The oil-producing characteristics of Bang-Bang control for 2D reservoir

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Abstract. The Bang-Bang control is one of the primary control methods to increase the cumulative oil production for water-flooding reservoir. The on-off state and operating time of the injection wells are adjusted to enhance the efficiency. In this article, the oil-producing characteristics of Bang-Bang control for optimization problems of 2D reservoir is studied. The efficiency of Bang-Bang control in different stages during the whole oilfield development process has been concluded. The regularity of the Bang-Bang control’s strategy is summarized. The numerical simulation results show that, in early stage of oilfield development, Bang-Bang control may be the optimal solution amongst two-segment regulation strategies. In middle period, Bang-Bang control gains less oil production than the simultaneous optimization scheme. In high water-cut period, Bang-Bang control’s oil production is approaching simultaneous scheme. For Bang-Bang control’s optimal strategies, according to the distance between the production well and injection well, the farther injection well is supposed to open first, then it will be closed and nearer well will open to operate in the next segment. With the total production time become longer, the proportion of farther well’s operating time will increase and stabilize at certain value finally. For engineering practice, it indicate that the optimal regulation strategies need to be formulated according to reservoir’s development stages in order to improve the efficiency of oil production.

1. Introduction

With the development of the technology of intelligent oilfield, the regulation strategies using Bang-Bang control for improving the oil production efficiency has received much attention [1, 2].

The prerequisite of implementing Bang-Bang control is dividing the production process into several segments. It has been pointed out that, without the segmentation, water phase does not move uniformly towards the production well and in certain regions the oil phase will not be fully drained, which will lead to poor sweeping efficiency [3]. The Bang-Bang type control, coming from the optimal control theory, adjusted the on-off state and operating time of the oil wells to delay the breakthrough time of the water-flood front. Therefore it can improve the water sweeping efficiency [4]. It was proposed that Bang-Bang type has the maximum displacement efficiency until the time of the breakthrough under the condition that the water injection rates of every wells have the upper and lower bounds [5]. The numerical simulations of the actual three-dimensional reservoir examples indicated that Bang-Bang type...
optimization method can significantly increase the amount of oil production and enhance oil recovery factor [6].

However, the study of whether Bang-Bang control is the optimal scheme amongst segmented regulation strategies is insufficient. The article is organized as follows. In Section 2, the mathematical expressions for Bang-Bang control are presented. For comparison, the simultaneous optimization of both segmented time and injection rate are also provided. In Section 3, numerical example is performed to test oil production efficiency of Bang-Bang control in different development stages. The regularity of the Bang-Bang control’s optimal strategy is analyzed. Conclusions are given in Section 4.

2. The Bang-Bang control and comparison scheme

2.1. Bang-Bang control with time optimization method

Consider the water flooding reservoir with two injection wells and one production well. The total water injection rate $Q_{total}$ is fixed. Take Bang-Bang control as the optimal valve settings for each injection well, which suggest that only one injection well should operate in the same period. When moving to next production process, the on-off state of injection wells will be changed. Divide the production process into $n$ segments with the length of time $t_i$ ($1 \leq i \leq n$). $T_{total}$ represents the total production time. Select the cumulative oil production $Q_o = \sum_{i=1}^{n} Q'_o$ as the objective function. Let $Q'_o$ denote the oil production in the segment $i$ ($1 \leq i \leq n$). The expression of optimization problem using Bang-Bang control and its constraint condition are shown in eq.1.

$$\max_{(t_i)} \sum_{i=1}^{n} Q'_o \quad \text{subject to:} \quad \sum_{i=1}^{n} t_i = T_{total}, \quad t_i \geq 0$$ (1)

The elimination method and steepest descent method is used to solve the constrained optimization problem.

2.2. Simultaneous scheme optimizing both segmented time and injection rate

Select the cumulative oil production $Q_o = \sum_{i=1}^{n} Q'_o$ as the objective function. The control variables include $q'_j$, which denote the water injection rate of the injection well $j$ ($1 \leq j \leq 2$) in the segment $i$ ($1 \leq i \leq n$), and the segmented time $t_i$ ($1 \leq i \leq n$). The expression of the optimization problem and constraint condition is shown in eq.2.

$$\max_{(q'_j, t_i)} \sum_{i=1}^{n} Q'_o \quad \text{subject to:} \begin{cases} \sum_{j=1}^{2} q'_j = Q_{total}, \quad q'_j \geq 0 \\ \sum_{i=1}^{n} t_i = T_{total}, \quad t_i \geq 0 \end{cases}$$ (2)

3. Numerical simulation results

In this section, we use 2D oil reservoir with its size 337.5m×225m, shown in Figure 1, to test Bang-Bang control’s oil producing efficiency in different development stages. For comparison purpose, the calculations of simultaneous scheme, listed in section 2, is also performed in the example. The reservoir has two injection wells and one production well. The outer boundary is imposed with impermeable boundary conditions. The absolute permeability distribution and the initial water saturation distribution are homogeneous. The total producing time ranges from 14 days to 700 days and the total fluid
production rate is fixed with 97.2 m$^3$/day. Other parameters are listed in Table 1. The calculations of cumulative oil production and optimization results of Bang-Bang control and simultaneous scheme are listed in Table 2.

![Image 216x584 to 287x598]

**Figure 1.** 2D reservoir and well’s location

### Table 1. Numerical model parameters

| Mesh size                     | $\Delta x = \Delta y = 7.5m$ | Initial pressure | $P^0 = 2.0\text{MPa}$ |
|------------------------------|-------------------------------|-------------------|------------------------|
| Porosity                     | $\phi = 0.3$                  |                   |                        |
| Absolute permeability        | $K = 0.5D$                    |                   |                        |
| Initial water saturation     | $s_w^0 = 0.3$                 |                   |                        |
| Irreducible water saturation | $s_{w0} = 0.3$                |                   |                        |
| Residual oil saturation      | $s_o = 0.2$                   |                   |                        |
| Production well pressure     | $P_w = 1.0\text{MPa}$         |                   |                        |
| Viscosity                    | $\mu_w = 1\text{mPa} \cdot \text{s}$ |                   |                        |
| Relative permeability        | $K_w = (\frac{s_o - s_{w0}}{1-s_{w0} - s_o})^2$ |                   |                        |

### Table 2. Comparison of cumulative oil production and optimization results

| Cases | Time ($T_{\text{total}}$) | $Q_0$, $t_1$, $t_2$ | $Q_0$, $t_1$, $t_2$ | $Q_0$, $t_1$, $t_2$ | $q_1$, $q_2$, $q_3$, $q_4$ |
|-------|---------------------------|----------------------|----------------------|----------------------|-----------------------------|
| 1     | 14                        | 1360.8 14 0          | 1360.8 14 0         | 97.2 0 0            | 97.2                        |
| 2     | 15                        | 1458.0 14 1          | 1458.0 14 1         | 97.2 0 0            | 97.2                        |
| 3     | 25                        | 2430.0 19 6          | 2430.0 19 6         | 97.2 0 0            | 97.2                        |
| 4     | 50                        | 4414.4 33 17         | 4415.1 31 19        | 97.2 0 0            | 12.2 85.0                   |
| 5     | 75                        | 5440.6 53 22         | 5442.1 50 25        | 97.2 0 0            | 20.9 76.3                   |
| 6     | 100                       | 6115.7 73 27         | 6118.8 69 31        | 97.2 0 0            | 24.8 72.4                   |
| 7     | 125                       | 6614.1 94 31         | 6617.9 88 37        | 97.2 0 0            | 31.1 66.1                   |
| 8     | 150                       | 7005.0 114 36        | 7009.1 107 43       | 97.2 0 0            | 33.0 64.2                   |
| 9     | 175                       | 7323.6 134 41        | 7327.5 125 50       | 97.2 0 0            | 33.2 64.0                   |
| 10    | 200                       | 7590.3 155 45        | 7594.0 145 55       | 97.2 0 0            | 33.3 63.9                   |
| 11    | 225                       | 7817.7 174 51        | 7821.0 164 61       | 97.2 0 0            | 33.5 63.7                   |
| 12    | 250                       | 8014.8 195 55        | 8017.7 182 68       | 97.2 0 0            | 33.0 64.2                   |
| 13    | 275                       | 8187.6 215 60        | 8190.2 202 73       | 97.2 0 0            | 32.7 64.5                   |
| 14    | 300                       | 8340.6 234 66        | 8343.1 221 79       | 97.2 0 0            | 31.4 65.8                   |
| 15    | 400                       | 8813.3 313 87        | 8815.3 296 104      | 97.2 0 0            | 30.6 66.6                   |
| 16    | 500                       | 9144.4 393 107       | 9145.9 374 126      | 97.2 0 0            | 29.2 68.0                   |
| 17    | 600                       | 9394.4 472 128       | 9395.5 449 151      | 97.2 0 0            | 28.0 69.2                   |
| 18    | 700                       | 9588.5 550 150       | 9589.3 528 171      | 97.2 0 0            | 23.8 73.4                   |

*Units: $T_{\text{total}}, t_i$ (days), $Q_0$ (m$^3$), $q_j$ (m$^3$/day)*
From table 2 and figure 2.a, it is shown that when the total producing time is less than 25 days, while water breakthrough has not occurred, the optimization result and oil production of Bang-Bang control are exactly same with the simultaneous optimization scheme. It indicate that Bang-Bang control may be the optimal solution amongst two-segment regulation strategies at this stage. And simultaneous scheme shows integrated Bang-Bang properties. With the increase of total producing time, in the middle stage of oilfield development, the Bang-Bang-type’s oil production is fewer than the simultaneous scheme. The distinction of oil production between these two schemes gradually increases. For simultaneous scheme, only the first segment has the Bang-Bang properties. While in the second segment, both injection wells are opened and water injection rates are distributed according to a certain proportion. In addition, the optimized time results are also different from Bang-Bang control. In that case, simultaneous scheme’s Bang-Bang properties are deteriorated in the middle stage. Nevertheless after the day 150 when the reservoir development comes into high water-cut period, the Bang-Bang-type oil production is still less than the simultaneous scheme, but the difference between two schemes is gradually shrinks. On the day 700, Bang-Bang-type can drill out nearly the same amount of crude oil compared with simultaneous scheme. And simultaneous scheme’s Bang-Bang properties are recovered to some extent.

From table 2 and figure 2.b, by observing the time optimization results of the Bang-Bang control, the optimal regulation strategies of Bang-Bang control can be summarized. According to the distance between the production well and injection well, the order of on-off state of injection wells can be decided. The farther injection well is supposed to open first and nearer well will operate in the next segment. On the other hand, with the increase of total production time, the proportion of farther well’s operating time will increase, and it will stabilize at around 79% finally. In the practical engineering, the Bang-Bang control’s regulation strategy can be operating time of each injection well can be formulated according to this regularity.

4. Conclusion
In this article, the Bang-Bang control’s oil production efficiency of 2D water-flooding reservoir is studied. By calculating optimization results in different development stages, a novel regularity including Bang-Bang control’s optimal characteristics and rules of Bang-Bang control’s regulation strategy are summarized. It is suggested that in early stage of development, Bang-Bang control may be the optimal solution amongst two-segment regulation strategies. In the middle stage of oilfield development, the Bang-Bang-type’s oil production is fewer than the simultaneous scheme. In high water-cut period, the Bang-Bang-type oil production is approaching the simultaneous scheme. According to the distance between the production well and injection well, the farther injection well is supposed to open first, then it will be closed and nearer well will open to operate in the next segment. With the total production time become longer, the proportion of farther well’s operating time will increase and stabilize at certain value.
finally. In the engineering practice, it indicate that the optimal regulation strategies need to be formulated according to reservoir’s development stages in order to improve the efficiency of oil production.

Acknowledgments
This work was financially supported by National Science and Technology Major Project of China (2017ZX05072005).

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