Accelerating ray tracing simulation using tensor completion for 3D radio map

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Abstract: We proposed a ray-tracing (RT) acceleration method using the tensor completion technique for a three-dimensional (3D) radio map. Internet of things (IoT) has become widespread recently, and analysis of indoor radio wave propagation has become important in optimizing the installation location of the IoT devices. Generally, RT is used for radio wave propagation analysis. However, RT is time-consuming to simulate. The simulation time required for RT is even larger when calculating the receiving points in 3D space with many objects. Hence, in the proposed method, we reduced the number of receiving points at which the RT calculation is performed, and we interpolated by tensor completion the received power not calculated by RT.

Keywords: IoT, ray tracing, TVNM, tensor completion, SPC

1 Introduction

Recently, there is an expansion of the Internet of things (IoT) in three-dimensional (3D) space, such as IoT in smart factories and parks [1]. In a factory environment...
where disconnection of communication is unacceptable, investigation of the radio propagation before installing a wireless system is necessary, and ray tracing (RT) is generally used to simulate the radio propagation. RT requires a long computation time because it calculates the radio wave’s entire path from the transmission point to the reception point. Before developing a 3D wireless network, it is necessary to simulate the 3D propagation; however, RT calculations for all receiving points in a 3D space require an enormous amount of computation time, making it impractical to perform large-area or high-resolution simulations.

One approach to accelerate RT is by reducing the number of receiver points where the received power is calculated and interpolating the received power at the receiver points that are not calculated by RT. One such method is the total variation norm minimization (TVNM) based RT acceleration method [2]. TVNM can accurately interpolate the power fluctuation caused by shadowing. This method is based on the interpolation of 2D received power data as a matrix; thus, it is not suitable for interpolating 3D received power data. In this letter, we newly treat 3D received power data as a 3D tensor and propose to apply tensor completion technique to the interpolation problem of 3D received power data. To achieve higher accuracy than TVNM, we apply smooth PARAFAC tensor completion (SPC) [3], a tensor interpolation based on PARAFAC decomposition (PD). To confirm the effectiveness of the proposed method, we conducted simulations in an indoor environment with several obstacles and we confirmed that the proposed method could estimate 0.6 dBm higher accuracy than the existing method.

2 Problem statement

In this letter, we assume the received power at some received points is calculated by RT and the other is interpolated by tensor completion. The 3D tensor data of received power is denoted by $P \in \mathbb{R}^{I_1 \times I_2 \times I_3}$, and the received power at $(i_1, i_2, i_3)$ point is denoted by $p_{i_1, i_2, i_3}$, where $i_n \in \{1, 2, \cdots, I_n\}$ and $n \in \{1, 2, 3\}$. The index sets $\Omega$ and $\overline{\Omega}$ represent the positions of some receiver points where RT is performed and not performed. Therefore, the received power $p_{i_1, i_2, i_3}$ at $(i_1, i_2, i_3) \in \Omega$ is regarded as known, and the received power $p_{i_1, i_2, i_3}$ at $(i_1, i_2, i_3) \in \overline{\Omega}$ is unknown and must be estimated. The index for the position of an obstacle, such as shelf or desk, is denoted by $\Omega_0$, and $\Omega_0 \subset \Omega$. The received power $p_{i_1, i_2, i_3}$ at $(i_1, i_2, i_3) \in \Omega_0$ is treated as zero. In our approach, the interpolation process is introduced instead of most part of the RT calculations. The tensor interpolation can be performed with smaller complexity than RT. Thus, the total simulation time is reduced. We denote missing rate $M$ as $M = |\overline{\Omega}|/|\Omega - \Omega_0 + \Omega|$. Since tensor interpolation is much faster than RT computation, the total simulation time can be accelerated by a factor of $1/(1 - M)$.

3 Proposed method

In this letter, we consider the interpolation problem of the 3D received power data. For such problem, the TVNM based interpolation method, which is based on a 2D matrix’s interpolation, does not match the problem treated in this letter because the contribution in the height direction cannot be utilized. In this study, we regard the 3D received power data as a 3D tensor and then applied the tensor interpolation
method to achieve more accurate interpolation taking into account the influence of data in the height direction. Moreover, TVNM interpolates to minimize the total variation (TV) norm and can accurately estimate discontinuous variations such as shadowing; however, it cannot accurately interpolate continuous variations such as fading. To solve these problems, we focus on tensor interpolation methods that can appropriately interpolate 3D data. We also propose to apply SPC, which minimizes quadratic variation (QV) instead of TV and leads to smooth interpolation.

SPC is a tensor interpolation based on the PD with a smoothing constraint. This constraint makes NP-hard PD computationally feasible. The PD of a 3D tensor can be formulated as follows:

\[
Z = \sum_{r=1}^{R} g_r u_r^{(1)} \circ u_r^{(2)} \circ u_r^{(3)},
\]

where \( \circ \) is the outer product, \( u_r^{(n)} \) is the \( n \)th dimensional feature vector, and \( g_r \) is the scaling factor. \( R \) is the tensor rank, the smallest number that satisfies Eq. (1). SPC with fixed tensor rank is formulated as follows:

\[
\min_{G, U^{(1)}, U^{(2)}, U^{(3)}} \frac{1}{2} \| X - Z \|_F^2 + \frac{R}{2} \sum_{r=1}^{R} \rho_r \| L^{(n)} u_r^{(n)} \|_F^p,
\]

\[\text{s.t.} \quad Z = \sum_{r=1}^{R} g_r u_r^{(1)} \circ u_r^{(2)} \circ u_r^{(3)}, \]

\[X_\Omega = P_\Omega, X_\Pi = Z_\Pi, \| u_r^{(n)} \|_2 = 1, \]

\[\forall r \in \{1, \ldots, R\}, \forall n \in \{1, 2, 3\}\]

where \( X \) is an estimation of 3D received power data, and \( Z \) is an auxiliary tensor. \( \rho = [\rho^{(1)}, \rho^{(2)}, \rho^{(3)}] \) is a parameter vector that controls the effect of smoothing. \( L^{(n)} \in \mathbb{R}^{(I_n-1) \times I_n} \) denotes the matrix that takes the difference between the elements. \( G \) represents the super-diagonal tensor, such that each super-diagonal element is \( G_{rr \ldots r} = g_r \), and \( U^{(n)} = [u_1^{(n)}, u_2^{(n)}, \ldots, u_R^{(n)}] \). The first and second terms in Eq. (2) represent the mean squared error of \( P_\Omega \) and \( Z_\Omega \) and the smoothing constraint, respectively. In the second term, \( \| L^{(n)} u_r^{(n)} \|_F^p \) is \( \| L^{(n)} u_r^{(n)} \|_F^p = \sum_{i=1}^{I_n-1} |u_r^{(n)}(i) - u_r^{(n)}(i+1)|^p \), representing TV (SPC-TV) when \( p = 1 \) and QV (SPC-QV) when \( p = 2 \). Eq. (2) can be alternatively solved by hierarchical alternating least squares (HALS) [4].

In SPC, the tensor rank \( R \) is gradually increased to find an optimal rank [3], such as 1, 2, \( \cdots \). For each rank candidate, the Eq. (2) is solved by HALS. When the condition expressed in Eq. (3) is satisfied, the calculation of HALS with \( R \) is stopped, and the calculation is performed again with \( R + 1 \).

\[
\frac{|\mu_t - \mu_{t+1}|}{|\mu_{t+1} - \mu_t|} < \nu
\]

where \( \mu_t = \| Z_t^\Omega - P_\Omega \|_F^2 \) and \( Z_t^\Omega \) represent the PD model at iteration \( t \). The \( \nu = 10^{-\text{SNR}/10} \| P_\Omega \|_F^2 \) and \( \nu > 0 \) represent the threshold of the stopping condition of the iterative process. The signal-to-noise (SNR) represents the threshold of error. When \( \mu_t < \nu \) is satisfied, complete the SPC calculation.
4 Experiment

4.1 Experiment setting

We conducted RT by RapLab [5] for an indoor environment simulation. As shown in Fig. 1(a), we set up a model with 10 steel desks (1.2 m × 0.7 m × 0.72 m) and 8 wooden shelves (1 m × 0.35 m × 1.769 m) against a wall in a 10 m × 6 m × 3 m indoor space surrounded by concrete and glass. We then set up a transmission point (Tx) on top of the installed shelves. We assumed the transmit antenna to be a Dipole and set the transmit frequency to 2.4 GHz, which is used in Wi-Fi. The receiver points (Rx) are allocated in 4 m × 4 m × 3 m 3D grid with 5 cm interval. The Rx is also equipped with a Dipole antenna. We also set $I_1 = 81$, $I_2 = 81$, and $I_3 = 61$. Thus, the total number of receiving points is 400,221. In radio propagation simulations,
it is usual to allocate the receiver points as a grid. This grid allocation makes the known points emerge periodically, which results that the SPC may select feature vectors with extremely high frequency. Therefore, in this paper, the locations of the known receiver points were chosen randomly. Interpolation by TVNM [2] and tensor interpolation of SPC were performed using MATLAB R2019b. For RT simulation, we used the computer with an Intel Core i9-9820X 3.30 GHz of CPU and 64 GB of RAM.

The parameters used in the calculation of SPC were set as follows: \( \nu = 0.001 \), SNR = 30, and \( \rho = [0.01, 0.01, 0.01] \) for \( p = 1 \); \( \nu = 0.001 \), SNR = 30, and \( \rho = [1.0, 1.0, 1.0] \) for \( p = 2 \). In [3], the threshold \( \nu \) is recommended to be set to 0.01. The lower \( \nu \) leads to higher accuracy at the expense of the increase of computational complexity. In this paper, we set \( \nu \) to 0.001 for the accurate interpolation. The increase of the complexity due to this setting is sufficiently small compared with the ray tracing calculation. The data size \((I_1, I_2, I_3)\) assumed in this paper is larger than that in [3]. Therefore, the error between the ground truth and the estimation may become large. To tolerate the larger error, we set SNR to 30 dB. This is smaller than the setting (50 dB) in [3]. \( \rho \) is set to the same value as in [3]. The maximum iteration number was set to 3,000 for both cases.

As a performance measure of the interpolation accuracy, we use mean absolute error (MAE) which defined as

\[
MAE = \frac{1}{|\Omega|} \sum_{i \in \Omega} |\hat{p}_i - p_i|
\]

where \( \hat{p}_i \) is the estimated received power value, and \( p_i \) is the received power value obtained by RT.

4.2 Evaluation results

We compared the MAE between SPC and TVNM with different missing rates (Fig. 2). In the figure, the two interpolations using SPC are both better than the interpolation using TVNM. The reason of this is that TVNM uses only vertical and horizontal information; thus, it cannot take into account the effect of receiving points located close to each other in the height direction. Also, QV is superior to TV in the choice of smoothing constraints in SPC because QV is smoothly interpolated for continuous variations, such as fading Fig. 1(b). For SPC-QV, the accuracy of MAE was improved by approximately 1 dBm over TVNM for the missing rate from 10% to 70%. For the missing rate from 80% to 90%, the accuracy was improved.

![Fig. 2. Comparison of the MAE performance between the proposed method and the conventional methods for different missing rates.](image-url)
by approximately 0.6 dBm.

Figure 1 shows the received power maps after interpolation at the heights $i_3 = 20$ with the missing rate 90%. Fig. 1(c) and 1(d) are the received power map obtained by RT simulation with the missing rate 0% and by TVNM interpolation, respectively. Fig. 1(e) and 1(f) are the received power maps estimated by SPC-TV interpolation and by SPC-QV interpolation, respectively. Although TVNM can roughly distinguish between regions with high and low received power due to shadowing, it does not appropriately interpolate variations within each region well. On the other hand, SPC-QV can interpolate both shadowing and fading compared to the TVNM.

Table I shows the simulation time and interpolation time for RT. Although SPC-QV took longer than the TVNM, the interpolation time of both methods were much less than the simulation time of RT. The total time required for simulation and interpolation was approximately 1/10 for the missing rate 90% compared with the time required for the missing rate 0%. In Figs. 1 and 2, it is confirmed that the SPC-QV can accelerate RT approximately 10-times faster with an error of approximately 4 dBm.

Table I. Simulation time and interpolation time for ray-tracing

| Missing rate (%) | Ray tracing (s) | Interpolation (s) | Total (s) |
|------------------|----------------|-------------------|-----------|
|                  |                | TVNM | SPC-QV |
| 0                | 495,495        | 0    | 0      |
| 10               | 445,946        | 17   | 1,294  |
| 50               | 247,748        | 17   | 1,913  |
| 80               | 99,099         | 17   | 1,853  |
| 90               | 49,550         | 16   | 1,738  |

5 Conclusion

To accelerate the RT simulation, we proposed an accelerating method by interpolation using SPC. We compared and evaluated the interpolation accuracies of TVNM and SPC by obtaining the received power map data in an indoor environment using a radio wave propagation simulator. The interpolation accuracy of the proposed method is higher than that of TVNM by approximately 0.6–1 dBm at the missing rate of 10%–90%. We found that the simulation time can be significantly reduced by tensor interpolation if a few dBm errors are allowed.