A rapid performance evaluation approach for lower mobility hybrid robot based on gravity-center position

Chensheng Wang¹,², Fang Su², Yanqin Zhao¹, Hongda Liu¹, Yonghao Guo¹ and Wentie Niu¹,³

¹Key Laboratory of Modern Mechanisms and Equipment Design of the State Ministry of Education, Tianjin University, Tianjin, China
²School of Mechanical Engineering, Shanxi Datong University, Datong, China
³School of Mechanical Engineering, Tianjin University, Tianjin, China

Abstract
This article develops a rapid performance evaluation approach for lower mobility hybrid robot, which provides guidance for manipulator evaluation, design, and optimization. First, a general position vector model of gravity center for the lower mobility hybrid robot in the whole workspace is constructed based on a general inverse kinematic model. A performance evaluation index based on gravity-center position is then proposed, where the coordinates pointing to the supporting direction are selected as the evaluation index of the robot performance. Furthermore, the credibility of the evaluation approach is verified from a 5-DOF hybrid robot (TriMule) by comparing with the condition number and the first natural frequency. Analysis results demonstrate that the evaluation index can not only reflect the performance spatial distribution in the whole workspace but also is sensitive to the performance difference caused by mass distribution. The proposed performance evaluation approach provides a new index for the rapid design and optimization of the cantilever robot.

Keywords
Evaluation approach, gravity-center position, lower mobility, hybrid robot, evaluation index

Corresponding author:
Wentie Niu, School of Mechanical Engineering, Tianjin University, No. 135, Yagyuan Road, Jinnan District, Tianjin 300350, China.
Email: niuwentie@tju.edu.cn

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Introduction

Hybrid robot has advantages of compact structure, high rigidity, and large bearing capacity, which has a broad application prospect in aerospace manufacturing, and in high-precision and high-intensity processing.\textsuperscript{1–3} Performance evaluation is one of the key issues in robot design and application. It is of great significance to select the evaluation approaches and index, which could precisely constrain design variables in the design process and embody the real performance of robot in the application.\textsuperscript{4,5}

At present, there are many evaluation approaches and indices for robot performance, including static structure symmetry, terminal stiffness, bearing capacity, force transfer, geometric accuracy, singularity, redundancy, and workspace, or dynamic precision, speed, dynamic stiffness, and mode shape.\textsuperscript{6–9} Among them, the most typical evaluation indices of the local performance are the kinematic evaluation index based on the condition number ($\kappa$) of Jacobian matrix\textsuperscript{10} and the dynamic stiffness evaluation index based on the low order natural frequency,\textsuperscript{11} both of which are closely related to the specific pose of the robot. Salisbury and Craig\textsuperscript{10} used the condition number as the evaluation index for the local performance of the robot. The smaller the condition number, the better the performance of the whole machine in this pose. Based on the Jacobian matrix, the problem of different physical quantities with different dimensions should be solved first, and then it should be normalized. Jacobian matrix depends on the pose of the robot, which is a local performance index. In order to evaluate the dexterity of robots in general, Gosselin and Angeles\textsuperscript{12} proposed Global Conditioning Index. Huang et al.\textsuperscript{13–15} applied this performance evaluation index to robot design, optimization, and evaluation. In the process of studying the performance of the robot, Dong et al.\textsuperscript{16,17} and Wu et al.\textsuperscript{18} assessed and evaluated its local performance with the help of the first natural frequency ($f_1$) of the robot. This approach has been widely used in the performance research and optimization design of various mechanisms. But acquisition of the first natural frequency of the mechanism mostly depends on the whole machine’s numerical analytical dynamics model or finite element model. The accuracy of the model determines the reliability of the index. At present, most of the high-precision analytical models or finite element models have some errors, which limit the exploration of the real performance of the mechanism. The experimental method is the most accurate means to obtain the first natural frequency of the mechanism. However, as the complexity of the mechanism increases, the complexity of the experiment also increases, making it difficult to quickly evaluate and study the mechanism.

The position of gravity center has a significant impact on the performance of the robot in the whole workspace,\textsuperscript{19–21} and gravity compensation has been a very important issue.\textsuperscript{22–24} Wu et al.\textsuperscript{25} studied the dynamic characteristics of the mechanism in consideration of gravity. Ma et al.,\textsuperscript{26} in order to improve the performance of the robot, designed a barycenter balance mechanism to solve the influence caused by the barycenter in the process of work. As known to all, the gravity center of the mechanism is closely related to the pose of the robot. Moreover, the acquisition of
the gravity center is relatively simple, which only relies on the kinematics model and the fine 3D model. The gravity center has a high sensitivity to the difference of positions and orientations between the different poses. However, there is no systematic research on the relationship between the gravity-center position and the performance of the manipulator, even though the gravity-center position is introduced into the robot design and optimization.

In this article, a performance evaluation approach of the lower mobility hybrid robot based on the position of gravity center is presented, aiming to put forward a simple index which could be used for the rapid evaluation, design, and optimization of the robot. A general inverse kinematic model of the lower mobility hybrid robot is constructed, following a general position vector model of gravity center. Furthermore, the coordinates pointing to the supporting direction are selected as the performance evaluation index. The credibility of the evaluation approach is then verified from the 5-DOF hybrid robot (TriMule).

The remainder of the article is organized as follows. Section “Evaluation method based on gravity-center position” describes a general kinematics model and gravity-center position solution for the lower mobility hybrid robot. Thereafter, the evaluation index based on gravity-center position is established. In section “Performance evaluation of 5-DOF hybrid robot,” the inverse kinematics solution model and the gravity-center vector model of the 5-DOF robot TriMule are deduced. Furthermore, the validity and reliability of the evaluation approach are verified by respectively comparing the gravity-center position with the condition number and the first natural frequency. Section “Application” discusses the selection of evaluation index based on the position of gravity center. Conclusions are made in last section.

**Evaluation method based on gravity-center position**

**Kinematic analysis**

The general model of a lower mobility hybrid robot is shown in Figure 1, which includes a fixed base, a moving platform, \(i\) limbs, and an end effector. One end of the limb is connected to the fixed base, and the other end is connected to the moving platform. It is customary to define the global coordinate frame \(B-xyz\) rigidly with the fixed base, and the moving coordinate frame \(A-uvw\) is fixed with the moving platform, and the cutter tool coordinate frame \(P'-x'_c y'_c z'_c\) is defined at the principal axis, respectively.

Then, the rotational transformation matrix of \(A-uvw\) with respect to \(B-xyz\) can be represented by \(R_A\)

\[ R_A : \text{Trans}(A - uvw \rightarrow B - xyz) \] (1)

The rotational transformation matrix of \(P' - x'_c y'_c z'_c\) relative to \(A-xyz\) can be expressed by \(R_C\)

\[ R_C : \text{Trans}(P' - x'_c y'_c z'_c \rightarrow A - uvw) \] (2)
The length and the direction vector of each limb or the rotational angle of end effector can be obtained through the inverse kinematic solution for the specific tool center point C.

**Performance index based on gravity-center position**

Based on the inverse kinematics model and the 3D model of the lower mobility hybrid robot, the parts are grouped according to the following principles:

1. The whole machine is grouped with joints as nodes;
2. When grouping, it is insisted that the relative position of the parts in the group does not change during the working;
3. The position vector of each part in $B$-xyz, $A$-uvw, or $P'$-x$_c$y$_c$z$_c$ is easily obtained as a priority, because the direction vector of each limb and joint need to be deduced in the inverse kinematic model.

Assuming that the manipulator is divided into $i$ components, the vector $i$th($x, y, z$) in the specific coordinate frame and the mass can be checked in the 3D model according to the grouping. The corresponding transformation matrix $R_i$ can be derived from the kinematic model. Then the vector of each gravity center in $B$-xyz can be expressed as

$$G_i = R_i j(x, y, z)$$ (3)

Furthermore the gravity-center vector of the manipulator could be deduced as

$$G = \frac{\sum_{i=1}^{n} m_i G_i}{\sum_{i=1}^{n} m_i}$$ (4)
According to the cantilever direction of the mechanism, the evaluation index $R_G$ is based on the position of the gravity center as

$$R_G = |G_{x/y/z}|$$  \( \text{(5)} \)

$R_G \geq 0$, and the larger the value, the greater the distance between the gravity center and the supporting surface, that is the worse the performance in the corresponding position.

Compared with other performance evaluation indices, such as $\kappa$ and $f_1$, $R_G$ is more intuitive, easier to obtain, and more operable. It is a convenient and rapid performance evaluation approach.

**Performance evaluation of 5-DOF hybrid robot**

The 5-DOF hybrid robot TriMule\textsuperscript{27} is shown in Figure 2, which is composed of a spatial parallel module (3UPS&UP) and an A/C type wrist (2R) module attached to the moving platform. The 3UPS&UP consists of four limbs, wherein three active universal-prismatic-spherical (UPS) limbs have the same structure and are equally distributed within 360 degrees in space. One end of the UPS limb is connected to the rotational frame through universal joint, and the other end is connected to the moving platform through spherical joint. The passive universal-prismatic (UP) limb is arranged in the middle of limb 1 and limb 2, in which one end is connected to the fixed base by U joint, and the other section is consolidated with the moving platform. The 2R module is connected to the moving platform via a trust bearing, which has two rotational DOF, rotating along A-axis and C-axis, respectively.

According to the grouping principle mentioned in section 2, TriMule is grouped as UPS outer tubes assembly (OTASM), UPS limbs inner tubes assembly (INASM), UP assembly (UPASM), C-axis assembly (CASM), and A-axis assembly (SPASM).

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{3-D_model_of_TriMule.png}
  \caption{3-D model of TriMule.}
  \end{figure}
Kinematics analysis

Inverse kinematic model of the 3UPS&UP. The schematic of TriMule is shown in Figure 3. Herein, \(B_i\) is supporting point, and \(A_i\) is the junction between moving platform and the \(i\)th limb. \(A\) and \(B\) are the junctions of the UP limb with the moving platform and the rotational frame, respectively. Point \(P\) is the junction of C-axis rotational shaft and \(A\)-axis rotational shaft. \(P'\) is the intersection of spindle rotational shaft and C-axis rotational shaft. Point C is the tool center point.

The translational matrix \(R_A\) between the moving platform body and fixed coordinate frame \((A-uvw)\) relative to the global coordinate frame \((B-xyz)\) can be obtained through first rotating an angle \(\psi\) about the \(x\)-axis, then rotating an angle \(\theta\) about \(v\)-axis, which can be expressed as

\[
R_A = \text{Rot}(x, \psi) \text{Rot}(v, \theta) = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
\sin \psi \sin \theta & \cos \psi & -\sin \psi \cos \theta \\
-\cos \psi \sin \theta & \sin \psi & \cos \psi \cos \theta \\
\end{bmatrix}
\]

(6)

The translational matrix \(R_C\) that \(P'\)-\(x_c y_c z_c\) relative to \(A\)-\(xyz\) can be obtained through first rotating an angle \(\delta\) about the \(w\)-axis, then rotating an angle \(\beta\) about \(u\)-axis, which can be represented as

\[
R_C = \text{Rot}(w, \delta) \text{Rot}(u, \beta) = \begin{bmatrix}
\cos \delta & -\sin \delta \cos \beta & \sin \delta \sin \beta \\
\sin \delta & \cos \delta \cos \beta & -\cos \delta \sin \beta \\
0 & \sin \beta & \cos \beta \\
\end{bmatrix}
\]

(7)

If point A has been given, the inverse solution analysis of the 3-UPS&UP is equivalent to calculating the length and vector of each limb.
The position vector of point A in $B$-xyz can be calculated as

$$r_A = b_i + q_i w_i - a_i, \quad (i = 1, 2, 3)$$  \hspace{1cm} (8)$$

$$r_A = q_4 w_4$$  \hspace{1cm} (9)

where, $q_i$ represents the length of the $i$th limb, $w_i$ represents the unit vector of the $i$th limb in $B$-xyz, $q_4$ represents the length of the UP limb, $w_4$ represents the unit vector of the UP limb in $B$-xyz, $a_i$ represents the position vector of $A_i$ in $A$-uvw, $b_i$ represents the position vector of $B_i$ in $B$-xyz.

The position vector of $A_i$ in $A$-uvw can be calculated as

$$a_i = R a_{io}$$  \hspace{1cm} (10)$$

$$a_{io} = a_i(\cos \gamma_i, \sin \gamma_i, 0)^{T}, \quad \gamma_i = \begin{cases} -\pi/2, & i = 1 \\ 0, & i = 2 \\ \pi, & i = 3 \end{cases}$$  \hspace{1cm} (11)

Where, $a_i = \|AA_i\|$, $\gamma_i$ is the angle between $AA_2$ and the $x$-axis.

The position vector of $B_i$ in $B$-xyz can be expressed as

$$b_i = b_i(\cos \beta_i, \sin \beta_i, 0)^{T}, \quad \gamma_i = \begin{cases} -\pi/2, & i = 1 \\ 0, & i = 2 \\ \pi, & i = 3 \end{cases}$$  \hspace{1cm} (12)

Where, $b_i = \|BB_i\|$, $\gamma_i$ is the angle between $AA_2$ and $x$-axis.

When $\psi = \theta = 0$

$$w_4 = R_A w = \begin{bmatrix} \sin \theta \\ -\sin \psi \cos \theta \\ \cos \psi \cos \theta \end{bmatrix}$$  \hspace{1cm} (13)$$

$$w_4 = \frac{r_A}{q_4} = \frac{(x, y, z)^{T}}{\sqrt{x^2 + y^2 + z^2}} = \begin{bmatrix} S_{Ax} \\ S_{Ay} \\ S_{Az} \end{bmatrix}^{T}$$  \hspace{1cm} (14)$$

Combining equations (13) and (14) yields

$$\theta = \arcsin(S_{Ax}), \psi = \arctan\left(-\frac{S_{Ay}}{S_{Az}}\right)$$  \hspace{1cm} (15)$$

There is the transformation matrix $R$ between $A$-uvw and $B$-xyz, and then substituting equation (15) into equations (8) and (9) leads to

$$q_i = |a_i - b_i + r_A|, \quad w_i = \frac{(a_i - b_i + r_A)}{q_i}$$  \hspace{1cm} (16)

**Inverse kinematic solution of the 2R model.** The position vector $r_c$ of the tool center point $C$ in $B$-xyz can be expressed as
\[ r_C = l_C z_c + l_B y_c + r_p \]  \hspace{1cm} (17)

\[ r_p = (q_4 + l_A) w_4 \]  \hspace{1cm} (18)

where \( l_A = \|AP\|, l_B = \|PP'\|, l_C = \|P'C\|, y_c \) represent the tool-axis vector, which can be derived through rotating \( \delta \) about \( w \)-axis first, and then rotating \( \beta \) about \( u \)-axis

\[ y_c = \text{Rot}(y, \beta) \text{Rot}(w, \delta) \]

One thing to point out is that tool-axis does not intersect with the \( A \)-axis. Therefore, in order to determine \( r_p \) from equation (17), it is necessary to determine \( z_c \) first. For a given \( r_c \), construct the unit vector as

\[ n = \frac{r_c}{|r_c|} \]  \hspace{1cm} (20)

There \( u \) is perpendicular to the plane spanned by \( w \) and \( n \), and when \( |w \times n| \neq 0 \), construct the unit vector as

\[ u' = \pm \frac{w \times n}{|w \times n|}, \quad n = w \times u' \]  \hspace{1cm} (21)

When \( |w \times n| = 0 \), \( w \) is collinear with \( n \), \( u' \) has infinitely many solutions. Therefore, the robot is in a singular configuration. When \( |w \times n| \neq 0 \), \( u' \) has two solutions, the \( u' \) where \( |r_p| \) is a smaller value should be adopted to obtain good kinematic performance and avoid mechanical interference.

So \( r_A \) can be expressed as

\[ r_A = r_c - l_C z_c - l_B y_c - l_A w \]  \hspace{1cm} (22)

Hence, taking norm on both sides of equation (18) leads to

\[ q_4 = |r_p| - l_A, \quad w_4 = \frac{r_p}{q_4 + l_A} \]  \hspace{1cm} (23)

Substituting equation (23) into equations (14)–(16), the rotational angle \( \theta, \psi \), the length \( q_i \), and the unit vector \( w_i \) of the \( i \)th UPS can be obtained, respectively.

From \( y_c = R_C R_A y, z_c = R_C R_A z \), there is

\[ R_C^T y_c = \begin{bmatrix} -\sin \delta \sin \beta \\ \cos \delta \sin \beta \\ \cos \beta \end{bmatrix}, \quad R_C^T z_c = \begin{bmatrix} \sin \delta \cos \beta \\ -\cos \delta \cos \beta \\ \sin \beta \end{bmatrix} \]

According to equation (24), two rotational angles can be calculated

\[ \beta = \arctan \left( \frac{s_9}{s_6} \right) \]  \hspace{1cm} (25)
\[
delta = \begin{cases} \arctan\left(-\frac{s_4}{s_5}\right), & \beta \neq 0, \pi \\ \arctan\left(-\frac{s_7}{s_8}\right), & \beta = 0, \pi \end{cases}
\]  

(26)

Take an attention, when \( \beta = \pi \), the robot is in the interference position
\[
\delta = \arctan\left(-\frac{s_4}{s_5}\right), \beta \neq \pi
\]

(27)

**Gravity-center position solution**

Relative to the position vector of the tool center point \( C \), the direction vector of the gravity center of the \( i \)th limb is the same as the vector \( w_i \) of the \( i \)th limb in the \( B-xyz \). The gravity-center position of the limbs 1–3 OTASM in the \( B-xyz \) can be expressed as

\[
G_{wi} = R_i q_w - b_i \quad (i = 1, 2, 3)
\]

(28)

where \( q_w \) represents the gravity-center vector of OTASM in \( U \) frame.

The gravity-center vector of INASM in the \( B-xyz \) can be expressed as

\[
G_{ni} = R_i (q_n + q_i w_i) + r_A \quad (i = 1, 2, 3)
\]

(29)

where \( q_n \) represents the gravity-center vector of INASM in \( S \) frame.

The gravity-center vector of UPASM in the \( B-xyz \) can be expressed as

\[
G_4 = R_A q_4 + r_A
\]

(30)

where \( q_4 \) represents the gravity-center vector of UPASM in \( A-uvw \).

The mess of each assembly is expressed as \( m_{wi} \) (\( i = 1, 2, 3 \)), \( m_{ni} \) (\( i = 1, 2, 3 \)), \( m_4 \), respectively. The gravity-center vector \( G_b \) of the 3UPS&UP is derived as

\[
G_b = \sum_{i=1}^{3} m_{wi} G_{wi} + \sum_{i=1}^{3} m_{ni} G_{ni} + m_4 G_4
\]

\[
\begin{bmatrix}
G_{bx} \\
G_{by} \\
G_{bz}
\end{bmatrix}
\]

(31)

The whole mechanism is cantilever supported in the \( B-xyz \). Therefore, the \( z \)-axis coordinate \( G_b \) is selected as the performance evaluation index of the 3UPS&UP

\[
R_G = |G_{bz}|
\]

(32)

The gravity-center position vector of the CASM in the \( B-xyz \) can be represented by

\[
G_A = R_A q_c + r_A
\]

(33)

Where, \( q_c \) represents the gravity-center vector of CASM in \( A-uvw \).

The gravity-center position vector of the SPASM in the \( B-xyz \) can be expressed as
\[
G_C = R_A (R_C (q_A + PP') + AP) + r_A
\]  
(34)

where \( q_A \) represents the gravity-center vector of SPASM in \( P' -x'y'z' \).

The messe of CASM and SPASM are \( m_A, m_C \), respectively. The gravity-center vector \( G_h \) of TriMule in the \( B-x'y'z' \) is derived as

\[
G_h = \frac{\sum_{i=1}^{3} m_{wi} G_{wi} + \sum_{i=1}^{3} m_{ni} G_{ni} + m_A G_A + m_C G_C}{\sum_{i=1}^{3} m_{wi} + \sum_{i=1}^{3} m_{ni} + m_A + m_C} = \begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix}
\]  
(35)

The whole mechanism is cantilever supported in the \( B-x'y'z' \). Therefore, the \( z \)-axis coordinate value \( G_h \) is selected as the performance evaluation index of the mechanism

\[
G_R = |G_{hz}|
\]  
(36)

Assembly parameters of TriMule are acquired by 3D model, as shown in Table 1. There are four things to add.

1. The length of UPS limb inner tubes assembly is \( 940 \times 10^{-3} \) mm, in which gravity center away from S joint is \( 415.2 \times 10^{-3} \) mm.
2. The distance between the spindle axis and the point A in moving platform is \( 342 \times 10^{-3} \) mm.
3. \( P' P = 120 \times 10^{-3} \) mm, the vector of point P in the \( A-uvw \) is \( \{0, 0, 342 \times 10^{-3} \} \) mm, the vector of point C in \( P' -x'y'z' \) is \( \{0, -320 \times 10^{-3} \text{ mm}, 0\} \);
4. The workspace of TriMule600: \( z = [600 \text{ mm}:1200 \text{ mm}], x^2 + y^2 \leq 600^2 \) mm.

**Comparison with the kinematic index**

The conditional number (\( \kappa \)) is an important evaluation approach for kinematics performance of manipulator. Literature of the previous works analyzed kinematics performance of the 3UPS&UP of TriMule based on \( \kappa \). In order to verify the validity of the gravity-center evaluation approach, two kinds of evaluation results of the 3UPS&UP were compared and analyzed in the same workspace. In addition, since many studies on TriMule have been carried out in published literature, its performance has a high consistency on the distribution rule in the whole workspaces. Herein, the gravity-center position index \( R_G \) is calculated within the plane of \( z = 800 \text{ mm} \) plane, wherein \( x^2 + y^2 \leq 600^2 \) mm.

Figure 4(a) is the spatial distribution of \( \kappa \), and Figure 4(b) is the spatial distribution of \( R_G \). It can be seen from Figure 4 that \( \kappa \) and \( R_G \) have the same distribution trend within the plane of \( z = 800 \text{ mm} \).

Figure 5 shows the contour distribution of \( \kappa \) and \( R_G \) within the plane of \( z = 800 \text{ mm} \). As show in Figure 5(a), \( \kappa \) is symmetric about the \( x \)-axis and asymmetric about the \( y \)-axis. The minimum point of \( \kappa \) is \( (0, 0.04) \). It can be seen from Figure 5(b) that
R_G is symmetric about the x-axis and asymmetric about the y-axis. The center of contour line distributions of R_G is lower than the y-axis, and the minimum point of R_G value is (0, 0.04). Furthermore, it is very clear that the two evaluation methods are highly consistent with the evaluation results of the 3UPS&UP.
In order to further investigate the difference between the two evaluation approaches, the contour envelope area corresponding to the two evaluation approaches can be obtained through color filling, identification, and calculation. Figure 6(a) is the stacked diagram of the two evaluation indices, and Figure 6(b) is the proportion of contour envelope area corresponding to the two evaluation indices.

As shown in Figure 6(a), the contour line shapes of the two evaluation indicators are not the same, but the core areas covered by them are consistent and have good correspondence. That is to say, the optimal workspace based on the two indicators is same. In particular, Figure 6(b) shows that the two contour lines envelope areas that are almost equal, as the 0.05 of $R_G$ corresponds to the 2.2 of $\kappa$, and the
0.055 of $R_G$ corresponds to the 2.3 of $\kappa$, and so on, which finely demonstrates the consistency of the evaluation results of the two evaluation methods. Therefore, $R_G$ is effective for the kinematics performance evaluation of the mechanism.

**Comparison with the dynamic index**

To further verify the validity of $R_G$, as mentioned earlier, $z = 800$ mm plane was selected for comparative analysis of $R_G$ and the first-order natural frequency ($f_1$).

Figure 7 compares $f_1$ from the previous works literature and $R_G$ for the 3UPS&UP, in the workspace of $x^2 + y^2 \leq 300^2$ mm. It can be seen that the spatial distribution of $f_1$ and $R_G$ have the same distribution trend within the plane of $z = 800$ mm. Figure 7(a) is the spatial distribution of $f_1$, in which the direction of...
z-axis is reverse for easy comparison, and Figure 7(b) is the spatial distribution of $R_G$, respectively.

Contour line distributions of $f_1$ and $R_G$ within the plane of $z = 800$ mm as shown in Figure 8, where the shape of the two contour lines is ellipse. As show in Figure 8(a), $f_1$ is symmetric about the $x$-axis and asymmetric about the $y$-axis. The minimum point of $f_1$ is (0, 0.04), and the center of contour line distributions is lower than the $y$-axis. As shown in Figure 8(b), $R_G$ is symmetric about the $x$-axis and asymmetric about the $y$-axis. The center of contour line distributions of $R_G$ is lower than the $y$-axis, and the minimum point of $R_G$ value is (0, 0.04). Therefore, it is very clear that the two evaluation methods are highly consistent with the evaluation results of the 3U&PS&UP.

In order to further investigate the difference between the two evaluation approaches, the contour line envelope areas corresponding to the two evaluation approaches can be obtained through color filling and identification. Figure 9(a) is the stacked diagram of the two evaluation indices, and Figure 9(b) is the proportion of contour line envelope areas corresponding to the two evaluation indices.

Figure 9(a) shows the core areas covered by the contour line shapes of the two evaluation indicators are consistent. The results show the two evaluation indicators have a good correspondence. The difference is that the contour lines of $f_1$ have a certain displacement along the $y$-axis compared with the contour lines of $R_G$.

As shown in Figure 9(b), although there is a certain difference in 0.045 and 32, the overall contour line envelope areas are very consistent, which finely demonstrates the consistency of the evaluation results based on the two evaluation methods. All aforementioned proofs indicate the effectiveness of $R_G$ on the performance evaluation for manipulator. From the foregoing analysis, $\kappa$, $f_1$, and $R_G$ are consistent in the performance evaluation of cantilever robot. However, $R_G$ is related to the dimensional parameters, and the quantitative expression of the relationship is related to the specific topology structure of the robot.
Analyses in the previous sections demonstrate enough credibility on $R_G$ for cantilever robot. However, the most key problem is the determination of $R_G$ in application. This requires enough research to establish corresponding relationship between the performance and $R_G$. The upper boundary of $R_G$ is further determined, which is used for performance evaluation, design, and optimization.

**Application**

Analyses in the previous sections demonstrate enough credibility on $R_G$ for cantilever robot. However, the most key problem is the determination of $R_G$ in application. This requires enough research to establish corresponding relationship between the performance and $R_G$. The upper boundary of $R_G$ is further determined, which is used for performance evaluation, design, and optimization.

**The choice of $R_G$ for performance evaluation**

The gravity-center position of TriMule in whole workspace was analyzed. The results are shown in Figures 10–13, where $z = 600$ mm in Figure 10, $z = 800$ mm in
Figure 11. $R_G$ distribution of TriMule within the plane of $z = 800$ mm: (a) spatial distribution and (b) contour distribution.

Figure 12. $R_G$ distribution of TriMule within the plane of $z = 1000$ mm: (a) spatial distribution and (b) contour distribution.

Figure 11, $z = 1000$ mm in Figure 12 and $z = 1200$ mm in Figure 13. In addition, (a) is the spatial distribution, and (b) is the corresponding contour distribution.

As shown in Figures 10–13, the distribution of $R_G$ is not symmetry about $x$-axis in the whole workspace; moreover, the asymmetry increases as the $z$ increases. The reason is that the mass distribution of 2R module is not symmetry about $x$-axis, which is bound to show that the performance of TriMule is not symmetry about $x$-axis in the whole workspace. Furthermore, the bigger $z$, the greater the impact on the performance of the distribution. However, $\kappa$ and $f_1$ cannot accurately reflect the difference due to some simplification considerations in the modeling process.

In addition, Figures 10–13 show that the distribution of $R_G$ in the whole workspace is very consistent. However, the increase of $z$ is not equal to the increase of $R_G$ for TriMule. According to literature, the accuracy of TriMule is maintained in workspace of $x = [-400$ mm, 400 mm], $y = [-600$ mm, 300 mm] within the
plane of $z = 800$ mm. As the spatial correspondence, the value of $R_G$ can be set as 0.26, to obtain the optimal performance space of TriMule in the whole workspace, as shown in Figure 14.

**Figure 13.** $R_G$ distribution of TriMule within the plane of $z = 1200$ mm: (a) spatial distribution and (b) contour lines.

**Figure 14.** Optimal workspace based on $R_G$.

plane of $z = 800$ mm. As the spatial correspondence, the value of $R_G$ can be set as 0.26, to obtain the optimal performance space of TriMule in the whole workspace, as shown in Figure 14.

**$R_G$ value choice for design and optimization of manipulator**

In the process of robot design and optimization, there are many traditional constraint variables such as kinematics constraints, force constraints,\textsuperscript{31} dimensional constraints, pressure angle constraints,\textsuperscript{32} and dynamic characteristics constraints.\textsuperscript{33} However, $R_G$ can be introduced instead of kinematics and dynamic constraints at the same time, which will effectively constrain the performance of the robot. Meanwhile, $R_G$ can be utilized for the trajectory optimization constraints of manipulator.
If there is the constraint as

\[ R_G < \text{constant} \]  \hspace{1cm} (37)  

Where, the \textit{constant} denotes the upper boundary of \( R_G \).

Predictably, \( \kappa \), dimensional parameters \( (D_i) \) and \( f_1 \) will be constrained within the limits

\[ R_G(\kappa; D_i; f_1) < \text{constant} \]  \hspace{1cm} (38)  

In conclusion, \( R_G \) is a convenient and fast performance evaluation method, which can be well applied to the design, optimization, and application evaluation of the robot.

**Conclusions**

This article presents a performance evaluation approach of the lower mobility hybrid robot based on the vector of gravity center for rapid evaluation, design, and optimization of the robot. The following conclusions are drawn:

1. A general position vector model of gravity center for the lower mobility hybrid robot in the whole workspace is constructed based on a general inverse kinematic model.
2. A rapid performance evaluation index \( R_G \) for cantilever robot based on gravity-center position is proposed, which has high sensitivity on mess distribution.
3. The validity of \( R_G \) is verified by comparing with the conditional number and the first natural frequency of 5-DOF hybrid robot. The results indicate that \( R_G \) have enough credibility, and it can perform performance evaluation accurately for TriMule. It is proved that the gravity-center position vector is closely related to the performance of the manipulator, which is of great significance.
4. The boundary selection of \( R_G \) is discussed, which can be used for design and optimization of lower mobility hybrid robot. If \( R_G \) is introduced during design and trajectory optimization, the performance of the robot in the whole workspace will be improved.

The line distributions between the conditional number, the first natural frequency, and \( R_G \) have been presented. Further work needs to be carried out to perform the relationship between position accuracy, cutting stability, and \( R_G \). These issues, however, deserve to be addressed in separate articles.

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ORCID iD
Wentie Niu https://orcid.org/0000-0002-4188-088X

References
1. Neumann K-E. Robot. Patent No. 4,732,525, USA, 1988.
2. Dorries Scharmann Technologie GmbH- SprintZ3 [EB/OL]. [2006-12-10]. Available at: http://www, ds-technologie, de/v3/products/index, php? id = 31& group_id = 9.
3. Chanal H, Duc E and Ray P. A study of the impact of machine tool structure on machining processes. Int J Mach Tools Manuf 2006; 46(2); 98–106.
4. Liu H, Huang T, Kecskeméthy A, et al. A generalized approach for computing the transmission index of parallel mechanisms. Mech Mach Theory 2014; 74: 245–256.
5. Barnfather JD, Goodfellow MJ and Abram T. A performance evaluation methodology for robotic machine tools used in large volume manufacturing. Robot Comput Integr Manuf 2016; 37: 49–56.
6. Joshi S and Tsai L-W. A comparison study of two 3-DOF parallel manipulators: one with three and the other with four supporting legs. IEEE Trans Rob Autom 2003; 19(2): 200–209.
7. Zhang D, Wang L and Lang SYT. Parallel kinematic machines: design, analysis and simulation in an integrated virtual environment. J Mech Design 2005; 127(4): 580–588.
8. Ma Y, Niu W, Luo Z, et al. Static and dynamic performance evaluation of a 3-DOF spindle head using CAD-CAE integration methodology. Robot Comput Integr Manuf 2016; 41: 1–12.
9. Liu H, Huang T, Kecskeméthy A, et al. Force/motion transmissibility analyses of redundantly actuated and overconstrained parallel manipulators. Mech Mach Theory 2017; 109: 126–138.
10. Salisbury JK and Craig JJ. Articulated hands: force control and kinematic issues. Int J Rob Res 1982; 1(1): 4–17.
11. Yu S, Xie M, Wu H, et al. Design and control of a piezoactuated microfeed mechanism for cell injection. Int J Adv Manuf Technol 2019; 105(12): 4941–4952.
12. Gosselin C and Angeles J. A global performance index for the kinematic optimization of robotic manipulators. J Mech Design: ASME 1991; 113: 220–226.
13. Huang T, Whitehouse D and Wang J. The local dexterity, optimal architecture and design criteria of parallel machine tools. CIRP Ann Manuf Technol 1998; 47(1): 346–350.
14. Huang T, Wang J and Whitehouse D. Design of hexapod-based machine tools with specified orientation capacity and well-conditioned dexterity. In: Proceedings of the tenth world congress on the theory of machines and mechanism, vol. 3, Oulu, 20–24 June 1999.
15. Huang T, Wang J and Whitehouse D. Theory and methodology for kinematic design of Gough-Stewart platforms. Sci China E: Technol Sci 1999; 42(4): 425.
16. Dong C, Liu H, Yue W, et al. Stiffness modeling and analysis of a novel 5-DOF hybrid robot. Mech Mach Theory 2018; 125: 80–93.
17. Dong C, Liu H, Huang T, et al. A screw theory-based semi-analytical approach for elastodynamics of the tricept robot. *J Mech Rob* 2019; 11(3): 031005.
18. Wu L, Wang G, Liu H, et al. An approach for elastodynamic modeling of hybrid robots based on substructure synthesis technique. *Mech Mach Theory* 2018; 123: 124–136.
19. Wang J and Gosselin CM. Static balancing of spatial four-degree-of-freedom parallel mechanisms. *Mech Mach Theory* 2000; 35(4): 563–592.
20. Li Y, Wang J, Liu X, et al. Dynamic performance comparison and counterweight optimization of two 3-DOF parallel manipulators for a new hybrid machine tool. *Mech Mach Theory* 2010; 45(11): 1668–1680.
21. Lin P, Shih W and Chen D. Design of a gravity-balanced general spatial serial-type manipulator. *J Mech Rob* 2010; 2(3): 031003.
22. Russo A, Sinatra R and Xi F. Static balancing of parallel robots. *Mech Mach Theory* 2005; 40(2): 191–202.
23. Fattah A and Agrawal SK. On the design of reactionless 3-DOF planar parallel mechanisms. *Mech Mach Theory* 2006; 41(1): 70–82.
24. Eckenstein N and Yim M. Modular advantage and kinematic decoupling in gravity compensated robotic systems. *J Mech Rob* 2013; 5(4): 041013.
25. Wu J, Wang J, Wang L, et al. Dynamic analysis and counterweight optimization of the 2-DOF parallel manipulator of a hybrid machine tool. *J Comput Nonlinear Dyn* 2007; 2(4): 344.
26. Ma Y, Zhang J, Dong C, et al. Kinetostatic modelling and gravity compensation of the TriMule robot. In: *IFToMM world congress on mechanism and machine science*, Krakow, June 30–4 July 2019. Cham: Springer.
27. Huang T, Dong C, Liu H, et al. A simple and visually orientated approach for type synthesis of overconstrained 1T2R parallel mechanisms. *Robotica* 2019; 37(7): 1161–1173.
28. Wang C, Zhao Y, Dong C, et al. Kinematic performance comparison of two parallel kinematics machines. In: *IFToMM world congress on mechanism and machine science*, Krakow, June 30–4 July 2019. Cham: Springer.
29. Zhao Y, Wang C, Niu W, et al. Stiffness modeling and analysis of a 3-DOF parallel kinematic machine. In: *IFToMM world congress on mechanism and machine science*, Krakow, June 30–4 July 2019. Cham: Springer.
30. Tian W, Mou M, Yang J, et al. Kinematic calibration of a 5-DOF hybrid kinematic machine tool by considering the ill-posed identification problem using regularisation method. *Robot Comput Integr Manuf* 2019; 60: 49–62.
31. Glorieux E, Franciosa P and Ceglarek D. End-effector design optimisation and multi-robot motion planning for handling compliant parts. *Struct Multidiscipl Optim* 2018; 57(3): 1377–1390.
32. Mei J, Zhang X, Zhang J, et al. Optimization design using a global and comprehensive performance index and angular constraints in a type of parallel manipulator. *Adv Mech Eng* 2018; 10(7): 1687814018878068.
33. Xu H, Xie S, Wang Z, et al. Dynamic optimization design method for high-speed serial industrial robot. In: *Proceedings of the 2018 IEEE international conference on real-time computing and robotics (RCAR)*, Kandima, Maldives, 1–5 August 2018.
**Author biographies**

Chensheng Wang is a PhD Candidate at Tianjin University, China. And her research interests are dynamics and control of machine tools.

Fang Su is a college lecturer at Shanxi Datong University, China. And her research interests are dynamics and control of machine tools.

Yanqin Zhao is a PhD Candidate at Tianjin University, China. And her research interests are stiffness, dynamics and control of parallel kinematic machines.

Hongda Liu is a PhD Candidate at Tianjin University, China. And her research interests is dynamics of machine tools.

Yonghao Guo is a PhD Candidate at Tianjin University, China. And her research interests is dynamics and control of parallel kinematic machines.

Wentie Niu is an associated professor at School of Mechanical Engineering, Tianjin University, China. His research interests are intelligent agricultural equipments and robots.