Monitoring Process Variability and Root Cause Analysis in Paper Box Production

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Abstract
In this paper, monitoring procedure for process variability in multivariate setting based on individual observations which is a combination of (i) Hotelling’s $T^2$ control chart in detecting out of control signal and (ii) implementation of Mason, Young and Tracy (MYT) decomposition and structure analysis technique for root cause analysis is introduced. The advantages of this procedure will be shown by using the case of a paper box production process in one of the Malaysian manufacturing companies. The successful application of this multivariate approach could act as a stimulant for most industries to imitate in process monitoring. Moreover, the computation efficiency in root cause analysis enables quality’s multiple characteristics to be monitored simultaneously. Based on the findings, the core issue that needs to be a matter of concern by the management team is the closure tap of the box. This process variation should be solved immediately to avoid the products’ quality from further deteriorating.

Keywords: Hotelling’s $T^2$; Multivariate Statistical Process Control; MYT decomposition.

1. Introduction
Malaysia’s economy has experienced a historic transformation since the 1970s where the focus on agricultural based changed dramatically to an industrial based. This tremendous alteration in the direction of economic development has greatly impacted the country. One of the major contributors to the rapid economic growth is the manufacturing sector. Its prominent achievements are apparent over the time. According to the annual report by [1], the manufacturing sector has recently accomplished an annual sales of RM 61.5 billion in 2016 with a significant growth of 10.6% as compared to 2015. It was further highlighted by [2] that Malaysia was the 17th most competitive country for manufacturing location in 2016. Moreover, manufacturing sector has gained much attention throughout the world due to its potential for further growth besides boosting economic performance [3, 4, 5]. These remarkable attainments encourage a research to be conducted in the manufacturing industry in order to improve the quality of outputs.

In the manufacturing industry, most processes involved at least two quality characteristics that need to be monitored concurrently for quality control and improvement [6-9]. Based on the same studies, application of an advanced approach which refers to the multivariate statistical process control (MSPC) is said to be vital in monitoring and analysing the multivariate data that contain several quality dimensions. However, the study by [10] revealed that Malaysian organizations strongly prefer to use simple control charts as a quality tool in process monitoring. The traditional execution of multiple univariate techniques in monitoring correlated variables independently often leads to a misleading result. It is clearly stated by [11] that an out of control signal might be detected through the using of multivariate approach even though none of the observations shows abnormality when the interrelated quality characteristics are being individually monitored. It is also supported by [12, 13] where the researchers presented that, to simply ignoring the correlated variables in multivariate data analysis will result in wrong interpretation to process variation.

In the multivariate setting, the interpretation of out of control signals and investigation on assignable causes require high analytical skills and knowledge. This is because the presence of correlated variables will frequently trigger a false alarm [14-16]. In addition, [15] and [16] also revealed that reduction of multivariate data to independent univariate data will conceal the actual leading factors for existing process variation. Thus, the consideration of both individual and joint contribution of variables is a must. This is to ensure that the signals of process variation could precisely be determined. Since interrelated quality characteristics are the main concern in multivariate analysis, appropriate multivariate techniques should be chosen and executed to obtain accurate results. According to the study by [16], it is stated that the use of Hotelling’s $T^2$ control chart in detecting the multivariate process variation is rapidly growing as one of the most effective tools in monitoring phase. This method is also strongly proposed by many researchers in the past [17-19]. However, there are some limitations in multivariate control charts. It could be seen from the aspect that the influential causes of the respective out of control signals are not presented directly to quality practitioners after the alerts on process variation are detected.

It is essential for quality practitioners to implement root cause analysis in order to investigate the root causes of these signals. Referring to [20] and [21], root cause refers to the actual underlying cause(s) with symptoms, either positive or negative in a pro-
cess; and when the problem is solved, the symptoms will be reduced or removed. For instance, in the industrial setting, when an out of control signal is detected in the multivariate process, a root cause analysis should be able to eliminate the influences of those causes when the problem is solved. The application of root cause analysis is vital when there is a need to improve output of processes through the detection and removal of the root causes as shown in the studies by [22, 23]. The successful application of root cause analysis is so significant in the way that it could effectively prevent recurrence of the same problem, improve outputs’ quality and minimize losses due to unstable process.

The study by [24] proposed an approach that decomposes Hotelling’s $T^2$ statistic in detecting the out of control variable, which is called Mason, Young and Tracy decomposition method (MYT decomposition). Decomposition of Hotelling’s $T^2$ value will be conducted after the out of control signal is detected in the control chart. Therefore, the root causes of such signals could be identified. This approach has gained much acceptance among many studies and researchers due to its effectiveness in detecting the contributing variables in out of control situation [13], [18], [25] and [26]. As the number of variable increases, the number of decomposition models increases significantly too. However, this is not an obstacle that stops researchers from applying this effective method in root cause analysis.

In this research, it aims to determine the out of control signals in multivariate data and further identify the root causes by using both multivariate and univariate approaches. This research is conducted as a case study in monitoring the production process of a bottom or side opening box set in an organization which is located in Selangor. The name of the organization will be kept hidden to protect the confidentiality of the respective company. The following section briefly discusses the methodology of the analysis and section 3 presents the results and discussions. Later on, the conclusions of the study are summarized and highlighted at the end of the paper.

2. Methodology

The data collection process is executed along the stages of a die-cutting process which produces pack-aging boxes. This process involves die-cut, line-crease, and perforation stages. In this study, the data set consists of measurements from a bottom or side opening box set. This type of box set is produced from 2 plies of 300gsm C1S art cards. All samples are taken within a single shift of production process and a machine operator is assigned to draw out 1 to 2 pieces of packaging sheets approximately every 30 seconds to ensure that samples are extracted randomly. In the production process which involves the quantity of 10000 to 50000 pieces, 100 samples are required for the purpose of quality checking. All samples will be inspected by taking the accurate measurements using digital vernier callipers. The quality of box set is measured based on the characteristic of of $p = 2$ interrelated dimensions. These dimensions are also referred as the critical to quality (CTQ). The 2 quality characteristics involved are:

i. Length of closure tap ($Q_1$)

ii. Length of closure hole ($Q_2$)

The two quality characteristics, $Q_1$ and $Q_2$, are illustrated in the box set drawing in Figure 1. This type of box set is designed with measurements of 50mm for $Q_1$ and 51mm for $Q_2$. The tolerance limit for both measurements is within 1 millimetre ($\pm 1 mm$). However, other measurements of this box set will be concealed as a protection of confidentiality of respective organization. For information, the closure taps and closure holes are crucial dimensions. These dimensions are fulfilled. Firstly, a preliminary inspection instead of implementing proper statistical process control to monitor the quality of box set produced. Thus, the reject proportion was relatively high; and leads to the allocation of extra resources for the purpose of reworking or re-producing the box set. In this research, a multivariate statistical process control technique will be applied by using the data set obtained. Before conducting further analysis on the data set, several testing is vital to ensure that the assumptions are fulfilled. Firstly, a preliminary analysis will be conducted for assumptions’ checking. Then, Hotelling’s $T^2$ control chart will be applied to detect the out of control signal(s), while MYT Decomposition technique and structure analysis are implemented to determine the precise root cause(s).

2.1. Preliminary Analysis

The study by [13] mentioned that a process is said to be multivariate when more than one variable is monitored simultaneously. However, not all the multivariate data set is appropriate to be analysed using multivariate analysis techniques. This is because there are some fundamental assumptions to follow as to ensure the execution of the selected technique would generate accurate result [27, 28]. The checking of fundamental assumptions before constructing the multivariate control chart using Hotelling’s $T^2$ statistics includes; (i) variables are correlated, (ii) observations are independent of each other, and (iii) data follow a multivariate normal distribution.

2.2. Hotelling’s $T^2$ Control Chart

Hotelling’s $T^2$ control chart is one of the most effective and popular tools in monitoring multivariate data [16]. The wide application of Hotelling’s $T^2$ control chart can be seen in many previous research [13, 26, 29]. Phase I control chart which is also known as controlling phase, will then be constructed when all the assumptions are fulfilled. The data collected will be analysed to identify whether any out of control signal is causing the process to be unstable. In the manufacturing sector, some industries conducted individual observation in monitoring the multivariate process. For this kind of data, the $T^2$ statistic for each sample is computed using the Equation 1 by considering the value of mean and the inverse of the covariance matrix.

$$T^2 = (x_i - \bar{x})^T S^{-1} (x_i - \bar{x})$$

(1)

The Hotelling’s $T^2$ control chart in Phase I is based on Beta distribution, $\beta$. An approximation of upper control limit (UCL) with false alarm rate, $\alpha$ is as follows:

$$UCL = \frac{(n-1)^2}{n} \beta \left( \frac{1}{\gamma} - \frac{1}{\gamma' - 1} \right)$$

(2)

The $T^2$ statistics and UCL are then plotted in the control chart. When one or more points located beyond the upper control limit, the points which are due to assignable causes will be eliminated. In addition, the control chart is revised by recalculating the mean,
covariance matrix, and $T^2$ value. Thereafter, the control chart will be redrawn until all the points are located within the control limit, in which this represents a stable process. On the contrary, when none of the points exceeds the UCL, then proceed to Phase II for monitoring process.

$T^2$ statistics are computed for all the new samples used in Phase II by using the same mean and covariance matrix calculated in stable controlling process. For Phase II, the approximation for UCL follows the $F$-distribution. The UCL for monitoring phase is as follows:

$$ UCL = \frac{\bar{x}^2}{n} + \sum_{j=1}^{p} \sum_{i=1}^{n} \frac{1}{n} \left( \frac{x_i - \bar{x}}{s_j} \right)^2 $$

(3)

Any point that exceeds the UCL needs to be a matter of concern and the root cause is to be investigated.

### 2.3. MYT Decomposition

MYT Decomposition technique is frequently used to decompose $T^2$ statistics for the detection of root cause of out of control signals [24]. For a data set with $p$ quality characteristics, the $T^2$ statistics will be decomposed into $p$ orthogonal components; which are also referred to as conditional terms and unconditional terms. Firstly, all the possible forms of MYT models are generated. One of the MYT decom-position models is as below.

$$ T^2 = T^2_1 + T^2_2 + T^2_{1,2} + \ldots + T^2_{1,2,\ldots,p-1} = T^2_1 + \sum_{j=2}^{p} T^2_{1,2,\ldots,j-1} $$

(4)

$T^2$ is the first term and is also referred to as the unconditional term of Hotelling’s $T^2$ while the other terms are the conditional terms. For computing the first term, $T^2_1$ for all possible models, the general formula is defined as equation 5.

$$ T^2_1 = \left( \frac{x_j - \bar{x}_j}{s_j} \right)^2 $$

(5)

for $j = 1, 2, \ldots, p$ and where $\bar{x}_j$ and $s_j$ represent the sample mean of and standard deviation of $j^{th}$ variable, respectively. The unconditional term follows $F$-distribution and is compared with the UCL computed as below:

$$ UCL = \left( \frac{n+1}{n} \right) F_{(n,1,k-1)} $$

(6)

In other words, when $T^1$ exceeds the UCL, it means that the out of control point deviates significantly from the sample mean of the $j^{th}$ variable. The variable that generates significant $T^1$ value will be removed and ignored from the checking of the association of the variable with other variables. After that, the conditional terms are computed as shown in Equation 7. Before that, the conditional mean and conditional variance are calculated.

$$ T^2_{1,2,\ldots,j-1} = \left( \frac{x_j - \bar{x}_{1,2,\ldots,j-1}}{s^2_{1,2,\ldots,j-1}} \right)^2 $$

(7)

for $j = 1, 2, \ldots, p$, where $\bar{x}_j$ is the conditional mean vector and $s^2_j$ is the conditional variance. These unconditional terms also follow $F$-distribution and are compared with the UCL

$$ UCL = \left( \frac{n+1}{n} \right) F_{(n,1,k-1)} $$

(8)

where $k$ is the number of conditional variables. These conditional terms could effectively be used in identifying whether the relationship of $j^{th}$ variable conforms to other variables as established in the process in Phase I due to the reason that these adjusted observations have greater sensitivity in the changes of covariance structure [9], [13] and [30]. The pair variables with significant $T^2$ value are removed from consideration, which refers to the $T^2$ value that exceeds the upper control limit. These steps are repeated for triple relationship between three variables, and the respective $T^2$ value is computed until no significant value being detected.

### 2.4. Structure Analysis

In this case, the main component in structure analysis is the variance structure. For variance structure, it is further separated into variance shift and total variance shift. The wide application of structural models could be seen in many research such as [31] and [32]. Besides, the effectiveness of structure analysis as a graphical approach to illustrate and summarize the shift in parameters is shown in the study by [33]. The root cause of out of control signal could be investigated by implementing structure analysis. Although the application of structure analysis could be seen in the research by [33], the respective algorithm for computational structure analysis has not been written in a proper manner. In the next sub-chapters, detailed algorithms in the computation of variance shift and total variance shift in monitoring phase will be developed.

#### 2.4.1 Variance Shift

In the analysis of variance shift, the covariance matrix computed using historical data in stable condition (Phase I) will be used as an indicator for comparison. To identify the variance shift for each observation in augmented data set (Phase II), the procedure is as shown below.

1. **Step 1:** Insert the observation for first sample in monitoring phase into the $n$ observations in stable controlling phase.
2. **Step 2:** Compute the new variance by using equation 3 for all quality characteristics in the new data set with $n + 1$ observations.
3. **Step 3:** Calculate the difference between new variance for $n + 1$ observations and variance for $n$ observations in stable controlling phase for every variable.
4. **Step 4:** Remove the first observation from the previous data set and form a new data set with $n + 1$ observations by combining the observation for next sample in monitoring phase.
5. **Step 5:** Repeat step 2 to step 4 to count the difference in variance for every sample in monitoring phase individually.
6. **Step 6:** Plot a run chart which consists of the baseline of 0 and difference in variance for each quality characteristic.
7. **Step 7:** The point that represents out of control signal in monitoring phase is checked whether it is located significantly far from the baseline. Extreme large value for absolute difference in variance indicates that the respective signal is due to a shift in variance structure. However, when there exists in-control point with larger difference in variance as compared to the signal, it indicates that the process variation might be a result of other root cause rather than a deviation in variance.

#### 2.4.2 Total Variance Shift
For total variance shift, it acts as another indicator in structure analysis. The computation of total variance is similar to the procedure in variance shift above. Step 1 to step 5 above are repeated to obtain the new variance of each quality characteristic for every observation in monitoring phase. Before constructing the run chart of total variance in step 6, the summation of new variance for all variables is computed for each observation. Next, step 7 is same as above where the point that represents out of control signal is referred and investigated for the existence of deviation in terms of total variance.

3. Results and Discussions

3.1. Hotelling’s $T^2$ Control Chart

There are 2 different phases in constructing the Hotelling’s $T^2$ control chart. The beginning phase refers to the controlling phase and followed by the monitoring phase. These control charts are used to determine the existence of out of control signal which triggers as an alert of process variation. Since the data have subgroup size of 1, Hotelling’s $T^2$ control chart for individual observation was applied.

3.1.1 Phase I Operation (Controlling Phase)

The first 50 samples are used in controlling phase. The estimated mean vector, covariance matrix and inverse covariance matrix are computed by considering the significance level of 0.0027. Besides, the upper alpha percentage of Beta distribution that will be taken to compute the $T^2$ statistics is 0.2225.

\[
\bar{X} = \begin{bmatrix} 50.01 \\ 51.102 \end{bmatrix}, \ S = \begin{bmatrix} 0.0666 & 0.0614 \\ 0.0614 & 0.0651 \end{bmatrix}, \ S^{-1} = \begin{bmatrix} 114.8744 & -108.3632 \\ -108.3632 & 117.5826 \end{bmatrix}
\]

The UCL at 0.0027 significance level with 50 observations for Phase I control chart is 10.6619. Hotelling’s $T^2$ statistics were then calculated for each and every observation in the controlling phase. For instance, the $T^2$ value for the first observation was computed as shown below:

\[
T_i^2 = (x - \bar{X})' S^{-1} (x - \bar{X})
\]

\[
= \begin{bmatrix} 50.2 - 50.01 \\ 51.3 - 51.102 \end{bmatrix}' \begin{bmatrix} 114.8744 & -108.3632 \\ -108.3632 & 117.5826 \end{bmatrix} \begin{bmatrix} 50.2 - 50.01 \\ 51.3 - 51.102 \end{bmatrix}
\]

\[
= 0.6034
\]

The Phase I Hotelling’s $T^2$ control chart is plotted (Figure 2) and it illustrates that the points are fluctuating from sample to sample. The fluctuations from sample 1 to sample 25 show smaller amplitude while the fluctuations from sample 27 to sample 50 display greater amplitude. It indicates that process variation might occur after sample 26. A point that represents the 26th sample is located beyond the UCL with $T^2$ statistics of 13.2063. To ensure that the controlling phase is under a stable process, the control chart is revised without considering the said 26th sample.

3.1.2 Phase II Operation (Monitoring Phase)

When the process is in-control, then proceed to Phase II operation which is also referred to as monitoring phase. The same value of statistics computed such as mean and covariance matrix calculated in controlling phase will be kept constant in monitoring future process. Besides that, all observations that lead to out of control signals are detected to further identify the root causes.

All $T^2$ statistics are computed for the remaining 50 samples before Hotelling’s $T^2$ control chart (Figure 4) is plotted. It should be noted that the upper control limit for monitoring phase follows F-distribution rather than the previous Beta distribution. Hence, the UCL for Phase II control chart is 13.9674 at significance level of 0.0027. In Figure 4, it can be seen that the points are fluctuating randomly. There is a significant point which is located beyond the UCL. The out of control signal is triggered from sample 35 with $T^2$ statistics of 15.0379.

Nevertheless, the limitation is that the root cause of such variation could not be shown directly through the Hotelling’s $T^2$ control chart. Although it is known that something has occurred in the production process, the factors that lead to such problem must be revealed. Therefore, another statistical approach is required to further determine the root cause of this detected signal.

3.2. Root Cause Analysis

Root cause analysis is vital in identifying the root cause of out of control signal. By taking sample 35 as the out of control signal in monitoring phase, MYT decomposition and structure analysis are conducted.
3.2.1 MYT Decomposition

The $T^2_{2}$ statistics of sample 35, which is 15.0379 is decomposed into unconditional terms and conditional terms as tabulated in Table 1. The UCL for unconditional terms is 10.1884 while the UCL for conditional terms is 10.4238. It shows that both the unconditional terms are smaller than the respective UCL. This indicates that the 35th observation did not deviate significantly from both of the sample means of individual quality characteristic.

Table 1: Decomposition of Hotelling’s $T^2$ statistics (Phase II)

| Terms  | Values |
|--------|--------|
| $T^1_1$ | 7.2319 |
| $T^1_2$ | 2.5851 |
| $T^2_1$ | 7.8059 |
| $T^2_2$ | 12.4528 |

Nevertheless, $T^2_2$ exceeds the respective UCL for conditional terms. It shows that the first quality characteristic ($Q_1$) has violated the relationship with second quality characteristic ($Q_2$) as computed in controlling phase. In other words, the root cause that leads to the out of control signal in monitoring phase was due to deviation in length of closure tap of box.

As a justification to the results above, the signal generated at controlling phase which refers to the 26th sample undergoes MYT Decomposition too for root cause analysis. The decomposition to unconditional and conditional terms are recorded in Table 2. Since the conditional term of is beyond the UCL, it leads to similar results that the process variation is due to deviation in the first quality characteristic ($Q_2$).

Hence, proper adjustment should be implemented to resolve the existing problem which is related to the length of closure tap of box. Furthermore, structure analysis is executed as univariate technique in root cause analysis.

3.2.2 Structure Analysis

In structure analysis, the root cause for out of control signal is determined from variance shift and total variance shift. All these indicators are applied to illustrate the root cause for out of control signal.

3.2.2.1 Variance Shift

For structure analysis in terms of variance shift, the out of control signal at monitoring phase is used to determine the root cause. The run chart of variance shift (Figure 5) presents that most of the observations have small positive or negative variance shift. Observation 8 shows that the respective sample has relatively large variance shift in the first quality characteristic. At the same time, observation 19 also displays a large variance shift in variance for both dimensions. However, these two points do not trigger any signal of out of control in the monitoring phase. Hence, these two points are said to have insignificant variance shift.

Next, the structure analysis is conducted in terms of total variance shift in the monitoring phase. A run chart of total variance shift is plotted as shown in Figure 6. It can be observed that most of the observations have small total variance shift within the absolute range of 0.003. Samples 8 and 19 present a large total variance shift with values of 0.0072 and 0.0081 respectively; but these observations are not the out of control signals in Phase II.

In addition, it is clearly seen that sample 35 has the largest total variance shift of 0.0098 as compared to other observations. It indicates that the respective signal is due to the shift in total variance structure.

4. Concluding Remarks

The data obtained have fulfilled the assumptions of correlated variables and observations are independent of each other. Moreover, it is assumed that the data are multivariate normal. By applying the Hotelling’s $T^2$ control chart in both controlling and monitoring phases, the out of control signals are successfully detected. The signals are those points that exceed the UCL.

The root cause analysis shows its effectiveness in identifying the precise causes that lead to process variation. Through MYT Decomposition, it presents that the signal in monitoring phase is due to the deviation in the length of closure tap of box ($Q_1$). In addition, the structure analysis also indicates similar results where the out of control signal is caused by a significant shift in both of the total variance and the variance for first quality characteristic (length of closure tap of box). It is interesting that the root cause analysis using both multivariate analysis and univariate analysis display the same results.

In this research, the successful application of Hotelling’s $T^2$ control chart in both controlling and monitoring phases enables the out of control signals of process variation to be detected accurately. The root cause analysis summarizes that the occurrence of process
variation is definitely due to the deviation in length of closure tap ($Q_i$). Thus, this indicates that more focus should be paid on the first quality characteristic in future production.

Although both the multivariate technique and univariate technique present similar results in root cause analysis, the MYT Decomposition approach shows higher computation efficiency in process monitoring. On the contrary, the computation of structure analysis is more complex, but it acts as a great graphical tool in illustrating the structure shift for each observation. In Malaysian manufacturing industry, it is strongly proposed that the application of MYT Decomposition in process monitoring could significantly assist in improving the outputs’ quality and minimizing quality issues in production process. From this research, there are two major recommendations to the said organization.

i. The company needs to replace the current visual inspection approach by implementing multivariate statistical process control (MSPC) in its mass production process. Since the quality characteristics are multivariate in monitoring process variation, the use of MSPC is vital and the advantages of successful MSPC application are proven as shown in the analysis above.

ii. The company has to invest in quality controlling and monitoring technologies. Qualified outputs can be produced through continuous quality monitoring. By producing high quality products, it indirectly leads to gaining more business advantages and improving the company’s competitiveness in the world market.

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