How a Quantum Theory Based on Generalized Coherent States
Resolves the EPR and Measurement Problems

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Abstract

It is shown that the quantum theory can be formulated on homogeneous spaces of generalized coherent states in a manner that accounts for interference, entanglement, and the linearity of dynamics \textit{without using the superposition principle}. The coherent state labels, which are essentially instructions for preparing states, make it unnecessary to identify properties with projectors in Hilbert space. This eliminates the so called “eigenvalue-eigenstate” link, and the theory thereby escapes the measurement problem. What the theory allows us to predict is the distribution in the outcomes of tests of \textit{relations} between coherent states. It is shown that quantum non-determinism can be attributed to a hidden variable (noise) in the space of relations without violating the no-go theorems (e.g. Kochen-Specker). It is shown that the coherent state vacuum is distorted when entangled states are generated. The non-locality of the vacuum permits this distortion to be felt everywhere without the transmission of a signal and thereby accounts for EPR correlations in a manifestly covariant way.
If a theorist wishes to instruct an experimentalist to prepare a specific quantum state, it will be of little use to the experimentalist if he is given a list of complex numbers that are to be its components in some Hilbert space basis. What the experimentalist requires is a *recipe* for preparing the state, e.g. to turn a Stern-Gerlach magnet in a prescribed way or to turn on a prescribed laser-generating current. In general the recipes specify a transformation $g$ to be applied to a reference state, and the transformations form a group $\mathcal{G}$ that is characteristic of the system. For experiments with Stern-Gerlach magnets $\mathcal{G}$ is the rotation group $SO_3$, and for laser states it is the Weyl-Heisenberg (WH) group expressed by exponentials of bose operators in which the current driving the laser appears.

States described by such recipes are called *generalized coherent states*. The Weyl-Heisenberg states were first introduced into optics by Glauber$^1$, and the concept was subsequently generalized by Perelomov$^2$ who, with his co-workers, discovered most of what we know about them.

If generalized coherent states are all that experimentalists ever prepare, one is motivated to apply Occam’s razor to the theory, so that they alone appear in it. But although the coherent states are complete (in fact over-complete), linear combinations of coherent states are not in general coherent states. Hence, if we are to permit the coherent states themselves and no others, we shall have to abandon the superposition principle.

The superposition principle is a pillar of orthodox quantum theory and is also the source of the measurement problem. I will show that the phenomena which are taken to justify the superposition principle (interference, entanglement, linear dynamics) can be correctly accounted for in a model, which I will call the coherent state (CS) model, which never requires us to introduce superpositions of coherent states. We then will not be required
to identify properties with lattices of subspaces in Hilbert space, and the measurement problem will literally go away.

The construction of the CS model begins with the hypothesis that for each system there is a characteristic group $G$ which will be called the “coherence group” of the system. No restriction is placed on $G$ other than that it be locally compact, a restriction quite in keeping with experimental implementability. By assuming it we shall have the benefit of an invariant (Haar) measure $d\mu$ by which we can integrate over the group when we have to.

Let $g \in G \rightarrow U(g)$ be an irreducible, unitary representation on a Hilbert space $\mathcal{H}$. Let $|0\rangle$ and $\langle 0'|$ be reference states for systems and detectors respectively. Let $G_o$ and $G_o'$ be the stability subgroups for the system and detector reference states, i.e. the subgroups that leave them invariant.

A set $\mathcal{F}$ of generalized coherent system states and a set $\mathcal{F}'$ of generalized coherent detector states are defined by

$$|g\rangle = U(g)|0\rangle, \quad \langle g'| = \langle 0'|U^\dagger(g),$$

in which we select one element $g$ from each left coset $gG_o$ for $\mathcal{F}$ and one element $g$ from each right coset $G_o'g$ for $\mathcal{F}'$. Thus $\mathcal{F} = G/G_o$ and $\mathcal{F}' = G_o'\backslash G$ are homogeneous spaces. Note that $\langle g|$ is not the dual of $|g\rangle$ unless it happens that the two reference states are duals of one another.

The dramatic effect of restricting the allowed states to generalized coherent states is that the group structure of $G$ is imparted to quantum mechanical amplitudes in the following way:

$$\langle g_1|g_2\rangle = \langle 0'|U^\dagger(g_1)U(g_2)|0\rangle = \langle 0'|U^{-1}(g_1)U(g_2)|0\rangle = \langle 0'|U(g_1^{-1})U(g_2)|0\rangle = \langle 0'|U(g_1^{-1}g_2)|0\rangle = \langle 0'|U(g)|0\rangle \equiv f(g), \quad g = g_1^{-1}g_2. \quad (2)$$

All predictions will be obtained from the amplitude $f(g)$ which is a function on $G$ that is constant on each double coset $G_o'\backslash G/G_o$. The double cosets...
partition \( G \) just as left and right cosets do\(^3\), and so we may treat the set \( S \) of double cosets as the set of possible relations between system and detector states. Each pair \( g_1, g_2 \) for which \( g_1^{-1} g_2 = g \) belongs to the same double coset may be regarded as different manifestations of the relation \( g \).

We see then that when restricted to coherent states the quantum theory can be regarded as a theory of relations rather than states, and these relations have a group theoretic structure. In particular the fundamental rule, that \( |\langle g_2 | g_1 \rangle|^2 \) is the probability for a system in state \( |g_1 \rangle \) to pass a detector in state \( \langle g_2 | \rangle \), now becomes a rule for computing the probability that the relation \( g \) “holds”, and we write:

\[
p(g) = |f(g)|^2.
\]

This leads us to the next benefit of our restriction to coherent states, for we observe that the probability function has a suggestive geometric structure. Since \( \langle g_1 | g_2 \rangle \) is a scalar product, the function

\[
s(g_1, g_2) \equiv \sqrt{1 - |\langle g_1 | g_2 \rangle|^2}
\]  

(4)

is a metric on \( G \). This distance between \( g_1 \) and \( g_2 \) is the same as the distance of \( g = g_1^{-1} g_2 \) to the identity of the group. Thus we can interpret

\[
s(g) \equiv s(g, e) = \sqrt{1 - |f(g)|^2}
\]  

(5)

as the “size” of the relation, and its square as a “cross section” for the relation. Thus one may think of experiments that test the relation in the way one thinks of scattering experiments. We “throw” a random relation \( h \) at \( g \) and say that \( g \) holds if it is smaller than \( h \), i.e. if \( s(g) < s(h) \). Thus the probability that \( g \) holds is the probability that we have thrown an \( h \) which is bigger than \( g \). If we throw \( h \)’s with a distribution such that the probability of having \( s(h) < r \) is the cross-section of a disk of radius \( r \), then
the probability that $g$ holds is just the value $1 - (s(g))^2 = |f(g)|^2$ required by quantum mechanics.

We define a random variable with values in $\mathcal{S}$ that is distributed in this way as the “relational hidden variable” for $\mathcal{G}$ coherent states. It should be noted that there is no conflict with no-go theorems because relations are intrinsically non-local. The existence of this kind of hidden variable was observed long ago by Bell for spin-1/2 systems. What we see here is that there is a natural generalization of the idea to all coherent state systems.

We pause here to examine the two most important examples for experimental applications.

Example 1 — Detection of spin-1/2 particles with Stern-Gerlach magnets: $\mathcal{G} = SO_3$. $U$ is the two dimensional representation. Taking any state as the reference state, the stability subgroup is the rotation about that direction. Thus $\mathcal{F} = SO_3/SO_2 = SU_2/U_1$ which is the 2-sphere. Thus states are labeled by their coordinates $(\theta, \phi)$, and if the reference state for both system and detector is the north pole we find that $s(\theta, \phi) = \sin(\theta/2)$ (the chord metric). Since the fraction of the area of a sphere lying within polar latitude $\theta$ of the pole is $\sin^2(\theta/2)$, we see that the relational hidden variable distribution is (happily) uniform over the area of the sphere.

Note that in this example every ray corresponds to a coherent state. This will be the case whenever $\mathcal{G}$ is the full unitary group $U_N$ in $N$-dimensions because any unit vector can be obtained from any other unit vector by a unitary transformation. One can still not say that the set of coherent states is linearly closed because of the necessity of normalizing linear combinations to make them state vectors.

The groups $U_N$ for arbitrarily large but finite $N$ are compact, but for $N \to \infty$ the group is not even locally compact. Our restriction to locally
compact groups thus recognizes the increasing difficulty of experimentally implementing the full unitary group as its dimension increases.

**Example 2** — Detection of the complex amplitude $\lambda$ of a single mode laser beam by the photocurrent it produces: The coherent states are the Glauber states. $\mathcal{G}$ is the Weyl-Heisenberg group (WH) in which the group elements have the composition law

$$g = e^{i\theta} U(\lambda), \quad \lambda \in \mathbb{C}, \quad U(\lambda_1)U(\lambda_2) = e^{i\text{Im}(\lambda_2^* \cdot \lambda_1)} U(\lambda_1 + \lambda_2). \quad (6)$$

In coherent superpositions it is the $\lambda$’s that are added, so the important lesson learned from (6) is that **coherent superpositions are described by group multiplication not state vector addition**.

By the Stone-von Neumann Theorem there is only one irreducible representation up to equivalence, the so-called Fock representation. In this representation $U$ is expressed in terms of bose operators $a, a^\dagger$ with $[a, a^\dagger] = 1$:

$$U(\lambda) = e^{\lambda a^\dagger - \lambda^* a}. \quad (7)$$

The reference state $|0\rangle$, called the Fock vacuum, is annihilated by $a$, and its stability subgroup is $U_1$ (phase multiplication). The space $\mathcal{F} = WH/U_1$ is the single mode phase space, i.e. the homogeneous space is the complex plane. Thus the coherent system states are

$$|\lambda\rangle = U(\lambda)|0\rangle, \quad \lambda \in \mathbb{C} \implies \langle 0|U(\lambda)|0\rangle = e^{-|\lambda|^2/2}. \quad (8)$$

In the coherent detector states $\langle \lambda|$, the parameter $\lambda$ is the detected photocurrent.

The invariant measure on WH is the area measure in the complex plane, so the measure on the group for which $\lambda$ is in the annulus of width $d|\lambda|$ at $|\lambda|$ is $dA = \pi d|\lambda|^2$. If the distribution of a trial value $\mu$ with respect to the
measure is \(\pi^{-1}e^{-|\mu|^2}d\lambda\), then the probability for \(|\mu| > |\lambda|\) is \(e^{-|\lambda|^2}\) which is the squared modulus of (8). It follows that the relational hidden variable for WH groups is Maxwellian i.e. it represents “thermal” noise in the space of relations.

The WH group is non-compact but is locally compact. The direct product \(WH_N\) of an arbitrarily large but finite number \(N\) of WH groups (which describes an \(N\)-mode laser) is also locally compact. We may represent it by simply replacing the complex number \(\lambda\) with an \(N\) component complex vector and understand \(\lambda \cdot a\) to be \(\lambda_1a_1 + \cdots + \lambda_Na_N\), in which the \(a_j\)’s are commuting bose operators, and the composition law (6) holds with \(\lambda_2^* \cdot \lambda_1\) being the complex scalar product of the vectors \(\lambda_1\) and \(\lambda_2\).

Up to this point we have said nothing about dynamics, and it is the linearity of quantum dynamics that has been the principle justification for the superposition principle. In order to keep dynamics linear when we restrict the states to be coherent, we can only allow those linear transformations that preserve the group structure (automorphisms) and are smooth (homeomorphisms) to preserve the topological structure of the homogeneous spaces \(\mathcal{F}\) and \(\mathcal{F}'\). Such transformations will be implemented in \(\mathcal{H}\) by unitary transformations \(A(\gamma)\) such that

\[
A(\gamma)U(g)A(\gamma)^{-1} = U(\gamma(g)), \quad \gamma \in \text{Aut}(\mathcal{G}).
\]

(9)

If \(A(\gamma)\) is not one of the \(U(g)\)’s we can enlarge the group \(\mathcal{G}\) to include it. Indeed if \(\gamma\) belongs to a subgroup \(\Gamma\) of \(\text{Aut}(\mathcal{G})\) then, by virtue of (9), all possible products of \(A(\gamma)\)’s and \(U(g)\)’s can be written in the form \(U(g)A(\gamma)\) with some choice of \(\gamma\) and \(g\). In cases of interest this decomposition will be unique (i.e. the group will be a semi-direct product). We can then adopt the “Schrödinger picture” and say that dynamical relations occur between detectors \(\langle g \rangle\) and systems that evolve according to the dynamical law \(|0\rangle \rightarrow \cdots \).
\(A(\gamma)|0\rangle\) as \(\gamma\) follows a one-parameter subgroup of \(\text{Aut}(\mathcal{G})\). Thus amplitudes will have the form \(\langle g|\gamma \rangle = \langle 0'|U(g)A(\gamma)|0\rangle\).

Since WH groups are generated by the exponentials of linear forms in the bose operators, one sees from the algebra of bose operators that the exponentials of \textit{quadratic} forms in the bose operators all generate automorphisms. Indeed all of the linear canonical transformations are produced in this way, and we are therefore able to account for all linear dynamical processes within \(\text{Aut}(\mathcal{G})\).

The following example of such a dynamical process is of great interest because it illustrates the mechanism by which entanglement is generated. Let us think of detectors for a pair of laser modes \(a, b\) as a single detector \(\langle \lambda| = \langle 0|U^\dagger(\lambda), \lambda = (\lambda_a, \lambda_b)\). The coherence group is \(\mathcal{G} = WH_2\), and among the canonical automorphisms are those defined by

\[
\mathcal{D}(\xi) \equiv e^{\xi a^\dagger b^\dagger - \xi^* a b}, \quad \xi \in \mathcal{C}.
\]

One can show\(^5\) that \(\mathcal{D}\) has the normal ordered form:

\[
\mathcal{D}(\xi) = \beta e^{\zeta a^\dagger b^\dagger} e^{-\zeta^* a b}, \quad \zeta = e^{i \arg \xi \tanh |\xi|}, \quad \beta = \sqrt{1 - |\zeta|^2},
\]

so that

\[
\mathcal{D}(\xi)|0\rangle = R_\zeta|0\rangle \equiv |\zeta\rangle, \quad R_\zeta = \beta e^{\zeta a^\dagger b^\dagger}.
\]

The states \(|\zeta\rangle\) are two-photon laser states (not to be confused with two mode laser states). A simple manipulation then gives the following amplitude for the relation between the detector and the reference state as the two photon laser is “turned on” by making \(\zeta\) non-zero:

\[
\langle \lambda|\zeta \rangle = \beta e^{-(|\lambda_a|^2 + |\lambda_b|^2)/2} e^{\xi \lambda_a \lambda_b}.
\]
As $\zeta$ is turned on the presence of the last factor causes the size of the relation to change in a manner that depends both on the modulus and phase of $\zeta$. For fixed non-zero $\zeta$ the size of the relation does not depend on $\lambda_a$ and $\lambda_b$ independently — there is a correlation which becomes more and more sensitive to the phase of $\zeta$ as $\zeta$ approaches the boundary of the unit disc, i.e. $\zeta \to e^{i\phi}$. In this limit the probability tends to

$$p = \beta^2 e^{-|\lambda_a + e^{i\phi} \lambda_b|^2}. \tag{13}$$

We cannot actually attain the limit because $\beta$ also tends to zero.

The phase of $\zeta$ is influenced by the space-time locations $x_a, x_b$ associated with the two laser photons. If their 4-momenta are $k_a, k_b$ the space-time translation automorphism generated by the 4-momentum operator $P = k_a a^\dagger a + k_b b^\dagger b$ will multiply the bose operators by phases $e^{ik_a \cdot x_a}$ and $e^{ik_b \cdot x_b}$ which, as one sees from (11), is equivalent to multiplying $\zeta$ by the product of the phases. If the two photons separate along any ray in space-time the phase of $\zeta$ will change by a factor $e^{ik \cdot x}$ where $x$ is the separation between them. Thus the correlation will change with $x$ regardless of whether the direction of $x$ is space-like, light-like, or time-like. The EPR problem of how to understand the correlations when the ray is space-like is now seen within this manifestly covariant framework to be no different than how to understand them when the ray is time-like.

To achieve this understanding let us first remark that $\langle \lambda \rangle$ are coherent states of $WH_2 = WH_1 \otimes WH_1$. While coherent states cannot be added to produce coherent states, they can always be tensored to form coherent states. To see this simply note that the coherence group will be the direct product, and the representation $U$ will be the tensor product. The reference state, however, may or may not be the tensor product of the two reference states. If it is the tensor product, one will easily check that amplitudes will
factorize, and there will be no correlation. What happens to produce the correlation in (12) is that for non-zero \( \zeta \) the state \( |\zeta\rangle \) no-longer factorizes.

To understand how this non-factorizability can be felt between arbitrary space-time points, observe the remarkable fact that \( |\zeta\rangle \) is a distorted form of the Fock vacuum. To see why note that while it is not annihilated by \( a, b \), it is annihilated by the commuting bose operators

\[
a(\zeta) = \beta^{-1}(a - \zeta b^{\dagger}), \quad b(\zeta) = \beta^{-1}(b - \zeta a^{\dagger}).
\]

The vacuum is “non-local” in the following precise sense. It is known from a powerful theorem due to Perelomov\(^6\) that the set of WH coherent states \( |\lambda\rangle \) with \( \lambda \) on the lattice \( n + im \) will be overcomplete by one, i.e. there is exactly one relation of linear dependence. This can be expressed as as an expansion of the vacuum \( |0\rangle \) as a linear combination of all of the others, and in this expansion all of the coefficients have the same modulus. (That such a state can be normalized is possible because the coherent states are not mutually orthogonal.) Thus the vacuum state is uniformly spread over the Hilbert space. Distortions in the vacuum therefore express themselves in a uniform manner everywhere in the space of coherent states.

The distortion becomes greater as \( \zeta \) moves toward the boundary of the unit circle. The transformations produced by \( D(\zeta) \) are isomorphic to the \( 1 - 1 \) Lorentz group with \( \zeta \) acting like a velocity parameter which reaches lightspeed on the unit circle. In some sense one can say that the two photon laser “boosts” the vacuum, and the resulting distortion of the reference (analogous to Fitzgerald contraction) alters the sizes of relations regardless of the space-time locations of the detection events.

One may observe that as \( \zeta \) approaches the boundary the distorted reference states become sharply distinguishable from one another in the following
The scalar product is found to be

$$\langle \zeta' | \zeta \rangle = \frac{(1 - |\zeta'|^2)^{1/2}(1 - |\zeta|^2)^{1/2}}{1 - \zeta'^* \zeta}$$  \hspace{1cm} (15)

which vanishes if either $\zeta$ or $\zeta'$ tend to the boundary, unless $\zeta$ and $\zeta'$ are equal, in which case it tends to unity. Thus such states with infinitesimally different phases are sharply distinguished, an indicator of classical behavior. This also means that the relative phase of $\lambda_a$ and $\lambda_b$ can be sharply measured by the correlation. Our ability to exploit this, however, is limited by the vanishing of $\beta$ at the boundary.

Because $D(\xi)$ is an automorphism, the stability subgroup of $|\zeta\rangle$ is isomorphic to that of the Fock vacuum. We are naturally led to consider what sort of entanglement will result if the stability subgroup of the reference state for a direct product is non-isomorphic to that of the factors. Bell states such as the Bohm-Aharonov singlet arise in this way, and indeed states of this kind are associated with coherent states of every compact $\mathcal{G}$ in the following way:

Observe first that the unit operator can be written

$$I = \int_S |g\rangle\langle g| \, d\mu,$$  \hspace{1cm} (16)

where $\langle g|$ and $|g\rangle$ are duals of one another). This may be deduced from Schur’s Lemma noting that it commutes with every element $U(g)$ of an irreducible representation. This also establishes the (over) completeness of the coherent states noted earlier. Now suppose we have an anti-unitary map

$$|g\rangle \rightarrow |g^*\rangle \implies \langle g^*| h^* \rangle = \langle h|g\rangle,$$  \hspace{1cm} (17)

and define the tensor product state

$$|B\rangle \equiv \int_{\mathcal{F}} d\mu |g\rangle \otimes |g^*\rangle.$$  \hspace{1cm} (18)
The stability subgroup for this state is not the direct product of those of the constituents. Now the state is invariant under all transformations of the form \((h, h^*)\) (with the same \(h\) in each component). Consider the effect of a change in reference state from \(|0\rangle = |0_1\rangle \otimes |0_2\rangle\) to \(|\mathcal{B}\rangle\rangle\) on the relation between the reference state and a detector state \(\langle g_1, g_2^* \mid |0\rangle \otimes |g_2^*\rangle\). We see that

\[
\langle g_1, g_2^* \mid 0 \rangle = \langle g_1 \mid 0_1 \rangle \langle g_2^* \mid 0_2 \rangle \implies \langle g_1, g_2^* \mid |\mathcal{B}\rangle\rangle = \int_{\mathcal{F}} d\mu \langle g_1 \mid g \rangle \langle g_2^* \mid g^* \rangle = \langle g_1 \mid g_2 \rangle,
\]

thus producing the same correlation we would have if the two constituents of the detector belonged to the same Hilbert space. In particular the probability is unity for the relation to hold when \(g_1 = g_2\) i.e. the state \(|\mathcal{B}\rangle\rangle\) always passes a pair of detectors in conjugate states.

Since

\[
\langle \langle \mathcal{B} \mid |\mathcal{B}\rangle \rangle = \int_{\mathcal{F}} d\mu,
\]

we see that the generalized Bell state has a finite norm if and only if \(\mathcal{F}\) is compact. We shall therefore be able to construct generalized Bell states in this way for compact groups only. Note that the construction shows clearly why anti-unitary transformations are always present in the construction of Bell states.

The generalized Bell state computed from (18) for \(\mathcal{F} = SU_2/U_1\) is found to be the Bohm-Aharonov singlet. The anti-unitary transformation \(\ast\) is time reversal. Although one cannot directly apply (18) to the \(WH_2\) group because it is not compact, it is noteworthy, that the states \(|\zeta\rangle\) can be produced by inserting a gaussian convergence factor parametrized by \(\zeta\) into (18).

I have now demonstrated that the CS model can account for the phenomena which have been taken to justify the superposition principle. In the CS model the classical proposition that a dynamical variable lies in a certain phase space neighborhood is replaced by the proposition that a relation \(g\)
in a certain neighborhood of \( S \) “holds”. We have seen that the CS model admits a relational hidden variable, i.e. a Kolmogorov probability space that accounts for the statistical distribution of outcomes by a simple kind of noise in the space of relations.

In the CS model there is no sharp separation between system and detector. A relation \( g_1^{-1}g_2 \) does not have a unique factorization, and the complete symmetry between system and detector is seen in the invariance of probabilities under complex conjugation which exchanges their roles in amplitudes.

One may contrast these features of the CS model with the orthodox (Born-Dirac-von Neumann) model in which “properties” are identified with lattices of subspaces in Hilbert space\(^9\). The no-go theorems, e.g. Kochen-Specker \(^{10}\) tell us that we cannot assign values (hidden variables) to account for the observed distribution of outcomes of measurements made to determine such properties. The familiar paradigm for the interaction of a system with a measuring device produces an entangled state which must quickly “collapse” if a determinate value for the property is to be obtained. The failure of the orthodox interpretation to account for the interaction without a non-linear modification to produce collapse is called the measurement problem. In the CS model the notion of “property” is replaced by the \( g \)-value of a relation, and we then have a hidden variable account of what may happen in a test of the relation to explain the statistical distribution of outcomes.

The CS model recognizes entangled states as distorted vacuum states which can evolve in a manner that is completely consistent with special relativity but nonetheless induce correlations between space-like events. The fact that a distortion of the vacuum effects all event pairs regardless of whether they are time-like, light-like, or space-like to one another is explained by the uniform non-locality of the vacuum.
The basic question we must answer before claiming that we have resolved the measurement and EPR problems is whether a coherence group exists for all quantum phenomena. The similarity of the double cosets structure to vacuum expectation values and the field theoretic form of the general tensor products of WH groups suggests that the way to find the coherence group for any system is to second quantize. Thus it is plausible to suppose that the way to guarantee that there is no measurement problem is to frame the theory as one of relations between regulated quantum fields.

References

1. R. Glauber, 131,2766,(1963)
2. A. Perelomov, “Generalized Coherent States and their Applications”, Springer-Verlag (1986)
3. M. Hall, Jr., “The Theory of Groups” p. 14, The Macmillan Co. (1959).
4. J.S. Bell, Physics 1, 195 (1964)
5. A. Perelomov, op. cit. p. 73
6. A. Perelomov, op. cit. p. 25
7. A. Perelomov, op. cit. p. 74
8. D. Fivel, UMBC Workshop on Fundamental Problems in Quantum Theory. “The Relationship Between Coherence and Entanglement” (1999).
9. J. Bub, “Interpreting the Quantum World”, p. 24 and p. 31 et. seq. Cambridge University Press, (1997)
10. S. Kochen an E.P. Specker, J. Math. Mech. 17, 59, (1967)