Solar Neutrino Solutions in Non-Abelian Flavor Symmetry

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We have studied the large mixing angle MSW solution for the solar neutrinos in the non-abelian flavor symmetry. We predict the MNS mixing matrix taking account of the symmetry breakings.

1 LMA-MSW Solution in Non-Abelian Flavor Symmetry

Recent data in S-Kam favor the large mixing angle MSW (LMA-MSW) solution. How does one get the LMA-MSW solution as well as the maximal mixing of the atmospheric neutrinos in theory? It is not easy to reproduce the nearly bi-maximal mixings with LMA-MSW solution in GUT models[1,2,3].

The non-abelian flavor symmetry $S_{3L} \times S_{3R}$ or $O_{3L} \times O_{3R}$ leads to the LMA-MSW solution naturally[4,5]. The flavor symmetry is broken explicitly by $\phi$-triplet of the Higgs potential $H$. Then, neutrinos have the symmetric traceless tensor 5's of $O(3)$.

$$H \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad M_{\nu} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  (1)

The orthogonal matrix diagonalizes $M_{\nu}$ is

$$F = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}.$$  (1)

The MNS mixing matrix is given as $U_\nu \approx F^T$, and so we predict $\sin^22\theta_{\odot} = 1$ and $\sin^22\theta_{\text{atm}} = 8/9$ in the symmetric limit.

In this talk, we discuss masses and flavor mixings of quarks/leptons in the non-abelian flavor symmetry with the SU(5) GUT[6]. We consider $O(3)_{5^*} \times O(3)_{10} \times Z_6$ symmetry. Our scenario for fermion masses is

- Neutrinos have degenerate masses.
- Quarks/charged-leptons are massless.
- Symmetry breakings give $\Delta m^2$ and other fermion masses.

2 $O(3)_{5^*} \times O(3)_{10} \times Z_6$ Symmetry

Quarks and leptons belong to 5* and 10 of the $SU(5)$ GUT and 3 of the $O(3)$ symmetry. Higgs $H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ belong to 5 (5*) of the $SU(5)$ and 1 of the $O(3)$. Then, neutrinos have the $O(3)_{5^*} \times O(3)_{10}$ invariant mass term

$$\frac{<H>^2}{\lambda} \nu_L \nu_L.$$  (2)

The $Z_6$ symmetry forbids $\psi_{10}(3)\psi_{10}(3)H$, which gives degenerate up-quark masses[6].

The flavor symmetry is broken explicitly by $\Sigma^{(i)}_{5^*}(5,1)$, $\Sigma^{(i)}_{10}(1,5)$ ($i = 1,2$), which transform as the symmetric traceless tensor 5's of $O(3)$. Dimensionless breaking parameters are given as

$$\sigma^{(1)}_{10, 5^*} = \frac{\Sigma^{(1)}_{10, 5^*}}{M_{\phi}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \delta_{10, 5^*},$$

$$\sigma^{(2)}_{10, 5^*} = \frac{\Sigma^{(2)}_{10, 5^*}}{M_{\phi}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \epsilon_{10, 5^*}.$$  (1)

Neutrinos get Majorana masses from a superpotential

$$W = \frac{H^2}{\lambda}(1 + \alpha_1 \sigma^{(i)}_{5^*})\ell,$$  (3)

which yields a diagonal neutrino mass matrix. In order to get the charged lepton masses, we introduce $O(3)_{5^*}$-triplet $\phi_{5^*}(3,1)$ and $O(3)_{10}$-triplet $\phi_{10}(1,3)$. These VEV’s are determined by the superpotential

$$W = Z_{5^*}(\phi_{5^*}^2 - 3v_{5^*}^2) + Z_{10}(\phi_{10}^2 - 3v_{10}^2) + X_{5^*}(a_{(i)}\phi_{5^*}\sigma^{(i)}_{5^*}\phi_{5^*}) + X_{10}(a_{(i)}\phi_{10}\sigma^{(i)}_{10}\phi_{10}) + Y_{5^*}(b_{(i)}\phi_{5^*}\sigma^{(i)}_{5^*}\phi_{5^*}) + Y_{10}(b_{(i)}\phi_{10}\sigma^{(i)}_{10}\phi_{10}).$$
where $Z_{10}$, $5^*$, $X_{10}$, $5^*$, $Y_{10}$, $5^*$ are all singlets of $O(3)_{5} \times O(3)_{10}$. Minimizing the potential, we get
\[
< \phi_{5^*} > \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_{5^*}, \quad < \phi_{10} > \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_{10}.
\]

Masses of charged leptons arise from a superpotential
\[
W = \frac{\kappa_E}{M_f^2} (\bar{\phi}_{10} \phi_{5^*} \ell) \mathcal{H},
\]
which is the realization of "Democratic Mass Matrix",
\[
M_E \propto \begin{pmatrix} v_{5^*} v_{10} \\ M_f^2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.
\]

Adding the superpotential containing the flavor symmetry breaking parameters $\sigma_{5^*}^{(i)}$, $\sigma_{10}^{(i)}$, we get the charged lepton mass matrix:
\[
M_E^H \equiv F^T M_E F = \kappa_E \left( \frac{v_{5^*} v_{10}}{M_f^2} \right) \begin{pmatrix} \epsilon_5 \epsilon_{10} & \epsilon_5 \delta_{10} & \epsilon_{10} \delta_{10} \\ \epsilon_{10} \delta_{10} & \epsilon_5 \delta_{10} & \epsilon_{10} \delta_{10} \\ \epsilon_{10} \delta_{10} & \epsilon_{10} \delta_{10} & \epsilon_{10} \delta_{10} \end{pmatrix},
\]
in which order one coefficients are omitted. The mass ratios are given as
\[
\frac{m_\mu}{m_\nu} \simeq \mathcal{O}(\delta_5 \delta_{10}), \quad \frac{m_e}{m_\tau} \simeq \mathcal{O}(\epsilon_5 \epsilon_{10}).
\]

The quark/lepton masses and mixings fix
\[
\delta_{10} \simeq \lambda^2, \quad \epsilon_{10} \simeq \lambda^3 \sim \lambda^4, \quad \delta_5 \simeq \lambda, \quad \epsilon_5 \simeq \lambda^2.
\]

3 Neutrino Masses and Mixings

Neutrino masses are given as
\[
\begin{align*}
m_1 & \simeq c_\mu (1 + \alpha_1 \delta_5^* + \alpha_2 \epsilon_5^*), \\
m_2 & \simeq c_\mu (1 + \alpha_1 \delta_5^* - \alpha_2 \epsilon_5^*), \\
m_3 & \simeq c_\mu (1 - 2 \alpha_1 \delta_5^*), \quad c_\mu = \sqrt{\frac{H^2}{\Lambda}}
\end{align*}
\]
which leads to (with $\delta_5^* \simeq \lambda, \quad \epsilon_5^* \simeq \lambda^2$)
\[
\left| \frac{\Delta m_{32}^2}{\Delta m_{31}^2} \right| = \left| \frac{2 \alpha_2 \epsilon_5^*}{3 \alpha_1 \delta_5^*} \right| \simeq \frac{1 + \alpha_2 \epsilon_5^*}{1 - \frac{1}{2} \alpha_1 \delta_5^*} \simeq \lambda^2 \sim \lambda.
\]
Putting $\Delta m_{32}^2 = 3 \times 10^{-3} \text{eV}^2$, we predict $\Delta m_{31}^2 \approx (\text{factor}) \times 10^{-4} \text{eV}^2$, which is consistent with the LMA-MSW solution. Flavor mixings come from the charge lepton mass matrix since the neutrino one is diagonal. The charged lepton mass matrix is diagonalized by $V_L^L M_E^H V_L$, in which
\[
V_L \simeq \begin{pmatrix} 1 & \lambda & \lambda^2 \\ -\lambda & 1 & \lambda \\ -\lambda^2 & -\lambda & 1 \end{pmatrix}.
\]
The neutrino mixing matrix is given by $V_L^L F$. We predict
\[
\sin^2 2\theta_\odot = (1 - \frac{4}{3}\lambda^2)^2 \simeq 0.87
\]
\[
\sin^2 2\theta_{\text{atm}} = \frac{8}{9} (1 - \lambda^2)(1 + \frac{1}{\sqrt{2}} \lambda - 2\lambda^2)^2 
\]
\[
\simeq 0.95
\]
\[
|U_{e3}| = \frac{2}{\sqrt{6}} \lambda (1 - \frac{1}{\sqrt{2}} \lambda) \simeq 0.14.
\]

4 Summary

It is remarked that:
\begin{itemize}
\item The solar neutrino mixing $\sin^2 2\theta_\odot$ deviates from the maximal mixing ($\sim 0.87$).
\item The atmospheric neutrino mixing $\sin^2 2\theta_{\text{atm}}$ deviates from $8/9$ depending phase of $\lambda$.
\item $U_{e3}$ is near to the experimental bound of CHOOZ ($\simeq 0.16$).
\end{itemize}

Neutrino masses are degenerated within a factor 2. For example, we get $m_1 \simeq 0.030\text{eV}, m_2 \simeq 0.033\text{eV}, m_3 \simeq 0.058\text{eV}$, which is consistent with $\beta\beta_0$ decay bound.

References

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