Consistency between the monopole strength of the Hoyle state determined by structural calculation and that extracted from reaction observables

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We analyze the $\alpha$-$^{12}$C inelastic scattering to the $0^+_2$ state of $^{12}$C, the Hoyle state, in a fully microscopic framework. With no free adjustable parameter, the inelastic cross sections at forward angles are well reproduced by the microscopic reaction calculation using the transition density of $^{12}$C obtained by the resonating group method and the nucleon-nucleon $g$ matrix interaction developed by the Melbourne group. It is thus shown that the monopole transition strength obtained by the structural calculation is consistent with that extracted from the reaction observable, suggesting no missing monopole strength of the Hoyle state.

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The $0^+_2$ state of $^{12}$C, the so-called Hoyle state, has intensively been studied theoretically and experimentally [1–13]. Despite a rather clear understanding of its three-$\alpha$ structure, the description of the Hoyle state appeared in reaction observables, the $(\alpha, \alpha')$ inelastic scattering cross section in particular, has not been achieved. It was reported in many studies [14–17] that the $(\alpha, \alpha')$ cross section theoretically obtained with using the transition density of $^{12}$C from the ground state to the $0^+_2$ state significantly overshot the observed cross section. This puzzle is called the missing monopole strength of the Hoyle state [15].

In these preceding studies, however, a semi-microscopic treatment of the distorting potential between $\alpha$ and $^{12}$C as well as the coupling potentials for the excitation of $^{12}$C was adopted. This suggests some ambiguities in the distorting and coupling potentials for the excitation of $^{12}$C. The description of the Hoyle state appeared in reaction observables [14–17] that the $(\alpha, \alpha')$ cross section theoretically obtained with using the transition density of $^{12}$C from the ground state to the $0^+_2$ state significantly overshot the observed cross section. This puzzle is called the missing monopole strength of the Hoyle state [15].

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We consider $\alpha$ as a projectile (P) and $^{12}$C as a target nucleus (T). The $\alpha$ particle is assumed to stay in the ground state, whereas the transition of $^{12}$C between the $0^+_1, 2^+_1, 0^+_2, 3^+_1, 2^+_2$, and $0^+_3$ states are explicitly taken into account. The coupled-channel (CC) equation to be solved is given by

$$
\left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} + \frac{\hbar^2}{2\mu} \frac{L(L+1)}{R^2} + F_{\gamma \gamma L}(R) + U_{\text{Coul}}(R) - E_{\gamma} \right] \chi_{\gamma L}(R) = - \sum_{\gamma' \neq \gamma L} F_{\gamma \gamma' L}(R) \chi_{\gamma' L}(R),
$$

where

$$
F_{\gamma \gamma' L}(R) = \frac{1}{4\pi} \sum_{\lambda} \int dR \rho_{\gamma}(r_T) \rho_{\gamma'}(r_T) g^{(dr)}(s; \rho) Y_{\lambda}(\hat{R}) \otimes Y_{\lambda}(\hat{r}_T) \left[ Y_{\lambda}(\hat{R}) \otimes Y_{\lambda}(\hat{r}_T) \right]_{00} \chi_{\gamma L}(R)
$$

$$
\times \left\{ \int dR d\rho_T \rho_T(r_T) g^{(dr)}(s; \rho) \left[ Y_{\lambda}(\hat{R}) \otimes Y_{\lambda}(\hat{r}_T) \right]_{00} \chi_{\gamma L}(R) \right. + \int dR d\rho_T d\rho_{\gamma'} \rho_{\gamma'}(r_T) g^{(dr)}(s; \rho) \chi_{\gamma' L}(R) \right\},
$$

where $\chi_{\gamma L}(R)$ is the radial part of the P-T scattering wave function in the $(\gamma L)$ channel; $\gamma$ specifies the state of T and L is the orbital angular momentum between P and T. The total spin of T in the $\gamma$ state is denoted by $I$. The definition of the coordinates is given in Fig. 1. $E_{\gamma}$ is defined by $E_{\gamma} = E - \varepsilon_{\gamma}$ with $E$ the incident energy of P in the center-of-mass system and $\varepsilon_{\gamma}$ the the excitation energy of T. $M$ is defined by $M = M_P M_T / (M_P + M_T)$, where $M_P (M_T)$ is the mass number...
of \( P (T) \). The Coulomb potential between \( P \) and \( T \) is denoted by \( U^{\text{ Coul}}(R) \). \( Y_\lambda \) is the spherical harmonics, \( j_\alpha \) is the spherical Bessel function, \( L(0\lambda 0\lambda) \) is the Clebsch–Gordan coefficient, and \( V(LS'S'|J\lambda) \) is the Racah coefficient.

This coupled-channel approach with a \( g \) matrix has widely been adopted so far \[22–25\]; the possible double-counting for the coupling to non-elastic channels are expected to be negligible as discussed in Ref. \[23\]. Equation \( 4 \) contains two key ingredients. One is the nuclear transition density \( \rho_{\gamma'\gamma} \), and the other is the \( g \) matrix \( g_{\text{dr/ex}} \); the superscript \( \text{dr} \) (ex) indicates the direct (exchange) part of \( g \), an explicit form of which is shown in, e.g., Ref. \[26\].

We adopt the transition density of \( ^{12}\text{C} \) obtained by the resonating group method (RGM) based on a three-\( \alpha \) model \[2\]; the \( 0^+_1 \), \( 1^+_2 \) (4.44 MeV), \( 2^+_2 \) (7.65 MeV), \( 3^+_1 \) (9.64 MeV), \( 2^+_3 \) (9.84 MeV) \[16\], and \( 0^+_2 \) (10.3 MeV) states are considered as mentioned above. These densities are shown to reproduce the elastic and inelastic form factors for electron scattering and are thus highly reliable. In the CC calculation of the \((\alpha, \alpha')\) process, we include all the six states listed above; we use the experimental values of the excitation energies. For the ground state density of \( \alpha \), \( \rho_\alpha \), we take the phenomenological one determined from electron scattering \[27\] in which the finite-size effect of proton charge is unfolded with a standard procedure \[28\].

As for \( g \), we use the Melbourne \( g \)-matrix interaction \[29\], which has been highly successful, with no free parameters, in describing various nucleon-nucleus elastic and inelastic cross sections in a wide range of incident energies. The use of a \( g \)-matrix interaction having a predictive power is one of the most essential features of the present study. As mentioned, we use the TDA for evaluating the argument \( \rho \) in \( g_{\text{dr/ex}} \), i.e., \( \rho = \rho_{\gamma'\gamma} \rho_{rm} \), where \( r_m \) denotes the midpoint of the interacting two nucleons. For the nondiagonal potentials, we take the average of the densities in the initial and final states, i.e., \( \rho_T = [\rho_{\gamma'\gamma} \rho_{rm} + \rho_{\gamma\gamma} \rho_{rm}] / 2 \).

Figure 2 shows the differential cross sections of \(^{12}\text{C}(\alpha, \alpha')^{12}\text{C}(0^+_2)\) at 172.5, 240, and 386 MeV, compared with the experimental data \[14\] \[30\] \[32\]. One sees that with no free adjustable parameters the inelastic cross section to the \( 0^+_2 \) state of \(^{12}\text{C} \) is reproduced well at forward angles at these three energies. At larger angles the calculation slightly overshoots the experimental data. However, there seems to be no room for the missing monopole strength; if it could exist, the inelastic cross section would decrease at all angles and the good agreement at forward angles would be lost, though it is not so clear at 172.5 MeV because of the lack of data at very forward angles. We show the results for the elastic scattering in Fig. 3 in the same way as in Fig. 2; a very good agreement between the calculated cross section and the experimental data is obtained, which confirms the reliability of the microscopic reaction calculation adopted.

In a recent work \[33\] it was shown that the monopole transition strength, \( 4.5 \pm 0.5 \text{ e fm}^2 \), to the \( 0^+_2 \) state determined by comparing the theoretical result with the experimental data is consistent with that deduced from electron scattering. However, the antisymmetrized molecular dynamics calculation adopted in the reaction analysis gives a larger value (6.6 e fm\(^2\)) of the transition strength. Therefore in Ref. \[33\] the structural input and the reaction observables still have a gap of about 30%. Furthermore, the interaction strength was adjusted so as to reproduce the elastic scattering data; the renormalization factor for the real (imaginary) part is 1.05 (1.27) and 1.24 (1.38) for the analysis at 240 and 386 MeV, respectively.

Finally, we show by the solid (dashed) line in Fig. 4 the real (imaginary) part of the coupling potential between the \( 0^+_1 \) and \( 0^+_2 \) states of \(^{12}\text{C} \) for the \( \alpha \) inelastic scattering at 172.5 MeV. An important characteristic of the coupling potential is that it has a peak at the origin and well concentrated in the nuclear interior region. In Ref. \[17\] it was shown phenomenologically that this behavior of the coupling potential is essential to reproduce the absolute value of the \(^{12}\text{C}(\alpha, \alpha')^{12}\text{C}(0^+_2)\) cross section. It should be noted, however, that the origin of this behavior in the present study is completely different from that...
FIG. 3: (Color online) Same as in Fig. 2 but for the elastic scattering (the ratio to the Rutherford cross section). The experimental data are taken from Refs. [16, 30, 31].

FIG. 4: (Color online) Coupling potential between the $0^+_1$ and $0^+_2$ states of $^{12}$C for the $(\alpha, \alpha')$ scattering at 172.5 MeV. The solid (dashed) line corresponds to the real (imaginary) part of the coupling potential.

In summary, we have calculated the $\alpha^{12}$C inelastic cross section to the $0^+_2$ of $^{12}$C with a microscopic coupled-channels method using the RGM transition density of $^{12}$C and the Melbourne $g$-matrix interaction. We have obtained a good agreement between the calculated and measured values of the inelastic cross section at forward angles, as well as that of the elastic cross section. It suggests that the monopole transition strength obtained by the RGM calculation is consistent with the value extracted from the reaction observable. Thus, it is concluded that there is no missing monopole strength of the Hoyle state.

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