Duality Invariance of the Hawking Temperature and Entropy

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Abstract

We consider solutions to low energy string theory which have a horizon and a spacelike symmetry. Each of these solutions has a geometrically different dual description. We show that the dual solution has a horizon with exactly the same Hawking temperature (surface gravity) and entropy (area) as the original solution.

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One of the fundamental differences between theories based on point particles and those based on strings is that the latter have a new type of symmetry called duality. Geometrically different spacetimes can be shown to be physically equivalent solutions to string theory. The simplest example consists of flat spacetime with one direction compactified to form a circle of radius $R$. This solution turns out to be equivalent to the one with radius $1/R$. More generally, it has been shown that any solution with a spacelike symmetry with compact orbits has an equivalent dual description. (If there are $d$ symmetry directions, there are many equivalent descriptions which are related to each other by $O(d, d, Z)$ transformations.)

Dual solutions can in general have quite different geometric properties. For example, solutions with curvature singularities can be equivalent to ones without. In a sense, certain aspects of the spacetime geometry are pure gauge. In this paper, we examine the effect of duality on Hawking evaporation. Since the thermodynamic properties of spacetimes with horizons, such as their temperature and entropy, are related to geometric properties of the solution, it is not at all obvious that dual solutions will describe Hawking evaporation in the same manner. Nevertheless, we will show that this is the case: If a solution to low energy string theory has a horizon and a spacelike symmetry, then the dual solution also has a horizon with exactly the same temperature. (More precisely, we prove that this is the case for solutions with a certain type of horizon called a bifurcate Killing horizon, which will be defined below.) Furthermore, the area of the horizon is invariant under duality. This is rather surprising in light of the fact that duality essentially replaces the radius $R$ with $1/R$. However, this description is appropriate for a metric which does not have the standard Einstein action. It is the area in the conformally related Einstein metric which is associated with the entropy and turns out to be duality invariant. Combining these results with the fact that the spectrum of the string is the same in dual solutions, one concludes that evaporation will occur at the same rate.

Our results clearly apply to solutions with both a horizon and a spacelike symmetry. Of most interest are the black string solutions, where the symmetry is an asymptotic translation. In this case, the original solution and its dual are both asymptotically flat, and the mass is duality invariant. The invariance of the Hawking temperature and entropy under duality also applies to higher dimensional extended objects surrounded by an event horizon, i.e., black p-branes. We will also consider the three dimensional black hole, where the spacelike symmetry is a rotation. This solution is dual to a three dimensional black string. This example is somewhat unusual since the black hole is not
asymptotically flat, but rather asymptotically anti-de Sitter. As a result, there is some ambiguity in the definition of the Hawking temperature. Unfortunately, our results do not apply to four (or higher) dimensional black holes since the rotational Killing field now vanishes on the axis, which causes the horizon in the dual spacetime to become singular.

The low energy action that arises in string theory is

$$S = \int d^D x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla \phi)^2 - \frac{1}{12} H^2 \right] ,$$

where $\phi$ is the dilaton and $H_{\mu\nu\rho}$ is the three form. Since $H$ is closed, one can define a two form potential $B_{\mu\nu}$ by $H = dB$. Consider an asymptotically flat $D$ dimensional solution with two commuting Killing vectors $\xi$ and $\eta$, where $\xi$ is timelike asymptotically and $\eta$ is spacelike asymptotically with compact orbits. We will assume that the solution has a bifurcate Killing horizon [11]. Roughly speaking, this means that the horizon is qualitatively like Schwarzschild, with both a past and future component. The precise requirement is simply that there is a constant $\Omega$ such that $\chi \equiv \xi + \Omega \eta = 0$ on a compact $D - 2$ dimensional spacelike surface $\Sigma$. The bifurcate Killing horizon is defined to be the future and past light cone of $\Sigma$. One can show that $\chi$ is normal to this light cone, i.e., it is null and tangent to this surface. $\Omega$ is called the velocity of the horizon. Our final assumption is that $\eta^\mu \eta_\mu \neq 0$ on the horizon.

Bifurcate Killing horizons are, in fact, quite common. The event horizons in all known solutions to low energy string theory are of this type, and this is probably true for all solutions as well. The reason is the following. Since the dilaton is invariant under all the symmetries of the solution, and a horizon is conformally invariant, one can consider the rescaled metric with the standard Einstein action. This has the advantage that the matter fields now satisfy the dominant energy condition. Under certain assumptions, Hawking has shown [12] that the event horizon of every stationary black hole must be a Killing horizon, i.e., a null hypersurface to which a Killing field is normal. (This is trivially true for static solutions.) Since the dominant energy condition is satisfied, the surface gravity $\kappa$ of every Killing horizon must be constant [13]. It then follows [14] that if $\kappa \neq 0$, the horizon is in fact a bifurcate Killing horizon. Since most of these results are local, they should apply to black strings as well as black holes.

Let us introduce coordinates so that $\eta = \partial/\partial x$ and $\xi = \partial/\partial t$. We now derive two properties which will be needed for the discussion of duality. We will show that everywhere on a bifurcate Killing horizon,

$$A_t \equiv \frac{g_{tx}}{g_{xx}} = -\Omega, \quad \text{and} \quad B_{tx} = 0$$

(2)
On the surface $\Sigma$, $\xi = -\Omega \eta$. Taking the inner product with $\eta$ yields $g_{tx} = -\Omega g_{xx}$, which implies $A_t \equiv g_{tx}/g_{xx} = -\Omega$ on $\Sigma$. By antisymmetry, $B_{tx} \equiv B_{\mu\nu} \xi^\mu \eta^\nu = B_{\mu\nu} \chi^\mu \eta^\nu$. Since $\chi = 0$ on $\Sigma$, we conclude that $B_{tx} = 0$ on this surface. Thus the two properties (2) hold on $\Sigma$. Now consider $A_t$ and $B_{tx}$ at an arbitrary point on the horizon. Since the null generators of the horizon are an isometry, $A_t$ and $B_{tx}$ must be constant along these null curves. But one can get arbitrarily close to $\Sigma$ by this isometry. It then follows by continuity that, at any point on the horizon, $A_t$ and $B_{tx}$ must be equal to their values on $\Sigma$. It should be noted that for spherically symmetric solutions, if $r$ is the area coordinate, $H_{txr}$ need not vanish on the horizon, since the vector $\partial/\partial r$ diverges at the bifurcation surface and compensates for the vanishing of $\chi$.

Since we are considering solutions having a spacelike symmetry with compact orbits, there is a dual solution which is physically equivalent in string theory [3]. The duality transformation takes a simple form if one writes the string metric as follows

$$ds^2 = g_{xx}(dx + A_\alpha dy^\alpha)^2 + g_{\alpha\beta} dy^\alpha dy^\beta$$

where $\alpha$ and $\beta$ run over all coordinates except $x$, and all metric components are independent of $x$. Under duality, $\tilde{g}_{\alpha\beta}$ is invariant, and the other fields transform as

$$\tilde{g}_{xx} = 1/g_{xx}, \quad \tilde{A}_\alpha = B_{x\alpha}, \quad \tilde{B}_{xx} = A_x, \quad \tilde{B}_{\alpha\beta} = B_{\alpha\beta} - 2A_{[\alpha} B_{\beta]x}$$

$$\tilde{\phi} = \phi - \frac{1}{2} \ln g_{xx}$$

The statement that the vector $\chi$ vanishes on the surface $\Sigma$ is independent of the metric and other fields on the spacetime. Thus the dual solution will have a bifurcate Killing horizon provided that the future and past light cones of $\Sigma$ are nonsingular. But this follows immediately from the fact that $B_{\mu\nu}$ is smooth and $g_{xx}$ is nonzero in a neighborhood of the horizon in the original spacetime. Thus duality preserves a bifurcate Killing horizon. Since it also preserves the two translational symmetries it might appear that the velocity $\Omega$ is duality invariant. However this is not the case. The problem is that $\Omega$ is defined by the condition $\chi = \xi + \Omega \eta = 0$ on $\Sigma$, where $\xi$ is chosen to be the time translation symmetry orthogonal to $\eta$ at infinity. But the duality transformation (3) can change the metric asymptotically so that $\xi$ is no longer orthogonal to $\eta$. ($\eta$ is fixed to be the symmetry with compact orbits and is invariant under duality.) When $\chi$ is expressed in terms of the vector which is orthogonal to $\eta$, the velocity may be different.
The duality transformation (4) has some gauge freedom. One can add the gradient of a function to $A_\alpha$, and the curl of a vector to $B_{\alpha\beta}$. Adding $\nabla_\alpha f$ to $A_\alpha$ corresponds to the coordinate transformation $x = \hat x + f$. One must sometimes use this freedom to insure that the dual fields are smooth. For example, we have seen that $A_t = -\Omega$ on the horizon. Using (4) we would find that the dual solution has $\tilde B_{xt} = -\Omega$ on the horizon which contradicts (2). To avoid this, we must first define a new coordinate

$$\hat x = x - \Omega t$$

In terms of the coordinates $(\hat x, t, y^i)$ the vector $\chi$ is simply $\partial/\partial t$ and $\hat A_t \equiv A_t + \Omega = 0$ on the horizon. In the dual solution $\tilde B_{\hat x t}$ will then vanish on the horizon.

We now consider the temperature of the horizon, which is equal to the surface gravity divided by $2\pi$. Set $\lambda = \chi^\mu\chi_\mu$. Then the surface gravity $\kappa$ is defined by

$$\nabla_\alpha \lambda = -2\kappa \chi_\alpha$$

In terms of the coordinates $(\hat x, t, y^i)$ we have just introduced, the vector $\chi$ is simply $\partial/\partial t$ so (6) reduces to $\nabla_\alpha g_{tt} = -2\kappa g_{\alpha t}$. Using the form of the metric in (3) and the definition of $\hat x$ (3), we have $g_{tt} = g_{xx}\hat A_t^2 + \bar g_{tt}$ and $g_{\alpha t} = g_{xx} A_\alpha \hat A_t + \bar g_{\alpha t}$. But $\hat A_t$ vanishes at the horizon and hence the definition of the surface gravity depends only on the duality invariant part of the metric, $\bar g_{\alpha\beta}$. We conclude that the Hawking temperature is duality invariant. (This argument is valid even though $t$ is not well behaved at the horizon, since replacing $t$ by e.g. an ingoing null coordinate $v$ does not change the vector $\partial/\partial t = \partial/\partial v$.)

To discuss the entropy, it is convenient to first rescale the metric in (4) so that one has the standard Einstein action. In $D$ dimensions, this is accomplished with the conformal factor $e^{-4\phi/(D-2)}$. Thus the Einstein metric can be expressed

$$ds_E^2 = e^{-4\phi/(D-2)}[g_{xx}(dx + A_\alpha dy^\alpha)^2 + \bar g_{\alpha\beta}dy^\alpha dy^\beta]$$

1 Recall that a coordinate basis vector is defined to be tangent to the curves obtained by holding the other coordinates fixed. Thus changing $x$ to $\hat x = x - \Omega t$ changes $\partial/\partial t$ but $\partial/\partial x = \partial/\partial \hat x$. In particular, $g_{\hat x\hat x} = g_{xx}$ and dualizing with respect to $\hat x$ is equivalent to dualizing with respect to $x$.

2 Since $\lambda = 0$ on the horizon, its gradient must be normal to the surface and hence some multiple of $\chi$. In writing the index $\alpha$ we are using the fact that $\lambda$ is independent of $x$ and $\chi_\mu\eta^\mu = 0$ on the horizon.
It is worth noting that the surface gravity computed from the Einstein metric is identical to that computed from the (string) metric we have been using previously. This is because $\kappa$ is invariant under arbitrary conformal rescaling as long as $\chi^\mu$ is kept fixed \[15\]. Since we now have the standard Einstein action, the entropy should be proportional to the area\[^3\] of the horizon. The metric induced on a cross section of the horizon takes the form \( (7) \) where the indices $\alpha, \beta$ run over the $D - 3$ angular variables. The area is thus

$$A_H = \int e^{-2\phi} \sqrt{g_{xx}} \sqrt{\bar{g}}$$ \quad (8)$$

But from \( (4) \), $e^{-2\phi} \sqrt{g_{xx}}$ is duality invariant. So the area is also duality invariant. Notice that it is the full area of the horizon and not the “area at fixed $x$” which is invariant. In fact, it is crucial for this argument that the horizon cross section is $D - 2$ dimensional. If we restrict the $y^\alpha$ to an $m$ dimensional subspace, it would have area

$$A_m = \int e^{-2(m+1)\phi/(D-2)} \sqrt{g_{xx}} \sqrt{\bar{g}}$$ \quad (9)$$

which is not duality invariant. On the other hand, the only properties of the horizon that we have used are its dimension and invariance under translation of $x$: The area of every $D - 2$ dimensional, $x$ invariant surface is unchanged under duality.

Another approach to showing that the thermodynamic properties should be duality invariant is to use the analogy with Kaluza-Klein theory \[16\] and pass to a $D - 1$ dimensional description. Since we are assuming all fields are independent of $x$, the original action \[1\] can be written

$$S = \int d^{D-1}y e^{-2\phi} \sqrt{g_{xx}} \sqrt{-\bar{g}} [R(\bar{g}) + \cdots]$$ \quad (10)$$

where the dots denote terms involving the “matter fields” $A_\alpha, g_{xx}$ as well as $\phi$ and $B_{\mu\nu}$. (Their specific form can be found e.g. in \[17\] but will not be important here.) Since $e^{-2\phi} \sqrt{g_{xx}}$ is duality invariant, the rescaled $D - 1$ dimensional Einstein metric is also duality invariant. In other words, in this formulation the duality transformation acts only on the matter fields and all geometric properties of the solution are duality invariant. The difficulty is that this approach obscures the connection with $D$ dimensional quantities. Since we have not assumed that the symmetry direction has a small “radius”, these are the quantities of most physical interest. For example, the area of any $m$ dimensional surface computed with the reduced metric is duality invariant, but is not simply related to

\[^3\] By “area” we mean the $D - 2$ dimensional volume.
the area of the corresponding \( m + 1 \) dimensional surface computed with the \( D \) dimensional metric. In addition, certain properties such as the fact that the velocity of the horizon can change under duality, are harder to see in the reduced formulation.

As one example, we consider a black string solution in dimension \( D > 4 \). The simplest black string is the product of the \( D - 1 \) dimensional Schwarzschild solution and \( S^1 \). The boosted form of this solution, with velocity \( v = \tanh \alpha \), is

\[
ds^2 = -\left(1 - \frac{r^n_0 \cosh^2 \alpha}{r^n} \right) dt^2 + \left(1 + \frac{r^n_0 \sinh^2 \alpha}{r^n} \right) dx^2 - \frac{2r^n_0 \sinh \alpha \cosh \alpha}{r^n} dx dt \tag{11}
\]

\[
+ \left(1 - \frac{r^n_0}{r^n}\right)^{-1} dr^2 + r^2 d\Omega_{n+1}^2
\]

\[B_{\mu\nu} = 0, \quad \phi = 0\]

where \( r_o \) is a constant related to the original Schwarzschild mass, \( n = D - 4 \) and \( x \) is identified with \( x + 1 \). This solution can be put into the form (3) with

\[
g_{xx} = \left(1 + \frac{r^n_0 \sinh^2 \alpha}{r^n} \right) \quad A_t = -\frac{r^n_0 \sinh \alpha \cosh \alpha}{r^n + r^n_0 \sinh^2 \alpha} \tag{12}
\]

\[
\bar{g}_{\alpha\beta} dy^\alpha dy^\beta = -\left(\frac{r^n - r^n_0}{r^n + r^n_0 \sinh^2 \alpha} \right) dt^2 + \left(1 - \frac{r^n_0}{r^n}\right)^{-1} dr^2 + r^2 d\Omega_{n+1}^2
\]

The surface \( r = r_o \) is a bifurcate Killing horizon. The velocity of the horizon is \( \Omega = -A_t(r = r_o) = \tanh \alpha \), which is just the velocity of the boost, as expected. Introducing the new coordinate \( \hat{x} = x - \Omega t \) we find that \( \partial / \partial t \) is tangent to the horizon. The surface gravity (3) must be calculated in coordinates which are good on the horizon. The result is \( \kappa = n(2r_o \cosh \alpha)^{-1} \). The area of the horizon is \( c_{n+1} r_o^{n+1} \cosh \alpha \) where \( c_{n+1} \) is the area of a unit \( n + 1 \) sphere.

Dualizing this solution gives the charged black string in \( D \) dimensions \[6\]

\[
ds^2 = -\frac{r^n - r^n_0}{r^n + r^n_0 \sinh^2 \alpha} dt^2 + \left(1 + \frac{r^n_0 \sinh^2 \alpha}{r^n} \right)^{-1} d\hat{x}^2 + \left(1 - \frac{r^n_0}{r^n}\right)^{-1} dr^2 + r^2 d\Omega_{n+1}^2 \tag{13}
\]

\[
\hat{B}_{\hat{x}t} = \tanh \alpha \left(\frac{r^n - r^n_0}{r^n + r^n_0 \sinh^2 \alpha} \right), \quad \hat{\phi} = -\frac{1}{2} \ln \left(1 + \frac{r^n_0 \sinh^2 \alpha}{r^n} \right)
\]

The surface \( r = r_o \) is still a bifurcate Killing horizon, but the velocity \( \Omega \) is now zero. The velocity has changed since, in terms of the new coordinate \( \hat{x} \), \( \hat{A}_t \) does not vanish at infinity in the solution (12). Since \( B_{\mu\nu} = 0 \), the duality transformation changes the metric.
at infinity. The surface gravity is the same as the boosted string \( \kappa = n(2r_o \cosh \alpha)^{-1} \). To compare the entropy, we must first obtain the Einstein metric by multiplying (13) by 
\[ e^{-4\hat{\phi}/D-2} = (1 + \frac{r_o^n \sinh^2 \alpha}{r_o^n})^{2/(n+2)} \]
The area of the horizon is then \( c_{n+1} r_o^{n+1} \cosh \alpha \) which agrees with the dual solution.

As another example, we consider the three dimensional black hole solution. The metric is
\[ ds^2 = -\left( \frac{r^2}{l^2} - M \right) dt^2 + \left( \frac{r^2}{l^2} - M + \frac{J^2}{4r^2} \right)^{-1} dr^2 + r^2 d\theta^2 - J dt d\theta \]  
(14)
where \( M \) and \( J \) are the mass and angular momentum of the black hole. This solution can be obtained by identifying points of three dimensional anti-de Sitter space. The curvature is thus constant and depends only on \( l, R_{\mu\nu} = -\frac{2}{l^2} g_{\mu\nu} \). The solution has two bifurcate Killing horizons at \( r = r_\pm \) where \( r_\pm^2 = \frac{M^2}{2} \left( 1 \pm \sqrt{1 - \frac{J^2}{M^2 l^2}} \right) \).

Unlike the case of asymptotically flat solutions, there is no preferred normalization of the timelike Killing vector. This is important since the surface gravity (13) depends on this normalization. If one sets \( \xi = \partial/\partial t \), then the velocity of the horizon is \( \Omega = J/2r_+^2 \) and the vector tangent to the horizon is \( \chi = \partial/\partial t + \Omega \partial/\partial \theta \). The surface gravity of the black hole is then
\[ \kappa = \frac{r_+^2 - r_-^2}{l^2 r_+^2} \]  
(15)
This has the interesting property that it decreases as the mass decreases. It is also reduced by the presence of angular momentum. It is important to note that \( T = \kappa/2\pi \) is not the temperature seen by observers at infinity. This is because the black hole (14) is asymptotically anti-de Sitter. For this metric, there is an infinite redshift between any finite radius and infinity. So the temperature at infinity always vanishes. Of course, just because no energy reaches infinity, does not mean that the black hole cannot evaporate. The mass of the black hole can still be converted into thermal radiation.

In [9] [10] it was shown that the black hole metric (14) along with the fields \( B_{\theta t} = r^2/l, \phi = 0 \), is a solution to string theory. To insure that \( B_{\mu\nu} \) is regular on the horizon, we must perform a gauge transformation so that \( B_{\theta t} = (r^2 - r_+^2)/l \). To insure that the dual antisymmetric tensor \( \tilde{B}_{\mu\nu} \) is also regular we introduce the shifted coordinate \( \hat{\theta} = \theta - \Omega t \). The result of dualizing on \( \hat{\theta} \) is then
\[ ds^2 = -\left( \frac{r^2 - r_+^2}{r^2 l^2} \right) dt^2 + \frac{2}{r^2 l} (r^2 - r_+^2) dt d\hat{\theta} + \frac{d\hat{\theta}^2}{r^2} + \frac{r^2 l^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 \]  
(16)
\[ \tilde{B}_{\hat{t}\hat{t}} = \frac{J}{2r_+^2} - \frac{J}{2r_-^2}, \quad \tilde{\phi} = -\ln r \]

where we have used the fact that \( M = (r_+^2 + r_-^2)/l^2 \) and \( J = 2r_+r_-/l \). The dual solution can be put in the form of a black string \[18\]

\[ d\tilde{s}^2 = -\left(1 - \frac{M}{\hat{r}}\right) d\hat{t}^2 + \left(1 - \frac{Q^2}{M\hat{r}}\right) d\hat{x}^2 + \left(1 - \frac{Q^2}{M\hat{r}}\right)^{-1} \left(1 - \frac{M}{\hat{r}}\right)^{-1} \frac{l^2 d\hat{r}^2}{4\hat{r}^2} \] (17)

by making the coordinate transformation \( t = l(r_+^2 - r_-^2)^{-1/2}(\hat{t} + \hat{x}), \hat{\theta} = (r_+^2 - r_-^2)^{1/2}\hat{x}, r^2 = \hat{r} \), and by the identification of parameters \( M = r_+^2/l, Q = -J/2 \). This is simpler than the transformation needed if the shift in \( B_{\theta t} \) is not made \[9\].

The black string (17) has a bifurcate Killing horizon at \( \hat{r} = \mathcal{M} \) which corresponds to \( r = r_+ \), the same location as the black hole horizon (14). The surface gravity of the black string is \( \kappa = \frac{1}{4}(1 - \frac{Q^2}{\mathcal{M}^2})^{1/2} \) which corresponds to

\[ \kappa = \frac{(r_+^2 - r_-^2)^{1/2}}{lr_+} \] (18)

This does not agree with (15). In particular, when \( Q = 0, r_- = 0 \) and \( \kappa \) is independent of the mass. The reason for the difference can be traced back to the ambiguity in the normalization of the timelike Killing field for the three dimensional black hole. The black string is asymptotically flat and so its surface gravity is unambiguous. However, in order to obtain the standard form of the metric (17) from the dual of the black hole (16), we had to rescale the time coordinate. Since the Killing field has been taken to be \( \partial/\partial t \), this results in a change in the surface gravity. In other words, the unambiguous timelike Killing field in the black string solution (17) does not correspond, under duality, to \( \partial/\partial t \) in (14), but rather some multiple of this.

The area of the horizon is defined unambiguously in both the black hole solution and its dual. We now show that these two areas are equal, as required by our general argument. The area (length) of the horizon of the three dimensional black hole (14) is clearly \( 2\pi r_+ \). To compute the area of the horizon for the black string (17), we first multiply by \( e^{-4\hat{\phi}} = \hat{r}^2 l^2 \) to obtain the Einstein metric. The area is then \( A = \int \hat{r} l(1 - \frac{Q^2}{\mathcal{M}^2})^{1/2} d\hat{x} \). But the horizon is at \( \hat{r} = \mathcal{M} \) and from the relation between \( \hat{x} \) and \( \hat{\theta} \) we have \( \int d\hat{x} = 2\pi/(r_+^2 - r_-^2)^{1/2} \). Therefore

\[ A = 2\pi l \left( \frac{\mathcal{M}^2 - Q^2}{r_+^2 - r_-^2} \right)^{1/2} = 2\pi r_+ \] (19)
where we have used the above expressions for $\mathcal{M}$ and $Q$ in terms of $r_{\pm}$. Thus even though the asymptotic structure changes under duality, the horizon area is invariant.

To summarize, we have shown that certain thermodynamic properties of solutions to low energy string theory with horizons are invariant under duality. This is strong evidence that the concepts of temperature and entropy remain physically meaningful in string theory. It is rather surprising that the duality invariant entropy is simply proportional to the area. For solutions to the low energy theory (1), the entropy is indeed given by the area. But dual solutions are certainly not physically equivalent as solutions to this theory. It is only when viewed in the full context of string theory that one can establish this equivalence. But the full string equations involves higher curvature terms, and it has recently been shown that for theories of this type, the entropy is not simply proportional to the area [19]. Thus, one might have expected that the duality invariant entropy would be a complicated expression involving integrals of curvature terms over the horizon. We have seen that this is not the case: The simplest expression is already invariant under the low energy duality transformation. This is the first step toward finding the exact expression for entropy in string theory.

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