Search for the glueball content of hadrons in $\gamma p$ interactions at GlueX

Thomas Gutsche,1 Serguei Kuleshov,2 Valery E. Lyubovitskij1,3,4 and Igor T. Obukhovsky5

1Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany
2Departamento de Física y Centro Científico Tecnológico de Valparaíso (CCTVal), Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile
3Department of Physics, Tomsk State University, 634050 Tomsk, Russia
4Laboratory of Particle Physics, Mathematical Physics Department, Tomsk Polytechnic University, Lenin Avenue 30, 634050 Tomsk, Russia
5Institute of Nuclear Physics, Moscow State University, 119991 Moscow, Russia

We suggest a theoretical framework for the description of double photon and proton-antiproton photoproduction off the proton $\gamma + p \rightarrow 2\gamma + p$ and $\gamma + p \rightarrow \bar{p}p + p$ which takes into account the contribution of the scalar mesons $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. These scalars are considered as mixed states of a glueball and nonstrange and strange quarkonia. Our framework is based on the use of effective hadronic Lagrangians that phenomenologically take into account two-gluon exchange effects governing the $\gamma + p \rightarrow 2\gamma + p$ and $\gamma + p \rightarrow \bar{p}p + p$ processes. Present results can be used to guide the possible search for these reactions by the GlueX Collaboration at JLab.

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I. INTRODUCTION

The lowest-lying glueballs with quantum numbers $J^{PC} = 0^{++}, 2^{++}$ might give significant contributions to hadronic production cross sections. In this respect useful reactions could be a proton-antiproton pair and photon photoproduction off a proton target in exclusive experiments (e.g., the GlueX experiment at JLab [1]) set up for the production of states with positive $P$ and $C$ parity, e.g. in two-particle channels with $J^{PC} = 0^{++}, 2^{++}$:

$$\gamma + p \rightarrow p + G \rightarrow p + (2\gamma, 2\eta, 2\eta', \pi\pi, KK, \bar{p}p, \ldots).$$

(1)

When focusing on events without mesons in the final state, the detection of the glueball or a glueball component in a hadron is significantly simplified. In this case only the two-body glueball decay channels $G \rightarrow 2\gamma$ and $G \rightarrow \bar{p}p$ will be present. One should stress that the mode $G \rightarrow 2\gamma$ is forbidden when the initial state is pure glueball and this process can only proceed via mixing of the glueball with nonstrange and strange quarkonia components [2,3]. Here the $\bar{p}p$ pair is in a definite partial wave: $^3P_0$ or $^3P_2$ in correspondence with the quantum numbers of the glueball $J^{PC} = 0^{++}$ or $2^{++}$.

Due to the constraints mentioned above, the mechanism of the reaction

$$\gamma + p \rightarrow p + G \rightarrow p + 2\gamma$$

(2)

or

$$\gamma + p \rightarrow p + G \rightarrow p + \bar{p}p$$

(3)

proceeding through the intermediate glueball $G$ could correspond to the search for the glueball content of the scalar $f_0$ and tensor $f_2$ mesons.

Note that the initial photon has the set of quantum numbers $J^{PC} = 1^{--}$; therefore, it could excite resonances with $J^{PC} = 0^{++}$ and $2^{++}$ only if it is absorbed by a hadron with the same quantum numbers, i.e., vector mesons, $\rho^0$ and $\omega$ from the meson cloud of the proton target. The Feynman diagrams corresponding to these processes are shown in Figs. [1](a) and [1](c). The QCD diagrams in the left panel of Fig. [1] can effectively be understood in terms of diagrams on the hadronic level as displayed in the right panel [Figs. [1](b) and [1](d)]. In the present manuscript we will concentrate on the analysis of the role of scalar mesons. The contribution of tensor mesons will be studied in a forthcoming paper.

The diagrams contributing to the photoproduction of $2\gamma$ and $\bar{p}p$ in the right panel can be calculated on the hadronic level using effective Lagrangians including photons [2,4]. These methods were proposed and successfully applied...
in Refs. [2]-[5] for the description of the mass spectrum and decay rates of scalar, pseudoscalar, and tensor mesons (including radially excited states) and three-body decays of $J/\psi$. In particular, our approach is based on the hypothesis of a mixed structure of the scalar $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ mesons, which involve the nonstrange and strange quarkonia and glueball components [2]-[3]. The use of such effective Lagrangians for light hadrons composed of light $(u,d,s)$ quarks in the energy region up to a few GeV is well justified — it is generalized chiral perturbation theory involving chiral fields, low-lying baryons and their resonances.

Note that taking into account vector meson ($\rho$ and $\omega$) exchange does not give a precise description of the amplitudes under study. Indeed one should include the Regge poles that will dominate in the physical amplitudes. As is well-known, Regge phenomenology is successfully used in the description of new highly excited meson and baryon -precision data on photo- and electroproduction of mesons of nucleons and nuclei [8]-[11] at energies $\sqrt{s} \geq 2–3$ GeV, allowing the emission of new highly excited meson and baryon resonances. For the present kinematical regime, Regge phenomenology, which is justified for asymptotically high energies $s \gg m^2$, allows for a description of photo- and electroproduction on a qualitative level. By fitting free parameters, such as little-known coupling constants and form factors, one can also obtain a quantitative description of data.

In the photoproduction of mesons with positive $P$- and $C$-parity, which could mix with the possibly existing low-lying glueball with $J^{PC} = 0^{++}$, the differential cross section is defined by exchange diagrams with mesons having negative charge parity and $P = \pm$. Note, that because low-lying scalar and pseudoscalar mesons with $J^{PC} = 0^{\pm}$ do not exist, the leading contribution to the matrix elements gives exchange by vector and axial-vector mesons with $J^{PC} = 1^{\pm}$. It is sufficient that a Reggeization of these meson exchanges leads to an amplitude qualitatively different from the corresponding Born amplitude in Born approximation (Feynman diagram). In our approach we consider specific processes for which we make an assumption that the $\rho$, $\omega$ Regge amplitude has a pole at $t \simeq -0.6$ GeV$^2$; i.e., in this range of values of the $t$ variable it is strongly suppressed in comparison with the Born amplitude. Note that such an assumption is not unique. For instance, in Ref. [25] in the consideration of pion-nucleon elastic scattering, there are two amplitudes involving the $\rho$ Regge poles. One has a zero for $t \sim -0.1$ GeV$^2$.

Reggeization of the diagrams corresponding to the $\rho$ and $\omega$ meson exchange effectively leads to a replacement of vector mesons poles in the propagators by poles lying on the linear angular-momentum $J = \alpha(t)$ Regge trajectories $\alpha_V(t) = \alpha_{0V} + \alpha'_V t$ (see details below). Axial-vector meson exchange is Reggeized in analogy, but it does not change the final result very much because in the reaction under consideration axial-vector exchange does not interfere with the vector one. A sizable influence on the final result comes from the uncertainty related to poor knowledge of meson-nucleon coupling constants. In the present manuscript we will consider two scenarios for the choice of the set of vector couplings: 1) Variant I, the standard “soft set” of vector couplings and 2) Variant II, the “hard set” of vector coupling constants usually used in the Regge approach to photo- and electroproduction of mesons [8]-[11]. In both cases the parameters of the Regge trajectories will also be different. In both scenarios we will use the same specific mixing scheme of scalar mesons obtained in Ref. [3] from the analysis of $J/\psi$ decays.

The paper is organized as follows. In Sec. II, we derive the formalism for the description of photoproduction off the proton target. We present the phenomenological hadronic Lagrangian, including photons. Then we calculate the matrix elements for the process under study. In Sec. III we discuss our numerical results for the differential and double-differential cross sections for two-photon and nucleon-antinucleon photoproduction. Finally, we give a brief conclusion.

II. FORMALISM

We start with the definition of the phenomenological Lagrangians. We indicate the terms that are relevant for the description of photon-proton collisions, including the resonance contributions of the scalar mesons $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. The full interaction Lagrangian [2,3], relevant for the description of the photoproduction processes $\gamma + p \rightarrow 2\gamma + p$ and $\gamma + p \rightarrow \bar{p}p + p$ is given by a sum of interaction Lagrangians

$$\mathcal{L}_{\text{full}}(x) = \mathcal{L}_{VVNN}(x) + \mathcal{L}_{f_0NN}(x) + \mathcal{L}_{f_0\gamma\gamma}(x) + \mathcal{L}_{f_0V\gamma}(x)$$

(4)
\[
\mathcal{L}_{VNN}(x) = g_{\mu NN} \bar{N}(x) \gamma^\mu \bar{p}_\mu(x) \tau N(x) + \frac{f_{\rho NN}}{4M_N} \bar{N}(x) \sigma^{\mu\nu} \bar{R}_{\mu\nu}(x) \tau N(x)
\]
\[
+ g_{\omega NN} \bar{N}(x) \gamma^\mu \omega_\mu(x) N(x) + \frac{f_{\omega NN}}{4M_N} \bar{N}(x) \sigma^{\mu\nu} W_{\mu\nu}(x) N(x),
\]
\[
\mathcal{L}_{f_0 NN}(x) = F_{\mu\nu}(x) F^{\mu\nu}(x) \sum_i g_{f_i NN} f_i(x),
\]
\[
\mathcal{L}_{f_0 \gamma \gamma}(x) = \frac{e^2}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \sum_i g_{f_i \gamma \gamma} f_i(x),
\]
\[
\mathcal{L}_{f_0 V \gamma}(x) = \frac{e}{2} F_{\mu\nu}(x) \sum_i \left[ g_{f_i \rho \gamma} f_i(x) R^{\rho \mu\nu}(x) + g_{f_i \omega \gamma} f_i(x) W^{\mu\nu}(x) \right].
\]

Here we introduce the following notation: \( f_i \) is the set of scalar mesons — \( f_1 = f_0(1370), f_2 = f_0(1500), \) and \( f_3 = f_0(1710) \); \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, R^{\rho\mu\nu}_{\mu\nu} = \partial_\mu p_\rho - \partial_\rho p_\mu, \) and \( W_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) are the stress tensors of the electromagnetic field and \( \rho \) and \( \omega \) mesons, respectively.

The scalar fields are considered as mixed states of the glueball \( G \) and nonstrange \( N \) and strange \( S \) quarkonia \( \text{[2, 3]} \) (see more details in the Appendix and the review \( \text{[12]} \) for a discussion on different interpretations of these states):

\[
f_i = B_{i1} N + B_{i2} G + B_{i3} S.
\]

The \( B_{ij} \) are the elements of the mixing matrix rotating bare states \( (N, G, S) \) into the physical scalar mesons \( (f_0(1370), f_0(1500), f_0(1710)) \). In Refs. \( \text{[2, 3]} \) we studied in detail different scenarios for the mixing of \( N, G, \) and \( S \) states. Here we proceed with the scenario fixed in Ref. \( \text{[3]} \) from a full analysis of strong \( f_0 \) decays and radiative decays of the \( J/\psi \) with the scalars in the final state:

\[
B = \begin{pmatrix}
0.75 & 0.60 & 0.26 \\
-0.59 & 0.80 & -0.14 \\
-0.29 & -0.05 & 0.95
\end{pmatrix}.
\]

The coupling constants involving scalar mesons are given in terms of the matrix elements \( B_{ij} \) and the effective couplings \( c^e_i, c^s_i, c^g_i, \) and \( c^f_i \) of \( \text{[3]} \):

\[
g_{f_i \gamma \gamma} = \frac{80}{9\sqrt{2}} c^e_i B_{i1} + \frac{32}{\sqrt{3}} c^g_i B_{i2} + \frac{16}{9} c^s_i B_{i3},
\]

\[
g_{f_i \rho \gamma} = 3g_{f_i \omega \gamma} = B_{i1} c^e_i + B_{i2} \sqrt{\frac{2}{3}} c^g_i.
\]

The effective couplings \( c^e_i = 0.056 \text{ GeV}^{-1}, c^s_i = 0.003 \text{ GeV}^{-1}, c^g_i = 1.592 \text{ GeV}^{-1}, \) and \( c^f_i = 0.078 \text{ GeV}^{-1} \) are fixed from data involving the scalar mesons \( f_i \). In case of the \( f_i NN \) couplings we suppose that they are dominated by the coupling of the nonstrange component to the nucleon,

\[
g_{f_i NN} \approx B_{i1} g_{NNN}.
\]

The coupling \( g_{NNN} \) can be identified with the coupling of the nonstrange scalar \( \sigma \) meson to nucleons,

\[
g_{NNN} = g_{\sigma NN} \approx 5
\]

which plays an important role in phenomenological approaches to the nucleon-nucleon potential generated by meson exchange \( \text{[13]} \).

Next we write down the standard expressions for the invariant amplitudes \( \mathcal{M}_{\gamma \gamma}^{\gamma \gamma} \) and \( \mathcal{M}_{\gamma \gamma}^{pp} \) sketched in Figs. \( \text{[1]} \) (b) and (d):

\[
\mathcal{M}_{\gamma \gamma}^{\gamma \gamma} = \sum_V \sum_i G_{\gamma f f}^\gamma(i, V) \bar{u}(p') \left[ \gamma^\mu G_V + \frac{i\sigma^{\mu\nu} k_\nu}{2M_N} \right] u(p) \frac{-g_{\mu\nu} + k_\mu k_\nu/M_N^2}{M_N^2 - k^2 + i\Gamma_V M_V} \times \left( k q \delta^{\alpha\beta} - k^\beta q^\alpha \right) \frac{1}{M_{\tilde{f}}^2 - \mathbf{l}^2 + i\Gamma_{\tilde{f}} M_{\tilde{f}}} \left[ q_1^\alpha q_2^\beta - g^\alpha\beta q_1 q_2 \right] \epsilon_\beta(q) \epsilon^*_\rho(q_1) \epsilon^*_\sigma(q_2)
\]

(14)
and
\[
M_{\text{inv}}^{pp} = \sum_{V} \sum_{i} G_{\text{eff}}^{pp}(i, V) \bar{u}(p') \left[ \gamma^\mu G_V + \frac{i\sigma^{\mu\nu}k_\nu}{2M_V} F_V \right] u(p) \frac{-g_{\mu\alpha} + k_\mu k_\alpha/M_V^2}{M_V^2 - k^2 + i\Gamma_V M_V} \times (kq g^{\alpha\beta} - k^\beta q^\alpha) \frac{1}{M_{T_i}^2 - q^2 + i\Gamma_{T_i} M_V} \epsilon_\beta(q) \bar{u}(q_1) v(q_2). \tag{15}
\]

The momenta \( p \) and \( p' \) are of the initial and final proton; \( q \) is the photon momentum in the initial state; \( q_1 \) and \( q_2 \) are the momenta of two photons or the nucleon and antinucleon in the final state; and \( k \) and \( l \) (\( i = k + q \)) are the intermediate vector and scalar meson momenta, respectively. Here the constants \( G_{\text{eff}}^{\gamma\gamma}(i, V) = g_{f_i \gamma\gamma} g_{f_i \gamma\gamma} \) and \( G_{\text{eff}}^{pp}(i, V) = g_{f_i \gamma\gamma} g_{f_i \gamma\gamma} \) are the effective couplings for \( \gamma \gamma \) and \( pp \) photoproduction.

The expression for the twofold differential cross section of the double photon production,
\[
\frac{d\sigma_{\gamma\gamma}}{ds dt_1} = \frac{\alpha^3}{4} \frac{1}{(s - m_N^2)^2} |M_{\gamma\gamma}|^2
\]
implies the summation over contributions from intermediate mesons, \( V = \rho, \omega \) and \( f_i, i = 1, 2, 3 \), to the amplitude. The spin-averaged square of the amplitude is given by
\[
|M_{\gamma\gamma}|^2 = \frac{1}{4} \sum_{\lambda_i = \pm 1} \frac{1}{2} \sum_{s_i = \pm 1/2} \sum_{V} \sum_{i} M_{\gamma\gamma}^{V,V} = -s_2 \sum_{V,V'} \sum_{i,j=1} G_{fV}(i, j; V'V') T_{V,N,N}(V, V', s, s, t_1) D_V(V, V'; t_1) D_f(i, j, s_2), \tag{17}
\]
where \( s = (p + q)^2, s_2 = (q_1 + q_2)^2 \), and \( t_1 = (p' - p)^2 \) are Mandelstam variables, corresponding to the total energy and transverse momentum squared to the two-photon pair and to the vector meson. The values of \( s_2 \) and \( t_1 \) are kinematically constrained to the intervals
\[
0 \leq \sqrt{s_2} \leq \sqrt{s} - m_N, \quad t_{\text{min}} \leq t_1 \leq t_{\text{max}}, \tag{18}
\]
where
\[
t_{\text{max/min}} = 2m_N^2 - \frac{1}{2s}[(s + m_N^2)(s - s_2 + m_N^2) \mp (s - m_N^2)\lambda^{1/2}(s, s_2, m_N^2)], \tag{19}
\]
and \( \lambda = 1/137.036 \) is the fine-structure constant.

The vertex functions \( G_{fV}(i, j; V, V') \), \( T_{V,N,N}(V, V', s, s, t_1) \), \( D_V(V, V'; t_1) \), and \( D_f(i, j, s_2) \) are defined as
\[
G_{fV}(i, j; V, V') = g_{f_i \gamma\gamma} g_{f_j \gamma\gamma} g_{f_i \gamma\gamma} g_{f_j \gamma\gamma},
\]
\[
T_{V,N,N}(V, V', s, s, t_1) = (G_V + F_V) (G_{V'} + F_{V'}) \frac{t_1(t_1 - s_2)^2}{4} + \left( G_V G_{V'} - \frac{t_1}{4m_N^2} F_V F_{V'} \right)
\times \left( \frac{m_N^2}{4} (t_1 - s_2)^2 + \frac{1}{4} (m_N^2 + t_1 - s_2) \right),
\]
\[
D_V(V, V'; t_1) = \frac{(t_1 - M_V^2)(t_1 - M_{V'}^2) + M_V M_{V'} (t_1 - M_V^2 + M_{V'}^2)}{[(t_1 - M_V^2)^2 + M_V^2 (t_1 - M_{V'}^2)] [(t_1 - M_{V'}^2)^2 + M_{V'}^2 (t_1 - M_V^2)]},
\]
\[
D_f(i, j; s_2) = \frac{(s_2 - M_f^2)(s_2 - M_f^2) + M_f^2 (s_2 - M_f^2)}{[(s_2 - M_f^2)^2 + M_f^2 (t_1 - M_{f_i}^2)] [(s_2 - M_{f_i}^2)^2 + M_{f_i}^2 (t_1 - M_f^2)]}, \tag{20}
\]
where the indices \( i, j = 1, 2, 3 \) number the scalar \( f_0(1370), f_0(1500), \) and \( f_0(1710) \) mesons and indices \( V, V' \) correspond the \( \rho \) and \( \omega \) vector mesons; \( g_{f_i \gamma\gamma} \) and \( f_{V,N,N} \) are the Dirac and Pauli coupling constants of vector mesons to nucleons, respectively; and \( g_{f_i \gamma\gamma} \) are the couplings of vector and scalar mesons with photons.

Note that for GlueX energies the leading contribution to the matrix element of \( 2\gamma \) production scales as \( G_V G_{V'} t_1 s_2^2 \), where the factor \( t_1 s_2^2 \) comes from the helicity flip in the \( \gamma - f_0 \) vertex squared.

The meson parameters can be grouped as follows:
(1) Parameters involving masses, widths and couplings of scalar mesons have been fixed from data with [3]

\[ M_{f_1} = 1.432 \text{ GeV}, \quad M_{f_2} = 1.510 \text{ GeV}, \quad M_{f_3} = 1.720 \text{ GeV}, \]
\[ \Gamma_{f_1} = 0.350 \text{ GeV}, \quad \Gamma_{f_2} = 0.109 \text{ GeV}, \quad \Gamma_{f_3} = 0.135 \text{ GeV}, \]
\[ g_{f_1 \gamma \gamma} = 0.30 \text{ GeV}^{-1}, \quad g_{f_2 \gamma \gamma} = -0.21 \text{ GeV}^{-1}, \quad g_{f_3 \gamma \gamma} = -0.01 \text{ GeV}^{-1}, \]
\[ g_{f_1 \gamma \gamma} = 1.24 \text{ GeV}^{-1}, \quad g_{f_2 \gamma \gamma} = -0.90 \text{ GeV}^{-1}, \quad g_{f_3 \gamma \gamma} = -0.47 \text{ GeV}^{-1}, \]
\[ g_{f_1 NN} = 3.75, \quad g_{f_2 NN} = -2.95, \quad g_{f_3 NN} = -1.45. \]

(2) Parameters involving masses and widths of vector mesons taken are from the PDG compilation [14]

\[ M_\rho = 0.7755 \text{ GeV}, \quad M_\omega = 0.7866 \text{ GeV}, \]
\[ \Gamma_\rho = 0.149 \text{ GeV}, \quad \Gamma_\omega = 0.0085 \text{ GeV}, \]
\[ G_\rho = 2.3, \quad G_\omega = 3G_\rho, \]
\[ F_\rho = 3.66G_\rho, \quad F_\omega = -0.07G_\omega. \]

For the process \( \gamma p \rightarrow p + p \bar{p} \) one has correspondingly

\[
\frac{d\sigma_{p\bar{p}}}{ds_2 dt} = \frac{\alpha}{32\pi^2 (s - m_N^2)} |M_{pp}|^2, \quad 2m_N \leq \sqrt{s_2} \leq \sqrt{s} - m_N, \quad t_{\text{min}} \leq t \leq t_{\text{max}}.
\]

where

\[
|M_{pp}|^2 = \frac{1}{2} \sum_{\lambda_\rho = \pm 1} \frac{1}{2} \sum_{s_\rho = \pm 1/2} \left|\sum_V \sum_i M_{pp}^{\rho V}, \sum_{i,j} M_{inv}^{\rho V, i,j} \right|^2
\]

\[ = -(s_2 - 4m_N^2) \sum_{V,V'} \sum_{i,j=1}^3 G_{fVVN(i,j;V,V')} T_{VVNN(V,V';s,s_2,t_1)} D_{V}(V,V';t_1) D_f(i,j,s_2) \]

and

\[
G_{fVVN(i,j;V,V')} = g_{f_1 NN} g_{f_2 NN} g_{f_3 \gamma \gamma} g_{f_1 \gamma \gamma}.
\]

As we stressed already in the Introduction, the vector meson exchanges in the \( t \) channel are Reggeized. In particular, Reggeization of the diagrams corresponding to the \( \rho \) and \( \omega \) meson exchange effectively leads to the replacement of Feynman propagators by contributions of poles lying on the angular-momentum \( J = \alpha(t) \) linear Regge trajectories \( \alpha_V(t) = \alpha_0 + \alpha_V' t \) as

\[
\frac{1}{t - m_V^2} \rightarrow \left( \frac{s}{80} \right)^{\alpha_V(t) - 1} \left( -\alpha_0' \right) \Gamma(1 - \alpha_V(t)) \frac{-1 + e^{i\pi \alpha_V(t)}}{2}.
\]

As before, in the present manuscript we consider two scenarios for the choice of the set of vector couplings

(1) Variant I, the standard “soft set” of \( G_\rho, G_\omega, F_\rho, F_\omega \) couplings and

(2) Variant II, the “hard set” of vector coupling constants \( G_\rho = 3.4, G_\omega = 15, F_\rho = 20.7, \) and \( F_\omega = 0 \) usually used in the Regge approach to photo- and electroproduction of mesons [8,11].

The parameters of the Regge trajectories will also be different in both cases: \( \alpha_{0\rho} = 0.53, \alpha_{0\omega} = 0.85 \text{ GeV}^{-2}, \) \( \alpha_{0\omega} = 0.4, \)
\( \alpha_{0\omega} = 0.85 \text{ GeV}^{-2} \) in the case of Variant I and \( \alpha_{0\rho} = 0.55, \alpha_{0\omega} = 0.8 \text{ GeV}^{-2}, \) and \( \alpha_{0\omega} = 0.44, \alpha_{0\omega} = 0.9 \text{ GeV}^{-2} \) in the case of Variant II. In both scenarios we will use the same specific mixing scheme of scalar mesons obtained in Ref. [3]. The couplings of scalar mesons to vector mesons and photons [see Eq. (21)] have been fixed in Ref. [3] from the analysis of \( J/\psi \) decays.

III. RESULTS

In Fig. 2 we present the results of our calculation for the differential cross section \( d\sigma/dt \) (as an integral of the double differential cross section \( d\sigma_{\gamma\gamma}/dtds_2 \) of \( 2\gamma \) photoproduction over the Mandelstam variable \( ds_2 \)) for two values for the energy of the initial photon: \( E_\gamma = 5 \) and 9 GeV. We compare results for these two variants of the coupling constants (“soft” and “hard”). Also for a comparison we present results of calculations using Feynman diagrams with vector meson exchange and taking into account its finite width.
The obtained results can be useful for an estimate of a signal from a mixed ($f_0 + G$) scalar meson in the $2\gamma$ channel. In this case the contributions of other subprocesses into $2\gamma$ photoproduction [for example via photoproduction of $\pi^0$, $f_0/\rho_0(980)$ or other mesons of positive charge parity] we consider as background, which can be estimated using known calculations of meson photoproduction \cite{8-11}. In Fig.3 we display the double-differential cross section $d\sigma_{\gamma\gamma}/dt ds_2$ at energies 5 and 9 GeV for our basic scenario (Variant I).

In Fig. 4 the results on $d\sigma/dt$ and $d\sigma/dtds_2$ for $pp$ photoproduction at energy $E_\gamma = 9$ GeV are also displayed for Variant I. Due to the threshold of $pp$ photoproduction, which lies above the resonance peaks related to the production of the scalar mesons $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, the differential cross section of the $\gamma + p \rightarrow p + G \rightarrow p + pp$ reaction has no resonance structure over the physical range of the $s_2$ variable. In the two-photon photoproduction corresponding resonance peaks play a very important role as seen from the direct comparison of Figs. 2, 3 and 4.

The absolute value of the cross section is defined in our approach by the parameters and coupling constants \cite{10}-\cite{13}, which have been fixed before in the analysis of data on scalar mesons \cite{2, 3}. The $s_2$ dependence of the differential cross section is generated by the contribution of the scalar meson propagators and the $V_{pp}$ vertex. For a model dependence of the coupling constants in the $V_{pp}$ vertex we have considered two variants, “soft” (Variant I) and “hard” (Variant II). The hard variant results in differential cross sections that are several times greater than the soft case. Nevertheless both variants predict a magnitude of the differential cross section $d\sigma/dt$ of about 10 nb/GeV$^2$ or greater in the region of the peak at $-t \approx 0.1-0.2$ GeV$^2$. These values are tentative, but they may be used as a guide for further research.

It should be noted that the Regge pole amplitude is suppressed (because of the zero at $t \approx -0.6$ GeV$^2$) when compared to the contribution of the vector-meson exchange diagrams (dotted-dashed curves in Figs. 2 and 4). Recently it was shown that the contribution of the Regge cut can considerably compensate this drawback at least in meson photoproduction for energies of $E_\gamma = 2-3$ GeV. It seems reasonable that the Regge-pole terms should be used only as a rough approximation in the considered region of small values of $\sqrt{s} \approx 2-4$ GeV.

It is important to mention scalings of the cross sections considered in the present manuscript on $s$ and $s_2$ variables. The differential cross sections scales at large $s$, $s_2$ as $O(1)$ for two-photon photoproduction and $O(s)$, $O(s_2)$ for proton-antiproton photoproduction.

In conclusion, we proposed the approach for the description of double photon and proton-antiproton photoproduction off the proton, which takes into account the contribution of the scalar mesons $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. The mesons are considered as mixed states of a glueball and nonstrange and strange quarkonia. Our framework is based on the use of effective hadronic Lagrangians that take into account two-gluon exchange effects governing the $\gamma + p \rightarrow 2\gamma + p$ and $\gamma + p \rightarrow pp + p$ processes. We presented results that can be used for a possible search for these reactions by the GlueX Collaboration at JLab.

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Appendix A: Mixing of scalar mesons

Here we briefly discuss the mixing of the glueball with the scalar-isoscalar quarkonia states $2\otimes 2$ and octet ($S^8$) scalar quarkonia states.

We start with the Lagrangian involving the scalar mesons and glueball,

$$\mathcal{L} = \frac{1}{2} \langle D_{\mu}\phi D^{\mu}\phi - M_\phi^2 \phi^2 \rangle + \frac{1}{2} \langle \partial_{\mu}G\partial^{\mu}G - M_G^2 G^2 \rangle \quad \text{(A1)}$$

where $S = \sum_{i=0}^{8} S_i \lambda_i / \sqrt{2}$. Singlet $S^0$ and octet $S^8$ components are linear combinations of nonstrange $N$ and strange $S$ components with the respective flavor content $|uu + dd|/\sqrt{2}$ and $ss$

$$S^0 = \sqrt{2/3} N + \sqrt{1/3} S, \quad S^8 = \sqrt{1/3} N - \sqrt{2/3} S. \quad \text{(A2)}$$
In Eq. (A1) the scalar nonet states have the same mass $M_S$, corresponding to the flavor and large $N_c$ limits. Deviations from this configuration (i.e., from the large $N_c$ limit) are encoded in $L_{mix}^S$, including quarkonia-glueball mixing, leading to different masses for the scalar mesons (in [13] this is explicitly shown for the scalar nonet, but no glueball is included; the glueball mixing is also a deviation from large $N_c$, since in this limit the glueball and the quarkonia states decouple). As a result, the scalar-isoscalar sector can be described by the most general Klein-Gordon (KG) Lagrangian, including mixing among the $N$, $S$, and the bare glueball $G$ configurations [16-20]:

$$L_{KG} = -\frac{1}{2} \sum_{\Phi=N,G,S} \Phi^\dagger \left( \partial^\mu \partial_\mu - \frac{1}{M_\Phi^2} \right) \Phi - f_{GS} - \sqrt{2} f_{rGN} \epsilon_{NS},$$

(A3)

where $\square = \partial^\mu \partial_\mu$. The parameter $f$ is the quarkonia-glueball mixing strength, analogous to the parameter $z$ of Refs. [16-20]; $z$ refers to the quantum mechanical case where the mass matrix is linear in the bare mass terms, $f$ in turn is related to the quadratic Klein-Gordon case. The connection between $f$ and $z$ discussed in Refs. [16,21] leads to the approximate relation $f \approx 2z M_G$. If $r = 1$, then the glueball is flavor blind and mixes only with the quarkonium flavor singlet $S^0$ (this is the case in Refs. [16,19,20]); a value $r \neq 1$ takes into account a possible deviation from this limit. However, as deduced from the analyses of Refs. [17,18,21] $r$ is believed to be close to unity. In the following we will use the limit $r = 1$; i.e., we restrict to a flavor blind mixing.

The parameter $\epsilon$ induces a direct mixing between $N$ and $S$ quarkonia states. This effect is neglected in [16,18,20], where flavor mixing is considered a higher-order effect. However, a substantial $N-S$ mixing in the scalar sector is the starting point of the analysis of Refs. [22,23]. The origin of quarkonia flavor mixing is, according to [22,23], connected to instantons as in the pseudoscalar channel, but with opposite sign (see also [24]): the mixed physical fields $\Phi = \sum_{\alpha=0}^{\infty} \Phi^\alpha$, $\Phi^\alpha \equiv \Phi^\alpha - \Phi^0$, $\Phi^0 \equiv \Phi_{N(S)}$ (this is the case in Refs. [16,19,20]); a value $\epsilon < 0$, $\epsilon > 0$, or $\epsilon = 0$ leads to the phase structure as in Refs. [22,23], but the quantitative results and interpretation will differ. The physical scalar states $|i\rangle$ are identified with $i = f_1 \equiv f_0(1370)$, $f_2 \equiv f_0(1500)$, and $f_3 \equiv f_0(1710)$, which are set up as linear combinations of the bare states: $|i\rangle = B^0(N) + B^0(G) + B^0(S)$. The amplitudes $B^0$ are the elements of a matrix $B$ that diagonalize the mass matrix of bare states including mixing, which in turn gives rise to the mass matrix of physical states $\Omega' = \text{diag}(M_1^2, M_2^2, M_3^2)$. In the following we use [2,3] a best fit of the parameters entering in Eqs. (A1) and (A3) to the experimental averages of masses and decay modes listed by PDG [14]. In the present manuscript we use the latest set of the parameters fixed in Ref. [3] with accuracy $\chi^2_{tot} = 19.82$ [see Eq. (21)].

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FIG. 1: QCD → Effective hadronic Lagrangian correspondence for $\gamma p$ reactions.

FIG. 2: Differential cross section $d\sigma/dt$ (in units of nb/GeV$^2$) for $\gamma\gamma$ photoproduction off the proton in the model based on the mixed structure of the scalar $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ mesons. The initial photon energies are $E_\gamma = 5$ GeV (left panel) and 9 GeV (right panel). Solid and short-dashed curves: Theoretical predictions in the glueball model with the Reggeized vector meson exchange for soft (Variant I) and hard (Variant II) $VNN$ coupling constants, respectively. Dotted-dashed curves: Results in the same model, but without Reggeization of the vector meson exchange.
FIG. 3: Double-differential cross sections $d\sigma/ds_2 dt_1$ (in units of nb/GeV$^4$) for $\gamma\gamma$ photoproduction off the proton. The same model as in Fig.2 for the soft set of $V_{NN}$ coupling constants (Variant I). Left: $E_{\gamma} = 5$ GeV. Right: 9 GeV.

FIG. 4: Differential (left panel) and double-differential (right panel) cross sections, $d\sigma/dt_1$ and $d\sigma/ds_2 dt_1$, for $p\bar{p}$ photoproduction off the proton. The same model as in Figs.2-3 with the soft (Variant I) and hard (Variant II) sets of $V_{NN}$ coupling constants. The same notations as in Figs. 2 and 3. $E_{\gamma} = 9$ GeV.