CP Violation in Dual Dark Matter

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In this paper we study the consequences of extending dual symmetry (DuSy) to include a generic $C_\mu$ vector field as a dual partner of the photon $A_\mu$. A new combined field, the complex $Z'_\mu$, is obtained from $C_\mu$ and $A_\mu$. The promotion of dual symmetry to a local symmetry for $Z'_\mu$ implies in the inclusion of an extra complex vector field $W_\mu$ with a complex gauge transformation. A dual dark matter (DM) Lagrangian $\mathcal{L}_{DDM}$ is obtained from the general DuSy invariant Lagrangian $\mathcal{L}_{DuSy}$. Our tentative conjecture is to interpret $W_\mu$ as the actual weak interaction charged gauge boson $W^{\pm}_\mu$, which leads us to speculate about a possible extra CP violation scenarios for future calculations.

I. INTRODUCTION

The enigma of dark matter (DM) remains unsolved, a mystery which has been puzzling scientists for more than 80 years. Still one of the most evasive and fascinating mysteries in physics, the problem of the DM in the Universe stirs the imagination of most astronomers, cosmologists and particle physicists, that are convinced that at least 90% of the mass of the Universe is due to some non-luminous form of matter.

In 1933, the astronomer Fritz Zwicky (Zwicky 1933) has observed the radial velocities of eight galaxies in the Coma cluster, providing the first evidence for DM. He has found an unexpected large velocity dispersion of these galaxies which would indicate that the visible mass of (luminous) matter in the Coma cluster was much smaller than its total mass. From these observations, he has found that the mean density of the Coma cluster would have to be 400 times greater than the one derived from luminous matter. Zwicky then concluded that DM (non luminous matter) would be present and that the large velocity dispersion in Coma (and other groups of galaxies) represents an unsolved problem.

In 1970, Rubin and Ford (Rubin & Ford 1970) obtained the strongest evidence up to that time for the existence of DM which became the most plausible explanation for the anomalous rotation curves in spiral galaxies.

Cosmology and astrophysics have provided historically, many convincing evidence for the existence of DM. The primordial observations which revealed the lack of mass to explain the internal dynamics of clusters of galaxies and the rotation of galaxies, which date back as we have seen the years 30 and 70, have been followed, more modernly, by observations that have provided substantial and consistent evidence for the existence of mass effects in regions of the universe where no luminous mass is observed.

Although the existence of DM is widely accepted, several other explanations of these discrepancies have also been proposed. Still, besides these signs of gravitational effects, there are other types of searches for dark matter, as the identification of explicit manifestation of DM particles and indirect searches involving annihilation or decay of DM particles in the flow of cosmic rays (Cirelli 2013; Bertone 2010).

Past all these years after the original suggestion related to its existence, we still do not know the composition of dark matter or its nature. Global fits to gravitational signal effects of DM at the cosmological and extra galactic scale for a large range of data sets (cosmic background radiation, large-scale structure of the universe, type Ia supernovae) determine with great accuracy the amount of cold DM in the overall content of matter energy in the universe at $\Omega_{DM} h^2 = 0.1123 \pm 0.0035$ (Cirelli 2013), where $\Omega_{DM} h^2$ represents the density of cold DM matter and $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$ is the Hubble parameter. Moreover, the observations after nine years release have placed stringent constraints to the abundance of baryons (B) and dark energy (DE) (Bennett et al. 2003; Spergel et al. 2003; Komatsu et al. 2009): $\Omega_B h^2 = 0.233 \pm 0.023$ ; $\Omega_{DE} h^2 = 0.721 \pm 0.025$ where $\Omega_B h^2$ is the physical baryon density and $\Omega_{DE} h^2$ is the dark energy density.

Modernly, there are many plausible candidates for dark matter, as weakly-interacting massive particles (WIMPs), SM neutrinos (Weinheimer 2003), sterile neutrinos (Dodelson & Widrow 1994), axions (Rosenberg & Bibber 2000), supersymmetric candidates (neutralinos, sneutrinos, gravitinos, axinos (Falk, Olive, & Srednicki 1994; Feng, Rajaraman & Takayama 2003; Goto & Yamaguchi 1992), light scalar DM (Lee & Weinberg 1977), DM from little Higgs models (Cheng & Low 2003), Kaluza-Klein particles (Agashe & Servant 2004), superheavy DM (Griest & Kamionkowski 1990). An excellent review in theoretical and experimental aspects of DM can be found in Bertone, Hooper & Silk (2005). In general they are present in theories of weak-scale physics beyond the Standard Model (SM) and give rise to appropriate relic abundance. Calculations have shown that stable WIMPs can remain from the earliest moments of the Universe in sufficient number to account for a significant fraction of relic DM density. This raises the hope of detecting relic WIMPs directly by observing their elastic scattering on targets. In the DM zoo many different types of particles have been introduced and their properties theoretically studied.

Many types of models that explore the physics beyond SM, share in common the presence of new $U(1)$ vector bosons. These new bosons are introduced basically in two ways: (i) minimal coupling; (ii) Stueckelberg mechanism. A new vector gauge boson would be massless if a new $U(1)$ symmetry should remain unbroken. This would imply in a long range force if it were to couple to ordinary matter, unless the coupling were incredibly small. This case would be allowed if
the primary coupling were to a hidden sector and connected only by higher-dimensional operators or alternatively by kinematic mixing with the photon. In the case of kinetic mixing, this scenario would induce a small fractional electric charge for hidden sector particles. Usually this class of models the extra gauge boson is called the $Z'$. Now, the experimental discovery of a $Z'$ would be exciting, but the implications would be much greater than just the existence of a new vector boson. Breaking the $U(1)'$ symmetry would require, for example, an extended Higgs, with significant consequences for collider physics and cosmology (Langacker 2009).

Following a different approach than the usual models with extra $U(1)$ sectors in the SM, we study the consequences of the introduction of new complex vector bosons $Z'$ and $Z''$ subject to dual symmetry (DuSy) requirements. An interesting conjecture is to apply this theory to CP violation. It is well known that if CP violation in the lepton sector is experimentally determined to be too small to account for matter-antimatter asymmetry, some new Physics beyond the Standard Model would be required to explain additional sources of CP violation. Fortunately, it is generally the case that adding new particles and/or interactions to the Standard Model introduces new sources of CP violation. In what follows we use the Minkowski space-time $x^{\mu} = (t, \mathbf{r})$ with signature $(-, +, +, +)$, and assume natural units $\epsilon_0 = \mu_0 = c = 1$.

## II. DUAL SYMMETRY

### A. A brief review

Since seminal works of Dirac on magnetic monopoles (Dirac 1931, 1948) a great deal of theoretical interest has appeared, despite no confirmed experimental evidence of their existence up to the present. Dirac’s conclusions about the possibility of the existence of magnetic monopoles were based on a logical conclusion. In a paper published in 1931 (Dirac 1931), Dirac showed that if a magnetic particle interacts with an electrically charged particle, according to the laws of quantum mechanics, the electric charge of the particle must necessarily be quantized. From a complementary reciprocal reasoning, as we know that electric charges are quantized Dirac concluded that magnetic monopoles must be taken seriously.

In the article published in 1948 (Dirac 1948), Dirac sets up a general theory of charged particles and magnetic poles in interaction through the medium of the electromagnetic field. Using the four components coordinates $x^{\mu}$ with $\mu = 0, 1, 3, 4$ to fix a point in space and time and the velocity of light to be unit, the starting point of Dirac’s formulation was the ordinary electromagnetic field, $F^{\mu\nu}$ at any point:

$$F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad A = (V, \mathbf{A}), \quad (1)$$

where $A_\mu$ represents the electromagnetic 4-potential.

In vacuum, Maxwell’s equations

$$\partial \cdot \mathbf{E} = 0; \quad \partial \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}; \quad \partial \cdot \mathbf{B} = 0; \quad \partial \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}, \quad (2)$$

exhibit a high degree of symmetry: they are invariant under conformal Lorentz transformation and under electromagnetic duality, $(\mathbf{E}, \mathbf{B}) \rightarrow (\mathbf{B}, -\mathbf{E})$. This discrete symmetry is a particular case of a continuous symmetry $E \rightarrow E \cos \theta + B \sin \theta$ and $B \rightarrow B \cos \theta - E \sin \theta$, where $\theta$ is a pseudoscalar. A consequence of this symmetry is that all fundamental properties of free electromagnetic fields, such as energy, momentum, angular momentum, among others, are symmetric with respect to this transformation.

Lorentz invariance can be made manifest by introducing additionally to the electromagnetic field strength $F^{\mu\nu}$, its dual $^{*}F^{\mu\nu}$. Since $F^{\mu\nu}$ is an antisymmetric 0-vector, $F^{\rho\sigma} = -F^{\sigma\rho}$, it can be shown that the unique way to construct a dual Lorentz-invariant four-tensor involving the electromagnetic field that is independent of the original field strength tensor is

$$^{*}F^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}, \quad (3)$$

where $\varepsilon^{\mu\nu\rho\sigma}$ represents a tensor equivalent to the Levi-Civita four-vector. In Minkowski space, $^{*}F^{\mu\nu} = 1$. The duality transformation corresponds to $F^{\mu\nu} \rightarrow ^{*}F^{\mu\nu}$, $^{*}F^{\mu\nu} \rightarrow -F^{\mu\nu}$.

The inner product $F^{\mu\nu} F_{\mu\nu}$ gives the Lorentz invariant

$$F^{\mu\nu} F_{\mu\nu} = 2 (B^2 - E^2), \quad (4)$$

where $E$ and $B$ represent respectively the electric and magnetic fields. The product of $^{*}F^{\mu\nu}$ with its dual $^{*}F^{\mu\nu}$ allows to obtain the pseudoscalar Lorentz invariant

$$^{*}F_{\gamma\delta} F^{\gamma\delta} = \frac{1}{2} \varepsilon^{\alpha\beta\gamma\delta} F^{\alpha\beta} F_{\gamma\delta} = -4 E \cdot B. \quad (5)$$

In the absence of field sources, it can be shown, that the divergence of the dual field strength tensor is $\partial_\mu ^{*}F^{\mu\nu} = 0$. We can then express the physical content of Maxwell’s equations in a form which is elegant and manifestly Lorentz-invariant, in the absence of field sources, as:

$$\partial_\nu ^{*}F^{\mu\nu} = 0; \quad \partial_\nu ^{*}F^{\mu\nu} = 0. \quad (6)$$

In Minkowski space, $\partial_\mu ^{*}F^{\mu\nu} = 0$ implies Eq. (1). Similarly $\partial_\nu ^{*}F^{\mu\nu} = 0$ implies

$$^{*}F_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu, \quad \hat{A} = (\hat{V}, \hat{A}), \quad (7)$$

where $\hat{A}_\mu$ represents the electromagnetic dual 4-potential. The duality transformation which relates $A_\mu$ and $\hat{A}_\mu$ is nonlocal. In the presence of sources, duality invariance is preserved provided we include in Eqs. (7) electric ($j^{\mu}_e$) and magnetic sources ($j^{\mu}_m$) so that

$$\partial_\nu F^{\mu\nu} = j^{\mu}_e; \quad \partial_\nu ^{*}F^{\mu\nu} = j^{\mu}_m, \quad (8)$$

and the duality transformation is supplemented by a corresponding transformation of the sources

$$j^{\mu}_e \rightarrow j^{\mu}_m; \quad j^{\mu}_m \rightarrow -j^{\mu}_e. \quad (9)$$

The imposition of Lorentz invariance and invariance under duality and parity transformations establishes severe restrictions on the mathematical composition of the free Lagrangian...
density $\mathcal{L}$: there are only three possibilities: $F_{\mu\nu} F_{\mu\nu}^*$, $F_{\mu\nu}^* F_{\mu\nu}^*$ and $F_{\mu\nu}^* F_{\mu\nu}$, at our disposal. The third term is proportional to $E, B$ and is not invariant under parity. The second term corresponds to the dual sector of the original electromagnetic field and is not our first option. We then write

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (10)$$

In this equation, the factor $-1/4$ is a matter of convention; any multiplicative factor would have given the same equations of motion.

In order to obtain a more complete Lagrangian density one might try the naive choice

$$\mathcal{L} = -\frac{1}{8} (F_{\mu\nu} F^{\mu\nu} + *F_{\mu\nu}^* F^{\mu\nu}) \quad (11)$$

where the two fields $F_{\mu\nu}$ and $*F_{\mu\nu}$ are considered independent. But since in reality they actually describe the same electromagnetic field, $\mathcal{L} = 0$. The second expression in Eq. (11) is actually a Bianchi identity.

Another important property is that Maxwell’s equations are invariant under the following dual global transformation

$$F_{\mu\nu} \rightarrow \cos \theta F_{\mu\nu}^* + \sin \theta *F_{\mu\nu} \quad (12)$$

$$*F_{\mu\nu} \rightarrow -\sin \theta F_{\mu\nu}^* + \cos \theta *F_{\mu\nu}$$

where $\theta$ is the dual angle. The inclusion of matter, i.e. electric ($e$) and magnetic ($m$) charges, implies that the zeros in (1) are replaced by electric $j^e_{\mu}$ and magnetic $j^m_{\mu}$ current densities that obey the following transformations

$$j^e_{\mu} \rightarrow \cos \theta j^e_{\mu} + \sin \theta j^m_{\mu}$$

$$j^m_{\mu} \rightarrow -\sin \theta j^e_{\mu} + \cos \theta j^m_{\mu} \quad (13)$$

which guarantees that dual symmetry is preserved.

The Lorentz force written in a covariant notation

$$f^\nu = j^e_{\mu} F_{\mu\nu}^* + j^m_{\mu} *F_{\mu\nu} \quad (14)$$

is invariant under the combined transformations (12) and (13), so there is no fundamental restrictions on the values of the dual angle $\theta$, which can be set to any convenient value, as for instance,

$$\theta = \arctan \left( \frac{m}{e} \right). \quad (15)$$

Due to the duality transformation, one cannot decide whether a particle has electric charge, magnetic charge or both just observing and comparing its behavior with the predictions of Maxwell’s equations. It is only a convention, not a requirement of Maxwell’s equations, that electrons have electric charge, but do not have magnetic charge; a $\pi/2$ transformation of the dual angle may originate exactly the opposite result. The fundamental empirical fact is that all observed particles always have the same ratio of magnetic charge and electric charge; transformations of the dual angle can change the ratio to any arbitrary numerical value but can not change the fact that all particles have the same proportion of electric and magnetic charges. Thus, there must be a transformation for which this reason equals to zero, so this particular choice makes the magnetic charge equal to zero. This particular choice is the conventional adopted definition in electromagnetism. For example, one can choose a dual transformation that sets

$$j^e_{\mu} \rightarrow -\sin \theta j^e_{\mu} + \cos \theta j^m_{\mu} = 0 \quad (16)$$

which results in non-symmetrical Maxwell’s equations

$$\partial_\nu F^{\mu\nu} = \frac{q}{e} j^e_{\mu} \quad \partial_\nu *F^{\mu\nu} = 0$$

with $q^2 = m^2 + e^2$. In summary, applying a dual transformation on symmetrical Maxwell equations (with non-zero $j^e_{\mu}$ and $j^m_{\mu}$), one can obtain non-symmetrical equations (with only one $j^e_{\mu}$ current) in which the resulting particles carry both electric and magnetic charges. For a particle with explicit magnetic charge to be observable, its ratio $m/e$ must be different than those of other particles.

### B. Global extended dual symmetry

A more consistent dual-symmetric formalism may be defined by the introduction of a new tensor field

$$C^{\mu\nu} = \partial^\mu C^\nu - \partial^\nu C^\mu \quad (17)$$

which is independent of $F_{\mu\nu}$ (Singleton 1995; Kato & Singleton 2002; Bliokh, Bekshaev & Nori 2013). $F_{\mu\nu}$ and $C^{\mu\nu}$ then transform as (see Eq. (12)):

$$F_{\mu\nu} \rightarrow \cos \theta F_{\mu\nu}^* + \sin \theta C^{\mu\nu}$$

$$C^{\mu\nu} \rightarrow -\sin \theta F_{\mu\nu}^* + \cos \theta C^{\mu\nu} \quad (18)$$

Transformations (18) reduce to (12) when the dual constraint is imposed

$$C^{\mu\nu} \rightarrow *F^{\mu\nu} \quad (19)$$

The dual-symmetric formalism acquires a particularly abbreviated form if we introduce the complex Riemann-Silberstein four-potential $Z^{\mu}$ and the field tensor $G^{\mu\nu}$

$$Z^{\mu} = A^{\mu} + i C^{\mu} \quad G^{\mu\nu} = F^{\mu\nu} + i C^{\mu\nu} \quad (20)$$

With these definitions, instead of the Lagrangian density (11) we may define

$$\mathcal{L}_0 = -\frac{1}{8} G^{\mu\nu} G^{\mu\nu}_{\mu\nu} \quad (20)$$

with the duality constraint $*G^{\mu\nu} = -i G^{\mu\nu}$, which reduces Maxwell’s equations now to a single equation

$$\partial_\nu G^{\mu\nu} = 0 \quad (21)$$

The dual transformation becomes now a simple $U(1)$ gauge transformation

$$Z^{\mu} \rightarrow e^{-i \theta} Z^{\mu} \quad G^{\mu\nu} \rightarrow e^{-i \theta} G^{\mu\nu} \quad (22)$$
The Lagrangian density \( \mathcal{L} \) is invariant under this transformation. From Noether theorem the conserved current \( J^\mu \) may be obtained
\[
J^\mu = \frac{1}{2} \text{Im} (G^a_{\mu\nu} Z^{a}_\nu) ; \quad \partial_{\mu} J^\mu = 0 ; \tag{23}
\]
in this expression the second equation represents helicity conservation in Maxwell’s equations.

C. Dark matter

The extension of the previous treatment to include DM is straightforward. To this end, we introduce a complex four-vector current
\[
J^\mu = j^\mu + i j^\mu \tag{24}
\]
where \( j^\mu \) represents the Standard Model current densities, associated to quarks and leptons, and \( j^\mu \) describes current densities of DM fermions only accessible by the extra gauge field \( C^\mu \). Extended dual symmetry transformations which include DM may be synthesized as
\[
Z^{\mu} \rightarrow e^{-i \theta} Z^{\mu} ; \quad C^{\mu\nu} \rightarrow e^{-i \theta} G^{\mu\nu} ; \quad J^{\mu} \rightarrow e^{-i \theta} J^{\mu} . \tag{25}
\]
The corresponding Lagrangian density may be written as
\[
\mathcal{L} = -\frac{1}{8} C^{\mu\nu} C^{*}_{\mu\nu} + \frac{1}{2} \left[ J^{\mu} Z^{\mu} + J^{*}_{\mu} Z^{\mu} \right] , \tag{26}
\]
where the first term in \( \mathcal{L} \) is the free field Lagrangian \( \mathcal{L}_0 \) and the second and third terms correspond to the interaction Lagrangian density \( \mathcal{L}_{\text{int}} \).

III. DUAL DARK MATTER THEORY

In the following we study the consequences of promoting the extended global extended version of dual symmetry (DuSy) to a local gauge symmetry, which always come together with some boson gauge field. This implies to make the substitution \( \theta \rightarrow g \theta(x) \) in Eq. \( \mathcal{L} \), where \( g \) represents a coupling constant. Infinitesimal variations of the \( Z^{\mu} \) field and its partial derivatives result in
\[
\delta Z^{\mu} = -ig \theta Z^{\mu} ; \quad \delta(\partial_\nu Z^{\mu}) = -ig \partial_\nu (\theta Z^{\mu}) ; \quad \delta Z^{*\mu} = ig \theta Z^{*\mu} ; \quad \delta(\partial_\nu Z^{*\mu}) = ig \partial_\nu (\theta Z^{*\mu}) . \tag{27}
\]
From these calculations it is straightforward to show that
\[
\delta G_{\mu\nu} = -ig \theta G_{\mu\nu} - ig \left[ (\partial_\nu \theta) Z^{\mu}_\nu - (\partial_\nu \theta) Z^{\mu}_\nu \right] ; \quad \delta G^{*}_{\mu\nu} = ig \theta G^{*}_{\mu\nu} + ig \left[ (\partial_\nu \theta) Z^{*\mu}_\nu - (\partial_\nu \theta) Z^{*\mu}_\nu \right] . \tag{28}
\]
To examine whether the local DuSy is a symmetry of the gauge field Lagrangian, the variation of \( \mathcal{L}_0 \) in \( \mathcal{L} \) must result in zero. This is easily calculated
\[
\delta \mathcal{L}_0 = -\frac{1}{8} \left( \delta G_{\mu\nu} G^{*\mu\nu} + G_{\mu\nu} \delta G^{*\mu\nu} \right) = \frac{ig}{4} \left( \partial^\nu \theta \right) \left[ Z^{\nu\mu} G^{*}_{\mu\nu} - Z^{*\nu\mu} G_{\mu\nu} \right] , \tag{29}
\]
revealing that \( \delta \mathcal{L}_0 \neq 0 \).

To assure that this symmetry is preserved, we define a counter-term \( \mathcal{L}_1 \) and a new complex field \( W^{\mu} \), such that \( \delta(\mathcal{L}_0 + \mathcal{L}_1) = 0 \), resulting in
\[
\mathcal{L}_1 = \frac{g}{4} \left[ W^{\mu\nu} G^{*}_{\mu\nu} + W^{\mu\nu*} Z^{\nu\mu} G_{\mu\nu} \right] . \tag{30}
\]
The complex field \( W^{\mu} \) is subject to the following transformation properties under local DuSy
\[
W^{\mu} \rightarrow W^{\mu} - i \partial^\mu \theta ; \quad W^{\mu*} \rightarrow W^{\mu*} + i \partial^\mu \theta . \tag{31}
\]
The variation \( \delta \mathcal{L}_1 \) results in
\[
\delta \mathcal{L}_1 = -\frac{ig}{4} \left( \partial^\mu \theta \right) \left[ Z^{\nu\mu} G^{*}_{\mu\nu} - Z^{*\nu\mu} G_{\mu\nu} \right] + \frac{ig^2}{4} \left( \partial^\mu \theta \right) \left[ W^{\mu\nu} Z^{\nu\mu} G_{\mu\nu} - W^{\mu\nu*} Z^{*\nu\mu} G_{\mu\nu} \right] + \frac{ig^2}{4} \left( \partial^\mu \theta \right) \left[ Z^{\mu\nu*} W^{\nu\mu} Z^{\nu\mu} G_{\mu\nu} - Z^{*\mu\nu} W^{\nu\mu*} Z^{\nu\mu} G_{\mu\nu} \right] . \tag{32}
\]
The first term in \( \delta \mathcal{L}_1 \) cancels \( \delta \mathcal{L}_0 \), but two extra terms proportional to \( g^2 \) appear. Now in order to cancel the second term in \( \delta \mathcal{L}_1 \), a new a counter-term \( \mathcal{L}_2 \) must be introduced:
\[
\mathcal{L}_2 = -\frac{g^2}{4} W^{\mu\nu} Z^{\nu\mu} Z^{\mu\nu} . \tag{33}
\]
The variation \( \delta \mathcal{L}_2 \) produces
\[
\delta \mathcal{L}_2 = -\frac{g^2}{4} \left[ (\delta W^{\mu}) W^{*\nu} Z^{\nu\mu} Z^{\mu\nu} - W^{\mu}(\delta W^{\mu*}) Z^{\nu\mu} Z^{\mu\nu} - W^{\mu*}(\delta W^{\mu}) Z^{\nu\mu} Z^{\mu\nu} \right] . \tag{34}
\]
It is easy to prove that
\[
\delta \left( Z^{\mu\nu*} \right) = \left( \delta Z^{\mu\nu*} \right) Z^{\nu\mu} + \left( \delta Z^{\nu\mu} \right) \left( \delta Z^{\nu\mu} \right) = -i g \theta Z^{\nu\mu} Z^{\nu\mu} + i g \theta Z^{\nu\mu} Z^{\nu\mu} = 0 ,
\]
which shows that the last term in Eq. \( \delta \mathcal{L}_2 \) is zero. Using Eq. \( \delta \mathcal{L}_2 \), one obtains for the remaining terms of Eq. \( \delta \mathcal{L}_2 \),
\[
\delta \mathcal{L}_3 = \frac{g^2}{8} \left[ W^{\mu\nu} W^{\nu\mu} Z^{\nu\mu} + W^{\nu\mu} W^{\mu\nu} Z^{\nu\mu} \right] . \tag{36}
\]
The variation of the first term in Eq. \( \delta \mathcal{L}_3 \) results in
\[
\delta (W^{\mu\nu*} Z^{\nu\mu}) = i \partial_\mu W^{\nu\mu*} Z^{\nu\mu} - i \partial_\nu W^{\nu\mu*} Z^{\nu\mu} + W^{\nu\mu*} \delta (Z^{\nu\mu}) . \tag{37}
\]
From \( \delta \mathcal{L}_3 \), the last term in Eq. \( \delta \mathcal{L}_3 \) is zero, resulting in
\[
\delta (W^{\mu\nu*} Z^{\nu\mu}) = i \partial_\mu W^{\nu\mu*} Z^{\nu\mu} - i \partial_\nu W^{\nu\mu*} Z^{\nu\mu} . \tag{38}
\]
The transformation of the second term in Eq. (36) by exchanging \( \mu \leftrightarrow \nu \)

\[
\delta (W^\mu W^\nu Z_\mu^* Z_\nu^*) = \delta \theta \left( W^\mu Z_\mu^* Z_\nu \right) - \delta \theta \left( W^\nu Z_\mu^* Z_\nu^* \right) .
\]

(39)

Combining (36), (38) and (39) allows one to calculate \( \delta \mathcal{L}_3 \)

\[
\delta \mathcal{L}_3 = -\frac{i g^2}{4} \left( \delta \theta \right) |Z_i^* W^\nu - Z_i W^\nu Z_i^*|^2.
\]

(40)

which cancels the third term in (32). The total invariant DuSy Lagrangian corresponds to the sum \( \mathcal{L}_{\text{DuSy}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 \), which can be written in a very compact form

\[
\mathcal{L}_{\text{DuSy}} = -\frac{1}{8} |G_{\mu \nu} - g K_{\mu \nu}|^2 + m_2^2 |Z_\mu|^2 - \frac{1}{2} F_{W_{\mu \nu}} F_{W_{\mu \nu}}^* + |m_W W_\mu - \delta \mu |^2 .
\]

(41)

where

\[
K_{\mu \nu} = Z_i^* W_{\mu -} - Z_i W_{\mu -} ; \ F_{W_{\mu \nu}} = \partial_\nu (W_{\mu -} - i \partial_\mu ) - \partial_\mu (W_{\mu -} - i \partial_\mu ) = F_{W_{\mu \nu}} .
\]

(42)

The transformation of \( F_{W_{\mu \nu}} \) under DuSy transformation (31) is invariant, as can be seen

\[
F_{W_{\mu \nu}} \rightarrow \partial_\nu (W_{\mu -} - i \partial_\mu ) - \partial_\mu (W_{\mu -} - i \partial_\mu ) = F_{W_{\mu \nu}} ,
\]

(43)

which proves the invariance of \(-\frac{1}{2} F_{W_{\mu \nu}} F_{W_{\mu \nu}}^* \) term in (41). The two mass terms where included in (41) for the \( Z_\mu \) and \( W_\mu \) bosons, which at a first glance seem to break the DuSy symmetry. The \( m_Z \) mass term depends on \( |Z_\mu|^2 \) which is trivially invariant under the dual transformation. The invariance of the \( m_W \) mass term is not so trivial. It requires the introduction of a complex \( \sigma \) Stueckelberg field in (41). Stueckelberg’s wonderful trick relies in the introduction of an extra scalar field \( \sigma \) (in our case a complex scalar), in addition to the four components \( W_\mu \). This results in a total of five fields, to describe coherently the three polarizations of a massive vector field. The Stueckelberg mechanism, not only manifests Lorentz covariance, but also, gauge invariance. In this sense the Stueckelberg field restores the gauge symmetry which had been broken by the mass term (Ruegg & Ruiz-Altaba 2004), (Körs & Nath 2004, 2005), (Feldman, Körs & Nath 2007), (Cheung & Yuan 2007), (Zhang, Wang, & Wang 2008). Under dual transformation \( W_\mu \) transforms according to (31) and \( \sigma \) as

\[
\sigma \rightarrow - \sigma + i m_W \theta .
\]

(44)

Under the transformations (31) and (44), the mass term of the \( W_\mu \) field transforms as

\[
m_W W_\mu - \delta_\mu \sigma \rightarrow m_W (W_\mu - i \partial_\mu ) - \partial_\mu (\sigma - i m_W \theta ) = m_W W_\mu - \delta_\mu \sigma .
\]

(45)

proving its invariance under DuSy transformation. The \( W_\mu \) and \( \sigma \) fields decouple if one introduces an extra gauge fixing term to the Lagrangian (41)

\[
\mathcal{L}_{gf} = -\frac{1}{\xi} |\partial_\mu W^\mu + \xi m_W \sigma|^2 .
\]

(46)

Then, the \( \sigma \) field decouples as we can see by the following development

\[
| m_W W_\mu - \delta_\mu \sigma |^2 + \mathcal{L}_{gf} = m_W^2 |W_\mu|^2 + \frac{1}{\xi} |\partial_\mu W^\mu|^2 - m_W (W_\mu \partial_\mu \sigma + W_\mu^* \partial_\mu \sigma ) + m_W (W_\mu \partial_\mu \sigma + W_\mu^* \partial_\mu \sigma ) + |\partial_\mu \sigma|^2 - \xi m_W^2 |\sigma|^2 .
\]

(47)

The mixing term \( \delta_\mu W^\mu \sigma \) that appeared in \( \mathcal{L}_{gf} \) cancels the corresponding mass term. This leads to the decoupling of the auxiliary complex scalar \( \sigma \) and the vector field \( W_\mu \). A mass proportional to a random parameter \( \sqrt{\xi} \) is then given to the unphysical \( \sigma \) field which has no influence on the vector field \( W_\mu \). Finally the complete Lagrangian in Dual DM theory is given by

\[
\mathcal{L}_{\text{DDM}} = \mathcal{L}_{\text{DuSy}} + \mathcal{L}_{gf} + \frac{1}{2} \left( J^\mu Z_\mu^* + J^*_{\mu} Z^\mu \right) .
\]

(48)

As we have seen, promoting dual symmetry to a local symmetry for the the complex four-potential \( Z_\mu^* \) implies in the inclusion of an extra complex vector field \( W_\mu \) with a complex gauge transformation. Our conjecture is the following: an interesting and possible connection of this dark sector with the SM can be established if one assumes that \( W_\mu \) represents the electroweak charged gauge bosons \( W_\mu^\pm \). This assumption is not in contradiction with the known fact that \( W_\mu^\pm \) is part of the \( SU(2) \) electroweak symmetry. As a novelty, we assume that \( W_\mu^\pm \) has an additional dual symmetry associated to \( Z_\mu^* \).

A. Perspectives in a new CP violation scenario

If the extended dual symmetry is a fundamental symmetry of physics, then our tentative conjecture to interpret \( W_\mu \) as the weak interaction charged vector field \( W_\mu^\pm \) leads us to speculate about a possible extra CP violation in \( W_\mu \) in electroweak interaction. In the SM, quark mixing is the source of CP violation. The charged currents interactions for quarks, i.e., the \( W_\mu \) interactions, are given by

\[
- \mathcal{L}_{W^\pm} = \sqrt{\frac{2}{3}} \bar{u}_L \gamma^\mu (V_{CKM})_{ij} d_{Lj} W_\mu^\pm + \text{h.c.} ,
\]

(49)

where the The Cabibbo-Kobayashi-Maskawa mixing matrix is defined as

\[
V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} .
\]

(50)

While a general \( 3 \times 3 \) unitary matrix depends on three real angles and six phases, the freedom to redefine the phases of the quark mass eigenstates can be used to remove five of the phases, leaving a single physical phase, the Kobayashi-Maskawa phase, that is responsible for all CP violation in meson decays in the Standard Model. As a consequence of this
Consider in the following particles $A$ and $B$ and their antiparticles $\bar{A}$ and $\bar{B}$ for which to the general symmetry transformation process $A \rightarrow B$ corresponds an antiparticle transformation symmetry process $\bar{A} \rightarrow \bar{B}$. We denote the amplitudes of these transformation processes as $M$ and $\bar{M}$ respectively. If CP violation occurs in these processes, these terms must correspond to the same complex number. We can separate the magnitudes and phases associated to these processes by writing $M = |M|e^{i\phi}$. If a phase term is introduced from the CKM matrix, call it $e^{i\theta}$, then we have the two amplitudes written as: $\bar{M} = |M|e^{i\theta}; M = |M|e^{i\phi}e^{-i\theta}$. Now, consider that there are two different decay channels for $A \rightarrow B$: $M = |M_1|e^{i\theta_1}e^{i\phi_1} + |M_2|e^{i\theta_2}e^{i\phi_2}; \bar{M} = |M_1|e^{i\theta_1}e^{-i\phi_1} + |M_2|e^{i\theta_2}e^{-i\phi_2}$. Combining these expressions in a CP violation scenario we get
\[ |M|^2 - |\bar{M}|^2 = -4|M_1||M_2|\sin(\theta_1 - \theta_2)\sin(\phi_1 - \phi_2), \]
where a complex phase gives rise to processes that proceed at different rates for particles and antiparticles. As a test of this picture, the presence of the complex $Z'_\mu$ could lead to an extra phase dependence that in future calculations could be evaluated. Another test could be, for instance, the one related to a very similar SM interaction, with top and bottom quarks, described by the corresponding $Wtb$ vertex. In this case, one can write the equivalent interaction of the $Z'_\mu$ field with top and bottom quarks as (Najafabadi 2010):
\[ -\mathcal{L} = \frac{g}{\sqrt{2}} \tilde{\tau}_\mu^\nu (a_L P_L + a_R P_R) b Z'_\mu + \text{h.c.} \] (51)
where $P_L$ ($P_R$) are the left-handed (right-handed) projection operators; $a_L$ and $a_R$ are the complex coefficients, where CP violating effects will appear. This interaction of the complex $Z'_\mu$ with top and bottom quarks consists of $V - A$ and $V + A$ structures and could produce an electric dipole moment for the top quark at one loop level, considering the Lagrangian defined in (51). The relevant Feynman diagrams are shown in figures (1). This could be one of the many scenarios where this complex $Z'_\mu$ theory could be tested.

IV. CONCLUSIONS

In this paper we have studied the consequences of extending dual symmetry to include a generic $C_\mu$ vector field as a dual partner of the photon $A_\mu$ and represented these fields by a complex Riemann-Silberstein-like four-potential $Z'_\mu$. The promotion of dual symmetry to a local symmetry for the the complex four-potential $Z'_\mu$ implies in the inclusion of an extra complex vector field $W_\mu$ with a complex gauge transformation. A Dual DM Lagrangian $\mathcal{L}_{DDM}$ is obtained from the general DuSy invariant Lagrangian $\mathcal{L}_{DuSy}$. Our tentative conjecture is to interpret $W_\mu$ as the actual weak interaction charged $W_\mu^\pm$, lead us to speculate about a possible extra CP violation scenarios for future calculations.

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As is well known, the CP transformation combines charge conjugation, represented by the operator $C$ with the parity transformation symbolized by $P$. Under charge transformation $C$, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, as for instance $Q \rightarrow -Q$ in case of electromagnetic charge. Under parity operation $P$, the handedness of space is reversed, $\vec{x} \rightarrow -\vec{x}$. In this case, a left-handed electron for instance $e_{-L}^-$ is transformed under CP into a right-handed positron, $e_{+R}^+$. CP could be violated also by the strong interaction. However, the smallness of the non-perturbative parameter associated to the strength CP violation in QCD, $\theta_{QCD}$, constitutes a theoretical puzzle, known as the strong CP problem.