Interacting dark matter and modified holographic Ricci dark energy induce a relaxed Chaplygin gas

Luis P. Chimento\textsuperscript{1} and Martín G. Richarte\textsuperscript{1}

\textsuperscript{1}Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina

(Dated: December 30, 2011)

We investigate a model of interacting dark matter and dark energy given by a modified holographic Ricci cutoff in a spatially flat Friedmann-Robertson-Walker (FRW) space-time. We consider a nonlinear interaction consisting of a significant rational function of the total energy density and its first derivative homogeneous of degree one and show that the effective one-fluid obeys the equation of state of a relaxed Chaplygin gas. So that, the universe is dominated by pressureless dark matter at early times and undergoes an accelerated expansion in the far future driven by a strong negative pressure. We apply the $\chi^2$-statistical method to the observational Hubble data and the Union2 compilation of SNe Ia for constraining the cosmological parameters and analyze the feasibility of the modified holographic Ricci cutoff. By using the new $\Omega_m$ diagnostic method, we find that the effective model differs substantially from the $\Lambda$–CDM one, because it gets the accelerated expansion faster than the $\Lambda$–CDM model. Finally, a new model with a third component decoupled from the interacting dark sector is presented for studying bounds on the dark energy at early times.

PACS numbers:
Keywords: interaction, modified Ricci’s cutoff, dark energy, dark matter, relaxed Chaplygin gas

I. INTRODUCTION

The holographic principle states that the maximum number of degrees of freedom in a volume should be proportional to the surface area \[\rho_L \leq \Lambda L^2\], where $\rho_L$ corresponds to the quantum zero-point energy density \[\rho_L = \frac{\hbar^2}{8\pi G L^2}\] and $M_\text{Pl} = 8\pi G^{-1/2}$. Using the effective quantum field theory it was shown that the zero-point energy of a system with size $L$ should not exceed the mass of a black hole with the same size, thus $L^3 \rho_L \leq \Lambda M^2_\text{Pl}$, where $\rho_L$ corresponds to the quantum zero-point energy density \[\rho_L\] and $M^2_\text{Pl} = 8\pi G$. The latter relation establishes a link between the ultraviolet cutoff, define through $\rho_L$, and the infrared cutoff which is encoded by the scale $L$. By applying this novel principle within the cosmological context, one can assume that the dark energy density of the universe $\rho_L$ takes the same form of the vacuum energy, $\rho_L = \rho_x$. Taking the largest $L$ as the one saturating the above inequality, it turns out to be the holographic dark energy is given by $\rho_x = 3c^2M^2_\text{Pl}L^{-2}$, where $c$ is a numerical factor. Here, the infrared cutoff will be considered as a function of the cosmic time so the holographic dark energy will evolve dynamically and in order to be taken as a good candidate for occupying the place of the cosmic fuel called dark energy it should be able to mimic the current accelerated expansion phase of the Universe. So far in the literature, the IR cutoff has been taken as the large scale of the universe, Hubble horizon [6, 7], particle horizon, event horizon [7] or generalized IR cutoff [8, 9, 10, 11, 12].

An interesting property of the modified Ricci cutoff [10] refers to the fine-tuning problem, that is, due to the holographic dark energy density is based on spacetime scalar curvature without involving a Planck or high physical energy scale the fine-tuning problem might be avoided, further the coincidence problem seems to disappear within this context [8]. For all these reasons, we will focus on a holographic model where the dark energy density corresponds to the modified Ricci cutoff [10, 13, 14, 15], extending the recently investigated linear interaction model [16] to the case of nonlinear ones.

Another central aspect of the modern cosmology is associated with the exchange of energy between the dark components, i.e., the dark matter not only can feel the presence of the dark energy through a gravitational expansion of the Universe but also can interact between them. Because the nature of the dark sector still remains unknown there is no a fundamental reason to specify the coupling between dark energy and dark matter. In fact, one follows a phenomenological approach by studying the properties hidden in each type of interaction and then one confronts the theoretical model with the available observational data. Reference [17] investigated several linear and nonlinear interactions in the dark sector, generalizing previous works on the literature. It was introduced an effective one-fluid description of the dark components and shown that interacting and unified models are related to each other. Also, a generic nonlinear interaction induces an effective equation of state resembling a variable Chaplygin gas model, giving rise to the “relaxed Chaplygin gas model” [17]. Within this framework, we are going to explore an holographic model and its relation to relaxed Chaplygin gas.

The paper is outlined as follows. In Sec. II we introduce the model and discuss about the exchange of energy in the dark sector where the modified Ricci cutoff is taken into account. For the selected nonlinear interaction term,
we show that the effective one-fluid obeys the equation of state of a relaxed Chaplygin gas. In Sec. III we find the most probable value of the cosmological parameters by applying the \( \chi^2 \) minimization method to the Hubble data and the Union2 compilation of SNe Ia. We use the best–fit value of the parameters for confronting our model with the standard Λ–CDM and apply the \( Om \) diagnostic method. Finally, in Sec IV we summarize our findings.

II. THE MODEL

We present an holographic cosmological model, where we associate the IR cutoff with the modified Ricci’s radius method. Finally, in Sec.IV we summarize our findings.

Finally, from Eqs. (6) and (8), we obtain the source equation \( \text{(17)} \) for the energy density \( \rho \)

\[
\rho'' + (\alpha + \beta)\rho' + \alpha \beta \rho = Q_M \Delta \gamma. \tag{10}
\]

Now we consider cosmological models where the interaction \( Q_M \) between both dark components is nonlinear and includes a set of terms which are homogeneous of degree 1 in the total energy density and its first derivative \( \frac{d}{d\eta} \). Introducing the new variable \( y = \rho^{(1+\nu)} \) into the latter equation one gets a second order linear differential equation \( y'' + (2 + \nu)y' + (1 + \nu)y = 0 \), whose solutions allow us to write the energy density

\[
\rho = \rho_0 a^{-3 + (1+\nu)} \tag{13}
\]

with \( \Delta \gamma = \alpha - \beta \), while the total pressure \( p = p_c + p_x \) is given by

\[
p(\rho, \rho') = -\rho - \rho', \tag{7}
\]

From now on we will use the MCE \( \text{(4)} \) instead of the WCE with variable \( \omega_c \) because it is simpler, and introduce an interaction between both dark components through the term \( 3HQ_M \) into the MCE \( \text{(4)} \) with constant coefficients, so

\[
\rho' + \alpha \rho = -Q_M, \tag{8}
\]

\[
\rho_x' + \beta \rho_x = Q_M. \tag{9}
\]

Finally, from Eqs. (6) and (8), we obtain the source equation \( \text{(17)} \) for the energy density \( \rho \)

\[
\rho'' + (\alpha + \beta)\rho' + \alpha \beta \rho = Q_M \Delta \gamma. \tag{10}
\]

Now we consider cosmological models where the interaction \( Q_M \) between both dark components is nonlinear and includes a set of terms which are homogeneous of degree 1 in the total energy density and its first derivative \( \frac{d}{d\eta} \). Introducing the new variable \( y = \rho^{(1+\nu)} \) into the latter equation one gets a second order linear differential equation \( y'' + (2 + \nu)y' + (1 + \nu)y = 0 \), whose solutions allow us to write the energy density

\[
\rho = \rho_0 a^{-3 + (1+\nu)} \tag{13}
\]

where \( \rho_{10} \) and \( \rho_{20} \) are positive constants. From Eqs. (6) and (7) we have both dark energy densities and the total pressure

\[
\rho_c = \frac{-\rho}{\alpha - \beta} \left[ \frac{\beta - 1 + \frac{\nu}{(1+\nu)(1 + \frac{\rho_{20} a^{-3(1+\nu)} / \rho_{10})}}}{1 + \frac{\nu}{(1+\nu)(1 + \frac{\rho_{20} a^{-3(1+\nu)} / \rho_{10})}} \right], \tag{14}
\]

\[
\rho_x = \frac{\rho}{\alpha - \beta} \left[ \frac{\alpha - 1 + \frac{\nu}{(1+\nu)(1 + \frac{\rho_{20} a^{-3(1+\nu)} / \rho_{10})}}}{1 + \frac{\nu}{(1+\nu)(1 + \frac{\rho_{20} a^{-3(1+\nu)} / \rho_{10})}} \right], \tag{15}
\]

\[
p = -\frac{\nu \rho_{10}}{1 + \frac{\nu}{\rho}}. \tag{16}
\]

From these equations we see that an initial model of interacting dark matter and dark energy can be associated with an effective one-fluid description of an unified cosmological scenario where the effective one-fluid,
with energy density $\rho = \rho_c + \rho_x$ and pressure 

$\frac{\rho}{\rho} = \frac{\omega}{\omega + 1}$, obeys the equation of state of a relaxed Chaplygin gas

$p = b \rho + f(a)/\rho^\nu$, where $b$ is a constant. The effective barotropic index $\omega = p/\rho = \omega (\rho_c/\rho)$ reads

$$\omega = -\frac{\nu \rho_{10}}{(1 + \nu) \rho_{10} + \rho_{20} a^{-3\nu}}.$$  \hfill (17)

At early times and for $\nu > 0$, the effective energy density $\rho_{\text{eff}}$ behaves as $\rho \approx a^{-3}$, the effective barotropic index $\gamma = 1$ and the effective fluid describes an Universe dominated by nearly pressureless dark matter. However, a late time accelerated Universe i.e., $\omega < -1/3$ with positive dark energy densities require that $\nu > 1/2$, $\beta < 1$ and $\alpha > 1$. From now on we adopt the latter restrictions.

In the case $\nu = 1$, one gets an equation of state similar to a variable Chaplygin gas as the one studied in [18, 19]. In these works the authors have used the gold sample of type Ia supernova as data as well as the x-ray gas mass fractions in galaxy clusters to obtain some observational constraints of the model.

III. OBSERVATIONAL HUBBLE DATA CONSTRAINTS

Now, we are going to find the observational constraints on the parameter space of the MHRDE using the observational $H(z)$ data. The function $H(z)$ plays a crucial role in understanding the properties of the dark energy, since its value is directly obtained from the cosmological observations. Using the absolute ages of passively evolving galaxies observed at different redshifts, one obtains the differential ages $dz/dt$ and the function $H(z)$ can be measured through the relation $H(z) = -(1 + z)^{-1} dz/dt$. We take the data coming from the Gemini Deep Deep Survey [20], archival data [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31], and compare it with the $H(z)$ to attempt to obtain the constraints for the current cosmological models. The 12 observational $H(z)$ data from [28] are listed in Table I. Here, $H_{\text{obs}}(z_i)$ and $H_{\text{obs}}(z_k)$ are uncorrelated because they are obtained from the observations of galaxies at different redshifts.

The probability distribution for the $\theta$-parameters is given by a Gaussian density (see e.g. [29])

$$P(\theta) = Z e^{-\chi^2(\theta)/2},$$

$$\chi^2(\theta) = \sum_{k=1}^{12} \frac{|H(\theta, z_k) - H_{\text{obs}}(z_k)|^2}{\sigma(z_k)^2},$$  \hfill (18)

where $Z$ is a normalization constant, $H_{\text{obs}}(z_k)$ is the observational $H(z)$ data at the redshift $z_k$, $\sigma(z_k)$ is the corresponding 1$\sigma$ uncertainty, and the summation is over the 12 observational $H(z)$ data of Table I. Since we are interested in obtaining the bounds for the model parameters, we have adopted as a prior $H_0 = 72.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The Hubble expansion of the model becomes:

$$H(\theta; z) = H_0 \left\{ B(1 + z)^3 + (1 - B)(1 + z)^{3(\nu + 1)} \right\}^{-1/2},$$

$$B[\theta] = \frac{\nu + 1}{\nu} \left\{ \alpha(\Omega_x - 1) + (1 - \beta \Omega_x) \right\}$$  \hfill (19)

and

$$\Delta H^2_0 = \chi^2(\theta) - \chi^2_{\text{min}}(\theta) \leq 2.30.$$  \hfill (20)

Besides, 68.3% confidence contours in the $(\nu, \Omega_x)$ plane are made of the random data sets that satisfy the inequality $\Delta H^2_0 \leq 6.17$ while the data within 99.73% confidence contours with a 3$\sigma$ error bar are accommodated in the domain defined by $\Delta H^2_0 \leq 11.8$. After performing this analysis we are in position to get confidence contours in the $(\nu, \Omega_x)$ plane, thus using the $\chi^2(\alpha_f, \beta_f, \nu, \Omega_x)$ distribution one can find the 68.3%, 95.4%, and 99.73% confidence contours respectively [see Fig. (II)]. We have taken the point of reference $(\alpha_f, \beta_f) = (1.01, 0.15)$ but it is possible to show a wide set of admissible values for $\alpha$ and $\beta$ which leads to a good fit [see Table (IV)]. Thus, from Fig. (II) we get the best fit at $c = (\nu, \Omega_x) = (1.19 \pm 0.12, 0.61 \pm 0.02)$. It corresponds to a local minimum $\chi^2_{\text{min}} = 7.86$ leading to a good fit with $\chi^2_{\text{ dof}} = 0.786$ per degree of freedom. The Ricci’s cutoff $(\alpha, \beta) = (4/3, 1)$ does not guarantee the convergence of the minimization process. However, the values of $(\nu, \Omega_x)$ obtained from an holographic dark energy $\rho_x \propto R$, namely $(4/3, \beta)$, fulfills the goodness condition $\chi^2_{\text{ dof}} < 1$. The values of $\Omega_x$, which varies from 0.58 to 0.69, do not deviate significantly from the observational limits provided by the WMAP-7 project [31] with $\Omega_x = 0.73$ [see Table (III)]. Comparing the Ricci model with the one arising from MHRDE for $(\alpha = 1.01, \beta = 0.15)$, the former gives $(\nu, \Omega_x) = (1.19, 0.69)$, whereas the latter yields $(\nu, \Omega_x) = (1.19, 0.61)$, so the Ricci model seems to be
statistically favored by $H(z)$ data showing a $\Omega_x0$ closer to the observational bound reported by the WMAP-7 project \cite{31}.

We estimate the best value of $H_0$ and $\Omega_x0$ for the $\Lambda$–CDM model using the Hubble data as well as the Union2 data for SNe Ia \cite{32}. The former dataset leads to $H_0 = 73.60 \pm 3.18$ km s$^{-1}$ Mpc$^{-1}$ and $\Omega_x0 = 0.730 \pm 0.04$ with $\chi^2_{dof} = 0.770$ whereas the latter one gives $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ and $\Omega_x0 = 0.73$ along with $\chi^2_{dof} = 0.978$. Now, in order to make possible a comparison with the $\Lambda$–CDM model we need to estimate the same types of parameters so we take as priors $\nu = 1.19$, $\alpha = 1.01$, and $\beta = 0.15$ but we allow $H_0$ and $\Omega_x0$ as free parameters to be found under the minimization process. It turns out that $H_0 = 73.011 \pm 2.97$ km s$^{-1}$ Mpc$^{-1}$ and $\Omega_x0 = 0.617 \pm 0.001$ with $\chi^2_{dof} = 0.779 < 1$ [see Fig. (2)], then both models give cosmological bounds of the pair $(H_0, \Omega_x0)$ very consistent with the values reported in \cite{31}.

In order to compare the Hubble data (12 points) with the Union2 compilation of 557 SNe Ia \cite{32} we proceed as follows; thus, we took as priors $H_0 = 72.2$ km s$^{-1}$ Mpc$^{-1}$, $\alpha = 1.01$ and $\beta = 0.15$ in both cosmological data. We found the best–fit values of $\nu$ and $\Omega_x0$ for both sets, focusing on the existence of some tighter constraints coming from the SNe Ia data. For the Hubble data we obtained $\alpha = 1.19$ and $\Omega_x0 = 0.61$ with $\chi^2_{dof} = 0.786$ whereas the SNe Ia data lead to $\nu = 1.5$ and $\Omega_x0 = 0.70$ with $\chi^2_{dof} = 0.812 < 1$; in broad terms the tighter constraints seems to be found with the Hubble data. Of course, these results can vary according to the parameter regions taken into account in the minimization process.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
$z_i$ & $H_{obs}(z_i)$ & $1\sigma$ uncertainty \\
\hline
0.00 & 74.2  & $\pm 3.6$  \\
0.10 & 69   & $\pm 12$  \\
0.17 & 83   & $\pm 8$   \\
0.27 & 77   & $\pm 14$  \\
0.40 & 95   & $\pm 17$  \\
0.48 & 97   & $\pm 60$  \\
0.88 & 90   & $\pm 40$  \\
0.90 & 117  & $\pm 23$  \\
1.30 & 168  & $\pm 17$  \\
1.43 & 177  & $\pm 18$  \\
1.53 & 140  & $\pm 14$  \\
1.75 & 202  & $\pm 40$  \\
\hline
\end{tabular}
\caption{The observational $H(z)$ data \cite{28}.}
\end{table}

Now, using the best–fit model parameters $\theta_c = (\nu, \Omega_x0) = (1.19 \pm 1.13, 0.61 \pm 0.02)$ we would like to compare the model having a MHRDE with the standard $\Lambda$–CDM scheme composed of baryonic matter and a constant dark energy $\Omega_{x0} = 0.73 \pm 0.04$. In Fig. (3) we show the $H_{obs}(z)$ data with their error bars, and the theoretical curves corresponding to the $\Lambda$–CDM and MHRDE models, respectively. At small redshift $0 \lesssim z \lesssim 0.2$ both models seem to be in excellent agreement with the observational $H(z)$ data, while at large redshift $z > 0.2$ the model shows a slight difference with the $\Lambda$–CDM. However, the observational data show that in the interval $z \in [0, 1]$ both models have a good adjustment. As $-1 \leq \omega(z) \leq 0$, the equations of state of the effective fluid and dark energy do not cross the phantom line, at least for the best–fit model parameters used previously [see Fig. (1)]. The expression of $\omega_x$ at present

\begin{equation}
\omega_x0 = \frac{\alpha - \beta}{\alpha - 1}B \frac{\nu(\alpha - \beta)B}{(\alpha - 1)(1 + \nu) + \nu B},
\end{equation}

becomes $\omega_x0 = -0.88$ when evaluating at the best–fit values $\theta_c = (\nu, \Omega_x0, \alpha, \beta) = (1.19, 0.61, 1.01, 0.15)$. It is close to the value reported by WMAP-7, $\omega_x0 = -0.93$, when the joint analysis of the WMAP+BAO+$H_0$+SN data \cite{31} for constraining the present-day value of the equation of state for dark energy is made.
We also show the observational data $H_0 - \Omega_{\text{de}}$ plane for the MHRDE model considering as priors $\nu = 1.19$, $\alpha = 1.01$ and $\beta = 0.15$. The best fit value of $(H_0, \Omega_{\text{de}})$ is used to compare with the $\Lambda$CDM model.

![FIG. 2: We show the confidence contour in the $H_0 - \Omega_{\text{de}}$ plane for the MHRDE model considering as priors $\nu = 1.19$, $\alpha = 1.01$ and $\beta = 0.15$. The best fit value of $(H_0, \Omega_{\text{de}})$ is used to compare with the $\Lambda$CDM model.](image)

At $z = 0$ for the former model, the critical redshift where the acceleration starts, one obtains $q_0 = -0.27$ while for the $\Lambda$CDM model one obtains $q_0 = -0.59$. The critical redshift where the acceleration starts, turns to be $z_{\text{acc}} = 1.06$ for the best–fit values $\theta_c$, then our model enters the accelerated regime earlier than the $\Lambda$CDM one with $z_{\text{acc}} = 0.75$.

In Fig. (3) we plot $H(z)$ for the $\Lambda$CDM with $H_0 = 73.60$ km/s Mpc and MHRDE model with prior $H_0 = 72.22$ km/s Mpc. We also show the observational data $H(z)$ with their error bars.

![FIG. 3: We plot $H(z)$ for the $\Lambda$CDM with $H_0 = 73.60$ km/s Mpc and MHRDE model with prior $H_0 = 72.22$ km/s Mpc. We also show the observational data $H(z)$ with their error bars.](image)

So far, we have discussed some cosmological constraints coming from the observational $H(z)$ data at different redshifts. Let us take into account an alternative method, which is beyond parameter constraints, that can be implemented to distinguish the $\Lambda$–CDM from other dark energy models. The new diagnostic of dark energy called $O_m(z)$ is an interesting one because it does not involve the cosmic equation of state [33]. The $O_m(z)$ function is constructed from the Hubble parameter $H$ and defined as

$$O_m(z) = \frac{H^2 - H_0^2(z + 1)^{3 - 1}}{H_0^2[(z + 1)^3 - 1]}, \quad (24)$$

Fig. (4) shows the equations of state for the effective fluid $\omega(z)$ and the dark energy $\omega_x(z)$.

![FIG. 4: We show the equations of state for the effective fluid $\omega(z)$ and the dark energy $\omega_x(z)$.](image)

Since the $O_m(z)$ only depends upon the scale factor and its first derivative, one considers the present diagnostic method as a geometrical one. Furthermore, for the $\Lambda$–CDM model one finds that $O_m(z) = \Omega_{\text{m}0}$ so only if the dark energy is given by a cosmological constant the $O_m(z)$ remains constant. For our model we find that $O_m(z)$ can be written as

$$O_m(z) = \frac{B(1 + z)^3 + (1 - B)(1 + z)^{3(\nu + 1)}}{(z + 1)^3 - 1},$$

Now, we are going to analyze some important regimes. In the far future, $z \to -1$, Eq. (25) reaches the value

$$O_m(z) = \frac{B(1 + z)^3 + (1 - B)(1 + z)^{3(\nu + 1)}}{(z + 1)^3 - 1}, \quad (25)$$
accelerated expansion (present-day scenario) and, in both
time dark energy that makes the Universe exhibit an
dark matter followed by an era dominated by the holo-
transition of the Universe between a stage dominated by
1. For the present time, \( z \to 0 \), the Eq. (25) gives
\[ [B + (1 - B)(\nu + 1)]/(\nu + 1) \]
which is positive for \( \nu > 0 \). In addition, using the constrained values of \( (\nu, \Omega_x) = (1.19, 0.61) \)
coming from the \( H_{\text{obs}} \) data, we plot the evolution of \( Om(z) \) for both the MHRDE and the \( \Lambda \)-CDM
models [see Fig. 7]. Interestingly enough, in the interval \( z \in [0, 1] \) the \( Om_{\text{MHRDE}} > \Omega_m \), the inequality is
reversed at redshift, \( z \geq 1 \), as it happens with a phantom dark energy model. Thus the same behavior occurs
with a quintessence model [38].

As a closing comment, we would like to address a dis-
cussion concerning the values of \( \alpha \) and \( \beta \) taken into ac-
count through this section. Here we have focused on the
transition of the Universe between a stage dominated by
dark matter followed by an era dominated by the holo-
graphic dark energy that makes the Universe exhibit an
accelerated expansion (present-day scenario) and, in both
stages a nonlinear interaction in the dark sector has been
taken into account. In order to estimate the parameter
\( \nu \) and \( \Omega_x \), we have used the values of \( \alpha \) and \( \beta \) which are
consistent with the \( \chi^2 \)-statistical analysis because they
fulfill the condition \( \chi^2_{\text{obs}} < 1 \). Now, we are going to ex-
plore a modification on the aforesaid model by adding a
third component, say \( \rho_m \), which does not interact with
\( \rho_c \) and \( \rho_x \). The total energy density reads as
\[ \rho = \rho_m + \rho \]
with \( \rho = \rho_c + \rho_x \) and the MCE is split as
\[ \rho' + \alpha \rho - (\alpha - \beta) \rho_x = 0, \]  
\[ \rho_m' + \alpha \rho_m = 0 \]  
so (26) is equal (2) while (27) shows that the third com-
ponent is decoupled from the interacting dark sector, indicating that \( \rho_m \) feels the presence of the dark sec-
tor through the gravitational expansion of the Universe.
From (27) one finds that \( \rho_m = \rho_m a^{-3\alpha} \) whereas the
behavior of dark matter and dark energy with the scale
factor can be obtained from (14) and (15), respectively.
Using (14), (15) and (27) one gets the Hubble parameter in
term of the redshift \( x = 1 + z \) and the relevant

\[ \frac{H(z)}{H_0} = \left[ (1 - \Omega_m) B x^3 + (1 - B) x^{3(\nu + 1)} \right]^{1/2} + \Omega_m x^{-3\alpha} \]  
(28)

As a closing comment, we would like to address a dis-
cussion concerning the values of \( \alpha \) and \( \beta \) taken into ac-
count through this section. Here we have focused on the
transition of the Universe between a stage dominated by
dark matter followed by an era dominated by the holo-
graphic dark energy that makes the Universe exhibit an
accelerated expansion (present-day scenario) and, in both

\[ B[\theta] = \frac{\nu + 1}{\nu} \left[ 1 + \frac{\alpha - \beta}{\Omega_x + \Omega_m} + (1 - \alpha) \right] \]  
(29)
in the radiation era should fulfill the stringent bound \( \Omega_r(z \approx 1100) < 0.1 \) in order for the model be consistent with the big bang nucleosynthesis (BBN) data. For the priors \( (H_0 = 72.2, \nu = 1.19, \Omega_{x0} = 0.7, \alpha = 4/3) \) the Hubble data give as the best–fit values \( \beta = 0.1 \) and \( \Omega_{m0} = 1.7 \times 10^6 \) along with \( \chi^2_{dof} = 0.78 < 1 \) with a fraction of dark energy \( \Omega_x(z \approx 1100) = 0.2 \) nearly close to the BBN's bound. Second, employing the Hubble data for \[25\] we estimate the best–fit value of \( \Omega_{m0} \) and \( \alpha \). Taking as priors \( (H_0 = 72.2, \nu = 1.19, \Omega_{x0} = 0.61, \beta = 0.15) \) the \( \chi^2 \)-analysis yields as the best–fit values \( \alpha = 1.01 \) and \( \Omega_{m0} = 9.9 \times 10^{-5} \) together with a \( \chi^2_{dof} = 0.79 < 1 \). Moreover, the latter case leads to an early dark energy \( \Omega_x(z \approx 1100) = 0.01 < 0.1 \) which is consistent with the bounds reported in \[34\] or with the future constraints achievable by Planck and CMBPol experiments \[35\]. Therefore, taking the third component as the radiation term or a nearly radiation contribution, has helped to validate the first model, indicating that the value of the cosmological parameters selected are consistent with BBN constraints.

IV. CONCLUSION

We have investigated the interacting dark sector with a MHRDE, where the IR cutoff is provided by the modified Ricci scalar in a form of a linear combination of \( \dot{H} \) and \( H^2 \), so that \( \rho_c = 2(\dot{H} + 3aH^2)/2(\alpha - \beta) \). This appealing cutoff yields a MCE in the dark sector with constant barotropic indexes, identified with \( \alpha \) and \( \beta \). We have introduced an interaction between the dark matter and dark energy densities, homogeneous of degree 1 in the variables \( \rho \) and \( \dot{\rho} \), and solved the source equation for the total energy density of the mix. The equation of state of this effective fluid is that of the relaxed Chaplygin gas. The latter interpolates between a matter dominated phase at early times and an accelerated expanding phase dominated by the MHRDE at late times.

We have used the observational Hubble data to constrain the cosmological parameters of the model and to compare with the \( \Lambda \)–CDM model. Taking as a reference point \( (\alpha f, \beta f) = (1.01, 0.15) \) we get the best fit at \( \theta_c = (\nu, \Omega_{x0}) = (1.19, 0.61) \) with \( \chi^2_{\min} = 7.86 \) leading to a good fit with \( \chi^2_{dof} = 0.786 < 1 \) per degree of freedom [see Fig.(1)]. We have established that a model with a holographic dark energy \( \rho_c \propto R \) leads to \( 0.59 < \Omega_{x0} < 0.69 \) which is close to the bounds \( \Omega_{x0} = 0.73 \) provided by WMAP-7 \[31\]. For \( \beta = 0.1 \), we have shown that the Ricci cutoff \( (\alpha = 4/3) \) is consistent with other values of \( \alpha \) because it fulfills the goodness condition \( \chi^2_{dof} < 1 \). In addition, we have obtained the allowed range of \( (\nu, \Omega_{x0}) \) when one varies \( \alpha \) and \( \beta \) [see Table(II)]. Properly estimating the \( H_0 \) and \( \Omega_{x0} \) with the Hubble data we have confronted the \( \Lambda \)–CDM with the MHRDE model; thus, both models give some bounds of the pair \( (H_0, \Omega_{x0}) \) consistent the those reported in \[31\]. Besides, we have taken into account the SNe Ia with the Union2 data for calculating the best–fit values of \( \nu \) and \( \Omega_{x0} \). It led to \( \nu = 1.5 \) and \( \Omega_{x0} = 0.70 \) with \( \chi^2_{dof} = 0.812 < 1 \) while the Hubble data gave \( \nu = 1.19 \) and \( \Omega_{x0} = 0.61 \) with \( \chi^2_{dof} = 0.786 \).

We have found that the equations of state of the dark energy equation and the unified fluid, at the best–fit values \( \theta_c \), do not cross the phantom divide line [see Fig.(4)] while the present value of the equation of state for the dark energy is \( \omega_{x0} = -0.88 \).

From the deceleration parameter [see Fig.(5)] and the best fit values \( \theta_c \), we have obtained that the acceleration starts at \( z_{acc} = 1.06 \) hence, the model with a MHRDE enters the accelerated regime earlier than the \( \Lambda \)–CDM with \( z_{acc} = 0.75 \). We have shown that the density parameters \( \Omega_c \), \( \Omega_x \), and its ratio \( r(z) \) in Fig.(6) seem to alleviate the coincidence problem. It is related to the decreasing behavior of the interaction with the redshift and its vanishing limit in the far future [see Fig.(6)]. We have applied the geometrical \( Om \) diagnostic method to our model which seems include several aspects different from the \( \Lambda \)–CDM model.

Finally, we have analyzed a new model with a third fluid decoupled from the dark interacting sector. The latter scheme allowed us to examine the issue of the early dark energy at \( z \approx 1100 \) as well as to check the match of the models with the BBN constraints \[34\], \[35\].

Acknowledgments

We would like to thank the referee for making useful suggestions, which helped improve the article. LPC thanks the University of Buenos Aires for their support under Project No. X044 and the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET) through the research Project PIP 114-200801-00328. MGR is partially supported by CONICET.

[1] G. ’t Hooft, arXiv:gr-qc/9310026.
[2] L. Susskind, J. Math. Phys. 36 (1995) 6377.
[3] W. Fischler, L. Susskind, arXiv:hep-th/9806039
[4] R. Bouso, Rev.Mod.Phys. 74 825-874 (2002).
[5] A. Cohen, D. Kaplan, A. Nelson, Phys. Rev. Lett. 82 (1999) 4971.
[6] S. D. H. Hsu, Phys. Lett. B 594 13 (2004).
[7] M. Li, Phys. Lett. B 603, 1 (2004).
[8] Gao, F. Q. Wu, X. Chen and Y. G. Shen, Phys. Rev. D 79, 043511 (2009).
[9] Linsen Zhang, Puxun Wu, Hongwei Yu, Eur. Phys. J. C 71, 1588 (2011); R.-J. Yang, Z.-H. Zhu, F.
[10] Wu, Int. J. Mod. Phys. A 26 (2011) 317-329.
[11] Shinichi Nojiri, S. D. Odintsov, Gen. Rel. Grav. 38, 1285-1304 (2006).
[12] J. C. Fabris, R. Herrera, W. Zimdahl, arXiv:1103.3441v2.
[13] L. N. Granda, W. Cardona, A. Oliveros, Eur. Phys. J. C (2011) 71: 1588.
[14] I. Duran and D. Pavon, Phys. Rev. D 81, 023504 (2011).
[15] L. Zhang, P. Wu, and H. Yu, Eur. Phys. J. C (2011) 71: 1588.
[16] S. del Campo, J. C. Fabris, R. Herrera, W. Zimdahl, arXiv:1103.3441v2.
[17] L. Zhang, P. Wu, and H. Yu, Eur. Phys. J. C (2011) 71: 1588.
[18] L. P. Chimento, M. Forte and M. G. Richarte, arXiv:1106.0781.
[19] L. P. Chimento, Phys. Rev. D 81, 043525 (2010).
[20] I. Duran and D. Pavon, Phys. Rev. D 81, 023504 (2011).
[21] R. G. Abraham et al., Astron. J. 127, 2455 (2004).
[22] Zong-Kuan Guo, Yuan-Zhong Zhang, Phys. Lett. B 645, 326-329 (2007).
[23] Zong-Kuan Guo, Yuan-Zhong Zhang, arXiv:astro-ph/0509790.
[24] Zong-Kuan Guo, Yuan-Zhong Zhang, arXiv:astro-ph/0509790.
[25] R. Amanullah et al., Astrophys. J. 716, 712 (2010).
[26] T. Treu, M. Stiavelli, S. Casertano, P. Moller and C. Heavens, Mon. Not. Roy. Astron. Soc. 341, 464 (2003).
[27] T. Treu, M. Stiavelli, S. Casertano, P. Moller and C. Heavens, Mon. Not. Roy. Astron. Soc. 341, 464 (2003).
[28] T. Treu, M. Stiavelli, S. Casertano, P. Moller and C. Heavens, Mon. Not. Roy. Astron. Soc. 341, 464 (2003).
[29] Press, W.H., et al., Numerical Recipes in C. Cambridge University Press, Cambridge (1997).
[30] A. G. Riess et al., Astrophys. J. 699, 539 (2009).
[31] E. Komatsu, et al., arXiv:1001.4538 [astro-ph.CO].
[32] R. Amanullah et al., Astrophys. J. 716, 712 (2010).
[33] V. Sahni, A. Shafieloo, A.A. Starobinsky, Phys. Rev. D 78 (2008) 103502.