Relativistic Nonlocality in experiments with successive impacts

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Abstract

Relativistic Nonlocality is applied to experiments in which one of the photons impacts successively at two beam-splitters. It is discussed whether a time series with 2 non-before impacts can be produced with beam-splitters at rest and such an experiment may allow us to decide between Quantum Mechanics (QM) and Relativistic Nonlocality (RNL).

Keywords: relativistic nonlocality, multisimultaneity, timing-dependent joint probabilities, 2 non-before impacts.

1 Introduction

Relativistic Nonlocality (RNL) is an alternative nonlocal description which unifies the relativity of simultaneity and superluminal nonlocality, avoiding superluminal signaling. Its main feature is Multisimultaneity, i.e. each particle at the time it impacts on a beam-splitter, in the referential frame of this beam splitter, takes account of what happens to the other “entangled” particles. Multisimultaneity implies rules to calculate joint probabilities which are unknown in QM, and deviates from the time insensitivity of the QM formalism: In RNL which rule applies to calculate probabilities depends not only on indistinguishability but also on the timing of the impacts at the beam-splitters [1, 3].

In previous articles RNL has been applied to experiments with fast moving beam-splitters. As well for experiments with 2 before impacts [4], as for such with 2 non-before impacts [2] RNL leads to predictions conflicting with QM.

The possibility of testing time insensitivity with beam-splitters at rest has also been suggested [6]. In this article we explore more in depth this possibility. In an experiment in which one of the particles impacts successively at two beam-splitters before getting detected, three different time series can be arranged, one of them exhibiting 2 non-before impacts. It is argued that for this case results contradicting QM cannot be excluded, and therefore it may be a profitable endeavour to perform the corresponding experiment.

2 Experiments with photons impacting successively at two beam-splitters

Consider the gedankenexperiment represented in Fig. 1. Two photons emitted back-to-back in a “Bell state”, can travel by alternative pairs of paths from the source S to either one of the left-hand detectors D1(+1) and D1(−1) and either one of the right-hand detectors D2(+1) and D2(−1). Before they are getting detected photon 1 impacts on beam-splitter BS11, and photon 2 impacts successively on beam-splitters BS21 and BS22. The phase parameters are labeled φ11, φ21 and φ22. The beam-splitters are supposed at rest in the laboratory frame.

By displacing the mirrors M11 it is possible to

![Figure 1: Experiment with photon 2 impacting successively at resting beam-splitters BS21 and BS22.](image-url)
achieve three different Time Series in the laboratory frame:

1. The impact on BS$_{22}$ occurs before the impact BS$_{11}$.
2. The impact on BS$_{11}$ occurs before the impact on BS$_{21}$.
3. The impact on BS$_{21}$ occurs before the impact on BS$_{11}$ the impact on BS$_{11}$ occurs before the impact on BS$_{22}$.

Unless stated otherwise, we assume in the following these two indistinguishability conditions:

**Condition 1:** Through detection of photon 1 after BS$_{11}$ and detection of photon 2 between BS$_{21}$ and BS$_{22}$ it is in principle impossible to know to which input sub-ensemble a particle pair belongs.

**Condition 2:** Through detection of photon 1 after BS$_{11}$ and detection of photon 2 after BS$_{22}$ it is in principle impossible to know which path photon 2 did travel, neither before its arrival at BS$_{21}$, nor before its arrival at BS$_{22}$.

In the following sections we discuss the three Time Series considered above, first according to QM and thereafter according to RNL.

### 3 The QM description

The conventional application of the quantum mechanical superposition principle considers all three time series as being equivalent. The relative time ordering of the impacts at the beam-splitters does not influence the distribution of the outcomes; in this respect only indistinguishability matters: if it is impossible to obtain path information the sum-of-probability-amplitudes rule applies. Accordingly for all three time series QM predicts:

\[
P^{QM}(u_{11}, u_{22})_{\sigma \omega} = \frac{1}{4} + \frac{\sigma \omega}{8} \left( \cos(\phi_{11} - \phi_{21} - \phi_{22}) - \cos(\phi_{11} - \phi_{21} + \phi_{22}) \right),
\]  

where $\sigma, \omega \in \{+, -\}$, and $P^{QM}(u_{11}, u_{22})_{\sigma \omega}$ denote the quantum mechanical joint probabilities for the four possible outcomes obtained through detections after BS$_{11}$ and BS$_{22}$ under the indistinguishability condition 2. From Eq. (1) follows the correlation coefficient:

\[
E^{QM} = \sum_{\sigma, \omega} \sigma \omega P^{QM}(u_{11}, u_{22})_{\sigma \omega} \\
= \frac{1}{2} \left( \cos(\phi_{11} - \phi_{21} - \phi_{22}) - \cos(\phi_{11} - \phi_{21} + \phi_{22}) \right).
\]  

### 4 The RNL description

The basic principles and theorems of RNL presented in [1] are now extended to experiments with successive impacts. We discuss experiments with moving beam-splitters involving multisimultaneity (i.e. several simultaneity frames) and, as particular cases, the three possible time series in the experiment of Fig. 1 with beam-splitters at rest (i.e., involving only one simultaneity frame).

At time $T_{ik}$ at which particle $i$, $(i \in \{1, 2\})$, arrives at beam-splitter BS$_{ik}$ we consider in the inertial frame of this beam-splitter which beam-splitters BS$_{jl}$ particle $j$, $(j \in \{1, 2\}, j \neq i)$ did already reach, i.e. we consider whether the relation ($T_{ik} < T_{j1}$)$_{ik}$ holds, or there is a BS$_{jl}$ such that the relation ($T_{jl} \leq T_{ik} < T_{jl+1}$)$_{ik}$ holds, the subscript $ik$ after the parenthesis meaning that all times referred to are measured in the inertial frame of BS$_{ik}$.

#### 4.1 Timing $(b_{11}, b_{22})$

If $(T_{11} < T_{21})_{11}$, then we consider the impact on BS$_{11}$ to be a before one, and we label it $b_{11}$.

If $(T_{21} < T_{11})_{21}$, we consider the impact on BS$_{21}$ to be a before one, and label it $b_{21}$.

If $(T_{22} < T_{11})_{22}$ and $(T_{21} < T_{11})_{21}$, then we assume the impact on BS$_{22}$ to be a before one, and we label it $b_{22}$). However, if $(T_{22} < T_{11})_{22}$, but $(T_{21} \geq T_{11})_{21}$, the impact on BS$_{22}$ would be a non-before one.

**Principle I** of RNL implies:

\[
P(b_{11}, b_{22})_{\sigma \omega} = P^{QM}(d_{11}, d_{21})_{\sigma \omega} = \frac{1}{4},
\]  

where $P^{QM}(d_{11}, d_{21})_{\sigma \omega}$ denotes the joint probabilities predicted by standard QM if the particles are detected after BS$_{11}$ and BS$_{21}$, and it is possible to know which path photon $i$ travels before entering BS$_{i1}$, i.e., to which of the two prepared sub-ensembles the photon pair belongs.

Eq. (3) leads to the correlation coefficient:
Similarly, if \( (a_{11}, b_{21}) \), the photon pair belongs.

\[ E(b_{11}, b_{21}) = \sum_{\sigma, \omega} \sigma \omega P(b_{11}, b_{21})_{\sigma \omega} \]

Similarly, we assume that the photons of a pair undergoing impacts \( b_{11} \) and \( b_{22} \) produce values taking into account only local information, i.e., photon \( i \) does not become influenced by the parameters photon \( j \) meets at the other arm of the setup. Therefore Principle I of RNL implies that:

\[ P(b_{11}, b_{22})_{\sigma \omega} = P_{QM}(d_{11}, d_{22})_{\sigma \omega} = \frac{1}{4}, \]

where \( P_{QM}(d_{11}, d_{22})_{\sigma \omega} \) denotes the joint probabilities predicted by standard QM if the particles are detected after BS\(_{11}\) and BS\(_{22}\), and it is possible to know which polarization photon \( i \) has before entering BS\(_{11}\), i.e., to which of the two prepared sub-ensemble the photon pair belongs.

Accordingly one is led to the correlation coefficient:

\[ E(b_{11}, b_{22}) = \sum_{\sigma, \omega} \sigma \omega P(b_{11}, b_{22})_{\sigma \omega} \]

\[ = \frac{1}{4} \sum_{\sigma, \omega} \sigma \omega = 0. \quad (4) \]

\[ \text{4.2 Timing} \quad (a_{11[22]}, b_{22}) \quad \text{(e.g. Series 1)}, \quad \text{and} \quad (b_{11}, a_{22}) \quad \text{ (e.g. Series 2)} \]

If \( (T_{22} > T_{11} \geq T_{21})_{11} \), we assume the impact on BS\(_{11}\) to be a non-before one with relation to the impact on BS\(_{21}\), and label it as \( a_{11[21]} \). If \( (T_{11} \geq T_{22})_{11} \), we assume the impact on BS\(_{11}\) to be a non-before one with relation to the impacts on BS\(_{22}\), and label it as \( a_{11[22]} \).

Similarly, if \( (T_{21} \geq T_{11})_{21} \), or \( (T_{22} \geq T_{11})_{22} \), we assume the impact on BS\(_{22}\) to be a non-before one with relation to the impacts on BS\(_{11}\), and we label it \( a_{22[11]} \), or simply \( a_{22} \) since no ambiguity results.

First of all consider an experiment \( (a_{11[21]}, b_{21}) \) in which the photons are detected after leaving BS\(_{11}\) and BS\(_{21}\). As stated in \( \text{(Principle II)} \) RNL considers the correlations to reveal causal links, and assumes the values \( (a_{11[21]} \sigma) \) to depend on the values \( (b_{21})_{\omega} \) as follows:

\[ P(a_{11[21]}, b_{21})_{\sigma \omega} = P_{QM}^{\sigma}(u_{11}, u_{21})_{\omega}, \quad (7) \]

what yields the correlation coefficient:

\[ E(a_{11[21]}, b_{21}) = \cos(\phi_{11} - \phi_{21}). \quad (8) \]

Principle \( \text{II} \) can be extended straightforward to experiments \( (a_{11[22]}, b_{22}) \) and \( (b_{11}, a_{22}) \) as follows:

\[ P(a_{11[22]}, b_{22})_{\sigma \omega} = P(b_{11}, a_{22})_{\sigma \omega} = P_{QM}^{\sigma}(u_{11}, u_{22})_{\omega}. \quad (9) \]

Obviously, time series 1 corresponds to an experiment \( (a_{11[22]}, b_{22}) \), and time series 2 to a \( (b_{11}, a_{22}) \) one, and therefore, taking Eq. \( \text{(8)} \) into account, one is led to the following correlation coefficient:

\[ E(a_{11[22]}, b_{22}) = E(b_{11}, a_{22}) = \frac{1}{2} \left( \cos(\phi_{11} - \phi_{21} - \phi_{22}) - \cos(\phi_{11} - \phi_{21} + \phi_{22}) \right). \quad (10) \]

Eq. \( \text{(10)} \) and the preceding Eq. \( \text{(8)} \) can be considered the translation into mathematical terms of Bell’s claim: "Correlations cry out for explanation".

\[ \text{4.3 Timing} \quad (a_{11[22]}, a_{22}): \quad \text{Need for conditional probabilities} \]

We consider now an experiment in which the impact on BS\(_{11}\) is non-before with relation to the impact on BS\(_{22}\), and the impact on BS\(_{22}\) is non-before with relation to the impact on BS\(_{11}\). As discussed in \( \text{(III)} \), it would be absurd to assume together that the impacts on BS\(_{22}\) take into account the outcomes of the impacts on BS\(_{11}\), and the impacts on BS\(_{11}\) take into account the outcomes of the impacts on BS\(_{22}\). That is why RNL assumes that photon \( i \) undergoing an \( a_{ik[j]l} \) impact always takes account of the values \( (b_{ij})_{\omega} \) photon \( j \) had produced in a before impact, but not necessarily of the values \( (a_{ij}[k]l)_{\sigma} \) photon \( j \) actually produces.

To put this principle into an equation requires the introduction of conditional probabilities. We denote by \( P\left(a_{ik[j]l} \sigma | (b_{ij}, b_{jl})_{\sigma \omega}\right) \) the probability that a particle pair that would have produced the outcome \( (\sigma, \omega) \) in a \( (b_{ij}, b_{jl}) \) experiment, produces the outcome \( (\sigma', \omega) \) if the experiment is a \( (a_{ik[j]l}, b_{jl}) \) one. Then it holds that:

\[ P(a_{11[22]}, a_{22})_{\sigma \omega} = \sum_{\sigma, \omega} P(b_{11}, b_{22})_{\sigma \omega} \times P\left(a_{11[22]} \sigma | (b_{11}, b_{22})_{\sigma \omega}\right) \times P\left(a_{22} \omega | (b_{11}, b_{22})_{\sigma \omega}\right). \quad (11) \]
Equation (11) corresponds to the Principle IV proposed in [1].

**4.4 Avoiding to multiply causal links needlessly**

Applying "Occam’s razor" RNL tries to account for the phenomena without multiplying causal links beyond necessity, and assumes:

\[
P(\sigma, \omega) = P(\sigma) \times P(\omega) = P(\sigma | \omega) \times P(\omega | \sigma).
\]

Eq. (12) is an straightforward application of Principle III in [1], and can be further extended in a natural way through the following two arrays of equalities:

\[
P(\sigma, \omega) = P(\sigma | \omega) \times P(\omega | \sigma).
\]

4.6 **Timing \((a_{11[21]}, a_{22})\), e.g., Series 3.**

Time series 3 clearly corresponds to an experiment in which the impact on BS\(_{11}\) is a non-before one with relation to the impact on BS\(_{21}\), and the impact on BS\(_{22}\) is a non-before one with relation to the impact on BS\(_{11}\).

Application of the rule expressed in Eq. (11) to this case yields

\[
P(a_{11[21]}, a_{22}) = \sum_{\sigma, \omega} P(b_{11}, b_{21}) \times P(a_{11[21]} | \sigma) \times P(a_{22} | \omega),
\]

and taking account of Eq. (12) and (14), one gets the corresponding the 2 non-before impacts theorem:

\[
E(a_{11[21]}, a_{22}) = E(b_{11}, b_{21}) \times E(a_{11[21]}, b_{21}) E(b_{11}, a_{22}).
\]

Then substitutions according to Eq. (3) and (4) yield:

\[
E(a_{11[21]}, a_{22}) = 0.
\]

5 **Other possible versions of RNL**

To this point we would like to stress that in case of the experiment \((a_{11[21]}, a_{22})\) one is not led into absurdities if one assumes a dependence of the values \((a_{22})_{\omega'}\) on the values \((a_{11[21]})_{\sigma}\) for the values \((a_{11[21]})_{\sigma}\) are assumed to depend on \((b_{21})_{\omega}\), and not on \((a_{22})_{\omega'}\).

Therefore a multisimultaneity theory in which it holds that

\[
P(a_{11[21]}, a_{22}) = P^{QM}(u_{11}, u_{22}) \sigma_{\omega},
\]

cannot be excluded in principle, at least at the present stage of analysis. Obviously, this would mean to assume (apparently without necessity) a dependence of the value \((a_{22})_{\omega'}\) on the value \((b_{21})_{\omega}\) through the bias of the twofold dependence of \((a_{22})_{\omega'}\) on \((a_{11})_{\sigma'}\) and \((a_{11})_{\sigma'}\) on \((b_{21})_{\omega}\). Accordingly Eq. (3) would fail, and neither theorem (3) follows from relation (3), nor theorem (3) from relation (7).
Furthermore, the version of RNL presented in Section 4 assumes that the joint probabilities in experiments $(b_{11}, a_{22})$, $(a_{11}[22], a_{22})$ and $(a_{11}[21], a_{22})$, do not depend on whether the impact on BS$_{21}$ is a $b_{21}$ or an $a_{21}$ one. The possibility of an alternative multisimultaneity theory with esthetically more appealing rules has been suggested in [4].

### 6 Real experiments

A real experiment can be carried out arranging the setup used in [5] in order that one of the photons impacts on a second beam-splitter before it is getting detected. For the values:

\[
\phi_{11} = 45^\circ, \phi_{21} = -45^\circ, \phi_{22} = 90^\circ, \tag{21}\]

Eq. (2) and Eq. (19) yield the predictions:

\[
E_{QM}(u_{11}, u_{22}) = 1 \\
E(a_{11}[21], a_{22}) = 0. \tag{22}\]

Hence, for Time Ordering 3 and settings according to (21) the experiment represented in Fig. 1 allow us to decide between QM and the version of RNL proposed in Section 4 through determining the experimental quantity:

\[
E = \sum_{\sigma,\omega} \frac{\sigma \omega R_{\sigma \omega}}{\sum_{\sigma,\omega} R_{\sigma \omega}}, \tag{23}\]

where $R_{\sigma \omega}$ are the four measured coincidence counts in the detectors.

However the experiment does not allow us to decide between QM and other versions of RNL based on (20).

### 7 Conclusion

We have discussed an experiment with successive impacts and beam-splitters at rest which makes it possible to test Quantum Mechanics vs Multisimultaneity theories. Although the experiment requires only minor variations of standard setups, it has not yet been carried out. If the results uphold QM one had taken an important bifurcation on the Multisimultaneity road: a particular version of RNL had been ruled out, and one should follow other possible ones at the price of multiplying causal links; moreover since the experiment fulfills the conditions for both first order interferences and entanglement, it had offered a nice confirmation of the superposition principle in a new situation. If the results contradict Quantum Mechanics superluminal nonlocality and relativity had unified into Multisimultaneity. In both cases the experiment promises interesting information.

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