Heterotic Coset Models

of

Microscopic Strings and Black Holes

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Abstract

Following a recent conjecture by Lapan, Simons and Strominger, we revisit and discuss an intrinsically heterotic class of conformal field theories, emphasizing their Lagrangian construction as asymmetrically gauged WZW models, which may be useful in several applications to the study of supersymmetric strings and black holes in heterotic and type II string theory compactified on $T^6$ and $K3 \times T^2$ respectively. In these cases, the leading supergravity geometry is singular, but higher order corrections remove this singularity in a way that is consistent with, for example the non–zero entropy for the black holes that these strings form after wrapping on an additional circle. The conformal field theories have the right structure to capture the features of the supergravity analysis, and possess precisely the microscopic target spaces required. We describe in detail the model with $\text{AdS}_3 \times S^2$ geometry, which is conjectured by Lapan et. al. to represent a fundamental heterotic string in five dimensions, and then propose conformal field theories which are potential candidates for the microscopic geometry of heterotic strings in $D$ dimensions, with target space $\text{AdS}_3 \times S^{D-3}$. We also discuss some conformal field theories that give microscopic AdS target spaces in various dimensions.
1 Introduction

In recent times, supersymmetric black hole solutions of $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supergravity in four dimensions have been of considerable interest. The latter, the initial focus of this paper, can be studied as D4–D0 bound states in type II string theory compactified on $K3 \times T^2$ (the D4–brane world–volume wraps the K3), or in heterotic string theory compactified on $T^6$, where the black holes arise as fundamental heterotic strings\[^{[1] [2] [3]}\] wrapped on one of the circles of the compactification with the inclusion of Kaluza–Klein momentum. These two–charge black holes, while classically having horizons of zero area (and in fact are naively singular at their core), turn out to have much more structure when studied beyond the leading order in the small $\alpha'$ expansion. The corrections remove the singularity and yield a smooth $\text{AdS}_2 \times S^2$ geometry at the core\[^{[4]}\]. The microstate counting (leading to the entropy, including the leading contribution and a family of corrections) for these black holes on the heterotic string side is remarkably successful\[^{[5]}\], and suggests (as was the case for other successful microstate studies\[^{[6] [7]}\]) that the conformal field theory whose properties control the correspondence between the heterotic counting and the black hole geometry has an holographic dual spacetime.

A very natural question to ask (as emphasized by Strominger recently in a talk at Strings 2007\[^{[8]}\]) is whether there is an accessible description of the geometry of the fundamental heterotic string which itself becomes the black hole. This is motivated by the analogy with the D1–D5–momentum system which is used for the successful microstate counting of the three–charge five–dimensional black holes in type II string theory\[^{[6] [7]}\] — there is a spacetime at the core which is asymptotically $\text{AdS}_3 \times S^3$, which is holographically dual to the $(1+1)$–dimensional theory (derived from the D1–D5 world–volume) doing the state counting. The nature of this new spacetime is particularly interesting, since while the leading order geometry of the fundamental string (the source that plays the role of the D–branes in the situation in hand) has a null singularity at the core, quite tantalizingly the string coupling is weak there. As a reminder, the form of the geometry for $N$ strings lying along the direction $x_1$ in $D$ non–compact directions is:

$$
\begin{align*}
    ds_h^2 &= H^{-1}(-dt^2 + dx_1^2) + dr^2 + r^2 d\Omega_{D-3}^2 + ds_{T^{10-D}}^2; \\
    e^{-2(\Phi - \Phi_0)} &= H; \\
    B_{01} &= -H^{-1}; \\
    H &= 1 + N \left(\frac{T_h}{r}\right)^{D-4}; \\
    r_h^{D-4} &= \frac{r_1^6}{V_{10-D}}; \\
    r_1^6 &= T \frac{16\pi G_{10}}{6\Omega_7},
\end{align*}
$$

(1)

where $r$ is the radial coordinate in $D$ dimensions, $d\Omega_{D-3}^2$ is the metric on the round unit $S^{D-3}$, and $r_1^6$ is the standard constant that ensures that the solution has $N$ units of fundamental
heterotic string tension $T = (2\pi \alpha')^{-1}$. Here, $G_{10}$ is Newton’s constant in ten dimensions, $\Omega_7$ is the volume of a unit seven–sphere, and $V_{10-D}$ is the volume of the torus, $T^{10-D}$, on which we’ve compactified. From this it is clear that as we move to the singular core at $r = 0$, the string coupling goes as:

$$g_s \sim \frac{1}{N \pi \alpha'} \left( \frac{r}{r_h} \right)^{\frac{D-4}{2}},$$

(2)

which is generically quite small (and can be tuned as small as one desires by sending $N$ large, although this is not necessary in this example). The curvature is clearly blowing up in this near–core limit, and so one should expect $\alpha'$ corrections to the solution to be important, and can be expected to modify the story.

The $\alpha'$ corrections have been shown to reveal an $\text{AdS}_3 \times S^2$ corrected geometry[9], for the unwrapped $T^5$ case of the infinite straight fundamental heterotic string. We need to go beyond supergravity to the full heterotic string theory to fully study this situation. There is additional hope for success since the heterotic string theory, our arena of study, has only NS–NS sources, which are readily accessible in conformal field theory, in contrast to the R–R sources of type II.

So in view of these encouraging signs it is prudent to seek a tractable conformal field theory representing this microscopic heterotic geometry, capturing the physics of the string theory in this regime[4]. This conformal field theory is logically distinct from the conformal field theory on the world–volume of the stretched or wrapped heterotic strings that become the black hole. The latter is the one that would be the holographic dual of the microscopic spacetime at the core. However, there is evidently a trilogy of dictionaries allowing translations between any two of the three systems. One is the standard world–sheet/space–time correspondence of string theory, while another would be of the now familiar AdS/CFT holographic type. The third one which is implied is a new correspondence that does not seem to involve gravity.

For the simpler case of the unwrapped string, a recent conjecture of Lapan, Simons, and Strominger (announced by the latter at Strings 2007[8]) suggested[2] that the (apparently) difficult $S^2$ part of the conformal field theory is a special uncharged case of the asymmetric orbifold presented[12] by Giddings, Polchinski and Strominger (GPS) in 1993, in the context of four dimensional non–supersymmetric magnetically charged black holes in heterotic string theory. It turns out that there is a non–trivial conformal field theory for the angular sector even when the charge of those black holes vanishes, and this is the special case that Lapan et. al. conjecture

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1While this manuscript reporting our results was being completed, we learned of a paper[10] by Dabholkar and Murthy which presents results in this area which may be related.

2Since an earlier version of this manuscript appeared, a paper by Lapan, Simons, and Strominger has now been submitted to the arXiv[11].

3
can then be tensored with an $SL(2,\mathbb{R})$ conformal field theory in order to represent the straight fundamental heterotic string in the $T^5$ compactification.

This conjecture is compelling, and while several aspects of the supergravity aspects of this study are still being considered by Lapan et. al., (but see footnote 2) we wish to firmly discuss the conformal field theory aspects further in this paper, and confirm that the suggestion makes sense. Furthermore, the idea leads us naturally to present several more conformal field theories and make some natural conjectures and suggestions concerning them.

The first mission of this paper is to emphasize that the full $S^2$ conformal field theory (and others in its class) is in fact quite easy to define in a full path integral formalism (it is arguably more natural to present it this way than as an orbifold) which may well be useful for future computations in this context. We present this in section 2, and this is entirely a review of the “heterotic coset model” construction presented\cite{13} by the author in 1994, the prototype of which was shown to be the GPS model. Next, in section 3, insights gained from the method immediately lead us to new conformal field theories that we conjecture represent fundamental heterotic strings in heterotic string theory compactified on $T^{10-D}$ to $D$ dimensions, where the “microscopic” geometry (its radii are again of order $\alpha'$) is AdS$_3 \times S^{D-3}$. The cosets in these cases are non–Abelian. In section 4 we discuss briefly the analogous constructions for conformal field theories with AdS$_{p+2}$ target spaces, where $p \geq 0$. Again their radius is frozen to be of order $\alpha'$. It is not clear what their role is in the current context, but since they may turn out to be relevant, we present them here. Generalizations to cases with fewer supersymmetries, where the non–angular geometry has several higher order corrections in $\alpha'$, and a non–trivial radial dependence for the dilaton, are also discussed in the concluding section 5.

2 The GPS Model from a Lagrangian Perspective

As originally suggested in ref.\cite{13}, the GPS monopole model\cite{12} model is very naturally defined as a gauged WZW model\cite{14, 15, 16} with a number of (relatively) unusual features that give it an intrinsically heterotic characteristic. The construction works as follows (we will write relatively few formulae and instead emphasize the concepts since the original paper\cite{13} is quite explicit). Starting with an $SU(2)$ WZW with coordinates $(z, \bar{z})$, and field $g(z, \bar{z}) \in SU(2)$, which has an $SU(2)_L \times SU(2)_R$ global symmetry, gauge a purely right–acting $U(1)$ subgroup, $g \rightarrow gh$, where $h \in U(1)_R$. The resulting model is classically anomalous, which is to say that a gauge invariant Lagrangian cannot be written for the model. Nevertheless, it is fruitful to exploit the freedom\cite{17} to introduce a two dimensional gauge field with components $A_z, A_{\bar{z}}$, 4
and couple it to $g$ in such a way that under a $U(1)_R$ gauge transformation, the Lagrangian changes by an amount proportional to:

$$\delta I = \frac{1}{8\pi} \int d^2\bar{z} F_{z\bar{z}}.$$  \hfill (3)

In fact, all two dimensional gauge anomalies that we will consider, classical or quantum (as will arise from fermions), can be written in this way, which is key. In the conventions we will choose here, the constant of proportionality for the gauging of the $U(1)_R$ is precisely $k$, the level of the WZW. This is because we choose the $U(1)_P$ to be generated by $i\sigma_3/2$, where $\sigma_3$ is the standard (real, diagonal) third Pauli matrix, and the constant is, more generally, $k\text{Tr}[\sigma_3^2]/2$. (For a more general discussion, see ref.[18].)

To complete the model, we must introduce fermions. As this is the heterotic string, the left movers must be coupled in a way that gives us left–moving supersymmetry. (Note that we have exchanged left and right here relative to the choices made in ref.[13]. This will give us a positive value for $k$ at the special point we are interested in.) There are two such fermions, and they are naturally defined as taking their values on the coset $SU(2)/U(1)$. This fixes very specifically the coupling of those fermions, determining their charge under the $U(1)_R$. They are of course anomalous under the $U(1)_R$, and in our conventions the anomaly is simply $-2$, a contribution of $-1$ for each fermion. We may couple in some number of right–moving fermions, with charge $Q$ under the $U(1)_R$, where we are much more free to choose the value of $Q$ since we are not constrained by the requirement to get a right–moving world–sheet supersymmetry. Picking the most natural quantity of fermions, two, their anomaly is simply $2Q^2$, where there is a contribution of $Q^2$ for each fermion, and the opposite sign is due to the opposite chirality.

The complete $(0, 2)$ model then has the three sectors. We can cancel the quantum and classical anomalies against each other if we satisfy the equation:

$$k = 2(1 - Q^2) = -2Q_+Q_-,$$  \hfill (4)

where $Q_\pm = Q \pm 1$. Here $k$ will not go negative since we will be choosing $Q$ to vanish shortly. For $Q > 1$, one can change the sign by simply exchanging the left and right moving fermions, or by acting on the left with the gauge action. To make things simple, we can henceforth write $k = 2|Q_+Q_-|$ in general formulae.

The complete model is now written as a Lagrangian definition, with $g$, the fermions from the left and the right, and the gauge field $A_z, A_{\bar{z}}$ all coupled together. It is gauge invariant, and defines a consistent conformal field theory. While it is a $(2, 0)$ model on the world–sheet (as is guaranteed because this is an $SU(2)/U(1)$ coset, which is Kahler[19]), it is not spacetime
supersymmetric in general. Modular invariance requires $Q$ to be integer in our units (matching the fact that it is a $U(1)$ monopole in spacetime), but this is inconsistent with the world–sheet condition on charges that promotes $(2,0)$ supersymmetry to spacetime supersymmetry. For the black hole application that GPS had in mind, (and the various generalizations to a host of interesting spacetimes in refs.[20, 21]), this model is combined with a radial direction $\sigma$ and a time direction $t$ defining a non–trivial “radial sector” $(\sigma, t)$ conformal field theory, also arising from a gauged WZW (usually based on $SL(2, \mathbb{R})/U(1)$, as in refs.[22, 23]). The latter sector has a level $k'$ which is linearly related to $k$ by the overall condition on the central charge of the total model. The spacetime geometry of the black hole can be reliably read off (carefully — see ref.[13] for subtleties involving fermionic back–reaction arising from cancelling a classical anomaly against a quantum one) the resulting heterotic sigma model in the large $k$ (small $\alpha'$) limit, or equivalently (because of equation 4), when the charge $Q$ is large.

We are not interested in spacetime solutions that have $U(1)$ gauge (as opposed to Kaluza–Klein) monopole charge $Q$ in spacetime, however. That $U(1)$ is a subgroup of the full heterotic string gauge group. This is not of interest to us here, and so we should in fact set $Q = 0$. Interestingly, there is a non–trivial solution (dubbed the “remnant”, by GPS) to the equation, $k = 2$. This system is therefore not appropriate for describing a solution which has a large (as compared to $\alpha'$) geometrical footprint in spacetime. It is a geometry that is microscopic, from the supergravity perspective. Happily (looking at just this angular sector on its own for now), the model still has the geometry of an $S^2$. There are no $\alpha'$ corrections, as can be seen in various ways, the most straightforward among them being: (1) the whole model can be written (by fixing a gauge and bosonizing the fermions into a single extra bosonic field) as an asymmetrically acting $\mathbb{Z}_2$–orbifold of an $SU(2)$ WZW — the original presentation of GPS for $Q = 0$, and (2) the model is spacetime supersymmetric when $Q = 0$.

To be slightly more explicit, the round $S^2$ that results from the construction has the familiar metric:

$$ds^2 = k(d\theta^2 + \sin^2 \theta d\phi^2) ,$$

written in terms of the the standard angles $\theta$ and $\phi$, which originate as part of the set of Euler angles $(\theta, \phi, \psi)$ of the $S^3 = SU(2)$, where a group element can be written:

$$g = e^{i\phi\sigma_3/2}e^{i\theta\sigma_2/2}e^{i\psi\sigma_3/2} , \quad 0 \leq \theta \leq \pi , \quad 0 \leq \phi \leq 2\pi , \quad 0 \leq \psi \leq 4\pi .$$

The angle $\psi$ is fibred over the $S^2$ in the standard Hopf manner to make an $S^3$. The right $U(1)$ action we discussed earlier is entirely on $\psi$, and after writing the gauged model, a natural gauge in which to work while studying the physics is $\psi = \mp \phi$, (where the “$\mp$” choice refers to the
Northern or Southern hemispheres of the $S^2$) which leaves the two scalar fields $X^1 = \phi, X^2 = \theta$, together with the left and right moving fermions, which are equivalent after bosonization to another scalar we can call $X^3$. The action is, after integrating out the world–sheet gauge fields (a procedure which is exact since there is no field dependence in the $A_z A_{\bar{z}}$ term):

$$I = \frac{k}{4\pi} \int d^2 z \left\{ G^{S^2}_{\mu\nu} \partial_\bar{z} X^\mu \partial_z X^\nu + \frac{1}{Q_+} (\partial_\bar{z} X^3 - 2Q_+ A^M_\mu \partial_z X^\mu) (\partial_z X^3 - 2Q_+ A^M_\mu \partial_\bar{z} X^\mu) \\
- \frac{2A^M_\mu}{Q_+} (\partial_\bar{z} X^3 \partial_z X^\mu - \partial_z X^3 \partial_\bar{z} X^\mu) \right\},$$

where $G^{S^2}_{\mu\nu}$ is the metric on the unit round $S^2$, $k = 2|Q_+ Q_-|$ and the spacetime background $A^M_\mu$ has only one non–zero component given by $2A^\phi_\mu = \pm 1 - \cos \theta$, where the “±” choice refers to the Northern or Southern hemispheres of the $S^2$. In a fermionic presentation of the content of $X^3$, the left–movers couple covariantly to $A^M_\mu$ with charge unity and the right–movers with charge $Q$, the latter being the gauge monopole charge that we will set to zero for our purposes, as already stated. A quick way to see that this, fermions and all, is also equivalent (as shown by GPS) to a bosonic $SU(2)$ WZW (up to a discrete identification to match the periodicity of the fields to the $2\pi$ of $X^3$) is to write $X^3 = Q_+ (\psi \pm \phi)$, from whence some algebra will return one to the standard WZW action with $S^3$ in the metric and a torsion term induced by the $S^3$ volume form $H \sim \sin \theta \, d\theta \wedge d\phi \wedge d\psi$ which can be locally written as $H = dB \sim (\pm 1 - \cos \theta) \, d\phi \wedge d\psi$.

The power and clarity obtained from recasting the GPS model in this way as a heterotic coset with a Lagrangian definition (with the explicit formulae for the world–sheet gauge couplings given in ref.[13]) should not be underestimated. It allows for many more models to be easily and quite intuitively defined by simply picking subgroups to gauge as dictated by one’s geometrical requirements (for example, the freedom to leave an entire $SU(2)_L$ untouched in the prototype GPS model guarantees the rotational invariance of the whole model, and hence a round $S^2$ metric) and then coupling in fermions in a manner which allows one to cancel the total anomaly. Furthermore, various quantities in the conformal field theory are readily extracted (see e.g. ref.[24], for these types of model), sometimes more easily than other methods, by using the path integral definition of the conformal field theory afforded by the method. It is to be expected that this may well be likely useful in the current black hole application.

Returning to the GPS monopole theory, the suggestion of Lapan et. al. that it should be tensored with an $SL(2, \mathbb{R})$ conformal field theory as a candidate for the conformal field theory of the microscopic AdS$_3 \times S^2$ geometry of the (corrected) core of the stretched heterotic string in five dimensions makes perfect sense. There are some pleasant consequences to be derived
when this is all put together: The level of \( SL(2, \mathbb{R}) \), \( k' \) is again fixed by the total condition on the central charge. We add three fermions (in \( Lie(SL(2, \mathbb{R})) \)) on the left for supersymmetry, and the trivial \( T^5 \) sector. The central charge in each sector is of the form

\[
c = \frac{k \text{dim} G}{k + g},
\]

where \( g \) is the Coxeter number of the group (which is 2 here for each factor). As is by now familiar\(^{[22, 23]} \), there is a continuation \( k' \to -k' \) for \( SL(2, \mathbb{R}) \) to achieve a \((-+++)\) signature. The condition on the left that \( c_L = 15 \) yields the same equation that the condition on the right that \( c_R = 26 \), which is:

\[
\frac{3k'}{k' - 2} + \frac{3k}{k + 2} = 6,
\]

which is solved by \( k' = k + 4 \). Since \( k = 2 \), we must have \( k' = 6 \). The AdS\(_3\) geometry of the angular sector is again microscopic, and uncorrected due to being a group manifold.

Finally, it is worth noting that since the microscopic squared radius of the \( S^2 \) part of the microscopic string is \( k = 2 \), we have the result that the area of the resulting four dimensional black hole (obtained by wrapping the string on an additional circle) is \( 8\pi \) in dimensionless units, in string frame. To get to Einstein frame, we need the value of the four dimensional dilaton, which contains the information about \( N \), the number of wrapped strings. According to ref.\(^{[4]} \), the attractor equations\(^{[27]} \) give \( e^{-2\Phi} = N^{1/2} \), and so in Einstein frame, we get the area to be \( A = 8\pi \sqrt{N} \), which after dividing by two (not four in this case\(^{[4]} \)) is indeed the quantum corrected result for the entropy, as can be computed in the holographically dual conformal field theory on the world–sheet of the \( N \) wrapped strings (with one unit of Kaluza–Klein momentum) or in the supergravity using the Wald entropy formula\(^{[5, 28, 29]} \). That this conformal field theory gives precisely the right value for the radius, and leaves room for no other, is encouraging.

### 3 Fundamental Heterotic Strings in Other Dimensions

It would be neglectful to stop at this point, since there is a very natural set of heterotic cosets to explore which may be of relevance to stretched heterotic string sources. The point is that in \( D \) dimensions, the relevant sphere surrounding a string is \( S^{D-3} \). It is again very easy to see how to design the conformal field theory having an exact microscopic geometry of such spheres,

\(^3\)In the first version of this manuscript, we incorrectly shifted\(^{[25]} \) the level \( k \) by \( g \) in the formula following, which led to an inconsistency. We thank Atish Dabholkar and Sameer Murthy for a question about this issue.
using the same heterotic coset method, and the nice fact that:

\[
\frac{SO(n)}{SO(n-1)} \sim S^{n-1}.
\] (10)

Simply gauge according to this coset, with a pure a right action, and the model will be guaranteed to have the required \(SO(n)\) global symmetry arising from the untouched \(SO(n)_L\) symmetry of the conformal field theory.

Our example of section 2 was the case \(n = 3\), for which the gauge group is Abelian. Here, the gauging is non–Abelian, which is a significant difference from the lower dimensional case. As before, however, the classically anomalous gauging must be compensated for by the quantum anomaly of the left–moving fermions one would add for supersymmetry. This will result in a specific value for the level \(k\) as we shall discuss further shortly. This is all then to be combined with the radial sector. For the radial sector, one would just use the \(SL(2,\mathbb{R})\) conformal field theory at level \(k'\) again, for the stretched string’s presumed AdS\(_3\) microscopic geometry.

Again, we can check the central charge, which will give us a condition linking \(k'\) and \(k\). For \(c_L = 15\) or \(c_R = 26\) (with \(n = D - 2\)):

\[
\frac{3k'}{k' - 2} + \frac{n(n-1)}{2} \frac{k}{k+g} - \frac{(n-1)(n-2)}{2} + \frac{1}{2} \times 3 + \frac{1}{2} \times (n-1) = \frac{3}{2} \times (n+2),
\] (11)

where \(g = n-2\) or \(n-1\) (the Coxeter number of the group \(SO(n)\) for \(n\) even or odd, respectively) and so we get the condition, after some algebra:

\[
k' = 2 + \frac{12(k+g)}{n(n-1)g}.
\] (12)

The consistency condition from the anomaly equation from asymmetric gauging to give the sphere gives \(k = A_n\), where \(A_n\) is a pure number which depends on \(n\) in a manner which results from the details of the non–Abelian embedding of the action of the gauge group and how the left–moving supersymmetry fermions in the coset couple. There are \(\dim(S^{n-1}) = n - 1\) such fermions, and so we expect that their anomaly (which has coefficient \(-A_n\)) will not grow any faster than linearly in \(n\). (It is tempting to simply write \(A_n = n - 1\), assuming a contribution of \(-1\) for each fermion in the normalization we’ve been using, but this should be proven.)

So finally, we can define a consistent model (at least as far as gauging and total central charge is concerned) which it is natural to suggest supplies a conformal field theory definition of the

\footnote{The case \(n = 4\) is interesting since then the target is \(S^3\). One might have imagined that simply using an \(SU(2)\) WZW would have sufficed for this example, but there would be no reason to have its size be of order \(\alpha'\) since no anomaly condition would arise to restrict \(k\).}
stretched heterotic string in $D$ dimensions by setting $k = A_n$ and

$$k' = 2 + \frac{12(A_n + g)}{n(n - 1)g}.$$ \hspace{1cm} (13)

Our model has the expected microscopic target space $\text{AdS}_3 \times S^{D-3}$, since the radii are both of order $\alpha'$. The dilation will again be as small as desired, with no coordinate dependence since the quadratic terms in the gauge fields for the non–Abelian gauge action defining the conformal field theory can be written in a way that does not involve any of the other fields[17]. The Jacobian which induces the dilaton coupling in the sigma–model will therefore be field (and hence spacetime coordinate) independent. For the same reason, we expect that the target space geometry is exact.

4 Higher Dimensional Anti–de Sitter Geometries

Given what we did in the previous sections, it is not hard to see how to design conformal field theories which have microscopic $\text{AdS}_{p+2}$ geometries, for $p \geq 0$. It follows quite straightforwardly since anti–de Sitter spacetimes are easily obtained by analytically continuing spheres, and thus the coset of equation (10), appropriately continued, will serve to define the desired spacetimes ($p = n - 3$). It is not clear what the role of such target spaces is, (nor what the detailed spectrum of the conformal field theories thus defined is), but the construction is quite natural. The case $p = 0$, $\text{AdS}_2$, is actually a special case of one of several models proposed in ref.[13], and of a model independently obtained from the GPS–type orbifold perspective in ref.[26]. We gauge a spacelike $U(1)_R$ subgroup of the $SL(2, \mathbb{R})$ WZW (with right–moving fermions) in a manner analogous to the procedure we carried out for $SU(2)$.

The construction works most simply for this example by analytically continuing much of the discussion in section 2 of the $S^2$ conformal field theory. By analytically continuing $SU(2)$ to $SL(2, \mathbb{R})$, or otherwise, it is easy to see that our $U(1)$ generators are again naturally written in terms of Pauli matrices (divided by two), and so we will choose $\sigma_3/2$ as our generator. An example of a continuation is:

$$\theta \rightarrow i\sigma, \phi \rightarrow it, k' \rightarrow -k',$$ \hspace{1cm} (14)

yielding in the heterotic sigma model the $\text{AdS}_2$ metric:

$$ds^2 = k'(d\sigma^2 - \sinh^2 \sigma dt^2).$$ \hspace{1cm} (15)
Other continuations will yield metrics on AdS$_2$ that cover different coordinate patches as desired.

In the same natural normalization as we used before (where gauge quantities and fermions are naturally living in the Lie algebra of the group, or in the case of cosets, the difference between the Lie algebras of the group and the subgroup), the classical anomaly due to gauging is now $k'$, while the quantum anomaly for the supersymmetry fermions (there are two, naturally living in the coset, which is two dimensional) is again $-2$. Therefore, we can again construct a consistent model with $k' = 2$. By analogy with the $S^2$ case, it is easy to see that the theory (after fixing a gauge) can be rewritten as a bosonic $SL(2, \mathbb{R})$ WZW (up to an identification), for which there are no corrections to the metric.

5 Conclusion

This construction of the heterotic conformal field theories with target AdS$_3 \times S^2$ is a very natural description of the microscopic heterotic string, as suggested by Lapan et. al.\cite{8}. By formulating it here as a heterotic coset model, the generalizations of section 3 to the important cases of stretched strings in higher dimensions are quite straightforward. As we have noted, for the higher dimensional models the coset construction is non–Abelian.

There are other models in ref.\cite{13} which can be revisited in the light of this proposal as models of (generically non–supersymmetric) microscopic heterotic strings. There are several two dimensional $SL(2, \mathbb{R})$ models with gauge actions on both the left and right which were still asymmetric. The anomaly from this can again be cancelled against the anomaly from the supersymmetry fermions, and the whole model tensored with the $Q = 0$ GPS monopole representing the $S^2$ theory. This would give a model with non–trivial $\alpha'$ corrections, and a dilaton that varies in the $\sigma$–direction. This might be a model of a wrapped heterotic string in four dimensions which becomes a non–supersymmetric black hole, and as such might make contact with some of the discussions of refs.\cite{30, 31, 32, 33}.

Many other interesting models present themselves for reconsideration, such as those that arose from the twisting together of the angular and radial sectors\cite{13}, resulting in a non–trivial stringy Taub–NUT solution (it came with dyonic charges and non–trivial NUT– and $H$–charge from $G_{t\phi}$ and $B_{t\phi}$ components). A search for a consistent microscopic solution of the various anomaly equations that ensured the consistency of that solution seems to yield interesting solutions that deserve further study.
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