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Algebraic approach for $\alpha$-cluster structure in $^{12}$C and $^{16}$O

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Abstract. We investigate the properties of the states observed around the $\alpha$-cluster breakup threshold region in $^{12}$C and $^{16}$O. We apply the orthogonality condition model (OCM), where the model space is given by the $SU_3$ algebra. From bases of this $SU_3$ configuration, two algebraic approaches of the symplectic (sp) method and $\mu$-truncation method are introduced. In $^{12}$C, we show that the $sp$-quanta are important to understand the mechanism of the appearance of the large $E0$ transition strength. With these methods, both strong- and weak-coupling configurations are taken into account at the same time. Therefore, the properties of gas-like and linear-chain states are investigated easily. In $^{16}$O, a simpler truncation utilizing the $\mu$ quanta is applied. Here, the energy levels and the linear-chain (L. C.) configurations are investigated.

1. Introduction

Recently, the $\alpha$-cluster structure of the states observed around the $\alpha$-cluster breakup threshold energy region has been investigated from various viewpoints. For instance, the large iso-scalar monopole ($E0$) transition strength has been observed in the vicinity of the $\alpha$-cluster threshold for many light nuclei such as $^{12}$C, $^{16}$O and $^{24}$Mg [1, 2]. These states are the candidates of the cluster states. Some of them include dilute $\alpha$-cluster configurations, in which each $\alpha$-particle moves with a relatively weak interaction. One of the remarkable states is $\alpha$-cluster gas state in the excited energy region of $^{12}$C [3]. It is expected that such a kind of exotic state can be observed in wider mass and energy regions due to the development of experimental technique or theoretical framework.

From the theoretical viewpoint, the $\alpha$-cluster structure like a gas-like state [3] and coherent $\alpha$-cluster configuration associated with the large $E0$ transition strength have been studied [4, 5]. We present our recent analyses of the excited states of $^{12}$C and $^{16}$O by using many $\alpha$-cluster model. The symplectic ($sp$) structure $Sp(2,R)_2$ [6, 7, 8, 5] has been found to be important in understanding the large $E0$ strength. Because the $sp$ algebraic approach becomes complicated in the analysis when the number of $\alpha$-particles increases, we introduce the $\mu$-truncation method in the study of $^{16}$O.
2. Results

In our three-\(\alpha\)-cluster model, the \(SU(3)\) model wave function is given by \(V^{N(\lambda,\mu)J,\kappa}_{N_1,N_2}(\vec{r},\vec{R})\), where \(\lambda = N_z - N_y\) and \(\mu = N_y\) are evaluated from the total harmonic oscillator (H. O.) quanta for each 3D axis. The quantum number \(\kappa\) indicates the orthonormalized \(K\) quantum number for the intrinsic frame, and \((\vec{r},\vec{R})\) are Jacobi-coordinates to describe three-\(\alpha\)-clusters. The use of the Pauli-allowed states can be performed in order to employ orthogonality condition model [9]. The Pauli-allowed state is obtained by the following transformation:

\[
U_i(\vec{r},\vec{R}) = \sum_{N_1 + N_2 = N} A^{(\lambda,\mu)\rho}_{N_1,N_2} V^{N(\lambda,\mu)J,\kappa}_{N_1,N_2}(\vec{r},\vec{R}),
\]

where \(i\) denotes a set of \((\lambda,\mu)\rho, J, \kappa\) quanta. The \(\rho\) shows the multiplicity of the Pauli-allowed state [10]. This \(\rho\) quanta is converted to the more physical basis state as follows.

\[
w_\alpha(\vec{r},\vec{R}) = \sum_{\rho} C^{\Lambda}_{\rho} U_i(\vec{r},\vec{R}).
\]

Here the coefficients \(C^{\Lambda}_{\rho}\) and the quantum number \(\Lambda\) are obtained as eigenvectors and eigenvalues, respectively of the Casimir operator of \(Sp(2,R)\) [8]. In the transformed basis, each \(\Lambda\) is classified with respect to the band-head states that is connect to the specific \(\Lambda\) quantum numbers by the ladder operator.

In fig. 1 we show the energy levels obtained by experiments and the present calculation. Here we use the phenomenological inter-three-\(\alpha\) interaction, which is optimized to reproduce the ground-band energy levels [11]. In the figure we show the energy levels, which are stable with respect the maximum H. O. quanta. The energy convergence of the Hoyle state is very slow as the number of H. O. quanta increases. The states in parentheses has slower energy convergence in comparison with that of the Hoyle state.

The \(E2\) transition strength from the Hoyle state to the \(2_1^+\) state is in accordance with the large observed value. The calculated \(E0\) and \(E2\) transition are in a good agreement with observed values with error ranges. The other \(E2\) strength from \(2_2^+\) to the Hoyle state becomes enormous \((232 \text{ fm}^4)\). From this analysis, we can see that the study to investigate the \(E0\) transition strength can be extended to other transition modes.

To investigate \(^{16}\text{O}\) in a four-\(\alpha\) model, we use the \(\mu\)-truncation method. The Pauli-allowed basis state is obtained similarly to the three-\(\alpha\) case by using three Jacobi-coordinates, \((\vec{r},\vec{R},\vec{\mathcal{R}})\) to describe four-\(\alpha\)-clusters. In the \(\mu\)-truncation procedure the eigenstate \(u_k(\vec{r},\vec{R},\vec{\mathcal{R}})\) calculated...
within subspace which is truncated by specific $\mu$ quanta and eigenenergies smaller than an appropriate cutoff energy. These eigenstates are expressed by using the original basis as follows,

$$u_k(\vec{r}, \vec{R}, \vec{\mathcal{R}}) = \sum_i D_k^i U_i(\vec{r}, \vec{R}, \vec{\mathcal{R}}).$$

Here the model space is constructed with H. O. quanta, $N \leq 32$. We obtained good energy convergence with respect to the energy cutoff. In fig. 2, the L. C. state energy region is also estimated. The pure L. C. state is specified by the condition $\mu = 0$ and $N = \lambda$ in the case of four-$\alpha$-cluster model. The horizontal axes show the $0^+$ energy from the four-$\alpha$ threshold. The vertical axes show the probability of L. C. configuration. The cutoff parameter changes from 50 MeV (lower panel) to 200 MeV (upper panel). The colored bars present large L. C. strength. The strength is estimated by the overlap between the wavefunctions obtained by the $\mu$-truncation with $\mu \leq 16$ quanta. The results show that the L. C. strength is about 10 % for each cutoff parameter. The strength is smaller than that for the L. C. states in $^{12}$C, which have about 50 % pure L. C. configuration.

In both nuclei the algebraic model is found to work well. However, to confirm the existence of L. C. as the same accuracy as the gas-like states, larger $N$ quanta are needed especially for $^{16}$O.

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