Strongly Interacting Light Dark Matter

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In the presence of approximate global symmetries that forbid relevant interactions, strongly coupled light Dark Matter (DM) can appear weakly coupled at small-energy and generate a sizable relic abundance. Fundamental principles like unitarity restrict these symmetries to a small class, where the leading interactions are captured by effective operators up to dimension-8. Chiral symmetry, spontaneously broken global symmetries and non-linearly realized supersymmetry are examples of this. Their DM candidates (composite fermions, pseudo-Nambu–Goldstone Bosons and Goldstini) are interesting targets for LHC missing-energy searches.

Studies of processes with missing energy at the LHC constitute an important part of the Dark Matter (DM) research program, in particular for light DM, \( m_{\text{DM}} \lesssim 10 \text{ GeV} \), below the threshold for direct detection experiments. In this case, parameterizing the thermally-averaged annihilation cross section as \( \langle \sigma v_{\text{rel}} \rangle \sim \frac{\alpha_{\text{DM}}^2}{m_{\text{DM}}^2} \) with \( m_{\text{DM}}, \Omega_{\text{DM}} \) the DM mass and coupling to the Standard Model (SM) fields – the WIMP miracle

\[
\Omega_{\text{DM}} h^2 \approx 10^{-26} \text{cm}^3/\text{s} \approx 0.1 \left( \frac{0.01}{\alpha_{\text{DM}}} \right)^2 \left( \frac{m_{\text{DM}}}{100 \text{ GeV}} \right)^2,
\]

seems to provide a convincing hint that light DM originates from weakly coupled dynamics \( \alpha_{\text{DM}} \ll 1 \), in order to reproduce the observed value \( \Omega_{\text{DM}} h^2 \approx 0.1 \) [1]. In this letter we want to explore how solid this indication is and study the viability of light DM associated with a new strong coupling, which we call \( g_s \lesssim 4\pi \). The core aspect of our analysis is approximate symmetries, which forbid relevant (renormalizable) SM-DM interactions, but allow irrelevant (non-renormalizable) interactions of dimension \( D \). Referring to \( M \) as the physical scale suppressing the latter, the amplitude for \( 2 \to 2 \) annihilation would scale as

\[
\alpha_{\text{DM}} \sim \frac{g_s^2}{4\pi} \left( \frac{E}{M} \right)^{D-4}, \tag{1}
\]

where \( E \) denotes the collision energy. At low energies \( E \ll M \), such as those relevant at freeze-out, these interactions appear weak, despite their strongly coupled nature at high-energy: this reconciles strong coupling with the WIMP miracle. For instance, for \( D=6 \), considering that in the relevant non-relativistic limit \( E \sim m_{\text{DM}} \),

\[
\Omega_{\text{DM}} h^2 \approx 0.1 \left( \frac{4\pi}{g_s} \right)^{4} \left( \frac{5 \text{ GeV}}{m_{\text{DM}}} \right)^{2} \left( \frac{M}{3 \text{ TeV}} \right)^{4}. \tag{2}
\]

This is particularly important for the LHC which, operating at high-energy, has direct access to the strongly coupled regime, as we anticipate in Fig. 1, where we compare the LHC reach in the \( (g_s, M) \)-plane with relic density (RD) expectations. As a matter of fact the combination of a large scale and strong coupling provide one of the few examples where the use of a DM effective field theory (EFT) [2–4] is well motivated even to parametrize LHC DM searches; we discuss this in detail in a companion paper [5].

So, what symmetries are compatible with irrelevant operators only? For scalars a well-known example is the shift symmetry associated with Nambu–Goldstone bosons (NGBs) of a global symmetry \( G \), spontaneously broken to a subgroup \( H \) by strong dynamics. As we will see below, depending on \( G \), the leading interactions appear at \( D=6 \) or \( D=8 \) [6]. For Dirac fermions, on the other hand, chiral symmetry and the absence of gauge interactions are enough to guarantee \( D \geq 6 \). Alternatively, for Majorana fermions (in analogy with NGBs), non-linearly
realized supersymmetry (SUSY) ensures that $D \geq 8$. Indeed the leading interactions of Goldstini from spontaneously broken SUSY only exhibit higher-derivative interactions in the limit where all other SUSY particles are heavy [7]. The question now is whether, beyond these examples, we can find an infinite set of symmetries such that the low-energy amplitude is suppressed by higher and higher powers of energy, i.e. where $D \geq 10$ constitute the only interactions allowed in the limit of exact symmetry. As a matter of fact the answer is negative. Fundamental principles based on analyticity, unitarity and crossing symmetry of the $2 \to 2$ amplitude provide strict positivity constraints for some of the coefficients of $D=8$ operators, so that there is no limit in which a symmetry that protects operators with four fields and $D \geq 10$, forbidding $D \leq 8$, can be considered exact [8].

So the complete set of scenarios with a naturally light strongly coupled DM, that however appears weakly coupled at small $E$ (and therefore fulfills the WIMP miracle) is given by the above examples\(^1\) and is captured by operators of $D \leq 8$. In this letter we rely on simple power counting rules to build the generic EFT describing the low-energy physics of these scenarios, and discuss the implications.

**Scalar Dark Matter.** Naturally light scalars originate as pseudo-NGBs of the spontaneously symmetry breaking (SSB) pattern $G/H$, in analogy with the QCD pions; if the sector responsible for SSB is strong, NGB interactions become strong at high-$E$. These scenarios are particularly interesting in association with the hierarchy problem [10–16], but also independently from it [17, 18]. Qualitatively different cases of interest can be identified, depending on the particular group structure being considered and the interplay with Higgs physics. First, a light scalar DM can be associated with an abelian $U(1) \to \mathbb{Z}_2$ breaking pattern, while a light composite Higgs originates from e.g. $G/H = SO(5)/SO(4)$ [19]. Alternatively, the DM originates from a non-abelian, e.g. $SU(2) \to U(1)$ or larger, symmetry breaking patterns [14–17]. Finally, both the Higgs and DM can arise together from a non-factorizable group $G$, such as $SO(6)/SO(5)$ [10–12, 20]. The very power of EFTs is that, at low-$E$, large groups of theories fall in the same universality classes: in our case the generic EFTs that we will now build to describe the above-mentioned scenarios can be matched to any model with approximate symmetries.

In all these cases, the NGB interactions are described by the CCWZ construction [6]: the light degrees of freedom $\phi^a$ are contained in the coset representative $U = \exp(i \phi^a t^a / f) \in G/H$ and appear in the Lagrangian only through the building blocks $d_{\mu}^a$ and $\varepsilon_{\mu}^a$ in $U^{-1} \partial_{\mu} U = i d_{\mu}^a t^a + i \varepsilon_{\mu}^a T^A$, where $f^a(T^A)$ are the broken (unbroken) generators in $G$, $f$ is the analog of the pion decay constant and is related to the mass and couplings of resonances from the (strong) sector that induces SSB through the naive dimensional analysis estimate $f = M / g_s$. In particular,

| $G/H$ | $\phi$ | $d_{\mu}^a / f$ | $\varepsilon_{\mu}^a$ |
|---|---|---|---|
| $\frac{U(1)}{\mathbb{Z}_2}$ | $\phi \in \mathbb{R}$ | $\frac{\partial_{\mu} \phi}{f}$ | 0 |
| $SU(2)/U(1)$ | $\phi \in \mathbb{C}$ | $\frac{(1 + |\phi|^2 + \ldots) \partial_{\mu} \phi}{f}$ | $\frac{\phi \partial^\mu \partial_{\mu} \phi + \ldots}{f}$ |
| $SO(5)/SO(4)$ | $H^i, \phi \in \mathbb{R}$ | $\frac{(1 + |\phi|^2 + |H|^2 + \ldots) \partial_{\mu} \phi}{f}$ | $\frac{H^i \partial^\mu \partial_{\mu} H + \ldots}{f}$ |

where dots denote higher order terms in $1/f$. Under a transformation $g \in G$, $U \to g U h(\phi, g)^{-1}$, where $h(\phi, g) \in H$. Then $d_{\mu}^a \equiv d_{\mu}^a t^a$ and $\varepsilon \equiv \varepsilon^a T^A$ transform under $G$ respectively in the fundamental representation of $H$ and shift as a connection, so that $D_{\mu}^a \equiv \partial_{\mu} + i \varepsilon_{\mu}^a$ is the covariant derivative. Then the low energy Lagrangian describing the canonically normalized light scalars only, is simply $\mathcal{L}^{2-11} = M^2 f^2 \mathcal{L} (d_{\mu}^a / f M, D_{\mu}^a / M)$ with the additional requirement of $H$ invariance: this automatically guarantees also $G$ invariance.

Clearly DM cannot be an exact massless NGB: the global symmetry must be broken explicitly. We keep track of this breaking by weighting interactions that violate the CCWZ construction with $m_h^2 / M^2$: an assumption that reflects to good extent the expectations in explicit models (see for instance [10]). We further assume the most favorable case in which, to the extent possible, the SM itself is part of the strong dynamics, as discussed in Ref. [21],\(^1\) so that DM-SM interactions do not introduce further symmetry breaking effects (we discuss below cases where only some species take part in the new dynamics). This implies in particular that we assume the new dynamics respects the SM (approximate) symmetries: custodial symmetry, CP, flavor symmetry (broken only by the SM Yukawas [25]) and baryon and lepton numbers. Finally we assume the new dynamics can be faithfully described by a single new scale $M$ and coupling $g_s$ [22]. Compatibly with these assumptions, the most general Lagrangian at the leading order in the $1/M$ expansion, and the maximum coefficient that we can ex-

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1 Ref. [9] proposes a somewhat different realization of the same principle, where symmetries imply suppression of the $2 \to 2$ amplitude in favor of the $3 \to 2$, which decouples fast as the DM density dilutes. Alternatively, selection rules in the UV could imply p- or d-wave suppressions in the non-relativistic limit, also satisfying our high-energy/strong-coupling, low-energy/weak-coupling dichotomy (yet the implied crosssection suppression is mild $\sim 0.2 \div 0.1$ [5]).

2 We will assume that anomalies and the Wess-Zumino-Witten term, that might lead to DM decay in similarity to $\pi \to \gamma \gamma$ in QCD, vanish.

3 This implies that the Higgs is itself a PNVGB [19, 22], SM fermions are partially composite [19, 23], and the transverse polarizations of gauge bosons have strong multipolar interactions [21] – constraints on these possibilities, independent of the new sector couplings to DM, will be studied in [24].
pect following the appropriate power-counting rules is \(^4\)

\[
6\mathcal{L}_{\text{eff}}^{DM} = \mathcal{C}_V \frac{g_s^2}{M^2} \phi^\dagger \partial_\mu \phi \psi \psi^\dagger \psi^\dagger \partial_\mu \phi^\dagger \partial_\mu \phi B^{\mu\nu} + \mathcal{C}_B \frac{g_s^2}{M^2} \partial_\mu \phi^\dagger \partial_\mu \phi B^{\mu\nu} \\
+ \mathcal{C}_H \frac{g_s^2}{M^2} \partial_\mu \phi^3 |H|^2 + \mathcal{C}_H \frac{g_s^2 m^2}{M^2} \phi^2 |H|^2 \\
+ \mathcal{C}_\psi \frac{g_s^2 g_y^2}{M^2} \phi^3 |\phi|^2 |\psi|^2 H
\]

and at \(D=8\), focussing on operators that contribute to \(2 \to 2\) scattering,

\[
8\mathcal{L}_{\text{eff}}^{DM} = \mathcal{C}_V \frac{g_s^2 m^2}{M^2} |\phi|^2 V^a_{\mu\nu} V^a_{\mu\nu} + \mathcal{C}_\psi \frac{g_s^2 g_y^2}{M^2} |\phi|^2 |\psi|^2 H \\
+ \mathcal{C}_H \frac{g_s^2}{M^2} |\partial^\mu \phi|^2 V^a_{\mu\nu} V^a_{\mu\nu} + \mathcal{C}_H \frac{g_s^2}{M^2} |\partial^\mu \phi|^2 |D^\nu H|^2 \\
+ \mathcal{C}_T \frac{g_s^2}{M^2} |\partial^\mu \phi^3 |\partial^\nu \phi^3 \partial^\lambda \partial^\rho \phi |D^\sigma \phi D^\mu \phi D^\nu \phi D^\lambda \phi D^\rho \phi D^\sigma \phi |
\]

with \(V^a_{\mu\nu} = B_{\mu\nu}, W^a_{\mu\nu}, C^a_{\mu\nu} \), for \((1+1) \times SU(2)_{L} \times SU(3)_{C}\) gauge bosons, and \(\psi, H\), the SM fermions and Higgs. We use a notation based on left-handed Weyl fermions, which carry additional internal indices to differentiate left-handed \(\psi\) and right-handed \((\psi^c)\) components of Dirac fermions \(\psi\); the Wilson coefficients \(c, C\), associated to the \(D = 6, 8\) Lagrangians respectively, carry these indices, and are expected to be \(O(1)\), unless otherwise stated, see table below.

Of course there are more operators that contribute to \(2 \to 2\) scattering, but these can either be eliminated through partial integration, field redefinitions (that eliminate operators proportional to the equations of motion), Bianchi or Fierz identities \(^5\), or they violate some of the linearly realized symmetries that we assume (CP, custodial), as we now discuss. Operators antisymmetric in the Higgs field, such as

\[
c^\ast_{\mu} \frac{g_s^2}{M^2} \partial^\mu \phi H^1 D^\mu H
\]

transform as \((1,3)\) under custodial symmetry \(SU(2)_L \times SU(2)_H\): their coefficient is expected to be generated first at loop level by custodial breaking dynamics, involving for instance \(g'\), which satisfies the required transformation rules \(c^\ast_{\mu} \sim g'^2/16\pi^2\). On the other hand at \(D=8\),

\[
\partial^\mu \phi^3 \partial_\mu \phi H^1 D^\mu H, \quad \partial^\mu \phi^3 \partial_\nu \partial_\mu \phi \psi^\dagger \partial^\nu \phi, \quad \partial^\mu \phi^3 \partial_\nu \partial_\mu \phi \psi^\dagger \partial^\nu \phi
\]

share the same symmetries (among the linearly and nonlinearly realized ones that we have presented \(^6\)) as operators in \(6\mathcal{L}_{\text{eff}}^{DM}\) and contribute to the same observables;

\(^4\) The scaling in powers of the coupling \(g_s\) can be unambiguously determined from a bottom-up perspective by restoring \(h \neq 1\) in the Lagrangian \([22, 26, 27]\): the coefficient \(c_{i}\) of an operator \(O_{i}\) with \(n\) fields scales as \(c_{i} \sim (coupling)^{n-2}\).

\(^5\) We eliminate structures involving \(\sigma^\mu\) in favor of structures that can be generated by tree-level exchange of scalars or vectors.

\(^6\) Technically the set of infinite symmetries of the free Lagrangian for this reason their contribution is expected to be always suppressed by \(\sim E^2/M^2 \ll 1\) in the amplitude and we neglect them. Similarly, \(m_s^2 |\phi|^2 |H|^4\) and \(\partial_\mu \phi^3 \partial_\nu \partial_\mu \phi |H|^2\) give a subleading (by a factor \(g^2 v^2/M^2 \lesssim 1\)) contribution w.r.t. \(c^S_H\) and \(c^H_H\), in processes with 2 longitudinal vectors or Higgses and can only be distinguished in processes with three or more external longitudinal vector bosons/Higgses. Finally operators of the form \(|\phi|^2 \times 6\mathcal{L}_{\text{eff}}^{SM}\), where \(6\mathcal{L}_{\text{eff}}^{SM}\) is the \(D=6\) SM Lagrangian (see \([28]\)) but also includes total derivatives, are generally further suppressed by \(m_s^2/M^2\) and count as \(D=10\) effects in our perspective.

The important aspect that is emphasized by our analysis is that both the \(D=6\) and \(D=8\) Lagrangians can be important, as symmetries can suppress the expected leading interactions in favor of higher order ones. Indeed, the structures \(c^\ast_H\) vanishes for antisymmetry if DM has a single real degree of freedom; on the other hand the structures \(c^S_H\) and \(c^H_H\) are unsuppressed only when the generators associated with \(\phi\) and \(H\) do not commute (such as in the \(SO(6)/SO(5)\) model \([10, 20]\)), but will be suppressed by \(\sim m_s^2/H^2\) in other cases. In this table we summarize situations in which one or more of the above operators are suppressed:

| \(\psi\) | \(c^V\) | \(c^B\) | \(c^S_H\) | \(c^H_H\) | \(c^S_T\) | \(c^T\) |
|---|---|---|---|---|---|---|
| \(\psi_{\text{elem}}\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) |
| \(\psi_{\text{elem}}\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) |
| \(U(1)/Z_2\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) |
| \(SU(2)/U(1)\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) |
| \(SO(6)/SO(5)\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) | \(\times\) |

We denote with \(\psi_{\text{elem}}\) the limit where SM fermions are only partially composite, where the operators marked with \(\times\) are suppressed by the degree of compositeness \(\xi_s^2\) \([19, 23]\) — a favorable situation is when only the right-handed top quark is fully composite \([31]\); see Ref. \([32]\) for a discussion of DM in this case. With \(\psi_{\text{elem}}\) we denote instead the familiar case where the transverse polarizations of vectors are genuinely elementary (as opposed to having strong multipolar interactions \([21]\)): in this case the interactions denoted by \(\times\) are suppressed by \(g^2_v/g_s^2\) (notice that our power-counting only keeps track of symmetry selection rules and powers of couplings – see footnote \(4\); properties of the underlying theory, such as minimal coupling, can lead to additional suppressions \(\sim g^2_v/16\pi^2\), that we neglect in this analysis).

In Fig. 1 we quantify, for the scalar case, the fact that approximate symmetries imply small(large) crossections at small(large) energies, thus providing visible LHC effects compatibly with a non-vanishing RD. For \(D = 6\) (e.g. DM as a PNGB of \(SU(2)/U(1)\)) this dichotomy is
in fact remarkable, as we show in Fig. 1 for the case in which SM fermions are composite and gauge fields are not (see [5] for a more complete discussion and details of the analysis – in particular we have use data from [33] and the techniques of [34, 35] to ensure a consistent use of the EFT for LHC). As shown in the above table, the same reasoning reveals examples where D=8 represent the leading effect at high-E (this includes models where DM has a single degree of freedom, e.g. $U(1)/Z_2$ and $SO(6)/SO(5)$, but also models where the DM dominantly couples to gluons only), the relative LHC constraints are also shown in Fig. 1 with a dashed curve. Notice that here, while the E-growing cross sections implied by our symmetry structure clearly dominate at LHC energies $M \gtrsim E \gg m_{DM}$, they might be comparable to symmetry breaking $m_{DM}$-suppressed interaction at lower- $E$, relevant at freeze-out. In other words, the complementarity between different DM experiments is partially lost in this setup – we discuss this issue further in [5].

**Fermionic Dark Matter.** As mentioned above, if DM is a strongly interacting fermion $\chi$ there are two structurally robust situations in which its mass and low-energy interactions might appear small: chiral symmetry for Dirac fermions or non-linearly realized SUSY for Majorana fermions. The first case is familiar: interactions involving the product $\chi^\dag \sigma^\mu \xi_\mu$ preserve chiral symmetry, while $\chi^\dag \chi$ break it and are expected to be weighed by $m_{DM}/M$.

In the second case, DM fermions are Goldstini of non-linearly realized SUSY. There are different motivations to discuss this scenario. First of all, a supersymmetric version of the equivalence theorem [36] implies that in the high-energy limit $E \gg m_{3/2}$, the gravitino behaves effectively like a Goldstino (in this case, however, the relation $m_{3/2} \sim F/M_{Pl}$ implies – for a SUSY breaking sector at $\sqrt{F} \sim$ TeV, necessary to have sizable LHC effects – a very light gravitino), Goldstini are even more interesting in scenarios where $N = 1$ SUSY is spontaneously broken in $n > 1$ nearly sequestered sectors [37]: in this case $n - 1$ approximate Goldstini appear in the light spectrum and are good DM candidates (their mass being independent from the strength of their interactions). More generally, in an EFT perspective, we can consider the case of approximate $N = 1$ SUSY that, when spontaneously broken, includes light Goldstini in the spectrum [38], and these are good DM candidates.

We work in the simplified limit where all SUSY partners are heavy $m_{susy} \approx \sqrt{F}$ so that the physics of Goldstini at $E \ll \sqrt{F}$ can be described in a formalism that parallels the CCWZ construction [39, 40], adapted to the breaking of spacetime symmetries [41–43]. The coset representative can be written as

$$U = e^{i P x} e^{i \frac{\chi}{2} Q} e^{i \frac{\hat{Q}}{2}}$$

where $Q$ and $\hat{Q}$ are the SUSY generators, $\chi$ the Goldstino, and the presence of momenta $P$ is a peculiarity of spontaneously broken space-time symmetries (it can be somehow thought as due to the fact that translations themselves are realized through coordinates shifts, in a way that mimics non-linear realizations [43]). The Maurer-Cartan form is now

$$U^{-1} \partial_i U = i \left( \delta^a_{\mu} + \frac{i}{2} \partial_\mu \sigma^a \chi^1 \right) P_a + i \frac{1}{2} \partial_\mu \chi^1 Q + \frac{i}{2} \partial_\mu \chi^1 \hat{Q}$$

Here the important building block of the low-energy Lagrangian is $E^a_{\mu} = \delta^a_{\mu} + \frac{i}{2} \partial_\mu \sigma^a \chi^1$, which transforms as a vierbein and plays the analogous rôle as $e_\mu$ for NGBs, rendering a Poincaré-invariant action, written in terms of these building blocks, into one invariant under (non-linear) SUSY. In particular, $\int dx^4 F^2 \det E^a_{\mu} = i \chi^1 \sigma^a \partial_\mu \chi + \cdots$, contains the kinetic term for the canonically normalized Goldstino [44], while interactions with light matter can be described through the vierbein $E^a_{\mu}(\chi)$ and metric $g_{\mu\nu}(\chi) \equiv E^a_{\mu}(\chi) E^a_{\nu}(\chi)$. For our purpose, the important result is that interactions with light fermions $\psi$, scalars $\phi$ or gauge field strengths $F_{\mu\nu}$ are captured by the following $D=8$ operators:

$$\frac{1}{F^2} \chi^1 \bar{\psi}_{\mu} \partial_\mu \chi \bar{\psi}_{\rho} \psi^\rho \quad \frac{1}{F^2} \chi^1 \bar{\psi}_{\mu} \partial_\mu \psi^\rho \bar{\psi}_{\rho} \psi \
\frac{i}{F^2} \chi \bar{\psi}_{\mu} \psi^\mu \bar{\psi}_{\rho} \psi^\rho$$

(8)

An explicit Goldstino mass can only be associated with explicit SUSY breaking (or departures from exact sequestering in [37]), which will generate operators different from Eq. (8), suppressed by $m_{10DM}/M$. Similarly to the scalar case above, we use this fact and power-counting arguments to write the most general effective Lagrangian weighed by the strongest possible interaction that can be achieved in the scenarios under scrutiny, and postpone more restrictions below. At leading order in the $1/M$ expansion,

$$s_{\text{eff}}^{DM} = c^V_\psi \frac{g_{\rm V}^2}{M^2} \chi \bar{\psi}_{\mu} \psi^\mu \bar{\psi}_{\rho} \psi^\rho + c_H \frac{g_{\rm D}^2 M^2}{M^2} \chi \bar{\psi}_{\mu} \psi^\mu \bar{\psi}_{\rho} \psi^\rho + c_D \frac{g_{\rm D}^2 M^2}{M^2} \chi \bar{\psi}_{\mu} \psi^\mu \bar{\psi}_{\rho} \psi^\rho$$

(9)

where the coefficient of $c_H$ reflects the fact that it does not respect the Higgs NGB symmetry (recall that in order for the Higgs to take part in the strong dynamics and be light, it is expected to arise as a PNGB [22]) and can only arise via effects involving SM symmetry breaking couplings, that we denote generically as $g_{\text{SM}}$. At $D=8$ we find,

$$s_{\text{eff}}^{DM} = C^V_\psi \frac{m_\chi y_\psi g_{\rm V}^2}{M^4} \chi \bar{\psi}_{\mu} \psi^\mu \bar{\psi}_{\rho} \psi^\rho + C_H \frac{g_{\rm D}^2 M^2}{M^4} \chi \bar{\psi}_{\mu} \psi^\mu \bar{\psi}_{\rho} \psi^\rho + C_D \frac{g_{\rm D}^2 M^2}{M^4} \chi \bar{\psi}_{\mu} \psi^\mu \bar{\psi}_{\rho} \psi^\rho + C_D' \frac{g_{\rm D}^2 M^2}{M^4} \chi \bar{\psi}_{\mu} \psi^\mu \bar{\psi}_{\rho} \psi^\rho$$

(10)

For generic Wilson coefficients, Eqs. (9,10) represent the most general $D=6,8$ contributions to $2 \rightarrow 2$ on-shell scattering at $D \leq 8$ (for $D=6$ see also [45]). Other structures
either violate underlying symmetries or can be eliminated as described in the scalar case above. In particular it can be shown that only three hermitian operators of the form $D^2\psi^4$ exist at $D=8$ and one, corresponding to the imaginary part of $C_\psi$ in Eq. (10), violates CP and we neglect it. Similarly operators antisymmetric in $H \leftrightarrow H^\dagger$, like $\chi^i \bar{\sigma}^\mu \chi H^{\dagger} D_\mu H$ that plays a rôle in mono-Higgs searches [46], violate custodial symmetry and we neglect them. Moreover, operators of the form $|H|^2 \times g \mathcal{L}_{\text{eff}}$ also appear at $D=8$, but, similarly to the scalar DM case they are are expected to be small (if the Higgs is also a PNGB) and moreover they only affect processes with additional $h$.

So, for composite Dirac fermions, only the $D = 6$ $c_\psi^V$ is important (also, for light DM $c_B^{\text{disp}}$ and $c_H^S$ are constrained by constraints from $Z$ and $h$ decays) in Fig. 2 we show that the LHC is here providing the most important piece of information, accessing the region in parameter space that reproduces the observed RD.\footnote{In addition to our $E$-scaling, renormalization group effect can play a rôle in the precise comparison between LHC and RD probes (see e.g. [47]).} Nevertheless, if $\chi$ is a Goldstino, then the $D = 6$ Lagrangian vanishes in the limit of exact SUSY, and the first strong interactions appear at $D = 8$. In this case only $C_V$ and $C_\psi^{(l)}$ are important for mono-jet analyses. This is an example (similar to the $U(1)/\mathbb{Z}_2$ PNGB) where approximate symmetries, that were invoked to hide strong coupling at small energy, go as far as suppressing the first order $1/M^2$ terms but allow the $1/M^4$ ones. Even in this case the LHC contains important information (dashed line of Fig. 2).

**Outlook.** In Summary, we have discussed natural situations in which light DM originates from a strongly-coupled sector but its interactions are small at low-energies because of approximate symmetries, that forbid relevant interactions and allow only irrelevant (higher-derivative) ones. Prime principles dictate that such symmetries are consistent only with $D=6$ and $D=8$ operators for $2 \rightarrow 2$ scattering, corresponding to DM as PNGB, strongly coupled fermions or Goldstini: we have built their EFT and identified the most relevant effects. These provide a class of models in which the LHC high-$E$ reach plays an important rôle with respect ot other types of experiments (such as RD indications and direct detection) and contains genuinely complementary information. Moreover, in these scenarios the DM EFT is not only consistent with LHC analysis (due to the underlying strong coupling, as shown in Figs. 1 and 2), but also necessary, as the underlying dynamics is uncalculable. Our characterization provides a well-motivated context to model missing transverse-energy distributions at the LHC, in mono-jet, mono-W,Z,\gamma or mono-Higgs searches, with a handful of relevant parameters and yet a clear and consistent microscopic perspective. To the question of what we have learned from LHC DM searches, these models provide one answer.

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