A Wind Climate Dynamic Modeling and Control Using Weibull and Extreme Value Distribution System

Tim Chen¹, Alfred Hausladen², Jonathan Sstamler³, Dneil Granger⁴, Abu Hurayraasiv Khanand⁵, Johney Cheng⁶, *, Cwc Chen⁷, ⁸, Chariklia Ageorgopoulou Kyriakos⁸

¹Laboratoire d’Energies Renouvelables, École Supérieure Polytechnique de Dakar, Dakar, Senegal
²NAAM Research Group, King Abdulaziz University, Jeddah, Saudi Arabia
³Department of Physiological Sciences, College of Medicine, Alfaisal University, Riyadh, Saudi Arabia
⁴Nuclear Power Corporation of India Limited, Mumbai, India
⁵Department of Electrical and Computer Engineering, Northsouth University, Taka, Bangladesh
⁶Department of Electronic and Automatic Engineering, Covenant University, Ota Ogun State, Nigeria
⁷Parallel CFD and Optimization Unit, Lab. of Thermal Turbomachines, School of Mechanical Engineering, National Technical University of Athens, Athens, Greece
⁸Department of Electrical and Computer Engineering, Asia University, Mssbaai, Taiwan, China

Email address: jc343965@gmail.com (Johncy Cheng)
*Corresponding author

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Abstract: The dynamics of wind velocity data modeling plays a crucial role for the estimation of wind load and wind energy. Apart from these, the same modeling must also be used in the load cycle analysis of fatigue failure in slender structures to address periodic vortex shedding. Most authors fitted wind velocities of various locations using Weibull model. However, they did not check the validity of the model in describing the range of extreme wind velocity, which is not clear from the usual graphical representation. In this work, the validity of Weibull model for describing parent as well as extreme hourly mean wind velocity data for four places on the east coast of India has been checked; Weibull model has been found to become inappropriate for describing wind velocity in the range of extremes.

Keywords: Weibull Distribution, Wind Velocities, Non-Exceedance Probability, Gumbel Distribution, Chauvenet’s Criterion, Probability Factor

1. Introduction

In recent years, modeling wind velocities by appropriate probability distributions has found great importance in many practical applications which include air pollution modeling, the analysis of wind loading to structures and determination of wind power potential (Zaharim et al., 2009) [1]. Therefore, an appropriate probability distribution is required for wind velocity data analysis. The vortex induced vibration on slender structures is another important phenomenon of concern. Due to the formation of Von Karman’s vortex street, slender structures are subjected to fatigue failure owing to cross wind vibrations. This vibration would be severe if there is resonance when the natural frequency of the slender structure is the same as that of this vortex shedding. Wind velocities, for which these frequencies are same, are called critical wind velocities. The fatigue failure would be caused due to this vortex shedding which is periodic in nature. The number of stress cycles which are expected in the service life of the structure can be determined from the probable annual number of hours in the critical wind velocity range. The annual number of hours which can be expected in the
critical wind velocity range can be determined by the integration of probability density distribution of hourly wind velocity data over the range of critical wind velocities. It is worthy to mention that wind power density varies as the cube of the wind velocity. Hence, probability density distribution of wind velocity data is required for the estimation of wind power density. Apart from these two important purposes, wind velocity data modelling is also necessary for the specification of probability factor or risk coefficient which is further required for the specification of the design wind velocity leading to the codification of design wind load.

Many authors, namely Sarkar et al. (2017) [2], Gupta (1986) [3], Rehman et al. (1994) [4], Deaves and Lines (1997) [5], Garcia et al. (1998) [6], Lun and Lam (2000) [7], Sulaiman et al. (2002) [8], Bivona et al. (2003) [9], Celik (2004) [10], Zaharim et al. (2009) [11], Chang (2011) [12], Costa Rocha et al. (2012) [13] and Harris and Cook (2014) [14] have fit the lower wind velocity range for different locations using Weibull models. The common methods for determining parameters of Weibull distribution, as employed by the above cited articles, are the frequently used graphical or least-square method, method of moments and maximum likelihood method. Other authors like Seguro and Lambert (2000) [15], Chang (2011) [16], Costa Rocha et al. [17], (2012) have compared various methods to determine the best method using different statistical techniques (K-S test, R. S. M. E. test, Chi Square test, etc.). However, the conclusions of such studies have not been unique. Notably, all such studies found that the least-square method performed poorly. For example, Seguro and Lambert (2000) [18] compared methods such as the maximum likelihood method, the modified maximum likelihood method and the frequently used least-square method for the determination of Weibull parameters of wind velocity data modeling by employing two tests. The first test used simulated wind velocity data set from which Weibull parameters were determined and compared with the known values whereas in the latter, performances of different methods were assessed based on the wind energy output which can be determined from Weibull parameters. The least-square method was found to be less accurate and less robust. However, Cook (2001) [19] stated that the conclusion that the traditional least-square method is less accurate than the maximum likelihood method and modified maximum likelihood method is incorrect. The reason for incorrect conclusion is the authors’ use of an incorrect interpretation of cumulative probability distribution. Cook showed that the result obtained by the corrected least-square method is very close to the two modern methods mentioned above. In line with this work, Chang (2011) [20] and Costa Rocha et al. (2012) [21] compared six methods (the method of moment, empirical method, least-square method, maximum likelihood method, modified maximum likelihood method and energy pattern factor method) for determining Weibull parameters. However, unfortunately, both studies used the same incorrect definition regarding the cumulative distribution function, as mentioned by Cook (2001) [22]. That is why, the above authors have again found the least-square method to be less appropriate. From these discussions it is clear that no single method is superior to any other method, as far as the determination of Weibull parameters is concerned. Hence, in this article, Weibull parameters for a chosen station have been estimated by all six methods; the best method is subsequently determined by the K-S test and tabulated. The details of the various methods and the underlying calculations involving determination of the best method are not a subject of this work.

Though appropriateness of Weibull model for fitting wind velocity distribution has been well-established, some other models have also been used to fit the same. Akdag et al. (2010) [23] proposed the use of a two-component mixture Weibull distribution involving five parameters. Similarly, Celik et al. (2013) [24], Thiw et al. (2010), Fadare (2010) [24] and Jung et al. (2013) [25] suggested the use of an artificial neural network (ANN) for wind velocity data modeling. Chang (2011) [26] proposed two new mixture probability functions, i.e., the mixture Gamma-Weibull function (GW) and the mixture truncated normal function (NN) to estimate wind power density. Zhang et al. (2014) [27] proposed the use of the maximum entropy distribution for estimating wind power density. Ust et al. (2012) [28] used a skewed generalized error distribution (SGED) and a skewed t-distribution (STD) for this purpose. Za’rate-Minano et al. (2013) [29] and Zhou et al. (2013) [30] used stochastic models, whereas Carvalho et al. (2013) [31] used meso-scale and micro-scale modeling. Soukissian (2013) [32] used the Johnson SB distribution in addition to the classical Weibull distribution to model wind velocity data. Lujano-Rojas et al. (2012) [33] proposed a mathematical model for stochastic simulations of small wind energy systems. Beccalli et al. (2010) [34] developed a methodology for estimating spatial wind velocities over complex terrain and, subsequently, wind energy. Carapellucci et al. (2013) [35] proposed a new methodology for synthetically generating hourly wind velocity data for any location by incorporating a physical-statistical approach based on readily available statistical input parameters, such as the mean, average and maximum wind velocities, on a monthly or yearly basis. Similarly, Morales et al. (2010) [36] presented a methodology for characterizing the stochastic process associated with wind velocity by specifying a set of plausible scenarios that can describe the uncertainties in the wind velocities at different geographical locations. Douak et al. (2013) [37] introduced an approach to active learning in the field of wind energy forecasting. Calif (2012) considered the use of parametric probability density functions (PDFs) to fit empirical wind velocity fluctuation distributions. Chen et al. (2014) [39] proposed a new hybrid model for predicting wind velocity and wind energy that is useful only for short-term forecasting. However, Harris and Cook (2014) [40] established that simple Weibull distribution is the best for wind velocity data modeling.

In spite of the suitability of Weibull model for fitting wind velocity data, it has been derived from the weakest link theory.
which indicates that it is used to fit minimum data though sometimes it is used to fit maximum data as well (Castillo et al., 2005) [41]. Hence, there may be a threshold wind velocity below which Weibull distribution can be considered suitable. However, for the analysis of the wind power density it is worthy to mention that the cut-in and cut-out wind velocities for most of the wind turbines are 5 and 25 m/s respectively (EI-Wakil, 2002) [42]. Hence, for the correct prediction of wind power density, wind velocity probability distribution should be able to model wind velocities till 25 m/s whereas Weibull distribution has been found to be suitable to model velocities till the maximum range of 14 m/s [43](Sarkar et al., 2017). Apart from this, fatigue failure due to cross wind vibration is important to consider for the wind velocity with an annual exceedance probability of 1/50. Hence, the modelling of upper tail is also important for this case. In addition, the probability factor was specified in IS: 875 Part III(2015) by modelling maximum daily gust wind velocities. It would obviously be better if this factor can be specified by the modelling of the same hourly mean wind velocity data. However, based on existing plotting techniques for wind velocity distributions, it cannot be predicted if the extreme of wind velocity data follows a Weibull model. Therefore, in this work, the authors are interested in verifying the suitability of Weibull model for modeling extreme wind velocities using suitable plotting techniques. If Weibull model is inappropriate to describe the extremes, then the limiting velocity up to which it can be considered appropriate must be determined. In this case, it is also necessary to determine the best theoretical estimator to fit wind data beyond this threshold value. In the present work, the details of wind velocity data are given in section 2; various probability distributions are described in section 3; determination of parameters and wind climate modeling using Weibull distribution are performed in section 4; the threshold value of the wind velocity up to which Weibull model is valid is analytically and graphically determined, the results of which are compared in section 5; the extreme value limit distributions for modeling are obtained in section 6; the determination of the probability factor is discussed in section 7. Section 8 presents the chief conclusions.

2. Wind Velocity Data

Wind velocity data have been obtained from the Indian Meteorological Department, Pune, measured at an altitude of 10 m. The instrument used to measure the data is Dyne Pressure Tube Anemograph (DPTA). The data were supplied in a time series format which contains hourly mean wind velocities measured every hour of a day. Though the available wind velocity data are in a class width of 1 km/h, the most appropriate class width that must be chosen for this purpose will be discussed later in this section. The frequency of each class of wind velocity is given in this format. The scope of this study includes four stations: Gopalpur, Kolkata, Madras Harbour and Meenambakkam. The latitude, longitude and observation period for these stations have been mentioned in Table 1.

| Station Name   | Latitude (°N) | Longitude (°E) | Observation Period   |
|----------------|---------------|----------------|----------------------|
| Gopalpur       | 19.27         | 84.92          | January 1969- December 1980 |
| Kolkata        | 22.57         | 88.37          | January 1969- December 2000 |
| Madras Harbour | 13.08         | 80.29          | January 1969- June 1987 |
| Meenambakkam   | 12.98         | 80.18          | January 1969- December 2005 |

The probability vs. hourly mean wind velocity histograms with a 1 km/h class width show that many flaws are subtly introduced with this class width and that the histograms are not stable. The unstable histogram with sampling error has been simulated in Figure 1.

![Figure 1. Simulated histogram with sampling error.](image-url)

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The flaws in wind velocities are noticeable in the graph of the relative frequency versus wind velocity in the example of data from Gopalpur which is depicted in Figure 2(a). However, this problem appears when the wind velocity...
is initially measured in one unit and then converted into another unit via multiplication by a factor and subsequently rounding to an integer. Initially, the wind velocities are generated in knots and then converted to km/h via multiplication by a factor of 1.852 and subsequent rounding to integer values. Because km/h and knots have no common integer multiple, the error due to sampling would vanish when the bin size is increased (Kasperski, 2010). However, increasing the class width results in a loss of accuracy. In this article, an optimum class width of 2 km/h, which is neither too low to introduce bias nor too high to sacrifice accuracy, is chosen for all calculations (Sarkar et al., 2017). The corresponding histogram is plotted in Figure 2(b), which is essentially free of distortions.

3. Weibull and Generalized Extreme Value Distribution

Weibull function, a continuous probability density function with three parameters, may be mathematically expressed as:

\[ f(v) = \frac{k}{s} \left(\frac{v-\varepsilon}{s}\right)^{k-1} \exp\left[-\left(\frac{v-\varepsilon}{s}\right)^k\right] \]  

Where \( v \) represents wind velocity, \( k \) is the dimensionless shape parameter, \( s \) is the scale parameter and \( \varepsilon \) is the location parameter which represents the minimum value of \( v \). The cumulative distribution function of Weibull model can be determined by integrating Eq. (1) which is as follows:

\[ F(v) = 1 - \exp\left[-\left(\frac{v-\varepsilon}{s}\right)^k\right] \]  

The location parameter \( \varepsilon \) can be equated to 0 as it is the minimum wind velocity. Therefore, 3-parameter Weibull distribution would become a 2-parameter distribution as given by Eq. (3).

\[ F(v) = 1 - \exp\left[-\left(\frac{v}{s}\right)^k\right] \]  

Extreme wind velocities can be modelled by using extreme value limit distributions. Gumbel (1958) was the first person who discussed extreme value theory in his comprehensive.
textbook. He basically discussed three different extreme value limit distributions namely Gumbel (type I), Fréchet (type II) and reverse Weibull (type III). Type I distribution appears to be a straight line when it is plotted in Gumbel probability paper. A probability paper is a graph paper where the axes are ruled in such a way that the transformed distribution with respect to the reduced variate appears to be a straight line in the graph (Kasperski, 2009). II and III appear as curves when plotted on a Gumbel probability paper. The curves for type II distributions are concave, whereas a clear convex characteristic has been observed for the curves corresponding to type III distributions, as shown in Figure 3.

4. Wind Speed Data Modeling Using Weibull Distribution

At the outset of wind data modeling, it should be noted that Weibull distributions cannot predict actual probability for calm hours, which is clear from Eq. (3). Hence, calm hours are eliminated from the wind velocity data. Then, Weibull parameters are estimated by the six methods, mentioned earlier in the introduction. The K-S test is then performed to ascertain the best method for the chosen station. From Weibull parameters, the theoretical probability density distributions (PDFs) of the wind velocity data for three of the above-mentioned locations in India are plotted along with the observed histograms. For all these stations, the wind velocity density distribution can be approximated by a Weibull model for lower wind velocity ranges (0-12 m/s). However, based on these graphical representations, it is not clear if the extreme wind velocities follow a Weibull distribution.

Another representation of wind velocity data modeling is to plot observed and theoretical probability distributions on a Weibull probability paper wherein ln(ln(1 − f(v))) is plotted with respect to ln(v) so that Weibull distribution would appear as a straight line in that paper. It is to be noted that the suitable simplification of Eq. (3) leads to the slope-intercept form of the straight-line equation. On this paper, the lowest probability has been taken as 0.00001 whereas the highest probability has been taken as 0.99999. The fit of the data to a Weibull distribution can be seen using a Weibull probability paper wherein ln(−ln(1 − f(v))) is plotted against ln are plotted on this new probability paper for the theoretical distributions, which in turn may be used to ascertain the appropriateness of the theoretical Weibull model in the range of extreme wind velocity data. The theoretical and observed cumulative probabilities of the wind velocity against ln are plotted on this new probability paper for the four stations.

5. Conclusions

Wind climate modeling has been conducted for both the lower and extreme wind velocity data. Following are the conclusions: Weibull distribution can properly model the lower tail; however, after a particular threshold value, Weibull distribution fails to model the wind climate for the upper tail. This threshold value can be determined analytically. Beyond this threshold value, the wind velocity distribution can be fitted by extreme value limit distributions.

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