BFKL Pomeron: modeling confinement

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E.L. and S. Tapia: “BFKL Pomeron: modeling confinement”
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Large $b$ dependence of the BFKL Pomeron

\begin{itemize}
  \item $N(r_1, r_2; Y, b) = \int \frac{d\gamma}{2\pi i} \phi^{(0)}_{in}(\nu) e^{\omega(\gamma = \frac{1}{2} + i\nu, 0)} Y$
  \item $\times \left\{ b_\nu (ww^*)^{\frac{1}{2}} + i\nu + b_{-\nu} (ww^*)^{\frac{1}{2}} - i\nu \right\} \rightarrow \frac{r_1 r_2}{b^2} e^{\omega_0 Y}$
  \item $ww^* = \frac{r_1^2 r_2^2}{\left( \vec{b} - \frac{1}{2} (\vec{r}_1 - \vec{r}_2) \right)^2 \left( \vec{b} + \frac{1}{2} (\vec{r}_1 - \vec{r}_2) \right)^2}$
\end{itemize}

$N(r_1, r_2; Y, b) \leq 1$ for $b^2 \leq r_1 r_2 e^{\omega_0 Y}$

Violation of Froissart theorem: (Kovner & Wiedemann)

$\int d^2b \ N(r_1, r_2; Y, b) \propto s^{\omega_0} \gg Y^2 = \ln^2 s$

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Lessons from numerical solutions and theory considerations:

(Kovner & Wiedemann, McLerran and Iancu, Golec-Biernat & Stasto, Gotsman et al, Berger & Stasto, 2011)

- The confinement of quarks and gluon have to be included in the BFKL kernel (to include in the initial conditions is not enough);

- Suppressing large sizes of the produced dipoles in the decay one dipole $\rightarrow$ two dipoles we reproduce correct $b$-dependence;

- Since at large $b$ the amplitude is small we do not need to take into account the non-linear corrections;
Corrections from confinement have to be included in the kernel of the BFKL equation

\[ \frac{\partial N(x_{10}, b, Y)}{\partial Y} = \bar{\alpha}_S \int d^2 x_{12} K(x_{12}, x_{20}|x_{10}) \times \left\{ 2 N(x_{12}, \vec{b} - \frac{1}{2} \vec{x}_{02}; Y) - N(x_{10}, b; Y) \right\} \]

Modified BFKL kernel:

\[ K(x_{12}, x_{20}|x_{10}) = \frac{x_{10}^2}{x_{12}^2 x_{02}^2} e^{-B(x_{12}^2 + x_{02}^2)} \]
Results

1. \( N(x_{10},b,Y) \xrightarrow{B b^2 \gg 1} e^{-4 B b^2} \leftrightarrow \text{expected} \)

2. \( N(x_{10},b,Y) \xrightarrow{Y \gg 1} e^{\omega_0 Y} \text{ with } \omega_0 = \omega_{\text{BFKL}} \leftrightarrow \text{!!!} \)

3. \( \langle |b^2| \rangle = \text{Constant (Y)} \leftrightarrow \text{expected} \)

4. Saturation scale \( Q_s^2 \propto e^{\lambda Y} \quad \lambda = \lambda_{\text{BFKL}} \leftrightarrow \text{expected} \)

5. The modified BFKL Pomeron looks similar to the Pomeron in N=4 SYM and high energy phenomenology: \( \Delta_P \sim 0.3; \alpha'_P = 0 \leftrightarrow \text{!!!} \)
Gribov’s diffusion:  \[ \Delta b \, p_T \sim 1 \]

\[ \langle b^2 \rangle = \Delta b^2 n \]

\[ \Delta b^2 = 1/\langle p_T^2 \rangle \]
Numerical calculations

\[
\frac{\partial \bar{N}(x_01; Y)}{\partial Y} = \frac{\partial}{\partial Y} \int d^2b \, b^2 \, N(x_01; Y) \quad \left( \langle |b^2| \rangle = \bar{N}(x_01; Y) / N(x_{12}; Y) \right)
\]

\[
= \bar{\alpha}_S \int d^2b' \, d^2x_{12} \left( \vec{b}' + \frac{1}{2} \vec{x}_{12} \right) \frac{1}{x_{12}^2} \left\{ 2 \bar{N}(x_{12}, \vec{b}'; Y) - \frac{x_{01}^2}{x_{12}^2} \bar{N}(x_{01}, b; Y) \right\}
\]

\[
= \bar{\alpha}_S \int d^2x_{12} \frac{1}{x_{12}^2} \left\{ 2 \bar{N}^{BFKL}(x_{12}; Y) - \frac{x_{01}}{x_{12}^2} \bar{N}(x_{01}; Y) \right\} + \frac{1}{2} \bar{\alpha}_S \int d^2x_{12} N(x_{12}; Y)
\]

\[
+ \left\{ \frac{1}{2} \bar{\alpha}_S \int d^2b' \, d^2x_{12} \, \vec{b}' \cdot \vec{x}_{12} \frac{1}{x_{12}^2} 2 \bar{N}(x_{12}, \vec{b}' \equiv \vec{b} - \frac{1}{2} \vec{x}_{12}; Y) = 0 \right\}
\]
Pomeron intercept $\omega_0$

- Remnant of conformal symmetry: $x_{ik} \rightarrow \bar{x}_{ik} = \sqrt{B} x_{ik}$

$$
\int d^2 x_{12} \frac{x_{10}^2}{x_{12}^2 x_{02}^2} e^{-B(x_{12}^2 + x_{02}^2)} \rightarrow \int d^2 \bar{x}_{12} \frac{\bar{x}_{10}^2}{\bar{x}_{12}^2 \bar{x}_{02}^2} e^{-(\bar{x}_{12}^2 + \bar{x}_{02}^2)}
$$

- $N(x_{01}; Y) = \int \frac{d\omega}{2\pi i} e^{\omega Y} N_\omega(x_{01})$

- $\omega N_\omega(x_{01}) = -\bar{\alpha}_S \mathcal{H} N_\omega(x_{01})$ or $E N_\omega(x_{01}) = \mathcal{H} N_\omega(x_{01})$

- $N_\omega(x_{01}) \xrightarrow{x_{01} \ll 1/B} \nu^{\text{BFKL}}(x_{01}) = \left( \frac{1}{x_{01}^2} \right)^{1/2 + i\nu}$; $N_\omega(x_{01}) \xrightarrow{x_{01} \gg 1/B} \text{Const}$

- $E(\nu) = 2\psi(1) - \psi(-\frac{1}{2} + i\nu) - \psi(\frac{1}{2} - i\nu)$

- $\int d^2 x_{01} N_\omega^*(x_{01}) N_{\omega'}(x_{01}) < \infty \quad \int d^2 x_{01} N_{\nu}^{\text{BFKL}*}(x_{01}) N_{\nu'}^{\text{BFKL}}(x_{01}) = \delta(\nu - \nu')$
Variational method

- \( E_{\text{ground}} \equiv -\omega_0 \leq F[\{N\}] = \frac{\langle N^*(x_{01}) | \mathcal{H} | N(x_{01}) \rangle}{\langle N^*(x_{01}) | N(x_{01}) \rangle} \)

- Our choice: \( \{N\} = \{N^{BFKL}\} \)

- \( \mathcal{H} N^{BFKL}_{\gamma=-\frac{1}{2} + i\nu}(x_{10}) = \chi(\gamma, x_{10}) N^{BFKL}_{\gamma=-\frac{1}{2} + i\nu}(x_{10}) \)

- \( \chi(\gamma; \tilde{x}_{12}) = \int_{0}^{1} dt \frac{t^{\gamma-1}}{1-t} + \int_{1}^{1/\tilde{x}_{12}^2} dt \frac{t^{\gamma-1}}{t-1} - \int_{0}^{1/\tilde{x}_{12}^2} \frac{1}{t} \left[ \frac{1}{|t-1|} - \frac{1}{\sqrt{4t^2 + 1}} \right] \)

- \( \chi(\gamma; \tilde{x}_{12}) = \chi^{BFKL}(\gamma) + \arccsch\left(\frac{2}{\tilde{x}_{12}^2}\right) - B(\tilde{x}_{12}; 1 - \gamma, 0) - \ln\left(1 - \tilde{x}_{01}^2\right) \)
$\omega(\gamma) \geq \omega_{\text{BFKL}}$
Semi-classical approach

\[ N(Y; l) = e^{S(Y, l)} = e^{\omega(Y, l)Y + (\gamma(Y, l) - 1)l} \]

where \[ \omega(Y, l) = \frac{\partial S(Y; l)}{\partial Y} ; \quad \gamma(Y, l) - 1 = \frac{\partial S|_Y(Y; l)}{\partial l} \]

with smooth functions \( \omega(Y, l) \) and \( \gamma(Y, l) \) and \( Y = \bar{\alpha}_S Y \).

Equation:

\[ \omega(Y, l) - \chi(\gamma, x_{12}^2) = 0 \]

\[ F(Y, l, S, \gamma, \omega) = 0 \]

(1.) \( \frac{dl}{dt} = F_\gamma = -\frac{d\chi(\gamma, 0, l)}{d\gamma} \)

(2.) \( \frac{dY(t)}{dt} = F_\omega = 1 \)

(3.) \( \frac{dS}{dt} = \gamma F_\gamma + \omega F_\omega = -(\gamma - 1) \frac{\partial \chi(\gamma, 0, l)}{\partial \gamma} + \omega \)

(4.) \( \frac{d\gamma}{dt} = -(F_l + \gamma F_S) = \frac{\partial \chi(\gamma(t), 0, l(t))}{\partial l} \)

\[ \frac{d\gamma}{dl} = -\frac{\partial \chi(\gamma(t), l(t))}{\partial l} \]

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\[ \omega \leq \omega_{\text{BFKL}} \]
Diffusion approximation

\[
\tilde{N}(\nu, l) = e^{\frac{1}{2}l} N(\nu, l) = \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\omega}{2\pi i} e^{\omega\nu} \tilde{n}(\omega, l)
\]

\[
= \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\omega}{2\pi i} \int_{i\epsilon-i\infty}^{i\epsilon+i\infty} \frac{d\nu}{2\pi i} \tilde{n}(\omega, \nu) e^{\omega\nu + ivl}
\]

\[
\tilde{n}(\omega, l') = \tilde{n}(\omega, l) + \frac{\partial \tilde{n}(\omega, l')}{\partial l'} |_{l'=l} (l' - l) + \frac{1}{2} \frac{\partial^2 \tilde{n}(\omega, l')}{\partial l'^2} |_{l'=l} (l' - l)^2 + \ldots
\]

\[
\Delta(l) = \chi \left( \frac{1}{2}, e^{\frac{1}{2}l} \right); \quad d1(l) = -i \frac{\partial \chi \left( \frac{1}{2} + iv, e^{\frac{1}{2}l} \right)}{\partial \nu} |_{\nu=0};
\]

\[
d2(l) = - \frac{\partial^2 \chi \left( \frac{1}{2} + iv, e^{\frac{1}{2}l} \right)}{\partial \nu} |_{\nu=0}
\]

\[
(\omega - \Delta(l)) \tilde{n}(\omega, l) - d1(l) \frac{\partial \tilde{n}(\omega, l)}{\partial l} - \frac{1}{2} d2(l) \frac{\partial^2 \tilde{n}(\omega, l)}{\partial l^2} = 0
\]
\[ \bar{n}(\omega, l) = \exp(\phi(\omega, l)); \quad \text{and} \quad \gamma = \frac{\partial \phi(\omega, l)}{\partial l} \]

\[ (\omega - \Delta(l)) = d1(l) \gamma(\omega, l) + \frac{1}{2} d2(l) \left( \frac{\partial \gamma(\omega, l)}{\partial l} + \gamma^2(\omega, l) \right) \]

\[ \gamma(\omega, l) \xrightarrow{|l| \gg 1} \sqrt{(\omega - \omega_{\text{BFKL}})/(2D_0)} \]
Numerical calculations of $Y$ dependence

Two problems:

1. The kernel is not Fredholm type

   \[ \int d^2 x_{01} d^2 x_{12} K (x_{12}, x_{02} | x_{01}) \rightarrow \infty \]

2. The kernel is singular at $x_{12} \rightarrow x_{01}$
Checks:

1. Dependence on $x_{min}$ and $x_{max}$;

2. Numerical solution to the BFKL equation coincide with the analytic one;

3. Independence on value of the regulator $R$;

$$\int d^2 x_{13} K_R^B (x_{13}, x_{32}|x_{12}) \mathcal{N} (x_{12}; Y) \equiv$$

$$\int d^2 x_{13} \frac{e^{-B (x_{13}^2 + x_{32}^2)}}{x_{32}^2 + R^2} \left\{ 2 \mathcal{N} (x_{13}; Y) - 2 \frac{x_{12}^2}{x_{13}^2 + x_{23}^2 + 2 R^2} \mathcal{N} (x_{12}; Y) \right\}$$

4. Independence on value of $B$;
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\[ \omega = \omega_{\text{BFKL}} \]
Saturation momentum

Saturation momentum can be found from linear equation:

\[
\frac{4}{Q_s^2(Y)} N_{BFKL} \left( \frac{2}{Q_s(Y)} ; Y \right) = N_0 \leq 1 \quad \text{where} \quad N_0 = \text{Const}
\]

For BFKL theoretical prediction:

\[
l_s(Y) \equiv \ln \left( \frac{Q_s^2(Y)}{Q_s^2(Y_0)} \right) = \\
\frac{\omega(\gamma_{cr})}{1 - \gamma_{cr}} (Y - Y_0) - \frac{3}{2(1 - \gamma_{cr})} \ln(Y/Y_0) \\
- \frac{3}{(1 - \gamma_{cr})^2} \sqrt{\frac{2 \pi}{\omega''(\gamma_{cr})}} \left( \frac{1}{\sqrt{Y}} - \frac{1}{\sqrt{Y_0}} \right)
\]

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\[ l_s[Y] = \ln \left( \frac{Q_s^2}{B} \right) \]
Next steps:

- Check that different models for large $b$ dependence in the BFKL kernel lead to $\omega = \omega_{\text{BFKL}}$
  
  $K(x_{12}, x_{20}|x_{10}) = \frac{x_{10}^2}{x_{12}^2 x_{02}^2} e^{-\mu(x_{12} + x_{02})}$;

- BFKL in gauge theories with the Higgs mechanism of mass generation (E.L., L. Lipatov and M. Siddikov)

\[ E\phi(\kappa) = \frac{\kappa + 1}{\sqrt{\kappa} \sqrt{\kappa + 4}} \ln \frac{\sqrt{\kappa + 4} + \sqrt{\kappa}}{\sqrt{\kappa + 4} - \sqrt{\kappa}} \phi(\kappa) \]

\[ \text{kinetic energy} \]

\[ - \int_0^\infty \frac{d\kappa' \phi(\kappa')}{\sqrt{(\kappa - \kappa')^2 + 2(\kappa + \kappa') + 1}} + \frac{N_c^2 + 1}{2N_c^2} \frac{1}{\kappa' + 1} \int_0^\infty \frac{\phi(\kappa')}{\kappa' + 1} d\kappa' \]

\[ \text{contact term} \]

- Build Pomeron calculus, based on modified BFKL Pomerons;

- Obtain non-linear equations for amplitude;
Hope:

- The global features of the BFKL Pomeron does not depend on confinement;

- We will be able to build the self-consistent theoretical CGC/saturation approach with correct $b$ behaviour of the scattering amplitude;
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THANK YOU