Non-standard interactions and the resolution of ordering of neutrino masses at DUNE and other long baseline experiments

Mehedi Masud⋆a and Poonam Mehta‡b

a⋆ Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211 019, India
b‡ School of Physical Sciences, Jawaharlal Nehru University, New Delhi 110067, India

September 13, 2016

Abstract

In the era of precision neutrino physics, we study the influence of matter NSI on the question of neutrino mass ordering and its resolution. At long baseline experiments, since matter effects play a crucial role in addressing this very important question, it is timely to investigate how sub-leading effects due to NSI may affect and drastically alter inferences pertaining to this question. We demonstrate that the sensitivity to mass ordering gets significantly impacted due to NSI effects for various long baseline experiments including the upcoming long baseline experiment, Deep Underground Neutrino Experiment (DUNE). Finally we draw a comparison of DUNE, with the sensitivities offered by two of the current neutrino beam experiments NOvA and T2K.

Email: masud@hri.res.in
Email: pm@jnu.ac.in
1 Introduction

A series of experiments using solar, atmospheric, accelerator and reactor neutrinos in the past several decades have established beyond doubt that neutrinos oscillate among the three flavors while conserving the lepton number. Oscillation experiments are sensitive to two mass-squared differences, three mixing angles, and the value of the Dirac type CP violating phase, $\delta$. The latest global fit \cite{1,2} to the world oscillation data leads us to two intriguing aspects concerning mass and mixing in the neutrino sector. The best-fit values \cite{1,2} of the two mass-squared splittings and the angles are

- **Mass splittings**:
  \[
  \delta m_{21}^2 = 7.5 \times 10^{-5} \text{eV}^2; \quad \delta m_{31}^2 = 2.457(-2.449) \times 10^{-3} \text{eV}^2, \quad (1)
  \]

- **Mixing pattern**:
  \[
  \sin^2 \theta_{12} = 0.304; \quad \sin^2 \theta_{23} = 0.452(0.579); \quad \sin^2 \theta_{13} = 0.0218(0.0219). \quad (2)
  \]

$\delta m_{31}^2 > 0$ is required from solar neutrino data while $\delta m_{31}^2$ can be either positive or negative. So far there is no constraint on $\delta$, and it can lie in $[-\pi, \pi]$. Additionally if $\theta_{23}$ differs from maximal mixing as is hinted by recent global analyses of data, one would like to pin down the correct octant of this angle. The emerging goals of neutrino oscillation physics are therefore to address the question of neutrino mass ordering which refers to ascertaining whether $\delta m_{31}^2 > 0$ (normal ordering, NO) or $\delta m_{31}^2 < 0$ (inverted ordering, IO), to measure the CP violating phase and to find the correct octant of $\theta_{23}$ if $\theta_{23}$ turns out to be non-maximal.

It is crucial to settle the issue of neutrino mass ordering as it would allow us to get closer towards determining the underlying structure of the neutrino mass matrix by being able to discriminate between theoretical models giving rise to neutrino masses \cite{3}. Knowledge of neutrino mass ordering would also have an important bearing upon the neutrinoless double beta decay searches which would allow us to probe the nature of neutrinos \cite{4}. It is also intimately related to the measurement of CP violating phase, $\delta$. In a large class of theoretical models, the neutrino mass ordering also impacts the effectiveness of leptogenesis scenario which can explain the matter-antimatter asymmetry of the Universe \cite{5}.

Wolfenstein in his seminal paper \cite{6,7} pointed out that neutrinos could experience flavour-dependent refraction in matter which can modify the neutrino oscillation probabilities. In case of standard interactions (SI), the flavour dependent refraction arises due to the coherent forward scattering of $\nu_e$ with electrons present in matter via charged current (CC) processes while the NC contribution is flavour universal. However, in presence of NSI, one could have additional CC (see for example, \cite{8}) or neutral current (NC) contributions. It turns out that only the NC processes play a role in the propagation of neutrinos and we will focus on these NC NSI in the present work.

The matter-induced modification of neutrino oscillation probabilities was found to be different for neutrino and antineutrino channels and a function of neutrino mass ordering. If we consider the $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ appearance channels, one sees an enhancement in

\footnote{The bracketted values correspond to IO.}
$P_{\mu e}$ and suppression in $\bar{P}_{\mu e}$ if ordering is normal while if the ordering is inverted, one gets the reverse. Since matter effects differ for neutrino and anti-neutrinos and are sensitive to ordering, they aid in the determination of the neutrino mass ordering. In presence of subdominant new physics effects such as non-standard interactions (NSI) during propagation, the resolution of neutrino mass ordering gets severely affected even if we take conservative values of the NSI parameters (for reviews, see [9,10]).

Recently, we have studied the influence of NSI during propagation (including both flavour non-universal and flavour changing interactions) on the CP violation sensitivity at a future super beam experiment, Deep Underground Neutrino Experiment (DUNE) [11–15] where we considered the so called platinum channel ($\nu_\mu \rightarrow \nu_e$) [16] (see also [17–23]). In Ref. [24], we discussed the role of disappearance channel ($\nu_\mu \rightarrow \nu_\mu$) in addition to the platinum channel and contrasted the CP sensitivities offered by some of the current and upcoming long baseline experiments such as Tokai to Kamioka (T2K) [25], NuMI off-axis $\nu_e$ appearance (NOvA) [26,27], DUNE as well as Tokai to Hyper-Kamiokande (T2HK) [28].

Apart from using the super beam experiments which are well-suited to exploit the platinum channel, the question of neutrino mass ordering can be addressed utilising sensitivity offered by other channels such as $\bar{\nu}_e \rightarrow \bar{\nu}_e$ in reactor experiments (e.g. Jiangmen Underground Neutrino Observatory (JUNO) [29,30]), muon disappearance channel ($\nu_\mu \rightarrow \nu_\mu$) using atmospheric neutrinos in conjunction with an iron calorimeter detector (e.g. India-based Neutrino Observatory (INO) [31]), a combination of oscillation channels and exploring matter resonances in the multi megaton ice detector Oscillation Research with Cosmics in the Abyss (ORCA) or Precision IceCube Next Generation Upgrade (PINGU) [32–35] using atmospheric neutrinos and precision cosmology [36].

In the present work, we include subdominant NSI effects (see [37–39] for models giving rise to NSI) and evaluate the sensitivities and discovery potential offered by some of these long baseline accelerator experiments and assess their roles in addressing the question of neutrino mass ordering [40]. We consider the following experiments: T2K (295 km), NOvA (800 km) and DUNE (1300 km).

T2K [25] and NOvA [26,27], are currently running long baseline experiments while DUNE [11–15] is one of the most promising upcoming long baseline experiments. The baseline of DUNE is chosen such that the experiment is expected to deliver optimal sensitivity to CP violation and is well-suited to address the question of neutrino mass ordering [40]. The energy at which the neutrino flux peaks ($\sim 2.5$ GeV) and where the peak in $P_{\mu e}$ occurs match. In the region of peak flux, this asymmetry in neutrino and antineutrino channels is expected to be nearly 40% which in turn implies that both the mass ordering and CP phase can be determined unambiguously with high confidence within the same experiment.

While sensitivity studies have been carried out in presence of NSI in the context of DUNE, we would like to stress that none of them deal with the precise impact of NSI on the standard sensitivity to mass ordering at long baseline experiments and it is timely to investigate extensively the impact of NSI on the question of mass ordering of neutrino states. We consider NSI terms whose strengths lie in the presently allowed limits (along with the phases associated which are presently unconstrained) and study the impact of individual and collective NSI terms on the CP violation sensitivity using the following channels: $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_\mu$. In a recent work, the authors have discussed the mass ordering
asymmetries in the $\nu_\mu \to \nu_\tau$ channel in presence of NSI and how it would impact the question of neutrino mass ordering at DUNE and the Long Baseline Neutrino Oscillations (LBNO) experiment \[41\].

The paper is organised as follows. Sec. 2 gives the framework for the present work. Sec 2.1 comprises of a brief introduction to NSI in propagation and how the NC NSI terms enter the oscillation framework. We then review $P_{\mu e}$ and $P_{\mu\mu}$ in Sec. 2.2 and highlight the dependence on mass ordering as well as $\delta_{CP}$. In Sec. 2.3 we give our analysis procedure using the dependence of probabilities on mass ordering in Sec. 2.2. We then go on to describe our results in Sec. 3 where we show how mass ordering sensitivity at DUNE gets affected due to individual and collective NSI terms (Sec. 3.1 and 3.3). We also show dependence on true values of standard oscillation parameters in Sec. 3.2 and compare the results obtained at DUNE with other long baseline experiments in Sec. 3.3. Using event rate plots for NO and IO, we depict degenerate and non-overlapping regions as a function of energy in Sec. 3.3. The impact of NSI on the mass ordering fraction is shown as a function of exposure and baseline in Sec. 3.5 and 3.6. We conclude with a discussion in Sec. 4.

2 Framework

2.1 Neutrino interactions and the Earth matter effects

In presence of NSI, the propagation of neutrinos is governed by an effective Schrödinger equation in the ultra-relativistic limit with the effective Hamiltonian in flavour basis given by

$$H_f = H_v + H_{SI} + H_{NSI}$$

$$= \lambda \left \{ U \begin{pmatrix} 0 & r_\lambda & 1 \\ r_\lambda & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} U^\dagger + r_A \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right \}, \quad (3)$$

where

$$\lambda \equiv \frac{\delta m_{31}^2}{2E}; \quad r_\lambda \equiv \frac{\delta m_{21}^2}{\delta m_{31}^2}; \quad r_A \equiv \frac{A(x)}{\delta m_{31}^2}. \quad (4)$$

$A(x) = 2\sqrt{2}EG_F n_e(x)$ is the standard CC potential due to the coherent forward scattering of neutrinos in Earth matter and $n_e$ is the electron number density. $U$ is the three flavour neutrino mixing matrix and is responsible for diagonalizing the vacuum part of the Hamiltonian. In the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) parametrization \[42\], $U$ is given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{13} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}, \quad (5)$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. Additionally, if neutrinos are Majorana particles, one can have two additional Majorana-type phases but those are irrelevant as far as neutrino oscillations are concerned. In the standard paradigm, we note that there is only one parameter, the Dirac CP phase $\delta$ that is responsible for genuine CP violating effects and SI
with Earth matter can introduce additional CP effects (referred to as fake CP effects) due to the fact that matter is CP asymmetric.

$H_{NSI}$ comprises of off-diagonal ($\alpha \neq \beta$) and diagonal ($\alpha = \beta$) parameters. The off-diagonal NSI parameters, $\varepsilon_{\alpha\beta} (\equiv |\varepsilon_{\alpha\beta}| e^{i\varphi_{\alpha\beta}})$ are complex while the diagonal ones are real due to the hermiticity of the Hamiltonian. In addition to $\delta$ appearing in $H_f$, we now have three other phases as $\varphi_{\mu\mu}, \varphi_{\tau\tau}, \varphi_{\mu\tau}$.

Let us briefly mention the constraints on the NC NSI parameters (for more details, see Ref. [9,43,44]). We use the following constraints on the NC NSI parameters

$$|\varepsilon_{\alpha\beta}| < \begin{pmatrix} 4.2 & 0.3 & 0.5 \\ 0.3 & 0.068 & 0.04 \\ 0.5 & 0.04 & 0.15 \end{pmatrix}. \tag{6}$$

The phases are unconstrained presently and can lie the allowed range, $\varphi_{\alpha\beta} \in (-\pi, \pi)$ (see Table. 1).

2.2 Probability level discussion: dependence on the neutrino mass ordering and the CP phase at long baseline experiments

Let us first describe the explicit dependence of probabilities on two of the unknowns - neutrino mass ordering and CP phase $^2$. As we will see, the dependence of probabilities on these two parameters is strongly interlinked. We discuss the appearance ($\nu_\mu \rightarrow \nu_e$) and disappearance ($\nu_\mu \rightarrow \nu_\mu$) channels that are relevant for accelerator-based neutrino oscillation experiments. The expressions given below are strictly valid when $r_\lambda \lambda L/2 \ll 1$, i.e. $L$ and $E$ are far away from the region where lower frequency oscillations dominate which is generally satisfied for long baseline experiments. For the case of DUNE, we have

$$r_\lambda \lambda L/2 = 0.05 \left[ 1.267 \times \frac{\delta m_{21}^2}{7.6 \times 10^{-5} \ eV^2} \frac{L}{1300 \ km} \frac{2.5 \ GeV}{E} \right] < 1. \tag{7}$$

For the case SI with matter (as well as in vacuum in the limit, $r_A \rightarrow 0$) we can express the probabilities for NO and IO in a compact form as given below (see [45]).

1. Normal mass ordering:

$$P^{NO}_{\mu e} = x^2 + y^2 + 2xy \cos(\delta + \lambda L/2), \tag{8a}$$

$$P^{NO}_{\mu e} = \bar{x}^2 + y^2 + 2\bar{y}x \cos(\delta - \lambda L/2), \tag{8b}$$

$$P^{NO}_{\mu\mu} = a + b + c + e - y^2 - d^2 - 2yd \cos \delta, \tag{8c}$$

$$P^{NO}_{\mu\mu} = a + b + \bar{c} + \bar{e} - y^2 - \bar{d}^2 - 2\bar{y}d \cos \delta, \tag{8d}$$

2. Inverted mass ordering:

$$P^{IO}_{\mu e} = \bar{x}^2 + y^2 - 2\bar{y}x \cos(\delta - \lambda L/2), \tag{9a}$$

For non-maximal $\theta_{23}$, the octant of $\theta_{23}$ also needs to be taken into account. Here we take $\theta_{23} = \pi/4$ for simplicity.
\[ P_{\mu e}^{IO} = x^2 + y^2 - 2xy \cos(\delta - \lambda L/2), \] (9b)
\[ P_{\mu \mu}^{IO} = a - b + \bar{c} + \bar{e} - y^2 - \bar{d}^2 + 2y\bar{d} \cos \delta \] (9c)
\[ P_{\mu e}^{IO} = a - b + c + e - y^2 - d^2 + 2yd \cos \delta, \] (9d)

where
\[
x = s_{2\times13} s_{2\times23} \sin \{(1 - r_A) \lambda L/2\} (1 - r_A),
\]
\[
y = r_A s_{2\times12} c_{23} \sin(r_A \lambda L/2)/r_A,
\]
\[a = 1 - s_{2\times23}^2 \sin^2(\lambda L/2) - r_A^2 \sin^2(2\lambda L/2) \cos \lambda L,
\]
\[b = r_A c_{12} s_{2\times23}^2 (\lambda L/2) \sin \lambda L,
\]
\[c = \frac{1}{2r_A} r_A^2 s_{2\times12} s_{2\times23} \sin(\lambda L/2) \sin(r_A \lambda L/2)/r_A \cos \{(1 - r_A) \lambda L/2\} - (\lambda L/4) \sin \lambda L,
\]
\[d = 2s_{13} s_{23} \sin(1 - r_A) \lambda L/2(1 - r_A),
\]
\[e = \frac{2}{1 - r_A} s_{13}^2 s_{23}^2 \sin(\lambda L/2) \cos(r_A \lambda L/2) \sin(1 - r_A) \lambda L/2)(1 - r_A) - (r_A \lambda L/4) \sin \lambda L.
\] (10)

where \(s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij}, s_{2\times i j} = \sin 2\theta_{ij}\) with \(i, j = 1, 2, 3\). In addition to the vacuum oscillation frequency \(\lambda L/2\) given by
\[\lambda L/2 \approx 1.57 \left[ 1.267 \times \frac{\delta m^2_{31}}{2.5 \times 10^{-3}eV^2} \frac{L}{1300 km} \times 2.5 GeV \right],\] (11)
which is \(E\)-dependent, matter (SI and NSI) introduces phase shifts such as \(r_A \lambda L/2\)
\[r_A \lambda L/2 \approx 0.4 \left[ 1.267 \times 0.756 \times 10^{-4} \frac{\rho}{3.0 g/cc} \frac{L}{1300 km} \right],\] (12)
which are \(E\)-independent. Given the probability expressions for the neutrino channel for a given ordering of neutrino mass states, we can get the corresponding expressions for the antineutrino channel by replacing \(\delta \rightarrow -\delta\) and \(r_A \rightarrow -r_A\). For IO, we need to substitute \(r_A \rightarrow -r_A, \lambda \rightarrow -\lambda\) and \(r \lambda \rightarrow -r \lambda\) in the expression for NO.

In what follows, we shall see that Eqs. 8a-8d and Eqs. 9a-9d serve as useful guide to understand the impact of neutrino mass ordering.

As far as \(P_{\mu e}\) is concerned (see Eqs. 8a-8b and Eqs. 9a-9b), \(x\) is the dominant contributor which depends upon the choice of the neutrino mass ordering. Close to the 1-3 resonance condition\(^3\)
\[\rho E_R[GeV \ g/cc] = \frac{\delta m^2_{31} [eV^2]}{0.76 \times 10^{-4} [eV^2]} \times \cos 2\theta_{13} \approx 30 GeV \ g/cc,\] (13)

\(^3\)The resonance condition is sensitive to the mass ordering (NO or IO), for antineutrinos resonance occurs for IO. The dependence on \(\delta m^2_{31}\) is strong while the dependence on \(\theta_{13}\) is very mild.
this term can be especially enhanced due to matter effects since $r_A \to 1$. $y$ on the other hand, is suppressed by $r_\lambda$. The interference terms in Eqs. 8a-8b and Eqs. 9a-9b involving $x$ and $y$ depend upon two of the unknowns - the CP phase ($\delta$) as well as the choice of mass ordering (via the $\lambda$ term). The interplay of these two dependencies can lead to degenerate solutions.

In Fig. 1, for a fixed $L (= 1300 \text{ km})$, we depict the $\nu_e$ appearance probability as a function of energy. The two bands correspond to variation in the values of phases for NO and IO. For the SI case, we note that the two bands corresponding to NO and IO are non-overlapping in the energy range around the position of the first vacuum peak (around $E \sim 2.5 \text{ GeV}$, see Eq. 11) of $P_{\mu e}$. The top boundary of the band corresponds to $\delta \sim -\pi/2$ for NO and IO while the bottom boundary of the band corresponds to $\delta \sim \pi/2$ (see Eq. 8a and 9a). For the NSI case, if we consider $\varepsilon_{ee} > 0$, the degree of separation between the two bands is more in comparison to SI case and also this extends to larger range of energies around the peak of $P_{\mu e}$. If we consider $\varepsilon_{e\mu}$ and $\varepsilon_{e\tau}$, we note that the two bands are overlapping at all energies with the degree of overlap decreasing around the vacuum peak (for similar discussion for the SI case in the context of NoVA and T2K see [46]). This naively tells us that for DUNE the region around 2.5 GeV is the most relevant region for an unambiguous determination of mass ordering as the overlap between the NO and IO bands is minimal around this energy.

Let us now visualize the dependence of probability as a function of $\delta$ for fixed $E$ and $L$ as shown in Fig. 2. Our choice of fixed energies correspond to the first vacuum peak (2.5 GeV),

\[ \delta \text{ for SI and also relevant } \varphi_{\alpha\beta} \text{ for NSI} \]
second vacuum peak (0.8 GeV) and a higher value of energy beyond the first peak (6 GeV). An understanding of the explicit \( \delta \) dependence will help us interpret the plots for sensitivity to mass ordering which are usually plotted as a function of the unknown standard Dirac CP phase \( \delta \). The solid (dashed) magenta and blue curves are for neutrinos (antineutrinos) for NO and IO respectively. The SI case is shown in the top row for three different fixed values of energies while the NSI case is depicted in the bottom row for those specific choices of energies. At \( E = 0.8 \) GeV from Fig. 1, we see that the two bands corresponding to NO and IO are overlapping. This can also be seen in Fig. 2 (top row, left plot) as the red and blue curves touch each other on the \( \delta > 0 \) side. At \( E = 2.5 \) GeV from Fig. 1 the two bands are non-overlapping and therefore we notice that the red and the blue curves are not crossing over in Fig. 2 (top row, middle plot) for all values of \( \delta \). For higher energies \( E = 6 \) GeV, we can see that there is an overlap in Fig. 1 and therefore the red and the blue curves are crossing over at two points on both sides of \( \delta = 0 \) (top row, right panel of Fig. 2).

Also, note that in Eqs. 8a-9d, the two frequencies involved are : \( \lambda L/2 \) which is energy-dependent and \( r_A \lambda L/2 \) which is energy-independent (see Eq. 11 and 12). By comparing the relative oscillation frequencies, we note that the energy-independent contribution should be the same for all energies and the \( E \)- dependent contribution is expected to rise with the decrease in energy for a fixed value of \( L \). This is evident from Fig. 2 as we go from right to left.

As far as \( P_{\mu\mu} \) is concerned, the plots versus \( \delta \) are shown in Fig. 3 and the features can be understood from Eqs. 8c and 9c. The peak (dip) conditions for \( P_{\mu\mu} \) are shifted in \( \delta \) w.r.t. \( P_{\mu e} \). The amount of this shift depends upon the value of the energy and the baseline involved. Around 2.5 GeV, we note that the position of peak of \( P_{\mu\mu} \) is shifted by \( \pi/2 \) w.r.t. \( P_{\mu e} \). At a given energy, we can see the value of \( P_{\mu\mu} \) is highest around 6 GeV and not at 2.5 GeV where the \( P_{\mu e} \) takes its maximum value and therefore one would expect that the contribution will be dominated by \( P_{\mu e} \) as the flux is the largest there. Overall, from Fig. 3
we note that $P_{\mu\mu}$ is not very sensitive to NSI effects (the curves in the top and bottom panel are similar). We can see that the curves for NO and IO in the bottom panel (NSI) of Fig. 3 appear different only (small relative difference) in the case of 2.5 GeV but one should notice that the overall value of probability scale is smaller by at least two orders of magnitude in this case. The SI and NSI curves for NO and IO are very similar both at 0.8 and 6 GeV. Nevertheless the small contribution from $P_{\mu\mu}$ can be understood from the plots given in Fig. 3.

All the plots presented in this paper are obtained by using General Long baseline Experiment Simulator (GLoBES) and related software [47–50] which numerically solves the full three flavour neutrino propagation equations using the PREM [51] density profile of the Earth and the latest values of the neutrino parameters as obtained from global fits [1, 53, 54]. Unless stated otherwise, we assume NO as the true hierarchy in all the plots.

2.3 Analysis procedure and interpretations based on probability expressions

The question of ordering of neutrino mass eigenstates is a binary one: the true ordering can be NO or IO. In order to obtain the sensitivity to neutrino mass ordering we need to ask the following question - what is the sensitivity with which a particular experiment can distinguish between NO and IO. We explore this question as a function of the Dirac CP phase $\delta$ as it is the only unknown parameter in the PMNS mixing matrix.

In order to understand the features of the sensitivity plots considering the true ordering as

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$P_{\mu\mu}(P_{\mu\mu})$ as a function of the CP phase, $\delta$ for three values of energy ($E = 0.8, 2.5, 6$ GeV) and fixed baseline ($L = 1300$ km) in presence of SI (top row) and NSI (bottom row).

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$^5$We use the matter density as given by PREM model. In principle, we can allow for uncertainty in the Earth matter density in our calculations but it would not impact our results drastically. 

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| Parameter | True value | Marginalisation range |
|-----------|------------|-----------------------|
| $\theta_{12}$ [deg] | 33.5 | - |
| $\theta_{13}$ [deg] | 8.5 | - |
| $\theta_{23}$ [deg] | 45 | - |
| $\delta m^2_{31}$ [eV$^2$] | $7.5 \times 10^{-5}$ | - |
| $\delta m^2_{31}$ (NH) [eV$^2$] | $+2.45 \times 10^{-3}$ | $(+2.25 - 2.65) \times 10^{-3}$ |
| $\delta m^2_{31}$ (IH) [eV$^2$] | $-2.46 \times 10^{-3}$ | $-(2.25 - 2.65) \times 10^{-3}$ |
| $\delta$ | $[-\pi : \pi]$ | $[-\pi : \pi]$ |

Table 1: SI and NSI parameters used in our study. For latest global fit to neutrino data see [2].

NO, we give a statistical definition of $\chi^2$ as follows:

$$
\chi^2_{NO}(\delta_{tr},|\varepsilon_{tr}|,\varphi_{tr}) \equiv \min_{\delta_{te},|\varepsilon_{te}|,\varphi_{te}} \sum_{i=1}^{x} \sum_{j=1}^{2} \sum_{k=1}^{2} \frac{\left[N_{NO}^{i,j,k}(\delta_{tr},|\varepsilon_{tr}|,\varphi_{tr}) - N_{IO}^{i,j,k}(\delta_{te},|\varepsilon_{te}|,\varphi_{te})\right]^2}{N_{NO}^{i,j,k}(\delta_{tr},|\varepsilon_{tr}|,\varphi_{tr})},
$$

where $N_{NO}^{i,j,k}$ and $N_{IO}^{i,j,k}$ are the number of NO and IO events in the $\{i, j, k\}$-th bin respectively. The NSI parameters are expressed in terms of moduli, $|\varepsilon| \equiv \{|\varepsilon_{\alpha\beta}|; \alpha, \beta = e, \mu, \tau\}$ and phases, $\varphi \equiv \{\varphi_{\alpha\beta}; \alpha, \beta = e, \mu, \tau\}$. The indices $i, j$ correspond to energy bins ($i = 1 \rightarrow x$, the number of bins depends upon the particular experiment - for DUNE, there are $x = 39$ bins of width 250 MeV in $0.5 - 10$ GeV, for T2K and T2HK, there are $x = 20$ bins of width 40 MeV in $0.4 - 1.2$ GeV, for NOvA, there are $x = 28$ bins of width 125 MeV in $0.5 - 4$ GeV) and the type of neutrinos i.e. neutrino or antineutrino ($j = 1 \rightarrow 2$).

$^{N_{o}} = \sqrt{\Delta \chi^2}$, $\Delta \chi^2 = \chi^2$ as we have not included any fluctuations in simulated data. This is the Pearson’s definition of $\chi^2$ [55]. For large sample size, the other definition using log-likelihood also yields similar results.
stands for the channels considered i.e. appearance and disappearance \((k = 1 \rightarrow 2)\). The \(\chi^2\) is computed as given in Eq. [14] for a given set of true values by minimizing over the test parameters and this procedure is repeated for all possible true values listed in Table 1. We do not marginalise over the standard oscillation parameters except \(\delta\) whose true value is unknown. As we are investigating the role of NSI in the present study, we marginalize over the allowed ranges of moduli and phases of the relevant NSI parameters. Our choice of range of NSI parameters is consistent with the existing constraints (Eq. 6). While the variation corresponding to the true value of \(\delta\) is depicted along the \(x\)-axis, the variation of the true values of \(\varphi_{e\mu}, \varphi_{e\tau}\) (within the allowed ranges) lead to the vertical width of the sensitivity bands which show the maximum variation in the \(\chi^2\) for each value of \(\delta\) (true).

For the sake of clarity, we have retained only statistical effects and ignored systematic uncertainties and priors in the above expression in eq. 14 (see [56] for the full expression of \(\chi^2\) including systematics and priors). It turns out that the above expression suffices to explain the gross behaviour of \(\chi^2\) curves in connection to the issue of mass ordering. In practice, we have indeed included the effects due to systematics for different experiments and also marginalised over systematic uncertainties while calculating the \(\chi^2\) using GLoBES. We have assumed that the standard oscillation parameters are known with infinite precision i.e. we have not included any priors.

The theoretically expected differential event rate is given by [12],

\[
\frac{dN_{\nu e}^{app}(E, L)}{dE} = N_{\text{target}} \times \Phi_{\nu\mu}(E, L) \times P_{\mu e}(E, L) \times \sigma_{\nu e}(E)
\]

(15)

where \(N_{\text{target}}\) is the number of target nucleons per kiloton of detector fiducial volume, \(N_{\text{target}} = 6.022 \times 10^{32} \text{ N/kt}\). \(P_{\mu e}(E, L)\) is the appearance probability for \(\nu_\mu \rightarrow \nu_e\) in matter, \(\Phi_{\nu\mu}(E, L)\) is the flux of \(\nu_\mu\), \(\sigma_{\nu e}(E)\) is the charged current (CC) cross section of \(\nu_e\) given by

\[
\sigma_{\nu e} = 0.67 \times 10^{-42} (m^2/\text{GeV}/N) \times E , \quad \text{for} \quad E > 0.5 \text{ GeV}
\]

(16)

For the disappearance channel, \(P_{\mu e}\) is to be replaced by \(P_{\mu \mu}\) and \(\sigma_{\nu e} \rightarrow \sigma_{\nu \mu}\). Note that \(\sigma_{\nu e} \sim \sigma_{\nu \mu}\) for the considered energy range [57]. For antineutrinos, \(\nu_\mu \rightarrow \bar{\nu}_\mu\) and \(\nu_e \rightarrow \bar{\nu}_e\) and \(P_{\mu e} \rightarrow \bar{P}_{\mu e}\).

The \(\chi^2\) for the question of mass ordering considering the appearance channel is obtained by adding the neutrino \((\nu_\mu \rightarrow \nu_e)\) and antineutrino \((\bar{\nu}_\mu \rightarrow \bar{\nu}_e)\) contributions in Eq. 15 which is cast in terms of difference between the true and the test event rates. The difference in true and test event sample can be related to corresponding probability differences between NO and IO at different fixed values of energies as shown in Fig. 2. For any true fixed value of \(\delta\) in case of NO i.e. lying on the magenta solid curve, one can easily see from Fig. 2 that the difference between the true fixed NO and test IO (all values of \(\delta\) on the blue solid curve) is
The numerator of the neutrino part in Eq. 17 can be decomposed into three terms which are plotted separately (dashed) as well as combined (solid). This demonstrates that the overall shape of the $\chi^2$ curves for mass ordering sensitivity depends upon the baseline and energy.

$\chi^2_{NO,app}(\delta_{tr}) = \chi^2_{\nu_\mu \to \nu_e} + \chi^2_{\bar{\nu}_\mu \to \bar{\nu}_e}$, 

$\sim \min_{\delta_{te} \in [-\pi, \pi]} \left\{ \frac{[P_{\mu e}^{NO}(\delta_{tr}) - P_{\mu e}^{IO}(\delta_{te} \sim -\pi/2)]^2}{P_{\mu e}^{NO}(\delta_{tr})} \times \Phi_{\nu_\mu} \sigma_{\nu_e} + \frac{[P_{\mu e}^{NO}(\delta_{tr}) - P_{\mu e}^{IO}(\delta_{te} \sim -\pi/2)]^2}{P_{\mu e}^{NO}(\delta_{tr})} \times \bar{\Phi}_{\nu_\mu} \sigma_{\bar{\nu}_e} \right\}$,

$\sim \left\{ [(x^2 - \bar{x}^2)^2 + 4y^2(x \sin \delta_{tr} + \bar{x})^2 + \{-4y(x \sin \delta_{tr} + \bar{x})(x^2 - \bar{x}^2)\}] \frac{1}{x^2 + y^2 - 2xy \sin \delta_{tr}} \times \Phi_{\nu_\mu} \sigma_{\nu_e} + \bar{\nu}_{\text{term}} \right\}.$

(17)

Since the antineutrino cross-section and flux is smaller than that of neutrinos, the gross behaviour of $\chi^2$ plots can be understood by looking at the neutrino contribution in Eq. 17.

The numerator contains three terms which are plotted in Fig. 4. For the case of DUNE, the first term $(x^2 - \bar{x}^2)^2$ is a measure of matter effects and is independent of $\delta$ (shown as blue dashed line). The second term $(4y^2(x \sin \delta_{tr} + \bar{x})^2)$ is shown as green dashed curve which peaks at $\delta_{tr} \sim \pi/2$ and is flat elsewhere. The third term $(-4y(x \sin \delta_{tr} + \bar{x})(x^2 - \bar{x}^2))$ is shown as brown dashed curve which has a peak at $\delta_{tr} \sim -\pi/2$ and a dip at $\delta_{tr} \sim \pi/2$. Both the second and third terms are $\delta$-dependent. The overall numerator is shown as red solid curve which has a peak at $\delta \sim -\pi/2$ for NO. For shorter baselines, T2K and NOvA,

\[10\]For true fixed IO, $\delta(\text{test}) = \pi/2$ for $E = 2.5 \text{ GeV}$ leads to the minimum difference.
the contributions from the three terms at the respective peak energies are different which lead to an overall feature of a primary peak at \( \delta \sim -\pi/2 \) and a secondary peak around \( \delta_{tr} \sim \pi/2 \) for NO which was absent in case of DUNE and which can be attributed to the contribution coming from the third term at different baselines. This simple decomposition of the net \( \chi^2 \) into terms appearing in the probability serves as a useful guide to the expected shape of the \( \chi^2 \) curves and contrasting different baselines. To the best of our knowledge, this baseline-dependent characteristic shape of the \( \chi^2 \) for mass ordering sensitivity has not been deduced from a simple probability level discussion in literature so far.

For the true IO (blue solid curve) and test NO (magenta solid curve) in Fig. 2, we have \( \delta_{te} \sim \pi/2 \) (selected by marginalisation) for \( E = 2.5 \) GeV, 

\[
\chi^2_{\text{IO,app}}(\delta_{tr}) = \chi^2_{\nu_{\mu} \rightarrow \nu_e} + \chi^2_{\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e},
\]

\[
\sim \min_{\delta_{te} \in [-\pi, \pi]} \left\{ \frac{[P^{\text{IO}}_{\mu e}(\delta_{tr}) - P^{\text{NO}}_{\mu e}(\delta_{te})]^2}{P^{\text{IO}}_{\mu e}(\delta_{tr})} \times \Phi_{\nu_{\mu}} \sigma_{\nu_e} + \frac{[\bar{P}^{\text{IO}}_{\mu e}(\delta_{tr}) - \bar{P}^{\text{NO}}_{\mu e}(\delta_{te})]^2}{\bar{P}^{\text{IO}}_{\mu e}(\delta_{tr})} \times \bar{\Phi}_{\nu_{\mu}} \sigma_{\bar{\nu}_e} \right\},
\]

\[
\sim \left\{ \frac{[(x^2 - \bar{x}^2)^2 + 4y^2(\bar{x} \sin \delta_{tr} - x)^2 + 4y(\bar{x} \sin \delta_{tr} - x)(x^2 - \bar{x}^2)]}{\bar{x}^2 + y^2 - 2\bar{x}y \sin \delta_{tr}} \times \Phi_{\nu_{\mu}} \sigma_{\nu_e} + \bar{\nu} \text{ term} \right\}. \tag{18}
\]

3 Results

3.1 Impact of individual NSI terms on mass ordering sensitivity at DUNE

In order to clearly understand the impact of the NSI terms, we first consider only one parameter non-zero at a time and discuss the role of appearance (\( \nu_{\mu} \rightarrow \nu_e \)) as well as the disappearance (\( \nu_{\mu} \rightarrow \nu_{\mu} \)) channels in addressing the question of ordering.

Before we describe the impact of a particular NSI parameter (i.e. \( \varepsilon_{\mu\mu} \)) we would like to point out that there are two effects responsible for altering the value of the \( \chi^2 \) which compete with each other as was first mentioned in Ref. [24]:

(a) **Decrease in the \( \chi^2 \) due to additional test values** - NSI introduces more number of parameters in the sensitivity analysis. If marginalization is carried out over more number of test parameters (the difference from Ref. [24] is that \( \varphi_{\alpha\beta} \) now takes more number of test values in the full range \([-\pi, \pi]\) as opposed to 0, \( \pi \) only in the CP sensitivity case), it results in a decreased value of \( \chi^2 \). This is purely a statistical effect.

(b) **Increase in the \( \chi^2 \) due to additional true values** - For the mass ordering sensitivity, all the NSI parameters (diagonal and off-diagonal ones) would serve as additional true values. In case of off-diagonal parameters, the variation over the NSI phases (\( \varphi_{e\mu}, \varphi_{e\tau} \)) tends to broaden the grey band provided the moduli (\(|\varepsilon_{e\mu}|, |\varepsilon_{e\tau}|\)) of the relevant NSI term is large. For the diagonal parameters (e.g. \( \varepsilon_{ee} \)), if the value of the parameter is
Figure 5: Impact of individual NSI terms on the mass ordering sensitivity. The top panel is for $\varepsilon_{e\mu}$, the middle panel is for $\varepsilon_{e\tau}$ and the last panel is for the three diagonal NSI parameters.
large enough (inducing more matter dependence which was indicated in Fig. 11), one can get a contribution to the $\chi^2$ which can be larger than SI case.

Let us now describe the impact of individual NSI parameters on the mass ordering sensitivity expected for DUNE as shown in Fig. 5. The black solid curve depicts the case of SI while the black dashed curve is for NSI with zero NSI phases. The shape of the black solid curve for the appearance channel for the baseline of DUNE follows from the discussion in Sec. 2.3 and Fig. 4. The impact of true non-zero NSI phases can be seen as grey bands for the choice of moduli of the NSI terms as mentioned in the legend.

For the case of $\varepsilon_{e\mu}$, the mass ordering sensitivity using both appearance and disappearance channels is shown (separately as well as combined) in Fig. 5(a). It can be seen that the disappearance channel contributes very little\textsuperscript{12} and the appearance channel is the main contributor to the total $\chi^2$. If $|\varepsilon_{e\mu}| = 0.01$ as shown in the top row, the overall $\chi^2$ decreases due to effect (a) for all values of $\delta$ while (b) is insignificant. However, for somewhat larger value such as $|\varepsilon_{e\mu}| = 0.07$ as shown in the second row of Fig. 5(a), we note that the net $\chi^2$ again decreases for all values of $\delta$ but the non-negligible impact of effect (b) can be inferred from the broadening of the grey bands.

The impact of $\varepsilon_{e\tau}$ is shown for three different values for the combined (appearance + disappearance) case in Fig. 5(b). The effect is similar to that of $\varepsilon_{e\mu}$ described above but the widening of grey bands due to the effect of relevant true phase variation ($\varphi_{e\tau}$) is somewhat more pronounced in comparison to Fig. 5(a). The widening of grey bands depends on the value of $|\varepsilon_{e\tau}|$. As we go from smaller to larger value of $|\varepsilon_{e\tau}|$, we see that NSI can drastically alter the $\chi^2$ at all values of $\delta$. However, the $\chi^2$ always stays below the SI expectation for the values considered.

The impact of all the three diagonal NC NSI parameters is shown in Fig. 5(c) for positive and negative values of the relevant NSI parameters. In general we note that for diagonal NSI parameters, the effect (a) is somewhat reduced due to the fact that there are no phases i.e. lesser parameters being marginalised over. This easily allows effect (b) to overtake (a) if the value of NSI parameter is large enough. The effect of $\varepsilon_{ee}$ is shown on the left for positive and negative values of $\varepsilon_{ee}$. For $\varepsilon_{ee} = 0.7$, the effect (b) overtakes (a). The case of negative NSI parameter is also shown for comparison. For $\varepsilon_{\mu\mu}$, there is hardly any impact since $\varepsilon_{\mu\mu}$ is constrained very well. For $\varepsilon_{\tau\tau}$, we note that effect (a) dominates the sensitivity plots and the general trend is opposite to that seen for $\varepsilon_{ee}$.

### 3.2 Dependence on the value of $\theta_{23}$ and $\delta m^2_{31}$

We depict the dependence of the mass ordering sensitivity on the choice of true values of $\theta_{23}$ and $\delta m^2_{31}$ in Fig. 6. The top row shows the impact of $\theta_{23}$, we note that the SI curves are consistent with Ref. \[13\] i.e. larger the value of $\theta_{23}$, higher will be the sensitivity to neutrino mass ordering. In presence of NSI, the sensitivity goes down but still remains above $5\sigma$ for most values of $\delta$ for the choice of parameters considered in the figure in accordance with

\[11\] The separation between NO and IO bands is visibly more pronounced (diminished) than the SI case when $\varepsilon_{ee} = +0.7 \ (-0.7)$ for $P_{\mu e}$.

\[12\] This can be attributed to the small matter dependence in $P_{\mu\mu}$ at the baselines considered \[52\] (see the previous section also).
the discussion in the preceding section. Again as far as $\theta_{23}$ dependence is concerned, the trend is similar to that in case of SI.

The bottom row shows the impact of choice of value of $\delta m^2_{31}$. Here again, the SI curves are consistent with Ref. [13] i.e., a larger value of $\delta m^2_{31}$ aids in determination of the neutrino mass ordering. We see a similar trend in case of NSI among the values of $\delta m^2_{31}$ considered. The overall mass ordering sensitivity is lower than the SI case (while still above 5\(\sigma\)) as expected from the discussion in the previous section.

Also, in both the panels for SI and NSI, we see a trend that is opposite to that seen in the CP sensitivity plots [24].

### 3.3 Comparison with other long baseline experiments

Below we give a very brief description\(^\text{13}\) of the ongoing and future experiments that would be sensitive to the question of neutrino mass ordering using the two oscillation channels considered in the present work.

**T2K:** The T2K (Tokai to Kamioka) experiment aims neutrinos from the Tokai site to the SK detector 295 km away (see [25]). The peak energy is 0.6 GeV. We consider a runtime of $3\nu + 3\bar{\nu}$ and a 22.5 kton Water Cherenkov (WC) detector.

**NOvA:** The NuMI Off-axis $\nu_e$ Appearance (NOvA) experiment has a baseline of 810 km, and the detector is exposed to an off-axis (0.8 degrees) neutrino beam produced from 120 GeV protons at Fermilab (see [26][27]). We consider a runtime of $3\nu + 3\bar{\nu}$ and a 14 kton Totally Active Scintillator Detector (TASD).

\(^{13}\)for more details, see [24].
### Table 2: Total number of signal events (SI/NSI) summed over all energy bins for each experiment using the oscillation parameters given in Table 1. For NSI, we show the collective case when the NSI parameters $|\varepsilon_{e\mu}| = 0.07, |\varepsilon_{e\tau}| = 0.07, |\varepsilon_{ee}| = 0.7, \varphi_{e\mu} = 0$ and $\varphi_{e\tau} = 0$ are considered.

| Experiment | Appearance channel | Disappearance channel |
|------------|---------------------|-----------------------|
|            | $\nu_\mu \rightarrow \nu_e$ | $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ | $\nu_\mu \rightarrow \nu_\mu$ | $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ |
| DUNE       |                     |                       |                     |                       |
| NO, $\delta = -\pi/2$ | 1610/1779 229/214 | 11431/11418 7841/7840 |                     |                       |
| NO, $\delta = 0$     | 1350/1803 292/257  | 11401/11313 7805/7826 |                     |                       |
| NO, $\delta = \pi/2$ | 1028/1182 309/279  | 11431/11417 7841/7840 |                     |                       |
| IO, $\delta = -\pi/2$ | 861/740 357/392  | 11052/11036 7611/7612 |                     |                       |
| IO, $\delta = 0$     | 610/383 416/529  | 11110/11188 7631/7612 |                     |                       |
| IO, $\delta = \pi/2$ | 468/399 499/540  | 11052/11036 7611/7609 |                     |                       |
| NOvA        |                     |                       |                     |                       |
| NO, $\delta = -\pi/2$ | 90/95 17/15     | 142/143 46/47        |                     |                       |
| NO, $\delta = 0$     | 78/93 25/20     | 141/140 45/46        |                     |                       |
| NO, $\delta = \pi/2$ | 57/62 30/27     | 142/143 46/47        |                     |                       |
| IO, $\delta = -\pi/2$ | 63/58 27/29     | 130/129 41/41        |                     |                       |
| IO, $\delta = 0$     | 46/36 33/40     | 131/132 42/41        |                     |                       |
| IO, $\delta = \pi/2$ | 36/33 42/45     | 130/129 41/41        |                     |                       |
| T2K         |                     |                       |                     |                       |
| NO, $\delta = -\pi/2$ | 127/130 20/19   | 372/371 130/129      |                     |                       |
| NO, $\delta = 0$     | 111/119 29/27   | 367/365 127/128      |                     |                       |
| NO, $\delta = \pi/2$ | 80/82 33/32     | 372/371 130/129      |                     |                       |
| IO, $\delta = -\pi/2$ | 113/110 23/24  | 335/336 116/117      |                     |                       |
| IO, $\delta = 0$     | 83/77 28/31     | 340/341 119/118      |                     |                       |
| IO, $\delta = \pi/2$ | 68/66 37/38     | 335/336 116/117      |                     |                       |

**DUNE:** The DUNE experiment has a baseline of 1300 km and the $\nu_\mu$ beam peaks at 2.5 GeV (see [13]). We consider a 35 kton fiducial liquid argon detector placed at 1300 km from the source, on-axis with respect to the beam direction. We consider a runtime of $5\nu + 5\bar{\nu}$ years.

In Table 2, we give the energy integrated events$^{14}$ for the mentioned experiments and channels for neutrinos as well as antineutrinos for the two possible orderings. As expected, the events for the disappearance channel are much larger than the appearance channel. The event rates for T2K and NOvA are comparable due to the larger detector size of T2K which compensates for the shorter baseline when compared with NOvA with a longer baseline. The event rates are higher for DUNE due the bigger detector size in comparison to NOvA even though the baseline for DUNE is larger than NOvA.

Let us look at the impact of NSI at the level of event rates for the choice of parameters considered.

$^{14}$The energy range for the various experiments is mentioned in Sec. 2.3
Variation of the neutrino and the antineutrino event rate histograms (for the appearance channel) are shown at DUNE for both SI (cyan) and the NSI (grey) scenario due to the variation in the relevant parameters as indicated in the legend. The dashed (solid) black curve indicates the case when all the relevant NSI (SI) phases are zero. 5 yrs. of neutrino and 5 yrs. of antineutrino runs were considered. The left (right) panel depicts the case of normal (inverted) mass ordering.

We show the event rate histograms at DUNE for the $\nu_e$ appearance (Fig. 7) and the $\nu_\mu$ disappearance (Fig. 8) channels. The cyan bands show the variation of events due to the full variation of the standard phase $\delta$ in the SI scenario. The grey bands indicate the variation in the NSI event spectra when the dominant NSI parameters ($\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$ and $\varepsilon_{ee}$) have been varied in addition to the variation of standard $\delta$ within the ranges indicated in the legends of the plots. We can make the following observations from Figs. 7 and 8:

- In Fig. 7, the grey bands (NSI) span much further on both sides of the cyan (SI) bands and this feature is most prominent around the peak ($\sim 2 − 5$ GeV). This implies that a significant surfeit or dearth of appearance events around the peak of the energy is a signature of the presence of new physics. On the other hand, Fig. 8 shows that the effect of NSI for the disappearance channel is very small and is manifested as the much thinner grey bands. This is because the matter effect in the disappearance channel is very small. Consequently the relative change in the disappearance event rate due to the presence of NSI is much smaller than that of the appearance events shown in Fig. 7.

- We note that both the NSI and the SI event spectra in Fig. 7 fall very close to zero around energies $\gtrsim 6.5$ GeV onwards, because of the rapidly falling flux as well as the probability $P_{\mu e}$ (or $\bar{P}_{\mu e}$). But, for the disappearance channel in Fig. 8 even if the flux

\[15\text{The matter effect in the } P_{\mu\mu} \text{ channel becomes significant only when the baseline chosen is very long.}^{52}\]
falls rapidly, the tail of the event spectra, unlike Fig. 7, is significantly high and falls much slowly. This is due to the large value of $P_{\mu\mu}$ (or $\bar{P}_{\mu\mu}$) even at energies $\sim 6 - 10$ GeV.

- The dashed black curve (NSI event spectra corresponding to zero phases) in Fig. 7 is greater or less than the solid black curve (SI event spectra) depending on the mass ordering and polarity. This trend is also similar for Fig. 8 although due to the much smaller effect of NSI, the dashed black curves are very close to the solid ones.

The expected mass ordering sensitivity offered by different experiments is illustrated in Figs. 9 and 10 for true NO and true IO respectively considering the relevant NSI parameters collectively.

In Fig. 9 (Fig. 10), we show a comparison of mass ordering sensitivity for the three experiments with and without NSI for the case of true NO (IO). We can immediately notice that the (solid black) curves for SI resemble the characteristic shape based on the statistical definition of $\chi^2$ described earlier (see Sec. 2.3 and Fig. 4). In presence of NSI, the baseline-dependent characteristic shape of the $\chi^2$ for mass ordering sensitivity is spoiled depending upon the baseline and the size of the NSI term. This distortion in shape is expectedly more for the longer baselines considered. In addition, due to the effect (a) mentioned in subsection 3.1, there are effects such as suppression in the value of $\chi^2$ for the values of NSI parameters considered.

For T2K and NOvA individually, we note that the mass ordering sensitivity almost never reaches $3\sigma$ (it barely touches $\sim 1.1\sigma$ (for T2K) and $\sim 3.2\sigma$ (for NOvA)) when SI is present (see Fig. 9, first and second row). This means that these two current experiments considered
in isolation are not so much interesting as far as mass ordering sensitivity is concerned. This does not come as a surprise as T2K and NOvA are shorter baseline experiments (compared to DUNE) and hence do not give the best sensitivity to the neutrino mass ordering. Even combining the data from T2K and NOvA would be a futile exercise for most values of \( \delta \) for SI. In presence of NSI, the value of \( \chi^2 \) undergoes a suppression in general for all values of \( \delta \).

If we look at the mass ordering sensitivity expected from DUNE (see Fig. 9, third row), we note that it is much above 5\( \sigma \) for all values of true \( \delta \) in presence of SI which means that it is not needed to combine data from the T2K and NOvA to DUNE. In presence of NSI, there is a suppression in the value of \( \chi^2 \) for all values of \( \delta \). However for most values of \( \delta \), it stays above 5\( \sigma \) if the NSI parameter \( \varepsilon_{ee} \) is positive. If \( \varepsilon_{ee} < 0 \), the cyan band shows that the \( \chi^2 \) can get further suppressed and can lie in the range 3 – 5\( \sigma \) if the NSI terms are large enough.

In Fig. 10 the mass ordering sensitivity is shown for the case of IO. The characteristic shapes of the SI curves can also be predicted from the discussion in Sec. 2.3. The impact
3.4 Degeneracy and the issue of mass ordering

Let us analyse the information contained in Fig. 1 in terms of event rate histograms for the appearance channel at DUNE for the off-diagonal and diagonal NSI parameters respectively (Figs. 11 and 12). These plots provide another perspective to understand the impact of NSI on the question of mass ordering. The NO band in Fig. 11 (true data set corresponding to Fig. 5) is obtained by fixing the NSI parameter (mentioned in the plot) is obtained by varying the relevant parameters $\delta$ and $\varphi_{e\mu}$ (or $\varphi_{e\tau}$) in each panel. The IO band (test data set) in Fig. 11 is obtained by varying the relevant parameters $\delta$ and $\varphi_{e\mu}$ (or $\varphi_{e\tau}$) as well as $|\varepsilon_{e\mu}|$ (or $|\varepsilon_{e\tau}|$) in each panel. This shows the maximum possible variation in the event rates for NO and IO in Fig. 11. The NO (true) band is for a fixed value of the $|\varepsilon_{e\mu}|$ (or $|\varepsilon_{e\tau}|$) and of NSI is similar to that for the case of NO.
Figure 11: Variation of the neutrino event rate histograms for the appearance channel at DUNE for the NSI scenario (taken one off-diagonal NSI parameter at a time) for NO (true) and IO (test) data set. The magenta (blue) shaded region corresponds to the NO (IO) variation due to the variation of the relevant parameters (see text for details) as shown in the legend. 5 yrs. of neutrino run was considered. The different panels are for different values of the true NSI parameter considered.

The IO (test) band corresponds to the case where $|\varepsilon_{e\mu}|$ (or $|\varepsilon_{e\tau}|$) are also allowed to vary.

Figure 12: Same as Fig. 11 but for different diagonal NSI parameter, $\varepsilon_{ee}$.

The $\Delta \chi^2$, as shown in Fig. 5 and explained in subsection 2.3, is roughly a difference between the NO (true) and the IO (test) events for two fixed values of true $|\varepsilon_{e\mu}| = 0.01, 0.07$. The
possibility of distinction between NO and IO depends upon the overlap between NO and IO bands and increases as the overlap becomes small. The NO bands in Fig. 11 which are shown as magenta shaded regions correspond to fixed true values of $|\epsilon_{e\mu}|$ or $|\epsilon_{e\tau}|$. Clearly, as the modulus of the relevant parameter increases, the shaded region becomes wider. The IO bands are shown as blue shaded regions and incorporate the entire range of $|\epsilon_{e\mu}|$ or $|\epsilon_{e\tau}|$.

Thus, as we go from left to right, these bands remain unchanged. As the true value of $|\epsilon_{e\mu}|$ or $|\epsilon_{e\tau}|$ increases, the NO band widens while the IO band remains unchanged. As a result, the overlap between NO and IO bands increases. This would imply that the sensitivity to mass ordering is impacted severely when relevant NSI parameter takes a larger value. This is also illustrated by means of sensitivity curves in Fig. 5 since the change in sensitivity is proportional to the magnitude of the NSI parameter considered.

For the diagonal NSI parameter ($\epsilon_{ee}$), we can make similar observations from Fig. 12. Note that in Fig. 5 (left panel) a positive value of $\epsilon_{ee}$ can enhance the sensitivity to mass ordering whereas a negative value of $\epsilon_{ee}$ suppresses it. Similar conclusions are also indicated in Fig. 12 i.e. the NO and the IO bands are closer together for a negative $\epsilon_{ee}$ and get further apart as $\epsilon_{ee}$ increases in the positive direction.

### 3.5 Optimal exposure for mass ordering discovery

All the plots shown thus far were obtained by keeping the total exposure fixed for a given experimental configuration. It is pertinent to ask what would be the optimal exposure for a given experiment that would be required to decipher the neutrino mass ordering.

We can define a useful quantity called the **mass ordering fraction** $f^{MO}(\sigma > 5)$ that can be used to quantify the ability of a given experiment to determine the neutrino mass ordering. $f^{MO}(\sigma > 5)$ refers to the fraction of true $\delta$ values for which mass ordering can be determined above a particular significance (here, $5\sigma$). Being a fraction, $f^{MO}(\sigma > 5)$ naturally lies between 0 and 1.

The choice of optimal exposure for discovery of mass ordering is guided by the value when $f^{MO}(\sigma > 5)$ reaches its maximum value. In Fig. 13 we show the mass ordering fraction for which the sensitivity to mass ordering exceeds $5\sigma$ ($f^{MO}(\sigma > 5)$) as a function of exposure.

For the SI case (shown as solid black lines), we note that $f^{MO}(\sigma > 5)$ rises initially but saturates to its maximum value of unity as we go to exposures beyond $\sim 115$ kt.MW.yr. So, in case of SI, the choice of optimal exposure is expected to be around 115 kt.MW.yr.

Let us now discuss the impact of NSI on the choice of the optimal exposure. For the NSI case, the panels of Fig. 5 correspond to the three different NSI terms (taken in isolation) and we can use these to understand the main features in Fig. 13. There are three coloured regions (blue, green, red) corresponding to the off-diagonal NSI terms in Fig. 13 which correspond to the three values of moduli of NSI parameters along with their respective phase variation (analogous to the grey bands seen in Fig. 5(a) and 5(b)). For the diagonal NSI terms, there are three dashed lines (blue, green, red) corresponding to three different positive values of diagonal NSI parameter $\epsilon_{ee}$ and a solid (red) line to depict the case of $\epsilon_{ee} < 0$ (see Fig. 5(c)).

The left panel in Fig. 13 depicts the impact of $\epsilon_{e\mu}$. The coloured bands stay below the solid black line mostly except around exposure of 100 kt.MW.yr for the considered values
of $\varepsilon_{e\mu}$. This is due to the dominating statistical effect (a) and the effect (b) that operates in a small regime mentioned in Sec. 3.1. For different values of $\varepsilon_{e\mu}$ ($|\varepsilon_{e\mu}| = 0.01, 0.04, 0.07$), $f^{MO}(\sigma > 5)$ gets distributed over a larger range of values as can be seen from the bands.

Incorporating the phase variation of the NSI parameter changes the value of exposure when the value of $f^{MO}(\sigma > 5)$ reaches its maximum value of $\sim 1$ i.e. it changes the optimal exposure. The optimal exposure for $\varepsilon_{e\mu}$ varies between $\sim 100 – 125$ kt.MW.yr. Similar effects are seen for the other off-diagonal parameter, $\varepsilon_{e\tau}$ which is shown in the middle panel. The optimal exposure in case of $\varepsilon_{e\tau}$ varies between $\sim 100 – 150$ kt.MW.yr.

For the diagonal NSI parameter $\varepsilon_{ee}$, $f^{MO}(\sigma > 5)$ (blue, green and red dashed lines) can be on the either side of solid black curve for a given choice of systematics (see also Fig. 5). This is again due to interplay of the two kinds of effects. For negative value of this parameter, it is below the SI case and the optimal exposure shifts to $\sim 125$ kt.MW.yr. If $|\varepsilon_{ee}| = 0.7(0.4)$, the choice of optimal exposure changes drastically from 115 to 50(75) kt.MW.yr.

If we take the true ordering as IO instead of NO, the impact of NSI shown in Fig. 13 is similar.

### 3.6 Systematics

The impact of different assumptions on systematics can be seen in Fig. 14. The black solid curve represents our nominal choice of systematics while the blue solid curve is for an optimal choice mentioned in the legend. The green (magenta) band corresponds to NSI case for off-diagonal parameters $\varepsilon_{e\mu}, \varepsilon_{e\tau}$ with full phase variation for nominal (optimal) choice of systematics. The green (magenta) dashed curve is for $\varepsilon_{ee}$ for nominal (optimal) choice of systematics.

It can be seen that $f^{MO}(\sigma > 5)$ nearly reaches its maximum ($\sim 1$) possible value at around 1150 km for SI (see Fig. 14). This implies that for the given configuration of the far detector planned for DUNE, the optimal distance to be able to infer the mass ordering for the largest fraction of the values of the CP phase is $\geq 1150$ km. Clearly, in case of SI, better systematics does not significantly change the optimal baseline for mass ordering determination above 5$\sigma$. For the SI case, therefore the optimal baseline choice for mass ordering sensitivity remains
the same for either choice of systematics. In case of NSI, the green (magenta) band shows the effect of two choices of systematics and there is an overlap between them as well as with the SI values. These aspects play a crucial role in altering the choice of best baseline for mass ordering sensitivity. However, in presence of NSI, for the choice of NSI phases representing the top (bottom) edge of the green or magenta band (we have used the dashed green or magenta lines to depict the diagonal NSI terms), the optimal choice of baseline that maximizes the mass ordering fraction changes as a function of systematics.

4 Summary

We have entered an era of precision neutrino oscillation physics. Among the few open questions that need to be addressed, the resolution of neutrino mass ordering is an important one. Also, the unprecedented precision offered by some of the experiments allow us to probe the subdominant effects due to new physics such as NC NSI as considered in the present work. We have earlier studied the impact of NSI on the CP measurements at DUNE analytically as well as numerically at the level of probability and event rates in Ref. [16] and at the level of \( \chi^2 \) using both appearance and disappearance channels in Ref. [24]. Here we analyse the impact of NSI on the mass ordering sensitivity at the level of probability, event rates and \( \chi^2 \) at long baseline experiments which include T2K, NOvA and DUNE.

Before we go on to summarise our main results, we would like to point out that we have explicitly shown (in subsection 2.3 and Fig. 4) that shape of the \( \chi^2 \) curves for mass ordering sensitivity depends on the baseline, using statistical definition of \( \chi^2 \) and probability expressions for the case of SI. For NSI case, the probability expressions are cumbersome so a simple-minded shape characterisation is not readily feasible.

Our main conclusions are

- Among the two channels considered, \( P_{\mu e} \) corresponding to the platinum channel contributes most to the sensitivity to neutrino mass ordering while \( P_{\mu\mu} \) has tiny effect.

The gross behaviour of sensitivity plots is dictated by \( P_{\mu e} \). On introducing propaga-
tion NSI, the largest impact in the $P_{\mu e}$ channel comes from the NSI parameters $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$ and $\varepsilon_{ee}$ and thus these parameters modify the sensitivity of long baseline experiments to neutrino mass ordering.

- From the probability expressions (Eqs. 8a-9d), we notice that there is a degeneracy pertaining to sign of $\delta m^2_{31}$ and the CP phase $\delta$ in the case of SI. Near the peak of $P_{\mu e}$ (around 2.5 GeV) for DUNE, we note that $\lambda L/2 \sim \pi/2$ and we can also note from Fig. 1 that the NO and IO bands are non-overlapping around this energy. This allows us to determine the mass ordering unambiguously near the peak above $5\sigma$ irrespective of the value of $\delta$. This does not hold for T2K or NOvA and the ordering can be determined in favourable half plane in $\delta$ only.

- The dependence of mass ordering sensitivity at DUNE on the true values of $\theta_{23}$ and $\delta m^2_{23}$ is shown in Fig. 6. Both $\theta_{23}$ and $\delta m^2_{31}$ change the sensitivity for SI as well as NSI. The effect is opposite in direction to CP sensitivity [24].

- The impact of NSI on the mass ordering fraction $f^{MO}(\sigma > 5)$ at DUNE as a function of exposure is shown in Fig. 13. The choice of optimal exposure changes in presence of NSI.

- The impact of change in systematics on $f^{MO}(\sigma > 5)$ at DUNE as a function of baseline is shown in Fig. 14. It is shown that the choice of optimal baseline changes when we include effects due to NSI.
It is shown that DUNE is sensitive not only to the ordering of neutrino masses due to the SI with matter \cite{12} but also to additional NSI induced matter effects arising due to moduli and phases of the NSI parameters. In particular, in addition to the off-diagonal NSI parameters $\varepsilon_{e\mu}$ and $\varepsilon_{e\tau}$, the diagonal NSI parameter $\varepsilon_{ee}$ also impacts the mass ordering sensitivity drastically as can be seen from Fig. 1.

In the present work, we have focussed on one of the outstanding questions in present day neutrino physics - the determination of neutrino mass ordering. We examine in detail the sensitivity offered by the long baseline experiments to this crucial question and how the presence of subdominant effects such as NSI spoils the sensitivity at these experiments. We have highlighted the importance of disentangling the new physics scenario in a reliable manner to be able to make clean inference regarding mass ordering of neutrino states. This can be achieved via a fine near detector for DUNE \cite{14, 58} so that the systematics be brought under control as much as possible to the extent required to cleanly determine the neutrino mass ordering and CP violation.

Acknowledgements

It is a pleasure to thank Raj Gandhi for useful discussions and critical comments on the manuscript. We acknowledge the use of HRI cluster facility to carry out computations in this work. We would like to thank the organisers of Nu Horizons VI at HRI for the warm hospitality during the initial stages of the present work. MM would like to acknowledge support from the DAE neutrino project at HRI. PM would like to thank HRI for a visit and the organisers of the Center for Theoretical Underground Physics and Related Areas (CETUP 2016) for the kind hospitality during the finishing stages of this work. PM acknowledges support from University Grants Commission under the second phase of University with Potential of Excellence at JNU and partial support from the European Unions Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant No 674896.

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