Half-Duplex Energy Harvesting Relay Network over Different Fading Environment: System Performance with Effect of Hardware Impairment

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Abstract: In this paper, we introduce a half-duplex (HD) energy harvesting (EH) relay network over the different fading environment with the effect of hardware impairment (HI). The model system was investigated with the amplify-and-forward (AF) and the power splitting (PS) protocols. The system performance analysis in term of the outage probability (OP), achievable throughput (AT), and bit error rate (BER) were demonstrated with the closed-form expressions. In addition, the power splitting (PS) factor was investigated. We verified the analytical analysis by Monte Carlo simulation with all primary parameters. From the results, we can state that the analytical and simulation results match well with each other.

Keywords: amplify-and-forward (AF); relay network; achievable throughput (AT); outage probability (OP); BER; energy harvesting (EH)

1. Introduction

Nowadays, wireless powered communication networks (WPCNs) have shown significant advantages in industry and living. The major benefits of WPCNs mainly come from battery charging operations through the air without physical cable connections and recharging, and thus, replacing the battery. As such, the maintenance, servicing, and charging of many battery-powered devices deployed in networks are crucially simplified, especially for future applications and technology [1–5]. Nowadays, there are three primary wireless energy harvesting (EH) and transfer techniques in the two main types of wireless charging—radiative (or RF based) and non-radiative (or coupling based) are employed in practice in WPCNs—which are feasible using the following techniques. In the first method, inductive coupling based on magnetic field induction can be used for transferring electrical energy over distances ranging from a few millimeters to a few centimeters. The efficiency of this method, from around 6% to 90%, is suitable for cell phone charging, contactless smart cards, and passive RFID cards [3,4]. Magnetic Resonant coupling is the second method, which is based on one vane scent wave coupling by making two separate coils resonate at the same frequency. Its efficiency ranges from 30% to around 90% and is suitable for plug-in hybrid electric vehicles and cell phone charging [3,4]. The last method is RF energy transfer, which is suitable for wireless body and wireless sensor networks. From this point of view, RF energy transfer WPCNs are suitable for long-distance transfers [1–5]. Some papers have
presented the process of EH through the RF signals in cooperative wireless networks using a MIMO relay system, and have investigated multi-user and multi-hop systems for simultaneous information and power transfer with a dual-hop channel [6–18]. In these previous papers, the authors have focused on WPCNs using only the Rayleigh or the Rician fading channel. However, to date not many papers concentrate on using both different fading channels. Hardware impairment (HI) suffers from phase noise, I/Q imbalance, and high-power amplifier nonlinearities [19–22]. HI is rarely studied in the literature on relay WPCNs.

In a wireless network, the source and destination may not communicate to each other directly, because the distance between the source and destination is greater than the transmission range of them both, hence the need for an intermediate node(s) to relay. Relaying is an effective way to combat the performance degradation caused by fading, shadowing, and path loss. In relay networks, the relay nodes help to boost the information exchange between source nodes and destination nodes, by forwarding (with or without decoding) the information-bearing radio frequency signals from sources to destinations [1–5]. In this paper, we introduce a half-duplex (HD) energy harvesting (EH) relay network over the different fading environment with the effect of hardware impairment (HI). The model system was investigated with the amplify-and-forward (AF) and the power-splitting (PS) protocols. The system performance analysis, in term of the outage probability (OP), achievable throughput (AT), and the bit error rate (BER), was analyzed and demonstrated with the closed-form expressions. Additionally, the power splitting (PS) factor was investigated. We verified the analytical analysis using a Monte Carlo simulation with all primary parameters. The research results showed that the analytical and simulation results matched well with each other. The main contributions are summarized as follow:

1. An HD EH relay network over the different fading environment (Rayleigh and Rician Fading Channel) with the HI effect of HI is introduced and investigated.
2. The closed form of OP, AT, and BER of the proposed system was analyzed and derived in connection with the main primary system parameters.
3. The correctness of the analytical expression was demonstrated by Monte Carlo simulation.

The remainder of this paper is introduced as follows. Section 2 introduces the system model with EH and information transmission (IT) phases. OP, AT, and BER are derived in Section 3. Section 4 shows and discusses the numerical results. Finally, Section 5 provides some conclusions.

2. System Model Network

An HD EH relay network over the different fading environment (Rayleigh and Rician Fading Channel), with the effect of HI, is illustrated in Figure 1. This system model is working in AF mode and PS protocol, in which the source (S) and destination (D) can exchange their signal via the helping Relay (R), as shown in Figure 1. The PS protocol of the system model is plotted in Figure 2. Here, T denotes the block time for EH and IT processes in the PS protocol. In the first half interval time $T/2$, the S simultaneously transfers information and energy to the R with the PS factor $\rho \in (0, 1)$. $\rho P$ is used for energy harvesting at the R and $(1-\rho)P$ is used for transmitting information to the R node. So, the remaining $T/2$ interval time is used for transferring information from the R to the D [17–20].

![Figure 1. System model.](image-url)
2.1. Energy Harvesting (EH)

The signal is transferred from the S to the R in the first-half $T/2$ and can be formulated as

$$y_r = h(x_s + \mu_s) + n_r$$

(1)

The received RF signal at the input of the energy harvesting unit can be calculated as

$$y_{hr} = \sqrt{\rho} \times y_r = \sqrt{\rho} h x_s + \sqrt{\rho} h \mu_s + \sqrt{\rho} n_r$$

(2)

In this equation, $x_s$ is the energy-transmitted signal with $\mathbb{E}\{|x_s|^2\} = P_s$, $n_r$ is the zero-mean additive white Gaussian noise (AWGN) with variance $N_0$, and $\mu_s$ denotes the distortion error caused by hardware impairment at the source node, which is modeled as a zero-mean Gaussian random variable with variance $P_s \sigma_1^2$ with $\mathbb{E}\{\left|\mu_s\right|^2\} = P_s \sigma_1^2$. Here $\mathbb{E}\{\cdot\}$ denotes the expectation operation.

From Equation (2), the harvested energy at R in the first interval $T/2$ can be formulated as the following

$$E_h = \eta \rho P_s |h|^2 (T/2)$$

(3)

Therefore, the transmitted power at R can be calculated as

$$P_r = \frac{E_h}{T/2} = \frac{\eta \rho P_s |h|^2 (T/2)}{T/2} = \eta \rho P_s |h|^2$$

(4)

where $\eta$ is denoted the energy conversion efficiency of the proposed system.

2.2. Information Transmission (IT)

In this model, the IT phase is divided into two equal-length subintervals with the length $T/2$. In the first interval, we can calculate the received signal as

$$y_r = \sqrt{1-\rho} h (x_s + \mu_s) + n_r = \sqrt{1-\rho} h x_s + \sqrt{1-\rho} h \mu_s + n_r$$

(5)

where $x_s$ is the transmitted signal, which satisfies $\mathbb{E}\{|x_s|^2\} = P_s$, $\mu_s$ denotes the distortion error caused by hardware impairment at R, which is modeled as a zero-mean Gaussian random variable with variance $P_s \sigma_2^2$ and $\mathbb{E}\{\left|\mu_s\right|^2\} = P_s \sigma_2^2$, and $n_r$ is the AWGN noise at R node.

Here, we use the amplify-and-forward (AF) protocol for our model. Then, the received signal at R is amplified by a factor $\beta$, which is given by Equation (6)

$$\beta = \frac{P_r}{\sqrt{(1-\rho) |h|^2 P_s + (1-\rho) |h|^2 P_s \sigma_2^2 + N_0}}$$

(6)

In the remaining $T/2$ interval time, R transfers the information to D. Hence, the received signal at D node is formulated by
\[ y_d = g(x_r + \mu_r) + n_d = g\lambda x_r + g\mu_r + n_d \]
\[ \eta = gb(\sqrt{1-\rho}h x_r + \sqrt{1-\rho}h \mu_s + n_r) + g\mu_r + n_d \]
\[ \eta = gb\sqrt{1-\rho}h x_r + \sqrt{1-\rho}h \mu_s + g\beta n_r + g\mu_r + n_d \]

(7)

Here \( n_d \) is the noise at the destination, which is assumed to have the same power as \( n_r \). The end-to-end signal-to-noise ratio (SNR) at D node can be given by

\[ \gamma_{e2e} = \frac{E\{signal\}^2}{E\{noise\}^2} = \frac{(1-\rho)|g|^2 \beta^2 \mu_f^2 P_s}{|g|^2 \beta^2 |h|^2 P_s \sigma_1^2 + |g|^2 \beta^2 N_0 + |g|^2 P_s \sigma_2^2 + N_0} \]

(8)

We denote \( \phi_1 = |h|^2 \), \( \phi_2 = |g|^2 \) and replacing this in Equation (8), we have

\[ \gamma_{e2e} = \frac{(1-\rho)\gamma_1 \gamma_2 \beta^2 P_s}{\phi_1 \phi_2 \beta^2 P_s \sigma_1^2 + \phi_2 \beta^2 N_0 + \phi_2 P_s \sigma_2^2 + N_0} \]

(9)

\[ \gamma_{e2e} = \frac{(1-\rho)\gamma_1 \gamma_2 P_s}{\phi_1 \phi_2 P_s \sigma_1^2 + \phi_2 N_0 + \phi_2 \sigma_2^2 + N_0} \]

\[ = \left( (1-\rho)\gamma_1 \gamma_2 P_s \right)/\left( \phi_1 \phi_2 P_s \sigma_1^2 + \phi_2 N_0 + \phi_2 \sigma_2^2 + N_0 \right) \]

\[ + \phi_2 N_0 \sigma_2^2 + \frac{N_0 (1-\rho) \gamma_1}{\eta_p} + \frac{N_0 (1-\rho) \gamma_2}{\eta_p} \]

(10)

Because \( N_0 \ll P_r \), then we can reformulate Equation (10) as the following equation

\[ \gamma_{e2e} = \left( (1-\rho)\gamma_1 \gamma_2 P_s \right)/\left( \phi_1 \phi_2 P_s \sigma_1^2 + \phi_2 N_0 + \phi_2 \sigma_2^2 + N_0 \right) \]

\[ + \phi_2 N_0 \sigma_2^2 + \frac{N_0 (1-\rho) \gamma_1}{\eta_p} + \frac{N_0 (1-\rho) \gamma_2}{\eta_p} \]

\[ + \gamma_{e2e} = \left( (1-\rho)\gamma_1 \gamma_2 P_s \right)/\left( \phi_1 \phi_2 P_s \sigma_1^2 + \phi_2 N_0 + \phi_2 \sigma_2^2 + N_0 \right) \]

\[ + \phi_2 N_0 \sigma_2^2 + \frac{N_0 (1-\rho) \gamma_1}{\eta_p} + \frac{N_0 (1-\rho) \gamma_2}{\eta_p} \]

(11)

(12)

where we denote \( \kappa = \frac{1-\rho}{\eta_p}, \gamma_0 = \rho_0, \phi_1, \phi_2 \).

In the next section, we analyze the achievable throughput (AT), outage probability (OP) and BER in AF mode with the PS protocol [6–8].

3. System Model Performance

In this section, we investigate the system performance of the relay network with the PS protocol [10–13,23–27]. In this analysis, we consider two Scenarios: (1) S-R link is the Rayleigh Fading Channel and R-D link is Rician Fading Channel, and (2) S-R link is the Rician Fading Channel and R-D link is the Rayleigh Fading Channel.

3.1. Scenario 1: S-R link Is Rayleigh Fading Channel, R-D link Is Rician Fading Channel

As in previous studies [6–9], the probability density function (PDF) of a random variable (RV) \( \varphi_1 \) can be written as the following equation

\[ f_{\varphi_1}(x) = \lambda_k e^{-\lambda_k x} \]

(13)

Here \( \lambda_k \) is the mean value of RV \( \varphi_1 \).
The cumulative density function (CDF) of RV $\varphi_1$ can be written as

$$F_{\varphi_1}(x) = 1 - e^{-\lambda_{e}x}$$  \hspace{1cm} (14)

Similarly, the PDF of RV $\varphi_2$ can be obtained as in [26], giving

$$f_{\varphi_2}(x) = \frac{(K+1)e^{-K}}{\lambda_{g}}e^{-\frac{(K+1)x}{\lambda_{g}}} I_0\left(2 \sqrt{\frac{K(K+1)x}{\lambda_{g}}}ight)$$  \hspace{1cm} (15)

where $\lambda_{e}$ is the mean value of RV $\varphi_2$, $K$ denotes the Rician K-factor, and $I_0(\bullet)$ is the zero-th order modified Bessel function of the first kind [26].

Then the Equation (14) can be reformulated as the following

$$f_{\varphi_2}(x) = a \sum_{l=0}^{\infty} \frac{(bK)^{l}}{(l!)^{2}} x^{l} e^{-bx}$$  \hspace{1cm} (16)

where we denote $a = \frac{(K+1)e^{-K}}{\lambda_{g}}$, $b = \frac{K+1}{\lambda_{g}}$ and $I_0(x) = \sum_{l=0}^{\infty} \frac{x^{2l}}{2^{2l}(l!)^{2}}$ [24].

The cumulative density function (CDF) of RV $\varphi_2$ can be computed like in [27]

$$F_{\varphi_2}(c) = \int_{0}^{c} f_{\varphi_2}(x)dx = 1 - a \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{K^{l+m}}{l!m!} c^{m} e^{-bc}$$  \hspace{1cm} (17)

After that, the OP of the proposed system can be computed as

$$P_{out} = F_{\gamma_{2c}}(\gamma) = \Pr(\gamma_{2c} < \gamma)$$  \hspace{1cm} (18)

If we denote $\gamma = 2^{K} - 1$ to be the lower threshold for SNR at both R and D, and R is fixed transmission rate at S, then Equation (18) can be reformulated as the following

$$P_{out} = \Pr\left\{ \varphi_1 \varphi_2 \gamma_0 [1 - \rho - \gamma \sigma_1^2 - \gamma(1 - \rho) \sigma_2^2] - \gamma(1 - \rho) \sigma_1^2 \sigma_2^2 < \varphi_2 (\gamma + \gamma \sigma_1^2 + \gamma \rho \sigma_2^2) \right\}$$  \hspace{1cm} (19)

Here we denote $c_1 = \gamma + \gamma \sigma_1^2$, $c_2 = \gamma \rho \sigma_2^2$, and $c_3 = \gamma_0 [1 - \rho - \gamma \sigma_1^2 - \gamma(1 - \rho) \sigma_2^2]$. We assume that $c_3$ is positive, because if $c_3$ is negative, the OP of the system is always equal to 1.

a. Outage Probability (OP)

$$P_{out} = \Pr\left\{ \varphi_1 < \frac{c_1 \varphi_2 + c_2}{c_3 \varphi_2} \right\} = \int_{0}^{\infty} F_{\varphi_1}\left(\frac{c_1 \varphi_2 + c_2}{c_3 \varphi_2}\right)f_{\varphi_2}(\varphi_2)d\varphi_2$$  \hspace{1cm} (20)

Combining Equation (20) with Equations (14) and (17) we have

$$P_{out} = 1 - \int_{0}^{\infty} e^{-\lambda_{e}x} e^{-\frac{\lambda_{e}x}{c_3}} x^{l} e^{-bx} dx = 1 - a e^{-\lambda_{e}c_3} \sum_{l=0}^{\infty} \frac{(bK)^{l}}{(l!)^{2}} \varphi_2 e^{-\frac{bK\varphi_2}{c_3}}$$  \hspace{1cm} (21)

Using Table of Integral Equation [3.471,9] in [24], Equation (21) can be given as

$$P_{out} = 1 - 2ae^{-\lambda_{e}c_3} \sum_{l=0}^{\infty} \frac{(bK)^{l}}{(l!)^{2}} \left(\frac{\lambda_{e}c_2}{c_3}\right)^{\frac{l+1}{2}} \times K_{l+1}\left(2 \sqrt{\frac{\lambda_{e}c_2 b}{c_3}}\right)$$  \hspace{1cm} (22)
where \( K_0(\bullet) \) is the modified Bessel function of the second kind and \( \nu \)th order.

b. Achievable Throughput (AT)

Here, the average throughput of the relay network system can be computed regarding the OP as in Equation (23)

\[
\tau = (1 - P_{out}) \frac{R}{2} = 2a e^{-\frac{\lambda_{b1}}{2s}} \sum_{l=0}^{\infty} \frac{(bK)^l}{(l!)^2} \left( \frac{\lambda_{b2}}{c_3 b} \right)^{\frac{l+1}{2}} \times K_{l+1} \left( 2 \sqrt{\frac{\lambda_{b2} c_3}{c_3}} \right) \times \frac{R}{2} \quad (23)
\]

c. The Bit Error Rate (BER)

The BER of the proposed system can be formulated from the expression of the OP as the following equation

\[
BER = E[\omega Q(\sqrt{2\theta x})] \quad (24)
\]

where \( Q(t) = \frac{1}{\sqrt{2\pi}} \int_{t}^{\infty} e^{-x^2/2} dx \) is the Gaussian Q-function, \( \omega \) and \( \theta \) are constants which are specific for modulation type. Here, we use \( (\omega, \theta) = (1, 2) \) for BPSK and \( (\omega, \theta) = (1, 1) \) for QPSK. Hence, we begin rewriting the BER expression in Equation (24) directly regarding OP at S by using integration as in the equation below

\[
BER = \frac{\omega \sqrt{\theta}}{2 \sqrt{\pi}} \int_{0}^{\infty} e^{-\theta x} F_{\gamma/2} (x) dx \quad (25)
\]

### 3.2. Scenario 2: S-R Link Is the Rician Fading Channel, R-D Link Is the Rayleigh Fading Channel

Similar to scenario 1, the CDF of RV \( \varphi_1 \) and PDF of RV \( \varphi_2 \) can be formulated as

\[
F_{\varphi_1}(z) = \int_{0}^{z} f_{\varphi_1}(x) dx = 1 - \frac{a}{b} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{K_{l} b^m m!}{l!m!} e^{-b c z} \quad (26)
\]

where \( a = \frac{(K+1)e^{-K}}{\lambda_{b}}, \quad b = \frac{K+1}{\lambda_{b}} \)

\[
f_{\varphi_2}(x) = \lambda_{e} e^{-\lambda_{e} x} \quad (27)
\]

a. Outage Probability (OP)

\[
P_{out} = \Pr \left\{ \varphi_1 < \frac{c_1 \varphi_2 + c_2}{c_3 \varphi_2} \right\} = \int_{0}^{\infty} F_{\varphi_1} \left( \frac{c_1 \varphi_2 + c_2}{c_3 \varphi_2} \right) f_{\varphi_2} (\varphi_2) d\varphi_2 \quad (28)
\]

\[
P_{out} = 1 - \int_{0}^{\infty} e^{-\frac{a}{b} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{K_{l} b^m m!}{l!m!} \left( \frac{c_1 \varphi_2 + c_2}{c_3 \varphi_2} \right)^{m}} \lambda_{e} e^{-\lambda_{e} \varphi_2} d\varphi_2 \quad (29)
\]

We apply the equation \( (x + y)^m = \sum_{n=0}^{m} \binom{m}{n} x^{m-n} y^n \), and change the variable by setting, and Equation (27) can be reformulated as the following

\[
P_{out} = 1 - \lambda_{e} e^{\frac{b c_3}{3}} \int_{0}^{\infty} \int_{0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \sum_{n=0}^{m} \frac{m!}{n!} \frac{K_{l} b^m m!}{l!m!} \frac{c_{2}^{m-n} c_{2}^{n}}{c_{3}^{m-n} c_{3}^{n}} e^{-\lambda_{e} \varphi_2} dt \quad (30)
\]
\[ P_{\text{out}} = 1 - e^{-\lambda g^{\frac{1}{c_3}}} b \sum_{l=0}^{\infty} \sum_{m=0}^{l} \sum_{n=0}^{m} K_l b^{m-n} c_1^{n-m} \tfrac{\lambda e c_3^{m-n}}{c_2} \tfrac{\lambda e c_3^{n-m}}{c_2} \tfrac{\lambda e c_3^{m-n}}{c_3} \tfrac{\lambda e c_3^{n-m}}{c_3} \int_{0}^{\infty} t^{\alpha - 2} e^{-\lambda g^{\frac{1}{c_3}}} e^{-\tau dt} \] (31)

Using the Table of Integral Equation [3.471,9] in [24], the above equation can be given as

\[ P_{\text{out}} = 1 - 2ae^{-\lambda g^{\frac{1}{c_3}}} b \sum_{l=0}^{\infty} \sum_{m=0}^{l} \sum_{n=0}^{m} K_l b^{m-n} c_1^{n-m} \tfrac{\lambda e c_3^{m-n}}{c_2} \tfrac{\lambda e c_3^{n-m}}{c_2} \tfrac{\lambda e c_3^{m-n}}{c_3} \tfrac{\lambda e c_3^{n-m}}{c_3} \int_{0}^{\infty} t^{\alpha - 2} e^{-\lambda g^{\frac{1}{c_3}}} e^{-\tau \rho} \] (32)

\[ P_{\text{out}} = 1 - 2ae^{-\lambda g^{\frac{1}{c_3}}} b \sum_{l=0}^{\infty} \sum_{m=0}^{l} \sum_{n=0}^{m} K_l b^{m-n} c_1^{n-m} \tfrac{\lambda e c_3^{m-n}}{c_2} \tfrac{\lambda e c_3^{n-m}}{c_2} \tfrac{\lambda e c_3^{m-n}}{c_3} \tfrac{\lambda e c_3^{n-m}}{c_3} \int_{0}^{\infty} t^{\alpha - 2} e^{-\lambda g^{\frac{1}{c_3}}} e^{-\tau \rho} \] (33)

where \( K_\nu(*) \) is the modified Bessel function of the second kind and \( v \)th order.

b. Achievable Throughput (AT):

\[ \tau = (1 - P_{\text{out}}) \tfrac{R}{2} = 2ae^{-\lambda g^{\frac{1}{c_3}}} b \sum_{l=0}^{\infty} \sum_{m=0}^{l} \sum_{n=0}^{m} K_l b^{m-n} c_1^{n-m} \tfrac{\lambda e c_3^{m-n}}{c_2} \tfrac{\lambda e c_3^{n-m}}{c_2} \tfrac{\lambda e c_3^{m-n}}{c_3} \tfrac{\lambda e c_3^{n-m}}{c_3} \int_{0}^{\infty} t^{\alpha - 2} e^{-\lambda g^{\frac{1}{c_3}}} e^{-\tau \rho} \tfrac{R}{2} \] (34)

c. The Bit Error Rate (BER)

Similar to scenario 1, BER can be calculated with the equation below

\[ \text{BER} = \frac{\omega \sqrt{\theta}}{2\sqrt{\pi}} \int_{0}^{\infty} e^{-\beta x} F_{\gamma,\nu,\alpha}(x) dx \] (35)

3.3. Optimal Power-Splitting (PS) Factor

In this section, we can calculate the optimal value \( \rho^* \) by solving the equation \( \frac{d\tau}{d\rho} = 0 \), using the AT expression in Equations (23) and (34). Here, we use the Golden section search algorithm as in [25,28–30], which is popularly used in many global optimization problems in communications.

4. Results and Discussion

In this section, we investigate the system performance in terms of OP, AT, BER, and the PS factor in connection with the main system parameters: \( \eta, \rho, P_1/N_0 \) and \( \sigma_1, \sigma_2 \). The primary system simulation parameters are listed in Table 1.

| Symbol | Name | Values |
|--------|------|--------|
| \( \eta \) | Energy harvesting efficiency | 0.7 |
| \( \lambda_b \) | Mean of \( |h|^2 \) | 0.5 |
| \( \lambda_g \) | Mean of \( |g|^2 \) | 0.5 |
| K | Rician K-factor | 3 |
| \( \gamma_{th} \) | SNR threshold | 7 |
| \( P_1/N_0 \) | Source power-to-noise ratio | 0–30 dB |
| \( \sigma_1 \) | Distortion error | 0.01 |
| \( \sigma_2 \) | Distortion error | 0.05 |
| R | Source rate | 3 bit/s/Hz |
Figures 3 and 4 plot the OP and AT versus the energy conversion efficiency $\eta$. The effect of $\eta$ was investigated in both scenarios, and we vary $\eta$ continuously from 0 to 1. From the figures, we can see that the OP decreased and the AT increased crucially with $\eta$ varying from 0 to 1; and the OP and AT in the second case is better than in the first case. Furthermore, the results show the correctness of the simulation and analytical expressions. Moreover, Figures 5 and 6 plot the impact of the ratio $P_s/N_0$ on the OP and AT, while the ratio $P_s/N_0$ increases from 0 to 30 dB. From the research results, we can state that the OP decreased and AT increased with the rising of the ratio $P_s/N_0$, and the analytical results match very well with the analytical values.
Figure 3. Outage probability versus $\eta$.

Figure 4. The throughput versus $\eta$.

Figure 5. The outage probability versus ratio of $P_s/N_0$.

In addition, the OP and AT versus the PS factor $\rho$ are shown in Figures 7 and 8. It can be observed that the AT increased and OP fell at the D with factor $\rho$ from 0 to 0.6. After that, the OP and AT had the opposite effect when ratio $\rho$ from 0.6 to 1.0. We can find the optimal value of factor $\rho$ from 0.6 to 0.7 in this situation. It can be observed that when $\rho$ is too small, R cannot harvest enough energy from S to operate reliably, but when $\rho$ is too large then the reliability of the communications link from S to R is impaired. Furthermore, the OP and AT of the model system versus $\sigma_1 = \sigma_2$ varied from 0 to 0.2 and are plotted in Figures 9 and 10. In the same way, OP increased and AT decreased with the increasing of $\sigma_1 = \sigma_2$. In all the above figures, the simulation and analytical results agree well with each other.
In addition, the OP and AT versus the PS factor $\rho$ are shown in Figures 7 and 8. It can be observed that the AT increased and OP fell at the D with factor $\rho$ from 0 to 0.6. After that, the OP and AT had the opposite effect when ratio $\rho$ from 0.6 to 1.0. We can find the optimal value of factor $\rho$ from 0.6 to 0.7 in this situation. It can be observed that when $\rho$ is too small, R cannot harvest enough energy from S to operate reliably, but when $\rho$ is too large then the reliability of the communications link from S to R is impaired. Furthermore, the OP and AT of the model system versus $\sigma_1 = \sigma_2$ varied from 0 to 0.2 and are plotted in Figures 9 and 10. In the same way, OP increased and AT decreased with the increasing of $\sigma_1 = \sigma_2$. In all the above figures, the simulation and analytical results agree well with each other.

**Figure 7.** The outage probability versus $\rho$ for the PS protocol.

**Figure 8.** The throughput versus $\rho$ for the PS protocol.

**Figure 9.** The outage probability versus $\sigma_1 = \sigma_2$.
Figures 11 and 12 show the optimal PS factor versus the ratio $P_s/N_0$ in both scenarios. As shown in the figures, the optimal power splitting factor increased while the ratio $P_s/N_0$ varied from 0 to 30 dB. Moreover, Figure 13 plots the dependent of BER on the ratio $P_s/N_0$. The results show that BER decreased significantly with $P_s/N_0$ from 0 to 20 dB. After that, BER slightly decreased.

Figure 8. The throughput versus $\rho$ for the PS protocol.

Figure 9. The outage probability versus $\sigma_1 = \sigma_2$.

Figure 10. The throughput versus $\sigma_1 = \sigma_2$.

Figure 11. The optimal PS factor versus $P_s/N_0$ for the first case.

Figure 12. The optimal PS factor versus $P_s/N_0$ for the second case.

Figure 13. The dependent of BER on the ratio $P_s/N_0$. The results show that BER decreased significantly with $P_s/N_0$ from 0 to 20 dB. After that, BER slightly decreased.
In this paper, we propose and investigate the system performance of an AF wireless network over the different fading channel. The system model is considered under the effect of hardware impairment with the PS protocols. Firstly, the closed-form expressions of OP, AT, and BER of the proposed system are analyzed and derived. After that, we used the Monte Carlo simulation to verify the correctness of the analytical expressions in connection with the primary system parameters. Finally, the numerical results show that the analytical mathematical expression and the simulation results using the Monte Carlo method are in agreement with each other. In addition, the optimal PS factor was investigated in both cases.

5. Conclusions

In this paper, we propose and investigate the system performance of an AF wireless network over the different fading channel. The system model is considered under the effect of hardware impairment with the PS protocols. Firstly, the closed-form expressions of OP, AT, and BER of the proposed system is analyzed and derived. After that, we used the Monte Carlo simulation to verify the correctness of the analytical expressions in connection with the primary system parameters. Finally, the numerical results show that the analytical mathematical expression and the simulation results using the Monte Carlo method are in agreement with each other. In addition, the optimal PS factor was investigated in both cases.

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