Input-Output Relation and Performance of RIS-Aided OTFS 
With Fractional Delay-Doppler

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Abstract—This letter presents an early investigation of reconﬁgurable intelligent surfaces (RIS)-aided orthogonal time-frequency space (OTFS) in high-Doppler channels. We derive the end-to-end delay-Doppler (DD) domain input-output relation of a RIS-aided OTFS system, considering rectangular pulses and fractional delay-Doppler values. We also consider a Zak receiver for RIS-aided OTFS that converts the received time-domain signal to DD domain in one step using Zak transform, and derive its end-to-end input-output relation. Our simulation results show that i) RIS-aided OTFS performs better than OTFS without RIS, ii) Zak receiver performs better than a two-step receiver, and iii) RIS-aided OTFS achieves superior performance compared to RIS-aided OFDM.

Index Terms—RIS-aided OTFS, delay-Doppler domain, end-to-end input-output relation, Zak receiver, RIS-aided OFDM.

I. INTRODUCTION

In the recent literature, reconﬁgurable intelligent surface (RIS)\cite{11}, \cite{2}, and orthogonal time-frequency space (OTFS) modulation\cite{3}, \cite{4}, \cite{5} have emerged as promising physical layer techniques. RIS technology is a power-eﬃcient form of wireless communication, and OTFS technology is a means for providing reliable high-mobility support. RIS aids communication between the transmitter and receiver by smartly controlling the propagation environment with tunable reﬂecting elements\cite{6}.

The newly introduced OTFS modulation\cite{3} performs signiﬁcantly better than OFDM in high-mobility environments. OTFS exploits the idea of multiplexing information symbols in delay-Doppler (DD) domain instead of time-frequency (TF) domain. Also, OTFS views time-varying channels in the DD domain where the time variations are slow, which simpliﬁes channel estimation.

Recent studies have investigated the use of RIS in OFDM systems, showing that RIS can enhance OFDM performance\cite{8}, \cite{9}, \cite{10}. Also, next-generation wireless systems will need to provide ubiquitous tetherless connectivity in high-mobility scenarios with increased power/spectral eﬃciency. Therefore, a combination of RIS and OTFS can simultaneously oﬀer the beneﬁts of both power eﬃciency (due to RIS) and robustness in high mobility (due to OTFS). Recently, \cite{11} considered a RIS-aided OTFS system, derived its input-output relation, and showed performance gains. While this study showed the beneﬁt of using RIS in OTFS, it considered ideal bi-orthogonal pulses and integer DD values. On the other hand, practical pulse shapes do not obey bi-orthogonality condition, and fractional DD values will be encountered in practice. This letter focuses on RIS-aided OTFS with rectangular pulses and fractional DD values, which are practically more relevant. The new and novel contributions in this letter are as follows.

• We derive the end-to-end DD domain input-output relation of a RIS-aided OTFS system with rectangular pulses at the transmitter and receiver and fractional DD values. We carry out this derivation for two types of OTFS receivers, namely, 1) a two-step receiver, where the received time domain (TD) signal is converted into a DD domain signal in two steps, viz., TD to time-frequency (TF) domain using Wigner transform and TF domain to DD domain using symplectic ﬁnite Fourier transform, and 2) a single-step Zak receiver, which uses Zak transform to directly convert the received TD domain signal to DD domain.

• Our results show that 1) RIS improves the performance of OTFS, 2) single-step Zak receiver performs better than two-step receiver, and 3) RIS-OTFS achieves superior performance compared to RIS-aided OFDM.

II. RIS-AIDED OTFS WITH TWO-STEP RECEIVER

A RIS-aided OTFS system consists of an OTFS transmitter, an OTFS receiver, and a RIS as shown in Fig. 1. The RIS has $K$ reﬂecting elements, whose phases can be controlled dynamically to enhance desired parameters at the receiver. The adjacent elements with highly correlated channel gains are grouped to a sub-surface to reduce the overhead in the reﬂection phase design. Therefore, RIS has $L$ sub-surfaces, where each sub-surface consists of $N_s = K/L$ adjacent elements. Let the reﬂection coeﬃcient at the $r$th sub-surface be deﬁned as $\phi_r = \gamma_r e^{j\theta_r}, r = 1, \cdots, L$, where $\gamma_r \in [0, 1]$, $\theta_r \in [-\pi, \pi]$ are the reﬂection amplitude and phase of the $r$th sub-surface, respectively.

Information symbols $x[k,l]$ from a modulation alphabet $\mathcal{A}$, $k = 0, \cdots, N-1, l = 0, \cdots, M-1$ are multiplexed in the DD domain at the OTFS transmitter, where $N$ and $M$ are the number of Doppler and delay bins, respectively, in the DD grid. The OTFS transmitter operations involve inverse symplectic ﬁnite Fourier transform (ISFFT) and Heisenberg transform to convert a DD domain signal to TF domain signal and TF signal to TD signal, respectively. The TD domain signal obtained through ISFFT is given by

\[
X[n,m] = \frac{1}{MN} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x[k,l] e^{2\pi j \left(\frac{n k}{N} - \frac{m l}{M}\right)},
\]  

(1)

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where $\rho_{pq}$, $\nu_{pq}$ denote the channel gain, delay, and Doppler indices, respectively, and $\alpha$, $\beta$ are the corresponding fractional parts. Similarly, the received TD signal at the receiver associated with $r$th sub-surface is given by

$$y^r(t) = \phi_t \sum_{q=1}^{P_3} g_q^r x(t - \tau_{pq}^r)e^{j2\pi \nu_{pq}^r(t - \tau_{pq}^r)},$$

where $P_3$ is the number of paths in the RIS-to-receiver link (i.e., $2$nd link), $g_q^r$, $\tau_{pq}^r$, and $\nu_{pq}^r$ are the channel gain, delay, and Doppler of the $q$th path associated with $r$th sub-surface in the $2$nd link, respectively. The number of paths $P_1, P_2$ depend on the number of dominant reflectors in the environment, and typical values of $P_1, P_2$ in standards range from 4 to 10 [15]. The total number of channel coefficients of interest, therefore, is $P_1P_2L$. Substituting for $\tilde{Z}(t)$, (4) can be written as

$$y^r(t) = \phi_r \sum_{q=1}^{P_3} g_q^r h_p^r \rho_{pq}^r x(t - \tau_{pq}^r)e^{j2\pi (\nu_{pq}^r(t - \tau_{pq}^r))},$$

In (5), multiplying and dividing by $e^{j\pi/4}$, we can write

$$y^r(t) = \psi_r \sum_{q=1}^{P_3} g_q^r h_p^r \rho_{pq}^r x(t - \tau_{pq}^r)e^{j2\pi (\nu_{pq}^r(t - \tau_{pq}^r))},$$

where $\rho_{pq}^r \Delta \nu_{pq}^r e^{j2\pi \nu_{pq}^r(t - \tau_{pq}^r)}, \nu_{pq}^r \Delta \nu_{pq}^r + \nu_{pq}^r/2$. The received discrete TF signal is obtained through Wigner transform of the TD signal, which is given by

$$Y^r(t, n) \Delta Y^r(t, nT) e^{-j2\pi \Delta f(t-nT)} dt,$$

$m = 0, \ldots, M-1, n = 0, \ldots, N-1$, where $g_{rx}(.)$ is the receive pulse. Substituting for $y^r(t)$ from (6) in (7), we get

$$Y^r(t, n) = \int_{-\infty}^{\infty} g_{rx}^r(t - nT) e^{-j2\pi \Delta f(t-nT)} dt.$$ 

Substituting for $x(t)$ from (2) in (8), we obtain in (9), as shown at the bottom of the next page.

Rearranging the terms in (9), we write in (10), as shown at the bottom of the next page.

The DD domain signal is obtained by applying SFFT on (10), as

$$y^r[k', l'] \Delta Y^r[k', l'] e^{j2\pi \nu_{pq}^r(t - \tau_{pq}^r)}.$$ (10)

Assuming $g_{rx}(t) = g(t)$ be the rectangular pulses [13], and substituting for $X[n', m']$ from (1) and evaluating the integration in (10), we get

$$y^r[k', l'] = \phi_r \sum_{k=0}^{N-1} x[k, l] \left( \hat{h}_1^r[k', l', k, l] + \hat{h}_2^r[k', l', k, l] \right),$$

where

$$\hat{h}_1^r[k', l', k, l] = \sum_{q=1}^{P_3} \sum_{p=1}^{P_2} g_q^r h_p^r \rho_{pq}^r e^{-j2\pi \nu_{pq}^r(t - \tau_{pq}^r)} \left(1 - \frac{\tau_{pq}^r}{T} \right)$$

and

$$\hat{h}_2^r[k', l', k, l] = \sum_{q=1}^{P_3} \sum_{p=1}^{P_2} g_q^r h_p^r \rho_{pq}^r e^{-j2\pi \nu_{pq}^r(t - \tau_{pq}^r)} \left(1 - \frac{\tau_{pq}^r}{T} \right).$$
Vectorizing (12), we can write

$$y^r = \phi_r H^r x,$$   \hspace{1cm} (13)

where $H^r \in \mathbb{C}^{MN \times MN}$ is the effective cascaded channel matrix for the $r$th sub-surface with the element in its $(l' + k'M + 1)$th row and $(l + kM + 1)$th column being $h[i][k', l', k, l] + h[i][k', l', k, l]$. Finally, the overall end-to-end input-output relation is obtained by adding the reflected signals from all the sub-surfaces at the receiver, as

$$y = \sum_{r=1}^{L} \phi_r H^r x + n = H_{\text{eff}} x + n, \hspace{1cm} (14)$$

where $H_{\text{eff}} = \sum_{r=1}^{L} \phi_r H^r$, $y$ is the combined received signal vector from all the sub-surfaces, and $n$ is the additive noise signal vector at the receiver.

### III. Single-Step Zak Receiver for RIS-Aided OTFS

In this section, we derive the end-to-end input-output relation for the single-step Zak receiver. A motivation for the combination of ISFFT-based transmitter and Zak-based receiver is as follows. At the transmitter side, the ISFFT-based approach can retain the advantage of building the OTFS transmitter as an overlay on existing multcarrier transmitters. On the other hand, at the user end, the receiver can optionally be implemented using either the SFIT_2 approach or the Zak approach, and therefore a comparison between them is of interest. The Zak representation of a signal $s(t)$ is defined as [13], [14]

$$Z_s(\tau, \nu) \triangleq \sqrt{T} \sum_{k=-\infty}^{\infty} s(t + kT)e^{-j2\pi k\nu T}, \hspace{1cm} (15)$$

The Zak representation of the received signal $y^r(t)$ is

$$Z_{y^r}(\tau, \nu) = \sqrt{T} \sum_{k=-\infty}^{\infty} y^r(t + kT)e^{-j2\pi k\nu T}. \hspace{1cm} (16)$$

Substituting for $y^r(t)$, (16) can be written as

$$Z_{y^r}(\tau, \nu) = \sqrt{T} \phi_r \sum_{k=-\infty}^{\infty} \left\{ \sum_{q=1}^{P_2} \sum_{p=1}^{P_1} \rho^q_{pq} g^r_{pq} h^r_{pq} e^{j2\pi \nu_p (\tau - \tau^r_{pq} + kT)} \right\} e^{-j2\pi k\nu T}. \hspace{1cm} (17)$$

which can be further written as

$$Z_{y^r}(\tau, \nu) = \phi_r \sum_{q=1}^{P_2} \sum_{p=1}^{P_1} \rho^q_{pq} g^r_{pq} h^r_{pq} e^{j2\pi \nu_p (\tau - \tau^r_{pq})} \times \sqrt{T} \sum_{k=-\infty}^{\infty} x(t - \tau^r_{pq} + kT)e^{-j2\pi (\nu - \nu^r_{pq}) kT}$$

$$= \phi_r \sum_{q=1}^{P_2} \sum_{p=1}^{P_1} \rho^q_{pq} g^r_{pq} h^r_{pq} e^{j2\pi \nu_p (\tau - \tau^r_{pq})} Z_x \times (\tau - \tau^r_{pq}, \nu - \nu^r_{pq}), \hspace{1cm} (18)$$

where $Z_x(\cdot)$ denotes the Zak transform of $x(t)$ with delay and Doppler shift of $\tau^r_{pq}$ and $\nu^r_{pq}$ along the delay and Doppler axis, respectively. The Zak transform of the rectangular pulse $g(t)$ is given by [13]

$$Z_g(\tau, \nu) = e^{j2\pi \nu (\frac{\tau}{T})^T}, \hspace{1cm} \tau, \nu, \in (-\infty, \infty). \hspace{1cm} (19)$$

where $[\cdot]$ denotes the floor function. The Zak transform of the transmitted OTFS signal $x(t)$ in (2) is given by [13]

$$Z_x(\tau, \nu) = \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x[k,l] \Psi_{k,l}(\tau, \nu),$$

$$\Psi_{k,l}(\tau, \nu) \triangleq \frac{Z_g(\tau, \nu)}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left[ e^{-j2\pi m T (\nu - \nu^r_{pq} - \frac{1}{2} f)} \times e^{j2\pi m \Delta f (\tau - \frac{\tau^r_{pq}}{T})} \right], \hspace{1cm} \hspace{1cm} (20)$$

where $Z_g(\tau, \nu)$ is the Zak transform of $g(t)$ defined in (19).

In (20), $\Psi_{k,l}(\tau, \nu)$ can be viewed as basis signals in the DD domain. Substituting (19) and (20) in (18), we get

$$Z_{y^r}(\tau, \nu) = \phi_r \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x[k,l] \left[ \sum_{q=1}^{P_2} \sum_{p=1}^{P_1} \rho^q_{pq} g^r_{pq} h^r_{pq} \right]$$

$$\times e^{j2\pi \nu_p (\tau - \tau^r_{pq})} \times \left( \frac{1}{N} \sum_{n=0}^{N-1} e^{-j2\pi n T (\nu - \nu^r_{pq} - \frac{1}{2} f) T} \right)$$

$$\times \left( \frac{1}{M} \sum_{m=0}^{M-1} e^{j2\pi m \Delta f (\tau - \frac{\tau^r_{pq}}{T})} \right). \hspace{1cm} (21)$$

In converting the TD signal to DD domain in the Zak receiver, the DD domain signal is obtained by sampling the Zak
transform of $y^r(t)$ at discrete points ($\tau = \frac{l'T}{M}, \nu = k'\Delta f/N$), for $l' = 0, \ldots, M - 1$ and $k' = 0, \ldots, N - 1$. Therefore, the sampled DD domain signal in the Zak receiver is given by

$$y^r[k', l'] \triangleq \mathcal{Z}y^r \left( \tau = \frac{l'T}{M}, \nu = k'\Delta f/N \right).$$

Substituting (21) in (22), we get

$$y^r[k', l'] = \phi_r \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x[k, l] \hat{g}^r[k', l', k, l],$$

where

$$\hat{g}^r[k', l', k, l] = \sum_{q=0}^{P_2} \sum_{p=1}^{P_1} \rho_{pq} g^r_p h^r_p e^{j2\pi \left(\frac{\nu k + \nu k'}{N} - \frac{\nu n}{N} \right)} \times e^{-j2\pi n T \left(\frac{l' k + l' k'}{M} - \frac{l' n}{M} \right)} \times \left(1 - \frac{1}{N} \sum_{n=0}^{N-1} e^{-j2\pi n T \left(\frac{l' k + l' k'}{M} - \frac{l' n}{M} \right)} \times \left(1 - \frac{1}{M} \sum_{m=0}^{M-1} e^{j2\pi m \Delta f \left(\frac{l' k + l' k'}{M} - \frac{l' m}{M} \right)} \right) \right),$$

for $k' = 0, \ldots, N - 1, l' = 0, \ldots, M - 1$. Vectorizing (23), we get

$$y^r = \phi_r G^r x,$$

where $G^r \in \mathbb{C}^{MN \times MN}$ is the effective cascaded channel matrix for the Zak receiver associated with the $r$th sub-surface with the element in its $(l' + k'M + 1)_{th}$ row and $(l + kM + 1)_{th}$ column being $g^r[k', l', k, l]$. The overall input-output relation is obtained by adding the received signals $y^r$ reflected from all the sub-surfaces at the receiver, as

$$y = \sum_{r=1}^{L} \phi_r G^r x + n.$$

where $G_{eff} = \sum_{r=1}^{L} \phi_r G^r$, $y$ is the combined received vector from all the sub-surfaces and $n$ is the additive noise vector.

### IV. Results and Discussions

In this section, we present the simulation results on the bit error rate (BER) performance of RIS-aided OTFS with two-step receiver and single-step Zak receiver. A carrier frequency ($f_c$) of 4 GHz, a subcarrier spacing ($\Delta f$) of 3.75 kHz, a maximum Doppler of 1.2 kHz, BPSK modulation, and MMSE detection are considered. The channel fade coefficients $h^s_p$ and $g^s_p$ are assumed to be i.i.d and distributed as complex Gaussian with zero mean and variance $1/P$, where $P$ denotes the number of DD channel paths in the corresponding link. The Doppler shift corresponding to $l$th tap is generated using Jakes’s formula, i.e., $\nu_l = \nu_{max} \cos(\psi_l)$, where $\nu_{max}$ denotes the maximum Doppler shift and $\psi_l$ is uniformly distributed in $[-\pi, \pi]$. The delay corresponding to $l$th tap in each link is generated uniformly in $[0, T_{\nu}]$, where $T_{\nu} = 1/\nu_{max}$. For OTFS without RIS, $P$ is taken to be 4 and the delay corresponding to $l$th tap is generated uniformly in $[0, MT_{\nu}]$ and Doppler is generated using Jakes’s formula. At the RIS, reflection phase vector $\Theta = [\theta_1 \theta_2 \cdots \theta_L]$ is chosen such that Frobenius norm of the effective end-to-end DD channel matrix of the RIS-aided OTFS system is maximized [11].

Figure 2 shows the BER performance of RIS-aided OTFS for $M = N = 16$ and $P_1 = P_2 = 4$. We have plotted the BER for two-step receiver and Zak receiver for the following three cases: 1) without RIS, 2) with RIS for $L = 1$, and 3) with RIS for $L = 5$. SNR is defined as the ratio between the transmit power and noise variance, $\sigma^2$. Specifically, SNR is defined as $\frac{P}{\sigma^2}$, and the beamforming effect of the RIS will boost the signal power by a factor proportional to $L^2$, i.e., the SNR at the receiver is $\propto \frac{L^2}{\sigma^2}$ [1]. It is seen that RIS with OTFS offers significantly better performance compared to OTFS without RIS. This performance gain with RIS is due to the boost in the received SNR offered by reflections from the RIS sub-surfaces whose phases are tuned to enhance the SNR at the receiver. The effect of $L^2$ boost in SNR can be seen in Fig. 2 by comparing the performance of $L = 1$ and $L = 5$, where there is a SNR gap of about $10 \log_{10}(5^2) = 13.9$ dB at a BER of $10^{-3}$. It can also be seen that the Zak receiver performs better than the two-step receiver by about 2 dB at $10^{-3}$ BER.

Figure 3 shows the performance comparison between RIS-aided OTFS and RIS-aided OFDM for $M = 12$, $N = 7$, which corresponds to the smallest resource block in LTE. A carrier frequency of 4 GHz, subcarrier spacing of 15 kHz, maximum Doppler shift of 1.85 kHz, and Jakes Doppler spectrum are considered. The plots show that RIS-aided OTFS with Zak receiver achieves the best performance, followed by
RIS-aided OTFS with two-step receiver. RIS-aided OFDM has the least performance.

Figure 4 shows the effect of the number of sub-surfaces on the BER performance of RIS-aided OTFS with two-step receiver and Zak receiver for the parameters mentioned in Fig. 3 at SNRs of −15 dB and −10 dB. It can be seen that, as expected, the BER improves as the number of sub-surfaces \( L \) increases. For example, at an SNR of −10 dB, the improvement in BER is two orders when \( L \) is increased from 5 to 25. Also, increasing \( L \) is observed to yield diminishing returns in BER performance, which can be explained as follows. For any two values of number of sub-surfaces \( L_1 \) and \( L_2 \), the performance gap is proportional to \( \left( \frac{L_1}{L_2} \right)^2 \), and the ratio \( \frac{L_1}{L_2} \) diminishes as \( L_1 \) and \( L_2 \) are increased keeping \( L_1 - L_2 \) fixed, e.g., for \( L_1 = 15, L_2 = 10, \frac{L_1}{L_2} = 1.5 \) and for \( L_1 = 25, L_2 = 20, \frac{L_1}{L_2} = 1.25 \), where \( L_1 - L_2 = 5 \) in both cases. Also, the Zak receiver is found to perform better than the two-step receiver.

Next, we provide an explanation for the better performance of the Zak receiver over the two-step receiver. For this, consider the case of zero delay (\( \tau^p_g = 0 \)) and non-zero Doppler with \( \nu_q = c \Delta f, c \in \{0, 1, \ldots, M - 1\} \). From (12), the effective channel gain terms for the two-step receiver become
\[
\hat{h}_{\nu_q}[k', l', k, l] = 0 \quad \text{and} \quad \hat{h}_{\nu_q}[k, l', k, l] = \hat{h}_{\nu_q}[g_{r}e^{j2\pi f_{c}l'/L}] \delta[k - k'] \left( \frac{1}{M} \sum_{m=0}^{M-1} c^{2m} e^{j2\pi m(l'/L)} \right),
\]
where \( \delta[n] \) is the Kronecker delta function. Likewise, from (23), the effective channel gain term for the Zak receiver becomes \( \hat{g}^{\nu_q}[k', l', k, l] = \hat{h}_{\nu_q}[g_{r}e^{j2\pi f_{c}l'/L}] \delta[k - k'] \delta[l - l'] \). It is clear that \( \hat{h}_{\nu_q}[k', l', k, l] \) and \( \hat{g}^{\nu_q}[k', l', k, l] \) are equal when \( c = 0 \) (i.e., zero-Doppler case). Whereas, they are not same when \( c \neq 0 \), i.e.,
\[
\left( \frac{1}{M} \sum_{m=0}^{M-1} c^{2m} e^{j2\pi m(l'/L)} \right) \neq \delta[l - l']
\]
is not equal to the delta function \( \delta[l - l'] \). This means, while there is no leakage to other bins in the case of Zak receiver, there is leakage into other bins in the case of two-step receiver. This leakage contributes to poorer performance of the two-step receiver. This point is illustrated through simulations for different values of Doppler, \( \nu_q = c \Delta f \) for \( c = 0, 2, 8 \), in Fig. 5. The BER plots in Fig. 5 show that the performance of Zak and two-step receivers are the same when \( c = 0 \). But, for \( c = 2, 8 \), the performance of two-step receiver degrades because of the leakage into other bins. This leakage increases for increasing Doppler and hence the performance with \( c = 8 \) is worse than that with \( c = 2 \). Whereas, Zak receiver retains its performance close to its performance with \( c = 0 \). This shows Zak receiver to be more resilient to Doppler than two-step receiver. Finally, some key challenges in RIS-aided OTFS that can be taken up for future work include DD channel estimation, development of efficient RIS phase optimization techniques, and proof-of-concept implementations and testbeds. The effect of correlation among sub-surfaces on performance can also be investigated.

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