Effect of Two-Dimensional Re-Entrant Honeycomb Configuration on Elastoplastic Performance of Perforated Steel Plate

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Abstract: Perforated steel plates with regularly shaped holes are already widely employed as steel dampers, which dissipate seismic energy through plastic deformation of steel. As a typical auxetic structure, two-dimensional (2D) re-entrant honeycomb configurations have characteristics of large deformation and good energy absorption. However, research on the effects of these configurations on the mechanical performance of steel is limited. This paper investigated the auxetic properties of perforated steel plates with re-entrant hexagon holes. Repetitive units are controlled by three parameters, hole ratio, re-entrant angle, and chamfer radius. Elastoplastic behavior and damage under large deformation were studied via tension tests and finite element (FE) analysis based on a micromechanics-based ductile fracture model. The effects of different parameters on mechanical properties of configurations were analyzed and discussed. The static performance of the perforated steel plates obtained in this study provides a good basis for its further dynamic study under large deformation.

Keywords: auxetic; re-entrant hexagon; elastoplastic; ductile fracture; finite element analysis; structural steel

1. Introduction

Materials and structures with negative Poisson’s ratios (NPRs) have attracted significant scientific interest because of their unique mechanical behaviors, such as increased shear modulus, energy absorption ability, fracture toughness, and vibration control [1–5]. NPRs are also called “auxetics” by Evans [6]. From theoretical analysis, these forms should have good performance under both static and dynamic loading cases, as energy absorbing and antiknock components. In recent decades, since Lakes [7] and Evans [6] found the early forms, this field has been intensively investigated. Almgren [8] generated the first mechanical model with a NPR configuration. Wojciechowski [9] proposed the first simulation of two-dimensional model molecules [10] and published a rigorously solved model (soft cyclic hexamers) with a stable isotropic phase. Well-known planar structures with chiral [11] and rotation [12] behaviors were studied. Simulations of auxetic materials [13], auxetic structures [14], auxetic two-dimensional hard body systems [15], auxetic nano-composite models in two dimensions [16] and in three dimensions [17], and auxetic metamaterials [18] have been performed. Theoretical and numerical modeling of auxetics has also been conducted [19–21].

Two-dimensional auxetic forms are always periodic and with perforated configurations [22–24], leading to auxetic behaviors by unit bending or rotating under tension or compression [25–27]. One of the most important 2D structures is the 2D re-entrant honeycomb form [28–30]. When fabricated
from materials with high ductility, 2D re-entrant honeycomb exhibits good auxetic behavior and load-carrying capacity under static loading. It also shows high stability and ductility under dynamic tensile or compressive loading [31–35].

Although 2D auxetic structures have been investigated for many years, their applications in civil engineering are still limited. The main reason is that the existing auxetic material strength and stiffness cannot meet the requirements under both static and dynamic loading, e.g., anti-seismic and anti-explosion. In recent years, in order to improve stiffness and strength, auxetic structures have gradually evolved from elastic foam materials to metallic structures. An important characteristic of these metallic structures is that the plasticity of the material is fully utilized to achieve auxetic performance of the structure, since large deformation of metal is commonly developed in the plastic state [36–38]. Thus, definition of the Poisson’s ratio is also extended from the elastic stage to the plastic one. At present, existing metallic auxetic structures are made of brass and aluminum, which are very expensive.

On the other hand, structural steel commonly has good ductility, and it can be perforated easily to achieve re-entrant configurations. Although a solid steel structure obviously has stronger bearing capacity under static loading, in dynamic situations such as anti-seismic and anti-explosion, strength is not the only mechanical index. Perforated members can have more plastic deformation to absorb energy under dynamic loading, which can protect the whole structure. There are already applications of perforated steel plates in civil engineering, using its plastic behaviors to dissipate seismic energy and needing to be replaced after a strong earthquake, such as slotted steel plates in shear walls and steel dampers in anti-seismic applications, which rely on the plastic deformation of perforated steel plates [39,40]. Generally, their holes are well designed and in regular shapes.

The novelty of this study is the realization of a metallic auxetic structure using a 2D re-entrant honeycomb form through the perforation of steel plates.

Firstly, steel has high strength and plastic deformation capacity, and it is widely used. However, there are few related studies on auxetic structures made of steel. Since the 2D re-entrant honeycomb structure is a porous form, it is possible to configure it through perforation of solid steel plates. From the viewpoint of technical realization, the rapid development of laser cutting technology has made it more convenient to cut holes in steel plates. High-temperature laser cutting machines can quickly cut steel plates below 25 mm.

Secondly, steel is a material with a high positive Poisson’s ratio of 0.5 under the plastic range, so it is also an innovation of this work to achieve auxetic performance even under a large plastic strain range.

Thirdly, the 2D re-entrant honeycomb form has been reported to have good energy absorption capacity and auxetic performance, and the potential to achieve good mechanical properties through perforated steel plates. Compared with other high-ductility materials (such as brass), perforated steel plates can be widely employed in the field of structural engineering as steel dampers and stiffeners in spatial structures. Corresponding achievements can be further employed to develop high-performance metallic dampers, as shown in References [40,41].

In this study, the static mechanical properties of steel plates with re-entrant honeycomb configurations, such as load–displacement curves, damage states, variation of the Poisson’s ratio, etc. were examined. The properties of specimens with re-entrant honeycomb structures were expected to be fully obtained by both experimental research and finite element analysis. This also lays a foundation for further study of these structures under dynamic loading.

2. Experimental Section

In order to investigate the effects of configurations on the static performance of steel plates with re-entrant honeycomb perforations, a series of monotonic tension tests were conducted. Tested specimens with typical re-entrant honeycomb structures were made via mechanical manufacturing. In this study,
the mechanical manufacturing method was employed for its minor effect on material properties of specimens, low cost, and high efficiency compared with the 3D printing method.

2.1. Design of Specimens

In total, seven specimens with the re-entrant honeycomb configuration illustrated in Figure 1 were designed and manufactured. All the specimens were cut from the same 16 mm thick steel plate made of Q235B. Three corresponding coupon tests were conducted to obtain the mechanical properties of the base metal. Long and narrow steel plate specimens were designed. The middle parts of the specimens consisted of staggered re-entrant honeycomb units, and the two end parts had no perforations and were employed to clamp the specimen to a testing machine. The total length of each specimen was 256 mm. The length of the perforated area had a fixed value of 76 mm, and the width \( b \) was a variable, which changed with the hole ratio of perforated part. The perforated area was a typical concave re-entrant honeycomb structure. For all seven specimens, five concave units were alternately embedded in the same horizontal direction. The geometrical configuration of the re-entrant honeycomb units was clear, and was determined by the following three critical parameters, as illustrated in Figures 1 and 2 in this study.

1. Re-entrant angle, \( \alpha \), defined as the angle between the horizontal and vertical edges of the re-entrant honeycomb unit. This parameter determines the bending degree of the whole configuration, and from mechanical common sense, it affects the stiffness and ductility of the tensile direction.
2. Hole ratio, \( h \), defined as the ratio of perforated area to the original one, can be calculated using the following equation.

\[
h = \frac{S_1}{S_2},
\]

where \( S_1 \) and \( S_2 \) are the perforated area and original area of cross section, respectively. This parameter can have a great effect on the strength and stiffness of re-entrant honeycomb specimens. In this experiment, the length of the perforated area \( l_y \) was a fixed value, and the width of a re-entrant honeycomb specimen, \( b \), is a variable. In this study, specimens with different hole ratios had the same cross-sectional area to ensure that the experimental results were comparable, and the hole ratio was the single factor affecting performance of the three specimens with different hole ratios. Thus, the width \( b \) was affected by the hole ratio.
3. Chamfer radius, \( R \). The re-entrant configurations had sharp angles at the corners of each honeycomb unit, which could lead to stress and strain concentration under loading. Thus, chamfering was conducted to reduce the concentration effect, and the chamfering radius was selected as a design parameter in this study.

Seven specimens were designed in order to study the aforementioned three critical parameters, each with three different values: hole ratio \( h = 30\%, 40\%, \) and \( 50\% \); concave angle \( \alpha = 75^\circ, 80^\circ, \) and \( 85^\circ \); and chamfer radius \( R = 0, 0.5, 1 \) mm. In terms of specimen numbering, Specimen H40A80R0.5 indicates a test specimen with a hole ratio of 40\%, a re-entrant angle of 80\°, and a chamfer radius of 0.5 mm. Specimen H40A80R0.5 was also a standard test specimen, and the other ones were compared with this specimen by fixing two parameters and changing the other one. Table 1 gives a breakdown of all dimensions of the seven tested specimens. Figure 2a shows the schematic configurations of all the tested specimens, and Figure 2b shows the actual manufactured ones. The sample design was as follows. Firstly, the lengths of the tested specimens were the same. Secondly, each parameter was investigated with three specimens while the other two parameters were kept the same. For example, the cross-sectional areas of the three specimens with different hole ratios had the same re-entrant angles and chamfering radii. The same rule was applied to the other two parameters. Thus, the experimental results for the specimens were comparable.
The engineering definition of Poisson’s ratio was presented by Strek et al. [23], and the variability of the Poisson’s ratio for specimens has been evaluated in the literature using the following formula.

\[ \nu = -\frac{\varepsilon_x}{\varepsilon_y}, \]  

(2)

where \( \varepsilon_x \) is the transverse strain and \( \varepsilon_y \) is the longitudinal strain. In this study, \( \varepsilon_x = \Delta l_x/l_x \) is the average strain in the transverse direction, \( \Delta l_x \) is the lateral displacement, and \( l_x \) is width of the section; \( \varepsilon_y = \Delta l_y/l_y \) is the average strain in the longitudinal direction and \( \Delta l_y \) is the longitudinal displacement. \( l_y \) is the length of the perforated area, which was 76 mm. Both convex and concave positions were considered, i.e., Point A and Point B in Figure 1, in order to observe the Poisson’s ratio in different sections.

Table 1. Design parameters of tested specimens.

| No. | Specimens     | h (%) | \( \alpha \) (°) | R (mm) | b (mm) | \( l_y \) (mm) | \( l_s \) (mm) | t (mm) |
|-----|---------------|-------|-----------------|-------|-------|--------------|------------|-------|
| 1   | H30A85R0.5    | 30    | 85              | 0.5   |       | 40.3         |            |       |
| 2   | H40A85R0.5    | 40    | 85              | 0.5   |       | 50.3         |            |       |
| 3   | H50A85R0.5    | 50    | 85              | 0.5   |       | 70.3         |            |       |
| 4   | H40A75R0.5    | 40    | 75              | 0.5   |       | 53.0         | 76         | 90    | 16   |
| 5   | H40A80R0.5    | 40    | 80              | 0.5   |       | 51.6         |            |       |
| 6   | H40A85R0      | 40    | 85              | 0     |       | 50.3         |            |       |
| 7   | H40A85R1      | 40    | 85              | 1.0   |       | 50.3         |            |       |

Figure 1. Design parameters of specimens.
2.2. Test Setup

The tests were conducted using the MTS testing machine shown in Figure 3 at room temperature, at a quasi-static speed of 0.015 mm/s. The capacity of the loading machine was 250 kN and the displacement capacity was ±75 mm. Specimens were clamped to the bottom loading head, and enforced displacement was applied to the top movable loading head. An extensometer with a gage length of 50 mm was employed to measure the net deformation of the central segment of the coupon, as shown in Figure 3a. Two vertical linear variable differential transformers (LVDTs) were set at the top and bottom of each specimen to measure the net displacement of the perforated segment shown in Figure 3b. A horizontal LVDT was set at the center–right to measure the horizontal expansion of specimens, which was used to obtain the variation of the Poisson's ratio with increasing vertical deformation.
3. Experimental Results

All the specimens failed in a ductile fracture mode, and Figure 4 shows the rupture modes of the tested specimens under monotonic tension. Two specimens, H30A85R0.5 and H40A75R0.5, failed prematurely due to unexpected manufacturing defects, which was due to the complicated configurations and small sizes of the specimens. Experimental results were obtained for the five well-manufactured specimens. Cracking first initiated at an edge of a re-entrant honeycomb unit, and propagated rapidly to the other neighboring honeycomb units. All the cracks occurred in the perforated segment, and were also away from the ends of the perforated segment. In most of the specimens, cracks propagated diagonally along the width direction except for in Specimen H40A85R0.5, as shown in Figure 4. Figure 5 gives experimental load–displacement curves of the specimens. According to the load–displacement curves, all members reached the yield force and entered the plastic state under a tensile force of 106.91 kN to 110.32 kN, as listed in Table 2, indicating minor differences among the specimens. Loads of the specimens continued to increase after initial yielding, and the five specimens ruptured at maximum tensile forces ranging from 187.06 kN to 210.75 kN. The ultimate displacement of the five specimens ranged from 14.83 mm to 17.26 mm; the specimen with the maximum ultimate displacement was H40A80R0.5, and the minimum was H40A85R1. The maximum difference of the rupture displacement was about 14.07%, which was not apparent.

Comparing the specimens before and after the test, it appeared that the re-entrant honeycomb elements in the specimens had significant shape changes, mainly axial stretch, and the concave angles were stretched. Searching for the occurrence of structural failure, most of the fractures appeared in the middle of the opening holes, and no fractures occurred in the corner end. The fractures were generally 45° inclined. Testing piece H40A85R0.5 was a little special, and the fracture appeared to be horizontal.

Correlation is a coefficient that indicates a statistical relationship between the FE analysis result and the experimental one, and it is defined by the following equation.

$$\text{Correlation}(X, Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}[X]\text{Var}[Y]}}$$  \hspace{1cm} (3)$$

where Cov(X,Y) is the covariance of X and Y, and Var[X] and Var[Y] are the variance of X and variance of Y in the statistics.
Figure 4. Failure modes of tested specimens.

Table 2. Experimental and numerical results of specimens.

| Sample | Yield Force $F_y$ (kN) | Yield Force FE/$F_y$ | Peak Load (kN) | Ultimate Force $F_u$ (kN) | Ultimate Force FE/$F_u$ | Ultimate Displacement $d_u$ (mm) | Ultimate Displacement FE/$d_u$ |
|--------|------------------------|----------------------|---------------|---------------------------|-------------------------|----------------------------------|----------------------------------|
| H40A85R0.5 | 106.91 | 115.29 | 1.08 | 187.06 | 197.41 | 1.06 | 15.24 | 17.04 | 1.12 |
| H50A85R0.5 | 109.53 | 116.89 | 1.07 | 197.94 | 197.70 | 1.00 | 17.00 | 16.13 | 0.95 |
| H40A80R0.5 | 110.06 | 119.65 | 1.09 | 210.75 | 216.04 | 1.03 | 17.26 | 17.38 | 1.01 |
| H40A85R0 | 107.07 | 115.80 | 1.08 | 187.19 | 197.54 | 1.06 | 16.98 | 15.78 | 0.93 |
| H40A85R1 | 110.32 | 115.37 | 1.05 | 190.76 | 199.04 | 1.04 | 14.83 | 15.45 | 1.04 |

Correlation is a coefficient that indicates a statistical relationship between the FE analysis result and the experimental one, and it is defined by the following equation.

$$ \text{Correlation}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}[X] \cdot \text{Var}[Y]}} $$

Where $\text{Cov}(X, Y)$ is the covariance of $X$ and $Y$, and $\text{Var}[X]$ and $\text{Var}[Y]$ are the variance of $X$ and variance of $Y$ in the statistics.

Figure 5. Load–displacement curves of tested specimens for experiments with (a) H40A85R0.5, (b) H50A85R0.5, (c) H40A80R0.5, (d) H40A85R0 and (e) H40A85R1.
Table 2. Experimental and numerical results of specimens.

|                  | Yield Force $F_y$ (kN) | $F_y\text{,test}/F_y\text{,FE}$ | Peak Load (kN) | $F_u\text{,test}/F_u\text{,FE}$ | Ultimate Displacement $d_u\text{,mm}$ | $d_u\text{,test}/d_u\text{,FE}$ |
|------------------|------------------------|----------------------------------|----------------|-------------------------------|-------------------------------------|------------------------------------|
|                  | Test                   | FE                               | Test           | FE                           | Test                                | FE                               |
| H40A80R0.5       | 106.91                 | 115.29                           | 1.08           | 187.06                       | 197.41                               | 1.06                              | 15.24                             | 17.04                             | 1.12                             |
| H50A85R0.5       | 109.53                 | 116.89                           | 1.07           | 197.94                       | 197.70                               | 1.03                              | 17.00                             | 16.13                             | 0.95                             |
| H40A80R0.5       | 110.06                 | 119.65                           | 1.09           | 210.75                       | 214.04                               | 1.03                              | 17.26                             | 17.58                             | 1.01                             |
| H40A85R0        | 107.07                 | 115.80                           | 1.08           | 187.19                       | 197.54                               | 1.06                              | 16.98                             | 15.78                             | 0.93                             |
| H40A85R1        | 110.32                 | 115.37                           | 1.05           | 190.76                       | 199.04                               | 1.04                              | 14.85                             | 15.45                             | 1.04                             |

Correlation 0.50 0.90 0.28 0.90 0.28

Notes: $F_y\text{,FE}$ and $F_y\text{,test}$ denote the yield force of the FE analysis result and the experimental result, respectively; $F_u\text{,FE}$ and $F_u\text{,test}$ denote the ultimate force of the FE analysis result and the experimental result, respectively; $d_u\text{,FE}$ and $d_u\text{,test}$ denote the ultimate displacement of the FE analysis result and the experimental result, respectively; the ultimate displacement refers to the instant when the force decreased to 85% of the peak load.

4. Numerical Simulation

4.1. FE Modeling

Three-dimensional FE models using solid elements were established in ABAQUS to simulate the experimental results of the specimens under tension. The FE models employed the element type C3D8R with a reduced integration scheme for its accuracy and efficiency. The same boundary conditions were applied to the FE models, with the bottom end fixed and enforced displacement applied to the top end. Elements with small sizes were employed in the perforated segment to capture stress and strain concentration there, while the two ends were meshed with relative large sizes. The number of elements for the mesh of the typical specimen H40A85R1 was 22,660, the FE model is shown in Figure 6. The whole failure process was simulated using FE analysis, and both plasticity and fracturing of the specimens were considered through proper mechanical models, which are detailed in the following section. To successfully simulate the whole failure process, an explicit integration scheme was employed to ensure convergence of the simulations until rupture of all the specimens. Meanwhile, a large enough step time was required to ensure that all analyses remained quasi-static, which was the same as the loading condition of the experiments.

![Figure 6. FE model of Specimen H40A85R1.](image)

4.2. Plasticity and Fracture Models

An isotropic hardening rule was employed, since no cyclic plasticity was involved. The true stress–true strain data were obtained from the corresponding coupon test results, and post-necking
The yield force, peak load, and ultimate displacement of the five specimens are compared in Table 2. The numerical results generally overestimated the experimental loads a bit, and the average values for the ratios of the numerical results to the experimental ones for the yield strength, the peak load, and the ultimate displacement were 1.07, 1.04, and 1.01. Values for the correlation were 0.50, 0.90, and 0.28, respectively. The relative errors of the numerical results of the yield strength and the peak load ranged from 5% to 9% and 0% to 6%, respectively; values for the corresponding coefficient of variation (CoV) were $1.84 \times 10^{-4}$ and $4.96 \times 10^{-4}$; The ultimate displacement of the specimens was also predicted with good accuracy, with an minimum average value of 1.01 for the ratio of the FE value to corresponding experimental value, where the corresponding relative error ranged from $-7$ to 12%. The corresponding CoV was $4.60 \times 10^{-3}$, implying that the ductile fracture model could effectively predict failure of the steel specimens. The load–displacement curves of the experimental results are compared with the corresponding numerical results in Figure 7. Based on the comparison results, it was found that loads and failure displacements were predicted with good accuracy. Cracking and final rupture of the specimens were also successfully captured by the FE analyses. Comparison of the rupture modes is given in Figure 8, indicating that the fracture model could generally reproduce the whole cracking processes. Due to the randomness of cracking in nature and the high degree of symmetry of configuration of the specimens, the cracking paths of the FE results were a bit different from the experimental ones, especially for Specimens H40A80R0.5, H40A85R0, and H40A85R1. Many factors can affect the crack path, such as manufacturing deviations and loading eccentricity. Ideally, the crack paths should be symmetrical since the loading conditions and the specimens were symmetrical. However, none of the specimens cracked symmetrically in the experimental or numerical results. This indicates randomness of the crack paths of the specimens. In addition, all the fracture modes were ductile fracture, and the random crack paths had no significant effect on the global seismic performance of the specimens.
Comparison of the rupture modes

Cracking and final rupture of the specimens were also successfully captured by the FE analyses. Due to the randomness of cracking in nature and the high degree of symmetry of configuration of the specimens, the cracking paths of the FE results are a bit different from the experimental ones, especially for Specimens H40A80R0.5, H40A85R0 and H40A85R1. Many factors can affect the crack path, such as manufacturing deviations and loading eccentricity.


dates, it was found that loads and failure displacements were predicted with good accuracy.

The relative errors of the peak load range, load values for the corresponding coefficient of variation (CoV) were 0.9 and 0.12%. The corresponding CoV was determined with good accuracy, with a minimum value of variation (CoV) of 0.9.

Table 2. The numerical results generally overestimate the load and ultimate displacement of the specimens a bit, and the average value of 1.01 for the ratio of the FE predictions to the corresponding experimental loads is reasonable. This indicates randomness of the crack paths of the specimens. In addition, all the fracture modes were generally reproduced by the FE analyses. Due to the randomness of cracking in nature and the effective for loading conditions and cyclic incremental loading, the ductile fracture model was found to be effective for crack initiation prediction of monotonic loading.

Figure 7. Comparisons of load–displacement curves between experimental and numerical results with (a) H40A85R0.5, (b) H50A85R0.5, (c) H40A80R0.5, (d) H40A85R0 and (e) H40A85R1.
5. Discussions

To clarify the mechanisms of influence of each parameter on the elastoplastic performance of specimens, numerical analyses were conducted using the aforementioned FE models. Seven specimens with the same configurations as those use in the experiments were constructed and compared with each other, including the two specimens failing prematurely.

Load–displacement curves of the 2D re-entrant honeycomb specimens with different hole ratios are compared in Figure 9, and indicated that the three specimens had almost the same load–displacement curves. The specimens with different hole ratios were designed with the same width for each edge of the re-entrant honeycomb unit, meaning that the strength and stiffness values of the three specimens were almost the same. To investigate the effect of hole ratio on the stress and strain conditions of specimens, equivalent plastic strain, stress triaxiality, and damage index contour plots at an average strain of about 10% were compared as shown in Figures 10–12, respectively. Figure 10 shows that values of maximum stress triaxiality of the three specimens with different hole ratios were close to each other, but the maximum equivalent plastic strain of Specimen H50A85R0.5 is much larger than that of the other two specimens, leading to early crack initiation of the specimen.
The Poisson’s ratio versus average strain curves are given in Figure 13, where the average strain is defined as the ratio of the relative vertical displacement of the perforated segment to its length, and the Poisson’s ratios were calculated from the horizontal and vertical average strain data. Horizontal displacement data of the two typical points shown in Figure 13, i.e., convex point (blue) and concave point (red), were obtained from the numerical results. Comparing Figure 13a–c, it can be seen that specimens with different hole ratios had a similar tendency. For Specimens H30A85R0.5, H40A85R0.5, and H50A85R0.5, the Poisson’s ratio was positive at the initial loading stage, and then changed to negative quickly before 0.15%. The Poisson’s ratios changed to positive values with an increasing vertical average strain, and achieved the maximum value. The Poisson’s ratio then decreased while still maintaining a positive value. Generally, the hole ratio had some effect on variation of the Poisson’s ratio, especially for Specimen H30A85R0.5, which was quite different from the other two specimens.
Figure 11. Effect of hole ratio on equivalent plastic strain with (a) H30A85R0.5 (b) H40A85R0.5 and (c) H50A85R0.5.

Figure 12. Effect of hole ratio on damage index with (a) H30A85R0.5 (b) H40A85R0.5 and (c) H50A85R0.5.

Figure 13. Effect of design parameters on Poisson’s ratio.

Comparing Figure 13, it can be seen that the concave angle had a great effect on the Poisson’s ratio. For Specimen H40A75R0.5, the Poisson’s ratio range from about −0.5 to 0, and was almost always below 0 throughout the whole loading history until rupture. However, the Poisson’s ratio range from about −0.2 to 0.2 and −0.2 to 0.3 for Specimens H40A80R0.5 and H40A85R0.5, respectively. As the concave angle decreased, the Poisson’s ratio tended to decrease, and it is expected that an angle below 75° can ensure that the Poisson’s ratio of a specimen will remain negative throughout the whole loading process. This conclusion was inferred from the geometric configuration of the specimens. The concave edges of each cell were gradually straightened in the process of stretching, so that the opening areas showed a trend of changing from concave hexagons to rectangles. In this process, expansion perpendicular to the stretching direction gradually occurred. Therefore, it is advantageous for negative Poisson’s ratio performance to reduce the concave angle and for the structure to have more straightened space. From another point of view,
Comparing Figure 13b,d,e, it can be seen that the concave angle had a great effect on the Poisson’s ratio. For Specimen H40A75R0.5, the Poisson’s ratio ranged from about −0.5 to 0, and was almost always below 0 throughout the whole loading history until rupture. However, the Poisson’s ratio ranged from about −0.2 to 0.2 and −0.2 to 0.3 for Specimens H40A80R0.5 and H40A85R0.5, respectively. As the concave angle decreased, the Poisson’s ratio tended to decrease, and it is expected that an angle below 75° can ensure that the Poisson’s ratio of a specimen will remain negative throughout the whole loading process. This conclusion was inferred from the geometric configurations of the specimens. The concave edges of each cell were gradually straightened in the process of stretching, so that the opening areas showed a trend of changing from concave hexagons to rectangles. In this process, expansion perpendicular to the stretching direction gradually occurred. Therefore, it is advantageous for negative Poisson’s ratio performance to reduce the concave angle and for the structure to have more straightened space. From another point of view, the straightened edges bear both tension and bending under uniaxial tension of the specimen, and a concave angle that is too small may lead to higher edge damage. In fact, from the present experimental and numerical results, failure points also occurred on these edges due to tension and bending. Therefore, finding a reasonable bending angle with which to achieve the best negative Poisson’s ratio performance was the goal of the next stage.

Comparing Figure 13b,f,g, it was found that the chamfering radius also had a certain effect on the Poisson’s ratio. For Specimen H40A85R0, the Poisson’s ratio ranged from about −0.3 to 0.3, while it ranged from about −0.2 to 0.3 for Specimen H40A85R0.5. The Poisson’s ratio for Specimen H40A85R1 ranged from −0.4 to 0.3. It was interesting to note that the peak value of the Poisson’s ratio at the convex point was always close to 0.3, while the minimum values changed greatly with varying chamfering radius.

The results showed that the 2D re-entrant honeycomb structure itself did not guarantee a negative Poisson’s ratio in practice, which can be explained by the following two factors. Firstly, the auxetic performance needs to be realized under the condition of large deformation. In contrast to non-metallic materials of linear elasticity and hyperelasticity, large deformation of steel needs to be realized in the plastic stage, and damage and failure of the material should be considered. Therefore, the negative Poisson’s ratio phenomenon is not easy to achieve, since the Poisson’s ratio of steel in the plastic stage is close to 0.5. In addition, according to the discussion in last paragraph, the auxetic performance is also closely correlated with the concave angle. In this study, the concave angle in the design of the structure was relatively large to realize high stiffness of the specimens under static loading, so auxetic performance of the structures could not be achieved in the full strain range.

Load–displacement curves of the specimens with different concave angles are compared in Figure 14, indicating that strength increased with a decreasing concave angle. This is mainly due to the fact that each edge of the re-entrant honeycomb unit was designed to be the same for all the specimens, and this led to the fact that the minimum cross-sectional area of the specimens increased with decreasing concave angle. Likewise, equivalent plastic strain, stress triaxiality, and damage index contour plots at an average strain of about 10% are compared in Figures 15–17, respectively. Figure 15 shows that values of the maximum stress triaxiality of the three specimens with different concave angles were close to each other, but the maximum equivalent plastic strain decreased with increasing concave angle. Thus, the maximum damage index given in Equation (1) decreased with increasing concave angle for the three specimens.
The Poisson's ratio needs to be determined for steel. Figure 19 shows that values of the maximum stress triaxiality and equivalent plastic strain contour plots at an average strain of about 10% are compared in Figures 19–21, almost the same. This can be easily understood since the cross-sectional areas of specimens with different chamfering radii are close to each other, indicating a finding that strength and stiffness values of the specimens before about 12.5 mm were almost the same. This can be easily understood since the cross-sectional areas of specimens with different chamfering radii were close to each other, indicating a close effect of chamfering radius on the ductility of the specimens, as shown in Figure 21.

Figure 14. Effect of concave angle on load–displacement curves.

Figure 15. Effect of concave angle on stress triaxiality with (a) H40A75R0.5 (b) H40A80R0.5 and (c) H40A85R0.5.
Figure 15. Effect of concave angle on stress triaxiality with (a) H40A75R0.5 (b) H40A80R0.5 and (c) H40A85R0.5.

Figure 16. Effect of concave angle on equivalent plastic strain with (a) H40A75R0.5 (b) H40A80R0.5 and (c) H40A85R0.5.

Figure 17. Effect of concave angle on damage index with (a) H40A75R0.5 (b) H40A80R0.5 and (c) H40A85R0.5.
Figure 17. Effect of concave angle on damage index with (a) H40A75R0.5, (b) H40A80R0.5 and (c) H40A85R0.5.

Load–displacement curves of the specimens with different chamfering radii are compared in Figure 18, indicating that strength and stiffness values of the specimens before about 12.5 mm were almost the same. This can be easily understood since the cross-sectional areas of specimens with different chamfering radii were close to each other. Likewise, equivalent plastic strain, stress triaxiality, and damage index contour plots at an average strain of about 10% are compared in Figures 19–21, respectively.

Figure 18. Effect of chamfering radius on load–displacement curves.

Figure 19. Effect of chamfering radius on stress triaxiality with (a) H40A85R0, (b) H40A85R0.5 and (c) H40A85R1.

In the design of re-entrant honeycomb structures, the behavior could also be affected by the proportion of solid area. In the original design, the total length of the specimen was 256 mm. The length of the perforated area was a fixed value of 76 mm, so the length of the solid area $l_s$ at both ends was 90 mm. The two long un-perforated edges were expected to achieve enough holding forces at the chucks. This length was mainly to ensure that the MTS machine chucks had enough clamping space. Nevertheless, although this paper focused on the mechanical performance of the perforated area, the large solid area may have affected the mechanical properties and Poisson’s ratios of the specimens.

In order to evaluate this effect, two additional FE models were established according to Specimen H40A85R0.5, with different lengths of solid area $l_s$ of 20 mm and 50 mm, to compare with the original lengths of 90 mm, as shown in Figure 22. Among the three models, the ratio of the perforated area to the surface area of the specimen was 29.7%, 43.2% and 65.5%, respectively. Explicit integration schemes were employed for these models with the same boundary conditions.

Figure 20. Effect of chamfering radius on equivalent plastic strain with (a) H40A85R0, (b) H40A85R0.5 and (c) H40A85R1.
According to the load–displacement curves shown in Figure 23a, the curves matched well before reaching the failure point and were similar in their failure process, which indicated that the bearing capacity and ductility of perforated area was almost unaffected by the length of solid area $l_s$. Figure 23b shows the fracture model with almost the same damage and cracking behaviors. On the other hand, the curves of Poisson’s ratio at the convex and concave points, were obtained from the numerical results, shown in Figure 24. Generally, the specimens with a small solid region showed better negative Poisson’s ratio performance. The maximum negative Poisson’s ratio of the model with $l_s$ of 20 mm and 50 mm was $-0.3$, while the maximum negative Poisson’s ratio of the original model was $-0.2$. After the strain exceeded 0.15, the negative Poisson’s ratio of the model with a solid area length of 20 mm and 50 mm gradually approached 0.1, while the original model gradually approached 0.2. The Poisson’s
ratio of the model with a length of 20 mm was smaller than that of the model of 50 mm in general when the strain exceeded 0.2.

Figure 22. H40A85R0.5 specimens with different lengths of solid area.

Figure 23. (a) Load–displacement curves and (b) failure modes of specimens with different solid areas.
The Poisson's model.

The rupture displacement of the original model was 20 mm, and the ones for \( l_y \) were 25 mm and 33 mm, respectively. On the other hand, Figure 26b shows almost the same damage which could restrain lateral expansion of the perforated part. Therefore, a shorter length of the solid region was 0.5 in the plastic range. The unperforated edges had a positive Poisson's ratio of 0.5, which could also affect the mechanical behaviors of the specimens. In order to evaluate this effect, two additional FE models were established based on Specimen H40A85R0.5. Different lengths of \( l_y \) of 114 mm and 152 mm were designed to compare with the original length of 76 mm. The arrangements of the re-entrant honeycomb units in the three models were \( 5 \times 8, 5 \times 6, \) and \( 5 \times 4 \), as shown in Figure 25.

Load–displacement curves of the three models are shown in Figure 26a, which indicated that the ultimate bearing capacities were similar, while the rupture displacement increased with an increasing \( l_y \). The rupture displacement of the original model was 20 mm, and the ones for \( l_y \) of 114 mm and 152 mm were 25 mm and 33 mm, respectively. On the other hand, Figure 26b shows almost the same damage and cracking behaviors in the fracture model. The equivalent plastic strains of the models were almost the same when rupture occurred.

Figure 24. Poisson's ratio curves of specimens with different solid areas with (a) \( l_y = 20 \) mm, (b) \( l_y = 50 \) mm and (c) \( l_y = 90 \) mm.

The reason for the above result is easy to understand. The Poisson’s ratio of the steel in the solid region was 0.5 in the plastic range. The unperforated edges had a positive Poisson's ratio of 0.5, which could restrain lateral expansion of the perforated part. Therefore, a shorter length of the solid area led to better auxetic performance.

During design of the specimens, the length of the perforated area \( l_y \) was 76 mm, which was mainly due to the displacement capacity of the displacement sensors. From the perspective of parametric analysis, the length of perforated area \( l_y \) could also affect the mechanical behaviors of the specimens. In order to evaluate this effect, two additional FE models were established based on Specimen H40A85R0.5. Different lengths of \( l_y \) of 114 mm and 152 mm were designed to compare with the original length of 76 mm. The arrangements of the re-entrant honeycomb units in the three models were \( 5 \times 8, 5 \times 6, \) and \( 5 \times 4 \), as shown in Figure 25.

Load–displacement curves of the three models are shown in Figure 26a, which indicated that the ultimate bearing capacities were similar, while the rupture displacement increased with an increasing \( l_y \). The rupture displacement of the original model was 20 mm, and the ones for \( l_y \) of 114 mm and 152 mm were 25 mm and 33 mm, respectively. On the other hand, Figure 26b shows almost the same damage and cracking behaviors in the fracture model. The equivalent plastic strains of the models were almost the same when rupture occurred.
Figure 25. H40A85R0.5 specimens with different lengths of perforated area.

Figure 26. (a) Load–displacement curves and (b) failure modes of specimens with different perforated areas.
The curves of the Poisson’s ratio at the convex and concave points were obtained from the numerical results and are shown in Figure 27. When the length $l_y$ was 114 mm, the maximum negative Poisson’s ratio of the specimen reached $-0.34$, and it gradually converged to 0.1 when the strain exceeded 0.15. When the length $l_y$ was 152 mm, the maximum negative Poisson’s ratio was $-0.4$, and this showed the best auxetic performance in the three models.

![Figure 27. Poisson’s ratio curves of specimens with different perforated areas with (a) $l_y = 76$ mm, (b) $l_y = 114$ mm and (c) $l_y = 152$ mm.](image)

Based on the above results, increasing the length of perforated area $l_y$ was beneficial to achieve better auxetic performance. The reason for this is easy to understand, mainly owing to less constraint of the solid area to the central perforated area, and the larger perforated areas had a better trend of lateral expansion.

In addition, a homogenization method to estimate the effective properties of perforated NPR specimens could be an alternative approach [44], although it is still not easy to achieve that goal within a large plastic range.

6. Conclusions

As an initial study for steel members with 2D re-entrant honeycomb configurations, a quasi-static experimental study was conducted to investigate the effects of different critical geometrical parameters, i.e., hole ratio, concave angle, and chamfering radius, on the static elastoplastic performance of specimens. Numerical cracking simulation using micromechanics-based ductile fracture model was also employed. Based on the above studies, the following conclusions were drawn.

1. The concave angle was found to have a great effect on the Poisson’s ratio of specimens compared with the other two parameters. It is predicted that an angle less than $75^\circ$ can ensure that the Poisson’s ratio will always be negative throughout the whole loading process until rupture.
2. For the investigated range of parameters, the critical parameters had minor effects on the ductility of the specimens under quasi-static loading.

3. The micromechanics-based ductile fracture model was able to capture rupture of specimens with acceptable accuracy.

4. According to the study of specimens with different lengths of solid area and lengths of perforated area, a shorter length of the solid region or a larger length of perforated area can lead to better auxetic performance, since solid edges with a positive Poisson’s ratio of 0.5 can restrain lateral expansion of the perforated region, and the re-entrant honeycomb configuration in the perforated area had a better trend of lateral expansion in the plastic range. Moreover, it is necessary to investigate the strain rate effect on the dynamic performance of specimens, which is of interest in practical applications.

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References
1. Saxena, K.K.; Das, R.; Calius, E.P. Three decades of auxetics research—materials with negative Poisson’s ratio: A review. Adv. Eng. Mater. 2016, 18, 1847–1870. [CrossRef]
2. Milton, G.W. Composite materials with Poisson’s ratios close to -1. J. Mech. Phys. Solids 1992, 40, 1105–1137. [CrossRef]
3. Evans, K.E.; Alderson, A. Auxetic materials: Functional materials and structures from lateral thinking. Adv. Mater. 2000, 12, 617–628. [CrossRef]
4. Choi, J.B.; Lakes, R.S. Fracture toughness of re-entrant foam materials with a negative Poisson’s ratio: Experiment and analysis. Int. J. Fract. 1996, 80, 73–83. [CrossRef]
5. Chen, C.P.; Lakes, R.S. Micromechanical analysis of dynamic behavior of conventional and negative Poisson’s ratio foams. J. Eng. Mater. Technol. 1996, 118, 285–288. [CrossRef]
6. Evans, K.E. Auxetic polymers: A new range of materials. Endeavour 1991, 15, 170–174. [CrossRef]
7. Lakes, R.S. Foam structures with a negative Poisson’s ratio. Science 1987, 235, 1038–1040. [CrossRef]
8. Almgren, R.F. An Isotropic 3-Dimensional Structure with Poisson Ratio=-1. J. Elast. 1985, 15, 427–430.
9. Wojciechowski, K.W. Constant thermodynamic tension Monte Carlo studies of elastic properties of a two-dimensional system of hard cyclic hexamers. Mol. Phys. 1987, 61, 1247–1258. [CrossRef]
10. Wojciechowski, K.W. Two-dimensional isotropic system with a negative Poisson ratio. Phys. Lett. A 1989, 137, 60–64. [CrossRef]
11. Prall, D.; Lakes, R.S. Properties of a chiral honeycomb with a Poisson’s ratio of -1. Int. J. Mech. Sci. 1997, 39, 305–307. [CrossRef]
12. Grima, J.N.; Evans, K.E. Auxetic behavior from rotating squares. J. Mater. Sci. Lett. 2000, 19, 1563–1565. [CrossRef]
13. Grima, J.N.; Winczewski, S.; Mizzi, L.; Grech, M.C.; Cauchi, R.; Gatt, R.; Attard, D.; Wojciechowski, K.W.; Rybicki, J. Tailoring graphene to achieve negative Poisson’s ratio properties. Adv. Mater. 2015, 27, 1455–1459. [CrossRef]
14. Hoover, W.G.; Hoover, C.G. Searching for auxetics with DYNA3D and ParaDyn. Phys. Status Solidi (B) 2005, 242, 584–594. [CrossRef]
15. Tretiakov, K.V.; Wojciechowski, K.W. Poisson’s ratio of simple planar ‘isotropic’ solids in two dimensions. Phys. Status Solidi (B) 2007, 244, 1038–1046. [CrossRef]
16. Bilski, M.; Wojciechowski, K.W. Tailoring Poisson’s ratio by introducing auxetic layers. Phys. Status Solidi (B) 2016, 253, 1318–1323. [CrossRef]
17. Piglowski, P.M.; Wojciechowski, K.W.; Tretiakov, K.V. Partial auxeticity induced by nanoslits in the Yukawa crystal. *Phys. Status Solidi (RRL)*–Rapid Res. Lett. 2016, 10, 566–569. [CrossRef]

18. Ho, V.H.; Ho, D.T.; Kwon, S.Y.; Kim, S.Y. Negative Poisson’s ratio in periodic porous graphene structures. *Phys. Status Solidi (B)* 2016, 253, 1303–1309. [CrossRef]

19. Scarpa, F.; Ruzzene, M.; Alderson, A.; Wojciechowski, K.W. Auxetics in smart systems and structures. *Smart Mater. Struct.* 2013, 22, 080201. [CrossRef]

20. Wojciechowski, K.W.; Scarpa, F.; Grima, J.N.; Alderson, A. Auxetics and other systems of anomalous characteristics. *Phys. Status Solidi (B)* 2017, 254, 1770266. [CrossRef]

21. Wojciechowski, K.W.; Scarpa, F.; Grima, J.N.; Alderson, A. Auxetics and other systems of “negative” characteristics. *Phys. Status Solidi (B)* 2015, 252, 1421–1425. [CrossRef]

22. Overvelde, J.T.B.; Shan, S.; Bertoldi, K. Compaction through buckling in 2D periodic, soft and porous structures: Effect of pore shape. *Adv. Mater.* 2012, 24, 2337–2342. [CrossRef] [PubMed]

23. Taylor, M.; Francesconi, L.; Gerendás, M.; Shanian, A.; Carson, C.; Bertoldi, K. Low porosity metallic periodic structures with negative Poisson’s ratio. *Adv. Mater.* 2014, 26, 2365–2370. [CrossRef] [PubMed]

24. Shan, S.; Kang, S.H.; Zhao, Z.; Fang, L.; Bertoldi, K. Design of planar isotropic negative Poisson’s ratio structures. *Extrem. Mech. Lett.* 2015, 4, 96–102. [CrossRef]

25. Mullin, T.; Willshaw, S.; Box, F. Pattern switching in soft cellular solids under compression. *Soft Matter* 2013, 9, 4951–4955. [CrossRef]

26. Strek, T.; Maruszewski, B.; Naroczynski, J.W.; Wojciechowski, K.W. Finite element analysis of auxetic plate deformation. *J. Non-Cryst. Solids* 2008, 354, 4475–4480. [CrossRef]

27. Strek, T.; Jopek, H.; Wojciechowski, K.W. The influence of large deformations on mechanical properties of sinusoidal ligament structures. *Smart Mater. Struct.* 2016, 25, 054002. [CrossRef]

28. Masters, I.G.; Evans, K.E. Models for the elastic deformation of honeycombs. *Compos. Struct.* 1996, 35, 403–422. [CrossRef]

29. Reis, F.D.; Ganghofer, J.F. Equivalent mechanical properties of auxetic lattices from discrete homogenization. *Comput. Mater. Sci.* 2012, 51, 314–321. [CrossRef]

30. Shokri Rad, M.; Prawoto, Y.; Ahmad, Z. Analytical solution and finite element approach to the 3D re-entrant structures of auxetic materials. *Mech. Mater.* 2014, 74, 76–87. [CrossRef]

31. Boldrin, L.; Hummel, S.; Scarpa, F.; DiMaio, D.; Lira, C.; Ruzzene, M.; Remillat, C.D.L.; Lim, T.C.; Rajasekaran, R.; Patias, S. Dynamic behaviour of auxetic gradient composite hexagonal honeycombs. *Compos. Struct.* 2016, 149, 114–124. [CrossRef]

32. Hajmohammad, M.H.; Nour, A.H.; Zarei, M.S.; Zarei, M.S.; Kolahchi, R. A new numerical approach and visco-refined zigzag theory for blast analysis of auxetic honeycomb plates integrated by multiphase nanocomposite facesheets in hygrothermal environment. *Eng. Comput.* 2018. [CrossRef]

33. Imbalzano, G.; Tran, P.; Ngo, T.D.; Lee, P.V. A numerical study of auxetic composite panels under blast loadings. *Compos. Struct.* 2016, 135, 339–352. [CrossRef]

34. Imbalzano, G.; Linforth, S.; Ngo, T.D.; Lee, P.V.S.; Tran, P. Blast resistance of auxetic and honeycomb sandwich panels: Comparisons and parametric designs. *Compos. Struct.* 2018, 183, 242–261. [CrossRef]

35. Jin, X.C.; Wang, Z.H.; Ning, J.G.; Xiao, G.S.; Liu, E.Q.; Shu, X.F. Dynamic response of sandwich structures with graded auxetic honeycomb cores under blast loading. *Compos. Part B Eng.* 2016, 106, 206–217. [CrossRef]

36. Ren, X.; Shen, J.; Ghaedizadeh, A.; Tian, H.; Xie, Y.M. A simple auxetic tubular structure with tuneable mechanical properties. *Smart Mater. Struct.* 2016, 25, 065012. [CrossRef]

37. Ren, X.; Shen, J.; Ghaedizadeh, A.; Tian, H.Q.; Xie, Y.M. Experiments and parametric studies on 3D metallic auxetic metamaterials with tuneable mechanical properties. *Smart Mater. Struct.* 2015, 25, 065012. [CrossRef]

38. Shen, J.; Zhou, S.; Huang, X.; Xie, Y.M. Simple cubic three-dimensional auxetic metamaterials. *Phys. Status Solidi (B)* 2014, 251, 1515–1522. [CrossRef]

39. Ghabraie, K.; Chan, R.; Huang, X.; Xie, Y.M. Shape optimization of metallic yielding devices for passive mitigation of seismic energy. *Eng. Struct.* 2010, 32, 2258–2267. [CrossRef]

40. Jia, L.J.; Dong, Y.; Ge, H.; Kondo, K.; Xiang, P. Experimental study on high-performance buckling-restrained braces with perforated core plates. *Int. J. Struct. Stab. Dyn.* 2018, 19, 1940004. [CrossRef]

41. Jia, L.J.; Xie, J.Y.; Wang, Z.; Kondo, K.; Ge, H.B. Initial studies on brace-type shear fuses. *Eng. Struct.* 2020, 208, 110318. [CrossRef]
42. Jia, L.J.; Kuwamura, H. Ductile fracture model for structural steel under cyclic large strain loading. *J. Constr. Steel Res.* **2015**, *106*, 110–121. [CrossRef]

43. Xiang, P.; Jia, L.J.; Shi, M.; Wu, M. Ultra-low cycle fatigue life of aluminum alloy and its prediction using monotonic tension test results. *Eng. Fract. Mech.* **2017**, *186*, 449–465. [CrossRef]

44. Kaminski, M.; Schrefler, B.A. Probabilistic effective characteristics of cables for superconducting coils. *Comput. Methods Appl. Mech. Eng.* **2000**, *188*, 1–16. [CrossRef]

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