Dynamical CP Violation in Composite Higgs Models
- A Review -

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Abstract

In composite Higgs models it was pointed out that there is a possibility to violate CP symmetry dynamically. We demonstrated a simple model of dynamical CP violation in composite Higgs models. We calculated the neutron electric dipole moment in our model and the constraint for our model is discussed below.

Talk presented at the Third KEK Topical Conference on CP Violation, 16-18 November, 1993. The main part of this talk is based on the work in collaboration with S. Hashimoto and T. Muta [1].
1 Introduction

In the standard theory CP violating phenomena are described by phases appearing in the Kobayashi-Maskawa matrix \([2]\). The CP violating phases in KM matrix are introduced as free parameters. This situation is not satisfactory for the fundamental theory of quarks and leptons. We would like to see what the origin of CP violation is.

One possibility to explain the origin of CP violation was pointed out by T. D. Lee \([3]\). The idea is the following: If the vacuum expectation value of Higgs fields is not real, the vacuum state is not symmetric under CP transformation. Thus CP symmetry is broken spontaneously. This mechanism suggests that the spontaneous electroweak symmetry breaking has something to do with the origin of CP violation.

On the other hand there is a possibility that the Higgs fields may be constructed as bound states of more fundamental fermions. In this case the electroweak symmetry is broken down dynamically by the vacuum expectation value of composite fields constructed by fermions and anti-fermions. Eichten, Lane and Preskill applied the idea of the spontaneous CP violation to one of the composite Higgs models, the technicolor model \([4]\). They pointed out that through phases of the vacuum expectation value of the composite Higgs field the CP symmetry is broken. If the vacuum expectation value of composite fields has the complex phase, CP symmetry is broken dynamically. We call this mechanism the dynamical CP violation \([5]\). As a result of the dynamical CP violation in composite Higgs models CP violating four-fermion interactions generally show up. In this talk first I will briefly explain a mechanism of the dynamical CP violation in composite Higgs models. Then I will construct a simple model of the dynamical CP violation. Finally I will discuss the neutron electric dipole moment in our model.
2 Mechanism of Dynamical CP Violation

In composite Higgs models the Higgs particle appears as a bound state of fundamental fermion $Q$.

$$\phi \sim \bar{Q}Q.$$  \hfill (2.1)

There is a variety of composite Higgs models including the technicolor model [1], top-quark condensation model [7], top-color model [8], fourth-generation model [9] and color-sextet quark model [10]. The Lagrangian for the theory considered now is described as

$$L_0 = L_{QCD} + L_{EW} + L_{dyn}$$  \hfill (2.2)

where $L_{QCD}$ is the QCD Lagrangian without mass terms and $L_{EW}$ is the electroweak Lagrangian without the elementary Higgs fields and $L_{dyn}$ is the Lagrangian describing the unknown dynamics among fundamental fermions. We assume that the Lagrangian has the global flavor symmetry.

To induce the electroweak symmetry breaking dynamically we require that the vacuum expectation value of the composite fields is non-vanishing. Thus we assume that $L_{dyn}$ generates the fermion-antifermion condensation. As is well-known, $L_{QCD}$ also generates the quark-antiquark condensation.

$$\langle \bar{Q}Q \rangle \neq 0, \quad \langle \bar{q}q \rangle \neq 0,$$  \hfill (2.3)

where $Q$ denotes the fundamental fermion and $q$ denotes the ordinary quark. These fermion-antifermion condensations break the chiral symmetry included in the global flavor symmetry. Thus we get the theory with highly degenerate vacua. Out of these degenerate vacua we can choose the vacuum for which all vacuum expectation values of the composite fields is real. Therefore the CP symmetry is not broken.
We need the flavor symmetry breaking terms $\mathcal{L}'$ to introduce the mass hierarchy of ordinary quarks in the theory. Of course in the case of standard model such terms are described by the Yukawa interactions $\mathcal{L}_Y$.

$$\mathcal{L}_Y = y^u \bar{q}_L u_R \phi + y^d \bar{q}_L d_R \tilde{\phi} + h.c.$$  \hspace{1cm} (2.4)

If we replace the elementary scalar field $\phi$ by the composite field $\bar{Q}Q$ we generate four-fermion interactions.

$$\mathcal{L}_4 = G \bar{q}_L q_R \bar{Q}_R Q_L + h.c.$$  \hspace{1cm} (2.5)

These terms are the flavor symmetry breaking terms in the ordinary composite Higgs models. We suppose that these terms correspond to the low energy effective Lagrangian stemming from the more fundamental Lagrangian at extremely high energy scale.

Let us see the vacuum state is symmetric or not under CP transformation when these small flavor symmetry breaking terms are added to the flavor symmetric Lagrangian $\mathcal{L}_0$. We require the CP invariance of the full Lagrangian. Then all the coupling $G$ of the four-fermion interactions should be real. In this case the degeneracy of the ground state is solved by the flavor symmetry breaking terms $\mathcal{L}'$ and the unique vacuum is determined. We no longer have the degree of freedom to choose the vacuum for which the vacuum expectation value of the composite fields is real. Hence the vacuum expectation value of composite fields has a complex phase. This means that for the vacuum the CP symmetry is broken. We need to make a transformation on the field under the global flavor symmetry to make the vacuum expectation value real. By this transformation the form of the flavor symmetry breaking terms $\mathcal{L}'$ are modified so that CP violating terms, in general, show up in $\mathcal{L}'$. Thus the dynamical CP violation occurs. In the following section I would like to present a simple model of the dynamical CP violation in composite Higgs models.
3 A simple model

To see the characteristics of the dynamical CP violation in composite Higgs models we construct a simple model here. We assume the presence of two generations of extra-fundamental fermions and three generations of ordinary quarks.

\[ Q \sim \begin{pmatrix} U \\ D \end{pmatrix}, \begin{pmatrix} C \\ S \end{pmatrix}, \quad (3.6) \]

\[ q \sim \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}. \quad (3.7) \]

Our Lagrangian here is divided into two parts. The full Lagrangian does not violate the CP symmetry.

\[ L_0 \] is symmetric part under the global flavor symmetry \( G_F \). The flavor symmetry is

\[ G_F = U^Q_L(2) \otimes U^Q_R(2) \otimes U^q_L(3) \otimes U^q_R(3). \quad (3.8) \]

The part \( U^Q_L(2) \otimes U^Q_R(2) \) is the flavor symmetry of fundamental fermions and the other part \( U^q_L(3) \otimes U^q_R(3) \) is the flavor symmetry of ordinary quarks. We consider only the up-type fermions for simplicity below. Of course our model equally applies to the system of the down-type quarks. The composite operators of fermion-antifermion develop the non-vanishing vacuum expectation values under the Lagrangian \( L_0 \).

\[ \langle \bar{U}_L U_R \rangle = \langle \bar{C}_L C_R \rangle \neq 0, \]
\[ \langle \bar{u}_L u_R \rangle = \langle \bar{c}_L c_R \rangle = \langle \bar{t}_L t_R \rangle \neq 0. \quad (3.9) \]

CP symmetry is not broken dynamically under the flavor symmetric part \( L_0 \). We can take the vacuum for which all of these vacuum expectation values are real. The complex phases appear by the effect of the flavor symmetry breaking part \( L' \). By a transformation under the global flavor symmetry \( G_F \), arbitrary complex phases are generated in the vacuum
expectation values.

\[ Q_L \rightarrow W_Q^L Q_L, \quad Q_R \rightarrow W_Q^R Q_R, \]
\[ q_L \rightarrow W_q^L q_L, \quad q_R \rightarrow W_q^R q_R, \]

(3.10)

where \( W_Q^L, W_Q^R, W_q^L \) and \( W_q^R \) are elements of flavor symmetries \( U_Q^L(2), U_Q^R(2), U_q^L(3) \) and \( U_q^R(3) \). We can cancel out the phases using this transformation. However the flavor symmetry breaking part \( L' \) is not symmetric under this transformation, then the form of \( L' \) will be modified.

In composite Higgs models the flavor symmetry breaking part of the Lagrangian is described by the four-fermion interactions.

\[
L' = G_1^Q \bar{U}_L U_R + G_2^Q \bar{C}_L C_R + G_3^C \bar{q}_L q_R + \bar{t}_R t_L
\]

(3.11)

Here we neglect the difference between up quark and charm quark. As we require the CP invariance, all of the couplings \( G \) have to be real. If we take all of the couplings to be positive or all of the couplings to be negative, the CP symmetry is not broken dynamically. Hence we suppose that all of \( G \) are positive and take a minus sign only for a coupling \( G_4^{Qq} \). To find the true vacuum state of the full Lagrangian, we start from one of the vacua for the flavor symmetric part \( L_0 \), transform the vacuum by \( W_L \) and \( W_R \) to find \( \langle -L' \rangle \) under the transformed vacuum, and then try to minimize \( \langle -L' \rangle \) by changing \( W_L \) and \( W_R \). To calculate this energy \( \langle -L' \rangle \) it is convenient to parametrize \( W_L \) and \( W_R \) in this way.

\[
(W_Q^L W_Q^{Q_1}) = w_i^Q \exp(i\theta_i^Q),
(W_q^L W_q^{q_1}) = w_i^q \exp(i\theta_i^q).
\]

(3.12)
Using this parametrization \( \langle -\mathcal{L}' \rangle \) is described as a function of \( w \) and \( \theta \).

\[
\langle -\mathcal{L}' \rangle = - \left( G_1^Q (w_1^Q)^2 + G_2^Q (w_2^Q)^2 \right) \Delta^Q \\
- G_1^{Qq} (w_1^Q w_1^q e^{i(\theta_1^Q - \theta_1^q)}) + w_1^Q w_2^q e^{i(\theta_1^Q - \theta_2^q)}) \Delta^{Qq} \\
- G_2^{Qq} (w_2^Q w_1^q e^{i(\theta_2^Q - \theta_1^q)}) + w_2^Q w_2^q e^{i(\theta_2^Q - \theta_2^q)}) \Delta^{Qq} \\
- G_3^{Qq} w_1^Q w_3^q e^{i(\theta_1^Q - \theta_3^q)}) \Delta^{Qq} \\
+ G_4^{Qq} w_2^Q w_3^q e^{i(\theta_2^Q - \theta_3^q)}) \Delta^{Qq} \\
+ O(r^2). \tag{3.13}
\]

In the vacua for \( \mathcal{L}_0 \) we may express the amplitude

\[
\langle \bar{Q}_L Q_R \bar{Q}_R Q_L \rangle = \Delta^Q, \\
\langle \bar{Q}_L Q_R \bar{q}_R q_L \rangle = \Delta^{Qq}, \\
\langle \bar{q}_L q_R \bar{q}_R q_L \rangle = \Delta^q. \tag{3.14}
\]

We try to find which \( w \) and \( \theta \) minimize the energy \( \langle -\mathcal{L}' \rangle \). To find the solution we assume

\[
r \sim \frac{\Delta^{Qq}}{\Delta^Q} \sim \frac{\Delta^q}{\Delta^{Qq}} \sim O(10^{-9}), \tag{3.15}
\]

and in the following we neglect the terms of \( O(r^2) \).

After some calculations we find that these relations have to be satisfied to minimize the energy \( \langle -\mathcal{L}' \rangle \).

\[
w_i^Q = w_i^q = 1, \tag{3.16}
\]

\[
\frac{G_1^{Qq}}{G_3^{Qq}} = - \frac{2 \sin(\theta_1^Q - \theta_3^q)}{\sin(\theta_1^Q - \theta_1^q)}, \\
\frac{G_2^{Qq}}{G_3^{Qq}} = \frac{2 \sin(\theta_2^Q - \theta_3^q)}{\sin(\theta_2^Q - \theta_1^q)}, \tag{3.17}
\]

\[
\frac{G_1^{Qq}}{G_2^{Qq}} = - \frac{\sin(\theta_1^Q - \theta_1^q)}{\sin(\theta_1^Q - \theta_1^q)}. 
\]
Non-vanishing $\theta$ means that the complex phase appears in the expectation value of the composite field and the CP symmetry is broken. So we would like to see if there are any couplings $G$ to satisfy these relations for non-vanishing $\theta$.

To find a realistic solution we assume that the strong CP violation should be absent for ordinary quarks. This means that

$$\theta_1^q + \theta_2^q + \theta_3^q = 2\theta_1^q + \theta_3^q = 0.$$  \hspace{1cm} (3.18)

Here we neglect the difference between the up quark and the charm quark and we take $\theta_1 = \theta_2$. We take into account that the top quark mass is 100 times heavier than the up quark mass and the charm quark mass. This means that the four-fermion couplings relevant to the top quark is larger than those for the up and the charm quark.

$$\frac{G_1^{Qq}}{G_2^{Qq}} \sim O(1/10),$$  
$$\frac{G_2^{Qq}}{G_2^{Qq}} \sim O(1/10),$$  \hspace{1cm} (3.19)

Here we notice that there are three equations and four parameters to be determined so that we have the degree of freedom to satisfy the relation (3.18). It should be noted that we have only one degree of freedom and so we can apply this relation to the ordinary quark sector only. Thus the strong CP problem is not resolved in the fundamental fermion sector.

After some calculation we get the solution which satisfies these relations.

$$\theta_1^Q = -\epsilon_1 + \frac{2}{3} \theta,$$
$$\theta_2^Q = \pi + \epsilon_2 + \frac{2}{3} \theta,$$
$$\theta_3^q = -\frac{1}{3} \theta,$$
$$\theta_3^q = \frac{2}{3} \theta.$$  \hspace{1cm} (3.20)

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Parameters $\epsilon_1$, $\epsilon_2$ and $\theta$ are defined by
\[
\begin{align*}
\frac{G_1^{Qq}}{G_3^{Qq}} &= \frac{\epsilon_1}{\sin \theta}, \\
\frac{G_2^{Qq}}{G_4^{Qq}} &= \frac{\epsilon_2}{\sin \theta}, \\
\frac{G_1^{Qq}}{G_2^{Qq}} &= 1 + (\epsilon_1 + \epsilon_2) \frac{\cos \theta}{\sin \theta}.
\end{align*}
\tag{3.21}
\]

The condition that the top quark is 100 times heavier than the up and the charm quark lead to
\[
\epsilon_1 \sim \epsilon_2 \sim O(1/10), \quad \sin \theta \sim O(1).
\tag{3.22}
\]

And this solution shows that
\[
W_R^Q W_L^{Q\dagger} = \left( e^{i(-\epsilon_1 + \frac{2}{3} \theta)} \quad e^{i(\pi + \epsilon_2 + \frac{2}{3} \theta)} \right),
\tag{3.23}
\]
\[
W_R^q W_L^{q\dagger} = \left( e^{-i\frac{2}{3} \theta} \quad e^{-i\frac{1}{3} \theta} \quad e^{i\frac{2}{3} \theta} \right).
\tag{3.24}
\]
These phases correspond to the phases of the vacuum expectation value of the composite operators. Accordingly the CP symmetry is broken.

We make inverse-transformations to obtain the real vacuum expectation value of the composite fields.

\[
\begin{align*}
Q_L &\longrightarrow W_L^{Q\dagger} Q_L, \quad Q_R \longrightarrow W_R^{Q\dagger} Q_L, \\
q_L &\longrightarrow W_L^{q\dagger} q_L, \quad q_R \longrightarrow W_R^{q\dagger} q_L.
\tag{3.25}
\end{align*}
\]

The form of the four-fermion terms $\mathcal{L}'$ is modified and we get the CP violating four-fermion interactions.

\[
\mathcal{L}' = G_1^{Qq} \bar{U}_L U_R \bar{U}_R U_L + G_2^{Qq} \bar{C}_L C_R \bar{C}_R C_L
\]
\[
+ (G_1^{Qq} e^{i(\theta - \epsilon_1)} \bar{U}_L U_R + G_2^{Qq} e^{i(\pi + \theta + \epsilon_2)} \bar{C}_L C_R) (\bar{u}_R u_L + \bar{c}_R c_L)
\]
\[
+ (G_3^{Qq} e^{i\epsilon_1} \bar{U}_L U_R - G_4^{Qq} e^{i(\pi + \epsilon_2)} \bar{C}_L C_R) \bar{t}_R t_L
\]
\[
+ G_1^{q1111} \bar{u}_L u_R \bar{u}_R u_L + \cdots
\]
\[
+ G_1^{q1313} e^{i\theta} \bar{u}_L t_R \bar{u}_R t_L + \cdots.
\tag{3.26}
\]
As we neglect the difference between the up quark and the charm quark, no relative phase appears between up and charm. And the parts describing the four-fermion interaction between ordinary quarks are important for the low energy phenomena.

We confirm that the dynamical CP violation occurs in our simple model. Our model is too simple to explain the Kobayashi-Maskawa matrix and should be elaborated to reproduce the standard theory as a low energy effective theory. If our model has something to do with nature, it has to be consistent with the existing experimental observations. For this purpose we calculate the neutron electric dipole moment under our Lagrangian.

4 Neutron electric dipole moment in our model

It is well-known that the neutron electric dipole moment calculated in the standard theory is extremely small \([11]\). Thus it is possible to observe an extra effect to the neutron electric dipole moment coming from the CP violating four-fermion interactions. Here we calculate the neutron electric dipole moment from the CP violating four-fermion Lagrangian of ordinary quarks obtained in the last section.

\[
\mathcal{L}' = G_{ijkl} \bar{q}_i \gamma_m q_j \bar{q}_k \gamma_m q_L ,
\]

(4.27)

where indices \(i, j, k, l\) represent flavor of \(q\), i.e., \(u, d, c, s, t, b\) and coupling constants \(G'\) generally have complex phases. The neutron electric dipole moment \(d_n\) is given in terms of the quark dipole moments \(d_u\) and \(d_d\) in the naive quark model such that

\[
d_n = \frac{1}{3} (4d_d - d_u) \tag{4.28}
\]

The upper bound for the neutron electric dipole moment \(d_n\) imposed by the experimental observation is \(d_n < 10^{-25}\text{e cm}\). The electric dipole moment of quarks is calculated through the following term in the quark electromagnetic form factor at zero-momentum transfer

\[
- \bar{q}_u \sigma_{\mu \nu} \gamma_5 q^{\nu} u_q
\]

(4.29)
where $q''$ is the momentum transfer for quarks (momentum carried by the virtual photon) and $u$ is the Dirac spinor for quark $q$.

At one loop-level, the diagrams shown in Fig.1 contribute to the quark electromagnetic form factor. However these diagrams do not contribute to the electric dipole moment. At two loop level, we show the diagrams which contribute to the quark electromagnetic form factor in Fig.2. Because of the quadratically divergent parts of the amplitude, the diagrams shown in Fig. 2(c) have the largest contribution to the quark electric dipole moment.

We calculate the quadratically divergent parts of the amplitude and find the main contribution to the quark electric dipole moment.

\[
d_u = \frac{2}{3} e \frac{\Lambda^2}{(4\pi)^4} \sum_{i,j,k} \text{Im}(G_{jiu}'G_{kjiu}')m_j \left(2\ln\frac{\Lambda^2}{m^2_j} - 2.01\right),
\]

where $\Lambda$ is the cut-off parameter. Here we assume

\[
|G_{jiu}'G_{kjiu}'| \sim \frac{g^4}{4\Lambda^4} \sim \frac{4\pi^2}{4\Lambda^4},
\]

where $g$ is the coupling constant for the fundamental theory at high energy scale $\Lambda$ and
we assume
\[ \frac{g^2}{4\pi} \sim 1. \] (4.32)

And the top quark mass is much heavier than the other quark masses. Only the term proportional to the top quark mass has a large contribution. Thus we find
\[ d_u \sim e^{-\frac{m_t}{48\pi^2\Lambda^2}} \sin \theta \left( 2\ln \frac{\Lambda^2}{m_t^2} - 2.01 \right). \] (4.33)

If we suppose that the top quark mass is about 140 GeV and use the upper bound of the neutron electric dipole moment $d_n$ from the experiments and the constraint shown Eq.(3.22) we obtain the lower bound of the cut-off scale $\Lambda$.

\[ \Lambda > 800 \text{TeV}. \] (4.34)

This lower bound of the cut-off scale $\Lambda$ is of the same order as the one set by the flavor changing neutral current in kaon physics [12].

5 CONCLUSION

We succeeded in finding a simple model of dynamical CP violation. And to satisfy the experimental constraint from neutron electric dipole moment in our model the cut-off scale of the theory has to be higher than 800 GeV.

There are some remaining problems. To explain the actual KM matrix our simple model has to be further elaborated. The strong CP problem in the sector of fundamental fermions is still an open problem. We are aware of the cosmological domain wall problem with regard to the dynamical breaking of the discrete symmetry and leave it to the future research.
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