\( \eta N \) interactions in the nuclear medium. \( \eta \)-nuclear bound states

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Abstract. We report on our recent study of in-medium \( \eta N \) interactions and \( \eta \)-nuclear quasi-bound states. The \( \eta N \) scattering amplitudes considered in the calculations are constructed within coupled-channel models that incorporate the \( S_{11} N^*(1535) \) resonance. The implications of self-consistent treatment and the role played by subthreshold dynamics are discussed.

1 Introduction

The \( \eta N \) attraction generated by the \( N^*(1535) \) resonance near threshold seems to be strong enough to allow binding of the \( \eta \) meson in nuclei. However, in-medium modifications and strong energy dependence of the \( \eta N \) scattering amplitudes have to be carefully taken into account. This contribution briefly summarizes systematic treatment of energy dependence within self-consistent calculations of \( \eta \) quasi-bound states in selected nuclei (see [1–3] for more details).

2 Methodology

The \( \eta N \) scattering amplitudes are highly model dependent as illustrated in Fig. 1 for meson-baryon interaction models GW [4], CS [5], M2 [6], and GR [7]. The \( \eta N \) amplitudes differ below as well as above the \( \eta N \) threshold, except perhaps common value \( \text{Im} F_{\eta N} \approx 0.2 \) – 0.3 fm at threshold.

The in-medium amplitudes which serve as an input in our many-body self-consistent calculations are obtained from the free-space amplitudes GW and M2 by applying the multiple scattering approach [8] (see Ref. [2] for details). In the GR and CS models, the Pauli principle restricts integration domain in the Green’s function which enters the underlying Lippmann-Schwinger equations [2].

The strong energy dependence of the \( \eta N \) scattering amplitudes \( F_{\eta N}(\sqrt{\text{s}}) \) has to be treated self-consistently [1, 2]. The argument \( \sqrt{\text{s}} \) in the scattering amplitudes is given by

\[
\sqrt{\text{s}} = \sqrt{(\sqrt{\text{s}_{\text{th}}} - B_\eta - B_N)^2 - (\vec{p}_\eta + \vec{p}_N)^2} \leq \sqrt{\text{s}_{\text{th}}},
\]

where \( \sqrt{\text{s}_{\text{th}}} = m_\eta + m_N \) and \( B_\eta \) and \( B_N \) are \( \eta \) and nucleon binding energies. In the nuclear medium (for \( A \gg 1 \) approximated by the lab system) the momentum dependent term causes additional downward energy shift, since \( (\vec{p}_\eta + \vec{p}_N)^2 \neq 0 \), which can be approximated as [2]

\[
\delta \sqrt{\text{s}} \approx -B_N \frac{\rho}{\rho_0} - \xi_N T_N\frac{\rho}{\rho_0} - \xi_N \frac{\sqrt{\text{s}}}{\omega N E_N} 2\pi \text{Re} F_{\eta N}(\sqrt{\text{s}}, \rho) \rho, \tag{2}
\]

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Figure 1. Energy dependence of the real (left panel) and imaginary (right panel) parts of the free $\eta N$ scattering amplitude in interaction models GW [4] (dashed), CS [5] (solid), M2 [6] (dot-dashed), and GR [7] (dotted). The vertical line denotes the $\eta N$ threshold.

where $\xi_{N(\eta)} = m_{N(\eta)}/(m_N + m_\eta)$, $T_N = 23.0$ MeV at $\rho_0$, $B_N \approx 8.5$ MeV is the average nucleon binding energy, and $\bar{\rho}$ is the average nuclear density. For attractive scattering amplitudes, all terms in Eq. 2 are negative definite, providing substantial downward energy shift. A variant of Eq. 2 was used in $\eta$-nuclear three- and four-body calculations (see Ref. [3] for details).

In few-body $\eta NN$ and $\eta NNN$ systems, the $\eta$-nuclear cluster wave functions were expanded in a hyperspherical basis and the ground-state binding energies were calculated variationally. For the $NN$ interaction, the Minnesota central potential [9] and the Argonne AV4’ potential [10] were used. The $\eta N$ interaction was described by energy dependent local $\eta N$ potentials that reproduce the $\eta N$ scattering amplitudes below threshold in considered interaction models [3].

The conversion widths were evaluated through the expression $\Gamma/2 \approx \langle \Psi_{gs} | -\text{Im} V_{\eta N} | \Psi_{gs} \rangle$, where $V_{\eta N}$ sums overall pairwise $\eta N$ interactions \(^1\).

The interaction of the $\eta$ meson with the nuclear many-body system was described by the Klein–Gordon (KG) equation of the form

$$[ \nabla^2 + \tilde{\omega}_\eta^2 - m_\eta^2 - \Pi_\eta(\omega, \rho) ] \psi = 0 ,$$

where $\tilde{\omega}_\eta = \omega_\eta - i \Gamma_\eta/2$ is complex energy of $\eta$, $\omega_\eta = m_\eta - B_\eta$, and $\Gamma_\eta$ is the width of the $\eta$-nuclear bound state. The self-energy operator $\Pi_\eta(\sqrt{s}, \rho) \equiv 2 \omega_\eta V_\eta = -(\sqrt{s}/E_N)4\pi F_{\eta N}(\sqrt{s}, \rho)\rho$ was constructed self-consistently using the RMF density distributions in a core nucleus.

It is to be stressed that $\text{Re} F_{\eta N}(\sqrt{s})$ and $B_\eta$ appear as arguments in the expression for $\delta \sqrt{s}$ (Eq. 2), which in turn serves as an argument for $F_{\eta N}$ and thus for the self-energy $\Pi_\eta$. Therefore, a self-consistency scheme in terms of both $\Pi_\eta$ and $B_\eta$ is required in calculations.

3 Results

Our few-body calculations of the $\eta NN$ system found no bound states in the considered coupled-channel models. For $\eta NNN$, a relatively broad and weakly bound state (with $\eta$ separation energy below 1 MeV) was found for the Minnesota $NN$ potential and one particular variant of the $\eta N$ potential

\(^1\)We found an error in normalization in Ref. [3], which made the calculated widths about factor of 2 larger.
that reproduced the GW scattering amplitudes (see Ref. [3] for details). No $\eta NNN$ bound states were found using more realistic $NN$ interaction models.

Figure 2. Binding energies (left) and widths (right) of the $1s$ $\eta$-nuclear states in selected nuclei calculated using the GR $\eta N$ scattering amplitude [7] with different procedures for subthreshold energy shift $\delta \sqrt{s}$.

Figure 2 illustrates the role of the energy dependence of $\eta N$ scattering amplitudes in self-consistent evaluations of $\eta$ nuclear-states in many-body nuclear systems. A comparison is made for the in-medium GR amplitude: our self-consistency scheme based on $\delta \sqrt{s}$ of Eq. 2 (marked $\delta \sqrt{s}$) reduces considerably the GR binding energies and widths with respect to the original calculations of Ref. [7] that used the $\delta \sqrt{s} = -B_\eta$ procedure (marked $-B_\eta$). However, even the reduced GR widths are still quite high, suggesting that $\eta$-nuclear states will be extremely difficult to resolve if the GR model is the realistic one.

The model dependence of the $\eta N$ scattering amplitudes shown in Fig. 1 manifests itself in the calculations of $\eta$-nuclear states. Figure 3 presents binding energies $B_\eta$ and widths $\Gamma_\eta$ calculated for the $1s$ $\eta$-nuclear states in selected nuclei using the above $\eta N$ amplitudes.

Figure 3. Binding energies (left) and widths (right) of $1s$ $\eta$-nuclear states in selected nuclei calculated self-consistently using the M2, GR, CS, and GW $\eta N$ scattering amplitudes (see text).
The left panel of Fig. 3 demonstrates that for each of the $\eta N$ amplitude models the binding energy increases with $A$ and tends to saturate for large values of $A$. The hierarchy of the curves reflects the strength of $\text{Re} F_{\eta N}(\sqrt{s})$ in the subthreshold region (see Fig. 1). The M2 amplitude is too weak to produce the $1s$ $\eta$ bound state in $^{12}\text{C}$. In contrast, $\text{Re} F_{\eta N}(\sqrt{s})$ of the GW model is strong enough to bind $\eta$ in $^{12}\text{C}$ and even in lighter nuclei, e.g., it predicts the $1s$ $\eta$ bound state in $^{4}\text{He}$ with $B_{\eta} = 1.2$ MeV and $\Gamma_{\eta} = 2.3$ MeV (calculated using a static $^{4}\text{He}$ density).

The right panel shows substantial differences between the widths $\Gamma_{\eta}$ calculated using the above mentioned models. The CS and GW models yield relatively small uniform widths of order 2 and 4 MeV, respectively. On the other hand, the GR and M2 models predict much larger widths which increase with $A$. This reflects partly the energy dependence of $\text{Im} F_{\eta N}(\sqrt{s})$ in the subthreshold region and partly the difference in the in-medium renormalization stemming from $\text{Re} F_{\eta N}(\sqrt{s})$. For instance, the large downward energy shift due to the subthreshold amplitude in the GW model (57 MeV at $\rho_{0}$) causes a particularly large reduction in the strength of the $\text{Im} F_{\eta N}(\sqrt{s})$ input.

The widths calculated here do not include contributions from two-nucleon processes which are estimated to add a few MeV. We may therefore conclude that $\eta$-nuclear states could in principle be observed if the CS and GW models turn out to be realistic ones, provided a suitable production/formation reaction is found. Other models give either too large widths or are too weak to generate $\eta$-nuclear bound states in lighter nuclei.

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