Numerical modelling of meniscus deformations and flows in a liquid bridge subjected to axial vibrations

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Abstract. Flows and deformations of free surface of isothermal liquid bridge under the influence of axial vibrations of finite amplitude and frequency in microgravity conditions are studied numerically by Volume of Fluid method. Investigation is carried out in the framework of full non-average approach. Numerical data on instantaneous and average velocity fields and instantaneous and average shapes of free surface of fluid at different vibration frequencies and amplitudes are obtained.

Vibrations are simple and low-energy means of control of inhomogeneous fluids behavior. One of important manifestation of vibration influence on fluids is generation of average flows. Investigation of generation of average flows was started by the work [1] of Rayleigh, where generation of average flow in the Stokes boundary layer near rigid surface during excitation of standing acoustic wave in the channel was studied. Schlichting obtained ([2]) effective boundary conditions for the tangential component of average velocity on rigid boundary based on the analysis of viscous boundary layer near oscillating rigid surface. In work [3] by Longuet-Higgins the generation of average flows near free surface due to the one-dimensional waves propagating on the free surface was studied. Generalization of the results of Longuet-Higgins for the case of two-dimensional wave fields was carried out in [4]. Theoretical, numerical and experimental study of flows in isothermal fluid near vibrating surface is carried out in [5], where method of averaging of vibrational flow during theoretical analyze is suggested, averaged solution of momentum equations is obtained, comparison with experimental results is performed. Zharikov et al. [6] performed an experimental study of the liquid motion near vibrating bodies imitating different techniques of growth from the liquid phase; in particular, the growth of NaN₃ crystal subject to axial vibrations, in Cz-configuration, with small amplitudes (from 0.005 to 0.5 mm) and frequencies up to 100 Hz. This experimental study showed that vibration-driven convection has an important effect on the shape of the liquid-solid interface, and demonstrated that the vibrational flow would be able to counteract the buoyancy-driven flow. Works [7-10] are devoted to the investigation of the vibration effect on flows and heat mass transfer during crystal growth out of melt by floating zone method. In these papers pulsational and average flows, induced by small-amplitude high-frequency axial vibrations of one or both rods and the interaction of this flows with thermocapillary flow induced by the dependence of surface tension on temperature were studied on the basis of average approach. In the present work the flows and deformations of free surface of isothermal liquid bridge under the influence of axial vibrations of finite amplitude and frequency in microgravity conditions are investigated numerically by Volume of Fluid method.
1. Problem statement
Let us consider the interface deformations and flows in a cylindrical liquid bridge, surrounded by a cylindrical layer of air, situated between two horizontal rigid plates (Fig. 1). The system is subjected to periodic forcing with finite amplitude $a$ and finite circular frequency $\omega$ along the bridge axis (periodically changing gravity force or synchronous vibrations of both rigid plates). Gravity force is absent. Isothermal problem is under consideration.

![Figure 1 Problem configuration](image)

2. Governing equations
Equations, describing the motion of isothermal viscous uncompressible fluid under the influence of axial vibrations in cylindrical coordinate system have the following form:

For radial component of velocity $u$:

$$
\frac{\partial}{\partial t}(\rho u) + \frac{1}{r} \frac{\partial}{\partial z}(r \rho u v) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho u^2) - \rho \frac{w^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial z}\left[r \mu \left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)\right] + \frac{2}{r} \frac{\partial}{\partial r}\left[r \mu \frac{\partial u}{\partial r}\right] - 2 \mu \frac{u}{r^2}
$$

(1)

For azimuthal velocity $w$:

$$
\frac{\partial}{\partial t}(\rho w) + \frac{1}{r} \frac{\partial}{\partial z}(r \rho u w) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho v w) = \frac{1}{r} \frac{\partial}{\partial z}\left[r \mu \frac{\partial w}{\partial z}\right] + \frac{1}{r^2} \frac{\partial}{\partial r}\left[r^3 \mu \frac{\partial w}{\partial r}\right] - \rho \frac{w w}{r}
$$

(2)

For axial component of velocity $v$:

$$
\frac{\partial}{\partial t}(\rho v) + \frac{1}{r} \frac{\partial}{\partial z}(r \rho v^2) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho v u) = \frac{\partial p}{\partial z} + \frac{2}{r} \frac{\partial}{\partial r}\left[r \mu \frac{\partial v}{\partial z}\right] + \frac{1}{r} \frac{\partial}{\partial r}\left[r \mu \left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right] - a \omega^2 \rho \sin \omega t
$$

(3)

Here $\rho$ is the density, $\mu$ is the dynamic viscosity of the melt.

Continuity equation reads:

$$
\nabla \cdot \vec{v} = \frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r} = 0
$$

(4)

3. Numerical method
The calculations were performed using Volume of Fluid method where the description of multiphase fluxes is carried out by introducing volume fraction of each fluid; volume fractions represent the space, occupied by each phase and conservation of mass and momentum laws are fulfilled for each phase.

The $q^{th}$ fluid volume fraction in the cell is denoted as $\alpha_q$, then the following three conditions are possible:
\[ \alpha_q = 0: \text{The cell is empty (of the } q^{th} \text{ fluid).} \]
\[ \alpha_q = 1: \text{The cell is full (of the } q^{th} \text{ fluid).} \]
\[ 0 < \alpha_q < 1: \text{The cell contains the interface between the } q^{th} \text{ fluid and one or more other fluids.} \]

The properties appearing in the transport equations are determined by the presence of the component phases in each control volume. In a two-phase system, for example, if the phases are denoted by the subscripts 1 and 2, and if the volume fraction of the second of them is being tracked, the density in each cell is given by \[ \rho = \alpha_2 \rho_2 + (1 - \alpha_2) \rho_1. \]

The volume fraction equation will not be solved for the primary phase; the primary-phase volume fraction will be computed based on the following constraint: \[ \sum_{q=1}^{n} \alpha_q = 1. \]

The tracking of the interface(s) between the phases is carried out by the solution of a continuity equation for the volume fraction of one (or more) of the phases. For the \( q^{th} \) phase, this equation has the following form:
\[
\frac{\partial}{\partial t} (\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \mathbf{v}_q) = \sum_{p=1}^{n} (\dot{m}_{pq} - \dot{m}_{qp})
\]  
(5)

where \( \dot{m}_{pq} \) is the mass transfer from phase \( q \) to phase \( p \) and \( \dot{m}_{qp} \) is the mass transfer from phase \( p \) to phase \( q \).

The no-slip condition is imposed for the at the rigid boundaries:
\[ r = R_{in}, \; z = 0, L: \; u = v = 0 \]

For volume fractions on the rigid boundaries a fixed value for fluid and for air, conditions of fixed contact line are imposed:
\[ z = 0, L: \; 0 < r < R_c: \; \alpha_{fluid} = 1, \; \alpha_{air} = 0 \]
\[ r > R_c: \; \alpha_{fluid} = 0, \; \alpha_{air} = 1 \]  
(6)

The calculations were carried out using the commercial package ANSYS Fluent. The analysis was limited by axisymmetric regimes and so 2ddp version of package was used. Implicit scheme of the second order of discretization by time was used. For better resolution the computational grid was condensed near free surface and rigid boundaries. In order to determine the parameters of computational grid, optimal from the view point of computation precision and computational time expenses the test runs with the use of different grids were carried out. On the basis of these computations the grid 100x100 was chosen for main computations.

The influence of vibrations was taken into account by introducing a periodically varying volumetric force in Source Terms. The fields of average velocity and other average quantities were calculated by averaging the functions over the five periods of vibrations in User Defined Function.

4. Numerical results
Calculations were carried out for parameters of fluid, corresponding to growth of silicon crystals: density was \( \rho = 2.53 \text{ g/cm}^3 \), dynamic viscosity was \( \mu = 8.855 \times 10^{-3} \text{ g/cm/s} \), surface tension was \( \gamma = 720 \text{ dyn/cm} \). Bridge radius was \( R_c = 0.5 \text{ cm} \), height \( L = 1 \text{ cm} \). Vibration parameters were varied.

In Figs. 2 - 4 the average velocity vector fields and average shapes of free surface at the vibration amplitude 0.15 mm and different vibration frequencies are presented.
The calculations show that the vibrations generate the waves on free surface, propagating from the oscillating rods toward zone center, the amplitude and number of waves depend on the amplitude and frequency of vibrations. The average flow, generated by the surface waves, is directed near free surface from the rigid rods to zone center. At frequency 10 Hz (Fig. 2) the average flow in the bridge consists of two toroidal surface-wave-induced vortices of large size and intensity with the flow direction near free surface from the rigid rods toward zone center and two small toroidal vortices, located near the rigid rods close to the axis, these vortices are generated by the Schlichting mechanism of generation of average flow.

With the increase of vibration frequency, the number of waves on the free surface increases, the flow becomes more intensive, the Schlichting vortices are suppressed by the vortices induced by surface waves.

![Figure 2](image1.png) ![Figure 3](image2.png) ![Figure 4](image3.png)

**Figure 2** Average velocity vector fields and meniscus deformations for the vibration amplitude equal to 0.15 mm and frequency 10 Hz

**Figure 3** Average velocity vector fields and meniscus deformations for the vibration amplitude equal to 0.15 mm and frequency 50 Hz

**Figure 4** Average velocity vector fields and meniscus deformations for the vibration amplitude equal to 0.15 mm and frequency 100 Hz
In Fig. 5 the evolution of instantaneous shape of free surface during the vibration period is shown. As one can see, the waves propagate along the free surface.

![Figure 5](image)

**Figure 5** Evolution of instantaneous shape of free surface during the vibration period

### 5. Conclusion

Meniscus deformations and flows in isothermal liquid bridge subjected to axial vibrations of finite amplitude and frequency in conditions of microgravity have been studied numerically. The calculations were performed in non-averaged approach for fluid parameters which correspond to silicon melt. Numerical data on instantaneous and average velocity fields and instantaneous and average shapes of the meniscus are obtained for different frequencies and amplitudes of vibrations. It is found that under the influence of vibrations the average flow in the form of toroidal vortices of Schlichting origin with the direction near rigid rods from the free surface toward the bridge axis is generated near the rigid rods. Additionally, the rods vibrations generate the waves on the free surface, propagating from the rods toward the bridge center. The average flow induced by surface waves is directed near free surface from oscillating rods toward the cavity center. With the increase of vibration frequency the relative contribution of the Schlichting mechanism of average flow generation decreases, and that of the surface waves mechanism increases. During crystals growth by the floating zone method with ring heater located in the zone center, the thermo-capillary flow develops in the zone; it has the form of two toroidal vortices with the direction of flow near free surface from the heated center of zone toward the cold rigid rods. The average flow, induced by surface waves mechanism of generation, represent two toroidal vortices with the direction, opposite to thermocapillary flow. This makes promising the use of axial vibrations for the improvement of quality of grown crystals.

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