THE MOST MASSIVE OBJECTS IN THE UNIVERSE

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ABSTRACT

We calculate the expected mass of the most massive object in the universe, finding it to be a cluster of galaxies with total mass $M_{200} = 3.8 \times 10^{15} M_\odot$ at $z = 0.22$, with the 1σ marginalized regions being $3.3 \times 10^{15} M_\odot < M_{200} < 4.4 \times 10^{15} M_\odot$ and $0.12 < z < 0.36$. We restrict ourselves to self-gravitating bound objects and base our results on halo mass functions derived from $N$-body simulations. The mass and redshift distribution of the largest objects in the universe are potentially interesting tests of $\Lambda$CDM, probing the initial conditions, non-Gaussianity, and the behavior of gravity on large scales. We discuss A2163 and A370 as candidates for the most massive cluster in the universe, although uncertainties in their masses preclude definitive comparisons with theory. We find that the three most massive clusters in the South Pole Telescope (SPT) 178 and 2500 deg$^2$ catalogs match predictions. Since the mass function evolves steeply with redshift, we also investigate the most unlikely clusters in the universe. We find that SPT-CL J2106–5844 is 2σ and XMMU J2235.3–2557 is 3σ inconsistent with $\Lambda$CDM, considering their respective redshifts and survey sizes. Our findings motivate further observations of the highest mass end of the mass function, particularly at $z > 1$, where a number of anomalously massive clusters have been discovered. Future surveys will explore larger volumes, and both the most massive object and the most unlikely object in the universe may be identified within the next decade.

Key words: cosmology: theory – galaxies: clusters: general

1. INTRODUCTION

Our universe has a finite observable volume, and therefore within our universe there is a unique most massive object. This object will be a cluster of galaxies. Theoretical studies of the growth of structure have now matured, and the mass of the most massive objects can be robustly predicted to the level of a few percent. Furthermore, we are in the midst of a revolution in our ability to conduct volume-limited samples of high-mass clusters, with X-ray and ground- and space-based Sunyaev-Zel’dovich (SZ) surveys able to provide complete samples at mass $> 5 \times 10^{14} M_\odot$ out to $z > 1$. The masses of the most massive clusters in the universe are therefore a robust prediction of $\Lambda$CDM models, as well as a direct observable of our universe.

The cluster mass function is already being utilized as a probe of cosmology, and, in particular, of the dark energy equation of state (Holder et al. 2001; Haiman et al. 2001; Weller et al. 2002) and non-Gaussianity (Matarrese et al. 1986; Verde et al. 2001; Grossi et al. 2007). What additional value is there in singling out the tail end of the mass function? First, we note that these systems are in many ways the easiest to find, as they are among the largest and brightest objects. Furthermore, because candidates for the most massive cluster constitute a very small sample, it is possible to perform coupled SZ, X-ray, and weak lensing measurements, and thus the masses of these objects will be particularly well determined. Although a larger sample of clusters is always preferable, we argue that a small sample of the highest-mass objects, thoroughly studied with direct mass determinations, is of complementary value to a large statistical sample of cluster masses derived from mass-observable relations. Although the statistics of rare events are often poorly understood, in the case of cosmological structure formation the distribution of outliers has been well characterized (Hu & Kravtsov 2003; Hu & Cohn 2006; Davis et al. 2011). If the most massive object at any given redshift is found to have too large or too small a mass, this single object will provide a strong indication of the breakdown of $\Lambda$CDM (Chiu et al. 1998; Bahcall & Fan 1998), independent of selection effects, completeness, and systematic errors at the level of tens of percent. We note that cosmic microwave background measurements of non-Gaussianity probe much larger scales (Smidt et al. 2010), and that clusters provide valuable new constraints at Mpc scales. Although much work has focused on using halo statistics as a probe of cosmology, here we focus on using the high-mass tails of precision mass functions to make explicit predictions for current and future observations.

2. MASS FUNCTION

In recent years the expected number density of dark matter halos as a function of mass and redshift has been established to better than 5% (Warren et al. 2006; Reed et al. 2007; Tinker et al. 2008) from cosmological dark matter $N$-body simulations. We restrict ourselves to the spherical overdensity (SO) approach to finding dark matter halos in simulations (Tinker et al. 2008). It is to be noted that the most massive clusters are often the least spherical (e.g., Foley et al. 2011), and this might argue that an alternative halo-finding algorithm might yield a better match to observations. The primary issue is whether observers can reliably measure the corresponding quantity, and, for our purposes, what observers designate as the mass of a spherical region should, in principle, correspond closely to the SO halos found in simulations.

We are also interested in the expected scatter in the number density of massive halos. At the high-mass end of the mass function, where the number density satisfies roughly one per volume of interest and the statistics are dominated by shot noise,
we assume that the distribution of halos is given by Poisson statistics. Although this remains to be firmly established by simulations on > Gpc scales, it has been shown that the tails of the appropriate Gumbel distributions are negligibly different from Poisson in the regime of interest (Hu & Kravtsov 2003; Hu & Cohn 2006; Davis et al. 2011). Another cause for concern is that the largest modes in any simulation box are poorly represented (Tormen & Bertschinger 1996), and this could lead to errors at the high-mass end of the mass function (although we note that the simulations underlying our mass function include a large, 1.28 h^{-1} Gpc box).

We use the mass function presented in Tinker et al. (2008), which gives the expected number density of dark matter halos, \(dn/dM\), in units of h^{-1} Mpc, where \(h\) is the Hubble constant and volume is measured in comoving Mpc^3. This mass function describes the abundance of SO dark matter halos and is accurate to \(\lesssim 10\%\) over the redshift range of interest (0 < z < 2) and for overdensity values in the range 200 < \(\Delta\) < 2300. \(\Delta\) is the overdensity in a spherical region compared to the mean background matter density at the given redshift, and \(M_{200}\) corresponds to the mass enclosed for \(\Delta = 200\). This mass function has been calibrated for \(M_{200} \lesssim 4 \times 10^{15} M_\odot\); our extreme high-mass results rely on (modest) extrapolation. We consider the best-fit ΛCDM model from WMAP7, with \(h = 0.710 \pm 0.025\), \(\Omega_m = 0.264 \pm 0.029\), \(\Omega_{\Lambda} = 0.734 \pm 0.029\), and \(\sigma_8 = 0.801 \pm 0.030\) (Komatsu et al. 2011). We generalize the mass function by using a transfer function and \(\sigma(M)\) appropriate for these parameters, and marginalize over the uncertainties. We find that the errors due to shot noise dominate over those due to uncertainties in the cosmological parameters, and that cosmological parameter uncertainty will become irrelevant in the Planck era (Colombo et al. 2009).

Ongoing refinements (e.g., Klypin et al. 2011; Prada et al. 2012) may change the shape and amplitude of the theoretical mass function. To get a sense of the importance of this, we have calculated that it would take a fractional change in the amplitude parameter, \(A\), of the Tinker mass function in the range 0.6–10 to broaden the 1σ contours into the 2σ ones (see Figure 1). This is to be compared to the \(\sim 10\%\) determination of the amplitude through normalization to simulations (Tinker et al. 2008). Similarly, we would have to introduce a fractional change in the Tinker cutoff scale parameter, \(c\), in the range 0.8–1.05 to cause significant broadening of the contours. This is again ruled out by simulations; e.g., a 5% increase in \(c\) causes an almost 50% decrease in the number of \(4 \times 10^{15} M_\odot\) halos. The effects of baryons may also lead to changes in the number density of the most massive objects (and potentially larger effects on halo density profiles, and therefore on the observational determinations of halo masses), although these effects are unlikely to be greater than 30% in the mass function in the regime of interest (Stanek et al. 2009).

Because the uncertainties in the mass of the most massive cluster will be large (cf. the large contours [in log \(M\)] in Figure 1), small (\(\lesssim 30\%\)) errors in the mass function (e.g., from the halo finding algorithm or problems resolving the largest mode, or an inadequate fitting function), in the cosmological parameters (e.g., errors in \(\sigma_8\)), or in the observational mass determination (e.g., determining the X-ray mass of a disturbed cluster) will leave our results substantially unchanged.

3. THE MOST MASSIVE OBJECT (THEORY)

We are interested in determining the mass of the most massive object in our universe. Figure 1 shows the expected distribution of the highest mass halos in the universe, assuming the Tinker mass function and Poisson statistics. We calculate the contours using two techniques: (1) a Monte Carlo approach, where we generate distributions of halo masses for a universe from the mass function, determine the most massive objects in that realization, and repeat many times to calculate the distributions of the most massive objects, and (2) an analytic approach, where we use the mass function to directly calculate the probability that there is at least one halo with a given mass, but no halos with greater mass, as a function of redshift. Results from these two approaches are indistinguishable. The most massive object in the universe is expected at \(z = 0.22\) with a mass \(M_{200} = 3.8 \times 10^{15} M_\odot\). The marginalized 1σ range in mass is \(3.3 \times 10^{15} < M_{200} < 4.4 \times 10^{15}\), while in redshift it is \(0.12 < z < 0.36\). If the most massive object falls outside the region \(2 \times 10^{15} M_\odot < M_{200} < 2 \times 10^{16} M_\odot\), we can conclude with high confidence that ΛCDM is broken; either the primordial initial conditions have significant excess power on cluster scales, the initial conditions are non-Gaussian, or the growth of structure deviates from the predictions of general relativity.

Figure 1 includes contours of the second and third most massive halos in the universe. As we go down in rank order (e.g., from the second to the third most massive), the contours rapidly converge due to the exponentially steep mass function; this demonstrates the power of just a few halos to constrain cosmology. We note that the contours are not centered on the most likely point; there is much larger scatter to high mass, with a sharp lower mass limit. The predicted masses of the first and second most massive halos are not independent, since...
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**Figure 2.** Expected number of halos at redshift $z > z_{\text{min}}$ with mass $M > M_{200,\text{min}}$ for a full sky survey. Each contour line represents a value of $\log_{10}(\langle N \rangle)$. For a survey with fractions, $f$, of the full sky, the expected numbers of halos are diminished by the factor $f$. The dotted (red) line represents the $\langle N \rangle = 1$ contour for $\Delta = 200\sigma$ mass function, with overdensity compared to the critical density instead of the mean matter density. Note that this agrees with the fiducial “0” line ($\Delta = 200$) at high redshift, as the universe becomes matter dominated. The data points are the same as in Figure 1. In particular, the two highest-redshift data points are more than $2\sigma$ inconsistent with $\Lambda$CDM, while the rest of the points are in accordance with expectations. The highest-redshift (red) asterisk is SPT J2106—the expected number of clusters of at least this mass and redshift in the survey ($2500\,\text{deg}^2$; $f = 0.06$) is $\langle N \rangle \sim 10^3 f = 0.06$. Fitting forms for the curves are provided in the text.

(A color version of this figure is available in the online journal.)

if the most massive object has an unusually low mass, it is assured that the subsequent few halos will also be underestimated. We have performed Monte Carlo studies which show that the correlations are weak, however, and the distribution of separations is well approximated by assuming the likelihoods are drawn independently. Figure 1 also shows the contours for the first and second most massive objects from the South Pole Telescope (SPT) 178 deg$^2$ survey (Vanderlinde et al. 2010), from the SPT 2500 deg$^2$ survey (Williamson et al. 2011), as well as the contours for the archival XMM-Newton survey which discovered XMMU J2235.3–2557 (hereafter XMM-2235; Mullis et al. 2005).

Figure 2 shows contours of the expected number of halos greater than a given mass found beyond a minimum redshift: $\langle N \rangle (z > M_{500}, > z)$. The contours are roughly linear in the range $0.2 < z \lesssim 3$ and are well approximated (to better than 5% for $z < 2.5$, and 10% for $2.5 < z < 3$) by the family of lines:

$$a(N) = 15.7 - 0.13N - 0.014N^2 - 0.0014N^3$$

$$b(N) = -0.533 + 0.0022N^2,$$

where $N = \log_{10} \langle N \rangle$. For the redshift range $z < 0.2$, the results are well represented by the values at $z = 0$, which are given (to better than 2%) by

$$\log_{10}(\langle N \rangle) = 15.6 - 0.143N - 0.014N^2 - 0.0011N^3.$$

These expressions can be utilized to calculate the expected number of objects above a given minimum mass and redshift in the mass range $10^{14} M_\odot < M_{200} < 10^{16} M_\odot$ and redshift range $0 < z \lesssim 2$, for any survey size. For a volume-limited sample we are interested in $\langle N \rangle (z < M_{200}, < z)$. These contours start at $0$ at $z = 0$ and rapidly rise to their maximum values, flattening by $z \sim 0.2$ at the values given by Equation (2). Since we have assumed Poisson statistics, the probability of finding at least one halo is given by $1 - e^{-\langle N \rangle}$; if the expected number of halos is 1, then the probability of finding at least one halo is 0.63. Note that Equations (1) and (2) assume the WMAP7 value of the Hubble constant. To explicitly put in the $h$ dependence, $M_{200}$ and $\langle N \rangle$ can be rescaled by $(h/0.71)$ and $(0.71/h)^3$, respectively.

4. THE MOST MASSIVE OBJECT (OBSERVATIONS)

The most massive object in the universe is likely to have already been detected by ROSAT, potentially even if it lies behind the galactic plane (Kocevski et al. 2007). Reliably measuring the masses of candidate ROSAT sources remains challenging, however, and therefore the specific identity and mass of the most massive object is unknown at present. Perhaps the most compelling candidate is A2163 at $z = 0.203$, which has an X-ray mass measurement of $M_{500} = 3.4 \pm 0.8 \times 10^{15} M_\odot$ (Mantz et al. 2010; A. Mantz 2010, private communication; where “500c” indicates $\Delta$ with respect to $\rho_{\text{crit}}$ rather than $\rho_{\text{mean}}$). We expect 0.02 (0.002/0.2) clusters with at least this mass and redshift in the entire universe, where the numbers in parentheses are the $1\sigma$ lower and upper bounds on $\langle N \rangle$. An alternative weak lensing measurement of the mass yields a lower value of $M_{500} = 2.0 \pm 0.3 \times 10^{15} M_\odot$ (Radovich et al. 2008), which has expectation 1.4 (0.5/4) (Vikhlinin et al. 2009 and Maurogordato et al. 2008 find consistent values).

A370 at $z = 0.375$ is another compelling candidate, with a weak lensing mass of $M_{\text{vir}} = 2.93^{+0.36}_{-0.32} \times 10^{15} h^{-1} M_\odot$ (Broadhurst et al. 2008; Richard et al. 2010), and an expectation of 0.02 (0.005/0.05). We note that the uncertainties in the mass measurements are presently comparable to the $1\sigma$ contours; significant improvement in mass determination will be required before definitive comparisons with $\Lambda$CDM will be possible.

The figures also show the three most massive clusters from the SPT 178 deg$^2$ survey (Vanderlinde et al. 2010) and the SPT 2500 deg$^2$ survey (Williamson et al. 2011), where we have added the statistical and systematic errors in quadrature. Figure 1 shows that both surveys agree well with predictions, with the most massive clusters nestled neatly within their respective $1\sigma$ contours (with one found within $2\sigma$).

We note that the most massive cluster in a survey may not necessarily represent the most unlikely outlier due to the evolution of the mass function with redshift (Hoyle et al. 2011; Cayón et al. 2011). A moderately massive cluster at high redshift (e.g., $M = 10^{15} M_\odot$ at $z = 2$) would be significantly more unlikely, and therefore significantly more interesting as a probe of $\Lambda$CDM, than a more massive cluster at lower redshift (e.g., $M = 2 \times 10^{15} M_\odot$ at $z = 0$). An interesting question for any survey is, in addition to the most massive cluster, which cluster is the most unlikely? It is to be emphasized that an outlier is not subject to improved statistics; if a single cluster has a robust mass estimate of $M_{200} > 5 \times 10^{15} M_\odot$ at $z > 1$, $\Lambda$CDM is ruled out at $>5\sigma$.

There are a large number of recently discovered high-redshift massive clusters (Brodwin et al. 2010; Fassbender et al. 2011a, 2011b). These clusters are generally consistent with $\Lambda$CDM, with a few notable exceptions. In the figures we plot XMM-2235, with a weak-lensing mass of $M_{200,\text{vir}} = (7.3 \pm 1.3) \times 10^{14} M_\odot$ at $z = 1.4$ (Jee et al. 2009). This cluster was found in an 11 deg$^2$ survey ($f = 0.0003$). From Figure 2 we would expect to find a few thousand objects with at least this mass in the entire universe ($z > 0$) and only 10 such objects at $z \geq 1.4$ on the entire sky. The expected number of clusters in an 11 deg$^2$ survey with this minimum mass and redshift is...
A conservative lower limit of $M_{200c} = 5 \times 10^{14} M_\odot$ is quoted in Jee et al. (2009), which leads to an expectation of $6 \times 10^3$ such clusters in such a small survey (Jimenez & Verde 2009; Sartoris et al. 2010). From Figure 1 we see that XMM-2235 is a $3\sigma$ outlier for an 11 deg$^2$ survey. Although this is an extremely unlikely cluster given the small survey area, it is not an unlikely cluster for the universe as a whole (we expect ~10 of at least this mass and redshift on the entire sky), and subsequent studies suggest that a larger sky area, and 1.5 $(0.8/3.2)$ in the 2500 deg$^2$ survey. Finally, we plot SPT-CL J2106–5844 (Foley et al. 2011), recently discovered by SPT at $z = 1.132$ with an estimated mass of $M_{200c} = (1.27 \pm 0.21) \times 10^{15} h^{-1}_0 M_\odot$. We would expect 0.02 (0.02) clusters with at least this mass and redshift in a survey of 2500 deg$^2$, and only one cluster with this mass (or greater) at $z \geq 1.132$ in the entire universe. The mass of SPT-J2106 would have to be reduced by 2$\sigma$ to become consistent with ACDM expectations. Since SPT’s sky coverage is relatively well understood, SPT-J2106 is arguably the unlikeliest high-redshift massive cluster seen to date.

Current data argues for further exploration of the highest-mass end of the mass function, both at low and high redshift. We find that the three most massive clusters in both the SPT 178 and 2500 deg$^2$ catalogs match predictions, while SPT-CL J2106–5844 is $2\sigma$ and XMUM J2235.3–2557 is $3\sigma$ inconsistent with ACDM. Although far from definitive, these may be hints of an excess of massive clusters at high redshift. It would be particularly difficult, theoretically, to account for excessively massive clusters at $z > 1$, while having agreement at lower redshift. We expect to dramatically improved complete high-redshift cluster surveys with which to test ACDM in the near future, including more refined analyses from the SPT and the Atacama Cosmology Telescope, the Dark Energy Survey (5000 deg$^2$), Planck (all-sky), and eventually LSST (20,000 deg$^2$). In particular, Planck is expected to provide a relatively complete, all-sky survey of all massive clusters in the near future (Planck Collaboration et al. 2011), although with a higher mass threshold than smaller ground-based surveys, and therefore less reach to high redshift. If results from these cluster surveys disagree with the predictions outlined above, the ACDM paradigm for the growth of structure will need to be revisited.

In contrast to previous work, we focus on making the first explicit predictions for the expected mass of the most massive object in the universe in ACDM. We demonstrate that the results are relatively insensitive to uncertainties in the theoretical mass function, and our results incorporate the latest SPT data, including SPT-CL J2106. Following the posting of this Letter on the arXiv, there have been a number of papers building on our results. Mortonson et al. (2011) present an extensive development of the basic idea. They focus on constraining quintessence models and also investigate important practical concerns when attempting a “most massive cluster” measurement (e.g., Eddington bias). Their results are fully consistent with ours—their fitting function (presented in Appendix A) agrees with ours to better than 10% (although theirs is for general ACDM models, while ours is pinned to the WMAP7 values). Our results for the most massive clusters are therefore virtually indistinguishable. A number of groups have fit for the amount of non-Gaussianity required to explain recent observations (Sartoris et al. 2010; Hoyle et al. 2011; Cayón et al. 2011), finding that non-Gaussianity is marginally preferred over Gaussian ACDM. It is to be noted that non-Gaussianity has difficulty explaining an excess of high-mass clusters at high redshift coupled with a low-redshift mass distribution consistent with ACDM.

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