HIGH-ENERGY SPECTRAL COMPONENTS IN GAMMA-RAY BURST AFTERGLOWS

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ABSTRACT

We investigate two high-energy radiation mechanisms, the proton-synchrotron and the electron inverse Compton emission, and explore their possible signatures in the broadband spectra and in the keV–GeV light curves of gamma-ray burst afterglows. We develop a simple analytical approach, also allowing for the effects of photon-photon pair production, and explore the conditions under which one or the other of these components dominates. We identify three parameter-space regions in which different spectral components dominate: (1) a region where the proton-synchrotron and other hadron-related emission components dominate, which is small; (2) a region in which the electron inverse Compton component dominates, which is substantial; and (3) a third substantial region in which electron-synchrotron emission dominates. We discuss the prospects and astrophysical implications of directly detecting the inverse Compton and the proton high-energy components in various bands, in particular, in the GeV band, with future missions such as the Gamma-Ray Large Area Space Telescope (GLAST) and in the X-ray band with Chandra. We find that regime II parameter space is the most favorable regime for high-energy emission. The inverse Compton component is detectable by GLAST within hours for bursts at typical cosmological distances and by Chandra within days if the ambient density is high.

Subject headings: gamma rays: bursts — gamma rays: theory — radiation mechanisms: nonthermal

1. INTRODUCTION

Gamma-ray burst (GRB) afterglows have been detected mainly at longer wavebands, from X-rays to radio, extending up to months after the burst triggers (see, e.g., van Paradijs, Kouveliotou, & Wijers 2000). These long-lived afterglow emissions are generally interpreted within the fireball shock model by the synchrotron emission of external shock-accelerated relativistic electrons. The broadband electron-synchrotron spectrum (Sari, Piran, & Narayan 1998) has proven to be a useful paradigm for studying afterglow light curves in the X-ray, optical, and radio bands and to constrain various unknown parameters. On the other hand, there are other high-energy emission components whose role in determining the long wavelength afterglows may be secondary or of limited duration but which may, for some time, dominate the high-energy, X-ray–to–GeV spectrum of the afterglows. These include the synchrotron self-inverse Compton (IC) emission of the electrons (Mészáros, Rees, & Pophamanniou 1994; Waxman 1997; Panaitescu & Mészáros 1998; Wei & Lu 1998, 2000; Dermer, Böttcher, & Chiang 2000a; Dermer, Chiang, & Mitman 2000b; Panaitescu & Kumar 2000; Sari & Esin 2001) and the proton-synchrotron emission (Vietri 1997; Böttcher & Dermer 1998; Totani 1998), as well as some other hadron-related emission components (Böttcher & Dermer 1998). The emission from such high-energy mechanisms in the GeV band may have been detected by EGRET in GRB 940217 (Hurley 1994) and probably also in some other GRBs and will be detectable by next-generation GeV γ-ray missions such as the Gamma-Ray Large Area Space Telescope (GLAST). In the X-ray band, the electron IC component has been pointed out as being potentially detectable at a later time (Sari & Esin 2001).

Previous afterglow snapshot spectral fits to multi-wavelength data up to X-rays using the synchrotron emission component (see, e.g., Wijers & Galama 1999; Panaitescu & Kumar 2000; van Paradijs et al. 2000; Freedman & Waxman 2001) have constrained some of the fireball shock parameters (e.g., the electron and magnetic field energy equipartition parameters $e_e$ and $e_B$, which are found to vary over a wide range. Because of the relative looseness of these fits, there is a need to tighten the model constraints. A promising way to do this is by extending upward the frequency range over which the snapshot spectra are fitted and to investigate the relative importance and additional constraints imposed by the high-energy radiation components, over a wider region of parameter phase space, which is the subject of this paper. For this purpose, we use a simple analytic approach to describe the various spectral components. Following a brief general treatment of the particle distribution and the synchrotron spectrum of the electrons (§ 2), we discuss the proton-synchrotron spectral component (§ 3) and the electron IC component (§ 4) and compare their importance relative to the electron-synchrotron emission in the frequency regime below the electron-synchrotron cutoff. Above this cutoff, the relative importance between the electron’s IC and the hadron-related components is also discussed. In § 5 we incorporate the high-energy absorption due to $\gamma$-$\gamma$ pair production in an analytic manner. We then present several examples of the broadband spectra of GRB afterglows within different parameter regimes in § 6 and discuss the detectability of these high-energy spectral components within various bands, especially in the X-ray and GeV band. Finally, we summarize our findings in § 7.

2. PROTON AND ELECTRON DISTRIBUTIONS

AND COOLING

We assume that for negligible radiative losses both electrons and protons are shock-accelerated to a single relativistic power-law distribution of index $p$. Denoting the particle species by the subindex $x$ (which indicates $e$ for electrons and $p$ for protons), after a certain time the particle distribution becomes a broken power law depending on the
relative ordering between the injection minimum energy $\gamma_{m,e}$ and the energy at which the particles cool radiatively in an expansion timescale $\gamma_{c,e}$. Given a maximum particle energy $\gamma_{m,e}$ at which the acceleration time equals the energy loss time, the particle distribution is $N(\gamma) \propto \gamma^{-p}$ for $\gamma_{m,e} < \gamma < \gamma_{c,e}$ and $N(\gamma) \propto \gamma^{-p-1}$ for $\gamma < \gamma_{m,e}$. If $\gamma_{m,e} < \gamma_{c,e}$ (slow-cooling regime) and is $(N(\gamma) \propto \gamma^{-2}$ for $\gamma_{c,e} < \gamma < \gamma_{m,e}$ and $N(\gamma) \propto \gamma^{-p-1}$ for $\gamma_{m,e} < \gamma < \gamma_{c,e}$ (fast-cooling regime). For $p > 2$, the mean particle energy is $\gamma_{m,e} = [(p-1)/(p-2)]^2 \gamma_{c,e}$. Hereafter, we assume for simplicity the same power-law index $p$ for both electrons and protons. Assuming that $\zeta_p$ and $\zeta_e$ are the injection fractions of the protons and electrons, respectively, where $\zeta_p$ and $\zeta_e$ are the injection fractions and $n$ is the number density of the preshock thermal medium, one can define the energy portion contained in the power-law–distributed electrons and protons with respect to the total energy behind the shock to be $\epsilon_p = \zeta_p m_p n_p/\gamma_{c,e}$ and $\epsilon_e = \zeta_e m_e n_e/\gamma_{c,e}$, respectively, where $\Gamma$ is the bulk Lorentz factor of the blast wave. Consequently, the minimum energies for the power-law–distributed electrons and the protons are $\gamma_{m,e}$ and $\gamma_{m,p}$, respectively, where

$$\gamma_{m,e} = \frac{\epsilon_p m_p}{\epsilon_e m_p} p - 2 \frac{\Gamma}{\zeta_e m_p} p - 1 \Gamma, \quad \gamma_{m,p} = \frac{\epsilon_p}{\epsilon_e} p - 1 \frac{\Gamma}{\zeta_p m_p} p - 1 \Gamma. \quad (1)$$

The above equations are for $p > 2$, which, for simplicity, will be assumed to be valid. For $p$ getting closer to 2, the $(p-2)/(p-1)$ factor is no longer precise, and when $p = 2$, it is replaced by $\ln^{-1}(\gamma_{m,e}/\gamma_{m,p})$. However, the important features of equation (1) at $p = 2$ do not influence the precision of the discussions in § 3 (see, e.g., eq. [2]) since correction factors for both the proton and the electron components are canceled out. The cooling energy $\epsilon_{c,e}$ is defined by equating the comoving adiabatic expansion time $t_{ad} \sim r/T_c \sim \Gamma t_s$ [where $\Gamma$ is the bulk Lorentz factor of the blast wave at the radius $r$ and $t_s \sim \Gamma^{-1}(r/c)$ is the expansion time in the observer frame] to the comoving radiation cooling time $t_c = [t_{ad}]^{-1} + (t_{ad})^{-1} = t_s/(1 + Y)$, where $t_{ad}$ and $t_{ad}$ are the synchrotron and the IC cooling time, respectively, and $Y = t_{ad} t_{ad} = [-1 + (1 + 4\ell e_{e}/e_{b})^{1/2}]^{-2}$ is the Compton factor (see, e.g., Panaitescu & Kumar 2000; Sari & Esin 2001). Here redshift corrections have been ignored for simplicity, $\eta = \min[1, (\gamma_{m,e}/\gamma_{c,e})^{p-2}]$ is the fraction of the particle energy that is radiated away via both synchrotron and IC emission (Sari & Esin 2001), and $e_b$ is the fraction of the magnetic energy density with respect to the total energy density behind the shock, so that the comoving magnetic field $B = \Gamma c(32 \eta m_p e_b)^{1/2}$. For $e_{e}/e_b \ll 1$, one has $Y \sim e_{e}/e_b \ll 1$ and $t_s = t_{ad}$, and the IC cooling is not important. Alternatively, when $\eta e_{e}/e_b \gg 1$, IC cooling is important and $t_{ad} \sim (\eta e_{e}/e_b)^{-1/2} t_{ad} \sim (\eta e_{e}/e_b)^{-1/2} t_{ad}$ (Sari & Esin 2001). The critical energy above which the particle species $x$ cool in an expansion time $\gamma_{c,x} = (1 + Y)^{-1}(6m_c/n_\Gamma \sigma_{T,x} B^2)$, where $t_{ad} = (\gamma_{c,x} m_x^2)/[(4/3)\Gamma m_x c^2 (/3)]$ is used, and $\sigma_{T,x} = (8\pi/3)(e^2/m_x e_b)^2$ is the Thomson cross section for particles of mass $m_x$, and $\Gamma_{T,x} \sim (m_x/m_p)^2$. A caveat is that, by adopting $Y = [-1 + (1 + 4\ell e_{e}/e_{b})^{1/2}]^{-2}$, one has implicitly assumed $L_{IC}/L_{tot} \sim U_{\gamma}/U_B$, where $U_{\gamma}$ and $U_B$ are the synchrotron radiation and magnetic field energy densities, respectively. Strictly speaking, this is valid only when the IC cooling occurs in the classical Thomson regime. In practice, high-energy electrons may cool in the Klein-Nishina (KN) regime, especially at the early afterglow phase. The cooling frequency in the above analytic treatment is then a crude approximation if the energy at which the electron cool is in the KN regime, and a more careful treatment should be made. As discussed more in § 4, the analytic treatment presented in this paper is adequate at 1 hr, or even earlier, after the burst trigger but may be crude for the prompt phase.

The maximum energy $\gamma_{m,e}$ is defined by equating the comoving acceleration time $t_{ad} = (2\pi a/c) m_p c/e_b$ (where $a_L$ is the Larmor radius and $a$ is a factor of the order of unity for relativistic shocks) to the minimum of the comoving adiabatic cooling time $t_{ad}$ and the comoving radiation cooling time $t_c$. This gives $\gamma_{m,e} \approx \min\{(2\pi a/c) m_p c/e_b(\Gamma t_s)/[30\ell a/c^2], (1 + Y)^{-1/2}\}$, where, for protons, the first part of the equation applies, while for the electrons, it is the latter.

The electron-synchrotron emission spectrum is a broken power law separated by three characteristic frequencies: the self-absorption frequency $v_{\nu,0}$, the characteristic frequency for the minimum energy particles $v_{m,e} \sim (4/3)\Gamma(3/4\pi e_b/m_e c^2)$, and the cooling frequency $v_{c,e} \sim (4/3)\Gamma(3/4\pi e_b/m_e c^2)$. Similar expressions apply for protons, except that synchrotron absorption and cooling are less important than in electrons. For both $p$ and $e$, the maximum particle energy $\gamma_{m,e}$ defines a synchrotron cutoff frequency $v_{m,e} \sim (4/3)\Gamma(3/4\pi e_b/m_e c^2)$. In the above expressions, the factor $4/3$ gives a more precise conversion from the comoving frame to the observer frame (Wijers & Galama 1999). The spectral indices of the four segments, ordering from low to high in frequency, are $2, \frac{3}{2}, -1, -p/2$ for the fast-cooling ($v_{c,e} < v_{\nu,0}$) regime and $2, \frac{3}{2}, -1, -p/2$ for the slow-cooling ($v_{c,e} > v_{m,e}$) regime (Sari et al. 1998).

3. SYNCHROTRON COMPONENTS

A potentially interesting high-energy emission component of GRB afterglows is the synchrotron emission from the shock-accelerated relativistic protons (Vietri 1997; Böttcher & dermer 1998; Totani 1998, 2000). Compared to electrons, the protons are inefficient emitters because of their much larger mass. The ratio of the characteristic synchrotron frequencies of protons to electrons is

$$\frac{v_{m,e}}{v_{m,e}} = \left(\frac{\gamma_{m,e}}{\gamma_{m,e}}\right)^2 \frac{m_p}{m_p} = \frac{m_p}{m_p} \left(\frac{\gamma_{m,e}}{\gamma_{m,e}}\right)^2 \left(\frac{m_p}{m_p}\right)^3, \quad (2)$$

where equation (1) is used. The peak synchrotron flux for the emission of particle $x$ is $F_{x,\nu,0} \propto n_x P_{x,\nu,0} \propto n_x m_e$, where $P_{x,\nu,0} = \phi(x/3e^2B/m_e c^2)$ and $\phi(0 \sim 0.6$ (Wijers & Galama 1999). This is independent of whether the peak is at $v_{m,e}$ or $v_{c,e}$. Thus, the ratio of the peak flux of the two particle species is

$$\frac{F_{x,\nu,0}}{F_{y,\nu,0}} = \frac{\gamma_{m,e}}{\gamma_{m,e}} \frac{m_p}{m_p}. \quad (3)$$

We see that both $v_{m,e}$ and $F_{x,\nu,0}$ are much smaller than $v_{m,e}$ and $F_{x,\nu,0}$, respectively, indicating that the proton

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1 We note that the larger minimum proton energy estimates $\gamma_{m,p}$ adopted, e.g., by Vietri (1997), Böttcher & dermer (1998), and Totani (1998), when $p$ is close to 2 as assumed in those references, lead to larger proton-synchrotron flux levels than those obtained using eq. (1) because of the higher $\gamma_p$ involved.
The emission component is usually buried under the electron emission component. However, the ratio of the cooling frequencies is

\[ \frac{v_{e,c}}{v_{e,c}} = \left( \frac{1 + Y_e}{1 + Y_p} \right) \left( \frac{m_e}{m_p} \right)^2 \]  

(see § 2), which means that protons barely cool, while electrons cool rapidly. This allows, in some cases, the proton component to dominate over the electron component at high frequencies.

We consider now a certain observation band Δν around v, which is above both the cooling frequency and the characteristic synchrotron frequency of the electrons but is below the electrons’ synchrotron cutoff frequency, i.e., v_{e,c} (and v_{m,c}) < v < v_{α,c}. Regardless of the relative ordering of v_{m,c} and v_{e,c} (slow cooling or fast cooling, respectively), the electron-synchrotron flux at this frequency is

\[ F_{e,ν}(v) = \frac{(v_ν/v_{m,c})^{(p-1)/2}F_{ν,α,max}v_{e,c}/v_{ν}}{F_{ν,α,max}} \]  

If v < v_{ν,α} is also satisfied; the flux of the proton-synchrotron component at the same frequency is

\[ F_{p,ν}(v) = \frac{(v_ν/v_{m,p})^{(p-1)/2}F_{ν,α,max}p/v_{ν}}{F_{ν,α,max}} \]  

where v_{p,c} and usually v_{p,c} ≥ v_{α,ν} p. Thus,

\[ \frac{F_{p,ν}(v)}{F_{e,ν}(v)} = \left( \frac{ε_p}{ε_e} \right)^{(p-1)/2} \left( \frac{v}{v_{e,c}} \right)^{(p-2)/2} \times \left( \frac{m_e}{m_p} \right)^{(3p-1)/2} \left( \frac{v}{v_{e,c}} \right)^{1/2} \]  

(5).

We see that ν must be >> v_{e,c} to make the proton component show up in the spectrum and that a large ε_{e} and a small ε_{p} favor the proton component. The dependence on the injection parameters ζ_{e} and ζ_{p} is weak for values of p close to 2.

For the purposes of numerical examples, below we use a standard scenario (see, e.g., Mészáros & Rees 1997) in which a blast wave with total energy per solid angle δ = E/Ω = 10^{52} ergs sr^{-1} δ_{52} expands into a uniform interstellar medium of particle number density n cm^{-3} and Ω is a putative jet opening solid angle. (In what follows, we will not discuss the jet dynamics. This does not influence our discussions on the early GeV afterglows but may quantitatively, although not qualitatively, change the discussions of the late X-ray afterglows if the transitions occur after the jet breaks. A windlike external medium with n \propto r^{-2} may also be incorporated in a similar way, but we do not discuss that here). For the uniform external medium case, the blast wave starts to decelerate at a radius r_{dec} \sim (3δ_{52}/4πn_{0}m_{p}c^{2}Γ_{0}^{1/3}) \sim 2.6 \times 10^{16} cm δ_{52}^{-1/3} n_{0}^{1/3} 10^{50} cm at an observer time t_{dec} \sim \left( r_{dec}4πn_{0}c/(1+z) \right) \sim 2.4 s δ_{52}^{-1/3} n_{0}^{-1/3} 10^{50} (1+z). Here Γ_{0} = 300Γ_{50} 10^{30} is the initial bulk Lorentz factor of the shocked materials at t = t_{dec} and the factor (1+z) reflects the cosmological time dilation effect. After collecting enough interstellar medium materials, the blast wave will be decelerated self-similarly, and its advance is measured by \nu(t) \sim [12δc/(1+z)]^{1/3} 4πn_{0}m_{p}c \Gamma_{0}^{1/3} \sim 1.6 \times 10^{17} cm δ_{52}^{-1/4} [t_{ν}(1+z)]^{1/4}, while the bulk Lorentz factor decays as \Gamma(t) \sim [3δ(1+z)^{2}/4πn_{0}m_{p}c^{3}]^{1/3} \sim 19.4δ_{52}^{-1/8} t_{ν}(1+z) \sim 3/8. Here r = 4T^{2}c/(1+z) has been adopted, and t_{ν} is the Earth observer time in units of hours. The comoving magnetic field is B = 7.5 Ge_{B}^{1/2} δ_{52}^{1/8} n_{0}^{1/8} [t_{ν}(1+z)]^{-3/8} G. The breaks in the electron energy distribution are then given by

\[ \gamma_{m,e} = 5.9 \times 10^{3}(e_{θ}/ζ_{θ})^{(δ_{52}/n)^{1/8}} [t_{ν}(1+z)]^{-3/8} \]  

(6)

\[ \gamma_{e,c} = 1.96 \times 10^{3}(1 + Y_e)^{-1}e_{θ}^{-1} δ_{52}^{3/8} n^{-5/8} \times [t_{ν}(1+z)]^{-1/2} \]  

(7)

In equation (6) and hereafter p = 2.2 for both electrons and protons has been adopted to calculate numerically the coefficients. The p dependences in the power indices, if necessary, will be still retained. The observed characteristic electron-synchrotron frequency is then \nu_{m,e} = 2.9 \times 10^{16} Hz (e_{θ}/ζ_{θ})^{2/3} δ_{52}^{4/3} t_{ν}^{-1} (1+z) \sim 2.4 s δ_{52}^{-3/2} n^{-3/2} (1+z)^{-1/2}. The proton-synchrotron frequency is

\[ \nu_{c,e} = 3.1 \times 10^{13} Hz (1 + Y_p)^{-2} e_{θ}^{3/2} δ_{52}^{-1/2} n^{-1} t_{ν}^{-1/2} \times (1+z)^{-1/2} \]  

(8)

and the cutoff maximum frequencies for the electrons and the protons are

\[ \nu_{α,e} = 2.3 \times 10^{23} Hz \sim (1 + Y_p)^{-1} (δ_{52}/n)^{1/8} t_{ν}^{-3/8} \times (1+z)^{-5/8} \]  

(9)

\[ \nu_{α,p} = 2.8 \times 10^{23} Hz \sim (2ε_{θ}/ζ_{θ})^{3/4} δ_{52}^{1/2} n^{-1/4} t_{ν}^{-1/4} \times (1+z)^{-3/4} \]  

(10)

Finally, the maximum electron-synchrotron flux is

\[ F_{ν,α,max} = [(4π/3)^{1/3} ζ_{θ} nΓ_{ν,m}/4πD^{2}](1+z) \]  

\[ = (φ_{0} √3/e^{θ} c^{2} nMC^{2} ΓB^{2}(1+z) \]  

\[ = 29 \mu Jy ε_{θ}^{1/4} δ_{52}^{1/2} n_{0}^{1/2} 10^{28} cm D_{25}^{25} (1+z) \]  

where D \equiv D(z) = 10^{28} cm D_{25} is the proper distance of the source (which, depending on the cosmological model, is also redshift-dependent). Similarly, the maximum proton-synchrotron flux is

\[ F_{ν,α,max} = 15.7 \mu Jy ε_{θ}^{1/4} δ_{52}^{1/2} n_{0}^{1/2} 10^{22} cm D_{25}^{25} (1+z) \]  

For ν < ν_{α,e}, the condition for the proton-synchrotron component to overcome the electron component is

\[ F_{ν,p}(ν_{α,p}) > F_{ν,e}(ν_{α,e}) \]  

Using equations (5), (8), and (10), this translates into

\[ (1 + Y_p)^{1/3} e_{θ} > 594(e_{θ}/ζ_{θ})^{2(p-1)/3} (ζ_{θ}/ζ_{θ})^{2} (p-2)/3 \times \sim (2π/3) δ_{52}^{7/12} n^{-7/12} [t_{ν}(1+z)]^{-1/12} \]  

(11)

which is shown as line 1 in Figure 1.

Another condition for the competition between the p and e components that might be thought to be relevant is ν_{α,p} > ν_{α,e}, which corresponds to \nu_{α,p}/e_{θ} > (0.88/2ε_{θ}/ζ_{θ})^{3/2} n^{-3/2} [t_{ν}(1+z)]^{-1/12}. However, this condition may not be essential since, as shown by Böttcher & Dermer (1999), there can be other proton-induced electromagnetic signals extending above this energy of a level comparable to or lower than the proton-synchrotron component. These include the synchrotron radiation from the positrons produced by π^{+} decay and the γ-rays produced directly from π^{0} decay. These components may be regarded as an extension to the proton-synchrotron component, which would stick out above the electron-synchrotron component even if ν_{α,p} ≤ ν_{α,e}. However, these proton components will compete with the electron IC component, which we discuss in § 4.
rate at $v > \gamma_m^2 v_m, e$ since electrons with a range of Lorentz factors between $\gamma_m$ and $\gamma_e$ contribute equally to the emission at each frequency. For the convenience of the following discussions, we will still adopt the broken power-law approximation to perform order-of-magnitude estimations, bearing in mind that more accurate expressions would be necessary in more detailed calculations.

In this approximation, the IC spectral component can be represented (Sari & Esin, 2001) by a four-segment broken power law with power indices, ordered from low to high frequency, of $[1, \frac{3}{2}, -(p - 1)/2, -p/2]$ in the slow-cooling regime and $[1, \frac{3}{2}, -2, -p/2]$ in the fast-cooling regime. The break frequencies are $v_{e, IC} = \gamma_m^2 v_e, e$ and $v_{m, IC} \approx \gamma_m^2 v_e, m, e$, and the IC cooling frequency is $v_{c, IC} \approx \gamma_e^2 \epsilon v_c, e$. The maximum flux of the IC component is roughly a factor of $(u_{\phi, IC}/u_{\phi, 0})^{v_{m, IC}/v_{e, IC}}$ of that of the synchrotron component, where $u_{\phi, IC} \approx (4/3) c \sigma_T, e u_B \gamma^2 (r/T_0)^4 (4\pi^2 n)$ and $u_B = B^2/8\pi$ are the comoving synchrotron photon and magnetic field energy densities, respectively. This gives

$$\frac{F_{\nu, max, IC}}{F_{\nu, max}} = 16 \frac{3}{3} \sigma_T, \epsilon \gamma \nu r \approx 3.5 	imes 10^{-7} \gamma \epsilon \nu r \gamma \gamma . \tag{12}$$

This shows that generally the IC component can only overtake the synchrotron component beyond the synchrotron component’s cooling break but before the IC component’s cooling break. In the slow-cooling regime, for a frequency $\nu$ satisfying $\nu_{c, IC} \leq \nu \leq \nu_{e, IC}$, and $\nu_{c, IC} \leq \nu \leq v_{m, IC}$, the flux ratio of the IC to the synchrotron components is

$$\frac{F_{\nu, IC}}{F_{\nu, IC}} = \left( \frac{\nu_{e, IC}}{\nu_{m, IC}} \right)^{(p-1)/2} \left( \frac{\nu}{\nu_{e, IC}} \right)^{1/2} . \tag{13}$$

Alternatively, in the fast-cooling regime, for a frequency $\nu$ satisfying $v_{m, IC} \leq \nu \leq v_{e, IC}$ and $v_{m, IC} \leq \nu \leq v_{m, IC}$, the flux ratio of the IC to synchrotron components is

$$\frac{F_{\nu, IC}}{F_{\nu, IC}} = \left( \frac{\nu_{m, IC}}{\nu_{e, IC}} \right)^{(p-1)/2} \left( \frac{\nu}{\nu_{m, IC}} \right)^{1/2} . \tag{14}$$

Similar to the proton-synchrotron case, the conditions under which the IC component overcomes the synchrotron component is $F_{\nu, IC}(v_{e, IC}) > F_{\nu, IC}(\nu_{e, IC})$ for the slow-cooling case and $F_{\nu, IC}(v_{m, IC}) > F_{\nu, IC}(\nu_{m, IC})$ for the fast-cooling case, and both conditions can be simplified to

$$16 \frac{3}{3} \sigma_T, \epsilon \gamma \nu r \gamma > 1 . \tag{15}$$

Using equations (6) and (7), this IC dominance condition over electron-synchrotron can be reexpressed as

$$(1 + Y_e) \epsilon < 3.8 (\epsilon/\gamma_e)^{(p-1)/2} \epsilon (d/2/n)^{p-2/8} \times [\nu(1+z)^{-3(p-2)/8} . \tag{16}$$

This is line 2 in Figure 1. We see that a large $\epsilon$, or a small $e_B$ makes the IC component more prominent since a large $\epsilon$ enhances $\gamma_{e, IC}$ and a small $e_B$ tends to increase $\gamma_{c, IC}$ because of the inefficient synchrotron cooling. A denser medium also favors the IC component (eq. [12]). One comment is that if one takes into account the flux increase above $v_{m, IC}$ due to the logarithmic term from the scattering contributions of the different electrons (Sari & Esin 2001), the IC dominance condition is less stringent than equation (16).

The cutoff energy in the IC component is defined by $v_{\nu, IC}^{\nu} = \min (\epsilon v_{\nu, IC}/\nu, v_{\nu, IC})$, where $v_{\nu, IC}$ is the Klein-
Nishina limit. A rough estimate of this frequency is given by \( \nu_{KN, e} \approx \gamma_{KN, e}^2 \nu_{KN, e} \), where \( \nu_{KN, e} \approx (4/3) \Gamma(3/4 \pi) \left( eB/m_e c \right)^2 / c^2 \) and \( \gamma_{KN, e} \sim mc^2 / \left( h\nu_{KN, e} \right) \). This can be translated into synchrotron emission dominates over electron-synchrotron emission.

The second case in the slow-cooling regime is for \( \nu > \nu_c, p \), and here the competition between the two components is more difficult to quantify. The fact that usually \( \nu_c, p < \nu_c, e \) would seem to indicate that the proton component is not important. However, as Böttcher & Dermer (1998) have shown, photo-meson interactions between the relativistic protons and the low-energy photon spectrum lead to additional hadron-related spectral components at high energies, with a \( \nu F_\nu \) level that can be a substantial fraction of that of the proton-synchrotron component but extending to much higher energies. For our purposes, it is not necessary here to compute these components in detail, assuming instead as a rough estimate that beyond the maximum proton-synchrotron frequency \( \nu_{c, p} \), the extended hadron component \( \nu F_\nu \) flux level represents a fraction \( k = 0.1k-1 \) of the proton-synchrotron flux level at the proton-synchrotron cutoff \( \nu_{c, p} \). This is consistent with Figure 1 of Böttcher & Dermer, and in general, \( k = 0.1k-1 \) represents an overestimate of the flux level from these hadron-related components. We then compare \( F_{\nu, c, p} \) to \( F_{\nu, e} \) \( \nu \nu_{c, e} \) and \( F_{\nu, c, e} \) \( \nu \nu_{c, e} \), noticing their dependence can be written out:

\[ F_{\nu, c, p}(\nu) = F_{\nu, c, e}(\nu) \left( \nu_{c, p} / \nu_{c, e} \right)^{p-2} \left( m_{c, p} / m_{c, e} \right)^{(3p-1)/2} \left( 16/3 \sigma_{\nu, c, e} \right) \left( \nu / \nu_{c, p} \right)^{(p-1)/2}. \]

After some further derivations, the condition \( F_{\nu, c, p}(\nu) > F_{\nu, c, e}(\nu) \) can be translated into

\[ (1 + Y)^{0.85} \beta_E > 215k^{0.43} \epsilon_{p, e}^{1.02} \epsilon_{p, e}^{0.51} \gamma^{0.17} A_9^{0.39} = 0.34 \]

\[ \times \sigma_{\nu, c, e}^{0.27} n^{0.35} T_h^{0.10} (1 + z)^{0.01} \nu_{c, e}^{0.24}. \]

These are the lines labeled 4 (calculated for \( k = 0.1 \) and 4 for \( k = 1 \)) in Figure 1. In the above expression, we have adopted \( p = 2.2 \) for both electrons and protons in order to avoid an unnecessarily complicated expression. (The explicit spectral index \( p \) dependence can be written out straightforwardly and is presented in the Appendix). Notice that this criterion is mildly dependent on \( \nu \), and we have adopted \( \nu = 10^{26} \) Hz \( z \), for the typical frequency in equation (18). Above this frequency (several TeV), the self-absorption due to \( \gamma-\gamma \) pair production becomes important (§ 5), and the comparison is no longer meaningful. For lower frequencies, the constraint on the hadron-dominant region of parameter space is more stringent.

In summary, the competition between various components in the \( \epsilon_{p, e} \) \( \beta_E \) diagram can be read off from Figure 1: (1) The first case, applicable to frequencies \( \nu < \nu_{c, e} \), is shown by the solid lines (eqs. [11] and [16]) that divide the space into three regions, in which the proton-synchrotron

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3 Here we have assumed \( \gamma = 1 \) for the proton component extending to infinity. When \( \gamma > 1 \), this is an overestimate of the other hadron-related components (Böttcher & Dermer 1998). Thus, the real criterion is even more stringent than equation (17).
component competes with the electron-synchrotron component and the electron-synchrotron competes with the electron IC component, respectively. Regions I and II are the regions where the proton component and the electron IC component dominate in the spectrum, respectively, while in region III, neither of these high-energy spectral components can overcome the electron-synchrotron component. (2) The second case, applicable to frequencies $v > v_{\nu c}$, is shown by the dashed lines dividing the parameter space into two regions, in which the proton-synchrotron and the electron IC components compete with each other. Region I is where the proton component as well as other hadron-related components may overcome the electron IC emission component, while region II is the IC-dominated region. The dashed line 3 (eq. [17]) is the separation line for the case of $v < v_{\nu c}$, and the dashed line 4 (eq. [18]) is the separation line for the case of $v > v_{\nu c}$, calculated for a typical frequency $v \sim 10^{26} \text{Hz}$. For lower frequencies, this line moves leftward, causing the hadron-dominated phase-space region to shrink. It can be seen that the hadron-related components are usually masked by the electron IC component, unless $\epsilon_e$ is very small. We note that both equations (17) and (18) are derived under conditions that are maximally favorable for the proton components, which includes also adopting the extreme case of $\epsilon_p = 1$, which may be an overestimate (Vietri 1997). Bearing all these facts in mind, we expect that the actual proton-dominated regimes (regions I and I') could be even smaller than what is indicated in Figure 1. On the other hand, in both the low-energy band ($v < v_{\nu c}$) and in the high-energy band ($v > v_{\nu c}$), the IC component is important in a much larger portion of the parameter phase space (regions II and II').

5. $\gamma\gamma$ Pair Attenuation

For the higher energy GRB photons in the observer frame, the comoving frame photon energy exceeds $mc^2$, and for sufficient high photon densities, a photon with energy $E$ in the observer frame may be attenuated by pair production through interaction with softer photons whose energy (also in the observer frame) is equal to or greater than $E_{\nu s} = (\Gamma mc^2)^2/E(1 + z)^2$, depending on the impact angle between the two photons. This may greatly degrade the high-energy fluence level, and the corrections due to this $\gamma\gamma$ absorption process needs to be taken into account. The $\gamma\gamma$ absorption in the GRB prompt phase has been studied by several authors (e.g., Krolik & Pier 1991; Fenimore, Epstein, & Ho 1993; Woods & Loeb 1995; Baring & Harding 1997; Lithwick & Sari 2001). Here, however, instead of the prompt phase, we concentrate on the afterglow phase.

To treat the absorption in the afterglow phase, we adopt an analytical approach similar to the one developed for internal shocks by Lithwick & Sari (2001; their eq. [2]), which we adapt here to the external shock scenario. Instead of using $\delta T$ (the temporal variation timescale in the internal shock scenario), we use, in our case, an emission timescale $t/(1 + z)$, which is the expansion time as viewed by the Earth observer with the cosmological time dilation effect correction. Assuming that the emission spectrum around $E_{\nu s} = h\nu_{\nu s}$ is $L_{\nu}(v) = L_{\nu,0} [(v/v_{\nu s})^{-\delta}$, the total photon number with $E > E_{\nu s}$ can be estimated as $N_{\nu E_{\nu s}} \sim \int_{v_{\nu s}}^{\infty} L_{\nu,0}/h\nu (v_{\nu s})^{-\delta} d\nu t/(1 + z) = L_{\nu,0} [t/(1 + z)] h\beta$. We adopt an averaged $\gamma\gamma$ cross section given by $C \sigma_{\nu \gamma \nu}$, where $C$ is a constant dependent on the photon spectral index. Svensson (1987) gives an analytic expression leading to $C = 11/180$ for a photon energy density index of $-1$ (photon number density index of $-2$), also used by Lithwick & Sari (2001). More detailed calculations by, e.g., Coppi & Blandford (1990) and Böttcher & Schlickeiser (1997) lead to a slightly larger $C$, on the order of 0.1. Noticing that $L_{\nu}(v_{\nu s}) = F_{\nu}(v_{\nu s})4\pi D^2/(1 + z)$, the attenuation optical depth is

$$
\tau_{\gamma\gamma}(v) = \frac{C \sigma_{\nu \gamma \nu} N_{\nu > E_{\nu s}}}{4\pi [4\Gamma^2 ct/(1 + z)]^2} = \frac{C \sigma_{\nu \gamma \nu} F_{\nu}(v_{\nu s}) D^2}{16\Gamma^2 c^2 h\beta t}, \quad (19)
$$

where, in terms of the quantities $t$ and $F_{\nu}(v_{\nu s})$ as measured by an Earth observer, the redshift factor has canceled out.

It is seen from equation (19) that the dominant spectral dependence of $\tau_{\gamma\gamma}(v)$ is on $F_{\nu}(v_{\nu s})$. For $E \sim 1$ TeV (where absorption becomes important), we find $E_{\nu s} \sim 2.6$ keV $\Gamma_2^2 (1 + z)^{-2}$, which is above $h\nu_{\gamma \nu}$. In this band, the electron-synchrotron emission component dominates in a large region of the phase space (Fig. 1), and even if the IC component may potentially dominate, this happens at a later time (§6) when the GeV–TeV emission is not important. Thus, for the regime we are interested in, we can approximate $F_{\nu}(v_{\nu s}) = F_{\nu, \max}(v_{\nu c}^e, v_{\nu c}^p)^{-p/(p - 1)/2}(v_{\nu s}/v_{\nu c}^e)^{-p/2}$ and take $\beta = p/2$. For the model of the dynamics adopted here and for $p = 2.2$, equation (19) then reads

$$
\tau_{\gamma\gamma}(v) \sim 0.56C_{-1}(1 + Y_\gamma)^{-1} (1 + z)^{(7p - 8)/8} (\epsilon_p^{p - 2})^4 (\epsilon_{p} - 1) \times e^{-v_{\gamma} - (p - 2) v_{\nu s}/4} \left(1 + 4/3 v_{\nu s}/4h T_{e}^{I} - 4/3v_{\nu s}/4h T_{e}^{I} - 1/2 v_{\nu s}^2 /2 \right), \quad (20)
$$

where $C_{-1} = C/0.1$ and the dependence of $(1 + Y_\gamma)^{-1}$ may be dropped for the regime I bursts ($\epsilon_e \ll \epsilon_p$ and $Y_\gamma \sim 0$). Notice the mild dependence on $v$ and the weak dependence on $t$. The absorption becomes important only when $\nu$ approaches 1 TeV ($\sim 2.4 \times 10^{26}$ Hz). This simple treatment is in qualitative agreement with the more detailed simulations of Dermer et al. 2000b (their Fig. 3).

An approximate expression for the final spectral flux including $\gamma\gamma$ attenuation is given by the flux escaping from a skin depth of unit optical depth, or

$$
F_{\nu}^e(v) = F_{\nu}^{\text{tot}}(v)/(1 + \tau_{\gamma\gamma}), \quad (21)
$$

where $F_{\nu}(v)^{\text{tot}}$ includes the contributions from all the spectral components discussed above.

6. RESULTS AND IMPLICATIONS FOR HIGH-ENERGY OBSERVATIONS

With a simple numerical code that includes the three spectral components discussed in this paper, as well as the $\gamma\gamma$ attenuation effect modeled through equation (19), we have investigated the spectra and the light curves in various energy bands for different choices of the most relevant parameters, especially $\epsilon_e$ and $\epsilon_p$. Because of the relatively small value of $\tau_{\gamma\gamma}$, for most energies of interest here, we have treated this as a simple absorption process and have neglected for simplicity the effects of the secondary pairs that it produces. The results generally confirm the division of the $\epsilon_e, \epsilon_p$ phase space sketched in Figure 1. Below we present some examples and explore the detectability of the proton component and the IC component in the afterglow phase in various bands, in particular, at GeV energies with future missions such as GLAST and in the X-ray band with the Chandra X-Ray Observatory. The signatures of the IC and proton components may be detected at high energies in at least two ways. One is through snapshot spectral fits, which require a wide and well-sampled energy coverage,
including the MeV–GeV band. Such simultaneous measurements may be achieved in the Swift and GLAST era. Another simpler way is to study the light curves at some fixed high-energy band, looking for a possible hardening of the light curve, the details of which we discuss below.

6.1. Detectability of the Proton-Synchrotron Component

To explore the detectability of the proton-synchrotron component, we choose as typical parameters for the regimes I and II the values \( \epsilon_p = 0.5 \epsilon_{B,0.5} \), \( \epsilon_e = 10^{-5} \epsilon_{e,-3} \), and \( n = 10^3 n_2 \). Notice that \( \epsilon_p = 0.5 \epsilon_{B,0.5} \) and \( \epsilon_e = 10^{-5} \epsilon_{e,-3} \) would fall outside region I for the density \( n \) ~ 1 assumed in our Figure 1, but adopting here a higher density \( n_2 \sim 1 \), these parameters are appropriate for region I. A denser medium will reduce \( \epsilon_e \), (eq. [8]) and consequently enlarges region I (eq. [11]). We do not explore an even lower \( \epsilon_e \), since it is unlikely to have values \( \epsilon_e \lesssim m_e/m_p \sim 0.5 \times 10^{-3} \).

For the frequency range \( v < v_{\nu, e} \), the critical time \( t_c \) at which the proton component overtakes the electron component can be derived by making equation (5) greater than unity. From equation (8), and taking a certain band, e.g., \( v = 10^{23} \) Hz \( v_{23} \), one gets

\[
t_p = 1.1 \text{ hr} \left( \epsilon_{B,0.5} \right) \left( \epsilon_{e,-3} \right)^{1/4} \frac{1}{p-1} \frac{p}{p_e} \left( \frac{v}{v_{\nu,e}} \right)^{2(p-2)} \times \frac{v_{23}^{} n_2^{-1}}{(1+z)^{-3} v_{23}^2}.
\]

The dependence on \( n \) is steep, so that for a lower density medium the overtaking time \( t_c \) could be too late for observational purposes. The dependence on the frequency is also very steep, so that the overtaking time for slightly higher \( v \) shifts to a much shorter time. For example, for \( v_{23} = 10 \) (4 GeV), the overtaking time moves down to \( t_c \sim 40 \) s.

Equation (22) indicates that for the above set of parameters, the proton-synchrotron component will show up in the GeV band. The possibility of directly detecting this component is interesting for several reasons. One is in possibly providing a constraint on the ratio of the proton to the electron injection fractions into the acceleration process \( \epsilon_p/\epsilon_e \), which is of interest for the shock physics. This would also have important implications for the overall energetics of the fireball. A detection might also provide information on whether the index \( p \) is the same for electrons and protons, as assumed here for simplicity (but departures from which would be interesting). We note, however, that the absolute luminosity of this proton GeV emission is too faint to be detected by GLAST, even for bursts at a distance of \( z \sim 0.1 \), unless \( \delta_{s2} \gtrsim 150 \). It may, however, be detectable for \( \delta_{s2} \gtrsim 1 \) by future larger effective area ground-based GeV telescopes from some close regime I bursts (see § 6.4 and Figure 4).

In the X-ray band, in which Chandra is sensitive (\( v \sim v_{18} \)), because of the steep frequency dependence \( t_p \propto v^{-3} \), the overtaking time always occurs too late, when the flux is low. Thus, it appears impossible to detect the proton-synchrotron component in the X-ray band. Figure 2a shows the snapshot spectra including the X-ray and GeV region for the typical parameter set representative of regime I, with a relatively dense external medium (\( n_1 \sim 1 \)). One remark is that the condition for the proton-synchrotron component to dominate a certain high-energy band is that \( v_{\nu,e} \) must decrease with time to make the electron component relatively less prominent (see eq. [5]). This is not the case for the afterglow evolution in a windlike external medium with \( n \propto r^{-2} \) (Chevalier & Li 1999). Thus, a detection of the proton component in the GeV band light curve would provide a diagnostic for an approximately constant external medium. A nondetection, however, would not necessarily be an argument against the constant medium since the phase-space region for the proton component detection is small.

6.2. Detectability of the Electron IC Component

To explore the detectability of the IC component, we choose the typical parameters for regions II and III to be \( \epsilon_p = 10^{-4} \epsilon_{B,-2} \), \( \epsilon_e = 0.5 \epsilon_{e,0.5} \), and \( n = 1 \), which are similar to Sari & Esin (2001). Following the same procedure to derive equation (22), one can derive the critical time \( t_{IC} \) when the IC component overtakes the synchrotron component at a typical frequency \( v < v_{\nu,e} \).

In the X-ray band, the overtaking time of the IC component usually occurs in the slow-cooling regime, which we will assume in the following discussions (see below). The crossing point between the synchrotron spectral component and the IC spectral component \( \nu_{IC} \) (Sari & Esin 2001) could, in principle, be either above or below \( \nu_{\nu,e} \). More generally, the overtaking occurs when in equation (13) could be used directly, and the overtaking condition is \((16/3) \tau_{\nu,e} n_p r_{p,e} \nu_{\nu,e}^2 \geq (1 + z)^{1/2} > 1 \). The complication in comparing regions I and II is that the Compton cooling factor \((1 + Y)\) in the expression of \( \nu_{\nu,e} \) (eq. [8]) can no longer be neglected. For \( (\epsilon_p/\epsilon_e) \gtrsim 1 \), one has \((1 + Y) \sim (\epsilon_p/\epsilon_e)^{1/2} \), where the slow-cooling phase \( n_2 = (\delta_{s2}^{1/2})^{1/2} (v/v_{\nu,e})^{2(p-1)} \). We have derived the overtaking time for this case, where \( t_{IC} \propto v^{4(4-p)/(3p^2-23p+36)} \) (see also Sari & Esin 2001). For reasonable values of \( p \) (e.g., 2.2–2.4), the quantity \((3p^2-23p+36) \) is close to zero, causing a very sharp, and probably unphysical, dependence of \( t_{IC} \) on all parameters. However, the value of \( t_{IC} \) derived in this manner is not generally useful for our purposes here. The reason is that, to ensure \( \nu_{\nu,e} > \nu_{IC} \), \( \epsilon_e \) and \( \epsilon_p \) should be close to the values near the boundary between regions II and III (eq. [16]), so that both values are comparable. In such a regime, the approximation of \((1 + Y) \approx (\epsilon_p/\epsilon_e)^{1/2} \) no longer holds, and one cannot get a simple analytic expression for \( t_{IC} \).

More generally, the overtaking occurs when \( v_{\nu,e} < \nu_{IC} \) in situations where \( \epsilon_e \gtrsim \epsilon_p \), as is the case for the typical values adopted above (and in Sari & Esin 2001). Noticing that in this regime \( \nu_{IC}^2 \left( \nu \right) = F_{\nu, max}^2 \left( \nu/v_{\nu,e} \right) \left( v/v_{\nu,e} \right)^{1/3} \approx F_{\nu, max} \left( \nu/v_{\nu,e} \right) \left( v/v_{\nu,e} \right)^{1/3} \), using equation (13) the overtaking condition can be written as \( (16/3) \tau_{\nu,e} n_p r_{p,e} \nu_{\nu,e}^2 \left( v/v_{\nu,e} \right)^{1/2} \left( v/v_{\nu,e} \right)^{1/3} \geq (1 + z)^{1/2} \). This finally leads to

\[
t_{IC} = 3.4 \text{ days} \left( \epsilon_{e,0.5} \right)^{0.08} \left( \epsilon_{B,-2} \right)^{1.63} \delta_{s2}^{-0.06} n \left( v_{18} \right)^{-0.66} \times (1 + z)^{-0.36} v_{18}^{0.68}
\]

for \( p = 2.2 \) and \( v = 10^{18} \) Hz \( v_{18} \) (X-ray band). The \( p \) dependence in the above expression is more cumbersome and is given in the Appendix. This result is in general agreement with Sari & Esin (2001), and, in addition, here we have explicitly presented the \( \epsilon_{e,0.5} \), \( \delta_{s2} \), and \( v_{18} \) dependences, which

4 The numerical indices in the eq. (23) are in agreement with the results of Sari & Esin for \( p = 2.2 \), except the index for \((1 + z) \), where we have included an additional factor from the frequency redshift correction, i.e., \(-0.36 = 0.32 - 0.68 \).
FIG. 2a

Fig. 2.—Temporal evolution of the broadband spectra of GRBs. The thick solid curves are the final spectra for various observer times, starting from (top) the onset of the afterglow, 1 minute, 1 hr, 1 day to (bottom) 1 month, respectively. The sharpness of the breaks and cutoffs is an artifact of the analytical approximations; in reality, these would be smoother transitions. For the top curve, contributions from the various radiation components are also plotted. Long-dashed lines are electron-synchrotron emission, short-dashed lines are proton-synchrotron emission, and dotted lines are electron IC emission. The thin solid line is the total energy flux level without c-absorption correction, while the thick solid line is the energy flux level after the c-self-absorption correction. The intergalactic absorption, which also becomes important around \( l \sim 10^{26} \), is distance-dependent and has not been included in this graph. Here all plots are calculated for standard parameters \( z = 1 \) (flat, \( \Lambda = 0 \) universe), \( \zeta_p = \zeta_e = 1 \), \( \epsilon_p = 1 \), \( \sigma_{\text{IC}} = 1 \), \( x = 1 \), \( \beta = 1 \), \( p = 2.2 \), and \( \Gamma_x = 300 \), while \( \epsilon_e, \epsilon_p \), and \( n \) vary for the different regimes. (a): Typical regime I burst: \( \epsilon_e = 10^{-3}, \epsilon_p = 0.5 \), and \( n = 100 \text{ cm}^{-3} \). (b): Typical regime II burst: \( \epsilon_e = 0.5, \epsilon_p = 0.01 \), and \( n = 1 \text{ cm}^{-3} \). (c): Typical regime III burst: \( \epsilon_e = 0.01, \epsilon_p = 0.1 \), and \( n = 1 \text{ cm}^{-3} \).

are absent in their paper. A factor of \( \sim 2 \) difference on the overtaking time (\( \sim 7.7 \) days in their case) may be caused by slightly different coefficients adopted in both works for \( v_m, \epsilon_e, v_e, \epsilon_p, F_s, \text{ max } \epsilon_e \), etc. Nonetheless, this confirms the finding of Sari & Esin that in a reasonably dense medium the IC component can be directly detected by Chandra a couple of days after the burst trigger. We note that a substantial flattening of the X-ray light curve for GRB 000926 has been detected by Chandra (Piro et al. 2001). Since the proton component cannot show up in the X-ray band under any circumstances, as we argued in § 6.1, such a flattening may indicate a direct detection of the IC emission of the elec-
trons. An alternative interpretation is advanced in Piro et al. (2001).

Extensive efforts have been made to determine key fireball parameters such as $\delta, \epsilon_p, \epsilon_e$, and $n$ using snapshot spectral fits extending from radio to X-rays on well-studied GRB afterglows (see, e.g., Galama et al. 1998; Wijers & Galama 1999; Panaitescu & Kumar 2001). A cautionary point that needs to be stressed about these analyses is that the spectrum that is observed does not need to be, as is generally assumed, solely due to electron-synchrotron radiation, especially when using late-time ($\gtrsim$ a couple of days) data in the fitting. According to our results in this paper, as long as the $\epsilon_e, \epsilon_p$ phase space is in regions I or III, the standard fitting assumption (i.e., electron-synchrotron dominance) is safe since there are no high-energy (proton or IC) spectral components appearing in the X-ray band. However, in region II of parameter space (which includes values of $\epsilon_p$ and $\epsilon_e$ often derived from such fits), the analysis may not be self-consistent since the X-ray data points may be due the IC component. This caution applies to even earlier snapshot spectral fits if the burst happens to occur in a denser medium [notice the negative dependence of $n$ on the $-(2/3)(5p - 26)/(3p^2 - 8p - 12)$ index].

The negative dependence on $v_{\text{ic}}$ of $F_{\text{IC}}$ [the explicit index is $-(2/3)(3p + 2p - 4)/(3p^2 - 8p - 12)$; see also Sari & Esin 2001] indicates that for energy bands above X-rays, the overtaking time is much earlier. In fact, in the GeV band, the IC component dominates almost throughout the entire afterglow phase. For the typical parameter set corresponding to the IC-dominated region II, Figure 2b shows the time evolution of the snapshot spectra. For completeness, we present also in Figure 2a the snapshot spectra for the typical parameter set in the proton-dominated region I, while Figure 2c shows the snapshot spectra for parameters in the electron-synchrotron-dominated region III. We can see that in this latter region neither of the two high-energy components are prominent below ~GeV energies (although at ~GeV, there is a lower level IC component). Two explanations should be made about our code. First, to avoid adding up an unphysical component in the electron-synchrotron self-absorption band, we have arbitrarily defined the self-absorption cutoff in the proton component. The self-absorption segment in the electron IC component is still plotted with a slope of 2 rather than 1 (Sari & Esin 2001) for the convenience of code developing, which does not influence the final broadband spectrum. Second, at the cutoff frequency of each component, we have adopted a sharp cutoff, while a more realistic cutoff should be exponential. The same applies for the sharp jumps in the light curves presented in Figure 4.

In Figure 3, we present the X-ray light curves for the typical bursts in the three different regimes. While the regime I and III afterglows show a monotonous decay in this band, we show that the regime II afterglows can show interesting bump features because of the dominance of the IC component at a later time, which is in qualitative agreement with Panaitescu & Kumar (2000) and Sari & Esin (2001).

6.3. GeV Afterglows and GRB 940217

The extended GeV emission 1.5 hr after the trigger of GRB 940217 detected by EGRET (Hurley 1994) indicates that a high-energy spectral component can extend into the GeV band for a long period of time, at least in some bursts. In principle, this could be due to either the proton-synchrotron emission in regime I or the electron IC emission in regime II or even the electron-synchrotron emission in regime III. We will show below that the regime II IC-dominated origin is the more plausible explanation.

The peak energy flux expected from various scenarios can be estimated straightforwardly. For the proton-synchrotron in regime I, we have

$$vF_{\nu,p}(\text{GeV}) = vF_{\nu,\text{max}} p(\nu/\nu_{\text{ic}})^{(p-1)/2}$$

$$\sim 1.4 \times 10^{-14} \text{ ergs s}^{-1} \text{ cm}^{-2} \times \epsilon_p^{-1} \epsilon_e^{2-3p} \epsilon_{\text{B}}^{0.5} \gamma^{2/3} D_2^{28} (1 + z)^{(9-p)/4} B^{-3(p-1)/4} \nu_{\text{max}}^{3-p}/2 \nu_{\text{ic}}^{3-p}/2$$

(see also Fig. 2). This is more than an order of magnitude below the calculated level of Böttcher & Dermer (1998) as well as the analytical estimate of Totani (1998). The main discrepancy with both of these results is due to their having adopted $\gamma_{\text{max}} = \Gamma$ rather than the more accurate lower value we adopted in equation (1) and also due to the fact that we calculate the flux coefficient using $p = 2.2$ while they

5 After this paper was submitted, we noticed that Harrison et al. (2001) has performed a detailed fit to the snapshot spectra of the afterglow data of this burst at 2 and 10 days. The X-ray emission data are consistent with an IC emission component. The best-fit parameters they derived lie in our regime II. This agrees with the discussions presented here.

6 At higher energy bands, $v_{\nu_c} < v_{\text{IC}}$, may be no longer satisfied, and the overtaking time may occur in the fast-cooling regime. Nonetheless, the negative dependence on $v$ of $F_{\text{IC}}$ generally holds.
The energy flux has been integrated within the range from MeV to GeV, with the index $p = 2$, which further enhances the discrepancy between $\gamma_{m,p}$ and $\Gamma$ and which gives a milder $v$ dependence. To test this, we have substituted $\gamma_{m,p} = \Gamma$ and $p = 2$ in our code, and this reproduces the results of Böttcher & Dermer. We conclude that the correction introduced by using here the more realistic $\gamma_{m,p}$ (eq. [24]) is essential and that the previous rough estimates using $\gamma_{m,p} = \Gamma$ can considerably overestimate the proton-synchrotron flux level. Another feature, which can be seen from equation (24), is the negative temporal decay of the flux level with the index $-3(p-1)/4 \sim -0.9$, which indicates that even if the proton-synchrotron emission flux level is detectable in the early afterglow phase, it will drop with time. For the regime III electron-synchrotron–dominated case the trend is similar, with a steeper temporal index of $-(3p-2)/4 \sim -1.15$.

In the IC-dominated regime II, contrary to the proton component, the IC component itself has a bump peaking at $v_{\text{IC},\max}$ (for slow cooling) or $v_{\text{IC},\max}^2$ (for fast cooling) in the $F_{\ell}$ plot. The peak will sweep the band $v = 10^{23}$ Hz $v_{\ell,3}$ at $t = t_{\text{max}} \sim 0.3 \text{ hr} \epsilon_{16}^{-1/12} r_{-16}^{-1/12} \epsilon_{B,2}^{-1/2} n_{-16}^{-1/12} (1 + z)^{1/2} \epsilon_{\phi,2} \delta_{2}^{1/2}$ in the slow-cooling regime. The temporal index before the flux reaches its peak is 1 (slow cooling) or $(8 - 3p)/3(4 - p) \sim 0.26$ (fast cooling), and it is $(11 - 9p)/8 \sim -1.1$ (slow cooling, which is usually the case) after the flux has passed its peak. The peak energy flux can be estimated as $v_{\ell, \text{IC}, \max} f(t = t_{\text{max}}) \sim 3.0 \times 10^{-10} \text{ ergs s}^{-1} \text{ cm}^{-2}$ for $\epsilon_{B} \sim 0.01$. This temporal evolution is mild, which allows for a substantial GeV emission component lasting hours after the GRB trigger.

The EGRET flux sensitivity above 100 MeV is $\sim 10^{-7} \text{ ph s}^{-1} \text{ cm}^{-2}$ for point-source observations over a period of 2 weeks in directions away from the Galactic plane. Correcting for an average effective on-source observing fraction of 45% (D. J. Thompson 2001, private communication), the flux threshold may be estimated as $\sim 10^{-7} T(t/ T)^{1/2} \sim 5 \times 10^{-5} t^{1/2} \text{ ergs cm}^{-2}$ at an average energy of 400 MeV, where $T = 14 \times 8400 \times 45\%$ and $t$ is integration time in seconds. This flux sensitivity may be extrapolated down to integration times such that at least, say, 5 photons are collected. For even shorter integration times, the flux sensitivity may be defined, e.g., by the criterion that at least 5 photons are collected, which is $\sim 5/(A_{\text{eff}} t) \text{ ph s}^{-1} \text{ cm}^{-2}$. For EGRET, $A_{\text{eff}} \sim 1500 \text{ cm}^2$, giving an estimated flux threshold in the low integration time regime of $\sim (5/150) \times 400 \text{ MeV} \sim 2.1 \times 10^{-6} \text{ ergs cm}^{-2}$. As shown in Figure 4, for $\epsilon_{2} \sim 1$, the EGRET flux threshold level is not reached by the regime I or III bursts, but this level is attainable for a regime II burst located at closer distances (e.g., $z = 0.1$) or in a higher density environment. Thus, the late ($\sim 1 \text{ hr}$) GRB 940217 afterglow was most likely dominated by the electron IC emission from a nearby or dense medium regime II burst. This agrees with Mészáros & Rees (1994), and Dermer et al. (2000b) drew a similar conclusion by detailed simulations using a specific set of parameters, i.e., $\epsilon_e = 0.5$ and $\epsilon_\phi \lesssim 10^{-4}$, which lies in our regime II. GLAST, which is currently under construction, will have a flux sensitivity of $\sim 1.6 \times 10^{-12} \text{ ergs s}^{-1} \text{ cm}^{-2}$ for long-term observations (Gehrels & Michelson 1999), roughly 40 times more sensitive than EGRET in the point mode. In the low-flux regime, given the effective area of $\sim 8000 \text{ cm}^2$ (Gehrels & Michelson 1999), it is only about a factor of 5 more sensitive than EGRET. The flux threshold for GLAST is roughly $\sim 1.2 \times 10^{-9} t^{1/2} \text{ ergs cm}^{-2}$ for a long integration time regime and $\sim 4.0 \times 10^{-8} \text{ ergs cm}^{-2}$ for a short integration time regime (again, assuming that at least 5 photons are collected). This will make most regime II burst afterglows detectable at a typical cosmological distance and in a moderate density medium (Fig. 4).

In Figure 4 we show the GeV light curves for bursts typical of the three different parameter regimes and $\epsilon_{2} \sim 1$. For comparison with future observations, we have integrated over the 400 MeV–200 GeV band to get the total energy fluence that GLAST can collect during a certain time duration $t$. The sensitivity thresholds of EGRET and GLAST are indicated. One sees that regime II burst afterglows would be generally detectable by GLAST within hours after the burst trigger. Bursts in regimes I and III are generally nondetectable by GLAST, even for burst at $z = 0.1$. Increasing the total energy budget ($\epsilon_{2}$) or ambient density ($n$) can increase the detectability of these bursts. Since for transient events, the key factor of sensitivity is the effective collecting area, some future ground-based larger area GeV telescopes, such as the 5/6.5 5 GeV energy threshold array of imaging atmospheric Cerenkov telescopes at a 5 km altitude (Aharonian et al. 2001), may have better chance to detect the regime I and III bursts (at the prompt phase) and, of course, to collect more photons from regime II bursts. A criterion to differentiate between nearby regime I and III bursts is that the light curve for the regime I burst is flatter. In any case, we conclude that an extended GeV
afterglow is a diagnostic of a regime II (IC-dominated) burst.

Recently, an energy flux upper limit $J(E > 760$ GeV) $< 9.4 \times 10^{-12}$ ergs cm$^{-2}$ s$^{-1}$ in GRB 010222 was obtained in a 4 hr measurement with the stereoscopic High-Energy Gamma-Ray Array Cerenkov telescope system, 19 hr after the burst trigger (Goetting & Horns 2001). Given the cosmological distance of $z \geq 1.477$ (Jha et al. 2001), this is consistent with our model prediction in this paper (see Fig. 2), even for the most favorable regime II case (Fig. 2b).

6.4. Prospects from Broadband Observations in the Swift-GLAST Era

The GLAST mission will be launched in 2005, with a sensitivity range of 20 MeV–300 GeV, complemented by the GLAST Burst Monitor, whose energy range extends from a few keV to 30 GeV. Another broadband GRB mission, Swift, will be launched in 2003 and will be sensitive in the optical, X-ray, and γ-rays up to $\lesssim 140$ keV. At the same time, ground-based experiments such as Milagro, HESS, Veritas, MAGIC, and Cangaroo-III may provide $\geq 0.5$ TeV, or in some cases $\geq 30$ GeV, data or upper limits. In the Swift-GLAST era, simultaneous broadband observations at the very earliest stages of the GRB afterglows will become possible, which will bring invaluable information about GRB shock physics and the central engine. Here we note the following interesting issues that can be addressed in the Swift-GLAST era:

1. Sari & Esin (2001) pointed out that because of the IC cooling, there are two possible solutions of the unknown fireball and shock parameters for a same set of observables from the low-energy afterglow fits. For high-density mediums, these two scenarios may be distinguished from the late-time X-ray observations, but for low-density mediums, the two solutions are degenerate and indistinguishable with the present data. However, we note that the two sets of solutions lie within regime III and regime II, respectively. This provides a natural way to distinguish between the two scenarios by using the GeV afterglow data.

If an extended GeV afterglow is detected by GLAST, then the parameter space should be in regime II, in which the IC component dominates in the high-energy band. In this case, equations (4.17)–(4.20) of Sari & Esin (2001) will apply. Otherwise, the parameter space should be in regime III, in which no prominent emission component shows up in the GeV band. This is the case in which equations (4.13)–(4.16) of Sari & Esin may apply.

2. It has been proposed by Waxman (1995) and Vietri (1995) that GRBs are likely sites to produce ultra–high-energy cosmic rays (UHECRs) and that the $10^{15}$ eV excess UHECRs detected are of GRB origin. This hypothesis is subject to debate (cf. Stecker 2000; Mannheim 2001; Scully & Stecker 2001). We note that future GeV observations with GLAST may be able to add important criteria to the debate, at least for the external shock scenario. To accelerate protons to ultra–high energies around $10^{20}$ eV, $\epsilon_B$ must be close to unity (Waxman 1995; Vietri 1995; Rachen & Mészáros 1998). If substantial extended GeV afterglows are common among GRBs and if the redshift measurements from the low-frequency afterglow observations indicate that the GRBs are around $z \sim 1$, this will impose severe constraints on the UHECR acceleration theory by the external shock scenario since regime II generally favors a small $\epsilon_B$.

(However, our calculations do not apply to a possible UHECR acceleration in internal shocks). On the other hand, if long-duration GeV afterglows are not common and, for a few nearby bursts, a GeV (prompt) light curve hardening is detected by GLAST or some other future telescope, this could be attributable to proton-synchrotron emission, in which case both $\epsilon_e$ and $\epsilon_B$ are close to unity. This would provide support to the theory of UHECR origin in GRBs. Since these components are not masked by the electron-synchrotron and the IC components, this would also hint at a small $\epsilon_e$ (e.g., due to a weak coupling between electrons and protons, as argued by Totani 1998, 2000).

3. Although most present snapshot spectral fits assume that the unknown shock parameters ($\epsilon_e$, $\epsilon_p$, and $\epsilon_B$) are constant with time, there is no a priori reason for this simplest assumption. Evolutions of one or more of these parameters are, in principle, possible (see, e.g., Dermer et al. 2000b). Present observations are too crude to explore these possibilities, but future broadband observations will provide the opportunity to explore this important issue. In Figure 1, we present generically the parameter space of various regimes as well as their dependences on some other parameters including the observation time. In the discussions in § 6, we have assumed the constant equipartition parameters. If there exist substantial evolutions of these parameters, the blast wave parameters will change in the $\epsilon_e$, $\epsilon_p$, $\epsilon_B$ space and may, in some cases, switch the regime they belong to as time goes by. This will bring some additional interesting signatures in the light curves in various bands, and further, more detailed, studies may provide diagnostics on the possible evolutionary effects.

7. SUMMARY

We have studied GRB afterglow snapshot spectra and light curves over a broader band than usual by including the canonical electron-synchrotron emission as well as two other high–energy spectral components, i.e., the proton-synchrotron emission component and the electron-synchrotron self-IC emission component. We have, in particular, concentrated on the X-ray to GeV–TeV ranges, including the effect of attenuation by $\gamma\gamma$ pair formation. This investigation has the advantage, relative to prior ones, of bringing together in a single coherent treatment the effects of these various high–energy mechanisms, which hitherto had been mostly treated singly or in twos, within the context of a specific GRB afterglow dynamical model. This is carried out over a wider range of parameter phase space than previously to allow a global view of the relative importance of the various spectral components.

For the frequency range below the electron’s synchrotron cutoff $v < v_{e, \epsilon}$, there is a competition between the electron-synchrotron component on the one hand and the proton-synchrotron component or the electron IC component on the other, which can affect the higher energy bands including X-rays or above. This competition divides the $\epsilon_e$, $\epsilon_B$ phase space into three regimes (Fig. 1). We have explored the range of validity of these regimes and discussed the conditions for which these high–energy spectral components would show up in various bands, especially in the GeV and the X-ray band. The conclusion is that the IC component is likely to be important in a relatively large region of param-
eter space, while the conditions for which the proton-synchrotron component is important involve a small, but nonnegligible, region of parameter space. One interesting consequence is that there is a substantial region (regime III) in which neither of the two high-energy components are important. Above the electron-synchrotron cutoff, the competition is between the electron IC component and the hadron-related photo-meson decay components, which we treat as a reduced extension of the proton-synchrotron component. Again, the phase-space region in which the latter effects are important in the afterglow is small. We also find that for the external shock and the afterglow phase the $\gamma$-$\gamma$ absorption is not important below the TeV range.

A general conclusion is that the most likely origin for an extended high-energy afterglow component at GeV energies is from the electron IC component. Not only is the phase-space region where the IC component dominates (regions II and II$'$) much larger than that where the proton component dominates (regions I and I$'$; see Fig. 1), but its intensity is also much higher than that of the hadron components, and the time scale during which an appreciable flux level is maintained in the GeV band is much longer than for the hadronic components. In the parameter regime favorable for the IC emission, this component is observable at and above the X-ray band. In the X-ray band, it will lead to a flattening of the light curve at late times, as long as the medium density is not too low (see also Sari & Esin 2001). Above the X-ray band, the time after which IC emission becomes dominant appears earlier, and the IC component dominates the GeV band emission almost from the onset of the afterglow phase. In general, a high external density medium favors the detectability of the IC component. Such an IC component is likely to have been responsible for the GeV photons detected from GRB 940217 with EGRET, and similar events should in the future be detectable by GLAST if successful, would imply, depending on its strength, fireball and shock parameters that are more extreme than currently commonly assumed. In particular, it would provide a diagnostic for a high $\epsilon_p$ and $\epsilon_B$ and/or a low $\epsilon_e$.

We point out a simple way to break the current parameter-space degeneracy that currently, from low-frequency observations alone (X-rays and below), prevents the unambiguous determination of the unknown fireball and shock parameters, through the use of snapshot spectral measurements extending into the GeV range. We also suggest a way to test the hypothesis of a GRB origin for UHECR using combined Swift and GLAST data. Such observations may also provide diagnostics for the presence of a quasi-homogeneous versus a windlike inhomoegensous external medium.

Finally, we note that the condition for the spectral IC component to be prominent (eq. [16] and Fig. 1) usually covers the parameter regime in which IC cooling is important ($\eta \epsilon_e / \epsilon_B > 1$; Sari & Esin 2001). Thus, the current snapshot spectral fits should be made with caution for the X-ray data points when the IC cooling is important, especially for the data at later times and for cases that may involve a high external medium density.

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**APPENDIX A**

**EXPLICIT $p$ DEPENDENCES OF EQUATIONS (18) AND (23)**

In this appendix, we explicitly present the $p$-dependent indices in equations (18) and (23). For equation (18),

$$
(1 + Y_0)^{(8/(16 - 3p))} \epsilon_B > 215k^{-1} \times \frac{(p - 1)(16 - 3p)}{(16 - 3p)} \frac{\epsilon_p (4(p - 1)/(16 - 3p))}{\epsilon_B} \frac{\epsilon_B (4(p - 1)/(16 - 3p))}{\epsilon_p} \frac{(8(p - 2)/(16 - 3p))}{\nu_e} \frac{(16 - 3p)/(16 - 3p)}{\nu_e} \times \delta_{S_2}^{-(13 - 4p)/(2(16 - 3p))} \nu_e^{-(11 - 2p)/(2(16 - 3p))} \nu_e^{-(16 - 3p)/(2(16 - 3p))} \left(1 + \frac{1}{2}\right)^{(19 - 4p)/(2(16 - 3p))} \nu_e^{-(16 - 3p)/(2(16 - 3p))}.
$$

(A1)

For equation (23),

$$
t_{IC} = 3.4 \text{ days} \times \delta_{S_2}^{(4(3p^2 - 8p - 7)/3(3p^2 - 8p - 12))} \times \nu_e^{(2(3p^2 - 10p + 4)/3(3p^2 - 8p - 12))} \nu_e^{(4(3p^2 - 5p - 22)/3(3p^2 - 8p - 12))} \times \delta_{S_2}^{(2p^2 - 4p + 1)/(2(3p^2 - 8p - 12))} \frac{1}{\nu_e} \times \nu_e^{-(2(3p + 2)/p - 4)/3(3p^2 - 8p - 12))}.
$$

(A2)
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