On O+ Ion Heating by BBELF Waves at Low Altitude: Test Particle Simulations

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Abstract We investigate mechanisms of wave particle heating of ionospheric O+ ions resulting from broadband extremely low frequency (BBELF) waves using numerical test particle simulations that take into account ion-neutral collisions, in order to explain observations from the Enhanced Polar Outflow Probe (e-POP) satellite at low altitudes (~400 km) (Shen et al., 2018, https://doi.org/10.1002/2017JA024955). We argue that in order to reproduce ion temperatures observed at e-POP altitudes, the most effective ion heating mechanism is through cyclotron acceleration by short-scale electrostatic ion cyclotron (EIC) waves with perpendicular wavelengths $\lambda_\perp \leq 200$ m. The interplay between finite perpendicular wavelengths, wave amplitudes, and ion-neutral collision frequencies collectively determine the ionospheric ion heating limit, which begins to decrease sharply with decreasing altitude below approximately 500 km, where the ratio $\nu_c/f_{\text{ci}}$ becomes larger than $10^{-3}$, $\nu_c$ and $f_{\text{ci}}$ denoting the O+-O collision frequency and ion cyclotron frequency. We derive, both numerically and analytically, the ion gyroradius limit from heating by an EIC wave at half the cyclotron frequency. The limit is $0.28\lambda_\perp$. The ion gyroradius limit from an EIC wave can be surpassed either through adding waves with different $\lambda_\perp$ or through stochastic “breakout,” meaning ions diffuse in energy beyond the gyroradius limit due to stochastic heating from large-amplitude waves. Our two-dimensional simulations indicate that small-scale (<1 km) Alfvén waves cannot account for the observed ion heating through trapping or stochastic heating.

1. Introduction

The importance of broadband extremely low frequency (BBELF) waves for transverse ionospheric ion heating and ion outflow in the high-latitude auroral region has been well established (Bonnell et al., 1996; Kintner et al., 1996; Knudsen et al., 1998; Lynch et al., 2002). Based on observations from sounding rockets and satellites, a number of wave modes may exist in the broadband emission from several Hz up to several kHz, including Alfvén waves below the ion cyclotron frequency (Chaston et al., 2004; Stasiewicz, Khodyatsev et al., 2000), ion acoustic waves (Wahlund et al., 1998), electrostatic waves near the ion cyclotron frequency (EIC waves) (Bonnell et al., 1996; Kintner et al., 1996), and electromagnetic ion cyclotron (EMIC) waves (Erlandson et al., 1994).

Although it is agreed upon that perpendicular electric fields, either left-hand circularly polarized or linearly polarized, are essential for ion acceleration (Chang et al., 1986; Chaston et al., 2004; Lysak et al., 1980), the ion heating mechanism, dominant wave modes, and perpendicular wavelengths that are responsible for ion heating are still not well known (Paschmann et al., 2003). Until recently, BBELF heating was thought to occur well above the ionosphere where collisional effects are negligible (e.g., X.-Y. Wu et al., 1999; Zeng et al., 2006). Shen et al. (2018) reported multiple examples of BBELF-induced heating at altitudes as low as 350 km and at spatial scales as narrow as 2 km. These observations call for a more comprehensive and realistic ion heating model in the BBELF wave environment, including the effects of both ionospheric ion-neutral collisions and finite perpendicular wavelengths. There has been no such model in the literature yet.

Previous studies have suggested different mechanisms of ion heating from auroral plasma waves. Ions can be resonantly accelerated by lower-hybrid waves when the perpendicular wave phase velocity is comparable with the ion velocity, leading to a heated tail (Lynch et al., 1996; Tsurutani & Lakhina, 1997). On the other hand, particles in the bulk of the ion population can be accelerated simultaneously by long-wavelength...
(much larger than the ion gyroradius $\rho_i$) ion cyclotron waves through quasi-linear cyclotron resonance (Chang et al., 1986). Ball and André (1991) used test particle simulations to study ion heating by BBELF waves and found that the heating rate increases as the wave frequency approaches the ion cyclotron frequency.

A similar, but nonlinear, acceleration mechanism is the coherent ion trapping by EIC waves (Lysak et al., 1980; Ram et al., 1998). Lysak et al. (1980) found that, in a single EIC wave, ions will be trapped in an effective potential well. The energy, or equivalently the gyroradius, of the ion is limited by the first zeros of the Bessel function of the first kind, governed by the parameter $k_\perp \rho_i$, where $k_\perp$ is the perpendicular wavenumber. Lysak et al. (1980) also showed that the presence of additional waves with different wavenumbers can detract the ion, and the gyroradius barrier can be surpassed. Lysak (1986) extended this theory to harmonics of EIC waves, in which case the upper limits of the ion gyroradius are specified by the first zeros of $J_n(k_\perp \rho_i)$, where $n$ is the harmonic integer order. In this paper, we extend these previous results of gyroradius limits to EIC waves with the frequency of $\Omega_i$, which has not been reported yet. We shall provide consistent analytical and numerical calculations for the gyroradius limits.

Another important type of acceleration mechanism is stochastic ion heating, in which ion energy increases due to random kicks in phase space, increasing temperature. Stochastic ion heating has been studied in detail for EIC waves and lower-hybrid waves (Karney, 1978; Lysak, 1986; Papadopoulos et al., 1980). The onset condition is

$$\frac{E_\perp}{B_0} \approx \frac{1}{4} \frac{\Omega_i}{k_\perp}$$

where $E_\perp$ is the amplitude of perpendicular wave electric field, $B_0$ the magnitude of magnetic field, $\Omega_i$ the ion cyclotron frequency, and $\omega$ the wave angular frequency.

Stochastic ion heating due to large-amplitude Alfvén waves has also been investigated by several subsequent studies (Bailey et al., 1995; Chaston et al., 2004; Chen et al., 2001; Johnson & Cheng, 2001; McChesney et al., 1987; Stasiewicz, Lundin, & Marklund, 2000). Chaston et al. (2004) found that, for transverse wave amplitude $E_\perp$ satisfying $\frac{E_\perp}{B_0} < \frac{\Omega_i}{k_\perp}$, ion motion in the wave field is coherent and the ion may become trapped by Alfvén waves in a manner similar to that proposed for EIC waves. The ion may be accelerated up to a gyrodiameter roughly equivalent to the perpendicular wavelength of the wave. When $\frac{E_\perp}{B_0} > \frac{\Omega_i}{k_\perp}$, the ion can be stochastically accelerated, and the gyroradius limit can be surpassed. In addition, in the presence of a large spatial gradient of electric fields, significant ion acceleration and orbit chaotization may take place (Cole, 1976; Stasiewicz, Lundin, & Marklund 2000). However, amplitudes of Alfvén or EIC waves observed in the low-altitude ionosphere are generally insufficient to initiate stochastic ion heating, according to the onset condition of the stochastic behavior (Bonnell et al., 1996; Chaston et al., 2004; Karney, 1978).

Ion-neutral collisions, due to large DC electric fields in the ionosphere, may generate a significant amount of O$^+$ ion heating and sometimes lead to anisotropic and toroidal ion distributions (Lorance & St. Maurice, 1994; St-Maurice & Schunk, 1979; Wilson, 1994). However, collisions can also put a limit on the ion heating in the ionosphere by restricting the achievable speed relative to neutral gases (Schunk & Nagy, 2009). Few models of ionospheric ion heating by plasma waves take into account both the effects of finite perpendicular wavelengths and ion-neutral collisions. Using a multispecies particle simulation, Providakes and Seyler (1990) showed that collisional current-driven EIC waves may be unstable in the bottomside ionosphere (<300 km) and that they are able to generate transverse bulk acceleration of the heavy ions to energies of more than a few keV in this region. However, the required critical field-aligned current for the cyclotron instabilities is at least 50 $\mu$A/m$^2$, which is seen rarely at best and is significantly higher than values reported by Shen et al. (2018). Burleigh (2018) used an anisotropic fluid model to study ion upflows in the ionosphere from drivers of both quasi-linear wave ion heating and collisions. But this fluid model simplifies wave ion heating processes and does not incorporate finite wavelength effects.

In this paper, we shall explore, using test particle simulations, whether and how cyclotron and stochastic ion heating from Alfvén (planar or nonplanar) or EIC waves with different perpendicular wavelengths, along with ion-neutral collisions, might contribute to ionospheric O$^+$ ion heating to observed levels, as exemplified by those found from the Enhanced Polar Outflow Probe (e-POP) at 410 km altitude (Shen et al., 2018). The test particle simulations we use are admittedly simple but allow us to understand the microphysical (kinetic)
processes in some detail. In the following, we shall first examine two different ion heating mechanisms—stochastic acceleration and coherent trapping—and then investigate the role of BBELF waves along with ion-neutral collisions in explaining O⁺ ion heating under the constraints of e-POP observations.

2. Stochastic Ion Acceleration by a Monochromatic Electrostatic Wave

We assume a coherent monochromatic electrostatic wave in a uniform background magnetic field \( \vec{B} = B_0 \hat{z} \):

\[
\vec{E} = E_0 \cos(kx - \omega t) \hat{x}
\]

where \( B_0 \) is the magnitude of magnetic field, \( E_0 \) the amplitude of the electrostatic wave, \( k \) the wavenumber, and \( \omega \) the wave angular frequency.

We use the Hamiltonian method to describe the charged particle’s motion perpendicular to the background magnetic field. The Hamiltonian reads

\[
H = T_{\text{kinetic}} + V_{\text{potential}} = \frac{1}{2m}[(P_y + qB_0x)^2 + P_x^2] - \left( \frac{qE_0}{k} \right) \sin(kx - \omega t)
\]

where \( P_x = mv_x \) and \( P_y = mv_y \) are the ion momentum terms. Following previous similar studies (Bailey et al., 1995; Karney, 1978), we can normalize the physical quantities as

\[
t' = \Omega_i t, \quad x' = kx, \quad P_y' = P_y' = \frac{k}{m\Omega_i},
\]

\[
\alpha = \frac{E_0 k}{B_0 \Omega_i}, \quad \nu = \frac{\omega}{\Omega_i},
\]

where the primed variables indicate normalized quantities. \( \Omega_i = \frac{qB_0}{m} \) is the ion cyclotron frequency, \( \alpha \) the normalized wave potential, and \( \nu \) the normalized wave frequency. The Hamiltonian takes this new form:

\[
H = \frac{1}{2}(P_y' + x')^2 + \frac{1}{2}P_x'^2 - \alpha \sin(x - \nu t)
\]

where we have dropped the prime symbol. The equations of motion for the particle can be found as

\[
\dot{x} = \frac{\partial H}{\partial P_x} = P_x
\]

\[
\dot{P}_x = -\frac{\partial H}{\partial x} = -P_y - x + \alpha \cos(x - \nu t)
\]

\[
\dot{y} = \frac{\partial H}{\partial P_y} = P_y + x
\]

\[
\dot{P}_y = -\frac{\partial H}{\partial y} = 0
\]

where we can identify that \( P_y \) is a constant of motion; therefore, the dynamics in terms of \( x \) and \( P_x \) are exclusively determined by each other. The symmetric equations can be conveniently integrated numerically using the fourth-order symplectic integrator (Forest & Ruth, 1990). Such an integrator is phase space area conserving and more suitable for long-term integration. It also takes less computing time than the classic Runge-Kutta scheme. The orbit of a particle in phase space is often represented by a Poincaré surface of section plot, constructed by marking the particle’s trajectory when it passes a constant plane in phase space, for example, when the wave phase equals \( 2\pi \) in our case. We use 30 test particles that are uniformly distributed in velocity space to explore their trajectories under the influence of the wave electric field. The accuracy of the fourth-order symplectic integrator has been tested on a simple harmonic oscillator so that the relative trajectory error is less than 0.01% for 10 million integration steps. A step size of 0.01 (equivalent to \( \frac{1}{100} \Omega_i^{-1} \)) is chosen as we numerically calculate the particle’s motion over two million steps and construct the Poincaré plots.
In the following, we define the off-resonance case when \( \nu = 0.1 \) and the on-resonance case when \( \nu = 1 \). Linearly polarized waves having a frequency much less than the ion cyclotron frequency (\( \nu = 0.1 \)) may represent Alfvén waves in the sheet-like current structures in the auroral region. In the case described here, when the perpendicular wave electric field and wave vector are approximately parallel, Alfvén waves are in the shear mode. Throughout the paper, the term “Alfvén wave” therefore refers to a linearly polarized wave with a frequency much lower than the ion cyclotron frequency. We ignore the magnetic perturbation, which is justifiable because magnetic perturbations at low altitudes (\(~400 \text{ km}\)) are insignificant compared with the background magnetic field. The on-resonance case corresponds to an EIC wave. Both cases are investigated for different wave potentials or amplitudes \( \alpha \), in order to understand the ion stochastic motion in the Alfvénic and ion cyclotron regimes.

Figure 1 shows Poincaré surface of section plots for both off- (Figures 1a and 1b) and on-resonance (Figures 1c–1f) cases when the wave amplitude \( \alpha \) is set to different values. In Alfvén waves, there is no net ion energy increase when the wave amplitude is small (\( \alpha = 0.1 \)). However, when \( \alpha \) approaches 0.8, or equivalently \( E_i k B_0 \Omega_i \approx 0.8 \), the ion can undergo stochastic acceleration. Figure 1b shows such a phenomenon, where ions start to randomly occupy the velocity space within a specific circle, which represents the energy limit that ions can gain. The stochastic onset condition for Alfvén wave ion heating is consistent with Chaston et al. (2004). Within the stochastic region, ions have access to all the available phase space region, or, equivalently, ions can be accelerated to any energy level accessible.

An ion trajectory that follows a circle in phase space means that its energy does not increase. However, for EIC waves, the ion’s energy increases and returns back to its original level cyclically (Figure 1c) if the ion stays in the system for a sufficiently long time. In this case, the ion is trapped within wave potential wells. The energy limit is determined by harmonic potential structures of the EIC wave, which can be represented by expansion of the wave potential in terms of the Bessel function \( J_n(k\psi) \) of the first kind (Gibelli et al., 2010; Lysak et al., 1980). Similar to Figure 1b, when \( \alpha \) gradually increases from 0.05 to 2.0, we observe that stochastic ion motion emerges from the inner circle and expands into outer circles. The largest circle is specified by \( J_n(k\rho_i) = 0 \) (Lysak et al., 1980). The stochastic onset condition for EIC waves was numerically determined by Karney (1978) as \( \frac{E_i}{B_0} \approx 0.25 \frac{(\Omega_i)}{k} \frac{1}{k_1} \). The numerical constant we found instead is 0.4, which is similar to 0.25 found earlier using different scenarios and parameters. The formula derived from Karney (1978) concerns electrostatic waves with frequencies much larger than the ion cyclotron frequency.

3. Coherent Ion Cyclotron Acceleration by EIC Waves

To understand ion cyclotron acceleration from EIC waves with different frequencies and perpendicular wavelengths, we perform another numerical simulation using the classic Runge-Kutta integrator to resolve ions dynamics. This is because we aim later to investigate how different frequency and wavenumber spectra may affect ion bulk heating, as applicable to the BBELF-heating scenario. The symplectic integrator has no advantage in this situation since the problem is not symmetric. We shall report two phenomena of ion acceleration in the single wave case, that is, the ion gyroradius limit and the stochastic “breakout,” where at a certain critical point an ion’s energy can diffuse beyond the gyroradius limit through stochastic acceleration.

The ion dynamics perpendicular to the background magnetic field are determined by the Lorentz equation:

\[
m \frac{\partial \vec{v}'}{\partial t} = q(\vec{E} (x, t) + \vec{v}' \times \vec{B})
\]

(8)

where \( \vec{B} \) only has a \( \zeta \) component and \( \vec{E} (x, t) \) is the wave electric field in the \( \zeta \) direction. For a single wave case, \( \vec{E} (x, t) = E_0 \cos(\frac{2\pi}{\lambda_x}x - \omega t) \dot{x} \). The equations of motion in the perpendicular directions are

\[
v_x = \frac{q}{m} E(x, t) + \Omega_z v_y
\]

(9)

\[
v_y = -\Omega_z v_x
\]

(10)
where $\Omega_i$ is the ion gyrofrequency. The equation can be numerically integrated using the fourth-order Runge-Kutta integrator. The accuracy of the integrator has been tested by comparing numerical results with the analytical solution of the $E \times B$ drift. A step size of 0.01$T_{gyro}$ is chosen to limit the relative $E \times B$ trajectory error to be within $10^{-6}$ over 10 million integration steps.

In the presence of a single cyclotron wave, an ion is accelerated over the entire gyro-orbit; the ion’s velocity has a component in the same direction of the wave electric field. Figures 2a–2c show the gyroradius evolution for $\omega = \frac{\Omega_i}{2}$, $\Omega_i$, and $2\Omega_i$. The $\frac{\Omega_i}{\lambda_L}$ limits are approximately 0.27, 0.6, and 0.83 for the half, fundamental, and double cyclotron frequency waves, respectively. By comparing these three cases, we observe that ion acceleration by higher-order cyclotron harmonics is generally much less effective than by the wave in the fundamental mode, as it takes a longer time for ions to reach the gyroradius limit in the former cases. The numerically calculated gyroradius limits for integer-harmonic cyclotron waves are consistent with those

![Alfvenic off-resonance](image1.png)

![Cyclotron on-resonance](image2.png)

**Figure 1.** Poincaré surface of section plots for both Alfvenic off-resonance (a and b) and electrostatic ion cyclotron (EIC) wave on-resonance (c–f) cases for different values of wave amplitude $\alpha$. For Alfven waves, there is no net ion energy increase when the wave amplitude is small ($\alpha = 0.1$) (a). However, when $\alpha$ approaches 0.8, the ions can undergo stochastic acceleration as shown in (b). For the on-resonance cases, when $\alpha$ gradually increases from 0.05 to 2.0, we observe that stochastic motion emerges from the inner circle and expands into outer circles. The largest circle is specified by $J_1(k\rho_i) = 0$. 
predicted by Lysak (1986). The limits are determined by the first zeros of $J_n(k\rho_i)$ for cyclotron harmonics. However, no calculation has been reported in the literature for the case of $\omega = \Omega_2$. In Appendix A1, we show an analytical derivation of the ion gyroradius limit with the emphasis on this half cyclotron frequency. The analytical predictions are consistent with the numerical results presented.

Figure 2d shows the result for waves with two frequencies whose difference is the cyclotron frequency. The ion gyroradius limit still holds for waves with multiple frequencies but with a single wavelength. For Figure 2e, there are two waves having different $\lambda_\perp$, each with half of the original wave amplitude. As a result, the ion gyroradius exceeds the limits set by both perpendicular wavelengths. The ion can be accelerated to a gyroradius more than 2.5 times of the maximum $\lambda_\perp$. This is the detrapping effect discussed in Lysak et al. (1980). Note that waves with multiple frequencies that add up to the cyclotron frequency can also accelerate ions (not shown here), which is consistent with previous studies (e.g., Temerin & Roth, 1986).

One way to break the ion gyroradius limit is to have multiple perpendicular wavelengths in the system as shown in Figure 2e. The other way, which we found through test particle simulations, is to increase the amplitude of the wave electric field to a degree that ions eventually diffuse out of the perpendicular
wavelength limit due to stochastic ion heating. Figure 3 shows how the stochastic “breakout” initiates when the EIC wave amplitude increases from 10 to 120 mV/m (a–d), corresponding to $\frac{E_0}{B_0} = 0.041V_{\text{phase}}$ to $0.493V_{\text{phase}}$. When $E_0 = 105$ mV/m, as shown in (c), or equivalently when $\frac{E_0}{B_0}$ approaches $0.43V_{\text{phase}}$ of the wave, the ion gyroradius can be larger than $0.6\lambda_{\perp}$. The magnitude of $B_0$ in the simulation is 40,000 nT.

Figure 3. Stochastic “breakout”—ions diffusing beyond the gyroradius limit due to stochastic ion heating—initiates when the EIC wave amplitude increases from 10 to 120 mV/m (a–d), corresponding to $\frac{E_0}{B_0} = 0.041V_{\text{phase}}$ to $0.493V_{\text{phase}}$. When $E_0 = 105$ mV/m, as shown in (c), or equivalently when $\frac{E_0}{B_0}$ approaches $0.43V_{\text{phase}}$ of the wave, the ion gyroradius can be larger than $0.6\lambda_{\perp}$. The magnitude of $B_0$ in the simulation is 40,000 nT.

### 4. e-POP Observations

One of the major objectives of this paper is to investigate essential wave properties, effective perpendicular wavelengths, and microscopic heating mechanisms that are responsible for BBELF wave ion heating as observed from the e-POP satellite. The e-POP scientific payload is part of the multipurpose CASSIOPE satellite (Yau & James, 2015), launched on 29 September 2013 into a polar elliptical orbit plane with an inclination of 81°, a perigee of 325 km, and an apogee of 1,500 km. In this section, we show the magnetic field, electric field, and ion observations from the wave ion heating event on 18 May 2015. Further details on this event can be found in Shen et al. (2018). Relevant instruments onboard e-POP are described separately in Knudsen et al. (2015), Wallis et al. (2015), and James et al. (2015). The field and particle observations put important constraints on wave electric field amplitudes and ion heating scales, which serve as baselines for the numerical test particle simulations in the next sections.

Figure 4 presents a summary of the field observations from e-POP at 410 km altitude between 22:29:47.4 and 22:30:28.4 UT. Figures 4a and 4b show the magnetic perturbations $B_x$ and $B_y$ in the spacecraft frame, corresponding to the along-track (+x, to the south) and cross-track (+y, to the west) components, respectively. Magnetic fields are measured at 160 samples per second (sps) and bandpass filtered to be within the
frequency range of 3–80 Hz. Figure 4c shows wave electric fields measured from the Radio Receiver Instrument (RRI) in the cross-track (y) direction. Electric fields have been low-pass filtered with a cutoff of 80 Hz. Time series electric fields are underestimated at frequencies below 7 Hz due to limitation of the instrument dynamic range. Note that magnetic fluctuations do not always accompany wave electric fields, meaning the waves are sometimes electrostatic. In fact, Shen et al. (2018) showed that electric fields within BBELF waves measured by e-POP in this region are mostly linearly polarized perpendicular to the magnetic field with frequencies up to 1 kHz (in their Figure 6). One clear signature of O+ ion cyclotron waves is present at the exact location of observed ion heating in this case, which will be shown in detail later. Figure 4d displays the calculated AC Poynting flux from $B_x$ and $E_y$. Alfvénic magnetic fluctuations up to 300 nT, perpendicular electric fields up to 8 mV/m, and Poynting fluxes up to 0.8 mW/m² are observed near 22:30:08 UT. These field fluctuations are colocated with strong O+ ion heating, indicated within the red-shaded region.

To demonstrate microstructures of ion heating, BBELF wave spectra, Alfvén wave, and cyclotron wave characteristics, we present a 1.7 s zoomed-in view of the measurements in Figure 5. Figure 5a shows a heated two-dimensional ion energy-angle distribution measured from the Suprathermal Electron/Ion Imager (SEI) instrument at 22:30:08.8 UT. The maximum O+ ion temperature, represented by the width of the distribution at 55° pitch angle (the white dashed line), is determined to be approximately 4.3 eV, which has been validated through a Monte Carlo charged particle ray tracing simulation (Burchill et al., 2010). The noteworthy feature in the image is that most of the ion signal lies within the energy of approximately 100 eV, as indicated by the red pixels (not saturated). This is one of the observables we use to compare
simulations with observations. In addition, statistical observations from e-POP have shown that ion heating by BBELF waves is associated with ion downflows in the low-altitude (325–730 km) auroral downward current region (Shen et al., 2018). Shen et al. (2018) applied the "pressure cooker" ion heating model with down-pointing electric fields in the return current region (Gorney et al., 1985) to explain the low-altitude
e-POP observations. Based on these results, we conclude that O$^+$ ions can be forced to remain within BBELF-heating regions long enough for their energies to saturate.

Figure 5b presents the $E_y$ and $B_x$ amplitude-frequency spectra calculated within a 0.4 s measurement period inside an ion heating region. We can correct the magnitude measurement of $E_y$ below 10 Hz to account for the instrument filter response. This is plotted as the black solid line in Figure 5b. This frequency spectrum up to approximately 120 Hz is fitted with a power law function $E_y = 4.95 \left( \frac{\nu}{1} \right)^{\frac{1}{14}}$. After taking into account the other component of perpendicular electric field that we do not measure, we assume much larger total electric fields as $E = 8 \left( \frac{\nu}{1} \right)^{\frac{1}{14}}$. This is another important baseline setting for simulations. The O$^+$ cyclotron frequency is about 38 Hz as indicated by the vertical orange line. In Figure 5b, the ratios of the corrected $E_y/B_x$ in the frequency range of 3–10 Hz suggest that the electromagnetic fluctuations have phase speeds of 400–700 km/s. Assuming O$^+$ dominated plasma with an electron density of $10^{11}$ m$^{-3}$ near 400 km altitude, the expected Alfvén speed is approximately 700 km/s, which is consistent with the observations.

Figure 5c compares the measurements of $E_y$ and $B_x$ within 1.7 s of the ion heating period. In this case, the electric field fluctuations are low-pass filtered with a cutoff of 150 Hz. Although time series electric field measurements deteriorate at frequencies lower than 10 Hz, we can see in-phase oscillations of electric field and magnetic field perturbations, with an oscillation period of approximately 0.2 s, which is identifiable near +0.5 and +1.5 s UT on the time axis. The fairly detailed correlation between $E_y$ and $B_x$, inferred Alfvén speeds, and the macroscopic Poynting fluxes as shown in Figure 4d strongly suggest that Alfvén waves are present within the measured BBELF wave spectrum. Most interestingly, right at the time location of O$^+$ ion heating (indicated by the dark-red dotted grid line), $E_y$ shows fluctuations with a frequency of approximately 100 Hz. These waves are not observed outside the ion heating time intervals (within 0.2 s centered around +0.5 s UT on the time axis). The waves are linearly polarized perpendicular to the background magnetic field based on two-component electric field ($E_y$ and $E_z$) hodogram analysis (provided in supporting information [SI]). These features are consistent with electrostatic O$^+$ ion cyclotron waves with amplitudes of up to 5 mV/m and a perpendicular wavelength of approximately 150 m. Such a wave scale is in accord with the gyroradius (180 m) of 100 eV O$^+$ ions. A zoomed-in view of these cyclotron waves is shown in Figure 5d. Therefore, our observations show that both Alfvén waves and EIC waves are present within the BBELF wave spectrum and are associated with ion heating observed by e-POP.

Based on previous studies, ion cyclotron waves may be generated by velocity shear-driven instabilities (Ganguli et al., 1994), current-driven instabilities (Kindel & Kennel, 1971), ion beam-driven instabilities (Hendel et al., 1976), or nonlinear breaking of Alfvén waves (Seyler et al., 1998). The exact nature of how these kinetic processes produce EIC waves is beyond the scope of this study.

5. Test Particle Simulation: Ion Heating From BBELF Waves

Wave frequency measurements from a single spacecraft can be complicated to interpret as to whether they are temporal or spatial structures. This is especially true as the e-POP satellite moves at a speed of 7.8 km/s, which translates to 7.8 Hz for static structures with a perpendicular wavelength of 1 km. In this section, we examine wave ion heating from BBELF waves, which are either Alfvén waves ($\nu = 0.01$, different from the earlier simulations to be consistent with common observations in the ionosphere) or EIC waves ($\nu = 1$) with varying wavenumber spectra. BBELF waves contain many wave modes, including EIC waves that are slightly above the ion gyrofrequency and ion acoustic waves that are below the ion gyrofrequency (Bonnell et al., 1996; Kintner et al., 1996; Wahlund et al., 1998). Here we term any electrostatic wave as EIC wave so far as it has a frequency near the ion gyrofrequency. We only consider wave ion heating in the two-dimensional plane perpendicular to the background magnetic field. Our objective is to find the asymptotic limit of perpendicular ion heating from plasma waves, which justifies our neglecting the effects of parallel transport, including the mirror force and others, that tend to remove hot ions from the heating region. Potential parallel electric fields in Alfvén waves, when their perpendicular wavelengths are comparable to the electron inertial length (e.g., Goertz & Boswell, 1979; Lysak & Lotko, 1996), are ignored in our simulations because ion parallel velocities (on the order of 1 km/s) are much less than the Alfvén wave...
Table 1

| Parameters for the Eight Test Particle Simulation Runs |
|--------------------------------------------------------|
| **Number** | **Wave mode**  | $\omega_{\text{Doppler-shifted}}$ | $\lambda_\perp$ | Initial $k_i\rho_i$ | **Integration time** | **Heating feature** |
| Run-1       | Temporal limit $k = 0$ | $0.1 \Omega_i - 4 \Omega_i$ | $\infty$ m | 0 | $200 T_{\text{gyro}}$ | no heating |
| Run-2       | Spatial limit $\omega = 0$ | $0.1 \Omega_i - 4 \Omega_i$ | $50 - 2,000$ m | 0.02–0.8 | $200 T_{\text{gyro}}$ | no heating |
| Run-3       | 1-D Alfvén 0.5 Hz | $0.1 \Omega_i - 4 \Omega_i$ | $50 - 2,400$ m | 0.017–0.8 | $200 T_{\text{gyro}}$ | no heating |
| Run-4       | 1-D Alfvén 0.5 Hz | $0.1 \Omega_i - 0.1 \Omega_i$ | $2,400 - 65,600$ m | 0.006–0.017 | $1,000 T_{\text{gyro}}$ | trapping |
| Run-5       | 1-D Alfvén 0.5 Hz stochastic | $0.2 \Omega_i - 4 \Omega_i$ | $50 - 1,000$ m | 0.04–0.8 | $1,000 T_{\text{gyro}}$ | small stochastic heating |
| Run-6       | Cyclotron $\Omega_i$ | $0.1 \Omega_i - 2 \Omega_i$ | $70 - 200$ m | 0.22–0.6 | $1,600 T_{\text{gyro}}$ | strong cyclotron heating |
| Run-7       | Cyclotron $\Omega_i$ + Collision | $0.1 \Omega_i - 2 \Omega_i$ | $70 - 200$ m | 0.22–0.6 | $1,600 T_{\text{gyro}}$ | limited cyclotron heating |
| Run-8       | 2-D Alfvén with $10^4\Omega_\perp$ | $0.42 \Omega_i, 0.69 \Omega_i, 1.4 \Omega_i$ | $500, 300, 150$ m | 0.04–0.8 | $1,000 T_{\text{gyro}}$ | no heating |

parallel phase velocity (on the order of 1,000 km/s) so that ions cannot be resonantly accelerated, and their perpendicular temperature is insignificantly affected by parallel electric fields (Chaston et al., 2004).

In the simulation, there are in total 20,000 particles, having an initial two-dimensional Maxwellian distribution in velocity space with a temperature of 0.2 eV, and exhibiting a uniformly distribution in the spatial domain $x = [0,2,000]$ m and $y = [-100,100]$ m. We use the fourth-order Runge-Kutta integrator, as discussed earlier, with a step size of 0.01 $T_{\text{gyro}}$ to evolve the 20,000-particle system numerically. Note that there exists no spatial boundary for an ion’s trajectory to evolve in the simulation. The ion temperature is calculated for the whole ion population and is expressed as $T_i = \frac{m_i}{2k_B} \left( v_y - <v_x> \right)^2 + \left( v_y - <v_y> \right)^2$. Ion temperatures are recorded every 500 steps, corresponding to every $5 T_{\text{gyro}}$. This ensures that we investigate ion dynamics at a constant ion gyrophase, indicating secular variations. The wave electric field is generally in the form $E_x = \sum_{40k} E_0 \cos(k_\perp x - \omega t + \phi_{\text{rand}})$, where 40 different $k$ modes are utilized with random phases $\phi_{\text{rand}}$. The wave amplitudes are $E_0 = 8 \left( \frac{\lambda}{\theta} \right)^{1.14}$.

In addition, many previous studies suggest that Alfvén waves become increasingly nonplanar and display Doppler shifts, perpendicular wavelengths, durations of numerical integration, and ion heating features in the eight test runs we performed. Each test run has been assigned a number indicated in the first column. Broadband emission can be interpreted either as temporal or Doppler-shifted spatial signals in the spacecraft frame, and both contributions will affect the spectrum we observe (Chaston et al., 2004; Stasiewicz, Khotyaintsev et al., 2000). We assume that much of the broadband...
frequency spectrum is due to Doppler shift ($k_{\perp} \cdot \mathbf{V}$) from a satellite passing through waves with finite $\lambda_{\perp}$ at a relative speed of 7.8 km/s. The only exception is Run-1 (temporal limit $k = 0$), where broadband emissions from 0.1$\Omega_i$ to 4$\Omega_i$ are all temporal variations with an infinitely long wavelength. Run-2 (spatial limit $\omega = 0$) treats all fluctuations as stationary spatial structures without time oscillations. No heating is observed for these two cases. The movies showing phase space evolution of all the test runs are in SI.

Figure 6 shows snapshots of the ion distributions (Figures 6a, 6c, 6e, and 6g) and temperature evolution (Figures 6b, 6d, 6f, and 6h) for the test runs of Run-3, Run-4, Run-5, and Run-8. In the case of short-scale
shear Alfvén waves, we observe no ion acceleration or heating as demonstrated in Figures 6a and 6b. On the other hand, large-scale Alfvén waves give rise to only a 0.2 eV ion temperature increase through coherent trapping. The periodic oscillation in the ion temperature is due to time oscillation of 0.5 Hz Alfvén waves. Short-scale planar shear Alfvén waves only generate a small amount of stochastic ion heating up to less than 0.6 eV (Figures 6e and 6f) even as we increase the wave amplitudes tenfold. Ion heating is still insignificant, as shown in Figures 6g and 6h, when we adopt a different field topology with equal \( k_x \) and \( k_y \) and with wave amplitudes increased tenfold (approximately 20, 10, and 5 mV/m for the three wavelengths included).

Our result of 2-D Alfvén wave ion heating differs from Chaston et al. (2004), who suggested that nonplanarity of small-scale Alfvén waves can facilitate ion energization up to several keV through coherent trapping at low altitudes and stochastic ion heating at higher altitudes. The difference can be explained by the fact that electric field amplitudes of Alfvén waves with a perpendicular wavelength of 800 m observed from e-POP (Doppler-shifted to 10 Hz in the satellite frame) are approximately 2 mV/m, which is 2 orders of magnitude smaller than 200 mV/m electric fields used by Chaston et al. (2004) for the same wavelength. It is generally not likely to observe a 200 mV/m electric field at 10 Hz from low-altitude (<1,000 km) satellites (Ball & André, 1991; Basu et al., 1988; J. Wu et al., 2020). As we artificially increase the wave amplitudes of nonplanar 2-D Alfvén waves in our simulation to 200 mV/m (\( \lambda_\perp \sim 500 \) m), we observe significant ion heating up to 20 eV (not shown here), consistent with the \( O^+ \) ion temperature obtained by Chaston et al. (2004). The temperature increase will drop to 1 eV with an electric field amplitude of 100 mV/m. Therefore, the relatively weak electric fields at low altitudes limit the role of either planar or 2-D Alfvén waves in explaining the ion heating we observed through either coherent trapping or stochastic ion heating.

The insignificant amount of ion heating from small-scale Alfvén waves from our simulations is also in contrast to the study of Stasiewicz, Lundin, and Marklund (2000), who suggested that stochastic ion heating from small-scale (of the order of 100 m perpendicular wavelength) Alfvén waves can explain the bulk ion heating observed at Freja and FAST altitudes (>1,700 km). Similarly, that study used large wave amplitudes of 100 mV/m within tens of meter spatial structures to initiate stochastic ion heating. Again, the field amplitudes observed from e-POP and used in the simulations in our paper are an order of magnitude smaller and thus not enough to create significant stochastic ion heating as Stasiewicz, Lundin, and Marklund (2000) does. Alfvén waves may play an indirect role in ion heating cases at low altitudes, which needs to be investigated in the future. In addition to the test runs described here, we also run cases that take into account waves with multiple frequencies below the cyclotron frequency. No significant difference is observed compared with the single-frequency Alfvénic cases.

Figure 7 presents results of the other test runs (Run-6 and Run-7) for short-scale (\( \lambda_\perp \sim 7–200 \) m) cyclotron wave ion heating with (Figures 7a and 7b) and without (Figures 7c and 7d) ion-neutral collisions. In the absence of collisions, small-scale cyclotron waves with wavelengths of less than 200 m effectively heat ions to temperatures of larger than 50 eV within 200 gyroperiods (Figure 7b), corresponding to approximately 5 s. Figure 7a illustrates that accelerated ions obtain energies well above the 100 eV observed limit (the red contour line) due to cyclotron heating. However, many ions are trapped at energies less than 100 eV, which can be attributed to the finite wavelength effect. When collisions are present, ions are isotropically restricted within the 100 eV limit and the temperature surges to a steady state of 26 eV within 50 gyroperiods (near 1 s). Figures 7c and 7d demonstrate that the effect of collisions is to limit as well as to heat the system to a steady-state ion temperature. Compared with the other test runs, small-scale cyclotron wave ion heating regulated by ion-neutral collisions can be most effective in heating \( O^+ \) ions at ionospheric altitudes where e-POP operates.

6. Test Particle Simulation: Cyclotron Ion Heating Regulated by \( E^2, \lambda_\perp, \) and Collisions

In this section, we use test particle simulations to explore how ion-neutral collisions, together with \( \lambda_\perp \) and wave amplitudes, affect ion heating from cyclotron waves in the ionosphere. For multiple test particle simulations, we specify a single cyclotron wave with either an infinite wavelength or a wavelength of 200 m, and with varying amplitudes of 1, 2, 4, or 6 mV/m. Note that the 200 m wavelength is chosen to represent the finite wavelength effect of cyclotron ion heating. We vary the collisional frequency by changing the atomic oxygen \( O \) neutral density from \( 5.7 \times 10^7 \) cm\(^{-3}\), which is calculated from the MSIS00 model (Hedin, 1991) at
410 km altitude, to half and tenth of this value, calculated at an altitude of 450 and 540 km, respectively. A linearly polarized wave can be decomposed into right-handed and left-handed circularly polarized waves with equal amplitudes and only left-handed polarized waves can interact with left-handedly rotating ions in the magnetic field through cyclotron resonance. Thus, only the left-handed polarized power of cyclotron waves are taken into account for comparison with the model of quasi-linear cyclotron heating according to Chang et al. (1986), who suggested that the ion heating rate from cyclotron waves with an infinite wavelength can be expressed as

\[
\frac{dW}{dt} = \frac{q^2 E^2}{2m_i} \tag{11}
\]

where \(E^2\) is the power spectral density of the left-hand component of electric fields. The cases of cyclotron heating with an infinite wavelength are termed “temporal” in the following.

Figure 8 presents the simulation results comparing the magnitude of, and the integration time to, a steady-state ion temperature from cyclotron heating with varying wavelengths, wave powers, and collisional frequencies at altitudes from 410 to 540 km. Figures 8a–8c show increasing steady-state temperatures, from several eV to tens of eV for \(E^2 = 0.5\) (mV/m)\(^2\), with decreasing collision frequencies by tenfold for a 100 km change in altitude. Ion heating limited by the finite wavelength effect (the black dots and lines) has a steady-state temperature smaller than that of temporal cyclotron heating (red triangles and lines) from wave electric fields with the same power. The finite wavelength effect is not discernible when collision frequencies are relatively high and wave powers are relatively weak but dramatic when collision frequencies are small and wave powers strong, as shown in Figure 8a and 8c. For temporal cyclotron ion heating, the temperature limits rise by more than 10 times, from 4.3 to 49 eV when \(E^2 = 0.5\) (mV/m)\(^2\), as the collision frequency is lowered by a factor of 10 from 410 to 540 km. In this case, the parameter \(\frac{v_c}{f_{ci}}\) decreases from about \(10^{-2}\) to \(10^{-3}\), where \(v_c\) is the collision frequency between neutral O and heated O\(^+\) ions in the steady state and \(f_{ci}\) is the ion...
cyclotron frequency in Hz. Overall, collisions play a critical role in regulating ion heating limits in the ionosphere. We observe drastic variations in the ion heating limit with even a 100 km change in altitude for the same wave electric fields.

The ion heating limit from temporal cyclotron heating with collisions can be theoretically calculated by solving the energy equation:

\[
\frac{dW}{dt} = \frac{q^2 E^2}{2m_i} - \nu c_1^2 v^2 = 0 \tag{12}
\]

where \( v \) is the two-dimensional root-mean-square speed in the steady state. Note that \( \nu c \) is constant and can be approximated by \( n(O)Q(v)\nu \) in a steady state, where \( Q(v) \) is the total collision cross section. This equation implies that for each effective resonant \( O^+\)-O collision taking place, the momenta of the two colliding particles exchange completely. The loss of energy due to collisions is balanced by the addition of energy due to ion heating. After we obtain a steady-state root-mean-square speed \( v \) based on Equation 12, we derive the steady-state ion temperature of a two-dimensional Maxwellian distribution based on the assumed relation:

\[
v = \sqrt{\frac{2kT_i}{m_i}} \tag{13}
\]

We display the theoretically calculated steady-state temperatures for cyclotron waves with different powers as the cyan triangles in Figure 8a. The simulated (red) and theoretically calculated (cyan) temporal cyclotron heating limits agree well when \( E^2 \) is below 2 (mV/m)^2. When the wave power becomes larger, the calculated Maxwellian temperatures become much greater than, and deviate from, the simulated limits. This is because the simulated ion distribution becomes increasingly non-Maxwellian and more flat-shaped due to larger electric fields, rendering Equation 13 incorrect.

Figures 8d–8f display integration times to reach a steady-state temperature as the collision frequency declines. As \( E^2 \) and the collision frequency increase, ions reach a steady-state temperature more rapidly,
generally in tens of to hundreds of cyclotron periods, corresponding to approximately 1 to 10 s. Finite wavelength effects heat ions to a steady-state temperature more efficiently when collision frequencies are small (Figures 8e and 8f). In Figure 8d, temporal cyclotron heating accelerates ions more effectively than with a wave having a finite wavelength, and collisions prevail over finite wavelengths in limiting the system to a steady state. Comparing Figure 8d with Figure 8e, as the collision frequency decreases by half, we observe a crossover between collisional and finite wavelength effects in heating ions to a steady-state temperature. In general, the effects of $\lambda_\perp$ wave power, and collision collectively determine the magnitude and efficiency (time to reach a steady-state temperature) of ion heating in the ionosphere.

Note that our neglecting of the magnetic mirror force may overestimate the ion heating limits only slightly, since within the period of 10 s, cold ionospheric ions cannot travel far along the field line and transfer of energy from perpendicular to parallel via the mirror force is quite limited. For example, ionospheric O$^+$ ions with a parallel temperature of 1 eV have parallel speeds of the order of 3 km/s. On a time scale of 10 s, an O$^+$ ion flowing upward due to the magnetic mirror force will travel a vertical distance of 30 km. The perpendicular energy of the ion with an initial value of 10 eV will decrease by only 0.1 eV. In addition, in light of evidence of a low-altitude “pressure cooker” effect (Shen et al., 2018), ions may well be trapped in the heating region long enough to reach the heating limits derived here under the assumption of negligible field-aligned transport.

7. Summary and Conclusions

In this paper, we have examined the physics of ion heating from Alfvén waves ($\omega \ll \Omega_i$) and cyclotron waves ($\omega = \Omega_i$) using test particle simulations, with an objective to explain how BBELF waves heat ionospheric ions as observed from the e-POP satellite in the low-altitude (~400 km) ionosphere. We have investigated the effects of finite perpendicular wavelengths, wave amplitudes, and ion-neutral collisions, on ionospheric ion heating. The results of our test particle simulations in this paper show the following:

1. The numerical onset thresholds of stochastic ion heating from the EIC wave and Alfvén wave, as illustrated by the Poincaré surface of section plots, are consistent with previous studies (Bailey et al., 1995; Chaston et al., 2004; Karney, 1978; Lysak, 1986). One important characteristic of stochastic ion heating is that ions undergoing stochastic acceleration can still be limited by the large potential structure of the wave or similarly by the perpendicular wavelength.

2. The ion gyroradius limit for the EIC wave with half of the cyclotron frequency is $0.28\lambda_\perp$, which has been both numerically and analytically derived. The ion gyroradius limit from a single EIC wave can be surpassed either through adding waves with different $\lambda_\perp$, or through stochastic “breakout,” when the wave electric field amplitude satisfies $E_0/B_0 \geq 0.43V_{\text{phase}}$, where $V_{\text{phase}}$ is the wave phase velocity.

3. In contrast to previous studies focusing on higher altitudes (Chaston et al., 2004; Stasiewicz, Lundin & Marklund 2000), our simulations indicate that both planar and nonplanar small-scale (<1 km) Alfvén waves cannot explain ion heating in our case. In order to reproduce ion distributions observed at e-POP altitudes, the most effective ion heating mechanism from BBELF waves is through collisional cyclotron heating by short-scale EIC waves with $\lambda_\perp \lesssim 200$ m.

4. The interplay between finite perpendicular wavelengths, wave amplitudes, and ion-neutral collision frequencies collectively determine the ionospheric ion heating limit, which begins to decrease sharply with increasing altitude below approximately 500 km altitude, where the ratio $\frac{V_\perp}{V_{\text{sat}}}$ becomes significant ($>10^{-3}$) with decreasing altitude.

Our model of ion heating by Alfvén waves is limited to the perpendicular plane. Ion dynamics in the direction parallel to the magnetic field is not included; as a result, the magnetic mirror force and parallel electric fields are not included. While effects of parallel dynamics on ion heating are not included, they are expected to be small in the low-altitude (410 km) ionosphere as discussed above.

Appendix A: Ion Gyroradius Limit From a Single EIC Wave

Lysak (1986) has given the derivation of the ion gyroradius limit for EIC waves with $\omega = n\omega_c$. Here we present an analytical derivation of the ion gyroradius limit using the gyro-orbit-averaging method for EIC waves with half of the cyclotron frequency, since this has not been reported elsewhere in the literature.
Following the main text, we assume a single coherent EIC wave with a form $E = E_0 \cos(k_\perp x - \frac{\Omega_i}{2} t)$ in a background magnetic field $\vec{B}$ in the $\hat{z}$ direction. Since the wave is electrostatic in nature, the wave scalar potential may be expressed as $\Phi = -\frac{E_0}{k_\perp} \sin(k_\perp x - \frac{\Omega_i}{2} t)$. The ion can be viewed as being trapped by the wave potential; therefore, the total energy of the wave ion system is conserved. The maximum of the ion’s kinetic energy corresponds to the minimum of the wave potential energy, which translates to finding zeros of the wave electric field. Considering secular increase of the ion’s gyroradius, we shall obtain a wave electric field that is averaged over one ion gyro-orbit. The electric field is first rewritten in the reference frame of the guiding center:

$$E = E_0 \cos(k_\perp x_0 + k_\perp \rho_i \cos(\Omega_i t - \frac{\Omega_i}{2} t))$$  \hspace{1cm} (A1)

where $x_0$ is guiding center coordinate and $\rho_i$ is the ion gyroradius. From trigonometric identities, this can be expressed as

$$E = E_0 \cos(k_\perp x_0) \cos(\frac{\Omega_i}{2} t) \cos(k_\perp \rho_i \cos(\Omega_i t)) + \cos(k_\perp x_0) \sin(\frac{\Omega_i}{2} t) \sin(k_\perp \rho_i \cos(\Omega_i t))$$  

$$- \sin(k_\perp x_0) \cos(\frac{\Omega_i}{2} t) \sin(k_\perp \rho_i \cos(\Omega_i t)) + \sin(k_\perp x_0) \sin(\frac{\Omega_i}{2} t) \cos(k_\perp \rho_i \cos(\Omega_i t))$$  \hspace{1cm} (A2)

This form can be further expanded using the Bessel function identities equations $\cos(\cos\theta) = J_0(\theta) + 2 \sum_{n=1}^{\infty} \cos(2n-1)\theta \cos((2n-1)\theta)$. After inserting these Bessel identities into the equation and integrating the electric field within one ion gyro-orbit $< E > = \frac{1}{2\pi} \int \theta < E \ d(\Omega_i t)$, we have the wave electric field in the final form:

$$< E > = E_0 \cos(k_\perp x_0) \left\{ \frac{-4}{3\pi} J_1(k_\perp \rho_i) + \frac{4}{35\pi} J_3(k_\perp \rho_i) - \frac{4}{99\pi} J_5(k_\perp \rho_i) + \ldots \right\}$$

$$+ E_0 \sin(k_\perp x_0) \left\{ \frac{2}{\pi} J_0(k_\perp \rho_i) + \frac{4}{15\pi} J_2(k_\perp \rho_i) - \frac{4}{63\pi} J_4(k_\perp \rho_i) + \ldots \right\}$$  \hspace{1cm} (A3)

The maximum magnitude of the averaged electric field is found when setting $\sin(k_\perp x_0) = \cos(k_\perp x_0) = \frac{\sqrt{2}}{2}$ in which case the wave potential magnitude is at its largest. The maximum ion kinetic energy, or equivalently the ion gyroradius limit, is obtained from the first zero of $< E >$. Since the terms after $J_5(k_\perp \rho_i)$ are negligibly small, we can truncate the series and find the solution to be $k_\perp \rho_i = 1.8$, which translates to the ratio $\frac{\rho_i}{\lambda_i} = 0.28$.

This analytical solution is consistent with the numerical result shown in Figure 2a.

Using the same procedure and changing the wave frequency to $\omega = \Omega_o$, $2\Omega_o$, $3\Omega_o$, and $4\Omega_o$, the resulting averaged wave electric fields are, respectively, $-E_0 \sin(k_\perp x_0) J_1(k_\perp \rho_i)$, $-E_0 \sin(k_\perp x_0) J_3(k_\perp \rho_i)$, $E_0 \sin(k_\perp x_0) J_1(k_\perp \rho_i)$, and $E_0 \cos(k_\perp x_0) J_2(k_\perp \rho_i)$. The first zeros of $J_5(k_\perp \rho_i)$ correspond to $\frac{\rho_i}{\lambda_i} = 0.61, 0.82, 1.0, 1.2$ for $n = 1, 2, 3, 4$. This result is in agreement with Lysak (1986).

**Appendix B: Simulating O⁺-O Collisions**

We use an approximate method of $O^+$-O collisions, which is essentially that used by Barakat et al. (1983) and Wilson (1994). The probability that a particle suffers no collision in the time interval $t$ overall follows Poisson statistics $P_{nc}(t) = e^{-\nu t}$. The probability of collision is therefore $P_c(t) = 1 - e^{-\nu t}$. $\nu$ depends on the neutral O density $n(O)$, RCE collisional cross section, and the relative speed $v$ between $O^+$ and O. Ideally, the overall probability of collision during the time $dt$ can be written in a general form as $P_c(t) = 1 - e^{-dn(O) \sigma(g) \rho v^2 \Delta t}$, where $\sigma(g)$ is the differential cross section of RCE collisions and $f(g)$ is normalized so that $\int f(g) \, dg = 1$. We
adopt the same approximate method as that used by Wilson (1994), where the integral in the exponential is replaced by \( n(O)Q(g)g \) (\( Q(g) \) is the total cross section) and \( g \) is found at each time step. We assume the background neutral O gas is near stationary so that for each RCE collision the O\(^+\) velocity is set to 0 with a probability of 0.5. The neutral O density is obtained from MSIS00 model (Hedin, 1991) using geophysical parameters relevant to the ion heating event reported by Shen et al. (2018). If \( v_r \) is a constant, the relationship between successive collisional time interval \( dt \) and \( P_i(t) \) can be simplified as

\[
\frac{1}{n(O)Q(g)g} \ln(1 - P_i(dt)).
\]

To take into account speed dependence of the collision time \( dt \), we use the “null collision” method (Lin & Bardsley, 1977; Winkler et al., 1992). The simulation steps are as follows:

1. Specify a very small constant \( dt \), which represents the maximum \( v_r \) possible to occur. We use a \( dt \) of about 0.03 s in this paper compared with 0.05 s used by Wilson (1994).
2. Assume collisions occur at this constant \( dt \). Calculate the impact parameter \( b \) based on

\[
\frac{-\ln(1 - r)}{\pi n(O)gd} = b,
\]

where \( r \) is a pseudorandom number between 0 and 1 representing the collision probability within time \( dt \).
3. Calculate a critical impact parameter \( b_{cr} \), which is determined by the experimental \( Q(g) \) measured for RCE collisions. We use the same phenomenological RCE model as in Wilson (1994). \( b_{cr} \) is calculated for each \( g \) such that the total cross section for RCE satisfies \( Q(g) = (10.995 - 0.95\log_{10}(g)) \times 10^{-16} \text{ cm}^2 \). The probability of charge exchange to occur during each collision is 0.5.
4. Compare \( b \) with \( b_{cr} \). If \( b \leq b_{cr} \), a real charge exchange collision occurs during this time interval \( dt \) and the O\(^+\) ion velocity is set to 0; if \( b > b_{cr} \), the assumed collision is “null” and the O\(^+\) ion velocity is not changed.
5. Proceed to the next \( dt \) and repeat the above process.

Using this approach, we find that each test particle within a heated (e.g., 4 eV) ion population experiences approximately 10 collisions within the integration time of 1,000 gyroperiods. This magnitude of O\(^+\)-O collision frequency is comparable with values estimated by Schunk and Nagy (2009) assuming a reduced temperature of approximately 2 eV.

**Data Availability Statement**

e-POP data are accessible online (through http://epop-data.phys.ucalgary.ca/). Simulation data and codes necessary to reproduce the results are available from PRISM Dataverse at University of Calgary’s Data Repository (through https://doi.org/10.5683/SP2/PWYYZQ).

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