Study on dynamic response characteristics of high and steep layered rock mass slopes using a modal analysis method

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Abstract. The common geological bodies found in southwest China are layered mass slopes. The modal analysis method is performed on four finite element numerical models of rock slopes, including the homogeneous slope, horizontally layered slope, bedding slope, and toppling slope to investigate the dynamic deformation characteristics of the layered slopes. Their dynamic deformation characteristics are systematically studied according to the modal characteristics of slopes. By using modal analysis, the numerical results show that a series of natural frequencies and vibration modes of the slopes can be obtained. The type of structural planes affects the natural frequency of the slopes. The values of the natural frequency of the slopes are as follows: homogeneous slope > toppling slope > horizontally layered slope > bedding slope. Weak structural planes also have an amplification effect on the dynamic deformation of the slopes according to the analyses of the relative displacement (U) of the first vibration modes of the slopes. The natural frequency has a significant impact on the dynamic deformation characteristics of the slopes. Low-order natural frequency mainly induces the overall deformation of the top slope and slope surface area, whereas the high-order natural frequency mainly induces the local deformation of the slope. Modal analysis can provide a new idea for further study of dynamic characteristics of rock slopes.

1. Introduction
The mountainous areas in western China are characterized by complex terrain [1-3], and frequent earthquakes induce a large number of landslides, which have become one of the major earthquake disasters in western China [4,5]. Seismic landslides have caused huge losses in western China recently [6,7]. The Wenchuan earthquake that occurred on May 12, 2008 caused the largest number of landslides in Chinese history [8]. Also, a massive landslide occurred in Xinmo Village, Sichuan province, China, on June 24, 2017, which is closely related to several major earthquakes in Chinese history [9]. It can be seen that an earthquake has a long-term effect on a landslide; therefore, seismic landslides have attracted research attention. Hence, it is important to study the dynamic response of seismic landslides, which is of great scientific significance for seismic disaster reduction.

Many scholars have performed much research recently on the dynamic response of rock slopes. Che et al. [10] studied the seismic dynamic response of a rock slope containing discontinuous joints by using a shaking table test and discussed the influence of joint distribution on its dynamic response. Li an Lin [11] used a shaking table test to study the dynamic response characteristics of the slope under different external loads and to investigate the influence of discontinuity on the dynamic response of rock slope. By using a shaking table test, Fan et al. [12] studied the law of seismic dynamic response of layered rock slope. To investigate the dynamic response characteristics of rock slope with weak structural surfaces, Song et al. [13] used shaking table test and discussed the influence of the structure on the
seismic failure mode of the slope. Zhu et al. [14] utilized a shaking table test on the rock slope with a cohesive structural surface and studied the propagation characteristics and dynamic evolution law of the seismic wavefield. In the study of dynamic response characteristics of rock slope by numerical simulation and shaking table model test, it can be seen that many achievements have been made. The dynamic response characteristics of slopes have been complicated due to the complex geological structure of rock mass slopes [15-19]. Modal analysis is mainly used at present to study the dynamic characteristics of structures and mechanical engineering [20,21]. Although there are few studies on the dynamic response characteristics of the rock slope in complex geological structure, modal analysis can better reflect the dynamic deformation characteristics of the engineering entity.

Four numerical models are established in this study using the finite element method (FEM), including homogeneous slope, horizontally layered slope, bedding slope, and anti-dip slope. The relationship between the natural frequency of layered slopes and their dynamic deformation characteristics is investigated following the modal analysis of the slopes, and the influence of weak structural planes on their dynamic response characteristics is analyzed. Moreover, by analyzing the relative displacement ($U$) change of the first-order mode, the dynamic amplification effect of the layered slope is analyzed, the influence of different types of structural planes on the dynamic response characteristics of slopes is studied, and the influence of natural frequency on the dynamic failure mechanism of the layered slope is also discussed. This study provides a new way to investigate the dynamic response of rock slopes.

**Figure 1.** The intensity of the 1996 Lijiang earthquake and its induced landslide hazard.

**Figure 2.** Geological generalization model of the slopes: (a) model 1, (b) model 2, (c) model 3, and (d) model 4.

**Table 1.** Physico-mechanical parameters of material parameters of the slope.

| Material parameters      | Density $\rho$ (g/cm$^3$) | Poisson ratio $\mu$ | Elastic modulus $E$ (MPa) | Friction angle $\phi$ (°) | Cohesive force $c$ (kPa) |
|--------------------------|----------------------------|---------------------|---------------------------|--------------------------|-------------------------|
| Rock mass                | 2.4                        | 0.16                | 375.0                     | 35.0                     | 1,200                   |
| Weak structural planes   | 18.4                       | 0.35                | 35.0                      | 23.0                     | 14                      |
2. Regional engineering geology overview
The location of the study area is in the middle reaches of the Jinsha river in China. The extension direction of rivers and mountains in the region is generally consistent with the direction of structural lines. Figure 1 shows the geographical location of the study area. Between the Eurasian plate and the Indian plate, the study area is located in the suture zone and belongs to the Sichuan–Yunnan block of the Qinghai–Tibet plateau. Several active faults pass through the periphery of the study area, and the tectonic activity of the regional tectonic block in the area is strong. Strong earthquakes induced a large number of landslides near the study area like the 1996 Lijiang earthquake [20]. The geological model generalization of the homogeneous slope, bedding slope, horizontally layered slope, and the anti-dipping slope is used in this work to compare the influence of the weak structure on the dynamic response of the slope, as shown in Figure 2. The elevation of the three model slopes is 40 m, which mainly comprises weak structural planes and rock mass. The rock mass is silty mudstone, and the weak structural plane is mainly composed of clay. In Table 1, the physical and mechanical parameters of the slope generalization model are shown.

3. Principle of modal analysis
Mode means that when the system vibrates freely according to its natural frequency, the displacement of each particle in the system deviated from its original equilibrium position satisfies a certain proportional relation. This also refers to the shape of vibration when the system vibrates under a natural frequency, in which the first mode is the dominant mode. An important method of dynamic structural analysis and a basic type of dynamic frequency analysis is modal analysis. FEM, in particular, is a commonly used method for modal analysis. The dynamic control equation of modal analysis is as follows, based on the principle of elasticity [20,21]:

\[ [M][\ddot{U}]+[C][\dot{U}]+[K][U]+[F] = 0 \]  \hspace{1cm} (1)

where \([M]\), \([C]\), and \([K]\) are mass matrix, damping matrix, and stiffness matrix, respectively. \([F]\) is the load function of external force with time; \([\ddot{U}]\) and \([\dot{U}]\) are the acceleration vector and velocity vector of the model, respectively; and \([U]\) is the displacement vector of the model, which is used to describe the modal analysis of the vibration mode. The influence of external force and damping effect is not considered in the modal analysis. The dynamic control equation of modal analysis can be expressed as [15,16]

\[ [M][\ddot{U}]+[K][U] = 0 \]  \hspace{1cm} (2)

The characteristic equation of Eq. (2) is as follows:

\[ ([K]−\omega^2[M])[U] = 0 \]  \hspace{1cm} (3)

where \(\omega_i\) is the \(i\)th natural circular frequency.

The natural frequency \(f_i\) obtained is as follows:

\[ f_i = \frac{\omega_i}{2\pi} \]  \hspace{1cm} (4)

The eigenvector corresponding to the eigenvalue is \([U]\)_i, which is the mode shape of vibration at the \(f_i\). The first mode is the main mode, and the low-order modes mainly control the dynamic characteristics of the model. Only the first few modes are considered in the modal analysis.

4. Finite element modal analysis

4.1. Numerical model
According to the original size of the high and steep rock mass slopes, four finite element numerical models were established: model 1 (homogeneous slope), model 2 (anti-dip slope), model 3 (horizontally layered slope), and model 4 (bedding slope), as shown in Figure 3. The size of the models is 170 m (length) × 80 m (height), and the gradient of the slopes is approximately 70°. Using a quadrangular grid, we modeled the rock mass and structural planes. The model mainly comprises weak structural planes and rock mass. Table 1 shows the physical and mechanical parameters of the
rock mass of the slopes. The deformation of the slopes is a small strain problem in finite element modal analysis. The boundary conditions of the slopes are infinite domain in the actual situation. The boundary conditions on both sides and the bottom of the models are set as infinite element boundaries to make the boundary conditions of the models conform to the actual situation. Therefore, the model grid mainly comprises two parts: the finite element meshes of rock mass and the infinite element meshes at boundary conditions. In this work, the linear perturbed analysis step in ABAQUS implicit solution function is used for FEM modal analysis.

Figure 3. Finite element numerical model: (a) model 1, (b) model 2, (c) model 3, and (d) model 4.

Figure 4. Natural frequencies of the four models.

4.2. Natural frequency analysis

Using FEM, we performed the modal analysis of the four models. Generally, the first few modes can reflect the dynamic deformation characteristics of the system. The first eight modes and natural frequencies of the models are selected for analysis in this work, and Figure 4 shows the first eight natural frequencies. The first four natural frequencies of model 1 are 2.96, 7.84, 11.18, and 18.06 Hz, respectively, as shown in Figure 4. The natural frequencies of the first four orders of model 2 are 2.86, 7.78, 10.89, and 17.73 Hz, respectively. The natural frequencies of the first four orders of model 3 are 2.83, 7.75, 10.69, and 17.33 Hz, respectively. The natural frequencies of the first four orders of model 4 are 2.81, 7.73, 10.41, and 17.10 Hz, respectively.

The natural frequencies of the four models increase with the increase of mode order, as shown in Figure 4, and their first four natural frequencies are similar. However, the natural frequencies of the three models are different after the fifth mode order. The natural frequency of model 1 is significantly greater than that of the other three models, whereas the natural frequency of model 2 is significantly larger than that of model 3, and that of the model 4 is the largest of all. This phenomenon indicates that due to that the existence of the weak structural surface reduces the stiffness of the slope, the natural frequency of the homogeneous slope is larger than that of the layered slopes on the whole. The decrease of stiffness results in the decrease of the natural frequency of the slope decreases accordingly. Hence, the natural frequency of the other models is as follows: anti-dip slope > horizontally layered slope > bedding slope, which indicates that the bedding weak structural planes have the most adverse effect on the stiffness of the slope and the anti-dip structural planes have less impact on it, whereas the adverse impacts of horizontally layered structural planes on its stiffness are the smallest of all.

4.3. Analysis of the dynamic deformation characteristics

The first few modes are mainly considered in the modal analysis, and low-order modes play a decisive role in the dynamic characteristics of the model [21]. The analysis of the vibration mode can provide a reference for the dynamic failure mode of the slope, and the vibration mode of the slope corresponds
to its natural frequency [22,23]. In Figures 5–8, the four-order modes of the four models are shown. In
the first vibration mode of the four models, the relative displacement $U$ of the top slope area is
relatively large, and the $U_{\text{max}}$ is concentrated on the slope crest, which indicates that the top slope area
is the most prone to deformation and damage under earthquake, as shown in Figures 5–8. The $U$ at the
slope surface area is greater than that in the internal slope, and the $U_{\text{max}}$ is mainly concentrated in the
slope surface area, as shown in the second-order mode. The third-order and fourth-order modes show
that the $U_{\text{max}}$ is concentrated in the slope crest area. Therefore, the $U_{\text{max}}$ of the first four-order modes of
models 1–4 mainly appeared in the top slope and slope surface area, which indicated that the dynamic
deformation characteristics of the top slope and slope surface area were the most obvious.

**Figure 5.** Vibration mode of the modal analysis of the model 1: (a) first order, (b) second order,
(c) third order, and (d) fourth order.

**Figure 6.** Vibration mode of the modal analysis of the model 2: (a) first order, (b) second order,
(c) third order, and (d) fourth order.

**Figure 7.** Vibration mode of the modal analysis of the model 3: (a) first order, (b) second order,
(c) third order, and (d) fourth order.

**Figure 8.** Vibration mode of the modal analysis of the model 4: (a) first order, (b) second order,
(c) third order, and (d) fourth order.

Moreover, it can be seen that an obvious difference of the $U$ on both sides of the structural planes
in models 2–4 can be found compared with the vibration mode of model 1, according to Figures 5–8.
Hence, there is an obvious deformation phase shift on both sides of the structural planes. In particular,
the phase shift is more obvious in the mode above the third order. This phenomenon indicates that
the structural planes have an obvious influence on the dynamic deformation characteristics of the slopes,
especially in a high-frequency band. Also, the dynamic deformation response characteristics of the
four models are closely related to the natural frequency of the slopes, as also shown in Figures 5–8.
The first natural frequency mainly induces the overall deformation of the top slope area, and the
second natural frequency mainly induces the overall deformation of the sloped surface area. The
natural frequency mainly induces local micro-deformation at the top of the slope after the third natural
frequency. It can be seen that the low-order natural frequency mainly induces the overall deformation of the top of the layered slope and slope surface, whereas the high-order natural frequency \( (f > 10 \text{ Hz}) \) mainly induces the local deformation of the slope crest or other local area. In particular, local deformation has an important influence on the seismic cumulative failure effect of the slopes in the rock mass slopes containing complex geological structures. Sliding failure gradually occurs with the accumulation of local deformation, and the deformation of the slopes under earthquake starts from local deformation. Therefore, the influence of the high-frequency band on dynamic deformation characteristics of slopes deserves further attention.

4.4. Dynamic amplification effect analysis

The first mode is the main mode of the slopes, which can reflect the main dynamic deformation characteristics of the slopes [24]. Some measuring points at different elevations of the slope were taken as the research object to study further the dynamic amplification effect of the slopes (Figure 2). In Figure 9, the change rule of the relative displacement \( U \) of the first-order mode slope with the elevation is shown. The \( U \) of the four models increases gradually with the increase of elevation, which indicates that the dynamic amplification effect in the slope increases with the increase of elevation, as shown in Figure 9a. Figure 9b shows that the \( U \) of the four model slope surface increases with the increase of elevation, which also indicates that the slope surface has an elevation amplification effect. The \( U \) of the slope surface and internal slope in model 1 (homogeneous slope) increases overall performance for increasing linear trend with slope elevation, whereas the \( U \) of the other three models (layered slopes) shows a nonlinear increase with the increase of elevation, as shown in Figure 9. This indicates that weak structural planes have a great influence on the dynamic amplification effect of the slopes. Also, it can be seen that the \( U \) of the slope surface is significantly larger than that in the internal slope by comparing Figure 9a,b, which indicates that the dynamic amplification effect of the slope surface is much larger, which is consistent with the shaking table test results [12]. Relative displacement ratio \( (U_S/U_I) \) between the slope surface and the internal slope is shown in Figure 10 to study this phenomenon further. Figure 10 shows that the \( U_S/U_I \) values of the four models are generally between 1.2 and 2.0; in particular, the \( U_S/U_I \) value of the homogeneous slope is significantly greater than that of the layered slopes. This indicates that the dynamic magnification effect of the slope...
surface is much larger than that of the internal slope, especially compared with that of the layered slope; in particular, compared with the layered slopes, the slope surface magnification effect of the homogeneous slope is more obvious.

4.5. Dynamic deformation mechanism of slope based on modal analysis
The seismic failure mechanism of layered slopes is closely related to their natural frequencies based on the aforementioned modal analysis. The low-order and high-order natural frequencies mainly cause the overall and local deformation of the slopes, respectively. Seismic waves are rich in many complex frequency components, and the seismic instability failure mechanism of rock slope is mainly related to the predominant frequency of seismic waves and the natural frequency of the slope. The resonance between the low-order predominant frequency of the seismic wave and the low-order natural frequency of the slopes is the main inducement to trigger the landslide. The high-order natural frequency of the slope mainly causes local deformation of the slopes. Overall sliding failure will occur under the action of low-order natural frequency as the local deformation of the slope surface accumulates when it reaches a certain value. By using shaking table tests, the seismic failure mode of the layered slope is mainly manifested as sliding failure in the slope crest area, which is consistent with the failure phenomenon of the layered slopes [12]. Hence, the deformation and failure of the slope are accumulating continuously with the increase of ground motion intensity, and the failure scale is gradually enlarged, which is related to the high-order natural frequency.

5. Conclusion
Modal analysis of different types of layered rock slopes is carried out by the FEM. The following conclusions can be drawn. By using FEM modal analysis method, the natural frequencies of the four models can be obtained. The natural frequency of layered rock slope increases with the increase of the order. The first four natural frequencies of the four models are approximately 2–3 Hz, 7–8 Hz, 10–11 Hz, and 17–18 Hz, respectively. The order of natural frequency values is as follows: anti-dip slope > horizontally layered slope > bedding slope. The dynamic deformation characteristics of the top slope and slope surface area were the most obvious according to the $U_{\text{max}}$ of the first four-order modes of the slopes. Weak structural planes have an impact on the dynamic deformation characteristics of the slopes. Elevation has an amplification on the dynamic response of the slopes, and the dynamic amplification of the slope surface is larger than that of the internal slope. Structural planes influence the dynamic amplification of slopes, and their order is as follows: bedding slope > horizontally layered slope > anti-dip slope > homogeneous slope. The seismic failure of layered slopes is closely related to its natural frequency. Low-order natural frequency mainly induces the overall deformation of the top slope and slope surface area, whereas the high-order natural frequency mainly induces the local deformation of the slope.

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