Gravitational-wave bursts from soft gamma-ray repeaters: Can they be detected?

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In this letter we suggest a scenario for simultaneous emission of gravitational-wave and γ-ray bursts (GRBs) from soft gamma-ray repeaters (SGRs). We argue that both of the radiations can be generated by a super-Eddington accreting neutron stars in X-ray binaries. In this model a supercritical accretion transient takes back onto the remnant star the disk leftover by the hydrodynamic instability phase of a low magnetized, rapidly rotating neutron star in a X-ray binary system. We estimate the rise timescale \( \Delta t_c = 0.21 \text{ ms} \), minimum mass accretion rate needed to trigger the γ-ray emission, \( \dot{M}_\gamma = 4.5 \times 10^{28} \text{ g} \), and its effective associated temperature \( T_{\gamma} = 740 \text{ keV} \), and the timescale for repeating a burst of γ-rays \( \Delta \tau_R = 11.3 \text{ yr} \). Altogether, we find the associated GW amplitude and frequency to be \( h_c = 2.7 \times 10^{-23} / (Hz)^{1/2} \) and \( f_{gw} = 966 \text{ Hz} \), for a source distance \( \sim 55 \text{ kpc} \). Detectability of the pulses by the forthcoming GW antennae is discussed and found likely.

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With the advent of the gravitational-wave astronomy a new window in physics is about to be opened. The first GW signal, whenever detected with the new generation of observatories such as LIGO, VIRGO, GEO600, TAMA [1], and/or the fourth generation of resonant-mass detectors like TIGAs [2–6], will mark a breakthrough in modern astrophysics [7,8]. Most theoretical studies to address the prospects have been concentrated in sources like coalescing NS binaries [9,10] and rapidly rotating NSs [11,12,14]. Statistical estimates of event rates, for both sources, point to an occurrence rate \( \sim 3 \text{ yr}^{-1} \), in the best cases, for distances up to the Virgo cluster [10,13]. On the other hand, neutron star hydrodynamical instabilities, like that one studied by Houser, Cen- tremilla & Smith (1994) (HCS’94) and Lai & Shapiro (1994) [12,14], should occur quite frequently in nature and we focus on the post-hydrodynamical instability phase in such systems, with the perspective of modeling SGRs, which may also be interesting GW sources [13].

Because of the physics underlying the exact nature of soft gamma-ray repeaters (SGRs) is still an unsolved puzzle in high energy astrophysics, the present study is a novel ingredient in our search for understanding these objects. Together with, it is the realization that the foreseeable detection of gravitational waves (GWs), as discussed above, may help inpinning various open problems in both lines of research, yet. The possible confluence of these widely different trends in modern astronomy is an interesting perspective. Motivated by such a worthy possibility, we propose that a neutron star accretes matter from a companion in a X-ray binary system until becoming a millisecond spinning star. Then, centrifugal effects drive equatorial mass ejection, and form a thick disk surrounding the NS. Various instabilities in this dense structure forces it to fragment into clumps that are later on capture back onto the star. Our model allows us to estimate basic important features of SGRs as its risetime, minimum mass accretion rate needed to trigger the γ-ray emission and its effective associated temperature. The time interval to becoming a repeating γ-ray burster is a worthy by-product of our scenario. as shown below our results are in quite well agreement with observations.

Since most of SGRs observed fluxes indicate a luminosity \( \sim 10^{41–42} \text{ erg/s} \) (or probably higher, \( \sim 10^{44} \text{ erg/s} \)) [13]; well above the Eddington luminosity, it is natural to consider these binary systems in a supercritical accretion regime. In our SGRs model we envision a disk-like structure as being a donut-type thick encircling disc with characteristic dimensions, (see Table-1): \( \Delta w = 2 \times R_{ns} \) as the disk width; the difference between the external radius \( R_d \); assumed to be that one derived for the final structure in HCS’94 simulations, \( R_d \sim 5R_{ns} \), and the internal radius, supposed to be the tidal radius, \( R_{ts} \sim 3R_{ns} \). We also assume that the disk density is near that one for the HCS’94 remnant disk-like structure, i.e., \( \rho_d \sim 5.7 \times 10^{12} \text{ g/cm}^3 \) [12]. For parameters of our NS model see Table-2 [13].

Fluid structures, like this accretion disk-type object, may undergo various instabilities, depending on the dynamical conditions (Jeans, Rayleigh-Taylor instability, viscous). Provided the system is driven by Jean’s instability, clumps of matter around the central compact body are produced. The lengthscale of these clumps follows from the Jeans’ wavenumber

\[
K_j^2 = \frac{4\pi G \rho_d}{a_d^2}
\]

where \( K_j \) is the wavenumber of the perturbation in
the parameter $\beta$ which tells us about the efficiency of viscosity in governing the structure. Hence, assuming that $\rho_d$ being the disc density, $T_{\text{peak}}$ is the radiation temperature, and $M_{\text{remn}}$ and $R_{\text{remn}}$ represent the mass and radius of the remnant compact star. We define $M_{\lambda} \approx \frac{M_{\text{tmp}}}{\tau_{ff}}$, being $\tau_{ff} = (G\rho)^{-1/2} \approx 548 \text{ms}$. The equivalent accretion rate per year yields $M_{\lambda} \sim 1.1 \times 10^5 M_\odot/\text{yr}$. Assuming that the hard component of GRB 790305 b is that one characteristic of all SGRs, it can be seen that most of SGRs observed thus far would have had associated mass accretion rates, $M_{\lambda} \sim 4.5 \times 10^{28} g/\text{s}$. We shall assume a value of the parameter $\beta_{\text{acc}} \sim 0.70$ for the case of accretion onto NSs.

TABLE II.

| Parameter | Value |
|-----------|-------|
| $P$ [ms]  | 0.900 | 0.98 |
| $M_{\text{ns}}$ [$M_\odot$] | 1.47 | 1.42 |
| $R_{\text{ns}}$ [cm] | $1.37 \times 10^6$ | $1.41 \times 10^6$ |
| $J$ [g cm$^2$/s] | $9.84 \times 10^{49}$ | $8.78 \times 10^{49}$ |
| $\beta$ | 0.2738 | 0.27 |

By using parameters derived from our scenario (see Table 1), we obtain for the burst peak temperature a value $T_{\text{peak}} \sim 2.6 \times 10^{40}$ K, or equivalently, $T_{\text{peak}} \sim 2.2$ MeV.

Observations of SGRs [22] have shown that in GRB 790305 b there was a soft spectral component which contained most of the energy and 90% of the photons between 30 and 2000 keV. Nevertheless, for blackbody models of the emission processes the effective temperature, i.e., that one is measured by the detectors, $T_{eff} \approx T_{\text{peak}}/3$. Thus, we infer from our model the value 740 keV which is comparable to the hard component observed in SGR 790305 b, 520 ± 100 keV [21]. This leads us to conclude that the scales of temperatures predicted and observed agree with each other. Higher accretion rate would yield in stronger $\gamma$-ray bursts. That it would be the case, for example, for the expected accretion rate, $\sim 8.9 \times 10^{30} g/\text{s}$, in the HCS’94 ejected disk-like matter; we expect the maximum of the emission in that situation to peak at $\sim 3.8$ MeV and the associated GW pulse in that event to be slightly stronger than the one computed for SGRs in the present estimate (see below).

Viscosity torques remove angular momentum forcing the disk eventually to fall down onto the star after a time $\Delta t_{\text{frag}} \sim R_d^2/\nu \geq 10^6 \text{ s}$, from the instant of its
formation. The compact body recovers angular momentum, and rotates more quickly. The amount of angular momentum restored, after the accretion of all the clumps of matter, reads

$$\beta_{\text{acc}} \times N \times M_{\text{\Lambda}} \times V_{\text{orb}} \times R_{\text{orb}} \sim \Delta J_{\text{rec}}$$  \hspace{1cm} (0.4)

where $N$ represents the number of clumps, $\sim K_j \times 2\pi R_{\text{orb}} \sim 5$, $R_{\text{orb}}$ defines the radius for bound orbiting, $\sim 4.0 R_{\text{ns}}$ (HCS’94), and $\Delta J_{\text{rec}}$ the recovered angular momentum. With our assumed parameters for the accreted material we get $\Delta J_{\text{rec}} \sim 5.2 \times 10^{48}$ gcm$^2$s$^{-1}$.

The timescale to repeat a $\gamma$-ray burst can be obtained from the amount of the angular momentum to be accreted from the main accretion disk (the donor star) in order to reentering into a new unstable phase. To estimate such contribution we first calculate the value of the energy parameter, $\beta$, after the transient. This yields

$$\beta = \frac{J_{\text{stable}}^2}{GM_{\text{stable}}^2/R_{\text{stable}}} = 0.27,$$  \hspace{1cm} (0.5)

where we have introduced the factor $f$ to measure the departure of the star from the dynamical instability, which lies the interval $0.3 \leq f \leq 0.6$, and parametrizes the actual star’s non-sphericity and polytropic index of its equation of state in the post-hydrodynamic instability phase. Here the subscript word stable stands for secular phase. The lower limit for $f$ indicates the star has encountered the secular instability. The higher one guarantees the achieving of the dynamical limit. In the present situation we expect this factor $f$ to be $\sim 0.6$. The $\beta$ value for the stable phase, in eq.(5), determines the amount of angular momentum to be gained from the primary star at the characteristic rate of accretion in such systems $\dot{M} \sim (10^{-7} - 10^{-5}) M_{\odot}/yr$ [21].

The next step is to determine $\Delta \tau_R$, the time to repeat a burst. Within a burster phase, we use the relation between the inward angular momentum, $J_+^+$, and the amount of angular momentum to be injected from the main accretion disk, $\Delta J_R$ (so that the star becomes unstable again at $\beta \sim 0.2738$), to write

$$J_+^+ \times \Delta \tau_R \simeq \beta_{\text{acc}} \times \Delta J_R,$$  \hspace{1cm} (0.6)

where $\Delta \tau_R$ defines the timescale for bursting again, and $J_+^+ \equiv M_\text{\Lambda} (GM_{\text{stable}} R_{\text{acc}})^{1/2}$ [21]. Here $M_\text{\Lambda}$ defines the rate of mass accretion from the main disk, $R_{\text{acc}}$ corresponds to the distance the accreted matter will come from, the Alfvén radius for the low $B$ NS in the system, $\sim 10^3 R_{\odot}$. Thus, if we impose for our model NS an accretion rate $\dot{M}_\Lambda \sim 10^{-6} M_{\odot}/yr$ we find $\Delta \tau_R \sim 11.3 \text{ yr}$. This result is in the observed ballpark figure of SGRs active phase timescales $\sim 10 \text{ yr}$. Particularly, the source SGR 1900+14 was observed for the first time in 1979 [24]. The SGR 1806-20 was observed in 1987 [20]. Both became active again in the nineties as confirmed by observations from BATSE (CGRO) and ASCA [23,24,25].

SGR 0526-66 was detected in December 1981 and April 1983, according to observations from the spacecrafts Ven- era 13 and 14 [24]. In this picture, the accretion rate must become strongly supercritical in the last source, $\dot{M} \sim 10^{-4} M_{\odot}/yr$, so that the time scale would turn out to be $\sim 0.11 \text{ yr}$, close to observed time intervals of gamma-ray recurrences in some SGRs, which are $\sim 39 \text{ days}$ [27]. This result is along the lines of King et al. (1997) model [25], and suggests that it can find SGRs systems undergoing episodes of irradiation-driven supercritical mass transfer (IDSMT) during short time spans, depending on binary orbital parameters, with recurrences occurring at time steps as short as $\sim 40 \text{ days}$.

Now, for the case of the theoretical HCS’94 NS remnant, and using the same distance and rate of accretion as above, with a stable mass for the NS $\sim 1.38 M_{\odot}$, we obtain for $\Delta \tau_R \sim 2.4 \times 10^3 \text{ yr}$. Hence, if the accretion process were supercritical, as in systems undergoing IDSMT, the resulting timescale for repeating a burst would be $\sim 24 \text{ yr}$, what is nearly two orders of magnitude longer than that one for Soft Gamma-Ray Repeater Low Mass X-Ray Binaries (SGR-LMXBs), just estimated in the last paragraph.

Finally, we wish to determine the characteristic GW amplitude associated with the burst of $\gamma$-rays. In our model, it is the sudden accretion of matter what triggers the emission of a pulse of GWs during the transition phase to a stable rotating NS, through the fluid modes $f$. To estimate the GW burst amplitude we can compare the GW flux received at Earth to the total gravitational luminosity radiated away by the source [3], extracted from the gravitational potential energy of the orbiting disk-like structure

$$\frac{c^3}{16\pi G} |h|^2 = \frac{1}{4\pi D^2} \frac{\Delta E_{\text{GW}}}{\Delta t}$$  \hspace{1cm} (0.7)

where $\Delta E_{\text{GW}}$ represents the energy flowing into GWs, and is defined as $\Delta E_{\text{GW}} = \Delta L_{\text{GW}} \times \Delta t_e$, with $\Delta t_e$ defined in eq.(2). The gravitational luminosity $\Delta L_{\text{GW}}$ is given by

$$\Delta L_{\text{GW}} = K \times \frac{GM_\text{\Lambda}^2 L^4}{5c^5(\Delta t_e)^6}$$  \hspace{1cm} (0.8)

where $K$ is a constant: $10^{-2} \leq K \leq 10$ [22]. We assume the value $K = 0.5$, the mass being $M_\text{\Lambda} = 3.6 \times 10^{30} \text{ g}$; the one that triggers the emission of the highest energy photons, and the parameter $L$ is the radius of the orbit of that mass around the remnant, $\sim 4R_{\text{ns}}$. Introducing these figures in the equation

$$h_e = \left[ \frac{4G}{c^3 D^2} \left( \Delta L_{\text{GW}} \times (\Delta t_e)^2 \right) \right]^{1/2}$$  \hspace{1cm} (0.9)

we get for the GW amplitude the value $h_e \sim 8.25 \times 10^{-22}$, for a source distance, $D$, of 55 kpc.
In order to be detected by the TIGAs network, the SGRs characteristic GW amplitude, when normalized to its frequency Fourier transform (the square root of the characteristic spectral frequency distribution), our estimated GW-amplitude reduces to \( h_c = 2.7 \times 10^{-23}/(Hz)^{1/2} \). We propose that the GW-frequency should be correlated with the timescale during which half of the infalling clumps of matter impact the NS surface, i.e., \( f_{gw} \sim 1/(5 \times \Delta t_c) = 966 \) Hz. This timescale specifies the elapsed time until a "clean" GW signal from the system is settled down in the GW detector. The frequency sensitivity bandwidth being \( \Delta f_{gw} = 2\mu f_{gw} \sim 30 \) Hz, with \( \mu \) the inverse of the two-mode transducer mass ratio. Such frequency bandwidth is expected to be achieved for most of the planned resonant-mass detectors\(^{[3]}\). The frequency-averaged GW amplitude is certainly under the quoted TIGA’s sensitivity cut off (see Figure 1).

As a check of the calculation just done, we can compute the GW characteristic amplitude with the Waggoner’s (1984) equation\(^{[11]}\) rescaled for the actual accretion mass rate, \( \dot{M}_\lambda \), in our scenario

\[
h = \sqrt{5} \times 10^{-26} \left( \frac{1 \text{kpc}}{D} \right) \left( \frac{k \text{Hz}}{f_{gw} \text{m}} \right)^{1/2} \left( \frac{\dot{M}_\lambda}{10^{-8} \text{M}_\odot/\text{year}} \right)^{1/2}.
\]

(0.10)

This yields \( h_c = 3.0 \times 10^{-23}/(Hz)^{1/2} \). The calculation for the whole range of frequencies (1-3000) Hz are presented in Figure-1. All of the plotted values were normalized for the frequency spectral distribution \((Hz)^{-1/2}\) of the outgoing GW burst, in order to be compared to the TIGAs network and LIGO sensitivities, as shown in Figure-1.

Our model is based on the hypothesis that there exists a peculiar class of LMXBs where higher accretion rates are at work. The burst duration in SGR-LMXBs should be associated with the total accreted mass for the transient. As shown above, the time span for re-bursting can be inferred to from the burst temperature. During quiescent stages, assuming an Eddington’s accretion rate and a source distance \( \sim 55 \) kpc, their X-ray fluxes must be around \( 3.4 \times 10^{-16} \text{erg/(cm}^2\text{s)} \). These fluxes are marginally detectable by Beppo-SAX or ASCA X-ray detectors. For galactic distances, \( \sim 10 \) kpc, those sources would certainly be detectable by such capabilities. For comparison, observed values for the SGRs X-ray luminosities are \( L_x = 7 \times 10^{35} \) erg/s and \( L_x = 3 \times 10^{38} (D/8 \text{kpc}) \text{erg/s, for SGR 1900+14 and 1806-20, respectively} \).

As a point aside, there are left two further pathways for the thick disk to evolve. Firstly, it may not fragment apart anyway. Thus, the whole disk infalls at once onto the NS triggering, therefore, more powerful bursts of both of the radiations. Secondly, had fragments it produces very small lengthscales, then, instead of being observed as a \( \gamma \)-rays train of pulses it should appear as persistent bursts of hard or even soft X-rays, with characteristic timescale \( \sim 1 \) month, and time recurrences far too short, \( \sim 3 \) mins, alike to the ones observed by BATSE from the source GRO J1744-28\(^{[29,30]}\). Sources such as these may prove interesting astrophysical labs to test this scenario. Also do numerical simulations of thick dense disk neutron star interactions taking into account relativistic effects, or instead observations of accretion X-ray systems in which accretion rates \( \sim 10^{-8} \text{M}_\odot/\text{yr} \) are seemingly common\(^{[2]}\), or even such systems as those simulated by Nomoto (1986) and Saio & Nomoto (1998)\(^{[31,32]}\). Our main result is displayed in Figure-1 which shows the "xilophone" of TIGAs, with a bandwidth \( \sim 30 \) Hz, may observe GWs through the NS fluid modes from SGRs, whenever they radiate at frequencies lower than \( 1.4 \) kHz, for distances as far as the LMC. As an example of the detectability of these signals by resonant-mass detectors it is plotted the pulse computed with eq.(7), marked with an (X), which corresponds to \( h_c = 2.7 \times 10^{-23}/(Hz)^{1/2} \) and \( f_{gw} = 966 \) Hz. Interferometric GW detectors like LIGO, for example, may observe such systems provided they emit at mid frequencies, \( \leq 640 \) Hz (first LIGO), and up to 2000 Hz (advanced LIGO), for the same distances. Given the potential implication for the physics of both SGRs and GWs, detailed studies

![Figure 1](image-url)
of the model would be needed.

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† Note that this choice guarantees us that the dynamically unstable NS gets rid of sufficient angular momentum so as to becoming itself secularly unstable (see also Shapiro & Teukolsky 1983).
‡ More details on tables 1 and 2, and also on suggested modelling and observational work will be given in a work in preparation.