Massive bosons and the dS/CFT correspondence

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Abstract

We compute the boundary two point functions of operators corresponding to massive spin 1 and spin 2 de Sitter fields, by an extension of the “S-Matrix” approach developed for bulk scalars. In each case the two point functions are of the form required for conformal invariance of the dual boundary field theory. We emphasise that in the context of dS/CFT one should consider unitary representations of the Euclidean conformal group, without reference to analytic continuation of the boundary theory to Lorentzian signature.

1 Introduction

The holographic principle is an attractive and fruitful way of thinking about quantum gravity, which has proven spectacularly successful in its application to the physics of AdS spacetimes. Fuelled by this success it is natural to attempt to find a similar holographic description of spacetimes with positive cosmological constant. However in spite of considerable recent activity [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], the proposed dS/CFT correspondence remains very much the poor relation of its AdS counterpart. Numerous conceptual issues remain to be addressed, chief among which is the delicate problem of observables in a spacetime with cosmological horizons. The absence of a controlled de Sitter solution of string theory means that we currently have no concrete input on the CFT side of the correspondence. The precise nature of the correspondence with the as yet unknown CFT has also remained vague; in particular, there are difficulties in attempting to define an AdS-like prescription [9], [11]. Furthermore though much work has been done on bulk scalars, a CFT description of higher spin bulk fields has remained unexplored.

Here we will address the last issue. Tests of the proposed correspondence can only be performed with our current level of knowledge from the bulk side, and this provides the motivation for seeking a dual description of higher spin bulk fields. In Section 2 we will briefly review the calculation of CFT two point functions corresponding to bulk scalars. In sections 3 and 4 we extend the “S-Matrix” approach to massive spin 1 bulk fields in any dimension and spin 2 bulk fields in \( d = 4 \), and find that in each case the two point functions obtained are of the form required for conformal invariance. Section 5 contains a discussion of our results, the issue of unitarity of the boundary CFT, and interesting future directions.

In what follows we will use two coordinatisations of \( dS_d \). The first which we will refer to as global coordinates is given by

\[
ds^2 = -d\tau^2 + \cosh^2 \tau d\Omega_{d-1}^2\]

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with $dΩ^2_{d-1} = γ_{ij}dω^i dω^j$. $γ_{ij}$ the round metric on $S^d$. We have set the de Sitter length $l = 1$. These coordinates cover all of $dS_d$. The second set which we will refer to as planar coordinates is given by

$$ds^2 = -dt^2 + e^{-2t}d\mathbf{x}^2$$

These cover only half the spacetime, namely the causal past of a geodesic observer. We also define the function

$$z(x, x') = \cos^2 \frac{\mu(x, x')}{2}$$

where $\mu(x, x')$ is the geodesic distance between two points.

## 2 Bulk Scalars

First we consider bulk scalar fields. We will for simplicity take $d = 3$. This has been extensively studied in [3], [8], [9], and we will review their results. The equation of motion is

$$(\nabla^2 - m^2)φ = 0$$

The limiting behaviour of $φ$ in global coordinates as $τ → -\infty$ is

$$\lim_{τ → -\infty} φ(τ, Ω) = φ^\text{out}_+ (Ω)e^{h_+τ} + φ^\text{in}_- (Ω)e^{h_-τ}$$

where $h_± = 1 ± ν, ν = \sqrt{1 - m^2}$. The Wightman function $G(x, x')$ is vacuum dependent and is a linear combination of the hypergeometric functions

$$F(h_+, h_-; \frac{3}{2}; z), F(h_+, h_-; \frac{3}{2}; 1 - z)$$

We consider the amplitude

$$(φ, φ)_{--} = \lim_{τ → -\infty} \int \int d^2Ω d^2Ω' \left[ √γ\sqrt{γ'} \left( φ(x) \overleftarrow{∂}_τ G(x, x') \overrightarrow{∂}_τ φ(x') \right) \right]_{τ = τ'}$$

and find on choosing a particular vacuum

$$(φ, φ)_{--} = \int \int d^2Ω d^2Ω' \left( b_+ φ^\text{in}_+(Ω)Δ_-(Ω, Ω') φ^\text{in}_-(Ω') + b_- φ^\text{in}_-(Ω)Δ_+(Ω, Ω') φ^\text{in}_+(Ω') \right)$$

with $b_±$ vacuum dependent constants (which vanish in the “in” vacuum). The kernels $Δ_±(Ω, Ω')$ are the correlation functions of CFT operators of conformal dimension $h_±$ on an $S^2$. The constants may be absorbed by a field redefinition for two point functions, though three and higher point functions depend nontrivially on the choice of vacuum [9]. Similarly one may evaluate $(φ, φ)_{-+}$, $(φ, φ)_{+-}$ and $(φ, φ)_{++}$. In particular with one point on $I^-$ and one point on $I^+$, we define $φ^\text{out}$ with an antipodal inversion relative to $φ^\text{in}$:

$$\lim_{τ → -\infty} φ(τ, Ω_A) = φ^\text{out}_+(Ω)e^{-h_+τ} + φ^\text{out}_-(Ω)e^{-h_-τ}$$

and then

$$(φ, φ)_{+-} = \int \int d^2Ω d^2Ω' \left( b_- φ^\text{out}_-(Ω)Δ_-(Ω, Ω') φ^\text{in}_+(Ω') + (- ↔ +) \right)$$

When evaluating CFT correlators with one point on each boundary, there is a wider choice of bulk Green functions that may be used in (7), as the bulk Schwinger function is then nonzero. In [8] the asymptotic behaviour of the bulk Wightman function with one point on each boundary is considered, while in the S-Matrix interpretation of [9] it is the Feynman propagator that is used. The dependence of CFT correlators on the bulk vacuum is encoded in the asymptotic behaviour of the bulk Hadamard function, as the Schwinger function is vacuum independent.
3 Massive Spin 1 Bulk Fields

We will apply a similar analysis to massive spin 1 bulk fields. The two point functions of the boundary operators may be obtained up to a constant factor as the kernels of the bilinear forms obtained from the amplitude (in planar coordinates)

$$(A, A) = \lim_{t \to -\infty} \int \int d^{d-1}x d^{d-1}x' \left[ \sqrt{-g} \sqrt{-g'} g^{\mu \nu} g^{\mu' \nu'} \left( A_\mu(t, x) \vec{\nabla}_t G_{\nu \nu'}(t, x, t', x') \vec{\nabla}_{t'} A_{\mu'}(t', x') \right) \right]_{t=t'}$$

(11)

where $G_{\mu \nu'}(t, x, t', x')$ is the Wightman function of $A_\mu(t, x)$

$$G_{\mu \nu'}(t, x, t', x') = \langle 0 | A_\mu(t, x) A_{\nu'}(t', x') | 0 \rangle$$

(12)

with $|0\rangle$ a de Sitter invariant vacuum state. The equation of motion for $A_\nu$ is

$$(\nabla^2 - (d - 1) - m^2) A_\nu = 0$$

(13)

subject to the constraint

$$\nabla_\mu A^\mu = 0$$

(14)

We solve these equations asymptotically. The constraint equation yields

$$\partial_t A_t - (d - 1) A_t = e^{2t} D_i A^i$$

(15)

(13) yields

$$\partial_t^2 A_t - e^{2t} D^2 A_t - 2e^{2t} D_i A^i - (d - 1) \partial_t A_t + m^2 A_t = 0$$

(16)

$$\partial_i^2 A_i - e^{2t} D^2 A_i - (d - 3) \partial_t A_i - 2D_i A_t + m^2 A_i = 0$$

(17)

Combining (15) and (16) we see that as $t \to -\infty$

$$A_t(t, x) \to e^{(r_- + 2)t} A_{-t}(x) + e^{(r_+ + 2)t} A_{+t}(x)$$

(18)

where

$$r_\pm = \frac{d - 3 \pm \rho}{2}$$

$$\rho = \sqrt{(d - 3)^2 - 4m^2}$$

(19)

Then from (17)

$$A_i(t, x) \to e^{r_- t} A_{-i}(x) + e^{r_+ t} A_{+i}(x)$$

(20)

with all corrections suppressed by at least two powers of $e^t$. The Wightman function $G_{\mu \nu'}(x, x')$ satisfies

$$(\nabla^2 - (d - 1) - m^2) G_{\mu \nu'}(x, x') = 0$$

(21)

subject to

$$\nabla_\mu G^\mu_{\nu'}(x, x') = 0$$

(22)

When a maximally symmetric vacuum state is chosen $G_{\mu \nu'}$ is a maximally symmetric bivector. This means that it may be expressed as the sum of products of a set of preferred geometric objects [12]: the geodesic...
distance $\mu(x, x')$ (equivalently $z(x, x')$); the unit tangents to the geodesic at $x$ ($\nabla_\alpha \mu(x, x') \equiv n_\alpha(x, x')$) and at $x'$ ($\nabla_\alpha' \mu(x, x') \equiv n_{\nu'}(x, x')$); and the parallel propagator along the geodesic $g_{\nu'}(x, x')$. $g_{\mu \nu'}$ satisfies

$$g_{\mu'}(x, x')g_{\nu' \sigma}(x', x) = g_{\mu \sigma}(x)$$

$$g_{\mu'}(x', x)g_{\nu' \sigma}(x, x') = g_{\mu' \nu'}(x')$$

with $g_{\mu \nu'}$, $g_{\mu' \nu'}$ the metric at $x$, $x'$. $g_{\sigma \nu'}$ may be expressed in terms of $z$, $n_\sigma$ and $n_{\nu'}$ as

$$g_{\sigma \nu'} = -2\sqrt{z(1-z)}\nabla_\sigma n_{\nu'} - n_\sigma n_{\nu'}$$

We will now choose the Euclidean vacuum state. This is the unique de Sitter invariant vacuum state where the Wightman function is singular for coincident points but nonsingular for antipodal points on the de Sitter hyperboloid. It would be interesting to examine the rôle of other vacua, but such issues are not explored here. Then the Wightman function is given by [12]

$$G_{\sigma \nu'}(x, x') = f(z)g_{\sigma \nu'} + g(z)n_\sigma n_{\nu'}$$

$$f(z) = \left(-\frac{2}{d-1}z(1-z)\frac{d}{dz} + 2z - 1\right)u(z)$$

$$g(z) = f(z) - u(z)$$

$$u(z) = qF(r_+ + 2, r_- + 2; \frac{d + 2}{2}; z)$$

$$q = \frac{(1-d)\Gamma(r_+ + 2)\Gamma(r_- + 2)}{2^{d+1}\pi^{d/2}\Gamma(d+2)m^2}$$

The constant $q$ is chosen such that the short distance singularity matches that of flat space. Expressing in terms of $P = 2z - 1$ and retaining only the terms which contribute to (11) as $t \to -\infty$ we find

$$G_{\sigma \nu'}(x, x') = \left[\left(\frac{P^2}{d-1} \frac{d}{dP} + P\right)u(P)\right]\left[-\frac{\nabla_\sigma P \nabla_\nu P}{P} + \nabla_\sigma \nabla_\nu P\right]$$

As $t \to -\infty$, $P(x, x') \to -\frac{e^{-(t+t')}}{2}|x - x'|^2$ and hence

$$-\frac{\nabla_i P \nabla_j P}{P} + \nabla_i \nabla_j P \to e^{-(t+t')} \left(\delta_{ij} - \frac{2(x - x')_i(x - x')_j}{|x - x'|^2}\right)$$

$$\equiv e^{-(t+t')} I_{ij'}(x - x')$$

Furthermore the coefficient of the leading $e^{-(t+t')}$ term in $-P^{-1}\nabla_i P \nabla_\nu P + \nabla_i \nabla_\nu P$ is zero, and hence $G_{\nu'}(x, x')$ is suppressed by at least two powers of $e^t$, $e^{t'}$ relative to $G_{ij'}(x, x')$. The asymptotic behaviour of $G_{ij'}(x, x')$ is determined by the transformation properties of the hypergeometric function (see appendix). We find that

$$G_{ij'}(x, x') \to \left(\frac{a_- e^{r_-(t+t')}}{|x - x'|^{2(r_+ + 1)}} + \frac{a_+ e^{r_+(t+t')}}{|x - x'|^{2(r_+ + 1)}}\right)I_{ij'}$$

$a_\pm$ constants. Then inserting (18), (20) and (30) into (11) and ignoring contact terms, we find

$$(A, A) = \int \int d^{d-1}x d^{d-1}x' \left(\frac{\dot{\hat{A}}_+ A_-^* (x) A_-^* (x')}{|x - x'|^{2(r_+ + 1)}} + \frac{\dot{\hat{A}}_- A_+^* (x) A_+^* (x')}{|x - x'|^{2(r_- + 1)}}\right)I_{ij'}$$

where $A_\pm^* = \delta^{ij} A_{\pm ij}$ and

$$\dot{\hat{a}}_\pm = -\frac{q\rho^2(r_\pm - 1)\Gamma(\mp \rho)\Gamma((d + 2)/2)4^{r_\pm + 2}}{6\Gamma(r_\pm + 2)\Gamma((1 \mp \rho)/2)}$$

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Hence we see that $A_{i+}^\mu$ correspond to a pair of CFT operators $V_{+}^i$ whose two point functions are up to a constant the kernels of (31):

$$< V_{+}^i(x)V_{+}^j(x') > = \frac{I^{ij}(x - x')}{|x - x'|^{2(r_{+}+1)}}$$

which is of the form required for conformal invariance\(^1\). The conformal dimensions are

$$\frac{1}{2}(d - 1 \pm \sqrt{(d - 3)^2 - 4m^2})$$

In global coordinates it may readily be verified that the asymptotic behaviour of $A_i$ is given by

$$\lim_{\tau \to -\infty} A_i(\tau, \Omega) = e^{\tau} A_{i+}^{\in}(\Omega) + e^{\tau} A_{i-}^{\in}(\Omega)$$

with $A_i(\tau, \Omega)$ suppressed by two powers of $e^{-\tau}$ as before. As $\tau, \tau' \to -\infty$ we find that $P(x, x') \to -\frac{e^{-(\tau+\tau')}}{2}(1 - \cos \theta)^2$, where $\theta(\Omega, \Omega')$ is the geodesic distance on the $S^{d-1}$. Also we see that

$$- \frac{\nabla_i P \nabla_j P}{P} + \nabla_i \nabla_j P \to \frac{e^{-(\tau+\tau')}}{4} \left( \nabla_i \theta \nabla_j \theta - \sin \theta \nabla_i \nabla_j \theta \right)$$

$$\equiv \frac{e^{-(\tau+\tau')}}{4} I_{ij}$$

$I_{ij}$ generalises the flat space inversion tensor $I_{ij}$ to the unit sphere [13]. We define $\sigma = \frac{1 - \cos \theta}{2}$, the generalisation of the flat space $|x - x'|^2$. Then evaluating the analogue of (11) in global coordinates as $\tau, \tau' \to -\infty$ we find

$$(A, A)_{--} = \frac{1}{2d+1} \int \int d^{d-1}\Omega d^{d-1}\Omega' \left( \frac{\hat{a}_- A_{i+}^{\in}(\Omega) A_{i+}^{\in}(\Omega')} {\sigma^{(r_{+}+1)}} + \frac{\hat{a}_+ A_{i-}^{\in}(\Omega) A_{i-}^{\in}(\Omega')} {\sigma^{(r_{-}+1)}} \right) I^{ij}$$

where $I_{jk} = \gamma^{ij} I_{ij}$. The kernels are proportional to the two point functions of the CFT operators and are just (33) conformally rescaled to the sphere. Similarly one may evaluate $(A, A)_{++}$, $(A, A)_{-+}$ and $(A, A)_{+-}$, in particular finding the familiar antipodal inversion when one point is on $I^+$ and one on $I^-$.\(^2\)

4 Massive Spin 2 Fields

We follow a similar procedure for massive spin 2 fields, which were discussed in [14]. In four dimensions the equation of motion is

$$(\nabla^2 - (m^2 + 2)) \chi_{\mu\nu} = 0$$

subject to the constraints

$$\nabla_\mu \chi^\mu_\nu = 0, \ g^{\mu\nu} \chi_{\mu\nu} = 0$$

and $\chi_{\mu\nu}$ symmetric. Bulk unitarity requires $m^2 \geq 2$, with $m^2 = 2$ giving partial masslessness (in general dimension partial masslessness occurs at $m^2 = d - 2$ [15]). We assume $m^2 > 2$. In global coordinates, (38)

\(^1\)We note that this derivation is valid in any dimension if the mass of the bulk field is such that $\rho$ is purely imaginary or $\rho$ is real and $\rho < 2$. If $\rho > 2$ (as may occur if $d \geq 6$) then corrections to the leading asymptotic behaviour of the field could in principle contribute to (11), and a more detailed analysis is required. These corrections are nevertheless not expected to change the result. The cases when $\rho$ is an integer require a case by case treatment and are explicitly excluded from the discussion.
Defining the functions $G_{\mu\nu\alpha'\sigma'}(x,x')$ in terms of a traceless basis of bitensors

$$G(x,x') = X(\mu)T^1 + Y(\mu)T^2 + Z(\mu)T^3$$

where

$$T^1_{\mu\nu\alpha'\sigma'} = \frac{1}{16}g_{\mu\nu}g_{\alpha'\sigma'} + n_\mu n_\nu n_\alpha n_\sigma - \frac{1}{4}(g_{\mu\nu}n_\alpha n_\sigma + n_\mu n_\nu g_{\alpha'\sigma'})$$

$$T^2_{\mu\nu\alpha'\sigma'} = (g_{\mu\alpha'}g_{\nu\sigma'} + g_{\mu\sigma'}g_{\nu\alpha'}) - \frac{1}{2}g_{\mu\nu}g_{\alpha'\sigma'}$$

$$T^3_{\mu\nu\alpha'\sigma'} = 4n_{(\mu}g_{\nu)(\alpha'}n_{\sigma')} + 4n_\mu n_\nu n_\alpha n_\sigma$$

and the equations of motion and constraints yield, in terms of the variable $z$,

$$\left(z(1-z)\frac{d^2}{dz^2} + [c - (1 + a + b)z] \frac{d}{dz} - ab\right)W = 0$$
and

\[ U = \frac{3}{16} \left( z(z - 1)\frac{dW}{dz} + 2(2z - 1)W \right) \]
\[ Y = \frac{3}{40} \left( 8z(z - 1)\frac{dU}{dz} + 16(2z - 1)U - W \right) \]

where \( a = q_- + 2, b = q_+ + 2 \) and \( c = 4 \), so in the Euclidean vacuum, \( W = F(q_- + 2, q_+ + 2; 4; z) \) (up to a constant) and \( X, Y \) and \( Z \) are completely determined. It is convenient for our purposes to change basis, and express the Wightman function as

\[ G(x, x') = \frac{1}{16}(X - 8Y)Q_1 + (X + 2Y)Q_2 + YQ_3 - \frac{1}{4}XQ_4 + UQ_5 \]  

(48)

The new basis bitensors are

\[ Q_{\mu\nu\sigma'}^{1} = g_{\mu\nu}g_{\alpha'\sigma'} \]
\[ Q_{\mu\nu\sigma'}^{2} = n_{\mu}n_{\nu}n_{\alpha'\sigma'} \]
\[ Q_{\mu\nu\sigma'}^{3} = 4z(1 - z)(\nabla_{\mu}n_{\alpha'}\nabla_{\nu}n_{\sigma'} + \nabla_{\mu}n_{\sigma'}\nabla_{\nu}n_{\alpha'}) \]
\[ Q_{\mu\nu\sigma'}^{4} = g_{\mu\nu}n_{\alpha'\sigma'} + n_{\mu}n_{\nu}g_{\alpha'\sigma'} \]
\[ Q_{\mu\nu\sigma'}^{5} = 8\sqrt{z(1 - z)}n_{\mu}(\nabla_{\nu}n_{\alpha'}n_{\sigma'}) \]

(49)

The leading asymptotic contribution to the Wightman function comes from the \( ijk'l' \) component of the \( Q_1, Q_3 \) terms. We find

\[ \left( \frac{1}{16}(X - 8Y)Q_1 + YQ_3 \right)_{ijk'l'} \rightarrow \left( \frac{c_-e^{(q_- - 2)(\tau + \tau')}}{\sigma_-} + \frac{c_+e^{(q_+ - 2)(\tau + \tau')}}{\sigma_+} \right) \left( I_{ik'}I_{jl'} + I_{il'}I_{jk'} - \frac{2}{3} \gamma_{ij}\gamma_{kl'} \right) \]
\[ \equiv c_-e^{(q_- - 2)(\tau + \tau')}H_{ijk'l'} + (- \leftrightarrow +) \]

(50)

with all corrections and other components suppressed by at least two powers, and \( c_{\pm} \) constants. (50) is traceless at the boundary \( (\gamma^{ij}I_{ik'}I_{jl'} = \gamma_{kl'}) \) since \( \chi^{ij}_{\bar{ij}} \) are traceless there (41). The CFT two point functions are given by the kernels of

\[ (\chi, \chi)_{-} = \lim_{\tau \to -\infty} \int d^3\Omega d^3\Omega' \left[ \sqrt{g(x)g(x')}g^{\mu\nu}g^{\alpha'\sigma'} \left( \chi_{\alpha\sigma}(x) \nabla^\omega G_{\mu\nu\alpha'\sigma'}(x, x') \nabla^\tau \chi_{\mu'\nu'}(x') \right) g^{\alpha'\mu'}g^{\sigma'\nu'} \right] \]

(51)

A careful counting of powers reveals that (50) provides the only contribution:

\[ (\chi, \chi)_{-} = \int d^3\Omega d^3\Omega' \left( e_+ \chi^{+ij}(\Omega)H^{-}_{ijk'l'}(\Omega, \Omega')\chi^{+k'l'}(\Omega') + (- \leftrightarrow +) \right) \]

(52)

Thus the CFT correlators are determined up to a constant as

\[ < T_{ij}^{\pm}(\Omega)T_{k'l'}^{\pm}(\Omega') > = H_{ijk'l'}^{\pm}(\Omega, \Omega') \]

(53)

which are the two point functions required by conformal invariance for traceless symmetric tensor fields of conformal dimension \( \frac{1}{2}(3 \pm \sqrt{9 - 4m^2}) \) on an \( \mathbb{S}^3 \) (the flat space form is given in eg [17]). The calculation may be trivially modified to planar coordinates, or to involve one point on each boundary. In general we expect conformal dimensions \( \frac{1}{2}(d - 1 \pm \sqrt{(d - 1)^2 - 4m^2}) \).
5 Discussion

In conclusion we have tested dS/CFT for higher spin massive bulk bosons. The symmetric traceless tensor representations of $SO_0(d, 1)$ (the identity component of the Euclidean conformal group in $d - 1$ dimensions) are labelled by $[s, \Delta]$, where $s$ is the tensor rank and $\Delta$ the conformal dimension. It is well known [18], [19] that some of these representations may be continued to unitary representations of the universal covering of $SO_0(d - 1, 2)$. This is possible if $\Delta$ is real and

$$\Delta \geq \frac{d - 3}{2}, \quad s = 0$$

$$\Delta \geq d - 3 + s, \quad s \geq 1$$

(54)

That these conditions are violated in the context of dS/CFT by all but a small range of scalar representations has led to the suggestion that the putative CFT dual is nonunitary. However as the boundary CFT is a priori Euclidean, it is more natural to demand the unitary realisation of $SO_0(d, 1)$ symmetry with no reference made to analytic continuation to Lorentzian signature. Apart from certain exceptional, topologically reducible representations, the unitary symmetric traceless tensor representations of $SO_0(d, 1)$ (discussed in detail in [19]) are irreducible and unique up to equivalence. They are given by the following.

The principal series:

$$s = 0, 1, 2... ; \quad \Delta = \frac{d - 1 + i\sigma}{2}, \quad \sigma \text{ real}$$

(55)

The complementary series:

$$s = 0; \quad 0 < \Delta < d - 1 \quad (d - 1 \geq 2)$$

$$s \geq 1; \quad 1 < \Delta < d - 2 \quad (d - 1 \geq 3)$$

(56)  (57)

In the bulk the principal and complementary series of representations of $SO_0(d, 1)$ are distinguished by the value of the mass. Taking for example the scalar representations, in $d$ bulk dimensions we have $m^2 > (d - 1)^2/4$ for the principal series and $0 < m^2 < (d - 1)^2/4$ for the complementary series. Principal (complementary) series representations in the bulk correspond to principal (complementary) series representations on the boundary. Similar considerations hold for spin 1 and spin 2 ([20], [21] and references therein). In particular we note that the bulk unitarity bound on the mass of the spin 2 field has a holographic reflection in (57). We conclude that the group theoretic content of dS/CFT is that bulk and boundary representations of $SO_0(d, 1)$ are unitarily equivalent.

It would be interesting to extend the calculations of CFT correlators to the exceptional representations mentioned above. These should correspond to bulk photons, gravitons and partially massless spin 2 fields, among others. However there are a number of difficulties which need to be overcome: there are subtleties in a de Sitter invariant treatment of massless particles and also a problem with infrared divergences of the Green functions [22], [23]. Another problem is to obtain a better understanding of field theories based on unitary representations of the Euclidean conformal group (this has been examined in two dimensions in [11], where novel hermiticity conditions were introduced). It would also be of great interest to find a more precise definition, going beyond the current S-Matrix formalism, of what is meant by dS/CFT.

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7 Appendix

Here we relate properties of the hypergeometric function used in the text.

\[
F(a, b; c; z) = \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}(-z)^{-a}F(a, 1-c+a; 1-b+a; z^{-1})
+ \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)}(-z)^{-b}F(b, 1-c+b; 1-a+b; z^{-1})
\]

\[
\lim_{z \to \infty} F(a, b; c; z^{-1}) = 1 + O(z^{-1})
\]

\[
\frac{d^n}{dz^n}F(a, b; c; z) = \frac{(a)_n(b)_n}{(c)_n}F(a+n, b+n; c+n; z)
\]

\[
(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}
\]

References

[1] C. Hull, JHEP 9807 021, 1998
[2] E. Witten, hep-th/0106109
[3] A. Strominger, JHEP 0110 034, 2001
[4] V. Balasubramanian, J. de Boer and D. Minic, Phys. Rev. D65 (2002) 123508
[5] D. Klemm, Nucl. Phys. B625 (2002) 295
[6] A. Strominger, JHEP 0111 049, 2001
[7] R-G. Cai, Phys. Lett. B525 (2002) 331
[8] R. Bousso, A. Maloney and A. Strominger, Phys. Rev. D65 (2002) 104039
[9] M. Spradlin and A. Volovich, Phys. Rev. D 65 (2002) 104037
[10] E. Abdalla, B. Wang, A. Lima-Santos, W. G. Qiu, Phys. Lett. B538 (2002) 435
[11] V. Balasubramanian, J. de Boer and D. Minic, hep-th/0207245
[12] B. Allen and T. Jacobsen, Comm. Maths. Phys. 103 (1986) 669-692
[13] H. Osborn and G. M. Shore, Nucl. Phys. B571 (2000) 287
[14] S. Deser and A. Waldron, Nucl. Phys. B607 (2001) 577
[15] I. L. Buchbinder, D. M. Gitman and V. D. Pershin, Phys. Lett. B492 (2000) 161
[16] B. Allen and M. Turyn, Nucl. Phys. B292 (1987) 813
[17] E. S. Fradkin and M. Ya. Palchik, hep-th/9712045
[18] E. S. Fradkin and M. Ya. Palchik, Conformal Quantum Field Theory in D-dimensions, Kluwer Academic Publishers, 1996

[19] V. K. Dobrev, G. Mack, V. B. Petkova, S. G. Petrova and I. T. Todorov, Lecture notes in Physics, v.63, Springer-Verlag, 1977

[20] J-P. Gazeau, J. Renaud and M. Takook, Class. Quant. Grav. 17 (2000) 1415

[21] J-P. Gazeau, M. V. Takook, J. Math. Phys. 41 (2000) 5920

[22] A. J. Tolley and N. Turok, hep-th/0108119

[23] I. Antoniadis and E. Mottola, J. Math. Phys. 32 (4) (1991) 1037