Anomalous flux quantization in a Hubbard ring with correlated hopping.

Liliana Arrachea*, †, A. A. Aligia and E. Gagliano*.

Centro Atómico Bariloche and Instituto Balseiro
Comisión Nacional de Energía Atómica
8400 Bariloche, Argentina
† Permanent address: Departamento de Física, Universidad Nacional de La Plata,
1900 La Plata, Argentina.

Abstract

We solve exactly a generalized Hubbard ring with twisted boundary conditions. The magnitude of the nearest-neighbor hopping depends on the occupations of the sites involved and the term which modifies the number of doubly occupied sites \( t_{AB} = 0 \). Although \( \eta \)-pairing states with off-diagonal long-range order are part of the degenerate ground state, the behavior of the energy as a function of the twist rules out superconductivity in this limit. A small \( t_{AB} \) breaks the degeneracy and for moderate repulsive \( U \) introduce superconducting correlations which lead to “anomalous” flux quantization.

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One of the most interesting problems of the physics of highly correlated electronic systems is the characterization of the metallic, insulating and superconducting phases, as well as the transitions among them. Kohn has shown that the Drude weight $D_c$ is the adequate quantity to identify the metal-insulator transition (MIT) \cite{1}, while Yang introduced the concept of off-diagonal long-range order (ODLRO) to characterize the superconducting nature of a metallic phase \cite{2}. ODLRO in all relevant low-energy eigenstates implies a periodicity of $h/2e$ in the free energy as a function of a magnetic flux threading a system with annular topology, which is referred to as “anomalous” flux quantization (AFQ) \cite{3,4}. In other words, AFQ in the ground-state (GS) means $E(\Phi + \pi) = E(\Phi)$, where $E$ is the GS energy and $\Phi$ is the twist angle. It is a necessary but not sufficient condition for superconductivity \cite{3}. Since $D_c \sim \partial^2 E/\partial \Phi^2$, the function $E(\Phi)$ gives crucial information about the metallic and superconducting character of the system \cite{1,4,5}.

Exactly solvable highly correlated models displaying a MIT or ODLRO are good laboratories to investigate the nature of the MIT and electronic mechanisms of superconductivity. The Bethe ansatz solution with twisted boundary conditions (i.e. arbitrary flux) of the one dimensional (1D) Hubbard model \cite{7}, allowed to apply Kohn’s ideas to the MIT in this model \cite{7,8}. Very few exact results exist, for electronic models exhibiting superconductivity (or dominant superconducting correlations at long distances in 1D). Several of them are related with the so called $\eta$-pairing mechanism, which allows to construct eigenstates with ODLRO \cite{3,12}. In particular, the widely studied \cite{13} Hubbard model with correlated hopping,

$$
H = H_U + H_t = U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_{<ij>\sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c) \{t_{AA} (1 - n_{i\sigma})(1 - n_{j\sigma}) + t_{BB} n_{i\sigma} n_{j\sigma} + t_{AB} [n_{i\sigma}(1 - n_{j\sigma}) + n_{j\sigma}(1 - n_{i\sigma})]\},
$$

(1)

has been exactly solved recently in 1D in the limit $t_{AB} = 0$ for open \cite{10} and periodic \cite{11,12} boundary conditions. It has been shown that, due to an SU(2) pseudospin symmetry, $\eta$-pairing states with ODLRO are part of the degenerate GS for moderate on-site repulsion $U$ and arbitrary band filling. Unfortunately, the function $E(\Phi)$ has not been obtained and
then, the AFQ and $D_c$ were not studied. Our main interest in this study is motivated by the following two facts: first, the superconducting character of the degenerate GS is not obvious, even when states with ODLRO are part of the GS manifold. Second, the GS was found to be a Mott insulator for $U > U_{MI} = 2D(|t_{AA}| + |t_{BB}|)$ at half-filling (density of particles $n = 1$), in a simple cubic lattice in $D$ dimensions, with a MIT for $D > 1$ \cite{10,18}. Strictly in $D = 1$, however, we find that for $n = 1$, $D_c = 0 \forall U$, in spite of a vanishing charge gap for $U < U_{MI}$. The possibility of an insulating phase with this feature was first remarked by Kohn and we think that this is, to our knowledge, the first non-trivial realization of that kind of insulators.

In this Letter, we solve exactly the model (1) with $t_{AB} = 0$, for twisted boundary conditions $\Phi_\uparrow$ ($\Phi_\downarrow$) for spin up (down) fermions. This allows us to calculate the Drude weight $D_c$ and the spin stiffness $D_s$ and to discuss the nature of the MIT as $n \to 1$. The behavior of $E(\Phi_\uparrow, \Phi_\downarrow)$, rules out superconductivity in the model for $t_{AB} = 0$, at least in the one-dimensional case we study here. In addition, we go one step further and show how the GS degeneracy is broken in favor of a state with dominant superconducting correlations when a finite $t_{AB}$ is allowed, for moderate repulsive $U$.

We first consider the Hamiltonian (1) with $-t_{AA} = t_{BB} = t > 0$, $t_{AB} = 0$. The other possible choices of the sign of $t_{AA}$ and $t_{BB}$ lead to an equivalent model \cite{19}. At each site $i$, we introduce two fermions $f_{i\sigma}$ and two bosons $b_{i\sigma'}$, where $b_{i+} \equiv e$ (empty) and $b_{i-} \equiv d$ (doublon). The fermions (bosons) transform according to an irreducible representation of the local spin (pseudospin) local $SU(2)$ symmetry that $H_t$ possesses for open boundary conditions \cite{10}. In this representation, $H_t$ of (1) in a ring of $L$ sites with twists $\Phi_\sigma$ for particles with spin $\sigma$ reads

$$H_t = \sum_{i=1}^{L-1} H_{i,i+1} + H_{L,1} = -t \sum_{i=1,\sigma\sigma'}^{L-1} (f_{i+1\sigma}^\dagger f_{i\sigma} b_{i+1\sigma'}^\dagger b_{i\sigma'} + \text{h.c.})$$

$$-t \sum_{\sigma} [f_{1\sigma}^\dagger f_{L\sigma} (e^{i\Phi_\sigma} b_{1+}^\dagger b_{L+} + e^{-i\Phi_{\sigma'}} b_{1-}^\dagger b_{L-}) + \text{h.c.}].$$

The numbers $N_{\sigma} = \sum_i f_{i\sigma}^\dagger f_{i\sigma}$, $N_e = \sum_i e_{i\sigma}^\dagger e_{i\sigma}$, $N_d = \sum_i d_{i\sigma}^\dagger d_{i\sigma}$ are all conserved when $t_{AB} = 0$. In each subspace with fixed $N_{\uparrow}, N_{\downarrow}, N_e, N_d$, any state has the form

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\[ \prod_{m=1}^{N_b} b_{i(m)\sigma'}(m) \prod_{j=1}^{N_f} f_{i(j)\sigma(j)}^\dagger(0), \]

where \( j \) labels the \( N_f = N_\uparrow + N_\downarrow \) fermions from left to right and \( i(j), \sigma(j) \) denote the position and the spin of the \( j \)th fermion. Similarly \( i(m), \sigma'(m) \) (with \( i(m+1) > i(m) \)) are the position and the pseudospin \( [\uparrow] \) of the \( m \)th boson. The number of bosons is \( N_b = N_e + N_d = L - N_f \). Because of the completeness relation

\[ \sum_{\sigma} f_{i\sigma}^\dagger f_{i\sigma} + b_{i\sigma}^\dagger b_{i\sigma} = 1, \]

\( \{i(j)\} \) and \( \{i(m)\} \) are complementary sets.

For the periodic case (\( \Phi_\sigma = 0 \)), the Hamiltonian is invariant under cyclic permutations of the fermions and bosons and it is convenient to work in the basis of the irreducible representation of the direct product group \( \mathcal{C}_{N_f} \otimes \mathcal{C}_{N_b} \). Our idea is to use appropriate weighted representations \( [20] \) to cancel out the difference in phases in \( H_{1,L} \) in order to map the problem into one of spinless fermions with twisted boundary conditions. With this objective in mind, we think the ring as a periodic system in which

\[ f_{i+L\uparrow}^\dagger = f_{i\uparrow}^\dagger, \]

\[ e_{i+L\downarrow}^\dagger = e_{i\downarrow}^\dagger, \]

but

\[ f_{i+L\downarrow}^\dagger = e^{-i(\Phi_\uparrow - \Phi_\downarrow)} f_{i\downarrow}^\dagger, \]

\[ d_{i+L\downarrow}^\dagger = e^{-i(\Phi_\uparrow + \Phi_\downarrow)} d_{i\downarrow}^\dagger. \]

Using these boundary conditions, it can be verified that

\[ H_{L,1} = e^{i\Phi_\uparrow} \sum_{\sigma\sigma'} f_{i+L\sigma}^\dagger f_{L\sigma} b_{i+L\sigma'}^\dagger b_{L\sigma'}, \]

as we want.

We look for a basis of many particle states transforming as irreducible representations of \( \mathcal{C}_{N_f} \otimes \mathcal{C}_{N_b} \) under the above mentioned boundary conditions. The part of these states which describes the singly occupied sites can be constructed using the operators

\[ F^\dagger(\{i(j)\}, \{k_l\}) = \prod_{k_l\uparrow}^{N_f} \tilde{f}_{k_l\uparrow}^\dagger \prod_{l=1}^{N_f} f_{k_l\downarrow}^\dagger, \]

\[ f_{k\downarrow}^\dagger = \frac{1}{\sqrt{N_f}} \sum_{j=1}^{N_f} e^{-ikj} f_{i(j)\downarrow}^\dagger, \]

\[ \tilde{f}_{k\uparrow}^\dagger = \frac{1}{\sqrt{N_f}} \sum_{j=1}^{N_f} e^{-i\tilde{k}j} f_{i(j)\uparrow}^\dagger (1 - f_{i(j)\downarrow}^\dagger f_{i(j)\downarrow}), \]

where in contrast to the wave numbers \( k_l \), the \( \tilde{k}_l \) are not shifted (\( \tilde{k}_l N_f / 2\pi \) is integer), and the set of \( k_l \) is fixed and chosen in such a way that \( \sum_l \tilde{k}_l = 0 \) (\( \pi \)) for \( N_f \) odd (even). The \( \nu_l \) are \( N_\downarrow \) different integers lying in the interval \([0, N_f - 1]\), and each of the \( N_f!/(N_\uparrow!N_\downarrow!) \) possible
choices of the set of $\nu_l$ define a spin configuration. It is easy to see that under cyclic permutation $C_{N_f}$, which carries each fermionic position to the right $C_{N_f} F^\dagger = -(-1)^{N_f} \exp(i \sum_i k_i) F^\dagger$. In a similar way, using a transformation that interchanges spin and pseudospin [21], the pseudospin configuration can be described by an operator $B^\dagger((\{i(m)\}, \{k'_l\}))$, such that $C_{N_b} B^\dagger = \exp(i \sum_l k'_l) B^\dagger$, with the $N_d$ different $k'_l = [2\pi \nu' - (\Phi_\uparrow + \Phi_\downarrow)]/N_b$. (details will be given elsewhere). The (non-orthonormal) basis states that we use are denoted by $|\psi\{i(j)\}, \{k\}, \{k'\} = B^\dagger F^\dagger |0\rangle$.

$H_t$ permutes a fermion and a nearest-neighbor boson. The cyclic orders of fermions and bosons are conserved. Thus, the numbers $\{k\}$ and $\{k'\}$ are conserved. We drop these indices for simplicity. $H_{l,l+1}|\psi\{i(j)\}\rangle = 0$ unless one and only one of the sites $l$ and $l + 1$ is contained in $\{i(j)\}$. We restrict ourselves to these states in the following discussion. It can be seen that for $l < L$, $H_{l,l+1}|\Psi(\{i(j)\})\rangle = -t|\Psi(\{i'(j)\})\rangle$, where $\{i'(l)\}$ differs from $\{i(j)\}$ in the position of one fermion only, which is shifted from site $l$ to $l + 1$ or conversely. If $i(N_f) = L$, then $H_{L,1}|\Psi(\{i(j)\})\rangle = -te^{i\Phi_t}C_{N_f} C_{N_b}^{-1}|\psi\{i'(j)\}\rangle = -t(-1)^{N_f} \exp[i(\sum_{l=1}^{N_f} k_l - \sum_{l=1}^{N_b} k'_l + \Phi_t)]|\psi\{i'(j)\}\rangle$, where $i'(1) = 1$, and for $j < N_f$, $i'(j + 1) = i(j)$. When $N_f = N_d = 0$, $H_t$ takes the form of a problem of spinless fermions with flux $\Phi_t$. The above equations show that in the general case, the problem takes the same form, with an effective flux:

$$\Phi_{eff} = \Phi_\uparrow + \sum_{l=1}^{N_f} k_l - \sum_{l=1}^{N_d} k'_l = (\frac{N\uparrow}{N_f} - \frac{N\downarrow}{N_b})\Phi_\uparrow + (\frac{N\downarrow}{N_f} - \frac{N\uparrow}{N_b})\Phi_\downarrow + \frac{2\pi}{N_f} \sum_{l=1}^{N_f} \nu_l - \frac{2\pi}{N_b} \sum_{l=1}^{N_d} \nu'_l.$$ (5)

The energy of the system is given by

$$E = -2t \sum_{j=1}^{N_f} \cos\left(\frac{2\pi \nu''_j + \Phi_{eff}}{L}\right) + UN_d,$$ (6)

where the $N_f$ integer numbers $\nu''_j$ should be different and can be chosen in the interval $[-L/2 + 1, L/2]$. When $N_b = \Phi_\uparrow = \Phi_\downarrow = 0$, the energy takes a similar form as that derived by Ogata and Shiba [22] for the infinite $U$ Hubbard model from the Bethe ansatz equations. Note, however, that our basis states do not correspond to a definite total spin in general. Also, our basis is complete, and (6) describes the energy of any state, while the regular Bethe ansatz states do not form a complete basis [23].
We are in position now to examine the ground state energy as a function of the fluxes. For fixed \( N_f = n_f L \) and total number of particles \( N = nL = N_f + 2N_b \), minimization of (6) leads to

\[
E_g(\Phi^\uparrow, \Phi^\downarrow) = UN_d - 2t \frac{\sin(n_f \pi)}{\sin(\pi/L)} \cos(\varphi),
\]

where \( \varphi = \Phi_{eff}/L \) for \( N_f \) odd and \( \varphi = (\Phi_{eff} - \pi)/L \) for \( N_f \) even. The value of \( U \) determines \( N_d \) for the GS. For each \( \Phi^\uparrow, \Phi^\downarrow \), the numbers \( N^\uparrow, N^\downarrow \), as well as \( \{\nu\}, \{\nu'\} \) should be chosen to minimize \(|\varphi|\) (module 2\( \pi \)). It can be easily seen that in the simplest case \( \Phi^\uparrow = \Phi^\downarrow = 0 \) and \( U = 0 \) that the GS is highly degenerate (many choices of quantum numbers lead to \( \varphi/2\pi \) integer). For \( U > U_c = -4t \cos(\pi n) \), it was found \([10–12]\) that the double occupation is forbidden in the GS (\( N_d = 0 \)), and we recover the solution of the \( U = +\infty \) Hubbard model with twisted boundary conditions. In this case \( E(\Phi^\uparrow + 2\pi/N, \Phi^\downarrow + 2\pi/N) = E(\Phi^\uparrow, \Phi^\downarrow) \), since the shift in \( \Phi_\sigma \) can be absorbed decreasing one of the \( \nu \) by 1, what is always possible if \( 0 \neq N^\uparrow \neq N_f \). For \( \Phi^\uparrow = \Phi^\downarrow \), this result has been obtained previously \([24]\). Similarly, for the more interesting case with \( N_d \neq 0 \), a change in both \( \Phi_\sigma \) by \( 2\pi/|N_b - N_e| \) can be counterbalanced by a change in the \( \{\nu'\} \), leading to

\[
E_g(\Phi^\uparrow + \frac{2\pi}{|L - N|}, \Phi^\downarrow + \frac{2\pi}{|L - N|}) = E_g(\Phi^\uparrow, \Phi^\downarrow),
\]

for \( L \neq N \), whereas for a half-filled system \( E_g \) depends only on the difference \( \Phi^\uparrow - \Phi^\downarrow \), a behavior typical of an insulator. For \( U < -4 \cos(\pi n) \), \( N_d > 0 \) and \( \eta \)-pairing states with ODLRO are present in the GS \([10, 12]\). However, we do not find AFQ, but a periodicity which depends on the particle-density \( n \). An example for finite chains is shown in Figs. 1 (a-b). The number of peaks of \( E_g(\Phi^\uparrow = \Phi^\downarrow = \Phi) \) for \( 0 \leq \Phi \leq 2\pi \) is at least \( L|n - 1| \), diverging in the thermodynamic limit, while the height of each peak decreases as \( 1/L^3 \), as in the \( U = +\infty \) Hubbard model. The response of the system to the flux is like that of a single particle with charge \( L - N \) or larger. One might ask whether a collection of weakly coupled chains behaves like a superfluid of these particles. However, since the compressibility diverges in the interesting regime \([10]\), charge can be transferred between chains without cost.
of energy, and the response to the flux of different chains does not add coherently. Thus, the system does not show the Meissner effect [2–4]. We should also note that the SU(2) \( \eta \)-symmetry which allows for the construction of eigenstates with ODLRO is broken in the presence of a flux. It can be seen from (5) that a twist of the form \( \Phi^\uparrow = -\Phi^\downarrow \), couples with the spin degrees of freedom in the same fashion as a twist \( \Phi^\uparrow = \Phi^\downarrow \) affects the pseudospin ones. While in the first case, the total spin invariance is broken, remaining only \( S^z \) as a good quantum number, the total pseudospin invariance is broken in the second case, with \( \eta^z = 1/2(L - N) \) fixed by \( U \) and the chemical potential. Thus, the \( \eta \)-pairing states do not necessarily give rise to superconducting currents in the presence of an external flux. The physics in the region with \( U < -4t \), where \( n_f = 0 \) [10–12] is more obvious. In this case, there are also \( \eta \)-paired states with ODLRO in the degenerate GS. However, these states are static, and from (7) \( E(\Phi^\uparrow, \Phi^\downarrow) = U N_d \) for all \( \Phi_{\sigma} \). This result might be anticipated from the form of \( H_t \). This demonstrates that the ODLRO of the \( \eta \)-paired states is not a sufficient condition for the existence of superconductivity. This fact has not been noted in previous related works.

The computations of Drude weight and the spin stiffness lead to

\[
D_c = \frac{L}{2} \frac{\partial^2 E(\Phi, \Phi)}{\partial \Phi^2} |_{\varphi=0} = \frac{t}{\pi} \frac{(1 - n)}{(1 - n_f)^2 \sin(\pi n_f)}
\]

\[
D_s = \frac{L}{2} \frac{\partial^2 E_g(\Phi, -\Phi)}{\partial \Phi^2} |_{\varphi=0} = \frac{t}{\pi} \frac{(n^\uparrow - n^\downarrow)}{n_f^2 \sin(\pi n_f)}, \tag{9}
\]

where \( n_f \) is a function of \( U/t \) and \( n \) that can be obtained minimizing (7) [10,12]. For half-filling, \( D_c = 0, \forall U \). The result is not surprising for \(|U| > 4t\), where the system is a Mott insulator, but rather unexpected otherwise. The symmetry between spin and pseudospin degrees of freedom [21], becomes explicit by replacing \( n_e - n_d = 1 - n \), \( n_b = 1 - n_f \) in the expression of \( D_c \). \( D_s \) vanishes in the sector \( S^z = 0 \) and with it, the inverse of the magnetic susceptibility [4], as a consequence of the spin degeneracy. Analogously, for \(|U| < 4t\), not only \( D_c \) vanishes at half-filling, but also the inverse of the charge compressibility. The system is an insulator, in spite of a vanishing charge gap. The behavior of the Drude weight for \( n \to 1 \) is the same as that of a system of \( n_f \) carriers with effective mass diverging as
\((1 - n_f)^2/(1 - n^2)\).

The particular behavior of the energy as a function of flux, is a consequence of the existence of excitations associated to the fermionic charge, spin and pseudospin degrees of freedom, which is explicit in Eqs. (9) and (4). This is similar to the case of the infinite \(U\) Hubbard model where spin and charge decouple. The ground state is highly degenerate as a consequence of the rich symmetry structure of Eq. (1) when \(|t_{AA}| - |t_{BB}| = t_{AB} = 0\).

In particular, for an open chain there is a local spin and pseudospin symmetry at each site [10]. Since \(t_{AB} = 0\) is an accident rather than a generic feature of any one-band model, it is very important to discuss the effect of a finite \(t_{AB}\), particularly taking into account that this term lifts the GS degeneracy. When \(t_{AB} \neq 0\), the sign of \(t_{AB}\) or those of \(t_{AA}\) and \(t_{BB}\) simultaneously, can be changed using symmetry properties [17], but models with different \(t_{AA}t_{BB}\) are not equivalent. In the following, we consider the case \(t_{AA} = t_{BB} = -t\), which interpolates between two exactly solvable cases: the one considered above and the Hubbard model. This case preserves the SU(2) pseudospin symmetry when \(t_{AB} \neq 0\) [16,17]. Previous numerical studies suggest that the 1D system for small \(U\) and \(n \sim 1\) is a Tomonaga-Luttinger liquid with dominant superconducting correlations at large distances [10,13]. We have studied numerically \(E(\Phi_\uparrow = \Phi_\downarrow = \Phi)\) for finite systems with fixed densities \(n \neq 1\), \(n_e, n_d \neq 0\) at \(t_{AB} \neq 0\). An example is shown in Fig. 1 (c). We find that a small \(t_{AB}\) gives rise to AFQ in the finite systems. The cusp near \(\Phi = \pi/2\) decreases with increasing length of chain. This behavior is typical of a system with power law superconducting correlations at long distances rather than a state with true ODLRO.

What is the origin of the superconducting correlations? Is it related with the \(\eta\)-pairing? To answer these questions let us begin by noting that the (nondegenerate) GS for \(t_{AB} \to 0\) is exactly known in the case \(N_eN_b = 0\) (for \(U > -4t\cos(\pi n)\) and \(U > 0\) [10,12]). In this case, the low energy physics of the model becomes equivalent to that of a Hubbard model with interaction \(U_H = t^2U/t_{AB} \to \infty\) (as can be easily seen eliminating in (2) the term in \(t_{AB}\) through the standard canonical transformation). In this limit Ogata and Shiba [22] have shown that the GS wave function can be factorized in two parts: one describing
the position and the other the spin of the $N_f$ fermions. The first factor corresponds to the GS of a Heisenberg chain with $N_f$ sites. This GS wave function can be mapped into the corresponding one for $U < 0$ and a magnetic field high enough to ensure $N_\uparrow N_\downarrow = 0$ using the transformation that interchanges spin and pseudospin [21]. It is natural to expect that the effect of a small $t_{AB}$ is to introduce antiferromagnetic correlations between spins and pseudospins in the general case. A straightforward generalization of the above analyzed two cases, led us to propose an ansatz for the GS in the limit $t_{AB} \to 0$ consisting of three factors, describing the positions of fermions and bosons and the spin and pseudospin variables. The first two factors are those of the GS Bethe ansatz solution of the $U = +\infty$ Hubbard model. The last one is the GS of a Heisenberg model for the pseudospin variables, which is also the GS of the large negative $U$ Hubbard model in a system with $N_b$ sites and $2N_d$ particles [22]. We have computed the overlap of our ansatz with the exact GS obtained from exact diagonalization in different chains (up to $L = 12$ sites), and we found that it is equal to $(1 - \alpha^2 t_{AB}^2)^{1/2}$, with $\alpha \sim 2 - 4$, for $0 \neq t_{AB} < 0.1$, confirming our conjecture for $t_{AB} \to 0$. It is easy to verify with our ansatz that in the thermodynamic limit, for $L \to \infty$, the pair correlation function $C(l) = \langle c_{i+\uparrow}^{\dagger} c_{i+\downarrow}^{\dagger} c_{i\downarrow} c_{i\uparrow} \rangle$ can be expressed in terms of the corresponding correlation function of the large $|U|$ attractive Hubbard model $C_H(l)$ for density $(n - n_f)/(1 - n_f)$ as $C_H(l) = (1 - n_f)[C_H(l')]_{av}$, where the average $l'$ is centered around $L(1 - n_f)$. Thus, the superconducting properties of the system are essentially those of the large $U$ attractive Hubbard model with dilute superfluid density. The superconducting properties of the model are not related with the $\eta$-pairing. For $t_{AA} = t_{BB}$, the generators of the total pseudospin algebra are $\eta^- = \sum_i (-1)^i c_{i\downarrow}^{\dagger} c_{i\uparrow}^{\dagger}$, $\eta^+ = (\eta^-)^\dagger$ and $\eta^z = (1/2) \sum_i (1 - \sum_\sigma n_{i\sigma})$. The $\eta$-pairing mechanism applies $\eta^-$ to an eigenstate with $\eta \neq 0$ to obtain eigenstates with $\eta^z < \eta$ which possess ODLRO [9–12]. However, since by construction, our ansatz for the GS has $\eta^z = \eta$, and the true (non-degenerate) ground state has the same quantum numbers, it cannot be the result of applying $\eta^-$ to any eigenstate. Exact diagonalization results show that this is also the case in 2D, even for $t_{AB} = 0$ [17].

In summary, we have shown that at least in the 1D generalized Hubbard model (1) for
$|t_{AA}| - |t_{BB}| = t_{AB} = 0$, the $\eta$-pairing does not lead to superconductivity. The possibility of constructing eigenstates with ODLRO using the SU(2) symmetry does not guaranty the existence of superconducting currents giving rise anomalous flux quantization and Meissner effect. The ODLRO must be analyzed in the presence of a finite magnetic flux threading the ring. This fundamental fact is in the spirit of the proposals of Refs. [2–4]. We have also examined the character of the metal-insulator transition near half filling and we have presented strong evidence that the GS degeneracy is broken in favor of a GS with dominant superconducting correlations in 1D when a small $t_{AB}$ is turned on.

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Figure Captions

1. Ground state energy as a function of twist angle (a) for $|t_{AA}| = |t_{BB}| = 1$, $t_{AB} = 0$, density $n = 2/3$ and $U = 0$; (b) same as (a) with $U > 4$; (c) for $t_{AA} = t_{BB} = -1$, $t_{AB} = -0.2$, $U = 0$, $n = 2/3$
