Towards gauge coupling unification in left-right symmetric $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times U(1)_X$ theories

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We consider the possibility of gauge coupling unification within the simplest realizations of the $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times U(1)_X$ gauge theory. We present a first exploration of the renormalization group equations governing the “bottom-up” evolution of the gauge couplings in a generic model with free normalization for the generators. Interestingly, we find that for a $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times U(1)_X$ symmetry breaking scale $M_X$ as low as a few TeV one can achieve unification in the presence of leptonic octets. We briefly comment on possible grand unified theory frameworks which can embed the $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times U(1)_X$ model as well as possible implications, such as lepton flavour violating physics at the LHC.

I. INTRODUCTION

There is no doubt, at least among theorists, that there must be new physics, unaccounted for by the Standard Model successfully describing most of the phenomena in particle physics. From the observational side, the theory lacks neutrino masses, needed to describe oscillations [1–3]. It also fails in accounting for the current cosmological puzzles, as well as the need to ultimately include gravity as part of the fundamental theory. From a more aesthetical point of view the Standard Model does not offer a basic understanding of many theoretical issues, such as the origin of parity violation. Indeed, the chiral nature of the weak interactions remains a mystery at a fundamental level. As a matter of fact in higher unified theories it constitutes an arbitrary input, by no means automatic. Likewise, the Standard Model gives no input on the number of fermion families nor any understanding of the flavor problem – neither an understanding of fermion masses themselves, nor of the fermion mixing patterns – both of which remain mysteries. And the list goes on [4].
Frustratingly enough, so far there is no strong hint of new physics from the Large Hadron Collider at CERN, despite the floods of data already obtained. No sign yet of supersymmetry, the leading candidate theory in terms of which to understand theoretical issues such as the consistency of the electroweak breaking mechanism, now vindicated by the celebrated discovery of the Higgs boson [5, 6].

Indeed, by establishing the existence of scalar particles in nature, the discovery of the Higgs boson has paved the way for probing extensions of the Standard Model (SM) at accelerators. One of the extensions of the Standard Model consists of left-right symmetric schemes which can give a dynamical basis for describing parity violation in weak interactions and shed light on the origin of neutrino mass [7]. An interesting alternative is the 331 model, which has the special feature that it is not anomaly free in each generation of fermions, but becomes anomaly free only when all the three generations of fermions are included in the theory [8], a feature which may shed light on the flavor puzzle, since one family of quarks must be distinguished from the others.

Both approaches provide a solution to neutrino mass problem either through the seesaw mechanism [7, 9-12] or through other alternative neutrino mass generation mechanisms [13, 14]. Both frameworks may yield new physics that can be observed at the LHC or the next generation accelerators [15-17]. Which alternative is right, if any, we currently do not know.

Recently a realistic theory framework based on the extended $G_{3C3L3R1X} = SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_X$ gauge group has been proposed. It requires the number of families to match the number of colors, while encompassing the idea of left-right symmetry. The new framework admits both high [18] as well as low scale [19] realizations of the seesaw mechanism of neutrino mass generation, and brings in a plethora of possible new physics at accessible accelerator energies. The low energy phenomenology like neutrinoless double beta decay has been explored recently in [20] where fermion masses including light neutrinos are governed by universal seesaw with extra vector like fermions and without having scalar bitriplet. An attractive feature is that, depending on how the symmetry breaks to the Standard Model, one recovers either a conventional left-right symmetric theory, or a 331 symmetry as the “next” step towards new physics.

However, as noted in [21], the fact that different multiplets of the $G_{3C3L3R1X} = SU(3)_C \times SU(3)_L \times U(1)_X$ group appear with different multiplicities makes it difficult to unify the model within Grand Unified Theories (GUTSs) using canonical routes 1. Here we perform the first step towards a possible unification of the gauge couplings within this class of left-right symmetric theories. This consists of a first exploration of the renormalization group equations governing the “bottom-up” evolution of the gauge couplings in a generic model with free normalization for the generators.

The article is organized as follows. In Section [II] we discuss the basic structure of the $G_{3C3L3R1X}$, also called 3331 model, for short. We consider models with sextet Higgs scalars to account for neutrino masses. In Section [III] we analyze the resulting renormalization group (RG) running of the gauge couplings in the model with and without additional octet states,

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1 For this reason possible string completions have been suggested in [22].
discussing in detail the necessary conditions for gauge coupling unification. For simplicity we assume direct breaking to the Standard Model. We conclude that, in the presence of octets, unification is possible while leaving a light "gauge boson portal" which may open access to new physics at collider energies [23]. The latter may allow one to probe physics beyond the Standard Model in a novel way. We also comment briefly on the possible embedding of the model within a higher unification group.

II. THEORY FRAMEWORK

The gauge group of the left-right symmetric model considered for the present work is given by

$$G_{3c3L3R1X} = SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_X,$$

where the electric charge relation is given by the following formula

$$Q = T_{3L} + T_{3R} + \beta (T_{8L} + T_{8R}) + X.$$  \(\text{(2)}\)

and the fermion assignments are given as

$$\Psi_{aL} = \begin{pmatrix} \nu_{aL} \\ \ell_{-aL}^{\gamma} \\ \chi_{aL} \end{pmatrix} \sim (1, 3, 1), \quad \Psi_{aR} = \begin{pmatrix} \nu_{aR} \\ \ell_{-aR}^{\gamma} \\ \chi_{aR} \end{pmatrix} \sim (1, 1, 3, \frac{q-1}{3}),$$

$$Q_{aL} = \begin{pmatrix} d_{aL} \\ u_{aL} \\ J_{-aL}^{-q+1/3} \end{pmatrix} \sim (3, 3^*, 1), \quad Q_{aR} = \begin{pmatrix} d_{aR} \\ u_{aR} \\ J_{-aR}^{-q+1/3} \end{pmatrix} \sim (3, 1, 3^*, \frac{-q}{3}),$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ J_{3L}^{q+2/3} \end{pmatrix} \sim (3, 3, 1), \quad Q_{3R} = \begin{pmatrix} u_{3R} \\ d_{3R} \\ J_{3R}^{q+2/3} \end{pmatrix} \sim (3, 1, 3, \frac{q+1}{3}).$$  \(\text{(3)}\)

Notice that the electric charge of third component of lepton triplet is related to \(\beta\) parameter in the following way

$$\beta = -\frac{2q + 1}{\sqrt{3}},$$  \(\text{(4)}\)

and its value is restricted by the fact that the SU(3)\(_{L,R}\) and U(1)\(_X\) coupling constants \(g_L = g_R = g\) and \(g_X\) must comply with the relation

$$\frac{g_X^2}{g^2} = \frac{\sin^2 \theta_W}{1 - 2(1 + \beta^2) \sin^2 \theta_W},$$

which implies that \(\beta^2 < -1 + 1/(2 \sin^2 \theta_W)\), hence the choice \(\beta = \sqrt{3}\) is excluded by consistency of the model [2].

[2] Other possibilities such as \(\beta = \pm \frac{2}{\sqrt{3}}\) and \(\beta = 0\) will not be considered since they lead to exotic fractionary charges for fermions.
The spontaneous symmetry breaking of $\mathcal{G}_{3c^3L^3R^1x}$ is implemented through scalar sextets $\Delta_{L,R}$ (will be associated with type-I and type-II seesaw) as well as by the usual scalar bitriplet $\Phi$ with their matrix representation given as follows

$$\Phi = \begin{pmatrix} \phi^0_{11} & \phi^{+}_{12} & \phi^{-q}_{13} \\ \phi^0_{21} & \phi^{0}_{22} & \phi^{-q}_{23} \\ \phi^{q+1}_{31} & \phi^{q}_{32} & \phi^{0}_{33} \end{pmatrix},$$  
$$\rho = \begin{pmatrix} \rho^0_{11} & \rho^{0}_{12} & \rho^{q+1}_{13} \\ \rho^0_{21} & \rho^{0}_{22} & \rho^{q}_{23} \\ \rho^{q+1}_{31} & \rho^{q}_{32} & \rho^{0+1}_{33} \end{pmatrix},$$ (6)

$$\Delta_L = \begin{pmatrix} \Delta^0_{11} & \Delta^{-1}_{12}/\sqrt{2} & \Delta^{q-1}_{13}/\sqrt{2} \\ \Delta^{-1}_{21}/\sqrt{2} & \Delta^{q-1}_{22} & \Delta^{q-2}_{23}/\sqrt{2} \\ \Delta^{q}_{31}/\sqrt{2} & \Delta^{q-2}_{32}/\sqrt{2} & \Delta^{q}_{33} \end{pmatrix}_L,$$  
$$\Delta_R = \begin{pmatrix} \Delta^0_{11} & \Delta^{-1}_{12}/\sqrt{2} & \Delta^{q-1}_{13}/\sqrt{2} \\ \Delta^{-1}_{21}/\sqrt{2} & \Delta^{q-1}_{22} & \Delta^{q-2}_{23}/\sqrt{2} \\ \Delta^{q}_{31}/\sqrt{2} & \Delta^{q-2}_{32}/\sqrt{2} & \Delta^{q}_{33} \end{pmatrix}_R,$$ (7)

We now turn to the Yukawa interactions of the theory. These are similar to the ones present in the most popular left-right symmetric models, namely

$$\mathcal{L}_y = \sum_{\alpha,\beta=1}^2 \left( h^{Q}_{\alpha 3} \bar{Q}_L^\alpha \Phi^\dagger Q_R^\beta \right) + \sum_{\alpha=1}^2 \left( h^{Q}_{\alpha 3} \bar{Q}_L^\alpha \rho^\dagger Q_R^\beta + h^{Q}_{\alpha 3} \bar{Q}_L^\alpha \rho^\dagger Q_R^\beta \right) + h^{Q}_{\alpha 3} \bar{Q}_L^\alpha \Phi Q_R^\beta + \text{h.c.}$$ (8)

with $h^Q = (h^Q)^\dagger$. The corresponding spontaneous symmetry breaking pattern is given by:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \text{diag}(k_1, k_2, n), \quad \langle \Delta_L \rangle = \frac{1}{\sqrt{2}} \text{diag}(v_L, 0, 0), \quad \langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \text{diag}(v_R, 0, 0),$$ (9)

$$\langle \rho \rangle = \begin{pmatrix} 0 & k_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here $\rho$ is the responsible for generating a realistic CKM matrix. Moreover, one assumes, for consistency, that $n, v_R \gg k_1, k_2, k_3, v_L$.

After spontaneous symmetry breaking, the first line in Eq. (14) produces the following Dirac mass matrices for the Standard Model and exotic quarks:

$$M^u = \frac{1}{\sqrt{2}} \begin{pmatrix} h^{Q}_{11} k_2 & h^{Q}_{12} k_2 & 0 \\ h^{Q}_{21} k_2 & h^{Q}_{22} k_2 & 0 \\ -h^{Q}_{31} k_3 & -h^{Q}_{32} k_3 & h^{Q}_{33} k_1 \end{pmatrix}, \quad M^d = \frac{1}{\sqrt{2}} \begin{pmatrix} h^{Q}_{11} k_1 & h^{Q}_{12} k_1 & h^{Q}_{13} k_3 \\ h^{Q}_{21} k_1 & h^{Q}_{22} k_1 & h^{Q}_{23} k_3 \\ 0 & 0 & h^{Q}_{33} k_2 \end{pmatrix},$$ (10)

$$M^{J-\frac{q}{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} h^{Q}_{11} n & h^{Q}_{12} n \\ h^{Q}_{21} n & h^{Q}_{22} n \end{pmatrix}, \quad M^{J+\frac{q}{2}} = h^{Q}_{33} n. \quad (11)$$

For leptons we have:

$$m^e_{ab} = \frac{k_2}{\sqrt{2}} h^e_{ab},$$ (12)
and the new leptons $\chi_{L,R}^q$ form heavy Dirac pairs with masses

$$m_{ab}^\chi = \frac{n}{\sqrt{2}} h_{ab}^\ell.$$  \hspace{1cm} (13)

Concerning neutrinos, the structure of their mass matrix will involve crucially the Higgs sextets, as explained below.

A. Model with sextets and seesaw mechanism

We now turn to the Yukawa interactions for neutrinos. These are similar to the ones present in the most popular left-right symmetric models, namely

$$\mathcal{L}_y = \sum_{a,b=1}^3 \left[ h_{ab}^\ell \psi_a^L \Phi^b \psi_b^R + f_{ab} \left( \bar{\psi}_a^L \Delta^1_L \psi_b^R + \bar{\psi}_a^R \Delta^1_R \psi_b^R \right) \right] + \text{h.c.}$$ \hspace{1cm} (14)

where $h^\ell = (h^\ell)^\dagger$ and $\Delta_{L,R}$ denote the Higgs sextets. As a result the neutrino mass matrix can be written as

$$m_\nu = \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix},$$ \hspace{1cm} (15)

where

$$M_L = 2 f_{ab} v_L, \quad m_D = h_{ab}^\ell k_1, \quad M_R = 2 f_{ab} v_R.$$ \hspace{1cm} (16)

Thus we obtain the standard combination of type I and type II seesaw mechanisms \[11\]:

$$m_1 \approx M_L - m_D M_R^{-1} m_D^T, \quad m_2 \approx M_R.$$ \hspace{1cm} (17)

for which the diagonalizing matrices can be obtained systematically in perturbation theory as in \[24\].

B. The SVS model: $\beta = \frac{-1}{\sqrt{3}}$

In this section we will fix the $\beta$ parameter (see equation \[2\]) to the value $\beta = \frac{-1}{\sqrt{3}}$ which correspond to the SVS model \[8\]. Notice that in this case the electric charge of the third component of the leptonic triplet is $q = 0$. In general the electric charge relation is given by the following formula

$$Q = T_{3L} + T_{3R} - \frac{1}{\sqrt{3}} (T_{8L} + T_{8R}) + X.$$ \hspace{1cm} (18)

$$\Psi_{aL} = \begin{pmatrix} \nu_{aL} \\ \ell_{aL}^- \\ N_{aL} \end{pmatrix} \sim (1, 3, 1, \frac{-1}{3}), \quad \Psi_{aR} = \begin{pmatrix} \nu_{aR} \\ \ell_{aR}^- \\ N_{aR} \end{pmatrix} \sim (1, 1, 3, \frac{-1}{3}),$$

$$Q_{aL} = \begin{pmatrix} d_{aL} \\ u_{aL} \\ D_{aL} \end{pmatrix} \sim (3, 3^*, 1, 0), \quad Q_{aR} = \begin{pmatrix} d_{aR} \\ u_{aR} \\ D_{aR} \end{pmatrix} \sim (3, 1, 3^*, 0),$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ U_{3L} \end{pmatrix} \sim (3, 3, 1, \frac{1}{3}), \quad Q_{3R} = \begin{pmatrix} u_{3R} \\ d_{3R} \\ U_{3R} \end{pmatrix} \sim (3, 1, 3, \frac{1}{3}).$$ \hspace{1cm} (19)
The spontaneous symmetry breaking pattern is now more general, since there are more scalars that can develop a vev. We will assume the simplest case, e.g. only $\Delta_{33}$ for $\Delta_{L,R}$ will develop a non-zero vev in order to give Majorana masses to exotic neutrinos:

$$\langle \Delta_L \rangle = \frac{1}{\sqrt{2}} \text{diag}(v_L, 0, \Lambda_L), \quad \langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \text{diag}(v_R, 0, \Lambda_R). \quad (20)$$

Let us also assume $\Lambda_R \sim v_R$ and $\Lambda_L \sim k_1$. In this case the neutrino mass matrix, in the basis $(\nu_L, \nu_R, N_L, N_R)$, is given by:

$$m_\nu = \begin{pmatrix} M_L & m_D & 0 & 0 \\ m_D^T & M_R & 0 & 0 \\ 0 & 0 & M'_L & M'_D \\ 0 & 0 & M_D & M'_R \end{pmatrix}, \quad (21)$$

With:

$$M_L = 2f_{ab} v_L, \quad m_D = h^{\ell}_{ab} k_1, \quad M_R = 2f_{ab} v_R,$$

$$M'_L = 2f_{ab} \Lambda_L, \quad M'_D = h^{\ell}_{ab} n, \quad M'_R = 2f_{ab} v_R. \quad (22)$$

We also note that in Eq. (11) there are two exotic vector-like down-type quarks and an exotic vector-like up-type quark.

### III. GAUGE COUPLING UNIFICATION

It is interesting to note that the spontaneous symmetry breaking of the $\mathcal{G}_{3331}$ model to the low energy theory can be implemented through three possible ways

A : $\mathcal{G} \overset{M_{T}}{\rightarrow} \mathcal{G}_{3c3L3R1X} \overset{n \simeq v_R \simeq \Lambda_R}{\rightarrow} \mathcal{G}_{3c2L1Y} \overset{k_i}{\rightarrow} \mathcal{G}_{3c1Q}$,

B : $\mathcal{G} \overset{M_{T}}{\rightarrow} \mathcal{G}_{3c3L3R1X} \overset{v_R}{\rightarrow} \mathcal{G}_{3c3L1Y1X'} \overset{n}{\rightarrow} \mathcal{G}_{3c2L1Y} \overset{k_i}{\rightarrow} \mathcal{G}_{3c1Q}$,

C : $\mathcal{G} \overset{M_{T}}{\rightarrow} \mathcal{G}_{3c3L3R1X} \overset{n}{\rightarrow} \mathcal{G}_{3c2L2R1X} \overset{v_R \simeq \Lambda_R}{\rightarrow} \mathcal{G}_{3c2L1Y} \overset{k_i}{\rightarrow} \mathcal{G}_{3c1Q}.$ \quad (23)

In this work we will focus only on the renormalization group study of Case-A, and leave the detailed analysis of cases -B and -C for a follow up study. The symmetry breaking chain that we will consider is as follows

$$SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_X$$

$$\downarrow n \simeq v_R \simeq \Lambda_R$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\downarrow \langle \phi(2, \frac{1}{2}, 1) \rangle \subset \Phi$$

$$SU(3)_C \times U(1)_Q.$$

Without presuming any underlying group for grand unification we will first study the RGEs in this section to explore whether unification of the three gauge couplings can be obtained in the $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ theory at a certain scale $M_T$. Using the RGEs
we express the hypercharge (and X) normalization and the unification scale as a function of \( SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X \) breaking scale. Next we study the allowed range of \( SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X \) breaking scale such that one can obtain a guaranteed unification of the gauge couplings. First we discuss the case of the minimal models discussed in section II B. Then, we study the impact of adding three generations of leptonic octet representations \((1,8_{L,R},0)\) that can give gauge coupling unification for a TeV scale \( SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X \) breaking.

The evolution for running coupling constants at one loop level is governed by the RGEs

\[
\mu \frac{\partial g_i}{\partial \mu} = \frac{b_i}{16\pi^2} g_i^3,
\]

which can be written in the form

\[
\frac{1}{\alpha_i(\mu_2)} = \frac{1}{\alpha_i(\mu_1)} - \frac{b_i}{2\pi} \ln \left( \frac{\mu_2}{\mu_1} \right),
\]

where \( \alpha_i = g_i^2/4\pi \) is the fine structure constant for \( i \)-th gauge group, \( \mu_1, \mu_2 \) are the energy scales with \( \mu_2 > \mu_1 \). The beta-coefficients \( b_i \) determining the evolution of gauge couplings at one-loop order are given by

\[
b_i = -\frac{11}{3} C_2(G) + \frac{2}{3} \sum_{R_f} T(R_f) \prod_{j \neq i} d_j(R_f) + \frac{1}{3} \sum_{R_s} T(R_s) \prod_{j \neq i} d_j(R_s).
\]

Here, \( C_2(G) \) is the quadratic Casimir operator for the gauge bosons in their adjoint representation,

\[
C_2(G) \equiv \begin{cases} 
N & \text{if } SU(N), \\
0 & \text{if } U(1).
\end{cases}
\]

On the other hand, \( T(R_f) \) and \( T(R_s) \) are the Dynkin indices of the irreducible representation \( R_{f,s} \) for a given fermion and scalar, respectively,

\[
T(R_{f,s}) \equiv \begin{cases} 
1/2 & \text{if } R_{f,s} \text{ is fundamental}, \\
N & \text{if } R_{f,s} \text{ is adjoint}, \\
0 & \text{if } R_{f,s} \text{ is singlet},
\end{cases}
\]

and \( d(R_{f,s}) \) is the dimension of a given representation \( R_{f,s} \) under all gauge groups except the \( i \)-th gauge group under consideration. An additional factor of \( 1/2 \) is multiplied in the case of a real Higgs representation.

The charge equation is given in Eq. [2], where the generators (Gell-Mann matrices) are normalized as \( \text{Tr}(T_i T_j) = \frac{1}{2} \delta_{ij} \). We define the normalized hypercharge operator \( Y_N \) and \( X_N \) as

\[
Y = n_Y Y_N, \quad X = n_X X_N,
\]

such that we have

\[
n_Y^2 = (1 + 2\beta^2) + n_X^2,
\]
and the normalized couplings are related by

\[ n_Y^2 (\alpha_Y^N)^{-1} = \beta^2 \alpha_{3L}^{-1} + (1 + \beta^2) \alpha_{3R}^{-1} + \left[ n_Y^2 - (1 + 2\beta^2) \right] (\alpha_X^N)^{-1}, \tag{31} \]

at the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X symmetry breaking scale, where

\[ \alpha_Y^N = n_Y^2 \alpha_Y, \quad \alpha_X^N = \left[ n_Y^2 - (1 + 2\beta^2) \right] \alpha_X, \quad \alpha_{3L} = \alpha_{2L}. \tag{32} \]

Furthermore, to keep things simple and minimal we will assume \( \alpha_{3L} = \alpha_{3R} \) and the left-right symmetry of the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X model ensures that \( b_{3L} = b_{3R} \).

The RG running for the phase between the electroweak symmetry breaking and the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X symmetry breaking is described by the SU(3)_C coefficient \( b_s \), the SU(2)_L coefficient \( b_{2L} \) and the U(1)_Y unnormalized coefficient \( b_Y^{UN} \).

Likewise, in the unbroken SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X phase, the running coefficients for the SU(3)_C, SU(3)_L, SU(3)_R and unnormalized U(1)_X components are \( b_{3C}^X, b_{3L}, b_{3R} \) and \( b_X^{UN} \), respectively. The scale \( M_Z \) corresponds to the Z boson-pole, the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X symmetry breaking scale is denoted by \( M_X \) and \( M_U \) is the scale of unification for the normalized gauge couplings. From the above set of equations the unification scale \( M_U \) and \( n_Y^2 \) can be expressed as a function of \( M_X \).

### A. The minimal SVS Model with sextet Higgs sector

The first case of interest is the minimal scenario described in section [III]. The relevant gauge quantum numbers for the fermions and the scalars relevant for the RG running of the beta-coefficients in different phases have been tabulated in Table I. For the phase between the electroweak symmetry breaking and the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X symmetry breaking the one-loop beta-coefficients are given by \( b_{2L} = -19/6, b_Y^{UN} = 41/6, b_{3C} = -7 \), while for the phase between the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X symmetry breaking and gauge coupling unification the one-loop beta-coefficients are given by \( b_{3L} = -31/6, b_X^{UN} = 43/9, b_{3C}^X = -5 \).

| Group \( G_I \)   | Fermions                                   | Scalars                                    |
|-------------------|--------------------------------------------|--------------------------------------------|
| \( G_{321} \) \( (M_Z \leftrightarrow v_R) \) | \( Q_{aL}(3, 2, 1/6) \), \( u_{aR}(3, 1, 2/3), d_{aR}(3, 1, -1/3) \) | \( \phi(1, 2, \frac{1}{2}) \) |
| \( G_{331} \) \( (v_R \leftrightarrow M_U) \) | \( \Psi_{aL}(1, 3, 1, \frac{q-1}{3}), \Psi_{aR}(1, 1, 3, \frac{q-1}{3}) \) | \( \Phi(1, 3, 3^*, 0) \) |
|                   | \( Q_{aL}(3, 3, 1, \frac{q^2}{3}), Q_{aR}(3, 1, 3, \frac{q^2}{3}) \) | \( \rho(1, 3, 3, \frac{2q+1}{3}) \) |
|                   | \( Q_{3L}(3, 3, 1, \frac{q+1}{3}), Q_{3R}(3, 1, 3, \frac{q+1}{3}) \) | \( \Delta_L(1, 6, 1, \frac{2(q-1)}{3}) \) |
|                   | \( \Delta_R(1, 1, 6, \frac{2(q-1)}{3}) \) |                                            |

**TABLE I**: Table showing the gauge quantum numbers for the fermions and the scalars, and the beta-coefficients for the renormalization group evolution in different phases of gauge symmetry.
FIG. 1: The hypercharge normalization factor $n_Y^2$ as a function of $M_X$, the scale of $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking, such that there is a guaranteed unification at some scale $M_U$: $M_X \leq M_U \leq 10^{18}$ GeV. The solid line gives $n_Y^2$ as a function of $M_X$ and the dot-dashed line shows the lower limit $n_Y^2 = \frac{5}{3}$ for the allowed value of $n_Y^2$ from Eq. (30).

As shown in Fig. 1 we plot the hypercharge normalization factor $n_Y^2$ as a function of $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking scale $M_X$ such that there is a guaranteed unification. We have chosen the $M_X$ scale such that there is a guaranteed unification at some scale $M_U$: $M_X \leq M_U \leq 10^{18}$ GeV. The dashed horizontal red line represents the lower limit $n_Y^2 = \frac{5}{3}$ for the allowed value of $n_Y^2$ from eq. (30). Interestingly, $n_Y^2 = \frac{5}{3}$ is also the standard $SU(5)$ normalization. It can be seen from the figure, for the $M_X$ range allowed by the condition that there is a guaranteed unification at some scale $M_U$: $M_X \leq M_U \leq 10^{18}$ GeV, the hypercharge normalization $n_Y^2$ is almost constant $\approx 1.3$ and is clearly below the allowed lower limit (implying a negative $n_X^2$, which is unphysical). Thus it is not possible to obtain gauge coupling unification for the minimal scenario described in section III. Interestingly, we find that adding sequential left- and right-handed fermionic octets for each generation it is possible to obtain a consistent unification for this model while having the $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ scale and the octet scale within the reach of the LHC.

B. The SVS Model with fermionic octets

In addition to the field content discussed above, we now include three generations of fermion octets $\Omega_{L,R}$ with the assignments under the $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ group given by

$$\Omega_L \equiv [1, 8, 1, 0], \quad \Omega_R \equiv [1, 1, 8, 0].$$  \hspace{1cm} (33)

In order to make the renormalization group evolution analysis general we will keep the octet mass scale $M_8$ as a separate scale from the $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking scale and assume it to lie somewhere in between the $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking scale and the unification scale. For the phases between the Standard Model and $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking scale, and the $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking and the octet scale $M_8$ the one-loop beta-coefficients remain the same as discussed in
The black dashed, blue dotted, and brown dot-dashed curves correspond to the cases $M_8/M_X = 1, 3, \text{ and } 3 \times 10^3$ respectively. The orange solid line shows the lower limit $n_2^2 = \frac{5}{3}$ for the allowed value of $n_2^2$ from Eq. (30) which is also the standard SU(5) normalization. (right) Plot showing the allowed range for $M_X$ for which unification is guaranteed at a scale $M_X \leq M_U \leq 10^{18}$ GeV. The orange lower boundary corresponds to $M_X \leq M_U$ and the upper red boundary corresponds to $10^{18}$ GeV. The black dashed, blue dotted, and brown dot-dashed curves correspond to the cases $M_8/M_X = 1, 3, \text{ and } 3 \times 10^3$ respectively.

In Fig. 2 (left) we plot the hypercharge normalization factor $n_2^2$ as a function of the SU(3)$_c$ \times SU(3)$_L$ \times SU(3)$_R \times U(1)_X$ symmetry breaking scale $M_X$ such that there is a guaranteed unification at some scale $M_U$: $M_X \leq M_U \leq 10^{18}$ GeV. The black dashed, blue dotted, and brown dot-dashed curves correspond to the cases $M_8/M_X = 1, 3, \text{ and } 3 \times 10^3$ respectively. The orange solid line shows the lower limit $n_2^2 = \frac{5}{3}$ for the allowed value of $n_2^2$ from Eq. (30) which is also the standard SU(5) normalization.

Note that in this case one can have a TeV scale SU(3)$_c$ \times SU(3)$_L$ \times SU(3)$_R \times U(1)_X$ symmetry breaking and octet mass scale consistent with unification with the hypercharge normalization $n_2^2$ is well above the allowed lower limit. This justifies the addition of the octets to the spectrum and makes the scenario testable in the current and near future collider experiments.

In Fig. 3 we show an example of gauge coupling evolution with the previous section. For the phase between the octet scale $M_8$ and gauge coupling unification the one-loop beta-coefficients are given by $b_{3L} = 5/6$, $b_{UN}^X = 43/9$, $b_{8c}^X = -5$.

In Fig. 2 (right) we plot the allowed range for $M_X$ for which unification is guaranteed at a scale $M_X \leq M_U \leq 10^{18}$ GeV. Interestingly, in this scenario we find that for a SU(3)$_c$ \times SU(3)$_L$ \times SU(3)$_R \times U(1)_X$ symmetry breaking scale $M_X$ as low as a few TeV it is possible to achieve unification.

We will show the plot for three distinct and physically interesting values of the ratio of the octet mass scale to the SU(3)$_c$ \times SU(3)$_L$ \times SU(3)$_R \times U(1)_X$ symmetry breaking scale $M_8/M_X$. In Fig. 2 (left) we plot the hypercharge normalization factor $n_2^2$ as a function of the SU(3)$_c$ \times SU(3)$_L$ \times SU(3)$_R \times U(1)_X$ symmetry breaking scale $M_X$ such that there is a guaranteed unification at some scale $M_U$: $M_X \leq M_U \leq 10^{18}$ GeV. The black dashed, blue dotted, and brown dot-dashed curves correspond to the cases $M_8/M_X = 1, 3, \text{ and } 3 \times 10^3$ respectively. The orange solid line shows the lower limit $n_2^2 = \frac{5}{3}$ for the allowed value of $n_2^2$ from Eq. (30) which is also the standard SU(5) normalization.

Note that in this case one can have a TeV scale SU(3)$_c$ \times SU(3)$_L$ \times SU(3)$_R \times U(1)_X$ symmetry breaking and octet mass scale consistent with unification with the hypercharge normalization $n_2^2$ is well above the allowed lower limit. This justifies the addition of the octets to the spectrum and makes the scenario testable in the current and near future collider experiments.

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3 We should emphasize that we here do not specify the GUT group and thus the limit coming from the non-observation of proton decay is beyond the scope of the current discussion.
FIG. 3: Gauge coupling running in the SVS Model adding three generations of leptonic octets with $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking scale at $M_X = 3000$ GeV, and the octet mass scale at $M_8 = 3 \times 10^3 M_X$ demonstrating successful gauge unification at the scale $M_U = 10^{16}$ GeV with $n_Y^2 = 2.26$.

$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking scale at $M_X = 3000$ GeV, and the octet mass scale at $M_8 = 3 \times 10^3 M_X$ demonstrating successful gauge coupling unification at a scale $M_U = 10^{16}$ GeV with $n_Y^2 = 2.26$. Thus, from the perspective of a low $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ symmetry breaking scale within the reach of accelerator experiments like the LHC ($\mathcal{O}(\text{TeV})$) this model is a very interesting candidate leading to a successful gauge coupling unification. In addition to the new gauge bosons, the model can also have a number of new states associated to the new exotic fermions as well as extra Higgs bosons which can be searched for at the LHC and other near future accelerator experiments.

IV. COMMENTS ON PHENOMENOLOGY

In order to achieve gauge coupling unification at a reasonable scale with the $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_X$ breaking scale around the few TeV scale the leptonic octets play a crucial role. Since they are in the $SU(3)$ adjoint representation they introduce no anomalies, but because of group theoretic considerations they can produce, though in a minor way, changes in the relevant mass matrices. From $SU(3)$ group theory we know that

\begin{align}
3 \times 3^* &= 1 + 8, \\
3^* \times 3^* &= 3_a + 6^*_s, \\
6 \times 3 &= 8 + 10, \\
6 \times 3^* &= 3 + 15, \tag{34}
\end{align}
thus there are some allowed couplings involving the octets $\Omega$:

$$3 \times 3^* \times 8,$$

$$3 \times 6 \times 8,$$

$$8 \times 8.$$ (35)

The first term is relevant for the low-scale seesaw version of the model [19], while the second is relevant for the high-scale seesaw case considered here. The third term will be always present and gives the octets a bare mass denoted by $M_8$ in our construction (see III.B). A more detailed analysis can show that with our quantum numbers the sextet mass term in equation (34) is actually not allowed by hypercharge conservation, so that the octets decouple from the neutrino mass matrix. This is in sharp contrast with the scenario considered in Ref. [14] in the context of the simpler 3-3-1 model, where the octets were responsible for radiative neutrino mass generation. Thus in our present case the neutrino masses are simply given by the seesaw mechanism, and the phenomenology remains unaffected by adding these new leptonic octets.

Before closing this discussion we note that, as seen above, in the presence of octets, unification can take place for relatively low values of the extended electroweak scale. This implies that, say, a new $Z'$ lying around the few TeV scale should be produced in a Drell-Yan process and act as a portal to access the messenger of neutrino mass generation. In the present version of the seesaw mechanism, this requires small Yukawa couplings in order to get small neutrino masses.

Thus in such scenario there would be no measurable lepton flavour violation at low energies, yet sizeable lepton flavour violation at the high energies accessible at the LHC would be expected [15, 23]. Moreover, such $Z'$ would have flavor changing neutral current (FCNC) couplings to quarks, leading to $Z'$-induced $K$, $D$ and $B$ neutral meson mass differences, and hence to transitions in the $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ meson systems. For a study of complementarity between FCNC and dilepton resonance searches at the LHC see Ref. [16].

V. COMMENTS ON GRAND UNIFICATION

In this section we comment on various possible ways of embedding our model within a grand unification scheme. Notice that the group $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_X$ has rank 7, the same as the rank of $\text{SU}(8)$. However, none of the subgroups of $\text{SU}(8)$ contains three $\text{SU}(3)$, hence the minimal $\text{SU}(N)$ group which can embed the $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_X$ model is actually $\text{SU}(9)$. A possible symmetry breaking chain to to obtain the $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_X$ model from $\text{SU}(9)$ is given by

$$\text{SU}(9) \rightarrow \text{SU}(6) \times \text{SU}(3) \times \text{U}(1)' \rightarrow \text{SU}(3) \times \text{SU}(3) \times \text{SU}(3) \times \text{U}(1).$$ (36)

In general $\text{SU}(N)$ groups have anomalies that one must cancel in order to keep gauge invariance. One can compute [25] the contribution of the $m$-rank antisymmetric representation of $\text{SU}(N)$ as:

$$\mathcal{A} = (N - 2m) \frac{(N - 3)!}{(m - 1)!(N - m - 1)!}.$$ (37)
so that the anomaly contributions for different SU(9) antisymmetric representations are given by

\[ A[9] = 1, \ A[36] = 5, \ A[84] = 9, \ A[126] = 5. \]  

Note that one can also introduce symmetric representations like the 45, but due to its large anomaly contribution \( A[45] = 13 \) we will not consider it. The anomaly contribution for larger symmetric representations grows rapidly. When constructing a Grand Unified Theory one must not only cancel anomalies, but also to make sure that the theory is able to generate the observed Standard Model chirality \[26\]. Some anomaly free sets capable of reproducing Standard Model chirality are given by

\[ 15(9) + 3(36), \]
\[ 9(\bar{9}) + 84, \]
\[ 3(\bar{36}) + 3(126), \]
\[ 14(\bar{9}) + 2(36) + 84 + \bar{126}, \]
\[ 3(\bar{9}) + 3(\bar{36}) + 2(84), \]
\[ 5(\bar{9}) + \bar{36} + 2(126), \]
\[ 4(\bar{9}) + 2(\bar{36}) + 84 + 126, \]  

We note that these sets contain the Standard Model fermion content plus vector-like representations. It is important to notice that combinations like \( 5(9) + 36 \) are anomaly free but only reproduce one family. This can be seen when decomposing all given combinations in terms of the \( SU(9) \) subgroup: \( SU(5) \) \[27\]. It is also interesting that the combination given by

\[ 1 + \bar{9} + 36 + \bar{84} + 126, \]  
is anomaly free but does not reproduce the required Standard Model chirality, since it corresponds to the SO(18) spinor representation 256. Despite being complex and chiral \[28\], the latter decomposes into a vector-like set of representations of its subgroups.

The next question that one should address is how to match acceptable multiplicities of the different representations of the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X model. An interesting possibility is to rely upon F-theory GUTs in order to obtain the required representations with the correct multiplicity \[29,32\].

An interesting alternative is to use the SO(18) gauge group \[9,28,33,35\] which can also embed our SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X model. A possible symmetry breaking chain to the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X model from SO(18) is given by

\[ SO(18) \rightarrow SU(9) \times U(1)'' \rightarrow SU(6) \times SU(3) \times U(1)' \rightarrow SU(3) \times SU(3) \times SU(3) \times U(1). \]  
The choice of SO(18) is particularly interesting, as it may potentially unify all of the fermionic fields within a single spinor representation 256. However, the problem of having unwanted mirror families contained in the 256 is still an open question. Another interesting choice of gauge group for embedding the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X model is an extended Pati-Salam like gauge group SU(4) × SU(3) × SU(3). A detailed study of the GUT embedding of the SU(3)_c × SU(3)_L × SU(3)_R × U(1)_X model is beyond the scope of
this article. Here we have taken a phenomenological approach to the problem, keeping the generator normalization as a free parameter. Once a particular GUT embedding is chosen and the field content of the \( \text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_X \) model is fully specified in correct representations, the hypercharge normalization can be determined and Fig. 2 can be readily used in order to check whether a few TeV scale \( \text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_X \) breaking scale is consistent with the corresponding GUT embedding.

**VI. SUMMARY**

In this work we have considered the possibility of gauge coupling unification in a simple model realization of the left-right symmetric \( \text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_X \) gauge theory. Our “bottom-up” analysis of the renormalization group equations for the SVS model with sextets (minimal model) shows that gauge coupling unification with the 3331 scale at \( \mathcal{O}(\text{TeV}) \) is possible in the presence of leptonic octets. Interestingly, unlike in the chiral 3-3-1 model, in the minimal \( \text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_X \) model these octets do not affect neutrino mass generation. Consequently, the neutrino masses arise exclusively from the seesaw mechanism in this model. It is also interesting that, due to hypercharge normalization requirements, one can not achieve unification without these octets. We have also briefly commented on possible phenomenological implications of this model and on possible grand unified theory frameworks which can embed the \( \text{SU}(3)_c \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_X \) model as an intermediate symmetry.

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