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Highlights:

- Whole-system optimisation model for wind-electricity-hydrogen networks
- Determines the number, size and location of conversion, storage and transport technologies
- Electricity versus hydrogen transmission
- Determines hourly operation of the whole network over an entire year
- Role and value of existing wind turbines, underground storage and hydrogen pipelines
Optimal design and operation of integrated wind-hydrogen-electricity networks for decarbonising the domestic transport sector in Great Britain

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Abstract

This paper presents the optimal design and operation of integrated wind-hydrogen-electricity networks using the general mixed integer linear programming energy network model, STeMES \cite{1}. The network comprises: wind turbines; electrolysers, fuel cells, compressors and expanders; pressurised vessels and underground storage for hydrogen storage; hydrogen pipelines and electricity overhead/underground transmission lines; and fuelling stations and distribution pipelines.

The spatial distribution and temporal variability of energy demands and wind availability were considered in detail in the model. The suitable sites for wind turbines were identified using GIS, by applying a total of 10 technical and environmental constraints (buffer distances from urban areas, rivers, roads, airports, woodland and so on), and used to determine the maximum number of new wind turbines that can be installed in each zone.

The objective is the minimisation of the total cost of the network, subject to satisfying all of the demands of the domestic transport sector in Great Britain. The model simultaneously determines the optimal number, size and location of each technology, whether to transmit the energy as electricity or hydrogen, the structure of the transmission network, the hourly operation of each technology and so on. The cost of distribution was estimated from the number of fuelling stations and length of the distribution pipelines, which were determined from the demand density at the 1km level.

Results indicate that all of Britain’s domestic transport demand can be met by on-shore wind through appropriately designed and operated hydrogen-electricity networks. Within the set of technologies considered, the optimal solution is: to build a hydrogen pipeline network in the south of England and Wales; to supply the Midlands and Greater London with hydrogen from the pipeline network alone; to use Humbly Grove underground storage for seasonal storage and pressurised vessels at different locations for hourly balancing as well as seasonal storage; for Northern Wales and England and Scotland to be self-sufficient, generating and storing all of the hydrogen locally. These results may change with the inclusion of more technologies such as electricity storage and electric vehicles.

Keywords: wind-hydrogen-electricity networks; renewable energy networks; energy integration; MILP; optimisation; site suitability.
1. Introduction

The advantages of hydrogen as an environmentally clean fuel can be fully realised when it is produced from renewable energy sources. Hydrogen is a flexible energy storage medium that can be used for both short- and long-term storage applications, in addition to being a versatile intermediate that can be converted to electricity, heat and transport fuel. Hydrogen, like electricity, can complement renewable sources particularly well – both are energy carriers that can transmit energy from primary energy sources to end-users.

It is widely accepted that hydrogen may have a role to play in decarbonising the transport sector, which still relies almost exclusively on oil. In Great Britain (GB), for example, the domestic transport sector is a major oil user and is responsible for approximately 20% of total GB carbon dioxide emissions [2]. Indeed, decarbonising this sector is a main driver behind the development of fuel cell and electric vehicles. On a more positive note, GB has a very strong potential for wind power; in fact, it is considered one of the best locations in the world and the best in Europe [3]. Converting wind energy to either electricity or hydrogen that can be used in electric or fuel cell cars results in zero emissions (or low emissions if the emissions in manufacturing and installing the network components are considered), which can help to achieve future emissions targets [4, 5, 6].

Since both demands and wind availability are distributed in space and vary with time, there is no guarantee that wind power will be available where and when it is needed. Therefore, a network of technologies that is sufficiently flexible to deal with the mismatch between the intermittent supply and demand for energy is needed. It is then natural to ask what the network will look like. How many wind turbines will be needed and where will they be located? What role will electricity networks have to play: should the energy generated by wind turbines be transmitted as electricity or converted in-situ to hydrogen and transmitted through pipelines? Even at the national level, where different regions are interconnected by transmission lines, there may not be sufficient wind power to meet peak demands. Therefore, storage technologies are expected to play a role; their type, size and location need to be determined. Conversion technologies, such as electrolysers and fuel cells, to interconvert electricity and hydrogen may also be necessary.

The questions posed above are extremely difficult to answer without using mathematical models, because the networks can be highly complex and integrated with many alternative options. Several models for hydrogen networks have been developed and typically fall into one of the following categories: focus on design with simplified operation; focus on operation with fixed design; and focus on design and/or operation for a single location (i.e. no spatial representation, therefore transport of energy was not modelled).

A widely used MILP model from the first category was presented by Almansoori and Shah [7]. It is a multi-period, spatially-resolved multi-echelon model considering the production of hydrogen, for fuel cell vehicles, from natural gas, electricity, biomass and coal. The model represents GB spatially using 34 square cells and considers the transition from 2015 to 2044 using 6 5-year periods. The focus of the model is on the hydrogen network alone: it determines the location of the hydrogen-production technologies and the transport of hydrogen between the cells but the raw materials cannot be transported. The operation of the network is based on a user-specified daily demand for hydrogen in each cell and each period that represents the average demand over a 5-year period. While others have used variants of this model in a number of studies, e.g. [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18], they have similarly focused on a long-term horizon
and not considered any finer time resolution. As the model was designed only to consider these 5-year periods, it is not suitable for the problem described above, where it is necessary to account for hour-by-hour operation of the network including a proper inventory balance for storage. The model is also unable to convert hydrogen back to electricity, due to its multi-echelon nature, and is therefore not sufficiently flexible to cover all possible routes from wind power to hydrogen-powered transport.

Conversely, there are models that focus on the operation of the network for a pre-determined design (the locations and size of the technologies are fixed), an example of which is that of Chaudry and co-workers [19, 20]. The original model [19] was developed to optimise the operation of gas-electricity networks over 30 one-day intervals. It has recently been extended to include wind generation and hydrogen injection into the natural gas grid and the operation of the network was optimised over 24 one-hour intervals [20].

Finally, the last group of models can determine the design and/or operation of stand-alone renewable energy networks but because there are no spatial representation in these models, the transport of energy between technologies installed at different locations cannot be modelled. Examples of such models include [21, 22, 23, 24, 25, 26, 27].

In the context of modelling and optimising integrated energy networks that include intermittent renewables with a detailed representation of storage, which will almost certainly be needed to increase the contribution of renewables on the grid, the models described above lack one or more of the elements required. Although the hydrogen supply chain model of Almansoori and Shah [7] was not designed for the short time scales required to model intermittent renewables and does not include an inventory balance for energy storage, it is also not sufficiently flexible to allow all possible pathways to be modelled, especially circular ones (e.g. conversion of electricity to hydrogen and back again). The model of Chaudry and co-workers [19, 20] does not consider a long enough time horizon to account for seasonality and the model is restricted to natural gas and electricity. The last category of models is unable to represent the spatial dependence of the energy system: a capability that is required for modelling transmission of resources, which is a crucial feature of the energy system to capture, given the distributed nature of many of the natural resources and energy demands.

This paper presents the simultaneous design and operation of integrated wind-hydrogen-electricity networks using the general spatio-temporal modelling framework, STeMES, the full and detailed mathematical formulation of which appears in [1]. STeMES is a mixed integer linear programming (MILP) model, which is implemented in AIMMS [28] and solved with the CPLEX solver [29]. It can be used to model any networks comprising technologies for conversion, storage and transport. It takes into account the spatial distribution of system properties such as demands and resource availability and determines decisions relating to space such as location of technologies. It is a dynamic model, which is necessary in order to model intermittency and dynamics of energy storage. It determines the optimal structure of the network and its operation considering simultaneously the short-term dynamics and a long-term planning horizon.

In this study, STeMES was applied to determine the design and operation of networks of technologies required to decarbonise the domestic transport sector in GB. Although significant uncertainty still remains in technologies that comprise the network, particularly hydrogen storage, it is useful to begin exploring the potential optimal network configurations in order to obtain some insights into its impact on society, environment and economy. It is important to understand the interactions between the different components of the network and how different configurations affect system-level performance and costs. In addition, it is
useful to identify and bring together data from different sources that are relevant to wind-hydrogen-electricity networks. Considering the existing literature in the field of energy systems modelling, this is an application that is newly addressed and beyond the scope of previous energy systems models.

In our previous publication [1], the focus was on the mathematical formulation and the solution procedure, providing an example in which the model was used to determine the optimal design and operation of a hydrogen network (not integrated with the electricity network) for a hypothetical island. In this paper, the focus is on the application of the model to more realistic scenarios, considering Great Britain, using actual data available from the open literature and governmental and industrial sources. Where actual data were not available, existing models that can determine the required data were used. For example, the characteristics of compressors and expanders were determined via simulations in gPROMS ProcessBuilder [30], the properties of the hydrogen pipeline were obtained using the pipeline model in gCCS [31] (whereby properties for hydrogen were specified instead of those for CO$_2$), and the time series wind data for GB were obtained from the Virtual Wind Farm Model [32]. Moreover, technologies associated with electricity networks were added to the database and the mathematical formulation was extended to account for the land area occupied by technologies (which for wind turbines was considered significant). Finally, a more detailed model of the hydrogen network was included, where hydrogen is generated and stored at a high pressure and transmitted and distributed at a lower pressure. The compressors and expanders required to move the hydrogen from one pressure level to another were also included in the model of the hydrogen network, along with their interaction with the electricity network (i.e. electricity consumed by the compressors and electricity generated by the expanders).

To date, to the authors’ knowledge, the model presented in this paper is the first MILP model in the literature for integrated wind-hydrogen-electricity networks that can simultaneously determine the design and operation of the network while considering both spatial and temporal aspects in detail so that transport and storage of energy can be modelled more accurately.

The paper is structured as follows: Section 2 defines the problem and Section 3 characterises the properties of the system based on the spatial and temporal discretisation used in the model. The wind farm siting analysis, performed using GIS, is also presented in Section 3. Section 4 describes the structure and components of wind-hydrogen-electricity networks, the mathematical model for which is presented in Section 5. The modelling of the distribution network is discussed in Section 6. The results of the case studies are presented in Section 7. Finally, some concluding remarks are made in Section 8.

2. Problem statement

The problem to be solved is briefly summarised below.

Given:

- The hourly hydrogen demand at different locations
- The hourly availability of wind power at different locations
The characteristics of each technology, e.g. efficiency and unit costs (capital, operating and maintenance costs)

Determine:

- The optimal number, size and location of wind turbines, electrolysers, hydrogen storage, fuel cells, compressors and expanders
- Whether to transmit the energy as electricity or hydrogen or both
- The structure of the transmission network
- The hourly operation of each technology and the transmission infrastructure

Subject to:

- The available land area for the wind turbines
- Satisfying the demands for hydrogen in all locations at all times

Objective:

- Minimise total network costs

3. Spatio-temporal representations

This section characterises the properties of the system according to the spatial and temporal representations used in the model.

3.1. Spatial discretisation

STeMES models the spatial dependencies of the system by dividing the study region into a number of zones, each of which is assumed to have uniform properties (e.g. resource demand and availability) and may host a certain number of technologies for generation/conversion and storage. Infrastructures for transporting resources may connect each zone to each of its neighbours.

In this study, Great Britain was divided into 16 zones based on the National Grid Seven Year Statement (NG SYS) 17 study zones [33]. Figure 1 shows the 16 transmission zones considered in this study: Z1 to Z3 correspond to the same zones in the NG SYS; NG SYS’s zones 4 and 5 were combined to form Z4 because zone 5 is much smaller than the other zones and keeping the number of zones to a minimum helps to reduce the computational burden of the model; and Z5 to Z16 correspond to NG SYS’s zones 6 to 17.
One advantage of using a similar spatial discretisation to that of the National Grid is that a significant amount of data, in both zonal and national form, are available from sources such as [34]. Where more detailed data are available, they are aggregated for each zone.

![Spatial discretisation of Great Britain into 16 transmission zones.](image)

**Figure 1:** Spatial discretisation of Great Britain into 16 transmission zones.

### 3.2. Wind turbine siting constraints

In each zone, only a certain amount of land is available for siting of wind turbines. The maximum land area in each zone was determined by applying a number of technical and environmental constraints. The technical constraints take into account the site’s wind speed, topography and accessibility, whereas the environmental constraints consider the landscape impacts and planning restrictions [35, 36, 37]. In this work, the following 10 criteria were used to determine the total land area in each zone suitable for siting wind turbines using GIS.

1. **Wind speed**
   An average annual wind speed of at least 5m/s measured 45m above ground level is needed to justify the installation of wind turbines on economic grounds [35, 36, 37].

2. **Slope**
   Sites with slope of less than 15 percent were considered suitable for wind turbines to ensure that the parts can be safely transported for installation on steep mountainous areas [38].
3. Access
A maximum distance of 500m from the minor road network is imposed to allow entry of construction vehicles, delivery of materials and general access for supplies and staff [35, 36].

4. Connectivity to National Grid
The wind turbines need to connect to an energy distribution/transmission network. For simplicity, it was assumed that National Grid lines closely follow the road and that new distribution/transmission lines, if they are to be built, will be laid along existing lines. Therefore, a suitable site should not be more than 1500m from the main road [35, 36]. For safety, a buffer of 200m from the main road was also applied.

5. Planning restrictions
Locations that are nationally designated as nature and science protection areas were excluded. This was done by excluding the areas classified as Sites of Special Scientific Interest (SSSI) [35, 36].

6. Population impacts
For safety and to minimise noise intrusion, a buffer of 500m from developed land used area (DLUA) was applied [35, 36].

7. Water pollution
To minimise the impact on wildlife in and around water courses and to decrease the risk of flooding of the wind turbine during winter and spring, only sites that are more than 200m from a river were considered suitable [35, 36].

8. Interference
To minimise impact on wildlife and to prevent wind interference, the suitable sites should be more than 250m from woodland [35, 36].

9. At least 5km from airports for safety [37]

10. Exclude land occupied by existing wind turbines including spacing between turbines of 5 rotor diameters [37]
In the model, the existing wind turbines are specified but only the land area occupied by the new wind turbines contributes to the land requirements. Thus the available area excludes the area occupied by the existing turbines.

The required geospatial data were obtained from the following sources: wind speed from the database of Department of Trade and Industry [39]; slope, major and minor roads, DLUA, rivers and woodlands from Ordnance Survey’s Meridian 2 [40], SSSI in England from Natural England [41], SSSI in Wales from Natural Resources Wales [42], SSSI in Scotland from Scottish Natural Heritage [43], airports from ShareGeo Open [44], and existing wind turbines from the Virtual Wind Farm Model [32].

The total available areas were determined by taking the union of the suitable areas defined by constraints 3 and 4 and then intersecting with those of the other 8 constraints. Figure 2 shows the available area for wind turbines in each zone, which defines the land footprint constraints in the model. For comparison, Table 1 gives the available areas before and after application of the 10 constraints.
Available area after applying constraints 1 to 10

Figure 2: The available area for wind turbines as determined by taking the union of the suitable areas defined by constraints 3 and 4 and then intersecting with those of the other 8 constraints.
Table 1: The total available area for wind turbines in each zone.

| Zone | Unconstrained (km²) | After application of the 10 constraints (km²) |
|------|---------------------|---------------------------------------------|
| Z1   | 34,151.72           | 74.19                                       |
| Z2   | 7,200.84            | 140.69                                      |
| Z3   | 5,132.40            | 7.11                                        |
| Z4   | 14,587.03           | 147.49                                      |
| Z5   | 19,610.47           | 420.41                                      |
| Z6   | 14,159.24           | 259.75                                      |
| Z7   | 15,169.65           | 355.71                                      |
| Z8   | 21,754.03           | 601.46                                      |
| Z9   | 7,091.72            | 168.25                                      |
| Z10  | 10,842.75           | 281.37                                      |
| Z11  | 21,047.13           | 445.71                                      |
| Z12  | 26,165.55           | 616.90                                      |
| Z13  | 4,345.66            | 61.34                                       |
| Z14  | 5,048.83            | 71.40                                       |
| Z15  | 10,225.73           | 159.29                                      |
| Z16  | 17,869.68           | 444.21                                      |

3.3. Temporal discretisation

The model needs to take into account simultaneously the long-term strategic decisions as well as short-term operational issues. The challenge in modelling is the need to represent energy storage using short time scales (i.e. not coarser than hourly intervals) to capture its dynamics. Considering a whole year planning horizon and formulating the model using contiguous hourly intervals result in a computationally intractable model. Several methods to overcome this problem were discussed in [1]. The non-uniform hierarchical time discretisation and a decomposition method were applied in this paper to solve the model within an acceptable time.

Instead of representing time as contiguous hourly intervals, different time layers can be used: yearly intervals to model investment decisions, seasonal intervals to model seasonal variations (e.g. in demand and wind availability), daily intervals to capture the difference between weekdays and weekends and hourly intervals for system balancing. This approach allows for a more efficient representation of time by exploiting the periodicity inherent in some of the system’s properties. For example, instead of using 7 daily intervals per week, the periodicity in the demand data can be utilised, e.g. weekdays are likely to have similar demand profiles, which are likely to be different from those for weekends. Therefore, a particular day type can be repeated a certain number of times and the demands can be represented as a sequence of repeated profiles, e.g. 5 repetitions of a weekday profile followed by 2 repetitions of a weekend profile can represent a week’s worth of data. Since energy storage is one of the technologies being modelled, the storage inventories within and between time levels have to be linked, thus requiring additional variables and constraints. In this work, a single year was considered and a time discretisation of 4 different seasons in a year (spring, summer, autumn, winter), 2 different day types in a week (weekday and weekend) and 24 hours in a day was used.

3.4. Spatio-temporal domestic road transport demand

The energy demand for transport in the domestic sector on a 1km square grid was estimated by Wang et al. [45], the data for which can be downloaded from [46]. They disaggregated the data for GB using the
statistical data from the Living Costs and Food Survey [47] and census data. The data are “home-based” rather than “road-based”, i.e. the data were assigned to the homes of the drivers of the cars and vans rather than the stretch of road where the emissions were produced. This assumption does not affect the suitability of the data for this study because the 1km data were aggregated to the 16 transmission zones considered in this study. The 1km domestic transport data provided by Wang et al. were assumed to be demands for petrol. These were converted to demands for hydrogen by using the average fuel economies of petroleum cars and fuel cell cars, the former being 49 mpgge and the latter being 79 mpgge [48]. Figure 3 presents the hydrogen demands for domestic transport at the 1km level and the aggregation to the 16 transmission zones used in the model.

Figure 3: Hydrogen demands for domestic transport (a) at the 1km level [46] with the cross mark indicating the location of the centre of demand in each zone, and (b) the aggregation to each of the 16 zones with the circles representing the magnitude of the demand.

The temporal distribution of demands was estimated from the statistical data set on traffic flows, which can be downloaded by month (TRA0305), by day of the week (TRA0306) and by time of day (TRA0307) from the Department for Transport website [49]. These data were used to disaggregate the average hydrogen demand in each zone, shown in Figure 3(b), into 4 different seasons, 2 different day types (weekday and weekend) and 24 hours in a day. An example result of the disaggregation is shown Figure 4, which presents
the temporal distribution of hydrogen demand in zone 13; similar graphs exist for the other zones but are not shown in the interest of space. The demands are highest in summer and lowest in winter. The weekday profiles show a distinct bimodal shape, corresponding to the morning and evening rush-hour periods, and the demands in the weekends peak around noon.

3.5. Spatio-temporal wind availability

The Department of Trade and Industry wind speed database [39] provides estimates of the annual wind speed throughout GB. Figure 5 shows the locations with annual average wind speed of at least 5m/s at 45m above ground level.
In this study, the spatio-temporal wind speed data were obtained from the Virtual Wind Farm Model [32]. Historic weather data from 2014 were used to produce wind speeds for 8,760 hours for 10 different locations in each zone, which were then aggregated to determine the representative wind speed in each zone. In order to match the temporal discretisation used in the case studies, a representative profile for each day type and each season is required. Averaging the profiles over all days of the same day type over all weeks in the same season resulted in the lose of variability in the data which may lead to an under-designed network. Therefore, for a conservative design, the daily profiles were chosen to be the most variable (defined as the one with the largest difference between the maximum and minimum wind speed) among all of the different days of the same day type over all weeks in the same season and, where possible, profiles for different day types were chosen to be different to each other. Figure 6 shows the resulting wind speed profiles for zone 13; the profiles for other zones are not shown in the interest of space. Given the large uncertainty in the behaviour of the wind, the actual operation of the network will change to accommodate different scenarios as they occur but the network design must of course remain fixed. Using different wind profiles in the optimisation may result in different network designs, e.g.: flatter wind profiles, or ones that match more closely the demands, may result in less installed storage capacity; the more the wind profiles are variable or incompatible with the demands, the higher the installed storage capacity is expected to be.

Figure 5: Sites with annual average wind speed of at least 5m/s at 45m above ground level [39].
Figure 6: Temporal distribution of wind speed in zone 13 (similar graphs exist for the other zones).

4. Network structure

As already mentioned, in this study GB is divided into 16 transmission zones. To illustrate the structure of the network, Figure 7 shows two example zones. In each zone, a number of wind turbines may be installed in order to generate electricity. This can be converted to hydrogen, using electrolysers, which is used to fulfil transport demand. The hydrogen produced by the electrolysers is assumed to be at 20MPa, which is also the pressure at which it can be stored in underground caverns and pressurised vessels. Therefore, the hydrogen produced by electrolysers is labelled “Hydrogen at 20MPa” in Figure 7 and only this state can be stored in “Underground storage” and “Pressurised vessels”. The hydrogen produced by the electrolysers or withdrawn from storage must ultimately be delivered to customers via underground distribution pipelines to fuelling stations. The hydrogen supplied to customers may have been produced locally or imported into the zone from another zone within GB. Transport of hydrogen from one zone to another takes place in transmission pipelines, which based on the data provided by Yang and Ogden [50], have a maximum inlet pressure of 7MPa and it is also assumed that the distribution network requires hydrogen at this pressure. Therefore a second hydrogen state, “Hydrogen at 7MPa”, is produced when the “Hydrogen at 20MPa” is expanded using the “Expanders” technology, which also produces some electricity. This lower-pressure hydrogen can then be distributed to customers within the zone (it is assumed that the fuelling stations are equipped with compressors to dispense hydrogen at a pressure required by fuel cell cars) or transmitted to another zone (where the demand may be higher but with fewer wind turbines available). Any hydrogen received from other zones may also be stored but this requires the hydrogen to be compressed from 7MPa back to 20MPa by using compressors, which also consume electricity. It was assumed that the pressure drop in the pipeline is negligible to restrict the number of hydrogen pressure levels to two, which reduces the size of the model; the validity of this assumption was confirmed when the results of the case studies were obtained (see Section 7.1, para. 3). Finally, the hydrogen may also be converted to electricity in fuel cells, where a maximum inlet pressure of 7MPa is assumed. Energy may also be transmitted between zones in the form of electricity: transmission lines (high voltage alternating current (HVAC) and high voltage direct current (HVDC) overhead lines (OHL) and underground cables (UC) are considered in this example). Therefore, when there is excess electricity generation from the wind turbines, the model is able to choose whether to transmit this electricity to other zones (where the demand would be higher), convert it to hydrogen for
storage or convert it to hydrogen and transmit the hydrogen to another zone; alternatively, the model may choose to reduce the excess electricity production by disengaging some or all of the wind turbines.

Figure 7 is the Resource-Technology Network (RTN) diagram for the problem being modelled, which represents all of the possible energy pathways in the system. An RTN comprises 2 nodes: resources (usually drawn as circles) to represent any distinct material state, e.g. having a particular composition, temperature and pressure, and technologies (usually drawn as rectangles) to represent processes that convert a set of input states to a different set of output states. More discussions about RTN can be found in [1, 51]. In this work, resources such as electricity, hydrogen at 20MPa and hydrogen at 7MPa, can be transformed from one form to another via any of the conversion technologies (e.g. electrolysers, fuel cells, compressors and expanders). The resources can also be moved to different locations via different transport technologies: transmission technologies (e.g. pipelines, electricity cables) for movement between different zones and distribution technologies (e.g. underground pipeline and fuelling stations) for delivery to customers within a zone. To balance resource availability and demand (over time and space), the resources can be put into or retrieved from storage technologies. The model formulation presented in this paper can accommodate storage of any resources but in this study only storage of hydrogen, specifically into underground storage or pressurised vessels, was considered. Each technology (be it conversion, transport or storage) is characterised by its efficiency (which is specified through the conversion factors in the model), minimum and maximum capacities and unit costs (e.g. capital cost, as well as fixed and variable operating costs). By defining the conversion factors appropriately, resource requirements and losses associated with each technology can be modelled.

4.1. Production/conversion technologies

This section describes the characteristics of the wind turbines, electrolysers, fuel cells, compressors and expanders that were considered in the case studies.
4.1.1. Wind turbines

Figure 8 shows that there is already a significant number of wind farms installed throughout GB. The optimisation will choose to include any of the existing wind turbines as part of the network if it is cost-effective to do so. From the data in [32], the average diameter of existing turbines in GB was calculated to be 70m. If utilised, only the O&M costs of the existing wind turbines are included in the total cost of the network and not their capital cost. If new wind turbines were to be installed, for simplicity, the model considers only one type of wind turbine, which represents a standard modern on-shore wind turbine with a rotor diameter of 100m and an efficiency (i.e. power coefficient) of 35%. The footprint of each turbine is calculated assuming a minimum spacing of 5 rotor diameters between turbines [37]. The unit capital cost of a turbine of this size (i.e. 1.23MW$_{el}$ at a wind speed of 9m/s) was assumed to be £1.09M, based on the estimates given in [52, 53, 54]. The annual operation and maintenance (O&M) costs, which include insurance, regular maintenance, repair, spare parts and administration, were assumed to be 5% of the capital cost [52].

![Wind farm capacity (MW)](image)

Figure 8: Capacity of existing on-shore wind farms [32].

4.1.2. Electrolysers

The electrolyser data were based on the report by the National Renewable Energy Laboratory (NREL) [55]. The maximum production rate is 50 tonnes H$_2$ per day (69.38MW$_{hy}$) and the efficiency is 50kWh electricity
per kg H\(_2\) (i.e. 67%). The unit capital cost is £31.56M and the annual O&M costs are 5% of the capital cost. The operating pressure of the electrolyser was assumed to be 20MPa, therefore the generated hydrogen can be directly put into storage but needs to be expanded to 7MPa for transmission or distribution.

### 4.1.3. Fuel cells

This work considers a 41.63MW\(_{el}\) solid oxide fuel cell (SOFC) with an efficiency of 60%. The cost of which was estimated from a 200kW\(_{el}\) SOFC unit manufactured by Bloom Energy using a sizing exponent of 0.85. The unit capital cost is £44.86M and the annual O&M costs as a fraction of capital cost per year is 6%.

### 4.1.4. Compressors

The electricity required to compress hydrogen from 7MPa to 20MPa was calculated to be 0.56kWh/kg H\(_2\) by simulating a compressor train with interstage cooling in gPROMS ProcessBuilder [30]. Compressors of 7 different sizes were considered – the size of each one was determined based on the maximum injection rate of each storage device (discussed in Section 4.3). The capital cost of each compressor was estimated using the equation presented by Yang and Ogden [50]: £9000 \((\text{S} \times 10)^{0.9}\), where \(S\) is the compressor size in kW\(_{el}\). The annual O&M costs were assumed to be 5% of the capital cost, which include changing oil regularly, replacing valves when needed among others. Table 2 summarises the characteristics of the compressors considered in the case studies; the storage IDs given in the third column are defined in Tables 6 and 7.

| ID    | Size (MW\(_{el}\)) | Storage technology for which it is sized | Unit capital cost (£M) | Unit O&M (£k/yr) |
|-------|-------------------|----------------------------------------|------------------------|------------------|
| COMP1 | 25.44             | CGH2S - L                               | 10.45                  | 522.53           |
| COMP2 | 2.54              | CGH2S - M                               | 1.32                   | 65.78            |
| COMP3 | 0.25              | CGH2S - S                               | 0.17                   | 8.28             |
| COMP4 | 154.15            | US - Ald                                | 52.89                  | 2,644.68         |
| COMP5 | 63.76             | US - Hum                                | 23.90                  | 1,194.89         |
| COMP6 | 168.17            | US - Rou                                | 57.20                  | 2,860.10         |
| COMP7 | 81.98             | US - War                                | 29.96                  | 1,498.16         |

### 4.1.5. Expanders

Similar to compressors, expanders of seven different sizes were considered; the size of each one was based on the maximum withdrawal rate of each storage technology (discussed in Section 4.3). The electricity that can be recovered from the expansion of hydrogen from 20MPa to 7MPa was calculated to be 0.29kWh/kg H\(_2\) by simulating a train of expanders with interstage heating in gPROMS ProcessBuilder [30]. The capital cost was estimated from an expander with a rated output of 1MW\(_{el}\) described in [56] using a sizing exponent of 0.80. The annual O&M costs were assumed to be 5% of the capital cost. The properties of the expanders used in the case studies are given in Table 3.
Table 3: Characteristics of the expanders used in the case studies

| ID  | Size (MW_{el}) | Storage technology for which it is sized | Unit capital cost (£M) | Unit O&M (£k/yr) |
|-----|----------------|------------------------------------------|------------------------|------------------|
| EXP1 | 13.17          | CGH2S - L                                | 17.30                  | 865.19           |
| EXP2 | 1.32           | CGH2S - M                                | 2.74                   | 137.12           |
| EXP3 | 0.13           | CGH2S - S                                | 0.43                   | 21.73            |
| EXP4 | 94.20          | US - Ald                                 | 83.50                  | 4,174.75         |
| EXP5 | 28.67          | US - Hum                                 | 32.23                  | 1,611.72         |
| EXP6 | 165.10         | US - Rou                                 | 130.81                 | 6,540.27         |
| EXP7 | 10.52          | US - War                                 | 14.46                  | 722.94           |

4.2. Transmission technologies

There have been a number of different approaches to calculating the distance between regions in order to model the transmission network. For those studies that used a grid of square cells, the centres of the squares were considered as the points where the transmission lines from different zones meet [7, 17, 57, 58]. The most common approach is to divide the study region into administrative regions and to take the centroid (centre of area) of each region as the point where the transmission lines meet [8, 9, 10, 11, 12, 13, 15, 18]. Finally, Sabio et al. [14, 16] used autonomous regions in Spain along with the locations of their capital cities. In this study, since the demand density at the 1km level is available, it was assumed that the zones are connected by their centres of demand, obtained from the spatially-distributed demand at the 1km level and equations 1 and 2:

\[
x_z = \frac{\sum_{i \in I_2} x_i \bar{D}_i}{\sum_{i \in I_2} \bar{D}_i} \quad \forall z
\]

\[
y_z = \frac{\sum_{i \in I_2} y_i \bar{D}_i}{\sum_{i \in I_2} \bar{D}_i} \quad \forall z
\]

where \(x_z\) is the x-coordinate of the demand centre of zone \(z\), \(x_i\) is the x-coordinate of the centroid of 1km cell \(i\), which is in zone \(z\) if \(i\) is in the set \(I_2\), and \(\bar{D}_i\) is the average demand in 1km cell \(i\); the y-coordinate is calculated similarly. The centres of demand are indicated by the cross-marks in Figure 3(a).

Since the underground storage facilities are not located at the centre of demand of each zone, the additional cost of transporting between the centre of demand and the underground storage was included in the cost of underground storage. The number of fuelling stations depend on the total demand in each zone whereas the length of the distribution network in each zone is estimated from the centres of demand and the distribution of demand at the 1km level (discussed in Section 6).

4.2.1. Hydrogen pipeline

Yang and Ogden [50] provided the following properties of hydrogen transmission pipelines: maximum inlet pressure of 7MPa, outlet pressure of 3.55MPa and diameter of 100cm. Given these conditions, and the length and angle of inclination of the pipeline, it is possible to calculate the maximum flow of hydrogen.
Through the pipeline. The maximum steady-state flowrate occurs when the pressure difference across the pipeline is balanced by the frictional forces at the wall and the gravitational forces (for inclined pipelines). The frictional forces depend on the length of the pipeline, the roughness of the wall and the square of the velocity of the fluid. Thus the force balance gives the velocity of the fluid in the pipeline and the maximum flowrate is obtained from \( F \rho H^2 = \rho H^2 u_H^2 A_{\text{pipe}} \), where \( u_H \) is the velocity of the fluid, \( \rho \) is the density of the fluid and \( A_{\text{pipe}} \) is the cross-sectional area of the pipe.

There is an additional restriction on the throughput of the pipeline due to the velocity of the fluid, which should be lower than the erosional velocity. Typically, the pipeline is operated to ensure that the velocity is always lower than the erosional velocity by a specified margin (the erosional velocity margin). In this study, the velocity was restricted to be no more than 70% of the erosional velocity, calculated as: \( u_{\text{er}} = 121.99/\sqrt{\rho H^2} \).

Since the density of the hydrogen in the pipeline depends on the pressure, which drops from 7MPa at the inlet to 3.55MPa at the outlet, a more accurate approach to calculating the flowrate is to derive and solve a coupled set of partial differential and algebraic equations from differential mass, momentum and energy balances, along with an appropriate equation of state to describe the physical properties of hydrogen. A number of commercial packages are available that can perform pipeline simulations using this approach. In this study, gCCS [31] was used to perform the detailed simulations of the pipelines in order to calculate the maximum flowrate. The pipe model in gCCS comprises dynamic mass, momentum and energy balances distributed in the axial direction of the pipe. Wall friction is calculated assuming fully-turbulent flow. The physical properties of the fluid are calculated using gSAFT, Process Systems Enterprise’s proprietary implementation of the SAFT \( \gamma \)-Mie equation of state [59, 60].

Since the distances between the zones are all different, simulations of the longest and shortest pipeline length in the network were performed. Each simulation starts with an inlet pressure of 4MPa, which is increased until the velocity in the pipe reaches 70% of the erosional velocity or until the inlet pressure reaches 70MPa. The results of the two simulations to determine the maximum flowrate are summarised in Table 4. The shorter pipeline reaches its maximum flowrate (98kg/s) with an inlet pressure of just 4.73MPa, which indicates that the velocity in the pipe has reached 70% of the erosional velocity. Similar results can be seen for the longer, 230km, pipeline, where the simulation ended with the inlet pressure at 6.77MPa and a maximum flowrate of 82kg/s. Taking the lower flowrate of 82kg/s and multiplying by the lower heating value of hydrogen gives, as a conservative estimate, a maximum energy throughput of the transmission network of 9.81GW\(_{\text{hy}}\). It may be noted that the erosional velocity is lower in the 230 km pipe than it is in the 48km pipe. This is due to the higher average pressure (since a much higher pressure drop is required to achieve the same flowrate as in the 48 km pipe), which results in a higher density and thus a lower erosional velocity.

| Pipe length (km) | Max flow rate (kg/s) | Inlet pressure (MPa) | Outlet pressure (MPa) | Erosional velocity (m/s) | Max velocity in pipe (m/s) |
|-----------------|----------------------|----------------------|-----------------------|--------------------------|--------------------------|
| 48              | 97.56                | 4.73                 | 3.55                  | 63.38                    | 44.36                    |
| 230             | 81.87                | 6.77                 | 3.55                  | 53.36                    | 37.35                    |

The capital cost of the pipeline comprises cost of materials, installation, right-of-way and miscellaneous. Similar to the approach used in [50, 61], the cost of materials was calculated as a function of the pipeline
diameter, which is approximately £1.74M/km for pipelines with a diameter of 100cm. The pipelines were assumed to be installed next to the existing pipelines, hence the rights of way cost can be neglected. The installation cost is higher in urban locations than in rural places but in this study an average installation cost of £270k/km was used. Since the operating pressures of the electrolyser and hydrogen storage are much higher than the maximum pressure of the pipeline, it was assumed that no compression would be required for transmission of hydrogen. A turbine at the pipeline inlet may be used to reduce the pressure to 7MPa and generate electricity. The annual O&M costs were assumed to be 2% of the capital cost.

4.2.2. Electricity transmission lines

Different high-voltage electricity transmission options were considered in the case study: alternating current overhead lines (HVAC OHL) and underground cables (HVAC UC) and direct current overhead lines (HVDC OHL) and underground cables (HVDC UC). The properties of these technologies, which were estimated from [62, 63], are summarised in Table 5. The capital cost includes equipment, installation, engineering, auxiliaries, civil works among others. HVAC transmission assets include overhead lines, cables, transformers, switchgear/substations, reactive compensators etc., while those for HVDC transmission comprises overhead lines, cables, converter stations among others. Overhead lines equipment costs include conductors, pylons/towers, foundations, clamps and related devices. Costs of converter stations equipment include valves, converter transformers, filters, control, switchyard etc. [62]. Other components that may contribute to the costs are land acquisition and local compensations, which were not included in the estimates used in the case studies. The third column in Table 5 gives the power rating of the technologies, which would only ever need to operate at full load under emergency conditions – the maximum load should not exceed 50% of the capacity [63]. The electrical energy losses during operation were estimated from the values reported by Parsons Brinckerhoff and associates. Figure 3-7 in [63] compares the energy losses between overhead lines and underground cables. For 1,500MVA-rated lines, for example, below the transfer power of approximately 900MVA, the losses from overhead lines are lower than those from underground cables; the reverse is true above this transfer power.

Table 5: Characteristics of the high-voltage electricity transmission lines considered in the case studies, estimated from [62, 63].

| Type                                      | Voltage level | Power rating | Unit capital cost (£M/km) | Unit O&M costs (£k/km/yr) |
|-------------------------------------------|---------------|--------------|---------------------------|---------------------------|
| HVAC OHL, single circuit                  | 400kV         | 1,500MVA     | 0.40                      | 7                         |
| HVAC OHL, double circuit                  | 400kV         | 2×1,500MVA    | 0.75                      | 13                        |
| HVAC underground XLPE cable, single circuit| 400kV         | 1,000MVA     | 2.00                      | 15                        |
| HVAC underground XLPE cable, double circuit| 400kV         | 2×1,000MVA    | 3.5                       | 26                        |
| HVDC OHL, bipolar                         | ±400kV        | 1,500MW      | 0.5                       | 8.71                      |
| HVDC underground cable pair               | ±350kV        | 1,100MW      | 1.75                      | 20.36                     |

4.3. Storage technologies

Storage technologies have three important characteristics:

1. Maximum available capacity,
2. Injectability, which is the maximum rate at which gas can be injected into storage, and
3. Deliverability, which is the maximum rate at which gas can be withdrawn from storage.

In the case studies, overground and underground hydrogen storage facilities of different sizes were considered and their properties are described in the following sections.

4.3.1. Pressurised storage vessels

One of the advantages of this type of storage is its simplicity: the only requirement is a compressor and a pressure vessel. One of the disadvantages, however, is the low storage density, which depends on the storage pressure. In general, higher pressure means higher capital and operating costs [64]. In the case studies, storage tanks of 3 different sizes, operating up to 20MPa, were considered, the characteristics of which are summarised in Table 6. Since compressors are considered as conversion technologies rather than components of storage technologies, the capital cost only includes the vessel and the cushion gas requirement. Cushion gas is the volume of gas required to be kept in the facility in order to maintain the operating pressure and cannot be recovered until the facility stops operation. For compressed gas storage, the cushion gas requirement or “heel” was assumed to be 7.5% of the total capacity [65]. The annual O&M costs, which cover personnel and maintenance costs, were assumed to be 2% of the capital cost, which is the average of the values reported in [65].

Table 6: Characteristics of the pressurised storage tanks used in the case studies, estimated from [64, 65].

| ID        | Maximum available capacity (MWh) | Injectability (MW) | Deliverability (MW) | Unit capital cost (£M) | Unit O&M costs (£k/yr) |
|-----------|----------------------------------|--------------------|---------------------|------------------------|------------------------|
| CGH2S - L | 36,300                            | 1,512.50           | 1,512.50            | 135.32                 | 2,706                  |
| CGH2S - M | 3,630                             | 151.25             | 151.25              | 23.45                  | 469                    |
| CGH2S - S | 363                               | 15.13              | 15.13               | 4.07                   | 81                     |

4.3.2. Underground storage

Potential candidates for underground storage include salt caverns, depleted oil/gas fields and aquifers. There are a significant number of underground storage facilities, both at the operational and planning stage, in GB (cf. Figure 1 in the fact sheet provided by British Geological Survey [66]). Only 4 underground storage facilities, for which data are available in the literature, were considered in the case studies. Table 7 gives a summary of the characteristics of the 2 salt caverns (Aldborough and Warmingham) and 2 depleted oil/gas fields (Humbly Grove and Rough) considered in the case studies. The data for the maximum available capacity, injectability and deliverability were obtained from the National Grid [67]. The capital costs, which were estimated from the report by Thistlethwaite et al. [68], include land/depleted field acquisition, cavern construction (leaching and brine disposal) for salt caverns, wells and above ground treatment facilities, connecting pipelines, cushion gas and so on. The cushion gas requirement was assumed to be 20% for salt caverns and 45% for depleted oil/gas fields [65]. The annual O&M costs were assumed to be 2% of the capital cost [65]. Since the location of the underground storage does not coincide with the centre of demand of the zones where they are located, the additional cost for transporting the hydrogen between the underground storage and the centre of demand was also included in the costs. Other components that can further contribute to the costs but were not considered in the estimates include sub-surface analysis (e.g. seismic data), control systems and planning and environmental approvals.
Table 7: Characteristics of the underground storage considered in the case studies, estimated from [65, 67, 68].

| ID    | Location | Type                  | Transmission zone | Max. available capacity (TWh) | Injectability (MW) | Deliverability (MW) | Unit capital cost (£M) | O&M costs (£M/yr) |
|-------|----------|-----------------------|-------------------|------------------|-------------------|-------------------|---------------------|------------------|
| US - Ald | Aldborough | Salt cavern           | 7                 | 3.3              | 9,167             | 10,817            | 429                 | 8.58             |
| US - Hum | Humbly grove | Depleted oil/gas field | 15                | 3.05             | 3,792             | 3,292             | 61                  | 1.22             |
| US - Rou | Rough     | Depleted oil/gas field | 7                 | 34               | 10,000            | 18,958            | 280                 | 5.6              |
| US - War | Warmingham | Salt cavern           | 8                 | 1.08             | 4,875             | 1,208             | 199.8               | 4.00             |
5. Model formulation

The model presented in this section is based on the general spatio-temporal energy systems model, STeMES, developed by Samsatli and Samsatli [1]. This section briefly describes the model and the adaptation of the general framework to include integrated wind-hydrogen-electricity networks.

5.1. Objective Function

The objective is to minimise the total annual costs comprising operating and capital costs of the network. The operating costs have fixed and variable components. The fixed component includes the annual operating and maintenance (O&M) costs, which are typically reported in the literature as a fraction of the capital cost. The variable component of the operating costs relates to the costs incurred on a per-rate basis or the costs associated with converting/storing/transferring one unit (e.g. 1MWh) of resource: it typically includes the costs of feedstock and energy requirements for operating the technology. However, when using STeMES (and other RTN-based models), the variable operating cost should *not* include the cost of any raw material appearing as a resource in the RTN. This is because its cost is directly accounted for by the model: if it is a primary resource, then it may need to be purchased locally (which in this paper is treated as imports from outside of the network boundary, e.g. conventional “grid”, but would be easy to implement explicitly) or imported from abroad, which incurs a cost directly; or if it is an intermediate resource then the cost of its production is obtained from the variable operating cost of the technologies producing it and the cost of their raw material inputs, and so on until all true inputs to the energy system are counted. The capital costs, on the other hand, are one-time costs associated with the establishment of technologies, which are annualised using a capital charge factor, $\gamma$, taken to be 3 in the case studies. The objective function is given by equation 3:

$$Z = \sum \left[ \left( W + I_P + I_S + I_Q + I_W + I_P + I_S + I_Q \right) \right]/\gamma$$

(3)

where $W$, $I_P$, $I_S$ and $I_Q$ are the fixed operating costs of wind turbines, production plants, storage facilities and transmission infrastructures, respectively. $I_P$, $I_S$ and $I_Q$ are the variable operating costs of production plants, storage facilities and transmission infrastructures, respectively. Since these are all zero in the case studies (because all of their feedstocks are modelled directly) their formulations, which appear in [1] and can be easily included if necessary, are not given in this paper. $I_W$ and $I_S$ are the respective costs of importing and exporting resources outside of the network boundary. $W$, $I_P$, $I_S$ and $I_Q$ are the capital costs of the wind turbines, production plants, storage facilities and transmission infrastructures, respectively.

The total O&M costs can be calculated by multiplying the number of technologies by their annual O&M unit costs, as given by equations 4 to 6 for wind turbines, production technologies and storage facilities, respectively. Annual O&M unit costs for transport infrastructures are per unit distance, so the total O&M costs are given by the number of infrastructures multiplied by the unit O&M cost and the length of the connection (equation 7).
\[ J^w = \varsigma \sum_z F_{WT} \left( N_{EWT}^z + N_{NWT}^z \right) \] (4)

\[ J^{fp} = \varsigma \sum_{pz} F^{P}_{p} N_{pz}^P \] (5)

\[ J^{fs} = \varsigma \sum_{sz} F^{S}_{s} N_{sz}^S \] (6)

\[ J^{fq} = 0.5\varsigma \sum_{bzz'} F^{B}_{b} N_{bzz'}^B d_{zz'} \] (7)

\( F_{WT}, F^{P}_{p}, F^{S}_{s} \) and \( F^{B}_{b} \) are the annual O&M costs of a wind turbine with a rotor diameter of 100m, production technology \( p \), storage facility \( s \) and per unit distance of transmission infrastructure \( b \), respectively; \( N_{EWT}^z \) is the number of existing wind turbines in zone \( z \) that are utilised as part of the network; \( N_{NWT}^z \) is the number of new wind turbines in zone \( z \); \( N_{pz}^P \) is the number of conversion technologies of type \( p \) in zone \( z \); \( N_{sz}^S \) is the number of storage facilities of type \( s \) in zone \( z \); and \( N_{bzz'}^B \) is the number of infrastructure connections of type \( b \) between zones \( z \) and \( z' \). \( N_{EWT}^z, N_{NWT}^z, N_{pz}^P, N_{sz}^S, N_{bzz'}^B \) are all integer variables. The parameter \( d_{zz'} \) represents the distance between the demand centres of zones \( z \) and \( z' \). Since the transmission lines, i.e. pipelines and cables, considered in the case studies are both bidirectional, when such lines are present between zones \( z \) and \( z' \), \( N_{bzz'}^B \) has the same values for both directions (i.e. \( N_{bzz'}^B = N_{bz'z}^B \)). Therefore, to avoid double-counting, only half of the cost is considered per connection because the summation in equation 7 is over all combinations of \( z \) and \( z' \). The factor \( \varsigma \) converts cost from £ to £M.

The annual costs of importing and exporting resources (e.g. buying and selling of electricity from the conventional grid) are given by equations 8 and 9, respectively,

\[ J^m = \varsigma \sum_{rzhdt} V^M_r M_{rzhdt} n^d_h d^w n^t_l \] (8)

\[ J^x = \varsigma \sum_{rzhdt} V^X_r X_{rzhdt} n^d_h d^w n^t_l \] (9)

where \( V^M_r \) and \( V^X_r \) are the unit import and export costs of resource \( r \), respectively (for exports, the cost is negative if the resource has value and can be sold); \( M_{rzhdt} \) and \( X_{rzhdt} \) are positive variables representing the rates of import and export, respectively, of resource \( r \) in zone \( z \) at hour \( h \), day \( d \) and season \( t \); \( n^d_h \) is the duration of hourly interval \( h \), \( n^d_w \) is the number of times day \( d \) occurs in a week and \( n^t_s \) is the number of repeated weeks in season \( s \).

The total capital costs of the technologies can be determined from the product of the number of technologies and their respective unit capital cost. These are given by equations 10 to 13 for wind turbines, production technologies, storage facilities and transport infrastructures, respectively,
where \( C_{\text{WT}} \), \( C_P \), \( C_S \) and \( C_B \) are the capital costs of a single wind turbine, production technology \( p \), storage technology \( s \) and per unit distance of transmission infrastructure \( b \), respectively (all £/technology installed except for \( C_B \) which is £/connection/km).

### 5.2. Constraints

#### 5.2.1. Resource balance

The operation of the network is governed by a resource balance given by constraint 14:

\[
U_{rzhdt} + M_{rzhdt} + P_{rzhdt} + S_{rzhdt} + Q_{rzhdt} \geq D_{rzhdt} + X_{rzhdt} \quad \forall r \in \mathbb{R}, z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T} \quad (14)
\]

where \( U_{rzhdt} \) is the rate of utilisation of naturally-occurring energy resource \( r \), \( M_{rzhdt} \) is the rate of import of resource \( r \) from outside of the network into zone \( z \) (e.g. purchase of electricity from the conventional grid), \( P_{rzhdt} \) is the net rate of production of resource \( r \) due to the operation of conversion technologies, \( S_{rzhdt} \) is the net rate of utilisation of resource \( r \) from storage in zone \( z \), \( Q_{rzhdt} \) is the net rate of transmission of resource \( r \) into zone \( z \) from other zones, \( D_{rzhdt} \) is the demand for resource \( r \) (which, in the case studies, is zero for all resources except for hydrogen at 7MPa) and \( X_{rzhdt} \) is the rate of export of resource \( r \) from zone \( z \) to outside of the network (e.g. selling of electricity to the conventional grid).

#### 5.2.2. Resource availability

The electrical power that the turbine generators can extract from the wind depends on the efficiency of the wind turbines, which cannot be greater than the Betz Limit of 59.3% [69], the rotor sweep area and the wind speed. Equation 15 gives the wind power potential, in MW\textsubscript{el}, in zone \( z \) at hour \( h \), day \( d \) and season \( t \),

\[
u_{\text{Elec},zhdt}^{\max} = 0.5 \times 10^{-6} \eta_{\text{Eff}} \pi \left[ N_{z}^{\text{EWT}} \left( R_{\text{EWT,ave}}^\text{EWT} \right)^2 + N_{z}^{\text{NWT}} \left( R_{\text{NWT}}^\text{NWT} \right)^2 \right] v_{zhdt}^3 \quad \forall z \in \mathbb{Z}, h \in \mathbb{H}, d \in \mathbb{D}, t \in \mathbb{T} \quad (15)
\]
where \( \eta \) is the efficiency of the wind turbine (power coefficient), \( \rho_{\text{air}} \) is the air density (taken to be \( 1.23 \text{kg/m}^3 \)), \( R_{\text{EWT,ave}} \) is the average radius of the existing wind turbines, which was taken to be 35m from the data given in [32], \( R_{\text{NWT}} \) is the radius of the new wind turbines (50m) and \( v_{\text{zhdt}} \) is the wind speed, which varies with both location and time (see Section 3.5). In the case studies, \( u_{\text{max}}^{\text{rzhdt}} \) is zero for all other resources.

The wind power potential, \( u_{\text{Elec, zhdt}}^{\text{max}} \), is the upper bound on the rate of utilisation of electricity, \( U_{\text{Elec, zhdt}} \). That is, one cannot utilise more electricity than can be generated from the wind with the number of wind turbines installed in the cell. This is expressed more generally below.

\[
U_{rzhdt} \leq u_{rzhdt}^{\text{max}} \quad \forall r \in R, \ z \in Z, \ h \in H, \ d \in D, \ t \in T \tag{16}
\]

Constraint 17 ensures that the number of existing turbines that are used as part of the network does not exceed the total number of existing turbines:

\[
N_{r\text{EWT}}^{z} \leq N_{r\text{EWT, tot}}^{z} \quad \forall z \in Z \tag{17}
\]

where the parameter \( N_{r\text{EWT, tot}}^{z} \) represents the total number of existing wind turbines in zone \( z \).

The land footprint constraint guarantees that there is sufficient land area suitable for the new wind turbines assuming a spacing between turbines of 5 rotor diameters:

\[
\pi (5R_{\text{NWT}})^2 N_{r\text{NWT}}^{z} \leq A_{r\text{max}}^{z} \quad \forall z \in Z \tag{18}
\]

where \( A_{r\text{max}}^{z} \) is the total available area in zone \( z \) obtained by applying the 10 constraints discussed in Section 3.2.

5.2.3. Conversion technologies

The net rate of production of resource \( r \), \( P_{rzhdt} \), is defined as:

\[
P_{rzhdt} = \sum_{p} \mathcal{P}_{pzhdt} \alpha_{rp} \quad \forall r \in R, \ z \in Z, \ h \in H, \ d \in D, \ t \in T \tag{19}
\]

where \( \mathcal{P}_{pzhdt} \) is the rate of operation of technology \( p \) and \( \alpha_{rp} \) is the conversion factor, which is the net production of resource \( r \) per unit operation of technology \( p \) (it is positive if \( r \) is produced and negative if \( r \) is consumed).

The production rate of conversion technology \( p \) in zone \( i \) is limited by the number of technologies of type \( p \) that are installed in zone \( i \) and the minimum and maximum capacities of a single technology:

\[
N_{pzhdt}^{z} \alpha_{p\text{min}}^{z} \leq \mathcal{P}_{pzhdt} \leq N_{pzhdt}^{z} \alpha_{p\text{max}}^{z} \quad \forall p \in P, \ z \in Z, \ h \in H, \ d \in D, \ t \in T \tag{20}
\]
The number of technologies that can be established every year in the whole study region (i.e. build rate) can also be constrained as follows:

\[ \sum_{z} N_{pz}^p \leq BR_p \quad \forall p \in P \]  

where \( BR_p \) is the maximum build rate for technology \( p \).

5.2.4. Storage technologies

Figure 9 shows the three stages involved in storing hydrogen: charging, maintaining and discharging, which are considered in the model as three storage tasks: “put”, “hold” and “get”. Similar to a conversion technology, the efficiency, resource requirements and losses of each storage task can be specified through its conversion factor. It is also necessary to define the direction of the flow of resources, i.e. the source and the destination (abbreviated in the formulation as “src” and “dst”, respectively). The “put” task transfers the hydrogen from zone \( z \) (source) to the store (destination). The “hold” task maintains the hydrogen in the store; if there are any losses while it is being maintained (e.g. hydrogen gas escaping to the atmosphere or in the case of electricity storage, the losses are in the form of heat) then the source is the store and the destination is the zone. The “get” task retrieves the hydrogen from storage (source) and delivers it to zone \( z \) (destination).

In this work, compressors and expanders are explicitly modelled as conversion technologies. Therefore it is not necessary to define the electricity requirement (or electricity generation in the case of expanders) of the “put” and “get” tasks.

The net rate of utilisation of resource \( r \) from storage is given by the rates of operation of all of the “put”, “hold” and “get” tasks multiplied by their conversion factors for resource \( r \):

\[ S_{rzhdt} = \sum_{s} \left( \mathcal{I}_{rzhdtsrc} \sigma_{rzhdtsrc}^\text{put} + \mathcal{I}_{rzhdtsrdst} \sigma_{rzhdtsrdst}^\text{hold} + \mathcal{I}_{rzhdtsrdst} \sigma_{rzhdtsrdst}^\text{get} \right) \quad \forall r \in R, \ z \in \mathbb{Z}, \ h \in \mathbb{H}, \ d \in \mathbb{D}, \ t \in T \]  

Figure 9: Three stages (“tasks”) for storing hydrogen: “put”, “hold” and “get”. 

The net rate of utilisation of resource \( r \) from storage is given by the rates of operation of all of the “put”, “hold” and “get” tasks multiplied by their conversion factors for resource \( r \):
The rates of charging and discharging the store cannot exceed the injectability, $s_{s}^{\text{put,max}}$, and deliverability, $s_{s}^{\text{get,max}}$, respectively.

\[ I_{szhdt}^{\text{put}} \leq N_{sz}^{S} s_{s}^{\text{put,max}} a_{sz} \quad \forall s \in S, z \in Z, h \in H, d \in D, t \in T \]  
(23)

\[ I_{szhdt}^{\text{get}} \leq N_{sz}^{S} s_{s}^{\text{get,max}} a_{sz} \quad \forall s \in S, z \in Z, h \in H, d \in D, t \in T \]  
(24)

$N_{sz}^{S}$ is the number of storage facility $s$ in zone $z$, $s_{s}^{\text{hold,max}}$ is the maximum capacity of a single storage facility $s$ and the binary parameter $a_{sz}$ can be assigned a value of 1 if storage facility $s$ can be established in zone $z$ and 0 otherwise (e.g. this can be used to indicate the location of caverns for underground storage).

The inventory balance for the store also depends on the rates of the three tasks, but this time multiplied by the conversion factor for the opposite flow direction:

\[ I_{szhdt} = n_{h}^{d} \sum_{r} (I_{szhdt}^{\text{put}} \sigma_{sr,dst}^{\text{put}} + I_{szhdt}^{\text{hold}} \sigma_{sr,src}^{\text{hold}} + I_{szhdt}^{\text{get}} \sigma_{sr,src}^{\text{get}}) \quad \forall s \in S, z \in Z, h \in H, d \in D, t \in T \]  
(25)

The rate of operation of the “hold” task is the inventory level from the previous time interval, divided by the length of the time interval:

\[ I_{sz,1,dt}^{\text{hold}} = I_{sz,1,dt}^{0,\text{sim}} / n_{h}^{d} \quad \forall s \in S, z \in Z, d \in D, t \in T \]  
(26)

\[ I_{sz,h-1,dt}^{\text{hold}} = I_{sz,h-1,dt} / n_{h}^{d} \quad \forall s \in S, z \in Z, h > 1 \in H, d \in D, t \in T \]  
(27)

Here, $I_{sz,1,dt}^{0,\text{sim}}$ is the initial inventory level for the start of the “simulated cycle” for day $d$ in season $t$. It is calculated (equation 29) so that the inventory levels in the simulated cycle will correspond to the average inventory levels over all occurrences of day $d$ and all weeks in season $t$, so that costs and resource requirements, which depend on the inventory levels, are calculated correctly. See the appendix in [1] for a detailed derivation.

Equations 25 and 27 can be rearranged to give the inventory balance in a more familiar form:

\[ \frac{I_{szhdt} - I_{sz,h-1,dt}}{n_{h}^{d}} = \sum_{r} (\sigma_{sr,dst}^{\text{put}} a_{sr,dst}^{\text{put}} (1 - \sigma_{sr,src}^{\text{hold}} I_{sz,h-1,dt}^{0,\text{sim}} / n_{h}^{d}) + \sigma_{sr,src}^{\text{get}} I_{szhdt}^{\text{get}} \sigma_{sr,src}^{\text{get}}) \quad \forall s \in S, z \in Z, h > 1 \in H, d \in D, t \in T \]  
(28)
The left hand side is the rate of change of the inventory level, the first and third terms on the right hand side are rates of addition and withdrawal from storage and the second term on the right hand side is the rate of loss of resource from storage.

The variable $I_{szdt}^{0,\text{sim}}$ in equation 26 represents the initial inventory at the start of the “simulated cycle” for day $d$ in season $t$. In order to make the model more efficient, only one of the identical days of each day type is considered and only one of the identical weeks of each season is considered. As the costs and resource requirements for storage depend on the inventory levels, the day and week that is included in the optimisation, the “simulated cycle”, should correspond to a day with inventory levels equal to the average inventory levels over all of the day types and all of the weeks in the particular day type, $d$, and season, $t$. If $I_{szdt}^{0,\text{act}}$ is the initial inventory for the first occurrence of day type $d$ and the first week of season $t$, then the initial inventory for the simulated cycle is given by:

$$I_{szdt}^{0,\text{sim}} = I_{szdt}^{0,\text{act}} + \left[ (n_{dw}^d - 1) \delta_{szdt}^d + (n_{wt}^t - 1) \delta_{szt}^t \right] / 2 \quad \forall s \in S, z \in Z, d \in D, t \in T \quad (29)$$

$\delta_{szdt}^d$ is the change in inventory over one day, for day type $d$ in season $t$ and $n_{dw}^d$ is the number of identical days in day type $d$. Therefore adding integer multiples of $\delta_{szdt}^d$ to $I_{szdt}^{0,\text{act}}$ gives the initial inventory on subsequent days (of the same day type) and adding $(n_{dw}^d - 1)/2$ multiples of $\delta_{szdt}^d$ shifts the initial inventory to the average level over all occurrences of that day type. $\delta_{szt}^t$ and $n_{wt}^t$ are defined as the change in inventory over a week and the number of weeks in season $t$; they are combined similarly to shift the inventory forward to the central week in season $t$. These relationships are derived in more detail in the appendix of [1]. $\delta_{szdt}^d$ and $\delta_{szt}^t$ are defined below.

$$\delta_{szdt}^d = I_{sz|h|dt} - I_{szdt}^{0,\text{sim}} \quad \forall s \in S, z \in Z, d \in D, t \in T \quad (30)$$

$$\delta_{szt}^t = \sum_d \delta_{szdt}^d n_{dw}^d \quad \forall s \in S, z \in Z, t \in T \quad (31)$$

The change in inventory over the whole year can also be calculated and used to ensure that there is no accumulation of inventory over a year:

$$\delta_{sz}^y = \sum_t \delta_{szt}^t n_{wt}^t \quad \forall s \in S, z \in Z, t \in T \quad (32)$$

$$\delta_{sz}^y = 0 \quad \forall s \in S, z \in Z \quad (33)$$

The initial inventories for each day type are also related: adding $n_{dw}^d$ multiples of $\delta_{szdt}^d$ to the initial inventory for day type $d$ gives the initial inventory of day type $d + 1$. The equations below link the initial inventories for subsequent day types and seasons.
\[ I_{szdt}^0 = I_{sz,d-1,t}^0 + n_{d-1}^{dw} \delta_{sz,d-1,t}^d \quad \forall s \in S, z \in Z, d > 1 \in D, t \in T \] (34)

\[ I_{sz,1,t}^0 = I_{sz,1,t-1}^0 + n_{t-1}^{wt} \delta_{sz,t-1}^t \quad \forall s \in S, z \in Z, t > 1 \in T \] (35)

The following set of constraints ensures that the inventory does not exceed the maximum capacity of storage, \( s_{sz}^{\text{hold,max}} \). For computational efficiency, the periodicity in system properties can be exploited, i.e. the inventory increases or decreases by the same amount, \( \delta_{szdt}^d \), each repeated day in day type \( d \) in season \( t \) and by the same amount, \( \delta_{sz,t}^t \), each repeated week in season \( t \). Therefore, instead of writing the constraints for every contiguous hour, it is sufficient to write the constraints only for the first and last instance of each repeated day type and the first and last week of each season, as given by constraint 36.

\[ s_{sz}^{\text{hold,min}} N_{sz}^S \leq s_{sz}^{\text{hold}} \leq \left( (n_{d}^{dw} - 1) \delta_{szdt}^d \pm (n_{t}^{wt} - 1) \delta_{sz,t}^t \right) / 2 \leq s_{sz}^{\text{hold,max}} N_{sz}^S \quad \forall s \in S, z \in Z, h \in H, d \in D, t \in T \] (36)

Note that the above is a short hand for the four constraints formed by using either a positive or negative sign for each of the \( \pm \) symbols.

In the case studies, it was assumed that each storage should be matched with a dedicated compressor and expander, each of which is specifically sized for each storage (see Tables 2 and 3). Therefore, two constraints were added for each row in Tables 2 and 3. Those for the first row (i.e. for the large H\textsubscript{2} storage tank) are shown below.

\[ N_{\text{COMP}1,z}^P \leq N_{\text{CGH2S-L},z}^S \quad \forall z \in Z \] (37)

\[ N_{\text{EXP}1,z}^P \leq N_{\text{CGH2S-L},z}^S \quad \forall z \in Z \] (38)

5.2.5. Transport technologies

The net rate of transport of resource \( r \) into zone \( z \) from other zones, \( Q_{rzhdt} \), is the difference between the incoming and outgoing flow rates given by the first and second terms on the right hand side of equation 39, respectively:

\[ Q_{rzhdt} = \sum_{z'|\nu_{z'}=1} \sum_{l \in L} \left[ (\bar{\tau}_{lr,dst} + \hat{\tau}_{lr,dst} d_{z'}^l) Q_{lzz'}^h \right] + \sum_{z'|\nu_{z'}=1} \sum_{l \in L} \left[ (\bar{\tau}_{lr,src} + \hat{\tau}_{lr,src} d_{z'}^l) Q_{lzz'}^h \right] \quad \forall r \in R, z \in Z, h \in H, d \in D, t \in T \] (39)

\( Q_{lzz'}^h \) is the rate of operation of transport mode \( l \) from zone \( z \) to zone \( z' \) during hour \( h \), of day type \( d \) in season \( t \). The conversion factors, \( \bar{\tau}_{lr,f} \), are the net flow of resource \( r \) into the source (\( f = \text{src} \)) and destination (\( f = \text{dst} \)).
(f = dst) zones per unit rate of operation of the transport mode; they are negative if the flow is out of the zone and positive if the flow is into the zone. \( \tilde{\tau}_{lr,f} \) are defined similarly but are also per unit distance between the zone, hence they are multiplied by the distance between the zones, 
\[
d_{zz'} = \sqrt{(x_z - x_{z'})^2 + (y_z - y_{z'})^2},
\]
where \( x_z \) and \( y_z \) are the x- and y-coordinates of the centre of demand of zone \( z \), and the rate of operation of the transport mode; they are mainly used to represent distance-dependent losses. By convention, \( \bar{\tau}_{lr,dst} \) is set to 1 for the resource being transported so that \( Q_{lzz'hdt} \) is the rate of resource \( r \) entering zone \( z \) from zone \( z' \) and \( q_{l}^{\text{max}} \) is the maximum rate of transport of the resource, see equation 40 below). For a lossless transport mode, with no resource requirements, \( \bar{\tau}_{lr,src} \) is set to \( -1 \) for the transported resource and all other elements are set to zero.

The rate of operation of transport mode \( l \), \( Q_{lzz'hdt} \), is limited by the maximum rate, \( q_{l}^{\text{max}} \), and the number of infrastructures of type \( b \), \( N_{bzz'}^{B} \), established between zones \( z \) and \( z' \):

\[
Q_{lzz'hdt} \leq \sum_{b \in B} q_{l}^{\text{max}} N_{bzz'}^{B} \quad \forall l \in L; z, z' \in Z; h \in H, d \in D, t \in T \tag{40}
\]

Here, \( LB_{lh} \) is a binary parameter with a value of 1 if transport mode \( l \) is supported by infrastructure \( b \), 0 otherwise. For computational efficiency, the infrastructure links can be only built between adjacent (or neighbour) zones. The transport of resource \( r \) over long distances can be achieved by making several neighbour-to-neighbour transfers along the route between the source and destination zones. The binary parameter, \( \nu_{zz'} \), is used to indicate neighbour zones: \( \nu_{zz'} = 1 \) if zone \( z \) is adjacent to zone \( z' \), 0 otherwise.

The total flow rate of all resources being transported along an infrastructure is also constrained by its maximum capacity, \( b_{b}^{\text{max}} \):

\[
\sum_{l \in L} Q_{lzz'hdt} LB_{lh} \leq b_{b}^{\text{max}} N_{bzz'}^{B} \quad \forall b \in B; z, z' \in Z; h \in H, d \in D, t \in T \tag{41}
\]

Finally, the following constraint applies for bidirectional transmission lines, such as the pipelines and cables considered in this study:

\[
N_{bzz'}^{B} = N_{bz'z}^{B} \quad \forall b \in B, z, z' \in Z \tag{42}
\]

5.2.6 Import and export

The rates of import and export can be constrained by specifying the maximum rates of import and export, \( m_{rz}^{\text{max}} \) and \( \chi_{rz}^{\text{max}} \), respectively.

\[
M_{rzhdt} \leq m_{rz}^{\text{max}} \quad \forall r \in R, z \in Z, h \in H, d \in D, t \in T \tag{43}
\]

\[
X_{rzhdt} \leq \chi_{rz}^{\text{max}} \quad \forall r \in R, z \in Z, h \in H, d \in D, t \in T \tag{44}
\]
6. Distribution network

It is assumed that customers will purchase hydrogen from a number of fuelling stations distributed throughout each zone, much as they currently do for petrol and diesel. Indeed, it would make sense for some existing petrol stations to convert to hydrogen, in which case the distribution network can be designed in detail. In this study, the exact locations of the fuelling stations have not been considered so the properties of the distribution network must be approximated from the spatial distribution of demands. In the earlier study of Almansoori and Shah [57, 58], the fuelling stations were assumed to be uniformly distributed within each square cell and the hydrogen was assumed to be transported to the fuelling stations via road tankers. The network distance was calculated assuming an outward and return journey from the centroid of the cell to each of the fuelling stations, with the number of fuelling stations determined from the total demand in each cell and the capacity of a single fuelling station. The network distance is therefore twice the number of fuelling stations multiplied by the average distance from the centre of the square cell to all other points in the cell, which they assumed to be half the length of the square [57]. These assumptions were not appropriate for the following reasons: first, Figure 3(a) shows that the demand for hydrogen is far from uniform; second, it can be shown that the average distance from the centre of a square to all points in the square is in fact $L_{sq}^{2}(\sqrt{2} + \ln(1 + \sqrt{2}))/6 \approx 0.383L_{sq}$, where $L_{sq}$ is the length of the square. Figure 10(a) compares the effect that the assumption of uniform demand has on the estimated length of the distribution network: generally, the length is overestimated and in one zone the error is roughly 90%. In this study, it was assumed that the hydrogen is distributed by pipeline, so the total distribution network distance in each zone can therefore be estimated from the 1km demand data using equation 45:

$$L_{z,\text{network}} = \sum_{i \in I_z} ((x_i - x_z)^2 + (y_i - y_z)^2)^{0.5} \bar{D}_i/FS \quad \forall z$$ \hspace{1cm} (45)

where $L_{z,\text{network}}$ is the length of the distribution network in km, $\bar{D}_i$ is the demand density at the 1km level and $FS$ is the capacity of a single fuelling station, which was assumed to be 1,500 kg/day (2.08MW$_{hy}$) and is the size of a facility described in the report by NREL [70].

The total network distance is required to cost the distribution network and the demand-weighted average distance from the centre of demand to all of the points in the cell ($L_{z,\text{network}}/FS$) can be used to estimate distribution losses in cases where the losses are proportional to distance from the supply point to the customer (one can easily perform more general calculations to arrive at a representative number to use in the loss constraints).

The number of fuelling stations in each zone, $N_{z,\text{FS}}$, can be determined by dividing the total demand in each zone by the capacity of a single fuelling station and then rounding up to the nearest integer (equation 46), the results of which are shown in Figure 10(b).

$$N_{z,\text{FS}} = \left[ \sum_{i \in I_z} \bar{D}_i/FS \right] \quad \forall z$$ \hspace{1cm} (46)
6.1. Sizing

The pipeline diameter is initially assumed to be 20cm but it is important to ensure that the capacity of the distribution network exceeds the peak demand in each zone. A detailed design of the distribution networks is beyond the scope of this study but it is possible to perform a similar analysis as was given in section 4.2.1 using some simple assumptions about the structure of the networks based on the distribution of demands in the zones. It can be seen from Figure 3 that there are two distinct patterns for the demands: a small number of large clusters close to the centre of demand (e.g. zones 5, 6, 7, 8, 10) and a more uniform distribution of demands throughout the zone, either as many small clusters (e.g. zones 9, 11, 12, 14–16) or as in zone 13, where the demands are concentrated in the centre. The demands in zones 1 to 4 are so low that the 20 cm pipeline is expected to be more than sufficient for the peak, which was indeed verified by the simulations. In the former demand pattern, it can be envisaged that the distribution network will comprise a small number of pipelines from the centre of demand to the large local clusters, from which a number of secondary pipes form a spoke-like structure. In this case, a significant portion of the demand will flow through each branch (a single pipeline) of the network. The flowrate of hydrogen and the length of each branch of pipeline is estimated and used in a simulation of the pipe to determine whether a 20 cm diameter is sufficient. In the latter case, one can imagine the distribution network being a simple hub-and-spoke structure, with the hub at the centre of demand and the spokes travelling to the small clusters throughout the zone. In this case, one can assume that the flow through each pipeline is the total peak demand divided by the number of clusters and the length of the pipeline to simulate is the average distance from the centre of demand to all demand points. In the case of zone 13, where the demand is more uniformly distributed, the flow is taken as the peak demand divided by the number of fuelling stations in the zone. The results of the simulations (not presented) indicate that most zones are well within the capacity of the distribution network. The fluid velocity is the limiting factor, with the velocity reaching 27%, 28% and 37% of the erosional velocity in zones 5, 12 and 6, respectively. This means that the demands could almost double before reaching the network capacity. All other zones have a much lower capacity factor than these, with the exception of zones 7, 8 and 10, which are at 69%, 144% and 96% of the erosional velocity and therefore require larger-diameter pipes to connect the demand clusters to the centre of demand. Increasing the pipe diameter to 30 cm brings these numbers down to 26%, 44% and 34% respectively. For additional robustness, the pipes in zone 8 were sized at 35 cm in order to bring the velocity down below half of the 70% limit, in line with all of the other zones.
which can support a doubling of the hydrogen demand.

6.2. Costing

The total annualised cost of the distribution network is estimated from the network length and the number of fuelling stations determined by equations 45 and 46, as well as the diameter of the pipeline as determined above and a standard capital charge factor of 3. The unit cost of the pipe was estimated to be £348k/km, £437k/km and £498k/km for 20, 30 and 35cm diameter pipes using a similar approach to that of Yang and Ogden [50] and Parker [61] and the unit cost of the fuelling station was approximated to be £3.03M based on the values reported in an NREL report [70]. This yields a total annualised cost for the distribution network of £17.1bn/yr. Although the cost may be reduced by optimising the distribution network, due to the very large size of the model that will result if the distribution is to be included in the optimisation problem, which is unlikely to be solvable using available computing resources, only networks comprising generation/conversion, storage and transmission technologies are optimised and the estimated cost for the distribution is simply added to the total cost.

7. Results and discussion

A number of different scenarios were considered, using the data described above, in order to investigate the different optimal configurations of the network. The base case includes all of the technologies but does not use existing wind turbines. In subsequent runs, the value of various network components was determined by excluding them from the optimisations and comparing the resulting cost to the base case. Another scenario also allows the existing wind turbines to be used. Although it is straightforward to consider imports and exports, as discussed in Section 5.2.6, none of the scenarios allowed them.

The overall conclusion is that both storage and transmission are required to obtain a feasible solution, i.e. without either of these, the energy demands cannot be met at all times. Transmission is needed because some zones in the south of England do not contain sufficient suitable land area for wind turbines in order to meet the demands independently, even with storage, i.e. energy must be supplied by other zones to meet the demands. Storage is required even if all zones are connected by unlimited transmission capacity because the intermittency of the wind availability is too high to meet the total demand at all times.

Of course, these results are to be expected but the interesting questions that the model is designed to answer are: how much storage is required and where should it be located; what type of transmission is required and which zones should be connected? Naturally, in addition to these, the model determines the number and location of all of the other technologies, such as electrolysers, fuel cells etc.

The different cases examined are described in the subsequent sections.
7.1. Base case

For the base case, all of the technologies in the database were considered but without the availability of the existing wind turbines, assuming that their generation capacity is already allocated to satisfying other demands. The optimal structure of the resulting network is shown in Figure 11(a). The symbols, with numbers, in each zone represent the type and number of technologies in the zone and the lines represent connections between the zones for transport of resources. Note that the location of the symbols does not represent the actual location of the technologies within the zone. Zones 1 to 6, 8 and 14 are all self-sufficient, each with its own set of technologies including wind turbines, electrolysers, storage tanks and expanders. In contrast, zones 9 to 13 have no technologies and completely rely on zones 7, 15 and 16 to meet their hydrogen demands. A pipeline transmission network is built in the south of England and Wales connecting zones 7, 9 to 13, 15 and 16. The Humbly Grove underground storage in zone 15 is effectively being shared by the zones that are connected to the pipeline transmission network.

The cost breakdown is shown in Figure 11(b), where it can be seen that the capital cost of the electrolysers and wind turbines dominates. The capital cost of the hydrogen transmission network is the next largest cost, about a quarter of the electrolyser CAPEX. Next are the storage CAPEX and the O&M costs of the electrolysers and wind turbines, all of about the same order of magnitude. Finally, the capital cost of the expanders and compressors is about 1% of the total cost and the remaining components are almost negligible. To these costs should be added the distribution network costs of £17.1bn/yr, as estimated in Section 6.2. However, since the distribution costs are the same in all cases, they are not included in the comparisons made in the following subsections.

The model also determines the hourly operation of each technology. For example, Figure 12 shows the operation of the pipeline transmission network at different times during weekdays in summer. At times of high demand, zone 15, where the Humbly Grove underground storage is located, supplies a large amount of hydrogen to other zones that are connected to the transmission pipeline. Zones 7 and 16, which have significant generating capacity, supply hydrogen to other zones through the transmission pipeline at some times and also receive hydrogen from other zones through the pipeline at other times. Zones 9 to 13 satisfy all of their domestic transport demand through the pipeline at all times. The maximum flow of hydrogen through the pipeline at peak time is $2.85 \text{ GW}_{hy}$ and the pressure drop was calculated using gCCS to be 0.06 MPa. Therefore, the decision not to model the booster compressors in the pipeline was justified.
Figure 11: Results of the base case: (a) optimal network structure and (b) breakdown of cost. The cost of the optimised network is £4,720M/yr plus distribution costs of £17,100M/yr, resulting in a total cost of £21,820M/yr.
Figure 12: Operation of the pipeline transmission network during weekdays in summer: (a) 07:00, (b) 13:00, (c) 19:00 and (d) 22:00.

During times of low demand, the excess production of hydrogen is being stored in the underground storage; an example of such instance is presented in Figure 13, which shows the operation of the transmission pipeline at 01:00h during weekdays in spring. The hydrogen being generated in zones 7 and 16 is being transmitted through the pipeline to zone 15. Because the transmitted hydrogen is at a lower pressure of 7MPa, a compressor is required to raise it up to the storage pressure of 20MPa. As can be seen in Figure 13, the Humbly Grove underground storage in zone 15 is equipped with both a compressor and an expander, the former is needed when the hydrogen from transmission is stored while the latter is required when the hydrogen from storage is transmitted to other zones or distributed within the zone to meet demands.
Figure 13: Snapshot of the operation of the network at 01:00h during weekdays in spring, when the Humbly Grove underground storage is being charged by the pipeline transmission network.

Figure 14 shows the hourly inventory of hydrogen for a whole year in various storage facilities located in different zones. The Humbly Grove underground storage is effectively being used for seasonal storage although the changes in inventory within each day can still be seen from Figure 14(a). The level of hydrogen in the facility is high during spring and gradually increases until it reaches the full capacity at the start of summer. The hydrogen level then continuously decreases throughout the summer and stays at a very low level during autumn. The storage is replenished during winter so that it reaches the inventory at the start of the year – this is due to the cyclic constraint (equation 33), which ensures that the overall change in inventory over a year is zero. The overground storage tanks, on the other hand, are being used for hourly balancing as well as for seasonal storage: a strong hourly variation in the inventory profile can be seen in Figures 14(b)-(d) but a trend in storage usage can still be observed in each season. Figure 14(b) shows the inventory of the large storage tank in zone 16; note that there are also 2 medium storage tanks in zone 16 but their inventory profiles are not shown in the interest of space. Although its hourly fluctuation is stronger, its seasonal trend is similar to that of the Humbly Grove underground storage: the inventory is high and slowly increasing in spring, decreasing during summer, low in autumn and being filled up again in winter. The storage is only ever full during a few days at the start of summer and most of the year it is below its full capacity. In zone 4, there are 1 medium and 3 small storage tanks; Figure 14(c) gives the inventory profile for the medium tank. Although hydrogen is being added or removed on an hourly basis throughout the year, this storage is effectively being replenished during spring and emptied throughout summer and autumn. The last example is for zone 8, where there are 1 large and 1 small storage tanks. In Figure 14(d), the hydrogen level in the small storage tank is high and there are many instances where it reaches its full capacity during spring and summer; it decreases in the autumn and is replenished in the winter.

7.1.1. Computational statistics

The model was implemented in AIMMS 3.12 and the decomposition method described in [1] was employed with each iteration being solved to a relative tolerance of 3% (i.e. the objective function of the final integer solution should be within 3% of the fully-relaxed LP) using CPLEX 12.5 on a PC with an Intel Xeon CPU at 3.19 GHz and 48 GB of RAM. The complex problem, which may be intractable for practical problems, is decomposed into three stages. The first stage solves the full design and operation of the energy network with a simplified temporal discretisation; stages 2 and 3 both optimise the operation of the whole network but iterate between designing the conversion and storage technologies with a fixed transport infrastructure.
Figure 14: Hourly inventory of hydrogen in storage for a whole year at different locations in Great Britain: (a) Humbly Grove underground storage in zone 15; (b) large storage tank in zone 16; (c) medium storage tank in zone 4; and (d) small storage tank in zone 8. The underground storage is being used effectively for seasonal storage, whereas the overground storage tanks are being used for hourly balancing as well as for seasonal storage.
Table 8: Computational statistics for the base case.

| Iteration | Stage | No. of variables | No. of constraints | CPU time (s) | Objective function value (£M/yr) |
|-----------|-------|------------------|-------------------|-------------|----------------------------------|
| 1         | 1     | 37,331 (502 integer) | 102,047         | 1,388       | 4,944                             |
| 2         | 2     | 207,205 (315 integer) | 498,019         | 8,982       | 4,720                             |
| 3         | 3     | 207,293 (403 integer) | 498,173         | 12,430      | 4,727                             |

and designing the transport infrastructure with a fixed set of conversion and storage technologies. The decomposition algorithm terminates when either stage 2 or 3 no longer improves the objective function. Table 8 shows the computational statistics for each iteration of the decomposition procedure. As the objective function does not improve in stage 3, the decomposition algorithm terminates at the third iteration and the solution for stage 2 is taken to be the optimal solution.

As can be seen from Table 8, stage 1 contains less than a quarter of the variables and constraints of the other two stages, which is because the temporal discretisation is simpler. However, there are more integer variables because it is solving the design of the conversion and storage technologies as well as designing the transport infrastructure. While first stage solves in about 25 minutes, the other stages take over an hour each with an overall solution of about 7 hours. As the problem solved in this paper was focused only on hydrogen vehicles and for a fully-developed hydrogen infrastructure, it will be necessary to solve problems of a much larger scale and this will result in much longer solution times or intractability. Therefore, depending on the problem being solved, there will be trade-offs in the different levels of resolution in the model. Increasing the spatial resolution is likely to increase the problem size more than any other enhancement because at higher resolutions, there are many more possible connections between zones. In the current example, many zones share a border with just one other; at higher resolutions, zones could be connected to four or five others, thus increasing the problem size in a non-linear way. However, if higher spatial resolutions are required, an additional decomposition method could be applied. More prosperous enhancements would be to increase the number of technologies or to include yearly intervals, in order to evaluate the transition from the current state of the energy system to whatever future state is determined by the model. These extensions would result in a roughly linear increase in the size of the model. Of course, the solution time is likely to increase faster than this due to the underlying properties of MILP solution algorithms.

7.2. The value of existing wind turbines

In case 2, the existing wind turbines can be used as part of the network. As can be seen in Figure 15(a) the pipeline transmission network is the same as that of the base case but in zones 9 to 13, existing wind turbines are utilised and conversion and storage technologies are established in those zones. There are no new wind turbines built in zones 1 to 3, 5 and 11: instead, existing wind turbines are used to generate electricity. In the remaining zones, both existing and new wind turbines are needed for generation. There are no existing wind turbines in zone 15 but the optimal solution indicates that 998MW\text{el} (at 9m/s) of new wind capacity should be installed in that zone to take full advantage of the Humbly Grove underground storage.

Figure 15(b) gives the breakdown of cost, where it can be seen that compared to the base case, the cost of the wind turbines in this network is 22% lower (as stated in Section 4.1.1, it was assumed that the capital
Figure 15: Results of the different case studies: (a) optimal network structure and (b) breakdown of cost for case 2, in which the use of existing wind turbines was permitted; (c) optimal network structure and (d) breakdown of cost for case 3, in which the use of underground storage was not allowed. The total costs (including distribution) are £21,473 M/yr for case 2 and £23,020 M/yr for case 3. (Continued on next page.)
Figure 15: (Continued.) Results of the different case studies: (e) optimal network structure and (f) breakdown of cost for case 4, in which hydrogen pipeline transmission was not allowed; and (g) optimal network structure and (h) breakdown of cost for case 5, in which only underground electricity transmission lines were allowed. The total costs (including distribution) are £22,325M/yr for case 4 and £23,574M/yr for case 5.
cost of the existing wind turbines does not contribute towards the total cost; only their O&M costs do), the
conversion (electrolysers, expanders and compressors) and transmission costs are similar (i.e. within the 3%
relative tolerance used as the terminating criterion when solving each stage of the model) and the cost of
storage is 20% higher. The cost of this network is 7% lower than that of the base case.

7.3. The value of underground storage

This case is the same as the base case except that the use of underground storage is not allowed. Figure
15(c) shows the resulting optimal network structure. Zones 1 to 5, 8 and 14 are still self-sufficient, having
the same number of technologies as in the base case. A similar pipeline transmission network is built in the
south of England and Wales but with zone 6 also connected to the network. In zone 15, a large storage tank
is used instead of the Humbly Grove underground storage. Zones 6, 7, 11 and 15, being the suppliers of
hydrogen to the other zones that are connected to the pipeline, have a large number of wind turbines; each
contains a number of electrolysers and 1 large storage tank equipped with an expander. Zone 16, which used
to be a main supplier of hydrogen in the base case, does not have any generation or storage capacity and
completely relies on other zones for the satisfaction of its demand. In contrast, zone 11, which does not have
any technology in the base case, now has a significant capacity for generation and storage and also supplies
hydrogen to its neighbouring zones through the transmission pipeline. As can be seen in Figure 15(d),
without underground storage the cost of the network is 25% higher than that of the base case: the wind
turbines, conversion, storage and transmission are more expensive by 29%, 23%, 32% and 20%, respectively.

7.4. The value of pipeline transmission

In this case, the effect of not being able to transmit hydrogen through pipelines was examined. Figure 15(e)
presents the resulting optimal network structure, in which it can be seen that all zones except zone 3 are now
interconnected by HVAC overhead lines – both single-circuit and double-circuit lines are used. In the base
case, all of the zones in Scotland are self-sufficient, but in this case, 4 zones are now sharing facilities and as
a result a smaller total capacity of wind turbines, electrolysers, storage tanks and expanders is needed. The
savings from having fewer of these facilities are offset by the investment in the overhead transmission lines.
In England and Wales, all of the zones are connected by overhead lines, with double-circuit electric cables
being used to link zones that have either high demand (e.g. zone 13) or high generation (e.g. zone 16) or
storage capacity (e.g. zone 15). Although all of the zones have electrolysers, only zones 7, 8, 11 and 14 to 16
have wind turbines; the rest of the zones generate hydrogen using the electricity obtained from the overhead
transmission lines. Similar to the base case, the Humbly Ground underground storage is also utilised but
1.42GW of fuel cell capacity is required to convert the hydrogen from storage to electricity for transmission
to other zones. In the base case, zones 9 to 13 do not require any conversion or storage capacity because they
satisfy their demands using hydrogen from the transmission pipeline. Without the hydrogen pipeline, these
zones now need electrolysers to generate hydrogen using either the electricity from the local wind turbines
(e.g. zone 11) or the overhead electricity cables.

Figure 15(f) shows the cost breakdown for this network. Compared to the base case, the cost of transmission
is lower by 31% but the cost of wind turbines, conversion technologies and storage facilities are higher by
7%, 17% and 57%, respectively. Overall, this network is 11% more expensive than that of the base case.
7.5. The cost of underground electricity cables

Overhead electricity lines have a negative visual impact on the landscape, therefore in this case, the additional cost of using underground electricity cables as the only option for transmission was determined. The resulting optimal network structure is presented in Figure 15(g). All zones have electrolysers and storage facilities that are equipped with expanders. Except for zone 9, which obtains its electricity from the transmission line and converts it to hydrogen, all zones have wind turbines. The type and number of technologies built in Scotland are the same as those in the base case. Compared to the network in case 4, where all zones are interconnected except for one, the links in this network are limited to only a few zones because of the higher cost of the transmission cables. Two groups of zones in England and Wales are connected by HVAC underground cables: one group consists of zones 7 to 9 and 11 and the other comprises zones 12 to 15. The Humbly Grove underground storage, located in zone 15, is used along with 957MW_{el} fuel cell capacity to convert the hydrogen from storage to electricity for transmission.

Figure 15(h) gives the breakdown of cost for this network. Compared to that of the base case, the total cost of this network is 37% higher: the costs of wind turbines, conversion, storage and transmission are more expensive by 36%, 38%, 83% and 8% respectively.

8. Conclusions

This paper presented an MILP model for the optimal design and operation of integrated wind-hydrogen-electricity networks. The general modelling framework, STeMES, was extended to include a footprint constraint for wind turbines and a more detailed representation of the available wind energy. The area available for siting wind turbines was determined, using GIS, by applying 10 environmental and technical constraints, such as buffer distances from urban areas, rivers, roads, airports, woodland and so on. The model was applied to the problem of satisfying all of the domestic transport demands, assuming 100% penetration of hydrogen fuel cell vehicles, in Great Britain using only on-shore wind power. Wind speed data were obtained from the Virtual Wind Farm Model for each of the 16 transmission zones (based on National Grid’s SYS 17 study zones) at the hourly level for a whole year. The energy system model was represented using the Resource-Technology Network (RTN) in which wind turbines, electrolysers, fuel cells, compressors and expanders were modelled along with underground storage (salt caverns and depleted oil/gas fields) and pressurised storage tanks for hydrogen storage, pipelines for hydrogen transmission and HVAC and HVDC overhead and underground transmission lines for electricity. The number of fuelling stations and the distribution network size were estimated from the 1km demand density.

Several case studies were presented in which various aspects of the energy network were excluded in order to determine their value. The results indicate that all of the domestic transport demand can be met by using only on-shore wind through an appropriately designed and operated network. In all of the cases, both transmission and storage are required to meet the demands, due to the intermittent nature of the wind.

The optimal structure of the network, for the base case, in which existing wind turbines are not used, involves building a hydrogen pipeline network in the south of England and South Wales, which connects South Wales, the Midlands, East Anglia and Greater London (all of which are completely reliant on the hydrogen pipeline...
network) to the wind generating capacity in the south of England, including the Humbly Grove underground storage in South East England, and to Yorkshire and the Humber. North Wales, Northern England and Scotland are all self-sufficient, with each transmission zone containing wind turbines, electrolyzers, storage tanks and expanders. The Humbly Grove underground storage is being used mainly for seasonal storage, while the storage tanks sited throughout Britain are typically used for both hourly balancing and seasonal storage.

If the use of the existing wind turbines is permitted, the total cost of the network is 7% cheaper than the base case, despite the cost of wind turbines being 22% lower. This indicates that not all of the existing wind farms are in the ideal location and require more expensive storage in order to utilise them in the energy network. Without any underground storage, the optimal network is 25% more expensive than the base case. This indicates how important large, seasonal storage facilities are. Without hydrogen transmission pipelines, the optimal network, which uses HVAC overhead lines, is 11% more expensive than the base case. The transmission is less expensive but fuel cells and more storage are required, more than offsetting the savings in the transmission network cost. Lastly, if transmission is restricted to underground electricity cables, the cost of the network is 37% higher than the base case.

The results of the case studies are naturally dependent on the quality of the input data used, especially the costs of the technologies, which were assumed to be independent of location but could vary widely across Great Britain due to a number of factors such as installation costs. It is straightforward to include these dependencies in the model but the availability of data is the main difficulty. While every effort was made to obtain reliable data, there is still significant uncertainty in their values. Therefore, a useful future step would be to perform sensitivity analyses. Moreover, the results are of course limited by the technologies that are currently in the database and they may change with the inclusion of more technologies, such as batteries and electric vehicles – these are all planned future extensions to the model. In addition, the model will be extended to include other networks such as natural gas and heat along with the demands from other sectors. Pipeline storage and hydrogen injection into the natural gas grid will also be investigated.

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Authors’ contributions

SS and NJS developed the mathematical model, STeMES. NJS performed the simulations in gPROMS ProcessBuilder and gCCS and calculated the distribution network lengths. SS collected the demand and technology data; performed all GIS analyses; designed and executed the case studies; and analysed the results. IS provided the wind data from the Wind Virtual Farm Model. SS and NJS wrote the manuscript, which was proofread by IS. All authors read and approved of the final manuscript.
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Nomenclature

Indices and sets

$b \in B$ Transport infrastructures
$d \in D$ Daily intervals
$f \in F$ Flow directions
$h \in H$ Hourly intervals
$i \in I_z$ 1km square cells in transmission zone $z$
$l \in L$ Transport technologies
$p \in P$ Conversion technologies
$r \in R$ Resources
$s \in S$ Storage technologies
$t \in T$ Seasonal intervals
$z \in Z$ Transmission zones

Parameters

$A_{z}^{\text{max}}$ Available area for wind farms in zone $z$ [m$^2$]
$a_{sz}$ Binary parameter: 1 if storage $s$ is allowed in zone $z$; 0 otherwise
$BR_p$ Maximum total number of technologies $p$ that can be built in a year (maximum build rate)
$b_{l}^{\text{max}}$ Maximum capacity of infrastructure $b$ [MW]
$C_{b}^B$ Unit capital cost for infrastructure $b$ [£/km]
$C_{p}^P$ Unit capital cost of technology $p$ [£]
$C_{s}^S$ Unit capital cost of storage technology $s$ [£]
$C_{WT}$ Unit capital cost of wind turbine [£]
$D_{i}$ Hydrogen demand in 1km square cell $i$ [MW]
$D_{rzhdt}$ Demand for resource $r$ in zone $z$ during hour $h$, day $d$ and season $t$ [MW]
$d_{z,z'}$ Distance between mean demand centres of zones $z$ and $z'$ [km]
$F_{b}^B$ Annual operating and maintenance costs of transport infrastructure $b$ [£/km/yr]
$F_{p}^P$ Annual operating and maintenance costs of conversion technology $p$ [£/yr]
$F_{s}^S$ Annual operating and maintenance costs of storage technology $s$ [£/yr]
$F_{WT}$ Annual O&M costs of a single wind turbine [£]
$LB_{lb}$ Binary parameter: 1 if transport technology $l$ can use infrastructure $b$; 0 otherwise
$m_{rz}^{\text{max}}$ Maximum rate of import of resource $r$ in zone $z$ [MW]
$N_{EWT,\text{tot}}^{z}$ Total number of existing wind turbines in zone $z$
$n_{dw}^{d}$ Number of times day type $d$ occurs in a week
$n_{hl}^{h}$ Duration of hourly interval $h$ [h]
$n_{wt}^{t}$ Number of repeated weeks in season $t$
$p_{p}^{\text{max}}$ Maximum production rate of technology $p$ [MW]
$p_{p}^{\text{min}}$ Minimum production rate of technology $p$ [MW]
$q_{l}^{\text{max}}$ Max transfer rate for each transport mode $l$ [MW]
$R_{EWT,\text{ave}}^{z}$ Average radius of existing wind turbines (35m)
$R_{NWT}^{z}$ Radius of the new wind turbines (50m)
$s_{s}^{\text{hold, max}}$ Maximum storage capacity of a single storage facility $s$ [MWh]
Maximum rate of addition to the storage $s$ [MW]

Maximum rate of withdrawal from the storage $s$ [MW]

Unit cost of importing resource $r$ [£/MWh]

Unit cost of exporting resource $r$ [£/MWh]

Wind speed in zone $z$ during hour $h$ of day $d$ in season $t$ [m/s]

$x_i$ x-coordinate of the centroid of 1km square cell $i$

$x_z$ x-coordinate of the centre of demand of transmission zone $z$

$y_i$ y-coordinate of the centroid of 1km square cell $i$

$y_z$ y-coordinate of the centre of demand of transmission zone $z$

Conversion factor or resource $r$ in technology $p$

Capital charge factor

Efficiency of the wind turbines

Binary parameter, 1 if zone $z$ is adjacent to zone $z'$

Air density [kg/m$^3$]

Conversion factor when withdrawing resource $r$ from storage $s$

Conversion factor when holding resource $r$ in storage $s$

Conversion factor when putting resource $r$ into storage $s$

Factor that converts cost from £ to £M ($10^{-6}$)

Distance-independent conversion factor for transport mode $l$ transporting resource $r$

Distance-dependent conversion factor for transport mode $l$ transporting resource $r$

Maximum rate of export of resource $r$ in zone $z$ [MW]

Inventory of storage $s$ in zone $z$ during hour $h$, day $d$ and season $t$ [MWh]

Inventory of storage $s$ in zone $z$ at the start of day $d$ and season $t$ [MWh]

Inventory of storage $s$ in zone $z$ at the start of the simulated cycle for day $d$ and season $t$ [MWh]

Total O&M costs of conversion technologies [£M/yr]

Total O&M costs of storage facilities [£M/yr]

Total O&M costs of transport technologies [£M/yr]

Total capital cost of conversion technologies [£M]

Total capital cost of transport infrastructures [£M]

Total capital cost of storage facilities [£M]

Total capital cost of wind turbines [£M]

Total cost of importing resources from outside the network boundary [£M/yr]

Total variable operating cost of conversion technologies [£M/yr]

Total variable operating cost of transport technologies [£M/yr]

Total variable operating cost of storage facilities [£M/yr]

Total O&M costs of wind turbines [£M/yr]

Total cost of exporting resources to outside the network boundary [£M/yr]

Rate of import of resource $r$ in zone $z$ during hour $h$, day $d$ and season $t$ [MW]

Total utilisation rate of technology $p$ in zone $z$ during hour $h$, day $d$ and season $t$ [MW]
\(Q_{lzz'hd} \) Rate of transport via mode \( l \) from zone \( z \) to zone \( z' \) during hour \( h \), day \( d \) and season \( t \) [MW]

\( \mathcal{S}_{szhd}^{\text{get}} \) Rate at which resources is withdrawn from storage \( s \) in zone \( z \) during hour \( h \), day \( d \) and season \( t \) [MW]

\( \mathcal{S}_{szhd}^{\text{hold}} \) Rate at which inventory is held in storage \( s \) in zone \( z \) during hour \( h \), day \( d \) and season \( t \) [MWh]

\( \mathcal{S}_{szhd}^{\text{put}} \) Rate at which resource is added to storage \( s \) in zone \( z \) during hour \( h \), day \( d \) and season \( t \) [MW]

\( U_{rzhdt} \) Utilisation of resource \( r \) in zone \( z \) during hour \( h \), day \( d \) and season \( t \) [MW]

\( n_{rzhdt}^{\text{max}} \) Maximum availability of resource \( r \) in zone \( z \) during hour \( h \), day \( d \) and season \( t \) [MW]

\( X_{rzhdt} \) Rate of export of resource \( r \) in zone \( z \) during hour \( h \), day \( d \) and season \( t \) [MW]

Free variables

\( P_{rzhdt} \) Net rate of production of resource \( r \) in zone \( z \) during hour \( h \), day \( d \) and season \( t \) [MW]

\( Q_{rzhdt} \) Net rate of transport of resource \( r \) into zone \( z \) from all other cells during hour \( h \), day \( d \) and season \( t \) [MW]

\( S_{rzhdt} \) Net rate storage of resource \( r \) in zone \( z \) during hour \( h \), day \( d \) and season \( t \) [MW]

\( Z \) Objective function

\( \delta_{szd}^d \) Accumulation of inventory \( s \) in zone \( z \) during day \( d \) and season \( t \) [MWh]

\( \delta_{szt}^s \) Accumulation of inventory \( s \) in zone \( z \) during season \( t \) [MWh]

\( \delta_{sz}^y \) Accumulation of inventory \( s \) in zone \( z \) during the year [MWh]

Integer variables

\( N_{B_{bz}}^B \) Number of transport infrastructure \( b \) built between zones \( z \) and \( z' \)

\( N_{EWT}^E \) Number of existing wind turbines in zone \( z \) that are used as part of the network

\( N_{pz}^P \) Number of technologies \( p \) in zone \( z \)

\( N_{sz}^S \) Number of storage facilities \( s \) in zone \( z \)

\( N_{z}^{\text{NWT}} \) Number of new wind turbines in zone \( z \)

Abbreviations

CGH\(_2\) Compressed gaseous hydrogen

CGH\(_2\)S Compressed gaseous hydrogen storage

HVAC High voltage alternating current

HVDC High voltage direct current

OHL Overhead lines

UC Underground cables

Subscripts

el Electricity

hy Hydrogen