Nonstandard implementation of the standard Higgs boson

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Abstract

We discuss an alternative implementation of the Higgs boson within the Standard Model which is possible if the renormalizability condition is relaxed. Namely, at energy scale \( \Lambda \) the Higgs boson interacts at tree-level only with matter fermions, while the full gauge invariance is still maintained. The interactions with the electroweak gauge bosons are induced at low energies through the radiative corrections. In this scenario the Higgs boson can be arbitrarily heavy, interacting with the Standard Model fields arbitrarily weakly. No violation of unitarity in the scattering of longitudinal electroweak bosons occurs, since they become unphysical degrees of freedom at energies \( \Lambda \sim \text{TeV} \).
The Higgs particle \([1]\) is widely expected to be found in high energy experiments at LHC. A theoretical motivation for the existence of such a particle is related to a "bad" high energy behaviour of the electroweak theory with purely massive vector bosons. Namely, scattering amplitudes of longitudinal modes (\(W_L\) and \(Z_L\)) of the massive electroweak vector bosons scale (at tree level) as \(\sqrt{s}\) with the center-of-mass energy \(E_{\text{CoM}} = \sqrt{s}\), and thus quickly saturate the unitarity bound at about \(\sqrt{s} \sim 800\) GeV. A relatively light (with a mass less than the unitarity bound) Higgs boson, which couples to the electroweak gauge bosons in gauge invariant way, automatically provides cancellation of the bad energy behaviour of the tree-level (and also higher order) amplitudes, regaining the unitarity of the theory. The converse statement is also true: any unitary and renormalizable theory of massive electroweak bosons must involve gauge invariant interactions with the Higgs particle \([2]\). If so, than the coupling of the Higgs particle to electroweak vector bosons is fixed by the gauge invariance. This fact is crucial in searching for the Higgs particle in actual experiments, e.g. in associate production of the Higgs and vector bosons.

In this paper we would like to explore unortodox model of spontaneous electroweak symmetry breaking with nonstandard interactions of the Higgs particle and electroweak vector bosons, which, to the best of our knowledge, has not been discussed previously. The model we are going to present is based on the observation that, contrary to a widespread belief, gauge invariance per se does not uniquely fix the interactions of the Higgs boson. The standard interactions of the Higgs boson with electroweak vector fields implied in the Standard Model, follow only if, in addition to the gauge invariance, one demands the renormalizability of the theory. However, from a modern perspective, the Standard Model is widely viewed as an effective low-energy theory, and thus its renormalizability perhaps is not very well justified requirement. If one relaxes the renormalizability requirement, one can write down an alternative to the Standard Model theory which is also perfectly unitary and gauge invariant, but the interactions of the Higgs boson are entirely different. The key idea behind such a theory is the following. If the coupling of the Higgs boson to the electroweak gauge bosons is absent, then the longitudinal modes are non-propagating degrees of the freedom due to the gauge invariance in the classical theory at certain energy scale \(\Lambda\). The interactions with the electroweak gauge bosons and their masses are induced at low energies through the radiative corrections involving fermionic loops. No violation of unitarity in the scattering of longitudinal electroweak bosons occurs, since they are unphysical degrees of freedom at energies \(\Lambda \sim \text{TeV}\). The remarkable thing then is that the unitarity bound is decoupled from the Higgs boson mass. That is to say, the Higgs boson can be arbitrarily heavy, providing it interacts arbitrarily weakly with the Standard Model fields. In this regime, the Higgs boson might not be observable at LHC.

\[1\] In \([3]\) we have discussed the model with the explicit breaking of the electroweak symmetry where all the components of the electroweak doublet are not propagating degrees of freedom at the classical level.
We start by considering the electroweak doublet field $\Phi(x)$ in the "polar" paramaterization,

$$
\Phi(x) = \frac{1}{\sqrt{2}} H(x) \mathcal{X}(x), \quad \mathcal{X}(x) = e^{-i\xi^a(x)\tau^a + i\xi^3(x)} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv U(\xi) \begin{pmatrix} 0 \\ 1 \end{pmatrix},
$$

where $\tau^a$ ($a = 1, 2, 3$) are the half-Pauli matrices, and $I = \text{diag}[1/2, 1/2]$. The field $H(x) = (2\Phi^\dagger \Phi)^{1/2}$ is a modulus of the doublet field $\mathcal{X}$ and thus it is an $SU(2) \times U(1)_Y$-invariant component of $\Phi(x)$. The physical Higgs boson is associated with quanta of $h(x)$, where $h(x)$ is a fluctuation over the background vacuum expectation value $\langle H \rangle = v$, i.e., $H(x) = v + h(x)$.

Since the field $H(x)$ is $SU(2) \times U(1)_Y$-invariant, we can write a gauge invariant Lagrangian solely for the $H(x)$ component of the electroweak doublet field $\mathcal{X}$ without invoking $SU(2) \times U(1)_Y$ covariant derivatives as follows:

$$
\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \partial_\mu H \partial^\mu H - V(H),
$$

where,

$$
V(H) = \frac{1}{2} m_0^2 H^2 + \frac{\lambda_0}{4} H^4,
$$

is the usual Higgs potential with tachyonic mass term, $m_0^2 < 0$. The couplings with fermionic matter are given by the standard Lagrangian,

$$
\mathcal{L}_{\text{Higgs–Yukawa}} = y_{ij}^{(u)} Q^i_L \tilde{\Phi} u^i_R + y_{ij}^{(d)} \tilde{Q}^i_L \Phi d^i_R + y_{ij}^{(l)} \tilde{L}^i_L \Phi e^i_R + \text{h.c.}
$$

which describes the gauge-invariant interactions of the electroweak Higgs doublet $\mathcal{X} | \tilde{\Phi} = -i\tau^2 \Phi^* \rangle$ with the Standard Model up and down quarks ($Q^i_L = (u^i_L, d^i_L)^T$, $u^i_R$, $d^i_R$; $i, j = 1, 2, 3$ are the generation indices) and leptons ($L^i_L = (\nu^i_L, e^i_L)^T$, $e^i_R$). The total Lagrangian includes also the usual gauge-invariant kinetic terms for fermions and for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge bosons. So the theory is indeed gauge invariant.

There are two crucial differences between the standard theory and the theory described by (2):

- Interactions of the Higgs boson $H(x)$ with the electroweak gauge bosons are absent in (2);
- The "polar" degrees of freedom $\xi^a(x)$ are not propagating degrees of freedom at the classical level, since they can be removed from the total classical Lagrangian by $SU(2)_L \times U(1)_Y$ gauge transformations.

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2The field $\mathcal{X}$ is unimodular, $\mathcal{X}^\dagger \mathcal{X} = 1$.

3$SU(2) \times U(1)_Y$-invariant kinetic term for $H(x)$ can be written also in terms of $\Phi(x)$:

$$
\frac{1}{2} \partial_\mu H \partial^\mu H \equiv \partial_\mu \Phi^\dagger \partial^\mu \Phi + \frac{J_\Phi \mu J_\Phi^\mu}{4(\Phi^\dagger \Phi)}, \quad \text{where}, \quad J_\Phi^\mu = \Phi^\dagger (\partial^\mu \Phi) - (\partial^\mu \Phi^\dagger) \Phi.
$$
Indeed, even though the Higgs field develops the (tree-level) vacuum expectation value,

\[ v_0 = \sqrt{-\frac{m_0^2}{\lambda_0}} \neq 0, \tag{5} \]

and quarks and leptons acquire their masses in the standard fashion through (4), the electroweak gauge bosons remain massless classically. This means that, the would-be longitudinal degrees of freedom \( \xi^a(x) \) still can be rotated away. Now, since the gauge bosons are massless (no longitudinal modes), tree-level scatterings do not violate unitarity. However, the theory, as it stands, i.e., with massless electroweak gauge bosons, is obviously an incorrect theory.

The masses for the electroweak gauge bosons emerge radiatively, through the fermionic loops, and thus in the full quantum theory \( \xi^a(x) \) represent propagating degrees of freedom. The mismatch of degrees of freedom in classical and corresponding quantum theory reflects the fact that our model is not renormalizable, and must be treated as an effective theory. Let us see how this happens explicitly. We consider the tree-level Lagrangian to be valid at a certain high energy scale \( \Lambda \). At lower energies \( \mu < \Lambda \), the theory gets modified. By computing the leading log one-loop contribution from the dominant top-Higgs-Yukawa interaction, we obtain for the Higgs part of the total Lagrangian,

\[
\mathcal{L}_{\text{Higgs-(1-loop)}} = \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{Z}{2(1 + Z)} H^2(D_\mu \mathcal{X})^\dagger(D_\mu \mathcal{X}) - \frac{m_H^2}{2(1 + Z)} H^2 - \frac{\lambda}{4(1 + Z)^2} H^4, \tag{6}
\]

where \( D_\mu = \partial_\mu - i A_\mu \) is the \( SU(2) \times U(1)_Y \) covariant derivative with \( A_\mu = g A^a_{\mu} \tau^a + g' B_{\mu} \mathbf{I} \), and

\[
Z = \frac{3y_t^2}{(4\pi)^2} \log \left( \frac{\Lambda^2}{\mu^2} \right), \tag{7}
\]

\[
m_H^2 = m_0^2 - \frac{6y_t^2}{(4\pi)^2} \left( \Lambda^2 - \mu^2 \right), \tag{8}
\]

\[
\lambda = \lambda_0 + \frac{3y_t^4}{(4\pi)^2} \log \left( \frac{\Lambda^2}{\mu^2} \right). \tag{9}
\]

In (6) we have rescaled \( H(x) \to H(x)/\sqrt{1 + Z} \) to canonically normalize its kinetic term\(^4\). In principle, we can renormalize the mass and the self-interaction coupling to remove the cut-off dependence from the Higgs potential part of the Lagrangian (6). Note, however, we are not free to renormalize all the radiatively induced terms in (6). The terms not present in (2) are log-divergent, i.e., have an explicit dependence upon \( \Lambda \). This is, of course, the manifestation of nonrenormalizability of our model.

\(^4\)We keep the same notation for the rescaled and original fields.
Now let us use the above Lagrangian (6) to calculate the spectrum of the low energy theory. Fixing the unitary gauge, we obtain:

\[ v = \sqrt{-\frac{m_H^2}{\lambda}}(1 + Z), \]

\[ m_h^2 = -\frac{2m_H^2}{1 + Z}, \]  

\[ m_t = \frac{1}{\sqrt{2}}y_t v \text{ (and similar for other fermions)}, \]

\[ m_W^2 = \frac{Z}{4(1 + Z)}g^2v^2 = \frac{3g^2m_t^2}{(32\pi^2)(1 + Z)} \log \left(\frac{\Lambda^2}{\mu^2}\right), \]

and \( m_Z^2 = m_W^2 \sec^2 \theta_W \), where \( \tan \theta_W = \frac{g'}{g} \) is the weak mixing angle. We can also easily obtain interactions of the Higgs boson with the gauge bosons:

\[ \frac{Zg^2}{4(1 + Z)}(2vh + h^2) \left[ W_\mu^+ W^-_\mu + \frac{Z_\mu Z^\mu}{2\cos^2 \theta_W} \right], \]

where \( W_\mu^\pm = (A_\mu^1 \mp A_\mu^2)/\sqrt{2} \) and \( Z_\mu = \cos \theta_W A_\mu^3 - \sin \theta_W B_\mu \) (in the unitary gauge). This all looks similar to the standard theory except \( Z \)-factors entering in the above equations. These differences give a new twist.

Observe, that when the typical energy of the process (e.g. \( \sqrt{s} \) in the scattering of longitudinal \( W_L \) and \( Z_L \)), approaches the cut-off, \( \mu \to \Lambda \), the mass of the gauge bosons (as well as the interactions in (14)) go to zero, while the Higgs expectation value (and hence the Higgs and fermion masses) remains non-zero, \( v \to v_0 \). Now, if we assume that \( \Lambda \sim \text{TeV} \), the violation of unitarity in scatterings of the longitudinal electroweak bosons can be avoided simply because the longitudinal modes become unphysical at \( \Lambda \), while the vacuum expectation value and the mass of the Higgs boson can be arbitrarily large! This, of course requires Yukawa couplings to be correspondingly small in order to fit experimentally observed fermion masses, that is \( Z \) is small. If so, the Higgs boson can indeed be a very massive particle weakly interacting with Standard Model fields, and thus it won’t be seen at the LHC or TEVATRON.

Unfortunately, the minimal model we have described above does give a wrong prediction for the mass ratio:

\[ m_W/m_t \approx 0.12, \]

(we have taken \( \Lambda = 1 \text{ TeV} \) and \( \mu = m_t = 173 \text{ GeV} \), while the experimental value is \( (m_W/m_t)_{\exp} \approx 0.46 \). To improve the prediction for the gauge boson masses we can introduce a set of fermions coupled to the full electroweak Higgs doublet which give a larger than the top-quark contribution to the gauge boson masses. Obviously, there are many different types of extra hypothetical fermions which can do the job. One of the simplest anomaly-free set is

\[ F_L^i = (1, 2, 0), \quad U_R^i = (1, 1, 1), \quad D_R^i = (1, 1, -1), \]
where in parenthesis, $SU(3)$, $SU(2)$ and $U(1)_Y$ quantum numbers are indicated. Also, we have included a generation index $i = 1, \ldots, N$ to allow for $N$ families of such fermions. These fermions can have Higgs-Yukawa interactions,

$$y^i_U F^i_L \Phi U^i_R + y^i_D \bar F^i_L \Phi D^i_R + \text{h.c.},$$  \quad (17)

where we have gone to a diagonal basis in family space. These fermions, together with the top quark contribution, will radiatively generate the $W, Z$ masses of the correct magnitude if

$$Z_{\text{extra}} = \sum_i y^2_U + \frac{y^2_D}{(4\pi)^2} \log \left( \frac{\Lambda^2}{\mu^2} \right) \approx 13.5 Z_{\text{top}},$$  \quad (18)

where $Z_{\text{extra}}, Z_{\text{top}}$ are the 1-loop contribution to the $Z$ factor due to the exotic fermions and top quark respectively. This requires $\sum_i m^2_U + m^2_D \approx 40 m_t^2$. Note that the exotic fermions can also have electroweak invariant masses,

$$\mathcal{L} = M_{ij} F^i_L (F^j_L)^c + M'_{ij} \bar U^i_R (D^j_R)^c$$  \quad (19)

If the electroweak invariant masses are larger than the electroweak violating masses, then the oblique electroweak radiative corrections due to the exotic fermions become suppressed [4], which means that the model will be phenomenologically viable for a range of parameters.

If the Higgs mass is indeed large, this would solve the hierarchy problem, since the radiative corrections to the "bare" mass would be negligibly small. We see, that the necessity of the low cut-off $\Lambda \sim \text{TeV}$ in our scenario is not related with a resolution of the hierarchy problem, but is linked to an entirely different physics. At present we are not certain what kind of theory completes our effective description beyond the scale $\Lambda$. However, our scenario can clearly be distinguished from the standard candidates for a TeV scale physics, such as supersymmetry and technicolour, in high energy experiments, especially through the studies of high energy scatterings of the longitudinal modes of the massive gauge bosons. We hope that such a new physics will be revealed in future experiments.

Since in our scenario the vacuum expectation value is disassociated with the electroweak scale, the Higgs boson can play other roles. For example, weakly coupled Higgs boson can play the role of the inflaton. Also, with a large vacuum expectation value it can spontaneously generate Newton's gravitational constant when coupled to a scalar curvature.

To conclude we have suggested a nonstandard implementation of the Higgs boson within the Standard Model framework. In our scenario the Higgs boson at some energy scale $\Lambda$ couples only with matter fermions, while its interactions with electroweak gauge bosons are induced at lower energies radiatively. We show that the Higgs boson can be arbitrarily heavy interacting with the Standard Model fields arbitrarily weakly, while the unitarity in the scatterings of the longitudinal electroweak gauge bosons is maintained if $\Lambda \sim \text{TeV}$.
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