Analysis on the optimal damper quantity of energy dissipation structure

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Abstract. In practical application, the design of energy dissipation usually adopts the concept design, that is, to estimate damper quantity by repeating calculation. However, few studies have quantitatively analyzed the energy dissipation structure. This paper proposed two analysis methods to analysis the damper quantity of energy dissipation structure, the multiple-yield-strength method, and the damping-performance-curve method. Both of them can calculate the optimal damping quantity of the structure by adding metal dampers. The multiple-yield-strength method refers to that the yield strength of the metal damper is set by the multiple of the yield strength of the original structure. The optimal damper quantity of metal dampers can be analyzed by time history analysis. The damping-performance-curve method refers to that the target story displacement of the original structure is set. According to the relationship between the target displacement and the shear force in the damping-performance-curve, the stiffness of the original structure to achieve the target story displacement angle is derived, the stiffness is taken as the optimal damping of the metal damper. The optimal damping quantity is added to the original structure for comparative study which is calculated by the two methods. Both of them have reference value, and it could be beneficial for the promotion of energy dissipation.

1. Introduction
The earthquake has caused immeasurable losses to human beings. It is very important to improve the seismic performance of buildings in areas which are in high earthquake incidence. Therefore, people's lives and property are guaranteed to be safe, and it can mitigate the disaster.

The principle of energy dissipation structure to reduce seismic response is that: firstly, the system period has been reduced by the addition of dampers. The damper consumes most of the energy when an earthquake occurs, thus it protects the original structure. Then, the viscous property of the damper
can increase the damping of the structure [1]. Therefore, the damper structure has good seismic performance, which is compared with the traditional seismic structure.

The application of energy dissipation technology has been widely used. However, it is mostly conceptual design. How many dampers should be installed in the structure? It is usually obtained by estimating and checking again and again. There is no quantification of that, how much damping should be set for a given structure [2]. The optimal damping quantity is added to the original structure for comparative study which is calculated by the two methods in this paper. Both of them have reference value, and it could be beneficial for the promotion of energy dissipation [3].

2. The example model

The example model is a 30-storey spatial steel frame structure in this paper. It has 7 trusses of horizontal frame and 4 trusses of longitudinal frame. The bottom layer is 5.5 meters high, the others are 4 meters high (See Figure 1). The column has a box section. The length and width of the column is 600 millimeters, and wall thickness is from 19 millimeters to 50 millimeters, the wall thickness decreases with the height of the floor. The beam has H-shaped section, the size is 792×300×14×22. The steel model is Q345. The modulus of elasticity is 206GPa. The mass of each layer is assumed to be 980kg·m⁻². The schematic diagram of structure with metal dampers has been showed in Figure 2.

In order to research the internal relationship between the damper quantity and the seismic performance of the structure, the energy dissipation damping effect of shear frame has been only discussed under the action of lateral ground motion in this paper. The storey shear force and storey displacement curves are obtained by using pushover analysis method [4]. The three-line standard model is used to represent the restoring force characteristics, and the parameters of the restoring force characteristics of each storey are shown in Table 1. This is just a way of simplifying the calculation, 𝛿₁, 𝛿₂ are respectively representative displacement of the structure, 𝑘₁, 𝑘₂ , 𝑘₃ are respectively representative stiffness of the structure.

![Figure 1. The original structure](image1)

![Figure 2. The structure with metal dampers](image2)
3. Oscillatory differential equation and the earthquake motion

The vibration differential equation of multi-particle system model is as follows under seismic action.

\[
[M][\ddot{x}] + [C + C_f][\dot{x}] + [K + K_f][x] = -[M][\ddot{x}_g],
\]

In the formula, \([M]\) represents mass matrixes of structure; \([C]\) represents viscous damping matrixes of structure, and \([C] = \alpha[M] + \beta[K];\) \([C_f]\) represents the damping coefficient matrix of additional viscous dampers, and it is only considered in a linear state. \([K]\) represents the structural stiffness matrix, which is came by pushover analysis; \([K_f]\) represents the structural stiffness matrix of additional viscous dampers; \([x], [\dot{x}], [\ddot{x}]\) respectively represent displacement, velocity, acceleration response vectors of the structure; \([x_g]\) represents seismic acceleration.

4. Seismic waves

In this paper, two artificial seismic waves are used to analyze the elastoplastic time history of the structure, which are respectively called the KOBE wave, the TOHOKU wave. Their curve of acceleration time history is shown in Figure 3. The time interval of seismic wave was 0.01s, and the seismic fortification intensity were 8 degrees. The design earthquake group was the first group, and the site classification is II.

In this paper, the Wilson-θ method [5] is used for the seismic response of the structure by time-history analysis. The acceleration conversion value is set as 70cm/s² under frequent earthquake; and the acceleration conversion value is set as 400cm/s² under rare earthquake. The time interval of seismic waves is 0.01s, and it lasts for 30s.

### Table 1. Dimensions of structural elements

| story | \(\delta_1\) (m) | \(\delta_2\) (m) | \(k_1\) (N·m⁻¹) | \(k_2\) (N·m⁻¹) | \(k_3\) (N·m⁻¹) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1     | 0.021           | 0.028           | 8.2×10⁸         | 2.2×10⁸         | 6×10⁷           |
| 2     | 0.024           | 0.034           | 7.0×10⁸         | 1.7×10⁸         | 4×10⁷           |
| 4     | 0.026           | 0.037           | 6.3×10⁸         | 1.8×10⁷         | 2×10⁷           |
| 9     | 0.030           | 0.070           | 5.6×10⁸         | 2.8×10⁷         | 8×10⁶           |
| 12    | 0.030           | 0.060           | 5.2×10⁸         | 4.0×10⁷         | 6×10⁶           |
| 15    | 0.029           | 0.100           | 5.0×10⁸         | 1.9×10⁷         | 5×10⁶           |
| 18    | 0.030           | 0.100           | 4.3×10⁸         | 9.0×10⁶         | 2×10⁶           |
| 21    | 0.026           | 0.100           | 3.9×10⁸         | 1.3×10⁷         | 3×10⁶           |
| 26    | 0.023           | 0.100           | 2.5×10⁸         | 3.7×10⁶         | 9×10⁶           |
| 28    | 0.020           | 0.100           | 1.5×10⁸         | 8.8×10⁶         | 2×10⁶           |
| 30    | 0.010           | 0.000           | 9.1×10⁷         | 2.3×10⁷         | 6×10⁶           |
The empirical formula of the basic natural vibration period for high-rise structures is that $T = (0.1-0.15)n$, when $n=30, T=(3-4.5)$ seconds [6]. The original structure before setting dampers is named structure-O, its fundamental period is $T=5.645$ seconds by modal analysis. The difference between the analysis result and the empirical value is very small. The calculation model is reasonable.

### 5. The multiple-yield-strength method

The ratio is set as $\lambda$, which represents the ratio of the yield strength of each layer of additional metal damper to the original structure. The cases of $\lambda=0.1, 0.2, 0.3, 0.4$ and 0.5 were selected for analysis.

The energy dissipation structures are respectively called structure-A, structure-B, structure-C, structure-D and structure-E, when $\lambda$ is respectively 0.1, 0.2, 0.3, 0.4, 0.5. The solid line represents the yield strength distribution of the structure-O, while the other dashed lines represent the yield strength distribution of the metal dampers set in the structures -A to E in figure 4.

The yield displacement of the metal damper is set as 0.003m [7], which is approximately 1/10 of the yield displacement of the structure-O. The lateral stiffness of energy dissipation structure is composed of lateral stiffness of the main structure and the metal damper, which is showed in Figure 5.
Time history analysis is used to research the damping effect of the six example models from structure-O to structure-E under the rare earthquake action of two kinds of seismic waves, and the analysis results are as follows.

Figure 6 shows the absolute displacement of storeys. The displacements of each layer of the structure gradually decrease from showing on the figure. The displacement values of the corresponding calculation model are not significantly different with λ=0.3, 0.4 and 0.5.

Figure 7 shows the shear force of their original structures, which is not included the shear force on the damper. The storey shear force of the original structure floor decreases obviously, with the
increasing of the input damping quantity. It indicates that the damper has damping effect.

It concluded that there is little difference in response values under the action of two kinds of waves, when $\lambda$ is 0.3, 0.4 and 0.5. However, the response value of the structure-C model is much smaller than the structure-O model, and the damping effect is obvious compared with the original structure. Thus, the optimal damping can be determined as $\lambda = 0.3$.

6. Damping-performance-curve
The principle of setting dampers to reduce seismic response is based on the shortening of system period due to the additional stiffness of damper. The other side, the absorbing energy of dampers can reduce the seismic response of the structure [8-11]. The damping-performance-curve method is that the equivalent period and the equivalent damping ratio of the system can be deduced by earthquake response spectrum.

Then, the maximum displacement of the energy dissipation system can be obtained by multiplying the reduction rates of displacement by the maximum displacement of the original structure. The maximum shear force of the energy dissipation system can be obtained by multiplying the reduction rates of acceleration by the maximum shear forces of the original structure. The damping-performance-curve method can calculate the optimal damping of the structure with additional dampers without time history analysis.

The energy dissipation damping system is composed of the main structure, supports and dampers. The supports and dampers constitute the structural additional system. The additional system and the original structure constitute the energy dissipation system. The stiffness of the additional system is $K_a$, the stiffness of the original structure is $K_f$ and the stiffness of energy dissipation system is $K$. The stiffness of brace is $K_b$, and the stiffness of damper is $K_d$. The relationship between them is shown in Equation (2), (3) below.

$$K_a = \frac{1}{1/K_f + 1/K_d} \quad (2)$$

$$K = K_a + K_f \quad (3)$$

6.1 Damping-performance-curve
The damping performance curve is a function graph of displacement reduction rate ($R_d$) and acceleration (shear force) reduction rate ($R_a$) expressed by two basic parameters of the system, which are ductility coefficient $\mu$ and additional system elastic stiffness ratio $K_a/K_f$, as shown in figure 8.

The relationship is as follows:

$$D_h = \frac{1 + 4\xi_0}{1 + 4\xi_0} \quad (4)$$

$$\xi_{eq} = \xi_0 + \frac{2}{\mu p} \ln \frac{1 + p(\mu - 1)}{\mu}, p = \frac{1}{1 + K_a/K_f} \quad (5)$$

$\xi_0$ represents the initial damping ratio of the original structure.

$$T_{eq} = T_f \sqrt{\frac{\mu}{\mu + K_a/K_f}} \quad (6)$$

$$R_a = D_h \frac{T_{eq}}{T_f} \quad (7)$$
$T_f$ represents natural period of structure.

$$R_a = R_1 \left( \frac{T_f}{T_m} \right)^2 \tag{8}$$

**Figure 8.** Damping performance curve

6.2 Computational procedure
Firstly, the elastic stiffness of the original structure should be calculated, and the calculation results have been shown in the table 1. The Initial damping ratio $\xi_0$ is set as 0.02.

Secondly, the natural period of the original structure is 5.645 seconds. Then it is simplified to an equivalent point system. The displacement response value of the structure is 1.38 meters by referring the displacement response spectrum (Figure 9). The storey displacement angle is 1/60.

The target storey displacement angle is set as 1/150, so the target storey displacement reduction rate is 0.4, which is obtained by the ratio between target storey displacement angle and the storey displacement angle of the original structure.

**Figure 9.** Displacement response spectrum
Thirdly, two parameters are given initial values, which are ductility coefficient $\mu$ and additional system elastic stiffness ratio $K_a/K_f$. The displacement reduction rate is calculated according to formula (7). The two parameters are adjusted continuously until the target displacement reduction rate and the calculated displacement reduction rate are approximately zero. At this point, $R_d=0.40$, $R_d=0.657$. The maximum displacement of the energy dissipation system is the values using the reduction rates of displacement to multiply the maximum displacement of the original structure. The maximum shear force of the energy dissipation system is the values using the reduction rates of acceleration to multiply the maximum shear force of the original structure.

At last, the elastic stiffness of each layer can be calculated by formula (9). This stiffness is determined as the optimal damping of the additional metal damper of the structure.

$$K_{ai} = \frac{Q_i}{h_i} \left( \sum_{i=1}^{n} \frac{(K_i h_i)}{(\mu + \frac{K_a}{K_f}) - \mu K_f} \right)$$

6.3 Damping effect analysis

The optimal damping quantity is added to the original structure to analyze damping effect, which is calculated by the damping-performance-curve method. The results are shown in the figures below. The dashed lines represent the results based on damping-performance-curve method, and the solid line represents the damping effect of the original structure without dampers. As can be seen, it has a good damping effect on the original structure.

![Figure 10. Absolute displacements of floors](image)

![Figure 11. The velocity of floors](image)
7. Compare the two methods

The optimal damping quantity is added to the original structure for comparative study which is calculated by the two methods. Their seismic response values are compared in Figure 15 to 18. The response value of damping-performance-curve method is smaller than that of the multiple-yield-strength method.

The optimal damping quantity can be determined as $\lambda = 0.3$, which was calculated by the multiple-yield-strength method. When $\lambda = 0.3$, the ductility coefficient $\mu$ is 8.67, and
additional system elastic stiffness ratio $K_a/K_f$ is 0.657. At this point, $R_d=0.784,R_a=0.81$. When the damping-performance-curve method is used to calculate the optimal damping, the ductility coefficient $\lambda$ is 4, and the elastic stiffness ratio of the additional system $K_a/K_f$ is 0.657. At this point, $R_d=0.40,R_a=0.657$. The optimal damping can be determined by as $\mu=4$. So the response value of damping-performance-curve method is smaller than the multiple-yield-strength method.

Figure 15. Absolute displacements of storeys

Figure 16. The velocity of storeys

Figure 17. The plastic ratio of storeys
8. Conclusions
When the economy and damping effect were synthetically considered, The optimal damping quantity of the structure is $\lambda=0.3$, which was calculated by the multiple-yield-strength method. The optimal damping quantity only corresponds to a set of solutions by deducing from the damping performance curve.

When the damping-performance-curve method is used to calculate the optimal damping quantity, the ductility coefficient $\mu$ is 4, and the elastic stiffness ratio of the additional system $K_d/K_f$ is 0.657. At this point, $R_d=0.40$, $R_d=0.657$. The optimal damping quantity corresponds to several groups of solutions by deducing from the damping performance curve. According to different ductility coefficients and additional elastic stiffness values of the system, it can be calculated multiple sets of solutions, which were all satisfied the target displacement reduction rate. Specific solutions are selected according to actual needs. Both of them have reference value, and it could be beneficial for the promotion of energy dissipation.

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