The interplay between electron-electron interactions and impurities in one-dimensional rings

Hiroyuki Mori
Department of Physics, Indiana University, Bloomington, IN 47405
(July 14, 2021)

The persistent current and charge stiffness of a one-dimensional Luttinger liquid on a ring threaded by a magnetic flux are calculated by Monte Carlo simulation. By changing the random impurity potential strength and the electron-electron interaction, we see a crossover behavior between weak and strong impurity limits. For weak impurity potentials, interactions enhance impurity effects, that is, interactions decrease the current and the stiffness. On the other hand, interactions tend to screen impurities when the impurity potential is strong. Temperature dependence of the persistent current and the charge stiffness shows a peak at a characteristic temperature, consistent with a recent single impurity study.

72.10.-d, 71.27.+a, 72.15.Rn

Recent technological progress now allows us to produce nearly ideal low-dimensional systems, and it is becoming more important to understand how disorder and electron-electron interactions control physics in such systems. In one-dimensional systems, Giamarchi and Schultz derived renormalization group (RG) equations for systems with weak but finite density of impurities, and showed that the critical impurity strength is zero for the Luttinger liquid parameter $g > 2/3$ and is finite for $g < 2/3$, where $g > 1$ and $g < 1$ correspond to repulsive and attractive interactions, respectively. Scaletti et al. confirmed a part of the phase diagram given by Giamarchi and Schultz, using quantum Monte Carlo (MC) simulation of the boson Hubbard model. For finite-size systems, current can flow even in the presence of impurities for any $g$ because of the finite size. One interesting example of a finite-size system is a one-dimensional ring threaded by magnetic flux, which creates persistent current as first predicted by Byers and Yang and later by Büttiker et al. When an electron gas moves in a dirty ring, we expect the persistent current to be suppressed as the impurity potential becomes stronger. What then will happen if electron-electron interactions are turned on? Krive et al. calculated persistent current of Wigner crystal on a ring with a single impurity in certain limits. From their expressions we can see the current decreases as we increase particle-particle repulsive interactions. This study indicates that interactions effectively enhance the impurity strength, although their system has only one impurity. For a finite density of weak impurities, we can use RG equations obtained by Giamarchi and Schultz, placing a cutoff in the length scale since the system is finite. Then the effective impurity strength $\tilde{W}$ is given by $\tilde{W} \sim L^{3-2/g}$, where $L$ is the system size and $g$ is the Luttinger liquid parameter which becomes large as repulsive interactions increase. Here we can see again that interactions would enhance the effective impurity strength.

There have been several exact diagonalization studies done for the ring problem. These calculations were performed in diffusive regime, i.e., for relatively weak impurities, and showed qualitatively the same results as the above analytical works.

This interplay between interactions and impurities can be understood in the following way. When impurities are weak enough, the system still resides in the Luttinger liquid regime. Luttinger liquids have charge-density wave (CDW) correlations which grow as the repulsive interaction becomes strong. Since a CDW would be easily pinned by impurities, stronger interactions make the system pinned more easily. Therefore the persistent current would decrease when interactions increase.

Now the question is what will happen with strong impurities. It seems clear that interactions would screen the impurities, which is opposite to what is expected in the weak impurity limit. When the impurities are very strong, particles are almost localized in the low potential regions. If we turn on repulsive interactions between particles, it will move the particles away from each other, and therefore the interactions would effectively weaken the impurity strength.

In this letter we will show the crossover behavior between these weak and strong impurity limits. We have calculated the persistent current and charge stiffness of a spinless Luttinger liquid on a ring with a finite density of impurities, using Monte Carlo simulation. One advantage of working with a Luttinger liquid instead of a microscopic lattice model is that we can separate out the lattice (periodic potential) effect, which is another important factor changing those quantities, so we can concentrate on the interplay between impurities and interactions only. We also show the temperature dependence of persistent current and charge stiffness. These have peaks at certain temperatures, as previously argued for the single impurity problem.

The problem of Luttinger liquid on an impurity-free ring penetrated by a magnetic flux was solved by Loss. He showed that the Lagrangian is given at $T \ll \pi g_s/2L$ by

$$\mathcal{L} = \frac{g_u}{8\pi} (\partial_\mu \theta)^2 + (N - 1 + 2\phi/\phi_0) \frac{i}{2L} \partial_\tau \theta, \quad (1)$$
where $\mu = \tau$ and $x$. The boundary condition of $\theta$ field is $\theta(x + L, \tau + \beta) = \theta(x, \tau) + 2n\pi$, where $n$ is a winding number. The derivatives of $\phi$ give the particle density and current: $\partial_\mu \theta = -2\pi \partial_\mu \phi$. Defining $\bar{\theta} \equiv \theta - 2n\pi/\beta$, we obtain the partition function $Z$ in the following form,

$$Z = \sum_n \int d\bar{\theta} \left( -1 \right)^{(N-1)n} \cos(2\pi n \phi/\phi_0) e^{-\beta \bar{\theta}^2/2} e^{-\int \bar{\theta}}$$

where $\bar{\theta} = (g\mu/8\pi)(\partial_\mu \bar{\theta})^2$, and $T_0 = 2/g_L L \pi$. In the following calculation we assume the particle number $N$ is odd, since we can get the physical quantities for the cases where $N$ is even by using the relation $Z(N \text{ even}); \phi/\phi_0) = Z(N \text{ odd}; \phi/\phi_0 + 1/2)$. Haldane [4] calculated the relation of $g_\tau$ and $g_{\bar{\theta}}$ to the microscopic parameters of a spinless fermion system with nearest neighbor interactions. $g_{\bar{\theta}}$ varies with density but does not change very much with interaction as long as the interaction is not too large, while $g_{\bar{\theta}}$ changes with both density and interaction. The sound velocity $v_s$ is given by $\sqrt{g_{\tau}/g_{\bar{\theta}}}$.

The impurity term is written in the microscopic model as $\int dx V_{imp}(x) \rho(x)$. Following Ref. [5] we obtain

$$\int V_{imp}(x) \rho(x) = \int \frac{1}{2\pi} V_{imp}(x) \bar{\theta} + \int V_{imp}(x) \rho_0 e^{i2k_F x} e^{i\bar{\theta}}$$

+ h.c. + higher orders,

where we have omitted the tilde from $\bar{\theta}$ and we used $k_F = \pi \rho_0$. Since the forward scattering is irrelevant [3], we took into account only the first and third terms in our calculations.

In order to do MC simulations, we do not want to have the Aharonov-Bohm phase factor $\cos(2\pi \phi/\phi_0)$ in Eq. (2) because it can give a negative weight. Fortunately we can express persistent current for finite $\phi$ in terms of $\langle n^k \rangle|_{\phi=0}$ as will be shown below, so that we just need to perform MC simulations at $\phi = 0$ which causes no negative weight problem.

Here we show how to calculate persistent current at finite $\phi$ using quantities at $\phi = 0$. The persistent current $I$ on a ring is given by $I(\phi) = -(1/\beta) \text{Im} \ln Z(\phi)/\phi_0 = -(2\pi i/\beta \phi_0) \langle n \rangle$. We expand $I(\phi)$ in a Fourier series: $I(\phi) = \sum_n I_n \sin(2\pi n \phi/\phi_0)$, in other words, $\langle n \rangle(\phi) = \beta \langle n \phi \rangle(2\pi) \sum_n I_n \sin(2\pi n \phi/\phi_0)$. Taking derivatives of $\langle n \rangle$ with respect to $\phi$ and setting $\phi = 0$, we obtain,

$$\frac{\partial^2(n)}{\partial \phi^2} \langle n \rangle|_{\phi=0} = (-)^k \beta \langle \frac{\phi_0}{2\pi} \rangle^{2k} \sum_n I_n n^{2k+1},$$

and $\partial^2 (n)/\partial \phi^2|_{\phi=0} = 0$ On the other hand, from the definition of $\langle n \rangle$ we have

$$\frac{\partial \langle n \rangle}{\partial \phi}|_{\phi=0} = \frac{2\pi i}{\phi_0} \langle n^2 \rangle|_{\phi=0},$$

$$\frac{\partial^3 \langle n \rangle}{\partial \phi^3}|_{\phi=0} = \frac{2\pi i}{\phi_0} \beta \langle n^4 \rangle - 3 \langle n^2 \rangle^2 |_{\phi=0},$$

and $\partial^2 \langle n \rangle/\partial \phi^2|_{\phi=0} = 0$. Consequently, we have

$$\frac{2\pi}{\phi_0} \langle n^2 \rangle|_{\phi=0} = \beta \sum_n I_n n,$$

$$2\pi \langle (n^4) - 3 \langle n^2 \rangle^2 |_{\phi=0} = \beta \sum_n I_n n^3.$$
When $W$ is small, both $I_1$ and $\rho_s$ are suppressed as $g_s$ increases, i.e., as repulsive interactions increase, whereas they increase with $g_s$ in the strong impurity region ($W > 2$). We mentioned earlier that the crossover should occur because the system resides in the Luttinger liquid regime for weak impurities whereas the particles are localized with strong impurities. Therefore we believe the crossover occurs when the localization length reaches the system size.

The temperature dependence of $I_1$ and $\rho_s$ are shown in Figs. 2 and 3. We can see that $I_1$ and $\rho_s$ have peaks at certain temperatures. This peak behavior comes from two competing effects: depinning by thermal activation and temperature destruction of quantum coherence. The former effect is important at rather low temperature but the latter dominates at higher temperature. This behavior has been seen in a single impurity problem for a Wigner crystal $^{[1]}$, and here we find the essentially the same feature in the Luttinger liquid in the presence of finite density of impurities. The peak position $T^*$ is expected to be determined by those competing effects, that is, $T^*$ should be controlled by the effective impurity strength. As discussed above, the effective impurity strength in the weak impurity region is enhanced by interactions. Therefore $T^*$ should be larger for larger $g_s$. Actually this is what we can see in Figs. 3 (a) and (b).

Figures 3 (a) and (b) show that, in strong impurity region, $T^*$ is shifted to the lower side as $g_s$ increases, because interactions would reduce the impurity strength.

Another interesting feature is the role of interactions at rather high temperature. For weak impurities, interactions always suppress the persistent current and the stiffness in the whole temperature range. When we have strong impurities, however, interactions would screen the impurities only at low temperature, and would suppress the current and the stiffness at high temperature. This could be because electrons are already delocalized by the thermal activation at high temperature but the CDW correlation still remains, in other words, the localization length becomes shorter than the system size above some temperature although the CDW correlation length is large. Therefore the role of interactions in strong impurity region at high temperature is expected to be the same as in weak impurity region.

In summary, we have reported on the first MC simulation of the 1D ring problem to see the interplay between disorder and electron-electron interactions. We have shown a crossover behavior from the weak-impurity to strong-impurity region. When the impurities are weak, interactions try to suppress the persistent current and charge stiffness. On the other hand, interactions enhance the current and the stiffness in the presence of strong impurities. In other words, interactions increase the effective strength of weak impurities, and decrease the effective strength of strong impurities. We gave a physical picture of these opposite behaviors, and we believe the crossover occurs when the localization length becomes order of the system size. The temperature dependences of persistent current and charge stiffness are also calculated. We showed they have peaks at certain temperatures and the peaks shift as we change interaction strength. The shifts are consistent with the change of the effective impurity strength by interactions. At high temperature the thermal activation would screen the impurity effect and interactions act in the same way as they do with weak impurities.

The author would like to acknowledge useful discussions with S. M. Girvin, C. Hanna, K. Moon, H. Weber, and G. S. Currin. The work was partially supported by NSF Grant No. DMR-9416906.

[1] On leave from Department of Materials Science, Faculty of Science, Hiroshima University, Higashi-Hiroshima 724, Japan.
[2] T. Giamarchi and H. J. Schulz, Phys. Rev. B 37, 325 (1988).
[3] R. T. Scalettar, et al., Phys. Rev. Lett. 66, 3144 (1991);
[4] See also C. A. Doty and D. S. Fisher, Phys. Rev. B 45, 2167 (1992); K. J. Runge and G. T. Zimanyi, (preprint).
[5] N. Byers and C. N. Yang, Phys. Rev. Lett. 46, 7 (1981);
[6] M. Büttiker et al., Phys. Lett. A 96, 365 (1983).
[7] H-F. Cheung et al., Phys. Rev. Lett. 62, 587 (1989); G. Montambaux et al., Phys. Rev. B 42, 7647 (1990); F. von Oppen and E. K. Riedel, Phys. Rev. Lett. 66, 84 (1991); B.L. Altshuler et al., ibid 88 (1991).
[8] V. Ambegaokar and U. Eckern, Phys. Rev. Lett. 65, 381 (1990); A. Schmid, ibid 66, 80 (1991); R. A. Smith and V. Ambegaokar, Europhys. Lett. 20, 161 (1992); F. Kopietz, Phys. Rev. Lett. 70, 3123 (1993).
[9] I. V. Krive et al., *Coulomb and Interference Effects in Small Electronic Structures*, D. C. Glattli and M. Suenner Eds., (Editions Frontieres, Gif-sur-Yvette, 1994) (in press); Physica Scripta, (1994) (in press).
[10] G. Bouzerar, D. Poiblanc, and G. Montambaux, Phys. Rev. B 49, 8258 (1994); H. Kato and D. Yoshioka, Phys. Rev. B 50, 4943 (1994).
[11] This is true since our system is finite. In infinite chains, however, electrons are localized no matter how weak impurities are, at least when interactions are repulsive (See Ref. 3).
FIG. 1. (a) Impurity and interaction dependence of the first harmonics $I_1$ of persistent current. $g_x$ is a renormalized parameter which is an increasing function of the microscopic particle-particle interactions. (b) The same dependence of charge stiffness $\rho_s$.

FIG. 2. Temperature dependence of $I_1$ (a) and $\rho_s$ (b) in the weak-impurity region (W=1). The dashed lines are for a Luttinger liquid with no impurity.

FIG. 3. Temperature dependence of $I_1$ (a) and $\rho_s$ (b) in the strong-impurity region (W=3).
Fig. 1
Fig. 2
no impurity

$g_x = 1$

$g_x = 3$

Fig. 3