1. Introduction

Turbomachines driven by built-in motor, such as canned motor pumps (TEIKOKU Electric Mfg Co Ltd, 2015), compressors, and centrifugal blood pumps (Ranjit, 2008), are extremely important systems worldwide. However, the rotor dynamics of turbomachines change throughout their service life and also change with respect to internal and external conditions. Therefore, industry is investing heavily into realizing real-time health monitoring during rotation, with the aim of extending performance and maintaining the machines so that they can be operated with high stability, reliability, and durability for long periods.

With traditional inspection methods, the displacement, velocity, and acceleration of turbomachines are measured. However, the information may be insufficient for diagnosis and prognosis. Alternatively, using an excitation force to vibrate the rotor and measure the frequency response function (FRF) is a common method to monitor the dynamic characteristics, such as the natural frequency and damping ratio, of the rotor system, which can then be used to estimate the margin of stability and determine the origin of vibrations.

In FRF measurements, a few types of exciters are used. Conventionally, impact hammering and electromagnetic actuators (Aenis et al., 2002; Takahashi et al., 2012; Tsunoda et al., 2016, 2017a) are generally used, but are undesirable. Hammering a rotating body is not simple and is unrealistic because of safety. Electromagnetic exciters are able to generate the required sweep excitation force to measure the FRF from the displacement of the rotor, enabling faults to be detected
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(Aenis et al., 2002), estimates of the stability margin to be made (Tsunoda et al., 2016, 2017a), and eigenvalues to be measured (Takahashi et al., 2012). However, installation of electromagnetic actuators is inconvenient and expensive, increasing the production costs. Moreover, the motor needs a larger footprint and longer rotor-shaft. Therefore, a low-cost, space-saving, safe and real-time monitoring system for turbomachines is required.

Here, the built-in motor that supplies the torque for the turbomachine is considered as an excitation source. An extra radial force, called the unbalanced magnetic force (UMF), developed as a result of the eccentricity of the built-in motor, can be utilized. In a previous study, C. I. Lee showed the characteristics of the UMF of a brushless DC motor (Lee and Jang, 2008). Factors, such as the non-uniform magnetization of a permanent magnet (PM) and the eccentricity were considered. Farhad Rezaee-Alam has calculated and analyzed the UMF in eccentric surface-mounted permanent-magnet motors under various eccentricity conditions (Rezaee-Alam et al., 2016). In this paper, the UMF is utilized as an excitation force in order to measure the dynamic characteristics of a turbomachine.

We propose utilizing electromagnetic excitation in order to make FRF measurements as shown in Fig. 1. Although various types of built-in motors are used to drive turbomachinery, in this method, we assume the built-in motor to be a permanent magnet (PM) synchronous motor. In general, direct-current (DC) added to the d-axis is usually used for flux-weakening control (Mwasilu et al., 2016; Nguyen et al., 2013; Liu et al., 2018). We assume the PM motor has static eccentricity. An alternating-current (AC) is injected into the d-axis to generate a sinusoidal radial excitation force, which causes the PM bias magnetic flux (Zhang and Foo, 2015) between the stator and rotor core to vary, leading to changes in the UMF in the PM motor. The varying UMF causes the rotor to vibrate in the radial direction. By measuring the displacement signal using displacement sensors, while sweeping the d-axis current, the FRFs can be obtained. This method has some advantages. First, the cost can be reduced and the mechanical components, except the non-contact displacement sensors, can be maintained. Second, the method is safe and effective. To verify our proposed method under various experimental conditions, a test rig with a bearingless motor (BELM) having motor windings and suspension windings was used. The proposed method was verified by comparing the FRFs measured using the motor winding with reference FRFs measured using the suspension windings. Rotor centricity is mainly divided into static and dynamic eccentricity. In previous research (Tsunoda et al., 2018, 2019b), we discussed measurement of the FRF of a rotor system with dynamic eccentricity. The effect of the eccentricity and the rotational speed on the FRF was not evaluated.

In the study reported in this paper, we made FRF measurements of a rotor system with static eccentricity. The rest of this paper is organized into five sections. Section 2 introduces the principle of the proposed method, and we carry out a theoretical analysis of the radial excitation force generated when using a rotor with static eccentricity. Section 3 presents the experimental setup. Section 4 describes the experimental method including simulation of the rotation of a rotor with static eccentricity and FRF measurements under various experimental conditions. Section 5 summarizes the results of the proposed method under the various test conditions and further discusses the proposed method. Section 6 gives the conclusions.
2. Nomenclature

| Symbol | Description |
|--------|-------------|
| $A$    | Constant of fitted gain curve |
| $c$    | Bearing damping coefficient |
| $F_d$  | Magnetic motive force induced by the d-axis current |
| $F_{\text{motor},x}$ | Attractive force in the x-direction |
| $F_{\text{motor},y}$ | Attractive force in the y-direction |
| $F_{\text{pm}}$ | Magnetic motive force induced by the permanent magnet |
| $F_y$  | Radial force in the y-direction |
| $g_0$  | Nominal air gap |
| $I_{d0}$ | Bias d-axis current |
| $I_{\text{exc}}$ | Excitation current in suspension winding |
| $I_q$  | q-axis current |
| $I_y$  | Current in the suspension winding in the y-direction |
| $I_{\text{ref}}$ | Reference current in suspension winding in the y-direction |
| $I_{2u}$ | U-phase suspension current |
| $I_{2v}$ | V-phase suspension current |
| $I_{2w}$ | W-phase suspension reference current |
| $I_{2w\text{ref}}$ | W-phase motor reference current |
| $I_{B_u}$ | U-phase motor current |
| $I_{B_w}$ | W-phase motor current |
| $I_{B_w\text{ref}}$ | W-phase motor reference current |
| $k_d$  | Eccentricity-current-force coefficient of motor winding |
| $k_s$  | Stiffness of suspension winding |
| $L$    | Rotor length |
| $m$    | Rotor mass |
| $P_i$  | Permeance in each section |
| $R_i$  | Magnetic resistance of the air gap in each section |
| $t$    | Time |
| $X_{\text{ref}}$ | Reference position in the x-direction |
| $Y_{\text{ref}}$ | Reference position in the y-direction |
| $\varepsilon$ | Static rotor eccentricity in the x-direction |
| $\varepsilon_0$ | Initial rotor eccentricity |
| $\varepsilon_{\text{amp}}$ | Oscillation amplitude |
| $\theta$ | Initial angle between the centerline of one pair of PMs and x-axis |
| $\mu_{\text{pm}}$ | Permeability of the permanent magnet |
| $\phi_0$ | Rotational angle |
| $B_i$  | Magnetic flux density in each section |
| $f_n$  | Natural frequency of rotor system |
| $F_i$  | Attractive force induced by the motor in each section |
| $F_x$  | Radial force in the x-direction |
| $g$    | Air gap for a rotor with static eccentricity |
| $I_d$  | d-axis current |
| $I_{d1}$ | d-axis current amplitude |
| $I_{\text{sus}}$ | Amplitude of excitation current in suspension winding |
| $i_x$  | Current in the suspension winding in the x-direction |
| $I_{x\text{ref}}$ | Reference current in suspension winding in the x-direction |
| $I_{2u\text{ref}}$ | U-phase motor reference current |
| $I_{2w\text{ref}}$ | W-phase motor reference current |
| $k$    | Bearing stiffness |
| $k_i$  | Current-force coefficient of suspension winding |
| $k_w$  | Stiffness of motor winding |
| $L_{\text{pm}}$ | Thickness of the permanent magnet |
| $N_d$  | Number of turns in the d-axis winding |
| $R$    | Rotor radius |
| $R_{\text{pm}}$ | Magnetic resistance of each permanent magnet |
| $\nu$  | Angular frequency of the excitation current |
| $x$    | Radial displacement of the rotor in the x-direction |
| $y$    | Radial displacement of the rotor in the y-direction |
| $\varepsilon_1$ | Rotor deformation |
| $\varepsilon_{\text{st}}$ | Static rotor eccentricity in the bearingless motor |
| $\zeta$ | Damping ratio of rotor system |
| $\mu_0$ | Permeability of air gap |
| $\phi$ | Absolute angle |
| $\phi_{\text{m}}$ | Oscillation phase |
3. Theoretical analysis of the excitation force for a rotor with static eccentricity

3.1 Model for radial motion of a rigid spindle with static eccentricity

The equation of radial motion of a rigid rotor supported by elastic bearings can be expressed by Eq. (1):

\[
\begin{align*}
mx + cx + kx &= F_x \\
my + cy + ky &= F_y
\end{align*}
\]

(1)

where \(m\), \(k\), and \(c\) are the mass, stiffness and damping coefficient, respectively, \(x\) and \(y\) are the radial displacements of the rotor in the \(x\)- and \(y\)-directions, respectively. The supporting stiffness and damping are isotropic in the radial direction. \(F_x\) and \(F_y\) are the radial forces. The rigid rotor is assumed to rotate with static eccentricity.

3.2 Generation of radial excitation force

In the proposed method, an AC current is injected into the \(d\)-axis of the motor to generate a radial excitation force. The bias magnetic flux generated by the PMs is changed by the DC \(d\)-axis current to extend the speed range. A consequent-pole-type motor with four PMs is used in the rotor system. Figure 2 shows a cross-sectional view of the rotor, which has static eccentricity \(\epsilon\) in the \(x\)-direction. The black lines show the magnetic flux from the PMs and the red lines show the magnetic flux from the motor windings. As shown in Fig. 3, the line between the centers of one pair of PMs has an initial angle \(\theta\) with respect to the \(x\)-axis. The rotational angle \(\phi_0\) is obtained from the product of the rotational speed \(\Omega\) and time \(t\). The absolute angle \(\phi\) is obtained by adding these together, as in the following equation:

\[
\phi = \phi_0 + \theta
\]

(2)

As shown in Fig. 4, the motor model is divided into eight 45-degree sections, and the sections are labeled in turn in the counterclockwise direction. We assume that the magnetic flux is uniformly distributed within each section. In each section, the magnetic resistance of the air gap between the stator and rotor core is \(R_i\) \((i = 1, 2, \ldots, 8)\), the magnetic flux in the air gap is \(\psi_i\), and the attractive force induced by the motor is \(F_i\). The magnetic resistance of each permanent magnet can be calculated from the following Eq. (3):

\[
R_{pm} = 4L_{pm}/(\mu_{pm}RL\pi)
\]

(3)

where \(L_{pm}\) and \(\mu_{pm}\) are the thickness and permeability of the permanent magnet, respectively. \(R\) and \(L\) are the radius...
and length of the rotor, respectively.

The air gap for a rotor with static eccentricity is given by Eq. (4):

$$
g = g_0 \left\{ 1 - \frac{\epsilon}{g_0} \cos(\theta + \phi_0) \right\} \quad (4)
$$

where $\epsilon$ is much smaller than the nominal air gap $g_0 \ (1 \gg \epsilon/g_0)$. The permeance $P_i \ (i = 1, 2, \ldots, 8)$ in each section can be obtained from Eq. (5):

$$
P_i = \frac{\mu_0 R L}{g_0} \int_{(-\pi/8)+(i-1)\pi/4}^{(\pi/8)+(i-1)\pi/4} \left\{ 1 - \frac{\epsilon}{g_0} \cos(\theta + \phi_0) \right\} d\theta \quad (i = 1, 2, \ldots, 8)
$$

Based on the magnetic flux path in Fig. 2, the equivalent magnetic circuit is obtained as shown in Fig. 5. $F_{pm}$ and $F_d \ (= N_d I_d)$ represent the magnetic motive force induced by the permanent magnet and the d-axis current, respectively. The magnetic flux $\psi_i \ (i = 1, 2, \ldots, 8)$ in the air gaps in the respective sections is given by Eq. (6):

$$
\begin{align*}
&F_d \quad F_d \quad F_d \quad F_d \\
&R_1 \quad R_3 \quad R_5 \quad R_7 \quad R_2 \\
&D_{pm} \quad D_{pm} \quad D_{pm} \quad D_{pm} \\
&\psi_1 \quad \psi_3 \quad \psi_5 \quad \psi_7 \\
&\psi_2 \quad \psi_4 \quad \psi_6 \quad \psi_8
\end{align*}
$$

Fig. 5 The equivalent circuit for the magnetic flux path.
The magnetic flux is divided by the area to obtain the magnetic flux density $B_i = \frac{\psi_i}{R_i L_\pi 4/\pi}$. The attractive force $F_i = (R_i L_\pi 4/\pi) B_i^2 \mu_0$ can be calculated utilizing the magnetic flux density. Finally, the attractive forces in the x- and y-directions ($F_{motor,x}, F_{motor,y}$) can be obtained from the following equations:

\[
\begin{align*}
F_{motor,x} &= \sum_{i=1}^{8} F_i \cos \left( \frac{\pi}{4} (i - 1) + \phi_0 \right) \\
F_{motor,y} &= \sum_{i=1}^{8} F_i \sin \left( \frac{\pi}{4} (i - 1) + \phi_0 \right)
\end{align*}
\]  

(7)

The above calculation is performed using Mathematica® and an analytical solution for the attractive forces in the x- and y-directions can be obtained using Eq. (8):

\[
\begin{cases}
F_{motor,x} = -4(-1)^{5/8} \left\{ 1 + (-1)^{3/4} \left( F_{pm} + F_d \right)^2 R L \mu_0 \frac{\epsilon}{\theta_0} \right\} + O \left( \frac{\epsilon}{\theta_0} \right)^3 \\
F_{motor,y} = 0
\end{cases}
\]  

(8)

where $k_w$ and $k_d$ are the initial negative stiffness and the eccentricity-current-force coefficient, respectively. $O \left( \frac{\epsilon}{\theta_0} \right)^3$ can be ignored. In this case, since $\phi_0$ is not in the above equation, the attractive force is independent of the rotational angle of the rotor. Since $F_{pm} \gg F_d = N_d I_d$, the formula can be simplified into a linear relationship.

### 3.3 Motor radial force model

With a static eccentricity $\epsilon$ in the x-direction, the attractive force in the y-direction is zero and the equation of motion in the x-direction is given by Eq. (9)

\[
m \ddot{x} + c \dot{x} + k x = F_{motor,x}
\]  

(9)

The center of the stator is set as the origin of the coordinates. When the initial eccentricity of the rotor in the x-direction is $\epsilon_0$, the rotor has a small deformation $\epsilon_1$ that generates a restoring force that balances the attractive force induced by the motor, which is given by Eq. (10):

\[
k \epsilon_1 = k_w \epsilon + k_d I_d \epsilon
\]  

(10)

where $\epsilon$ is the static eccentricity, which is equal to the sum of the initial eccentricity $\epsilon_0$ and the deformation of the rotor $\epsilon_1$ as shown in Fig. 6.
We assumed that the displacement of the rotor from the initial eccentricity $\epsilon_0$ is $x = \epsilon_1 + \Delta x$ and the d-axis current is $I_d = I_{d0} + I_{d1}\cos vt$, in which the AC component excites the vibration. The displacement, attractive force, and d-axis current can be combined with Eqs. (9) and (10) resulting in the following equation of motion:

$$m(\ddot{\epsilon}_1 + \ddot{\Delta} x) + c(\dot{\epsilon}_1 + \dot{\Delta} x) + k(\epsilon_1 + \Delta x) = k_w(\epsilon + \Delta x) + k_d I_{d0}(\epsilon + \Delta x) + k_d I_{d1}(\epsilon + \Delta x)\cos vt$$  \hspace{1cm} (11)

Since $k_w \gg k_d I_{d0}$ and $\epsilon \gg \Delta x$, the motor radial force model can be simplified to Eq. (12) by substituting Eq. (10) into Eq. (11):

$$m\ddot{\Delta} x + c\dot{\Delta} x + (k - k_w)\Delta x = k_d I_{d1}\epsilon \cos vt$$  \hspace{1cm} (12)

This equation describes the dynamic model of the rigid rotor and we can define the excitation force as $F_{exc} = k_d I_{d1}\epsilon \cos vt$. The excitation force can be controlled by the d-axis current.

4. Experimental test rig
4.1 Test rig with a bearingless motor

A compact and simple test rig with a BELM was used as shown in Fig. 7 (Tsunoda et al., 2019a, 2019b). The rotor is supported by two circular oil-film bearings and rotated by a consequent-pole-type BELM (Chiba et al. 2005) placed at the midpoint of the rotor-shaft. The oil viscosity is 49 mPa·s and the 1st bending frequency of the rotor-shaft is 42 Hz. Table 1 shows the dimensions of the test rig. Displacement sensors (PU-05, Applied electronics Crop.) are mounted symmetrically on both sides of the BELM in the x- and y-directions. A non-contact encoder is placed at the end of the rotor-shaft to detect the rotational speed and angle. To avoid any rotor-stator contact during magnetic suspension, two touchdown bearings made of polyacetal are located on both sides of the BELM. Figure 8 shows a cross-sectional view of the consequent-pole-type BELM, which has motor windings and suspension windings in one stator. Table 2 summarizes the dimensions of the BELM. The motor windings, which are three phase and have eight poles, are used for rotation. The suspension windings are three phase and have two poles, and are used for positioning the rotor. Table 3 shows the configurations and characteristics of the two types of windings.
As illustrated in Fig. 8, the radial force can be generated by the flux from the suspension windings, which can strengthen or weaken the bias magnetic fluxes on both the right and left hand sides. Therefore, the position of the rotor can be controlled by the current in the suspension windings. In previous simulations and experiments (Tsunoda et al., 2017b), the radial forces in the x- and y-directions were measured. These can be expressed by the following equations:

\[
\begin{align*}
F_x &= k_i i_x - k_s x \\
F_y &= k_i i_y - k_s y
\end{align*}
\]

where \( k_i = 27.0 \text{ N/A} \) and \( k_s = -971 \text{ N/mm} \) are the current-force coefficient and the negative stiffness, respectively. \( i_x \) and \( i_y \) are the currents in the suspension winding in the x- and y-directions. \( x \) and \( y \) are the displacements of the rotor in the x- and y-directions.

### Table 1 Dimensions of test rig.

| Rotor | Oil-film bearing |
|-------|------------------|
| Mass  | 2.6 [kg]         |
| Diameter | 25 [mm]       |
| Diameter | 15 [mm]         |
| Clearance | 0.1 [mm]     |
| Length | 720 [mm]        |
| Length | 25 [mm]         |

### Table 2 Dimensions of consequent-pole-type BELM.

| Rotor diameter | PM outer diameter |
|----------------|-------------------|
| 70 [mm]        | 69 [mm]           |
| Stator diameter | PM thickness    |
| 160 [mm]       | 5 [mm]            |
| Clearance      | PM arc            |
| 1 [mm]         | 45 °              |
| Length         | Length of teeth   |
| 50 [mm]        | 29 [mm]           |
| Slot number    | Teeth width       |
| 24             | 4.8 [mm]          |

### 4.2 Suspension and rotation control systems

Proportional-integral-derivative (PID) controllers were used in the suspension control system. The values of the PID parameters were tuned experimentally. Figure 9 shows the suspension control system. The reference currents \( I_{c xref} \) and
The reference currents $I_{reref}$ are outputted from the PID controller. These are transformed from two-phase into three-phase currents $I_{2uref}$, $I_{2vref}$, and $I_{2wref}$, and then inputted into three current amplifiers to generate the radial force on the BELM. These controls are carried out using a DSP system (MicroLabBox, dSPACE, GmbH.) with a sampling frequency of 20 kHz. Three PWM single-phase amplifiers (Junus JSP-180-20, Copley Controls Corp.) are used for the suspension control system and the bandwidth of current feedback system $I_{2u}/I_{2uref}$ is about 600 Hz.

| The design of coils | Copper, Ø0.8 mm, Max. current 5 A |
|---------------------|----------------------------------|
| Motor windings      |                                  |
| Parallel wires      | 2                                |
| Series turns per slot | 25                             |
| Average eccentricity-current-force coefficient $k_d$ | 18.5 [N/A·mm] |
| Suspension windings |                                  |
| Single wire         | 1                                |
| Series turns per slot | 10                             |
| Current-force coefficient $k_i$ | 27.0 [N/A] |
| Negative stiffness $k_s$ | -971 [N/mm] |

Table 3 Configurations of windings and characteristics of BELM.

Figure 10 shows the rotational control system. A PI controller is used and the coordinates are transformed from direct-quadrature to three-phase to control the rotational speed. The absolute rotor angle is measured by the non-contact encoder and the rotational speed $\Omega$ is calculated using a frequency-to-voltage (FV) converter through the sawtooth wave of the rotor angle. The differences between the reference rotational speed $\Omega_{reref}$ and the measured rotational speed $\Omega$ is inputted into the PI controller and the amplitude of the q-axis current $I_q$ is calculated by the PI controller, whose parameters are 0.5 A/rps for the proportional gain and 0.5 A/rps·s for the integral gain. $I_q$ and $I_d$ are transformed to three-phase reference currents and the motor currents $I_{8u}$, $I_{8v}$ and $I_{8w}$ act on the motor windings to generate the motor torque through the amplifiers. The same DSP system used for the suspension control system is applied. Current amplifiers (7425 AC, Copley Controls Corp.) are used and the bandwidth of the current feedback system $I_{8u}/I_{8uref}$ is approximately 1000 Hz.

5. Experimental method

5.1 Realization of rotation with static eccentricity

5.1.1 Motor center

In this experiment, the center of the motor is defined when the DC current through the suspension windings is zero. The static eccentricity of the rotor is defined utilizing the distance and direction from the motor center. To find the center of the motor, the zero-power controller was used by first setting the reference position $X_{reref}$ and $Y_{reref}$ to (0 μm, 0 μm).
Due to the roundness error of the rotor, the position of the motor center depends on the rotational angle. Therefore, the average position (-20 μm, 70 μm) during the rotation is defined as the motor center.

5.1.2 Elimination of whirling

When the rotor rotates at a speed of 40 rps, the amplitude of whirling is approximately 95 μm. To eliminate this, the reference positions in the suspension control system were sinusoidally changed synchronously with the rotation to generate a radial force to counteract the unbalanced force (Tsunoda et al., 2017b, 2019a). Furthermore, the static eccentricity $\epsilon_{st}$ was added to the reference positions in the suspension control system as follows:

$$
\begin{align*}
X_{ref} &= \epsilon_{amp} \cos(\Omega t + \phi_m) + \epsilon_{st} \\
Y_{ref} &= \epsilon_{amp} \sin(\Omega t + \phi_m)
\end{align*}
$$

Fig. 11  Simulation of the static eccentricity rotation.

Note that $\epsilon_{amp}$ is the amplitude of the oscillation and $\Omega$ is the rotational speed, and $\phi_m$ is the phase, which depends on the position of the center of mass. Figure 11 shows the amplitude of whirling is reduced to 20 μm and the static eccentricity rotation is fixed at 100 μm.

5.2 FRF measurement using the suspension windings and the motor windings

5.2.1 FRFs measured utilizing the suspension windings

To verify the proposed method, FRF measurements utilizing the radial excitation force generated by the suspension windings of the BELM were used as a reference. Figure 9 shows the configuration of the FRF measurements using the suspension windings. The excitation current $I_{exc} = I_{sus} \cos vt$ from a frequency response (FR) analyzer (FRA5022, NF Corp.) was inputted into the reference current in the x-direction, where $I_{sus}$ is the amplitude of the excitation current in the suspension windings and $v$ is the angular frequency of the excitation current. The amplitude of excitation current $I_{sus}$ was set to 0.5 A. The FRFs from the excitation current $I_{sus}$ to the displacement of the rotor were measured by a FR analyzer from 1 Hz to 300 Hz during rotation.

5.2.2 FRFs measured utilizing motor windings

Figure 10 shows the configuration of the FRF measurement using the motor windings. An AC current signal of $I_{d1} \cos vt$ from the FR analyzer was added to the d-axis current $I_d$. During rotation, the d-axis current was as follows:

$$I_d = I_{d0} + I_{d1} \cos vt$$

Note that $I_{d0}$ is the bias d-axis current for flux control during rotation and the value of the bias d-axis current depends on the rotational speed due to the flux-weakening control. $I_{d1}$ is the amplitude of the excitation current, which was set to 2.5 A. The FRFs from the current command $I_{d1}$ to the displacement of the rotor were also measured by the FR analyzer.

5.3 Test conditions

To more completely verify the feasibility of the proposed method, some of the rotational conditions, such as the
eccentricity and rotational speed, were varied. Furthermore, the rotor dynamics were also changed by tuning the PID parameters of the BELM controller. In these experiments, three sets of the PID controller parameters were implemented as shown in Table 4. The controller parameters were different in the x- and y-directions to ensure the same closed-loop in both directions due to the position of the levitation having a non-uniform air gap, with the zero-power controller used to balance the force due to gravity. In the PID controller, parameters 1, 2, and 3 had the highest, middle and lowest derivative gains, respectively, to simulate the changes in the dynamic characteristics of the rotor system. The proportional and integral gains remain constant to set the natural frequency response of the first shaft bending mode at approximately 50-70 Hz.

Table 4  PID controller parameters.

| Parameter 1 (High) | Parameter 2 (Medium) | Parameter 3 (Low) |
|---------------------|-----------------------|-------------------|
| $K_p$ [A/m]         | 40,000                |                   |
| $K_i$ [A/m·s]       | 13,100                |                   |
| $K_d$ [A·s/m]       | 52                    | 36                | 20                |

Table 5 lists the experimental eccentricities and rotational speeds. As shown in Eq. (12), the excitation force is proportional to the eccentricity and the eccentricities of rotational machines are different due to imprecise nature of assembly. Thus, the effect of eccentricity on the measurement accuracy of the FRF was tested. The values of the static eccentricities were set to 100 $\mu$m, 50 $\mu$m, and 25 $\mu$m in the negative x-direction, respectively.

The dynamic characteristics of the rotor system also depend on the rotational speed due to the stiffness and damping variation of the oil-film bearings. The effect of the rotational speed on the proposed method needs to be considered. The rotational speed in our experiments was set to 40 rps, 30 rps, and 20 rps and the $I_{d0}$ bias d-axis current was set to -4.5 A, -2.5 A, and -2.0 A, respectively.

Table 5  Experimental eccentricities and rotational speeds.

| Eccentricities [$\mu$m] | 100 | 50 | 25 |
|-------------------------|-----|----|----|
| Rotational speeds [rps] | 40  | 30 | 20 |

6. Experimental results

6.1 Electromagnetic excitation

Various rotational speeds, eccentricities, and rotor dynamics were changed in the experiments. Figure 12 shows the waveform of a typical experiment utilizing the motor windings. The rotational speed was set to 40 rps and the static eccentricity $\epsilon_{st}$ was fixed as 100 $\mu$m in the negative x-direction. The offset of the d-axis current $I_{d0}$ and the amplitude of the d-axis current $I_{d1}$ were set to -4.5 A and 2.5 A, respectively. Parameter 3, having a low damping ratio, was chosen due to the obvious peak of the natural frequency. With an eccentricity of 100 $\mu$m in the negative x-direction and excitation signal at 1 Hz, the amplitude of the rotor calculated utilizing a FFT at 1 Hz was 9.7 $\mu$m. The average q-axis current used to generate the torque was 0.68 A. The measured rotational speed had an error of 1 rps and 0.1 rps with and without excitation, respectively.
6.2 FRF measurement
6.2.1 Effect of rotor dynamics

Figure 13 shows the measured gain plots of the FRFs with different PID parameters and fitted lines from 1 Hz to 300 Hz under typical experimental conditions. The unit of gain is mm/A. The red and black lines were measured utilizing excitation forces generated by the motor windings and the suspension windings, respectively, and the three pairs of red and black gain curves are similar, except for an offset of 20 dB owing to the different current-force coefficients. The gain curves, shown by the blue lines, have been fitted utilizing the least squares method to the measured curves using Eq. (16):

\[ G_p(s) = \frac{A}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]  \hspace{1cm} (16)

Note that \( A \) is a constant, \( \zeta \) is the damping ratio, and \( \omega_n \) is the natural angular frequency, respectively. The natural frequency \( f_n \) is obtained as \( f_n = \omega_n / 2\pi \).

Table 6 lists the identified damping ratio, natural frequency, and the percentage differences in the values obtained using the motor windings compared to those obtained using the suspension windings. The difference between the identified natural frequencies and the difference between the damping ratios for the two different FRFs are less than 5.2%, proving the validity of the proposed method.

Fig. 12  (a) Waveform of displacement and FFT analysis of displacement in the negative x-direction at 1 Hz excitation in a typical experiment. (b) Motor current at 1 Hz excitation in a typical experiment. (c) Rotational speed with and without 1 Hz excitation in a typical experiment.
6.2.2 Effect of eccentricity

Figure 14 illustrates the frequency responses from 1 Hz to 300 Hz and the results of the displacement in the x-direction with static eccentricities of 25 \( \mu \)m, 50 \( \mu \)m, and 100 \( \mu \)m. The rotational speed was 40 rps, the bias d-axis current was -4.5 A and the amplitude of the d-axis current was 2.5 A. In Fig. 14 (a), the different offsets in the gain plots measured by the motor windings are a result of the different excitation forces caused by the different eccentricities. In Fig 14 (b), the amplitudes of displacement at 1 Hz are 4.3 \( \mu \)m, 6.1 \( \mu \)m, and 9.7 \( \mu \)m with the static eccentricities fixed at 25 \( \mu \)m, 50 \( \mu \)m, and 100 \( \mu \)m, respectively, in which the eccentricity is positively correlated with the amplitude of the displacement. The peak at 40 Hz is due to the unbalanced vibration of the rotor at 40 rps. Table 7 summaries the identified natural frequencies, damping ratios, and percentage differences. The differences are less than 6.3%. Therefore, even when the eccentricity is 25 \( \mu \)m, the FRF measurement can be achieved using the proposed method.
6.2.3 Effect of rotational speed

Table 8 lists the identified natural frequencies, damping ratios, and the differences at different rotational speeds. The eccentricities are 100 µm and the amplitudes of the d-axis currents are 2.5 A. The natural frequencies and damping ratios were changed slightly depending on the rotational speed due to the variation in stiffness and damping of the oil-film bearings. However, the differences between the identified parameters measured using the suspension windings and the motor windings are less than 7.7%. Therefore, the FRF can be measured using the proposed method.

| Eccentricity | $f_n$ [Hz] Suspension windings | $f_n$ [Hz] Motor windings | Differences (%) |
|--------------|--------------------------------|---------------------------|----------------|
| 25 µm        | 69                              | 66                        | 4.4            |
|              | 0.20                            | 0.19                      | 5.0            |
| 50 µm        | 68                              | 66                        | 2.9            |
|              | 0.16                            | 0.15                      | 6.3            |
| 100 µm       | 68                              | 67                        | 1.5            |
|              | 0.15                            | 0.15                      | 0              |

6.2.4 Discussion

In the experimental results, many of the natural frequencies measured by the suspension winding excitation are higher than those measured by the motor winding excitation. The natural frequency of the rotor system is affected by the bias d-axis currents, as shown in Eq. (11). However, the same bias d-axis currents were set for the rotational control of the motor both in the suspension and motor winding excitations. Another possibility of having a difference in the measured natural frequencies is that the excitation force is not the same in both excitation methods. It may cause the difference in frequency responses due to nonlinear dynamic of the rotor system. For more accurate analysis, further research is required about the effect of the rotor non-linearity.

7. Conclusion

This paper proposes a practical method of electromagnetic excitation utilizing the motor windings of a built-in motor to evaluate the dynamic characteristics of turbomachinery. The principle of generating an excitation force utilizing an AC d-axis current in a rotor with static eccentricity of the rotor is presented. The theoretical equations show that the radial excitation force is not affected by the rotational angle but is proportional to the eccentricity and the d-axis current. A test rig with a BELM was used to verify the feasibility of the proposed method. The FRFs measured utilizing the suspension windings and the motor windings of BELM were compared. The natural frequencies and damping ratios were identified from the two types of FRF. The effectiveness of the proposed method with different rotor dynamics and rotational conditions, such as the eccentricity and rotational speed, was studied. FRF measurements with three different sets of parameters for the rotor dynamics were made and the percentage differences between measurements made with the two different FRFs were less than 5.2%. The proposed method was verified for different rotational conditions. Even when the
eccentricity was 25 \mu m and the rotor dynamics were changed by varying the rotational speed, the differences between the natural frequencies and the differences between the damping ratios obtained from the two different FRFs were less than 7.7%.

Future work will focus on extending the proposed electromagnetic excitation, including evaluations using canned motor pumps or artificial blood pumps. Furthermore, sensorless measurement for the displacement will be considered.

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