The robust-yet-fragile nature of interdependent networks

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Interdependent networks have been shown to be extremely vulnerable based on the percolation model. Parshani et. al further indicated that the more inter-similar networks are, the more robust they are to random failure. Our understanding of how coupling patterns shape and impact the cascading failures of loads in interdependent networks is limited, but is essential for the design and optimization of the real-world interdependent networked systems. This question, however, is largely unexplored. In this paper, we address this question by investigating the robustness of interdependent ER random graphs and BA scale-free networks under both random failure and intentional attack. It is found that interdependent ER random graphs are robust-yet-fragile under both random failures and intentional attack. Interdependent BA scale-free networks, however, are only robust-yet-fragile under random failure but fragile under intentional attack. These results advance our understanding of the robustness of interdependent networks significantly.

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I. INTRODUCTION

We live in a modern society supported by many critical networked infrastructures such as power grids, communication and transportation systems [4]. The robustness of systems has been undoubtedly the central issue [2–4]. Over the past decade, cascades on complex networks have thus been widely explored [3, 6]. It has been revealed that scale-free networks are robust to random failure but fragile to intentional attacks, whereas the random graph is robust to both [5]. Xia et al. further indicated that the robust-yet-fragile (RYF) property in cascades scenario is principally associated with the heterogeneity of network betweenness distribution rather than degree distribution [6]. Small-world networks, for example, are also robust-yet-fragile although its degree distribution is homogenous.

The aforementioned conclusions, however, are mainly derived based on the framework of isolated networks. Actually, various networked infrastructures interact with each other significantly. Some coupled network models have been thus established to further capture real-world networked systems [4, 7, 8]. Recently, interdependent networks have been formalized to explore the robustness of the systems in which two different subsystems have to depend on each other [4]. In other words, the failure of nodes in one network will trigger the counterpart ones in the other network to collapse accordingly. Buldyrev et al. has showed that such dependency between networks significantly increases the vulnerability of networks so that interdependent networks are even vulnerable to random failure. Based on the percolation theory, deeper studies have focused on the effect of coupling pattern [10, 11] and network structures [12] on the robustness of interdependent networks. Such conceptual breakthrough shift from the isolation towards the interaction allows us to understand real-world networks more readily. As another form of interaction, interconnected networks have also been developed to capture the coupling between two originally isolated networks [5, 13]. Different from dependency links between two parts for interdependent networks, the interconnected links across two networks play as the same role as the connectivity within networks. For cascades of loads in interconnected networks, the optimal coupling pattern and/or coupling probability could be found.

Recently, traffic overload has been extended to the cascades on interdependent networks. Undoubtedly, such dependency will aggravate cascading failures of loads in interdependent networks [14]. Nevertheless, is the extent to which network robustness against cascades of loads is undermined by the dependency relation always large enough? In this paper, we will try to answer this question by considering network structures, attack strategies and the coupling mechanisms.

II. MODELS

A. Network model

For simplicity and clarity of the results, our model considers two networks A and B with the same size \( N = N_A = N_B \) and same average degree \( \langle k \rangle = \langle k_A \rangle = \langle k_B \rangle \). Here networks A and B are assumed to fully coupled, and each node has only one interdependent link. Different from interconnected networks, the coupling links just refer to the dependency relationships between two networks. As mentioned in previous studies, the way in which the coupling links are established has great impact on the robustness of coupled networks. As we focus on how traffic overload leads to cascading failures, three coupling patterns are thus described based on load dis-
B. Traffic model

In this paper, we adopt the data-packet transport scenario based on the shortest-path routing \[3, 13\]. The node betweenness can thus approximate the traffic load. To be concrete, the betweenness of node \( k \) is denoted as \[15\] 

\[ B_k = \sum_{s \neq k \neq t} \frac{n_{st}^k}{g_{st}}, \quad (1) \]

where \( g_{st} \) is the total number of possible shortest topological paths from node \( s \) to node \( t \) and \( n_{st}^k \) denotes the number of such paths running through node \( k \). \( n_{st}^k / g_{st} \) is stipulated to be zero, if \( n_{st}^k = g_{st} = 0 \).

The capacity of a node is the maximum load that the node can handle. The capacity of node \( k \) is thus set to be proportional to its initial load \( L_k \) \[8, 3, 16\]

\[ C_k = (1 + \alpha) L_k, \quad (2) \]

where the constant \( \alpha \) is the tolerance parameter. The network is in the free-flow state (without overload) if \( \alpha \geq 0 \).

When one node is removed from the network (due to a failure or attack), it will trigger two kinds of effects. On the one hand, the removal affects the shortest paths between some other nodes within the attacked network (e.g., network A). Then the packets from these nodes have to adjust their paths. In this way, loads of many nodes are changed. Those nodes with loads greater than their capacities will fail, then it triggers further load adjustment over and over until loads of all remaining nodes are less than their capacities. It is obvious that the tolerance parameter \( \alpha \) affects the effect of cascading failures. On the other hand, the removed node in network A will cause the dependent counterpart node in network B to fail. The failure of this node will cause the same cascades within network B. Actually, these two effects interact with each other in cascading failures. Note that nodes outside the giant component are assumed to be failed. Therefore, the dependency relationship makes the cascading failure process more complicated compared to isolated networks.

In this paper, two attack strategies will be explored, i.e., random failure and intentional attack. We use the relative size of the giant component \( G \) to characterize the robustness of networks. In our model, the remaining giant component after cascading failures in two subnetworks share the same size. Thus, \( G \) is the ratio between the remaining giant component size and original network size.

III. RESULTS

In this section, we will carry out numerical simulations on coupled ER random graphs and coupled BA scale-free networks. We mainly explore four kinds of situations, i.e., (1) coupled ER random graphs under random failure, (2) coupled ER random graphs under intentional attack, (3) coupled BA scale-free networks under random failure, (4) coupled BA scale-free networks under intentional attack.

We first investigate the robustness of interdependent ER random graphs under random failure. As ER random graph is robust to random failure, and BA scale-free network is fragile to intentional attack, we view them as the baselines. First, we find that the dependency relationship between two networks reduces the network robustness. As shown in fig. 1, under random failure, the robustness of interdependent networks under three coupling mechanisms is worse than that of the single networks. Second, the coupling patterns influence the robustness of interdependent networks remarkably. It is shown that the assortative > the random coupling > the disassortative coupling, in terms of network robustness. Note that the robustness of interdependent random graphs with assortative coupling approaches that of the single ER random graph under random failure. Interdependent ER random graphs with the assortative coupling can thus be regarded as robust. Nevertheless, it is obvious that the disassortative coupling is fragile to random failure. They
FIG. 1. (Color online) Cascading failures of coupled ER random graphs under random failure. The network size $N_A = N_B = 1000$ and average degree $\langle k_A \rangle = \langle k_B \rangle = 6$. The relative size of the giant component $G$ is shown as a function of the tolerance parameter $\alpha$ with different types of coupling patterns, namely, assortative coupling (■), random coupling (▲) and disassortative coupling (●). Cascading failures of the single ER random graph under random failure (♦) and the single BA scale-free network under intentional attack (•). It is obvious that the disassortative coupling is fragile to intentional attack. They are even more vulnerable than the single BA scale-free networks under intentional attack. Consider, for example, the tolerance $\alpha = 0.25$ in which coupled networks are still fragmented. When $\alpha$ reaches 0.4, $G$ is still less than 0.9. Thus, similarly to random failure, interdependent ER random graphs remains robust-yet-fragile under intentional attack.

Take the above two situations together, under both random failure and intentional attack, interdependent ER random graphs are robust for the assortative coupling but fragile for the disassortative coupling. Nevertheless, it is obvious that the disassortative coupling is fragile to intentional attack. They are even more vulnerable than the single BA scale-free networks under intentional attack. Consider, for example, the tolerance $\alpha = 0.2$ in which coupled networks are still fragmented. When $\alpha$ reaches 0.4, $G$ is still less than 0.9. Thus, similarly to random failure, interdependent ER random graphs remain robust-yet-fragile under intentional attack.

Take the above two situations together, under both random failure and intentional attack, interdependent ER random graphs are robust for the assortative coupling but fragile for the disassortative coupling. Thus, interdependent ER random graphs are robust-yet-fragile. We further explore the robustness of interdependent BA scale-free networks under random failures. As BA scale-free network is robust to random failure but fragile to intentional attack, we view them as the baselines. First, we find that the dependency relationship between two networks reduces the network robustness. As shown in fig. 2, under intentional attack, the robustness of interdependent ER random graphs under three coupling mechanisms is worse than that of the single ER random graph. Second, the coupling patterns influence the robustness of interdependent networks remarkably. It is shown that the assortative > the random coupling > the disassortative coupling, in terms of network robustness. Note that the robustness of the interdependent network with assortative coupling approaches that of the single network under intentional attack. Interdependent ER random graphs with the assortative coupling can thus be regarded as robust. Nevertheless, it is obvious that the disassortative coupling is fragile to intentional attack. They are even more vulnerable than the single BA scale-free networks under intentional attack.

FIG. 2. (Color online) Cascading failures of coupled ER random graphs under intentional attack. The network size $N_A = N_B = 1000$ and average degree $\langle k_A \rangle = \langle k_B \rangle = 6$. The relative size of the giant component $G$ is shown as a function of the tolerance parameter $\alpha$ with different types of coupling patterns, namely, assortative coupling (■), random coupling (▲) and disassortative coupling (●). Cascading failures of the single ER random graph (♦) and BA scale-free network (•) under intentional attack.
FIG. 3. (Color online) Cascading failures of coupled BA scale-free networks under random failure. The network size $N_A = N_B = 1000$ and average degree $\langle k_A \rangle = \langle k_B \rangle = 6$. The relative size of the giant component $G$ is shown as a function of the tolerance parameter $\alpha$ with different types of coupling patterns, namely, assortative coupling (■), random coupling (▲) and disassortative coupling (∗). Cascading failures of the single BA scale-free network under random failure (♦) and intentional attack (•).

FIG. 4. (Color online) Cascading failures of coupled BA scale-free networks under intentional attack. The network size $N_A = N_B = 1000$ and average degree $\langle k_A \rangle = \langle k_B \rangle = 6$. The relative size of the giant component $G$ is shown as a function of the tolerance parameter $\alpha$ with different types of coupling patterns, namely, assortative coupling (■), random coupling (▲) and disassortative coupling (∗). Cascading failures of the single BA scale-free network under random failure (♦) and intentional attack (•).

Collecting these four situations (as shown in the Ta-
ables I, II, and III, we find that interdependent ER random graphs are robust-yet-fragile under both random failure and intentional attack. Interdependent BA scale-free networks are robust-yet-fragile under random failure but fragile under intentional attack. Note that these results are different from the extreme vulnerability of interdependent networks based on the percolation theory.

IV. CONCLUSIONS

In this paper, we have explored cascades of loads in interdependent networks. The interplay of traffic overload and dependency makes the robustness of interdependent networks more complicated than the single network. For interdependent ER random graphs under both random failure and intentional attack, they are robust for the assortative coupling but fragile for the disassortative coupling. For interdependent BA scale-free networks, under random failure, they are robust for the assortative coupling but fragile for the disassortative coupling, whereas under intentional attack, they are fragile for both. Theoretically, the robust-yet-fragile property can refresh our understanding of the robustness of interdependent networks. Furthermore, our results can provide useful insights for the design and optimization of real-world interdependent networked systems.

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