Designing Inventory Models to Minimize Total Inventory Costs by Using Mixed Integer Linear Programming (MILP) in the Warehouse of MRO Materials

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Abstract. In this study, the problem that happen in the object of research is the company's target which is Inventory Turn Over. These problems occur because the companies tend to decide to store materials in large quantities so there is no potential for material shortages that can disrupt maintenance activities. However, the large amount of storage creates risks such as high holding costs and the risk of damage. One way to solve inventory management problems is to make efficient decisions on order quantity (Q) and message time (T) so that a minimum total inventory cost is obtained without hampering maintenance activities. To achieve that, the method used in this study is ABC classification to determine the material classification that needs to be prioritized and Mixed Integer Linear Programming (MILP) to get a minimum total inventory cost. From this research, it is known that there are 9 types of materials that fall into category A out of 150 types. In addition, the total quantity of order (Q) and order time (T) results of the model is able to minimize the total inventory cost to IDR127,610,000

1. Introduction
Maintenance activities are important things carried out by companies to manage machines. In carrying out maintenance, many aspects need to be considered and prepared beforehand, one of which is the availability of material Maintenance, Repair, and Operation (MRO). MRO material is material that is deliberately stored by the company in order to support maintenance activities whenever needed. MRO materials can be in the form of office equipment, business service support, cleaning tools, or spare parts [1]. But in the many company, the management of these materials is not easy and can cause various problem because the amount that is stored affects the company in such a way. If the company stored too many materials, it can cause a high inventory cost and hold capital that can be invested in another aspect or value-added activity. But on another hand, if the company stored too little materials it may cause downtime or breakdown that leads to a bigger loss [2]

Procurement of MRO material in the object of this research involves the maintenance, procurement, and warehousing division. The procurement and maintenance division carries out planning regarding maintenance activities and the amount of material that needs to be purchased in order to support all activities within one year. In the process, coordination with the warehouse is also needed, because all MRO materials are stored and managed in a warehouse.

This cross-division process needs good material planning and management because all divisions with different roles need to work together to meet the company's targets which is value of inventory
turnover. The purpose of inventory turnover is to see how fast the product or goods flow relative to the average amount stored as inventory [3]. This target is used as an indicator whether the material stored in the warehouse is piling up or not.

Based on the company historical data, in one year, there were several times that the monthly target of inventory turnover was not full filled. This indicates that the inventory costs were high because the material stored in warehouse was more than it was supposed to be. Therefore, it can be said that the current system is not optimal to support companies in getting high inventory turnover so improvement is needed.

According to Chikán [4] and Yang et al. [5], one of the factors that can increase the value of inventory turnover is the Just In Time (JIT) adaptation theory in which the time to order materials is adjusted to the time of materials usage. Therefore, this study will create a mathematical model that can help the company to minimize its total inventory cost by getting the optimal amount of material ordered (Q) and ordering time (T) but still considering the constraints from the stakeholders involved in the process.

2. Literature Review

Maintenance is all activities carried out to maintain the condition of an item or equipment, or return it to certain conditions [6]. The maintenance system is divided into two types, namely: Corrective maintenance and preventive maintenance. Corrective maintenance is maintenance that only carried out when the machine / tool breaks down. While preventive maintenance, maintenance is carried out at intervals (schedule) or criteria that have been determined before to reduce the possibility of damage from the machine / equipment [7]. In carrying out maintenance, various supporting factors are needed so that activities can run smoothly, one of which is MRO (Maintenance, Repair, and Operation) material. There are few types of MRO material such as: construction, material handling, lab and chemical, tools and cleaning, safety and security, measuring and testing, heavy equipment and vehicle. All of these materials need to be managed so it will not create risks that can distracting maintenance activities or causing excessive expenditure in inventory cost. Therefore we need a good inventory management.

According to Jacobs & Chase [8], inventory management is a set of policies that control, monitor, and determine the level of inventory that must be maintained, the right time to fill inventory, and how many orders must be done. Meanwhile, according to Ramuna and Mahmudy [9], inventory management is a major requirement for companies to be able to win fierce global competition. The quality of inventory management in the object of this study is measured by the value of inventory turnover.

There are two main components in Inventory Turnover, the first is stock purchasing and the second is sales. However, because maintenance is a scheduled activity and sudden damage is very much avoided, the sale or use of material is not easily increased. Thus, a factor that can be improved in maintenance is the process of purchasing and storing materials. The amount of material stored becomes very important because by decreasing the amount of inventory, the value of inventory turnover will increase. But this can trigger a material shortage due to uncertainty of demand and stockouts are not allowed in maintenance because of the crucial impact.

Therefore, the company needed an optimal amount of storage in the form of safety stock. According to Santoso & Dhiana [10], safety stock is the minimum amount that must be available and can only be used during an emergency. The calculation of the value of the safety stock used is the following formula [11]:

\[
\text{Safety stock} = (\text{Maximum Usage} - \text{Average Usage}) \times \text{Lead Time}
\]

To design an appropriate inventory model, this research tries to create an optimization model using Mixed Integer Linear Programming (MILP). The advantage of using MILP as a method is because the basic formula of MILP can be modified according to the condition and is often used in research aimed
at reducing costs. Several studies [12,13] used the MILP method as a solution to improve the procurement and inventory systems in the manufacturing industry. These studies have same goal as what this research are trying to solve which is a minimum inventory cost with an optimal amount of inventory in warehouse.

3. Methodology

3.1. ABC Classification
To be able to solve the problems in this study, the authors used two main methods, ABC Classification and Mixed Integer Linear Programming (MILP) Optimization Model. The reason for choosing the ABC classification method is because it is one of the most widely used techniques in organizations and also easy to use or understood by all parties. In this study, 9 materials were categorized as category A, 9 categories of B material, and 132 categories C material from the total 150 types of material in the warehouse. Any material that is included in category A and will be used as a research sample can be seen in Table 1.

Table 1. ABC classification in this study is based in demand

| Material | Demand | | | Total | Pareto |
|----------|--------|---|---|-------|--------|
|          | Jan-March | April-June | July-Sept | Oct-Dec |
| 1        | 24200    | 8600       | 0          | 14000   | 46800  | 41.69% |
| 2        | 0        | 0          | 10000      | 0       | 10000  | 50.59% |
| 3        | 1800     | 600        | 5000       | 1200    | 8600   | 58.25% |
| 4        | 0        | 0          | 0          | 5210    | 5210   | 62.89% |
| 5        | 0        | 64         | 4300       | 12      | 4376   | 66.79% |
| 6        | 84       | 112        | 0          | 3465    | 3661   | 70.05% |
| 7        | 2600     | 0          | 1000       | 0       | 3600   | 73.26% |
| 8        | 3600     | 0          | 0          | 0       | 3600   | 76.46% |
| 9        | 51       | 52         | 0          | 2416    | 2519   | 78.71% |

3.2. Model Formulation
This model formulation start with creating a mathematical model that can represent the real condition of research object. This model, which aim to minimizing the total cost of inventory, is a modification from a previous study that being customized to match what this research need. The components of this model can be seen below:

- Limitation
  - The number of periods on the planned horizon is known to be 52 periods in which 1 period = 1 week
  - Costs in all periods are considered fixed and do not depend on the amount
  - All demands must be fulfilled. Shortage is not allowed
  - Each material comes from a different supplier so every orders are separated

- The indicators used in this study are:
  - $i$: type of material classification A ($i = 1, ..., 9$)
  - $t$: time period ($t = 1, ..., t$)
  - $t'$: order time period ($t' = 1, ..., t'$)

- The parameters used in the model formulation include:
  - $h_i$: Lead time material $i$
  - $g_i$: Price of material $i$
  - $a_i$: Ordering costs $i$
  - $h_i$: Holding costs $i$
Safety stock needed

da : Demand for material i in period t

- The decision variable sought is:
  \( Y_{itt'} \): \{1,0\} - binary variable
  1: Place an order for period t
  0: Do not place an order for period t

\( Q_{itt'} \): The amount of material i ordered at t' and comes at t

- The initial inventory level of material i in period t

- The objective function of this research is to minimize the total inventory costs. This function is formulated into the mathematical formula as follows:

\[
\begin{align*}
\text{Min } Z &= \sum_{t=1}^{g} \sum_{t'=l_i+1}^{t-l_i} a_i Y_{itt'} + \sum_{t=1}^{g} \sum_{t'=l_i+1}^{t-l_i} \left( \frac{h_i}{2} \times Q_{itt'} \right) + h_i X_{it} \\
\end{align*}
\]

\[ Min \ Z = \text{Ordering Cost} + \text{Holding Cost} \]  

- The model in this study has several constraint functions to describe the system. These functions include:

\[
X_{it} = X_{i(t-1)} + \sum_{t'=l_i+1}^{t-l_i} Q_{itt'} - d_{it} \quad \forall i; \forall t \tag{3}
\]

\[
X_{it} \geq SS_{it} \quad \forall i; \forall t \tag{4}
\]

\[
Q_{itt'} \leq MY_{itt'} \quad \forall i; \forall t; \forall t' \tag{5}
\]

\[
Y_{itt'} = \begin{cases} 
1 & \forall i; \forall t; \forall t' \\
0 & \end{cases} \tag{6}
\]

\[
Q_{itt'} \in \mathbb{Z}^+ \quad \forall i; \forall t; \forall t' \tag{7}
\]

\[
Y_{itt'} \in \mathbb{Z}^+ \quad \forall i; \forall t; \forall t' \tag{8}
\]

\[
X_{it} \in \mathbb{Z}^+ \quad \forall i; \forall t; \tag{9}
\]

The mathematical model then translated into the LINGO programming in order to get a verified model and be able to provide results needed, minimal inventory costs. If the mathematical model is correctly translated, the type of model obtained in LINGO software is Mixed Integer Linear Programming (MILP) with an global optimal solution.

4. Results and Discussion

After the model on LINGO is verified and validated, the model is being run to get the decision variables and total inventory costs. Tables 2 and Table 3 below will show the total cost of the model for all types of materials.

| Table 2. Components of total research inventory costs |
|-----------------------------------------------|
| No | Cost Structure | Results of Model |
|----|----------------|-----------------|
| 1  | Holding Cost (IDR) | 126,621,000 |
| 2  | Ordering Cost (IDR)  | 989,000 |
From Table 2 it is known that the holding cost is the largest component in the total cost of inventory. Whereas in Table 3 it is known that the order quantity of model (Q) obtained is equal to the actual demand. This indicates that the model has been able to represent the real condition and also able to fulfill all demand for one year. The same amount of order quantity and demand also shows that the company will not have excess inventory due to errors in the number of purchases.

After getting the results, the writer tries to do an analysis of these results by conducting a sensitivity analysis. Sensitivity analysis is conducted to find out which parameter has the most influence on the change in total inventory cost. This is done to provide a suggestion to the company about which factors are most important. The analysis in this study involves two parameters, which is the price of materials and demand. The reason for choosing these two parameters is because in the real world these two factors are influenced by outside parties and have a higher chance to change. To see the difference more clearly, sensitivity test conducted in this study took four materials with the highest and lowest demand (material 1 and 9) and materials with the highest and lowest purchase prices (material 5 and 4) where all four materials are also materials with the highest and lowest total costs. Figure 4.1 below shows the effect of changes in the price and demand parameters to the total inventory costs for the four types of material if reduced or increased by 15%, 10%, and 5%. From Figure 1, the relationship between the parameters of the purchase price and demand for the total cost is directly proportional. When the result is compared, it is known that the purchase price parameter is more significant than the demand parameter.
Figure 1. The relationship between the parameters the total cost

5. Conclusions and Future Work
This research has succeeded in creating a MILP model using LINGO software that is able to reduce the total inventory cost of the research object. The proposed inventory system that using safety stock and making adjustment for order quantity (Q) and order time (T) can reduce the problems faced by the company, which is excess inventory due to the low value of inventory turn over. From the nine materials that being studied, a total cost of IDR127,610,000 was obtained. To keep the total cost to a minimum, the company needs to pay attention to is the price of material and demand because the changes in both parameters are sensitive to the result of total inventory costs.

For further research, we can try to calculate all types of material and not just classification A to get more comprehensive results. In addition, the model can also be modified to include probabilistic parameters to represent the real changing conditions in industries.

Acknowledgment
This work is supported by the 2020 PUTI Proceedings Grant funded by the University of Indonesia DRPM Number: NKB-1147 / UN2.RST / HKP.05.00 / 2020.

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