Reliability analysis of Unsaturated Slope Stability Considering Spatial Variability in Hydraulic Parameters

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Abstract: Reliability analysis of slope stability provides a rational framework for slope safety evaluation, which plays a key role in geotechnical decision making for landslide disaster prevention. However, previous studies mainly focused on the slope reliability analysis accounting for uncertainties in shear strength parameters. How to perform reliability analysis of unsaturated slope stability considering the spatial variability of hydraulic parameters remains an open question. This paper aims to systematically investigate the effects of spatial variability of hydraulic parameters on the unsaturated slope reliability. The random fields of hydraulic parameters are derived from the random void ratio field, instead of direct generation, for considering the spatial variability of hydraulic parameters in a rational manner. It is found that the coefficient of variation (COV) and scale of fluctuation (SOF) of void ratio affects the unsaturated slope failure probability significantly. Additionally, the failure probability is sensitive to the selection of auto-correlation function (ACF) in the unsaturated slope reliability analysis.

1. Introduction
Landslide caused by slope failure has become one of the major geological disasters in the world. Slope safety evaluation is a significant prerequisite for geotechnical decision making in landslide disaster prevention. The reliability analysis approach provides a feasible means to evaluate the stability of geotechnical structures from a probabilistic perspective, which has been successfully applied to slope reliability analysis [1-5]. It is widely recognized that rainfall is the main factor leading to slope failure in geotechnical engineering practice. Thus, it is necessary to perform reliability analysis of unsaturated slope stability under rainfall.

Besides shear strength parameters and hydraulic conductivity, soil–water characteristic curve (SWCC) is also a necessity in the unsaturated slope stability analysis and subsequent reliability analysis. Because direct measurements of SWCC is relatively time-consuming and costly [6], only a limited number of test data rather than a complete SWCC over the entire suction range are typically obtained. Accordingly, SWCC uncertainty is unavoidable [7]. However, previous studies on the reliability analysis of unsaturated slope stability mainly focused on the spatial variations of shear strength parameters and saturated hydraulic conductivity [8-10], neglecting the uncertainty in SWCC. Although Chiu et al. [11] and Le et al. [12] attempted to explore the effects of hydraulic parameter uncertainty on the failure probability of unsaturated slope, the influences of the spatial variability of hydraulic parameters on the unsaturated slope reliability have not been systematically explored.

This paper aims to investigate the effects of spatial variability of hydraulic parameters (i.e., SWCC model parameter and saturated hydraulic conductivity) on the unsaturated slope reliability. Overall, this
paper is organized as follows. Firstly, the simulation of random field is introduced. Subsequently, a reliability analysis approach for unsaturated slope is developed. Finally, the proposed approach is applied to the reliability analysis of an unsaturated slope subjected to rainfall.

2. Random field characterization of hydraulic parameters

2.1. Relationship between the hydraulic parameters

Unsaturated slope stability analysis necessitates determination of soil-water characteristic curve (SWCC). In the past decades, several parametric models have been proposed to describe the SWCC. For illustration, the van Genuchten-Mualem model (VGM) [13-14] is used in this study, which can be written as:

\[
S_e = \left[1 + \left(\frac{\psi}{\alpha_{sg}}\right)^{n_{sg}}\right]^{-m_{sg}}, \quad m_{sg} = 1 - 1/n_{sg}
\]

where \( S_e \) is the effective degree of saturation; \( \psi \) is the matric suction (kPa); \( \alpha_{sg}, n_{sg}, \) and \( m_{sg} \) are fitting parameters.

Le et al. [12] developed a relationship between \( \alpha_{sg} \) and saturated hydraulic conductivity \( k_s \) based on the void ratio \( e \), which is given by:

\[
\alpha_{sg} = \alpha_{sg0} \exp\left[\eta \left(\frac{e_0}{1+e_0} - \frac{e}{1+e}\right)\right] \quad \text{and} \quad k_s = k_{s0} \left(\frac{e^3}{1+e}\right) \left(\frac{1+e}{e_0^3}\right)
\]

where \( \alpha_{sg0}, e_0 \) and \( k_{s0} \) are the initial value of \( \alpha_{sg}, e \) and \( k_s \), respectively; \( \eta \) is a scaling parameter. For the sake of brevity, interested readers are referred to Le et al. [12] for detailed theoretical explanation of the above relationship.

As indicated by equation (2), after obtaining the random void ratio field, the random fields of \( \alpha_{sg} \) and \( k_s \) can be conveniently derived according to the above relationship, as discussed in the next subsection.

2.2. Discretization of random void ratio field

Midpoint method is adopted in this study, and the generation procedures of random void ratio field through midpoint method are outlined. Firstly, the slope domain is divided into a number of random field elements (i.e., \( N_e \)). Based on Cholesky decomposition technique, the autocorrelation matrix \( C \) obtained from a prescribed autocorrelation function (ACF) can be further factored into:

\[
L L^T = C
\]

where \( L \) is the lower triangular matrix.

Then, the correlated standard Gaussian random field of void ratio is given by:

\[
\hat{E}_t^G = L \bar{\xi}_e, \quad t = 1, 2, \ldots, N_{MC}
\]

where \( \hat{E}_t^G \) is the \( t \)-th realization of the correlated standard Gaussian random field among a total of \( N_{MC} \) Monte Carlo (MC) simulations; \( \bar{\xi}_e \) is a vector of independent standard normal random samples with a dimension of \( N_e \times 1 \).

Finally, the corresponding non-Gaussian random field of void ratio can be obtained by performing isoprobabilistic transformation:

\[
\hat{E}_t^{NG} = F^{-1}[\Phi(\hat{E}_t^G)], \quad t = 1, 2, \ldots, N_{MC}
\]

where \( \hat{E}_t^{NG} \) is the \( t \)-th realization of the correlated non-Gaussian random field; \( F^{-1}(\cdot) \) is the inverse function of marginal cumulative distribution of void ratio; \( \Phi(\cdot) \) is the standard normal cumulative distribution function.

Based on the random void ratio field, the random fields of \( \alpha_{sg} \) and \( k_s \) can be generated according
to their relationships (i.e., equation (2)). Subsequently, these random fields of $\alpha_g$ and $k_s$ are incorporated into unsaturated slope reliability analysis.

3. Reliability analysis of unsaturated slope stability

The slope failure probability $P_f$ is generally used as a quantitative index to evaluate the slope safety, and MC simulation provides an efficient tool to solve the $P_f$. After performing a total of $N_{MC}$ simulations, the $P_f$ can be calculated as:

$$P_f = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} [FS_i^{min}(\tilde{E}_i) < 1]$$  \hspace{1cm} (6)

where $FS_i^{min}$ is the minimum factor of safety (FS) corresponding to the critical slip surface; $I[\cdot]$ is an indicator function for unsaturated slope failure. If $FS_i^{min} < 1$, $I[\cdot] = 1$; otherwise, $I[\cdot] = 0$.

4. Illustrative example

For illustration, the proposed approach is applied to performing reliability analysis of an unsaturated slope adapted from Le et al. [12]. As shown in figure 1, the slope example has a height of 10m and slope angle of 26.6°, and is underlain by a 20m thick base. The initial groundwater table is at the elevation of 15m. A prescribed rainfall intensity of $5.0 \times 10^{-7} \text{m/s}$ over 10 days is simulated by applying a flux boundary to the ground surface of the slope. More detailed description of this slope example can be found in Le et al. [12]. Then, the SEEP/W and SLOPE/W [15] are used to perform seepage and stability analysis in this example, respectively. The $FS$ (i.e., 1.283) associated with the corresponding critical slip surface at the end of rainfall (i.e., day 10) is also plotted in figure 1, which agrees well with the result (i.e., 1.315) reported in Le et al. [12].

![Figure 1. Geometry of the unsaturated slope example.](image)

Following Le et al. [12], the mean of $e$ is equal to 0.5, and the coefficient of variation (COV) is taken as 0.8. Besides, the horizontal and vertical scale of fluctuation (SOF) are $\delta_h=8\text{m}$ and $\delta_v=4.0\text{m}$, respectively. These parameters are regarded as a baseline case in this study.

4.1. Effect of COV on the failure probability

To examine the effect of COV of void ratio on the unsaturated slope reliability, five values (i.e., $\text{COV}_e = 0.80$, 1.00, 1.20, 1.40, and 1.60) are used in this study. Figure 2 plots the variations of failure probability $P_f$ for various $\text{COV}_e$. In general, the $P_f$ of unsaturated slope tends to grow linearly with the increase of $\text{COV}_e$, which ranges from 0.039 to 0.523. In particular, the largest value is about 13
times larger than the smallest value, this indicates that the COV of void ratio affects the \( P_f \) significantly.

\[ \text{Figure 2. Effect of COV}_e \text{ on the unsaturated slope failure probability.} \]

4.2. Effect of SOF on the failure probability
To explore the effect of SOF of void ratio on the unsaturated slope reliability, different vertical and horizontal SOFs (i.e., \( \delta_h = 2, 4, 8, 16, \) and \( 32m \), \( \delta_v = 1, 2, 4, \) and \( 8m \)) are adopted in this study. As \( \delta_h \) increases from 2m to 16m, the failure probability \( P_f \) increases significantly from about 0.002 to 0.061 (see figure 3). When \( \delta_h \) is larger than 16m, the effect of \( \delta_h \) on \( P_f \) begins to diminish, and \( P_f \) varies slightly as \( \delta_h \) further increases. As shown in figure 3, the failure probability \( P_f \) increases monotonically with increasing \( \delta_v \). As \( \delta_v \) increases from 1m to 8m, the \( P_f \) increases considerably from about 0.010 to 0.063. In general, both the horizontal and vertical SOF (i.e., \( \delta_h \) and \( \delta_v \)) of void ratio have significant effects on the unsaturated slope failure probability.

\[ \text{Figure 3. Effect of SOF on the unsaturated slope failure probability.} \]

4.3. Effect of ACF on the failure probability
Five theoretical ACFs are used in this study to explore the choice of ACF on the unsaturated slope reliability, including single exponential, squared exponential, second-order Markov, cosine exponential and binary noise autocorrelation functions [4], which are denoted as SECF, SQECF, SMCF, CECF, and BNCF, respectively. Figure 4 plots the probability density function (PDF) of \( FS \) given different ACFs,
and provides the corresponding results of unsaturated slope failure probability. It is shown that the $P_f$ is sensitive to the selection of ACF in the unsaturated slope reliability analysis, which ranges from 0.039 (i.e., SECF) to 0.092 (i.e., SQECF). Generally, it is difficult to determine the ACF in geotechnical engineering due to the limited site-specific test data [16], it is advisable to develop a method for determining an appropriate ACF of hydraulic parameters in the future.

![Figure 4. Effect of ACF on the PDF of $F_S$.](image)

5. Summary and Conclusions
This paper investigated the effects of uncertainties of hydraulic parameters on the unsaturated slope reliability via Monte Carlo simulation, which simultaneously considers the spatial variability of SWCC model parameter and saturated hydraulic conductivity. Sensitivity analysis is performed to evaluate the influences of spatial variability of void ratio on the unsaturated slope reliability. Several conclusions drawn from this study are given below:

(1) The coefficient of variation (COV) of void ratio affects the unsaturated slope reliability significantly, and there exists an approximately linear relationship between them. Besides, both the horizontal and vertical scale of fluctuation (SOF) of void ratio have significant effects on the unsaturated slope reliability.

(2) The failure probability of unsaturated slope is highly dependent on the selection of auto-correlation function (ACF) in the unsaturated slope reliability analysis. Among five theoretical ACFs, the single exponential auto-correlation function gives the smallest value of failure probability. This indicates that if the single exponential auto-correlation function is used in the unsaturated slope reliability analysis, it is likely to lead to an unconservative estimate of unsaturated slope failure probability.

(3) Despite the fact that the spatial variability of void ratio (i.e., COV, SOF, and ACF) has a significant influence on the unsaturated slope failure probability. However, it is difficult to obtain the corresponding COV, SOF and ACF in geotechnical engineering practice due to the limited site-specific test data. Further research on this is warranted.

Acknowledgements
This work was supported by the Chongqing Engineering Research Center of Disaster Prevention & Control for Banks and Structures in Three Gorges Reservoir Area (Nos. SXAPGC18ZD01 and SXAPGC18YB03) and Natural Science Foundation of Chongqing, China (cste2018jeyjAX0632). The financial support is gratefully acknowledged.

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