Comment on Experiments Related to
the Aharonov-Bohm Phase Shift

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Abstract

Recent experiments undertaken by Caprez, Barwick, and Batelaan should clarify the connections between classical and quantum theories in connection with the Aharonov-Bohm phase shift. It is pointed out that resistive aspects for the solenoid current carriers play a role in the classical but not the quantum analysis for the phase shift. The observed absence of a classical lag effect for a macroscopic solenoid does not yet rule out the possibility of a lag explanation of the observed phase shift for a microscopic solenoid.
A. Introduction

For the past thirty-five years, it has been suggested repeatedly that the experimentally observed Aharonov-Bohm phase shift\(^1\)\(^2\)\(^3\) may arise from a classical electromagnetic lag effect involving velocity changes for charged particles passing a solenoid.\(^4\)\(^5\)\(^6\)\(^7\)\(^8\)\(^9\)\(^10\)\(^11\)\(^12\) In recent years, these suggestions have been dismissed as being without merit by the editors at the leading physics journals. The suggestions are dismissed despite the fact that these semiclassical energy arguments predict precisely the observed phase shift and despite the fact that the classical electromagnetic interaction of a point charge and a solenoid remains poorly understood.

Within the past year, some of the ideas associated with the proposed classical lag effect have come under experimental test from work by Caprez, Barwick, and Batelaan.\(^13\) These experimenters observe no lag effect for electrons passing a macroscopic solenoid.\(^13\) Of course, the Aharonov-Bohm phase shift has never been observed for such a macroscopic solenoid, so this experiment does not yet rule out a classical lag effect as the basis for the Aharonov-Bohm phase shift.

Indeed, the experiment with a macroscopic solenoid emphasizes the complete contrast between the theoretical treatment of the Aharonov-Bohm phase shift now appearing in all the recent quantum textbooks\(^14\) and the classical electromagnetic analysis suggesting a lag effect. According to presently accepted quantum theory, the Aharonov-Bohm phase shift arises from an enclosed magnetic flux with no need to discuss any interaction between the passing charges and the sources of the magnetic flux. In contrast, the classical lag analysis depends crucially upon the details of the interaction between the passing charged particles and the current-carriers of the magnetic flux.\(^11\)

One may ask how two such totally different points of view can lead to the same experimental prediction. The relative success of the contrasting points of view may reflect a situation analogous to that when a charged particle passes a conductor. As long as the conductor is a good conductor, the forces on a passing charged particle due to image charges will involve only the physical shape of the conductor and not the detailed composition of the conductor. However, if the conductivity is imperfect or if the induced charges correspond to dielectric behavior, then more information regarding the interaction of the passing charge and the material will be required to account for the behavior of a passing charged particle.
In a similar vein for the Aharonov-Bohm phase shift, it may well be that when the resistive energy loss of a solenoid is small, then all solenoids behave in the same way regarding energy conservation for a passing charged particle. However, when the resistive energy loss is large, then the energy-conserving interaction becomes negligible and the interaction of the particle and solenoid becomes quite different.

B. Summary of the Classical Aspects of the Aharonov-Bohm Situation

The magnetic energy of interaction of a charged particle $q$ passing a solenoid with constant currents is given by

$$U = \frac{q}{c} v_q \cdot A(r_q)$$  \hspace{1cm} (1)

where $v_q$ is the velocity of the passing charge and $A(r_q)$ is the vector potential in the Coulomb gauge of the solenoid evaluated at the position of the passing charge. If this magnetic interaction energy is compensated by a change in the kinetic energy of the passing charge $mv_q \Delta v_q = -U$, then there is a relative spatial lag

$$\Delta Y = \frac{q \Phi}{mv_q c}$$  \hspace{1cm} (2)

between particles passing on opposite sides of the long solenoid of flux $\Phi$, and the associated semiclassical phase shift is exactly that predicted by Aharonov and Bohm

$$\Delta \phi = \frac{mv_q \Delta Y}{\hbar} = \frac{q \Phi}{c \hbar}$$  \hspace{1cm} (3)

Associated with this spatial lag is also a relative time lag

$$\Delta t = \frac{\Delta Y}{v_q} = \frac{q \Phi}{mv_q^2 c}$$  \hspace{1cm} (4)

between particles passing on opposite sides of the solenoid. However, the currently accepted dogma is that the Aharonov-Bohm phase shift occurs without any velocity changes for the passing charged particles. The experiments recently undertaken by Herman Batelaan may allow one to distinguish between these two competing points of view regarding the experimentally observed phase shift. The classical electromagnetic description based upon kinetic energy changes predicts the time delay in Eq. (4) for charges passing on opposite sides of the solenoid. In the first, and easiest experiment, Caprez, Barwick and Batelaan,
looked for a time delay for electrons passing a macroscopic solenoid. No delay was found although the experimental accuracy was quite sufficient to detect a delay of the magnitude in Eq. (4) predicted by energy conservation.\cite{13}

If one accepts the current quantum point of view that all solenoids should behave the same way regarding passing charged particles, then this experiment already implies the failure of the classical lag explanation. However, classical physics indicates that not all solenoids will interact in the same way with passing charges. The relative magnitude of frictional (resistive) effects plays an important role in classical descriptions of multiparticle systems.

C. Analogue Involving Electromagnetic Induction

In order for the observed Aharonov-Bohm phase shift to be based upon a change in kinetic energy for a passing charge, we must have an electric field back at the passing charge generated by induced currents in the solenoid which are caused by the electric field of the passing charged particle.\cite{9} Since a long constant-current solenoid has no electric or magnetic fields outside its winding, any such back electric field must arise from accelerations of the solenoid charges in response to the electric field of the passing charge. Exactly such behavior has been calculated for the interaction of a passing charge and a classical hydrogen atom\cite{11} where the electromagnetic fields correspond to those of the Darwin Lagrangian\cite{17}

\[
E = \frac{q}{r^2} \left( \frac{1}{2} \frac{v^2}{c^2} - \frac{3}{2} \frac{(v \cdot \hat{r})^2}{c^2} - \frac{q}{2c^2 r} (a + a \cdot \hat{r} \hat{r}) \right)
\]

\[
B = \frac{q}{c} \frac{v}{r^2} \times \frac{\hat{r}}{r^2}
\]

(5)

(6)

giving the interaction of charged particles accurately through order $1/c^2$ when radiation is neglected. However, there has been no classical calculation for a charged particle passing a multiparticle solenoid.

We may come closest to understanding what is involved for a solenoid by examining (as an analogy) a situation involving electromagnetic induction where the accelerations of charged particles are again the crucial element. Suppose we had a magnetic dipole $\vec{\mu} = \hat{k} \mu$ oriented along the $z$-axis and moving with initial velocity $\vec{v} = \hat{k} v$ along the $z$-axis. The magnetic dipole moves toward a set of $N$ equally-spaced charged particles in the plane $z=0$, each of charge $e$ and mass $m$, which are constrained to move on a circular loop of radius $r$ with
center on the z-axis. The moving magnetic moment $\vec{\mu}$ generates an electric field acting on each charge $e$ given by

$$E_\phi = -[\vec{v}_\mu \times \vec{B}_\phi / e] = -\frac{3 v_\mu}{c} \frac{r z}{(z^2 + r^2)^{5/2}}$$  \hspace{1cm} (7)$$

The nonrelativistic equation of motion for each charge $e$ (neglecting radiation) is

$$mr \frac{d^2 \phi}{dt^2} = -m \gamma r \frac{d \phi}{dt} + \sum_{j=2}^{j=N} e^2 \frac{d^2 \phi}{dt^2} \left( 2 \sin \left( \frac{\pi j}{N} \right) \right)^{-1} \left[ \cos \left( \frac{2 \pi j}{N} \right) + \sin^2 \left( \frac{2 \pi j}{N} \right) \right] + eE_\phi$$  \hspace{1cm} (8)$$

where $mr(d^2 \phi/dt^2)$ is the particle mass times acceleration, $-m \gamma r(d\phi/dt)$ represents the frictional force on the particle due to the constraining ring, the term with the sum involves the electric acceleration fields in Eq. (5) of all the other charges on the ring acting on the charge whose acceleration is being discussed, and $eE_\phi$ is the electric force in Eq. (7) due to the approaching magnetic dipole. The motions of the charges $e$ around the ring in turn cause a magnetic field $\vec{B}_e$, which along the z-axis is given by

$$\vec{B}_e = \sum \frac{e v_e \times \hat{r}}{c} = \hat{k} N \frac{e}{c} \frac{r^2}{(z^2 + r^2)^{3/2}} \frac{d \phi}{dt}$$  \hspace{1cm} (9)$$

which acts back on the approaching magnetic dipole $\vec{\mu}$ with a force

$$F_x = \frac{\partial}{\partial z} (\vec{\mu} \cdot \vec{B}_e) = -3 \mu N \frac{e}{c} \frac{2 r^2}{(z^2 + r^2)^{4/2}} \frac{d \phi}{dt}$$  \hspace{1cm} (10)$$

Here we have a clear physical situation involving accelerations of the particles on the ring which in turn put a force back on the approaching magnetic moment. The situation has an analogy to the classical interaction of a point charge and a solenoid; both situations depend on the electric field associated with the moving object to produce accelerations of the charges of a multiparticle system, and then the force back on the moving object arises due to changes in the multiparticle motions.

The energies for the situation of Eqs. (7)-(10) involve the kinetic energy of the magnetic dipole $\vec{\mu}$, the kinetic energy of the ring particles $e$, the magnetic energy of the ring particles, and the magnetic interaction energy between the magnetic field of the ring particles and the magnetic field of the magnetic dipole $\vec{\mu}$. If the particles $e$ on the ring are initially at rest, then any energy transferred to these particles must come from the approaching magnetic dipole. If the magnetic dipole passes through the center of the circular ring of particles, then we expect that the magnetic moment will be slowed down during its passage and hence
that there will be a relative time delay associate with the interaction of the particles on the ring compared to a magnetic dipole $\vec{\mu}$ which did not encounter the ring.

We notice immediately that the acceleration $d^2\phi/dt^2$ in Eq. (8) appears as a multiplicative factor both for the particle mass $m$ and for sum term arising from the electric fields due to the accelerations of the other charges $e$. Thus if there are many charges so that this sum term is enormous, then one can neglect the particle masses $m$ compared to the self-inductance effects associated with the mutual interactions of the ring charges $e$. Similarly, the particle kinetic energy and the magnetic energy of self-inductance both depend upon the square of the velocities $r^2(d\phi/dt)^2$of the charges $e$. We can see that we expect different behavior when the friction constant $\gamma$ is large compared to when it is small. If the friction constant $\gamma$ is very small, then we have a situation involving energy conservation during the interaction of the magnetic moment and the particles $e$ of the ring. We expect that the the particles $e$ of the ring will accelerate in response to the electric field $E_\phi$ of the approaching magnetic dipole $\vec{\mu}$, so that kinetic energy will be removed from the magnetic dipole on its approach, and then, after the magnetic dipole passes through the ring, the kinetic energy of the magnetic dipole will be restored. The interaction of the magnetic dipole and the ring will be evident in a relative time lag compared to a magnetic dipole which continued with constant velocity. On the other hand, if the friction constant $\gamma$ is very large, then from Eq. (8) we see that the velocities $r d\phi/dt$ of particles on the ring will be very small since the velocities depend inversely on $\gamma$. But the force back on the approaching magnetic dipole depends upon the velocity $r d\phi/dt$ of the ring particles, and hence the force back on the approaching magnetic moment will be negligible for large $\gamma$. Hence no time lag for the passing magnetic dipole will be observed for a large friction constant $\gamma$. For a single ring particle ($N = 1$), the relative size of the friction requires a comparison between the value of $\gamma$ and the inverse passage time $v_\mu/r$ of the magnetic moment through the circular ring. If the passage time is small (large inverse passage time $\gamma << v_\mu/r$), then we expect $mr(d^2\phi/dt^2)$ to dominate $mr\gamma(d\phi/dt)$. When many charges $e$ are present on the ring ($N >> 1$), then the energy conservation requires that the characteristic time $L/R$ of the ring as an $LR$ circuit should be long compared to the passage time.

The same qualitative features arising in this example of a magnetic dipole interacting with the charges on a ring will also occur for the interaction of a charge particle passing a solenoid. In this case the accelerations of the solenoid particles are not symmetrical and are
causes by the electric field of the passing charge. When the frictional forces are very small, we expect to find a time lag associated with the energy-conserving interaction between the charged particle and the solenoid. On the other hand, if the frictional forces (solenoid resistance) is large, then we do not expect to find the time lag because the accelerations of the solenoid charges will be small and hence the back forces at the passing charge will be small.

In the experiments of Moellenstedt and Bayh, where the Aharonov-Bohm phase shift is clearly present, the passage times of electrons past the solenoids is of the order of $10^{-13}$ sec (for a 40 keV electron passing 20 microns from the center of the solenoid). This time is not much longer than the collision time $10^{-14}$ sec in the Drude model for conductivity of a metal. Indeed, Jackson gives $\gamma$ as of the order of $10^{13}$ inverse seconds where $-m\gamma v$ is the resistive damping of a particle of mass $m$ and speed $v$. Thus it is possible that the conservation of energy involving magnetic fields holds in the short-time regime where Moellenstedt and Bayh’s experiments were performed, yet would not hold for the much longer passage times for the slower electrons passing the much larger solenoid in the experiment of Caprez, Barwick, and Batelaan.

D. Crucial Experimental Tests

The crucial test involving time delay corresponds (in a single experiment) to observing the Aharonov-Bohm phase shift and yet not observing the time delay of Eq. (4). Such an observation would rule out the classical lag interpretation of the Aharonov-Bohm phase shift. For the regimes where the Aharonov-Bohm phase shift has been observed, the time delays would be extraordinarily small. Thus for the experiments of Moellenstedt and Bayh, the time delay in Eq. (4) is of the order of $10^{-21}$ sec.

It is the shifts of particle interference patterns which allow the measurement of extraordinarily small quantities. Thus use of the shift of the interference pattern by a large flux seems the best way realistically to test for the absence of a velocity change. According to the currently accepted quantum point of view, the Aharonov-Bohm phase shift is a purely topological shift, and there is no velocity change and hence no spatial lag for charged particles passing on opposite sides of a solenoid. Accordingly, one can increase the flux in a solenoid by an arbitrary amount and never break down the particle interference pattern. On
the other hand, in the classical lag point of view, a sufficiently large solenoid flux will cause a relative lag which is larger than the coherence length associated with the charged particle and at this point the observed particle interference pattern should break down. Caprez, Barwick, and Batelaan are hoping to achieve appropriate conditions to observe this possible breakdown.

In the classical analysis, the observed Aharonov-Bohm phase shift due to a solenoid (a line of *magnetic* dipoles) is analogous to the observed Mattucci-Pozzi phase shift due to a line of *electric* dipoles. This is not at all the view of currently accepted quantum theory, which accepts the idea that the electrostatic phase shift is due to a classical lag but claims that the magnetic phase shift is due to a new quantum mechanical effect having no classical analogue. Thus ideally one would like to see both these phase shifts (electric and magnetic) tested at the same time with the same coherence length for the electron beam. From both classical and quantum analyses of the electrostatic situation, we believe that we know exactly what is going on involving electrostatic forces leading to a relative lag effect producing the Mattucci-Pozzi electrostatic phase shift. If the interference pattern breaks down for large *electric* dipole magnitude but not for large *magnetic* dipole magnitude, then (in contradiction to the classical analysis) the mechanisms for these two interference pattern shifts are quite different, as is indeed claimed by currently accepted quantum theory. However, if the interference pattern breaks down for both the electric and the magnetic phase shifts, the currently accepted quantum view is in error.

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