Gravitational Wave Background from Phantom Superinflation

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Recently, the early superinflation driven by phantom field has been proposed and studied. The detection of primordial gravitational wave is an important means to know the state of very early universe. In this brief report we discuss in detail the gravitational wave background excited during the phantom superinflation.

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Now a number of detectors for gravitational waves are (or expected to start) operating, one of whose aim is searching a stochastic background of gravitational wave. This background is expected to have different components with different origins. The primordial gravitational wave among them will be especially interesting, since it would carry information about the state of the very early universe. The basal mechanism of generation of primordial gravitational wave in cosmology has been discussed in Ref.\textsuperscript{[14, 15, 16, 17, 18, 19]}, the initial perturbation during the phantom superinflation will leave the horizon, and can reenter the horizon during the radiation/matter domination, which may be regarded as our late time observable universe. The phantom superinflation model leads to a blue or strong blue spectrum of primordial tensor perturbations, which is different from our late time observable universe. The phantom superinflation can reenter the horizon during radiation/matter domination, which may be responsible for the structure formation of our observable universe. Further it may be convenient to define

\[ h \equiv \frac{a'}{a^2} = \frac{1}{(\epsilon - 1) a^\eta}, \]  \hspace{1cm} (2)

where the prime denotes the derivative with respect to \(\eta\). The perturbations leaving the horizon during the phantom superinflation can reenter the horizon during radiation/matter domination, which may be responsible for the structure formation of our observable universe. Further it may be convenient to define

\[ \mathcal{N} \equiv \ln \left( \frac{a_e h_e}{a_i h_i} \right), \]  \hspace{1cm} (3)

which measures the efolding number that the perturbation with the present horizon scale leaves the horizon before the end of the phantom phase, where the subscript \(e\) and \(i\) denote the end time of the phantom superinflation and the time that the perturbation with the present horizon scale leaves the horizon, respectively, and thus \(a_e h_e = a_i h_i\), where the subscript \(0\) denotes the present time, see Ref.\textsuperscript{[21]}. From (1) and (2), we obtain

\[ a \sim \left( \frac{1}{(1 - \epsilon) a^\eta} \right)^{\frac{1}{1 - \epsilon}}. \]  \hspace{1cm} (4)

Thus we have

\[ \frac{a_e}{a_i} = \left( \frac{a_e h_e}{a_i h_i} \right)^{\frac{1}{1 - \epsilon}} = e^{\mathcal{N} \epsilon}. \]  \hspace{1cm} (5)

We can see that during the phantom superinflation, the change of scale factor is dependent on \(\epsilon\). For the negative enough \(\epsilon\), the change \(\Delta a/a = (a_e - a_i)/a_i \approx \mathcal{N}/(1 - \epsilon)\) of \(a\) can be very small. Taking the logarithm in both sides of (4), we obtain\textsuperscript{2}

\[ \ln \left( \frac{1}{a h} \right) = (\epsilon - 1) \ln a. \]  \hspace{1cm} (6)

We plot Fig.1 to further illustrate the characters of the phantom superinflation. We assume, throughout this

\textsuperscript{1} Note that it is also possible to get phantom energy in scalar-tensor theories of gravity without non canonical kinetic terms. \textsuperscript{2} The equation can be shown and actually also applied for the expansion with arbitrary constant \(\epsilon\).
phantom superinflation is given by, accounting for both

\[ k \eta \]

where \( \epsilon < 0 \), we have \( 1/2 < \nu < 3/2 \).

The amplitude of gravitational wave after the end of phantom superinflation is given by, accounting for both polarizations,

\[
P_t^{1/2}(k, \eta) = \frac{k^{3/2}}{\pi} \frac{a_e |u_e(-k\eta_e)|}{a_e} \epsilon^{1/2-\nu} k^{3/2-\nu}.
\]  

(10)

We obtain, from (2) and (3),

\[
P_t^{1/2}(k, \eta) = \frac{1}{\pi a_e} \frac{1}{(1-\epsilon)\alpha e \eta_e} \epsilon^{1/2-\nu} k^{3/2-\nu}
\]

\[
= \frac{h_e}{\pi} \frac{1}{(1-\epsilon)^{1/2-\nu}} \frac{k_0}{k} \epsilon^{1/2-\nu} \left( \frac{k}{k_0} \right)^{3/2-\nu}
\]

\[
= h_e e^{-N(3/2-\nu)} \frac{k}{k_0} \epsilon^{1/2-\nu} \left( \frac{k}{k_0} \right)^{3/2-\nu}.
\]  

(11)

We can see that for fixed \( N \) and \( h_e \), when \( \nu \approx 3/2 \), which corresponds to \( \epsilon \approx 0 \), \( P_t^{1/2}(k, \eta) \approx h_e/\pi \) is scale invariant, which is the usual result of inflation models, while when \( \nu < 3/2 \), which corresponds to \( \epsilon < 0 \), the spectrum of \( P_t^{1/2}(k, \eta) \) is blue tilted and has the exponentially suppressed amplitude \( \sim h_e e^{-N(3/2-\nu)}/\pi \) at the largest scale \( k \approx k_0 \). The limit of \( \nu \to 1/2 \) is the solution of Ref. 14, in which the scale factor is nearly unchanged and the Hubble parameter \( h \) experiences an instantaneous “jump”, which results in that the spectrum is strong blue tilt

\[
P_t^{1/2}(k, \eta) \approx h_e e^{-N(k/k_0)/\pi}.
\]  

(12)

To convert from the primordial spectrum to the present spectrum, we need to know the transfer function \( T(k) \),

\[
P_t^{1/2}(k, \eta) = T(k)P_t^{1/2}(k, \eta_e).
\]  

(13)

In general, \( P(k, \tau) \) is roughly time independent outside the horizon and decay as \( a^{-1} \) as long as the corresponding mode reenters the horizon. Therefore, based on the fact that \( h \sim a^{-2} \) during the radiation domination and \( h \sim a^{-3/2} \) during the matter domination, the numerical fitting gave 22, 23

\[
T(k) \approx \left( \frac{k_0}{k} \right)^2 \left( 1 + \frac{4}{3} \frac{k}{k_{eq}} + \frac{5}{2} \frac{k}{k_{eq}}^2 \right)^{1/2},
\]  

(14)

where \( k_{eq} = a_{eq} h_{eq} \) denotes the modes entering the horizon at the time of matter-radiation equality. Eq. (14) actually only applies for \( k > k_0 \) and below this \( T(k) = 1 \). We have, from (5),

\[
\frac{k_{eq}}{k_0} = \frac{a_{eq} h_{eq}}{a_0 h_0} = \frac{a_0}{a_{eq}} \epsilon^{-1}.
\]  

(15)

For the matter domination, \( \epsilon = 3/2 \). Thus \( k_{eq}/k_0 = \sqrt{1 + z_{eq}} \).

To characterize the spectrum of the stochastic gravitational wave signal, an useful quantity \( \Omega_{gw}(k) \) is introduced, see 24, 25, 26, which is the gravitational wave energy per unit logarithmic wave number in the units of the critical density

\[
\Omega_{gw}(k) = \frac{k}{\rho_{cr}} \frac{d\rho_{gw}}{dk},
\]  

(16)
We can see that for is $\Omega$ at the large angular scale. The present bound we will neglect $\tilde{h}_0$ existing experimental uncertainty. However, for simplicity we will neglect $\tilde{h}_0$ in the following. In general [23],

$$\Omega_{gw}(k) \simeq \frac{1}{6} \left( \frac{k_0}{k} \right)^2 P_t(k, \eta_0)$$

$$\simeq \frac{1}{6} \left( \frac{k_0}{k} \right)^2 \left( 1 + \frac{4}{3} \frac{k}{k_{eq}} + \frac{5}{2} \frac{k}{k_{eq}} \right) P_r(k, \eta_0)$$

where [14] has been used. Fig.2 shows the results of $\Omega_{gw}(k)$ for the phantom superinflation with different $\epsilon$. The spectrum is cut off at the wave number $k_{max} = k_c$. For the instantaneous reheating in which the phantom superinflation is immediately followed by a phase of radiation domination, $k_{max} = a_u h_c \simeq a_0 T_0 h_c^{-1/2}$, where $T_0$ is the temperature of present CMB. The overall amplitude $\sim h_0^2$ of $\Omega_{gw}(k)$ should be given by the CMB observation at the large angular scale. The present bound is $\Omega_{gw}(k) < 10^{-11} k_0/k$ at $10^{-18}$Hz $< k < 10^{-16}$Hz [27]. We can see that for $\epsilon \simeq 0$, $\Omega_{gw}(k)$ decays with $k$ up to $k \simeq k_{eq}$, and after $k \gg k_{eq}$ it enters a nearly long plateau

$$\Omega_{gw}(k) \gg k_{eq}, \epsilon \simeq 0 \simeq \frac{5 h_c^2}{12 \pi^2} \left( \frac{k_0}{k_{eq}} \right)^2, \quad (18)$$

which is the character of inflation model [28], while for $\epsilon < 0$ the case is distinctly different, in which at $k \gg k_{eq}$, $\Omega_{gw}(k)$ will increase with $k$ up to $k \simeq k_{max}$. The slope of increasing is very dependent of $\epsilon$, and the extreme one is $\sim k^2$ corresponding to $\epsilon \rightarrow -\infty$ [14]. However, since at low frequency region, the amplitude of $\Omega_{gw}(k)$ has a strong suppression from the folding number, see [11], which is relevant with $\epsilon$, at the case that the reheating temperature $T_r \simeq h_d^{1/2}$ is same we can obtain at $k \gg k_{eq}$

$$\Omega_{gw}(k_{max}) \simeq (1 - \epsilon)^{2v - 1} \frac{5 h_c^2}{12 \pi^2} \left( \frac{k_0}{k_{eq}} \right)^2$$

$$\simeq (1 - \epsilon)^{2v - 1} \frac{5 h_c^2}{12 \pi^2} \left( \frac{k_0}{k_{eq}} \right)^2$$

$$\simeq \Omega_{gw}(k) \gg k_{eq}, \epsilon \simeq 0). \quad (19)$$

Thus though for $\epsilon < 0$ there is an interesting increasing of $\Omega_{gw}(k)$ amplitude at high $k$, when the scale $h_c$ (or reheating scale) is taken as same, the largest value obtained at $k \simeq k_{max} = k_c$ is always around the plateau $\Omega_{gw}(k \gg k_{eq}, \epsilon \simeq 0)$ of the inflation. This result is not dependent on the value of the reheating scale.

In summary, we study in detail the gravitational wave background excited during the phantom superinflation. For $\epsilon \simeq 0$, the amplitude of spectrum is very close to that of inflation, and can be expected to be detected in the future. But for $\epsilon \ll -1$, the amplitude is very low at the large scale (low $k$) and hardly seen, however, in high frequency region the amplitude increases $\sim k^{3-2v} = k^{2v/(\epsilon - 1)}$ up to the plateau of gravitational wave background of the inflation, thus there may be some significant and different signals around $k_{max}$. These results may be interesting for the present and planned experiments detecting gravitational wave.

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