Estimation of aircraft coordinates and speed by telemetry information clock pulses

The study introduces a new mathematical method for time difference measurements by telemetry information clock pulses recorded during aircraft field tests. We developed the algorithms and software for determining the aircraft trajectory parameters by telemetry information clock pulses and verified the algorithms and software by mathematical simulation and real information.

Keywords: aircraft, trajectory parameters, telemetry information clock pulses, measuring tool, time difference method, range, range rate.

When carrying out flight experiments, the conditions of information ambiguity, caused by the absence of trajectory measurements on certain segments of flight or over its entirety, may occur if:

- there is no technical possibility for applying ground-based facilities;
- the telemetered parameters lack trajectory measurements made by means of satellite navigation equipment (SNE);
- loss of SNE tracking of navigation satellites over considerable time intervals has occurred.

Under conditions of said information ambiguity, it is proposed to determine aircraft (A/C) trajectory parameters by the telemetry information (TMI) clock pulses, applying the time difference method.

The use of a telemetry signal in this method is quite cost-efficient, as a system of signal propagation and recording (transmitter – receiver) already exists. The telemetry signal itself is organised such that it contains recurring characteristic marker events (clock pulses) for determining logic structure of the telemetry data. In the receiving telemetry equipment, the markers are identified and recorded in a telemetry data file.

In this way, when processing a telemetry file, it remains to determine the time of marker recording at different stations, using the high-precision Coordinated Universal Time (UTC) system, and obtain time differences, which convert into the differences of ranges from A/C to telemetry stations.

It is shown in this paper that the telemetry information clock pulses can serve not only to experimentally determine trajectory parameters, but also to make a lower-bound estimate of A/C velocity modulus by a single measuring tool. With information from three or more measuring tools available, it can be possible to determine speed components by the coordinates.

Papers [1, 2] describe the technique for determining A/C trajectory parameters by the TMI clock pulses using the time difference method. The accuracy of determining trajectory parameters by the proposed method is conditioned by the magnitude of UTC random error, clock pulse time calculation procedure error, possibility of accounting for UTC systematic errors, and the factor of measuring tools arrangement geometry.

Fig. 1 shows the structure of a telemetry frame.

The event selected for determining marker recording time is the telemetry frame start. Having set the frame start time at each telemetry station, it is possible to find time differences for any pair of stations, and hence, range differences.

It is possible to compute two range difference types: at a selected instant of measurements on several measuring tools, and range increments for a selected pair of measurement instants on each measuring tool.
The method for determining range differences of the first type is shown in Fig. 2.

The differences in frame start recording time are calculated by the formula

$$T_{ji} = t_j^{(i)} - t_i^{(i)},$$

and the differences in measured ranges –

$$\Delta D_{ji} = cT_{ji} = D_j^{(i)} - D_i^{(i)},$$

where $t_j^{(i)}$ – telemetry frame start time;

$c$ – speed of light;

$D_j^{(i)}$ – distance from A/C to measuring tool;

$j$ – measuring tool number;

$i = 1, \ldots, N$ – measurement number;

$N$ – quantity of measurements.

For each measuring tool, a difference in ranges (increments) can be determined:

$$\Delta D_{ji} = c(t_j^{(i)} - t_i^{(i)}) = D_j^{(i)} - D_i^{(i)}, \quad i = 2, \ldots, N. \quad (3)$$

A diagram of determining range increments is shown in Fig. 3.

Range difference $\Delta D_{ji}^{(i)}$ in accordance with (3) is implemented as follows:

$$\Delta D_{ji}^{(i)} = c(t_j^{(i)} - t_i^{(i)}) - c(t_j^{(i)} - t_i^{(i)}) =
\Delta D_{ji}^{(i)} = c(t_j^{(i)} - t_i^{(i)}) - c(t_j^{(i)} - t_i^{(i)}), \quad (4)$$

where $t_j^{(i)}$ – event receiving time for the $i$-th trajectory point;

$t_i^{(i)}$ – event transmitting time for the $i$-th trajectory point;

$t_j^{(i)}$ – event receiving time for the first trajectory point;

$t_i^{(i)}$ – event transmitting time for the first trajectory point.

The magnitude of $t_j^{(i)} - t_i^{(i)}$ is defined by the expression

$$t_j^{(i)} - t_i^{(i)} = ndt,$$

where $Round$ – function of real argument rounding to the nearest integer;

$dt$ – on-board cycle of telemetry frames transmission.

The range and range rate are determined (omitting the measuring tool index) by the formulas

$$D = \sqrt{x^2 + y^2 + z^2}; \quad D' = \frac{v_x x + v_y y + v_z z}{D}. \quad (5)$$
Denoting A/C coordinate vector and speed vector, respectively, as
\[
\mathbf{q} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix},
\]
we can write down
\[
DD' = (\mathbf{q}, \mathbf{v}) = ||\mathbf{q}|| \cdot ||\mathbf{v}|| \cos \varphi,
\]
where \( \varphi \) – angle between vectors \( \mathbf{q} \) and \( \mathbf{v} \).

Hence, \( D \cdot ||D'|| \leq ||\mathbf{q}|| \cdot ||\mathbf{v}|| \).

Taking into account that \( D = ||\mathbf{q}|| \), a lower-bound estimate for the A/C velocity modulus value is obtained:
\[
||\mathbf{v}|| \geq ||D'||.
\]

With the data from three measuring tools available, it is possible to obtain estimates of the range rates for each one of them:
\[
D'_j(t_i) = \frac{\Delta D'_j}{(t_i - t_{i-1})},
\]
where \( j = 1, 2, 3 \) – measuring tool number, \( i = 2, \ldots, N \).

The error of range rates in accordance with (7) is conditioned by the error of determining range increments and the error of numerical differentiation. In their turn, the errors of determining range increments depend on the error of determining the starting time of frames and the on-board cycle of telemetry frames transmission in accordance with formula (4).

In this case it is possible to obtain estimates of the A/C velocity vector components.

The trajectory coordinates are computed by three ranges according to the algorithm given in [3].

In the Greenwich coordinate system (GCS) the measurement equations have the view
\[
D_j = \sqrt{(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2};
\]
\[
D'_j = \frac{v_x (x - x_j) + v_y (y - y_j) + v_z (z - z_j)}{D_j},
\]

In accordance with [3], after determining matrix
\[
\mathbf{A} = \begin{pmatrix} (x - x_1) & (y - y_1) & (z - z_1) \\ (x - x_2) & (y - y_2) & (z - z_2) \\ (x - x_3) & (y - y_3) & (z - z_3) \end{pmatrix},
\]
where \( (x_j, y_j, z_j) \) – GCS coordinates of the measuring tools, the equation for computing speed components has the view
\[
Av = \mathbf{u},
\]
where \( \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_j \end{pmatrix}, \quad u_j = D'_j D_j.
\]

As a result, velocity vector is calculated by the formula
\[
\mathbf{v} = A^{-1} \mathbf{u}.
\]

A diagram of determining trajectory parameters from range increments, based on application of telemetry information clock pulses, is given in Fig. 4.

This paper, as distinct from [1, 2], proposes the second-type method for computing range differences, namely, that of range increments over time by each of the measuring tools, with subsequent determination of ranges. This method demonstrates the best results when TMI recording station is located close to the launch position, or under small differences of the actual and reference A/C trajectories on the initial segment of flight.

To reduce the influence of random errors on range increments \( \Delta D^{(i)}_j \), determined in accordance with (3), the obtained increments are smoothed out by polynomials on each measuring tool.

Smoothing of ranges and range rates is done with the use of polynomials \( P(t) = a_0 + a_1 t + \ldots + a_m t^m \) based on the least squares method (LSM):
\[
F(a_0, a_1, \ldots, a_m) \rightarrow \min \left\{ \sum_{i=1}^{n} (q_j (t_i) - P_j (t_i))^2 \right\},
\]
where \( \{t_1, \ldots, t_n\} \) – set of measurement instants for which approximation is performed;
\( m \) – polynomial order.
In so doing, the following condition is checked:
\[ |(t_n - t_l) - n\Delta t| \leq \varepsilon, \]  
(12)
where \( n \) – number of smoothing operations;
\( \Delta t \) – measurement resolution;
\( \varepsilon \) – permissible level of skipping some measurements in the smoothing interval.

These magnitudes and the polynomial order are the controlling parameters of the program.

Smoothing is performed independently by each component of vector \( s = \{D_1, D_2, D_3, D_1', D_2', D_3'\}^T \).

When computing inequality (12) in accordance with formula (11), the summary value in a given sampling is determined for the smoothing interval midpoint. The next sampling is formed by excluding the first value and adding a new measurement. The smoothing procedure is repeated in accordance with formulas (12), (11) until reaching the end of measurements. If expression (12) is not executed, as well as for the first and last measurements, a special smoothing procedure is provided.

For calculation of smoothing polynomial coefficients from condition (11), it is necessary to solve a system of linear algebraic equations (SLAE) which may turn out poorly conditioned. Computation of such SLAE belongs to ill-posed problems with their solving procedures considered in [4–7].

Therefore, the software for solving task (11) employs singular decomposition [4, 5] of the linear equation matrix for finding polynomial coefficients.

Given below are the simulation results.

To determine velocities, numerical differentiation is performed as per design trajectory coordinates. The coordinates and velocities are taken for true values. These parameters serve for calculating ranges for three measuring tools. A pseudorandom number sensor introduces random errors with normal distribution into the ranges. The random errors of range measurements are simulated with root-mean-square deviation of 20 m. From the obtained 'measured' ranges, 'measured' range rates are determined by numerical differentiation. The 'measured' ranges and range rates are smoothed out.

According to formulas (8)–(10) and the paper [3], A/C coordinates and speeds are determined from the 'measured' and smoothed ranges and range rates.

Given in Fig. 5, a–f, are deviations from the 'true' values of A/C coordinates and speeds.
determined from the 'measured' and smoothed ranges and range rates. The horizontal axis features a time scale in seconds, and the vertical axis shows deviations of A/C coordinates and speeds in km and km/s, respectively.

The plots in Fig. 5, a–f, demonstrate effectiveness of the smoothing procedure.

In the given example, a timewise increase in the errors of A/C motion parameters determination is caused by an increase of geometry factor in mutual arrangement of the measuring tools and the A/C.

The method for determining A/C speed and trajectory parameters by telemetry information clock pulses was verified through mathematical simulation and by real information.

It should be pointed out that in emergencies and abnormal situations, even in the absence of information in the TMI structure, the described method allows to determine range differences, and hence, the A/C trajectory parameters, by the TMI frame structure.

Conclusions

1. A mathematical method for determining range differences by telemetry information clock pulses has been developed.

2. Algorithms and software for calculating A/C speed trajectory parameters by telemetry information clock pulses have been created.

3. The algorithms and software for calculating A/C speed trajectory parameters by telemetry information clock pulses were verified
by the results of mathematical simulation and flight test data.

4. This mathematical method makes it possible to estimate A/C motion trajectory parameters and determine its crash site in case of emergencies and abnormal situations.

5. The main advantage of the method proposed in this paper is a possibility to determine A/C trajectory by three stations and trajectory trace in the horizontal plane by two stations, as well as to obtain a lower-bound estimate of A/C speed by a single station.

Bibliography

1. **Kisin Yu. K.** Methods of passive radiolocation of airborne vehicles by the signals of telemetering systems // Marine Radio-electronics [Morskaya radioelektronika]. 2008. No. 2. P. 30–35. (Russian)

2. **Kisin Yu. K.** Opredeleniye koordinat letatel’nykh apparatov po raznostno-dal’nomernym izmereniyam pri nalichiyi individual’nykh sistemacheskih pogreshnostei // Stokhasticheskaya optimizatsiya v informatike. 2013. T. 9. Vyp. 1. S. 59–67. (Russian)

3. **Zhdanyuk B. F.** Osnovy statisticheskoy obrabotki traecktornych izmereniy. M.: Sovetskoie radio, 1978. 384 s. (Russian)

4. **Tikhonov A. N., Ufimtsev M. V.** Statistical processing of experimental results. M.: Izd-vo Moskovskogo universiteta, 1988. 174 p. (Russian)

5. **Forsythe J., Malcolm M., Mowler C.** Computer Methods for Mathematical Calculations. [Transl. from Eng.]. M.: Mir, 1984. 280 p.

6. **Lawson C., Hanson R.** Solving least squares problems. [Transl. from Eng.]. M.: Nauka, Gl. red. fiz.-mat. lit., 1986. 232 p.

7. **Tikhonov A. N., Arsenin V. Ya.** Methods for solving ill-posed problems. M.: Nauka, Gl. red. fiz.-mat. lit., 1979. 288 p. (Russian)

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Оценка координат и скорости летательного аппарата по синхроимпульсам телеметрической информации

Предложен новый математический метод построения разностно-дальномерных измерений по синхроимпульсам телеметрической информации, регистрируемой в ходе натурных испытаний летательных аппаратов. Разработаны алгоритмы и программное обеспечение определения параметров траектории летательных аппаратов по синхроимпульсам телеметрической информации. Алгоритмы и программное обеспечение проверены математическим моделированием и по реальной информации.

**Ключевые слова:** летательный аппарат, параметры траектории, синхроимпульсы телеметрической информации, измерительное средство, разностно-дальномерный метод, дальность, скорость изменения дальности.

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