1. Editor’s note

Some good news in short:

(1) The published version of Moore’s paper *A solution to the L space problem* is now available online at

http://www.ams.org/journal-getitem?pii=S0894-0347-05-00517-5

(2) Maharam’s Problem was solved by Talagrand (see below).

(3) In the last Jerusalem Logic Seminar, Shelah reported on a new try to solve the minimal tower problem (see Issue 5).

(4) The BEST meeting and the satellite meeting on SPM are very nearby.

Contributions to the next issue are, as always, welcome.
2. Research announcements

2.1. Hurewicz-like tests for Borel subsets of the plane. Let $\xi \geq 1$ be a countable ordinal. We study the Borel subsets of the plane that can be made $\Pi_0^\xi$ by refining the Polish topology on the real line. These sets are called potentially $\Pi_0^\xi$. We give a Hurewicz-like test to recognize potentially $\Pi_0^\xi$ sets.

http://www.ams.org/journal-getitem?pii=S1079-6762-05-00152-6

Dominique Lecomte

2.2. Ordered Spaces, Metric Preimages, and Function Algebras. We prove some results about compact Hausdorff spaces which have scattered-to-one maps onto compact metric spaces, along with two types of consequences of these results for compact LOTses (totally ordered spaces). The first type shows that many products of $n + 1$ LOTses cannot be embedded into any product of $n$ LOTses. The second involves the Complex Stone-Weierstrass Property (CSWP), which is the complex version of the Stone-Weierstrass Theorem. If $X$ is a compact subspace of a product of three LOTses, then $X$ has the CSWP if and only if $X$ has no subspace homeomorphic to the Cantor set.

http://arxiv.org/abs/math.GN/0512553

Kenneth Kunen

2.3. On the independence of a generalized statement of Egoroff’s theorem from ZFC, after T. Weiss. We consider a generalized version (GES) of the well-known Severini-Egoroff theorem in real analysis, first shown to be undecidable in ZFC by Tomasz Weiss. This independence is easily derived from suitable hypotheses on some cardinal characteristics of the continuum like $b$ and $\mathfrak{s}$, the latter being the least cardinality of a subset of $[0, 1]$ having full outer measure.

http://arxiv.org/abs/math.LO/0512546

Roberto Pinciroli

2.4. Forty annotated questions about large topological groups. This is a selection of open problems dealing with “large” (non locally compact) topological groups and concerning extreme amenability (fixed point on compacta property), oscillation stability, universal minimal flows and other aspects of universality, and unitary representations.

http://arxiv.org/abs/math.GN/0512564

Vladimir Pestov

2.5. Strong compactness and a partition property. We show that if Part($\kappa$, $\lambda$) holds for every $\lambda \geq \kappa$, then $\kappa$ is strongly compact.

http://www.ams.org/journal-getitem?pii=S0002-9939-05-08206-7

Pierre Matet
2.6. Countable Borel equivalence relations and quotient Borel spaces. We consider countable Borel equivalence relations on quotient Borel spaces. We prove a generalization of the Feldman-Moore representation theorem, but provide some examples showing that other very simple properties of countable equivalence relations on standard Borel spaces may fail in the context of nonsmooth quotients.

\texttt{math.LO/0512626}

Roberto Pinciroli

2.7. Decisive creatures and large continuum. For \( f, g \in \omega^\omega \) let \( c_{f,g}^\forall \) be the minimal number of uniform trees with \( g \)-splitting needed to \( \forall^\infty \)-cover a uniform tree with \( f \)-splitting. \( c_{f,g}^\exists \) is the dual notion for the \( \exists^\infty \)-cover.

Assuming \( \text{CH} \) and given \( \aleph_1 \) many (sufficiently different) pairs \( (f_\epsilon, g_\epsilon) \) and cardinals \( \kappa_\epsilon \) such that \( \kappa_\epsilon^{\aleph_0} = \kappa_\epsilon \), we construct a partial order forcing that \( c_{f_\epsilon,g_\epsilon}^\exists = c_{f_\epsilon,g_\epsilon}^\forall = \kappa_\epsilon \).

For this, we introduce a countable support semiproduct of decisive creatures with bigness and halving. This semiproduct satisfies fusion, pure decision and continuous reading of names.

\texttt{http://arxiv.org/abs/math.LO/0601083}

Jakob Kellner and Saharon Shelah

2.8. Models of real-valued measurability. Solovay’s random-real forcing (1971) is the standard way of producing real-valued measurable cardinals. Following questions of Fremlin, by giving a new construction, we show that there are combinatorial, measure-theoretic properties of Solovay’s model that do not follow from the existence of real-valued measurability.

\texttt{http://arxiv.org/abs/math.LO/0601087}

Sakaé Fuchino, Noam Greenberg, and Saharon Shelah

2.9. Hausdorff ultrafilters. We give the name \textit{Hausdorff} to those ultrafilters that provide ultrapowers whose natural topology (\( S \)-topology) is Hausdorff, e.g. selective ultrafilters are Hausdorff. Here we give necessary and sufficient conditions for product ultrafilters to be Hausdorff. Moreover we show that no regular ultrafilter over the “small” uncountable cardinal \( u \) can be Hausdorff. (\( u \) is the least size of an ultrafilter basis on \( \omega \).) We focus on countably incomplete ultrafilters, but our main results also hold for \( \kappa \)-complete ultrafilters.

\texttt{http://www.ams.org/journal-getitem?pii=S0002-9939-06-08433-4}

Mauro Di Nasso and Marco Forti

2.10. Block combinatorics. In this paper we extend the block combinatorics partition theorems of Hindman and Milliken-Taylor in the setting of the recursive system of the block Schreier families (\( B^\xi \)), consisting of families defined for every countable ordinal \( \xi \). Results contain (a) a block partition Ramsey theorem for every countable ordinal \( \xi \) (Hindman’s Theorem corresponding to \( \xi = 1 \), and the Milliken-Taylor Theorem to \( \xi \) a finite ordinal), (b) a countable ordinal form of the block Nash-Williams partition theorem, and (c) a countable ordinal block partition theorem for sets closed in the infinite block analogue of Ellentuck’s topology.

\texttt{http://www.ams.org/journal-getitem?pii=S0002-9947-06-03864-5}
2.11. **Maharam’s problem.** We construct an exhaustive submeasure that is not equivalent to a measure. This solves problems of J. von Neumann (1937) and D. Maharam (1947).

http://arxiv.org/abs/math.FA/0601689

**Michel Talagrand**

2.12. **Universality of uniform Eberlein compacta.** We prove that if \( \mu^+ < \lambda = \text{cf}(\lambda) < \mu^{80} \) for some regular \( \mu > 2^{80} \), then there is no family of less than \( \mu^{80} \) \( c \)-algebras of size \( \lambda \) which are jointly universal for \( c \)-algebras of size \( \lambda \). On the other hand, it is consistent to have a cardinal \( \lambda \geq \aleph_1 \) as large as desired and satisfying \( \lambda^{<\lambda} = \lambda \) and \( 2^{\lambda^+} > \lambda^{++} \), while there are \( \lambda^{++} \) \( c \)-algebras of size \( \lambda^+ \) that are jointly universal for \( c \)-algebras of size \( \lambda^+ \). Consequently, by the known results of M. Bell, it is consistent that there is \( \lambda \) as in the last statement and \( \lambda^{++} \) uniform Eberlein compacta of weight \( \lambda^+ \) such that at least one among them maps onto any Eberlein compact of weight \( \lambda^+ \) (we call such a family universal). The only positive universality results for Eberlein compacta known previously required the relevant instance of \( GCH \) to hold. These results complete the answer to a question of Y. Benyamini, M. E. Rudin and M. Wage from 1977 who asked if there always was a universal uniform Eberlein compact of a given weight.

http://www.ams.org/journal-getitem?pii=S0002-9939-06-08189-5

**Mirna Dzamonja**

2.13. **Linearly ordered compacta and Banach spaces with a projectional resolution of the identity.** We construct a compact linearly ordered space \( K \) of weight aleph one, such that the space \( C(K) \) is not isomorphic to a Banach space with a projectional resolution of the identity, while on the other hand, \( K \) is a continuous image of a Valdivia compact and every separable subspace of \( C(K) \) is contained in a 1-complemented separable subspace. This answers two questions due to O. Kalenda and V. Montesinos.

http://arxiv.org/abs/math.FA/0602628

**Wieslaw Kubis**

2.14. **Steinhaus Sets and Jackson Sets.** We prove that there does not exist a subset of the plane \( S \) that meets every isometric copy of the vertices of the unit square in exactly one point. We give a complete characterization of all three point subsets \( F \) of the reals such that there does not exists a set of reals \( S \) which meets every isometric copy of \( F \) in exactly one point. A finite set \( X \) in the plane is Jackson iff for every subset \( S \) of the plane there exists an isometric copy \( Y \) of \( X \) such that \( Y \) does not meets \( S \) in exactly one point. These results are related to the open problem (Steve Jackson): Is every finite set \( X \) in the plane of two or more points Jackson?

http://www.math.wisc.edu/~miller/res/jack.pdf

**Su Gao, Arnold W. Miller, and William A. R. Weiss**
3. Problem of the Issue

For a set of reals $X$, $\mathcal{B}_\omega$ denotes the collection of all countable Borel $\omega$-covers of $X$, and $\mathcal{B}_\gamma$ denotes the collection of all countable Borel $\gamma$-covers of $X$. $S_1(\mathcal{B}_\omega, \mathcal{B}_\gamma)$ is equivalent to $\left(\mathcal{B}_\omega \upharpoonright \mathcal{B}_\gamma\right)$, that is, “every countable Borel $\omega$-cover of $X$ contains a $\gamma$-cover of $X$”.

The following problem is posed in Miller’s paper A Nonhereditary Borel-cover $\gamma$-set (Real Analysis Exchange 29 (2003/4), 601–606).

**Problem 3.1.** Does Martin’s Axiom imply the existence of an uncountable set of reals satisfying $S_1(\mathcal{B}_\omega, \mathcal{B}_\gamma)$?

It is known that the Continuum Hypothesis can be used for that (see e.g. Miller’s mentioned paper). There are unpublished notes of Todorcevic on this problem, contact me directly for a copy.

Boaz Tsaban
4. Problems from earlier issues

Issue 1. Is \( (\Omega_1)^{\Gamma_1} = (\Omega_1)^{\Omega_1} \)?

Issue 2. Is \( U_{\text{fin}}(\Gamma, \Omega) = S_{\text{fin}}(\Gamma, \Omega) \)? And if not, does \( U_{\text{fin}}(\Gamma, \Gamma) \) imply \( S_{\text{fin}}(\Gamma, \Omega) \)?

Issue 4. Does \( S_1(\Omega, \Gamma) \) imply \( U_{\text{fin}}(\Gamma, \Gamma) \)?

Issue 5. Is \( p = p^* \)? (See the definition of \( p^* \) in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying \( S_1(\mathcal{B}_\Gamma, \mathcal{B}) \)?

Issue 8. Does \( X \not\in \text{NON} (\mathcal{M}) \) and \( Y \not\in \text{D} \) imply that \( X \cup Y \not\in \text{COF} (\mathcal{M}) \)?

Issue 9. Assume \( CH \). Is \( \text{Split}(\Lambda, \Lambda) \) preserved under finite unions?

Issue 10. Is \( \text{cov}(\mathcal{M}) = \text{od} \)? (See the definition of \( \text{od} \) in that issue.)

Issue 11. Does \( S_1(\Gamma, \Gamma) \) always contain an element of cardinality \( b \)?

Issue 12. Could there be a Baire metric space \( M \) of weight \( \aleph_1 \) and a partition \( \mathcal{U} \) of \( M \) into \( \aleph_1 \) meager sets where for each \( \mathcal{U}' \subset \mathcal{U} \), \( \bigcup \mathcal{U}' \) has the Baire property in \( M \)?

Issue 14. Does there exist (in ZFC) a set of reals \( X \) of cardinality \( \mathfrak{d} \) such that all finite powers of \( X \) have Menger’s property \( U_{\text{fin}}(\mathcal{O}, \mathcal{O}) \)?

Issue 15. Can a Borel non-\( \sigma \)-compact group be generated by a Hurewicz subspace?

Issue 16. Does \( \text{MA} \) imply the existence of an uncountable \( X \subseteq \mathbb{R} \) satisfying \( S_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma) \)?

Previous issues. The first issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, on http://arxiv.org/abs/math.GN/x, where \( x \) is 0301011, 0302062, 0303057, 0304087, 0305367, 0312140, 0401155, 0403369, 0406411, 0409072, 0412305, 0503631, 0508563, 0509432, and 0512275, respectively, for issues number 1 to 15.

Contributions. Please submit your contributions (announcements, discussions, and open problems) by e-mailing us. It is preferred to write them in LATEX. The authors are urged to use as standard notation as possible, or otherwise give the definitions or a reference to where the notation is explained. Contributions to this bulletin would not require any transfer of copyright, and material presented here can be published elsewhere.

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