Multifractal structure of Lyα clouds: An example with the spectrum of QSO 0055–26

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ABSTRACT

Lyα forests are usually associated with intergalactic gas clouds intercepting the quasar sight–line. Using as an example the Lyα forest of QSO 0055–26 (zem = 3.66), we show that the probability of observing a line at the velocity difference Δv has an intermittent structure in the redshift space. On small scales (70 km s⁻¹ ≤ Δv ≤ 1580 km s⁻¹) the signature of intermittency appears as a self–similar structure of spikes systematically visible at the same redshift on all scales. This behaviour can be interpreted as due to the presence of clustering of the Lyα lines, which appear as singularities of the density of the probability measure. From a direct measurement of the generalized dimensions, we show that the intermittency can be described by a multifractal structure which is due to a Fourier phase correlation of the signal. The multifractal structure disappears at scales larger than Δv ≃ 1580 km s⁻¹.

Key words: intergalactic matter – quasar absorption lines – large scale structure of Universe

1 INTRODUCTION

The numerous absorption lines observed in the quasar spectra, and commonly referred as the Lyα forest, are usually ascribed to intergalactic gas clouds which intercept the quasar sight–line (Sargent et al., 1980). In this sense Lyα lines represent good tracers of the intergalactic matter distribution. Up to now data analysis of Lyα forests is essentially based on the traditional statistical methods, like for example the two–point correlation function. However it would be quite interesting to perform an analysis where the traditional methods are replaced, or complemented, by the investigation of the scaling properties of the Lyα column densities. In fact, the two–point correlation function has failed to highlight without ambiguity the presence of clustering in the Lyα forest on low resolution data (Sargent et al. 1980; Bechtold et al. 1987; Webb & Barcons 1991). High resolution observations (Chernomordik 1995; Cristiani et al. 1996) has revealed clustering up to scales of about 300 km s⁻¹.

We attempt to apply a method where spurious effects due to the finite size of the sample and to the poorly known column density distribution, can be controlled. The aim is to unambiguously recognize the presence of clusters or voids in the sample without losing information on their localization. In fact gravitational clustering should manifest itself as structures localized in redshift, detectable on all the dynamically interesting scales. The statistical analysis which we would like to perform is based on the idea that clustering is due to a Fourier phase correlation, which generates intermittency. The scaling behaviour of a probability measure derived from the intermittent signal is singular and can be described by the multifractal geometry. A quite similar approach has recently been used also by Pando & Fang (1995) who analyzed the scaling behaviour in the structure of the QSOs Lyα lines by using the wavelets decomposition analysis. Starting from the work by Frisch & Parisi (1985), multifractals have been invoked to describe many physical phenomena where anomalous scaling laws are present. Examples include chaotic dynamical systems, the rate of energy dissipation in fluid flows etc (Halsey et al., 1986, and the reviews by Paladin & Vulpiani, 1987; Meneveau & Sreenivasan, 1991, and references therein). Multifractal geometry in astronomy has been used mainly to describe the mass distribution of galaxies (among others see for example Martinez et al., 1990; Jones et al., 1992; Coleman & Pietronero, 1992; Borgani et al., 1993, 1994; Martinez & Coles, 1994), the apparent luminosity field of galaxies (Garrido et al., 1996), the cosmic microwave background radiation (Pompilio et al., 1995) and the intermittency in the Solar Wind turbulence (Carbone, 1993). However the literature on the subject has increased very rapidly in the last few years, and the multifractal analysis is now a standard way to recover anomalous scaling laws, if any exist, from random signals (Borgani, 1993; Vainshtein et al., 1994).
2 DATA ANALYSIS: THE MULTIFRACTAL STRUCTURE

The aim of the present paper is to study the scaling behaviour of the HI column densities $N(z)$ of Lyα clouds by using the absorption line list of quasars. As an example of the suitability of the method to capture the clustering due to phase correlations, we show the results relative to the high resolution spectrum (FWHM = 14 km s$^{-1}$) of the $z_{em} = 3.66$ QSO 0055–26 (Cristiani et al. 1995). The observations in the interval $4750 < \lambda < 6300$ Å cover a Lyα forest in the redshift range $2.958 < z < 3.654$. The Lyα lines have HI column densities in the interval $12.8 \leq \log_{10} N \leq 15.2$.

The data acquisition, reduction and analysis are described in Cristiani et al. (1995). The distribution of the Lyα lines with column density $N$ along the redshift direction is shown in Figure 1.

Two properties are evident from Figure 1: (i) $N(z)$ looks like a stochastic variable; (ii) this variable has an intermittent nature. Intermittency here is used in the same sense as it is used in fluid dynamics for the dissipation of kinetic energy (Meneveau & Sreenivasan, 1991), that is regions of large activity are interspersed between those where the absorption line density is relatively depressed. This phenomenon, in the framework of galaxy distribution, has been named “heterotopic intermittency” (Jones et al., 1992). As a reference the reader can note the striking analogy between our Figure 1 and the Figure 1 of Meneveau & Sreenivasan (1991) showing the typical time evolution of the one–dimensional energy dissipation rate in turbulent flows. This behaviour can be thought of as representing the near–singular characteristics of the phenomenon. To see this, we make a scaling analysis of $N(z)$ in the velocity space, introducing the velocity difference $\Delta v$ between two redshifts $z_1$ and $z_2$ (Sargent et al., 1980)

$$\frac{\Delta v}{\epsilon} = \frac{(z_1 + 1)^2 - (z_2 + 1)^2}{(z_1 + 1)^2 + (z_2 + 1)^2}.$$ (1)

For each scale $\Delta v$ we define a probability measure by dividing the redshift range into disjoint subsets $\Omega_i$ from equation (1). This measure $P_i(\Delta v)$ is defined as the total column density in the $i$–th subset characterized by a velocity separation $\Delta v$, normalized to the total column density in the spectrum. This can be related to the probability of occurrence of a certain amount of gas in the $i$–th box at a certain scale $\Delta v$. In Figure 2 we show $P_i(\Delta v)$ for different scales, as function of the velocity $v$ in the redshift range $2.958 < z < 3.654$. As can be noted the general behaviour of the measure on a given scale appears to be similar to that on another scale. What is interesting is that strong recurrent peaks with the same velocity are systematically detected on all scales $\Delta v$, indicating highly localized clouds with a strong scale–independent probability for their column density. These structures are now identified as the signature of clusters of Lyα absorption lines within the forest.

To investigate the multifractal structure of the measure,
we use the usual box-counting method (Halsey et al., 1986), by defining the generalized partition function
\[ \chi^q(\Delta v) = \sum_i [P_i(\Delta v)]^q, \] (2)
where the sum is extended to all the subsets \( \Omega_i \) at a given scale \( \Delta v \). High values of \( q \) enhance the strongest singularities (say the most intense clusters), while small values of \( q \) evidence the regular regions. It is evident that negative values of \( q \) in the partition function emphasizes the low density regions where the probability \( P_i(\Delta v) \) is low. The information relative to the multifractal structure can be recognized by calculating the generalized Renyi dimensions \( D_q \) from the scaling law
\[ \chi^q(\Delta v) \sim [\Delta v]^{(q-1)D_q} \] (3)
or the singularity spectrum \( f(\alpha) \) (Halsey et al., 1986). This last quantity represents the dimension associated with each singularity of strength \( \alpha \) defined through \( P_i(\Delta v) \sim (\Delta v)^\alpha \). The singularity spectrum is nothing but the Legendre transform of \( (q - 1)D_q \), so that it can be recognized from the measurement of \( D_q \). For a monofractal, \( D_q \) is constant for each \( q \). By contrast, if the intermittency can be described by multifractality we have \( D_p < D_q \) for \( p > q \). For example \( D_0 \) is the dimension of the support of the measure, which indicates the degree of fullness of the observed redshift range. The absorption lines are concentrated asymptotically on a set of dimension \( D_1 \), while \( D_2 \) is the correlation dimension (Grassberger & Procaccia, 1983).

Some care must be used in performing the analysis just described, in fact some effects can induce evidence of spurious multifractal structures. This is due to the lack of statistics, for high \( q \), in samples with a finite number of points. To check the genuine multifractal structure of the absorption lines, we examine the properties of their Fourier phases (see for example Pompilio et al., 1995). The idea comes from the fact that intermittency, and thus localized clusters, are due to Fourier phase correlations of the signal. For this reason we built up a set of 2000 “fake” sequences of column densities \( N_f(z) \) obtained from the true signal \( N(z) \) by a process of phase randomization. In other words we have calculated the Fourier coefficients for the observed sequence \( N(z) \), then the Fourier amplitudes of these coefficients are used to generate a new sequence of coefficients by using the same amplitudes but random phases. Finally, by the inverse Fourier transform, we obtain a given sequence \( N_f(z) \). Each fake sequence, which has the same spectrum as the observed one, is then used to perform the same multifractal analysis we have described so far and compared with that obtained with the true line distribution.

In Figure 3 we show the values of \( \log_{10} \chi^{(2)}(\Delta v) \) vs. \( \log_{10} \Delta v \). The interesting behaviour is the fact that there exist two distinct ranges where a linear relation can be found, and in these ranges we obtain two different values of \( D_q \) from equation (4). This happens for all the values of \( q \). The presence of two distinct linear ranges is due to a “lacunarity effect”. In fact the absorption lines are very localized in redshift, the error for the determination of a line being actually of the order of \( 10^{-5} \) (Cristiani et al., 1995). As a consequence, large gaps are present in between the absorption lines, and the support of the probability measure forms a random Cantor dust. The same effect is visible in the mass distribution of galaxies (Martinez & Coles, 1994), and is obviously visible for each sequence \( N_f(z) \). The separation of the scaling behaviour, for all our sequences, happens at about \( \Delta v \approx 1580 \) km s\(^{-1} \), a scale which physically separates two different regimes. In the following we will distinguish between the large–scales where \( \Delta v > \Delta v_* \) and the small–scales where \( \Delta v < \Delta v_* \).

In Table 1 we report the values of the generalized dimensions \( D_q \) and \( D_q^f \) (calculated as the mean of the 2000 fake values), obtained through equation (4). Some features must be discussed, the most evident being the remarkable difference which we found between the small–scales and the large–scales. By looking at the small–scales we can see that \( D_q \) is not constant, that is it behaves as a nonlinear function of \( q \). This indicates the presence of a multifractal structure. To see that the differences between the various values of \( D_q \) are meaningful, we compare these values with \( D_q^f \). As can be seen \( D_q < D_q^f \) for each \( q \), which allows us to conclude that a multifractal structure, due to phase correlation, underlies the observed sequence. The decrease of \( D_q^f \) as \( q \) increases, indicates a spurious multifractality not due to phase correlations which would tend to disappear for richer samples.

**Table 1.** The values of the generalized dimensions for both the small–scales (70 km s\(^{-1} \) \( \leq \Delta v \leq 1580 \) km s\(^{-1} \)) and for large–scales (\( \Delta v > 1580 \) km s\(^{-1} \)). \( D_q \) and \( D_q^f \) represent the generalized dimensions obtained by using respectively the spectrum of QSO 0055–26 and the average value obtained from the set of 2000 fake sequences.

| \( q \) | \( D_q \) | \( D_q^f \) | \( D_q \) | \( D_q^f \) |
|-----|-----|-----|-----|-----|
| 0   | 0.72 ± 0.02 | 0.72 ± 0.02 | 1.00 ± 0.00 | 1.00 ± 0.00 |
| 1   | 0.50 ± 0.01 | 0.65 ± 0.03 | 0.94 ± 0.01 | 0.93 ± 0.02 |
| 2   | 0.39 ± 0.02 | 0.60 ± 0.05 | 0.90 ± 0.01 | 0.90 ± 0.03 |
| 3   | 0.27 ± 0.03 | 0.55 ± 0.07 | 0.88 ± 0.03 | 0.88 ± 0.03 |
| 4   | 0.19 ± 0.05 | 0.49 ± 0.10 | 0.87 ± 0.05 | 0.86 ± 0.06 |
of lines. However the genuine multifractality of $N(z)$, due
to phase correlations, is evident. As regards the large-scale
behaviour we can see that the generalized dimensions ob-
tained both from the true and from the fake samples show
the same behaviour, with the same values. This should be
interpreted as the absence of intermittency and the lack of
a true multifractal structure beyond the scale $\Delta v_r \approx 1580$
km s$^{-1}$. Even in this case the residual spurious multifrac-
tality could be due to the limited number of data points in
our sample.

3 DISCUSSION

In order to better understand the structure of the Universe
and to find, if any, the scale at which it becomes homoge-
eous, a large sample of objects with a large spatial distribu-
tion is required. In this respect, the QSO absorption spec-
tra available to the scientific community represent a valu-
able test, being distributed in a much more extended region
(which can reach a space coverage of several hundred h$^{-1}$
Mpc in comoving distances) with respect to the observed
galaxy distribution, and forming a more unbiased sample
because the absorbing objects are not selected according to
luminosity criteria.

In the present paper we performed a statistical analysis of
the Lyo absorption lines based on the multifractal for-
malism. The main motivation to do this analysis is the fact
that the behaviour of the column density $N(z)$ with redshift
shows a highly intermittent structure where singularities can
be detected. Since in the multifractal formalism different re-
gions at a given scale are weighted in different ways, as $q$
is increased the regions where the strongest column densi-
ties $N(z)$ lie are weighted differently from the regions where
$N(z)$ are small or absent. Through a scaling analysis we
are then able to detect the signature of clustering due to
gravitational effects. The presence of a multifractal struc-
ture in the behaviour of $N(z)$ should indicate the presence
of a hierarchy of intergalactic Lyo clouds, probably gener-
ated through contraction effects (Muck et al., 1995). We
found this structure at smaller scales, say in the range 70
km s$^{-1} \leq \Delta v \leq 1580$ km s$^{-1}$ (corresponding to comoving
distances of about $0.5 \leq r \leq 7.6$ h$^{-1}$ Mpc). At scales larger
than $7.6$ h$^{-1}$ Mpc the multifractal structure disappears and
the structure is homogeneous. It is interesting to note that
the maximum scale of clustering we obtain is larger than
that obtained from preliminary results with the two-point
correlation function (Chernomordik 1995; Cristiani et al.,
1996), which in fact found clusters up to a scale $\Delta v \approx 300$
km s$^{-1}$.

High resolution spectroscopy of quasars has recently re-
ceived a boost by observations with the HIRES spectrograph
of the Keck telescope (Fan & Tytler 1994; Hu et al. 1995).
It is important to see whether the scaling behaviour is also
present in other available spectra. Preliminary results have
shown that the scaling properties persists in different quasar
line–lines. The generalized dimension for small scales is
consistent with what found for QSO 0055–26, but the char-
acteristic scale $\Delta v_r$ shows a redshift dependence. A detailed
discussion will be presented in a forthcoming paper (Savaglio
& Carbone, 1996).

A comparison of our results with those obtained from
the same analysis when applied to galaxy catalogues (CfA1,
CfA2 and QDOT redshift survey) and to the Abell and ACO
catalogs of galaxy clusters, should allows us to obtain a more
unified picture of the mass distribution. In fact from the CfA
survey one obtains significantly smaller values for the gen-
eralized dimensions, say in a three-dimensional embedding
space $D^{(3)}_0 \approx 2.1$, and $D^{(3)}_2 \approx 1.3$ for a wide range of scales
(the relation between the one–dimensional cut and the d-
dimensional cut is simply $D^{(d)}_0 = (d - 1) + D^{(1)}_0$). On the
contrary the Abell and ACO clusters (Borgani et al., 1994)
show a scale–invariant multifractal structure only in a lim-
ited range of scales, for Abell the range is $15 - 60$ h$^{-1}$ Mpc,
while for ACO it extends to smaller scales. In both cases
$D^{(2)}_2 \approx 2.2$, and the picture of a pure scale invariant frac-
tal structure extending to larger distances is disproved by
these analyses. Finally the multifractal scaling properties of
the QDOT redshift survey (Martinez & Coles, 1994) extend
over two well defined scaling ranges, for $10 - 50$ h$^{-1}$ Mpc
and for $1 - 10$ h$^{-1}$ Mpc. In both cases $D^{(3)}_0 \approx 2.9$ while
$D^{(2)}_2 \approx 2.77$ at large scales and $D^{(2)}_2 \approx 2.25$ at small scales.
These last results are not really different from those obtained
in our case. As regards the CfA surveys and the galaxy clus-
ters, our results would highlight the fact that galaxies and
the Lyo cloud have different behaviour in space. This would
be natural in a universe where the baryonic dark matter is
distributed in a homogeneous way, as most of the cosmolo-
mical models imply and the COBE results on the cosmic
microwave background radiation show.

So far we have calculated the values of $D_q$ for positive
values of $q$, because this part of the spectrum emphasizes
the regions characterized by high densities of the measure
(clusters). Even if there is a lack of statistical significance,
due to the fact that the statistics of rare events requires very
long data sets, we found two scaling regimes with multifrac-
tal scaling for $\Delta v \times \Delta v_r$. By using the clustering paradigm
(Martinez et al., 1990; Martinez & Coles, 1994), this implies
the presence of clusters localized at these scales. On the con-
trary the part of the spectrum $D_q$ with negative values of $q$
characterizes the low density regions. In this way we should
be able to recognize the presence, if any, of voids. Unfortu-
nately (Borgani et al., 1993) discreteness effects and lack of
statistics heavily affect the evaluation of $D_q$ for negative $q$,
and this effect is stronger than the lack of statistics which
affects the positive part of the spectrum.

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