Dynamic Modeling and Simulation of Quick-Setting Slurry with Spatiotemporal Rheological Properties

Pengda Cheng¹, Shaohui Deng², Bao lin Liu³, Zhanqing Xing⁴, Peizhou Yang⁵, Chun Feng¹*

1 Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China
2 Yalong River Hydropower Development Co., Ltd., Chengdu 610051, Sichuan, China
3 CREEC(Chongqing) Survey, Design and Research Co., Ltd., Chongqing, 400023, China
4 China Institute of Water Resources and Hydropower Research (IWHR), Beijing 100038, China
*Corresponding author.
E-mail address: fengchun@imech.ac.cn
ORCID: 0000-0001-6592-144X

Abstract. A major concern in underground infrastructures is how to sufficiently seal the area from water ingress. To achieve this, grout needs to be spread adequately in the surrounding fractures. Cement-based composite grouting is probably the best method for this purpose because of its lower costs and reduced environmental impacts. Chemical reaction accompanied by the flow is a prominent feature of cement-based composites. Rheological properties, especially yield stress and viscosity, are non-uniformly distributed in time and space. In this paper, the rheological properties of quick-setting slurry of cement-based composites were measured over time, and the rheology constitutive equation including time was established based on non-Newtonian fluids. Considering the rheological properties, the Lagrangian method was introduced to track their space–time distribution, and then a dynamic model of quick-setting slurry was established based on continuity equation and momentum equation. The relationship between time-varying rheological characteristics and flow field characteristics was obtained through a numerical simulation of equal pressure injection and equal flux injection. The simulation results were compared with the experimental results in the references, thereby verifying the reliability and accuracy of the model. Results show that the local erosion state and local sealing effect of grouting directly depend on the yield stress of different positions in the flow field. The yield stress of quick-setting slurry increases rapidly with time, which is more likely to affect the flow field, and the viscosity of the slurry has a small effect on the flow field.

Keywords. grouting, quick-setting slurry, spatiotemporal rheological property, yield stress, dynamic model

1. Introduction
The grouting method is progressing rapidly as an effective means of improving the mechanical properties of rocks and preventing groundwater leakage in construction projects[1–3]. Cement-based composite grouting is probably the best method for this purpose because of its lower costs and reduced...
environmental impacts. Chemical reaction accompanied by the flow is a prominent feature of cement-based composites. Rheological properties are non-uniformly distributed in time and space, especially yield stress and viscosity. Many scholars studied the flow characteristics of viscosity changing with time through theoretical derivation laboratory tests and numerical calculation[4], [5]. Previous studies showed that the change of rheological properties with time and space will significantly affect the slurry flow field[6], [7]. In previous studies, rheological properties were generally considered as the change of viscosity with time and space. In fact, the rheological properties of cement-based composites with time and space depend on the rate of chemical reaction. When the chemical reaction speed is fast, such as that of quick-setting slurry, the yield stress increases rapidly with time, and its effect on local shear stress is much greater than that of slurry viscosity. The rapid increase of yield stress leads to more obvious change of the flow field, which is also the key for the quick-setting slurry to resist water erosion. In this study, the Lagrangian method was introduced to track the temporal and spatial distribution of the slurry rheological properties. On the basis of the continuity equation and the momentum equation, a dynamic model of the quick-setting slurry was established, and the flow law of the quick-setting slurry was studied under hydrodynamic conditions.

2. Experiment on slurry rheology
On the basis of the rotational speed and torque recorded by a torque rheometer, the variation law of yield stress and viscosity in the Bingham model or the Herschel–Bulkley model can be obtained by using Couette transformation (Reiner–Riwlin equation)[8]. The main materials of quick-setting slurry are cement, water, and setting accelerator, as shown in Table 1.

| Materials (m)     | Weight (g) |
|-------------------|------------|
| Cement            | 1000       |
| Water             | 500        |
| Setting accelerator| 75         |
| Dispersant        | 3          |
| Nano silica particles | 5       |

Table 1. Mix materials of the quick-setting slurry

Figure 1 shows the relationship between time and yield stress of the quick-setting slurry as obtained by an experiment. The results show that the yield stress increases rapidly with time and then tends to stabilize gradually. The yield stress remains steady when the solidification process is almost over. Within three minutes, the maximum yield stress is nearly 209 Pa. Figure 2 shows the relationships between shear time and shear stress of the quick-setting slurry as obtained by an experiment. In Experiments 1, 2, and 3, the solidification time was 3 minutes, 1 minute, and 30 seconds, respectively. The shear stress of slurry with different solidification times decreases rapidly with the shear time and tends to achieve equilibrium gradually. Fifteen seconds after the shear started, the shear stress of the slurry decreased to 4.5 Pa and reached the shear equilibrium state. In the shear equilibrium state, shear stress is related to slurry viscosity and shear rate. The shear stress in the equilibrium state is nearly 50 times lower than the yield stress. In this study, the rheological properties of the quick-setting slurry, especially yield stress, are more likely to affect the flow field, whereas the slurry viscosity has less influence on the flow field.
3. Numerical model

3.1. Governing equations

The flow process of quick-setting slurry is accompanied by a chemical reaction, and the physical parameters of slurry change with the flow. With the use of Lagrangian time points to record the start time of local chemical reactions, considering the uneven temporal and spatial distribution of the physical properties of the slurry, as well as the density and velocity of each phase\cite{9}, the dynamic model of quick-setting slurry and water flow can be established. The model hypothesis is as follows:

1. Slurry and water are incompressible fluids.
2. The density of each phase is approximately constant\cite{10}.
3. Both phases share the same pressure field\cite{11}.
4. The yield stress of quick-setting slurry increases with the chemical reaction time.
5. There exists no slip between fluid and rock surface.
6. The fractures in rock mass are horizontal and have a certain spacing.

For phase $k$, the mixture density and the mixture velocity of slurry water are defined as follows:

$$\rho_m = \sum_{k=1}^{\phi_k \rho_k}$$ \hspace{1cm} (1)

$$u_m = \frac{1}{\rho_m} \sum_{k=1}^{\phi_k \rho_k} u_k$$ \hspace{1cm} (2)

where $\phi_k$ is the volume fraction of a phase; $\rho_k$ is the density of a phase; $u_k$ is the velocity of a phase; $\rho_m$ is the mixture density of the grout–water; and $u_m$ is the mixture velocity of grout–water. $\rho_m$ varies even though the component densities are constants.

The grout–water mixture can be considered a continuous medium. Thus, the continuity equation of the suspension can be obtained as follows:

$$\frac{\partial \rho_m}{\partial t} + \rho_m \nabla \cdot u_m = 0$$ \hspace{1cm} (3)

The momentum equation for the suspension flow is obtained by summing over the phases.

$$\frac{\partial}{\partial t} \rho_m u_m + \nabla \cdot (\rho_m u_m u_m) = -\nabla p_m + \nabla \cdot \tau_m + \nabla \cdot \tau_{m_{\Omega}} + \rho_m \mathbf{g} + \mathbf{M}_m$$ \hspace{1cm} (4)

where $p_m$ is the mixture pressure. In practice, the phase pressures are often regarded equal. $\tau_m$ is the sum of the shear stress, the shear stress of time-varying rheology grout is written as a function of the
local time and Lagrange time, and $\tau_m = \tau_0(t^*-t) + \mu_m(t^*-t)$ holds for a non-Newtonian fluid. $\mu_m$ is the viscosity, and $\gamma_m = \nabla u_m + \nabla u_m^T$. Furthermore, $M_m$ is the influence of the surface tension force on the mixture. To simulate the flow fields, the standard $k$-$\varepsilon$ turbulence model was used[12]. The model constants were then determined from experimental data[13]: $C_\mu = 0.09$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, $C_{\mu 1} = 1.44$, and $C_{\mu 2} = 1.92$.

$\tau_{Dm}$ is defined by the following relation:

$$\tau_{Dm} = -\sum_{k=1}^{n} \phi_k \rho_k u_{Mk} u_{Mk}$$  \hspace{1cm} (5)

where $u_{Mk}$ is the diffusion velocity, i.e., the velocity of phase $k$ relative to the center of the mixture mass, i.e., $u_{Mk} = u_k - u_m$.

The transport equation for phase in the flow is determined as follows:

$$\frac{\partial}{\partial t} \phi_k + \nabla \cdot (\phi_k u_m) = -\nabla \cdot N_k$$  \hspace{1cm} (6)

where $N_k$ is the total diffusive flux of a phase resulting from two different mechanisms, namely, the flux generated by the gradients in collision, $N_{ke}$, and the flux generated by gradients in suspension viscosity, $N_{kl}$. The effect of $N_{ke}$ is opposite that of the strain rate and particle concentration gradients, whereas the effect of $N_{kl}$ leads to the migration of particles in the opposite direction of the suspension viscosity gradient.

The transport equation for the Lagrange time in the flow is determined as follows:

$$\frac{\partial \tau^*}{\partial t} + \mu_m \cdot \nabla \tau^* = 0$$  \hspace{1cm} (7)

where $\tau^*$ is the Lagrange time point.

3.2. Geometric model

A narrow parallel plate model is used to analyze the transport process of quick-setting slurry with a chemical reaction in a single fracture. In the numerical simulation, the geometric model used refers to the experiment[1], [4]. The model size was set to $3 \text{ m} \times 3 \text{ m}$, and the fracture width was $5 \text{ mm}$ (Figure 3). The grouting hole was located at the geometric center of the model and had a diameter of $4 \text{ cm}$. The selection of all the above parameters was based on recommendations in the literature[4].

![Figure 3. Computational domain for simulation](image)

3.3. Boundary and initial conditions

3.3.1 Flow field boundary conditions

The grouting hole is the inlet boundary and is located in the geometric center of the model. The inlet boundary is set as the velocity boundary condition or pressure boundary condition. The upper and lower surfaces of the narrow parallel plate model are no-slip boundaries.
\[ u_m = 0 \]  

The pressure boundary condition is around the narrow parallel plate model

\[ p = p_0 \]  

### 3.3.2 Transport field boundary conditions

The grouting hole is located in the geometric center of the model and is the inlet boundary of the phase- and Lagrange time-transport field.

For the phase- and Lagrange time-transport field, the upper and lower surfaces of the narrow parallel plate model are no-flux boundaries

\[
\begin{align*}
    n \cdot (u_m \varphi_k \cdot D \nabla \varphi_k) &= 0 \\
    n \cdot (u_m^* \cdot D \nabla t^*) &= 0
\end{align*}
\]  

(10)

The outlet boundary conditions are around the narrow parallel plate model in the phase- and Lagrange time-transport field.

### 3.3.3 Initial conditions

The initial velocity and pressure, the initial quick-setting slurry volume fraction, and the initial Lagrange time in water are 0.

The densities of water and quick-setting slurry are 997 and 1900 kg/m³, respectively, and the viscosity of the overlying water is \(1 \times 10^{-3}\) Pa·s. The relationship between the solidification time \((t - \tau)\) and yield stress is based on experimental results (Figure 1).

### 3.4. Numerical method

In this study, the flow of the mixture of quick-setting slurry and water was analyzed by using the multiphase CFD solver in the open-source package OpenFOAM[14]. The mixture flow in the narrow parallel plate was modeled in three dimensions by using the RANS equations with the \(k-\varepsilon\) turbulence closure scheme, as solved by the finite volume method. The computational domain was discretized into a series of non-repeated control volumes, namely, cells. In this domain, each cell is represented by a node that is located at the center, and the flow field information is stored at this node. The PIMPLE Algorithm is a combination of PISO (Pressure Implicit with Splitting of Operator) and SIMPLE (Semi-Implicit Method for Pressure-Linked Equations), which was used to solve the pressure–velocity coupling problem in the incompressible Navier–Stokes equations. PIMPLE is mainly inherited from the PISO algorithm. With regard to the numerical discretization schemes described in this paper, the Euler scheme was utilized for time integration, the Gauss limited linear V1 scheme was used for the advective term, and the Gauss linear corrected scheme was used for the Laplacian terms.

### 4. Model validation

Previous researchers performed corresponding laboratory tests and theoretical and numerical experiments in static water conditions. They found that the time dependent characteristics of the viscosity of cement-sodium silicate (C-S) slurry were in accordance with the power function distribution. By deriving the spatial and temporal distribution function of the viscosity in a slurry diffusion zone, Zhang et al.[3] established the horizontal fracture grouting diffusion theoretical model considering the time dependent viscosity characteristics during constant grouting rate conditions. The relationship between the grouting time, grouting pressure, and slurry diffusion radius was obtained. In this study, when the yield stress of quick-setting slurry is not considered, the slurry can be simplified as Newtonian fluid, and the viscosity of slurry is time dependent. For numerical simulation, after the grid independence test, the 60 grids were divided within fracture width, and the grids were refined with eight layers at the surface. The model validation was based on the reference data of the slurry (C:S = 1:1, \(\mu = 0.003182 \times t^{2.23} + 0.04\)) diffusion[3]. The fracture width was 5 mm, the grouting hole was located at the geometric center of the model and had a diameter of 4 cm, the grouting time was 60 seconds, and the grouting pressure was 30 kPa. As shown in Figure 4(a), the slurry viscosity
increases with distance at the same time, while the slurry viscosity increases with time at the same position. The velocity of the slurry–water interface gradually decreases with time, and the grouting diffusion radius reaches the maximum. As shown in Figure 4(b), the pressure gradually decreases with distance, but the nonlinearity of the pressure gradient is more obvious in the initial stage. The numerical simulation results are in good agreement with reference data, thereby verifying the rationality of the dynamic model of quick-setting slurry and water flow.

![Graph 1](image1.png)  ![Graph 2](image2.png)

**Figure 4.** Viscosity and pressure change with distance at different times (comparison of numerical results and reference data)

5. Results
On the basis of experimental measurements for quick-setting slurry, the relationship between solidification time is shown in Figure 1, and the relationship between shear time and shear stress is shown in Figure 2. The maximum yield stress $\tau_0$ is nearly 209 Pa. The shear stress in equilibrium state is nearly 50 times lower than the yield stress. The rheological properties of the quick-setting slurry, especially the yield stress, are more likely to affect the flow field characteristics, whereas the slurry viscosity has less influence on the flow field. In this study, the relationship between the grouting time, grouting pressure, and slurry diffusion radius was calculated under different cases. Two kinds of boundary conditions, i.e., pressure inlet and flow inlet boundary conditions, were adopted in calculation in Table 2, and the grouting time is 60 seconds.

| Table 2. Calculation parameters for grouting conditions |
|--------------------------------------------------------|
| Inlet conditions | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 |
| Pressure (Pa) | $4 \times \tau_0$ | $8 \times \tau_0$ | $16 \times \tau_0$ | - | - | - |
| Flux (L·min$^{-1}$) | - | - | - | 1.5 | 3.0 | 4.5 |

The change in the yield stress with diffusion distance under different conditions when grouting for 60 seconds is shown in Figure 5. An obvious interface exists between quick-setting slurry and water because of the different yield stress. Under constant pressure (Figure 5[a]), the slurry yield stress changes little with distance at 60 seconds, thereby showing that the velocity of the flow field decreases rapidly with the increase in the slurry yield stress. Under constant flux (Figure 5[b]), the yield stress of slurry increases rapidly with distance at 60 seconds, and the yield stress is the lowest at the inlet and the highest at the slurry–water interface. The local flow field characteristics change because the physical properties of quick-setting slurry change rapidly with the flow. Under different hydrodynamic
conditions, the relationship between local pressure and slurry yield stress has an important influence on the flow field.

Under constant pressure (Cases 1, 2, and 3), the variation of yield stress with distance at 1, 2, 10, and 60 seconds are shown in Figures 6(a), 6(b), 6(c), and 6(d), respectively. In the early stage of grouting (Figures 6[a] and 6[b]), the slurry yield stress increases rapidly with distance and time. After grouting for a period of time (Figures 6[c] and 6[d]), the slurry yield stress increases rapidly with time, but changes slowly with distance. This finding shows that the yield stress is low in the early stage, the injection pressure can quickly drive the slurry flow in the fracture after overcoming the yield stress, and the slurry–water interface moves faster. After a period of time, the slurry gradually solidifies, the injection pressure could not overcome the yield stress of the slurry, and the slurry–water interface almost stops moving. The diffusion distance of slurry can be obtained from the position of the slurry–water interface. In Cases 1, 2, and 3, the grouting pressure increases by 2 and 4 times, the final position of the slurry–water interface changes nonlinearly, and the slurry diffusion distance increases by 0.65 and 1.64 times, respectively.
Figure 6. Variation of yield stress (Pa) with distance different times in Cases 1, 2, and 3

Under constant pressure (Cases 1, 2, and 3), the variation of pressure with distance at 1, 2, 10, and 60 seconds are shown in Figures 7(a), 7(b), 7(c), and 7(d), respectively. The pressure decreases rapidly with the increase in distance, but the pressure decrease with time is significantly different. In combination with the position of the slurry–water interface, the pressure drop in the slurry is much greater than the pressure drop in the water in the early stage of grouting (Figures 7[a] and 7[b]). After grouting for a period of time (Figures 7[c] and 7[d]), the pressure drop in the slurry and water remains almost constant. In the process of slurry flow, the maximum pressure gradient is near the slurry–water interface, which is related to the maximum yield stress at the interface. During the solidification process, the maximum pressure gradient is near the inlet, which is related to the gradually consistent yield stress in the flow field.
Figure 7. Variation of pressure (Pa) with distance different times in Cases 1, 2, and 3

Under constant pressure (Cases 1, 2, and 3) and constant flux (Cases 4, 5, and 6), the variations of yield stress and pressure with distance at 60 seconds are shown in Figures 8(a) and 8(b), respectively. After 60 seconds of grouting, the maximum yield stress of slurry in different conditions tends to be the same. Under constant pressure conditions, the slurry gradually solidifies and stops flowing, the yield stress of the slurry changes little with the distance, and the pressure drop decreases rapidly with the distance and drops to a minimum near the slurry–water interface. Under constant flux conditions, the slurry is always in a flowing state, and its yield stress is the smallest at the inlet and the largest at the slurry–water interface. The pressure increases significantly to keep the slurry flowing. When the diffusion distance of slurry is about 0.5 m (Cases 1 and 6), the pressure required by constant flux condition is about 400 times greater than that required by constant pressure condition. Considering the fracture width and diffusion radius, the energy of injected slurry can be obtained by integration. The energy required by the constant flux condition is about 420 times greater than that required by the constant pressure condition. For the quick-setting slurry containing chemical reaction, the rheological properties change with flow. Thus, we can infer that increasing the pressure will significantly affect the diffusion distance at the early stage of grouting.

Figure 8. Variation of yield stress and pressure with distance at 60 seconds

6. Conclusions
Chemical reaction accompanied by the flow is a prominent feature of cement-based composites. Rheological properties, especially yield stress and viscosity, are non-uniformly distributed in time and space. Considering time-varying rheological properties, the Lagrangian method was introduced to track their space–time distribution, and then the dynamic model of quick-setting slurry was established. The relationships between time-varying rheological characteristics and flow field characteristics were obtained based on experimental and coupled numerical simulation results. The main conclusions of this work are as follows:

1. For quick-setting slurry, the yield stress increases rapidly in a short time. The shear stress of slurry with different solidification time decreases rapidly with the shear time and tends to achieve equilibrium gradually. The shear stress in equilibrium state is much lower than the yield stress. Therefore, the yield stress has more influence on the flow field than the slurry viscosity.

2. Under constant pressure conditions, the injected energy remains constant. The yield stress of the slurry changes little with the diffusion distance, and the pressure drop decreases rapidly with the distance and drops to a minimum near the slurry–water interface. Under constant flux conditions, the injection energy increases significantly to keep the slurry flowing. The slurry is always in a flowing state, and its yield stress is the smallest at the inlet and the largest at the slurry–water interface.

3. Under different hydrodynamic conditions, the relationship between local pressure and slurry yield stress has an important influence on the flow field. The local erosion state and local sealing effect of grouting depend directly on the yield strength of different positions in the flow field. According to the position of the slurry–water interface, the slurry diffusion distance varies nonlinearly with injection pressure or injection flux, which is closely related to the rheological characteristics of uneven temporal and spatial distribution of slurry, especially the change in the yield strength of slurry. For the quick-setting slurry containing chemical reaction, its rheological characteristics change with flow, and the diffusion distance is greatly affected by the hydrodynamic conditions of injection. We can infer that selecting different injection conditions is conducive to optimizing the grouting design of different projects.

The study of a dynamic model considering time-varying rheological characteristics can help provide a deep and systematic understanding of the flow field change and functional response of grouting. Knowledge of the coupling transport mechanism may offer important insights into grouting engineering designs. An important research direction for the future is to elucidate the interactions of quick-setting slurry, water, and solid. Unlike in the case of single fracture, the local opening or closing of a fracture network (discontinuous structure) should be considered under the grouting pressure, which may lead to a large change in the flow field in local areas.

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7. References
[1] Wang, Z. F., Shen, S. L. & Modoni, G. 2019. Enhancing discharge of spoil to mitigate disturbance induced by horizontal jet grouting in clayey soil: theoretical model and application. Computers and Geotechnics, 111, 222–228.
[2] Algin, H. M. 2016. Optimised design of jet-grouted raft using response surface method. Computers & Geotechnics, 74, 56–73.
[3] Liu, H., Zhou, H., Kong, G., Qin, H. & Zha, Y. 2017. High pressure jet-grouting column installation effect in soft soil: theoretical model and field application. Computers & Geotechnics, 88, 74–94.
[4] Deng, S., Wang, X., Yu, J. et al. 2018. Simulation of Grouting Process in Rock Masses Under a Dam Foundation Characterized by a 3D Fracture Network. Rock Mechanics and Rock Engineering, 51, 1801–1822.

[5] Zhang Q S, Zhang L Z, Liu R T, et al. 2017. Grouting mechanism of quick setting slurry in rock fissure with consideration of viscosity variation with space. Tunnelling and Underground Space Technology, 70, 262–273.

[6] Li, S. Pan, D. Xu, Z. Lin, P. & Y Zhang. 2020. Numerical simulation of dynamic water grouting using quick-setting slurry in rock fracture: the sequential diffusion and solidification (sds) method. Computers and Geotechnics, 122, 103497.

[7] Cheng P D, Li Lu, Tang J, et al. 2011. Application of time-varying viscous grout in gravel-foundation anti-seepage treatment. Journal of Hydrodynamics, 23(3), 391–397.

[8] Dimitri, Feys, Jon, et al. 2013. Extension of the Reiner–Riwlin equation to determine modified Bingham parameters measured in coaxial cylinders rheometers. Materials & Structures, 46, 178–183.

[9] Cheng, P.D.; Wang, X.Q.; Feng, C. 2020. Numerical simulation of phosphorus release from resuspended sediment. Acta Mechanica Sinica, 36, 1191–1201.

[10] Stickel, J.J., & Powell, R.L. 2005. Fluid mechanics and rheology of dense suspensions. Annual Review of Fluid Mechanics, 37, 129–149.

[11] Hinch, E.J. 2011. The measurement of suspension rheology. Journal of Fluid Mechanics, 686, 1–4.

[12] Launder, B.E., & Spalding, D.B. 1974. The numerical computation of turbulent flows. Computer Methods in Applied Mechanics and Engineering, 3, 269–289.

[13] Wilcox, D.C. 2010. Turbulence Modeling for CFD (3rd edn.). DCW Industries Inc., La Canada Flintridge, USA.

[14] OpenFOAM User Guide. 2021. CFD Direct, Architects of OpenFOAM. https://cfd.direct/openfoam/user-guide/.