Fuzzy Precompact Space

Saad Mahdi Jaber

University of Wasit/ Education College for Pure Science/ Department of Mathematics

s.jaber@uowasit.edu.iq

Abstract. In this paper we study and introduce the concept of preompactness of a fuzzy topological space and also, we attain a number of important characterizations of a fuzzy precompact space. The notion of precompactness that can be extended to arbitrary fuzzy sets. So, this paper explains the relationship between fuzzy precompact space and fuzzy precompact subspace. Finally, we give necessary and sufficient conditions for a fuzzy pre regular space to be fuzzy precompact.

Key words and phrases: Fuzzy precompact space, Fuzzy pre q-nbd, Fuzzy pre cluster point.

1. Introduction

The fuzzy concept has invaded almost all branches of mathematics, since the introduction the fundamental concept of fuzzy sets by Zadeh [9] in 1965. Chang [4] in 1968, introduced the definition of fuzzy topological spaces and extended in a straightforward manner some concepts of crisp topological spaces to fuzzy topological spaces. The fuzzy topology was originating with Chang's article [9] in 1968, also may be considered as a new branch of mathematics, then many additional structures were studied by using fuzzy sets and the related problems in pure and applied mathematics. While Wong [16] in 1974 discussed and generalized some properties of fuzzy topological spaces. Ming, p.p. and Ming, L.Y. [11] in 1980 used fuzzy topology to define the neighborhood structure of fuzzy point. Shahna A. S. Bin [13] in 1991 defined the concept of pre open in fuzzy topological space.

In what follows, a fuzzy topological space \((X,T)\) as defined by Chang [4], we shall denote for its by a \(fts (X,T)\) or simply by a \(fts X\). The concepts closure [4], interior [4] and complement [15] of a set \(A\) in a fuzzy topological space \((X,T)\) are denoted by \(cl(A), int(A)\) and \(1 - A\) respectively. A fuzzy set \(A\) in \(X\) is said to be fuzzy pre open if \(A \subseteq int(cl(A))\). The fuzzy pre closed \(1 - A\) is a complement of a fuzzy pre open set \(A\). The notation \(pcl(A)\) stands for the fuzzy pre closure, which is the union of all fuzzy points \(x_{\alpha}\), when any fuzzy pre open set \(U\) containing \(x_{\alpha}\) with \(A \cup U \neq 0\), every fuzzy open in a fts \(X\) is fuzzy pre open.

2. Preiminaries

First, we recall the following definitions, theorems, propositions, corollaries, remarks and lemmas that are needed in the next section.

2.1. Definition [10, P.211-220]

Let \(X \neq \emptyset\) and let \(I\) be the unite interval, that means \(I = [0,1]\). A fuzzy set \(A\) in \(X\) is a function from \(X\) into the unit interval \(I\). (that means \(A: X \rightarrow [0,1]\) be a function).

A fuzzy set \(A\) in \(X\) can be explain by the set of pairs: \(A = \{(x,A(x)) : xeX\}\). The notation \(I^X\) stand for the family of all fuzzy sets in \(X\).
2.2. Definition [10, P.211-220]
Let \( f \) be a fuzzy mapping from a set \( X \) into \( Y \). Let \( A \in I^X \) and \( B \in I^Y \).

a- The image of \( A \) under \( f \), \( f(A) \) is a fuzzy set in \( Y \) defined by for each \( y \in Y \),

\[
[f(A)](y) = \begin{cases} 
\sup A(x) & \text{if } f^{-1}(y) \neq \emptyset \\
0 & \text{otherwise,}
\end{cases}
\]

Where \( f^{-1}(y) = \{ x \in X | f(x) = y \} \).

b- The inverse image of \( B \) under \( f \), \( f^{-1}(B) \) is a fuzzy set in \( X \) defined by for each \( x \in X \),

\[
[f^{-1}(B)](x) = B(f(x)).
\]

2.3. Definition [10, P.211-220],[3]
A fuzzy point \( x_\alpha \) in \( X \) is fuzzy set defined as follows:

\[
x_\alpha(y) = \begin{cases} 
\alpha & \text{if } y = x \\
0 & \text{if } y \neq x
\end{cases}
\]

Where \( 0 < \alpha \leq 1 \); \( \alpha \) is called its value and \( x \) is support of \( x_\alpha \).
The set of all fuzzy points in \( X \) will be denoted by \( FP(X) \).

2.4. Definition [10, P.211-220], [11]
A fuzzy point \( x_\alpha \) in \( X \) is said to belong to a fuzzy set \( A \) (denoted by: \( x_\alpha \in A \)) if and only if \( \alpha \leq A(x) \).

2.5. Definition [10, P.211-220],[11]
A fuzzy set \( A \) in \( X \) is called quasi–coincident with a fuzzy set \( B \) in \( X \) denoted by \( A \sqcap B \) if and only if \( A(x) + B(x) > 1 \), for some \( x \in X \). If \( A \) is not quasi–coincident with, then \( A(x) + B(x) \leq 1 \), for every \( x \in X \) and denoted by \( A \sqcup B \).

2.6. Lemma [3, P.137-150]
Let \( A \) and \( B \) are fuzzy sets in \( X \). Then:

(a) If \( A \sqcap B = 0 \), then \( A \sqcap B \) B
(b) \( A \sqcap B \) if and only if \( A \leq B^c \)

2.7. Proposition [3, P. 137-150]
If \( A \) a fuzzy set in \( X \), then \( x_\alpha \in A \) if and only if \( x_\alpha A^c \).

2.8. Definition [4, P.182-190]
A fuzzy topology on a set \( X \) is a collection \( T \) of fuzzy sets in \( X \) satisfing:

1. \( 0 \in T \) and \( 1 \in T \),
2. If \( A \) and \( B \) belong to \( T \) then \( A \sqcap B \in T \),
3. If \( A_i \) belong to \( T \) for each \( i \in I \), then so does \( \bigvee_{i \in I} A_i \)

If \( T \) is a fuzzy topology on \( X \), then the pair \((X, T)\) is called a fuzzy topological space. Members of \( T \) are called fuzzy open sets. Fuzzy sets of the forms \( 1 - A = A^c \), where \( A \) is fuzzy open set are called fuzzy closed sets.

2.9. Definition [13, P.303-308], [17]
Let \((X, T)\) be a fuzzy topological space. Then:

i) The fuzzy interior of \( A \), denoted by \( \text{int}(A) \) is the union of all fuzzy open sets in \( X \) which are contained in \( A \). (\( \text{int}(A) = \bigvee \{ B : B \sqsubseteq A, B \in T \} \))

ii) The fuzzy pre closure of \( A \), denoted by \( \text{cl}(A) \) is the intersection of all fuzzy closed sets in \( X \) contains \( A \). (\( \text{cl}(A) = \bigwedge \{ B : A \sqsubseteq B, B^c \in T \} \))
2.10. Definition [10, P.211-220]
A fuzzy set $A$ in $fts(X)$ is called quasi-neighborhood of fuzzy point $x_\alpha$ in $X$ if and only if there exists $B \in T$ such that $x_\alpha q B \subseteq A$.

2.11. Definition [10, P.211-220]
Let $(X, T)$ be a fuzzy topological space and $x_\alpha$ be a fuzzy point in $X$. Then the family $N_{x_\alpha}^q$ consisting of all quasi-neighborhood (q-nbd) of a fuzzy point $x_\alpha$ is said to be the system of quasi-neighborhood of $x_\alpha$.

2.12. Theorem [13, P.303-308], [17]
Let $(X, T)$ be a fuzzy topological space and $A, B$ are two fuzzy sets in $X$. Then:

i) $0 = cl(0)$,

ii) $cl(A \cup B) = cl(A) \cup cl(B)$ and $cl(A \cap B) \leq cl(A) \cap cl(B)$,

iii) $int(A \cap B) = int(A) \cap int(B)$ and $nt(A \cup B) \geq int(A) \cup int(B)$,

iv) $cl(int(A)) = cl(A)$ and $nt(int(A)) = int(A)$,

v) $nt(A) \leq A \leq cl(A)$,

vi) If $A \subseteq B$ then $int(A) \subseteq int(B)$ and $cl(A) \subseteq cl(B)$.

2.13. Remark
Let $A, B$ are two fuzzy sets in $fts(X)$, then:

a- $int(A) = 1 - cl(1 - A)$,

b- $pint(A) = 1 - pcl(1 - A)$.

Proof: a- It is straightforward. b- It is straightforward.

2.14. Definition [4, P.182-190]
Let $(X, T)$ be a fuzzy topological space and let $A$ be any fuzzy set in $X$. $A$ is called fuzzy pre open set if $A \subseteq int(cl(A))$. The complement of a fuzzy pre open set is called fuzzy pre closed set. The family of all fuzzy pre open sets in $X$ will be denoted by $FPO(X)$.

2.15. Definition [2, P.131-139]
A fuzzy set $A$ in $fts(X)$ is said to be pre quasi-neighborhood (pre q-nbd) of $x_\alpha \in FP(X)$ if and only if there exists $B \in FPO(X)$ such that $x_\alpha q B \subseteq A$.

2.16. Definition [6, P.303-312]
A fuzzy set $A$ in $fts(X)$ is said to be fuzzy pre quasi-neighborhood (pre q-nbd) of $x_\alpha \in FP(X)$, if there is a fuzzy pre open set $B$ in $X$, such that $x_\alpha q B \subseteq A$. The family of all pre quasi-neighborhood of fuzzy point $x_\alpha$ is said to be the system of pre quasi-neighborhood of $x_\alpha$ and denoted by $N_{x_\alpha}^{pq}$.

2.17. Proposition
Let $A$ be a fuzzy set in $fts(X)$. Then:

iii) The fuzzy pre interior of $A$, denoted by $pint(A)$ is the union of all pre open subsets of $X$ which are contained in $A$.

iv) The fuzzy pre closure of $A$, denoted by $pcl(A)$ is the intersection of all fuzzy pre closed subset of $X$ contains $A$.

2.18. Proposition [12, P.1601-1608]
Let $(X, T)$ be a fuzzy topological space and $A, B \subseteq X$. Then:

i) $int(A) \subseteq pint(A) \subseteq A$,

ii) $A \subseteq pcl(A) \subseteq cl(A)$,

iii) $A$ is a fuzzy pre closed iff $pcl(A) = A$,

iv) $pcl(pcl(A)) = pcl(A)$.
If $A \leq B$, then $pcl(A) \leq pcl(B)$.

$V_{i \in I} pcl(U_i) \leq pcl(V_{i \in I}(U_i))$.

$x_\alpha \in pcl(A)$ iff $U \wedge A \neq 0, \forall U \in FPO(X), x_\alpha \in U$.

2.19. Remark

If $A, B$ are fuzzy pre open sets, then $A \wedge B$ is fuzzy pre open.

Proof: It is clear.

2.20. Remark [8, P.111-12]

Let $A$ be a fuzzy set in $fts(X)$. Then $A$ is a fuzzy pre open if and only if $A$ is a fuzzy pre quasi-neighborhood of its fuzzy points.

2.21. Proposition

Let $A$ be a fuzzy set in $fts(X)$. Then a fuzzy point $x_\alpha \epsilon pcl(A)$ if and only if every fuzzy pre open $B \epsilon FPO(X)$, if $x_\alpha qB$ then $AqB$.

Proof: $\Rightarrow$ Suppose that $B$ be a fuzzy pre open set in $X$ such that $x_\alpha qB$ and $AqB$. Then $A \leq (1-B)$, but $x_\alpha \epsilon (1-B)$ (since $x_\alpha qB$, then $\alpha \geq (1-B)(x)$) and $1-B$ is a fuzzy pre closed set in $X$. Thus $x_\alpha \epsilon pcl(A)$.

$\Leftarrow$ Suppose that $x_\alpha \epsilon pcl(A)$, then there exists a fuzzy pre closed set $B$ in $X$ such that $A \leq B$ and $x_\alpha \epsilon B$, therefore by (2.7. Proposition), we have $x_\alpha q1 - B$. Since $A \leq B$, then by (2.6.ii. Lemma), $Aq1 - B$. Hence $x_\alpha \epsilon pcl(A)$ if $x_\alpha qB$ and $AqB$.

2.22. Definition [8, P.111-121]

In a $fts(X)$, a mapping $S: D \rightarrow FP(X)$ is said to be a fuzzy net and denoted by $\{S(n): n \in D\}$, $D$ is directed set. If $S(n) = x^n_{\alpha_n}$ where $x \in X$, $n \in D$ and $\alpha_n \in (0,1]$, then we shall denote it by $\{x^n_{\alpha_n}: n \in D\}$ or simply $\{x^n_{\alpha_n}\}$.

2.23. Definition [8, P.111-121]

A fuzzy net $\Xi = \{y^m_i: m \in E\}$ in $X$ is called a fuzzy subnet of fuzzy net $S = \{x^n_{\alpha_n}: n \in D\}$ if and only if there is a mapping $f: E \rightarrow D$ such that:

(a) $\Xi = Sof$, that is $y^m_i = x^{f(i)}_{\alpha_{f(i)}} \forall i \in E$.

(b) $\forall n \in D$ there is some $m \in E$, such that $f(m) \geq n$.

A fuzzy sub net of a fuzzy net $\{x^n_{\alpha_n}: n \in D\}$ denoted by $\{x^{f(m)}_{\alpha_{f(m)}}: m \in E\}$.

2.24. Definition [8, P.111-121]

Let $S = \{x^n_{\alpha_n}: n \in D\}$ be a fuzzy net in a fuzzy topological space $(X, T)$ and $A \in I^X$, then:

i- $S$ is said to be eventually with $A$ if and only if $\exists m \in D$ such that $x^n_{\alpha_n}qA, \forall n \geq m$.

ii- $S$ is said to be frequent with $A$ if and only if $\forall n \in D, \exists m \in D, m \geq n$ and $x^m_{\alpha_m}qA$.

2.25. Definition [8, P.111-121]

Let $S = \{x^n_{\alpha_n}: n \in D\}$ be a fuzzy net in a fuzzy topological space $(X, T)$ and $x_\alpha \in FP(X)$, then:

(i) $S$ is said to be convergent to $x_\alpha$ and denoted by $S \rightarrow x_\alpha$, if $S$ is eventually with $A, \forall A \in N^Q_{x_\alpha}$, $x_\alpha$ is called a limit point of $S$.

(ii) $S$ is said to be has a cluster point $x_\alpha$ and denoted by $S \alpha x_\alpha$, if $S$ is frequently with $A, \forall A \in N^Q_{x_\alpha}$. 
2.26. Definition
Let \( S = \{ x^p_{\alpha} : \alpha \in D \} \) be a fuzzy net in a fuzzy topological space \((X, T)\) and \( x_\alpha \in FP(X) \). Then:

(i) \( S \) is said to be p-convergent to \( x_\alpha \) (denoted by: \( S \xrightarrow{p} x_\alpha \)), if \( S \) is eventually with \( A, \forall A \in N^p_{x_\alpha} \), \( x_\alpha \) is called a pre limit point of \( S \).

(ii) \( S \) is said to be called has a fuzzy p-cluster point \( x_\alpha \) (denoted by: \( S^p_{\alpha} x_\alpha \)), if \( S \) is frequently with \( A, \forall A \in N^p_{x_\alpha} \).

2.27. Definition \([8, P.111-121]\]
A fuzzy filterbase on \( X \) is a non-empty subset \( F \) of \( I^X \) such that:

(1) \( 0 \notin F \).

(2) If \( A_1, A_2 \in F \), then \( \exists A_3 \) such that \( A_3 \leq A_1 \wedge A_2 \).

2.28. Definition \([8, P.111-121]\]
A fuzzy point \( x_\alpha \) in a fuzzy topological space \((X, T)\) is said to be a fuzzy pre cluster point of a fuzzy filterbase \( F \) on \( X \) if \( x_\alpha \in pcl(F) \), for all \( F \in F \).

2.29. Definition
A fuzzy topological space \((X, T)\) is called fuzzy pre Hausdorff or pre \( T_2 \)-space if and only if for every pair of distinct fuzzy points \( x_\alpha, y_\beta \) in \( X \), there exist \( A \in N^p_{x_\alpha}, B \in N^p_{y_\beta} \) such that \( A \wedge B = 0 \).

2.30. Definition
Let \( B \) be a fuzzy set in a fuzzy topological space \((X, T)\), then \( T_B = \{ A \wedge B : A \in \tau \} \) is called a fuzzy relative topology and \((B, T_B)\) is said to be a fuzzy topological subspace of \( X \).

2.31. Theorem
In a fuzzy topological space \((X, T)\), if \( V \) is a fuzzy open set, then \( V \wedge cl(A) \leq cl(V \wedge A) \) for any fuzzy set \( A \) in \( X \).

Proof: Let \( x_\alpha \in FP(X) \) and \( V \) is a fuzzy open in \( X \). If \( x_\alpha \in V \wedge cl(A) \), then \( x_\alpha \in V \) and \( U \wedge A \neq 0 \), \( \forall U \in T, x_\alpha \in U \). Since \( U \wedge V \) is fuzzy open set, therefore \( U \wedge (V \wedge A) \neq 0 \) and \( x_\alpha \in cl(V \wedge A) \). Hence \( V \wedge cl(A) \leq cl(V \wedge A) \).

2.32. Definition
In a fuzzy topological space \((X, T)\), if \( A \leq B < X \), then a fuzzy set \( A \) is called fuzzy pre open in \( B \) if there exist a fuzzy pre open \( H \) in \( X \) such that \( A = H \wedge B \).

2.33. Proposition
In a fuzzy topological space \((X, T)\), if \( A \leq B < X \), then a fuzzy set \( A \) is a fuzzy pre open in \( B \), if \( A \) is a fuzzy pre open in \( X \).

Proof: We have \( A = A \wedge B \), but \( A \) is fuzzy pre open in \( X \). Hence, by (2.32. Definition) \( A \) is a fuzzy pre open in \( B \).

2.34. Proposition
Let \( A \leq B < X \), where \((X, T)\) is a fuzzy topological space and \( B \) is a fuzzy pre open in \( X \). Then \( A \) is a fuzzy pre open in \( B \) if and only if \( A = S \wedge B \), where \( S \) is a fuzzy open in \( X \).

Proof: \( \Rightarrow \) To prove \( A \) is a fuzzy pre open in \( B \), we must prove \( S \wedge B \) is a fuzzy pre open in \( X \) (i.e. \( S \wedge B \leq int(cl(S \wedge B)) \)).

Since \( S \wedge B \leq S \wedge int(cl(B)) = int(int(S)) \wedge int(cl(B)) \),

by (2.31. Theorem) \( \leq int(cl(int(S) \wedge B)) \). Thus \( S \wedge B \) is pre open in \( X \). Hence \( A \) is a fuzzy pre open in \( B \) by (2.33. Proposition).

\( \Leftarrow \) We have \( A = S \wedge B \), since \( S \) is fuzzy open in \( X \), then \( S \) is a fuzzy pre open. Hence by (2.30. Definition) \( A \) is a fuzzy pre open in \( B \).
2.35. Definition [2, P.131-139]
A family \( V \) of fuzzy sets has the finite intersection property if and only if the intersection of the members of each finite subfamily of \( V \) is a non-empty.

2.36. Definition [8, P.111-121]
A family \( B \) of a fuzzy sets in a fuzzy topological space \((X, T)\) is said to be a fuzzy pre open cover of a fuzzy set \( A \) if and only if \( A \subseteq \bigvee \{G : G \in B\} \) and each member of \( B \) is pre open fuzzy set. A sub cover of \( B \) is a sub family which is also cover.

2.37. Definition [5, P.39-49]
In a fuzzy topological space \((X, T)\), a fuzzy set \( D \) is said to be fuzzy dense if there exists no fuzzy closed set \( B \) in \( X \), such that \( D \cap B \neq \emptyset \). That is, \( cl(D) = 1 \).

2.38. Definition [5, P.39-49]
In a fuzzy topological space \((X, T)\), a fuzzy set \( D \) is said to be fuzzy pre dense if there exists no fuzzy pre closed set \( B \) in \( X \), such that \( D \cap B \neq \emptyset \). That is, \( pcl(D) = 1 \).

2.39. Theorem
Let \((X, T)\) be a fuzzy topological space. A fuzzy set \( D \) in \( X \) is a fuzzy dense if and only if it is a fuzzy pre dense, with \( int(D) \neq \emptyset \).

Proof: (\( \Rightarrow \)) Suppose that \( D \) is a fuzzy dense in \( X \). Let \( x_\alpha \in FP(X) \) and \( x_\alpha \in cl(D) \). but \( x_\alpha \notin pcl(D) \). So \( x_\alpha \in (1 - pcl(D)) \) by (2.13.Remark) implies that \( x_\alpha \in pint(1 - D) \subseteq (1 - D) \subseteq cl(1 - D) \). So \( x_\alpha \in (1 - int(D)) \) by (2.13.Remark). Thus, \( x_\alpha \notin int(D) \) so there is no fuzzy open \( U \) (containing \( x_\alpha \)) such that \( U \subseteq D \), and so \( U \cap D = \emptyset \), a contradiction that \( D \) is a fuzzy dense set. Therefore \( x_\alpha \notin pcl(D) \) and \( 1 \leq pcl(D) \). Hence, \( pcl(D) = 1 \).

(\( \Leftarrow \)) It is straightforward.

2.40. Definition [6, P.303-312]
A fuzzy topological space \((X, T)\) is said to be fuzzy pre regular if for each fuzzy point \( x_t \) and each fuzzy pre q-nbd \( U \) of \( x_t \), there exists a fuzzy pre open set \( V \) in \( X \) such that \( x_t \in V \) and \( \supseteq \) 1.

2.41. Theorem
In a \( fts(X) \), \( x_\alpha \in FP(X) \) and \( A \subseteq \bigvee \). Then \( x_\alpha \in pcl(A) \) if and only if there exists a fuzzy net in \( A \) pre converge to \( x_\alpha \).

Proof: (\( \Rightarrow \)) Suppose that \( x_\alpha \in pcl(A) \), then for every \( B \in N_{x_\alpha}^{p} \), there is

\[
x_B(y) = \begin{cases} A(x_\alpha) & \text{if } y = x_B \\ 0 & \text{if } y \neq x_B \end{cases}
\]

such that \( A(x_\alpha) + B(x_\alpha) > 1 \).

Notice that \( (N_{x_\alpha}^{p}, \supseteq) \) is directed set, therefore \( S; N_{x_\alpha}^{p} \to FP(X) \) is a fuzzy net in \( A \) and defined as \( S(B) = x_B \). To show that \( S \to x_\alpha \). Let \( W \in N_{x_\alpha}^{p} \), then there is \( F \subseteq T \) such that \( x_\alpha \in F \) and \( F \subseteq W \).

Since \( F(x_F^A) + x_F^A > 1 \) and \( F \subseteq W \), then \( x_F^A \subseteq W \). Let \( E \subseteq F \), therefor \( E \subseteq F \).

Since \( x(x_F^A) + x_F^A > 1 \) and \( F \subseteq W \), then \( x(x_F^A) + x_F^A > 1 \). Thus \( x_F^A \subseteq W \). Let \( E \subseteq F \). Therefore \( S \to x_\alpha \).

(\( \Leftarrow \)) Suppose that \( \{x_n^A : n \in D\} \) is a fuzzy net in \( A \) where \( (D, \supseteq) \) is directed set, such that \( x_n^A \to x_\alpha \). Then for every \( \in N_{x_\alpha}^{p} \), there exists \( m \in D \) such that \( x_m^A \subseteq W \) for all \( n \geq m \). Since \( x_n^A \in A \), then by (2.7.Proposition) \( x_n^A \subseteq A^c \), thus \( A \subseteq W \) and \( x_\alpha \in pcl(A) \).
2.42. Lemma
In a $f(s) X$, a fuzzy point $x_\alpha$ is a fuzzy pre cluster point for the fuzzy net $\{S(n) : n \in D\}$ with a directed set $(D, \geq)$ if and only if it has a fuzzy subnet which fuzzy pre converges to $x_\alpha$.

**Proof:** $\implies$ Suppose that a fuzzy net $\{x_{\alpha n}^n : n \in D\}$ has the pre cluster point $x_\alpha$. Let $N_{x_\alpha}^{pq}$ be the collection of all fuzzy pre q-nbd of $x_\alpha$. Thus, for any $W \in N_{x_\alpha}^{pq}$ there exists $\{x_{\alpha n}^n\}$ such that $\{x_{\alpha n}^n\}qW$.

All ordered pairs $(n, W)$ with the above character forms the set $O$, that means $n \in D, W \in N_{x_\alpha}^{pq}$ and $\{x_{\alpha n}^n\}qW$. Now, we will define a relation $^p\mathcal{E}$ on $O$ given by $(m, U) \mathcal{E}(n, V)$ iff $m \geq n$ in $D$ and $U \subseteq V$, then $(O, \mathcal{E})$ is a directed set and it is clear to see that $\mathcal{S} : O \rightarrow FP(X)$ given by $\mathcal{S}(m, U) = \{x_{\alpha m}^m\}qU$ is a fuzzy subnet of the assumed fuzzy net $W$. A pre q-nbd of $x_\alpha$ thus, there exists $n \in D$ and therefor $\{x_{\alpha n}^n\}qW$ when $(n, W) \in O$. Now, $(m, W) \in O$ and $(m, U) \mathcal{E}(n, W)$ $\implies$ $\mathcal{S}(m, U) = \{x_{\alpha m}^m\}qU$ and $U \subseteq W \implies \mathcal{S}(m, U)qW$. Hence $\mathcal{S}$ is pre converges to $x_\alpha$.

$\implies$ Suppose that a fuzzy net $\{x_{\alpha n}^n : n \in D\}$ has not a pre cluster point. Therefor, for every fuzzy point $x_\alpha$ there exists a pre q-nbd of $x_\alpha$ such that $x_{\alpha n}^n \not\triangleright U$ for all $m \geq n, n \in D$. Hence, clear no fuzzy net pre converge to $x_\alpha$.

2.43. Proposition
In a fuzzy pre Hausdorff space $X$, any pre convergent fuzzy net has a unique limit point.

**Proof:** $\implies$ Suppose that $x_{\alpha n}^n$ is a fuzzy net on $X$ with directed set $D$, such that $x_{\alpha n}^n \rightarrow x_\alpha, x_{\alpha n}^n \rightarrow y_\beta$ and $x \neq y$. Since $x_{\alpha n}^n \rightarrow x_\alpha$, we have $\forall A \in N_{x_\alpha}^{pq}, \exists m_1 \in D$, such that $\{x_{\alpha n}^n\}qA, \forall n \geq m_1$. Also, $x_{\alpha n}^n \rightarrow y_\beta$, we have $\forall B \in N_{y_\beta}^{pq}, \exists m_2 \in D$, such that $\{x_{\alpha n}^n\}qB, \forall n \geq m_2$. Now, then there exists $m \in D$, such that, $m \geq m_1$ and $m \geq m_2$ then $\{x_{\alpha n}^n\}q(A \land B), \forall n \geq m$. Therefore $A \land B \neq 0$. Hence $X$ is not fuzzy pre Hausdorff.

$\implies$ Let $X$ be a not fuzzy pre Hausdorff space, then there is $x_\alpha, y_\beta \in FP(X)$, such that $x \neq y$ and $A \land B \neq 0, \forall A \in N_{x_\alpha}^{pq}, \forall B \in N_{y_\beta}^{pq}$. Put $N_{x_\alpha,y_\beta}^{pq} = \{A \land B : A \in N_{x_\alpha}^{pq}, N_{y_\beta}^{pq}\}$. Therefore $\forall D \in N_{x_\alpha,y_\beta}^{pq}$, there exists $x_DqD$, then $\{x_D\}_{D \in N_{x_\alpha,y_\beta}}$ is a fuzzy net in $X$. To prove that $x_D \rightarrow x_\alpha$ and $x_D \rightarrow y_\beta$. Let $W \in N_{x_\alpha}^{pq}$, then $W \in N_{x_\alpha,y_\beta}^{pq}$ (since $W = W \land X \neq 0$). Thus $x_DqW, \forall D \geq W$, thus $x_D \rightarrow x_\alpha$ and $x_D \rightarrow y_\beta$. Hence, $\{x_D\}_{D \in N_{x_\alpha,y_\beta}}$ has two fuzzy limit point.

2.44. Definition
A fuzzy space $X$ is called fuzzy precompact if every fuzzy pre open of cover $X$ has finite sub cover.

2.45. Theorem [8, P.111-121]
A fuzzy topological space $(X, T)$ is a fuzzy compact if and only if every fuzzy filter base on $X$ has a fuzzy cluster point.

3. Fuzzy precompact space

3.1. Theorem
A fuzzy topological space $(X, T)$ is a fuzzy precompact, if and only if any collection $\{B_j : j \in J\}$ of fuzzy pre closed sets in $X$ having the finite intersection property.

**Proof:** $\Rightarrow$ Suppose that $X$ is fuzzy precompact space and $\{B_j : j \in J\}$ is collection of fuzzy pre closed sets of $X$ with the finite intersection property. To show $\{B_j : j \in J\}$ has a non-empty intersection (i.e to show $\bigwedge_{j \in J} B_j \neq 0$).
Assume that $\bigwedge_{j \in J} B_j = 0$, then $\bigvee_{j \in J} B_j^c = 1$ and each $B_j^c$ is fuzzy pre open set, thus there exist $j_1, j_2, ..., j_n$ such that $\bigwedge_{i=1}^m B_{j_i} = 1$ by (2.44.Definition), therefore $\bigwedge_{i=1}^m B_{j_i} = 0$ which is contradiction and therefore $\bigwedge_{j \in J} B_j \neq 0$.

($\Leftarrow$) Conversely, let $\{A_j: j \in J\}$ be a fuzzy pre open cover of $X$ and every collection of fuzzy pre closed sets in $X$ with the finite intersection property has a non-empty. To show that $X$ is a fuzzy precompact space. Since $\bigvee_{j \in J} A_j = 1$, then $\bigwedge_{j \in J} A_j^c = 0$ and each $A_j^c$ is fuzzy pre closed set which implies that $\{A_j^c: j \in J\}$ collection of fuzzy pre closed sets with empty intersection and so by hypothesis this collection does not have the finite intersection property. Thus, there exist a finite member of fuzzy sets $A_{j_i}^c, i = 1, 2, ..., n$, such that $\bigwedge_{i=1}^m A_{j_i}^c = 0$, which implies $\bigvee_{i=1}^m A_{j_i} = 1$ and $\{A_{j_i}: i = 1, 2, ..., n\}$ is finite sub cover of the space $X$ belong to a fuzzy pre open cover $\{A_j: j \in J\}$. Hence, $X$ is a fuzzy precompact space.

3.2. Theorem
A fuzzy topological space $(X, T)$ is a fuzzy precompact, if and only if for every fuzzy filterbase on $X$ has a fuzzy pre cluster point.

Proof: $\Rightarrow$ Suppose that $X$ is a fuzzy precompact and $F = \{F_\alpha: \alpha \in A\}$ is a fuzzy filterbase on $X$ having no fuzzy pre cluster point. Let $x \in X$, then for each $n \in N(n \text{ is natural number})$, there exists a pre q-nbd $U^0_n$ of $x \in FP(X)$ and $F^0_n \in F$ such that $U^0_n \cap F^0_n \neq \emptyset$. Now, $U^0_n(x) > 1 - \frac{1}{n}$, since we have $U^0_n(x) = 1$, where $U_n = \bigvee\{U^0_n: n \in N\}$. Therefore, $O = \{U^0_n: n \in N, x \in X\}$ is a fuzzy pre open cover of $X$. When $X$ is fuzzy precompact, then there exists $U^1_{x_1}, U^2_{x_2}, ..., U^k_{x_k}$ of $O$ such that $\bigwedge_{i=1}^k U^i_{x_i} = 1$. If $F \in F$ such that $F \leq F^1_{x_1} \land F^2_{x_2} \land ... \land F^k_{x_k}$, then $F \nexists 1$. Consequently, $F \nexists 0$ and this contradicts the definition of a fuzzy filterbase.

($\Leftarrow$) Suppose that every fuzzy filterbase have a fuzzy pre cluster point. To prove that $X$ is fuzzy precompact. A collection of fuzzy pre closed sets $\beta = \{F_\alpha: \alpha \in A\}$ having finite intersection property. Now, the set of finite intersections of members of $\beta$ forms a fuzzy filterbase $F$ on $X$. By assumed condition $F$ has a fuzzy pre cluster point, which is $x_\alpha$. Thus, $x_\alpha \in \bigwedge_{\alpha \in A} \text{pcl}(F_\alpha) = \bigwedge_{\alpha \in A} F_\alpha$ and $\bigwedge\{F: F \in F\} \neq 0$. Hence by (3.1.Theorem), $X$ is a fuzzy precompact.

3.3. Theorem
A fuzzy topological space $(X, T)$ is a fuzzy pre compact if and only if for every fuzzy net in $X$ has a fuzzy pre cluster point.

Proof: $\Rightarrow$ Suppose that $X$ is a fuzzy precompact and $\{S(n): n \in D\}$ is a fuzzy net in $X$ which has no pre cluster point. Thus, for any fuzzy point $x_\alpha$, there is a fuzzy pre q-nbds $U_{x_\alpha}$ of $x_\alpha$ and an $n_{U_{x_\alpha}} \in D$ such that, for each $m \in D$, $S_m \cap U_{x_\alpha}$ with $m \geq n_{U_{x_\alpha}}$. Since $x_\alpha \in U_{x_\alpha}$ then $S_m \neq \emptyset, \forall m \geq n_{U_{x_\alpha}}$. Let $U$ be a symbol for the collection of all $U_{x_\alpha}$ and $x_\alpha$ is symbol for all fuzzy points $FP(X)$. Now, to show that $V = \{1 - U_{x_\alpha}: U_{x_\alpha} \in U\}$ is a family of fuzzy pre closed sets in $X$ having finite intersection property. At first notice that there exists $k \geq U_{x_{a_1}}, U_{x_{a_2}}, ..., U_{x_{a_m}}$ such that $S_p \cap U_{x_{a_i}}$ for $i = 1, 2, ..., m$ and for all $p \geq k (p \in D)$, that means $S_p \cap U_{x_{a_i}} = \bigwedge_{i=1}^m (1 - U_{x_{a_i}})$ for all $p \geq k$. Hence $\bigwedge_{i=1}^m (1 - U_{x_{a_i}}; i = 1, 2, ..., m) \neq 0$. Since $X$ is a fuzzy precompact, then by (3.1.Theorem), there is $y_{\beta} \in FP(X)$ such that, $y_{\beta} \in \bigwedge\{1 - U_{x_{a_i}}; U_{x_{a_i}} \in U\} = 1 - \bigvee\{U_{x_a}: U_{x_a} \in U\}$. Therefor, $y_{\beta} \in 1 - U_{x_{a}}$, for all $U_{x_a} \in U$ and $y_{\beta} \in 1 - U_{y_{\beta}}$, which means $y_{\beta} \cap U_{y_{\beta}}$. Since, for each fuzzy point $x_\alpha$, there is $U_{x_\alpha} \in U$ such that $x_\alpha \in U_{x_\alpha}$, then we get a contradiction.

($\Leftarrow$) By (3.2.Theorem) we prove the converse, since every fuzzy filterbase on $X$ has a fuzzy pre cluster point. Let $F$ be a fuzzy filterbase in $X$, then for each $0 \neq F \in F$, we can select $x_F \in FP(X)$ such that $x_F \in F$. Let $S = \{x_F: F \in F\}$ with the relation "$\geq" be defined as follows $F_\alpha \geq F_\beta$ if and only if $F_\alpha \geq F_\beta$ in $X$, for $F_\alpha, F_\beta \in F$. Thus $(F, \geq)$ is directed set. Thus, $S$ is a fuzzy net when $(F, \geq)$ is directed
set for its. From assumption, \( S \) has a cluster point \( x_t \). Therefore, for every fuzzy pre \( q \text{-nbd} N \) of \( x_t \) and for each \( F \in \mathcal{F} \), there is \( G \in \mathcal{F} \) with \( G \supseteq F \) such that \( x_q \in G \). As \( x_q \subseteq G \subseteq F \). It follows that \( F q N \) for each \( F \in \mathcal{F} \), then by (2.21.\textit{Proposition}), \( x_t \in pcl(F) \). Hence \( x_t \) is a fuzzy pre cluster point of \( \mathcal{F} \).

### 3.4. \textbf{Corollary}

A fuzzy topological space \((X, T)\) is a fuzzy precompact if and only if for every fuzzy net in \( X \) has a pre convergent fuzzy subnet.

\textbf{Proof}: By (2.42. \textit{Lemma}) and (3.3. \textit{Theorem}).

### 3.5. \textbf{Proposition}

Let \((X, T)\) be a fuzzy topological space. If \( G \) and \( H \) are two fuzzy precompact in \( X \), then \( \bigcup G \) is also fuzzy precompact.

\textbf{Proof}: Suppose that \( \{ A_j : j \in I \} \) is a fuzzy pre open cover of \( \bigcup G \), then \( \bigcup G \subseteq \bigcup A_j \). Since \( G \subseteq \bigcup G \) and \( H \subseteq \bigcup G \), thus \( \{ A_j : j \in I \} \) is a fuzzy pre open cover of \( G \) and fuzzy pre open cover of \( H \). But \( G \) and \( H \) are two fuzzy precompact sets, thus there exists a finite sub cover \( \{ A_{j_1}, A_{j_2}, \ldots, A_{j_n} \} \) of \( \{ A_j : j \in I \} \) which covering \( G \) and a finite sub cover \( \{ A_{j_{k_1}}, A_{j_{k_2}}, \ldots, A_{j_{k_n}} \} \) of \( \{ A_j : j \in J \} \) which covering \( H \) such that \( G \subseteq \bigcup_{i=1}^{m} A_{j_i} \) and \( H \subseteq \bigcup_{k=1}^{n} A_{j_k} \), therefore, \( \bigcup G \subseteq \bigcup_{i=1}^{m+n} A_{j_i} \). Hence \( \bigcup G \) is fuzzy precompact.

### 3.6. \textbf{Proposition}

Every fuzzy precompact space is a fuzzy compact.

\textbf{Proof}: Suppose that \( \mathcal{A} = \{ A_j : j \in I \} \) is a fuzzy open cover of fuzzy space \( X \) and \( X = \bigcup A_j \). But, every fuzzy open set in \( X \) is a fuzzy pre open and \( X \) is a fuzzy precompact space, then there exists \( j_1, j_2, \ldots, j_n \in I \) such that \( X = \bigcup_{i=1}^{n} A_{j_i} \), thus \( X \) is fuzzy compact space.

### 3.7. \textbf{Corollary}

Let \((X, T)\) be a fuzzy topological space. If \( G \) is a fuzzy precompact in \( X \), then \( G \) is fuzzy compact.

\textbf{Proof}: It is straightforward.

### 3.8. \textbf{Proposition}

Let \((X, T)\) be a fuzzy topological space. If \( B \) is a fuzzy set in \( X \) and \( A \subseteq B \), then \( A \) is a fuzzy precompact in \( X \) if and only if \( A \) is a fuzzy precompact in \( B \).

\textbf{Proof}: \( \Rightarrow \) Suppose that \( \mathcal{A} = \{ A_j : j \in I \} \) is a fuzzy cover of \( A \) by pre open sets in \( B \). By (2.32.\textit{Definition}), \( A_j = S_j \wedge B \) for each \( j \in I \), where \( S_j \) is a fuzzy pre open in \( X \). Thus \( \mathcal{D} = \{ S_j : j \in I \} \) is a fuzzy cover of \( A \) by pre open sets in \( X \), but \( A \) is a fuzzy pre compact in \( X \), so there exists \( j_1, j_2, \ldots, j_n \in I \) such that \( A \subseteq \bigcup_{i=1}^{n} (S_{j_i} \wedge B) = \bigcup_{i=1}^{n} (A_{j_i}) \). Hence, \( A \) is a fuzzy precompact in \( B \).

\( \Leftarrow \) It is straightforward.

### 3.9. \textbf{Proposition}

Let \((X, T)\) be a fuzzy topological space. If \( B \) is a fuzzy pre open set in \( X \) and \( A \subseteq B \), then \( A \) is a fuzzy compact in \( X \) if and only if \( A \) is a fuzzy precompact in \( B \).

\textbf{Proof}: \( \Rightarrow \) Suppose that \( \mathcal{A} = \{ A_j : j \in I \} \) is a fuzzy pre open cover of \( A \) in \( B \). By (2.34.\textit{Proposition}), \( A_j = S_j \wedge B \) for each \( j \in I \), where \( S_j \) is a fuzzy open in \( X \). Thus \( \mathcal{D} = \{ S_j : j \in I \} \) is a fuzzy cover of \( A \) by fuzzy open sets in \( X \), but \( A \) is a fuzzy compact in \( X \), so there exists \( j_1, j_2, \ldots, j_n \in I \) such that \( A \subseteq \bigcup_{i=1}^{n} (S_{j_i} \wedge B) = \bigcup_{i=1}^{n} (A_{j_i}) \). Hence, \( A \) is a fuzzy precompact in \( B \).

\( \Leftarrow \) Suppose that \( \mathcal{D} = \{ S_j : j \in I \} \) is a fuzzy open cover of \( A \) in \( X \). Then \( \mathcal{A} = \{ S_j \wedge B : j \in I \} \) is a fuzzy cover of \( A \). But, \( S_j \) is a fuzzy open in \( X \) for all \( j \in I \) and \( B \) is a fuzzy pre open in \( X \), then by (2.34.\textit{Proposition}) \( S_j \wedge B \) is a fuzzy pre open in \( B \) for all \( j \in I \). By assumption \( A \) is a fuzzy precompact in \( B \), then there exists \( j_1, j_2, \ldots, j_n \in I \) such that \( A \subseteq \bigcup_{i=1}^{n} (S_{j_i} \wedge B) \leq \bigcup_{i=1}^{n} (S_{j_i}) \). Hence, \( A \) is a fuzzy compact in \( X \).
3.10. Proposition
Let \( (X, T) \) be a fuzzy topological space. If \( B \) is a fuzzy pre open set in \( X \) and \( A \leq B \), then \( A \) is a fuzzy compact in \( X \) if and only if \( A \) is a fuzzy compact in \( B \).

Proof: By (3.8. Proposition), (3.9. Proposition) and (3.7. Corollary).

3.11. Proposition
Let \( (X, T) \) be a fuzzy topological space. If \( B \) is a fuzzy set in \( X \) and \( A \leq B \), then \( A \) is a fuzzy compact in \( X \) if \( A \) is a fuzzy compact in \( B \).

Proof: Suppose that \( S = \{S_j: j \in J\} \) is a fuzzy open cover of \( A \) in \( X \). Since \( A \leq B \) and \( A \leq S_j \), then \( \mathcal{A} = \{S_j \wedge B: j \in J\} \) is a fuzzy cover of \( A \). But, \( S_j \) is a fuzzy open in \( X \) for all \( j \in J \), then by (2.30. Definition) \( S_j \wedge B \) is a fuzzy open in \( B \) for all \( j \in J \); by assumption \( A \) is a fuzzy compact in \( B \), so there exists \( j_1, j_2, \ldots, j_n \in J \) such that \( A \leq \bigvee_{i=1}^{n} (S_{j_i} \wedge B) \leq \bigvee_{i=1}^{n} (S_{j_i}) \). Hence, \( A \) is a fuzzy compact in \( X \).

3.12. Proposition
A fuzzy pre closed subset of a fuzzy pre compact space \( (X, T) \) is a fuzzy pre compact.

Proof: Suppose that \( G \) is a fuzzy pre closed subset of a fuzzy pre compact space \( X \) and \( \{A_j: j \in J\} \) is a fuzzy open cover of \( G \) in \( X \), which implies that \( G \leq \bigvee_{j \in J} A_j \). Thus, \( G \) has a fuzzy pre open cover \( \{A_j: j \in J\} \). Since \( G^c \) is pre open, then the family \( \{A_j: j \in J\} \) is a fuzzy pre open cover of \( X \), which is a fuzzy pre compact space. Thus there exists \( j_1, j_2, \ldots, j_n \) such that \( \bigvee_{i=1}^{n} A_{j_i} \) is finite subcover of \( X \) and \( G \leq \bigvee_{i=1}^{n} A_{j_i} \). Since \( \{A_{j_1}, A_{j_2}, \ldots, A_{j_n}, G^c\} \) is finite subcover of \( X \) and \( G \leq \bigvee_{i=1}^{n} A_{j_i} \), but \( G \neq G^c \), therefor \( G \leq \bigvee_{i=1}^{n} A_{j_i} \). Hence, \( G \) is a fuzzy pre compact.

3.13. Corollary
A fuzzy closed subset of a fuzzy pre compact space \( (X, T) \) is fuzzy pre compact.

Proof: It is clear.

3.14. Corollary
A fuzzy closed subset of a fuzzy pre compact space \( (X, T) \) is fuzzy compact.

Proof: It is clear.

3.15. Theorem
Every fuzzy pre compact subset of a fuzzy pre Hausdroff topological space is fuzzy pre closed.

Proof: Suppose that \( x_{\alpha} \in pcl(A) \), then by (2.41. Theorem) there exists a fuzzy net \( x_{\alpha}^{n} \) such that \( x_{\alpha}^{n} \rightarrow x_{\alpha} \). Since \( A \) is fuzzy pre compact and \( X \) is fuzzy pre Hausdroff space, then by (3.4. Corollary) and (2.43. Proposition), we have \( x_{\alpha} \in A \) which implies that \( pcl(A) \leq A \). Hence \( A \) is fuzzy pre closed set.

3.16. Theorem
In any fuzzy space, the intersection of a fuzzy pre compact set with a fuzzy pre closed set is fuzzy precompact.

Proof: Suppose that \( A, B \) are two fuzzy sets such that \( A \) is a fuzzy precompact and \( B \) is a fuzzy pre closed. We must prove that \( A \wedge B \) is a fuzzy precompact. Let \( x_{\alpha}^{n} \) is fuzzy net in \( A \), since \( A \) is fuzzy pre compact, then by (3.4. Corollary), \( x_{\alpha}^{n} \xrightarrow{p} x_{\alpha} \) for some \( x_{\alpha} \in FP(X) \) and by (2.41. Proposition), \( x_{\alpha} \in pcl(A) \). Since \( B \) is fuzzy pre closed, then \( x_{\alpha} \in B \). Hence \( x_{\alpha} \in A \wedge B \). Thus \( A \wedge B \) is fuzzy precompact.

3.17. Definition
In a fts \( X \), a fuzzy set \( G \) is said to be pre compactly fuzzy pre closed if \( G \wedge K \) is fuzzy precompact, for every fuzzy pre compact set \( K \) in \( X \).
3.18. Proposition
Every fuzzy pre closed subset of a fuzzy topological space $X$ is precompactly fuzzy pre closed.

Proof: Suppose that $G$ is a fuzzy pre closed subset of a fuzzy space $X$ and let $K$ be a fuzzy precompact set. Then by (3.16.Theorem), $G \cap K$ is a fuzzy precompact. Thus $G$ is a precompactly fuzzy pre closed set.

3.19. Theorem
In a fuzzy pre Hausdorff space $X$, a fuzzy set $G$ is precompactly fuzzy pre closed if and only if $G$ is fuzzy pre closed.

Proof: $\Rightarrow$ Suppose that $G$ is a precompactly fuzzy pre closed and $x_{\alpha} \in pcl(G)$. Then, by (2.41.Proposition), there is a fuzzy net $x_{\alpha}^{n}$ in $G$, such that $x_{\alpha}^{n} \xrightarrow{p} x_{\alpha}$, then by (3.4.Corollary), $B = \{x_{\alpha}^{n}, x_{\alpha}\}$ is a fuzzy precompact set. But $G$ is precompactly fuzzy pre closed, then $G \cap B$ is a fuzzy precompact set, also $X$ is a fuzzy pre Hausdroff space by assumption, then by (3.16.Theorem), $G \cap B$ is fuzzy pre closed. Since $x_{\alpha}^{n} \xrightarrow{p} x_{\alpha}$ and $x_{\alpha}^{n} \in G \cap B$, then by (2.41.Theorem) $x_{\alpha} \in G \cap B$, so $x_{\alpha} \in G$. Therefore, $pcl(G) \subseteq G$. Hence $G$ is a fuzzy pre closed set.

$\Leftarrow$ By (3.18. Proposition).

3.20. Theorem
A fuzzy pre regular space $X$ is a fuzzy precompact if and only if there exist a fuzzy dense $D$ of $X$ such that any fuzzy filterbase in $D$ have a fuzzy pre cluster point in $X$, with int$(D) \neq 0$.

Proof: $\Rightarrow$ By (3.2.Theorem).

$\Leftarrow$ we prove if there exist a fuzzy dense $D$ in $X$ such that any fuzzy filterbase in $D$ have a fuzzy pre cluster point in $X$, then $X$ is a fuzzy precompact. Let $D$ be a fuzzy dense set and $X$ is not fuzzy precompact, then there exist a cover $\{U_{j}: j \in J\}$ of fuzzy pre open set in $X$ with no finite fuzzy subcover.

Since $X$ is a fuzzy pre regular, then there exists fuzzy pre open cover $\{V_{i}: i \in I\}$ of $X$ such that for each $j$ there exist $i$ such that $pcl(V_{i}) \subseteq U_{j}$. By (2.39.Theorem) $X = Pcl(D)$. Now, $\{V_{i}: i \in I\}$ is a fuzzy pre open cover of $pcl(D)$ with no finite subcover. Therefore, the collection $B = \{D \cap (1 - V_{i_{k}}), k = 1, 2, \ldots, n\}$ is a fuzzy filterbase in $D$. But, $B$ has a fuzzy pre cluster point $x_{\alpha}$. Then $x_{\alpha} \in pcl(D)$ implies $x_{\alpha} \in V_{i}$ for some $i$ and so $V_{i}$ is a fuzzy pre open set containing $x_{\alpha}$. Then $(D \cap (1 - V_{i})) \cap V_{i} = 0$ contradicts the fact that $x_{\alpha}$ is a fuzzy pre cluster point of $B$. Hence $pcl(D) = X$ is a fuzzy precompact.

3.21. Corollary
A fuzzy pre regular space $X$ is a fuzzy compact if and only if there exist a fuzzy dense $D$ of $X$ such that any fuzzy filterbase in $D$ have a fuzzy pre cluster point in $X$, with int$(D) \neq 0$.

Proof: $\Rightarrow$ By (2.45. Theorem) and (2.39. Theorem).

$\Leftarrow$ By (3.20.Theorem) and (2.7. Corollary).

References
[1] A. A. Nouh, " On convergence theory in fuzzy topological spaces and its applications ", J. Dml. Cz. Math., 55(130)(2005), 295-316.
[2] Anjana Bhattacharyya, $p^{*}$-Closure Operator and $p^{*}$-Regularity in Fuzzy Setting, Mathematica Moravica.Vol. 19-1 (2015), 131–139.
[3] B. Sikin, " On fuzzy FC-compactness ", Korean. Math. Soc, 13(1)(1998), 137-150.
[4] C. L. Chang, " Fuzzy topological spaces ", J. Math. Anal. Appl, 24(1968), 182-190.
[5] G. Thangaraj and E. Poongothai, On Fuzzy Preσ-Baire Spaces, Research India Publications. ISSN 0973-533X Volume 11, Number 1 (2016), pp. 39-49.
[6] Jin Han Park and B.H. Park, Fuzzy preirresolute mappings, Pusan-Kyongnam Math. J. 10 (1995) 303-312.
[7] Jin Han Park and H.Y. Ha, Fuzzy weakly preirresolute and fuzzy strongly preirresolute mappings, J. Fuzzy Math. 4 (1996) 131-140.
[8] Kareem, H. Reyadh, N "Fuzzy Compact and Coercive Mappings ", Journal of Karbala University, 10(3), 2012, 111-121.
[9] L. A. Zadeh, ” fuzzy sets “, Information and control, 8(1965), 338-353.
[10] M. H. Rashid and D. M. Ali, " Separation axioms in maxed fuzzy topological spaces " Bangladesh, J. Acad. Scie, 32(2)(2008), 211-220.
[11] Ming, P. P. and Ming, L. Y., “Fuzzy Topology I. Neighborhood Structure of a Fuzzy point and Moor-smith Convergence”, J. Math. Anal. Appl., Vol. 76, PP. 571-599, 1980.
[12] Rubasri.M and Palanisamy.M,"On fuzzy pre-α-open sets and fuzzy contrapre-α- continuous functions in fuzzy topological space", IJARIIE-ISSN (O)-2395-4396,vol-3 Issue-4, PP.1601-1608, 2017.
[13] Shahna A. S. Bin (1991), "On fuzzy strongly semi-continuity and fuzzy pre continuity", Fuzzy sets and Systems, vol. 44, 303-308.
[14] S. M. AL-Khafaji, " On fuzzy topological vector spaces ", M. Sc., Thesis, Qadisiyah University, (2010 ).
[15] S. P. Sinha and S. Malakar, On s-closed fuzzy topological spaces, J. Fuzzy Math. 2(1) (1994), 95–103.
[16] Wong, C. K., “Fuzzy Points and Local Properties of Fuzzy Topology”, J. Math. Anal. Appl., Vol. 46, PP. 316-328, 1974.
[17] X. Tang, "Spatial object modeling in fuzzy topological space", PH. D. dissertation, University of Twente, The Netherlands, (2004).