Transitions from small to large Fermi momenta in a one-dimensional Kondo lattice model

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We study a one-dimensional system that consists of an electron gas coupled to a spin-1/2 chain by Kondo interaction away from half-filling. We show that zero-temperature transitions between phases with “small” and “large” Fermi momenta can be continuous. Such a continuous but Fermi-momentum-changing transition arises in the presence of spin anisotropy, from a Luttinger liquid with a small Fermi momentum to a Kondo-dimer phase with a large Fermi momentum. We have also added a frustrating next-nearest-neighbor interaction in the spin chain to show the possibility of a similar Fermi-momentum-changing transition, between the Kondo phase and a spin-Peierls phase, in the spin isotropic case. This transition, however, appears to involve a region in which the two phases coexist.

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I. INTRODUCTION

One of the key issues in the theory of heavy fermion materials is the size of the Fermi surface.

It is traditional to think that each spin-1/2 localized moment, through the Kondo effect, contributes a Kondo resonance. These Kondo resonances, which are fermionic, combine with the conduction electrons to form (very heavy) quasiparticles. In this way, the local moments ultimately become a part of the low-energy electronic fluid. This is reflected in the size of the Fermi surface: the Fermi momentum $k_F^*$ is “large” in the sense that the Fermi surface encloses a volume that counts the number of both conduction electrons and local moments. This is in contrast to a “small” Fermi momentum $k_F$, which would correspond to a Fermi surface that has a volume counting only the number of conduction electrons. While some questions have been occasionally raised, including the exhaustion problem noted by Nozières, this picture has been thought to apply everywhere in the heavy fermion phase diagram.

Recently, the question about the size of the Fermi surface has gained renewed importance in the light of the anomalous behavior observed in the vicinity of the antiferromagnetic quantum critical point (QCP). Particularly noteworthy are the experiments in CeCu$_6$Si$_2$, which observed non-Gaussian behavior. These experiments have inspired the development of locally critical quantum phase transitions and related theoretical pictures. In these pictures, the paramagnetic heavy fermion metal phase, as usual, contains fully developed Kondo resonances, with a large Fermi momentum $k_F^*$. It is argued that the Kondo effect can be destroyed as the system approaches a QCP and goes into an antiferromagnetic metal phase, by some competing processes: the local moments in such a lattice setting are also coupled to some fluctuating magnetic field, which impedes the Kondo effect. Once the Kondo effect is destroyed, the local moments are no longer a part of the electron fluid and the Fermi surface becomes small. Other theoretical works have proposed paramagnetic phases in which the Kondo effect is destroyed due to the fractionalization taking place in the localized-spin component. One of the important questions that these theoretical developments raised is whether quantum transitions between phases with very different Fermi momenta is necessarily first order or can be continuous. Only in the latter case can transitions between such phases serve as a mechanism for the non-Fermi liquid behavior observed in the heavy fermion metals.

In this paper we address this issue in models in one dimension, where they are amenable to controlled computation. We show that continuous transitions from small to large Fermi momenta can indeed take place in a one dimensional Kondo lattice model away from half-filling.

We now discuss a few additional background issues before we go into the details of our analysis.

A. Small and large Fermi momenta

We prefer to speak about small and large Fermi momenta as opposed to small and large Fermi volumes.

The situation in the paramagnetic phase is straightforward and is standard in the heavy fermion literature. Whenever the Kondo effect takes place, the localized spins are part of the low-energy electron fluid. The Fermi momentum in this case is the same as that when each localized spin is replaced by an electron orbital and, in addition, with all two-particle interactions turned off (i.e., a non-interacting Anderson lattice model). We call this Fermi momentum a large Fermi momentum. On the other hand, in the absence of the Kondo effect, localized spins — being charge neutral — are not part of the electron fluid. The Fermi momentum is that of the conduction electrons alone with all interactions turned off. This is defined as a small Fermi momentum. Clearly the Fermi volume (more precisely, Fermi length in one dimension and Fermi area in two dimensions) is different in the two cases. A transition between a large-Fermi-
momentum phase and a small-Fermi-momentum phase involves a jump in Fermi volumes. (This is distinct from other electronic topological transitions. For example, in the case of Lifshitz transitions in metals, the Fermi volume changes continuously, while the behavior of the density of states is anomalous.)

The situation is more subtle in an ordered phase that breaks translational invariance. This includes states with either an antiferromagnetic order or a spin-Peierls order. In the case of even-integer-fold breaking of discrete translational invariance of the Hamiltonian, the Fermi volumes (Fermi area in two dimensions and Fermi length in one dimension) in the reduced Brillouin zone would be the same regardless of whether the local moments are a part of the electron fluid or not.

However, for commensurate ordering, large Fermi momentum and small Fermi momentum can still be defined as follows. The key distinction is again whether local moments are a part of the electron fluid. We define a large Fermi momentum to be one that is associated with each localized spin being replaced by an electron orbital, all two-particle interactions turned off, but in the presence of an infinitesimal static field that characterizes the symmetry breaking. (In a commensurate antiferromagnetic metal phase, this field would be a static staggered magnetic field.) Likewise, we define a small Fermi momentum to be that associated with conduction electrons alone, with all two-particle interactions turned off, and in the presence of the same infinitesimal static field. In dimensions higher than one, the corresponding large Fermi surface and small Fermi surface have different topologies and, hence, represent distinct quantum phases. In one dimension, on the other hand, the only distinction between the two is the actual location of the Fermi momentum; the two phases may not necessarily be distinct if there are no additional order parameters that differentiate them.

B. The one-dimensional Kondo lattice

The system of interest consists of a one-dimensional electron gas (1DEG) coupled to a one-dimensional spin-1/2 chain via Kondo interaction. The spins in the spin chain interact with each other by nearest-neighbor (NN) and next-nearest-neighbor (NNN) interactions. The 1DEG is characterized by Fermi energy $\epsilon_F$ and Fermi momentum $k_F$. We take the lattice constants for both the conduction electrons and spin chains to be equal to $a$. Away from half-filling, $k_F \neq \pi/2a$ so that the 1DEG and the spin chain are incommensurate with each other.

The nature of Fermi momentum in various phases of such a one-dimensional Kondo lattice has been addressed in the past. That the Fermi momentum in the Kondo phase is indeed $k_F$ is supported by a generalized Luttinger’s theorem derived in a way analogous to that for the Lieb-Schultz-Mattis theorem, and by the numerical calculations. However, indications for phases with small Fermi momentum have been shown in more recent density matrix renormalization group calculations and the effects of spin dimerization in this context have been recently discussed.

In absence of Kondo interaction, the 1DEG and the spin chain are completely decoupled. In order to study the low-energy excitations, one can take the continuum limit so that 1DEG will be described as a Luttinger liquid.

For large NN interaction, the spin chain will also become a chargeless Luttinger liquid, with NNN interaction serving as a perturbation. When the perturbation becomes sufficiently large, the system undergoes a phase transition into the spin-Peierls phase. When the NNN interaction becomes much larger than the NN interaction, the spin-Peierls ordering persists, but it becomes more convenient to describe the system in terms of a “zigzag” model in which the odd spins and the even spins form two separate chargeless Luttinger liquids and NN interaction introduces a weak coupling between them.

The spin-Peierls phase is characterized by staggered bond order parameter, $(\tau_j \cdot \tau_{j+1}) \sim (-1)^j$. Thus, the spins form dimers, the translational symmetry is broken, and in certain limits one can even represent the ground state as a chain of singlets formed by nearest-neighbor spins. This phase has an exponentially small spin gap. The 1DEG remains a Luttinger liquid with Fermi momentum $k_F$.

Non-vanishing Kondo interaction introduces a number of novel phases in the system. However, we will be primarily interested in limit where the Kondo interaction is weak. In this case and for an incommensurate lattice, a relevant Kondo coupling leads to a phase in which the electrons and some of the spins form Kondo singlets $s_j \cdot \tau_j$, where $s_j \equiv \frac{1}{2} \psi_j^\dagger (x) \sigma \psi_j (x)$ and $\psi_j (x)$ are the annihilation and the creation operators in the 1DEG. The Kondo singlets are further dimerized so that the phase is characterized by an order parameter $\Phi_K$.

Due to the coupling to the charge sector, this order is associated with gapless charge-density modes $\mathcal{O}$ at $2k_F^+$,

$$\mathcal{O} (x_j) = s_j \cdot \tau_j,$$

so that the expectation value of this operator varies as $\langle \mathcal{O} \rangle \sim e^{i2k_F^+ x_j}$, where $k_F^+ = k_F + \pi/2a$ [a more detailed expression will be given in Eqs. (21-22) later in the text]. For these modes the charge and spin quantum numbers are both zero. Note that the charge-density modes are described by a local operator, however, due to the fact that they embed the Kondo singlets, the order parameter $\Phi_K$ becomes nonlocal. As a result, the translational symmetry of the system is not broken and the order is hidden. Furthermore, there is an exponentially small spin gap which exists due to the fact that the order parameter is nonlocal and one cannot distinguish paired spins from unpaired ones. The existence of the gapless charge-density modes at $2k_F^+$ implies that the system...
should be described as one with a large Fermi momentum.

We will argue in the following for a new co-existence phase. In the Kondo dimer phase the spins can further dimerize, producing a mixed phase in which the Kondo dimer order coexists with spin-Peierls order. This phase has both a broken translational symmetry and a large Fermi momentum.

In addition, we will show that, away from the spin-isotropic case, the decoupled Luttinger liquid phase has a finite range of stability. This phase is paramagnetic and has a small Fermi momentum $k_F$.

We now proceed to the quantitative analysis of the model, which appears to possess all the mentioned phases.

II. MODEL

The Hamiltonian of the system is

$$\mathcal{H} = \mathcal{H}_{1\text{DEG}} + \mathcal{H}_{\text{nnn}} + \mathcal{H}_{\text{nnn}} + \mathcal{H}_{\text{anis}} + \mathcal{H}_K,$$

where $\mathcal{H}_{1\text{DEG}}$ is the kinetic term of non-interacting one-dimensional electron gas (1DEG) with Fermi energy $\epsilon_F$, $\mathcal{H}_{\text{nnn}}$ is Heisenberg nearest-neighbor interaction between the spins in the one-dimensional 1/2-spin chain,

$$\mathcal{H}_{\text{nnn}} = J_1 \sum_j \mathbf{\tau}_j \cdot \mathbf{\tau}_{j+1},$$

(3)

$\mathcal{H}_{\text{nnn}}$ is Heisenberg next-nearest-neighbor interaction in the spin chain,

$$\mathcal{H}_{\text{nnn}} = J_2 \sum_j \mathbf{\tau}_j \cdot \mathbf{\tau}_{j+2},$$

(4)

$\mathcal{H}_{\text{anis}}$ introduces anisotropy in the spin chain, whose exact form is not essential provided that the spin chain by itself ($\mathcal{H}_{\text{nnn}} + \mathcal{H}_{\text{nnn}} + \mathcal{H}_{\text{anis}}$) is gapless. We can, for instance, take $\mathcal{H}_{\text{anis}} = -J_3 \tau_j \mathbf{\tau}_{j+1}$, with $0 < \lambda < 1$, which is an easy-plane ($XY$) anisotropy. [For a discussion of the role of anisotropy, see Appendix A] Finally, $\mathcal{H}_K$ is Kondo interaction between the 1DEG and the spin chain,

$$\mathcal{H}_K = J_K \sum_j \mathbf{s}(x_j) \cdot \mathbf{\tau}_j,$$

(5)

We will focus on the antiferromagnetic case ($J_K > 0$). As a reminder, here $\mathbf{\tau}_j$ are the spins in the spin chain and $s(x) = \frac{1}{2}\psi^\dagger(x) \sigma \psi(x)$ is the spin density of the electron gas at $x$. We decompose $\psi(x)$ into right- and left-moving components, $\psi_\sigma(x) = R_\sigma(x) e^{ik_F x} + L_\sigma(x) e^{-ik_F x}$, $\sigma = \uparrow, \downarrow$, so that the spin density becomes

$$s(x) = J_R^R + J_L^L + \mathbf{n}^s,$$

(6)

where $J_{R}^R = \frac{1}{2} R^\dagger R \sigma R$, $J_{L}^L = \frac{1}{2} L^\dagger L \sigma L$, and $\mathbf{n}^s = \frac{1}{2} R^\dagger R e^{-2ik_F x} + \text{h. c.}$

We will study this model in the limit $J_K, J_2 \ll J_1 \ll \epsilon_F$. Then we can take a continuum limit for the spin chain and describe it also as a one-dimensional free chargeless fermion system with Fermi energy of order of $J_1$. Hence, we can perform a similar decomposition of $\mathbf{\tau}_j$ into right- and left-directed currents $\mathbf{J}_R^j$ and $\mathbf{J}_L^j$, as well as the staggered spin $\mathbf{n}^j = (-1)^j \mathbf{n}^s$:

$$\mathbf{\tau}_j = \mathbf{J}_R^j + \mathbf{J}_L^j + \mathbf{n}_j^s.$$  

(7)

Likewise, in the limit $J_1 \ll J_2$, the continuum limit of the spin chain can be taken using a system of two mutually noninteracting fermion gases characterized by the Fermi energies of order of $J_2$ (the “zigzag” model), with $J_1$ serving as a perturbation.

For the single impurity Kondo problem and for the commensurate Kondo lattice problem, the most relevant part of Kondo interaction is $\mathbf{n}_j^s \cdot \mathbf{n}^j$. However, for the incommensurate case — which is what happens away from half-filling — this interaction becomes irrelevant due to the oscillating factors, and the only component that is not irrelevant is the forward-scattering one,

$$\mathcal{H}_K^f = J_K \sum_j (\mathbf{J}_R^j + \mathbf{J}_L^j) (\mathbf{J}_R^j + \mathbf{J}_L^j),$$

(8)

which is marginally relevant in the SU(2)-symmetric case.

Our next step is Abelian bosonization in which we represent the fermionic fields of 1DEG as

$$R_\sigma(x) = (F_\sigma/\sqrt{2\pi a}) \exp \{-i\sqrt{\pi} [\theta_\sigma - \phi_\sigma]\}$$

and

$$L_\sigma(x) = (F_\sigma/\sqrt{2\pi a}) \exp \{-i\sqrt{\pi} [\phi_\sigma + \phi_\sigma]\},$$

where $\phi_\sigma(x)$ are the bosonic fields, $\theta_\sigma(x) = \int_{-\infty}^{\infty} \Pi_{\sigma}(x') dx'$ are the fields dual to $\phi_\sigma$, $\Pi_{\sigma}(x)$ are the conjugate fields satisfying $[\phi_{\sigma}(x), \Pi_{\sigma}(x')] = -i \delta_{\sigma,\sigma} \sigma(x - x')$, and $F_\sigma$ are the Klein factors satisfying $\frac{1}{\lambda} F_{\sigma} F_{\sigma'} = F_{\sigma} F_{\sigma'} = 1$ and, for $\sigma \neq \sigma'$, $F_{\sigma} F_{\sigma'} = -F_{\sigma} F_{\sigma'}$ and $F_{\sigma} F_{\sigma'} = -F_{\sigma} F_{\sigma'}$. The bosonic fields are further re-expressed in terms of the spin fields $\phi^\sigma(x) = (\phi_\uparrow + \phi_\downarrow)/\sqrt{2}$ and charge fields $\phi^c(x) = (\phi_\uparrow + \phi_\downarrow)/\sqrt{2}$, with similar expressions for their duals $\theta^\sigma$ and $\theta^c$. The spin chain is bosonized in a similar manner and we will denote its spin fields as $\phi^s(x)$.

The charge sector of the model separates from the spin one. Since localized spins do not carry charge, only 1DEG contributes to this sector, which is described by a Gaussian model,

$$L_c = \frac{1}{2K_c v_c} \int dx \left[ (\partial_r \phi^c)^2 + v_c^2 (\partial_x \phi^c)^2 \right].$$

(9)

Therefore, we will focus on the spin sector. We will study the system in the general case when the spin chain can be anisotropic. Although we assume that the microscopic Kondo interaction is isotropic (we will comment on the more general picture later), it is necessary to keep track of its diagonal component $J_K^{f_2}$ separately from the orthogonal component $J_K^{f_1}$. As we will show, in the process of the renormalization-group (RG) flow, the
originally isotropic Kondo interaction usually becomes anisotropic and restores the isotropy only near the fixed point of the Kondo phase.

The resulting Lagrangian becomes

$$\mathcal{L}^{\text{spin}} = \mathcal{L}_0 + \mathcal{L}_{\text{SP}} + \mathcal{L}^{Lz}_K + \mathcal{L}^{L\perp}_K,$$  \hspace{1cm} (10)

where

$$\mathcal{L}_0 = \frac{1}{2Ks^2} \int dx \left[ (\partial_x \phi^s)^2 + v_s^2 (\partial_x \phi^s)^2 \right] + \frac{1}{2Kv_{\tau}} \int dx \left[ (\partial_x \phi^\tau)^2 + v_{\tau}^2 (\partial_x \phi^\tau)^2 \right], \hspace{1cm} (11a)$$

$$\mathcal{L}_{\text{SP}} = \frac{J_{SP}}{(\pi a)^2} \int dx \cos \left( \sqrt{8\pi} \phi^\tau \right), \hspace{1cm} (11b)$$

$$\mathcal{L}^{Lz}_K = \frac{J^L_k}{2m} \int dx \phi^s \partial_x \phi^s \partial_x \phi^\tau, \hspace{1cm} (11c)$$

$$\mathcal{L}^{L\perp}_K = \frac{2J^L_k}{(\pi a)^2} \int dx \cos \left( \sqrt{2\pi} \phi^s \right) \cos \left( \sqrt{2\pi} \phi^\tau \right) \times \cos \left[ \sqrt{2\pi} \left( \theta^s - \theta^\tau \right) \right], \hspace{1cm} (11d)$$

$v_s$ and $v_{\tau}$ are the Fermi velocities of the 1DEG and the spin chain, respectively, $a$ is the lattice constant, and $J_{SP}$ is the backscattering interaction, which favors spin-Peierls phase when it is positive. For the spin-isotropic case (when $\lambda = 0$), $v_{\tau} = \pi J_1 / 2$, $J_{SP} = f (J_2 - J_3) \approx c_1 (J_2 - J_3)$, where $c_1 \approx 1.72 \pi^2$ and $J_2 \approx 0.24 J_1/2$. Thus, we still assume $J_2 \ll J_1$. If we increase $J_2$ to the point when $J_K$, $J_1 \ll J_2$, it will be more appropriate to describe the system using the “zigzag” representation so that for both spin sub-chains $v_s \sim J_2$ and $J_{SP} \sim J_1$. In addition, $\lambda$ modifies the expressions for both $v_s$ and $J_{SP}$.

In general, the value of $K_\perp$ reflects the fact that the spin channel is anisotropic (we will assume that it has XY anisotropy) and will generalize to Ising anisotropy in Appendix A. This substantially affects the scaling dimensions of the terms in Eq. (10), in particular, the backscattering and transverse Kondo interactions are no longer marginal.

The Kondo part of this Lagrangian contains a term $\cos \sqrt{2\pi} (\phi^s - \phi^\tau) \cos \sqrt{2\pi} (\theta^s - \theta^\tau)$, which is strongly irrelevant. Thus, we can omit it in $\mathcal{L}^{L\perp}_K$, leading to:

$$\mathcal{L}^{L\perp}_K = \frac{J^L_k}{(\pi a)^2} \int dx \cos \left( \sqrt{2\pi} \phi^s \right) \cos \left( \sqrt{2\pi} \theta^\tau \right).$$ \hspace{1cm} (12)

This expression is quite remarkable. We observe that exchanging the bosonic fields with their duals $\phi^s \leftrightarrow \theta^\tau$, $\phi^\tau \leftrightarrow -\theta^s$ and simultaneously replacing $K_{s,\tau}$ with $1/K_{s,\tau}$ leaves the Lagrangian $\mathcal{L}_0 + \mathcal{L}^{L\perp}_K$ invariant, but induces changes in $\mathcal{L}^{Lz}_K$ and $\mathcal{L}_{\text{SP}}$. This should manifest itself not only in the RG equations, but also in the low-energy physics.

Although interaction $J^L_k$ alone is marginal and $J^{L\perp}_K$ alone is marginal or irrelevant, these constants effectively modify the scaling dimensions of each other so that they both can become relevant.

### III. RENORMIZATION-GROUP EQUATIONS

In order to treat the combined effects of the various interaction terms appearing in the model Eqs. (10), we carry out an RG analysis. We will use a Coulomb-gas expansion in which the interactions associated with the cosine term $\mathcal{L}^{L\perp}_K$ and $J_{SP}$ appear as fugacities, and those associated with the quadratic terms $\mathcal{L}^{Lz}_K$ and $K_{s,\tau}$ specify stiffness constants. The quadratic parts of the Lagrangian Eqs. (11a,11c) can be diagonalized.

The general aspects of the derivation of the RG equations are standard and will not be detailed here. What is nontrivial here, however, is the fact that we have two kinds of fugacities (those associated with $J^L_k$ and $J_{SP}$), and the way they are coupled with each other is the key to the determination of the phase diagram. Below, we document some of the details regarding this coupling.

The RG equations are found to be

$$\frac{dy_\perp}{dl} = \left( \frac{2 - K_s}{2} - \frac{K_\tau}{2} - \frac{1}{2K_s} - \frac{1}{2K_\tau} \right) y_\perp + u K_{s,\tau} y_\perp, \hspace{1cm} (13a)$$

$$\frac{dy_z}{dl} = u K_{s,\tau} y_z, \hspace{1cm} (13b)$$

$$\frac{dy_{SP}}{dl} = 2 (1 - K_\tau) y_{SP}, \hspace{1cm} (13c)$$

$$\frac{dK_{s,\tau}}{dl} = (1 - K_s^2) y_\perp^2 - 2K_{s,\tau}^2 y_{SP}^2, \hspace{1cm} (13d)$$

$$\frac{dK_s}{dl} = (1 - K_s^2) y_z^2, \hspace{1cm} (13e)$$

where $u = \sqrt{2\pi} v_s / (v_s + v_{\tau})$ and we have introduced the dimensionless coupling constants $y_\perp = J^L_k / (2\pi)^{3/2} v_s$, $y_z = J^L_k / (2\pi)^{3/2} v_s$, and $y_{SP} = J_{SP} / 4\pi^{3/2} v_{\tau}$. Note that the coefficients of the $y_{SP}$ term must preserve the invariance of the equations (for $y_{SP} = y_z = 0$) with respect to the change of variables $K_s \rightarrow 1/K_s$ and $K_\tau \rightarrow 1/K_\tau$ due to the symmetry of Eq. (12) discussed above. At the spin-chain isotropic point with $y_{SP} = 0$, the flow is along $K_s = K_\tau = 1$. Far away from this point the Kondo interaction is irrelevant and its flow is dominated by the first term in Eq. (13a).

There is no relevant correction due to $y_z$ or $y_\perp$ on the right-hand side of Eq. (13c). In the third order of perturbation we have found the following term there:

$$-2\pi^2 v_s^2 (v_s + v_{\tau}) K_s K_{s,\tau}^2 \frac{v_{\tau}}{(v_s + v_{\tau})^3} y_{SP}^2.$$ \hspace{1cm} (14)
A similar expression has been obtained for a different spin-Peierls order as well. The fact that this correction makes the backscattering interaction more irrelevant means that there is a competition between the Kondo-dimer and spin-Peierls phases and not a mutual enhancement. If there was enhancement (as in the case of melting in two dimensions), the two primary phases would be one with both kinds of order being present and one with no order. In our case, the mixed phase can only be intermediate, while the pure Kondo and spin-Peierls phases play major roles. Remarkably, in the second order of perturbation even the direct competition vanishes.

In absence of either the backscattering $J_{SP}$ or transverse Kondo $J_{K}^{\perp}$ interactions, the remaining $z$-component of Kondo interaction $J_{K}^{z}$ is strictly marginal. However, the theory is still Gaussian and, by using a linear transformation of fields $\phi^s$ and $\phi^z$, we have obtained (see Appendix B) an exact solution describing two decoupled liquids characterized by new spin wave velocities $v'_s$, $v'_r$, with corrections of order of $(J_{K}^{z})^2$ for $J_{K}^{z} / 2\pi \ll v'_s, v'_r$. Thus, we still have a Luttinger-liquid state with four “Fermi points” in the spin sector, two at $\pm k_F$ and two at $\pm \pi/2a$, which means that small $J_{K}^{z}$ does not change the behavior of the system qualitatively. In each of the decoupled liquids the particles are partially in the spin sector of the original electron liquid and partially in the original localized-spin liquid. Increasing $J_{K}^{z}$ enhances this mixing until one encounters a singularity, which could be a signature of a transition into the “Toulouse point phase” with Fermi momentum $k_F^{z}$ or the development of quasi-long-range spin-density-wave order.

There is a process that is not being taken into account in Eqs. (13). During the RG flow, a term $\partial_\tau \theta^s \partial_\tau \theta^s$ is generated. This term has a physical meaning of an interaction between the $z$-components of physical spin currents of the electrons and the localized moments. Its coupling constant (let us denote it $\bar{C}$) from the initial condition that the Kondo interaction is anisotropic as a result of $y$. As long as $x$, $y \ll 1$, this correction only negligibly affects the flow of $x$. However, in the Kondo phase all three coupling constants become approximately equal to each other and renormalize to infinity. Taking $y$ into account rigorously might affect certain numerical factors in the critical exponents, but it should not change the critical behavior qualitatively, therefore it has been neglected in Eqs. (13).

Let us now focus on the equations involving only Kondo interaction in presence of anisotropic spin chain (which also corresponds to the case when $y_{SP} = 0$ and $K_x = 1$). We aim at determining the separatrix which divides the region where the Kondo coupling ($y_{\perp}$) is irrelevant from the region where it is relevant. For small values of the bare Kondo coupling, we will see that the anisotropy $x = K_y - 1$ is small on the separatrix. For small $x$, the RG equations simplify to the following:

$$\frac{dy_{\perp}}{dl} = -\frac{1}{2} x^2 y_{\perp} + uy_{\perp} y_{z}, \quad (15a)$$

$$\frac{dy_{z}}{dl} = uy_{\perp}, \quad (15b)$$

$$\frac{dx}{dl} = -2xy_{\perp}. \quad (15c)$$

These equations have the following first integrals:

$$y_{\perp}^2 - y_{z}^2 + x^2 / 4 = C_1, \quad (16a)$$

$$xe^{y_{\perp}/a} = C_2. \quad (16b)$$

The physical meaning of the constant $C_1$ can be derived from the initial condition that the Kondo interaction is isotropic at the beginning of the flow, when $x = x_0$ and $y_{\perp} = y_{z} = y_0$, thus, $C_1 = x_0^2/4$. While $C_2$ can be understood as the value of $x = \bar{x}$ such that $y_2(\bar{x}) = 0$, it is only a formal definition, since $y_2$ never vanishes or changes sign for the mentioned initial conditions. Therefore, $C_2 = \bar{x}$ actually determines the value of $y_0$.

The equations Eq. (16) can be written in a more convenient form,

$$y_{\perp} = \pm \left( x^2 + \frac{x_0^2}{4} - \frac{x_0^2}{4} \right)^{1/2}, \quad (17a)$$

$$y_{z} = \frac{u}{2} \ln \frac{x}{\bar{x}}, \quad (17b)$$

which represents the lines of the flow of the RG charges $y_{\perp}$ and $y_{z}$ (Fig. 11). As we see, $dy_{\perp}/dl$ (and consequently, $dy_{\perp}/dx$) vanishes at $x = \bar{x}$, where

$$\bar{x} = \frac{u}{\sqrt{2}} \left( 1 + \frac{4x_0^2}{u^2} - 1 \right)^{1/2} \quad (18)$$

For $x_0 \ll u$, one can approximate $\bar{x} \simeq x_0 - x_0^3/2u^2$. There is also a flow line that separates the lines ending at $y_{\perp} = 0$ from the lines flowing towards $y_{\perp} = \infty$. This separatrix is determined by

$$\bar{x}_{sep} = \bar{x} \exp \left( \frac{1}{u} \sqrt{\frac{x_0^2}{u^2} - \bar{x}^2} \right) \quad (19)$$

which is $\bar{x}_{sep} \simeq x_0 + x_0^3/2u^2$ for $x_0 \ll u$.

It follows that in general, even if initially $y_{\perp} = y_{z} = y_0$, the Kondo interaction becomes anisotropic as a result of the RG flow such that $y_{\perp} < y_{z}$. In particular, at the line of Luttinger-liquid fixed points $\bar{x} = x, y_{\perp} = 0$, the diagonal component of Kondo interaction $y_{2}$ is finite. However, if the lines flow towards the fixed point of the Kondo phase $x = 0, y_{\perp} = \infty$, the isotropy is eventually restored, $y_{z} \rightarrow y_{\perp}$.

A nontrivial consequence of the presented calculation is that the flow along the separatrix ends at a point where not only Kondo interaction is anisotropic, but also
spin-chain $XY$ anisotropy is characterized by a nonzero value $\bar{x}$. This value is nonuniversal and is related to the strength of the $z$-component of the Kondo interaction at the point. These features are unique to our problem and have to do with the fact that both $y_z$ and $K_\tau$ affect the conditions for the development of the Kondo phase. However, since this transition involves the change of the behavior of one fugacity, we expect that the universal properties will be of the Kosterlitz-Thouless type. Indeed, by introducing $\epsilon = \frac{1}{2}x^2 - wy_z$, the RG equations (15) can be expressed in the standard Kosterlitz-Thouless form.

IV. PHASES AND FERMI MOMENTA

First, we consider the quantum phases as characterized by the fixed points of the RG flow.

A. Decoupled Luttinger liquid with small Fermi momentum

For spin-isotropic systems, the decoupled Luttinger liquid phase is unstable. The situation is drastically different when we go to the anisotropic case. For simplicity, we consider $K_s = 1$ corresponding to the case in which the 1D electron gas is non-interacting. If initially $y_K \ll y_{SP}$ and $K_\tau \gg 1$, the backscattering interaction $y_{SP}$ rapidly renormalizes to zero (Fig. 2). After that we only need to consider the flow of the $y_K$. For $K_\tau > 1$, there is a finite range of the Kondo interactions (both $J_{K}^{1z}$ and $J_{K}^{1\perp}$) over which $y_K$ renormalizes to zero.

The impedance to the Kondo effect here comes from the formation of a Luttinger liquid among the localized spins. The latter changes the scaling dimension of the Kondo interaction and, over an appropriate region of the interaction parameter space, renders the Kondo coupling irrelevant in the RG sense. While the details of this mechanism differ from those of either Ref. 11 (fluctuating magnetic field) or Refs. 13 (gapped spin liquid), the effect is similar: they lead to the destruction of the Kondo effect.

The only subtlety in the calculation of correlation functions in this phase is the $J_{K}^{1z}$-coupling, which is marginal. It mixes the spin degrees of freedom from the spin chain and from 1DEG. This mixing can be straightforwardly treated by introducing a new basis, leading to two decoupled spin branches with renormalized velocities. The details are given in Appendix B. The spin excitations are gapless at both $2k_F$ and $\pi/a$, which is due to the fact that $J_{K}^{1z}$ does not induce a magnetic order and, consequently, does not create new spin excitations.

The charge sector is completely independent of the $J_{K}^{1z}$-coupling, therefore, the number of particles in this sector does not change at all. Consequently, the Fermi momentum is not affected and the charge excitations are gapless only at $2k_F$. In addition, the single-electron excitations are gapless only at $k_F$: while $J_{K}^{1z}$ changes the shape of the spectral function, it does not induce gapless single-electron excitations at other wavevectors. In this sense, the Fermi momentum is small and is equal to $k_F$.

We should stress that there is no violation of the “generalized Luttinger theorem” as specified in Ref. 18. This theorem states that the twist operation ($U$) will introduce a gapless state, $U|gs\rangle$, whose momentum differs from that of the ground state $|gs\rangle$ by $2k_F + \pi/a$. In the decoupled Luttinger liquid, a part of the momentum ($2k_F$) measures the change to the 1DEG and the other part ($\pi/a$) measures the change to the spin chain.
B. The Kondo phase with large Fermi momentum

Another RG fixed point is given by $J_{K}^{\perp}=\infty$. As discussed earlier, this corresponds to the spin-gap phase with $\sqrt{2\pi}(\phi^{\sigma}+\phi^{\pi})=(2n_{1}+1)\pi$, $\sqrt{2\pi}(\theta^{\sigma}-\theta^{\pi})=2\pi n_{2}$, where $n_{1,2}$ are integers, or with $\sqrt{2\pi}(\phi^{\sigma}+\phi^{\pi})=2\pi n_{1}$, $\sqrt{2\pi}(\theta^{\sigma}-\theta^{\pi})=(2n_{2}+1)\pi$. This phase is precisely the state in which Kondo singlets $n_{\uparrow}^{\ast} \cdot n_{\downarrow}^{\ast}$ form an order parameter

$$\Phi_{K} = \left\{ \cos \sqrt{2\pi} (\theta^{\sigma}-\theta^{\pi}) - \cos \sqrt{2\pi} (\phi^{\sigma}+\phi^{\pi}) \right\}. \quad (20)$$

This order parameter is associated with the gapless charge-density mode Eq. (11) at $2k_{F}^{\perp}$:

$$O(x) \sim \Phi_{K} e^{i\sqrt{2\pi} \phi^{\pi}(x) + i2k_{F}^{\perp}x}. \quad (21)$$

In the left-hand side of Eq. (21), one can replace $O(x_{j})$ with $O(x_{j})O(x_{j+1})$, which follows from $J^{s+1}n^{s+1} \sim n^{s+2}$ and similar relations. Physically this new formula means that the spin sector of the Kondo phase contains the dimers of the Kondo singlets. However, the order parameter $\Phi_{K}$ also includes a contribution from the charge sector $exp(-i\sqrt{2\pi}\phi^{\pi})$.

Only relative values of the phase bosonic fields attain nonvanishing expectation values, while the fields themselves remain fluctuating. This picture does not depend on whether the original Kondo interaction was anisotropic or not, thus, Kondo dimer phase restores the isotropy of the Kondo interaction.

C. The spin-Peierls phase with a small Fermi momentum

Yet another RG fixed point is given by $y_{SP}=+\infty$ and $y_{K}=0$, corresponding to the spin-Peierls phase with $\sqrt{2\pi}\phi^{\pi}=\pi n$, where $n$ is an integer. Here, the Kondo effect is destroyed due to the spin gap induced by the spin-Peierls ordering in the spin chain. As a result, the Fermi momentum is $k_{F}$.

D. A coexisting Kondo and spin-Peierls phase

An interesting possibility is described by the RG fixed point with $y_{K}=y_{SP}=+\infty$. This is the regime in which the system is simultaneously in the Kondo-dimer and spin-Peierls phases. Then all fields $\phi^{\pi}, \theta^{\sigma}$ attain nonvanishing expectation values and the spectrum of spin excitations becomes gapped. There is still a gapless charge-density mode at $2k_{F}^{\perp}$ in this state, though.

Because of the broken translational symmetry, the coexisting phase and the purely spin-Peierls phase have the same Fermi volume (length). They are, however, distinct phases due to the absence (presence) of the Kondo-dimer order parameter and the spin gap in the spin-Peierls (coexistence) phase.

If we extend our study towards Ising anisotropy of the spin chain (see Appendix A), we will have to add Ising phase to the total picture as well. However, this phase is of little interest in the context of our problem.

V. QUANTUM PHASE TRANSITIONS

We now consider in some detail the transitions between the phases. For simplicity, we will again set $K_{s}=1$.

A. Transition from the decoupled Luttinger liquid to the Kondo phase

First, we will study the transition from the Luttinger-liquid phase to the Kondo-dimer one. For $y_{SP}<x$, $x=K_{s}-1$, the backscattering interaction $y_{SP}$ always renormalizes to zero. There is, however, a separatrix between the regions where $y_{\perp}$ is relevant and where it is irrelevant (Fig. 1). By substituting Eq. (14) into Eq. (12c), we find that the equation of the separatrix is

$$y_{K}^{\text{sep}} = \frac{u}{2} \ln \left( \frac{x}{2x} \right) + \frac{1}{2} \sqrt{x^{2}-x^{*2}}, \quad (22)$$

where $x^{*2}=x_{0}^{2}-y_{SP}^{2}$. Thus, the phase boundary corresponding to this transition is determined by the initial conditions for isotropic Kondo interaction $y_{\perp}=y_{z}=y_{K}$ of the form $y_{K}=y_{K}^{\text{sep}}(x_{0})$. An approximate equation of the phase boundary is $y_{K} \simeq x^{*2}/4u$ for $x^{*}\ll u$, in agreement with the result obtained at the end of Sec. III.

When Kondo interaction is stronger than $y_{K}^{\text{sep}}$, the flow of the constants $y_{\perp}$ and $y_{z}$ eventually becomes relevant and $x$ approaches zero. Suppose that initially $y_{K}=y_{K}^{\text{sep}}(x_{0})+t$, where $t \ll x^{*}$. Then the flow of $x$ is determined by the following differential equation:

$$\frac{dx}{dt} = \frac{x}{2} \left[ u^{2} \ln \left( \frac{x}{x_{0}} \right)^{2} + x^{2} - x^{*2} \right. \left. + 4u t \ln \left( \frac{x}{x_{0}} \right) \right]. \quad (23)$$

By expanding it about $x=\bar{x}$ and assuming that $x^{*}\ll u$, we derive

$$\frac{dx}{dt} = -\frac{2t}{u} x^{*3} - \frac{u^{2}}{x^{*2}} (x-\bar{x})^{2}. \quad (24)$$

The magnitude of the spin gap is determined by the value of the parameter $t$ when $y_{K}$ becomes of order of unity, $\Delta \sim \exp(-t^{1})$. Thus, we find that in the Kondo-dimer phase near the transition the spin gap is exponentially small,

$$\Delta_{K} \sim \exp \left[ -\frac{\pi}{x^{*} \sqrt{2u(y_{K}-y_{K}^{\text{sep}})}} \right]. \quad (25)$$
Therefore, the transition from Luttinger-liquid phase to the Kondo-dimer phase is continuous and, indeed, belongs to the Kosterlitz-Thouless type. However, the exponent is not universal due to the dependence on $x_0$. The formula above is correct only for $t = y_K - y_{K}^{\text{sep}} \ll x^*$ and it becomes invalid as $x^* \to 0$, where it is $\Delta_K \sim \exp(-1/ut)$.

Now consider the behavior of the correlation functions at the transition. We observe that the correlation functions $\langle s(x, \tau) \cdot s(0) \rangle$ and $\langle \tau(x, \tau) \cdot \tau(0) \rangle$ are nonuniversal. Indeed, the uniform part of the former decays as $r^{-\gamma_v}$ and the uniform part of the latter is $r^{-\gamma_r}$, where $r = x \pm v\tau$ and

$$\gamma_v = 2 + \pi \frac{v_s(v_s + 2v_\tau)}{(v_s + v_\tau)^2} K_r z^2, \quad (26a)$$

$$\gamma_r = K_r + \frac{1}{K_r} + \frac{\pi^2}{v_s(v_s + v_\tau)^2} \frac{K_r^2}{y_z^2} z^2 . \quad (26b)$$

Here $y_z = y_z(\bar{x})$ on the separatrix, $y_z \simeq x_0^2/2u$. The staggered part of the localized-spin susceptibility decays as $r^{-1/K_r}$, as it involves only $\theta_r$ and no $\phi_r$. The corresponding local spin susceptibility, $\chi_\omega(x, \omega = 0)$, is divergent. (The divergence at the QCP becomes logarithmic in the spin-isotropic limit, which is consistent with an extrapolation towards the QCP of the result obtained inside the Kondo-dimer phase of the spin-isotropic model using the form-factor technique \cite{37}.)

In order to construct a correlation function that is characterized by a universal critical exponent at the transition, we will introduce the following operator, defined in terms of right- and left-moving fields:

$$P = \sum_\sigma R_\sigma^+ L_\sigma + L_\sigma^+ R_\sigma + \text{h. c.} \quad (27)$$

The fields $R_\sigma$ and $L_\sigma$ are the spin pieces of the right- and left-moving fermions in the 1DEG, introduced earlier, so that $R_\sigma$, $L_\sigma \sim \exp \left[ \frac{\sigma i \sqrt{\pi/2}}{2} (\pm \phi^* \mp \theta^*) \right]$. This operator mixes the spinons of the spin chain and those of the conduction electron gas. While the spin does not flip in the process of such transformation, the momentum is not conserved and changes by $k_{FF}$. Judging by the representation of $P$ in the bosonic fields, one could formally identify it with the square root of the Kondo singlet, $P \sim (s_+ + \tau_z)^{1/2}$. The susceptibility associated with $P$, $\chi_P(x, \tau) = \langle P(x, \tau) P(0) \rangle$, decays as a power law $r^{-1-\epsilon/2}$, therefore, the corresponding local susceptibility at the transition (where $\epsilon = 0$) has universal behavior

$$\chi_\omega(\omega; x = 0) \sim \ln \left( \frac{i\Lambda}{\omega} \right) , \quad \omega \to 0 , \quad (28)$$

where $\Lambda$ is a cutoff.

B. Transition between the Kondo, spin-Peierls, and coexistence phases

If we start at the separatrix Eq. (22) and begin to decrease $K_r$, we will get deeper and deeper inside the Kondo phase. By iterating the RG equations, we find that there are two additional regions of the phase diagram.

For $K_r$ sufficiently smaller than the separatrix value, we find that $y_\perp$ renormalizes to zero while $y_{SP}$ renormalizes to larger and larger values. This yields the pure spin-Peierls phase discussed in Sec. IV C.

Between the Kondo phase and the spin-Peierls phase, we find a finite region of parameters over which both $y_\parallel$ and $y_{SP}$ renormalize to larger values and reach order unity simultaneously. We interpret this as meaning that we enter a state in which the Kondo dimers coexist with spin-Peierls phase.

The exact locations of the corresponding phase boundaries are impossible to determine from the RG equations, since the latter become invalid when $y_\perp \sim 1$ or $y_{SP} \sim 1$. However, we can determine an approximate location of the coexistence phase from the condition that both $y_\parallel$ and $y_{SP}$ reach $O(1)$ at certain finite value of $l$. This line has the following shape for finite $y_K$:

$$y_{SP} = x + \left( \frac{\pi}{2} y_K \right)^2 , \quad x > 0 ,$$

$$\sim \exp \left( \frac{2}{uy_K} |x| \right) , \quad x_c < x < 0 ,$$

$$\sim \exp \left[ - \left( \frac{2\pi^2}{u(y_K - x_c^2/4u)} \right)^{1/2} \right] , \quad x \to x_c ,$$

where $x_c = -2\sqrt{uy_K}$. The dependence of the spin-gap in the spin-Peierls phase near $x_c$ is $\Delta_{SP} \sim \frac{1}{4} y_{SP} \sqrt{\text{m}_{SP}}$.

The same line for finite $y_{SP}$ ends at the point $x = y_{SP}$, at which all three phase boundaries merge and which separates spin-Peierls phase from Luttinger liquid when $y_K$ vanishes.

C. Phase diagram

These results specify the phase diagram. It is hard to show the diagram in the full parameter space ($J_K$, $J_{SP}$, $K_r$), as it would require a three-dimensional graph. Instead, we plot two typical cross-sections, corresponding to the cut at a finite constant value of $J_{SP}$ (Fig. 3) and at a finite constant value of $J_K$ (Fig. 4).

One can also easily generalize these phase diagrams to the case when $J_{SP} < 0$ (see Appendix A). There is a complete symmetry with respect to the change of the sign of $J_{SP}$, except that one will have to replace spin-Peierls phase with Ising one on the phase diagrams.
FIG. 3: Phase diagram for fixed finite $J_{SP}$. The solid line shows the continuous transition and the shaded area shows the possible location of the coexistence region. The dashed line has been defined in Eq. (29), but the exact shape of the area is unknown yet. The lines merge at a single point. The brackets label the Fermi momentum of each phase.

FIG. 4: Phase diagram for fixed finite $J_K$. The lines and the shaded area have the same meaning as in Fig. 3.

VI. CONCLUSIONS

We have studied a one-dimensional Kondo lattice model that consists of a one-dimensional electron gas coupled to a spin-$1/2$ chain through Kondo interaction. The phase diagram is quite rich and contains phases with either a large Fermi momentum or a small Fermi momentum.

When both nearest-neighbor and a significant next-nearest-neighbor interactions are present in the isotropic spin chain and Kondo interaction is small, the system is in the spin-Peierls phase, which is characterized by broken translational symmetry due to the presence of spin dimers and by small Fermi momentum $k_F$ of conduction electrons.

Increasing the strength of Kondo coupling constant, we have found indications that the system enters a coexistence phase, containing both the dimers of Kondo singlets and the dimers of the spins in the spin chain. In this phase the translational symmetry is also broken, but the conduction electrons are part of the system with large Fermi momentum $k_F$. Additionally, the spin gap encompasses conduction electrons as well.

As we further increase Kondo interaction, the system enters a pure Kondo-dimer phase with no translational symmetry broken and with large Fermi momentum $k_F$. We were unable to determine the precise nature of the transitions either between the spin-Peierls phase and the coexistence phase or between the coexistence phase and the pure Kondo phase, due to the inherent limitations of the renormalization-group approach.

We have also studied the model in which the spin chain contains an XY anisotropy. For sufficiently weak Kondo interaction, the system is always in a Luttinger-liquid state, characterized by small Fermi momentum $k_F$ and no breaking of translational invariance. Increasing Kondo interaction triggers a continuous Kosterlitz-Thouless transition into the Kondo phase. We have calculated the dependence of the spin gap on Kondo interaction near the transition and have found that it is exponentially small. We have also identified a correlation function that logarithmically diverges at zero frequency at the transition.

The possibility of a continuous transition from small to large Fermi momenta looks puzzling at first sight. In our case, the weight of the Kondo resonance is characterized by the Kondo-dimer order parameter. The fact that the transition is continuous reflects the continuous onset of this weight at the transition. However, even infinitesimally small order parameter is still a macroscopic quantity. The electron count includes the localized spins as soon as the Kondo-dimer order parameter is developed, but does not include them as long as we are in the Luttinger liquid phase. A jump in the Fermi momentum then takes place at the transition point.

In dimensions higher than one, Fermi-momentum-changing transitions have been discussed in the literature. One such transition arises in an extended dynamical mean field treatment of the Kondo lattice model between a large-Fermi-momentum paramagnetic metal phase and a small-Fermi-momentum antiferromagnetic metal phase. Here the transition is continuous and is accompanied by a logarithmically divergent local spin susceptibility. The transition is locally quantum critical, in the sense that Kondo resonances are destroyed at the transition. A related Fermi-momentum-changing transition has been discussed in certain large-$N$ limit of frustrated Kondo lattice systems between a large-Fermi-momentum paramagnetic metal phase and a fractional-
ized small-Fermi-momentum paramagnetic phase. The asymptotically exact study we have carried out in the one-dimensional Kondo lattice model reveals a Fermi-momentum-changing transition that bears strong similarities to the local quantum criticality proposed earlier for Kondo lattice systems in dimensions higher than one.

We close by noting that some of the phases we have discussed for the purely one-dimensional Kondo lattice may be of direct experimental significance. For instance, the organic material $(\text{perylene})_2\text{[Pt(S}_2\text{C}_2\text{(CN)}_2)_2]}$ is believed to be a realization of a quasi-one-dimensional Kondo lattice\textsuperscript{34,35} the system also displays spin-Peierls ordering. It would be very interesting to study the Fermi-surface properties of this material, as well as the phase transitions of the system by tuning, say, pressure.

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**APPENDIX A: XY AND ISING ANISOTROPY.**

In our study of the Lagrangian Eq. (10), we were assuming that the spin chain had $XY$ anisotropy. In this appendix we will generalize our results for Ising anisotropy as well.

First of all, let us determine the boundary between the regions with $XY$ and Ising anisotropy, i.e., the area with $SU(2)$ symmetry. In the full phase diagram in the space of $J_{SP}$, $J_K$, and $K_\tau$ this area would be a surface. However, if we look at the cross-section for fixed $J_K$, this area reduces to a line (Fig. 5).

For $J_K = 0$, this line $\alpha$ actually consists of two pieces. One $\alpha_1$ that separates regions $III_a$ and $IV_b$ has a stable fixed point at the origin, corresponding to the isotropic Luttinger liquid. One $\alpha_2$ that separates regions $I$ and $II_a$, has a stable fixed point at infinity, corresponding to spin-Peierls phase. The regions $I$ and $II$ (both with $J_{SP} > 0$) flow towards spin-Peierls fixed point, the region $III$ flows towards Ising fixed point on the line $\gamma_2$ at infinity, which corresponds to Néel order with broken Ising symmetry, and the region $IV$ flows towards the line of Luttinger-liquid fixed points $\beta_2$. This is a standard Kosterlitz-Thouless picture\textsuperscript{34,35}.

As $J_K \to \infty$, the line with $SU(2)$ symmetry rotates so that it overlaps with $J_{SP}$ axis. Now in all regions the flow is towards the line of Kondo-phase fixed points along $J_{SP}$ axis. For finite values of $J_K$, spin-Peierls and Kondo fixed points still possess $SU(2)$ symmetry, while Luttinger-liquid and Ising fixed points break it.

The continuous transition from the Luttinger-liquid to the Kondo phase is always in the $XY$ region and it exists for any sign of $J_{SP}$, but only for $K_\tau > 1$, which is why most of the calculations in this paper have been performed assuming $XY$ anisotropy. The line segment $\gamma_1$ that was separating Luttinger liquid from spin-Peierls phase for $J_K = 0$ now splits into two boundaries that surround the Kondo phase (Fig. 5).

As for the transition from Kondo to spin-Peierls phase, its study is more complicated due to the mentioned possibility of the coexistence region. However, it is clear that the finite $J_K$ makes this transition (or the coexistence region) extend towards a finite range within region $I$ as well. We have determined the location of the coexistence region for Ising anisotropy with $J_{SP} > 0$ and $K_\tau < 1$, which is reflected in Eq. (26).

What will happen for the remaining region with Ising anisotropy ($III$)? Let us observe that the RG equations Eq. (13) are invariant with respect to the change of the sign of $J_{SP}$. This means that both of our phase diagrams Figs. 3 and 4 should remain the same, except that now $J_{SP}$ will need to be replaced with $-J_{SP}$ and Ising order will take place of spin-Peierls one. In the coexistence region (now for Kondo and Ising phases) the electrons not only will become spin-gapped, but also will break $SU(2)$ invariance so that Ising order will effectively expand over the entire system.

![FIG. 5: XY and Ising anisotropy (cross-section at $J_K = 0$). The line with $SU(2)$ symmetry is $\alpha$, the regions $II$ and $IV$ have XY anisotropy, and the regions $I$ and $III$ have Ising anisotropy. For $J_K \to \infty$, the line with $SU(2)$ symmetry becomes $J_{SP}$ axis.](image-url)
APPENDIX B: SMALL FERMI MOMENTUM IN
THE DECOUPLED LUTTINGER LIQUID

In this appendix, we consider the decoupled Luttinger liquid phase in which the transverse component of the Kondo coupling ($J_{Kz}^z$) is irrelevant. We wish to establish that the Fermi momentum is $k_F$. This statement is obviously true if the longitudinal component of the Kondo coupling ($J_{Kz}^\perp$) is absent, in which case the conduction electrons would be completely decoupled from the spin chains when we reach the fixed points. Below, we show that it remains valid even for a finite $J_{Kz}^z$-coupling.

The $J_{Kz}^z$ term describes the forward-scattering interaction between the $z$-components of the spins of the conduction electrons and those of the spin-chain [cf. Eq. (113)]. It is marginal in the RG sense when $y^\perp = 0$ [cf. Eq. (135)]. In addition, both $K_\tau$ and $K_s$ are also marginal since, in the decoupled Luttinger liquid phase, $y_{SP} = 0$ as well. We can then introduce the following diagonalization (mentioned already in Section III):

\[
\begin{align*}
\phi'_s &= a_s \phi_s + b_s \phi_\tau, \\
\phi'_\tau &= a_\tau \phi_s + b_\tau \phi_\tau, \\
v'_s &= c_s v_s + d_s v_\tau, \\
v'_\tau &= c_\tau v_s + d_\tau v_\tau.
\end{align*}
\]

(B1a) (B1b) (B1c) (B1d)

The transformation coefficients can be straightforwardly derived for arbitrary values of $J_{Kz}^z$. For simplicity, however, we will write down the expressions that are valid only up to the second order in $J_{Kz}^z$:

\[
\begin{align*}
a_s &= 1 - \frac{v_r}{4v_s (v_s^2 - v_\tau^2)} K_s K_\tau \left( \frac{J_\tau}{2\pi} \right)^2, \\
b_s &= \frac{v_s K_\tau}{v_s - v_\tau} \left( \frac{J_\tau}{2\pi} \right), \\
c_s &= 1 + \frac{v_s v_\tau}{(v_s^2 - v_\tau^2)} K_s K_\tau \left( \frac{J_\tau}{2\pi} \right)^2, \\
d_s &= -\frac{v_s^2 + v_\tau^2}{2 (v_s^2 - v_\tau^2)} K_s K_\tau \left( \frac{J_\tau}{2\pi} \right)^2.
\end{align*}
\]

(B2a) (B2b) (B2c) (B2d)

The expressions for $a_\tau$, $b_\tau$, $c_\tau$, and $d_\tau$ have the same forms, except that the subscripts $s$ and $\tau$ are exchanged.

The single electron Green’s function, $G_\sigma(x,t) \equiv -\langle T_{c_\sigma \dagger}(x,t) c_\sigma(0,0) \rangle$, can now be calculated using the bosonization form: $c_\sigma(x) = \sum_{r=\pm} c_{r\sigma}(x)$, where $r = \pm$ labels the right/left moving parts and

\[
c_{r\sigma}(x) = \frac{1}{\sqrt{2\pi a}} e^{ik_F x} F_{r\sigma} \exp \{-ir\Phi_{r\sigma}(x)\}.
\]

(B3)

Here, $\Phi_{r\sigma}(x) \equiv \sqrt{\pi} [\phi_\sigma + r\theta_\sigma]$. For definiteness, we consider the Green’s function of a right-moving electron, $G_{+\sigma}(x,t) \equiv -\langle T_{c_\sigma \dagger}(x,t) c_\sigma(0,0) \rangle$. Using $\Phi_{+\sigma} = (1/\sqrt{2})\Phi_{c,+} + (\sigma/\sqrt{2})\Phi_{s,+}$ and the properties of the Klein factors described in the main text, we can see that $G_{+\sigma}(x,t)$ factorizes into a charge part and a spin part,

\[
\begin{align*}
G_{+\sigma}(x,t) &= -\frac{1}{2\pi a} e^{ik_F x} G_{c,+}(x,t) G_{s,+\sigma}(x,t), \\
G_{c,+}(x,t) &= \left\langle T_t \exp \left\{ -\frac{i}{\sqrt{2}} \Phi_{c,+}(x,t) \right\} \right. \\
&\times \exp \left\{ \frac{i}{\sqrt{2}} \Phi_{c,+}(0,0) \right\}, \\
G_{s,+\sigma}(x,t) &= \left\langle T_t \exp \left\{ -\frac{\sigma}{\sqrt{2}} \Phi_{s,+}(x,t) \right\} \right. \\
&\times \exp \left\{ \frac{\sigma}{\sqrt{2}} \Phi_{s,+}(0,0) \right\}.
\end{align*}
\]

(B4)

The charge part, $G_{c,+}(x,t)$, is determined by the 1DEG alone. Because of the $J_{Kz}^z$ coupling, the spin part, $G_{s,+\sigma}(x,t)$, does involve the spins of both the 1DEG and the spin-chain. This $J_{Kz}^z$ coupling, however, can be handled by going to the diagonalized basis $\phi'_s$ and $\phi'_\tau$ introduced in Eq. (B1). Since the diagonalization affects only the $q \sim 0$ spin component of the spin chain, going to the primed basis does not introduce any oscillatory factor in the spatial dependence of $G_{s,+\sigma}(x,t)$. As a result, the single-electron Green’s function $G_{+\sigma}(x,t)$ is a product of $e^{ik_F x}$ and two factors which decay algebraically in $x - v_c t$ and $x - v_\tau t$, respectively. The Fermi momentum, hence, is $k_F$.

The key to the above reasoning is that only the forward scattering component of the longitudinal Kondo interaction is marginal. All other components, including those that involve either the $q = \pm \pi$ mode of the spin chain or the $q = \pm 2k_F$ spin mode of the 1DEG [cf. Eqs. (B1)] are irrelevant. In particular, the irrelevant nature of the interactions involving the staggered moment of the spin chain leaves no room for the single electron Green’s function to contain any Fermi momentum other than $k_F$.

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40 As we increase $J_K$ beyond $\epsilon_F$ (keeping $J_1, J_2 \ll \epsilon_F$), we will reach a “Toulouse point” phase which still has a spin gap and which is characterized not only by the Kondo dimer order parameter but also with ordinary charge-density wave (with a gapless mode at $2k_F$).

41 Another quantity that can be dimerized is $\Delta^s \cdot n^s_j$, where $\Delta^s = iL^s (\sigma_2 \sigma_1) R^s$ is triplet superconductivity pairing; the associated gapless modes $\Phi_{KD} \exp [i\theta_c (x) + i\pi x/a]$ have momentum $\pi/a$.