MODELS OF ELECTROWEAK INTERACTIONS IN NON-COMMUTATIVE GEOMETRY: A COMPARISON∗)

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Abstract

Alain Connes’ construction of the standard model is based on a generalized Dirac-Yukawa operator and the K-cycle \((\mathcal{H}, D)\), with \(\mathcal{H}\) a fermionic Hilbert space. If this construction is reformulated at the level of the differential algebra then a direct comparison with the alternative approach by the Marseille-Mainz group becomes possible. We do this for the case of the toy model based on the structure group \(U(1) \times U(1)\) and for the \(SU(2) \times U(1)\) of electroweak interactions. Connes’ results are recovered without the somewhat disturbing \(\gamma_5\)-factors in the fermion mass terms and Yukawa couplings. We discuss both constructions in the same framework and, in particular, pinpoint the origin of the difference in the Higgs potential obtained by them.

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The minimal standard model of strong and electroweak interactions is in perfect agreement with experiment (see e.g. [1]). As experiment allows for only small deviations from its predictions there is not much freedom for extended or alternative descriptions of the fundamental interactions. Recently, in the framework of non-commutative geometry, several constructions were proposed which either are very close to or yield exactly the standard model [2]-[6]. The non-commutative geometry approach extends and generalizes the framework of Yang-Mills theories but, in the end, yields the same Yang-Mills-Higgs Lagrangian. A most noteworthy virtue of this new approach is the progress in understanding better some of the qualitative features of the standard model. For instance, the Higgs fields appear naturally as part of a (super)connection. Spontaneous symmetry breaking receives a new geometric interpretation [6, 7]. This should be compared to the usual, more quantitative, phenomenology of the Higgs sector which concentrates on determining its parameters from experiment [8].

The aim of this note is to compare the construction of the standard model proposed by Connes [2] and elaborated further by Kastler [3], to the approach developed by the Marseille group and the Mainz group [4]-[7]. In the sequel, we shall refer to the former as construction (I), and to the latter as construction (II), for the sake of brevity.

Connes’ construction (I) is formulated in terms of abstract algebraic objects which generalize, in the sense of non-commutative geometry, the notion of differential forms and, as a result, the notion of connection (gauge potential). The generalized differential forms are realized within a certain fermionic representation space and by means of a (generalized) Dirac-Yukawa operator. After a lengthy calculation an action is obtained which is very close to, albeit not identical with, the one of the standard model. When taken at face value and at the classical level, the Lagrangian obtained in (I) fixes some of the parameter ratios of the model [9]. The first formulation of (I) contained auxiliary, adynamical fields which had to be eliminated by minimization and which made the comparison with (II) difficult. Since then Connes proposed a modified construction of the underlying differential algebra [10] which renders such a comparison much easier.

The Marseille-Mainz construction (II) rests on less sophisticated mathematics. It is formulated in the $\mathbb{Z}_2$-graded space of matrix-valued differential forms, equipped with a multiplication law inherited, in a straightforward manner, from the tensor product of matrix multiplication and the wedge product for differential forms. It makes use of a generalized differential which is composed of the usual exterior (Cartan) derivative and a matrix derivative. Construction (II) leads to a Lagrangian which coincides exactly with the Lagrangian of the standard model, including spontaneous symmetry breaking and in the correct “shifted” phase of the neutral Higgs field. Again, some ratios of the model’s parameters seem to be fixed but they can be modified even at the classical level [11]. Furthermore, there is nothing to protect these ratios under quantization [12].

With the new formulation of (I) referred to above a detailed comparison of the two approaches is now within reach because Connes’ modified differential algebra and ours are very similar. In what follows we work out this comparison in the simplest case of one
generation of leptons. The results of this comparison are interesting and, to some extent, surprising. We start with a short summary of (II). We then present the main steps which lead to (I) in a framework which is very close to the one of (II). In this way we achieve not only a direct and transparent comparison but we are enabled to perform calculations within Connes’ framework (I) which are considerably simpler than in the original work [3]. In order to facilitate the understanding we present our detailed calculations only for the case of the structure group \( U(1) \times U(1) \), considered in [4]. We make this restriction because in the framework of (I) calculations are simplified considerably as compared to the more realistic case of the structure group \( SU(2) \times U(1) \). The conclusions are exactly the same. We show that the \( \gamma_5 \) factor in the fermionic mass terms, not present in the standard model and also not present in (II), can in fact be avoided in the approach (I). Finally, we comment on the question of parameter ratios. We show that the situation is very similar in either construction, i.e. that the conclusions reached in (II) also apply to (I).

2. In summarizing (II) we closely follow [4]. The basic mathematical object is the \( \mathbb{Z}_2 \)-graded algebra \( \Omega^*_M(X) \) of matrix-valued forms as obtained from the skew tensor product of the matrix algebra \( M(n, \mathbb{C}) \) and the algebra \( \Lambda^*(X) \) of differential forms over spacetime \( X \). The matrix algebra is is taken to be \( \mathbb{Z}_2 \)-graded, and \( \Lambda^* \) carries its \( \mathbb{N} \)-grading. So we have

\[
\Omega^*_M(X) = M(n, \mathbb{C}) \hat{\otimes} \Lambda^*(X).
\]

Here we take \( n = 2 \). In this case the \( \mathbb{Z}_2 \)-(matrix) grading distinguishes the diagonal part, which is even, and the off-diagonal part, which is odd, and can be defined with the grading automorphism \( \Gamma = \text{diag}(1,-1) \). Homogeneous elements of \( \Omega^*_M \) are written as \( a \otimes \alpha \), with \( a \) a matrix and \( \alpha \) a differential form. In a simplified notation the grade of this element is

\[
\partial(a \otimes \alpha) = \partial a + \partial \alpha \pmod{2},
\]

and the multiplication law reads

\[
(a \otimes \alpha) \cdot (b \otimes \beta) = (-)^{\partial a \cdot \partial b} a \cdot b \otimes \alpha \wedge \beta.
\]

The generalized differential is given by

\[
d(a \otimes \alpha) = (d_M a) \otimes \alpha + (-)^{\partial a} a \otimes d_C \alpha,
\]

with \( d_C \) denoting the usual (Cartan) exterior derivative in \( \Lambda^* \), and \( d_M \) the matrix derivative in \( M(2, \mathbb{C}) \) defined by its action on the even and odd parts \( a_0 \) and \( a_1 \) of \( a \), respectively, [6]

\[
d_M(a) = [\eta, a_0] + i\{\eta, a_1\},
\]

with \( \eta \) denoting the odd element of \( M(2, \mathbb{C}) \)

\[
\eta = i \begin{pmatrix} 0 & c \\ \overline{c} & 0 \end{pmatrix}.
\]

(1)
In an obvious notation the structure of the algebra $\Omega^*_M$ is summarized by writing symbolically

$$(\Omega^*_M(X), \bullet, d\mathbb{Z}_2).$$

(2)

The generalized potential (superconnection) $A$ is an element of $\Omega^1_M(X)$ and reads explicitly\(^1\)

$$A = i \left( \begin{array}{cc} A & c\Phi/\mu \\ c\overline{\Phi} \end{array} \right) B$$

(3)

with $A = A_\mu dx^\mu, B = B_\mu dx^\mu$, $\Phi$ a scalar field, $c$ a constant and $\mu$ a parameter of dimension mass. The field strength (supercurvature) is obtained, e.g., form the structure equation

$$F = dA + A \bullet A.$$  

Without loss of generality we may set $c = 1$ and $\mu = 1$, (the dimensionless $c$ can be absorbed in the mass parameter $\mu$ while the latter sets the mass scale). The explicit form of $F$ is then

$$F = i \left( \begin{array}{ccc} d_C A - (\Phi + \overline{\Phi} + \Phi\overline{\Phi}) & -d_C \Phi - i(A - B)(\Phi + 1) \\ -d_C \overline{\Phi} + i(A - B)(\Phi + 1) & d_C B - (\Phi + \overline{\Phi} + \Phi\overline{\Phi}) \end{array} \right).$$

(4)

The Lagrangian is calculated from $\mathcal{L} = -\text{tr}(F^\dagger F)$ and takes the form

$$\mathcal{L} = -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} - \frac{1}{4} F^B_{\mu\nu} F^{B\mu\nu} + 2D\overline{\Phi}D\Phi - V(\Phi)$$

(5)

with $D\Phi = D\Phi + i(A - B)$, $D\overline{\Phi} = d_C \Phi + i(A\Phi - \Phi B)$, and $V(\Phi) = 2(\Phi + \overline{\Phi} + \Phi\overline{\Phi})^2$. It is important to notice that the Higgs potential stems from the diagonal part of $F$, while the mass term of the boson field $Z = A - B$ originates from the off-diagonal. It is evident that (5) is the exact analogue of the standard model Lagrangian. In particular, through the structure equation it includes spontaneous symmetry breaking and places the Higgs-like field $\Phi$ in the right “shifted” phase. As discussed in detail in \cite{3, 4, 5} an analogous result is obtained in the case of the structure group $SU(2) \times U(1)$ and of the graded Lie algebra $su(2|1)$.

3. We now describe approach (I) in a formulation which follows closely the discussion of (II) given above. Regarding the specific question of constructing the standard model in noncommutative geometry, we thus provide a third derivation of (I), after Connes’ original work \cite{2, 10} and Kastler’s detailed account of (I) \cite{3}, which is considerably simpler and more transparent than the original formulation.

Denote by $\mathcal{F}$ the space of complex functions on spacetime $X$ and, for the case of $U(1) \times U(1)$, write

$$\mathcal{M} = \left( \begin{array}{cc} \mathbb{C} & 0 \\ 0 & \mathbb{C} \end{array} \right).$$

\(^1\)We choose conventions such that $A$ and $\mathcal{F}$ are antihermitean, cf. \cite{7}
Connes’ approach (I) is based on the algebra 
\[ A = \mathcal{M} \otimes \mathcal{F} \] 
and the corresponding universal differential envelope \( (\Omega^*(A), \delta) \) that is generated by the formal elements (“words”) \( A_0 \delta A_1 \ldots \delta A_n \in \Omega^n(A) \) and the operator \( \delta \) obeying the Leibniz rule \( \delta(AB) = (\delta A)B + A(\delta B) \). This algebra is realized by means of a K-cycle (Dirac-Kasparov cycle) \( (\mathcal{H}, D) \) over \( \mathcal{A} \), where \( \mathcal{H} \) is a Hilbert space and \( D \) a Dirac-Yukawa operator, and a representation \( \pi \) of \( \Omega^*(A) \) on that Hilbert space. The Dirac-Yukawa operator has the form \( D = i\gamma^\mu \partial_\mu + D_M \) where \( D_M = \mu \eta \), with \( \eta \) as given in the construction of (II) above, cf. eq. (1). \( D_M \) may be understood to be a fermionic mass matrix. Note that such an interpretation was not made in (II) because it is unnecessary in that framework. The representation \( \pi \) of the universal envelope on the space \( \mathcal{L}(\mathcal{H}) \) of bounded linear operators over \( \mathcal{H} \) is given by

\[
\begin{align*}
\pi : \Omega^*(A) \rightarrow & \mathcal{L}(\mathcal{H}) \\
A_0 \delta A_1 \ldots \delta A_n \rightarrow & A_0[D, A_1] \ldots [D, A_n] .
\end{align*}
\]

In the original version of Connes’ construction the gauge potential and the field strength were taken to be elements of \( \pi(\Omega^*(A)) \). This led to the appearance of auxiliary or ady-namic fields (fields without kinetic energy) in the Lagrangian which had to be eliminated by minimization \([3, 13]\). At that stage a direct comparison with other approaches such as (II) was impossible.

In the more recent version of (I) given in \([10]\) one goes one step further by considering the space \( \Omega^*_D(A) \), obtained from \( \Omega^*(A) \) by dividing out the ideals \( J^k(A) = (K^k + \delta K^{k-1}) \), where \( K^k := \ker \pi \cap \Omega^k \), viz.

\[
\Omega^*_D(A) = \Omega^k(A)/J^k(A) ,
\]

or, equivalently,

\[
\Omega^*_D(A) = \pi(\Omega^k(A))/\pi(J^k(A))
\]

In contrast to \( \pi(\Omega^*(A)) \) the space \( \Omega^*_D \) is an N-graded differential algebra (like the universal object \( \Omega^* \)). Therefore, \( \Omega^*_D(A) \) is the space which should be compared to the space \( \Omega^*_M(X) \) of the approach (II) discussed in sect. 2 above. The multiplication law is defined by the ordinary multiplication in \( \mathcal{L}(\mathcal{H}) \) and by taking the quotient. We denote it by the symbol \( \odot \). The differential, denoted by \( \delta \), is given by commutation with the Dirac-Yukawa operator and by taking the quotient as above. In obvious analogy to (2) we may summarize the structure of this algebra as follows

\[
(\Omega^*_D(A), \odot, \delta)_{\mathbb{N}} .
\]

The explicit construction of the space \( \Omega^*_D(A) \) in the most general case, to the best of our knowledge, has not been given in the literature. There is, however, important progress in this direction that will be published elsewhere \([14]\). For the example of \( U(1) \times U(1) \) (and likewise for the case of \( SU(2) \times U(1) \)) the explicit calculation can be performed and the results for \( \Omega^*_D, \odot, \) and \( \delta \) can be given in simple terms. This is what we set out to do next.
For the purposes of physics we need to know only the spaces \( \pi(\Omega^k) \) for \( k = 0, 1, \) and 2. Thus we have to determine the projected ideals \( \pi(J^k) \) for these three values of \( k \). They are found to be, respectively,

\[
\pi(J^0) = \{0\}, \quad \pi(J^1) = \{0\}, \quad \pi(J^2) = \mathcal{M}_0 \otimes \Lambda^0(X),
\]

so that we have

\[
\Omega^0_D = \begin{pmatrix} \Lambda^0 & 0 \\ 0 & \Lambda^0 \end{pmatrix} = \mathcal{M}_0 \otimes \Lambda^0(X),
\]

\[
\Omega^1_D = \begin{pmatrix} \Lambda^1 & 0 \\ 0 & \Lambda^1 \end{pmatrix} + \begin{pmatrix} 0 & \Lambda^0 \\ \Lambda^0 & 0 \end{pmatrix} \equiv \mathcal{M}_0 \otimes \Lambda^1(X) + \mathcal{M}_1 \otimes \Lambda^0(X),
\]

\[
\Omega^2_D = \frac{\mathcal{M}_0 \otimes \Lambda^2 + \mathcal{M}_1 \otimes \Lambda^1 + \mathcal{M}_0 \otimes \Lambda^0}{\mathcal{M}_0 \otimes \Lambda^0} \approx \mathcal{M}_0 \otimes \Lambda^2(X) + \mathcal{M}_1 \otimes \Lambda^1(X)
\]

\[
= \begin{pmatrix} \Lambda^2 & 0 \\ 0 & \Lambda^2 \end{pmatrix} + \begin{pmatrix} 0 & \Lambda^1 \\ \Lambda^1 & 0 \end{pmatrix}.
\]

The construction for grades 0, 1, and 2 is fairly obvious\(^2\). For arbitrary grade \( k \in \mathbb{N} \), \( \Omega^k_D \) can be shown to be given by the pattern visible already in (10) and (11), viz.

\[
\Omega^k_D(\mathcal{A}) = \begin{pmatrix} \Lambda^k(X) & 0 \\ 0 & \Lambda^k(X) \end{pmatrix} + \begin{pmatrix} 0 & \Lambda^{k-1}(X) \\ \Lambda^{k-1}(X) & 0 \end{pmatrix}.
\]

Regarding the multiplication law we derive the example of the product \( \Omega^1_D \times \Omega^1_D \rightarrow \Omega^2_D \). The general case can easily be guessed from this example. We have, in an obvious notation,

\[
\left[ \begin{pmatrix} \Lambda^1 & 0 \\ 0 & \Lambda^1 \end{pmatrix} + \begin{pmatrix} 0 & \Lambda^0 \\ \Lambda^0 & 0 \end{pmatrix} \right] \odot \left[ \begin{pmatrix} \Lambda^1 & 0 \\ 0 & \Lambda^{k-1}(X) \end{pmatrix} + \begin{pmatrix} 0 & \Lambda^0 \\ \Lambda^0 & 0 \end{pmatrix} \right]
\]

\[
= \begin{pmatrix} \Lambda^2 & 0 \\ 0 & \Lambda^2 \end{pmatrix} + \begin{pmatrix} 0 & \Lambda^1 \\ \Lambda^1 & 0 \end{pmatrix}.
\]

In particular, it is important to note that

\[
\begin{pmatrix} 0 & \Lambda^0(X) \\ \Lambda^0(X) & 0 \end{pmatrix} \odot \begin{pmatrix} 0 & \Lambda^0(X) \\ \Lambda^0(X) & 0 \end{pmatrix} = 0 \in \Omega^2_D.
\]

\(^2\)A more detailed derivation of eq. (11) is given in [14]. Note that for more than one generation \( \Omega^k_D \) would also contain zero forms.
It is easy to work out the action of the differential \( \delta \) on \( \Omega^*_D \). Again we give only the example of grade \( k = 1 \), \( \delta : \Omega^1_D \rightarrow \Omega^2_D \):

\[
\delta \left[ \begin{pmatrix} \Lambda^1 & 0 \\ 0 & \Lambda^1 \end{pmatrix} + \begin{pmatrix} 0 & \Lambda^0 \\ \Lambda^0 & 0 \end{pmatrix} \right] = d_C \begin{pmatrix} \Lambda^1 & 0 \\ 0 & \Lambda^1 \end{pmatrix} - d_C \begin{pmatrix} 0 & \Lambda^0 \\ \Lambda^0 & 0 \end{pmatrix} + \{ \eta, \begin{pmatrix} \Lambda^1 & 0 \\ 0 & \Lambda^1 \end{pmatrix} \} + i \{ \eta, \begin{pmatrix} 0 & \Lambda^0 \\ \Lambda^0 & 0 \end{pmatrix} \}. \tag{13}
\]

Note that for the same reasons as in the multiplication law the anticommutator on the right-hand side vanishes,

\[
\{ \eta, \begin{pmatrix} 0 & \Lambda^0 \\ \Lambda^0 & 0 \end{pmatrix} \} = 0 \in \Omega^2_D.
\]

Thus, we have at our disposal an explicit construction of the space (7).

The construction of the generalized potential and field strength proceeds along the same lines as for (II), cf. sect. 2, keeping track of the modified multiplication and differential. As the spaces of grade 1 are the same in both frameworks, \( \Omega^1_M(X) = \Omega^1_D(A) \), the gauge potential (3) is the same. The field strength given by

\[
\mathcal{F} := \delta A + A \odot A,
\]

however, is different. A straightforward calculation using the multiplication rule and the differential given above leads to the result

\[
\mathcal{F} = i \left( \begin{pmatrix} d_C A \\ -d_C \overline{\Phi} + i(A - B)(\Phi + 1) \\ -d_C \Phi - i(A - B)(\Phi + 1) \\ \overline{d_C B} \end{pmatrix} \right). \tag{14}
\]

The most noticeable difference to eq. (4) is that the Higgs potential has disappeared from eq. (14). Indeed, the corresponding Lagrangian is given by

\[
\mathcal{L} = -\frac{1}{4} F^A_{\mu \nu} F^{A \mu \nu} - \frac{1}{4} F^B_{\mu \nu} F^{B \mu \nu} + 2 \overline{D \Phi} D \Phi. \tag{15}
\]

This is a consequence of the fact that we considered one generation of fermions (leptons) only. Thus, with this assumption, (I) leads to a trivial Higgs potential while (II) yields the correct potential and spontaneous symmetry breaking in the correct “shifted” phase. If one adds one or more generations then the Higgs potential appears also in (I) provided the fermion masses are not degenerate (see below).

4. We conclude by sketching the analogous calculation in the more realistic case of \( SU(2) \times U(1) \). The Marseille-Mainz construction (II), described in sect. 2 above, comes closest to Connes’ result if \( \eta \) is chosen as in eq. (1) above with

\[
c = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes M
\]
and $M$ a fermionic mass matrix, possibly containing more than one generation. At the same time, the ansatz for the (super)connection $A$ is enlarged by tensorizing $\Phi$ with $M$, $\overline{\Phi}$ with $M^\dagger$. Repeating the calculation of the field strength $F$ whose details are found in \cite{4, 7} one finds the expressions given in \cite{7, 13} with the following modifications. Denote for a moment by $F^{(0)}$ the field strength obtained with the choice $c = (1, 0)^T$ in eq. (1). In the diagonal elements $F_{11}^0$ and $F_{22}^0$ of $F$ the terms in the Higgs fields are tensorized with $M M^\dagger$ and $M^\dagger M$, respectively, while the off-diagonal terms are tensorized with $M$ and $M^\dagger$, respectively,

$$F_{12} = F^{(0)}_{12} \otimes M \quad F_{21} = F^{(0)}_{21} \otimes M^\dagger.$$ 

As a consequence the kinetic energy of the Higgs field in the Lagrangian (5) is multiplied by $\text{tr}(M M^\dagger)$, while the Higgs potential is multiplied by $\text{tr}(M M^\dagger)^2$.

Considering Connes’ construction (I), the essential difference with (II) lies in the diagonal elements of $F$. Before taking the quotient, they contain terms of the type $F_{\mu\nu}^\gamma \gamma^\mu \gamma^\nu$ and terms of the form $(\Phi + \overline{\Phi} + \Phi \overline{\Phi}) M M^\dagger \mathbb{1}$. The division by the ideal $\mathcal{J}^2$, in essence, leads to replacement of $M M^\dagger$ by

$$(M M^\dagger)_\perp = M M^\dagger - \frac{1}{n} \text{tr}(M M^\dagger),$$

where $n$ is the dimension of the matrix $(M M^\dagger)$. As a result, the Higgs potential is proportional to $\text{tr}(M M^\dagger)_\perp^2$ and vanishes whenever $(M M^\dagger)$ is proportional to the unit matrix, i.e. when the masses of equally charged fermions are degenerate \cite{16}. This is the main difference in the two approaches.

In conclusion we have developed a simplified, somewhat more algebraic construction of Connes’ algebra which allows for a direct comparison with the Marseille-Mainz approach. Unlike Connes’ approach we did not start from the generalized Dirac-Yukawa operator. Thereby we avoid the somewhat disturbing factors $\gamma_5$ in the off-diagonal elements of that operator which lead to fermionic mass matrices and to Yukawa couplings which are not those of the standard model. By deriving both (I) and (II) in the same algebraic spirit we exhibit more clearly the similarities and localize the differences in the underlying algebras (2) and (7). Our explicit calculation of the parts with grade 0, 1, and 2 of these algebras shows that while in (II) the Higgs potential is independent of fermionic masses altogether, the approach (I) yields a non-vanishing potential only if these masses are not degenerate.

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References

[1] L. Okun, Proc. Europhysics Conference on High-Energy Physics, Marseille, 1993
[2] A. Connes, and J. Lott, Nucl. Phys. B (Proc. Suppl.) 18 B (1990) 29
[3] D. Kastler; preprints Marseille-Luminy CPT-91/p. 2610, CPT-92/P.2814
[4] R. Coquereaux, G. Esposito-Farèse, and G. Vaillant, Nucl. Phys. B 353 (1991) 689
[5] R. Coquereaux, G. Esposito-Farèse, and F. Scheck, Int. J. Mod. Phys. A 7 (1992) 6555
[6] R. Häußling, N.A. Papadopoulos, and F. Scheck, Phys. Lett. B 260 (1991) 125
[7] R. Coquereaux, R. Häußling, N.A. Papadopoulos, and F. Scheck, Int. J. Mod. Phys. A 7 (1992) 2809
[8] B.A. Kniehl, Higgs phenomenology at one loop in the standard model, DESY 93-069, 1993
[9] D. Kastler, and Th. Schücker, Theor. Math. Phys. 92 (1992) 522
[10] A. Connes, Les Houches Lectures (1992)
[11] R. Häußling, N.A. Papadopoulos, and F. Scheck, Phys. Lett. B 303 (1993) 265
[12] E. Álvarez, J.M. Gracia-Bondía, and C.P. Martin, Phys. Lett. B 309 (1993) 55
[13] A.H. Chamseddine, G. Felder, and J. Fröhlich, Phys. Lett. B 296 (1992) 109
[14] W. Kalau, N.A. Papadopoulos, J. Plass, and J.M. Warzecha, Differential algebras in non-commutative geometry, preprint MZ-TH/93-27, Mainz 1993
[15] F. Scheck, Phys. Lett. B 284 (1992) 303
[16] J.C. Várilly, and J. M. Gracia-Bondía, J. Geometry and Physics (in print)