Efficient Long-distance Quantum Communication Using Microtoroidal Resonators

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Based on the interaction between a three-level system and a microtoroidal resonator, we present a scheme for long-distance quantum communication in which entanglement generation with near 0.5 success probability and swaps can be implemented by accurate state detection via measuring about 100 photons. With this scheme the average time of successful entanglement distribution over 2500 km with high fidelity can be decreased to about only 30 ms, by 7 orders of magnitude smaller than that through DLCZ protocol. This is in the same order of magnitude as the time scale for the light traveling over this distance. This scheme can produce results comparable to those in [17] but avoid its complex entanglement purification process. Recent advances in the observation of strong interaction between photons and single atoms through microscopic optical resonators [21, 22] lay the foundation for this scheme.

The schematic description of the generation of entanglement between two qubits in a basic segment is shown in Fig. 1. A microtoroidal resonator has two internal counterpropagating modes $a$ and $b$ with a common frequency $\omega_c$. These two modes are coupled owing to the scattering [21, 22]. The intracavity fields are coupled to a tapered fiber with high efficiency $\varepsilon > 0.99$ [23]. The evanescent intracavity fields coherently interact with the ground state $|g\rangle$ and the excited state $|e\rangle$ with energy $\omega_c$ of a three-level atom which is near the external surface of the resonator with well-defined azimuthal phase $\theta = \pi/2$ [22]. A metastable $|s\rangle$ of the atom does not interact with the field of modes $a$ and $b$ due to being off resonance with the field. Such a $\Lambda$ system may be provided by a donor atom in doped silicon basis [24], where the qubit states $|g\rangle$ and $|s\rangle$ are encoded onto electron Zeeman sublevels and the excited state is provided by the lowest bound-exciton state, or other examples such as the hyperfine structure of trapped ions [18].

In this paper we consider the situation where the input field and the resonator are impedance-matched (critical coupling), which can be reached through careful choice of the point of contact between the surface of the microtoroid and the tapered optical fiber and applying the input probe pulse with frequency $\omega_p = \omega_c$ [21, 22]. Under this condition, there are two cases in the forward output of the tapered fiber: First, the output will drop to zero due to the interference between the cavity field $a$ and the input field $\delta a_{in}$ when the atom is in state $|s\rangle$ (dark state); Second, many single-photons come through the resonator one by one with average interval time $\tau_p$ [22], when the atom is in state $|g\rangle$ (bright state) and is in resonance with the cavity $\omega_c = \omega_c$. The qubits in nodes L and R are initialized in state $|g\rangle +$
of a microtoroidal resonator with two modes. The dark count in this case is excluded and the propagation photons are combined through such as a convex lens and measured at the midpoint of the optical length between nodes R and L. The mode of output photon measured by the single-photon detector D measures the photon L, which contains many photons, we may assume the mode of state 1 of output photon measured by the single-photon detector D at the midpoint of the optical length between atoms I1 and I2. Therefore, if D records n = |40, 120| photons, i.e., one of the atoms I1 and I2 is bright, the entanglement swap succeeds with the success probability p1 = 0.5P1. Otherwise, the entanglement fails to extend, and the previous entanglement generating and swapping process needs to be repeated, till the protocol is successful at last. The detector D measures a† a with a = (aL + aR)/√2. Successful measurement will project the state |ψ(φ(1))⟩⟨ψ(φ(2))| into |Ψ(φ)⟩ = (|gs⟩|gs⟩|I1,L,R⟩ + eφ|sg⟩|sg⟩|I1,L,R⟩)/√2 with φ = φ1 + φ2. Then, atoms I1,2 are manipulated to make a unitary transform |s⟩I1 → |e⟩I1,2 by applying π pulses and decay back to the ground state |s⟩I1,2 due to spontaneous emission. Finally, we obtain the state |ψ(φ(1))⟩L. This protocol for entanglement swap can be repeated to extend the communication length. We have the success probability pl = 0.5P1, (i = 1, 2, ..., m) for the rth entanglement swap. Considering the time for the signal traveling from D to L and R (see Fig. 1) to tell the controller whether or not to start the next process, the average total time required for successful distribution of entanglement over distance Lr = Lm = 2nL0 is

\[ T = \frac{T_0}{\prod_{i=0}^{m-1} P_i} = \frac{2^{m+1}T_0}{P_{m+1}}, \]

where T0 = L0/c + tkeL0/c is the time needed to establish entanglement within the basic links with c being the light speed in the optical fiber. Note that T increases near linearly with the channel length.
With the entangled state $|\psi(\phi)\rangle$ between two distant sites in hand, we can apply the entanglement to quantum communication protocols, such as quantum teleportation, cryptography, and Bell inequality test directly. For quantum cryptography and Bell inequality test (see Fig. 4A), we first make a local phase shift to transform the state $|\psi(\phi)\rangle_{LR}$ to state $|\psi\rangle_{LR} = |gs\rangle_{LR} - |sg\rangle_{LR}$ \[10\]. Then, two Raman beams are simultaneously applied to atoms L and R to make $\phi_L$ and $\phi_R$ rotation about axis X, respectively. Finally, two probe pulses of duration $t_\alpha = 100\tau_B$ are applied simultaneously to the resonators to measure the states of atoms $R$ and $L$ with the results 0 for the forward propagating photon number $n < 40$ recorded by detector $D_{1,2}$ and 1 for $n \geq 40$. According to the Ekert protocol for quantum cryptography \[6\], $\phi_L$ and $\phi_R$ are chosen randomly and independently from the set $[0, \pi/2]$, the measurement results becomes the shared secret key if the two sides get information through the classic communication that they have chosen the same rotation. For the Bell inequality test, we obtain the correlations $E(\phi_L, \phi_R) = \frac{N_s(\phi_L, \phi_R) - N_s(\phi_L, \phi_R)}{N_s}$, where $N_s$ denotes the number of measurements with two same (different) results, and the result $E(0, \phi_L, \phi_R) = |E(0, \phi_L, \phi_R) + E(\phi_L, \phi_R) - E(\phi_L, \phi_R)| = 2\sqrt{2}$ violates the CHSH inequality $E(0, \phi_L, \phi_R) \leq 2$ \[27\]. Note that the outcome of every Bell inequality experiment is used due to the unity state detection efficiency. Combined with the space-like measurements, this scheme can be used for a loophole-free test of the Bell inequality \[28\].

To faithfully transfer unknown quantum states $\varphi_i = \alpha|g\rangle_i + \beta|s\rangle_i$ with arbitrary complexes $\alpha$ and $\beta$ satisfying $|\alpha|^2 + |\beta|^2 = 1$, we can use quantum teleportation protocol through the entangled state $|\psi(\phi)\rangle_{LE}$ (see Fig. 4B). Two identical probe pulses of duration $t_\gamma = 100\tau_B$ simultaneously enter the microtoroids, the forward propagating photons are mixed together and measured by the single-photon detector D, which records the photon $a_i^\dagger a_i$ with $a_i = (a_i + e^{i\phi}a_i)/\sqrt{2}$. If the detector records $n \in [40, 120]$ photons, the protocol succeeds with a probability of $0.5P_1$ and the state $|\varphi_i\rangle_{LE}$ is projected into a state $a_s|gs\rangle_{LE} + \beta s|sg\rangle_{LE}$. Then, atom $I$ and $L$ are manipulated to make a unitary transformation $|s\rangle_{IL} \rightarrow |e\rangle_{IL}$ and decay to the ground state $|g\rangle_{IL}$. Finally, we obtain the state $|\varphi_R\rangle = \alpha|g\rangle_R + \beta|s\rangle_R$.

Because of the unambiguous detection of the state of qubits with unity detection efficiency through measuring many single-photon events rather than one arising from the atom bright, the errors from, e.g., the dark count, low detection efficiency, and the imperfection that the single-photon detector may not distinguish between one and two photons can be overcome in this scheme. This results in the increase of the quantum communication speed and the corresponding fidelity. Under the Born approximation, the influence arising from the lost photons in the optical fiber and from the spontaneous emission in stages of entanglement extension and teleportation may be negligible. Owing to the limited coherence time $t_c$, atoms may decay from metastable state $|s\rangle$ to the ground state $|g\rangle$. To solve this problem, we may store the spin state in nuclear memory which has the longest decoherence time in all quantum systems so far. Fast electron-nuclear double resonance (ENDOR) pulse techniques may be used for prompt storage and retrieve of the electron-spin state \[17, 28\]. If two atom $I_1$ and $I_2$ in Fig. 5 decay from $|s\rangle$ to $|g\rangle$, detection D measures $n > 120$ photons, thus, the entanglement swap fails. In a similar way, we can show that in the whole process of distributing entanglement over long distance, there may exist only one atom decaying with at most probability $0.5P_1(1 - e^{-t/c})$ that we cannot exclude through detection of $n$ photons to ensure only one atom bright, except for the two atoms apart with a distance $L_t$. The decoherence arising from the spontaneous decay can be written as $DF \sim 3(1 - e^{-t/c})$. Assuming $L_t = 2^6L_0 = 2500 km$, $c = 2.0 \times 10^8 m/s$, $L_0 = 22 km$ \[14\], from equation \[1\] we have $T = 30 ms$, which is on the same order of the time $L_t/c = 12.5 ms$. Compared with the corresponding time $T = 650000 s$ using the DLCZ protocol for expected fidelity $F = 0.9$ provided that many atoms excitations are the only imperfection in the experiment \[14\], the total average time for successful distributing entanglement over that distance can be reduced by 7 orders of magnitude. Assuming $\tau_B = 6 ns$ \[22\], $t_\gamma = 6 ms$, we have $DF < 0.0018$.

Stationary and non-stationary phase shifts from stationary and non-stationary channels and set-up asymmetries, respectively, are main decoherence source in our scheme. The non-stationary phase shifts increase with the length by the random-
walk rule $\sqrt{L_m/L_0}$ and can be reduced to a negligible degree [10]. Because the total average time $T$ decreases significantly by several orders of magnitude, this phase shifts in our scheme can be overcome much easier than that in DLCZ protocol [16, 29]. The stationary phase shifts are easier to handle than the non-stationary ones are. The mechanism for photon blockade used in our scheme is very robust against many experimental imperfections [22].

As a summary, we present a microtoroidal resonator-based scheme for efficient long-distance quantum communication. Moreover our scheme shows high fidelity and good robustness against many experimental imperfections. To trap single atoms near the surface of the microtoroidal resonator is still a technical challenge. However, with the rapid advances in the relevant technologies it is no doubt that in near future the obstacle will be overcome. This scheme may open up realistic probability of efficient long-distance quantum communication.

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