A multi-rate ISM approach for robust vehicle stability control during cornering

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Abstract: In this paper a Model Predictive Control (MPC) architecture is proposed to achieve a smooth and progressive stabilization of the vehicle in limit cornering conditions. Integral Sliding Mode (ISM) is effectively exploited, in a multi-rate structure, in order to enhance the robustness with respect to changes in the tire/road friction characteristic curve and modelling errors. These typically arise due to the linearizations and discretizations needed in the controllers implementation. The performance is verified in simulation using IPG-CarMaker, a high-fidelity vehicle dynamics simulation software. The results obtained evidence the effectiveness of the proposed approach, also compared to an LQR based architecture.

Keywords: Automotive control, Integral Sliding Mode, MIMO Control, predictive control, steady state cornering

1. INTRODUCTION

Automotive chassis control systems have shown, over the years, their ability to increase safety and driving comfort. With their continuous evolution, it is natural to consider the possibility of a less aggressive and invasive action, able to control the stability of the vehicle even in limit conditions (e.g. limit cornering, Beal and Gerdes (2013)). Control architecture have been proposed, mainly for rear-driven electric/hybrid vehicles (HEV, see e.g. Siampis et al. (2013)), to mitigate the terminal understeer during cornering at the limit of stability, controlling the yaw rate and the velocity. Model Predictive Control (MPC) has been effectively used for the stabilization of Rear Wheel Drive (RWD) vehicles. Example are Filho and Wolf (2014), in which situations at the limit of handling are considered and Carvalho et al. (2013), in which the focus is on autonomous vehicles path-following and obstacle avoidance. In that paper, the nonlinear MPC algorithm is suitably tailored with the aim to reduce computational time and allow a longer prediction horizon. Other configurations, such as 4 Wheel Drive (4WD) or Front Wheel Drive (FWD) are considered for instance in Her et al. (2016), where an integrated chassis control (ICC) algorithm is proposed for differential braking, front/rear traction torque and active roll moment control. The experimental approach in that work is to increase stability during high-speed cornering.

When designing MPC/LQR controllers for this purpose, especially when dealing with stabilized systems, the deviation between utilized model and actual system might pose serious problems for the stability of the controlled system, in particular for growing values of the wheels side slip angle. In order to reduce the effect of model uncertainties on the MPC/LQR controllers, the adoption of Sliding Mode Control (SMC) can be a valid choice. This methodology is capable to guarantee good performance of the controlled system even in case of nonlinear systems affected by a significant class of uncertainties and disturbances. Several applications of SMC techniques have been proposed in literature, with reference to vehicle lateral dynamics control. Examples of SMC solutions can be found in Amodeo et al. (2010). In Goggia et al. (2014) an application, with experimental validation, of Integral Sliding Mode (ISM) is also presented.

The adoption of a multi-rate strategy for the application of ISM control law together with MPC has been proposed in Rubagotti et al. (2011), with particular focus on nonlinear predictive controllers (NMPC). In that work, it has been proven that the region of attraction of the controller benefits from the ISM controller when disturbances are present.

In all the mentioned works which propose methods for overall vehicle state control, the problem of designing a controller, which is robust in conditions different from the nominal ones, is not yet explored. Nevertheless, when working at the limits of stability of the vehicle, such a property is of paramount importance. Relying on these considerations, in this paper, a vehicle control system based on MIMO controllers of LQR and MPC type is investigated and applied to maintain the vehicle stability during limit cornering events, also in the presence of disturbances. These are majorly related to temporary changes in the road grip. In order to improve the performance of the MIMO controllers even with unknown tire/road characteristics, a continuous time ISM component at high frequency is introduced. The choice of this specific multi-rate control structure is due to the increased robustness which it guarantees with a minor increase in the computational burden.

The paper is structured as follows: in Section 2 the vehicle model and control problem are presented. The description of the control architecture is presented in Section 3, while the results obtained with the simulation software IPG CarMaker can be found in Section 4. Finally, some conclusions are drawn in Section 5.

2. PRELIMINARIES

2.1 Vehicle Model

The single-track vehicle model (see Fig. 1) considered is a system of 3 nonlinear equations

\begin{equation}
\begin{align*}
\dot{\psi} &= \psi + \frac{F_{y,TOT}}{m}, \\
mv(\dot{\psi} + \dot{\omega}) &= F_{y,f} + F_{y,r}, \\
J_{z}\dot{\omega} &= l_{f}F_{y,f} - l_{r}F_{y,r} + M_{\psi}.
\end{align*}
\end{equation}

In (1) the terms \(l_{f}, l_{r}\) correspond to the distances from the front and rear axles to the vehicle center of gravity respectively, \(J_{z}\) is...
the vehicle inertia around the z-axis. The vehicle mass is denoted by \( m \), and the lateral tire forces by \( F_{y,i} \), with \( i \in \{ f, r \} \) for the front and rear axles. Longitudinal dynamics are neglected, so that the approximation \( T_{ij} = \tau_{ij} \) \( F_{x,i} \) is made, where \( T_{ij} \) is the torque applied to the \( j \)-th wheel, \( F_{x,i} \) the longitudinal contact force and \( R_{i} \) the effective radius. This is a reasonable assumption, as long as a traction control/anti-lock system (TC/ABS) is in place, and so is the wheel speed variation. Under this hypothesis, the slip ratio \( \lambda \) can be neglected in the computation of the lateral forces \( F_{y,i} \). The latter can therefore be obtained from

\[
\dot{x}_i = A_{x}(x_i, u_i)x_i + B_{x}(x_i, u_i)u_i, 
\]

where the subscript \( c \) denotes the continuous time. Linearized system (9) has time-varying matrices which depend on the states and inputs, as well as on the cornering stiffness coefficients \( c_f, c_r \). Assuming the wheels side slip angle is always in the linear region of its characteristic, the cornering stiffness coefficients can be calculated from the estimations of lateral forces \( F_{y,f}, F_{y,r} \) (see e.g. Re-golin et al. (2017b)) and wheels side slip angle \( \alpha_f, \alpha_r \). Similarly, it is assumed that the state value is available from sensors measurement \((v, \psi)\) or online estimation \((\beta)\). Note that, in the discrete case, the steering input \( \delta \), needed for the definition of the matrices value, is assumed to be the control input calculated at the previous cycle. The values for \( A_{x}(x_i, u_i) \) and \( B_{x}(x_i, u_i) \) obtained are therefore:

\[
\begin{align*}
A_x &= \begin{bmatrix} 2a_1 \frac{v \sigma_{\beta}}{\lambda} + (c_f + c_r) \frac{\beta v}{\lambda} & -\sigma \frac{c_f \gamma \dot{v}}{\lambda} & -\sigma \frac{c_r \gamma \dot{v}}{\lambda} & -a_1 \frac{v}{\lambda} - \frac{a_1 \dot{v}}{\lambda} \\
0 & b_1 & b_1 & b_1 \\
(2) & b_f & b_f & b_f \\
(3) & b_r & b_r & b_r \\
\end{bmatrix}, \\
B_x &= \begin{bmatrix} c_f \frac{v}{\lambda^2} & 0 & 0 & 0 \\
0 & c_r \frac{v}{\lambda^2} - \frac{b_f}{2\lambda R_w} \\
(0) & 0 & 0 & 0 \\
(c_f + c_r) \frac{v}{\lambda^2} - \frac{b_r}{2\lambda R_w} \\
\end{bmatrix}.
\end{align*}
\]

2.2 Problem Formulation

The problem considered in this paper is the tracking of a predefined trajectory in conditions at the limits of stability. In order to achieve this, the tracking of a specific dynamic state of the vehicle is performed. In particular, an autonomous vehicle is considered, where the wheels steering angle \( \delta \) is an output of the controller, and it is possible to apply wheel torque \( T_{ij} \) independently to the 4 wheels, with only negative torque applied to the rear ones, due to the FWD configuration.

The specific nature of the reference generation is such that no feedback is present with the information about the actual position of the vehicle. Therefore, for the evaluation of the performance, only the state tracking is considered rather than the following of an ideal trajectory. The main reason behind this choice is that, while the final application of the proposed controllers has to be framed within the larger scope of autonomous driving (and thus path tracking), the purpose of this work is limited to presenting specific controllers with the capability of robust tracking in the presence of low grip portions of the road. In particular, the focus is on the behavior in presence of a change of surface which is not associated with a change in the maximum grip (which is in general easily detectable), but instead in the tire/road friction characteristic shape.

3. CONTROL SCHEME DESIGN

The multi-rate control loop (see Fig. 2) is composed of a reference generation device (Section 3.1), a MIMO controller which works at a lower sampling rate (Sections 3.3 and 3.4), and the ISM component working at higher frequency (Section 3.5). Preliminary considerations concerning the implementation of a multi-rate controller are gathered in Section 3.2.

3.1 State Reference

The reference generator relies on the information, ideally transmitted by an optical sensor, regarding the road curvature radius \( R_{\text{curv}} \) of a point located 5\( m \) ahead of the current vehicle.
COG, along the vehicle’s path. In particular, in this paper we are interested to design a control scheme capable of maintaining the vehicle in a stability condition during cornering at the maximum possible lateral acceleration. This is why we use a reference generator system which in practice provides the reference signal for such specific vehicle behavior. In Fig. 2 we call this system “maximum acceleration generator”. Possible ways to realize such a generator are those illustrated in Acosta et al. (2017), Siampos et al. (2013).

The reference calculated with such device, includes the vehicle dynamic states based on model (1) (side slip angle $\beta^*$, vehicle velocity $v^*$ and yaw rate $\psi^*$) and the nominal steering angle which is used as a feed-forward component $\delta_{ff} = \delta^*$ in the control structure, in order to improve the system response. This procedure allows to determine an offline mapping which provides reference setpoints for different road geometries.

Note that in general, in order to utilize a reference set consistent with the actual tire/road friction forces generated, an algorithm for the identification of the surface type, such as the one proposed in Regolin and Ferrara (2017), should be utilized, so that the correct set of maps is adopted.

### 3.2 Multi-rate Control Considerations

The proposed control structure (Fig. 2) presents a hybrid approach with continuous (ISM) and discrete (LQR/MPC) techniques. It is therefore appropriate to express some considerations on this aspect. When dealing with a continuous time system description, such as the one in (1), it is a defensible choice to design a continuous time controller, as long as the adopted sampling frequency is justified by the actual bandwith of the system. Nonetheless, since solving an optimization problem in continuous time would be practically unfeasible due to computational complexity, MPC algorithms which use continuous-time models with sampled data systems, e.g. in Rubagotti et al. (2011), result in being an appropriate choice. In this work, following such approach, the optimization problem is solved in a discrete time setting, thus generating piecewise-constant control signals, while the continuous-time control law of the ISM is added to the optimization based control signal. In order to preserve symmetry in the approach, which is useful for the comparison of the performance, the LQR controller is also implemented in discrete time, so that it can be considered a degenerate case of the MPC implementation, with no constraints considered in the design and infinite prediction horizon.

### 3.3 Discrete time LQR

Given the discretized version of system (9)

$$ x(k+1) = A(x(k),u(k))x(k) + B(x(k),u(k))u(k) $$

the purpose is to minimize the tracking error $x$ of the target reference $v^*$, $\beta^*$ and $\psi^*$:

$$ x(k) = [v^*(k) - v(k), \beta^*(k) - \beta(k), \psi^*(k) - \psi(k)]^T $$

by controlling the control inputs:

$$ u(k) = [\delta(k) \ T_{fl}(k) \ T_{fr}(k) \ T_{vl}(k) \ T_{vr}(k)]^T $$

the gain $K(k)$ of the LQR controller, at each iteration, is calculated by finding the solution $P$ of the Riccati equation

$$ A^TPA - P - (A^TPB)(B^TPB + R)^{-1}(B^TPA) + Q = 0 $$

where $A = A(x(k),u(k)), B = B(x(k),u(k))$, and $Q$ and $R$ are the weight matrices (assumed to be constant) in

$$ J_{LQR} = \sum_{k=1}^{\infty} x^T(k)Q x(k) + u^T(k)R u(k) $$

Finally, one obtains the $3 \times 5$ gain matrix $K(k)$

$$ K(k) = (B(k)^TP(k)B(k) + R)^{-1}B(k)^TP(k)A(k) $$

which defines the feedback law $u(k) = -K(k)x(k)$.

The output defined by Equation (19) is saturated, in order to limit the maximum amount of wheel torque request. This device allows to reduce the risk of activating the ABS/TC system, in case of reduced tire/road friction. A rate limitation is not included, in the attempt of not affecting excessively the LQR control action. Note that, due to the FWD configuration, the controller can only request positive torque from the front axle, and this has to be reflected in the saturation.

### 3.4 Linear MPC

In order to include constraints on the control input and on its rate, system (13) is extended to the form

$$ \begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} = \begin{bmatrix} A(k) \\ 0 \end{bmatrix} \begin{bmatrix} B(k) \\ l \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \Delta u(k) \quad \text{for} \quad \tau(k) $$

An MPC controller with prediction horizon $N$ is implemented which adopts the same cost function (17) as in the LQR case, with an additional cost term $R_{\Delta}$ for the rate of the inputs. $J_{MP_3}$, based on the extended state $\tau(k)$ defined in (20), is

$$ J_{MP_3} = \sum_{k=1}^{N-1} \pi^T(k)Q\pi(k) + \pi^T(k)R\pi(k) $$

where $Q$, $R$ are defined as follows:

$$ Q = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}, \quad \pi = R_{\Delta} $$

Hard constraints are imposed on the control input, of the form

$$ u_{\text{min}} \leq u(k) \leq u_{\text{max}}, \Delta u_{\text{min}} \leq \Delta u(k) \leq \Delta u_{\text{max}} $$

while no constraints are considered for the state $x(k)$, in order not to compromise the feasibility. The $k$-th step control input is then $u(k) = u(k-1) + \Delta u(k)$.

### 3.5 ISM Control

The purpose of adding the ISM controller to the LQR/MPC control scheme is to reduce the uncertainty (Rubagotti et al. (2011) introduced by the linearization and discretization which has led to model (13), in particular when wheel slip angles increase, and approximations (8) and (3) are not representative of the actual behavior. For this reason, the ISM technique in its formulation for chatter alleviation (see Utkin and Shi (1996) as reference, Fig. 3) is adopted, with reference to the 2-states linearized bicycle model ($\mathbf{z} = \begin{bmatrix} \beta \psi \end{bmatrix}^T, \bar{\mathbf{z}} = \begin{bmatrix} \beta_{err} \psi_{err} \end{bmatrix}^T$).
The 2-states model is preferred to the 3-states one used for the design of the LQR/MPC controllers, due to the different sampling frequency. In fact, the MIMO controllers cannot be implemented at high sampling frequency, since, at each iteration, the control unit has to solve the Riccati equation (16) and, most of all, the QP problem defined by (20), (21) and (23). Conversely, the higher sampling time of the ISM component allows to approximate the vehicle velocity as constant, from the viewpoint of the controller.

At a low sampling frequency this approximation would generate a problematic mismatch between model (25) and the actual controlled system, which might lead to a negative effect of the ISM component on the overall control signal. Moreover, in consideration of the application to the steady-state cornering problem, the natural choice of the sliding surface is a linear combination of the states in model (25), which allows to minimize the deviation of side slip angle and yaw rate from the region of stability.

In the structure considered (see Fig. 3), the Sliding Mode component is only added to the wheel steering angle signal (and not to the wheel torques), so that the resulting control input is:

$$\ddot{\theta} + \beta \dot{\psi} + \varphi = \frac{c_j + c_e}{T_e - \frac{c_j}{c_e}} \left( \frac{q_{\mu} - \mu_x}{J} \right)$$

(25)

Fig. 4. (a) Route followed in the simulations. (b) Tire/road friction characteristics of the three considered surfaces.

Table 1. Nominal Tire/Road friction models

| Surface       | Type     | B   | C   | D   | E   |
|---------------|----------|-----|-----|-----|-----|
| Terrain 1 Wet Asphalt | 11.415 | 1.4601 | 0.6 | 0.20939 |
| Terrain 2 Dirt Road | 15.289 | 1.0901 | 0.6 | 0.86215 |
| Terrain 3 Gravel | 1.5289 | 1.0901 | 0.6 | 0.95056 |

this time in the case where the tire is rolling on a soft surface (e.g. gravel) that contributes to the reduction of cornering/longitudinal stiffness, reduction in the maximum friction and absence of a visible peak (monotonic behavior). With such modelling, the presence of the terrains 2 and 3 in the trajectory, simulates the crossing of a puddle or the presence of dirt on an already wet road. The setpoint calculation does not change during the transition on terrains 2 and 3: it is assumed that the algorithm for the identification of the surface is not able to recognize the change in such a short time span. The vehicle considered for the simulation is a generic FWD compact car, with parameters specified in Table 2.

Table 2. Vehicle Parameters

| m         | l_f       | l_r       | h       | J_z       |
|-----------|-----------|-----------|---------|-----------|
| 519.3kg   | 1.283m    | 1.307m    | 0.589m  | 647kg.m²  |

4.2 Controllers tuning

Having defined the weight matrices in (17), (22) as:

$$Q = \text{diag}\{q_1, q_2, q_3\}$$

(31)

$$R = \text{diag}\{r_1, r_2, r_3, r_4\}$$

(32)

$$R_A = \text{diag}\{r_5, r_6, r_7, r_8, r_9\}$$

(33)

the tuning of the controllers parameters, for all simulations, is reported in Table 3. At each iteration k, the matrices A(k), B(k)

Table 3. Controllers Parameters

| LQR/MPC | \(q_c = 0.5\) | \(q_\theta = 100\) | \(q_\omega = 20\) |
|---------|---------------|-----------------|-----------------|
| N       | \(r_T = 10^{-3}\) | \(r_\theta = 2 \cdot 10^{-3}\) |
| ISM     | \(T_{\max, f} = 100N\text{m}\) | \(T_{\max, r} = 0\text{N}\text{m}\) | \(T_{\min} = 500\text{N}\text{m}\) |
| \(r_\tau = 0.01\) | \(r_\phi = 0.1\) | \(\delta_{\max, \Delta} = 0.5\) |
| Constraint | \(r_\phi = 0.1\) | \(\delta_{\min, \Delta} = 0.1\) |

and A(k), B(k) are calculated based on the vehicle parameters (Table 2), the states \(v, \beta, \phi\) (which are supposed to be known), and the cornering stiffness \(c_f, c_r\) which is derived from (8), based on
the lateral forces \( F_{y,f}, F_{y,r} \) estimated by means of a Suboptimal Second Order Sliding Mode observer (see Regolin et al. (2017b)). Note that, the different limits in maximum (positive) wheel torque, for the front \( T_{max,f} \) and rear \( T_{max,r} \) axle, are due to the chosen FWD configuration of the vehicle.

4.3 Performance Evaluation

The simulation results of the tests presented in the previous section are now illustrated. A graphic rendering of the performance is provided, in terms of tracking, for the side slip angle (Fig. 5), yaw rate (Fig. 6), vehicle velocity (Fig. 7), as well as the overall control signal output for wheels steering angle (Fig. 8) and wheels torque (Fig. 9, front left wheel). The graphics on the left side (a) correspond to the simulation where the entire track surface corresponds to the nominal tire/road characteristic used for the calculation of the reference. In this usecase, due to the absence of disturbances, the controllers considered are LQR and MPC, both running at the sampling time \( T_s = 200\,ms \). In order to highlight the fact that the reference set used corresponds to a situation close to the limit of stability, both cases are compared against the default IPG Driver (referred to as “no ctrl” in the graphs), configured with maximum lateral acceleration \( a_y,IPG = 5.7m/s^2 \) which is below the maximum lateral acceleration \( a_y,\text{max} = 5.9m/s^2 \) measured in the controlled cases. One can see how, although the driving condition is below the limit, the driver can’t keep the trajectory without reducing the velocity (Fig. 7(a)) considerably for the front left wheel.

In case of the LQR controller this happens around \( 25s \), while for the MPC it happens at \( 35s \). By reducing the sampling time \( T_s = 20\,ms \), the effect obtained is that the tracking improves marginally, at the cost of great oscillations, although at higher vehicle velocity. This behavior is also associated with the saturation of the control signals (Fig. 8(b) -9(b)), in case of LQR, with the MPC one also causing the input signals being constantly at the limit value for long portions of time.

When disturbances are added, the nominal MIMO controllers at \( T_s = 200\,ms \) are not able to keep the yaw rate and side slip angle limited, so that the trajectory exceeds the maximum distance \( d_{\text{max}} = 6m \) from the ideal trajectory allowed by the simulation. Conversely, a considerable improvement is obtained by applying the ISM strategy at a sampling time of \( T_{s,\text{ISM}} = 1\,ms \), combined with the MIMO controllers at \( T_s = 200\,ms \), in the multi-rate structure illustrated in Section 3. In the case with disturbance, the ISM based controllers show improved tracking for all vehicle states. In particular, the ISM-MPC one, shows the best performance and reduced/smoker control signals, which never approach the saturation limits. The presented results are confirmed by the evaluation of the tracking error root mean square (RMSE) values, reported in Table 4 and Figure 10 for the usecase “with disturbances”.

Fig. 5. Side slip angle \( \beta \) tracking.

Fig. 6. Yaw rate \( \dot{\psi} \) tracking.

Fig. 7. Vehicle velocity \( v \) tracking.

Fig. 8. Wheels steering angle \( \delta \). Control requests (before saturation) compared against the feedforward (nominal) component alone.

Fig. 9. Wheel torque request \( T_{qFL} \) for the front left wheel.
Table 4. States tracking RMSE

|       | LQR 0.2s | LQR 0.02s | LQR (ISM) 0.2s | MPC 0.2s | MPC (ISM) 0.02s |
|-------|----------|-----------|----------------|----------|-----------------|
| $\psi_{err}$ | 0.1007 | 0.1133 | 0.1105 | 0.0863 | 0.1155 | 0.0387 |
| $\beta_{err}$ | 0.1292 | 0.1267 | 0.1675 | 0.1238 | 0.1608 | 0.0805 |
| $\varphi_{err}$ | 2.0361 | 2.2303 | 1.4008 | 1.6589 | 1.3977 | 1.0826 |

Fig. 10. Normalized RMSE.

Fig. 11. Detail of the phase portrait $\beta$, tire parameters correspond to terrain 1 (normal road) in Table 1 Velocity 17 m/s, steering angle $\delta = 0.1 \text{rad}$. Stable and unstable trajectories are green and magenta respectively.

The above considerations are confirmed when evaluating the results in the phase plane (Fig. 11). The so called $\beta - \dot{\beta}$ phase plane analysis (Inagaki et al. (1995)) is adopted to evaluate the effect of the controllers on the dynamics of the side slip angle. For a specific set of inputs $v, \delta$, the phase portrait of model (1) (with reference to the tire force representation of equations (2), (5)), starting from different initial conditions $([\delta_0, \psi_0])$, is drawn. Vehicle parameters are those reported in Table 2. Depending on the initial condition, the natural system trajectories can be stable or unstable. One can obtain a visual rendering of the robustness of the different control strategies, by considering the evolution of the trajectories when the system is perturbed. The presented trajectories show the system behavior when the 3rd disturbance occurs, at $\approx 16s$: in this situation the ISM-MPC multi-rate strategy allows to keep the system behavior closest to the equilibrium point, without transitioning in working points associated with unstable trajectories.

5. CONCLUSIONS

In this paper a robust multi-rate controller for the stabilization of steady state cornering when transitioning on road sections with reduced grip is presented. Simulations performed on IPG CarMaker show that, when controlling the vehicle close to the limits of stability with linearized optimal control methods, such as LQR and MPC, the reduced friction has a great impact on the overall stability of the system. In these situations, an ISM component at higher sampling frequency provides better robustness than an increase in the sampling frequency of the main controller. Moreover, the multi-rate implementation facilitates the adoption of a reduced state model for the ISM control loop, and therefore a simplified implementation.

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