Queuing theory based part-flow estimation in a pick-and-place task with a multi-robot system

Yaniang HUANG***, Ryosuke CHIBA***, Tamio ARAI****, Tsuyoshi UEYAMA*****,
Xianmin ZHANG* and Jun OTA**

Abstract
This paper addresses the problem of estimating the maximum part-flow or part feed rate for a pick-and-place task with a multi-robot system. An appropriate part-flow is important to guarantee productivity and robustness in the presence of pattern variation, that is, when the timing and position of the parts on the conveyor are presented in random. To determine the appropriate part-flow for a multi-robot system quickly, we divided the multi-robot system into several single-robot systems and estimated the part-flow for each single-robot system based on M/M/s queuing model with impatient customers. Then, the task completion rate for the ensemble of robots was computed. Simulations were used to compare the proposed method to a method based on a Monte Carlo Strategy (MCS) and a method based on an M/M/1 queuing model. The simulation results show that the proposed method can guarantee the required productivity and the task-completion success rate. The task-completion success rate determined using the proposed method reached 98.9% for 10000 randomly generated patterns. The computation time for the proposed method satisfies computation time constraints and is less than 1/10 of that for the MCS method.

Keywords : Queuing theory, Part-flow, Multi-robot system, Pick-and-place task, Pattern variation

1. Introduction

Multi-robot systems have been used to perform a variety of pick-and-place tasks, such as palletizing tasks and packaging tasks (Bozma and Kalahoglu, 2012, Li and Masood, 2008). As shown in Fig. 1, a multi-robot system for a pick-and-place task commonly consists of several manipulators, a moving conveyor, a part feeding device (part feeder) and part collection devices (e.g., packaging boxes or returning conveyor). The part feeder feeds the parts onto one end of the conveyor. A robot picks up parts from a moving conveyor moving through its workspace, and then places them at an appointed location. The productivity of the multi-robot system depends on the capability of an individual robot and the rate at which parts are fed by the part feeder. We define the number of parts fed per unit time by the part feeder as the part-flow (i.e., the part feed rate). Normally, the capacity of an individual robot is constant when the type and the base location of the robot are given. For a multi-robot system, the part-flow can be increased to improve productivity. However, when the part-flow exceeds the capability of the multi-robot system, many parts may be left on the conveyor without being picked. In this case, additional devices (e.g., a collecting box or a returning conveyor) are required to handle the missed parts. Conversely, a low part-flow results in reduced system productivity. Since the flow of parts onto the conveyor can be considered to follow a probability distribution in a real application, the pattern variation (i.e., the process of feeding parts onto the conveyor at random times with a given distribution but using a different random seed for each experiment) makes the analysis of the relationship between part-flow and the performance of the
A pattern is defined as the process of feeding parts onto the conveyor following a given distribution where different patterns are generated using different random seeds, as shown in Fig. 2. The pattern varies when the random seed for the given distribution is changed, but all the patterns have the same distribution. For a given multi-robot system, an appropriate part-flow should be set to improve productivity without leaving parts on the conveyor for all the varying patterns. To quickly implement a multi-robot system for a pick-and-place task with an appropriate part-flow, it is important to estimate the value within a short amount of time. For example, the part-flow needs to be changed in a real application when some picking tools or some robot arms in the multi-robot system cannot work well. In this case, an appropriate part-flow should be determined in a reasonable amount of computation time to increase the adaptability of the multi-robot system. For a real application, a reasonable amount of computation time is approximately 10 seconds based on the discussion with the engineers in the manufacturing company.

In previous studies, most researchers focused on coordination architectures or algorithms for multi-robot systems. A framework and software architecture were proposed to control a multi-robot system in an unstructured and unknown environment (Alur et al., 2002). In this architecture, each robot was assumed to have a finite set of behaviors and the programming language was used to formally specify a set of conditions. In (Kaminka and Frenkel, 2007), a multi-robot behavior based architecture using the micro-kernel integration approach was proposed to coordinate the robots. An agent based multi-robot architecture was proposed to coordinate multiple robots in a manufacturing system (Panescu and Varvara, 2006). In (Kaminka, 2010), the game theory was used to coordinate the actions of multiple robots. In (Li and Latombe, 1997), an on-line manipulation planning system was proposed for two robot arms to complete a task in a

Fig. 1 A pick-and-place task with a multi-robot system

Fig. 2 Patterns in part arrival
dynamic environment. In (Galante and Passannanti, 2006), a practical approach and explanatory Gantt plots were used to minimize the cycle time of manufacturing lines with multiple dual-gripper robots. The previous studies described above aimed to improve the productivity of the multi-robot systems. In some other studies, researchers focused on the design of a flexible part feeding system to identify or reorient the fed parts (Vilan et al., 2009, Tay et al., 2005, Causey et al., 1999). The relationship between part-flow and the capacity of the multi-robot system was not taken into account in the previous studies.

Some researchers developed models to describe or analyze the manufacturing flow line. The major classes of models, features, properties and the relationships among different models were discussed in (Dallery and Gershwin, 1992). Gebenini and Grassi designed a buffer for a manufacturing line to enable the line to work at two different capacities (Gebenini and Grassi, 2015). The relationship between the buffer and the performance of the manufacturing line was analyzed. To improve the productivity of an assembly line, Manitz proposed an approximation procedure based on queuing theory to determine system throughput for an assembly task (Manitz, 2008). A continuous approximation model for a manufacturing system was proposed to analyze the dynamics of the system (Berg et al., 2008). Li and Masood proposed a queuing theory based method to describe a robotic palletizing process with two robots (Li and Masood, 2008). In this system, the two robots shared the same workspace. In a recent study (Huang et al., 2015), we predicted the part-flow for a multi-robot manufacturing system using a Monte Carlo Strategy (MCS), which is robust against the varied patterns of part-flow. In the MCS method, the part-flow is calculated based on statistical inference with several samplings of part-flow. Each sampling of part-flow is obtained through a simulation. A large amount of computation time is required to obtain sampling data from the simulations, so the MCS based method proposed in our previous study is time-consuming.

In this study, our goal is to quickly determine an appropriate part-flow for a multi-robot system executing a pick-and-place task. In a pick-and-place task, the process of feeding parts follows a given distribution with different random seeds (i.e., pattern variation), which will make the analysis of the relationship between the part-flow and the performance of the multi-robot system complex. It is time-consuming to use a simulation-based statistical method, such as the MCS, to obtain an appropriate part-flow for a given multi-robot system. To determine the appropriate part-flow quickly, we modeled the pick-and-place process of a multi-robot system using queuing theory. Queuing theory is a mathematical method for solving queuing problems and it has been extended to a variety of applications (Govil and Fu, 1999, Shanthikumar et al., 2007). In this paper, we analyzed the queuing model of multi-server queues with impatient customers, and modified the queuing model to describe the pick-and-place task with a multi-robot system. A numerical calculation with multiple loops was used to quickly estimate the part-flow. The method was compared with a statistical method through simulations. The difference between this study and our previous study (Huang et al., 2015) is as follows: (1) a multi-server queue with impatient customers is proposed and modified to describe the pick-and-place task with a multi-robot system; (2) numerical calculations with multiple loops are proposed to estimate the part-flow.

The remaining parts of this paper are organized as follows. Section 2 formulates the problem. The proposed method is described in Section 3. In Section 4, the simulation, results, and discussion are presented. Section 5 provides the conclusions.

2. Problem formulation

This section describes the problem of part-flow estimation for a pick-and-place task with a multi-robot system. An analysis of the problem, the assumptions which were made, the input parameters, performance index, and constraints are described based on a real application.

2.1. Problem analysis

As shown in Fig. 3, the multi-robot system considered in this paper consists of multiple manipulators, several packaging boxes and a moving conveyor. The parts on the moving conveyor are randomly fed by the part-feeder without a buffer. Therefore, the process of feeding parts is a stochastic process. The position and orientation of parts on the moving conveyor are varied. To improve productivity, a picking tool with multiple suction-cups is used to grasp the parts in the real applications (Kakamu et al., 2005). This complicates the multi-robot system. Due to the use of multiple suction-cups and the stochastic process of the part feeder, it is difficult to estimate the part-flow of the system to maximize the productivity of the system in the presence of pattern variation. To solve the problem of estimating the maximum part-flow, we made some assumptions and set the input parameters as follows.
2.2. Assumptions

- The process of feeding parts onto the moving conveyor follows a known distribution (e.g., Poisson distribution or normal distribution). The parts on the moving conveyor do not overlap. The moving conveyor moves at a given constant velocity that will not lead the parts to slide on the conveyor.
- The multi-robot system is comprised of identical robots. Each robot is lined up on one side of the moving conveyor. The distance between a robot’s base and the center-line of the moving conveyor is 70% of the maximum reach of the robot (Gueta et al., 2011). The packaging boxes are positioned between the robots’ bases and the moving conveyor. Each robot has an individual workspace (i.e., a non-overlapping workspace).
- The position and orientation of the parts in each robot’s workspace is obtained by a camera. The robots inherently know the gripping point on the part (e.g., the center point of the top surface of the part).
- A four suction-cups tool is attached to the end-effector of the robot. The robot picks up each part one-by-one. After all the suction-cups are occupied, the robot moves to the packaging box, places the four parts into the box, and then moves to pick up the next part. After picking up one part, the suction-cup that holds a part can be raised to avoid colliding with the moving conveyor or other parts.
- The part that enters the robot’s workspace first will be picked up first. In other words, all the robots follow the part-dispatching rule of first-in-first-out (FIFO). The picking order for the four suction-cups is predetermined.

2.3. Input parameters

- Specifications of the multi-robot system, including the number of robots, the Denavit–Hartenberg (DH) parameters for each robot, the maximum reach and velocity of each robot, the location of the robot base, the conveyor size, the velocity of conveyor, and the position of packaging boxes. The detailed values for the specifications of the multi-robot system are provided in Section 4.
- Specifications of the parts on the moving conveyor, including the shapes and sizes of the parts, and the probability distribution for feeding the parts to the conveyor. The position and orientation of the parts and the sequence of picking order need to be specified. The detailed values for the specifications of parts will be provided in Section 4.

2.4. Performance index and constraints

Normally, the part-flow and picking rate (i.e., the rate that is calculated using the number of parts picked divided by the number of parts fed) are two important indexes used to evaluate the performance of a multi-robot system. Typically, high part-flow will result in a low picking rate. As the number of fed parts per unit time increases, more parts will be left on the conveyor without being picked up due to the limitations of the handling capability of the multi-robot system. Although a low part-flow can guarantee a high picking rate, it will result in some robots being idle, reducing the productivity of the system. Therefore, for the multi-robot system to be highly productive it must have both a high part-flow and a high picking rate. To simplify the problem, it is reasonable to maximize the part-flow subject to a
desired picking rate (e.g., 99%). Due to pattern variation, the picking rate derived under the maximal part-flow will not satisfy the desired picking rate all the time. Therefore, in this paper, we set the part-flow as the performance index and try to obtain the appropriate part-flow. The desired picking rate and pattern variation are set as constraints. The picking rate determined under the appropriate part-flow has to satisfy the desired picking rate against the pattern variation. We can describe the picking rate constraint as follows:

\[ R_{act} \geq R_{req} \]  

(1)

\[ R_{act} = \frac{\sum N_i}{N_{fed}} \]  

(2)

where, \( R_{act} \) is the picking rate derived by a multi-robot system under a give part-flow. \( R_{req} \) is the desired picking rate defined by the user. \( N_i \) is the number of parts picked by the \( i^{th} \) robot. \( j \) is the number of robots in the multi-robot system. \( N_{fed} \) is the number of parts fed on the moving conveyor.

To determine whether the picking rate under the derived part-flow can satisfy the desired picking rate against the pattern variation, we proposed a task-completion success rate to evaluate the derived part-flow. A pick-and-place task is considered to be completed if the picking rate under the derived part-flow can satisfy the desired picking rate. The details of task-completion success rate are described in Section 4.2.

When the product types and the configuration of multi-robot system are defined, the industrialist hopes the multi-robot system can work quickly and with high productivity as soon as possible. Therefore, the appropriate part-flow should be determined for the multi-robot system within a computation time constraint. Normally, a reasonable amount of computation time to determine the maximum part flow is about 10 seconds.

3. Proposed approach

3.1 Overview of the proposed approach

Because the position and orientation of each part in the robot’s workspace is variable, the processing time (i.e., servicing time) for each part is also variable. It is assumed to follow an exponential distribution (Li and Masood, 2008). Therefore, the instantaneous number of parts in the multi-robot system is directly related to the stochastic nature of arrival and servicing. The parts on the moving conveyor can be considered as customers in a queue. We thus utilize a queuing model to evaluate the process for the pick-and-place task with a multi-robot system. Because the conveyor is moving with a constant speed, the parts on the conveyor will be lost if they cannot be picked up within the workspace of the robot. In this case, the parts on the moving conveyor can be considered as the customers with impatience of constant duration. The constant duration depends on the length of the robot’s workspace and the velocity of the moving conveyor. Based on the above analysis, a queuing model with impatient customers can be used in this study.

In the multi-robot system considered in this paper, the workspace for each robot is not overlapped. Additionally, the robots in the multi-robot system are independent. Since the arrival and servicing of the parts on the moving conveyor can be considered a Poisson process and the robots are independent, the additive property of a Poisson process can be utilized (Tang and Xu, 2007). As a result, the parts fed to each robot can be considered to be following an exponential distribution. Due to the identical specifications of each robot, it is reasonable to treat the multi-robot system as several single-robot systems. There are several suction-cups attached to the end-effector of robot. Each suction-cup can be considered as a server for picking up a part, as shown in Fig. 4. Therefore, the parts on the moving

![Fig. 4 Picking tool with four suction-cups](image-url)
conveyor and the robot with multiple suction-cups can be modeled by the M/M/s queuing model. The first M means the part arrival follows a Poisson process, the second M means the part processing time (i.e., service time) follows an exponential distribution, and s is the number of servers (i.e., the number of suction-cups in this paper). We will describe the M/M/s queuing model with impatient customers and its application to the estimation of part-flow in the Section 3.2 and 3.3.

3.2 M/M/s queuing model with impatient customers

In the M/M/s queuing model with impatient customers of constant duration, the customers are served in the order of arrival, if not lost. According to the epochs on the deadlines of a customer’s impatience, there are two types of impatience systems (Choi et al., 2001, Brandt and Brandt, 1999, Movaghar, 1998). One is called an impatient model with deadlines until the beginning of service. Here a customer who waits in the queue becomes lost if the service cannot be acquired within a constant time. Another one is called an impatient model with deadlines until end of service. Here a customer becomes lost if their sojourn time in the system (including the waiting time and service time) exceeds the constant time. Once the part on the moving conveyor is picked up, it will be placed in the packaging box. Therefore, the impatient model with deadlines until beginning of service is used in this study. Later in this section, we described the probability of loss in the impatient model with deadlines until beginning of service.

Considering that the customer arrival follows a Poisson process with rate \( \lambda \), the service time follows an exponential distribution with mean \( \mu \). The number of servers is \( s \). The constant waiting time is \( T \). According to reference (Choi et al., 2001), we can derive the probability \( q_i \) that there are \( i \) customers in the queue. Then, the loss rate \( p(x) \) in steady state can be described as follows:

\[
q_i = \frac{1}{i!} \left( \frac{\lambda}{\mu} \right)^i q_0, \quad 0 \leq i \leq s
\]

\[
p(x) = \frac{\lambda}{s!} \left( \frac{\lambda}{\mu} \right)^x e^{-(\lambda-\mu)x} q_0, \quad 0 \leq x \leq T
\]

where, \( q_0 \) is the probability that no part enters the robot workspace at the steady state. Based on the normalization condition, the \( q_0 \) can be calculated as follows:

\[
q_0 = \begin{cases} 
\sum_{i=0}^{s-1} \frac{1}{i!} \left( \frac{\lambda}{\mu} \right)^i + \frac{s}{s!} \left( \frac{\lambda}{\mu} \right)^s \frac{1 - e^{-(\lambda-\mu)T}}{s - \lambda}, & \text{if } \lambda \neq s\mu \\
\sum_{i=0}^{s-1} \frac{1}{i!} + \frac{s\lambda T}{s!}, & \text{if } \lambda = s\mu
\end{cases}
\]

The loss probability \( p_{\text{loss}} \) in the M/M/s queuing model with impatient customer of constant duration can be defined as the ratio of the loss rate and arrival rate in the steady state, as follows:

\[
p_{\text{loss}} = \frac{p(T)}{\lambda} = \frac{1}{s!} \left( \frac{\lambda}{\mu} \right) e^{-(\lambda-\mu)T} q_0
\]

For a detailed derivation of the steps from Eq. (3) to Eq. (6), the reader can refer to (Choi et al., 2001).

3.3 Implementation of M/M/s queuing model in estimation of part-flow

Due to the additive property of the Poisson process, a multi-robot system performing a pick-and-place task can be treated as several single-robot systems, as shown in Fig. 5. The part-flow for the multi-robot system can be calculated as follows:

\[
f_{\text{flow}} = \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_j
\]

Here, \( f_{\text{flow}} \) is the part-flow for the multi-robot system. \( \lambda_j \) is the part-flow for the \( j \)th robot. \( j \) is the number of robots in the multi-robot system.

Consider a single robot with a multiple suction-cups tool performing the pick-and-place task. The M/M/s queuing model can be used to model the execution of the task. Because the parts on the conveyor are lost when they move
through the robot’s workspace without being picked up, the waiting time that a part waits for until being picked up depends on the velocity of the moving conveyor and the length of the robot’s workspace over the moving conveyor. The impatient time can be calculated as follows:

\[ T_j = \frac{l_j}{v_c} \quad (8) \]

where, \( T_j \) is the waiting time for the parts that will be handled by the \( j \)th robot, \( l_j \) is the length of the \( j \)th robot’s workspace on the moving conveyor, \( v_c \) is the velocity of the moving conveyor.

The loss probability for each single-robot system has to satisfy the following inequality:

\[ p_{\text{loss, } j} \leq 1 - R_{\text{req}} \quad (9) \]

Substituting Eq. (6) into Eq. (9), the following inequality can be derived:

\[ \frac{1}{s!} (\frac{\lambda_j}{\mu_j})^{(s-\mu_j)} e^{-(\lambda_j-\mu_j)T_j} q_{0,j} \leq 1 - R_{\text{req}} \quad (10) \]

where \( \lambda_j \) is the part-flow for the \( j \)th robot; \( \mu_j \) is the processing rate of the \( j \)th robot; and \( \mu_j \) is defined as the number of parts that can be picked up by the \( j \)th robot per second in this study. Normally, the processing rate of the robot depends on the capability of the robot (Mattone et al., 2000, Nof and Hanna, 1989). Once the specifications and the location of the base of the robot are determined, the capability of the robot is determined. In this study, we calculated the processing rate and the waiting time for a given part-flow as a test. The given part-flow for the test was determined based on a trial-and-error method and experience.

After determining the processing rate and waiting time, the part-flow \( \lambda_j \) in Eq. (10) is unknown. Due to the complexity of Eq. (10), we used a numerical method to derive the \( \lambda_j \) instead of using analytic methods. The part-flow \( \lambda_j \) depends on the processing rate and waiting time for the \( j \)th robot. If the processing rate and waiting time for the \( j \)th robot is the same as that for the \( (j+1) \)th robot, the value of \( \lambda_j \) will be the same with \( \lambda_{j+1} \). The process for determining the \( \lambda_j \) is shown in Fig. 6. The estimation of part-flow can be considered as an optimization problem. As the search space (i.e., the design space) is enlarged, the computation time needed to estimate part-flow increases. Especially with the integrated design of a multi-robot system to perform a pick-and-place task, when considering the types of robot, the layout of each robot, and the number of robots, the design space is greatly enlarged. A large amount of computation time is required to realize the integrated design. We performed a numerical calculation with multiple loops and found the computation time for the proposed method was shorter than that for the simulation-based statistical inference method. This is because when running the numerical method, only one simulation is conducted. Using the simulation-based statistical inference method, \( n \) simulations are conducted to obtain the \( n \) samplings of part-flow.

4. Simulation, results and discussion

4.1 Comparative methods

To verify the effectiveness of the proposed method, we compared it with a statistical inference method based on the MCS method and a method based on the M/M/1 queuing model with an impatient customer (abbreviated as M/M/1 model). By using the MCS method, the part-flow was estimated based on the sampling data and three-sigma rule.
Details about the MCS method can be found in (Huang et al., 2015). When using the M/M/1 model, the multi-robot system is considered as a server. While in the proposed method, the robot in the multi-robot system is considered as a server. In other words, there are \( s \) servers if there are \( s \) robots in the multi-robot system. This is the difference between the proposed method and the method based on M/M/1 model. In the M/M/1 model, the processing rate and waiting time are calculated by considering the complete multi-robot system instead of just a single-robot system. For details about the M/M/1 model applied in the estimation of part-flow for a multi-robot system, see reference (Huang et al., 2013).

![Flowchart](image)

**Fig. 6** Process for determining part-flow. The `init_flag` is set by a test; \( p_{\text{loss,cur}} \) is the current loss probability; \( p_{\text{loss,thre}} \) is the loss probability threshold value; \( \lambda_{\text{cur}} \) is the current part-flow; \( R_{\text{req}} \) is the desired picking rate; \( \text{step}_s \) is a small step of part-flow; \( \text{step}_l \) is a large step of part-flow.

### 4.2 Evaluation of derived solutions

To evaluate the solutions determined by the different methods, we defined a task-completion success rate \( (R_s) \). Task-completion success means that the picking rate under the defined part-flow with a given pattern can satisfy the picking rate constraint. The \( R_s \) is calculated as the number of patterns for which the task is completed divided by the number of total patterns, as follows:

\[
R_s = \frac{N_{\text{com}}}{N_{\text{total}}} \tag{11}
\]

where \( N_{\text{com}} \) is the number of patterns that enables the task to be completed and \( N_{\text{total}} \) is the number of total patterns. In this study, we empirically set \( N_{\text{total}} \) as 10000 by generating 10000 random seeds.

Normally, the task-completion success rate should be 100%. Considering both the productivity and the pattern variation, the 3-sigma rule can be accepted from a real applications viewpoint. In this study, we set the value of sigma to 1% based on discussions with engineers at the robotics company.
4.3 Simulation parameter setting

In the simulation, we used a multi-robot system with three identical robots, three identical packaging boxes, and a moving conveyor, to evaluate the proposed method. The multi-robot system was as shown in Fig. 3. All the robots are lined up on one side of the conveyor so they have non-overlapping workspaces. To avoid collisions between two adjacent robots, the intervals between two adjacent workspaces was set to 0.2 m. The specifications of the robots and conveyor are shown in Table 1. The robot manipulators VS-6577 produced by DENSO WAVE INCORPORATED make up the multi-robot system. Detailed specifications on the robot manipulators can be obtained from the robotics company (DENSO, 2016). Each robot has four suction-cups attached to the end-effector. Therefore, $s$ is 4 in the M/M/s queuing model. The position of each packaging box is 0.375 m from the center line of the conveyor and is set on the same side as the robot’s base. The position of each robot’s base is 0.6 m from the center line of the conveyor. Because the specifications for each robot is the same, the part-flow for each robot is the same in the M/M/s queuing model. Although the number of suction-cups will affect the processing rate for one robot and the number of robots will affect the derived part-flow for the multi-robot system, the part-flow for the multi-robot system is equal to the part-flow for one robot multiples by the number of robots based on our proposed M/M/s model, this is because the specifications of each robot and the part-feed distribution for each robot are set to the same in this study. When the number of robots or the number of suction-cups change, the part-flow derived by the proposed M/M/s model will change linearly. Therefore, we set the number of robots as 3 and set the number of suction-cups as 4 for one robot to evaluate the proposed method in this study. In the simulation, the MCS method is conducted under the same constraints with M/M/s model.

The specifications of part flow are shown in Table 2. All the parts have the same shape and size with dimensions $0.03 \times 0.015 \times 0.005$ m. The desired picking rate was set to 99%. For the estimation of part-flow based on the queuing model, $p_{loss,\text{thr}}$ is set to 0.2, $step_s$ is set to 0.1; $step_l$ is set to 0.5. When using the MCS method to estimate the part-flow, the sample size was set to 24. The number of parts in the simulation was set to 100. The simulation was conducted by using a computer with an Intel Core (TM) 2.5 GHz processor and 4 GB of memory.

4.4 Results and discussion

The simulation results are shown in Table 3. The part-flow determined by the proposed M/M/s model is 9.36 pieces/s, and the task-completion success rate is 98.9% for the given 10000 patterns. Almost all tasks with varied patterns can be completed for the computed part-flow or part feed rate. That means the proposed M/M/s model can obtain the appropriate part-flow for the multi-robot system in a pick-and-place task to guarantee both the task-completion success rate and system productivity. The task-completion success rate satisfies the requirement of a real application. From the equation (10), we know that the part-flow is affected by the processing rate and waiting time by using the proposed method. It is difficult to obtain the exact processing rate for a robot. This is the reason why the task-completion success rate under the part-flow derived by the proposed method cannot reach 100%. When considering the results determined by the MCS method, the computed part-flow is smaller than that computed by the

| Parameter value | 0.85 | 7.6 | 0.3 | 0.5 |
|------------------|------|-----|-----|-----|
| Maximum reach of robot (m) | Maximum composite speed of robot (m/s) | Width of conveyor (m) | Velocity of conveyor (m/s) |
| Time interval between two consecutive parts(s) | Location of part in normal direction of conveyor (m) | Orientation of part (rad) |
| Distribution | $E(\lambda)$ | $U[-0.125,0.125]$ | $U[-\pi, \pi]$ |

Note: $E$ is exponential distribution; $U$ is uniform distribution.
Table 3

| ESTIMATED PART-FLOW, COMPUTATION TIME, AND TASK-COMPLETION SUCCESS RATE |
|---------------------------------------------------------------|
| **Part-flow (piece/s)** | MCS method | M/M/1 model | M/M/s model |
|-------------------------|------------|-------------|-------------|
| Computation time (s)    | 60.8       | 4.7         | 5.6         |
| Task completion success rate | 100%     | 83.3%       | 98.9%       |

This is because the three-sigma rule is used to calculate the lower limit of the estimated part-flow. This method guarantees the picking rate constraint by sacrificing productivity. When considering the results derived by M/M/1 model, the part-flow is larger than that derived by M/M/s model. This is because the multi-robot system is considered as a server in the M/M/1 model; a part on the moving conveyor may be picked up by the next robot even if it is missed by the previous robot. In this case, the waiting time for the part is prolonged, which can affect the estimation of the part-flow. Due to the difficulty of obtaining the exact waiting time, it is difficult to obtain the appropriate part-flow using the M/M/1 model. This is the reason why the task-completion success rate derived by the M/M/1 model is low.

In terms of computation time, the computation time for M/M/s model is similar to that of the M/M/1 model, and it is less than 1/10 of that for MCS method. This is because the calculation of part-flow is performed numerically in the queuing model. For the MCS method, part-flow is calculated based on the sampling simulation. A large amount of computation time is required to obtain the sampling data. Due to the computation time constraint, the proposed M/M/s model is a better choice for estimating part-flow for a real pick-and-place application. Although the computation time for the other methods is not large in this case study, when the number of robots and the number of parts on the moving conveyor increases, the computation time will increase sharply.

To investigate the processing rate for each robot under different part-flows, we conducted a simulation for the computed part-flow with 500 parts. The number of picked parts, the processing time and the processing rate for each robot under the different computed part-flows are shown in Table 4. The processing rate for each robot under the different part-flows is shown in Table 5. From the results, we can find that when the parts on the moving conveyor is sufficient, the processing rate of each robot is similar, such as the processing rate for robot 1 and 2 shown in Table 4 and 5. These results show that it is reasonable to divide the multi-robot system into several single-robot systems. When the parts on the moving conveyor are insufficient, the processing rate of the robot is smaller than its actual capability, such as the processing rate for robot 3 shown in Table 4 and 5. However, such phenomenon is reasonable because the last robot in the multi-robot system is usually used to guarantee the picking rate against the pattern variation of part-flow.

By using the M/M/s model to estimate the part-flow for a pick-and-place task with a multi-robot system, the processing rate for each robot can affect the estimation of part-flow. Normally, the processing rate for the robot can be obtained exactly based on a long-time simulation or by experiment. However, the simulation requires a large amount of computation time. To satisfy changes to requirements in a real application quickly, it is reasonable to obtain an approximate part-flow within a short amount of computation time.

Considering the application of the proposed method, the process of a pick-and-place task where both the part feeding rate and the part processing rate follow exponential distributions can be described by the M/M/s queuing model. When the distribution of the part feeding rate, or the distribution of part processing rate changes, the proposed queuing model should be modified. When considering the pick-and-place task with a multi-robot system, it is difficult to model the system as a queuing model with multiple servers due to the non-overlapping workspace. In this case, the pick-and-place task with a multi-robot system can be divided into several pick-and-place tasks with single robot systems, as in our proposed method. Because the estimated part-flow by using the proposed method is affected by the variates (e.g., number of robots, number of suction-cups, and location of robot base), the appropriate part-flow can be quickly estimated when the variates changed. Due to the short computation time, the proposed method can be used to solve a complex problem: the integrated design of multi-robot systems for pick-and-place tasks.
In this paper, we solved the problem of estimating part-flow for a pick-and-place task with a multi-robot system. To solve the problem within a short amount of computation time, we viewed the multi-robot system as several single-robot systems, and proposed the M/M/s queuing model to estimate the part-flow for each individual robot. The proposed method was compared with a simulation-based MCS method and an M/M/1 queuing model. The simulation results showed that the part-flow derived by the proposed method can both guarantee the productivity and the task-completion success rate. Compared to the M/M/1 queuing model, the task-completion success rate derived by the proposed method is improved to 98.9%, which can satisfy the requirements of a real application. Compared to the simulation-based MCS method, the computation time for the proposed method is less than 1/10 of that for the MCS method. The processing rate for each robot under different part-flow rates was investigated.

The queuing model has been applied to the pick-and-place task with a multi-robot system under some assumptions in this paper. The proposed method was evaluated through simulation, without being implemented in a real-world application. However, the simulation settings were set based on a real-world application. In future work, the feasibility of applying the queuing model to a complex multi-robot system with various robot types and overlapping workspaces will be examined.

### Table 4
**NUMBER OF PICKED PARTS, PROCESSING TIME AND PROCESSING RATE UNDER DERIVED PART-FLOW**

| Method          | Part-flow | Robot 1 | Robot 2 | Robot 3 |
|-----------------|-----------|---------|---------|---------|
| Number of picked parts (pieces) | 223       | 192     | 85      |
| Processing time (s) | 57.4      | 48.9    | 26.7    |
| Processing rate (piece/s) | 3.89      | 3.93    | 3.18    |

| Method          | Part-flow | Robot 1 | Robot 2 | Robot 3 |
|-----------------|-----------|---------|---------|---------|
| Number of picked parts (pieces) | 203       | 195     | 95      |
| Processing time (s) | 51.6      | 48.5    | 29.2    |
| Processing rate (piece/s) | 3.94      | 4.02    | 3.25    |

| Method          | Part-flow | Robot 1 | Robot 2 | Robot 3 |
|-----------------|-----------|---------|---------|---------|
| Number of picked parts (pieces) | 213       | 203     | 82      |
| Processing time (s) | 54.6      | 52.8    | 25.1    |
| Processing rate (piece/s) | 3.90      | 3.84    | 3.27    |

### Table 5
**PROCESSING RATE UNDER DIFFERENT PART-FLOW (UNIT: PIECE/S)**

| Part-flow (piece/s) | Robot 1 | Robot 2 | Robot 3 |
|---------------------|---------|---------|---------|
| 8                   | 3.80    | 3.69    | 2.91    |
| 9                   | 3.90    | 3.70    | 3.41    |
| 10                  | 3.91    | 3.70    | 3.48    |
| 11                  | 3.89    | 3.71    | 3.63    |
| 12                  | 3.90    | 3.77    | 3.56    |
| 15                  | 3.81    | 3.85    | 3.87    |
| 18                  | 3.90    | 3.81    | 3.87    |
| 22                  | 3.90    | 3.85    | 3.89    |

## 5. Conclusion

In this paper, we solved the problem of estimating part-flow for a pick-and-place task with a multi-robot system. To solve the problem within a short amount of computation time, we viewed the multi-robot system as several single-robot systems, and proposed the M/M/s queuing model to estimate the part-flow for each individual robot. The proposed method was compared with a simulation-based MCS method and an M/M/1 queuing model. The simulation results showed that the part-flow derived by the proposed method can both guarantee the productivity and the task-completion success rate. Compared to the M/M/1 queuing model, the task-completion success rate derived by the proposed method is improved to 98.9%, which can satisfy the requirements of a real application. Compared to the simulation-based MCS method, the computation time for the proposed method is less than 1/10 of that for the MCS method. The processing rate for each robot under different part-flow rates was investigated.
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