On the Distribution of Neutral and Charged Pions
through the Production of a Classical Pion Field

A. A. Anselm *

Petersburg Nuclear Physics Institute, Gatchina, 188 350 St. Petersburg, Russia

Myron Bander†

Department of Physics, University of California, Irvine, California 92717, USA

Abstract

High energy reactions may produce a state around the collision point that is best described by a classical pion field. Such a field might be an isospin rotated vacuum of the chiral $\sigma$-model or, as discussed in this work, a solution of the equations of motion resulting from the coupling of fields of this model to quarks produced in the collision. In such configurations all directions in isospin space are allowed leading to a sizable probability of events with, essentially, only charged particles (Centauros) or all neutral particles (anti-Centauros). (In more common statistical models of multiparticle production, the probability of such events is suppressed exponentially by the total multiplicity.) We find that the isospin violation due to the mass difference of the up and down quarks has a significant effect on these distributions and enhances the production of events consisting predominantly of neutral particles.

*Electronic address: anselm@lnpi.spb.su
†Electronic address: mbander@funth.ps.uci.edu
In recent years several authors \[1–6\] have suggested that the celebrated Centauro events \[7\], in which no \(\pi^0\)'s have been observed versus a large number of charged hadrons, might be explained by the production at these high energies of a classical pion field; an interesting example is the "disoriented chiral condensate" \[4,5\]. The idea is that such a process, considered event-by-event, would correspond to the field being along a given Cartesian isospin direction. In events where the isospin is oriented (almost) parallel to the 3-rd axis one would expect mainly neutral pions while in events where the isospin lies in the perpendicular plane predominantly charged pions would be produced. Let \((\pi_1, \pi_2, \pi_3)\) be the three Cartesian isotopic amplitudes of the classical pion field. As all the orientations are equivalent, the distribution in the amplitude \(\pi_3\) is

\[
\text{d}w \sim \text{d}\pi_3; \quad \pi^2 = \pi_1^2 + \pi_2^2 + \pi_3^2 = \text{const}.
\]  

(1)

The number of neutral pions, \(n_0\), is proportional to \(\pi_3^2\) while the total number of produced pions, \(n = n_0 + n_+ + n_- \sim \pi^2\). With \(f = n_0/n\), the fraction of neutral pions, one has from (1),

\[
\text{d}w = \frac{\text{d}f}{2\sqrt{f}};
\]

(2)

this distribution is normalized to unity.

Obviously (2) predicts many more events with a small number of neutrals than do usual statistical mechanisms for pion production. In the latter case one expects \(\text{d}w/\text{d}f\) to peak at \(f = 1/3\) \((n_+ = n_- = n_0 = 1/3 \text{ as } n \to \infty))\) and to decrease exponentially with \(n\) as \(f\) deviates from this value. The distribution (2) corresponds to the limit \(n \to \infty\) and gives for the relative number of events with the fraction of neutrals less than \(f\)

\[
P(f) = \int_0^f \frac{\text{d}w}{\text{d}f'} \, df' = \sqrt{f}.
\]

(3)

For a typical Centauro event \(f \sim 1/100\) and \(P \sim 10\%\). This seems to be a reasonable number as the five "classic" Centauros represent about 1\% of events with appropriate energies \[7\].

At the other end of the spectrum, near \(f = 1\), the probability of an event having an anomalously large fraction of \(\pi^0\)'s is
1 - P(f) = 1 - \sqrt{f} \sim \frac{1}{2}(1 - f). \quad (4)

We do not have the square root enhancement exhibited in (3) and instead we find a linear
dependence at the end of the spectrum; however, there still is a finite probability of finding
events with a large number of \pi^0's. It is possible that such “anti-Centauro” events have
been observed \cite{8} and we shall present a mechanism for enhancing their probability over
that of (4).

The distribution (2) results from exact isospin symmetry. At the quark level this sym-
mmetry is rather strongly violated due to the up-down quark mass difference, \( m_u \neq m_d \). In
this Letter we shall demonstrate that this mass inequality can enhance the probability of
anti-Centauros.

2. A class of solutions for the pion field whose dynamics are governed by a non-linear
chiral Lagrangian was presented in Ref. [1]. The results of that work may be understood in
the following simple way. The Lagrangian is

\[ \mathcal{L} = \frac{f_\pi^2}{2} \text{Tr} \left( \partial_\mu U \partial^\mu U^\dagger \right), \quad (5) \]

where \( f_\pi = 93 \text{ MeV} \) and the unitary matrix \( U \) is connected to the pion fields by

\[ U = \exp \left( \frac{i \tau \cdot \pi}{f_\pi} \right). \quad (6) \]

For the particular form

\[ U = \exp[i \tau_3 \theta(r, t)] \quad (7) \]

the Lagrangian (3) leads to the free equation of motion

\[ \partial^2 \theta = 0. \quad (8) \]

For constant unitary matrices \( V_L \) and \( V_R \) a generalization of (7) is

\[ U = V_L^\dagger \exp(i \tau_3 \theta)V_R; \quad (9) \]
this is a general class of solutions which has been studied in Ref. [1]. All other known
solutions [3,9] are particular cases of (9).

At large distances from the collision point we require the normal structure of the vacuum,
i.e. $U = 1$. Likewise we will pick solutions in which $\theta(r, t) \to 0$ as $r \to \infty$. This forces
$V_L = V_R$ and the solutions (3) reduce to isotopic rotations of (7). In other words, (9) takes
the form

$$U = \exp[i \tau \cdot n \theta(x)]$$

(10)

for some direction $n$ in isotopic spin space. A possible scenario for the production of a
classical pion field discussed in [4,5] is that inside a certain volume around the collision
point a state corresponding to a constant (in the volume) $\theta$ is produced. This state is
degenerate with the normal vacuum (in the limit $m_\pi = 0$) but is rotated with respect to it
in isotopic spin space. In [4,5] this situation is referred to as “disoriented chiral condensate”.
It follows from (10) that any solution of (8) describes chiral dynamics.

We now introduce interactions of pions with quarks keeping in mind that the pion field is
the chiral phase of the quark field [10]. In the presence of pion fields the quark fields should
be modified

$$q_L(x) \to \exp[i \frac{1}{2} \tau \cdot n \theta(x)] q_L$$

(11)

$$q_R(x) \to \exp[-i \frac{1}{2} \tau \cdot n \theta(x)] q_R.$$

The quark mass terms give rise to the quark-pion interaction Hamiltonian

$$\mathcal{H} = m_u \bar{u}u + m_d \bar{d}d \to \bar{q} \exp\left(i \frac{1}{2} \tau \cdot n \theta\right)(m_+ + m_- \tau_3) \exp\left(i \frac{1}{2} \tau \cdot n \theta\right) q + \text{h. c.},$$

(12)

where $m_\pm = \frac{1}{2} (m_u \pm m_d)$. For the solution (10)

$$\mathcal{H} = \bar{q}(m_+ + m_- \tau_3) q - (1 - \cos \theta)\bar{q} (m_+ + m_- \tau_3 \cdot n) q + \sin \theta \bar{q} i \gamma_5 (m_+ \tau \cdot n + m_- n_3) q.$$

(13)
In the normal vacuum (13) accounts for the pion mass term through the existence of the chiral condensate $\langle \bar{q}q \rangle \neq 0$. From (13) one sees that 
\[ m_\pi^2 = -m_+ \langle \bar{q}q \rangle / f_\pi^2, \quad \pi = f_\pi \theta. \]

The distributions, in the parameters $\theta$ and $n$, of a classical pion field produced in a high energy collision are expected to depend on a production temperature $T$ and have the form
\[
dw \sim \int \prod_x d\theta(x) \exp \left( -\frac{1}{T} \int d^3x \mathcal{H} \right) d\mathbf{n} \delta(n^2 - 1). \tag{14}
\]

If the quark density in the collision is not too high $\langle \bar{q}q \rangle$ should be set equal to its usual vacuum value. Expanding around $\theta = 0$ (14) becomes
\[
dw \sim \int \prod_x d\theta(x) \exp \left( -\frac{m_+ |\langle \bar{q}q \rangle|}{2T} \int d^3x \theta^2 \right) d\mathbf{n} \delta(n^2 - 1). \tag{15}
\]

For $T = T_c \sim 140$ MeV [11] and a volume $V \sim 100$ fm$^3$ the above is $\exp[-4 < \theta^2 >]$; large values of $\theta$ will not be excited. However, after the functional $\theta$ integration the distribution in isospin directions remains uniform leading immediately to (2).

3. Our critical assumption is that in the high density medium created by such collisions the quark density and other bilinears in $q, \bar{q}$ acquire classical values that may be comparable to or larger than the vacuum chiral condensate $\langle \bar{q}q \rangle \simeq -(250$ MeV)$^3$. From the explicit dependence of (13) on $n_3$ we see that isospin rotation symmetry is broken. We consider two possibilities: either $I(x) = \langle \langle \bar{q} \gamma_5 q \rangle \rangle \neq 0$ or $P(x) = \langle \langle \bar{q} \gamma_5 q \rangle \rangle \neq 0$, in addition to $S(x) = \langle \langle \bar{q}q \rangle \rangle \neq 0$ and are sizable. $\langle \langle \cdots \rangle \rangle$ denotes the averaging over quantum fluctuations and we allow for a smooth (on the microscopic scale) position dependence. The value of $S(x)$ may differ significantly from the vacuum value of $\langle \bar{q}q \rangle$.

We first consider the first case, $I(x) \neq 0$; although it has less interesting consequences it is simpler to analyze. The functional integration over $\theta(x)$ in (14) (in the quadratic approximation) yields
\[
dw \sim \frac{1}{\sqrt{|m_+ S(x) + m_- I(x) n_3^2|}} dn_3. \tag{16}
\]

For the dependence of the above on $n_3$ to be significant it is necessary for the second term in the square root to be comparable in magnitude to the first one. This is, however, unlikely as their ratio is (even for $f = n_3^2 = 1$)
\[
\frac{m_- I(x)}{m_+ S(x)} = \frac{m_u - m_d}{m_u + m_d} \langle \langle u\bar{u} - d\bar{d} \rangle \rangle;
\]

(17)

with \(m_u - m_d/m_u + m_d \sim -0.3\) and the second factor less than unity the \(n_3\) dependence will be insignificant. We reach the same conclusion if we allow other components of \(\bar{q}\tau q\) to acquire some classical value.

The situation is significantly different if we assume that \(P(x)\) has a sizable value. Below, we shall return to see whether this is feasible, but first discuss the consequences of this assumption. We are now asked to evaluate

\[
dw \sim \int \prod_x d\theta(x) \exp \left\{ \frac{1}{T} \int d^3x \exp [m_+ S(x)(1 - \cos \theta) - m_- n_3 P(x) \sin \theta] \right\} dn_3(n^2 - 1).
\]

(18)

The exponent has a minimum for a non-zero \(\theta\) obtained from \(\tan \theta = m_- n_3 P(x)/m_+ S(x)\). The functional integral can be done (again in a quadratic approximation) and, aside from a prefactor, yields

\[
dw \sim \exp \frac{1}{T} \int d^3x \left[ + \sqrt{m^2_+ S^2(x) + m^2 P^2(x)n_3^2} + m_+ S(x) \right] dn_3.
\]

(19)

Although we could analyze this result it is simpler to consider the situation where \(|m_- P/m_+ S| < 1\). Keeping only the first term in the expansion of the square root we obtain (ignoring, in the case \(S(x)\) is positive, terms not depending on \(n_3\))

\[
dw \sim \exp \left[ \frac{1}{2T m_+} \int d^3x \frac{P^2(x)}{|S(x)|} n_3^2 \right] dn_3.
\]

(20)

Remembering that \(f = n_3^2\) we find

\[
dw = N(A) e^{Af} \frac{df}{2\sqrt{f}},
\]

(21)

where

\[
A = \frac{1}{2T m_+} \int d^3x \frac{P^2(x)}{|S(x)|},
\]

(22)

and the normalization factor
\[ N^{-1}(A) = \int_0^1 dx e^{Ax^2}. \] (23)

Evidently the change in the distribution is important only if \( A \) is large enough. In this case the distribution (21) has a minimum at \( f = 1/2A \) and, contrary to the situation described by (2), grows as \( f \) approaches 1. For \( A >> 1 \) an approximate evaluation of (23) yields

\[ dw \simeq A e^{-(1-f)A} \frac{df}{\sqrt{f}}. \] (24)

This distribution has a peak at \( f = 1 \) and is enhanced near that value by a factor \( 2A \) over that of (2) making anti-Centauros more probable.

We shall now try to estimate possible values for \( A \). While \( |S(x)| \) presumably coincides with the quark density \( \rho(x) \), \( P(x) \) can be represented in the form

\[ P(x) = \xi R\sigma(x) \cdot \nabla \rho(x). \] (25)

Here \( \sigma(x) \) is some spin density, \( R \) is a characteristic linear size of the effective volume (or characteristic time before hadronization) and \( \xi \) is a constant, probably smaller than one.

Integrating (22) we get

\[ \int d^3x \frac{P^2(x)}{|S(x)|} = 4\pi R^2 r \frac{\xi^2 \rho^2 R^2}{r^2} \frac{1}{\rho} = \frac{4}{3} \pi R^3 \frac{3R}{r} \xi^2 \rho. \] (26)

We use \( r \) as a characteristic length for the gradient; this variation in density is likely to be confined to the surface of the quark matter produced in the collision. We assume that the volume over which \( P(x) \) does not vanish is \( 4\pi R^2 r \). The spin densities are averaged approximately to unity. Thus for the parameter \( A \) we have:

\[ A = \frac{\xi^2}{2T} \frac{m^2}{m_+} \frac{3R}{r} N \simeq \frac{1}{70} \frac{R}{r} \xi^2 N. \] (27)

Here \( N = 4\pi R^3 \rho/3 \) is the number of quarks produced. We believe one could expect \( N \geq 200 \) in a sphere of \( R \simeq 3 \) fm (note that for the vacuum \( \rho = <\bar{q}q> = 2 \) fm\(^{-3} \), so that \( N \simeq 200 \)). For \( R/r \simeq 5 \) we find \( A \sim 15\xi^2 \) and for \( \xi \geq 0.25 \) \( A \) will be sizable enough to enhance the probability of anti-Centauros. Note that \( \xi \leq 0.8 \) is required for the approximation in going
from (19) to (20). We are well aware of the crudeness of these estimates and the purpose of this exercise was only to show that values of $A \geq 1$ are not excluded.

The whole change in the distribution of neutrals is due to the violation of isotopic spin invariance; the parameter $A$ in (21) is proportional to $(m_u - m_d)^2$. Can we claim that the anti-Centauro events are caused by the mass difference of light quarks?

We would like to thank J. D. Bjorken for interesting discussions. One of us (A. A.) would like to thank the Physics Department of the University of California at Irvine for warm hospitality. This research was supported in part by the National Science Foundation under Grant PHY-9208386.
REFERENCES

[1] A. A. Anselm, Pis'ma Zh. Eksp. Teor. Fiz. 48 (1988) 49 [JETP Lett 48 (1988) 51]; Phys. Lett. B217 (1989) 169.

[2] A. A. Anselm and M. G. Ryskin, Phys. Lett. B266 (1991) 482.

[3] J.-P. Blaizot and A. Krzywicki, Phys. Rev. D46 (1992) 246.

[4] J. D. Bjorken, Int. J. Mod. Phys. A7 (1992) 4189; Acta Phys. Polonica B23 (1992) 561.

[5] J. D. Bjorken, K. L. Kowalski and C. C. Taylor, SLAC-PUB-6109 and to appear in the Proceedings of Les Rencontres de Physique de la Vallée D’Aoste La Thuile, March, 1993.

[6] K. Rajagopal anf F. Wilczek, PUPT-1347, IASSNS-HEP-92/60 (unpublished).

[7] C. M. G. Lattes, Y. Fujimoto and S. Kasegawa, Phys. Rep. 65 (1980) 151.

[8] J. Lord and J. Iwai, University of Washington preprint (paper 515 submitted to the International Conference on High Energy Physics, Dallas, Texas, August, 1992); J. Iwai (JACEE Collaboration), UWSEA 92-06.

[9] M. M. Enikova, V. I. Karlonkovski and C. I. Velcher, Nucl. Phys. B151 (1979) 172.

[10] H. Georgi, Weak Interactions and Modern Particle Theory, (Benjamin/Cummings Pub. Co., Menlo Park, California, 1984).

[11] C. Bernard et. al., Phys. Rev. D45 (1992) 3854.