The Relevance of the Dipole Penguin Operators in $\epsilon'/\epsilon$

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Abstract

The standard model contribution to $\epsilon'/\epsilon$ of the magnetic- and electric-dipole penguin operators $Q_{11} = \frac{g_s}{16\pi^2} m_s \sigma \epsilon^{\mu \nu} G_{\mu \nu} (1 - \gamma_5) d$ and $Q_{12} = \frac{eQ_q}{16\pi^2} m_s \sigma \epsilon^{\mu \nu} F_{\mu \nu} (1 - \gamma_5) d$ is discussed. While the electromagnetic penguin $Q_{12}$ seems to have a vanishingly small matrix element, we find that the gluonic dipole operator $Q_{11}$ may give a contribution to $\epsilon'/\epsilon$ comparable to that of other operators so far considered, and should therefore be consistently included in the analysis.

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The determination of the $\Delta S = 1$, CP-violating parameter $\epsilon'/\epsilon$ in neutral kaon decays has attracted considerable experimental and theoretical interest (for a complete list of references see, for instance, ref. [1]). The possibility of drastic cancellations among QCD- and electroweak-induced operators [2], occurring for large values of the top quark mass, has spurred new and more accurate theoretical investigations [3, 4]. On the experimental side, the present sensitivity of 1 part in $10^3$ [5] is not conclusive and an effort in improving it by one order of magnitude is under way.

The theoretical framework for the study of $\epsilon'/\epsilon$ is the effective field theory. The local Hamiltonian for $\Delta S = 1$ transitions can be written, for $\mu < m_c$, as [3]

$$H = \frac{G_F}{\sqrt{2}} \lambda_u \sum_i \left[ z_i(\mu) + \tau y_i(\mu) \right] Q_i(\mu).$$

(1)

The list of the effective operators $Q_i$ ($i = 1 - 10$) is reported in refs. [3, 4], whose notation we follow closely and where the reader may find a complete discussion of the basic tools we use in the present analysis. We just recall that $Q_{1,2}$ stand for the “tree-level” $W$-induced current-current operators, $Q_{3-6}$ for the QCD penguin operators and $Q_{7-10}$ for the electroweak penguin (and box) ones.

The Wilson coefficients $z_{1,2}(\mu)$ run from $m_W$ to $m_c$ via the corresponding $2 \times 2$ sub-block of the $10 \times 10$ anomalous dimension matrix, while $z_i(\mu) = 0$ for $i = 3 - 10$. From $\mu = m_c$ down, as the charm-induced penguins come into play, $z_i(\mu)$ evolve, given the proper matching conditions, with the full anomalous dimension matrix. The Wilson coefficients $v_i(\mu)$ ($y_i(\mu) = v_i(\mu) - z_i(\mu)$) arise at $m_W$ due to integration of the $W$ and top quark fields. They coincide with $z_i(\mu)$ for $i = 1, 2$, the information about the top quark being encoded in the $i = 3 - 10$ components. Finally, $\lambda_i \equiv V_{td} V_{ts}^{*}$, where $V$ is the Kobayashi-Maskawa (KM) matrix, and $\tau \equiv -\lambda_t/\lambda_u$.

Let us mention that, according to standard conventions, $\text{Im} \lambda_u = 0$ and the short distance component of $\epsilon'/\epsilon$ is thus determined by the Wilson coefficients $y_i$. Notice that, following the approach of ref. [3], $y_1(\mu) = y_2(\mu) = 0$ and the effect of $Q_{1,2}$ appears only through the linearly dependent operators $Q_{4,9,10}$.

In this letter, we would like to discuss the possible relevance of two extra operators that have so far been neglected, namely

$$Q_{11} = \frac{g_8}{16\pi^2} m_s \bar{s} \sigma_{\mu\nu} t^a G^{\mu\nu}_a (1 - \gamma_5) d,$$

(2)

and

$$Q_{12} = \frac{e Q_t}{16\pi^2} m_s \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1 - \gamma_5) d,$$

(3)
which account for the magnetic and electric dipole part of, respectively, the QCD and electromagnetic penguin operators. In eq. (2), \( t^a \) are the \( SU(3)_c \) generators normalized as \( \text{Tr}(t^at^b) = (1/2)\delta^{ab} \), while in eq. (3) \( Q_d = -1/3 \) is the charge of the down quarks; \( \sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu] \).

These operators have been left out in the past, together with \( Z \) penguins and electroweak box diagrams, because their Wilson coefficients exhibit, for light quark masses, a power-like GIM suppression \[6\], to be compared with the leading soft logarithmic behavior of the “monopole” component of the gluonic and photonic penguins. However, for a heavy top quark, there is no reason a priori to discard these contributions, which should be considered on the same grounds as \( Z \) penguins and electroweak box diagrams. In addition, \( Q_{11} \) and \( Q_{12} \) receive a large QCD renormalization due to the mixings with the other operators, as has been shown in the case of \( b \to s\gamma \) \[7\].

The potential relevance of the dipole components of the gluonic and photonic penguins for \( \epsilon'/\epsilon \) has been emphasized by one of us a few years ago \[8\]. Only recently has a quantitative attempt to estimate the relevance of \( Q_{11} \) appeared \[9\]. However, in ref. \[9\], only the multiplicative renormalization—which is by no means the leading effect—is taken into account. We also disagree on the evaluation of the hadronic matrix element that, we believe, is overestimated in ref. \[9\] by more than one order of magnitude (by a factor 8 using their method).

The evaluation of the matrix elements is in fact the crucial point in determining the relevance of the two additional operators. As we shall see, while the contribution of the electromagnetic penguin \( Q_{12} \) seems to be vanishingly small, that of the gluonic \( Q_{11} \) should not be neglected. In fact, its Wilson coefficient turns out to be sizeably enhanced by mixing and the matrix element is not dramatically suppressed with respect to the other ten operators.

2. In order to compare meaningfully the relevance of the new contributions with the traditional ones, from now on we strictly follow the analysis presented in ref. \[8\]. The parameter \( \epsilon'/\epsilon \) can be obtained by means of the Hamiltonian in eq. (1), which yields \[8\]

\[
\frac{\epsilon'}{\epsilon} = 10^{-4} \left[ \frac{\text{Im} \lambda_i}{1.7 \times 10^{-4}} \right] \left[ P^{(1/2)} - P^{(3/2)} \right],
\]

(4)

where

\[
P^{(1/2)} = r \sum y_i \langle 2\pi, I = 0 | Q_i | K^0 \rangle (1 - \Omega_{\eta+\eta'})
\]

(5)

\[
P^{(3/2)} = \frac{r}{\omega} \sum y_i \langle 2\pi, I = 2 | Q_i | K^0 \rangle .
\]

(6)
We take, as input values for the relevant quantities, the central values given in appendix C of ref. [3]. This will allow us to reproduce in the ten-operator case the central values of the leading order (LO) results, as given in appendix B of ref. [3]. In particular, we take

\[ r = 1.7 \frac{G_F \omega^2}{2 |\epsilon| \text{Re} A_0} \simeq 594 \text{ GeV}^{-3}, \quad \omega = 1/22.2, \quad \Omega_{\eta+\eta'} = 0.25; \]  

(7)

\[ \text{Im} \lambda_t \text{ is determined from the experimental value of } \epsilon \text{ as an interpolating function of } m_t. \text{ For instance, taking the KM phase } \delta_{KM} \text{ in the first quadrant, we find, for the central value,} \]

\[ \text{Im} \lambda_t = 2.77 \times 10^{-4} x_t^{-0.365}, \]  

(8)

where \( x_t = m_t^2/m_W^2. \)

The value of the Wilson coefficients \( y_{11} \) and \( y_{12} \) at the hadronic scale of 1 GeV can be found by means of the renormalization group equations. Denoting generically by \( \vec{C}(\mu) \) the vector of Wilson coefficients, its scale dependence is given by

\[
\left[ \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right] \vec{C} \left( \frac{m_W^2}{\mu^2}, g^2, \alpha \right) = \gamma^T(g^2, \alpha) \vec{C} \left( \frac{m_W^2}{\mu^2}, g^2, \alpha \right),
\]  

(9)

where \( \beta(g) \) is the QCD beta function and \( \alpha \) the electromagnetic coupling (the running of \( \alpha \) is being neglected). At the leading order we have

\[
\gamma(g^2, \alpha) = \frac{\alpha_s}{4\pi} \gamma_s^{(0)} + \frac{\alpha}{4\pi} \gamma_e^{(0)},
\]  

(10)

where \( \gamma_s^{(0)} \) governs the QCD and \( \gamma_e^{(0)} \) the electromagnetic running.

The 10×10 mixing matrix of the anomalous dimension for \( Q_{1-10} \) has recently been evaluated at the next-to-leading order in refs. [3, 4]. On the other hand, the matrix of the strong anomalous dimensions of \( Q_{11} \) and \( Q_{12} \), and their QCD-induced mixing with \( Q_{1-6} \), can be borrowed from the existing calculations for the \( b \to s\gamma \) decay [7] (we will use the most recent and complete analysis of ref. [10]). Assembling the known results, we thus write the leading order 12×12 anomalous dimension matrix \( \gamma^{(0)} T \) as
active quarks; finally, \( N \) is the number of colors, \( n_f \) is the number of active flavors, and \( \bar{n}_f = n_d + (Q_u/Q_d)n_u \), with \( Q_u = 2/3 \), \( Q_d = -1/3 \), \( n_u \) (\( n_d \)) being the number of up (down) active quarks; finally, \( C_f \equiv (N^2 - 1)/2N \).

A few comments are in order. The 10 \times 10 part of the matrix (11) is identical to that used in refs. [3, 4] for the leading one-loop order result. The two extra columns and rows represent the mixing of the first ten operators with the two new ones, which takes place first at the two-loop level. We have taken for these entries the ‘t Hooft-Veltman (HV) scheme results [3, 4]. In this way, no finite additional contributions to the renormalization of \( y_{11} \) and \( y_{12} \) arise at the various quark thresholds (for a discussion see ref. [10]). The entries labelled with \( X_{7-10}, Y_{7-10} \) represent the mixings of, respectively, \( Q_{11} \) and \( Q_{12} \) with the electroweak penguins, which have not yet been computed. We set them equal to zero, expecting that the leading contribution to the running of the Wilson coefficients \( y_{11} \) and \( y_{12} \) arises from the mixing with the current-current operators and the gluonic penguins.

Since we want to fully reproduce the leading-order results for the 10 \times 10 operator system, we have included, in the running of the Wilson coefficients, the effect of
the electromagnetic renormalization following the procedure described in ref. [3]. To the 10 × 10 electromagnetic anomalous dimension matrix, which can be found in refs. [3, 4], we have simply added two rows and two columns of zeros, thus neglecting the electromagnetic running of $Q_{11}$ and $Q_{12}$.

The fact that the mixing of $Q_{11}$ and $Q_{12}$ with the other operators arises at the two-loop level introduces an explicit scheme dependence in our analysis. From the next-to-leading results of refs. [3, 4], it appears however that the inclusion of next-to-leading effects reduces the values of $\epsilon'/\epsilon$ compared to the leading-order result. Since the presence of the two extra operators does not affect the running of $y_{1-10}(\mu)$ (see eqs. (9)–(11)), we conclude that our simplified leading-order procedure will at most underestimate the effective weight in $\epsilon'/\epsilon$ of the new contributions.

For the purpose of comparison, we take as initial conditions for $v_{1-10}(m_W)$ the leading-order values of ref. [3], which coincide with the HV results when neglecting $O(\alpha_s)$ corrections to $v_{1,2}(m_W)$ (strictly speaking at the leading order one should set $v_{3-10}(m_W) = 0$). For what concerns the new coefficients $v_{11,12}(m_W)$ we have

$$v_{11}(m_W) = -E'(m_t^2/m_W^2)$$
$$v_{12}(m_W) = -D'(m_t^2/m_W^2)/Q_d,$$

where

$$E'(x) = \frac{3x^2}{2(1-x)^4} \ln x - \frac{x^3 - 5x^2 - 2x}{4(1-x)^3}$$
$$D'(x) = \frac{x^2(2-3x)}{2(1-x)^4} \ln x - \frac{8x^3 + 5x^2 - 7x}{12(1-x)^3}.$$  (12)

In Table 1 we report the results for $z_{1-12}(1 \text{ GeV})$ and $y_{3-12}(1 \text{ GeV})$ (remember that $y_{1,2}(\mu) = 0$) compared to their initial values, for $m_t$ equal to 130 and 170 GeV and $\Lambda_{QCD}^{(4)}$ equal to 200 and 300 MeV. As expected, a comparison between our results and those of Table 1 of ref. [3] shows that we fully agree on the values of the renormalized coefficients for the first ten operators. Regarding $v_{11,12} (= y_{11,12} + z_{11,12})$, we note that the effect of operator mixing induces a renormalization that is a factor of 4–5 larger than that induced by multiplicative running alone (which roughly reduces by a factor of 2 the initial Wilson coefficients).

In order to ascertain the importance of $Q_{11}$ and $Q_{12}$ in the estimate of $\epsilon'/\epsilon$, we now have to address the question of the evaluation of their hadronic matrix elements.
Table 1: Wilson coefficients at $\mu = 1$ GeV, in the leading order, as explained in the text ($\alpha = 1/128$). The corresponding values at $\mu = m_W$ are given in parenthesis. In addition, at $\mu = m_c$ we have $z_{3-12}(m_c) = 0$. 

| $\Lambda_{QCD}^{[4]}$ | 0.2 GeV | 0.3 GeV |
|----------------------|---------|---------|
| $m_t$ 130 GeV 170 GeV | 130 GeV 170 GeV |
| $z_1$ | -0.587 (0.0) | -0.715 (0.0) |
| $z_2$ | 1.319 (1.0) | 1.409 (1.0) |
| $z_3$ | 0.004 | 0.005 |
| $z_4$ | -0.010 | -0.014 |
| $z_5$ | 0.003 | 0.004 |
| $z_6$ | -0.011 | -0.015 |
| $z_7/\alpha$ | 0.005 | 0.008 |
| $z_8/\alpha$ | 0.001 | 0.001 |
| $z_9/\alpha$ | 0.005 | 0.009 |
| $z_{10}/\alpha$ | -0.001 | -0.001 |
| $z_{11}$ | -0.038 | -0.045 |
| $z_{12}$ | 0.431 | 0.582 |
| $y_3$ | 0.027 (0.0003) | 0.028 (0.0004) |
| $y_4$ | -0.048 (0.0018) | -0.049 (0.0014) |
| $y_5$ | 0.011 (0.0006) | 0.012 (0.0005) |
| $y_6$ | -0.078 (0.0018) | -0.080 (0.0014) |
| $y_7/\alpha$ | -0.025 (0.091) | 0.025 (0.151) |
| $y_8/\alpha$ | 0.053 (0.0) | 0.109 (0.0) |
| $y_9/\alpha$ | -1.160 (-0.793) | -1.554 (-1.094) |
| $y_{10}/\alpha$ | 0.488 (0.0) | 0.663 (0.0) |
| $y_{11}$ | -0.328 (-0.172) | -0.340 (-0.193) |
| $y_{12}$ | 2.070 (0.965) | 2.161 (1.158) | 2.166 (0.965) | 2.245 (1.158) |
3. Most of the uncertainties in the estimate of $\epsilon'/\epsilon$ arise from the evaluation of the hadronic matrix elements. For the operators $Q_{1-10}$, we follow the strategy of ref. [3] where the various matrix elements are evaluated by means of the $1/N$ expansion and soft-meson methods. Overall coefficients $B_i^{(1/2)}$ and $B_i^{(3/2)}$ ($i = 1 - 10$) parameterize our level of ignorance of their normalization scale, scale dependence and error estimate. Since some of the matrix elements are phenomenologically determined using $CP$-conserving data ($\Delta I = 1/2$ rule), and further relations among the $B_i$'s are advocated, the final result is parameterized in terms of two coefficients: $B_6^{(1/2)}$ and $B_8^{(3/2)}$, which take 1 as central value. The inclusion of $Q_{11}$ and $Q_{12}$ requires three additional effective parameters: $B_6^{(1/2)}$, $B_8^{(3/2)}$, and $B_{12}^{(3/2)}$. In order to evaluate $\langle \pi^+\pi^- | Q_{11} | K^0 \rangle$ and $\langle \pi^+\pi^- | Q_{12} | K^0 \rangle$, we use effective chiral lagrangian techniques. The lowest order chiral representation for these operators vanishes because of the cancellation between the direct transition and the pole diagram induced by a non-vanishing $\langle 0 | Q_{11} | K^0 \rangle$ matrix element [11]. The leading next order ($O(p^4)$) contribution arises from the bosonization of $Q_{11}$ into $L_{Q_{11}}$, where

$$L_{Q_{11}} = G_{Q_{11}} \text{Tr} \left\{ \Sigma^\dagger M_q \lambda_+ + \lambda_+ \Sigma M_q^\dagger \right\} \partial_\mu \Sigma^\dagger \partial^\mu \Sigma \right\} .$$

(15)

In the formula above $\lambda_+ = (\lambda_6 + i \lambda_7)/2$ is the octet $\Delta S = 1$ projector, $M_q$ is the quark current mass matrix, $\Sigma = \exp \left( \sqrt{2} i \Pi/F_\pi \right)$ and $\Pi = \sum_{a=1}^{8} \lambda^a \pi^a$ is the usual matrix of chiral $SU(3)$ Goldstone bosons. The constant $G_{Q_{11}}$ can be computed by considering a model for the QCD effective action at long distances [12]; it is found to be [13]

$$G_{Q_{11}} = -\frac{11}{64\pi^2} \langle 0 | \bar{q}q | 0 \rangle ,$$

(16)

where $\langle 0 | \bar{q}q | 0 \rangle$ is the quark condensate that we take to be $-F_K^2 m_K^2 / [2(m_s + m_u)]$. More details on this computation will be given elsewhere [14]. The final result can be written as

$$\langle \pi^+\pi^- | Q_{11} | K^0 \rangle = \frac{1}{16\pi^2} \frac{11}{2} \frac{m_s}{m_s + m_u} \frac{F_K^2}{F_\pi} m_\pi^2 B_{11}^{(1/2)} ,$$

(17)

with $B_{11}^{(1/2)} = 1$. The corresponding matrix element for $Q_{12}$ is very much suppressed because it is proportional to the photon condensate and it must therefore be very small, if not vanishing; we will neglect it altogether in what follows. Nevertheless, we must bear in mind the possibility that the same electromagnetic dipole penguin operator could give
a sizeable contribution, for instance, once saturated with an external quark current and written as a four-fermion operator. Such a possibility is under investigation.

Notice in eq. (17) the presence of the factor $1/16\pi^2$, reminding us that the leading-order mixing between $Q_{1-10}$ and $Q_{11}$ appears at the two-loop level.

Since $\langle \pi^+\pi^-|Q_{11}|K^0\rangle = \langle \pi^0\pi^0|Q_{11}|K^0\rangle$ we finally obtain

$$
\begin{align*}
\langle 2\pi, I = 0 | Q_{11} | K^0 \rangle &= \sqrt{\frac{3}{2}} \langle \pi^+\pi^-|Q_{11}|K^0\rangle \\
\langle 2\pi, I = 2 | Q_{11} | K^0 \rangle &= 0,
\end{align*}
$$

which enter in the determination of $\epsilon'/\epsilon$ (see eq. (4)).

We feel that our estimates for the hadronic matrix elements of the operator $Q_{11}$ and $Q_{12}$ are consistent with those in ref. [3] for the other ten operators.

4. We can now discuss the effect of the various operators in determining the size of $\epsilon'/\epsilon$, and identify the role of the new contributions. We follow here the analysis of ref. [3], where the hadronic matrix elements are evaluated using the $1/N$ expansion and soft-meson methods. As previously mentioned, the final results are written in terms of effective coefficients $B_{1,LO}^{(1/2)}$ and $B_{1,LO}^{(3/2)}$, which encompass our lack of knowledge on scale normalization, scale dependence and goodness of the method. The matrix elements of $Q_1$ and $Q_2$ can however be determined phenomenologically from the experimental values of $\text{Re} \, A_0$ and $\text{Re} \, A_2$, so as to reproduce the $\Delta I = 1/2$ rule. In particular, in ref. [3] it is found that $B_{2,LO}^{(1/2)} \approx 5.8$, which is about three times larger than the $1/N$ result. Related to this coefficient is the value of $B_{1,LO}^{(1/2)}$, which we find equal to 19.3 and 13.5 for $\Lambda_{QCD}^{(4)} = 0.2$ and 0.3 GeV, respectively. Correspondingly, $B_{1,LO}^{(3/2)} = 0.48$ and 0.50. Relations among the other coefficients are advocated in ref. [3], depending on the relevance and the role of the various operators, so as to reduce, in the ten-operator case, the description of the hadronic sector to two effective parameters: $B_{6}^{(1/2)}$ and $B_{8}^{(3/2)}$, whose $1/N$ value is 1.

Since the determination of $B_1$ and $B_2$ is best achieved at $\mu = m_c$ [3], all the hadronic matrix elements are assumed to be evaluated at that scale and renormalized down to 1 GeV via their anomalous dimension matrix.

We proceed analogously by setting $B_{11}^{(1/2)} = 1$ and $B_{12}^{(1/2)} = B_{12}^{(3/2)} = 0$ at $\mu = m_c$ and using the $12 \times 12$ QCD and electromagnetic evolution matrices to evolve all the hadronic matrix elements to the 1 GeV scale. Since the anomalous dimension matrices, which govern the evolution of the hadronic matrix elements, are the transpose of
those evolving the Wilson coefficients, we now find that the presence of $Q_{11,12}$ affects the renormalization of the first ten operators. On the other hand, the evolution of $Q_{11,12}$ is determined solely by their $2 \times 2$ anomalous dimension matrix, which imply that the matrix element of $Q_{12}$ remains vanishing.

As a consequence of the previous remarks, our results for the individual contributions of the operators $Q_{1-10}$ to $\epsilon'/\epsilon$ differ (slightly) from those reported in ref. [3]. We have however checked that, in the ten-operator case, we reproduce their leading-order results exactly.

We have chosen to illustrate numerically our conclusions as tables. Tables 2, 3 and 4 show the contributions to $\epsilon'/\epsilon$ of each operator, for different choices of $\Lambda_{QCD}$, $B_6^{(1/2)}$ and $B_8^{(3/2)}$. The first ten contributions are also partially grouped in a “positive” gluonic component versus a “negative” electroweak component, which shows the “superweak” behavior of $\epsilon'/\epsilon$ within the standard model, as the top mass increases. The total effect in the twelve-operator case is then compared with the ten-operator case result (we have filled in those data that are not explicitly available from ref. [3]).

These tables suggests that $Q_{11}$ should be consistently included in any estimate of $\epsilon'/\epsilon$. Its sign is the same as that of $Q_6$ and therefore makes $\epsilon'/\epsilon$ larger; for large $m_t$, the “super-weak” regime is accordingly shifted to higher values.

Let us conclude by mentioning the importance of completing the leading-order (two-loop) calculation of the anomalous dimension matrices—which is important also for the $\Delta B = 1$ processes. Moreover, a consistent discussion of the short-distance part of the present analysis should include the next-to-leading order. This however implies computing the mixings of $Q_{11,12}$ with the other operators at the three-loop level—a truly formidable task.

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Table 2: Anatomy of $\epsilon'/\epsilon$ for $\Lambda_{QCD}^{(4)} = 300$ MeV, $B_6^{(1/2)} = B_8^{(3/2)} = B_{11}^{(1/2)} = 1$. The contribution of each operator is shown at $\mu = 1$ GeV, together with partial grouping of the gluonic and electroweak sectors. The contribution of $Q_{12}$ is being neglected. The total effect is compared with the corresponding leading-order result of the ten-operator case (last line).
\[ \epsilon' / \epsilon \times 10^4 \text{ (Leading Order)} \]

| \(m_t\) | 130 GeV | 170 GeV | 200 GeV | 230 GeV |
|-------|---------|---------|---------|---------|
| \(Q_3\) | 0.4 | 0.3 | 0.3 | 0.3 |
| \(Q_4\) | -5.1 | -4.2 | -3.8 | -3.5 |
| \(Q_5\) | -0.6 | 6.7 | 5.6 | 0.4 | 5.1 | -0.4 | 4.6 |
| \(Q_6\) | 12.0 | 10.0 | 9.0 | 8.2 |
| \(Q_7\) | 0.4 | -0.3 | -0.7 | -1.2 |
| \(Q_8\) | -2.7 | -4.6 | -5.9 | -7.3 |
| \(Q_9\) | 2.9 | -0.3 | 3.2 | -2.7 | 3.5 | -4.2 | 3.7 | -5.9 |
| \(Q_{10}\) | -0.9 | -1.0 | -1.1 | -1.1 |
| \(Q_{11}\) | 0.4 | 0.4 | 0.3 | 0.3 |
| \(Q_1 - Q_{12}\) | 6.8 | 3.3 | 1.0 | -1.1 |
| Ref. [3] | 6.4 | 3.0 | 0.7 | -1.4 |

Table 3: Same as in Table 2 for \(\Lambda_{QCD}^{(4)} = 200\text{ MeV}, B_6^{(1/2)} = B_8^{(3/2)} = 1\).
\[ \epsilon'/\epsilon \times 10^4 \text{ (Leading Order)} \]

| \( m_t \) | 130 GeV | 170 GeV | 200 GeV | 230 GeV |
|--------|---------|---------|---------|---------|
| \( Q_3 \) | 0.5 | 0.4 | 0.4 | 0.4 |
| \( Q_4 \) | -5.2 | -4.4 | -4.0 | -3.6 |
| \( Q_5 \) | -0.4 | 5.8 | -0.4 | 4.8 | -0.3 | 4.1 | -0.3 | 3.9 |
| \( Q_6 \) | 11.0 | 9.2 | 8.2 | 7.5 |
| \( Q_7 \) | 0.2 | -0.3 | -0.6 | -0.8 |
| \( Q_8 \) | -2.6 | -4.3 | -5.6 | -6.8 |
| \( Q_9 \) | 3.0 | -0.5 | 3.4 | -2.4 | 3.6 | -3.6 | 3.9 | -5.1 |
| \( Q_{10} \) | -1.1 | -1.2 | -1.3 | -1.4 |
| \( Q_{11} \) | 0.5 | 0.4 | 0.3 | 0.3 |
| \( Q_{12} \) | 5.7 | 2.8 | 0.8 | -1.0 |
| Ref. | 5.3 | 2.4 | 0.5 | -1.3 |

Table 4: Same as in Table 2 for \( \Lambda_{QCD}^{(4)} = 300 \text{ MeV} \), \( B_{6}^{(1/2)} = B_{8}^{(3/2)} = 0.75 \).
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