I. CAVITY-MAGNON COUPLING ASSUMING A UNIFORM PRECESSION MODE

The Hamiltonian of the cavity-magnon system consists of its cavity and magnon parts, $H_c$ and $H_m$, respectively, as well magnon-cavity interaction, $H_{int}$:

$$H = H_c + H_m + H_{int}$$

$$H = \hbar \omega_c c^\dagger c + \hbar \omega_m b^\dagger b + H_{int}, \quad (1)$$

where $c^\dagger (c)$ is a creation (annihilation) operator for photon, $b^\dagger (b)$ is a magnon creation (annihilation) operator, $\omega_c$ is the cavity frequency, $\omega_m$ is magnon frequency and $\hbar$ is the reduced Planck’s constant. These expressions can be found from first principles. The interaction term can then be evaluated by the Zeeman energy of the ferrimagnetic sample:

$$H_{int} = -\mu_0 \int_{V_m} M \cdot H_c dV,$$  \hspace{1cm} (2)

where $M$ is the magnetisation, $H$ is the cavity mode auxiliary magnetic field, $\mu_0$ is the permeability of free space and the integral is performed over the magnetic sample volume $V_m$. The form of the magnetic field in the cavity mode can be found by the usual methods of quantisation:

$$H_c = \frac{1}{\mu_0} \sqrt{\frac{\hbar}{2\omega_c \epsilon_0 \epsilon_{r,c}}} (c + c^\dagger) \nabla \times U,$$  \hspace{1cm} (3)

where $\epsilon_0$ is the permittivity of free space and $\epsilon_{r,c}$ is an average relative permittivity experienced by the cavity mode defined as:

$$\epsilon_{r,c} = \int_{V_c} \epsilon_r U \cdot UdV.$$  \hspace{1cm} (4)

Finally, $U$ solves the wave equation and is orthonormal with other cavity modes as follows, respectively:

$$\frac{1}{\epsilon_r} \nabla \times \nabla \times U_n - \omega^2_{0} U_n = 0,$$  \hspace{1cm} (5)

$$\int_{V_c} U_n \cdot U_m dV = \delta_{nm},$$  \hspace{1cm} (6)

where $c$ is the speed of light and $\delta_{nm}$ is the Kronecker delta.

The uniform precession mode of the magnetic sample can also be quantised by introducing a macrospin $S = \frac{M_{5/2}}{2}$. If we assume the direction of the DC magnetic field which saturates the magnetic material is in the $z$ direction we can then introduce raising and lowering operators $(S^\dagger = S_z \pm i S_y)$ followed by the Holstein-Primakoff transformations:

$$S^+ = \sqrt{2S - b^\dagger b} b,$$

$$S^- = b^\dagger (\sqrt{2S - b^\dagger b}), \quad (7)$$

$$S_z = S - b^\dagger b,$$

where $S$ is the total spin number of the macrospin operator. This number is determined by $S = \frac{\mu}{g\mu_B} N_s$, where $\mu$ is the magnetic moment of the magnetic sample, $\mu_B$ is the Bohr magneton, $g$ is the g-factor ($g = 2$) and $N_s$ is the number of spins in the sample (given by $N_s = n_s V_m$ with $n_s$ as spin density and $V_m$ as volume).

For low excitation numbers ($\langle b^\dagger b \rangle \ll 2S$), the macrospin operators may be approximated by $S^+ \approx \sqrt{2S}b$ and $S^- \approx \sqrt{2S}b^\dagger$. If we substitute eqn. 3 and these transformations into eqn. 2, we arrive at the interaction Hamiltonian:

$$H_{int}/\hbar = g_{cm}(c + c^\dagger)(b + b^\dagger) + g_{cm}(c + c^\dagger)(b - b^\dagger) + g_{cm}(c + c^\dagger)b^\dagger b + \Omega^2(c + c^\dagger)$$  \hspace{1cm} (8)

where the coupling rates are defined as:

$$g_{cm}^x = -\frac{\gamma}{2V_m} \sqrt{\frac{\hbar}{\omega_c \epsilon_{r,c} \epsilon_0}} \int_{V_m} (\nabla \times U) \cdot \hat{d} dV,$$

$$g_{cm}^y = \frac{i \gamma}{2V_m} \sqrt{\frac{\hbar}{\omega_c \epsilon_{r,c} \epsilon_0}} \int_{V_m} (\nabla \times U) \cdot \hat{y} dV,$$

$$g_{cm}^z = \frac{\gamma}{V_m} \sqrt{\frac{\hbar}{2\omega_c \epsilon_{r,c} \epsilon_0}} \int_{V_m} (\nabla \times U) \cdot \hat{z} dV,$$

$$\Omega^2 = -\frac{\gamma S}{V_m} \sqrt{\frac{\hbar}{2\omega_c \epsilon_{r,c} \epsilon_0}} \int_{V_m} (\nabla \times U) \cdot \hat{z} dV.$$  \hspace{1cm} (9)

To aid in the evaluation of these expressions the following relations can be used:

$$U = \frac{E}{\sqrt{\int_{V_c} |E|^2 dV}},$$

$$\nabla \times U = \frac{\sqrt{\epsilon_r \omega_c}}{c} \frac{H}{\sqrt{\int_{V_c} |H|^2 dV}}.$$  \hspace{1cm} (10)

These expressions, which are normalised to cavity energy, allow the use of calculated field shapes to evaluate
the coupling rates. The first two terms in the interaction
hamiltonian are standard coupled mode terms due to
cavity RF field perpendicular to the external DC field
coupling to the magnon mode. This will be the focus of
this analysis. The third term is a parametric term due
to the cavity RF field parallel to the external DC field
modulating the magnon frequency and the final term is
a result of the DC saturation magnetisation generating
field in the cavity. These last two terms can be neglected
under a rotating wave approximation or by ensuring the
cavity field is perpendicular to the external DC field.
That is the case in this work.

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