Quantum Lift of Non-BPS Flat Directions

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Abstract

We study $N = 2$, $d = 4$ attractor equations for the quantum corrected two-moduli prepotential $\mathcal{F} = st^2 + i\lambda$, with $\lambda$ real, which is the only correction which preserves the axion shift symmetry and modifies the geometry.

In the classical case the black hole effective potential is known to have a flat direction. We found that in the presence of $D0 - D6$ branes the black hole potential exhibits a flat direction in the quantum case as well. It corresponds to non-BPS $Z \neq 0$ solutions to the attractor equations. Unlike the classical case, the solutions acquire non-zero values of the axion field.

For the cases of $D0 - D4$ and $D2 - D6$ branes the classical flat direction reduces to separate critical points which turn out to have a vanishing axion field.
1 Introduction

The attractor mechanism was firstly described in the seminal papers [1]-[5] and is now the object of intense studies (for a comprehensive list of references, see e.g. [6]). While originally this mechanism was discovered in the context of extremal BPS black holes, later it was found to be present even for non-BPS ones. Differently from the BPS black holes, such new attractors do not saturate the BPS bound and thus, when considering a supergravity theory, they break all supersymmetries at the black hole event horizon [7].

Attractor mechanism equations are given by the condition of extremality [5]

\[
\phi_H(p,q) : \left. \frac{\partial V_{BH}(\phi,p,q)}{\partial \phi^a} \right|_{\phi=\phi_H(p,q)} = 0 \tag{1}
\]
of the so-called black hole potential \(V_{BH}\), which is a real function of the moduli \(\phi^a\) and magnetic \(p^A\) and electric \(q_A\) charges.

The crucial condition for a critical point \(\phi_H(p,q)\) to be an attractor in the strict sense is that the Hessian matrix

\[
\mathcal{H}_{ab}(p,q) = \nabla_a \nabla_b V_{BH}\big|_{\phi=\phi_H} = \partial_a \partial_b V_{BH}\big|_{\phi=\phi_H} \tag{2}
\]
of \(V_{BH}\) evaluated at the critical point (1) be positive definite.

In \(N = 2, d = 4\) Maxwell-Einstein supergravities based on homogeneous scalar manifolds, the Hessian matrix has in general either positive or zero eigenvalues. The latter ones correspond to massless Hessian modes, which have been proven to be flat directions of \(V_{BH}\) [8, 9].

The presence of flat directions does not contradict the essence of the attractor mechanism: although the moduli might not be stabilized, the value of the entropy does not change when the moduli change along the flat directions of \(V_{BH}\). Indeed, in \(N = 2, d = 4\) supergravity, the black hole entropy is related to its potential through the formula [5]

\[
S_{BH}(p,q) = \pi V_{BH}(\phi,p,q)\big|_{\phi=\phi_H}. \tag{3}
\]
Therefore, whether the flat directions are present or not, it does not affect the value of the entropy. Consequently, one may allow the eigenvalues of the Hessian matrix to be zero, as well.

Actually, this phenomenon always occurs in \(N > 2\)-extended, \(d = 4\) supergravities, also for \(1/3\)-BPS configurations, and it can be understood through an \(N = 2\) analysis, as being due to \(N = 2\) hypermultiplets always present in these theories [8, 6].

In \(N = 2, d = 4\) supergravity with more than one vector multiplet coupled to the supergravity one, the black hole potential \(V_{BH}\) has flat directions provided that the critical points exist [9, 10]. They correspond to non-BPS states with non-vanishing central charge.

The simplest model possessing a flat direction is that with two vector multiplets, i.e. the so-called \(st^2\) model. The latter we treat in this paper which might be thought of as a continuation of the investigation started in an earlier paper [11], where we found an effect of multiplicity of the attractors in the presence of quantum corrections. This effect is related to the fact that when quantum corrections are introduced, the scalar manifold is not simply connected any more.

Even in the classical case, solutions for the attractor equations are known just for quite a few models. For example, in the framework of special Kähler d-geometry, supersymmetric
attractor equations are solved in \[12\]. Non-supersymmetric ones are solved completely both for the $t^3$ model \[13\] and for the $stu$ one \[14\], taking advantage of the presence of a large duality symmetry. States with vanishing central charge are investigated in \[15, 14\].

As it has been already mentioned, in the paper \[11\] we began the study of a quantum $t^3$ model of $N = 2$ $d = 4$ supergravity with the prepotential

$$F(X) = \frac{(X^1)^3}{X^0} + i\lambda X^0)^2 = (X^0)^2 \left(t^3 + i\lambda\right), \quad \lambda \in \mathbb{R}.$$  

There it was argued that this is the only possible correction preserving the axion shift symmetry and that it cannot be reabsorbed by a field redefinition \[16, 17\]. The black hole potential of this model does not possess any flat direction, nevertheless, the appearance of the quantum contribution reveals an effect of multiplicity of the attractors. This effect is similar to that observed in \[18\]. Due to this effect other ones arise such as “transmutations” and “separation” of attractors. In $st^2$ model they appear as well, but here we are mostly concerned with another phenomenon, not present in $t^3$ model, – namely, how the flat direction of the $st^2$ model undergoes the insertion of quantum corrections.

The quantum corrected $st^2$ model that we consider is based on the holomorphic prepotential

$$F(X) = \frac{X^1 (X^2)^2}{X^0} + i\lambda (X^0)^2 = (X^0)^2 \left(st^2 + i\lambda\right), \quad \lambda \in \mathbb{R}. $$

The complex moduli $s$ and $t$ span the rank-2 special Khler manifold $(SU(1,1)/U(1))^2$. When $\lambda = 0$ this formula gives classical expression for the prepotential, which we start the next section with.

Knowing the superpotential, one may easily calculate the corresponding black hole potential \[5\]

$$V_{BH} = e^K \left[WW + g^{ab}\nabla_a W \nabla_b \bar{W}\right]$$

in terms of the superpotential $W$ and the Khler potential $K$

$$W = q_\Lambda X^\Lambda + p^A F_\Lambda, \quad K = -\ln \left[-i \left(X^\Lambda F_\Lambda - X^A F_\Lambda\right)\right].$$

2 $D0 - D4$ branes

This brane configuration corresponds to vanishing charges $q_a$ and $p^0$. The quartic invariant in this case is given by

$$I_4 = 4q_0 p^1 \left(p^2\right)^2.$$  

1In general, $\lambda$ is related to perturbative quantum corrections at the level of non-linear sigma model, computed by 2-dimensional CFT techniques on the world-sheet. For instance, in Type IIA CY$_3$-compactifications \[19, 20, 21\]

$$\lambda = -\frac{\chi \zeta(3)}{16 \pi^3},$$

where $\chi$ is the Euler character of CY$_3$, and $\zeta$ is the Riemann zeta-function. Within such a framework, it has been shown that $\lambda$ has a 4-loop origin in the non-linear sigma-model \[22, 23, 19\].

2Generally, the indices $a, b, c, \ldots$ run from 1 to $n$, while $\Lambda, \Sigma, \ldots$ – from 0 to $n$, with $n = 2$ for the $st^2$ model
Reon and corresponds to a non-BPS state. The black hole entropy \( S_{BH} \) turns out not to depend y moduli in complete agreement with the attractor mechanism paradigm.

This result differs from that present in the classical case (7). With this assumption, the attractor mechanism equations become

\[
\begin{align*}
\text{Im } s &= \pm \sqrt{-\frac{q_0}{p^1} \left( \frac{\text{Re } t}{p^2} \right)^2 - \frac{q_0}{p^1}}, \\
\text{Re } s &= \frac{p^1 q_0}{p^2} \left( \frac{\text{Re } t}{p^2} \right)^2 - \frac{q_0}{p^1}, \\
\text{Im } t &= \pm \sqrt{-\frac{q_0}{p^1} - \left( \frac{\text{Re } t}{p^2} \right)^2}
\end{align*}
\]

parameterized, for instance, by the real part of the modulus \( t \). Naturally, it solves the criticality condition of the black hole potential (4) evaluated when \( \lambda = 0 \)

\[
\frac{\partial V_{BH}}{\partial s} = 0, \quad \frac{\partial V_{BH}}{\partial t} = 0
\]

and corresponds to a non-BPS state. The black hole entropy (3) turns out not to depend on \( \text{Re } t \)

\[
S_{BH} = \pi \sqrt{-I_4} = 2\pi \sqrt{-q_0 p^1 (p^2)^2}
\]

in complete agreement with the attractor mechanism paradigm.

When switching the quantum parameter \( \lambda \) on, it is convenient to pass to the rescaled moduli \( y^1, y^2 \) and the quantum parameter \( \alpha \)

\[
s = p^1 \sqrt{-\frac{q_0}{p^1 (p^2)^2}} y^1, \quad t = p^2 \sqrt{-\frac{q_0}{p^1 (p^2)^2}} y^2, \quad \lambda = q_0 \sqrt{-\frac{q_0}{p^1 (p^2)^2}} \alpha
\]

in order to factorize the dependence of \( W \) and \( V_{BH} \) on the charges

\[
W = q_0 \left[ 1 - 2 y^1 y^2 - (y^2)^2 \right], \quad V_{BH} = \frac{1}{2} \sqrt{-I_4} v(y, \bar{y}) = \sqrt{-q_0 p^1 (p^2)^2} v(y, \bar{y}).
\]

The expression for the black hole potential is quite cumbersome and not too illustrative, so we restricted ourselves to writing down explicitly only the superpotential. The function \( v(y, \bar{y}) \) is a rational one with the numerator being a polynomial of ninth degree and the denominator – of eighth degree on \( y^a \) and \( \bar{y}^a \). So at the moment it is quite improbable to resolve attractor mechanism equations (8) analytically. Nevertheless, numerical simulations show that all solutions to eqs. (8) have vanishing values of the axion fields

\[
\text{Re } y^1 = \text{Re } y^2 = 0.
\]
where for the sake of brevity we denoted $t_\alpha = \text{Im} y^\alpha$. Depending on the value of the parameter $\alpha$, the number of the solutions to the eqs. (13) and their stability change. The stable solutions have all eigenvalues of the Hessian matrix positive, while for the unstable ones – one of them is negative. In what follows we consider only stable solutions.

Substituting stable solutions of (13) into eq. (3) one gets the following behaviour of the entropy with respect to the quantum parameter (Fig. 1). One can easily see that for $\alpha > 2/(3\sqrt{3})$ there are two solutions to the attractor equations. The one having no classical limit is a 1/2-BPS solution. Such an effect – i.e. the appearance of a BPS solution when the quartic invariant $I_4$ is negative was also observed in a quantum $t^3$ model [11].

An interesting property is exhibited by the non-BPS solution with positive value of the quantum parameter $\alpha$ (Fig. 2). Being evaluated on this solution, the superpotential does not depend on the parameter $\alpha$ and is equal to

$$W = 2q_0.$$  

Generally, the superpotential without axion fields (12) has the form

$$W = q_0 \left[ 1 - 2 \text{Im} y^1 \text{Im} y^2 - (\text{Im} y^2)^2 \right].$$  

By equating it to $2q_0$ one obtains the following relation between the moduli:

$$\text{Im} y^1 = -\frac{1 + (\text{Im} y^2)^2}{2 \text{Im} y^2},$$  

which is consistent with the criticality condition (8) provided

$$\alpha = 1 \text{Im} y^2 + (\text{Im} y^2)^3 = 0.$$  

Obviously, such an algebraic equation has either one or three real solutions depending on the value of $\alpha$. When $\alpha$ is positive, only one of the solutions is stable. When it is negative, there is no stable solution of (16).

Just for the sake of mentioning it, we should say that there is another solution yielding $\alpha$-independent prepotential, namely $W = 0$, so that the central charge vanishes as well. It holds provided

$$D_t W = \lambda \frac{D_s W}{(\text{Im} t)^3},$$

which is nothing but a “quantum” generalization of the zero central charge condition found in [15]. Since this solution turns out to be unstable, we do not consider it further.
To conclude this section we consider a case when the quartic invariant $I_4$ is positive. Classically, it is known to correspond to 1/2-BPS solutions and thus in this case the black hole potential has no flat direction. Although it is unlikely that a flat direction might appear when introducing quantum corrections, we consider this case as well and list the results:

1. there exists one 1/2-BPS solution, which is, naturally, stable \[5\]. This solution pertains to the $t^3 + i\lambda$ model \[11\].

2. there exists a stable non-BPS $Z = 0$ solution with $\text{Im} y^1 = \text{Im} y^2$, which does not have a classical limit.

3. there exists a stable non-BPS $Z = 0$ solution, having its classical limit as found in \[15\].

4. there exist two unstable non-BPS solutions, having no classical limit. They correspond to $\alpha$-independent values of the superpotential: either $2q_0$ or zero.

The behaviour of the function $v$ related to the black hole potential via (11) is presented in Fig. 3. The properties of the three solutions depicted here might be easily traced from the list above (remember that only stable solutions are depicted).

To summarize, in the presence only of $D0 - D4$ branes the flat direction of the classical black hole potential gets removed.

Unlike the classical case, there appear as well 1/2-BPS quantum solutions with $I_4 < 0$ and non-BPS ones with $I_4 > 0$. This fact was observed in \[15\]. Another state not observed before is the $\alpha$-independent one of the superpotential. A question to be yet clarified is a correlation between the ground states of BPS and non-BPS solutions. As one sees from Fig.1 once both states –BPS and non-BPS – are present simultaneously for $I_4 < 0$, the ground state energy of the BPS one is lower than that of the non-BPS one. This is not valid anymore for $I_4 > 0$: from Fig.3 one sees that there exists a value of $\alpha$ when the BPS state has a lower energy than that of the non-BPS state, and there exists as well a value of $\alpha$ when the relation between energies is opposite.

3 $D2 - D6$ branes

Let us consider now a situation when only $D2$ and $D6$ branes are present. The quartic invariant $I_4$ in this case is equal to

$$I_4 = -p^0 q_1 q_2^2.$$  \(17\)

Let us start with $I_4 > 0$, when there exist classically 1/2-BPS and non-BPS $Z = 0$ attractors. The black hole potential exhibits no flat directions. In the presence of $D6$ branes, switching the quantum correction $\alpha$ on, the attractor eqs. \[8\] are hard to be solved even numerically, due to the presence of the $D6$-brane charge $p^0$ \[5\]. Thus, the question whether there is a quantum critical solution with positive $I_4$ remains open.
Considering the case $I_4 < 0$, which classically admits only non-BPS attractors, one can simplify the analysis by defining rescaled moduli $y^a$ and a quantum parameter $\alpha$ as follows:

\[
s = \frac{y^1}{p^0 q_1} \sqrt{-I_4}, \quad t = \frac{y^2}{p^0 q_2} \sqrt{-I_4}, \quad \lambda = \frac{\alpha}{(p^0)^2} \sqrt{-I_4}.
\]  

(18)

In the following treatment we choose $p^0$ to be positive. Classically, the black hole potential exhibits a non-compact flat direction

\[
\text{Re} y^1 = -\frac{\text{Re} y^2}{1 + (\text{Re} y^2)^2}, \quad \text{Im} y^1 = -\frac{1}{2} - (\text{Re} y^2)^2, \quad \text{Im} y^2 = \pm \sqrt{1 - (\text{Re} y^2)^2},
\]  

(19)

which spans a one-dimensional manifold $SO(1,1)$ [9]. The BH potential evaluated on eq. (19) gives as usual

\[
V_{BH} = \sqrt{-I_4} = \sqrt{p^0 q_1 q_2}.
\]  

(20)

Introducing quantum effects destroys the classical flat direction (19), and the critical solutions are reduced to a discrete set of points (usually one or two) having the axion fields $\text{Re} y^a$ equal to zero. The domain of positivity of the metric is defined by condition

\[
[\alpha - \text{Im} y^1 (\text{Im} y^2)^2] [\alpha + 2 \text{Im} y^1 (\text{Im} y^2)^2] < 0.
\]  

(21)

In the considered case $I_4 < 0$ with only $D2$ and $D6$ branes present, the dependence of the minimum value of the quantum corrected black hole potential $V_{BH}$ on the quantum parameter $\alpha$ is presented in Fig. 4. Numerical analysis does not support any evidence for the existence of solutions with $|\alpha| > 1$, because such solutions fall outside the domain of positivity defined by eq. (21).

The plot for $\alpha < 0$ and the short curve for $\alpha > 0$ in Fig. 4 correspond to solutions to attractor equations with $\alpha$-independent covariant derivatives of the superpotential

\[
D_a W = q_a, \quad a = 1, 2.
\]  

(22)

4 \hspace{1em} \textbf{D0 – D6 branes}

Let us now briefly analyze the $D0 – D6$ brane configuration in the $st^2$ model. The corresponding charges $p^0$ and $q_0$ are those associated to the Kaluza-Klein vector arising through dimensional reduction from $d = 5$ to $d = 4$ [24].

In this framework, the quartic invariant $I_4$ is negative definite

\[
I_4 = -(p^0 q_0)^2.
\]  

(23)
In order to perform the analysis, it is once again convenient to introduce rescaled moduli $y^a$ and the quantum parameter $\alpha$

\[
s = \sqrt[3]{\frac{q_0}{p^0}} y^1, \quad t = \sqrt[3]{\frac{q_0}{p^0}} y^2, \quad \lambda = \frac{\alpha q_0}{2p^0}.
\]  

(24)

The black hole potential has a flat direction

\[
\text{Re } y^1 = \text{Re } y^2 = 0, \quad \text{Im } y^1 = \pm \frac{1}{(\text{Im } y^2)^2},
\]  

(25)

corresponding to non-BPS states. The sign plus is to be taken for $p^0 q_0 < 0$ and the sign minus – otherwise. This flat direction is characterized by a minimum of the black hole potential which turns out to be equal to

\[
V_{BH} = \sqrt{-I_4} = |p^0 q_0|.
\]  

(26)

Notice that, consistently with the analysis of [24], the $D0 - D6$ configuration admits axion-free solutions at the classical level, and they are actually general solutions.

Performing a thorough numerical analysis, an unexpected evidence emerges: the classical non-BPS $Z \neq 0$ flat direction of the $st^2$ model in the $D0 - D6$ brane configuration seemingly survives the considered quantum correction. This fact deeply distinguishes the $D0 - D6$ configuration from the others treated above, when the quantum correction always lifts the flat direction. Another feature of this quantum flat direction is the presence of non-vanishing axion fields.

Thus, one can conclude that the axion-free classical non-BPS $Z \neq 0$ flat direction is kept by the quantum corrections, but it gets distorted and acquires non-zero values of the axion fields. Naturally, the black hole potential takes a minimal value along the flat direction and the dependence of this value on the quantum parameter is presented in Fig. 5.

![Figure 5: Plot of the minimum of $V_{BH}$ in the $D0 - D6$ brane configuration](image)

Figure 5: Plot of the minimum of $V_{BH}$ in the $D0 - D6$ brane configuration

5 Conclusion and Outlook

We addressed the issue of the fate of the unique non-BPS flat direction of $st^2$ model in the presence of the most general class of quantum perturbative corrections consistent with continuous axion-shift symmetry [16].
We performed our analysis in $D_0 - D_4$, $D_2 - D_6$ and $D_0 - D_6$ brane configurations. For the first two cases we showed that the classical flat direction gets lifted at the quantum level. The same behavior one may expect for the unique non-BPS $Z = 0$ flat direction of the third element of the cubic reducible sequence $SU(1,1)/U(1) \times SO(2) \times SO(n)$ [9].

On the other hand, the analysis performed in the $D_0 - D_6$ brane configuration yielded a somewhat surprising result: the classical flat direction gets modified at the quantum level, acquiring a non-zero value of the axion fields. The origin of such a deep difference among the brane configurations, which is expected to hold in other models as well, are yet to be understood, and we leave the study of this issue for future work.

Clearly, in the considered two moduli quantum corrected special Kähler $d$-geometry based on a holomorphic prepotential, the phenomena of “separation” and “transmutation” of attractors, firstly observed in [11], also occur, with a richer case study, due to the presence of non-BPS $Z = 0$ attractors.

By generalizing the results obtained in the present paper to the presence of more than one flat direction, one would thus be led to state that only a few classical attractors do remain attractors in a strict sense at the quantum level. Consequently, at the quantum level the set of actual extremal black hole attractors should be strongly constrained and reduced.

As a final remark, it is worth pointing out that in $N = 8$ $(d = 4)$ supergravity “large” $1/8$-BPS and non-BPS BHs exhibit 40 and 42 flat directions, respectively [25] [8]. If $N = 8$ supergravity is a finite theory of quantum gravity (see e.g. [20] and Refs. therein), it would be interesting to understand whether these flat directions may be removed at all by perturbative and/or non-perturbative quantum effects.

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