SOME STYLISED FACTS ABOUT THE EXCHANGE RATE BEHAVIOUR OF CENTRAL EUROPEAN CURRENCIES

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Abstract
The paper investigates developments of exchange rate time series of Central European currencies and tries to find evidence of some stylised facts. Statistical methods and an econometric approach to the univariate time series modelling of high-frequency data, i.e., daily, are used. The main conclusions are as follows: (1) All the CE nominal exchange time series are not stationary: nevertheless, stationarity of all the return time series was confirmed. (2) Volatility clustering was proven and the GARCH modelling approach was successfully applied, including asymmetric modelling of volatility. (3) The more flexible an exchange rate regime is, the more volatile the respective currency. This is true for both nominal and real exchange rates. While nominal volatility is lower than real volatility in a system of fixed or less flexible exchange rates, the opposite is true for flexible systems: exchange rate volatility is higher in nominal terms than in real terms.

Keywords: exchange rate, volatility, time series analysis, GARCH models

JEL Classification: C22, F31

Introduction
Financial markets behave in such a way that the time series of prices (or yields) of financial market instruments often demonstrate some behaviour or characteristics that can be generalised for a similar type of univariate or multivariate time series. Observations or empirical results that are generally accepted as being true are called stylised facts. Sewell [2011] defines stylised facts as empirical findings that are so consistent that they are accepted as true. The dictionary on About.com adds that stylised facts are observations that have been made in so many contexts that they are widely understood to be empirical truths to which theories must adhere.

We will focus on some stylised facts of the exchange rate time series behaviour in this article. Thus, we will not follow mainstream macro or micro exchange rate

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models but rather models based on the behavioural finance framework. The key theoretical assumption is that agents have cognitive limitations, i.e., their decisions are based on the rule of thumb (heuristic forming of expectations). However, it does not mean that agents behave irrationally: they are willing to learn and adjust their behaviour [De Grauwe and Kaltwasser, 2012].

Stylised facts of exchange rate development are mainly researched for major currencies. We will focus on the behaviour of Central European currencies (CE currencies) in this paper. Central European countries went through a long period of transition to market-oriented economies and a wide range of exchange rate systems. The aim of the paper is to investigate the (statistical) behaviour of the respective currencies. Specifically, the typical behaviour of the Czech, Polish, Hungarian and Slovak currencies quoted against the euro, which is the reference currency for all the Central European countries, and other indicators linked to exchange rates will be investigated.

The structure of the paper is as follows: after a brief description of the data used (Chapter 1), we will focus on the distribution characteristics of exchange rate returns (Chapter 2). Then we will move on to heteroskedasticity and volatility modelling in Chapter 3 and a discussion of differing volatility in different exchange rate regimes (Chapter 4). Chapter 5 summarises the final conclusions.

1. Description of data used

Daily nominal exchange rate data for EUR/CZK, EUR/HUF, EUR/PLN and EUR/SKK were taken from the DataStream database for the period from 1999 to 2014. In some cases, data before 1999 were used and are described in the respective chapters of this paper. Monthly time series of real effective exchange rates of CZK, HUF, PLN and SKK are also taken from the DataStream database; the primary source is the OECD database.

2. Normal versus fatter-tail distribution of exchange rate returns

In this section, we will focus on pure time series of exchange rate data (univariate time series analysis). Financial time series, as well as exchange rate time series, are characterised by properties and shapes given by the microstructure of the financial market (or FX market, respectively). As a result, the dynamics of such a time series are more influenced by non-systematic factors, leading to relatively high volatility unstable in time, resulting in a conditional non-stationarity of the time series [Arlt and Arltová, 2001].

Not only time-varying variability but also the long-term trend is behind the non-stationarity of CE currencies (this is clearly visible in Figure 1), which is the result of convergence processes of the Central European region toward the eurozone’s economic levels (and is even more visible in time series of real exchange rates of CE currencies; see Figure 2).

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2 A review of the traditional macro or micro approach to the exchange rate modelling can be found, for example, in James et al. [2012].
3 For example, Lothian [1998] focuses on exchange rate behaviour of OECD currencies.
4 Currency pairs are quoted in terms of units of the respective Central European currency per unit of EUR.
5 Actually, real exchange rate movements are what investors follow. However, high-frequency data are available only in nominal terms.
Figure 1 | Exchange rate development of CE currencies (4 Jan 1999 = 100), in nominal terms

Source: Author's calculations based on data from Bloomberg

Figure 2 | Exchange rate development of CE currencies (January 1995 = 100), in real terms

Source: Author's calculations based on data from the DataStream database
The summary statistic of the EUR/CZK exchange rate (first row of Table 1 – summary statistic) highlights a slight negative skewness and low kurtosis. The EUR/CZK time series is not normal at the 99% confidence interval according to the Jarque-Bera statistic. Consequently, the Augmented Dickey Fuller Test (Table 2 – unit root tests) clearly shows that we cannot reject the hypothesis of a unit root, i.e., the EUR/CZK time series is non-stationary and some transformation is needed. The conclusion of non-stationarity is also valid for the EUR/PLN, EUR/HUF and EUR/SKK time series at the 5% significance level.

### Table 2: Summary statistics

| Exchange Rate | Sample Period | Number of Observations | Mean   | Median  | Maximum | Minimum | Std. Dev. | Skewness   | Kurtosis   | Jarque-Bera | Probability |
|---------------|---------------|------------------------|--------|---------|---------|---------|-----------|------------|------------|-------------|-------------|
| EUR/CZK       | 1/1/99 – 31/12/14 | 4171                   | 29.15987 | 28.21   | 38.633  | 22.945  | 3.884591  | 0.568583   | 2.187653   | 339.4241    | 0           |
| EUR/HUF       | 4/1/99 – 31/12/14 | 4109                   | 266.513  | 260.94  | 320.78  | 228.16  | 20.54391  | 0.732029   | 2.474301   | 414.2945    | 0           |
| EUR/PLN       | 4/1/99 – 31/12/14 | 4109                   | 4.058215 | 4.0812  | 4.9346  | 3.2053  | 0.299227  | -0.100005  | 3.296354   | 21.8654     | 0.000018    |
| EUR/SKK       | 4/1/99 – 31/12/08 | 2580                   | 39.51241 | 40.91   | 47.484  | 30.126  | 4.154182  | -0.769149  | 2.540462   | 274.9372    | 0           |

Source: Author’s calculations based on data from the Bloomberg database

### Table 2: Unit root tests

| Exchange Rate | Augmented Dickey-Fuller Test statistic | Phillips-Perron Test statistic |
|---------------|----------------------------------------|--------------------------------|
|               | t-Statistic | Probability*                    | Adjusted t-Statistic | Probability* |
| EUR/CZK       | -1.138541* | 0.2264                           | -1.157815* | 0.2258    |
| EUR/HUF       | -3.597025** | 0.0301                           | -3.257382** | 0.0736    |
| EUR/PLN       | -2.618106*** | 0.0893                          | -2.446589*** | 0.1291    |
| EUR/SKK       | -2.158702** | 0.5120                           | -2.272655** | 0.4483    |

Mackinnon (1996) one-sided p-values, * without trend and intercept, ** with trend and intercept, *** with intercept

Source: Author’s calculations based on data from the DataStream database

The natural method of stationarisation is a transformation of the exchange rate time series into the rates of return time series. 6 Table 3 clearly shows that the EUR/CZK, EUR/HUF, EUR/PLN and EUR/SKK return time series are stationary, as the Augmented Dickey Fuller Test and Phillips-Perron Test statistics confirm.

6 Actually, we used another common method of transformation: the logarithmic transformation, i.e., \( rt = \ln St - \ln St-1 \).
One of the assumptions in the classical analysis of time series is that the return transformation has a normal distribution.

The summary statistics for the EUR/CZK returns presented in Figure 4 illustrate a negligible negative skewness (big negative returns are slightly more probable than big positive returns) and a very high kurtosis (log returns are significantly more peaked than normal distribution). The later is in compliance with findings of Bubáč.
thus, it is not a surprise that the Jarque-Bera test confirms non-normality of the EUR/CZK return series. As the PLN, HUF and SKK behave in a similar way, we can conclude that a feature of Central European exchange rates shows the same characteristics that are typical for high-frequency financial time series data, i.e., exchange rate returns' distributions have fat tails, implying that the probabilities of extremely high and extremely low returns are higher than in the case of normal distribution. The PLN and HUF showed a positive skewness, implying that big positive returns are more probable than big negative returns.

Table 4 | Summary statistics of the exchange rate return time series

| Exchange rate | Number of observations | Mean   | Median   | Maximum  | Minimum  | Std. Dev. | Skewness | Kurtosis | Jarque-Bera | Probability |
|---------------|------------------------|--------|----------|----------|----------|-----------|----------|----------|-------------|-------------|
| Return of EUR/CZK | 4168                   | -5.57e-05 | -0.00011 | 0.071966 | -0.072525 | 0.004446 | 0.227101 | 44.55131 | 299873.3 | 0.000        |
| Return of EUR/HUF | 4066                   | 6.57e-05 | -3.90e-05 | 0.050693 | -0.033885 | 0.005717 | 0.750415 | 11.99828 | 14099.11 | 0.000        |
| Return of EUR/PLN | 4066                   | 1.39e-05 | -0.00212 | 0.041636 | -0.036798 | 0.006363 | 0.458517 | 8.17698  | 4683.028 | 0.000        |
| Return of EUR/SKK | 2528                   | -0.000135 | -0.000165 | 0.0185 | -0.031801 | 0.003031 | -0.19063 | 11.89614 | 8351.536 | 0.000        |

Source: Author’s calculations based on data from the DataStream database

Figure 4 | Summary statistic for EUR/CZK returns

Source: Author’s calculations based on data from the DataStream database

3. Heteroskedasticity modelling of CE exchange rates movements

The other key assumption of classical time series analysis is that returns behave like a white noise process, i.e., that returns are not correlated, identically distributed with zero means and constant variance (or a strict white noise process, requiring not only non-correlation but independence).
The EUR/CZK return time series\(^7\) is presented in Figure 3 (upper left graph). It is clearly seen that a common characteristic of the return time series is not constant but time-changing volatility (variability). Thus, the time series is heteroskedastic. It changes in clusters [Mandelbrot, 1963], i.e., sometimes volatility changes in very short time periods and sometimes it is stable for a longer period of time.

Changing volatility should be modelled using a group of so-called Generalised Autoregressive Conditional Heteroskedasticity models (GARCH models).\(^8\) GARCH models are based on the Box-Jenkins approach to volatility modelling. Every GARCH model consists of two equations: (1) a conditional mean equation, and (2) a conditional variance equation. An example of the mean equation is as follows:

\[ Y_t = c + X'_t \theta + e_t, \]  

i.e., the dependent variable \( Y_t \) (which will be the return \( r_t \) in our case) is modelled as a function of the explanatory variables \( X'_t \) and an error term \( e_t \) equal to \( \sigma \varepsilon_t \), where \( \varepsilon_t \) it is the \( iid \) variable with a zero mean and unity variance, while \( c \) is a constant. Volatility \( \sigma_t^2 \) is not constant and is modelled by the conditional variance equation:

\[ \sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i e_{t-i}^2, \]  

which is an expression of the GARCH\((p,q)\) model. \( \omega \) represents a constant term, \( \sigma_{t-j}^2 \) is \( t-j \) period lagged volatility (the so-called GARCH term) and \( e_{t-i}^2 \) (the ARCH term) is a variable measuring the last forecasting error or capturing news about volatility from the \( t-j \) period.

We have applied the GARCH approach to the time series of CE currency returns. In all the cases, the GARCH \((1,1)\) model was the most suitable. The results are depicted in Table 5.

Taking a look at Panel A of Table 5, it is quite obvious that the GARCH \((1,1)\) model is suitable for modelling daily returns of all the CE currencies as all the coefficients in the conditional variance equations are statistically significant and comply with the assumptions. Regarding the conditional mean equations, the constant is not statistically significant in the case of the EUR/CZK and EUR/HUF time series. Thus, the daily return of exchange rate time series can be decrypted using the following expressions:

- **EUR/CZK:** \( r_t = e_t + \sigma_t \varepsilon_t \),  
  \( \sigma_t^2 = 0.0000000064 + 0.137602 \sigma_{t-1}^2 + 0.832790 \sigma_{t-1}^2; \)

- **EUR/HUF:** \( r_t = e_t + \sigma_t \varepsilon_t \),  
  \( \sigma_t^2 = 0.000000331 + 0.059899 \sigma_{t-1}^2 + 0.930985 \sigma_{t-1}^2; \)

- **EUR/PLN:** \( r_t = -0.000150 + e_t + \sigma_t \varepsilon_t \),  
  \( \sigma_t^2 = 0.000000388 + 0.090082 \sigma_{t-1}^2 + 0.902635 \sigma_{t-1}^2; \)

- **EUR/SKK:** \( r_t = -0.000120 + e_t + \sigma_t \varepsilon_t \),  
  \( \sigma_t^2 = 0.000000251 + 0.105633 \sigma_{t-1}^2 + 0.877734 \sigma_{t-1}^2. \)

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7 However, the return coming from the spot exchange movement is a result of the spot-spot speculation, i.e., intraday speculation. For longer duration of the opened positions (more than one day), interest rate differential must be taken into account. Theoretically, market players follow uncovered interest rate parity condition.

8 GARCH models introduced by Bollerslev [1986] and Taylor [1986] generalised the Autoregressive Conditional Heteroskedasticity model (ARCH model) of Engle [1982].
Table 5 | Application of the GARCH approach

Conditional mean equation:  
\[ r_t = c + \varepsilon_t; \]
Conditional variance equation:  
\[ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (\alpha_0 > 0, \alpha_1, \beta_1 \geq 0, \alpha_1 + \beta_1 < 1). \]

| Coefficient (standard error) | EUR/CZK | EUR/HUF | EUR/PLN | EUR/SKK |
|-----------------------------|---------|---------|---------|---------|
| **c**                      | $-6.37 \times 10^{-5}$ ($5.01 \times 10^{-5}$) | $0.000103^* (0.000102)$ | $-0.000150 (7.07 \times 10^{-5})$ | $-0.000120 (5.03 \times 10^{-5})$ |
| **\(\alpha_0\)**          | $6.14 \times 10^{-2}$ ($1.56 \times 10^{-2}$) | $3.31 \times 10^{-2}$ ($1.59 \times 10^{-2}$) | $3.88 \times 10^{-2}$ ($8.01 \times 10^{-2}$) | $2.51 \times 10^{-2}$ ($1.09 \times 10^{-2}$) |
| **\(\alpha_1\)**          | $0.137602 (0.016059)$ | $0.058899 (0.0286051)$ | $0.090082 (0.011508)$ | $0.105633 (0.020471)$ |
| **\(\beta_1\)**           | $0.532191 (0.015804)$ | $0.930988 (0.021692)$ | $0.902635 (0.010135)$ | $0.877734 (0.024862)$ |

**Panel B – ARCH test**

| F-Statistic | EUR/CZK | EUR/HUF | EUR/PLN | EUR/SKK |
|-------------|---------|---------|---------|---------|
| Prob. F     | 0.062216 | 0.006280 | 0.532384 | 0.001005 |
| Obs. *R-squared | 0.082254 | 0.006283 | 0.532952 | 0.001006 |
| Prob. Chi-squared(f) | 0.7743 | 0.9368 | 0.1115 | 0.9747 |

**Panel C – Normality test of standardised residuals**

| Jarque-Bera | EUR/CZK | EUR/HUF | EUR/PLN | EUR/SKK |
|-------------|---------|---------|---------|---------|
| Probability | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

Source: Author’s calculations based on data from the DataStream database, *not significant

Q-tests of the standardised models verify the respective GARCH models. Also, the ARCH-LM test (see Panel B in Table 5) shows that there is no other ARCH structure in the respective models. Only the distributions of standardised residuals are not normal according to the Jarque-Bera tests (see Panel C in Table 5) because of too high kurtosis.

The GARCH model is the first step in modelling volatility with its own assumptions and limitations. There are more and more models based on the GARCH approach that relax the origin assumptions and develop models further. For a comprehensive discussion, see Green [2008] or Enders [2010].

Modelling exchange rate volatility represents a growing area in literature, including the Central European region. Exchange rate volatility is modelled using simple GARCH models or extended or modified GARCH models; modelled volatility is then explained by various determinants such as different FX regimes [Kočenda and Valachy, 2006 – using T-GARCH model9], estimated target exchange rate [Fidrmuc and Horváth, 2008 – using GARCH and extended TARCH models], or optimal currency area criteria [Horváth, 2005]. The GARCH (1,1) process for EUR/CZK exchange rate returns was used by Fišer & Horváth [2010] for examining the effects of the Czech National Bank’s

9 Or changes between volatility regimes, respectively [Frömmel, 2010 – using Markow-Switching GARCH model].
communication, macroeconomic news and exchange rate differentials on exchange rate volatility.

One of the advantages of the above-mentioned T-GARCH model (or threshold model or GRJ model) is that it allows us to model asymmetric behaviour, where positive and negative errors can have a different impact on volatility. The T-GARCH (1,1) model should have the following form:

\[ r_t = c + e_t, \quad e_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 e_{t-1}^2 I_{t-1}, \quad I_{t-1} = 1 \text{ for } e_{t-1} < 0, \text{ 0 others.} \]  

(3)

Having applied the T-GARCH (1,1) model to CE currencies, we have found that the model was suitable for modelling the volatility of EUR/HUF and EUR/PLN time series only. The outcome is depicted in Table 6 below:

Table 6: Application of the T-GARCH approach

| Coefficient (standard error) | EUR/CZK | EUR/HUF | EUR/PLN | EUR/SKK |
|-----------------------------|---------|---------|---------|---------|
| \( c \) \( (-7.53 \times 10^{-3}) \) | 0.000148** (9.92 \times 10^{-5}) | 0.000131 (4.91 \times 10^{-5}) |
| \( \alpha_0 \) \( (6.17 \times 10^{-7}) \) | 4.63 \times 10^{-7} \( (2.62 \times 10^{-7}) \) | 4.56 \times 10^{-7} \( (9.89 \times 10^{-8}) \) |
| \( \alpha_1 \) \( (0.123438) \) | 0.102564 (0.027937) | 0.118993 (0.015790) |
| \( \beta_1 \) \( (0.829299) \) | 0.916798 (0.016850) | 0.903780 (0.009828) |
| \( \gamma_1 \) \( (0.035321**) \) | -0.068331 (0.022715) | -0.069007 (0.019175) |

Panel B – ARCH test

| F-Statistic | Prob. F | Obs.*R-squared | Prob. Chi-squared(1) |
|-------------|---------|----------------|---------------------|
| 0.068774 | 0.7931 | 0.068806 | 0.7931 |
| 0.003610 | 0.9521 | 0.033161 | 0.9521 |
| 0.743809 | 0.3885 | 0.744309 | 0.3884 |
| 0.007284 | 0.9320 | 0.007290 | 0.9320 |

Panel C – Normality test of standardised residuals

| Jarque-Bera | Probability |
|-------------|-------------|
| 65943.50 | 0.0000 |
| 170445.6 | 0.0000 |
| 577.0782 | 0.0000 |
| 8176.070 | 0.0000 |

* Note: quasi-maximum likelihood (QML) covariances and standard errors are presented using the methods described by Bollerslev & Wooldridge [1992].

* not significant at 5%, ** not significant at 10%

Source: Author’s calculations based on data from the DataStream database

10 Maybe surprisingly, the leverage effect is not significant for the Czech and Slovak currencies, i.e., for countries where the nominal convergence has gone through the nominal exchange rate appreciation.
The leverage factor is present in the EUR/HUF and EUR/PLN time series; however, the coefficients are not very high. Q-tests of the standardised models verify these two T-GARCH models. Also the ARCH-LM test (see Panel B in Table 6) shows that there is no other ARCH structure in the respective models. Again, only the distributions of standardised residuals are not normal according to Jarque-Bera tests (see Panel C in Table 6) because of too high kurtosis. Thus, the daily return of the EUR/HUF and EUR/PLN exchange rate time series can be described using the following expressions:

**EUR/HUF:**
\begin{align*}
r_t &= e_t, \\
e_t &= \sigma_{e_t}, \sigma_e^2 = 0.000000463 + 0.102564e_{t-1}^2 + 0.916798\sigma_{e_{t-1}}^2 - 0.068331e_{t-1}^2I_{t-1}, \\
I_{t-1} &= 1 \text{ for } e_{t-1} < 0, \text{ 0 others.}
\end{align*}

**EUR/PLN:**
\begin{align*}
r_t &= -0.0000663 + e_t, \\
e_t &= \sigma_{e_t}, \sigma_e^2 = 0.000000456 + 0.118993e_{t-1}^2 + 0.903780\sigma_{e_{t-1}}^2 - 0.069007e_{t-1}^2I_{t-1}, \\
I_{t-1} &= 1 \text{ for } e_{t-1} < 0, \text{ 0 others.}
\end{align*}

While the leverage factor is assumed to be quadratic in the T-GARCH model, another approach to modelling the asymmetry was proposed by Nelson [1991]. He proposed an exponential leverage effect. That is why the model is called the Exponential GARCH or E-GARCH model. The E-GARCH (1,1) model should have the following form [Cipra, 2008]:
\begin{equation}
r_t = c + e_t, \quad e_t = \sigma_{e_t}, \quad \ln \sigma_e^2 = \alpha_t + \alpha_1 \frac{e_{t-1}}{\sigma_{e_{t-1}}} + \beta_t \ln \sigma_{e_{t-1}}^2 + \gamma_t \frac{e_{t-1}}{\sigma_{e_{t-1}}}.
\end{equation}

Asymmetry is present in the case of $\gamma_t \neq 0$ for the special case of the leverage effect $\gamma_t < 0$. Application of the E-GARCH approach to CE currencies is summarised in Table 7. The asymmetry was confirmed for all the considered currency pairs with the exception of EUR/SKK, where the parameter $\gamma$ is not statistically significant. For EUR/CZK, the leverage effect was confirmed. Thus, based on the E-GARCH modelling, the rate time series of CE exchange rates can be described using the following expressions:

**EUR/CZK:**
\begin{align*}
r_t &= -0.000101 + e_t, \quad e_t = \sigma_{e_t}, \quad \ln \sigma_e^2 = -0.458947 + 0.205607 \frac{e_{t-1}}{\sigma_{e_{t-1}}} + 0.973208 \ln \sigma_{e_{t-1}}^2 - 0.015848 \frac{e_{t-1}}{\sigma_{e_{t-1}}}.
\end{align*}

**EUR/HUF:**
\begin{align*}
r_t &= 0.000131 + e_t, \quad e_t = \sigma_{e_t}, \quad \ln \sigma_e^2 = -0.356978 + 0.145435 \frac{e_{t-1}}{\sigma_{e_{t-1}}} + 0.976797 \ln \sigma_{e_{t-1}}^2 + 0.043238 \frac{e_{t-1}}{\sigma_{e_{t-1}}}.
\end{align*}

**EUR/PLN:**
\begin{align*}
r_t &= e_t, \quad e_t = \sigma_{e_t}, \quad \ln \sigma_e^2 = -0.359555 + 0.183930 \frac{e_{t-1}}{\sigma_{e_{t-1}}} + 0.979199 \ln \sigma_{e_{t-1}}^2 + 0.058809 \frac{e_{t-1}}{\sigma_{e_{t-1}}}.
\end{align*}

**EUR/SKK:**
\begin{align*}
r_t &= -0.000147 + e_t, \quad e_t = \sigma_{e_t}, \quad \ln \sigma_e^2 = -0.614646 + 0.228885 \frac{e_{t-1}}{\sigma_{e_{t-1}}} + 0.962271 \ln \sigma_{e_{t-1}}^2.
\end{align*}

11 It can be explained by higher EUR/PLN and EUR/HUF exchange rate volatility in comparison with EUR/CZK and EUR/SKK and/or by higher interest rates on both currencies, which support investors’ taking position in carry trades, i.e., long in the high-yielding currency / short in EUR.
Table 7 | Application of the E-GARCH approach

Conditional mean equation: \( r_t = c + \epsilon_t \)
Conditional variance equation: \( \ln \sigma_t^2 = \alpha_0 + \alpha_1 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \gamma_1 \).

Panel A

| Coefficient (standard error) | EUR/CZK       | EUR/HUF       | EUR/PLN       | EUR/SKK       |
|-----------------------------|---------------|---------------|---------------|---------------|
| \( c \)                    | \(-0.000101\) | \(0.000131\)  | \(-0.000117^*\) | \(-0.000147\) |
| \( (4.24.10^{-5}) \)      | \(4.29.10^{-5})\) |               |               |               |
| \( \alpha_0 \)             | \(-0.458947\) | \(-0.356978\) | \(-0.359555\) | \(-0.614646\) |
| \( (0.038470) \)           | \(0.021161\)  |               |               |               |
| \( \alpha_1 \)             | \(0.205607\)  | \(0.145435\)  | \(0.183930\)  | \(0.228885\)  |
| \( (0.013673) \)           | \(0.007114\)  |               |               |               |
| \( \beta_1 \)              | \(0.973208\)  | \(0.976797\)  | \(0.979199\)  | \(0.962271\)  |
| \( (0.002916) \)           | \(0.003387\)  |               |               |               |
| \( \gamma_1 \)             | \(-0.015848\) | \(0.043238\)  | \(0.058809\)  | \(0.004729^*\) |
| \( (0.008005) \)           | \(0.005817\)  |               |               |               |

Panel B – ARCH test

| F-Statistic | 0.001422 | 0.075989 | 2.073401 | 0.024682 |
| Prob. F     | 0.9699   | 0.7828   | 0.1500   | 0.8752   |
| Obs. R-squared | 0.001423 | 0.076025 | 2.073364 | 0.024701 |
| Prob. Chi-squared(1) | 0.9699 | 0.7828 | 0.1499 | 0.8751 |

Panel C – Normality test of standardised residuals

| Jarque-Bera | 56392.10 | 1167198 | 614.7385 | 14348.07 |
| Probability | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

Note: Assumed Generalised error distribution (GED) of \( \epsilon_t \) with the parameter equal to 1.5.
* not significant

Source: Author’s calculations based on data from the DataStream database

4. Volatility of CE exchange rate movements versus exchange rate systems

Up to now, volatility properties of unilateral time series of exchange rates or volatility clustering in high-frequency time series (daily in our case) were discussed, respectively. Let us now move on to an investigation of the relationship between the volatility of the nominal and real exchange rates on the one hand and the regime of the respective exchange rate on the other. It is another stylised fact that when exchange rates are flexible, they tend to be more volatile.12 We have investigated the volatility of CE currencies in different exchange rate regimes.

Since the early 1990s, CE countries have undergone different exchange regimes over the past two decades on the way from planned to market-oriented economies. For all the CE countries, a move from non-convertible to convertible currencies is typical for the first few years of transition. Regarding exchange rate regimes, a move from fixed or less flexible systems (a wide range of peg systems used to be applied quite often) to the much more flexible or even float system typical of the recent years. The only exception

12 A conclusion pioneered by Mussa [1986].
is Slovakia, as it entered the euro area in 2009, i.e., after applying the managed float system between 1998 and 2005, the SKK spent the following three years under the less flexible ERMII regime. A different approach can also be seen in the case of the CZK. In November 2013, the Czech central bank decided to use the exchange rate as a monetary policy instrument resulting in a move from a managed float system to a less flexible intervention system with a set floor at the level of EUR/CZK 27. Above the floor, the exchange rate is market-determined.

How the volatility of nominal exchange rates of CE currencies changes through different exchange rate regimes is shown in Table 8 below:

| EUR/CZK | EUR/PLN | EUR/HUF | EUR/SKK |
|---------|---------|---------|---------|
| ER regime | volatility | ER regime | volatility | ER regime | volatility | ER regime | volatility |
| 03/03/1993– 29/02/1996 | 0.371738 | 02/08/1991– 15/03/1995 | 0.570721 | 02/08/1991– 05/03/1995 | 4.377472 | 01/01/1997– 30/09/1998 | 1.892928 |
| basket peg, band +/-0.5% | Data not available | basket peg, band +/-1.5% | Data not available | basket peg, band +/-0.6% | basket peg, band +/-2.25% | Data not available | basket peg, band +/-15% |
| 01/03/1996– 26/05/1997 | Data not available | 16/03/1995– 30/04/2001 | Data not available | 6/03/1995– 15/05/1995 | 5.445230 | 16/05/1995– 24/02/1998 | 8.821312 |
| basket peg, band +/-0.5% | basket peg, band +/-3.0% | crawling peg, band +/-2.25% | crawling peg, band +/-2% | crawling peg, band +/-7% | Data not available | crawling peg, band +/- 7% | Data not available |
| 27/05/1997– 06/11/2013 | Data not available | 01/05/2001– 30/09/2001 | 5.557278 | 25/02/1998– 11/04/2000 | 0.136005 | 01/10/2001– 25/2/2008 | 0.306003 |
| managed float | basket peg, band +/-5.0% | crawling peg, band +/-15% | crawling peg, band +/-2% | crawling peg, band +/- 15% | Data not available | horizontal peg, 100% EUR, band +/-15% | Data not available |
| 07/11/2001– present | Data not available | 01/10/2001– 25/2/2008 | 5.821312 | 01/01/2009 EMU | 0.0 | 12/04/2000– present | 0.0 |
| Intervention regime (floor set at around 27.0) | basket peg, band +/-7.0% | horizontal peg, 100% EUR, band +/-15% | Data not available | Data not available | Data not available | Data not available | Data not available |
| 01/10/1998– 24/11/2005 | 2.662918 | 19.27760 | 0.306003 |
| managed float | Data not available | Data not available | Data not available |
| 25/11/2005– 31/12/2008 | 1.892928 | 26/02/2008– present | 0.0 |
| peg 100% EUR +/-15% (ERM II) | Data not available | present free float | Data not available |
| 01/10/2009 EMU | 2.662918 | 19.27760 | 0.306003 |

Note: Volatility is measured as a standard deviation (based on daily figures).

Note: The EUR/CZK exchange rate before euro introduction was recalculated from DEM/CZK or the basket of respective former national currencies.

Source: Author’s calculations based on data from the DataStream database.
The results clearly confirm a very intuitive conclusion that the more flexible an exchange rate regime is, the more volatile the respective currency. This hypothesis is valid for all the currencies under investigation.

However, taking into account the mainstream view that nominal volatility does not have any impact in the long term, the question that arises is whether volatility of the real exchange rates is determined by the flexibility of the exchange rate system. Volatilities of respective real effective exchange rates of CE currencies under different exchange rate regimes are represented in Table 9 below:

| ER regime | volatility | ER regime | volatility | ER regime | volatility | ER regime | volatility |
|-----------|------------|-----------|------------|-----------|------------|-----------|------------|
| 03/03/1993–29/02/1996 basket peg, band +/-0.5% | 1.925061 | 14/07/1994–31/12/1995 basket peg, band +/-1.5% | 0.616748 | 02/08/1991–15/03/1995 adjustable peg (irregular devaluations) band +/-0.3% – +/-2.25% | 2.035759 | 14/10/1991–05/03/1995 crawling peg, band +/- 0.6% | 1.789789 |
| 01/01/1996–26/05/1997 basket peg, band +/-7.5% | 1.816578 | 01/01/1996–30/07/1996 basket peg, band +/-3.0% | 0.315851 | 16/03/1995–30/04/2001 crawling peg, band +/-2.25% | 3.797941 | 06/03/1995–15/05/1995 crawling peg, band +/- 2% | 1.151535 |
| 27/05/1997–06/11/2013 managed float | 13.90059 | 31/07/1996–31/12/1996 basket peg, band +/-5.0% | 0.221420 | 01/05/2001–30/09/2001 crawling peg, band +/-15% | 1.353359 | 16/05/1995–24/02/1998 crawling peg, band +/- 7 % | 3.184114 |
| 07/11/2013–present Intervention regime (floor set at around 27.0) | 0.996485 | 01/01/1997–30/09/1998 basket peg, band +/-7.0% | 1.329397 | 01/10/2001–25/2/2008 horizontal peg, 100% EUR, band +/-15% | 6.002546 | 25/02/1998–11/04/2000 crawling peg, band +/- 15% | 2.836713 |
| | | | | | | | |
| 01/10/1998–24/11/2005 managed float | 8.513663 | 26/02/2008–present free float | 5.314608 | 12/04/2000–present free float | 6.612350 |
| 25/11/2005–31/12/2008 peg 100% EUR +/-15% (ERM II) | 7.362501 |
| 01/01/2009 EMU | 1.779245 |

*Note: Volatility is measured as a standard deviation (based on monthly figures), effective exchange rates are taken from the OECD database.

Source: Author’s calculations based on data from the DataStream database.

Based on the results in Table 9, it is clear that exchange rate volatility increases with a more flexible exchange rate system not only in the case of nominal terms but also in real terms. Nevertheless, while nominal volatility is lower than real volatility in
a system of fixed or less flexible exchange rates, the opposite is true for flexible systems: exchange rate volatility is higher in nominal terms than in real terms. The reason behind this is the fact that in the free-float system, the reaction of the exchange rate to changes in demand/supply is immediate. On the contrary, consumer prices (used as a deflator for real exchange rate calculations) are more sticky than flexible.

**Conclusion**

The main goal of this paper was to investigate developments of exchange rate time series of Central European currencies and find evidence of some stylised facts.

First of all, we found that all the CE nominal exchange time series are not stationary, which is a confirmation of the convergence process in all the respective economies. This is even more pronounced in real terms. Turning to return time series, we confirmed the stationarity of all the series. However, normality was not proven, confirming the fat-tail distribution of exchange rate returns.

Volatility of exchange rates was investigated from several points of views. First of all, we found a time-changing volatility in all the series, i.e., heteroskedasticity was confirmed. We tried to model the volatility using GARCH models. The result is that the GARCH (1,1) model was the most suitable for modelling daily returns of all the CE currencies. We showed that the T-GARCH (1,1) model, taking asymmetry into account, is also possible for the EUR/HUF and EUR/PLN time series. However, better results were achieved using the E-GARCH (1,1) modelling approach.

Finally, we investigated the behaviour of nominal and real exchange rates under different exchange rate regimes in Central European countries. Our results confirmed that the more flexible an exchange rate regime is, the more volatile the respective currency is. This is true for both nominal and real exchange rates. We found that while nominal volatility is lower than real volatility in a system of fixed or less flexible exchange rates, the opposite is true for flexible systems: exchange rate volatility is higher in nominal terms than in real terms.

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