Computer Graphic Analysis of Construction of Planar Dynamic Systems with Truncation Function

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Abstract. To analyze the characteristics of the truncation function. The paper made up some planar dynamic systems. They are constructed with truncation functions. There are many parameters in the functions. Monte Carlo search was used to search the parameters. The Lyapunov exponent was used to judge the characteristics of dynamical systems. We construct chaotic graphics in the cyclic windows. We can generate dozens of planar arrangement graphics of equidistant. Experimental results show the effectiveness of the proposed approach.

Keywords: Isometric Arrangement, Cyclic Windows, Truncation Function, Full Julia Set

1. Introduction

During the practice, periodic functions can already study many problems, but the continuity of the above functions is usually very poor. So it is necessary to find simple and smooth types of functions. The truncation function has the required characteristics. The theory to solve the problem is the truncation function analysis method. From a mathematical point of view, the Fourier transform is a special integral transform. It can give expression to the functions that meet certain conditions as linear combinations or integers of sine basis functions.

Chaos dynamics and fractal geometry are the frontiers of current scientific research. They are also an important part of nonlinear science. With the development of science and technology, especially the rapid progress of computer software and hardware technology, the computer graphics visualization research of the chaotic characteristics of the system has also been further developed. Chaos dynamics and fractal geometry have discovered new laws of computer graphics, studied new phenomena, and can generate a large number of beautiful and symmetrical graphics. The computers are used to study chaotic modes. The study has attracted the research interest of researchers in many fields such as mathematics, computers, and art design [1-2]. The research results obtained in these fields have made people more aware of the inseparable relationship between science and art. Research experience tells us that the emergence of fractal geometry can become the main catalyst for the development of art. Therefore, fractal geometry heralds another new development in art.
In the computer visualization research of nonlinear science, except the graphic research of various analytic mappings, people began to keep a watchful eye on the computer graphic research of non-analytic mappings. Clifford uses non-analytic mapping to construct 17 Isometric arrangement planar crystal groups. This paper analyzes the characteristics of the standard plane crystal group map, constructs a nonlinear system corresponding using different truncation functions, and generates a good deal of chaotic Isometric arrangement figures on the plane.

2. Truncation Function

2.1. Definition of Truncation Function
Let \( f(x) \) be a periodic function, and its period is \( 2p \). And it can be expanded into a trigonometric series with trigonometric functions:

\[
f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)
\]

Among them:

\[
a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx \quad (n = 0, 1, 2, 3, L )
\]

\[
b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx \quad (n = 1, 2, 3, L )
\]

If the integrals in formulas (2) and (3) exist, the coefficients \( a_0, a_1, b_1, a_2, b_2, L \) in formula (1) are determined by them. Equations (2) and (3) are called the Fourier coefficients of function (1). If they are substituted into formula (1), the resulting function is a trigonometric series with Fourier transform characteristics.

2.2. Iterative Mapping of Truncation Function
There is a \( P1 \) symmetrical dynamic system in the plane crystal group. It is the simplest plane power system on the Euclidean plane. The \( P1 \) dynamic system only has independent periodicity in the x and y directions. For the planar arrangement of the square grid, for each periodic window, the \( P1 \) dynamic system requires that the left and right boundaries and the upper and lower boundaries of each periodic window are continuous. Literature [2] constructed an iterative function with the characteristics of \( P1 \) symmetrical dynamic system. Combined with the iterative function constructed in [2], this paper uses different forms of truncation function to construct the following iterative mapping. The formula for iterative mapping is as follows:

\[
f_{A_i}(x, y) = (1, \cos x, \cos 2x, \cos 3x, \sin x, \sin 2x, \sin 3x)
\]

\[
\cdot A_i \cdot (1, \cos y, \cos 2y, \cos 3y, \sin y, \sin 2y, \sin 3y)^T \mod \left( \begin{array}{l} 2\pi \\ 2\pi \end{array} \right)
\]

Among them: \( A_n = (a_{jk})(n = 1, 2, 3) \). The subscript i represents the x direction of the truncation function; the subscript j represents the output coordinates of the truncation function; the subscript k represents the y direction of the truncation function. The mapping defined by formula (4) has periodic characteristics in the x and y directions, so the formula includes the result of the rest of the calculation result. And \( A_i \) is a \( 7 \times 2 \times 7 \) matrix of coefficients.
3. Decorative Pattern Constructed by Truncation Function

3.1. Graphical Truncation Function

First, literature [3] proposed the iterative function to randomly select dynamic parameters as the corresponding parameters of the system. Then select a point around the origin of the coordinates as the initial iteration point of the system. Calculate the corresponding Lyapunov exponent \(L\), record the dynamic parameters of the Lyapunov exponent \(L > 0\), and finally select the parameter structure of the dynamic system to draw the corresponding graph of the chaotic attractor using the computer's graphical function. However, in the actual operation of the program, we found that if the fixed initial iteration point is not within the attraction domain of the attractor corresponding to the dynamic system, no matter how many iterations it is, the graph corresponding to the chaotic attractor cannot be generated, so it is impossible to use the fixed point. To obtain the correct \(L\) value corresponding to the dynamic system brings great difficulties to the experiment. This paper adopts a new effective method to determine the initial iteration point. First, calculate the Jacobian matrix determinant of the corresponding dynamic system after randomly selecting the parameters. Then through calculation, the point where the value of the Jacobian matrix determinant is zero is used as the initial iteration point of the dynamic system [4-7]. Therefore, the method of judging the dynamic characteristics of the dynamic system in the literature [4-7] is adopted, and the parameters are randomly selected, And construct the complete Julia set of the dynamic system with \(L < 0\).

Formula (4) contains 98 parameters. The coefficient matrix \(A_i\) is represented by a vector \(\vec{a} = (a_1, a_2, \ldots, a_{98})^T\). We use the above method to study the dynamic system corresponding to formula (4), and use the computer to effectively form the corresponding graphical study of the dynamic system. First, we use the Monte Carlo search method to search for the parameters of the dynamic system corresponding to formula (4). Then calculate the value of the Jacobian matrix determinant corresponding to the dynamic system under the selected parameters. Set the point where the determinant of the Jacobian matrix is 0, that is, the extreme point as the initial iteration point. Then, recalculate the system after 5000 iterations. When \(L<0\), in 50 million iterations after the transient is removed by statistics, we plot the number of fills in the Julia set with the number of access points, which is represented by the 256×256 pixels of the limit point track. The equidistantly arranged Julia set pattern of the plane symmetric dynamic system of the truncation function is constructed by the translation of the pattern on the basic domain grid of the pixel matrix. The corresponding parameters of the power system of formula (4) are given in Table 1. Figure 1 shows the full Julia set diagram of the isometric arrangement of the plane-symmetrical dynamic system of the dynamic system corresponding to the parameters in Table 1.

3.2. Decorative Pattern Constructed by Truncation Function

We use architectural decoration to protect the main structure of the building, improve the physical properties of the building, use functions and beautify the building. We use decoration materials or accessories to perform various treatment processes on the internal and external surfaces and spaces of buildings. Decorative pattern refers to the design concept expressed by the designer in the form of a picture according to the purpose, object shape, pattern, material configuration and process treatment. The reasonable application of decorative patterns can bring unlimited vitality and artistic charm to the building and its environment, and give people the enjoyment of beauty [8-9].
Table 1. The dynamical system parameter of constructing fig.1

| i  | a_{11} | a_{12} | a_{13} | a_{14} | a_{15} | a_{16} | a_{17} | a_{21} | a_{22} | a_{23} | a_{24} | a_{25} | a_{26} | a_{27} |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1  | -0.25  | 0.025  | -0.175 | 0.15   | 0.025  | -0.025 | -0.075 | 0.175  | 0.15   | 0.10   | -0.1    | 0.175  | 0.15   | 0.10   |
| 2  | -0.10  | -0.25  | -0.225 | -0.075 | -0.20  | -0.175 | 0.225  | -0.05  | -0.20  | -0.25  | -0.25  | -0.075 | 0.00   | 0.025  |
| 3  | 0.05   | 0.05   | -0.175 | 0.075  | -0.025 | -0.075 | 0.225  | 0.05   | 0.125  | 0.15   | 0.00   | -0.1    | 0.175  | 0.10   |
| 4  | 0.225  | 0.20   | 0.00   | -0.20  | -0.025 | -0.15  | 0.175  | -0.15  | 0.125  | 0.15   | 0.225  | 0.225  | 0.05   | -0.075 |
| 5  | -0.125 | 0.15   | -0.25  | -0.075 | 0.225  | -0.075 | 0.225  | -0.25  | 0.20   | -0.125 | 0.125  | 0.025  | 0.075  | 0.10   |
| 6  | 0.15   | 0.10   | -0.025 | -0.15  | 0.10   | -0.025 | 0.025  | 0.20   | 0.10   | -0.20  | 0.025  | -0.075 | 0.10   | 0.05   |
| 7  | 0.025  | -0.075 | 0.175  | -0.15  | -0.05  | 0.15   | 0.00   | 0.225  | 0.125  | 0.10   | -0.175 | 0.075  | -0.025 | 0.225  |

Figure 1. The filled-in sets of the dynamical systems

4. Conclusion
In this paper, combining literature [2], using different combinations of truncation function to construct an iterative map. Study the dynamic characteristics of the plane symmetrical dynamic system corresponding to the iterative map, use the Monte Carlo search method to randomly search for parameters, and determine the dynamic characteristics of the dynamic system through the Lyapunov exponent, and draw the corresponding dynamic system of the iterative map. Chaotic fractal graphics in a periodic window. This paper proposes a method for constructing a symmetrical dynamic system with Isometric arrangement. The method proposed in this paper can construct a large number of chaotic patterns arranged in a square plane, thus providing a large number of novel and unique materials for architectural decoration patterns.

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