THE HUBBLE CONSTANT FROM TYPE Ia SUPERNOVAE CALIBRATED WITH THE LINEAR AND NONLINEAR CEPHEID PERIOD-LUMINOSITY RELATIONS

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ABSTRACT

It is well known that the peak brightness of Type Ia supernovae calibrated with Cepheid distances can be used to determine the Hubble constant. The Cepheid distances to the host galaxies of the calibrating supernovae are usually obtained using the period-luminosity (P-L) relation derived from Large Magellanic Cloud (LMC) Cepheids. However, recent empirical studies provide evidence that the LMC P-L relation is not linear. Here we determine the Hubble constant using both the linear and nonlinear LMC Cepheid P-L relations as calibrating relations to four galaxies that hosted Type Ia supernovae. Our results suggest that the obtained values of the Hubble constant are different. However, a typical error of ~0.03 mag has to be added (in quadrature) to the systematic error for the Hubble constant when the linear LMC P-L relation is used, assuming that the LMC P-L relation is indeed nonlinear. This is important to minimize the total error on the Hubble constant in the era of precision cosmology. The Hubble constants calibrated from the linear and nonlinear LMC P-L relations are $H_0 = 74.92 \pm 2.28$ (random) ± 5.06 (systematic) km s$^{-1}$ Mpc$^{-1}$ and $H_0 = 74.37 \pm 2.27$ (random) ± 4.92 (systematic) km s$^{-1}$ Mpc$^{-1}$, respectively. Hubble constants calculated using the Galactic P-L relation are also presented and briefly discussed.

Subject headings: Cepheids — distance scale — galaxies: distances and redshifts

1. INTRODUCTION

Because of their intrinsic brightness at maximum, it is well known that Type Ia supernovae (SNe Ia) can be used to obtain the Hubble constant ($H_0$). Furthermore, the peak brightness of SNe Ia is regarded as a standard candle after one corrects for their light-curve shape (using, e.g., $\Delta m_{15}(B)$, Phillips 1993; the $s$-factor, Perlmutter et al. 1997; MLCS, Riess et al. 1996; CMAGIC, Wang et al. 2003). Nevertheless, the peak brightnesses of the supernovae after light-curve shape (and extinction) corrections still need to be calibrated with nearby samples before they can be used to derive the Hubble constant.

A common way to calibrate the peak brightness of SNe Ia is by using the period-luminosity (P-L) relation from Cepheid variables in the galaxies that host the supernovae (see, e.g., Gibson et al. 2000; Saha et al. 2001; Riess et al. 2005; references therein). At present, the most widely used P-L relation is derived from Large Magellanic Cloud (LMC) Cepheids. The LMC P-L relation has long been regarded as linear in log $P$, where $P$ (in days) is the pulsation period of the Cepheids. However, recent empirical studies have implied that the LMC P-L relation is not linear: the LMC P-L relation can be broken into two relations, for short-period ($\log P < 1.0$) and long-period LMC Cepheids, respectively (Tammann & Reindl 2002; Kanbur & Ngeow 2004; Sandage et al. 2004; Ngeow et al. 2005; Kanbur & Ngeow 2006). Various rigorous statistical tests have been performed, and the results strongly suggest that this nonlinearity is real and not due to other factors such as extinction errors or a small number of long-period Cepheids. 1 Therefore, it is of great interest to examine how the nonlinear LMC P-L relation affects the calibration of the Hubble constant.

Currently, there are two studies that deal with this problem: Ngeow & Kanbur (2005, hereafter NK05) and Riess et al. (2005, hereafter R05). In the former study, the linearity of the LMC Wesenheit function, a linear combination of P-L and P-C relations, was examined. The Wesenheit function is frequently applied to derive Cepheid distances because it is reddening-free (see, e.g., Freedman et al. 2001; Saha et al. 2001; Kanbur et al. 2003; Leonard et al. 2003). NK05 found that the Wesenheit function for LMC Cepheids is linear because the nonlinearities of the LMC P-L and P-C relations almost cancel out. NK05 also suggested that the effect of a nonlinear LMC P-L relation in distance scale applications is minimal (at the ~0.03 mag level). However, that work did not go a step further to compare the Hubble constant calibrated from the linear and nonlinear LMC P-L relations and show that this is indeed the case. In contrast, R05 used the long-period part of the nonlinear LMC P-L relation (their “OGLE+10” P-L relation) to calibrate the peak brightness of the SNe Ia and hence derive the Hubble constant. But their study lacks a detailed comparison of the effect of linear versus nonlinear P-L relations in distance scale applications. Therefore, the main purpose of this Letter is to bridge the gap between these two studies by comparing the Hubble constants calibrated from the linear and nonlinear LMC P-L relations.

2. DATA AND ANALYSIS

Following the prescription given in R05, the Hubble constant from SNe Ia can be obtained with the following equation:

$$\log H_0 = 0.2M_V^a(t_{\max}) + 5 + a_s,$$  (1)

where $M_V^a(t_{\max})$ is the extinction-corrected absolute magnitude (in bandpass $\lambda$) at peak brightness and $a_s \equiv \log c_z - 0.2M_V^a(t_{\max})$ is the distance scale–free intercept parameter determined from “distant” supernovae that are located well within the Hubble flow (see, e.g., Jha et al. 1999; Riess et al. 2005). Here we adopt the same value of $a_s = 0.697 \pm 0.005$ as in R05. This value was determined from 38 SNe Ia in the “gold”
sample of Riess et al. (2004). Therefore, once the value of $M_V(t_{\text{max}})$ is calibrated with a Cepheid distance, the Hubble constant can be obtained in a straightforward manner.

R05 also listed four “ideal” SNe Ia for the purpose of calibrating $M_V(t_{\text{max}})$: SN 1994ae (in NGC 3370), SN 1998aq (in NGC 3982), SN 1981B (in NGC 4536), and SN 1990N (in NGC 4639). Since $m_V(t_{\text{max}}) - M_V(t_{\text{max}}) = \mu_{\text{Ceph,0}}$, where $m_V(t_{\text{max}})$ has been corrected for the extinction and light-curve shape of individual supernovae, and because we are only interested in changes in $\mu_{\text{Ceph,0}}$, this equation can be rewritten as $M_V(t_{\text{max}}) + \mu_{\text{Ceph,0}} = m_V(t_{\text{max}}) = \text{const}$ (with the constant term uniquely determined from observations of individual supernovae). Table 13 of R05 gives the values of $M_V(t_{\text{max}})$ and $\mu_{\text{Ceph,0}}$ for these four SNe Ia; hence, the change of $M_V(t_{\text{max}})$ due to the recalculation of $\mu_{\text{Ceph,0}}$ is just $M_V(\text{new}) = M_V(\text{R05}) + \mu_{\text{Ceph,0}(\text{R05})} - \mu_{\text{Ceph,0}(\text{New})}$.

To obtain $\mu_{\text{Ceph,0}(\text{new})}$, we use four sets of LMC P-L relations, as given in NK05.\(^2\) Each set of P-L relations contains both the linear and nonlinear (i.e., the long-period P-L relation) versions of the LMC P-L relation. The Cepheid data for these four galaxies are adopted from the following sources: NGC 3370 from R05, NGC 3982 from Stetson & Gibson (2001), and NGC 4536 and NGC 4639 from Gibson et al. (2000). As in Freedman et al. (2001), we apply a period cut to Cepheids in NGC 3982, NGC 4536, and NGC 4639 to avoid the incompleteness bias at the faint end of the Cepheid P-L relation (there is no need to do this for NGC 3370; see R05). After fitting the P-L relations to the data, we obtain the distance modulus through the Wesenheit function (see, e.g., references in NK05), $\mu = \mu_V - 2.45(\mu_I - \mu_V)$. Metallicity corrections to $\mu_V$ were applied in the same manner as in R05 (i.e., using the values in their Table 8). Finally, a CTE (charge transfer efficiency) correction of $-0.07$ mag is applied to NGC 4536 and NGC 4639 (Gibson et al. 2000; Freedman et al. 2001). No CTE correction is needed for NGC 3370 and NGC 3982. Our results for $\mu_{\text{Ceph,0}(\text{new})}$, $M_V(\text{new})$, log $H_0$ from equation (1), and $H_0$ are summarized in Table 1.

The random errors that contribute to the Hubble constant include the random error in the Cepheid distance modulus and the error from the supernova light-curve fit. Since the random errors in distance moduli, as given in Table 1, are almost identical when using either the linear or the nonlinear P-L relation (among the four sets of P-L relations), we adopt a single value for the error for each galaxy. The random errors from supernova light-curve fits are given in R05 with values of 0.12 mag for each calibrator. The adopted values of the random errors for the four galaxies/calibrators are listed in Table 2. The systematic errors are discussed below and are summarized in Table 2 as well. These systematic errors are as follows:

1. **Distance to the LMC.**—The P-L relations given in NK05 are based on an LMC distance of $18.50 \pm 0.10$ mag, as adopted by Freedman et al. (2001); hence, we continue to adopt the conservative value of 0.10 mag as the uncertainty in the LMC distance.

2. **Linear versus nonlinear P-L relation.**—Since there is growing evidence that the LMC P-L relation is nonlinear, we assume that the nonlinear LMC P-L relation is the true underlying P-L relation. There is an additional $\pm 0.03$ mag systematic error for the derived distance modulus when using the linear version of the LMC P-L relation (NK05).\(^3\) This error is not applicable to the distance modulus (or the Hubble constant) when using the nonlinear P-L relation.

3. **Hubble flow.**—R05 determined the error in the Hubble

$^2$ Except for the “KNB05” relations, which we have updated in Kanbur & Ngeow (2006) and refer to as KN06 in this Letter.

$^3$ This can be seen from, e.g., Table 1. For a given set of P-L relations, the distance moduli from the linear P-L relation are systematically closer or farther than the distance moduli, by $-0.01$ to $-0.03$ mag, from the nonlinear P-L relation among the four calibrators.
flow from their “gold” sample to be about 0.025 mag; we adopt this value as well.

4. **HST zero points.**—The zero-point uncertainties in the V and I bands for the Hubble Space Telescope cameras (both ACS and WFPC2) are 0.03 mag, and the total uncertainty in Cepheid distance measurements is \((1.45 \sigma_r)^2 + (2.45 \sigma_i)^2)^{1/2} = 0.086\) mag for all four galaxies.

5. **Metallicity correction.**—Uncertainties in the metallicity corrections are adopted from Table 8 of R05. Note that this uncertainty is considered to be a systematic error (Freedman et al. 2001; Leonard et al. 2003) and not a random error.

The random errors for the four calibrators are used as the weights when calculating the weighted mean for the Hubble constant. This procedure also combines the random errors for individual calibrators into the overall random error on \(H_0\). The overall systematic error is taken as a straight average of the systematic errors from the four calibrators. The results are shown in Table 3 for use of the four sets of linear and nonlinear P-L relations.

### Table 3: The Hubble Constant from Various Sets of LMC P-L Relations

| P-L Set | Linear LMC P-L (km s\(^{-1}\) Mpc\(^{-1}\)) | Nonlinear LMC P-L (km s\(^{-1}\) Mpc\(^{-1}\)) | Variation (%) |
|---------|---------------------------------------------|----------------------------------------------|--------------|
| TR02    | 74.96 ± 2.28 ± 5.07                        | 74.02 ± 2.26 ± 4.90                         | 1.3          |
| STR04   | 75.75 ± 3.51                               | 76.47 ± 2.33                               | 0.9          |
| KN06    | 75.13 ± 2.93                               | 75.44 ± 2.30                               | 0.4          |
| KN06    | 74.92 ± 2.28 ± 5.06                        | 74.37 ± 2.27 ± 4.92                        | 0.7          |

Note.—The first and second errors listed are random and systematic, respectively.

### Table 2: Error Budget for the Hubble Constant

| Source            | NGC 3370/SN 1994ae | NGC 3982/SN 1998aq | NGC 4536/SN 1981B | NGC 4639/SN 1990N |
|-------------------|--------------------|--------------------|-------------------|--------------------|
| **Random Error**  |                    |                    |                   |                    |
| R1: Cepheid distance | 0.034              | 0.065              | 0.044             | 0.077             |
| R2: SN light-curve fit | 0.12               | 0.12               | 0.12              | 0.12              |
| Total random      | 0.125              | 0.136              | 0.128             | 0.143             |
| **Systematic Error** |                    |                    |                   |                    |
| S1: LMC distance | 0.10               | 0.10               | 0.10              | 0.10              |
| S2: Linear vs. nonlinear P-L | 0.03               | 0.03               | 0.03              | 0.03              |
| S3: Metallicity correction | 0.03               | 0.03               | 0.05              | 0.08              |
| S4: Hubble flow (=5\(a_j\)) | 0.025            | 0.025             | 0.025             | 0.025             |
| S5: HST camera zero point | 0.086            | 0.086             | 0.086             | 0.086             |
| Total systematic, linear P-L | 0.141             | 0.141             | 0.146             | 0.159             |
| Total systematic, nonlinear P-L | 0.138            | 0.138             | 0.143             | 0.156             |

* Only applicable when using the linear LMC P-L relation.

The nonlinear LMC P-L relations are consistent with the Hubble constant obtained from the two methods is \(\sim 0.03\) mag or \(\sim 1\%–2\%\) level, on distance scale studies for other (within the total error). The difference in the value of the Hubble constant from observations (to less than the few-percent level) to break this degeneracy. In this Letter, we concentrate on studying the contribution to the error only from the form of the P-L relation (linear vs. nonlinear) and find that this could introduce an additional error at the \(\sim 1\%–2\%\) level to the total. Table 3 suggests that although the Hubble constants and the associated random errors are similar when using the linear versus the nonlinear P-L relation, the systematic errors are larger when using the linear P-L relation if the LMC P-L relation is indeed nonlinear. Hence it is important to eliminate this additional error to improve the measurement of the Hubble constant to within the few-percent level. However, there are other systematic errors that contribute to the total error, such as the uncertainty of the LMC’s distance (0.10 mag), which remains one of the largest systematic errors in estimating the Hubble constant. Further refinement of all these systematic errors is clearly desirable.

NK05 described a way to derive Cepheid distances with nonlinear P-L relations when the target galaxy consists of both short- and long-period Cepheids. This can be done by using

\[ \mu = N_{short}^{-1} \sum \mu_{short} + N_{long}^{-1} \sum \mu_{long}, \]

where \(\mu_{short}\) and \(\mu_{long}\) are the distance moduli for short- and long-period Cepheids, respectively. Most of the HST-observed galaxies only contain long-
period Cepheids, and hence the long-period part of the non-linear P-L relation can be used. In addition to distance scale studies, the existence of a nonlinear P-L relation in the LMC is very important to investigate the underlying physics behind nonlinear LMC P-L relations (see, e.g., Kanbur & Ngeow 2004, 2006; Ngeow et al. 2005).

We emphasize that the P-L relations used in this Letter are the (linear and nonlinear) LMC P-L relations only. There are recent studies suggesting that Galactic Cepheids follow a different P-L relation than do LMC Cepheids (Tammann et al. 2003; Ngeow & Kanbur 2004; Sandage et al. 2004), presumably as a consequence of metallicity effects (but see, e.g., Gieren et al. 2005 for the opposite point of view). Kanbur et al. (2003) compared the Cepheid distances to 25 HST-observed galaxies using both the (linear) LMC and Galactic P-L relations and found that there is, on average, a negligible (~0.001 mag) difference in the distance moduli when appropriate metallicity corrections are applied to the distance moduli from both the LMC and Galactic P-L relations. A similar result was found recently by Saha et al. (2006). This may suggest that the use of the Galactic versus the LMC P-L relation could have a minimal effect on the value of the Hubble constant if the metallicity correction is applied (again, the contribution of using correct vs. incorrect P-L relations to the systematic error may be more important in reducing the total error on the Hubble constant). To see the effect of using the Galactic P-L relation, we have applied the same data and methodology as in § 2, except that we replace the LMC P-L relations with the Galactic P-L relations. We adopt the recent Galactic P-L relation from Ngeow & Kanbur (2004) and the updated version from Sandage et al. 2004; Saha et al. 2006), which includes various uncertainties from the distance measurements to individual Galactic Cepheids (e.g., the open cluster fitting method, the infrared surface brightness method, and parallax measurements). The resulting Hubble constants are $H_0 = 70.91 \pm 2.16$ (random) $\pm 4.69$ (systematic) km s$^{-1}$ Mpc$^{-1}$ and $H_0 = 69.60 \pm 2.12$ (random) $\pm 4.60$ (systematic) km s$^{-1}$ Mpc$^{-1}$ with the Ngeow & Kanbur (2004) Galactic P-L relations and Sandage et al. (2004) Galactic P-L relations, respectively. The values of the Hubble constant from the Galactic P-L relation are lower than those obtained from the LMC P-L relations; however, they are still consistent with each other within 1 $\sigma$ of the total errors.

The apparent discrepancy between our results and the results presented in Kanbur et al. (2003) is due to the number of galaxies (25 vs. 4) available in each study. Kanbur et al. (2003) found that if the average log $P$ from the Cepheids in a given galaxy is close to $-1.4$, then the difference in the distance moduli from the LMC and the Galactic P-L relations will be negligible after the metallicity correction (see eqs. [5] and [6] of Kanbur et al. 2003). This is also true for an ensemble of galaxies. The average log $P$ for all 25 galaxies in Kanbur et al. (2003) is indeed $-1.4$ (see their Table 14); therefore, an average $\approx 0.001$ mag difference in Cepheid distances was obtained there. For each of the four calibrating galaxies in this Letter, the average log $P$ is greater than 1.46, which increases the Cepheid distances obtained from the Galactic P-L relation. Hence, the Hubble constant obtained with Galactic P-L relations will be lower than that obtained with LMC P-L relations.

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## TABLE 4

### RESULTS OF USING THE GALACTIC P-L RELATION

| Galaxy/SN | $N_{gal}$ | $\mu_{gal}$ | $M'_{(t_{cand})}$ | $log H_0$ | $H_0$ | $\mu_{gal,0}$ | $M'_{(t_{cand,0})}$ | $log H_0$ | $H_0$ |
|-----------|--------|-----------|-------------|---------|------|-----------|-------------|---------|------|
| NGC 3370/SN 1994a | 64 | 32.324 ± 0.035 | -19.214 | 1.854 | 71.4 | 32.363 ± 0.035 | -19.253 | 1.846 | 70.1 |
| NGC 3826/SN 1980a | 29 | 31.728 ± 0.069 | -19.258 | 1.845 | 70.0 | 31.768 ± 0.069 | -19.298 | 1.837 | 68.7 |
| NGC 4536/SN 1981B | 35 | 30.902 ± 0.039 | -19.252 | 1.847 | 70.3 | 30.943 ± 0.039 | -19.293 | 1.838 | 68.9 |
| NGC 4639/SN 1990N | 14 | 31.839 ± 0.075 | -19.199 | 1.857 | 71.9 | 31.877 ± 0.075 | -19.237 | 1.850 | 70.8 |

Note.—Errors in $\mu_0$ are random errors only. $H_0$ is in units of km s$^{-1}$ Mpc$^{-1}$.