Massive String Theories From M-Theory and F-Theory

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ABSTRACT

The massive IIA string theory whose low energy limit is the massive supergravity theory constructed by Romans is obtained from M-theory compactified on a 2-torus bundle over a circle in a limit in which the volume of the bundle shrinks to zero. The massive string theories in 9-dimensions given by Scherk-Schwarz reduction of IIB string theory are interpreted as F-theory compactified on 2-torus bundles over a circle. The M-theory solution that gives rise to the D8-brane of the massive IIA theory is identified. Generalisations of Scherk-Schwarz reduction are discussed.
There is a massive version of the ten dimensional type IIA supergravity due to Romans [1] and it has long been a mystery as to whether it has an eleven dimensional origin, in which the mass might arise from an explicit mass in eleven dimensions, or from a parameter in the dimensional reduction ansatz, as in Scherk-Schwarz dimensional reduction in which the fields have non-trivial dependence on the coordinates of the internal dimensions [2]. In [3], it has been argued, subject to certain assumptions, that no covariant massive deformation of 11-dimensional supergravity is possible, which would mean that the massive IIA supergravity cannot come from a conventional reduction of such a massive theory. Dimensionally reducing the Romans theory on a circle gives a massive 9 dimensional theory which can also be obtained from the type IIB theory by a Scherk-Schwarz reduction [4], but the Romans theory cannot be obtained by a Scherk-Schwarz reduction of 11-dimensional supergravity; a different 10-dimensional massive supergravity theory, for which there is no action, was proposed in [5], and obtained via a Scherk-Schwarz reduction of 11-dimensional supergravity in [16]. In [7], the Romans supergravity was lifted to a massive deformation of 11-dimensional supergravity where the terms in the 11-dimensional action depending on the mass parameter $m$ also depend explicitly on the Killing vector used in the dimensional reduction to 10-dimensions, and so this 11-dimensional theory is not fully covariant.

The IIA supergravity is the field theory limit of the IIA superstring, and the strong coupling limit of the IIA superstring is M-theory, which has 11-dimensional supergravity as its field theory limit. There is a massive version of type IIA string theory [8] whose field theory limit is the Romans supergravity theory (see for instance [4]), and the question arises as to how this massive IIA string theory arises from M-theory. Our purpose here is to argue that although the Romans supergravity theory may not be derivable from 11-dimensional supergravity, or any covariant massive deformation thereof, the massive IIA superstring, whose low energy limit is the Romans theory, can be obtained from M-theory.

The type IIB supergravity theory also cannot be obtained from 11-dimensional supergravity, but the type IIB string theory can be obtained from M-theory by
compactifying on a 2-torus and taking a limit in which the area of the torus tends
to zero while the modulus $\tau$ tends to a constant, the imaginary part of which is the
string coupling constant of the IIB string theory [9]. The massive IIA string theory
compactified on a circle of radius $R$ is T-dual to a Scherk-Schwarz compactification
of the IIB superstring on a circle of radius $1/R$, with mass-dependent modifications
of the usual T-duality rules [4]. Thus the massive IIA string can be obtained from
M-theory by first reducing on a 2-torus that shrinks to zero size to obtain the IIB
string, and then using a ‘twisted’ T-duality to obtain the massive IIA string, by
making a Scherk-Schwarz reduction on a circle and then shrinking the radius to
zero size. Moreover, we shall argue that the Scherk-Schwarz compactification of
the IIB superstring has a natural formulation in terms of F-theory. The Scherk-
Schrödinger reduction of the IIB string theory can then be obtained from a limit of a
compactification of M-theory, using the relation between F-theory and M-theory,
and it will be shown that the massive IIA string can be obtained by reducing
M-theory on a torus bundle over a circle and taking a limit in which the bundle
shrinks to zero size, with all three radii tending to zero. It will be seen that this
relates the D8-brane, which only occurs in the IIA string with non-vanishing mass,
to a brane-like solution of M-theory, which might be thought of as an M9-brane,
and to a related 12-dimensional F-theory ‘solution’.

The Scherk-Schwarz mechanism and its generalisations [2,10-17] introduces
mass parameters into toroidal compactifications of supergravities and string theo-
ries. If the original theory has a global symmetry $G$ acting on fields $\phi$ by $\phi \rightarrow g(\phi)$,
then in a generalised Scherk-Schwarz reduction or twisted reduction the fields are
not independent of the internal coordinates, but are chosen to depend on the torus
coordinates $y$ through an ansatz

$$\phi(x^\mu, y) = g_y(\phi(x^\mu))$$  \hspace{1cm} (1)

for some $y$-dependent symmetry transformation $g_y = g(y)$ in $G$. In many cases this
leads to a spontaneous breaking of supersymmetry [2], while in others it results
in the gauging of certain symmetries of the conventionally reduced theory, and 
the introduction of a scalar potential and cosmological constant [13,14,17]. Here, 
we will restrict ourselves to compactifications on a circle, with periodic coordinate 
y \sim y + 1. For example, for reducing a theory with a linearly realised \( U(1) \) 
symmetry on a circle, a massless field \( \phi \) of charge \( q \) can be given a \( y \) dependence 
\( \phi(x, y) = e^{2\pi iqmy}\phi(x) \), so that the field \( \phi(x) \) is given a mass of \( qm \).

The map \( g(y) \) is not periodic, but has a monodromy

\[
\mathcal{M}(g) = g(1)g(0)^{-1}
\]  

(2)

for some \( \mathcal{M} \) in \( G \). We will consider here maps of the form

\[
g(y) = \exp(My)
\]  

(3)

for some Lie algebra element \( M \), so that the monodromy is

\[
\mathcal{M}(g) = \exp M
\]  

(4)

Then

\[
M = g^{-1}\partial_y g
\]  

(5)

is proportional to the mass matrix of the dimensionally reduced theory and is 
independent of \( y \) [17].

The next question is whether two different choices of \( g(y) \) give inequivalent 
theories. The ansatz breaks the symmetry \( G \) down to the subgroup preserving 
\( g(y) \), consisting of those \( h \) in \( G \) such that \( h^{-1}g(y)h = g(y) \). Acting with a general 
constant element \( k \) in \( G \) will change the mass-dependent terms, but will give a \( D-1 \) 
dimensional theory related to the original one via the field redefinition \( \phi \rightarrow k(\phi) \). 
This same theory could have been obtained directly via a reduction using \( k^{-1}g(y)k \) 
instead of \( g(y) \), so two choices of \( g(y) \) in the same conjugacy class give equivalent 
reductions (related by field-redefinitions). As a result, the reductions are classified 
by conjugacy classes of the mass-matrix \( M \).
The map \( g(y) \) is a local section of a principal fiber bundle over the circle with fibre \( G \) and monodromy \( \mathcal{M}(g) \) in \( G \). Such a bundle is constructed from \( I \times G \), where \( I = [0,1] \) is the unit interval, by gluing the ends of the interval together with a twist of the fibres by the monodromy \( \mathcal{M} \). Two such bundles with monodromy in the same \( G \)-conjugacy class are equivalent. Only those monodromies \( \mathcal{M} \) that can be written as \( e^M \) for some \( M \) arise in this way, and for those monodromies that are in the image of the exponential map, there are in general an infinite number of possible choices of mass-matrix. Indeed, if \( M, M' \) are two such mass matrices for a given monodromy such that \( e^M = e^{M'} = \mathcal{M} \), then \( e^M e^{-M'} = 1 \) and so there is a \( \lambda \) satisfying \( e^\lambda = 1 \) such that \( M - M' - \frac{1}{2}[M, M'] + \ldots = \lambda \). The general solution of \( e^M = \mathcal{M} \) is then of the form \( M = M' + \lambda + \frac{1}{2}[M', \lambda] + \ldots \) where \( M' \) is a particular solution and \( \lambda \) is any solution of \( e^\lambda = 1 \). The algebra elements \( \lambda \) with \( e^\lambda = 1 \) fall into adjoint orbits, as, for any group element \( g \), \( \lambda' = g\lambda g^{-1} \) satisfies this condition if \( \lambda \) does. The set of all Lie algebra elements \( \lambda \) with \( e^\lambda = 1 \) is given by the adjoint orbits of all points in the dual of the weight lattice of the maximal compact subalgebra \( H \) of \( G \), sometimes called the integer lattice.

Of particular interest are the \( D \)-dimensional supergravity theories with rigid duality symmetry \( G \) and scalars taking values in \( G/H \) \cite{18,19}, which can be Scherk-Schwarz-reduced on a circle to \( D - 1 \) dimensions. The reduction requires the choice of a map \( g(y) \) of the form (3) from \( S^1 \) to \( G \), which then determines the \( y \)-dependence of the fields through the ansatz (1), and any choice of Lie algebra element \( M \) is allowed. In the quantum theory, the symmetry group \( G \) is broken to a discrete subgroup \( G(\mathbb{Z}) \) \cite{20}. A consistent twisted reduction of a string or M-theory, whose low-energy effective theory is the supergravity theory considered above, then requires that the monodromy be in the U-duality group \( G(\mathbb{Z}) \). (In the classical supergravity theory, any element of \( G \) can be used as the monodromy.) Then the choice of \( M \) is restricted by the constraint that \( e^M \) should be in \( G(\mathbb{Z}) \). As before, if \( M = kM'k^{-1} \) where \( k \) is in \( G \), the theories are related by field redefinitions. However, only if \( k \) is in \( G(\mathbb{Z}) \) will the redefinition preserve the charge lattice \cite{20}. Once the conventions for the definitions of charges are fixed, it is necessary to restrict to conjugation
by elements of $G(\mathbb{Z})$, and so reductions are specified by $G(\mathbb{Z})$ conjugacy classes of maps (3) with monodromy (4) in $G(\mathbb{Z})$.

Here we will concentrate on the examples relevant to the massive IIA superstring. The type IIB supergravity theory has $G = SL(2, \mathbb{R})$ global symmetry and any element $M$ of the $SL(2, \mathbb{R})$ Lie algebra can be used in the ansatz (1),(3) to give a Scherk-Schwarz reduction to 9-dimensions to obtain a class of massive 9-dimensional supergravity theories. Such reductions for particular elements of $SL(2, \mathbb{R})$ were given in [4,15,16], and the general class of $SL(2, \mathbb{R})$ reductions of IIB supergravity was obtained in [17]. Note that this ansatz does not allow the monodromy to be an arbitrary $SL(2, \mathbb{R})$ group element, but requires it to be in the image of the exponential map. Acting with an $SL(2, \mathbb{R})$ transformation leaves the mass-independent part of the theory unchanged but changes the mass matrix by $SL(2, \mathbb{R})$ conjugation, and so there are three distinct classes of inequivalent theories, corresponding to the hyperbolic, elliptic and parabolic $SL(2, \mathbb{R})$ conjugacy classes, represented by monodromy matrices of the form

\[
\begin{pmatrix}
a & 0 \\
0 & a^{-1}
\end{pmatrix}, \quad \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}, \quad \begin{pmatrix}
1 & a \\
0 & 1
\end{pmatrix}
\]

(6)

respectively. The details of the reduction of the bosonic sector of the supergravity theory for general $M$ were given in [17].

In the quantum theory, only an $SL(2, \mathbb{Z})$ symmetry remains [20]. The quantum-consistent Scherk-Schwarz reductions of this theory to 9 dimensions are those for which the monodromy is in $SL(2, \mathbb{Z})$, and are defined up to $SL(2, \mathbb{Z})$ conjugacy. The fact that the monodromy must be in $SL(2, \mathbb{Z})$ implies a quantization of the masses.

The IIB supergravity scalars take values in $SL(2, \mathbb{R})/U(1)$ and can be represented by a complex scalar $\tau = C_0 + i e^{-\Phi}$ transforming under $SL(2)$ by fractional linear transformations, so that $g \in SL(2)$ acts as

\[
g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : \tau \rightarrow \tau g = \frac{a\tau + b}{c\tau + d}
\]

(7)
The Scherk-Schwarz ansatz, \( \tau(x,y) = \tau(x)g(y) \), gives a complex scalar \( \tau(x)g(y) \) of the reduced theory and for fixed \( x \), \( \tau(x)g(y) \) depends on \( y \) and is a section of the bundle over the circle with fibre \( SL(2,\mathbb{R})/U(1) \) obtained as a quotient of the principle \( SL(2,\mathbb{R}) \) bundle by \( U(1) \).

For the IIB string theory, the monodromy must be restricted to lie in \( SL(2,\mathbb{Z}) \). If \( g(y) \) has \( SL(2,\mathbb{Z}) \) monodromy, the local section \( \tau(y) = \tau_g(y) \) can be used to construct a torus bundle over the circle in which \( \tau \) is the \( T^2 \) modulus, and depends on the position on the circle. The total space of the torus bundle is a 3-dimensional space \( B \) with metric

\[
ds^2_B = R^2 dy^2 + \frac{A}{Im(\tau)} |dz_1 + \tau(y)dz_2|^2
\]

where the fibre is a \( T^2 \) with real periodic coordinates \( z_1, z_2, z_i \sim z_i + 1 \), constant area modulus \( A \) and complex structure \( \tau(y) \), which depends on the coordinate \( y \) of the circular base space, and this has circumference \( R \). The Scherk-Schwarz reduction of the IIB superstring with an ansatz \( \tau(y) = \tau_g(y) \) associated with a particular torus bundle \( B \) is precisely what is meant by F-theory compactified on the three dimensional total space \( B \) [21-24].

This generalises; for theories in which the global symmetry is \( G = SL(n,\mathbb{R}) \) with quantum symmetry \( SL(n,\mathbb{Z}) \), a twisted reduction on an \( m \)-torus in which all monodromies are in \( SL(n,Z) \) corresponds to a torus bundle with fibres \( T^m \) over a base \( T^m \). For \( m = 1 \), this gives a \( T^m \) bundle over a circle. Certain torus bundles over a circle are also circle bundles over a torus, and the latter was the interpretation used in [16]. However, the torus bundle over a circle is both more general and more useful, as it has an F-theory interpretation. For example, the 7-dimensional maximal supergravity theory has \( G = SL(5,\mathbb{R}) \) symmetry, while the 7-dimensional type II string theory has \( SL(5,\mathbb{Z}) \) U-duality. The general twisted reduction from 7 to 6 dimensions would involve a map \( g(y) : S^1 \to SL(5,\mathbb{R}) \) with \( SL(5,\mathbb{Z}) \) monodromy, which is also the data for a \( T^5 \) bundle over \( S^1 \). Then the general \( SL(5) \) Scherk-Schwarz reduction can be re-interpreted as a reduction of
the $F'$-theory of [24] on a $T^5$ bundle over $S^1$. (The $F'$-theory is an analogue of F-theory, also in 12 dimensions, which can be compactified on spaces admitting a $T^5$ fibration [24].)

Any twisted reduction of the IIB string to 9 dimensions can be recast as the reduction of F-theory on a bundle $B$ which is a $T^2$ bundle over $S^1$. One can also consider compactifications of M-theory on $B$, and the two are related by fibre-wise duality as follows. For M-theory compactified on $B$ in which the $T^2$ fibres have a constant area $A$, the limit $A \to 0$ keeping the modulus $\tau(x, y)$ fixed gives F-theory compactified on $B$ with fixed torus area $A = 1$, say. For a trivial bundle, this follows from the fact that M-theory compactified on $T^2$ becomes, in the limit in which the torus shrinks to zero size, the IIB string theory, and the generalisation to non-trivial bundles follows from the adiabatic argument [25].

Consider the Scherk-Schwarz reduction using the map $S^1 \to SL(2, \mathbb{R})$

$$g(y) = \begin{pmatrix} 1 & my \\ 0 & 1 \end{pmatrix}, \quad M = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix}$$

so that (1) leads to the linear ansatz

$$\tau(x, y) = \tau(x) + my$$

(10)

The monodromy is

$$\mathcal{M} = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}$$

(11)

and in the quantum theory this must be in $SL(2, \mathbb{Z})$ so that $m$ must be an integer, and the mass is quantized, as it is proportional to $m$. This is precisely the reduction studied in [4], and is T-dual to the massive IIA string theory, with mass parameter $m$, conventionally compactified on $S^1$. The bundle $B$ has a metric given by (8),(10),
which takes the simple form

\[ ds^2 = dy^2 + (dz_1 + mydz_2)^2 + dz_2^2 \]  

(12)

if \( \tau_0 = i, A = R = 1 \). This 3-space \( B \) is also a circle bundle over a 2-torus with fibre coordinate \( z_1 \), base-space coordinates \( y, z_2 \) and connection 1-form \( A = mydz_2 \) [16].

The massive IIA string theory arises from M-theory as follows. Let \( B(A, R) \) be the the torus bundle over a circle of radius \( R \), where the torus has modulus \( \tau \) depending on the \( S^1 \) coordinate \( y \) through

\[ \tau = \tau_0 + my \]  

(13)

for some constant \( \tau_0 \), and \( y \)-independent area \( A \). Compactifying M-theory on \( B(A, R) \) and taking the limit \( A \to 0 \) gives F-theory compactified on \( B(1, R) \), or equivalently the Scherk-Schwarz reduction of the IIB string on a circle of radius \( R \) using the ansatz (10). This is T-dual to the massive IIA string with mass parameter \( m \) compactified on a circle of radius \( 1/R \), and so the uncompactified massive IIA string is obtained by taking the limit \( R \to 0 \). Putting this together, we obtain the massive IIA string by compactifying M-theory on \( B(A, R) \) and taking the zero-volume limit \( A \to 0, R \to 0 \). The bundle also depends on \( \tau_0 \) and \( m \), and is trivial if \( m = 0 \), in which case \( B \) is a 3-torus, and M-theory on a 3-torus indeed gives, in the limit in which the torus shrinks to zero size, the massless IIA string theory or M-theory, depending on the value of the string coupling. The IIB string coupling constant \( g_B \) is given by the imaginary part of \( \tau_0 \), \( g_B = 1/Im(\tau_0) \), and the coupling constant \( g_A \) for the T-dual IIA theory is related to this by \( g_A = g_B/R \), so that

\[ g_A = \frac{1}{Im(\tau_0)R} \]  

(14)

Then if \( Im(\tau_0) \to \infty \) as \( R \to 0 \) so that \( Im(\tau_0)R \) remains fixed, the massive IIA theory at finite string coupling (14) is obtained. The massive IIA string theory
can also be obtained from F-theory on $B(1, R)$ by taking the limit $R \to 0$, keeping $Im(\tau)R$ fixed.

The massive IIA supergravity theory doesn't have a Minkowski or (anti) de Sitter solution, and there is no maximally supersymmetric solution. There is a D8-brane solution which preserves half of the supersymmetries, however [4]. The string-frame metric is

$$ds^2 = H^{-1/2}d\sigma_{8,1}^2 + H^{1/2}dx^2$$

(15)

where $d\sigma_{p,1}^2$ is the $p + 1$ dimensional Minkowski metric on $\mathbb{R}^{p,1}$. There is an 8+1 dimensional longitudinal space and a one-dimensional transverse space with coordinate $x$. The function $H(x)$ is harmonic, $H'' = 0$, and the solution

$$H = \begin{cases} 
  c + m'|x| & \text{for } x < 0 \\
  c + m|x| & \text{for } x > 0 
\end{cases}$$

(16)

for some constant $c$ represents a domain wall at $x = 0$, separating regions with two different (integer) values of the mass parameter, $m$ and $m'$. If one of the longitudinal coordinates, $y$ say, is made periodic, a T-duality in the $y$-direction leads to the circularly symmetric IIB D7-brane solution of [4], with string-frame metric

$$ds^2 = H^{-1/2}d\sigma_{7,1}^2 + H^{1/2}(dx^2 + dy^2)$$

(17)

and

$$e^{-\phi} = H, \quad C'_0 = H'$$

(18)

where $\phi$ is the dilaton and $C_0$ is the RR scalar. In Einstein frame, the metric is

$$ds^2 = d\sigma_{7,1}^2 + H(dx^2 + dy^2)$$

(19)

Dimensional reduction in the $y$ direction of the D8-brane (15) or D7-brane (17) leads to the 7-brane solution [13] of the massive 9-dimensional theory (obtained by
twisted reduction of the IIB theory using (10)) with metric

\[ ds^2 = H^{-1/2}d\sigma_{7,1}^2 + H^{1/2}dx^2 \]  \hspace{1cm} (20)

Conventional dimensional reduction of 11-dimensional supergravity on a 2-torus gives massless 9-dimensional type II theory with scalars in the coset space \( \mathbb{R}^+ \times SL(2, \mathbb{R})/U(1) \), which is the moduli space of the torus [26]. A Scherk-Schwarz reduction of this to 8-dimensions using the ansatz (10) for the complex scalar in \( SL(2, \mathbb{R})/U(1) \) gives a massive type II supergravity in 8-dimensions [13] and this theory has a 6-brane solution [13] with metric

\[ ds^2 = H^{2/3} \left( H^{-1/2}d\sigma_{6,1}^2 + H^{1/2}dx^2 \right) \]  \hspace{1cm} (21)

However, this massive 8-dimensional theory arises directly from reduction from 11-dimensions on the torus bundle \( B \), and we will now check that the 6-brane solution arises from an 11-dimensional solution reduced on the torus bundle \( B \). The moduli \( \tau, A, R \) of the bundle become scalar fields in the dimensionally reduced theory, and for the 11-dimensional oxidation of the solution (21), these moduli can be expected to be functions of transverse coordinate \( x \). The 11-dimensional oxidation of (21) was given in [13,16,27], with metric

\[ ds^2 = d\sigma_{6,1}^2 + Hdx^2 + H(dy^2 + Adz_2^2) + AH^{-1}(dz_1 + mydz_2)^2 \]  \hspace{1cm} (22)

where \( A \) is a constant that can be absorbed into a rescaling of \( z_1, z_2 \). This can be rewritten in the form

\[ ds^2 = H^{1/2} \left( H^{-1/2}d\sigma_{6,1}^2 + H^{1/2}dx^2 \right) + ds_B^2 \]  \hspace{1cm} (23)

where \( ds_B^2 \) is a \( B \)-metric of the form (8),(10), but where the moduli \( \tau, R \) depend
on $x$ as well as $y$:

$$R = H^{1/2}, \quad \tau = my + iH,$$

(24)

The metric is of the form $\mathbb{R}^{6,1} \times M_4$ where $M_4$ is of the form $\mathbb{R} \times B$ with coordinates $x, y, z_1, z_2$ and Ricci-flat metric

$$ds^2 = Hdx^2 + ds_B^2$$

(25)

with the moduli of $B$ given by (24). The 11+1 dimensional space $\mathbb{R}^{7,1} \times M_4$ is Ricci-flat and is the F-theory ‘solution’ that gives rise to the Einstein-frame 7-brane solution (19), which can be reduced further to the 9-dimensional 7-brane (20). Note that for domain walls separating regions of mass $m, m'$, as in (16), then there are two different bundles $B, B'$ arising on either side of the wall, one with monodromy (11) and one with monodromy given by (11) with $m$ replaced by $m'$.

Now taking the limit in which the total spaces $B, B'$ shrink to zero size, the solution (23) becomes the D8-brane solution of the massive IIA string, while taking the limit in which the $T^2$ fibres shrink to zero size ($A \to 0$) gives the circularly symmetric D7-brane (17). This can be seen in a number of ways. For example, first dimensionally reducing in the $z_1$ direction and Weyl rescaling to obtain the IIA string-frame metric, (22) becomes the D6-brane solution

$$ds^2 = H^{-1/2}d\sigma^2_{6,1} + H^{1/2}(dx^2 + dy^2 + Adz_2^2)$$

(26)

where the harmonic function depends only on $x$, so that this can be thought of as a D6-brane ‘smeared’ over the $y$ and $z_2$ directions. Thus regarding $B$ as a circle bundle over $T^2$ with fibre coordinate $z_1$, we can shrink the fibre to obtain the smeared D6-brane solution of the IIA theory with charge proportional to $m$. Now the limit $A \to 0$ is obtained by T-dualising in the $z_2$ direction, using the rules of [26], gives the circularly symmetric D7-brane (17) of the IIB theory. A further T-duality in the $y$ direction gives the D8-brane solution (15). Then taking the limit
of (22) in which the $T^2$ fibres are shrunk is given by first reducing on $z_1$ to obtain (26) and then T-dualising in the $z_2$ direction to obtain the D7-brane (17), while the limit in which the total space shrinks is given by making a further T-duality in the $y$ direction to obtain the D8-brane (15).

In [28], it was argued that there should be an ‘M9-brane’ that gives rise to the D8-brane of the IIA theory, arising as a domain wall in M-theory, and in [29,30], such branes were considered further. In particular, in [30] it was shown that such branes could not be $SO(9,1)$ invariant, but that one of the directions was special, in the same way that the KK monopole solution giving rise to the D6-brane is not $SO(7,1)$ invariant, and has a special compact direction corresponding to the Taub-NUT fibre. The solution (22) is a domain wall solution of M-theory that gives the D8-brane of the massive IIA theory in the limit in which the 3-space $B$ shrinks to zero size, and so might be thought of as a type of M9-brane, with three special compact directions.

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