Evidence that the pseudogap (PG) in a near-optimally doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ sample destroys the BCS logarithmic pairing instability [1] raises again the question of the role of the PG in the high-temperature superconducting cuprates [2]. The elimination of the BCS instability is consistent with the view that the PG competes with superconductivity. However, as noted in [1], the onset of superconductivity with a $T_c \sim 90$ K suggests an alternative scenario in which the PG reflects the formation of short range pairing correlations. Here, we report results obtained from a dynamic cluster quantum Monte Carlo approximation (DCA) for a 2D Hubbard model and conclude that (1) the PG, like the superconductivity, arises due to short-range antiferromagnetic correlations and (2) contrary to the usual case in which the pairing instability arises from the Cooper instability, here, the strength of the spin-fluctuations increases as the temperature decreases leading to the pairing instability.

The superconducting transition temperature can be determined from the Bethe-Salpeter gap equation

$$-\frac{T}{N} \sum_{n',k'} \Gamma_{\text{irr}}^{pp}(k,\omega_n, k', \omega_{n'}) G(k',\omega_{n'}) G(-k', -\omega_{n'}) \times \phi_\alpha(k',\omega_{n'}) = \lambda_\alpha \phi_\alpha(k,\omega_n).$$ (1)

Here $G(k,\omega_n)$ is the dressed single particle Green’s function, $\Gamma_{\text{irr}}^{pp}$ the irreducible particle-particle pairing vertex and $k$ and $\omega_n = (2n + 1)\pi T$ are the usual momentum and Matsubara frequencies, respectively. The temperature at which the leading eigenvalue of Eq. (1) goes to 1 gives $T_c$ and the corresponding eigenfunction $\phi_\alpha(k,\omega_n)$ determines the symmetry of the gap. In spin fluctuation theories the pairing vertex is approximated by an effective interaction

$$V_{\text{eff}}(q,\omega_m) = \frac{3}{2} \bar{U}^2 \chi(q,\omega_m)$$ (2)

with $\chi(q,\omega_m)$ the spin susceptibility and $\bar{U}$ a coupling strength. Various groups have used experimental data to model $\chi(q,\omega_m)$, $G(k,\omega_n)$ and $\bar{U}$ in order to determine whether a spin-fluctuation pairing interaction is consistent with the observed $T_c$ values.

Dahm et al. [3] used inelastic neutron scattering (INS) measurements for YBa$_2$Cu$_3$O$_{6.6}$ to model the spin susceptibility $\chi(q,\omega_m)$ and a one-loop self-energy approximation to determine $G$. $\bar{U}$ was an adjustable parameter estimated from INS and ARPES data. Using the resulting $G$ and $V_{\text{eff}}$ in Eq. (1), they concluded that a spin-fluctuation interaction had sufficient strength to account for the observed $T_c$. Nishiyama et al. [4] used inelastic neutron scattering results for $\chi(q,\omega)$ and solved the Eliashberg equations for the heavy fermion compounds CeCuSi$_2$ and CeIrIn$_3$. For reasonable values of $\bar{U}$, they found $T_c$ values which were again consistent with the notion that antiferromagnetic spin fluctuations were responsible for pairing in these materials. In a recent paper Mishra, et al. [5] used angular resolved photoemission spectroscopy (ARPES) data for a slightly underdoped BSCCO ($T_c = 90K$) sample to examine the effect of the pseudogap (PG) on the superconducting transition temperature and to determine whether a spin-fluctuation pairing mechanism could account for the observed $T_c$. They found that the usual BCS logarithmic divergence associated with the propagators in Eq. (1) was destroyed by the pseudogap and the leading eigenvalue $\lambda_\alpha(T)$ remained small, and was essentially independent of temperature. This raises old questions regarding the interplay between the PG and superconductivity [2] which continue to be of interest [6-8]. Here, using the dynamic cluster approximation (DCA), we explore spin-fluctuation pairing in a Hubbard model which exhibits a PG.

The two-dimensional Hubbard model we will consider has a near neighbor hopping $t$, a next near neighbor hopping $t'/t = -0.15$, an onsite Coulomb interaction $U/t = 7$ and a filling $\langle n \rangle = 0.92$. We will work in energy units where $t = 1$. The DCA calculations [9] were carried out on a 4 x 4 cluster and employed both continuous-time, auxiliary-field (CT-AUX) quantum Monte Carlo (QMC) [10] and Hirsch-Fye (HF) QMC methods to solve the effective cluster problem [12]. In the DCA approximation, where $\Gamma_{\text{irr}}^{pp}$ depends only on a finite set of cluster momenta $K$, the $k$-sum in Eq. (1) gives [13]

$$-\frac{T}{N_c} \sum_{n',K'} \Gamma_{\text{irr}}^{pp}(K,\omega_n, K', \omega_{n'}) \chi_0^{pp}(K',\omega_n') \phi_\alpha(K',\omega_{n'})$$

$$= \lambda_\alpha \phi_\alpha(K,\omega_n).$$ (3)

Here $N_c = 16$ is the cluster size and the pairing kernel

\[ \text{Pairing in the Presence of a Pseudogap} \]

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\( G(k, \omega_n)G(-k, -\omega_n) \) has been coarse-grained (averaged) over the momenta \( k' \) of the DCA patches

\[
\bar{\chi}_0^{pp}(K, \omega_n) = \frac{N_c}{N} \sum_{k'} G(K + k', \omega_n)G(-K - k', -\omega_n) \, .
\]

For the parameters we have chosen, the uniform static susceptibility \( \chi(q = 0, T) \) versus temperature, shown in Fig. 1a, exhibits a peak at \( T^* = 0.22 \) below which it decreases as \( T \) is reduced [14]. This behavior, seen in measurements of the magnetic susceptibility [15] and Knight shifts [16] of underdoped (hole) cuprates, reflects the opening of a pseudogap. ARPES experiments [17, 18] find that this gap is anisotropic, opening in the antinodal regions of the Fermi surface. This behavior has also been seen in DCA calculations of the single-particle spectral weight [13, 19]. In Fig. 1b, the temperature dependence of the leading eigenvalue of the Bethe-Salpeter equation [3] is shown as the circles. Its eigenfunction has \( d \)-wave symmetry and \( \lambda_d(T) \) approaches 1 at low temperatures. Thus this model system has a pseudogap that opens below \( T^* \) and a \( d \)-wave eigenvalue that increases towards 1 as \( T \) decreases.

In addition to suppressing the \( q = 0 \) spin susceptibility, we find that the opening of the pseudogap destroys the low temperature BCS logarithmic divergence of the \( d \)-wave projection of the pairing kernel

\[
P_{0d}(T) = -\frac{T}{N_c} \sum_{K, \omega_n} \phi_d(K, \omega_n) \bar{\chi}_0^{pp}(K, \omega_n) \phi_d(K, \omega_n) \, .
\]

Here, \( \bar{\chi}_0^{pp}(K, \omega_n) \) is defined in Eq. (4) and \( \phi_d(k, \omega_n) \) is the \( d \)-wave eigenfunction, which is approximated as

\[
\phi_d(k, \omega_n) \sim \begin{cases} 
(\cos k_x - \cos k_y) & |\omega_n| < J \\
0 & \text{otherwise}
\end{cases}
\]

with \( J \sim 4t^2/U \). A plot of \( P_{0d}(T) \) versus \( T \) is shown in Fig. 2a, and one can see that below \( T^* \), \( P_{0d}(T) \) is suppressed as the pseudogap opens [20, 21]. Here we have normalized \( P_{0d}(T) \) to its value at a temperature \( T = 0.5t \) above \( T^* \). For comparison, the solid squares in Fig. 2a show \( P_{0d}(T) \) for \( \langle n \rangle = 0.85 \) which does not have a pseudogap and one sees the usual BCS logarithmic behavior (dashed curve).

The absence of the BCS divergence in \( P_{0d}(T) \) when there is a pseudogap is consistent with the finding of Mishra et al. [22]. However, as noted, they found that with this suppression, the spin-fluctuation pairing interaction failed to give a superconducting transition. Based on this, they suggested that the pseudogap reflects the absence of short-range antiferromagnetic correlations which grow below \( T^* \) and become coherent at \( T_c \). This behavior could be likened to the magnetic response of the large \( U \) half-filled Hubbard model. In this case, the formation of local moments when the temperature drops below \( \sim U/2 \) is seen in an increase in the expectation value of the square of the local moment \( \langle S^2 \rangle = (\langle \frac{1}{2} (n_{\uparrow} - n_{\downarrow}) \rangle)^2 \). In a similar way one can look for the onset of local pair formation as \( T \) decreases below the pseudogap temperature \( T^* \). Here with \( \Delta^{\uparrow,\downarrow}_{\ell+x,\ell} = c^\dagger_{\ell+x}\uparrow \epsilon_{\ell+x}\uparrow - c^\dagger_{\ell}\uparrow c_{\ell}\downarrow \) and \( \Delta_d = \Delta^{\uparrow,\downarrow}_{\ell+x,\ell} + \Delta^{\uparrow,\downarrow}_{\ell+y,\ell} \), we have calculated \( \langle \Delta_d \rangle \) versus temperature. As shown in Fig. 2a, this correlation function does increase as the temperature decreases. However, the four near neighbor pairfield correlations

\[
\langle \Delta^{\uparrow,\downarrow}_{\ell+x,\ell} \Delta^{\uparrow,\downarrow}_{\ell+x,\ell} \rangle = \frac{1}{2} \langle n_{\ell}n_{\ell+x} \rangle - \langle S^\dagger_{\ell} S_{\ell+x} \rangle,
\]

contribute the dominant contribution to this increase as shown in Fig. 2a. These results suggest that the pseudogap is more closely related to the formation of short range...
antiferromagnetic correlations than to local pair correlations in agreement with earlier ideas of Johnston and more recent theoretical results. This identification of the PG with the development of short-range AF spin correlations is also consistent with the increase of the spin-susceptibility \(\chi(Q = (\pi, \pi), \omega_m = 0)\) as shown in Fig. 3 and as seen experimentally.

Returning to the question of whether the spin-fluctuation interaction, Eq. (2), can lead to superconductivity when the logarithmic singularity of the BCS kernel is suppressed, we use DCA results for \(G(k, \omega_n)\) to construct \(V_{\text{eff}}(q, \omega_m)\). Here, following Mishra et al., an

\[\chi_{\text{RPA}}(Q, \omega_m) = \frac{\chi_0(Q, \omega_m)}{1 - U\chi_0(Q, \omega_m)}\]

with

\[\chi_0(Q, \omega_m) = \frac{T}{N_c} \sum_K \bar{G}(K + Q, \omega_n + \omega_m)\bar{G}(K, \omega_n),\]

where \(\bar{G}(K, \omega_n) = N_c/N \sum_{k'} \bar{G}(K + k', \omega_n)\) is the DCA coarse-grained Green’s function. The coupling \(U\) is estimated from the approximate fit of \(\chi_{\text{RPA}}\) to \(\chi_{\text{DCA}}\) shown in Fig. 3.

Then, replacing \(\Gamma_{\text{pp}}\) by \(V_{\text{eff}}\) and using DCA Green’s functions, we solve the Bethe-Salpeter equation. Results for \(\lambda_d(T)\) are shown (solid squares) in Fig. 2. We conclude that the increase in the strength of the pairing interaction \(V_{\text{eff}}\) leads to an increasing \(\lambda_d(T)\) similar to that which is found using \(\Gamma_{\text{pp}}\) determined from the DCA calculation. Thus, in spite of the absence of the BCS logarithmic increase in \(P_{0d}(T)\), we find that the increase in the strength of the spin-fluctuations leads to an increase in \(\lambda_d(T)\) as the temperature is lowered. This differs from the results of reference 11 and we speculate that this difference arises from a failure of their parametrization of \(G(k, \omega_n)\) by ARPES data taken at 140 K as the temperature is lowered.

To summarize, we have used DCA calculations for an under (hole) doped 2D Hubbard model, which exhibits a pseudogap, to see whether a spin-fluctuation interaction provides a reasonable approximation of the irreducible pairing interaction. In this calculation, the dynamic

\[\chi_{\text{DCA}}(Q = (\pi, \pi), \omega_m = 0)\] from the DCA calculation (circles) and the RPA fit, Eq. (8), with \(U = 6.7\) (squares). The AF response continues to increase as \(T\) decreases below \(T^*\) leading to an increase of the spin-fluctuation interaction so that even though the BCS logarithmic increase of \(P_{0d}(T)\) is suppressed, the \(d\)-wave eigenvalue \(\lambda_d(T)\) increases as seen in Fig. 2.
mean-field cluster is such that charge density and strip-
ing instabilities are suppressed, leaving antiferromagnetic
and $d$-wave pairing as the dominant correlations. We
find that while the pseudogap eliminates the usual BCS
logarithmic divergence of the pairing kernel, a pairing
instability arises from an increase in the strength of the
spin-fluctuation interaction as the temperature decreases.

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