The Evolution of Nuclear Multifragmentation in the Temperature-Density Plane

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Abstract

The mean transverse kinetic energies, $\langle KE_\perp \rangle$, of fragments formed in the interaction of 1 A GeV Au+C have been determined. An energy balance argument indicates the presence of a collective energy which increases in magnitude with increasing multiplicity and accounts for nearly half of the measured $\langle KE_\perp \rangle$. The radial flow velocity associated with the collective energy yields estimates for the time required to expand to the freeze-out volume. Isentropic trajectories in the temperature-density plane are shown for the expansion and indicate that the system goes through the critical region at the same multiplicities as deduced from a statistical analysis. Here, the expansion time is $\sim 70$ fm/c.

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Recently, there has been renewed experimental [1–4] and theoretical interest [5–12] in the phenomenon known as nuclear multifragmentation (MF). With sufficient excitation energy, a large nucleus will disassemble into nucleons, light fragments, and several intermediate mass fragments (IMF) $3 \leq Z_f \leq 30$. A recent experiment [1] conducted at the Lawrence Berkeley Bevalac by the EOS Collaboration has provided a large sample of events in which the total charge of the projectile, 1 GeV per nucleon gold incident on a carbon target, has been completely reconstructed. Such events have permitted a detailed fragment yield analysis that points strongly to the connection between MF and critical behavior in a finite system [1]. The implication of this statistical analysis is that the system is substantially equilibrated at the time of fragment formation. Further support for equilibration comes from a study of MF using different beam energies and projectile-target combinations [13]. In a recent paper [14], the notion of thermal and chemical equilibrium was employed to calculate an initial and final temperature of the system undergoing MF. Here the evolution of MF in the temperature-density $(T - \rho)$ plane is explored as a function of the total charged particle multiplicity.
It has long been a goal to understand the evolution of highly excited nuclear systems in the $T-\rho$ plane. Theoretical efforts addressing this issue have tended to focus on the behavior of infinite nuclear matter \[15\]. It is much more difficult to solve this problem for finite size systems. One theoretical approach often taken is to circumvent the complete dynamical description and use statistical methods to form the fragments \[7,11\]. Such methods describe the statistical aspects of MF reasonably well, but fail, as shown below, to describe the dynamics satisfactorily. An alternative method is to follow the dynamical process from the beginning. These approaches, however, do not adequately describe the statistical aspects \[16\]. With simple assumptions and the experimental data, another approach is presented which describes the dynamical evolution of the fragmenting system.

The experimental apparatus was described by Gilkes et al \[1\]. Two essential features of the experiment, reverse kinematics and the large mass asymmetry between target and projectile, permit an unambiguous identification of the source of all projectile fragments of charge $Z > 2$. Particles with $Z \leq 2$ can be separated into two categories, those associated with the heavier nuclear fragments, which are emitted in the second stage of the reaction, and those emitted in the primary collision stage. The source of the heavier fragments is termed the remnant and travels at close to beam velocity. In a recent paper \[14\], the remnant mass and excitation energy were determined as a function of the measured total charged particle multiplicity, $m$. To convert the initial excitation energy of the remnant to an initial temperature, $T_i$, the state of the remnant was assumed to be that of an equilibrated Fermi gas at a density, $\rho_i$, determined by the remnant mass and the unexpanded gold nuclear volume, $V_i$. A freeze-out temperature, $T_f$, was determined using a double ratio of observed isotopic yields \[17\].

Figure 1 shows $\langle KE_\perp \rangle$, where $KE_\perp = p^2_\perp / 2M$, for fragments of charge 3-6 for 5 different multiplicity intervals. Individual fragments had $KE_\perp$ values which ranged up to 100 MeV. Only statistical errors are shown. Systematic errors are discussed in reference \[18\]. A significant evolution has occurred in going from low $m$ to high $m$ events. For comparison, the predictions of the intranuclear cascade calculation ISABEL \[19\] coupled to the statistical
multifragmentation model (SMM) are included in Fig. 1. No radial flow was included in the SMM-generated data. These two calculations model the first and second stages of the reaction, respectively. Substantial differences between data and this model develop as a function of increasing multiplicity. For the lowest \( m \) interval, the simulation is in reasonable agreement with the data, both in magnitude and as a function of charge. However, as \( m \) increases, the simulation predicts that a given fragment’s average transverse kinetic energy decreases, while the data show an increase in \( \langle KE_\perp \rangle \) with \( m \). Similar trends are also found for the heavier fragments.

The behavior in SMM can be understood as follows. SMM produces spectra by randomly placing fragments in a volume that increases with multiplicity. Fragments are given thermal energy and are then evolved classically under the influence of the Coulomb force. The decrease in a fragment’s kinetic energy in the simulation with increasing \( m \) is due to both the increase in volume and the decrease in the charge of the largest fragment. The average charge of the largest fragment as a function of \( m \) in data and SMM are in very good agreement, so this is not the source of the discrepancy. Likewise, no reasonable adjustment of the distance between fragments can lead to resolution of the discrepancy. The excitation energy and mass spectra of the remnant predicted by ISABEL are also in reasonable agreement with the data [20].

It will be assumed that the initial hot remnant undergoes expansion to freeze-out with no decrease in nucleon number. Thus, at freeze-out, not only the IMFs, but all particles associated with the breakup of the remnant must be taken into account. Energy conservation relates the initial excitation energy of the remnant, \( E_{i}^* \), to the energy at freeze-out. This latter energy can be decomposed into four contributions:

\[
E_{i}^* = E_{\text{th}} + E_{C} + Q_f + E_X ,
\]

where \( E_{\text{th}} \) is the total thermal contribution to the spectra, \( E_{C} \) is the Coulomb contribution, \( Q_f \) is the sum of the Q-value contribution for each final state particle, and \( E_X \) is what remains when the other three terms are subtracted from \( E_{i}^* \). Within the spirit of the calculation
of the final state temperature, each final state particle originating in the breakup of the remnant will contribute \(\frac{3}{2}T_f\) to \(E_{th}\). The total Coulomb energy available for doing work is given by the difference in the self-energy of the remnant and the sum of the self-energies of all the second stage particles. Thus,

\[
E_C = \frac{3}{5} \epsilon^2 \left( \frac{Z_R^2}{R_{Au}} - \sum \frac{Z_i^2}{R_f} \right).
\]  

(2)

In Eq. 1, the contribution of the unobserved neutrons are estimated as in [14].

With Eq. 1, \(E_X\) can be calculated for the experimental data. This quantity, displayed in Fig. 2 as a function of \(m\), increases from 1 to 6 MeV/nucleon with increasing \(m\), thus accounting for nearly half of \(\langle KE_{\perp}\rangle\) that is not explained by the thermal, Coulomb, and Q-value terms of Eq. 1. For the simulated data, a freeze-out temperature was determined using the Albergo procedure [17]. This freeze-out temperature is in good agreement with that obtained from the experimental data [20]. When the same energy conservation is applied to the ISABEL-SMM generated spectra, a value of \(E_X\) of approximately zero is obtained. The nature of the energy, \(E_X\), in the data is now addressed.

Two characteristics of the kinetic energy spectra of IMFs have long been noted: the Coulomb energy contribution is much smaller than that estimated assuming surface emission from a normal density remnant system, and the inverse slope parameter characterizing the high energy tail of the spectra is much larger than the thermal energy inferred from isotopic yields [21,22]. The low Coulomb energies support the view that fragment formation occurs in an expanded system [23,24]. However, there is no general consensus on the interpretation of the kinetic energy spectra [22,25,26].

A possible resolution to the discrepancy between the freeze-out temperature and the \(\langle KE\rangle\) may lie in the quantity \(E_X\). It will be assumed that entropy is generated during the rapid cascade stage of the collision process and that thereafter the remnant expands at constant entropy [28,29]. The quantity \(E_X\) may then represent the collective motion generated during the expansion [27,30]. Statistical models such as SMM must add collective energy explicitly since they do not perform a true dynamical evolution of the remnant
system. The addition to SMM of collective energy of the same magnitude as indicated in
Fig. 2 substantially diminishes the discrepancies of the trends and magnitudes between the
simulation and the data [31].

If the remnant expands isentropically, the freeze-out volume can be estimated using the
initial temperature and volume of the remnant, $T_i$ and $V_i$, and the freeze-out temperature,
$T_f$, [14]. For a non-dissipative Fermi gas, $V_f = V_i(T_i/T_f)^{3/2}$. Figure 3 shows the freeze-out
density, obtained by dividing the remnant nucleon number by the freeze-out volume. This
yields a density about 1/3 normal nuclear matter in the multiplicity interval identified as
the critical region by a statistical analysis of this data [1].

The assumption of an isentropic expansion can be checked by estimating the entropy per
particle before and after expansion of the remnant. The initial entropy is taken to be that
of a Fermi gas at the temperature and density of the remnant and ranges from 1 to 2.5 per
particle as a function of $m$. The entropy in the expanded state is computed using the SMM
prescription with the freeze-out temperature and volume taken to be $T_f$ and $V_f$, respectively.
The final entropy is systematically lower than the initial entropy by about 0.2 per particle.
However, because it was assumed that all of the initial excitation energy was thermalized,
the initial entropy estimated represents an upper bound. Reducing the initial excitation
energy per nucleon by only 5% brings the initial and final entropies into agreement. The
significant point is that no additional entropy is generated in the fragmentation process,
supporting the assumption of an isentropic expansion.

For an isotropically expanding remnant, the average time required for the system to
expand to freeze-out can be estimated by dividing the increase in radius by the mean flow
velocity, $\langle v_f \rangle = v_f/2$. This estimate yields a short expansion time of about 70 fm/c, in
good agreement with expansion times obtained from fragment-fragment correlation studies
[32,33] and theoretical predictions [11]. The time is not strongly dependent on multiplicity.
Thus, multifragmentation is a very fast process which suggests a simultaneous disassembly
of the remnant. The presence of a significant component of directed sideward flow could
affect this time estimate. However, it is unlikely that such flow is of importance in the very
asymmetric Au+C collisions. It is known that directed flow is maximal at intermediate multiplicities [34] whereas the observed value of $E_X$ becomes largest at the highest $m$.

Fig. 4 shows the evolution of the remnant system from initial density and temperature to freeze-out density and temperature. The remnant system is driven towards lower $\rho$ and $T$ via expansion. Multiplicities in the critical region, as deduced from a statistical analysis of the data, are indicated by bold lines which follow the $VT$ relationship for an isentropic expansion of a Fermi gas. The region in the $T - \rho$ plane for which critical behavior is expected for a finite charged nucleus [15] is about 1/3 normal nuclear density and approximately between 5-8 MeV in temperature, i.e. $T_c \sim E_B$, the binding energy/nucleon for a finite nucleus [15,33]. This dynamical analysis indicates that as the initial excitation energy is increased the system begins to enter the critical region for the same multiplicities (excitation energies) as deduced from the statistical analysis. This conclusion is based on the assumption of energy and entropy conservation.

In summary, an examination of the behavior of the mean transverse kinetic energies for fragments of charge 3-6 for five different multiplicity intervals and an exercise in energy conservation have shown evidence for collective motion increasing with multiplicity and thus, with excitation energy. Time scales for expansion agree well with both previous experimental determinations and theoretical estimates. Isentropic trajectories in the neighborhood of the critical point in the temperature-density plane have been determined from experimental data for the first time. Together with the statistical analysis of the data, a strong case can be made for thermalization in multifragmentation.

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Figures

Figure 1. Average transverse kinetic energy as a function of multiplicity bin for fragments of charge 3-6. Circles are data and triangles are results of ISMM (ISABEL+SMM) simulation without the addition of flow. Multiplicity intervals are: 1-10, 11-20, 21-30, 31-40, 41+.

Figure 2. Energy per remnant nucleon versus total charged particle multiplicity. Squares are the remnant excitation energy; circles are sum of thermal, Coulomb and Q-value contributions. The energy $E_X$, the difference between the two quantities, is shown as triangles.

Figure 3. Initial (open squares) and final (filled squares) densities as a fraction of normal nuclear density versus $m$.

Figure 4. Trajectories in the temperature-density plane shown for different multiplicities. Trajectories in the neighborhood of the critical region (as determined from the statistical analysis in [1]) are in boldface.
Density ratio $\rho / \rho_0$

Temperature (MeV)

- $T_i, \rho_i$
- $T_f, \rho_f$

$E_i^*/A = 13.2 \text{ MeV/A}, \ S/A = 2.6$

$E_i^*/A = 10.1 \text{ MeV/A}, \ S/A = 2.1$

$E_i^*/A = 5.9 \text{ MeV/A}, \ S/A = 1.5$

$E_i^*/A = 4.0 \text{ MeV/A}, \ S/A = 1.2$

Density ratio $\rho / \rho_0$