Femtoscopic study of coupled-channel $N\Xi$ and $\Lambda\Lambda$ interactions

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The momentum correlation functions of $S = -2$ baryon pairs ($p\Xi^{-}$ and $\Lambda\Lambda$) produced in high-energy $pp$ and $pA$ collisions are investigated on the basis of the coupled-channel formalism. The strong interaction is described by the coupled-channel HAL QCD potential obtained by the lattice QCD simulations near physical quark masses, while the hadronic source function is taken to be a static Gaussian form. The coupled-channel interaction, the threshold difference, the realistic strong interaction, and the Coulomb interaction are fully taken into account for the first time in the femtoscopic analysis of baryon-baryon correlations. The characteristic features of the experimental data for the $p\Xi^{-}$ and $\Lambda\Lambda$ pairs at LHC are reproduced quantitatively with a suitable choice of non-femtoscopic parameters and the source size. The agreement between theory and experiment indicates that the $N\Xi$ ($\Lambda\Lambda$) interaction is moderately (weakly) attractive without having a quasi-bound (bound) state.

I. INTRODUCTION

Dibaryons in the strangeness $S = -2$ sector have long been attracted theoretical and experimental attention [1-3]. Among others, the $H$ (uuddss) dibaryon with $(I,J^P) = (0,0^+)$ was suggested to be a possible bound state below the $\Lambda\Lambda$ threshold [4]. However, discovery of the double $\Lambda$ hypernuclei [5-6] ruled out the deeply bound $H$ with the binding energy of $B_H > 6.91$ MeV. Also, recent femtoscopic studies disfavor the existence of a bound state below the $\Lambda\Lambda$ threshold [7-9]. Moreover, the latest $(2+1)$-flavor lattice QCD simulations indicate that there is no bound state below $\Lambda\Lambda$ [10].

Hence the current interest in the $S = -2$ dibaryons is shifted to the region around the $N\Xi$ threshold in the $(I,J^P) = (0,0^+)$ channel where the $N\Xi$ interaction is moderately attractive as indicated by the existence of the $\Xi$-hypernucleus $^{15}\Sigma_{C}$ [11,12], by the femtoscopic studies of the $N\Xi$ interaction [14,15], by the chiral effective field theory calculation [16,17], and by the lattice QCD simulation [10]. Therefore, it is of great importance to make a quantitative comparison between theoretical analysis and the experimental data with the $N\Xi$-$\Lambda\Lambda$ coupled channel framework and the state-of-the-art baryon-baryon interactions (e.g. [18,19]). Such studies are also crucial for identifying the role of $\Lambda$ and $\Xi^{-}$ in neutron star matter at several times the nuclear matter density in relation to the so-called hyperon puzzle in neutron star structure originally pointed out in Ref. [20], as well as to the observed constraints on mass and radius of neutron stars [21].

It has been known that the correlation function in high-energy collisions is sensitive to the interaction when the absolute value of the scattering length ($a_0$) is comparable to or larger than the emission source size $R$ of hadronic pairs, where $R \sim 1-5$ fm depending on the reactions ($pp$, $pA$ or $AA$) [7,22-23]. It has been also argued that the source size dependence of the correlation function is useful to deduce the existence or non-existence of hadronic bound states [27,28].

In the present paper, we focus on the momentum correlations of $N\Xi$ and $\Lambda\Lambda$ in $pp$ and $pA$ collisions. Recent experimental measurements of such correlations have opened a new way to probe the hyperon interactions which are not accessible in the standard scattering experiments [29]. Theoretically, the correlation function can be described by the convolution of the source function and the relative wave function in the pair rest frame [30-34].

We consider the coupled-channel formalism ($p\Xi^{-}-n\Xi^{0}$-$\Lambda\Lambda$ for $J = 0$ and $p\Xi^{-}-n\Xi^{0}$ for $J = 1$) with the latest HAL QCD coupled-channel potential in the s-wave obtained from the $(2+1)$-flavor lattice QCD simulations at almost physical quark masses [10]. The threshold differences and the Coulomb interaction are taken into account simultaneously. For the source function in $pp$ and $pA$ reactions, we take a static and spherically symmetric Gaussian form with a source size $R$. Our theoretical calculations are then compared with the experimental data of $p\Xi^{-}$ and $\Lambda\Lambda$ correlation functions in $pp$ and $pA$ collisions at LHC [8,9,14,15]. Similar analysis with all ingredients (coupled channel, threshold difference, realistic strong interaction, and Coulomb interaction) was recently performed for the $S = -1$ meson-baryon system ($K\Sigma^{-}\pi\Sigma^{-}\pi\Lambda$) for the first time [28].

This article is organized as follows. In Sect. [11] we briefly review the $S = -2$ baryon-baryon potential from lattice QCD...
In Sect. [11] the theoretical framework to calculate the \( p\Xi^- \) and \( \Lambda\Lambda \) correlation functions in the coupled-channel framework is discussed in detail. In Sect. [12] we show the determination of the phenomenological parameters from the experimental data at LHC on the basis of the formalism in the previous section. In Sect. [13] our theoretical results of \( p\Xi^- \) and \( \Lambda\Lambda \) correlation functions and the experimental data are compared. Section [14] is devoted to summary and concluding remarks. The low energy scattering parameters from a modified HAL QCD potential, the location of the virtual pole near the \( N\Xi \) threshold, and an analytic model of the correlation function with Gamow factor are discussed in Appendix A, B, and C, respectively.

II. \( S = -2 \) COUPLED-CHANNEL POTENTIAL FROM LATTICE QCD

Throughout this paper, we employ the state-of-the-art coupled-channel \( N\Xi-\Lambda\Lambda \) potential below the \( \Sigma\Sigma \) threshold obtained by the (2+1)-flavor lattice QCD simulations near the physical point (\( m_{\pi} = 146 \text{ MeV} \) and \( m_{K} = 525 \text{ MeV} \) [10]). It is the local and energy-independent potential in the leading-order of the derivative expansion at low energies [35, 36]. The coupled-channel \( N\Xi-\Lambda\Lambda \) potential is fitted in terms of a combination of Gaussian, Yukawa and squared-Yukawa functions with the pion and kaon masses on the lattice mentioned above [10]. Shown in Fig. 1 are the results of the fitted potentials in the isospin-spin basis with the notation, \( 2^{I+1}I+1L_{IJ} \) with the isospin \( I \) and the spin \( s \). The statistical error of the potentials originating from the Monte Carlo simulations is evaluated by the standard jackknife method and is denoted by the colored shadows, while the systematic error originating from the truncation of the derivative expansion is estimated by the \( t \)-dependence of the potentials with \( t \) being the temporal distance between source and sink operators in the lattice unit [10]. The important features of the HAL QCD potential are (i) a large attraction in the \( I = s = 0 N\Xi \) channel (the upper left panel), (ii) a weak mixing between \( N\Xi \) and \( \Lambda\Lambda \) (the upper middle panel) at low energy, and (iii) a weak attraction in the \( \Lambda\Lambda \) channel (the upper right panel).

As low energy constants characterizing the strong interaction, we calculate the scattering length \( a_0 \) and the effective range \( r_{\text{eff}} \) in the \( s \)-wave by solving the Schrödinger equation with the HAL QCD potential in Fig. 1 without the Coulomb interaction. Here we take the nuclear and atomic physics conventions, where the \( s \)-wave phase shift at low energies is given by

\[
q \cot \delta_0(q) = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \cdots ,
\]

with \( q \) being the relative momentum. Table 1 summarizes the results where the central values of \( a_0 \) and \( r_{\text{eff}} \) are obtained from \( t = 12 \) with the statistical errors evaluated by the jackknife method and the systematic errors estimated from \( t = 11 \) and 13. Unlike the procedure in Ref. [10] where baryon masses measured on the lattice are used in the kinetic part of the the Schrödinger equation, we use the experimental baryon masses of \( p, \pi, \Lambda, \Xi^- \), and \( \Xi^0 \) [1].

Note that \( a_0 \) in \( \Lambda\Lambda(J = 0) \) and \( n\Xi^0(J = 1) \) channels in Table 1 are strictly real since there are no two-baryon states below, while those in \( p\Xi^- (J = 0) \) and \( n\Xi^0(J = 0) \) channels are complex due to the coupling to the lower \( \Lambda\Lambda \) channel. Also, \( a_0 \) in the \( p\Xi^- (J = 1) \) channel is complex in principle due to the coupling to the lower \( n\Xi^0(J = 1) \) channel.

Solving the Schrödinger equation, we find that neither bound \( H \) dibaryon below the \( \Lambda\Lambda \) threshold nor a quasi-bound state below the \( N\Xi \) threshold are allowed with the HAL QCD potential, although the interactions in both channels are attractive. Also, the large \( |a_0| \) in the \( n\Xi^0(J = 0) \) channel indicates that this system is close to the unitary regime. In fact, there appears a virtual pole in the complex energy plane (see Appendix B). The imaginary part of \( a_0 \) in the \( p\Xi^- (J = 1) \) channel is essentially zero, which implies that the transition between \( p\Xi^- \) to \( n\Xi^0 \) is very weak: This is partly due to the fact that the \( N\Xi \) potential in \( I = 0 \) (the lower middle panel of Fig. 1) and that in \( I = 1 \) (the lower right panel of Fig. 1) are very close to each other.

III. COUPLED-CHANNEL CORRELATION FUNCTION WITH COULOMB INTERACTION

In high-multiplicity events of \( pp \) and \( pA \) collisions as well as in high-energy AA collisions, the hadron production yields are well described by the statistical model, which implies that the hadrons are produced independently. In such a situation, the momentum correlations between outgoing particles are generated by the the quantum statistics and the final state interactions. Consider two particles, \( a \) and \( b \), with relative momentum \( q = (m_a p_a - m_b p_b)/(m_a + m_b) \) observed in the final state. Let this two-particle state be fed by a set of coupled channels, each denoted by \( j \). In the pair rest frame of the two measured particles, their correlation function \( C(q) \) is given by [34]

\[
C(q) = \int d^3r \sum_j \omega_j S_j(r)|\Psi_j^{(-)}(q; r)|^2 ,
\]

where the wave function \( \Psi_j^{(-)} \) in the \( j \)-th channel is written as a function of the relative coordinate \( r \) in that channel, with going out boundary condition on the measured channel. \( S_j(r) \) and \( \omega_j \) are the normalized source function and its weight in the \( j \)-th channel, respectively; \( \int d^3r S_j(r) = 1 \) and \( \omega_1 = 1 \), where we label the measured channel as channel 1. The latter normalization of the source weight follows from the fact that the correlation function must be unity for any momentum \( q \).

1 In Appendix A we show the results of \( a_0 \) and \( r_{\text{eff}} \) with the experimental baryon masses in the kinetic term and a modified HAL QCD potential in which \( m_{\pi,K} \) in the fitted potential are replaced by the isospin-averaged experimental values of the pion and kaon masses. The results in this procedure are consistent with those of Table 1 within statistical and systematic errors.
in the non-interacting limit $V_{ij} \rightarrow 0$. In this study, we use the static Gaussian $S_R(r) \equiv \exp(-r^2/R^2)/(4\pi R^2)^{3/2}$ with source size $R$ for the hadron source function. In this case, the correlation function only depends on $q = |q|$. Thus the correlation function contains information on both the hadron source and the hadron-hadron interactions. We call $R$, the Koonin–Pratt–Lednicky–Lyuboshits–Lyuboshits (KPLL) formula after the series of works [30,34].

There are essentially four theoretical ingredients to fully utilize the KPLL formula and to compare with the experimental data: (i) the coupled channel wave functions, (ii) threshold differences, (iii) the modern hadron-hadron interactions, and (iv) the Coulomb interaction. A comprehensive analysis with all these ingredients has been recently carried out for the first time in the case of the $K^{-}p$ correlation function in high-energy nuclear collisions on the basis of the $KN-\pi\Sigma-\pi\Lambda$ coupled-channel framework [28]. In the subsections below, we generalize this approach applicable to the $N\Xi-\Lambda\Lambda$ system.

### A. Coupled channel formalism

Let us first illustrate some features of the coupled-channel wave function for non-identical particles. We focus on the small momentum region and assume that the strong interaction modifies only the $s$-wave part of the wave function. The coupled-channel wave function $\Psi^{(-)}_j(q; r)$ with the outgoing boundary condition can be written as

$$
\Psi^{(-)}_j(q; r) = (\phi(q; r) - \phi_0(qr))\delta_{1j} + \psi^{(-)}_j(q; r),
$$

where $r = |r|$, $\phi(q; r)$ is the wave function without the strong interaction, $\phi_0(qr)$ is its $s$-wave component, and $\psi^{(-)}_j(q; r)$ is the total wave function in the $s$-wave affected by the strong interaction.

The wave function $\psi^{(-)}_j(q; r)$ in Eq. 3 can be obtained by solving the coupled-channel Schrödinger equation,

$$
\sum_j \left( -\nabla^2 + V_j(r) \right) \psi_j(q; r) = E\psi_j(q; r),
$$

where $V_j(r)$ is the interaction potential and $E$ is the energy. This equation can be solved numerically, and the solution provides the wave function $\psi^{(-)}_j(q; r)$.

### Table I

| total spin | baryon pair | $a_0$ [fm] | $r_{eff}$ [fm] |
|------------|-------------|------------|---------------|
| $J = 0$    | $p\Xi^-$   | -1.22(0.13)(-0.08) - i1.57(0.35)(-0.23) | 3.7(0.3)(+0.1) - i2.7(0.2)(+0.3) |
|            | $n\Xi^0$   | -2.07(0.39)(+0.28) - i0.14(0.08)(+0.01) | 1.5(0.3)(-0.0) - i0.2(0.0)(-0.0) |
|            | $\Lambda\Lambda$ | -0.78(0.22)(+0.08) | 5.4(0.8)(+0.3) |
| $J = 1$    | $p\Xi^-$   | -0.35(0.06)(-0.07) - i0.00            | 8.3(1.0)(+0.2) + i0.0(0.1)(-0.0) |
|            | $n\Xi^0$   | -0.35(0.06)(+0.09)            | 8.4(1.0)(+0.3) |
where \( E_i = E - \Delta_i \) with \( \mu_i \) and \( \Delta_i \), representing the reduced mass in channel \( i \) and the threshold energy difference between channel \( i \) and channel 1, respectively. Since \( \Delta_1 = 0 \), we have \( E = E_1 \) and \( q \equiv \sqrt{2\mu_1E} = q_1 \). Note that \( E_{i>1} \) can be positive or negative depending on the scattering energy, while \( E \geq 0 \) for physical scattering.

Unlike the case of the standard scattering problem where the flux of the incoming wave is normalized, the outgoing wave in the measured channel needs to be normalized in the present case under the boundary condition;

\[
\psi_j^{(-)}(q; r) \xrightarrow{r \to \infty} \frac{1}{2iq_j} \left[ \delta_{1j} u_j^{(+)}(qjr) - A_j(q) u_j^{(-)}(qjr) \right].
\]

(5)

Here \( q_j = \sqrt{2\mu_jE_j} \) for open channels \( (E_j \geq 0) \) and \( q_j = -i\nu_j = -i\sqrt{2\mu_j(-E_j)} \) for closed channels \( (E_j < 0) \). Through these relations, all the momenta \( q_j \) can be expressed as functions of \( q \). Also, \( u_j^{(\pm)}(qjr) \) denotes the outgoing (+) or incoming (−) asymptotic wave; it is the spherical wave \( e^{\pm iq_jr} \) for channels without the Coulomb force, while the Coulomb wave function needs to be used for charged particles, \( u_j^{(\pm)}(qjr) = \pm e^{\mp iq_jr} [F(qjr) \pm G(qjr)] \) with \( \sigma_j = \arg\Gamma(1 + i\mu_j), \eta_j = -\mu_j\alpha/q_j \) and \( F(x) \), \( G(x) \) being the regular (irregular) Coulomb wave function.

In the following subsections, we discuss the coupled channel treatment with \( \Xi^- \) and \( \Lambda\Lambda \) as measured channels.

### B. \( \Xi^- \) correlation function

Let us consider the \( \Xi^- \) correlation function and assign the channel indices \( i = 1, 2, \) and \( 3 \) as \( \Xi^- \), \( n\Sigma^0 \), and \( \Lambda\Lambda \), respectively. For the \( \Xi^- \) pair, there are two \( s \)-wave channels, spin 0 (singlet) and spin 1 (triplet). The former couples to the singlet \( n\Sigma^0 \) and \( \Lambda\Lambda \) channels, while the latter couples only to the triplet \( n\Sigma^0 \) channel. What we observe in experiments is the spin-averaged correlation function given by

\[
C_{\Xi^-}(q) = \frac{1}{4} C_{\Xi^-}^{s\text{inglet}}(q) + \frac{3}{4} C_{\Xi^-}^{s\text{triplet}}(q).
\]

(6)

For \( \Xi^- \), it is necessary to treat the Coulomb interaction carefully because it distorts the wave function significantly in the small momentum region. We introduce the Coulomb potential \( V_C(r) = -\alpha/r \) to the diagonal component of the \( \Xi^- \) channel as \( V^{\Xi^-}_{\Xi^-}(r) = V^{\Xi^-}_{\Xi^-}(r) + V_C(r) \). Since the long-range Coulomb force affects all the partial waves while the short-range strong force affects only the \( s \)-wave at low energies, the wave function in channel 1 (\( \Xi^- \)) in Eq. (3) should be written as

\[
\Psi_1^{(-)}(q; r) = (\phi_C(q; r) - \phi_0^{(\pm)}(q; r)) + \psi_1^{(-)}(q; r),
\]

(7)

where \( \phi_C(q; r) \) is the free Coulomb wave function and \( \phi_0^{(\pm)}(q; r) \) is its \( s \)-wave component. The boundary condition for \( \psi_1^{(-)}(q; r) \) must be given by the the Coulomb wave function

\[ u_j^{(\pm)}(q;r) \] for \( j = 1 \) and the spherical wave \( e^{\pm iq_jr} \) for \( j = 2 \) and 3 in Eq. (5). Then the KPLL formula can be written as

\[
C(q) = \int d^3r S_1(r) [\phi_C^2(q;r) - \phi_0^2(q;r)] + \sum_{j=1}^3 \int_0^\infty 4\pi r^2 dr \omega_j S_j(r) \psi_j^{(-)}(q;r)^2.
\]

(8)

In Fig. 2, we show the fully coupled-channel results of the \( \Xi^- \) correlation function with and without the Coulomb attraction (the solid line and the dashed line, respectively), together with the case of pure Coulomb attraction (the dotted line). Here we use the \( N\Sigma^-\Lambda\Lambda \) coupled-channel potential at \( t = 12 \) given in Fig. 1. To see the qualitative behavior of \( C_{\Xi^-}(q) \), we take a common source function of Gaussian shape for all channels \( S_j(r) = S_R(r) \) with \( R = 1.2 \text{ fm} \) and \( \omega_j = 1 \) for all \( j \). The error bars for the solid and dashed lines estimated by the jackknife method reflect the statistical errors of the lattice QCD data. Compared to the pure Coulomb case, the correlation function shows a large enhancement by the strong interaction in the low momentum region, \( q < 100 \text{ MeV} \).

To see the individual contribution in the \( j \)-sum in Eq. (8), we plot in the left panel of Fig. 3 the three cases for \( C_{\Xi^-}(q) \) with the same parameters as Fig 2, \( j = 1 \ (\Xi^- \text{ only}), j = 1 \) and 3 \( (\Xi^- + \Lambda\Lambda) \), and \( j = 1, 2, \) and 3 \( (\Xi^- + n\Sigma^0 + \Lambda\Lambda) \). For simplicity, the statistical errors are not shown. One finds that the major enhancement of \( C_{\Xi^-}(q) \) over the pure Coulomb case comes from the \( N\Sigma^- \) attraction, while the channel coupling to \( \Lambda\Lambda \) is negligible. Further decomposition into spin-singlet \( C_{\Xi^-}^{s\text{inglet}}(q) \) and spin-triplet \( C_{\Xi^-}^{s\text{triplet}}(q) \) are shown in the middle and right panels of Fig. 4, respectively. Due to
the larger negative scattering length in the spin-singlet channel, its enhancement is stronger, although the spin degeneracy factor is smaller. Also, we find that the contribution from the $n\Xi^0$ channel source to the singlet correlation function gives a small enhancement, while the $\Lambda\Lambda$ source is almost negligible. For the triplet correlation function, the contribution from the $n\Xi^0$ source is almost invisible.

**C. $\Lambda\Lambda$ correlation function**

To study the $\Lambda\Lambda$ correlation function, we assign the channel indices $i = 1, 2,$ and $3$ to $\Lambda\Lambda$, $n\Xi^0$, $p\Xi^-$, respectively. For identical particles, the wave function (3) is distorted by the quantum statistical effect. Then the wave function in channel 1 ($\Lambda\Lambda$) can be decomposed in the even parity (spin-singlet) and the odd parity (spin-triplet) components as

$$
\Psi_{1,E}^{(-)}(q; r) = \frac{1}{\sqrt{2}} \left[ \Psi_1^{(-)}(q; r) + \Psi_1^{(-)}(q; -r) \right]
$$

$$
= \sqrt{2} \left[ \cos (q \cdot r) - \phi_0(qr) \right] + \psi_1^{(-)}(q; r),
$$

$$
\Psi_{1,O}^{(-)}(q; r) = \frac{1}{\sqrt{2}} \left[ \Psi_1^{(-)}(q; r) - \Psi_1^{(-)}(q; -r) \right]
$$

$$
= \sqrt{2} i \sin (q \cdot r).
$$

Since we consider only the $s$-wave distortion by the strong interaction, the scattering wave function $\psi_1$ appear only in the even parity part. Thus the even and odd parity correlation functions are given by

$$
C_{E}(q) = \int d^3r \sum_{j=1}^{3} \omega_j S_j(r) |\psi_{j,E}^{(-)}(q; r)|^2
$$

$$
= 1 + \exp(-4q^2 R^2)
$$

$$
+ 2 \int d^3r \sum_{j=1}^{3} \omega_j S_j(r) \left[ |\psi_{j}^{(-)}(q; r)|^2 - |\phi_0(qr)|^2 \delta_{j1} \right],
$$

$$
C_{O}(q) = \int d^3r \sum_{j=1}^{3} \omega_j S_j(r) |\Psi_{j,O}^{(-)}(q; r)|^2
$$

$$
= 1 - \exp(-4q^2 R^2).
$$

Taking into account the spin degrees of freedom, the final form of the $\Lambda\Lambda$ correlation function reads

$$
C_{\Lambda\Lambda}(q) = \frac{1}{4} C_{E}(q) + 3 \frac{1}{4} C_{O}(q)
$$

$$
= 1 - \frac{1}{2} \exp(-4q^2 R^2)
$$

$$
+ 2 \int d^3r \sum_{j=1}^{3} \omega_j S_j(r) \left[ |\psi_{j}^{(-)}(q; r)|^2 - |\phi_0(qr)|^2 \right].
$$

The $\Lambda\Lambda$ correlation is always suppressed by $(1/2) \exp(-4q^2 R^2)$ by the quantum statistical effect which is independent of the interactions.

We note here that, if the energy is above the $p\Xi^-$ threshold ($E_3 > 0$), $p\Xi^-$ is an open channel and the asymptotic wave function is given by the Coulomb wave function as

$$
\psi_3^{(-)}(q; r) \rightarrow \frac{A_3(q)}{2iq_3} \frac{u_3^{C(-)}(qr)}{r}.
$$

If the energy is less than the $p\Xi^-$ threshold ($E_3 < 0$), $q_3$ should be replace by $-i\kappa_3$ in the above expression, so that we have $u_3^{C(-)}(qr) = e^{i\pi |q_3|/2} W_{|q_3|+1/2}(2\kappa_3 r)$ with $W_{k,\ell+1/2}(z)$ being the Whittaker function [37] [38].

In Fig. 3, we show the fully coupled-channel result of the $\Lambda\Lambda$ correlation function (the solid line) together with the case of pure quantum statistics contribution (the dotted line). The
coupled-channel potentials at $t = 12$ given in Fig. 7 are employed, and a common source function of Gaussian shape is assumed for all channels as in the case of Fig. 2. The error band for the solid line reflecting the statistical errors of the lattice QCD data is estimated by the jackknife method.

Compared to the case of pure quantum statistics, $C_{\Lambda \Lambda}(q)$ shows a strong enhancement by the strong interaction in the low momentum region: $q < 100$ MeV. Also, two cusps corresponding to the $n\Xi^0$ threshold at 2254 MeV and the $p\Xi^-$ threshold at 2260 MeV are found as previously pointed out in Ref. [26]. Such a threshold cusp is indeed found experimentally in the $K^- p$ correlation function [26, 28, 39]. In the present case, these cusps are rather moderate due to the weak coupling between $\Lambda \Lambda$ and $N\Xi$, and it would be a challenging problem to find them experimentally.

To see the individual contribution in the $j$-sum in Eq. (18), we plot in the left panel of Fig. 5 the three cases for $C_{\Lambda \Lambda}(q)$ with the same parameters as Fig. 4: $j = 1$ ($\Lambda \Lambda$ only), $j = 1$ and 2 ($\Lambda \Lambda + n\Xi^0$), and $j = 1$, 2, and 3 ($\Lambda \Lambda + n\Xi^0 + p\Xi^-`). For simplicity, the statistical errors are not shown. The figure shows that the $n\Xi^0$ and $p\Xi^-$ sources only affect the cusp region, and make little contribution to the other momentum region. Nevertheless, solving the coupled-channel Schrödinger equation [4] is important to take into account the extra $\Lambda \Lambda$ attraction due to the coupling with $N\Xi$ states.

**IV. DETERMINATION OF PARAMETERS**

A. Source function and weight

For $pp$ and $pA$ collisions, a spherical and static Gaussian source function works well to reproduce the data, while the analysis of $AA$ collisions requires more detailed information on the source function, e.g. asymmetrical distribution shape and flow effects [7]. In the present analysis of the $pp$ collisions we adopt the static Gaussian source function;

$$S_j(r) = \frac{1}{(4\pi R_j^2)^{3/2}} \exp\left(-\frac{r^2}{4R_j^2}\right). \quad (20)$$

Here, the effective source size $R_j$ would depend on hadron pairs and reactions. In experiments, the source size has been studied by using the correlation function of the $pp$ pairs for which the elaborated strong interaction potential is available. For the $pp$ pairs, the ALICE collaboration has previously determined $R_{pp}$ to be $R_{pp}^{ALICE}(pp) = 1.182 \pm 0.008$ (stat) $^{+0.005}_{-0.007}$ (syst) fm in $pp$ collisions at 13 TeV and $R_{pp}^{ALICE}(pPb) = 1.427 \pm 0.007$ (stat) $^{+0.014}_{-0.013}$ (syst) fm in $pPb$ collisions at 5.02 TeV [8, 14]. On the other hand, smaller source sizes are reported for $p\Xi^-$ and $p\Omega^-$ pairs; $R_{p\Xi^-}^{ALICE}(pp) = 1.02 \pm 0.05$ fm and $R_{p\Omega^-}^{ALICE}(pp) = 0.95 \pm 0.06$ fm [15]. In the present paper, we assume that the source sizes of $N\Xi$ pairs and $\Lambda \Lambda$ pairs are the same ($R_j = R_s$), since their total masses are close to each other and the contribution from the coupled channel sources is not large.

In the theoretical analysis in Sect. III we set $\omega_j = 1$ for simplicity. In actual high energy collisions, the source weights depend on the channel and the reaction. In general, the ratio of the source weights in channels $i$ and $j$ is written in terms of the particle yields $N$ as $\omega_i/\omega_j = (\alpha_i N(i_1)N(i_2))/\alpha_j N(i_1)N(i_2)$ with $i_1, i_2, j_1$ and $j_2$ being the labels of particles in each channel and $\alpha_i$ representing the ratio of the number of particle pairs assigned to channel $i$ among $N(i_1) \times N(i_2)$ pairs. For the $K^- p$ correlation function analyses in Ref. [28], a statistical model [40, 41].
TABLE II. The pair purity $\lambda$, non-femtoscopic parameters $a$ and $b$, and the effective source size $R$ in the fitting function $C_{th}(q)$. The parameters $a$ and $b$ in $pp$ ($\Lambda\Lambda$ pairs) and $pPb$ ($p\Xi^{-}$ and $\Lambda\Lambda$ pairs) collisions and $R$ in $pp$ collisions are the actual fitting parameters. Numbers with references are taken from Refs. [9,14,15], and the number with ($*$) is estimated from other other parameters. See the text for details.

| collision | pair     | $\lambda$ | $a$          | $b (\text{MeV}/c^{-1})$ | $R$ [fm] |
|-----------|----------|------------|--------------|--------------------------|----------|
| $pp$ (13 TeV) | $p\Xi^{-}\Lambda\Lambda$ | 0.338 [9] | 1            | 0.95                     | $1.28 \times 10^{-4}$ | 1.05     |
| $pPb$ (5.02 TeV) | $p\Xi^{-}\Lambda\Lambda$ | 0.513 [14] | 1.09         | $-2.56 \times 10^{-4}$  | $1.27^{(*)}$  |

2 Eq. [71] gives a slightly different relative weight $\omega_{N\Xi}/\omega_{\Lambda\Lambda}$ from the statistical model due to the approximation that holds for $m_{i_1,i_2}/(m_{i_1,m_{i_2}}) \sim 1$. We have checked that this factor does not change the following qualitative results and the pictures.

V. COMPARISON WITH EXPERIMENTAL DATA

A. $p\Xi^{-}$ correlation function

In the upper panels of Fig. [6], our final results of the $p\Xi^{-}$ correlation functions are compared with the $p\Xi^{-}$ data in $pp$ collisions at 13 TeV (the left panel) and in $pPb$ collisions at 5.02 TeV (the right panel) [14,15]. The solid lines denote our final results with statistical and systematic errors of the HAL QCD potential. The former is estimated by the jackknife method with the $t = 12$ data, and the latter is estimated by the potentials for $t = 11$ and 13. The dotted green lines are the results with the Coulomb potential only. Shown by the shaded region is the larger one among the statistical and systematic errors.

The solid lines explain not only the strong enhancement at small $q$ but also the $q$ dependence of $C_{th}(q)$. The enhancement over the pure Coulomb potential implies the attractive nature of the strong $N\Xi$ interaction. Such an observation has been already reported in the previous works [14,15,24,26]. However, our paper provides for the first time the coupled-channel analysis with the threshold difference, the strong interaction, and the Coulomb interaction taken into account.
FIG. 6. Experimental and theoretical correlation functions of the $p\Xi^-$ pairs (the upper panels) and the $\Lambda\Lambda$ pairs (the lower panels). The blank squares are the ALICE data taken from Refs. [9, 14, 15]: The statistical error and systematic error are denoted by the vertical line and the shaded bar, respectively. Solid lines are the theoretical results with statistical and systematic uncertainties represented by the shaded region. The left (right) panels correspond to the results in $pp$ collisions at 13 TeV ($pPb$ collisions at 5.02 TeV). The dotted lines show the results with only Coulomb interaction (only quantum statistics) for the $p\Xi^-$ ($\Lambda\Lambda$) correlation functions. The dash-dotted lines show the correlation function calculated with the LL formula.

(Neither the coupled channel effect nor the threshold difference has been considered in Refs. [14, 15, 24], while the Coulomb interaction was not considered in Ref. [26].) We note that the agreement of the correlation function in Refs. [14, 15] and that in the present work comes from the fact that the coupled-channel effects are not significant in the $p\Xi^-$ correlation function due to weak transition between $p\Xi^-$ and $\Lambda\Lambda$.}

$p\Xi^-$ (corrected)
B. ΛΛ correlation function

In the lower panels of Fig. 6 our final results of the ΛΛ correlation functions are compared with the ΛΛ data in pp collisions at 13 TeV (the left panel) and in pPb collisions at 5.02 TeV (the right panel) \(^6\). The solid lines denote our final results with statistical and systematic errors of the HAL QCD potential. The dotted green lines are the results with only the quantum statistics effect. Although there are large uncertainties of the experimental data at small \(q\) region, the agreement of the solid line with the data indicates a weak attraction in the ΛΛ channel without a deep bound state. This is consistent with the conclusions in Refs. \(^8, 9\).

The correlation functions calculated with the Lednicky-Lyuboshits (LL) formula for identical spin-half baryon pairs \(^33\) are also plotted in the lower panels of Fig. 6 by the dash-dotted line:

\[
\begin{align*}
C(q) &= 1 - \frac{1}{2} e^{-4q^2 R^2} + \frac{1}{2} \Delta C(q), \tag{23} \\
\Delta C(q) &= \frac{|f(q)|^2}{2R^2} F_3 (\frac{r_{\text{eff}}}{R}) + \frac{2Re\{f(q)\}}{\sqrt{\pi R}} F_1 (2qR) \\
&\quad - \frac{1}{R} \frac{Im\{f(q)\}}{F_2 (2qR)}, \tag{24}
\end{align*}
\]

where \(F_1(x) = \int_0^x dt e^{-t^2}/x, F_2(x) = (1 - e^{-x^2})/x, F_3(x) = 1 - x/2\sqrt{\pi}\), and we make the effective range expansion of single channel ΛΛ scattering amplitude \(f(q)\) with \(a_0 = -0.78\) fm and \(r_{\text{eff}} = 5.4\) fm given in Table 1. The same non-femtoscopic parameters and the pair purity listed in Table 1 are used. We find that the single-channel LL formula gives a good approximation to the fully coupled-channel results for wide range of \(q\) in both pp and pPb collisions. It would be interesting to see whether high precision data for \(C_{\Lambda\Lambda}(q)\) in the future may reveal cusp structures at the \(n\Xi^0\) and \(p\Xi^-\) thresholds as expected from the coupled channel effect.

C. System size dependence

The enhancement of \(C(q)\) for fixed \(R\) alone cannot conclude whether bound or quasi-bound state is generated by the strong interaction. This can be demonstrated by using an analytic model for neutral and non-identical particles \(C(q) = 1 + \Delta C(q)\) with \(r_{\text{eff}} = 0\) which is obtained from Eq. (24) as

\[
\Delta C(q) = \frac{1}{x^2 + y^2} \left[ \frac{1}{2} - \frac{2y}{\sqrt{\pi}} F_1 (2x) - x F_2 (2x) \right], \tag{25}
\]

with \(x = qR\) and \(y = R/a_0\). Shown in Fig. 7 is a contour plot of \(C(q)\) in the \(x-y\) plane. The strongly enhanced region \(C(q) > 2\) indicated by the white area extends to both negative and positive sides of \(y\) for \(x < 0.5\). (Even if one introduces the Coulomb attraction such as the case of \(p\Xi^-\), this situation does not change qualitatively as discussed in Appendix C.)

Scanning through the \(y\)-axis by changing the system size \(R\) would provide further experimental information on the sign of \(y\). To demonstrate this, we show the \(p\Xi^-\) and ΛΛ correlation functions for several different source sizes \((R = 0.9, 1.2, 1.5,\) and \(3\) fm) in Fig. 8 with the HAL QCD potential (the thick lines) and without the HAL QCD potential (the thin lines).

For the \(p\Xi^-\) correlation function, Fig. 8 implies that the enhancement of \(C(q)\) due to strong interaction over the pure Coulomb attraction is significant around \(R = 1\) fm but is gradually reduced toward the larger values of \(R\). This is consistent with the fact that we are in the negative \(y\) region as indicated by Fig. 7. If the scattering length is in the bound region \((y = R/a_0 > 0)\), we would expect that \(C(q)\) undershoots the Coulomb contribution and may form a dip as a function of \(x = qR\). Thus the experimental studies of the \(p\Xi^-\) correlation function in heavy-ion collisions corresponding to larger \(R\) are of particular interest.

For the \(\Lambda\Lambda\) correlation function, Fig. 8 shows that the enhancement of \(C(q)\) due to strong interaction over the pure quantum statistics has characteristic non-monotonic behavior for \(q\) smaller than the \(N\Xi\) threshold. However, to make quantitative discussions for large \(R\) corresponding to the heavy-ion collisions, more realistic source shape as well as the flow effect need to be taken into account \(^7\), since the effect of quantum statistics is particularly important in the ΛΛ correlation.

We note here that a high-momentum tail of the ΛΛ correlation function above the \(N\Xi\) threshold was observed in Au+Au collisions at RHIC \(^42\), and a residual source having a small size \((R_{\text{res}} \approx 0.5\) fm) was introduced in previous works \(^7, 22, 42\). Although it was suggested in Ref. \(^43\) that the coupled-channel effects may explain the high-momentum tail in Au+Au collisions, the present analysis shows that such a tail does not appear unless \(R\) is smaller than 1 fm as shown in Fig. 8. Thus this issue is still left open for future studies.
Coulomb interaction at the same time to analyze the perfect, the threshold difference, the strong interaction, and the interactions of hadron pairs, we considered the coupled-channel effects (pure quantum statics cases) are shown for the calculation. For comparison, the calculations with the pure Coulomb potential produces a virtual pole below the \( n\Xi^0 \) and \( p\Xi^- \) thresholds due to channel coupling, which would be interesting to be seen in future high precision data.

Studies with femtoscopic techniques in different collision systems will help us to unravel the physics of hadron-hadron interactions further. For example, it is interesting to examine the \( N\Xi \) correlation function in nucleus-nucleus collisions by changing the impact parameter, so that one can utilize the idea of the “small-to-large ratio” to extract the strong interaction effect without much contamination from the Coulomb interaction \( [23] \). A femtoscopic study of the hadron-deuteron correlation functions \( [44–47] \) is another feasible and valuable direction to pursue. The production of the \( S = -2 \) system through the \((K^-, K^+)\) reaction with nuclear target is also an alternative and promising approach to study the \( N\Xi - \Lambda\Lambda \) system in a controlled fashion \( [48–50] \).

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**Appendix A: Low energy constants from modified HAL QCD potential**

The HAL QCD potential used in the text is constructed at \( m_\pi \approx 146 \text{ MeV} \) and \( m_K \approx 525 \text{ MeV} \) which are slightly away from the physical point \( [10] \). To estimate the effect of this discrepancy, we replace \( m_\pi \) and \( m_K \) in the parametrization of the HAL QCD potential by the isospin-averaged physical masses, 137.3 MeV and 495.6 MeV, respectively. Resulting scattering lengths and effective ranges are shown in Ta-
The numbers are consistent with those given in Table I within the errors, although the central values of the scattering length $a_0$ are slightly larger due to slight increase of the attraction by the smaller pion and kaon masses.

### Appendix B: Virtual pole and $N\Xi$ interaction

In the single-channel problem with a sufficiently attractive $s$-wave interaction, a bound state pole lies on the imaginary momentum $q$ axis in the upper half of the complex $q$ plane, as shown in Fig. 9. This pole goes down along the imaginary $q$ axis to the lower half plane (virtual pole), and eventually goes off the imaginary $q$ axis to the right half plane with decreasing attraction $|q_{pole}|$. Eventually, the physical eigenstate emerges as a resonance in the case where the eigen-momentum of pole $q_{pole}$ satisfies the condition, $\text{Re } q_{pole} > -\text{Im } q_{pole}$. When there are decay channels, the relation between the interaction and the pole position is more complicated compared to the single-channel case. Nevertheless, we can find a similar behavior for a pole lying nearby the threshold energy.

As indicated by the negative scattering length in Table I in the case of HAL QCD potential with $t = 12$, the strong interaction does not generate bound or quasi-bound states near the $\Lambda\Lambda$, $n\Xi^0$ and $p\Xi^-$ thresholds. Instead, we find a virtual pole lying at $E_{pole} = 2250.5 - i0.3$ MeV in the $(+, -, +)$ sheet in the $J = 0$ channel: The real part of the energy is just below the $n\Xi^0$ threshold by $-3.93$ MeV, while the sign of the imaginary part of the eigen-momenta of $\Lambda\Lambda$, $n\Xi^0$, and $p\Xi^-$, are $+$, $-$, and $+$, respectively. If the $N\Xi$ quasi-bound state would emerge, the corresponding pole should have appeared below the $n\Xi^0$ threshold in the $(-, -, +)$ sheet. The near-threshold virtual pole in the $(+, -, +)$ sheet still contributes to the enhancement of the scattering length in the $n\Xi^0$ channel. We note here that, if we use the modified HAL QCD potential associated with Table III the virtual pole moves closer to the threshold energy of $n\Xi^0$, $E_{pole} = 2251.8 - i0.2$ MeV. This is due to the fact that the attraction becomes slightly stronger in this case and the virtual pole moves toward right direction in the complex $E$ plane as seen from Fig. 9.

### Appendix C: LL model with Gamow factor

When the Coulomb attraction operates on top of the strong interaction, $C(q)$ is enhanced in the low $q$ region and the suppression found in Fig. 7 with $a_0 > 0$ (without the Coulomb potential) is expected to appear as a dip of $C(q)$ when the source size $R$ is comparable to $a_0$. This is illustrated in Fig. 10 where the Coulomb effect is considered qualitatively by multiplying the Gamow factor given as $A_{\text{Gamow}}(\eta) = 2\pi\eta/(\exp(2\pi\eta) - 1)$. On the other hand, in the negative $a_0$ region without the bound state, the dip structure is not expected in $C(q)$ for wide range of $R = 1-5$ fm. Recent preliminary data from Au+Au collisions [62] seem to show no dip in the $p\Xi^-$ correlation function, which is consistent with the HAL QCD potential where there is no quasi-bound state of $p\Xi^-$ generated by the strong interaction.
FIG. 9. A schematic picture of the $s$-wave pole position generated by the strong interaction in the complex energy and momentum space. As the attractive interaction becomes weaker from the bound region, the bound pole becomes the virtual pole first, and then moves on to the resonance pole.

FIG. 10. Same as Fig. 7 but with Gamow factor. For given $(x, y) = (qR, R/a_0)$, $\eta = -\mu \alpha / q$ is calculated as $\eta(x, y) = -\alpha |\mu y a_0| / x$ where we adopt $\mu a_0 = \mu_{\text{p}} - a_0^{(J=0)} = -3.32$.

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