Monte Carlo Simulations of Photospheric Emission in Relativistic Outflows

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Abstract

We study the spectra of photospheric emission from highly relativistic gamma-ray burst outflows using a Monte Carlo code. We consider the Comptonization of photons with a fast-cooled synchrotron spectrum in a relativistic jet with a realistic photon-to-electron number ratio $N_e/N_p = 10^5$, using mono-energetic protons that interact with thermalized electrons through Coulomb interaction. The photons, electrons, and protons are cooled adiabatically as the jet expands outward. We find that the initial energy distributions of the protons and electrons do not have any appreciable effect on the photon peak energy $E_{\gamma,\text{peak}}$ and the power-law spectrum above $E_{\gamma,\text{peak}}$. The Coulomb interaction between the electrons and the protons does not affect the output photon spectrum significantly as the energy of the electrons is elevated only marginally. $E_{\gamma,\text{peak}}$ and the spectral indices for the low- and high-energy power-law tails of the photon spectrum remain practically unchanged even with electron-proton coupling. Increasing the initial optical depth $\tau_{in}$ results in a slightly shallower photon spectrum below $E_{\gamma,\text{peak}}$ and fewer photons at the high-energy tail, although $f_e \propto \nu^{-0.5}$ above $E_{\gamma,\text{peak}}$ and up to $\sim 1$ MeV, independent of $\tau_{in}$. We find that $E_{\gamma,\text{peak}}$ determines the peak energy and the shape of the output photon spectrum. Finally, we find that our simulation results are quite sensitive to $N_e/N_p$, for $N_e = 3 \times 10^5$. For almost all our simulations, we obtain an output photon spectrum with a power-law tail above $E_{\gamma,\text{peak}}$ extending up to $\sim 1$ MeV.

Key words: gamma-ray burst: general – methods: numerical – radiation mechanisms: thermal – radiative transfer – scattering

1. Introduction

The radiation mechanism responsible for the prompt emission of gamma-ray bursts (GRBs) is still not very well understood. The observed spectra are generally modeled using the Band function (Band et al. 1993), which is a smoothly connected broken power law with an observed peak energy of $E_p \sim 300$ keV and nonthermal power laws below and above the peak (in some cases up to $\sim$GeV energies; Kaneko et al. 2006 and Preece et al. 2000). The two most widely explored models to explain the GRB spectrum are the internal dissipation model and the photospheric model (Rees & Meszaros 1994; Piran 2004; Kumar & Zhang 2015).

In the internal dissipation model, the energy is dissipated either by internal shocks (Rees & Meszaros 1994) or by magnetic reconnection in a Poynting-dominated jet (Zhang & Yan 2011). The prompt radiation is due to the synchrotron emission from nonthermal electrons gyrating in a shock-generated magnetic field (Meszaros et al. 1994; Piran 1999; Lloyd & Petrossian 2000). This model suffers from low radiation efficiency because only the kinetic energy associated with the differential motion of the shells can be dissipated, not the kinetic energy associated with the bulk motion of the jet (Kobayashi et al. 1997; Kumar 1999; Lazzati et al. 1999; Guetta et al. 2001; Kino et al. 2004). However, the observations confirm high efficiencies up to a few tens of percentage (Zhang et al. 2007). Moreover, synchrotron emission cannot explain hard GRB spectra at low energies (Preece et al. 1998; Ghirlanda et al. 2003), and the spectra are directly related to the radiation mechanism involved rather than interaction with the baryons and leptons in the jet.

Owing to these shortcomings of the internal dissipation model, many researchers have recently focused on the photospheric emission model (Meszaros & Rees 2000; Rees & Meszaros 2005; Lazzati & Begelman 2010; Ito et al. 2014; Chhotray & Lazzati 2015; Santana et al. 2016). Unlike the internal dissipation model, the photospheric model can explain the observed high radiation efficiencies. The shape of the spectrum is determined by the interaction of photons with matter in the jet, which is through Compton scattering, and hence should be independent of the emission mechanism. There have been many successful attempts to explain the high-frequen nonthermal tails using subphotospheric dissipation (Giannios 2006; Pe’er et al. 2006; Lazzati & Begelman 2010; Vurm et al. 2011; Ito et al. 2014; Chhotray & Lazzati 2015; Santana et al. 2016); however, nonthermal tails at low energies still cannot be obtained (Pe’er & Ryde 2011; Chhotray & Lazzati 2015).

In this paper, we study the Comptonization of seed photons produced by the synchrotron emission of fast-cooling electrons below the photosphere (Granot et al. 2000; Ghisellini et al. 2010). The electrons and protons are accelerated to relativistic energies by a dissipation mechanism such as internal shocks (Lazzati & Begelman 2010; Toma et al. 2011; Lazzati et al. 2013) or magnetic reconnection (Thompson 1994; Giannios 2006, 2012) at an optical depth of a few or larger. The choice of the synchrotron spectrum over the thermal spectrum for photons is justified as there are not enough scatterings at relatively small optical depths $\tau \lesssim 10$ to thermalize the photon spectrum (Begue et al. 2013). Most of the energy in the jet is carried by the protons because of their large mass, and the average energy of the electrons is assumed to be much larger as compared to the average energy of the photons. We consider the subphotospheric heating of electrons, which occurs as a result of the continuous injection of energy from the protons through the Coulomb interaction and is more
physically motivated than the episodic injection of energy (Giannios 2006; Pe’er et al. 2006; Lazzati & Begelman 2010; Santana et al. 2016). Photons undergo multiple scatterings with electrons and gain energy until the outflow becomes optically thin and the photons escape the photosphere. Unlike many previous photospheric Monte Carlo (MC) simulations (Lazzati & Begelman 2010; Chhotray & Lazzati 2015), we also include the adiabatic cooling of electrons, protons, and photons due to the expansion of the relativistic jet (Santana et al. 2016).

Almost all photospheric MC simulations performed previously used a relatively small photon-to-electron ratio $N_g/N_e \sim 10^1$–$10^4$ (Lazzati & Begelman 2010; Chhotray & Lazzati 2015), which leads to unrealistically low radiation efficiencies contradicting GRB observations (Zhang et al. 2007; Santana et al. 2016). In this work, we use $N_g/N_e = 10^3$, which gives a radiative efficiency of $\eta \sim 10\%$ (consistent with observations) in addition to incorporating electron heating in a more realistic way to determine if the high-energy GRB prompt emission spectral index can be reproduced using the photospheric emission model. For all our simulations, we use Maxwell–Boltzmann (MB) electrons and mono-energetic protons as the respective seed distributions.

This paper is organized as follows. In Section 2, we describe the physics and implementation of our MC photospheric code. We present our simulation results in Section 3 and discuss the interpretation of these results in Section 4. Finally, we present our conclusions in Section 5.

2. Implementation of the Photospheric Code

In this section, we describe the implementation of our MC code and give an overview of the basic physics included. We discuss how the energy and velocity distributions of the electrons, protons, and photons are initialized and how they are affected by adiabatic cooling, Coulomb interaction, and scattering events. The scattering events between the electrons and photons are performed one at a time in our MC code.

Throughout this paper, primed quantities are in the jet-comoving frame while unprimed quantities are in the lab frame.

2.1. Input Parameters

Here we describe the input parameters used for our MC simulations.

1. Isotropic equivalent luminosity of the jet, $L$: we consider $L = 10^{52}$ erg s$^{-1}$ for all our simulations (Liang et al. 2007; Wanderman & Piran 2010).

2. Bulk Lorentz factor of the jet, $\Gamma$: for all our simulations, we consider $\Gamma = 300$ (Xue et al. 2009; Liang et al. 2010).

3. Number of electrons in a simulation, $N_e$: as in previous photospheric simulations (Lazzati & Begelman 2010; Chhotray & Lazzati 2015; Santana et al. 2016), we consider $N_e = 10^8$. In Figure 1, we show that it is enough to use $10^5$ electrons for accurately simulating the GRB jet.

4. Number of photons in a simulation, $N_g$: we consider $N_g = 10^5$ for all our simulations (Santana et al. 2016). This was done to ensure that $N_g/N_e = 10^3$.

5. Number of protons in a simulation, $N_p$: we consider $N_p = 10^5$ as $N_p = N_e$ due to the charge neutrality of the jet.

6. Number of photons collected for the output spectrum, $N_{\text{collect}}$: as in previous simulations (Lazzati & Begelman 2010; Santana et al. 2016), we consider $N_{\text{collect}} = N_g/3$ as it gives us a time-averaged representation of the GRB spectrum by allowing for enough photon-electron scatterings to accurately represent the output spectrum.

7. Initial optical depth, $\tau_0$: the initial optical depth determines the distance from the central engine where all the electrons, photons, and protons are injected. We consider $\tau_0 = 2, 4, 8, 16$ in this work.

8. Seed photon spectrum: we consider the synchrotron spectrum for fast-cooling electrons where the energy distribution is given by smoothly connected power laws
(Granot et al. 2000; Piran 2004):

\[ f_p = \begin{cases} \left( \frac{\nu}{\nu_{\text{sa}}} \right)^{11/8}, & \nu_1 < \nu < \nu_{\text{sa}} \\ \left( \frac{\nu}{\nu_{\text{sa}}} \right), & \nu_{\text{sa}} < \nu < \nu_{\text{ac}} \\ \left( \frac{\nu}{\nu_{\text{ac}}} \right)^{-1/2}, & \nu_{\text{ac}} < \nu < \nu_m \\ \left( \frac{\nu}{\nu_m} \right)^{-p/2}, & \nu_m < \nu < \nu_r, \end{cases} \]

where \( f_p \) is the flux per unit frequency in the lab frame. Throughout this paper we consider \( h\nu_{\text{e}} = 3 \times 10^{-9} \text{ eV}, \) \( h\nu_{\text{p}} = 2 \times 10^{-2} \text{ eV}, \) \( h\nu_{\text{sa}} = 2 \text{ eV}, \) \( h\nu_{\text{ac}} = 1 \text{ keV}, \) and \( h\nu_{\text{m}} = 30 \text{ keV}, \) which are justified by the choice of our parameters and the typical values of other parameters: \( \epsilon_R = 0.1, \epsilon_e = 0.1, \) \( N = 10^5, \) and \( T = 10 \text{ s} \) (Granot et al. 2000). \( f_p \) is peak normalized and the high-energy spectral index \( p = 2.5 \) (Kumar & Zhang 2015).

9. Electron distribution: we consider the MB distribution of electrons with the initial \( \gamma'_{\text{e, in}} \) as the input parameter. For our simulations, \( \gamma'_{\text{e, in}} = 25, 50, 75, \) and 100.

10. Proton distribution: for our simulations, we consider the mono-energetic distribution of protons with the initial \( \gamma'_{\text{p, in}} \) as the input parameter. We perform the simulations using \( \gamma'_{\text{p, in}} = 1.5, 2, 5, \) and 10.

2.2. Initialization of Electrons, Protons, and Photons

At the beginning of our photospheric MC code, we initialize the directions and energies of all the electrons, protons, and photons.

2.2.1. Direction and Energy of Electrons and Protons

The initial directions of the velocities of \( N_e \) electrons and \( N_p \) protons are chosen randomly in the comoving frame of the jet (see Appendix B1 of Santana et al. 2016). For the initial energies of the electrons, \( \gamma'_{\text{e}} \) is chosen from the relativistic MB distribution corresponding to the temperature \( T_{\text{e, in}}' \), which is given by (see Appendix B2.1 of Santana et al. 2016)

\[ k_B T_{\text{e, in}}' = (\gamma'_{\text{ad,e}} - 1)(\gamma'_{\text{e}} - 1) m_e c^2, \]

where \( \gamma'_{\text{ad,e}} \approx (4\gamma'_{\text{e}} + 1)/(3\gamma'_{\text{e, in}}') \) is the electron adiabatic index. For the mono-energetic protons, \( \gamma'_{\text{p}} = \gamma'_{\text{p, in}} \). We assume that initially all the \( N_e \) electrons and \( N_p \) protons are distributed uniformly in the comoving frame of the jet.

2.2.2. Direction, Energy, and Position of Photons

The initial directions of the velocities of \( N \) photons are chosen randomly in the comoving frame of the jet (see Appendix C1 of Santana et al. 2016). The initial energies of the photons in the comoving frame of the jet are chosen from the synchrotron radiation distribution of fast-cooling electrons as given in Equation (1) (see Appendix A for algorithm).

The positions of the \( N \) photons are assigned randomly and are uniformly distributed within a cone with a solid angle \( 1/\Gamma \) pointing toward the observer. The initial distance from the central engine (in the lab frame) where the photons are injected is given by

\[ R_{\text{in}} = \frac{L\sigma_T}{8\pi m_pc^3\beta^3\Gamma^3 n_{\text{in}}}. \]

where \( \beta = \sqrt{1 - (1/\Gamma^2)} \) and \( \sigma_T \) is the Thomson cross-section.

2.3. Adiabatic Cooling of Electrons, Protons, and Photons

Due to adiabatic cooling, the energies of the electrons, protons, and photons decrease as the jet expands outward, and the energy of the electrons (protons) decreases by a factor of \( R^{-2(\gamma'_{\text{ad,e}} - 1)} [R^{-2(\gamma'_{\text{ad,p}} - 1)}] \), where \( \gamma'_{\text{ad,e}} \approx (4\gamma'_{\text{e}} + 1)/(3\gamma'_{\text{e, in}}') \) is the adiabatic index of the electron (proton) and \( R \) is the radial distance the jet has traveled from the central engine. For the photons, the drop in energy is by a factor of \( R^{-2/3} \). These expressions are valid because the electron density \( n_{\text{e}} \) drops by a factor of \( R^2 \) as the relativistic outflow expands outward and the radial width of the jet remains unchanged. After each scattering event, the energies of the electrons, protons, and photons are modified due to adiabatic cooling as

\[ \frac{\gamma'_{\text{e,f}} - 1}{\gamma'_{\text{e,i}} - 1} = \left( \frac{R_{\text{in}} + (t_{\text{e}} + \Delta t_{\text{e}})\beta c}{R_{\text{in}} + t_{\text{e}}\beta c} \right)^{-2(\gamma'_{\text{ad,e}} - 1)} \]

\[ \frac{\gamma'_{\text{p,f}} - 1}{\gamma'_{\text{p,i}} - 1} = \left( \frac{R_{\text{in}} + (t_{\text{e}} + \Delta t_{\text{e}})\beta c}{R_{\text{in}} + t_{\text{e}}\beta c} \right)^{-2(\gamma'_{\text{ad,p}} - 1)} \]

\[ \frac{E'_{\text{e,f}}}{E'_{\text{e,i}}} = \left( \frac{R_{\text{in}} + (t_{\text{e}} + \Delta t_{\text{e}})\beta c}{R_{\text{in}} + t_{\text{e}}\beta c} \right)^{-2/3} \]

where \( R_{\text{in}} \) is given by Equation (3). The subscripts \( i \) and \( f \) are used to denote the energies before and after the photon has traveled a distance \( s' \) in the comoving frame of the jet. The total time elapsed in the lab frame for the photon and the electron (which undergo scattering) is given by \( t_{\text{e}} \) and \( t_{\text{e}} \), respectively. The time needed by the photon to travel a distance \( s' \) in the lab frame is given by \( \Delta t_{\text{e}} \), (see Appendix C3 of Santana et al. 2016 for the Lorentz transformation). The proton is considered to be moving with the electron and hence can be represented by the same time \( t_{\text{e}} \) as it is practically unaffected by the photon–electron scattering event. After the photon travels a distance \( s' \), the electron and the photon reach the same final radial position where they interact by inverse-Compton/Compton scattering.

2.4. Coulomb Interaction

In addition to adiabatic cooling, the energies of the electrons and the protons are also affected by the Coulomb interaction between them. As the protons have much larger energies as compared to the electrons, electrons are always heated due to the energy transfer from the protons. Moreover, the electrons exchange energy between themselves and attain MB distribution after reaching equilibrium. Below we discuss how the electron and proton energies are affected due to these interactions.
2.4.1. Electron–Proton (e–p) Interaction

The timescale for the Coulomb cooling of protons in the jet-comoving frame is (Schlickeiser 2002)

$$t'_{\text{p, Coul}} = \frac{(\gamma_p' - 1) m_p c^2}{5 \times 10^{-19} n'_p} \left(8.3 \times 10^{-15} T_e'^{5/2} + \beta_p'^3\right) / \beta_p'^2,$$

(7)

where $n'_p$ is the electron density in the jet-comoving frame, $T_e'$ is the temperature of the electrons in the jet-comoving frame, and $\beta_p'$ is the speed of the protons divided by the speed of light. The electron density $n'_e$ is given by

$$n'_e = \frac{L}{4\pi (R_{\text{in}} + t_\gamma / c)^2 m_p c^3 \Gamma^2}.$$  

(8)

The energies of the protons and electrons are modified due to a Coulomb interaction after each scattering event. The expressions used to update the $\gamma'_e$ of an electron and the $\gamma'_p$ of a proton due to a Coulomb interaction are

$$\gamma'_{e,i} = \gamma_{e,i} + \frac{5 \times 10^{-19} n'_e}{\Gamma m_e c^2} \frac{\beta_{p,i,\text{avg}}^2 (t_\gamma + \Delta t - t_e)}{(8.3 \times 10^{-15} T_{\text{e,avg}}'^{5/2} + \beta_{p,i,\text{avg}}^3)},$$

$$\gamma'_{p,j} = \gamma_{p,j} + \frac{5 \times 10^{-19} n'_e}{\Gamma m_p c^2} \frac{\beta_{p,i,\text{avg}}^2 (t_\gamma + \Delta t - t_e)}{(8.3 \times 10^{-15} T_{\text{e,avg}}'^{5/2} + \beta_{p,i,\text{avg}}^3)}.$$  

(9)

(10)

As before, the subscripts $i$ and $f$ are used to denote the energies before and after the photon travels by a distance $s'$ in the jet-comoving frame. The factor of $1/\Gamma$ is included to transform the time from the lab frame to the jet-comoving frame. As the electrons experience Coulomb heating due to the average proton distribution around them and vice versa, we include the averaged quantities $\beta'_{p,i,\text{avg}}$ and $T'_{e,i,\text{avg}}$ which denote the speed of protons averaged over $N_p$ protons in the jet-comoving frame divided by the speed of light and the temperature of electrons corresponding to $\gamma'_e$ averaged over $N_e$ electrons in the jet-comoving frame (see Equation (2)), respectively. Thus after each scattering event the electrons gain some energy from the protons that is determined by their respective energy distributions.

2.4.2. Electron–Electron (e–e) Interaction

In addition to interacting with the protons, the electrons also exchange energy between themselves. The energy distribution of the electrons at thermal equilibrium is given by an MB distribution with the peak temperature $T'_{\text{e, avg}}$ determined by $\gamma'_{\text{e,avg}}$ (see Equation (2)). As the nature of the interaction between the electrons is the same as that with the protons, the timescale for this interaction can be obtained by simply replacing the proton parameters with the electron parameters from Equation (7):

$$t'_{\text{e, Coul}} = \frac{(\gamma'_{\text{e,avg}} - 1) m_e c^2}{5 \times 10^{-19} n'_e} \left(8.3 \times 10^{-15} T_{\text{e,avg}}'^{5/2} + \beta'_{\text{e,avg}}^3\right) / \beta'_{\text{e,avg}}^2.$$  

(11)

where $\beta'_e$ is the speed of the electron in the jet-comoving frame divided by the speed of light and all the electron parameters are averaged over all the $N_e$ electrons. After each photon–electron scattering event, the average (over the $N_e$ electrons) total time elapsed in the lab frame is evaluated for the electrons, which is denoted by $t_{\text{e,avg}}$. Whenever $t_{\text{e,avg}}$ exceeds any multiple of $t_{\text{e, Coul}} = T'_{\text{e, Coul}}$, the electron distribution is reinitialized to an MB distribution with $T'_{\text{e, avg}}$ determined by $\gamma'_{\text{e, avg}}$ at that point of the simulation.

Note that the electron distribution between consecutive scattering events can deviate from the MB distribution for large values of $N_e / N_p$ (see Figure 7). In that case, the electron temperature $T'_{\text{e, avg}}$ evaluated from Equation (2) using $\gamma'_{\text{e, avg}}$ may not exactly correspond to that of the MB distribution with the same energy. However, Equations (7) and (11) can still be used to model the Coulomb interactions fairly well as long as (1) the quasi-MB distribution is unimodal with a peak energy close to that of the approximated MB distribution, and (2) the timescale at which the electrons are reinitialized to the MB distribution is comparable to the electron-photon scattering timescale. Both these conditions are satisfied for all our simulations, and the electron distribution need not be updated after every scattering event, which is computationally very expensive.

2.5. Main Photospheric Code

At the beginning of the simulation, the distance $s'$ that each photon travels in the comoving frame of the jet before scattering an electron is drawn randomly using the formula $s' = -\ln(n_{\text{scat}} / n_{\text{ph}}) - \ln(\alpha)$ (Santana et al. 2016). Here $n_{\text{scat}} = 1/(\Gamma n_e \sigma_T)$ is the mean free path of the photons in the jet-comoving frame and $\alpha$ is a uniformly distributed random number within 0 and 1. Once the $s'$ for all $N_p$ photons is drawn, the photons are propagated and their new positions are Lorentz-transformed to the lab frame (see Appendix C3 of Santana et al. 2016) and compared with the photospheric distance $R_{\text{ph}}$ (for the $R$ that corresponds to $\tau = 1$ in Equation (3)) to check if any photon escapes the photosphere without interacting with an electron. For the photons that escape the photosphere, the energies are Doppler-boosted to the lab frame and are stored. All other photons are placed in a priority queue $(t_{\gamma, i}, l)$, where $t_{\gamma, i}$ denotes the total time elapsed in the lab frame for the photon with the index $l$. The photon properties, such as position, direction, and energy, can be accessed using their respective photon index $l$. The priority queue structure allows for the propagation of all photons that are considered to be scattered first (Santana et al. 2016).

Next we propagate the first photon in the priority queue using the corresponding $s'$. One of the $N_p$ protons is chosen randomly while one of the $N_e$ electrons is selected by sampling the electron–photon scattering probability distribution function (see Appendix B for the algorithm) given by

$$P_{\text{scatt}}(\beta'_e, \theta'_e) = \frac{1}{4\pi \beta'_e^2} (1 - \beta'_e \cos \theta'_e),$$  

(12)

where $\theta'_e$ is the angle between the electron and photon directions before the scattering event in the jet-comoving frame and $\beta'_e$ is the speed of the electron in the jet-comoving frame divided by the speed of light.
Next, Equations (4)–(6) are used to update the energies after adiabatic cooling and Equations (9)–(10) are used to update the energies after Coulomb interaction. Once the energies are determined, the dimensionless photon energy in the electron rest frame \( z_i \) and the scattering cross-section \( \sigma(z_i') \) are calculated (see Appendix D of Santana et al. 2016). A uniformly distributed random number \( 0 \leq \alpha_i \leq 1 \) is drawn and is compared with the scattering probability \( \sigma(z_i')/\sigma_T \) to determine whether the electron–photon scattering event actually happens. If \( \alpha_i \leq \sigma(z_i')/\sigma_T \) is satisfied, then the scattering event takes place and the direction and energy of the photon are updated along with the direction and energy of the electron (see Appendices D and E of Santana et al. 2016).

Finally, regardless of whether the photon is scattered or not, a new \( s' \) is drawn at its current location and the photon is propagated as was done at the beginning of the simulation. The distance traveled by the photon is Lorentz-transformed to the lab frame and it is determined whether the current location of the photon \( R \) exceeds \( R_{ph} \) or not. If \( R > R_{ph} \), then the energy of the photon is Doppler-boosted to the lab frame and is stored. Otherwise the photon is again placed in the priority queue with the updated total elapsed time in the lab frame \( t_{e,l} \) and the whole process is repeated until \( N_e \) collect photons escape the photosphere.

### 2.6. Photospheric Code Tests

We first try to reproduce the equilibrium distributions for electrons and photons undergoing Compton scatterings to check the validity of our simulations. In the left panel of Figure 2, we present the results of simulations in which MB electrons are held fixed at an energy of \( \gamma_{e,in} = 1.001 \) while they scatter blackbody photons with \( k_B T_{e,in} = 1000 \, \text{eV} \) for \( N_e/N_\gamma = 10^2 \) and \( \Gamma = 300 \). The electrons and the photons are not cooled due to adiabatic expansion of the jet as the initial optical depth is varied, \( \tau_{in} = 100, 300, \) and 500. For photons interacting with electrons kept at a constant temperature bath, the equilibrium distribution at a large \( \tau_{in} \) approaches a Bose–Einstein distribution with a nonzero chemical potential while the electrons attain an MB distribution.

We find that the equilibrium distribution (within uncertainty) for photons \( f_{\nu} \propto \nu^3 \) at low energies and \( f_{\nu} \propto e^{-\nu} \) at high energies) and electrons \( f_{\nu} \propto \nu^2 \) at low energies and \( f_{\nu} \propto e^{-\nu} \) at high energies) is obtained close to \( \tau_{in} \sim 500 \). It should be noted that the spectral indices obtained from the power-law fits in the left panel of Figure 2 (and the rest of the figures in this paper) have statistical uncertainties due to the nonzero energy bin width, \( \Delta E_{bin} = 1 \, \text{eV} \), and the relatively small number of photons at low energies \( (N_\gamma/10^3) \) in our simulation. However, these uncertainties are typically very small for the parameters that we consider and can be ignored.

In the right panel of Figure 2, we compare our simulation results with those of Figure 1 of Chhotray & Lazzati (2015) for the same input parameters: Wien photons with \( T_{e,in} = 10^6 \, \text{K} \), \( N_e/N_\gamma = 10^3 \), and no adiabatic cooling for Maxwell–Juttner electrons with \( T_{e,in} = 6.5 \times 10^9 \, \text{K} \) and \( \tau_{in} = 5 \) and 75.

We then check whether \( N_e = 10^3 \), \( N_\gamma = 10^3 \), and \( N_\gamma = 10^5 \) are appropriate choices for representing the electron, proton, and photon distributions in the relativistic jet. MC photospheric simulations have been performed previously with \( N_e = 10^5 \) (Lazzati & Begelman 2010; Chhotray & Lazzati 2015), but smaller \( N_e/N_\gamma \sim 10^{-1} \text{--} 10^{-4} \) were considered for those simulations. Simulations have also been performed with thermal photons as the seed spectrum to show that \( N_e = 10^3 \) is enough to represent the electron distribution for \( N_e/N_\gamma = 10^3 \) (Santana et al. 2016), although e–p and e–e interactions were neglected in those simulations. We perform simulations with \( N_e = 10^8 \), \( N_\gamma = 10^3 \), and \( N_\gamma = 10^3 \) and compare them with \( N_e = 4 \times 10^8 \), \( N_e = 4 \times 10^8 \), and \( N_\gamma = 4 \times 10^3 \) in Figure 1.
For both panels, $t = 2$ in, $g' = 100$ in, and $g' = 1.5$ in are considered and the seed photon distribution is given by Equation (1). The left panel shows simulations performed without considering Coulomb interactions whereas the right panel shows simulations where both e–p and e–e Coulomb interactions were considered. The very good agreement between the simulation results suggests that $N_e / N_p = 10^5$ is enough for accurately representing relativistic jets for $N_e / N_p = 10^5$.

3. Simulation Results

In this section, we present the results of our photospheric MC simulations. In all the figures, the photon energy spectrum and the electron kinetic energy spectrum are in the lab frame (Doppler-boosted from the jet-comoving frame by multiplying with $\Gamma$) at the end of each simulation. Unless stated otherwise, e–p and e–e interactions are considered for the simulations.

In Figure 3, we present the simulation results for different combinations of $\gamma'_{e,\text{in}} (=25, 50, 75, 100)$ and $\gamma'_{p,\text{in}} (=1.5, 2, 5, 10)$ at $t = 4$ when e–p and e–e interactions are considered. In comparing the different panels, we see that $\gamma'_{p,\text{in}}$ does not have any effect on the output spectra. However, the electrons are more energetic at the end of the simulation for larger $\gamma'_{e,\text{in}}$:

$\gamma'_{e,\text{in}} = 50$ for $\gamma'_{e,\text{in}} = 25$, $\gamma'_{e,\text{in}} = 75$, and $\gamma'_{e,\text{in}} = 100$.

Unlike previous simulations for $N_e / N_p = 10^5$ (Santana et al. 2016), our output photon spectrum does not have a sharp drop in $f_\gamma$ after $E_{\gamma,\text{peak}}$. The photon spectra show a power law, $n_{\gamma} \propto E_{\gamma}^{-0.5}$, from $E_{\gamma,\text{peak}}$ up to $\sim 10^4$ keV. After $\sim 10^4$ keV, the $f_\gamma$ for photons drops sharply by $\sim 2$ orders of magnitude. This is because the average electron energy $\Gamma (\gamma'_{e,\text{avg}} - 1) m_e c^2$ is $\sim 10^4$ keV, beyond which enough photons cannot be upscattered by the electrons. The photon spectrum extends to higher energies for a larger $\gamma'_{e,\text{in}}$ ($\sim 4 \times 10^5$ keV for $\gamma'_{e,\text{in}} = 25$ to $\sim 4 \times 10^6$ keV for $\gamma'_{e,\text{in}} = 100$) as the highest energy to which a photon with energy $E_{\gamma,\text{peak}}$ can be upscattered after one scattering is $\sim E_{\gamma,\text{peak}} \Gamma_{E,\gamma}$. More energetic electrons (with a larger $\gamma'_{e,\text{in}}$) can transfer more energy to...
the photons, which results in a higher $f_\nu$ at large energies of $\sim 10^4 - 10^7$ keV.

In Figure 4, we present the simulation results for two different combinations of $\gamma'_e,\text{in}$ and $\gamma'_p,\text{in}$ for $\tau_m = 2, 4, 8, \text{and } 16$ when Coulomb (e–p and e–e) interaction is considered. As $\tau_m$ increases, the peak energy of the electron and the photon output spectrum shifts to lower energies, which is due to adiabatic cooling (see Equations (4) and (6)). The energy of the electrons at the end of the simulation drops from $\gamma'_e \sim 1.196$ ($\gamma'_e \sim 1.130$) for $\tau_m = 2$ to $\gamma'_e \sim 1.052$ ($\gamma'_e \sim 1.046$) for $\tau_m = 16$ when $\gamma'_p,\text{in} = 1.5$ and $\gamma'_e,\text{in} = 100$ ($\gamma'_p,\text{in} = 5$ and $\gamma'_e,\text{in} = 50$). While there are smaller number of photons at higher energies for a larger $\tau_m$, the photon spectrum becomes slightly shallower at energies below $E_{\gamma,\text{peak}}$. As in the previous case considered, the photon spectrum shows a power law of $f_\nu \propto \nu^{-0.5}$ right after $E_{\gamma,\text{peak}}$ and up to $\sim 10^3$ keV even though the $\tau_m$ changes considerably. The output photon spectrum becomes shallower below $E_{\gamma,\text{peak}}$ for a larger $\tau_m$: it changes from $f_{\nu} \propto \nu^{1.4}$ for $\tau_m = 2$ to $f_{\nu} \propto \nu^{1.2}$ for $\tau_m = 16$. Multiple scatterings become more probable with increasing $\tau_m$, which results in photons getting scattered a different number of times by the electrons before escaping out of the photosphere (Pozdnyakov et al. 1983). As a result, the output photon spectrum broadens and becomes shallower below $E_{\gamma,\text{peak}}$ (see below in Figure 8).

In Figure 5, we present the simulation results for two different combinations of $\gamma'_e,\text{in}$ and $\gamma'_p,\text{in}$ for $\tau_m = 4$ when Coulomb (e–p and e–e) interaction is considered and $E'_{\gamma,\text{peak}} = h\nu'_{\text{in}} = 0.2$ eV, 2 eV, and 20 eV; for $\tau_m = 4$; and with Coulomb interaction (e–p and e–e). Left panel: for $\gamma'_p,\text{in} = 1.5$ and $\gamma'_e,\text{in} = 100$. Right panel: for $\gamma'_p,\text{in} = 10$ and $\gamma'_e,\text{in} = 25$. The electron temperature at the end of the simulation is almost unaffected by the choice of $\gamma'_e,\text{in}$ in the photon seed spectrum.
and can thus cool down the electrons more slowly. This results in more protons being upscattered to larger energies as the average number of scatterings per photon is higher. As a result, the photon spectrum is shallower above the peak for a smaller $E_{\gamma,\text{peak}}$: $f_{\gamma} \propto \nu^{-0.5}$ for $E_{\gamma,\text{peak}} = 20 \text{ eV}$, $f_{\gamma} \propto \nu^{-0.5}$ for $E_{\gamma,\text{peak}} = 2 \text{ eV}$, and $f_{\gamma} \propto \nu^{-0.4}$ for $E_{\gamma,\text{peak}} = 0.2 \text{ eV}$. At photon energies smaller than $E_{\gamma,\text{peak}}$, $f_{\gamma} \propto \nu^{-2} - \nu^{-1.4}$, which becomes steeper for larger values of $E_{\gamma,\text{peak}}$. The photons have a lower peak energy at the end of the simulation as they cool down adiabatically.

In Figure 6, we present the evolution of $\gamma'_e$ ($\gamma'_p$) of three randomly selected electrons (protons) for $\tau_{\text{in}} = 8$, $\gamma'_{e,\text{in}} = 75$, and $\gamma'_{p,\text{in}} = 2$ when (1) both e–p and e–e interactions are not considered, (2) only e–p interaction is considered, and (3) both e–p and e–e interactions are considered. The spikes in $\gamma'_e$ correspond to the instances when the electron interacts with either a proton or a highly energetic photon resulting in a large transfer of energy to the electron. After each such instance, the energy of the electron falls back quickly to nonrelativistic energies when the electron upscatters a photon to transfer almost all the kinetic energy that was gained earlier. The electrons cool down very fast from $\gamma'_{e,\text{in}}$ to $\gamma'_e \sim 1$ as the IC timescale is much smaller than the dynamical timescale $t_{\text{dyn}} = R/(c\Gamma c)$ and the electron heating timescale $[\gamma'_e - 1)m_e/(\gamma'_p - 1)m_p]t_{\text{p,Coul}}$ for a large $\gamma'_p$ (see Equation (13) in Section 4). It can be seen that the three electrons experience a different number of scatterings, which is expected as the electrons that are moving toward the photons are more likely to be scattered by the photons than the electrons that are moving away from the photons (Equation (12)).

We compare the $\gamma'_e$ at the end of the simulation for the electron that experiences the largest number of scatterings for each of the three cases to find that e–p and e–e interactions do not have a very significant effect on $\gamma'_e$: $\gamma'_e = 1.048$ without e–p and e–e, $\gamma'_e = 1.062$ with e–p and without e–e, and $\gamma'_e = 1.059$ with e–p and e–e. The electrons are cooled down faster to a small $\gamma'_e$ ($\sim \gamma'_p$) without Coulomb interaction when e–e is included in addition to e–p as $t_{\text{p,Coul}} \sim \beta_\gamma/\Delta t_\gamma$ times smaller than $[\gamma'_e - 1)m_e/(\gamma'_p - 1)m_p]t_{\text{p,Coul}}$. The protons have $\gamma'_p \sim 2$ for $\sim 10^9$ scatterings, beyond which their energy drops significantly due to adiabatic cooling. The protons cool down to $\gamma'_p = 1.123$ for all three cases (regardless of whether Coulomb interaction is considered) as $t_{\text{p,Coul}} \sim t_{\text{dyn}}$ is much smaller than $t_{\text{p,Coul}}$ for a large $R$ toward the end of the simulation ($t_{\text{p,Coul}} \propto R$ whereas $t_{\text{p,Coul}} \propto R^2$).

In Figure 7, we present the simulation results for two different combinations of $\gamma'_{e,\text{in}}$ and $\gamma'_{p,\text{in}}$ for $\tau_{\text{in}} = 2$ when Coulomb (e–p and e–e) interaction is considered and $N_e/N_\gamma = 3 \times 10^9/3 \times 10^3$, $3 \times 10^9/3 \times 10^3$, and $3 \times 10^9/3 \times 10^3$. We find that the electrons are considerably hotter at the end of the simulation for a smaller $N_e/N_\gamma$: $\gamma'_e = 1.391$ (1.261) for $N_e/N_\gamma = 10^3$, $\gamma'_e = 1.196$ (1.144) for $N_e/N_\gamma = 10^4$, and $\gamma'_e = 1.130$ (1.091) for $N_e/N_\gamma = 10^5$ when $\gamma'_{e,\text{in}} = 100$ and $\gamma'_{p,\text{in}} = 1.5$ ($\gamma'_{e,\text{in}} = 50$ and $\gamma'_{p,\text{in}} = 5$). This is expected as the electrons cool down faster when there are more photons available to be upscattered. As a result, the photon spectrum becomes shallower above $E_{\gamma,\text{peak}}$ as more photons are upscattered by the slowly cooling electrons to higher energies for a smaller $N_e/N_\gamma$: $f_\gamma \propto \nu^{-0.5}$ for $N_e/N_\gamma = 10^3$, $f_\gamma \propto \nu^{-0.4}$ for $N_e/N_\gamma = 10^4$, and $f_\gamma \propto \nu^{-0.2}$ for $N_e/N_\gamma = 10^5$ from the peak energy up to $\sim 10^3 \text{ keV}$.

In Figure 8, we present the simulation results for the broadening of mono-energetic and blackbody seed photon spectra with $E_{\gamma,\text{peak}} = 20 \text{ eV}$, $\gamma'_{e,\text{in}} = 100$, and $\gamma'_{p,\text{in}} = 1.5$ when $\tau_{\text{in}} = 2$, 4, and 8. As expected, the electrons have a smaller energy at the end of the simulation for a larger $\tau_{\text{in}}$: $\gamma'_{e,\text{in}} = 1.130$ ($\gamma'_{e,\text{in}} = 1.117$) for $\tau_{\text{in}} = 2$, $\gamma'_{e,\text{in}} = 1.065$ ($\gamma'_{e,\text{in}} = 1.091$) for $\tau_{\text{in}} = 4$, and $\gamma'_{e,\text{in}} = 1.052$ ($\gamma'_{e,\text{in}} = 1.052$) for $\tau_{\text{in}} = 8$ when $E_{\gamma,\text{peak}}$ is broadened for a smaller $\tau_{\text{in}}$ and 8. As expected, the electrons have a smaller energy at the end of the simulation for a larger $\tau_{\text{in}}$: $\gamma'_{e,\text{in}} = 1.130$ ($\gamma'_{e,\text{in}} = 1.117$) for $\tau_{\text{in}} = 2$, $\gamma'_{e,\text{in}} = 1.065$ ($\gamma'_{e,\text{in}} = 1.091$) for $\tau_{\text{in}} = 4$, and $\gamma'_{e,\text{in}} = 1.052$ ($\gamma'_{e,\text{in}} = 1.052$) for $\tau_{\text{in}} = 8$ when $E_{\gamma,\text{peak}}$ is broadened for a smaller $\tau_{\text{in}}$ and 8. Unlike in previous simulations, the power law above $E_{\gamma,\text{peak}}$ only extends up to $E_{\gamma} \sim 10^2 \text{ keV}$.

To summarize, we studied the effect of $\gamma'_{e,\text{in}}$ $\gamma'_{p,\text{in}}$ $\tau_{\text{in}}$, $E_{\gamma,\text{peak}}$, $N_e/N_\gamma$, and Coulomb (e–p and e–e) interaction on the output spectrum of the photons and the electrons. In Figure 3, we show that $\gamma'_{e,\text{in}}$ and $\gamma'_{p,\text{in}}$ do not have any significant effect on $E_{\gamma,\text{peak}}$ and the power law above peak energy except that the high-energy tails in the photon spectrum extend to larger energies for a larger $\gamma'_{e,\text{in}}$. In Figure 4, we find that an increasing $\tau_{\text{in}}$ slightly flattens the photon spectrum at low energies although $f_\gamma$ drops faster at higher energies. In Figure 5, we find that the peak energy of the seed photon spectrum determines the peak energy and shape of the output photon spectrum. In Figure 6, we track $\gamma'_e$ to establish that e–p and e–e interactions do not affect the electron energies significantly, which is in good agreement with the previous simulations (Figures 3–5). In Figure 7, we find that although $E_{\gamma,\text{peak}}$ is unaffected by the decrease in $N_e/N_\gamma$, $f_\gamma$ increases significantly above $E_{\gamma,\text{peak}}$, resulting in a shallower photon spectrum. In Figure 8, we find that the output photon spectrum is broadened for a large $\tau_{\text{in}}$ regardless of the choice of the seed photon spectrum. This implies that the output photon spectrum for a high $\tau_{\text{in}}$ few tens–hundred will be in good agreement with the observed Band spectrum.

4. Discussion of Results

In this section, we first discuss the simulation parameters that significantly affect the output photon spectrum. Then we discuss the energy constraint that the electrons must satisfy in order to transfer enough energy to the photons so that a power law can be produced above $E_{\gamma,\text{peak}}$. Although this constraint is a necessary condition, it is not a sufficient condition to ensure a power-law spectrum for photons at high energies (Santana et al. 2016). Next we discuss the evolution of energies for the photons, protons, and electrons due to processes such as Comptonization, adiabatic cooling, and Coulomb interaction (e–p and e–e) during the expansion of the relativistic jet. We also evaluate the equilibrium $\gamma'_e$ when the electrons are cooled due to IC and are heated by the protons due to e–p interaction. Finally, we discuss the effect of $N_e/N_\gamma$ on our simulation results.
Figure 6. Evolution of $\gamma_e$ for three electrons and $\gamma_p$ for three protons, for photons with the seed spectrum given by Equation (1), and $\tau_m = 8$, $\gamma_{p,\text{in}} = 75$, and $\gamma_{p,\text{in}} = 2$. Top left and top right panels: without e–p and e–e interactions. Middle left and middle right panels: with e–p and without e–e interaction. Bottom left and bottom right panels: with e–p and e–e interactions.
4.1. Effect of Simulation Parameters on Output Spectrum

In our simulations, the parameters that mainly affect the output photon spectrum are $\gamma_{\text{e,in}}$, $\gamma_{\text{p,in}}$, $E_{\gamma,\text{peak}} = h\nu_{\text{sat}}$, $\tau_{\text{in}}$, and $N_p/N_e$. The e–p interaction slightly elevates the energy of the electrons but does not change the photon spectrum and the proton energies appreciably. The e–e interaction plays an important role in redistributing energy among the electrons after they are heated by e–p interactions.

$\gamma_{\text{e,in}}$ and $\gamma_{\text{p,in}}$ determine the amount of energy that the electrons can transfer to the photons through Comptonization and the amount of energy that the protons can transfer to the electrons through e–p interactions, respectively. A higher $\gamma_{\text{p,in}}$ can also result in more energetic photons as the electrons will gain more energy from the protons to transfer energy to the photons. However, for most of the simulations, $\langle (\gamma'_{\text{e}} - 1) m_e c / (\gamma'_{\text{p}} - 1) m_p \rangle_{\text{IC}}$ is of the same order as the IC timescale in the jet-comoving frame, which is given by

$$t'_{\text{IC}} = \frac{3}{4} \frac{(\gamma'_{\text{e}} - 1) m_e c}{U'_r \sigma_T \gamma'_{\text{e}}^2 \beta_P^2},$$

(13)

where $U'_r = L_{\gamma} / (4\pi R^2 \Gamma^2 c)$ is the radiation energy density. Hence the electrons attain an equilibrium $\gamma'_{\text{e}}$ after a certain number of scatterings, and Coulomb interaction is relatively unimportant in determining the shape of the output photon spectrum.

$E'_{\gamma,\text{peak}}$ is also an important parameter that affects the shape of the output photon spectrum. However, for almost all our simulations (except Figure 5), we fix the seed photon spectrum (as given by Equation (1)) to study the effect of other parameters and interactions better. As a photon can be upscattered to an energy of $\sim E'_{\gamma,\text{peak}} \Gamma'_{\text{e,in}}$ after one scattering, more energetic photons (with higher $E'_{\gamma,\text{peak}}$) cool the electrons faster after multiple scatterings. We do not consider electron–positron pair production for our simulations as $E'_{\gamma,\text{peak}} \sim 0.2–20$ eV is much less than the rest mass energy of the electrons in the jet-comoving frame (see Appendix C for more details).

The average number of scatterings experienced by a photon before escaping the photosphere is $\sim 2\tau_{\text{in}}$ (Begue et al. 2013) and hence $\tau_{\text{in}}$ also determines the shape of the output photon spectrum. For a larger $\tau_{\text{in}}$, the electrons and the protons cool down more adiabatically (see Equations (4) and (5)). The photons are scattered multiple times, thus increasing the probability of different photons being scattered a different number of times before escaping the photosphere. This results in the broadening of the output photon spectrum, which looks shallower below $E_{\gamma,\text{peak}}$. This broadening of the output photon spectrum at a large $\tau_{\text{in}}$ is independent of the choice of the seed photon spectrum (see Figure 8).

Another parameter that affects the photon and electron energies is $N_p/N_e$. For a smaller $N_p/N_e$, there are more electrons to upscatter the photons to higher energies. Moreover, $N_p = N_{\gamma}$ implies that there are more protons to transfer energy to the electrons. Hence the output photon spectrum has more photons at higher energies, resulting in a shallower spectrum above $E_{\gamma,\text{peak}}$.

Note that unlike previous simulations (Lazzati & Begelman 2010; Santana et al. 2016), we do not reaccelerate the electrons back to their initial distribution after every few scattering events. Rather the electrons are redistributed to an MB distribution whose peak temperature is determined using $\gamma'_{\text{e,avg}}$ after each scattering event. We do not consider any external dissipation events for electron heating except the energy transfer from the protons to the electrons.

4.2. Energy Constraint for Power Law above $E'_{\gamma,\text{peak}}$

Now we discuss the constraint that $\gamma'_{\text{e,in}}$ and $\gamma'_{\text{p,in}}$ need to satisfy in order to have a power-law spectrum above $E'_{\gamma,\text{peak}}$. The total initial kinetic energy of the electrons and the protons at the beginning of the simulation is $(\gamma'_{\text{e,in}} - 1) N_e m_e c^2$ and $(\gamma'_{\text{p,in}} - 1) N_p m_p c^2$, respectively. The energy available to the electrons should be at least as large as the energy gain that is required by the photons to populate the high-energy tail. The energy transferred from the protons to the electrons in the course of the jet expansion is $\sim (t'_{\text{dyn}} / t'_{\text{p,Cou}}) (\gamma'_{\text{p,in}} - 1) N_p m_p c^2$. In
order to have a power-law spectrum above $E'_{\gamma,\text{peak}}$, the energy of a fraction $\sim f$ of the photons near the peak-energy $E'_{\gamma,\text{peak}}$ has to increase by a factor of $\sim f$. Assuming that most of the photons have energies close to the photon peak-energy $E'_{\gamma,\text{peak}}$, the electron $\gamma_{e,\text{in}}'$ and the proton $\gamma_{p,\text{in}}'$ should satisfy the energy constraint given by

$$\left(\gamma_{e,\text{in}}' - 1\right) N_e m_e c^2 + \frac{t_{\text{dyn}}}{t_{\text{p,Coul}}} (\gamma_{p,\text{in}}' - 1) N_p m_p c^2 \gtrsim \frac{f N_e}{f} E'_{\gamma,\text{peak}}.$$  

(14)

For our simulations, $E'_{\gamma,\text{peak}} = 2 \text{ eV}$, $N_e = 10^8$, and $N_p = N_e = 10^3$. $t_{\text{dyn}}$ and $t_{\text{p,Coul}}$ are evaluated when most of the scatterings occur with $\gamma_{e,\text{in}}' \sim 1$ and $\gamma_{p,\text{in}}' \sim \gamma_{p,\text{in}}'$ (see Figure 6). For this choice of parameters, the above condition is satisfied for $\gamma_{e,\text{in}}' \sim 25$–100 and $\gamma_{p,\text{in}}' \sim 1.5$–10 that we have considered. This explains the power-law spectrum from $E_{\gamma,\text{peak}}$ up to $\sim 10^3 \text{ keV}$ in all our simulations.

### 4.3. Energy Evolution for Photons, Protons, and Electrons

We discuss the evolution of energy for the photons, protons, and electrons to explain our simulation results.

#### 4.3.1. Photons

The photons gain energy from the electrons through Comptonization and cool due to adiabatic expansion. The IC timescale is much smaller compared to $t_{\text{ad}} \sim t_{\text{dyn}}$ until the electrons cool down to nonrelativistic energies ($\gamma_{e}' \sim 1$). Although some photons are upscattered to high energies through Comptonization when the electrons are hot, the peak of the photon spectrum is affected only by adiabatic cooling and not IC cooling. This is expected as most of the scatterings occur after the electrons already cool down to nonrelativistic energies, making Comptonization unimportant for determining $E'_{\gamma,\text{peak}}$ in the output photon spectrum.

The peak energy of the output photon spectrum can be obtained using Equations (3) and (6),

$$\frac{E'_{\gamma,\text{peak},f}}{E'_{\gamma,\text{peak},i}} = \left(\frac{R_{\text{ph}}}{R_{\text{in}}}ight)^{-2/3} = \tau_{\text{in}}^{-2/3},$$

(15)

where $E'_{\gamma,\text{peak},i}$ ($E'_{\gamma,\text{peak},f}$) is the peak energy of the initial (final) photon spectrum in the jet-comoving frame. This is in good agreement with our simulation results for a different $\tau_{\text{in}}$ and $E'_{\gamma,\text{peak}}$ in Figures 4 and 5.

#### 4.3.2. Protons

The protons lose energy to the electrons through $e$–$p$ interaction in addition to cooling adiabatically as the jet expands. The proton cooling timescale $t_{\text{p,Coul}}$ is much larger compared to $t_{\text{ad}}$ when the electrons are relativistic (see Equation (7)). After the electrons cool down to nonrelativistic energies, $t_{\text{p,Coul}} \propto (\gamma_{p}' - 1)(\beta_p'/\gamma_{p}')^2 \propto R^2$, which increases faster compared to $t_{\text{ad}} \propto R$ with the expansion of the jet. Thus $e$–$p$ interaction is relatively unimportant in determining the final energy of the protons, which is actually determined by adiabatic cooling. From Figure 6, we can see that $\gamma_p' \sim 2$ for most part of the simulation. Thus we can write using Equation (5),

$$\frac{\gamma_{p,f}'}{\gamma_{p,i}'} - 1 \sim \left(\frac{R_{\text{ph}}}{R_{\text{in}}}ight)^{-1} = \tau_{\text{in}}^{-1},$$

(16)

as $\gamma_{p,\text{ad},p}' \sim 3/2$. For $\tau_{\text{in}} = 8$ and $\gamma_{p,\text{in}}' = 2$ as used in the simulations in Figure 6, $\gamma_{p,f}' = 1.125$, which is in very good agreement with the $\gamma_p'$ value that we find by tracking the protons in Figure 6.

#### 4.3.3. Electrons

While the electrons gain energy from the protons through $e$–$p$ interaction, they also lose energy due to adiabatic expansion of the jet and Comptonization. The Comptonization of electrons no longer decreases the energy of the electrons
significantly after $\gamma'_e$ drops to $\gamma'_{e,\text{Comp}} = 1 + 1/(8\tau_{in})$ (Santana et al. 2016). Here we estimate the change in the energy of the electrons due to adiabatic cooling, Comptonization, and e–p interaction, after the electrons have cooled down to $\gamma'_{e,\text{Comp}}$ to explain our simulation results in Figure 6.

The evolution of $\gamma'_e$ due to adiabatic cooling is given by Equation (4). After the electrons have already cooled down to $\gamma'_e \sim 1$, the energy change due to adiabatic cooling is

$$\frac{\gamma'_e - 1}{\gamma'_{e,\text{Comp}} - 1} \sim \frac{R_{ph}}{R_{in}} = \tau_{in}^{-4/3},$$

as $\gamma'_{ad,e} \sim 5/3$. For $\tau_{in} = 8$ and $\gamma'_{e,i} = \gamma'_{e,\text{Comp}} \sim 1.016$ as used in simulations in Figure 6, $\gamma'_{Comp, e} \sim 1.001$. To estimate the change in $\gamma'_e$ due to Comptonization and Coulomb heating by the protons, we first evaluate the corresponding timescales along with the dynamical timescale (all timescales are averaged over $R$) for $\tau_{in} = 8$. For $\gamma'_e = \gamma'_{e,\text{Comp}}$ and $\gamma'_p \sim 2$, the $R$-averaged timescales are

$$\langle t'_{\text{dyn}} \rangle = \frac{R}{\Gamma c} = 0.05 \text{ s}$$
$$\langle t'_{\text{IC}} \rangle = \frac{3}{R} \frac{m_e c}{8U'_p \sigma_T \gamma'_{e,\text{Comp}}} = 1.25 \times 10^{-4} \text{ s}$$
$$\langle t'_{\text{Coul}} \rangle = \frac{5 \times 10^{-19} n'_e \beta_p^2}{m_e c^2} = 2.82 \times 10^{-5} \text{ s.}$$

The final energy of the electrons after $t = \langle t'_{\text{dyn}} \rangle$ due to IC is given by

$$E'_{e,\text{IC}} = E'_e e^{-\langle t'_{\text{dyn}} \rangle / \langle t'_{\text{IC}} \rangle},$$

which reduces to

$$\gamma'_{e,\text{IC}} = 1 + (\gamma'_{e,\text{Comp}} - 1)e^{-400} \sim 1.$$  

The energy gain rate for the electrons gives

$$\frac{5 \times 10^{-19} n'_e \beta_p^2}{8.3 \times 10^{-15} [(\gamma'_e - 1)m_e c^2/k_{\text{B}}]^3/2 + \beta_p^3} = \frac{4}{3} U'_p \sigma_T (\gamma'_e - 1)c.$$  

Using the expressions for $n'_e$, $U'_p$, and $\gamma'_p = 1.123$, we obtain

$$39.07(\gamma'_e - 1)^{3/2} + 1 = \frac{0.273}{\gamma'_e^2 - 1},$$

which gives $\gamma'_e = 1.074$. Thus the equilibrium $\gamma'_e$ is close to $\gamma'_e = 1.062$ obtained in Figure 6 when e–p interaction is considered. The equilibrium $\gamma'_e$ is slightly higher than the $\gamma'_e$ obtained at the end of the simulations for Figure 6, which is expected as we neglect adiabatic cooling and e–e interaction for our equilibrium calculations.

In our analysis, we have assumed that the electrons always cool due to IC and neglect the possibility that an energetic photon can transfer energy back to the electrons. However, there are about $\sim 10$ instances in each of the three cases (without e–p and e–e, with e–p, and with e–p and e–e) when the electron energy increases to $\gamma'_e \sim 2$. As a result, more photons are upscattered to higher energies and the power law $f_p \propto \nu^{-0.5}$ extends to $\sim 10^3$ keV for almost all our simulations. In addition, the Compton-Y parameter for subrelativistic electrons (Rybicki & Lightman 1979) is

$$Y = 2\tau_{in} \frac{4kT'_p}{m_e c^2} \sim 8\tau_{in} \times (\gamma'_e - 1),$$

which is $\sim 4$ at the end of the simulation for $\gamma'_e \sim 1.062$ and $\tau_{in} = 8$ (see Figure 6)—large enough to upscatter most of the photons by a factor of two in energy and populate the high-energy tail of the photon spectrum. The considerably large value of the Compton-Y parameter accounts for the upscattering of photons near $E_{\text{peak}}$ to the high-energy power-law region of the photon spectrum.

4.5. Effect of $N_i/N_e$ on Simulation Results

In this subsection we discuss the simulation results in Figure 7 that were performed at $\tau_{in} = 2$ and different values of $N_i/N_e = 10^1, 10^4$ and $10^5$ for two different combinations of $\gamma'_{e,\text{in}}$ and $\gamma'_{p,\text{in}}$. The number of electrons $N_e = 3 \times 10^3$ is kept constant for the simulations and $N_i (= 3 \times 10^8, 3 \times 10^7$ and $3 \times 10^6)$ is varied. We find that the electrons are hotter and the photon spectrum is shallower for a smaller $N_i/N_e$ for both combinations of $\gamma'_{e,\text{in}}$ and $\gamma'_{p,\text{in}}$ considered in Figure 7. Rewriting Equation (14) using Equation (7) and the fact that $N_p = N_e$,

$$(\gamma'_{e,\text{in}} - 1)m_e c^2 + \frac{5 \times 10^{-19} n'_e}{\beta_p^3} R \Gamma c \geq \frac{N_i}{N_e} E_{\text{peak}}.$$  

Thus for a given $\gamma'_{e,\text{in}}$, $\gamma'_{p,\text{in}}$, and $E_{\text{peak}}$, the electrons cannot transfer enough energy to the photons to populate the higher-energy power-law tail for a larger $N_i/N_e$. As a result, the photon spectrum falls down faster at higher energies for larger values of $N_i/N_e$. Our simulations show that the photon spectrum is significantly affected by the choice of $N_i/N_e$, and
it is important to perform the simulations with realistic values of $\eta_c/\eta_a = 10^3$. For all three values of $\eta_c/\eta_a$ we have more photons just above the peak photon energy as compared to previous simulations (Lazzati & Begelman 2010; Santana et al. 2016), due to smaller $E'_{\gamma,\text{peak}}$ resulting in slower cooling of the electrons by the photons.

5. Conclusions

We studied photospheric emission for GRB prompt emission using an MC code with a photon-to-electron number ratio of $N_e/N_\gamma = 10^5$, which is close to the expected value for a typical GRB if the radiation efficiency is $\sim 10\%$. Our objective was to find out whether photospheric emission can explain the observed nonthermal low- ($f_\nu \propto \nu^0$) and high-energy ($f_\nu \propto \nu^{-1.2}$) spectrum of GRB prompt emission. For all our simulations, we considered the Comptonization of seed photons with the synchrotron spectrum in a fast-cooling regime. The electrons are continuously heated by the monoenergetic protons as the electrons interact with the protons (e-p) and other electrons (e-e) through Coulomb interaction. In all our simulations, we also considered the energy change for electrons, photons, and protons due to the adiabatic expansion of the jet. We found that the output photon spectrum exhibits a power law extending up to $\sim 10^3$ keV from $E'_{\gamma,\text{peak}}$ for the parametric space of the initial electron energy ($\gamma_{\text{E,ini}}$), initial proton energy ($\gamma_p$), and initial optical depth ($\tau_\text{in}$) that we considered in this work.

We found that the output photon spectrum becomes slightly shallower below $E'_{\gamma,\text{peak}}$ as the initial optical depth $\tau_\text{in}$ increases. This is expected as photons are scattered by electrons a different number of times before escaping the photosphere as $\tau_\text{in}$ increases. This can possibly result in an output photon spectrum, which is in good agreement with the observed low-energy spectrum $f_\nu \propto \nu^0$ of the prompt emission, especially at a large $\tau_\text{in} \sim$ few tens-hundred. The flattening of the output photon spectrum below $E'_{\gamma,\text{peak}}$ for a large $\tau_\text{in}$ is independent of the choice of seed photon spectrum. We found that the peak energy and shape of the output photon spectrum are also determined by the peak energy $E'_{\gamma,\text{peak}}$ of the seed photon spectrum. The peak energy in the output spectrum reduces by a factor of $\sim \gamma_{\text{E,ini}}^{-2/3}$ compared to the seed spectrum because of the adiabatic cooling of photons. As expected, the photon spectrum is broader around the peak energy for a smaller $E'_{\gamma,\text{peak}}$ because the photons are less energetic in the jet-comoving frame and can cool the electrons more slowly, resulting in more scatterings.

We tracked the electrons and the protons to study the effect of Coulomb (e-p and e-e) interaction on the electron and proton energies and the output photon spectrum. We found that the electron energies are slightly elevated in the presence of Coulomb interaction and that the protons cool down considerably by the end of the simulation due to adiabatic expansion of the jet for the optical depths that we considered. The presence of Coulomb interaction does not affect $E'_{\gamma,\text{peak}}$ and the shape of the output photon spectrum (both below and above $E'_{\gamma,\text{peak}}$) in general. We evaluated $\gamma'_\gamma$ at equilibrium due to IC and e-p interactions and found that the Compton-Y parameter $\sim 4$ at the end of the simulation—which is large enough to populate the high-energy power-law tail of the photon spectrum.

We also performed simulations for different $N_e/N_\gamma$ and found that the photon spectrum becomes shallower above $E'_{\gamma,\text{peak}}$ and does not exhibit a power-law tail at high energies for a smaller $N_e/N_\gamma$. This shows the importance of performing simulations with a realistic $N_e/N_\gamma$ and thus a radiation efficiency of $\eta$. We found that the Comptonization of seed photons with the synchrotron spectrum in a fast-cooling regime cannot explain the high-energy power-law dependence ($f_\nu \propto \nu^{-1.2}$) and the peak energy of the observed GRB prompt emission spectrum. However, $f_\nu \propto \nu^0$ for the photon spectrum below $E'_{\gamma,\text{peak}}$ can be successfully explained using a fast-cooling synchrotron seed photon spectrum at very large optical depths ($\tau_\text{in} \sim 100$).

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Appendix A

Initialization of Photon Energy

Here we describe the algorithm that we implemented to draw seed photons from the synchrotron spectrum for fast cooling electrons. The energy distribution is given by Equation (1) with break energies of $E_{\gamma,1} = h\nu_1$, $E_{\gamma,2} = h\nu_2$, $E_{\gamma,3} = h\nu_3$, $E_{\gamma,4} = h\nu_m$, and $E'_{\gamma,5} = h\nu_\gamma$. We denote the spectral indices between the break energies using $p_1 = -1.0$, $p_2 = 2.0/3.0$, $p_3 = 1.5$, and $p_4 = 4.5/2.0$, where the photon spectrum is given by $f_\nu \propto \nu^{p_4 - p_1}$.

We first evaluate,

$$C_1 = \frac{E_1^{-p_1} - E_2^{-p_1}}{p_1 - 1}$$

$$C_2 = \left(\frac{E_1^{-p_1} - E_3^{-p_1}}{p_2 - 1}\right)E_2^{p_2 - p_1}$$

$$C_3 = \left(\frac{E_3^{-p_1} - E_4^{-p_1}}{p_3 - 1}\right)E_2^{p_3 - p_1}E_3^{p_4 - p_2}$$

$$C_4 = \left(\frac{E_4^{-p_1} - E_5^{-p_1}}{p_4 - 1}\right)E_2^{p_4 - p_1}E_3^{p_5 - p_2}E_4^{p_6 - p_3}$$

to find $K_1 = C_1/(C_1 + C_2 + C_3 + C_4)$, $K_2 = (C_1 + C_2)/(C_1 + C_2 + C_3 + C_4)$, and $K_3 = (C_1 + C_2 + C_3)/(C_1 + C_2 + C_3 + C_4)$. Next, we draw two random numbers $\xi_1$ and $\xi_2$ between 0 and 1 to set

$$E'_{\gamma,1} = \begin{cases} \left[\xi_2(E_{\gamma,2}^{-p_1} - E_{\gamma,1}^{-p_1}) + E_{\gamma,1}^{-p_1}\right]^{-1/p_1}, & 0 < \xi_1 < K_1 \\ \left[\xi_2(E_{\gamma,3}^{-p_1} - E_{\gamma,2}^{-p_1}) + E_{\gamma,2}^{-p_1}\right]^{-1/p_1}, & K_1 < \xi_1 < K_2 \\ \left[\xi_2(E_{\gamma,4}^{-p_1} - E_{\gamma,3}^{-p_1}) + E_{\gamma,3}^{-p_1}\right]^{-1/p_1}, & K_2 < \xi_1 < K_3 \\ \left[\xi_2(E_{\gamma,5}^{-p_1} - E_{\gamma,4}^{-p_1}) + E_{\gamma,4}^{-p_1}\right]^{-1/p_1}, & K_3 < \xi_1 < 1 \end{cases}$$

Appendix B

Selection of Electron–photon Scattering

In this appendix, we describe the algorithm to select an electron for scattering with a photon using the scattering probability $P_{\text{scatter}}$. We denote the angle between the propagation directions of a particular electron among the $N_e$ electrons and the photon (already selected from the priority queue, see Section 2.5) in the jet-comoving frame before scattering...
by $\theta'_e$. The differential number of scatterings experienced by the photon in time $dt'$ in jet-comoving frame is then given by

$$dN_{\text{scatt}}' = dN_{\text{scatt}}' \sigma_{\gamma e}(1 - \beta'_e \cos \theta'_e) dt',$$

where $dN_{\text{scatt}}'$ is the differential number density corresponding to the electron number density in the jet-comoving frame. $f(\beta'_e, \gamma_e)$ corresponds to the energy distribution of the electrons, which is MB, and $d^3\beta'_e d\gamma_e / (\gamma_e^2 \sin \theta'_e d\theta'_e d\phi'_e)$ is the differential element in the velocity space of the electrons. The probability of scattering between an electron and the photon is

$$P_{\text{scatt}}(\beta'_e, \theta'_e) \propto \frac{d\nu_{\text{scatt}}'}{d\nu_e d\beta'_e d\gamma_e} = f(\beta'_e, \theta'_e) \sigma_{\gamma e}(1 - \beta'_e \cos \theta'_e),$$

where $d\nu_{\text{scatt}}'$ is the differential frequency of electron-photon scattering. $P_{\text{scatt}}$ is independent of $\phi'_e$ because of the azimuthal symmetry of the scattering event in the jet-comoving frame. Assuming that the electron distribution remains isotropic between scattering events,

$$P_{\text{scatt}}(\beta'_e, \theta'_e) \propto e^{-c\beta'_e}(1 - \beta'_e \cos \theta'_e),$$

where $c$ is a constant determined by the temperature of the electrons. The normalized probability can then be written as

$$P_{\text{scatt}}(\beta'_e, \theta'_e) = \frac{1}{4\pi \beta'_e^2}(1 - \beta'_e \cos \theta'_e).$$

The cumulative distribution function corresponding to the above probability distribution is

$$F_{\text{scatt}}(\beta'_e, \theta'_e) = \frac{1}{2} \beta'_e \left[ 1 - \cos \theta'_e \right] + \frac{1}{4} \beta'_e \left( \cos^2 \theta'_e - 1 \right),$$

which is zero for $\theta'_e = 0$ and $\beta'_e$ for $\theta'_e = \pi$. Next we draw a random number $\xi_3$ between 0 and $N_e - 1$ and evaluate $[\xi_3 - N_e F_{\text{scatt}}(\theta'_e)]$ for all $N_e$ electrons. The electron selected for scattering with the photon is the one with a minimum value of $[\xi_3 - N_e F_{\text{scatt}}(\theta'_e)]$.

### Appendix C

**Pair Production and Annihilation**

The fraction of photons with sufficient energy, $E_{\gamma} \sim m_e c^2 \Gamma \sim 1.5 \times 10^5$ keV, needed to create pairs in the jet is $f_\gamma \sim 10^{-4}$ for $\tau_m = 4$ (see Figure 3). Let $\eta$ be the fraction of photons that are close to the peak photon energy, $E_{\gamma, \text{peak}} = \Gamma h \nu_\gamma$, and within an energy range of $E_{\gamma, 1} = \Gamma h (0.75 \nu_\gamma)$ to $E_{\gamma, 2} = \Gamma h (1.25 \nu_\gamma)$. Then the number of photons with sufficient energy to produce pairs is $N_{\gamma, \text{MeV}} \sim 10^{-4} \eta N_{\gamma, \text{tot}}$, where $N_{\gamma, \text{tot}} = 10^8$ is the total number of photons in the jet.

The optical depth for pair production is

$$\tau_{\gamma \gamma, \text{MeV}} \sim \left( N_{\gamma, \text{MeV}} \sigma_{\gamma \gamma, \text{avg}} \right) / (4\pi R^2),$$

where

$$\sigma_{\gamma \gamma, \text{avg}} = \int_{\nu_{\gamma, \text{min}}}^{\nu_{\gamma, \text{max}}} \sigma_{\gamma \gamma}(y)(f_{\gamma} / y) dy \int_{\nu_{\gamma, \text{min}}}^{\nu_{\gamma, \text{max}}} (f_{\gamma} / y) dy$$

is the average pair production cross-section with $y^2 = \nu_{\gamma, \text{min}} \nu_{\gamma, \text{max}} (1 - \cos \theta)$. Here $\nu'_E$ denotes the energy of the incoming photons and $\theta$ is the angle between them. Assuming an isotropic photon distribution, i.e., $f(\cos \theta = 0)$, we have $\nu_{\gamma, \text{min}} \sim 1, \nu_{\gamma, \text{max}} \sim 0.7 (E_{\gamma, \text{max}} / \Gamma m_e c^2) \sim 50$, and $f_{\gamma} \propto y^{-1.25}$ as the maximum possible photon energy $E_{\gamma, \text{max}} \sim 10^3$ keV from our simulation results. The average pair production cross-section is then (Pozdnyakov et al. 1983)

$$\sigma_{\gamma \gamma, \text{avg}} \sim 0.16 \sigma_{\gamma \gamma}. \quad (28)$$

Substituting $N_{\gamma, \text{MeV}}$ and $\sigma_{\gamma \gamma, \text{avg}}$,

$$\tau_{\gamma \gamma, \text{MeV}} \sim 10^{-4} \left( N_{\gamma, \text{tot}} \sigma_{\gamma \gamma, \text{avg}} N_{\gamma, \text{tot}} \right) / (4\pi R^2) \sim 10 \eta \times 10^8 \tau_m \sim 1.6 \eta \tau_m \eta,$$  

where we used $N_{\gamma, \text{tot}} / N_{\gamma, \text{tot}} = 10^8$ and $\tau_m \sim (N_{\gamma, \text{tot}} / \sigma_{\gamma \gamma}) / (4\pi R^2)$. Therefore, $\tau_{\gamma \gamma, \text{MeV}} \lesssim 1$ is satisfied as long as $\eta \lesssim 1/6$ (for $\tau_m \sim 4$).

For the synchrotron seed spectrum of fast-cooled electrons, $f_{\gamma} \propto 1.25 \nu_{\gamma}^{-3/2} / (\nu_{\gamma, \text{min}} \nu_{\gamma, \text{max}}) \approx 1/6$, which gives $\tau_{\gamma \gamma, \text{MeV}} \sim 1$ (from Equation 29). Note that here we make a conservative (although arbitrary) choice for the energy bin width $\Delta E_{\gamma} = \Gamma h (0.5 \nu_{\gamma})$ as $\eta$ strictly corresponds to photons with energies very close to $\Gamma h \nu_{\gamma}$.

The optical depth for pair annihilation is $\tau_{\gamma e^\pm} \sim \left( N_{\gamma e^\pm} \sigma_{\gamma e^\pm} \right) / (4\pi R^2) \sim (3/8) (N_{\gamma e^\pm} / N_{\gamma, \text{tot}}) (\tau / \beta_e^2)$, where $N_{\gamma e^\pm}$ is the total number of pairs in the jet and $\sigma_{\gamma e^\pm} \sim (3/8) (\sigma_{\gamma \gamma} / \beta_e^2)$ is the pair annihilation cross-section. Equating the pair production and annihilation rates at equilibrium

$$\frac{\sigma_{\gamma e^\pm} N_{\gamma e^\pm} \beta_e^2}{4\pi R^2} = \frac{\sigma_{\gamma \gamma, \text{avg}} N_{\gamma, \text{MeV}}}{4\pi R^2}$$

gives

$$N_{\gamma e^\pm} \sim 4.2 N_{\gamma, \text{tot}} \eta \sim 0.7 N_{\gamma, \text{tot}}.$$  

for $\eta \sim 1/6$. Hence the number of pairs $N_{\gamma e^\pm}$ in the jet is always less than the total number of electrons $N_{\gamma, \text{tot}}$. 


We have not explored $\tau_{\text{in}} \lesssim 4$ while evaluating $N_{\text{e}^{-}\text{e}^{+}}$ as the photons at such low optical depths do not experience enough scatterings for Comptonization to modify the seed photon spectrum appreciably. Although $f_{\text{e}}$ can be larger by a factor of $\gtrsim 5$ for $1 \lesssim \tau_{\text{in}} \lesssim 2$, the number density of pairs and thus the pair annihilation optical depth, $\tau_{\text{e}^{-}\text{e}^{+}} \sim \sigma_{\text{e}^{-}\text{e}^{+}} N_{\text{e}^{-}\text{e}^{+}}$, is also larger by the same factor. This increases the probability of the additional pairs being annihilated very quickly, and the number of pairs is comparable to that obtained in Equation (32). For larger values of $\tau_{\text{in}} \gtrsim 4$, $f_{\text{e}} \lesssim 10^{-5}$, which means that the number of pairs in the jet is even smaller. Thus the effect of pairs can be ignored for the present work.

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**References**

Bland, D., Matteson, J., Ford, L., et al. 1993, ApJ, 413, 281

Begue, D., Siutsou, I. A., & Vereshchagin, G. V. 2013, ApJ, 767, 139

Chhotray, A., & Lazzati, D. 2015, ApJ, 802, 132

Ghirlanda, G., Celotti, A., & Ghisellini, G. 2003, A&A, 406, 879

Ghisellini, G., Ghirlanda, G., Nava, L., et al. 2010, MNRAS, 403, 926

Giannios, D. 2006, A&A, 457, 763

Giannios, D. 2012, MNRAS, 422, 3092

Granot, J., Piran, T., & Sari, R. 2000, ApJL, 534, L163

Guetta, D., Spada, M., & Waxman, E. 2001, ApJ, 557, 399

Ito, H., Nagataki, S., Matsumoto, J., et al. 2014, ApJ, 789, 159

Keane, Y., Preece, R. D., Briggs, M. S., et al. 2006, ApJS, 166, 298

Kino, M., Mizuta, A., & Yamada, S. 2004, ApJL, 611, 1021

Kobayashi, S., Piran, T., & Sari, R. 1997, ApJ, 490, 92

Kumar, P. 1999, ApJL, 523, L113

Kumar, P., & Zhang, B. 2015, PhR, 561, 1

Lazzati, D., & Begelman, M. C. 2010, ApJ, 725, 1137

Lazzati, D., Ghisellini, G., & Celotti, A. 1999, MNRAS, 309, L13

Lazzati, D., Morsony, B. J., Margutti, R., et al. 2013, ApJ, 765, 103

Liang, E., Zhang, B., Virgili, F., et al. 2007, ApJ, 662, 1111

Liang, E.-W., Yi, S.-X., Zhang, J., et al. 2010, ApJ, 725, 2209

Lloyd, N. M., & Petrovian, V. 2000, ApJ, 543, 722

Meszaros, P., & Rees, M. J. 2000, ApJ, 530, 292

Meszaros, P., Rees, M. J., & Papanastasiou, H. 1994, ApJ, 432, 181

Pe'er, A., Meszaros, P., & Rees, M. J. 2006, ApJ, 642, 995

Pe'er, A., & Ryde, F. 2011, ApJ, 732, 49

Piran, T. 1999, PhR, 314, 575

Piran, T. 2004, RevMP, 76, 1143

Pozdnyakov, L. A., Sobol, I. M., & Syunyaev, R. A. 1983, ASPRv, 2, 189

Preece, R. D., Briggs, M. S., Mallozi, R. S., et al. 1998, ApJL, 506, L23

Preece, R. D., Briggs, M. S., Mallozi, R. S., et al. 2000, ApJS, 126, 19

Rees, M. J., & Meszaros, P. 1994, ApJL, 430, L93

Rees, M. J., & Meszaros, P. 2005, ApJ, 628, 847

Rybicki, G. B., & Lightman, A. P. 1979, Radiative Processes in Astrophysics (New York: Wiley)

Santana, R., Crumley, P., Hernandez, R. A., et al. 2016, MNRAS, 456, 1049

Schlickeiser, R. 2002, Cosmic Ray Astrophysics (New York: Springer)

Thompson, C. 1994, MNRAS, 270, 480

Toma, K., Wu, X.-F., & Meszaros, P. 2011, MNRAS, 415, 1663

Vurm, I., Beloborodov, A. M., & Poutanen, J. 2011, ApJ, 738, 77

Wanderman, D., & Piran, T. 2010, MNRAS, 406, 1944

Xue, R.-R., Fan, Y.-Z., & Wei, D.-M. 2009, A&A, 498, 671

Zhang, B., & Yan, H. 2011, ApJ, 726, 90

Zhang, B., Liang, E., Page, K. L., et al. 2007, ApJ, 655, 989