The microscopic picture of the integer quantum Hall regime

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Abstract

Computer modelling of the integer quantum Hall effect based on self-consistent Hartee-Fock calculations has now reached an astonishing level of maturity. Spatially-resolved studies of the electron density at near macroscopic system sizes of up to $1 \mu m^2$ reveal self-organized clusters of locally fully filled and locally fully depleted Landau levels depending on which spin polarization is favoured. The behaviour results, for strong disorders, in an exchange-interaction induced $g$-factor enhancement and, ultimately, gives rise to narrow transport channels, including the celebrated narrow edge channels. For weak disorder, we find that bubble and stripes phases emerge with characteristics that predict experimental results very well. Hence the HF approach has become a convenient numerical basis to quantitatively study the quantum Hall effects, superseding previous more qualitative approaches.

Keywords: Localization, quantum Hall effect, Hartree-Fock interactions, non-equilibrium network model, spatially-resolved charge density, exchange-enhanced $g$-factor, local filling factor, bubble and stripe phases, Knight shift

1. Introduction

Our current understanding of the (integer) quantum Hall effects rests on three main pillars. The groundwork for them was laid in the 1980s and 1990s and the scientific papers connected with each pillar continue to be cited by the quantum Hall community and continue to inspire new experiments within the quantum Hall setting and beyond.

The first pillar represents the famous scaling theory of the integer quantum Hall effect (IQHE) \cite{1}. It is based on the so-called localization picture of the IQHE and has been underpinned experimentally and theoretically many times, for example by providing a consistent explanation of the existence of plateaus in the transversal transport characteristics as well as leading to the estimation of critical exponents for transitions between plateaus \cite{2}. The theoretical basis for this approach are narrow quantum channels of non-interacting electrons that are created at the Fermi level in strongly disordered electron systems. The paradigmatic model for this scaling scenario is provided by the famous Chalker-Coddington model \cite{3}. Equally well, a tight-binding Anderson model with onsite disorder and Peierls phases modelling the perpendicular magnetic field gives similar answers for universal properties \cite{4}.

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The existence of narrow transport channels had to be assumed without detailed microscopic justifications. The second pillar of the IQHE provides this detail by also including electron-electron interactions at the single-particle level. This delivers a much more realistic screening behavior in the smoothly disordered bulk and edge potentials. The main result is that the narrow channel picture has to be modified and, instead, the associated screening behavior creates wide channels at the Fermi level \([5]\). At this point wide, so-called compressible and incompressible, stripes have been born. These became a real focus of IQHE research and the existence of compressible and incompressible stripes has been confirmed experimentally by local probe measurements albeit not their spatial widths.

Going beyond screening effects leads to the third pillar of IQHE physics. First theoretical approaches used completely disorder-free systems and successfully delivered, for example, insights into the surprising spontaneous appearance of ordered charge-density waves observed at lower disorder and weaker magnetic fields. These density modulations appears as stripes or bubbles in high mobility samples at larger filling factors \([6]\).

More than 20 years after introduction of these seminal discoveries and more then 40 years after the discovery of the IQHE itself, one might expect that by now all those milestones have merged into a well understood unifying picture of the IQHE as a whole. However, a look at the literature delivers an quite unexpected and different situation: each of the "pillar" papers cited above has about 500, and some times even more citations. However, they share in citations much less often. Pillar 1 and 2 papers share only 15 papers citing them both, pillar 2 and 3 only 8, while pillar 1 and 3 have just 6 such joint citations. All three pillar papers together share only two joint citations \([7]\), namely, an older paper on using percolation-type physics to explain QH effects in anti-dot lattices \([8]\) and a recent review in Russian \([9]\). Indeed, analysing for shared Phys. Rev. Lett. citations — as a proxy for generally accepted importance — gives an even bleaker picture: only pillar 1 and pillar 2 paper share 2 papers in Phys. Rev. Lett. citing them both. This is clearly a surprising and somewhat disconcerting finding; von Klitzing himself mentions “a microscopic picture of the quantum Hall effect for real devices with electrical contacts and finite current flow is still missing” \([10]\).

Surely it should be possible to explain the experimental observations of IQHE physics within a unified understanding. We believe that we have recently made great progress towards such an understanding. In a series of papers, using a self-consistently converged numerical Hartree-Fock approach, together with an effective non-equilibrium model for the finite current flow, we have been able to model (i) the IQHE plateau-plateau transitions, (ii) the appearance of bulk and edge channels as well as (iii) the emergence of the bubble and stripe phases. Within this model, the only parameters changing across all three areas were the strength of the magnetic field \(B\) and the characteristic parameters of the smooth disorder potential such a dopant density and potential strength.

In the following, we shall provide some of the details of our approach, focusing on hitherto unpublished results and information which at best appeared in supplemental materials accompanying the original publications. This paper should hence usefully be read as a companion source to those previous publications.

2. Method

In order to model a high-mobility heterostructure in the QH regime, we consider a two-dimensional electron system (2DES) in the \((x, y)\)-plane subject to a perpendicular magnetic field
\( \vec{B} = B \vec{e}_z \). The Hamiltonian

\[
H_{2\text{DES}}^{\sigma} = \hbar \sigma + V_{\text{Coulomb}} = \left( \vec{p} - e \vec{A} \right)^2 / 2m^* + g^* \mu_B B + V_{\text{impurities}}(\vec{r}) + V_{\text{Coulomb}}(\vec{r}, \vec{r}'),
\]

(1)
describes spin degree of freedom, \( \sigma = \pm 1 \), a smooth random potential \( V_{\text{impurities}} \) modeling the effect of the electron-impurity interaction, the electron-electron interaction \( V_{\text{Coulomb}} \) and with \( m^* \), \( g^* \), and \( \mu_B \) the parameters for effective electron mass, \( g \)-factor, and Bohr magneton, respectively. The electron-impurity interaction is modeled electrostatically and describes a remote impurity density separated from the plane of the 2DES by a spacer-layer of thickness \( d \), as found for instance in modulation-doped GaAs-GaAlAs heterojunctions. This creates a random, spatially correlated potential with a typical length scale \( d \) within the plane of the 2DES. We use \( N_1 \) Gaussian-type "impurities", randomly distributed at \( \vec{r}_i \), with random strengths \( w_i \in [-W, W] \), and a fixed width \( d \). The areal density of impurities is given by \( n_i = N_i / L^2 \) \(^{11} \). For the system’s many-body state, \( |\Phi \rangle \), we use the ansatz \(^{12},^{13} \) of an anti-symmetrized product of single particle wave-functions, chosen as a linear combination of Landau states \(^{11} \). The number of flux quanta piercing the 2DES is given by \( N_B = L^2 / 2\pi l_c^2 \), yielding a total number of \( M = N_{\text{LL}} N_B \) states per spin direction. The filling of the system is characterized by the filling factor \( \nu = \nu_\uparrow + \nu_\downarrow \) and \( N_\uparrow \) number of electrons in the system and areal density \( n_\uparrow = N_\uparrow / L^2 \). In terms of \( N_\uparrow \) spin-up and \( N_\downarrow \) spin-down electrons, we can hence write \( \nu = \nu_\uparrow + \nu_\downarrow \) and \( N_\uparrow = N_\uparrow \downarrow + N_\downarrow \). The total LL density is given by \( n_0 = eB / h \) and \( l_c = \sqrt{\hbar/eB} \) the magnetic length.

3. Results

3.1. Transport results

The hallmark of the IQHE is of course the quantization observed for the transversal conductance/resistance of a 2DES at finite perpendicular \( B \) field with steps in multiples of \( e^2 / h \ (h/e^2) \) \(^{14} \). Such behaviour can already very well be modelled using non-interacting approaches \(^2 \). In Ref. \(^{15} \), it was shown previously that, with HF interactions, a straightforward application of the Kubo formula also captures this integer quantization, but has large fluctuations in the vicinity of the plateau-plateau transitions due to finite-size effects. In Fig. \(^1 \) we show transport results also obtained from our HF calculations, but now constructed after coupling with a non-equilibrium network model (NNM) \(^{16},^{17} \) which allows to reach larger effective sizes. While the results in Fig. \(^1 \) exhibit much less fluctuations around the transitions, it must be emphasized that this is due to effectively averaging over many more nodes in the network. Furthermore, for the implementation of the procedure, an ad-hoc assumption has to be made about how to go from the computed charge densities in the HF approach to the transmittivity of a dissipative saddle-point in the NNM \(^{17},^{18},^{19},^{20} \). Effectively, this rules out the determination of the critical properties of the QH transition \(^1 \) from the NNM. In order to generate transport data, a very large number of step by step calculations are required \(^{17} \) in the NNM as well as in the HF procedure. Hence, for keeping within the available computing time, the transport simulations have been performed for sample sizes around \( 500 \times 500 \) nm\(^2 \) while the underlying HF simulations were performed for a typical IQHE set-up with relatively strong disorder potential fluctuations of \( V_{\text{impurities}} \in 10 \) mV. As shown in Fig. \(^1 \) (b), the approach can also very well capture the difference in transport between hard and easy directions in the stripe phase of the QH system \(^{21},^{20} \). The only change of substance in the simulation parameters is a reduction of the fluctuations of \( V_{\text{impurities}} \) to range from 0.43 mV to −0.50 mV. Hence the self-consistent HF
approach demonstrates its powers in capturing the large variety of transport properties observed across the realm of IQHE physics. In the following, we shall show that this power also extends to spatially resolved properties, again for larger disorders, cp. sections 3.2 and bubble and stripe phases in sections 3.3 and 3.4.

3.2. Charge densities

In Fig. 2 we show the spatial distribution of the filling factors $\nu_\uparrow$ and $\nu_\downarrow$, with $\nu = \nu_\uparrow + \nu_\downarrow$, obtained from the HF calculation as well as the associated non-equilibrium chemical potentials $\mu$ resulting from the NNM. The data were computed for the disorder potential strength $V_{\text{impurities}}$ of maximally $\pm 10 \text{ mV}$ used to generate Fig. 1. Our results for $\nu_\uparrow$ have been shown before [20], while the $\nu_\downarrow$ only appeared in the supplement. The direct comparison between $\nu_\uparrow$ and $\nu_\downarrow$ makes it clear that regions which are populated for $\nu_\downarrow$ are still largely unpopulated for $\nu_\uparrow$. Hence, instead of observing an overall, spatially homogeneous, increase or decrease of the carrier density, we find shrinking or growing clusters of fully filled spin-up LL at $\nu_\uparrow = 2$ and growing or shrinking areas of depleted spin-up LL at $\nu_\uparrow = 1$. On average this results in a continuous change of the spatially averaged $\nu_\uparrow$ and $\nu_\downarrow$. Therefore a combined $\nu_\uparrow = 1.5$ is made up by half of the area taken up by clusters of $\nu_\uparrow = 1$ and the other half taken up by $\nu_\uparrow = 2$. This is of course similar for e.g. $\nu_\downarrow = 1.5$ in the case of an average total filling factor $\nu = 2.5$.

In Fig. 3 we show how $\nu_\uparrow$ (left) and $\nu_\downarrow$ (right) change when $B$ is varied while Fig. 4 shows a situation analogous to Fig. 3 but now instead of $B$, we vary the density $\rho$. The behaviour in these figures has already be discussed in detail in Refs. [20, 21]. Here, we simply note that the aforementioned shrinking or growing clusters of fully filled and the implied growing or shrinking areas of depleted areas for the two spin directions in both figures follows the same exchange-enhanced $g$-factor "rules". Furthermore, we can study the temperature dependence, following from the broadening of the Fermi distribution, of $\nu_\uparrow$ and $\nu_\downarrow$ in Fig. 5. As should be expected, results for $T = 20$ Kelvin show a much less structured distribution for $\nu_\uparrow$ and $\nu_\downarrow$ even for the HF results.

Last, we can also directly study the different responses of the electron system when changing the underlying interaction. In Fig. 6 we show the situation without interaction in the top row. There is no visible difference between the behaviour of $\nu_\uparrow$ and $\nu_\downarrow$ anymore since the chosen $g$-factor is too small and the exchange-interaction-based enhancement is gone. We also note that the spatial variations are much less pronounced and for the same values of $\rho = 2 \times 10^{11} \text{ cm}^{-2}$ as before, the electron gas remains much more localized in the centre of the system, which leads to significantly larger local filling factors and hence, an additional conductance step in Fig. 1a. Before, the electron gas remains much more localized in the centre of the system, which leads to significantly larger local filling factors and hence, an additional conductance step in Fig. 1a. Hence, in partial summary, we have shown that our self-consistent HF modelling can reproduce the "classic" features associated with the IQHE such as the quantization of transport properties and the existence of network-like transport channels, along edges between plateau transitions and in the bulk directly at the transitions. In addition, we have full spatial resolution of local filling distributions $\nu(\vec{r}) = \nu_\uparrow(\vec{r}) + \nu_\downarrow(\vec{r})$ and find a microscopic behaviour in line with...
exchange-enhanced CDWs. However, spatially resolved experimental information does not yet exist of these phases due to the intrinsic challenge of using local scanning probes in low temperatures for such remotely doped systems [22, 23, 24]. Hence fully resolved modelling as shown here can provide a very valuable insight into the microscopic physics of the IQHE.

### 3.3. Bubble and stripe phases

Further examples of such microscopic detail are given by so-called "bubble" and "stripe" phases [25]. Their underlying geometric anisotropies have been identified, e.g., by transport experiments in higher Landau levels (LLs) of ultra-high mobility samples [26, 27, 28] and are characterized as stripes (strong transport anisotropies) or bubbles (reentrance effects). It is believed that the phases correspond to density modulations with characteristic geometric non-uniformities due to the interplay of Coulomb interaction and the wave functions in higher Landau levels. Recent theoretical modelling has mostly concentrated on transport signatures of these phases [29, 30, 31, 32] while the original papers either assumed uni-directional charge-density waves (CDWs) [33, 6] or showed the consistency of mean field treatments with anisotropic phases in a Fermi liquid [34, 35].

As we have shown in Ref. [36] the same HF approach used in the preceding sections for the strongly disordered QH liquid, also reproduces the stripe and bubble phases that emerge at weak disorder as similarly self-consistent solutions of the Hartree-Fock (HF) equations, i.e. in the experimentally relevant regime and without any ad hoc assumptions beyond a weak smooth disorder. For example, the quantitative prediction of the period of the stripes finds \( \delta = aR_c = \alpha \sqrt{2n + 1}/\hbar B/e \) with \( \alpha = 2.9 \pm 0.1 \) [6, 22, 23, 24, 26]. This shows a perhaps still underappreciated strength of the HF approach. To simulate ultra-high mobility samples, the random potential strength is kept low (cp. Fig. 7). The random potential is generated by Gaussian impurity potentials of radius 40 nm, the number of impurities is \( N = 2000 \), their random placement results in a fluctuating potential of \( V_{\text{max}} = 0.43 \text{mV} \) and \( V_{\text{min}} = -0.50 \text{mV} \), considerably less than the \( V_{\text{impurities}} \in [-10 \text{mV}, 10 \text{mV}] \) used in the preceding sections. At a total filling factor of, e.g., \( \nu = 6.54 \) (\( \nu_1 + \nu_\uparrow = 3.54 + 3.0 \)) and \( B = 1.5 \text{Tesla} \), the present low-disorder situation still corresponds to more than 2000 electrons. The filling factor and the lateral system size are made as large as possible with respect to the available computing power in order not to unduly influence the stripe and bubble formation by possible boundary effects. Configurations up to \( 1 \mu m^2 \) are achievable as shown in Fig. 8 with spatial resolution of \( \sim 4.4 \text{nm} \) well below \( l_B \) for a magnetic field \( B \) varying from, e.g., \( 1 \text{Tesla} (l_B \sim 26 \text{nm}) \) to \( 6.5 \text{Tesla} (l_B \sim 10 \text{nm}) \). In Fig. 8 we show the computed variation in \( \nu(\vec{r}) \) for three different densities \( \rho \) at fixed magnetic field \( B \) for \( \nu_1(\vec{r}) \) but also \( \nu_\uparrow(\vec{r}) \) in the left column. The choice of colors is as in Refs. [21, 20]. Clearly, depending on the value of \( \rho \), "bubble" and "stripe" phase have emerged self-consistently without prior intentional induction of any spatial anisotropy — except of course that which is induced by the randomness of the disorder. "Bubble" and "stripe" phases are hence stable phases of the HF ground state, no more "interaction" is needed for their emergence. For a detailed discussion of the features shown in Fig. 8 we refer the reader to Ref. [36].

In the top row of Fig. 9 we show the situation analogous to Fig. 8 but now computed without the Coulomb interaction. Clearly, there is no stripe formation. The \( \nu(\vec{r}) \) modulation rather closely follows the random potential of Fig. 7. The charge densities in the spin-up and spin-down levels do not influence each other and follow nearly identically the disorder potential because of the missing interaction. As we have chosen to keep the same colors as in Fig. 8 it is clear that there is comparatively very little spatial variation for \( \nu_1(\vec{r}) \) and \( \nu_\uparrow(\vec{r}) \). In Fig. 9 one can also see that for pure Hartree interaction there is no stripe formation. The density modulation in \( \nu(\vec{r}) \) is much
less than in Fig. 8 and roughly follows the random potential shown in Fig. 7. Furthermore, the charge density modulation in the spin-up and spin-down levels slightly "repel" each other due to the Hartree interaction.

3.4. Knight shift

In Ref. [24], Friess et al. investigated the Knight shift $I_N(f)$ in nuclear magnetic resonance (NMR) spectra and modelled it with a semiclassical model [39, 24] based on superpositions of the single electron densities obtained from the Landau basis functions. A periodic variation of the filling factor, due to the presence of stripes and bubbles, should result in characteristic peaks, due to the implied regions with different spin polarizations for $\nu_\uparrow(\vec{r})$ and $\nu_\downarrow(\vec{r})$. While their method provides direct information about the area fractions, there is no direct information about geometry and periodicity.

In order to show that the patterns which we observe here within HF for $\nu_\uparrow(\vec{r})$ and $\nu_\downarrow(\vec{r})$, in particular the presence of the remnants of the LL wave functions around each stripe and bubble, are significant, we perform the calculations of $I_N(f)$ [36] also for three test patterns for $\nu(\vec{r})$. The results are given in Fig. 10. We find that the variation given by $\nu(\vec{r})$ as calculated with HF can reproduce global features of the experimental NMR results presented in Ref. [24], while all test density patterns have essential departures from the experimental results.

We note that normally stripes appear only starting with filling factor $\nu = 4.5$. This is known also experimentally, but for experimental reasons the authors of Ref. [24] could not go to that filling factor. Instead, they used filling factor $\nu = 2.5$ and forced, by using an in-plane component of the magnetic field, the electron system to form a stripe pattern. Clearly, there is no need for our simulations to also model this experimental “trick”. In order to compare the effect of stripe patterns on the NMR Knight shift, we therefore use the stripe pattern in the "correct" range $\nu = 4–5$. The result in Fig. 11 (b) has striking similarities but seems indeed a bit richer in features than the experimental curve for $\nu = 2–3$. However, since the Knight shift spectrum looses its local information due to the spatial integration, we can also evaluate the range $\nu = 2–3$ as shown in Fig. 11. Indeed the agreement with the experiments of Ref. [24] becomes even better in this filling factor range, although no stripes at all are formed in our simulations in this filling factor range (see Fig. 11c). For example, in Fig. 12 around $\nu \approx 2.5$, we can still see in total 3 peaks, two of them clearly separated and a third one as a shoulder on the high frequency flank, just as shown for the experiments in Fig. 2b of Ref. [24].

4. Conclusions

In conclusion, we have shown that a computational, fully-self consistent HF calculation reproduces much of the physics of the IQHE regime, across many LLs, disorder strengths, magnetic fields and temperatures. Near macroscopic system sizes of up to $1 \mu m^2$ can be reached. When coupled with a NNM, the approach is able to model the observed quantization of $R_{xy}$ at strong disorders as well as to reproduce the transport anisotropies for stripe and bubble phases at low disorders. This success relies in a fundamental way on the underlying basis set of LL wave functions. When these fail to be good descriptors of the physical situation, then the predictions of the HF approach become less reliable. This is clearly the case for $B \to 0$, i.e. when $l_c \sim L$. On the other hand, for strong magnetic fields at low disorders, one expects to see fractional quantum Hall physics to appear [40, 2]. In principle, the LL basis should still be able to capture such behavior. However, our results up to $\nu = 6$ suggest that this requires a much larger number of LLs.
than just those needed to accommodate all electrons. So far this appears to be beyond available computing resources.

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7
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Figure 1: (a) Two-point conductance $G$ versus carrier density $n$ at fixed magnetic field of $B = 3$ T for the non-interacting ($\times$), the Hartree-interacting (+) and the Hartree-Fock-interacting model (○) at strong disorder. The horizontal dotted lines indicate integer multiples of $e^2/h$ while the vertical dashed lines indicate the three density values of $n = 1.9, 2.0, 2.15 \times 10^{11}$ cm$^{-2}$. (b) Longitudinal resistance $R_{xx}$ and Hall resistance $R_{xy}$ versus density for a HF interacting system at weak disorder, exhibiting transport anisotropies due to the formation of stripe phases. Easy and hard directions are indicated by different colors.
Figure 2: Top row: Spatial distribution of filling factors $\nu_\\uparrow$ (left) and $\nu_\\downarrow$ (right). The colors represent the filling factor, where blue means the first LL for $\nu_\\uparrow,\downarrow = 0 \rightarrow 1$, green the second LL for $\nu_\\uparrow,\downarrow = 1 \rightarrow 2$ and red the third. The filling factor range close to $\nu_\\uparrow,\downarrow = 1.5$ is highlighted in light gray in order to identify the stripes appearing close to the half filled top LL. They appear only for $\nu_\uparrow$. Bottom row: Spatial distribution of non-equilibrium chemical potential $\mu$ (colors) shown on top of the corresponding $\nu_\\uparrow,\downarrow$ distributions (grey heights) as in the top row. The colors represent $\mu$ in arbitrary units with overall clock-wise propagating potential reducing from the high potential supplied to the current contact on the left from red to orange and yellow while the low potential is supplied to the current contact on the right and is indicated as increasing from blue to cyan to green. The parameters in both rows are identical, i.e. constant carrier density $\rho = 2 \times 10^{11}$ cm$^{-2}$ and $B = 3$ Tesla. The situation corresponds to the $\nu = 3 \rightarrow 4$ plateau transition of Fig. [11].
Figure 3: Lateral distribution of filling factors $\nu^\uparrow$ (left) and $\nu^\downarrow$ (right) when (top row) decreasing or (bottom row) increasing the magnetic field $B$ at fixed $\rho = 2 \times 10^{11}$ cm$^{-2}$. Colors are as in Fig. 2.
Figure 4: Lateral distribution of filling factors $\nu_\uparrow$ (left) and $\nu_\downarrow$ (right) when (top row) decreasing or (bottom row) increasing the density $\rho$ at fixed $B = 3$ Tesla. Colors are as in Fig. 2.

Figure 5: Distribution of filling factors $\nu_\uparrow$ (left) and $\nu_\downarrow$ (right) when increasing the temperature to $T = 20$ Kelvin (2 meV). Parameters are again $\rho = 2 \times 10^{11} \text{ cm}^{-2}$ and $B = 3$ Tesla. We use the same colors scheme as in Fig. 4.
Figure 6: Distributions of $\nu_\uparrow$ and $\nu_\downarrow$ for (top row) the interaction-free single particle approximation, (centre row) the Hartree approximation and (bottom row) the same Hartree results, but now without color-highlighting of the half-odd integer filling. The parameters correspond to $\rho = 2 \times 10^{11}$ cm$^{-2}$ and $B = 3$ Tesla as in Fig. 2. The colors represent $\nu_\uparrow$ and $\nu_\downarrow$ as in Fig. 2 with additional colors light blue and yellow denoting LLs 4 and 5. The filling factor range close to $\nu_\uparrow$, $\nu_\downarrow$ = half-odd integer is highlighted in light gray from LL2 onwards in order to identify possible stripes appearing close to the half filled top LL.
Figure 7: Lateral random disorder potential $V(r)$ visualized in a false color plot. The lines denote equipotentials while the potential energies are indicated by the colors as in the color scale provided.
Figure 8: Distributions of $\nu_\uparrow$ and $\nu_\downarrow$ for (top row) $\nu = 6.20$, (central row) $\nu = 6.54$ and (bottom) $\nu = 4.54$ with the left column corresponding to $\nu_\uparrow$ and the right column to $\nu_\downarrow$ in all three rows. Colors are as given before, i.e. as in Fig. 4 and others above. The parameters now correspond to $\rho = 10^{11}$ cm$^{-2}$ and $B = 1.5$ Tesla.
Figure 9: Lateral charge density of the partly filled top Landau levels at total filling factor $\nu = 4.5$. The top row shows results for the non-interacting system while the bottom row corresponds to a pure Hartree interaction with left column corresponding to $\nu_\uparrow$ and the right column to $\nu_\downarrow$ in all two rows. The color shades represent the filling factor $\nu(\vec{r})$, i.e. as in Fig 4 and other above.
Figure 10: Calculations of the NMR intensity $I(f)$ for 4 different stripe-like variations of $\nu(\vec{r})$ at $\nu = 4.5$. Rows 1-4 corresponds to (a) a simply square modulation, (b) a sinusoidal modulation, (c) a sinusoidal variations with left and right shoulders, (d) a modulation as computed from HF. The first column shows the spatial variations in $\nu(\vec{r})$ as given by the color scales. The second column represent a typical cross-section for each situation and the third column shows the estimated NMR intensity $I_{4.5}(f)$.
Figure 11: NMR intensities $I_{\nu}(f)$ for (a) $\nu = 2–3$ and (b) $\nu = 4–5$ and local $\nu(\vec{r})$ at (c) $\nu = 2.5$ and (d) $\nu = 4.5$ in left and right columns, respectively. (b+d) The right column shows results as in the bottom row of Fig. 10. The color shades represent $I_{\nu}(f)$ and $\nu(\vec{r})$ as given by the scales. Lines in (c+d) connect equal height in $\nu(\vec{r})$. 
Figure 12: NMR intensities $I_{\nu}(f)$ as shown in Fig. [13] for selected filling factors in range $\nu = 2 - 3$. The style is similar to the one used in Ref. [24].