Determinant of the $^{3}\text{He} + \alpha \rightarrow ^{7}\text{Be}$ asymptotic normalization coefficients (nuclear vertex constants) and their application for extrapolation of the $^{3}\text{He}(\alpha, \gamma)^{7}\text{Be}$ astrophysical $S$-factors to the solar energy region

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Abstract

A new analysis of the precise experimental astrophysical $S$-factors for the direct capture $^{3}He(\alpha, \gamma)^{7}$Be reaction [B.S. Nara Singh et al., Phys.Rev.Lett. 93 (2004) 262503; D. Bemmerer et al., Phys.Rev.Lett. 97 (2006) 122502; F.Confortola et al., Phys.Rev. C 75 (2007) 065803 and T.A.D.Brown et al., Phys.Rev. C 76 (2007) 055801] populating to the ground and first excited states of $^{7}$Be is carried out based on the modified two-body potential approach in which the direct astrophysical $S$-factor, $S_{24}(E)$, is expressed in terms of the asymptotic normalization constants for $^{3}\text{He} + \alpha \rightarrow ^{7}\text{Be}$ and two additional conditions are involved to verify the peripheral character of the reaction under consideration. The Woods–Saxon potential form is used for the bound ($\alpha + ^{3}\text{He}$)-state and the $^{3}\text{He}$-scattering wave functions. New estimates are obtained for the "indirectly measured" values of the asymptotic normalization constants (the nuclear vertex constants) for $^{3}\text{He} + \alpha \rightarrow ^{7}\text{Be}$(g.s.) and $^{3}\text{He} + \alpha \rightarrow ^{7}\text{Be}$(0.429MeV) as well as the astrophysical $S$-factors $S_{34}(E)$ at $E \leq 90$ keV, including $E=0$. The values of asymptotic normalization constants have been used for getting information about the $\alpha$-particle spectroscopic factors for the mirror $(^7\text{Li}^7\text{Be})$-pair.

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1 Introduction

The $^{3}\text{He}(\alpha, \gamma)^{7}\text{Be}$ reaction is one of the critical links in the $^{7}\text{Be}$ and $^8B$ branches of the $pp$-chain of solar hydrogen burning [1–3]. The total capture rate determined by processes of

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this chain is sensitive to the cross section $\sigma_{34}(E)$ (or the astrophysical $S$-factor $S_{34}(E)$) for the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction and predicted neutrino rate varies as $[S_{34}(0)]^{0.8}$ [2, 3].

Despite the impressive improvements in our understanding of the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction made in the past decades (see Refs [4–10] and references therein), however, some ambiguities connected with both the extrapolation of the measured cross sections for the aforesaid reaction to the solar energy region and the theoretical predictions for $\sigma_{34}(E)$ (or $S_{34}(E)$) still exist and they may influence the predictions of the standard solar model [2, 3].

Experimentally, there are two types of data for the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction at extremely low energies: i) six measurements based on detecting of $\gamma$-rays capture (see [4] and references therein) from which the astrophysical $S$-factor $S_{34}(0)$ extracted by the authors of those works changes within the range $0.47 \leq S_{34}(0) \leq 0.58$ keV b, which yield a weighted mean of $S_{34}(0) = 0.507 \pm 0.016$ keV b [4], and ii) five measurements based on detecting of $^7\text{Be}$ (see [4] references therein as well as [6–10]) from which $S_{34}(0)$ extracted by the authors of these works changes within the range $0.53 \leq S_{34} \leq 0.63$ keV b, which yield weighted means of $S_{34}(0) = 0.572 \pm 0.026$ keV b [4], $S_{34}(0) = 0.53$ keV b [6], $S_{34}(0) = 0.547 \pm 0.017$ keV b [7, 8], and $S_{34}(0) = 0.560 \pm 0.017$ keV b [9] and $S_{34}(0) = 0.595 \pm 0.018$ and 0.596 \pm 0.021 keV b [10].

All of these measured data have a similar energy dependence for the astrophysical $S$-factors $S_{34}(E)$ but the extrapolation of each of the measured data from the observed energy ranges to low experimentally inaccessible energy regions, including $E = 0$, gives a value of $S_{34}(0)$ with an uncertainty exceeding noticeably the experimental one. The recent aforesaid values of $S_{34}(0)$ recommended in Refs [6–9] and [10] have nevertheless been obtained from the analysis of the precisely measured data for $S_{34}^{\exp}(E)$ by means of the artificial renormalization of the energy dependence of the R-matrix calculation [11] and of the resonating-group method calculation [12] for $S_{34}(E)$ to the corresponding experimental data, respectively.

The theoretical calculations of $S_{34}(0)$ performed within different methods also show considerable spread [11–14]. The aforesaid resonating-group method calculations of $S_{34}(0)$ performed in Ref. [12] show considerable sensitivity to the form of the effective NN interaction used and the estimates have been obtained within the range of $0.312 \leq S_{34}(0) \leq 0.841$ keV b. Calculations performed in microscopic single-channel ($\alpha + ^3\text{He}$) and two-channel ($^3\text{He} + \alpha$) and ($p + ^6\text{Li}$) cluster models gave the values of $S_{34}(0) = 0.56$ keV b [13], $S_{34}(0) = 0.52$ keV b [14] and of $S_{34}(0) = 0.83$ keV b [14], respectively, that is, the estimate of the value of the $S_{34}(0)$ strongly changes when the model space is expanded. The calculations performed by the authors of Refs [13] and [16] in the two-body potential model with different forms of the two-body potential gave the values of $S_{34}(0) = 0.516$ and about of 0.5 keV b, respectively, although different values of 1.174 in [13] and 1.0 in [16] have been used for the spectroscopic factor for the ($\alpha + ^3\text{He}$)-configuration in $^7\text{Be}$. Calculations performed in the variational Monte-Carlo technique (VMCT) with seven-particle wave functions derived from realistic NN interaction gave $S_{34}(0) \approx 0.40$ keV b [17]. But, as it was emphasized in paper [17], serious problems occur with the normalization for the calculated astrophysical $S$-factor $S(E)$ in respect to the experimental data. The estimation of $S_{34}(0) = 0.52 \pm 0.03$ keV b [18] also should be noted. The latter has been obtained within the framework of the asymptotic method developed in [19, 20] based on the idea proposed in paper [21]. This idea is based on the assumption about the fact that low-energy direct radiative captures in light nuclei ($A(a, \gamma)B$) proceed mainly in
regions well outside the range of the internuclear interactions. But in Ref. [13] the contribution from the nuclear interior \((r < 4 \text{ fm})\) to the amplitude was assumed to be negligibly small. In this assumption from the analysis of the experimental astrophysical \(S\)-factors for the direct capture \(^3\text{He}(\alpha, \gamma)^7\text{Be}(\text{g.s.})\) and \(^3\text{He}(\alpha, \gamma)^7\text{Be}(0.429 \text{ MeV})\) reactions in the energy range \(180 \lesssim E \lesssim 500 \text{ keV} \) [22] the values of the nuclear vertex constants (NVC) for the virtual decays \(^7\text{Be}(\text{g.s.}) \rightarrow \alpha + ^3\text{He}\) and \(^7\text{Be}(0.429\text{MeV}) \rightarrow \alpha + ^3\text{He}\) [23] (or the respective asymptotic normalization coefficients (ANC) for \(^3\text{He} + \alpha \rightarrow ^7\text{Be}(\text{g.s.})\) and \(^3\text{He} + \alpha \rightarrow ^7\text{Be}(0.429 \text{ MeV})\)) obtained were then used for calculations of the astrophysical \(S\)-factors for the same reactions at \(E < 180 \text{ keV}\), including \(E=0\). However, the experimental astrophysical \(S\)-factors for the direct capture \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) reactions [22] used in [13] for the analysis have considerable spread. Consequently, the values of the ANC’s \(^3\text{He} + \alpha \rightarrow ^7\text{Be}\) and the \(S_{34}(0)\) obtained in [13] may not be enough accurate. Therefore, determination of precise experimental values of the ANC’s for \(^3\text{He} + \alpha \rightarrow ^7\text{Be}(\text{g.s.})\) and \(^3\text{He} + \alpha \rightarrow ^7\text{Be}(0.429 \text{ MeV})\) is highly desirable since it has direct effects in the correct extrapolation of the \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) astrophysical \(S\)-factor at solar energies [21][24].

In this work new analysis of the highly precise experimental astrophysical \(S\)-factors for the direct capture \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) reaction at extremely low energies \((\gtrsim 90 \text{ keV})\) [6-10] is performed within the modified two-body potential approach [24] to obtain “indirectly measured” values both of the ANC’s (the NVC’s) for \(^3\text{He} + \alpha \rightarrow ^7\text{Be}(\text{g.s.})\) and \(^3\text{He} + \alpha \rightarrow ^7\text{Be}(0.429)\), and of \(S_{34}(E)\) at \(E \lesssim 90 \text{ keV}\), including \(E=0\). In the present work we show that one can extract ANC’s for \(^3\text{He} + \alpha \rightarrow ^7\text{Be}\) directly from the \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) reaction where the ambiguities inherent for the standard two-body potential model calculation of the \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) reaction being connected with the choice of the geometric parameters (the radius \(r_o\) and the diffuseness \(a\)) for the Woods–Saxon potential and the spectroscopic factors, can be reduced in the physically acceptable limit, being within the experimental errors for the \(S_{34}(E)\).

The contents of this paper are as follows. In Section 2 basic formulae of the modified two-body potential approach to the direct radiative capture \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) reaction are given. There the analysis of the precise measured astrophysical \(S\)-factors for the direct radiative capture \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) reaction is performed (Subsections 2.2-2.4). The conclusion is given in Section 3.

## 2 Analysis of \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) reaction

### 2.1 Basic formulae

Here we give the formulae specialized for the \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) astrophysical \(S\)-factor. Let us write \(l_f\) \((j_f)\) for the relative orbital (total) angular moment of \(^3\text{He}\) and \(\alpha\)-particle in nucleus \(^7\text{Be}(\alpha+^3\text{He})\), \(l_i\) \((j_i)\) for the orbital (total) angular moment of the relative motion of the colliding particles in the initial state, \(\lambda\) for multipole order of the electromagnetic transition, \(\eta_f(\eta_i)\) for the Coulomb parameter for the \(^7\text{Be}(\alpha+^3\text{He})\) bound \(^3\text{He}\)-scattering state and \(\mu\) for the reduced mass of the \((^3\text{He}\alpha)\)-pair. For the \(^3\text{He}(\alpha, \gamma)^7\text{Be}\) reaction populating the ground and first excited \((E^*=0.429 \text{ MeV}; J^*=1/2^-)\) states of \(^7\text{Be}\), the values of \(j_f\) are taken to be equal to \(3/2\) and \(1/2\), respectively, the value of \(l_f\) is taken to be equal to \(1\) as well as \(l_i=0, 2\) for the \(E1\)-transition and \(l_i=1\) for the \(E2\)-transition.
According to \cite{23,24}, for fixed \( l_f \) and \( j_f \) we can write the astrophysical \( S \)-factor, \( S_{l_f j_f}(E) \), in the following form
\[
S_{l_f j_f}(E) = C_{l_f j_f}^2 \mathcal{R}_{l_f j_f}(E, C_{l_f j_f}^{(sp)}).
\] (1)

Here, \( C_{l_f j_f} \) is the ANC for \(^3\)He + \( \alpha \to ^7\)Be, which determines the amplitude of the tail of the \(^7\)Be nucleus bound state wave function in the \((\alpha + ^3\)He\)-channel and is related to the NVC \( G_{l_f j_f} \) for the virtual decay \(^7\)Be \( \to \alpha + ^3\)He and to the spectroscopic factor \( Z_{l_f j_f} \) for the \((^3\)He + \( \alpha \))-configuration with the quantum numbers \( l_f \) and \( j_f \) in the \(^7\)Be nucleus as \cite{23}
\[
G_{l_f j_f} = -i^{l_f + \eta_f} \sqrt{\frac{\mu}{E}} C_{l_f j_f} \tag{2}
\]
and
\[
C_{l_f j_f} = Z_{l_f j_f}^{1/2} C_{l_f j_f}^{(sp)}, \tag{3}
\]
respectively, and
\[
\mathcal{R}_{l_f j_f}(E, C_{l_f j_f}^{(sp)}) = \frac{\tilde{S}_{l_f j_f}(E)}{\langle C_{l_f j_f}^{(sp)} \rangle^2}, \tag{4}
\]

where \( \tilde{S}_{l_f j_f}(E) = \sum_{\lambda} \tilde{S}_{l_f j_f \lambda}(E) \) is the single-particle astrophysical \( S \)-factor \cite{5} and \( C_{l_f j_f}^{(sp)} \) is the single-particle ANC, which determines the amplitude of the tail of the single-particle wave function of the bound \(^7\)Be(\( \alpha + ^3\)He) state. In (2) the factor taking into account the nucleon’s identity \cite{23} is absorbed in the \( C_{l_f j_f} \). The single-particle bound state wave function, \( \varphi_{l_f j_f}(r) \), is determined by the solution of the radial Schrödinger equation with the phenomenological Woods–Saxon potential for the given quantum numbers \( n \) (\( n \) is the nodes of \( \varphi_{l_f j_f}(r) \)), \( l_f \) and \( j_f \) as well as geometric parameters of \( r_o \) and \( a \), and with depth adjusted to fit the binding energy of the \(^7\)Be bound state with respect to the \((\alpha + ^3\)He\)-channel. Note that in Eq. (1) the dependence of the function \( \mathcal{R}_{l_f j_f}(E, C_{l_f j_f}^{(sp)}) \) on the free parameter \( C_{l_f j_f}^{(sp)} \) also enters through the single-particle wave function \( \varphi_{l_f j_f}(r; C_{l_f j_f}^{(sp)}) \equiv \varphi_{l_f j_f}(r) \) \cite{26}, and the single-particle ANC \( C_{l_f j_f}^{(sp)} \) in turn is itself a function of the geometric parameters of \( r_o \) and \( a \), i.e., \( C_{l_f j_f}^{(sp)} = C_{l_f j_f}^{(sp)}(r_o, a) \).

According to \cite{24}, the peripheral character for the direct capture \(^3\)He(\( \alpha, \gamma \))^7Be reaction is conditioned by
\[
\mathcal{R}_{l_f j_f}(E, C_{l_f j_f}^{(sp)}) = f(E) \tag{5}
\]
as a function of the \( C_{l_f j_f}^{(sp)} \) within the energy range \( E_{\text{min}} \leq E \leq E_{\text{max}} \), where the left hand side (l.h.s.) of Eq. (5) must not depend on \( C_{l_f j_f}^{(sp)} \) for each fixed \( E \) from the aforesaid energy range, and by
\[
C_{l_f j_f}^2 = \frac{S_{l_f j_f}(E)}{\mathcal{R}_{l_f j_f}(E, C_{l_f j_f}^{(sp)})} = \text{const} \tag{6}
\]
for each fixed \( E \) and the function of \( \mathcal{R}_{l_f j_f}(E, C_{l_f j_f}^{(sp)}) \) from \cite{5}.

As it was previously shown in \cite{24} and \cite{27} for the direct capture \( t(\alpha, \gamma)^7\)Li and \(^7\)Be(\( p, \gamma \))\(^8\)B reactions, respectively, fulfillment of the conditions (5) and (6) enables one also to obtain
valuable information about the experimental ("indirectly measured") value of the ANC $(C_{ijjj}^{\exp})^2$ for $^3$He + α → $^7$ Be by using $S_{ijjj}^{\exp}(E)$ instead of $S_{ijjj}(E)$ in the right hand side (r.h.s.) of Eq. (6):

$$(C_{ijjj}^{\exp})^2 = \frac{S_{ijjj}^{\exp}(E)}{R_{ijjj}(E; C_{ijjj}^{(sp)})}.$$  

(7)

Then the value of the ANC, $(C_{ijjj}^{\exp})^2$, obtained from Eq. (7) together with the condition (5) can be used for calculation of $S_{ijjj}(E)$ at energies of $E < E_{min}$ by using the following expression:

$$S_{ijjj}(E) = (C_{ijjj}^{\exp})^2 R_{ijjj}(E; C_{ijjj}^{(sp)}).$$  

(8)

Note that the total astrophysical $S$-factor for the $^3$He(α, γ)$^7$Be(g.s. + 0.429MeV) reaction is given by

$$S_{34}(E) = \sum_{j_1,1/2,3/2} S_{ijjj}(E) = C_{11/2}^2 R_{11/2}(E, C_{11/2}^{(sp)}) + C_{13/2}^2 R_{13/2}(E, C_{13/2}^{(sp)})$$  

(9)

$$= C_{13/2}^2 R_{13/2}(E, C_{13/2}^{(sp)})[1 + R(E)]$$  

(10)

$$= C_{11/2}^2 R_{11/2}(E, C_{11/2}^{(sp)})[1 + R^{-1}(E)]$$  

(11)

$$= C_{13/2}^2 R_{13/2}(E, C_{13/2}^{(sp)}) + \lambda_C R_{11/2}(E, C_{11/2}^{(sp)})]$$  

(12)

where $R(E) = S_{11/2}(E)/S_{13/2}(E) = C_{11/2}^2 R_{11/2}(E, C_{11/2}^{(sp)})/C_{13/2}^2 R_{13/2}(E, C_{13/2}^{(sp)})$ is a branching ratio and $\lambda_C = (C_{11/2}/C_{13/2})^2$.

Values obtained in such a way for the $(C_{ijjj}^{\exp})^2$ and $S_{ijjj}(E)$ at energies of $E < E_{min}$ can be considered as an "indirect measurement" of the ANC (or NVC) for $^3$He + α → $^7$ Be and of the astrophysical $S$-factor for the direct capture $^3$He(α, γ)$^7$Be reaction at $E < E_{min}$, including $E = 0$. It should be noted that the expressions (1) and (11)-(12) allow one to determine both the absolute value of ANC (or NVC) for $^3$He + α → $^7$ Be and that of the astrophysical S-factor $S_{ijjj}(E)$ for the peripheral direct capture $^3$He(α, γ)$^7$Be reactions at extremely low experimentally inaccessible energy regions by means of the analysis of the same precisely measured values of the experimental astrophysical $S$-factors, $S_{ijjj}^{\exp}(E)$ and $S_{34}^{\exp}(E)$.

2.2 The asymptotic normalization coefficients for $^3$He + α → $^7$ Be

To determine the ANC values for the $^3$He + α → $^7$ Be(g.s) and $^3$He + α → $^7$ Be(0.429 MeV) the experimental astrophysical $S$-factors, $S_{ijjj}^{\exp}(E)$, for the $^3$He(α, γ)$^7$Be reaction populating the ground ($l_f = 1$ and $j_f=3/2$) and first excited ($E^*\approx 0.429$ MeV; $J^* = 1/2^-$, $l_f = 1$ and $j_f=1/2$) states are reanalyzed based on the relation (1), the conditions (2) and (4), and the relations (7) and (8). As it was mentioned above, the experimental data have been obtained by different authors, which have considerable spread with experimental uncertainty being more than 10%. Recently, S.B. Nara Singh et al. [6], D. Benmerer et al. [7], Gy. Gyüky et al.
F. Confortola et al. [2] and T.A.D. Brown et al. [10] have apparently performed the most accurate direct measurement of the total astrophysical $S$-factor for the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction, covering the energy ranges $E=92.9-168.9$ keV [7–9], 420–951 keV [8] and 327–1235 keV [10] with absolute uncertainty not exceeding 5%. However, one should note that the experimental astrophysical $S$-factors for the the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction populating to the first and excited states of the residual nucleus have been separated only for the energies of $E=92.9$, 105.6 and 147.7 keV in [9] and for all experimental points of $E$ from the aforesaid energy region in [10]. Whereas, in [10] the experimental astrophysical $S$-factors were measured by using two different experimental approaches: the detection of the delayed $\gamma$ ray from $^7\text{Be}$ (the activation) and the measurement of the prompt $\gamma$ emission (the prompt). So, in our analysis we naturally use $S_{34}^{exp}(E)$ most recently independently measured in Refs.[6–9] and [10], since the reaction under consideration is nonresonant and, consequently, proceeds mainly in regions well outside the internuclear interaction range [21].

The Woods–Saxon potential split with a parity ($l$-dependence) for the spin-orbital term proposed by the authors of Refs. [28–30] is used here for the calculations of both bound state radial wave function $\varphi_{ljf}(r)$ and scattering wave function $\psi_{ljf}(r)$. It should be emphasized that the choice of this potential is based on the following considerations. Firstly, this potential form is justified from the microscopic point of view because it makes it possible to take into account the Pauli principle between nucleons in $^3\text{He}$- and $\alpha$-clusters in the $(\alpha+^3\text{He})$ bound state by means of inclusion of deeply bound states forbidden by the Pauli exclusion principle, i.e. without an explicit introduction of a repulsive core at small distance. The latter imitates the additional node ($n$) arising in the wave functions of $\alpha-^3\text{He}$ relative motion in $^7\text{Be}$. Secondly, this potential describes well the phase shifts for $^3\text{He}$-scattering in the wide energy range [29–30].

The test of the peripheral character of the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction for the aforesaid energy range has been made by means of verifying the conditions (5) and (6), and by changing the geometric parameters (radius $r_o$ and diffuseness $a$) of the adopted Woods–Saxon potential using the procedure of the depth adjusted to fit the binding energies , as it was done in Ref. [21]. According to Ref. [24], we vary $r_o$ and $a$ in the physically acceptable ranges ($r_o$ in 1.62–1.98 fm and $a$ in 0.63–0.77 fm) in respect to the standard values ($r_o=1.80$ fm and $a=0.70$ fm [29–30]). Such a choice of the $r_o$ and $a$ parameters variation limit allows us to provide fulfillment of the conditions (5) and (6) in the aforesaid energy range within the experimental errors for the $S_{ljf}^{exp}(E)$.

As an illustration, Fig.1 shows plots of the $R_{ljf}(E, C^{(sp)}_{ljf})$ dependence on the single-particle ANC, $C^{(sp)}_{ljf}$ for $l_f=1$ and $j_f=3/2$ and $1/2$ only for the two values of energy $E$. The width of the band for these curves is the result of the weak “residual” ($r_o, a$)-dependence of the $R_{ljf}(E, C^{(sp)}_{ljf})$ on the parameters $r_o$ and $a$ (up to ±2%) for the $C^{(sp)}_{ljf} = C^{(sp)}_{ljf}(r_o, a) = \text{const}$ [24, 21]. The same dependence is also observed at other energies. It is seen that for the calculated values of $R_{ljf}(E, C^{(sp)}_{ljf})$ the dependence on the $C^{(sp)}_{ljf}$ values is rather weak (no more than ±5.0% ) in the interval of $3.205 \leq C^{(sp)}_{13/2} \leq 4.397$ fm$^{-1/2}$ (2.788 $\leq C^{(sp)}_{11/2} \leq 3.763$ fm$^{-1/2}$) for the $^3\text{He}(\alpha, \gamma)^7\text{Be}(\text{g.s.})$ ($^3\text{He}(\alpha, \gamma)^7\text{Be}(0.429 \text{MeV})$) reaction, which corresponds to the parameters
Figure 1: The dependence of $R_{I_{f}j_{f}}(E, C^{(sp)}_{I_{f}j_{f}})$ as a function of the single-particle ANC, $C^{(sp)}_{I_{f}j_{f}}$, for the $^3\text{He}(\alpha, \gamma)^7\text{Be}(\text{g.s.})$ $((l_{f},j_{f})=(1,3/2))$ and $^3\text{He}(\alpha, \gamma)^7\text{Be}(0.429 \text{ MeV}) ((l_{f},j_{f})=(1,1/2))$ reactions at different energies E.

of the adopted Woods–Saxon potential $r_{o}$ ranging from 1.62–1.98 fm and $a$ in the range of 0.63–0.77 fm. It follows from here that the condition (5) is satisfied for the considered reaction within the uncertainties not exceeding the experimental errors of $S_{I_{f}j_{f}}^{\text{exp}}(E)$.

We also calculated the $^3\text{He}$-elastic scattering phase shifts by variation of the parameters $r_{o}$ and $a$ in the same range for the adopted Woods–Saxon potential. The results of the calculations corresponding to $s$- and $p$-waves are presented in Fig.2 in which the width of the bands corresponds to a change of phase shifts values with respect to variation of values of the $r_{o}$ and $a$ parameters. As it is seen from Fig.2 the experimental phase shifts [31] are well reproduced within uncertainty of about $\pm$ 5%.

This circumstance allows us to test the condition (9), which is no less essential for the peripheral character of these reactions, at the energies of $E= 92.9, 105.6$ and 147.7 keV for which the $^3\text{He}(\alpha, \gamma)^7\text{Be}(\text{g.s.})$ and $^3\text{He}(\alpha, \gamma)^7\text{Be}(0.429 \text{ MeV})$ astrophysical $S$-factors were separately measured in [9]. As an illustration, for the same energies $E$ as in Fig.1 we present in Fig.3 (the upper panels) the results of $C_{I_{f}j_{f}}^{2}$-value calculation given by Eq. (5) $((l_{f},j_{f})=(1,3/2)$ and $(1,1/2))$ in which instead of the $S_{I_{f}j_{f}}(E)$ the experimental $S$-factors for the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction populating the ground and first excited states of $^7\text{Be}$ were taken. It is also noted that the same dependence occurs for other considered energies. It is seen from this figure that the obtained $C_{I_{f}j_{f}}^{2}$ values also weakly depend on the $C^{(sp)}_{I_{f}j_{f}}$ value. However, the values of the spectroscopic factors $Z_{1\,3/2}$ and $Z_{1\,1/2}$ corresponding to the $(\alpha+^3\text{He})$-configuration for
Figure 2: The energy dependence of the $^3$He-o-elastic scattering phase shifts for different partial waves. The experimental data are from [31]. The bands are our calculated data. The width of the bands for fixed energies corresponds to the variation of the parameters $r_o$ and $a$ of the adopted Woods-Saxon potential within the intervals of $r_o=1.62$ to 1.98 fm and $a=0.63$ to 0.77 fm.
$^7\text{Be}(g.s.)$ and $^7\text{Be}(0.429\text{keV})$, respectively, change strongly (see, the lower panels in Fig.3). The

![Graphs showing dependence of ANC's $C_{l_{j_{ff}}}$ (upper band) and spectroscopic factors $Z_{l_{j_{ff}}}$ (lower band) on single-particle ANC $C_{l_{j_{ff}}}^{(sp)}$ for $^3\text{He}(\alpha,\gamma)^7\text{Be}(g.s.)$ (left column, $(l_f, j_f)=(1,3/2)$) and $^3\text{He}(\alpha,\gamma)^7\text{Be}(0.429\text{MeV})$ (right column, $(l_f, j_f)=(1,1/2)$) reactions at different energies $E$.](image)

Figure 3: The dependence of the ANC's $C_{l_{j_{ff}}}$ (upper band) and the spectroscopic factors $Z_{l_{j_{ff}}}$ (lower band) on the single-particle ANC $C_{l_{j_{ff}}}^{(sp)}$ for the $^3\text{He}(\alpha,\gamma)^7\text{Be}(g.s.)$ (the left column, $(l_f, j_f)=(1,3/2)$) and $^3\text{He}(\alpha,\gamma)^7\text{Be}(0.429\text{MeV})$ (the right column, $(l_f, j_f)=(1,1/2)$) reactions at different energies $E$.

calculation shows that the uncertainty in $\mathcal{R}_{l_{j_{ff}}}(E, C_{l_{j_{ff}}}^{(sp)})$ and $C_{l_{j_{ff}}}^2$ is up to ±5.0% relative to the central values of $\mathcal{R}_{l_{j_{ff}}}(E, C_{l_{j_{ff}}}^{(sp)})$ and $C_{l_{j_{ff}}}^2$, obtained for the standard values of $r_o = 1.80$ fm and $a = 0.60$ fm, for the $(r_o, a)$-pair varying in the above mentioned intervals for $r_o$ and $a$, while the uncertainty in the $Z_{l_{j_{ff}}}$ is about ±30%. It should be noted that the uncertainty in the $C_{l_{j_{ff}}}^2$ values becomes even less when in one uses the experimental astrophysical $S$-factors corresponding to smaller energies. Thus, the peripheral character of the reactions under consideration allows one to determine the $C_{1/3/2}^2$ and $C_{1/1/2}^2$ values for the $\alpha +^3\text{He} \rightarrow^7\text{Be}(g.s.)$
and $\alpha^3\text{He} \rightarrow ^7\text{Be}(0.429\text{ keV})$, respectively, with a maximal uncertainty of about $\pm 5.0\%$ when the geometric parameters $r_o$ and $a$ are varied within the aforesaid ranges and the experimental data are used at the aforesaid three energies in the analysis.

For different energies $E$ we also estimate a relative contribution of the nuclear interior ($r < r_N$) to the astrophysical $S$-factors for the $^3\text{He}(\alpha,\gamma)^7\text{Be}$ reaction populating the ground and first excited states in dependence on the variation $C_{l_{ij}j}^{(sp)}$ (or $r_o$ and $a$) introducing the cutoff radius $r_{cut}$ ($r_{cut} \approx r_N$) in the lower limit of integration of the radial integral (10) of Ref.[24] entering in the amplitude of the reaction under consideration. With this aim one considers the ratio $\Delta(E, C_{l_{ij}j}^{(sp)}; r_{cut}) = |R_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)}; r_{cut}) - \tilde{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)}; r_{cut})|/|\mathcal{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)})|$, where $\tilde{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)}; r_{cut})$ is given by Eqs.(10) and (13) of Ref.[24], but in the radial integral (10) of Ref.[24] the integration over $r$ is performed in the interval $r_{cut} \leq r \leq \infty$, i.e. $\mathcal{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)}; 0) = \mathcal{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)})$. The $\mathcal{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)})$ and $\tilde{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)}; r_{cut})$ functions were calculated for different values of the single-particle ANC $C_{l_{ij}j}^{(sp)}$ (or the parameters $r_o$ and $a$). A value of the cutoff radius is taken as in Ref. [32], that is $r_{cut} = r_{cut}^o = 1.36(4/3 + 3/3)^2=4.12$ fm, as well as $r_{cut}=4.00$ fm and 4.25 fm. The calculation of $\Delta(E, C_{l_{ij}j}^{(sp)}; r_{cut})$ performed at different energies shows that the quantities of $\Delta(E, C_{l_{ij}j}^{(sp)}; r_{cut})$ change from $5.4\%$ up to $15.5\%$ under variation of $C_{l_{ij}j}^{(sp)}$ and $r_{cut}$. The calculation shows that the contribution of the nuclear interior ($r < r_N$) to the astrophysical $S$-factors calculated for different sets of geometric parameters $r_o$ and $a$ of the Woods–Saxon potential, and values of $r_{cut}$ does not exceed about $15.5\%$ and this small quantity is due mainly to oscillations observed in the integrand of the radial integral (10) of Ref.[24]. As an illustration of this fact, in Fig.1 we show a dependence of the integrand of the radial integral (10) of Ref.[24] on geometric parameters $r_o$ and $a$ of the Woods–Saxon potential and values of $r_{cut}$ for the $^3\text{He}(\alpha, \gamma)^7\text{Be}(\text{g.s.})$ reaction at different energies. As one can see from Fig.1 the integrand of the radial integral changes with the variation of the geometric parameters $r_o$ and $a$, which is associated with changes of the calculated bound ($\alpha + ^3\text{He}$) state wave function and the calculated $^3\text{He}o$-scattering wave function, and these wave functions indeed reached simultaneously their asymptotic form for $r \gtrsim 5.0$ fm. Such a change leads to calculated $S_{l_{ij}j}(E)$ that vary by 1.75 times over the energy region $92.9 \leq E \leq 1200$ keV, while the calculated values of the function $\mathcal{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)})$ change by only $\pm 5\%$ with respect to the value of $\mathcal{R}_{l_{ij}j}(E, C_{l_{ij}j}^{(sp)})$ corresponding to the standard values of $r_o=1.80$ fm and $a=0.70$ fm. Besides, it is seen from Fig.1 that the behavior of the integrand in the radial integral (10) of Ref.[24] over a wide energy range provides a strong suppression of the contribution only from the part of the nuclear interior with $0 \leq r \lesssim 2.0$ fm to the integral (10) of Ref.[24]. But a noticeable change of the integrand of the aforesaid integral is observed with the variation of the parameters $r_o$ and $a$ in the range $2.0 \lesssim r \lesssim 6.0$ fm. The similar situation occurs for the $^3\text{He}(\alpha, \gamma)^7\text{Be}(0.429$ MeV) reaction. Therefore, the choice of the cutoff radius $r_{cut}$ becomes ambiguous since a fitted value of $r_{cut}$ also becomes dependent on the parameters $r_o$ and $a$. In this connection one would like to note the following. In paper [18] the calculation of the astrophysical $S$-factors for the reactions under consideration has been carried out using the expression (11) but introducing the cutoff radius $r_{cut}$ in the lower limit of integration in the radial integral (10) of Ref.[24] and replacing
the bound state wave function $\varphi_{IJJ}(r)$ with its asymptotic form starting from $r = r_{cut}$. At this the best fitting of the calculated $S_{IJJ}(E)$ to the experimental ones [22] was reached when the cutoff radius was $r_{cut} = 4.0$ fm. It is seen from here that in paper [18] the contribution of the nuclear interior to the calculated astrophysical $S$-factors was indeed underestimated [2], since contribution of the nuclear interior $0 < r \leq 4.0$ fm to the calculated astrophysical $S$-factors, which is up to about 14%, has not been taken into account. Here, firstly, the contribution of the nuclear interior ($r \leq r_N$) to the calculated astrophysical $S$-factors is taken into account in a correct way by means of the appropriate choice of the adopted potential both for the initial state and for final state of the reactions under consideration. Secondly, the problem of the ambiguity connected with the strong $(r_o, a)$-dependence of the calculated astrophysical $S$-factors is removed by inclusion of the information about ANC (or NVC). The latter reduces this ambiguity to minimum. At last, in the present work the more precise experimental data for the $^3\text{He}(\alpha, \gamma)^7\text{Be}(g.s.)$ reaction within the considered energy ranges is peripheral.

For each energy $E$ experimental point ($E=92.9$, 105.6 and 147.7 keV) the values of the ANC’s are obtained for the $\alpha + ^3\text{He} \rightarrow ^7\text{Be}(g.s.)$ and $\alpha + ^3\text{He} \rightarrow ^7\text{Be}(0.429 \text{ MeV})$ by using the corresponding experimental astrophysical $S$-factor ($S_{1\frac{3}{2}}^{exp}(E)$ and $S_{1\frac{1}{2}}^{exp}(E)$, (the activation))

\[\text{Figure 4: The integrand of the radial integral (10) of Ref. [24] for the }^3\text{He}(\alpha, \gamma)^7\text{Be(g.s.) reaction at energies } E=0.1056 \text{ and } 0.506 \text{ and } 0.951 \text{ MeV for different sets of } (r_o; a)\text{-pairs:}(1.62 \text{ fm}; 0.63 \text{ fm}) \text{ (dashed line), (1.80 fm}; 0.70 \text{ fm}) \text{ (solid line) and (1.98 fm}; 0.77 \text{ fm}) \text{ (dotted line).} \]

\[\text{It should be noted that there is a misprint in the line 36 upper of section 3 of [18]. There the phrase "no more than 1\% to" must be written as "no more than 10\% to".} \]
in the ratio of the r.h.s. of the relation \( \mathcal{R}_{ljj}(E) \) instead of the \( S_{ljj}(E) \) and the central values of \( \mathcal{R}_{ljj}(E, C_{ljj}^{(sp)}) \) corresponding to the adopted values of the parameters \( r_0 \) and \( a \). The results of the ANC’s, \((C_{13/2}^{\exp})^2\) and \((C_{11/2}^{\exp})^2\) for these three energy \( E \) experimental points are displayed in Figs. 5a and 5b (filled circle symbols), and the second and third columns of Table 1. The

Figure 5: The values of the ANC’s, \( C_{13/2}^2 \), for the \( \alpha + ^3\text{He} \rightarrow ^7\text{Be}(g.s.) \) for each energy \( E \) experimental point. The opened triangle and diamond symbols (filled star and square symbols) are data obtained by using the total (separated) experimental astrophysical \( S \)-factors from \cite{6} and \cite{7–9} (a) from \cite{10}, the activation (b) and the prompt (c), respectively, while filled circle symbols are data obtained from the separated experimental astrophysical \( S \)-factors from Refs.\cite{7–9}. The symbols in (d) are data obtained from all experimental astrophysical \( S \)-factors. The solid lines present our results for the weighted means. Everywhere the width of each of the band is the weighted uncertainty.

uncertainties pointed in this figure correspond to those found from \cite{6} (averaged square errors (a.s.e.)), which include the total experimental errors in the corresponding experimental astrophysical \( S \)-factor and the aforesaid uncertainty in the \( \mathcal{R}_{ljj}(E, C_{ljj}^{(sp)}) \). One should note that
the same results for the ANC’s, $C^{2}_{1\ 3/2}$ and $C^{2}_{1\ 1/2}$, (the opened symbols in Figs.5b and 6a) are obtained when $S^{exp}_{34}(E)$ (or $S^{exp}_{34}(E)$ and $R^{exp}(E)$ [7] [9] are used in Eq.(9) (or in Eq.(10) and (11)) instead of $S_{34}(E)$ ($S_{34}(E)$ and $R(E)$). Then in Eq.(12), inserting the weighted means of

\[ \lambda_{C} (\lambda_{C}=0.666), \]

obtained from the three data (the filled circle symbols) plotted in Figs.5b and 6a, and replacing of the $S_{34}(E)$ in the l.h.s. of Eq.(12) with $S^{exp}_{34}(E)$ for the others, the two experimental points of energy $E$ ($E=126.5$ and 168.9 keV) from [7], four one $E$ ($E=420.0$, 506.0, 615.0 and 951.0 keV) from [6] and the three one $E$ ($E=93.3$, 106.1 and 170.1 keV) from [9] can also determine values of ANC’s, $C^{2}_{1\ 3/2}$ and $C^{2}_{1\ 1/2}$. The results of the ANC’s are also displayed in Figs.5b and 6a (the opened diamond and triangle symbols) and the second and third columns of Table 1. The same way the values of the ANC’s ($C^{2}_{1\ 3/2}$ and $C^{2}_{1\ 1/2}$) are obtained by using the separated experimental astrophysical $S$-factors ($S^{exp}_{1\ 3/2}$ and $S^{exp}_{1\ 1/2}$ [10] obtained by using the experimental astrophysical $S$-factors (the activation (b) and the prompt (c)). The results for ANC’s are presented in Figs.5b (the activation) and 5f (the prompt) as well as Figs.6b and 6r. The weighted means of the ANC-values and their uncertainties, deduced separately from each

Figure 6: The same as Fig.5 for the $\alpha +^3$He $\rightarrow$ $^7$Be(0.429 MeV).
experimental data, are displayed by the solid lines and the band widths, respectively, in these figures and also presented in the second and fourth columns of Table 2. The corresponding NVC-values are also presented in the third and fifth columns of Table 2. Besides, in Figs. 3 and 4 (the solid line) as well as the second and third columns of Table 2, the weighted means of the ANC-values derived from all of the experimental points of Figs. 3 (a, b and c) and Figs. 4 (a, b and c) are also presented. As it is also seen from Figs. 3 and 4 (the second and fourth columns of Table 1) (Table 2)) the values of the ANC’s obtained from the analysis of the experimental astrophysical S-factors measured by the authors of Refs.[6–10] in different intervals of energy E, agree rather well with each other within about 12%. Also, it is seen from here that the ratio in the r.h.s. of the relation (7) does not practically depend on the energy E, although absolute values of the corresponding experimental astrophysical S-factors for the reactions under consideration depend noticeably on the energy and change by up to about 1.7 times when E changes from 92.6 keV to 1200 keV.

This fact allows us to conclude that the energy dependence of the experimental astrophysical S-factors [6–10] is well determined by the calculated function \( \mathcal{R}_{ij} = (E, C^{(sp)}_{ij}) \) and \( \mathcal{R}_{13/2} = E, C^{(sp)}_{13/2} + \lambda C \mathcal{R}_{11/2} = E, C^{(sp)}_{11/2} \). Hence, the corresponding experimental astrophysical S-factors can be used as an independent source of reliable information about the ANC’s for the \( \alpha + ^3\text{He} \to ^7\text{Be}(g.s.) \) and \( \alpha + ^3\text{He} \to ^7\text{Be}(0.429 \text{ MeV}) \).

The weighted means of the ANC-values recommended by us for \( ^3\text{He} + \alpha \to ^7\text{Be}(g.s.) \) and \( ^3\text{He} + \alpha \to ^7\text{Be}(0.429 \text{ MeV}) \), obtained from all of the experimental data presented in Figs. 3 and 4, are equal to \( (C^{\text{exp}}_{1/2})^2 = 23.19 \pm 1.37 \text{ fm}^{-1} \), \( (C^{\text{exp}}_{1/2})^2 = 15.73 \pm 1.02 \text{ fm}^{-1} \), \( (C^{\text{exp}}_{3/2})^2 = 4.82 \pm 0.14 \text{ fm}^{-1/2} \), and \( (C^{\text{exp}}_{3/2})^2 = 3.97 \pm 0.13 \text{ fm}^{-1/2} \). One should note that the values of \( C_{1/2} \) and \( C_{1/2} \) should not be equal, in contrast with the assumption made in Ref.[11]. The corresponding values of the NVC’s are \( |G_{1/2}|^2 = 1.11 \pm 0.07 \text{ fm} \) and \( |G_{1/2}|^2 = 0.75 \pm 0.05 \text{ fm} \). As noted earlier in paper [12], the values ANC’s (NVC’s) \( C^2_{1/2} = 18.19 \text{ fm}^{-1} \) and \( C^2_{1/2} = 15.02 \text{ fm}^{-1} \) ( \( |G_{1/2}|^2 = 0.86 \text{ fm} \) and \( |G_{1/2}|^2 = 0.71 \text{ fm} \)) were obtained from the experimental data analysis [22]. But we mentioned above, in paper [12], firstly, the contribution of the nuclear interior \( (r < 4.0 \text{ fm}) \) to the calculated astrophysical S-factors was not included and, secondly, the values of ANC’s were obtained from the analysis of the experimental data [22], which has considerable spread. It is seen that taking into account the contribution of the nuclear interior and use the experimental data more accurate than those in Ref.[12] one can strongly influence the extracted values of the ANC’s for \( ^3\text{He} + \alpha \to ^7\text{Be}(g.s.) \) and \( ^3\text{He} + \alpha \to ^7\text{Be}(0.429 \text{ MeV}) \). A comparison of the present result and that obtained in paper [12] shows that the underestimate of the contribution both of the nuclear interior and of the nuclear exterior indeed occurs in [12] since the present value of ANC \( C^2_{1/2} \) obtained from the analysis of the more accurate experimental astrophysical S-factor [6–10] is larger than that obtained in [12].

The resulting ANC (NVC) values obtained by us are in good agreement with the values \( C^2_{1/2} = 20.52 \text{ fm}^{-1} \) and \( C^2_{1/2} = 15.23 \text{ fm}^{-1} \) ( \( |G_{1/2}|^2 = 0.97 \text{ fm} \) and \( |G_{1/2}|^2 = 0.72 \text{ fm} \) ) [33]. However, the results recommended by us for these ANC’s differ noticeably from the values \( C^2_{1/2} = 12.60 \pm 1.07 \text{ fm}^{-1} \) and \( C^2_{1/2} = 8.41 \pm 0.58 \text{ fm}^{-1} \) \( (C_{1/2} = 3.55 \pm 0.15 \text{ fm}^{-1/2}, C_{1/2} = 2.90 \pm 0.10 \text{ fm}^{-1/2}, |G_{1/2}|^2 = 0.60 \pm 0.05 \text{ fm} \) and \( |G_{1/2}|^2 = 0.40 \pm 0.03 \text{ fm} \) ) [17] as well as those \( C^2_{1/2} = C^2_{1/2} = 12.36 \text{ fm}^{-1} \) \( (C_{1/2} = C_{1/2} = 3.79 \text{ fm}^{-1/2} \) and \( |G_{1/2}|^2 = |G_{1/2}|^2 = 0.68 \text{ fm} \) ) [11]. In this
connection one would like to draw attention to the following. The bound state wave functions and the initial state wave functions in [17] were computed with different potentials and, so, these calculations were not self-consistent. Besides, the values of the binding energies for the bound states of $^7\text{Be}$ calculated in Ref.[17] differ from the experimental ones. Therefore, the calculated value of the binding energy for the bound state of $^7\text{Be}$ (g.s.) in the $(\alpha + ^3\text{He})$-channel (4.73 MeV, see Table I in Ref.[17]) is also not in agreement with the experimental one (1.59 MeV). Since the ANC’s (or NVC’s) for $^3\text{He} + \alpha \rightarrow ^7\text{Be}$ are sensitive to the form of the NN potential [12], it is desirable, firstly, to calculate the wave functions of the bound state using other forms of the NN potential, and, secondly, in order to guarantee the self-consistency, the same forms of the NN potential should be used for such calculation of the initial wave functions. Besides, one would also like to note the recent result of Ref.[14] obtained for $C_1 3/2$ and $C_1 1/2$ from the analysis of the experimental $^3\text{He}(\alpha, \gamma)^7\text{Be}$ astrophysical S-factors performed within the R-matrix method, where the contribution from the internal part of the amplitude was simulated by the background for a single pole. But there to reduce the number of free parameters the assumption about equality of the ANC’s $(C_1 3/2 = C_1 1/2)$ was used, and the best fitting of the data was reached at $C_1 3/2 = 3.79$ fm$^{-1/2}$ and the channel radius $r_c = 3.0$ fm. It follows from here that in reality the values of the ANC’s, $C_1 3/2$ and $C_1 1/2$, should not be equal. Moreover, the calculation shows that the asymptotic behavior of the bound $(\alpha + ^3\text{He})$ state and $^3\text{He}$-scattering wave functions is reached, as it was mentioned above, simultaneously only at $r_c \gtrsim 5.0$ fm and, so, at $r_c \geq 3.0$ fm their substitution for these wave functions in the external part of the amplitude in Ref.[14] is not correct.

2.3 $\alpha$-particle spectroscopic factors for the mirror $(^7\text{Li}^7\text{Be})$-pair

The “indirectly measured” values of the ANC’s for $^3\text{He} + \alpha \rightarrow ^7\text{Be}$ obtained in the present work and those for $\alpha + t \rightarrow ^7\text{Li}$ deduced in Ref.[21] can be used for obtaining information on the ratio $R_{Z;jf} = Z_{1;jf}(^7\text{Be})/Z_{1;jf}(^7\text{Li})$ for the virtual $\alpha$ decays of the bound mirror $(^7\text{Li}^7\text{Be})$-pair, where $Z_{1;jf}(^7\text{Be})/(Z_{1;jf}(^7\text{Li}))$ is the spectroscopic factor for $^7\text{Be}$ $(^7\text{Li})$ in the $(\alpha + ^3\text{He})/(\alpha + t)$-configuration. For this aim we can easily derive the following from Eq.[3]

$$R_{Z;jf} = \frac{R_{C;jf}}{R_{C^{(sp)};jf}},$$

(13)

where $R_{C;jf} = \left(C_{1;jf}(^7\text{Be})/C_{1;jf}(^7\text{Li})\right)^2 (R_{C^{(sp)};jf} = \left(C_{1;jf}^{(sp)}(^7\text{Be})/C_{1;jf}^{(sp)}(^7\text{Li})\right)^2)$ is the ratio of squares of the ANC’s (single-particle ANC’s) for the bound mirror $(^7\text{Li}^7\text{Be})$-pair and $j_f=3/2(1/2)$ for the ground (first excited) state of the mirror nuclei. It should be noted that in Eq.[13] by using the values of the ANC’s for the $t + \alpha \rightarrow ^7\text{Li}$ and $^3\text{He} + \alpha \rightarrow ^7\text{Be}$ obtained in Ref.[21] and in the present work, respectively, one can verify a validity of the approximation ($R_{C;jf} \approx R_{C^{(sp)};jf}$; i.e., $R_{Z;jf} \approx 1$) used in Refs.[21, 35] for the mirror $(^7\text{Li}^7\text{Be})$ conjugated $\alpha$ decays. For the bound (first excited) state of the mirror $(^7\text{Li}^7\text{Be})$-pair the ratio $R_{C^{(sp)};3/2}/R_{C^{(sp)};1/2}$ changes by only about $1.5\% (6\%)$ under the variation of the geometric parameters ($r_o$ and $a$) of the adopted Woods–Saxon potential [29, 30] within the aforesaid ranges. The ratios are equal to $R_{C^{(sp)};3/2}=1.37\pm 0.02$ and $R_{C^{(sp)};1/2}=1.40\pm 0.09$. These ratios can be determined by using the
values of the single-particle ANC’s for the mirror ($^7Li^7Be$)-pair obtained in the present work and deduced in Ref.\textsuperscript{[24]}. The ratios for the ANC’s are $R_C;3/2=1.85\pm 0.11$ and $R_C;1/2=1.75\pm 0.10$. From \textsuperscript{[13]} the values of the ratio $R_{Z;j_f}$ are equal to $R_{Z;3/2}=1.35\pm 0.08$ and $R_{Z;1/2}=1.25\pm 0.11$ for the ground and the first excited states, respectively. These values differ noticeably from those of $R_{Z;3/2}=0.995\pm 0.005$ and $R_{Z;1/2}=0.99$ calculated in Ref.\textsuperscript{[35]} within the microscopic cluster model. One of the reasons of these differences can be associated with the aforesaid approximation used in Ref.\textsuperscript{[35]}, which is hardly valid for the mirror ($^7Li^7Be$)-pair \textsuperscript{[36]}.

Thus, as it is seen from here that, in fact, the magnitudes of $R_{Z;j_f}$ differ noticeably from unity both for the ground state and for the first excited state of the mirror ($^7Li^7Be$) -pair.

2.4 Astrophysical S-factor for the $^3$He($\alpha, \gamma$)$^7$Be reaction at solar energies

The equation \textsuperscript{[8]} and the weighted means of the ANC’s obtained for the $^3$He+$\alpha$ $\rightarrow$ $^7$Be(\text{g.s}) and $^3$He + $\alpha$ $\rightarrow$ $^7$Be(0.429 MeV) can be used for calculating the $^3$He($\alpha, \gamma$)$^7$Be astrophysical S-factor for capture to the ground and first excited states as well as the total astrophysical S-factor at solar energies ($E \leq 25$ keV). At first, we tested again the fulfilment of the condition \textsuperscript{[1]} in the same way as it is done above for $E \geq 90$ keV. Similar results plotted in Fig.\textsuperscript{[4]} are also observed for the $R_{l;j_f}(E, C_{l;j_f}^{(sp)})$ function dependence on the single-particle ANC, $C_{l;j_f}^{(sp)}$, at energies of $E < 90$ keV.

The astrophysical S-factors for the $^3$He($\alpha, \gamma$)$^7$Be(g.s.) and $^3$He($\alpha, \gamma$)$^7$Be(0.429 MeV) reactions are displayed in Figs.\textsuperscript{[7]} and \textsuperscript{[8]} respectively, as well as presented in Table \textsuperscript{[1]}. In Figs.\textsuperscript{[7]} and \textsuperscript{[8]}, the opened diamond and triangle symbols show our result obtained from the analysis of the total experimental astrophysical S-factors of \textsuperscript{[7-9]} and \textsuperscript{[6]} by using the corresponding values of the ANC’s for each energy $E$ experimental point presented in Table \textsuperscript{[1]}. In Figs.\textsuperscript{[7]} and \textsuperscript{[8]} \textsuperscript{(a, b and c)} the experimental data plotted by the filled circle symbols (filled star and square symbols) are taken from \textsuperscript{[9]} (from \textsuperscript{[10]}). The symbols in Figs.\textsuperscript{[7]} \textsuperscript{l} and \textsuperscript{8} \textsuperscript{l} are data of all experiments \textsuperscript{[9] [10]} and our ones. The opened circle symbols in these figures are the results of the extrapolation recommended in the present work. The solid lines present our calculations performed with the standard values of geometric parameters $r_o=1.80$ fm and $a=0.70$ fm both for the bound ($\alpha + ^3$He) state and for $^3$He-$\alpha$-scattering state. Everywhere the width of each of the band is the weighted uncertainty, which includes the uncertainties in the “indirectly measured” values of ANC’s and the aforesaid uncertainty in the $R_{l;j_f}(E, C_{l;j_f}^{(sp)})$. 

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Figure 7: The astrophysical S-factors for the $^3$He($\alpha,\gamma$)$^7$Be(g.s.):$^\ell_f, j_f=1,3/2$) reaction. The opened diamond and triangle symbols (a) are our result separated from the total experimental astrophysical $S$-factors of Refs.[7-9] and [10], respectively. The filled circle symbols (filled star and square symbols) are experimental data of Ref. [9] (Ref. [10], the activation (b) and the prompt (c)). The opened circle symbols are our results of the extrapolation. The symbols in d are data of all experiments [9,10] and the present work. The solid lines present our calculations performed with the standard values of geometric parameters $r_0=1.80$ fm and $a=0.70$ fm both for the bound ($\alpha + ^3$He) state and for $^3$He-$\alpha$-scattering state. Everywhere the width of each of the band is the weighted uncertainty.
Figure 8: The same as Fig.7 for the $^3\text{He}(\alpha, \gamma)^7\text{Be}(0.429 \text{ MeV})((l_f, j_f)=(1, 1/2))$ reaction.
Table 1: The "indirectly measured" values of the asymptotic normalization constants \((C_{13/2}^{exp})^2\) and \((C_{11/2}^{exp})^2\) for \(^3\text{He}+\alpha \rightarrow ^7\text{Be}\), the experimental astrophysical S-factors (\(S_{ij}^{exp}\) and \(S_{34}^{exp}(E)\)) and branching ratio (\(R^{exp}(E)\)) at different energies \(E\).

| \(E\) (keV) | \((C_{13/2}^{exp})^2\) (fm\(^{-1}\)) | \(S_{13/2}^{exp}\) (keV b) | \(S_{34}^{exp}(E)\) (keV b) | \(R^{exp}(E)\) |
|-------------|---------------------------------|-----------------|-----------------|--------------|
| j\(_f=3/2\) | j\(_f=1/2\) | j\(_f=3/2\) | j\(_f=1/2\) | j\(_f=3/2\) | j\(_f=1/2\) | j\(_f=3/2\) | j\(_f=1/2\) | j\(_f=3/2\) | j\(_f=1/2\) | j\(_f=3/2\) | j\(_f=1/2\) |
| 92.9        | 22.02±1.84 | 14.04±1.16 | 0.387±0.03 \(\text{[9]}\) | 0.147±0.012 \(\text{[9]}\) | 0.534±0.023 \(\text{[9]}\) | 0.380±0.03 \(\text{[9]}\) |
| 93.3        | 21.41±1.38 | 14.66±0.94 | 0.374±0.02 | 0.153±0.009 | 0.527±0.03 \(\text{[9]}\) | 0.409±0.03 |
| 105.6       | 20.95±1.79 | 14.55±1.21 | 0.365±0.03 \(\text{[9]}\) | 0.151±0.012 \(\text{[9]}\) | 0.516±0.03 \(\text{[9]}\) | 0.415±0.03 \(\text{[9]}\) |
| 106.1       | 21.23±1.28 | 14.55±0.88 | 0.368±0.02 | 0.150±0.009 | 0.518±0.03 \(\text{[9]}\) | 0.408±0.02 |
| 126.5       | 21.23±0.87 | 14.58±0.59 | 0.365±0.01 | 0.149±0.006 | 0.514±0.02 \(\text{[7]}\) | 0.408±0.02 |
| 147.7       | 20.84±1.13 | 14.60±0.74 | 0.352±0.02 \(\text{[9]}\) | 0.147±0.007 \(\text{[9]}\) | 0.499±0.02 \(\text{[9]}\) | 0.417±0.02 \(\text{[9]}\) |
| 168.9       | 20.57±0.81 | 14.09±0.55 | 0.343±0.01 | 0.139±0.006 | 0.482±0.02 \(\text{[7]}\) | 0.405±0.02 |
| 170.1       | 21.90±1.21 | 15.00±0.83 | 0.362±0.02 | 0.148±0.008 | 0.510±0.02 \(\text{[9]}\) | 0.409±0.03 |
| 420.0       | 21.41±1.67 | 14.66±1.14 | 0.297±0.02 | 0.123±0.009 | 0.420±0.03 \(\text{[6]}\) | 0.414±0.05 |
| 506.0       | 20.91±1.88 | 14.32±1.29 | 0.266±0.02 | 0.113±0.010 | 0.379±0.03 \(\text{[6]}\) | 0.424±0.05 |
| 615.0       | 21.50±1.35 | 14.73±0.92 | 0.254±0.02 | 0.108±0.006 | 0.362±0.02 \(\text{[6]}\) | 0.425±0.04 |
| 951.0       | 22.74±1.21 | 15.58±0.83 | 0.220±0.01 | 0.096±0.005 | 0.316±0.01 \(\text{[6]}\) | 0.436±0.03 |
The results for the total astrophysical $S$-factor $S_{34}(E)$ and the branching ratio for the reaction under consideration are presented by Figs. 9 and 10 respectively. The opened circle symbols in Figs. 9 are our result of extrapolation in which each of the quoted uncertainties is the a.s.e., which involves the uncertainties both for the ANC’s adopted and that in $R_{i}^{(a)}(E,C_{i}^{(sp)})$. The solid lines and the width of each of the band are the same as in Figs. 7 and 8. As it is seen from Fig. 9, the equation (9) allows us to perform a correct extrapolation of the corresponding astrophysical $S$-factors at solar energies when the corresponding ANC-values are known. In particular, the values of the total astrophysical $S$-factor $S_{34}(E)$ at solar energies are presented in Table 2 and those recommended by us are $S_{34}(0) = 0.610 \pm 0.037$ keV b and $S_{34}(23$ keV)$=0.599 \pm 0.036$ keV $b^{2}$.

Figure 9: The same as Fig. 7 for the $^{3}$He$(\alpha,\gamma)^{7}$Be(g.s.+$0.429$ MeV) reaction.

$^{2}$The energy of $E=23$ keV corresponds to the Gamow one.
Figure 10: The branching ratio. The filled square, opened circle and square symbols are experimental data taken from Refs. [37], [9] and [10], respectively, and the opened triangle symbols are our results. The straight line and width of band are our results for the weighted mean and its uncertainty, respectively.
Table 2: The weighted means of the ANC-values \((C_{\exp}^{\text{exp}})^2\) for \(^3\text{He} + \alpha \rightarrow ^7\text{Be}\), NVC’s | \(G_{1/2}^{\text{exp}}\) and the calculated values of \(S_{3,4}(E)\) at energies \(E=0\) and 23 keV

| Exp. | \((C_{13/2}^{\text{exp}})^2, \text{fm}^{-1}\) | \(|G_{13/2}|^2_{\text{exp}}, \text{fm}\) | \((C_{11/2}^{\text{exp}})^2, \text{fm}^{-1}\) | \(|G_{11/2}|^2_{\text{exp}}, \text{fm}\) | \(S_{3,4}(0), \text{keV b}\) | \(S_{3,4}(23 \text{ keV}), \text{keV b}\) |
|------|---------------------------------|----------------|---------------------------------|----------------|----------------|----------------|
| 10   | \(23.98\pm0.82\)                | \(1.14\pm0.04\) | \(16.25\pm0.62\)               | \(0.77\pm0.03\) | \(0.630\pm0.022\) | \(0.619\pm0.022\) |
| 10   | \(23.72\pm1.01\)                | \(1.13\pm0.05\) | \(16.09\pm0.94\)               | \(0.76\pm0.05\) | \(0.624\pm0.029\) | \(0.612\pm0.019\) |
| 6    | \(21.31\pm0.61\)                | \(1.06\pm0.03\) | \(14.59\pm0.40\)               | \(0.69\pm0.02\) | \(0.562\pm0.016\) | \(0.552\pm0.015\) |
| 6 7 9| \(23.19\pm1.37\)                | \(1.11\pm0.07\) | \(15.73\pm1.02\)               | \(0.75\pm0.05\) | \(0.610\pm0.037\) | \(0.599\pm0.036\) |
Comparison of our results with those of the authors of Refs. [18] and [17] shows that a noticeable discrepancy between the present results and those of Refs. [18] [17] occurs. This circumstance is apparently connected with the underestimated value of $C_{13/2}^2$ and $C_{11/2}^2$ (or $|G_{13/2}|^2$ and $|G_{11/2}|^2$) obtained in Ref. [18] [17] in respect to our result. However, our result is also in a good agreement with that recommended in Refs. [4, 6-10] and differs slightly from that recommended in Refs. [11, 6] ($S_{34}(0)=0.51\pm0.04$ keV b [11] and 0.53 keV b [6]).

Besides, we observe that the value of $S_{34}(0)=0.56$ keV b [13] obtained within the microscopical ($\alpha+^3\text{He}$)-cluster approach is also in agreement with our result about $1.4\sigma$ level. It follows from here that the mutual agreement between the results obtained in the present work and in [13], which is based on the common approximation about the cluster ($\alpha+^3\text{He}$) structure of the $^7\text{Be}$, allows one to draw a conclusion about the dominant contribution of the ($\alpha+^3\text{He}$) clusterization to the low-energy $^3\text{He}(\alpha, \gamma)^7\text{Be}$ cross section both in the absolute normalization and in the energy dependence [6-10]. Therefore, single-channel ($\alpha+^3\text{He}$) approximation for $^7\text{Be}$ [13] is quite appropriate for this reaction in the considered energy range.

One notes also that the ratios of the “indirectly measured” astrophysical $S$-factors, $S_{13/2}(0)$ and $S_{11/2}(0)$, for the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction populating to the ground and first excited states obtained in the present work to those for the mirror $t(\alpha, \gamma)^7\text{Li}$ reaction populating to the ground and first excited states deduced in Ref. [24] are equal to $R_S^{(g.s.)}=6.4\pm0.8$ and $R_S^{(exc.s.)}=6.1\pm0.7$, respectively. These values are in a good agreement with those of $R_S^{(g.s.)}=6.6$ and $R_S^{(exc.s.)}=5.9$ deduced in Ref. [35] within the microscopic cluster model.

Fig. [10] shows a comparison between the branching ratio $R_{\text{exp}}(E)$ obtained in the present work (the opened triangle symbols) and that recommended in Refs. [37] (the filled square symbols), in [9] (the opened circles) and [10] (the opened squares). There the solid line and the width of the band present the weighted mean $\bar{R}_{\text{exp}}$ of the $R_{\text{exp}}(E)$ (the opened triangle symbols) and the weighted uncertainty obtained by us, respectively, which is equal to $\bar{R}_{\text{exp}}=0.41\pm0.01$. As it is seen from Fig. [10] the branching ratio obtained in the present work and in [7, 10] is in a good agreement with that recommended in Ref. [37] although the underestimation occurs for the $S_{34}^{\text{exp}}(E)$ obtained in Ref. [37]. Such a good agreement between two of the experimental data for the $R_{\text{exp}}(E)$ can apparently be explained by the fact that there is a reduction factor in [37], being overall for the $^3\text{He}(\alpha, \gamma)^7\text{Be}(\text{g.s.})$ and $^3\text{He}(\alpha, \gamma)^7\text{Be}(0.429$ MeV) astrophysical $S$-factors. The present result for $\bar{R}_{\text{exp}}$ is in a good agreement with those of $0.43\pm0.02$ [37] and $0.43$ [13, 38] but is noticeably larger than $0.37$ [17] and $0.32\pm0.01$ [39].

Thus, it follows from here that the overall normalization of the astrophysical $S$-factors at extremely low energies for the reactions under consideration is mainly determined by the ANC values for the $^3\text{He} + \alpha \rightarrow^7\text{Be}(\text{g.s.})$ and $^3\text{He} + \alpha \rightarrow^7\text{Be}(0.429$ MeV), which can be determined rather well from the model independent analysis of the precise experimental astrophysical $S$-factor [6-10], and the values of the ANC’s allow us to perform correct extrapolation of the astrophysical $S$-factors for the direct radiative capture $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction at solar energies, including $E=0$, and to predict the separated experimental astrophysical $S$-factors for the $^3\text{He}(\alpha, \gamma)^7\text{Be}(\text{g.s.})$ and $^3\text{He}(\alpha, \gamma)^7\text{Be}(0.429$ MeV) reactions at low experimentally acceptable energy regions ($126.5 \leq E \leq 951$ keV) obtained by using the total experimental astrophysical $S$-factors measured in Refs. [6-9].
3 Conclusion

The analysis of the experimental astrophysical $S$-factors, $S^\text{exp}_{34}(E)$, for the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction, which were precisely measured at energies $E=92.9$-$1235$ keV [6–10], has been performed within the modified two-body potential approach proposed recently in Ref. [21]. The scrupulous quantitative analysis shows that the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ reaction within the considered energy ranges is peripheral and the parameterization of the direct astrophysical $S$-factors in terms of ANC’s for the $^3\text{He} + \alpha \rightarrow ^7\text{Be}$ is adequate to the physics of the peripheral reaction under consideration.

It is demonstrated that the experimental astrophysical $S$-factors of the reaction under consideration measured in the aforesaid energy region can be used as an independent source of getting the information about the ANC’s (or NVC’s) for $^3\text{He} + \alpha \rightarrow ^7\text{Be}$. The weighted means of the ANC’s (NVC’s) for $^3\text{He} + \alpha \rightarrow ^7\text{Be}$ are obtained. They have to be $(C^\text{exp}_{1/2})^2=23.19\pm 1.37$ fm$^{-1}$ and $(C^\text{exp}_{3/2})^2=15.73\pm 1.02$ fm$^{-1}$ for $^3\text{He} + \alpha \rightarrow ^7\text{Be}$(g.s) and $^3\text{He} + \alpha \rightarrow ^7\text{Be}$(0.429 MeV), respectively. The corresponding values of the NVC’s are $|G_{1/2}|^2 = 1.11\pm 0.07$ fm and $|G_{3/2}|^2 = 0.75\pm 0.05$ fm. The uncertainty in the ANC (NVC )-values includes the experimental errors for the experimental astrophysical $S$-factors, $S^\text{exp}_{34}(E)$, and that of the used approach. Besides, the values of ANC’s were used for getting the information about the $\alpha$-particle spectroscopic factors for the mirror $(^7\text{Li}^7\text{Be})$-pair.

The obtained values of the ANC’s were also used for obtaining the experimental $^3\text{He}(\alpha, \gamma)^7\text{Be}$ astrophysical $S$-factors for capture to the ground and first excited states, the branching ratio at the six experimental points of energy $E$ ($E \geq 126.5$ keV) [6–9] and for their extrapolation at energies less than 90 keV, including $E=0$. In particular, for the weighted mean of the branching ratio $\bar{R}^\text{exp}$ and the total astrophysical $S$-factor $S_{34}(0)$ the values of $\bar{R}^\text{exp}=0.41\pm 0.01$ and $S_{34}(0) = 0.610\pm 0.037$ keV b have been obtained, respectively. The latter is noticeably larger than the result of $S_{34}(0)=0.507\pm 0.016$ keV b, deduced in Ref. [11] from the measurements of capture $\gamma$ ray, and is in an agreement with those of $S_{34}(0)=0.572\pm 0.026$ keV b [11], deduced in Ref. [11] from the measurements of $^7\text{Be}$ activity, and $S_{34}(0)=0.56$ keV [13] obtained within the microscopical $(\alpha + ^3\text{He})$-cluster model.

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