Flow investigation of second grade micropolar nanofluid with porous medium over an exponentially stretching sheet

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Abstract
This article mainly focuses on the influence of heat and mass transportation of micropolar second grade nanofluid toward porous medium of an exponentially stretched surface. The significance of activation energy and viscous dissipation with magnetic effect are taken into deliberated. Furthermore, to analyze the heat and mass transport scrutiny the concentration and thermal slip boundary conditions are assessed on the surface of the sheet. The convenient similarity variables are adopted to transfer the non-linear governing PDEs into the dimensionless ODEs and their corresponding boundary conditions also transformed. The nonlinear coupled ODEs are numerically solved by the usage of BVP4C MATLAB technique. The obtained numerical estimations are displayed graphically to display the significance of the various parameters against the velocity, temperature, microrotation, concentration distributions. It is noticed that larger estimations of micropolar and second grade parameter improves the fluid velocity consequently, while opposite trend is found for the higher estimation of porous medium parameter. Further, it is observed that the skin friction rate is boosted by the increment of \( \varepsilon \) and \( \beta \), whereas opposite trend is noted against the mass transfer rate and heat transfer rate.

Keywords
Second grade micropolar nanofluid, slip boundary conditions, activation energy, porous medium, exponential stretching sheet

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Introduction
The non-Newtonian fluids are such kinds of fluids which do not follow the Newtonian’s law of viscosity. For examples lava, honey, gums, toothpaste, blood, shampoo, and ketchup etc. In these fluids, the fundamental relations for the apparent viscosity are usually more complicated. The apparent viscosity constantly decreases versus the applied shear stresses. Therefore, for non-Newtonian fluids the relation between shear stress and shear rate is non-linear. The mathematical characterizations of a non-Newtonian fluid have to a constitutive equation which governs the rheological fluid properties. These fluids are commonly distributed into three types, that is, the differential type, the rate type, and the integral type. The elementary subclass of the differential fluid is the second order fluid model which describes the viscous and elastic properties with specific
conditions. This type of fluid is used in engineering and industrial procedures for examples, metal spinning, in production of paper, glass blowing, and extrusion of plastic sheets. Mostly researchers are working on the investigation of the non-Newtonian liquids. The examination of the flow of the non-Newtonian liquids on a stretchable sheet has a greater consciousness. Nowadays, the laminar boundary layer flow toward a sheet by virtue of stretching exponentially is a subject of sufficient studies for the sake of its perseverance in the distinct industrial and engineering techniques like the aerodynamic expulsion of plastic, the metallic cooling sheets in a cooling bath, the production of paper and glass fiber, the plastic sheet drawings and films, rubber sheets, and the annealing and thinning of copper wires etc. Initially, Sakiadis\textsuperscript{1} used equations of the boundary layer for axisymmetric two-dimensional flow and Crane\textsuperscript{2} deliberated the flow behavior through a stretching surface. Carragher and Crane\textsuperscript{3} analyzed the mass and heat transportation toward a continuously stretching sheet. Further, Magyari and Keller\textsuperscript{4} examined the stable boundary layers of a fluid flow on a continuously stretched sheet and analyzed the mass and heat transmission numerically and analytically. The flow of second grade fluid with the significance of the viscous dissipation on an exponentially stretchable surface was scrutinized numerically by Khan and Sanjaynanad.\textsuperscript{5} Prabhaker et al.\textsuperscript{6} scrutinized numerically with the consequences of MHD and chemical reaction of the flow behavior of non-Newtonian (Casson fluid) having nanoparticles by virtue of an exponentially stretching surface. Tili\textsuperscript{7} compare the hybrid nanofluid and nano-fluid flow between concentric pipes and discussed the rheological behavior and thermophysical characteristic under the consequence of thermal conductivity. Tili et al.\textsuperscript{8} scrutinized computationally for the flow of Casson fluid under the existence of Joule heating, magnetic field, Souret or Dufort, thermophoresis, and Brownian motion impacts caused by a linearly stretching sheet. Aladabesh et al.\textsuperscript{9} investigated the axisymmetric flow between parallel stretching disks of Casson nanofluid with magnetic field, Brownian motion, thermophoresis, thermal radiation impacts. Nayak et al.\textsuperscript{10} examined the Darcy-Forchheimer flow of nanofluid into the carbon nanotubes with Cattaneo-Christov, non-linear thermal radiation and magnetic field effects, and discussed Entropy optimization. Khan et al.\textsuperscript{11} discussed the Bio-convection model for the convective flow of a fluid having nanoparticles under the consequences of activation energy, magnetic field, thermophoresis, and Brownian motion impacts by virtue of a stretching sheet. A few explorations at the laminar boundary layer flow by virtue of the exponentially stretching surfaces are marked in Refs.\textsuperscript{12–16}

The energy needed for the chemical reaction to come out is familiar as activation energy. In a chemical reaction, the entire atoms or molecules possess either kinetic energy or potential energy. Firstly, the activation energy was disclosed by Svabte Arrhenius (a Swedish scientist). Some compounds and elements respond simultaneously the existence of fixed energy amount. Awad et al.\textsuperscript{17} reported the significance of the chemical reaction and activation energy on the flow of mass and heat through rotating incompressible viscous fluid by virtue of an unsteady flow on a stretchable sheet. Zaib et al.\textsuperscript{18} analyzed the process of heat and mass transmission through the consequences of activation energy, thermophoresis, chemical reaction, and Brownian motion. Majeed et al.\textsuperscript{19} evaluated numerically the significance of the activation energy to the flow and mass and heat transportation in the presence of the chemical reaction along MHD toward an exponentially stretched sheet. Irfan et al.\textsuperscript{20} investigated the significances of activation energy for the mass and heat transportation of unsteady Carreau fluid having nanoparticles with the impacts of thermal radiation, viscous dissipation, and Joule heating toward the stretching sheet. Majeed et al.\textsuperscript{21} used Darcy-Forchheimer model to report the significances of the chemical reaction, activation energy, second order slip, and magnetic field for the viscous fluid flow with a linear stretched sheet. Shah et al.\textsuperscript{22} introduced Casson nanofluid to analyze the mass and heat transportation under the magnetic field, activation energy, joule heating, and thermal radiation effects with the convective boundary conditions. Khan et al.\textsuperscript{23} investigated the influence of activation energy, magnetic field, motile microorganism, and non-linear thermal radiation effect on the flow of micropolar nanofluid toward a permeable rotating disk. Song et al.\textsuperscript{24} scrutinized the heat and mass transportation into the micropolar nanofluid in bioconvection flow with the Darcy law, thermal radiation, and activation energy effects by virtue of rotating disk. Chu et al.\textsuperscript{25} considered the influence of activation energy, thermophoresis, Brownian motion, magnetic field, chemical reaction, and bioconvection effects for the flow of third grade fluid flow with Buongiorno model of nanofluid induced by a stretching surface. Ramzan et al.\textsuperscript{26} scrutinized the heat and mass transfer for the flow of tangent hyperbolic fluid having nano materials under the influence of activation energy, magnetic field, Hall, ion slip, and microorganism effects with in the presence of Cattaneo-Christov heat flux model due to a bidirectional stretchable sheet. Waqas et al.\textsuperscript{27} study the consequences of melting process, non-linear thermal radiation, and activation energy for the Falkner-Skan transient bioconvection flow of Cross fluid having nano materials via of a stretching wedge. Xia et al.\textsuperscript{28} discussed the heat and mass transmission into the flow of Eyring-Powell fluid through permeable medium having nanoparticles under the impact of Bio-convection and microorganism and discussed entropy analysis with a continuously stretching cylinder. Ramesh et al.\textsuperscript{29} analyzed the convective flow of Maxwell fluid with Buongiorno model under the effects of Bioconvection, non-linear thermal radiation, and activation energy over a Riga plate. Moreover details, seen in the Refs.\textsuperscript{30–35}

Micropolar fluids are such type of fluid which can undergo rotation and follow the basic equations of the
deliberated non-Newtonian fluid model with an intimacy of the asymmetrical stress tensor. In fact, such types of fluids characterize the fluids including of arbitrarily acquainted particles suspended in a viscid medium. The fluid model with micropolar perhaps useful to describe the flow of suspension solutions, animal blood, liquid crystals, colloidal fluids, and so forth. Additionally, these fluids are strongly exaggerated by spin inertia, here the deformation of the particles is neglected. Initially Eringen\textsuperscript{36} suggested the theory of the micropolar fluids. Hassanien and Gorla\textsuperscript{37} used the micropolar fluids and scrutinized the mass and heat transmission into a microporous medium toward an exponentially stretchable surface. The solutal and thermal energy equations in the form PDEs has transformed the flow of suspension solutions, animal blood, liquid crystals, colloidal fluids, and so forth. Additionally, these micropolar fluid model with micropolar perhaps useful to describe the concentration and thermal energy transmission past a Riga plate. The physical flow chart and the coordinates.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{flow_chart.pdf}
\caption{The physical flow chart and the coordinates.}
\end{figure}

The motivation of current investigation is to inquiry the heat and mass transport analysis of micropolar second grade nanofluid across the exponentially stretchable sheet with slip effects. The MHD and activation energy effect is also investigated. The main concern of study from the novelty point of view is to discuss the micropolar second grade nanofluid flow with porosity and irregular heat sources/sink effect induced by stretching surface, which are not studied yet in the literature. The obtained flow equation and energy equations in the form PDEs has transformed into coupled ODEs with the support of similarity variables. These coupled ODEs are tackled numerically by the usage of BVP4C MATLAB approach. The consequence of the various emerging parameters is discussed graphically and tabulated data. Moreover, comparison of current investigation is presented with available literature and shows good harmony between two.

\section*{Construction of the problem}

Here, we consider 2D steady, incompressible, boundary layer flow of second grade micropolar nanofluid with the significances of porous medium toward an exponentially stretchable surface. The solutal and thermal energy equation has been modified with the impact of viscous dissipation and activation energy. The concentration and thermal slip effect also consider at the boundary of the sheet. The magnetic field $B_0$ effect applied normal to the fluid flow.

The stretching velocity is $u_w = U_0 e^t$. Where $U_0$ is the reference velocity, with $U_0 > 0$. The $x$-axis is imagined by the side of the sheet and $y$-axis is vertical to it. The physical flow chart and the coordinates are depicted by the Figure 1. The fluid temperature and concentration is denoted by $T$ and $C$ respectively. Further, the wall temperature and concentration is given as $T_w$ and $C_w$ respectively, and far-off the wall they are signified by $T_\infty$ and $C_\infty$ respectively.

Under boundary layer approximation and above-mentioned suppositions, the 2D steady laminar boundary layer flow and energy equations are followed as,

\begin{equation}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\end{equation}

\begin{equation}
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \frac{\mu + k^*}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{k^*}{\rho} \frac{\partial N_1}{\partial y}
+ \frac{\alpha_1}{\rho} \left( \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} \right) + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} u \frac{\mu \phi}{k_4},
\end{equation}

\begin{equation}
u \frac{\partial N_1}{\partial x} + v \frac{\partial N_1}{\partial y} = \left( \frac{j^*}{j \rho} \right) \frac{\partial^2 N_1}{\partial y^2} - \frac{k^*}{j \rho} \left( 2N_1 + \frac{\partial u}{\partial y} \right),
\end{equation}

\begin{equation}
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left( \frac{D_h}{T_w} \frac{\partial C}{\partial y} \right)
+ \frac{\alpha_1}{\rho C_p} \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) \right)
+ \frac{\mu + k^*}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{1}{\rho C_p} \frac{\partial q^m}{\partial y}.
\end{equation}
By using equation (7), the equations (2)–(5) are reduced in the following form,

\[
(1 + K) f''' + \Phi_1' + f f'' - 2 f'^2 + \beta \left( 2 f'' f''' + 5 f f'' - 2 f'^1 \right) \exp \left( -\frac{E}{kT} \right) = 0,
\]

\[
1 + \frac{K}{2}, \Phi_1' + 3 f' \Phi_1' - K (2 \Phi_1 + f') = 0,
\]

\[
\theta^* + P_r N_b \theta^2 - P_r (f \theta' - 2 f' \theta) + P_r N_b \Phi_0' + (A^* f' + B^* f') \exp \left( \frac{\theta}{\eta} \right) = 0,
\]

\[
\phi^* - P_r L_c f' \phi - P_r L_c f' \phi' - N_b \theta^* \exp \left( \frac{\theta}{\eta} \right) = 0,
\]

Related boundary conditions are followed as,

\[
f = 0, f' = 1, \Phi_1 = -nf^*, \theta = 1 + S \theta', \phi = 1 + S \phi', \text{at} \eta = 0,
\]

\[
f' = 0, \Phi_1 = 0, \theta = 0, \phi = 0 \text{as} \eta \to \infty.
\]

Here, the symbols are denoted the second grade fluid parameter, Prandtl number, the parameter magnetic field, the parameter of activation energy, parameter of Brownian motion, porous medium parameter, thermophoresis parameter, micropolar parameter, Lewis number, Eckert number, temperature difference parameter, the reaction rate constant, the thermal slip parameter, and the parameter of concentration slip respectively.

**Physical quantities**

The physical parameters of interest such as Sherwood number, skin friction and Nusselt number are follows as:
C_{fi} = \frac{1}{2} \rho u^2 \frac{x_{q_w}}{k(T-T_w)} , N_{ax} = \frac{xh_w}{D_w(\phi_w-\phi_a)} \quad (14)

\tau_w = \left\{ \begin{array}{l}
\frac{\alpha_t}{\rho} \left[ \frac{\partial^2 u}{\partial \eta^2} - 2 \frac{\partial u}{\partial \eta} \frac{\partial \xi}{\partial \eta} + \frac{\partial^2 u}{\partial \xi \partial \eta} \right] \\
+ \left( \frac{\mu+k^*}{\rho} \right) \frac{\partial u}{\partial \eta} + k^* N_1 \end{array} \right|_{\eta=0} \quad , \quad (15)

q_w = \left. \left( -\frac{\partial T}{\partial \eta} \right) \right|_{\eta=0} , S_{ht} = -\frac{x}{(C_w-C_{\infty})} \left( \frac{\partial C}{\partial \eta} \right) \big|_{\eta=0} \quad . \quad (16)

In the non-dimensional form, the skin friction, Nusselt number, and Sherwood number are follows as,

\begin{align*}
C_{f(\eta=0)} &= \left[ \beta (3 f'(0) f''(0) - f'''(0)) \right] \left( \frac{R_{ex}}{2} \right)^{\frac{1}{2}} , \\
R_{ex}^{\frac{1}{2}} \sqrt{2} N_{ax} &= -\theta'(0) , \\
S_{ht} \sqrt{2} R_{ex}^{\frac{1}{2}} &= -\phi'(0) .
\end{align*}

Where \( R_{ex} \) is Reynolds number.

**Results and discussion**

In a prevailing research article, the two-dimensional second grade micropolar nanofluid in the occurrence of magnetic field and slip conditions past an exponentially stretched sheet is analyzed. The mass and heat transportation inspection are examined with the activation energy and viscous dissipation impacts. The consequences of the different arising parameters on the micropolar distribution, velocity distribution, temperature distribution, and concentration distribution are scrutinized via graphs. The Figure 2(a) displays the increasing behavior of \( f'(\eta) \) by the higher estimations of \( (\beta) \). It is noted that, the velocity distribution degenerates by improving the variations of the 2-grade fluid parameter \( (\beta) \). Physically, the momentum diffusivity diminishes for the stronger variations of the 2-grade fluid parameter. Figure 2(b) illustrates the consequence of micropolar parameter on the sketch of the velocity of fluid profile. For the higher variations of \( K \) the \( f'(\eta) \) of the fluid enhances. Actually, the increasing values of \( (K) \) source to enhance acceleration of the micropolar fluid flow, consequently, rises the fluid velocity. The Figure 2(c) indicates the impact of the \( (M) \) to the \( f'(\eta) \). From the graph it is observed that the \( f'(\eta) \) slows down by the enhancing values of \( (M) \). In fact, with the increasing in the magnetic field, as a result, a resistive force provided by the Lorenz force that resists the motion of the fluid. So, the thickness of the velocity boundary layer reduces caused in the \( f'(\eta) \) declines by an enhancing in values of \( (M) \). Figure 2(d) shows the difference in the \( f'(\eta) \) against the \( (\varepsilon) \) of the medium through which the fluid flows. It is depicted that the \( f'(\eta) \) reduces by higher values of \( (\varepsilon) \). As the values of \( \varepsilon \) increase the medium porosity, that resists the fluid flow, cause to improved deceleration of the flow. The influence of increasing variations of \( (K) \) at the \( \Phi(\eta) \) is displayed in the Figure 3(a). Here depicted that, as parameter \( (K) \) boosts the \( \Phi(\eta) \) also increases. With the higher in the values of \( (n) \) tends to boost the \( \Phi(\eta) \) as seen in the Figure 3(b). The Figure 4(a) shows the \( \theta(\eta) \) for the increasing values of \( (E_c) \). The impact of \( (E_c) \) is to boost in the \( \theta(\eta) \) in the fluid flow region. Physically, with rising the values of \( (E_c) \), in the K.E of the fluid flow rises upshot the \( \theta(\eta) \) distribution increases. The behavior of \( (N_t) \) against \( (\phi(\eta)) \) is shown in the Figure 4(b). It is felt that the \( \theta(\eta) \) enhances by the stronger estimations of \( (N_t) \). Enlarging in \( (N_t) \) undergoes in the direction of the faster arbitrary motion of the nanofluids fluid flow whatever leads an expansion in the thickness of the thermal boundary layer, therefore the \( \theta(\eta) \) rises. An increasing result is perceived for growing values of \( (N_t) \) to the \( \theta(\eta) \) is seen in Figure 4(c). Indeed, in the action of the \( (N_t) \), more excited nanofluid particles close to the surface move off from the warm regions to the colder region in the flow field that leads to expansions in the \( \theta(\eta) \) and the mutual temperature of the entire system arises. The significance of \( (P_r) \) on \( \theta(\eta) \) is allotted in Figure 4(d). Apparently, the \( \theta(\eta) \) diminishes by increasing the estimations of \( (P_r) \). From the description of the Prandtl number, variations of Prandtl number increase means the thermal diffusivity of the liquid deteriorates that leads to reduce in the heat dissipation. Consequently, spreading the thermal boundary layer thickness and the temperature of the nanofluid are both depreciate in the region of the fluid. Figure 4(e) declares the variance in \( \theta(\eta) \) against the thermal slip parameter \( S_t \). Indeed, the thermal and the wall temperature \( \theta(\eta) \), the thermal boundary layer density declines by the enhancing in values of \( (\beta) \). Figure 4(f) depicts the significance of \( (\beta) \) for the \( \theta(\eta) \). For enhancing values of \( (\beta) \), both the thermal boundary layer density and the \( \theta(\eta) \) are abated. Figure 4(g) depicts the influence of the parameter of the space-dependent heat generation/sinks on the temperature sketch. It is detected that for the stronger estimations of \( A' \), the temperature profile decreases by virtue of the thickening of the thermal boundary layer.
Figure 2. (a–d) Behavior of $\beta$, $K$, $M$, and $\varepsilon$ on the velocity distribution respectively.

Figure 3. (a and b) Behavior of $K$ and $n$ on the micropolar distribution respectively.
thickness. Figure 4(h) shows the consequence of parameter of the different dependent $B^*$ on the temperature sketch. It is illustrated that, for stronger variations of $B^*$, the temperature profile decreases. Physically, the energy is loosed for the increasing values of $B^*$. Therefore, the temperature sketch declines. The impacts of $(E_c)$ to the $\phi(\eta)$ is revealed in Figure 5(a). It is cleared that, the higher values of $(E_c)$ increases the $(\phi(\eta))$. The boosting behavior is observed of activation energy on the $(\phi(\eta))$. The influence of $(E)$ to the $(\phi(\eta))$ is detected in Figure 5(b). The $(\phi(\eta))$ boosts for the greater estimations of $(E)$ of the nanofluid flow. Figure 5(c) demonstrates the changing of nanoparticles concentration against the variation in the $(L_e)$ increases upshot the mass transfer rate boosts. Also, the concentration rate at the surface of the sheet enhances. Further, the concentration of the nanofluids to the sheet surface declines with the stronger estimations of $(L_e)$ increases. The signifies of $(N_p)$ on the $(\phi(\eta))$ is expressed in Figure 5(d). It can be identified that for the higher values of $(N_p)$ enhances the concentration boundary layer density is declining. Moreover, the values of the concentration rate to the sheet surface boosts with the enhancing in the values of $(N_p)$ rise. Therefore, the $(−\phi(0))$ which shows the mass transportation gradient on the surface rises by the values of $(N_p)$ increase. In fact, that $(N_p)$ decreases the rate of the mass transportation of the nanofluid, therefore, the mass transportation rate to the surface of the sheet boosts.
The reason for this is that different nanoparticles have different estimations \( N_b \) for the Brownian movement parameter that leads to an improvement in the rate of heat transfer. Figure 5(e) displays the impacts of the \( ( N_t ) \) to the \( \phi(\eta) \). The boundary layer density concentration built up with the rising values of the \( ( N_t ) \). Fact that the \( ( N_t ) \) enhances the rate of the mass transfer of the nanofluid. Figure 5(f) shows the variation in the \( \phi(\eta) \) for larger values of \( ( P_r ) \). The values of \( P_r \) increase the \( \phi(\eta) \). The variation in concentration profile against the concentration slip parameter \( ( S_2 ) \) is revealed in Figure 5(g). By boosting the values of the \( ( S_2 ) \), the \( \phi(\eta) \) decreases. The significance of \( \epsilon \) on the \( \phi(\eta) \) is shown in Figure 5(h). \( \phi(\eta) \) increases as the values of \( \epsilon \) increase. Table 1 authenticated that the variation in the \( -(f'')^0(0) \) for the distinct values of parameters such as

![Figure 5. (a–h) Behavior of \( \epsilon, E, L, N_i, N_t, S, S_2 \), and \( \epsilon \) on the concentration distribution respectively.](image-url)

**Table 1.** Computed estimations of Skin friction coefficient \( C_{f_h} \) for the different variations of \( \beta, K, M, \) and \( \epsilon \) .

| \( \beta \) | \( K \) | \( M \) | \( \epsilon \) | \( C_{f_h} \) |
|---|---|---|---|---|
| 0.1 | | | | 2.27962 |
| 0.2 | | | | 2.43023 |
| 0.3 | | | | 2.6134 |
| 0.1 | 2.27962 |
| 0.2 | 2.39411 |
| 0.3 | 2.50492 |
| 0.1 | 1.70159 |
| 0.2 | 1.75304 |
| 0.3 | 1.8024 |
| 0.1 | 1.4709 |
| 0.2 | 1.53278 |
| 0.3 | 1.59169 |
β, K, M and ε. It can be seen that β, K, M and ε have increasing behavior for the skin friction. Table 2 displays the computed estimations of Nusselt number and Sherwood number for distinct estimations of parameters such as Ec, Le, Nt, Nb, S2, β, E, P, ε. It is detected that from the table, by increasing values of Ec, Nb, β, E, P, and ε the (−θ(0)) declines and by enhancing the variations of Le, Nt, S2, and Pr (−θ'(0)) declines. Along the enhancing estimations of Ec, Nb, S2, and ε, the (−φ(0)) decreases and increases for Le, Nt, E, P, and S1. There is no impact of β on the (−φ(0)). Results of comparison Table 3 have good concurrent with the results of passed published article in Khan et al.48

### Table 2. Computed estimations of the (−θ'(0)) and the (−φ'(0)) for the different values of Ec, Le, Nt, Nb, S2, β, E, P, S1, and ε.

| Ec | Le | Nt | Nb | S2 | β | E | P | S1 | ε | θ'(0) | φ'(0) |
|----|----|----|----|----|----|----|----|----|----|------|------|
| 0.1 |    |    |    |    |    |    |    |    |    | 4.037 | 4.036 |
| 0.2 |    |    |    |    |    |    |    |    |    | 4.017 | 4.006 |
| 0.3 |    |    |    |    |    |    |    |    |    | 3.997 | 3.976 |
| 0.1 |    |    |    |    |    |    |    |    |    | 1.432 | 1.225 |
| 0.2 |    |    |    |    |    |    |    |    |    | 1.598 | 1.399 |
| 0.3 |    |    |    |    |    |    |    |    |    | 1.743 | 1.55  |
| 0.1 |    |    |    |    |    |    |    |    |    | 2.524 | 2.417 |
| 0.2 |    |    |    |    |    |    |    |    |    | 4.038 | 3.238 |
| 0.3 |    |    |    |    |    |    |    |    |    | 5.255 | 3.871 |
| 0.1 |    |    |    |    |    |    |    |    |    | 1.31462 | 1.809 |
| 0.2 |    |    |    |    |    |    |    |    |    | 1.30164 | 1.739 |
| 0.3 |    |    |    |    |    |    |    |    |    | 1.28878 | 1.687 |
| 0.4 |    |    |    |    |    |    |    |    |    | 1.31462 | 1.809 |
| 0.5 |    |    |    |    |    |    |    |    |    | 1.31758 | 1.785 |
| 0.6 |    |    |    |    |    |    |    |    |    | 1.32034 | 1.762 |
| 0.7 |    |    |    |    |    |    |    |    |    | 1.31462 | 1.809 |
| 0.8 |    |    |    |    |    |    |    |    |    | 1.31275 | 1.801 |
| 0.9 |    |    |    |    |    |    |    |    |    | 1.31091 | 1.800 |
| 1.0 |    |    |    |    |    |    |    |    |    | 1.31462 | 1.809 |
| 1.1 |    |    |    |    |    |    |    |    |    | 1.31461 | 1.809 |
| 1.2 |    |    |    |    |    |    |    |    |    | 1.31460 | 1.81  |
| 1.3 |    |    |    |    |    |    |    |    |    | 1.31462 | 1.809 |
| 1.4 |    |    |    |    |    |    |    |    |    | 1.31466 | 1.857 |
| 1.5 |    |    |    |    |    |    |    |    |    | 1.37318 | 1.904 |
| 1.6 |    |    |    |    |    |    |    |    |    | 1.31462 | 1.792 |
| 1.7 |    |    |    |    |    |    |    |    |    | 1.29318 | 1.80  |
| 1.8 |    |    |    |    |    |    |    |    |    | 1.27242 | 1.809 |
| 1.9 |    |    |    |    |    |    |    |    |    | 1.31462 | 1.809 |
| 2.0 |    |    |    |    |    |    |    |    |    | 1.31461 | 1.809 |

### Table 3. Comparison table of θ'(0) for P, Ec, Nt, and Nb.

| P | Ec | Nt | Nb | Nadeem et al.50 | Present |
|---|----|----|----|----------------|---------|
| 0.7 | -0.0535 | -0.0611 |
| 1.0 | -0.1008 | -0.1112 |
| 1.5 | -0.1681 | -0.1590 |
| 0.1 | -0.2001 | -0.1999 |
| 0.2 | -0.1841 | -0.1737 |
| 0.3 | -0.1681 | -0.1509 |
| 0.4 | -0.1658 | -0.1536 |
| 0.5 | -0.1628 | -0.1521 |
| 0.6 | -0.1599 | -0.1428 |
| 0.7 | -0.1570 | -0.1490 |
| 0.8 | -0.1520 | -0.1440 |
| 0.9 | -0.1480 | -0.1395 |

### Concluding remarks

The MHD flow of micropolar second grade nanofluid toward an exponentially stretchable sheet with the activation energy and non-uniform heat source/sink. The heat and mass transmission investigation is discussed with the impact of thermophoresis and Brownian motion. The BVP4C technique by MATLAB is applied to resolve the
coupled nonlinear ODEs. The vital outcomes of the present research study are followed as,

- The velocity of fluid boosts by the increment of second grade parameter due to occurrence of resistive force, whereas opposite trend is noticed for porous medium parameter.
- The linear and angular velocity improves for the larger variations of micropolar parameter.
- The fluid velocity enhances for the stronger estimations of magnetic parameter, but opposite trend is seen against the temperature profile.
- By the increment of thermophoresis parameter, both temperature and concentration improve.
- The temperature profile increase for larger estimation of Brownian motion parameter, while concentration profile reduces.
- Stronger estimations of the concentration and thermal slip decreases the temperature and concentration of fluid consequently.
- The larger estimations of the micropolar and second grade parameter boost the skin friction rate.
- The heat and mass transmission rate boosts for the larger values of the thermophoresis parameter and exhibits the opposite trend for the Brownian motion parameter.

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Appendix

Notations

\((u, v)\) Cartesian components of velocity respectively along \(x\) and \(y\) axes.

\((x, y)\) \(x\) and \(y\) axes

\(\mu\) Coefficient of viscosity

\(k_1\) Vortex viscosity

\(\rho\) Fluid density

\(N^*\) Micro-rotation component

\(\alpha_1\) Material parameter of second grade fluid

\(\sigma\) Electrical conductivity

\(B_0\) Strength of magnetic field

\(K\) Permeability of the porous medium

\(\gamma^*\) Spin gradient viscosity

\(j^*\) Micro inertia density

\(\tau\) Heat capacitance ratio

\(D_B\) Brownian diffusion coefficient

\(T\) Fluid temperature

\(T_{\infty}\) Free stream temperature

\(D_T\) Thermophoretic diffusion coefficient

\(C_p\) Specific heat

\(C\) Concentration component

\(C_\infty\) Ambient concentration

\(k_r\) Chemical reaction rate

\(n\) A constant

\(E_a\) Activation energy coefficient

\(U_0\) Exponential stretching factor

\(m\) Fitted rate constant

\(u_w\) Stretching velocity of the sheet

\(\lambda_1\) Thermal Slip coefficient

\(\lambda_2\) Concentration slip coefficient

\(k\) Microgyration Parameter

\(\beta\) Second grade fluid parameter

\(M\) Magnetic field parameter

\(P\) Porous medium parameter

\(P_r\) Prandtl Number

\(f\) Nondimensional stream function

\(f'\) Nondimensional velocity

\(\theta(\eta)\) Nondimensional temperature

\(h(\eta)\) Dimensionless micropolar

\(h'(\eta)\) Dimensionless micropolar velocity

\(N_b\) Parameter of Brownian motion

\(N_t\) Parameter of thermophoresis

\(E_c\) Eckert number

\(L_e\) Lewis number

\(\phi\) Dimensionless concentration

\(\epsilon_1\) Parameter of reaction rate

\(E\) Parameter of Activation energy

\(S_1\) Parameter of thermal slip

\(C_{fr}\) Concentration slip

\(-\theta'(0)\) Skin friction

\(R_{ex}\) Nusselt number

\(Sh_x\) Reynolds Number

\(Sh_x\) Sherwood number