perturbative calculation for light hadrons and heavy mesons [1] at intermediate rapidities $y \sim 2$ and $\sqrt{s} = 200$ GeV d+Au collisions at RHIC compared to PHENIX data [17].

4 Conclusions

Dynamical calculations of initial- and final-state elastic [2], inelastic [10,11] and coherent [4,5] multiple parton scattering have been incorporated in the framework of the perturbative QCD factorization approach to describe high $p_T$ and $E_T$ observables in collisions of heavy nuclei. They naturally account
for the process dependence of nuclear effects [1] and provide adequate unified description of seemingly disparate phenomena, such as the Cronin effect [3] and nuclear shadowing [6] in p+A/ℓ+A reactions and high-\(p_T\) jet quenching [7,12] and low \(p_T\) large-angle di-hadron enhancement [14] in A+A reactions. In cold nuclear matter, parton diffusion, nuclear enhanced power corrections and initial state energy loss are compatible with similar momentum transfer scales: \(\xi^2 A^{1/3} \approx 2\mu^2 L/\lambda\). In the QGP, jet quenching results are in agreement with the predicted energy and centrality \(N_{\text{part}}^{2/3}\) dependence of the non-Abelian parton energy loss.

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