Large rescaling of the Higgs condensate: theoretical motivations and lattice results

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In the Standard Model the Fermi constant is associated with the vacuum expectation value of the Higgs field, ‘the condensate’, usually believed to be a cutoff-independent quantity. General arguments related to the ‘triviality’ of $\Phi^4$ theory in 4 space-time dimensions suggest, however, a dramatic renormalization effect in the continuum limit that is clearly visible on the relatively large lattices available today. The result can be crucial for the Higgs phenomenology and in any context where spontaneous symmetry breaking is induced through scalar fields.

1. Introduction

To understand the scale dependence of the ‘Higgs condensate’ $\langle \Phi \rangle$ let us define the $\lambda \Phi^4$ theory in the presence of a lattice spacing $a \sim 1/\Lambda$. The basic problem is the relation between the bare “lattice” condensate $v_B(\Lambda) \equiv \langle \Phi_{\text{latt}} \rangle$ and its renormalized physical value $v_R \equiv \langle \Phi \rangle_{\text{phys}}$ in the continuum limit $\Lambda \to \infty$.

In the presence of spontaneous symmetry breaking, there are two basically different definitions: $Z \equiv Z_{\text{prop}}$ from the propagator of the bare shifted ‘Higgs’ field $h_B(x) \equiv \Phi_{\text{latt}}(x) - v_B$

$$G(p^2) \sim \frac{Z_{\text{prop}}}{p^2 + M_h^2}$$

and $Z \equiv Z_\varphi$ where $Z_\varphi$ is the rescaling of $v_B^2$ needed in the effective potential $V_{\text{eff}}(\varphi_B)$

$$\chi^{-1} = \frac{d^2V_{\text{eff}}}{d\varphi_B^2} \bigg|_{\varphi_B = v_B} = \frac{M_h^2}{Z_\varphi}$$

(2)

to match the quadratic shape at its absolute minima with the Higgs mass. The usual assumption $Z_\varphi \sim Z_{\text{prop}}$ is equivalent to require a smooth limit $p \to 0$.

This is not necessarily true in the presence of Bose condensation phenomena \textsuperscript{11} where one can have a very large particle density at zero-momentum that compensates for the vanishing strength $\lambda \sim 1/\ln \Lambda$ of the elementary two-body processes. In this case, one can have trivially free fluctuations so that $Z_{\text{prop}} \to 1$ and $h_B(x) = h_R(x) = h(x)$, but a non-trivial effective potential with a divergent $Z_\varphi \sim 1/\lambda \sim \ln \Lambda_{\text{lat}}$

(11). Therefore, the bare ratio $R_{\text{bare}} = \frac{M_h^2}{v_B} \to 0$ consistently with the rigorous results of Euclidean field theory \textsuperscript{23} but $R_\varphi = \frac{M_h^2}{v_R}$ remains finite and cannot be used to constrain the magnitude of $\Lambda$.

2. The lattice simulation

The one-component $(\lambda \Phi^4)_4$ theory becomes in the Ising limit

$$S = -\kappa \sum_x \sum_{\mu} (\phi(x + \hat{\mu})\phi(x) + \phi(x - \hat{\mu})\phi(x))$$

with $\phi(x) = \sqrt{2}\kappa \phi(x)$ and where $\phi(x)$ takes only the values $+1$ or $-1$.

We performed Monte-Carlo simulations of this Ising action using the Swendsen-Wang cluster algorithm. Lattice observables include: the bare magnetization $v_B = \langle |\Phi| \rangle$ where $\Phi \equiv \sum_x \Phi(x)/L^4$ is the average field for each lattice configuration), the zero-momentum susceptibility $\chi = L^4 \left\langle (|\Phi|^2) - (\langle |\Phi| \rangle)^2 \right\rangle$,

the shifted-field propagator

$$G(p) = \left\langle \sum_x \exp(ipx) (\Phi(x) - v_B)(\Phi(0) - v_B) \right\rangle ,$$

(4)

where $p_\mu = \frac{2\pi}{L} n_\mu$ with $n_\mu$ being a vector with integer-valued components, not all zero.

When approaching the continuum limit, one can compare the lattice data for $G(p)$ to the 2-parameter formula

$$G_{\text{fit}}(p) = \frac{Z_{\text{prop}}}{p^2 + m_{\text{latt}}^2}$$

(5)
where $m_{\text{latt}}$ is the dimensionless lattice mass and $ar{p}_\mu = 2 \sin \frac{p_\mu}{2}$. Actually, if “triviality” is true, there must be a region of momenta where Eq. (5) gives a good description of the lattice data and can be used to define the mass. However, since the determination of the mass is a crucial issue, it is worth to compare the results of the previous method with the determination in terms of “time-slice” variables [3]. To this end let us consider a lattice with 3-dimension $L^3$ and temporal dimension $L_t$ and the two-point correlator $C_1(t, 0; k)$. In this way, parameterizing the correlator $C_1$ in terms of the energy $\omega_k$, the mass can be determined through the lattice dispersion relation [4]

$$m_{\text{TS}}^2 = 2(\cosh \omega_k - 1) - 2 \sum_{\mu=1}^{3} (1 - \cos k_\mu). \quad (6)$$

3. Numerical results: symmetric phase

As a check of our simulations we started our analysis at $\kappa = 0.0740$ in the symmetric phase, where the high-statistics results by Montvay & Weisz [3] are available.

Fig. 1 displays the data for the scalar propagator suitably re-scaled in order to show the very good quality of the fit Eq. (5). The 2-parameter fit gives $m_{\text{latt}} = 0.2141(28)$ and $Z_{\text{prop}} = 0.9682(23)$. The value at zero-momentum is defined as $Z_{\varphi} = m_{\text{latt}}^2 \chi = 0.9702(91)$. Notice the perfect agreement between $Z_{\varphi}$ and $Z_{\text{prop}}$. We measure also the time-slice mass Eq. (6) at several values of the 3-momentum. Our results are in good agreement with the corresponding results of Jansen et al. [5]. We checked that or results for the magnetization and the susceptibility at $\kappa = 0.076$ are in excellent agreement with the corresponding results of Jansen et al. [6]. Typical data for the re-scaled propagator are reported in Fig. 2. Unlike Fig. 1, the fit to Eq. (5), though excellent at higher momenta, does not reproduce the lattice data down to zero-momentum. Therefore, in the broken phase, a meaningful determination of $Z_{\text{prop}}$ and $m_{\text{latt}}$ requires excluding the lowest-momentum points from the fit. The fitted $Z_{\text{prop}}$ is slightly less than one. This fact is attributable to residual interactions since we are not exactly at the continuum limit, so that the theory is not yet completely “trivial.” This explanation is reasonable since we see a tendency for $Z_{\text{prop}}$ to approach unity as we get closer to the continuum limit.

![Figure 1. The lattice data for the re-scaled propagator at $\kappa = 0.0740$ in the symmetric phase. The zero-momentum full point is defined as $Z_{\varphi} = m_{\text{latt}}^2 \chi$. The dashed line indicates the value of $Z_{\text{prop}}$.](image)

4. Numerical results: broken phase

We now choose for $\kappa$ three successive values, $\kappa = 0.076, 0.07512, 0.07504$, lying just above the critical $\kappa_c \simeq 0.0748$ [3]. Thus, we are in the broken phase and approaching the continuum limit where the correlation length $\xi$ becomes much larger than the lattice spacing. To be confident that finite-size effects are sufficiently under control, we used a lattice size, $L^4$, large enough so that $5 \lesssim L/\xi$ [3]. We checked that results for the magnetization and the susceptibility $Z_{\varphi}$ are well under control, we used a lattice size, $L^4$, large enough so that $5 \lesssim L/\xi$ [3]. We checked that results for the magnetization and the susceptibility at $\kappa = 0.076$ are in excellent agreement with the corresponding results of Jansen et al. [6]. Typical data for the re-scaled propagator are reported in Fig. 2. Unlike Fig. 1, the fit to Eq. (5), though excellent at higher momenta, does not reproduce the lattice data down to zero-momentum. Therefore, in the broken phase, a meaningful determination of $Z_{\text{prop}}$ and $m_{\text{latt}}$ requires excluding the lowest-momentum points from the fit. The fitted $Z_{\text{prop}}$ is slightly less than one. This fact is attributable to residual interactions since we are not exactly at the continuum limit, so that the theory is not yet completely “trivial.” This explanation is reasonable since we see a tendency for $Z_{\text{prop}}$ to approach unity as we get closer to the continuum limit. Moreover, we find good agreement between our result, $Z_{\text{prop}} = 0.9321(44)$, and the Lüscher-Weisz perturbative prediction $Z_{\text{pert}} = 0.929(14)$ [5] at $\kappa = 0.0760$. The comparison $Z_{\text{prop}} = 0.9566(13)$ with $Z_{\text{pert}} = 0.940(12)$ at $\kappa = 0.07504$ is also fairly good. The quantity $Z_{\varphi}$ is obtained from the product $m_{\text{latt}}^2 \chi$. 

- **Figure 1:** The lattice data for the re-scaled propagator at $\kappa = 0.0740$ in the symmetric phase. The zero-momentum full point is defined as $Z_{\varphi} = m_{\text{latt}}^2 \chi$. The dashed line indicates the value of $Z_{\text{prop}}$. 

- **Figure 2:** Typical data for the re-scaled propagator are reported in Fig. 2. Unlike Fig. 1, the fit to Eq. (5), though excellent at higher momenta, does not reproduce the lattice data down to zero-momentum. Therefore, in the broken phase, a meaningful determination of $Z_{\text{prop}}$ and $m_{\text{latt}}$ requires excluding the lowest-momentum points from the fit. The fitted $Z_{\text{prop}}$ is slightly less than one. This fact is attributable to residual interactions since we are not exactly at the continuum limit, so that the theory is not yet completely “trivial.” This explanation is reasonable since we see a tendency for $Z_{\text{prop}}$ to approach unity as we get closer to the continuum limit.

Moreover, we find good agreement between our result, $Z_{\text{prop}} = 0.9321(44)$, and the Lüscher-Weisz perturbative prediction $Z_{\text{pert}} = 0.929(14)$ [5] at $\kappa = 0.0760$. The comparison $Z_{\text{prop}} = 0.9566(13)$ with $Z_{\text{pert}} = 0.940(12)$ at $\kappa = 0.07504$ is also fairly good. The quantity $Z_{\varphi}$ is obtained from the product $m_{\text{latt}}^2 \chi$.
and is shown in Fig. 3. According to conventional ideas \( Z_\varphi \) should be the same as the wavefunction-renormalization constant, \( Z_{\text{prop}} \), but clearly it is significantly larger. Note that there was no such discrepancy in Fig. 1 for the symmetric phase. Moreover our data show that the discrepancy gets worse as we approach the critical \( \kappa \). Indeed Fig. 3 shows that \( Z_\varphi \) grows rapidly as one approaches the continuum limit (where \( m_{\text{latt}} \to 0 \)). Thus, the effect cannot be explained by residual perturbative \( \mathcal{O}(\lambda_R) \) effects that might cause \( G(p) \) to deviate from the form in Eq. (5); such effects die out in the continuum limit, according to “triviality.” The results accord well with the “two \( Z \)” picture \([1]\) in which, as we approach the continuum limit, we expect to see the zero-momentum point, \( Z_\varphi \equiv m_{\text{latt}}^2 \chi \), become higher and higher.

5. Conclusions

We have reported new numerical evidence that the re-scaling of the ‘Higgs condensate’ \( Z_\varphi \equiv m_{\text{latt}}^2 \chi \) is different from the conventional wavefunction renormalization \( Z \equiv Z_{\text{prop}} \). Perturbatively, such a difference might be explicable if it became smaller and smaller when taking the continuum limit \( \lambda_R \to 0 \). However, our lattice data shows that the difference gets larger as one gets closer to the continuum limit, \( m_{\text{latt}} \to 0 \). Our lattice data is consistent with the unconventional picture \([1]\) of “triviality” and spontaneous symmetry breaking in which \( Z_\varphi \) diverges logarithmically, while \( Z_{\text{prop}} \to 1 \) in the continuum limit. In this picture the Higgs mass \( M_h \) can remain finite in units of the Fermi-constant scale \( v_R \), even though the ratio \( M_h/v_B \to 0 \). The Higgs mass is then a genuine collective effect and \( M_h^2 \) is not proportional to the renormalized self-interaction strength. If so, then the whole subject of Higgs mass limits is affected.

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