Fluctuations of Goldstone modes and the chiral transition in QCD

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Abstract

We provide evidence for the influence of thermal fluctuations of Goldstone modes on the chiral condensate at finite temperature. We show that at fixed temperature, $T < T_c$, in the vicinity of the chiral transition temperature this leads to a characteristic dependence of the chiral condensate on the square root of the light quark mass ($m_l$), which is expected for 3-dimensional models with broken O(N) symmetry. As a consequence the chiral susceptibility shows a strong quark mass dependence for all temperatures below $T_c$ and diverges like $1/\sqrt{m_l}$ in the chiral limit.

Key words: QCD thermodynamics, chiral symmetry breaking, Goldstone modes
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1. Introduction

Establishing the properties of the chiral phase transition in QCD is one of the outstanding problems in lattice simulations of QCD thermodynamics. Although several attempts have been undertaken to establish the unique scaling behavior in the limit of vanishing light quark masses no clear-cut evidence for the expected $O(N)$ scaling, related to the underlying chiral symmetry of the QCD Lagrangian, has been found. This may have several reasons related to common problems in lattice calculations: cut-off effects, finite volume effects, explicit flavor symmetry breaking and/or too large quark masses.

We will discuss here yet another effect that influences the scaling behavior of thermodynamic quantities in the vicinity of the finite temperature chiral transition of QCD and may compete with the universal scaling in the vicinity of $T_c$. This is related to singularities in chiral observables, e.g. the chiral susceptibility, that are induced at ANY

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temperature in the chiral symmetry broken phase. The existence of these singularities induced by fluctuations of Goldstone modes are well known from the analysis of 3 and 4 dimensional statistical models and field theories with global $O(N)$ symmetries that are spontaneously broken at low temperature [1–4].

2. Chiral condensate and susceptibility for $T \lesssim T_c$

At vanishing temperature properties of QCD in the light quark sector are described by an effective 4-d, $O(4)$ symmetric Lagrangian; chiral symmetry is broken and, what is of interest here, at non-zero quark mass the light quark chiral condensate, $\langle \bar{\psi} \psi \rangle$, receives logarithmic corrections proportional to $m_l \ln m_l$ that are due to the presence of light Goldstone modes [2]. This leads to a logarithmic divergence of the chiral susceptibility, $\chi_m \sim d\langle \bar{\psi} \psi \rangle/dm$. In three dimensions the analog to these logarithmic terms are corrections that are proportional to the square root of the quark mass [1]. As a consequence $\chi_m$ will diverge in the chiral limit like $\chi_m \sim 1/\sqrt{m_l}$.

In the vicinity of the QCD phase transition a 3-d, $O(4)$ symmetric effective theory is expected to describe the universal properties of the transition [5] which will also control the properties of the chiral condensate for temperatures close to $T_c$. We thus expect that in the chiral limit $\chi_m$ will diverge for all values of the temperature $T \leq T_c$,

$$\chi_m \sim \begin{cases} m_l^{-1/2} & , T \leq T_c \\ m_l^{1/\delta - 1} & , T = T_c \end{cases}$$

(1)

where $1/\delta - 1 \approx 0.79$ for 3-d $O(4)$ as well as $O(2)$ symmetric spin models.

The appearance of a square root singularity has, in fact, been established in the vicinity of the chiral phase transition of an $SU(3)$ gauge theory coupled to adjoint fermions [6] where it is apparent already for moderately small values of the quark mass. Similar evidence for the role of light Goldstone modes in QCD, however, has been missing so far.

In Fig. 1 we show the light quark chiral condensate, $\langle \bar{\psi} \psi \rangle$ calculated in (2+1)-flavor QCD with improved staggered fermions (p4-action) on lattices of size $N_\sigma^3 \times 4$ for various values of the gauge coupling. The figures show results for $\langle \bar{\psi} \psi \rangle$, plotted versus the light
Fig. 2. Disconnected part of the chiral susceptibility calculated on lattices of size $N^3 \times 4$ in (2 + 1)-flavor QCD for various values of the light quark mass.

The general features of the quark mass dependence of the disconnected part of the chiral susceptibility are also apparent in calculations currently performed by the hotQCD collaboration on lattice with temporal extent $N_\tau = 8$ [8], i.e. closer to the continuum limit. In Fig. 3 we show $\chi_{\text{dis}}/T^2$ and the rescaled susceptibility. The left hand part of the
Fig. 3. Disconnected part of the chiral susceptibility calculated on lattices of size $32^3 \times 8$ in (2+1)-flavor QCD for $\frac{m_l}{m_s} = 0.05$, 0.1 and 0.2, respectively.

The figure shows that for temperatures above the transition region the susceptibilities quickly become quark mass independent. This is expected as the chiral condensates will be linear in the quark mass in the chirally symmetric phase of QCD. Below the transition region we again observe a strong quark mass dependence that is consistent with $\sim 1/\sqrt{m_l}$.

The weak quark mass dependence of $\chi_m$ in the high temperature regime suggest that the susceptibility can already be described by the limiting form for the $m_l \rightarrow 0$ limit,

$$
\chi_m = \begin{cases} 
\infty, & T \leq T_c \\
A \left( \frac{T - T_c}{T_c} \right)^{-\gamma}, & T > T_c 
\end{cases}
$$

A consistent picture for the chiral susceptibility thus seems to emerge despite the missing evidence for $O(N)$ scaling for its peak height.

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