Can CCM law properly represent all extinction curves?

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Abstract. We present the analysis of a large sample of lines of sight with extinction curves covering wavelength range from near-infrared (NIR) to ultraviolet (UV). We derive total to selective extinction ratios based on the Cardelli, Clayton & Mathis (1989, CCM) law, which is typically used to fit the extinction data both for diffuse and dense interstellar medium. We conclude that the CCM law is able to fit most of the extinction curves in our sample. We divide the remaining lines of sight with peculiar extinction into two groups according to two main behaviors: a) the optical/IR or/and UV wavelength region cannot be reproduced by the CCM formula; b) the optical/NIR and UV extinction data are best fit by the CCM law with different values of $R_V$. We present examples of such curves. The study of both types of peculiar cases can help us to learn about the physical processes that affect dust in the interstellar medium, e.g., formation of mantles on the surface of grains, evaporation, growing or shattering.

1. Introduction
Interstellar grains affect starlight which passes through them by absorbing and scattering photons. These two physical processes produce the interstellar extinction which depends on the properties of dust grains, e.g., size distribution and composition. The description of the average extinction curve of our Galaxy as a function of the wavelength from the infrared to ultraviolet can be found in Savage [1]. Extinction curve shows some evident features: it rises in the infrared, it shows a slight knee in the optical, it is characterized by a bump at 2175 Å, and it rises in the far-ultraviolet. These features are common between different environments. The properties of interstellar grains are different in diffuse and dense interstellar medium and thus also the extinction changes. CCM [2] found an average extinction law valid over the wavelength range 0.125μm ≤ $\lambda$ ≤ 3.5μm, which is applicable to both diffuse and dense regions of the interstellar medium. This extinction law depends on only one parameter $R_V = A_V/E(B−V)$. The $R_V$ parameter ranges from about 2.0 to about 5.5 (with a typical value of 3.1) when one goes from diffuse to dense interstellar medium and thus $R_V$ characterizes the region that produces the extinction.

If one knows the value of $R_V$ along a particular line of sight, one can obtain the extinction curve from the infrared to ultraviolet using the CCM law (see Figure [1]):

$$\frac{A_\lambda}{A_V} = a(x) + b(x) \cdot R_V^{-1},$$

(1)
where $x = 1/\lambda$, and $a(x)$ and $b(x)$ are the wavelength-dependent coefficients.

There are different ways to obtain $R_V$ using the NIR or UV extinction data. Wegner [3] computed $R_V$ values for the sample of 597 OB stars with known near-infrared magnitudes. He assumed that, in the infrared spectral region, the normalized extinction curve is proportional to $\lambda^{-3}$ or $\lambda^{-4}$. Extrapolating the IR interstellar extinction curve to $1/\lambda = 0$ he derived $R_V$ as:

$$R_V = - \left[ \frac{E(\lambda - V)}{E(B - V)} \right]_{\lambda \to \infty}$$

(2)

Gnaciński & Sikorski [4] applied the $\chi^2$ minimization method to compute the $R_V$ values for a sample of ultraviolet (UV) extinction data using the linear relation (1). Geminale & Popowski [5] extended the analysis from [4] by using non-equal weights derived from observational errors to determine $A_V$ and $R_V$ values toward a sample of stars with known ultraviolet color excesses.

In this paper we use both optical/infrared ($URIJHKLM$) and UV extinction data to obtain $R_V$ values for a sample of 436 lines of sight. We arrive at two main conclusions: (i) there are lines of sight with extinction in the optical/IR or/and UV which generically don’t follow the CCM law, and (ii) there are lines of sight which show an extinction curve that cannot be reproduced with a single $R_V$ value in the whole wavelength range.

2. Theoretical Basis

We normalize the extinction in a standard way:

$$\epsilon(\lambda - V) = \frac{E(\lambda - V)}{E(B - V)}.$$  

(3)
The absolute extinction may be deduced from the relative extinction by using a total-to-selective extinction ratio $R_V$:

$$R_V = \frac{A_V}{E(B-V)}. \quad (4)$$

Then:

$$\epsilon(\lambda - V) = \frac{E(\lambda - V)}{E(B-V)} = \frac{A_\lambda - A_V}{E(B-V)} = R_V \left\{ \frac{A_\lambda}{A_V} - 1 \right\}. \quad (5)$$

For each individual band, equation (11) and (5) can be combined to derive an $R_V$ value. More generally, the $\chi^2$ minimization can be used to obtain the $R_V$ value that provides the best CCM fit to all observed extinction data. Gnaciński & Sikorski [4] suggested the following $\chi^2$, which can be minimized to derive $R_V$:

$$\chi^2 = \sum_{i=1}^{N_{\text{bands}}} \left\{ E(\lambda_i - V) - E(B-V) \cdot [R_V(a(x_i) - 1) + b(x_i)] \right\}^2, \quad (6)$$

where $a(x_i)$ and $b(x_i)$ are the coefficients of the CCM law. Following our previous work [5], we use an improved weighted formula to find $R_V$. We minimize the following $\chi^2$:

$$\chi^2 = \sum_{i=1}^{N_{\text{bands}}} w_\lambda_i \epsilon(\lambda_i - V) - [R_V(a(x_i) - 1) + b(x_i)]^2 E^2(B-V) \quad (7)$$

where $\omega_i \equiv 1/\sigma_i^2$ are the weights associated with each band.

Minimizing equation (7) with respect to $R_V$, we find:

$$R_V = \frac{\sum_{i=1}^{N_{\text{bands}}}(a(x_i) - 1) \cdot (\epsilon(\lambda_i - V) - b(x_i))/\sigma_i^2}{\sum_{i=1}^{N_{\text{bands}}}(a(x_i) - 1)^2/\sigma_i^2} \quad (8)$$

where here $\sigma_i$ values are taken from Wegner [6] and they were computed according to:

$$\sigma_i^2 \equiv \sigma^2[\epsilon(\lambda_i - V)] = \left[ \frac{1}{E(B-V)} \right]^2 \left\{ \sigma^2[E(B-V)] + [E(\lambda_i - V)]^2 \sigma^2[E(\lambda_i - V)] \right\} \quad (9)$$

We estimate the error in $R_V$ from:

$$\sigma(R_V) \equiv \sum_{j=1}^{N_{\text{bands}}} \left| \frac{\partial R_V}{\partial \epsilon(\lambda_j - V)} \right| \cdot \sigma_j = \frac{1}{\sum_{j=1}^{N_{\text{bands}}}(a(x_i) - 1)^2/\sigma_i^2} \cdot \sum_{j=1}^{N_{\text{bands}}} \left| a(x_j) - 1 \right|/\sigma_j \quad (10)$$

### 3. Data

We use a sample of 436 lines of sight with optical/IR and UV extinction data reported by Wegner [6]. In Wegner’s computation the ultraviolet photometry is taken from Wesselius et al. [7] and based on Astronomical Netherlands Satellite (ANS); infrared magnitudes in $J, H, K, L, M$ passbands are originate mostly from the catalog of Gezari, Schmitz and Mead [8] and Gezari et al. [9]. The $R$ and $I$ magnitudes with accuracy of $\pm 0.01$ mag are taken from Johnson [10] and Fernie [11]. The spectral classification and $UBV$ data which have the accuracy of $\pm 0.01$ mag come from the SIMBAD database.

The effective wavelengths of the optical/IR bands are: $\lambda_U = 0.36\mu m$, $\lambda_R = 0.71\mu m$, $\lambda_I = 0.97\mu m$, $\lambda_J = 1.25\mu m$, $\lambda_H = 1.65\mu m$, $\lambda_K = 2.2\mu m$, $\lambda_L = 3.5\mu m$, $\lambda_M = 4.8\mu m$; whereas the effective wavelengths of the UV bands are: 0.1549, 0.1799, 0.2200, 0.2493, and 0.3294 $\mu m$.

From the original sample, we exclude 20 lines of sight because of their negative value of $R_V$. Negative $R_V$ values are typically the result of noisy data for the lines of sight with small $E(B-V)$. 
Figure 2. Examples of extinction curves with $\chi^2/dof > 2.0$. The long dashed line represents the CCM curve which is the best fit to the IR data and the point-short dashed line represents the CCM curve which best follows the UV data.

4. Results
4.1. $\chi^2$ test
A useful method to test if all extinction data points are well fitted by the CCM law is to compute $\chi^2$ based on equation (7) normalized to the number of degrees of freedom, which is equal to the number of observed points minus the number of fitted parameters (in our case the only parameter is $R_V$). We note that our average $\chi^2/dof$ is not equal to the expected value of 1; since the errors are taken from Wegner (2002) and we do not want to modify them, we do not renormalize our errors requesting $<\chi^2/dof> = 1$. Therefore it is safer to treat our $\chi^2/dof$ as a measure of a relative rather than absolute quality of the CCM fit in the optical/IR and UV wavelength ranges. We select outliers considering the tail of our $\chi^2/dof$ distribution. We assume that the values $\chi^2/dof > 2.0$ are indicative of the lines of sight with extinction curves not well fitted by the CCM law. We find that 25% of the lines of sight of our sample shows this disagreement with the CCM law. Figure 2 displays two extinction curves representative of this group.

4.2. Test for the universality of CCM law
We further analyze the lines of sight with $\chi^2/dof < 2.0$ for which we expect the CCM law to fit well all observed extinction data from optical/IR to UV. The usual assumption is that the knowledge of the $R_V$ value obtained from the IR part of the extinction curve may be used to obtain the entire extinction curve by using the CCM law. We critically test this assumption using two sets of $R_V$ values for each line of sight: the infrared $R_V^{(IR)}$ and ultraviolet $R_V^{(UV)}$.
Figure 3. The comparison between the $R_V$ values obtained from optical/IR and UV extinction data. The points with error bars are those for which $|\delta| \geq 2.0$.

$(\text{UV}R_V)$. We compute the following statistic:

$$\delta = \frac{\text{UV}R_V - \text{IR}R_V}{\sqrt{\sigma^2[\text{UV}R_V] + \sigma^2[\text{IR}R_V]}}$$ (11)

Specifically, when $|\delta| \geq 2.0$ we classify the line of sight as anomalous in the sense that the CCM law is not able to reproduce the whole extinction curve with a single value of $R_V$.

Figure 3 shows the comparison between the $R_V$ values obtained from optical/IR and UV extinction data. There is a large scatter around the 1-to-1 relationship; however, only the points with the error bars shown are those which deviate from this relation significantly ($|\delta| \geq 2.0$). These 25 lines of sight need two different values of $R_V$ to match the optical/IR and UV part of the extinction curve properly. Therefore, in these cases we cannot universally use the CCM law to represent the entire extinction curve. Six cases out of 25 have the $\text{UV}R_V$ higher than the $\text{IR}R_V$ one, and the other 19 have the $\text{UV}R_V$ lower than the $\text{IR}R_V$ one. Figure 4 shows two examples of such anomalous extinction curves.

5. Conclusion
We use a $\chi^2$ minimization method to compute the $R_V$ values for a sample of 436 lines of sight. We exclude 20 cases because of their negative $R_V$ and analyze the final sample of 416 lines of sight. We derive our $R_V$ values assuming CCM law. This law aims at reproducing the entire extinction curve by using only one parameter: $R_V$. We analyze the goodness of the CCM fit for all lines of sight using the $\chi^2/dof$ statistic and test the universality of the CCM law by computing $R_V$ values separately for the optical/IR and UV part of the extinction curve. We find that for 69% of our original sample, the CCM law is able to fit well both the optical/IR and UV data. We divide the remaining 31% of cases into two groups, according to two main peculiarities: a) the optical/IR or/and UV extinction data points cannot be fitted by the CCM
law (25% of the entire sample); b) $R_V$ values are significantly different for the two spectral regions: optical/IR and UV (6% of the entire sample). Unless caused by faulty data, peculiar extinction curves result from unusual properties of dust grains. Therefore, theoretical modeling of these extinction curves (e.g., Mishchenko [12]; Saija et al. [13]; Weingartner & Draine [14]) may help us to understand the processes which modify the properties of interstellar grains.

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