Dispersion in the growth of matter perturbations

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Abstract

We consider the linear growth of matter perturbations on low redshifts in modified gravity Dark Energy (DE) models where $G_{\text{eff}}(z, k)$ is explicitly scale-dependent. Dispersion in the growth today will only appear for scales of the order the critical scale $\sim \lambda_{c,0}$, the range of the fifth-force today. We generalize the constraint equation satisfied by the parameters $\gamma_0(k)$ and $\gamma'_0(k) \equiv \frac{d\gamma(z,k)}{dz}(z=0)$ to models with $G_{\text{eff,0}}(k) \neq G$. Measurement of $\gamma_0(k)$ and $\gamma'_0(k)$ on several scales can provide information about $\lambda_{c,0}$. In the absence of dispersion when $\lambda_{c,0}$ is large compared to the probed scales, measurement of $\gamma_0$ and $\gamma'_0$ provides a consistency check independent of $\lambda_{c,0}$. This applies in particular to results obtained earlier for a viable $f(R)$ model.

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1 Introduction

The present accelerated expansion of the universe [1]is a major problem facing cosmologists [2]. If it is due to some smooth (isotropic) perfect fluid, its pressure should be sufficiently negative in order to induce an accelerated expansion. The simplest candidate, and maybe even the oldest one, is a cosmological constant \( \Lambda \). However its required magnitude can be viewed as unnaturally small in order to provide a satisfactory solution of the puzzle. It is then natural to look for other alternative models providing an effective cosmological constant in the late-time universe whose origin is dynamical. A more drastic step is to assign the present accelerated expansion to a modification of gravity on cosmic scales.

It was realized some time ago that the simultaneous study of the background expansion and of the matter perturbations growth could provide a mean to break the degeneracy between DE models based on different gravity theories [3], (see also e.g.[4]). For this reason the study of the matter perturbations growth has been the subject of many investigations in recent years. An important goal is to find appropriate characterizations of the perturbations growth allowing to discriminate between DE models based on General Relativity and those outside GR and a convenient such characterization is the \( \gamma \) formalism in which one writes the growth factor \( f \) as \( f = \Omega_m \) [5].

It was found that in models outside General Relativity, even on very low redshifts \( \gamma \) must be treated as a non-constant function [6]. In particular, it is possible to have a large slope \( \gamma'_{0} \equiv \frac{d\gamma}{dz}(z = 0) \) in some scalar-tensor models [7] and \( f(R) \) models [8] while this is not the case for standard DE models inside GR [6]. An additional interesting phenomenon occuring in some modified gravity models is the appearance of dispersion in the growth of matter perturbations even for scales well inside the Hubble radius. Of course such a dispersion is absent in all DE models inside GR. It is then important to investigate if and how this dispersion could manifest itself in the function \( \gamma \) on very low redshifts. This will the focus of this letter.

As we will show there is a straightforward way to check for the presence of dispersion and this method can be used to probe the parameter space of a given model to find which values will allow for dispersion. Precise measurements of the behaviour of \( \gamma(z, k) \) at small redshifts could then not only detect a departure from \( \Lambda \)CDM but also directly constrain the models’ parameters and the kind of modified gravity model under consideration. While we will use a fiducial model for the calculation of the matter perturbations growth, this model is just used for illustrative purposes. Our results will apply to a large class of modified gravity DE models for which the growth of matter perturbations exhibit a scale-dependence.

2 Modified gravity DE models

We describe first how modified gravity DE models can affect the growth of matter perturbations. We consider spatially flat Friedman-Lemaitre-Robertson-Walker (FLRW) universes with a time-dependent scale factor \( a(t) \) and a metric

\[
    ds^2 = g_{\mu\nu} \, dx^\mu \, dx^\nu = -dt^2 + a^2(t) \, dx^2.
\]
In many modified gravity DE models matter perturbations satisfy a modified equation of the type

$$
\ddot{\delta}_m + 2H \dot{\delta}_m - \frac{3}{2} \Omega_m H^2 \frac{G_{\text{eff}}(t, k)}{G} \delta_m = 0 ,
$$

(2)

where the quantity $G_{\text{eff}}(z, k)$ is given by the expression

$$
G_{\text{eff}}(z, k) = G \left( 1 + 2\beta^2 \frac{k^2}{a^2 m^2} \frac{1}{1 + \frac{k^2}{a^2 m^2}} \right).
$$

(3)

The mass $m$ is varying with time and $a m$ is typically decreasing very rapidly with the expansion, the details of this decrease is model-dependent. In some viable $f(R)$ models expression (3) (with constant $G$) applies only in the high curvature regime for which $F(R) \equiv \frac{df}{dR} \approx 1$ [9], [10], [8], and $m$ is the scalaron mass introduced in [11] while $\beta = \frac{1}{\sqrt{6}}$.

In some DE models outside GR like scalar-tensor models, $G_{\text{eff}}$ appearing in (2) does not depend on $k$ [12]. Equation (2) is a generalization to models where the effective gravitational constant depends not only on time (or on $z$) but also on the perturbations scales. This is how dispersion can appear in their growth and it is important to investigate when it can appear on small redshifts. Note that the scalar-tensor models considered in [7] do not correspond to expression (3) not even in the limit $k \to \infty$.

We will investigate equation (2) using a chameleon model with action

$$
S = \int d^4 x \sqrt{-g} \left( \frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) + S_m [\Psi_m; A^2(\phi) g_{\mu\nu}] ,
$$

(4)

with an exponential potential of the type [13]

$$
V = M^4 e^{\frac{\phi}{M_{\text{PL}}} n}.
$$

(5)

In our fiducial model the mass appearing in (3) satisfies $m^2 \equiv V_{,\phi\phi}$ so it is possible to change the mass scale $m$ by varying the parameter $n$ of the potential. Hence the parameter $n$ can be used in order to tune the critical scale $\lambda_c$ ($\lambda_c$ depends also marginally on $\beta$). For a conformal factor $A = e^{\beta \phi / M_{\text{PL}}}$, $\beta$ is a free parameter [13]. If we assume that the (chameleon) field $\phi$ sits in the minimum of the (effective) potential from the early stages of the universe on, then we have $\frac{\phi}{M_{\text{PL}}} \ll 1$ during the subsequent evolution until today. As a result the background evolution is nothing else than that of $\Lambda$CDM. Moreover the conformal factor $A(\phi) \equiv e^{\beta \phi / M_{\text{PL}}}$, with $M_{\text{PL}}^2 \equiv 8\pi G$, satisfies $A = 1$ to very high accuracy, so it will disappear from equations and does not have to be considered here.

We insist that our aim is not to explore chameleon models here. This specific model is used in order to illustrate the various ways in which modified gravity DE models can depart from $\Lambda$CDM through the growth of matter perturbations on small redshifts including dispersion.

The physical meaning of $G_{\text{eff}}$ is that the gravitational potential (per unit mass) in real space is modified through the presence of a fifth-force deriving from a Yukawa potential

$$
V(r) = -\frac{G}{r} \left( 1 + 2\beta^2 e^{-mr} \right).
$$

(6)
It is obvious that $\beta$ enters as the coupling constant of the fifth-force which has a range $L \sim m^{-1}$. If we introduce the characteristic scale $\lambda_c$

$$\lambda_c = \frac{2\pi}{m},$$  \hspace{1cm} (7)

we have obviously

$$G_{\text{eff}}(z, \lambda) = G \left( 1 + 2\beta^2 \frac{\lambda^2}{\lambda_c^2} \right),$$  \hspace{1cm} (8)

where we have set $\lambda \equiv a \frac{2\pi}{k}$. We have the asymptotic regimes

$$G_{\text{eff}} = G(1 + 2 \beta^2), \quad \lambda \ll \lambda_c,$$ \hspace{1cm} (9)

$$G_{\text{eff}} = G, \quad \lambda \gg \lambda_c,$$ \hspace{1cm} (10)

Equation (2) can be recast in the form

$$\frac{df}{dN} + f^2 + \frac{1}{2}(1 - 3 w_{\text{eff}}) f = \frac{3 G_{\text{eff}}}{2 G} \Omega_m.$$ \hspace{1cm} (11)

where $w_{\text{eff}} = -1 - \frac{2H}{\dot{3}H^2}$. In (11) we have defined $f = \frac{d\ln \delta}{d\ln a}$ and $N \equiv \ln a$. A well-known way to describe the growth of perturbations is through $\gamma$ defined as follows $f = \Omega_m(z)^\gamma(z)$. For a modified equation of the type (2) or (11), $f$ can be also scale dependent so that one must write

$$f(z, k) = \Omega_m(z)^\gamma(z, k).$$ \hspace{1cm} (12)

The analysis we perform here applies to many models. What will change is the microscopic origin of the quantities $\beta$ and $\lambda_c$ and the background expansion which does not have to be exactly that of $\Lambda$CDM. The parameter $\beta$ can be used to tune the change of $G_{\text{eff}}$ with respect to its value $G$ in GR. A gratifying property of our fiducial model is that for all model parameters values that we will consider the background evolution is that of $\Lambda$CDM so that it is completely fixed once $\Omega_m, 0$ is known. In this way we can straightforwardly relate $\gamma(z, k)$ to the behaviour of $G_{\text{eff}}(z, k)$ and compare with the growth in $\Lambda$CDM.

3 Growth of matter perturbations

As we will see some models can exhibit a dispersion in the growth of matter perturbations on low redshifts. This dispersion will appear in the function $\gamma(z, k)$. All behaviours can be understood from the form of $G_{\text{eff}}(z, k)$. It is clear from (11) that $\frac{3 G_{\text{eff}}}{2 G} \Omega_m$ is the source term driving the growth of $f$. By construction, all our fiducial models have the same background expansion as in $\Lambda$CDM. Therefore the function $\Omega_m(z)$ will be the same for all models with same $\Omega_{m,0}$ and differences in the functions $\gamma(z)$ are directly related to differences in $f$. More specifically, all changes can be immediately traced back to the behaviour of $\frac{G_{\text{eff}}}{G}$, at any redshift, meaning that if it is in the transition regime at a given redshift, then there will be dispersion in the growth at that redshift. As we have seen $\frac{G_{\text{eff}}}{G}$ has the two asymptotic regimes (9) and (10) and smaller scales are the first to undergo the transition from the large
scales regime (10) to the small scales regime (9). The smaller the scale, the higher the transition redshift.

We are interested in some interval of cosmic scales

\[ \lambda_{\text{min}} \leq a_0 \frac{2\pi}{k} \leq \lambda_{\text{max}} . \]  

(13)

We will often illustrate our results with the concrete values

\[ \lambda_{\text{min}} = 1 h^{-1}\text{Mpc} \]  

(14)

\[ \lambda_{\text{max}} = 30 h^{-1}\text{Mpc} . \]  

(15)

Three main situations can arise depending on the critical length \( \lambda_{c,0} \) today.

### 3.1 Small critical length \( \lambda_{c,0} \)

All cosmic scales in the interval (13) satisfy

\[ \frac{G_{\text{eff}}(z = 0, k)}{G} \equiv \frac{G_{\text{eff},0}(k)}{G} = 1 , \]  

(16)

in other words they are still today in the large scales regime (10). This will hold for any model for which the critical length \( \lambda_{c,0} \) is sufficiently small

\[ \lambda_{c,0} \ll 1 h^{-1}\text{Mpc} . \]  

(17)

or equivalently when the mass \( m_0 \) is sufficiently large that no modification of gravity is felt on these scales. Then the function \( \gamma(z) \) will be the same in our fiducial model as in \( \Lambda \)CDM for all low redshifts where (16) holds, it is quasi-constant and depends weakly on \( \Omega_{m,0} \). Then matter perturbations on these scales will not help to distinguish that model from a model inside GR with same background expansion and there is evidently no dispersion in the growth.

### 3.2 Large critical length \( \lambda_{c,0} \)

All scales in the interval (13) satisfy

\[ \frac{G_{\text{eff},0}(k)}{G} = 1 + 2\beta^2 . \]  

(18)

Physically this means that these scales had time to make the transition from the large-scales to the small-scales regime. This will happen for any model where the critical length satisfies

\[ \lambda_{c,0} \gg 30 h^{-1}\text{Mpc} . \]  

(19)

For these models the growth of matter perturbations will be affected on low redshifts allowing to distinguish them from the growth in GR. However the growth on low redshifts where (18) holds is the same for all scales, hence there is no dispersion in the growth and \( \gamma(z) \) does not depend on \( k \). Of course dispersion can appear on larger redshifts because different scales start their transition at different times. We will have the same parameter \( \gamma_0 \) for all scales in the interval (13) and depending on the value of \( 2\beta^2 \) this value can still differ significantly from 0.55 unless \( 2\beta^2 \ll 1 \). This situation is illustrated with Figure 1.
3.3 Intermediate critical length $\lambda_{c,0}$

Finally new features appear if a given scale is today just in the transition between both regimes. For such scales we have

$$1 < \frac{G_{\text{eff},0}(k)}{G} < 1 + 2\beta^2. \quad (20)$$

This will happen in models for which

$$\lambda_0 \sim \lambda_{c,0}. \quad (21)$$

The critical length should neither be much larger than $\lambda_{\text{max}}$ nor much smaller than $\lambda_{\text{min}}$. For these models we get a larger spectrum of behaviours as can be seen from Figure 2. We can have a pronounced dispersion in the growth of matter perturbations on small redshifts with scale-dependent $\gamma(z, k)$. Depending on whether $\frac{G_{\text{eff}}(z=0,k)}{G}$ is close to one or not, large departures from the value 0.55 are obtained for $\gamma_0(k)$. When $\frac{G_{\text{eff}}(z=0,k)}{G} \approx 1$ we have a small departure, while for $\frac{G_{\text{eff}}(z=0,k)}{G} \approx 1 + 2\beta^2$ large departures can be obtained. Of course, if $2\beta^2 \ll 1$, the difference is essentially irrelevant. It could be in principle that the scales $\lambda_{\text{min}}$ have completed their transition to the small-scales regime by today but not the larger scales. Higher (lower) values for $\gamma_0(k)$ are obtained for the larger (smaller) scales.
Figure 2: a) On the left panel, the behaviour of the quantity $G_{\text{eff},0}(k)$ is shown when the critical length today $\lambda_{c,0}$ satisfies $\lambda_{c,0} = 5h^{-1}\text{Mpc}$ while $\beta = 0.5$. Hence none of the scales displayed has completed its transition to the small-scale regime. In particular, the value $G_{\text{eff},0}(k)$ depends strongly on the scale and dispersion appears today in the growth of matter perturbations. b) On the right panel, the corresponding functions $\gamma(z)$ are shown for which dispersion is evident. For large scales today $\lambda \gg \lambda_{c,0}$, $\gamma(z)$ is undistinguishable from its value in $\Lambda$CDM, but for $\lambda \lesssim \lambda_{c,0}$ it can be significantly different and large slopes $\gamma_0'$ are possible. Note in this respect that for our fiducial model, even large departures of $\gamma_0$ from 0.55 can still yield a quasi-constant $\gamma(z)$ because this departure is compensated by a value for $G_{\text{eff},0}(k)$ significantly different from 1 (see also Figure 3).

4 Important signatures and properties

Let us consider all the characteristic signatures that can be obtained. An important point concerns the behaviour of $\gamma(z, k)$. As emphasized in [6], while $\gamma(z)$ is quasi-constant on low redshifts for standard DE models inside General Relativity (GR), this is no longer true for DE models outside GR [7,8]. Actually there is a constraint equation which relates $\gamma_0(k)$ to the other background parameters. Indeed it is easy to derive the following equation which is valid for any $\gamma$ (we drop here the arguments for compactness)

$$- (1 + z) \ln \Omega_m \gamma' + \Omega_m^\gamma + \frac{1}{2}(1 + 3(2\gamma - 1) w_{\text{eff}}) = \frac{3}{2} \frac{G_{\text{eff}}}{G} \Omega_m^{1-\gamma}.$$  \hspace{1cm} (22)

From (22), it is easy to derive in turn the constraint equation

$$\gamma_0' = \left[ \ln \Omega_m^{-1} \right]^{-1} \left[ -\Omega_m^{\gamma_0} - 3(\gamma_0 - \frac{1}{2}) w_{\text{eff},0} + \frac{3}{2} \frac{G_{\text{eff},0}}{G} \Omega_m^{1-\gamma_0} - \frac{1}{2} \right].$$  \hspace{1cm} (23)

It is seen that the value of $\gamma_0'$ is fixed by the other parameters, we have a constraint equation (putting back the k dependence)

$$C \left( \gamma_0(k), \gamma_0'(k), \Omega_m, w_{\text{eff},0}, \frac{G_{\text{eff},0}(k)}{G} \right) = 0.$$  \hspace{1cm} (24)

The constraint (24) generalizes the constraint found in [6] when $\frac{G_{\text{eff},0}}{G} = 1$. In (24), $\frac{G_{\text{eff},0}(k)}{G}$ is scale-dependent and can differ from one. We can parametrize this departure with a parameter
Figure 3: The quantity $\gamma_0'$ is shown as a function of $\gamma_0$ for several values of $B^2$ defined in (25) for the background parameters $\Omega_{m,0} = 0.29$ and $w_{DE,0} = -1$. Note that $B^2 \leq \beta^2$, the equality holds in the small-scales asymptotic regime (9). It is seen that as $B^2$ increases, quasi-static cases with $\gamma_0' \approx 0$ correspond to decreasing values of $\gamma_0$ significantly smaller than 0.55. The value $B^2 = 0$ corresponds to General Relativity. Note further that $B^2 = \frac{1}{5}$ corresponds to $f(R)$ models for scales in the small-scales asymptotic regime (9).

$B$ defined as follows

$$\frac{G_{\text{eff},0}(k)}{G} = 1 + 2B^2. \tag{25}$$

We have obviously $B^2 \leq \beta^2$, the equality is saturated for $\lambda_{c,0} \to \infty$, in practice this asymptotic regime is accurate up to 1% for scales satisfying $\lambda_{c} \gtrsim 10\lambda$. So (25) can be used in two ways. Either it is used for scales that are in the small-scales regime (9) and then we have $B = \beta$. Or it is used for scales in the intermediate regime and then $B^2$ measures the departure of $\frac{G_{\text{eff},0}(k)}{G}$ from 1 (GR), still different from $1 + 2\beta^2$. The constraint (24) is illustrated for representative background parameters in Figure 3.

The actual occurrence of the different possibilities must be addressed separately for each model and will be considered elsewhere but we can draw some general conclusions. The less interesting case is when $\lambda_{c,0}$ is so small that the growth on cosmic scales is like in GR. But in the other cases interesting signatures are obtained.

If the background is known accurately we can hope to pinpoint the quantities $\Omega_{m,0}$ and $w_{\text{eff},0}$ at the percent level. If the characteristic length $\lambda_{c,0}$ which defines the range of the fifth-force today is large enough, no dispersion will be seen on scales (13) if they satisfy $\lambda \ll \lambda_{c}$. Then the late-time growth on these scales takes place with a constant gravitational constant different from its value $G$ inside GR if the coupling $\beta$ is large enough. This was indeed found for Starobinsky’s $f(R)$ model [8], remember there $1 + 2\beta^2 = \frac{4}{3}$. For Starobinsky’s model it was also found in [8] that $\lambda_{c,0} \sim H_0^{-1}$. A large coupling $\beta$ will tend to lower the value of $\gamma_0$. For chameleon models, $\beta$ is a free parameter that can be recovered using (24) with $\frac{G_{\text{eff},0}(k)}{G} = 1 + 2\beta^2$. For $f(R)$ models, $\beta$ is fixed but one can recover $F_0 \equiv \frac{dF}{dR}(z = 0)$ using
\( G_{\text{eff},0}(k) = \frac{4}{3F_0} \) \cite{1}. If we know \( \Omega_{m,0} \) and \( w_{\text{eff},0} \) we can find \( F_0 \) from the constraint \cite{2}. On the other hand if we know in addition \( H_0 \) we can also find independently \( R_0 \) and we can check whether \( F_0 = F(R_0) \) if some \( f(R) \) model is assumed. Hence simultaneous measurement of \( \gamma_0 \) and \( \gamma'_0 \) can serve as a consistency check in chameleon models if \( \beta \) is known from other considerations or in \( f(R) \) models if some given model is assumed.

Finally in the intermediate case when \( \lambda_{c,0} \) is of the order of the scales that are probed dispersion will appear in a way which depends on the exact location of \( \lambda_{c,0} \) and it can be significant if \( 2\beta^2 \) is large. In the presence of dispersion, measurement of both \( \gamma_0(k) \) and \( \gamma'_0(k) \) on various scales can be used to reconstruct the quantity \( G_{\text{eff},0}(k) \). Such measurements can give information not only on \( \beta \) but also on \( \lambda_{c,0} \) as the constraint \cite{2} must be satisfied for all scales in the transition regime.

To summarize, in a situation where there is no dispersion in \( \gamma_0 \) and \( \gamma'_0 \) these quantities provide important consistency checks. Even when dispersion does not affect the parameter \( \gamma_0 \), it will still appear at large redshifts and induce a distortion of the power spectrum. On the other hand, a dispersion in the quantities \( \gamma_0(k) \) and \( \gamma'_0(k) \) could be a clear signature of the scale dependence of \( G_{\text{eff}}(k) \), and hence of a modification of gravity. It is however not clear whether it can appear in viable DE models.

In any case, the growth of matter perturbations provides clearly a powerful discriminative tool in order to investigate whether or not the origin of the present accelerated expansion is due to a modification of gravity on cosmic scales.

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