GSVD-Based Precoding in MIMO Systems with Integrated Services

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Abstract—This letter considers a two-receiver multiple-input multiple-output (MIMO) Gaussian broadcast channel model with integrated services. Specifically, we combine two sorts of service messages, and serve them simultaneously: one multicast message intended for both receivers and one confidential message intended for only one receiver. The confidential message is kept perfectly secure from the unauthorized receiver. Due to the coupling of service messages, it is intractable to seek capacity-achieving transmit covariance matrices. Accordingly, we propose a suboptimal precoding scheme based on the generalized singular value decomposition (GSVD). The GSVD produces several virtual orthogonal subchannels between the transmitter and the receivers. Subchannel allocation and power allocation between multicast message and confidential message are jointly optimized to maximize the secrecy rate in this letter, subject to the quality of communications (QoMS) constraints. Since this problem is inherently complex, a difference-of-concave (DC) algorithm, together with an exhaustive search, is exploited to handle the power allocation and subchannel allocation, respectively. Numerical results are presented to illustrate the efficacy of our proposed strategies.

Index Terms—Physical-layer service integration (PHY-SI), GSVD, broadcast channel (BC), secrecy capacity region

I. INTRODUCTION

HIGH transmission rate and secure communication are the basic demands for the future 5-Generation (5G) cellular networks. A heuristic way is to combine multiple coexisting services, e.g., multicast service and confidential service, into one integral service for one-time transmission, referred to as physical-layer service integration (PHY-SI). Traditionally, service integration techniques rely on upper-layer protocols to allocate different services on different logical channels, which is quite inefficient. On the contrary, PHY-SI enables coexisting services to share the same resources by exploiting the physical characteristics of wireless channels, thereby significantly increasing the spectral efficiency.

The study of PHY-SI can be traced back to Csiszár and Körner’s seminar work in [1], where the fundamental limitation of PHY-SI is established in a discrete memoryless broadcast channel (DMBC). In recent years, this kind of approach has gained renewed interest, especially that in various multi-antenna scenarios, such as Gaussian broadcast channels [2], [3], and bidirectional relay channels [4]. Nonetheless, these works merely handle the PHY-SI from the viewpoint of information theory, i.e., derive capacity results or characterize coding strategies that result in certain rate regions [5]. As to how to design the transmit strategies to achieve these capacity regions, there are few works.

In this letter, we handle the PHY-SI from the viewpoint of signal processing, i.e., design the precoding matrices of the transmitted messages. However, the resultant optimization problem is challenging to solve and it is physically difficult to eliminate the interference induced by the coupling of service messages. As a comprise, we pay our attention to some suboptimal but easy-to-implement transmit designs, e.g., the generalized singular value decomposition (GSVD). The basic merit of GSVD lies in its simplicity, since it yields several decoupled parallel subchannels between the transmitter and the receivers. In fact, GSVD-based precoding has been widely adopted in MIMO Gaussian broadcast channels for the purpose of confidentiality [6]–[8] or multicasting [9]–[11]. It is natural to extend these results to the case with concurrent transmission of multicast message and confidential message.

This letter considers a two-receiver MIMO broadcast channel with two sorts of messages: a multicast message intended for both receivers, and a confidential message intended for merely one authorized receiver. The confidential message needs to be kept perfectly secure from the unauthorized receiver. Both messages are precoded by the matrices generated from GSVD. The resulting optimization problem turns out to be a secrecy rate maximization problem with quality of multicast service constraints, which is nonconvex by nature. To handle the nonconvexity, a difference-of-concave algorithm is proposed to determine the power allocation scheme for each subchannel. Based on the results, an exhaustive search is performed to determine the subchannel allocation scheme. By this means, the GSVD secrecy rate region could be derived.

II. SYSTEM MODEL

We consider the downlink of a multiuser system in which a multi-antenna transmitter serves two receivers, and each receiver is equipped with multiple antennas. Both receivers have ordered the multicast service and receiver 1 (authorized receiver) further ordered the confidential service. The confidential message must be kept perfectly secure from receiver 2 (unauthorized receiver).

The received signal at receivers is modeled as

\[ y_1 = H_1 x + z_1, \quad y_2 = H_2 x + z_2, \]

where \( H_1 \) and \( H_2 \) are the channel matrices from the transmitter to receivers 1 and 2, respectively. \( x \) is the transmitted signal, and \( z_1 \) and \( z_2 \) are the additive white Gaussian noise (AWGN) vectors at receivers 1 and 2, respectively.
respectively, where $H_1 \in \mathbb{C}^{N_b \times N_t}$ (resp. $H_2 \in \mathbb{C}^{N_e \times N_t}$) is the channel matrix from the transmitter to receiver 1 (resp. receiver 2), $N_t$, $N_b$ and $N_e$ are the number of antennas employed by the transmitter, receiver 1 and receiver 2, respectively. $z_1$ and $z_2$ are independently identically distributed (i.i.d.) complex Gaussian noise with zero mean and unit variance. $x \in \mathbb{C}^{N_t}$ is the coded information, which consists of two independent components, i.e.,

$$x = x_0 + x_c,$$

where $x_0$ is the multicast message intended for both receivers, and $x_c$ is the confidential message intended only for receiver 1. We assume $x_0 \sim \mathcal{CN}(0, Q_0)$, $x_c \sim \mathcal{CN}(0, Q_c)$, where $Q_0$ and $Q_c$ are the transmit covariance matrices.

Denote $R_0$ and $R_c$ as the achievable rates associated with the multicast and confidential messages, respectively. Then the secrecy capacity region $C_s(H_1, H_2, P)$ is given as the set of nonnegative rate pairs $(R_0, R_c)$ satisfying

$$R_0 \leq \min_{k=1,2} \log \left| I + (I + H_k Q_c H_k^H)^{-1} H_k Q_0 H_k^H \right|,$$  

$$R_c \leq \log \left| I + H_k Q_c H_k^H \right| - \log \left| I + H_2 Q_c H_2^H \right|,$$

and $\text{Tr}(Q_0 + Q_c) \leq P$ with $P$ being the total transmit power budget at the transmitter.

With perfect CSI being available at the transmitter, to find capacity-achieving $Q_0$ and $Q_c$, we must solve the following secrecy rate maximization (SRM) optimization problem with quality of multicast service (QoS) constraints, i.e.,

$$g(r_{ms}) = \max_{Q_0, Q_c} \log \left| I + H_1 Q_c H_1^H \right| - \log \left| I + H_2 Q_c H_2^H \right|$$

s.t. $\log \left| I + (I + H_k Q_c H_k^H)^{-1} H_k Q_0 H_k^H \right| \geq r_{ms}, k = 1, 2$

and $\text{Tr}(Q_0 + Q_c) \leq P$, $Q_0 \succeq 0, Q_c \succeq 0$,

where $r_{ms}$ is predetermined requirement of the achievable multicast rate. To derive the boundary points of the secrecy capacity region $C_s(H_1, H_2, P)$, one should traverse all possible $r_{ms}$’s and store the corresponding optimal objective value $g(r_{ms})$, and then the rate pair $(r_{ms}, g(r_{ms}))$ is a boundary point of the secrecy capacity region.

However, the coupling of confidential message and multicast message renders problem nonconvex and thus intractable to solve. On the other hand, it makes the interference cancelation operation difficult for receivers. These facts motivate us to devise some simple but physically realizable alternatives. Naturally, the concept of GSVD-based precoding becomes attractive, since it can perfectly decouple all data streams.

### III. GSVD-BASED PRECODING SCHEME FOR PHY-SI

In this section, we will show our proposed GSVD-based precoding design for PHY-SI mathematically. First, let us introduce the GSVD via the following lemma.

**Lemma 1:** (GSVD transform, [7] Definition 1) Given two matrices $H_1 \in \mathbb{C}^{N_b \times N_t}$ and $H_2 \in \mathbb{C}^{N_e \times N_t}$, GSVD transform returns two unitary matrices $\Psi_r \in \mathbb{C}^{N_b \times N_b}$ and $\Psi_e \in \mathbb{C}^{N_e \times N_e}$, and non-negative diagonal matrices $C$ and $D$, and a matrix $A \in \mathbb{C}^{N_t \times q}$ with $q = \min(N_t, N_b + N_e)$, such that

$$H_1 A = \Psi_r C,$$

$$H_2 A = \Psi_e D.$$

The nonzero elements of $C$ are in ascending order while the nonzero elements of $D$ are in descending order. Moreover, $C^T C + D^T D = I$. Let $c_i$ and $d_i$ represent the $i$th diagonal elements of $C^T C$ and $D^T D$, respectively.

If matrices $H_1$ and $H_2$ represent the wireless channels as we have defined in (1), with the precoding matrices $\rho A$ at the transmitter and receiver reconstruction matrices $\Psi_r / \rho$ and $\Psi_e / \rho$ at the respective receiver, we will get $q$ noninterfering broadcast subchannels between the transmitter and the receivers. The coefficient $\rho$ denotes transmit power normalization. The gains of those subchannels are determined by the diagonal elements of $C$ and $D$. Let us define the subchannels with condition $c_i = 1$ (resp. $d_i = 1$) as private channels (PCs) of receiver 1 (resp. receiver 2), and the subchannels with condition $0 < c_i, d_i < 1$ as common channels (CCs) of both receivers.

The detailed number of CCs and PCs realized through GSVD under different system configurations is given in Table I as below. Herein we point out two cases under which the GSVD-based PHY-SI is infeasible. First, we know from Table I that when $N_t \geq N_b + N_e$, GSVD precoding will not generate any CCs. Thus, multicast message cannot be transmitted under this case. Second, if $c_i \leq d_i$ holds for all $i$, the achieved secrecy rate would always be zero even without multicasting [6] Claim 1, which invalidates the confidential message transmission. In this letter, we will only focus on the nontrivial cases where GSVD-based PHY-SI is feasible.

**TABLE I NUMBERS OF CCs AND PCs REALIZED THROUGH GSVD PRECODING [9]**

| System Configuration | #CC | Receiver 1 | Receiver 2 |
|----------------------|-----|------------|------------|
| C1: $N_t < N_b$, $N_e \leq N_t$ | $N_e$ | $N_t - N_e$ | 0           |
| C2: $N_t \geq N_b$, $N_e > N_t$ | $N_b$ | 0 | $N_t - N_b$ |
| C3: $N_t \geq N_b$, $N_e \leq N_t$ | $N_t$ | 0 | 0           |
| C4: $N_b < N_t$, $N_e < N_t$, $N_b + N_e - N_t$ | $N_b + N_e - N_t$ | $N_t - N_e$ | $N_t - N_b$ |
| C5: $N_b + N_e > N_t$ | 0 | $N_b$ | $N_e$ |

Next we define two sets $\Gamma_0$ and $\Gamma_c$ with cardinality $|\Gamma_0| = M$ and $|\Gamma_c| = N$, corresponding to the indices of subchannels allocated to the multicast message and confidential message, respectively. These two sets satisfy $\Gamma_0 \cup \Gamma_c = \{1, 2, ..., q\}$ and $\Gamma_0 \cap \Gamma_c = \emptyset$. Without loss of generality, we assume $\Gamma_0 = \{i_1, i_2, ..., i_M\}$ and $\Gamma_c = \{j_1, j_2, ..., j_N\} = \{1, 2, ..., q\} \setminus \Gamma_0$. Both receivers are assumed to be aware of $\Gamma_0$ and $\Gamma_c$.

Now, let the transmitted signal vectors $x_0$ and $x_c$ be constructed as

$$x_0 = A_0 s_0, \quad s_0 \sim \mathcal{CN}(0, P_0)$$

$$x_c = A_c s_c, \quad s_c \sim \mathcal{CN}(0, P_c),$$

where $A_0 = AE_0$, $A_c = AE_c$ with $E_0 \overset{\Delta}{=} [e_{i_1}, e_{i_2}, ..., e_{i_M}]$ and $E_c \overset{\Delta}{=} [e_{j_1}, e_{j_2}, ..., e_{j_N}]$, $e_l \in \mathbb{R}^q, l = 1, 2, ..., q$ represents the $l$th column vector of $I_q$, $A$ is obtained from the GSVD
Consequently, the secrecy rate $R_c$ in (3) can be expressed as

$$R_c = \log \left| \frac{I + \Psi_c CE_c P_c E_c^H C^H \Psi_r^H}{I + \Psi_c DE_c P_c E_c^H D^H \Psi_e^H} \right|$$

where $p_{c,n}$ is the $n$th diagonal element of $P_c$, equality (a) is due to the fact that $\Psi_c^H \Psi_r = \Psi_c^H \Psi_e = I$ and the Sylvester’s determinant theorem (12). i.e., $\det(I + UV) = \det(I + VU)$ for appropriate dimensions of $U$ and $V$, and equality (b) is due to the fact that any two columns of $C$ (or $D$) are orthogonal.

Since all subchannels are parallel and ideally non-interfering, the multicast message transmission is able to experience a clean link without the interference of confidential message. Thus, in the same way as (9), the achievable multicast rate w.r.t. receiver 1 and receiver 2 is given by

$$R_{0,1} = \sum_{m=1}^{M} \log(1 + p_{0,m}a_{m}^2), \quad R_{0,2} = \sum_{m=1}^{M} \log(1 + p_{0,m}d_{m}^2),$$

respectively, where $p_{0,m}$ is the $m$th diagonal element of $P_0$.

The transmit power allocated to multicast message and confidential message is therefore determined by

$$\text{Tr}(x_0x_0^H) = \text{Tr}(A_0P_0A_0^H) = \sum_{m=1}^{M} a_{0,m}p_{0,m},$$

$$\text{Tr}(x_cx_c^H) = \text{Tr}(A_cP_cA_c^H) = \sum_{n=1}^{N} a_{c,n}p_{c,n},$$

where $a_{0,m}$ is $m$th diagonal element of $A_0^H A_0$, and $a_{c,n}$ is $n$th diagonal element of $A_c^H A_c$. Hence, the resultant QoMS-constrained SRM problem is given by

$$\max_{\substack{\{p_{c,n}\}_{n=1}^{N}, \{p_{c,n}\}_{n=1}^{N}, \Gamma_c \in \Gamma}} \sum_{n=1}^{N} \log(1 + p_{c,n}e^2_{j,n}) - \sum_{n=1}^{N} \log(1 + p_{c,n}d^2_{j,n})$$

s.t. $\sum_{m=1}^{M} \log(1 + p_{0,m}a^2_{m}) \geq r_{ms}$, \hspace{1cm} (11a)

$\sum_{m=1}^{M} \log(1 + p_{0,m}d^2_{m}) \geq r_{ms}$, \hspace{1cm} (11b)

$\sum_{m=1}^{M} a_{0,m}p_{0,m} + \sum_{n=1}^{N} a_{c,n}p_{c,n} \leq P$, \hspace{1cm} (11c)

$p_{c,n} \geq 0, p_{0,m} \geq 0, \forall m, n$, \hspace{1cm} (11d)

$\Gamma_0 = \{i_1, i_2, \ldots, i_M\}$, $\Gamma_c = \{j_1, j_2, \ldots, j_N\}$.

IV. A Tractable Approach to the SRM Problem

Problem (11) decouples the confidential message and multicast message; however, it couples the subchannel allocation and power allocation to each subchannel. To solve (11), our strategy is to determine the power allocation scheme with a given subchannel allocation scheme. Then by exhausting all possible subchannel allocation schemes, we could find the maximum secrecy rate.

A. Subchannel Allocation Scheme

Although we resort to the exhaustive search to handle the subchannel allocation, the following criterions could help us reduce the computational complexity.

Claim 1: The PCs of unauthorized receiver should be discarded, for they cannot transmit either confidential message or multicast message.

Claim 2: The PCs of authorized receiver can only be used for the confidential message transmission.

Claim 3: For any CC satisfying $c_i \leq d_i$, it can only be used for the multicast message transmission.

Proof: For Claim 1 and 2, it is easy to see that the PCs must be invalid to the multicast message transmission, since only one receiver is able to receive the multicast message. Claim 3 could be verified by contradiction. Assume that the maximum secrecy rate can be achieved when CCs satisfying $c_i \leq d_i$ are used for confidential message transmission, then a larger secrecy rate will always be attained if we specify the power allocated to these subchannels as zero, which is contrary to the assumption.

As a result, when performing the exhaustive search, we can limit our searching scope to CCs with condition $c_i > d_i$.

B. Power Allocation Scheme

With a given subchannel allocation scheme, we need to solve the following optimization problem, i.e.,

$$\max_{\substack{\{p_{c,n}\}_{n=1}^{N}, \{p_{c,n}\}_{n=1}^{N}, \Gamma_c \in \Gamma}} \sum_{n=1}^{N} \log(1 + p_{c,n}e^2_{j,n}) - \sum_{n=1}^{N} \log(1 + p_{c,n}d^2_{j,n})$$

s.t. (11a)-(11d) satisfied.

However, problem (12) remains nonconvex because of its objective function. Moreover, due to the additional QoMS constraints, it is difficult to obtain its closed-form solutions by directly checking its Karush-Kuhn-Tucker (KKT) conditions, as [7], [8] did. To handle it, we propose a difference-of-convex (DC) approach to (12). Its basic idea is to locally linearize the nonconvex part $- \sum_{n=1}^{N} \log(1 + p_{c,n}d^2_{j,n})$ at some feasible point $(\{p_{c,n}\}_{n=1}^{N})$ via first-order Taylor expansion and iteratively solve the linearized problem, i.e.,

$$\max_{\substack{\{p_{c,n}\}_{n=1}^{N}, \{p_{c,n}\}_{n=1}^{N}, \Gamma_c \in \Gamma}} \sum_{n=1}^{N} \log(1 + p_{c,n}e^2_{j,n}) - \sum_{n=1}^{N} \log(1 + p_{c,n}d^2_{j,n})$$

s.t. (11a)-(11d) satisfied, (13)

where $g(\{p_{c,n}\}_{n=1}^{N}, \{p_{c,n}\}_{n=1}^{N}) \Delta \sum_{n=1}^{N} \log(1 + p_{c,n}d^2_{j,n}) - \sum_{n=1}^{N} d_{j,n} \log(1 + p_{c,n}d^2_{j,n}) - \sum_{n=1}^{N} d_{j,n} \log(1 + p_{c,n}d^2_{j,n})$. 
Subchannel and power allocation strategies for solving (11) objective secrecy rate returned by the interior-point method (IPM) [13]. Denoting the collection of all possible subchannel allocation schemes; {Φ}.

We summarize the above-developed DC approach to problem (12) together with the exhaustive search over subchannel allocation schemes, in Algorithm 1. Notice that in line 11 of Algorithm 1, we diminish the size of Φ by directly eliminating the infeasible subchannel allocation scheme.

Algorithm 1 Subchannel and power allocation strategies for solving (11)

1. Initiate \( r_{ms} = 0, \delta > 0 \) and \( \epsilon > 0 \), and let \( \Phi \) be the collection of all possible subchannel allocation schemes;
2. while \( \Phi \neq \emptyset \) do
3. \( k = 1 \), \( |\Phi| = K \) and \( \Phi = \{\Phi_1, \Phi_2, \ldots, \Phi_K\} \);
4. while \( k \leq K \) do
5. Fix \( \Gamma_0 \) and \( \Gamma_c \) by allocating subchannels to different service messages according to \( \Phi_k \);
6. Set \( i = 0 \), \( \Phi^{(k,i)} = 0 \) and \( \{p_{c,n}\}_{n=1}^N \) such that \( \sum_{n=1}^N a_{c,n}p_{c,n} \leq \Gamma_k \);
7. Repeat
8. \( i = i + 1 \);
9. Solve problem (13) via IPM and get \( p_{c,n}^{(k,i)} \);
10. if problem (13) is infeasible then
11. \( R^{k,i} = 0 \), \( \Phi = \Phi - \{\Phi_k\} \);
12. jump to line 10
13. end if
14. Compute \( R^{k,i} = \sum_{n=1}^N \log(1 + p_{c,n}^{(k,i)} g_{n}^2) - \sum_{n=1}^N \log(1 + p_{c,n} g_{n}^2) \);
15. Until \( |R^{k,i} - R^{k,i-1}| < \epsilon \)
16. \( R^k = R^{k,i} \)
17. \( k = k + 1 \)
18. end while
19. Let \( R(r_{ms}) = \arg \max_{k=1,2,\ldots,K} R^k \), and store the rate pair \( (r_{ms}, R(r_{ms})) \);
20. Update \( r_{ms} = r_{ms} + \delta \);
21. end while

V. NUMERICAL RESULTS

In this section, we provide numerical results to illustrate the secrecy rate region derived from our proposed GSVD-based scheme, compared with the secrecy capacity region obtained from exhaustive search over the set \( \{Q_c, Q_n\} | Q_0 \geq 0, Q_n \geq 0, Tr(Q_0 + Q_c) \leq P \} \), and the traditional time division multiple address (TDMA)-based service integration strategy, which assigns the confidential message and multicast message to two different orthogonal time slots.

For the fairness of comparison, the secrecy rate and multicast rate achieved by TDMA should be halved [4].

In the simulation, the channels are randomly generated from i.i.d. complex Gaussian distribution with zero mean and unit variance. The number of antennas at the transmitter, authorized receiver and unauthorized receiver are \( N_t = 3 \), \( N_h = 4 \) and \( N_e = 3 \), corresponding to C1 and C3 in Table I and the transmit power \( P \) is set as 20dB.

Fig. 1 plots the secrecy rate region achieved by different schemes. The secrecy capacity region serves as a reference indicating the performance loss the GSVD-based scheme would inevitably experience. One can notice that there exist a switching point at the boundary of the GSVD secrecy rate region. Actually, it is caused by the switch of different subchannel allocation schemes. From Fig. 1 we find that at low QoMS region, the GSVD-based scheme achieves identical performance to the secrecy capacity region. This is attributed to the near-optimality of GSVD-based precoding at high signal-to-noise ratio (SNR) in the confidential message transmission [6]. However, with the increase of QoMS, the gap between these two regions gradually expands. This performance degradation is due to the suboptimality of GSVD-based precoding to the multicast message transmission. Even so, our proposed scheme achieves significantly larger rate region than the TDMA-based one. Note that the motivations to use GSVD and TDMA are both to decouple the confidential message and multicast message. It follows that GSVD-based decoupling gives better performance than TDMA, which implies the inherent advantage of PHY-SI over traditional service integration.

In addition, we examined by simulations that our observations above also apply to C2 and C4 in Table I. The results are not shown here due to the page limit. This observation suggests that it is sound to use GSVD-based precoding for achieving service integration at PHY.

VI. CONCLUSION

In this letter, we consider a GSVD-based precoding design for two-receiver MIMO broadcast channel with PHY-SI. The GSVD precoding matrices of confidential message and multicast message are designed to maximize the achievable secrecy rate while satisfying the QoMS constraints. Since this QoMS-constrained SRM problem is simultaneously associated with the optimization of subchannel allocation and power allocation, we combine an exhaustive search over subchannel allocation schemes with a DC algorithm to solve it. Numerical results show that the GSVD-based scheme outperforms the traditional TDMA-based service integration and attains the boundary of secrecy capacity region at low QoMS region.
REFERENCES

[1] I. Csiszár and J. Körner, “Broadcast channels with confidential messages,” *IEEE Trans. Inf. Theory*, vol. 24, no. 3, pp. 339–348, May 1978.

[2] H. D. Ly, T. Liu, and Y. Liang, “Multiple-input multiple-output Gaussian broadcast channels with common and confidential messages,” *IEEE Trans. Inf. Theory*, vol. 56, no. 11, pp. 5477–5487, Oct. 2010.

[3] E. Ekrem and S. Ulukus, “Capacity region of Gaussian MIMO broadcast channels with common and confidential messages,” *IEEE Trans. Inf. Theory*, vol. 58, no. 9, pp. 5669–5680, Sep. 2012.

[4] R. Wyrembelski and H. Boche, “Physical layer integration of private, common, and confidential messages in bidirectional relay networks,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3170–3179, Sep. 2012.

[5] R. Schaefer and H. Boche, “Physical layer service integration in wireless networks: Signal processing challenges,” *IEEE Signal Process. Mag.*, vol. 31, no. 3, pp. 147–156, Apr. 2014.

[6] A. Khisti and G. W. Wornell, “Secure transmission with multiple antennas II: The MIMOME wiretap channel,” *IEEE Trans. Inf. Theory*, vol. 56, no. 11, pp. 5515–5532, Nov. 2010.

[7] S. A. A. Fakoorian et al., “Optimal power allocation for GSVD-based beamforming in the MIMO gaussian wiretap channel,” in *Proc. IEEE Int. Symp. Inf. Theory (ISIT’2012)*, Cambridge, MA, Jul. 2012, pp. 2321–2325.

[8] ——, “Dirty paper coding versus linear GSVD-based precoding in MIMO broadcast channel with confidential messages,” in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM 2011)*, Houston, TX, USA, Dec. 2011, pp. 1–5.

[9] D. Senaratne and C. Tellambura, “GSVD beamforming for two-user MIMO downlink channel,” *IEEE Trans. Veh. Technol.*, vol. 62, no. 6, pp. 2596–2606, Jul. 2013.

[10] ——, “Beamforming for physical layer multicasting,” in *Proc. IEEE WCNC*, Cancun, Quintana Roo, Mexico, Mar. 2011, pp. 1776–1781.

[11] ——, “Generalized singular value decomposition for coordinated beamforming in MIMO systems,” in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM’2010)*, Miami, FL, Dec. 2010, pp. 1–6.

[12] D. A. Harville, *Matrix algebra from a statistician’s perspective*. Springer, 1997, vol. 1.

[13] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, UK: Cambridge university press, 2009.

[14] G. R. Lanckriet and B. K. Sriperumbudur, “On the convergence of the concave-convex procedure,” in *Advances in Neural Information Processing Systems, 2009*, pp. 1759–1767.