Two Nucleon-States in a Chiral Quark-Diquark Model

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Abstract

We study the ground and first excited states of nucleons in a chiral quark-diquark model. We include two quark-diquark channels of the scalar-isoscalar and axial-vector-isovector types for the nucleon states. The diquark correlation violating the spin-flavor SU(4)$_{SF}$ symmetry allows to treat the two quark-diquark channels independently. Hence the two states appear as the superpositions of the two quark-diquark channels; one is the nucleon and the other is a state which does not appear in the SU(4)$_{SF}$ quark models. With a reasonable choice of model parameters, the mass of the excited state appears at around 1.5 GeV, which we identify with the Roper resonance $N(1440)$.

1 Introduction

Hadron spectrum should in principle be understood by Quantum Chromodynamics (QCD). However, due to the difficulties of non-perturbative nature of QCD, in practice, QCD oriented models are often used for the description of hadrons. The non-relativistic quark model is one of successful ones[1, 2, 3, 4]. Employing a confining potential which is usually taken as a harmonic oscillator, various hadrons are described as single particle states of two (for mesons) and three (for baryons) quark states. Being not always said explicitly, the SU(4)$_{SF}$ spin-(two) flavor symmetry is usually assumed to work well, and possible interactions such as spin-color and spin-flavor interactions are treated perturbatively.

Such residual interactions may play an essential role for hadron properties by determining symmetries of hadrons. One of efficient methods which takes into account such interactions is to consider diquark correlations. In fact, it is known that the spin-color and spin-flavor interactions lead to a strongly attractive correlation for a color, flavor and spin antisymmetric quark-quark pair (scalar diquark) [5, 6]. Contrary, the color symmetric states receive repulsive correlations. If such correlations are strong enough, the resulting hadron properties will be quite different from what are expected from the SU(4)$_{SF}$ quark models.

In this paper, we would like to consider the case in which diquarks receive sufficiently strong correlation such that they are treated as independent degrees of freedom. This is the SU(2)$_{L}$×SU(2)$_{R}$ chiral quark-diquark model which we studied previously[7, 8, 9, 10].
It was shown that there are two low lying diquarks with color $\bar{3}$, one is the scalar and isoscalar ($\equiv$ scalar) diquark, and the other is the axial-vector and isovector ($\equiv$ axial-vector) diquark\footnote{11}. In a quark model picture, they are both in the ground state but with different spin and isospin configuration; the former is spin-singlet and flavor-singlet and the latter is spin-triplet and flavor-triplet ones. Adding another quark to these diquarks we can construct two states having the nucleon quantum numbers $J^P = 1/2^+$, i.e. $[1/2, 1/2]^1_0, 1/2]$ and $[(1/2, 1/2)^1_1, 1/2]^{1/2}$. In the SU(4)$_{SF}$ quark models, such two states do not exist independently, but only their linear combination is realized (the sum of the $\rho$ and $\lambda$ symmetric states)\footnote{12}. If the diquark correlation exists, these two states may be treated independently.

Having the two states independently in the model set up, the two nucleon states emerge naturally. It is known that the scalar diquark is lighter than the axial-vector diquark, whose mass difference may be related to the mass difference between the nucleon and delta, which is of order 300 MeV. Hence we have the two nucleon states with the mass difference of this order to start with. Introducing a mixing interaction between them, the two states repel each other. One would then expect additional, but not too large, mass splitting due to the mixing interaction. Altogether, we expect a mass difference about 4-500 MeV. A nucleon excited state with an excitation energy of this amount is naturally identified with the Roper resonance $N(1440)$. The Roper state appears as a spin partner of the nucleon in the sense that it has a diquark component of different spin. Hence the mass splitting is generated by residual (hyper-fine) interactions and is likely not too large. This picture is very much different from the conventional picture of the Roper resonance of the excitation of $2\hbar\omega$ of the harmonic oscillator potential for confinement\footnote{13, 14} or breathing mode of the bag model or Skrymion\footnote{15, 16, 17, 18, 19, 20, 21}. In this paper, we demonstrate such a realization of the Roper resonance as a partner of the ground state of the nucleon in the quark-diquark model with a path-integral hadronization method.

This paper is organized as follows. In $\S$2 we introduce the quark-diquark model with scalar and axial-vector diquarks. Chiral symmetry of the model is briefly discussed in the nonlinear representation. In $\S$3 the model is hadronized to obtain an effective meson-baryon Lagrangian in the path-integral method. The obtained Hamiltonian is diagonalized in the two dimensional space of the two nucleons constructed by the scalar and axial-vector channels. In $\S$4 numerical results are presented and the higher nucleon state is identified with the Roper resonance. The final section is devoted to a summary.

2 Framework

We start from the SU(2)$_L \times$ SU(2)$_R$ chiral quark-diquark model of two diquarks $\S$10,

$$
\mathcal{L} = \bar{\chi}_c (i\not{\partial} - m_q) \chi_c + D_c^\dagger (\partial^2 + M_S^2) D_c + \vec{D}_\mu^\dagger \left[(\partial^2 + M_A^2)g_{\mu\nu} - \partial_\mu \partial_\nu \right] \vec{D}_\nu + \mathcal{L}_{int},
$$

(2.1)

where $\chi_c$, $D_c$ and $\vec{D}_\mu$ are the constituent quark, scalar diquark and axial-vector diquark fields, and $m_q$, $M_S$ and $M_A$ are the masses of them. The indices $c$ represent the color. Note that the diquarks microscopically correspond to the bi-linears of two quarks $\S$; $D_c \sim \epsilon_{abc} \chi^b \chi^c$, $\vec{D}_\mu \sim \epsilon_{abc} \chi^b \gamma_\mu \gamma_5 \vec{\tau} \chi^c$.\[2]
Here $\tilde{\chi} = \chi^T C \gamma_5 i \tau_2$ and $\epsilon_{abc}$ is the antisymmetric tensor, then both the diquarks belong to color anti-triplet. The term $\mathcal{L}_{\text{int}}$ is the quark-diquark interaction, which is written as

$$
\mathcal{L}_{\text{int}} = G_S \bar{\chi}^c D_\mu^c D^\mu \chi^c + v(\tilde{\chi}^c D_\mu^c \gamma^5 \tau^\dagger \bar{\chi}^c + \tilde{\chi}^c \gamma^5 \tau^\dagger \bar{\chi}^c D_\mu^c \chi^c D_{\nu c}^c \gamma^5 \tau^\dagger \bar{\chi}^c) + G_A \bar{\chi}^c \gamma^5 \tau^\dagger \bar{\chi}^c \gamma^5 \chi^c,
$$

(2.2)

where $G_S$ and $G_A$ are the coupling constants for the quark and scalar diquark, and for the quark and axial-vector diquark. The coupling constant $v$ causes the mixing between the scalar and axial-vector channels in the nucleon wave-functions.

Since chiral symmetry is important for hadron physics, we briefly discuss the properties of our model Lagrangian (2.1) under chiral transformation. In our formulation we employ the non-linear representation of chiral symmetry[8], therefore the constituent quarks are transformed as

$$
\chi^c \rightarrow h(x) \chi^c.
$$

(2.3)

Here $h(x)$ is a local transformation depending on the global SU(2)$_L \times$ SU(2)$_R$ transformation and the pion field at $x$. Then, the baryon field written as a product of a scalar diquark and a quark, and of an axial-vector diquark and a quark are transformed as (in detail see Appendix. A)

$$
D_\mu \chi^c \rightarrow h(x) D_\mu \chi^c,
$$

(2.4a)

$$
\bar{\chi} D^\mu \gamma^5 \chi^c \rightarrow h(x) \bar{\chi} D^\mu \gamma^5 \chi^c.
$$

(2.4b)

Note that the both baryon operators are transformed in the same way as the quark is under the chiral SU(2)$_L \times$ SU(2)$_R$ transformation. In the non-linear representation, the kinetic term of the quark in Eq. (2.1) contains the mesonic currents[4]. However, we do not show them, because they are not important in the present discussions.

### 3 Hadronization and the self-energies of nucleons

To perform the hadronization procedure, we introduce the auxiliary fields for baryons

$$
\mathcal{L} = \bar{\psi} (i \partial - m_q) \chi + D_\mu^\dagger (\partial^2 + M_S^2) D + \bar{\psi} \gamma^5 \chi^c [i (\partial^2 + M_A^2) g_{\mu \nu} - \partial_\mu \partial_\nu] \bar{\psi} + \bar{\psi} \bar{G} \psi - \bar{B} \hat{G}^{-1} B.
$$

(3.1)

From here, we omit the color indices for brevity. $B = (B_1, B_2)^T$ is a two component auxiliary baryon field, whose components correspond to scalar and axial-vector channels; $B_1 \sim D \chi$ and $B_2 \sim \bar{\tau} \cdot \bar{D} \tau^\mu \gamma^5 \chi$. In Eq. (3.1) we have introduced matrix notations as

$$
\psi = \begin{pmatrix} D \chi \\ \bar{D} \tau^\mu \gamma^5 \chi \end{pmatrix}, \quad \bar{\psi} = \begin{pmatrix} \bar{\chi} D^\dagger \\ \bar{\chi} \bar{D}^\dagger \tau^\mu \gamma^5 \chi \end{pmatrix},
$$

(3.2)

$$
\hat{G} = \begin{pmatrix} G_S & v \\ v & G_A \end{pmatrix}.
$$

(3.3)

Through the hadronization procedure[8, 22, 23], the quark and diquark fields are eliminated and a meson-baryon Lagrangian in the tr log form is obtained as

$$
\mathcal{L} = -\bar{B} \hat{G}^{-1} B + i \text{Tr} \ln(1 - A).
$$

(3.4)
Here the matrix $A$ is defined by

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix},$$

(3.5a)

$$a_{11} = \Delta^T \bar{B}_1 S B_1,$$

(3.5b)

$$a_{12} = \Delta^T \bar{B}_2 \tau^i \gamma^5 S B_1,$$

(3.5c)

$$a_{21} = (\Delta_{ij}^l \rho^\nu_\gamma) \bar{T} \bar{B}_1 S_{\gamma}^\mu \gamma^5 \tau^j B_2,$$

(3.5d)

$$a_{22} = (\Delta_{ij}^l \rho^\nu_\gamma) \bar{T} \bar{B}_2 \gamma^5 \tau^i S_{\gamma}^\mu \gamma^5 \tau^j B_2.$$

(3.5e)

where $S$, $\Delta$ and $\Delta_{\mu\nu}$ are the propagators of the quark, scalar diquark and axial-vector diquark, respectively. The expansion of the $\text{tr} \log$ in Eq. (3.4) gives various terms. The first term of the expansion gives the self-energies of the nucleons as

$$\mathcal{L} = \bar{B} \begin{pmatrix} \Sigma_S(p) & 0 \\ 0 & \Sigma_A(p) \end{pmatrix} B \frac{1}{|\hat{G}|} \bar{B} \begin{pmatrix} G_A & -v \\ -v & G_S \end{pmatrix} B,$$

(3.6)

where $|\hat{G}| = \det \hat{G} = G_S G_A - v^2$. The scalar and axial-vector diquark contributions to the self-energies, $\Sigma_S$ and $\Sigma_A$, are shown diagrammatically in Fig. 1 and are computed as

$$\Sigma_S(p) = -i N_c \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M_S^2} \frac{\not{p} - \not{k} + m_q}{(p - k)^2 - m_q^2},$$

(3.7a)

$$\Sigma_A(p) = -i N_c \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{k^2 - M_A^2} \frac{g^{\mu\nu}}{(p - k)^2 - m_q^2} \delta_{ij} \gamma^5 \tau_j \frac{\not{p} - \not{k} + m_q}{(p - k)^2 - m_q^2} \gamma_\mu \gamma_5.$$  

(3.7b)

Here $N_c$ is the number of colors. Since the self-energies Eqs. (3.7) are divergent, we regularize them by the three momentum cutoff scheme [10].

![Figure 1](image.png)

Figure 1: A diagrammatic representation of the quark-diquark self-energy. The single, double and triple lines represent the quark, diquark and nucleon respectively. The blobs represent the three point quark-diquark-baryon interactions.

The self-energies $\Sigma_S$ and $\Sigma_A$ may be decomposed into the scalar and vector parts as

$$\Sigma_S(p_0) - \frac{1}{|\hat{G}|} G_A = Z^{-1}_S (p_0 \gamma^0 - \alpha_S),$$

(3.8a)

$$\Sigma_A(p_0) - \frac{1}{|\hat{G}|} G_S = Z^{-1}_A (p_0 \gamma^0 - \alpha_A),$$

(3.8b)

where we employ the nucleon rest frame $p_\mu = (p_0, \bar{0})$. The bare baryon fields $B_{1,2}$ are now renormalized as

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_S} B'_1 \\ \sqrt{Z_A} B'_2 \end{pmatrix}.$$  

(3.9)
with which the Lagrangian \((3.6)\) is reduced to
\[
\mathcal{L} = \bar{B}'(p_0\gamma^0 - \hat{M})B',
\]
(3.10)
where the mass matrix \(\hat{M}\) is given as
\[
\hat{M} = \begin{pmatrix}
a_S & -\sqrt{Z_SZ_A} v_{|G|} \\
-\sqrt{Z_SZ_A} v_{|G|} & a_A
\end{pmatrix},
\]
(3.11)
When there is no mixing interaction \((v = 0)\), \(B'_1, B'_2\) become the physical baryon fields and \(a_{S,A}\) the physical masses. On the contrary, in the presence of the mixing, the physical states are obtained after the diagonalization of the mass matrix by an unitary transformation:
\[
B' = U^\dagger N,
\]
(3.12)
\[
U\hat{M}U^\dagger = \begin{pmatrix}
M_1 & 0 \\
0 & M_2
\end{pmatrix}.
\]
(3.13)
One finds
\[
\mathcal{L} = \bar{N}_1(p_0\gamma^0 - M_1)N_1 + \bar{N}_2(p_0\gamma^0 - M_2)N_2,
\]
(3.14)
where the physical eigenvalues \(M_{1,2}\) and eigenvectors \(N = (N_1, N_2)^T\) are obtained as
\[
M_{1,2} = \frac{1}{2} \left[ a_S + a_A \pm \sqrt{(a_S - a_A)^2 + 4Z_SZ_A \left( \frac{v}{|G|} \right)^2} \right],
\]
(3.15)
\[
N_1 = \cos \phi B'_1 + \sin \phi B'_2
\]
(3.16)
\[
N_2 = -\sin \phi B'_1 + \cos \phi B'_2,
\]
(3.17)
and the mixing angle \(\phi\) is given by
\[
\tan 2\phi = \frac{2\sqrt{Z_SZ_A} v}{(a_A - a_S)|G|}
\]
(3.18)

4 Numerical results

For numerical calculations, let us first discuss model parameters. The constituent mass of the \(ud\) quarks \(m_q\) and the three momentum cutoff \(\Lambda\) are determined in such a way that they reproduce meson properties in the NJL model\[24, 25\]. The masses of the diquarks may be also calculated in the NJL model\[24, 26\]. Here we choose \(m_q=390\) MeV, \(M_S=650\) MeV, \(M_A=1050\) MeV and \(\Lambda=600\) MeV, which are within the reasonable range known from the previous study of diquarks in the NJL model. The mass difference \(M_A - M_S\) may be related to that of the nucleon and delta. In the quark-diquark models, the delta is expressed as a bound state of an axial-vector diquark and a quark, while the nucleon is a superposition of the two components:
\[
|\Delta\rangle \sim \bar{D}_\mu \chi,
\]
\[
|N\rangle \sim \cos \theta D \chi + \sin \theta \tau \cdot \bar{D}_\mu \gamma^\mu \gamma_5 \chi.
\]
Therefore, in a simple additive picture, the $N - \Delta$ mass difference can be expressed as

$$M_\Delta - M_N = \cos^2 \theta (M_A - M_S).$$

(4.1)

In deriving this relation we have ignored possible interactions such as the binding effect of the quarks and diquarks, and the interaction between the two diquark channels. However, for a rough estimation we may use the relation (4.1) and the mixing angle $\cos^2 \theta > \frac{1}{2}$, assuming that the nucleon state is rather dominated by the scalar diquark component. This qualitatively justifies the mass difference $M_A - M_S \sim 400$ MeV that we adopt. The masses of the two nucleon states are then studied in the following two cases.

(i) In the first case, $G_S$ and $G_A$ are fixed such that the binding energies of the two quark-diquark bound states become 50 MeV when there is no coupling $v$. The resulting coupling strengths are $G_S \sim 54$ and $G_A \sim 5.9$, which generates the masses $M_1 = 0.99$ GeV and $M_2 = 1.39$ GeV at $v = 0$ GeV$^{-1}$. In the previous works, this binding energy was chosen in order to obtain a reasonable size of the nucleon. The masses and the mixing angle $\phi$ of the two nucleon states are then calculated as functions of the coupling strength $v$, which are shown in Figs. 2. The effect of the $v$-coupling appears not only in the off-diagonal elements but also in the diagonal elements of the mass matrix (3.11). The effect of the $v$-coupling on the diagonal elements is from the term of $|\hat{G}|$ in Eqs. (3.8), which is shown by dashed lines of Fig. 2. As is shown in the figure, the $v$-coupling acts repulsively to both the diagonal elements $a_{S,A}$. The non-zero off-diagonal elements then split $M_1$ and $M_2$ as is shown by the solid lines of Fig. 2. The off-diagonal coupling decreases the mass of the nucleon and increases that of the heavier state, while both the two diagonal elements increase as $v$ is increased. This is why these two contributions cancel each other for the nucleon, but they are enhanced for the heavier state. We find that $M_1 = 0.99$ GeV and $M_2 = 1.44$ GeV when $v \sim 9$ GeV$^{-1}$. The mass of the second nucleon is close to that of the Roper resonance $N(1440)$. At this strength of $v$, as is shown in the right panel of Fig. 2, the mixing angle is rather small, $\phi < 10$ degree. Even at the small mixing angle, the effect of the axial-vector component is significant, since the self-energy $\Sigma_A$ is much larger than $\Sigma_S$.

(ii) As have been mentioned, the masses $M_{1,2}$ depend not only on the off-diagonal elements of the mass matrix (3.11), but also on the diagonal elements. In the second choice we determine the parameters $G_S$ and $G_A$ such that they always produce the same amount of the binding energy of 50 MeV as the coupling strength $v$ is varied, or $a_S$ and $a_A$ are fixed at $a_{S,A} = m_q + M_{S,A} - 50$ MeV. In this way we can study the effect of the off-diagonal coupling $v$ just as in a simple two level problem with fixed values of the diagonal elements, which is shown in Fig. 3. In this case dependence of $M_{1,2}$ on the values of $v$ is larger than that of the case (i). We find that $M_1 = 0.94$ GeV, $M_2 = 1.44$ GeV and $\phi = 18$ degree at $v \sim 22$ GeV$^{-1}$.

In both the cases of (i) and (ii), we can obtain the reasonable mass $M_2 = 1.4 \sim 1.5$ GeV. Hence we identify the second state with the Roper resonance.

The present identification of the Roper resonance is very much different from the conventional picture; in the quark model, it is described as an excited state of $2\hbar \omega$ with $(n, l) = (1, 0)$, where $(n, l)$ are the principle and angular momentum quantum numbers of the harmonic oscillator. The
excitation energy of such a state is as high as $2\hbar\omega \sim 1\text{ GeV}$ for the oscillator parameter $\omega \sim 0.5\text{ GeV}$, and many mechanisms have been proposed to lower the energy \cite{13, 14}. In the present picture the two nucleons are described as quark-diquark bound states, but with different diquarks with different spins, isospins and masses. In a quark model picture, the two diquarks correspond to the quark pair of the $\rho\rho$ and $\lambda\lambda$ mixed symmetry (see Appendix B). In the limit of SU(4)$_{SF}$ symmetry, these state can not be independent degrees of freedom due to the Pauli principle when constructing the nucleon state of $J^P = 1/2^+$, only their symmetric combination is allowed. In the present case, because the strong correlation between the quarks violates the SU(4)$_{SF}$ symmetry, the two states can be independent. In the picture of the harmonic oscillator basis, the two quarks of the diquarks are in the ground state but with being correlated. The energy difference is therefore supplied not by the difference in the single particle energies, but by the residual but significant correlation between the quarks. In any event, the scale of the energy of such correlations is expected to be of order of a few to several hundreds MeV. Concerning the wave functions, the two nucleon states of the quark-diquark model may be given in terms of quarks as

$$
B_1' = D\chi \sim \left[ [1/2, 1/2]^0, 1/2 \right]^{1/2},
$$

$$
B_2' = D\chi \sim \left[ [1/2, 1/2]^1, 1/2 \right]^{1/2}.
$$

From these structure, we may say that the Roper resonance appears as a spin partner of the nucleon, which has different internal spin structure.

![Figure 2: The mixing interaction $v$ dependence of the masses $M_{1,2}$ (left) and the mixing angle $\phi$ (right) with the fixed values of the quark-diquark coupling constants $G_{S,A}$. In the left panel the solid lines represent $M_1$ and $M_2$ and the dashed lines represent $a_S$ and $a_A$. In this plot, the coupling constants $G_S$ and $G_A$ are fixed as $G_S=54\text{ GeV}^{-1}$ and $G_A=5.9\text{ GeV}^{-1}$ for all range of the values of $v$.](image)

5 Summary

In this paper, we have studied the nucleon and Roper resonance using the chiral quark-diquark model. It was shown that in the chiral symmetric construction of the model, there appear two
states as the physical states with the quantum numbers of the nucleon. It was also shown that if we identified the low-lying state with the nucleon, the mass of the higher state became about $1.4 \sim 1.5$ GeV, with model parameters chosen so as to reproduce the mass difference of the nucleon and delta. Hence we consider this explanation of the Roper resonance is natural, although our current result have some dependence on the model parameters. Although some authors have studied the Roper resonance by using the lattice QCD\cite{27,28,29,30,31} the nature of the Roper resonance is still puzzle. The explanation here is different from many conventional approach, it may reveal the nature of the Roper resonance. One interesting aspect is the size of the Roper resonance. In our picture, the nucleon is expressed as a superposition of the quark and scalar diquark bound state and the quark and axial-vector diquark bound state, while the Roper resonance is another superposition orthogonal to the nucleon state. Then, the difference between the wave-functions of the quark and scalar diquark bound state and that of the quark and axial-vector diquark bound state would make that of the nucleon and Roper resonance different. It is interesting to study the effect of the change in the wave-functions on various properties, such as radii, the decay width and so on.

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A Chiral transformation of baryon operators

Here we explicitly show the transformation of the baryon operators under the group $SU(2)_L \times SU(2)_R$. The constituent quark field is defined through the Weinberg rotation\cite{4, 8}

$$\chi = \xi_5 q,$$  \hspace{1cm} (A.1)
where $\xi_5 = \exp(i\gamma_5 \vec{\Phi}/2f_\pi)$ with the pion field $\vec{\Phi}$. Using the right- and left-handed spinors $q_R$ and $q_L$ of linear chiral representations, Eq. (A.1) is written as

$$\chi = \begin{pmatrix} \xi q_R \\ \xi^\dagger q_L \end{pmatrix}, \quad \text{(A.2)}$$

where $\xi = \exp(i\vec{\tau} \cdot \vec{\Phi}/2f_\pi)$. Here $\xi$ and $\xi_5$ are related to each other

$$\xi_5 + \xi_5^\dagger = \xi + \xi^\dagger, \quad \text{(A.3a)}$$

$$\xi_5 - \xi_5^\dagger = \gamma_5(\xi - \xi^\dagger). \quad \text{(A.3b)}$$

Under the group $SU(2)_L \times SU(2)_R$, $q_R$, $q_L$ and $\xi$ are transformed as

$$q_{R,L} \rightarrow g_{R,L} q_{R,L}, \quad \text{(A.4)}$$

$$\xi \rightarrow g_L \xi h^\dagger = h \xi g_R^\dagger. \quad \text{(A.5)}$$

then the transformations of the constituent quark field and its Dirac and charge conjugated fields are given as

$$\chi \rightarrow h \chi, \quad \text{(A.6a)}$$

$$\bar{\chi} \rightarrow \bar{\chi} h, \quad \text{(A.6b)}$$

$$\tilde{\chi} \rightarrow \tilde{\chi} h^\dagger. \quad \text{(A.6c)}$$

Then, the two diquark states are transformed as

$$\tilde{\chi}\chi \rightarrow \tilde{\chi}\chi, \quad \text{(A.7a)}$$

$$\tilde{\chi}\gamma_\mu\gamma_5 \tau^a \chi \rightarrow R^{ab}(x) \tilde{\chi}\gamma_\mu\gamma_5 \tau^b \chi. \quad \text{(A.7b)}$$

Here $R^{ab}(x)$ is the three dimensional rotation matrix, which is defined by $h^\dagger \tau^a h = R^{ab}(x) \tau^b$. Finally, the transformations of the baryon operators Eqs. (2.4) are obtained as

$$D\chi \rightarrow h(x) D\chi, \quad \text{(A.8a)}$$

$$D^{a_\mu\gamma_5 \tau^a}_\mu \chi \rightarrow h(x) D^{a_\mu\gamma_5 \tau^a}_\mu \chi. \quad \text{(A.8b)}$$

**B Non Relativistic limit**

It is useful to consider the connection to the non-relativistic quark model. The interaction Eq. (2.2) may be rewritten as

$$\mathcal{L}_{\text{int}} = a\bar{\chi}(\cos \theta D^\dagger + \frac{1}{3} \sin \theta \gamma^\mu \gamma^5 \vec{\tau} \cdot \vec{D}_\mu^\dagger) \left(\cos \theta D + \frac{1}{3} \sin \theta \vec{\tau} \cdot \vec{D} \right) \bar{\chi}(\cos \theta D + \frac{1}{3} \sin \theta \vec{\tau} \cdot \vec{D} \gamma^5) \chi$$

$$+ b\bar{\chi}(-\sin \theta D^\dagger + \frac{1}{3} \cos \theta \gamma^\mu \gamma^5 \vec{\tau} \cdot \vec{D}_\mu^\dagger)(-\sin \theta D + \frac{1}{3} \cos \theta \vec{\tau} \cdot \vec{D} \gamma^5) \chi, \quad \text{(B.1)}$$
with the relations between the parameters

\begin{align}
 a \cos^2 \theta + b \sin^2 \theta &= G_S, \\
 a \sin^2 \theta + b \cos^2 \theta &= 9G_A, \\
 (a - b) \sin \theta \cos \theta &= 3v.
\end{align}

(B.2a)

(B.2b)

(B.2c)

Here we note that the baryon fields, in terms of a quark and a diquark, are related to the nucleon wave-functions of the constituent quark model by

\begin{align}
 D\chi &= 2\phi\rho\chi\rho, \\
 \vec{D}\nu \cdot \vec{\tau}\gamma^0\gamma_5\chi &= 6\phi\lambda\chi\lambda,
\end{align}

(B.3a)

(B.3b)

in the non-relativistic limit, where \(\phi\rho, \phi\lambda\) and \(\chi\rho, \chi\lambda\) are the standard three quark spin and isospin wave-functions [4, 12]. By the use of these expressions, the interaction Eq. (B.1) is reduced to

\[
 L_{\text{int}} = 2a(\cos \theta \phi\rho\chi\rho + \sin \theta \phi\lambda\chi\lambda)(\cos \theta \phi\rho\chi\rho + \sin \theta \phi\lambda\chi\lambda)
 + 2b(-\sin \theta \phi\rho\chi\rho + \cos \theta \phi\lambda\chi\lambda)(-\sin \theta \phi\rho\chi\rho + \cos \theta \phi\lambda\chi\lambda).
\]

(B.4)

If we take tan \(\theta = 1\), we realize SU(4)\(\text{SF}\) symmetry for the first term in Eq. (B.4), while the second term become the SU(4)\(\text{SF}\) forbidden state. Since we employ the diquark correlations violating SU(4)\(\text{SF}\) symmetry, the SU(4)\(\text{SF}\) forbidden state can be included. Then, the two channel type interaction Eq. (2.2) gives the two bound states of a quark and a diquark.

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