Calculation of near-field scanning optical images of excitonic, charged excitation and multiexciton wavefunctions in self-assembled InAs/GaAs quantum dots

Lixin He,1 Gabriel Bester,2 Zhiqiang Su,1 and Alex Zunger2

1Laboratory of Quantum Information, University of Science and Technology of China, Hefei, Anhui 230026, Peoples Republic of China
2National Renewable Energy Laboratory, Golden, Colorado 80401

(Dated: March 23, 2022)

The near-field scanning optical microscopy images of excitonic wavefunctions in self-assembled InAs/GaAs quantum dot are calculated using an empirical pseudopotential method, followed by the configuration interaction (CI) treatment of many-particle effects. We show the wavefunctions of neutral exciton \( X^0 \), of different polarizations, and compare them to those of the biexciton \( XX \) and the charged excitons \( X^+ \) and \( X^- \). We further show that the exciton \( X(P_h \rightarrow S_e) \) transition which is forbidden in the far-field photoluminescence has comparable intensities to that of \( X(S_h \rightarrow S_e) \) transition in the near-field photoluminescence.

PACS numbers: 73.21.La, 71.35.-y, 78.67.Hc

I. INTRODUCTION

The 3D confinement and nearly perfect isolation from its environment given in self-assemble semiconductor quantum dot (QDs) leads to “atom like” electronic structure features manifested, among others, by \( \mu eV \) photoluminescence (PL) line widths and a long coherence time. Such narrow and isolated levels open the way for potential applications using as single-photon emitters1 and quantum-entangled sources.2 At the same time, this allows fundamental studies of \( eV \)-scale fine-structure splitting3,4 of exciton lines due to the electron-hole exchange interaction, spectral shift due to charged excitons5,6,7 as well as multi-exciton formation and decay. However, the spatial resolution of the traditional far-field spectroscopy is limited to \( \lambda/2 \), where \( \lambda \) is the wavelength of the probing light, much larger than the typical sizes of quantum dots (a few tens to about a hundred nanometers). Recently, it became possible to exam the inner structures of an exciton in the QDs, and map out its spatial distribution, using the near-field scanning optical microscopy (NSOM or SNOM)8,9,10. In the far field measurement, the transition intensity is modulated by the shape of the external field, i.e., proportional to \( |P(r)|^2 = |\psi_c(r)\psi_v(r)|^2 \). The resolution of NSOM can thus be much higher than \( \lambda/2 \), providing details on how the carriers are distributed in the quantum dots.

In a QD, the electron and hole wavefunctions can be approximated by products of envelope functions and Bloch functions. In the far field measurement, the interband transitions obey two kinds of selection rules: one related to the Bloch part of the wavefunctions, and one to the envelope component of the wavefunctions. For example, from the four possible \( S_h \rightarrow S_e \) excitonic transitions, two are bright and two are dark. The dark transitions are forbidden because the total angular momentum from the electron and hole Bloch functions parts sum up to two, and can hence not be carried out by a single photon. In contrast, all of the eight \( P_h \rightarrow S_e \) transitions are forbidden because a photon does not couple effectively to the envelope orbital angular momentum of the exciton. However, in NSOM, where the electromagnetic field is applied inhomogeneously only to a part of the dot, the global spatial symmetry is broken, thus altering the selection rules for the envelope functions and allowing transitions that are forbidden in far-field measurement.11,12 We show here that the \( P_h \rightarrow S_e \) transitions (which are far-field forbidden) have comparable intensities in NSOM to the \( S_h \rightarrow S_e \) transitions. Furthermore, NSOM provides additional far-field allowed information about the Bloch part of the wavefunctions, while the far field spectroscopy is lacking. For example, by comparing the transition intensities of different polarizations, one could determine the relative phase between \( |X \rangle \) and \( |Y \rangle \) components of the hole wavefunctions at each point. In contrast, in the far field measurements, only the total polarization is measured and the wavefunction phase information of the

\[
E(r) = \xi (r-r_0) E_0 , \quad (1)
\]
Bloch part is lost.

Recently, Matuda et al. have measured the near-field PL image of naturally occurring GaAs QDs formed spontaneously in narrow quantum wells. The enhanced NSOM spatial resolution is as high as 30 nm, much smaller than the measured QD sizes of ~200 nm. Unfortunately, this spatial resolution is still too low for self-assembled InAs/GaAs quantum dots. In this paper, we predict via atomistic pseudopotential method, the near-field PL images of the neutral exciton $X^0$, the biexciton $XX$ and the charged excitons $X^+, X^-$ in the much smaller self-assembled InAs/GaAs quantum dots, in anticipating of future NSOM measurements.

II. METHODS

A. Far-field vs NSOM transition elements

To obtain the NSOM images, one must first calculate the single-particle wavefunctions for electrons and holes by solving the Schrödinger equations,

$$\left[-\frac{1}{2} \nabla^2 + V_{eff}(\mathbf{r}) \right] \psi_i(\mathbf{r}) = \epsilon_i \psi_i(\mathbf{r}),$$

where $V_{eff}$ is the effective potential for the electrons in the quantum dots. For example, in an effective mass approximation (EMA), the effective potential is usually taken as two-dimensional harmonic potential, whereas in atomistic approaches, $V_{eff}$ is a superposition of screened atomic pseudopotentials. Once one has the electron and hole single-particle wavefunctions, it is possible to construct the many-particle wavefunctions of the excitonic states, e.g., via the configuration interaction (CI) method by expanding them as the linear combination of Slater determinants constructed from the single-particle electron and hole wavefunctions. The ground state can be described by a single determinant,

$$\Phi_0(\mathbf{r}_1, \sigma_1, \cdots, \mathbf{r}_N, \sigma_N) = \mathcal{A}[\psi_1(\mathbf{r}_1, \sigma_1) \cdots \psi_v(\mathbf{r}_v, \sigma_v) \cdots \psi_N(\mathbf{r}_N, \sigma_N)],$$

where $N$ is the total number of electrons in the system, $\sigma$ is the spin index, and $\mathcal{A}$ is the anti-symmetrizing operator. The Slater determinants of neutral excitons $\Phi_{v,c}$ are obtained by promoting an electron at the valence state $\psi_v$ to the conduction state $\psi_c$ from the ground state $\Phi_0$, i.e.,

$$\Phi_{v,c}(\mathbf{r}_1, \sigma_1, \cdots, \mathbf{r}_N, \sigma_N) = \mathcal{A}[\psi_1(\mathbf{r}_1, \sigma_1) \cdots \psi_v(\mathbf{r}_v, \sigma_v) \cdots \psi_c(\mathbf{r}_c, \sigma_c) \cdots \psi_N(\mathbf{r}_N, \sigma_N)].$$

The $\alpha$-th neutral exciton wavefunction can be written as,

$$\Psi^{(\alpha)}_X = \sum_{v=1}^{N_v} \sum_{c=1}^{N_c} C_{v,c}^{(\alpha)} \Phi_{v,c},$$

where $N_v$ and $N_c$ are the number of valence and conduction states included in the expansion of the exciton wavefunctions respectively. The coefficients $C_{v,c}^{(\alpha)}$ are obtained by diagonalizing the many-particle Hamiltonian $H$ in the basis set $\{\Phi_{v,c}\}$.

The far-field optical absorption of the $\alpha$ transition can be written as,

$$\sigma(\omega) \propto \sum_{\alpha} |M^{(\alpha)}|^2 \delta(h\omega - E^{(\alpha)}),$$

where, $M^{(\alpha)}$ are the dipole matrix elements between initial many-particle state $|\Psi_i\rangle$ and the final state $|\Psi_f\rangle$,

$$M^{(\alpha)} = \langle \Psi_f | \mathbf{r} | \Psi_i \rangle.$$  

$\omega$ and $E^{(\alpha)}$ are the frequency of the external E-field and the exciton transition energy, respectively. For a neutral exciton, the dipole matrix elements $M$ can be written in terms of single-particle orbitals as,

$$M^{(\alpha)} = \langle \Phi_0 | \mathbf{r} | \Psi_X \rangle = \sum_{v=1}^{N_v} \sum_{c=1}^{N_c} C_{v,c}^{(\alpha)} \langle \psi_v | \mathbf{r} | \psi_c \rangle,$$

where, $\mathbf{p} = \langle \psi_v | \mathbf{r} | \psi_c \rangle$ are the single-particle dipole matrix elements between the conduction state $\psi_c$ and valence state $\psi_v$. Similarly, one can calculate $M$ for charged excitons $X^+, X^-$ and biexciton $XX$.

For NSOM, the external E-field is applied locally at $\mathbf{r}_0$ modulated by the electromagnetic profile of the tip $\xi(\mathbf{r} - \mathbf{r}_0)$, which can be taken as a Gaussian function, e.g.,

$$\xi(\mathbf{r} - \mathbf{r}_0) = c e^{-|\mathbf{r} - \mathbf{r}_0|^2/a^2},$$

where $a$ is the width of the excitation field, much smaller than the dimension of the QD. For such local optical transition, we have to replace $M^{(\alpha)}$ in Eq. (8) by

$$M^{(\alpha)}(\mathbf{r}_0) = \sum_{v=1}^{N_v} \sum_{c=1}^{N_c} C_{v,c}^{(\alpha)} \langle \psi_v | \mathbf{r}_0 \xi(\mathbf{r} - \mathbf{r}_0) | \psi_c \rangle,$$

equivalent to replacing in Eq. (8) the single-particle dipole moment $\mathbf{p} = \langle \psi_v | \mathbf{r} | \psi_c \rangle$ by the local single-particle dipole moment $\mathbf{p}(\mathbf{r}_0) = \langle \psi_v | \mathbf{r}_0 \xi(\mathbf{r} - \mathbf{r}_0) | \psi_c \rangle$.

B. Evaluation of the NSOM transition elements: an atomistic approach

In the present work, we calculate the single-particle energy levels and wavefunctions in a atomistic screened
calculated exciton energies

FIG. 1: The exciton energies of the lowest neutral exciton $X^0$, the charged excitons $X^+$, $X^-$ and the biexciton $XX$ of self-assembled InAs/GaAs QD with base $b=27.5$ nm and height $h=3.5$ nm. The right hand side shows the schematic energy levels of fine structure splittings of each state. The solid lines ($B1$, $B2$, etc.) denote bright ($B$) states, whereas the dashed lines($D1$, $D2$, etc.) denote dark ($D$) states.

We use the lowest conduction bands for electrons, and three highest valence bands for holes. Instead of calculating the local dipole matrix elements using Eq. (11), we project $\psi(\mathbf{r})$ onto (strained) Bloch functions of InAs at the $\Gamma$ point in the Brillouin zone,

$$\psi(\mathbf{r}) = \sum_{m=1}^{N_p} f_m(\mathbf{r}) u_{\Gamma,m}(\mathbf{r}).$$  \hspace{1cm} (12)$$

To simplify the notation, we drop the material index $\lambda$ and strain index $\varepsilon$ in Eq. (12). Here, $N_p$ is the number of the projection bands, which is usually much larger than the number of bands used in the LCBB equation. We found that $N_p = 9$ provide accurate projections. Here, $f_m(\mathbf{x})$ is the envelope function for the $m$-th band. The envelope functions are slow varying functions, compared to the Bloch functions. We can therefore separate the Bloch functions and envelope functions by dividing the volume into small regions, e.g. the 8-atom unit cells. In each unit cell, the envelope functions $f_m(\mathbf{x})$ and $\xi(\mathbf{r})$ can be treated as constants, i.e.,

$$p(\mathbf{r}_0) = \int d\mathbf{r} \psi^*_0(\mathbf{r}) \psi(\mathbf{r}) \mathbf{r} \xi(\mathbf{r} - \mathbf{r}_0)$$ \hspace{1cm} (13)$$

where, $\mathbf{r}_i$ is the position of $i$-th cell and

$$d_{m,m'} = \frac{1}{V} \int d\mathbf{r} \ u^*_{\Gamma,m}(\mathbf{r}) \mathbf{r} u_{\Gamma,m'}(\mathbf{r}),$$ \hspace{1cm} (14)$$

is the dipole matrix element between bulk bands. In a periodic system, it is more convenient to use an alternative formula to calculate the dipole,$^{17}$

$$d_{m,m'} = \frac{1}{m} \int d\mathbf{r} \ u^*_{\Gamma,m}(\mathbf{r}) \mathbf{p} u_{\Gamma,m'}(\mathbf{r}).$$  \hspace{1cm} (15)$$

If we assume high spatial NSOM resolution, i.e., $\xi(\mathbf{r}_i - \mathbf{r}_0) = \delta(\mathbf{r}_i - \mathbf{r}_0)$, Eq. (14) can be further simplified as,

$$p(\mathbf{r}_0) \approx \sum_{m,m'} f^*_m(\mathbf{r}_0) f_m(\mathbf{r}_0) d_{m,m'}. $$  \hspace{1cm} (16)$$

Combining Eq. (10) and Eq. (15), we see that the shape of NSOM transition is the sum of the (multi-band) envelope functions of the exciton weighted by the (single-particle) dipole matrix elements, i.e.,

$$M^{(\alpha)}(\mathbf{r}_0) = \sum_{v=1}^{N_v} \sum_{c=1}^{N_c} C_v^{(\alpha)} \left[ \sum_{m,m'} f^*_m(\mathbf{r}_0) f_m(\mathbf{r}_0) d_{m,m'} \right].$$  \hspace{1cm} (17)$$

Previously, the NSOM images have been calculated via single-band effective mass approximation (EMA).$^{11,18,19}$ In this approximation, the dipole moment $\mathbf{d}$ is directly taken from bulk materials or from quantum wells,$^{18}$ while in our case, it is calculated for nine bands in Eq. (15) and is specific to the nanostructure considered. In the single-band EMA, only the envelope part of the information is
kept in the calculation of NSOM images, whereas detailed information associated with the Bloch functions, such as polarizations is lost. In the more advanced \(k \cdot p\) methods, one includes the heavy-hole, light-hole, and spin-orbit bands at \(\Gamma\) point of the Brillouin zone in the calculations. For the direct gap materials, these approximations give qualitative right results, but for indirect gap materials, such Si quantum dots, one need to include many more bands.

III. RESULTS AND DISCUSSION

A. The excitonic spectrum of the InAs/GaAs QD

We performed calculations on a lens-shaped InAs/GaAs quantum dot with 27.5 nm base diameter and 3.5 nm height. The calculated transition energies of the exciton \(X^0\), the charged excitons \(X^-\) and \(X^+\) and the biexciton \(XX\) of \(S_h \rightarrow S_e\) transitions are given in Fig. 1. The neutral \(X^0\) has the largest transition energy of 981 meV, while charged excitons and biexcitons have 1 - 3 meV red shifts relative to the neutral exciton, due to correlation effects\(^{20}\) in agreement with experiments.\(^{6,7}\) For the neutral exciton, the exciton line splits into four lines shown on the righthand side of Fig. 1 as a consequence of the asymmetric electron-hole exchange energies.\(^{21}\) The two low lying lines labeled \(D_1, D_2\) are “dark states” (forbidden due to the angular momentum selection rule of the Bloch part of the electron/hole states), whereas the upper two states \((B_1, B_2)\) are bright states. The calculated energy splitting between the bright and dark states is about 210 µeV. The energy splitting between two dark states \(D_1, D_2\) is about 1 µeV, whereas the splitting between two bright states \(B_1, B_2\) is about 10 µeV, in agreement with previous calculations.\(^{20}\) The low energy bright state \(B_1\) is polarized along the [110] direction, whereas the high energy state \(B_2\) is polarized along the [1 1 0] direction. Although the biexciton \(XX\) states themselves have no fine structures, the \(XX\) to \(X\) transition exhibit fine structures. The charged excitons \(X^+\) and \(X^-\) have no fine structures.

The energy of neutral exciton \(P_h \rightarrow S_e\) transition is calculated here 995 meV, also shown in Fig. 1. Like the \(S_h \rightarrow S_e\) transition, the \(P_h \rightarrow S_e\) transition also split into four fine structures. The two lower energy states are “forbidden” by the Bloch function angular momentum selection rule, whereas the two high energy states \(B_1\) and \(B_2\) are allowed by the Bloch part. In the far field measurement, the \(P_h \rightarrow S_e\) transition is forbidden for the lens-shaped QDs, due to the envelope function angular momentum selection rule (i.e., the overlap in-
FIG. 3: (Color online) The NSOM of excitonic $P_h \rightarrow S_e$ transitions (see Fig. 1) in self-assembled InAs/GaAs QD with $b=27.5$ nm and $h=3.5$ nm, calculated from Eq. (17) for (a) $B1$ and (b) $B2$ states. The unit of the color-bar is arbitrary but equal for all sub-figures. In each sub-figure, there are six panels showing the transition intensities of different polarizations. In the first row, the polarizations are along the [100], [010] and [001] direction respectively, and in the second row, the polarizations are along the [110] and [100] directions. The last panels show the sum of transition intensities of different polarizations. We magnify the transition intensities by the factors on the upper-right corner of each panel. The white circles (27.5 nm in diameter) show the boundaries the QDs.

The NSOM images of excitonic $P_h \rightarrow S_e$ transition of $X^0$ are shown in Fig. 2. We show the images of all fine structures components $B1$, $B2$, $D1$, $D2$ in Figs. 2(a), 2(b), 2(c) and 2(d) respectively. The color-bars indicate the magnitudes of transition intensities. We use arbitrary but equal units for the transition intensity in all sub-figures. Each sub-figure is composed of six panels showing the transition intensity of different polarizations. The first row in each sub-figure shows the transition of polarization along the [100], [010] and [001] directions, whereas the second row of each sub-figure shows the transition of polarization along the [110] and [100] directions. The final panel is the total transition intensity, being the sum of intensities over polarization along the [100], [010] and [001] directions. (The sum of intensities over the [100], [010] polarization equals the sum over the [110] and [110] polarization.) We magnified the transition intensity by the factor shown on the upper right corner of each panel, so the transition intensities in all sub-figures can be in similar scale.

Considering first the total transition intensities, we notice that the transitions originate from a relatively small region around the center of the dot. The transition intensities of dark states $D1$ [Fig. 2(a)] and $D2$ [Fig. 2(b)] are about 2 - 3 orders of magnitude lower than the those of bright exciton states $B1$ [Fig. 2(c)] and $B2$ [Fig. 2(d)], because they are “forbidden” due to angular momentum selection rule for the heavy-hole to conduction band transitions. Therefore the shapes of $D1$ and $D2$ are determined by the shape of higher bands [see Eq. (10)]. In contrast, the bright states $B1$ and $B2$ have $S$-like shape as expected. However, the NSOM of $B1$ and $B2$ are not perfectly round, even though the lens-shaped QD we choose has cylindrical symmetry: we see that $B1$ is elongated along the [110] direction, whereas $B2$ is elongated along the [010] direction.

We next discuss the NSOM images of different polarizations, focusing on the bright states. For far field excitation, the low energy bright exciton $B1$ is calculated to be polarized along the [110] direction, whereas the high energy bright exciton $B2$ is polarized along the [110] direction. This is clearly seen in the NSOM plots of Fig. 2 by comparing the [110] polarized plots to the [100] polarized plot. However, NSOM can provide much more information than the usual far field measurement: The dipole moments have the following relations, $p_{[100]} = (p_{[100]} + p_{[010]})/\sqrt{2}$ and $p_{[110]} = (p_{[100]} - p_{[010]})/\sqrt{2}$. Therefore, by comparing the NSOM images of different polarization, we can obtain the relative phases between $p_{[100]}$ and $p_{[010]}$ at each spatial point. For the bright states of $S_h \rightarrow S_e$ transitions, the relations are simple, i.e., $p_{[100]}$ and $p_{[010]}$ have almost fixed phases $\pi/2$ everywhere inside the QD. As we know, the $p_{[100]}$ corresponding to the $\langle X \rangle$ component in the hole Bloch functions, whereas $p_{[010]}$ corresponding to the $\langle Y \rangle$ component. We therefore obtain additional information about the character of the hole Bloch functions as a function of position inside the QDs.
Figure 2 also show the NSOM image of polarization along the [001] direction. For the bright exciton, the [001] components are significantly weaker than the components polarized in (001)-plane, and may not be detectable in practice. However, they are of theoretical interest. The polarization along the [001] direction come from the emission of light hole. Therefore, the [001] image shows how the light hole components distributed in the QDs.

C. NSOM \( P_h \rightarrow S_e \) transition for neutral exciton in the InAs/GaAs QD

Figure 3 depicts the \( P_h \rightarrow S_e \) transitions, showing the two bright states (\( B_1, B_2 \)) out of the four fine-structure split states. Similar to Fig. 2 we show the NSOM images with polarization along the [100], [010], [001] directions in the first row, and those with the [110], [1\( \bar{1} \)0] polarization and the total transition intensities in the second row. We see that the \( P_h \rightarrow S_e \) transitions have comparable intensities to those of the \( S_h \rightarrow S_e \) transitions. The shape of total transition intensity of the \( B_1 \) state somewhat differs from the intensity from the \( B_2 \) state. However, both states have ring like structures, and both states have two peaks in the [110] direction.

Each of the polarization-resolved plots looks like orbital-\( p \) functions, although the transition peaks are oriented in different directions. For example, for the \( B_1 \) state, the peaks of the [100] ([010]) polarized light are oriented along the [100] ([010]) direction, whereas the peaks of [110] ([1\( \bar{1} \)0]) polarized light are oriented in [110] ([1\( \bar{1} \)0]) direction. As discussed in Sec. III B, we can obtain the phase relations of \(|X\rangle\) and \(|Y\rangle\) components in hole wavefunctions. We see that unlike the previous case involving an \( S_h \) wavefunction, in the Bloch part of the \( P_h \) wavefunctions, \(|X\rangle\) and \(|Y\rangle\) components have different phases at each point of the QDs. Similar results are found for the \( B_2 \) state.

D. Comparison of NSOM of \( X^0, X^+, X^- \) and XX excitons in the InAs/GaAs QD

In Figs. 4 (b), (c), (d) we depict the NSOM images of the charged exciton \( X^+ \), \( X^- \) and the biexciton XX, respectively, and compare them to the \( X^0 S_h \rightarrow S_e \) transition from Fig. 2 (a). We also plot the NSOM transition intensities along the [110] direction in Fig. 4 for \( X^0, X^+, X^- \) and XX. The transition intensities of \( X^0, X^+, X^- \) and XX along [110] direction are very similar to those along the [110] direction, and therefore are not plotted.

![compare NSOM transition of exciton complexes](image-url)
FIG. 5: (Color online) The NSOM transition intensities of $X^0$ (black solid line), $X^+$ (red dashed), $X^-$ (green dashed) and $XX$ (blue dashed) along the [110] direction in self-assembled InAs/GaAs QD with $b=27.5$ nm and $h=3.5$ nm, calculated from Eq. (17). The origin is at the center of the dot.

We see that $X^-$ has the strongest transition intensity, followed by the $X^0$, $XX$ and $X^+$. Even though, the charged excitons and the biexciton have 1 - 3 meV red shifts relative to $X^0$, they have very similar shapes and sizes to $X^0$. The only noticeable difference being that $X^+$ has a slightly larger size than the other transitions. In Ref. 9, it was found experimentally and confirmed theoretically by single-band effective mass calculations\textsuperscript{18,19} that the radius of the biexciton NSOM signal is smaller than the radius of $X^0$. In this calculation, we did not find a significant difference in size and shape between $X^0$ and $XX$. However, the QDs used in the experiments (created by composition fluctuations in a quantum well) are much larger ($\sim 200$ nm in diameter) than the self-assembled QDs we studied here, therefore the correlation effects are stronger. On the other hand, we ignored the continuum states in the CI calculations, which may also underestimate the correlations effects to the wavefunctions.

IV. SUMMARY

We calculate the NSOM images of exciton complexes in self-assembled InAs/GaAs QD using an atomistic pseudopotential method followed by a configuration interaction treatment of the correlations. The NSOM images reveal the spatial structure of exciton and give information about the Bloch function character of the wavefunction, which remain unobserved in conventional far-field measurements. We show that the $P_h \rightarrow S_e$ transition, which is forbidden in the far field PL measurement, has comparable transition intensity to that of the $S_h \rightarrow S_e$ transition in NSOM measurements. We also calculate the NSOM image of charged exciton $X^+$, $X^-$, and biexciton $XX$, and compare them with the image of $X^0$. We found that the images obtained for charged excitons and the biexciton show very similar features to the ones obtained from the exciton decay.

Acknowledgments

This work is supported by the Chinese National Fundamental Research Program, the Innovation funds and “Hundreds of Talents” program from Chinese Academy of Sciences, and National Natural Science Foundation of China (Grant No. 10674124). The work done at NREL was funded by the U.S. Department of Energy, Office of Science, Basic Energy Science, Materials Sciences and Engineering, LAB-17 initiative, under Contract No. DE-AC36-99GO10337 to NREL.

1. P. Michler, A. Kiraz, C. Becher, W. V. Schoenfeld, P. M. Petroff, L. Zhang, E. Hu, and A. Imamoglu, Science 290, 2282 (2000).
2. N. Akopian, N. H. Lindner, E. Poem, Y. Berlatzky, J. Avron, D. Gershoni, B. D. Gerardot, and P. M. Petroff, Phys. Rev. Lett. 96, 130501 (2006).
3. D. Gammon, E. S. Snow, B. V. Shanabrook, D. S. Katzer, and D. Park, Phys. Rev. Lett. 76, 3005 (1996).
4. M. Bayer, A. Kuther, A. Forchel, A. Gorbunov, V. B. Timofeev, F. Schäfer, J. P. Reithmaier, T. L. Reinecke, and S. N. Walck, Phys. Rev. Lett. 82, 1748 (1999).
5. B. Urbaszek, R. J. Warburton, K. Karrai, B. D. Gerardot, P. M. Petroff, and J. M. Garcia, Phys. Rev. Lett. 90, 247403 (2003).
6. D. V. Regelman, E. Dekel, D. Gershoni, E. Ehrenfreund, A. J. Williamson, J. Shumway, A. Zunger, W. V. Schoenfeld, and P. M. Petroff, Phys. Rev. B 64, 165301 (2001).
7. A. Hartmann, Y. Ducommun, E. Kapon, U. Hohenester, and E. Molinari, Phys. Rev. Lett. 84, 5648 (2000).
8. G. W. Bryant, Appl. Phys. Lett. 72, 768 (1998).
9. K. Matsuda, T. Saiki, S. Nomura, M. Mihara, Y. Aoyagi, S. Nair, and T. Takagahara, Phys. Rev. Lett. 91, 177401 (2003).
10. F. Flack, N. Samarth, V. Nikitin, P. A. Crowell, J. Shi, J. Levy, and D. D. Awschalom, Phys. Rev. B 54, R17312 (1996).
11. U. Hohenester, G. Goldoni, and E. Molinari, Phys. Rev. Lett. 95, 216802 (2005).
A. J. Williamson, L.-W. Wang, and A. Zunger, Phys. Rev. B 62, 12963 (2000).
A. Franceschetti, H. Fu, L.-W. Wang, and A. Zunger, Phys. Rev. B 60, 1819 (1999).
P. N. Keating, Phys. Rev. 145, 637 (1966).
J. L. Martins and A. Zunger, Phys. Rev. B 30, R6217 (1984).
L.-W. Wang and A. Zunger, Phys. Rev. B 59, 15806 (1999).
D. Dragoman and M. Dragoman, Optical characterization of solids (Springer Verlag, 2002).
S. V. Nair and T. Takahara, Phys. Rev. B 55, 5153 (1997).
U. Hohenester, G. Goldoni, and E. Molinari, Appl. Phys. Lett. 84, 3963 (2004).
G. Bester and A. Zunger, Phys. Rev. B 68, 073309 (2003).
G. Bester, S. V. Nair, and A. Zunger, Phys. Rev. B 67, 161306 (2003).
G. A. Narvaez and A. Zunger, Phys. Rev. B 74, 045316 (2006).