MEASUREMENT OF THE SCALING PROPERTY OF FACTORIAL MOMENTS IN HADRONIC Z DECAY

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Both three- and one-dimensional studies of local multiplicity fluctuations in hadronic Z decay are performed using data of the L3 experiment at LEP. The normalized factorial moments in three dimensions exhibit power-law scaling, indicating that the fluctuations are isotropic, which corresponds to a self-similar fractal. A detailed study of the corresponding one-dimensional moments confirms this conclusion. However, two-jet subsamples have anisotropic fluctuations, corresponding to a self-affine fractal. These features are, at least qualitatively, reproduced by the Monte Carlo models JETSET and HERWIG.

1 Introduction

Dynamical fluctuations can be investigated using the normalized factorial moments (NFM),

$$F_q(M) = \frac{1}{M} \sum_{m=1}^{M} \frac{<n_m(n_m-1) \cdots (n_m-q+1)>}{<n_m>^q}$$

(1)

where $M$ is the number of bins in which momentum space is partitioned and $n_m$ is the multiplicity in the $m$th bin. If power-law scaling (intermittency),

$$F_q(M) \propto M^\phi_q$$

(2)

is observed then the corresponding hadronic system is fractal, as is expected from a branching process. Since particle production occurs in three-dimensional momentum space, we can distinguish two cases: If the scaling is observed when the space is partitioned equally (unequally) in each direction then the fractal is self-similar (self-affine). For two dimensions the degree of anisotropy is quantified by the so-called Hurst exponent, $H_{ab} = \ln M_a / \ln M_b$, where $M_a$ and $M_b$ are the number of bins in the two dimensions. Projected onto one dimension, $a$, the second-order NFM saturates and is given by

$$F_2^{(a)}(M_a) = A_a - B_a M_a^{-\gamma_a}$$

(3)

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$H_{ab}$ is related to the exponents $\gamma$ through $H_{ab} = (1 + \gamma_b)/(1 + \gamma_a)$. In three dimensions, the dynamical fluctuations are isotropic if $H_{ab} = 1$ for all pairs of axes, $ab$, or, equivalently if the $\gamma_a$ for all three axes, $a$, are equal.

Consequently, we measure the NFM’s in three dimensions to examine whether there is power-law scaling and to measure the intermittency indices, $\phi_q$. We also measure the one-dimensional $F_2$ to get the exponents $\gamma_a$ for the three directions.

To investigate possible dependence of the fluctuations on perturbative or non-perturbative scales, we also examine the $F_2$ for 2-jet events as a function of $y_{cut}$.

## 2 Method of analysis

Since we are studying dynamical fluctuations, the coordinate system should not depend on the dynamics of $q\bar{q} \to$ hadrons. So we choose a cylindrical coordinate system with $z$ axis along the $q\bar{q}$ direction and we partition the phase space in bins of $y$, $p_t$, $\phi$. The $y$ and $p_t$ directions are defined by the electro-weak process. But, we need an origin for $\phi$. Clearly, we should not use an axis, like major, which depends on the QCD dynamics. One possibility is to rotate the coordinate system around the thrust axis and to define a new $x$ axis as lying in the $z$-beam plane. However, it is easier simply to choose the origin at random.

![Figure 1. The 3D NFM as a function of the number of bins, $M$, compared with results of jetset and herwig.](image)

![Figure 2. The $F_2$ in one dimension as function of number of bins, $M$, in the randomized frame, compared to jetset and herwig.](image)
Table 1. The fit parameters of the three-dimensional NFM

| order | $\phi_q$ | $\chi^2$/dof | $\phi_q$ | $\chi^2$/dof |
|-------|----------|--------------|----------|--------------|
| 2     | 0.194 $\pm$ 0.003 $\pm$ 0.003 | 8/9 | 0.221 $\pm$ 0.003 $\pm$ 0.003 | 8/9 |
| 3     | 0.598 $\pm$ 0.011 $\pm$ 0.014 | 11/9 | 0.685 $\pm$ 0.011 $\pm$ 0.012 | 11/9 |
| 4     | 1.082 $\pm$ 0.013 $\pm$ 0.018 | 8/9 | 1.206 $\pm$ 0.022 $\pm$ 0.026 | 7/9 |
| 5     | 1.731 $\pm$ 0.024 $\pm$ 0.025 | 6/9 | 1.858 $\pm$ 0.028 $\pm$ 0.035 | 7/9 |

Experimentally, we estimate the $q\bar{q}$ direction by the thrust axis. We also examine the effect of correcting this direction to the $q\bar{q}$ direction using a correction factor determined from JETSET.

The data used in the analysis were collected by the L3 detector in 1994 at the Z pole. Hadronic events are selected and cuts are applied to obtain well measured tracks and calorimetric clusters. A total of about one million events satisfy the selection criteria.

The NFM's are calculated from the raw data and corrected for detector effects by a factor determined from generator level MC and detector level MC. Systematic errors are estimated from different selection cuts, different methods of event selection and different MC for detector correction.

3 Results of $F_q$ for the full sample

The three-dimensional $F_q$ for an isotropic partitioning of phase space, are shown in Fig. 1 together with the results of fits of Eq. 2, omitting the first point to eliminate the influence of momentum conservation. The results of the fit are also given in Table 1. The fits are good, as is expected if dynamical fluctuations are isotropic. Correcting from the thrust to the $q\bar{q}$ direction systematically increases the values of the $\phi_q$. Both JETSET and HERWIG are seen to agree well with the data.

The one-dimensional $F_2$ are plotted in Fig. 2 and the results of fits of Eq. 3 are shown in Table 2. The values of $\gamma$ for the three axes are equal, confirming that the fluctuations are isotropic. Correcting to the $q\bar{q}$ direction decreases slightly the $\gamma$ values. Again, JETSET and HERWIG agree with the data.

4 Results from 2-jet subsamples

We use the Durham algorithm to define 2-jet events for various values of the jet resolution parameter, $y_{cut}$. We fit the resulting one-dimensional $F_2$ using
Eq. 3 and plot the values of $\gamma$ vs. $k_t = \sqrt{y_{\text{cut}}}$ in Fig. 3. While $\gamma$ increases with $k_t$ (or $y_{\text{cut}}$) for each of the variables, the dependence on $y_{\text{cut}}$ is very different for the different variables, $y$, $p_t$, and $\phi$. JETSET and HERWIG behave qualitatively similarly to the data.

5 Conclusions

In three dimensions, the factorial moments exhibit power-law scaling when momentum space is partitioned isotropically. Fits to the three-dimensional $F_q$ vs. $M$ determine the intermittency indices $\phi_q$. In one dimension, fits to $F_2(y)$, $F_2(p_t)$, $F_2(\phi)$ confirm the isotropy. Both JETSET and HERWIG agree remarkably well with both the three-dimensional $F_q$ and the one-dimensional $F_2$. For 2-jet events, $F_2(y)$, $F_2(p_t)$, $F_2(\phi)$ depend very differently on the jet resolution parameter. Similar behavior is seen in JETSET and HERWIG.

Table 2. The fit parameters of the 1-D NFM.

| $y$  | $p_t$ | $\phi$ |
|------|-------|--------|
| $\gamma$ | $\chi^2$/dof | $\gamma$ | $\chi^2$/dof | $\gamma$ | $\chi^2$/dof |
| 0.992 $\pm$ 0.014 $\pm$ 0.028 | 36/36 | 0.986 $\pm$ 0.022 $\pm$ 0.032 | 29/37 | 0.993 $\pm$ 0.024 $\pm$ 0.033 | 25/35 |

Corrected to $q \bar{q}$

| $y$  | $p_t$ | $\phi$ |
|------|-------|--------|
| $\gamma$ | $\chi^2$/dof | $\gamma$ | $\chi^2$/dof | $\gamma$ | $\chi^2$/dof |
| 0.966 $\pm$ 0.019 $\pm$ 0.018 | 35/36 | 0.972 $\pm$ 0.042 $\pm$ 0.030 | 31/37 | 0.967 $\pm$ 0.034 $\pm$ 0.024 | 27/35 |

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Figure 3. The variation of $\gamma_i$ ($i = y, p_t, \phi$) with $k_t$ ($y_{\text{cut}}$) for data (left), JETSET (middle) and HERWIG (right).

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chengang: submitted to World Scientific on March 25, 2022
Jetset

Herwig

corrected to \( q\bar{q} \)

L3 data (preliminary)

\[ F_2(y) \]

\[ F_2(p_t) \]

\[ F_2(\phi) \]