Accuracy Improvement of Zenith Tropospheric Delay Estimation Based on GPS Precise Point Positioning Algorithm

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Abstract In the precise point positioning (PPP), some impossible accurately simulated systematic errors still remained in the GPS observations and will inevitably degrade the precision of zenith tropospheric delay (ZTD) estimation. The stochastic models used in the GPS PPP mode are compared. In this paper, the research results show that the precision of PPP-derived ZTD can be obviously improved through selecting a suitable stochastic model for GPS measurements. Low-elevation observations can cover more troposphere information that can improve the estimation of ZTD. A new stochastic model based on satellite low elevation cosine square is presented. The results show that the stochastic model using satellite elevation-based cosine square function is better than previous stochastic models.

Keywords precise point positioning; stochastic model; zenith tropospheric delay; cosine; GPS

Introduction

The zenith tropospheric delay (ZTD) plays a key role in numerical weather prediction, climate research, and atmosphere science, and hence, the acquisition of precise ZTD in GPS is very valuable and beneficial.

The least-squares (LS) estimation method is usually introduced in the processing of GPS observations. In order to use the LS method for ZTD estimation, both the functional model and the stochastic model of GPS measurements need to be properly defined.¹⁻⁴ The functional model, also called the mathematical model, describes the mathematical relationship between the GPS observations and the unknown parameters, while the stochastic model describes the statistical characteristics of the GPS observations.

Therefore, the stochastic model is dependent on the selection of the functional model. For the relative positioning mode, a double-differencing technique is commonly used for constructing the functional model, because it can eliminate many GPS biases, such as the satellite and receiver clock biases, and atmospheric biases. However, in PPP mode,⁵⁻⁷ some errors cannot be mitigated and will increase the difficulty of PPP-derived ZTD estimation. Theoretically, it is possible to mitigate systematic errors through using the precise GPS orbits, satellite clock corrections, precise error models, and so on. However, it is impossible to mitigate all systematic errors in the functional model without some understanding of the physical phenomena, which underpin these errors. For example, the model of the wet delay is difficult to build because of its large spatial and temporal variability. The
residual systematic errors will severely affect the accuracy of PPP-derived ZTD estimation. Therefore, any misspecifications of the stochastic model will result in unreliable unknown parameter estimation.

The undifferentiated GPS technique cannot eliminate most errors like the differenced GPS technique, and hence, the precise stochastic model is certainly critical for the accuracy of PPP-derived ZTD estimation. Over the past two decades, the functional models for GPS measurements have been investigated in considerable detail. However, accurate stochastic model for GPS measurements is still both a controversial topic and a difficult task to implement in practice.

Currently, a sine function based on satellite elevation is commonly used as the stochastic model. The model is based on the assumption that the noises at low elevation are higher than those at high elevation. However, the fact that the GPS signal at low elevation involves more information about troposphere is ignored in this model. Therefore, a cosine function based on low elevation is proposed in the paper.

1 Function model

In order to use the LS method for ZTD estimation, the functional model is first defined, namely, the mathematical relationships between GPS measurements and unknown parameters. In the current, the ionospheric-free model[5,8,9] is usually used in the PPP. In the PPP, the ionospheric-free combinations of dual-frequency GPS pseudorange observations are related to the user position, clock, and troposphere parameters according to the following simplified observation equations:

\[ l_p = \rho + c(dt - dT) + T_{\text{trop}} + \epsilon_p \]  (1)

Where \( l_p \) is the ionosphere-free combination of \( P_1 \) and \( P_2 \) pseudoranges, \( \rho \) is the geometrical range between the satellite and the station position, \( c \) is the vacuum speed of light, \( dt \) is the station receiver clock offset from the GPS time, \( dT \) is the satellite clock offset from the GPS time, \( T_{\text{trop}} \) is the signal path delay due to the troposphere, and \( \epsilon_p \) are the relevant measurement noise components.

The linearization of observation Eq. (1) and observations in the matrix form becomes

\[ L = Ax + V \]  (2)

Where \( A \) is the design matrix, \( x \) is the vector of corrections to the unknown parameters, \( V \) is the residual vector, and \( L \) is the misclosure vector. Using the LS method, the solution is given by

\[ \delta = -(P_{x^0} + A^T P_x A)^{-1} A^T P l \]  (3)

where \( P_x \) is the observation weight matrix, and \( P_{x^0} \) is the apriori weighted parameter constraints.

Then, the estimated parameters are

\[ \hat{X} = X^0 + \delta \]  (4)

where \( X^0 \) is the initialized value.

2 Stochastic model

The estimated unknown parameters are dependent on the stochastic model adopted for the measurements. Any misspecifications in the stochastic model may lead to inaccurate results. Some different stochastic models have been investigated. These models[9] can be divided into two categories. The first assumes that all GPS observations are statistically independent and have the same variance. The second assumes that all GPS observations are statistically independent and have the different variance based on the satellite elevation angle. The first assumption is unrealistic, which will inevitably degrade the accuracy of ZTD estimation. In fact, GPS measurement errors[10] are dominated by systematic errors, for example, multipath delay, ionospheric delay, and tropospheric delay. The effects of these error sources are different for each satellite. Therefore, the second assumption is more realistic. Because of the complexity of GPS measurements, the statistical characteristic can only be approximately expressed. In order to build the model, the statistical characteristic, the functions of satellite elevation angles are often used for the GPS measurement variance, namely,

\[ \sigma^2 = a^2 f^2(el) \]  (5)

where \( \sigma^2 \) is the GPS measurement variance, \( a \) is a constant, \( el \) is the satellite elevation, and \( f(el) \) is a function of satellite elevation angles.

Because of some unknown of the physical phenomena, it is impossible to mitigate all systematic errors. The residual systematic errors will affect the results of the unknown parameter estimation. There-
fore, the realistic stochastic model is important for the ZTD estimation. Different stochastic modeling methods are presented.

2.1 Sine model

The sine model based on the satellite elevation angle can be given by

$$\sigma^2 = \text{signmas}^2 / \sin(el)$$  \hspace{1cm} (6)

where $el$ is the satellite elevation, and $\text{signmas}$ is the standard deviation of observations. The model is similar to various models of the tropospheric mapping function, which have a $1/\sin e$ shape.\[^{[11]}\] This is based on assumption that the signal noise will increase with the decrease of elevation angle.

2.2 Sine square model

The sine square model\[^{[12,13]}\] based on the satellite elevation angle is given by

$$\sigma^2 = \text{signmas}^2 / \sin^2(el)$$  \hspace{1cm} (7)

Constructing this model is from the fact that GPS residuals can reveal a more swiftly increasing noise level for low elevation angles. This model is generally used in the precise positioning soft like Bernese\[^{[14]}\] and GAMIT.\[^{[15]}\]

2.3 Sine step model

The sine step model\[^{[16]}\] is also based on the satellite elevation.

$$\sigma^2 = \begin{cases} \text{signmas}^2, & \text{elev} > a \\ \text{signmas}^2 / \sin^2(el), & \text{elev} < a \end{cases}$$  \hspace{1cm} (8)

where $\alpha$ is the elevation threshold angle.

Compared with high satellite elevation observation, the model assumes there are more observation noises at low elevation. It might be appropriate to apply a step function using a uniform weight for high elevation angle observation and lower weight at low elevation angle. Deweighting observations at high elevation angles will lose valuable information.

2.4 Cosine square model

The cosine model is the equation as follows:

$$\sigma^2 = \text{signmas}^2 / \cos^2(el)$$  \hspace{1cm} (9)

The cosine model is based on the assumption that the relative measurement precision of observations at low elevation is higher than that at high elevation. The errors related to observations, such as the satellite clock offset, receiver clock offset, relativistic effect, and earth tide, are independent of satellite elevation. Of course, the measurement error is higher at low elevations, but the slant tropospheric delay increase is faster than the measurement noise with decreasing elevation. A high relative precision at low elevations is the consequence.\[^{[17]}\] Therefore, the observation weight at low elevation should be enhanced.

3 Test results and analysis

In order to determine the validity of cosine square function, GPS observations are dealt with by using the different models, such as sine model, sine square model, and sine step model. The corresponding results are compared with those of radiosondes.

The test data are from observations in IGS Shanghai station from May 2 to 6, 2008 and IGS Beijing station from June 2 to 6, 2008. The data sampling interval is 30 s, and the satellite cut-off elevation is set to 15°. Radiosonde data are from China Meteorological Data Sharing Service System. A Matlab-based precise point positioning soft is developed jointly by Xi’an University and China Research Institute of Radio Wave Propagation.

ZTD of different model in Shanghai and Beijing stations are shown in Figs. 1 and 3. Figs. 2 and 4 show the ZTD error of corresponding models. In addition, estimated coordinates of Shanghai and Beijing stations in different models are shown in Figs.5 and 6.

According to Figs. 1 and 3 shown, the results of cosine square function agree best with those of radiosondes in these functions. The results of sine step model are nearly the same with those of sine square model. In Figs. 2 and 4, it is found that the error of cosine square model is the smallest. According to Figs. 5 and 6, the convergence time and trend of different models is almost the same.

In order to explicitly analyze the errors of the different models, the mean error and root mean square error (RMS) are calculated, which are shown in Table 1. The standard deviation (STD) of coordinates is shown in Table 2.

According to Table 1, for Shanghai and Beijing stations, the RMS of cosine square model is about one
Fig.1  ZTD with different models in Shanghai station

Fig.2  Errors of ZTD of different models in Shanghai station

Fig.3  ZTD with different models in Beijing station

Fig.4  Errors of ZTD of different models in Beijing station

Fig.5  Estimation results of coordinate with different models in Shanghai station

Fig.6  Estimation results of coordinate with different models in Beijing station
Table 1 ZTD error of Shanghai and Beijing stations

| Station | cosine square | sine  | sine square | sine step |
|---------|--------------|-------|-------------|-----------|
| Mean    | -0.0012      | 0.0515 | 0.0738      | 0.0738    |
| RMS     | 0.0574       | 0.0819 | 0.1021      | 0.1021    |
| Mean    | 0.0017       | 0.0294 | 0.0220      | 0.0220    |
| RMS     | 0.0291       | 0.0462 | 0.0409      | 0.0409    |
| Beijing |              |       |             |           |

Table 2 Standard deviation of coordinates in Shanghai and Beijing station

| Station | cosine square | sine  | sine square | sine step |
|---------|--------------|-------|-------------|-----------|
| STD-x   | 0.0552       | 0.0434 | 0.0398      | 0.0398    |
| STD-y   | 0.0748       | 0.0636 | 0.0605      | 0.0605    |
| STD-z   | 0.0618       | 0.0298 | 0.0276      | 0.0276    |
| STD-x   | 0.0274       | 0.0220 | 0.0243      | 0.0243    |
| STD-z   | 0.0180       | 0.0322 | 0.0309      | 0.0309    |
| Beijing |              |       |             |           |
| STD-x   | 0.0160       | 0.0129 | 0.0128      | 0.0128    |
| STD-y   | 0.0274       | 0.0220 | 0.0243      | 0.0243    |
| STD-z   | 0.0160       | 0.0129 | 0.0128      | 0.0128    |

half of that of other model, and the mean value of cosine square model is the smallest. The mean and RMS of sine model are better than those of sine square and step model. It is consistent with the assumption of cosine square model. The observation weight at low elevation should be enhanced. In Table 2, it is clear that the STD of different models is nearly the same.

From the theoretical point of view, GPS observations are disposed in different aspects based on cosine and sine models. The sine model is based on the assumption that the observations at low elevation involve more noises, and hence, the weight should be decreased. The cosine model is based on the assumption that the observations at low elevation involve more the troposphere information, and so, the weight should be increased. Consequently, any misspecifications of the stochastic model will degrade the precision of the unknown parameter estimation.

4 Conclusion

In this paper, a new stochastic model based on low satellite elevation is described. Through comparison with previous stochastic models, it is found that the results of the cosine square model are in better agreement with those of radiosondes. Therefore, the information from low-elevation observations can also certify the estimation of ZTD. In addition, the high-accuracy ZTD plays a key role in Numerical Weather Prediction and atmosphere science. Therefore, the more suitable stochastic modeling for GPS precise point positioning should be further investigated.

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