Proposal of a Temperature Rise Estimation Method for Densely Mounted Components

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Abstract
In this paper, a method for estimating the temperature of densely mounted chip components is discussed. It is assumed that several to dozens of small chip components are mounted in a grid shape and the heat generation of each component is about 0.1 W or less. The temperature distribution and heat flows in the board under various board conditions were investigated by numerical experiments using CFD simulation. Based on the results, a simple estimation formula that can be used for temperature estimation for densely mounted components is proposed.

Keywords: Chip Components, Dense Mounting, Thermal Management, Surface Mount, Pad Dimensions, Thermal Network, Estimation Method

1. Introduction
In recent years, thermal design methods for electronic devices have been changing along with the increase in power density associated with downsizing of electronic devices. Figure 1 shows an image of transition of heat transfer paths in an electronic device. In early days, electronic device used to be mounted on large chassis with ventilation with only a few large components generating almost all heat. So the thermal design of such devices focused on heat dissipation by convection and ventilation. In contrast to this, the latest devices are placed on small sealed chassis, and tend to use many small high-heat-density components. It is important in thermal design for such devices to use heat dissipation by heat conduction and equalize the temperature inside the board and the chassis.

The recent situation about components is such that the rated power for small chip components (hereinafter referred to as small chips) has increased greatly along with demands for downsizing and high performance of devices. Since the surface area of such small chips is small and heat dissipation by convection and radiation from the surface is limited, heat dissipation of components is done mostly through heat conduction to the board. With small chips being used in large quantities, it is not practical to install a dedicated heat sink like a power semiconductor or MPU (Micro Processing Unit). Thus, many small chips today are put to use on the premise of heat dissipation to the board. Reflecting this situation, it has been proposed in the resistor industry that the reference temperature for determining the load power of the chip resistor be...
changed from the conventional ambient temperature \((T_a)\) to the terminal part temperature.\[1\] The term “terminal part temperature” refers to a connecting portion between component and board. This change in the thermal design method also affects the usage of the components.

Thus far, many studies have been done on the thermal design of electronic devices and electronic components. For example, there have been a number of proposals about heat spreading around large heat generating components on the board\[2\] and simplification of analysis methods taking convection inside enclosures into consideration.\[3–5\] For components, studies are being undertaken on the internal modeling of MPU.\[6\] The previous studies have commonly dealt with components with significant heat generation such as MPU and power semiconductors. On the other hand, there has been little discussion about small chip components whose heat generation is about 1 W or less. However, in an evident trend, the heat density of small chips has been dramatically increasing in recent years. There are, in fact, products for which the rated power of 0.5 W is specified for the chip size of 2.0 mm × 1.2 mm, and the heat density in such applications can reach a maximum of 20 W/cm². In order to safely use such small chips with high heat generation density, appropriate heat dissipation measures must be taken. As mentioned above, since the main heat dissipation paths for these components are through the board, it is essential to control the size of the heat dissipation pad (copper pattern) and the layout of the components (spacing of heat-generating components).

Then, we face a problem here. Generally, pattern design of a circuit board is a task of a circuit designer (or board designer). However, many of the designers are not experts in heat transfer engineering, and are unfamiliar with thermal design. When there are many components that require measures to dissipate heat, it is not easy to do the simulation. In view of such a situation, it is strongly desired that designing methods for heat dissipation pad that a circuit designer can easily use be proposed.

The authors of this paper have already proposed a simple method for determining the temperature rise in applications where a plurality of heat-generating chip components are densely mounted as a typical example in the design of a heat dissipation pad.\[7, 8\] The object analyzed was represented by a simple thermal network model and the resulting model was identified by a numerical experiment of CFD (Computational Fluid Dynamics) simulation for various pattern shapes. The result was in a simple estimation formula that can calculate temperature rise on the board from design parameters such as pad shape and board thermal conductivity. The authors confirmed the validity of these thermal models and estimation formulas by experimental facts.\[9\]

In the present paper, we summarize our research results of the temperature estimation of components densely mounted on the board which we have reported so far. Specifically, it covers the following:

- CFD simulations conducted under wider ranges of conditions including square and non-square component layout and the relationship determined between the thermal resistance of each part and the design parameters.
- The estimation formula of the temperature difference \(\Delta T_{cb}\) in the pad obtained for the square layout as discussed in the previous report.\[8\] In this paper, a more general estimation formula derived in consideration of non-square layout as well.

2. Modeling of Physical Phenomena and Investigation by Simulation

2.1 Description of the dense mounting of components on a board in a thermal network model

Figure 2 shows a thermal network model of heat gener-

![Thermal network model of densely mounted board](image)

Fig. 2 Thermal network model of densely mounted board.
ating components mounted densely. Components are placed in a grid pattern on copper pads arranged in the central portion of the board. Figure 2(a) shows the heat flows on the board. The heat flow spreads into the board through the copper pad, and then gets transferred by convection and radiation from the board surface to the ambient. Figure 2(b) shows the heat flows in a thermal network. A simplified thermal network model is shown in Fig. 2(c). The relationship between the temperature of each part and the heat flow is represented by equations (1) to (6).

\[
T_c - T_a = R_{th_{total}} \cdot Q_c \\
T_b - T_a = R_{th_{ba}} \cdot Q_b \\
T_c - T_b = R_{th_{cb}} \cdot Q_b \\
T_c - T_a = R_{th_{sa}} \cdot Q_a \\
Q_b = Q_c - Q_a \\
\frac{1}{R_{th_{total}}} = \frac{1}{R_{th_{sa}}} + \frac{1}{R_{th_{cb}}} + \frac{1}{R_{th_{ba}}} 
\]

In the previous report, relationships with design parameters such as pad shape and board thermal conductivity are discussed for \( R_{th_{ba}}, R_{th_{sa}} \) and \( R_{th_{cb}} \)[7, 8] for non-divided patterns is small from the comparison of the simulation result of them.[10]

### 2.2 Simulation model and conditions

Numerical experiments using CFD simulation were carried out to clarify the thermal resistance of each part of the board model proposed in the previous section. FloTHERM® Ver. 12.0 (manufactured by Mentor Graphics, based on finite volume method) is used for the simulation. Since the phenomenon analyzed in this study is the condition of low Reynolds number with low flow velocity, steady state analysis using 1/4 model is carried out taking flow symmetry into consideration. The simulation covered the influence of radiation as well.

Figure 3 shows the outline of the simulation model used in this study. The board is installed horizontally under natural convection conditions. The board is simplified as shown in the Fig. 3(a). Chip components are replaced with 1.3 mm × 1.3 mm × 0.5 mm ceramic equivalent to the mounting area assuming 1.6 mm × 0.8 mm size components. 1.6 mm × 0.8 mm is a typical size of chip components used in many electronic devices. Components are aligned at regular intervals in a matrix in the pad as shown in Fig 3(b). The copper pattern is simplified to a non-divided pattern. Although the actual copper pad is divided at the component mounting position, the heat flow from that position is symmetrical and can be regarded as a substantially adiabatic condition. It was confirmed that the difference in temperature distribution between divided and non-divided patterns is small from the comparison of the simulation result of them.[10]

In consideration of the influence of the pattern of the inner layers on the multilayer board, the thermal conductivity in the \( x \) and \( y \) directions of the board material (\( \lambda_{xy} \)) is set to 0.6 to 100. The board thickness \( L_y \) is set to 0.8 and 1.6 mm, which are generally used for glass epoxy boards. Other details of the shape of the model are the same as those used for the study in references.[7, 8] The central component temperature \( T_c \) to be derived by simulation is that at the center point of the pad which is the point with the highest temperature. In this study, which aims to estimate the board temperature, the pad temperature (not the component temperature) is evaluated. \( T_c \) can be used as the terminal part temperature of the component mounted in the center. \( T_b \) is the midpoint temperature on the edge of the mounting pad. When \( N_x \neq N_y, T_b \) is the temperature at the midpoint of the longer side of the pad. \( Q_b \) is obtained as the total amount of the heat flux flowing outward from the pad portion (actually the heat flux flowing out from the resin board portion under the copper pad). Table 1 shows the physical property values used in the simulation, and Table 2 is the list of shapes and analysis condition. Simulation is carried out for 52 types of shape conditions (number and pitch of components), two kinds of board thick-
3. Derivation of Thermal Resistance Estimation Formula \((R_{th_{ba}}, R_{th_{sa}})\)

This section shows the relationship between \(R_{th_{ba}}, R_{th_{sa}}\) calculated from simulation results and design parameters.

### 3.1 \(R_{th_{ba}}\): Thermal resistance of heat dissipation from the pad edge through the outer board

\(R_{th_{ba}}\) is a thermal resistance obtained from the pad edge temperature rise \(T_b - T_a\) and the heat flow \(Q_b\) through the board from the pad edge portion, and is calculated from equation (2). As mentioned in the previous report, \(R_{th_{ba}}\) is roughly inversely proportional to the pad perimeter.

\(R_{th_{ba}}\) is also inversely proportional to the 0.5th power of \(L_b \cdot \lambda_{xy}\). [7]

\(L_b\) is the board thickness, and \(\lambda_{xy}\) is the thermal conductivity in the \(x\) and \(y\) direction of the board. The relationship between the thermal resistance \(R_{th_{ba}}\) calculated from the simulation results and the pad perimeter \(L\) is shown in Fig. 4. \(R_{th_{ba}}\) is inversely proportional to the 0.7–0.9th power of \(L\). Figure 5 shows the relationship between \(R_{th_{ba}}\) and \(\lambda_{xy}\). \(R_{th_{ba}}\) is inversely proportional to the approximately 0.5th power of \(\lambda_{xy}\). Although omitted here, it has also been confirmed that \(R_{th_{ba}}\) is inversely proportional to the approximately 0.5th power of \(L_b\). In view of these tendencies, \(R_{th_{ba}}\) can be expressed as equation (7).

\[
R_{th_{ba}} = \frac{0.284 \cdot \lambda_{xy}^{-0.5} \cdot L_b^{-0.5} \cdot L^{-0.839}}{\bar{q}}
\]

(7)

where \(L\): Pad perimeter = \(2P_c \cdot (N_x + N_y)\)

Figure 6 shows the relationship of equation (7), mainly

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### Table 1 Properties of materials.

| Material          | Thermal conductivity [W·m\(^{-1}\)·K\(^{-1}\)] |
|-------------------|-----------------------------------------------|
| Glass epoxy       | \(\lambda_{xy} = 0.6, 1, 5, 10, 50, 100 (x, y direction), \lambda_z = 0.3 (z direction)\) |
| Copper            | 385                                           |
| Ceramic (components) | 25                                            |

Emmissivity of surface: 0.9

### Table 2 Simulation conditions.

| Board conditions          | 150 mm × 150 mm |
|---------------------------|-----------------|
| \(L_b\): Board thickness  | 0.8, 1.6 mm     |
| Copper pattern thickness  | 0.035 mm        |

| Pad and component dimensions |
|-----------------------------|
| \(N_x, N_y\): Number of components |
| \(N_x = N_y = 1, 2, 3, 4, 5, 7\) |
| \(N_x \times N_y = 7 \times 1, 7 \times 2, 7 \times 3, 7 \times 4, 7 \times 5, 5 \times 1, 3 \times 1, 3 \times 2, 2 \times 1\) |

| \(P_c\): Mounting pitch of components | 3, 5, 7 mm |
|--------------------------------------|------------|
| \(q_c\): Heat generation of single component | 0.05 W (\(P_c = 3\) mm), 0.1 W (others) |

| Components’ size                   |
|------------------------------------|
| Normally \(1.3\ mm × 1.3\ mm × 0.5\ mm\) (Length × Width × Height) |
| Heat generating components covering the entire pad. (Fig. 10, 11, 13, 14, 15) |
| Components size: \(\text{Length} = \text{Width} = P_c\), \(\text{Height} = 0.015\ mm\) |

| Others                             |
|------------------------------------|
| Component size changed with \(P_c\) constant (Fig. 8, 9) |
| Components’ size: \(\text{Length} = \text{Width} = 1, 1.4, 2.33, 4.67, 7\ mm\) |
| \(N_x = N_y = 5, P_c = 7\) |

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Fig. 4 Relationship between thermal resistance \(R_{th_{ba}}\) and pad perimeter \(L\).
formulated according to the tendency of L. It is shown that the relationship between $R_{th\_ba}$ and design parameters can be described uniquely by using equation (7). This study shows that the relationship presented in the previous report is valid for a wide range of conditions. The dependency of $R_{th\_ba}$ on the pad perimeter is easily understood by considering that the heat transfer surface from the pad portion to the outer board increases in proportion to the pad perimeter. The board discussed in this paper can be regarded as a type of heat dissipation fin from the form of heat dissipation. In general, the relationship between fin design parameters and fin efficiency in disk-shaped fins can be expressed by a dimensionless number $r_e \cdot \sqrt{h/\lambda}$ ($r_e$: fin base radius, $h$: heat transfer coefficient, $\lambda$: thermal conductivity of fin material, $t$: fin thickness).[11] The above-mentioned dimensionless number suggests the equivalence of the influence of the board thickness and the thermal conductivity in equation (7). In this way, it is thought that the relation between the heat dissipation from the board and the design parameters can be organized using the theory of fin efficiency. However, since more detailed and complicated discussion is necessary, we will discuss it in another paper.

### 3.2 $R_{th\_sa}$: Thermal resistance with convection and radiation from the pad portion

$R_{th\_sa}$ is a thermal resistance representing heat dissipation by convection and radiation from the pad surface. As is mentioned in the previous report, $R_{th\_sa}$ is approximately in inverse proportion to the pad area $S$ and is proportional to 0.1 to 0.2 of $\lambda_{xy}$.[7] From the simulation result, the relationship between the design parameter and $R_{th\_sa}$ is determined and equation (8) is derived.

$$R_{th\_sa} = 0.0708 \cdot \lambda_{xy}^{-0.143} \cdot S^{-0.929} \quad (8)$$

Figure 7 shows the relationship of equation (8), mainly formulated by pad area $S$. Equation (8) expresses the relationship between $R_{th\_sa}$ and design parameters is expressed uniquely. As for $R_{th\_sa}$, it has also been confirmed that the relation presented in the previous report is valid even within the range of the conditions of this study. The reason why $R_{th\_sa}$ increases with the increase in $\lambda_{xy}$ is as follows. As mentioned in section 3.1, $R_{th\_ba}$ decreases as $\lambda_{xy}$ increases. At this time, as the ratio of $Q_b$ increases, the ratio of $Q_{sa}$ decreases. As a result, $R_{th\_sa}$ increases.

### 4. Analysis of Temperature Rise Mechanism within Pad

In this section, we analyze the mechanism of temperature rise within the pad part in detail and discuss the relationship with each design parameter.

#### 4.1 Relationship between component size and temperature distribution (square-shaped pad)

For simplicity, cases where the chips are arranged in a square ($N_x = N_y$) are considered. Figure 8 shows the rela-
The temperature distribution around each component includes influence from heat received from surrounding heat-generating components. That is, when the component pitch is constant, the temperature influence of each component working on adjacent components can be considered almost constant regardless of chip size. Such temperature distribution suggests that the principle of superposition applies to this particular case of analysis. Generally, heat transfer phenomena including the effects of natural convection and radiation show temperature dependency and are nonlinear phenomena. Theoretically, the superposition principle cannot be applied. However, with regard to the temperature distribution in the pad of this analysis model, it can be considered that the linear heat transfer phenomenon is dominant and that the superposition principle can be applied. In this case, the phenomenon can be explained by considering the temperature distribution in an individual pad and the other temperature distribution separately. The temperature difference $\Delta T_{cb}$ can be expressed in the following relationship:

$$\Delta T_{cb} = \Delta T_{c0b} + \Delta T_{cns}$$

where $\Delta T_{c0b}$ is $\Delta T_{cb}$ when the chip size is equal to the individual pad size and $\Delta T_{cns}$ is temperature rise in the individual pad ($= \Delta T_{cb} - \Delta T_{c0b}$).

Figure 9 shows the relationship between chip size and $T_b$, $Q_{sa}$, $Q_b$. The chip size has no significant effect on these values. In other words, the magnitude of $\Delta T_{cns}$ has little impact on the thermal behavior of the entire board. For this reason, $T_{c0}$ can be used instead of $T_{cb}$ as the pad center temperature when describing the thermal behavior of the entire board. However, since $T_c$ is important for managing the component temperature, $\Delta T_{c0b}$ and $\Delta T_{cns}$ are estimated individually. Relationships with various design parameters are confirmed in the following sections for $\Delta T_{c0b}$ and $\Delta T_{cns}$.

### 4.2 Thermal resistance of the uniformly heating pad

In this section, we investigate the dependence of $\Delta T_{c0b}$ on the pad shape. The thermal resistance $Rth_{c0b}$ describing the relationship between $\Delta T_{c0b}$ and $Q_b$ is defined by equation (10).

$$Rth_{c0b} = \Delta T_{c0b} / Q_b$$

The relationship between $Rth_{c0b}$ and pad shape will be discussed in the following. First of all we need to think about the temperature rise of such a heating element and the tendency of thermal resistance theoretically. When the size of the pad is sufficiently larger than the thickness of the board, the pad portion and the board directly beneath it can be regarded as a uniform heat generating object. Moreover, by replacing the shape with a cylindrical shape.
(axisymmetric shape), the temperature distribution can be theoretically obtained by equation (11).[12]

\[ T(x) = T_{\text{max}} - \frac{1}{4\lambda} \frac{1}{x^2} q \]  

(11)

Where \( q \) is heat generation per volume and \( x \) is distance from the axis of symmetry.

In cases where \( x = r \) and pad area = \( S \), the thermal resistance is obtained from equation (11). The heat generation \( Q \) is \( q \cdot V \) (\( V = S \cdot L_p \)).

\[ R_{\text{th, c0ba}} = \frac{T_{\text{max}} - T(r)}{q \cdot S \cdot L_p} = \frac{1}{4\lambda_p} \frac{1}{S \cdot L_p} r^2 \]  

(12)

where \( R_{\text{th, c0ba}} \) is \( R_{\text{th, c0b}} \) analytically obtained, \( L_p \) is board thickness (including pad), \( S \) is pad area, and \( \lambda_p \) is thermal conductivity of the board (including the pad).

\( S \) in equation (12) is originally \( \pi r^2 \). However, since \( S \) is an amount that determines the volume of the heating element, \( N_x \cdot N_y \cdot P_x^2 \), which is the pad area, is given to reflect the heat generation of the pad portion. \( *r^2 \), which is the distance from the pad center to the edge, is \( N_x \cdot P_x / 2 \).

From these, using equation (12), \( R_{\text{th, c0ba}} \) is expressed as follows:

\[ R_{\text{th, c0ba}} = \frac{1}{4\lambda_p \cdot N_x \cdot N_y \cdot P_x^2 \cdot L_p} \left( \frac{N_x \cdot P_x}{2} \right)^2 = \frac{1}{16 \cdot \lambda_p \cdot L_p} \]  

(13)

Equation (13) shows that \( R_{\text{th, c0ba}} \), which represents the thermal resistance in the uniform heat generating pad, will be constant regardless of pad size. Figure 10 shows the relationship between thermal resistance \( R_{\text{th, c0b}} \) and pad shape for heat generation on the entire pad. In the graph, \( R_{\text{th, c0ba}} \) calculated by equation (13) is indicated by a broken line. Since \( \lambda_p \cdot L_p \) used for calculation of \( R_{\text{th, c0ba}} \) is the combined thermal conductance of the copper foil and the board and is therefore given by the equation (14).

\[ \lambda_p \cdot L_p = 385 \times 35.0 \times 10^{-6} + \lambda_{xy} \cdot L_b \]  

(14)

\( R_{\text{th, c0b}} \) converges on a constant value as the pad size increases, and its value does not differ greatly from \( R_{\text{th, c0ba}} \). In the range where the pad size is small, \( R_{\text{th, c0b}} \) increases as the pad size increases. This tendency is presumed to be caused by the heat flow from the chip spreading in the thickness direction of the board, with the thermal resistance decreasing. Figure 11 shows the relationship between \( R_{\text{th, c0b}} \) and \( \lambda_p \cdot L_p \) when the pad size is 70 mm. Data on \( L_b \) of 1.6 mm and 0.8 mm can be uniquely represented by \( \lambda_p \cdot L_p \).

\[ R_{\text{th, c0b}} = \frac{0.0579}{\lambda_p \cdot L_p} \]  

(15)

The above discussion covers the cases where the pad shape is square (\( N_x = N_y \)). But for application of more general conditions, the cases where the pad shape is not square are considered. When the pad shape is not a square, the heat flow from the wider side is dominant. In our study, the non-square pad shape has wider sides in the \( x \) direction so the heat flow \( Q_{by} \) in the \( y \) direction is dominant. Figure 12 shows the pad shape and the heat flows from respective sides. \( R_{\text{th, c0by}} \), which is the heat flow in the \( y \) direction, is defined as equation (16):

\[ R_{\text{th, c0by}} = \frac{T_{\text{by}} - T_b}{Q_{by}} \]  

(16)

\( R_{\text{th, c0by}} \) can be expressed by equation (17).

\[ R_{\text{th, c0by}} = \frac{1}{8 \cdot \lambda_p \cdot L_p \cdot N_x} \]  

(17)

\( R_{\text{th, c0by}} \) is analytically obtained \( R_{\text{th, c0by}} \), and is derived from the temperature distribution of one-dimensional uniform heat generating object as in the same way.
as in deriving the equation (13).

\[ N_y / N_x \] indicates the flatness of the pad. \( N_y = N_x \) means a square pad shape and in that case \( Rth_{c0by} \) is exactly twice the \( Rth_{c0ba} \). Figure 13 shows the relationship between \( Rth_{c0by} \) and the pad size. As the longer side length \( P_x \) increases, \( Rth_{c0by} \) tends to converge on a constant value. The tendency of \( Rth_{c0by} \) relative to the pad size is similar to the tendency shown in Fig. 10. When \( N_y / N_x \) is small, the pad is more flat and \( Rth_{c0by} \) is smaller. Figure 14 shows the relationship between \( Rth_{c0by} \) and equation (15) for \( Rth_{c0by} \) in applicable conditions. \( Rth_{c0by} \) shows a tendency of being proportional to \( N_y / N_x \). That tendency matches with the equation (17). Figure 15 shows a relationship between \( Rth_{c0by} / (N_y / N_x) \) (slope of \( N_y / N_x \) relative to \( Rth_{c0by} \) shown in Fig. 14) and \( \lambda_p \cdot L_p \). The \( Rth_{c0by} / (N_y / N_x) \) shows a tendency of being approximately inversely proportional to \( \lambda_p \cdot L_p \). This relation is represented by equation (18).

\[ Rth_{c0by} = \frac{0.115 \cdot N_y}{\lambda_p \cdot L_p} \quad (18) \]

\( Rth_{c0by} \) indicating the temperature distribution in the pad can be represented using \( N_y / N_x, \lambda_p, \) and \( L_p \). In cases where \( N_x = N_y \), the relationship of equation (19) is obvious since \( Q_{by} = Q_{by} \) from the symmetry of the shape.

\[ Rth_{c0by} = 2 \cdot Rth_{c0b} \quad (19) \]

Equation (20) can be obtained through comparison by substituting equation (18) for \( Rth_{c0by} \) and equation (15) for \( Rth_{c0b} \) into equation (19).

\[ \frac{0.115}{\lambda_p \cdot L_p} = \frac{0.0579}{\lambda_p \cdot L_p} \quad (20) \]

Equation (18) for \( Rth_{c0by} \) substantially coincides with equation (15) under the condition of \( N_x = N_y \). As it is necessary to derive \( Rth_{c0b} \) for arbitrary \( N_y / N_x \), when calculating the actual temperature rise, the relationship between \( Rth_{c0by}, Rth_{c0b}, \) and \( N_y / N_x \) is investigated. It is inferred that the following relationship exists in view of the conductance of the outer board as seen from the pad portion.

\[ Q_{by} / Q_{by} = N_y / N_x \quad (21) \]
The following equation is obtained from $Q_{by} + Q_{bx} = Q_{b}$

$$Q_{b} = Q_{by} (1 + N_y / N_x)$$  \hspace{1cm} (22)

Figure 16 is plotted using the values of $Q_{b}$ and $Q_{by}$ obtained from all our simulation results, taking both sides of equation (22) as the values of the $x$ axis and the $y$ axis. The correlation coefficient ($R^2$) on both sides of equation (22) is 0.9982.

Equation (24) is obtained from equations (18) and (23).

$$R_{th\_c0by} = R_{th\_c0b} \left(1 + \frac{N_y}{N_x}\right)$$  \hspace{1cm} (23)

In cases where $N_x = N_y = 1$, it is equal to equation (19). Equation (24) is obtained from equations (18) and (23).

$$R_{th\_c0b} = \frac{0.115}{\lambda_p L_p} \frac{N_y}{N_x + N_y}$$  \hspace{1cm} (24)

From the above discussion, it is possible to obtain an estimation formula of $R_{th\_c0b}$ in arbitrary $N_x$ and $N_y$.

4.3 Relationship between $R_{thp\_cns}$ and pad design parameters

Figure 17 shows the relationship between $R_{thp\_cns}$ and the component pitch $P_c$ and the number of components $N_x$ ($= N_y$). $R_{thp\_cns}$ is a thermal parameter obtained by dividing $\Delta T_{cns}$ by $q_c$. The relationship of each parameter is expressed by equation (25).

$$\Delta T_{cns} = R_{thp\_cns} \cdot q_c$$  \hspace{1cm} (25)

When $P_c$ is 3 mm, there is a tendency of $R_{thp\_cns}$ decreasing as $N_x$ increases. However, when $P_c$ is 5 mm or more, there is almost no influence due to the increase in $N_x$, and $R_{thp\_cns}$ depends only on $P_c$. $\Delta T_{cns}$ decreases when the adjacent components are close to each other due to the influence of the temperature gradient from surrounding heat-generating components. In case of $P_c = 3$ mm, as $N_x$ increases from 1 to 7, $R_{thp\_cns}$ decreases from 8 K / W to 2 K / W. If $q_c$ is about 0.1 W, the temperature drop is as small as 0.6°C. For this reason, discussion about the influence of $N_x$ when $P_c = 3$ mm is omitted this time.

Figure 18 shows the relationship between $R_{thp\_cns}$ and $P_c$ when $N_x = 1$. $R_{thp\_cns}$ tends to increase in proportion to the logarithm of $P_c$. In addition, with respect to the change in $\lambda_{xy}$, $P_c$ and $R_{thp\_cns}$ hardly change and the influence of $\lambda_{xy}$ is limited. The chip size is 1.3 mm $\times$ 1.3 mm in
the simulation model of this study. Therefore, the individual pad size and chip size are equal in the case of $P_c = 1.3$, and in principle $Rthp_c$ is zero. In consideration of these conditions, the relationship between $Rthp_c$ and $P_c$ can be described in a form proportional to the logarithm of $P_c$. This relationship is represented by equation (26).

$$Rthp_c = 9.94 \cdot \ln P_c + 66.1 \quad (26)$$

The above discussion applies to cases where $N_c = N_p$. In the following, $Rthp_c$ is contemplated for arbitrary $N_c$ and $N_p$. Figure 19 shows the trend of $Rthp_c$ when $\lambda_y = 0.6$, $N_c = 7$ and $N_p$ is varied to 1, 3, 5. The condition of $P_c$ is the same as in Fig. 17 (a). $Rthp_c$ shows the same tendency as Fig. 17 (a), and the values in cases where $P_c = 7$ mm are almost equal. Figure 20 shows the relationship between $Rthp_c$ and $P_c$ for the same condition as Fig. 19. Figure 18 shows variations of thermal conductivity in cases of square pad and Fig. 20 shows variations in the flatness of the pad. From this result, in cases where the pitch is wide and the thermal conductivity is low, we can see that $\Delta T_{cns}$ is determined only by the component size in the individual pad and the mounting pitch $P_c$, and is not related to the flatness of the pad. The relationship between $Rthp_c$ and $P_c$ obtained from Fig. 20 is represented by equation (27).

$$Rthp_c = 10.0 \cdot \ln P_c + 66.6 \quad (27)$$

$Rthp_c$ obtained from equations (26) and (27) has a difference of less than 1% within the range of the conditions employed in this study, which allows the use of either one of them. (In the verification in section 5, we use equation (27).)

5. Simplification of Thermal Resistance Estimation Formula

As discussed in section 4, the relationship between the temperature distribution inside the pad and the pad design parameters is complicated. There are many behaviors with strong nonlinearity such as thermal resistance converging on a fixed value against changes in $P_c$ and $N_c$. In addition, when the $\lambda_y$ is high, the dependence on the shape $(N_c, N_p$ and $P_c$) tends to decrease. When $\lambda_y$ is high, with thermal conduction in the board being dominant, it is considered that thermal resistance in the pad decreases. It is very complicated and cumbersome to formulate all relationships between such $Rth_c0b$ and $Rthp_c$ and design parameters. This is also a problem when using the estimation formula. The final purpose of this study is to propose a simple temperature rise estimation method that can be easily used by board designers. According to such a viewpoint, the priority for this estimation formula is not to strictly show the tendency, but to simply express it by appropriately using highly influential parameters. $Rth_{ba}$ is very dominant for the temperature rise of the densely mounted board covered by this study. $Rth_{ba}$ describes the relationship between the heat flow from the pad to the ambient through the board and the temperature rise, and most of the heat dissipation performance of the entire board depends on $Rth_{ba}$. As can be seen from the thermal network in Fig. 2, when $Rth_{ba}$ is very large with respect to $Rth_{cb}$, $T_b - T_a$ becomes much larger than $T_c - T_b$. In such a case, even if the estimation accuracy of $Rth_{cb}$ is poor, the influence on $T_c$ estimation accuracy is limited. On the other hand, when $Rth_{ba}$ is small, the estimation accuracy of $Rth_{cb}$ also affects the estimation accuracy of $T_c$. According to section 3.1, $Rth_{ba}$ is expressed by equation (7) using pad design parameters. As shown in equation (7), $Rth_{ba}$ is inversely proportional to pad perimeter and board thicknesses $L_b$ and $\lambda_y$. Therefore, attention must be paid to cases when the pad perime-
caused by the error of the ratio of large number of components. And these variations are lower than the simulation results. It is confirmed that estimated values indicated by a circle in a broken line are estimated value on the safe side as intended. Some estimation results under many conditions so it shows the estimated value. The estimated value is larger than the simulation results for \( \Delta T_{cb} \).

6. Verification of The Estimation Formulas

In this section, we verify the validity of the estimation formula obtained through the discussion up to the previous section.

6.1 Estimation accuracy verification of \( \Delta T_{cb} \)

Figure 21 shows the comparison of the simulation result with the estimated value (calculated from the estimation formula) for \( \Delta T_{cb} \). The simulation result is within \( \pm 2 \) K of the estimated value. The estimated value is larger than the simulation result under many conditions so it shows the estimated value on the safe side as intended. Some estimated values indicated by a circle in a broken line are lower than the simulation results. It is confirmed that these points are under conditions with large pad size and large number of components. And these variations are caused by the error of the ratio of \( Q_b / Q_a \) rather than the thermal resistance error \( Rth_{cb} \). Additionally, \( Q_b / Q_a \) depend on \( Rth_{sa} \) and \( Rth_{ba} \). So they affect the estimation accuracy of \( T_b \) and \( T_c \). This is discussed in the next section.

6.2 Verification of estimation accuracy of \( T_b \) and \( T_c \)

Figure 22 shows a comparison between the estimated values and simulation results for \( T_b \) and \( T_c \). The estimated values are generally seen to match the simulation results. The temperature estimation error of \( \Delta T_{cb} \) has no significant impact on the temperature estimation of \( T_c \) and \( T_b \). On the high temperature side, the estimated values show values slightly higher than the simulation result, which is a tendency also discussed in the previous report. This tendency is presumed to be due to priority placed on matching on the low temperature side (30°C. to 100°C. or less) where there are many data in calculating the estimation formula. However, in the application to the circuit board design, there is no practical problem since the upper limit of board temperature is around 100°C in many cases. From the above course of investigation, it was found that simulation results can be reproduced with sufficient accuracy using temperature estimation formulas based on \( Rth_{cb} \), \( Rth_{sa} \), \( Rth_{c0b} \) and \( Rth_{cns} \) as determined in this study.

7. Conclusion

As a conclusion of our investigation thus far, a simplified temperature estimation method of densely mounted components was established. Estimation formulas applicable to a wide range of conditions over thermal resistance \( Rth_{ba} \) and \( Rth_{sa} \) were obtained. For \( \Delta T_{cb} \) indicating the temper-
ature rise in the pad, we obtained estimation formulas applicable to arbitrary layouts including non-square pads. The main findings of ours are as follows:

- \( R_{th_{-ba}} \) is proportional to the \(-0.83\)th power of perimeter \( L \) and \(-0.5\)th power of \( L_b \) (substrate thickness) \( \cdot \lambda_{xy} \) (xy direction thermal conductivity of board).
- \( R_{th_{-sa}} \) is proportional to the \(-0.93\)th power of pad area \( S \) and \(0.14\)th power of \( \lambda_{xy} \).
- As for the temperature distribution inside the pad, the principle of superposition can be applied.
- \( R_{th_{-c0b}} \) that determines \( \Delta T_{c0b} \) varies depending on the pad size, but converges on a constant value when the pad size is large.
- \( R_{th_{-c0b}} \) in any pad shape including non-square pad shape can be described using \( \lambda_p \cdot L_p \cdot N_y \) and \( N_x \).
- The thermal parameter \( R_{thp_{-cns}} \) indicating the temperature rise \( \Delta T_{cns} \) around the component in the pad is not affected by the flatness of the shape, but it is proportional to the logarithm of the component pitch \( P_c \).
- The estimated values of the temperature rise by the proposed simplified estimation formula generally match the simulation results.

\( R_{th_{-ba}} \) is a dominant factor in determining the temperature distribution of the board. The future task is to conduct a more detailed and theoretical analysis on the estimation formula of \( R_{th_{-ba}} \). We will try a theoretical explanation using fin efficiency analysis.

**Nomenclature**

- \( L \): Pad perimeter (m)
- \( L_p \): Thickness of the pad portion (m)
- \( L_b \): Thickness of the board (m)
- \( N_x \): Number of components in x direction (pcs)
- \( N_y \): Number of components in y direction (pcs)
- \( P_c \): Mounting pitch of components (m)
- \( q_c \): Heat generation of single component (W)
- \( Q_c \): Total amount of heat generation by components (\( = N_x \cdot N_y \cdot q_c \)) (W)
- \( Q_{cb} \): Heat flow due to conduction from pad edge to surrounding board (W)
- \( Q_{sa} \): Heat flow from pad surface to ambient (W)
- \( R_{th_{-ba}} \): Thermal resistance between pad edge and ambient (K/W)
- \( R_{th_{-cb}} \): Thermal resistance between pad center and pad edge (K/W)
- \( R_{th_{-c0b}} \): Thermal resistance between pad center and pad edge (in case of uniform heat generation inside the pad) (K/W)

\[ R_{th_{-c0ba}}: R_{th_{-c0b}} \text{ obtained from the theoretical formula (K/W)} \]
\[ R_{th_{-c0by}}: R_{th_{-c0b}} \text{ for heat flow in the y direction (K/W)} \]
\[ R_{th_{-c0bxy}}: R_{th_{-c0by}} \text{ obtained from the theoretical formula (K/W)} \]
\[ R_{th_{-sa}}: \text{Thermal resistance between pad surface and ambient (K/W)} \]
\[ R_{thp_{-cns}}: \text{Thermal parameter of individual pad (}= \Delta T_{cns} / q_c \text{) (K/W)} \]

- \( S \): Pad area (m²)
- \( T_x \): Ambient temperature (°C)
- \( T_p \): Temperature of pad edge (°C)
- \( T_c \): Temperature of pad center (°C)
- \( T_{cb} \): Temperature of pad center (in case of uniform heat generation inside the pad) (°C)
- \( \Delta T_{c0} \): Board temperature rise (= \( T_x - T_c \)) (K)
- \( \Delta T_{cb} \): Board temperature rise (= \( T_p - T_c \)) (K)
- \( \Delta T_{c0b} \): Temperature rise of inside pad in case of uniform heat generation inside the pad (= \( T_{cb} - T_{c0} \)) (K)
- \( \Delta T_{cns} \): Temperature rise in individual pad (= \( \Delta T_{cb} - \Delta T_{c0b} \)) (K)

\[ \lambda_p \]: Thermal conductivity of pad portion (W/m/K)
\[ \lambda_{xy} \]: Thermal conductivity of board (xy direction) (W/m/K)
\[ \lambda_z \]: Thermal conductivity of board (z direction) (W/m/K)
\[ \varepsilon \]: Emissivity of board surface

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