Optimal Search and Discovery∗

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Abstract

This paper studies a search problem where a consumer is initially aware of only a few products. At every point in time, the consumer then decides between searching among alternatives he is already aware of and discovering more products. I show that the optimal policy for this search and discovery problem is fully characterized by tractable reservation values. Moreover, I prove that a predetermined index fully specifies the purchase decision of a consumer following the optimal search policy. Finally, a comparison highlights differences to classical random and directed search.

1 Introduction

Consumers typically first need to search for product information before being able to compare alternatives. The resulting search frictions have received considerable attention in the literature.1 Under the rational choice paradigm, the analysis of such limited information settings relies on optimal search policies that describe how a consumer optimally searches among all available alternatives. I add to this literature by developing and solving a sequential search problem that introduces a novel aspect: limited awareness of available products.

To fix ideas, consider a consumer looking to buy a mobile phone. Through advertising or recommendations from friends, the consumer initially is aware of a single available phone and

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1For example, Stigler (1961); Diamond (1971); Burdett and Judd (1983); Anderson and Renault (1999); Kuksov (2006); Choi et al. (2018); Moraga-González et al. (2017a,b) study search frictions in equilibrium models and Hortaçsu and Syverson (2004); Hong and Shum (2006); De Los Santos et al. (2012); Bronnenberg et al. (2016); Chen and Yao (2017); Zhang et al. (2018); Jolivet and Turon (2019) study implications of search empirically.
has some (but not all) information on what it offers. Given this basic information, the consumer can directly gather more detailed information on this alternative, for example by reading a review online. Besides, there are also phones available that the consumer is initially not aware of. For these alternatives, he knows neither of their existence, nor the features they offer. This precludes the consumer from directly inspecting these phones. Instead, he first needs to discover and become aware of them, for example by getting more recommendations from friends or through a search intermediary. Figure 1 depicts a possible choice sequence for this case.

![Choice Sequence Diagram]

**Figure 1** – Example of a choice sequence in the search and discovery problem.

The “search and discovery problem” introduced in this paper formalizes a consumer’s dynamic decision process in this and similar settings. The resulting framework allows to study settings that are difficult to accommodate in existing search problems. In particular, neither random (e.g. McCall, 1970) nor directed search (e.g. Weitzman, 1979) is well suited to study settings where rational consumers remain oblivious to some, while obtaining only partial information on other products. However, such settings are common in practice. For example, online retailers and search intermediaries present an abundance of alternatives on product lists that reveal partial information only for some products. Consumers then decide between clicking on products already discovered on the list to reveal full information, and browsing further to discover more products. More generally, in markets with a large number of alternatives, consumers will remain unaware of many alternatives unless they actively set out to discover more products. Similarly, in markets where rapid technological innovations lead to a constant stream of newly available alternatives, few consumers are aware of new releases without exerting effort to remain informed.

The contribution of this paper is to show that despite its complexity, optimal search decisions and outcomes in the search and discovery problem remain tractable if the consumer has stationary beliefs. First, I prove that the optimal policy is fully characterized by reservation values similar to the well-known reservation prices derived by Weitzman (1979). In each period, a reservation value is assigned to each available action, and it is optimal to always choose the
action with the largest value. Each of the reservation values is independent of any other available action and can be calculated without having to consider expectations over a myriad of future periods. Hence, reservation values remain tractable. This allows to determine optimal search behavior under limited awareness without using numerical methods.

Second, I prove that the purchase of a consumer solving the search and discovery problem is equivalent to the same consumer having full information and directly choosing products from a predetermined index. This result generalizes the “eventual purchase theorem” derived independently by Choi et al. (2018), Armstrong (2017) and Kleinberg et al. (2017) to the case of limited awareness. Similar to the eventual purchase theorem, my generalization allows to derive a consumer’s expected payoff and market demand without having to consider a multitude of possible choice sequences that otherwise make aggregation difficult.

This paper also highlights several implications of limited awareness through a comparison of stopping decisions, expected payoffs and market demand with classical random and directed sequential search. A first implication of limited awareness is that it leads to two distinct search actions which posits a novel question: Do consumers benefit more from making it easier to discover more alternatives (e.g. through search intermediaries), or from facilitating inspection by more readily providing detailed product information? For the case where a consumer discovers one product at a time, I show that there exists a (possibly small) threshold for the number of alternatives after which the expected payoff increases more when facilitating discovery instead of facilitating inspection. This highlights the relative importance of discovery costs in settings with many alternatives.

Moreover, limited awareness generates distinct patterns in the resulting market demand. In directed search, more consumers preferring a product based on partial product information increases its market demand. This need not be the case with limited awareness; if consumers remain unaware of a product, its market demand does not increase as it becomes the preferred option. Whereas the same holds with random search, not being able to use partial information to decide whether to inspect a product induces consumers to stop earlier if total costs of revealing full product information remain the same.

The search and discovery problem also provides an intuitive rationalization of ranking effects commonly observed in click-stream data (e.g. Ursu, 2018): as consumers stop search before having discovered all products, products that would be discovered later are less likely to be bought. I show that these ranking effects are independent of the number of available alternatives.

Choi et al. (2018) introduced the name and noted that “Our eventual purchase theorem was anticipated by Armstrong and Vickers (2015) and has been independently discovered by Armstrong (2017) and Kleinberg et al. (2017).”
and decrease as more products are discovered. This mechanism offers a meaningful interpretation of how advertising that provides partial product information is beneficial for a seller;\(^3\) if a seller’s marketing efforts make more consumers aware of a product before search or increase the probability of the product being discovered early on, ranking effects directly imply that they will increase the demand.

Finally, this paper adds to the empirical search literature by discussing implications of limited awareness for the estimation of structural search models. Besides highlighting differences in parameter estimates and counterfactual predictions across the three models, I show that a directed search model will lead to accentuated search cost estimates due to not accounting for limited awareness when rationalizing stopping decisions.

The remainder of this paper is organized as follows. First, I discuss related literature. Section 3 introduces the search and discovery (henceforth SD) problem. Section 4 provides the optimal policy and discusses several extensions as well as limitations. In Section 5, I generalize the eventual purchase theorem of Choi et al. (2018) and use this to derive a consumer’s expected payoff as well as market demand. Section 6 compares search problems and discusses empirical implications. Section 7 concludes. Throughout, proofs are deferred to the appendix.

2 Related Literature

The search and discovery problem introduced in this paper nests both classical random and directed sequential search as special cases. In random search, a searcher has no prior information, searches randomly across alternatives and decides when to end search (e.g. McCall, 1970; Lippman and McCall, 1976). In directed search, the searcher is aware of all available alternatives and uses partial product information to determine an order in which to inspect products and when to end search (e.g. Weitzman, 1979; Chade and Smith, 2006). In contrast, in the search and discovery problem, the consumer is aware of only a few products. Hence, he not only decides in what order to inspect products and when to end search, but also when to try to discover more alternatives.

To prove the optimality I use results from the multi-armed bandit literature to first determine that a Gittins index policy is optimal,\(^4\) and then introduce a monotonicity condition to show that the Gittins index reduces to simple reservation values. Specifically, I use the results of Keller and Oldale (2003) who proved that a Gittins index policy is optimal in their branching

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\(^3\)This relates to the “informative view” of advertising. See e.g. Bagwell (2007) for a summary and comparison to the “persuasive view”.

\(^4\)Gittins et al. (2011) provide a textbook treatment of multi-armed bandit problems and the Gittins index policy. As purchasing a product ends search, search problems correspond to stoppable superprocesses as introduced by Glazebrook (1979).
bandits framework. This framework differs from the standard multi-armed bandit problem in that taking an action will reveal information on multiple other actions. However, as an action branches off into new actions and reveals information only on those, the state of other available actions is never altered. Hence, the important independence assumption continues to hold.

Similar monotonicity conditions also apply in other multi-armed bandit problems where they simplify the otherwise difficult calculation of the Gittins index values (see e.g. Section 2.11 in Gittins et al., 2011). The present case differs in that monotonicity is only required for the action of discovering more alternatives, but does not hold when inspecting a product. In a recent working paper, Fershtman and Pavan (2019) independently derived a similar characterization of the optimal policy when applying a monotonicity condition in a general multi-armed bandit problem where a decision maker also extends a set of alternatives.

Moreover, monotonicity conditions also lead to the results in the literature on (random) search problems where a searcher learns about the distribution from which he is sampling (Rothschild, 1974; Rosenfield and Shapiro, 1981; Bikhchandani and Sharma, 1996). These authors determine priors and learning rules that satisfy a similar condition based on which they can derive an optimal policy that is myopic. The SD problem differs in that not all information about a product is revealed when it is discovered such that it entails two distinct search actions. As I show, this makes it difficult to find similar priors or learning rules that would lead to a myopic optimal policy in extensions to the SD problem that incorporate learning.

Several other contributions extend Weitzman’s (1979) seminal search problem in different directions. Adam (2001) studies the case where the searcher updates beliefs about groups of alternatives during search and finds a similar reservation value policy to be optimal. Olszewski and Weber (2015) generalize Pandora’s rule to search problems where the final payoff depends on all the alternatives that have been inspected, not only the best one. Finally, Doval (2018) analyzes the optimal policy when a searcher can directly choose alternatives without first inspecting them.

This paper also relates to the recent literature studying problems where a consumer gradually reveals more information on products (Branco et al., 2012; Ke et al., 2016; Ke and Villas-Boas, 2019). These problems are formulated in continuous-time and generally do not admit an optimal policy based on an index. The SD problem differs in that it assumes that a consumer cannot purchase a product before having revealed full information. This makes available actions independent such that a tractable reservation value policy is optimal. Furthermore, the SD problem allows that multiple products can be discovered at a time such that with one action, information on multiple products is revealed. Though Ke et al. (2016) also consider correlated payoffs, discovering multiple products differs in that the correlation structure of payoffs changes
after the discovery; inspecting one does not reveal information about other products discovered at the same time.

The SD problem also subsumes decision processes considered in the growing empirical literature estimating structural search models (e.g. Honka, 2014; Chen and Yao, 2017; Ursu, 2018). Most closely related are De los Santos and Koulayev (2017) and Choi and Mela (2019), who also model consumers that decide between inspecting and revealing more products. This paper differs in that I provide a tractable optimal policy for the decision problem, whereas these studies use simplifying assumptions and numerical methods to solve their models. The results presented in this paper can serve as a justification for some of these simplifying assumptions: Given that the optimal policy is myopic, the one-step look-ahead approach adopted by De los Santos and Koulayev (2017) yields optimal choices of search actions if monotonicity holds. Moreover, the optimal policy in the SD problem implies that as long as the consumer has not yet revealed the last alternative, it will never be optimal to go back and inspect a product that was discovered earlier if beliefs are stationary. Hence, the simplifying assumption made in Choi and Mela (2019) where consumers cannot go back and inspect a product revealed previously does not affect the estimation as it would not be optimal to do so.

Honka et al. (2017) and Morozov (2019) also consider limited awareness and assume that consumers cannot inspect products they are not aware of. However, in their models consumers cannot discover products beyond those they are initially aware of and the underlying search problem then is equivalent to directed search. Koulayev (2014) estimates a search model where consumers also decide whether to reveal more products, but assumes that revealing a product shows all information on that product. Hence, there is no need for inspecting a product as considered in this paper.5

Finally, related studies have highlighted other potential biases in search cost estimates. Jindal and Aribarg (2020) show how heterogeneous prior beliefs can lead to an overestimation of search costs, Ursu (2018) argues that an incomplete search history also accentuates search cost estimates, whereas Yavorsky et al. (2020) discuss the effects of normalizing search benefits.

3 The Search and Discovery Problem

A risk-neutral consumer with unit demand faces a market offering a (possibly infinite)6 number of products gathered in set $J$. Alternatives are heterogeneous with respect to their characteristics.

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5 Koulayev (2014) solves the dynamic decision problem using numerical backwards induction. For the case where costs are increasing in time (which is the case in his results), the present results suggest that a simple index policy also characterizes the optimal policy for his model.

6 The problems with infinitely many arms in a multi-armed bandit problem discussed by Banks and Sundaram (1992) do not arise in the present setting.
The consumer has preferences over these characteristics which can be expressed in a utility ranking. To simplify exposition and facilitate a comparison to existing models from the consumer search literature (e.g. Armstrong, 2017; Choi et al., 2018), I assume that the consumer’s ex post utility when purchasing alternative $j$ is given by

$$u(x_j, y_j) = x_j + y_j$$

where $x_j$ and $y_j$ are valuations derived from two distinct sets of characteristics. Note, however, that the results presented continue to hold for more general specifications that do not rely on linear additive utility.\(^7\) An outside option of aborting search without a purchase offering $u_0$ is available.

The consumer has limited information on available alternatives. More specifically, in periods $t = 0, 1, \ldots$ the consumer knows both valuations $x_j$ and $y_j$ only for products in a consideration set $C_t \subseteq J$. For products in an awareness set $S_t \subseteq J$, the consumer only knows partial valuations $x_j$. This captures the notion that if the consumer is aware of a product, he has received some information on the total valuation of the product. Finally, the consumer has no information on any other product $j \notin J \setminus (S_t \cup C_t)$.

During search, the consumer gathers information by sequentially deciding which action to take starting from period $t = 0$. If the consumer decides to discover more products, $n_d$ alternatives are added to the awareness set. If less than $n_d$ alternatives have not yet been revealed, only the remaining alternatives are revealed. For each of the $n_d$ alternatives, the partial valuation $x_j$ is revealed. To reveal the remaining characteristics of a product $j$, summarized in $y_j$, the consumer has to inspect the product. This reveals full information on the product and moves it from the awareness into the consideration set. The latter implies $S_t \cap C_t = \emptyset$.

The order in which products are discovered is tracked by positions $h_j \in \{0, 1, \ldots \}$, where a smaller position indicates that a product is discovered earlier, and $h_j = 0$ implies either $j \in C_0$ or $j \in S_0$. Without loss of generality, it is assumed that products are discovered in increasing order of their index.\(^8\)

Two precedence constraints on the consumer’s actions are imposed. First, the consumer can only buy products from the consideration set. Second, the consumer can only inspect products from the awareness set. Whereas the first constraint is inherent in most search problems and

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\(^7\)Specifically, suppose that when the consumer becomes aware of alternative $j$, he reveals a signal on the distribution from which the utility of $j$ will be drawn. Appropriately defining the distribution of signals and the distribution of utilities conditional on these signals then yields an equivalent search problem.

\(^8\)Note that in equilibrium settings, the order may be determined by sellers’ actions, requiring a careful analysis of how these will determine the consumer’s beliefs. For example, in online settings it is common for sellers to bid on the position at which their product adverts are shown (see e.g. Athey and Ellison, 2011).
implies that a product cannot be bought before having obtained full information on it, the latter is novel to the proposed search problem. It implies that a product cannot be inspected unless the consumer is aware of it. In an online setting where a consumer browses through a list of products, this constraint holds naturally: Individual product pages are reached by clicking on the respective link on the list. Hence, unless a product has been revealed on the list, it cannot be clicked on. In other environments, this precedence constraint reflects that, unless a consumer knows whether an alternative exists, he will not be able to direct search efforts and inspect the specific alternative. For example, if a consumer is not aware of a newly released phone model, he will not be able to directly acquire detailed information before discovering it.

Given the setting and these constraints, the consumer decides sequentially between the following actions:

i) Purchasing any product from the consideration set $C_t$ and end search.

ii) Inspecting any product from the awareness set $S_t$, thus revealing $y_j$ for that product and adding it to the consideration set.

iii) Discovering $n_d$ additional products, thus revealing their partial valuations $x_j$ and adding them to the awareness set.

The distinction between inspectsing and discovering products is novel in the SD problem. The two actions differ in three important ways. First, whereas the consumer can use product-specific information to decide the order in which to inspect products from the awareness set, the decision whether to discover more products is based solely on beliefs over products that may be discovered. Second, if $n_d > 1$, discovering products reveals information on multiple products. Finally, discovering products adds them into the awareness set, whereas inspecting a product moves it into the consideration set. In combination with the precedence constraints this implies that the actions that are available in the next period differ.¹⁰

These actions are gathered in the set of available actions, $A_t = C_t \cup S_t \cup \{d\}$, where $d$ indicates discovery. If a consumer chooses an action $a = j \in C_t$, he buys product $j$, whereas if he chooses an action $a = j \in S_t$, he inspects product $j$. To clearly differentiate between the different types of actions, this set can also be written as $A_t = \{b_0, b_3, s_4, \ldots, d\}$, where $b_j$ indicates purchasing and $s_j$ inspecting product $j$.

Both inspecting a product and discovering more products is costly. Inspection and discovery costs are denoted by $c_s > 0$ and $c_d > 0$ respectively. These costs can be interpreted as the cost

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⁹ Doval (2018) is a notable exception.

¹⁰ Note that the latter two points also imply that products that the consumer is not aware of cannot be modeled as a set of ex ante homogeneous products that differ in terms of beliefs and associated costs from the products in the awareness set.
of mental effort necessary to evaluate the newly revealed information, or an opportunity cost of
the time spent evaluating the new information. In line with this interpretation, I assume that
there is free recall: Purchasing any of the products from the consideration set does not incur
costs, and \( c_s \) is the same for inspecting any of the products in the awareness set.

The consumer has beliefs over the products that he will discover, as well as the valuation
he will reveal when inspecting a product \( j \). In particular, \( x_j \) and \( y_j \) are independent (across \( j \))
realizations from random variables \( X \) and \( Y \), where the consumer has beliefs over their
joint distribution. This implies that the consumer believes that in expectation, products are
equivalent. A generalization where the distribution of \( X \) depends on index \( j \) is discussed in
Section 4. Note that throughout, capital letters are used for random variables, lower case letters
are used for the respective realizations and bold letters indicate vectors.

The consumer also has beliefs over the total number of available alternatives. I assume
that the consumer believes that with constant probability \( q \in [0,1] \), the next discovery will
be the last.\(^{11} \) As shown in the next section, the optimal policy is independent of the number
of remaining discoveries that may be available in the future. Note, however, that this belief
specification implicitly assumes that the consumer always knows whether he can reveal \( n_d \) more
alternatives. An extension presented in Section 4 covers the case where the consumer does not
know how many alternatives will be revealed.

All information the consumer has in period \( t \) is summarized in the information tuple \( \Omega_t =
(\bar{\Omega}, \omega_t) \). The tuple \( \bar{\Omega} = \langle u(x,y), n_d, c_d, c_s, G_X(x), F_{Y|X=x}(y), q \rangle \) represents the consumer’s
knowledge and beliefs on the setting. It contains the utility function, how many products
are discovered, and the different costs. It also contains the consumer’s beliefs summarized in
the probability \( q \) and the cumulative densities \( G_X(x) \) and \( F_{Y|X=x}(y) \). The latter specifies the
cumulative density of \( Y \), conditional on the realization of \( X \), which is observed by the consumer
before choosing to inspect a product. As a short-hand notation, I use \( G(x) \) and \( F(y) \) for these
distributions. As a regularity condition, it is assumed that both \( G(x) \) and \( F(y) \forall x \) have finite
mean and variance.

During search, the consumer reveals valuations \( x_j \) and \( y_j \) for the various products. This
information is tracked in the set \( \omega_t \), containing realizations \( x_j \) for \( j \in S_t \cup C_t \) and \( y_j \) for
\( j \in C_t \). The set of available actions \( A_t \) and the information tuple \( \Omega_t \) capture the state in \( t \).
The consumer’s initial information on the alternatives are captured in \( \omega_0 \) which will contain

\(^{11}\)Note that one can translate beliefs over a specific number of available alternatives to this probability by assuming
it varies during search. For example, if \( n_d = 1 \) and the consumer believes that there are 3 alternatives in
total, then \( q_t = 0 \) when the consumer has not yet discovered the second alternative and \( q_t = 1 \) otherwise. A
specification like this (and any specification where \( q_t \leq q_{t+1} \forall t \)) also satisfies the monotonicity condition (30)
presented in Appendix C. Consequently, if it is assumed that the consumer knows \( |J| \), monotonicity continues to hold.
(partial) valuations of products in the initial awareness and consideration set. Figure 2 shows their transitions starting from period \( t = 0 \). The depicted example assumes that there are only two alternatives available and that products are discovered one at a time. If the consumer initially chooses the outside option \( (b_0) \), no new information is revealed, and no further actions remain. If the consumer instead reveals the first alternative, he can inspect it in \( t = 1 \).

\[
\begin{align*}
\Omega_0 &= \langle \Omega, \{x_0, y_0\} \rangle \\
A_0 &= \{b_0, d\} \\
\Omega_1 &= \langle \Omega, \{x_0, y_0, x_1\} \rangle \\
A_1 &= \{b_0, s_1, d\} \\
\Omega_2 &= \langle \Omega, \{x_0, y_0, x_1, x_2\} \rangle \\
A_2 &= \{b_0, s_1, s_2\} \\
\end{align*}
\]

Figure 2 – Transition of state variables \( \Omega_t \) (information tuple) and \( A_t \) (set of available actions) for \( n_d = 1 \) and \(|J| = 2\).

### 3.1 The Consumer’s Dynamic Decision Problem

The setting above describes a dynamic Markov decision process, where the consumer’s choice of action determines the immediate rewards, as well as the state transitions. The state in \( t \) is given by \( \Omega_t \) and \( A_t \). As the valuations \( x_j \) and \( y_j \) can take on any (finite) real values, the state space in general is infinite.\(^\text{12}\) Time \( t \) itself is not included in the state; given \( A_t \) and \( \Omega_t \), it is irrelevant to the agent’s choice, because beliefs (over valuations and termination of discovery) are time invariant.

The consumer’s problem consists of finding a feasible sequential policy, which maximizes the expected payoff of the whole decision process. A feasible sequential policy selects an action \( a_t \in A_t \) given information in \( \Omega_t \) in each period \( t \). Let \( \Pi \) denote the set containing all feasible policies. Formally, the consumer solves the following dynamic programming problem

\[
\max_{\pi \in \Pi} V(\Omega_0, A_0; \pi) \tag{2}
\]

\(^\text{12}\)An exception is when \( x_j \) and \( y_j \) are drawn from discrete distributions, which limits the number of possible valuations that can be observed.
where \( V(\Omega_t, A_t; \pi) \) is the value function defined as the expected total payoff of following policy \( \pi \) starting from the state in \( t \). Let

\[
[B_a V](\Omega_t, A_t; \pi) = R(a) + \mathbb{E}_t [V(\Omega_{t+1}, A_{t+1}; \pi)|a]
\]

(3)

denote the Bellman operator, where the immediate rewards \( R(a) \) either are inspection costs, discovery costs, or the total valuation of a product \( j \) if it is bought. Immediate rewards \( R(a) \) therefore are known for all available actions. \( \mathbb{E}_t [V(\Omega_{t+1}, A_{t+1}; \pi)|a] \) denotes the expected total payoff over the whole future, conditional on policy \( \pi \) and having chosen action \( a \). The expectations operator integrates over the respective distributions of \( X \) and \( Y \). A purchase in \( t \) ends search such that \( A_{t+1} = \emptyset \) and \( \mathbb{E}_t [V(\Omega_{t+1}, \emptyset; \pi)|a] = 0 \) whenever \( a \in C_t \). The corresponding Bellman equation is given by

\[
V(\Omega_t, A_t; \pi) = \max_{a \in A_t} [B_a V](\Omega_t, A_t; \pi)
\]

(4)

4 Optimal Policy

The optimal policy for the SD problem is fully characterized by three reservation values. In what follows, I first define these reservation values, before stating the main result. At the end of this section, I discuss possible extensions based on a monotonicity condition, as well as limitations.

As in Weitzman (1979), suppose there is a hypothetical outside option offering utility \( z \). Furthermore, suppose the consumer faces the following comparison of actions: Immediately take the outside option, or inspect a product with known \( x_j \) and end search thereafter. In this decision, the consumer will choose to inspect alternative \( j \) whenever the following holds:

\[
Q_s(x_j, c_s, z) \equiv \mathbb{E}_Y [\max\{0, x_j + Y - z\}] - c_s \geq 0
\]

(5)

\( Q_s(x_j, c_s, z) \) defines the expected myopic net gain of inspecting product \( j \) over immediately taking the outside option. If the realization of \( Y \) is such that \( x_j + y_j \leq z \), the consumer takes the hypothetical outside option after inspecting \( j \) and the gain is zero. When \( x_j + y_j > z \), the gain over immediately taking the hypothetical outside option is \( x_j + y_j - z \). The expectation operator \( \mathbb{E}_Y [\cdot] \) integrates over these realizations.

The search value of product \( j \), denoted by \( z_s^j \), then is defined as the value offered by a

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13In this formulation of the problem, the consumer does not discount future payoffs. This is in line with the consumer search literature, which usually assumes a finite number of alternatives without discounting. However, it is straightforward to show that the results continue to hold if a discount factor \( \beta < 1 \) is introduced. In this case, the search and discovery values defined in the next section need to be adjusted accordingly.
hypothesized outside option that makes the consumer indifferent in the above decision problem. Formally, $z_j^s$ satisfies

$$Q_s(x_j, c_s, z_j^s) = 0 \quad (6)$$

which has a unique solution (see Lemma 1 in Adam, 2001). The search value can be calculated as

$$z_j^s = x_j + \xi \quad (7)$$

where $\xi$ solves $\int_0^\infty \left[ 1 - F(y) \right] dy - c_s = 0$ (see Appendix B).

The *purchase value* of product $j$, denoted by $z_j^b$, is defined as the utility obtained when buying product $j$:

$$z_j^b = u(x_j, y_j) \quad (8)$$

Based on reservation values given by (6) and (8), Weitzman (1979) showed that it cannot be optimal to inspect a product that does not offer the largest search value, or to stop when the largest remaining search value exceeds the largest purchase value. Hence, for given $S_t$ and $C_t$, it is optimal to always inspect and buy in decreasing order of search and purchase values. However, this rule does not fully characterize an optimal policy in the SD problem, as the consumer can additionally discover more alternatives.

For this additional action, a third reservation value based on a similar myopic comparison is introduced. Suppose the consumer faces the following comparison of actions: Take a hypothetical outside option offering $z$ immediately, or discover more products and then search among the newly revealed products. The consumer will choose the latter whenever the following holds:

$$Q_d(c_d, c_s, z) \equiv E_X \left[ V \left( \langle \bar{\Omega}, \omega(X, z) \rangle, \{b_0, s_1, \ldots, s_{n_d}; \tilde{\pi} \} \right) \right] - z - c_d \geq 0 \quad (9)$$

where $\omega(X, z) = \{z, x_1, \ldots, x_{n_d}\}$ denotes the information the consumer has after revealing the $n_d$ more alternatives and $\tilde{\pi}$ is the policy that optimally inspects the $n_d$ discovered products. Note that with some abuse of notation, product indices were adjusted to the reduced decision problem, such that $j = 0, 1, \ldots, n_d$ indicates the hypothetical outside option and the newly revealed products.

$Q_d(c_d, c_s, z)$ defines the *myopic* net gain of discovering more products and optimally searching among them over immediately taking the outside option. It is myopic in the sense that it ignores the option to continue searching beyond the products that are discovered. In particular, note that $V \left( \langle \bar{\Omega}, \omega(X, z) \rangle, \{b_0, s_1, \ldots, s_{n_d}; \tilde{\pi} \} \right)$ is the value function of having an outside option offering $z$ and optimally inspecting alternatives for which partial valuations in $X$ are known. Possible
future discoveries and any products in \( S_t \) or \( C_t \) are excluded from the set of available actions in this value function. This implies that the discovery value does not depend on the consumer’s beliefs over whether the next discovery will be the last. Finally, \( \mathbb{E}_X \) defines the expectation operator integrating over the joint distribution of the partial valuations in \( X \). Formal details on the calculation of the expectations and the value function are provided in Appendix B.

As for the search value, let the discovery value, denoted by \( z^d \), be defined as the value of the hypothetical outside option that makes the consumer indifferent in the above decision. Formally, \( z^d \) is such that

\[
Q_d(c_d, c_s, z^d) = 0
\]

(10)

which has a unique solution. In the case where \( Y \) is independent of \( X \), the discovery value can be calculated as

\[
z^d = \mu_X + \Xi(c_s, c_d)
\]

(11)

where \( \mu_X \) denotes the mean of \( X \) and \( \Xi(c_s, c_d) \) solves (10) for an alternative random variable \( \tilde{X} = X - \mu_X \). Further details for the calculation are provided in Appendix B.

Theorem 1 provides the first main result. It states that the optimal policy for the search problem reduces to three simple rules based on a comparison of the search, purchase and discovery values. In particular, the rules imply that in each period \( t \), it is optimal to take the action with the largest reservation value defined in (6), (8), and (10). Hence, despite being fully characterized by myopic comparisons to a hypothetical outside option, these reservation values rank the expected payoffs of actions over all future periods.

**Theorem 1.** Let \( \tilde{z}^b(t) = \max_{k \in C_t} u(x_k, y_k) \) and \( \tilde{z}^s(t) = \max_{k \in S_t} z^s_k \) denote the largest search and purchase values in period \( t \). An optimal policy for the search and discovery problem is characterized by the following three rules:

**Stopping rule:** Purchase \( j \in C_t \) and end search whenever \( z^b_j = \tilde{z}^b(t) \geq \max \{ \tilde{z}^s(t), z^d \} \).

**Inspection rule:** Inspect \( j \in S_t \) whenever \( z^s_j = \tilde{z}^s(t) \geq \max \{ \tilde{z}^b(t), z^d \} \).

**Discovery rule:** Discover more products whenever \( z^d \geq \max \{ \tilde{z}^b(t), \tilde{z}^s(t) \} \).

The proof of Theorem 1 relies on results from the literature on multi-armed bandit problems, specifically the branching bandits framework of Keller and Oldale (2003). These authors show that in a multi-armed bandit problem where taking an action branches off into new actions, a Gittins index policy is optimal. Importantly, as an action branches off, it cannot be taken again in its original state. This ensures that available actions are independent in the sense that taking one does not alter the state of any other available action. The imposed precedence constraints
combined with the fact that the consumer cannot discover a product for a second time imply the same branching structure in the SD problem, and the results of Keller and Oldale (2003) therefore imply that a Gittins index policy is optimal. Introducing a monotonicity condition I then show that the Gittins index is equivalent to the simple reservation values defined above.

Based on Theorem 1, optimal search behavior can be analyzed using only (6), (8) and (10). Weitzman (1979) showed that search values decrease in inspection costs and increase if larger realizations \( y_j \) become more likely through a shift in the probability mass of \( Y \). The same applies to the discovery value. It decreases in discovery costs and increases if probability mass of \( X \) is shifted towards larger values. The discovery value also depends on inspection costs and the conditional distribution of \( Y \) through the value function; it decreases in inspection costs and increases if larger values of \( Y \) are more likely.

To see the latter, consider the case where alternatives are discovered one at a time. In this case, the myopic net gain of discovering more products reduces to

\[
Q_d(c_d, c_s, z) = \mathbb{E}_X \left[ \max \left\{ 0, Q_s(X, c_s, z) \right\} \right] - c_d
\]

(12)

For any \( c'_s > c_s \), it holds that \( Q_s(x, c'_s, z) \leq Q_s(x, c_s, z) \) for all finite values of \( x \) and \( z \), implying that \( Q_d(c_d, c'_s, z) \leq Q_d(c_d, c_s, z) \) for all \( z \). As \( Q_d(c_d, c_s, z) \) is decreasing in \( z \) (see Appendix A), it follows that the respective discovery values satisfy \( z_d^{d'} \leq z_d^d \).

The optimal policy being fully characterized by simple rules leads to straightforward analysis of optimal choices for any given awareness and consideration sets. For example, consider a period \( t \) where \( \max \{ z_d^d, z^s(t) \} < z^b(t) \) such that the consumer stops searching. When decreasing inspection costs sufficiently in this case, the inequality reverts and the consumer will instead either first discover more products, or inspect the best product from the awareness set.

4.1 Monotonicity and Extensions

For the reservation value policy of Theorem 1 to be optimal, the discovery value needs to fully capture the expected net benefits of discovering more products, including the option value of being able to continue discovering products. The monotonicity condition used in the proof of the theorem ensures that this holds. It states that the expected net benefits of discovering more products do not increase during search. Hence, whenever the consumer is indifferent between taking the hypothetical outside option and discovering more products in \( t \), he will either continue to be indifferent or take the outside option in \( t + 1 \). Whether the consumer can continue to discover products in \( t + 1 \) thus does not affect expected net benefits in \( t \), and the
discovery value fully captures the expected net benefits.\textsuperscript{14}

In the baseline SD problem, several assumptions directly imply that the monotonicity condition holds. Specifically, (i) the consumer believes that product valuations are independent and identically distributed, (ii) \( q \) remains constant and (iii) \( n_d \) is known. However, these assumptions can be relaxed to capture a wider range of settings. Below, three related extensions are presented. Formal results and further details are presented in Appendix C.

**Ranking in distribution:** In some settings, the consumer’s beliefs are such that the distribution of partial valuations depends on the position at which a product is discovered. Monotonicity will be satisfied if beliefs are such that the mean of \( X_j \) decreases in a product’s position \( h_j \), or more generally if beliefs are such that \( X_j \) first-order stochastically dominates \( X_k \) if \( h_j \leq h_k \). The optimal policy then continues to be characterized by Theorem 1, the only difference being that the discovery value is based on the position-specific beliefs and decreases during search, making it optimal to recall products in some cases. This could result in a market environment where sellers of differentiated products compete in marketing efforts for consumers to become aware of their products early on. If sellers offering better valuations have a stronger incentive to be discovered first, they will increase marketing efforts.\textsuperscript{15} Consumers’ beliefs then will reflect this ordering such that monotonicity holds and the simple optimal policy can be used to characterize equilibria. Similarly, online stores often use algorithms to first present products that consumers may like more. This again satisfies monotonicity such that the tractable optimal policy can be used to rationalize search behavior in click-stream data from such stores.

**Unknown \( n_d \):** In other environments, a consumer may not know how many alternatives he will discover. For example, a consumer may believe that there are still alternatives he is not aware of and thus try to discover them, only to realize that he already is aware of all the available alternatives. In such cases, a belief over how many alternatives are going to be discovered needs to be specified. The reservation value policy continues to be optimal if these beliefs are such that monotonicity is satisfied. This will be the case if beliefs are constant, or if (more realistically) the consumer expects to discover fewer alternatives the more alternatives he already has discovered.\textsuperscript{16} The only difference to the baseline is that in \( Q_d(c_d, c_s, z) \), expectations are additionally based on beliefs over how many alternatives will be revealed.

\textsuperscript{14}For the search and purchase values, no monotonicity condition is required. This follows from the fact that in the independent comparison to the hypothetical outside option, both actions do not provide the option to continue searching. After buying a product, search ends, and after having inspected a product, the only option that remains is to either buy the product or choose the hypothetical outside option. Consequently, for inspection and purchase, at most one future period needs to be considered to fully capture the respective net benefits over immediately taking the outside option.

\textsuperscript{15}See, for example, the discussion on non-price advertising and the related references cited in Armstrong (2017).

\textsuperscript{16}This would reflect the case where the consumer expects it to become harder to discover alternatives the fewer alternatives have not yet been discovered. Alternatively, this could be modeled as either \( q \) or \( c_d \) to increase with each discovery, which also satisfies monotonicity.
**Multiple discovery technologies:** Consumers may also have multiple discovery technologies at their disposal. In an online setting, for example, each technology may represent a different online shop offering alternatives. Moreover, advertising measures may separate products into different product pools. In such settings, the consumer also decides which technology to use to discover more alternatives. By assigning each of the discovery technologies a different discovery value, the optimal policy can be adjusted to accommodate this case.\(^{17}\)

### 4.2 Limitations

Though the optimal policy applies to a broad class of search problems, two limitations exist. The first is that in the dynamic decision process, all available actions need to be independent of each other; performing one action in \(t\) should not affect the payoff of any other action that is available in \(t\). This is required to guarantee that the reservation values fully capture the effects of each action. Recall that each reservation value does not depend on the availability of other actions. If independence does not hold, however, the availability of other actions also influences the expected payoff of an action. Choosing actions based only on reservation values that disregard these effects therefore will not be optimal. Alternative search problems that violate this independence assumption are presented in the appendix.

The second limitation is that the monotonicity condition discussed above needs to hold for the discovery value to be based on *myopic* net benefits. If this condition does not hold, then the discovery value does not fully capture the expected net benefits of discovering more products. However, as long as independence of the available actions is satisfied, a Gittins index policy remains optimal (see proof of Theorem 1). Hence, the optimal policy when monotonicity fails consists of comparing the search and purchase values from equations (7) and (8) with the Gittins index value for discovery that explicitly accounts for future discoveries.

One interesting case where this fails is if the consumer learns about the distribution of \(X\) or the number of alternatives he will discover during search. So far, it was assumed that independent of the information the consumer reveals during search, his beliefs remain unchanged. This will be the case if either the consumer has rational expectations and hence knows the underlying distributions, or simply does not update beliefs. With learning, the consumer updates his beliefs based on partial valuations or number of products revealed in a discovery.

\(^{17}\)An interesting extension for future research is to model the case where a consumer can choose the order in which products are revealed based on a product characteristic such as price. This requires modeling beliefs that reflect this ordering through updating the support of the price distribution; in an ascending order the minimum price that can be discovered needs to increase with every discovery. Chen and Yao (2017) incorporate choices of such search refinements in their empirical model. However, in their model, a consumer simultaneously decides on the refinement and which position to inspect. In contrast, if such choices are modeled as a SD problem the consumer would sequentially decide between a discovery technology and whether to inspect a product. This is more closely done by De los Santos and Koulayev (2017), who also model sequential choice of search refinements and clicks, but use simplifying assumptions and do not derive the optimal policy.
Similar learning models have been studied in the context of classic search (and stopping) problems, where the consumer learns about the distribution he is sampling from (e.g. Rothschild, 1974; Rosenfield and Shapiro, 1981; Bikhchandani and Sharma, 1996; Adam, 2001). Whereas these studies determine prior beliefs or learning rules such that the optimal policy is based on myopic reservation values, similar conditions do not guarantee that monotonicity holds in a SD problem where a consumer learns about the distribution of $X$ or the number of alternatives he will discover. The reason is that in classic search problems a consumer reveals full information when inspecting a product. Hence, if a product turns out to be a good match, the value of stopping increases along with the value of continuing search, where the learning rule guarantees that this is such that the expected net benefits of continuing search over stopping with the current best option weakly decrease with each inspection. In contrast, in the SD problem, discovering either more or better partial valuations does not necessarily increase the value of the best option in the consideration set. For example, the consumer can discover many products that look very promising based on partial valuations, but after inspection realize that these products are a bad match after all. In this case, the value of stopping remains the same, whereas beliefs are shifted such that the consumer expects to find better or more products in future discoveries.

Extending the SD problem to the case where the consumer learns about the distribution of $X$ or the number of alternatives therefore comes at the cost of losing tractability of the discovery value; a tractable expression for the Gittins index value for the discovery action (henceforth denoted by $z_t^L$) is difficult to obtain as it is necessary to determine the value function of a dynamic decision process that includes many future periods. Moreover, whereas the discovery value in Theorem 1 remains constant throughout search, $z_t^L$ changes whenever the consumer updates beliefs. Consequently, the optimal policy when the consumer updates beliefs becomes more complex in that the discovery value changes with each discovery and explicitly includes future periods.

Whereas $z_t^L$ is not tractable and computationally expensive to obtain, it is possible to derive bounds on this value that are easier to compute and can serve as an approximation. First, $z_t^L$ can be approximated from below through $k$-step look-ahead values. The 1-step look-ahead value is defined by (10), where the expectation operator is adjusted to account for the consumer’s beliefs in $t$. As $k$ increases, more future discoveries are considered in (10), leading to a more precise approximation of $z_t^L$ up to the point where $z_t^L$ is calculated precisely. Second, a result

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18The SD problem is equivalent to these learning problems in the case where $c_s = 0$ and the consumer updates beliefs about the distribution of the random variable $X + Y$.

19See e.g. Theorem 1 in Rosenfield and Shapiro (1981).

20If the consumer learns about the distribution of $Y$ conditional on $X$, then discovering more alternatives with similar $X$ can increase the value of the best option. Analyzing this mechanism provides an interesting avenue for future research.
of Kohn and Shavell (1974) can be used to derive an upper bound. These authors show that the expected value of continuing search when the consumer fully resolves uncertainty on the underlying distributions in the next period exceeds the true continuation value in a classic search problem where a consumer samples from an unknown distribution. The same logic directly applies in the extension to the SD problem and the upper bound then can be computed using the results provided in the next section. A formal treatment of these bounds is provided in the appendix.

5 Eventual Purchases, Consumer’s Payoff, and Demand

In an environment where consumers sequentially inspect products, a consumer’s expected payoff and the market demand results from integrating over different possible choice sequences leading to eventual purchases. Conceptually, this poses a major challenge, as the number of possible choice sequences grows extremely fast in the number of available alternatives.\textsuperscript{21}

Theorem 2 allows to circumvent this difficulty. It states that the purchase outcome of a consumer solving the search problem is equivalent to a consumer directly buying a product that offers the highest effective value. Importantly, a product’s effective value does not depend on the various possible choice sequences leading to its purchase.

**Theorem 2.** Let

\[
w_j \equiv \begin{cases} 
    u_j & \text{if } u_j < z^d \text{ and } j \in C_0 \\
    \tilde{w}_j & \text{if } \tilde{w}_j < z^d \text{ or } j \in S_0 \\
    z^d + f(h_j) + \varepsilon \tilde{w}_j & \text{else}
\end{cases}
\]

be the effective value for product $j$ revealed on position $h_j$ where $\tilde{w}_j \equiv \min\{z^s_j, z^b_j\} = x_j + \min \{\xi, y_j\}$, $f(h_j)$ is a non-negative function and strictly decreasing in $h_j$ and $\varepsilon$ is an infinitesimal. The solution to the search and discovery problem with initial consideration set $C_0$ and awareness set $S_0$ leads to the eventual purchase of the product with the largest effective value.

This result is based on and generalizes the “eventual purchase theorem” of Choi et al. (2018) (and independently Armstrong, 2017; Kleinberg et al., 2017) to the case where the consumer has limited awareness. The value $\tilde{w}_j$ used in the theorem is equivalent to the effective value defined by Choi et al. (2018), and the proof follows the same logic; as a product (incl. out outside option) is always bought, the proof only needs to establish that the optimal policy never prescribes to buy a product that does not have the largest effective value.

\textsuperscript{21}For example, with only one alternative and an outside option, there are four possible choice sequences. With two alternatives, the number of possible choice sequences increases to 20, and with three alternatives, there are already more than 100 possible choice sequences.
The generalization to the case of limited awareness follows from the following implication of the optimal policy: Whenever both the inspection and the purchase value of a product in the awareness set exceed the discovery value, the consumer will buy the product and end search. Hence, when \( \tilde{w}_j \geq z^d \), the consumer never discovers products on positions beyond \( h_j \). This is captured in the effective values by the term \( z^d + f(h_j) \), which ranks alternatives based on when during search they are discovered, yielding a larger effective value if a product is discovered earlier. The infinitesimal in the last condition additionally is necessary to rank products that are revealed on the same position. Suppose we have \( \tilde{w}_j > \tilde{w}_k \geq z^d \) for two products discovered on the same position. Without the infinitesimal, the effective value would be \( w_j = w_k \), implying the consumer would be indifferent between buying either of the two products. This contrasts the optimal policy, which for \( \tilde{w}_j > \tilde{w}_k \) will never prescribe to buy \( k \) if both \( j \) and \( k \) are in the awareness set. If \( n_d = 1 \), the infinitesimal is not required.

The result continues to hold for extensions of the SD problem, as long as the discovery values are predetermined. The only difference then is that in the effective value of an alternative \( j \), the discovery value depends on the position at which \( j \) is revealed.

### 5.1 Expected Payoff

Based on these results, it is now possible to derive a simple characterization of a consumer’s expected payoff, as summarized in Proposition 1. In this expression, the expected payoff does not explicitly depend on inspection and discovery costs; they affect the expected payoff only through the discovery and search values. As the proof shows, this follows from the definition of these values, which relate expected payoffs and costs (as in Choi et al., 2018). Based on this characterization, it is only necessary to derive the distribution of the effective values without having to explicitly consider different choice sequences. Note also that as the effective value is adjusted, the expected payoff does not depend on the choice of function \( f(h) \) which ranks alternatives based on their position in the effective value.

**Proposition 1.** A consumer’s expected payoff in the SD problem is given by

\[
V(\Omega_0, A_0; \pi) = \mathbb{E}_{\tilde{W}} \left[ \max_{j \in J} \tilde{W}_j \right]
\]

where \( \mathbb{E}_{\tilde{W}} \left[ \cdot \right] \) integrates over the distribution of \( \tilde{W} = [\tilde{W}_0, \ldots, \tilde{W}_{|J|}]' \), with \( \tilde{w}_j \) being the effective value adjusted with \( \tilde{w}_j = u_j \forall j \in C_0, \tilde{w}_j = \hat{w}_j \forall j \in S_0 \), and \( f(h_j) = \varepsilon = 0 \forall h_j \). If \( |J| = \infty \), \( V(\Omega_0, A_0; \pi) = z^d \).

Whereas it is clear that making either inspection or discovery easier leads to an increase in the expected payoff, it is not obvious which of these two changes is more beneficial for a consumer.
For the case where $n_d = 1$, Proposition 2 shows that if the number of alternatives exceeds some threshold, then the consumer benefits more from facilitating the discovery of additional products.\footnote{Note that this threshold can be zero. For example, this is the case when $u_0 = 0$, $c_s = 0.1$ and $c_d = 0.1$, and the valuations are drawn from standard normal distributions.}

**Proposition 2.** If $n_d = 1$, there exists a threshold $n^*$ such that whenever $|J| > n^*$, a consumer benefits more from a decrease in discovery costs than a decrease in inspection costs. This threshold decreases in the value of the alternatives in the initial consideration and awareness set.

Whereas the proof is more involved, the intuition is that when there are only few alternatives available, the consumer is more likely to first discover all alternatives and then start inspecting alternatives. Hence in expectation, he pays the inspection costs relatively often and a reduction in inspection costs will be more beneficial. Similarly, when the value of the outside option is large, the consumer is likely to inspect fewer of the products he discovers, leading to relatively small benefits of a reduction in inspection costs.

For settings where $n_d > 1$, it becomes difficult to obtain similarly general results. In particular, for some distributions and $n_d$, it is possible that decreasing inspection costs increases the discovery value $z^d$ by more than decreasing the discovery costs by the same amount. In such cases, the consumer will benefit more from making inspection less costly. Nonetheless, the general intuition remains the same in such settings; a reduction in inspection costs is more beneficial, the more likely it is that the consumer inspects relatively many alternatives.

### 5.2 Market Demand

Using Theorem 2, it is straightforward to derive a market demand function when heterogeneous consumers optimally solve the SD problem. In particular, let the effective value $w_{ij}$ for each consumer $i$ be a realization of the random variable $W_j$ and gather the random variables in $W = [W_0, \ldots, W_{|J|}]'$. For a unit mass of consumers the market demand for a product $j$ then is given by

$$D_j = \mathbb{E}_h [\mathbb{P}_W (W_j \geq W_k \forall k \in J \setminus j)]$$

(13)

where the expectations operator $\mathbb{E}_h [\cdot]$ integrates over all permutations of the order in which products are discovered by a consumer.

As the effective value decreases in the position at which a product is discovered, (13) reveals that the demand for a product depends on the probability of each position at which it is displayed. Specifically, the demand for a product exhibits ranking effects; products that are more likely to be discovered early are more likely to be bought. As discussed in detail in the next section,
this follows from the structure of the SD problem. As search progresses, it becomes less likely that a consumer has not yet settled for an alternative; hence, fewer consumers become aware of products that would be revealed later, leading to a lower demand for such products.

6 Comparison of Search Problems

To highlight implications of limited awareness and how the SD problem differs from existing approaches, I compare it with the two classical sequential search problems; directed search as in Weitzman (1979) and random search as in McCall (1970). Both these search problems are nested within the SD problem. Directed search results if the consumer initially has full awareness (i.e. \( S_0 = J \)) such that the consumer knows all partial valuations prior to search and does not need to discover products. Random search results if discovering a product reveals full information on this product, hence the consumer always both inspects and discovers a product, precluding him to use partial product information to only inspect promising products.\(^{23}\)

For clarity, I focus the comparison on the case where products are discovered one at a time \((n_d = 1)\) and where the consumer initially only knows an outside option \((S_0 = \emptyset)\). Furthermore, valuations \(x_j\) and \(y_j\) are assumed to be realizations of mutually independent random variables \(X\) and \(Y\), where the consumer has rational expectations such that beliefs are correct. Assumptions specific to each search problem are described below.

**Search and Discovery (SD):** The consumer searches as described in Section 3, incurring inspection costs \(c_s\) and discovery costs \(c_d\). Without loss of generality, I assume that the consumer discovers products in increasing order of their index, making subscripts for position \(h\) and product \(j\) interchangeable.

**Random Search (RS):** When discovering a product \(j\), the consumer reveals both \(x_j\) and \(y_j\); hence does not have to pay a cost to inspect the product. Costs to reveal this information are given by \(c^{RS}\). In this case, the consumer optimally stops and buys product \(j\) if \(x_j + y_j \geq z^{RS}\).

The reservation value is given by \(z^{RS} = \mu_X + \mu_Y + \tilde{\xi}\), where \(\tilde{\xi}\) is the same as in (7) but defined over the joint distribution of demeaned \(X\) and \(Y\). Products are discovered in the same order as in SD. Furthermore, I assume \(u_0 < z^{RS}\) to ensure a non-trivial case.

**Directed Search (DS):** The consumer initially observes \(x_j \forall j\), based on which he chooses to search among alternatives following Weitzman’s (1979) reservation value policy. Costs to inspect product \(j\) are given by a function \(v_{DS}^{DS} = v_{DS}(c_s, h_j)\), where \(c_s\) are baseline costs that are adjusted for the position through a function \(v_{DS}: \mathbb{R}_+^2 \to \mathbb{R}_+\) which is assumed to be strictly

\(^{23}\)Directed search also results if discovery costs are zero such that the consumer first discovers all products and only then starts inspecting, whereas random search also results if inspection costs are zero and the consumer inspects any products he discovers.
increasing in a product’s position $h_j$. As costs vary across products, reservation values are given by $z_j^* = x_j + \xi_j$, where $\xi_j$ is the same as in (7) with product-specific inspection costs. The assumption on $v_{DS}(c_s, h_j)$ implies that $\xi_j$ decreases in $j$. I impose this functional form restriction as otherwise the DS problem does not generate similar patterns, as discussed in Section 6.2.

### 6.1 Stopping decisions

In search settings, consumers’ stopping decisions determine which products consumers consider and buy. Stopping decisions therefore shape how firms compete in prices, quality or for being discovered early during search. Hence, comparing stopping decisions across the different search problems provides important insights on how well existing approaches are able to capture the more general setting where consumers are not aware of all alternatives and use partial information to determine whether to inspect products.

In the SD problem, a consumer always stops search at a product $k$ whenever the product is both promising enough to be inspected and offers a large enough valuation to not make it worthwhile to continue discovering more products. Formally, this is given by the condition $x_k + \min\{y_k, \xi\} \geq z^d$. The probability that a consumer will stop searching before discovering product $j$ therefore is given by

$$\mathbb{P}_{X,Y}(X_k + \min\{Y_k, \xi\} \leq z^d \forall k < j) = 1 - \mathbb{P}_{X,Y}(X + \min\{Y, \xi\} \leq z^d)^{j-1}$$ (14)

Similarly, in the RS problem, a consumer will always stop search at a product $k$ whenever $x_k + y_k \geq z^{RS}$, hence the probability of stopping search before discovering product $j$ is given by

$$\mathbb{P}_{X,Y}(X_k + Y_k \leq z^{RS} \forall k < j) = 1 - \mathbb{P}_{X,Y}(X + Y \leq z^{RS})^{j-1}$$ (15)

In both search problems, a consumer may stop search before discovering a product $j$. Consequently, stopping decisions in the SD and the RS problem imply the same feature: Products that a consumer initially has no information on may never be discovered and bought, independent of how the consumer values them.

However, as the consumer has the option of not inspecting products with low partial valuations, stopping probabilities differ. In particular, in the case where the total cost to reveal all information about a product are the same, stopping probabilities are smaller in the SD problem. This is highlighted in Proposition 3 and follows from the fact that not having to inspect alternatives with small partial valuations allows to save on inspection costs. This increases the expected benefit of discovering more products, which implies a smaller probability of search stopping, and that on average, more products will be discovered in the SD problem.
Proposition 3. If costs in the RS problem are given by $c^{RS} = c_s + c_d$, a consumer on average ends search at earlier positions in the RS than in the SD problem.

In contrast, stopping decisions are different in the DS problem. As the consumer initially knows of the existence of all products and can order them based on partial information, there is no stopping decision in terms of discovering products. Instead, the consumer directly compares all partial valuations and the different inspection costs, based on which he decides the order in which to inspect products. Hence, he can directly inspect highly valued products even when they are presented at the last position.

This difference arises from the different assumptions on consumers’ initial information and is paramount in the analysis of search frictions. Consider an equilibrium setting where horizontally differentiated alternatives are supplied by firms that compete by setting mean partial valuations (e.g. by setting prices as in Choi et al., 2018). If consumers are aware of all alternatives and search as in the DS problem, all firms will compete directly with each other. In contrast, in a SD problem, the firm that is discovered first initially competes only with the option of discovering potentially better products. This difference is further illustrated in Appendix G, and as it determines how firms compete, will lead to different equilibrium dynamics.  

6.2 Ranking Effects

The above analysis already suggests that the demand structure differs across the three search problems. To provide further details, I focus on a particular pattern that is generated by all three search problems: Market demand for a product decreases in its position. Such ranking effects are important as they determine how fiercely sellers compete for their products to be revealed on early positions, for example through informative advertising or position auctions (e.g. Athey and Ellison, 2011). Furthermore, they have received considerable attention in the marketing literature, which has produced ample empirical evidence that suggests their importance in online markets (e.g. Ghose et al., 2014; De los Santos and Koulayev, 2017; Ursu, 2018).

To compare the mechanism producing ranking effects across the search problems, I use the following definition: The ranking effect for a product is the difference in market demand of the product being revealed at position $h$ and at $h + 1$, with the corresponding exchange of the product previously revealed at position $h + 1$. Formally, this is given by

$$r_k(h) \equiv d_k(h) - d_k(h + 1)$$  (16)

To give an example, Anderson and Renault (1999) and Choi et al. (2018) model a similar environment, with the difference that in the former, consumers initially are not aware of any alternatives, whereas in the latter they are aware and observe prices of all alternatives. Whereas in the former, decreasing inspection costs lowers the equilibrium price in a symmetric equilibrium, the opposite holds in the latter environment.
where \( d_k(h) \) denotes the market demand for a product when revealed at position \( h \) in search problem \( k \in \{ SD, RS, DS \} \). For clarity, product specific subscripts are either omitted or exchanged with position subscripts in the following. The former is feasible as effective values are assumed to be independent realizations of a random variable \( W \).

To investigate ranking effects, it is first necessary to derive the market demand at a particular position \( h \). For a unit mass of consumers with independent realizations of effective values, it is given by

\[
d_{SD}(h) = P\left(W < z^d\right)^{h-1} \left[ P\left(W \geq z^d\right) + P\left(W < z^d\right) \right] - P\left(W < z^d\right)^{h-1} P\left(W \geq \max_{k \in J} W_k | W_k < z^d \forall \right)
\]

(17)

The expression follows from Theorem 2 which implies that if a consumer discovers a product with \( w_j \geq z^d \), he will stop searching and buy a product \( j \). The consumer will only discover and have the option to buy a product on position \( h \) if \( w_j < z^d \) for all products on better positions. In contrast, when \( w_j < z^d \), the consumer will first discover more products, and only recall \( j \) if he discovers all products and \( j \) is the best among them.

In the latter case, a product’s position does not affect market demand; once all products are discovered, products are equivalent in terms of their inspection costs and the order in which they are inspected is only determined based on partial valuations. This implies that the ranking effect in the SD problem is independent of the number of alternatives and simplifies to

\[
r_{SD}(h) = P\left(W \geq z^d\right) \left[ P\left(W < z^d\right)^{h-1} - P\left(W < z^d\right)^{h} \right]
\]

(18)

This expression reveals that the ranking effect in the SD problem solely results from the difference in the probability of a consumer reaching positions \( h \) or \( h + 1 \) respectively. Besides the distribution of valuations and the inspection and discovery costs, Proposition 4 shows that the ranking effect is determined by the position \( h \) to which the product is moved. When \( h \) is large, fewer consumers will not have already stopped searching before reaching \( h \). Hence, the later a product is revealed, the smaller is the increase in demand when moving one position ahead.

The demand in a random search problem is derived similarly. In RS, a consumer will only be able to buy a product if he has not stopped searching before, which requires that \( x + y < z^{RS} \) for all products on better positions. Furthermore, a consumer will also only recall a product if he has inspected all alternatives. Similar to the SD problem, this implies that the ranking effect
in the RS problem is given by

\[ r_{RS}(h) = P_{X,Y} (X + Y \geq z^{RS}) \left[ P_{X,Y} (X + Y < z^{RS})^{h-1} - P_{X,Y} (X + Y < z^{RS})^h \right] \tag{19} \]

Comparing (18) with (19) reveals that ranking effects in the RS problem are produced by the same mechanism as in the SD problem. In both search problems; fewer consumers buy products at later positions due to the increasing the probability of having stopped searching before discovering these products. It follows that in both search problems, ranking effects decrease in the position and are independent of the total number of alternatives.

Though their extent generally differs, Proposition 4 additionally shows that at later positions, ranking effects will be larger in the SD problem. The result is a direct implication of Proposition 3; as a consumer is more likely to reach a product at a later position in the SD problem, ranking effects at later positions will be larger.

**Proposition 4.** *The ranking effect in both the SD and the RS problem decreases in position* \( h \) *and is independent of the number of alternatives. Furthermore, if* \( c^{RS} = c_s + c_d \), *there exists a threshold* \( h^* \) *such that* \( r_{SD}(h) \geq r_{RS}(h) \) *for all* \( h > h^* \).

Given the different stopping decisions, ranking effects in directed search do not result from consumers having stopped searching before reaching products revealed at later positions. Instead, they result from differences in the cost of inspecting products at different positions. To see this, write the ranking effect in the DS problem as

\[ r_{DS}(h) = \mathbb{E}_{\tilde{W}_h} \left[ \prod_{k \neq h} P(\tilde{W}_k \leq \tilde{W}_h) \right] - \mathbb{E}_{\tilde{W}_{h+1}} \left[ \prod_{k \neq h+1} P(\tilde{W}_k \leq \tilde{W}_{h+1}) \right] \tag{20} \]

This expression reveals that the ranking effect results from two sources in the DS problem. First, by moving a product \( j \) one position ahead, the product previously on position \( h \) is now more costly to inspect, making it more likely that \( j \) is bought for any \( \tilde{w}_j \). Second, by making it less costly to inspect \( j \), the distribution of \( \tilde{w}_j \) shifts such that larger values \( \tilde{w}_j \) become more likely.

In contrast to RS and SD, the ranking effect in the DS problem depends on the number of available alternatives. In RS and SD, ranking effects result from the decreasing probability of a consumer having stopped searching before reaching a particular position, which does not depend on how many alternatives there are in total. In DS, however, a consumer directly compares all

\[^{25}\text{Alternatively, ranking effects could be modeled in a DS problem by assuming that the consumer initially has full information on some products. In this case, the model effectively has only 2 positions (full and partial information), and hence would not be able to explain the decrease in demand across all positions resulting from the SD problem.}\]

25
alternatives based on partial valuations. Adding more alternatives thus will affect the demand on each position.

Specifically, Proposition 5 shows that ranking effects in the DS problem will be smaller if there are many alternatives. The reason is that as the number of alternatives increases, each product is less likely to be bought and differences in the position-specific market demand decrease. Note, however, that in cases where the probability of consumers buying products on the last positions is very small or exactly zero (e.g. when inspection costs are large), adding more alternatives will not affect ranking effects in the DS problem.

**Proposition 5.** The ranking effect in the DS problem is weakly decreasing in the number of alternatives.

A second difference to the RS and SD problems is that the ranking effect does not necessarily decrease in position. This is possible as there are two counteracting channels through which position affects the ranking effect in a DS problem. First, as there is lower demand for products at later positions, differences between them will be smaller. Second, if \( v_{DS}(c_s, h) \) is such that \( \xi_h \) decreases in \( h \) at an increasing rate, the difference in the purchase probability at \( h \) instead of at \( h + 1 \) increases in the position. When the latter dominates, the ranking effect will first increase in position.

The above comparison highlights that the mechanism producing ranking effects in the DS problem is distinct from the one in the SD and RS problems, leading to a different demand structure. In the former, ranking effects result from differences in inspection costs relative to differences in partial valuations. Hence, a better partial valuation is a substitute for moving positions ahead. In contrast, in a SD or RS problem, a product’s large partial valuation does not affect consumers that stop search before discovering it. Hence, offering a larger partial valuation does not substitute for being discovered early in a SD or RS problem.\(^{26}\)

Moreover, the size of ranking effects determines how important it is for products to be revealed on an early position. As ranking effects are independent of the number of alternatives in SD and RS, so are sellers’ incentives to have their products revealed early during search. In contrast, in DS, the demand increase of moving positions ahead becomes smaller when the number of alternatives increases. Hence, sellers can have smaller incentives to be revealed on early positions when there are many, relative to when there are only few alternatives.

Finally, the above comparison between the number of alternatives and ranking effects also suggests the existence of an empirical test to distinguish the search modes in some settings. If

\(^{26}\)Note, however, that in an equilibrium setting, offering larger partial valuations may indirectly serve as a substitute for being discovered early by raising consumers’ expectations and induce them to search longer.
data is available that allows to test whether ranking effects depend on the number of alternatives, then it will be possible to empirically determine whether a DS problem, instead of a RS or SD problem provides a framework that better captures ranking effects in a particular setting. Furthermore, if data is available that allows to test whether a product’s partial valuation has an effect on whether it is inspected, it will be possible to distinguish between RS and SD.

6.3 Expected Payoff

If costs are specified such that the total costs of revealing all product information remain the same, then the three search problems differ only in the information the consumer can use during search. A comparison of a consumer’s expected payoff based on such a specification therefore provides some insight into whether it is always to the consumer’s benefit to provide information that helps to direct search towards some alternatives.

For total costs of revealing full information about a product on position \( h \) to be the same in the three search problems, inspection costs in the RS and DS problem are specified as \( c_{RS} = c_s + c_d \) and \( c_{DS}^j = c_s + h_j c_d \) respectively.

The SD problem extends the RS problem by additionally providing the consumer with the option to not inspect products depending on their partial valuations. This allows the consumer to save on inspection costs by not inspecting products with small partial valuations. As stated in Proposition 6, this increases the expected payoff which implies that providing product information across two layers, as done for example by online retailers or search intermediaries, is beneficial for consumers.

**Proposition 6.** If \( c_{RS} = c_s + c_d \), then a consumer’s expected payoff in the SD problem is larger than in the RS problem.

In contrast to the SD problem, the consumer can use all partial valuations to direct search in the DS problem. Hence, if inspection costs for all products are the same in both problems (i.e. \( c_{DS}^j = c_s \forall j \)), a consumer will have a larger expected payoff in the DS problem as he can directly inspect products with large partial valuations. However, under the assumption that total costs of revealing full information are the same in both search problems, a more detailed analysis is necessary to determine which search problem offers a larger expected payoff.

Denote a consumer’s expected payoff in a search problem \( k \) as \( \pi_k \) for \( k \in \{SD, DS\} \). Proposition 1 implies that expected payoffs are given by

\[
\begin{align*}
 p_{SD} &= \mathbb{E}\hat{W}\left[ \max\{u_0, \max_{j \in J} \hat{W}_j\} \right] \\
 p_{DS} &= \mathbb{E}\tilde{W}\left[ \max\{u_0, \max_{j \in J} \tilde{W}_j\} \right]
\end{align*}
\]
Furthermore, let $H_k(\cdot)$ denote the cumulative density of the respective maximum value over which the expectation operator integrates in problem $k$. The difference in expected payoffs of the SD and the DS problem then is given by

$$p_{SD} - p_{DS} = \int_{-\infty}^{\infty} H_{DS}(w) - 1dw + \int_{u_0}^{z^d} H_{DS}(w) - H_{SD}(w)dw$$

(21)

The first expression in (21) is negative, capturing the advantage of observing partial valuations for all products and being able to directly inspect a product at a later position. Given $H_{DS}(w) \leq H_{SD}(w)$ on $w \in [u_0, z^d]$, the second expression in (21) is positive, revealing that directly observing all partial valuations $x_j$ does not only yield benefits.

The latter stems from the difference in how inspection and discovery costs are taken into consideration in the two dynamic decision processes. In DS, the total cost of inspecting a product $j$ at a later position is directly weighed against its benefits given the partial valuations. In contrast, in SD, the consumer first weighs the discovery costs against the expected benefits of discovering a product with a larger partial valuation. Once product $j$ is revealed, the accumulated cost paid to discover $j$ ($j_{cd}$) is a sunk cost and does not affect the decision whether to inspect $j$.

Hence, in cases where products on early positions have below-average partial valuations $x_j$, the optimal policy in SD tends to less often prescribe to inspect these products compared to the direct cost comparison in DS. In some cases, the former can be more beneficial, leading to a larger expected payoff.\(^{27}\) Directly revealing all partial valuations therefore does not always improve a consumer’s benefit, if the consumer continues to incur the same total costs to reveal the full valuation of any given product. \(^{28}\)

**6.4 Empirical Implications**

Differences in the underlying search problem also have implications for the estimation of structural search models. For example, a structural search model will use price differences across all products to inform parameter estimates if it abstracts from limited awareness and assumes that consumers observe all prices prior to search. Consumers not inspecting low-price products they are unaware of then may be spuriously attributed either to a small price sensitivity or large inspection costs. Whereas there are many applications of structural search models and an ubiquity of settings where consumers remain unaware of some alternatives, the sensitivity of results from structural search models to limited awareness remains unclear.

\(^{27}\)For example, this is the case if $X \sim N(0, \frac{1}{4}), Y \sim N(0, \frac{2}{3})$, $c_s = c_d = 0.05$ and $|J| = 10$.

\(^{28}\)No threshold result as in Proposition 2 applies in this case. The first expression in (21) decreases whereas the second expression increases in the number of alternatives.
I therefore investigate the implications of estimating either a random or directed search model in a setting where consumers instead solve the search and discovery problem. I focus on a scenario where preference and cost parameters are estimated using data on consumers’ consideration sets and purchases; a common case as consideration sets are observable in click-stream or survey data. Using a simple specification, I first analyze how the different models attribute observed stopping decisions to structural parameters. A numerical exercise then reveals that this can lead to sizable differences in parameter estimates and counterfactual predictions.

**Empirical setting:** The data consist of consumers’ consideration sets and purchases, as well as a number of characteristics for each of the available products. The utility of purchasing product \( j \) is specified as \( u_j = \mathbf{x}_j' \beta + y_j \), where \( \mathbf{x}_j \) is a vector containing the observed product characteristics, \( \beta \) is a vector of preference parameters and \( y_j \) is an idiosyncratic unobservable taste shock with mean zero. Depending on the model, consumers are assumed to reveal \( \mathbf{x}_j \) when either discovering \( j \) (SD) or inspecting \( j \) (RS), or know \( \mathbf{x}_j \) prior to search (DS). \( y_j \) is revealed after inspecting \( j \) in all three models.

Given this setting, Table 1 shows sufficient or necessary conditions for the purchase of product \( j \) across the three models, conditional on \( j \) being the best product inspected and (ii) the observed consideration set not coinciding with the set of all available alternatives. The condition for the SD problem shows that a purchase of product \( j \) can be independent of realized valuations of products that the consumer is not aware of in the purchase period \( \bar{t} \). \( j \) only needs to offer “good enough” characteristics relative to the mean and to products the consumer is aware of at the time of purchase. The RS model features the same structure; a consumer will end search and buy product \( j \) if its valuation exceeds the reservation value. However, \( \Xi \) and \( \tilde{\xi} \) depend differently on the underlying costs and distributions of characteristics in \( \mathbf{x}_j \) and \( y_j \). Through these non-linear functions, a RS model will attribute observed limited consideration sets differently to preference and cost parameters.

In the DS model rationalizing the purchase of \( j \) requires that the valuation of the purchased product is larger than the search values of all uninspected products. If, for example, \( \mathbf{x}_k > \mathbf{x}_j \) for an uninspected product, the DS model will require either relatively small preference parameters, or relatively large inspection costs. Hence, depending on the characteristics of the uninspected products, rationalizing limited consideration sets in a DS model will require a combination of large inspection costs and attenuated preference parameters, as the estimation procedure will

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29 The empirical literature extends the simple specification for a range of settings, for example by introducing heterogeneous preferences. The main rationale continues to hold in such settings.

30 The simulated data also contains consumers with an empty consideration set, i.e. those that did not search any alternatives. This corresponds to an ideal setting where the whole population of consumers is observed.

31 The condition is sufficient but not necessary. Alternatively, the consumer can first become aware of all alternatives, before then purchasing \( j \).
try to fit an inequality for each uninspected product.

**Table 1 – Purchase conditions**

|   | Condition                                                                 |
|---|---------------------------------------------------------------------------|
| SD | \((x_j - \mu_X)'\beta + y_j \geq \Xi \) \& \((x_j - x_k)'\beta + y_j \geq \xi \forall k \in A_t \) (sufficient) |
| RS | \((x_j - \mu_X)'\beta + y_j \geq \xi \) (necessary)                       |
| DS | \((x_j - x_k)'\beta + y_j \geq \xi \forall k \notin C_t \) (necessary)     |

*Notes:* Sufficient or necessary conditions for purchase of product \(j\) conditional on \(u_j \geq u_k \forall k \in C_t\) and \(J \notin C_t\). \(t\) denotes the purchase period.

To investigate the extent to which this influences results from structural search models, I perform simulations for this setting. First, I simulate consumers solving a SD problem with the given utility specification and under the assumption that consumers initially aware of one product. Using these data, I then estimate structural parameters in search models based on either the RS and DS problem. For the DS problem, two specifications are estimated. DS1 is a baseline where inspection costs are parameterized as \(c_{DS1}^j = c_s\). DS2 introduces an additional cost parameter such that inspection costs increase in position \(h_j\) with \(c_{DS2}^j = c_s + c_d h_j\). This specification additionally uses data on the order in which products are discovered by consumers. For all three models, the estimation fits inequalities based on the conditions of Table 1, as well as other inequalities coming from continuation and purchase decisions. Details on the maximum likelihood estimation are provided in the appendix. As comparison, I also present estimates of a full information (FI) model.

Results of such a simulation are presented in Table 2. Parameters for this particular simulation are shown in the same table and were chosen to reflect a setting with relatively few searches, as is often the case in click-stream data.\(^{32}\) To account for the fact that assuming the distribution of \(y_j\) is a normalization in the empirical context, estimates are presented as a ratio to the coefficient of the second characteristic. Given its negative coefficient, this characteristic will be interpreted as a product’s price.

The results show that both DS specifications are able to match the number of purchases, as well as the relative preference coefficients relatively well, despite the price coefficient being strongly attenuated and the number of searches being underestimated. However, inspection costs are strongly accentuated in both DS models. This offers a novel explanation for the large estimates of baseline costs estimated with some DS models (e.g. Chen and Yao, 2017; Ursu, 2018): By not accounting for consumers not being aware of some alternatives, a DS model spuriously attributes consumers not inspecting products they are not aware of to large inspection costs.

\(^{32}\) For example, Ursu (2018) reports an average of 1.12 clicks per consumer and two thirds of consumers ending up booking a hotel. Chen and Yao (2017) reports an average of 2.3 clicks per consumer using data only on consumers that ended up booking a hotel.
Table 2 – Estimated Coefficients and Search Set Size

|            | #Searches | Purchases (%) | $\beta_2$ | $\beta_1/|\beta_2|$ | $\beta_3/|\beta_2|$ | $c_s/|\beta_2|$ | $c_d/|\beta_2|$ |
|------------|-----------|---------------|-----------|----------------------|----------------------|----------------|----------------|
| SD         | 1.35      | 63.70         | -1.00     | 1.00                 | 3.50                 | 0.03           | 0.06           |
| DS1        | 1.18      | 65.48         | -0.19     | 1.01                 | 2.58                 | 1.79           |                |
| DS2        | 1.18      | 65.22         | -0.19     | 1.01                 | 2.72                 | 1.58           | 0.01           |
| RS         | 1.00      | 72.85         | -0.82     | 1.28                 | 5.21                 | 0.05           |                |
| FI         |           | 60.54         | -0.62     | 1.00                 | 5.01                 |                |                |

Notes: Estimation from a simulated dataset with 2,000 consumers and 30 products per consumer. Characteristics are independent draws (across consumers and products) from $x_{1j} \sim N(2.3,0.0)$, $x_{2j} \sim N(3.5,1.0)$, and $y_j \sim N(0,1)$. The third characteristic is an outside dummy. The data is generated based on the SD model with $n_d = |A_0| = 1$, with parameters in the estimated models denoted by $c_{RS} = c_s$, $c_{DS1} = c_s$, and $c_{DS2} = c_s + c_d h_j$. The first two columns are based either on the generated data (SD) or estimated by generating 5,000 search paths for each consumer.

costs.\textsuperscript{33} This continues to occur in the DS2 model that could rationalize ranking effects produced by the SD model through inspection costs that increase in the position at which a product is discovered. However, the results show that instead the DS2 model estimates only a small increase in inspection costs across positions and also strongly overestimates baseline inspection costs.

The RS model underestimates inspection costs; they are less than the combined inspection and discovery costs. Moreover, the ratio of preference parameters deviates from the true values. The large differences in the estimated coefficient for the outside option result from how the different models interpret consumers not inspecting or not buying. Whereas in the DS problem this occurs from large inspection costs, the RS model attributes the lack of search mainly to a good outside option.

Differences in the structural search models also influence results from counterfactual simulations. Table 3 shows the results of two different counterfactuals for each of the models. For each counterfactual scenario, parameters from Table 2 are used for each model to simulate consumer surplus ($CS$) and the demand for the outside option ($D_0$), as well as for products shown on the first ($D_1$) and fifth ($D_5$) position. Throughout, results are expressed in percentage deviations from the baseline scenario.

The first counterfactual consists of removing all search costs, which can be used to gauge the effects of removing the search friction. For both DS models, accentuated baseline inspection costs lead to a larger increase in consumer surplus compared to the SD model with which the data was generated. Moreover, removing costs in the DS models makes consumers more likely to purchase any product, independent of their position. In contrast, demand in the SD and RS model decreases for both products listed. This stems from the inherent ranking effects where products on early positions are bought more frequently as consumers stop search early. In this

\textsuperscript{33}Other explanations for large search search cost estimates are incomplete search histories (e.g. Ursu, 2018) and heterogeneous prior beliefs (Jindal and Aribarg, 2020).
case, removing all costs moves demand to later positions.

The second counterfactual scenario analyzes the effects of a one percent price decrease of products discovered on the fifth position. The change in demand for the first product highlights an important difference in the substitution pattern. In the data-generating SD model, the demand for products on the first position decreases by little, as the price decrease of a product on a later position does not affect choices of consumers that stopped search before becoming aware of the product. In contrast, in a DS (or FI) model, consumers who were previously buying products on the first few positions observe the price decrease and can directly substitute to the fifth product. This translates into more substitution from the first few positions as a response to a price decrease of a product on a later position. The predicted changes in the demand for the fifth product further highlight that the different models lead to different predictions for consumers’ responses to price changes; whereas the DS1 and RS models underestimate, the DS2 model overestimates the increase in demand in response to the price change.

| Remove costs | ∆p₅ = -1% |
|--------------|------------|
|              | ∆CS | ∆D₁ | ∆D₅ | ∆CS | ∆D₁ | ∆D₅ |
| **SD**       | 28.60 | -37.35 | -2.32 | 0.02 | -0.01 | 1.81 |
| **DS1**      | 85.06 | 38.04 | 43.11 | 0.01 | -0.04 | 1.72 |
| **DS2**      | 81.38 | 15.53 | 29.19 | 0.01 | -0.03 | 2.75 |
| **RS**       | 18.73 | -25.36 | -11.78 | 0.01 | -0.02 | 1.49 |
| **FI**       | 0.00 | 0.00 | 0.00 | 0.01 | -0.05 | 1.91 |

Notes: Results from simulated counterfactuals based on Table 2, where (i) all costs are set to zero and (ii) the price for the 5th product is reduced by 1% for each consumer. All changes are expressed in % relative to the baseline. Demand and consumer surplus are calculated by averaging across 5,000 simulated search paths for each consumer.

Though results from only a single simulation are presented, I obtained qualitatively similar results across a wide range of parameter values. Throughout, DS models overestimate inspection costs and all estimated models can lead to sizable differences in parameters and results from counterfactual predictions. Nonetheless, the SD problem will be more similar to the DS problem if consumers are aware of many alternatives when they end search (e.g. due to small discovery costs). Similarly, if consumers inspect most products they discover independent of their characteristics, the SD problem will be more similar to the RS problem. When estimating search models, researchers should therefore carefully consider the degree to which limited awareness plays a role in the specific setting they are studying and which model is appropriate.

34 These results can be replicated with the supplementary material.
To this end, the results of Propositions 4 and 5 can be used to empirically differentiate the search modes in some settings. If data are available that allow to test whether ranking effects depend on the number of alternatives, it will be possible to empirically determine whether a DS problem, instead of a RS or SD problem provides a framework that better captures ranking effects. Furthermore, if data are available that allow to test whether a product’s partial valuation has an effect on whether it is inspected, it will be possible to distinguish between RS and SD.

7 Conclusion

This paper introduces a search problem that generalizes existing frameworks to settings where consumers have limited awareness and first need to become aware of alternatives before being able to search among them. The paper’s contribution is to provide a tractable solution for optimal search decisions and expected outcomes for this search and discovery problem. Moreover, a comparison with classical random and directed search highlights how limited awareness and the availability of partial product information determine search outcomes and expected payoffs.

A promising avenue for future research is to build on this paper’s results and study limited awareness in an equilibrium setting. This could yield novel insights into how consumers’ limited information shapes price competition. Furthermore, the search and discovery problem can serve as a framework to analyze how firms compete for consumers’ awareness. For example, informative advertising can make it more likely that consumers are aware of a seller’s products from the outset. Ranking effects derived in this paper already suggest that it will be in a seller’s best interest to make consumers aware of his product, but further research is needed to determine equilibrium dynamics.

Another avenue for future research entails incorporating the search and discovery problem into a structural model that is estimated with click-stream data. The available actions in the search and discovery problem closely match how consumers scroll through product lists (discovery) and click on products (inspection) on websites of search intermediaries and online retailers. By accounting for the fact that consumers initially do not observe entire list pages, such a model could improve the estimation of consumers’ preferences, inspection costs and ranking effects relative to models that abstract from consumers not observing the whole product list.
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APPENDIX

A PROOFS OF MAIN THEOREMS AND PROPOSITIONS

A.1 Theorem 1

Let \( \Theta(\Omega_t, A_t, z) \) denote the value function of an alternative decision problem, where in addition to the available actions in \( A_t \), there exists a hypothetical outside option offering value \( z \). As the SD problem satisfies that taking an action does not change the state of another available action and has the same branching structure, Theorem 1 of Keller and Oldale (2003) states that a Gittins index policy is optimal and that the following holds:

\[
\Theta(\Omega_t, A_t, z) = b - \int_{z}^{b} \Pi_{a \in A_t} \frac{\partial \Theta(\Omega_t, \{a\}, w)}{\partial w} dw
\]

(22)

where \( b \) is some finite upper bound of the expected immediate rewards.\(^{35}\) The Gittins index of action \( d \) (discovering products) is defined by \( g^d_t = \mathbb{E}_X \left[ \Theta(\Omega_{t+1}, A_{t+1} | A_t, g^d_t) \right] \). Suppose the consumer knows the total number of alternatives \( |J| \), and consider a period \( t \) in which more discoveries will still be available in \( t + 1 \) with certainty. In this case we have

\[
g^d_t = \mathbb{E}_X \left[ \Theta(\Omega_{t+1}, \{d, s_1, \ldots, s_{kd}\}, g^d_{t+1}) \right] - c_d
\]

\[
= \mathbb{E}_X \left[ b - \int_{g^d_t}^{b} \partial \Theta(\Omega_{t+1}, \{d\}, w) \prod_{k=1}^{n_d} \partial \Theta(\Omega_{t+1}, \{s_k\}, w) dw \right] - c_d
\]

(23)

where \( s_k \in S_{t+1} \setminus S_t \forall k \). \( \Theta(\Omega_t, \{s_k\}, z) \) is the value of a search problem with an outside option offering \( z \) and the option of inspecting product \( k \) (with known partial valuation \( x_k \)). \( \Theta(\Omega_{t+1}, \{d\}, w) \) is the value of a search problem with an outside option offering \( z \), and the option to discover more products. Finally, \( \mathbb{E}_X [\cdot] \) is the expectation operator integrating over the beliefs over the \( n_d \) random variables in \( X = [X, \ldots, X] \), which does not depend on time.

Optimality of the Gittins index policy then implies that when \( z \geq g^d_{t+1} \), the consumer will choose the outside option in \( t + 1 \). Hence \( \Theta(\Omega_t, \{d\}, w) = w \forall w \geq g^d_{t+1} \) which yields \( \frac{\partial \Theta(\Omega_t, \{d\}, w)}{\partial w} = 1 \forall w \geq g^d_{t+1} \). This implies that for \( g^d_t \geq g^d_{t+1} \), \( g^d_t \) does not depend on whether more products can be discovered in the future, and the optimal policy is independent of the beliefs over the number of available alternatives. As a result, as long as the Gittins index is weakly decreasing during search, i.e. \( g^d_t \geq g^d_{t+1} \forall t \), it is independent of the availability of future discoveries and beliefs \( q \).

It remains to show that \( g^d_t \geq g^d_{t+1} \forall t \) holds in the proposed search problem. When \( |J| = \infty \), \( g^d_t = g^d_{t+1} \) is immediately given by the fact that in both periods infinitely many products remain to be discovered and that the consumer has stationary beliefs (i.e. \( q \) is constant and valuations

\(^{35}\) Compared to the baseline branching framework discussed by Keller and Oldale (2003), the SD problem does not have discounting, and purchasing a product is a “terminal” action. Note also that whereas not explicitly stated by the authors, their framework accommodates the case where it is not known ex ante to how many “children” an available action branches into. This will be the case in the SD problem if the consumer does not know the number of products he will discover.

\(^{36}\) Expected immediate rewards are in \([- \max\{c_s, c_d\}, \mathbb{E}[X + Y]]\), hence assuming finite mean of \( X \) and \( Y \) guarantees that they have a finite upper bound.
are independent and identically distributed). For \( |J| < \infty \), backwards induction yields that this condition holds: Suppose that in period \( t + 1 \), no discovery action is available as all products have been discovered. In this case, the Gittins index is given by

\[
g_{t+1}^d = \mathbb{E}_X \left[ b - \int_{g_{t+1}}^{b} \prod_{k=1}^{n_d} \frac{\partial \Theta (\Omega_{t+1}, \{s_k\}, w)}{\partial w} \right] - c_d
\]  

(24)

As \( 0 \leq \frac{\partial \Theta (\Omega_t, \{d\}, w)}{\partial w} \leq 1 \) and \( \frac{\partial \Theta (\Omega_t, \{s\}, w)}{\partial w} \geq 0 \), it holds that

\[
\mathbb{E}_X \left[ b - \int_{g_{t+1}}^{b} \prod_{k=1}^{n_d} \frac{\partial \Theta (\Omega_{t+1}, \{s_k\}, w)}{\partial w} \right] \leq q \mathbb{E}_X \left[ b - \int_{g_{t+1}}^{b} \prod_{k=1}^{n_d} \frac{\partial \Theta (\Omega_{t+1}, \{s_k\}, w)}{\partial w} \right] +
(1 - q) \mathbb{E}_X \left[ b - \int_{g_{t+1}}^{b} \prod_{k=1}^{n_d} \frac{\partial \Theta (\Omega_t, \{d\}, w)}{\partial w} \prod_{k=1}^{n_d} \frac{\partial \Theta (\Omega_t, \{s_k\}, w)}{\partial w} \right]
\]

(25)

which implies \( g_t \geq g_{t+1} \).

Finally, \( \Theta(\Omega_{t+1}, \{d, s_1, \ldots, s_{n_d}\}, g^d_t) = V((\Omega, \omega(x, z)), \{0, s_1, \ldots, s_{n_d}\}; \tilde{\pi}) \) in (9) implies \( z^d = g^d_t \). Similarly, the definition of the inspection and purchase values (in (6) and (10)) are equivalent to the definition of Gittins index values for these actions and it follows that the reservation value policy is the Gittins index policy.

A.2 Theorem 2

Proof. As a product always is bought, it suffices to show that the optimal policy never prescribes to buy product \( j \) if there exists another product \( k \) with \( w_k \geq w_j \). To account for the case where \( C_0 \neq \emptyset \), define \( z_k^* = \infty \forall k \in C_0 \) which implies \( \tilde{w}_k \equiv \min \{ z_k^*, z_k^\tilde{b} \} = z_k^\tilde{b} \forall k \in C_0 \). First, consider the case where \( k \) is revealed before \( j \) \( (h_k \leq h_j) \). In this case, \( w_k \geq w_j \) if and only if either (i) \( \tilde{w}_k \geq z^d \) or (ii) \( z^d > \tilde{w}_k > \tilde{w}_j \). In the former, the optimal policy prescribes to not discover products beyond \( k \), hence not to buy product \( j \). This follows as \( z_k^* \geq z^d \) and \( z_k^\tilde{b} \geq z^d \) imply that the optimal policy prescribes that search ends with buying \( k \) before discovering \( j \). In the latter, \( w_j = w_j < w_k = \tilde{w}_k \), and the optimal policy prescribes to continue discovering such that both products are in the awareness set. The eventual purchase theorem of Choi et al. (2018) then applies, and hence the optimal policy does not prescribe to buy product \( j \). Second, consider the case where \( k \) is discovered after \( j \) \( (h_k > h_j) \). In this case, note that \( w_j > w_k \) if and only if \( \tilde{w}_j > z^d \). Hence, \( w_k > w_j \) if and only if \( z^d > \tilde{w}_k > \tilde{w}_j \), which is the same as (ii) above. Finally, consider the case where \( k \) is discovered at the same time as \( j \) \( (h_k = h_j) \). Then \( w_k > w_j \) if and only if \( \tilde{w}_k > \tilde{w}_j \), which follows from the construction of the effective values. This again is the same as (ii) above and hence the optimal policy does not prescribe to buy \( j \). □

A.3 Proposition 1

Proof. The proof follows a similar structure as the proof of Corollary 1 in Choi et al. (2018). To simplify exposition, the following additional notation is used: Let \( \tilde{w}_j \equiv x_j + \min \{ y_j, \xi_j \} \) as in Theorem 1, and \( \tilde{w}_j \) equal to the effective value from Theorem 2, with the adjustment that \( f(h_j) = \varepsilon = 0 \). Furthermore, let \( \tilde{w}_r \equiv \max_{k \in J_0, r-1} \tilde{w}_k \forall r \geq 1 \), \( \tilde{r} = \max_{k \in J_r, \tilde{r}} \tilde{w}_k \) and \( \tilde{r}_{j, r} \equiv \max_{k \in J_{r, \tilde{r}}} \tilde{w}_k \) where \( J_{a, b} \) denotes the set of products discovered on position \( r \in \{ a, \ldots, b \} \), and \( J_r \) is short-hand for \( J_{r, r} \). Finally, let \( 1(\cdot) \) denote the indicator function and \( \tilde{h} \) the maximum position.
The payoff of a consumer given realizations $x_j$ and $y_j$ for all $j$ is given by

$$
\sum_{r=1}^{k} 1(\bar{w}_r < z^d) \left[ \sum_{j \in J_r} 1(\bar{w}_j \geq \max \{z^d, \bar{w}_{r,j}\})(x_j + y_j) - 1(x_j + \xi_j \geq \max \{z^d, \bar{w}_{r,j}\}) c_s \right] + 1(\bar{w}_0 \geq z^d) \nu_0 - \sum_{r=1}^{k} 1(\bar{w}_r < z^d) c_d + 1(w_h < z^d) \nu
$$

which follows from the optimal policy and Theorem 2: (i) If $\bar{w}_0 \geq z^d$, the stopping rule implies that the consumer does not discover any products beyond the initial awareness set. Conditional on not discovering any additional products, the payoff then is equal to $\nu_0$, which denotes the payoff of a directed search problem over products $k \in S_0$ and an outside option offering $\bar{u}_0 = \max_{k \in C_0} u_k$. (ii) If $\bar{w}_r < z^d$, the continuation rule implies that the consumer continues beyond position $r - 1$, i.e. discovers products on position $r$ and pays discovery costs $c_d$. (iii) Conditional on discovering $j$, when $\bar{w}_j \geq \max \{z^d, \bar{w}_{r,j}\}$, the stopping and inspection rules imply that the consumer buys $j$, gets utility $x_j + y_j$ and does not continue beyond position $r$. (iv) Conditional on discovering $j$, when $x_j + \xi_j \geq \max \{z^d, \bar{w}_{r,j}\}$, the inspection rule implies that the consumer inspects $j$ and incurs costs $c_s$. (v) If $w_h < z^d$, the continuation rule implies that the consumer discovers all products, whereas the inspection rule implies that he inspects all products $\{j \mid x_j + \xi_j \geq z^d\}$. Conditional having discovered all products, the consumer therefore has the payoff of a directed search problem over products $\{j \mid x_j + \xi_j \geq z^d\}$ with outside option $\bar{u}_0 = \max \{u_0, \max_{k \in \{j \mid x_j + \xi_j \geq z^d, x_j + y_j \leq \xi_j\}} x_k + y_k\}$. This is denoted by $\nu$.

Let $E[\cdot]$ integrate over the distribution of $X_j, Y_j \forall j \in J$, and substitute inspection and discovery costs by $c_s = E\left[1(Y_j \geq \xi_j)(Y_j + x_j - z^*_j)\right] = \forall j$ (note that $z^*_j = x_j + \xi_j$) and $c_d = $
The second-to-last step substitutes \( \nu \), which directly follows from Corollary 1 in Choi et al. (2018). The last step combines the expressions of the three mutually exclusive cases using the definition of the adjusted effective value \( \hat{w} \).

To prove the second claim, note that the \( Q_d(c_d, c_s, z^d) \) requires that \( \mathbb{P}(\hat{w}_j > z^d) > 0 \), as otherwise \( Q_d(c_d, c_s, z^d) > 0 \). Hence with \( |J| = \infty \), \( \mathbb{P}\left( \max_{j \in J} \hat{w}_j < z^d \right) = 0 \) such that \( \mathbb{E}\left[ \max_{j \in J} \hat{w}_j \right] = z^d \).

### A.4 Proposition 2

**Proof.** Consider a situation where we decrease costs \( c_s \) and \( c_d \) to either \( c_s' = c_s - \Delta \) or \( c_d' = c_d - \Delta \), while keeping the other cost constant. Let \( H_1(\cdot) \) and \( H_2(\cdot) \) denote the cumulative density of \( \tilde{W} = \max\{\tilde{w}_0, \max_{k \in S_0} \tilde{w}_k\} \) in the former and the latter case respectively, where \( \tilde{w}_0 = \max \{\max_{k \in C_0} u_k, \max_{k \in S_0} \tilde{w}_k\} \) is the value of the alternatives in the initial consideration and awareness sets. Similarly, let \( z_1^d \) and \( z_2^d \) denote the associated discovery values. Given \( n_d = 1 \), we have \( \frac{\partial Q_d(c_d, c_s, z^d)}{\partial c_d} < \frac{\partial Q_d(c_d, c_s, z^d)}{\partial c_s} \); hence \( \frac{\partial z_1^d}{\partial c_d} > \frac{\partial z_2^d}{\partial c_s} \) and \( z_1^d > z_2^d \). Moreover, note that the definition of the adjusted effective value \( \hat{w}_j \) implies \( H_1(w) = 1 \forall w \geq z_1^d \), and \( H_2(w) = 0 \forall w \leq \tilde{w}_0 \).

Conditional on \( \tilde{w}_0 < z_1^d \), the difference in a consumer’s expected payoff across the two changes
therefore can be written as

\[
\int_{z_1^d}^{z_2^d} 1 - H_2(w)dw - \int_{\tilde{w}_0}^{z_1^d} H_2(w) - H_1(w)dw \tag{27}
\]

Whereas the first part is strictly positive, the second part is negative. The latter follows as for \( w \in [\tilde{w}_0, z_1^d] \), \( \tilde{W} = \max_{j \in J, \eta \in \mathcal{S}_0} X_j + \min\{Y_j, \xi\} \) and \( \frac{\partial \tilde{r}_S}{\partial c_s} < 0 \) such that \( H_1(w) \leq H_2(w) \). As valuations are independent across products, we have \( H_k(w) = \mathbb{P}_{X,Y} (X + \min\{Y, \xi\} \leq w)^{|J|} \); hence, as \(|J|\) increases, \( H_2(w) - H_1(w) \) and \( H_2(w) \) decrease for \( w \in [\tilde{w}_0, z_1^d] \). Consequently, for all \( \Delta > 0 \) there exists some threshold \( n^* \) for \(|J|\) such that the difference in the expected payoff conditional on \( \tilde{w}_0 < z_1^d \) is positive, i.e.

\[
\int_{\tilde{w}_0}^{z_1^d} 1 - H_2(w)dw > \int_{\tilde{w}_0}^{z_1^d} H_2(w) - H_1(w)dw \tag{28}
\]

Conditional on \( \tilde{w}_0 \geq z_1^d \), having \( z_2^d > z_1^d \) immediately implies that the expected payoff increases by at least as much when decreasing discovery costs. Note also that when \( z_1^d < \tilde{w}_0 \), neither change affects the expected payoff. Finally, integrating over the realizations \( y_k \) for \( k \in \mathcal{S}_0 \) that determine \( \tilde{w}_0 \) yields the unconditional expected payoff as a combination of these cases, which implies the first result.

Increasing the value of the alternatives in the initial consideration and awareness set then makes larger values of \( \tilde{w}_0 \) more likely. This implies the second result, as it makes both the case \( \tilde{w}_0 \geq z_1^d \) more likely, as well as decrease the right-hand-side of (28).

A.5 Proposition 3

\[ h \]

Proof. At \( c_s = 0 \), we have \( z^d = z^{RS} \). \( \left| \frac{\partial z^{RS}}{\partial c_s} \right| \geq \left| \frac{\partial z^d}{\partial c_s} \right| \) then implies \( z^d \geq z^{RS} \). Using this in (14) and (15) immediately yields the result.

A.6 Proposition 4

\[ h \]

Proof. The first two statements immediately follow from (18) and (19). To see the latter, rewrite (18) as \( \mathbb{P}_{W} (W < z^d)^{h-1} \mathbb{P}_{W} (W \geq z^d)^2 \), and (19) in a similar way. \( c^{RS} = c_s + c_d \) then implies \( z^d \geq z^{RS} \). Hence, \( \mathbb{P}_{W} (W < z^d) = \mathbb{P}_{X,Y} (X + \min\{Y, \xi\} < z^d) \geq \mathbb{P}_{X,Y} (X + Y < z^{RS}) \) which directly implies the existence of the threshold.

A.7 Proposition 5

\[ h \]

Proof. Write the first expression in (20) (demand at position \( h \)) as \( \mathbb{E}_{\tilde{W}_h} \left[ \mathbb{P} \left( \tilde{W}_{h+1} \leq \tilde{W}_h \right) \prod_{k \notin \{h, h+1\}} \mathbb{P} \left( \tilde{W}_k \leq \tilde{W}_h \right) \right] \). When \(|J|\) decreases, this expression decreases through the product term, which is weighted by the first term \( \mathbb{P} \left( \tilde{W}_{h+1} \leq \tilde{W}_h \right) \). As \( \mathbb{P} \left( \tilde{W}_{h+1} \leq t \right) \geq \mathbb{P} \left( \tilde{W}_h \leq t \right) \forall t \), the first expression in (20) decreases by more than the second one when the number of alternatives increases.

\[ h \]

Note that if \( \mathbb{P}_{X,Y} (X + \min\{Y, \xi\} \leq w) \) is large, then \( H_1(w) - H_2(w) \) will first increase in \(|J|\), before starting to decrease.
A.8 Proposition 6

Proof. The RS problem is equivalent to a policy in the SD problem that commits on inspecting every product that is discovered, conditional on which the consumer chooses to stop optimally. However, as the optimal policy in the SD problem is not this policy, it must yield a (weakly) larger payoff.

A.9 Uniqueness of discovery value

Proposition 7. (10) has a unique solution.

Proof. \( Q_d(c_d, c_s, z) \) with respect to \( z \) yields (see Appendix B)

\[
\frac{\partial Q(c_d, c_s, z)}{\partial z} = \begin{cases} 
+H(z) - 1 & \text{if } z < 0 \\
-2 + H(z) & \text{else}
\end{cases}
\]

(29)

where \( H(\cdot) \) denotes the cumulative density of the random variable \( \max_{k \in J} \tilde{W}_k \). This implies \( \frac{\partial Q_d(c_d, c_s, z)}{\partial z} \leq 0 \), which combined with continuity, \( Q_d(c_d, c_s, \infty) = -c_d \) and \( Q_d(c_d, c_s, -\infty) = \infty \) imply that a solution to (10) exists. Finally, uniqueness requires \( Q_d(c_d, c_s, z) \) to be strictly decreasing at \( z = z^d \). \( \frac{\partial Q_d(c_d, c_s, z^d)}{\partial z} = 0 \) would require that \( H(z^d) = 1 \), which contradicts the definition of the discovery value value \( z^d \) in (10), as it implies \( Q_d(c_d, c_s, z^d) \leq -c_d < 0 \).

B Further details on Search and Discovery Values

The search value of a product \( j \) is defined by equation (6) and sets the myopic net gain of the inspection over immediately taking a hypothetical outside option offering utility \( z \) to zero. This myopic net gain can be calculated as follows:\(^{38}\)

\[
Q_s(x_j, c_s, z) = E_Y \left[ \max \{0, x_j + Y - z\} \right] - c_s \\
= \int_{z-x_j}^{\infty} (x_j + y - z) dF(y) - c_s \\
= \int_{z-x_j}^{\infty} \left[ 1 - F(y) \right] dy - c_s
\]

Substituting \( \xi_j = z - x_j \) then yields (7).

The discovery value is defined by equation (10) and sets the expected myopic net gain of discovering more products over immediately taking a hypothetical outside option offering utility

---

\(^{38}\)The second steps holds as with a change in the order of integration we get \( \int_{z-x_j}^{\infty} [1 - F(y)] dy = \int_{z-x_j}^{\infty} \int_{y}^{\infty} f_Y(t) dt dy = \int_{z-x_j}^{\infty} \int_{y}^{\infty} f_Y(t) dtdy = \int_{z-x_j}^{\infty} [y f_Y(t)]_{y=\infty}^{y=t} dt. \)
z to zero. Corollary 1 in Choi et al. (2018) and similar steps as the above then imply that:

\[
Q_d(c_d, c_s, z) = \mathbb{E}_{X,Y} \left[ \max \left\{ z, \max_{k \in \{1, \ldots, n_d\}} \tilde{W}_k \right\} \right] - z - c_d
\]

\[
= \mathbb{E}_{X,Y} \left[ \max \left\{ 0, \max_{k \in \{1, \ldots, n_d\}} \tilde{W}_k - z \right\} \right] - c_d
\]

\[
= \int_{z}^{\infty} 1 - H(w) \, dw - c_d
\]

where \( H(\cdot) \) denotes the cumulative density of the random variable \( \max_{k \in J} \tilde{W}_j \). The above also implies that in the case where \( Y \) is independent of \( X \), a change in variables yields that the discovery value is linear in the mean of \( X \), denoted by \( \mu_X \):

\[
z^d = \mu_X + \Xi(c_s, c_d)
\]

where \( \Xi(c_s, c_d) \) solves (10) for an alternative random variable \( \tilde{X} = X - \mu_X \).

\section{Monotonicity and Extensions}

Monotonicity of the Gittins index values \( g^d_t \geq g^d_{t+1} \forall t \) is satisfied whenever the following holds:

\[
0 \leq \mathbb{E}_{X,Y,n_d,q,t} \left[ \Theta(\Omega_{t+1}, \hat{A}_{t+1}, g^d_t) \right] - \mathbb{E}_{X,Y,n_d,q,t+1} \left[ \Theta(\Omega_{t+2}, \hat{A}_{t+2}, g^d_{t+1}) \right]
\]

(30)

where \( g^d_t \) is the Gittins index of discovering products (defined by (23)), and \( \hat{A}_{t+1} \equiv \{ d, s_1, \ldots, s_{n_d} \} \) is the set of actions available in \( t + 1 \) containing the newly revealed products and (if available) the possible future discoveries. The expectation operator \( \mathbb{E}_{X,Y,n_d,J,t} [\cdot] \) integrates over the following random realizations, where the respective joint distribution now can be time-dependent: (i) Partial valuations drawn from \( X = [X_1, \ldots, X_{n_d}] \); (ii) conditional distributions \( F_{Y|X=x}(y) \); (iii) the number of revealed alternatives \( (n_d) \); (iv) whether more products can be discovered in future periods determined by the belief \( q \).

It goes beyond the scope of this paper to determine all possible specifications of beliefs which satisfy this condition. However, Proposition 8 provides two specifications that can be of interest and for which (30) holds (see also Section 4).

\textbf{Proposition 8.} (30) holds for the below deviations from the baseline model:

i) \( Y \) is independent of \( X \). Beliefs are such that the revealed partial valuations in \( X \) are i.i.d. with time-dependent cumulative density \( G_t(x) \) such that \( G_t(x) \leq G_{t+1}(x) \forall x \geq z^d - \xi \).

ii) The consumer does not know how many alternatives he will discover. Instead, he has beliefs such that with each discovery, at most the same number of alternatives are revealed as in previous periods \( (n_{d,t+1} \leq n_{d,t}) \).

\textbf{Proof.} Each part is proven using slightly different arguments.
i) Let \( \tilde{x} \equiv \max_{k \in \{1, \ldots, n_d\}} x_k \). If \( \tilde{z}^s = \tilde{x} + \xi \leq z^d \), \( \Theta(\Omega_{t+1}, \tilde{A}_{t+1}, z^d) = 1 \), whereas for \( \tilde{x} > z^d - \xi \), \( \partial \Theta(\Omega_{t+1}, \{e, s_{1}, \ldots, s_{n_d}\}, g^d_t) \geq 0 \). Independence implies that the cumulative density of the maximum \( \tilde{x} \) is \( \tilde{G}_{t+1}(x) = G_t(x^{n_d}) \). Consequently, whenever the distribution of \( X \) shifts such that \( G_t(x) \leq G_{t+1}(x) \forall x \geq z^d - \xi \), larger values of \( \Theta(\Omega_{t+1}, \tilde{A}_{t+1}, g^d_t) \) become less likely in \( t+1 \), and hence (30) holds.

ii) Since \( \frac{\partial \Theta(\Omega_{t+1}, \{s_k\}, w)}{\partial w} \leq 1 \), we have \( \frac{\partial \Theta(\Omega_{t+1}, \tilde{A}_{t+1}, g^d_t)}{\partial n_d} \geq 0 \). Hence (30) holds given \( n_{d,t+1} \leq n_{d,t} \). □

Based on this monotonicity condition, Proposition 9 generalizes Theorem 1. It implies that whenever (30) holds, the discovery value can be calculated based on the expected myopic net gain of discovering products over immediately taking the hypothetical outside option. Hence, whenever (30) holds, the optimal policy continues to be fully characterized by reservation values that can be obtained without having to consider many future periods.

**Proposition 9.** Whenever (30) is satisfied, Theorem 1 continues to hold (with appropriate adjustment of the discovery value’s time-dependence).

**Proof.** Follows directly from the proof of Theorem 1. □

### D Violations of Independence Assumption

**Costly recall:** Consider a variation to the search problem, where purchasing a product in the consideration set is costly unless it is bought immediately after it is inspected. If in period \( t \) product \( j \) is inspected, then inspecting another product or discovering more products in \( t+1 \) will change the payoff of purchasing product \( j \) by adding the purchase cost. In the context of a multi-armed bandit problem, this case arises if there are nonzero costs of switching between arms. Banks and Sundaram (1994), for example, provide a more general discussion on switching costs and the nonexistence of optimal index-based strategies. The same reasoning also applies in a search problem where inspecting a product is more costly if the consumer first discovers more products. The exception is if there are infinitely many alternatives. In this case, the optimal policy never prescribes to recall an alternative.

**Learning:** Independence is also violated for some types of learning. Consider a variation of the search problem, where the consumer updates his beliefs on the distribution of \( Y \). In this case, by inspecting a product \( k \) and revealing \( y_{ik} \), the consumer will update his belief about the distribution of \( Y \), thus affecting the expected payoffs of both discovering more and inspecting other products. Independence therefore is violated and the reservation value policy is no longer optimal.\(^{39}\) Note, however, that as long as learning is such that only payoffs of actions that will be available in the future are affected, independence continues to hold. This is for example the case when the consumer learns about the distribution of \( X \) as discussed in Section 3.2.

\(^{39}\)Adam (2001) studies a similar case where independence continues to hold across groups of products. However, his results do not extend to the case with limited awareness, as the beliefs of \( Y \) also determine the expected benefits of discovering more products.
**Purchase without inspection:** A final setting where independence does not hold is when a consumer can buy a product without first inspecting it. In this case, the consumer has two actions available for each product he is aware of. He can either inspect a product, or directly purchase it. Clearly, when the consumer first inspects the product, the information revealed changes the payoff of buying the product. Independence therefore is violated and the reservation value policy is not guaranteed to be optimal. Doval (2018) studies this search problem for the case where a consumer is aware of all available alternatives, and characterizes the optimal policy under additional conditions.

## E Learning

Several studies consider priors or learning rules under which the optimal policy is myopic when searching with recall (Rothschild, 1974; Rosenfield and Shapiro, 1981; Bikhchandani and Sharma, 1996; Adam, 2001). A sufficient condition for the optimal policy to be myopic is given in Theorem 1 of Rosenfield and Shapiro (1981): Once the expected net benefits of continuing search over stopping with the current best option are negative, they remain so. Hence, whenever it is optimal to stop in $t$, it is also optimal to stop in all future periods. The monotonicity condition used in this paper directly imposes that this is satisfied; expected benefits of discovering more products remain constant or decrease during search. A fairly general assumption underlying learning rules that satisfy this condition is Assumption 1 in Bikhchandani and Sharma (1996). This assumption requires that beliefs are updated such that values above the largest value revealed so far become less likely.\(^{40}\) Hence, whenever a better value is found than the current best, finding an even better match in the future becomes less likely.

In the SD problem, similar learning rules that satisfy this condition are difficult to find. When the consumer learns about the number of products that are revealed with each discovery, expected benefits of discovering more products increase if many products are revealed, but the value of stopping remains the same if all these products are bad matches. Hence, a learning rule would need to guarantee that beliefs shift such that the expected benefits of discovering more products do not increase, as opposed to only the net benefits over stopping. Similarly, if the consumer learns about the distribution of partial valuations $X$, the value of stopping need not increase even if partial information indicates a good match leading the consumer to shift beliefs towards larger values; after inspecting a promising product, the consumer may still realize that the product is worse than the previously best option.

Though the optimal policy is not myopic with learning, it is still based on the Gittins index, where the search and purchase values are as in the baseline SD problem. The main difficulty is calculating the index value for discovering more products, denoted by $z_L^t$. Whereas calculating this value precisely would require accounting for learning in future periods, it is possible to derive bounds on this value that are easier to calculate and can be used to judge how far off a myopic policy is.

To show this, I focus on the case where the consumer learns about the distribution of partial valuations. In particular, consider the following variation of the search and discovery problem:

\(^{40}\)Note that Bikhchandani and Sharma (1996) consider search for low prices.
Let the distribution of partial valuations in $X$ be characterized by a parameter vector $\theta$ and denote its cumulative density by $G_{\theta}(\cdot)$. The consumer initially does not know the true parameter vector and Bayesian updates his beliefs in period $t$ given some prior distribution. Denoting the consumers’ beliefs on $\theta$ with cumulative density $P_t(\cdot)$, the consumers’ beliefs about $X$ drawn in the next discovery are characterized by the cumulative density $\bar{G}_t(X) = \int G_{\theta}(X) dP_t(\theta)$.\(^{41}\)

Denote a $k$-step look-ahead value as $z^t_k(k)$ and define it as the value of a hypothetical outside option that makes the consumer indifferent between stopping immediately, and discovering more products after which at most $k - 1$ more discoveries remain. For example, $z^t_1(1)$ satisfies the myopic comparison in (10), where expectations are calculated based on period $t$ beliefs $\bar{G}_t(\cdot)$. The definition of $z^t_d(1)$ then implies that it is equal to the expected value of continuing to discover products if no future discoveries remain. As the consumer can stop and take this hypothetical outside option in $t + 1$, allowing for more discoveries after $t + 1$ can only increase the expected value, hence $z^t_d(1) \leq z^t_d(2) \cdots \leq z^t_d$. $z^t_d(1)$ therefore provides a lower bound on $z^t_d$, and $z^t_d$ can be approximated with increasing precision through $k$-step look-ahead values.

To derive an upper bound, consider the case where the consumer learns the true $\theta$ in $t + 1$, if he chooses to discover more products in $t$. The value of discovering more products in $t$ when the true $\theta$ is revealed in $t + 1$ then is larger compared to the case where the consumer continues to learn. This is formally derived by Kohn and Shavell (1974) for a search problem where a consumer samples from an unknown distribution. Intuitively, when the true $\theta$ is revealed, the consumer is able choose the action in $t + 1$ that maximizes the expected payoff going forward for each realization of $\theta$. In contrast, if the consumer does not learn the true $\theta$ in $t + 1$, he cannot choose the maximizing action for each realization of $\theta$, but only the action that maximizes expected payoff on average across possible $\theta$.

An upper bound therefore is given by the value $z^t_d$ such that the consumer is indifferent between stopping and taking a hypothetical outside option offering $z^t_d$, and discovering more products after which the true $\theta$ is revealed. Formally, $z^t_d$ satisfies

$$z^t_d = \int \int \bar{V}(\Omega_{t+1}, A_{t+1}, \bar{z}; \theta) dP_{t+1}(\theta) d\bar{G}_t(X) \quad (31)$$

where $\bar{V}(\Omega_{t+1}, A_{t+1}, z^t_d; \theta)$ denotes the expected value of a search and discovery problem with known $\theta$ and an outside option offering $z^t_d$. Proposition 1 then directly allows to calculate this value without having to consider all the possible search paths.

Proposition 10 summarizes these results. A similar result can also be derived for the case where the consumer learns about a distribution from which the number of products that are discovered is drawn.

**Proposition 10.** In the search and discovery problem with Bayesian learning about an unknown distribution of partial valuations $X$, it is optimal to:

1) continue whenever $\max_{k \in C_t} u_k \leq z^t_d(1)$

---

\(^{41}\)For example, consider the case of sampling from a Normal distribution with unknown mean and known variance, and assume $n_d = 1$. If the consumer believes in $t$ that the mean is distributed normally with $\theta \sim N(\mu_t, \sigma^2_t)$, then $\bar{G}_t(x) = \Phi(\frac{x - \mu_t}{\sigma_t})$, where $\Phi(\cdot)$ is the standard normal cumulative density (see e.g. Theorem 1 in DeGroot, 1970, Ch. 9.5).
ii) stop whenever \( \max_{k \in C_i} u_k \geq z^d_t \)

## F Estimation Details

To estimate the three models I use a simulated maximum likelihood approach based on a kernel-smoothed frequency simulator. Using numerical optimization, parameters are found that maximize the simulated likelihood given by:

\[
\max_{\gamma} \sum_i L_i(\gamma) = \sum_i \log \left( \frac{1}{N_d} \sum_{d=1}^{N_d} \frac{1}{1 + \sum_{k=1}^{N_k} \exp(-\lambda \kappa_{kdi})} \right)
\]

where \( \gamma \) is the parameter vector, \( N_d \) is the number of simulation draws, \( \lambda \) is a smoothing parameter and \( \kappa_{kdi} \) is one of \( N_k \) inequalities resulting from the optimal policy in the respective model evaluated for draw \( d \). All three models are estimated with \( \lambda = 10 \) and \( N_d = 500 \). At these values, parameters are recovered well when data is generated with the same model.

**DS conditions** These conditions are the same as in Ursu (2018), who provides further details on how they relate the optimal policy in the DS problem. The difference to her specification is that inspection costs are linear, and that in DS1 there are no positions. For observed consideration set \( C_i \) for consumer \( i \), a given draw \( d \) for the unobserved taste shocks \( y_j(d) \) which defines product utilities \( u_j(d) \) as well as the utility of the purchased option \( u^*_i(d) \), there are multiple purchase, and stopping conditions expressed in inequalities:

Stopping: \( \kappa_{kdi} = \max_{j \in C_i} u_j(d) - z_m \forall m \notin C_i \)

Continuation \( \kappa_{kdi} = z_{m+1} - \max_{j \in C_i(m)} u_j(d) \forall m = 1, 2, \ldots, N_{is} - 1 \)

Purchase: \( \kappa_{kdi} = u^*_i(d) - u_j(d) \forall j \in C_i \)

In the continuation conditions, \( N_{is} \) denotes the number of observed inspections, \( z_{m+1} \) is the search value of the next inspection, \( C_i(m) \) is the consideration set of \( i \) after \( m \) inspections. Note that the last relies on observing the order in which products are inspected; if this order were not observed, the method proposed by Honka and Chintagunta (2017) could be used to integrate over possible search orders. The stopping condition only applies if not all products are inspected, the continuation condition only applies if \( i \) inspected at least one product.

**RS conditions** The conditions in the RS model are similar to the ones in the DS model. However, the stopping and continuation conditions now are based on the reservation value \( z^{RS} \), which follows directly from the optimal policy:

Stopping: \( \kappa_{kdi} = \max_{j \in C_i} u_j(d) - z^{RS} \)

Continuation \( \kappa_{kdi} = z^{RS} - \max_{j \in C_i(m)} u_j(d) \forall m = 1, 2, \ldots, N_{is} - 1 \)

Purchase: \( \kappa_{kdi} = u^*_i(d) - u_j(d) \forall j \in C_i \)
**FI conditions** In the FI model, standard purchase conditions apply:

\[ \kappa_{kdi} = u_i^*(d) - u_j(d) \forall j \]

**G Sellers’ decisions**

To illustrate the difference in sellers’ decision making across the SD and DS problem, we can compare the market demand generated by the SD problem with the one from the DS problem when there are infinitely many alternatives. Given a unit mass of consumers, market demand for a product discovered at position \( h \) is given by

\[
d_{SD}(h) = \mathbb{P}_{W} \left( W_k < z^d \forall k < h \right) \mathbb{P}_{W_h} \left( W_h \geq z^d \right)
\]

where \( W_h \) is the random effective value of a product on position \( h \). The expression immediately follows from the stopping decision which implies that if a consumer discovers a product with \( w_j \geq z^d \), he will stop searching and buy a product \( j \). Hence, the consumer will only discover and have the option to buy a product on position \( h \) if \( w_h < z^d \) for all products on earlier positions.

For the DS problem, Choi et al. (2018) showed that the market demand is given by

\[
d_{DS}(h) = \mathbb{P}_{\hat{W}} \left( \hat{W}_h \geq \max_{k \in J} \hat{W}_k \right)
\]

where \( \hat{W}_k = X_k + \min \{ Y_k, \xi_k \} \).

Now suppose that the seller of a product on position \( h \) sets the mean of \( X_h \), for example by choosing a price. In the SD problem, this is equivalent to choosing \( \mathbb{P}_{W_h} \left( W_h \geq z^d \right) \); the probability that the consumer inspects and then stops search by buying the seller’s product. Importantly, this does not directly depend on partial valuations of both products at earlier, and products at later positions. This results from the stopping decisions, and given the infinite number of products a consumer will never recall a product discovered earlier.

In contrast, in the DS problem, choosing the mean of \( X_h \) influences demand through the joint distribution of all products. As consumers are aware of all products, they compare all partial valuations. Hence, each seller’s choice of partial valuations affects all other sellers demand, and sellers do not make independent decisions.