Structure of the $\Lambda(1405)$ baryon resonance from its large $N_c$ behavior

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We study the behavior with the number of colors ($N_c$) of the two poles associated to the $\Lambda(1405)$ resonance obtained dynamically within the chiral unitary approach. The leading order chiral meson-baryon interaction manifests a nontrivial $N_c$ dependence for SU(3) baryons, which gives a finite attractive interaction in some channels in the large $N_c$ limit. As a consequence, the SU(3) singlet $(\bar{K}N)$ component of the $\Lambda(1405)$ survives in the large $N_c$ limit as a bound state, while the other components dissolve into the continuum. The $N_c$ dependence of the decay widths shows different behavior from the general counting rule for a $qqq$ state, indicating the dynamical origin of the two poles for the $\Lambda(1405)$ resonance.

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The quest for the understanding of the nature of the hadronic spectrum is one of the most challenging issues in particle physics. Especially the structure of hadronic resonances is one of the cornerstones in this field. Recently it has been suggested that multiquark and/or hadronic components of some resonances in intermediate energies prevail over the simple mesonic $qq$ and baryonic $qqq$ components. For instance, the light scalar mesons have been investigated in the four-quark picture 1, in the mesonic molecular picture 2, and in lattice QCD 3. Methods to clarify the internal structure of the hadrons are called for. This is one of the main aims of the present work, in the baryonic sector.

Despite the success of QCD as the theory for the strong interactions, the inherent confinement properties of quarks and gluons make its direct applicability to investigate the hadron structure far from being feasible. One of the most powerful tools to investigate the properties of hadrons are the effective theories of QCD, such as chiral perturbation theory (ChPT) 4 and its recent properties of hadrons are the effective theories of QCD, such as chiral perturbation theory (ChPT) 4 and its recent approximation to QCD valid for the whole energy region, and enables us to investigate the qualitative features of hadrons 12. In past years the dependence on $N_c$ of the resonance properties within the chiral unitary approach has shown up as a powerful tool to discriminate the quark structure for particular mesonic resonances 13,14. The study of the $N_c$ scaling allows for a clear identification of $qq$ mesonic states based on the general scaling properties of their masses as $O(1)$ and the widths as $O(1/N_c)$ 12. These $N_c$ behaviors were compared to the chiral unitary approach predictions in order to clarify the internal structure of the resonances. Recent interest in the large $N_c$ argument of low-energy hadrons is also related to the holographic QCD 15.

In this paper, we present the first study of the $N_c$ behavior of the physical baryon resonances in the chiral unitary approach, focusing on the structure of the $\Lambda(1405)$. The study of the baryon resonances has an important difference from the meson sector: the nontrivial $N_c$ dependence of the leading order chiral interaction 16. The $N_c$ dependence stems from the change of the flavor representation of the baryons with $N_c$, when the number of flavors is larger than 2 17. As a consequence, the meson-baryon interactions for some channels remain finite in the large $N_c$ limit, leading to nontrivial consequences for the generated resonances. The study of the $N_c$ behavior also tells us about the quark structure of the generated resonances. The general $N_c$ counting rule for ordinary $qqq$ baryons indicates the scaling of the decay width as $\Gamma \sim O(1)$, the mass $M_{br} \sim O(N_c)$ and the excitation energy $\Delta E \sim O(1)$ 18. Hence any significant deviation from these behaviors indicates that the molecular, or dynamical, component of the $\Lambda(1405)$ resonance dominates over the $qqq$ contribution. The $N_c$ behavior of the baryon resonances has been studied in Ref. 19 in the SU(6) symmetric limit. Here we consider physical $\Lambda(1405)$ resonance including the flavor symmetry break-
The $\Lambda(1405)$ in the chiral unitary approach is dynamically generated in the $s$-wave meson-baryon scattering with $S = -1$ and $I = 0$. Based on the N/D method \[6\], the coupled-channel scattering amplitude $T_{ij}$ is given by the matrix equation

$$T = [1 - VG]^{-1}V,$$  

(1)

where $V_{ij}$ is the interaction kernel and the function $G_i$ is given by the dispersion integral of the two-body phase space $\rho_i(s) = 2M_i \sqrt{(s - s_i^+)(s - s_i^-)/(8\pi s)}$ in a diagonal matrix form by

$$G_i(W) = -\hat{a}(s_0) - \frac{s - s_0}{2\pi} \int_{s_i^+}^{\infty} ds' \frac{\rho_i(s')}{(s' - s)(s' - s_0)},$$  

(2)

where $W$ is the center-of-mass energy, $s_0$ the subtraction point, $\hat{a}(s_0)$ the subtraction constant, and $M_i$ ($m_i$) is the mass of the baryon (meson) of the channel $i$. The degree of freedom introduced by the subtraction constant is equivalent to the one from the cutoff of the loop integral in the scattering equation, and is fixed following the prescription given in Refs. \[6\]. Namely, we impose the condition $G_i(\mu) = 0$, where $\mu$ is the matching scale of the full amplitude $T_{ij}$ to the interaction kernel $V_{ij}$ and we take $\mu = M_i$. For the physical scattering case, the scale $\mu$ is taken to be $M_\Lambda$. We will also examine the three-momentum cutoff scheme later.

The interaction kernel $V_{ij}(W)$ in Eq. (1) is the driving force to generate the resonances. The leading order term of ChPT provides the $s$-wave meson-baryon interaction for energy $W$ as

$$V_{ij}(W) = -C_{ij} \frac{1}{4f^2} (2W - M_i - M_j) \eta_i \eta_j,$$  

(3)

where $\eta_i = \sqrt{(M_i + E_i)/(2M_i)}$, $C_{ij}$ expresses the coupling strength, $f$ is the pseudoscalar meson decay constant, and $E_i$ is the energy of the baryon in channel $i$. Equation (3) is known as the Weinberg-Tomozawa (WT) term derived using current algebra \[20\].

The $N_c$ dependence of the parameters is introduced as $M_i \propto N_c$, $m_i \propto 1$, and $f \propto \sqrt{N_c}$ \[21\]. In the following, we will discuss the $N_c$ dependence in the coupling strengths $C_{ij}$ in SU(3) and isospin basis.

Let us first consider a simple case in the SU(3) symmetric limit. The coupling strengths $C_{ij}$ are determined by group theoretical arguments, including the $N_c$ dependence \[10\]. In the SU(3) basis, the relevant coefficients for $S = -1$ and $I = 0$ are given by the diagonal matrix

$$C_{ij}^{SU(3)}(N_c) = \text{diag} \left( \frac{9 + N_c}{2}, 3, 3, \frac{1 - N_c}{2} \right),$$  

(4)

with the channels being $1$, $8$, $8'$, and $27$. At $N_c = 3$, we find attractive interaction for the singlet and the two octet channels. It has commonly been considered that the WT term (3) behaves as $1/N_c$ because of the $1/f^2$ factor \[21\]. Here we would like to emphasize that, in the baryon case, the linear $N_c$ dependence on the coupling strength $C$ indicates an $O(1)$ attractive (repulsive) interaction in the singlet (27-plet) channel even at the large $N_c$ limit. This is not contradictory to the general counting rule of meson-baryon scattering \[12, 21\].

The derivation of Eq. (4) is based on the standard $N_c$ extension for the baryon, that is, the irreducible representation $[p, q]$ of the flavor SU(3) at $N_c = 3$ is extended to $[p, q + (N_c - 3)/3]$ for an arbitrary $N_c$. In this prescription, the spin, isospin, and strangeness of the baryon are the same with those at $N_c = 3$, while the baryon has different charge and hypercharge from those at $N_c = 3$. Here we adopt this standard $N_c$ extension, since it is convenient for the flavor SU(3) breaking. There are two more extensions \[17\]: $[p, q] \rightarrow [p + (N_c - 3)/3, q + (N_c - 3)/3]$ and $[p, q] \rightarrow [p + N_c - 3, q]$. These extensions have some advantages, but the baryons constructed in these ways have unphysical strangeness and spin. We have confirmed that it is also the case in these $N_c$ extensions that the singlet (27-plet) channel has positive (negative) $N_c$ dependence as seen in Eq. (4).

Since in the SU(3) basis the scattering equation (1) is a set of single-channel problems, the existence of bound states can be studied by comparing the coupling strengths (4) with the critical coupling $C_{\text{crit}}$ introduced in Refs. \[16\]:

$$C_{\text{crit}}(N_c) = \frac{2[f(N_c)]^2}{m_i [-G(M_i(N_c) + m_i)].}$$  

(5)

If the coupling strength of the channel $i$ is larger than this critical value, a bound state is generated. Studying the $N_c$ dependence, we find that $C_{\text{crit}}(N_c)$ increases more slowly than the $N_c/2$ growth of the coupling strength in the singlet channel. This means that the bound state at $N_c = 3$ in the singlet channel \[6\] still survives in the large $N_c$ limit. This is a nontrivial consequence of the $N_c$ dependence of the WT interaction for the baryon. On the other hand, the attraction in the octet channels becomes smaller than $C_{\text{crit}}(N_c)$ at larger values of $N_c$ and the bound states will disappear in the large $N_c$ limit. The $N_c$ dependence of the coupling strengths given in Ref. \[10\] shows that $\Lambda(1405)$ is the only example of hadron bound states of $qqq$ baryon and the Nambu-Goldstone boson in the large $N_c$ limit.

Now we turn to the central problem of the physical $\Lambda(1405)$. We introduce SU(3) breaking in the masses of the particles to discuss physical $\Lambda(1405)$ and work in the isospin basis. The coupling strengths (4) can be translated into the isospin basis by the SU(3) Clebsch-Gordan coefficients with $N_c$ dependence \[22\] as

$$C_{ij}^I(N_c) = \begin{pmatrix}
\frac{N_c + 3}{2} & -\sqrt{N_c - 3}/2 & \sqrt{N_c - 3}/2 \\
-\sqrt{N_c - 3}/2 & 0 & \sqrt{N_c - 3}/2 \\
\sqrt{N_c - 3}/2 & -\sqrt{N_c - 3}/2 & 0
\end{pmatrix}^{\frac{9 + N_c}{2}, 3, 3, \frac{1 - N_c}{2}}.$$  

(6)
with the channels being $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, and $K\Xi$.

The strengths in diagonal $\bar{K}N$ and $K\Xi$ channels are $O(N_c)$, and the negative $N_c$ dependence in the $K\Xi$ channel changes the sign of the interaction from attractive to repulsive for $N_c > 9$. On the other hand, the off-diagonal elements (and diagonal $\pi\Sigma$ and $\eta\Lambda$ ones) are $O(\sqrt{N_c})$ or $O(1)$, so that any transitions among these channels vanish either as $1/\sqrt{N_c}$ or $1/N_c$. This means that the meson-baryon scattering in this sector becomes essentially a set of single-channel problems in the large $N_c$ limit, even with the SU(3) breaking. The coupling strength of $\bar{K}N$ channel in the large $N_c$ limit is the same as the one of the singlet channel in the SU(3) basis. Therefore, following the same argument as in the SU(3) symmetric case, we conclude that there is one bound state in the $\bar{K}N$ channel in the large $N_c$ limit. It was found in Ref. 10 that the $\bar{K}N$ interaction develops a bound state at $N_c = 3$, when the transition to other channels is switched off. Thus, as in the SU(3) singlet channel, the $\bar{K}N$ bound state found at $N_c = 3$ remains in the large $N_c$ limit in contrast to the mesonic resonances, while the other states, such as a resonance in $\pi\Sigma$ channel, will disappear.

To study the $N_c$ scaling of the generated resonances, it is interesting to observe how the positions of the resonance poles evolve when $N_c$ increases. At $N_c = 3$, we have found two poles associated with the physical $\Lambda(1405)$ at $z_1 = -20 - 15i$ MeV and $z_2 = -61 - 66i$ MeV [6, 8], of which real parts correspond to the excitation energy measured from the $\bar{K}N$ channel threshold and the imaginary part expresses the half width. We show in Fig. 1 the trajectories of the pole positions from $N_c = 3$ to 12. As $N_c$ increases, the pole $z_1$ approaches the real axis with reducing the width, while $z_2$ moves to higher energy region and the imaginary part increases with $N_c$. The substantial change of the pole $z_2$ between $N_c = 4$ and 5 is due to the fact that the pole crosses the $\bar{K}N$ threshold and hence the width suddenly increases because the important $\bar{K}N$ decay channel opens.

An important finding in Fig. 1 is that, as $N_c$ increases, one resonance tends to become a bound state while the other tends to dissipate by moving away from the physical axis. This implies that the width of the resonance associated to the pole $z_1$ ($z_2$) decreases (increases) as $N_c$ increases, which is in obvious contradiction to the dominance of the $qqq$ component, whose decay widths should scale as $\Gamma \sim O(1)$ [18]. This observation supports the idea of the dynamical origin of the $\Lambda(1405)$ resonance against a $qqq$ dominant composition.

As discussed in Ref. 14, the cutoff scale may have an $N_c$ dependence. We have examined the cutoff regularization scheme for the loop function (2) with two different $N_c$ dependences of the three-momentum cutoff, $q_{\text{max}} \sim 1$ and $q_{\text{max}} \sim \sqrt{N_c}$. The results among the different regularization schemes are qualitatively similar [23]. Hence, the conclusions drawn here are independent on the regularization procedure and its $N_c$ dependence.

In order to illustrate the properties of the dynamically generated resonances in large $N_c$, it is very interesting to discuss the coupling strengths, $g_i$, to the different states, which are evaluated by the residues of the scattering amplitude at the pole positions. In Fig. 2 we show the couplings of the poles to the different isospin states normalized to the $\bar{K}N$ channel for reference: $|g_i|/|g_{\bar{K}N}|$. We can see that the pole $z_1$ (upper panel), which tends to the bound state, couples dominantly to the $\bar{K}N$ state. On the contrary, the pole $z_2$ (lower panel), which dissolves into the continuum for large $N_c$, couples dominantly to $\pi\Sigma$ and $K\Xi$ component becomes less important. Analogously, the one resonance tends to become a bound state while the other tends to dissipate by moving away from the physical axis. This implies that the width of the resonance associated to the pole $z_1$ ($z_2$) decreases (increases) as $N_c$ increases, which is in obvious contradiction to the dominance of the $qqq$ component, whose decay widths should scale as $\Gamma \sim O(1)$ [18]. This observation supports the idea of the dynamical origin of the $\Lambda(1405)$ resonance against a $qqq$ dominant composition.

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gously, by studying the residues of the poles in the SU(3) basis, we find that the $z_1$ pole becomes dominantly the flavor singlet state, while the $z_2$ pole is dominantly by the other components different to the singlet. These analyses of the coupling strengths indicate that the dominant component of the pole becoming bound state is flavor singlet ($\bar{K}N$) in the SU(3) (isospin) basis, whereas such component in the dissipating resonance becomes less important. Hence, we expect that the pole $z_1$ is smoothly connected to the bound state in the idealized large $N_c$ limit.

In conclusion, we have addressed the fundamental problem of the structure of the $\Lambda(1405)$ baryonic resonance studying the $N_c$ behavior of the two poles associated to it in the framework of the chiral unitary approach. Based on the consideration of the standard $N_c$ dependence of the parameters of the theory (the meson decay constant and the hadron masses), we have discussed important and nontrivial $N_c$ dependence in the Weinberg-Tomozawa interaction, which leads to an $O(1)$ attraction in the large $N_c$ limit for the flavor singlet and $\bar{K}N$ channels in the SU(3) and isospin bases, respectively. The attraction in these channels is strong enough to create a baryonic bound state. The $N_c$ scaling of the $\Lambda(1405)$ resonance shows that one of the two poles tends to become a bound state as $N_c$ increases, while the other pole eventually dissolves into the scattering states. The $N_c$ scaling of the decay widths of the poles turns out to be at odds with usual QCD predictions for the $N_c$ dependence for genuine $qqq$ baryons, indicating that the structures of these poles are not predominantly $qqq$ states. Thus these findings reinforce the dynamically generated nature of the $\Lambda(1405)$ resonance. Both the results obtained here and the methodology used about the $N_c$ behavior of the baryonic resonance go one step forward in the understanding of the connection with the underlying QCD degrees of freedom and can be used to shed light into the structure of other baryonic resonances.

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