New Epistemological Perspectives on Quantitative Methods: An Example Using Topological Data Analysis

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Background: Education researchers use quantitative methodologies to examine generalizable correlational trends or causal mechanisms in phenomena or behaviors. These methodologies stem from (post)positivist epistemologies and often rely on statistical methods that use the means of groups or categories to determine significant results. The results can often essentialize findings to all members of a group as truth knowable within some quantifiable error. Additionally, the attitudes and beliefs of the majority (i.e., in engineering, White cis men) often dominate conclusions drawn and underemphasizes responses from minoritized individuals. In recent years, engineering education research has pursued more epistemologically and methodologically diverse perspectives. However, quantitative methodologies remain relatively fixed in their fundamental epistemological framings, goals, and practices.

Purpose: In this theory paper, we discuss the epistemic groundings of traditional quantitative methods and describe an opportunity for new quantitative methods that expand the possible ways of framing and conducting quantitative research—person-centered analyses. This article invites readers to re-examine quantitative research methods.

Scope: This article discusses the challenges and opportunities of novel quantitative methods in engineering education, particularly in the limited epistemic framings associated with traditional statistical methods. The affordances of person-centered analyses within different epistemological paradigms and research methods are considered. Then, we provide an example of a person-centered method, topological data analysis (TDA), to illustrate the unique insights that can be gained from person-centered analyses. TDA is a statistical method that maps the underlying structure of highly dimensional data.

Discussion/Conclusions: This article advances the discussion of quantitative methodologies and methods in engineering education research to offer new epistemological approaches. Considering the foundational epistemic framings of quantitative research can expand the kinds of questions that can be asked and answered. These new approaches offer ways to conduct more interpretive and inclusive quantitative research.

Keywords: Epistemology; Quantitative Methods; Topological Data Analysis; Person-Centered Analysis; Latent Diversity

Introduction

Broadly, the purpose of research is to develop new knowledge or insight regarding a specific topic. As such, researchers and research communities must reflect on how they theorize and frame knowledge (i.e., their epistemologies and methodologies) and their processes to build that knowledge (i.e., their methods). This reflection not only facilitates alignment between research questions, theory, methodology, and methods but also can identify new opportunities for expanding the kinds of questions that can be asked and approaches to conducting research. In this theory paper, we explore emerging epistemic possibilities for quantitative research in the context of engineering education, particularly regarding person-centered analyses. These possibilities may offer ways to conduct more interpretive and inclusive quantitative research.

Engineering education research is practiced within a community that is shaped by the very engineering education systems being studied (Kant & Kerr, 2019). Two major discourses in engineering education research methodologies have emerged...
from this history: rigor and methodological diversity (Beddoes, 2014). Rigor discourse historically focused on legitimating engineering education as an emerging research field. This discourse has resulted in a history of engineering education research that has emphasized objective and generalizable research methods (Jesiek et al., 2009; Riley, 2017). Similarly, this discourse has been critiqued as enforcing limited epistemic framings of what counts as high-quality engineering knowledge and perpetuating inequity (Beddoes, 2014; Riley, 2017). More recently, methodological diversity discourse has created calls for and value of varied research approaches, particularly in qualitative research methodologies (Douglas et al., 2010). Researchers have faced challenges with qualitative methods in their inculation into engineering education research due to boundary spanning between engineering and social science (Douglas et al., 2010). However, in recent years, engineering education has seen a surge in published qualitative papers with methodological diversity (Walther et al., 2017). There have been dedicated conversations to clarifying methodological rigor (Streveler et al., 2006), epistemic foundations (Baille & Douglas, 2010; Douglas et al., 2010), and a holistic framework for qualitative inquiry in engineering education (Walther et al., 2013, 2015, 2017). However, there has been little reflection on the epistemic norms of qualitative research. Targeting this reflection towards quantitative studies can situate current scholarship in engineering education as well as identify new possibilities that move beyond research methods aligned with a postpositivist epistemology (i.e., truth is knowable within some margin of error) that may be currently overlooked due to norms in the field (Baille & Douglas, 2014; Koro-Ljungberg & Douglas, 2008).

The purpose of this paper is to outline a discussion that invites readers to re-examine quantitative research methods and provides reflections on how an emerging set of quantitative methods—person-centered analyses (PCA)—can expand how we frame research in engineering education. Approaches that employ PCA treat the individual as a unique, holistic entity and work to maintain their whole response in the analysis, as opposed to traditional variable-centered approaches. We also provide an example of a person-centered analysis in engineering education to illustrate the possibilities of this approach. This paper does not provide an exhaustive review of all possible ways that quantitative research can be reconsidered beyond the epistemic norms of (post)positivism. We use a research example to support the arguments made rather than present this example as a set of research findings or specific implications. Instead, we outline a gap in current methodological approaches to quantitative research and invite dialogue around embedded assumptions and norms within quantitative research.

Epistemologies in Social Science and Educational Research

Epistemology refers to beliefs about knowledge and how knowledge is constructed. It is one part of the philosophical assumptions that influence which methodologies and methods researchers consider appropriate (Crotty, 1998; Lather, 2006). All aspects of the research process are informed by one’s epistemology, from embedded assumptions about what is known to the development of theories, research questions, and study designs (Pallas, 2001; Collins, 1990). Upon the dissemination of findings, epistemologies also influence how research is interpreted and understood within a research community (Pallas, 2001). In social science research, common terms have been developed to describe general categories of epistemologies. We describe three of these categories in this paper: (post)positivism, constructivism, and critical theory. We do not present these categories to continue the “Paradigm Wars” between quantitative and qualitative research as incompatible research approaches (see Bryman, 2008). Instead, we present the categories to provide context to the proposed discussion of quantitative methods and non-positivist approaches.

Postpositivism refers to a set of beliefs characterized by the assumption that reality can be known within some degree of certainty. Historically, postpositivism emerged as a response to positivism, an epistemology that was popular in early social science work (Reed, 2010). Positivism takes a narrow view on knowledge production, focusing only on what can be measured and observed, with a strict focus on causality and the separation between knowledge and observer. Postpositivism allows for the role of human perspective and error, but still maintains a commitment to objective measurement and observation. Researchers leveraging a postpositivist perspective are often concerned with determining averages and trends in the dataset, attempting to minimize or control variation from these trends, and generalizing results to a larger population. Quality or validity is traditionally focused on measurement, generalization, and controlling variables to reduce bias (Hammersley, 2008). While quantitative research is not a monolith, few studies have taken epistemological framings different from positivism or postpositivism (Bryman, 2008).

In contrast, constructivism is often concerned with how an individual develops a socially negotiated and personal understanding of reality (Baille & Douglas, 2014). This understanding is varied for each individual, leading the researcher to study complexity and shared reality. Research leveraging constructivism recognizes individuals’ perspectives and the constellation of factors that may shape their lived experiences. It also acknowledges that research is a co-production between

1 In this article we use (post)positivism to refer to the family of epistemologies related to positivism. For concision, we use the term non-positivist to refer to epistemologies outside of this family.
the researcher and participant(s). Thus, constructivism focuses on the subjective experience and its value for knowledge production.

Similarly, critical approaches emphasize the subjective reality of lived experiences to reveal power and oppression within social contexts with aims for social transformation (i.e., move away from (re)producing knowledge laden with inequity). Critical paradigms include feminist scholarship, Critical Race Theory, and disability studies or Crip Theory, among many others (Lather, 2006). Critical epistemologies acknowledge that conceptions of knowledge are not value-neutral and that marginalized forms of knowledge must be valued and studied. This epistemological approach opposes how postpositivism imposes structural laws and theories that do not fit marginalized individuals or groups and posits that constructivism does not adequately address needed action against oppressive social structures.

Even though epistemologies are not tied to specific research methods, the affordances and foci of these common epistemological paradigms have resulted in historically bifurcated research approaches, where quantitative methods are typically associated with (post)positivism and qualitative methods are typically associated with constructivist, critical, or other non-positivist epistemologies (Tuli, 2010). For instance, education researchers often use quantitative methodologies to study generalizable correlational trends or causal mechanisms. They typically rely on traditional statistics that use the means of groups (e.g., engineers versus non-engineers or women versus men) to determine statistically significant differences between groups or average effects of a variable on an outcome (i.e., variable-centered approaches). Research findings typically report means, line or bar graphs, p-levels, or Bayes factors. These methodologies often result in essentializing results of analyses to all members of a group as truth (a [post]positivist approach) and perpetuate a problematic dichotomy of identity.

As an alternative to such essentializing approaches, this theory paper focuses on the links between novel quantitative research methods in person-centered analyses and non-positivist epistemologies. However, we acknowledge that epistemology informs other components of the research process besides methodology, such as theory and dissemination. Douglas, Koro-Ljungberg, and Borrego (2010) argued against approaching theory, method, and epistemology separately or decontextualizing the framing of research (p. 255). Thus, despite a focus on methods of analysis, this work also demonstrates the potential need for alternatives to traditional conceptions of quantitative research that are reformulated from the epistemic foundations.

Epistemic Standpoint of Research Team

We are a team of researchers engaged in mixed-methods research focused on identity and diversity in engineering education. Some of us specialize more deeply in quantitative or qualitative paradigms, but together we recognize the value in each paradigm to answer particular kinds of questions, and an added richness in combining research approaches. As such, we approach our research and this discussion from a pragmatic epistemology. Pragmatism emerged in the late 19th century (Maxcy, 2003), and is a set of philosophical tools rather than solely a philosophical standpoint (Biesta, 2010), which focus on research choices that result in anticipated or desired outcomes (Tashakkori & Teddlie, 2008). Pragmatism holds that knowledge is individual and socially constructed; nevertheless, it also posits that much of this knowledge is socially shared and research can begin to examine these shared realities (Morgan, 2014). Pragmatism has been used recently in social science as the epistemology guiding mixed and multiple methods (Creswell & Clark, 2011; Johnson & Onwegbuzie, 2004) as it “rejects traditional philosophical dualism of objectivity and subjectivity” (Kaushik & Walsh, 2019, p. 4). With a focus on meaningful research that has utility for action for making purposeful difference in practice, pragmatism is also consistent with action for social justice (Morgan, 2014).

One of the challenges in mixed methods research is synthesizing research findings from qualitative or quantitative paradigms. In this process, we have begun to engage in newer quantitative methods that provide additional nuance and the ability to preserve individuals’ responses within the data. We have found these practices both demanding and rewarding. From this standpoint, we open discussion of considering research questions and approaches in the quantitative paradigm from non-positivist epistemologies.

Traditional Methodological Approaches in Quantitative Research

Stemming out of (post)positivism, most quantitative methodologies emphasize objectivity, replicability, and causality. Most quantitative studies in social science research were designed to address research questions using variable-centric methods. Variable-centered approaches (i.e., correlations, regressions, factor analysis, and structural equation models) are appropriate for addressing inquiries concerned with “how variables, observed or latent, relate to each other” (Wang et al., 2013, p. 350) and generate outcomes based on an averaged set of parameters. In engineering education, the study population is often cis-gender, White, male, upper-middle-class, able-bodied, continuing generation, and heterosexual (Pawley, 2017). Historically, this population has been accepted as the default in engineering education research, resulting in findings and implications for practice that are often decontextualized from the social reality of individuals’ backgrounds and experiences. By conducting research with demographic homogeneity, the understanding of phenomena for individuals who are not the default is
limited and warrants a need for researchers to justify their rationale for generating theory based on individuals with a dom-
inant presence in engineering (Slaton & Pawley, 2018; Pawley, 2017). For our research, particularly in focusing on diversity
in engineering education, traditional quantitative methods have provided useful answers to important questions; however,
they also present challenges in adequately representing all students.

To illustrate these challenges and highlight how variable-centric statistical methods can reinforce dominant norms,
we provide an example related to research on gender segregation in science, technology, engineering, and math (STEM)
professions. This example, drawing on common, and well-known phenomena, illustrates the ability of variable-centered
approaches to ask nuanced questions while still essentializing the findings of an individual to a group. Thus, even as this
approach provides valuable and important research findings, it also shows the ways in which even carefully constructed
quantitative studies that meet standards of quality still align with (post)positivism.

The phenomenon in question emerges from studies comparing the future goals and outcome expectations of men and
women that find women are more interested in person-oriented or altruistic roles. Engineering, as a male-dominated
and thing-oriented field, is not consistent with this characterization (e.g., Ngambeki et al., 2011; Su & Rounds, 2015).
Therefore, studies conclude that misaligned orientations are a key reason for women’s lack of representation in engineering
(Bairaktarova & Pilotte, 2020; Cejka & Eagly, 1999; Miller, Eagly, & Linn, 2015; National Academy of Engineering, 2008; Su
& Rounds, 2015). These studies give some important general characterization of how engineering culture is gendered, and
their findings are consistent across repeated studies and cultural contexts.

However, the limits of this variable-centered approach emerge when we explore the question from an alternate di-
rection. For example, a study of women in engineering disciplines with above-average (i.e., biomedical, industrial, etc.)
and below-average female enrollment (i.e., mechanical, electrical, etc.) indicate different patterns, with women in the
below-average female enrollment group having less interest in stereotypically feminine outcome expectations (Verdín et al.,
2018). This study points to the reality that not all women follow general findings about interests and goals. Thus, even with
careful explanation by researchers that quantitative results are true for most women, the nuance of individual differences
is not captured by these approaches. Indeed, most social science studies focus on variation between groups and make con-
clusions based on statistically significantly different average effects (Fanelli, 2010). However, differences between groups,
even with so-called large effect sizes, can occur even when two groups are much more similar than different (Hanel et al.,
2019). Additionally, the attitudes and beliefs of the majority (i.e., in engineering, White men) dominate conclusions drawn
and underemphasizes responses from minoritized individuals.

Slaton and Pawley (2018) argued that it is not sufficient for scholars to justify the exclusion of individuals based on tradi-
tional quantitative norms of sampling and large-n studies. Instead, engineering education must create and learn new meth-
ods that empower researchers to learn from small numbers. The number of participants or lack thereof in a study is not an
excuse to generate theory based on homogenous populations and perpetuate limited standards of representation (Pawley,
2018; Slaton & Pawley, 2018). There is a need for epistemic shifts to advance our understanding and challenge what counts
as adequately representative in engineering education research (Slaton & Pawley, 2018). Otherwise, engineering education
researchers reinforce systemic inequities through our logic and methods, unconsciously or otherwise.

Pawley and colleagues have offered small-n qualitative studies as a valuable solution to large quantitative studies’ impor-
tant criticisms. The purpose of these studies is to capture and highlight the experiences of individuals often minoritized in
engineering and sometimes (but not always) identify patterns across participants (Merriam & Tisdell, 2016). These studies
also can leverage the complexity and power of intersectionality studies to reveal inequities in engineering education.
Through the thick description of individuals’ experiences, these qualitative studies lead to a richer and more nuanced
understanding of phenomena otherwise left ignored or masked in studies that prioritize large-n studies. However, the level
of detail often precludes the breadth of participants seen in quantitative studies. While this focus is a feature of qualitative
research rather than a problem, it does constrain the kinds of questions that qualitative research can and cannot answer.
There is still a need to conduct quantitative studies that are generalizable, are inclusive, and do not essentialize results to a
single average or group.

As a result, in addition to qualitative studies that provide valuable insight into individual lived experiences, new quanti-
tative methodological approaches have emerged in the social sciences that also begin to address the critiques raised about
(post)positivist quantitative paradigms. These new approaches can introduce epistemologically novel ways to approach
quantitative research questions that fill a gap not addressed by qualitative, mixed methods, or traditional quantitative
research alone. New quantitative approaches do not need to replace traditional methods, but instead offer additional ways
of understanding and querying a phenomenon. We describe some of these approaches below before focusing on per-
son-centered analyses.

**New Methodological Approaches in Quantitative Research**

*Multi-Modal Approaches*

Emerging scholarship in engineering education has begun to re-examine quantitative methods, particularly in using mul-
ti-modal approaches to understand cognition and emotion in authentic contexts. We provide a few but not exhaustive
examples of these approaches. Villanueva, Di Stefano, Gelles, Vicioso Osoria, and Benson (2019) conducted a study with multi-modal approaches to data collection, including interviews and electrodermal activity sensor data, from 12 women students to study psychophysiological responses to academic mentoring. This approach treated inequity issues as core to participants’ experiences rather than moderating quantitative analysis variables. The quantitative data were analyzed using MANOVA and representative response profiles before synthesizing the findings with qualitative data. This approach allowed for both conscious (interview responses) and unconscious (electrodermal activity sensor data) to be examined simultaneously. This multi-modal approach has also been applied to an experimental study of students’ emotional experiences during testing with electrodermal activity sensor data saliva testing during a practice exam (Villanueva et al., 2019).

Other researchers have used similar multi-modal protocols to study design thinking. Gero and Milanovic (2020) proposed a framework for design thinking that involves design cognition, design physiology, and design neurocognition. Gero and Milanovic (2020) provided a detailed description of prior studies and various measurement methods for these dimensions (i.e., brain imaging, electrodermal activity, eye movements, protocol analysis, surveys, interviews, etc.). These measurements are combined to inform a larger understanding of these processes in contexts that are often studied separately (i.e., affect and emotion or cognition). These data are examined using traditional statistical techniques but also using novel approaches like linkography to examine relationships between design moves (Goldschmidt, 2014), Markov modeling to examine probable transitions in design reasoning or processes (Gero & Peng 2009; Kan & Gero 2010), and correspondence analysis to describe the degree and extent of relationships between categories (Greenacre & Hastie, 1987).

These multi-modal approaches offer new ways to examine complex phenomena and provide ways to integrate the strengths of quantitative and qualitative data. Two of the biggest challenges of multi-modal approaches are the effort (i.e., time, cost, etc.) associated with data collection and synthesis of heterogeneous data. As such, these studies are often conducted with small sample sizes and most studies rely on traditional statistical methods such as the correlation of quantitative results (where qualitative data streams are coded into quantitative frequencies or patterns; Gero & Milanovic, 2020). These approaches have strength in examining the underlying mechanisms in rich and nuanced ways.

The novelty of these methods is predominantly in data collection tools and integration of results of these tools to generate new insights and questions in educational research. Fewer studies have deeply examined the epistemic and statistical methods of solely quantitative research for the same goal. We believe that person-centered statistical analyses offer ways to reimagine quantitative educational research using more common numeric data collection approaches such as surveys and observations. This approach re-imagines how student responses are characterized and understood in context through statistical methods.

**Person-Centered Approaches**

Person-centered approaches sit in contrast to traditional variable-centric approaches and assume that the population under study is heterogeneous. The results of such studies focus on preserving the variation in individual’s responses resulting in authentic groupings of individuals, as opposed to imposing superficial characterizations of groups (Laursen & Hoff, 2006; Morin et al., 2018). In a variable-centered approach, individual differences are treated as outliers from a mean value, or even erased, due to low sample size, a decision that disproportionately impacts minoritized individuals. While these approaches are not a panacea for all challenges with quantitative methods, especially concerning measurement and fairness (Douglas & Purzer, 2015), they do open new avenues for quantitative inquiry beyond (post)positivist epistemologies. In doing so, they provide new avenues of research and potentially more equitable approaches to quantitative methodologies.

Person-centered analyses are a relatively young methodological approach arising alongside the increased availability of computing resources (Laursen & Hoff, 2006). As with all innovations, they occupy an ill-defined space with concepts that both overlap and differ in key ways. Consequently, a call for increased use of person-centered analyses requires some discussion for readers to navigate this confusing morass of shared terminology. A central area of overlap and potential confusion that new researchers will likely encounter is between the terms *person-centered analysis* and *data-driven approach*. For instance, discussions of specific techniques (e.g., cluster analysis or mixture modeling) occur in both spheres, and both approaches rely on modern computational power and sprawling datasets (also called Big Data; Lazer et al., 2009; Gillborn, Warmington, & Demack, 2018).

A data-driven approach rejects traditional formulations of the scientific method that begin and end with theory developments. Instead, it lets the data “tell their own story,” independent of researchers’ assumptions and preconceptions, and then reconcile findings and theories once the analysis is complete (Qiu et al., 2018). Data-driven approaches thus utilize bottom-up frameworks centered on relationships instead of top-down frameworks driven by explanations and causality (Qiu et al., 2018). It is not surprising that data-driven approaches have increased in popularity as more and more data is created as part of our daily lives (Gero & Milanovic, 2020; Villanueva, Di Stefano, et al., 2019), which also lessens the need for experiments that control for confounds and the influence of covariates. Instead, data-driven approaches accommodate for the lack of control in data generation and collection through sheer numbers and advanced computational power (Lazer et al., 2009).

Person-centered analyses, in contrast, challenge assumptions about group homogeneity, variable effects, and the generalizability of conventional inferential analyses (e.g., linear regression; Eye & Wiedermann, 2015). The mean of a dataset is not
always the best way to describe or represent a population—not only can it be distorted by a small number of outliers (e.g., the average net worth in the United States where wealth is concentrated among a relatively small group of individuals), but it may also represent an impossible or otherwise inaccurate value (e.g., the average of 2.5 children per American household; Eye & Wiedermann, 2015). Similarly, variable-centered analyses estimate the effects of individual variables by controlling for, or removing the effects of, other variables in the model, although this separation cannot occur in real life (e.g., attempting to attribute an outcome to racism or socioeconomic inequality when these experiences exist in a state of mutual or spiraling causality; McCall, 2002). Thus, person-centered analyses utilize the identification of underlying groups (i.e., latent profile/class analysis; Jack et al., 2018), hidden clusters or structures (i.e., cluster analysis, Topological Data Analysis, Principal Component Analysis, Self-Organizing Maps, and Multidimensional Scaling; Chazal & Michel, 2017; Everitt et al., 2011), or mixture components (i.e., mixture modeling; Jack et al., 2018) when examining the relationships of individual response patterns within the data. This approach preserves heterogeneity instead of masking or minimizing it. In other words, person-centered analyses adopt a data-driven approach and use this approach to identify subpopulations not readily visible to the naked eye and use these subpopulations to improve the clarity and accuracy of predictions and explanations. Although person-centered analyses incorporate data-driven approaches, not all data-driven approaches are person-centered; many other exploratory and Big Data techniques, including Classification and Regression Trees (CART; Breiman et al., 1984), still foster variable-centered approaches that aim to reconcile variables with predefined (and thus potentially biased or inaccurate) categories. We provide a description, but not an exhaustive list, of these different analyses in Table 1.

Table 1: Examples of person-centered and data-driven analyses.

| Analysis                      | Description                                                                                           | Reference                    |
|-------------------------------|-------------------------------------------------------------------------------------------------------|------------------------------|
| Topological Data Analysis     | Used to identify geometric patterns in multivariate data. Continuous structures are built on top of the data and geometric information is extracted from the created structures and used to identify groups. For more information, see the example from engineering education provided below. | Chazal & Michel, 2017       |
| Cluster Analysis              | Used to create groups according to similarity between observations in a dataset, often through the algorithm K-means clustering. Groups are created according to their distance from the center of a cluster and group assignment is not probabilistic. | Garcia-Dias et al., 2020     |
| Gaussian Mixture Modeling     | Used to create groups according to similarity between observations in a dataset. Unlike cluster analysis, this technique accounts for variance in the data, and thus allows for more variability in group shape and size while providing probabilistic assignment to groups. | McNicholas, 2010             |
| Latent Profile/Class Analysis | Used to recover hidden groups from multivariate data. Falls within the larger umbrella of mixture modeling. Can be used with continuous or categorical data, and results in probability-based assignment to groups. | Oberski, 2016               |
| Growth Mixture Modeling       | Similar to latent profile/class analysis but used with longitudinal data. Can be used to identify groups and then track individual movement across group lines or can be used to identify groups that emerge over time. | Ram & Grimm, 2009           |
| Artificial Neural Networks    | A machine-learning classical algorithm that performs tasks using methods derived from studies of the human brain. Can be used to recognize patterns or classify data. Self-Organizing Maps (Saxxo, Motta, You, Bertolazzio, Carini, & Ma, 2017) are a form of person-centered neural networking that can be used to convert complex multivariate data into two-dimensional maps that emphasize the relationships between observations. | Abiodun et al., 2018         |
| Principal Component Analysis  | Used to collapse correlated multivariate data into smaller composite components that maximize the total variance (aka dimension reduction). Often used to reduce a large number of variables to a more manageable number. For non-continuous data, categorical principal component analysis can be used. Data-driven but not person-centered. | Kherif & Latypova, 2020      |
| Multidimensional Scaling      | Another form of dimension reduction, but with a focus on graphics and the visual analysis of data. Multivariate data is collapsed into two dimensions by computing the distance between variables and plotting the resulting output. Data-driven but not person-centered. | Hout et al., 2013            |
| Exploratory Factor Analysis   | Used to identify latent factors or variables in correlated multivariate data. Often used in scale development or when analyzing constructs that cannot be measured directly. Data-driven but not person-centered. | Sellbom & Tellegen, 2019     |
Person-centered analyses are not necessarily associated with a particular epistemological paradigm. The techniques associated with person-centered analysis may be used to make (post)positivist claims, such as clustering engineering students based on learning orientations and study strategies, then evaluating the study success of each cluster (e.g., GPA; Tynjälä et al., 2005). However, a benefit of person-centered analyses is that it disrupts some of the assumptions typically associated with (post)positive, variable-centered approaches. Below, we provide an example of one kind of person-centered analysis that takes a non-positivist viewpoint.

An Example of Person-Centered Analysis from Engineering Education

We use a research project that employed Topological Data Analysis (TDA) to demonstrate the kinds of knowledge afforded by a specific type of person-centered analysis. This empirical example was a part of a study titled, CAREER: Actualizing Latent Diversity: Building Innovation through Engineering Students’ Identity Development (NSF Grant No. 1554057), focused on understanding first-year engineering students’ latent diversity through a national survey and longitudinal narrative interviews. Latent diversity refers to students’ underlying attitudes, mindsets, and beliefs that are not readily visible in engineering classrooms yet have the potential to contribute to innovation in engineering solutions (Godwin, 2017). This latent diversity is often undervalued or unacknowledged in engineering education with an emphasis on particular ways of being, thinking, and knowing aligned with rigid norms and expectations centered in engineering’s historic lack of diversity (Benedict et al., 2018; Danielak et al., 2014; Foor et al., 2007). We hypothesized that these cultural norms force students to conform to these expectations, thus reducing capacity for innovation and creating identity conflict that results in a lack of belonging and, ultimately, attrition. The goal of this project was to characterize latent diversity in incoming students to understand different subpopulations in engineering and how their experiences within the dominant culture of engineering affected their development as engineers to provide more inclusive ways of educating engineering students. The Purdue University Internal Review Board approved this study under protocol number 1508016383.

This study was executed in three consecutive phases: 1) instrument development; 2) characterization of latent diversity from a nationally representative sample; 3) longitudinal narrative interviews. For more details about the survey development, see Godwin et al. (2018). We used TDA to identify six data progressions among engineering students’ attitudinal profiles. These groups were later used to identify and recruit students to participate in bi-annual longitudinal narrative interviews designed to capture student identity trajectories. Our example focuses on the second phase of research focused on characterizing latent diversity. It demonstrates the type of person-centered characterizations that can be conducted in engineering education research.

Data Sources

We recruited U.S. institutions to participate based on a stratified sample of small (7,750 or fewer), medium (7,751 to 23,050), and large (23,051 or more) institutions in the United States (Godwin et al., 2018). We chose this sampling approach to ensure there was equal representation among the institution types (i.e., small, medium, and large), instead of an overrepresentation of large, public engineering institutions. The survey instruments were administered in common first-year engineering courses via paper-and-pencil format at 32 ABET-accredited institutions during the Fall 2017 semester. This timing captured students’ incoming latent diversity before being influenced by the process and culture of engineering education and captured students interested in a wide range of engineering disciplines. The data were digitized and cleaned by removing indiscriminate responses resulting in 3,711 valid responses.

Study Participants

Students indicated their self-reported demographics at the end of the survey instrument. These measures were designed to include a wide range of identities and included a multi-select question (Fernandez et al., 2016). The majority of participants identified as men (n = 2150), with other students identifying as a woman (n = 720), transgender (n = 70), agender (n = 14), or genderqueer (n = 49). Some students used the self-identify write-in option to indicate a gender not listed (n = 75), and some did not respond (n = 782). The majority of the students identified as White (n = 2089). The remaining students identified as Asian (n = 380), Latino/a or Hispanic (n = 347), African American/Black (n = 209), Middle Eastern or Native African (n = 65), Pacific Islander or Native Hawaiian (n = 34), Native American or Alaska Native (n = 49), used the self-identify write-in option to indicate another race/ethnicity not listed (n = 72), or did not respond (n = 793). We note that a large portion of students did not report demographics; often, students do not complete surveys due to fatigue, lack of time, or loss of interest. The survey was extensive, and some students dropped off in responding at the end of the survey. These reasons may account for students who did not report a gender identity or race/ethnicity, which were asked at the end of the survey. Students were allowed to select all that applied regarding their gender and race/ethnicity with which they identified. For example, out of the 2,089 (56%) students who identified as White, 291 (14%) of them also identified with another race/ethnicity. Additionally, students were asked to report their home ZIP code. These ZIP codes were plotted on the U.S. map to provide a geographic distribution of the overall first-year engineering student sample in the dataset, Figure 1.
An Overview of Topological Data Analysis

Generally, the field of topology refers to an area of mathematics, persistent homology, that relies on the study of shapes and structures to make sense of the world. However, more recently, topological data analysis (TDA) has emerged as a person-centered analysis that allows quantitative researchers to take an exploratory approach to draw insights from complex high-dimensional datasets (see Wasserman, 2018 for a detailed review). These shapes or structures allow the researcher to identify subgroups that may not have been considered when using traditional pairwise comparative methods that rely on researchers’ predetermination of groups (Lum et al., 2013). TDA differs from other person-centered approaches (i.e., Principal Component Analysis, multidimensional scaling, and clustering methods) based on its capabilities to capture geometric patterns that may have been ignored by other statistical methods (Lum et al., 2013). Instead, TDA provides a mapping of the data into a two-dimensional representation while maintaining the complex structure of the data. The resulting map is constructed from the shape and proximity of the data to itself, rather than a reference or seed point. As such, the mapping is not influenced by the measurement scale or random generation of multiple possible models. Topological methods are capable of handling the data by compressing the infinite data points into a finite, manageable network of nodes (Lum et al., 2013).

TDA has proven useful for wide-ranging applications in fields such as natural science, social science, and other computational fields. Studies have identified subgroups within breast cancer patients for targeted therapy (Lum et al., 2013), real-time air detection of bacterial agents (McGuirl et al., 2020), stratification of basketball positions above the traditional five characterizations of players (Lum et al., 2013), and player and team performance of football data (Perdomo Meza, 2015). Despite such broad and useful applications, TDA has been underutilized among engineering education and social science research except for two studies. Of the two studies, the first focused on distinguishing between normative and non-normative attitudinal profiles among incoming engineering students at four institutions (n = 2,916; Benson et al., 2017). In that study, TDA was useful for identifying groupings of students based on latent constructs rather than demographic variables. This study also provided evidence that some students’ attitudes differ from the normative group, especially in terms of feeling recognized as an engineer (Benson et al., 2017). The second study is the example used below. The specific results from this study have been published previously (see Godwin et al., 2019 for more detailed discussions of the specific study and TDA analysis); here, we focus on highlighting the ways in which the study illustrates the contributions afforded by person-centered approaches.

Analysis Steps in Topological Data Analysis

The process for conducting TDA for the example provided, including the sensitivity of these parameters is discussed in detail in our previous work (Godwin et al., 2019), but we highlight key details here for context. Before conducting TDA, several considerations must be made to minimize error and bias. First, methods to estimate missing data must be used to address potential errors when computing distance between points within the metric space (Lum et al., 2013; Godwin et al., 2019).

Figure 1: The map represents students’ self-reported home Zip Codes from a national survey. Each dot may represent more than one student. This image was generated in R (R Core Team, 2018) using the ggplot2 package (Wickham, 2009).
This specific consideration is especially important in social science research, where missing data are common. Next, if using latent variable measures, a typical practice in engineering education survey methods, a valid factor space must be created. This step involves verifying the study measurements through confirmatory factor analysis and generating factor scores based on the results of this factor analysis. Finally, the TDA algorithm parameters must be tuned to detect the underlying structure of the data. These parameters include the filtering method, clustering method, number of filter slices (n), amount of overlap of individuals, and cut height.

Interpreting TDA Maps
TDA generates a rich graphical representation of the data structure that consists of nodes and edges. The nodes represent multiple students, and the edges represent the overlap of student membership with other nodes. The size of the node indicates the number of students present in that area of the map. The color indicates the density of student responses within the node. Density indicates how similar student response patterns are across all dimensions. The resulting map is descriptive rather than inferential in group determination and differences between groups. It is particularly important to emphasize how TDA results are not a defined group but a representation of the structure of interconnectedness and difference within the data (Laubenbacher, 2019). This approach contrasts with other statistical methods that rely on specifying a probability at which a group is considered different or forcing data into deterministic groups (as in clustering and latent profile analysis). This approach allows for more nuanced relationships and patterns to be identified between groups and individuals while also preserving the individual’s response within the study. The resulting map shows data progressions, which are groupings of students and their relation to one another—the groupings were determined visually by the researchers from this descriptive method rather than from the method’s results.

Results
We created a 17-dimensional factor space based on the items used to measure students’ attitudes, mindsets, and beliefs concerning their STEM role identities (physics, mathematics, and engineering), motivation beliefs (control and autonomous regulation), epistemic beliefs, sense of belonging (engineering and engineering classroom), and two personality dimensions (neuroticism and conscientiousness). The results of TDA indicate six data progressions (i.e., A–F) for the characterization of latent diversity (Figure 2).

Figure 2: TDA map generated from the analyses, including groupings based on the distribution of the network of nodes. The colors shown in the map above represent the density of the map. The blue nodes denote a population of approximately 200 students, while the red nodes denote a smaller population of approximately three to five students. Our final parameters included a k-nearest neighbors filtering method, a single-linkage hierarchical agglomerative clustering method, 35 filter slices (n), a 50% overlap in data, and a 4.0 cut height (ε).
The resulting data progressions show descriptive differences across various factors, as shown in Figure 3. We provide these descriptive differences to illustrate the utility of this approach in producing data progressions that indicate unique student groupings and relationships within the dataset. We avoid conducting traditional variable-centered comparisons that reduce these data progressions to finite groups or clusters to avoid the knowledge claims we have critiqued in this paper. The discussion that follows provides the description of these data progressions as evidence for pragmatic validation or the utility of this method to reveal structure in complex, noisy data while still maintaining individual student responses (Walther et al., 2013).

First-year engineering students’ incoming attitudes and beliefs vary across the dimensions, but students also share similarities between the groups. Group A has the largest number of students \( n = 952 \) with moderately strong STEM role identities, motivation beliefs, epistemic beliefs, and a sense of belonging. In contrast, students in Group E \( n = 144.5 \), average partial membership because edges in Figure 2 are shared membership) shared moderately low beliefs about their STEM role identities and indicated low emotional stability. These qualities of Group E were similar to students identified in groups A, B \( n = 517 \), C \( n = 21 \), and D \( n = 27 \). Interestingly, students in Group F \( n = 51.5 \) had high emotional stability, STEM role identities, and a sense of belonging, but indicated low motivation beliefs (i.e., Controlled Regulation).

While additional similarities and differences can be drawn about each progression, such discussion is outside the scope of this paper. Rather, this paper focuses on the utility of person-centered approaches and how the results assert the assumptions of person-centered analysis. Thus, through our example, we wish to highlight how multiple subpopulations can exist among a sample and to explicitly draw attention to the power of taking an exploratory approach to data analysis, as opposed to methods that require defined hypotheses. By relying on the shape of the data, we were able to draw meaningful insights about the landscape of students’ attitudes, beliefs, and mindsets rather than binning students into groups based on demographic variables. Some data progressions show strong common patterns with small sample sizes (for example, Groups C and D). Many statistical techniques would ignore these groups in inferential testing because of this limitation. TDA allows these patterns to be detected and placed within the large dataset structure.

Figure 3: Spider plot of average student responses on factors within TDA. Measures include disciplinary role identity constructs: Math_Int = mathematics interest; Math_PC = mathematics performance/competence beliefs; Math_Rec = mathematics recognition; Phys_Int = physics interest; Phys_PC = physics performance/competence beliefs; Phys_Rec = physics recognition; Eng_Int = engineering interest; Eng_PC = engineering performance/competence beliefs; Eng_Rec = engineering recognition. Two factors from the Big Five Personality measure were used: Ocean_NC = conscientiousness and Ocean_Neu = neuroticism. Belonging was measured in two contexts: Bel_Fac1 = in the engineering classroom and Bel_Fac2 = in engineering as a field. Students’ motivation was captured by Motiv_CR1 = controlled regulation for engaging in courses; Motiv_CR2 = controlled regulation for completing course requirements; and Motiv_AR2 = autonomous regulation for completing course requirements. Students’ epistemic beliefs (Epis_Fac4) captured the certainty of engineering knowledge (i.e., absolute to emergent).
Implications of TDA Example

The TDA map (Figure 2) illustrates a wide variation among students’ attitudes, beliefs, and mindsets in engineering education. Students’ incoming latent diversity in U.S. engineering programs is not homogeneous. Additionally, results from this work often reveal small groups of student attitudes that would not emerge using variable-centered methods. This approach also allows new ways of framing research questions to understand general positions of students’ multidimensional attitudes, beliefs, and mindsets in relation to one another rather than forcing students into rigidly defined groupings based on probability. Importantly, this approach highlights how a one-size-fits-all approach to engineering education cannot adequately support the variation of students entering engineering programs with differing ways of seeing themselves in STEM. This variation includes students’ motivation to engage in courses and assignments, personalities, and beliefs about knowledge. Teaching all students in the same way or portraying a stereotype of the kind of person that becomes an engineer can communicate dominant norms that push students out of engineering (Benedict et al., 2018; Cech, 2015). This finding indicates how non-positivist epistemologies help frame research questions aimed at understanding how students build their understanding and knowledge of the world. In answering these questions, engineering educators can create experiences and reflection opportunities that support the diversity of students in the classroom.

Comparison to Traditional Methods

To further illustrate the contributions of TDA specifically and person-centered analyses generally, we compared the TDA results to more traditional statistical methods. For example, we examined the demographic representation of students within each data progression by gender identity and race/ethnicity individually and, where possible based on sample sizes, at the intersection of race and gender (i.e., White women, Black women, Asian women, Latinas, White men, Black men, Asian men, and Latinos). We did not find any differences in representation across data progressions using a chi-square test with a Holm-Bonferroni correction for gender, race/ethnicity, and intersectional groups of gender and race/ethnicity at the alpha value of 0.1. In this comparison, we emphasize that these tests rely on traditional statistical tests and do not consider individual responses with small numbers, particularly non-binary students across racial/ethnic categories and Native Hawaiian, Alaska Native, Native American, or other Pacific Islander students within the dataset.

However, when examining the data by traditional demographic groups using a Kruskal-Wallis test with a follow-up Dunn’s test, we did find statistically significant differences across the majority of the 17 factors. For example, we found that students’ controlled regulation motivation for engaging in engineering courses (Mov_CR1) showed significant differences by intersectional gender and race/ethnicity ($H(7) = 93.787, p < 0.001$) with a small effect size ($\eta^2 = 0.023; Cohen, 1988$) as shown in Figure 4. A post hoc Dunn’s test indicated that Black men and Latinos reported statistically significantly lower controlled regulation motivation ($p < 0.01$) than all other groups and that Black women and Latinas reported statistically significantly higher scores than all-male groups ($p < 0.001$).

From these results, one might conclude that Black and Latinx groups show average differences (i.e., lower motivation from external sources) by gender and race/ethnicity. However, a focus on demographics as explanations for student outcomes treats minoritized groups as homogeneous and often implicitly suggests race or gender as a causal variable for differences rather than other structural issues (Holland, 2008). Other analyses focused on investigating differences in latent constructs.

Figure 4: Differences in controlled regulation for classroom engagement by intersectional gender and race/ethnicity groups. Groups with large enough samples for comparisons include: WW = White women, AW = Asian women, BW = Black women, LW = Latinas, WM = White men, AM = Asian men, BM = Black men, and LM = Latinos.
by demographic characteristics often bin together groups of minoritized students to satisfy sample size requirements (i.e., all underrepresented racial and ethnic groups in engineering). This practice assumes that the experiences of minoritized students are a monolith and ignores the context as to why certain norms and inequities exist in engineering education.

Our TDA results, in contrast, indicate that these conclusions, based on a traditional approach to understanding gender and racial/ethnic diversity within our sample, oversimplify students’ responses within the data. Black and Latinx men and women have a wide range of attitudes and are equally represented in the data progressions within our results. This person-centered analysis allows for individual student differences to exist in complex large datasets. Additionally, the person-centered analysis allows for students who do not meet the sample size requirements for traditional statistical comparisons to be included within data analysis. Even with a large social science sample greater than 3,000 responses, many intersectional groups with small numbers were excluded from the demographic analyses presented. A person-centered analysis allows for inclusive representation where data analysis and conclusion include all responses rather than only those with dominant group status. Finally, this approach allows the structure and connections within the data to be uncovered.

Our example illustrates how engineering education researchers might reframe research questions and approaches from non-positivist epistemologies. Engineering culture and structures have been constructed as raced, classed, and gendered, and negatively affect all students. Engineering culture emphasizes and perpetuates demographic normativity of Whiteness, masculinity, competition, and emphasis on technical solutions (Akpanudo et al., 2017; Secules et al., 2018; Slaton, 2015; Uhlar & Secules, 2018).

Challenges and Opportunities for Person-Centered Analysis

Person-centered analysis can provide ways to ask research questions outside of the “to what extent” research questions or hypotheses often tested with quantitative research in (post)positivist paradigms. In our example, we examined the data structure with no a priori hypotheses about how gender, race/ethnicity, or other demographic factors might influence students’ incoming underlying attitudes, beliefs, and mindsets in engineering. TDA allowed us to find the emergent structure of relationships among student responses within the dataset and make generalized and descriptive conclusions about our results. This statistical approach provided ways to re-think the types of questions we asked of our data and the assumptions we brought to our analysis.

Additionally, these methods do not replace the need for qualitative, mixed methods, and multi-modal studies that have different purposes for generating knowledge. However, research methods focused on retaining the integrity of the individual within the dataset do provide opportunities to ask more complex and potentially novel research questions than the ones traditional quantitative methods can address. Person-centered analyses can help reveal relationships and patterns between large amounts of information by allowing discovery to be emergent. This approach aligns more closely with constructivist or even critical epistemologies. As discussed previously, many of our approaches to knowledge are implicitly biased, influenced by an epistemological racism and discrimination woven into the fabric of our social history (Scheurich & Young, 1997). While it is necessary to address these biases and acknowledge the reality of research, traditional variable-centric methods are often framed as “objective” and researchers often do not interrogate the assumptions of statistical tests, prohibiting them from making these types of considerations. Person-centered analysis alleviates some of the systemic discrimination within our research paradigms by challenging or eliminating a priori knowledge necessary for quantitative research methods. More importantly, these new approaches provide new insight and knowledge to bolster our current understanding.

Critical Alternatives to Person-Centered Approaches

While person-centered analyses can address many systemic issues embedded within traditional quantitative research methods, there remain related problems that person-centered analyses still cannot solve. As an option for other research approaches, we discuss critical methodologies, which are approaches that do not distinguish between the methodologies/methods and epistemologies used. Instead, these approaches frame methods and epistemologies in critical studies as inextricably linked. These approaches often used person-centered analysis in conjunction with qualitative data and have specific tenants and framings that make them unique from general person-centered methods.

Critical quantitative methodological approaches are quantitative methodological approaches consistent with critical epistemologies. There are numerous books and excellent studies that give a complete discussion of these approaches (see McCall, 2002; Oakley, 1998; Sprague & Zimmerman, 1989; Sprague, 2005; and a special issue by Gillborn, 2018). Nevertheless, we still include basic descriptions of these methodologies to illustrate other methodological framings of quantitative inquiry that directly challenge, refute, or build upon (post)positivist approaches to research. There are many bodies of critical quantitative research; here, we focus on just two that are consistent with Feminist and Critical Race Theory: FemQuant and QuantCrit. These two bodies formed separately with FemQuant forming and developing much earlier than the other. Both bodies have similar underlying tenets that provide ways to frame and conduct quantitative research critically.
Feminist-specific or not, critical quantitative approaches build upon general ideas of the feminist paradigm or feminist ethics, assuming systemic power relations beyond gender rule all aspects of social life through the organization of institutions, structures, and practices (Jagger, 2014). This organization of resources results in an unequal system of advantages and disadvantages (Acker, 1990; Ray, 2019). The feminist paradigm requires that research and praxis be positioned to promote a more just and equitable society (Collins & Bilge, 2016). In this approach, all methodologies—created and used by researchers who are also social participants—influence and can be influenced by the hierarchical social system in which research is situated (Oakley, 1998). This framing contrasts (post)positivist epistemology, which situates context (including the positional and influence of the researcher if this context is even acknowledged) as a weakness to the supposed objectivity of quantitative research (Hundleby, 2012; Sprague & Zimmerman, 1989). Harding (2016) wrote that reflexive incorporation actually makes quantitative research more objective or strong. She and others emphasized that the doing of research is messy, unpure, and laden with power relations, and the acknowledgment of these dynamics is essential (Harding, 2016; Hesse-Biber & Piatelli, 2012). Quantitative researchers need to explore, and make explicit, how their methodological use is complicit in that larger system of hierarchical power relations.

FemQuant and QuantCrit are based in these same basic epistemological framings but also advance their individual ethical positions to focus on race and racism (QuantCrit) and gender and sexism (FemQuant). Both approaches acknowledge the intersectional nature of multiple identities and different power relations associated with them. Still, each has developed from different historical and theoretical roots. QuantCrit maintains primary adherence to the first tenet of Critical Race Theory, that racism is a normal and ordinary component of daily life (Delgado & Stefancic, 2012), and that other power relations such as gender and class are used to support a larger racist project (Gillborn et al., 2018). FemQuant centers Feminist Theory with the incorporation of post-modern and post-feminist Intersectionality Theory (Codiroli Mcmaster & Cook, 2019), a partnership that highlights the many ways in which gender inequality exists and is enacted through the unique interactions of inequality due to gender, race, class, sexuality, disability, and more (Bowleg, 2008). While FemQuant and QuantCrit’s moral commitments and directions are different, their underlying reflexive methods and feminist philosophy are the same.

We present a very brief summary of these complex ideas here. In addition, we provide multiple brief engineering education-specific examples to situate our summary. Generally, the methodological and epistemological commitments of approaches can be summarized in six tenets (Major, Godwin, & Kirn, 2021) adapted from prior work (Bowleg, 2008; Gillborn et al., 2018; Hesse-Biber & Piatelli, 2012; Oakley, 1998; Sigle-Rushton, 2014; Sprague & Zimmerman, 1989):

1. Naturality – Domination is a central component of society that is not natural but rather is socially constructed and supported through multiple dimensions of difference or categories that quantitative research cannot be absent from. For example, accepted government categories of race and ethnicity that are typically recognized and used in quantitative research, such as in engineering education, have changed over time according to changing U.S. and broader global political motivations, not for natural reasons (Omi & Winant, 2014). Such motivations directly impact the ways in which racially diverse populations in engineering education are represented numerically.

2. Neutrality – Numbers cannot be neutral, but are rather numerically constructed representations of domination based on locally or globally rectified meanings relating to differences in human bodies. As such, neutrality often parallels naturalness in that what is deemed natural is often connected to political ideology (Oakley, 1998). In a similar example to that of naturality, the gender identity of students, such as those in engineering education, is often assumed according to physical traits such as the existence of sexual organs, or according to social performances of gender that relate to name, hair length and color, and even symbolic expressions of femininity or masculinity (Connell, 2009; Akpanudo et al., 2017). These considerations conflate sex and gender. Thus, like race/ethnicity, numerical representations of gender, and their relation to ones’ ability to be an engineer or participate in engineering education, are tied to non-neutral local or global beliefs about gender identity and gender performance.

3. Intersectionality – Inequality exists beyond one’s social position. In addition, inequality is multiplicative for persons experiencing multiple inequalities, and that multiplicative effect is not representable by simple variable positions, or identities. Rather, Intersectionality must be acknowledged and quantified as the unique experience it is, including its implications in engineering education, specifically. As one identity-specific example, one may want to consider the unique gendered-raced experiences of Black women as a combined numerical category rather than consider the additive or interactional effects that one who is Black or a woman might experience. In another more inequality-specific example, one instead may want to consider measures of the causes and implications of socioeconomic inequality itself rather than income itself (Major & Godwin, 2019).

4. Humanity – Data cannot speak for itself or act anthropomorphically in any other way. Rather, data is interpreted by researchers through their scientific understandings and global enculturation. There are thus implications to ones’ interpretations. For example, if researchers have results in which a control for race/ethnicity or gender is significant, they must consider the social processes associated with the tenets of naturality and neutrality. The data may suggest that race/ethnicity or gender creates statistical difference, but these are not casual variables. Instead, the researcher
should identify and discuss the systems of hierarchy and oppression that benefits White and male identified individuals (Holland, 2008; Gillborn, Warmington, & Demack, 2018).

5. Counter-Majority – Quantification unduly supports assumptions that there is an average, or dominant, group from which marginalized and minoritized individuals simply differ, and that quantification must also seek out counter-stories (quantitative or qualitative) which concurrently challenge those assumptions. Results of person-oriented methodologies, such as those we discuss in this work, may identify narratives that are counter to what may be extracted from traditional variable-oriented engineering education work. Similarly, small-n qualitative accounts of student experience may also identify quantitative components which have gone unaccounted or wrongly accounted (such as identity rather than inequality) in traditional accounts (Sigle-Rushton, 2014).

6. Reflexivity – Research is inherently political, biased, and essentialized, as shown through prior tenets. As such, disseminated research containing and striving for the equitable participation of diverse people, such as in engineering education, must be vocal about its association with a socially just political direction. It must also articulate how its data, methods, or results might otherwise support an oppositional direction. For example, one may want to openly disseminate details regarding their political directionality and positionality more broadly, and more specifically as it relates to methods of quantifying experience.

These tenets provide additional epistemic guidance for how quantitative research should be conducted from a critical epistemology. In this paper, we have focused on person-centered analyses as a novel quantitative method that could be used across non-positive paradigms. In conducting work aligned with critical epistemology and theory, person-centered methods may be used but must be grounded in these tenants and supplemented with other research methods.

Conclusions
In writing this paper, our goal is not to replace research traditions in qualitative methodologies with quantitative ones nor to indicate that all quantitative analyses must be person-centered. While methodologies and methods such as TDA, FemQuant, QuantCrit, and others provide more robust and nuanced understandings of relationships, groupings, experiences, and qualities within a dataset, ultimately, there are still individuals who can be misrepresented or unnoticed. As person-centered analyses are used to search for generalizable patterns among large, sprawling information, there remains space for over-generalizations or lack of representation in research findings. Even though the results from person-centered analyses are not restricted to a small number of dimensions or rigid relationships, an individual still may only partially fit within a pattern. Thus, results can give insight into a portion of their experience but may not fully capture the lived experiences of individuals.

We offer this discussion as a way to ask the engineering education research community to evaluate what we can ask and conclude from research aligned with non-positivist epistemologies. We hope that this discussion can expand the conceptualizations and operationalizations of new quantitative methods aligned with non-positivist epistemologies within engineering education research and open new frontiers within the field to serve students better and more inclusively.

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Competing Interests
The authors have no competing interests to declare.

Authors Contributions
Regarding this manuscript, AG conceptualized the idea for research, supervised all aspects of the research, conducted post-TDA analyses, wrote portions of each of the sections, and edited the document for flow and consistency. AG also wrote the sections describing the TDA analyses and results. JR wrote the introduction and epistemology section, as well as contributed throughout to link person-centered analysis to particular epistemological framings. In the example project described in this article, AT led and AG and JR assisted with data analysis and interpretation. BB contributed to the sections focused on
new methodological approaches in quantitative research and the example of TDA used in engineering education. BB also contributed to the data collection and interpretation of the national survey data, as well as the data collection and analysis of the longitudinal narrative interviews. HP wrote sections on person-centered analyses. JM wrote sections on critical quantitative methodologies. RC contributed to the challenges and opportunities associated with person-centered analysis. RC also contributed to the data collection and analysis of the longitudinal narrative interviews. SC edited the document, found references for claims made in the paper, and properly cited all references used.

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