Using the Theory of Games to Modelling the Equipment and Prices of Car Parking

Waldemar Parkitny

1 Cracow University of Technology, Faculty of Civil Engineering, Institute of Building and Transport Management, ul. Warszawska 24, 31-155 Cracow, Poland
wpark@usk.pk.edu.pl

Abstract: In large cities there are two serious problems connected with increasing number of cars. The first problem is the congestion of vehicles' movement. The second one is too small number of car parks, especially in centres of the cities. Authorities of cities and management of municipal streets introduce limitations in vehicles' movement and reduce the number of car parks to minimalize streets crowd. That acting seems logical, but this is only the one point of view. From the other point of view municipal governments should aim to improve the level of the occupants' life and assure the financial incomes, which enable to cover indispensable expenses. From this point of view, the municipal car parks are needed and bringing the profits element of municipal infrastructure. Cracow, which is one of the largest cities in Poland (about 760 thousands of occupants, and Cracovian agglomeration is about 1.4 million persons), was chosen as the object of the investigations. The zone of paid parking in Cracow, administered by the company belonging to city, has possessed 28837 parking places in 28.01.2016. In the zone there are assigned car parks or parking places near to the curbs and on pavements. The zone operates from Monday to Friday, from 10.00 to 20.00. Assuming using car parks only in 50% and fare of about 0.7 euro per hour, we receive incomes figuring out about 740000 euro/month. The purpose of the investigations was the identification of technical parameters of car parks being preferred by drivers. The investigations had been executed by method of questionnaires. Next the mathematical model of competition was made. The model was executed basing on the theory of games. Strategies of "Player 1" were prices and technical equipment of car parks and parking places lying in the zone of paid parking, administered by municipal company. Strategies of "Player 2" were prices and technical equipment of car parks belonging to private owners and two commercial centres in the city center. There were assumed that from the city's point of view it is one rival, independently on the actual ownership of private car parks on whose strategy of acting the city do not have an influence. It is consistent with the basic foundations of the game theory. Both players compete for consumers - drivers using car parks, with such parameters like: price, distance from the destination, car parks' equipment. The built model allows to indicate the best strategy. Knowing that strategy, one can form prices and equipment of car parks. That model may be used by municipal governments and companies which administer car parks. However, one should remember about limitations, which occur in reality, e.g. law restrictions referring to maximum prices for parking. The increasing pressure of cities' authorities in Poland for changing those regulations, and examples of such solutions received, e.g. in USA, which concern varying prices for parking according to the demand on different car parks, and different hours, permit to have hopes, that the proposed model will be possible to use practically, soon.

1. Introduction

In large cities there are two serious problems connected with increasing number of cars. The first problem is the congestion of vehicles' movement. The second one is too small number of car parks,
especially in centres of the cities. Authorities of cities and management of municipal streets introduce limitations in vehicles' movement and reduce the number of car parks to minimalize streets crowd. That acting seems logical, but this is only the one point of view. From the other point of view municipal governments should aim to improve the level of the occupants' life and assure the financial incomes, which enable to cover indispensable expenses. From this point of view, the municipal car parks are needed and bringing the profits element of municipal infrastructure.

As the objects of the investigations, car parks in Cracow were chosen. Cracow, which belongs to the largest cities in Poland (about 760 thousands of occupants, and Cracovian agglomeration is about 1,4 million persons), was chosen as the object of the investigations.

The zone of paid parking in Cracow, administered by the company belonging to city, has possessed 28837 parking places in 28.01.2016. Taking into account working hours of the zone of paid parking and assuming using car parks only in 50% and fare of about 0,7 euro per hour, we receive incomes figuring out about 740000 euro/month. Nevertheless the facilitations in parking in centres of cities generate the bigger movement of vehicles, as well as influence on roads crowd, pollution and noise, from the financial point of view it is worth to think about the way of competing with private car parks. The mathematical model of car parks can be a tool to analysis. The model of competition built on the base of theory of games is presented below.

One can find different models of car parks in literature. M. Nourinejad, M. J. Roorda produced the model of setting parking prices [1]. L. Guo, S. Huang, J. Zhuang, A. W. Sadek in their paper write, among others, about the role of drivers’ psychological proprieties in process of choice of parking places [2]. Those authors paid attention to defining environmental costs connected with seeking parking places [3]. Parkitny W. introduced logit models to predict the behaviour of personal vehicles’ drivers with reference to car parks choice [4]. F. Zong, Y.-S. Zhang, Z.-Z. Wang, Z.-Y. Li show the manner of using the theory of games to describe relation between car parks’ administrators and drivers [5]. Authors of the publication [6] paid attention to data’s credibility of models of information systems about parkings. W. Young, R. G. Thompson, M. A. P. Taylor, performed a review of urban car parking models [7]. Another car parks models, including those that use the theory of games, may be found e.g. in papers [8, 9, 10, 11, 12, 13, 14, 15].

2. Description of objects of investigations, aim and method of the investigations

Cracow is the voivodeship city in which there is the seat of the Małopolska Voivodeship. There are many universities in the city. There are also different scientific research centres and service centres of the largest global corporations. This is also the former historical capital of Poland. The city is well-known for its relics and art. It is estimated that about 10 million tourists come there during a year. Such a great number of people in the city and attendant upon it the vehicles' movement had caused, that many years ago there has been taken the decision about the ban of vehicles' movement in the city centre which means in the Old Town. There have been introduced the zones of limited movement on the areas adjoining the Old Town, too. Within a dozen or so months the zones of paid parking have been also augmented. In the zone there are assigned car parks or parking places near to the curbs and on pavements. The zone operates from Monday to Friday, from 10.00 to 20.00 [16, 17]. The introduced limitations in movement of vehicles cause that the occupants and tourists can get to the Old Town only on foot, walking several hundred metres from stops of collective transportation or car parks, which filling by vehicles in some hours comes up to 100%.

The investigations included 5 parkings. Four of them were the private car parks (their actual names have been changed in further description). Two of them are multistoreyed ones, belonging to shopping malls (Shopping Mall "A" and Shopping Mall "Z"). The shopping malls are placed near to the exact centre of the city about historical and touristic character. And yet the car parks are intended for customers of shops and service points of the shopping malls, one may observe, that some people use those car parks in different aims, among others as a good place to parking vehicles while visiting the Old Town. The third car park is the big square meant to parking vehicles, laid near to the Old Town. The car park in Shopping Mall "Z" was free a short time ago. The payment was introduced in connection with the augmentation of the zone of paid parking in the city centre. There were assumed that from the city's point of view those parkings are one rival (independently on the actual ownership of private car parks), and the city do not have an influence on the strategy of private car parks' acting.
Those car parks were compared with the administered by municipal company place to parking located in the zone of paid parking near the Old Town in Garncarska Street, between Karmelicka and Łobzowska Streets. Vehicles can park there standing at an angle to sidewalk.

The purpose of the investigations was the identification of parameters of individual car parks, identification and working out the ranking of car parks’ features being the most preferred by drivers, working out the coefficients of preferences as well as construction of the model of car parks’ competition basing on the theory of games. The identification of parking parameters was executed on the ground of ground investigations and observations, and identification of the drivers’ preferences – on the base of questionnaire investigations.

3. Using the theory of games in choosing parking strategies, results and discussions
The procedure of car parks’ estimation and allotting importance to the model of car parks’ strategies estimation based on the game theory, consists of several stages:
1. stage I – evaluation of position, price and technical equipment of particular car parks,
2. stage II – awarding of points to particular car parks,
3. stage III – definition of coefficients of car parks’ choice preferences by drivers’
4. stage IV – identification of present strategies as well as choice of alternative strategies,
5. stage V – building of matrix of payments according to the game theory,
6. stage VI – choice of the optimum strategy.

| Parameter                  | Car park Garncarska Street | Car park C | Open space car park - Shopping Mall „A” | Multistoreyed car park - Shopping Mall „Z” |
|----------------------------|----------------------------|------------|----------------------------------------|-------------------------------------------|
| distance                   | 685 m                      | 780 m      | 1390 m                                 | 1210 m                                    |
| parking fee                | up to 20 min. = 1 zł, the   | up to 20   | up to 20 min. = 1 zł                   | up to 20 min. = 1 zł                      |
|                           |    hour = 3 zł, the second    | min. =     | the first hour = 0 zl                   | the first hour = 0 zl                     |
|                           |    hour = 3.50 zł, the third | 3.50 zł/h  | the second hour = 2 zł                 | the second hour = 2 zł                    |
|                           |    hour = 4.10 zł, the fourth |            | every next hour = 4 zł                  | every next hour = 4 zł                    |
|                           |    and next hours = 3 zł     |            | daily fee = 50 zł                      | maximum daily fee = 50 zł                 |
| enclosure                  | no                         | yes        | yes                                    | yes                                       |
| monitoring                 | no                         | at the gate| partly                                 | partly                                    |
| workers’ presence          | no                         | yes        | yes                                    | yes                                       |
| pavement                   | asphalt                     | asphalt,   | asphalt/concrete                        | asphalt/concrete                           |
|                           | holes                       | ground,    |                                        | asphalt/concrete                           |
| number of places           | 65                         | 250 + 15   | 558                                    | 830                                       |
| mark of single place for   | no                         | no         | yes                                    | yes                                       |
| a vehicle                  |                            |            |                                        |                                           |
| numbering of place for a   | no                         | no         | no                                     | yes                                       |
| vehicle                    |                            |            |                                        |                                           |
| opening hours              | 7 days/24 h                 | 7 days/24 h| 7 days/24 h                            | Mon.-Sat.: 8.00-23.00, Sun.: 9.00-22.00 |
| lighting                   | yes                        | yes        | yes                                    | yes                                       |

Stage I and II – Distance from the geometrical centre of every car park to the middle of Sukiennice on the Main Market Square in Cracow was taken to the calculation. Sukiennice is situated in the
middle of the Main Maerket Square, with museums of art and the Main Market Square, souvenirs shops, cafes and restaurants inside. There were shops in the Middle Ages. The distance was reckoned not in a straight line, but along the shortest track of pedestrian walking from particular car parks to Sukiennice. The technical parameters of individual car parks were presented in Table 1, and points awarded to parkings for technical parameters – in Table 2.

The parameter "chance for finding a vacant stand " is connected with number of places on car parks, and the "lack of limitations of parking time" – with opening hours of studied car parks. The size of delimited individual parking place concerns only those car parks, which have the marks of individual stand for vehicle painted on road. Such a mark orders the setting of vehicles on a car park. Car parks that possessed them, received 50 points.

There were granted 100 points for asphalt/concrete surfaces, 50 points for pavements made of concrete bricks, 0 points for ground surfaces, and (-50) for those with holes.

Numbering of individual place for a vehicle makes easier its finding, especially in case of large car parks next to hypermarkets or multistoreyed car parks. Among analysed car parks only one had such a mark. Because of that it received additional 50 points.

The author’s investigations proved that 75% of drivers accepts the distance from of a car park to the aim of trip up to 300 m. The following score was assumed: up to 300 m = 100 points, up to 600 m = 75 points, up to 900 m = 50 points, up to 1200 m = 25 points, over 1500 m = 0 points.

The average preferred max admissible price for 1 hour of parking amounted to 1.82 zł, quarter 1 = 0 zł, quarter 2 = 2 zł, quarter 3 = 2.25 zł. It was presumed that the price for 1 h of parking equal 0 zł = 100 points, 2 zł = 50 points, 2.25 zł = 25 points itd., 5 zł = 5 points (for less than 5% of investigated that price is acceptable).

Safeguard of car park, system of monitoring and enclosure combine to the safety of vehicles left on a car park. It was admitted 33.33 points for every of these elements. In case of car park without workers, 0 points were granted for factor "safeguard of car park". 3.33 points were granted for car parks with full monitoring of all parking stands, 17 points for those ones with partly monitoring, and 8.5 points for those with monitoring only at entrance gates.

### Table 2. Points granted to car parks for technical parameters presented in Table 1.

| Parameter                        | Car park Garncarska Street | Car park „C“ - Shopping Mall „A“ | Multistoreyed car park - Shopping Mall „A“ | Multistoreyed car park - Shopping Mall „Z“ |
|----------------------------------|-----------------------------|---------------------------------|---------------------------------------------|---------------------------------------------|
| $k$                              | 668 m                       | 780 m                           | 1390 m                                      | 1210 m                                      | 1930 m                                      |
| distance                         | 67.92 points                | 9,17 points                      | 24.17 points                                | 0 points                                    | 1930 m                                      |
| price for 6 hours of parking     | 19.60 zł = 3.27 z/h → 18 points | 21 zł/h = 3.50 z/h → 16 points | 18 zł/h = 3 z/h → 20 points                | 18 zł/h = 3 z/h → 20 points                | 14 zł = 2.33 zł/h → 24 points               |
| enclosure                        | 0                           | 33.33 points                     | 33.33 points                                | 33.33 points                                | 33.33 points                                |
| monitoring                       | 0                           | 8.5 points                       | 17 points                                  | 17 points                                   | 17 points                                   |
| workers’ presence                | 0                           | 33.33 points                     | 33.33 points                                | 33.33 points                                | 33.33 points                                |
| pavement                         | 100 points                   | - 50 points                      | 100 points                                  | 100 points                                  | 100 points                                  |
| number of places                 | 3.64 points                  | 14 points                       | 31.25 points                                | 46.48 points                                | 100 points                                  |
| mark of single place for a vehicle | 0 points                   | - 50 points                      | 100 points                                  | 62.55 points                                | 50 points                                   |
| lighting                         | 100 points                   | 100 points                       | 100 points                                  | 100 points                                  | 100 points                                  |
| opening hours                    | 100 points                   | 50 points                        | 100 points                                  | 100 points                                  | 50 points                                   |
The chance of finding a free parking place - depends on location, hour of parking and car park’s size. To simplify, one assumed to calculation that the biggest car park having 1800 places, receives 100 points, which gives 0.056 points/1 parking place. Limitation of parking time – for 24 h = 100 points, which gives 4.17 points/h.

Data to the stage II – evaluation of position, price and technical equipment of particular car parks, are in table 1: Technical parameters of car parks and the distance from Sukiennice on the Main Market Square in Cracow.

Stage III – definition of coefficients of car parks’ choice preferences by driver’s \( f \) was made on the ground of questionnaire investigations being executed among drivers. The investigations have been performed among users of car parks in Cracow, practising questionnaire method.

On the ground of answers given by drivers, the ranking of car parks’ features being the most desirable by drivers, was performed (Table 3).

| Standing in ranking | % of answers | Coefficient of preferences \( f \) | Feature of car park |
|---------------------|--------------|-----------------------------------|--------------------|
| 1                   | 85.71%       | 85.71                             | distance of car park to destination of journey |
| 2                   | 77.55%       | 77.55                             | price for parking  |
| 3                   | 73.06%       | 73.06                             | chance for finding a vacant stand              |
| 4                   | 63.67%       | 63.67                             | lack of limitations of parking time           |
| 5                   | 58.37%       | 58.37                             | safety of vehicles on a car park              |
| 6                   | 19.18%       | 19.18                             | size of delimited individual parking place     |
| 7                   | 15.10%       | 15.10                             | lighting                                       |
| 8                   | 13.88%       | 13.88                             | sort of pavement of a car park                |

Next the mathematical model of competition was made (stage IV, V i VI). The model was executed basing on the theory of games [18].

It assumed that there are two players: one of them is a city, represented by a company, which administers the earlier described car park that is in the zone of paid parking. This the "Player 1". He has some number of ways of acting being called "Strategies of Player 1".

"Player 2" are enterprises managing private car parks (3 car parks in shopping malls „A and Z”, and 1 private car park „C” on a square eliminated to parking). There were assumed that from the city’s point of view it is one rival, independently on the actual ownership of private car parks on whose strategy of acting the city do not have an influence.

The lack of influence of “Player 1” on the action of “Player 2” is consistent with the basic foundations of the game theory. Another assumptions ensuing from the game theory are: players make a decision independently, players know all strategies of adversary, players make decisions at the same time.

Both players compete for consumers - drivers using car parks, with such parameters like: price, distance from the destination, car parks' equipment.

Strategies of "Player 1" were prices and technical equipment of car parks and parking places lying in the zone of paid parking, administered by municipal company. Strategies of "Player 2" were prices and technical equipment of car parks belonging to private owners and two commercial centres in the city center. The results of applying different strategies by every „Player“ are put in so called matrix of payments. Lines in this matrix concern strategy of „Player 1”. Columns in this matrix concern "Player’s 2" strategy. The matrix of payments shows advantages or profits of "Player 1", which are also the losses for "Player 2". In the presented model assumed that „Player 1” wants to maximalize his victory, and "Player 2" wants to minimize his loss simultaneously. It presumed that this is two - person game about zero sum. In such games advantages of one player are always equal the second player's losses.

The following signs are presupposed in the mathematical model:

\( z_m \) – strategies of „Player 1”

\( l_n \) – strategies of „Player 2”
$W$ – matrix of payments
$z_d$ – predominant strategy
$z_s$ – dominated strategy

The matrix of payments is noted by general formula:

$$
W = \begin{bmatrix}
W_{11} & W_{12} & W_{13} & \cdots & W_{1n} \\
W_{21} & W_{22} & W_{23} & \cdots & W_{2n} \\
W_{31} & W_{32} & W_{33} & \cdots & W_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
W_{m1} & W_{m2} & W_{m3} & \cdots & W_{mn}
\end{bmatrix}
$$

The value of the game is a payment which will be received by both players while using their optimum strategies.

$r_i$ – probability of applying particular strategies $z_i$ by „Player 1” for $i = 1, 2, \ldots, m$
$d_j$ – probability of applying particular strategies $z_j$ by „Player 2” for $j = 1, 2, \ldots, n$

Expected payments of “Player 1”, when he plays his optimum strategy, will be not smaller than the value of the game $v$ (formula 2):

$$
\begin{bmatrix}
W_{11} & W_{12} & W_{13} & \cdots & W_{1n} \\
W_{21} & W_{22} & W_{23} & \cdots & W_{2n} \\
W_{31} & W_{32} & W_{33} & \cdots & W_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
W_{m1} & W_{m2} & W_{m3} & \cdots & W_{mn}
\end{bmatrix}
\begin{bmatrix}
r_1 \\
r_2 \\
r_3 \\
\vdots \\
r_m
\end{bmatrix}
\geq
\begin{bmatrix}
v \\
v \\
v \\
\vdots \\
v
\end{bmatrix}
$$

Expected payments of “Player 2”, when he plays his optimum strategy, will be not bigger than the value of the game $v$ (formula 3):

$$
\begin{bmatrix}
W_{11} & W_{12} & W_{13} & \cdots & W_{1n} \\
W_{21} & W_{22} & W_{23} & \cdots & W_{2n} \\
W_{31} & W_{32} & W_{33} & \cdots & W_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
W_{m1} & W_{m2} & W_{m3} & \cdots & W_{mn}
\end{bmatrix}
\begin{bmatrix}
d_1 \\
d_2 \\
d_3 \\
\vdots \\
d_n
\end{bmatrix}
\leq
\begin{bmatrix}
v \\
v \\
v \\
\vdots \\
v
\end{bmatrix}
$$

„Player 1” applies the maxmin principle. From elements of matrix of payments $W$ he chooses the number called the bottom value of game:

$$
v_z = \max_i (\min_j w_{ij})
$$

„Player 2” applies the minmax principle. From elements of matrix of payments $W$ he chooses the number called the upper value of game:

$$
v_l = \min_j (\max_i w_{ij})
$$

If the equation is accomplished:

$$
v_z = v_l \quad \text{so:} \quad \max_i (\min_j w_{ij}) = \min_j (\max_i w_{ij})
$$
then such an element of matrix of payments, for which the above mentioned condition is accomplished, we call the saddle point, the game has a solution among pure strategies the solution, and its value equals \( v \):

\[
v = v_z = v_f
\]

(7)

Players should then apply strategies which correspond with a saddle point. A game which has a saddle point, is called the closed game, and a game without a saddle point, is the open game.

The question of optimization for "Player 1" resolves itself into working out equations and inequalities written below:

Boundary conditions:

\[
x_i \geq 0, \quad \text{for } i = 1, 2, ..., m
\]

(8)

Limitations:

\[
\sum_{i=1}^{m} w_{ij} \cdot x_i \geq 1, j = 1,2,\ldots,n
\]

(9)

Function of purpose:

\[
CG1 = \frac{1}{v} = \sum_{j=1}^{n} x_j \to \text{min.}
\]

(10)

The question of optimization for "Player 2" resolves itself into working out equations and inequalities written below:

Boundary conditions:

\[
y_j \geq 0, \quad \text{for } j = 1, 2, ..., n
\]

(11)

Limitations:

\[
\sum_{j=1}^{n} w_{ij} \cdot y_j \leq 1, i = 1,2,\ldots,m
\]

(12)

Function of purpose:

\[
CG2 = \frac{1}{v} = \sum_{j=1}^{n} y_j \to \text{max.}
\]

(13)

For the car park in Garncarska Street there have been assumed following alternative strategies:

1) strategy \( z_2 \) for „Player 1“ – abatement of price to 2 zł / 1 hour of parking (proconsumer strategy),
2) strategy \( z_3 \) for „Player 1“ – rise of price to 9 zł / 1 hour of parking (proecological strategy),
3) strategy \( z_4 \) for „Player 1“ – installation of cameras to observe cars on a car park (strategy of enlargement of safety),
4) strategy \( z_5 \) for „Player 1“ – extension of individual parking stands for cars by painting horizontal marks on street every 2,70 m (strategy of convenience).

Those strategies, apart from advantages, have also expected disadvantages:

1. strategy \( z_2 \) – bigger crowd and pollution of the city,
2. strategy \( z_3 \) – smaller incomes to city budget,
3. strategy \( z_4 \) – costs of installation and service of cameras,
4. strategy \( z_5 \) – smaller number of parking places (55 places).

Elements of matrix of payments \( w_{mn} \) were counted according to the formula (14):

\[
w_{mn} = \sum_{p=1}^{b} (f_p \cdot LP_{p}^{ab2,G_1}) - \sum_{p=1}^{b} (f_p \cdot LP_{p}^{ab2,G_2,k}) \quad \text{for: } m = 1,\ldots,5; \quad n = 1,\ldots,4; \quad k=1,\ldots,4
\]

(14)
LP \text{tab2, G1} – number of points for particular parameters of parking in Garncarska Street presented in Table 2 (for „Player 1"
LP \text{tab2, G2, k} – number of points for particular parameters of parking presented in Table 2 (for „Player 2”
f_c – appropriate coefficients of preferences to particular car parks’ features presented in Table 3
p – parameters of car parks (Table 2), where: \( p=1, 2, \ldots, b \)
b – number of car parks’ parameters (Table 2)
m – number of strategies
n – number of car parks competitive to car park in Garncarska Street
k - coefficient of competitive car park

Table 4. Corrected matrix of payments with counted number of points for particular strategies after conversion of minus values to positive values

| Strategy                  | Number of strategy | \( l_1 \)  | \( l_2 \)  | \( l_3 \)  | \( l_4 \)  |
|---------------------------|--------------------|------------|------------|------------|------------|
| without change            | \( z_1 \)         | 10621.38   | 9115.39    | 2798.15    | 1395.9     |
| proconsumer               | \( z_2 \)         | 13102.98   | 11596.99   | 5279.75    | 3877.5     |
| proecological             | \( z_3 \)         | 9225.48    | 7719.49    | 1402.25    | 0          |
| increase of safety        | \( z_4 \)         | 12566.85   | 11060.86   | 4743.62    | 3341.37    |
| comfort                   | \( z_5 \)         | 10610.46   | 9104.47    | 2787.23    | 1384.98    |

After creating the matrix of payments it was corrected. The correction consistent in choosing from the matrix of payments the smallest value, calculating its absolute value and adding it to all the elements of the matrix of payments. The aim of that was to simplify further computations. The corrected matrix of payments is in Table 4.

On the base of the matrix of payments for present strategies and alternative strategies (Table 4), after eliminating dominated strategies, the optimum strategies were chosen. For data presented above the best strategy for "Player 1" is proconsumer strategy \( z_2 \). Although the car park of "Player 1" is the nearest the aim of trip, which is the most important factor for drivers, then his small size (65 stands in comparison with e.g. 1800 places of car park "Z") affects smaller chance for finding a vacant place to parking, and so also smaller effectiveness of "Player's 1" strategy. Therefore, as one of conclusions one may say, that evaluating players' strategies it is important to analyse comprehensively all factors considering as well as the suitable selection of importance of coefficients occurring in formulas presented above.

4. Conclusions
The built model allows indicate the best strategy with the counted probability. Knowing that strategy one can form prices and equipment of car parks. That model may be used by municipal governments and companies which administer car parks. However, one should remember about limitations, which occur in reality.

The model postulates that all drivers pay for parking. In fact, a big number of drivers may avoid parking charges. There has been introduced a quite developed system of subscriptions for using zones of paid parking. The brought in subscriptions are: accessible for everyone, for inhabitants of zone, for handicapped, for microenterprises or owners being simultaneously administrators of the reality, institutional, for owners of electric or hybrid vehicles, for reserved halting stands, supplementary, duplicates of subscriptions. The extent of subscriptions figures out from 2.50 zł to 850 zł for 30 succeeding days (about 0.58 euro to 197.70 euro for 30 succeeding days) [17]. The necessity of regard in general number of vehicles parking on analysed car park that belongs to the zone of paid parking, those vehicles to which some kind of subscription was earlier taken out, is one of the limitations of applicability of described method.

One should also remember, that the range of applied technical solutions which can attract drivers or discourage them, is small. The elements of infrastructure have durable character mostly. One can improve the quality of car park’s pavement, install monitoring cameras or additional lighting, but those elements of infrastructure are durable and they usually will not be modified for several years.
Lately one can notice the tendency to enlargement of external dimensions of passenger cars. The width of individual place to parking influences the comfort of parking. In case of multistoreyed car parks the extension of the individual place's width is often difficult because of constructional elements of car park, e.g. poles. For plane car parks it is easier. In both types of car parks the extension of individual stand is, however, connected with diminution of general capacity of car park, and so with reduction of the chance for finding a vacant place.

The element, which can be shaped more easily, is the price for parking. Fixing its amount is not a problem in case of private car parks, and it depends only on the prices on competitive car parks. In case of car parks administered by city, which are situated in zone of paid parking, prices depend at present on legal regulations, defining their maximum extent. Thus administrator of such a car park can not shape them in elastic way. The increasing pressure of cities’ authorities in Poland for changing those regulations, and examples of such solutions received, e.g. in USA, which concern varying prices for parking according to the demand on different car parks, and different hours, permit to have hopes, that the proposed model will be possible to use practically, soon. Examples of dynamic fixing of parking prices are presented in references, too [19].

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