Higher-Order Nonclassicality in PhotonAddedandSubtractedQuditStates

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Higher-order nonclassical properties of r photon added and t photon subtracted qudit states (referred to as rPAQS and tPSQS, respectively) are investigated here to answer: How addition and subtraction of photon can be used to engineer higher-order nonclassical properties of qudit states? To obtain the answer, higher-order moment of relevant bosonic field operators is first obtained and subsequently used to study the higher-order nonclassical properties (e.g., higher-order antibunching, higher-order squeezing, and higher-order sub-Poissonian photon statistics) of the corresponding states. These witnesses establish that rPAQS and tPSQS are highly nonclassical. To quantitatively establish this observation and to make a comparison between rPAQS and tPSQS, volumes of the negative part of Wigner function are computed. Finally, for the sake of verifiability of the obtained results, optical tomograms are also reported. Throughout the study, a particular type of qudit state named as a new generalized binomial state is used as an example.

1. Introduction

Various exciting applications of nonclassical properties of quantum states have recently been proposed and realized. Specifically, squeezed states have been used in the LIGO experiment for the detection of gravitational waves[1] and in continuous variable quantum key distribution;[2–5] entangled states have been used in quantum teleportation and quantum cryptography[6–8] moreover antibunching is found to be useful in characterizing single photon sources.[9,10] In quantum state engineering[11–14] and quantum computation,[15–16], nonclassical properties of any quantum state are getting much attention.[10,18,19] This is so because nonclassical states have no classical analogue and can thus be useful in realizing tasks that are impossible in the classical world. In other words, nonclassical states which are characterized by the negative values of Glauber–Sudarshan P-function can only establish quantum supremacy.[22,23] Examples of nonclassical properties are squeezing, antibunching, and entanglement. In summary, nonclassical features of quantum states are very important and the same has been studied for various families of quantum states.[20,24]

One such family of quantum states is called Qudit (d-level states) states which may be viewed as finite superposition of Fock states in d-dimensional Hilbert space.[12,24] A particular subclass of qudit states is the set of finite dimensional intermediate states.[10,18,19] These states are interesting because any state of this family can be reduced to various other quantum states at different limits of the state parameters. Usually, intermediate states correspond to states having photon number distribution analogous to a well-defined statistical distribution function like binomial function, negative binomial function, and many more. The intermediate state is named in accordance with statistical distribution that represents the photon number distribution of the state. One such intermediate state named as new generalized binomial state (NGBS) was introduced by Fan et al.,[25] and we have recently reported higher-order nonclassicality in NGBS.[24] Here, we aim to extend the work and check how addition and subtraction of photon can engineer the higher-order nonclassical properties of this state. In short, in what follows, we aim to study higher-order nonclassical properties of photon added and subtracted NGBS. The nonclassical properties we aim to study, can be witnessed by some well-defined inequality in terms of moments of creation and annihilation operators under the framework of second quantization. These criteria can have lower-order and higher-order versions. Study of lower-order nonclassicality of a quantum state is reported in literature since the early days of quantum optics, but interest in higher-order nonclassicality is relatively new and promising for the experiments point of view also as it can detect weaker nonclassicality which are hard to observe using lower-order criteria.[26,27]

We have already mentioned that in what follows, we wish to study higher-order nonclassical properties of photon added and subtracted NGBS. Now the question arises, how can we generate these (photon added and/or subtracted) nonclassical states? Here, to address this question, we wish to note that any nonclassical state can be generated by two kinds of operations, unitary and nonunitary operations. In unitary operation, nonclassical state is generated under control of a Hamiltonian and the example of nonunitary operation is like photon addition and subtraction to a quantum state. Here, in Figure 1, we have provided the possible scheme for experimental realization of photon addition and subtraction to a quantum state. Operation of photon
subtraction and addition is depicted by using a beam splitter (Figure 1a) and a nonlinear crystal (Figure 1b), respectively. Here, it would be apt to note that a few interesting experiments involving qudits have been performed in the recent past. The number of such successful experiments are small compared to experiments involving qubits, but the experiments are related to various aspects of physics including but not restricted to superconducting phase qudit,\cite{128} quantum football,\cite{299} quantum state tomography of large nuclear spin,\cite{300} and time-domain grating with a periodically driven qutrit.\cite{31}

Above-mentioned techniques in Figure 1a,b can be used to generate engineered quantum states. Interesting examples of such engineered nonclassical states are Fock state, photon added/subtracted coherent state, displaced Fock state, intermediate state like binomial state (BS), and NGBS. In our earlier works, we have reported higher-order nonclassicality in different quantum systems.\cite{10,18,20,24} However, the effect of multiple photon addition and subtraction on qudits in general and NGBS in particular have not yet been studied. Although it can reveal higher-order nonclassical properties of a family of quantum states in different limits as photon added and subtracted, NGBS can be reduced to various states in different limits. We will return to this point later. Here, we just wish to note that this particular possibility, the above-mentioned features and aspects of higher-order nonclassicality have motivated us for the present study, and in what follows, we will study higher-order nonclassical properties of r photon added qudit state (rPAQS) and t photon subtracted qudit state (tPSQS).

Higher-order nonclassicality can be witnessed through various operational criteria (inequalities); most of them are expressed as a function of moments of annihilation and creation operators. Keeping that in mind, in what follows, we have first expressed rPAQS and tPSQS in general as Fock superposition states (FSSs), and subsequently that to obtain expressions for a general moment of annihilation and creation operator (say, $\langle a^{\dagger}a^{\dagger} \rangle$) which in turn provides us analytic expression for the nonclassicality witnessing parameters for various higher-order nonclassical phenomena, for example, higher-order antibunching (HOA), higher-order squeezing (HOS-Hillery type), higher-order sub-Poissonian photon statistics (HOSPSS), etc. The systematic study revealed that the depth of nonclassicality witnessing parameter increases with the number of added photon, but no conclusive decision can be made from them as none of the witnesses of nonclassicality can yield a quantitative measure. So we looked back to a quasi-distribution function, namely Wigner function whose negative parts illustrate the presence of nonclassicality and volume of the negative part quantifies nonclassicality. The quantification helped us to establish that nonclassicality indeed increases with the addition of photon; it further helped us compare rPAQS and tPSQS. This was consistent with the witness-based observation, but still an issue remained—Wigner function is not measurable in general. So to complete the work, we have computed optical tomograms which are the special cases of symplectic tomograms\cite{32} for the quantum states of our interest. Optical tomograms can be produced experimentally, and thus, they can be used to verify our results and subsequently Radon transform can be used to obtain Wigner function from optical tomogram. This part makes the predictions of present analytic study experimentally verifiable. For more details of such kind of tomogram study, readers may refer more general form of tomograms as symplectic tomograms.\cite{33} Explicit relations for the tomograms of photon-added and photon-subtracted squeezed coherent states and squeezed number states are reported in ref. [33].

Before we proceed to the more technical part of the paper, it will be apt to note that in the first part of the paper a general construction of the problem is done in terms of rPAQS and tPSQS, but the higher-order nonclassicality witness are illustrated by considering a particular type of qudit state only (namely NGBS state).\cite{25} Thus, in what follows, higher-order nonclassicality will be witnessed and quantitatively measured for r photon added NGBS and t photon subtracted NGBS. Earlier, we reported higher-order nonclassicality in NGBS\cite{24} clearly, those results will be obtained as special cases of the present results with $r = 0$ and $t = 0$. Further, in an earlier study, nonclassical properties of single photon added and subtracted in binomial state\cite{34} were studied. One can easily understand that the present results would be so general that all such existing results will be reducible from the present results at different limits. In addition to our earlier works, a large number of works have been performed to elaborate on the relevance and importance of single photon and multi photon addition and subtraction in different quantum states (see refs.\cite{35–40} and references therein).

Similarly, many works on nonclassicality in different systems are reported in literature. For example, unified derivation for multimode nonclassicality,\cite{41} sudden vanishing and reappearance of nonclassical effects in a system,\cite{42} increasing relative nonclassicality quantified by standard entanglement potentials,\cite{43,44} and more recently quantifying the nonclassicality of pure dephasing are reported in literature.\cite{45,46} Most of these works were focused to specific quantum states, and that has set the motivation of the present work where aim to approach the problem from a much more general perspective as far as the state to be considered and the expression for moment of field operators are concerned. This paper is organized in five sections. In Section 2, we present the mechanism of photon addition and subtraction in general qudit state and further, we provide expressions of higher-order moment for studying various higher-order nonclassical phenomena. In Section 3, various criteria of witness of higher-order nonclassicality are explored. Section 4 provides quantitative analysis of higher-order nonclassicality in the form of nonclassical volume. Finally, Section 5 concludes our results.
2. Photon Addition and Subtraction in Qudit State

A qudit state of radiation field in Fock basis can be expressed as

$$|\psi\rangle = \sum_{n=0}^{M} C_n |n\rangle$$

(1)

where $C_n$ is the probability amplitude and $|n\rangle$ represents a Fock state having $n$ photon. If we add $r$ photon to this state through creation operator, we obtain a new qudit state as $r$ photon added qudit state and can be expressed as

$$a^\dagger |\psi\rangle = |r\rangle PAQS = N_r \sum_{n=0}^{M} C_n \left[ \frac{(n+r)!}{n!} \right]^{1/2} |n+r\rangle$$

(2)

where $N_r$ is normalization constant and is defined as

$$N_r = \left[ \sum_{n=0}^{M} C_n \left[ \frac{(n+r)!}{n!} \right]^{1/2} \right]^{-1/2}$$

(3)

Similarly, a $t$ photon subtracted qudit state can be obtained by repeatedly applying annihilation operator and can be expressed as

$$a^\dagger |\psi\rangle = |t\rangle PSQS = N_t \sum_{n=0}^{M} C_n \left[ \frac{n!}{(n-t)!} \right]^{1/2} |n-t\rangle$$

(4)

where $N_t$ is normalization constant and is defined as

$$N_t = \left[ \sum_{n=0}^{M} C_n \left[ \frac{n!}{(n-t)!} \right]^{1/2} \right]^{-1/2}$$

(5)

Now, to study nonclassical properties of these states, we would require analytic expressions of the moments of the relevant field operators. For $r$ photon added qudit state, a bit of computation would yield

$$\langle a^{\dagger r} a^r \rangle_{PAQS} = |N_r|^2 \sum_{n=0}^{M} C_n^2 \left[ \frac{(n+r)!}{n!(n-l+k)!} \right]^{1/2}$$

(6)

where

$$\langle a^{\dagger r} a^r \rangle_{PAQS} = |N_r|^2 \sum_{n=0}^{M} C_n^2 \left[ \frac{(n+r)!}{n!(n-l+k)!} \right]^{1/2}$$

and

$$\langle a^{\dagger t} a^t \rangle_{PSQS} = |N_t|^2 \sum_{n=0}^{M} C_n^2 \left[ \frac{(n+t)!}{n!(n-l+k)!} \right]^{1/2}$$

(7)

Similarly, we can get analytic expressions of the moments for $t$ photon subtracted qudit state

$$\langle a^{\dagger t} a^t \rangle_{PSQS} = |N_t|^2 \sum_{n=0}^{M} C_n^2 \left[ \frac{(n-t)!}{n!(n-l+k)!} \right]^{1/2}$$

(8)

$$\langle a^{\dagger t} a^t \rangle_{PSQS} = |N_t|^2 \sum_{n=0}^{M} C_n^2 \left[ \frac{(n-t)!}{n!(n-l+k)!} \right]^{1/2}$$

(9)

Here, we have considered NGBS as a particular example of qudit state, whose probability amplitude $C_n$ is defined as

$$C_n = \left[ \frac{I}{1 + Mq (M - n)!} \right]^{1/2} \left( 1 - \frac{p + nq}{1 + Mq} \right)^{M-n}$$

(10)

Interestingly for $q = 0$, NGBS converted to BS, which can further be reduced to Fock state (most nonclassical) and coherent state (most classical) with different limits of depending parameters $M$ and $n$. Here, Equation (10) is a more general form of probability amplitude of BS so defined as NGBS. More details for same are already addressed in our recent paper.[24] After application of $t$ photon addition and $t$ photon subtraction, NGBS is described as rNGBS and tNGBS. In what follows, we will see that the above analytic expression will essentially lead to analytic expression for various witness of nonclassicality. In the next section, we wish to apply various witness criteria over rNGBS and tNGBS and want to observe the effect of photon addition and subtraction over higher-order nonclassical phenomena.

3. Nonclassical Properties: Witness Criteria

A quantum mechanical state $|\psi\rangle$ has $n^{th}$ order nonclassicality with respect to any arbitrary quantum mechanical operator $A$ if the $n^{th}$ order moment of $A$ in that state reduces below to the value of the $n^{th}$ order moment of $A$ in a Poissonian state, that is, the condition of $n^{th}$ order nonclassicality with respect to the operator $A$ is given by $(\Delta A)^n |\psi\rangle < (\Delta A)^n$|coherent state$\rangle$, where $(\Delta A)^n$ is the general $n^{th}$ order moment. This is a general criterion of higher-order nonclassicality in any state originated by uncertainty principle (see ref. [18], and references there in). Depending upon the operator form of $A$, we may have different criteria of nonclassicality. All these criteria fall under two categories: 1) witness; 2) quantifier. Here, in Section 3, we wish to study, witness criteria for rPAQS and tPSQS. Specifically, we study HOA, HOS (Hillery type), HOPS in the next subsections.

3.1. Higher-Order Antibunching

Phenomenon of antibunching is related to photon statistics of a state. Using antibunching criterion, one can describe statistical property of the radiation field. This phenomenon ensures that in
3.2. Higher-Order Squeezing

The phenomenon squeezing originates from the Heisenberg uncertainty relation, in which the product of fluctuation of two non-commuting operators in Heisenberg uncertainty relation (uncertainty product) has a minimum value. At this point, both the quadrature variance are equal. If the variance of one of the quadrature goes below this equal value (on the cost of increase in other quadrature), the corresponding quadrature is squeezed. The higher-order counterpart of the squeezing is higher-order squeezing. In literature, we have three types of higher-order squeezing criteria, Hong–Mandel Squeezing,[50] Hillery-type squeezing,[51] and amplitude squared squeezing in matrix form given by Vogel. Here, we have studied Hillery type squeezing which is described as,

\[ A_{i,a} = \langle \Delta Y_{i,a} \rangle^2 - \frac{1}{2} |\langle [Y_{1,a}, Y_{2,a}] \rangle| < 0 \]  

(12)

where \( Y_{1,a} = \frac{a + a^\dagger}{\sqrt{2}} \) and \( Y_{2,a} = \frac{ip[a - a^\dagger]}{\sqrt{2}} \) are amplitude powered quadrature. In this article, we have calculated HOS by amplitude square squeezing, that is, for \( l = 2 \). The result is exhibited in Figure 4a–d, from where we can easily conclude that photon subtraction is more effective than photon addition for getting squeezing and also squeezing is decreasing with number of photon subtraction.

3.3. Higher-Order Sub-Poissonian Photon Statistics

Phenomenon of sub-Poissonian photon statistics (SPS) is again described: the statistical property of any qudit state. Lower-order SPS is equivalent to normal antibunching, but the higher-order criterion is different than that of HOA. Higher-order
Figure 3. a–c) HOA for rNGBS with $l = 3$, $M = 10$, and different values of $q = 0.01$ and $q = -0.01$ with $r = 1, 3$, and $5$, respectively. d–f) HOA for tNGBS with same parameters. It is found that again photon addition is more prominent than photon subtraction for HOA, and state with negative $q$ is more nonclassical.

Figure 4. Variation of HOS (Hillery type) for photon added and subtracted NGBS is shown here with probability $p$. a,b) HOS for rNGBS with $q = 0.01$ and $q = -0.01$, respectively. c,d) HOS for tNGBS with $q = 0.01$ and $q = -0.01$, respectively. Photon subtraction is more effective for existence of HOS as high negative values are shown in (c) and (d).
Sub-Poissonian photon statistics HOSPS is given by the following criterion:

$$D_h((l-1)) = \sum_{r=0}^{l-1} \sum_{k=0}^{r} S(r,k)\mathcal{C}_r(-1)^r D(k-1)(N)^{r-k} < 0$$  \hspace{1cm} (13)$$

where $S(r,k)$ is the Stirling number of second kind. The inequality in Equation (13) is the condition for the $(l-1)$th order non-classicality, and for $l \geq 3$, it is the condition for HOSPS. When higher-order moment of the photon number is less than that of the Poissonian level, that is, $\langle (\Delta N)^l \rangle < \langle (\Delta N)^l \rangle_{\text{Poissonian}}$, the state shows HOSPS. We have obtained an analytic expression for the inequality in (13) by using Equations (6)–(9); the corresponding results are shown in Figure 5a–d where the negative parts in figure ensure the HOSPS in NGBS. In Figure 5a,b, we have observed that the depth of the witness of HOSPS is increasing with number of photon addition and in Figure 5c,d, we have observed that the depth of the witness of HOSPS decreases with number of photon subtraction.

3.4. Wigner Function

In our previous work,[24] we have derived a compact form of Wigner function of finite dimensional FSS and also reported the Wigner function of NGBS, which is the special form of FSS.

$$|\psi\rangle = \sum_{n=0}^{N} c_n |n\rangle$$  \hspace{1cm} (14)$$

The final expression of the Wigner function of FSS is shown below.

$$W(x, p) = \frac{1}{\pi^2} \sum_{n,n'=0}^{N} c_n^* b_n^* c_{n'} b_{n'} e^{-((x^2+p^2))} (1)^{n'} 2^{n'} n! |(ip-x)|^{n'} (r_n^{n'-n}$$

By using Equations (6–9 and 15), we can easily calculate Wigner function of $r$PAQS and tPSQS. The obtained Wigner function for rNGBS and tNGBS are shown in Figures 6 and 7. From results, it is clear that in both the cases, the depth of Wigner function is increasing with increase in the number of photon addition or subtraction. It is also clear that photon subtraction is more effective than photon addition in the state. In the next subsection, we wish to show optical tomograms as it show probabilistic measurement of Wigner function.

3.4.1. Optical Tomogram

Though there exist some proposals for the direct measurement of Wigner function, as it has probabilistic nature direct measurement of Wigner function is not possible, but through data processing, we may measure it experimentally. Tomogram gives the probabilistic description of the quantum state which is accessible for direct measurement. For any quantum state $|\psi\rangle$, the optical propagation
Figure 6. Wigner function for $r$ photon added NGBS with $M = 10$, $q = -0.01$, and $p = 0.8$. Subparts (a), (b), and (c) represent effect of 1 photon, 3 photon, and 5 photon addition, respectively. Here, the depth of Wigner function increases with the addition of photon as for 5 photon maximum depth is -0.0748.

Figure 7. Wigner function for $t$ photon subtracted NGBS with $M = 10$, $q = -0.01$, and $p = 0.8$. Subparts (a), (b), and (c) represent effect of 1 photon, 3 photon, and 5 photon subtraction, respectively. Here, the depth of Wigner function increases with the subtraction of photon as for 5 photon maximum depth is -0.0891. Means photon subtraction is more effective than photon addition at least in NGBS.

tomogram $w_{\psi}(X, \theta)$ is reported earlier as

$$w_{\psi}(X, \theta) = \frac{e^{-X^2}}{\sqrt{\pi}} \left[ \sum_{n=0}^{N} \frac{|c_n|^2}{2^n n!} H_n^2(X) \right. + \sum_{n<k} \frac{|c_n||c_k| \cos((n-k)\theta - (\phi_n - \phi_k))}{\sqrt{2^{n+k-2}n!k!}} H_n(X)H_k(X) \bigg]$$

(16)

where $c_j = |c_j|e^{i\phi_j}$ and $H_j$ is the Hermite polynomial of degree $j$. Optical tomogram of $r$PAQS and $t$PSQS is calculated by using Equations 6–9 and 16. The results are shown in Figures 8 and 9. In the next section, we wish to present quantifier criterion of nonclassicality as quantitative measurement of the state with the help of nonclassical volume.

4. Nonclassical Volume: Quantifier Criterion

For quantitative analysis of the state, we study nonclassical volume which is essentially volume of the negative value region of Wigner function for $r$ photon added NGBS and $t$ photon subtracted NGBS. Kenfack and Zyckowski introduced negative volume of Wigner function as quantifier of nonclassicality in 2004. In fact, the negative volume of the Wigner function is not only a quantifier of nonclassicality but it is a proper rigorous monotone of a resource theory of Wigner negativity and non-Gaussianity. In spite of the negative Wigner volume, there are several other methods to calculate the amount of nonclassicality like Hillery’s distance-based measure of nonclassicality, Lee’s idea of the nonclassical depth, Asboth et al. idea of using measures of entanglement as a measure of nonclassicality, and Vogel’s work. In this particular measure, the volume of the negative part of the Wigner function is considered as the quantitative measure of nonclassicality. To be precise, the nonclassical volume associated with a quantum state $|\psi\rangle$ is

$$\delta(\psi) = \int \int |W_{\psi}(p, q)| \, dqdp - 1,$$

(17)

where $W_{\psi}(p, q)$ is the Wigner function of a quantum state $|\psi\rangle$.

In Table 1, we have shown variation of $\delta(\psi)$ with number of photon addition or subtraction. It is clear that nonclassical volume is increased with the number of photon addition or subtraction, but more effective is photon subtraction as is shown greater volume for the same number of photon. Results are shown in tabular form.
**Figure 8.** Tomogram for $r$ photon added NGBS with $M = 10$, $q = -0.01$, and $p = 0.8$. Different figures (a), (b), and (c) are representing effect of 1 photon, 3 photon, and 5 photon addition respectively.

**Figure 9.** Tomogram for $t$ photon subtracted NGBS with $M = 10$, $q = -0.01$, and $p = 0.8$. Subparts (a), (b), and (c) represent effect of 1 photon, 3 photon, and 5 photon subtraction, respectively.

| Number of photon added and subtracted | Nonclassical volume for photon added state | Nonclassical volume for photon subtracted state |
|--------------------------------------|------------------------------------------|-----------------------------------------------|
| 1                                    | 0.255922                                 | 0.260153                                      |
| 3                                    | 0.31384                                  | 0.335625                                      |
| 5                                    | 0.363856                                 | 0.482082                                      |

Table 1. Nonclassical volume of $r$ photon added and $t$ photon subtracted NGBS for $M = 10$, $q = -0.01$, and $p = 0.8$.

Nonclassical volume increases with the number of photon addition or subtraction but it is more effective in photon subtracted NGBS. The results are consistent with our observation in the study of Wigner function that photon subtraction creates more nonclassicality when compared to photon addition. Overall photon addition or subtraction to a qudit state enhances the nonclassicality of state.

**5. Conclusion**

In the above, we have presented a rigorous study on higher-order nonclassicality of $r$PAQS and $t$PSQS with a general structure which is valid for any qudit states. However, the illustrative examples are given for a particular type of qudit state named as new generalized binomial state (NGBS). First, we describe the analytical form of higher-order moment of $r$PAQS and $t$PSQS; then by using general form of moment, we study many criteria of witness of nonclassicality like higher-order antibunching, higher-order squeezing (HOS - Hillery type), higher-order sub-Poissonian photon statistics (HOSPS), Wigner function, and optical tomogram. Effect of photon addition and subtraction fairly affect nonclassical properties, which are shown in the results. In case of HOA and HOSPS, depth of nonclassical witness increased by addition of number of photon and decreases by subtraction of photon in NGBS state. In HOS, it is shown that photon subtraction is more effective than photon addition. The Wigner quasi-probability distribution function of the $r$NGBS and $t$NGBS are reported for photon addition and subtraction. As Wigner function cannot be measured directly in general but same can be obtained by optical tomogram with the help of Radon transform. Here, we also computed optical tomogram for $r$NGBS and $t$NGBS. To make a quantitative analysis, we further calculate amount of nonclassicality in the form of nonclassical volume, which is essentially the volume of negative part of the Wigner function. It is found that nonclassical volume is increases with the number of photon addition or subtraction and photon subtraction gives more nonclassicality as a comparison. From all our observations, it can be concluded that photon addition and subtraction are useful nonclassicality enhancing or inducing operations. These studies and earlier experimental studies related to tomograms for photon addition and subtraction in squeezed coherent state and number states\cite{33} open up the possibility of experimental verification of our results for photon added and
subtracted qudit states. Therefore, we conclude this paper with the hope that characterization of higher-order nonclassicality in multi-photon added and subtracted NGBS will soon be experimentally verified and will be useful in the applications of quantum information processing. It will also open the ways to study photon added and subtracted version of other qudit states.

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Conflict of Interest
The authors declare no conflict of interest.

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