Dark Matter Indirect Detection with Gamma Rays

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Abstract. Searches for weakly interacting massive particle (WIMP) dark matter with gamma-ray instruments are a way to get a unique observational handle on the particle nature of dark matter. I will discuss the details of how to perform these searches, both for annihilating and decaying WIMPs. I will discuss the calculation of the gamma-ray flux from possible sources of dark matter annihilation or decay and show examples of limits which have been calculated using these techniques.

1. Evidence for Dark Matter
Since its discovery in the 1930s, the evidence for dark matter has been plentiful. Dark matter was first discovered by observations that rotation curves in galaxies and clusters differed significantly from the rotation curve expected if the gravitational potential was dominated by the observed luminous matter [1]. The luminous matter of galaxies is largely contained within the inner few kiloparsecs of the galaxy. According to the law of gravitation, in the outer regions of a galaxy, the circular velocity of observed bodies should therefore follow:

\[ F_G = \frac{GmM(< r)}{r^2} = \frac{m v_{\text{cir}}^2}{r} \]

(1)

\[ v_{\text{cir}}(r) \propto \sqrt{\frac{M(< r)}{r}} , \]

(2)

with \( M(< r) \) the mass contained with radius from the galactic center \( r \) and \( v_{\text{cir}} \) the circular velocity at radius \( r \). \( G \) is the universal gravitational constant and \( m \) is a test mass, which cancels out in the second equation. However, the circular velocity tends to flatten out at large radii rather than decreasing, consistent with

\[ M(< r) \propto r \]

\[ \rho_M \propto r^{-2} . \]

(3)

This indicates an additional mass component to galaxies which is not luminous and has a density profile \( \rho_M \sim r^{-2} \) which continues out to large radii. This additional, non-luminous mass is referred to as “dark matter”. Further evidence for the dark matter comes from anisotropies in the cosmic microwave background (CMB), which has shown that the dark matter makes up 26.7% of the energy budget of the universe, but ordinary baryonic matter makes up only 4.9% [2]. The case for dark matter has also been made based on large-scale structure of the universe [3, 4] and observations of galaxy clusters [5, 6].
Although dark matter has only been observed through its gravitational interactions, the properties of dark matter are well constrained. Lack of sufficient gravitational microlensing have determined that the dark matter is made up of particles with masses $M_\chi \lesssim 10^{-7}M_\odot$, and baryonic clumps of matter cannot explain the majority of the dark matter observed in the Galaxy [7, 8, 9].

The most popular dark matter model is the weakly interacting massive particle (WIMP), with a dark matter particle of mass $M_\chi \sim 100$ GeV which interacts through a force of similar strength to the weak force. WIMPs were formed in the early universe in equilibrium with other particles at high temperature. As the universe cooled, WIMPs collided and annihilated into standard model particles. However, once the expansion of the universe exceeded the annihilation rate of the WIMPs, they stopped annihilating and the number density of WIMPs “froze out”, giving the dark matter number density seen today. The so-called “WIMP Miracle” is that for a WIMP with weak-scale cross-section $(\sigma \sim 1$ pb) and a weak-scale mass $(M_\chi \sim 100$ GeV), the relic density of thermal WIMPs is $\Omega_{WIMP} \approx 0.2$, consistent with the energy density for the dark matter. A detailed calculation of the thermal cross-section is found in Ref. [10].

Though the WIMP model is often motivated by Supersymmetry, other theories of beyond-the-Standard-Model physics predict WIMPs as well, such as models with compact extra dimensions. Most of the current models predict WIMP masses from 10 GeV-5 TeV, but WIMPs as heavy as several PeV could exist as well. The typical velocity-weighted WIMP annihilation cross-section is $\langle \sigma v \rangle = 2.2^{−26} cm^3/s$ [10]. If the WIMPs are not produced as thermal relics but are produced non-thermally though decays of heavier particles, or if the cross-section is boosted through Sommerfeld enhancement (discussed in Section 3), then the cross-section today could be much different from this value [11].

2. Calculations of the Dark Matter Annihilation Flux

2.1. Particle Flux from Dark Matter Annihilations

The number of dark matter annihilations per unit volume is $\rho(\vec{x})^2/(2M_\chi^2) \langle \sigma_A v \rangle$, for dark matter mass $M_\chi$ and dark matter number density $\rho/M_\chi$. $\langle \sigma_A v \rangle$ is the annihilation cross-section times the relative velocity between interacting dark matter particles, averaged over their velocity distribution, and the factor of two comes from symmetry concerns. The number of dark matter collisions per unit time per unit observational area integrated along a particular line-of-sight is then

$$\frac{d^2N}{dt dA} = \frac{\langle \sigma_A v \rangle}{2M_\chi^2} \int \frac{d^2r d\Omega}{4\pi r^2} \rho(\vec{x})^2 = \frac{\langle \sigma_A v \rangle}{2M_\chi^2} \int \frac{dr d\Omega}{4\pi} \rho(\vec{x})^2.$$

(4)

Observationally, we detect the number of Standard Model particles per unit energy produced by dark matter annihilations rather than the annihilations themselves. For an annihilation final-state $f$, the branching fraction $Br_f$ is the fraction of time a dark matter annihilation will end up in that final state. For a particular Standard Model particle $i$, $dN_f^{(i)}/dE$ is the number of particles $i$ per unit energy produced in final-state $f$. The details of which final states occur with which branching fractions depends on the precise dark matter model under consideration, though often, for simplicity, a particular dark matter annihilation channel is assumed to dominate and its branching fraction is set to unity. To simulate hadronization and decay of standard model particles into final states, analytical expressions or simulations are used (see section 3.1). The flux of gamma rays per units energy and solid angle coming from a region of dark matter annihilations is given by

$$\frac{d^2F_\gamma}{dEd\Omega} = \frac{\langle \sigma_A v \rangle}{2} \sum_f \frac{dN_f^{(i)}}{dE} Br_f \int_0^\infty \frac{dr \rho(\vec{x})^2}{4\pi M_\chi^2}. $$

(5)
Here, $r$ is the integration over the line-of-sight toward the source. $|\vec{x}| = \sqrt{R^2 - 2rR\cos(\theta) + r^2}$ is the distance from a differential volume of the source to the source center, where $R$ is the distance from the Earth to the source and $\theta$ is the observation angle from the source center to the differential volume.

### 2.2. Dark Matter Distributions

Dark matter in galaxies exists in a roughly spherically symmetric “halo” that approximately falls off as $\rho(|\vec{x}|) \propto |\vec{x}|^{-2}$ with the distance from the galactic center. Numerical N-body simulations have been done considering both pure dark matter and dark matter with baryons [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. The most widely adopted dark matter profile is the Navarro-Frenk-White (NFW) profile [18], parameterized as

$$\rho(|\vec{x}|) = \frac{\rho_s}{(|\vec{x}|/r_s)(1 + |\vec{x}|/r_s)^2}$$

with scale radius $r_s \approx 20$ kpc for our Galaxy. A somewhat shallower dark matter profile, the Einasto profile, is parameterized as [25, 26]

$$\rho(|\vec{x}|) = \rho_s \exp \left[ -\frac{2}{\alpha} \left( \left( \frac{|\vec{x}|}{r_s} \right)^\alpha - 1 \right) \right],$$

with scale radius $r_s \approx 20$ kpc and $\alpha = 0.1 - 0.2$ for Milky-Way-type halos [19, 23]. The scale density for these Galactic dark matter profiles, $\rho_s$, is fixed such that the total integrated dark matter density of the object gives its measured dark matter mass.

### 3. Cross-section and Substructure Boosts

The dark matter cross-section can be larger than thermal if the dark matter couples to light gauge bosons. The exchange of the bosons by the annihilating dark matter particles can create a resonance and increase the annihilation cross-section today by orders of magnitude relative to the value at the time of freeze-out. This is called “Sommerfeld enhancement”, after a similar effect due to photons in electron scattering. Sommerfeld enhancement goes as [27]

$$\epsilon_v \equiv \frac{v}{\alpha},$$

$$\epsilon_\phi \equiv \frac{m_\phi}{\alpha M_\chi},$$

$$S \approx \frac{\pi}{\epsilon_v \cosh \left( \frac{2\pi \epsilon_v}{\pi^2 \epsilon_\phi/\mu} \right)} - \cos \left( 2\pi \sqrt{\frac{1}{\pi^2 \epsilon_\phi/\mu} - \frac{\epsilon_\phi^2}{\pi^2 \epsilon_\phi/\mu}} \right),$$

where $\alpha \sim 1/35$ is the weak coupling, $m_\phi$ is the gauge boson mass, $M_\chi$ is the dark matter mass, $v$ is the relative velocity of the dark matter particles, and $S$ is the enhancement to the cross-section. Because the Sommerfeld enhancement increases at low velocities, and the relative velocity of dark matter in the Galaxy is $\sim 300$ km s$^{-1}$, much smaller than the $\sim c/4$ velocities in the early universe, the dark matter cross-section during thermal freeze out can be much less than the cross-section today. The shape of Sommerfeld enhanced cross-section limits have multiple peaks in them, corresponding to the combinations of velocity + dark matter mass + gauge boson mass which lie directly at resonance, not merely near it. Usually, Sommerfeld enhancement is achieved using a dark-sector light boson. However, for the heavy dark matter masses relevant to HAWC, Standard Model W and Z bosons are light enough to cause this enhancement. Especially for WIMPs which annihilate primarily into $W^+W^-$ or $Z^+Z^-$ states,
they are guaranteed to couple to Standard Model gauge bosons and naturally have Sommerfeld enhancement.

The flux can also be boosted due to dark matter substructure. In all numerical simulations of dark matter, most of the dark matter is not in a smooth halo but is found in smaller subhalos, which can have masses as low as $10^{-6} M_⊙$. Because the flux is proportional to the dark matter density squared, and the local density in these subhalos is much larger than from the smooth halo, these substructures can increase the dark matter flux from 2-1000 times, depending on the object. Most of the subhalos are far from the center of the smooth halo [28]. Therefore, in observations of the Galactic center, substructure boosts do not contribute to the flux significantly. For dwarf galaxies, which are actually large, virialized substructures themselves, the substructure boost does not contribute much because the objects are small in extent and do not have that many orders of magnitude difference between the smooth halo mass and the size of the substructure. However, for extragalactic dark matter sources, especially other galaxies and galaxy clusters, the substructure boost can be large.

3.1. Calculation of Dark Matter Spectra
To calculate the photon spectrum for a particular WIMP annihilation channel, programs like PYTHIA are used to simulate the photon radiation of charged particles as well as decays of particles such as the $\pi^0$ [29]. Because even fairly stable particles decay within a short distance on astrophysical scales, all possible particles are decayed, including pions, muons, and neutrons. Using a large sample of events for each final state and each value of $M_\chi$, the number of photons in the final state in a given logarithmic energy bin is counted and averaged over the number of events, yielding the average number of photons in that energy bin per annihilation event.

4. Dark Matter Decay
The procedures for getting dark matter limits from decay are very similar to those from dark matter annihilation, with a few small changes. First, the center-of-mass energy available for decay channels in the dark matter mass, $M_\chi$ for decay, instead of $2M_\chi$ in annihilation. This is because a decaying particle has only one particle-mass to covert into final states instead of two. Second, the source shape and extension for sources of dark matter decay are different than for annihilation. The flux for annihilation goes as the density squared ($\rho^2$) whereas for decay the flux goes as the density ($\rho$). Third, for dark matter annihilation, the limit is an upper limit on the velocity-weighted dark matter cross-section $\langle \sigma_v \rangle$. For decay, it is a lower limit on the dark matter lifetime $\tau$. These differences are discussed in more detail below.

4.1. Particle Flux from Dark Matter Annihilations
The number of dark matter decays per unit volume per unit time is $\rho(\vec{x})/(M_\chi \tau)$, for dark matter mass $M_\chi$ and dark matter number density $\rho/M_\chi$. $\tau$ is the dark matter lifetime, which must be much longer than the age of the universe. The number of dark matter collisions per unit time per unit observational area integrated along a particular line-of-sight is then

$$\frac{d^2 N}{dt dA} = \frac{1}{M_\chi \tau} \int \frac{r^2 dr d\Omega}{4\pi r^2} \rho(\vec{x}) = \frac{1}{M_\chi \tau} \int \frac{dr d\Omega}{4\pi} \rho(\vec{x}) .$$  \hspace{1cm} (11)

Note that for decay, the total center-of-mass energy is $M_\chi$, rather than $2M_\chi$ for annihilation. This yields a flux equation similar to that for annihilation, but depending linearly on the dark matter density $\rho$ instead of quadratically. The flux of gamma rays per units energy and solid angle coming from a region of dark matter annihilations is given by

$$\frac{d^2 F_\gamma}{dE d\Omega} = \frac{1}{\tau} \sum_f dN_f(\gamma)/dE Br_f \int_0^\infty \frac{dr}{4\pi} \frac{\rho(\vec{x})}{M_\chi} .$$ \hspace{1cm} (12)
Figure 1. Sample dark matter annihilation and decay limit plots for two dark matter channels, bottom quarks and tau leptons. Information about the data on these figures can be found in Ref. [30].

The dark matter spatial distributions for decaying dark matter are the same as for annihilating dark matter. Because dark matter decay involves only one dark matter particle, enhancements such as Sommerfeld enhancement do not apply. Additionally, because the flux goes as the density rather than the density squared for decay, dark matter substructure does not give additional boosts to the flux over the smooth dark matter halo.

5. Calculating Dark Matter Limits
To get limits on the WIMP annihilation cross-section (decay lifetime), the dark matter annihilation (decay) channel and the dark matter mass are set to fixed values. This leaves the annihilation cross-section (decay lifetime) as the only free parameter in the flux. A $\chi^2$ value or likelihood test-statistic is calculated which compares a model with the dark matter gamma-ray source to a model without it. The flux is scaled by changing the cross-section or lifetime until the $\chi^2$ (test statistic) is minimized (maximized). Then the value of the annihilation cross-section (decay lifetime) is increased (decreased) until the $\chi^2$ or test statistic changes from its optimal value by 2.71 [31]. This is the value of the cross-section (lifetime) which is constrained at 95% confidence level, which is the standard for dark matter limits. This process is repeated for all dark matter masses and channels that are being considered. The results are then plotted as several log-log plots, one for each dark matter channel, with the dark matter mass on the x-axis and the cross-section (lifetime) on the y-axis. Some example figures from Ref. [30] are shown as Figure 1 for illustration. Note that all cross-section above the exclusion line are excluded while all decay lifetimes below the exclusion line are excluded. It is also common to show limits from more than one experiment on the same figure for comparison or to show expected values from dark matter models (e.g. the thermal dark matter cross-section) on these figures as well.
way, you can tell the relevance of a particular experiment to dark matter searches.

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