Implementation of lagrangian optimization model for optimal power flow in power system

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Abstract
This paper proposed the Lagrangian optimization model for Optimal Power Flow (OPF) problem. It is designed by relaxing the constraints from the Quadratic Programming (QP) problem. The objective of this model is to minimize the total cost of active power generation. The solution of QP is obtained by different optimization techniques like Particle Swarm Optimization (PSO) method, Genetic Algorithm (GA), Differential Evolution (DE) algorithm. In this paper, the optimum value of QP is obtained by the proposed model and it is compared with other methods PSO, GA, DE. The results of the methods have been tested through the standard IEEE 30 bus system. Based on numerical calculations and graphical representation, the optimal generation cost for OPF can be achieved.

Keywords: Optimal power flow, Genetic algorithm, Particle swarm optimization, Lagrangian model.

Introduction
Carpentier (1962) discussed the Optimal Power Flow (OPF). OPF has been widely used in power system and management. After restructuring the electricity sector OPF is a tool, which is used to minimize the power production cost by adjusting the power system control variables. The objective of OPF is to minimize the generation cost and / or transmission losses. The optimal operation of power system is to determine the power schedule so that the total cost of operation is minimized with respect to operating constraints. The constraints involved are the physical laws governing the power generation - transmission systems and operating limitations of the equipment.

The power flow study is required for planning the operation of power systems with respect to existing conditions and its future expansion. The load flow studies in essential for future system expansion to meet the increased load demand. Operation of the power grid at steady state is one of the most fundamental requirement of proper operation of a power system. The steady state operation of the power network is principally governed by the system voltage at the two ends, the transfer reactance of the line and the power angle between the two buses (Fig.1).

In past research, Chung and Shaoyun (1997) have discussed recursive linear programming which minimizing line losses and finding the optimal capacitor allocation in a distribution system. Laboto et al. (2001) proposed LP based OPF for minimization of transmission losses and generator reactive margins of the Spanish power system.

Chen and Chen (1997) have designed a new algorithm based on Newton-Raphson (NR) method in order to solve emission dispatch in real tune. Tong et al. (2005) presented semi smooth Newton-type algorithms for solving OPF problems. These algorithms separated inequality and bounded constraints.

Momoh (1989) has discussed the extension of basic Kuhn-Tucker conditions and generalized quadratic-based model for OPF. Grudinin (1998) has designed a reactive power optimization model which is based on successive QP (SQP) methods. These methods used to test 30 bus and 278 bus systems. Feasibility, convergence and optimal. Execution time is reduced. SQP methods provide more fast and reliable optimization.

Pudjianto et al. (2002) used LP and NLP based reactive OPF for allocating reactive power among competing generators in a deregulated environment. Torres and Quintana (2002) proposed the methods to calculate the price of reactive power support service in a multi-area power system. Methods which are based on Cost Benefit Analysis (CBA) and linear convex network flow programming.

LP method calculated the overall cost associated with the system reactive requirement. It gives reasonably accurate. NLP gives a faster computation speed and accuracy for the solution. The reactive power support benefits with respect to power delivery increases of tie lines, Generators individual commitments vary. The convergence could not be guaranteed for every condition.

Ding Xiaoqing et al. (2002) have discussed an Interior Point Branch and Cut Method (IPBCM) to solve decoupled OPF problem. The Modern Interior Point Algorithm (MIPA) is used to solve Active Power Sub Optimal Problem (APSOP) and use IPBCM to iteratively solve linearization of Reactive Power Sub Optimal Problem (RPSOP). Wei Yan et al. (2006) presented the solution of the optimal reactive power flow (ORPF) problem by the Predictor Corrector Primal Dual Interior Point Method (PCPDIPM). ORPF was designed as a model in rectangular formal the Hessian matrices in this model are constants, it has been evaluated only once in the entire optimal process.

The variables and constraints of RPSOP are less than that of original OPF problem, which gives the fast calculation speed.

Iwan Santoso and Tan (1990) have discussed a two-stage Artificial Neural Network (ANN) to control in real time the multi tap capacitors installed on a non conforming load profile such that the system losses are minimized.
David and Sheble (1992) applied a genetic algorithm (GA) to solve an economic dispatch problem for valve point discontinuities. Chung & Li (2001) have proposed a Hybrid Genetic Algorithm (GA) method to solve OPF in corporation FACTS devices.

GA is integrated with conventional OPF to select the best control parameters to minimize the total generation fuel cost and keep the power flows within the security limits. It converged in a few iterations.

Yoshida et al. (2000) have discussed a particle swarm optimization (PSO) for reactive power and voltage / VAR control (CCV) considering voltage security assessment. It determined an online VVC strategy with continuous and discrete control variables, Cui Ru Wang et al. (2005) presented a modified particle swarm optimization (MPSO) algorithm to solve economic dispatch problem.

Yu et al. (2001) have proposed a novel cooperative agents approach, Ant colony search algorithm (ACS) based scheme, for solving a short-term generation scheduling problem of thermal power systems.

Somasundaram et al. (2004) have discussed an algorithm for solving security constrained optimal power flow problem through the application of EP. Maheswari et al. (2011) have analyzed the optimal power flow by Lagrangian Relaxation technique. Optimization Model for Electricity Distribution System Control using Communication System by Lagrangian Relaxation Technique was proposed by Maheswari et al. (2011).

The controllable system quantities in the base case state are optimized to minimize some defined objective function subject to the base-case operating constraints fitness function converges smoothly without any oscillations.

Many researchers have discussed the solution of OPF by different optimization techniques. In this paper, the proposed model gives the optimum solution for OPF with respect to penalty factors and the Lagrangian multipliers used for faster convergence.

Optimization model for OPF

The mathematical formulation for OPF is based on the control variables and operating conditions (or) constraints.

Control variables
(a) Generators active power outputs
(b) Generator bus voltages
(c) Controllable reactive compensation elements
(d) Transformers tap positions.

Constraints
Equality constraints: The equality constraints are the active and reactive power balance equations at all the bus bars in each and every bus which are itself the load flow equations.

Inequality constraints: The equality constraints are basically operating limits and physical limits of each equipment. That is active and reactive power limits, lines and transformers, transmission reactive power injection limits in the controlling tension bars and injection of active power in the reference bar.

Parameters
i, j – Number of buses.
NG – Total number of buses.
PG – Generated active power output at bus i.
QG – Generated reactive power output at bus i.
a, b, c – Unit costs curve for ith generator.
gij – Conductance between buses i and j.
bij – Susceptance between buses i and j.
V – Voltage magnitude at the bust i.
θ – Voltage phase angle difference between i and j.
VG – Generator voltage magnitude at the bust i.

Objective function

Min \( \sum_{i=1}^{NG} (a_i + b_i PG_i + c_i PG_i^2) \)

Subject to

\[ \sum_{j=1}^{NG} V_j V_i (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) = PG_i - PD_i, i = 1, ..., NG \] (1)

\[ \sum_{j=1}^{NG} V_j V_i (g_{ij} \sin \theta_{ij} + b_{ij} \cos \theta_{ij}) = QG_i - QD_i, i = 1, ..., NL \] (2)

\[ VG_{\min} \leq VG_i \leq VG_{\max}, \quad i = 1, ..., NG \] (3)

\[ PG_{\min} \leq PG_i \leq PG_{\max}, \quad i = 1, ..., NG \] (4)

\[ QG_{\min} \leq QG_i \leq QG_{\max}, \quad i = 1, ..., NG \] (5)

\[ T_{\min} \leq T_i \leq T_{\max}, \quad i = 1, ..., NT \] (6)
Solution for OPF by different methodologies: Quadratic Programming model can be solved by Particle Swarm Optimization (PSO) method, Genetic Algorithm (GA), Differential Evolution (DE) algorithm. These methodologies have discussed in Maheswari et al. (2011).

**Proposed lagrangian optimization model**

Lagrangian Objective function is formulated by relaxing the power flow equations from the QP model which minimizes the real power generation cost. This model is obtained by using Lagrangian Relaxation (LR) method.

Lagrangian function

Relaxing Equations (1) and (2),

$L[PG, QG, PD, QD, \lambda_{PG}, \mu_{QG}]$

$$= \sum_{i=1}^{NG} (a_i + b_i PG_i + c_i PG_i^2) + \lambda_{PG} [PG_i - PD_i]$$

+ $\mu_{QG} [QG_i - QD_i]$

Subject to

$$VG_i^{min} \leq VG_i \leq VG_i^{max}, \quad i = 1,...,NG \quad (1)$$

$$PG_i^{min} \leq PG_i \leq PG_i^{max}, \quad i = 1,...,NG \quad (2)$$

$$QG_i^{min} \leq QG_i \leq QG_i^{max}, \quad i = 1,...,NG \quad (3)$$

$$T_i^{min} \leq T_i \leq T_i^{max}, \quad i = 1,...,NT \quad (4)$$

$$QC_i^{min} \leq QC_i \leq QC_i^{max}, \quad i = 1,...,NC \quad (5)$$

Here $\lambda_{PG}, \mu_{QG}$ are penalty factors of active and reactive power at the buses. Lagrangian Relaxation replaces the original problem with an associated Lagrangian problem whose optimal solution provides a bound on the objective function of the problem.

This is achieved by eliminating (relaxing one or more) constraints of the original model and adding these constraints, multiplied by an associated Lagrangian multiplier in the objective function.

The main objective of this method is to relax the constraints that will result in a relaxed problem. When it gives the values of multipliers, it is much easier to solve optimally. The role of these multipliers is to derive the Lagrangian problem towards a solution that satisfies the relaxed constraints.

The Lagrangian relaxation approach replaces the problem of identifying the optimal values of all the decision variables with one of finding optimal or good values for the Lagrangian multipliers. Most Lagrangian-based heuristics use a search heuristic to identify the optimal multipliers. A major benefit of Lagrangian-based heuristics is that they generate bounds (i.e., lower bounds on minimization problems and upper bounds on maximization problems) on the value of the optimal solution of the original problem. For any set of values for the Lagrangian multipliers, the solution to the Lagangian model is less than or equal to the solution to the original model. Therefore, the Lagrangian solution is a lower bound on the solution to the original problem.

The solution to the Lagrangian problem for any given values of the Lagrangian multipliers will generally violate...
Optimization model, the equality constraints (power flow equations) and inequality constraints (Generation operating conditions) are tested through the data sets which are available in IEEE-30 bus system (Fig.1 and Table 1).

Based on Table 2 and Fig.3, the minimum real power generation cost is achieved by LR Method with respect to the Lagrangian Multipliers. The solution of the model is obtained by PSO, GA, DE and LR by using the algorithmic approach which is implemented in MATLAB 7.0 and these are performed in acer p.c. The convergence speed for PSO and DE are 15, 28 seconds respectively. The minimum value achieved by LR is 9 seconds in 144 iterations.

### Conclusion

In this paper, Lagrangian optimization model is designed for Optimal Power Flow (OPF) problem. This model is obtained from the optimization model Quadratic Programming (QP) by using Lagrangian Relaxation method. Lagrangian function gives the optimum value for QP problem. Based on the numerical calculations and graphical representation, the minimum active power generation cost is achieved by Lagrangian Relaxation method. For convergence criteria, the execution time of LR is faster than other soft computing techniques. This model helps to maintain the system stability and minimize the losses in the power system.

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