Three-Body Elastic and Inelastic Scattering at Intermediate Energies
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Abstract: The Faddeev equation for three-body scattering at arbitrary energies is formulated in momentum space and directly solved in terms of momentum vectors without employing a partial wave decomposition. For identical bosons this results in a three-dimensional integral equation in five variables, magnitudes of relative momenta and angles. The cross sections for both elastic and breakup processes in the intermediate energy range up to about 1 GeV are calculated based on a Malfliet-Tjon type potential, and the convergence of the multiple scattering series is investigated.

1. Introduction

Traditionally three-nucleon scattering calculations are carried out by solving Faddeev equations in a partial wave truncated basis. That means an angular momentum decomposition replaces the continuous angle variables by discrete orbital angular momentum quantum numbers, and thus reduces the number of continuous variables needed to be discretized in a numerical treatment. For low projectile energies the procedure of considering orbital angular momentum components appears physically justified due to arguments related to the centrifugal barrier. If one considers three-nucleon scattering at a few hundred MeV projectile energy, the number of partial waves needed to achieve convergence proliferates, and limitations with respect to computational feasibility and accuracy begin to appear. The amplitudes acquire stronger angular dependence, which is already visible in the two-nucleon amplitudes, and their formation by an increasing number of partial waves not only becomes more tedious but also less informative. It appears therefore natural to avoid a partial wave representation completely and work directly with vector variables.

2. Selected Three-Body Scattering Observables

In its simplest form the Faddeev equation with scalar particles is a three-dimensional integral equation in five variables, which is numerically solved below and above the break-up threshold. From its solution the scattering amplitude is obtained as function of vector Jacobi momenta. As a simplification we neglect spin and isospin degrees of freedom and treat three-boson scattering. The interaction employed is of Malfliet-Tjon type, i.e. consists of a short range repulsive and intermediate range attractive Yukawa
force. The parameters of the potential are adjusted so that a bound state at $E_d = -2.23$ MeV is supported. The numerical feasibility and stability of our algorithm for solving the Faddeev equations, especially the treatment of the logarithmic singularities with a spline based semi-analytic method is demonstrated in Refs. [1, 2].

The scattering amplitude is then used to calculate either elastic scattering observables, total and differential cross sections, as well as break-up observables, i.e. exclusive and inclusive inelastic scattering cross sections. In Fig. 1 the elastic differential cross section is shown as function of center-of-mass (c.m.) scattering angle for energies between 0.2 and 1.0 GeV.

![Fig. 1: The elastic differential cross section (c.m.) at $E_{lab} = 0.2$ GeV, 0.5 GeV, 0.8 GeV, and 1.0 GeV projectile energy as function of the scattering angle $\theta_{c.m.}$. All calculations are solutions of the full Faddeev equation.](image)

The semi-exclusive cross section $d\langle N, N' \rangle$ for scattering at 1 GeV is given in Fig. 2 for the emission angles 15° and 33°, where the full Faddeev calculation together with the lowest orders in the multiple scattering series are displayed.

![Fig. 2: The semi-exclusive cross section at 1.0 GeV laboratory incident energy and at 15° angle (left panels) and 33° angle of the emitted particle (right panels). In both cases the upper panel displays the high energy range of the emitted particle, whereas the lower panel shows the low energy range. The full solution of the Faddeev equation is given by the solid line in all panels, The contribution of the lowest orders of the multiple scattering series added up successively is given by the other curves as indicated in the legends.](image)

The peak at the highest energy of the emitted particle is the so called final state interaction (FSI) peak, which only develops if rescattering terms are taken into account. This peak is a general feature of semi-exclusive scattering and is present at all energies. The next peak is the so called quasi-free (QFS) peak, and one observes that at both angles one needs at least rescattering up to the 3rd order to come close to the full result. At
both angles the very low energies of the emitted particle exhibit a strong peak in first order, which is considerably lowered by the first rescattering. Here a calculation up to 3rd order in the multiple scattering series is already sufficient. For small ejectile angles there is interference between the FSI and QFS peak resulting in a shift of the QFS peak to higher ejectile energies when higher orders in the multiple scattering series are taken into account. This phenomenon is not present once the angle of the ejected particle gets larger.

In an exclusive breakup process, two of the outgoing particles are measured in coincidence, resulting in the five-fold differential break-up cross section. As illustration we give in Figs. 3 and 4 specific configurations at $E_{lab} = 1.0$ GeV and consider the exclusive breakup cross section $d^5\sigma_{br}/d\Omega_p d\Omega_q dE_q$ given in the c.m. frame. The energy of the outgoing particle is given by $E_q = \frac{3}{4m}q^2$ and takes for $E_{lab}=1.0$ GeV the value $\frac{3}{4}E_{lab} + E_d \approx 664$ MeV.

In Fig. 3 we show two specific configurations illustrating quasi-free (QFS) scattering conditions. The QFS condition assumes one particle at rest in the laboratory frame, e.g. $k_1 = 0$. This is equivalent to $q = -\frac{1}{2}q_0$, which means that the cosine of the angle between $q$ and $q_0$, $x_q = -1$. The energy $E_q$ corresponding to the QFS condition is approximately one quarter of the total energy, leading to the peak at $\approx 168$ MeV. The left panel indicates very clearly that if one of the scattered particle is parallel to the incoming beam, the first order is dominant, whereas the rescattering terms significantly reduce this first order peak.

The situation is quite different if we consider socalled star configurations, where the three outgoing particles have equal energies and leave with angles of 120° to each other in the c.m. frame. If the plane spanned by the three outgoing particles is orthogonal to the beam direction, the configuration is named space star, if the beam lies in the plane it is called coplanar star. These two special configurations are shown in Fig. 4.
Fig. 4: The exclusive differential cross section at $E_{lab} = 1.0 \text{ GeV}$ for the space star (left panel) and the coplanar star configuration (right panel). The full solution of the Faddeev equation is given by the solid line. The contributions of the lowest orders in the multiple scattering series added up successively are given by the other curves as indicated in the legend.

Since the momenta of the outgoing particles are equal for a given beam energy (here 1 GeV), the energy of a single particle in the star configuration is approximately $(\frac{3}{2}E_{lab} + E_d)/2 = 332 \text{ MeV}$. One important feature of the star configuration is clearly seen, the first order calculation does essentially not contribute to the cross sections. The peak around 330 MeV is completely developed by rescattering contributions. Fig. 4 also shows that for the space star the rescattering contributions shown increase the cross section, whereas the full calculation is lower. This indicates that the multiple scattering series converges very slowly. The situation is similar for the coplanar star, where adding higher orders lets the cross section oscillate around the final result. In the coplanar configuration FSI peaks develop at the highest and lowest energy $E_q$ when rescattering terms are taking into account.

The study of exclusive breakup processes shows very clearly that for the specific configurations considered here, even at an energy as high as 1 GeV the full solution of the Faddeev calculation is needed to obtain a converged result. Further studies scanning the complete three-body phase space are underway. This will be important in order to shed light on previous theoretical analyses of p(d,ppn) reactions which relied on low order reaction mechanisms. It will be also important, to investigate of there are regions in phase space where low order calculations are valid.

3. Relativistic Effects in First Order Calculations

As stated in the introduction, the key advantage of our three-dimensional formulation lies in its applicability at higher energies. At these energies relativistic effects are expected to become important, and their influence on the observables needs to be studied. In our approach we want to identify relativistic effects within the framework of Poincaré invariant quantum mechanics. Here Poincaré invariance is an exact symmetry that is realized by a unitary representation of the Poincaré group on a few-particle Hilbert space [3]. The equations we use have the same operator form as the non-relativistic Faddeev equations,
however the ingredients are quite different.

**Fig. 5:** The semi-exclusive differential cross section at $E_{lab} = 495$ MeV and $\theta_{lab} = 18^\circ$ calculated with the first order term in the multiple scattering series. The dotted line represents the non-relativistic calculation. Using relativistic kinematics leads the long dashed line (r-kin), whereas a full relativistic calculation (r-dyn) gives the solid line. The data represent the spin averaged differential cross section for the (p,n) charge exchange process [4].

First, there are the kinematic effects, which account for the Lorentz transformations between the laboratory and c.m. frame of the three-body system, and different phase space factors in the cross sections. Further Lorentz transformations occur when considering the definitions of Jacobi momenta in the three-body system, which lead to a significantly more involved expression for the permutation operator for identical particles. Further relativistic effects arise from the propagators. Since in a first step we only concentrate on a calculation based on the first order term in the multiple scattering expansion, we only need to consider the two-body propagator in a relativistic Lippmann-Schwinger (LS) equation. To compare with a non-relativistic calculation, this relativistic LS equation needs a potential as driving term which is phase shift equivalent to the non-relativistic one. Last, since in a relativistic formulation the two-body LS equation depends on the two-body total momentum, it must be boosted. We follow here the scheme described in Ref. [5]. In Fig. 5 we present first results for the semi-exclusive break-up cross section in comparison with a non-relativistic calculations. Taking into account relativistic kinematics puts the position of the QFS peak consistent with experimental information. Since in QFS kinematics one particle is assumed to be a spectator, dynamic relativistic effects are expected to be small, as is confirmed by the calculation. All calculations shown are in first order, and rescattering effects are still important at this energy, no statement concerning the height of the peaks should be made. However, we can see, that at higher energies relativistic effects will be quite visible, and further investigation is under way.

**Acknowledgments:** We thank T. Lin for allowing to use her relativistic result for comparison. This work was supported in part by the U.S. DOE under contract no. DEFG02-93ER40576. We thank the Supercomputer Centers NERSC and OSC for the use of their facilities.

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