SHARP ESTIMATES FOR THE ZEROS OF THE DERIVATIVE OF OSCILLATING POLYNOMIALS WITH LAGUERRE WEIGHT

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Abstract. Denote by $V_n(\lambda)$ the set of all weighted polynomials of the form $f(x) = e^{-\lambda x} p(x)$ ($\lambda > 0$), where $p$ is an algebraic polynomial of degree $n$ which has $n$ simple real zeros. Given $f \in V_n(\lambda)$, let $x_1 < \cdots < x_n$ and $t_1 < \cdots < t_n$ be the zeros of $f$ and $f'$, correspondingly. Set $h_k := x_{k+1} - x_k$, $k = 1, \ldots, n-1$. We prove sharp estimates of the forms

$$x_k + c_1 h_k \leq t_k \leq x_{k+1} - d_k h_k, \quad k = 1, \ldots, n-1,$$

and

$$x_n + c_n h_{n-1} \leq t_n \leq x_n + d_n h_{n-1},$$

with explicit expressions for the coefficients, depending on $\lambda$. Known estimates of the same type for algebraic polynomials can be obtained by letting $\lambda \to 0$.

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