Baryogenesis via leptogenesis

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Abstract

We discuss how leptogenesis can explain the observed baryon asymmetry and summarize attempts of testing leptogenesis. We first perform estimates and discuss the main physics, and later outline the techniques that allow to perform precise computations.

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1 Introduction

The universe contains various relict particles: $\gamma$, $e$, $p$, $\nu$, $^4\text{He}$, Deuterium, . . . , plus likely some Dark Matter (DM). Their abundances are mostly understood, with the following main exception:

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{n_B}{n_\gamma} = (6.15 \pm 0.25) \cdot 10^{-10}$$  \hspace{1cm} (1)

where $n_\gamma$ and $n_B$ and $n_{\bar{B}}$ are the present number densities of photons and baryons (anti-baryons have negligible density). This is the problem of baryogenesis. Before addressing it, let us briefly summarize the analogous understood issues.

As suggested by inflation, the total energy density equals the critical energy density, discussed later. Next, almost all relative abundances can be understood assuming that these particles are thermal relics of a hot Big-Bang phase. The number densities of electrons and protons are equal, $n_e = n_p$, because nothing violated electric charge. The relative proton/neutron abundancy was fixed by electroweak processes such as $n\nu_e \leftrightarrow p e$ at $T \sim \text{few MeV}$. Neutrons get bound in nuclei at $T \sim 0.1 \text{MeV}$: the measured nuclear primordial nuclear abundances agree with predictions: $n_{^4\text{He}}/n_p \approx 0.25/4$, $n_D/n_p \approx 3 \times 10^{-5}/2$, etc. The neutrino density, predicted to be $n_{\nu_e,\mu,\tau} = n_{\bar{\nu}_e,\mu,\tau} = 3 n_\gamma/22$, is too low to be experimentally tested: the baryon asymmetry problem is more pressing than the analogous lepton asymmetry problem because we do not know how to measure the lepton asymmetry. Finally, the DM abundancy suggested by present data is obtained if DM particles are weakly-interacting thermal relics with mass

$$m \sim \sqrt{T_{\text{now}} \cdot M_{\text{Pl}}} \sim \text{TeV}$$  \hspace{1cm} (2)

where $T_{\text{now}} \approx 3 \text{K}$ is the present temperature and $M_{\text{Pl}} \approx 1.2 \times 10^{19} \text{GeV}$ is the Planck mass. The LHC collider might soon produce DM particles and test this speculation. It is useful to digress and understand eq. (2), because the necessary tools will reappear, in a more complicated context, when discussing leptogenesis.

1.1 The DM abundancy

First, we need to compute the expansion rate $H(t)$ of the universe as function of its energy density $\rho(t)$. Let us study how a homogeneous $\rho(t)$ made of non-relativistic matter evolves according to gravity. A test particle at distance $R$ from us feels the Newton acceleration

$$\ddot{R} = -\frac{G M(R)}{R^2} = -\frac{4\pi G \rho(t)}{3} R$$  \hspace{1cm} (3)

where $M(R)$ is the total mass inside a sphere of radius $R$ and $G = 1/M_{\text{Pl}}^2$ is the Newton constant. By multiplying both sides of eq. (3) times $\dot{R}$ and integrating taking into account that $\rho(t) \propto 1/R^3$ one obtains as usual the ‘total energy’ constant of motion, here named $k$:

$$\frac{d}{dt} \left[ \frac{1}{2} \dot{R}^2 - \frac{4\pi}{3} G \rho R^2 \right] = 0 \quad \text{so that} \quad H^2 \equiv \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3} \rho - \frac{k}{R^2}. \hspace{1cm} (4)$$

Let us discuss the special case $k = 0$. It is obtained when the density $\rho$ equals the ‘critical density’ $\rho = \rho_{\text{cr}} \equiv 3H^2/8\pi G$. $k = 0$ is special because means zero ‘total energy’ (the negative gravitational potential energy compensates the positive matter energy): a universe with critical density that expands getting big for free could have been theoretically anticipated since 1687. Today we abandoned prejudices for a static universe, and more advanced theories put the above discussion on firmer grounds. In general relativity eq. (4) holds for more general sources of energy (relativistic particles, cosmological
Figure 1: (a) Main reactions that determine primordial nuclear abundances. (b) How CMB anisotropies depend on the baryon abundance $\Omega_B = \rho_B/\rho_c$, compared with data.

constant,...), and the constant $k$ gets the physical meaning of curvature of the universe. The inflation mechanism generates a smooth universe with negligibly small $k$.

Second, we need to know that a gas of particles in thermal equilibrium at temperature $T \gg m$ has number density $n_{\text{eq}} \sim T^3$ and energy density $\rho_{\text{eq}} \sim T^4$: one particle with energy $\sim T$ per de-Broglie wavelength $\sim 1/T$. The number density of non relativistic particles ($T \ll m$) is suppressed by a Boltzmann factor, $n_{\text{eq}} \sim (mT)^{3/2} e^{-m/T}$, and their energy density is $\rho_{\text{eq}} \simeq m n_{\text{eq}}$.

We can now understand eq. (2), by studying what happens to a DM particle of mass $m$ when the temperature $T$ cools below $m$.

Annihilations with cross section $\sigma(\text{DM DM} \rightarrow \text{SM particles})$ try to maintain thermal equilibrium, $n_{\text{DM}} \propto \exp(-m/T)$. But they fail at $T \lesssim m$, when $n_{\text{DM}}$ is so small that the collision rate $\Gamma$ experienced by a DM particle becomes smaller than the expansion rate $H$:

$$\Gamma \sim n_{\text{DM}} \sigma \lesssim H \sim T^2/M_{\text{Pl}}.$$ 

As illustrated in the picture, annihilations become ineffective, leaving the following out-of-equilibrium relic abundance of DM particles:

$$\frac{n_{\text{DM}}}{n_\gamma} \sim \frac{m^2/M_{\text{Pl}} \sigma}{m^2} \sim \frac{1}{M_{\text{Pl}} \sigma m} \quad \text{i.e.} \quad \frac{\rho_{\text{DM}}(T)}{\rho_\gamma(T)} \sim \frac{m}{T} \frac{n_{\text{DM}}}{n_\gamma} \sim \frac{1}{M_{\text{Pl}} \sigma T}. \quad (5)$$

Inserting the observed DM density, $\rho_{\text{DM}} \sim \rho_\gamma$ at $T \sim T_{\text{now}}$, and a typical cross section $\sigma \sim g^4/m^2$ gives eq. (2) for a DM particle with weak coupling $g \sim 1$. A precise computation can be done solving Boltzmann equations for DM.

1.2 The baryon asymmetry

Let us summarize how the value of the baryon asymmetry in eq. (1) is measured. The photon density directly follows from the measurement of the CMB temperature and from Bose-Einstein statistics: $n_\gamma \sim T^3$. Counting baryons is more difficult. Direct measurements are not accurate, because only
some fraction of baryon formed stars and other luminous objects. Two different indirect probes point
to the same baryon density, making the result trustworthy. Each one of the two probes would require a
dedicated lesson:

1. Big-Bang-Nucleosynthesis (BBN) predictions depend on \( n_B/n_\gamma \). Fig. 1a illustrates the main
reactions. The important point is that the presence of many more photons than baryons delays
BBN, mainly by enhancing the reaction \( pn \leftrightarrow D\gamma \) in the \( \leftarrow \) direction, so that \( D \) forms not
when the temperature equals the Deuterium binding energy \( B \approx 2 \text{ MeV} \), but later at \( T \approx B/\ln(n_B/n_\gamma) \approx 0.1 \text{ MeV} \), giving more time to free neutrons to decay. A precise computation
can be done solving Boltzmann equations for neutrino decoupling and nucleosynthesis.

2. Measurements of CMB anisotropies [1], among many other things, allow us to probe acous-
tic oscillations of the baryon/photon fluid happened around photon last scattering. A precise
computation can be done evolving Boltzmann equations for anisotropies, assuming that they
are generated by quantum fluctuations during inflation: fig. 1b illustrates how the amount of
anisotropies with angular scale \( \sim 1/\ell \) depends on \( n_B/n_\gamma \). Acoustic oscillations have been seen
also in matter inhomogeneities [1], at \( \approx 3\sigma \) level.

2 Baryogenesis

The small baryon asymmetry \( n_B/n_\gamma \ll 1 \) can be obtained from a hot big-bang as the result of a small
excess of baryons over anti-baryons. We would like to understand why, when at \( T \lesssim m_p \) matter almost
completely annihilated with anti-matter, we survived thanks to the ‘almost’:

\[ n_B - n_\bar{B} \propto 1000000001 - 1000000000 = 1. \]

This might be the initial condition at the beginning of the big-bang, but it would be a surprisingly
small excess. In inflationary models it is regarded as a surprisingly large excess, since inflation erases
initial conditions.

In absence of a baryon asymmetry an equal number of relic baryons and of anti-baryons survive
to annihilations at \( T \lesssim m_p \). This process is analogous to DM annihilations studied in the previous
section, so we can estimate the relic baryon density by inserting \( m = m_p \) and a typical \( pp \) cross section
\( \sigma \sim 1/m_p^2 \) in eq. (5), obtaining \( n_p/n_\gamma \sim m_p/M_{Pl} \sim 10^{-19} \). Therefore this is a negligible contribution.

Assuming that the hot-big-bang started with zero baryon asymmetry at some temperature \( T \gg m_p \), can the baryon asymmetry can be generated dynamically in the subsequent evolution? Once that
one realizes that this is an interesting issue (this was done by Sakharov), the answer is almost obvious: yes, provided that at some stage [3]

1. baryon number \( B \) is violated;
2. \( C \) and \( CP \) are violated (otherwise baryons and antibaryons behave in the same way);
3. the universe was not in thermal equilibrium (we believe that CPT is conserved, so that particles
and antiparticles have the same mass, and therefore in thermal equilibrium have the same
abundance).

Having discussed in section 1.1 a concrete out-of-equilibrium situation, it should be clear what the
general concept means.
2.1 Baryogenesis in the SM?

A large amount of theoretical and experimental work showed that, within the SM, Sakharov conditions are not fulfilled. At first sight one might guess that the only problem is 1.; in reality 2. and 3. are problematic.

1. Within the SM $B$ is violated in a non-trivial way [3], thanks to quantum anomalies combined with extended SU(2)$_L$ field-configurations: the anomalous $B$ and $L$ symmetry are violated, while $B - L$ is a conserved anomaly-free symmetry. The basic equation is $\partial_\mu J_B^\mu \sim N_{\text{gen}} F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}$, and implies that there are many vacua that differ by their $B, L$ content, separated by a potential barrier of electroweak height. At temperatures $T \ll 100 \text{ GeV}$ transition between different vacua are negligible because suppressed by a quantum-tunneling factor $e^{-2\pi/\alpha^2}$. If $N_{\text{gen}} = 1$ this would imply proton decay with an unobservably slow rate; since there are 3 generations and all of them must be involved, proton decay is kinematically forbidden. This suppression disappears at high temperature, $T \gtrsim 100 \text{ GeV}$, and the space-time density of $B, L$-violating ‘sphaleron’ interactions is $\gamma \sim \alpha^3_3 T^4$, faster than the expansion rate of the universe up to temperatures of about $T \sim 10^{12} \text{ GeV}$ [3].

3. SM baryogenesis is not possible due to the lack of out-of-equilibrium conditions. The electroweak phase transition was regarded as a potential out-of-equilibrium stage, but experiments now demand a higgs mass $m_h \gtrsim 115 \text{ GeV}$, and SM computations of the Higgs thermal potential show that, for $m_h \gtrsim 70 \text{ GeV}$, the higgs vev shifts smoothly from $\langle h \rangle = 0$ to $\langle h \rangle = v$ when the universe cools down below $T \sim m_h$ [3].

2. In any case, the amount of CP violation provided by the CKM phase would have been too small for generating the observed baryon asymmetry, because it is suppressed by small quark masses. Indeed CP-violation would be absent if the light quarks were massless.

Many extensions of the SM could generate the observed $n_B$. ‘Baryogenesis at the electroweak phase transition’ needs new particles coupled to the higgs in order to obtain a out-of-equilibrium phase transition and to provide extra sources of CP violation. This already disfavored possibility will be tested at future accelerators. ‘Baryogenesis from decays of GUT particles’ seems to conflict with non-observation of magnetic monopoles, at least in simplest models. Furthermore minimal GUT model do not violate $B - L$, so that sphaleron processes would later wash out the eventually generated baryon asymmetry.

The existence of sphalerons suggests baryogenesis through leptogenesis: lepton number might be violated by some non SM physics, giving rise to a lepton asymmetry, which is converted into the observed baryon asymmetry by sphalerons.

This scenario can be realized in many different ways [3]. Majorana neutrino masses violate lepton number and presumably CP, but do not provide enough out-of-equilibrium processes. The minimal successful implementation [4] needs just the minimal amount of new physics which can give the observed small neutrino masses via the see-saw mechanism [2]: heavy right-handed neutrinos $N$ with masses $M$. ‘Baryogenesis via thermal leptogenesis’ [4] proceeds at $T \sim M$, when out-of-equilibrium (condition 3) CP-violating (condition 2) decays of heavy right-handed neutrinos generate a lepton asymmetry, converted in baryon asymmetry by SM sphalerons (condition 1).

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A real understanding of these issues needs advanced quantum field theory. This kind of theoretical studies lead to one observed consequence: the $\eta'$ mass, that is related to some QCD analogous of the SU(2)$_L$ effects we are considering. Therefore there should be no doubt that $B, L$ are violated, and this is almost all what one needs to know to understand leptogenesis quantitatively.
3 Thermal leptogenesis: the basic physics

We now discuss the basic physics, obtaining estimates for the main results. The SM is extended by adding the heavy right-handed neutrinos suggested by see-saw models. To get the essential points, we consider a simplified model with one lepton doublet $L$ and two right-handed neutrinos, that we name $N_1$ and $N_{2,3}$, with $N_1$ lighter than $N_{2,3}$. The relevant Lagrangian is

$$
\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{N}_1 i \not\!\partial N_1 + \lambda_1 N_1 H L + \frac{M_1}{2} N_1^2 + \\
+ \bar{N}_{2,3} i \not\!\partial N_{2,3} + \lambda_{2,3} N_{2,3} H L + \frac{M_{2,3}}{2} N_{2,3}^2 + \text{h.c.}
$$

By redefining the phases of the $N_1, N_{2,3}, L$ fields one can set $M_1, M_{2,3}, \lambda_1$ real leaving an ineliminable CP-violating phase in $\lambda_{2,3}$.  

\subsection*{3.1 CP-asymmetry}

The tree-level decay width of $N_1$ is $\Gamma_1 = \lambda_1^2 M_1 / 8 \pi$. The interference between tree and loop diagrams shown in fig. 2 renders $N_1$ decays CP-asymmetric:

$$
\varepsilon_1 \equiv \frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \bar{L}\bar{H})}{\Gamma(N_1 \to LH) + \Gamma(N_1 \to \bar{L}\bar{H})} \sim \frac{1}{4 \pi} \frac{M_1}{M_{2,3}} \text{Im} \lambda_{2,3}^2
$$

In fact

$$
\Gamma(N_1 \to LH) \propto |\lambda_1 + A \lambda_{2,3}^* |^2, \quad \Gamma(N_1 \to \bar{L}\bar{H}) \propto |\lambda_1^* + A \lambda_{2,3} |^2
$$

where $A$ is the complex CP-conserving loop factor. In the limit $M_{2,3} \gg M_1$ the sum of the two one loop diagrams reduces to an insertion of the $(LH)^2$ neutrino mass operator mediated by $N_{2,3}$; therefore $A$ is suppressed by one power of $M_{2,3}$. The intermediate states in the loop diagrams in fig. 2 can be on shell; therefore the Cutkosky rule guarantees that $A$ has an imaginary part. Inserting the numerical factor valid in the limit $M_{2,3} \gg M_1$ we can rewrite the CP-asymmetry as

$$
\varepsilon_1 \simeq \frac{3}{16 \pi} \frac{M_1 \text{Im} \tilde{m}_{2,3}}{v^2} = 10^{-6} \frac{\text{Im} \tilde{m}_{2,3}}{0.05 \text{eV}} \frac{M_1}{10^{10} \text{GeV}}
$$

where $\tilde{m}_{2,3} \equiv \lambda_{2,3}^2 v^2 / M_{2,3}$ is the contribution to the light neutrino mass mediated by the heavy $N_{2,3}$.

The operator argument implies that eq. (7) holds in any model where particles much heavier than $M_1$ mediate a neutrino mass operator with coefficient $\tilde{m}_{2,3}$. In the past it was debated about if only the ‘vertex’ diagram in fig. 2 or also the ‘self-energy’ diagram should be included when computing the CP asymmetry: the operator argument makes clear that both diagrams contribute, since in the limit $M_{2,3} \gg M_1$ the two diagrams reduce to the same insertion of the $(LH)^2$ operator.
The final amount of baryon asymmetry can be written as
\[
\frac{n_B}{n_\gamma} \approx \frac{\varepsilon_1 \eta}{g_{SM}}
\]  \tag{8}
where \(g_{SM} = 118\) is the number of spin-degrees of freedom of SM particles (present in the denominator of eq. \(8\) because only \(N_1\) among the many other particles in the thermal bath generates the asymmetry) and \(\eta\) is an efficiency factor that depends on how much out-of-equilibrium \(N_1\)-decays are.

### 3.2 Efficiency

We now discuss this issue. If \(N_1 \rightarrow LH\) decays are slow enough, the \(N_1\) abundancy does not decrease according to the Boltzmann equilibrium statistics \(n_{N_1} \propto e^{-M_1/T}\) demanded by thermal equilibrium, so that late out-of-equilibrium \(N_1\) decays generate a lepton asymmetry. Slow enough decay means \(N_1\) lifetime longer than the inverse expansion rate. At \(T \sim M_1\) one has

\[
R \equiv \frac{\Gamma_1}{H(M_1)} \approx \frac{\tilde{m}_1}{\tilde{m}^*} \quad \text{where} \quad \tilde{m}^* = \frac{256\sqrt{g_{SM} v^2}}{3M_{Pl}} = 2.3 \times 10^{-3} \text{eV}
\]

is fixed by cosmology. All the dependence on the mass and Yukawa couplings of \(N_1\) is incorporated in \(\tilde{m}_1 \equiv \lambda^2 v^2/M_1\), the contribution to the light neutrino mass mediated by \(N_1\). Unfortunately \(\tilde{m}_1\) and \(\tilde{m}_{2,3}\) are only related to the observed atmospheric and solar mass splittings in a model-dependent way. Unless neutrinos are almost degenerate (and unless there are cancellations) \(\tilde{m}_1\) and \(\tilde{m}_{2,3}\) are smaller than \(m_{\text{atm}} \approx 0.05\text{eV}\).

If \(R \ll 1\) (i.e. \(N_1\) decays strongly out-of-equilibrium) then \(\eta = 1\).

If instead \(R \gg 1\) the lepton asymmetry is only mildly suppressed as \(\eta \sim 1/R\). The reason is that \(N_1\) inverse-decays, which tend to maintain thermal equilibrium by regenerating decayed \(N_1\)’s, have rates suppressed by a Boltzmann factor at \(T < M_1\): \(R(T < M_1) \approx R \cdot e^{-M_1/T}\). The \(N_1\) quanta that decay when \(R(T) < 1\), i.e. at \(T < M_1/\ln R\), give rise to unwashed leptonic asymmetry. At this stage the \(N_1\) abundancy is suppressed by the Boltzmann factor \(e^{-M_1/T} = 1/R\). In conclusion, the suppression factor is approximately given by

\[
\eta \sim \min(1, \tilde{m}^*/\tilde{m}_1) \quad \text{(if } N_1 \text{ initially have thermal abundancy).} \tag{9}
\]

Furthermore, we have to take into account that virtual exchange of \(N_{1,2,3}\) gives rise to \(\Delta L = 2\) scatterings (see fig. 3) that wash-out the lepton asymmetry. Their thermally-averaged interaction rates are relevant only at \(M_1 \gtrsim 10^{14}\text{GeV}\), when \(N_{1,2,3}\) have large \(\mathcal{O}(1)\) Yukawa couplings. When relevant,
these scatterings give a strong exponential suppression of the baryon symmetry, because their rates
are not suppressed at $T \lesssim M_1$ by a Boltzmann factor (no massive $N_1$ needs to be produced).

So far we assumed that right-handed neutrinos have thermal initial abundancy. Let us discuss
how the result depends on this assumption. If $\tilde{m}_1 \gg \tilde{m}^*$ (in particular if $\tilde{m}_1 = m_{\text{atm}}$ or $m_{\text{sun}}$) the
efficiency does not depend on the assumed initial conditions, because decays and inverse-decays bring
the $N_1$ abundancy close to thermal equilibrium. For $\tilde{m}_1 \lesssim \tilde{m}^*$ the result depends on the unknown
initial condition.

• If $N_1$ have negligible initial abundancy at $T \gg M_1$ and are generated only by the processes
previously discussed, their abundancy at $T \sim M_1$ is suppressed by $\tilde{m}_1/\tilde{m}^*$. Therefore the
efficiency factor is approximatively given by

$$\eta \sim \min(\tilde{m}_1/\tilde{m}^*, \tilde{m}^*/\tilde{m}_1) \quad \text{(if } N_1 \text{ initially have zero abundancy).} \quad (10)$$

• Finally, in the opposite limit where $N_1$ initially dominate the energy density of the universe,
the suppression factor $1/g_{\text{SM}}$ in eq. (8) no longer applies, and the efficiency factor can reach
$\eta \sim g_{\text{SM}}$.

$$\eta \sim \min(g_{\text{SM}}, \tilde{m}^*/\tilde{m}_1) \quad \text{(if } N_1 \text{ initially have dominant abundancy).} \quad (11)$$

These estimates agree with the results of a detailed numerical computation, shown in fig. 4a. Notice
that if $\tilde{m}_1 \gg \tilde{m}^*$ the $N_1$ abundancy gets close enough to thermal equilibrium, such that the lepton
asymmetry generated by $N_1$ decays does not depend on the initial $N_1$ abundancy. Furthermore, $N_1$
decays and inverse-decays typically wash-out a possible pre-existing lepton asymmetry.

In the next section we describe how a precise computation can be done. This is not necessary for
understanding the final discussion of section 5.

4 Thermal leptogenesis: precise computation

As previously discussed many important computations in cosmology are done using Boltzmann equa-
tions. So it is useful to have this tool.

4.1 Boltzmann equations

In absence of interactions the number of particles in a comoving volume $V$ remains constant. Boltz-
mann equations allow to follow the effect of different interactions. Let us study e.g. how $1 \leftrightarrow 2 + 3$
decay and inverse-decay processes affect the number $n_1$ of ‘1’ particles (in the case of leptogenesis we
have $N_1 \leftrightarrow LH$):

$$\frac{d}{dt}(n_1 V) = V \int d\tilde{p}_1 \int d\tilde{p}_2 \int d\tilde{p}_3 (2\pi)^4 \delta^4(p_1 - p_2 - p_3) \times \quad (12)$$

$$\times [-|A_{1\to 23}|^2 f_1 (1 \pm f_2)(1 \pm f_3) + |A_{23\to 1}|^2 (1 \pm f_1)f_2f_3]$$

where $d\tilde{p}_i = d^3p_i/2E_i(2\pi)^3$ is the relativistic phase space, $|A_{1\to 23}|^2$ and $|A_{23\to 1}|^2$ are the squared
transition amplitude summed over initial and final spins, and $f_i$ are the energy (and eventually spin,
flavour, color,...) distributions of the various particles. To start we assume that CP violation can be
neglected, such that the direct and the inverse process have a common amplitude $A$.

In line of principle we should study the evolution of all $f$’s in order to obtain the total densities
$n = \sum \int f d^3p/(2\pi)^3$. In practice elastic scatterings (i.e. interactions that do not change the number of
particles) are typically fast enough that they maintain kinetic equilibrium, so that the full Boltzmann equations for \( f \) are solved by \( f(p) = f_{eq}(p)n/n_{eq} \) where \( f_{eq} = \left[ e^{E/T} \pm 1 \right]^{-1} \) are the Bose-Einstein and Fermi-Dirac distributions. Each particle species is simply characterized by its total abundancy \( n \), that can be varied only by inelastic processes.

The factors \( 1 \pm f_i \) in eq. \( (12) \) take into account Pauli-Blocking (for fermions) and stimulated emission (for bosons). Since the average energy is \( \langle E \rangle \sim 3T \) within 10% accuracy one can approximate with the Boltzmann distribution \( f_{eq} \approx e^{-E/T} \) and set \( 1 \pm f \approx 1 \). This is a significant simplification.

When inelastic processes are sufficiently fast to maintain also chemical equilibrium, the total number \( n_{eq} \) of particles with mass \( M \) at temperature \( T \) are

\[
n_{eq} = g \int \frac{d^3p}{(2\pi \hbar)^3} \frac{f_{eq}}{2\pi^2} K_2(M/T) = \begin{cases} \frac{gT^3}{\pi^2} & T \gg M \\ \frac{g(MT/2\pi)^{3/2}e^{-M/T}}{\pi^2} & T \ll M \end{cases}
\]

where \( g \) is the number of spin, gauge, etc degrees of freedom. A right handed neutrino has \( g_N = 2 \), a photon has \( g_\gamma = 2 \), the 8 gluons have \( g_{G^8} = 16 \), and all SM particles have \( g_{SM} = 118 \). The factor \( \hbar = h/2\pi \) has been explicitly shown to clarify the physical origin of the \( 2\pi \) in the denominator.

The Boltzmann equation for \( n_1 \) simplifies to

\[
\frac{1}{V} \frac{d}{dt} (n_1 V) = \int d\vec{p}_1 \int d\vec{p}_2 \int d\vec{p}_3 (2\pi)^4 \delta^4(p_1 - p_2 - p_3) \times |A|^2 \left[ -\frac{n_{1eq}}{n_1} e^{-E_1/T} + \frac{n_{eq} n_3}{n_2} e^{-E_2/T} e^{-E_3/T} \right]
\]

One can recognize that the integrals over final-state momenta reconstruct the decay rate \( \Gamma_1 \), and that the integral over \( d^3p_1/E_1 \) averages it over the thermal distribution of initial state particles; the factor
1/E_1 corresponds to Lorentz dilatation of their life-time. Therefore the final result is

$$\frac{1}{V} \frac{d}{dt} \langle n_1 \rangle = \langle \Gamma_1 \rangle n_1^{\text{eq}} \left[ \frac{n_1}{n_1^{\text{eq}}} - \frac{n_2}{n_2^{\text{eq}}} \frac{n_3}{n_3^{\text{eq}}} \right]$$

(14)

where $\langle \Gamma_1 \rangle$ is the thermal average of the Lorentz-dilatated decay width

$$\Gamma_1(E_1) = \frac{1}{2E_1} \int d\vec{p}_2 \, d\vec{p}_3 (2\pi)^4 \delta^4(p_1 - p_2 - p_3)|A|^2.$$  

(15)

Analogous results holds for scattering processes.

If $\langle \Gamma_1 \rangle \gg H$ the term in square brackets in eq. (14) is forced to vanish. This just means that interactions much faster than the expansion rate force chemical equilibrium, giving $n = n^{\text{eq}}$.

In the case of leptogenesis 2, 3 = L, H have fast gauge interactions. Therefore we do not have to write and solve Boltzmann equations for $L, H$, because we already know their solution: $L, H$ are kept in equilibrium. We only need to insert this result in the Boltzmann equation for $N_1$, that simplifies to

$$\dot{n}_1 + 3H n_1 = \langle \Gamma_1 \rangle (n_1 - n_1^{\text{eq}})$$

(16)

having used $\dot{V}/V = 3H = -\dot{s}/s$.

In computer codes one prefers to avoid very big or very small numbers: it is convenient to reabsorb the $3H$ term (that accounts for the dilution due to the overall expansion of the universe) by normalizing the number density $n$ to the entropy density $s$. Therefore we study the evolution of $Y = n/s$, as function of $z = m_N/T$ in place of time $t$ ($H dt = d\ln R = d\ln z$ since during adiabatic expansion $sV$ is constant, i.e. $V \propto 1/T^3$).

Using $Y(z)$ as variables, the general form of Boltzmann equations is

$$sH_2 \frac{dY_1}{dz} = \sum \Delta_1 \cdot \gamma_{\text{eq}}(12 \cdots \leftrightarrow 34 \cdots) \left[ \frac{Y_1}{Y_1^{\text{eq}}} \frac{Y_2}{Y_2^{\text{eq}}} \cdots - \frac{Y_3}{Y_3^{\text{eq}}} \frac{Y_4}{Y_4^{\text{eq}}} \cdots \right]$$

(17)

where the sum runs over all processes that vary the number of ‘1’ particles by $\Delta_1$ units (e.g. $\Delta_1 = -1$ for $1 \rightarrow 23$ decay, $\Delta_1 = -2$ for $11 \rightarrow 23$ scatterings, etc.) and $\gamma_{\text{eq}}$ is the spacetime density (i.e. the number per unit volume and unit time) in thermal equilibrium of the various processes.

Neglecting CP-violating effects, direct and inverse processes have the same reaction densities. For a scattering and for its inverse process one gets the previous result:

$$\gamma_{\text{eq}}(1 \rightarrow 23) = \int d\vec{p}_1 \, f_1^{\text{eq}} \int d\vec{p}_2 \, d\vec{p}_3 (2\pi)^4 \delta^4(p_1 - p_2 - p_3)|A|^2 = \gamma_{\text{eq}}(23 \rightarrow 1)$$

The thermal average of the decay rate can be analytically computed in terms of Bessel functions:

$$\gamma_{\text{eq}}(1 \rightarrow 23 \cdots) = \gamma_{\text{eq}}(23 \cdots \rightarrow 1) = n_1^{\text{eq}} \frac{K_1(z)}{K_2(z)} \Gamma(1 \rightarrow 23 \cdots)$$

(18)

For a 2-body scattering process

$$\gamma_{\text{eq}}(12 \leftrightarrow 34) = \int d\vec{p}_1 \, d\vec{p}_2 \, f_1^{\text{eq}} f_2^{\text{eq}} \int d\vec{p}_3 \, d\vec{p}_4 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)|A|^2$$

When there are $n$ identical particles in the final or initial states one should divide by a $n!$ symmetry factor. One can analytically do almost all integrals, and obtain

$$\gamma_{\text{eq}}(12 \rightarrow 34 \cdots) = \frac{T}{32\pi^4} \int_{s_{\text{min}}}^{\infty} ds \, s^{3/2} \lambda(1, M_1^2/s, M_2^2/s)\sigma(s) K_1(\sqrt{s}/T)$$

which is the thermal average of $v \cdot \sigma$, summed over initial and final state spins.
4.2 Boltzmann equations for leptogenesis

We now specialize to leptogenesis. The main process is $N_1 \leftrightarrow HL, \bar{H}\bar{L}$ decay and inverse decay. These processes are enough to generate a $N_1$ population that follows the Boltzmann distribution\(^2\), so that we can write the Boltzmann equation for the total $N_1$ abundance. We denote with $\gamma_D$ its equilibrium density rate, computed inserting $\Gamma(N_1) = \lambda_2^2 M_1 / 8\pi$ in eq. (18). The Boltzmann equation for the $N_1$ abundance is

$$sHz \frac{dY_{N_1}}{dz} = -\gamma_D \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right).$$

Fig. 5 shows how $Y_{N_1}$ evolves for different values of $\tilde{m}_1$. As expected if $\tilde{m}_1 \gg 10^{-3}$ eV one gets a result close to thermal equilibrium independently of the initial condition.

In order to get the Boltzmann equation for the lepton asymmetry one needs to take into account the small CP-violating terms. Let us start by including only the $\Delta L = 1$ CP-violating $N_1 \rightarrow HL, \bar{H}\bar{L}$ decays. We write the decay rates in terms of the CP-conserving total decay rate $\gamma_D$ and of the CP-asymmetry $\varepsilon_1 \ll 1$:

$$\gamma_{eq}(N_1 \rightarrow LH) = \gamma_{eq}(LH \rightarrow N_1) = (1 + \varepsilon_1)\gamma_D / 2,$$

$$\gamma_{eq}(N_1 \rightarrow \bar{LH}) = \gamma_{eq}(LH \rightarrow N_1) = (1 - \varepsilon_1)\gamma_D / 2.$$  

In this approximation the number of leptons $L$ and anti-leptons $\bar{L}$ evolve as

$$zHz Y'_{L} = \frac{\gamma_D}{2} \left[ \frac{Y_{N_1}}{Y_{N_1}^{eq}} (1 + \varepsilon_1) - \frac{Y_L}{Y_{L}^{eq}} (1 - \varepsilon_1) \right].$$

\(^2\)Intuitively, one would probably guess a different, incorrect, result: that $HL \rightarrow N_1$ inverse decays generate $N_1$ with the average energy of two particles, rather than with the average energy of one particle, since $E_{N_1} = E_L + E_H$, and that two particles have more energy than one. However, using $e^{-E_L/T} \cdot e^{-E_H/T} = e^{-E_{N_1}/T}$ one verifies that $\langle E_{N_1} \rangle = \langle E_L + E_H \rangle$: this is why Boltzmann found that the thermal distribution is exponential.
\[ zHsY_L' = \frac{\gamma_D}{2} \left[ \frac{Y_{N_1}^\text{eq}}{Y_{N_1}^\text{eq}} (1 - \varepsilon_1) - \frac{Y_L}{Y_L^\text{eq}} (1 + \varepsilon_1) \right]. \tag{22} \]

Here \( Y_{N_1}^\text{eq}, Y_L^\text{eq} \) and \( Y_L^\text{eq} \) are equilibrium densities each with 2 degrees of freedom. Ignoring \( \mathcal{O}(\varepsilon_1^2) \) terms, the lepton asymmetry \( \mathcal{L} = Y_L - Y_L \) evolves as

\[ sHs\mathcal{L}' = \varepsilon_1\gamma_D (\frac{Y_N}{Y_N^{\text{eq}}} + 1) - \frac{\mathcal{L}}{2Y_L^{\text{eq}}} \gamma_D. \tag{23} \]

The second term describes how \( \gamma_D \) tends to restore thermal equilibrium, washing out the lepton asymmetry. \textit{The first term makes no sense:} it would generate a lepton asymmetry even in thermal equilibrium, \( Y_{N_1} = Y_{N_1}^{\text{eq}} \), violating Sakharov conditions. An acceptable Boltzmann equation would contain \( Y_N/Y_N^{\text{eq}} = 1 \), but we made no sign error. Indeed, taking into account only decays and inverse decays, an asymmetry is really generated even in thermal equilibrium, since CPT invariance implies that if \( N_1 \) decays preferentially produce \( L \), then inverse decays preferentially destroy \( \bar{L} \) i.e. they have the same net effect.

**A subtlety: avoiding overcounting**

To obtain correct Boltzmann equations one must include all processes that contribute at the chosen order in the couplings. The CP-asymmetry is generated at \( \mathcal{O}(\lambda^4) \): at this order we must include also the \( \Delta L = 2 \) scatterings of fig. 3; we name their rates as

\[ \gamma_{N_1} \equiv \gamma_\text{eq}(LH \leftrightarrow \bar{L}\bar{H}), \quad \gamma_{N_1} \equiv \gamma_\text{eq}(LL \leftrightarrow \bar{H}\bar{H}), \tag{24} \]

and \( \gamma_{\Delta L=2} \equiv 2(\gamma_{N_1} + \gamma_{N_1}) \). At first sight it is enough to include these scatterings at tree level, obtaining CP-conserving reaction densities \( \gamma = \mathcal{O}(\lambda^4) \) that cannot correct our non-sensical CP-violating term. This is basically right, although the true argument is more subtle.

Indeed \( LH \leftrightarrow \bar{L}\bar{H} \) can be mediated by on-shell \( N_1 \) exchange (see fig. 3a): as usual in these situations (e.g. the \( Z \)-peak) resonant enhancement gives \( \sigma_{\text{peak}} \propto \lambda^0 \) in an energy range \( \Delta E \propto \lambda^2 \), so that \( \gamma_{N_1} \propto \sigma_{\text{peak}} \cdot \Delta E \propto \lambda^2 \). (We will soon obtain the exact result). Nevertheless one can prove that the reaction density remains CP-conserving up to one-loop order: unitarity demands \( \sum_j |M(i \rightarrow j)|^2 = \sum_j |M(j \rightarrow i)|^2 \), so

\[ \sigma(LH \rightarrow LH) + \sigma(LH \rightarrow \bar{L}\bar{H}) = \sigma(LH \rightarrow LH) + \sigma(\bar{L}\bar{H} \rightarrow LH) \]

(at higher order states with more particles allow a negligible CP asymmetry).

The key subtlety is that the \( LH \leftrightarrow \bar{L}\bar{H} \) scattering rate mediated by \( s \)-channel exchange of \( N_1 \) shown in fig. 3a, must be computed by subtracting the CP-violating contribution due to on-shell \( N_1 \) exchange, because in the Boltzmann equations this effect is already taken into account by successive decays, \( LH \leftrightarrow N_1 \leftrightarrow \bar{L}\bar{H} \). Since the on-shell contribution is

\[ \gamma_{\text{on-shell}}(LH \rightarrow \bar{L}\bar{H}) = \gamma_\text{eq}(LH \rightarrow N_1) \text{BR}(N_1 \rightarrow \bar{L}\bar{H}), \]

where \( \text{BR}(N_1 \rightarrow \bar{L}\bar{H}) = (1 - \varepsilon_1)/2 \), we obtain

\[ \gamma_{\text{sub}}(LH \rightarrow \bar{L}\bar{H}) = \gamma_{N_1} - (1 - \varepsilon_1)^2 \gamma_D/4, \tag{25} \]

\[ \gamma_{\text{sub}}(\bar{L}\bar{H} \rightarrow LH) = \gamma_{N_1} - (1 + \varepsilon_1)^2 \gamma_D/4. \tag{26} \]

Including subtracted scatterings at leading order in \( \varepsilon_1 \) gives the final Boltzmann equation [4]

\[ zHs\mathcal{L}' = \gamma_D \left[ \varepsilon_1 \left( \frac{Y_{N_1}^{\text{eq}}}{Y_{N_1}^{\text{eq}}} - 1 \right) - \frac{\mathcal{L}}{2Y_L^{\text{eq}}} \right] - 2\gamma_{\Delta L=2} \frac{\mathcal{L}}{Y_L^{\text{eq}}} \tag{27} \]

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where \( \gamma_{\Delta L=2} = \gamma_{\Delta L=2} - \gamma_D/4 \).

Equivalently, one can more simply not include the decay contribution \( \gamma_D \) to washout of \( \mathcal{L} \) because it is already accounted by resonant \( \Delta L = 2 \) scatterings. Then one gets

\[
zHs' = \gamma_D \varepsilon_1 \left( \frac{Y_{N_1}}{Y_{N_1}} - 1 \right) - 2\gamma_{\Delta L=2} \frac{\mathcal{L}}{\mathcal{L}_{N_1}},
\]

which is equivalent to (27). Using eq. (28) it is not necessary to compute subtracted rates. (Subtraction is performed incorrectly in works before 2003).

**Including sphalerons and Yukawas**

Finally, we have to include sphaleronic scatterings, that transmit the asymmetry from left-handed leptons to left-handed quarks, generating a baryon asymmetry. Similarly we have to include SM Yukawa couplings, that transmit the asymmetry to right-handed quarks and leptons.

In theory one should enlarge Boltzmann equations adding all these processes. In practice, depending on the value of \( M_1 \), during leptogenesis at \( T \sim M_1 \) these processes often give reaction densities which are either negligibly slower or much faster than the expansion rate: in the first case they can be simply neglected, in the second case they simply enforce thermal equilibrium. One can proceed by converting the Boltzmann equation for \( \mathcal{L} \) into a Boltzmann equation for \( B - \mathcal{L} \): since \( B - \mathcal{L} \) is not affected by these redistributor processes we only need to find how processes in thermal equilibrium relate \( B - \mathcal{L} \) to \( \mathcal{L} \). Sphalerons and the \( \lambda_{t,b,c,\tau} \) Yukawas are fast at \( T < \sim 10^{11-12} \text{GeV} \). At larger temperatures all redistribution processes are negligibly slow and one trivially has \( B - \mathcal{L} = -\mathcal{L} \). At intermediate temperatures one has to care about flavor issues, discussed later.

It is interesting to explicitly compute redistribution factors at \( T \sim \text{TeV} \) when all redistributor processes are fast. Each particle \( P = \{L, E, Q, U, D, H\} \) carries an asymmetry \( A_P \). Interactions equilibrate 'chemical potentials' \( \mu_P \equiv A_P/g_P \) as

\[
\begin{align*}
\text{ELH Yukawa} : & \quad 0 = \mu_E + \mu_L + \mu_H \\
\text{DQH Yukawa} : & \quad 0 = \mu_D + \mu_Q + \mu_H \\
\text{UQH Yukawa} : & \quad 0 = \mu_U + \mu_Q - \mu_H \\
\text{QQQL sphalerons} : & \quad 0 = 3\mu_Q + \mu_L \\
\text{No electric charge} : & \quad 0 = N_{\text{gen}}(\mu_Q - 2\mu_U + \mu_D - \mu_L + \mu_E) - 2N_{\text{Higgs}}\mu_H
\end{align*}
\]

Solving the system of 5 equations and 6 unknowns, one can express all asymmetries in terms of one of them, conveniently chosen to be \( B - \mathcal{L} \):

\[
B = N_{\text{gen}}(2\mu_Q - \mu_U - \mu_D) = \frac{28}{79} (B - \mathcal{L}), \quad \mathcal{L} = B - (B - \mathcal{L}) = -\frac{51}{79} (B - \mathcal{L}).
\]

The efficiency \( \eta \) is precisely defined such that the final baryon asymmetry is

\[
\frac{n_B}{s} = B = \frac{28}{79} \epsilon \eta Y_{N_1}^{\text{eq}}(T \gg M_1) \quad \text{i.e.} \quad \left. \frac{n_B}{n_\gamma} \right|_{\text{today}} = \frac{\varepsilon \eta}{103}. \tag{29}
\]

in agreement with the estimate (8).

Various extra processes give corrections of relative order \( g^2/\pi^2, \lambda_t^2/\pi^2 \sim \text{few \%} \). Some of these corrections have been already computed: scattering involving gauge bosons and/or top quarks. Others have not yet been included: three body \( N_1 \)-decays, one-loop correction to the \( N_1 \to LH \) decay and its CP-asymmetry. Thermal corrections have not been fully included. The fact that \( \gamma_D \) is the only really relevant rate makes a full inclusion of these subleading corrections feasible.
Including flavor

So far we studied the dynamics of leptogenesis in ‘one-flavor’ approximation, eq. (6). In the literature, flavor was fully included only after that neutrino data showed that flavor mixing among leptons can be large. The one-flavor approximation remains useful because including flavor adds so many unknown parameters that a precise discussion is impractical: e.g. we do not know which combination of \( L_e, L_\mu, L_\tau \) is the lepton doublet \( L \) coupled to \( N_1 \) in the see-saw Lagrangian, eq. (6). To include flavor, the Boltzmann equation for \( Y_L \) must be generalized to an evolution equation for the \( 3 \times 3 \) matrix density \( \rho \) of lepton asymmetries in each flavor. However, it simplifies to qualitatively different behaviors in different ranges of \( M_1 \):

- If \( M_1 \gtrsim 10^{11} \text{ GeV} \) all SM lepton Yukawa couplings induce scattering rates much slower than the expansion rate \( H \) at \( T \sim M_1 \): quantum coherence among different flavors survives undamped and the main new effect is that lepton asymmetries generated by \( N_2, N_3 \) decays can be washed-out by processes involving \( N_1 \) only along the combination of flavors to which \( N_1 \) couples. So, one must sum the contributions of all right-handed neutrinos produced after reheating.

- If \( M_1 \lesssim 10^9 \text{ GeV} \), the \( \lambda_\mu \) and \( \lambda_\tau \) Yukawa couplings induce scattering rates faster than \( H \) at \( T \sim M_1 \) and damp quantum coherence in \( \rho \): the matrix equation for \( \rho \) reduces to 3 Boltzmann equations for the asymmetries in the \( \ell = \{e, \mu, \tau \} \) flavors. Neglecting a mild flavor mixing (induced by sphaleronic scatterings) these equations have the following form: eq. (19) for \( Y_N \) remains unchanged, and eq. (28) for \( \mathcal{L}_e, \mathcal{L}_\mu, \mathcal{L}_\tau \) with the CP-violating term and the wash-out term restricted to each flavor, as intuitively expected. Therefore, in each flavor \( \ell \) one has a different CP asymmetry \( \varepsilon_{1\ell} \) and efficiency \( \eta_{\ell}(\tilde{m}_1, \tilde{m}_{1\ell}) \) that now depends on two parameters: the usual flavor independent \( \tilde{m}_1 \) that tells the total \( N_1 \) decay rate, and the flavor-dependent \( \tilde{m}_{1e,\mu,\tau} \equiv |\lambda_{1\ell}^2|v^2/M_1 \) that parameterize wash-out. A good approximation is [4]:

\[
\eta_{\ell}(\tilde{m}_1, \tilde{m}_{1\ell}) \approx \eta(\tilde{m}_{1\ell}).
\]

where \( \eta \) is the one-flavor efficiency, plotted in fig. 4a. The side figure shows the exact numerical result of \( \eta_{\ell} \) as function of \( \tilde{m}_{1\ell} \) for different values of \( \tilde{m}_1/\tilde{m}_{1\ell} = 1 \) (continuous line), 2 (red dotted line), 4 (blue dashed line), confirming that the result negligibly depends on \( \tilde{m}_1 \), especially if it is close to the values suggested by solar or atmospheric oscillations. Therefore eq. (8) gets replaced by

\[
\frac{n_B}{n_\gamma} \approx \sum_\ell \varepsilon_{1\ell} \cdot \eta(\tilde{m}_{1\ell})/g_{\text{SM}}.
\]

For large mixing angles one typically has \( \tilde{m}_1/\tilde{m}_{1\ell} \sim \mathcal{O}(2) \): the above approximation tells that taking flavor into account can enhance the efficiency by an \( \mathcal{O}(2) \) factor.

- Something intermediate happens if \( 10^9 \text{ GeV} \lesssim M_1 \lesssim 10^{11} \text{ GeV} \): quantum coherence stays undamped only among \( \mu \) and \( e \), such that one must take into account the asymmetry possibly generated by \( N_2, N_3 \) along \( e, \mu \).
5 Testing leptogenesis?

Unfortunately speculating that neutrino masses and the baryon asymmetry are produced by the seesaw mechanism and by thermal leptogenesis is much easier than testing them.

A direct test seems impossible, because right-handed neutrinos are either too heavy or too weakly coupled to be produced in accelerators.

What about indirect tests? We trust BBN because it predicts the primordial abundances of several nuclei in terms of known particle physics. Leptogenesis explains a single number, \(n_B/n_\gamma\), in terms of speculative physics at high energies: the see-saw model has 18 free parameters. Neutrino masses only allow to measure 9 combinations of these parameters, and thereby provide a too weak link. The situation might improve if future experiments will confirm certain supersymmetric models, in which quantum corrections imprint neutrino Yukawa couplings \(\lambda_{ij}N_iL_jH\) in slepton masses, inducing lepton flavour violating (LFV) processes such as \(\mu \to e\gamma\), \(\tau \to \mu\gamma\), \(\tau \to e\gamma\) with possibly detectable rates. Measuring them, in absence of other sources of LFV, would roughly allow us to measure the 3 off-diagonal entries of \(\lambda^\dagger \cdot \lambda\). Detectable LFV rates are obtained if \(\lambda > \sim 10^{-1÷2}\). In any case, reconstructing all see-saw parameters in this way is unrealistic. Maybe future experiments will discover supersymmetry, LFV in charged leptons, and will confirm that neutrino masses violate lepton number and CP, and we will be able to convincingly argue that this can be considered as circumstantial evidence for see-saw and thermal leptogenesis. Archeology is not an exact science.

Another possibility is that we might find a correctly predictive model of flavour. Presently three approaches give some predictions: symmetries, numerology, zerology. Symmetries can be used to enforce relations like \(\theta_{23} = \pi/4\), \(\tan^2 \theta_{12} = 1/2\), \(\theta_{13} = 0\) where \(\theta_{ij}\) are the neutrino mixing angles. Numerology can suggest relations like \(\theta_{12} + \theta_C = \pi/4\). Zerology consists in assuming that flavour matrices have many negligibly small entries; for example one can write see-saw textures with only one CP-violating phase. This scheme does not allow to predict the sign of CP-violation in neutrino oscillations in terms of the sign of the baryon asymmetry, because the sign of the baryon asymmetry also depends on which right-handed neutrino is the lightest one.

5.1 Constraints from leptogenesis

We here discuss testable constraints. Although this topic is tortuous, we avoid over-simplifications, at the price of obtaining a tortuous section.

As discussed in section 3, assuming that \(N_1\) is lighter enough than other sources of neutrino masses that their effects can be fully encoded in the neutrino mass operator, the CP-asymmetry \(\varepsilon_1\) is directly connected with it. Under the above hypothesis, one can derive constraints from leptogenesis. To do so, we need to generalize eq. (7) taking flavour into account. Denoting flavour matrices with boldface, we define \(\tilde{m}_i\) to be the contribution to the neutrino mass matrix mediated by the right-handed neutrino \(N_i\), so that \(m_\nu = \tilde{m}_1 + \tilde{m}_2 + \tilde{m}_3\). Then

\[
|\varepsilon_1| = \frac{3}{16\pi} \frac{M_1}{v^2} \frac{|\text{Im} \, \text{Tr} \tilde{m}_1^\dagger (\tilde{m}_2 + \tilde{m}_3)|}{\tilde{m}_1} \leq \frac{3}{16\pi} \frac{M_1}{v^2} (m_{\nu_3} - m_{\nu_1}) \tag{32}
\]

where \(m_{\nu_3} (m_{\nu_1})\) denotes the mass of the heaviest (lightest) neutrino. Rather than rigorously proving the constraint (32) [5], let us understand its origin and limitations in a simpler way.

1) Let us start considering the case of hierarchical neutrinos: \(m_{\nu_1} \ll m_{\nu_2}\): the constraint is obtained by substituting \(\tilde{m}_2 + \tilde{m}_3 = m_\nu - \tilde{m}_1\), and holds whatever new physics produces \(\tilde{m}_{2,3}\). Since \(|\varepsilon_1| \propto M_1\), one can derive a lower bound on the mass \(M_1\) of the lightest right-handed neutrino by combining eq. (32) with a precise computation of thermal leptogenesis in one-flavor
approximation and with the measured baryon asymmetry and neutrino masses:

\[
M_1 > \begin{cases} 
2.4 \times 10^9 \text{ GeV} & \text{if } N_1 \text{ has zero} \\
4.9 \times 10^8 \text{ GeV} & \text{if } N_1 \text{ has thermal initial abundancy} \\
1.7 \times 10^7 \text{ GeV} & \text{if } N_1 \text{ has dominant}
\end{cases}
\]

and assuming \( M_1 \ll M_{2,3} \). Including flavor as discussed previously relaxes the first constraint by a factor \( O(2) \).

2) The factor \( m_{\nu_3} - m_{\nu_1} \) is specific to the see-saw model with 3 right-handed neutrinos. To understand it, let us notice that 3 right-handed neutrinos can produce the limiting case of degenerate neutrinos \( m_{\nu_1} = m_{\nu_2} = m_{\nu_3} = m_{\nu} \) only in the following way: each \( N_i \) gives mass \( m_{\nu} \) to one neutrino mass eigenstate. Since they are orthogonal in flavour space, the CP-asymmetry of eq. (32) vanishes due to flavour orthogonality: this is the origin of the \( m_{\nu_3} - m_{\nu_1} \) suppression factor.

The bound (32) implies an upper bound on the mass of quasi-degenerate neutrinos: \( m_{\nu} \lesssim 0.2 \text{ eV} \) [5]. Indeed for large \( m_{\nu} \), both the efficiency and the maximal CP-asymmetry become smaller, because heavy neutrinos must be quasi-degenerate, and \( m_{\tilde{n}_1} \geq m_{\nu_1} \), \( m_{\nu_3} - m_{\nu_1} \simeq \Delta m^2_{\nu_3}/2m_{\nu} \). Furthermore, the bound can be improved up to \( m_{\nu_3} < 0.15 \text{ eV} \) [5] (3\( \sigma \) confidence level) by computing the upper bound on \( |\varepsilon_1| \) for given \( m_{\tilde{n}_1} \) and maximizing \( n_B \) with respect to \( m_{\tilde{n}_1} \) taking into account how the efficiency of thermal leptogenesis decreases for large \( m_{\tilde{n}_1} \). The leptogenesis constraint is very close to observed neutrino masses, and stronger than experimental bounds.

However, this leptogenesis constraint holds under the dubious assumption that hierarchical right-handed neutrinos produce quasi-degenerate neutrinos, while good taste suggests that quasi-degenerate neutrinos are more naturally produced by quasi-degenerate right-handed neutrinos. In general the constraint (32) evaporates if the particles that mediate \( \tilde{m}_{23} \) are so light that their effects cannot be encoded in \( \tilde{m}_{23} \): the CP-asymmetries becomes sensitive to the detailed structure of the neutrino mass model. Suppression due to flavor-orthogonality was the most delicate consequence of our initial assumption, and is the first result that disappears when they are relaxed, allowing \( M_{2,3} \) to be not much heavier than \( M_1 \). Correspondingly, the constraint on quasi-degenerate neutrino masses, that heavily relies on the factor \( m_{\nu_3} - m_{\nu_1} \), becomes weaker than experimental constraints.

Furthermore, in the extreme situation where right-handed neutrinos are very degenerate, \( M_2 - M_1 \ll \Gamma_{1,2} \) a qualitatively new effect appears: CP violation in \( N_1 \leftrightarrow N_2 \) mixing. This phenomenon is fully analogous to \( K^0 \leftrightarrow \bar{K}^0 \) mixing, and for \( M_2 - M_1 \sim \Gamma_{1,2} \) it allows a maximal CP-asymmetry, \( |\varepsilon_1| \sim 1 \). This means that with a tiny \( M_2 - M_1 \) one can have successful thermal leptogenesis even at the weak scale.

The constraint (33) is more robust, but we still have to clarify what the assumption \( M_{2,3} \gg M_1 \) means in practice. Surely one needs \( M_2 - M_1 \gg \Gamma_{1,2} \) such that only CP-violation in \( N_1 \) decay is relevant. The issue is: is \( M_{2,3}/M_1 \sim 10 \) (a hierarchy stronger than the one present in left-handed neutrinos) hierarchical enough to guarantee that the constraint holds, up to \( 1/10^2 = \% \) corrections? The answer is no: an operator analysis allows to understand how the constraint can be completely relaxed. The physics that above \( M_1 \) produces the dimension-5 neutrino mass operator \( (LH)^2/2 \) can also produce a related dimension-7 operator \( \Upsilon \equiv (LH)\partial^2(LH)/2 \), that contributes to \( \varepsilon_1 \). Since neutrino masses do not constrain \( \Upsilon \), it can be large enough to over-compensate the suppression due to its higher dimension. We make the argument more explicit, by considering the concrete case of see-saw models, where above \( M_1 \) there are two other right-handed neutrinos with masses \( M_2 \) and \( M_3 \). Including ‘dimension-7’ terms suppressed by \( M_1^2/M_{2,3}^2 \) and dropping inessential \( O(1) \) and flavour
factors, the CP-asymmetry becomes:

\[ \varepsilon_1 \sim \frac{3}{16\pi^2} v^2 \text{Im} \left[ \tilde{m}_2(1 + \frac{M_1^2}{M_2^2}) + \tilde{m}_3(1 + \frac{M_1^2}{M_3^2}) \right]. \]  

(34)

At leading order \( \varepsilon_1 \) depends only on \( \tilde{m}_2 + \tilde{m}_3 \), that, as previously discussed, cannot be large and complex. At higher order \( \varepsilon_1 \) depends separately on \( \tilde{m}_2 \) and \( \tilde{m}_3 \), that can be large and complex provided that their sum stays small. One can build models where this naturally happens, obtaining an \( \varepsilon_1 \) orders of magnitudes above the DI bound. The enhancement is limited only by perturbativity, \( \lambda_{2,3} \lesssim 4\pi \). In supersymmetric models, large Yukawa couplings lead to the testable effects previously discussed.

5.2 Leptogenesis and supersymmetry

Adding supersymmetry affects some \( O(1) \) factors. Eq. (29) remains almost unchanged, because adding spartners roughly doubles both the number of particles that produce the baryon asymmetry and the number of particles that share it. Ignoring small supersymmetry-breaking terms, right-handed neutrinos and sneutrinos have equal masses, equal decay rates and equal CP-asymmetries. Both \( \Gamma_{N_1} \) and \( \varepsilon_1 \) become 2 times larger, because there are new decay channels. As a consequence of more CP-asymmetry compensated by more wash-out, the constraints on right-handed and left-handed neutrino masses discussed in the non-supersymmetric case remain essentially unchanged.

The leptogenesis constraint (33) on \( M_1 \) acquires a new important impact: in many supersymmetric models the maximal temperature at which the Big-Bang started (or, more precisely, the ‘reheating temperature’ \( T_{\text{RH}} \)) must be less than about \( T_{\text{RH}} \lesssim 10^7 \text{ GeV} \), in potential conflict with the constraint (33).

Let us discuss the origin of the supersymmetric constraint on \( T_{\text{RH}} \). Gravitinos are the supersymmetric partner of the graviton: they have a mass presumably not much heavier than other sparticles, \( m_\tilde{G} \lesssim \text{TeV} \), and gravitational couplings to SM particles. Therefore gravitinos decay slowly after BBN (life-time \( \tau \sim M_{\text{Pl}}^2/m_\tilde{G}^3 \sim \text{sec} (100 \text{ TeV}/m_\tilde{G})^3 \)): their decay products damage the nuclei generated by BBN. The resulting bound on the gravitino abundancy depends on unknown gravitino branching ratios. The gravitino interaction rate is \( \gamma_\tilde{G}(T) \sim T^6/M_{\text{Pl}}^2 \), which means that gravitinos have been generated around the reheating temperature \( T_{\text{RH}} \), with abundancy \( n_\tilde{G}/n_\gamma \sim \gamma_\tilde{G}/Hn_\gamma \sim T_{\text{RH}}/M_{\text{Pl}} \). Therefore gravitinos suggest an upper bound on \( T \).

This is why in the previous section we carefully discussed specific scenarios that allow low-scale thermal leptogenesis. Supersymmetry suggests a new scenario named ‘soft leptogenesis’: complex soft terms in the see-saw sector give new contributions to the CP-asymmetry, \( \varepsilon_1 \sim \alpha_2 m_{\text{SUSY}}^2/M_1^2 \), which can be significant if \( M_1 \) is not much larger than the SUSY-breaking scale \( m_{\text{SUSY}} \) (presumed to be below 1 TeV). At larger \( M_1 \) ‘soft leptogenesis’ can still be relevant, but only in a fine-tuned range of parameters.

5.3 Leptogenesis in alternative neutrino-mass models

Generic neutrino masses can be mediated by tree-level exchange of three different kinds of new particles [2]: I) at least three fermion singlets; II) at least three fermion SU(2)_L triplets; III) one scalar SU(2)_L triplet (or of combinations of the above possibilities). Fig. 6 shows the relevant Feynman diagrams.

So far we studied case I). Can leptogenesis be used to distinguish between these possibilities? Leptogenesis can be produced in decays of \( P \), the lightest particle that mediates neutrino masses. The

\[^3\text{Gravitinos might be stable if they are the lightest supersymmetric particle. But in this case dangerous effects are produced by gravitational decays of the next-to-lightest SUSY particle into gravitinos.}\]
neutrino-mass contribution to its CP-asymmetry is given by expressions similar to eq. (32), with $\tilde{m}_1$ generalized to be the contribution to neutrino masses mediated by $P$, and $\tilde{m}_2 + \tilde{m}_3$ generalized to be the contribution of all heavier sources.

It was expected that the efficiency $\eta$ can be high enough only if $P$ is a right-handed neutrino: as discussed in section 3.2 it easily decays out-of-equilibrium giving

$$\eta(\text{fermion singlet}) \approx \min \left[ X, \frac{H}{\Gamma} \right]$$  \hspace{1cm} (35)

where $\Gamma$ is its decay rate, $H$ is the expansion rate at $T \sim M$ and

$$X = \begin{cases} 1 & \text{for thermal} \\ \frac{\Gamma}{H} & \text{for negligible initial abundancy.} \\ g_{SM} & \text{for dominant} \end{cases}$$  \hspace{1cm} (36)

A SU(2)$_L$ triplet (scalar or fermion) has gauge interactions that keep its abundancy close to thermal equilibrium so that the 3rd Sakharov condition cannot be fulfilled. This suppression is present, and a quantitative analysis is needed to see if/when it is strong enough. The Boltzmann equation for the triplet abundancy $Y$ has an extra term $\gamma_A$ that accounts for annihilations of two triplets into gauge bosons:

$$sHz \frac{dY}{dz} = -\gamma_D \left( \frac{Y}{Y_{eq}} - 1 \right) - 2\gamma_A \left( \frac{Y^2}{Y_{eq}^2} - 1 \right).$$  \hspace{1cm} (37)

The term $\gamma_A \sim g^4 T^4$ is dominant only at $T \gtrsim M$, where $M$ is the triplet mass; at lower temperatures it is strongly suppressed by a double Boltzmann factor $(e^{-M/T})^2$, because gauge scattering must produce 2 triplets. The resulting efficiency can be approximated as

$$\eta(\text{fermion triplet}) \approx \min \left[ 1, \frac{H}{\Gamma}, \frac{M}{10^{12} \text{GeV}} \max(1, \frac{\Gamma}{H}) \right].$$  \hspace{1cm} (38)

$\eta$ is univocally predicted, because at $T \gg M$ gauge interactions thermalize the triplet abundancy. At $T \sim M_1$ gauge scatterings partially annihilate triplets: in section 1.1 we learnt how to estimate how many particles survive to annihilations, and this is the origin of the factor $M/10^{12} \text{GeV}$ in $\eta$. The last factor takes into account that annihilations are ineffective if triplets decay before annihilating.

As illustrated in fig. 6 a scalar triplet separately decay to leptons, with width $\Gamma_L(T^* \rightarrow LL)$, and in Higgses, with width $\Gamma_H(T \rightarrow HH)$. Lepton number is effectively violated only when both processes are faster than the expansion of the universe, giving

$$\eta(\text{scalar triplet}) \approx \min \left[ 1, \frac{H}{\min(\Gamma_L, \Gamma_H)}, \frac{M}{10^{12} \text{GeV}} \max(1, \frac{\Gamma_L + \Gamma_H}{H}) \right].$$  \hspace{1cm} (39)
This means that a quasi-maximal efficiency $\eta \approx 1$ is obtained when $T^* \rightarrow LL$ is faster than gauge annihilations while $H \rightarrow TT$ is slower than the expansion rate. In conclusion, leptogenesis from decays of a SU(2)$_L$ triplet can be sufficiently efficient even if triplets are light enough to be tested at coming accelerators, $M \sim \text{TeV}$.

5.4 Leptogenesis and dark matter in loop-mediated neutrino-mass models

Neutrino masses mediated at one-loop level can be realized by exchanging various possible kinds of new particles; some of them are potential DM candidates. For example, one can introduce the usual right-handed neutrinos $N$ and couple them as $\lambda NLH'$, where $H'$ is not the usual Higgs scalar doublet, but another scalar doublet coupled to the Higgs as $\lambda (H'H')^2 + \text{h.c.}$ The Lagrangian is restricted to couplings invariant under the $Z_2$ symmetry $N \rightarrow -N$ and $H' \rightarrow -H'$, such that the lightest component of these particles is stable and is a DM candidate.

Neutrino masses are suppressed by a one loop factor $r \sim \lambda'/(4\pi)^2$ with respect to the standard see-saw. Leptogenesis depends on the coupling $\lambda$ and on $M_{1,2,3}$ in the usual way: therefore, at fixed values of neutrino masses, the CP asymmetry ($\propto m_{12,3}$, see eq. (7)) is enhanced by $1/r$ with respect to the standard case, while the efficiency factor $\eta$ (that depends on $m_1$ as in eq.s (9–11)) is suppressed by $r$ if $m_1 \sim m_{\text{sun, atm}}$. The constraints in eq. (33) on $M_1$ are relaxed by the factor $1/r$.

DM might be generated in the usual way discussed in section 1.1. Alternatively, a new interesting possibility is that both DM and leptogenesis might be generated by the same out-of-equilibrium process. In the example above, $N_1$ decays generate an asymmetry both in $L$ and $H'$: $H'$ could be DM with abundancy dominated by its asymmetry (like protons) rather than by the usual freeze-out relic abundancy discussed in section 1.1. If the DM asymmetry is not washed-out, this scenario leads to a precise testable prediction for the DM mass: $m_{\text{DM}} = cm_\rho\Omega_{\text{DM}}/\Omega_b \sim 10 \text{GeV}$, where $c$ is a model-dependent $O(1)$ factor, analogous to the 28/79 in eq. (29).

However the DM asymmetry survives only until $T \lesssim 1.2m_h$. Indeed DM is neutral under unbroken electromagnetism and a $Z_2$-like symmetry guarantees DM stability but does not protect the DM asymmetry. One can avoid wash-out by promoting $Z_2$ to a global U(1): we now discuss why this class of models seems not viable. In the model used as concrete example, imposing a global U(1) means setting $\lambda' = 0$, such that $H'$ is not affected when $H$ develops its vev. However, direct searches for DM told that DM must not only be neutral under electromagnetism, but also almost neutral under the $Z$: the DM $Z$ coupling must be $10 \pm 100$ times smaller than a typical electroweak coupling [6]. If the DM multiplet (either scalar or fermionic) lies in a complex representation of the electroweak gauge group (such that it can carry an asymmetry), reduced $Z$ couplings need $\lambda'$-like couplings. E.g., in the concrete $H'$ example, $\lambda'$ splits the complex neutral component of $H'$, $S + iA$, into a real scalar and pseudo-scalar, $S$ and $A$, with different masses. The $Z_\mu$ does not couple to the lightest DM mass eigenstate $S$ or $A$ but only to the off-diagonal combination $A \cdot \partial_\mu S - S \cdot \partial_\mu A$, without giving unseen DM signals if the mass difference $\Delta m \equiv |m_S - m_A|$ is bigger than about $20\text{keV}$ (the expected kinetic energy of DM around the earth). One can show that the needed $\Delta m$ is so big that oscillations among $S + iA$ and $S - iA$ destroy the DM asymmetry.\footnote{This needs a non-trivial analysis: Boltzmann equations for the density matrix of $S$ and $A$ show that DM-number-conserving gauge scatterings with rate $\Gamma \gg H$ synchronize oscillations among different moment and reduce their frequency from $\Delta m$ to $\Delta m^2/\Gamma$. These non-trivial features help a lot, but not enough.}

To avoid this conclusion one needs either $m_{\text{DM}} \approx 30m_h$ or models where DM is a complex neutral scalar singlet coupled to the Higgs: both possibilities look unattractive.

Acknowledgments

The original work presented in the last section was performed in collaboration with M. Raidal and V. Rychkov, that agreed on briefly summarizing here our no-go result. I thank
S. Davidson, A. Riotto and T. Kashti for useful discussions. Since these are lessons, the bibliography prefers later systematic works to pioneering imperfect works.

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