Studying method of microlensing effect estimation for a cluster of primordial black holes.

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Abstract. Primordial black holes (PBH) attracted especially great attention last time. It is possible candidate not only to dark matter, but to supermassive black holes, gravitational waves events from black hole merger and others. However, recently there appeared constraints on PBH abundance from different observations. The given work is devoted to the model of PBH cluster in which some constraints can be avoided. We investigate effect of gravitation microlensing for PBH cluster. Analysis of respective data could alleviate limitations coming from them, but also could make it possible to test this model discriminating between the models of single and clustered PBHs.

1. Introduction
There are several fundamental problems of physics and cosmology, which are now trying to be solved with the help of primordial black holes (PBH). These are

- the problem of dark matter [1, 2],
- the problem of the origin of supermassive black holes (early quasars) [3, 4, 5],
- the problem of explanation of the origin of the merged black holes recently registered by LIGO and Virgo [6, 7, 8].

Also, reionization of the Universe, nature of unidentified cosmic gamma-ray sources can be partially explained by PBH [9, 10, 11]. See recent review [12].

One of the most, possibly, viable model is the model of a cluster of PBH [12, 13]. Within this model, black holes are born in the early Universe at the stage soon after cosmological inflation in the result of phase transition. Such a cluster has certain PBH distributions density in space and in mass. Therefore, the restrictions obtained for single black holes (homogeneously distributed in space) with monochromatic functional mass distribution should be revised [1].

One of the strongest limitation comes from gravitational microlensing. It is received by observing stars from nearby galaxies such as Andromeda, Magelan Clouds and others. In the case, when the line of sight is intersected by a black hole in our Galaxy, an increase of the observed brightness of the star occurs.

Now there is a large set of data from different experiments on the observation of such microlensing events (MACHO, EROS, OGLE, HSC, Kepler and etc). If the cluster of PBHs crosses the line of sight, the effect can be very different.
2. Theoretic background
As is known, the trajectory of light near massive bodies differs from the straight line. The equation of the geodesic line, including one of the light, follows from variation of trajectory between two points [14, 15].

\[ \delta \int_{A}^{B} ds = \int_{A}^{B} \delta(ds). \]  

(1)

It leads to equations for the geodesic line

\[ \frac{d^2 x^\lambda}{ds^2} = -\Gamma_{\mu\nu}^{\lambda} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}, \]  

(2)

where

\[ \Gamma_{\mu\nu}^{\lambda} = \frac{g^{\lambda\sigma}}{2} \left( \frac{\partial g_{\sigma\nu}}{\partial x^\mu} + \frac{\partial g_{\mu\sigma}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right). \]  

(3)

In the simplest case, \( g_{\mu\nu} \) corresponds to the spherically symmetric metric of Schwarzschild for point-like mass

\[ ds^2 = \left( 1 - \frac{2GM}{c^2 r} \right) (dx^0)^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \]  

(4)

Since the PBH cluster also has spherical symmetry in the first approximation, one can use it.

Analyzing the geodesic in the Schwarzschild metric we find the angle of light deviation [16]:

\[ \phi = \frac{4GM}{c^2 R}, \]  

(5)

where \( R \) is the impact parameter.

For a spherically symmetric gravitational lens, we can use formula (5) if we replace \( M \) by \( M(R) \) — the mass enclosed by sphere of radius \( R \). So, one has [17, 16]

\[ \phi = \frac{4GM(R)}{c^2 R}. \]  

(6)

3. Calculation method
Modeling of light passing through the PBH cluster is done as follows. Rays are generated from a light source at different angles. Depending on the part of the cluster which the light crosses, effective mass \( M(R) \) and the impact parameter \( R \) are calculated. Then the formula (6) defines the angle of deflection \( \phi \) and a trajectory of light is built.

The number of beams arriving at the observation point gives the value of the amplitude of the signal at this point. Figure 1 shows the result one of the simulation of the beams coming through the cluster gravitation field.

Also the histogram figure 2 is drawn, which shows an example how the signal amplitude can vary when the lens moves. Using this, one can try to predict the expected signal change if the lens is PBH cluster at different parameters, and to make cross-check of the result obtained by other methods.

4. Result
We elaborated the method for the study of gravitational microlensing effect induced by the PBH cluster. It may open an opportunity to distinguish between the cases of a single PBH model and that of the cluster.
Figure 1. Simulation of light rays passing through the PBH cluster. The light source is at (0, 0), the lens is at the (0, 1.5) point.

Figure 2. This is example of the time profile of the signal amplitude for microlensed PBH cluster at the one of the chosen parameters.
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