Annihilation contribution and $B \to a_0 \pi, f_0 K$ decays.

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We analyze the decays $B^0 \to a_0^\pm \pi^\mp$ and $B^- \to f_0 K^{*-0}$ and show that within the factorization approximation a phenomenological consistent picture can be obtained. We show that in this approach the $O_6$ operator provides the dominant contributions to the suppressed channel $B^0 \to a_0^- \pi^+$. When the $a_0$ is considered a two quark state, evaluation of the annihilation form factor using Perturbative QCD implies that this contribution is not negligible, and furthermore it can interfere constructively or destructively with other penguin contributions. As a consequence of this ambiguity, the positive identification of $B^0 \to \pi^+ a_0^- \pi^-$ can not distinguish between the two or four quark assignment of the $a_0$.[6]. According to our calculation, a best candidate to distinguish the nature of $a_0$ scalar is $Br(B^- \to \pi^0 a_0^0)$ since the predictions for a four quark model is one order of magnitude smaller than for the two quark assignment. When the scalars are seen as two quarks states, simple theoretical assumptions based on $SU(2)$ isospin symmetry provide relations between different $B$ decays involving one scalar and one pseudoscalar meson.

INTRODUCTION

$B$ factories provides large samples of $B - \bar{B}$ mesons allowing the study of physical phenomena such as CP violation, the determination of the CKM mixing angles and the search for new physics[1, 2]. Clearly, hadronic physics will benefit of the high statistics achieved, and the study of processes with small branching ratios will be possible. The full understanding of the $B$ physics is still lacking as well as a systematic first principles description of the phenomena involved. Instead different theoretical approaches are compared to data and assumptions such as factorization, or estimation of the relative size of different contributions (tree level, annihilation, penguins, final state interactions) can be tested. This can be achieved in processes where the dominant contributions are suppressed by symmetry or accidental cancellations.

The BABAR and Belle collaborations already reported precise measurements of non-leptonic $B$ meson decays involving scalar mesons with branching ratio of order as low as $10^{-6}$. Thus for example, for the $B^0 \to f_0 K^0$ channel, besides the branching ratio the CP violating asymmetries are reported and, from the two pion spectrum, the authors are able to obtain the mass and width of the $f_0(980)$. This is not the case for the $B \to a_0(980)\pi$ where branching fraction for given final states are reported -in particular $a_0^- \pi^+$- however in this case it is not possible to separate the $B^0$ from the $\bar{B}^0$ decays, unless a dominant decay mechanism is assumed[4]. In this context it is worth remarking that the $B \to a_0(980)\pi$ decay was suggested as a place where $\alpha_1$, the weak mixing angle, could be measured through the CP violating asymmetries[4]. However, it was shown that the $B \to a_0^+ \pi^-$ is suppressed by $G$ parity and also by isospin, which implies that in the symmetry limit no CP violating asymmetry is expected to be experimentally accessible[4]. Thus, theoretical arguments support the idea that the $B^0 \to a_0^0 \pi^-$ is strongly suppressed, so that

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the reported branching ratio can be identified with the dominant $B^0 \to a_0^-\pi^+$ decay. As a by product, in [6] the author concludes that the positive identification of $B^0/\bar{B}^0 \to a_0^\pm\pi^\mp$ is an evidence against the four quark assignment of $a_0$ or else, for the breakdown of perturbative QCD. Both, the suppression of the $B^0 \to a_0^+\pi^-$ channel as well as the potential evidence in favor or against a four quark state are rather interesting observation that deserves further analysis.

The low lying scalar sector of QCD represents a major challenge [8]. From the experimental point of view, the determination of the nature of existing states has not been achieved while from the theory side no consistent interpretation of the experimental data exists [9]. This is so even though a number of processes involving the appropriated determination of the nature of existing states has not been achieved while from the theory side no consistent interpretation of the experimental data exists [9]. This is so even though a number of processes involving the appropriated determination of the nature of existing states has not been achieved while from the theory side no consistent interpretation of the experimental data exists [9]. This is so even though a number of processes involving the appropriated determination of the nature of existing states has not been achieved while from the theory side no consistent interpretation of the experimental data exists [9]. This is so even though a number of processes involving the appropriated determination of the nature of existing states has not been achieved while from the theory side no consistent interpretation of the experimental data exists [9].

Unfortunately, although different processes are included in the analysis, data is not good enough to provide a clear picture of the scalars. In fact no consensus exist even on the fundamental intrinsic properties (mass and width) of the low lying scalars. In fact no consensus exist even on the fundamental intrinsic properties (mass and width) of the low lying scalars. In fact no consensus exist even on the fundamental intrinsic properties (mass and width) of the low lying scalars. In fact no consensus exist even on the fundamental intrinsic properties (mass and width) of the low lying scalars. In fact no consensus exist even on the fundamental intrinsic properties (mass and width) of the low lying scalars.

The appropriated theoretical description of non-leptonic $B$ decays involving scalar mesons is important not only to understand the nature of the scalars but also because these must be considered as background to other processes of interest in $B$ physics. Since scalars, vectors and tensors couple to two pseudoscalars, the following decays lead to the scalar mesons have been considered by a number of authors. Thus for example in [14] the tree level Hamiltonian and quark model calculation are used to predict branching ratios, and central production involve the $\Delta S = 1$ weak Hamiltonian including QCD corrections to next to leading order. To evaluate the matrix elements we use values reported in the literature or model dependent estimates of the decay constants and form factors. In particular, the annihilation contribution is considered and evaluated using Perturbative QCD. This is important in estimating contributions previously neglected, and it is also relevant to quantify the statement in [6] regarding the four quark nature of the $a_0(980)$.

In this note we analyze the $B^0 \to f_0(980)K$ and $B \to a_0(980)\pi$ decays using the factorization approximation. To this end we use the $\Delta B = 1$ weak Hamiltonian including QCD corrections to next to leading order. To evaluate the matrix elements we use values reported in the literature or model dependent estimates of the decay constants and form factors. In particular, the annihilation contribution is considered and evaluated using Perturbative QCD. This is important in estimating contributions previously neglected, and it is also relevant to quantify the statement in [6] regarding the four quark nature of the $a_0(980)$.

Following the conventional approach [18, 19, 20, 21, 22], we start with the $\Delta B = 1$ effective Hamiltonian $H_{eff}(q = d, s)$

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left[ \lambda_{uq}(C_1O_1^u + C_2O_2^u) - \lambda_{tq} \left( \sum_{i=3}^{18} C_iO_i + C_qO_q \right) \right] + h.c. \quad (1)$$

where $\lambda_{q'q} = V_{q'q}V_{q'q}^*$, $q = d, s, q' = u, c, t, V_{ij}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The Wilson coefficients, $C_i$, including next to leading order QCD corrections, are evaluated at the renormalization scale $\mu \simeq m_B/2$. We use the conventions and $C_i$ values reported in [23]. It remains to evaluate the matrix elements between the states of interest.

$$A(B \to PS) = <PS|H_{eff}|B> \quad (2)$$

$P$ and $S$ stand for pseudoscalar and scalar meson respectively. In terms of the amplitude the branching ratio is given by:

$$Br(B \to PS) \simeq \tau_B \frac{G_F^2|A(B \to PS)|^2}{32\pi m_B} \quad (3)$$
with $\tau_B$ the appropriated B meson lifetime ($\tau_{B^+} = 1.65 \cdot 10^{-12}s, \tau_{B^0} = 1.56 \cdot 10^{-12}s$).

The matrix elements are evaluated using the assumption of factorization. In that approximation the matrix elements of interest are given by:

\begin{equation}
A_{B^0 \to \pi^- a_0^+} \simeq \lambda_{ud} (a_1 X_{B^0 a_0^+}^{\pi^-} + a_2 X_{B^0 a_0^+}^{\pi^+}) - \lambda_{td} \left[ (a_4 + a_{10} - \frac{(a_6 + a_8)m_2^2}{m(m_b + m_u)}) X_{B^0 a_0^+}^{\pi^-} \right.
\end{equation}

\begin{equation}
\left. + \left(2(a_3 - a_5) + a_4 + \frac{a_9 - a_7 - a_{10}}{2} \right) \frac{(a_6 - a_8/2)m_2^2}{m(m_b + m_d)} X_{B^0 a_0^+}^{\pi^-} \right] \end{equation}

\begin{equation}
A_{B^0 \to \pi^- a_0^-} \simeq \lambda_{ud} (a_1 X_{B^0 a_0^-}^{\pi^-} + a_2 X_{B^0 a_0^-}^{\pi^+}) - \lambda_{td} \left[ (a_4 + a_{10}) X_{B^0 a_0^-}^{\pi^-} - 2(a_6 + a_8) \tilde{X}_{B^0 a_0^-}^{\pi^-} \right]
\end{equation}

\begin{equation}
\left. + \left(2(a_3 - a_5) + a_4 + \frac{a_9 - a_7 - a_{10}}{2} \right) \frac{(a_6 - a_8/2)m_2^2}{m(m_b + m_d)} X_{B^0 a_0^-}^{\pi^-} \right] \end{equation}

\begin{equation}
A_{B^- \to \pi^- a_0^-} \simeq \lambda_{ud} a_1 (X_{B^- a_0^-}^{\pi^-} + X_{B^- a_0^-}^{\pi^+}) - \lambda_{td} \left[ (a_4 + a_{10}) \tilde{X}_{B^- a_0^-}^{\pi^-} \right]
\end{equation}

\begin{equation}
\left. - 2(a_6 + a_8) \tilde{X}_{B^- a_0^-}^{\pi^-} \right) \left( a_4 - \frac{3}{2} (a_9 - a_7) - \frac{1}{2} a_{10} + \frac{(a_6 + a_8)m_2^2}{m(m_b + m_d)} \right) X_{B^- a_0^-}^{\pi^-}
\end{equation}

\begin{equation}
\left. + \left( a_4 + a_{10} + \frac{(a_6 + a_8)m_2^2}{m(m_b + m_u)} \right) X_{B^- a_0^-}^{\pi^-} \right] \end{equation}

\begin{equation}
A_{B^- \to j^+ K^-} \simeq \lambda_{as} a_1 \left[ X_{B^- j^+}^{j^+} + X_{B^- j^+ K^-} \right] - \lambda_{ts} \left[ (a_4 + a_{10} - \frac{(a_6 + a_8)m_2^2}{m(m_b + m_u)}) X_{B^- j^+ K^-} \right]
\end{equation}

\begin{equation}
\left. + \left( a_4 + a_{10} - \frac{2(a_6 + a_8)m_2^2}{m(m_b + m_u)} \right) X_{B^- j^+ K^-} \right] \end{equation}

\begin{equation}
A_{B^0 \to j^+ K^-} \simeq - \lambda_{ts} \left[ \frac{(a_4 - a_{10} - \frac{2(a_6 - a_8)m_2^2}{m(m_b + m_d)})}{(m_b + m_d)(m_u + m_d)} \right] X_{B^0 j^+ K^-} \end{equation}

\begin{equation}
\left. + \left( a_4 - a_{10} - \frac{2(a_6 - a_8)m_2^2}{m(m_b + m_d)} \right) X_{B^0 j^+ K^-} \right] \end{equation}

where $\hat{m} = (m_u + m_d)/2$. For future reference in Table 1 we quote the numerical values of the $a_i$ coefficients. These expressions are obtained by inserting the vacuum between the currents in all possible ways, and a typical $X_{b,c}$ product of matrix elements is parameterized in terms of form factors and decay constant as:

\begin{equation}
X_{K^0 f_0}^{b,c} = \langle \bar{K}^0 f_0 | (s\bar{d})_L | 0 \rangle < \langle (\bar{d}b)_L | \bar{B}^0 > = f_B (m_{f_0}^2 - m_K^2) F_0^{\bar{b}c}(m_B^2)
\end{equation}

In the appendix we define in detail all of the $X_{b,c}$. Let us start by summarizing our knowledge about the decay constants and form factors entering the calculation. We can classify these in four categories: Pseudoscalar decay constants ($f_\pi, f_{K^0, f_B}$). The values of the two former decay constants are taken from [4] while for the later we use $f_B = 170$ MeV [23]. The second kind are the scalar decay constants ($f_{s, f_f}$). For these we use published values estimated using theoretical arguments [2]. We then have form factors of the type $F_0^{S, P}(m_B^2)$, evaluated at the $m_B$ scale, i.e. calculable with perturbative methods. In Table [11], we quote the values we use. Finally we need the form factor $F_0^{S, P}(m_B^2)$, $F_0^{P, S}(m_B^2)$ where $S$ and $P$ stand for a scalar or pseudoscalar meson. For the decay under
ANNIHILATION FORM FACTORS FROM PERTURBATIVE QCD.

At tree level the $B^0 \to \pi^+ a_0^-$ is strongly suppressed due to the absence of second class currents. To get an evaluation for such a decay an estimate of the contribution of the $B$ annihilation is necessary. The annihilation amplitude is proportional to $X_{(a_0^- \pi^+)}^{B^0}$ which itself is proportional to the $F_0^{a_0^- \pi^+} (m_B^2)$ form factor. At the scale $m_B^2$, perturbative QCD provides an adequate framework to evaluate this form factor. So, below we compute this form factor using the standard approach of perturbative QCD assuming the scalar meson $a_0^-$ to be a two quark state.

In order to fix the convention, we recall the form factor definitions:

\[
\begin{align*}
\text{Br}(B^0 \to \pi^+ a_0^-) & = (2.8^{+1.6}_{-1.2}) \times 10^{-6} \\
\text{Br}(B^0 \to \pi^- a_0^0) & = (3.6^{+2.6}_{-2.0}) \times 10^{-6} \\
\text{Br}(B^- \to K^- f^0) & = (13.5^{+3.6}_{-4.2}) \times 10^{-6} \\
\text{Br}(B^0 \to K^0 f^0) & = (8.8 \pm 2.38) \times 10^{-6}
\end{align*}
\]

**TABLE III:** Branching ratios of measured PS channel decays of $B$ mesons

\(^1\text{we used } Br(f^0 \to 2\pi) = 0.68\) in order to get the $Br(B^{*-0} \to K^{-0} f^0)$ from published results.
\[ \langle M_2(p_2)|L^u|M_1(p_1) \rangle = \left( p_1 + p_2 - \frac{m_1^2 - m_2^2}{q^2} \right) F_+^{M_2 M_1}(q^2) + \left( \frac{m_1^2 - m_2^2}{q^2} \right) q_u F_0^{M_1 M_2}(q^2), \]  

(11)

(12)

with \( q = p_1 - p_2 \). Projecting the amplitude on \( q \) one obtains:

\[ q_u \langle M_2(p_2)|L^u|M_1(p_1) \rangle = (m_1^2 - m_2^2) F_0^{M_1 M_2}(q^2) \]  

(13)

\[ q_u \langle M_2(p_2)|M_1(p_1)|L^u|0 \rangle = (m_2^2 - m_1^2) F_0^{M_2 M_1}(q^2) \]  

(14)

PQCD contributions to both amplitudes have exactly the same structure, so we compute \( q_u \langle M_2(p_2)|L^u|M_1(p_1) \rangle \) following ref. 27, then \( q_u \langle M_2(p_2)|L^u|M_1(p_1) \rangle \) is obtained just changing the sign of \( p_2 \). The form factors are expressed in terms of the distribution amplitudes:

\[ \Psi_{\pi}(x, p) = -i \frac{I_c}{\sqrt{2N_c}} \phi_{\pi}(x)(\hat{p} + m_{\pi}) \gamma_5 \]  

(15)

\[ \Psi_{a_0}(x, p) = \frac{I_c}{\sqrt{2N_c}} \phi_{a_0}(x)(\hat{p} + m_{a_0}) \gamma_5 \]  

(16)

where \( I_c \) is the identity in color space, \( \hat{p} = \gamma_\mu P^\mu \) and

\[ \int \phi_{\pi}(x)dx = \frac{1}{2\sqrt{2N_c}} f_{\pi} \]  

(17)

\[ \int \phi_{a_0}(x)dx = \frac{1}{2\sqrt{2N_c}} f_{a_0} \]  

(18)

The wave functions \( \phi_{\pi, a_0}(x) \) are given by 27.

\[ \phi_{\pi}(x) = \frac{2N_c}{2\sqrt{2N_c}} f_{\pi} x(1 - x) + \cdots \]  

(19)

\[ \phi_{a_0}(x) = \frac{2N_c}{2\sqrt{2N_c}} f_{a_0} x(1 - x) \left( 1 + B_1 C_1^{3/2}(2x - 1) \right) + \cdots \]  

(20)

where \( f_{\pi} = 130 MeV, f_{a_0} = 1 MeV, |B_1 f_{a_0}| \simeq 75 MeV, \) and \( C_1^{3/2}(2x - 1) \) is the Gegenbauer polynomial. Thus, the matrix element is expressed as:

\[ q_u \langle \pi(p_2)|L^\nu|a_0(p_1) \rangle = -C(R) \frac{T_v(I_C)}{2N_c} g_s^2 \int dx dy \phi_{a_0}(x) \phi_{\pi}(y) \left\{ \begin{array}{l} \text{Tr} \left[ \gamma_5 (\hat{p}_2 + m_{\pi}) \gamma_\nu \hat{P}_{1l} q_u L^\nu (\hat{p}_1 + m_{a_0}) \gamma_5 \right] \frac{k^2 P_{1l}^2}{k^2 P_{2l}^2} \\
\text{Tr} \left[ \gamma_5 (\hat{p}_2 + m_{\pi}) q_u L^\nu \hat{P}_{2l} \gamma_\nu (\hat{p}_1 + m_{a_0}) \gamma_5 \right] \frac{k^2 P_{2l}^2}{k^2 P_{1l}^2} \end{array} \right\} \]  

(21)

where \( C(R) = 4/3, (p_1 - p_2)^2 = q^2 = m_{\pi}^2, k = -xp_1 + (1 - y)p_2, P_{1l} = k + yp_2, P_{2l} = -k + (1 - x)p_1 \) and

\[ P_{1l}^2 = x^2 m_{a_0}^2 + m_{\pi}^2 + x(m_B^2 - m_{a_0}^2 - m_{\pi}^2), \]  

(22)

\[ P_{2l}^2 = (1 - y)m_B^2 + y m_{a_0}^2 - m_{\pi}^2 y(1 - y). \]  

(23)
FIG. 1: Allowed region for $F^\pm_a$ and $F^\mp_a$ at one $\sigma$ using experimental data on $Br(B^0 \to \pi^+ a^0_0)$ and $Br(B^- \to \pi^- a^0_0)$.

Integrating numerically, one gets

$$\left| F^\pm_0 \pi^+ \left( m_B^2 \right) \right| \approx 0.004$$

(24)

It is important to notice that the CP-conserving phase of the annihilation contributions is not fixed since we only known the absolute value of $B_1$.

GENERAL FRAMEWORK TO PREDICT $B \to PS$

One can proceed along similar lines for processes involving $a_0$ or $f_0$ scalar mesons. Instead, we use isospin, $SU(2)$ quark symmetry and the quark contents of the scalar mesons to obtain relation between the form factors. We also used available experimental data to obtain constraints on the form factor values. It turns out that the consistency of the two sets of values so obtained provide further confidence on the approach.

We assume the conventional quark content of the pseudo scalar mesons[2] and parameterize the mixing in the scalar sector in the strange-nonstrange basis as:

$$\sigma = \cos \phi_S \bar{n}n - \sin \phi_S \bar{s}s,$$

(25)

$$f_0 = \sin \phi_S \bar{n}n + \cos \phi_S \bar{s}s,$$

(26)

where $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$, and the singlet-octet mixing angle $\theta_S$ is related to $\phi_S$ by $\phi_S - \theta_S = \cos^{-1} \left[ 1/\sqrt{3} \right] \simeq 55^\circ$. A diagrammatic analysis of the contributions of the form factors, which involve the quark composition and isospin and $SU(2)$ symmetry between up and down quarks, lead the following relations:

$$\frac{\sqrt{2}}{\cos \phi_S} F^\beta_0^\pm = \frac{\sqrt{2}}{\sin \phi_S} F^\beta_0^0 \frac{F^\beta_0^0 f_0}{F_0^{a_0}} = F^\beta_0 a^+_0$$

(27)

From these relations it follows that $|F_0^{a_0 f_0}| < |F_0^{a_0 a^+_0}|/\sqrt{2}$. 

FIG. 2: Allowed region for parameters $F_{0}^{B_{0}f_{0}}$ and $F_{0}^{K_{0}f_{0}}$ at one $\sigma$

It is possible to obtain similar relations between the annihilation form factors ($F_{0}^{a_{0}\pi}$ and $F_{0}^{f_{0}K}$), however we will not use $SU(3)$ symmetry since large deviations from the symmetry limit are expected. Using the experimental results given in Table (III), the values for the scalar masses given in ref. [2] and the values given in table (II) for the other form factors appearing in the amplitudes, it is possible to determine separately for each scalars $a_{0}$ and $f_{0}$ the allowed regions for the values of the form factors $F_{0}^{a_{0}\pi}$ ($F_{0}^{f_{0}K}$) and $F_{0}^{B_{0}a_{0}}$ ($F_{0}^{BK}$) respectively.

The results are summarized in figures [1] and [2]. Assuming that Perturbative QCD leads us the right order of magnitude for $F_{0}^{a_{0}\pi}$, it follows that:

$$0.14 \leq |F_{0}^{B_{0}a_{0}}| \leq 0.21$$

(28)

We should note that this result is compatible (even if slightly smaller) with the predictions for $F_{0}^{B_{0}a_{0}}(0) = 0.55 \pm 0.22$ obtained in a model-dependent way. Using Eq. (27), it follows that

$$|F_{0}^{B_{0}f_{0}}| \leq 0.20$$

(29)

Figure [2] shows the values allowed by the experimental data when one standard deviation is considered. One observes that $|F_{0}^{B_{0}f_{0}}| \leq 0.20$ requires a large contribution from $|F_{0}^{f_{0}K}|$. In fact, the smallest value for $|F_{0}^{f_{0}K}|$ is around 0.05, which is more than one order of magnitude bigger than the PQCD evaluation of $|F_{0}^{a_{0}\pi}|$. It is interesting to note in this respect that:

$$\frac{|F_{0}^{f_{0}K}|}{|F_{0}^{a_{0}\pi}|} \approx \frac{m_{K}^{2}}{m_{\pi}^{2}} \approx 12$$

(30)

Scalar mesons as $qq\bar{q}q$ states.

Several models where the scalars are four quark states [13, 24, 30, 31] have been published but no model is favored at present time. We shall apply our method to one example, following [28] we assume that the quarks contents of the scalars is given by
FIG. 3: Branching ratio for $Br(B^0 \to \pi^+ a_0^-)$. The dot-dashed line correspond to the four quark assignment for $a_0$, and the space between both solid horizontal line is the value for $Br(B^0 \to \pi^+ a_0^-)$ expected from 2 quark models for $a_0$

\[
a_0^+ = uud\bar{s}, \quad a_0^- = dsu\bar{s}, \quad a_0^0 = \frac{1}{\sqrt{2}}(usu\bar{s} - dsd\bar{s}), \quad K_0^+ = udd\bar{s}, \quad K_0^0 = ud\bar{s}u, \quad K_0^- = dsd\bar{s}
\]

\[
f_0 = \frac{\cos \phi}{\sqrt{2}}(su\bar{s}u + sdsd\bar{s}) + \sin \phi ud\bar{s}u, \quad \sigma = -\frac{\sin \phi}{\sqrt{2}}(su\bar{s}u + sdsd\bar{s}) + \cos \phi ud\bar{s}u\]

(31)

where the mixing angle is obtained from the relation $\tan \phi = -0.19$ (for $m_\sigma = 0.47$ GeV), so $\phi = -5.4^\circ$ and $84.6^\circ$.

It is well-known that perturbative QCD predicts that the form factor will go like $1/q^2(n-1)$ where $n$ is the number of constituents of the hadron. If $n = 4$, $F_{0}^{a_0^-\pi^+}(m^2_{B})$ is strongly suppressed and annihilation can be neglected. Varying the experimental results within one $\sigma$ and using the fact that in four quark models for scalars the annihilation does not contribute to the processes ($F_{0}^{a_0^0\pi^+} = F_{0}^{Kf_0} = 0$), one concludes that:

\[
0.70 \leq F_{0}^{Bf_0} \leq 0.75
\]

(32)

Proceeding in the same way with processes $B \to a_0^0\pi^-, a_0^+\pi^+$, one gets

\[
0.15 \leq |F_{0}^{Ba_0}| \leq 0.20
\]

(33)

which are closed to the values of $|F_{0}^{Ba_0}|$ obtained assuming the scalars are two quark states.

Sub-dominant processes $Br(B^0 \to \pi^+ a_0^-)$ and annihilation

Once the allowed values for the form factors $F_{0}^{Bf_0}$ and $F_{0}^{Ba_0}$ have been constrained, we turn our attention to the subdominant processes $Br(B^0 \to \pi^+ a_0^-)$ which is strongly suppressed by G parity and isospin. In [6] the author concludes that a positive identification of this process is an evidence against the four-quark assignment of $a_0$ or else for breakdown of perturbative QCD. Using our estimates for annihilation contributions obtained using PQCD one can predict the values of $Br(B^0 \to \pi^+ a_0^-)$. The results are presented in figure (3). In the four quark models for $a_0$ where annihilation is strongly suppressed, one gets for $Br(B^0 \to \pi^+ a_0^-) \approx 10^{-7}$. In the two quark model for $a_0$, the
main source of uncertainty is the phase of the annihilation contributions which cannot be fixed using PQCD. Varying between 0 and \(\pi\) the CP-conserving phase for the annihilation, it follows that:

\[
10^{-9} \leq Br(B^0 \to \pi^0 a_0^\mp) \leq 4 \times 10^{-7}
\]

(34)

where the lower limit is obtained when annihilation interferes destructively and the upper limit when annihilation interferes constructively with the other contributions. Our conclusion is that this \(B\) decay cannot be used to distinguish between two and four quark assignment of the \(a_0\), unless one can obtain an independent determination of the annihilation phase.

Another channel suppressed by G parity is \(B^- \to \pi^0 a_0^\mp\). It is interesting that this channel is better to distinguish between the 2 or 4 quark models for \(a_0\). Indeed, in 4 quark models for \(a_0\), with the value for \(|F_0^{B_0}|\) determined in previous sections, one gets

\[
2 \times 10^{-9} \leq Br(B^- \to \pi^0 a_0^\mp) \leq 10^{-8}
\]

(35)

while in the 2 quark model:

\[
6.4 \times 10^{-8} \leq Br(B^- \to \pi^0 a_0^\mp) \leq 2.4 \times 10^{-7}
\]

(36)

CONCLUSIONS

In this paper we consider processes for which the dominant contribution is suppressed. Using the factorization approximation and available experimental data, we estimate the effect of annihilation penguins contribution to the processes \(Br(B^0 \to \pi^\pm a_0^\mp)\) and \(Br(B^{0,-} \to K^{0,-} f_0)\). We show that a consistent picture can be obtained when the scalars are described as two quark states, although one requires an important contributions from annihilation penguins to \(Br(B^{0,-} \to K^{0,-} f_0)\). Within our analysis the four quark models for \(f_0\) cannot be excluded.

Applying our estimates of the annihilation contributions to suppressed processes like \(Br(B^- \to \pi^0 a_0^-)\) and \(Br(B^0 \to \pi^+ a_0^-)\), we conclude that the positive identification of \(B^0 \to \pi^+ a_0^-\) cannot be taken as evidence for the four quark assignment of \(a_0\). This is in contrast with ref. [6], where the annihilation contribution is not quantified. Relevant for this conclusion is the ambiguity in the CP-conserving phase of the annihilation penguins contributions. Our best candidate process to distinguish the nature of \(a_0\) scalar is \(Br(B^- \to \pi^0 a_0^-)\) where the predictions for 4 quark models are typically one order of magnitude smaller than 2 quark models.

Using the mesons quark content, \(SU(2)\) quark symmetry and isospin we derive relations between the form factors \(F_0^{B_0}\) to \(F_0^{B_0a_0}\) for different charge states. One can extend the analysis to \(SU(3)\) however one expects large deviations from the symmetry limit. This restricts the applicability of our approach to the four quark states since in that kind of models scalars necessarily involve strange quarks.

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APPENDIX

Below we list the $X^0_{B_s}$ expressed in terms of the form factors. We quote only those needed to compute the branching ratios given in the paper:

$$X^{\pi^-}_{B_0^+a_0} = \langle \pi^- |(\bar{d}u)_L|0 > a_0^+ |(\bar{u}b)_L|\bar{B}_0^0 >= f_{\pi}(m_{B}^2 - m_\pi^2) F_0^{\pi a_0 a_0} (m_{\pi}^2)$$

$$X^{a_0}_{B_0^+\pi^+} = \langle a_0^- |(\bar{d}u)_L|0 > \pi^+ |(\bar{u}b)_L|\bar{B}_0^0 >= -f_0 (m_{B}^2 - m_\pi^2) F_0^{a_0 a_0 a_0} (m_{\pi}^2)$$

$$X^{a_0}_{B_\pi^-} = \langle a_0^- |(\bar{d}u)_L|0 > \pi^0 |(\bar{u}b)_L|\bar{B}_0^0 >= f_0 (m_{B}^2 - m_\pi^2) F_0^{a_0 a_0 a_0} (m_{\pi}^2)$$

$$X^{a_0}_{B_\pi^- a_0} = \langle a_0^0 |(\bar{d}u)_L|0 > a_0^+ |(\bar{u}b)_L|\bar{B}_0^0 >= \frac{f_0}{\sqrt{2}} (m_{B}^2 - m_\pi^2) F_0^{a_0 a_0 a_0} (m_{\pi}^2)$$

$$X^{\pi^-}_{B^- B} = \langle \pi^- |(\bar{d}u)_L|0 > S^0 |(\bar{u}b)_L|\bar{B}_0^0 >= f_{\pi}(m_{B}^2 - m_{s_{0}}^2) F_0^{\pi a_0 a_0} (m_{\pi}^2)$$

$$X^{B^-}_{S^0 a_0} = \langle S^0 |P^- |(\bar{d}u)_L|0 > 0 |(\bar{u}b)_L|\bar{B}_0^0 >= -f_B (m_{s_{0}}^2 - m_\pi^2) F_0^{S^0 a_0 a_0} (m_{\pi}^2)$$

$$X^{a_0 a_0 a_0} = \langle a_0^- |(\bar{d}u)_L|0 > a_0^0 |(\bar{u}b)_L|\bar{B}_0^0 >= -f_B (m_{a_0}^2 - m_\pi^2) F_0^{a_0 a_0 a_0} (m_{\pi}^2)$$

$$X^{K^0 S^0}_{B_0 S_0} = \langle K^0 |S^0 |(\bar{d}u)_L|0 > S^0 |(\bar{u}b)_L|\bar{B}_0^0 >= -f_{K^0} (m_{a_0}^2 - m_\pi^2) F_0^{K^0 S^0 a_0 a_0} (m_{\pi}^2)$$

$$X^{B^-}_{S_0 a_0} = \langle S^0 |K^- |(\bar{d}u)_L|0 > S^0 |(\bar{u}b)_L|\bar{B}_0^0 >= -f_B (m_{s_{0}}^2 - m_\pi^2) F_0^{S^0 K^0 a_0 a_0} (m_{\pi}^2)$$

$$X^{a_0}_{B_0^0 a_0} = \langle a_0^- |(\bar{d}u)_L|0 > a_0^0 |(\bar{u}b)_L|\bar{B}_0^0 >= -f_B (m_{a_0}^2 - m_\pi^2) F_0^{a_0 a_0 a_0} (m_{\pi}^2)$$

$$X^{B_0}_{a_0 a_0 a_0} = \langle a_0^- |(\bar{d}u)_L|0 > a_0^0 |(\bar{u}b)_L|\bar{B}_0^0 >= -f_B (m_{a_0}^2 - m_\pi^2) F_0^{a_0 a_0 a_0} (m_{\pi}^2)$$

$$X^{K^0}_{B_0 S_0} = \langle K^0 |S_0 |(\bar{d}u)_L|0 > S^0 |(\bar{u}b)_L|\bar{B}_0^0 >= -f_{K^0} (m_{a_0}^2 - m_\pi^2) F_0^{K^0 S^0 a_0 a_0} (m_{\pi}^2)$$

$$X^{a_0}_{B_0^0 a_0} = \langle a_0^- |(\bar{d}u)_L|0 > a_0^0 |(\bar{u}b)_L|\bar{B}_0^0 >= -f_B (m_{a_0}^2 - m_\pi^2) F_0^{a_0 a_0 a_0} (m_{\pi}^2)$$

$$\tilde{X}^{s_0}_{B^- a_0} = \langle s_0^0 |(\bar{d}d)_L|0 > \pi^- |(\bar{u}b)_L|\bar{B}_0^0 >= m_a f_{a_0} \frac{m_{s_{0}}^2 - m_\pi^2}{m_{b} - m_{d}} (m_{a}^2)$$

$$\tilde{X}^{a_0}_{B_- a_0} = \langle a_0^- |(\bar{d}d)_L|0 > \pi^- |(\bar{u}b)_L|\bar{B}_0^0 >= m_a f_{a_0} \frac{m_{a}^2 - m_\pi^2}{m_{b} - m_{d}} (m_{a}^2)$$

$$\tilde{X}^{s_0}_{B^- S_{0}} = \langle s_0^0 |(\bar{d}d)_L|0 > K^- |(\bar{u}b)_L|\bar{B}_0^0 >= m_a f_{a_0} \frac{m_{s_{0}}^2 - m_K^2}{m_{b} - m_{s}} (m_{s_{0}}^2)$$

$$\tilde{X}^{a_0}_{B^- K_{0}} = \langle a_0^- |(\bar{d}d)_L|0 > K^- |(\bar{u}b)_L|\bar{B}_0^0 >= m_a f_{a_0} \frac{m_s^2 - m_K^2}{m_{b} - m_{s}} (m_{s_{0}}^2)$$

where $S_0$ is a neutral scalar ($a_0^0$ or $f_0^0$).

FORM FACTORS DEFINITIONS AND CONVENTIONS.

In order to compute the amplitude using the factorization, we use the following parametrization of the form factors. The decay constants are defined as:
\[ \langle 0 | A_\mu | P(q) \rangle = i f_P q_\mu \]  
(38)

\[ \langle 0 | \bar{q}_1 \gamma_5 q_2 | P(q) \rangle \simeq \frac{-i f_P m_2^2}{m_1 + m_2} \equiv f_P m_P \]  
(39)

\[ \langle a_0^- | \bar{d} \gamma_\mu u | 0 \rangle = f_{a0} p_\mu \]  
(40)

\[ \langle a_0^- | \bar{d} u | 0 \rangle = m_{a0} \bar{f}_{a0} \]  
(41)

Using the equations of motion \(-i \partial^\mu (\bar{q}_1 \gamma_\mu \gamma_5 q_2) = (m_1 + m_2) \bar{q}_1 \gamma_5 q_2 + i \partial^\mu (\bar{q}_1 \gamma_\mu q_2) = (m_1 - m_2) \bar{q}_1 q_2 \) \cite{18, 20} on can show that \( f_S = m_S f_S / (m_1 - m_2) \) and that \( f_{S0} = 0 \) for a neutral scalar.

Form factors are defined as:

\[ < M_2(p_2) | L_\mu | M_1(p_1) > = \left( p_1 + p_2 - \frac{m_1^2 - m_2^2}{q^2} q \right) F_{M_1 M_2}^{M_1 M_2} + \frac{m_1^2 - m_2^2}{q^2} q_\mu F_{0}^{M_1 M_2}(q^2) \]  
(42)

\[ < M_2(p_2) M_1(p_1) | L_\mu | 0 > = \left( p_2 - p_1 - \frac{m_2^2 - m_1^2}{q^2} q \right) F_{+}^{M_2 M_1}(q^2) + \frac{m_2^2 - m_1^2}{q^2} q_\mu F_{0}^{M_2 M_1}(q^2) \]  
(43)

where \( L_\mu = \gamma^\mu \frac{1 - \gamma_5}{2} = \gamma^\mu P_L \). A factor of \(-i\) has to be added to the form factors in the case one of the mesons is scalar.