Handling multi-parametric variations in distributed control of cyber-physical energy systems through optimal communication design

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Abstract: Cyber physical systems like smart grid are largely migrating towards distributed control philosophy to achieve high reliability. The design of communication network between various sensors and controllers plays an important role in control of these systems. The design process involves examining a number of topological combinations, which increase exponentially with the number of nodes in the considered system. Moreover, for a practical system, the different characteristics and availability of various physical and communication resources in the network pose multiple constraints on this design. In this work, a generalised constraint-based sensor controller connection design methodology has been developed, which effectively reduces the number of combinations, to design more stable cyber-physical controllers. To handle variations in multiple parameters in physical and communication domain, different controllers have been developed for different operating conditions that are scheduled as per requirement. The methodology has been shown to stabilise bus voltages in a smart grid scenario under variations in load, communication delays and loss of communication links.

1 Introduction

Cyber physical systems (CPS) [1] research aims to provide integrated multi-disciplinary frameworks for understanding and manipulating complex systems. The surfeit of advances in multiple technological domains as sensing, communications, control systems and information technology and their massive deployment in everyday systems have provided sufficient motivation to envision and actualise the emergent properties resulting from their co-existence. CPS methodologies are dedicated to exploiting these properties to provide the existing systems with a fresh set of capabilities in areas [2] such as safety, utility, resiliency, security, adaptability, scalability, and reliability. CPS have already begun to broach many unlooked-for implementation in various fields such as smart grid [3], robotics [4], health care [5], data centres [6], and many more.

Today’s smart grids constitute a conflation of many technologies including renewable distributed generation, power electronics, communications, advanced sensing, and embedded technologies which make them viable candidates for positioning CPS technologies [7] so much so that a field called cyber physical systems (CPS) has advented. The fast acting renewable energy-based distributed generators (DGs) provide both active power and reactive power support making them resourceful in handling voltage drops [8], local active power requirement etc. The cyber-physical approach towards solving any problem in a smart grid would include models that would take inputs and provide outputs belonging to multiple fields including power, communication and computation.

For example, consider a smart grid scenario in Fig. 1. The grid in the picture consists of a buses where each bus is connected to a DG-like solar, wind etc. The voltage at each bus is sensed by a sensor and sent to DGs for determining their operating voltages. An established procedure for this problem would be to let each DG control the voltage of the respective bus with the help of data obtained from sensors placed at that particular bus. An alternate cyber-physical solution for the problem would be to design a set of distributed controllers using a central server based on forecast data and communicating them ahead to local DGs/controllers. These controllers can provide enhanced control using the information acquired from sensors present at multiple buses. Along with the perks obtained from incorporation of CPS into the grid, it is imperative that issues would arise pertaining to both, control theory and communications. Controllers tailored for such systems should be able to control voltage efficiently in scenarios like variation in loads and generation, while tackling communication constraints such as bandwidth, propagation delay [9], and loss of communication.

Ameliorating distribution systems with communication have been widely studied for quite some time [10]. The usage of distributed [11, 12] control framework with communication has gained importance over a centralised [13, 14] scheme as it enhances the reliability and scalability of the system. For instance, Xin et al. [15] talk about a cooperative control strategy for multiple solar plants using minimal communications. The authors in [16] proposed a distributed secondary control technique which enhances the working of traditional droop control in a microgrid by adding a ubiquitous communication framework. The work in [17] discusses a hybrid control scheme which works both in the presence and absence of a centralised communication scheme. However, these works mostly concentrate on system dynamics with the communication topology assumed a priori. It has also been noticed that design of communication network has considerable effect on the various physical parameters of the system. Works like [18, 19] develop communication routing algorithms and protocols to connect DGs and enhance power sharing in various microgrid configurations. The effect of different communication structures on distribution system protection performance has been studied in [20]. These, however, do not come up with any systematic strategy for control of a physical system variable based on communication topology design. The work in [21] proposed a simple optimisation technique to design the communication topology for voltage control in CPS. A greedy algorithm was used to route the connections between sensors and DGs based on controller stability. However, this approach fails to consider many topologies while evaluating the most stable controller for a particular situation. In addition, both the constraints and parameters in the domains of communication and control remain fixed. In practice, various physical parameters like load and...
communication parameters like delay keep changing simultaneously over the course of the day.

To overcome these issues, this paper proposes a generalised constraint-based sensor controller connection design (CBSCD) methodology to design controllers for CPES considering:

i. Connection constraints that can be specified to define the boundaries of the communication network.

ii. Resource constraints like communication bandwidth that can be specified to describe the connecting capabilities of various sensors and controllers.

iii. Utility-based constraints like cost can be specified, so as to accommodate required operational demands of power utilities.

iv. Physical variable constraints like load requirement on the buses can be specified.

The idea behind developing this framework is to handle changes in multiple parameters of the CPES more effectively. Appropriate constraints can be tuned to respond to any change in system. This way, it provides more options to respond to a change in system and this framework will be very useful if operator can select an appropriate constraint depending on his/her experience about the possible source of the problem. This way it is more effective than other conventional approaches. The organisation of the paper is as follows.

Section 2 gives a general idea of mathematical modelling and communication structure of a CPS. Section 3 describes the Lyapunov function-based optimisation frameworks adopted for designing stable controllers in the presence and absence of delays. This follows Section 4 which explains the CBSCD methodology to design the controllers with most appropriate communication structure of a CPS. Section 3 describes the physical system model adopted for representing such a system must be able to obtain information. The term p-neighbour node refers to the node in the previous layer which is situated in between the sensor-to-controller nodes placed at DGs which compute the control references in a distributed manner. Most of the equipment in the CPES is assumed to be equipped with wireless/wired communication interfaces. These interfaces can be clubbed into three types of nodes – sensor nodes, controller nodes, and the server nodes. The data flow among these nodes is assumed to be continuous, i.e. the sampling rate is high, quantisation error is low and there is no issue of packet loss. The routing between the nodes would be on a multicast [22, 23] basis.

It is to be noted that the generators in the power network are classified into two categories – peripheral generators and central generators. The peripheral generators are the generators located nearer to the boundary of the power network and which are capable of handling only lighter loads. The central generators, on the other hand, are located in the central region of the power network and possess capacity to handle higher loads. The value of connection constraint \( cc \) signifies the maximum number of \( p \)-neighbour nodes located on single side of the current bus from which a particular node can receive information. The term \( p \)-neighbour node refers to the node in the previous layer which is situated in between the sensor-to-controller nodes connected to a bus other than the current bus. For instance, if \( cc = 2 \), then the possible set of connections will be as shown in Fig. 2.

### 2.1 Physical system model
Let the CPES contain \( n_s \) number of nodes where sensors are placed and \( n_c \) number of distributed controllers. Each distributed controller may receive readings from various sensors placed at many buses to generate its local control input. In this case, the multi-input multi-output (MIMO) system model of the system can be considered in the form as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]

where \( x(t) \) is the state vector of length \( n_s \), \( u(t) \) is the control vector of length \( n_c \) and \( y(t) \) represents the observation vector from sensors of length \( n_c \). Thus

\[
A \in \mathbb{R}^{n_s \times n_s}, \quad B \in \mathbb{R}^{n_s \times n_c}, \quad C \in \mathbb{R}^{n_c \times n_s}
\]

Even, if the system may be non-linear, it can be linearised around its equilibrium point. The observations in the system are considered to be noiseless.

### 2.2 Communication structure
There exist two types of communication networks in the CPES. Network-1 consists of central server which sends routing data to the DGs and sensors along with controller gains to the controllers. Network-2 consists of interconnections between the sensors and controllers which will be operational on a regular basis. The \( n_s \) sensor nodes placed at individual buses collect the voltage data coming out of the sensors and send them to all/some of \( n_c \) controller nodes placed at DGs which compute the control references in a distributed manner. Most of the equipment in the CPES is assumed to be equipped with wireless/wired communication interfaces. These interfaces can be clubbed into three types of nodes – sensor nodes, controller nodes, and the server nodes. The data flow among these nodes is assumed to be continuous, i.e. the sampling rate is high, quantisation error is low and there is no issue of packet loss. The routing between the nodes would be on a multicast [22, 23] basis.

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The topology of the communication network is defined by the following communication constraints:

i. **Bandwidth constraint**: This constraint specifies the number of connections that must start from any node. For example, Fig. 2 shows a set of connections that can exist when \( bwc = 2 \) with \( bwc \) representing this constraint.

ii. **Connection constraint**: This constraint deals with the general topology of the communication network. It specifies the availability of a particular node to be connected to other nodes. The value of connection constraint \( cc \) signifies the maximum number of \( p \)-neighbour nodes located on single side of the current bus from which a particular node can receive information. The term \( p \)-neighbour node refers to the node in the previous layer which is situated in between the sensor-to-controller nodes connected to a bus other than the current bus. For instance, if \( cc = 2 \), then the possible set of connections will be as shown in Fig. 2.

iii. **Peripheral cost constraint**: This constraint \( prec \) represents the maximum number of sensors that can be connected to a generator located in the periphery of the power network. This...
has been defined as per the assumption that generators with less capacity will be installed on the boundary of the power network which can handle lesser loads compared with the central ones.

iv. Central cost constraint: This constraint cnc represents the minimum number of sensors that must be connected to central generator so that it can be operational. This means that this generator needs to be connected to at least cnc number of sensors to be switched on failing which it is shut down. This has been defined as per the assumption that generators with more capacity will be installed in the central region of the power network and that they can handle higher loads compared with the peripheral ones.

2.3 Distributed control scheme

A distributed output-feedback control scheme

\[ u(t) = K x(t) \]  

(2)

is used where \( K \in \mathbb{R}^{n_c \times n_s} \) takes a form based on the connection structure between controllers and sensors. If \( K_{ij} \) is non-zero, it represents the existence of connection between controller \( i \) and sensor \( j \). Substituting (2) in (1), results in

\[ \dot{x}(t) = \tilde{A} x(t) \]  

(3)

where \( \tilde{A} = A + BKC \) represents closed loop system matrix. Without inclusion of the controller, the system stability is defined by the eigenvalues of the matrix \( A \) whereas, upon inclusion of controller, the system's stability gets defined through eigenvalues of the matrix \( \tilde{A} \). If the real parts of all the eigenvalues are negative, then the system matrix \( \tilde{A} \) is stable.

2.4 Delay representation in the MIMO model

Delay existing in between various sensors and controllers has been included in the MIMO model as follows:

\[ x(t) = A x(t) + \sum_{(j,i) \in \mathbb{R}} b(j,i) \dot{K}(j,i) C x(t) - d_{ji} \]  

(4)

where \((j,i) \in \mathbb{R}\) means that the connection between sensor \( j \) and controller \( i \) is established, \( b(j,i) \) is the \( j \)th column of the matrix \( B \), \( K(j,i) \) is the \( j \)th row of matrix \( K \), and \( d_{ji} \) is the delay between sensor \( j \) and controller \( i \). A particular case of delay has been considered in this work where the delay is small. Here

\[ x(t) - d_{ji} \dot{x}(t) \approx x(t) - d_{ji} \tilde{A} x(t) + \sum_{(j,i) \in \mathbb{R}} b(j,i) \dot{K}(j,i) C x(t) \]  

(5)

By neglecting the higher order terms and substituting (5) into (4), the dynamics of the system with small delay can be written as

\[ \dot{x}(t) \approx \tilde{A} x(t) \]  

(6)

where \( \tilde{A} = (I - BDKC)(A + BKC) \) and \( D \) is the delay matrix formulated as

\[ D_{ji} = \begin{cases} d_{ji}, & \text{if } j \text{ and } i \text{ are connected} \\ 0, & \text{if } j \text{ and } i \text{ are not connected} \end{cases} \]  

(7)

3 Lyapunov-based optimisation formulations

This section deals with the description of optimisation formulations obtained using basic Lyapunov stability analysis for finding controllers for a given set of system parameters and communication design. Conditions assuring stability have been formulated into an optimisation framework using linear matrix inequalities (LMIs). Two cases will be looked into, namely the delay-free case and the small delay case.

3.1 System without delay

This formulation has been developed on the assumption that all the communication links are delay free. Considering the linear system model as in (1), and given a Lyapunov function \( V(x) = x^T P x \) \((P > 0)\), it is well known that equilibrium point goes to zero, if the following two inequalities hold simultaneously for all \( x \neq 0 \)

\[ V(x) > 0, \quad \dot{V}(x) < 0 \]  

(8)

The rate derivative of the Lyapunov function \( V \) for the system model (1) is obtained as

\[ \dot{V}(x) = x^T P x + x^T P \dot{x} = x^T (\tilde{A}^TP + P \tilde{A})x \]  

(9)

Given that the matrix \( P > 0 \), from (9) it can be inferred that \( \dot{V} < 0 \), if the following condition holds true:

\[ \tilde{A}^T P + P \tilde{A} < 0 \]  

(10)

For the computed \( \tilde{A} \) to be more stable than \( A \), the matrix \( P \) is computed by selecting an arbitrary positive value of \( \beta \) such that the following condition holds true:

\[ (A - \beta I)^T P + P(A - \beta I) = -I \]  

(11)

It should be noted that for different value of \( \beta \), the controller solution will be different. By introducing a design parameter \( \gamma \) into (3), the optimisation problem in the form of LMIs is obtained as
\[
\max_k \gamma \\
\text{s.t. } A^T P + PA + \gamma I < 0 \\
K_{ij} = 0, \text{ if link between sensor } j \text{ and DG } i \text{ is absent} \\
\| K \|_2 \leq \rho
\]

Note that the stability margin increases with \( \gamma \). The constraint

\[
\| K \|_2 \leq \rho
\]

is incorporated so that the controller gains are bounded, where \( \rho \) is a user defined scalar value.

### 3.2 Systems with delay

The communication networks can deliver the data in order of milliseconds, whereas the dynamics of DGs is generally in the order of tens to hundreds of milliseconds. Since, the time scales are comparable, the effect of delay should be included into the system model. The dynamics with small delay as given in (6) can be rewritten as

\[
\dot{x} = (A + BK\gamma)x + f(x)
\]

where \( f(x) = -BDK(A + BK\gamma)x \) is considered as the uncertainty due to delay and

\[
f^T(x)f(x) \leq \alpha x^T x
\]

The norm bound on \( \alpha \) can be found as

\[
\alpha = \sqrt{\max \rho} \| B \|_2 (\| A \|_2 + \| B \|_2 \| \rho \|_2 \max (d_{ij}))
\]

Now, assuming that the dynamics of this system be stabilised by a Lyapunov function \( V(x) = x^TPx \) \( (P > 0) \)

\[
\dot{V}(x) = x^TP(Ax + f(x)) + (x)^T A^T K + f^T(x)Px
\]

where \( A_K = A + BK\gamma \).

This equation can be rewritten as \( \dot{V}(x) = y^T F y \) where

\[
F = \begin{bmatrix}
A_K^T P + PA_K & P \\
P & 0
\end{bmatrix}
\]

and \( y = [x \ f(x)] \)

Also, (15) can be rewritten as: \( y^T G y \leq 0 \) where

\[
G = \begin{bmatrix}
-\alpha I & 0 \\
0 & I
\end{bmatrix}
\]

Now, \( T = y^TFy < 0 \) holds only if there exists a \( y^T Gy \leq 0 \) such that \( (F - \gamma G) < 0 \) and a positive \( \gamma \), which results in

\[
\begin{bmatrix}
A_K^T P + PA_K + \gamma \alpha I & P \\
P & -\gamma I
\end{bmatrix} < 0
\]

Thus, using the conditions as constraints (18) along with (13) to limit the controller gains, the optimisation formulation for this case can be written as follows:

\[
\max_k \gamma \\
\text{s.t. } \text{condition (18) holds} \\
\| K \|_2 \leq \rho
\]

### 4 CBSCD-based controller design methodology

The distributed control structure in (2) shows a clear dependence of control on communication topology. Using the optimisation frameworks described in Section 3, it is possible to find the controller gains with maximum stability for a particular communication topology. A particular operating condition of a CPES is dictated by a set of physical and communication parameters such as load, bandwidth constraint, and connection constraint. For any operating condition, there can exist many communication topologies. This section delineates the proposed CBSCD methodology which finds the most stable controllers with least set of operating communication resources for different operating conditions of the CPES during the course of the day. The controllers are designed based on forecast data, which will be scheduled as per the variation in operating conditions.

The overall methodology consists of two different parts which are explained in the following two subsections- the connection finding algorithm which finds the possible connections for a particular set of communication and cost constraints and the controller design procedure containing the resource minimisation logic.

#### 4.1 Connection finding algorithm

The sensor and the DG nodes need to be connected in the presence of communication constraints such as bandwidth \( bwc \) and connection constraints \( cc \) as well as operator requirements such as peripheral cost constraint \( prc \) and central cost constraint \( cnc \). Given a set of these constraints, the connection finding algorithm is used to find the possible connection topologies that satisfy them (see Fig. 3).

The different variables used in the algorithm have been listed in Table 1. It can be seen that assigning as \( a = 4 \), \( nc = 4 \), \( bwc = 2 \) and \( cc = 1 \), the possible connections for this configuration are shown in Fig. 5. Further selecting \( prc = 1 \) and \( cnc = 2 \) results in \( cost = 1331 \), \( cost-sortup = 1133 \) and \( count = 4 \). Figs. 4 and 5 show various stages of development of communication connections described in Steps 8 to 16 of the algorithm.

The set \( R \) contains all the final connection sets which satisfy all the constraints provided.

#### 4.2 Controller design procedure

This controller manipulates the communication constraints to handle all sorts of variation in parameters. The following procedure utilises the connection finding algorithm and Lyapunov-based optimisation formulation to arrive at the value of connection parameter (CP) that would suit a particular range of parameter variation we wish to address at a particular point of time.

1. Plot the eigenvalues of the open loop system matrix for the known range of parameters.
2. Segregate the range of eigenvalues into individual smaller zones.
3. Find the worst-case working system matrix for all \( n \) individual time zones \( A_1, A_2, \ldots, A_n \).
4. Select a particular zone of operation depending on the time of the day. Let it be \( A_i \).
5. List all the possible constraint combinations and represent them using the variable connection-parameter (CP). The CP of a connection gives an idea of various constraints imposed on the system. In short, \( CP = [bwc \ bwc \ cnc \ prc] \).
6. Find the possible sensor controller combinations for all the CP combinations as prescribed by the connection finding algorithm described in the previous subsection.
7. Find the \( \gamma \) values and the maximum eigenvalues for all the combinations found in Step (5) using the optimisation formulation mentioned in Section 3.
8. Define the \( \gamma \) tolerance value \( \epsilon \) for the chosen zone.
9. Find the least value of constraints that can satisfy the tolerances.

   - List the maximum \( \gamma \) for all bandwidths and find the highest one among all values.
1. Initialize connection set $R$ as empty set.
2. Initialize the constraints $bw_{c}$, $cc$, $prc$ and $cnr$ in the communication network.
3. Initialize set $Cost$ as all possible generator cost configurations for the given constraints.
4. for All configurations in $Cost$
   5. Initialize connection set $R_1$ as empty set.
   6. Sort the non-zero costs of individual generators in ascending order and store in cost-sortup.
   7. Store the size of cost — sortup in count.
   8. while $count > 0$
   9. Create set $v$ for the first generator in cost — sortup with all possible existing configurations generated for the given cost.
   10. if $count > bw_{c}$
      11. Add feasible connections in $v$ to each connection in $R_1$ and update $R_1$.
   12. else
      13. Add only the elements in $v$ which when added to $R_1$ result in a minimum of $bw_{c} - count + 1$ connections at all sensors and update $R_1$.
   14. end if
   15. Remove the current generator and update cost — sortup.
   16. $count := count - 1$.
   17. end while
   18. Add the connection in $R_1$ to $R$.
   19. Remove the configuration from $Cost$.
5. end for

Fig. 3 Algorithm 1: connection finding algorithm

Fig. 4 Connections available from DGs 1 and 4
(a) DG-1 connection-1, (b) DG-1 connection-2, (c) DG-4 connection-1, (d) DG-4 connection-2

- Find the lowest $bw_{c}$ whose maximum $\gamma$ lies within the range of tolerance value $\epsilon$.
- Within this bandwidth, search for the lowest connection constraint value $cc$ that contains its maximum $\gamma$ within the $\epsilon$ range.
- Within the selected $bw_{c}$ and $cc$, list all the CPs within the $\epsilon$ range and select the CP that has the least sum of its digits.
- Select the connection topology with the lowest maximum eigenvalue in the selected CP.
10. List the controller $K$ matrix for the obtained connection.
5. Application to voltage control in smart grids

This section describes the application of the general system model and control design previously developed for the purpose of controlling the bus voltages in a smart grid scenario.

5.1 Description of the smart grid system

The n-bus system can be represented as shown in Fig. 6. The bus voltages \( V_t = [V_{t1}, \ldots, V_{tn}]' \) are controlled with the help of DGs which are modelled as voltage sources \( V_c = [V_{c1}, \ldots, V_{cn}]' \). Each DG actually consists of a power electronic inverter with a DC side capacitor. The DG is connected to the power system with the help of an inductor. The control response of DGs is much faster than conventional sources due to low inertia and fast acting power electronic controllers.

Upon applying Kirchoff’s current law at each bus in Laplace domain and converting them to time domain it is possible to arrive at the state-space representation of the system represented in (1).

\[
\begin{align*}
\Delta V_t(t) & = A' \Delta V_c(t) + B' \Delta V_c(t) + M' \Delta V_c(t) \\
\end{align*}
\]

where \( \Delta V(t) = V_t - V_{ref} \) and \( \Delta V_c \) represents the change in DG voltages required for bus voltages to approach \( V_{ref} \). For the purpose
of voltage control, $\Delta V_c$ is the control input $u$ and the following state is chosen

$$x(t) = \Delta V_c(t) - M' \Delta V_c(t)$$

and applied into (23) to arrive at the state-space representation of the system represented in (1). Since the control input also holds a linear relation with the state as per (2), the bus voltages also approach their reference values if the state vector goes to zero. Thus every local controller obtains the information of all the bus voltages to generate a voltage magnitude command that is to be maintained by the DG so as to maintain the bus voltage magnitudes at $V_{ref}$.

### 5.2 Test system configuration

A sample four-bus system is used for demonstrating voltage control whose per unit description has been given in Table 2. The value of $R_4$ and $L_4$ depends on the amount of load resistance and reactance at a particular point of time. For all the cases $L_4$ has been taken to be 0.0148. Also, the value of $L_{cn} = 0.001$ where $n = 1, 2, 3, 4$.

The different system matrices used for different loading conditions $R_4 = 0.001$ and 0.35 are given as follows:

$$A_1 = \begin{bmatrix} -150.43 & -63.00 & -21.47 & -0.000 \\ -50.65 & -94.50 & -32.21 & -0.000 \\ 35.60 & 9.19 & -53.69 & -0.001 \\ 155.01 & 138.91 & 100.57 & -0.003 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -152.61 & -65.17 & -23.65 & -2.17 \\ -53.91 & -97.76 & -35.47 & -3.26 \\ 30.16 & 3.76 & -59.12 & -5.43 \\ 143.04 & 126.95 & 88.61 & -11.96 \end{bmatrix}$$

(25)

### 5.3 Controller design example

The following example has been provided to elucidate the process of controller design for the case of varying load resistance.

1. Plot the eigenvalues of the system matrix for the known range of output resistance variation.
2. Segregate the range of eigenvalues into individual smaller zones as in Fig. 7.
3. Find the worst-case working system matrix for all $n$ individual time zones $A_1, A_2, \ldots, A_n$. Details available for two zones in the previous subsection.
4. Select a particular zone of operation depending on the time of the day. Let it be $A_1$.
5. List all the possible constraint combinations and represent them using the variable connection parameter. The CP of a connection gives an idea of various constraints imposed on the system. In short, $CP = [bwc cc cnc prc]$.
6. Find the possible sensor controller combinations for all the CP combinations as prescribed by the algorithm in Section 4.
7. Find the $\gamma$ values and the maximum eigenvalues for all the combinations found in Step (5) using the optimisation formulation mentioned in Section 3. Table 3 reflects the output of this step.
8. Define the $\gamma$ tolerance for the chosen zone. Since $A_1$ is near the verge of instability, the $\epsilon_i$ will be chosen to be low like 0.1.
9. Find the least value of constraints that can satisfy the tolerances. This process has been explained in Fig. 7. Find the maximum $\gamma$ in a particular bandwidth. Here only $bwc = 3$ and $bwc = 4$ connections exist with respective maximum $\gamma$ values 4.302 and 4.304. The lower $bwc = 3$ gets selected. Further $cc = 3$ is chosen which supports this maximum $\gamma$ and within the purview of these two constraints the constraint configuration 3303 is chosen since the sum of its individual digits is least.
10. Find the $K$ matrix for this connection.

### Table 2 Grid parameters

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $R_1$     | 0.175 | $L_1$     | 0.0005|
| $R_2$     | 0.1667| $L_2$     | 0.0004|
| $R_3$     | 0.2187| $L_3$     | 0.0006|

### Table 3

| $B_1$ | $B_2$ |
|-------|-------|
| $\begin{bmatrix} 56.13 & -3.22 & -23.58 & -27.468 \\ -3.22 & 39.00 & -8.97 & -25.105 \\ 23.58 & -8.97 & 40.89 & -7.801 \\ -27.468 & -25.105 & -7.801 & 56.555 \end{bmatrix}$ | $\begin{bmatrix} 56.15 & -3.20 & -23.55 & -27.394 \\ -3.20 & 39.03 & -8.92 & -24.994 \\ 23.55 & -8.92 & 40.97 & -7.616 \\ -27.394 & -24.994 & -7.616 & 56.964 \end{bmatrix}$ |

C has been assumed to be an identity matrix.
Table 3  Step (7) result

| CP   | Max eigenvalue | \( \gamma \) | CP   | Max eigenvalue | \( \gamma \) |
|------|----------------|-------------|------|----------------|-------------|
| 3202 | -2.15          | 2.04        | 3332 | -3.64          | 3.574       |
| 3212 | -2.15          | 2.04        | 3342 | -3.64          | 3.574       |
| 3222 | -2.15          | 2.04        | 3303 | -4.44          | 4.302       |
| 3232 | -2.15          | 2.04        | 3313 | -4.44          | 4.302       |
| 3242 | -2.15          | 2.04        | 3323 | -4.44          | 4.302       |
| 3203 | -2.838         | 2.7         | 3333 | -4.44          | 4.302       |
| 3213 | -2.838         | 2.7         | 3343 | -4.2           | 4.01        |
| 3223 | -2.838         | 2.7         | 3304 | -4.44          | 4.302       |
| 3233 | -2.838         | 2.7         | 3314 | -4.44          | 4.302       |
| 3243 | -2.15          | 2.04        | 3324 | -4.44          | 4.302       |
| 3204 | -2.838         | 2.7         | 3334 | -4.44          | 4.302       |
| 3214 | -2.838         | 2.7         | 3344 | -4.2           | 4.01        |
| 3224 | -2.838         | 2.7         | 4301 | -4.381         | 4.304       |
| 3234 | -2.838         | 2.7         | 4314 | -4.381         | 4.304       |
| 3244 | -2.15          | 2.04        | 4324 | -4.381         | 4.304       |
| 3302 | -3.64          | 3.574       | 4334 | -4.381         | 4.304       |
| 3312 | -3.64          | 3.574       | 4344 | -4.381         | 4.304       |
| 3322 | -3.64          | 3.574       |      |                |             |

\[
K = \begin{bmatrix}
0 & -2.1345 & -5.3123 & 0.7135 \\
-1.8397 & -1.3090 & -4.1146 & 0 \\
2.0861 & 0 & -3.2176 & -9.001 \\
-7.5319 & -5.5769 & 0 & -1.0024
\end{bmatrix}
\]

6 Results
This section describes various results obtained while designing controllers using CBSCD methodology for voltage control in a four-bus system whose details are given in previous section. The system has been tested in various scenarios including change in load resistance, change in delay and failure in communication link. All of these studies have been carried out using the YALMIP toolbox [24]. These cases can further be combined for describing many more practical scenarios in the cyber-physical framework. These controllers have been found by determining the minimal set of communication constraints that can maximally stabilise the CPES for given sets of physical and communication parameters. The maximum value of these communication constraints is generally specified by the operator based on experience and the algorithm finds the least value within these bounds that can provide maximal stability.

6.1 Load variation
The load on the power system is modelled as a resistance and reactance. As described previously, two zones of operation \( A_1 \) and \( A_2 \) with different load resistances have been selected and respective controller set 1 has been designed, which can comfortably take care of voltage control within respective zones. The values of different \( K \) matrices are stored at the individual DGs and get updated according to the zone of operation. The topology of a \( K \) matrix is shown in Fig. 8. Table 4 and Fig. 9 summarise the results obtained for two scenarios of loading.

The following is a controller from controller set 1:

\[
K = \begin{bmatrix}
0 & -2.1345 & -5.3123 & 0.7135 \\
-1.8397 & -1.3090 & -4.1146 & 0 \\
2.0861 & 0 & -3.2176 & -9.001 \\
-7.5319 & -5.5769 & 0 & -1.0024
\end{bmatrix}
\]

6.2 Variation of delay
It is possible to map an approximate variation of the maximum delay value throughout the day based on forecast data. Thus, the controllers can be designed for the maximum delay predicted during different periods of the day. Along with this, the load variation is also an inevitable change. The controller set 2 designed using optimisation formulation for delay case has demonstrated that it can stabilise the bus voltages for simultaneous change in load and delay profile. The results in Table 5 and Fig. 9 summarise the outputs achieved when system moves from a region of no delay with load of \( R_i = 0.5 \) to a region containing maximum delay of 1 ms with load of \( R_i = 0.001 \). The \( K \) matrix given in (28) shows a design from controller set 2 intended to work for operating region \( A_1 \) and maximum delay of 1 ms. The topology for the same has been presented in Fig. 8.

\[
K = \begin{bmatrix}
0 & 0 & 0.5498 & 0 \\
0 & -3.7826 & 0 & 4.0082 \\
0 & -3.611 & 0 & 1.2577 \\
-4.1047 & -6.7700 & -3.3168 & 0
\end{bmatrix}
\]

6.3 Failure of communication link
The communication signals in the smart grid get attenuated due to both reflection from objects along the path of the wave and shadowing, which happens due to wave obstruction. When the signal attenuation exceeds a certain limit, the particular communication node needs to be shutdown for repair. Before shutting down the communication node, the controller \( K \) will be calculated and put into action which ensures smooth transition without compromise in stability between the system in which all the \( K \) values are present and the system in which a particular \( K \) value is absent due to the removed connection. Table 6 and Fig. 9 summarise the results obtained for various scenarios of load when there is a communication loss in the link between controller-3 and sensor-3. The \( K \) matrix in (29) shows a design from controller set 3 for a case where there has been communication failure between sensor-3 and DG-3. Moreover, the system parameters have also been changed due to change in load resistance from \( A_i \) to \( A_j \). Fig. 8 shows the output topology obtained in the presence of communication failure when operating the region \( A_i \) with \( R_i = 0.001 \). The \( K \) matrix further confirms the alternate route chosen by the algorithm in the absence of the link between sensor-3 and controller-3.

\[
K = \begin{bmatrix}
0 & 0 & -1.6398 & 0 \\
-2.2646 & 0.5854 & -0.3357 & 0 \\
0 & 0 & 0 & -3.9057 \\
-5.3527 & -2.9395 & 0 & -0.5783
\end{bmatrix}
\]
Fig. 8  Communication topologies for controllers in different cases
(a) Topology of a controller from controller set 1, (b) Topology of a controller from controller set 2, (c) Topology of a controller from controller set 3

Fig. 9  Response of the designed controllers in different test cases
(a) Comparison of bus voltages between grid with controller set 1 and uncompensated grid when load is changed at $t = 500$ ms, (b) comparison of bus voltages between the situation with controller set 1 designed only for load variation and the situation with controller set 2 designed for both load and delay variation when load and delay are changed at $t = 500$ ms, (c) Comparison of bus voltages between the grid with controller set 1 designed only for load variation and grid with controller set 3 designed for both load variation and communication link loss when link 3–3 is lost and load is changed at $t = 500$ ms
been successfully demonstrated in situations of load change, delay topologies in the presence of changing electrical and communication parameters. The voltage control problem in smart grid has been modelled using a distributed control framework. An important finding in this method is that even if the system works with lower values of bandwidth and other constraints, that is, with lower number of connections, comparable stabilities are achieved to that of highly connected cases. The summary of performance comparison of various controllers in different cases is tabulated in Table 7. The terms WoC refer to without controller and WC-1,2,3 refer to with controller-1,2,3 and so on. Similarly, the terms Max Var refer to maximum variation/overshoot and Acc Time refer to accommodation time in Table 7.

7 Conclusion and future work
This paper presents a framework for designing cyber-physical voltage control of a smart grid using variations in communication topologies in the presence of changing electrical and communication parameters. The voltage control problem in smart grid has been modelled using a distributed control framework. Optimisation frameworks using the theories of LMIIs and Lyapunov stability analysis have been used to develop robust controllers both in the presence and absence of delays. The proposed CBSCD methodology for controller design finds the set of communication links using minimal communication resources to obtain controllers providing maximal stability. The efficacy of this technique has been successfully demonstrated in situations of load change, delay change, and communication link failure.

A more detailed model of the smart grids with dynamic loads will be pursued as the future scope of this work. Currently, the routing scheme operates from the central server. The authors are working towards introducing a distributed routing framework so as to increase the reliability of the overall system. The framework should also contain a provision for considering the dispatchability of the DGs. The system should take into account network uncertainties such as packet loss and computational constraints as well as non-linearity of grid dynamics.

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