Deducing Security Goals From Shape Analysis Sentences

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Abstract

Guttman presented a model-theoretic approach to establishing security goals in the context of strand space theory. In his approach, a run of the Cryptographic Protocol Shapes Analyzer (cpsa) produces models that determine if a goal is satisfied.

This paper presents a method for extracting a sentence that completely characterizes a run of cpsa. Logical deduction can then be used to determine if a goal is satisfied. This method has been implemented and is available to all.

1 Introduction

This revision updates the strand-oriented protocol language described in Section 3 to one that has been shown to be useful in practice. The November 2014 revision of the January 2012 paper corrects several minor errors and adds a description of a node-oriented protocol language in Appendix [2].

A central problem in cryptographic protocol analysis is to determine whether a formula that expresses a security goal about behaviors compatible with a protocol is true. Following [6], a security goal is a quantified implication:

\[ \forall \vec{x} (\Phi_0 \supset \bigvee_{1 \leq i \leq n} \exists \vec{y}_i \Phi_i). \]  

(1)

The hypothesis \( \Phi_0 \) is a conjunction of atomic formulas describing regular (honest) behavior. Each disjunct \( \Phi_i \) that makes up the conclusion is also a conjunction of atomic formulas. When \( \Phi_i \) describes desired behaviors of
other regular participants, then the formula is an authentication goal. The goal says that each run of the protocol compatible with \( \Phi_0 \) will include the regular behavior described by one of the disjuncts. When \( n = 0 \), the goal’s conclusion is false. If \( \Phi_0 \) mentions an unwanted disclosure, (1) says the disclosure cannot occur, thus a security goal with \( n = 0 \) expresses a secrecy goal.

Guttman [6] presented a model-theoretic approach to establishing security goals in the context of strand space theory. In that setting, a skeleton describes regular behaviors compatible with a protocol. For skeleton \( k \) and formula \( \Phi \), he defined \( k, \alpha \models \Phi \) to mean that the conjunction of atomic formulas that make up \( \Phi \) is satisfied in \( k \) with variable assignment \( \alpha \).

A realized skeleton is one that includes enough regular behavior to specify all the non-adversarial part of an execution of the protocol. In a realized skeleton, its message transmissions combined with possible adversarial behavior explain every message reception in the skeleton.

In strand space theory, a homomorphism is a structure-preserving map \( \delta \) that shows how the behaviors in one skeleton are reflected within another. As skeletons serve as models, homomorphisms preserve satisfaction for conjunctions of atomic formulas.

The Cryptographic Protocol Shapes Analyzer (cpsa) constructs homomorphisms from a skeleton \( k_0 \) to realized skeletons [9]. If \( \text{cpsa} \) terminates, it generates a set of realized skeletons \( k_i \) and a set of homomorphisms \( \delta_i: k_0 \mapsto k_i \). These realized skeletons are all the minimal, essentially different skeletons that are homomorphic images of \( k_0 \) and are called the shapes of the analysis.

Guttman proposed a recipe for evaluating goal (1) based on the following two technical results.

- For any security hypothesis \( \Phi_0 \) there is a skeleton \( k_0 \) that characterizes it in the sense that for all \( k \):

  \[
  \exists \alpha \ k, \alpha \models \Phi_0 \iff \exists \delta \ : \ k_0 \mapsto k
  \]

- There exists a realized skeleton that is a counterexample to (1) iff there exists some shape in the analysis of \( k_0 \) that is a counterexample.

These two results justify the following procedure.

1. Construct a characteristic skeleton \( k_0 \) for \( \Phi_0 \).
2. Ask cpsa for the shapes produced by analyzing $k_0$.

3. As cpsa delivers shapes, check that each satisfies some disjunct $\Phi_i$.

4. If the answer is no, this shape is a counterexample to (1).

5. If cpsa terminates with no counterexample, then (1) is achieved.

**Implementing Security Goals.** cpsa now has support for security goals, but not as specified by Guttman. Part of the reason for the difference is that the details of the formalism that underlies the cpsa implementation [11] dictate changes to the logic of security goals. These details will be elaborated later in this paper.

The key difference is a change in perspective. Instead of finding a formula that characterizes a security hypothesis, cpsa includes a tool that extracts a sentence that characterizes a shape analysis. This so called *shape analysis sentence* is special in that it encodes everything that can be learned from the shape analysis.

Given a shape analysis sentence, a security goal is achieved if the goal can be deduced from the sentence. cpsa includes a Prolog program that translates shape analysis sentences into Prover9 [8] syntax. Typically, a goal that is a theorem is quickly proved by Prover9.

There is another advantage to this approach. It can be tedious to generate security goals. Realistic ones can be large and complicated. An easy way to create one is to modify a shape analysis sentence. This typically involves deleting parts of the conclusion.

There is a disadvantage to this approach. When a goal cannot be deduced from a shape analysis sentence, one cannot conclude that there is a counterexample. It could be simply that the sentence is not relevant to the security goal.

**Motivating Example.** The running example used throughout this paper is now presented. An informal version of the example is presented here, and the example with all of the details filled is in Section 4.

The following protocol is a simplified version of the Denning-Sacco key distribution protocol [4] due to Bruno Blanchet [1].

\[
\begin{align*}
A &\rightarrow B : \#\{s\}_{a-1}\|_{b}
\end{align*}
\]

\[
\begin{align*}
B &\rightarrow A : \#\{d\}_{s}
\end{align*}
\]
Alice (A) freshly generates symmetric key $s$, signs the symmetric key with her private uncompromised asymmetric key $a^{-1}$ and intends to encrypt it with Bob’s (B) uncompromised asymmetric key $b$. Alice expects to receive data $d$ encrypted, such that only Alice and Bob have access to it.

The protocol was constructed with a known flaw for expository purposes, and as a result the secret is exposed due to an authentication failure. The protocol does not prevent Alice from using a compromised key $b'$, so that Mallory (M) can perform this man-in-the-middle attack:

$$A \rightarrow M : \{ \{s\}_{a^{-1}} \}_{b'}$$
$$M \rightarrow B : \{ \{s\}_{a^{-1}} \}_b$$
$$B \rightarrow E : \{d\}_s$$

The protocol fails to provide a means for Bob to ensure the original message was encrypted using his key. The authentication failure is avoided with this variation of the protocol:

$$A \rightarrow B : \{ \{s, b\}_{a^{-1}} \}_b$$
$$B \rightarrow A : \{d\}_s \quad (2)$$

In strand space Theory, a strand is a linearly ordered sequence of events $e_0 \Rightarrow \cdots \Rightarrow e_{n-1}$, and an event is either a message transmission $\bullet \rightarrow$ or a reception $\bullet \leftarrow$. In CPSA, adversarial behavior is not explicitly represented, so strands always represent regular behavior.

Regular behavior is constrained by a set of roles that make up the protocol. In this protocol, Alice’s behaviors must be compatible with an initiator role, and Bob’s behaviors follow a responder role.

$$init \quad \bullet \rightarrow \{ \{s\}_{a^{-1}} \}_b \quad \{ \{s\}_{a^{-1}} \}_b \rightarrow \bullet$$
$$\bullet \leftarrow \{d\}_s \quad \{d\}_s \leftarrow \bullet \quad (3)$$

The important authentication goal from Bob’s perspective is that if an instance of a responder role runs to completion, there must have been an instance of the initiator role that transmitted its first message. Furthermore, assuming the symmetric key is freshly generated, and the private keys are uncompromised, the two strands agree on keys used for signing and encryption.
A CPSA analysis of the authentication goal requires two inputs, a specification of the roles that make up the protocol, as in Eq. 3, and a question about runs of the protocol. The question in this case is the hypothesis of Eq. 4 that an instance of the responder role ran to completion. In these diagrams, a strand instantiated from a role is distinguished from a role by placing messages above communication arrows, and \( \succ \) is used to assert an event occurred after another.

\[
\begin{align*}
\text{resp} & \not\rightarrow \{ d \}_{s} \quad \text{implies} \quad \text{init} \\
\langle s,a-1 \rangle & \not\rightarrow \{ d \}_{s}
\end{align*}
\]

CPSA produces the conclusion in Eq. 4 that an instance of the initiator role must have transmitted its first message, but it does not conclude that the strands agree on the key used for the outer encryption. When CPSA is run using the amended protocol in Eq. 2, the strands agree on the key, and the authentication goal is achieved.

The contribution of this paper is a method of formalizing security goals and the results of a CPSA analysis in first-order logic such that whenever a CPSA analysis demonstrates that a security goal is achieved, the logical sentence associated with the security goal will be deducible from the shape analysis sentence with the relevant CPSA analysis. The sentences associated with this example are presented in Section 4.

**Some Related Work.** This paper is the result of implementing security goals as described in Guttman [6]. The original motivation for extracting shape analysis sentences rather than following the procedure in [6] was ease of implementation. With shape analysis sentences, most of the work is performed by a post-processing stage, and there were only a few changes made to the core CPSA program. Only later it was realized the sense in which shape analysis sentences completely characterize a shape analysis.

The Scyther tool [2] integrates security goal verification with its core protocol analysis algorithm. Security goals are easy to state as long as they can be expressed using a predefined vocabulary, however, there is no sense in which Scyther goals characterize an analysis.

The Protocol Composition Logic [3] provides a contrasting approach to specifying security goals. It extends strand spaces by adding an operational
Sorts: $\top, A, S, D$

Subsorts: $A < \top, S < \top, D < \top$

Operations: $(\cdot \cdot) : \top \times \top \rightarrow \top$ Pairing

$\{\cdot\}((\cdot)) : \top \times A \rightarrow \top$ Asymmetric encryption

$\{\cdot\}((\cdot)) : \top \times S \rightarrow \top$ Symmetric encryption

$(\cdot)^{-1} : A \rightarrow A$ Asymmetric key inverse

$(\cdot)^{-1} : S \rightarrow S$ Symmetric key inverse

Equations: $(x^{-1})^{-1} = x$ for $x : A$

$y^{-1} = y$ for $y : S$

Figure 1: Simple Crypto Algebra Signature

semantics as a small set of reduction rules, and a run of a protocol is a sequence of reduction steps derived from an initial configuration. The logic is a temporal logic interpreted over runs. The logic is more expressive than what is described within this paper at the cost of added complexity.

Structure of this Paper. Section 2 describes the formalism on which cpsa is built, Section 3 presents the logic built upon that formalism, and Section 4 displays the example above in full detail.

Notation. A finite sequence is a function from an initial segment of the natural numbers. The length of a sequence $X$ is $|X|$, and sequence $X = \langle X(0), \ldots, X(n - 1) \rangle$ for $n = |X|$. If $S$ is a set, then $S^*$ is the set of finite sequences over $S$, and $S^+$ is the non-empty finite sequences over $S$. The prefix of sequence $X$ of length $n$ is $X \uparrow n$.

2 Message Algebras and Homomorphisms

The two details of cpsa’s formalism that dictate changes to the logic of security goals are the fact that in cpsa, a message algebra is an order-sorted quotient term algebra and homomorphisms are strand-oriented, not node-oriented. The issues surrounding homomorphisms will be described later.

An order-sorted algebra is a generalization of a many-sorted algebra in which sorts may be partially ordered [5]. The carrier sets associated with ordered sorts are related by the subset relation.
Figure 1 shows the simplification of the CPSA message algebra signature used by the examples in this paper. Sort $\top$ is the sort of all messages. Messages of sort $A$ (asymmetric keys), sort $S$ (symmetric keys), and sort $D$ (data) are called atoms. Messages are generated from the atoms using encryption $\{|\cdot|\}$ and pairing $(\cdot, \cdot)$, where the comma operation is right associative and parentheses are omitted when the context permits.

Each variable $x$ in an order-sorted term has a unique sort $S$. The declaration of $x$ is $x : S$. The set of variables that occur in term $t$ is $\operatorname{Vars}(t)$.

The quotient term algebra generated by declarations $X$ over the signature in Figure 1 is written $\mathfrak{A}_X$. It is the carrier set of sort $\top$. The canonical representative of each member of $\mathfrak{A}_X$ is the term with the fewest occurrences of the $(\cdot)^{-1}$ operation. Unification and matching can be implemented in such a way that only canonical terms are considered [10, Appendix B].

A message $t_0$ is carried by $t_1$, written $t_0 \sqsubseteq t_1$ if $t_0$ can be derived from $t_1$ given the right set of keys, that is $\sqsubseteq$ is the smallest reflexive, transitive relation such that $t_0 \sqsubseteq t_0$, $t_0 \sqsubseteq (t_0, t_1)$, $t_1 \sqsubseteq (t_0, t_1)$, and $t_0 \sqsubseteq \{|t_0|\}$. 

The use of a message algebra that is order-sorted dictates that the logic used to express the characteristic sentence associated with a shape analysis is also order-sorted. Furthermore, the signature for the logic must inherit the sorts and subsort relations from the message algebra.

Implementation-Oriented Strand Spaces. A run of a protocol is viewed as an exchange of messages by a finite set of local sessions of the protocol. Each local session is called a strand [12]. The behavior of a strand, its trace, is a non-empty sequence of messaging events. An event is either a message transmission or a reception. Outbound message $t \in \mathfrak{A}_X$ is written as $+t$, and inbound message $t$ is written as $-t$. A message originates in a trace if it is carried by some event and the first event in which it is carried is outbound.

A strand space $\Theta_X$ is a finite map from a set of strands to their traces. CPSA represents a set of strands as an initial segment of the natural numbers, therefore, a strand space is a sequence of traces. The nodes of a strand space are $\operatorname{nodes}(\Theta_X) = \{(s, i) \mid s \in \operatorname{Dom}(\Theta_X), 0 \leq i < |\Theta_X(s)|\}$. The event at node $n = (s, i)$ is $\operatorname{evt}_\Theta(s, i) = \Theta(s)(i)$.

In a strand space, a message that originates in exactly one trace is uniquely originating, and represents a freshly chosen value. A message that originates nowhere and is never used by the adversary to decrypt or encrypt a message is non-originating, and represents an uncompromised key.
A protocol $P$ is a finite set of traces, which are the roles of the protocol. Strand $s \in \text{Dom}(\Theta_X)$ is an elaboration of role $r \in P$ if $\Theta_X(s)$ is a prefix of the result of applying some substitution $\sigma$ to $r$. An example of a protocol is in Eq. (6) in Section 4.

Skeletons. A skeleton represents all or part of the regular portion of an execution. A skeleton contains a strand space, a partial ordering of its nodes, assumptions about uncompromised keys and freshly generated atoms, and role associations.

A skeleton $k = k_X(rl, P, \Theta_X, \prec, N, U)$, where $rl: \text{Dom}(\Theta_X) \to P$ is a role map, $\prec$ is a strict partial ordering of the nodes, $N$ is a set of atoms, none of which originate in a trace in $\Theta_X$, and $U$ is a set of atoms, all of which originate in no more than one trace in $\Theta_X$. In addition, $\prec$ must order the node for each event that receives a uniquely originating atom after the node of its transmission, so as to model the idea that the atom represents a value freshly generated when it is transmitted.

The above definition of a skeleton is useful for defining the semantics of shape analysis sentences, but it does not reflect the syntax used by CPSA. In CPSA syntax, the trace and the role associated with a strand is specified by an instance. An instance is of the form $i(r, h, \sigma)$, where $r$ is a role, $h$ specifies the length of a trace instantiated from the role, and $\sigma$ specifies how to instantiate the variables in the role to obtain the trace. Thus the trace associated with $i(r, h, \sigma)$ is $\sigma \circ r \upharpoonright h$, the prefix of length $h$ that results from applying $\sigma$ to $r$.

In the CPSA syntax, the role map and sequence of traces are replaced by a sequence of instances. So for skeleton $k_X(rl, P, \Theta_X, \prec, N, U)$, the CPSA syntax is $k_X(P, I, \prec, N, U)$, where for each $s \in \text{Dom}(\Theta_X)$, $I(s) = i(r, h, \sigma)$, $r = rl(s)$, and the trace of $i(r, h, \sigma)$ is $\Theta_X(s)$.

Two examples of skeletons are displayed in Figure 2 in Section 4.

Homomorphisms. Let $k_0 = k_X(rl_0, P, \Theta_0, \prec_0, N_0, U_0)$ and $k_1 = k_Y(rl_1, P, \Theta_1, \prec_1, N_1, U_1)$ be skeletons. There is a skeleton homomorphism $(\phi, \sigma): k_0 \mapsto k_1$ if $\phi$ and $\sigma$ are maps with the following properties:

1. $\phi$ maps strands of $k_0$ into those of $k_1$, and nodes as $\phi((s, i)) = (\phi(s), i)$, that is $\phi$ is in $\text{Dom}(\Theta_0) \rightarrow \text{Dom}(\Theta_1)$;

2. $\sigma: \mathfrak{A}_X \rightarrow \mathfrak{A}_Y$ is a message algebra homomorphism;
3. \( n \in \text{nodes}(\Theta_0) \) implies \( \sigma(evt_{\Theta_0}(n)) = evt_{\Theta_1}(\phi(n)) \);

4. \( n_0 \prec_0 n_1 \) implies \( \phi(n_0) \prec_1 \phi(n_1) \);

5. \( \sigma(N_0) \subseteq N_1 \);

6. \( \sigma(U_0) \subseteq U_1 \);

7. \( t \in U_0 \) implies \( \phi(O_{k_0}(t)) \subseteq O_{k_1}(\sigma(t)) \);

where \( O_k(t) \) is the set of nodes of events at which \( t \) originates. Item 7 says the node at which an atom declared to be uniquely originating is preserved by homomorphisms. Note that \( \phi \) is a strand mapping, not a node mapping as in [6].

3 Shape Analysis Sentences

Given the definitions in the previous section, the language \( \mathcal{L}(P) \) used for shape analysis sentences is quite constrained. The signature for terms extends the one used for the underlying message algebra with a sort \( N \), the sort of natural numbers, and two new operations, constant \( \text{zero} : N \), and the successor function \( \text{succ} : N \rightarrow N \). The text uses the usual numerals for natural numbers. Variables of this sort will denote strands.

Security goals make use of protocol specific and protocol independent predicates. For each role \( r \in P \), there is a protocol specific binary strand length predicate \( P[r] : N \times N \). For each variable \( x : S \) that occurs in \( r \), there is a protocol specific binary strand parameter predicate \( P[r,x] : N \times S \). The protocol independent predicate of arity four is \( \text{prec} : N \times N \times N \times N \). The protocol independent unary predicates are \( \text{non} : B \) and \( \text{uniq} : B \) for each atomic sort \( B \in \{A, S, D\} \), and the protocol independent ternary predicates are \( \text{orig} : B \times N \times N \). The predicate \( \text{false} \) has arity zero and, of course, equality is binary.

We define \( \mathcal{K}(k) = (Y, \Phi) \), where \( \Phi \) is \( k \)'s skeleton formula, and \( Y \) is the formula's declarations. Using the CPSA skeleton syntax presented in Section 2, let \( k = k_X(P,I,\prec,N,U) \). The declarations \( Y \) is \( X \) augmented with a fresh variable \( z_s : N \) for each strand \( s \in \text{Dom}(I) \). Let \( \prec^- \) be the transitive reduction of \( \prec \). The skeleton formula \( \Phi \) of \( k \) is a conjunction of atomic formulas composed as follows.

9
For each $s \in \text{Dom}(I)$, let $I(s) = i(r, h, \sigma)$. Assert $P[r](z_s, h)$. For each variable $x \in Vars(r \uparrow h)$ and term $t = \sigma(x)$, assert $P[r, x](z_s, t)$.

For each $(s, i) \prec (s', i')$ with $s \neq s'$, assert $\text{prec}(z_s, i, z_{s'}, i')$.

For each $t \in N$, assert $\text{non}(t)$.

For each $t \in U$, assert $\text{uniq}(t)$.

For each $t \in U$ and $(s, i) \in O_k(t)$, assert $\text{orig}(t, z_s, i)$.

Given a set of homomorphisms $\delta_i: k_0 \mapsto k_i$, its shape analysis sentence is

$$S(\delta_i: k_0 \mapsto k_i) = \forall X_0(\Phi_0 \supset \bigvee_i \exists X_i(\Delta_i \land \Phi_i)), \quad (5)$$

where $\mathcal{K}(k_0) = (X_0, \Phi_0)$. The same procedure produces $X_i$ and $\Phi_i$ for shape $k_i$ with one proviso—the variables in $X_i$ that also occur in $X_0$ must be renamed to avoid trouble while encoding the structure preserving maps $\delta_i$.

The structure preserving maps $\delta_i = (\phi_i, \sigma_i)$ are encoded in $\Delta_i$ by a conjunction of equalities. Map $\sigma_i$ is coded as equalities between a message algebra variable in the domain of $\sigma_i$ and the term it maps to. Map $\phi_i$ is coded as equalities between strand variables in $\Phi_0$ and strand variables in $\Phi_i$. Let $Z_0$ be the sequence of strand variables freshly generated for $k_0$, and $Z_i$ be the ones generated for $k_i$. The strand mapping part of $\Delta_i$ is

$$\bigwedge_{j \in \text{Dom}(\Theta)} Z_0(j) = Z_i(\phi_i(j)).$$

An example shape analysis sentence is displayed in Figure 3. A strand length predicate $P[r](z, h)$ is written $r(z, h)$ with the protocol left implicit, and similarly for the strand parameter predicates.

**Semantics of Skeleton Formulas.** Let $k = k_X(rl, P, \Theta, \prec, N, U)$. The universe of discourse is $\mathfrak{D} = N \cup \mathfrak{A}_X$. When formula $\Phi$ is satisfied in skeleton $k$ with variable assignment $\alpha: Y \to \mathfrak{D}$, we write $k, \alpha \models \Phi$. We write $\bar{\alpha}$ when $\alpha$ is extended to terms in the obvious way. When sentence $\Sigma$ is satisfied in skeleton $k$, we write $k \models \Sigma$.

For each strand length predicate $P[r]$, $k, \alpha \models P[r](y, h)$ iff $\alpha(y) \in \mathbb{N}$, $\alpha(h) \in \mathbb{N}$, and with $s = \alpha(y)$ and $i = \alpha(h)$,

1. $s \in \text{Dom}(\Theta)$,
2. $1 \leq i \leq |\Theta(s)|$, and
3. $\Theta(s) \uparrow i = \sigma \circ r \uparrow i$ for some $\sigma$.

In an interpretation, $rl(s)$ need not be $r$. The events that make up a strand’s trace is all that matters.

For each strand parameter predicate $P[r, x], k, \alpha \models P[r, x](y, t) \text{ iff } \alpha(y) \in \mathbb{N}, \tilde{\alpha}(x) \in \mathfrak{A}$, and when $s = \alpha(y)$ and $i$ is the index of the first event in $r$ in which $x$ occurs,

1. $s \in \text{Dom}(\Theta)$, and
2. $\Theta(s) \uparrow i + 1 = \sigma \circ r \uparrow i + 1$ for some $\sigma$ with $\sigma(x) = \tilde{\alpha}(t)$.

The interpretation of the protocol independent predicates is straightforward.

- $k, \alpha \models \text{prec}(w, x, y, z)$ iff $(\alpha(w), \alpha(x)) \prec (\alpha(y), \alpha(z))$.
- $k, \alpha \models \text{non}(t)$ iff $\tilde{\alpha}(t) \in \mathbb{N}$.
- $k, \alpha \models \text{uniq}(t)$ iff $\tilde{\alpha}(t) \in \mathfrak{U}$.
- $k, \alpha \models \text{orig}(t, y, z)$ iff $\tilde{\alpha}(t) \in \mathfrak{U}$ and $(\alpha(y), \alpha(z)) \in \mathcal{O}_k(\tilde{\alpha}(t))$.
- $k, \alpha \models y = z$ iff $\tilde{\alpha}(y) = \tilde{\alpha}(z)$.
- $k, \alpha \not\models \text{false}$.

**Theorem 1.** Let $K(k_0) = (X, \Phi)$ and $\Sigma = \exists X \Phi$. Sentence $\Sigma$ is satisfied in $k$ iff there is a homomorphism from $k_0$ to $k$, i.e. $k \models \Sigma \text{ iff } \exists \delta \vDash \delta : k_0 \mapsto k$.

This theorem corrects the first of the two main results from [6], as that paper omits the $\text{orig}$ predicate. A later paper includes the $\text{orig}$ predicate [7], using the symbol $\text{UnqAt}$.

**Proof.** For the forward direction, assume $\alpha$ is a variable assignment for the variables in $X$ such that $k, \alpha \models \Phi$, and let $Z$ be the sequence of strand variables constructed while generating $\Phi$ from $k_0$. Then the pair of maps $\delta = (\alpha \circ Z, \alpha)$ demonstrate a homomorphism from $k_0$ to $k$, i.e. each item in the definition of a skeleton homomorphism in Section 2 is satisfied.

For the reverse direction, assume maps $\delta = (\phi, \sigma)$ are such that $\delta : k_0 \mapsto k$. Then the desired variable assignment is

$$\alpha(x) = \begin{cases} \phi(Z^{-1}(x)) & x \in \text{Ran}(Z) \\ \sigma(x) & x \in \text{Dom}(\sigma). \end{cases}$$
Deducing Security Goals. A shape analysis $\delta_i : k_0 \mapsto k_i$ is complete if for each realized skeleton $k$, $\delta : k_0 \mapsto k$ iff $\exists i, \delta' : k_i \mapsto k$. There is an ongoing effort to show that whenever CPSA terminates it produces a complete shape analysis, however, preliminary analysis suggests that with the exception of specially constructed, artificial protocols, CPSA’s output is complete. See Appendix A for an example of a troublesome artificial protocol.

The next theorem captures the sense in which a shape analysis sentence characterizes a complete shape analysis.

**Theorem 2.** Let $\delta_i : k_0 \mapsto k_i$ be a complete shape analysis. Then the shape analysis sentence $\Sigma = S(\delta_i : k_0 \mapsto k_i)$ is satisfied in all realized skeletons $k$, i.e. $k \models \Sigma$.

*Proof.* Shapes are minimal among realized skeletons, so there is no realized skeleton in the image of $k$ that is not in the image of one of the shapes. Therefore, by Theorem 1, the negation of the hypothesis of the implication is satisfied in all realized skeletons that are not in the image of $k_0$, and the disjunction is satisfied in the remaining realized skeletons. \hfill $\blacksquare$

Let $\Sigma$ be the shape analysis sentence of a complete shape analysis and $\Psi$ be a security goal. If $\Sigma \supset \Psi$ is a theorem in order-sorted first-order logic, then $\Psi$ is satisfied in all realized skeletons and its protocol achieves this goal.

Since $\prec$ is transitive, transitivity of $\text{prec}$ can also be used to prove a protocol achieves a goal. That is,

$$\text{prec}(x, i, y, j) \land \text{prec}(y, j, z, k) \supset \text{prec}(x, i, z, k),$$

Furthermore, $\text{prec}(x, i, y, j)$ when $x = y$ and $i < j$.

4 Detailed Example

The simplified version of the Denning-Sacco key distribution protocol [4] due to Bruno Blanchet is now revisited.

$A \rightarrow B : \{\{s\}_{a^{-1}}\}_{b}$

$B \rightarrow A : \{d\}_s$

Symmetric key $s$ is freshly generated, asymmetric keys $a^{-1}$ and $b^{-1}$ are uncompromised, and the goal of the protocol is to keep data $d$ secret. This
\( k_0 = k_X(\{\text{init}(a_0, b_0, s_0, d_0), \text{resp}(a_1, b_1, s_1, d_1)\}), \)
\[ \langle i(\text{resp}, 2, \{a_1 \mapsto a, b_1 \mapsto b, s_1 \mapsto s, d_1 \mapsto d\}), \emptyset, \{a^{-1}, b^{-1}\}, \{s\}\rangle \]

\( k_1 = k_Y(\{\text{init}(a_0, b_0, s_0, d_0), \text{resp}(a_1, b_1, s_1, d_1)\}), \)
\[ \langle i(\text{resp}, 2, \{a_1 \mapsto a, b_1 \mapsto b, s_1 \mapsto s, d_1 \mapsto d\}), \]
\[ i(\text{init}, 1, \{a_0 \mapsto a, b_0 \mapsto b', s_0 \mapsto s\}) \]
\[ \{(1, 0) \prec (0, 0)\}, \]
\[ \{a^{-1}, b^{-1}\}, \]
\[ \{s\}\rangle \]

where \( X = a, b : A, s : S, d : D \)

\( \delta_1 = (\langle 0 \rangle, \{a \mapsto a, b \mapsto b', s \mapsto s, d \mapsto d\}) \)

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The protocol was constructed with a known flaw for expository purposes, and as a result the secret is exposed due to an authentication failure. The desired authentication goal is:

\[
\forall a, b : A, s : S, d : D, z_0 : N( \\
\text{resp}(z_0, 2) \land \text{resp}_a(z_0, a) \land \text{resp}_b(z_0, b) \land \text{resp}_s(z_0, s) \land \text{resp}_d(z_0, d) \land \\
\text{non}(a^{-1}) \land \text{non}(b^{-1}) \land \text{uniq}(s) \supset \exists z_1 : N(\text{init}(z_1, 1) \land \text{init}_b(z_1, b)))
\]

that is, when the responder (\( B \)) runs to completion, there is an initiator (\( A \)) that is using \( b \) for the encryption of its initial message.

To investigate this goal, we ask CPSA to find out what other regular behaviors must occur when a responder runs to completion by giving CPSA skeleton \( k_0 \) in Figure 2. CPSA produces shape \( k_1 \) that shows that an initiator must run, but it need not use the same key to encrypt its first message. The shape analysis sentence for this scenario is displayed in Figure 3. Needless to say, the authentication goal cannot be deduced from this sentence due to...
∀a₀, b₀ : A, s₀ : S, d₀ : D, z₀ : N(
resp(z₀, 2) ∧ resp_a(z₀, a₀) ∧ resp_b(z₀, b₀) ∧ resp_s(z₀, s₀) ∧ resp_d(z₀, d₀) ∧
non(a₀⁻¹) ∧ non(b₀⁻¹) ∧ uniq(s₀)
⊃
∃a₁, b₁, b₂ : A, s₁ : S, d₁ : D, z₁, z₂ : N(
  z₀ = z₁ ∧ a₀ = a₁ ∧ b₀ = b₁ ∧ s₀ = s₁ ∧ d₀ = d₁ ∧ resp(z₁, 2) ∧
  resp_a(z₁, a₁) ∧ resp_b(z₁, b₁) ∧ resp_s(z₁, s₁) ∧ resp_d(z₁, d₁) ∧
  init(z₂, 1) ∧ init_a(z₂, a₁) ∧ init_b(z₂, b₁) ∧ init_s(z₂, s₁) ∧ orig(s₁, z₂, 0) ∧
  prec(z₂, 0, z₁, 0) ∧ non(a⁻¹₁) ∧ non(b⁻¹₁) ∧ uniq(s₁))
)

Figure 3: Shape Analysis Sentence for Blanchet’s Protocol

the man-in-the-middle attack. If one repeats the analysis using the protocol in Eq. 2, the generated shape analysis sentence can be used to deduce the authentication goal.

5 Conclusion

This paper presented a method for extracting a sentence that completely characterizes a run of CPSA and showed that logical deduction can then be used to determine if a security goal is satisfied. To ensure the fidelity of the translation between CPSA output and a shape analysis sentence, an order-sorted first-order logic is employed. Furthermore, the first-order language used for formulas is dictated by the CPSA syntax for skeletons and the formalization of homomorphisms used by CPSA.

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## A Artificial Protocol

This section presents an example of a protocol that causes cpsa to fail to produce a complete shape analysis.

\[
\text{init}(a: A, d: D) = \langle +\{d\}_a, -d \rangle \\
\text{resp}(x: \top, d: D) = \langle -x, +d \rangle
\]  

(7)

The initiator in the protocol specifies half of a common authentication pattern. Assuming nonce \(d\) is freshly generated, and key \(a^{-1}\) is uncompromised, an execution of the protocol in which an instance of the initiator role runs to completion must include other regular behavior by a strand that possesses the decryption key \(a^{-1}\).

It’s the responder role that is artificial. Its first event is the reception of a message of any sort, and then it transmits a message of sort data. There are many ways in which an instance of the responder role can serve as the other half of the authentication pattern, such as:

\[
\begin{align*}
\text{init} &\quad \text{resp} \\
\text{init} &\quad \text{resp}
\end{align*}
\]  

(8)

Yet consider an operational interpretation of the responder strand in Eq. (8). The role states that it first receives a message without knowing its structure, but the strand interprets that message as something it can decrypt and extracts the nonce. Formalizations based on an operation semantics, such as what is used for the Protocol Composition Logic [3], exclude the executions in Eq. (8) but there in nothing in strand space theory that prohibits those executions.
B  Node-Oriented Shape Analysis Sentences

The language $\mathcal{L}(P)$ used for node-oriented shape analysis sentences more in tune with the language described in [7]. The signature for terms extends the one used for the underlying message algebra with a sort $N$, the sort for nodes.

Security goals make use of protocol specific and protocol independent predicates. For each role $r \in P$ and $i < |r|$, there is a protocol specific unary position predicate $P[r,i] : N$. For each role $r \in P$ and variable $x : S$ that occurs in $r$, there is a protocol specific binary parameter predicate $P[r,x] : N \times S$. The protocol independent unary predicates are non : $B$ and uniq : $B$ for each atomic sort $B \in \{A,S,D\}$. The remaining protocol independent predicates are binary, and are orig : $B \times N$, sprec : $N \times N$, prec : $N \times N$, and equality.

Soon we define $K(k) = (Y, \Phi)$, where $\Phi$ is $k$’s skeleton formula, and $Y$ is the formula’s declarations, but first we define the relevant nodes of a skeleton $N$. Let $k = k_X(P, I, \prec, \nu, \upsilon)$ and let $\prec^-$ be the transitive reduction of $\prec$. Recall that $\Theta_X$ is the strand space defined by $I$. The relevant nodes of $k$ are $N = N_s \cup N_\prec \cup N_\upsilon$, where

$$N_s = \{(s, i) \mid s \in \text{Dom}(\Theta_X) \land i = |\Theta_X(s)| - 1\}$$

$$N_\prec = \{(s, i) \mid (s', i') \in \text{nodes}(\Theta_X) \land s \neq s' \land ((s, i) \prec^- (s', i') \lor (s', i') \prec^- (s, i))\}$$

$$N_\upsilon = \{(s, i) \mid t \in \upsilon, (s, i) \in \mathcal{O}_k(t)\}$$

For $K(k) = (Y, \Phi)$, the declarations $Y$ is $X$ augmented with a fresh variable of sort $N$ for each node in $N$, and let $v(n)$ be the variable associated with node $n$.

The formula $\Phi$ is a conjunction of atomic formulas composed as follows.

- For each $(s, i) \in N$, assert $P[r,i](v(s,i))$, where $I(s) = i(r, h, \sigma)$.
- For each $s \in \text{Dom}(I)$, let $I(s) = i(r, h, \sigma)$. For each variable $x \in \text{Vars}(r \uparrow h)$ and term $t = \sigma(x)$, assert $P[r,x](v(s,h-1), t)$.
- For each $(s, i), (s, i') \in N$ such that $i < i'$, assert $\text{sprec}(v(s,i), v(s,i'))$.
- For each $(s, i) \prec^- (s', i')$ such that $s \neq s'$, assert $\text{prec}(v(s,i), v(s',i'))$.
- For each $t \in \upsilon$, assert $\text{non}(t)$.
For each $t \in \nu$, assert $\text{uniq}(t)$.

For each $t \in \nu$ and node $n$ such that $n \in \mathcal{O}_k(t)$, assert $\text{orig}(t, v(n))$.

Given a set of homomorphisms $\delta_i : k_0 \leftrightarrow k_i$, its shape analysis sentence is

$$S(\delta_i : k_0 \leftrightarrow k_i) = \forall X_0 \Phi_0 \supset \bigvee_i \exists X_i \Delta_i \land \Phi_i,$$

where $\mathcal{K}(k_0) = (X_0, \Phi_0)$. The same procedure produces $X_i$ and $\Phi_i$ for shape $k_i$ with one proviso—the variables in $X_i$ that also occur in $X_0$ must be renamed to avoid trouble while encoding the structure preserving maps $\delta_i$.

The structure preserving maps $\delta_i = (\phi_i, \sigma_i)$ are encoded in $\Delta_i$ by a conjunction of equalities. Map $\sigma_i$ is coded as equalities between a message algebra variable in the domain of $\sigma_i$ and the term it maps to. Map $\phi_i$ is coded as equalities between node variables in $\Phi_0$ and node variables in $\Phi_i$. Let $v_0$ be the node variables freshly generated for $k_0$, and $v_i$ be the ones generated for $k_i$. The strand mapping part of $\Delta_i$ is

$$\bigwedge_{(s,j) \in \text{Dom}(v_0)} v_0(s,j) = v_i(\phi_i(s), j).$$

**Semantics of Skeleton Formulas.** Let $k = k_X(P, I, \prec, \nu, v)$. The universe of discourse is $\mathcal{D} = (\mathbb{N} \times \mathbb{N}) \cup \mathfrak{A}_X$. When formula $\Phi$ is satisfied in skeleton $k$ with variable assignment $\alpha : Y \rightarrow \mathcal{D}$, we write $k, \alpha \models \Phi$. We write $\bar{\alpha}$ when $\alpha$ is extended to terms in the obvious way. When sentence $\Gamma$ is satisfied in skeleton $k$, we write $k \models \Gamma$.

- $k, \alpha \models P[r, i](y)$ iff $\alpha(y) \in \text{nodes}(\Theta_X)$, $\alpha(y) = (s, i)$, and for some $\sigma$,

$$\Theta_X(s) \uparrow i + 1 = \sigma \circ r \uparrow i + 1.$$

- $k, \alpha \models P[r, x](y, t)$ iff $\alpha(y) = (s, i) \in \text{nodes}(\Theta_X)$, $\bar{\alpha}(t) \in \mathfrak{A}_X$, $x$ occurs in $r \uparrow i + 1$, and for some $\sigma$ with $\sigma(x) = \bar{\alpha}(t)$,

$$\Theta_X(s) \uparrow i + 1 = \sigma \circ r \uparrow i + 1.$$

The interpretation of the protocol independent predicates is straightforward.
\[ \forall a_0, b_0 : A, s_0 : S, d_0 : D, n_0 : N \]
\[ \text{resp}_1(n_0) \land \text{resp}_a(n_0, a_0) \land \text{resp}_b(n_0, b_0) \]
\[ \land \text{resp}_s(n_0, s_0) \land \text{resp}_d(n_0, d_0) \]
\[ \land \text{non}(a_0^{-1}) \land \text{non}(b_0^{-1}) \land \text{uniq}(s_0) \]
\[ \supset \]
\[ \exists a_1, b_1, b_2 : A, s_1 : S, d_1 : D, n_1, n_2, n_3 : N \]
\[ n_0 = n_1 \land a_0 = a_1 \land b_0 = b_1 \land s_0 = s_1 \land d_0 = d_1 \]
\[ \land \text{resp}_1(n_1) \land \text{resp}_a(n_1, a_1) \land \text{resp}_b(n_1, b_1) \]
\[ \land \text{resp}_s(n_1, s_1) \land \text{resp}_d(n_1, d_1) \land \text{resp}_0(n_2) \]
\[ \land \text{init}_0(n_3) \land \text{init}_a(n_3, a_1) \land \text{init}_b(n_3, b_2) \land \text{init}_s(n_3, s_1) \]
\[ \land \text{orig}(s_1, n_2) \land \text{prec}(n_3, n_2) \land \text{sprec}(n_2, n_1) \]
\[ \land \text{non}(a_1^{-1}) \land \text{non}(b_1^{-1}) \land \text{uniq}(s_1) \]

Figure 4: Node-Oriented Shape Analysis Sentence for Blanchet’s Protocol

- \( k, \alpha \models \text{prec}(y, z) \) iff \( \alpha(y) < \alpha(z) \).
- \( k, \alpha \models \text{sprec}(y, z) \) iff \( \alpha(y) < \alpha(z) \), \( \alpha(y) = (s, i) \), and \( \alpha(z) = (s, i') \).
- \( k, \alpha \models \text{non}(t) \) iff \( \bar{\alpha}(t) \in \nu \).
- \( k, \alpha \models \text{uniq}(y) \) iff \( \bar{\alpha}(t) \in \nu \).
- \( k, \alpha \models \text{orig}(t, y) \) iff \( \bar{\alpha}(t) \in \nu \) and \( \alpha(y) \in \mathcal{O}_k(\bar{\alpha}(t)) \).
- \( k, \alpha \models y = z \) iff \( \bar{\alpha}(y) = \bar{\alpha}(z) \).

The node-oriented shape analysis sentence equivalent to the one in Figure 3 is in Figure 4.

Since \( < \) is transitive, transitivity of \( \text{prec} \) can be used to prove a protocol achieves a goal. That is,
\[ \text{prec}(x, y) \land \text{prec}(y, z) \supset \text{prec}(x, z). \]

Furthermore, \( \text{sprec}(x, y) \supset \text{prec}(x, y) \).