Mathematical modelling for didactical phenomenon of fireworks

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Abstract. This research is based on the lack of mathematics teaching materials that are appropriate to the context. The purpose of this study is to develop teaching materials for mathematics learning through the phenomenon of fireworks firing. The type of research used in this research is research and development. Based on the results of the study found that the temperature caused by combustion approaches the logarithmic function. So that the fireworks phenomenon can be used as teaching material for learning logarithmic functions.

1. Introduction
The American Association of Colleges for Teacher Education (AACTE) and the Partnership for 21st Century Skills realize that education needs to equip its citizens with the ability and skills that are critical and creative thinking, communication, collaboration in technology literacy [1]. It is important because we face global competition and various problems arising from the development of technology, communication, the speed of population, environmental and social issues. Teacher education institutions have an important role in addition to preparing teacher candidates as well as preparing proven methods to train the skills that are important for students as prospective citizens who are prepared. Some research has found that students who learn with teaching materials didactic phenomena are said to have better cognitive abilities mathematical connections, critical and creative thinking [2-5]. The others research findings are metamorphing teaching is combined with projects to build motivation and interest and increase cognitive activity to build student collaboration and communication [6]; development of instruments to measure 21st-century skills [7]; application of Project-Based Learning (PBL) to train 21st-century skills [8].

There have been many innovations in mathematics teaching, including research studies on realistic mathematics, which are very promising and can attract students' attention [9,10]; research on modeling showed that the students involved in the study felt there was something new in mathematics learning [11]. Previous research related to ethnomathematics opened special horizons in mathematics learning [12]; with the disclosure of cultural mathematics in this ethnomathematics framework, it is possible to create an appropriate context for Indonesians, so that this cultural context can be used as a material for the starting point of mathematics learning or as a vehicle for mathematical applications. However, it is felt that there is a need for mathematics teaching that can be built in its entirety to train 21st-century skills (critical and creative thinking skills, collaboration, communication and technology literacy), these skills are considered important and the development of ways to train these skills is still not optimal for the education system in Indonesia.
Through the experience of researching and other research outcomes, the researchers intend to develop a didactic design of teaching phenomena-based mathematics to train 21st-century skills in high school students. The didactic design was designed by involving students in science laboratory activities, observing the symptoms of science and finding mathematical models to explain the symptoms of science. The findings will be related to technology-nuanced mathematical projects, learning activities will be observed using videos so that we can found track records of critical and creative thinking, collaboration and communication. Therefore, the designed design can be developed and refined through video analysis. The purpose of this study was to develop teaching materials based on didactic phenomena for learning mathematics with the phenomenon of fireworks firing.

2. Methods
The research method used in this study is research and development. Development research is research that is based on an industrial development model and is used as a procedure for designing and developing quality products [13]. The procedure performed is an adaptation of the research procedure for the development of Borg and Gall as shown in Figure 1.

![Figure 1. The phase of research and development.](image)

The phenomenon of fireworks developed as teaching materials in this study will use several variables that can be observed and measured. Fireworks fires will be measured the amount of burning mass associated with the time of combustion. During combustion, the fireworks will produce heat so that the temperature of the combustion results will be measured. On the other hand, fireworks will emit the results of smoke combustion, this is related to environmental pollution, to get the amount of smoke produced in fireworks, we use a smoke sensor developed with a micro-controller by the Arduino program so that the impact of fireworks fires can be analyzed. Another gas sensor is CO$_2$ gas, measurement of CO$_2$ gas can be done with CO$_2$ gas sensors.

Fireworks phenomenon as a phenomenon that will be used as a mathematical model in this study relating to:

- The length of a burning mass as a function of time.
- Temperature behavior during the combustion process.
- Amount of combustion CO$_2$ gas
- The amount of smoke produced by the combustion process

3. Result and discussion
Based on the results of combustion on the fireworks, we have the data in table 1.
Table 1. The length of the mass of combustion with time (for one firework).

| No | t (s)   | y(cm)   |
|----|---------|---------|
| 1  | 0,00E+00 | 4,96E+00 |
| 2  | 3,34E-01 | 5,16E+00 |
| 3  | 6,67E-01 | 5,33E+00 |
| 4  | 1,00E+00 | 5,53E+00 |
| 5  | 1,33E+00 | 5,80E+00 |
| 6  | 1,67E+00 | 6,03E+00 |
| 7  | 2,00E+00 | 6,13E+00 |
| 8  | 2,34E+00 | 6,30E+00 |
| 9  | 2,67E+00 | 6,54E+00 |
| 10 | 3,00E+00 | 6,64E+00 |
| 11 | 3,34E+00 | 6,87E+00 |
| 12 | 3,67E+00 | 7,08E+00 |
| 13 | 4,00E+00 | 7,38E+00 |
| 14 | 4,34E+00 | 7,48E+00 |
| 15 | 4,67E+00 | 7,91E+00 |
| 16 | 5,01E+00 | 8,12E+00 |

Based on the data in table 1 above, a line diagram is made to get the right model as shown in Figure 2.

Based on the modeling with assistance with Ms Excel in Figure 1, the data connected with line diagrams tend to be in the form of a straight line. Therefore, modeling the mass burning of fireworks against time can be associated with straight line equations. In natural science, this phenomenon can be used to explain the irregular straight-motion material, when modeling is done by a computer or using an excel program. In mathematics, this phenomenon can be used in the matter of straight-line equations. The formula commonly used in determining the equation of a straight line is \( \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \). This phenomenon can also be used in the gradient topic, in this case as shown in the figure 1, that the gradient of the line equation produced by combustion of fireworks is a positive gradient. Based on the linear line model obtained, the line equation model that shows the relationship of mass to time can be determined based on two coordinates between the other points: (1.33, 5.80) and (4.00, 7.38) through manual calculation,
the equation is obtained. By using the formula of the line equation through two points, results are obtained: \( Y = 0.59 X + 5.015 \)

This means that a firework gives a gradient of 0.59 which indicates that every increase of 1 unit of seconds, the mass of the fireworks burned will increase by 0.59.

From the others experiments with seven fireworks, we have data in table 2.

Table 2. The connection between the temperature and the length.

| No. | T (°C) | Length (cm) | Length (m) | Data 1 |
|-----|--------|-------------|------------|--------|
| 1   | T1     | 2           | 0.02       | 153.3  |
| 2   | T2     | 5           | 0.05       | 331.6  |
| 3   | T3     | 8           | 0.08       | 365.3  |
| 4   | T4     | 11          | 0.11       | 367.9  |
| 5   | T5     | 14          | 0.14       | 379.3  |
| 6   | T6     | 17          | 0.17       | 340.6  |
| 7   | T7     | 20          | 0.2        | 379.7  |

Based on the data in table 2 above, a line diagram is made to get the model as shown in Figure 2.

![Figure 2. A line diagram of length (m) on temperature (°C).](image)

Based on figure 2, the relationship between the length of the fireworks burned to the temperature produced is shown in the line diagram. To see the relationship above, the predictor equation using the logarithmic equation is used, which shows that the relationship between the two variables (the length of the fireworks burned and the temperature) is 0.7474 with a high relationship category. The predictor equation is as follows:

\[ y = 86.39 \ln(x) + 540.76 \]

In learning mathematics, teachers sometimes experience confusion giving exam examples of phenomena related to logarithmic functions. The observations related to the firing of fireworks with the resulting temperature can be used as a material for the teacher to provide cases and scientific phenomena related to logarithmic functions.

The logarithmic function is a function defined by \( y = f(x) = \log_a x \) with a real number, \( a > 0, a \neq 1 \) and \( x > 0 \). \( x \) is a variable (independent variable) and \( a \) is a principal number or base. The form of logarithms, in general, is as follows: if \( a^b = c \) with \( a > 0 \) and \( a \neq 1 \) then \( \log_a c = b \), in this case \( a \) is called the base or principal of logarithms and \( c \) is a number which is logically fed. If the
exponent function expresses its function as \( y = a^x \), then the logarithm function has the form \( \log_y a = x \). Logarithmic function is a function whose free variable is a logarithmic form. The Logarithmic function is the inverse of the exponent function.

The material for logarithmic functions in the Indonesian curriculum is given in class ten of Senior High School [14]. Unlike linear functions where applied cases are easier to find in everyday life, to teach logarithmic functions, teachers often have difficulty in providing applied examples, because logarithmic functions are usually applied in astronomy. In astronomy another application of the logarithmic function is the calculation of the Richter scale earthquake events and the calculation of music frequencies. Logarithmic function is used to calculate the magnitude or scale of brightness. magnitude scale shows that the greater the magnitude number, the greater the star's brightness. on the contrary the smaller the magnitude, the more energy we receive on Earth. In the economic field, the logarithmic function is used to calculate population growth and compound interest.

In the introduction of logarithmic functions to students, some applied astronomy, seismology, geography, anthropology above can be conveyed to students. However, these concepts are far enough known or experienced by students, so that it might have an impact on students' ability to understand and the meaningfulness of the material learned for students. Therefore, it would be nice if the phenomenon conveyed to students is a phenomenon that is close to life and often experienced by students.

4. Conclusion
Some research has found that students who learn with teaching materials didactic phenomena are having better cognitive abilities in mathematical connections, critical and creative thinking. This study develops teaching materials for mathematics learning through the phenomenon of fireworks firing. The study found that the temperature caused by combustion approaches the logarithmic function. So that the fireworks phenomenon can be used as teaching material for learning logarithmic functions.

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