Mathematical simulation for compensation capacities area of pipeline routes in ship systems

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Abstract. In this paper, the authors considered the problem of manufacturability’s enhancement of ship systems pipeline at the designing stage. The analysis of arrangements and possibilities for compensation of deviations for pipeline routes has been carried out. The task was set to produce the “fit pipe” together with the rest of the pipes in the route. It was proposed to compensate for deviations by movement of the pipeline route during pipe installation and to calculate maximum values of these displacements in the analyzed path. Theoretical bases of deviation compensation for pipeline routes using rotations of parallel section pairs of pipes are assembled. Mathematical and graphical simulations of compensation area capacities of pipeline routes with various configurations are completed. Prerequisites have been created for creating an automated program that will allow one to determine values of the compensatory capacities area for pipeline routes and to assign quantities of necessary allowances.

1. Introduction
Improvement of methods for designing, manufacturing and installation of ship pipelines without fitting work and redetermination of dimensions on the ship is associated with solving two problems: accuracy in the design dimensions of the main pipes and obtaining a reliable design configuration of the “fit pipes” [1-8]. The technology of manufacturing and installation of the fit pipe includes actual measurements between two rigidly fixed connections and fitting work on the installation of welded joints [9-15]. Increasing a technological level of pipeline routes in ship systems is limited by the fact that during the installation, actual design dimensions and configuration of the fit pipes are not allowed to be changed.

In connection with the foregoing, in order to improve the processability of ship pipelines, the fit pipe must be manufactured according to design information without specifying the dimensions at the place on the vessel. The design documentation for the fit pipes will differ from the documentation for the main pipes only by the presence of technological allowances at the ends of the fit pipe. For the successful installation of trails with fit pipes, it is necessary to be able to move the route during the installation, thus compensating for the deviations of both pipes and adjacent structures in directions that cannot be compensated for by allowances on the fit pipe [16]. The pipeline route must be able to be moved at least in one coordinate direction.

2. Compensation of deviations by moving pipeline routes
The possibility of moving when installing routes from finished pipes was assumed earlier by the
existing regulatory documents due to the rotation of pipes in free connections. Turnings in connections can change the direction of the last section in the route, which will break the alignment along the angle with the axis of rigidly fixed connection that limits the route. For different paths, the allowed angle is 1 - 3 ° [16]. This tolerance does not allow applying turns to move the track if the last straight section of the route is not parallel to the selected section where the rotated connection is located. If there is a section in the path behind the rotatable connection that is parallel to the selected one and there is a free connection on it, then turning the part of the route remaining behind this connection to the same angle as the selected connection but in the opposite direction, the misalignment in the direction angle of the last segment of the route will be eliminated. Thus, the movement of the route is possible if there are parallel sections in the route and free connections located on these sections.

Rotating pairs of straight, mutually parallel sections allows the end of the route to be moved in a limited area. It is called a compensatory area of pipeline routes. The more the pairs of parallel sections there are in the path, the more the options there are for moving the end of the route, i.e. the more the compensatory possibilities of the route there are. The shape and dimensions of compensation area capacities in this case depend on the number and parameters of parallel section pairs in the pipeline route. To determine the possibilities of compensating pipeline routes, it is necessary to describe compensation area capacities in mathematical and graphical expressions.

In the process of rotating a parallel section pair in the route in free connections, the end of the route is moved along an arc of a circle in a rotated plane passing through the point of the end in the path perpendicular to the direction of a parallel section pair. In this case, its direction vector is normal in relation to the rotated plane. The radius is equal to the perpendicular dropped from any point of the first section to the axis of the second section. Mathematical description of the actions to determine compensation capacities of the route is related to solving the problem of arc surfaces. If the circle is moved along its axis, it is possible to get a cylindrical surface. If the part of the circle in the same way is moved, that is, its arc, it is possible to obtain a curvilinear surface - part of cylinder shell. If one moves a part of the arc not in the direction of its axis, but along the arc of another circle that intersects this circle at point A, it is possible to get a bent surface. If these two arcs intersect at the intersection point, the third arc has axis of the circle not parallel to the axes of the two previous arcs, then, by moving the bent surface along the arc of the third circle, it is possible to obtain a three-dimensional figure.

3. Mathematical simulation of the compensation area of pipeline routes

To determine the shape and magnitude of compensation area capacities in pipeline routes, it is necessary to set and solve the problem of arc surfaces.

There are \( k \) circles with centers \( C_1, C_2, \ldots, C_k \) and corresponding radius \( R_1, R_2, \ldots, R_k \). Circles belong to planes which are defined by unit normal vectors \( \vec{n}_1, \vec{n}_2, \ldots, \vec{n}_k \), i.e. vectors are perpendicular to planes of circles and their length is equal to 1.

All circles pass through the point. For each circle arc, \( M_iN_i \) is defined. It comprises point \( A \), where all circles intersect. There are angles \( \angle M_iC_iA = \alpha_i \) and \( \angle AC_iN_i = \beta_i \).

Arc \( I \) is moved in the direction of the second arc by means of parallel transferring and forms a curved surface. Curved surface \( S_2 \) is moved by means of parallel transferring along the direction of the third arc and forms curved volume \( S_3 \) [16].

**Task**

1. Draw up surface equation \( S_2 \), which is formed by moving arc \( M_1N_1 \) with parallel transfer along arc \( M_2N_2 \).

2. Draw up spatial area equation \( S_3 \), which is formed by moving surface \( S_2 \) with parallel transfer along arc \( M_3N_3 \).
3. Draw up spatial area equation $S_i$, which is formed by moving the area with parallel transfer along arc $M_iN_i$, $i = 4, 5, \ldots, k$.

**Arbitrary point coordinates of the i-th circle**

Arbitrary point coordinates for each circle need determination. For that task, the radius of the unit vector is adjusted, $\vec{e}_i = \frac{\overrightarrow{CA}}{R_i} = \frac{\overrightarrow{OA} - \overrightarrow{OC}_i}{R_i}$, i.e., a single centrifugal vector - to the given circle, where $O = [0, 0, 0]$ – the origin of coordinates.

Let $\overrightarrow{u}_i = [\overrightarrow{n}_i, \overrightarrow{e}_i]$ (here $[\overrightarrow{n}_i, \overrightarrow{e}_i] = [n_i, e_i] = [n_i e_3 - n_i e_2, n_i e_1 - n_i e_3, n_i e_2 - n_i e_1]$ be a vector product). Vector coordinates $\overrightarrow{u}_i$ are defined in the following way:

$$\overrightarrow{u}_i = [\overrightarrow{n}_i, \overrightarrow{e}_i] = \left[\begin{array}{c} n_2 \\ n_3 \\ e_2 \\ e_3 \\ n_1 \\ e_1 \\ n_1 \\ n_2 \end{array}\right] = \left(n_2 e_3 - n_1 e_2; n_1 e_3 - n_1 e_1; n_1 e_2 - n_1 e_1\right)$$

Vector length $|\overrightarrow{u}_i|$ is determined using the form of vector product:

$$|\overrightarrow{u}_i| = |[\overrightarrow{n}_i, \overrightarrow{e}_i]| = |\overrightarrow{n}_i| |\overrightarrow{e}_i| \sin(\overrightarrow{n}_i, \overrightarrow{e}_i) = 1$$

where $\angle(\overrightarrow{n}_i, \overrightarrow{e}_i) = 90^\circ$; $|\overrightarrow{n}_i| = 1$; $|\overrightarrow{e}_i| = 1$

Vector $\overrightarrow{u}_i$ is called a unit vector tangent to the appropriate circle.

Therefore, vectors $\overrightarrow{e}_i, \overrightarrow{u}_i, \overrightarrow{n}_i$ form an orthonormal basis of the circle considered (Figure 1).

![Figure 1. Determining arbitrary point coordinates of the circle.](image)

A parametric equation of arbitrary point coordinates for the i-th circle looks in the following way:

$$\overrightarrow{O\hat{F}(t_i)} = \overrightarrow{OA} + \overrightarrow{AF}_i,$$

where $F_i$ – arbitrary point in the $i$-th circle:

$$\overrightarrow{AF}_i = \overrightarrow{AF} \cos\left(\frac{t_i}{2}\right)\overrightarrow{u}_i - \overrightarrow{AF} \sin\left(\frac{t_i}{2}\right)\overrightarrow{e}_i,$$

where $\overrightarrow{AF} = 2R \sin\left(\frac{t_i}{2}\right)$.

Hence:

$$\overrightarrow{AF}_i = 2R \sin\left(\frac{t_i}{2}\right)\cos\left(\frac{t_i}{2}\right)\overrightarrow{u}_i - 2R \sin^2\left(\frac{t_i}{2}\right)\overrightarrow{e}_i;$$

$$\overrightarrow{AF}_i = R \sin(t_i)\overrightarrow{u}_i - R_i (1 - \cos(t_i))\overrightarrow{e}_i.$$  

(2)

Let us substitute (1) with (2) and obtain the position of an arbitrary point in the i-th circle:
\[ \overrightarrow{OF_i}(t_i) = \overrightarrow{OA} + R_i \sin(t_i) \overrightarrow{u_i} - R_i (1 - \cos(t_i)) \overrightarrow{e_i}. \]  
\hspace{1cm} (3)

Value \( t_i = 0 \) corresponds to point \( A \).

**Surface equation \( S_2 \)**

Position of arbitrary point \( F_i \), which belongs to the first circle \((C_i; R_i)\), is determined from the parametric equation:

\[ \overrightarrow{OF_i}(t_i) = \overrightarrow{OA} + R_i \sin(t_i) \overrightarrow{u_i} - R_i (1 - \cos(t_i)) \overrightarrow{e_i}; \quad -\alpha_i \leq t_i \leq \beta_i. \]  
\hspace{1cm} (4)

While moving arc \( M_1N_1 \) along arc \( M_2N_2 \), point \( F_i \) moves up to position \( F_2 \) along circle \((C'_2; R_2)\), where circle \((C'_2; R_2)\) is formed while moving circle \((C_2; R_2)\) with parallel transfer along vector \( \overrightarrow{AF_i} \), i.e. \( \overrightarrow{C_2C_2} = \overrightarrow{AF_i} \) (Figure 2). Basic vectors \( \overrightarrow{e_2}, \overrightarrow{u_2}, \overrightarrow{n_2} \) do not alter.

![Figure 2. Determining coordinates of the arbitrary point on surface \( S_2 \).](image)

The position of point \( F_2 \) is determined by the formula:

\[ \overrightarrow{OF_2}(t_2) = \overrightarrow{OF_i} + \overrightarrow{F_iF_2}, \]  
\hspace{1cm} (5)

where \( \overrightarrow{OF_i} \) is determined by formula (4); \( \overrightarrow{F_iF_2} \) in circle \((C'_2; R_2)\) is defined as vector \( \overrightarrow{AF_i} \) in circle \((C_i; R_i)\):

\[ \overrightarrow{F_iF_2} = R_2 \sin(t_2) \overrightarrow{u_2} - R_2 (1 - \cos(t_2)) \overrightarrow{e_2}. \]  
\hspace{1cm} (6)

Let us insert formulas (4) and (6) in (5) and find the equation of surface \( S_2 \) (Figure 3):

\[ \overrightarrow{OF_2}(t_1, t_2) = \overrightarrow{OA} + R_i \sin(t_1) \overrightarrow{u_i} - R_i (1 - \cos(t_1)) \overrightarrow{e_i} + R_2 \sin(t_2) \overrightarrow{u_2} - R_2 (1 - \cos(t_2)) \overrightarrow{e_2}; \]

\[ -\alpha_i \leq t_1 \leq \beta_1; \quad -\alpha_2 \leq t_2 \leq \beta_2. \]  
\hspace{1cm} (7)
Figure 3. Surface $S_2$ and forming arcs

Equation of spatial area $S_1$

While moving area $S_1$ along arc $M_1N_1$, point $F_2$ in formula (7) is transferred up to the position of $F_3$ along circle $(C_3; R_3)$, where circle $(C_3; R_3)$ is formed while moving circle $(C_1; R_1)$ with parallel transfer along vector $\overrightarrow{F_1F_2}$, i.e. $C_1C_3 = \overrightarrow{F_1F_2}$. Similar to formula (7), let us obtain the equation of area $S_1$ (Figure 4):

$$\overrightarrow{OF}(t_1, t_2, t_3) = \overrightarrow{OA} + R_1 \sin(t_1)\overrightarrow{u_1} - R_1 (1 - \cos(t_1))\overrightarrow{e_1} + R_1 \sin(t_2)\overrightarrow{u_2} - R_1 (1 - \cos(t_2))\overrightarrow{e_2} +$$

$$+ R_1 \sin(t_3)\overrightarrow{u_3} - R_1 (1 - \cos(t_3))\overrightarrow{e_3};$$

$$\alpha_1 \leq t_1 \leq \beta_1; -\alpha_2 \leq t_2 \leq \beta_2; -\alpha_3 \leq t_3 \leq \beta_3.$$  

(8)

Figure 4. Surface, bounding area region $S_3$ and forming arcs

Equation of spatial area $S_3$

Spatial area $S_3$ moves along arc $M_3N_3$ and forms area $S_4$. Since the motion of interior points of spatial area $S_3$ does not affect the shape of area $S_4$, it can be assumed that spatial area $S_4$ is obtained by motion of all boundary surfaces of area $S_3$. Using formula (8), it is possible to concurrently draw an equation of area $S_4$. 

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Areas $S_1, S_2, ..., S_k$ are obtained similarly. Thus, one shall define the general formula for the compensation capacities area of pipeline routes applying rotations of parallel section pairs:

$$
\overline{OF_n}(t_1, t_2, ..., t_n) = \overline{OA} + \sum_{i=1}^{n} \left[ R_i \sin(t_i) \overline{u_i} - R_i \left(1 - \cos(t_i)\right) \overline{e_i}\right].
$$

(9)

Formula 9 describes the compensation capacities area of pipeline routes using a mathematical apparatus. When pairs of parallel sections with free connections rotate, the end point of the route moves to the area bounded by equation 9. To facilitate determining coordinates for the end point of the path and the parameters for arcs of circles, let us apply certain coordinate systems for each route. Initial points of these systems coincide with positions of route beginning.

There is constant part $\Delta \overline{S} = \overline{AF} = \sum_{i=1}^{n} \left[ R_i \sin(t_i) \overline{u_i} - R_i \left(1 - \cos(t_i)\right) \overline{e_i}\right]$ in the mathematical description of the position of the route end when parallel section pairs rotate. This part is called an absolute compensation area in comparison with the position route end, characterized by absolute displacement of the route end when pairs of parallel pipe sections rotate. Magnitude and direction of this part do not depend on the theoretical position of the route endpoint and the chosen coordinate system. They depend only on parameters that define arcs of rotated circles, i.e. parallel section pairs of the route.

When the coordinate system is moved from the original point to the endpoint of the route, a vector expression is defined. It describes compensation capacity area of the route in the abbreviated form:

$$
\overline{OF_n}(t_1, t_2, ..., t_n) = \sum_{i=1}^{n} \left[ R_i \sin(t_i) \overline{u_i} - R_i \left(1 - \cos(t_i)\right) \overline{e_i}\right]
$$

(10)

If by the rotation of pipes in free connections, one obtains a compensation area that overlaps the figure of deviations parallelepiped, then the task is solved. The solution does not depend on the functional purpose of pipelines and systems. Based on this task, it is possible to construct an area of compensatory capabilities for pipeline routes with various configurations. The results of this study can be used in many industries with respect to objects equipped with pipelines.

4. Conclusion

In the course of research into compensation capacities of ship systems pipelines:

- theoretical bases of deviations compensation for pipeline routes using rotations of parallel section pairs of pipes are formed;
- mathematical and graphic simulations of the compensation capacities area for pipeline routes with various configurations were completed;
- prerequisites for creating an automated program have been created, which will allow determining values of compensation capacities for pipeline routes.

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