Probing individual topological tunneling events of a quantum field: Switching statistics of a superconducting nanowire

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Abstract. Phase slips are topological fluctuation events that carry the superconducting order-parameter field between distinct current carrying states and impart a non-zero resistance to superconducting nanowires. They play a fundamental role in determining the fate of superconductivity in nanowires. Conversely, superconducting nanowires provide an ideal setting for accessing non-trivial fluctuations driven by thermal activation and—at low temperatures—by quantum tunneling of a one-dimensional field. However, this potential has not been fully realized because resistance measurements, on the one hand, are capable of capturing only the averaged phase-slip behavior, and on the other hand, are incapable of pinning down the low temperature phase-slip behavior, as the measured resistance values drop below the noise floor. On going beyond the linear-response regime, the I-V characteristics show a hysteretic behavior. As the current is ramped up repeatedly, the state switches from a superconductive to a resistive one, doing so at somewhat random current values below the depairing critical current. The distribution of these switching currents was studied recently. I will report on the rather counter-intuitive temperature dependence of the distribution and its theoretical understanding via a stochastic model. It will be shown that although, in general, several phase-slip events are necessary to induce switching, there is an experimentally accessible temperature- and current-range for which a single phase-slip event is sufficient to switch the wire to the normal (resistive) state. I will conclude by arguing that switching-current statistics provide an effective probe to resolve individual phase-slip events and in addition offer unprecedented access to quantum phase-slip tunneling events.

1. Introduction
Quantum tunneling [1, 2, 3] is one of the most spectacular signatures of quantum physics. It clearly demarcates the quantum from the classical since it involves tunneling through classically forbidden regions. It lies at the heart of whole range of observed or proposed phenomena and encompasses all branches of modern physics that emanated from or metamorphized by the quantum revolution. Examples range from alpha decay of nuclei which is perhaps the most classic manifestation of quantum tunneling, to instanton solutions [4, 5] corresponding to the tunneling between different vacuua in quantum chromodynamics as well as the cosmological implications of the fate of a false vacuum [6, 7]. Examples in condensed matter systems abound [8] and quantum tunneling is essential also for experimental methods such as scanning tunneling microscopy and for technological applications.
The question of whether one can observe Macroscopic Quantum Tunneling (MQT) \cite{9,10} i.e. quantum tunneling on a macroscopic scale is intriguing and has spurred a whole area of research at the intersection of fields such as foundations of quantum mechanics, quantum measurement, quantum computation and information \cite{11,12,13}, condensed matter physics \cite{8} and ultra-cold atomic physics \cite{14}. Indeed, macroscopic systems exist for which the temperature can be lowered much below the frequency of small oscillations (about the metastable equilibrium) and are expected to furnish tunneling probabilities that are not unobservable. The coupling to the environment and dissipation has been argued to play a crucial role in dictating whether the observation of MQT is in fact possible \cite{15,16,17} or not.

In accordance with theoretical expectations \cite{18,16}, Josephson junction circuitry has proven to be an ideal setting for the observation of MQT, wherein the macroscopic degree of freedom corresponds to the phase difference of the superconducting order parameter across a Josephson junction or the flux in a Superconducting Quantum Interference Device (SQuID) \cite{19,20,21,22,8}. A current-biased Josephson junction can be effectively represented by a mechanical analog of a ‘particle’ moving in a tilted cosine (washboard) potential and the MQT corresponds to the particle escaping from a metastable minimum of the potential by tunneling through the barrier \cite{23}. In an underdamped Josephson junction, a single tunneling event can take the system from a zero-voltage state (corresponding to the phase point or the particle confined to a local minimum) to a finite voltage state (corresponding to a delocalized phase or the particle running through the washboard potential). It is important to note that it is not a single electron but the phase variable describing the collective (superconducting) state of a macroscopic number of electrons that is tunneling.

Encouraged by the observation of MQT of a quantum variable, one cannot but wonder about the next intriguing question: is it possible to experimentally observe MQT of a quantum field? If a Josephson junction is the ‘zero-dimensional’ case then one would like to consider the ‘one-dimensional’ case in which the phase is a function of position. Lo and behold, a superconducting wire (with length greater and diameter much less than the coherence length) precisely furnishes this desirable configuration: the superconducting order parameter is a complex field with a fixed amplitude and the phase winding along the length of the wire to form a helix. Furthermore, in a so-called phase slip event \cite{24}, the complex-valued order parameter field goes from one metastable minimum to another in which the number of phase-windings is reduced by one; i.e. the phase slips by $2\pi$ and the phase-difference across the wire is correspondingly lowered by $2\pi$. In going from one topologically distinct super-current carrying state to another, the order-parameter field essentially passes through a configuration in which its amplitude drops to zero somewhere along the wire (typically over a length-scale of the order of coherence length) where the phase slips by $2\pi$. Therefore, during the course of a phase-slip process the superconducting order loses its coherence and the wire acquires a non-zero resistance. In fact, such resistive phase-slip fluctuations are responsible for determining the very essence of superconductivity in quasi-one-dimensional wires \cite{25,23}.

Phase-slip fluctuations in which the order-parameter field jumps over the potential barrier via thermal activation are well-understood in the vicinity of the transition temperature. The LAMH theory \cite{26,27,28} for thermally-activated phase-slips (TAPS) derived within the (time-dependent) Ginzburg-Landau formalism provides good fits for the broad resistive transition (as against a sharp transition to the zero resistance state in bulk superconductors) in quasi-one-dimensional wires. The search for quantum phase slips (QPS) \cite{29,30} in which the order-parameter field tunnels through the potential barrier has been mostly elusive. In this proceeding I will report on a recent experiment \cite{31} and its theoretical interpretation \cite{32,33} which we believe provides evidence for the detection of a single quantum phase slip process and hence for an observation of the MQT of a quantum field.
Figure 1. Illustration of the system and the model: a. Schematic of an experimental set-up in which a superconducting nanowire is suspended between two thermal baths. b. Sketch showing the attenuation of the order parameter in the core of a phase-slip. c. Schematic of the simplified model. All phase slips are taken to occur in the central (i.e., shaded) segment of length $l$, which is assumed to be at a uniform temperature $T$; heat is carried away through the end segments, which are assumed to have no heat capacity. The temperature at the ends of the wire is fixed to be $T_b$. d. Sketch of a typical temperature profile.

2. System and Model
Recent advances in the fabrication of long and homogeneous superconducting nanowires—using carbon nanotube [34] or DNA templating [35]—has made it possible to access regimes in which the probability for QPS at low enough temperatures is in the potentially observable range. However, in the low temperature window in which QPS are expected to be significant enough, the measured resistance values drop below the noise floor, thereby making it difficult and challenging to unambiguously observe QPS. A recent experiment [31] seeks to address this problem by going beyond the linear-response regime and modulating the bias current as a periodic (triangular or sinusoidal) function of time. As the current is ramped up repeatedly, the state switches from a nominally superconductive (i.e., barring the exponentially suppressed yet non-zero resistance imparted by the phase slip fluctuations) to a nominally normal (i.e., resistive) one; doing so at somewhat random current values below the depairing critical current [36, 37, 23]. Since one expects phase-slip fluctuations to be responsible for the premature stochastic switching [30, 38, 39], the distribution of these switching currents and its temperature evolution promises to provide a useful and less ambiguous probe of phase-slip fluctuations.

To be able to theoretically understand the switching current distribution or alternatively the mean switching time $\tau_s$ of the bistable current-biased nanowire that can be extracted from
the switching-current statistics [40], one needs to first identify the physical mechanism that causes the wire to switch. Given that the experimental set-up consists of a free-standing wire connected to thermal baths at its two ends (see Figure 1a), it has been argued [38, 32] that if the Joule heating caused by resistive phase slip fluctuations is not overcome sufficiently rapidly by conductive cooling, it effectively reduces the depairing current, ultimately to below the applied current, thus causing switching to the highly resistive state.

To study the dynamics of switching, it is necessary to explicitly take into consideration the fact that the resistive fluctuations of the superconducting nanowire consist of discrete phase-slip events that take place at random moments of time and are centered at random spatial locations. Instead of solving a full 1+1 dimensional stochastic differential equation, it is reasonable to consider a simplified model (as shown in panels c and d of Figure 1) in which all the phase slips and hence the heating takes place within a central segment of length \( l \) to which a uniform temperature \( T \) is assigned and the heat is conducted away through the end segments (within which the heat capacity can be ignored). The dynamics of the nanowire is then given by a stochastic ordinary differential equation for the time-evolution of the temperature \( T \) and is dictated by the competition between discrete heating (since each phase slip heats the wire by a “quantum” of energy \( hI/2e \), for a given value of the applied current \( I \)) and continuous cooling.

3. Results and Conclusion

The recent experimental measurement (introduced in the previous section) of switching current statistics of a superconducting nanowire found that the width of the distribution increases as the temperature of the bath \( T_b \) is lowered [31]. Given that the stochasticity in the switching process stems from the inherent stochasticity of the phase slip fluctuations, one might naively assume that the mean switching time \( \tau_s \) for a given value of \( T_b \) and current \( I \) would correspond to the inverse of the phase-slip rate \( \Gamma(T_b, I) \). Under this assumption the observed temperature dependence is counter-intuitive since the phase-slip rate is expected to go down as the temperature is lowered (see the dashed contour lines in Figure 2a). Even after modifying \( \Gamma \) to incorporate quantum phase slips, one is not able to fit the \( \tau_s \) curves extracted from the observed switching-current distribution with those of \( \Gamma \).

The model discussed in the previous section indeed yield \( \tau_s \) that drastically deviate from \( \Gamma \) except in a certain range of \( T_b \) and \( I \) as shown in panels (a) and (c) of Figure 2. One can understand this as follows. ‘Thermal runaway’—heating by rare sequences of closely-spaced phase slips—constitutes the mechanism of superconductive-to-resistive switching within our model. The number of phase-slips in such a phase-slip sequence can be easily estimated by evaluating the number \( N(T_b, I) \) of phase-slips needed to cause switching in the absence of cooling. In the range of \( T_b \) and \( I \) in which \( N \) is greater than one, the value of \( \tau_s \) is of course expected to be larger than \( \Gamma^{-1} \). The larger the number of phase-slips in the sequence inducing the superconductive-to-resistive thermal runaway, the smaller the stochasticity in the switching process and, hence, the sharper the distribution of switching currents (see Figure 2b). Upon entering the regime in which a single phase slip is sufficient to switch the wire to the normal (resistive) state, \( \tau_s \) coincides with \( \Gamma^{-1} \) and the distribution width decreases upon lowering the temperature in accordance with the naive expectation for thermally activated phase slips.

The experimentally observed increase in the distribution width is thus explained by the model even if \( \Gamma(T_b, I) \), which is an input function in the model, is chosen to be determined solely by thermally activated phase slips (TAPS). One can, in fact, successfully fit the experimental curves for a wide range of \( T_b \) [31, 33]. However, contrary to the non-monotonic behavior of the distribution width expected for this choice of \( \Gamma \) (Figure 2), the observed distribution width does not reverse the trend. By choosing a form of \( \Gamma \) that phenomenologically incorporates quantum phase slips (QPS), it becomes possible to fit the experimental curves down to the lowest temperatures. Moreover, based on the comparison with theory, one can identify a temperature
Figure 2. Switching statistics a. Map showing $N(T_b, I)$ (see text) and the contour lines (solid lines) for the inverse of mean switching time, $\tau^{-1} = 1, 10^3, 10^6 \text{ s}^{-1}$; the contour lines (dashed lines) for the phase-slip rate $\Gamma$ are also shown. The thermodynamic (depairing) critical current (dashed-dotted line) is plotted for reference. b. Switching-current distributions $P_{SW}$ obtained at various values of $T_b$ and for $r = 58 \mu \text{A} / \text{s}$. c. The logarithms of $\tau^{-1}$ (colored lines) and of $\Gamma$ (thinner black lines) as a function of $I$, obtained for the same set of $T_b$ values as in panel (a). The colors of $\tau^{-1}$ plots correspond to different values of $N(T_b, I)$ [as indicated in the legend of panel (a)].
window in which the experiments are probing a single quantum phase slip event via its drastic consequence of switching the entire nanowire from the superconductive to the resistive state.

Fluctuations play an important role in determining the behavior of a superconducting nanowire, which provides an ideal laboratory not only for observation of MQT of a quantum field, but also for studying quantum phase transitions [34, 41, 42, 43, 44]. Experimental characterization of QPS is bound to enhance the understanding of various mechanisms for quantum phase transitions and their relevance for the experiments. A better understanding of low-temperature fluctuations has also implications for varied technological applications such as integration of superconducting wires in electronic circuitry (e.g., as controllable switching elements), implementation of nanowire-based devices [35, 45, 46], and the potential use of nanowires as qubits in quantum computers [47].

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