Thermalization time and specific heat of neutron stars crust

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Study of the cooling of a neutron star (NS) with fast cooling. Cooling time essentially determined by the properties of the inner-crust ie. :

- the thickness,
- the properties of the baryonic matter.

Composition of the inner-crust :

- ultrarelativistic electrons,
- unbound neutrons that can be superfluid,
- nuclear clusters, whose influence on the superfluid properties has to be taken into account.
Solve the relativistic heat equation in the whole NS using NSCool\(^1\) (D. Page),

- with a model of NS that is almost completely consistent (SLy4 nuclear interaction),
- using new calculations for the specific heat of unbound neutrons in the inner-crust.
- estimation of the cooling time.

\(^1\)available on http://www.astroscu.unam.mx/neutrones/NSCool/
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1$S_0$ neutron pairing

HFB-FT calculations

- Mean field : Skyrme force SLy4 (Chabanat et al. 1997),
- Nuclear clusters : WS cells from Negele & Vautherin (1973),
- Pairing correlations :

\[
V(r - r') = V_0 \left[ 1 - \eta \left( \frac{\rho(r)}{\rho_0} \right)^\alpha \right] \delta(r - r'),
\]

with $V_0$, $\eta$ and $\alpha$ simulating two pairing scenarios :
$^1S_0$ neutron pairing

**Introduction**

- $^1S_0$ neutron specific heat calculations
- Cooling model
- Neutron star model
- Heat equation
- Cooling
- Crust thermalization
- Scaling relations
- Conclusion

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**Weak pairing**

![Graph showing the weak pairing for different cells with temperature (T) vs specific heat ($C_v$) for various temperatures.]

**Strong pairing**

![Graph showing the strong pairing for different cells with temperature (T) vs specific heat ($C_v$) for various temperatures.]

- Weak pairing graphs for cells 1 to 5 and cells 6 to 10.
- Strong pairing graphs for cells 1 to 5 and cells 6 to 10.
Neutron star model

Equation of state (EoS):

- Core: Douchin & Haensel (2001)
  - based on the SLy4 effective nuclear interaction (the same as in the $C_V$ calculations),
  - $npe\mu\mu$ composition.
- Inner-crust: Negele & Vautherin (1973)
  - $4 \times 10^{11} \leq \rho \leq 1.6 \times 10^{14} \text{ g cm}^{-3}$
  - density functional,
  - Hartree-Fock calculations.
- Outer-crust: Haensel, Zdunik & Dobaczewski (1989)
  - Skyrme effective nucleon-nucleon interaction (Dobaczewski, Flocard & Treiner, 1984),
  - Hartree-Fock-Bogoliubov (HFB) calculations.
- Effective mass: Skyrme nuclear interaction.
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Introduction

Heat equation (Thorne, 1977)

\[
\frac{\partial}{\partial r} \left( \frac{Kr^2}{\Gamma(r)} e^\phi \frac{\partial}{\partial r} (Te^\phi) \right) = r^2 \Gamma(r) e^\phi \left( C_V \frac{\partial T}{\partial t} + e^\phi Q_\nu \right),
\]

- $\Gamma = \left( 1 - \frac{2Gm(r)}{rc^2} \right)^{-1/2}$, $\phi$ the gravitational potential,
- $K$ the thermal conductivity,
- $Q_\nu$ the neutrino emissivity,
- $C_V$ the specific heat.

Boundary conditions:

- $T(r, t = 0) = T_i$
- $\rho = 10^{10}$ g cm$^{-3}$, model of non-accreted envelope (Potekhin et al. 1997).
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Thermal conductivity

Core :
- electrons & muons (Shternin & Yakovlev, 2007)
- nucleons (Baiko et al. 2001)

Crust :
- electron-ion (Gnedin et al. 2001)
- electron-electron (Shternin & Yakovlev, 2006)
Neutrino emissivity (1)

Core:

- bremsstrahlung processes,
- MURCA,
- DURCA imposed for $\rho \geq 5 \times 10^{14}$ g cm$^{-3}$ → fast cooling.

Cooling model

- Neutrino emissivity (1)
- Core:
  - bremsstrahlung processes,
  - MURCA,
  - DURCA imposed for $\rho \geq 5 \times 10^{14}$ g cm$^{-3}$ → fast cooling.
Neutrino emissivity (2)

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Crust:
- plasmon decay,
- $e^- - e^-$, $e^- - Z$ & n-n bremsstrahlung.

Superfluidity:
- reduction of the emissivities,
- Cooper pair breaking and formation processes (PBF).

Cooling model

$T = 10^9$ K

$\log_{10} (Q, \text{[erg s}^{-1}\text{cm}^{-3}])$

$\log_{10} (\rho, \text{[g cm}^{-3}])$

$T = 10^9$ K

Plasmon
e$^- - e^-$ brems
e$^- - Z$ brems
n-n brems
PBF n $^1S_0$ weak
PBF n $^1S_0$ strong
PBF p $^3S_0$
Specific heat (1)

Electrons:
- \( C_V \) of a uniform, degenerate gas.

Ions in the crust:
- solid-liquid phase transition included,

Protons in the core:
- \(^1S_0\) pairing from Takatsuka (1973),
Specific heat (2)

**Unbound neutrons:**

- In the core:
  - $^3P_2$ pairing: model "a" from Page et al. 2004 with $T_c^{\text{max}} \sim 10^9$ K,

- In the inner-crust:
  - $^1S_0$ pairing: fits of the previous calculations.

![Graph showing specific heat vs. density and temperature](image)

- **Ions**
- **Electrons**
- **Protons**
- **Non-superfluid neutrons**
- **Weakly paired neutrons**
- **Strongly paired neutrons**

**T = 10^9 K**

- $\log_{10}(C_V [\text{erg cm}^{-3}\text{K}^{-1}])$
- $\log_{10}(\rho [\text{g cm}^{-3}])$
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M = 1.6 M_☉ & T_i = 5 \times 10^9 K

No pairing

Weak pairing

Strong pairing

\[ t(\text{yr}) = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5} \]

\[ t(\text{yr}) = 1, 5, 10, 15, 20 \]

\[ r \text{ [km]} = 10.5, 10.8, 11.1, 11.4 \]
Crust thermalization

Cooling curves & pairing scenarios - $M=1.6\, M_\odot$

Cooling time $t_w$ : $T_\infty(t = t_w)$ has its most negative slope.
Introduction

1\textsuperscript{S\textsubscript{0}} neutron specific heat calculations

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Thermalization time and specific heat of neutron stars crust

M \in [1.4, 2.0] \, M_{\odot} \, \& \, T_i = 5 \times 10^{9} \, K

\begin{align*}
\log (t_w [\text{years}]) & \quad \log (\alpha) \\
\begin{array}{c}
\text{Normal Neutrons} \\
\text{Strong Pairing} \\
\text{Weak Pairing}
\end{array}
\end{align*}

Lattimer et al. 1994, Gnedin et al. 2001

Scaling parameter: \( \alpha = \left( \frac{\Delta R_{\text{crust}}}{1 \, \text{km}} \right)^2 \left( 1 - \frac{2GM}{c^2R} \right)^{-3/2} \)
Conclusion (1)

New calculations of the specific heat of neutrons in the crust:
- HFB at finite temperature;
- inclusion of the effects of:
  - the temperature,
  - the nuclear clusters,
  - the pairing correlations.

Study the thermalization of NS crusts in the fast cooling scenario for an almost completely consistent model (SLy4).
Results

- The pairing correlations have a strong influence on cooling.
- The cluster structure of the inner-crust has a non-trivial influence.

Perspective

- Performing cooling calculations in WS cells calculated for the SLy4 force.
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Cooling curves & pairing scenarios - $M=1.6 \, M_\odot$

$T_i = 5 \times 10^9 \, K$

- no pairing
- weak pairing
- strong pairing
- weak pairing (NC)
- strong pairing (NC)

$\log_{10}(T^{\text{eff}}_\infty) [\text{K}]$

Time [years]

0 10 20 30 40 50 60 70 80 90 100
Crust thermalization

Cooling curves & pairing scenarios - M=1.6 M⊙

- no pairing
- weak pairing
- strong pairing
- weak pairing (NC)
- strong pairing (NC)

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\[ C^n_V = (1 - x_{cl})C^{cl}_V + x_{cl}RC^q_V \]

with:

- \( C^{cl}_V \) the specific of non-superfluid unbound neutrons in the classical regime,
- \( C^q_V \) the specific of non-superfluid unbound neutrons in the quantum regime,
- \( x_{cl} \) the factor describing the transition between classic and quantum behavior,
- \( R \) the factor simulating the reduction due to pairing correlations.

1S_0 neutron pairing
$^{1}S_0$ neutron pairing

Parametrization of $C^n_V$

$$C^n_V = x_{cl} R C^q_V + (1 - x_{cl}) C^{cl}_V$$

with:

- the factor describing the transition between classic and quantum behavior,
  $$x_{cl} = \left(1 + e^{5(\frac{\pi T}{\varepsilon_F} - 1)}\right)^{-1}$$

- with $\varepsilon_F = \hbar^2 k_F^2 / 2m^*_n$ the Fermi energy at zero $T$.

For normal, unbound neutrons:

$$C^q_V = \frac{1}{6} \left(\frac{2m^*_n}{\hbar^2}\right)^{3/2} \varepsilon_F^{1/2} T \times \left[1 - \frac{7}{40} \left(\frac{\pi T}{\varepsilon_F}\right)^2 - \frac{155}{896} \left(\frac{\pi T}{\varepsilon_F}\right)^4\right].$$
**Parametrization of $C^n_V$**

\[
C^n_V = x_{cl} R C^q_V + (1 - x_{cl}) C^{cl}_V
\]

with:

- **For classic neutrons:**
  \[
  C^{cl}_V = \frac{3}{2} \rho_{gas},
  \]

  with, for $T < T_{gas} = 5.5$ MeV,
  \[
  \rho_{gas} = \rho_n(T = 0) + \frac{T}{T_{gas}} (\rho_{max} - \rho_n(T = 0)),
  \]

  for $T > T_{gas} = 5.5$ MeV,
  \[
  \rho_{gas} = \rho_{max}.
  \]

- with $\rho_{max}$ for neutrons uniformly distributed in the cell.
$^1S_0$ neutron pairing

Parametrization of $C^n_V$

$$C^n_V = x_{cl} R C^q_V + (1 - x_{cl}) C^{cl}_V$$

with the factor simulating the reduction due to pairing correlations:

$$R = R_{YL}(u) f_1(T, \Delta_o, a_0, a_1, a_3) (1 - f_2(T, \Delta_o, a_0, a_2, a_3)),$$

where

- $R_{YL}(u)$ is the superfluid reduction factor for uniform neutron matter (Levenfish et al., 1994),
- $f_1$ & $f_2$ are two functions describing the normal/superfluid transition, depending on:
  - $\Delta_o$ the pairing energy gap in the neutron gas at $T=0$,
  - $a_0, a_1, a_2, a_3$ four parameters fitted to reproduce the results of the HFB calculations.
Scaling relations

\[ t_w = \alpha t_1 \text{ with } \alpha = \left( \frac{\Delta R_{\text{crust}}}{1 \text{ km}} \right)^2 \left( 1 - \frac{2GM}{c^2R} \right)^{-3/2} \]

\[ M = 1.5M_\odot \text{ & } T_i = 5 \times 10^9 \text{ K} \]

| Model of neutron superfluidity    | \( t_w \)   | \( t_1 \) |
|-----------------------------------|-------------|-----------|
| No superfluidity                  | 76.3        | 66.4      |
| Weak pairing                      | 43.1        | 37.4      |
| Strong pairing                    | 24.7        | 21.5      |