Renormalizable theory of massive spin two particle and new bigravity

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In this paper, we propose a renormalizable theory describing massive spin two particle. The coupling of the theory with gravity can be regarded as a new kind of bimetric gravity or bigravity. We show that the field of the massive spin two particle plays the role of the cosmological constant.

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I. INTRODUCTION

After the establishment of the free theory of massive gravity by Fierz and Pauli [1] (for a recent review, see [2]), any consistent interacting theory has not been found during three fourth centuries. One of the reasons is that there appears the Boulware-Deser ghost [3, 4] in general and another is the appearance of the van Dam-Veltman-Zakharov (vDVZ) discontinuity [5] in the massless limit, \(m \to 0\) although the discontinuity can be screened by the Vainstein mechanism [6] (see, for example, Ref. [7]).

After the elapse of seventy five years from the work by Fierz and Pauli [1], there have been remarkable progress in the study of the non-linear massive gravity and the ghost-free models without ghost, which are called the de Rham, Gabadadze, Tolley (dRGT) models, have been found in [8–10]. The models have non-dynamical background metric but the models have been extended to the models with dynamical metric [11–13], which are called as bigravity models. After that cosmology has been studied in the massive gravity models [14–17] and in the bimetric gravity models [18–25].

The absence of ghost was shown in the Hamiltonian analysis [13] by using the Arnowitt-Deser-Misner (ADM) formalism, where the metric is assumed to be

\[
\begin{align*}
g_{00} = -N^2 + g^{ij} N_i N_j, \
g_{0i} = N_i, \
g_{ij} = g_{ij}. 
\end{align*}
\]

(1)

Here \(i,j = 1, 2, 3\) and \(N\) is called as the lapse function and \(N_i\) as the shift function. We denote the inverse of \(g_{ij}\) by \(g^{ij}\). In the dRGT models, after the redefinition of the shift function \(N_i\), the Hamiltonian becomes linear to the lapse function \(N\) and in the expression of the new shift function given by solving the equation obtained from the variation of the new shift function, the lapse function \(N\) does not appear. Therefore the variation of \(N\) give a constraint on \(g_{ij}\) and their conjugate momenta. By combining this constraint with the secondary constraint derived from the constraint, an extra degree of freedom corresponding to the ghost is eliminated.

Recently in [34], it has been proposed possibilities of new non-linear ghost-free derivative interactions in massive gravity. After that, however, in [37], it has shown that a class of the derivative interactions includes ghost and a kind of no-go theorem prohibiting the derivative interactions has been claimed. In this paper, we show the existence of the non-linear derivative interactions which are not included in [37] although such derivative interactions generate ghost, unfortunately.

In this paper, we also propose a renormalizable model describing massive spin two particle. The coupling of the renormalizable model with gravity can be regarded as a new kind of bigravity. The effective cosmological constant can be generated by the field of the massive spin two particle, which may describe the inflation.

II. STILL NEW DERIVATIVE INTERACTION IN MASSIVE GRAVITY?

In [34], by using the perturbation \(h_{\mu\nu}\) from the flat metric

\[
h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu},
\]

(2)
as a dynamical variable, new ghost free interactions were proposed. The interaction terms have the following form:

\[
\mathcal{L}_{d,0} \sim \eta^{\mu_1 \nu_1 \cdots \mu_n \nu_n} h_{\mu_1 \nu_1} \cdots h_{\mu_n \nu_n},
\]

(3)
or terms including \(d\)-derivative, which is called pseudo linear terms (see also [35]),

\[
\mathcal{L}_{d,n} \sim \eta^{\mu_1 \nu_1 \cdots \mu_n \nu_n} \partial_{\mu_1} \partial_{\nu_1} h_{\mu_2 \nu_2} \cdots \partial_{\mu_{d-1}} \partial_{\nu_{d-1}} h_{\mu_d \nu_d} h_{\mu_{d+1} \nu_{d+1}} \cdots h_{\mu_{n+d/2} \nu_{n+d/2}}.
\]

(4)
Here $\gamma^{\mu_1\nu_1} \cdots \mu_n\nu_n$ is given by the product of $n$ $\eta_{\mu\nu}$ and anti-symmetrizing the indexes $\nu_1, \nu_2, \cdots, \nu_n$, for examples
\[
\eta^{\mu_1\nu_1} \mu_2\nu_2 = \eta^{\mu_1\nu_2} \eta^{\nu_1\nu_2} - \eta^{\mu_1\nu_2} \eta^{\nu_1\nu_2},
\]
\[
\eta^{\mu_1\nu_1} \mu_2\nu_3\nu_5 \equiv \eta^{\mu_1\nu_1} \mu_2\nu_3\nu_5 \eta^{\nu_2\nu_3} \eta^{\nu_4\nu_5} + \eta^{\mu_1\nu_1} \mu_2\nu_3\nu_5 \eta^{\nu_4\nu_5} \eta^{\nu_2\nu_3},
\]
\[
- \eta^{\mu_1\nu_1} \mu_2\nu_3\nu_5 \eta^{\nu_2\nu_3} \eta^{\nu_4\nu_5} - \eta^{\mu_1\nu_1} \mu_2\nu_3\nu_5 \eta^{\nu_2\nu_3} \eta^{\nu_4\nu_5}.
\]
(5)

It is evident that these terms are linear with respect to $h_{00}$, which could be a perturbation of the lapse function $N$ in the Hamiltonian and there do not appear the terms which include both of $h_{00}$ and $h_{ii}$. Therefore the variation of $h_{ii}$ gives a constraint for $h_{ij}$ and their conjugate momenta $\pi_{ij}$ and eliminate the ghost.

The non-linear counterparts for (8) is nothing but the mass terms and the interaction terms in the dRGT models,
\[
\eta^{\mu_1\nu_1} \cdots \mu_n\nu_n \frac{\partial}{\partial t} \frac{\partial}{\partial t} h_{\mu_1\nu_1} \cdots h_{\mu_n\nu_n} \approx \sqrt{-g} \gamma^{\mu_1\nu_1} \cdots \mu_n\nu_n \mathcal{K}_{\mu_1\nu_1} \cdots \mathcal{K}_{\mu_n\nu_n}.
\]
(6)

Here $\mathcal{K}_{\mu\nu}$ is defined by
\[
\mathcal{K}_{\mu\nu} \equiv \delta_{\mu\nu} - \sqrt{g^{-1}} f_{\mu\nu},
\]
(7)
and $f_{\mu\nu}$ is the fiducial metric and often chosen to be $f_{\mu\nu} = \eta_{\mu\nu}$.

In $D = 4$ space-time dimensions, a possible non-trivial term with 2-derivative is given by
\[
\mathcal{L}_{2.2} \approx \eta^{\mu_1\nu_1} \mu_2\nu_2 \nu_3 \nu_3 \frac{\partial}{\partial t} h_{\mu_1\nu_1} h_{\mu_2\nu_2},
\]
(8)
and
\[
\mathcal{L}_{2.3} \approx \eta^{\mu_1\nu_1} \mu_2\nu_2 \nu_3 \nu_3 \frac{\partial}{\partial t} h_{\mu_2\nu_2} h_{\mu_3\nu_3}.
\]
(9)

The non-linear counterpart of (8) could be nothing but the Einstein-Hilbert term. In case of the massive gravity, there is another candidate of the non-linear counterpart for (8) [33], which is
\[
\sqrt{-g} \gamma^{\mu_1\nu_1} \mu_2\nu_2 \nu_3 \nu_3 \mathcal{R}_{\mu_1\mu_2\nu_1\nu_2} \mathcal{K}_{\mu_3\nu_3}.
\]
(10)

The non-trivial, fully non-linear counterpart of (9) could be also given by
\[
\sqrt{-g} \gamma^{\mu_1\nu_1} \mu_2\nu_2 \nu_3 \nu_3 \mathcal{R}_{\mu_1\mu_2\nu_1\nu_2} \mathcal{K}_{\mu_3\nu_3} \mathcal{K}_{\mu_4\nu_4}.
\]
(11)

Here $\gamma^{\mu_1\nu_1} \cdots \mu_n\nu_n$ is, as in the definition of $\eta^{\mu_1\nu_1} \cdots \mu_n\nu_n$, given by the product of $n$ $g_{\mu\nu}$ and anti-symmetrizing the indexes $\nu_1, \nu_2, \cdots, \nu_n$.

In [37], however, it has been shown that the non-linear terms (10) and (11) generate the ghost by using the mini-superspace where
\[
N = N(t), \quad \dot{N} = 0, \quad g_{ij} = a(t)^2 \eta_{ij}.
\]
(12)

In fact, in the mini superspace [12], the terms (10) and (11) have the following form [37]:
\[
\sqrt{-g} \gamma^{\mu_1\nu_1} \mu_2\nu_2 \nu_3 \nu_3 \mathcal{R}_{\mu_1\mu_2\nu_1\nu_2} \mathcal{K}_{\mu_3\nu_3} \sim N a^3 \left[ \frac{\dot{a}^2}{a^2 N^2} - \frac{a^2}{a^3 N^2} + \frac{\dot{a}^2}{a^2 N^3} \right] \mathcal{K}_{\mu_3\nu_3}.
\]
(13)
\[
\sqrt{-g} \gamma^{\mu_1\nu_1} \mu_2\nu_2 \nu_3 \nu_3 \nu_4 \nu_4 \mathcal{R}_{\mu_1\mu_2\nu_1\nu_2} \mathcal{K}_{\mu_3\nu_3} \mathcal{K}_{\mu_4\nu_4} \sim N a^3 \left[ \frac{\dot{a}^2}{a^2 N^3} - \frac{\dot{a}^2}{a^3 N^2} + \frac{\dot{a}^2}{a^3 N^3} \right] \mathcal{K}_{\mu_3\nu_3} \mathcal{K}_{\mu_4\nu_4}.
\]
(14)

The expressions (13) and (14) tell that in the Hamiltonian, the terms (10) and (11) generate the terms which are not linear with respect to the lapse function $N$. Therefore the equation given by the variation of $N$ can be solved with respect to $N$ and does not give any constraint on $\dot{g}_{ij}$ or their conjugate momenta, which tells that the ghost cannot be eliminated.

We should note that the terms (10) and (11) are not unique terms reproducing (8) and (9), respectively. Another candidate reproducing (8) is
\[
\sqrt{-g} \gamma^{\mu_1\nu_1} \mu_2\nu_2 \nu_3 \nu_3 \left( \nabla_{\mu_1} \nabla_{\nu_1} \mathcal{K}_{\mu_2\nu_2} \right) \mathcal{K}_{\mu_3\nu_3},
\]
(15)
and a candidate for (9) is
\[
\sqrt{-g} \gamma^{\mu_1\nu_1} \mu_2\nu_2 \nu_3 \nu_3 \nu_4 \nu_4 \left( \nabla_{\mu_1} \nabla_{\nu_1} \mathcal{K}_{\mu_2\nu_2} \right) \mathcal{K}_{\mu_3\nu_3} \mathcal{K}_{\mu_4\nu_4}.
\]
(16)
In the mini superspace \([12]\), these terms can be expressed as

\[
\sqrt{-g} g^{\mu_1 \nu_1 \mu_2 \nu_2 \mu_3 \nu_3} (\nabla_{\mu_1} \nabla_{\nu_1} K_{\mu_2 \nu_2}) K_{\mu_3 \nu_3} \sim N a^3 \left[ \frac{a^2}{a^3 N^4} \right],
\]

(17)

\[
\sqrt{-g} g^{\mu_1 \nu_1 \mu_2 \nu_2 \mu_3 \nu_3 \mu_4 \nu_4} (\nabla_{\mu_1} \nabla_{\nu_1} K_{\mu_2 \nu_2}) K_{\mu_3 \nu_3} K_{\mu_4 \nu_4} \sim N a^3 \left[ \frac{a^2}{a^3 N^4} - \frac{\dot{a}^2}{a^2 N^4} \right],
\]

(18)

From the above expressions \([17]\) and \([18]\), however, we find that these terms \([15]\) and \([16]\) also generate the ghost. The ghost cannot be eliminated even if we consider the combinations in \([14]\), \([14]\), \([15]\), \([17]\), and \([18]\).

We should note that there is another candidate to reproduce \([8]\):

\[
\sqrt{-g} g^{\rho \nu \rho \sigma \sigma'} f^{\nu' \nu''} \nabla_\nu K_{\rho \rho'} \nabla_{\nu'} K_{\sigma \sigma'}. \tag{19}
\]

Here \(f^{\mu \nu} = \eta^{\mu \nu}\). In the mini superspace, however, this term has the following form:

\[
\sqrt{-g} g^{\rho \nu \rho \sigma \sigma'} f^{\nu' \nu''} \nabla_\nu K_{\rho \rho'} \nabla_{\nu'} K_{\sigma \sigma'} \sim N a^3 \left[ \frac{-6 \dot{a}^2}{a^3 N^4} + \frac{6 \dot{a}^2}{a^3 N^4} - \frac{6 \dot{a}^2}{N^4} \right],
\]

(20)

and therefore the ghost is not eliminated even if we consider any combination with other terms.

Then we consider the possibility of other classes of the no-ghost interactions by relaxing the assumption in \([37]\). In the argument so far, we have considered the terms which have invariance under the general coordinate transformation if the fiducial metric \(f_{\mu \nu}\) could be a dynamical tensor. We may relax this condition and require only the Lorentz invariance. Then we may consider the term given by replacing the covariant derivatives \(\nabla_\mu\) in \([18]\) by the partial derivative \(\partial_\mu\):

\[
\sqrt{-g} g^{\rho \nu \rho \sigma \sigma'} f^{\nu' \nu''} \partial_\nu K_{\rho \rho'} \partial_{\nu'} K_{\sigma \sigma'}. \tag{21}
\]

In the mini superspace \([12]\), this term is surely linear with respect to \(N\). Then we now check if the term \([21]\) could give interactions without ghost by using the full ADM formalism. Explicitly the term \([21]\) has the following form:

\[
\sqrt{-g} \delta_{\mu_1}^{[\nu_1} \delta_{\mu_2}^{\nu_2} \delta_{\mu_3}^{\nu_3}] g^{\mu_1 \rho \mu_2 \rho \mu_3 \rho} \partial_{\nu_1} K_{\mu_2}^{\nu_2} \partial_{\nu_3} K_{\sigma \sigma'}. \tag{22}
\]

In order that ghost could not appear, the term should be given in the form where the time-derivative of the lapse and shift functions do not appear. This kind of form might be obtained by the cancellations between several terms after the partial integration. Because this kind of the cancellation should occur between the terms including the same number of the time derivatives, we now consider the following terms:

\[
\sqrt{-g} \left[ - \left( \partial_0 \sqrt{g^{-1} \eta} \right)^2 + 2 \left( \partial_i \sqrt{g^{-1} \eta} \right) \left( \partial_j \sqrt{g^{-1} \eta} \right) \right] \tag{23}
\]

As in \([10]\), for convenience, we use the redefined shift function \(n^i\), which is given by

\[
N^i = (\delta^i_j + N D^i_j) n^j. \tag{24}
\]
The definition of $D^i_j$ is given by solving the following equation:

\[
(\sqrt{1 - n^i n} I_n) D = \sqrt{(\gamma^{-1} - D n n^T D^T)} I, \quad I = \delta_{ij}, \quad I^{-1} = \delta^{ij}.
\]  

(25)

By using $n^i$, we rewrite $\sqrt{g^{-1} \eta^i_{\nu}}$ as follows,

\[
\sqrt{g^{-1} \eta} = \frac{1}{N} A + B.
\]  

(26)

Here

\[
A = \frac{1}{\sqrt{1 - n^i n}} \begin{pmatrix} 1 & n^T I \\ -n & -n n^T I \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{(\gamma^{-1} - D n n^T D^T)} I \end{pmatrix}.
\]  

(27)

In order to simplify the notation, we define the following quantities:

\[
A := \frac{1}{\sqrt{1 - n^i n}} \quad B^i := n^i, \quad C^i_j := \sqrt{(\gamma^{-1} - D n n^T D^T)} I.
\]  

(28)

By using (28), $\sqrt{g^{-1} \eta^i_{\nu}}$ can be rewritten as

\[
\sqrt{g^{-1} \eta^i_{\nu}} = \begin{pmatrix} A/N & AB^i_0 n \\ -AB^i/N & -B^i B^k \delta_{kj} / N + C^i_j \end{pmatrix}.
\]  

(29)

and $\partial_0 \sqrt{g^{-1} \eta^i_{\nu}}$ can be expressed as follows,

\[
\partial_0 \sqrt{g^{-1} \eta} = \frac{B^i B^k \delta_{kj} \dot{N}}{N^2} - \frac{\ddot{B}^i B^k \delta_{kj}}{N} - \frac{B^i \ddot{B}^k \delta_{kj}}{N} + \dot{C}^i_j.
\]  

(30)

Therefore by using ADM variables, Eq. (28) has the following form:

\[
\sqrt{-g} \left[ -\left( \partial_0 \sqrt{g^{-1} \eta} \right)^2 + \left( \partial_0 \sqrt{g^{-1} \eta} \right) \left( \partial_0 \sqrt{g^{-1} \eta} \right) \right] = N \sqrt{|\gamma|} \left[ \frac{2 (B^i B^k \delta_{ik})^2}{N^2} - (\dot{C}^i_j)^2 + \frac{4 (B^i B^k \delta_{ik}) \dot{C}^i_j}{N} - \frac{2 (B^i B^k \delta_{ik}) N \ddot{C}^i_j}{N^2} - \frac{2 B^i B^k \delta_{kj} \dddot{C}^i_j}{N^2} + \frac{2 B^i B^k \delta_{kj} \dddot{C}^i_j + \dot{C}^i_j \dddot{C}^i_j}{N} \right].
\]  

(31)

From the expression (31), we find the time-derivatives of the lapse and shift functions cannot be canceled and therefore there could appear ghost.

### III. RENORMALIZABLE MODEL OF MASSIVE SPIN TWO PARTICLE

We now propose a power-counting renormalizable model of the massive spin two particle, whose Lagrangian density is given by

\[
\mathcal{L}_{h0} = \frac{1}{2} \epsilon^i_{\mu \nu} \epsilon^j_{\rho \sigma} \left( \partial_\mu \partial_\nu h_{\rho \sigma} + \partial_\rho \partial_\sigma h_{\mu \nu} - \partial_\mu \partial_\rho h_{\sigma \nu} + \partial_\sigma \partial_\nu h_{\rho \mu} \right)
\]

\[
- \frac{\mu}{3!} \delta^i_{\mu \nu} h_{\rho \sigma} \left( \partial_\mu \partial_\nu h_{\rho \sigma} + \partial_\rho \partial_\sigma h_{\mu \nu} - \partial_\mu \partial_\rho h_{\sigma \nu} + \partial_\sigma \partial_\nu h_{\rho \mu} \right)
\]

\[
- \frac{\lambda}{4!} \left( h^2 - 2 h_{\mu \nu} h_{\mu \nu} - \frac{\mu}{3} \left( h^3 - 3 h_{\mu \nu} h_{\mu \nu} + 2 h_{\mu \nu} h_{\sigma \nu} \right) \right)
\]

\[
- \frac{\lambda}{4!} \left( h^4 - 6 h_{\mu \nu} h_{\mu \nu} + 8 h_{\mu \nu} h_{\rho \sigma} h_{\mu \nu} - 6 h_{\mu \nu} h_{\rho \sigma} h_{\sigma \nu} + 3 \left( h_{\mu \nu} h_{\mu \nu} \right)^2 \right).
\]  

(32)
Here $m$ and $\mu$ are parameters with the dimension of mass and $\lambda$ is a dimensionless parameters. Therefore the model given by the Lagrangian is power-counting renormalizable. It is also clear that the model is free from ghost.

We should note, however, that the propagator is given by

$$D_{\alpha\beta, \rho\sigma}^{m} = \frac{1}{2(p^2 + m^2)} \left\{ P_{\alpha\rho}^{m} P_{\beta\sigma}^{m} + P_{\alpha\sigma}^{m} P_{\beta\rho}^{m} - \frac{2}{D-1} P_{\alpha\beta}^{m} P_{\rho\sigma}^{m} \right\},$$

(33)

$$P_{\mu\nu}^{m} \equiv \eta_{\mu\nu} + \frac{p_{\mu} p_{\nu}}{m^2}. \quad (34)$$

Then when $p^2$ is large $D_{\alpha\beta, \rho\sigma}^{m} \sim O(p^2)$ due to the projection operator $P_{\mu\nu}^{m}$, which makes the behavior for large $p^2$ worse and therefore the model should not be renormalizable.

There is a similar problem in the model of massive vector field, whose Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4} \left( \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \right) \left( \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} \right) - \frac{1}{2} m^2 A_{\mu} A^{\mu}. \quad (35)$$

The propagator $D_{\mu\nu}$ of the massive vector is given by

$$D_{\mu\nu} = -\frac{1}{p^2 + m^2} P_{\mu\nu}^{m}, \quad (36)$$

which is the inverse of

$$O^{\mu\nu} \equiv -\left( p^2 + m^2 \right) \eta^{\mu\nu} + p^{\mu} p^{\nu}, \quad (37)$$

and is

$$O^{\mu\nu} D_{\nu\rho} = \delta_{\mu}^{\rho}. \quad (38)$$

The expression (36) tells that for large $p^2$, $D_{\mu\nu}$ behaves as $O(1)$ and therefore the model (35) could not be renormalizable. If the vector field, however, couples only with the conserved current $J_{\mu}$ which satisfies the conservation law $\partial^{\mu} J_{\mu} = 0$, the term $\frac{p_{\mu} p_{\nu}}{m^2}$ in the projection operator $P_{\mu\nu}^{m}$ drops and the propagator behaves as $D_{\mu\nu} \sim O(1/p^2)$ and therefore the model may become renormalizable. Instead of imposing the conservation law, we may add the following term to the action:

$$2\alpha \phi \partial^{\mu} A_{\mu}, \quad (39)$$

and consider the inverse of the operator

$$O_{A\phi} = \begin{pmatrix} O^{\mu\nu} & -i\alpha p^{\mu} \\ i\alpha p^{\nu} & 0 \end{pmatrix}, \quad (40)$$

which is given by

$$D_{A\phi} = \begin{pmatrix} -\frac{1}{p^2 + m^2} P_{\nu\rho} & -i\frac{p_{\mu}}{\alpha p^{2}} \\ i\frac{p_{\nu}}{\alpha p^{2}} & \frac{m^2}{\alpha^2 p^{2}} \end{pmatrix}, \quad (41)$$

$$P^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{p^{\mu} p^{\nu}}{p^2}, \quad (42)$$

$$O_{A\phi} D_{A\phi} = \begin{pmatrix} \delta_{\mu}^{\rho} & 0 \\ 0 & 1 \end{pmatrix}. \quad (43)$$

The projection operator $P_{\mu\nu}$ is equal to the projection operator $P_{\mu\nu}^{m}$ on shell, $p^2 = -m^2$, but the behavior in large $p^2$ becomes different from each other. As a result, the propagator between two $A_{\mu}$'s behaves as $O(1/p^2)$ and therefore the model becomes renormalizable if the interaction terms are also renormalizable.

We may consider similar deformation of the model by adding the following new term to the Lagrangian density

$$\mathcal{L} = \mathcal{L}_{h0} + 4\alpha A^{\mu} \partial^{\nu} h_{\mu\nu}, \quad (44)$$

and the model becomes renormalizable if the interaction terms are also renormalizable.
and consider the following equation:

\[
\left( \frac{O^{\mu \nu, \alpha \beta}}{i \alpha (p^\alpha \eta^\beta + p^\beta \eta^\alpha)} - i \rho (p^\mu \eta^\nu + p^\nu \eta^\mu) \right) \left( D_{\alpha \beta, \rho \sigma} - i E_{\alpha \beta, \rho \sigma} \right) F_{\alpha \sigma} = \left( \frac{1}{2} \left( \delta^\rho_\sigma \delta^\nu_\alpha + \delta^\nu_\sigma \delta^\rho_\alpha \right) \right) \left( \begin{array}{cc} 0 & 0 \\ \delta^\rho_\sigma & 0 \end{array} \right). \tag{45} \]

We should note that \( O^{\mu \nu, \alpha \beta} \) in \( 51 \) can be rewritten as

\[
O^{\mu \nu, \alpha \beta} = - \left\{ \frac{1}{2} \left( P^{\mu \alpha} P^{\nu \beta} + P^{\mu \beta} P^{\nu \alpha} - P^{\mu \nu} P^{\alpha \beta} \right) \right\} \left( p^2 + m^2 \right) - \left\{ \frac{1}{2} \left( p^\alpha P^{\nu \beta} + p^\beta P^{\nu \alpha} + p^\alpha P^{\mu \beta} + p^\beta P^{\mu \alpha} \right) - P^{\mu \nu} P^{\alpha \beta} - p^\alpha p^\beta P^{\mu \nu} \right\} \frac{m^2}{p^2}. \tag{46} \]

Then we find

\[
D_{\alpha \beta, \rho \sigma} = - \frac{1}{2 (p^2 + m^2)} \left\{ P_{\rho \sigma} P_{\alpha \beta} + P_{\alpha \beta} P_{\rho \sigma} - \frac{2}{D-2} P_{\alpha \beta} P_{\rho \sigma} \right\}, \tag{47} \]

\[
E_{\alpha \beta, \rho \sigma} = \frac{1}{2 \alpha^2 p^2} \left\{ 2p_\rho P_{\alpha \sigma} + p_\sigma P_{\alpha \rho} - \frac{m^2 p_\alpha}{D-2} \frac{p_\rho}{p^2} P_{\rho \sigma} + \frac{P_{\rho \sigma} p_\rho}{p^2} \right\}, \tag{48} \]

\[
F_{\alpha \sigma} = \frac{m^2}{2 \alpha^2 p^2} P_{\alpha \sigma} + \frac{(D-1) m^4}{4 \alpha^2 (D-2) (p^2)^2 (p^2 + m^2)} \frac{P_{\rho \sigma}}{p^2}. \tag{49} \]

Because the propagator between two \( h_{\mu \nu} \)'s behaves as \( O(1/p^2) \), the model becomes renormalizable.

We should note that the coupling of \( h_{\mu \nu} \) with the energy-momentum tensor \( T_{\mu \nu}, \kappa^2 h_{\mu \nu} T_{\mu \nu} \), which appears in the general relativity, breaks the renormalizability because \( \kappa \) has the dimension of length. The coupling with a scalar field \( \phi \) or the Rarita-Schwinger field \( \psi_\mu \) can be, however, renormalizable,

\[
\phi \eta^{\mu_1 \nu_1 \mu_2 \nu_2 \mu_3 \nu_3}, \quad h_{\mu \nu} \bar{\psi}_\mu \psi_\nu, \tag{50} \]

which may appear when we supersymmetrize the action \( 52 \) or \( 41 \).

IV. NEW BIGRAVITY

The bigravity model can be regarded as a model where massive spin two field couples with gravity. Then we may consider the model where \( h_{\mu \nu}, \) whose Lagrangian is given by \( 52 \) or \( 41 \), couples with gravity

\[
S = \int d^4 x \sqrt{-g} \left\{ \frac{1}{2} g^{\mu_1 \nu_1 \mu_2 \nu_2} \nabla_{\mu_1} h_{\mu_2 \nu_2} h_{\mu_3 \nu_3} - \frac{1}{2} m^2 g^{\mu_1 \nu_1 \mu_2 \nu_2} h_{\mu_1 \nu_1} h_{\mu_2 \nu_2} - \mu \eta^{\mu_1 \nu_1 \mu_2 \nu_2 \mu_3 \nu_3} h_{\mu_1 \nu_1} h_{\mu_2 \nu_2} h_{\mu_3 \nu_3} - \frac{\lambda}{4!} g^{\mu_1 \nu_1 \mu_2 \nu_2 \mu_3 \nu_3 \mu_4 \nu_4} h_{\mu_1 \nu_1} h_{\mu_2 \nu_2} h_{\mu_3 \nu_3} h_{\mu_4 \nu_4} + 4 \alpha A^\mu \nabla_\mu h_{\mu \nu} \right\}, \tag{51} \]

which can be regarded as a new bigravity model because there appear two symmetric tensor fields \( g_{\mu \nu} \) and \( h_{\mu \nu} \). We should note that \( h_{\mu \nu} \) is not the perturbation in \( g_{\mu \nu} \) but \( h_{\mu \nu} \) is a field independent of \( g_{\mu \nu} \).

We may consider the cosmology given by the action \( 51 \) with the Einstein-Hilbert action:

\[
S_{EH} = \frac{1}{2 \kappa^2} \int d^4 x \sqrt{-g} R. \tag{52} \]

We assume the solution of equations given by the actions \( 51 \) and \( 52 \) is given by

\[
h_{\mu \nu} = C g_{\mu \nu}. \tag{53} \]

Here \( C \) is a constant. We can directly check that Eq. \( 53 \) satisfies the field equation given by the variation of \( h_{\mu \nu} \) and also the Einstein equation by properly choosing \( C \). By substituting \( 53 \) into the action \( 51 \), we find

\[
S = - \int d^4 x \sqrt{-g} V(C), \quad V(C) \equiv 6m^2C + 4\mu C^3 + \lambda C^4. \tag{54} \]
We should note $\nabla_\rho g_{\mu\nu} = 0$. The constant $C$ can be determined by the equation $V'(C) = 0$. We now parametrize $m^2$ and $\mu$ by

$$m^2 = \frac{\lambda}{3} C_1 C_2, \quad \mu = -\frac{\lambda}{3} (C_1 + C_2).$$

(55)

Then the solutions of $V'(C)$ are given by

$$C = 0, C_1, C_2,$$

(56)

and we find

$$V(C_1) = \frac{\lambda}{3} C_1^3 (-C_1 + 2C_2), \quad V(C_2) = \frac{\lambda}{3} C_2^3 (-C_2 + 2C_1).$$

(57)

Then we find $V(C)$ plays the role of cosmological constant. Let assume $0 < C_1 < C_2$ and $C_2 < 2C_1$. Then $V(C_1)$ is a local maximum and $V(C_2) > 0$ is a local minimum. Then $V(C_1)$ or $V(C_2)$ might generate the inflation.

It has been shown that the causality could be broken in the previous bigravity models [33] due to the existence of the superluminal mode. We should note that in the model given by the actions (51) and (52), the superluminal mode does not appear and therefore the causality could not be broken.

V. SUMMARY

In summary, we considered the non-linear derivative interactions which are not included in [37] but unfortunately we have shown that such derivative interactions generate ghost, unfortunately. We also investigated the possibility of other classes of the no-ghost interactions by only requiring the Lorentz invariance.

We have also propose a renormalizable theory describing massive spin two particle. The coupling of the theory with gravity can be regarded as a new kind of bimetric gravity or bigravity and we have shown that the field of the massive spin two particle plays the role of the cosmological constant.

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