Determination of the dynamical parameters of the Universe and its age

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ABSTRACT

We determine the density parameters \( \Omega_m \) of gravitating matter and \( \Omega_\Lambda \) of vacuum energy, by making a \( \chi^2 \) fit to nine independent astrophysical constraints. Paying rigorous attention to statistical detail, we find that the present best values are \( \Omega_m = 0.31 \pm 0.07, \Omega_\Lambda = 0.70 \pm 0.13 \), where these 1σ errors are approximately Gaussian (thus trivially convertible to whatever percentage confidence range desired). The total \( \chi^2 \) is 2.5 for 7 degrees of freedom, testifying that the various systematic errors included are generous. Since \( \Omega_m + \Omega_\Lambda = 1.01 \pm 0.15 \), it follows that the Einstein-de Sitter model is very strongly ruled out, that also any low-density model with \( \Omega_\Lambda = 0 \) is ruled out, and that a flat cosmology is not only possible, but clearly preferred. In the flat case we find \( \Omega_m = 0.31 \pm 0.04 \), from which it follows that the age of the Universe is \( t_0 = 13.7^{+1.2}_{-1.1}(0.68/h) \) Gyr.

1 INTRODUCTION

If the dynamical parameters describing the cosmic expansion were known to good precision, we would know whether the Universe is open or closed, or whether its geometry is in fact exactly flat as inflationary theory wants it. To know the answer we need at least (i) the Hubble constant \( H_0 \), usually given in the form \( H_0 = 100h \) km s\(^{-1}\) Mpc\(^{-1}\), (ii) the dimensionless density parameter \( \Omega_m \) of gravitating matter, comprising baryons, neutrinos and some yet unknown kinds of dark matter, and (iii) the density parameter \( \Omega_\Lambda \) of vacuum energy, related to the cosmological constant \( \Lambda \) by

\[
\Omega_\Lambda = \Lambda / 3H_0^2 .
\]  
A flat universe is defined by the condition

\[
\Omega_m + \Omega_\Lambda = 1 .
\]
In a previous publication (Roos & Harun-or-Rashid 1998) we tried to determine the preferred region in the \((\Omega_m, \Omega_\Lambda)\)-plane by combining three independent observational constraints and a value for \(H_0\). We then arrived at the conclusion that the standard Einstein-de Sitter model with \((\Omega_m, \Omega_\Lambda) = (1, 0)\) was ruled out, that the preferred density parameter ranges were \((0.2-0.4, 0.8-0.6)\), and that the geometry of the Universe therefore could be flat. Since then several other constrained fits with the same purpose have been published, however making use of only a small number of constraints (some recent ones are Efstathiou et al. 1998, Lahav & Bridle 1998; Lineweaver 1998; Tegmark 1998; Tully 1998; Waga & Miceli 1998; Webster 1998; White 1998).

The purpose of the present paper is to continue this pursuit of what kind of universe we are living in. In this study we make use of nine independent constraints of which only two are related to the constraints we used before, but they are now based on much larger data samples. As is certainly well known to the reader, the whole field is in a state of rapid expansion both in the quantity of observations made and in the quality of the results. As we shall see in the light of present data, our previous conclusions are being upheld, now indeed with higher confidence.

When \(H_0, \Omega_m\) and \(\Omega_\Lambda\) are known, the age of the Universe, \(t_0\), can be obtained from the Friedman-Lematre model as

\[
t_0 = \frac{1}{H_0} \int_0^1 dx \left[ (1 - \Omega_m - \Omega_\Lambda) + \Omega_m x^{-1} + \Omega_\Lambda x^2 \right]^{-1/2}.
\]

(3)

In Section 2 we describe the nine observational constraints entering our least-squares fit. In Section 3 we describe the results of our fit to these constraints, assuming that all reported observational errors as well as systematic errors are Gaussian (unless explicitly stated otherwise). Paying rigorous attention to statistical detail, we present our best value in the \((\Omega_m, \Omega_\Lambda)\)-plane as well as along the flat line Eq. (2). We then also use Eq. (3) to determine \(t_0\) in the plane and on the line. Section 4 contains a discussion of the effects of systematic errors and a comparison with related results not included in our fit. Section 5 summarizes our conclusions.

2 OBSERVATIONAL CONSTRAINTS

2.1 Cosmic Microwave Background Radiation

The observations of anisotropies in the CMBR are commonly presented as plots of the multipole moments \(C_\ell\) against the multipole \(\ell\), or equivalently, against the FWHM value
of the angular anisotropy signal. This power spectrum has most recently been analysed by Lineweaver (1998) and Tegmark (1998) in order to constrain possible cosmological models. In general, the theoretical models for the power spectrum may depend on up to 9 parameters.

What interests us here is the confidence region in the marginal subspace of the \((\Omega_m, \Omega_\Lambda)\)-plane. The 1\(\sigma\) region then appears to be approximately a wedge with straight-line edges and its apex \(A\) is at \((0.15, 0.77)\). The best fit point \(O\) is located asymmetrically within this wedge at \((0.45, 0.35)\) (Fig. 2 in Lineweaver 1998). However, the 1\(\sigma\) line plotted in this graph corresponds to \(\Delta \chi^2 = 1\) whereas it should be drawn at \(\Delta \chi^2 = 2.3\) in a two-dimensional parameter space. Denoting the distance from \(O\) to an arbitrary point \(P\) in the plane by \(r\), and the distance from \(O\) through \(P\) to the 1\(\sigma\) wedge line by \(r_0\), our constraint is therefore of the form

\[
\frac{r^2}{r_0^2}.
\]  

Fortunately the position of the point \(O\) with respect to the 1\(\sigma\) wedge lines is so remote from the range preferred by the collection of all constraints, that we do not have to worry about the asymmetry.

2.2 Gas fraction in X-ray clusters

Matter in an idealized, spherically symmetric cluster is taken to be distributed in two characteristic regions — a nearly hydrostatic inner body surrounded by an outer, infalling envelope. Outside this virial radius separating these regions, all matter is infalling with the cosmic mix of the components. A common definition of the virial radius is \(R_{500}\), the radius outside which the density drops below 500 in units of the critical density (Navarro, Frenk & White 1995) The baryonic component of the mass in galaxy clusters is dominated by gas which can be observed by its X-ray emission. Thus by measuring the gas fraction near the virial radius one expects to obtain fairly unbiased information on the ratio of \(\Omega_m\) to the cosmic baryonic density parameter \(\Omega_b\).

For this purpose Evrard (1997) has used a very large sample of clusters: the ROSAT compilation of David, Jones & Forman (1995) and the Einstein compilation of White & Fabian (1995). He has obtained a 'realistic' value of

\[
\frac{\Omega_m h^{-4/3}}{\Omega_b} \approx (11.8 \pm 0.7).
\]
This value includes a galaxy mass estimate of 20% of gas mass, and a baryon diminution
\( \Upsilon(500) = 0.85 \) at \( R_{500} \).

Taking \( \Omega_b = 0.024 \pm 0.006 h^{-2} \) from the low primordial deuterium abundance (Tytler, Fan & Burles 1996), and \( h = 0.68 \pm 0.05 \) from the analysis by Nevalainen & Roos (1998) who studied the metallicity effect of Cepheid-calibrated galaxy distances, one obtains

\[
\Omega_m = 0.36 \pm 0.09 ,
\]

which we use as our constraint. Note that the lower limit of \( \Omega_m \) only holds if dark matter is altogether non-baryonic. We shall come back to this problem in the Discussion.

### 2.3 Cluster mass function and the Ly\(\alpha\) forest

In theories of structure formation based on gravitational instability and Gaussian initial fluctuations, massive galaxy clusters can form either by the collapse of large volumes in a low density universe, or by the collapse of smaller volumes in a high density universe. This is expressed by the cluster mass function which constrains a combination of \( \Omega_m \) and the amplitude \( \sigma \) of mass fluctuations (normalized inside some volume). The amplitude \( \sigma \) is given by an integral over the mass power spectrum.

Weinberg et al. (1998) estimate \( \Omega_m \) by combining the cluster mass function constraint with the linear mass power spectrum determined from Ly\(\alpha\) data. For \( \Omega_{\Lambda} = 0 \) they obtain \( \Omega_m = 0.46^{+0.12}_{-0.10} \) and for a flat universe they obtain \( \Omega_m = 0.34^{+0.13}_{-0.09} \). In the region of interest of our fits, these results can be approximated by the relation

\[
\Omega_m + 0.18 \Omega_{\Lambda} = 0.46 \pm 0.08
\]

which we use as one constraint.

### 2.4 X-ray cluster evolution

Clusters of galaxies are the largest known gravitationally bound structures in the Universe. Since they are thought to be formed by contraction from density fluctuations in an initially fairly homogeneous Universe, their distribution in redshift and their density spectrum as seen in their X-ray emission gives precious information about their formation and evolution with time. Thus by combining the evolution in abundance of X-ray clusters with their luminosity-temperature correlation, one obtains a powerful test of the mean density of the Universe, \( \Omega_m \).
The results of Bahcall, Fan & Cen (1997) defines a $1\sigma$ band intersecting $\Omega_\Lambda = 0$ at $\Omega_m = 0.3 \pm 0.1$, and intersecting $\Omega_\Lambda = 1 - \Omega_m$ at $\Omega_m = 0.34 \pm 0.13$. This result can be summarized in the relation

$$\Omega_m = 0.195 \pm 0.11 + 0.071\Omega_\Lambda$$

which we use as one constraint.

The results of Eke et al. (1998) defines a $1\sigma$ band intersecting $\Omega_\Lambda = 0$ at $\Omega_m = 0.44 \pm 0.2$, and intersecting $\Omega_\Lambda = 1 - \Omega_m$ at $\Omega_m = 0.38 \pm 0.2$. This analysis uses data independent of Bahcall, Fan & Cen (1997) [we do not use Henry (1997) who analyses essentially the same data as Eke et al. (1998)]. The above result can be summarized in the relation

$$\Omega_m = 0.44 \pm 0.20 - 0.077\Omega_\Lambda$$

which we use as one constraint.

### 2.5 Gravitational lensing

Chiba & Yoshii (1999) have presented new calculations of gravitational lens statistics in view of the recently revised knowledge of the luminosity functions of elliptical and lenticular galaxies and their internal dynamics. They apply their revised lens model to a sample of 867 unduplicated QSOs at $z > 1$ taken from several optical lens surveys, as well as to 10 radio lenses. In sharp contrast to previous models of lensing statistics that have supported a high-density universe with $\Omega_m = 1$, they conclude that a flat universe with $\Omega_m = 0.3^{+0.2}_{-0.1}$ casts the best case to explain the results of the observed lens surveys.

The number of multiply imaged QSOs found in lens surveys is sensitive to $\Omega_\Lambda$. Models of gravitational lensing must, however, explain not only the observed probability of lensing, but also the relative probability of showing a specific image separation. The image separation increases with increasing $\sigma^*$, the characteristic velocity dispersion. Thus the results can be expressed as likelihood contour plots in the two-dimensional parameter space of $\sigma^*$ and $\Omega_m = 1 - \Omega_\Lambda$.

Instead of using the above quoted $\Omega_m$ value, we use the 68% likelihood contour in the two-dimensional parameter space of Fig. 8 of Chiba & Yoshii (1999), we integrate out $\sigma^*$, and we thus obtain the one-dimensional 68% confidence range

$$\Omega_\Lambda = 0.70 \pm 0.16$$
2.6 Classical double radio sources

There are two independent measures of the average size of a radio source, where size implies the separation of two hot spots: the average size of similar sources at the same redshift, and the product of the average rate of growth of the source and the total time for which the highly collimated outflows of that source are powered by the AGN. This outflow leads to the large scale radio emission. The two measures depend on the angular size distance to the source in different ways, so equating them allows a determination of the coordinate distance to the source which, in turn, can be used to determine pairs of $\Omega_m, \Omega_\Lambda$-values.

Daly, Guerra & Lin Wan (1998) have used 14 classical double radio galaxies with redshifts $z \leq 2$ to determine an approximately elliptical 68% confidence region in the $(\Omega_m, \Omega_\Lambda)$-plane centered at $(0.05, 0.32)$. In our fit this constraint is represented by a term in the $\chi^2$-sum of the form

$$[w^2 + z^2 - (\sigma_w^2 - \sigma_z^2)(w^2/\sigma_w^2)]/\sigma_z^2,$$

where $w$ and $z$ are the rotated coordinates

$$w = (\Omega_m - \Omega_{m,0}) \cos \theta + (\Omega_\Lambda - \Omega_{\Lambda,0}) \sin \theta$$

$$z = -(\Omega_m - \Omega_{m,0}) \sin \theta + (\Omega_\Lambda - \Omega_{\Lambda,0}) \cos \theta.$$  

The rotation angle is $\theta = 70^\circ.4$ and the $w$ and $z$ errors are $\sigma_w = 0.84, \sigma_z = 0.33$.

2.7 Supernovæ of type Ia

Type Ia supernovæ can be calibrated as standard candles, and have enormous luminosities (at maximum, $\sim 10^{10} L_\odot$). These two features make them a near-ideal tool for studying the luminosity-redshift relationship at cosmological distances. By calibrated standard candles, we mean there is some correctable dispersion among the magnitudes at light maximum. The light curves are very similar, but some are slightly wider and some narrower than average, with width being correlated with brightness at maximum light. The wider, intrinsically brighter objects are also bluer and have minor but recognizable spectral differences from the narrower, fainter, redder objects.

The factor relating brightness to redshift (as compared with 'nearby' SNe Ia) is a function of $\Omega_m$ and $\Omega_\Lambda$. If one has a set of nearby and a set of distant supernovæ all at about the same redshift, then a nearly-linear relationship between $\Omega_m$ and $\Omega_\Lambda$ can be
found. At a different redshift the slope is different (Goobar & Perlmutter 1995), and so for a set of SNe with various redshifts, say between 0.5 and 1.0, one can find a best-fit region in the $(\Omega_m, \Omega_\Lambda)$-plane.

The High-z Supernova Search Team (Riess et al. 1998) have used 10 SNe Ia in the redshift range 0.16 – 0.62 and 34 nearby supernovae to place constraints on the dynamical parameters and $t_0$. In the $(\Omega_m, \Omega_\Lambda)$-plane their 68% confidence region is an ellipse centered at (0.20, 0.65). In our fit this constraint is represented by a term in the $\chi^2$-sum of the same form as in Eq. (11) with the parameter values $\theta = 51^\circ.1$, $\sigma_w = 1.27$, $\sigma_z = 0.18$.

The Supernova Cosmology Project has so far published an analysis based only on their first discovered six supernovae (Perlmutter et al. 1998). Since then the team has discovered many more supernovae, and a preprint is now available describing the analysis and results of 42 supernovae in the high-redshift range 0.18 – 0.83. The redshifts have mainly been measured from the narrow host-galaxy lines, rather than the broader supernova lines. The light curves are compared to a standard 'template' time function, which is then time-dilated by a factor $1+z$ to account for the cosmological lengthening of the time scale. The conclusion in this paper that the Universe is today accelerating is very strongly influencing our fit. However, we don’t see it as our task to invent possible systematic errors affecting this conclusion, we take the results as they are presented.

In the $(\Omega_m, \Omega_\Lambda)$-plane the 68% confidence range is an ellipse centered at (0.75, 1.36). In our fit this constraint is represented by a term in the $\chi^2$-sum of the same form as in Eq. (11) with the parameter values $\theta = 54^\circ.35$, $\sigma_w = 1.3$, $\sigma_z = 0.15$.

3 RESULTS

We perform a least-squares fit to the above nine constraints using two free parameters $\Omega_m, \Omega_\Lambda$. The Hubble constant is not treated as a free parameter at this stage, but is fixed to the Nevalainen & Roos (1998) value. Actually it only enters when constraining the gas fraction in X-ray clusters, and there the $H_0$ error is negligible compared to the errors specific to this constraint.

For some of the constraints both a $1\sigma$ contour and a $2\sigma$ contour are given. This constitutes useful information on the relevant probability density function, which we take into account. When only a value and one error (of given confidence level) has been reported, we treat the probability density function as if Gaussian. When several error terms have been reported for one measurement, e.g. statistical and systematic, we always
add them quadratically.

To find the absolute minimum in the \((\Omega_m, \Omega_\Lambda)\)-plane and the 1\(\sigma\) and 2\(\sigma\) contours (at \(\Delta \chi^2 = 2.3\) and 6.2 up from the minimum, respectively, we use the standard minimization program MINUIT (James & Roos 1975). Although this program is widely used in high energy physics, we digress briefly in Sec. 4.4 to describe it to astronomers who might not know it.

The best fit value is then found to be

\[
\Omega_m = 0.31 \pm 0.07, \quad \Omega_\Lambda = 0.70 \pm 0.13, \quad \chi^2 = 2.5 .
\] (13)

In Fig.1 we plot the shape of the 1\(\sigma\) and 2\(\sigma\) contours. It is obvious from this figure that the errors of the parameters are Gaussian to a very good precision.

The above values can be added to yield

\[
\Omega_m + \Omega_\Lambda = 1.01 \pm 0.15 .
\] (14)

From this we conclude that (i) the data require a flat cosmology, (ii) the Einstein-de Sitter model is very convincingly ruled out, and (iii) also any low-density model with \(\Omega_\Lambda = 0\) is ruled out. Note that the results depend very strongly on the SNeIa data (Perlmutter et al. 1998). If we require exact flatness, the parameter values do not change noticeably but the errors are of course smaller in this one-dimensional space,

\[
\Omega_m = 0.31 \pm 0.04, \quad \Omega_\Lambda = 0.69 \pm 0.04, \quad \chi^2 = 2.5 .
\] (15)

Let us now substitute the above parameter values and the accurate Hubble constant value \(h = 0.68 \pm 0.05\) (Nevalainen & Roos 1998) into Eq. (3). [An equally accurate value, \(h = 0.69 \pm 0.05\), has been published by Giovanelli et al. (1997), obtained by using a Tully-Fisher template relation with a kinematical zero point from the best 12 galaxies in a sample of 555 galaxies in 24 clusters.]

Adding a 7.4\% \(H_0\) error quadratically to the density parameter errors in Eq. (13) propagated through the integral (3), we find as a value for the age of the Universe

\[
t_0 = 13.8 \pm 1.3 \ (0.68/h) \ \text{Gyr} .
\] (16)

For an exactly flat Universe only the errors change slightly to

\[
t_0 = 13.7^{+1.2}_{-1.1} \ (0.68/h) \ \text{Gyr} .
\] (17)
4 DISCUSSION

4.1 Systematic errors

With results as precise as those for the flat model, a question arising is, what about neglected systematic errors? To this we have a qualitative answer and a quantitative answer.

The constraints we use are indeed pulling the results in every direction. CMBR is orthogonal to the supernova constraints, the gravitational lensing constraint Eq. (10) is exactly orthogonal to the gas fraction in X-ray clusters Eq. (6), and the remaining constraints represent bands of several different directions. Thus if many systematic errors have been neglected, their total effect might largely be mutual cancellation.

This argument can be carried over to the systematic errors already included: they represent pulls in every direction, so their distribution must be quite random. Thus we think that it is justified to consider the total systematic error to be random, just like the total statistical error.

Let us now go to the quantitative answer. In the two-dimensional fit, Eq. (13), $\chi^2$ is 2.5 for 7 degrees of freedom, and in the one-dimensional fit, Eq. (15), $\chi^2$ is 2.5 for 8 degrees of freedom. These $\chi^2$ values are much too low for statistically distributed data. Thus we can conclude that the various errors quoted for our nine constraints are not statistical: they have been blown up unreasonably by the systematic errors added.

There are several reasons why this may have been done. One reason is that several of these constraints have come from fits in parameter spaces of higher dimension than just two. As a result, the quoted 1σ confidence regions in the $(\Omega_\text{m}, \Omega_\Lambda)$-plane which we use could be much too generous if they have not been properly rescaled to lower dimensionality.

Another reason is psychological. When one adds systematic errors one usually wants to have them big enough to play safe. But that means that one is really adding a 95% CL (Confidence Level) or 99% CL systematic error to a 68% CL statistical error. The result is then a too large total error, one not corresponding to a 68% CL error.

Note that under the assumption of a Gaussian probability density function, a 99% CL is not any 'safer' nor any more informative than a 68% CL. Both measures parametrize uniquely the width of one and the same function, and they translate into one another by the mere multiplication of a constant.

Thus we conclude that our errors are already very generous, and that there is no
motivation for blowing them up with further arbitrary systematic errors.

The most spectacular constraint is perhaps that of the Supernova Cosmology Project (Perlmutter et al. 1998). Consequently one may worry about whether unidentified systematic errors in those data might corrupt our conclusions. To study that, we left out the Supernova Cosmology Project constraint, and refitted the remaining 8 constraints. The result is then almost unchanged from Eq. (13),

$$\Omega_m = 0.30 \pm 0.09, \quad \Omega_\Lambda = 0.70 \pm 0.14, \quad \chi^2 = 1.7,$$

and the sum of the parameters is

$$\Omega_m + \Omega_\Lambda = 0.99 \pm 0.16.$$

Thus our result is robust.

4.2 Comparison with other data

There are some categories of data which we have not used, but to which it is nevertheless interesting to compare our results. Our selection consists only of observations quoting a value and an error, but in addition many interesting limits also exist.

Totani, Yoshii & Sato (1997) have tested cosmological models for the evolution of galaxies and star creation against the evolution of galaxy luminosity densities. They have found $\Omega_\Lambda > 0.53$ at 95% confidence in a flat universe. This is inside our one-sided 95% confidence limit $\Omega_\Lambda > 0.62$.

Willick et al. (1997) have been comparing Tully-Fisher data for 838 galaxies within $cz \leq 3000$ km/s from the Mark III catalog with the peculiar velocity and density fields predicted from the 1.2 Jy IRAS redshift survey. Taking $H_0$ in the range $55 \leq H_0 \leq 85$ (which is three times too wide a range) and the spectral index $n$ of the rms mass fluctuation power spectrum to be in the range 0.9–1.0, they conclude that $0.16 \leq \Omega_m \leq 0.40$, which covers our value in Eq. (15).

Danos & Pen (1998) have determined an upper limit to $\Omega_m$ from the apparent evolution of gas fractions in three rich clusters at $z > 0.5$. For the case of a flat universe they quote $\Omega_m < 0.63$ at 95% confidence. Our result from Eq. (15) is considerably more stringent, $\Omega_m < 0.38$ at 95% confidence.

Falco, Kochanek & Munoz (1998) have determined the redshift distribution of 124 radio sources and used it to derive a limit on $\Omega_m$ from the statistics of six gravitational
lenses. In a flat universe their best fit yields $\Omega_m > 0.26$ at 95.5% confidence, whereas our result (15) corresponds to the limit $\Omega_m > 0.24$.

Im, Griffiths & Ratnatunga (1997) use seven field elliptical galaxies to determine $\Omega_\Lambda = 0.64^{+0.15}_{-0.26}$, in good agreement with Eq. (13). We do not use this result as a constraint in the $\Omega_m, \Omega_\Lambda$-plane because it explicitly refers only to the flat model.

The data on X-ray clusters show enormous scatter and the various analyses published to date appear very model-dependent and mutually contradictory, divided into papers with low or intermediate $\Omega_m$-results and papers favoring the Einstein–de Sitter model with $\Omega_m = 1$. Above we made use of data from the lower set, quoting values ruling out $\Omega_m = 1$ by several standard deviations (Bahcall, Fan & Cen 1997; Henry 1997; Eke et al. 1998). The higher set we think is less useful because it exhibits very shallow likelihood functions weakly supporting almost any value in the range from 0.3 to 1.0 (within 90% CL) (Sadat, Blanchard & Oukbir 1998; Reichart et al. 1998; Blanchard & Bartlett 1998; Viana & Liddle 1998). An analysis of the possible reasons for the contradictions and the effects of known systematics can be found in Eke et al. (1998).

An upper limit to $\Omega_m$ can also be found by measuring the Sunyaev-Zel’dovich effect in rich clusters, which gives directly a lower limit to the total baryon fraction. Using the BBN prediction $\Omega_b h^2 = 0.013 \pm 0.002$ from the high deuterium abundance observation (Rugers & Hogan 1996), Myers et al. (1997) derive the upper limit $\Omega_m h \leq 0.21 \pm 0.05$. Here we do not understand what the limit means, because a limit cannot be a number with errors. If we take it to mean $\Omega_m h \leq 0.26$ at 68% CL, our corresponding limit using $h = 0.68 \pm 0.05$ is $\Omega_m h \leq 0.24$, thus in good agreement.

However, since the deuterium abundance is such a sensitive and contradictory issue, we would prefer to turn the logic of Myers et al. (1997) around. Let us take the value of $\Omega_m$ from Eq. (13), the value of $h$ from Nevalainen & Roos (1998), then the Sunyaev-Zel’dovich effect data can be used to derive a 68% confidence limit for $\Omega_b$,

$$\Omega_b h^2 > 0.015 \ .$$  \hspace{1cm} (20)

It may be interesting to go back to the Evrard (1997) analysis from which we quoted a value in Eq. (3) of Sec. 2.2. If instead, we derive a value for the corresponding quantity using the low deuterium abundance data of Tytler, Fan & Burles (1996), the value of $\Omega_m$ from Eq. (13), and the value of $h$ from Nevalainen & Roos (1998), we find

$$\frac{\Omega_m}{\Omega_b} h^{-4/3} = 10.0 \pm 2.9 \ .$$  \hspace{1cm} (21)
The large error reflects the uncertainty in the deuterium abundance, but given that we agree with Evrard’s value in Eq. (5), 11.8 ± 0.7.

Since we also determine a value for the age of the Universe, it is of interest to look at other recent \( t_0 \) determinations. Chaboyer (1998) refers to recent work on nucleochronology for an estimate of the oldest stars, \( t_{\text{stars}} = 15.2 \pm 3.7 \) Gyr, a value too inaccurate to be of much interest, and to several techniques which taken together determine the age of the oldest globular clusters (GC) to \( t_{GC} = 11.5 \pm 1.3 \) Gyr. Another estimate of the age of the oldest globular clusters is due to Jimenez (1998). He quotes the 99% confidence range \( t_{GC} = 13.25 \pm 2.75 \) Gyr, which translates into the 68% confidence range \( t_{GC} = 13.3 \pm 1.1 \) Gyr (in the abstract Jimenez (1998) calls this \( t_{GC} = 13.5 \pm 2 \) Gyr which we do not use). Thus there is some disagreement of the order of 1.8 Gyr between Chaboyer (1998) and Jimenez (1998), motivating us to take a mid-value and add a 0.9 Gyr systematic error,

\[
t_{GC} = 12.4 \pm 1.3 \pm 0.9 \text{ Gyr}.
\] (22)

In order to obtain \( t_0 \) from the GC value, however, one must add the time it took for the metal-poor stars to form. Estimates are poor due to the lack of a good theory, so Chaboyer (1998) recommends adding 0.1 to 2 Gyr. We presume that this is a 90% CL estimate, +1 ± 1 Gyr. Thus one arrives at the 68% confidence range

\[
t_0 = 13.4 \pm 1.3 \pm 0.9 \pm 0.6 \text{ Gyr} = 13.4 \pm 1.7 \text{ Gyr}.
\] (23)

Jimenez (1998) also finds an age of \( t_0 = 13 \pm 2 \) Gyr for the red elliptical galaxy 53W069 at \( z = 1.43 \). This value as well as Eq. (23) are in excellent agreement with the somewhat more accurate Eq. (17).

### 4.3 Cluster evolution and large-scale structure modelling

In the subsection on X-ray cluster evolution we pointed out that the data on X-ray clusters show enormous scatter and the various analyses published to date are very model-dependent and mutually contradictory. Although the authors of these studies defend the view that this is a good method to determine \( \Omega_m \), we think that the data would be better used if the value of \( \Omega_m \) would be taken from elsewhere, e.g. from our Eq. (17), in order to learn more about cluster evolution modelling.

The same comment applies to most of large-scale structure modelling. Not long ago Monte Carlo simulations of existing models were still used to defend the Einstein-de Sitter
model, which was disfavored or ruled out by almost every piece of information from other fields, and which we now rule out by some 16 standard deviations. The understanding of galaxy formation and cluster formation models would probably take a big step forward if one used as a starting point the dynamic parameters derived in this study.

4.4 Numerical analysis by MINUIT

MINUIT (James & Roos 1975) is conceived as a tool to find the minimum value of a multi-parameter function $F(X)$ in the space of the parameters $X$, and to analyze the shape of the function around the minimum. The principal application is foreseen for statistical analysis, working on $\chi^2$ or log-likelihood functions, to compute the best-fit parameter values and uncertainties, including correlations between the parameters. The space may be limited by physical restrictions on the allowed values of the parameters (constrained minimization). The function $F(X)$ need not be known analytically, but it is specified by giving its value at any point $X$. Minimization proceeds by evaluating $F(X)$ repeatedly at different points $X$ determined by the minimization algorithm used (there is a choice between different algorithms), until some minimum is attained (as defined by chosen convergence criteria). At that point the numerical covariance matrix found is used to give information on the errors (of chosen confidence) of the parameters. MINUIT is available from the CERN Program Library and is documented there as entry D506.

5 SUMMARY

With the purpose of determining the present best values of the density parameters $\Omega_m$ of gravitating matter and the density parameter $\Omega_\Lambda$ of vacuum energy, related to the cosmological constant $\Lambda$, we have made a $\chi^2$ fit to nine independent astrophysical constraints. These are:

(i) the anisotropies in the CMBR as expressed in terms of multipole moments (Lineweaver 1998; Tegmark 1998),

(ii) measurements of the gas fraction near the virial radius in X-ray clusters (Evrard 1997),

(iii) combining the cluster mass function with the linear mass power spectrum in Ly$\alpha$ data (Weinberg et al. 1998),

(iv-v) combining the evolution in abundance of X-ray clusters with their luminosity-
temperature correlation (Bahcall, Fan & Cen 1997; Eke et al. 1998),
(vi) gravitational lensing (Chiba & Yoshii 1998),
(vii) measurements of the average separation of hot spots in classical double radio sources
(Daly, Guerra & Lin Wan 1998),
(viii-ix) measurements of the luminosity distance of supernovae of type Ia via the magnitude-
redshift relation, as reported by the High-z Supernova Search Team (Riess et al. 1998) and
the Supernova Cosmology Project (Perlmutter et al. 1998).

Assuming that all reported observational errors as well as systematic errors are Gaussian (unless explicitly stated otherwise), and paying rigorous attention to statistical detail, we find that these constraints exhibit an agreement better than statistical, $\chi^2 = 2.5$ for 7 degrees of freedom. This indicates that the included systematic errors have been overestimated rather than vice versa. Our constrained fit finds the best point in the parameter space to be at

$$\Omega_m = 0.31 \pm 0.07, \quad \Omega_\Lambda = 0.70 \pm 0.13.$$  \hfill (24)

Fig. 1: The 1\(\sigma\) and 2\(\sigma\) statistical confidence regions in the ($\Omega_m, \Omega_\Lambda$)-plane are shown. The '+' marks the best fit: ($\Omega_m, \Omega_\Lambda$) = (0.31,0.70). The diagonal line corresponds to a flat cosmology.

In Fig.1 we plot the shape of the 1\(\sigma\) and 2\(\sigma\) contours. In an exactly flat cosmology the $\Omega_m$ value is the same, but the error is only 0.04.
Since $\Omega_m + \Omega_\Lambda = 1.01 \pm 0.15$, the important conclusions are that (i) the data strongly prefer a flat cosmology, (ii) the Einstein-de Sitter model is very convincingly ruled out, and (iii) also any low-density model with $\Omega_\Lambda = 0$ is ruled out.

Substituting the above parameter values into the Friedman-Lemaître model Eq. (3) for the case of a flat cosmology, and taking the Hubble constant to be known to an accuracy of 7.4% (Nevalainen & Roos 1998; Giovanelli et al. 1997), yields an age estimate for the Universe of

$$t_0 = 13.7^{+1.2}_{-1.1} \ (0.68/h) \ \text{Gyr} \ .$$

(25)

Note that this very accurate new result is purely cosmological, completely independent of estimates of the ages of stars or on the onset of star formation.

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