Beam deflection estimation by Monte Carlo simulation and Kalman filter based ultrasonic distance sensor

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Abstract
The beam deflection estimation is of primary importance on design stage and health monitoring stage as it provide an indication about the failure of the structure. The bending theory based model is a deterministic model that widely used to determine the beam deflection theoretically. Two significant factors are influenced the beam deflection estimation in bending theory, the applied load and cross section dimensions. Taking the uncertainty of these two factors into account provide an opportunity to understand the uncertainty of the beam deflection estimation. In this paper the so called Monte Carlo Simulation (MCS) is used to create stochastic bending model from bending theory model by generating random load and random cross section dimensions based on the accuracy of measurement devices. On the other hand, the beam deflection can be estimated experimentally within a specific accuracy depending on the accuracy of measuring device. A validation tool is needed to combine the experimental and theoretical results in one model to obtain a more accurate estimator than using experimental results alone or theoretical deterministic model alone. In the present work the Kalman filter (KF) is used to put together the experimental results from a cheap distance sensor (ultrasonic distance sensor) with bending theory results to estimate the deflection of simply supported beam. The KF algorithm taking into account the uncertainty of process and the uncertainty of the sensor, resulted in robust estimation of beam deflection.

Keywords: Beam deflection, Stochastic estimation, Kalman Filter, Bending Analysis, Ultrasonic sensor.

1. Introduction

Deflection of a beam refers to the displacement of the beam from its original position due to applied load on the beam. Beam deflection is usually within acceptable limits to prevent the failure or yield of the structure.
The deflection of a simply supported beam subjected to a concentrated load can be estimated either from bending theory or from experimental work. According to the deterministic model based on bending theory, the deflection of a beam depending on the weight and its position and on the cross section dimensions. Uncertainty of the model input leads to an uncertainty on the model output, which is the beam deflection in this case. On the other hand, the experimental results usually associated with the measurement error which depends on the measurement device accuracy. Taking the measurement accuracy into account can provide an indication about the uncertainty in the measurement of beam deflection.

Regarding of the prediction model uncertainty, the stochastic methods has been used for deterministic model simulation. Stochastic finite volume is adapted to understand the effect of the variation of system parameters on the deflection of a simply supported beam [1]. The study considered a stochastic modulus of elasticity and stochastic load. Monte Carlo simulation is used to generate random samples and the cost of estimation reduced by using Neural network as a dimension reduction algorithm.

Neural network is used to accelerate the Monte Carlo simulation, that used to create samples for one dimensional stochastic finite element[2].Numerical methods are widely used in stochastic manor such as stochastic finite difference methods [3], and stochastic finite element methods [4] and spectral stochastic finite element methods [5,6,7,8]. Most of the above methods need a method for sampling to create more examples. Monte Carlo method widely used to understand the stochastic behaviour of several systems in different fields. The method is based on generating stochastic random input variables within the variables variation to estimate the variation of the output.

Measurement uncertainty can provide another way to understand the beam deflection uncertainty. A cantilever is used to estimate the uncertainty in the measuring of static in plane and dynamic out of plane displacements based on digital image technique [9]. The uncertainties estimation is similar to the procedure introduced in [10] except the ability of applying in plane bending loads. The aim of the study in [10] was to establish a standard test and test specimen to improve the predictability of the optical sensor with improved strain estimation confidence.

[11] introduce a state estimation relating the temperature to the strain of a simple composite beam under static load to study the effect of temperature on the structure damage of the beam. Kalman filter was used to estimate accurately the neutral axis location as a health monitor for the beam.

A validation procedure for solid mechanics problem is developed [12]. The proposed approach based on the comparison between the experimental and computational model results. The computational model is image decomposition technique, that produce a feature vector representing the characteristic vector of strain field. The latter is used as an output of the computational model to compare with a corresponding measurements vector. A load monitoring of a simply supported beam is estimated by using Kalman Filter (KF) and fibre-optic strain sensor [13]. The applied static loads are limited to axial and bending loads, the KF results showed a better correlation than direct estimation of load monitoring. Kalman
filter widely used to improve the quality of estimation by reducing the variance of the predicted variable. Kalman filter is an uncertainty state estimation algorithm based on set of observations (measurements) and a state transition model [14,15]. The algorithm consists of a prediction step and update step. The prediction based on a deterministic model for estimation, whereas the update step aims to update the estimated value from measurements. The filter provides more accurate estimation by taking into account the variance of the measurements and the variance of the prediction model.

In the present work, a non parametric stochastic beam deflection two estimators are proposed. The first estimator is based on Monte Carlo simulation, whereas the second one is relied on Kalman filter algorithm. The Kalman filter is used to validate the experimental deflection resulted from ultrasonic distance sensor. The above two methods is used to understand the uncertainty of deflection estimation for simply supported steel beam.

2 Simply supported beam deflection estimation

In this work a simply supported steel beam is used as shown in Figure (1) below, according to the bending moment theory the deflection of the beam can be estimated theoretically from [16]:

\[ y = \frac{1}{EI} \left( \frac{WLx^2}{8} - \frac{Wx^3}{12} - \frac{WL^3}{48} \right) \]  

(1)

Where: \( y \) is the deflection, \( W \) is applied concentrated load, \( E \) is the modulus of elasticity, \( I \) is the second moment of area and \( x \) is the distance from centre as shown in Figure 1.

Experimentally, a contactless ultrasonic distance sensor type HC-SR04 is used to measure the beam deflection. The sensor has two transducers, transmitter and receiver. Transmitter sends a series of ultrasonic pulses, that run into the beam and reflect to the receiver. Ultrasonic signals travel at the speed of sound. The time delay between transmitting and receiving the signal is used to calculate the distance between the beam and the sensor.

\[ D = (\Delta t / 2) \times C \]  

(2)

Where: \( D \) is the distance between the sensor and the beam, \( \Delta t \) is the time delay between transmitting and receiving the signal and \( C \) is the speed of sound.

Speed of sound is a function of temperature and humidity of the air, which can be estimated from [17].

\[ C = 331.4 + (0.606T) + (0.0124H) \]  

(3)

In order to measure the temperature and humidity a DHT11 temperature and humidity sensor is added to the distance measuring system. An Arduino Uno board is used to read the data from DHT11 and HC-SR04 sensor and then estimate the distance according to equation 2 and
equation 3. The ultrasonic sensor located down the beam by 37 centimetres in the middle between the load and the support, as a result the deflection \( y \) will be equal to \( (37-D) \) in the position where \( x=18.75 \) as shown in Figure(1) and Figure(2).

Figure 1: Beam system (A) free body diagram, (B) schematic diagram

Figure 2: Experimental setup
2.1 Monte Carlo Simulation

Monte Carlo Simulation consider a deterministic model as the model in equation 1 to generate all the possible results by examining a range of values of the input. In the current work, the source of uncertainty comes from variation in the load \( W \) and variation in the dimensions. Algorithm 1 shows the Monte Carlo algorithm adaptation for simply supported beam with concentrated load based on variable load and variable dimensions:

Algorithm 1: Monte Carlo Simulation for beam deflection

| Stochastic Load | Stochastic Dimension | Stochastic Load&Dimension |
|-----------------|----------------------|---------------------------|
| \( W \leftarrow N(\mu_w, \sigma_w^2) \) | \( b \leftarrow \mu_b \) | \( W \leftarrow N(\mu_w, \sigma_w^2) \) |
| \( b \leftarrow \mu_b \) | \( h \leftarrow \mu_h \) | \( b \leftarrow N(\mu_b, \sigma_b^2) \) |
| \( h \leftarrow \mu_h \) | \( I \leftarrow \frac{bh^3}{12} \) | \( h \leftarrow N(\mu_h, \sigma_h^2) \) |

where: \( \mu_w \): mean value of the weight \( W \).
\( \sigma_w \): standard deviation of weight \( W \).

\[ y \leftarrow \frac{1}{EI} \left[ \frac{WLx^2}{8} - \frac{Wx^3}{12} - \frac{WL}{48} \right] \]

\[ y \leftarrow \frac{1}{EI} \left[ \frac{W^2}{8} - \frac{Wx^3}{12} - \frac{WL}{48} \right] \]

\[ y \leftarrow \frac{1}{EI} \left[ \frac{W^2}{8} - \frac{Wx^3}{12} - \frac{WL}{48} \right] \]

\( \mu_b \): mean value of dimension \( b \).
\( \sigma_b \): standard deviation of dimension \( b \).

\( \mu_h \): mean value of dimension \( h \).
\( \sigma_h \): standard deviation of dimension \( h \).

2.2 Kalman filter based ultrasonic sensor

Kalman Filter combines the deterministic model (deflection in equation 1) and the ultrasonic sensor reading to form a new estimator, which better than the estimation from measurement alone or from physical model alone [18]. The proposed system is time uni variate system and one dimension state variable (the deflection), in this case the kalman filter algorithm can be implemented as follows:

Algorithm 2: Kalman Filter algorithm

For \( j \leftarrow 1 \) to \( 8 \) do

\[ W \leftarrow W^j \]

\[ y_{i,j} \leftarrow \frac{1}{EI} \left[ \frac{W^jLx^2}{8} - \frac{W^jx^3}{12} - \frac{W^jL}{48} \right] \]

The deterministic model from bending theory
\[
P_{i,1} \leftarrow \text{initial value}
\]

\[P: \text{is the uncertainty of the deterministic model.}\]

\[\text{For } i \leftarrow 1 \text{ to } n \text{ do}\]

\[
\hat{y}_i \leftarrow \hat{y}_{i-1} + N(0, \nu) \quad \nu: \text{is the standard deviation of process equal to that of MCS.}
\]

\[P_i \leftarrow P_{i-1}\]

\[
\text{Measurement update}\]

\[K_i \leftarrow P_i / (P_i + R) \quad \text{compute the Kalman gain}\]

\[\hat{y}_i \leftarrow \hat{y}_i + K_i (x_i - \hat{y}_i) \quad \text{update the estimation}\]

\[P_i \leftarrow (1 - K_i)P_i \quad \text{update of } P_i\]

\[End\]

\[End\]

3. Results and discussion

In order to understand the behaviour of the system under uncertain load and uncertain dimensions. Algorithm 1 is used with the following parameters. The uncertainty of the load is corresponding to the uncertainty of the load measurement device which is equal to 0.5 g and the dimensions variation is equal to the vernier calliper accuracy which is 0.02 mm. Figures (3) and (4) show the deflection estimation for 8 different loads with stochastic load and stochastic dimension respectively, whereas Figure(5) display combined effect of stochastic load and dimension. It can be seen that the stochastic load has a very small effect on deflection estimation, which can be neglected. The combined effect of stochastic load and dimension in Figure(5) is equivalent to stochastic load only Figure (4). That can be understood from deterministic model (equation (1)), where the deflection proportional to load with power of 1, while it proportional to the dimension with power of 4. Another point to note is that the uncertainty of the weighting measuring device used in this work is small (0.5 g) compared to the application.

On the other hand, The deflection corresponding for load \( W \) is measured for \( n \) times resulted in \( n \) deflection estimation. The measured values for each deflection give an indication about the uncertainty of the ultrasonic distance sensor used in this work. To improve the quality of estimation of the deflection Kalman filter is used based on Algorithm 2. the Kalman filter in Algorithm 2 is used to estimate the deflection from ultrasonic measurement system explained in section 2. The initial covariance \( (P_{i,1}) \) is set to 1 as an initial value and then updated for each iteration in covariance update step. Each iteration corresponding to \( i \)th measurement from \( n \) measurements for weight \( W \). The process variance \( \nu \) is assumed equivalents to the variance of Monte Carlo simulation deflection. Figure (6) display the deflection obtained experimentally, by Kalman filter and by bending theory model for eight weights used in this work. Table (1) shows the mean value and variance of
measurement of Monte Carlo Simulation (MCS), Kalman Filter (KF) and Ultrasonic Distance Sensor (UDS). The deflection from KF is better than that obtained experimentally or from MCS, where KF taking the experimental and theoretical uncertainties into account rather than experimental alone or theoretical alone.

Figure (3) Deflection histogram based on stochastic weight
Figure(4) Deflection histogram based on stochastic dimension

Figure(5) Deflection histogram based on stochastic load & dimension
Figure (6) Kalman filter deflection estimation from experimental reading
Figure (7) Kalman gain for beam deflection estimation

Table 1: Standard deviation comparison

| Weight (kg) | Standard deviation (σ) of deflection | Mean (μ) of estimation (m) × 10⁻³ |
|-------------|--------------------------------------|-----------------------------------|
|             | MCS × 10⁻³ | KF × 10⁻³ | UDS | MCS | KF | UDS |
| 0.5         | 0.0166      | 0.1526    | 0.206 | 0.0011 | 0.0037 | 0.0040 |
| 1           | 0.0333      | 0.1665    | 0.146 | 0.0022 | 0.0046 | 0.0048 |
| 1.5         | 0.0499      | 0.2657    | 0.236 | 0.0033 | 0.0051 | 0.0053 |
| 2           | 0.0666      | 0.4631    | 0.263 | 0.0044 | 0.0069 | 0.0061 |
| 2.5         | 0.0832      | 0.4717    | 0.233 | 0.0056 | 0.0074 | 0.0079 |
| 3           | 0.0998      | 0.2049    | 0.219 | 0.0067 | 0.0086 | 0.0092 |
| 3.5         | 0.1165      | 0.1500    | 0.235 | 0.0078 | 0.0100 | 0.0099 |
| 4           | 0.133       | 0.3328    | 0.240 | 0.0089 | 0.0105 | 0.0111 |
4. Conclusion

Experimental and theoretical beam deflection estimation are not identical. Theoretically, the theoretical model is considered bending effect rather than shear effect, which can influence the accuracy of estimation. Another point to note is that the theoretical model is deterministic model without a stochastic sense. Experimentally, the measurement sensor has a measurement error resulted in a fluctuation in estimation. Monte Carlo simulation is used to understand the stochastic behaviour of the bending theory to improve the theoretical estimation. In the current study the uncertainty of two parameters are considered, the load uncertainty and dimension uncertainty. The obtained results showed that the load uncertainty is very small and can be neglected compared to the dimension uncertainty.

On the other hand, the Kalman filter is implemented to validate the experimental results based on correcting the experimental estimation from theoretical model. The KF taking into account the uncertainty of measurement sensor and the uncertainty of theoretical model resulted in Kalman gain of 0.18 for all eight loads studied in this work. The KF converges after 100 iterations with small uncertainty compared to sensor reading for one load. In sum, the KF showed a good ability by combining the experimental and theoretical model to improve the quality of estimation. The Monte Carlo simulation gives an indication about the
uncertainty in the theoretical model resulted from uncertain model parameters without taking into account the experimental reading. The proposed two methods should be considered in design and construction because of there are rationale behind each one, that can play a role in beam deflection estimation.

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