Drell-Yan Cross Section and $J/\psi$ Production in High-Energy Nucleus-Nucleus Collisions

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We use the Drell-Yan differential distribution $dN_{AB}^{DY}/dE_T$ in high-energy nucleus-nucleus collisions to obtain a relation between the transverse energy $E_T$ and the impact parameter $b$. Such a relation is then utilized to study the transverse-energy dependence of $J/\psi$ production in Pb-Pb collisions, using the $J/\psi$ absorption model presented previously. The anomalous Pb-Pb suppression data at 158A GeV can be explained if one assumes the occurrence of a new phase of strong $J/\psi$ absorption when the energy density exceeds 4.2 GeV/fm$^3$. The results are extended to make predictions for $J/\psi$ production at higher collision energies. It is found that $J/\psi$ survival probabilities reach the lowest survival limit when the nucleon-nucleon center-of-mass energies $\sqrt{s}$ is greater than about 35 GeV.

1 Introduction

Following the initial suggestion by Matsui and Satz\cite{1} that $J/\psi$ production is suppressed in a quark-gluon plasma, the NA50 experimental observation\cite{2-5} of an anomalous $J/\psi$ absorption in Pb-Pb collisions has led to a flurry of theoretical activity\cite{6-14}. The absorption is anomalous in the systematics of the total normalized yields as a function of the sum of the radii of the colliding nuclei. It is also anomalous in the differential $J/\psi$ survival probability, which shows a discontinuity as a function of the transverse energy. A central question is whether these discontinuities indicate the occurrence of a new phase of strong $J/\psi$ absorption, as expected when a quark gluon plasma is produced.

Theoretical studies on $J/\psi$ suppression require the relation between the transverse energy $E_T$ and the impact parameter $b$ of a collision. We shall first show how we can obtain such a relationship from the Drell-Yan distribution. We shall use the $J/\psi$ absorption model presented previously\cite{15} to study $J/\psi$ absorption in Pb-Pb collisions at $p_{lab} = 158$ A GeV ($\sqrt{s} = 17.3$ GeV). The results are then utilized to predict $J/\psi$ production at higher energies where we find interesting trends as the collision energy increases.

2 Drell-Yan Cross Section

In NA50 experiments, the transverse energy is measured in coincidence with the detection of Drell-Yan and $J/\psi$ dimuon pairs. Based on the assumption of a monotonic dependence, the relation between $E_T$ and $b$ can be easily obtained from the Drell-Yan differential cross section $d\sigma_{AB}^{DY}/dE_T$ or the Drell-Yan dis-
tribution \(dN^{AB}_{DY}/dE_T\) which is equal to \((d\sigma^{AB}_{DY}/dE_T)[N^{AB}_{DY}(total)/\sigma^{AB}_{DY}(total)]\). One relies here on the experimental nuclear mass dependences\(^{15}\) which indicate that initial- and final-state interactions have only very small effects on the Drell-Yan cross section in nuclear collisions. Neglecting initial- and final-state interactions, the Drell-Yan cross section element in the collision of nucleus \(A\) with nucleus \(B\) is given by

\[
d\sigma^{AB}_{DY} = AB \ T_{AB}(b) \ \sigma^{pp}_{DY} \ db,
\]

where \(T_{AB}(b)\) is the thickness function,

\[
T_{AB}(b) = \int db_A T_A(b_A) T_B(b - b_A),
\]

and \(T_A(b_A)\) and \(T_B(b_B)\) are the thickness functions for nucleus \(A\) and \(B\) respectively.\(^{16}\) Therefore, we have

\[
d\sigma^{AB}_{DY}/dE_T = AB \sigma^{pp}_{DY} T_{AB}(b) \ \frac{db}{dE_T}.
\]

From these relations, we can get a relation between \(E_T\) and \(b\) given by

\[
\Sigma(E_T) \equiv \frac{1}{\sigma^{AB}_{DY}(total)} \int_0^{E_T} d\sigma^{AB}_{DY}/dE_T dE_T = \int_0^\infty T_{AB}(b') db' = \Sigma(b),
\]

where \(\Sigma(E_T \to \infty) = \Sigma(b = 0) = 1\) and

\[
\sigma^{AB}_{DY}(total) = \int_0^\infty d\sigma^{AB}_{DY}/dE_T dE_T.
\]

If the Drell-Yan distribution \(dN^{AB}_{DY}/dE_T\) is measured instead, the function \(\Sigma(E_T)\) can be expressed in terms of \(dN^{AB}_{DY}/dE_T\) by

\[
\Sigma(E_T) = \frac{1}{N^{AB}_{DY}(total)} \int_0^{E_T} dN^{AB}_{DY}/dE_T dE_T
\]

where

\[
N^{AB}_{DY}(total) = \int_0^\infty dN^{AB}_{DY}/dE_T dE_T.
\]

To obtain \(E_T\) as a function of \(b\), one can use a graphical method where one obtains \(\Sigma(E_T)\) from experimental data and \(\Sigma(b)\) from \(T_{AB}(b)\) which can be calculated from the known geometry of the colliding nuclei. One then uses \(b\) and \(E_T\) as the abscissa, and plots \(\Sigma\) as the ordinate, resulting in the curves.
of $\Sigma(E_T)$ and $\Sigma(b)$. From Eq. (4), the horizontal line of constant $\Sigma$ then intercepts the two curves at $b$ and its corresponding $E_T(b)$. Alternatively, one can parameterize $E_T(b)$ as

$$E_T(b) = \frac{E_{T0}}{1 + \exp[(b - b_0)/a]},$$

which gives $db/dE_T$. One then uses Eq. (3) to determine $d\sigma_{DY}^{AB}/dE_T$. The Drell-Yan distribution for Pb-Pb collisions calculated with $E_{T0} = 135$ GeV, $b_0 = 6.7$ fm, and $a = 3$ fm is shown as the dashed curve in Fig. 1.

One notes that for a given value of $b$, the $E_T$ value cannot be a sharp distribution. One can consider the righthand side of Eq. (8) as defining the mean value $\bar{E}_T(b)$, and include the spreading of $E_T$ about this mean value by introducing a distribution function $D(b,E_T)$,

$$D(b,E_T) = \frac{1}{\sqrt{2\pi}\sigma(b)} e^{-[E_T - \bar{E}_T(b)]^2/2\sigma(b)^2}.$$  

Then, with the inclusion of the spreading of $E_T$, $d\sigma_{DY}^{AB}/dE_T$ is given by

$$\frac{d\sigma_{DY}^{AB}}{dE_T} = AB\sigma_{DY}^{pp} \int db ~ T_{AB}(b) ~ D(b,E_T).$$

From the results of Ramello et al., $\sigma(b)$ in Eq. (3) can be parameterized as

$$\sigma(b) = \frac{\sigma_0}{1 + \exp[(b - b_\sigma)/a_\sigma]},$$

Fig. 1. The Drell-Yan distribution as a function of the transverse energy in Pb-Pb collisions at 158A GeV. The data points are preliminary data from NA50.
where $\sigma_0 = 11$ GeV, $b_\sigma = 10$ fm, and $a_\sigma = 2$ fm. The Drell-Yan distribution with the inclusion of $E_T$ spreading is shown as the solid curve in Fig. 1, which agrees well with the preliminary NA50 data. The Drell-Yan distribution provides a relation between $E_T$ and $b$ which is needed to analyze the transverse-energy dependence of $J/\psi$ and $\psi'$ production.

3 Model of $J/\psi$ and $\psi'$ Absorption

We can use the $J/\psi$ absorption model presented previously in [15] to study $J/\psi$ and $\psi'$ absorption. In this model, a produced $J/\psi$ (or its precursor) meets the projectile and target nucleon at high energies and is absorbed with a cross section $\sigma_{abs}(J/\psi-N)$. It also collides with produced hadrons (comovers) at low relative energies and is absorbed with a cross section $\sigma_{abs}(J/\psi-h)$. In a nucleus-nucleus collision, each nucleon-nucleon collision is a possible source of $J/\psi$ and $\psi'$ precursors. It is also the source of a fireball of produced hadrons which can absorb $J/\psi$ and $\psi'$ precursors produced by other nucleon-nucleon collisions. One follows the space-time trajectories of precursors, baryons, and the centers of the fireballs of produced hadrons. Absorption occurs when the space-time trajectories of the precursors cross those of other particles. Using a row-on-row picture in the center-of-mass system and assuming straight-line space-time trajectories, we obtain the differential cross section for $J/\psi$ production in an $AB$ collision as

$$\frac{d\sigma_{j/\psi}^{AB}(\mathbf{b})}{\sigma_{j/\psi}^{NN}} = \int \frac{db_A}{\sigma_{abs}^{NN}(J/\psi-N)} \left\{ 1 - \left[ 1 - T_A(b_A)\sigma_{abs}(J/\psi-N) \right]^A \right\} \times \left\{ 1 - \left[ 1 - T_B(b - b_A)\sigma_{abs}(J/\psi-N) \right]^B \right\} F(b_A, b),$$

(12)

where $F(b_A, b)$ is the survival probability due to soft particle collisions. To calculate $F(b_A, b)$, we sample the target tranverse coordinate $b_A$ for a fixed impact parameter $b$ in a row with the nucleon-nucleon inelastic cross section $\sigma_{in}$. In this row, $BT_B(b - b_A)\sigma_{in}$ projectile nucleons will collide with $AT_A(b_A)\sigma_{in}$ target nucleons. We construct the space-time trajectories of these nucleons to locate the position of their nucleon-nucleon collisions. These collisions are the sources of $J/\psi$ and $\psi'$ precursors and the origins of the fireballs of produced particles. For each precursor source from the collision $j$ and each absorbing fireball from the collision $i$ at the same spatial location, we determine the time $t_{ij}^h$ when the precursor source coexists with the absorbers in the state of produced hadrons. The survival probability due to this combination of precursor source and absorber is then $\exp\{-k_{\psi h}t_{ij}^h\}$, where the rate constant $k_{\psi h}$ is

$$k_{\psi h} = \rho_h^{NN}v_h\sigma_{abs}(J/\psi-h),$$

(13)

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\( v_h \) is the average \((J/\psi)-h\) relative velocity, \( \rho_{NN}^h \) is the average produced hadron number density per \( NN \) collision, 

\[
\rho_{NN}^h = \frac{dN_{NN}^h}{dy} \frac{1}{\sigma_{in}(d/\gamma)} \tag{14}
\]

\( dN_{NN}^h/dy \) is the particle multiplicity per unit of rapidity at \( y_{CM} = 0 \) for an \( NN \) collision, \( d \approx 2.46 \) fm is the internucleon spacing, and \( \gamma = \sqrt{s/2m_{\text{nucleon}}} \) is the Lorentz contraction factor. We can use the parametrization of Eq. (2.4) in Reference [19] to obtain \( dN_{NN}^h/dy \) as a function of the collision energy.

When we include all possible precursor sources and absorbers, \( F(b_A, b) \) becomes

\[
F(b_A, b) = \sum_{n=1}^{N_>} a(n) \sum_{j=1}^{N_<} \exp \{ -\theta \sum_{i=1, i \neq j}^{n} k_{\psi h} t_{ij} \} \tag{15}
\]

where \( N_>(b_A) \) and \( N_<(b_A) \) are the greater and the lesser of the (rounded-off) nucleon numbers \( AT_A(b_A)\sigma_{in} \) and \( BT_B(b - b_A)\sigma_{in} \), \( a(n) = 2 \) for \( n = 1, 2, ..., N_< - 1 \), and \( a(N_<) = N_> - N_< + 1 \). The function \( \theta \) is zero if \( A = 1 \) or \( B = 1 \) and is 1 otherwise.

We assume that the Bjorken-type longitudinal expansion occurs after the last nucleon-nucleon collision takes place at that spatial point. The survival probability \( F \) can be determined from \( k_{\psi h} \) and

\[
t_{ij}^h = t_n + t_f - \text{Max}(t_i + t_h, t_j + t_{c\bar{c}}). \tag{16}
\]

Here we shall set the \( c\bar{c} \) production time \( t_{c\bar{c}} \) equal to 0.06 fm/c, the hadronization time \( t_h \) equal to 1.2 fm/c, and \( t_f \) is related to the freezeout time \( t_{\text{freezeout}} \) by

\[
t_f = t_h + t_h \ln(t_{\text{freezeout}}/t_h) = t_h + t_h \ln(\rho_{NN}^h / \rho_{\text{freezeout}}^N). \tag{17}
\]

We use a freezeout density of \( \rho_{\text{freezeout}}^N = 0.5 \) hadrons/fm\(^3\), corresponding to a produced hadron freezeout separation of 1.3 fm. The \( \psi' \) production cross section can be obtained from the above equations by changing \( J/\psi \) into \( \psi' \).

We have used this absorption model to study the experimental \( J/\psi \) and \( \psi' \) data in \( p-A \) and nucleus-nucleus collisions [10]. We found that \( J/\psi \) absorption by produced hadrons, as revealed by O-Cu, O-U, and S-U collisions, is small, and the experimental Pb-Pb yield is much smaller than the extrapolated results if one assumes \( J/\psi \) absorption by nucleons and produced hadrons only.

The deviation of the \( J/\psi \) data in Pb-Pb collisions from the conventional theoretical extrapolations of \( p-A \), O-A, and S-U collisions suggests that there is a transition to a new phase of strong absorption which sets in when the
local energy density exceeds a certain threshold. We have extended the absorption model to describe this transition in terms of the critical number of \( NN \) collisions at a spatial point \( i \). We can reformulate the model here in terms of the critical energy density. Evaluated in the nucleon-nucleon center-of-mass system, the energy density at the spatial point, which has had \( \nu \) prior nucleon-nucleon collisions, is approximately

\[
\epsilon = \nu \rho_{NN}^m t
\]

(18)

where \( m_t \) is the average transverse mass of a produced hadron.

Fig. 2. The \( J/\psi \) distribution as a function of the transverse energy in Pb-Pb collisions at 158A GeV. The data points are preliminary data from NA50.

We postulate that soft particles make a transition to a new phase with stronger \( J/\psi \) absorption at a spatial point if the energy density at that point exceeds \( \epsilon_{\text{crit}} \). When this happens, the matter in the new phase absorbs \( J/\psi \) with a greater strength. To keep track of the same matter density, we still use the same number of particles in the new phase as in the hadron matter, but the particles in the new phase absorb \( J/\psi \) with an effective cross section \( \sigma_{\text{abs}}(J/\psi-x) \). The quantity \( k_{\psi h}t_{ij}^h \) in Eq. (13) becomes \( k_{\psi h}t_{ij}^h + k_{\psi x}t_{ij}^x \), where the new rate constant \( k_{\psi x} \) is

\[
k_{\psi x} = \rho_{NN}^m v_x \sigma_{\text{abs}}(J/\psi-x),
\]

(19)

and the quantity \( t_{ij}^x \) is the time for a \( J/\psi \) produced in collision \( j \) to coexist at the same spatial location with the absorbing soft particles produced in collision
in the form of the new phase. The duration when the matter is in the new
phase can be inferred by assuming that the matter undergoes Bjorken-type
longitudinal expansion after all the nucleon-nucleon collisions have occurred
at that point. Because the soft particles are relativistic, we take all relevant
velocities $v$ in Eqs. (13) and (19) to be 1.

We obtain good agreement with Pb-Pb data using the following parame-
ters: $\sigma_{\text{abs}}(\psi-N) = \sigma_{\text{abs}}(\psi'-N) = 6.36$ mb, $\epsilon_{\text{crit}} = 4.2$ GeV/fm$^3$, $\sigma_{\text{abs}}(J/\psi-h) = 0.15$ mb, $\sigma_{\text{abs}}(J/\psi-x) = 3.7$ mb, and $\sigma_{\text{abs}}(\psi'-h) = 11$ mb. Note that $\sigma_{\text{abs}}(J/\psi-x) >> \sigma_{\text{abs}}(J/\psi-h)$. We show in Fig. 2 the $J/\psi$ distribution in Pb-Pb collisions at 158A GeV. The solid curve is the result when one assumes
a new phase and $E_T$ spreading, the dashed curve is the result with the new
phase but without $E_T$ spreading. The dashed-dot curve is the result without
the new phase, but the $E_T$ spreading is included. There is good agreement
of the theoretical curve with the preliminary NA50 data when one assumes a
new phase of strong $J/\psi$ absorption and takes into account the $E_T$ spreading.

In Fig. 3 we show $B\sigma(\psi)/\sigma(DY)$ for Pb-Pb collisions at 158A GeV as a function of $E_T$. The data points are preliminary data from NA50.

In Fig. 3 we show $B\sigma^{AB}(\psi)/\sigma^{AB}(DY)$ for Pb-Pb collisions at 158A GeV. The short-dashed curve gives the results when there is absorption by nucleons
only, while the long-dashed curve gives the results when one assumes absorption
by nucleons and produced hadrons. Depending on the critical energy density,
the ratio $B\sigma^{AB}(\psi)/\sigma^{AB}(DY)$ deviates from the long-dashed curve (for no
new phase) at different transverse energies. For $\epsilon_{\text{crit}} = 4.2$ GeV/fm$^3$, the break occurs at $E_T \sim 40$ GeV which is close to the break observed experimentally. For $\epsilon_{\text{crit}} = 3.0$ GeV/fm$^3$, it occurs at $E_T \sim 20$ GeV, and for $\epsilon_{\text{crit}} = 5.5$ GeV/fm$^3$, it occurs at $E_T \sim 90$ GeV. However, the discontinuity in the theoretical results is much smoother than the discontinuity in the preliminary NA50 data shown in Fig. 3.

4 Dependence of $J/\psi$ Survival Probability on Collision Energy

The above formulation allows one to calculate the $J/\psi$ survival probabilities, represented by $S(b) = \frac{[B\sigma^{AB}(J/\psi)/\sigma^{AB}(DY)]/[B\sigma^{pp}(J/\psi)/\sigma^{pp}(DY)]}{[B\sigma^{pp}(J/\psi)/\sigma^{pp}(DY)]}$, as a function of the impact parameter for different collision energies. The results are shown in Fig. 4. The short-dashed curve is the survival probability when $J/\psi$ is absorbed by nucleons only. The dash-dot and the solid curves are the results when there is a new phase. The dashed curves are the survival probabilities when there is no new phase. The numbers label the collision energy, $\sqrt{s}$.

![Graph](image)

Fig. 4. The ratio $[B\sigma^{AB}(J/\psi)/\sigma^{AB}(DY)]/[B\sigma^{pp}(J/\psi)/\sigma^{pp}(DY)]$ for Pb-Pb collisions at different collision energies as a function of the impact parameter.
Figure 4 shows that if a new phase of strong $J/\psi$ absorption is assumed, the $J/\psi$ survival probability reaches the “lowest survival” (LS) limit (the solid curve) when the nucleon-nucleon center-of-mass energy $\sqrt{s}$ reaches about 35 GeV. This is the limit at which the energy density of the produced matter is so large that no $J/\psi$ particles, except those produced at the peripheral region of the collision, can survive the nucleus-nucleus collision. If there is no phase transition, the lowest survival limit is reached at about $\sqrt{s} \sim 70$ GeV.

The existence of this lowest $J/\psi$ survival limit shows up as a universal curve when one plots the $J/\psi$ survival probability as a function of $b$ for different energies. This can be obtained by transforming $S(E_T)$ to $S(b)$, utilizing the method of Section 2 to make a correspondence between $E_T$ and $b$ by means of the Drell-Yan distribution. Upon verification of the existence of the lowest survival limit, the energy at which this LS limit is reached then gives a good indication as to whether the new phase of strong absorption occurs or not.

The above results have an important implication for the RHIC collider. At the energy $\sqrt{s} = 200$ GeV, one expects that the $J/\psi$ survival probability, as measured by $S(b) = [B \sigma^{AB}(J/\psi) / \sigma^{AB}(DY)]/[B \sigma^{pp}(J/\psi) / \sigma^{pp}(DY)]$, will be in the lowest survival limit. However, such a limit can also be reached by the absorption of $J/\psi$ by collisions with produced hadrons alone, as one can see from Fig. 4. It will be difficult to distinguish the two cases at such a high energy. It is therefore important to perform experiments at lower energies where such a distinction can be observed.

5 Conclusions

Using the relation between $E_T$ and $b$ obtained from the Drell-Yan distribution, we have analyzed the transverse-energy dependence of $J/\psi$ production. We found that the anomalous suppression of $J/\psi$ in Pb-Pb collisions can be explained by a model in which a new phase of strong absorption sets in when the local energy density exceeds 4.2 GeV/fm$^3$. The critical energy density $\epsilon_{\text{crit}}$ which corresponds to the onset of the new phase is close to the quark-gluon plasma energy density, $\epsilon_{c} \sim 4.17$ GeV/fm$^3$, calculated from the lattice gauge theory result of $\epsilon_{c}/T^4_c \sim 20$ with $T_c \sim 0.2$ GeV. Therefore, it is interesting to speculate whether the new phase of strong absorption may be the quark-gluon plasma. However, other signatures are needed to corroborate whether the quark-gluon plasma has been produced.

Our formulation of the absorption model in terms of produced hadron densities allows us to calculate $J/\psi$ survival probabilities at higher energies. We found interesting new results that the $J/\psi$ survival probability reaches the lowest survival limit when the collision energy $\sqrt{s}$ reaches about 35 GeV. At
the energy of \( \sqrt{s} = 200 \) GeV, it will become difficult to distinguish absorption by the new phase or absorption by produced soft particles, as both effects will lead to the lowest survival limit.

Acknowledgments

This research was supported by the Division of Nuclear Physics, U.S. D.O.E. under Contract DE-AC05-96OR22464 managed by Lockheed Martin Energy Research Corp.

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