Quenching of quantum Hall effect and the role of undoped planes in multilayered epitaxial graphene

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We propose a mechanism for the quenching of the Shubnikov de Haas oscillations and the quantum Hall effect observed in epitaxial graphene. Experimental data show that the scattering time of the conduction electron is magnetic field dependent and of the order of the cyclotron orbit period, i.e. can be much smaller than the zero field scattering time. Our scenario involves the extraordinary graphene n = 0 Landau level of the uncharged layers that produces a high density of states at the Fermi level. We find that the coupling between this n = 0 Landau level and the conducting states of the doped plane leads to a scattering mechanism having the right magnitude to explain the experimental data.

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Graphene is a two dimensional carbon material which takes the form of a planar honeycomb lattice of sp² bonded atoms. Its two dimensional character and the linear dispersion relation implies that, close to the charge neutrality point, the electrons obey an effective massless Dirac equation. The properties of electrons in graphene, deriving from the Dirac equation, are fundamentally different from those of electrons in standard semi-conductors which obey the Schrödinger equation. A remarkable example is the quantum Hall effect which is quantized with integer plus half values [1, 2] and has even been observed at room temperature [3]. Another major fact is the large electronic coherence [4, 5], that is observed even at room temperature, and gives the hope to produce devices with new properties.

Graphene can be produced by mechanical exfoliation of graphite [1, 2] which produces either single or few layer graphene. Another way of producing graphene is by thermal decomposition of the surface of SiC [4, 5, 6]. Epitaxial graphene is attractive because it grows on an insulating substrate, with a high structural quality and very smooth graphene layers [7]. Both methods produce graphene samples with spectacular electron coherence properties. Yet magnetotransport remains puzzling in epitaxial graphene. On one hand, at low magnetic field, the experimental results indicate electronic mean free paths of more than half a micron at 4K [3, 8]. On the other hand Shubnikov de Haas oscillations of the magnetoresistance are weak and the quantum Hall effect is not observed. Here we show that the small amplitude of the Shubnikov de Haas oscillations is not a signature of a low quality graphene. We show, by analysing the experimental data, that the results can be explained by a scattering time of the conduction electron which is magnetic field dependent, and reduced to the order of the cyclotron orbit period. We argue that the conducting states in a doped layer can couple to the zeroth Landau levels in an undoped layer, which is on top of the doped one. As a consequence the conducting electrons of the doped layer are subjected to a scattering mechanism that increases with magnetic field, because the number of states in the zeroth Landau level increases with magnetic field. At low magnetic field the scattering time τ can be long but at stronger field τ decreases in such a way that ωτ ≃ 1, where ω is the cyclotron frequency. This forbids the observation of strong Shubnikov de Haas oscillations. We show also that in some limits the magnetoresistance increases linearly with the magnetic field. This has been observed recently in epitaxial graphene multi-layers and the magnitude of the linear magnetoresistance is in quantitative agreement with our model.

The letter is organized as follows: we present first the experimental results, and show that they can be explained by a linear dependence of the scattering rate with the magnetic field. Based on experimental results we propose a simple model of electronic structure and derive the equations satisfied by the Green’s functions, in the Self-Consistent Born Approximation. Finally we analyse the electronic structure of this model and show that its magnetotransport properties are in good agreement with the experimental observations.

Summary of previous experimental results

Electronic properties of epitaxial graphene have been analyzed in great detail. They clearly point to the existence of a electron doped plane at the SiC/graphene interface [8, 10] carrying the main part of the current [5]. Since the screening length is of 1-2 interlayer spacing, only 2-3 planes are doped, and the other planes are quasi-neutral, as shown in infra-red spectroscopy [11] and thus comparatively poor conductors. The experimental evidence that one of the doped planes carries the main part of the current suggests that the other doped planes have a low conductivity due to the impurities spatial distribution. The doped and quasi-neutral planes all have the characteristics of an isolated graphene plane i.e. massless Dirac electrons [5, 3, 10, 11]. This was initially surprising...
and is so linear with $B$, we can deduce from experiments:

$$\bar{h}\tau(B) \simeq \bar{h}\tau(B = 0) + \gamma\bar{h}\omega$$

(1)

where $\gamma$ is a positive constant. Note that $1/\tau(B) > 1/\tau(B = 0)$, and $\omega\tau < 1/\gamma$ (with $\gamma$ equal to 0.4, 0.45 and 1.25 for the present $1\mu m \times 5\mu m$, $0.27\mu m \times 6\mu m$ and $100\mu m \times 1000\mu m$ samples). Moreover, we clearly see (figure 1) that $\omega\tau$ is directly related to the system’s ability to exhibit Shubnikov de Haas oscillations: At low $\omega\tau$, the system does not exhibit such oscillations, while higher values are concomitant with the appearance of well separated Landau-level in the electronic structure. A scattering rate $1/\tau$ linear with $B$ can thus explain both the unusual magnetoresistance behaviour and the quenching of Shubnikov de Haas oscillations. If one assumes that the coupling between the different planes is negligible it might seem quite difficult to explain a scattering rate that varies linearly with the magnetic field. Indeed, the conventional theory only predicts a quadratic correction to the magnetoresistance [18]. Yet, a scattering mechanism due to coupling between the doped and undoped planes will depend on the magnetic field since the electronic structure of the undoped planes depends on the magnetic field. Even in weak magnetic fields the Fermi level in the nearly neutral planes will be in the low indexes Landau levels and it will be in the zeroth Landau level as soon as $B > B_C = \hbar n_d / 2e$ where $n_d$ is the electron density. Infrared Landau spectroscopy for the nearly undoped planes indicate $n_d$ is less than $10^{10} cm^{-2}$, i.e. $B_C << 1T$ [11]. We will see that this scattering mechanism implies an increasing scattering rate with an order of magnitude of $\Delta(\bar{h}/\tau(B)) \simeq \bar{h}\omega$, with $\gamma$ close to 1, compatible with the experiments.

Model of electronic structure

We consider that the perfect rotationally stacked planes are essentially decoupled since the coupling between different states close to the Dirac point is of the order of $1meV$ (compared to $0.2 - 0.3eV$ for A-B stacking) [12]. It is important to note that the hopping matrix elements between neighboring orbitals of the two rotated planes are of the order of $0.2 - 0.3eV$, and that the effective electronic decoupling is due to an averaging specific to the perfect structure. This averaging is destroyed by disorder, and consequently the defects in one plane will introduce a scattering in that plane and a coupling with electronic states in the other plane. Defects that spread on both planes also introduce in-plane scattering and interplane coupling. Here we consider a model with one doped plane and one plane which is essentially neutral, for which the zeroth Landau level is half-filled at all values of the magnetic field. The two planes are decoupled in the absence of disorder but are coupled by disorder. In the following plane "1" is the doped plane and plane "2" the undoped one.

We consider also that the electronic states in plane "1" are coupled essentially to the zeroth order Landau level of plane "2" only. Indeed we note that for $B = 1T$ the index of the Landau levels of the doped plane is $n \simeq 30 - 40$ and there are about $2N_L \simeq 15$ Landau levels of the doped plane which are closer to the zeroth Landau level than to the other Landau levels of plane "2". In the following
we shall treat $N_L$ as a large number.

The model for the Green’s functions of the uncoupled and perfect planes ”1” and ”2” are $G_{1,0}(z)$ and $G_{2,0}(z)$:

$$G_{1,0}(z) = \sum_{n=-N_L}^{n=N_L} \frac{N(B)}{z - n\hbar\omega}, \quad G_{2,0}(z) = \frac{RN(B)}{z - E_{L0}} \quad (2)$$

with $\hbar\omega = \frac{eBV_F}{k_F}$ and $\frac{N(B)}{\hbar\omega} = n_0 = \frac{2k_F}{\sqrt{\pi}h}$, $k_F$ is the Fermi wavevector of the doped plane and $n_0$ its density of states at the Fermi energy, without magnetic field. In this model we assume that the Landau levels of plane ”1” are equally spaced by $\hbar\omega$ where $\omega$ is the cyclotron frequency (which is valid as long as their index is high). $E_{L0}$ is the energy of the zeroth Landau level ($E_{L0} << N_L\hbar\omega$ varies with $B$) so that the zeroth Landau level stays half-filled, and $R$ is the ratio between the number of states in the zeroth Landau level of plane ”2” and in a Landau level of plane ”1”. $R$ is equal to 1 if the two planes are equivalent (apart from the doping). We can also simulate the case where the doped plane is coupled to two undoped planes by taking $R = 2$. The degeneracy of a zeroth Landau level of a bilayer in AB stacking is also $R = 2$ [10]. Thus different configurations can lead to different values of $R$ and we let this parameter free in the following.

In order to treat the effect of in-plane scattering by disorder and intra-plane coupling by disorder we use a standard Self-Consistent Born Approximation (SCBA). We introduce the Green’s function $G_P(z)$ and the self-energies $\Sigma_P(z)$ of plane ”P” ($P = 1$ or $P = 2$). The density of states per unit surface in plane $P$ is given by $n_P(E) = -1/\pi Im(G_P(z = E + i\epsilon))$, where $\epsilon$ is an infinitely small positive real number. One get:

$$G_P(z) = G_{P,0}(z - \Sigma_P(z)) \quad (3)$$

$$\Sigma_P(z) = |V|^2 G_P(z) - i\frac{\hbar}{2\tau_{P,P}} \quad (4)$$

$P'$ is the plane coupled to plane $P$ ($P' = 1$ if $P = 2$ and vice-versa). $V^2$ is an average value of the square coupling between states in plane ”1” and in plane ”2”. The terms $\hbar/2\tau_{P,P}$ represent the effect of in-plane scattering for each plane $P$ ($\tau_{P,P}$ is the in-plane scattering time).

In [2] and [11] the two important parameters for the effect of the interplane coupling are $R$ and $V$. We analyse below the regimes of large and small $R$ or $V$. We show that in the large $V$ limit the experimental magnetic field dependence of the scattering rate (see figure [11]) are explained with a reasonable value of the parameter $R$. We emphasize that the physics described below presents some analogies with the coupling between localized $d$-orbitals with extended $sp$-states [20].

Large and small $R$ regimes

In this part, we focus on the case $z = E_{L0} + i\epsilon$. Since the zeroth Landau level is half-filled, $E_{L0}$ is always close to the Fermi level. Let us assume that $G_1(z = E_{L0} + i\epsilon) \approx -i\pi n_0$ and thus $n_1(E_{L0}) \simeq n_0$ (we show below that this corresponds to the large $R$ limit). After [2] and [11], this occurs for $|Im\Sigma_1(E_{L0} + i\epsilon)/\hbar\omega| >> 1$. In this limit, the real parts $ReG_2(E_{L0} + i\epsilon), ReG_1(E_{L0} + i\epsilon), Re\Sigma_2(E_{L0} + i\epsilon), Re\Sigma_1(E_{L0} + i\epsilon)$ are all negligible. Using the correspondence $\hbar/\tau = 2Im\Sigma$, where $\tau$ is an electron lifetime, one may write the SCBA equations in a form similar to the Fermi Golden Rule i.e. $\frac{\hbar}{\tau_P} = \frac{\hbar}{\tau_{P,P}} + 2\pi V^2 n_P$ with $n_1 \approx n_0$ and $n_2 = RN(B)\frac{2\pi}{\hbar}$ the densities of states at $z = E_{L0} + i\epsilon$ and then:

$$\frac{\hbar}{\tau_p} \approx \frac{\hbar}{\tau_{1,1}} + \frac{2R}{\pi} \frac{\hbar\omega}{1 + \alpha} \quad (5)$$

with $\alpha = \frac{\hbar/\tau_{2,2}}{2\pi V^2 n_0}$. Here $\hbar/\tau_{2,2}$ and $2\pi V^2 n_0$ are respectively the width of the zeroth Landau level due to disorder in plane 2 and to coupling with plane ”1” in the limit where its density is $n_0$. One sees that if disorder in plane ”2” (term $\hbar/\tau_{2,2}$) increases then the scattering rate $\hbar/\tau_1$ decreases. Indeed the scattering by plane ”2” is favored by a strong density of states in plane ”2”, whereas the term $\hbar/\tau_{2,2}$ tends to decrease this density. If $\alpha >> 1$, the coupling between planes ”1” and ”2” has essentially no effect (i.e. $\omega_{\tau_1} \simeq \omega_{\tau_1}$) but in the opposite limit $\alpha << 1$ the scattering rate for electrons in plane ”1” increases linearly with the magnetic field and $\omega_{\tau_1} \leq \pi/2R$.

We show now that one get the regime $G_1(z = E_{L0} + i\epsilon) \approx -i\pi n_0$ at sufficiently large values of $R$. For simplicity we consider the limit $\hbar/\tau_{2,2} = \hbar/\tau_{1,1} = 0$, but let us just note that $\hbar/\tau_{1,1}$ and $\hbar/\tau_{2,2}$ have opposite effects, since $\hbar/\tau_{1,1}$ favors the large $R$ regime while $\hbar/\tau_{2,2}$ favors a small $R$ regime. The SCBA equations can be written with dimensionless quantities $F_P = \frac{F_P}{\hbar\omega}, \frac{E_{L0}}{\hbar\omega}, \frac{2\pi V^2 n_0}{\hbar\omega}, R$, where $F_P = \frac{G_P(z)}{\hbar\omega}$. At $z = E_{L0} + i\epsilon$ one get for $G_1(z)$, after [2] and [11], $G_1(z) = G_{1,0}(z - R\frac{N(B)}{\hbar\omega})$. This equation is independent of the coupling parameter $V$, and in that case $n_1/n_0$ and $2Im\Sigma_1/\hbar\omega = 1/\omega_{\tau_1}$ are functions only of $E_{L0}/\hbar\omega$ and $R$.

We consider $E_{L0}/\hbar\omega = 0$ and $E_{L0}/\hbar\omega = 1/2$ for which by symmetry $E_{L0} = E_F$ and the real parts of the self-energies and Green’s functions are zero. The result is shown in figure [11]. For large $R$ typically $R \gtrsim 1.5$ one has $n_1(E_F)/n_0 \approx 1$ which is the criterion for the strong scattering regime. In that case one recovers the strong scattering limit $\hbar/\tau_1 \approx 2\hbar/\tau_0$ that is $\omega_{\tau_1} \approx \pi/2R$. Note also that in the large $R$ regime the results are essentially independent of $E_{L0}/\hbar\omega$. At $R = 1$ (i.e. not in the large $R$ regime) and for $E_{L0}/\hbar\omega = 0$ the density of states $n_1(E_F)$ diverges. Indeed as soon as $R < 1$ there
levels of plane "1" and "2" are only slightly hybridized.

We define the large and small $V$ regimes respectively by $2\pi V^2 n_0/\hbar \omega > 1$ and $2\pi V^2 n_0/\hbar \omega << 1$. After the dimensional analysis the width $W$ of the zeroth Landau level of plane "2" and thus the energy range on which the electronic structure is modified by the coupling satisfies $W/\hbar \omega = W(E_{L0}/\hbar \omega, V^2 n_0/\hbar \omega, R)$. In the large $V$ regime, as long as $|z - E_{L0}|/\hbar \omega \leq 2\pi V^2 n_0/\hbar \omega$ the term $|z - E_{L0}| << |V^2 G(z)|$ and one recovers $G(z) = G_{1,0}(z - R \frac{\Sigma(z)}{\omega z(z)})$. The dimensionless $G_{1}(z)/n_0$ and $\Sigma_{1}(z)/\hbar \omega$ depend on $z/\hbar \omega$ and $R$ but not on $E_{L0}/\hbar \omega$ and $2\pi V^2 n_0/\hbar \omega$. Note that the periodicity $\hbar \omega$ of $G_{1,0}(z = E + i\epsilon)$ implies the same periodicity for $G_{1}(z = E + i\epsilon)$ and $n_1(E)$ in this limit. For the density of states in plane "2" one has after $n_2(E)/n_0 = (R/\pi^2)(\hbar \omega/V^2 n_0)(n_0/n_1(E))$. Thus $n_2(E)/n_0$ presents the same periodicity (within the range $W$) and is very small. The spectral weight $R N(B)$ of the zeroth Landau level spreads on a width $W$ and thus since $n_2(E)/n_0 \approx (R/\pi^2)(\hbar \omega/V^2 n_0)$ one has $W/\hbar \omega \approx \pi^2 V^2 n_0/\hbar \omega$. Finally in the small$V$ regime $2\pi V^2 n_0/\hbar \omega << 1$ the width $W/\hbar \omega << 1$ and the Landau levels of plane "1" and "2" are only slightly hybridized.

Magnetotransport

When the density of states is uniform on an energy scale $W \gg \hbar \omega$, which is the case in the large $R$ and $V$ regime (see figure 3), we can apply the semiclassical theory of transport. The scattering time $\tau_1$ is given by $\hbar/\tau_1 = h/\tau_{1,1} + 2h \omega R/\pi(1 + \alpha)$ where $h/\tau_{1,1}$ is the in-plane scattering rate. As long as $\alpha$ is not too large we expect the term $2h \omega R/\pi(1 + \alpha)$ to be of order $\hbar \omega$ and the model is consistent with the experimental results presented above (see figure 1 and equation 1). Shubnikov de Haas oscillations can occur when the field dependent scattering studied here is destroyed, that is for $\alpha >> 1$. This can be due for example to disorder in plane "2" or even to a confinement effect as in a ribbon of finite width 21, as in figure 1. Shubnikov de Haas oscillations can thus be enhanced by disorder or by confinement effects. This spectacular effect is clearly observed in experiments 4.

To conclude we have shown that magnetotransport experiments on epitaxial graphene are consistent with a scattering time that is magnetic field dependent and is reduced to the order of the cyclotron period. This explains the unusual variation of the magnetoresistance, the quenching of the Shubnikov de Haas oscillations and of the quantum Hall effect. The magnitude of the field dependent scattering time is consistent with a mechanism where the conducting electrons of the doped plane are scattered due to their coupling with the zeroth Landau level of the undoped planes.

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