Best Performance of n⁺ - p Crystalline Silicon Junction Solar Cells at 300 K, Due to the Effects of Heavy Doping and Impurity Size. I

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Abstract: The effects of heavy doping and donor (acceptor) size on the hole (electron)-minority saturation current density \( J_{Eo}(J_{Bo}) \), injected respectively into the heavily (lightly) doped crystalline silicon (Si) emitter (base) region of n⁺ - p junction, which can be applied to determine the performance of solar cells, being strongly affected by the dark saturation current density: \( J_o = J_{Eo} + J_{Bo} \), were investigated. For that, we used an effective Gaussian donor-density profile to determine \( J_{Eo} \), and an empirical method of two points to investigate the ideality factor \( n \), short circuit current density \( J_{sc} \), fill factor (FF), and photovoltaic conversion efficiency \( \eta \), expressed as functions of the open circuit voltage \( V_{oc} \), giving rise to a satisfactory description of our obtained results, being compared also with other existing theoretical-and-experimental ones. So, in the completely transparent and heavily doped (P-Si) emitter region, CTHD(P-Si)ER, our obtained \( J_{Eo} \)-results were accurate within 1.78%. This accurate expression for \( J_{Eo} \) is thus imperative for continuing the performance improvement of solar cell systems. For example, in the physical conditions (PCs) of CTHD (P-Si) ER and of lightly doped (B-Si) base region, LD(B-Si)BR, we obtained the precisions of the order of 8.1% for \( J_{sc} \), 7.1% for FF, and 5% for \( \eta \), suggesting thus an accuracy of \( J_{Bo} \) (≤ 8.1%). Further, in the PCs of completely opaque and heavily doped (S-Si) emitter region, COHD(S-Si)ER, and of lightly doped (acceptor-Si) base region, LD(acceptor-Si)BR, our limiting \( \eta \)-results are equal to: 27.77%„„, 31.55%, according to the \( E_{gi} \)-values equal to: 1.12eV „„, 1.34eV, given in various (B,…, Tl)-Si base regions, respectively, being due to the acceptor-size effect. Furthermore, in the PCs of CTHD (donor-Si) ER and of LD(Tl-Si)BR, our maximal \( \eta \)-values are equal to: 24.28%„„, 31.51%, according to the \( E_{gi} \)-values equal to: 1.11eV „„, 1.70eV, given in various (Sb,…, S)-Si emitter regions, respectively, being due to the donor-size effect. It should be noted that these obtained highest \( \eta \)-values are found to be almost equal, as: 31.51%≈ 31.55%, coming from the fact that the two obtained limiting \( J_{Eo} \)-values are almost the same.

Keywords: Donor (Acceptor)-Size Effect, Heavily Doped Emitter Region, Ideality Factor, Open Circuit Voltage, Photovoltaic Conversion Efficiency

1. Introduction

The minority-carrier transport in the non-uniformly and heavily doped (NUHD), quasi-neutral, and uncompensated emitter region of impurity-silicon (Si) devices such as solar cells and bipolar transistors at temperature \( T(= 300 K) \), plays an important role in determining the behavior of many semiconductor devices [1-29]. It should be noted that the minority-carrier saturation current density, \( J_{Eo} \), injected into this emitter region strongly controls the common emitter current gain [4-8]. Thus, an accurate determination of this \( J_{Eo} \) or an understanding of minority-carrier physics inside heavily doped semiconductors is imperative for continuing the performance improvement of bipolar transistors, and that of solar cell systems, which is commonly characterized in terms of the parameters such as: the ideality factor \( n \), short circuit current density \( J_{sc} \), fill factor FF, and photovoltaic conversion efficiency \( \eta \), being
expressed as functions of the open circuit voltage $V_{oc}[4]$. Further, it should be noted that, in most fabricated silicon devices, the effective Gaussian donor-density profile $\rho(x)$, being proposed in next Equation (24), varies with carrier position $x$ in the emitter region of width $W$ [13, 18-20, 22], and it decreases with increasing $W$, being found to be in good agreement with that used by Essa et al. [13]. As a result, many other physical quantities, given in this NUHD n(p)-type thin emitter region such as [1-45]: the band gap narrowing (BGN), $\Delta E_g$, Fermi energy $E_F$, apparent band gap narrowing (ABGN), $\Delta E_{ga}$, minority-hole (electron) mobility $\mu_{b(e)}$, minority-hole (electron) lifetime $\tau_{b(e)}$, and minority-hole (electron) diffusion length $L_{b(e)}$, strongly depend on $p(x)$.

In the present paper, we determine an accurate expression for the minority-hole current density $J_{ho}$, injected into the NUHD emitter region of $n^+ - p$ junction silicon solar cells at 300 K, being also applied to determine the performance of such crystalline silicon solar cells.

In Section 2, we study the effects of impurity size [or compression (dilation)], temperature and heavy doping, affecting the energy-band-structure parameters such as: the intrinsic band gap $E_{gi}$, intrinsic carrier concentration $n_i$, band gap narrowing $\Delta E_g$, Fermi energy $E_F$, apparent band gap narrowing $\Delta E_{ga}$, and effective intrinsic carrier concentration $n_{ie}$. In Section 3, an accurate expression for the optical band gap (OBG), $E_{gi1}$, is investigated in next Equation (16), being accurate within 1.86%, as showed in Table 3. Some useful minority-carrier transport parameters such as: $\mu_b$ and $L_b$, being given in the heavily doped n-type emitter region, and $\mu_e$, $\tau_e$ and the minority-electron saturation current density $J_{ho}$, being given in the lightly doped p-type base region, are also presented in Section 4. Then, in Section 5, an accurate expression for the minority-hole saturation current density $J_{ho}$, injected into the heavily doped emitter region of $n^+ - p$ junction silicon solar cells at 300 K is established in Equation (39) or its approximate form given in Eq. (44), indicating an accuracy of the order of 1.78%, as seen in Table 4. Further, the total saturation current density: $J_0 = J_{ho} + J_{bo}$, where $J_{bo}$ [1, 7], determined in Equation (21), is the minority-electron saturation current density $J_{bo}$, injected into the lightly doped base region of $n^+ - p$ junction silicon solar cells, can be used to investigate the photovoltaic conversion effect, as presented in Section 6. Finally, some concluding remarks are given and discussed in Section 7.

### 2. Energy-Band-Structure Parameters in Donor (Acceptor)-Si Systems

Here, we study the effects of donor (acceptor) $[d(a)]$-size, temperature, and heavy doping on the energy-band-structure parameters of $d(a)$-Si systems, as follows.

#### 2.1. Effect of $d(a)$-Size

In $d(a)$-Si-systems at $T=0$ K, since the $d(a)$-radius $r_{d(a)}$, in tetrahedral covalent bonds is usually either larger or smaller than the Si atom-radius $r_o$, assuming that in the P(B)-Si system $r_{P(B)} = r_o = 0.117 \text{nm}$, with $1 \text{nm} = 10^{-8} \text{m}$, a local mechanical strain (or deformation potential energy) is induced, according to a compression (dilation) for $r_{d(a)} > r_o$ ($r_{d(a)} < r_o$), respectively, due to the $d(a)$-size effect. Then, in the Appendix A of our recent paper [42], basing on an effective Bohr model, such a compression (dilation) occurring in various $d(a)$-Si systems was investigated, suggesting that the effective dielectric constant, $\varepsilon(r_{d(a)})$, decreases with increasing $r_{d(a)}$. This $r_{d(a)}$-effect thus affects the changes in all the energy-band-structure parameters, expressed in terms of $\varepsilon(r_{d(a)})$, noting that in the P(B)-Si system $\varepsilon(r_{P(B)}) = 11.4$. In particular, the changes in the unperturbed intrinsic band gap, $E_{go}(r_{P(B)}) = 1.17 \text{eV}$, and effective $d(a)$-ionization energy in absolute values $E_{do}(r_{d(a)})(r_{P(B)}) = 33.58 \text{meV}$, are obtained in an effective Bohr model, as [42]:

$$E_{go}(r_{d(a)}) - E_{go}(r_{P(B)}) = E_{do}(r_{d(a)})(r_{d(a)}) - E_{do}(r_{P(B)}) = E_{do}(r_{d(a)})(r_{P(B)}) \times \left[\frac{\varepsilon(r_{P(B)})}{\varepsilon(r_{d(a)})} - 1\right]$$  \( (1) \)

Therefore, with increasing $r_{d(a)}$, the effective dielectric constant $\varepsilon(r_{d(a)})$ decreases, implying that $E_{go}(r_{d(a)})$ increase. Those changes, which were investigated in our previous paper [42], are now reported in the following Table 1, in which the data of the critical $d(a)$-density $N_{cn(cit)}(r_{d(a)})$ are also reported. This critical density marks the metal-to-insulator transition from the localized side (all the impurities are electrical neutral), $N(N_a) \leq N_{cn(cit)}(r_{d(a)})$, to the extended side, $N(N_a) > N_{cn(cit)}(r_{d(a)})$, assuming that all the impurities are ionized even at 0 K. However, at $T = 300$ K, for example, all the impurities are thus ionized and the physical conditions, defined by: $N(N_a) > N_{cn(cit)}(r_{d(a)})$ and $N(N_a) < N_{cn(cit)}(r_{d(a)})$, can thus be used to define the n(p)-type heavily and lightly doped Si, respectively.

| Donor  | Sb   | P    | As   | Bi   | Ti   | Te   | Se   | S    |
|--------|------|------|------|------|------|------|------|------|
| T=0 K  |      |      |      |      |      |      |      |      |
| $r_d$ (nm) | 0.1131 | 0.1170 | 0.1277 | 0.1292 | 0.1424 | 0.1546 | 0.1621 | 0.1628 |
| $\varepsilon(r_d)$ | 12.02 | 11.40 | 8.47 | 7.95 | 4.71 | 3.26 | 2.71 | 2.67 |
| $E_{go}(r_d)$ (eV) | 1.167 | 1.170 | 1.197 | 1.205 | 1.333 | 1.547 | 1.729 | 1.749 |
| $N_{do}(r_d) \times 10^{16}$ cm$^{-2}$ | 37 | 5.2 | 8.58 | 10.37 | 50 | 150.74 | 261.24 | 274.57 |

Table 1. The values of $r_{d(a)}$, $\varepsilon(r_{d(a)})$, and $E_{go}(r_{d(a)})$, and critical impurity density $N_{cn(cit)}(r_{d(a)})$, obtained in our previous paper [42], are reported here.
2.2. Temperature Effect

Being inspired from excellent works by Pässler [33, 34], who used semi-empirical descriptions of T-dependences of band gap of the Si by taking into account the cumulative effect of electron-phonon interaction and thermal lattice expansion mechanisms or all the contributions of individual lattice oscillations [33-35], we proposed in our recent paper [43] a simple accurate expression for the intrinsic band gap in the silicon (Si), due to the T-dependent carrier-lattice interaction-effect, $E_{gl}(T, r_d(a))$, by

$$E_{gl}(T, r_d(a)) = E_{go}(r_d(a)) - 0.071 \text{ (eV)} \times \left\{ 1 + \left( \frac{2T}{440.6913 K} \right)^{2.201} \right\}^{2/3} - 1 \tag{2}$$

where the values of $E_{go}(r_d(a))$ due to the d(a)-size effect are given in Table 1 and those of $E_{gl}(T = 300 K, r_d(a))$ tabulated in Table 2. Further, as noted in this Reference 43, in the (P, S)-Si systems, for $0 K \leq T \leq 3500 K$, the absolute maximal relative errors of this $E_{gl}$-result were found to be equal respectively to: 0.22% and 0.15%, calculated using the very accurate complicated results given by Pässler [34]. Then, in the n-type HD silicon at temperature T, the effective mass of the majority electron can be defined by [31, 32]:

$$m_e(T, r_d) = \left[ \frac{0.9163 \times \left( \frac{1.1905 \times E_{go}(r_d)}{E_{gl}(T, r_d)} \right)^{2/3}}{m_o} \right]^{1/3} \times m_o = \frac{m_{eo}}{m_{eo} \times \left( \frac{E_{go}(r_d)}{E_{gl}(T, r_d)} \right)^{2/3}} \tag{3}$$

which gives: $m_{eo} = m_e(T = 0 K) = 0.3216 \times m_o$, $m_o$ being the electron rest mass, and the effective mass of the minority hole yields [31, 32]:

$$n_i^2(T, r_d(a), g_c) \equiv N_c(T, r_d, g_c) \times N_v(T, g_v) \times \exp \left( \frac{-E_{gl}(T, r_d(a))}{k_bT} \right) \tag{5}$$

where, $N_{c(v)}$ is the conduction (valence)-band density of states, given by [31, 32]:

$$N_c(T, r_d, g_c) = 2g_c \times \frac{(m_e(T, r_d) / h)^{2/3}}{2\pi^2} \times \left( \frac{m_e(T, r_d) / h}{k_bT} \right)^{3/2} \text{ (cm}^{-3}) \tag{6}$$

and

$$N_v(T, g_c) = 2g_v \times \frac{((\pi / h)^{2/3})}{2\pi^2} \times \left( \frac{m_v(T, g_c) / h}{k_bT} \right)^{3/2} \text{ (cm}^{-3}) \tag{7}$$

where $h = \hbar / 2\pi$ is the Dirac’s constant, $k_b$ is the Boltzmann constant, and $g_c$ is the effective average number of equivalent conduction-band edges. Moreover, for $r_d \equiv r_p$ and at 300 K, some typical $n_i$-results obtained for different $g_c$-values, calculated using Equation (5), are given as follows.

(i) If $g_c = 6$, one then obtains: $n_i = 10.7 \times 10^9 \text{ cm}^{-3}$, being just a result investigated from a measurement of energy-band-structure parameters and intrinsic conductivity by Green [31].

(ii) If $g_c = 5$, one then obtains: $n_i = 9.77 \times 10^9 \text{ cm}^{-3}$, according to a result given from a capacitance measurement of a pin diode biased under high injection, by Misiakos and Tsamakis [37].

(iii) Finally, if $g_c = 4.9113$, one then gets: $n_i = 9.68 \times 10^9 \text{ cm}^{-3}$, according to a result proposed by Couderc et al. (C) as [38]:

$$n_i(g_c) = 1.541 \times 10^{13} \times T^{1.721} \times \exp \left( \frac{-E_{gl}}{2k_bT} \right) \text{ (cm}^{-3}) = 9.68 \times 10^9 \text{ cm}^{-3} \text{ for } T=300 \text{ K},$$

and $E_{gl}$ is the band gap, based on their updated fit of experimental data for the minority-carrier mobility and open-circuit voltage decay, which were given by Sproul and Green [36]. Further, from Equations (5, 2), in donor-Si systems and for $T=300 \text{ K}$, the numerical results of $n_i$ and $E_{gl}$, calculated for $g_c = 6, 5,$ and 4.9113, as functions of $r_d(a)$, are tabulated in Table 2.

Table 2. The values of intrinsic carrier concentration $n_i(T = 300 K, r_d(a), g_c)$ and intrinsic band gap $E_{gl}$ are calculated for $g_c = 6, 5,$ and 4.9113, using Equations (5, 2), respectively, as functions of $r_d(a)$.

| Donor | Sb | P | As | Bi | Ti | Te | Se | S |
|-------|----|---|----|----|----|----|----|---|
| $g_c = 6$ | $E_{gl}(300K)$ in eV | 1.1215 | 1.1245 | 1.1515 | 1.1595 | 1.2875 | 1.5015 | 1.6835 | 1.7035 |
| $n_i(300K)$ in $10^9 \text{ cm}^{-3}$ | 1.13 | 1.07 | 6.24 $\times 10^{-1}$ | 5.43 $\times 10^{-1}$ | 4.56 $\times 10^{-2}$ | 7.26 $\times 10^{-2}$ | 2.14 $\times 10^{-5}$ | 1.46 $\times 10^{-5}$ |
Having a same empirical form as that given in Equation (14).

2.3. Heavy Doping Effect

First of all, in the donor-Si system, we define the effective intrinsic carrier concentration \( n_{\text{te}} \), by

\[
\Delta E_{\text{ga}} = N \times p_o \equiv n_{\text{te}}^{3} \times \exp \left( \frac{\Delta E_{\text{ga}}}{k_B T} \right)
\]

(8)

where \( n_{\text{te}}^{3} \) is determined in Equation (5). Here, we can also define the “effective doping density” by [8]:

\[
N_{\text{Def}} = N/\exp \left( \frac{\Delta E_{\text{ga}}}{k_B T} \right)
\]

so that

\[
N_{\text{Def}} \times p_o = n_{\text{te}}^{3}
\]

Here, \( p_o \) is the density of minority holes at the thermal equilibrium and the ABGN is defined by:

\[
\Delta E_{\text{ga}(IR)}(N) = 8.5 \times 10^{-3} \times \left[ \ln \left( \frac{N}{3 \times 10^{14} \text{ cm}^{-3}} \right) + \left[ \ln \left( \frac{N}{3 \times 10^{14} \text{ cm}^{-3}} \right) \right]^{2} + 0.5 \right] \text{ (eV)}
\]

(10)

\[
\Delta E_{\text{ga}(KSG)}(N) = 6.92 \times 10^{-3} \times \left[ \ln \left( \frac{N}{3 \times 10^{14} \text{ cm}^{-3}} \right) + \left[ \ln \left( \frac{N}{3 \times 10^{14} \text{ cm}^{-3}} \right) \right]^{2} + 0.5 \right] \text{ (eV)}
\]

(11)

\[
\Delta E_{\text{ga}(ZA)}(N) = 18.7 \times 10^{-3} \times \ln \left( \frac{N}{7 \times 10^{17} \text{ cm}^{-3}} \right) \text{ (eV)}
\]

(12)

\[
\Delta E_{\text{ga}(SC)}(N) = 14 \times 10^{-3} \times \ln \left( \frac{N}{1.4 \times 10^{18} \text{ cm}^{-3}} \right) \text{ (eV)}
\]

(13)

\[
\Delta E_{\text{ga}(YC)}(N) = 4.2 \times 10^{-5} \times \left[ \ln \left( \frac{N}{10^{15} \text{ cm}^{-3}} \right) \right]^{3} \text{ (eV)}
\]

(14)

Then, in such the P-Si system at 300K, being inspired by the term: \( k_B T \times \ln \left( \frac{N}{N_{C}} \right) \) given in Equation (9), and also by the result: \( \Delta E_{\text{ga}(YC)}(N) \) given in Equation (14), we can now propose a modified (Mod.) YC-model for the ABGN so that its numerical results are found to be closed to those calculated by using Equation (9), as:

\[
\Delta E_{\text{ga}(Mod.YC)}(N, g_c) = 114.94 \times 10^{-6} \times \left[ \ln \left( \frac{2.7167 \times 10^{15}}{N_{C}} \times \frac{N_{C}}{g_c} \right) \right]^{3} \text{ (eV)} = 114.94 \times 10^{-6} \times \left[ \ln \left( \frac{N}{3 \times 10^{18} \text{ cm}^{-3}} \right) \right]^{3} \text{ (eV)}
\]

(15)

having a same empirical form as that given in Equation (14).
Now, for \( g_c = 6 \), in \( d \)-Si systems at 300 K, our numerical ABGN (\( \Delta E_{ga} \)) results are calculated, using Equation (9). First, ours, obtained for the P-Si system, are plotted as a function of \( N \) in Figure 1 (a), in which, for a comparison, the other ones, calculated using Equations (10-15), are also included. Secondly, in this P-Si system, the relative deviations between ours and the others are also plotted as functions of \( N \) in Figure 1 (b). Finally, in Figure 2 (\( c_1, c_2 \)), ours are plotted in donor-Si systems as functions of \( N \).

Here, one observes that:

(i) our numerical ABGN-results obtained using Equations (9, 15) are found to be closed together as seen in Figure 1 (a), and their absolute maximal relative deviation yields: 3.03%, which occurs at \( N = 1.2 \times 10^{20} \text{ cm}^{-3} \), as observed in Figure 1 (b), and

(ii) in Figure 2 (\( c_1, c_2 \)), for a given donor-Si system, due to the heavy doping effect, ours increase with increasing \( N \), and for a given \( N \), ours increase (\( \Delta \)) with increasing \( r_{dt} \), due to the donor-size effect.

Then, in the following, it is possible to define the optical band gap (OBG), expressed in terms of the ABGN (or BGN), suggesting a conjunction between the electrical-and-optical phenomena.

### 3. Conjunction Between Electrical-and-Optical Phenomena

First of all, we define the optical band gap (OBG) by [25]:

\[
E_{g1}(N, T, r_{dt}, g_c) = E_{g1}(T, r_{dt}) - \Delta E_g(N, T, r_{dt}, g_c) + E_F(N, T, r_{dt}, g_c)
\]  

where the intrinsic band gap \( E_{g1} \) is determined in Equation (2), the BGN \( \Delta E_g \) is investigated in Equation (A9) of the Appendix B, and the Fermi energy \( E_F \) is given in Equation (A3) of the Appendix A, suggesting that the optical phenomenon is represented by \( E_{g1} \). Furthermore, it is possible to establish a conjunction between the electrical and optical phenomena, obtained from Equations (9, 16), as:

\[
E_{g1(\text{Mod.YC})}(N, T, r_{dt}, g_c) = E_{g1}(T, r_{dt}) - \Delta E_{ga(\text{Mod.YC})}(N, g_c) + k_B T \times \ln \left( \frac{N}{N_C(T, r_{dt}, g_c)} \right)
\]  

which can be rewritten, for example, replacing \( \Delta E_{ga} \) by \( \Delta E_{ga(\text{Mod.YC})}(N) \) determined in Equation (15), as:

\[
E_{g1(\text{Mod.YC})}(N, T, r_{dt}, g_c) = E_{g1}(T, r_{dt}) - \Delta E_{ga(\text{Mod.YC})}(N, g_c) + k_B T \times \ln \left( \frac{N}{N_C(T, r_{dt}, g_c)} \right)
\]  

Now, in the P-Si system, our numerical OBG-results, calculated using Equations (16, 17) for \( g_c = 6, 5, 4.9113 \) and at \( T = 300 \text{ K} \), are tabulated in following Table 3, in which our numerical results of \( E_{g1} \) and \( E_{g1(\text{Mod.YC})} \), obtained for \( g_c = 6 \), are accurate within 1.86% and 1.9%, respectively, and found to be the best ones, compared with those obtained for \( g_c = 5, 4.9113 \). One notes that the relative deviations (RDs) between calculated \( E_{g1} \)-results and \( E_{g1}-\text{data} \) [44] are defined by:

\[
1 - \frac{\text{Calculated } E_{g1}-\text{results}}{E_{g1}-\text{data}}
\]

\( \text{Table 3. Our numerical results of optical band gap (OBG), expressed as functions of } N \text{ for } g_c = 6, 5, 4.9113, \text{ and their relative deviations.} \)

| \( N \left( 10^{18} \text{ cm}^{-3} \right) \) | 4     | 8.5   | 15    | 50    | 80    | 150   |
|-----------------|-------|-------|-------|-------|-------|-------|
| \( E_{g1}(eV) \)-data [44] | 1.020 | 1.028 | 1.033 | 1.050 | 1.056 | 1.059 |
| \( E_{g1}(eV) \) | 1.0390 | 1.0465 | 1.0496 | 1.0483 | 1.0463 | 1.0479 |
| RD(%) | -1.86 | -1.80 | -1.61 | 0.17  | 0.92  | 1.05  |
N (10^{18} \text{ cm}^{-2})

\begin{tabular}{cccccc}
4 & 8.5 & 15 & 50 & 80 & 150 \\
\hline
g_e = 5 & & & & & \\
E_p(eV) & 1.0411 & 1.0478 & 1.0501 & 1.0473 & 1.0462 & 1.0470 \\
RD(\%) & -2.07 & -1.92 & -1.66 & 0.25 & 0.92 & 1.14 \\
g_e = 4.9113 & & & & & \\
E_p(eV) & 1.0413 & 1.0479 & 1.0502 & 1.0473 & 1.0463 & 1.0468 \\
RD(\%) & -2.09 & -1.93 & -1.66 & 0.26 & 0.92 & 1.15 \\
\end{tabular}

Other OBG-results are obtained from Equation (17).

\begin{tabular}{cccccc}
\hline
g_e = 6 & & & & & \\
E_p(MOD.YOC)(eV) & 1.0394 & 1.0459 & 1.0489 & 1.0492 & 1.0469 & 1.0415 \\
RD(\%) & -1.90 & -1.74 & -1.54 & 0.08 & 0.86 & 1.66 \\
g_e = 5 & & & & & \\
E_p(MOD.YOC)(eV) & 1.0412 & 1.0470 & 1.0495 & 1.0485 & 1.0456 & 1.0394 \\
RD(\%) & -2.08 & -1.85 & -1.59 & 0.15 & 0.98 & 1.85 \\
g_e = 4.9113 & & & & & \\
E_p(MOD.YOC)(eV) & 1.0414 & 1.0471 & 1.0495 & 1.0484 & 1.0454 & 1.0392 \\
RD(\%) & -2.09 & -1.86 & -1.60 & 0.15 & 0.99 & 1.87 \\
\end{tabular}

The underlined |RD| values are the maximal ones.

In order to determine the minority-hole saturation-current density \(J_{Bo}\), injected into the heavily doped n-type emitter-region, we need to know an expression for the minority-hole mobility \(\mu_h\), being related to the minority-hole diffusion coefficient \(D_h\), by the well-known Einstein relation: \(D_h = \frac{kT}{\mu_h}\), where \(e\) is the positive hole charge. Here, in donor-Si systems at 300 K and for any \(g_e\), since the minority-hole mobility depends on \(N\) [10], and also on \(g_e\) and \(\varepsilon(r_d)\) [11], we can propose:

\[
\mu_h(N, T, r_d, g_e) = \left[130 + \frac{500-130}{1+(\frac{N}{10^{18}} \text{ cm}^{-2} + 1)^3} \right] \times \left( \frac{T}{300K} \right)^{3/2} \times \left( \frac{\varepsilon(r_d)}{\varepsilon(r_p)} \right)^2 \times \frac{1}{\varepsilon(p_d)} \times \frac{1}{\varepsilon(p_o)} \times \frac{N}{D_h} \times \frac{N}{D_h \times \exp(1360-92 \frac{N_2}{1+3.10^{-15} \text{ cm}^{-3}})}
\]

noting that as \(T = 300 K, g_e = 6, \) and \(r_d \equiv r_p,\) Equation (18) is reduced to that given by del Alamo et al. [10]. Moreover, Equation (18) indicates that, for a given \(N\) and with increasing \(r_d,\) \(\mu_h\) decreases, since \(\varepsilon(r_d)\) decreases as seen in Table 1, being due to the d-size effect, in good accordance with that observed by Logan et al. [9]. Further, from Equations (5, 8, 9, 15, 18), we can define the following minority-hole transport parameter \(F\) as [22, 25]:

\[
F(N, T, r_d, g_e) \equiv \frac{n_i^2}{n_p \times D_h} \equiv \frac{N_{Deff}}{D_h \times \exp(1360-92 \frac{N_2}{1+3.10^{-15} \text{ cm}^{-3}})} \times \left( \frac{\varepsilon(r_d)}{\varepsilon(r_p)} \right)^2 \times \left( \frac{T}{300K} \right)^{3/2} \times \frac{1}{\varepsilon(p_d)} \times \frac{1}{\varepsilon(p_o)} \times \frac{N}{D_h} \times \frac{N}{D_h \times \exp(1360-92 \frac{N_2}{1+3.10^{-15} \text{ cm}^{-3}})}
\]

being reduced to the result obtained by Slotboom and de Graaff [13, 16], as \(T=300 K\) and \(r_d = r_p,\) and \(\tau_e(N_2)\) is the
minority-electron lifetime, computed by [16, 25]:
\[ \tau_n(N_a)^{-1} = \frac{1}{2.5 \times 10^{-7}} + 3 \times 10^{-13} \times N_a + 1.83 \times 10^{-31} \times N_a^2. \] (23)

Furthermore, Equation (22) indicates that, for a given \( N_a \) and with increasing \( r_g, \mu_h \) decreases, since \( e(r_g) \)
decreases, as seen in Table 1, in good accordance with that observed by Logan et al. [9].

Then, in P(B)-Si systems at 300 K and for \( g_e = 6 \), Klaassen et al. confirmed, in Figures 1 and 2 of their paper [16], that the expressions (18, 22) for minority-hole (electron) mobility \( \mu_h(e) \) are simple and accurate.

In the following, we will determine the minority-hole saturation-current density \( J_{eo} \), injected into the heavily doped n-type emitter-region of the \( n^+ - p \) junction solar cells.

5. Minority-Hole Saturation Current Density

Let us first propose in the non-uniformly and heavily doped (NUHD) emitter region of donor-Si devices our expression for the effective Gaussian donor-density profile or the donor (majority-electron) density, defined in the emitter-region width \( W \), by:
\[ \rho(x) = N \times \exp \left\{ -\left( \frac{x}{W} \right)^2 \times \ln \left[ \frac{N(x)}{N_0(W)} \right] \right\} \equiv N \times \left[ \frac{N}{N_0(W)} \right]^{-\left( \frac{x}{W} \right)^2} \] (24)
where \( N_0(W) \equiv 7.9 \times 10^{-17} \times \exp \left\{ -\left( \frac{W}{0.1682 \mu m} \right)^{1.068} \right\} \) (cm\(^{-3}\)), 1 \( \mu m = 10^{-4} \) cm, decreases with increasing \( W \), in good agreement with the doping profile measurement on silicon devices, studied by Essa et al. [13]. Moreover, Equation (24) indicates that:
(i) at the surface emitter: \( x = 0 \), \( \rho(0) = N \), defining the surface donor density, and
(ii) at the emitter-base junction: \( x = W \), \( \rho(W) = N_0(W) \), which decreases with increasing \( W \), as noted above. Here, we also remark that \( N_{0(CVD)} = 7 \times 10^{17} \) cm\(^{-3}\) was proposed by Van Cong and Debiasi (CVD) [22], and \( N_{0(ZA)} = 2 \times 10^{16} \) cm\(^{-3}\), by Zouari and Arab (ZA) [17], for their Gaussian impurity density profile. Moreover, all the parameters given in Equation (24) were chosen such that the errors of our obtained \( J_{eo} \) –values are minimized, as seen in next Table 4, and our numerical calculation indicates that, from Equation (24), we can determine the highest value of \( W \), being equal here to 85 \( \mu m \).

Now, from Equations (8, 9) or Equation (19), taken for \( 0 \leq x \leq W \), and using Equation (24), the result: \( N_{def}(x = 0) \equiv N/\exp \left[ \frac{\Delta E_g(N)}{k_B T} \right] \) may be rewritten as:
\[ N_{def}(x) \equiv \rho(x)/\exp \left[ \frac{\Delta E_g(p(x))}{k_B T} \right] \] (25)
which gives at \( x = W \): \( N_{def}(W) \equiv \frac{N_0(W)}{\exp \left[ \frac{\Delta E_g(N_0(W))}{k_B T} \right]} \).

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-hole density by:
\[ p_0(x) \equiv \frac{n_f}{N_{def}(x)} \] (26)
and a normalized excess minority-hole density [or a relative deviation between \( p(x) \) and \( p_0(x) \)] by [22, 25]:
\[ u(x) \equiv \frac{p(x) - p_0(x)}{p_0(x)} \] (27)
which must verify the two following boundary conditions proposed by Shockley as [2]:
\[ u(x = 0) = -\ln(\frac{N_0(0)}{N_{def}(x = 0)}) \] (28)
\[ u(x = W) = \exp \left( \frac{V}{\mu(n(V)) \times \mu_T W} \right) - 1, \text{for small } W - \text{values} \] (29)

Here, \( n(V) \) is an ideality factor, \( \frac{S}{\mu n(V)} \) is the hole surface recombination velocity at the emitter contact, \( V \) is the applied voltage, \( \mu_T \equiv (k_B T/e)^{1/2} \) is the thermal voltage, and the minority-hole current density \( J_{ho}(x) \) being found to be similar to the Fick’s law for diffusion equation, is given by [8, 22]:
\[ J_{ho}(x) = -\frac{e n_f^2}{F(x)} \times \frac{du(x)}{dx} = -\frac{e n_f^2}{N_{def}(x)} \times \frac{du(x)}{dx} \] (30)

where \( F(x) \) is determined in Equation (19), in which \( N \) is replaced by \( \rho(x) \), proposed in Equation (24).

Further, the minority-hole continuity equation yields [8, 22]:
\[ \frac{dF_{ho}(x)}{dx} = -en_f^2 \times \frac{u(x)}{F(x) \times \mu_T^2} = -en_f^2 \times \frac{u(x)}{N_{def}(x) \times \mu_T(\rho(x))} \times -e \times \left[ \frac{p(x) - p_0(x)}{\mu_T(\rho(x))} \right] \times \frac{1}{\mu_T(N)} \times \frac{1}{\mu_T(N)} \] (31)

Then, from these two Equations (30, 31), one obtains the following second-order differential equation as [22]:
\[ \frac{d^2u(x)}{dx^2} - \frac{F(x)}{dx} \times \frac{du(x)}{dx} - \frac{u(x)}{\mu_T^2} = 0 \] (32)

Using the two boundary conditions (28, 29), one thus gets the general solution of this Equation (32) as [22]:
\[ u(x) = \left[ A(W) \times \sinh(P(W)) + B(W) \times \cosh(P(W)) \right] \times \left[ \exp \left( \frac{V}{\mu(n(V)) \times \mu_T} \right) - 1 \right] \] (33)

where \( A(W) \equiv \frac{1}{\sinh(P(W)) + (\mu(n(V)) \times \cosh(P(W)))} \), \( \mu(W, S) \equiv \frac{B(W)}{A(W)} \times \frac{D_{ho}(N_0(W))}{S \times F(x)(N_0(W))} \) and \( P(x) \equiv \int_0^x C \times F(x) \times dx \), since \( \frac{dP(x)}{dx} \equiv C \times F(x) \). Here, \( C = 10^{-17} \) (cm\(^4\)/s), as that chosen in Equation (20), and the hyperbolic sine-and-cosine functions are defined by: \( \sinh(x) \equiv 0.5 \times \left[ e^x - e^{-x} \right] \) and \( \cosh(x) \equiv 0.5 \times \left[ e^x + e^{-x} \right] \).

Further, from Eq. (33), as \( P(W) \ll 1 \) (or for small \( W \)) one has: \( A \approx \frac{1}{2} \) or \( B \approx 1 \), and one therefore obtains: \( u(W) = \left[ \exp \left( \frac{V}{\mu(n(V)) \times \mu_T} \right) - 1 \right] \), which is just the boundary condition given in Equation (29). Now, using Equations (30, 33), one gets:
where $I_{eo}$ is the minority-hole saturation current density, being injected into the heavily doped n-type emitter region for $0 \leq x \leq W$ and given by:

$$I_{eo}(x, N, T, r_d, g_c, S) = \frac{J_h}{n_s(x, N, T, r_d, g_c, S)} \left( 1 - \frac{v}{\sqrt{n_s(x, N, T, r_d, g_c, S)}} \right)$$ (34)

One also remarks that, from Equations (20, 33-35) and after some manipulations, one gets: $u(x = 0) = 1 - \frac{J_h(x = 0)}{eS \times \rho(x = 0)}$, being just the boundary condition given in Eq. (28). Now, using the P(x)-definition given in Equation (31), at T=300 K, one can define the inverse minority-hole effective diffusion length:

$$\frac{1}{l_{h_{eff}}(x = W, N, T, r_d, g_c, S)} = \frac{1}{\int_0^W dx \frac{W}{I_{eo}(x = W, N, T, r_d, g_c, S)} = \frac{1}{\int_0^W W \cos(x) dx} = P(x = W, N, T, r_d, g_c) / W$$ (36)

Now, from above Equations (38-43), some important results can be obtained and discussed below.

### 5.1. Very Large $S(= 10^{56} \text{ cm}^2)$ For Example $S \rightarrow \infty$ and $P \ll 1$ or $W \ll l_{h_{eff}}$

Here, various results can be investigated as follows.

(i) From Equations (38-40), since $I(W) = \frac{B_h(N_0(W))}{S \times l_{h_{eff}}}$, $\lim_{S \rightarrow \infty} I(W, N, r_d, g_c, S) = 0$ since $P \ll 1$, one then obtains $l_{h_{eff}}(x = W, N, r_d, g_c, S) \rightarrow \infty$. Therefore, from Equation (41), one obtains:

$$\frac{\tau_{th}(N, r_d, g_c, S) \rightarrow 0, \text{ suggesting a completely transparent emitter region (CTER).}}{\tau_{th}(N, r_d, g_c, S) \rightarrow 0, \text{ suggesting a completely transparent emitter region (CTER).}}$$

(ii) Further, from Equations (18-20, 39), since $1 \rightarrow 0$ and $P \ll 1$, the result (39) is now reduced to:

$$I_{eo}(x = W, N, r_d, g_c, S) \rightarrow 0 = \frac{en^2C}{\sinh(P) + 1} \times l_{h_{eff}}$$ (44)

being found to be independent of S and C, since $l_{h_{eff}}$ is independent of S and C as observed in Equations (20, 36), and noting that the ABGN-expression is determined by Equation (9) or by Equation (15).

Now, in the P-Si system, for $T = 300 K, r_d \equiv r_p$ and $g_c = 6, 5, 4.9113$, our two numerical $I_{eo}$-results are calculated, using Equations (44) and (44), and given in Table 4, in which the CTER -condition, $P \ll 1$ (or $l_{h_{eff}} \ll 1$), is fulfilled, and we also compare them with modeling and measuring $I_{eo}$-results investigated by del Alamo et al. (ASS) [10, 12]. One notes that their modeling $I_{eo}$-result [10], based only on two independent parameters: $N_{def}/D_h$ and $\tau_{th}$, can be obtained, for instance, by $\tau_{th} = W$, from our above result (44). This could explain a great difference between their modeling results [10, 12], being accurate within 36%, and ours, accurate within 1.78%, for $g_c = 6$, as those observed in the following Table 4.

| $N$ ($10^{19} \text{ cm}^{-2}$) | 2.1 | 3.3 | 4.6 | 12 |
|-------------------------------|-----|-----|-----|----|
| W [μm]                       | 0.20 | 1.00 | 2.43 | 0.20 |
| $I_{eo}(S \rightarrow \infty)$-data | $3.2 \times 10^{-12}$ | $8.3 \times 10^{-13}$ | $2.6 \times 10^{-12}$ | $1.1 \times 10^{-12}$ | $2.8 \times 10^{-12}$ |
| ASS- $I_{eo}(S \rightarrow \infty)$ | $3.6 \times 10^{-12}$ | $1.1 \times 10^{-12}$ | $2.6 \times 10^{-12}$ | $1.5 \times 10^{-12}$ | $2.8 \times 10^{-12}$ |
| RD(%)                        | -12.5 | -32.5 | 0 | -96 | -4.0 |
| $N_d$ ($\text{cm}^{-3}$)     | $2.65 \times 10^{17}$ | $1.82 \times 10^{15}$ | $2.22 \times 10^{17}$ | $1.60 \times 10^{16}$ | $2.65 \times 10^{17}$ |

Table 4. Our present results of $I_{eo}$ expressed as functions of $N$ for $g_c = 6, 5, 4.9113$, and their relative deviations (RDs), calculated by: RD(%) = $I_{eo}(N = 6, 5, 4.9113).$
Table 4 indicates that:

(i) the maximal relative deviations (RDs) in absolute values between our results (44, 9) and the $I_{\text{Eo}}$-data [10, 12] are found to be: 1.78% for $g_{\text{e},6}, 4.56\%$ for $g_{\text{e},5},$ and $5.07\%$ for $g_{\text{e},4}$, and $4.9113,$ and

(ii) the maximal RDs in absolute values between our results (44, 15) and the $I_{\text{Eo}}$-data [10, 12] are given by: 2.42% for $g_{\text{e},6}, 5.49\%$ for $g_{\text{e},5},$ and $5.75\%$ for $g_{\text{e},4}$, and $4.9113.$ It suggests that our numerical results (44, 9) for $g_{\text{e},6}$ are the best ones, since they are accurate within 1.78%. Further, one notes that our $\Delta E_{g_{\text{a}}}$ expression given in Equation (9) was obtained, taking into account all the physical effects such as: those of donor size, heavy doping and Fermi-Dirac statistics, while in Equation (15) our $\Delta E_{g_{\text{a}}(\text{Mod.YC})}$ expression is only an empirical one. So, in the following, we will choose: $g_{\text{e},6}, T=300 \text{ K},$ and our ABGN-expression (9), for all the numerical calculations.

(iii) Furthermore, in particular, for large $S$ and small $P$, from Equation (40) one gets:

$$\frac{I_{\text{Eo}}(x=0,N_g,S)}{I_{\text{Eo}}(x=W,N_g,S)} = \frac{1}{\cosh(P) \times \sinh(P)} \approx 1 - \frac{\Delta E_{g_{\text{a}}(N_g,W)}}{5 \times \Delta h(N_g,W)} \times P - \frac{P^2}{2}.$$ 

Then, from Equation (43), using Equations (20, 37) one obtains in the heavily doped case:

$$\tau_{\text{t eff}}(x=W,N,r_d\text{, }g_{\text{e},6}) \approx S \text{h} \times \left( \frac{D_h(N_g(W))}{S \times L_h(N_g(W))} \times P + \frac{(P)^2}{2} \right) \approx \frac{W}{S} \times \frac{L_h(N_g(W))}{L_{\text{h eff}}(N_g(W))} + \frac{W^2}{2D_h(N_g(W))} \times \left( \frac{L_h(N_g(W))}{L_{\text{h eff}}(N_g(W))} \right)^2, \text{ as } S \to \infty$$

(45)
\[
\frac{D_{W}(N_{d}(W))}{S} \rightarrow \infty, \quad \text{since} \ S \rightarrow 0. \quad \text{Therefore, from Equation (43), one obtains:} \quad \frac{\tau_{\text{eff}}(x=W,N_{d},r_{d},S)}{\tau_{b}} \rightarrow 1, \quad \text{suggesting a completely opaque emitter region (COER).}
\]

Now, our numerical results of \( J_{Eo}(x=W,N_{d},r_{d},S) \equiv J_{Eo} \) \( \frac{\tau_{\text{eff}}(x=W,N_{d},r_{d},S)}{\tau_{b}} \) for simplicity, are respectively computed, using Equations (39) and (43), and then plotted into Figures 3 (a), (b) and 4 (a), (b) as functions of \( N \), and Figures 3 (c) and 4 (c), as functions of \( S \), noting that in those figures we also include various physical conditions such as: \( S, W, r_{d} \) and \( N \).

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figures.png}
\caption{(a), (a) Our \( J_{Eo} \)-results obtained as functions of \( N \), with a condition: \( N > N_{d}(r_{d}) \), given in heavily doped donor-Si systems, as defined in Table 1, (b) ours obtained as a function of \( N \), and (c) ours obtained as a function of \( S \).}
\end{figure}
Some concluding remarks are obtained and discussed below.

(i) Figures 3(a1, a2) and 4(a1, a2) indicate that, since as $S \to \infty$ and $W = 1 \mu m$, $\tau_{\text{eff}} / \tau_b < 4 \times 10^{-8}$, according to the CTMR, and for a given $N$, due to the donor-size effect, both $J_{\text{eo}}$ and $\tau_{\text{eff}} / \tau_b$ decrease (i) with increasing $r_d$. Then, for a given $r_d$, at large values of $N \geq 3 \times 10^{20} \text{ cm}^{-3}$, due to the heavy doping effect, $J_{\text{eo}}$ (or $\tau_{\text{eff}} / \tau_b$) increases (or decreases) with increasing $N$.

(ii) Figures 3(b) and 4(b) show that, for a given $N$, $J_{\text{eo}}$ (or $\tau_{\text{eff}} / \tau_b$) decreases (or increases) with increasing $W$.

(iii) Figures 3(c) and 4(c), suggest that, for given $S$, $J_{\text{eo}}$ (or $\tau_{\text{eff}} / \tau_b$) decreases (or increases) with increasing $S$.

(iv) In particular, in Figure 4(c), as $S \to 0$ and $W = 85 \mu m$, $\tau_{\text{eff}} / \tau_b \to 1$, according to the COER.

Finally, it should be noted that in next Section 6 we must know the numerical results of dark saturation current density, defined by:

$$J_o(x = W, N, r_d, S, N_d, r_d) = J_{\text{eo}}(x = W, N, r_d, S) + J_{\text{bo}}(N_d, r_d) \quad (47)$$

where $J_{\text{bo}}$ and $J_{\text{eo}}$ are determined respectively in Equations (21, 39). Then, those are tabulated in the following Table 5, in which all the physical conditions are also presented.

Table 5. Our numerical results of $J_o = J_{\text{eo}} + J_{\text{bo}}$, calculated using Equation (47), where $J_{\text{bo}}$ and $J_{\text{eo}}$ are determined respectively in Equations (21, 39), and those are obtained in the three following cases.

| First case: In the heavily doped (HD) P-Si emitter region ($N = 10^{20} \text{ cm}^{-3}$), and in the lightly doped (LD) B-Si base region ($N_a = 10^{10} \text{ cm}^{-3}$) in which $J_{\text{bo}} = 6.0912 \times 10^{-13} (\Delta / \text{cm}) $. |
|---|
| For $S = 10^{10} \text{ cm/s}$ and $W = 0.206 \text{ nm},$ according to the completely transparent emitter region, one has: $J_{\text{eo}} = 2.483 \times 10^{-10} (\Delta / \text{cm}) \gg J_{\text{bo}}$ and $J_o = 2.483 \times 10^{-9} (\Delta / \text{cm}) = J_{\text{eo}}$. |
| For $S = 10^{10} \text{ cm/s}$ and $W = 4.4 \text{ mm},$ according also to the completely transparent emitter region, one has: $J_{\text{eo}} = 1.1645 \times 10^{-9} (\Delta / \text{cm}) \gg J_{\text{bo}}$ and $J_o = 1.1706 \times 10^{-10} (\Delta / \text{cm}) = J_{\text{eo}}$. |
| Second case: In the completely opaque HD S-Si emitter region ($N = 5 \times 10^{20} \text{ cm}^{-3}$, $S = 10^{-5} \text{ cm/s}$ and $W = 85 \mu m$), and in the lightly doped $a$-Si base region, in which $N_a = 10^{10} \text{ cm}^{-3}$. |
| $J_{\text{eo}} = J_{\text{bo}}$ |
| $J_o = J_{\text{eo}} + J_{\text{bo}}$ |
| $J_{\text{bo}} (\Delta / \text{cm})$ | $J_{\text{eo}} (\Delta / \text{cm})$ | $J_o (\Delta / \text{cm})$ |
| $1.8728 \times 10^{-29}$ | $1.8728 \times 10^{-29}$ | $1.8728 \times 10^{-29}$ |
| $6.0912 \times 10^{-10}$ | $6.0912 \times 10^{-10}$ | $6.0912 \times 10^{-10}$ |
| $5.3080 \times 10^{-17}$ | $5.3080 \times 10^{-17}$ |
| Third case: In the completely transparent HD d-Si emitter region ($N = 5 \times 10^{20} \text{ cm}^{-3}$, $S = 10^{-5} \text{ cm/s}$ and $W = 0.0000206 \mu m$), and in the lightly doped Ti-Si base region, in which $N_a = 10^{10} \text{ cm}^{-3}$ and $J_{\text{bo}} = 5.3080 \times 10^{-17} (\Delta / \text{cm})$. |
| $J_{\text{eo}} = J_{\text{bo}}$ |
| $J_o = J_{\text{eo}} + J_{\text{bo}}$ |
| $J_{\text{bo}} (\Delta / \text{cm})$ | $J_{\text{eo}} (\Delta / \text{cm})$ | $J_o (\Delta / \text{cm})$ |
| $2.7206 \times 10^{-9}$ | $2.7206 \times 10^{-9}$ | $2.7206 \times 10^{-9}$ |
| $5.3080 \times 10^{-17}$ | $5.3080 \times 10^{-17}$ | $5.3080 \times 10^{-17}$ |
Some important remarks are given and discussed below.

(i) In the first case, with decreasing S and increasing W, \( J_o \) thus decreases from the CTER to the COER, and one gets in this COER: \( J_o = J_{BO} \).

(ii) In the second case or in the COER-conditions, \( J_{BO} \) decreases with increasing \( r_a \), being due to the acceptor-size effect, and for given \( r_a \) one has: \( J_o = J_{BO} \) since \( J_{EO} = 0 \).

(iii) In the third case or in the CTER-conditions, \( J_{EO} \) decreases with increasing \( r_d \), being due to the donor-size effect, and for \( r_d \) one gets: \( J_o = J_{EO} = 5.3080 \times 10^{-17} \text{ cm}^2 \), calculated for \( (r_{BO}, r_{TT}) \).

It should be noted that these values of \( J_o \) will strongly affect the variations of various photovoltaic conversion parameters of \( n^-p \) junction silicon solar cells, such as: the ideality factor \( n \), short circuit current density \( J_{SC} \), fill factor FF, and photovoltaic conversion efficiency \( \eta \), being expressed as functions of the open circuit voltage, \( V_{OC} \) [4], as investigated in the following. Our empirical treatment method used is that of two points. The first point is characterized by [27]:

\[
V_{OC1} = 624 \text{ mV}, J_{SC1} = 36.3 \text{ mA cm}^{-2}, FF_1 = 80.1 \%
\]  

(48)

and the second one by [23, 28]:

\[
V_{OC2} = 740 \text{ mV}, J_{SC2} = 41.8 \text{ mA cm}^{-2}, FF_2 = 82.7 \%.
\]  

(49)

In the following, we will develop our empirical treatment method of two points, used to determine \( J_{SC} \) and FF, basing on accurate results given in Equations (48) and (49).

6. Photovoltaic Conversion Effect

The well-known net current density \( J \) at \( T=300 \text{ K} \), expressed as a function of the applied voltage \( V \), flowing through the \( n^-p \) junction of silicon solar cells, is defined by:

\[
J(V) \equiv J_{ph}(V) - J_o \times (e^{V} - 1), \quad V \equiv \frac{kT}{e} = 25.8543 \text{ mV}
\]  

(50)

Noting that \( J(V) = 0 \) at \( V = V_{OC} \), \( V_{OC} \) being an open circuit voltage, at which \( J_{ph}(V = V_{OC}) = J_{SC}(W, N, r_d, S, N_a, r_a, V_{OC}) \), where \( J_{SC} \) is the short circuit current density. Here, \( J_{ph} \) is the photocurrent density and \( J_o(W, N, r_d, S, N_a, r_a) \equiv J_{EO} + J_{BO} \) is the “dark saturation current density” or the \( n^-p \) junction leakage saturation current density in the absence of light, defined in Equation (47). Therefore, the photovoltaic conversion effect occurs, according to:

\[
J(V) \equiv J_{ph}(V) - J_o \times (e^{V} - 1), \quad V \equiv \frac{kT}{e} = 25.8543 \text{ mV}
\]  

(51)

where

\[
n(W, N, r_d, S, N_a, r_a, V_{OC}) = n_1(W, N, r_d, S, N_a, r_a, V_{OC1}, J_{SC1}) + n_2(W, N, r_d, S, N_a, r_a, V_{OC2}, J_{SC2}) \times \left( \frac{V_{OC}}{V_{OC1}} - 1 \right)^{y_n},
\]

\[y_n = 1.1248\]

(52)

which is valid for any \( W, N, r_d, S, N_a, r_a, V_{OC} \geq V_{OC1} \), and increases with increasing \( V_{OC} \) for given \( W, N, r_d, S, N_a \) and \( r_a \).

Further, the values of \( V_{OC1}, J_{SC1}, V_{OC2} \) and \( J_{SC2} \) are given in Equations (48, 49), and the numerical results of \( n_1(2) \) can be determined from Equation (51) by:

\[
n_{1(2)}(W, N, r_d, S, N_a, r_a, V_{OC1(2)}, J_{SC1(2)}) \equiv \frac{V_{OC1(2)}}{V_{R}} \times \frac{1}{\ln \left( \frac{n_{1(2)}}{n_1} + 1 \right)}
\]  

(53)

implying that both \( n_1(2) \) (or \( n \)) and \( J_o \) have the same variations for given \( W, N, r_d, S, N_a, r_a \)-variations, being found to be an important remark.

Furthermore, in Equation (52), for the CTER-conditions such as:

\[
W = 4.4 \text{ nm} = 0.0044 \mu \text{m}, N = 10^{20} \text{ cm}^{-3}, r_d = r_p, S = 10^{50} \text{ cm}^{-3}, N_a = 10^{16} \text{ cm}^{-3}, r_a = r_B
\]  

(54)

the exponent \( y_n = 1.1248 \) was chosen such that:

\[
n(W, N, r_d, S, N_a, r_a, V_{OC1(2)}(2) \equiv n_1(2)(W, N, r_d, S, N_a, r_a, V_{OC1(2)}, J_{SC1(2)}) = 1.2344 (1.4534) \text{ respectively.}
\]
For example, from the above remark given in Eq. (53) and from the first case reported in Table V, we can conclude that, with decreasing $S$ and increasing $W$, both $n$ and $J_{sc}$ decrease from the CTER to the COER. Therefore, from Equation (51), $J_{sc}$ thus increases from the CTER to the COER, since $J_{sc}$ is expressed in terms of $e^{-\frac{V_{oc}}{n_{oc}V_{oc}}}$.

Then, the values of the fill factor $FF$ for $V_{oc} = V_{oc1(2)}$ can be found to be given by:

$$FF_{1(2)}(W, N, r_d, S, N_p, r_a, V_{oc1(2)}) = \frac{v(N, r_d, S, N_p, r_a, V_{oc1(2)}) - \ln[v(N, r_d, S, N_p, r_a, V_{oc1(2)}) + 0.72]}{v(N, r_d, S, N_p, r_a, V_{oc1(2)}) + y_{FF}(a)}$$

(55)

where $z_{FF1(2)} = 1.1 (0.472)$ was chosen such that, under the above conditions (54), the values of $FF_{1(2)}$, calculated using Equation (55), are identical to the data given in Equations (48, 49): 80.1% (82.7%), respectively [27, 23].

Moreover, in the case where both series resistance and shunt resistance have a negligible effect upon cell performance, $z_{FF1(2)} = 1$, as proposed by Green [4].

Now, by applying a same above treatment method of two points, one has:

$$y_{FF} = 2.0559$$

(56)

which is valid for any $W, N, r_d, S, N_p, r_a, V_{oc} \geq V_{oc1}$, and increases with increasing $V_{oc}$, for given $W, N, r_d, S, N_p, r_a$. Here, the value of $y_{FF} = 2.0559$ was chosen such that, under the conditions (54), $FF(W, N, r_d, S, N_p, r_a, V_{oc1(2)}) \equiv FF_{1(2)}(W, N, r_d, S, N_p, r_a, V_{oc1(2)}) = 80.1\% (82.7\%)$, respectively [27, 23].

Then, the photovoltaic conversion efficiency $\eta$ can be defined by:

$$\eta(W, N, r_d, S, N_p, r_a, V_{oc}) \equiv \frac{J_{sc} \times V_{oc} \times FF}{P_{in}}$$

(57)

where $J_{sc}$ and FF are determined respectively in Equations (51, 56), being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ($P_{in} = 0.100 \frac{W}{cm^2}$) [27, 28].

In summary, all above parameters such as: $n$, $J_{sc}$, FF and $\eta$, defined in above, strongly depend on $J_{oc}$, determined in Equation (47), which is thus a central result of the present paper.

Now, for given physical conditions such as: $W, N, r_d, S, N_p$, and $r_a$, and by taking into account all remarks given in Table 5 and also in above Equation (53), our numerical results of $n$, $J_{sc}$, FF and $\eta$, expressed as functions $V_{oc}$, are respectively computed by using Equations (52, 51, 56), and reported in following Table 6 and Figures 7, 8 and 9.

In Table 6, in which, for $624 \leq V_{oc}(mV) \leq 750$ [23, 24, 27-29] the physical conditions used are:

$$W = 0.206 \text{nm}, N = 10^{20} \text{cm}^{-3}, r_d \equiv r_p, S = 10^{50} \text{cm}^{-s}, N_a = 10^{16} \text{cm}^{-3}, r_a \equiv r_B$$

(58)

according to the CTER, we get the precisions of the order of 8.1% for $J_{sc}$, 7.1% for FF, and 5% for $\eta$, calculated using the corresponding data [23, 24, 27-29], which is strongly affected by $J_0 = J_{oc} + J_{bo}$, as noted above, suggesting thus an accuracy of $J_{bo} \leq 8.1\%$, since $J_{bo}$ was accurate within 1.78%, as given in Table 4.

Table 6. With the physical conditions given in Equation (58), our present results (PR) of $n$, $J_{sc}$, $FF$($\%$), and $\eta$($\%$), calculated using Equations (52,51,56,57), being compared with corresponding data [23, 24, 27-29], and their relative deviations (RD), computed using the formula: $RD = |1 - (PR/\text{Data})|$. 

| Data (D) from References | $V_{oc}$ (mV) | n | $J_{sc}(PR)/(J_{sc}(D))$; RD | $FF(PR)/(FF(D))$; RD | $\eta_{PR}/(\eta(D))$; RD |
|--------------------------|-------------|---|-----------------------------|----------------------|--------------------------|
| [28]                     | 750         | 1.7474 | 40.24 (39.5); 1.9 | 80.58 (83.2); 3.1 | 24.32 (24.7); 1.5 |
| [23, 28]                 | 740         | 1.7222 | 40.01 (41.8); 1.9 | 80.11 (82.7); 3.1 | 24.31 (25.6); 5.0 |
| [28]                     | 738         | 1.7122 | 41.16 (40.8); 0.9 | 80.02 (85.3); 4.2 | 24.31 (25.1); 3.2 |
| [28]                     | 737         | 1.7146 | 42.33 (41.3); 0.2 | 80.00 (82.7); 3.3 | 24.30 (25.2); 3.6 |
| [28]                     | 718         | 1.6676 | 42.43 (42.1); 0.8 | 79.22 (82.3); 4.8 | 24.13 (25.1); 3.8 |
| [24]                     | 710         | 1.6481 | 42.82 (42.3); 1.2 | 78.95 (82.6); 4.4 | 24.00 (24.8); 3.2 |
| [28, 29]                 | 706         | 1.6384 | 42.98 (42.7); 0.6 | 78.82 (82.8); 4.8 | 23.91 (25.0); 4.3 |
| [24]                     | 705         | 1.6360 | 43.02 (42.2); 1.9 | 77.87 (83.1); 6.3 | 23.89 (24.7); 3.3 |
| [24]                     | 703         | 1.6312 | 43.08 (42.0); 2.6 | 78.73 (82.7); 4.8 | 23.84 (24.4); 2.3 |
| [28]                     | 695         | 1.6122 | 43.30 (40.2); 7.7 | 78.50 (80.5); 2.5 | 23.62 (22.5); 4.9 |
| [28]                     | 680         | 1.5772 | 43.37 (40.5); 7.1 | 78.14 (80.3); 2.7 | 23.05 (22.1); 4.3 |
| [29]                     | 671.7       | 1.5584 | 43.20 (40.5); 6.5 | 77.98 (80.9); 3.6 | 22.63 (22.0); 2.8 |
| [28]                     | 667         | 1.5479 | 43.01 (39.8); 8.1 | 77.91 (80.0); 2.6 | 22.35 (21.3); 4.9 |
| [27]                     | 665         | 1.5434 | 43.91 (42.2); 1.7 | 76.87 (78.7); 1.0 | 22.22 (22.1); 0.5 |
Table (D) from References

| Data (D) from References | $V_{oc}$ (mV) | $n$ | $I_{sc}(R_d)/I_{sc(0)}$: RD | $FF_{pp}(FF_{pp})$: RD | $\eta_{pp}(\eta_{pp})$: RD |
|--------------------------|--------------|-----|-----------------------------|------------------------|-----------------------------|
| [24]                     | 655          | 1.5217 | 42.21 (39.8); 6.1           | 77.74 (79.4); 2.1      | 21.50 (20.7); 3.8          |
| [28]                     | 643          | 1.4968 | 40.83 (39.3); 3.9           | 77.64 (83.6); 7.1      | 20.38 (21.1); 3.4          |
| [27]                     | 632          | 1.4758 | 38.80 (39.2); 1.0           | 77.59 (75.8); 2.4      | 19.02 (18.7); 1.7          |
| [27]                     | 624          | 1.4630 | 36.30 (36.3); 0.0           | 77.58 (80.1); 3.1      | 17.57 (18.1); 2.9          |

The underlined RD (%)-values are the maximal ones.

In Figures 5 (a), (b), (c) and (d), the physical conditions used are:

$$N = 10^{20} \text{ cm}^{-3}, r_d = r_p, N_a = 10^{16} \text{ cm}^{-3}, r_a = r_B,$$

and different $(S, W)$ values

which are given also in these figures, and in Table 5 for the first case. Here, for a given $V_{oc}$, and with decreasing S and increasing W, we observe that:

(i) in the Figure 5 (a), the function $n$ determined in Equation (52) (or the function $J_o$ given in Table 5) decreases from the CTER to the COER

(ii) in Figures 5 (b), 5 (c) and 5(d), the functions $J_{sc}$, FF and $\eta$ therefore increase from the CTER to the COER, and

(iii) in Figure 5 (d), for the physical functions: $W=85 \mu m$ and $S=10^{-50} \text{ cm/s}$, the function $\eta$ reaches a maximum equal to 27.77% at $V_{oc}=715 \text{ mV}$; here $1 \mu m = 10^{-6} \text{ m}$.

![Figure 5](image_url)

Figure 5. (a) Our $n$-results, (b) $J_{sc}(\mu m)$-results, (c) FF(%)-results, and (d) $\eta$(%)-results, plotted as functions of $V_{oc}$ and obtained with increasing $W$ and decreasing $S$ (or from the completely transparent emitter region to the completely opaque emitter region).

In Figures 6 (a), (b), (c) and (d), the physical conditions used are:

$$W = 85 \mu m, N = 5 \times 10^{20} \text{ cm}^{-3}, r_d = r_p, S = 10^{-50} \text{ cm/s},$$

$$N_a = 10^{16} \text{ cm}^{-3}, r_a,$$

and $E_{gl}(r_a)$ at 300 K (60) according to the COER, and they are also given in these
figures and in Table 5 for the second limiting case, in which $J_0 = J_{00}$, since $J_{00} = 0$. Thus, this simplifies the numerical calculation of functions $n$, $J_{sc}$, FF and $\eta$, using Equations (52, 51, 56, 57), where $J_0$ is replaced by $J_{00}$, determined by Eq. (21). Further, in Equation (60), the values of $E_{gl}(r_d)$ are given in Table 2. Then, for a given $V_{oc}$ and with increasing $r_d$-values, it should be concluded that, due to the acceptor-size effect, (i) in the Figure 6 (a), the function $n$ determined in Equation (52) (or the function $J_0$ given in Table 5) decreases (\(\downarrow\)), and (ii) in Figures 6 (b), (c), (d), the functions $I_{sc}$, FF and $\eta$ therefore increase (\(\uparrow\)), and in particular, in Figure 6 (d), for the completely opaque (Si) emitter-region conditions, where $J_{00} = 0$ or $J_0 = J_{00}$, the maximal $\eta$-values are equal to: 27.77 \%, ..., 31.55 \%, at $V_{oc} = 715 \text{ mV}$, ..., 703 mV, according to the $E_{gl}$-values equal to: 1.12 eV, ..., 1.34 eV, which are obtained in various lightly doped (B, ..., TI)-Si base regions, respectively, being due to the acceptor-size effect.

![Figure 6](image1)

Finally, in Figures 7 (a), (b), (c) and (d), the physical conditions used are:

$$W = 0.000206 \mu \text{m}, N = 5 \times 10^{20} \text{ cm}^{-3}, r_\text{a} S = 10^{50} \text{ cm} \text{s}^{-1},$$

$$N_a = 10^{16} \text{ cm}^{-3}, r_{\text{Tr}} \text{ and } E_{gl}(r_d) \text{ at } 300 \text{ K}$$

(61)

according to the CTER, and they are also given in Table 5 for the third case. Here, the values of $E_{gl}(r_d)$ at 300 K are given in Table 2. Then, the numerical results of $n$, $I_{sc}$, FF and $\eta$ are calculated, using Equations (52, 51, 56, 57). Further, for a given $V_{oc}$ and with increasing $r_a$-values, it should be concluded that, due to the donor-size effect, (i) in the Figure 7 (a), the function $n$ determined in Equation (52) (or the function $J_0$ given in Table 5) decreases (\(\downarrow\)), and (ii) in Figures 7 (b), (c), (d), the functions $I_{sc}$, FF and $\eta$ therefore increase (\(\uparrow\)), and in particular, in Figure 7 (d), in the conditions of completely transparent and heavily doped (donor-Si) emitter-and- lightly doped (TI-Si) base regions, the maximal $\eta$-values are equal to:
24.28 %,..., 31.51 %, at $V_{oc} = 748 \text{ mV},...703 \text{ mV}$, according to the $E_G$-values equal to: 1.11 eV,..., 1.70 eV, obtained in various (Sb,..., S)-Si emitter regions, respectively, being due to the donor-size effect, which can be compared with those given in Figure 6 (d).

### Figure 7.

For $N = 5 \times 10^{19} \text{ cm}^{-3}$ and $N_i = 10^{16} \text{ cm}^{-3}$, (a) our $n$-results, (b) $J_{sc}(\Omega/m^2)$-results, (c) FF(%)-results, and (d) $\eta(\%)$ - results, plotted as functions of $V_{oc}$ and obtained in the CTER-conditions.

### 7. Concluding Remarks

We have developed the effects of heavy doping and impurity size on various parameters at 300 K, characteristic of energy-band structure, as given in Sections 2 and 3, and of the performance of crystalline silicon solar cells, being strongly affected by the dark saturation current density: $J_0 \equiv J_{sc} + J_{BO}$, as given in Sections 4, 5 and 6. Then, some concluding remarks are obtained and discussed as follows.

1. Using the OPG ($E_G$)-data given by Wagner and del Alamo [44], our $E_G$-results, due to the heavy doping effect, and calculated using Equation (16), is found to be accurate within 1.86%, as observed in Table 3.

2. In the CTER-conditions, as those given in Table 4, and using the $J_{sc}$-data, given by del Alamo et al. [10, 12], by using Equation (44), our $J_{sc}$-results, obtained in the heavily doped and completely transparent (P-Si) emitter region, are found to be accurate within 1.78%, while the modeled $J_{sc}$-results, obtained by those authors, are accurate within 36% [10, 12].

3. For given physical conditions and using an empirical treatment method of two points, as developed and discussed in Section 6, both our two results ($n$ and $J_0$) have the same variations, which strongly affect other ($V_{oc}$, $J_{sc}$, FF, $\eta$)-results, as discussed in Eq. (53). Thus, $J_0$, determined in Equation (47), is a central result of our present paper.

4. In the CTER-conditions, as those given in Equation (58), and using various ($J_{sc}$, FF, $\eta$)-data [23, 24, 27-29], we get the precisions of the order of 8.1% for $J_{sc}$, 7.1% for FF and 5% for $\eta$, suggesting thus a probable accuracy of $J_{BO} \leq 8.1\%$, since our $J_{BO}$-results are accurate within 1.78%.

5. In the physical conditions of completely opaque and heavily doped (S-Si) emitter-and-lightly doped (acceptor-Si) base regions, as given in Eq. (60), and in the physical conditions of completely transparent and heavily doped (donor-Si) emitter-and-lightly doped (Tl-...
Si base regions, as given in Eq. (61), our obtained maximal η-values, due to the impurity-size effect, are found to be equal respectively to: 27.77%, ..., 31.55%, as seen in Figure 6 (d), and 24.28%, ..., 31.51%, as observed in Figure 7 (d), suggesting that our obtained highest η-values are found to be almost equal, as: 31.51% ≈ 31.55%, since the two corresponding limiting $J_\theta$-values are almost the same, as given in Table 5, for second and third cases.

In summary, being due to the impurity-size effects, our limiting value of η = 31.55%, as that given in Figure 6 (d), is thus obtained in the following limiting physical conditions as:

\[ W = 85 \, \mu m, \quad N = 5 \times 10^{20} \, cm^{-3}, \quad E_g(r_d = r_p) = 1.7035 \, eV, \quad S = 10^{-50} \, cm^3, \]

\[ N_a = 10^{16} \, cm^{-3}, \quad \text{and} \quad E_g(r_a = r_p) = 1.3415, \quad \text{at} \quad 300 \, K, \]

and η ≈ 27.77%, as that given in Figure 5 (d), is obtained in the following limiting physical conditions as:

\[ W = 85 \, \mu m, \quad N = 5 \times 10^{20} \, cm^{-3}, \quad E_g(r_d = r_p) = 1.1245 \, eV, \quad S = 10^{-50} \, cm^3, \]

\[ N_a = 10^{16} \, cm^{-3}, \quad \text{and} \quad E_g(r_a = r_p) = 1.1245, \quad \text{at} \quad 300 \, K. \]

Those limiting η₁,₂-results can be compared with that obtained by Richter et al. (R) [26], ηR = 29.43%, for a thick 100 μm solar cell made of un-doped silicon, as: η2 < ηR < η1.

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Appendix

Appendix A: Fermi Energy

The Fermi energy $E_F$, obtained for any T and donor density N, being investigated in our previous paper, with a precision of the order of 2.11 × 10⁻⁴ [39], is now summarized in the following. First of all, we define the reduced electron density by:

\[ u(N,T,r_d,g_c) \equiv \frac{N}{N_c(T,r_d,g_c)} \equiv F_{1/2}(\theta) \] (A1)

where $N_c$ is defined in Eq. (6), $\theta(u) \equiv \frac{E_F(u)}{k_B T}$ is the reduced Fermi energy, and $F_{1/2}(\theta)$ is the Fermi-Dirac integral, defined by [40]:

\[ F_1(\theta) \equiv \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{1}{1+e^{\theta x}} \, dx \equiv \frac{E}{k_B T} \] (A2)

which was calculated for any values of θ, with a precision of the order of 10⁻⁷, by Van Cong and Doan Khanh [40], using a theorem existence of Hermite interpolating polynomials.

Then, by a reversion method of u ≡ $F_{1/\gamma}(\theta)$ so useful to obtain $\theta(u)$, concerned with doped semiconductors at arbitrary N and T, our expression for reduced Fermi energy was found to be given by [39]:

\[ \theta(u) \equiv \frac{E_F(u)}{k_B T} = \frac{g(u)+A\mu_b^p(u)}{1+\mu_b^p}, \quad \text{with} \quad A = 0.0005372 \quad \text{and} \quad B = 4.82842262 \] (A3)

where, in the degenerate case or when $\theta(u) \gg 1 \to \infty$, Equation (A3) is reduced to:

\[ F(u) = au^2 \left( 1 + \frac{b^2}{3} + \frac{c^2}{1920} \right); \quad a = \frac{3\sqrt{\pi}}{4} \] ,

\[ b = \frac{1}{8} \left( \frac{\pi}{2} \right)^2 \], and $c = \frac{62.3739855}{1920} \left( \frac{\pi}{2} \right)^2$, in the non-degenerate case or when $\theta(u) \ll 1 \to 0$, to:

\[ G(u) \approx \ln(u) + \frac{3}{2} u \times e^{-du}, \quad d = \frac{1}{2} \frac{1}{\sqrt{27} - 3 \sqrt{3}} > 0 \]

Appendix B: Approximate Form for Band Gap Narrowing (BGN)

First of all, we will normalize the various energies by using the effective Rydberg energy R, as:

\[ R(T, r_d) = 13.605693 \times \frac{m_c(T,r_d)}{e^2(r_d)} \] (A4)

and we express the effective Wigner-Seitz radius $r_s$ characteristic of the interactions by:

\[ r_s(N,T, r_d, g_c) \equiv \frac{3g_c}{4\pi N} \times \frac{1}{a_B(T, r_d)} \]

Here, $a_B(T, r_d) = 5.2917715 \times 10^{-9} \times \frac{e(r_d)}{m_c(T,r_d)}$ (cm) is the Bohr radius. Therefore, one has:

\[ r_s(N, T, r_d, g_c) = 1.1723 \times 10^8 \times \left( \frac{R}{N} \right)^{1/3} \times \frac{m_c(T, r_d)}{e^2(r_d)} \] (A5)

Therefore, the ratio $R/r_s$ is thus proportional to:

\[ \frac{e(r_d)}{m_c} \times N_\Gamma^{1/3} \], where $N_\Gamma = \frac{6 \times 10^6}{g_c \times 9.999 \times 10^{17} \, cm^{-3}}$. Now, an empirical expression for BGN is proposed by:

\[ \Delta E_g(N, T, r_d, g_c) = -R \times \mu_c(r_s) - R \times \mu_e(r_s) - R \times \mu_h-\text{Cor}(r_s) - R \times \mu_h-d(r_s) - R \times \mu_h-\text{Cor}(r_s) + \Delta E_g(\text{LT}) \] (A6)

where, R and $r_s$ are defined above, and five first contributions of the spin-polarized chemical potential energy $\mu$ were determined in our previous paper [42], and sixth $\mu$-one by Lanyon and Tuft [6]. One notes here that the second $-R \times \mu_e(r_s)$-term of Equation (A6) represents the shift in majority conduction-band edge, due to the correlation (Cor) energy of an effective electron gas, $E_c(r_s)$, as [42]:

\[ E_c(N, T, r_d, g_c) = \frac{0.87533 + 0.87533}{0.09084 + r_s} \times \left( 2(1-\ln(2)) \times \ln(r_s) - 0.09328 \right) [37/37] \] (A7)

and that from the a Seitz’s theorem [42], one has:
being obtained with an accuracy of 1.87% for 6 in various donor-Si systems. Then, an approximate expression for the BGN is found to be given by:

\[
\Delta E_g(N, T, r, d, g_c) \approx a_1 \times \frac{e^{(r_p)}}{e^{(r)}} \times \frac{4}{r} \times \frac{N^{1/2}}{r} + a_2 \times \frac{N^{1/2}}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r}
\]

\[
(2.503 \times [-E_z(r_s) + r_1]) + a_3 \times \frac{e^{(r_p)}}{e^{(r)}} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r}
\]

\[
N^{1/2} \times \frac{m_0(T)}{m_e(T, r)} + a_4 \times \frac{m_0(T, r)}{m_e(T, r)} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r}
\]

\[
\left\{ 1 \times \frac{m_0(T, r)}{m_e(T, r)} + a_4 \times \frac{m_0(T, r)}{m_e(T, r)} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \right\}
\]

noting that, in the P-Si system for 300 K, these constants:

\[a_1 = 3.8 \times 10^{-3} \text{ (eV)}, \quad a_2 = 6.5 \times 10^{-4} \text{ (eV)}, \quad a_3 = 2.8 \times 10^{-3} \text{ (eV)}, \quad a_4 = 5.597 \times 10^{-3} \text{ (eV)}, \quad \text{and} \quad a_5 = 8.1 \times 10^{-4} \text{ (eV)}\],

\[\text{were chosen such that for} \quad \text{the resulting values are found to be accurate within 1.78%, as seen in Table 4.}\]

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