Measuring Forward-Backward Asymmetry of $t\bar{t}$ and $b\bar{b}$ at Electron-Positron Colliders

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Abstract

Motivated by the measurement on the asymmetry at TEVATRON and relevant theoretical interpretations, in this work we calculate the forward-backward asymmetry ($A_{FB}$) in processes $e^+e^- \to t\bar{t}$ and $b\bar{b}$ up to the next-to-leading order (NLO) in both the Standard Model (SM) and the little Higgs model (LHM) which is one of the models beyond the SM (BSM). We carefully analyze the contributions of the SM and LHM to the $A_{FB}$ for $t\bar{t}$ and $b\bar{b}$ and find that to reconcile the theoretical predictions on $A_{FB}$ and the data of LEP I and II, the model parameter $a$ (see the text to find its definition) is constrained in a rather narrow region. The $A_{FB}$ will be more precisely measured at the proposed ILC and Z-factory. By a comparison with data, we can testify the validity degree of LHM and the theoretical results may be useful for designing future experiments.
I. INTRODUCTION

As well recognized, the hadron colliders are machines for discovery. On other aspects, the electron-positron collider, muon-collider and even the proposed photon collider will provide detailed information about the discovered new physics candidates. Once some peculiar phenomena are observed at the hadron colliders such as TEVATRON or LHC beyond the expectation of the Standard Model (SM), one is tempted to associate them to new physics. Generally, making conformation is difficult, especially there are too many models about the new physics available and most of them can offer a plausible interpretation towards the new observation. One of the reasons is that by the data obtained at hadron colliders, it is difficult to study the details which are crucial for identifying the new interaction and/or new particles observed in the physical process accompanied by an enormous background. That is why people will turn to invoke high-energy lepton colliders after successful operation of hadron colliders, especially electron-positron colliders which are more favorable because the technique for building such machines are more mature.

To discover new physics, one is looking for phenomena beyond the SM expectation through experimental measurements carried at hadron colliders. Confirming or at least claiming existence of new physics needs to measure several characteristic quantities which do not meet the SM predictions.

The forward-backward asymmetry ($A_{FB}$) in top-antitop production at TEVATRON is one of such measurements. It is defined as

$$A_{FB} = \frac{N_Q(\cos \theta > 0) - N_Q(\cos \theta < 0)}{N_Q(\cos \theta > 0) + N_Q(\cos \theta < 0)},$$

(1)

where $\theta$ is the angle between the outgoing top quark and the injecting proton beam and $N_Q$ is the number of top quarks.

The data of TEVATRON at the Fermilab observed a large $A_{FB}$. The measurements of the CDF and D0 Collaborations yield $A_{FB} = 0.158 \pm 0.075$, $A_{FB} = 0.162 \pm 0.047$ and $A_{FB} = 0.196 \pm 0.065$, which are significantly larger than the SM prediction $A_{FB}^{SM} = 0.089$. This discrepancy would compose a hint of existence of new physics beyond SM. Numerous models beyond SM have been proposed to explain the deviation from the SM prediction, and we list a few of them in our references as examples.

We also showed that the deviation of the theoretical prediction and the data can be mended in the little Higgs model (LHM). Definitely, as commonly conjectured, since top
quark is much heavier than rest members of quark families, its mass could be close to the scale of new physics, so that observation on processes associated with top quark should be more favorable for discovering new physics.

However, on another aspect, it is also proposed to observe the $A_{FB}$ in $b\bar{b}$ production \cite{18}. Indeed, comparing the asymmetries for top ($A_{tFB}$) and bottom ($A_{bFB}$) productions would be interesting. If the new physics beyond SM indeed contributes to the $A_{FB}$, and the scale of new physics is high and close to the top quark, then the new physics should make a larger contribution to $A_{FB}$ in $t\bar{t}$ production than in $b\bar{b}$ production. Namely, if we calculate the $A_{FB}$ within the framework of only SM, the theoretical prediction on the $A_{FB}$ for bottom should be closer to the data than for top. This general analysis might be violated due to so far unknown behaviors of new physics BSM. Indeed all depends on the scales of new physics (see below in the context).

In this work, we study the $A_{FB}$ at $e^+e^-$ colliders within the framework of SM and LHM which is one of the models beyond SM. For a proposed Z factory the process under consideration is only $e^+e^- \rightarrow b\bar{b}$ because of the constraint of the phase space, while at ILC both the processes $e^+e^- \rightarrow t\bar{t}$ and $e^+e^- \rightarrow b\bar{b}$ would be accounted.

By a direct observation, the $A_{FB}$ is induced by the odd power of $\cos \theta$ in the amplitude square. Obviously, such terms imply parity in the process is violated (PV). In the SM, the parity violation in the process $e^+e^- \rightarrow b\bar{b}$ is due to $Z_0$ boson exchange, whose interaction with fermions has both vector and axial vector components. For next-to-leading order (NLO), the box diagrams also generate the asymmetry, because it is equivalent to a t-channel tree diagram, thus lead to odd powers of $\cos \theta$.

Similar to the case of $p\bar{p}$ collision at TEVATRON, the production angle $\theta$ at the $e^+e^-$ collider is defined as the angle between the outgoing bottom or top quark and the incoming electron beam. The difference of the rapidities of the $Q$ and $\bar{Q}$ which is Lorentz invariant is written in the $e^+e^-$ center-of-mass frame as

$$y_Q - y_{\bar{Q}} = 2\text{arctanh}(\sqrt{1 - \frac{4m_Q^2}{s}} \cos \theta),$$  

where $s = (p_1 + p_2)^2$ with $p_1$ and $p_2$ being the momenta of $e^-$ and $e^+$. Obviously, the sign of $y_Q - y_{\bar{Q}}$ is the same as $\cos \theta$, the asymmetry in Eq.(3) which is experimentally measurable, can be recast as

$$A_{FB} \equiv \frac{N_Q(y_Q - y_{\bar{Q}} > 0) - N_Q(y_Q - y_{\bar{Q}} < 0)}{N_Q(y_Q - y_{\bar{Q}} > 0) + N_Q(y_Q - y_{\bar{Q}} < 0)},$$  

(3)
The $b\bar{b}$ asymmetry $A_{FB}^b$ was theoretically predicted with the SM as $10.34 \pm 0.07\%$ \cite{19} and experimentally measured value at LEPI is $9.92 \pm 0.16\%$ \cite{19}. There indeed exists an observable distinction between the prediction and data. Moreover, the theoretical estimate did not involve the higher order corrections and interference with the photon contribution, when such corrections are taken into account, the situation becomes even worse, namely the theoretical prediction gets larger above the data (see the details in next sections). It implies that new physics beyond the standard model whose contribution to the asymmetry destructively interferes with that of SM is needed to reduce the value. Combining with the observation about $A_{FB}$ at TEVATRON where the SM prediction is also apart from the data, we would take this deviation as a hint of existence of new physics BSM.

Of course the distinction might be due to the measurement errors, but one cannot exclude a possible contribution from new physics beyond SM. The strategy of this work is to investigate the contributions of both SM and BSM to the asymmetries in $e^+e^- \rightarrow b\bar{b}$ and $e^+e^- \rightarrow t\bar{t}$ with a special BSM, i.e. the LHM which we used to explain the $A_{FB}'$ observed at TEVATRON. The energies we set are that of the Z-factory and ILC (or CLIC) respectively. Then we compare the asymmetries obtained for $t\bar{t}$ and $b\bar{b}$ to investigate their differences. Even though we employ a special model BSM, the obtained results can make sense about the role of BSM for the asymmetries. It is noted that there is an obvious difference in the two cases, even though we employ the same model: LHM. For the TEVATRON case, the main contribution is from an exchange of the heavy $Z$ boson $Z_H$ whose mass is generally believed to be around 400 to 500 GeV, whereas for the LEP cases, the main contribution comes from the heavy photon $A_H$ whose mass is within a range of a few tens of GeV to 100 GeV.

The future experiments will provide us more definite information. Especially, a comparison of theoretical predictions and the data for both TEVATRON and $e^+e^-$ collider may tell us consistency degree of the model and enrich our understanding of the nature, namely help us to search for new physics.

This paper is organized as follows. After this introduction, in Section II, we formulate the total scattering cross section as well as $A_{FB}^b$ to NLO within the frameworks of SM and LHM. The numerical results along with all the input parameters are shown in Section III. The obtained results are shown explicitly in several figures and tables, as possible interpretations on those curves are made. The last section is devoted to a simple discussion and conclusion.
II. THE CONTRIBUTIONS OF SM AND LHM TO THE ASYMMETRY UP TO NLO

In this section we formulate the contributions up to NLO to the $A_{FB}$ and total cross section of the $e^+e^- \rightarrow QQ$ system in the frameworks of SM and LHM. The derivation in SM is standard and straightforward, here we just repeat the calculation which was done by number of authors to check our programs. Then we turn to the contribution of new physics, concretely the LHM.

In LHM only two vector bosons $A_H$ and $Z_H$ can contribute asymmetry through s-channel. The relevant Lagrangian is

$$\mathcal{L}_{A_H} = A_H \bar{t}(g_{vt} + g_{at}\gamma^5)\gamma^\mu t + A_H \bar{b}(g_{vb} + g_{ab}\gamma^5)\gamma^\mu b + A_H \bar{u}(g_{vu} + g_{au}\gamma^5)\gamma^\mu u + A_H \bar{e}(g_{ve} + g_{ae}\gamma^5)\gamma^\mu e, \tag{4}$$

and

$$\mathcal{L}_{Z_H} = Z_H \bar{t}(g_{vt} + g_{at}\gamma^5)\gamma^\mu t + Z_H \bar{b}(g_{vb} + g_{ab}\gamma^5)\gamma^\mu b + Z_H \bar{u}(g_{vu} + g_{au}\gamma^5)\gamma^\mu u + Z_H \bar{e}(g_{ve} + g_{ae}\gamma^5)\gamma^\mu e. \tag{5}$$

In the LHM, the masses are separately $m_{A_H} \propto (a + \frac{1}{a})f$ and $m_{Z_H} \propto (36.73g'_{vu} + \frac{1}{g'_{vu}})f$ where $f$ is the vacuum exception value (vev) in LHM and $a$ is a model parameter equal to $\frac{s'}{c'}$ where $s' \equiv \sin\theta'$ and $c' \equiv \cos\theta'$ defined in Ref. [20]. The relations of the coupling constants are listed in the appendix of Ref. [20] and those numerical values are presented in next section. The tree and NLO Feynman diagrams are shown in Figs 1, 2 and 3 respectively.

The amplitude of the first two diagrams of Fig 1 is:

$$\mathcal{M}_1 = \left(\frac{-G_F m^2_{W}}{2\sqrt{2}}\right)\bar{u}(p_4)\gamma^\mu\left[-(1 - \frac{4}{3}\sin^2\theta_W) + \gamma^5\right]v(p_3) \times \frac{i}{(p_1+p_3)^2-m^2_Z}\bar{v}(p_2)\gamma^\mu\left[-(1 - 4\sin^2\theta_W) + \gamma^5\right]u(p_1) + \bar{u}(p_4)(-i\epsilon\gamma^\mu)v(p_3)\frac{i}{p_1+p_2}\bar{v}(p_2)(-i\epsilon\gamma^5\gamma^\mu)u(p_1), \tag{6}$$

where $\theta_W$ is the Weinberg angle, $p_1$ and $p_2$ respectively stand for the four-momenta of the initial electron and positron, $e_Q$ is the electric charge of the heavy quark ($b$ or $t$), and $p_3, p_4$ denote the four-momenta of the final $Q$ and $\bar{Q}$. Here, $p_1 + p_2 = p_3 + p_4$ is four-momentum of $Z_0$ or photon at s-channel. For the rest three diagrams, the amplitudes are similar, so we omit them for saving space of the text.

In Fig 2 all SM and LHM box diagrams are presented. The diagrams (c) and (d) where charged W-bosons are exchanged stand for $b\bar{b}$ and $t\bar{t}$ productions respectively. To explicitly
FIG. 1: The tree diagrams for the process of $e^+e^- \rightarrow Q\bar{Q}$.

FIG. 2: The box diagrams for the process of $e\bar{e} \rightarrow Q\bar{Q}$ system.

demonstrate the procedure of the derivation, let us present the amplitude of the first two
diagrams in Fig. 2 where only photons are exchanged as an example, that is:

\[ \mathcal{M}_2 = \int \frac{d^4 k}{(2\pi)^4} \bar{u}(p_4) \frac{-i}{k^2} (-i e Q \gamma^\mu \frac{i(p_4 + k + m_A)}{(p_4 + k)^2 - m_A^2})(-i e Q \gamma^\nu) v(p_3) \]

\[ \times \bar{v}(p_2)(-i e Q \gamma_\mu \frac{-i}{(p_1 + p_2 + k)^2} (-i e Q \gamma_\nu) \frac{i(p_1 + k + m_2)}{(p_1 + k)^2 - m_2^2}) u(p_1) \]

\[ + \int \frac{d^4 k}{(2\pi)^4} \bar{u}(p_4) \frac{-i}{k^2} (-i e Q \gamma^\mu \frac{i(p_4 + k + m_A)}{(p_4 + k)^2 - m_A^2})(-i e Q \gamma^\nu) v(p_3) \]

\[ \times \bar{v}(p_2)(-i e Q \gamma_\mu \frac{-i}{(p_1 + p_2 + k)^2} (-i e Q \gamma_\nu) \frac{i(p_1 + k + m_2)}{(p_1 + k)^2 - m_2^2}) u(p_1). \]

(7)

For the rest diagrams, the amplitudes are similar but the coupling constants are different. We carry out the complete calculation of four diagrams in Fig. 2 with the software LoopTools.

![Diagram](image)

FIG. 3: The vacuum polarization diagrams for the process of $e\bar{e} \rightarrow Q\bar{Q}$ system.

The regular vacuum fluctuation of the $Z$-boson or heavy photon is included in the Breit-Wigner propagator which is proportional to $\frac{(q_\nu - q_\mu)^2}{q^2 - M^2 + i\Gamma M}$ where $M$ and $\Gamma$ stand as the masses and total widths of $Z_0$, heavy photon and heavy $Z$ bosons respectively, whereas the diagram with a photon being on the left and a $Z_0$ on the right of the bubble or reversed in Fig. 3 is different. The loop includes all charged fermions (charged leptons and quarks) and possible bosons (for example $W^\pm$), here we only calculate the diagrams with two different propagators. As an example, we display the amplitude of Fig. 3 with photon on left and $Z_0$ on right. $M_{3a}$ corresponds to the contribution of fermion loop as:

\[ \mathcal{M}_{3a} = \int \frac{d^4 k}{(2\pi)^4} \frac{-G_F m_Z^2}{2\sqrt{2}} \bar{u}(p_4) \gamma^\mu \left[ -(1 - \frac{4}{3} \sin^2 \theta_W + \gamma^5) \right] \frac{-i}{(p_1 + p_2)^2 - m_Z^2} v(p_3) \]

\[ \times (-1) tr \{\gamma_\mu[\left[ -(1 - \frac{4}{3} \sin^2 \theta_W + \gamma^5) \frac{-i(p_1 + p_2 - \gamma + m)}{(p_1 + p_2 - k)^2 - m^2} \right] \frac{-i(\gamma + m)}{(k)^2 - m^2}) \}

(8)

\[ \times \bar{v}(p_2)(-i e Q \gamma_\mu \frac{-i}{(p_1 + p_2 + k)^2} (-i e Q \gamma_\nu) \frac{i(p_1 + k + m_2)}{(p_1 + k)^2 - m_2^2}) u(p_1), \]

where $m$ stands for the mass of the fermion in fermion loop, and $M_{3b}$ corresponds to the
boson loop

\[ M_{3b} = \int \frac{d^4k}{(2\pi)^4} \frac{-G_F m_Z^2}{2\sqrt{2}} \bar{u}(p_4)\gamma^\mu \left[ -(1 - \frac{4}{3}\sin^2\theta_W) + \gamma^5 \right] \]
\[ \times \frac{i}{(p_1+p_2)^2 - m_Z^2} v(p_3) (-4\cos^2\theta_W) \]
\[ \times \left[ (2k - p_1 - p_2)_{\mu} g_{\sigma\lambda} - (p_1 + p_2 + k)_{\sigma} g_{\lambda\mu} + (2p_1 + 2p_2 - k)_{\lambda} g_{\mu\sigma} \right] \]
\[ \times \frac{-i}{(p_1+p_2-k)^2 - m_W^2} (ie) \]
\[ \times \left[ (p_1 + p_2 - 2k)_{\nu} g_{\sigma\lambda} + (p_1 + p_2 + k)_{\sigma} g_{\lambda\nu} - (2p_1 + 2p_2 - k)_{\lambda} g_{\nu\sigma} \right] \]
\[ \times \bar{v}(p_2)(-ie\gamma^\mu) \frac{i}{(p_1+p_2)^2} u(p_1), \]

where \( m_W \) is the mass of W-boson.

After averaging the initial spin-states and summing over the final spin- and color-states, the differential cross section with respect to the production angle \( \theta \) is:

\[ \frac{d\sigma}{d\cos\theta} = 3 \times \frac{2\pi\sqrt{1 - \frac{4m_W^2}{s}}}{64\pi^2 s^4} \sum \left| M_1 + M_2 + M_3 \right|^2 \]
\[ \approx 3 \times \frac{2\pi\sqrt{1 - \frac{4m_W^2}{s}}}{64\pi^2 s^4} \left( |M_1|^2 + 2Re(M^*_1 M_2) + 2Re(M^*_1 M_3) \right). \]

The asymmetry is obtained by integrating over the positive and negative ranges of \( \cos\theta \) separately. We use the Lorentz invariant rapidity difference \( y_Q - y_{\bar{Q}} \) to calculate the asymmetry as shown in Eq. (2) and Eq. (3). The numerical results will be presented in next section.

### III. NUMERICAL RESULTS

In our numerical computation, the mass of charm, bottom and top quark are taken as 1.25, 5 and 175 GeV and the masses of light quarks (\( u, d, s \)) are neglected. In the center of mass frame, one has \( p_Q = p_{\bar{Q}} \) and \( p_{Q}^2 = m_{Q}^2 \), the kinematics is determined

\[ p_1.p_2 = \frac{s}{2}, \quad p_3.p_4 = \frac{s}{2} - m_Q^2, \]
\[ p_1.p_3 = p_2.p_4 = \frac{s}{4}(1 + \sqrt{1 - \frac{4m_Q^2}{s}\cos\theta}), \]
\[ p_1.p_4 = p_2.p_3 = \frac{s}{4}\left(1 - \sqrt{1 - \frac{4m_Q^2}{s}\cos\theta}\right). \]

For the energy corresponding to LEP I experiment, we set \( \sqrt{s} = 92.5 \) GeV and \( m_Z = 91.2 \) GeV, \( m_W = 80 \) GeV [22, 23]. The electromagnetic coupling constant and weak mixing angle
are running with energy, at different energy scales we take $\alpha_e = 1/128.878$, $\sin^2 \theta_W = 0.2316$ for $\sqrt{s} = 91.2$ GeV; $\alpha_e = 1/128.516$, $\sin^2 \theta_W = 0.2398$ for $\sqrt{s} = 500$ GeV; $\alpha_e = 1/128.369$, $\sin^2 \theta_W = 0.2444$ for $\sqrt{s} = 1$ TeV \[19, 26, 27\]. At the proposed Z factory the center-of-mass energy is located in vicinity of $Z_0$ mass, so the on-mass-shell resonance effect would be dominant and the Breit-Winger formulation should be adopted.

In the LHM\[20\], the relevant parameters depend on parameters $a$ and $b$ via the relations:

$$
g_{vu} = -0.0292\left(\frac{3}{a} - 2a\right); \quad g_{au} = -0.0175\left(\frac{3}{a} - 2a\right);
$$

$$
g_{vd} = 0.2742\frac{3}{a} + 0.245a; \quad g_{ad} = 0.0175\left(\frac{3}{a} - 2a\right);
$$

$$
g_{vt} = -0.0292\left(\frac{3}{a} - 2a\right) - 0.35\left(\frac{1}{a} + a\right)b;
$$

$$
g_{at} = -0.0175\left(\frac{3}{a} - 2a\right) - 0.35\left(\frac{1}{a} + a\right)b;
$$

$$
g_{ve} = 0.0525\left(\frac{3}{a} - 2a\right); \quad g_{ae} = 0.0175\left(\frac{3}{a} - 2a\right);
$$

and

$$m_{A_H} = 0.08138\left(\frac{1}{a} + a\right)f \text{ GeV} \[20\].$$

It is noted that for the heavy photon, all its couplings to fermions uniquely depends on parameters $a$ and $b$, which are not determined in the model, so that here we treat them as free parameters. The only way, so far before a more fundamental principle appears, to fix them is by fitting available experimental data. To fit the data of the measured asymmetry, we find that $a$ can only reside in a rather narrow range from 1.1 to 1.3. Thus we let $a$ vary from 1.0 to 1.4 and $b$ vary from 0 to 1. Instead, the mass of $Z_H$ is written as $m_{Z_H} = 0.0539(36.73g'_{ua} + \frac{1}{g_u})f$ GeV, and for simplicity we use the relation $g'_{vu} = -g'_{vd} = g'_{vt} = -g'_{ve} = -g'_{au} = g'_{ad} = g'_{at} = -g'_{ae}$ and they all vary from $-0.0165$ to $-0.33[20]$, thus the coupling constant $\alpha_l = \frac{g'^2_{ua}}{4\pi}$ varies from 0.00002 to 0.00867.

Fig.4 shows the dependence of $A_{FB}^b$ on $\sqrt{s}$ (the superscript $b$ refers to $b\bar{b}$ production) which is theoretically estimated by SM up to NLO, and the experimental data. The narrow band corresponds to the measurement error range. One notices that $A_{FB}^b$ predicted by only SM up to NLO is 12.78% above the data at $\sqrt{s} = 92.5$ GeV which is about 20% larger than the previous LO estimate $10.34 \pm 0.07%$ \[19\]. This change is mainly caused by the interference between photon and $Z_0$ and as well NLO corrections.
FIG. 4: The $A_{FB}^b$ for b quark versus the center of mass energy theoretically estimated by SM and the experimental data where the band corresponds to the measurement errors [28].

When the contribution of LHM is introduced, the theoretical prediction can be in agreement with experimental data as shown in Fig.5, Fig.6 and Fig.7.

Fig.5 and Fig.6 are the production rates of $b\bar{b}$ pair at LEP I and II vs the parameter $a$ predicted by LHM+SM. The results indicate that the parameter $a$ must fall into a narrow range from 1.1 to 1.3 by fitting the LEP I and II data. For a clear illustration, we set $a$ between 1.0 and 1.4 in the following.

Fig.7 shows that with the LHM, the theoretical prediction on the asymmetry $A_{FB}^b$ can coincide with experimental data as long as the model parameter $a$ exists in the narrow window. In Fig.8, we present dependence of $A_{FB}^b$ on $\sqrt{s}$ with $a$ being 1.26, 1.27, 1.28 and 1.29 respectively, where we only choose $\sqrt{s}$ around the center-of-mass energy of the proposed $Z$ factory. We do see that the predicted $A_{FB}^b$ overlaps with the data band.

Fig.9 and Fig.10 respectively illustrate the $A_{FB}^b$ and $A_{FB}^t$ vs the center-of-mass energy for ILC.

It is noted that in Fig.9, the curve of $A_{FB}^b$ evaluated with LHM+SM has a minimum near the $\sqrt{s} = 410$ GeV, this is understood as a destructive interference between contribution of $Z_H$ and that of SM bosons.

Fig.10 shows that $A_{FB}^t$ behaves quite differently when it is evaluated with SM-only and...
FIG. 5: The dependence of the total cross section of $b\bar{b}$ pair production theoretically evaluated by SM-only and SM+LHM on parameter $a$ at $\sqrt{s} = 91.2$GeV and the experimental data\cite{19}.

FIG. 6: The dependence of the total cross section of $b\bar{b}$ pair production evaluated by SM-only and SM+LHM respectively on parameter $a$ at $\sqrt{s} = 189$GeV and the experimental data\cite{29}.
FIG. 7: The dependence of the asymmetry $A_{FB}^b$ on the parameter $a$ at $\sqrt{s} = 92.5$ GeV.

FIG. 8: Dependence of $A_{FB}^b$ evaluated with LHM on the center-of-mass energy $\sqrt{s}$ of the proposed Z factory and the experimental data [28].
FIG. 9: $A_{FB}^b$ evaluated with SM-only and LHM+SM vs the center-of-mass energy $\sqrt{s}$ of ILC.

FIG. 10: $A_{FB}^t$ evaluated with SM-only and LHM+SM vs the center-of-mass energy $\sqrt{s}$ of ILC.
TABLE I: The total cross section of top and bottom quark pair production. (The range of the values predicted by LHM corresponds to a varying from 1.26 to 1.29)

LHM+SM. The behavior of $A_{tF}^t$ evaluated with LHM+SM has a bump peaked at $\sqrt{s} = 430$ GeV, this is also caused by a destructive interference between $Z_H$ and SM particles. While, $\sqrt{s}$ is above 500 GeV and below 400 GeV, theoretical prediction on $A_{tF}^t$ tends to be that determined by SM-only.

In table [I] we list the total cross sections up to NLO for top and bottom quark pair production within the frameworks of SM and LHM+SM. Table [II] is about a ratios of the production rate of $b\bar{b}$ over that of light quark pairs which are experimentally measured quantities, at various energies. In table [III] we list the $A_{tF}^t$ and $A_{bF}^b$ evaluated with SM and LHM+SM, and a comparison with experimental data if it is available. From table [III] we notice that introducing the LHM which is a model beyond standard model, the gap between the SM predictions on $A_{tF}^t$ and the data at LEP energies can be naturally remedied. Moreover, to fit the data, the parameter $a$ is required to fall into a rather narrow window, it seems to be slightly fine-tuning.
TABLE II: The ratio of the production rates of top and bottom quark pair over light quark pairs  
\[ R = \frac{\sigma_{e^+e^-\rightarrow \bar{t}t}}{\sigma_{e^+e^-\rightarrow \bar{q}q}}. \]  
(The range of the values predicted by LHM corresponds to \( a \) varying from 1.26 to 1.29)

|       | theoretical value(R) | experimental value(R) |
|-------|----------------------|-----------------------|
| \( t\bar{t} \) | ILC(500GeV) | 0.165956 | 0.142362 |
|       | ILC(1TeV) | 0.217092 | 0.205868 |
| \( b\bar{b} \) | ILC(500GeV) | 0.118978 | 0.149088 |
|       | ILC(1TeV) | 0.115201 | 0.124723 |
|       | Z-factory(92.5GeV) | 0.21406 | 0.21495-0.21493 |
|       | LEPI(91.2GeV) | 0.21576±0.00004[19] | 0.21580-0.21579 | 0.21629±0.00066[19] |
|       | LEPIII(189GeV) | 0.16035 | 0.15758-0.15757 | 0.163 ± 0.013(stat) ± 0.005(syst)[29] |

IV. DISCUSSION AND CONCLUSION

The observation of the asymmetry of top pair production \( A_{FB} \) at TEVATRON, which is obviously larger than the SM prediction, implies existence of new physics beyond standard model. Many authors tried to explain the discrepancy between theoretical predictions and data in terms of various models BSM. Moreover, some authors also studied asymmetries for b-quark pair production with those models.

Recollecting the theoretical predictions on the asymmetry \( A_{FB} \) at LEP I and II energies, one notices that there is also a gap between SM prediction and the measured value. It is natural to conjecture that such new physics BSM should contribute to \( A_{FB} \) at LEP energies.

In our previous work, we investigated possible contributions of the heavy Z-boson which exists in LHM, to \( A_{FB} \) and found that as its mass and coupling to SM particles are within a suitable range, the asymmetry observed at TEVATRON can be reasonably explained. Extending this scenario to study the asymmetry which can be observed at the \( e^+e^- \) colliders, we notice that the mass of heavy Z-boson is as high as more than 400 GeV, so does not make substantial contributions to \( A_{FB} \), instead in the LHM the heavy photon whose mass is around 100 GeV does play an important role at the LEP I and II energies. Thus, we employ the LHM to calculate the production rate and asymmetry for \( e^+e^- \) collisions at the
| Process                | theoretical value(%) | experimental value(%) |
|------------------------|-----------------------|-----------------------|
|                        | SM                    | LHM+SM                |
| Tevatron               | 8.9                   | 20                    | 19.6±6.5[4] |
| ILC(500GeV)            | 46.68                 | 51.86                 |
| ILC(1TeV)              | 56.63                 | 54.81                 |
| ILC(500GeV)            | 58.83                 | 70.64                 |
| ILC(1TeV)              | 55.06                 | 61.27                 |
| Z factory ($\sqrt{s} = 92.5$GeV)$^a$ | 12.78                 | 11.35-9.69            |
| $b\bar{b}$             |                       |                       |
| LEPI ($\sqrt{s} = 91.2$GeV)$^b$ | 10.27                 | 9.97-9.69             | 9.89±0.27±0.13[28] |
| LEPIII ($\sqrt{s} = 189$GeV) | 66.57                 | 55.29-51.21           | 61 ± 18(stat) ± 9(syst)[29] |
| $e^+e^- (\sqrt{s} = 58$GeV ) | -43[30]               | -57.25                | -20±16±1[30] |
| $e^+e^- (\sqrt{s} = 34.6$GeV ) | -25[31]               | -24.59                | -25±22[31] |

$^a$To make a reasonable calculation at the Z-factory, we set $\sqrt{s}$ at 92.5 GeV which is the mass of $Z^0$ boson plus half of its width.

$^b$Even though the LEP I was running at $\sqrt{s} = 92.5$ GeV, the data was normalized to the $Z^0$ pole of 91.2 GeV and all the available data on the asymmetry is corresponding to $\sqrt{s} = 91.2$ GeV.

TABLE III: Theoretical and experimental values of $A_{FB}^t$ for top and bottom quarks, in LHM+SM the theoretical value varies along with $a$ in a range of 1.26 ~ 1.29. There is no difference between SM and LHM+SM estimates at the energies of 58 and 34.6 GeV.

LEP I and II energies. We find that as the model parameter $a$ takes a value of 1.1 ~ 1.3, the theoretical predictions on the total cross sections of $b\bar{b}$ production and the asymmetry $A_{FB}^b$ can be well consistent with the data. This is somehow slightly fine-tuning.

By the same model, we further calculate the asymmetries $A_{FB}^t$ and $A_{FB}^b$ at the ILC energy of about $\sqrt{s} = 500$ GeV. Moreover, we also investigate $A_{FB}^b$ at the proposed Z-factory. All those values should be tested by the more precise experiments which will be carried out at the Z-factory and ILC. But our calculations also indicate that for the B-factory and charm-tau factory, which are running at much lower energy scales than that of LEP, such new physics does not apply. We are expecting the future high energy experiments to confirm or negate the LHM or make more rigorous constraints on its model parameters.
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