The Exchange of Mass and Angular Momentum in the Impact Event of Ice Giant Planets: 
Implications for the Origin of Uranus

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Abstract

Uranus has a tilted rotation axis, which is supposed to have been caused by a giant impact. In general, an impact event also changes the internal compositional distribution and drives mass ejection from the planet, which may provide the origin of satellites. Previous studies of the impact simulation of Uranus investigated the resultant angular momentum and the ejected mass distribution. However, the effect of changing the initial condition of the thermal and compositional structure is not studied. In this paper, we perform hydrodynamics simulations for the impact events of Uranus-size ice giants composed of a water core surrounded by a hydrogen envelope using two variant methods of the smoothed particle hydrodynamics. We find that the higher-entropy target loses its envelope more efficiently than the low-entropy target. However, the higher-entropy target gains more angular momentum than the lower-entropy target since the higher-entropy target has a more expanded envelope. We discuss the efficiency of angular momentum transport and the amount of the ejected mass and find a simple analytical model to roughly reproduce the outcomes of numerical simulations. We suggest the range of possible initial conditions for the giant impact on proto-Uranus that reproduces the present rotation tilt of Uranus and sufficiently provides the total angular momentum of the satellite system that can be created from the fragments from the giant impact.

Key words: planets and satellites: formation – planets and satellites: gaseous planets – planets and satellites: individual (Uranus)

1. Introduction

Our solar system has two ice giants, Uranus and Neptune, that are supposed to be mainly composed of a gas envelope, icy mantle, and solid core from the exterior to interior (e.g., Hubbard & MacFarlane 1980; Helled et al. 2011; Nettelmann et al. 2016). These two planets have similar mass and radius, while their spin axes, satellite systems, and intrinsic luminosities are different. For example, the obliquity of Uranus is 98°, while that of Neptune is 27°. Safronov (1966) pointed out that Uranus experienced a giant impact event to reproduce its tilted rotation axis. An impactor of several earth mass may have transported angular momentum to proto-Uranus via collision and tilted the rotation axis of proto-Uranus. Parisi & Brunini (1997) estimated the giant impact based on the conservation of angular momentum and energy, and they concluded that the minimum impactor mass is \( \sim 1 \sim 1.1 M_{\oplus} \). Such a large impact event may also have produced a circumplanetary disk around proto-Uranus, which might have been the origin of the small prograde satellites around Uranus (e.g., Parisi et al. 2008). A giant impact scenario is widely accepted as an explanation for terrestrial moon formation (Hartmann & Davis 1975; Cameron & Ward 1976) and was recently suggested to apply to the formation of Phobos and Deimos (e.g., Citron et al. 2015; Hyodo et al. 2017b). Since the impact event caused erosion of proto-Uranus, the angular momentum that impactor brought was redistributed to proto-Uranus, and it eroded the gas envelope and fragments around proto-Uranus. The internal compositional distribution after the impact is unknown because the mixing process by the impact event is a highly nonlinear problem to analyze theoretically. Moreover, the impact event is also essential to thermal evolution. Presently, Uranus has a low intrinsic luminosity compared to Neptune. Nettelmann et al. (2016) explained the luminosity of Uranus considering a thermal boundary layer between an outer H–He-rich envelope and an inner ice-rich layer. If a giant impact event occurs, the internal compositional distribution should change. If the mixing of icy material in the H2/He envelope is efficient, then thermal evolution is expected to be accelerated (Kurosaki & Ikoma 2017).

Giant impact simulations on rock-composed protoplanets have been systematically calculated (Marcus et al. 2009; Genda et al. 2012). We note, though, that those studies have not considered atmospheres. The atmosphere is eroded more efficiently than rock materials because the thermal pressure of gas is more sensitive than that of rock materials. Impact events on Uranus have been investigated by use of hydrodynamical simulation. Korycansky et al. (1990) studied a giant impact on an ice giant for the first time. They calculated one-dimensional spherically symmetric hydrodynamic simulations and investigated the H2/He envelope erosion due to the impact. They found a sharp transition between the cases of nearly complete retention and dispersal of the H2/He envelope, which depend on the amount of energy deposition on the envelope. Three-dimensional hydrodynamical simulations were completed by Slattery et al. (1992) and Kegerreis et al. (2018). Slattery et al. (1992) used the smoothed particle hydrodynamic simulation (hereafter SPH simulation) to constrain the angular momentum of proto-Uranus and eroded mass after the giant impact.

Previous work constrained the impactor mass to explain the amount of the present angular momentum of Uranus. In the context of thermal evolution of Uranus, the evolutionary stage at the time of the giant impact is also important. When the age of proto-Uranus was younger than 10^9 yr, the H2/He envelope still remained extended, and the efficiency of the gas envelope erosion would have been increased. Thus, it should be useful to study the effect of changing the structure of proto-Uranus on the result of a giant impact to constrain the evolutionary stage of proto-Uranus at the time of the giant impact event.
In this paper, we study the impact event on proto-Uranus. The aim of this paper is to investigate the envelope erosion and the efficiency of angular momentum transport to proto-Uranus. Since there are no constraints on the age of Uranus at the time of the impact event, we consider two extreme cases: an impact onto a young ($10^8$ yr) ice giant, when it was in a high-temperature state, and an impact onto a mature ice giant ($10^9$ yr), when it was in a low-temperature state.

Section 2 describes the methods and settings for our simulation. The results of our study are described in Section 3, and the discussion is presented in Section 4. We summarize the conclusion of our study in Section 5.

2. Method

We use Godunov-type smoothed particle hydrodynamical calculation, hereafter GSPH. (Inutsuka 2002; Sugiura & Inutsuka 2016) to solve the following hydrodynamic equations:

$$\frac{dtP}{dt} = -\rho \nabla \cdot \mathbf{v} \quad (1)$$

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla P + \nabla \int d\mathbf{x}' \frac{G \rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \quad (2)$$

$$\frac{dt\mathbf{u}}{dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} \quad (3)$$

$$P = P(\rho, \mathbf{u}) \quad (4)$$

where $\rho$, $P$, $\mathbf{v}$, and $\mathbf{u}$ are density, pressure, velocity, and specific internal energy, respectively. $t$ is the time, $\mathbf{x}$ is the position, and $G(=6.67408 \times 10^{-8}$ cm$^3$ g$^{-1}$ s$^{-2}$) is the gravitational constant. This method has advantages for tracing strong shock and contact discontinuity (Cha et al. 2010). We introduce the exact and spatially second-order Riemann solver for piece-wise polytropic gas to our GSPH. As for the equation of state (Equation (4)), we use the values from Saumon et al. (1995) for hydrogen and helium, and those of SESAME 7150 for water (Lyon & Johnson 1992). To implement non-ideal equation of states in GSPH, the effective heat ratio $\gamma_{eff}$ is calculated from the data table of non-ideal equation of state. We also take into account the gravity force on Riemann solver (Guo et al. 2018, submitted). We have implemented the acceleration modules for our SPH code with FDPS (Iwasawa et al. 2016) and FDPS Fortran interface (Namekata et al. 2018). We have tested our SPH code by reproducing the analytical solution for a shock tube and the Lane–Emden solution for a polytrope gas equilibrium spheres.

2.1. Initial Conditions for Target and Impactor

In this paper, we fix two parameters. We assume the masses of the target and impactor are $13 M_\oplus$ and $1 M_\oplus$, respectively. We assume the target is composed of 20% of hydrogen and 80% of water, and that the impactor is composed of 100% of water. The impact angle, hereafter $\theta_{imp}$, is assumed to be 15°, 30°, 45°, and 60°. The impact position is $(R_p + r_{imp}) \sin \theta_{imp}$, where $R_p$ and $r_{imp}$ are the radii of target and impactor, respectively. The impact velocity is assumed to be the escape velocity, which is represented by $v_{imp} = \sqrt{2GM_p/R_p}$, where $M_p$ is the target mass, and $m_{imp}$ is the impactor mass. The impact velocities are $1.68 \times 10^4$ m s$^{-1}$ and $1.85 \times 10^4$ m s$^{-1}$ for the HT and LT target discussed below, respectively. These impact velocities correspond to freefall motion from infinity. The escape velocity of the target depends on the target’s radius. Since the HT target has a larger radius than the LT target, its escape velocity is lower. Here we assume that both the target and impactor are not spinning before the collision, as in Slattery et al. (1992) and Kegerreis et al. (2018). We also investigate the effect of changing the temperature structure of the target on the outcome of the giant impact. Here we assume two types of targets. The high-temperature target, hereafter HT, is assumed to have entropy $S = S(100\text{bar}, 1000\text{K})$, while the low-temperature target, hereafter LT, is assumed to have entropy $S = S(100\text{bar}, 500\text{K})$. Table 1 shows the parameters for the target and impactor. Both the HT and the LT target are warmer than present-day Uranus. The age of the target that Kegerreis et al. (2018) adopted is equivalent to present Uranus. If the rotation period of the target is longer than ~100 hr, the target’s angular momentum is much smaller than present-day Uranus by an order of magnitude, and the initial target’s spin can be ignored. If the target’s angular momentum is comparable to present-day Uranus, the rotating H–He atmosphere changes its impact mass number and hence the resulting internal angular momentum distribution. However, this is beyond the scope of this study. We assume that the target’s angular momentum is much smaller than that provided by the impactor for simplicity. Hereafter, we discuss the spin of the target resulting from the impact.

We stop the numerical simulation at $t = t_{ff}$, where $t_{ff}$ is the freefall time of the target given by $t_{ff} = \sqrt{\pi^2 R_p^3/(8GM_p)}$. $t_{ff}$ for HT and LT are 0.95 hr and 0.71 hr, respectively. Thus we stop HT and LT simulations at 9.5 hr and 7.1 hr, respectively. In this case, the number of timesteps in our simulations are only on the order of $10^2$ thanks to the efficiency of GSPH.

3. Results

In this section, we show results of our impact simulations. Figure 1 shows four snapshots of the impact simulation for the HT whose impact angle is 30°, which represents the initial conditions ($t = 0$), $t = t_{ff}$, $t = 2 t_{ff}$, and $t = 8 t_{ff}$. When the impactor collides with the target, the impactor transfers its

| Mass (g) | Radius (cm) | Temperature (K) | $H_2$ (wt%) | $H_2O$ (wt%) | Particles |
|---------|-------------|-----------------|-------------|--------------|------------|
| HT      | $7.82 \times 10^{28}$ | $3.18 \times 10^9$ | 1000 K (100 bar) | 20 | 80 | 65,500 |
| LT      | $7.82 \times 10^{28}$ | $3.03 \times 10^9$ | 500 K (100 bar) | 20 | 80 | 65,500 |
| Impactor| $5.97 \times 10^{27}$ | $1.13 \times 10^9$ | 300 K (1 bar) | 0 | 100 | 5000 |
momentum to the hydrogen envelope of the target. In the case of $\theta = 30^\circ$, the impactor collides with the H$_2$O core of the target. After the collision, the impactor falls onto the target’s core and the hydrogen envelope is eroded due to the collision.

Here we introduce the definition for the eroded particle from the target at $t = 10t_{ff}$ after the collision. The mass, position, velocity, and internal energy of $i$th particle are $m_i$, $r_i$, $v_i$, and $u_i$, respectively. After the impact, the hydrogen gas expands around the target. Eroded gas particles have enough energy to escape from the target and their positions are outside of the target. In this study, the eroded particle condition is

$$\frac{1}{2}m_i|v_i|^2 - m_i \sum_{i \neq j} \frac{Gm_j}{|r_i - r_j|} > 0$$

$$|r_i - R_{t,c}| > R_p$$

where $R_{t,c}$ is the position of the center of the target.

Figures 2 and 3 show the eroded region after the collision for the high-temperature target and low-temperature target, respectively. The eroded regions for hydrogen and water are shown in orange and yellow, respectively. Those figures show the $z$-plane cross sections of the results by choosing the particles in the range of $z = [-0.1, 0.1]$. We find that the property of the mass erosion changes depending on whether the impactor collides with the target core or not. When the impactor collides with the core, the impactor changes its trajectory, and the hydrogen envelope is eroded along the impactor’s trajectory. In the cases where the impact angles are 15$^\circ$ and 30$^\circ$, the impactor collides with the water core of the target. On the other hand, the impactor does not collide with the water core in the cases where the impact angles are 45$^\circ$ and 60$^\circ$. After the collision, the impactor is captured in the target while part of the hydrogen envelope of the target receives energy and angular momentum from the impactor and escapes from the target.

Figure 5 shows the relationship between the eroded mass and impact parameter for the HT and LT cases. In the HT case, the eroded mass is larger than LT case. That is because the volume of the hydrogen envelope of the HT case is larger than that of the LT case, and the escaped region of the HT case is also larger than for the LT case. Moreover, the escape velocity at the surface of the HT is smaller than that of the LT, which

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Figure 1. Snapshots of the high-temperature target’s impact simulation, whose impact angle is $\theta = 30^\circ$. Green, purple, and blue dots represent the target’s hydrogen particles, target’s water particles, and impactor’s water particles, respectively. Those snapshots represent initial conditions ($t = 0$), $t = t_{ff}$, $t = 2t_{ff}$, and $t = 8t_{ff}$ from top-left, top-right, bottom-left, and bottom-right, respectively, where $t_{ff}$ is the freefall time.
promotes the ejection of particles from the target. After the collision, the minimum masses of H–He retained for the HT and LT cases are 80% and 90%, respectively, while Kegerreis et al. (2018) obtain 75%. This difference is due to the equation of states for hydrogen–helium. The adopted equations of states for hydrogen–helium (Saumon et al. 1995) in our calculation have a smaller heat capacity ratio compared to that of Hubbard & Macfarlane (1980). That is, in our calculation, the pressure response against the density is softer than in previous studies. We think this is the reason for the difference between our results and that of Kegerreis et al. (2018). The eroded mass of ice for a small impact angle is smaller than that of hydrogen by an order of magnitude. In the case of a low impact angle, both the HT and LT cases lose little water from the core and impactor. However, the eroded water mass is more than the total mass of the Uranian satellites. In the case of a high impact angle, the impactor escapes from the target and the mass loss of water increases.

4. Discussion

The eroded mass due to the collision can be understood by considering the heating area. Here we estimate the volume of the ejected mass in terms of the collision-induced erosion volume $V_{\text{col}}$ and the shock-induced erosion volume, $V_{\text{shk}}$, as

$$V_{\text{ej}} = V_{\text{col}} + V_{\text{shk}}$$

(7)

where

$$V_{\text{col}} = \sigma(R_p - R_c)\cos \theta$$

(8)

$$V_{\text{shk}} = \left[\frac{\pi}{6} h_1^2 (3R_p - h_1) - \frac{\pi}{6} h_2^2 (3R_c - h_2)\right]$$

(9)

$$-\left[\frac{\pi}{6} h_3^2 (3R_p - h_3) - \frac{2\pi R_p^3}{3} - \frac{4\pi}{3} (R_p^3 - R_c^3) \frac{\varphi}{2\pi}\right]$$

(10)

where $h_1 = R_p + R_c \cos \varphi$, $h_2 = R_c + R_e \cos \varphi$, $h_3 = R_p + R_e \varphi = \sin^{-1}[(R_p + r_{\text{imp}})\sin \theta/R_c]$, $R_c$ is the core radius of the...
target, and $r_{\text{imp}}$ is the radius of the impactor (see Figure 4). Thus, we can estimate the escaped mass as

$$M_{\text{esc,t}} = \overline{\rho_{\text{env}}} V_{\text{area}}$$

(11)

where $\overline{\rho_{\text{env}}}$ is the averaged density. Since there is a strong negative gradient of density in the hydrogen envelope, we adopt the interior density of the hydrogen envelope as the characteristic density. The density of hydrogen envelope $\approx 0.1 \, \text{g cm}^{-3}$ at the core-envelope boundary for the HT and LT. Here we assume $\overline{\rho_{\text{env}}} = 0.1 \, \text{g cm}^{-3}$. When the impact parameter is large, a part of the impactor does not collide with the envelope of the target, and then it does not fall onto the target. The impactor’s loss $m_{\text{esc,imp}}$ can be estimated as

$$m_{\text{esc,imp}} = \rho_{\text{imp}} \pi r_{\text{imp}}^3 \left[ \frac{2}{3} + \frac{(R_p + r_{\text{imp}}) \sin \theta - R_p}{r_{\text{imp}}} \right]$$

$$- \frac{1}{3} \left( \frac{(R_p + r_{\text{imp}}) \sin \theta - R_p}{r_{\text{imp}}} \right)^3$$

(12)

for $\sin \theta \geq \frac{R_p - r_{\text{imp}}}{R_p + r_{\text{imp}}}$. If $\sin \theta < \frac{R_p - r_{\text{imp}}}{R_p + r_{\text{imp}}}$, the entire impactor is expected to fall onto the target. That is, we set $m_{\text{esc,imp}} = 0$. Figure 5 also shows the analytical result. The analytical model can reproduce the trend of the relationship between the eroded mass and the impact parameter. However, the analytical model does not include the effect of the escape velocity, which should determine the difference between the HT and LT cases, quantitatively. We suggest that it will be important to determine the propagation of three-dimensional shock waves in the interior of the HT and LT cases.

The total angular momentum that is transferred from the impactor to the target via collision is $L_{\text{col}} = m_{\text{imp}} v_{\text{imp}} (R_p + r_{\text{imp}}) \sin \theta)$. After the collision, the eroded mass removes some fraction of the angular momentum that was given by the impactor. The angular momentum that is transferred thus

$$L_p = L_{\text{col}} - m_{\text{esc,t} \vee} R_e - m_{\text{esc,imp}} v_{\text{imp}}$$

$$\times (R_p + r_{\text{imp}}) \sin \theta$$

(13)
where \( L_i \) is transferred angular momentum to the target by the impactor, and \( v_{ej} \) is the ejecta velocity. Here we estimate \( v_{ej} \) by the impact ejecta scaling law (Melosh 1989; Richardson et al. 2005)

\[
v_{ej} = \frac{2 \sqrt{R_p g_s}}{1 + \varepsilon}
\]

(14)

where \( g_s \) is the surface gravity, and \( \varepsilon \) is a material constant. When all of the impactor particles exist in \( \theta > 0 \), all particles move counterclockwise after the impact, which means \( L_+ = L_{\text{tot}} \). When \((R_p + r_{\text{imp}}) \sin \theta < r_{\text{imp}}\) is satisfied, some particles whose positions are \( \theta < 0 \) impact and reduce \( L_+ \). Here we divide the impactor to the \( \theta > 0 \) and \( \theta < 0 \) regions. \( V_1, V_2 \) are the volumes of the \( \theta > 0 \) and \( \theta < 0 \) regions, respectively, and \( h_1, h_2 \) are the distances from the \( x-z \)-plane. Thus,

\[
L_+ = \rho_{\text{imp}} V_{\text{imp}} [V_1 (r_{\text{imp}} + (R_p + r_{\text{imp}}) \sin \theta - h_1) - V_2 (r - (R_p + r_{\text{imp}}) \sin \theta - h_2)].
\]

(15)

Figure 6 shows the relationship between the obtained angular momentum and impact parameter. It shows that the angular momentum gain of the target is related to the eroded mass. The trend of the obtained angular momentum can be understood by Equation (13). Moreover, the ejecta velocity is also understood by the ejecta scaling law (Equation (14)). The right panel of Figure 6 shows the relationship for the obtained angular momentum normalized by the present-day angular momentum of Uranus (Podolak & Reynolds 1987) and impact parameter. Our result suggests that the HT case explains the present-day angular momentum of Uranus even if the impactor’s mass is \( 1M_\oplus \). If the target is in the high-entropy state, the hydrogen envelope is expanded. Then the cross-section of proto-Uranus is enlarged and the angular momentum transported by the impactor is larger than in the LT case. We assumed that the initial hydrogen envelope is \( 20\% \), which is larger than for present-day Uranus. Venturini et al. (2016) implied that low- and intermediate-mass planets (mini-Neptunes to Neptunes) can be formed with total mass fractions of hydrogen up to \( 30\% \) considering the envelope polluted by ice materials. Thus, proto-Uranus might have had a more massive envelope than present-day Uranus because the target can lose envelope gas as a result of a giant impact.

### 4.1. Angular Momentum Transfer

Hyodo et al. (2017a) showed that the tidal disruption of a passing Kuiper Belt object is able to explain the formation of the current ring and inner regular satellites of Uranus. On the other hand, previous studies (Slattery et al. 1992; Kegerreis et al. 2018) and our simulation suggest that a giant impact is also able to supply material to form regular satellites of Uranus. The total mass of major regular satellites around Uranus (Miranda, Ariel, Umbriel, Titania, and Oberon) is \( \sim 8.8 \times 10^{24} \) g (Brown et al. 1991), which is equivalent to \( 10^{-4} \) by Uranus’s mass. Moreover, their total angular momentum is \( \sim 10^{-2} \times L_U \). In this section, we consider the condition for particles that are bounded gravitationally and do not fall onto the target after the impact. Those particles are supposed to be the origin of the circumplanetary disk or ring.

Here we propose the criterion that determines the ejected particles will reaccrete or not by using the particle’s orbital element. That is, they will not reaccrete if the pericenter distance of the \( i \)th particle is longer than the target radius;

\[
a_i (1 - e_i) > R_p
\]

(16)

where \( a_i \) and \( e_i \) are the semimajor axis and eccentricity of the \( i \)th particle, respectively. \( a_i \) and \( e_i \) are calculated by

\[
a_i = \left( \frac{2}{|x_i - x_{\text{e}}|^2} - \frac{|v_i - v_{\text{e}}|^2}{G (M_p + m_i)} \right)^{-1}
\]

(17)

\[
e_i = \sqrt{1 - \frac{|(x_i - x_{\text{e}}) \times (v_i - v_{\text{e}})|^2}{G (M_p + m_i) a_i}}
\]

(18)

where \( x_{\text{e}} \) and \( v_{\text{e}} \) are the position and velocity of the center of the gravity, respectively (see Murray & Dermott 1999). Figure 7 shows the particles that satisfy the condition of Equation (16). Our
Figure 6. This figure shows the relationship between the obtained angular momentum and impact positions. The purple and green lines represent the high-temperature target and low-temperature targets, respectively. The blue and orange lines represent the analytical solutions for the high-temperature target and low-temperature target derived by Equation (13). The left panel shows the efficiency of the transported angular momentum by a giant impact. The right panel shows the obtained angular momentum normalized by the present-day angular momentum of Uranus (Podolak & Reynolds 1987).

Figure 7. Particles that become the circumplanetary disk are shown with different colors in the initial condition. Only the particles close to the midplane (−0.1 < z < 0.1) are shown. The color code is the same as Figures 2 and 3 except that in addition, the red and black dots show hydrogen and water particles that satisfy Equation (16), respectively. In our simulation, most of the hydrogen particles that satisfy Equation (16) also satisfy Equation (5). That is, those particles should have eroded, and they are shown by orange dots. Thus, the number of red particles is very small in this representation. The left-top and right-top panels show the result of the HT target whose impact angles are 45° (left-top) and 60° (right-top), respectively. The left-bottom and right-bottom panels show the result of the LT target whose impact angles are 45° (left-bottom) and 60° (right-bottom), respectively.
result shows that the particles satisfying Equation (16) and being bounded gravitationally exist in the cases of $\theta = 45^\circ, 60^\circ$. Such particles mainly come from the impactor, and their composition is ice. Previous studies (Slattery et al. 1992; Kegerreis et al. 2018) also demonstrated the same conclusion.

4.2. Implication for the Satellite Formation

In this study, the impact velocity is fixed to the escape velocity of the target, which means that all particles are bounded in the system gravitationally. The hydrogen envelope is eroded due to the momentum exchange with impactor. After that, the hydrogen envelope is scattered by the impact. Particles gravitationally unbound will escape, while the others will accrete onto the target.

On the other hand, some fraction of the impactor material and a small amount of material blown out from the target do not accrete onto the target and remain in orbit around the target. During the impact event, hydrodynamical and tidal forces stretch the material, and then angular momenta are exchanged by self-gravitational and hydrodynamical forces. Figure 8 shows the time derivative of the particle’s angular momentum. Particles in the direction of motion transfer their angular momentum to particles behind them, and then the orbital distances of the latter increase. If $dL_z/dt$ are positive, particles should move outward because they gain angular momentum, while particles whose $dL_z/dt$ are negative fall onto the target. Consequently, materials from the impactor are left around the target.

Figure 8. The transport of angular momentum via self-gravity. This snapshot is taken 3 $t_i$ after the collision for the case of the impact angle of $60^\circ$ on the HT target. The color shows the time derivative of the angular momentum due to the gravitational force. Red and blue plus symbols show $dL_z/dt > 0$ and $dL_z/dt < 0$, respectively.

Figure 9 shows the particle mass that satisfies Equation (16). Our simulation shows that an impact event can supply a
sufficient amount of ices for the formation of the regular satellites. In the present simulation, we have adopted the impactor composed only of water, just for simplicity. In reality, the impactor should contain more or less rocky materials although the impactor’s ice-to-rock ratio is unknown. We expect that rocky material should also be ejected depending on the impact parameters, if we perform a simulation with the impactor that contains rocky material. However, the determination of the ejected material in the case of a rocky impactor is beyond the scope of the first paper in this line of our work. Comparing to Kegerreis et al. (2018), the mass in orbit according to our result is a factor of two lower. Our result also suggests that an LT target leads to a larger amount of material in orbit than does an HT target (Figure 9). Since the entropies of our targets are higher than assumed in Kegerreis et al. (2018), our results are consistent with that work.

5. Conclusion

In this paper, we have performed numerical simulations of a giant impact on a young Uranus-like ice giant using the Godunov SPH simulation with realistic structures composed of ice and hydrogen–helium gas in the case of no initial rotation of the target. We find that there is a relationship between the resultant mass loss and the angular momentum of the target. Our results suggest that a giant impact on proto-Uranus can explain the present value of the angular momenta of Uranus and its satellite system. We also find that if the target is in a high-entropy state, it obtains larger angular momentum and lose its envelope more efficiently than the low-entropy state, because the hydrogen envelope of the high-entropy target is significantly more extended. Our results also show that less mass remains gravitationally bound in the high-entropy target than in the low-entropy target. Our results may provide a step forward to understanding the origin of Uranus.

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Appendix A

Riemann Solver for Godunov SPH with Self-gravity

In this section, we introduce the Riemann solver for our GSPH (see also Guo et al. 2018). Here we consider the Riemann solver for $i$ and $j$ particles. The expression for the result of Riemann problem we use is

$$p^* = \frac{p_i / W_i + p_j / W_j - v_i + v_j}{1 / W_i + 1 / W_j}$$

(19)

$$v^* = \frac{v_i W_i + v_j W_j - p_i + p_j}{W_i + W_j}$$

(20)

where $p, v$ are pressure and velocity, respectively. $W_k$ abbreviates

$$W_k = \sqrt{p_k / p_k} \sqrt{1 + \frac{\gamma + 1}{2} \frac{p^* - p_k}{p_k}} \text{ for } p^* \geq p_k$$

(21)

$$W_k = \sqrt{p_k / p_k} \sqrt{1 + \frac{\gamma - 1}{2} \frac{1 - p^* / p_k}{\gamma - 1}} \text{ for } p^* < p_k$$

(22)

where $\gamma, \rho$ are the ratio of specific heats and density, respectively. To take into account the gravitational force $F_i$, on Equations (19) and (20), we replace $p_i$ and $p_j$ with $p_i'$ and $p_j'$ as shown in the following:

$$p_i' = p_i - \frac{1}{2} \rho_i C_s F_k \cdot \hat{s} \delta t$$

(23)

$$p_j' = p_j + \frac{1}{2} \rho_j C_s F_k \cdot \hat{s} \delta t$$

(24)

where $C_{s,k} = \sqrt{\gamma_k p_k / \rho_k}$, $\hat{s} = (x_i - x_j) / |x_i - x_j|$ and $\delta t$ is the time step.

Appendix B

Thermal Evolution of the Target

In this section, we briefly explain the thermal evolution of the target. The target is composed of 80% water core surrounded by 20% of hydrogen–helium atmosphere. We numerically integrate the thermal evolution. We assume the target consists of three layers in spherical symmetry and hydrostatic equilibrium, that include, from exterior to interior (1) a radiative–convective equilibrium atmosphere composed of hydrogen–helium, (2) a convective equilibrium envelope composed of hydrogen–helium, and (3) a convective equilibrium water–ice core. Details of the atmosphere, interior, and thermal evolution model are described in Kurosaki & Ikoma (2017). Figure 10 shows the thermal evolution of the target. The purple line shows an ice giant whose hydrogen–helium atmosphere is free from ice materials, while the green line shows the case where the atmosphere is mixed with ice materials 50% by mass. The temperature at 1 bar of present-day Uranus is ~80 K. Temperatures at 1 bar for the HT and LT targets are 270 K and 120 K, respectively. HT and LT correspond to $1.6 \times 10^8$ yr and $2.3 \times 10^8$ yr, respectively. Previous studies (Fortney et al. 2011; Nettelmann et al. 2016) indicated that Uranus could prove a strong barrier to interior convective cooling. Kurosaki & Ikoma (2017) suggested that
an ice-rich atmosphere can accelerate the cooling due to the effect of the condensation in the atmosphere, though it requires 50% of ice by mass. Our target assumes an ice giant with an ice-free atmosphere. In our model, the target in Kegerreis et al. (2018) may correspond to an evolved ice giant with an ice-rich atmosphere. Therefore, we are proposing that if the HT target’s interior is mixed with ice efficiently after the impact, subsequent thermal evolution of the HT can reproduce present-day Uranus. However, the detailed determination of the amount of mixing by the impact is beyond the scope of this study.

Appendix C
The Determination of Unbound Particles: The Effect of the Internal Energy

We check the effect of the internal energy on the determination of unbound particles. The condition that the particle’s velocity exceeds the escape velocity (hereafter CRT1) is Equation (5). On the other hand, the particle’s internal energy increases after the impact event. The larger internal energy causes a larger pressure on the particle. Thus, the pressure gradient may accelerate the particle and contribute to its escape. We include the internal energy into the condition in Equation (5):

\[ m_i u_i + \frac{1}{2} m_i |v_i|^2 - m_i \sum_{i \neq j} \frac{G m_j}{|r_i - r_j|} > 0, \]  

and hereafter we call this condition, CRT2. Figure 11 shows the relationship between the eroded mass and the impact parameter. We find that the eroded mass of CRT2 is larger than that of CRT1. When the impact occurs, the particle’s internal energy is increased because the impact event distributes the kinetic energy to the internal energy of the particles. After the impact, the hydrodynamic motion redistributes the particle’s internal energy to kinetic energy (see Genda et al. 2015). That is, the pressure gradient caused by the internal energy accelerates the particles. However, the difference of the eroded mass between CRT1 and CRT2 is less than 5%. Therefore, we adopt CRT1 as the condition of unbound particles.

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Figure 11. Relationship between the eroded mass and the impact parameter. Purple and green symbols represent the high-temperature target and low-temperature target, respectively, using criterion CRT1 (squares, circles) or CRT2 (crosses, pluses).