AN EXTENSION OF THE ELECTROWEAK MODEL WITH DECOUPLING AT LOW ENERGY

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ABSTRACT

We present a renormalizable model of electroweak interactions containing an extra $SU(2)'_L \otimes SU(2)'_R$ symmetry. The masses of the corresponding gauge bosons and of the associated Higgs particles can be made heavy by tuning a convenient vacuum expectation value. According to the way in which the heavy mass limit is taken we obtain a previously considered non-linear model (degenerate BESS) which, in this limit, decouples giving rise to the Higgsless Standard Model (SM). Otherwise we can get a model which decouples giving the full SM. In this paper we argue that in the second limit the decoupling holds true also at the level of radiative corrections. Therefore the model discussed here is not distinguishable from the SM at low energy. Of course the two models differ deeply at higher energies.
1 Introduction

In a previous paper [1] we have considered a model (degenerate BESS) of electroweak interactions describing, besides the usual $W^\pm$, $Z$ and $\gamma$ vector bosons, two new triplets of spin 1 particles, $V_L$ and $V_R$. These new states are degenerate in mass if one neglects their mixing to the ordinary vector bosons. The description of the model was based on a non-linear gauged $\sigma$-model and we refer to [1] for more details. The interest in the model was due to its decoupling properties: in the limit of infinite mass of the heavy vector bosons one gets back the SM. This is a rather non trivial property because one is dealing with a non-linear theory with couplings increasing with the heavy masses. In fact, the decoupling originates from an accidental global symmetry that the model possesses when the gauge couplings are turned off. This is also the symmetry from which the quasi-degeneracy of the heavy vector states arises.

It is an interesting question by itself to ask if a linear version of the model does exist. The original philosophy underlying the non-linear version was based on the idea that the non-linear realization would be the low-energy description of some underlying dynamics giving rise to the breaking of the electroweak symmetry. In this respect looking for a linear realization might appear as based on a completely different standpoint. However we are thinking of a scenario very close to the one arising in non-commuting technicolor models [2], where one has an underlying strong dynamics producing heavy Higgs composite particles. In this sense we are trying to describe the theory at the level of its composite states, vectors (the new heavy bosons), and scalars (Higgs bosons). That is, we are looking at a scale in which the Higgs bosons are yet relevant degrees of freedom. The advantage of this is that of dealing with a renormalizable theory. By that one is able to discuss if the decoupling holds at the level of radiative corrections. We will argue that the linear realization of the model decouples, and consequently that the high-energy physics we are talking about is not relevant at the LEP scale.

In the following we will describe the linearized version of the model showing that, at tree level, it coincides with the non-linear model of ref. [1]. We will then prove that, by diagonalizing the vector boson mass matrices and via a redefinition of the gauge couplings, all the couplings in the light and in the light-heavy sectors do not increase with the heavy masses. From this we can argue that the theory decouples also at the level of the radiative corrections. A detailed check of this point by means of an explicit calculation will be given in a more technical and complete paper [3]. We can therefore say that the model we present is identical to the standard model in its low energy manifestations, although at higher energies the differences can be rather dramatic [1].
2 The Model

The model we consider here is based on a gauge group $SU(2)_L \otimes U(1) \otimes SU(2)'_L \otimes SU(2)'_R$ and has a scalar sector consisting of scalar fields belonging to the following representations of the group $SU(2)_L \otimes SU(2)_R \otimes SU(2)'_L \otimes SU(2)'_R$

\[ \tilde{L} \in (2, 0, 2, 0) \quad \tilde{U} \in (2, 2, 0, 0) \quad \tilde{R} \in (0, 2, 0, 2) \quad (2.1) \]

that is with transformation properties

\[ \tilde{L}' = g_L \tilde{L} h_L \quad \tilde{U}' = g_L \tilde{U} g_R^\dagger \quad \tilde{R}' = g_R \tilde{R} h_R \quad (2.2) \]

where

\[ g_L \in SU(2)_L \quad g_R \in SU(2)_R \]
\[ h_L \in SU(2)'_L \quad h_R \in SU(2)'_R \quad (2.3) \]

We will see that with this system of scalar fields it is possible to break the gauge symmetries through the following chain

\[ SU(2)_L \otimes U(1) \otimes SU(2)'_L \otimes SU(2)'_R \]
\[ \downarrow u \]
\[ SU(2)_{\text{weak}} \otimes U(1)_Y \]
\[ \downarrow v \]
\[ U(1)_{\text{em}} \quad (2.4) \]

The two breakings are induced by the expectation values $\langle \tilde{L} \rangle = \langle \tilde{R} \rangle = u$ and $\langle \tilde{U} \rangle = v$ respectively. The first two expectation values make the breaking $SU(2)_L \otimes SU(2)'_L \rightarrow SU(2)_{\text{weak}}$ and $U(1) \otimes SU(2)'_R \rightarrow U(1)_Y$, whereas the second breaks in the standard way $SU(2)_{\text{weak}} \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$. In the following we will assume that the first breaking corresponds to a scale $u \gg v$. This chain of breaking is reminiscent of the one conjectured in non-commuting extended technicolor theories (NCETC) \cite{[3]}, where one has

\[ G_{ETC} \otimes SU(2)_{\text{light}} \otimes U(1) \]
\[ \downarrow f \]
\[ G_{TC} \otimes SU(2)_{\text{heavy}} \otimes SU(2)_{\text{light}} \otimes U(1)_Y \]
\[ \downarrow u \]
\[ G_{TC} \otimes SU(2)_{\text{weak}} \otimes U(1)_Y \quad (2.5) \]

where $G_{ETC}$ is the extended technicolor gauge group, and $G_{TC}$ the technicolor one. If we make the following identifications $SU(2)_L = SU(2)_{\text{light}}$, $SU(2)'_L = SU(2)_{\text{heavy}}$, one can think of an extension of the NCETC schemes such as

\[ G_{ETC} \otimes SU(2)_L \otimes U(1) \]
\[ \downarrow f \]
\[ G_{TC} \otimes SU(2)'_L \otimes SU(2)_L \otimes SU(2)'_R \otimes U(1) \quad (2.6) \]
After the $G_{ETC}$ has been broken the chain proceeds as in eq. (2.4). The original NCETC scheme has been here modified in order to allow for the gauge particles transforming under $SU(2)'_R$, ensuring, together with the vector bosons from $SU(2)'_L$, the decoupling at low energy.

Proceeding in a completely standard way, we can build up covariant derivatives with respect to the local $SU(2)_L \otimes U(1) \otimes SU(2)'_L \otimes SU(2)'_R$

\[
D\tilde{L} = \partial\tilde{L} + i\frac{\tau_2}{2} \cdot \vec{W}\tilde{L} - ig_2\tilde{L}\frac{\tau_2}{2} \cdot \vec{V}_L
\]

\[
D\tilde{R} = \partial\tilde{R} + ig_1\tau_3 Y\tilde{R} - ig_3\tilde{R}\frac{\tau_3}{2} \cdot \vec{V}_R
\]

\[
D\tilde{U} = \partial\tilde{U} + i\frac{\tau_2}{2} \cdot \vec{W}\tilde{U} - ig_1\tilde{U}\tau_3 Y
\]

(2.7)

where $\vec{V}_L$ ($\vec{V}_R$) are the gauge fields in $SU(2)'_L$ ($SU(2)'_R$), with the corresponding gauge couplings $g_2$, and $g_3$, whereas $g_0$, $g_1$, are the gauge couplings of the $SU(2)_L$ and $U(1)$ gauge groups respectively.

This model contains, besides the standard Higgs sector given by the field $\tilde{U}$, the additional scalar fields $\tilde{L}$ and $\tilde{R}$.

The lagrangian for the kinetic terms of these scalar fields is given by

\[
\mathcal{L}^h = \frac{1}{4} \left[ Tr(D_\mu \tilde{U})^\dagger(D^\mu \tilde{U}) + Tr(D_\mu \tilde{L})^\dagger(D^\mu \tilde{L}) + Tr(D_\mu \tilde{R})^\dagger(D^\mu \tilde{R}) \right]
\]

(2.8)

We have then to discuss the scalar potential which is supposed to break the original symmetry down to the $U(1)_{em}$ group. The most general potential invariant with respect to the group $SU(2)_L \otimes SU(2)_R \otimes SU(2)'_L \otimes SU(2)'_R$ is given by

\[
V(\tilde{U}, \tilde{L}, \tilde{R}) = \mu_1^2 Tr(\tilde{L}^\dagger \tilde{L}) + \frac{\lambda_1}{4}[Tr(\tilde{L}^\dagger \tilde{L})]^2 + \mu_2^2 Tr(\tilde{R}^\dagger \tilde{R}) + \frac{\lambda_2}{4}[Tr(\tilde{R}^\dagger \tilde{R})]^2 + m^2 Tr(\tilde{U}^\dagger \tilde{U}) + \frac{h}{4}[Tr(\tilde{U}^\dagger \tilde{U})]^2 + \frac{f_3}{2} Tr(\tilde{L}^\dagger \tilde{L}) Tr(\tilde{R}^\dagger \tilde{R})
\]

\[
+ \frac{f_1}{2} Tr(\tilde{L}^\dagger \tilde{L}) Tr(\tilde{U}^\dagger \tilde{U}) + \frac{f_2}{2} Tr(\tilde{R}^\dagger \tilde{R}) Tr(\tilde{U}^\dagger \tilde{U})
\]

(2.9)

In the following we will also require, for the scalar potential, the discrete symmetry $L \leftrightarrow R$, which implies

\[
g_3 = g_2
\]

\[
\mu_1 = \mu_2 = \mu
\]

\[
\lambda_1 = \lambda_2 = \lambda
\]

\[
f_1 = f_2 = f
\]

(2.10)
The total lagrangian is obtained by adding the kinetic terms for the gauge fields:

\[ \mathcal{L} = \mathcal{L}^h - V(\tilde{U}, \tilde{L}, \tilde{R}) + \mathcal{L}^{\text{kin}}(W, Y, V_L, V_R) \]  

where

\[ \mathcal{L}^{\text{kin}}(W, Y, V_L, V_R) = \frac{1}{2} \text{tr}[F_{\mu \nu}(W)F^{\mu \nu}(W)] + \frac{1}{2} \text{tr}[F_{\mu \nu}(Y)F^{\mu \nu}(Y)] \]

\[ + \frac{1}{2} \text{tr}[F_{\mu \nu}(V_L)F^{\mu \nu}(V_L)] + \frac{1}{2} \text{tr}[F_{\mu \nu}(V_R)F^{\mu \nu}(V_R)] \]

Notice that, when neglecting the gauge interactions, the lagrangian is invariant under an extended symmetry corresponding to \((SU(2)_L \otimes SU(2)_R)^3\). In fact, in this case, we are free to change any of the fields \(\tilde{U}, \tilde{L}, \tilde{R}\) by an independent transformation of a group \(SU(2)_L \otimes SU(2)_R\). As far as the fermions are concerned they transform as in the SM with respect to the group \(SU(2)_L \otimes U(1)\).

### 3 The scalar potential

Let us parameterize the fields as

\[ \tilde{L} = \rho_L L \quad \tilde{R} = \rho_R R \quad \tilde{U} = \rho_U U \]

with \(L^\dagger L = I, R^\dagger R = I\) and \(U^\dagger U = I\).

The scalar potential after these transformations can be rewritten as

\[ V(\rho_U, \rho_L, \rho_R) = 2\mu^2(\rho_L^2 + \rho_R^2) + \lambda(\rho_L^4 + \rho_R^4) + 2m^2\rho_U^2 + h\rho_U^4 \]

\[ + 2f_3\rho_L^2\rho_R^2 + 2f\rho_U^2(\rho_L^2 + \rho_R^2) \]

To study the minimum conditions, let us consider the first derivatives of the potential

\[ \frac{\partial V}{\partial \rho_L} = 4\rho_L(\mu^2 + \lambda\rho_L^2 + f_3\rho_R^2 + f\rho_U^2) \]  

\[ \frac{\partial V}{\partial \rho_R} = 4\rho_R(\mu^2 + \lambda\rho_R^2 + f_3\rho_L^2 + f\rho_U^2) \]  

\[ \frac{\partial V}{\partial \rho_U} = 4\rho_U(m^2 + h\rho_U^2 + f(\rho_L^2 + \rho_R^2)) \]

By considering the vacuum expectation values \(<\rho_U> = v\) and \(<\rho_L> = <\rho_R> = u\), the minimum conditions are

\[ \mu^2 + (f_3 + \lambda)u^2 + f v^2 = 0 \]
\[ m^2 + 2fu^2 + hv^2 = 0 \] (3.7)
from which we get the following solutions
\[ v^2 = -\frac{m^2}{h}(1 + \frac{f_3}{\lambda})(1 + \frac{f_3}{\lambda} - \frac{2f^2}{h\lambda})^{-1} \] (3.8)
\[ u^2 = -\frac{\mu^2}{\lambda}(1 - \frac{fm^2}{h\mu^2})(1 + \frac{f_3}{\lambda} - \frac{2f^2}{h\lambda})^{-1} \] (3.9)

By considering the second derivatives of the potential we can get the mass matrix for the three Higgs particles
\[
\begin{pmatrix}
\lambda u^2 & f_3 u^2 & fuv \\
f_3 u^2 & \lambda u^2 & fuv \\
fuv & fuv & hv^2
\end{pmatrix}
\] (3.10)
The mass eigenvalues are
\[
m_1^2 = 4[(f_3 + \lambda)u^2 + hv^2 - \sqrt{8u^2v^2f^2 + ((f_3 + \lambda)u^2 - hv^2)^2}] \\
m_2^2 = 8\lambda u^2(1 - \frac{f_3}{\lambda}) \\
m_3^2 = 4[(f_3 + \lambda)u^2 + hv^2 + \sqrt{8u^2v^2f^2 + ((f_3 + \lambda)u^2 - hv^2)^2}] \] (3.11)

Let us comment on the limitations on the parameters coming from the study of the positivity of the eigenvalues. Adding the requirement of \( u^2 > 0, v^2 > 0, \) with the hypothesis \( m^2, \mu^2 < 0 \) together with \( \lambda, h > 0 \) for the boundedness of the potential, we finally get
\[
\lambda - f_3 > 0, \quad h > \frac{m^2}{\mu^2} \] (3.12)
and
\[
\lambda + f_3 > 2f\frac{\mu^2}{m^2} \quad \text{for } f > 0 \quad \text{or} \quad \lambda + f_3 > 2f\frac{\mu^2}{h} \quad \text{for } f < 0 \] (3.13)

The limit we will be interested in the following is \( u \to \infty \) with \( v \) fixed. To define the limit one has to look carefully at the minimum conditions (3.6), (3.7). It follows from these equations that at least \( m^2 \) and \( \mu^2 \) must behave like \( u^2 \). Then, in order to keep \( v^2 \) finite, if follows from eq. (3.8) that one has also to send \( h \to \infty \), unless there is a cancellation between the leading behaviours of \( m^2 \) and \( \mu^2 \) with \( u^2 \). We can translate this reasoning in formal terms by requiring that for \( u \to \infty \)
\[
\mu^2 \to au^2, \quad m^2 \to bu^2, \quad h \to c\frac{u^2}{v^2} \] (3.15)
The minimum conditions (3.6), (3.7) give the following relations at the leading order

$$a = -(f_3 + \lambda), \quad b + c = -2f$$  \hspace{1cm} (3.16)$$

We can eventually require that $h$ goes like a constant for $u \to \infty$, by putting $c = 0$ at the end of our calculations. Notice that in this case one has to require $f > 0$. We can now evaluate the leading behaviour of the potential $V$ in terms of the displaced fields $\rho_L \to \rho_L + u$, $\rho_R \to \rho_R + u$, $\rho_U \to \rho_U + v$. By neglecting a term independent of the fields we get for $u \to \infty$

$$V \to 4\lambda u^2(\rho_L^2 + \rho_R^2) + 8f_3 u^2 \rho_L \rho_R + 4cu^2(\rho_U^2 + \frac{1}{v} \rho_U^3 + \frac{1}{4v^2} \rho_U^4)$$  \hspace{1cm} (3.17)$$

In this limit we can neglect the kinetic term, and the classical equations of motion are given by

$$\frac{\partial V}{\partial \rho_L} = \frac{\partial V}{\partial \rho_R} = \frac{\partial V}{\partial \rho_U} = 0$$  \hspace{1cm} (3.18)$$

that is

$$\rho_L = \rho_R = 0$$  \hspace{1cm} (3.19)$$

and

$$\rho_U(\rho_U^2 + 3v \rho_U + 2v^2) = 0$$  \hspace{1cm} (3.20)$$

The last equation has solutions $\rho_U = 0, -v, -2v$. The values $\rho_U = 0, -2v$ correspond to two degenerate minima, whereas $\rho_U = -v$ is a maximum of the potential. However the solution $\rho_U = -2v$ is not a physical one because it corresponds to a negative value of the original unshifted field, which by hypothesis is positive definite. Therefore the classical solution is $\rho_U = 0$. We see that in the case $c \neq 0$, the limiting procedure is equivalent to set the fields $\rho_L$, $\rho_R$ and $\rho_U$ at their minimum. The potential $V(\rho_L, \rho_R, \rho_U)$ is just a constant, whereas the kinetic term in (2.8) becomes

$$L^h = \frac{1}{4} \left[ v^2 Tr(D_\mu U)^\dagger (D^\mu U) + u^2 Tr(D_\mu L)^\dagger(D^\mu L) + u^2 Tr(D_\mu R)^\dagger(D^\mu R) \right]$$  \hspace{1cm} (3.21)$$

In this limit, and at tree level (we are considering the classical solutions), the linear model described by the lagrangian (2.8) coincides with the non-linear one discussed in ref [1], after identification of the gauge coupling constants

$$g_0 = \tilde{g}, \quad g_1 = \tilde{g}', \quad g_2 = \frac{g''}{\sqrt{2}}$$  \hspace{1cm} (3.22)$$

and of the parameter

$$a_2 = \frac{1}{2} \frac{u^2}{v^2}$$  \hspace{1cm} (3.23)$$
The limit $u \to \infty$ is equivalent to $a_2 \to \infty$. In this limit we have shown that the heavy vector fields $V_L$ and $V_R$ decouple and that the non-linear model reduces to the Higgsless standard model, after the following redefinition of the gauge couplings

$$
\frac{1}{g^2} = \frac{1}{\tilde{g}^2} + \frac{2}{g''^2} \quad \frac{1}{g'^2} = \frac{1}{\tilde{g}'^2} + \frac{2}{g''^2} \quad (3.24)
$$

or, in the present notations

$$
\frac{1}{g^2} = \frac{1}{g_0^2} + \frac{1}{g_1^2} \quad \frac{1}{g'^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2} \quad (3.25)
$$

In this form of the limit it is difficult to argue about the radiative corrections because the self-coupling of $\rho_U$ is increasing with $u$. The situation is different when $h$ is finite, that is $c = 0$. In fact, we see from eq. (3.17) that whereas $\rho_L$ and $\rho_R$ are set to their minimum, the field $\rho_U$ drops out from the leading term in the potential, and therefore it is not determined. So, after setting $\rho_L$ and $\rho_R$ to their minimum we get an extension of the non-linear model of ref. [1], in which the extra-field $\rho_U$ appears. The potential $V(\rho_L, \rho_R, \rho_U)$ coincides with the Higgs field potential in the standard model, and $\mathcal{L}^h$ becomes the standard model Higgs kinetic term supplemented by the non-linear pieces in the fields $L$ and $R$. Therefore, in this case, the limit $u \to \infty$ gives the standard model with a Higgs field light with respect to the scale $u$.

In this paper we are interested in showing that the linear model, presented here, satisfies our decoupling requirement also at the level of radiative corrections, and consequently we shall consider the limit $u \to \infty$ with $c = 0$ (that is the self-coupling $h$ fixed).

At the lowest order in this expansion we get for the Higgs masses

$$
m_1^2 \sim 8h v^2 \quad m_2^2 \sim 8u^2(\lambda - f_3) \quad m_3^2 \sim 8u^2(\lambda + f_3) \quad (3.26)
$$

4  Gauge vector boson spectrum and interactions

The vector boson mass spectrum can be studied in the unitary gauge $U = L = R = I$ by shifting the scalar fields as $\rho_U \to \rho_U + v$, $\rho_{L,R} \to \rho_{L,R} + u$. We get

$$
\mathcal{L}^h = \frac{1}{2} \left[ (\partial_\mu \rho_L)^2 + (\partial_\mu \rho_R)^2 + (\partial_\mu \rho_U)^2 \right]
$$
\[ 1 \{ (\rho_L + u)^2 [g_0^2 (W_3^2 + 2W^+ W^-) - 2g_0g_2 (W_3 V_{3L} + W^-V_L^+ + W^+V_L^-) \\
+ g_2^2 (V_{3L}^2 + 2V_L^+ V_L^-)] \\
+ (\rho_R + u)^2 [g_1^2 Y^2 - 2g_1g_2 V_{3R} Y + g_2^2 (V_{3R}^2 + 2V_R^+ V_R^-)] \\
+ (\rho_U + v)^2 [g_0^2 (W_3^2 + 2W^+ W^-) - 2g_0g_1 W_3 Y + g_1^2 Y^2] \} \] (4.1)

In ref. [1] we have studied the mass matrices of the vector bosons in the limit of small \( \tilde{g}/g'' \), for fixed mass eigenvalues. Here we are rather interested in the mass matrices for large mass eigenvalues of \( V_{L,R} \), at fixed \( \tilde{g}/g'' \). We will give here only some results of the diagonalization (see [3] for more details). First of all, it turns out to be convenient to re-express the results in terms of the parameters \( g \) and \( g' \) defined in equation (3.25). In fact, as we have said, these are the relevant parameters in the limit \( u \to \infty \). It is also convenient to introduce the angle \( \varphi \), such that

\[ s_\varphi = \frac{g}{g_2} = \sqrt{2} \frac{g}{g''} \] (4.2)

in terms of which

\[ g_0 = \frac{g}{s_\varphi} \] (4.3)

and

\[ \frac{g_1}{g_0} = \frac{\tilde{g}'}{\tilde{g}} \equiv \tan \tilde{\theta} = \frac{c_\varphi s_\tilde{\theta}}{\sqrt{P}} \] (4.4)

where

\[ \tan \theta = \frac{g'}{g} \] (4.5)

and

\[ P = c_\theta^2 - s_\varphi^2 s_\theta^2 \] (4.6)

The mass eigenvalues of the vector bosons are the following:

**Charged sector**

The fields \( V_R^\pm \) are unmixed and their mass is given by

\[ M_{V_R^\pm}^2 = \frac{1}{4} g_2^2 u^2 \equiv M^2 \] (4.7)

The absence of mixing terms is a consequence of the invariance of the lagrangian under the phase transformation \( V_R^\pm \to \exp(\pm i\alpha) V_R^\pm \). It will be also convenient to introduce the parameter

\[ r = \frac{1}{4} \frac{g_2^2 u^2}{M^2} = \frac{v^2 g_2^2}{u^2 g_2^2} \] (4.8)
which goes to zero for \( u \to \infty \). The remaining two eigenvalues, in the limit of small \( r \) are (we keep the same notation \( W^\pm, V_L^\pm \) also for the mass eigenvectors)

\[
M_{W^\pm}^2 = \frac{v^2 g^2}{4} (1 - r s_\varphi^2 + \cdots )
\]

\[
M_{V_L^\pm}^2 = \frac{v^2 g^2}{4} (\frac{1}{r c_\varphi^2} + \frac{s_\varphi^2}{c_\varphi^2} + r s_\varphi^2 + \cdots )
\]  
(4.9)

Notice that for \( r \to 0 \), \( M_{W^\pm}^2 \) coincides with the standard model expression for the \( W \) mass.

**Neutral sector**

In this sector there is a null eigenvector corresponding to the photon:

\[
\gamma = (s_\tilde{\theta} W_3 + c_\tilde{\theta} Y) \cos \psi + \frac{1}{\sqrt{2}} (V_{3L} + V_{3R}) \sin \psi
\]  
(4.10)

where

\[
\tan \psi = \sqrt{2} s_\tilde{\theta} \frac{g_0}{g_2} = \sqrt{2} \frac{s_\varphi^2 s_\theta}{\sqrt{1 - 2 s_\varphi^2 s_\theta^2}}
\]  
(4.11)

The remaining eigenvalues are, again in the limit of small \( r \),

\[
M_Z^2 = \frac{v^2 g^2}{4} (1 - r s_\varphi^2 \frac{1}{c_\varphi^2} + 2 c_\varphi^4 + \cdots )
\]

\[
M_{V_{3L}}^2 = \frac{v^2 g^2}{4} (\frac{1}{r c_\varphi^2} + \frac{s_\varphi^2}{c_\varphi^2} - r s_\varphi^2 \frac{c_\theta^2}{1 - 2 c_\theta^2} + \cdots )
\]

\[
M_{V_{3R}}^2 = \frac{v^2 g^2}{4} (\frac{1}{r P} + \frac{s_\varphi^2 s_\theta^4}{P} + r \frac{s_\varphi^2 s_\theta^8}{c_\theta^4 (1 - 2 c_\theta^2)} + \cdots )
\]  
(4.12)

Only for small \( \varphi \) the heavy vectors are degenerate in mass.

Let us now verify that there are no couplings which increase with \( u \) in the light and in the heavy-light sectors of \( \mathcal{L}^h \). From eq. (4.1), using the new couplings defined in eq. (3.25) and the diagonalized vector fields, we get for the light sector of the Higgs-vector interactions at the leading order in \( r \) an expression which coincides with the analogous one in the standard model

\[
\mathcal{L}_{\text{light}}^h = \frac{g^2}{4} (\rho_U^2 + 2 \rho_U v) (W^+ W^- + \frac{1}{2 c_\theta^2} Z^2)
\]  
(4.13)

and for the heavy-light sector

\[
\mathcal{L}_{\text{heavy-light}}^h = \frac{g^2}{4} (\rho_U^2 + 2 \rho_U v) [- \tan \varphi (W^+ V_L^- + W^- V_L^+ + \frac{1}{c_\theta} Z V_{3L})
\]

\[
+ \tan^2 \varphi V_L^+ V_L^- + \frac{s_\varphi \tan^2 \theta}{\sqrt{P}} Z V_{3R}]
\]  
(4.14)


We will now consider the couplings of the vector bosons to the fermions. We assume that the fermions have standard transformation properties under the group $SU(2)_L \otimes U(1)_Y$, and therefore the couplings to the heavy mesons arise only through the mixing:

**Charged sector:** At the first order in $r$ the couplings are given by

$$
L_{\text{fermions}}^{\text{charged}} = -(a_W W^-_{\mu} + a_L V^-_{L\mu}) J^\mu_L + h.c. \quad (4.15)
$$

with

$$a_W = \frac{g}{\sqrt{2}}(1 - s_\varphi^2 r) \quad (4.16)$$

and

$$a_L = -\frac{g}{\sqrt{2}}(1 + c_\varphi^2 r) \tan \varphi \quad (4.17)$$

and $J^\pm_L = \bar{\psi}_L \gamma^\mu r^\pm \psi_L$. Notice that there is no coupling of $V_R^\pm$ to fermions, because these particles do not mix with the $W^\pm$’s. Also, for $r = 0$ the couplings of $W^\pm$ to the fermions coincide with the standard ones.

**Neutral sector:** The couplings are defined by the expression

$$
L_{\text{fermions}}^{\text{neutral}} = -eJ_{em}\gamma - [AJ_{3L} + BJ_{em}] Z
- [CJ_{3L} + DJ_{em}] V_{3L} - [EJ_{3L} + FJ_{em}] V_{3R} \quad (4.18)
$$

with

$$e = gs_\theta \quad (4.19)$$

and

$$
A = \frac{g}{c_\theta}(1 - s_\varphi^2 s_\theta^4 + c_\theta^4 r) \\
B = \frac{g}{c_\theta}(-s_\theta^2 + s_\varphi^2 c_\theta^4 r) \\
C = \frac{g}{c_\theta}(-\tan \varphi c_\theta + c_\varphi s_\varphi c_\theta^3 c_\theta r) \\
D = \frac{g c_\varphi s_\varphi c_\theta^2 c_\theta}{c_\theta (2c_\theta^2 - 1)} \\
E = \frac{g}{c_\theta}(s_\varphi c_\theta^2 + s_\varphi s_\theta^6 \sqrt{P}) + \frac{s_\varphi s_\theta^6 \sqrt{P}}{c_\theta (1 - 2c_\theta^2) r} \\
F = \frac{g}{c_\theta}(-s_\varphi s_\theta^2 \sqrt{P} - s_\varphi s_\theta^6 \sqrt{P}) \quad (4.20)
$$

The expression for the electric charge is valid to all order in $r$, while the other coefficients in (4.20) are given only at first order in $r$. In particular the couplings to the $Z$ go
back to their standard model values for $r \to 0$. Again, there are no couplings increasing when $r \to 0$, both in the charged and in the neutral sector.

Finally we have to examine the gauge boson self-couplings. Let us define the following formal combination

$$AB^{-}C^{+} = A^{\mu\nu}B^{-}_{\mu}C^{+}_{\nu} + A^{\nu}(B^{-}_{\mu\nu}C^{+\mu} - B^{\mu+}_{\mu\nu}C^{\mu-})$$

(4.21)

where

$$A^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

(4.22)

and similar expression for $B^{\pm}_{\mu\nu}$. Then the trilinear gauge boson couplings in terms of the original fields are given by

$$\mathcal{L} = ig[\frac{1}{c_{\theta}}(2c_{\varphi}^{2} - 1)]ZV_{R}^{-}V_{L}^{+}$$

(4.23)

Using the redefinition of the couplings and the expressions for the mass eigenstates, we find, again at the first order in $r$, for the light sector

$$\mathcal{L}_{\text{light}} = ig[s_{\theta}^{\gamma}]W_{R}^{+}W_{L}^{-} + c_{\theta}ZW_{R}^{-}W_{L}^{+}$$

(4.24)

and for the heavy-light sector

$$\mathcal{L}_{\text{heavy-light}} = ig[s_{\theta}^{\gamma}]V_{R}^{-}V_{L}^{+} + c_{\theta}ZV_{R}^{-}V_{L}^{+}$$

(4.25)

The quadrilinear couplings are obtained starting from

$$-\frac{g^{2}}{2}S_{\mu\nu\rho\sigma} \left[ W_{R}^{+}W_{R}^{-}(W_{\rho}^{+}W_{\sigma}^{-} + W_{3\rho}W_{3\sigma}) + \frac{1}{\tan^{2}\varphi}V_{L\rho}^{+}V_{L\sigma}^{-}(V_{L\rho}^{+}V_{L\sigma}^{-} + V_{3L\rho}V_{3L\sigma}) + \frac{1}{\tan^{2}\varphi}V_{R\rho}^{+}V_{R\sigma}^{-}(V_{R\rho}^{+}V_{R\sigma}^{-} + V_{3R\rho}V_{3R\sigma}) \right]$$

(4.26)

with $S_{\mu\nu\rho\sigma} = 2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}$. 

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At the lowest order in $r$ one gets for the light part (in this case the corrections are of the order $r^2$)

$$
\mathcal{L}_{\text{light}} = -\frac{g^2}{2} S_{\mu\nu\rho\sigma} \left[ W_\mu^+ W_\nu^- (W_\rho^+ W_\sigma^- + c_\theta^2 Z_\rho Z_\sigma \\
+ 2 c_\theta s_\theta \gamma_\rho Z_\sigma + s_\theta^2 \gamma_\rho \gamma_\sigma) \right]
$$

and for the heavy-light part (here the corrections are of order $r$)

$$
\mathcal{L}_{\text{heavy-light}} = -\frac{g^2}{2} S_{\mu\nu\rho\sigma} \left[ W_\mu^+ W_\nu^- (V_{3L}^+ V_{3L}^- + V_{L}^+ V_{L}^-) \\
+ (W_\mu^+ V_{L\nu}^- + V_{L\mu}^+ W_{\nu}^-) (V_{L\rho}^+ W_\sigma^- + W_{\rho}^+ V_{L\sigma}^-) \\
+ 2 c_\phi^2 - \frac{1}{c_\phi s_\phi} (V_{L\rho}^+ V_{L\sigma}^- + V_{3L}^+ V_{3L}^-) \right] \\
+ V_{L\mu}^+ V_{L\nu}^- (W_\rho^+ W_\sigma^- + c_\theta^2 Z_\rho Z_\sigma + 2 c_\theta s_\theta \gamma_\rho Z_\sigma + s_\theta^2 \gamma_\rho \gamma_\sigma \\
+ 2 c_\phi^2 - \frac{1}{c_\phi s_\phi} (V_{L\rho}^+ V_{L\sigma}^- + V_{L\rho}^+ V_{L\sigma}^-) \\
+ 2 c_\theta V_{3L}^+ V_{3L}^- + 2 s_\theta V_{3L}^+ V_{3L}^-) \right) \\
+ V_{R\mu}^+ V_{R\nu}^- (s_\phi^4 c_\phi^2 Z_\rho Z_\sigma - 2 s_\phi^3 c_\phi \gamma_\rho Z_\sigma + s_\phi^2 \gamma_\rho \gamma_\sigma \\
- 2 s_\theta \sqrt{P} (s_\theta V_{3R}^+ V_{3R}^- Z_\sigma - c_\theta V_{3R}^+ V_{3R}^- \gamma_\sigma) \right]
$$

Both the trilinear and quadrilinear light parts of the lagrangian agree with the standard model results, and the heavy-light sectors do not show any coupling increasing with the heavy mass $M$.

### 5 Conclusions

In this work we have formulated a renormalizable model which can be suitable to describe at some intermediate energy a scenario of the kind considered in the non-commuting extended technicolor schemes. The gauge symmetries of the model are an extension of the SM symmetries by an extra $SU(2)_L' \otimes SU(2)_R'$ factor. In the limit in which the expectation value of the Higgs fields related to the new symmetries gets very large, one recovers a previously considered non-linear model [1]. The main property of the non-linear model was its decoupling for large values of the masses of the gauge bosons associated to the extra-symmetry factors, in spite of its non-linearity. The masses of the fields $V_L$ and $V_R$ go to infinity together with the expectation values of the related the Higgs fields, and
therefore one recovers in this limit the non-linear realization of the standard model, or
the Higgsless standard model.

In the present paper we have also considered a slight modification of this limiting
procedure which allows to the normal Higgs field (the one associated to the global sym-
mety $SU(2)_L \otimes SU(2)_R$) to remain light (with respect to the heavy scale). We can then
decompose the lagrangian into three pieces: the light sector, involving the SM fields in-
cluding the light Higgs, the heavy-light and the heavy sectors. The light sector part of
the lagrangian is identical to the lagrangian of the SM, and the heavy-light part does not
contain coupling increasing with the heavy scale. Having isolated all the big parameters
in the heavy part, one can conclude that at tree level there is decoupling (this has been
shown explicitly for the case of the non-linear model in ref. [1]). Furthermore, by the
arguments leading to the Appelquist-Carazzone [4] theorem, one can argue that the de-
coupling must hold also at the level of the radiative corrections. An explicit proof of this
statement will be given in a more technical paper [3]. As a consequence, in the low-energy
region the model cannot be distinguished by the SM. However, as shown in the case of
the non-linear model [1], when approaching the threshold for the production of the heavy
vector states it is possible to have big deviations, and more spectacular effects after the
threshold.

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