Abstract

In this talk I report on some recent calculations on the production of pseudoscalars from intense electromagnetic fields.

1Talk at QED2000, Trieste (October 2000)
1 Decay of classical background fields into pseudoscalars

We would like to calculate the effective action for the background electromagnetic $E$ and $B$ fields,

$$e^{iS_{\text{ef}}[E,B]} = \int \mathcal{D}\phi \ e^{iS[\phi,E,B]}$$

(1)

when integrating a pseudoscalar $\phi$ of mass $m$, that has an action $S$ with a coupling to the $E$ and $B$ fields of the form

$$S[\phi,E,B] = \int d^4x \ \frac{1}{2} \phi(x) \left[-\partial^2 - m^2 + f(x,E,B)\right] \phi(x)$$

(2)

Using the identities

$$i \frac{\partial S_{\text{ef}}[E,B]}{\partial m^2} = -\frac{\int \mathcal{D}\phi \ \phi^2 e^{iS[\phi,E,B]}}{\int \mathcal{D}\phi e^{iS[\phi,E,B]}}$$

$$= -\frac{1}{2} \int d^4x \ G(x,x;E,B)$$

$$= -\frac{1}{2} \int d^4x \ \int \frac{d^4p}{(2\pi)^4} \ G(p;E,B)$$

(3)

we can express the effective Lagrangian of the background fields in terms of the Green’s function $G(p)$ of $\phi$ propagating in these fields.

$$\mathcal{L}_{\text{ef}}[E,B] = i \frac{1}{2} \int dm^2 \ \int \frac{d^4p}{(2\pi)^4} \ G(p;E,B)$$

(4)

Our objective now is to determine 1) $G(p)$ and 2) $\mathcal{L}_{\text{ef}}$.

1) The action (1) contains the interaction Lagrangian

$$\mathcal{L}_I(x) = \frac{1}{2} f(x) \phi^2(x)$$

(5)

and leads to the following equation for the Green function

$$\left[\partial^2 + m^2 - f(x)\right] G(x,0) = \delta^4(x)$$

(6)

We will approximate $f(x)$ by its Taylor series near the reference point $x = 0$ up to second order,

$$f(x) = \alpha + \beta_{\mu} x^\mu + \gamma_{\mu\nu} x^\mu x^\nu$$

(7)
The equation (6) is then approximated by
\[
\partial^2 + m^2 - \alpha - \beta_{\mu}x^{\mu} - \gamma^{2}_{\mu\nu}x^{\mu}x^{\nu} \] \[G(x,0) = \delta^4(x) \] (8)
or, in momentum space,
\[
\left[ -p^2 + m^2 - \alpha + i\beta_\mu \frac{\partial}{\partial p_\mu} + \gamma^2_{\mu\nu} \frac{\partial}{\partial p_\mu} \frac{\partial}{\partial p_\nu} \right] G(p) = 1 \] (9)

As is shown in detail in [1], the solution for \( G(p) \) that satisfies the boundary conditions is
\[
G(p) = i \int_0^\infty ds \ e^{-is(m^2-i\epsilon)} e^{ip_\mu A_{\mu\nu}p_\nu + B_{\mu}p_\mu + C} \] (10)
where
\[
A = \frac{1}{2} \gamma^{-1} \cdot \tan(2\gamma s) \] (11)
\[
B = -\frac{i}{2} \gamma^{-2} \cdot [1 - \sec(2\gamma s)] \cdot \beta \] (12)
\[
C = i\alpha s - \frac{1}{2} \text{tr} [\ln \cos(2\gamma s)] + \frac{i}{8} \beta \cdot \gamma^{-3} \cdot [\tan(2\gamma s) - 2\gamma s] \cdot \beta \] (13)

2) The effective Lagrangian is obtained by substituting \( G(p) \) in (4) and carrying out the integration over \( m^2 \),
\[
L_{\text{eff}} = -\frac{i}{2} \int_0^\infty ds \int \frac{d^4 p}{(2\pi)^4} \ \exp \left\{ -ism^2 + ip \cdot A \cdot p + B \cdot p + C \right\} \] (14)
After evaluation of the Gaussian integral we finally get
\[
L_{\text{eff}} = -\frac{1}{32\pi^2} \int_0^\infty ds \frac{ds}{s^3} e^{-is(m^2-\alpha)} \left[ \det \left( \frac{2\gamma s}{\sin 2\gamma s} \right) \right]^\frac{1}{2} e^{i\ell(s)} \] (15)
where
\[
\ell(s) = \frac{1}{4} \beta \cdot \gamma^{-3} \cdot [\tan(\gamma s) - \gamma s] \cdot \beta \] (16)
When the effective Lagrangian has an imaginary part, there is particle production with a probability density given by
\[
w = 2 \ \text{Im} L_{\text{eff}}[E, B] \] (17)
In turn, a non-zero value for \( \text{Im} L_{\text{eff}} \) may arise depending on the sign of the \( \gamma \)-matrix eigenvalues.
2 Effective $F^2 \phi^2$ interactions

We assume the standard pseudoscalar-two photon coupling

$$\mathcal{L}_{\phi\gamma\gamma} = \frac{1}{8}g\phi \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$  \hspace{1cm} (18)

We shall now show that, for the purposes of calculating $\mathcal{L}_{\text{eff}}$ of external $E$ and $B$, we can use an interaction Lagrangian of the type displayed in (5).

We first calculate the two-photon two-pseudoscalar amplitude in momentum space,

$$4 \left( \frac{1}{4} g \phi \right)^2 \epsilon^{\mu\nu\rho\sigma} k_\mu \tilde{F}_{\sigma\rho} \frac{-ig_{\nu\nu'}}{k^2} \epsilon^{\mu'\nu'\rho'\sigma'} (-k_{\mu'}) \tilde{F}_{\rho'\sigma'}$$  \hspace{1cm} (19)

Due to the presence of the $k^2$ term in the denominator, the effective coupling (19) is non-local. However, when we calculate the effective action for the external electromagnetic field the momentum $k$ is integrated over. One can therefore make use of the identity

$$\int d^4k \ k_\mu k_{\mu'} g(k^2) = \int d^4k \ \frac{g_{\mu\nu} k^2}{4} g(k^2)$$  \hspace{1cm} (20)

to simplify (19). Thus, we can reduce the effective two-photon two-pseudoscalar to a local interaction vertex. Back in configuration space, it is given by

$$\mathcal{L}_{\text{I}} = -\frac{1}{4}g^2 \phi^2 F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}g^2 \phi^2 (E^2 - B^2)$$  \hspace{1cm} (21)

so that we can identify $f(x)$ in (5) with

$$f(x) = g^2 (E^2 - B^2)$$  \hspace{1cm} (22)

3 Pseudoscalar production in a cylindrical capacitor

In order to have a non trivial $\mathcal{L}_{\text{eff}}$, one needs non-zero second derivatives of the electromagnetic fields as they appear in expression (22), which imply a non-zero
\( \gamma \)-matrix. We illustrate it in the simple situation of the electric field inside a cylindrical capacitor.

The modulus of the electric field inside a cylindrical capacitor whose axis lies along the \( z \)-axis depends only on \( \rho = (x^2 + y^2)^{\frac{3}{2}} \),

\[
E(\rho) = \frac{\lambda}{2\pi} \frac{1}{\rho}
\]

(23)

with \( \lambda \) the linear electric charge density. It follows that

\[
f(\rho) = g_c^2 \left( \frac{1}{\rho^2} \right)
\]

(24)

where \( g_c \equiv \lambda g/2\pi \)

Expanding the fields near some reference point \((x_0, y_0, z_0)\) with \( \rho_0 = (x_0^2 + y_0^2)^{\frac{3}{2}} \), we identify

\[
\alpha = \frac{g_c^2}{\rho_0^2}
\]

(25)

\[
(\beta)_i = -\frac{2g_c^2}{\rho_0^2}(x_0, y_0)
\]

(26)

and

\[
(\gamma^2)_{ij} = \frac{g_c^2}{\rho_0^6} \left( \begin{array}{cc}
-y_0^2 + 3x_0^2 & 4x_0y_0 \\
-4x_0y_0 & -x_0^2 + 3y_0^2
\end{array} \right)
\]

(27)

(We only include the \( x - y \) entries). In diagonal form \( \gamma^2 \) reads

\[
(\gamma^2_D)_{ij} = \frac{g_c^2}{\rho_0^6} \left( \begin{array}{cc}
3 & 0 \\
0 & -1
\end{array} \right) \equiv \left( \begin{array}{cc}
a^2 & 0 \\
0 & -b^2
\end{array} \right)
\]

(28)

Introducing \( \alpha, \beta \) and \( \gamma \) in \( \mathcal{L}_{\text{eff}} \) as given in (15), we get the following expression

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{32\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-is(m^2-\alpha)} \sqrt{\frac{2as}{\sinh 2as}} \sqrt{\frac{2bs}{\sin 2bs}} e^{il(s)}
\]

(29)

where

\[
l(s) = \lambda (as - \tanh as) \quad \lambda = g_c^4 \rho_0^{-6} a^{-3} = \frac{g_c}{3 \sqrt{3}^3}
\]

(30)

One can perform the integration in (29) by extending \( s \) to the complex plane. The details of the integration can be found in [1], where it is found that

\[
\text{Im} \mathcal{L}_{\text{eff}} = \frac{a^3 b^3}{8\pi^2} \sum_{n=0}^\infty (-1)^n C_n e^{-\chi(2n+1)\pi}
\]

(31)
\[ C_n = \int_0^\pi du \frac{e^{-\chi u} e^{-\lambda \cot(u/2)}}{[u + (2n + 1)\pi]^2 \sin u \left[ \sinh \left( \frac{u}{\alpha} [u + (2n + 1)\pi] \right) \right]^{1/2}} \] (32)

(we can put \(b/a = 1/\sqrt{3}\)). We have defined

\[ \chi = \frac{m^2 - \alpha}{2a} + \frac{\lambda}{2} = \frac{1}{\sqrt{3}} \left( \frac{1}{2} \frac{m^2}{g_c} - \frac{1}{3} \bar{g}_c \right) \] (33)

Finally, the expression for the probability per unit volume and per unit time for pseudoscalar production inside a cylindrical capacitor is

\[ w = \frac{3 + \bar{g}_c^2}{4\pi^2 \rho_0^2} C_0 e^{-\chi \pi} \] (34)

were we kept only the leading \(n = 0\) term in (31).

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References

[1] J. A. Grifols, E. Massó, S. Mohanty and K.V. Shajesh, Phys. Rev. D60 097701 (1999);
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