The $\rho$ and $A$ mesons in strong abelian magnetic field in SU(2) lattice gauge theory.

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We calculated correlators of vector, axial and pseudoscalar currents in external strong abelian magnetic field according to SU(2) gluodynamics. The masses of neutral $\rho$ and $A$ mesons with various spin projections to the axis parallel to the external magnetic field $B$ have been calculated. We found that the masses of neutral mesons with zero spin $s = 0$ decrease in increasing magnetic field, while the masses of the $\rho$ and $A$ mesons with spin $s = \pm 1$ increase in the mentioned field. Also we performed extrapolation and renormalization of masses on the lattice.

I. INTRODUCTION

Quantum Chromodynamics in external magnetic field is an area that presents enormous interest for physicists. Strong magnetic fields could be associated with formation of the early Universe. It is assumed that magnetic fields $\sim 2$ GeV existed in the Universe during the electroweak phase transition [1]. It is known that cosmic space objects called magnetars or neutron stars possess magnetic field in their cores equal to $\sim 1$ MeV.

Recently there was a number of amazing effects experimentally discovered and theoretically approved. For instance, STAR collaboration detected chiral magnetic effect during non-central collisions of gold ions [2] and [3]. Later this effect was also observed in ALICE experiment at the LHC.

The values of magnetic fields in non-central heavy-ion collisions can reach up to $15m_e^2 \sim 290\text{MeV}^2$ [2], energy of hadronic scale. This magnetic field is caused by motion of ions, and it creates charge asymmetry of particles emitted from different sides of the reaction plane.

Strong magnetic field can be created in terrestrial laboratories, which makes it possible to explore quark-hadronic matter under extreme conditions.

Strong enough magnetic fields lead also to modifications of the QCD phase diagram. Phenomenological models show that the critical temperature of transition between phases of confinement and deconfinement varies with the increase of external magnetic field, and the phase transition becomes the one of the first order [4].

The increase of $T_c$ is also predicted in Nambu-Jona-Lasinio models: NJL, EPNJL, PNJL [7] and PNJLs [8], Gross-Neveu model [9, 10].

The first prediction of lattice simulations with two flavours in QCD regarding behavior of the deconfinement transition and the chiral symmetry restoration in increasing magnetic field was made in [11]. However lattice simulations with $N_f = 2 + 1$ [12] revealed that $T_c$ decreases under increasing field value. As a matter of fact, the chiral perturbation theory predicts the decrease of the transition temperature under increasing magnetic field value as well [13].

These effects were studied in the past but only for the case of a chromomagnetic (not usual abelian magnetic) field [14, 15]. External magnetic fields lead to the enhancement of the chiral symmetry breaking [17, 21]. The analytical calculations [22, 24] predict a linear increase of the chiral condensate under increasing magnetic field in the leading order. Lattice simulations show that chiral condensate depends on the strength of applied field as exponent function with $n = 1.6 \pm 0.2$ [25]. AdS/CFT approach shows the quadratic behaviour [26].

In the framework of the Nambu-Jona-Lasinio model it was shown that QCD vacuum becomes a superconductor [27, 28] along the direction of the magnetic field in the presence of sufficiently strong magnetic fields ($B_c = m^2_e / e \simeq 10^{10}$ Tl). This transition to superconducting phase is accompanied by condensation of charged $\rho$ mesons. Calculations on the lattice [29] indicate existence of the superconducting phase as well. We have investigated the behavior of masses of neutral $\rho$ and $A$ mesons with spin projection $s = 0, \pm 1$ to the axis of the magnetic field. In QCD condensation of neutral $\rho$ and $A$ mesons in external magnetic field would become an evidence of the superfluid phase existence. In [30] they calculated the mass of neutral vector $\rho$ meson according to the relativistic quark-antiquark model and found out that in the phase of confinement the mass of this neutral $\rho$ meson with zero spin increases under raising magnetic field which contradicts the results presented in [27]. This paper is organized in the following way. In Section II we describe technical and numerical specifications of our simulations. In Section III we discuss measured observables. Section IV is devoted to the methods of calculations. The results of our calculations are presented in Sections VI and VII.

II. DETAILS OF THE CALCULATIONS

To generate SU(2) gauge field configurations we use the tadpole improved Symanzik action

$$S = \beta_{\text{imp}} \sum_{pl} S_{pl} - \frac{\beta_{\text{imp}}}{2u_0^2} \sum_{rt} S_{rt},$$

where $S_{pl,rt} = (1/2)\text{Tr}(1 - U_{pl,rt})$ is the plaquette (denoted by $pl$) or $1 \times 2$ rectangular loop term ($rt$), $u_0 = (W_{1 \times 1})^{1/4} = (\langle 1/2 \text{Tr} U_{pl} \rangle)^{1/4}$ is the input tadpole factor computed at zero temperature [31]. This action suppresses ultraviolet dislocations, leading to non-physical near-zero modes of the Wilson-Dirac operator.
We calculated fermionic spectrum in the presence of $SU(2)$ gauge fields using the chiral-invariant overlap operator proposed by Neuberger [32]. This operator allows to explore the theory without chiral symmetry breaking and can be written as follows:

$$D_{ov} = \frac{\rho}{a} \left( 1 + D_W / \sqrt{D_W^t D_W} \right),$$  \hspace{1cm} (2)

where $D_W = M - \rho/a$ is the Wilson-Dirac operator with the negative mass term $\rho/a$, $a$ is the lattice spacing in physical units, $M$ is the Wilson hopping term with $r = 1$. Fermionic fields comply with periodic boundary conditions imposed on space and anti-periodic boundary conditions imposed on time. The sign function is calculated by the minmax polynomial approximation

$$D_W / \sqrt{D_W^t D_W} = \gamma_5 \text{sign} (H_W),$$  \hspace{1cm} (3)

where $H_W = \gamma_5 D_W$ is the hermitian Wilson-Dirac operator. To compute the sign function we use the 50 lowest Wilson-Dirac eigenmodes.

The Dirac operator in continuous space is $D = \gamma^\mu (\partial_\mu - iA_\mu)$ and the corresponding Dirac equation looks as follows:

$$D \psi_k = i\lambda_k \psi_k.$$  \hspace{1cm} (4)

The Neuberger overlap operator allows to calculate eigenfunctions $\psi_k$ and eigenvalues $\lambda_k$ for the test quark in external gauge field $A_\mu$. $A_\mu$ is the sum of $SU(2)$ gauge field and the external abelian uniform magnetic field. The eigenmodes of the Dirac operator make it possible to construct operators and correlators presented in Section 3.

Abelian gauge fields interact only with quarks. To include abelian magnetic field in the simulations we perform the following substitution:

$$A_{\mu ij} \rightarrow A_{\mu ij} + A_\mu^B \delta_{ij},$$  \hspace{1cm} (5)

$$A_\mu^B(x) = \frac{B}{2} (x_1 \delta_{\mu,2} - x_2 \delta_{\mu,1}).$$  \hspace{1cm} (6)

To match this exchange with lattice boundary conditions the twisted boundary conditions for fermions have been used [33].

Magnetic field in our calculations is directed along the third axes, its value is quantized

$$qB = \frac{2\pi k}{(aL)^2}, \quad k \in \mathbb{Z},$$  \hspace{1cm} (7)

where $q = -1/3e$ is the electric charge of the $d$-quark. There is one type of fermions in the theory. The quantization condition imposes a limit on the minimal value of the magnetic field. It equals to $\sqrt{qeB} = 476.13$ MeV for the lattice volume $18^4$ and lattice spacing $a = 0.0998$ fm. We are far from the saturation regime in magnetic field when $k/(L^2)$ is not small because we use the values of $k$ in the interval $0 - 6$ for $18^4$ lattice volume and $k = 0 - 14$ for $14^4$ and $16^4$ lattice volumes. For inversion of the mentioned operator we use the Gaussian source (with the radius $r = 1.0$ in lattice units in both spatial and time directions) and the point sink (a quark position smeared over the Gaussian profile).

Our calculations were performed on symmetric lattices with the lattice volumes $14^4$, $16^4$, $18^4$ and lattice spacings $a = 0.0681$, 0.0998 and 0.1383 fm. We use statistically independent configurations of the gluon field for each value of the quark mass in the interval $m_q a = 0.01 - 0.8$. For quark masses larger than 0.01 the inversion works well.

### III. OBSERVABLES

The following observables were calculated

$$\langle \psi_1^\dagger(x) O_1 \psi(x) \psi_2^\dagger(y) O_2 \psi(y) \rangle_A,$$  \hspace{1cm} (8)

where $O_1, O_2 = \gamma_5, \gamma_5 \gamma_\mu, \gamma_\mu$ are Dirac gamma matrices, $\mu, \nu = 1, \ldots, 4$ are Lorentz indices. In the Euclidean space $\psi^\dagger = \psi$ [34]. In presence of gauge field being the sum of gluonic fields and abelian constant magnetic field these current-current correlators in a meson channel can be expressed via Dirac propagators.

In the continuous field theory [35] can be written as follows:

$$\int DA_\mu e^{-S_{YM}[A_\mu]} \left[ \text{Tr} \left( \frac{1}{D + m - O_1} \right) \text{Tr} \left( \frac{1}{D + m - O_2} \right) \right] -$$

$$\int DA_\mu e^{-S_{YM}[A_\mu]} \left[ \text{Tr} \left( \frac{1}{D + m - O_1} \frac{1}{D + m - O_2} \right) \right].$$  \hspace{1cm} (9)

The first term in the numerator of (9) is the disconnected part, while the second one is connected. We checked that the disconnected part makes rather little relative contribution to correlators in comparison with the connected part in our model. Also this disconnected part does not effect the values of meson masses. Therefore we calculate only the connected parts of correlators [36].

Correlators [37] are defined by Dirac propagators. To calculate these correlators we should first determine the inverse matrix for the massive Dirac operator. For the $M$ lowest eigenstates of the Dirac operator this matrix is represented by the sum:

$$\frac{1}{D + m} (x,y) = \sum_{k,M} \frac{\psi_k(x) \psi_k^\dagger(y)}{i\lambda_k + m},$$  \hspace{1cm} (10)

where $M = 50$. Observables [38] have the following form

$$\langle \bar{\psi} O_1 \psi \bar{\psi} O_2 \psi \rangle_A =$$

$$\sum_{k,p,M} \frac{\langle k | O_1 | k \rangle \langle p | O_2 | p \rangle - \langle p | O_1 | k \rangle \langle k | O_2 | p \rangle}{(i\lambda_k + m)(i\lambda_p + m)}.$$  \hspace{1cm} (11)
The first term in (9) is not essential as we mentioned above. The mass of neutral $\rho$ meson was extracted from the correlator of vector currents $(j^V_\mu(x)j^V_\nu(y))_A$, where $j^V_\mu(x) = \psi^\dagger(x)\gamma_\mu\psi(x)$. We calculated the mass that has magnetic fieldwise projection of its spin equal to zero and it corresponds to $O_1, O_2 = \gamma_3$ in the expression (11). The mass of a neutral $A$ meson can be found from the correlator of axial currents $(j^A_\mu(x)j^A_\nu(y))_A$, where $j^A_\mu(x) = \psi^\dagger(x)\gamma_5\gamma_\mu\psi(x)$. The correlator $(j^{PS}_\mu(x)j^{PS}_\nu(y))_A$ enables us to compute the mass of $\pi$ meson, where $j^{PS} = \psi^\dagger(x)\gamma_5\psi(x)$ is the pseudoscalar current.

### IV. METHODS

To calculate masses we apply two methods. The first one is based on spectral expansion of the lattice correlation function

$$C(n_t) = \langle \psi^\dagger(\vec{0},n_t)O_1\psi(\vec{0},n_t)\psi^\dagger(\vec{0},0)O_2\psi(\vec{0},0)\rangle_A = \sum_k \langle 0|O_1|k\rangle\langle k|O_2^\dagger|0\rangle e^{-n_t\alpha E_k},$$

where $A$ is some constant value, $E_0$ is the energy of the lowest state. For a particle with average momentum equal to zero $(\vec{p}) = 0$ this energy is equal to its mass $E_0 = m_0$. $E_1$ is the energy of the first excited state, $\alpha$ is the lattice spacing, $n_t$ is the number of a lattice site in the line of time direction. From expansion (12) one can see that for large values $n_t$ the main contribution comes from the ground state energy.

Due to periodic boundary conditions the contribution of the ground state into meson propagator has the form

$$C_{fiu}(n_t) = A_0 e^{-n_t\alpha E_0} + A_1 e^{-n_t\alpha E_1} + \ldots,$$

where $A$ is some constant value, $E_0$ is the energy of the lowest state. For a particle with average momentum equal to zero $(\vec{p}) = 0$ this energy is equal to its mass $E_0 = m_0$. $E_1$ is the energy of the first excited state, $\alpha$ is the lattice spacing, $n_t$ is the number of a lattice site in the line of time direction. From expansion (13) one can see that for large values $n_t$ the main contribution comes from the ground state energy.

$$2A_0 e^{-N\tau\alpha E_0/2} \cosh((N\tau-n_t)\alpha E_0).$$

The value of the ground state mass can be obtained by fitting the function (14) to the lattice correlator (12). To minimize errors we take various $n_t$ values from the interval $4 < n_t < N\tau - 4$.

The second method we use is the Maximal Entropy Method (MEM) [25]. Euclidean correlator of the imaginary time $G(\tau,\vec{p}) = \int d^3 x (O(\tau,\vec{x})O^\dagger(0,\vec{0})) e^{-i\vec{p}\vec{x}}$ corresponds to the spectral function $\rho(\omega,\vec{p})$ as follows:

$$G(\tau,\vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho(\omega,\vec{p}).$$

In general case $\rho(\omega,\vec{p})$ contains all the properties of mesons and hadrons having desired quantum numbers. We presume $\langle\vec{p}\rangle = 0$ and do not consider any functions from it. The first peak in the spectral function corresponds to the energy of the ground state. The kernel in (15) can be expressed as follows:

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)},$$

where $T$ is the temperature, $\tau$ is the Euclidean time, $\omega$ is the frequency. To extract the spectral function we should perform an inversion of the equation (15).

This problem on the lattice is ill-defined as the correlator $G(\tau)$ can be calculated only numerically at discrete points $\tau_i = \tau_{min} + (i-1)\alpha$, $i = 1, \ldots, N\tau$, and $N\tau \sim 10^{-50}$. The integral was approximated by the discrete sum at points $\omega_n = n \Delta \omega$, $n = 1, \ldots, N\omega$ and $N\omega$ is usually $O(10^4)$. We cut the integral (15) off at some $\omega_{max}$. Nevertheless this inversion turns out to be impossible. Yet the ideas of Bayesian probability theory make it possible to overcome this problem.

The most probable spectral function $\rho(\omega)$ can be computed provided that we find the maximum of conditional probability $P[\rho|DHomm]$, where $D$ is the data, $H$ is our hypothesis, $\alpha$ is a real and positive parameter, $m = m(\omega)$ is a default model. This procedure is equivalent to the maximization of free energy $F = L - \alpha S$, where $S$ is Shannon entropy,

$$S = \int_0^\infty \omega \left[ \rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)} \right].$$

$L$ is the standard likelihood function. Detailed explanation of how to make a corresponding discretization on the lattice is offered in [25]. We take $\alpha \in [\alpha_{min}, \alpha_{max}]$ and average the data within this interval. This interval was chosen in such a way that the results may vary slightly (by approximately 10%).

The kernel (10) contains divergence at $\omega = 0$ leading to the unstable behaviour of the procedure under small energies. The Bryan’s key idea was to redefine the kernel and the spectral function

$$\tilde{K}(\omega,\tau) = \frac{\omega}{2T} K(\omega,\tau), \quad \tilde{\rho}(\omega) = \frac{2T}{\omega} \rho(\omega),$$

so that $K(\omega,\tau)\rho(\omega) = \tilde{K}(\omega,\tau)\tilde{\rho}(\omega)$ and apply the SVD theorem to the modified discretized kernel $\tilde{K}(\omega_n,\tau_i)$, see [26]. We use this modified algorithm to determine the spectral function in the following form

$$\tilde{\rho}(\omega) = \tilde{m}(\omega) \exp \sum_{i=1}^N c_i \tilde{u}_i(\omega).$$

The column vectors $u_i$, $(i = 1, \ldots, N)$ are normalized

$$\langle u_i|u_j \rangle = \sum_{n=1}^N u_i(\omega_n)u_j(\omega_n) = \delta_{ij},$$

$c_i$ are the coefficients and we consider $\tilde{K}(0,\tau) = 1$. 


Therefore, to reconstruct spectral function $\rho(\omega)$ we have to choose default model $\tilde{m}(\omega)$ correctly. Such a default model should describe high and low energy behaviours of the spectral function correctly. According to the analysis [37] we choose the following form of the model:

$$\tilde{m}(\omega) = m_a \omega + m_b, \quad m_a = \frac{G(N_c/2)}{T^2}, \quad m_b = a_H \frac{3}{8\pi^2},$$

where $a_H = 1$ for scalar and pseudoscalar channels, $a_H = 2$ for vector and axial vector channels [38]. We also try to apply other default models (constant function, $\sim \omega^2$, vary the $m_a$ and $m_b$), but the choice (21) gives the best convergence for the MEM.

V. VECTOR MESON MASS AT $B = 0$

In this section we present the calculation of neutral $\rho$ meson mass in $SU(2)$ gluodynamics under zero magnetic field. We compare our results to the previous ones to make sure that our method works properly. We measure the correlator of pseudoscalar currents $C^{PSPS}(n_t) = \langle j^{PS}(\vec{0}, n_t)j^{PS}(\vec{0}, 0)\rangle_A$, where $j^{PS}(\vec{0}, n_t) = \bar{\psi}(\vec{0}, n_t)\gamma_5\psi(\vec{0}, n_t)$ and calculate the mass of the lowest energy state of neutral $\pi$ meson for different quark masses, volumes and lattice spacings. Fig.1 and 2 demonstrate the linear dependence of the squared $\pi$ meson mass versus the bare quark mass. The chiral perturbation theory predicts linear dependence of squared pion mass on the renormalized quark mass $m_q^{ren}$, and this dependence can be expressed as follows:

$$f_\pi^2 m_{\pi}^2 = m_{q}^{ren} \langle \bar{\psi}\psi \rangle,$$

where $f_\pi$ is the pion decay constant, $\langle \bar{\psi}\psi \rangle$ is the chiral condensate. Provided that the mass of quark tends to zero, the pions are massless. Fig.1 and 2 show that mentioned functions are slightly shifted relative to the origin of the coordinates and that the shift of quark mass from zero corresponds to the renormalization of this quark mass on the lattice.

We calculate the mass of neutral vector $\rho$ meson under zero magnetic field via the correlators of vector currents. The symmetry between the different spatial directions was taken into account which improved the statistics significantly (thrice). The quark mass renormalization was not considered because its value is very small at zero field. The extrapolation of $\rho$ meson mass to the limit of infinite physical volume was performed for several values of quark mass and two lattice spacings 0.1383 fm and 0.1155 fm (Fig.3 and 4). After all we extrapolated $m_{\rho}$ to the quark mass $m_{q0}$ corresponding to the value of the $\pi$ meson mass equal to 135 MeV. We obtained $m_{\rho} \simeq 980 \pm 30$ MeV for
the lattice spacing $a = 0.1338$ fm and $1020 \pm 20$ MeV for $a = 0.1155$ fm in SU(2) gluodynamics.

VI. THE MASSES OF MESONS AT $B \neq 0$

On Fig. 6 the squared pion mass is depicted for $16^4$ lattice volume, various bare quark masses and different values of the magnetic field $H = \sqrt{eB}$. The observed linear dependence is legitimate for the chiral perturbation theory. Fig. 7 represents the $\pi$ meson mass versus the value of the squared magnetic field. The mass depends on the lattice volume not significantly. For the renormalized quark mass and magnetic fields less than 1 GeV we have obtained linear dependence of the mass on the magnetic field value; the slope of the graph of this function is negative, which agrees with results of A.Smilga obtained in the chiral perturbation theory [24]. The angle of the slope is obtuse and it differs from ChPT because we explore the SU(2) gauge theory without dynamical quarks. Thus SU(2) reveals qualitative properties of the theory correctly.

If the direction of external magnetic field is parallel to the 3rd coordinate axis then meson masses having magnetic fieldwise projections of their spins equal to zero are calculated from the expression (11) with $O_1, O_2 = \gamma_3$. The diagonal components of the correlators are not equal to zero while the nondiagonal ones equal to zero within the error bars. Fig. 8 shows the mass of neutral vector meson with zero spin calculated in accordance with the Maximal Entropy Method for different lattice volumes, spacings and bare quark masses. We have not performed mass extrapolations and renormalizations though. We found that the mass decreases under raising magnetic field for all the sets of data. We use the ensemble of $O(10)$ results of MEM procedure and calculate average
Fig. 8: Dependence of the mass of the neutral vector $\rho$ meson with zero spin $s = 0$ on the value of external magnetic field for the lattice volumes $14^4$, $16^4$, and lattice spacings $a = 0.0998$ fm, $0.1338$ fm calculated in accordance with the Maximal Entropy Method.

The lattice artefacts are not significant yet we are restricted by the small lattice spacing which is not fine enough and cannot explore the masses under strong magnetic fields. As we know the radius of the lowest Landau level

$$r_L = \frac{1}{\sqrt{eB}}.$$  \hspace{1cm} (23)

We may assume that lattice artefacts might appear under $r_L = a$. Therefore we make simple estimations and obtain the maximal value of magnetic field $eB = 3.9$ GeV$^2$ for the spacing $a = 0.0998$ fm, $2.9$ GeV$^2$ for the $a = 0.1155$ fm and $2.0$ GeV$^2$ for $0.1338$ fm lattice spacing.

VII. LATTICE EXTRAPOLATIONS AT $B \neq 0$

Fig. 10: Dependence of the mass of the neutral vector $\rho$ meson with spin $s = 0$ on the value of external magnetic field for the lattice volumes $16^4$, $18^4$ and lattice spacings $a = 0.0998$, $0.1155$ fm.

On Fig. 10, 11 and 12 we present the masses with various spin projections that were received by fitting the cosinus function to correlators and further extrapolation of quark masses.

The masses of the vector meson were calculated for various values of magnetic field.

We calculated these masses for nonzero magnetic field taking the quark mass renormalization $\delta m_{\text{quark}}$ into account. On the basis of the mentioned calculation we
can conclude that masses of vector mesons depend on that renormalization not significantly. Afterwards we performed ρ meson mass extrapolation on the mass of quark to its value under which the mass of π meson is equal to 135 MeV. This procedure was done taking the renormalization of the quark mass into account.

We calculate the mass of ρ meson for several values of \( m_q \) from the interval \( m_q = 0.01 \pm 0.8 \) perform fitting and find coefficients \( a_i \) and \( b_i \) in the equations

\[
m_{\rho}(s = 0) = a_0 + a_1 m_q,
\]

\[
m_{\rho}(s = 0) = b_0 + b_1 m_q.
\]

Then we extrapolate \( m_{\rho}(m_q) \) on physical values \( m_{\rho}(m_{\rho_0}) \) at \( m_q = m_{\rho_0} \) using (24) and (25).

Different components of the vector currents correlators were calculated, and it was found that diagonal components differ from zero essentially while nondiagonal ones are equal to zero within the error bars. Correlators of the vector currents perpendicular to the magnetic field are \( C_{11}^{\gamma V}(m_t) = \langle j_{11}^V(0, n_t) j_{11}^V(0, 0) \rangle_A \) and \( C_{22}^{\gamma V}(m_t) = \langle j_{22}^V(0, n_t) j_{22}^V(0, 0) \rangle_A \), where \( j_{11}^V(0, n_t) = \overline{\psi}(0, 0) \gamma_1 \psi(n_t) \) and \( j_{22}^V(0, n_t) = \overline{\psi}(0, 0) \gamma_2 \psi(n_t) \). The masses with spin \( s = \pm 1 \) are found from the relations \( C^{\gamma V}(s = 1) = (C_{11}^{\gamma V} + iC_{22}^{\gamma V})/\sqrt{2} \) and \( C^{\gamma V}(s = -1) = -(C_{11}^{\gamma V} - iC_{22}^{\gamma V})/\sqrt{2} \).

On Fig. 11 one can see the dependence of the mass of the neutral vector ρ meson with zero spin on the value of the field after the mass renormalization and extrapolation for the lattice volumes \( 16^4 \) and \( 18^4 \) and lattice spacings \( a = 0.0998, 0.1155 \) fm. For the purposes of visualization we connected the points by splines. The mass of vector meson decreases nonlinearly under raising magnetic field for all the lattices. We observe a weak dependence of masses from lattice volumes and spacings, but the qualitative behaviour for all the sets of data is the same.

Fig. 11 shows the dependence of the ρ meson mass with nonzero spin on the field value. Masses with spin \( s = \pm 1 \) increase under the raising field. These results were obtained after the quark mass extrapolation.

On Fig. 12 we see the mass of the neutral axial meson with fieldwise spin projections \( s = 0, \pm 1 \). Calculation of the axial meson mass requires much more statistics than the calculation of the vector meson mass especially in cases with nonzero spin components. One can observe that the mass of A meson with zero spin decreases while the masses with \( s = \pm 1 \) increase slowly.

Unfortunately the quantum numbers of mesons on the lattice in the presence of magnetic field are not precise. The mixing takes place due to the interaction between photons and vector (axial) quark currents and can occur between neutral pion and neutral \( \rho \) (or \( A \)) meson with zero spin. Now there are no rigorous methods to disentangle these two states in magnetic field, this is the topic for the further work. However, we have clear signs of the increase of the vector and axial mesons masses with \( s = \pm 1 \) in our \( SU(2) \) theory.

\section{VIII. CONCLUSIONS}

In this work we explore the behaviour of masses of the neutral pseudoscalar \( \pi \), vector \( \rho \) and axial \( A \) mesons in confinement phase in the presence of external magnetic field of hadronic scale. We observe that the masses with fieldwise spin projection equal to zero differ from the masses with spin projection \( s = \pm 1 \). The masses with \( s = 0 \) decrease with the growth of the magnetic field while the masses with \( s = \pm 1 \) increase in the same conditions. We consider this phenomena to be the result of the anisotropy created by the strong magnetic field. We do not observe any condensation of neutral mesons, so there are no evidences of superfluidity in the confinement phase. However, the presence of superconducting phase at high values of the magnetic field \( B \) [23] in QCD is a hot topic for discussions. Condensation of charged \( \rho \) mesons would be an evidence of the existence of superconductivity in QCD.

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