A natural solution to the \( \mu \)-problem in dynamical supergravity model

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Abstract

The Higgs mixing term coefficient \( \mu \) is calculated in the supersymmetric theory which possesses a non-anomalous \( U(1)_R \) symmetry in the limit of global supersymmetry. In this model, supersymmetry is assumed to be broken by gaugino condensation in the hidden sector when the supergravity effects are turned on. The soft breaking terms in the visible sector and the \( \mu \) term of order the weak scale are produced in a simple manner.
1 Introduction

The Standard Model (SM) of particle interactions enjoys overwhelming phenomenological successes, but it does not account for the gravitational interactions nor does it explain the origin or naturalness of the electroweak scale $M_W \ll M_{pl}$. These theoretical problems result in a brief that the high-energy physics should be described by supergravity. The hierarchy of mass scales can be naturally explained if supersymmetry is exact at high energies but becomes spontaneously broken, above $M_W$, by a non-perturbative mechanism. At low energies, this mechanism should decouple from the observable physics and supersymmetry would appear to be broken by explicit soft terms in the effective low-energy Lagrangian. 

From the low-energy point of view, these soft terms - which include the masses of the super-partners of all known particles - are simply independent input parameters, just like the gauge and the Yukawa couplings of the Standard Model, but from the high-energy point of view, they are calculable in terms of the supergravity couplings. Because of its non-renormalizability, supergravity itself has to be thought of as an effective theory, valid below the Planck scale $M_{pl}$. Currently, the best candidate for a consistent theory governing the physics of energies near the Planck scale is the superstring theory. However, in general, we have no reliable stringy mechanisms that lead to non-perturbative spontaneous breaking of supersymmetry. Instead, one generally assumes that the dominant non-perturbative effects emerges at energies well below $M_{pl}$. Gaugino condensation in an asymptotically free hidden sector of the effective supergravity is a prime example of this type of mechanisms. These models, however, face another problem of naturalness which we call the $\mu$-problem where $\mu$ is the coefficient of the $H_1 H_2$ term in the low energy superpotential and $H_1$ and $H_2$ denote the usual Higgs SU(2) doublet chiral superfields. In the minimal supersymmetric standard model (MSSM) the matter content consists of three generations of quarks and lepton superfields plus two higgs doublets $H_1$ and $H_2$ of opposite hypercharge. The most general effective observable superpotential has the form:

$$W^{MSSM} = W^0 + W^\mu_{\text{term}}$$

$$\begin{align*}
W^0 &= \sum_{\text{generations}}(h_u Q_L H_2 u_R + h_d Q_L H_1 d_R + h_e L_L H_1 e_R) \\
W^\mu_{\text{term}} &= \mu H_1 H_2
\end{align*}$$

(1.1)
where $h_i$ presents dimensionless Yukawa coupling constants. This includes the usual Yukawa couplings plus a possible supersymmetric mass term for the Higgses. In addition to these supersymmetric terms, we should include soft breaking terms of order 1Tev. For this model to work well, it is known that $\mu$ should not be too large and its value is estimated to be the same order as the soft breaking terms. Once it is accepted that the presence of the $\mu$-term is essential, immediately a question arises. Is there any dynamical reason why $\mu$ should be so small of the order of the electroweak scale? We should note that, to this respect, the $\mu$-term is different from the supersymmetry breaking terms so its origin should be different from the supersymmetry breaking mechanism. In principle the natural scale of $\mu$ would be $M_{pl}$, but this would re-introduce the hierarchy problem since the Higgs scalars get a contribution $\mu^2$ to their squared mass. Thus, any complete explanation of the electroweak breaking scale must justify the origin of $\mu$. This is the so-called $\mu$-problem and this has been considered by several authors[1]. In this letter we suggest a natural solution to the $\mu$-problem without introducing complicated non-renormalizable terms and any additional mechanisms other than dynamical supersymmetry breaking in the hidden sector.

2 A natural solution to the $\mu$-problem

The model of supersymmetry which we study is based on the continuous $U(1)_R$ symmetry that is extended from R-parity. We define this R-symmetry by giving the coordinate of superspace $\theta$ charge $+1/2$, all matter fields charge $+1/2$, and all Higgs superfields charge 0. Expansions of the superfields in terms of the component fields then show that all ordinary particles are R-neutral while all superpartners carry non-zero R-charge. For gauginos in the hidden sector, there is an R-symmetry which is spontaneously broken if a gaugino condensate forms and leads to a Goldstone mode. (Here we should note that this R-symmetry should be explicitly broken when we fine-tune the cosmological constant by adding a constant to the superpotential and turn on the supergravity effects. See also ?? for more detailed discussions on this point.) In this case the auxiliary field $\phi$ describing this would-be mode must be embedded in a chiral superfield $\Phi$ which is coupled in a supersymmetric way. (Here we can also consider the auxiliary field $\Phi$ as
the compensator superfield for the anomaly of the R-symmetry.) In this respect, we consider an auxiliary field $\Phi$ which also couples to the ordinary components of MSSM. This field $\Phi$ has R-symmetry $+1$, and its scalar component is an order parameter of gaugino condensation. (The construction of the Lagrangian is motivated by ref.\cite{3}) The general supersymmetric lagrangian is generally characterized by three functions; Kähler potential $K(z_i, z_i^*)$, superpotential $W(z_i)$, and kinetic function $f(z_i)$ for vector multiplets. Kähler potential $K(z_i, z_i^*)$ is a function of scalar fields $z_i$ and $z_i^*$, while superpotential $W(z_i)$ and the kinetic function $f(z_i)$ depend scalar fields with definite chirality. Using these functions, the general form of the supersymmetry lagrangian can be written in the superfield formalism,

$$L = \int d^4 \theta K + \int d^2 \theta \left[ W + \left\{ \frac{1}{4} f_{HS} W W \right\}_{H.S.} + \left\{ \frac{1}{4} f_{VS} W W \right\}_{V.S.} \right] + h.c.$$

(2.1)

where $f_{HS}$ and $f_{VS}$ denote the kinetic functions for the hidden and the visible sector. Here we simply assume that the hidden sector consists of $SU(N_c)$ supersymmetric pure Yang-Mills. Demanding that the effective theory has non-anomalous $U(1)_R$ invariance in the limit of global supersymmetry, which is compensated by the auxiliary field $\Phi$, the form of the $W$ and $f_{HS}$ are determined:

$$W = W(\Phi) + W^0$$

$$W(\Phi) = (m^2 + \lambda H_1 H_2) \Phi$$

$$f_{HS} = S - \xi \ln(\Phi/\mu)$$

(2.2)

where $m$ and $\mu$ are the mass parameters, $\xi$ is a dimensionless constant and $S$ is a dilaton superfield. $W$ is the total superpotential which can be written in the sum of the hidden, observable and mixing terms. One can determine $\xi$ by demanding that the low-energy effective Lagrangian is anomaly free under the non-anomalous R-symmetry transformation. Here, for simplicity, we neglect the gauge interactions in the observable sector because such interactions are not important when we consider the strong dynamics in the hidden sector. The Higgs fields $H_i$ does not have any $U(1)_R$ charge. In general, the explicit mass parameter $m$ should be the order of the Planck mass $M_{pl}$. Other contributions to the superpotential, such as the non-renormalizable couplings suppressed by the Planck
mass, are not important in this model. The classical equation of motion for the auxiliary component of $\Phi$ ($\phi, \chi, h$) yields:

$$\frac{\partial W}{\partial \phi} + \frac{1}{4} \frac{\partial f}{\partial \phi} \lambda \lambda = 0 \quad (2.3)$$

This gives the relation:

$$(m^2 + \lambda H_1 H_2) \phi = \frac{\xi}{4} \lambda \lambda. \quad (2.4)$$

Once gaugino condensate in the hidden sector, the right hand side of (2.4) becomes about $\Lambda_{HS}^3$ where $\Lambda_{HS}$ is the dynamical scale of the hidden sector ($\Lambda_{HS} \sim 10^{12}$GeV). For $H_1$ and $H_2$, it is obvious that their vacuum expectation values cannot become large once $\phi$ develops non-zero vacuum expectation value. As a result, from the equation (2.4) we obtain a solution $\phi \sim < \lambda \lambda > /m^2 \sim m_2^2$. For the superfield $H_i$, this induces a supersymmetric mass term of the electroweak scale.

$$W^{\mu-\text{term}} \sim m_2^2 H_1 H_2 \quad (2.5)$$

This explains the scale of the $\mu$-term in a natural way. The scalar component of $\Phi$ is determined mainly from the strong coupling effect which is relevant to the gaugino condensation in the hidden sector, and its non-zero value induces a supersymmetric mass term to the Higgs superfields which appears as the $\mu$-term in the observable sector.

3 Conclusion

The $\mu$-problem is the necessity of introducing by hand a small mass term($\mu$) of order the soft breaking mass scale in the observable sector which (in general) is not correlated to the breaking of supergravity. We have shown that this problem can be avoided if we consider an effective Lagrangian which possesses $U(1)_R$ symmetry which is compensated by an auxiliary superfield $\Phi$.

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