Compositional waves and variations in the atmospheric abundances of magnetic stars

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Abstract The stars of the middle main sequence often have relatively quiescent outer layers and spot-like chemical structures may develop in their atmospheres. Recent observations show that abundance peculiarities can change as stars evolve on the main sequence and the timescale of these changes lies in a wide range from million years to months. These observations imply that, perhaps, our understanding of diffusion processes at work in magnetic stars is incomplete and a more detailed analysis of these processes is required. In the present paper, we consider diffusion caused by a combined influence of the electric current and the Hall effect. Such diffusion has a number of very particular properties and, generally, can change the surface chemistry of stars in combination with other diffusion processes. For instance, current-driven diffusion is accompanied by a propagation of the special type of waves in which only the impurity number density oscillates. Propagation of such waves changes the shape and size of spots as well as chemical abundances within them. The period of compositional waves depends on the parameters of plasma (magnetic field, electric current, temperature, etc.) and can be different for the waves of different elements. Compositional waves exist in the regions where the magnetic pressure is greater than the gas pressure. These waves can be the reason of variations with different timescales in the abundance peculiarities of magnetic stars.

Keywords Stars: magnetic fields - stars: abundances - stars: chemically peculiar - stars: spots

1 Introduction

The stars of the middle main sequence have quiescent outer layers, and often chemical structures with abundance peculiarities may develop in their atmospheres. Likely, diffusion plays an important role in the formation of such structures (see, e.g., Khohlova (1985)) but many details of diffusion processes are still the subject of debate. The point is that diffusion in astrophysical bodies is influenced by a number of factors (gravity, radiative force, magnetic field, temperature gradient, etc.; see Spitzer (1998)) and is rather complicated. Therefore, chemically peculiar stars are excellent laboratories to study diffusive processes in plasma. Note that, apart from the diffusion model, some other models have been suggested to account for the observed abundance peculiarities in stars (e.g., nuclear evolution, mass transfer from a companion, selective accretion from ISM; see Landstreet (2014) for discussion) but all these models are much less developed than the diffusion model.

At present, little is known about the temporal evolution of chemical structures during the main sequence evolution of stars. Only recently, using high-resolution and high signal-to-noise radio spectra, changes in the atmospheric abundances of trace elements have been detected in chemically peculiar stars. For example, Briquet et al. (2010) discovered noticeable variations of Ti, Sr, and Y with the characteristic timescale ~ 65 days on the surface of HD 11753. Later on, this result was confirmed by Korhonen et al. (2013) who also detected variations of HD 11753 on a long timescale. Note that a slow evolution of chemical spots is also seen in AR Aur (Hubrig et al. (2011)). A temporal evolution has been studied through the main-sequence lifetime of magnetic peculiar stars also by Bailey et al. (2014). These authors have performed spectral analysis of 15 Bp stars that are members of open clusters and thus have well-known ages. Bailey et al. (2014) considered global average abundances in these stars rather
than chemical spots and discovered a systematic time
evolution of trace elements in these stars. The authors
have attempted to interpret the observed variations in
the context of radiatively driven diffusion theory but
not all abundances can be explained in the frame of
this theory. All these findings indicate the hitherto not
well understood physical processes that cause a rather
fast changes of the chemical spots.

Diffusion of trace elements in plasma can differ qualitatively from that in neutral gases because of the presence of electric currents. This particularly concerns hydrogen plasma where the rate of momentum exchange between electrons and protons is comparable to the rate of the momentum redistribution among protons. It was first clearly understood by Braginskii (1965) in his theory of transport phenomena in a high-temperature plasma. This point can be clarified by simple qualitative estimates. Indeed, the momentum of electrons is
\[ m_e c_e \] (\( c_e = \sqrt{k_B T/m_e} \) is the thermal velocity of electrons), and the rate of momentum transfer from electrons to protons is
\[ m_e c_e \nu_e \sim m_e c_e / \tau_e \] where \( \nu_e \) is the frequency of electron collisions with protons and \( \tau_e \) is the electron relaxation time. On the other hand, the momentum of protons is
\[ m_p c_p \] where \( c_p = \sqrt{k_B T/m_p} \) is the thermal velocity of protons and, correspondingly, the rate of momentum redistribution among protons is
\[ m_p c_p \tau_p \approx m_p c_p / \nu_p \] where \( \nu_p \) is the frequency of proton collisions. Comparing these estimates, we obtain that the rate of momentum exchange between electrons and protons is of the same order of magnitude: the momentum transferred in a collision by electrons is smaller but the frequency of collisions is higher. Therefore, the momentum taken from (or transferred to) the background plasma by trace elements is redistributed among both electrons and protons, and neglecting the electron contribution is unjustified in plasma with \( Z_0 \approx 1 \) where \( Z_0 \) is the charge of the background ions. However, if the background plasma consists of ions with \( Z_0 \gg 1 \), then one should replace \( \tau_p \) by the relaxation time of the background ions that is
\[ \tau_p / Z_0^2 \]. The rate of momentum transfer by ions turns out to be \( \sim Z_0^2 \) times greater than that by electrons. Therefore, if \( Z_0 \gg 1 \), the electron contribution to momentum transfer is small and can be neglected. Note also the crucial importance of the momentum transport by electrons in magnetized plasma. Electrons can be magnetized by an essentially weaker magnetic field than protons or impurities. Therefore, transport processes are influenced by a relatively weak field since electrons give a comparable to protons contribution to the momentum transport (see, Urpin (2015) for detail).

Chemical inhomogeneities can appear in quiescent plasma because of a number of reasons (e.g., diffusion in the magnetic field, nuclear evolution with non-spherical mixing, accretion from a companion or ISM, etc.). The model of diffusion in a star with the magnetic field seems to be most developed. The magnetic field can magnetize electrons and result in anisotropic transport and inhomogeneous distribution of trace elements. Anisotropy of diffusion is characterized by the Hall parameter, \( x_e = \omega_e/\tau_e \), where \( \omega_e = eB/m_e c \) is the gyrofrequency of electrons and \( \tau_e \) is their relaxation time; \( B \) is the magnetic field. In hydrogen plasma, \( \tau_e = 3\sqrt{m_e/(k_B T)}^{3/2}/4\sqrt{2\pi e^2 nA} \) where \( n \) and \( T \) are the number density of electrons and their temperature, respectively, \( A \) is the Coulomb logarithm (see, e.g., Spitzer (1953)).

In this paper, we consider one more diffusion process that can be responsible for the formation of chemical inhomogeneities and their evolution. This process is caused by a combined influence of electric currents and the Hall effect and can operate on a timescale shorter than the standard diffusion timescale (Urpin (2015)). Using a simple model, we show in Sec.2 that the interaction of the electric current with impurities leads to their diffusion in the direction perpendicular to both the electric current and magnetic field. We argue in Sec.3 that such diffusion in combination with the Hall effect can be the reason of a particular type of waves in which the impurity number density oscillates alone. These waves can be responsible for a change of the shape and size of chemical spots and even can lead to migration of spots under certain conditions (see discussion in Sec.3 and 4).

2 Diffusion velocity

Consider a cylindrical plasma configuration with the magnetic field parallel to the axis \( z \), \( \vec{B} = B(s) \hat{e}_z \) \((s, \varphi, z) \) and \((\hat{e}_s, \hat{e}_\varphi, \hat{e}_z) \) are cylindrical coordinates and the corresponding unit vectors. The electric current in such configuration is equal to \( j_\varphi = -(c/4\pi)(dB/ds) \). We use this simplified model to understand the main qualitative features of compositional waves. In some cases, however, the considered configuration can mimic the real magnetic fields. For example, the magnetic field near the magnetic pole is close to a cylindrical geometry (see, e.g., Urpin & van Riper (1993)). Note that \( B(s) \) cannot be arbitrary function of \( s \) because the magnetic configurations can be unstable for some dependences \( B(s) \) (see, e.g., Tayler (1973), Bonanno & Urpin (2008b)).

We assume that plasma is fully ionized and consists of electrons \( e \), protons \( p \), and small admixture of heavy ions \( i \). The number density of species \( i \) is small and does not influence dynamics of plasma. The partial momentum equations in multicomponent plasma
have been considered by a number of authors (see, e.g., Braginskii [1965]; Vekstein et al. [1975]; Urpin [1981]; Vekstein [1987]). For example, Urpin [1981] considers the hydrogen-helium plasma but the derived equations describe also the hydrogen plasma with an admixture of any heavy ions if their number density is small. If the hydrodynamic velocity of plasma is zero and only small diffusive velocities are non-vanishing, the partial momentum equation for the species \(i\) reads

\[
-\nabla p_i + Z_i e n_i \left( \vec{E} + \frac{\vec{V}_i}{c} \times \vec{B} \right) + \vec{R}_{ie} + \vec{R}_{ip} + \vec{F}_i = 0, \tag{1}
\]

where \(Z_i\) is the charge number of ions \(i\), \(p_i\) and \(n_i\) are their pressure and number density, \(\vec{E}\) is the electric field, and \(\vec{V}_i\) is the diffusion velocity. The force \(\vec{F}_i\) is the external force on species \(i\); in stellar conditions, \(\vec{F}_i\) is the sum of gravitational and radiative forces. Since diffusive velocities are usually very small, we neglect the terms proportional to \((\vec{V}_i \cdot \nabla)\vec{V}_i\) in the momentum equation (1). The forces \(\vec{R}_{ie}\) and \(\vec{R}_{ip}\) are caused by the interaction of ions \(i\) with electrons and protons, respectively. Note that forces \(\vec{R}_{ie}\) and \(\vec{R}_{ip}\) are internal and the sum of internal forces over all plasma components is zero in accordance with Newton’s third law.

For the sake of simplicity, we consider plasma with \(T=\text{const}\). If \(n_i \ll n\), then \(\vec{R}_{ie}\) is given by

\[
\vec{R}_{ie} = -(e_i Z_i n_i / n) \vec{R}_e \tag{2}
\]

where \(\vec{R}_e\) is the friction acting on the electron gas (see, e.g., Urpin [1981]). One can use for \(\vec{R}_e\) the expression for hydrogen plasma \(\text{Braginskii} [1965]\) since \(n_i \ll n\).

Generally, \(\vec{R}_e\) contains terms proportional to the temperature gradient and relative velocity of \(e\) and \(p\). If \(T=\text{const}\), then the expression for \(\vec{R}_e\) reads

\[
\vec{R}_e = -\alpha_\parallel \vec{u} - \alpha_\perp \vec{u}_\perp + \alpha_\lambda \vec{b} \times \vec{u}, \tag{3}
\]

where \(\vec{u} = -\vec{J}/en\) is the current velocity of electrons; \(\vec{b} = \vec{B}/B\); the subscripts \(\parallel\), \(\perp\), and \(\lambda\) denote the parallel, perpendicular, and so-called Hall components of the corresponding vector; \(\alpha_\parallel\), \(\alpha_\perp\), and \(\alpha_\lambda\), are the coefficients calculated by \text{Braginskii} [1965]. The force (3) describes a small friction force caused by a relative motion of electrons and protons in magnetized plasma.

In our model, \(\vec{u} = (c/4\pi e n)(d\vec{B}/ds)\vec{e}_c\) and \(\vec{B} \perp \vec{u} \) and \(\vec{u}_\parallel = 0\). In this paper, we consider diffusion only in a relatively weak magnetic field with \(x_e \ll 1\). Substituting \(\vec{u}\) into Eq.(3) and using coefficients \(\alpha_\perp\) and \(\alpha_\lambda\) with the accuracy in linear terms in \(x_e\), we obtain

\[
R_{iee} = Z_i^2 n_i \left( 0.51 \frac{m_e}{\tau_e} u \right), \quad R_{iee} = Z_i^2 n_i \left( 0.21 \frac{m_e}{\tau_e} u \right). \tag{4}
\]

The friction force \(\vec{R}_{ip}\) is proportional to the relative velocity of ions and protons. Like \(\vec{R}_e\), this force has a tensor character and depends on \(B\). However, the dependence of \(\vec{R}_{ip}\) on the magnetic field is insignificant if \(x_e \ll 1\) (see, e.g., Urpin [1981]). This is qualitatively clear because it is much more difficult to magnetize heavy ions than protons or electrons. The force \(\vec{R}_{ip}\) is particularly simple if \(A_i = m_i / m_p \gg 1\). Neglecting the influence of the magnetic field on \(\vec{R}_{ip}\) and taking into account that the velocity of the background plasma is zero, \(\vec{V}_p = 0\), the force \(\vec{R}_{ip}\) can be written as

\[
\vec{R}_{ip} = (0.42 m_i n_i Z_i^2 / \tau_i) ( -\vec{V}_i ), \tag{5}
\]

where \(\tau_i = 3 \sqrt{m_i (kB) T}^{3/2} / 4 \sqrt{2 \pi} e^4 n \Lambda\); \(\tau_i / Z_i^2\) is the timescale of \(i \rightarrow p\) scattering; we assume that \(\Lambda\) is the same for all collisions.

After the above simplifications, the cylindrical components of Eq.(1) yield

\[
- \frac{d}{ds} (n_i k_B T) + Z_i e n_i \left( E_x + \frac{\vec{V}_i}{c} \times \vec{B} \right) + R_{iee} + R_{ips} = 0, \tag{6}
\]

\[
Z_i e n_i \left( E_\varphi - \frac{\vec{V}_i}{c} \times \vec{B} \right) + R_{iee} + R_{ip\varphi} = 0, \tag{7}
\]

\[
- \frac{d}{dz} (n_i k_B T) + Z_i e n_i E_z + R_{iee} + R_{ipz} + F_{iz} = 0. \tag{8}
\]

In the chosen magnetic configuration, we have \(R_{iee} = 0\). Eqs.(6)-(8) depend on cylindrical components of the electric field. These components can be determined from the momentum equations of electrons and protons

\[
- \nabla (nk_B T) - en \left( \vec{E} + \frac{\vec{u}}{c} \times \vec{B} \right) + \vec{R}_e + \vec{F}_e = 0, \tag{9}
\]

\[
- \nabla (nk_B T) + en \vec{E} - \vec{R}_e + \vec{F}_p = 0. \tag{10}
\]

In these equations, we neglect collisions of electrons and protons with the ions \(i\) since \(n_i \ll n\). The sum of Eqs.(9) and (10) yields the equation of hydrostatic equilibrium. The difference of Eqs.(10) and (9) yields

\[
\vec{E} = - \frac{1}{2c} e \vec{B} + \frac{\vec{R}_e c}{en} - \frac{1}{2en} (\vec{F}_p - \vec{F}_e). \tag{11}
\]

Taking into account the friction force \(\vec{R}_e\) (Eq.(3)) and the coefficients \(\alpha_\perp\) and \(\alpha_\lambda\) calculated by \text{Braginskii} [1965], we obtain with accuracy in linear terms in \(x_e\)

\[
E_x = - \frac{u B}{2c} - \frac{1}{e} \left( 0.21 \frac{m_e u}{\tau_e} x_e \right), \quad E_\varphi = - \frac{1}{e} \left( 0.51 \frac{m_e u}{\tau_e} \right), \quad E_z = - \frac{1}{2en} (F_{pz} - F_{ez}). \tag{12}
\]

Substituting Eqs.(4) and (12) into Eq. (8), we obtain the following expression for the vertical velocity

\[
V_{iz} = - D \frac{d \ln n_i}{dz} + \frac{D}{n_i k_B T} F_{iz}, \tag{13}
\]
where \( D = 2.4c^2_t\tau_i/Z_i^2 \) is the diffusion coefficient, \( c_i^2 = k_B T/m_i \), and

\[
F_z^{(i)} = F_{iz} - \frac{Z_i n_i}{2n} (F_{pz} - F_{ez}).
\]

(14)

Usually, acceleration due to the radiative energy flux and gravitational settling give the main contribution to \( F_z^{(i)} \) (Michaud et al. 1973; Vauclair et al. 1979; Alecian & Stift 2006). The diffusion velocity caused by these forces can be relatively fast and, therefore, the vertical diffusion often is faster than diffusion parallel to the surface. As a result, the vertical distribution of trace elements reaches a quasi-steady equilibrium on a relatively short timescale. If the radiative and gravitational forces are of the same order of magnitude the velocity of radial diffusion can be estimated as \( V_r \sim q\tau_i \) (see Vauclair et al. 1979). Then, the chemical equilibrium in the vertical direction is reached approximately on a timescale \( \tau_{ver} \sim H/V_r \sim H/g\tau_i \) where \( H \) is the height of the atmosphere. Assuming \( H \approx 10^8 \) cm, \( T \sim 10^4 K \), \( g \sim 10^4 \) cm/s\(^2\), and \( n \sim 10^{14} \) cm\(^{-3}\), we obtain that \( \tau_{ver} \sim 10^3 - 10^4 \) yrs if \( m_i/m_p \sim 40 \).

This timescale is much shorter than the lifetime of peculiar stars. Note that this timescale can be substantially shorter for stars with a hot surface because the radiative acceleration in such stars can essentially exceeds gravity for some ions.

The tangential components of the diffusion velocity can be obtained from Eqs. (6)-(7). Taking into account Eq. (5) for \( \bar{R}_{ip} \), one can transform Eqs. (6)-(7) into

\[
V_{is} - qV_{i\varphi} = A, \quad V_{i\varphi} + qV_{is} = G,
\]

(15)

where

\[
A = \frac{D}{n_i k_B T} \left( -\frac{dp_i}{ds} + Z_i e n_i E_n + R_{res} \right),
\]

\[
G = \frac{D}{n_i k_B T} (Z_i e n_i E_\varphi + R_{\varphi res}), \quad q = 2A \frac{eB}{Z_i m_i c} \tau_i.
\]

(16)

Then, diffusion velocities in the \( s \)- and \( \varphi \)-directions are

\[
V_{is} = A + qG, \quad V_{i\varphi} = G - qA.
\]

(17)

The parameter \( q \sim (m_e/m_i)^{1/2}x_e \) is small even for magnetic fields of Ap-stars since \( q \ll x_e \ll 1 \). Therefore, \( V_{is} \approx A, V_{i\varphi} \approx G \). Substituting Eqs.(4) and (12) into Eqs.(16)-(17), we obtain for the diffusion velocities

\[
V_{is} = V_n + V_B, \quad V_{i\varphi} = D_B e \frac{dB}{ds};
\]

(19)

\[
V_{ni} = \frac{D}{n_i k_B T} \frac{d\ln n_i}{ds}, \quad V_B = D_B \frac{d\ln B}{ds};
\]

(20)

\( V_{ni} \) is the velocity of ordinary diffusion and \( V_B \) is the velocity caused by the electric current. The corresponding diffusion coefficients are

\[
D = 2.4c^2_t\tau_i/Z_i^2, \quad D_B = 2.4c^2_A\tau_i/Z_i A_i (0.21Z_i - 0.71),
\]

(21)

\[
D_B = 1.22 \sqrt{\frac{m_e c (Z_i - 1)}{m_i 4\pi n_i Z_i}}.
\]

(22)

where \( c_i^2 = k_B T/m_i \) and \( c_A^2 = B^2/(4\pi n_i m_p) \). Eqs. (19)-(20) describe the drift of ions \( i \) under the combined influence of \( \nabla n_i \) and \( j \). Note that our consideration of diffusion in plasma differs from that in astrophysical calculations (see, e.g., Chapman & Cowling 1970; Burgers 1969) by taking into account interaction of protons with electrons.

### 3 Compositional waves

The continuity equation for ions \( i \) reads in our model

\[
\frac{\partial n_i}{\partial t} + \frac{1}{s} \frac{\partial}{\partial s} (sn_i V_{is}) + \frac{1}{s} \frac{\partial}{\partial \varphi} (n_i V_{i\varphi}) = 0.
\]

(23)

Consider the behaviour of small disturbances in the impurity number density, \( n_i \), by making use of a linear analysis of Eq. (23). Assume that plasma is in equilibrium in the unperturbed state. Since the number density of trace elements is small, their influence on parameters of the basic state is negligible. We consider disturbances that do not depend on \( z \). Denoting the disturbances of \( n_i \) by \( \delta n_i \) and linearizing Eq.(23), we obtain

\[
\frac{\partial \delta n_i}{\partial t} + \frac{1}{s} \frac{\partial}{\partial s} \left( s n_i V_{is} \right) + \frac{1}{s} \frac{\partial}{\partial \varphi} \left( n_i D_B e \frac{dB}{ds} \right) = 0.
\]

(24)

We consider disturbances with the wavelength shorter than the lengthscale of \( B \). In this case, we can use the so called local approximation and assume that disturbances are \( \propto \exp(-iks - iM\varphi) \) where \( k \) is the wavevector, \( ks \gg 1 \), and \( M \) is the azimuthal wavenumber. Since the basic state does not depend on \( t \), \( \delta n_i \) can be represented as \( \delta n_i \propto \exp(\omega t - iks - iM\varphi) \) where \( \omega \) should be calculated from the dispersion equation. We consider two particular cases of the compositional waves, \( M = 0 \) and \( M \gg ks \).

#### Cylindrical waves with \( M = 0 \)

Substituting \( \delta n_i \) into Eq. (24), we obtain the dispersion equation in the case \( M = 0 \),

\[
i\omega = -\omega_R + i\omega_B, \quad \omega_R = Dk^2, \quad \omega_B = k DB(d\ln B/ds).
\]

(25)
This dispersion equation describes cylindrical waves in which only the number density of impurity oscillates. The quantity $\omega_R$ characterizes the decay of waves with the characteristic timescale $\sim (Dk^2)^{-1}$ typical for a standard diffusion. The frequency $\omega_B$ describes the oscillation of impurities caused by the combined action of electric current and the Hall effect. Note that this frequency can be of any sign but $\omega_R$ is always positive. The compositional waves are aperiodic if $\omega_R > |\omega_B|$ and oscillatory if $|\omega_B| > \omega_R$. This condition is equivalent to

$$c_A^2/c_s^2 > Z_i^{-1}|0.21Z_t - 0.71|^{-1}kL,$$

where $c_s$ is the sound speed, $c_A^2 = k_BT/m_p$. Compositional waves become oscillatory if the field is strong enough and the magnetic pressure is substantially greater than the gas pressure. The frequency of diffusion waves is higher in the region where the magnetic field has a strong gradient. The order of magnitude estimate of $\omega_I$ yields

$$\omega_I \sim kA(1/Z_iA_i)(c_A/c_s)(l_i/L),$$

where $l_i = c_i\tau_i$ is the mean free-path of ions $i$. Note that different impurities oscillate with different frequencies.

Non-axisymmetric waves with $M \gg ks$. In this case, the dispersion equation reads

$$i\omega = -\omega_R + i\omega_{B\varphi}, \quad \omega_{B\varphi} = (M/s)BD_{B\varphi}(d\ln B/ds).$$

Non-axisymmetric waves rotate around the cylindrical axis with the frequency $\omega_{B\varphi}$ and decay slowly on the diffusion timescale $\sim \omega_R^{-1}$. The frequency of such waves is typically higher than that of cylindrical waves. One can estimate the ratio of these frequencies as

$$\omega_{B\varphi}/\omega_R \sim (BD_{B\varphi}/DB) \sim (1/A_xx)(M/ks).$$

Since we consider only a weak magnetic field ($x_e \gg 1$), the period of non-axisymmetric waves is shorter for waves with $M > A_xx(ks)$. The ratio of the diffusion timescale and period of non-axisymmetric waves is

$$\omega_{B\varphi}/\omega_R \sim (1/x_e)(c_A^2/c_s^2)(Z_i/A_i)(1/kL)$$

and can be large. Hence, these waves can be oscillatory as well.

4 Conclusion

We have considered diffusion of heavy ions under the combined influence of electric currents and the Hall effect. The velocity of such diffusion can be larger than that caused by other diffusion mechanisms. The considered diffusion forms chemical inhomogeneities even if the magnetic field is relatively weak whereas other mechanisms require a stronger field. This type of diffusion is relevant to the Hall effect and, therefore, it leads to drift of heavy ions perpendicular to both the magnetic field and electric current.

The current-driven diffusion in combination with other diffusion processes can contribute essentially to the surface chemistry of various types of stars. Certainly, this type of diffusion may play an important role in the surface chemistry of Ap/Bp stars. These stars have a strong magnetic field (see, e.g., Khohlova (1984)) that can magnetize the atmospheric plasma and produce a strong Hall drift of electrons. These conditions seem to be suitable for the propagation of compositional waves considered in this paper and, likely, such waves can be the reason of variations in atmospheric abundances of these stars. The current-driven diffusion may be also important in HgMn stars. The magnetic field of HgMn stars is substantially weaker than that of Ap/Bp stars (see, e.g., Wade et al. (2004); Hubrig & Castelli (2001); Hubrig et al. (2006)) and, likely, it cannot magnetize heavy ions and form element spots by the mechanism based on the magnetization of such ions. The current-driven diffusion requires, however, a relatively weak magnetic field compared to other diffusion mechanisms (see discussion in Urpin (2013)). Perhaps, the considered diffusion mechanism can be important also in compact stars like white dwarfs or neutron stars. For instance, many white dwarfs and neutron stars have a strong magnetic field with complex topology and spot-like structures at the surface. Such magnetic configurations can lead to the formation of a spot-like element distribution at the surface. Particularly, this concerns the accretion phase in binary systems. Many white dwarfs and neutron stars in relatively close binaries pass through the accretion phase when plasma of a companion is accreted by the compact star. Such accretion phase is typical for binaries with various types of the companion (see, e.g., Urpin et al. (1998a)) and the evolution of compact stars is very complicated during this period. For example, a strong magnetic field can channel plasma of a companion onto the surface of the neutron star and form chemical spots there. The evolution of such spots on neutron stars can be very complicated because of strong magnetic field, high gravity and large luminosity. The chemical processes in these spots may influence the burst activity of neutron stars (see, e.g., Brown et al. (2002); Chang & Bildsten (2004)); their thermal evolution, etc. and, likely, the current-driven diffusion can play an important role in the evolution of spots.

Our study reveals that the particular type of waves (compositional waves) may exist in multicomponent
plasma in the presence of electric currents. Such waves exist only if the magnetic pressure is greater than the gas pressure. The compositional waves are slowly decaying and characterized by oscillations of the impurity number density alone. For example, in subsequent spectral observations, the propagation of these waves can manifest themselves by changes in the location of regions with a higher intensity in spectral lines (bumps). These bumps indicate chemical spots on the surface of peculiar stars and their motion can be caused by compositional waves. Note that, generally, bumps in different spectral lines can move with different velocities.

The frequency of compositional waves turns out to be different for different sorts of ions. Therefore, the abundances of different elements in peculiar stars can vary on different timescales. The period of waves can change in a wide range from that comparable to the lifetime of a star to a rather short value of the order of few months.

For instance, the compositional modes can be responsible for changes in the distribution of few elements on the surface of the HgMn star HD 11753 discovered by Briquet et al. (2010). Spectral line profile changes were detected by making use of the Doppler imaging technique and using two datasets separated by ~ 65 days. The results of this study revealed noticeable changes in the distribution of TiII, SrII, and YII indicating a rather fast chemical spots evolution. If this evolution is caused by a propagation of the compositional waves then one can expect spectral line profile changes on the timescale \( \tau \sim \frac{1}{\omega_B} \sim 10^{13} \sqrt{A_i} \frac{L_i}{B_i^2} \frac{1}{\rho_i^{2/10}} \frac{\Lambda_i}{\Lambda} \) days. (33)

This estimate is in a good agreement with the characteristic timescale measured by Briquet et al. (2010). Note that our model predicts that the timescale \( \tau \) can be different for different elements and \( \propto \sqrt{A_i} \). The square root of atomic numbers for TiII:SrII:YII is equal approximately to 6.9:9.4:9.4. Therefore, the timescales for SrII and YII should be approximately the same according to our model, whereas changes of TiII can occur on a timescale about 20-30% shorter. Since Briquet et al. (2010) compared only two datasets separated by 65 days, more detailed study of the abundance variations in HD 11753 is required.

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