Black Hole Thermodynamics, Casimir Effect and Induced Gravity

F. Belgiorno* and S. Liberati†

* Dipartimento di Fisica, Università di Milano, 20133 Milano, Italy
E-mail address: belgiorno@vaxmi.mi.infn.it
† SISSA/ISAS, Via Beirut 3-4, 34100 Trieste, Italy
E-mail address: liberati@sissa.it

(December 9, 1996)

An analogy between the subtraction procedure in the Gibbons-Hawking Euclidean path integral approach to Horizon’s Thermodynamics and the Casimir effect is shown. Then a conjecture about a possible Casimir nature of the Gibbons-Hawking subtraction is made in the framework of Sakharov’s induced gravity. In this framework it appears that the degrees of freedom involved in the Bekenstein-Hawking entropy are naturally identified with zero-point modes of the matter fields. Some consequences of this view are sketched.

PACS: 04.70.Dy, 04.62.+v, 04.20.Cv

I. INTRODUCTION

Semiclassical Euclidean quantum gravity techniques play a key role in the investigation of thermodynamics of black holes. Nonetheless their physical interpretation is still a matter of debate. In this paper we propose a new framework in which this interpretation is achieved in a natural way by focusing our attention on the dynamics of quantum vacuum fluctuations in curved spacetime. We stress that this paper has a programmatic nature. We mean to approach a more quantitative level in a future work.

As a starting point for our reasoning, we shall summarize the path integral approach procedure following the steps first delineated by Gibbons and Hawking [1]. Given the classical Einstein-Hilbert action for gravity and the action of classical matter fields, one formulates the Euclidean path integral by means of a Wick rotation. In a semiclassical (saddle point) expansion of the action around classical field configurations, one can find a non-trivial partition function even taking into account only the gravitational tree level action. Indeed, in the presence of static background manifolds with bifurcate Killing horizons, the requirement of non-singular behaviour for the solutions of the equations of motion implies the periodicity of imaginary time. The period corresponds to the inverse Hawking-Unruh temperature because of the relation between the periodicity of the Euclidean Green’s functions and the thermal character of the corresponding Green’s functions in Lorentzian signature. Hence, in these cases, the effective action is truly a free energy function \( \beta F \).

This method, schematically sketched above, contains a step not definitively understood, namely the “reference” action subtraction for the gravitational tree level contribution. The action on shell consists of the usual Einstein-Hilbert action, of a surface term related to the extrinsic curvature and a Minkowskian subtraction term (the “reference” action). The latter is introduced by requiring that in flat spacetime the gravitational action is zero and it is necessary in order to obtain a finite value when evaluated on shell. In the following section we will further analyze this topic.

II. GRAVITATIONAL ACTION SUBTRACTION

Hawking and Horowitz [2] have developed this subtraction scheme to the case of non-compact geometries. They considered the Lorentzian gravitational action for a metric \( g \) and matter fields \( \phi \):

\[
I(g, \phi) = \int_M \left[ \frac{R}{16\pi} + L_m(g, \phi) \right] + \frac{1}{8\pi} \oint_{\partial M} K. \tag{1}
\]

The surface term is needed to give rise to the correct equations under the constraint of fixed induced metric and matter fields on the boundary \( \partial M \). The action is not well-defined for non-compact geometries: one has in the latter case to choose a rather arbitrary background \( g_0, \phi_0 \). Indeed Hawking and Horowitz chose a static background solution of the field equations. Their definition of the physical action is then:

\[
I_{\text{phys}}(g, \phi) \equiv I(g, \phi) - I(g_0, \phi_0). \tag{2}
\]

The physical action for the background is thus zero. Further, it is finite for a class of fields \((g, \phi)\) asymptotically equal to \((g_0, \phi_0)\). For asymptotically flat metrics the background is \((g_0, \phi_0) \equiv (g, 0)\); the action so obtained is then equal to that of Gibbons and Hawking:
The last term is just the Minkowskian subtraction: $K_0$ is the trace of the extrinsic curvature of the boundary of the background spacetime. The above subtraction could be physically interpreted by requiring that it should represent that of a background contribution w.r.t. which a physical effect is measured.

There is a nontrivial point to be stressed about (2): it is implicitly assumed that the boundary metric $h$ on $\partial M$ induced by $g_0$ and $g$ is the same. In general it is not possible to induce a 3–metric $h$ from a given 4–metric $g_0$; the same problem arises for the induction of a generic $h$ by flat space $\mathbb{R}^3$. In the case where the asymptotic behaviour of the 4–metrics $g$ and $g_0$ is the same, one can assume that the 3–metrics, say $h$ and $h_0$, induced respectively by $g$ and $g_0$, become asymptotically equal [3]. More generally the requirement to get the same boundary induced metric by $g_0$ and $g$ can be thought as a physical constraint on the choice of a reference background for a given spacetime.

So far we have argued that the subtraction procedure is a fundamental step in the path-integral formulation of semiclassical quantum gravity. In what follows, we will recall some well–known facts about the Casimir effect [4–6], in order to suggest a formal similarity between Casimir subtraction and the above gravitational action subtraction.

### III. CASIMIR SUBTRACTION

We start discussing the problem of two parallel infinite conducting plates; the energy density is obtained by means of the subtraction of the zero–point modes energy in absence of the planes from the zero–point mode energy in the presence of the two planes. One can in general formally define the Casimir energy as follows [5]:

$$E_{\text{casim}}[\partial M] = E_0[\partial M] - E_0[0]$$

(4)

where $E_0$ is the zero–point energy and $\partial M$ is a boundary.

Boundary conditions in the Casimir effect can be considered [3] as idealizations of real conditions in which matter configurations or external forces act on a field. The most general formula for the vacuum energy is

$$E_{\text{casim}}[\lambda] = E_0[\lambda] - E_0[\lambda_0]$$

(5)

where $\lambda$ is a set of suitable parameters characterizing the given configuration (e.g. boundaries, external fields, nontrivial topology), and $\lambda_0$ is the same set for the configuration w.r.t. which the effect has to be measured. In the case that $\lambda$ represents an external field $A$, the vacuum energy distortion induced by switching on the external field is given by:

$$E_{\text{casim}}[A] = E_0[A] - E_0[0].$$

(6)

One can also take into account the finite temperature Casimir effects [7,8]: in this case matter fields are not in their vacuum state, there are real quanta excited which are statistically distributed according to Gibbs canonical ensemble. The Casimir free energy is:

$$F_{\text{casim}}[\beta, \lambda] = F[\beta, \lambda] - F[\beta, \lambda_0].$$

(7)

The zero–point contribution [7] to the finite temperature effective action is simply proportional to $\beta$ (so it doesn’t contribute to the thermodynamics).

The formal analogy of (2) with e.g. (7) consists just in the fact that in both cases there is a reference background to be subtracted in order to get a physical result. In particular, the subtraction (4) is analogous to that in (3): the obvious substitutions being $g_{\mu\nu}$ in place of $A$ and $\eta_{\mu\nu}$, in place of 0.

We stress that there are still substantial differences between (2) and (3) due to the fact that in (2) the field $A$ is external whereas in (3) the field $g_{\mu\nu}$ is the dynamical field itself; moreover, a deeper link of (2) with the Casimir effect would require a quantum field whose zero point modes are distorted by spacetime curvature. Note that in the latter case one could naively invoke a Casimir effect w.r.t. the background spacetime $(M_0, g_0)$:

$$F_{\text{casim}}[\beta, g]_M = F[\beta, g]_M - F[\beta, g_0]_{M_0}.$$  

(8)

We stress that (8) is purely formal and requires static manifolds $(M, g)$, $(M_0, g_0)$. For zero–temperature the idea underlying the Casimir effect, as seen above, is to compare vacuum energies in two physically distinct configurations. If the gravitational field plays the role of an external field, one can a priori compare backgrounds with different manifolds, topology and metric structure. The non triviality one finds in defining meaningfully a gravitational Casimir effect can be easily understood for example in terms of the related problem of choice of the vacuum state for quantum fields [3]. Moreover, in the presence of a physical boundary, the subtraction (8) is ill–defined in general because the same embedding problems exist for (3). Despite these problems, we assume that it is possible to give a physical meaning to (8). It is at least well known how to do this in the case of static spacetime with fixed metrics and topology like $R \times M^3$ where the spatial sections $M^3$ are Clifford-Klein space forms of flat, spherical or hyperbolic 3-spaces.

*In eq. 4 and in the following analogous equations concerning the Casimir effect a regularization of the right hand side terms is understood.
\( M^3 = R^3/\Gamma, S^3/\Gamma, H^3/\Gamma \) \[12\]

Then Sakharov’s conjecture [14] about the nature of the gravitational field represents a conceptual framework in which the analogy can be strongly substantiated.

**IV. INDUCED GRAVITY**

According to Sakharov’s ideas, the Einstein–Hilbert gravitational action is induced by vacuum fluctuations of quantum matter fields and it represents a type of elastic resistance (of constant \( G \)) of the spacetime to being curved. The qualitative basis of this statement [15] is the fact that the Einstein–Hilbert action density is given by the Ricci scalar \( R \) times a huge constant (order of the square of the Planck mass): curvature development requires a large action penalty [14] to be paid, that is there is an “elastic” resistance to curvature deformations. The fact that a long–wavelength expansion of quantum matter fields in curved spacetime contains zero point divergent terms proportional to the curvature invariants according to Sakharov suggests that zero point fluctuations induce the gravitational action. “Induction” means that no tree level action is considered: quantum matter fields generate it at a quantum level. Gravitational interaction in this picture becomes a residual interaction [16] of a more fundamental one living at high energy scales (Planck mass); there are various ways to implement such a fundamental theory [17].

The induced gravitational action should be given by the difference between the quantum effective zero–point action for the matter fields in the presence of the spacetime curvature and the effective action when the curvature is zero [18] i.e.

\[
S_{\text{induced gravity}} = \Gamma[R] - \Gamma[0].
\] (9)

The field \( g_{\mu\nu} \) actually appears, in this low energy regime, as an external field and not as a dynamical one. Then the induced gravity framework allows us to identify the Minkowskian subtraction as a Casimir subtraction. We note that there is a boundary term in [2] that is necessary in order to implement a Casimir interpretation of the subtraction and that is missing in the original idea of Sakharov. But if the manifold has a boundary it is natural to take into account its effects on vacuum polarization [1]. As in a renormalization scheme for quantum field theory in curved spacetime it is necessary to introduce suitable boundary terms in the gravitational action in order to get rid of surface divergences [13,14] so there should be boundary terms in the induced gravitational action [20]. Anyway we don’t know if taking into account boundary terms and suitable boundary conditions it is possible to produce a self–consistent theory of induced gravity [1]. The choice of the boundary conditions should be constrained in such a way to get an induced gravity action with a boundary term as in Hawking approach. In this paper we shall limit ourselves to a discussion of the case of a scalar field and to the divergent part of the effective action.

In a curved manifold \( M \) with smooth boundary \( \partial M \), the zero–point vertex functional for a scalar field depends on the curvature and is divergent:

\[
\Gamma[\phi = 0, g_{\mu\nu}] = \Gamma[g_{\mu\nu}].
\] (10)

The zero–point effective action [10] is comprised of divergent terms that, if \( D \) is the dimension of \( M \), correspond to the first \( l \leq D/2 \) \((l = 0, 1/2, \ldots)\) coefficients, \( c_l \), in the heat kernel expansion [13,8]. In our case \( D = 4 \) and \( l \leq 2 \). For a smooth boundary the coefficients \( c_l \) can be expressed as a volume part plus a boundary part:

\[
c_l = a_l + b_l.
\] (11)

The \( b_l \) depend on the boundary geometry and on the boundary conditions. The \( a_l \) coefficients vanish for \( l \) half integral and for integral values are equal to the Mi-nakshisundaram coefficients for the manifold \( M \) without boundaries.

If there is a classical (not induced but fundamental) gravitational action, the divergent terms in [12] can be renormalized [13] by means of suitable gravitational counterterms: that is by reabsorbing the divergences into the bare gravitational constants appearing in the action for the gravitational field:

\[
S_{\text{ren}}[g_{\mu\nu}] = S_{\text{ext}}[g_{\mu\nu}] + \Gamma_{\text{div}}[g_{\mu\nu}].
\] (12)

In an induced gravity framework, there is no classical (tree level) term like \( S_{\text{ext}}[g_{\mu\nu}] \) to be renormalized and so there should exist a dynamical cut–off rendering finite also the divergent terms. These terms give rise to the gravitational (effective) action, so we can call \( \Gamma_{\text{div}}[g_{\mu\nu}] \) “the gravitational part” of the effective action.

In the case of finite temperature field theory on a static manifold with boundary, the standard periodicity condition in the imaginary time \( \tau \) with period \( \beta \) can be implemented by means of the following image sum over a non–periodic heat kernel:

\[
K_\beta(x, y; s) = \sum_{n=-\infty}^{+\infty} K_\infty(x, y - nk\beta; s)
\] (13)

**For a wider discussion on this point see also [2].**
where $s$ is the usual “fifth coordinate” and $k$ is a four vector in the same direction as the periodic coordinate. The calculation of the partition function of the matter fields is meant to be carried out in the so called “on shell” approach, i.e., without introducing any conical defect in the manifold. The $n = 0$ term in (3) is ordinarily a zero temperature–term and it is the only divergent one (cf. 3). It corresponds, in the induced gravity framework, to the gravitational contribution.

We are mainly interested in the case of the Schwarzschild black hole: (4) could be then interpreted as a Casimir free energy contribution relative to the matter field zero–point modes. We consider the one–loop divergent contribution for a massive scalar field enclosed in a sphere with radius $r = R_{\text{box}}$.

We choose the boundary condition by looking at the structure of the boundary terms. For consistency, one would get in particular the Einstein–Hilbert action term (3) at the same order in the heat kernel expansion, so we point our attention to the boundary term $b_1$. It is possible to get the right form for the integrated coefficient

$$
\int_{\partial M} K \tag{14}
$$

both for Dirichelet and Neumann boundary conditions. The first one is selected on the physical grounds that for a sufficiently large box (infinite in the limit) the field should be zero on the boundary.

Of course in order to get ordinary gravitational dynamics, i.e. General Relativity, it is necessary that the couplings and the mass of the fundamental theory fulfill suitable renormalization constraints.

In our conjecture, the gravitational part of the free energy (3) becomes a Casimir free energy contribution arising by zero–point modes. What should it mean from a physical point of view? The most naive answer to this question is that black hole (equilibrium) thermodynamics becomes a thermal physics of quantum fluctuations that are initially in a spherically symmetric spacetime and then are thermally distorted by the formation of a black hole. In this view it seems that there is implicitly and then are thermally distorted by the formation of a black hole where a sort of dynamical Casimir effect prevents the formation of the event horizon.

We underline that the idea of a link between the Casimir effect, induced gravity and black hole evaporation was formulated by U.H. Gerlach in a model for an incipient black hole where a sort of dynamical Casimir effect prevents the formation of the event horizon.

V. THERMAL BEHAVIOUR OF ZERO POINT MODES

The induced gravity framework implies that black hole entropy can by explained in terms of Boltzmann’s counting of microstates by identifying the statistical mechanical degrees of freedom with the zero–point fluctuations of quantum matter fields. This particular view seems nonsense in the framework of standard (i.e. without the Hawking–Unruh effect) statistical mechanics, because in this case zero–point modes cannot contribute to the entropy, their contribution to the effective action being proportional to $\beta$. But there is no real contradiction: in the case of horizon’s thermodynamics, the subtracted gravitational action is not simply proportional to $\beta$. In black holes spacetime it is proportional to $\beta^2$. In this case there is a bifurcate Killing horizon that is related to the presence of thermal zero–point mode contribution. This means that vacuum fluctuations do contribute to the entropy. Note that this conclusion is independent of the induced gravity framework: matter fields give a thermal zero–point contribution (that has to be renormalized in the gravitational action outside of Sakharov’s viewpoint).

The topological structure of spacetime seems to be deeply linked to the thermal behaviour of quantum fluctuations. They are expected to have stochastic properties described by the structure of the $n$-th order correlation functions; a thermal spectrum is a subcase requiring both a Gaussian distribution (n-order correlations zero for $n \geq 3$) and a Planckian frequency spectrum (the Fourier transform of the 2-point correlation function) (2). A bifurcate Killing horizon in a static manifold seems to ensure a Gaussian behaviour that in other more general cases (e.g. non static manifolds without bifurcate Killing horizons) is substituted by a “standard” stochasticity.

\[\text{§§ This idea is pursued with different conceptual tools in the papers of Jacobson, of Frolov, Fursaev and Zelnikov and also of Gerlach. Zero–point modes could explain more generally the entropy appearing in Horizon Thermodynamics.}\]
In order to see that it is possible to find a thermalization the zero–point modes, we recall some facts concerning the Unruh effect in Rindler spacetime. An uniformly accelerated observer in a Rindler wedge of Minkowski spacetime perceives Minkowski vacuum as a thermal state satisfying a KMS condition, that is a detailed balance condition. The “particle” spectrum seen by the accelerated observer depends only on the acceleration and not on the velocity of the observer, and this fact means that actually zero–point fluctuations of Minkowski vacuum appear as particles thermally distributed in the accelerated detector. Moreover, at a formal level, the thermofield dynamics approach can explain why a pure state (Minkowski vacuum) appears as a mixed one (thermal) to a Rindler observer; there is a horizon hiding part of the information relative to the state. A similar thermofield scheme is also available for the Schwarzschild black hole, in which the information relative to a vacuum state defined on the Kruskal extension (the Hartle–Hawking state) appears to be thermal to a static observer in the Schwarzschild external region. Actually, it has to be stressed that the thermofield framework can be used, only if one can consider the maximal extension of a spacetime and even in these cases one can find, as in Kerr black holes (cf. [23]), that it is impossible to mimic the Unruh-Rindler scheme. So from the Unruh effect we have some insights about zero–point mode thermalization, but no general and definitive explanation available also in the cases of black holes arising from a realistic collapse, rotating.

A more general explanation of vacuum thermalization can be found in the framework of non-equilibrium horizon thermodynamics introduced by Sciama [31]. In Sciama’s approach the irreversible process of black hole evaporation can be interpreted as a progressive dissipation of the geometry against vacuum fluctuations. In his view, black hole and radiation are linked by a fluctuation-dissipation relation for their zero-point modes. In the static case a local equilibrium condition is achieved in which the only distribution of the quantum states that is independent of time and stable w.r.t weak interactions between the modes of the field does satisfy the KMS condition (i.e. it exists a thermal distribution). This is possible apparently only in presence of proper degrees of freedom for the black hole. As an atom can come into equilibrium with vacuum fluctuations of the electromagnetic field (its zero-point fluctuations being “enslaved” by those of the field) so the black hole would come into equilibrium only if endowed with proper degrees of freedom. Hence, in order to pursue the above analogy, one has to deal with the highly non trivial problem of identifying a physically acceptable notion of degrees of freedom for a black hole, a problem that is commonly believed to be associated to a full quantum gravity theory rather than to a semiclassical approach.

The interpretation we are proposing of gravitational action and horizon thermodynamics appears as a strong link to Sciama’s framework. Indeed it describes the geometry-matter system at the equilibrium as a sort of static Casimir effect and the non-equilibrium regimes are naturally interpreted as dynamical Casimir-like effects in the same way as the energy spent by the geometry in thermally exciting vacuum fluctuations can be interpreted in Sciama’s view as an infalling of negative energy into the black hole. Moreover, in our framework of induced gravity, black hole entropy is associated with a Casimir distortion of vacuum fluctuations. Finally, regarding the problem of a fluctuation–dissipation theorem for black holes, our proposal leads us to conjecture that one could bypass the request of internal degrees of freedom for the black hole (at least at this mesoscopic level) as follows. Geometry deformations w.r.t. the static case (like quasinormal modes of black hole) should be driven to a stationary state (static black hole). This means that the way geometry affects the quantum matter field spectrum should be such that the static black hole geometry, although quantum unstable (through Hawking radiation), is more stable than a deformed black hole geometry.

VI. CONCLUSIONS

We conclude with some comments on the implications of our framework. Firstly it can be cast in the recently sustained interpretation of General Relativity as an effective theory [23,24]. In Sakharov’s picture the Einstein–Hilbert action, being induced by quantum matter field fluctuations, should not be considered as the action of a fundamental theory. It can be interpreted as the (effective) long–wavelength action of a mesoscopic theory, as elasticity is a mesoscopic theory which is related to the fundamental theory of quantum electrodynamics. So quantizing gravity could be equivalent to quantizing phonons [24].

Our attempt consists in finding a link with Casimir physics suggested by the subtraction procedure in black hole thermodynamics. The subtraction in our view takes into account a physical process of adiabatic vacuum energy distortion and in a real collapse we expect nonadiabatic contributions. To be more specific about this point, we shall shortly summarize the general framework we choose.

Quantum matter fields coupled with geometry induce geometrodynamics action terms that correspond to zero point fluctuations and are both local (bulk part) and global (boundary part). One can also look for a more fundamental pregeometric theory living at the planckian scale (e.g. noncommutative geometry, string theory). In any case, the low energy theory gets from high energy

***See also [27] and references therein.
theory only a dynamical scale and the value of the couplings (via the renormalization group). In Sakharov’s view, one can conclude that the degrees of freedom involved in the gravitational action are vacuum fluctuations that can explain black hole entropy [24]. In particular the vacuum acts as a viscous (dissipative) medium whose distortion, due to a nontrivial topology and/or geometry configuration, gives rise to black hole entropy. Note that the Casimir interpretation of [6] involves only external degrees of freedom i.e. external vacuum fluctuations without involving any notion of internal states. In particular static black hole entropy is a static “Casimir” entropy. In a dynamical configuration such as gravitational collapse or black hole evaporation a dynamical Casimir effect would be involved.

About the prescription one has to follow in order to actually compute such an entropy, our framework implies a conceptually easy prescription. The gravitational free energy is now the free energy for the matter field zero point modes. Once one calculates the latter one can find the correspondent entropy by applying the usual formula \( F = \beta E - S \). In Schwarzschild case the internal energy is the black hole mass.

Finally, we think that an induced gravity framework could explain why General Relativity, as a classical theory, knows about a quantum phenomenon like the Hawking effect. Indeed, the so-called four laws of black hole mechanics were found at a classical level before of the discovery the Hawking effect (quantum level). Quantum radiation from black holes has given a semantic meaning to a surprising syntactic analogy between black hole mechanics and standard thermodynamics. But the meaning of thermodynamical behaviour of the classical level gravity is still mysterious. We think that imposing Einstein’s gravity to be an effect of order \( \hbar \) could represent a good bridge to such an understanding.

Of course, there are many open questions to be solved. Quantitative and further conceptual developments of our approach are deferred to a future publication.

ACKNOWLEDGEMENTS

The authors are particularly indebted with D.W. Sciama for several illuminating discussions. They wish to thank B. Bassett, S. Sonego, G. Immirzi, L. Pilo, G. Pollifrone, K. Yoshida and M. Martellini for useful comments and remarks.

1. G.W. Gibbons and S.W. Hawking, Phys. Rev. D 15, 2752 (1977)
2. S.W. Hawking, G.T. Horowitz, Class. Quant. Grav. 13, 1478 (1996)
3. S.W. Hawking, “The path integral approach to quantum gravity”, in “General Relativity. An Einstein Centenary Survey”, Cambridge University Press, Cambridge (1979)
4. N.D. Birrell and P.C.W. Davies, “Quantum Fields in Curved Space”, Cambridge University Press, Cambridge (1982)
5. A.A. Grib, S.G. Mamayev, V.M. Mostepanenko, “Vacuum Quantum Effects in Strong Fields”, Friedmann Laboratory Publishing, St.Petersburg (1994)
6. G. Plunien, B. Müller and W. Greiner, Phys. Rep. 134, No 2 & 3, 87 (1986)
7. G. Plunien, B. Müller and W. Greiner, Physica 145A, 202 (1987)
8. J.S. Dowker and G. Kennedy, J. Phys. A 11, 895 (1978)
9. J.S. Dowker and R. Banach, J. Phys. A 11, 2255 (1978)
10. B.S. DeWitt, C.H. Hart and C.J. Isham, Physica 96A, 197 (1979)
11. Y.P. Goncharov and A.A. Bytsenko, Class. Quantum Grav. 8, L211 (1991)
12. Y.P. Goncharov and A.A. Bytsenko, Nucl. Phys. B271, 726 (1986)
13. J.A. Wolf, “Spaces of constant curvature”, University of California, Berkeley (1967)
14. A.D. Sakharov, Sov. Phys. Dok. 12, 1040 (1968)
15. S.L. Adler, “Sakharov and Induced Gravitation”, Priroda, 1990, No 8
16. C.W. Misner, K.S. Thorne and J.A. Wheeler, “Gravitation”, Freeman, San Francisco (1973)
17. See e.g. S.L. Adler, Rev. Mod. Phys. 54, 729 (1982) and the conclusions.
18. A.D. Sakharov, Theor. Math. Phys. 23, 435 (1976)
19. G.Kennedy, R.Critchley and J.S.Dowker, Ann. Phys. 125, 346 (1980)
20. G. Denardo and E. Spallucci, Nuovo Cim. 69A, 151 (1982)
21. A.O. Barvinsky and S.N. Solodukhin, “Non–Minimal coupling, boundary terms and renormalization of the Einstein–Hilbert action and black hole entropy”, preprint gr-qc/9512047
22. F. Belgiorno and S. Liberati, Phys. Rev. D53, 3172 (1996)
23. F. Belgiorno and M. Martellini, Phys. Rev. D53, 7073 (1996)
24. F. Belgiorno and M. Martellini, “Equilibrium Thermodynamics for Quantum Fields on a Black Hole Background”, to be published in the proceedings of: Second International Conference “Astronomy, Cosmoparticle Physics”-COSMION ’96. May 25–June 6, 1996, Moscow.
25. U.H. Gerlach, Phys. Rev. D14, 1479 (1976)
26. T. Jacobson, Black Hole Entropy and Induced Gravity, preprint gr-qc/9404039
27. V.P. Frolov, D.V. Fursaev and A.I. Zelnikov, “Vacuum Structure in Intense Fields”, in "Vacuum Structure in Intense Fields"
Plenum Press, New York (1991);
N. Sanchez and B.F. Whiting, Phys. Rev. 34D, 1056 (1986)
[28] Y. Takahashi, H. Umezawa, Collective Phenomena 2, 55 (1975)
[29] R.M. Wald, “Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics”, The University of Chicago Press, Chicago and London (1994)
[30] W. Israel, Phys. Lett. 57A, 107 (1976)
[31] D.W. Sciama, “Thermal and quantum fluctuations in Special and General Relativity: an Einstein Synthesis”, in “Centenario di Einstein”, Editrici Giunti Barbera Universitaria (1979);
P. Candelas and D.W. Sciama, Phys. Rev. Lett. 38, 1372 (1977);
D.J. Raine and D.W. Sciama, to appear Class. Quantum. Grav. (1997);
D.W. Sciama, private communications
[32] S. Chandrasekhar, “The Mathematical Theory of Black Holes”, Oxford University Press, Oxford (1983);
J.W. York, Phys. Rev. D28, 2929 (1983)
[33] T. Jacobson, Phys. Rev. Lett. 75, 1260 (1995)
[34] B.L. Hu, “General Relativity as Geometric–Hydrodynamics”, preprint gr-qc/9607020