The test for suppressed dynamical friction in a constant density core of dwarf galaxies

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ABSTRACT

The dynamical friction problem is a long-standing dilemma about globular clusters (hereafter GCs) belonging to dwarf galaxies. GCs are strongly affected by dynamical friction in dwarf galaxies, and are presumed to fall into the galactic centre. But, GCs do exist in dwarf galaxies generally. A solution of the problem has been proposed. If dwarf galaxies have a core dark matter halo which has constant density distribution in its centre, the effect of dynamical friction will be weakened considerably, and GCs should be able to survive beyond the age of the Universe. Then, the solution argued that, in a cored dark halo, interaction between the halo and the GC constructs a new equilibrium state, in which a part of the halo rotates along with the GC (corotating state). The equilibrium state can suppress the dynamical friction in the core region. In this study, I tested whether the solution is reasonable and reconsidered why a constant density, core halo suppresses dynamical friction, by means of \(N\)-body simulations. As a result, I conclude that the true mechanism of suppressed dynamical friction is not the corotating state, although a core halo can actually suppress dynamical friction on GCs significantly.

Key words: methods: \(N\)-body simulations – galaxies: dwarf – galaxies: kinematics and dynamics – galaxies: star clusters – galaxies: structure.

1 INTRODUCTION

The world we live in is a hierarchical universe, in which galaxies are made by a myriad of merging events. A large-scale numerical simulation based on the cold dark matter (CDM) theory has been operated as a greatly declarative method for the hierarchical scenario. It indicated that the structure of the Universe develops from the small dark matter clumps which collapsed first, and result in the formation of large and massive dark matter haloes. In such a formation history, it is appropriate to consider that dwarf galaxies are fundamental ‘building-blocks’ and expected to be the oldest structures of the Universe. Dwarfs are believed to have important clues in understanding the hierarchical Universe.

In this paper, I will discuss the dynamical friction problem which refers to orbital motion of globular clusters (GCs) in dwarf galaxies. The drag force of dynamical friction is negligibly weak for GCs in the Milky Way. In contrast, it operates strongly in small systems like dwarf galaxies (see Chapter 8 of Binney & Tremaine 2008). Thus, the GCs in dwarfs are presumed to lose their orbital energy and fall into the galactic centre by strong friction force from the dark matter halo. According to results of both analytical and numerical studies, the time-scale for a GC to fall into the centre is of the order of \(\sim 1\) Gyr (Tremaine 1976; Hernandez & Gilmore 1998; Oh, Lin & Richer 2000; Vesperini 2000, 2001; Goerdt et al. 2006; Sánchez-Salcedo, Reyes-Iturbide & Hernandez 2006). Nevertheless, even in the present universe, these GCs still do exist and keep their orbital motions. For example, the Fornax dSph galaxy has five GCs which are metal poor and as old as the Universe, thus resembling the GCs of Milky Way (Buonanno et al. 1998, 1999; Mackey & Gilmore 2003; Strader et al. 2003; Greco et al. 2007).

However, by an analytical approach, Hernandez & Gilmore (1998) have discovered that a King model halo can significantly weaken the effect of dynamical friction in the core region. As for a cuspy halo (Navarro–Frenk–White profile (Navarro, Frenk & White 1995) or singular isothermal sphere), a GC is sucked into the galactic centre by the dynamical friction (see fig. 2 of Goerdt et al. 2006). The analytical approach of Hernandez & Gilmore (1998) was constructed on the Chandrasekhar dynamical friction formula (Chandrasekhar 1943). On the other hand, by \(N\)-body simulations, Goerdt et al. (2006) and Read et al. (2006) (hereafter R06) confirmed the cessation of dynamical friction on a GC in a core region of haloes. However, R06 concluded that an important key to this suppressed dynamical friction is a ‘corotating state’. They argued that a part of halo particles in the constant density core begin to rotate with the GC. The authors suggested that this dynamical state is a new equilibrium state including the GC. The dynamical friction ceases under this equilibrium; hence, the GC could survive beyond the age of the Universe. Goerdt et al. (2006) and R06 confirmed that these results do not depend on the mass of a GC, the orbital parameters (circular or elliptical orbit) of a GC or the size of core structure of a halo (core radius).

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However, the conclusions of these studies, Hernandez & Gilmore (1998) and R06, imply a discrepancy between them. The approach of Hernandez & Gilmore (1998) is based on the Chandrasekhar formula; hence, it cannot account of velocity anisotropy of the field particles, because the formula is based on the assumption of isotropic velocity state. But on the contrary, R06 concluded that the mechanism of suppressed dynamical friction is the very anisotropy in the velocity state: the corotating state. My aim in this paper is to assess the authenticity of the corotating state proposed by R06 as the mechanism of the suppressed dynamical friction.

Although the result of R06 appears to be convincing, the new equilibrium, the corotating state, will be vulnerable to perturbation on the GC. Once the GC orbit is perturbed and the orbital plane inclined, the corotating equilibrium state will be broken easily. This means that the system is no longer equilibrium: the dynamical friction force on the GC will be rejuvenated. R06 have also mentioned this fragile nature of the corotating equilibrium state. In R06, they studied single-GC cases only. But real dwarf galaxies do not necessarily have only one GC, but several or more (Durrell et al. 1996; Miller et al. 1998; Lotz, Miller & Ferguson 2004). The GCs in a dwarf will be perturbed by the other GCs. Then, I infer that some GCs would fall into the galactic centre by such orbital perturbation, and may merge and form a stellar nucleus cluster at the galactic centre (Miochci et al. 2006; Capuzzo-Dolcetta & Miocchci 2008). Actually, some dwarf galaxies have a nuclear stellar cluster at the centre, and some observational researchers have discussed that some of these nuclei may be remnants of GCs (Miller et al. 1998; Lotz et al. 2004). I conducted N-body simulations to examine whether the GCs in dwarfs fall into the galactic centre by perturbation from the other GCs, even in the case of cored halo structure.

This paper is organized as follows: in the next section, I will explain my simulation method and models in detail; in Section 3, I will show my simulation results and their analysis; and I will discuss my results, comparing them with preceding studies, and give my conclusion in Section 4.

2 THE SIMULATIONS

My simulational settings are almost the same as the N-body simulation of R06. The simulations are pure N-body simulations (no gas component). I use a Barnes–Hut modified tree code (Barnes & Hut 1986; Barnes 1990) in order to lighten the heavy burden of gravitational force calculation, setting an open angle of 0.5. A special-purpose calculator for collisionless N-body simulations, GRAPE-7 model 600, is used with the tree algorithm to accelerate gravity calculation (Makino 1991). The total number of time steps is 11,841 for the whole of a simulated period which corresponds to 10 Gyr in real time-scale. It takes roughly half a month to finish each simulation.

2.1 The setting of halo model

To imitate R06, I adopt the same spherical density distribution:

$$\rho(r) = \frac{\rho_0}{(r/r_s)^{\alpha} [1 + (r/r_s)^{\beta}]^{-\gamma/\beta}}; \quad r < r_{\text{vir}}$$

with $\alpha = 1.5, \beta = 3.0, \gamma = 0.0$. The density in the core $\rho_0$ is 0.10 $M_\odot$ pc$^{-3}$. The scale radius $r_s$ is set to 0.91 kpc. The density is nearly constant at the centre within 200–300 pc, which defines the core region. The virial mass of the halo is 2.0 $\times$ 10$^9$ $M_\odot$. I add an exponentially decaying envelope to prevent instability at the outer region caused by an artificial cut-off radius $r_{\text{vir}}$ (Springel & White 1999). Velocity dispersion of particles is given by the solution of Jeans equations as a function of radius,

$$\sigma^2(r) = \frac{1}{r^{2\beta} \rho(r)} \int_r^{\infty} dr' r'^{2\beta} \rho(r') \frac{d\Phi}{dr'},$$

where $\beta$ is the anisotropy parameter. Although the effect of dynamical friction is sensitive to the velocity distribution of the field particles, isotropic velocity state is supposed to be reasonable in the inner region of dwarf haloes (Mashchenko, Wadsley & Couchman 2008). In this paper, I assume the isotropic velocity state in the halo, setting $\beta = 0$ ($\sigma_r = \sigma_\theta = \sigma_\phi$). With this assumption, equation (2) reduces to

$$\sigma^2(r) = \frac{1}{\rho(r)} \int_r^{\infty} dr' \rho(r') \frac{d\Phi}{dr'}.$$  (3)

The velocity distribution is determined by the local Maxwellian approximation,

$$F(v, r) = 4\pi \left(\frac{1}{2\pi\sigma^2}\right)^{3/2} v^2 \exp\left(-\frac{v^2}{2\sigma^2}\right).$$  (4)

where $F(v, r)$ is a probability distribution function of velocity (Hernquist 1993). Equation (4) is normalized so that $\int_0^{\infty} F(v, r)dv = 1$.

Like R06 and Goerdt et al. (2006), I adopt a three-shell model (Zemp et al. 2008), which consists of finer grained particles in inner regions and coarser particles in outer regions. This technique enables it to reduce computational run-time, and resolve much smaller scales in the inner region. But this multishell model inevitably admits the heavier particles coming from the outer shells into the inner shells, and may induce two-body relaxation between these different mass particles. To avoid such an unfavourable artificial effect, I refine the particles in the outer shells depending on their orbit. From a set of initial positions and velocities, I can calculate the pericentre distance of a specific particle in the smooth potential given by the density profile (equation 1). By the pericentre distance, I detect the heavier particles which are supposed to intrude into the inner region. I divide these intruding particles into a set of particles which have the same mass resolution as the particles in the inner region. The new particles will have the same radial velocity component as the original particle, but a new random tangential component of the same magnitude as the original one. The divided particles are randomly placed on a sphere whose radius is the same as the initial galactocentric distance of the original particle. For a simple explanation, let us suppose that the halo consists of shells A, B and C, from inner to outer shell. The shells A, B and C consist of particles which have the mass of $m_A$, $m_B$ and $m_C$, respectively ($m_A < m_B < m_C$). For example, if a certain particle which has the mass of $m_C$ and an initial position in the shell C intrudes into the shell A, the particle will be divided into a set of new particles which have the mass of $m_A$, and the number of the new particles will be $m_C/m_A$ (for detail, see Zemp et al. 2008); therefore, not all particles in an outer shell have a uniform mass resolution, but two-body relaxation can be minimized in the innermost region. In the simulations of R06 or Goerdt et al. (2006), they have missed the particle-dividing step; therefore, at this point my simulations have an improvement over the preceding studies.

Specifically, the particle masses are $m_A = 17.8 M_\odot$, $m_B = 356 M_\odot$ and $m_C = 7118 M_\odot$. The inner most region, the shell A, is within 300 pc (to be accurate, the shell A is not a shell, but a sphere). The shell B, the middle shell, is the region from 300–1100 pc. The shell C, the outer most shell, is the entire region outside of the shell B. The number of particles is 8.31 $\times$ 10$^6$ in total.
The softened lengths of the particles, $m_A$, $m_B$ and $m_C$, are 3, 8 and 22 pc, respectively. I checked that my results were not sensitive to these values.

The three-shell model has coarse resolution in the outer shells. Such heavier particles may affect the nature of dynamical friction. For confirmation, I ran a simulation with a uniform mass particle model, and checked that the results were not sensitive to these model settings (see Appendix A).

By analytical calculation, Sánchez-Salcedo et al. (2006) has proposed that the dynamical friction induced by the stellar component is not negligible in a cored dwarf halo. But the purpose of this paper is to judge the corotating state proposed by R06 as the mechanism of the suppressed dynamical friction. For fair comparison with the simulation of R06 in which stellar component was excluded, I do not take the effect from stellar component into account in my simulations here.

2.2 The setting of globular clusters

In my simulations, each GC is represented by a point mass with $m_{GC} = 2.0 \times 10^4 M_\odot$. The softened length is 10 pc. Just to make sure, I ran another simulation in which a GC was resolved by many particles, and confirmed that tidal disruption did not destroy the GC. In this study, I do not consider mass loss from a GC, merging between GCs, or dynamical heating by halo potential. Although these effects may play important roles in the case of the resolved GCs (Fujii, Funato & Makino 2006; Miocchi et al. 2006; Esquivel & Fuchs 2007; Capuzzo-Dolcetta & Miocchi 2008), I consider the GCs as point-masses for the sake of comparison with R06 or Goerdt et al. (2006).

3 THE RESULTS

In this paper, I operate simulations for the cases of 1, 5 and 30 GCs. In single-GC cases, I give the GC a set of specific orbital parameters in each case. On the other hand, in multi-GC cases, I set their orbits at basically random as detailed in the following subsections.

3.1 The single-GC cases

To begin with, I conduct some single-GC cases. Initial orbit of the GC is circular and set to 600 pc or 1 kpc. In Fig. 1, I show two comparison cases with the Chandrasekhar dynamical friction formula (the derivation of the analytical result is described in Appendix B). As seen in the figure, the analytical results correspond to the $N$-body results before the GC enters into the core region (inside 200–300 pc). But when entering into the core, these results diverge abruptly: the analytic results continue to fall into the galactic centre, while the orbital shrinkage by dynamical friction stops in the $N$-body cases.

However, as I noted above, the effect of dynamical friction is sensitive to the velocity distribution of the field particles. But I confirmed that the core stalling of dynamical friction was not sensitive to details of the velocity distribution function with various anisotropy parameters $\beta$ in equation (2).

Next, I investigate the relation of the suppressed dynamical friction with orbital eccentricities. The initial orbit is circular, elliptical or radial and set to 600 pc. In the case of elliptical orbit, the rotational velocity of the GC is initially set to $0.6v_c$ ($v_c$ is the circular velocity at the initial position) and the radial velocity is 0. In the case of radial orbit, the GC is at rest initially. The results are indicated in Fig. 2. As shown in the figure, when the GC enters into the core region, the orbital shrinkage by dynamical friction stops in all cases regardless of their initial orbital eccentricities. After the cessation of dynamical friction, the orbit expands a little again. This phenomenon is called ‘kickback effect’, and a detailed investigation about the effect has been done by Goerdt et al. (2008). In this study, I do not treat this effect. These behaviours of the GC in the cored profile (the cessation of orbital shrinkage, the kickback effect) are consistent with R06 and Goerdt et al. (2006). The mass included in the halo core is heavier than a GC by two orders, which should be enough to operate dynamical friction. Goerdt et al. (2006) has confirmed independence of suppressed dynamical friction from the size of the core region, $r_c$. Moreover, I find that the density profile of the halo is scarcely changed by the GC (Fig. 3). The energy conservation rate of the system, $1 - E_{end}/E_{ini}$, is $6.46 \times 10^{-3}$.

This result confirms the suppressed dynamical friction on a GC in a cored halo. However, what is the cause of it? In R06, the authors have proposed that a part of halo particles in core region is made to rotate with the GC by gravitational interaction (see the fig. 4...
in R06). They called this dynamical state as ‘corotating state’ and concluded it to be the mechanism to suppress the dynamical friction on the GC. I will examine whether the corotating state is the true mechanism or not. To identify the corotating state, I follow the same manner as R06. In order to visualize the velocity state of the halo, I sieve out the particles which have the radial distance \( r_p = 150–300 \) pc, subject to a condition

\[
\frac{\mathbf{J}_p \cdot \mathbf{J}_c}{||\mathbf{J}_p|| ||\mathbf{J}_c||} = \cos \theta,
\]

(5)

where \( \mathbf{J}_p \) means the specific angular momentum of a halo particle and the GC, respectively. A criterion parameter \( \theta \) is set to 10\(^\circ\), which is the same value as R06. Because the halo potential is spherically symmetric and the GC is significantly heavier than any halo particles, the direction of \( \mathbf{J}_c \) is assumed to be constant in time. The condition (equation 5) screens out the field particles for which the direction of angular momentum vector coincides with that of the GC within \( \theta \). Fig. 4 indicates histograms of rotational velocity distributions in the case of a circular orbit for which the initial orbital radius is 1 kpc (the upper jaggy line in Fig. 1). The upper panel of Fig. 4 shows the initial state, the bottom panel is for \( t = 8.2 \) Gyr (after the cessation of dynamical friction). As seen in the bottom panel, the corotating state is constructed after the dynamical friction is suppressed. The velocity distribution becomes somewhat anisotropic: the fraction of prograde rotating particles seems to increase, whereas retrograde particles decrease. The figure is consistent with the result of R06. The over-plotted dashed lines in the histograms represent Gaussian fitting given by the minimum \( \chi^2 \) method. In the bottom panel (corotating state), the peak height of the fitting line for prograde side \( h_+ \) and the retrograde side \( h_- \) is 0.0132 and 0.0106, respectively. The residual fraction of these, \((h_+ - h_-)/h_-\), is 0.245. This value means that the peak of the prograde side is 24.5 per cent higher than that of the retrograde side. The minimum value of \( |\chi| \) for the bottom panel is 0.00175, and \( |\chi|/h_- \) is 0.165.

From this analysis, the existence of corotating state seems to be confirmed. But it may be marginal because the value of \((h_+ - h_-)/h_-\) is not significantly larger than \( |\chi|/h_- \). One point to note is that the direction of \( \mathbf{J}_c \), which is assumed to be constant, actually fluctuates due to the \( N \)-body nature of this simulation (i.e. a finite number of particles has been used). For the sake of more precise discussion, I evaluate \((h_+ - h_-)/h_-\) and \(|\chi|/h_-\) for slightly different directions of \( \mathbf{J}_c \). I re-analyse the velocity states, changing the inclination of the vector \( \mathbf{J}_c \) in equation (5) little by little. By this procedure, I draw contour maps of the value of \((h_+ - h_-)/h_-\) and \(|\chi|/h_-\). Fig. 5 indicates the results. As shown in the figure, the corotating state \((h_+ - h_-)/h_- > 0\) can be found within \( \sim 10^\circ \) from the maximal direction. But the value of \((h_+ - h_-)/h_-\) should be compared with the value of \(|\chi|/h_-\). The contour map of \(|\chi|/h_-\) is given in the bottom panel of Fig. 5. From the map, it is found that the range of the value is 0.12 < \(|\chi|/h_- < 0.2 \) for the entire area plotted. Therefore, the value of \((h_+ - h_-)/h_-\) is almost submerged under the fitting error, \(|\chi|/h_-\), in most of the area except the central region. This discussion means that the corotating state shown in the upper panel of Fig. 5 is statistically marginal and unimportant, maybe except the central region (\( \sim 3^\circ \) from the maximal direction). Such weak anisotropy could not be expected to affect the dynamical friction on a GC.

Finally, I inspect the corotating state for the dependence on radial distance from the galactic centre. So far, I have analysed the corotating state in the radial range of 150 pc < \( r < 300 \) pc. I additionally carry out the same analysis in other radial ranges, \( r < 150 \) pc, 300 pc < \( r < 450 \) pc and 450 pc < \( r < 600 \) pc, with \( \mathbf{J}_c \).
unchanged. Fig. 6 indicates the results. From the figure, it can be seen that the corotating velocity state is constructed only in 150 pc < r < 300 pc.

3.2 The five-GC case

From the discussion of the previous subsection, the corotating state cannot be influential in dynamical friction. Even if the corotating state suppresses the dynamical friction on a GC, because it would be fragile against orbital perturbation on the GC as proposed in R06, the dynamical friction would be rejuvenated by the presence of the perturbation. Actually, because real dwarf galaxies generally have some GCs (Miller et al. 1998), there are probably frequent perturbations on the GCs in real dwarfs. As an additional test for the authenticity of the corotating state, I perform the simulations of multi-GC cases.

In this subsection, I present the result of the five-GC case. The GCs are represented as point masses. The initial positions of the GCs are randomly determined in the radial range of 300 pc < r < 2 kpc. The energy of each GC is also given randomly in the range of $E < \Phi_{2kpc}$ with a random direction of the velocity vector ($\Phi_{2kpc}$ is the potential energy of the halo at $r = 2.0$ kpc). To facilitate direct comparison with single-GC cases, one GC (reference GC) is placed on a circular orbit with radius 600 pc.

The results for all GCs are shown in Fig. 7. The top left panel indicates the reference GC, and the other panels represent the other four GCs which have randomly chosen positions and velocities. As shown in Fig. 7, all orbits of the GCs are frequently perturbed. From the comparison with single-GC cases (Fig. 2), the perturbation is expected to be caused by mutual interaction among the GCs, because the difference between these simulations is the number of GCs only. Despite these perturbations, no GCs fall into the galactic centre, with all GCs surviving and keeping their orbital motions. This means that the dynamical friction is suppressed in a core region even under frequent perturbation. This implies the inconsistency with the description in R06: the vulnerability of the corotating state to perturbations. This result casts a doubt on the corotating state as the mechanism of suppressed dynamical friction, together with the analysis of the single-GC case. Actually, the corotating state cannot be found in velocity histograms of the five-GC case.

3.3 The 30-GC case

To confirm the result of the five-GC case, I investigate the case of 30 GCs. The number of GCs in this case is somewhat too large, because the actual number of GCs in any dwarf galaxies is ~20 at most (Miller et al. 1998). The settings for the initial condition is basically the same as the five-GC case: random positions and velocities are given except for one GC which is placed on a circular orbit with radius 600 pc (reference GC). Some simulation results of these GCs are shown in Fig. 8.

As seen from Fig. 8, these results are essentially the same as five-GC case. Dynamical friction is suppressed in the core region despite more frequent perturbation than the five-GC case. Moreover, the corotation state is not confirmed in this case also, and can be guessed to be broken by perturbation among numerous GCs. The result reinforces my argument.

4 DISCUSSION AND CONCLUSION

My simulation results indicate the following. On one hand, dynamical friction is indeed suppressed in a constant density core region, orbital shrinkage of a GC stops and the orbital motion is sustained. But on the other hand, this result does not depend on the number of GCs. This means that the dynamical friction does not work in a core, even if the corotating state is broken by frequent perturbation. Moreover, in single-GC cases, the corotating state seems to be too marginal to affect the dynamical friction. My main conclusion here is that the corotating state is not the true mechanism. There may be another reason why dynamical friction ceases in a constant density halo.
However, I confirmed that a cored halo structure certainly can weaken the dynamical friction force on a GC in the core region. This means that the cored structure can be the solution of the dynamical friction problem. Actually, as an observational fact, the rotation curves of low surface brightness (LSB) galaxies seem to be like a solid body rotation which means a nearly constant density distribution, although CDM cosmological N-body simulations indicate cuspy density distributions (Dalcanton & Bernstein 2000; de Blok et al. 2001). Moreover, by cosmological smoothed particle hydrodynamics simulation, Mashchenko et al. (2008) has discovered a dwarf galaxy which has a constant density core in the halo. They concluded that massive stars inject large amount of energy into the dark matter halo via supernova explosions, and the feedback induced by bursty star formation can turn the cuspy density distribution of the dark matter into a cored profile in the halo centre. The suppressed dynamical friction would reinforce the existence of a core structure in dwarfs. If it is the case that the actual dwarfs have a core region in the centre, all GCs belonging to a dwarf are included within the core region of the dwarf. Thus, it can be expected that in a dwarf the largest galactocentric distance of the GCs indicates the core region.
minimum value of the core radius. For instance, in the Fornax dSph, the furthest GC from the galactic centre has a projected distance of \( \sim 1.6 \) kpc (Mackey & Gilmore 2003). The core region of the Fornax would be larger than \( \sim 1.6 \) kpc.

The most fundamental examination of dynamical friction is embodied by the Chandrasekhar dynamical friction formula (Chandrasekhar 1943). However, several approximations are included in the derivation. Current astrophysicists realized that the formula is not always correct because of complex non-linear effects (Jiang & Binney 2000; Hashimoto, Funato & Makino 2003; Fujii, Funato & Makino 2006; Kim, Kim & Sánchez-Salcedo 2008). Probing the dark matter distribution of dwarfs requires more sophisticated approaches to the nature of dynamical friction.

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APPENDIX A: THE REASONABILITY OF THE THREE-SHELL MODEL

The three-shell model I used here is a complex model. To make sure that the heavier particles with large force softening lengths in the outer shells do not affect the dynamical friction unfavourably, I run a simulation with a uniform mass resolution model. The uniform mass model consists of particle \( m_0 \) described in Section 2.1. The total number of particles is \( 6.57 \times 10^6 \). The velocity state of the system. In this paper, I approximate that the orbit is circular. Therefore, \( L = M_{GC} r v_c \) and for the gravitational constan

\[
\frac{dL}{dr} = M_{GC} v_c \frac{dr}{dr}.
\]

Figure A1. The comparison of the uniform mass model with the three-shell model. The result of the three-shell model is the same as the case of the circular orbit from 1 kpc in Section 3.1.
Moreover, because dynamical friction drag force is parallel to \(v_c\), and therefore a torque
\[
\frac{dL}{dr} = M_{\text{GC}} \frac{v_c}{dr}.
\] (B3)
From these equations,
\[
\frac{dr}{dt} = \frac{r}{v_c} \frac{v_c}{dr}.
\] (B4)
Substituting equation (B1) for equation (B4), I calculate the time evolution of a GC orbit by numerical integration of equation (B4).
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