Comment on “Physics without determinism: Alternative interpretations of classical physics”

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Comment on “Physics without determinism: Alternative interpretations of classical physics”,
Phys. Rev. A, 100:062107, Dec 2019

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Abstract

The paper “Physics without determinism: Alternative interpretations of classical physics” [Phys. Rev. A, 100:062107, Dec 2019] defines finite information quantities (FIQ). A FIQ expresses the available information about the value of a physical quantity. We show that a change in the measurement unit does not preserve the information carried by a FIQ, and therefore that the definition provided in the paper is not complete.

The expression of the state of knowledge about a measurand as a probability distribution (or some summary of it, such as its mean and standard deviation) is the conventional approach for expressing a measurement result [1–4]. However, it does not intuitively parallel the much more immediate concepts of “certain” and “uncertain digits” that every experimentalist feels when taking note of a measurement outcome in the lab notebook.

In [5], Del Santo and Gisin introduce the concept of finite information quantities (FIQ). A FIQ ranging in the interval [0, 1] is expressed by the binary number \( Q = 0.Q_1Q_2Q_3 \ldots \), where the individual bits \( Q_k \) are Bernoulli random variables having propensities \( q_k \) for the realisation of the case \( Q_k = 1 \). A specific FIQ \( Q \) is thus defined by the vector of propensities \( q = [q_1, q_2, \ldots, q_k, \ldots, q_M, \frac{1}{2}, \frac{1}{2}, \ldots] \) of its bits \( Q_k \); it is assumed that \( q_k = \frac{1}{2} \) for \( k > M \), i.e., all bits beyond position \( M \) have a 50% propensity of being either 0 or 1 and therefore carry no information. Only a finite number \( M \) of propensities are needed to specify \( Q \).

The FIQ concept is very appealing and it is tempting to adopt it to express the value and uncertainty of a quantity as an alternative to probability distributions. However, for the concept of FIQ to become a practical alternative to the current way of representing the state of knowledge about a quantity, it is mandatory that calculations with them be possible and, hopefully, simple.

Consider for example the expression of the value of a quantity, traditionally written as \( Q = \{Q\} [U] \), where \( \{Q\} \) is the numerical value and [U] is the unit. Changing the unit to \( U' = U/L \), \( L \) being a constant, implies \( Q = \{Q'\} [U'] \), with \( \{Q'\} = L\{Q\} \). So, even such an elementary transformation as the change of measurement unit implies the multiplication of a FIQ by a constant.

Indeed, the FIQ definition suggests that it is possible to identify simple, practical calculation rules operating on the finite (and, intuitively, small) number of indeterminate bits and their propensities; rules suitable to be converted in efficient computation algorithms.
The arithmetic relevant to a unit change (Appendix A) shows that the transformation $Q' = LQ$ generates bits $Q'_k$ of $Q'$ which are not mutually independent even if the original $Q_k$ bits are independent. Therefore, expressing $Q'$ by providing only the propensities $q'_k$ of its individual bits deletes some of the original information.

Random variables $Q$ with independent binary digits $Q_k$ have been considered in mathematical literature [6–8]. In general, $Q$ has a ‘reasonable’ probability density function (pdf) only if the $q_k$ satisfy strict conditions, and in that case the pdf is necessarily an exponential [6]; otherwise, it becomes a fractal [7], hence difficult to associate with a physical quantity.

In conclusion, it appears that a specification of the state of knowledge about a quantity $Q$ by means of a FIQ should also include information on the dependencies among the $Q_k$, and therefore that, although the FIQ concept might be physically sound and useful, its definition as given in [5] is not complete, and deserves further development.

**Appendix A: Minimal FIQ maths**

A FIQ arithmetics can be established by generalizing operations on binary numbers. The sum $S = Q + R = 0.S_1S_2S_3 \ldots$ of two FIQs, $Q = 0.Q_1Q_2Q_3 \ldots$ and $R = 0.R_1R_2R_3 \ldots$, is given by the full adder rule, Tab. I.

| $Q_k$ | $R_k$ | $C_{k+1}$ | $S_k$ | $C_k$ |
|-------|-------|-----------|-------|-------|
| 0     | 0     | 0         | 0     | 0     |
| 0     | 0     | 1         | 1     | 0     |
| 0     | 1     | 0         | 1     | 0     |
| 0     | 1     | 1         | 0     | 1     |
| 1     | 0     | 0         | 1     | 0     |
| 1     | 0     | 1         | 0     | 1     |
| 1     | 1     | 0         | 0     | 1     |
| 1     | 1     | 1         | 1     | 1     |

**TABLE I.** Binary full adder truth table. $C_k$ is the carry bit.

If $q$ is the vector of propensities associated with $Q$, and $r$ with $R$, then under the as-
sumption of independence of $q_k$ and $r_k$, the propensity $s_k$ of each sum bit $S_k$ can be written as the sum of the four propensities of the $S_k = 1$ cases in Tab. I:

$$s_k = (1 - q_k)(1 - r_k)c_{k+1} + (1 - q_k)r_k(1 - c_{k+1})$$
$$+ q_k(1 - r_k)(1 - c_{k+1}) + q_k r_k c_{k+1}$$
$$= q_k + r_k + c_{k+1}$$
$$- 2(q_k r_k + q_k c_{k+1} + r_k c_{k+1}) + 4q_k r_k c_{k+1}$$

(A1)

and similarly the propensity $c_k$ of the carry bit $C_k$ is

$$c_k = q_k r_k + q_k c_{k+1} + r_k c_{k+1} - 2q_k r_k c_{k+1}$$

(A2)

For example for the case $c_{k+1} = \frac{1}{2}$, we have $s_k = \frac{1}{2}$ and $c_k = \frac{1}{2}(q_k + r_k)$: the information provided by $q_k$ and $r_k$ is transferred, through the carry bit $C_k$, to bit $S_{k-1}$.

Multiplication by a deterministic constant $L$ can be performed by repeated shifting and addition. Table II gives a simple example. If $P = LQ$, where $Q = [0, 0, q_3, \frac{1}{2}, \ldots]$ and $L = (11)_2 = (3)_{10}$, then

| 0. | 0 | 0 | $Q_3 \ldots$ |
|----|---|---|----------------|
| $\times$ | 1 | 1 |
| 0. | 0 | 0 | $Q_3 \ldots$ |
| + 0. | 0 | $Q_3$ | $Q_4 \ldots$ |
| = 0. | $P_1$ | $P_2$ | $P_3 \ldots$ |

TABLE II. Multiplication table, $P = LQ$ where $Q = 0.0Q_2Q_3\ldots$ and $L = (11)_2 = (3)_{10}$.

$L = (11)_2 = (3)_{10}$, then

$$p_1 = \frac{1}{2}q_3^2 + \frac{1}{4}q_3,$$

$$p_2 = q_3 - q_3^2 + \frac{1}{4},$$

$$p_3 = \frac{1}{2}, \ldots$$

(A3)

The propensity of occurrence of specific digit couples can also be computed. For example, denoting as $p_{12}$ the propensity of the event \{ $P_1 = 1, P_2 = 1$ \} we have $p_{12} = 0$ (to have $P_1 = 1$, it should occur that $Q_3 = 1$ and $C_3 = 1$ at the same time, hence $C_2 = 1$. However,
the case \( \{Q_3 = 1, C_3 = 1\} \) always generates \( P_2 = 0 \), so \( \{P_1 = 1, P_2 = 1\} \) is never possible. Since \( p_{12} \neq p_1 p_2 \), bits \( P_1 \) and \( P_2 \) are not independent.

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