Switch Control Between Different Speeds for a Passive Dynamic Walker

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Abstract To make a biped robot walk stably at various speeds, a novel switch control approach is proposed to make the gaits switch smoothly between different walking speeds. The switch controller is designed based on the Lyapunov stability theory and the sufficient condition is given to make the closed-loop system stable. This controller can allow the robot to reach the stable gaits corresponding to the various speeds and improve the robustness of switch process. Potential energy compensation control has been studied in the dynamic model of a passive dynamic walking robot with knees. The functional relationship between the initial states and the walking speed is obtained. Numerical simulations are provided to verify the effectiveness of the control strategy.

Keywords Passive Dynamic Walking, Walking Speed Switch, Potential Energy Compensation, Limit Cycle

1. Introduction

The idea of passive dynamic walking was pioneered by McGeer more than a decade ago[1]. A stable walking motion that does not require any external energy source except gravity effect is called passive dynamic walking. McGeer designed several unpowered biped robot prototypes and studied their gravity-induced passive walking down a shallow slope [2]. He demonstrated that the machines can attain a stable natural periodic gait and the passive walking has high energy efficiency. Since then, passive walking has been widely studied by several researchers [3, 4, 5]. However, the passive gaits exist for only shallow slopes and are sensitive to slope magnitude and initial conditions [6, 7, 8]. Thus, the active feedback control laws to yield a stable walking motion have been investigated by several researchers. These control laws were based on the passivity property of the biped robot. Spong has used the potential-shaping control to make the biped robot walk on any slope [9, 10]. Furthermore, the total energy-shaping control for the passive biped robot has been proposed to enlarge the basin of attraction and increase the rate of convergence [11]. A virtual gravity field is introduced to act as a driving force, which realizes the virtual passive walking on level ground [12]. The reinforcement learning-based method was used to control the robot to walk on an uneven floor [13, 14]. Some approaches to producing periodic walking motions while keeping a low energy control have been designed [15-17]. The emphasis in the literature is mainly on how to
enlarge the basin of attraction, increase the rate of convergence and improve the robustness to disturbances, therefore a wide range of behaviours has not been investigated.

In this paper, we are primarily interested in proposing a feedback control method which can make the gaits switch smoothly between different walking speeds. Then the biped robot can walk steadily at the various speeds and the range of walking speeds can be expanded. Russell and Granata have designed the virtual slope control algorithm to control the average walking velocity by modifying the controller coefficients [18]. Yamakita and Asano have combined a virtual passive walk with passive velocity field control (PVFC) to regulate the walking speed effectively [19]. In addition, Grizzle has designed a switching and event-based PI feedback control to regulate the average walking rate to a continuum of values [20, 21]. Holm and Spong have discussed kinetic energy-shaping as a means of controlling the speed while adjusting step length [22]. In addition, a time-scaling control law has also been proposed to regulate the walking speed and control the transition between limit cycles in a single step has been achieved for a biped robot [23, 24]. Other regulation methods which can lead to stable walking at various speeds have also been introduced in [25-27]. Methods in the previous research are effective in regulating the speeds, yet most of them are designed for a biped robot without knees or limitations on the range of walking speeds achievable are encountered. Therefore, the focus of this paper is to design a speed switch control for a biped robot with knees to regulate the speeds within a large range.

The paper is organized as follows: Section 2 presents a dynamic model of the biped robot with knees. Switch control between different speeds for the biped robot with knees is presented in Section 3. In this section, we describe the principle and structure of the switch controller. The stability analysis is also studied in Section 3. Section 4 is the numerical simulations that verify the effectiveness of the methods proposed in this paper. Finally, Section 5 is devoted to conclusions and future work.

2. Dynamic Model of a 2-D Biped Robot with Knees

Figure 1 shows a model of a 2-D biped robot with knees. This robot has no torso and consists of two legs without feet. Each leg has a thigh and a shank connected at the knee joint that has a knee stopper. With the knee stopper, the angle of the knee rotation is restricted like the human knee. Masses concentrate at three points: hip, thigh and shank. Table 1 lists the symbols and physical meanings of the configuration parameters in the model of the biped robot with knees. The walking cycle is divided into four stages, as shown in Figure 2[28]:

Stage I: The stance leg straightens out and the knee is locked, just like a single link. Meanwhile the swing leg with unlock knee comes forward, just like two links connected by a frictionless joint. This stage is called the unlocked swing stage.

Stage II: When the thigh and the shank of the swing leg have the same angle, the knee of the swing leg is locked. Then the swing leg straightens out and the knee-strike occurs. We make the standard assumption that the knee-strike is perfectly inelastic.

Stage III: The knee joint of the swing leg is locked after the knee-strike. Then, the swing leg keeps straight after the knee-strike. Thus, this stage is just like the swing phase of the compass-like robot. This stage is called the locked swing stage.

Stage IV: The swing leg impacts with the ground. This stage is called the heel-strike. The heel-strike is assumed to be inelastic and there is no slipping at the stance leg ground contact. Transfer of support between the swing leg and the stance leg is instantaneous.

Figure 1. Model of the biped robot with knees.

Figure 2. Four stages in one step cycle for the biped robot with knees.
2.1 Equation of the Unlocked Swing Stage

The dynamic equation of the unlocked swing stage obtained by the Euler-Lagrange approach is given as

\[ M_{i}(\Theta_{i})\ddot{\Theta}_{i} + C_{i}(\Theta_{i}, \dot{\Theta}_{i})\dot{\Theta}_{i} + g_{i}(\Theta_{i}) = B_{i}\mu \]  

(1)

where \( M_{i}(\Theta_{i}) \) is the \( 3 \times 3 \) inertia matrix, \( C_{i}(\Theta_{i}, \dot{\Theta}_{i})\dot{\Theta}_{i} \) represents the coriolis and centrifugal terms, and \( g_{i}(\Theta_{i}) \) is the gravity term. \( \Theta_{i} = [\theta_{1}, \theta_{2}, \theta_{3}]^{T} \) is the configuration vector of the robot. \( \mu = [u_{1}, u_{2}, u_{3}] \) is the torque vector. \( u_{1}, u_{2} \) and \( u_{3} \) are the motor torques at the hip joint, knee joint and ankle joint, respectively. The matrices \( M_{i}(\Theta_{i}), C_{i}(\Theta_{i}, \dot{\Theta}_{i}) \), \( B_{i} \) and the vector \( g_{i}(\Theta_{i}) \) are given as:

\[
M_{i}(\Theta_{i}) = \begin{bmatrix}
    m_{11} & m_{12} & m_{13} \\
    m_{21} & m_{22} & m_{23} \\
    m_{31} & m_{32} & m_{33}
\end{bmatrix},
\]

\[
C_{i}(\Theta_{i}, \dot{\Theta}_{i}) = \begin{bmatrix}
    c_{11} & c_{12} & c_{13} \\
    c_{21} & c_{22} & c_{23} \\
    c_{31} & c_{32} & c_{33}
\end{bmatrix},
\]

\[
B_{i} = \begin{bmatrix}
    -1 & -1 & 0 \\
    0 & 0 & 1 \\
    0 & 0 & 1
\end{bmatrix},
\]

\[
g_{i}(\Theta_{i}) = [g_{11}(\Theta_{i}) \quad g_{12}(\Theta_{i}) \quad g_{13}(\Theta_{i})]^{T},
\]

where

\[
m_{11} = m_{12}l_{1}^{2} + m_{2}l_{1}^{2} + (m_{1} + m_{2})l_{2}^{2},
\]

\[
m_{12} = -(m_{1}l_{2} + m_{1}l_{2} + m_{2}l_{2}) \cos(\theta_{2} - \theta_{1}),
\]

\[
m_{12} = m_{1}l_{1} \cos(\theta_{3} - \theta_{2}),
\]

\[
m_{13} = -m_{1}l_{1} \cos(\theta_{3} - \theta_{2}),
\]

\[
m_{21} = m_{1}l_{1} \cos(\theta_{3} - \theta_{2}),
\]

\[
m_{22} = m_{1}l_{1} \cos(\theta_{3} - \theta_{2}),
\]

\[
m_{23} = m_{1}l_{1} \cos(\theta_{3} - \theta_{2}),
\]

\[
c_{11} = 0, c_{12} = (m_{1}l_{2} + m_{1}l_{2} + m_{2}l_{2}) \sin(\theta_{2} - \theta_{1}),
\]

\[
c_{12} = m_{1}l_{1} \sin(\theta_{3} - \theta_{2}),
\]

\[
c_{13} = m_{1}l_{1} \sin(\theta_{3} - \theta_{2}),
\]

\[
c_{21} = -(m_{1}l_{2} + m_{1}l_{2} + m_{2}l_{2}) \sin(\theta_{2} - \theta_{1}),
\]

\[
c_{22} = 0,
\]

\[
c_{23} = -(m_{1}l_{2} + m_{1}l_{2} + m_{2}l_{2}) \sin(\theta_{2} - \theta_{1}),
\]

\[
c_{31} = m_{1}l_{1} \sin(\theta_{3} - \theta_{2}),
\]

\[
c_{32} = m_{1}l_{1} \sin(\theta_{3} - \theta_{2}),
\]

\[
c_{33} = 0.
\]

2.2 Knee-Strike Equation

In Stage II, the knee-strike results in an instantaneous change of the velocities of the joints, but the configuration of the biped robot remains invariant. Then the pre-impact and the post-impact configurations of the robot can be expressed as \( \Theta_{i}^{p} = \Theta_{i}^{f} \), where the index “p” means before the knee-strike and the index “f” means after the knee-strike. Based on the conservation of angular momentum, we obtain the knee-strike equation as follows:

\[
\begin{bmatrix}
    \Theta_{1}^{p} \\
    \Theta_{2}^{p} \\
    \Theta_{3}^{p}
\end{bmatrix} = H_{1}(\Theta_{i}^{f}) \dot{\Theta}_{i}^{f}
\]

(2)

The matrix \( H_{1}(\Theta_{i}^{f}) = Q_{1} \cdot Q_{1}^{T} \), where

\[
Q_{1} = \begin{bmatrix}
    Q_{11} & Q_{12} & Q_{13} \\
    Q_{21} & Q_{22} & Q_{23} \\
    Q_{31} & Q_{32} & Q_{33}
\end{bmatrix},
\]

\[
D = \begin{bmatrix}
    D_{11} & D_{12} \\
    D_{21} & D_{22}
\end{bmatrix},
\]

\[
Q_{11} = -m_{1}l_{2} \cos(\theta_{1} - \theta_{2}) - m_{1}l_{2} \cos(\theta_{1} - \theta_{2}) + (m_{2} + m_{1}l_{1})L^{2} + m_{2}l_{2}^{2} + m_{1}l_{3}^{2},
\]

\[
Q_{12} = -(m_{1}l_{2} + m_{1}l_{2} + m_{2}l_{2}) \cos(\theta_{1} - \theta_{2}) + m_{1}l_{1} \cos(\theta_{2} - \theta_{3}) + m_{2}l_{2}^{2} + m_{1}l_{2}^{2},
\]

\[
Q_{13} = -m_{1}l_{1} \cos(\theta_{1} - \theta_{2}) - m_{1}l_{1} \cos(\theta_{1} - \theta_{2}) + m_{1}l_{1}^{2},
\]

\[
Q_{21} = -(m_{2}l_{2} + m_{2}l_{2}) \cos(\theta_{2} - \theta_{3}) - m_{1}l_{1} \cos(\theta_{1} - \theta_{2}) + m_{2}l_{2}^{2} + m_{1}l_{2}^{2},
\]

\[
Q_{22} = m_{1}l_{1} \cos(\theta_{2} - \theta_{3}) + m_{2}l_{2}^{2} + m_{1}l_{2}^{2},
\]

\[
Q_{23} = m_{1}l_{2} \cos(\theta_{2} - \theta_{3}) + m_{2}l_{2}^{2} + m_{1}l_{2}^{2},
\]

\[
D_{11} = D_{21} + (l_{1} + a_{2})^{2} + (m_{1} + m_{2})L^{2} + m_{1}a_{2}^{2},
\]

\[
D_{12} = D_{21} + m_{1}(l_{2} + b_{1})^{2} + m_{2}b_{2}^{2},
\]

\[
D_{21} = -(m_{1}l_{1} + l_{2}) + m_{2}b_{2}L \cos(\theta_{1} - \theta_{2}) + m_{2}l_{2}^{2} + m_{1}l_{2}^{2},
\]

\[
D_{22} = m_{2}(l_{1} + b_{1})^{2} + m_{2}b_{2}^{2}.
\]
2.3 Equation of the Locked Swing Stage

In Stage III, the knee-joint is locked and the swing leg keeps straight after the knee-strike. The model in this stage is just like the compass-like robot. Thus, the swing equation of the compass-like robot can be used to describe the locked swing stage. The equation of the locked swing stage is

\[ M_2(\Theta_2)\ddot{\Theta}_2 + C_2(\Theta_2, \dot{\Theta}_2)\dot{\Theta}_2 + g_2(\Theta_2) = B_2u \]  

(3)

where \( M_2(\Theta_2) \) is the terms and \( g_2(\Theta_2) \) is the gravity terms. \( \Theta_2 = [\theta_1 \theta_2]^T, \quad B_2 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad u = [u_1 u_2]^T \) is the torque vector at the hip and ankle joints. Since the configuration vector just after the knee-strike is \([3 \times 1]\) and the configuration vector in the locked swing stage is \([2 \times 1]\), the initial state of Stage III satisfies

\[ \begin{bmatrix} \Theta_2(0) \\ \dot{\Theta}_2(0) \end{bmatrix} = F \cdot \begin{bmatrix} \Theta_1^T \\ \dot{\Theta}_1^T \end{bmatrix} \]

where \([\Theta_2(0) \dot{\Theta}_2(0)]^T\) is the initial state of Stage III,

\[ F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \]

The matrices \( M_2(\Theta_2), \ C_2(\Theta_2, \dot{\Theta}_2), \ g_2(\Theta_2) \) are given as

\[ M_2(\Theta_2) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \quad C_2(\Theta_2, \dot{\Theta}_2) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \]

\[ g_2(\Theta_2) = [g_{21}(\Theta_2) \ g_{22}(\Theta_2)]^T, \]

where

\[ m_{11} = m_1^2 + m_2(l_1 + a_2)^2 + (m_1 + m_2)L^2, \]

\[ m_{12} = -(m_2b_2L + m_1(L(l_2 + b_1)))(\cos(\theta_2 - \theta_1)), \]

\[ m_{21} = m_{12}, \quad m_{22} = m_2b_2^2 + m_1(l_2 + b_1)^2, \]

\[ c_{11} = 0, \quad c_{21} = 0, \]

\[ c_{12} = -(m_2b_2L - m_1(l_2 + b_1))\dot{\theta}_2 \sin(\theta_1 - \theta_2), \]

\[ c_{21} = (m_2b_2L + m_1(l_2 + b_1))\dot{\theta}_1 \sin(\theta_1 - \theta_2), \]

\[ g_{21}(\Theta_2) = -(m_1L + m_1 + m_2)gL\sin(\theta_1 - m_1g_1 \sin(\theta_1) - m_2g_2(l_2 + a_2) \sin(\theta_1), \]

\[ g_{22}(\Theta_2) = (m_2b_2L + m_1(l_2 + b_1))g \sin(\theta_2). \]

2.4 Heel-Strike Equation

Since the robot with knees can be regarded as the compass-like robot after the knee-strike, the heel-strike transition equation of the compass-like robot can be applied directly to the biped robot with knees. The heel-strike equation is listed as follows:

\[ \begin{bmatrix} \Theta_1^+ \\ \dot{\Theta}_1^+ \end{bmatrix} = \Theta_2 \]

\[ \dot{\Theta}_1^+ = H_2(\Theta_2)\dot{\Theta}_2 \]

(4)

where the index “-” means before the heel-strike and the index “+” means after the heel-strike. \( H_2(\Theta_2) = \left( P_2^+ \right)^{-1} P_2^- \),

\[ P_1^+ = \begin{bmatrix} P_{11}^+ & P_{12}^+ \\ P_{21}^+ & P_{22}^+ \end{bmatrix}, \]

\[ P_2^- = \begin{bmatrix} P_{11}^- & P_{12}^- \\ P_{21}^- & P_{22}^- \end{bmatrix}, \]

\[ P_{11}^+ = -(m_1(l_1 + a_2) + m_2b_2L)\cos(\theta_1 + m_2a_2^2), \]

\[ m_2(l_2 + b_1)^2 + m_2a_2^2, \]

\[ P_{12}^+ = -(m_1(l_1 + a_2) + m_2b_2L)\cos(\theta_1 + m_2a_2^2) \cos(\theta_1), \]

\[ m_2(l_2 + b_1)^2 + m_2a_2^2, \]

\[ P_{21}^+ = -(m_1(l_1 + a_2) + m_2b_2L)\cos(\theta_1), \]

\[ m_2(l_2 + b_1)^2 + m_2a_2^2, \]

After the heel-strike, the previous swing leg becomes the new stance leg and the previous stance leg becomes the new swing leg. Therefore, the initial state of Stage I satisfies

\[ \begin{bmatrix} \Theta_1(0) \\ \dot{\Theta}_1(0) \end{bmatrix} = E \cdot \begin{bmatrix} \Theta_2^+ \\ \dot{\Theta}_2^+ \end{bmatrix}, \]

where \([\Theta_1(0) \dot{\Theta}_1(0)]^T\) is the initial state of Stage I,

\[ E = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]
3. Switch Control between Different Walking Speeds

In this paper, the walking speed is defined as the average walking speed. Thus, the walking speed $\bar{v}$ can be easily computed as $\bar{v} = \frac{d}{T}$, where the step length $d$ is the distance between the ground contact points of the legs at the moment of impact; $T$ is the period of the single step.

In this section, we introduce the theory of the potential energy compensation control first. Then, the results of the potential energy compensation control are extended to the biped robot with knees to solve the stable walking at the ideal speed. Finally, we propose a switch control between different speeds to realize stable walking at the various speeds.

3.1 Potential Energy Compensation Control for the Biped Robot with Knees

Several researchers have shown that regulations of walking speeds may be accomplished via potential energy-shaping control. From a given passive limit cycle, we can obtain the active limit cycle with any desired speed. These results are given by a theorem proved in [29]. Contents of the theorem are given as follows:

**Theorem 1.** Suppose we are given a vector of initial state $X = X(\bar{v}_0) = (\dot{\theta}(0), \dot{\theta}(0))^T$. With this initial state, a stable passive limit cycle with speed $\bar{v}_0$ exists. For any desired speed $\bar{v}$, define $f = (\frac{\bar{v}}{\bar{v}_0})^2$. Then, with the control law:

$$u = g(\theta) - f \cdot g(\theta)$$  \hspace{1cm} (5)

where $g(\dot{\theta})$ is gravity term of the robot, there is a stable limit cycle with speed $\bar{v}$, and the initial state is $X = X(\bar{v}) = (\dot{\theta}(0), \sqrt{f} \cdot \dot{\theta}(0))^T$.

We suppose that the biped robot with knees has a passive walking gait at speed $\bar{v}_0$. Motivated by theorem 1, we propose the potential energy compensation control law for the biped robot with knees. The controller in the unlocked swing stage is designed as

$$u = B_1^{-1}(g_1(\Theta_1) - f \cdot g_1(\Theta_1))$$  \hspace{1cm} (6)

Then, the dynamic equation of the unlocked swing stage is transformed to be

$$M_1(\Theta_1)\ddot{\Theta}_1 + C_1(\Theta_1, \dot{\Theta}_1)\dot{\Theta}_1 + f \cdot g_1(\Theta_1) = 0$$  \hspace{1cm} (7)

Moreover, the torque vector of the locked swing stage is designed as

$$u = B_2^{-1}(g_2(\Theta_2) - f \cdot g_2(\Theta_2))$$  \hspace{1cm} (8)

The equation of the locked swing stage is transformed to be

$$M_2(\Theta_2)\ddot{\Theta}_2 + C_2(\Theta_2, \dot{\Theta}_2)\dot{\Theta}_2 + f \cdot g_2(\Theta_2) = 0$$  \hspace{1cm} (9)

With control laws (6) and (8), the biped robot with knees can walk stably at the desired speed $\bar{v}_0$.

Consequently, the numerical simulations are performed to demonstrate the effectiveness of the potential energy compensation controllers for the biped robot with knees. The values of parameters used in simulation are $m=1[kg]$, $m_1=0.1[kg]$, $m_2=1[kg]$, $a_0=0.375[m]$, $L=1[m]$, $b_1=0.125[m]$, $a_2=0.175[m]$, $b_s=0.325[m]$. $\Phi=0.0524$[radian]. For $u=0$, the biped robot with knees has a passive limit cycle with speed $0.9456m/s$. Initial states of the passive dynamic walking are

$$[0.1902, -0.2948, -0.2948, -1.1162, -0.0203, -0.0203]^T$$.

Thus, the initial states in the simulation are set to be

$$[0.1902, -0.2948, -0.2948, -1.1162, \sqrt{f}, -0.0203, \sqrt{f}, -0.0203, \sqrt{f}]^T$$.

We use MATLAB to carry out the simulations. Table 2 lists the speed values for the different limit cycles. Both of them illustrate that the potential energy compensation control for the biped robot with knees can drive the robot to walk at the desired speed. Figure 3 and Figure 4 show the curves of the kinetic energy and the potential energy for the different $f$ in a single cycle step. It is demonstrated that the potential energy and the kinetic energy all increase with accelerating the speeds. The bigger $f$ is, the greater the compensational potential energy. Then, there is more energy to compensate the lost energy during the knee-strike and heel-strike. Thus, the kinetic energy can be increased to accelerate the walking speed for the biped robot with knees.

| $f$ | 1  | 2  | 3  | 4  |
|-----|----|----|----|----|
| $\bar{v}$ | 0.9456 | 1.8911 | 2.8367 | 3.7823 |

**Table 1.** $f$ and the average walking speeds

![Figure 3. Kinetic energy in a single step](image)
Since switch theory, switching desired linearization. Compensating respectively. Above the given, the knees regulating the heel strike, we make the walk process. Considering the equation of the unlock swing stage in the $i$th step of the speed switching process

$$M_1(\Theta_1)\ddot{\Theta}_1 + C_1(\Theta_1, \dot{\Theta}_1)\dot{\Theta}_1 + f_1 \cdot g_1(\Theta_1) = B_1u$$

(10)

the torque vector in Eq. (10) is designed as

$$u = B_1^{-1}[M_1(\Theta_1)\ddot{\Theta}_1 + C_1(\Theta_1, \dot{\Theta}_1)\dot{\Theta}_1 + f_1 \cdot g_1(\Theta_1) + M_1(\Theta_1)K_1(\Theta_1^i - \Theta_1)]$$

(11)

Meanwhile, the equation of the locked swing stage in the $i$th step of the speed switching process is

$$M_2(\Theta_2)\ddot{\Theta}_2 + C_2(\Theta_2, \dot{\Theta}_2)\dot{\Theta}_2 + f_1 \cdot g_2(\Theta_2) = B_2u$$

(12)

We design the torque vector in Eq.(12) to be as

$$u = B_2^{-1}[M_2(\Theta_2)\ddot{\Theta}_2 + C_2(\Theta_2, \dot{\Theta}_2)\dot{\Theta}_2 + f_1 \cdot g_2(\Theta_2) + M_2(\Theta_2)K_2(\Theta_2^i - \Theta_2)]$$

(13)

where $K_1 = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}$, $K_2 = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}$. $(\Theta_1^i, \dot{\Theta}_1^i)$ and $(\Theta_2^i, \dot{\Theta}_2^i)$ are the state trajectories for walking at the speed $\overline{v}_2$.

Let $c_1 = \Theta_1 - \Theta_1^i$, $c_2 = \Theta_2 - \Theta_2^i$. The closed-loop system in the $i$th step of the speed switching process is

$$\begin{cases} \dot{c}_1 + K_1 \cdot c_1 = 0 \\ \dot{c}_2 + K_2 \cdot c_2 = 0 \end{cases}$$

(14)

Let $t_i^k$ be the starting time of the $i$th step in the speed switching process and $t_i^k$ be the time instant at which the knee-strike happens in the $i$th step of the speed switching process. Meanwhile, let $t_i^{h_1}$ be the time instant at which heel-strike happens in the $i$th step of the speed switching process and $t_i^{h_2}$ also be the starting time of the $(i+1)$th step in the speed switching process.

In the $i$th step of the speed switching process, the equations of the knee-strike and heel-strike are respectively

$$\begin{cases} \Theta_1(t_i^k) = \Theta_1(t_i^{h_1}) \\ \dot{\Theta}_1(t_i^k) = H_1(\Theta_1(t_i^{h_1}))\dot{\Theta}_1(t_i^{h_1}) \end{cases}$$

(15)

$$\begin{cases} \Theta_2(t_i^k) = \Theta_2(t_i^{h_1}) \\ \dot{\Theta}_2(t_i^k) = H_2(\Theta_2(t_i^{h_1}))\dot{\Theta}_2(t_i^{h_1}) \end{cases}$$

(16)

Then, the states just before and after the knee-strike satisfy

$$\begin{cases} c_i^1(t_i^{h_1}) = c_i^1(t_i^k) \\ c_i^1(t_i^{h_1}) = H_1(\Theta_1(t_i^{h_1})) - H_1(\Theta_1(t_i^k)) \cdot c_i^1(t_i^k) \end{cases}$$

(17)

$$\begin{cases} \dot{c}_i^1(t_i^{h_1}) = \dot{c}_i^1(t_i^k) \\ \dot{c}_i^1(t_i^{h_1}) = \dot{c}_i^1(t_i^{h_1}) \cdot \dot{c}_i^1(t_i^k) \end{cases}$$

The states just before and after the heel-strike satisfy

$$\begin{cases} c_i^2(t_i^{h_1}) = c_i^2(t_i^k) \\ c_i^2(t_i^{h_1}) = H_2(\Theta_2(t_i^{h_1})) - H_2(\Theta_2(t_i^k)) \cdot c_i^2(t_i^k) \end{cases}$$

(18)

$$\begin{cases} \dot{c}_i^2(t_i^{h_1}) = \dot{c}_i^2(t_i^k) \\ \dot{c}_i^2(t_i^{h_1}) = \dot{c}_i^2(t_i^{h_1}) \cdot \dot{c}_i^2(t_i^k) \end{cases}$$

If we select the appropriate gains $K_1$ and $K_2$, the errors exponentially decrease in the speed switching process. Since $H_1()$ and $H_2()$ are differentiable everywhere and locally Lipschitz, there are the constants $\alpha_1^i > 0$ and $\alpha_2^i > 0$, such that

$$\begin{cases} \|H_1(\Theta_1^i(t_i^k)) - H_1(\Theta_1^i(t_i^k))\cdot \dot{\Theta}_1(t_i^k)\| \leq \alpha_1^i \|\dot{\Theta}_1^i(t_i^k)\| \\ \|H_2(\Theta_2^i(t_i^k)) - H_2(\Theta_2^i(t_i^k))\cdot \dot{\Theta}_2(t_i^k)\| \leq \alpha_2^i \|\dot{\Theta}_2(t_i^k)\| \end{cases}$$

$$\begin{cases} \|H_1(\Theta_1^i(t_i^{h_1})) - H_1(\Theta_1^i(t_i^{h_1}))\cdot \dot{\Theta}_1(t_i^{h_1})\| \leq \alpha_1^i \|\dot{\Theta}_1(t_i^{h_1})\| \\ \|H_2(\Theta_2^i(t_i^{h_1})) - H_2(\Theta_2^i(t_i^{h_1}))\cdot \dot{\Theta}_2(t_i^{h_1})\| \leq \alpha_2^i \|\dot{\Theta}_2(t_i^{h_1})\| \end{cases}$$
Let $\beta_1 = \| H_1(\Theta(t_i^k)) \|$ and $\beta_2 = \| H_2(\Theta(t_i^k)) \|$. From Eq. (17) and Eq. (18), we have

$$
\begin{align*}
\| e_1^i(t_i^k) \|^2 &\leq \alpha_1 \| e_1^i(t_i^k) \|^2 + \beta_1 \| e_1^i(t_i^k) \|^2, \\
\| e_2^i(t_i^k) \|^2 &\leq \alpha_2 \| e_2^i(t_i^k) \|^2 + \beta_2 \| e_2^i(t_i^k) \|^2.
\end{align*}
$$

Since

$$
\begin{align*}
\begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 &\leq \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 + \alpha_1 \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2, \\
&+ \beta_1 \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 + 2\alpha_1 \beta_1 \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix},
\end{align*}
$$

and

$$
\alpha_1 \beta_1 \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 \leq (\alpha_1^2 \beta_1^2) \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2,
$$

thus

$$
\begin{align*}
\begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 &\leq \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 + 2\alpha_1^2 \beta_1^2 \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 + 2\alpha_1 \beta_1 \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}.
\end{align*}
$$

Denote $\alpha_i = \max(1 + 2(\alpha_1^2 \beta_1^2), 2(\beta_1^2))$,

where $\alpha_i$ depends on the states just before the knee-slip in the $i$th step of the speed switching process and the walking at speed $\bar{v}_2$.

From Eq. (19), we have

$$
\begin{align*}
\begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 &\leq \alpha_i \begin{bmatrix} e_1^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2
\end{align*}
$$

Similarly, a constant $\alpha_i > 0$ exists such that

$$
\begin{align*}
\begin{bmatrix} e_2^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2 &\leq \alpha_i \begin{bmatrix} e_2^i(t_i^k) \\ e_2^i(t_i^k) \end{bmatrix}^2.
\end{align*}
$$

where $\alpha_i$ depends on the states just before the heel-slip in the $i$th step of the speed switching process and the walking at speed $\bar{v}_2$.

In the $i$th step, Eq. (14) can be transformed as

$$
\begin{align*}
\frac{d}{dt} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} &\begin{bmatrix} I_2 & 0 \\ -K_1 & 0 \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} + \begin{bmatrix} e^2_1(t_i^{(i-1)}) \\ e^2_2(t_i^{(i-1)}) \end{bmatrix} \\
\begin{bmatrix} e_1(t_i^{(i)}) \\ e_2(t_i^{(i)}) \end{bmatrix} &= T_e \begin{bmatrix} e_i^k(t_i^k) \\ e_i^k(t_i^k) \end{bmatrix},
\end{align*}
$$

where $I_3$ is the third-order identity matrix and $I_2$ is the second-order identity matrix. Solutions of Eq. (22) and Eq. (23) can be represented as respectively

$$
\begin{align*}
\begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} &= \exp\left[ \begin{bmatrix} I_3 & 0 \\ -K_1 & 0 \end{bmatrix} (t-t_i^k) \right] \cdot \begin{bmatrix} e_1(t_i^{(i-1)}) \\ e_2(t_i^{(i-1)}) \end{bmatrix}, \\
\begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} &= \exp\left[ \begin{bmatrix} I_2 & 0 \\ -K_2 & 0 \end{bmatrix} (t-t_i^k) \right] \cdot \begin{bmatrix} e_i^k(t_i^k) \\ e_i^k(t_i^k) \end{bmatrix}
\end{align*}
$$

Denote

$$
\begin{align*}
A(t) &\triangleq \exp\left[ \begin{bmatrix} I_3 & 0 \\ -K_1 & 0 \end{bmatrix} (t-t_i^k) \right], \\
B(t) &\triangleq \exp\left[ \begin{bmatrix} I_2 & 0 \\ -K_2 & 0 \end{bmatrix} (t-t_i^k) \right].
\end{align*}
$$

When the gains $K_1$ and $K_2$ satisfy the following conditions, the biped robot with knees can realize the smooth switch between the different speeds.

**Theorem 2.** If $K_1$ and $K_2$ in the $i$th step of the speed switching process satisfy the following conditions simultaneously

\(1\) $\max_{j \neq k} \lambda_j \leq \frac{1}{\alpha_j^{k-1}} \| H \|^2$, where $\lambda_j$ is the eigenvalue of $A(t) \cdot A(t)$, $j = 1, 2, \cdots, 6$;

\(2\) $\max_{j \neq k} \lambda_j \leq \frac{1}{\alpha_j^{k-1}} \| A \|^2$, where $\lambda_j$ is the eigenvalue of $B(t) \cdot B(t)$, $j = 1, 2, 3, 4$.

Then

$$
\begin{align*}
\lim_{t \to \infty} e_1(t) &= 0, \quad \lim_{t \to \infty} e_1(t) = 0, \quad \lim_{t \to \infty} e_2(t) = 0, \quad \lim_{t \to \infty} e_2(t) = 0.
\end{align*}
$$
Proof: We assume that $V_1(t_{k_{i+1}})$ represents as the error values just before the knee-strike in the $(i+1)$ step, namely

$$V_1(t_{k_{i+1}}) = \|e_1(t_{k_{i+1}})\|^2 + \|c_1(t_{k_{i+1}})\|^2.$$  \hspace{1cm} (26)

Since

$$V_1(t_{k_{i+1}}) = \left\| \begin{bmatrix} e_1(t_{k_{i+1}}) \\ c_1(t_{k_{i+1}}) \end{bmatrix} \right\|^2 = A(t_{k_{i+1}}) \cdot E \cdot \left[ \begin{bmatrix} e_2(t_{k_{i}}) \\ c_2(t_{k_{i}}) \end{bmatrix} \right]^2,$$

then $V_1(t_{k_{i+1}}) \leq \frac{1}{\alpha_{k_{i}}} \left\| \begin{bmatrix} e_2(t_{k_{i}}) \\ c_2(t_{k_{i}}) \end{bmatrix} \right\|^2$.

From (20), (21) and (25), we have

$$V_1(t_{k_{i+1}}) \leq \frac{1}{\alpha_{k_{i}}} \cdot \alpha_{k_{i}} \left\| \begin{bmatrix} e_2(t_{k_{i}}) \\ c_2(t_{k_{i}}) \end{bmatrix} \right\|^2$$

$$\leq B(t_{k_{i}}) \cdot F \cdot \left[ \begin{bmatrix} e_1(t_{k_{i}}) \\ c_1(t_{k_{i}}) \end{bmatrix} \right]^2$$

$$\leq \alpha_{k_{i}} \left\| \begin{bmatrix} e_1(t_{k_{i}}) \\ c_1(t_{k_{i}}) \end{bmatrix} \right\|^2$$

$$\leq V_1(t_{k_{i}}).$$

Since $V_1(t_{k_{i+1}}) \leq V_1(t_{k_{i}})$ and $V_1(t_{k_{i}})$ is nonnegative, we have

$$\lim_{i \to \infty} V_1(t_{k_{i}}) = 0.$$  \hspace{1cm} (27)

From Eq. (27), we have

$$\lim_{i \to \infty} e_1(t_{k_{i}}) = 0, \quad \lim_{i \to \infty} c_1(t_{k_{i}}) = 0.$$  \hspace{1cm} (28)

That is to say that the state just before the knee-strike in the each step is asymptotically converged to the state just before the knee-strike of the walking at desired speed $p_2$.

Also from (20), we have $\lim_{i \to \infty} e_1(t_{k_{i}}) = 0, \lim_{i \to \infty} c_1(t_{k_{i}}) = 0$. It can be deduced from Eq. (25) that

$$\lim_{i \to \infty} e_2(t_{k_{i}}) = 0, \lim_{i \to \infty} c_2(t_{k_{i}}) = 0.$$  \hspace{1cm} (29)

Eq. (29) illustrates that the state just before the heel-strike eventually converges to the state just before the heel-strike of the walking at speed $p_2$. Hence the robot can walk at speed $p_2$. Gaits of walking at speed $p_2$ finally switch to the gaits of walking at speed $p_2$, namely,

$$\lim_{i \to \infty} e_1(t) = 0, \lim_{i \to \infty} c_1(t) = 0, \lim_{i \to \infty} e_2(t) = 0, \lim_{i \to \infty} c_2(t) = 0.$$  

4. Simulation Results

In this section, we describe an example of a biped robot with knees walking at the various speeds using our proposed approach. Adjustment of speeds is done from 0.8446m/s to 1.4628m/s. The model and parameters used in these simulations are the same as the ones in section 3.1. Figure 5 describes the changes of speeds. The biped robot walks at the speed of 0.8446m/s for ten steps, then regulates the speed to 1.4628m/s, and finally walks at the speed of 1.4628m/s for ten steps. Figure 6 is the angular velocity trajectories of the hip joint in this process. Figure 7 shows the energy changing results with regulating the speeds. Figure 8 presents the input torques with changing the speeds. Comparing with the desired speed, the results show that the speed from 0.8446m/s switches to 1.4628m/s successfully and speed switching process is smooth and natural.

![Figure 5. Walking speeds with changing the step numbers](image)

![Figure 6. The angular velocity trajectories of the hip joint with regulating the speeds](image)
Figure 7. The kinetic energy and the potential energy with regulating the speeds

Figure 8. Input torques in the simulation
5. Conclusions

In order to make the biped robot with knees walk at the various speeds, a control algorithm named the speed switch control is proposed in this paper. The speed switch control ensures smooth switching between the different gait speeds and also expands the range of regulating the speeds achievable. Based on the theory of Lyapunov stability, the sufficient condition is given, which ensures the closed-loop system is stable. In addition, the stability analysis has been done. Finally the effectiveness of the control algorithm is verified with the numerical simulations. It is shown that the speed switch controller can not only realize smooth switching between different speeds, it also has strong robustness.

How to use the speed switch control in a biped robot prototype is the subject for future research.

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