Rapid Aerodynamic Calculation Method for Hypersonic Gliding Vehicle

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Abstract. The accuracy and efficiency are always conflicted in the process of aerodynamic shape design, especially for hypersonic gliding vehicle (HGV). In this work, a rapid aerodynamic calculation method for hypersonic gliding vehicle is conducted, including shape parametric modelling, automatic mesh generation and rapid computational fluid dynamics (CFD) method. In details, the aerodynamic shapes of HGV are modelled by multi-section solids, which parameterized by class function/shape function transformation (CST) method and exponential function. Then, the computational meshes are generated automatically in two steps: one is generating surface mesh by Delaunay method, and another is generating volume mesh by adaptive Cartesian method. In order to minor the computational cost, the hypersonic aerodynamic are analysed by combining the Euler equation and the viscosity force. We have tested the accuracy and efficiency of this rapid aerodynamic calculation method in two flight states for a typical HGV. Compared with Navier Stokes (NS) equations, the results shown the errors of axial aerodynamic force are all less than 6%, but time costs are reduced in hundreds of times.

1. Introduction

At present, the HGV has become a research hotspot for its high speed and high mobility. The main flight altitude of HGV is in near-space of 20km-100km and the Mach number is greater than 5 [1]. Due to the special flight environment with high-altitude and high-speed, the aerodynamic simulation of HGV is difficult and costly, especially in time-consuming. In addition, the configurations of HGV are unique and complex, such as lifting body and wave-rider vehicle [2].

An accurate shape parametric model is the basis for the aerodynamic shape design of HGV, which plays a crucial role in its performance. When calculating the aerodynamic force in design of a lifting body, the accuracy and efficiency are always conflicted. With multiple iterations in the aerodynamic shape design works, it is important to minimize the computational consumption.

Consideration above challenges, this paper developed a rapid hypersonic aerodynamic calculation method for aerodynamic design of HGV, which contains three key points: shape parametric modeling, automatic mesh generation and rapid CFD method. Based on this rapid aerodynamic calculation method, the designer can predict the aerodynamic results for HGV with high efficiency and credible accuracy.

2. Shape parametric model
According to the characteristics of aerodynamic shape of HGV, the class function/shape function transformation (CST) method [3] is suitable to generate the section curve. Combined with exponential function generates the profile curve, the shape parametric model of HGV can be completed.

2.1. Section curve based on CST method

The CST method is a commonly used parametric geometry representation technique, which can be developed to generate the complicated three dimensional configurations of HGV. Before using the CST method to describe the section curve, the actual geometry coordinates \((x, y)\) should be normalized with parameterized coordinates \((\eta, \psi)\). The normalized function of section curve based on CST method can be written as:

\[
\eta(\psi) = C(\psi)S(\psi) \tag{1}
\]

where \(\eta \in [0,1]\) and \(\psi \in [0,1]\) are the normalized coordinates of section curve, \(C(\psi)\) is the class function and \(S(\psi)\) is the shape function.

In the expression of CST method, the class function of \(C(\psi)\) represents the classification and geometric properties of section curve, and the shape function of \(S(\psi)\) defines the figure of the curve more accurately.

2.2. Profile curve based on exponential function

Generally, the profile curve in the top view of HGV is a smoothed curve, which extending smoothly from the top to the bottom. Based on above consideration, we can use the exponential function to generate the profile curve, defined as:

\[
y = \frac{W}{2L^2} x^n \tag{2}
\]

where \(W\) is the maximum width of HGV, \(L\) is the total length of HGV, and \(n\) is an index that controls the curvature change of the profile curve.

2.3. Shape parametric model based on “Section-Profile” curve

Combined the “Section” curve based on CST method with the “Profile” curve based on exponential function, we can complete the shape parametric model for HGV based on “Section-Profile” curve. In details, the execution steps are as follows:

a) Provide the maximum width \(W\) and total length \(L\) of the HGV to define the outer outline;

b) Ascertain the number of section curves to express the geometry characteristics of HGV;

c) Ascertain the functions of \(C(\psi)\) and \(S(\psi)\) in each section, then generate the section curves based on the CST method;

d) Ascertain the value of index \(n\) to generate a profile curve based on exponential function;

e) Generate the shape parametric model of HGV based on above “section-contour” curves.

3. Automatic grid generation

When the shape model of HGV has been completed by above parametric method, the next work is to generate the computational grids, including the surface grid and volume grid. At present, there are three mainly used mesh types, such as structured grid, unstructured grid and Cartesian grid. On one hand, the unstructured grid is easy to generate surface grid without the constraints of geometry boundaries. On the other hand, the adaptive Cartesian grid can provide the automatic generation of volume grid, which based on the surface grid. Therefore, for the shape model HGV, the surface grid is generated based on the unstructured mesh, and the volume grid is generated based on the adaptive Cartesian method.

3.1. Surface grid based on Delaunay method
When generating the unstructured meshes, the Delaunay method is a commonly used and reliable method. The basis theory of Delaunay method is dividing a plane into some convex polygons based on the point sets we have known. The Delaunay method has some advantages: good quality, high efficiency, strict basis of mathematical theoretical, and can generate equilateral triangles as much as possible.

3.2. Volume grid based on adaptive Cartesian method
The data structure of Cartesian grid is based on the dynamic fork tree, such as a dynamic quadtree structure for two-dimensional grid and a dynamic octree structure for three-dimensional grid. On the basis of geometric shape and flow field characteristics, the adaptive Cartesian method is able to continuously generate mesh adaptive refinement in the local area.

The adaptive Cartesian mesh method has some advantages: high efficiency, no division of mesh topology, automation mesh generation, etc. Based on the adaptive Cartesian mesh method, a high-quality volume grid can be generated without human intervention.

4. Rapid CFD method
In the flight environment with high-altitude and high-speed, the viscous effect is the mainly factor interfering the aerodynamic force of HGV [4]. In order to balance the calculation efficiency and precision, a rapid CFD method for HGV is developed.

First, an Euler equation is used to calculate the non-viscous aerodynamic force. Then, the viscosity force is computed based on the friction coefficient. Combined the non-viscous with viscosity force, the final aerodynamic force of HGV can be predicted.

4.1. Euler equation
Generally, the Euler equation can calculate the three-dimensional compressible and non-viscous aerodynamic force. When the incoming fluid is considered as the perfect gas, the Euler equation can be written as:

$$\iiint \frac{\partial Q}{\partial t} dV = -\iiint (\nabla \cdot \hat{F}) \cdot \hat{n} ds$$  \hspace{1cm} (3)

where, $\hat{n}$ is a unit vector, $\hat{F}$ is the flux function, and $Q$ is the state vector, which consisting of flow variables, such as the density $\rho$ and the velocity vectors of $u, v, w$.

4.2. Viscosity force
The viscous correction is calculated by using the axial force viscous component based on the friction coefficient. First, the friction coefficient of the aircraft is calculated, and then the viscous force of the surface is integrated to obtain the viscous force component of the axial force.

For the turbulent flow, the definition of friction coefficient can be written as

$$C_f = \frac{0.37}{(\log Re^*)^{2.564}} \left( \frac{\rho^*}{\rho_c} \right)$$  \hspace{1cm} (4)

where, $C_f$ is the friction coefficient, $Re^*$ is the referenced Reynolds number at the edge of the boundary layer, $\rho^*$ is the referenced density, and $\rho_c$ is the density of incoming fluid.

The referenced values of $Re^*$, $\rho^*$ and $\mu^*$ can be calculated by the reference enthalpy method, which developed by Eckert [5]. In this paper, we take the Meador’s reference enthalpy method [6] to calculate the viscosity force:

$$\frac{H^*}{H_e} = 1 + 0.5 \left( \frac{H_w}{H_e} - 1 \right) + 0.16 \frac{Y-1}{2} Ma^2$$  \hspace{1cm} (5)

where, $H^*$ is the reference enthalpy, $H_w$ is the enthalpy on the wall, $H_e$ is the enthalpy at the edge
of the boundary layer, $Ma_e$ is the Mach number at the edge of the boundary layer, $r$ is the recovery factor that depends on the Prandtl number, and $\gamma$ is the ratio of specific heats.

5. Results
Integrated above shape parametric model, automatic grid generation and rapid CFD method, we can set up the rapid aerodynamic calculation method for hypersonic gliding vehicle. In order to verify the accuracy and efficiency of the proposed method, we demonstrate a typical example.

5.1. Shape parametric Model
Based on the shape parametric model based on “Section-Profile” curves, we take three section curves to shape the parametric model as a typical example of HGV, as shown in Fig. 1.

![Image of the shape parametric model of HGV.]

Table 1. Main values of shape parametric model in Fig. 1.

| Shape parameter | $L$   | $L_1$ | $L_2$ | $W$ | $H$ | $H_1$ | $H_2$ | $r$ | $\theta$ |
|-----------------|-------|-------|-------|-----|-----|-------|-------|-----|----------|
| Main values (mm)| 4000  | 3000  | 1000  | 1000| 560 | 400   | 160   | 10  | 18°      |

5.2. Automatic mesh generation
First, we generate the surface mesh for above shape parametric model of HGV based on Delaunay method, as show in Fig. 2.
5.3. CFD results
For the above typical HGV, we compute its axial aerodynamic force results under the flight height of 55km with two commonly used computational states of $Ma=8$, $\alpha =8^\circ$ and $Ma=15$, $\alpha = 10^\circ$. Taking the results of NS equations as the reference, we compare the calculation accuracy and efficiency between the rapid aerodynamic calculation method proposed in this paper and the NS equations.

The results between rapid aerodynamic calculation method and NS equations are shown in Table 1 and in detail, Rapid represents the results of the rapid aerodynamic calculation method, NS represents the results of the NS equations, and the Delta represents the relative errors between the results of rapid aerodynamic calculation method and the NS equation, which can be written as:

$$\text{Delta} = \frac{\text{Rapid} - \text{NS}}{\text{NS}} \times 100\%$$

As Table 2 shows, the results of Delta in aerodynamic force are all within 6% for the two computational states, which means the calculation accuracy of this rapid aerodynamic calculation method is credible. On the other hand, the computation time of NS equations is as 160 times as the rapid aerodynamic calculation method. What’s worse, the rapid aerodynamic calculation method just takes one central processing unit (CPU), but the NS equations need four CPUs. Otherwise the difference of time consumption is larger. In conclusion, the rapid aerodynamic calculation method not only equips with credible calculation accuracy, but also have higher efficiency.
### Table 2. Results between rapid aerodynamic calculation method and NS equation.

|                  | $H=40\text{km}$ | $Ma=6, \alpha=8^\circ$ | $Ma=10, \alpha=8^\circ$ |
|------------------|------------------|-------------------------|--------------------------|
| **Aerodynamic force** | **NS**           | 0.05795                 | 0.03981                  |
|                  | **Rapid**        | 0.04627                 | 0.04032                  |
|                  | **Delta**        | 4.42%                   | 1.29%                    |
| $H=55\text{km}$  |                  |                         |                          |
| **Aerodynamic force** | **NS**           | 0.07211                 | 0.05867                  |
|                  | **Rapid**        | 0.06933                 | 0.06218                  |
|                  | **Delta**        | -3.86%                  | 5.98%                    |
| **Time**         | **NS (4 CPU)**   | 80h                     | 80h                      |
|                  | **Rapid (1 CPU)**| 0.5h                    | 0.5h                     |

### 6. Conclusion

This paper introduces a rapid aerodynamic calculation method for hypersonic gliding vehicle, which contains shape parametric modeling, automatic mesh generation and rapid CFD method.

Based on this rapid aerodynamic calculation method, we have tested its accuracy and efficiency for a typical HGV. Compared the computational results of NS equations, the errors of axial aerodynamic force are all less than 6%, but time costs are reduced in hundreds of times.

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