The Relationship Between the Electromagnetic Field Amplitude and its Frequency in a Single Photon

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Abstract. The current theory in physics states that amplitude, namely the amplitude of the electric and magnetic fields, is continuous. It is proposed in this paper that these two types of amplitude can be quantized. This study reports that the amplitude of the electromagnetic field is discrete. Two methods were used to generate an equation of the discrete amplitude of the electromagnetic field, namely the Maxwell’s theory of Electromagnetic Field and the Einstein's theory of the photoelectric effect. According to the results of the theoretical analysis, the lowest amplitude of the electric field and the lowest amplitude of the magnetic field depend on the magnitude of frequency of the electromagnetic field. The study concludes that in a photon containing an electromagnetic field amplitude equal to the E-Goen or B-Goen constant is multiplied by the square of its frequency.

1. Introduction
There are two properties of the electromagnetic wave theories which have generated considerable recent research interest, namely (i) the Maxwell’s Theory of the electromagnetic wave, which focuses on a wave property [11], [12], [24], [29], and (ii) the Einstein’s Theory of the photoelectric effect, which focuses on a particle property. On one hand, the better theory that explained the electromagnetic phenomenon as a wave was first proposed by Maxwell in the 19th century [9]. As a wave, in the Maxwell’s Theory of Electromagnetic Field, it has been proven that the electric field can induce the magnetic field and the magnetic field can induce the electric field, which generates the Maxwell's equations of the electromagnetic field [16]-[18]. On the other hand, the better theory that explained the phenomenon of light which is part of electromagnetic waves as a particle was first proposed by Einstein [20] in the 20th century. In other cases on atoms, this quantum of action, emission spectrum of monochromatic light as electromagnetic wave, also implies that only certain energy levels are allowed, and values in between are forbidden [5]. As a particle, the Einstein’s Theory of the photoelectric effect proposed that a beam of light is not a wave propagating through space, but rather a collection of discrete wave packets (photons) [3]. Each photon, the particle of light [22], comes with energy \( hf \), where \( f \) is frequency arising from quantization of energy and \( h \) is known as the Planck constant [4]. Previous works of the energy analysis of the electromagnetic fields or photons have only focused either on a wave [14] or a particle [20], but not on the combination of both wave and particle together. The aim of this study is to combine the two theories so as to generate a theoretical analysis that the amplitude of the electromagnetic field is discrete. As a result, the analysis shows that the lowest amplitude of the electric field, as well as the lowest amplitude of the magnetic field, depends on the quadrate magnitude of the frequency of the electromagnetic field.

2. Literature Review
Between 1861 and 1862, the Scottish physicist and mathematician James Clerk Maxwell was proposed the theory that explained the electromagnetic phenomenon as a wave [10]. In 1900, Max Planck was working on black-body radiation, he suggested that the energy in electromagnetic waves could only be released in the form of the discrete energy [21]. Later, in 1905, Albert Einstein proposed that the property of electromagnetic waves were the discrete wave-packets [6]. Two properties of the energy of the electromagnetic wave theories, namely the theory that has the property of a wave and the property of a particle, have been generated [20]. As a wave, it is also known that the propagating of the electromagnetic field in the vacuum has the energy density per unit volume [8] which is equal to

\[ u = \frac{1}{2} \varepsilon_0 E_{\text{max}}^2 + \frac{1}{2} \mu_0 H_{\text{max}}^2 \]  

(1)

where [2] \( H = B/\mu_0, B = E/c, \) and \( c = 1/(\mu_0 \varepsilon_0)^{0.5}. \) The Eq. (1) above can be rewritten as:

\[ u = \varepsilon_0 E_{\text{max}}^2. \]  

(2)

For (1) and (2) the international unit is Joule/meter [3], \( \mu_0 \) and \( \varepsilon_0 \) are magnetic permeability and electrical permittivity in vacuum, while \( E_{\text{max}} \) is the amplitude of the electric field of the electromagnetic wave and \( B_{\text{max}} = \mu_0 H_{\text{max}} \) is the amplitude of the magnetic field of the electromagnetic wave [20].

As a particle, through Einstein's theory of photoelectric effect, it was known that the amount of energy carried by a photon was proportional to the magnitude of the frequency of the electromagnetic field. Newstead [20] point out that “… at all times the total energy of the electromagnetic wave is conserved; its energy is just transferred between electromagnetism and mass” (p.4). A photon has no electric charge [12] and mass [7]. Hence the equation, the amount of energy of the one photon [20] is formulated as follows:

\[ E_t = h f, \]  

(3)

where \( h \) is Planck's constant and \( f \) is the magnitude of the frequency of the electromagnetic field. Since the property of the photon is a particle, then the propagation of the photon has a maximum value of momentum \( p \), that is \( P_{\text{max}} = h/\lambda, \) where \( \lambda \) is the magnitude of the wavelength of electromagnetic waves and \( c = f \lambda \) is the propagation speed of electromagnetic wave in vacuum, so \( E_t = c \cdot P_{\text{max}}. \) In empty space, the photon moves at the speed of light and maximum of its energy \( (E_t) \) and momentum \( (P_{\text{max}}) \) are formulated [1] by \( E_t = c \cdot P_{\text{max}}. \) Relativity would be unaffected by the speed of light as a constant of nature which is the maximum speed in space-time [15].

If the propagation of the harmonic photon is assumed straight in one dimension \( x \), then the magnitude of the momentum of the photon will be dependent on the change of the distance \( x \) and time \( t. \) Mathematically, the equation of the momentum can be written as \( P(x,t) = P_{\text{max}} \cos(\omega t - k x), \) where \( \omega = 2\pi f \) is the angular velocity of the propagation of the electromagnetic wave and \( k = 2\pi/\lambda \) is the wave number.

The consequence of the linearity of Maxwell’s equations which enforce the linear spreading relation \( \omega = k \cdot c \) for the electromagnetic field has the relation, namely the maximum momentum of one photon \( P_{\text{max}} = h/\lambda \) with the maximum energy of one photon \( E_t = c \cdot P_{\text{max}}. \) Therefore, the function of the energy of one photon which is dependent on the value of momentum that is non-independent on the value of the distance \( x \) and the time \( t \) can be reformulated as

\[ E_t(x,t) = c \cdot P_t(x,t) = c \cdot P_{\text{max}} \cos(\omega t - k x), \]  

(4)

where \( c \) is the propagation speed of electromagnetic waves in vacuum. Maxwell has noted that speed of light is the same as the propagation speed of electromagnetic wave in vacuum then concluded that light is an electromagnetic wave [2].

The function of the energy density of the unit volume of one photon is the derivative of the function of the energy of one photon, Eq.(4) to change of the distance \( x \) as many as three times, that is:

\[ u_{\text{photon}}(x,t) = d^3 E_t(x,t)/dx^3 = c \cdot P_{\text{max}} k^2 \cos(\omega t - k x) = E_t(x,t)/(1/k^3), \]  

(5)
where $1/k^3$ is a unit volume of one or more photons, therefore the energy density of the unit volume of one photon is

$$u_{\text{photon}} = E_\varphi (1/k^3) = E_\varphi k^3.$$

(6)

However, the energy analysis of the electromagnetic fields or photons has only focused either on a wave Eq.(1) or a particle property Eq.(6), but not on the combination of both wave and particle properties together. This paper reports on a study that combines the energy of the two properties to generate a theoretical analysis that the amplitude of the electromagnetic field is discrete.

### 3. Result and Discussion

First, to analyze the discrete amplitude of the electromagnetic field is to compare the energy density of the unit volume of the electromagnetic fields, which implies the value of the elementary amplitude of the electric field $(E_{\text{max-elementary}})$. Furthermore, the number of photons of energy density per unit volume $(n)$ can be obtained by dividing the energy density per unit volume of the electromagnetic fields $(u)$ with the elementary energy density per unit volume of the electromagnetic field $(u_{\text{elementary}})$. It can therefore be analyzed that $n$ is square of the ratio between the amplitude of the electric field by the elementary amplitude of the electric field. Mathematically it can be written as:

$$n = u/u_{\text{elementary}} = (\varepsilon_0 E_{\text{max}}^2)/(\varepsilon_0 E_{\text{max-elementary}}^2) = (E_{\text{max}}/E_{\text{max-elementary}})^2.$$

(7)

Note that $n$ is an integer value and the lowest value starting from $n = 1$.

The lowest value of $n$ from Eq.(7) above is the number of one photon in the energy density of the unit volume of the electromagnetic fields, which implies the value of the elementary amplitude of the electric field $(E_{\text{max-elementary}})$. Furthermore, the number of photons of energy density per unit volume $(n)$ can be obtained by changing the value of $E_{\text{max-elementary}}$.

$$1 = (\varepsilon_0 E_{\text{max-elementary}}^2)/(E_\varphi k^3) = (\varepsilon_0 E_{\text{max-elementary}}^2)/(h f k^3),$$

(9)

so that

$$E_{\text{max-elementary}} = k (h f k/\varepsilon_0)^{0.5} = (2\pi f/\lambda)(2\pi f/h/\varepsilon_0 c)^{0.5},$$

(10)

or

$$E_{\text{max-elementary}} = f^2 (2\pi/c) (2\pi f/h/\varepsilon_0 c)^{0.5}.$$

(11)

From Eq.(11) above, let us redefine $(2\pi/c)(2\pi f/h/\varepsilon_0 c)^{0.5}$, namely the constant value of the elementary amplitude of the electric field $(E_{\text{max-elementary}} = E_{\text{me}})$ is only dependent on the square of the frequency of the electromagnetic field as follows

$$E_{\text{me}} = f^2 k_\varepsilon,$$

(12)

where it has been known that the value of the propagation speed of electromagnetic wave in vacuum $c = 2.99792458 \times 10^8$ m/s, the Planck's constant $h = 6.6260755 \times 10^{-34}$ J·s, and the electrical permittivity in vacuum or the absolute permittivity $\varepsilon_0 = 8.854187817 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$, so the constant value of the elementary amplitude of the electric field obtained or the E-Goen’s constant [27] is $k_\varepsilon = 2.624777975 \times 10^{-33}$ V·s²/m.
The electric field ($E$), also known as relationship between the magnetic field ($B^2$) which is $B = E/c$, can be seen also as the equation of the elementary amplitude of the magnetic field ($B_{max-elementary} = B_{me}$) which is only dependent on the square of the frequency of the electromagnetic field

$$B_{me} = f^2 \cdot k_B,$$

(13)

where $k_B$ is the constant value of the elementary amplitude of the magnetic field or the B-Goen’s constant, the $k_B$ constant value is the division between the constant value of the elementary amplitude of the electric field ($k_e$) by the constant value of the speed of the electromagnetic fields in a vacuum ($c$), $k_B = 8.755316903 \times 10^{-32} \text{T} \cdot \text{s}^2$.

The Eq.(12) and (13) above demonstrate that the amplitude of the electric and magnetic fields are discrete. The higher the frequency means that the lowest amplitude of the electric and magnetic fields will become higher. The full results above were only obtained by the theoretical analysis. Therefore, the experimental researchers in physics are encouraged to prove the results.

These results indicate that the amplitude of the electric and magnetic fields are not continuous. As one of the implications, in another way, the total number of photons of the electromagnetic waves can be identified from the comparison between the amplitude of the electric field and the lowest amplitude of the electric field. The assumption that there is a radio station [28] whose frequency specification is 110 MHz and whose power specification is 120 kW, the energy density per unit volume of the electromagnetic wave at a distance of 7 km from the station is measured by $u = 3.2 \times 10^{-10} \text{J/m}^3$. Therefore, from Eq.(2) the amplitude value of the electric field can be generated that is $E_{max} = (u / \varepsilon_0)^{0.5} = ((3.2 \times 10^{-10}) / (8.854187817 \times 10^{-12}))^{0.5} = 6.011746 \text{ V/m}$. Similarly, from Eq.(12) the value of the elementary amplitude of the electric field can also be generated that is $E_{max-elementary} = f^2 \cdot k_e = (1.1 \times 10^8)^2 \times 2.624777975 \times 10^{-23} = 3.175981 \times 10^{-7} \text{ V/m}$. Assuming that the electromagnetic waves are particles, the number of photons of the energy density per unit volume of the electromagnetic waves is $n$, from Eq.(8), $n = (E_{max}/E_{max-elementary})^2 = (6.011746 / (3.175981 \times 10^{-7}))^2 = 3.582988 \times 10^{14} \text{ photons}$. Of course the numbers of $3.582988 \times 10^{14}$ photons are very much larger than one and hence the quantum effects do not play a role. The electromagnetic waves by the radio station are well into the classical limit. The discrete amplitude of the electric fields in electromagnetic waves can explain that the same power of the radio stations [26], the higher the frequency of the radio stations causes the higher the magnitude of the elementary amplitude of the electric field Eq.(12) and the amount of photons produced fewer Eq.(8). Therefore the propagations of the same distance considered by the same distribution of scattering photons effect the measurement results of electric field strength will be lower, see table 1 [25], [27].

Table 1: The Measurement results of electric field strength ($E$) and calculating the amount of photons ($n$)

| No. | Location                      | $f=14.15$ MHz | $f=21.15$ MHz | $f=28.15$ MHz |
|-----|-------------------------------|---------------|---------------|---------------|
|     |                               | $E_{me} = 5.25 \times 10^{-9}$ | $E_{me} = 11.74 \times 10^{-9}$ | $E_{me} = 20.80 \times 10^{-9}$ |
|     |                               | V/m           | V/m           | V/m           |
| 1.  | Near bedroom doorway appx. 3-5 m from closest section of antenna located above ceiling | 14-17 (V/m) $n \approx (7.10-10.46) \times 10^{-18}$ | 10-14 (V/m) $n \approx (0.73-1.42) \times 10^{-18}$ | 7-12 (V/m) $n \approx (0.11-0.33) \times 10^{-18}$ |
| 2.  | Above bedroom doorway near heating/cooling vent (localized) appx. 2-3 m from antenna | 19 (V/m) $n \approx 13.07 \times 10^{18}$ | 17-19 (V/m) $n \approx (2.10-2.62) \times 10^{18}$ | 17 (V/m) $n \approx 0.67 \times 10^{18}$ |
| 3.  | Top of hall stairway & almost directly beneath antenna (appx. 1-2 m); localized near smoke alarm | 75 (V/m) $n \approx 203.66 \times 10^{18}$ | 30 (V/m) $n \approx 6.53 \times 10^{18}$ | 14-17 (V/m) $n \approx (0.45-0.67) \times 10^{18}$ |

4. Conclusion
Two theories are used to generate the elementary amplitude of the electric field and the magnetic field still leaves a gap question. This can happen because of the same phenomenon such as light evidently have two theories which have different properties namely the theory with the viewpoint as a wave and the theory with the viewpoint as a particle. After going through the synergy of both theories, it can be generated the elementary amplitude of the electric field and the elementary amplitude of the magnetic field. Therefore, in the future through this achievement will be expected capable as the stepping stone to combine the two major theories. The two major theories, The Maxwell's theory of the electromagnetic wave which focuses on a wave property and the Einstein’s Theory of the photoelectric effect which focuses on a particle property, will be able to be a single theory that is more elegant.

In this paper, to generate the equation of the elementary amplitude of the electric and magnetic field are used two theories namely the Maxwell’s Theory of Electromagnetic Field with the viewpoint as a wave and the Einstein’s Theory of the photoelectric effect with the viewpoint as a particle. The magnitude of the amplitude of the electric and magnetic fields in electromagnetic waves are not continuous but quantized of the sum of the elementary amplitude of the electric and magnetic fields which are equal to the square of the frequency of the electromagnetic fields. The constant value of the elementary amplitude of the electric field or the E-Goen’s constant is
\[ k_E = 2.624777975 \times 10^{-23} \text{ V} \cdot \text{s}^2/\text{m} \]
and the magnetic field or the B-Goen’s constant is
\[ k_B = 8.755316903 \times 10^{-32} \text{ T} \cdot \text{s}^2. \]

Through this knowledge, the discrete amplitude of the electromagnetic field, then our understanding of electromagnetic waves as the carrier of communication signals will be more detailed and accurate.

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