We propose and study a two-orbital lattice extension of the Sachdev-Ye-Kitaev model in the large-$N$ limit. The phase diagram of this model features a high-temperature isotropic non-Fermi liquid which undergoes first-order thermal transition into a nematic insulator or continuous thermal transition into a nematic metal phase, separated by a tunable tricritical point. These phases arise from spontaneous partial orbital polarization of the multiorbital non-Fermi liquid. We explore the spectral and transport properties of this model, including d.c. elastoresistivity, which exhibits a peak near nematic transition, as well as nonzero frequency elastocconductivity. Our work offers a useful perspective on nematic phases and transport in correlated multiorbital systems.

The interplay of non-Fermi liquid physics (nFL) with broken symmetry states of matter is a rich field of research in correlated electron systems. One approach to this physics is to study metallic quantum critical points (QCPs) where fluctuations associated with the onset of symmetry breaking can destroy quasiparticles on the Fermi surface (1–7). An equally important exploration is to ask how nFLs, which may arise more generically in correlated narrow-band materials including flat-band systems, become unstable to diverse broken symmetry phases as we lower temperature (8–11). Understanding these issues is of enormous interest for ongoing experiments on a wide range of quantum materials.

The electron nematic, a quantum liquid crystal, is a ubiquitous broken symmetry phase associated with the loss of lattice rotational symmetry. Nematicity and nFL physics have been explored in a host of correlated quantum materials including Moiré crystals such as twisted bilayer graphene with flat bands (12–14), iron pnictide and chalcogenide systems (15–20), doped cuprates (21–23), the bilayer strontium ruthenates (24–27), and quantum Hall fluids (28–32). Quantum criticality of uniform nematic order is also of great interest since it impacts electrons on the entire Fermi surface. Quantum Monte Carlo (QMC) simulations of sign-problem free models show signatures of nFL properties and an emergent superconducting dome near such nematic QCPs (6, 33). On the experimental front, a particularly useful tool to detect nematic fluctuations and symmetry breaking is elastoresistivity. This measures the impact of uniaxial strain on the resistive anisotropy, providing a transport probe of nematic susceptibility (34–38). While there has been progress in exploring elastoresistivity in strongly correlated Hubbard-type models (39), it is important to study extensions to multiorbital and multiband nFLs which are of relevance to diverse materials including the cuprates, FeSe, Sr$_3$Ru$_2$O$_7$, and Moiré crystals.

Recently, the theoretical study of nFLs has also seen significant progress. Starting with the formulation of the Sachdev-Ye-Kitaev (SYK) model (40–43) as a solvable example of nFL on a quantum dot, the field has grown to include several illuminating generalizations (44–49). In this context, lattice extensions of the SYK model are particularly interesting since they provide a controlled route to accessing several phenomena, including lattice nFLs (50, 51), FL to nFL crossovers (52–54), metal-insulator transitions (55), heavy fermion physics (56), and critical Fermi surfaces (54, 57). However, the question of how nematicity impacts a high-temperature nFL phase of an SYK lattice remains unexplored. In particular, is it possible to formulate a theoretically solvable microscopic lattice model with strong interactions that spontaneously manifests nematic phases and to study its transport properties?

Here, we address this question and other issues highlighted above by constructing a two-orbital extension of a lattice SYK model schematically depicted in Fig. 1. The two orbitals may be viewed as representing, for instance, $d_{x^2}$ and $d_{y^2}$ orbitals, each with a preferred hopping direction, which play a role in many quantum materials. The

**Significance**

Strong interactions can cause electronic fluids to lose their well-defined electron-like excitations and behave as non-Fermi liquids with anomalous transport properties. Understanding spontaneous symmetry breaking and its experimental consequences in such non-Fermi liquids is an important endeavor in condensed matter physics. Here, we study crystalline rotational symmetry breaking, also called nematic ordering, in a non-Fermi liquid using a paradigmatic theoretical model. We explore the resulting thermodynamic, spectral, and transport properties, including the impact of strain which acts as a probe as well as a tuning parameter. Our results are of broad interest for a broad variety of quantum materials including cuprate and iron pnictide superconductors and twisted bilayer graphene, which feature an interplay of non-Fermi liquid behavior and electronic nematic order.
underlying symmetry of this system is a $C_4$ lattice rotation followed by the exchange of the two orbitals. Uniform orbital polarization breaks this symmetry down to $C_2$, resulting in an Ising ferronematic, while staggered orbital polarization results in an Ising antiferronematic. Focusing on uniform orders, we study the complete phase diagram, thermodynamics, spectral functions, and transport, for this model in the large-$N$ limit. We show that this model exhibits a non-Fermi liquid phase at high temperature, which gives way to a nematic insulator or a nematic metal upon cooling. Depending on parameters, this thermal transition is first-order or continuous, with a tunable tricritical point. We examine the transport properties of this model, including the impact of uniaxial strain which breaks orbital degeneracy. We find that uniaxial $B_{1g}$ strain leads to a peak in the d.c. elastoresistivity anisotropy in the vicinity of the isotropic-to-nematic transition, and we also present results on the frequency-dependent elastoelectric conductivity. Finally, we extend our work to present preliminary results on checkerboard type antiferronematic order.

Two-Orbital SYK Lattice Model

The SYK model represents a single-site “dot” with $N$ fermionic modes having random all-to-all interactions which is exactly solvable when $N \to \infty$. We generalize the SYK model to a square lattice with each site having two “SYK dots,” representing two orbitals, and each orbital accommodating $N$ fermionic modes. Modes in the two orbitals hop anisotropically on the lattice, with a preferred direction as shown in Fig. 1: modes in orbital $i = +$ (red) hop along the $\hat{x}$ and $\hat{y}$ directions with respective amplitudes $t_1 = t + \delta t$ and $t_2 = t - \delta t$, with $\delta t > 0$, and vice versa for modes in orbital $i = -$ (blue). The kinetic energy is

$$H_{\text{kin}} = \sum_{\mathbf{k},s,i} \epsilon_s(k) c_{\mathbf{k},s,i}^\dagger c_{\mathbf{k},s,i},$$

where $\mathbf{k}$ is the momentum, $s = \pm$ is the orbital, $i = 1, \ldots, N$ denotes modes in each orbital, and the dispersion $\epsilon_{s\pm}(k) = -2t(\cos k_x + \cos k_y) \mp 2\delta t(\cos k_x - \cos k_y)$. The symbols $c_s^\dagger$, $c_s$ represent creation and annihilation operators for the fermions.

The interactions take on the SYK form, with two-body intraorbital and interorbital pair-hopping terms:

$$H_{\text{SYK}}^{\text{intra}} = \sum_{\mathbf{r},i,(ijkl)} J_{ijkl}^{(i)} (\mathbf{r}) c_{\mathbf{r},+,i}^\dagger c_{\mathbf{r},+,i} c_{\mathbf{r},-,i} c_{\mathbf{r},-,i},$$

$$H_{\text{SYK}}^{\text{inter}} = \sum_{\mathbf{r},(ijkl)} V_{ijkl} (\mathbf{r}) c_{\mathbf{r},+,i}^\dagger c_{\mathbf{r},+,j} c_{\mathbf{r},-,l} c_{\mathbf{r},-,l} + \text{H.c.},$$

where “H.c.” denotes Hermitian conjugate and $\mathbf{r}$ denotes the position of a lattice site. The couplings $J_{ijkl}^{(i)} (\mathbf{r})$, $V_{ijkl} (\mathbf{r})$ are uncorrelated random complex numbers having Gaussian distributions with zero-mean and satisfy $\langle J_{ijkl}^{(i)} (\mathbf{r}) J_{ijkl}^{(j)} (\mathbf{r}') \rangle = \delta_{ij} \delta_{\mathbf{r}\mathbf{r}'} (2N)^3$ and $\langle V_{ijkl} (\mathbf{r}) V_{ijkl} (\mathbf{r}') \rangle = \delta_{\mathbf{r}\mathbf{r}'} V^2 / (2N)^3$, respectively. Here, $\langle \cdots \rangle$ denotes the average over all disorder realizations, and $J^2 / (2N)^3$ and $V^2 / (2N)^3$ sets the variance. The couplings are properly antisymmetrized to obey $J_{ijkl}^{(i)} = -J_{ijkl}^{(j)}$ and a similar condition holds for $V_{ijkl}$ as well.

To solve for the phase diagram of this model, we go to the imaginary-time path-integral formulation and average over disorder realizations for $J_{ijkl}^{(i)}$ and $V_{ijkl}$ via the replica trick. Doing so results in a disorder-averaged action parameterized by $J^2$ and $V^2$. Next, we integrate out the fermion fields and rewrite the action using a replica-diagonal ansatz for the site-local imaginary-time Green’s function $G_s (\tau) \equiv - (1/N) \sum_i \langle T_{\tau} c_{s,i}^\dagger (\tau) c_{s,i} (0) \rangle$ for each orbital $s$ and their corresponding self-energies $\Sigma_s (\tau)$ (SI Appendix). Here, $\tau$ represents the imaginary-time coordinate, and $T_{\tau}$ is the time-ordering operator.

In the large-$N$ limit, the free energy functional $\Omega$ for our model can be obtained from the resulting action given by

$$\Omega = \sum_{s = \pm} \left[ -\frac{1}{\beta} \sum_{\text{ion}} \int d\varepsilon g_s (\varepsilon) \ln [i\omega_n + \mu - \varepsilon - \Sigma_s (i\omega_n)] + \int_0^\beta d\tau \Sigma_s (\tau) G_s (\beta - \tau) - J^2 / 4 \int_0^\beta d\tau G_s^2 (\beta - \tau) G_s^2 (\tau) \right] - \frac{V^2}{2} \int_0^\beta d\tau G_s^2 (\beta - \tau) G_s^2 (\tau),$$

where $g_s (\varepsilon) = \int \frac{d^2k}{(2\pi)^2} \delta (\varepsilon - \varepsilon_s (\mathbf{k}))$ is the lattice density of states for orbital-$s$, $\mu$ is the chemical potential, $\omega_n = (2n + 1)\pi / \beta$ represents the fermionic Matsubara frequencies, and $\beta = T^{-1}$ with $T$ denoting the temperature.
We note here since the dispersions for the +, −, orbitals obey $e_{\pm}(k_x, k_y) = e_{\pm}(k_y, k_x)$ (Eq. 1), both orbitals are described by the same density of states, so that $g_{\pm}(e) = g_-(e) = g(e)$ in Eq. 4. However, despite having the same $g(e)$, the interorbital SYK interaction $V$ can still drive a spontaneous symmetry breaking between the orbitals, as we demonstrate in the next section. The imaginary-time Green’s function $G_0(t)$ in Eq. 4 satisfies the boundary condition $G_0(-\tau) = G_0(\beta - \tau)$ and so does the imaginary-time self-energy $\Sigma_0(\tau)$. Setting $\delta\Omega/\delta G_0(\tau) = 0$ and $\delta\Omega/\delta \Sigma_0(\omega_n) = 0$ leads to

$$\Sigma_0(\tau) = -f^2 G_0^2(\tau) G_0(-\tau) - V^2 G_{\pm}(\tau) G_{\pm}(-\tau),$$

$$G_{\pm}(\omega_n) = \int d\varepsilon [\omega_n + \mu - \Sigma(\omega_n)]^{-1},$$

which we solve self-consistently (see ref. 51 and SI Appendix, SI 3). The solution is used to compute the equilibrium free energy and thermodynamic properties using Eq. 4. While these equations have been obtained starting from the SYK model in the large-$N$ limit, we may also view them as a type of self-consistent dynamical mean field theory of a two-orbital model although we caution that the lattice SYK equations, and results, differ from iterated perturbation theory for solving the Hubbard model within dynamical mean field theory (DMFT) (58, 59).

**Phase Diagram**

We begin by discussing the uniform nematic order which appears as a symmetry-breaking solution to the above equations with $\Sigma_+ \neq \Sigma_-$. This phase, driven by the interorbital interaction $V$, may be viewed as a lattice generalization of the flavor-imbalanced phase of a two-flavor SYK model (45, 60). The interorbital $V$ interaction hops a pair of electrons from modes of a single orbital $s$ to modes of the other orbital $t$ and is thus distinct from the original single SYK dot interaction which randomly hops electrons between any two pairs of fermionic modes. While the latter does not have a symmetry broken phase, the $V$ interaction can instead induce symmetry breaking. As shown in Fig. 2A, for $V = t = 1$, the nematic phase is separated from an isotropic nFL by a first-order nematic transition at small hopping amplitude $t$. Increasing $t$, we encounter a tricritical point beyond which the thermal-nematic transition becomes second order. The tricritical point can be tuned by the hopping anisotropy $\delta t$ as shown in Fig. 2B.

We note that the nematic phases appear in the regime $V > t$, which is the regime certainly relevant to flat-band systems. This is also reasonable for typical correlated oxides or chalcogenides where the hopping $t = 100$ to 300 meV, while the scale of electron–electron interactions is $\sim 1$ to 3 eV. We have also explored how the phase diagram changes as we tune $V/J$; increasing $V/J > 1$ does not qualitatively modify our results but leads to quantitative shifts in the phase boundaries, where the nematic transition occurs at higher temperatures as $V/J$ increases. This is explored in more detail in SI Appendix, Figs. S2 and S3 and SI 3.

We characterize the nematic order by the orbital polarization $P = \langle n_+ \rangle - \langle n_- \rangle$, where the orbital densities are computed as $\langle n_\pm \rangle = G_0(\tau = 0^-)$. As seen from Fig. 2C, $P$ increases sharply below the first-order transition, while it increases gradually below the continuous thermal transition. Everywhere in the nematic phase, the polarization remains below its maximal value $P_{\text{max}} = 1$.

These transitions also exhibit corresponding signatures in the entropy $S = -\delta\Omega/\delta T$ and the specific heat $C_\varepsilon = T^2 \partial S/\partial T$ (SI Appendix, Fig. S1). The existence of a tricritical point in the phase diagram can be qualitatively understood using a Landau–Ginzburg (LG) theory approach as discussed in SI Appendix, Fig. S7; also see previous work on two coupled SYK dots (45).

In the Outlook section, we discuss closely competing staggered nematic phases which result from a more challenging numerical solution of the lattice SYK equations on an enlarged unit cell, in a spirit similar to cellular DMFT (61, 62).

We next discuss the behavior of the electronic compressibility $\kappa = \langle n_\varepsilon \rangle^{-2} \partial^2 \Omega/\partial \mu^2 = \langle n_\varepsilon \rangle^{-2} \partial^2 (\omega_\varepsilon) / \partial \mu^2$, where, $\langle n_\varepsilon \rangle = -\delta\Omega/\delta \mu$ is the total number density of fermions set to half-filling by particle–hole symmetry ($\langle \varepsilon \rangle = 0.5$). As seen in Fig. 2D, the compressibility vanishes as $T \to 0$ at small $t$, but it remains nonzero at larger $t$. This allows us to distinguish insulating from metallic phases, which we also confirm below using spectral functions and transport. We observe that the nematic phase exhibits both metallic (NM) and insulating (NI) regimes; these appear to be separated by a continuous transition as $T \to 0$ and by the indicated crossover lines in Fig. 2A at nonzero $T$.

**Spectral Functions**

To explore the spectral properties across the phase diagram, we have analytically continued the self-consistent Eqs. 5 and 6 from the Matsubara frequencies $(i\omega_n)$ to the real frequency line ($\omega \in \mathbb{R}$) following the method presented in refs. 44 and 51 (also see SI Appendix, SI 2 for a summary). Solving these equations on the real frequency axis allows us to extract the onsite retarded Green’s functions $G^R_0(\omega)$ and the retarded self-energies $\Sigma^R_0(\omega)$ without resorting to any numerical algorithms for analytic continuation.

**Fig. 2.** (A) Phase diagram in terms of temperature ($T$) and hoppings ($t$ and $\delta t$), showing an isotropic non-Fermi liquid (nFL), a nematic metal (NM), and a nematic insulator (NI). We have set $J = V = 1$ and the density to half-filling. The isotropic and nematic phases are separated by first-order or continuous thermal transitions which meet at a tricritical point (filled circle). The NM and NI regimes are separated by a crossover at nonzero $t$. (B) Temperature at the tricritical point versus $\delta t$ showing that it could be potentially further tuned (see the solid line, guide to eye) to reach a quantum tricritical point in a more generalized model. (C) Polarization ($P$) or orbital density imbalance for a cross-section taken along the $t - T$ plane in (A) with $\delta t/t = 0.05$. (D) Compressibility ($\kappa$), distinguishing metallic (nFL and NM) from insulating phases (NI), for a $t - T$ plane cross-section with $\delta t/t = 0.05$. 

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such as Padé approximation or maximum entropy. We then utilize $G^R_{\alpha\alpha}(\omega)$ to compute the local (onsite) spectral function $A_{\alpha}(\omega)$ for each orbital using $A_{\alpha}(\omega) = -\text{Im} G^R_{\alpha\alpha}(\omega)/\pi$.

Fig. 3 A and B shows the total spectral function $A(\omega) = \sum_\alpha A_{\alpha}(\omega)$ in the regimes where the $T = 0$ phase is a) a nematic insulator and b) a nematic metal respectively. At high $T$, both regimes feature an orbital symmetric nFL regime with a peak at $\omega = 0$. Since orbital symmetry breaks below the nematic transition, the spectral functions for the two orbitals split and move away from $\omega = 0$, leading to twin peaks in $A(\omega)$, one from each orbital. Since $\langle n_i \rangle = \int_{-\infty}^{\infty} d\omega \, n_F(\omega) A_{\alpha}(\omega)$, this splitting correlates with the onset of nonzero polarization $P$ discussed earlier. Here, $n_F(\omega)$ denotes the Fermi function. The symmetries of the local spectral function in the isotropic phase, which include particle–hole symmetry, $A_{\alpha}(\omega) = A_{\alpha}(-\omega)$, and orbital-exchange symmetry, $A_{\alpha}(\omega) = A_{\alpha-\omega}(\omega)$, get reduced in the nematic phase to a combined symmetry under orbital exchange $\omega \rightarrow -\omega$ followed by orbital exchange $i \rightarrow -i$, so that $A_{\alpha}(\omega) = A_{\alpha-\omega}(\omega)$.

Fig. 3A. we find a regime of temperatures below the nematic transition where a spectral peak survives at $\omega = 0$, suggestive of an NM, before a (soft) gap opens up, leading to loss of low-frequency spectral weight and eventually a hard insulating gap at $T > 0$. This is consistent with a vanishing compressibility $\kappa(T \rightarrow 0)$ for small values of hopping $t$ in Fig. 2D.

Furthermore, since $\Sigma_{\alpha\alpha}(\omega)$ is $\mathbf{k}$-independent, the momentum-resolved spectral function $A_{\alpha}(\mathbf{k},\omega)$ is obtained from the lattice Green’s function $G^R_{\alpha\alpha}(\mathbf{k},\omega) = 1/(\omega - m_{\alpha}(\mathbf{k}) - \Sigma_{\alpha\alpha}(\omega))$ using $A_{\alpha}(\mathbf{k},\omega) = -\text{Im} G^R_{\alpha\alpha}(\mathbf{k},\omega)/\pi$. Representative plots for $A_{\alpha}(\mathbf{k},\omega)$ and related details are given in SI Appendix, SI 4 and Fig. S4.

We next turn to the quasiparticle residue $Z$ and effective mass $m^*$, dropping the orbital label since these observables turn out to be the same for both orbitals. We define $Z(T) = \left(1 - \frac{\partial \Sigma(\omega)}{\partial \omega} \right)^{-1}|_{\omega=0}$, so that the quasiparticle residue in a Fermi liquid ground state corresponds to $Z(T \rightarrow 0)$. Since the self-energy in our model is momentum independent, the temperature-dependent effective mass enhancement may be written as $m^*(T)/m = Z^{-1}(T)$. As discussed in detail in SI Appendix, SI 4, we find that upon cooling the isotropic nFL, $m^*(T)/m$ grows and appears to diverge. However, this growth is cut off below the nematic transition, leading to a finite mass enhancement and a correspondingly reduced $Z < 1$.

**Transport**

Given the spectral functions above, the real part of the conductivity (per flavor, i.e., scaled by $1/N$) is computed (along both directions $\alpha = x,y$)

$$\text{Re} \sigma_{\alpha\alpha}(\omega, T) = \frac{1}{\omega} \sum_i \int_{k\omega} v^2_{\alpha\alpha}(\mathbf{k}) A_{\alpha}(\mathbf{k}, \omega') A_{\alpha}(\mathbf{k}, \omega + \omega')$$

$$\times \left[ n_F(\omega) - n_F(\omega + \omega') \right],$$

where $\tilde{v}_\alpha(\mathbf{k}) = \tilde{v}_{\alpha\alpha}(\mathbf{k})$.

In the $N \rightarrow \infty$ limit, this disorder-averaged Kubo formula result has been shown to be exact, with no vertex corrections (63, 64), and with a separable product of disorder-averaged Green functions $G^G_{\alpha\alpha} = G^G_{\alpha\alpha}(54)$. We extract the d.c. conductivity as the slope of the $\omega \sigma_{\alpha\alpha}(\omega)$ curve for $\omega \rightarrow 0$, and invert it to obtain the d.c. resistivity $\rho_{\alpha\alpha}$. Fig. 3 C and D show the resistive nematicity $N' = (\rho_{xx} - \rho_{yy})/(\rho_{xx} + \rho_{yy})$ as a function of temperature, with the average resistivity plotted in the corresponding insets. The high-temperature nFL regime displays a characteristic $\rho \propto T$ resistivity. This nFL phase is a well-known result common to high-temperature lattice-SYK models (49).

As we enter the nematic phase at low temperature, the decrease in the spectral weight at the Fermi level (Fig. 3B) cuts off the effect of strong scattering from the SYK interactions and leads to an FL with $\rho \propto T^2$. We find that $N'(T)$ vanishes in the isotropic phase, while it displays a plateau in the NM, before rapidly increasing to $N' \sim O(1)$ deep in the NI.

**Elasto-transport**

It has been shown that transport in the presence of uniaxial strain can provide a sensitive probe of nematic fluctuations and the onset of nematic order (36, 39). In order to explore this, we assume that the uniaxial strain imposes a local orbital splitting which varies linearly with the strain; physically, this will arise due to a modification in the local crystal field environment. Given its strain-induced origin, we use $\epsilon$ to denote this splitting. For $B_{1g}$ strain, this leads to an additional term in the Hamiltonian $\sum_r (n_{r,+i} - n_{r,-i})$ which explicitly breaks the $C_4$ symmetry by favoring one of the two orbitals. To explore the effect of strain on transport, we solve the self-consistent large-$N$ equations with a nonzero $\epsilon$ and compute changes in the resistivity $\delta \rho_{\alpha\alpha}$. Fig. 4A shows the computed differential anisotropic elastoresistivity (34)

$$N'_{\epsilon} = \frac{1}{\epsilon} \left( \frac{\delta \rho_{xx}(\epsilon) - \delta \rho_{yy}(\epsilon)}{\rho_{xx}(0) - \rho_{yy}(0)} \right),$$

for different hoppings $t$, corresponding to different cuts through the phase diagram which pass through the nematic metal. Here, $\delta \rho_{\alpha\alpha} = (\rho_{\alpha\alpha}(\epsilon) - \rho_{\alpha\alpha}(0))$ represents the change in resistivity from its unstrained value due to a weak nonzero $\epsilon = 10^{-5}$. We find that $N'_{\epsilon}(T)$ shows a significant increase upon cooling toward the nematic transition, with a peak at the onset of nematic...
order. In this particle–hole symmetric model, we expect \( N_e \) to be dominated by changes in the orbital occupation rather than inducing orbital-dependent scattering rates, so that \( N_e \) reflects changes in the orbital polarization due to strain and is thus tied to the nematic susceptibility. Fig. 4B shows the strain dependence of the differential anisotropic elastoresistivity \( \delta N_e (T) \) for a fixed hopping \( t \), where we compute the resistivity \( \delta \rho_{xx} \) at two nearby strain values \( \epsilon \) (indicated in the plot) and \( \epsilon + d\epsilon \) with \( d\epsilon = 10^{-4} \). We see that with increasing \( \epsilon \), which imposes orbital symmetry breaking, the nematic phase transition gets rounded out. These results on elastotransport bear a striking resemblance to experimental observations on the iron-based materials (35, 37).

We have also studied the effects of \( B_{2g} \) strain on our model by introducing a hybridization term \( H_f = \gamma \sum_i e_i^s c_{i+} \) in the Hamiltonian (Eq. 1). We find that this off-diagonal term acts as a transverse field on the Ising orbital order (65), which can tune the nematic transition and tricritical point to lower temperatures, potentially leading to quantum critical and tricritical points (SI Appendix, Fig. S6). Additional details of this analysis and other results are discussed in SI Appendix, SI 5.

We finally turn to the frequency-dependent elastoconductivity for weak nonzero \( B_{1g} \) strain. Fig. 4C shows plots of \( \omega \Delta \sigma_e (\omega) \) as a function of temperature as we cool into the nematic insulator, where \( \Delta \sigma_e (\omega) = \text{Re}(\sigma_{xx} - \sigma_{yy})/\epsilon \) is the differential anisotropic elastoconductivity obtained from the change in conductivities due to \( \epsilon = 10^{-5} \). We find that \( \omega \Delta \sigma_e (\omega) \) exhibits a bump near \( \omega \sim 0.03 \) (at higher \( T \)), which shifts to lower frequency upon cooling, and is largest near \( T_c \). We expect the location of this peak to track the scattering rate, with the peak height tracking the nematic susceptibility. Indeed, in a simple Drude-like theory, with \( \sigma(\omega) = (ne^2 \tau/m^*)/(1 + \omega^2 \tau^2) \), \( \omega \sigma(\omega) \) peaks at \( \omega = 1/\tau \), with the peak height \( ne^2/2m \), which is independent of the scattering rate \( 1/\tau \). Here, \( n, e, \tau, \) and \( m^* \) refer to the carrier density, charge, lifetime, and effective mass for electrons. It is thus plausible that the peak in \( \omega \Delta \sigma_e (\omega) \) could also be a better measure of the nematic susceptibility, being independent of the scattering rate even in a more general setting where particle–hole symmetry is lost. The second peak in \( \omega \Delta \sigma_e (\omega) \), visible at higher frequency, reflects subtle features in the single-particle spectrum.

**Outlook**

We have proposed a two-orbital model for an nFL which undergoes a \( C_4 \rightarrow C_2 \) rotational symmetry-breaking transition to a nematic phase, and studied its thermodynamic, spectral, and transport properties. We have unveiled a rich phase diagram, where tuning the temperature and orbital anisotropy in the nFL leads to nematic critical or tricritical points. In addition, we have shown that strain fields with different symmetries (\( B_{1g} \) or \( B_{2g} \)) can be used to suppress the nematic ordering, potentially driving quantum critical or tricritical points in the nFL. Our results on the impact of strain on transport in an nFL are of broad interest for a wide range of quantum materials such as twisted bilayer graphene, iron-based superconductors, and underdoped cuprates. Our d.c. elastotransport results show a strong peak in the elastoresistivity as we approach the nematic transition that closely resembles experimental data obtained for the normal state of the iron chalcogenide superconductors (35, 37, 38). Furthermore, our work goes beyond a Boltzmann equation treatment of quasiparticles with impurity scattering near nematic symmetry breaking transitions which is applicable only in the weakly correlated regime (66, 67). Therefore, our work can serve as a useful point of comparison against experiments as well as

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*Fig. 4.* (A) \( N_e (T) = (\delta \rho_{x+} - \delta \rho_{y+})/\epsilon \) is the anisotropic differential elastoresistivity (SI Appendix for details) for \( \epsilon = 10^{-3} \) for various cuts through the nFL-NM transition; the peaks correspond to \( T_c \). (B) Strain dependence of \( \delta N_e (T) \) for \( t = 0.12 \) showing that the transition and hence \( \delta N_e (T) \) gets rounded with increasing strain. (C) \( \omega \Delta \sigma_e (\omega) \) vs. frequency for \( t = 0.12 \), where \( \Delta \sigma_e (\omega) \) is the anisotropic differential elastoconductivity at fixed small strain \( \epsilon = 10^{-3} \). Here, \( \epsilon \tau/\omega \) is the anisotropic differential elastoresistivity (SI Appendix for details) for \( \epsilon = 10^{-3} \) for various cuts through the nFL-NM transition; the peaks correspond to \( T_c \). (B) Strain dependence of \( \delta N_e (T) \) for \( t = 0.12 \) showing that the transition and hence \( \delta N_e (T) \) gets rounded with increasing strain. (C) \( \omega \Delta \sigma_e (\omega) \) vs. frequency for \( t = 0.12 \), where \( \Delta \sigma_e (\omega) \) is the anisotropic differential elastoconductivity at fixed small strain \( \epsilon = 10^{-3} \). Here, \( \epsilon \tau/\omega \) is the anisotropic differential elastoresistivity (SI Appendix for details) for \( \epsilon = 10^{-3} \) for various cuts through the nFL-NM transition; the peaks correspond to \( T_c \). (B) Strain dependence of \( \delta N_e (T) \) for \( t = 0.12 \) showing that the transition and hence \( \delta N_e (T) \) gets rounded with increasing strain. (C) \( \omega \Delta \sigma_e (\omega) \) vs. frequency for \( t = 0.12 \), where \( \Delta \sigma_e (\omega) \) is the anisotropic differential elastoconductivity at fixed small strain \( \epsilon = 10^{-3} \). Here, \( \epsilon \tau/\omega \) is the anisotropic differential elastoresistivity (SI Appendix for details) for \( \epsilon = 10^{-3} \) for various cuts through the nFL-NM transition; the peaks correspond to \( T_c \).
numerical techniques capable of accessing the strongly correlated regime, such as QMC studies of metals undergoing nematic ordering (6, 33). Our predictions for the frequency-dependent elastocconductivity could be tested in future experiments; since the peak in \(\omega \Delta \sigma_x(\omega)\) occurs at the scattering rate, exploring this physics may call for new THz spectroscopic probes in strained quantum materials.

We have also explored competing orders in our two-orbital SYK model via a numerical study using an expanded unit cell, in the spirit of cellular DMFT (61, 62). Since this is a far more challenging numerical computation, we have thus far explored only a limited set of parameters. We find that a staggered nematic metal state, with a “checkerboard” pattern of orbital polarizations, is nearly degenerate with the uniform ferronematic but with a free energy density which is very slightly lower \(\Delta \Omega \propto \epsilon^2/\epsilon^3\). Applying even a small \(B_{1g}\) strain, with \(\epsilon/t \sim 10^{-3}\), already tilts the balance in favor of the uniform ferronematic. These results are presented and discussed in more detail in SI Appendix, SI 6 and Fig. S7. Thus, even if the uniform nematic is a metastable state for the limited set of parameters we have explored, very small modifications to the Hamiltonian may be sufficient to render it the stable ground state. Moreover, going to larger clusters may also impact this competition. Resolving this issue is a topic for future investigation.

Turning to other future research directions, an important question is a careful theoretical understanding of strain-induced nematic quantum critical or tricritical points in the background of the nFL. Another important direction is to study the fate of nematic phases and phase transitions when the SYK couplings in our multiorbital model are made translationally invariant at the microscopic Hamiltonian level rather than in a disorder-averaged sense. This will lead to momentum-dependent self-energies, which could be of potential interest for exploring pseudogap physics in multiorbital systems.

Materials and Methods

Additional details of the analytic methods used for deriving Eq. 4 and the quantities presented in Eq. 7 are provided in SI Appendix. The numerical calculations were performed in Julia and Python using the NumPy library. Further numerical details corresponding to the figures are also included in SI Appendix.

Data, Materials, and Software Availability. All study data are included in this article and SI Appendix.

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Turning to other future research directions, an important question is a careful theoretical understanding of strain-induced nematic quantum critical or tricritical points in the background of the nFL. Another important direction is to study the fate of nematic phases and phase transitions when the SYK couplings in our multiorbital model are made translationally invariant at the microscopic Hamiltonian level rather than in a disorder-averaged sense. This will lead to momentum-dependent self-energies, which could be of potential interest for exploring pseudogap physics in multiorbital systems.

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