The Interactive Simulation of the Liquid in the Container with Spring-mass Model

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Abstract. With the development of the virtual reality industry, the demand for performance of virtual simulation is increasing. In the field of virtual simulation, fluid simulation has become one of the most challenging research directions in computer graphics. High-performance fluid simulation methods are still a problem which is worth studying. This paper presents a method based on grid and Spring-mass model interactions for simulating liquid in the container. Firstly, the mass-spring model is introduced to simulate the water surface. Create a flat grid, then add physical properties to the point on the triangular grid. Grids are used to connect the water proton. Secondly, change the current height of the particle through calculating the particle density on the surface of liquid to simulate the fluctuations of the water. Finally, realize the calculation method in unity, prove that the method has both performance and sense of reality and better than previous methods.

1. Introduction

At present, the virtual simulation has great commercial value. Virtual space can replace expensive real space to achieve low-cost entertainment and learning. Liquid as an important part of life, fluid simulation and interaction is indispensable, some existing methods can realize the calculation of real fluid, but also to face the plight of lower painting efficiency. A stable and interactive liquid physics simulation is an urgent problem to be solved in virtual reality.

Water simulation in the field of computer graphics, virtual reality and animation has a wide range of application and research achievements. Earlier at SIGGRAPH Alain Fournier put forward sea surface wave simulation based on small amplitude on BBS [1], but this method cannot interaction, simulation is relatively rigid. Later, the researchers proposed liquid simulation method based on the navier-stokes equation, such as the smooth particle hydrodynamics (SPH) method, this method is first put forward by scholars Gingold and Monaghan and Lucy independently [2], the method simulated the flow law of fluid actually, but has the problem of large amount of calculation. Scholars also proposed the euler grid method, the method to give a single grid as a big particles, based on particle grid method in 1971 [3], the amount of calculation is also not suitable for real-time interaction. In real-time simulation of fluid, some scholars put forward the method that make liquid in the container as half spherical pendulum model, this method is lower amount of calculation and can be used to simulate the effect of the sloshing of water, and stable finally, but the surface without any waves, lack of sense of reality.

This paper proposes a method of fluid particle properties combined with spring-mass model to establish water surface, the method can simulate the water surface in the container in real time.
mass model used in the simulation of the realistic cloth interaction, this model has good real-time and authenticity, have been widely used in the game virtual reality [4]. In the process of interaction with natural scene, such as wind and gravity, fabric can also be as generalized fluid, can be blowing the waves like liquid and tilt because of gravity, etc. Import the spring-mass model to simulate the liquid can pursue the balance of the real-time and authenticity. In this paper, the liquid level creation method based on grid, combined with spring-mass model in the case of conform to the liquid physical properties, rendering the surface grid model which has a fixed shape, realize the physical simulation of liquid. On the premise of guarantee the frames, comply with the motion law of fluid, achieve the goal of realize interaction with liquid fluid smooth and really.

2. Related work
In the aspect of spring-mass model application and liquid surface simulation, many researchers have made a body of researches and have gained great achievements, WenTao and others used mass-spring model on fire simulation [5], and obtained a certain result, make the real-time fire simulation does not depend on the particle system which take more resources to achieve real appearance and interaction capabilities at the same time, opens the spring-mass system application in the field outside of cloth simulation. Tang Yong and others used a spring-mass model to establish a water surface model [6], realized the simulation on the influence of wind particles on the water surface. Xu Guokai and others realized the use of spring-mass model to establish a water surface model with a high degree [7], realized the transmission of force between the object and the water surface, the feasibility and interactivity of the spring-mass model in liquid level simulation is confirmed. Zheng Xuan simulated the interaction between water surface and raindrops through spring-mass model [8]. On this basis, this paper simulates the motion of the liquid in the container caused by translation, gravity and inertia.

3. Establishment of water surface model

3.1. Application of spring-mass system
The Mass-Spring Model is a method of simulating the deformation of an object in accordance with Newton's law [9], First proposed by Haumann [10], is widely used in hair simulation and cloth[11]. This model is a \( m \times n \) grid of virtual particles. The masses are connected by a massless and non-zero natural length spring. The link relationship is divided into: stretch springs, Shear Springs, and bend springs [12]. As shown in Figure 1, these three springs are used for calculations related to structural forces (tension or pressure), shear force and bending, respectively. In this paper, the water surface model does not involve folding and bending, and the liquid surface edge is fixed in real time, omitting the calculation of bending, so only stretch springs and shear springs are considered.

![Figure 1. Mass-Spring Model](image)

3.2. The mathematical model of mass
Surface grid topology into a uniform regular grid structure, the point on the grid is treated as a proton. The non-edge proton unit consists of 4 horizontal structural springs, 4 shear springs and a vertical
structural spring, the horizontal position of the edge protons is fixed and is only affected by a vertical structural spring. Proton units have some properties of liquid particles: weight, velocity, acceleration (force $F$) and the current position coordinates $(x, y, z)$, spring properties: Damping, spring rate and resting length. According to figure 2, assume that there are $N$ particles. The quality is $m_i$, the position is $(x_i, y_i, z_i)$, the speed is $v_i$, $1 < i < N$, boolean parameters represent whether they are fixed. These particles are connected by eight sets of horizontal springs $S$, a set of vertical springs $S_o$, the horizontal spring parameter is set to $(N_i, N_j, L, k, k_d)$, $N_i, N_j$ are the spring points of the two ends of the connection, $k$ is the elastic coefficient, $k_d$ is the damping, $L$ is the resting length. Vertical spring parameter set to $(N_i, L_o, k_z, k_d)$, $N_i$ is the particle attached to the spring, $k_z$ is the elastic coefficient, $k_d$ is the damping, $L_o$ is the resting length.

![Figure 2. The mathematical model of the point on grid](image)

The force $F_i, F_j$ exert by the horizontal direction spring $S$ at the two vertices is calculated by Hooke's law:

$$F_i = -F_j = k \frac{x_j-x_i}{|x_j-x_i|} (|x_j-x_i| - L)$$ (1)

The force $F_z$ exert by the vertical direction spring $S_o$ to the particle:

$$F_z = k \frac{x_j-x_i}{|x_j-x_i|} (|x_j-x_i|-L_o) + mg$$ (2)

The damping $f$ of $S, S_o$:

$$f_i = k_d \frac{x_j-x_i}{|x_j-x_i|} (v_j - v_i)$$ (3)

$v_i$ is the velocity of the center particle, $v_j$ is the velocity of its neighboring particle, and $k_d$ is the damping coefficient. By adjusting $k_d$ and the magnitude of the spring stiffness coefficients $k, k_z$ in equations (1) and (2), the different viscosities of the liquid in the container can be simulated. The implicit Euler integral is used to solve the linear equations to get the next moment state.

3.3. The interaction of the container with water

The interaction of the liquid in the container is the transfer of external force to the container, which in turn causes the fluctuations of the protons at the edge of the container. Through the observation of real liquids, when the liquid is affected by an external force in the container, a force opposite to the direction of the external force is generated due to inertia, the shape of the liquid forward view like a exponential function in the container slides along the wall of the container. The ordinary real-time simulation of the liquid interaction in the container is simplified to the one shown in Figure 3. The liquid surface follows the physical properties of the pendulum and gradually stabilize by the loss of energy. In reality, when the liquid surface is shaken, the shape is as shown in Figure 4. The liquid at the edge is low and wide, and the liquid in the middle rises exponentially. As the liquid rises up a certain height along the walls of the container, the width of the liquid becomes narrower, and the higher the liquid, the narrower the width.
3.4. The water sloshing method in this paper

3.4.1. The establishment of the model. Create multiple rigid bodies have mass and collision and distribute them evenly across the plane. The particles are sorted by edge points (The blue dots in figure 5) and center points (The red dots in figure 5) and datum point (the blue dots at the bottom in figure 5), and placed into three arrays of three-dimensional positions in order. Edge particle fixed their $X$, $Y$, close to the edge of the container, the axis of particle in the middle is freedom, but their position does not exceed the edge particle. Add eight physical springs in the horizontal direction and one vertical spring to each center particle. The spring of the central particle links the particle in all directions around it, the vertical spring links the corresponding point on the reference surface. Connect all the particles to form a grid of triangular faces, and update the point position every time step.
Figure 6 shows the real-time effect when a plane with a face number of 200 added with a horizontal spring. Add a horizontal force to the left, liquid surface point move to the right under the inertia and internal friction, be pulled back to the original position by the spring and shaking. Due to the presence of internal friction, the force is lost in transmission and eventually returns to a static state. Add force in different directions to experiment, liquid protons can fluctuate and rotate in all directions, the physical performance close to the real world liquid level sway. The number of frames is basically the same as the maximum number of frames in an empty scene. When the number of faces is doubled, the number of frames does not change significantly, which is meet the requirement of frames.

3.4.2. The calculation method of the surface height. The ordinary spring-mass model can't perfectly simulate the height of the liquid surface when the sway is large. The spring mass system can simulate the slosh caused by the mutual squeeze of water molecules and the fluctuations caused by the interaction between the foreign object and the liquid surface, but can not express the volume change. The SPH method explains the fluid simulation, and the particles are stacked in one direction because of the inertia, causing the liquid level to rise. In this paper, a special computational idea was proposed. By changing the position of the corresponding particle on the reference surface, the height of the liquid surface is the Z-axis parameter of the point on the reference surface plus the resting length L of the vertical direction spring and then subtracting the length of spring pressed by weight ($L_0 = \frac{mg}{k}$ (4)), when the point on the reference surface rises or falls, the corresponding point on the liquid surface will also rise and fall. The position of the point on the datum plane determines the position of the point on the liquid surface. The Z-axis of the point on the datum plane is proportional to the distance between the horizontal coordinate position of the point corresponding to the reference plane and the horizontal coordinate position of its adjacent point. The denser the point-to-point distance, the higher the height of the point. Update and calculate at each time step, simulating the physical phenomenon that the volume of the liquid increases when the volume of the liquid particles is accumulated. The effect of the liquid level sloshing is simulated, and the calculation of the z-axis parameter of the point on the reference plane:

$$z_i = z_o + \frac{h}{j} \left( |x_{n} - x_i| - L_0 \right) + \left( |y_{n} - y_i| - L_0 \right)$$  \hspace{1cm} (5)

$$z_n = \sum_{i=1}^{j} z_i \hspace{1cm} (j \geq 2)$$  \hspace{1cm} (6)

$L_0$ in formula (5) is the resting length of the horizontal spring. In the static state $|x_{n} - x_i| = |y_{n} - y_i| = L_0$, $h$ is the adjustable height coefficient. The higher the value, The higher the fluctuation of the point on the reference plane caused by the distance of the adjacent point, the sum of the multiple directions $z_i$ in formula (6) is the current height of point n, the simulation effect is shown in Figure 7, the frame rate is maintained at about 200 frames. Meet the need for real-time rendering and interaction on the number of frames.
3.4.3. The boundary of the surface treatment. Under the action of inertia, the surface points will exceed the boundary. In order to limit the center particle beyond the boundary proton, record the horizontal position of the particle placed in the boundary proton array, and then detect the particle belonging to the central array, if the particle position exceeds the boundary particle position, the current velocity is immediately reversed according to the inelastic reflection law, causing it to move into the opposite direction.

4. Experimental results

The experiment was carried out under the Windows operating system, and the water surface scene simulation model was established using the Unity development platform and VS 2015. Hardware environment: Intel(R) Core(TM) i7-4790 processor with NVIDIA® GeForce® GTX 960.

In order to enhance the interactivity with the liquid surface, the force parameters $F$ that can change the level and direction in real time are designed, and an input elastic coefficient be designed to test the liquid simulation under different viscosities. The parameters that can be adjusted in the control panel are: elastic coefficient, the weight of mass point and friction coefficient. The experimental results of different force in the same direction, as shown in figure 8.

![Figure 7. (a) Simulation of shaking in this paper and (b) liquid sloshing in real world.](image)

![Figure 8. Under different stress wave effect](image)

This experiment created an edge-fixed mesh, simulates the left and right sway of the mesh in a container under force, and then restores the static state under the action of friction. The motion process under the physical laws and Mass-spring model is shown in Figure 9. While satisfying the realism, the frame rate is more than 200 frames, which can achieve real-time rendering effects. When the number of triangles reaches 12k, the frame rate drops below 100, but more realistic, the frame rate also meets real-time rendering requirements.
The amount of calculation in this method is related to the number $n$ of particle. The more the number of particles, the finer the dynamic drawing of the liquid surface, and the slower the drawing speed, as shown in Table 1.

Table 1. Part of performance when drawing

| Number of the proton | FPS     |
|----------------------|---------|
| 121                  | 200-270 |
| 441                  | 200-270 |
| 6561                 | 100-200 |
| 12100                | 30-90   |

In the usual interactive simulation, the surface fineness does not affect the overall trend of the physical dynamics of the liquid surface. The liquid surface flow can be compensated by dynamic mapping, particle sputtering, etc. So that the liquid surface visual fineness does not depend very much on the finesse of the mesh. The lower number of faces can basically meet most of the interactive simulation requirements. In the experiment, simulating different liquid level, such as the water in the cup, the water in the bathtub, the swimming pool, the sea surface, etc, the number of particle not have to be upgraded in the same level. If using traditional particle simulation methods, when the simulated area is increased, the required number of particles is increased by one to one, and almost no interactive system can undertake real-time calculation for the particle level of a bathtub level in the normal frame number. At the same time, this method will cause the particles to splash because of particle compression. Compared with the traditional particle simulation method, the method in this paper has larger optimization space.

5. Conclusion and Future works
In this paper, an interactive simulation method for liquids in containers based on grid and spring-mass model is proposed. By adding physical properties to the points on the grid, connected with massless springs, the current height of the particle is obtained by calculating the density of the particles on the liquid surface to simulate the sense of volume. Update the position of each point for each time step and draw a grid in real time to simulate the dynamics of the liquid surface under real-time stress conditions.

Compared with the previous method, this method has better performance and larger application range while ensuring the realism.
In the future work, there are two main areas that need to continue to study in depth. One is the processing of boundary particles, mainly the boundary treatment of irregular containers and the redistribution of edge particles when the container is tilted. One is the artistic effect on the liquid surface. For example, adding dynamic maps, particle sputtering and container edges collide to produce optimized image effects.

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