Phantom Thermodynamics

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This paper deals with the thermodynamic properties of a phantom field in a flat Friedmann-Robertson-Walker universe. General expressions for the temperature and entropy of a general dark-energy field with equation of state \( p = \omega \rho \) are derived from which we have deduced that, whereas the temperature of a cosmic phantom fluid \( (\omega < -1) \) is definite negative, its entropy is always positive. We interpret that result in terms of the intrinsic quantum nature of the phantom field and apply it to (i) attain a consistent explanation for some recent results concerning the evolution of black holes which, induced by accreting phantom energy, gradually lose their mass to finally vanish exactly at the big rip, and (ii) introduce the concept of cosmological information and its relation with life and the anthropic principle. Some quantum statistical-thermodynamic properties of the quantum phantom field are also considered that include a generalized Wien law and the prediction of some novel phenomena such as the stimulated absorption of phantom energy and the anti-laser effect.

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I. INTRODUCTION

Phantom energy remains for the moment a theoretical possibility [1] with potential application to describe super-accelerated evolution in both the primordial and late universe. But it is not just that. Concerning late evolution of the universe, the possibility that the equation of state governing current cosmic evolution would correspond to phantom energy is still not at all excluded [2]. Actually, if the accelerating expansion of the universe turned out to be not due to the existence of a positive cosmological constant, then presently supplied cosmic data appear to favor phantom energy over quintessence models for positive internal energy fields [3]. From a theoretical point of view, the study of phantom energy is by itself a very interesting subject. It actually bears some resemblance with the study of black holes first studied by Babichev, Dokuchaev and Eroshenko [8], interpreting it in terms of a new thermodynamic description of a phantom fluid which is developed and includes a negative temperature and a positive entropy. A
discussion of the properties and meaning of the cosmological information induced by the presence of dominating phantom energy in the universe, as well as the implications that it may have on the emergence of life and the anthropic principle, is included in Sec. III. By interpreting the phantom field as a radiation field, we consider some of the quantum statistical-thermodynamic properties of the resulting description, including a generalized Wien spectrum (Sec. IV) and a derivation of the generalized Planck law based on introducing novel radiative processes (Sec. V). We finally conclude and add some more comments in Sec. VI.

II. PHANTOM-ENERGY THERMODYNAMICS AND BLACK HOLES

Phantom energy has been criticized by several authors [10-12]. The main difficulty stemming from such criticisms concerns the issue of phantom stability and this could still be circumvented if an axion model is considered for the phantom field [7]. Indeed, the large amount of papers on the phantom subject that have appeared [4-6] before and after references [10-12] reflects the fact that most of the criticisms can actually be regarded as manifesting weird phantom properties that could nevertheless be accommodated into the current and future evolution of the universe without contradicting observations.

A. Accretion of dark energy onto black holes

Among such properties, an intriguing recent result is the discovery by Babichev, Dokuchaev and Eroshenko [8] that all black holes in a universe filled with a fluid violating the dominant energy condition (i.e. a phantom-energy fluid inducing a big rip singularity in the future) will steadily lose all of their mass to fully disappear, all at once, at the big rip, no matter their initial mass or the moment at which they were formed. In fact, by extending the classical theory of Michel [13], these authors found that, as a result of dark energy accretion, the mass $M$ of a black hole in a universe filled with a general quintessence scalar field describable by means of a fluid with equation of state $p = \omega \rho$, varies at a rate given by [8]

$$\dot{M} = 4\pi AM^2 (\rho + p),$$

(2.1)

where $A$ is a dimensionless positive constant and $\dot{} = d/dt$. Using then the constant equation of state $p = \omega \rho$ for the fluid described by the quintessence field, Eq. (2.1) can finally be re-written as

$$\dot{M} = 4\pi AM^2 (1 + \omega) \rho.$$

(2.2)

Thus, since in all conceivable situations $\rho > 0$ should be satisfied, accretion of dark energy onto a black hole leads the mass of this black hole to increase if $\omega > -1$ and, as a consequence from the negative value of the internal energy of the phantom field, to a mass loss in the case that $\omega < -1$. The stability of the Schwarzschild-de Sitter universe [14] to the dark-energy accretion process is ensured by the fact that a positive cosmological constant corresponds to $\omega = -1$ for which the rate $\dot{M} = 0$. Of course, in this case there will still be a continuous loss of black hole mass due to Hawking radiation, a process which is not included in the present formalism.

For the flat geometry which our universe appears to satisfy, it has been obtained [15] that the most general expression of the scale factor of a universe filled with such a general quintessence field is given by

$$a(t) = \left( a_0^{3(1+\omega)/2} + \frac{3}{2} (1 + \omega) t \right)^{2/[3(1+\omega)]},$$

(2.3)

in which $a(0) = a_0$ is taken as the initial value of the scale factor at the onset of the accelerating regime. This solution satisfies the Friedmann equation for flat geometry and the conservation law for cosmic energy. If we, moreover, re-interpret the constant $a_0$ as the value of the scale factor at the onset of the radiation domination era for $\omega = 1/3$, then re-scaling time $t$ so that $t \to \bar{t} = a_0^2 + 2t$, we in fact obtain from Eq. (2.3) the expression of the scale factor for a decelerating radiation-dominated universe, i.e. $a(\bar{t}) = \bar{t}^{1/2}$. Eq. (2.3) furthermore tells us that for the interval $-1/3 > \omega > -1$ the scale factor steadily increases in an accelerating fashion, with $t$ tending to infinity as $t \to \infty$. From the very onset of the phantom energy regime $\omega < -1$ [3] assumed to occur at $a = a_0$, the scale factor (2.3) predicts a super-accelerated expansion along that regime which finally reaches the so-called big rip curvature singularity as $t$ approaches the finite time value

$$t = t_\ast = \frac{2}{3(|\omega| - 1)a_0^{3(|\omega|-1)/2}}.$$

(2.4)
After the big rip, as \( t > t_\star \), the universe would start a period of continuous contraction where its size tended to vanish as \( t \to \infty \). The super-accelerated expanding phase and the contracting phase would however be disconnected to each other because the big rip is a true curvature singularity.

If, as it is assumed throughout this paper, we set \( \omega \) constant, the integration of the cosmic conservation law for energy, \( \dot{\rho} + 3\rho(1 + \omega)\dot{a}/a = 0 \) [16], in this case leads to an expression for the dark energy density

\[
\rho = \rho_0 a^{-3(1+\omega)},
\]

with \( \rho_0 \) a constant. Thus, the energy density consistently becomes constant for the cosmological constant case \( \omega = -1 \), it steadily decreases with the scale factor for \( \omega > -1 \), and rather surprisingly increases with \( a \) for \( \omega < -1 \). Once the scale factor \( a(t) \) and the energy density \( \rho(t) \) have been obtained, the field theory associated with the dark energy fluid through the definitions of the pressure and energy density, \( \rho = \dot{\phi}^2/2 + V(\phi) \), \( p = \omega \rho = \dot{\phi}^2/2 - V(\phi) \), can be solved (i.e. expressions for \( \phi(t) \) and \( V(\phi) \) can be obtained). We in this way get

\[
\phi(t) = \phi_0 + \frac{2}{3} \sqrt{\frac{\rho_0}{1+\omega}} \ln \left( a_0^{3(1+\omega)/2} + \frac{3}{2} (1+\omega) t \right)
\]

\[
V(\phi) = \frac{1}{2} (1 - \omega) \rho_0 e^{-3\sqrt{\frac{\rho_0}{1+\omega}}(\phi - \phi_0)}.
\]

The phantom regime is derived when we let \( \phi \to i\Phi \) and \( \omega < -1 \) [7]. It can be seen that both \( \Phi \) and \( V(\Phi) \) diverge at the big rip.

Inserting now Eqs. (2.3) and (2.5) into Eq. (2.2) and integrating we finally obtain the black hole mass time-evolution equation [17]

\[
M(t) = \frac{M_\star}{1 - \left(\frac{1+\omega}{1+3+\omega}\right)^{3/2} t^{3+\omega}}
\]

where \( M_\star \) is the initial mass of the black hole, \( \tilde{t} = a_0^{3(1+\omega)/2} \) and \( \tilde{M}_\star = (4\pi A\rho_0)^{-1} \). Eq. (2.6) actually shows the same predictions as those which were first deduced in Ref. [1]. That is: (i) for \( \omega > -1 \), \( M \) monotonically increases with time \( t \), tending to a maximum value

\[
M_{\text{max}} = \frac{M_\star}{1 - \frac{3M_\star t}{3M_\star t}}
\]

as \( t \to \infty \). This result comes about because quintessence fields with \( \omega > -1 \) have positive energy which, when accreted onto the black hole, makes the positive mass of this black hole to increase. (ii) If \( \omega = -1 \), the black hole mass remains constant, i.e. the black hole does not accrete any energy from vacuum simply because there then is anything like a quintessence vacuum field, but a cosmological constant instead. (iii) Finally, and more importantly, such as it was pointed out by Babichev, Dokuchaev and Eroshenko [8], if \( \omega < -1 \), then \( M \) steadily decreases with time \( t \) and tends to vanish as \( t \) approaches the big rip singularity at time \( t = t_\star \). In order for accounting for such a highly unconventional behaviour one must resort to the feature that the internal phantom energy is negative definite and therefore, when it is accreted onto black holes, it subtracts rather than adds on their total energy. Moreover, near the big rip singularity, we have that the mass of any black hole tends to be

\[
M \to \frac{\tilde{M}_\star \tilde{t} (\omega - 1) t}{|\omega| - 1},
\]

which remarkably does not depend on the initial mass of the black hole. According to Babichev, Dokuchaev and Eroshenko [8], that result means that all black holes in a universe filled with phantom energy will tend to be equal as the big rip is approached, and that phantom-energy accretion prevails over Hawking radiation, at least if quantum-gravity effects are not taken into account. In their original derivation, Babichev et al. [8] obtained exactly the same conclusions by using a simpler expression for the scale factor. One could still wonder, what happens with black holes after the big rip. According to Eq. (2.6), just after the big rip the mass of the black holes would start increasing and tend to the finite maximum value given by Eq. (2.7) as \( t \to \infty \), such as it happens in the case of dark energy with \( \omega > -1 \). The memory of the initial mass would thus be recovered, even though the regions before and after the big rip are mutually disconnected.
Further generalization of these results can be achieved by considering the case where a positive cosmological constant \( \Lambda = 3\lambda \) is added to the dark energy fluid. This situation would describe a Schwarzschild-de Sitter spacetime embedded in dark energy. If we set a vanishing initial time, \( t_0 = 0 \), the scale factor \( a(t) \) then reads [15]

\[
a(t) = \left( \frac{2\pi G}{3\lambda} \right)^{1/[3(1+\omega)]} \left( e^{3(1+\omega)\sqrt{\Lambda} t/2} - C e^{-3(1+\omega)\sqrt{\Lambda} t/2} \right)^{2/[3(1+\omega)]},
\]

where

\[
C = \frac{\sqrt{\lambda + 8\pi G a_0^{-3(1+\omega)/3} - \sqrt{\lambda}}}{\sqrt{\lambda + 8\pi G a_0^{-3(1+\omega)/3} + \sqrt{\lambda}}},
\]

with \( a_0 \) the assumed initial value of the scale factor at the onset of the phantom energy dominance. Since we always have \( 0 < C < 1 \), a big rip singularity for the phantom regime where \( \omega < -1 \) is also predicted in the presence of a cosmological constant. Actually, that singularity takes in this case place at a finite time given by

\[
t = t_* = -\frac{\ln C}{3|\omega|^{-1}\sqrt{\lambda}},
\]

which becomes shorter as the value of the cosmological constant is made larger. The field theory can also be solved in this case. Following the same procedure as for the scale factor (2.3) we now attain

\[
\phi(t) = \phi_0 + \sqrt{\frac{\rho_0}{6\pi G(1+\omega)\ln\left(\sqrt{C} - e^{3(1+\omega)\sqrt{\lambda} t/2}\right)}} \ln\left(\frac{\sqrt{C} - e^{3(1+\omega)\sqrt{\lambda} t/2}}{\sqrt{C} + e^{3(1+\omega)\sqrt{\lambda} t/2}}\right),
\]

\[
V(\phi) = \frac{1}{2}(1-\omega)\rho_0 \sqrt{\frac{3\lambda}{32\pi G C}} \sinh^2\left(\sqrt{\frac{6\pi G(1+\omega)}{\rho_0}}(\phi - \phi_0)\right).
\]

Also in this case the phantom regime can be obtained by introducing the conditions \( \phi \rightarrow i\Phi \) and \( \omega < -1 \). Once again both the resulting field and its potential tend to blow up as the big rip is approached.

Using solution (2.9) we can now obtain from Eqs. (2.2) and (2.5) that in the presence of a cosmological constant the accretion of dark energy onto a black hole makes its initial mass \( M_i \) to vary according to the law

\[
M(t) = \frac{M_i}{1 - \frac{M_i}{M_0} \left( \frac{e^{3(1+\omega)\sqrt{\lambda} t/2} - 1}{e^{3(1+\omega)\sqrt{\lambda} t/2} + 1} \right)},
\]

where now \( M_0 = (1 - C)/(2C\rho_0) \). Clearly, again \( M \) remains all the time equal to \( M_i \) if only a cosmological constant \( \omega = -1 \) is present. For \( \omega > -1 \), \( M \) again monotonically increases with time \( t \) towards a maximum \( M_{\text{max}} = M_i/(1 - M_i\sqrt{\lambda}/M_0) \), which also occurs at \( t = \infty \). For \( \omega < -1 \), Eq. (2.12) can be cast in the form

\[
M(t) = \frac{M_i}{1 + \frac{M_i}{M_0} \left( \frac{e^{3(1+\omega)\sqrt{\lambda} t/2} - 1}{e^{3(1+\omega)\sqrt{\lambda} t/2} + 1} \right)}.
\]

In this case \( M \) once again decreases with time \( t \), tending to vanish on the neighborhood of \( t = t_* \). Thus, all the qualitative behaviours obtained from solution (2.3) are matched when one uses solution (2.9). The effect induced by the presence of an extra cosmological constant is to make \( M \) evolve exponentially. Actually, it can be shown that in the phantom regime corresponding to all existing dark-energy models, the above result is always obtained, provided that regime will show a big rip singularity in the finite future. In fact, tachyon like models admit [18] a scale factor solution as given by Eq. (2.9), and therefore we obtain again for these models result (2.13). Since generalized Chaplygin phantom models do not show any big rip singularities [19], we are finally left only with k-essence models with non-canonical kinetic energy [20] for which there will be a big rip future singularity in the phantom region and this can be defined by [21]

\[
p + \rho = -3(1-\mu)H^2/\mu < 0,
\]
with \( \mu \) a constant satisfying \( 0 < \mu < 1 \), and \( H = \dot{a}/a \) the Hubble parameter defined from the scale factor

\[
a \propto (t - t_b)^{-2\mu/[3(1-\mu)]},
\]

where \( t_b \) denotes the arbitrary time at which big rip takes place. Inserting the latter two expressions into Eq. (2.1) we finally obtain after integration

\[
M = \frac{M_i}{1 + \frac{t M_i}{t_b M_i (t_b - t)}}
\]

in which now \( \dot{M}_0 = 3(1 - \mu)/(16\pi A \mu) \). Note that Eq. (2.16) is formally the same as the expression derived in Ref. [8] for a differently defined \( M_0 \), so confirming and generalizing to all phantom models characterized by the existence of a sudden big rip singularity in the finite future the result pointed out in that reference. We finally solve the field theory for this case. Once we have Wick rotated the field \( \phi \), we get

\[
\Phi = \Phi_0 - \sqrt{\frac{4\mu}{3(1-\mu)}} \ln(t - t_b)
\]

\[
V(\Phi) \propto \frac{\rho_0 (1+\mu)}{\mu} e^{-\sqrt{\frac{3(1-\mu)}{\rho}} (\Phi - \Phi_0)}.
\]

We note that in this case, whereas the phantom field diverges at the big rip, the phantom field potential tends to vanish at that singularity.

B. Thermodynamics of a dark-energy universe

The results on accretion of dark energy onto black holes first obtained by Babichev et al [8] and generalized above could however get into serious conflict with thermodynamics. First of all, one could always assume the current existence of primordial black holes which, at the onset of dark energy domination, could have reached a very small mass and hence a very large temperature due to a long process of Hawking thermal evaporation. This would, in principle, imply that Hawking radiation effects would by now prevail over the effects of the accretion of phantom fluid if this accretion was characterized by a lower positive temperature [22]. On the other hand, one should also expect that if the results derived in Subsec. II B hold, then the generalized entropy [23] defined for the whole universe, \( S = S_{\text{bh}} \) (where \( S \) and \( S_{\text{bh}} \propto M^2 \) are the entropy of the phantom fluid and the entropy of the black hole), should be expected to decrease with time, so violating the generalized second law [23]. In what follows we are going to show nevertheless that none of these two apparent difficulties actually matters because, contrary to a recent claim [22], the temperature of the phantom fluid is definite negative, and this will allow us to re-interpret results from a different, “quantum” standpoint.

Lima and Alcaniz have recently proposed a general thermodynamic theory for dark energy, according to which they argue [22] that, whereas the temperature for the phantom energy regime is always positive, its entropy is negative definite and that, therefore, a cosmic scenario in which the universe is filled with phantom energy should be ruled out. There exist however rather general thermodynamic arguments which are valid for all available cosmic phantom-energy models, which appear to prevent the holding of that conclusion. Thus, if the first law of thermodynamics is assumed to hold in every involved case, then it has been shown [24] that in a general Friedmann-Robertson-Walker flat universe filled with a dark energy that satisfies the equation of state \( p = \omega \rho \) (with \( \omega = \text{constant} \)), the entropy density \( s \) per comoving volume stays always constant [24], while the temperature of the universe turns out to be given by the expression

\[
T = (1 + \omega) \rho_0 a^{-\omega} / (s - s_0),
\]

with \( s_0 \) an integration constant. Thus, we can generally write for the temperature of a universe equipped with a dominating dark energy and equation of state \( p = \omega \rho \),

\[
T = \kappa (1 + \omega) a^{-3\omega},
\]

where \( \kappa \) is a positive constant. The interpretation of this result can obviously resort to the most commonly used concept of kinetic temperature which is a measure of the average translational kinetic energy of the system. In the case of a general dark-energy scalar field \( \phi \), the kinetic term \( \dot{\phi}^2 \) can be obtained from the definition of the field itself in terms of the pressure \( p \) and energy density \( \rho \), the equation of state \( p = \omega \rho \), and the expression of the energy density derived by integrating the conservation law for cosmic energy, i.e. Eq. (2.5). This gives \( \dot{\phi}^2 \propto (1 + \omega) a^{-3(1+\omega)} \). Now,
an estimate of the isotropic translational kinetic energy is \( a^3 \dot{\phi}^2 = K(1 + \omega) a^{-3\omega} \) (with \( K \) a numerical constant), which can in fact be made the same as Eq. (2.17).

Along the present paper we shall take the general expression (2.17) as the temperature that characterizes a universe dominated by dark energy. We next derive from Eq. (2.17) the thermodynamic functions of interest. Thus, Eqs. (2.5) and (2.17) will allow us to readily get the following generalized Stefan-Boltzmann law for a dark energy universe,

\[
\rho = \rho_0 \left( \frac{T}{\kappa (1 + \omega)} \right)^{1/\omega}.
\]  

(2.18)

We notice that, whereas for \( \omega = -1 \) \( \rho \) becomes a simple constant (which actually corresponds to a cosmological constant), for \( \omega = 1/3 \), Eq. (2.18) consistently reduces to the usual law for radiation, and for \( 0 > \omega > -1 \) the dark energy density decreases with the temperature. However, for the phantom regime where \( \omega < -1 \), in order to preserve \( \rho \) positive, we must necessarily take \( T < 0 \), which is a condition that really directly stems from Eq. (2.17) for \( \omega < -1 \).

It follows that \( \rho \) will always increase with \( |T| \) along the entire phantom regime. The energy of such a regime actually is bounded from above and allows therefore the occurrence of negative temperatures [25]. Since on the phantom regime \( \rho \) increases with the scale factor \( a(t) \) it also follows that \( |T| \) increases as the universe expands on that regime. These two characteristics are quite surprising actually and therefore they should be added to the increasing collection of phantom weird properties.

Finally, a general expression for the entropy of a dark energy universe can also be obtained by using the procedure of Ref. [22] which in the present case leads to

\[
S = C_0 \left( \frac{T}{1 + \omega} \right)^{1/\omega} V,
\]  

(2.19)

where \( C_0 \) is a positive constant and \( V \) is the volume of the considered portion within the dark energy fluid. Thus, contrary to the claim in Ref. [22], the entropy of a dark-energy universe is \textit{always} positive, even on the phantom regime. Actually, by inserting Eq. (2.17) into Eq. (2.19) one in fact attains that \( S \) becomes a constant when we take \( V \) to be the volume of the entire universe. For usual radiation corresponding to \( \omega = 1/3 \), entropy and temperature can be linked by means of the general relationship, entropy \( \propto \) energy/temperature. According to Eq. (2.19), however, that relationship should be generalized to read: entropy \( \propto (1 + \omega) \) energy/temperature, according to which entropy is positive along the entire interval of conceivable values of parameter \( \omega \).

Even though it is not very common in physics and could thereby be listed as just another more weird property of the phantom scenario, a negative temperature is not at all unphysical or meaningless [26]. Nuclear spin and other quantum Systems with negative temperatures have already been observed in the laboratory and interpreted theoretically. As referred to the case of phantom energy, the existence of a negative temperature would imply that the entropy of a phantom universe monotonically decreased if one would be able to supply energy to that universe. Hence, the onset at the coincidence time of the dominance of a phantom energy, \( \omega < -1 \), universe would imply the emergence of a necessarily "hotter" cosmic evolution regime in such a way that, if two copies of the current universe were taken, one with positive and other with negative temperature, and put them in thermal contact, then heat would always flow from the negative-temperature universe into the positive-energy universe. It could yet be argued that negative temperature is a quantum-statistical mechanics phenomenon and therefore cannot be invoked in the classical realm. However, the negative temperature for the universe given by Eq. (2.17) when \( \omega < -1 \) [16] can still be heuristically interpreted along a way analogous to how e.g. black hole or de Sitter temperature can be interpreted (and derived) without using any quantum-statistical mechanics arguments; that is by simply Wick rotating time, \( t \to i\tau \), so that the metric becomes positive definite, and checking that in the resulting Euclidean framework \( \tau \) is periodic with a period which precisely is the inverse of the Hawking temperature [27]. Thus, the Euclideanized black hole turns out to be somehow "quantized". Similarly in the present case, the phantom regime can be obtained by simply Wick rotating the classical real scalar field [28], \( \phi \to i\Phi \), which can be generally seen to be equivalent to rotating time so that \( t \to i\tau \), too (see Sec. III). It is in this sense that the phantom energy universe can be also regarded as a somehow "quantized" system and that the emergence of a negative temperature in the phantom regime can be interpreted in a consistent way.

Since a system with negative temperature is "hotter" than any other systems having positive temperature (i.e. energy will always flow from the former to the latter system), even if that positive temperature is \( +\infty \) [9,26], we see that the first of the two thermodynamical difficulties pointed out before becomes fully solved, no matter how high the black hole temperature could be. Concerning the second of the above difficulties, let us notice on the other hand that, if the universe is assumed to contain a black hole of mass \( M \), the entropy of the dark energy fluid will be smaller than that is given by (2.19) for \( V \propto a^3 \). In this case, the entropy of the universe can be written to be

\[
S = C_0 \kappa^{1/\omega} \left( 1 - \frac{V_{bh}}{V} \right),
\]  

(2.20)
where $V_{bh} \propto M^3$ is the volume occupied by the black hole and $V$ is the volume of the entire universe. Thus, for $\omega > -1$, $S$ decreases and, for $\omega < -1$, $S$ increases as dark energy is accreted onto the black hole. Moreover, after varying $S$ with respect to $V_{bh}$ and multiplying by $T$, one can obtain from Eq. (2.20)

$$\delta S \propto (1 + \omega) \frac{\delta E}{T},$$

in which $\delta E = \mp \rho \delta V_{bh}$ and the upper sign corresponds to $\omega > -1$ and the lower sign corresponds to $\omega < -1$. On the other hand, if on the black-hole spacetime we assume the time taken by a light signal to travel along the entire hole to be $t \propto M$, we can derive from Eq. (2.1)

$$\delta S_{bh} \propto (1 + \omega) \frac{\delta E}{T_{bh}},$$

where $S_{bh} \propto M^2$ and $T_{bh} \propto M^{-1}$ are the entropy and temperature of the black hole.

Let us first analyze the total balance of entropy in the case of dark energy with $\omega > -1$. Thus, from Eqs. (2.21) and (2.22) we see that the accretion of a given amount of dark energy onto the black hole leads to an increase of dark energy entropy which will exceed the corresponding decrease of black hole entropy, so preserving the second law, provided that $T > T_{bh}$, i.e. if $8\pi GM_{bh}T \geq 1$, a condition which appears to be of general applicability along the latest accelerating evolution of the universe. It would be in this case expected however that the competition between dark energy accretion and black hole thermal emission, both characterized by positive temperatures, will be governed by usual thermodynamic laws. Thus, if initially $8\pi GM_{i}M_{i} < 1$ (with $T_{initia} = \kappa(1 + \omega)a^{3\omega}$), then thermal emission will prevail over dark energy accretion and the black hole mass decreased initially. That situation could nevertheless be maintained only for a while because, during accelerating expansion, temperature (2.17) will increase at a much greater rate than black hole temperature did due to thermal evaporation. Thus, after a given time, both processes would first tend to balance one another, to inexorably allowing then for dominance of the dark energy accretion process, leading finally to a maximum black hole mass given by Eq. (2.7). If initially we already had $8\pi GM_{i}M_{i} > 1$, then dark energy accretion prevailed over thermal radiation along the entire evolution, so that the mass of the black hole steadily increased up to $M_{max}$.

As to the case of phantom energy for $\omega < -1$, we ought to recall that according to the Carnot equality the coupling between a negative-temperature system and a positive-temperature system leads to a Carnot engine of greater than 100 percent efficiency [29]. Thus, in the present case

$$F = 1 + \frac{T_{bh}}{|T|} > 1,$$

so that the generalized entropy $S + S_{bh}$ ought to inexorably decrease along phantom energy accretion, so violating the second law. In fact, Eqs. (2.21) and (2.22) tell us that in the phantom case the entropy $S$ increases by an amount which is smaller than the decrease of $S_{bh}$ provided the reasonable condition $|T| > T_{bh}$ is again applied. This violation of the second law was to be expected because, as it was mentioned earlier, negative temperatures are compatible with observationally checked “quantum” Carnot equalities violating the second law [29]. In any event, the implication that negative temperatures lead through the quantum Carnot engine to violation of the second law can be argued under several presumptions. Although discussing these arguments is clearly outside the scope of the present paper, we want to leave open the possibility that they might preserve the second law even in the case being considered.

One very remarkable property of phantom energy is worth mentioning here. Even though most of its properties might be weird, phantom energy seems to be able to elegantly solve one of the most conspicuous and debated paradoxes of all the physics. In fact, if all existing black holes will simultaneously disappear at the big rip leaving no Hawking radiation, then the information initially lost during formation of the black holes should somehow be recovered. Actually, the so-called quantum coherence loss paradox, long championed by Hawking [30] and according to which an initial pure state is transformed into a final mixed state, during the whole process of black hole formation and subsequent complete evaporation, is here naturally solved in at least the phantom-energy regime in the sense that neither Hawking radiation nor black holes are left in the final singular state where only the big rip singularity plus a naked black hole singularity could take place. Thus, even in the classical case, the information paradox appears to be solved.

C. Dark energy as a radiation field

At first sight, one could be tempted to dismiss Eq. (2.17) because a spacetime dominated by a cosmological constant has a nonzero temperature of the order of the Hubble constant $H = \dot{a}/a$ while, according to Eq. (2.17) the
temperature of a fluid with \( \omega = -1 \) vanishes. If by instance we take for the scale factor the expression (2.3), then as one approaches the value \( \omega = -1 \) that expression can be reduced to \( a \simeq a_0 \exp \left( \frac{a_0^{-3(1+\omega)/2}t}{t} \right) \), and hence it becomes \( a \simeq a_0 \exp(t) \) at the special case where \( \omega = -1 \). In such a case we have then \( H = 1 \), while for \( \omega \neq -1 \) the Hubble parameter is time-dependent and given by the expression \( H \equiv H(t) = a^{-3(1+\omega)/2} \). Thus, whereas in the first case we have a cosmological horizon of radius \( H^{-1} = 1 \) which is characterized by a surface gravity \( K_c = 1 \), no constant horizon can be defined for any \( \omega \neq -1 \) in which cases every point is subject nevertheless to an isotropic acceleration which can be given by \( q \propto -Ha^3/G \propto -(1 + \omega)a^{-3\omega} \). Each of these two cases has therefore a distinct expression for temperature: if \( \omega = -1 \) the surface gravity implies a temperature \( T_{\text{dS}} \propto H = K_c = 1 \) [27]; if however \( \omega \neq -1 \) then every accelerating point will be bathed by a thermal radiation at the quantum Unruh temperature [31] \( T_{\text{dS}} \propto -1 \propto q \), which is precisely the expression given by Eq. (2.17). It follows that, even though the Gibbons-Hawking temperature for event horizons and the Unruh temperature for accelerating systems are conceptually equivalent as the surface gravity is nothing but a measure of how hard you have to accelerate to stay a given short distance away from the horizon, de Sitter space must be endowed with a temperature \( T_{\text{dS}} \propto H \) but not with temperature \( T \propto -Ha^3 \), and those spacetimes with \( \omega \neq -1 \) have in turn a temperature \(-Ha^3 \) but not temperature \( H \). Thus, Eqs. (2.17)-(2.19) are valid only for \( \omega \neq -1 \). The above discussion makes it clear that these equations are not mere definitions devoid of physical significance, but they all possess a precise physical meaning.

We can finally easily convince ourselves of the relevance of temperature (2.17) if we take for the parameter of the equation of state the value \( \omega = 1/3 \) which obviously corresponds to usual radiation. In fact, if \( \omega = 1/3 \) Eqs. (2.17)-(2.19) reduce to \( T \propto a^{-1}, \rho \propto T^4 \) and \( S \propto T^3 \), which are just the values of the temperature, energy density and entropy for the Friedmann-Robertson-Walker universe dominated by usual radiation. Therefore, the equation of state \( \omega = p/\rho \), together with the conservation law for cosmic energy, suffice by themselves to determine the thermodynamic properties of a general relativistic fluid which is assumed to dominate in a Friedmann-Robertson-Walker universe, much as e.g. the gravitational characteristics of black holes or de Sitter space are now known to determine well-defined and precise laws of thermodynamics, provided quantum theory is taken into account.

As represented by a stuff governed by the conservation law for cosmic energy and satisfying a perfect fluid equation of state the way we have considered so far, dark energy can be looked at as being a generalized radiation field whose characteristics are fixed by the value of parameter \( \omega \). Thus, if \( \omega > -1 \) dark energy would account for a quintessence radiation, if \( \omega < -1 \) it would describe a phantom radiation, and finally if \( \omega = 1/3 \) dark energy described a conventional radiation field for which we obtain from Eqs. (2.3) and (2.17) that \( a \propto t^{1/2} \) (with the time \( t \) re-defined so that \( t \rightarrow \tilde{t} = a_0^2 + 2t \) and \( T \propto a^{-1} \). In field theory in curved space usual radiation can be represented by means of a homogeneous massless, scalar field \( \Phi \) which is conformally coupled to gravity. The action integral of the system formed by gravity plus a scalar field \( \Phi \) can be written as

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{\tilde{g}} \left[ (1 - 8\pi G \xi \Phi^2) \right] \]
\[ + \frac{1}{2} \int d^4x \sqrt{\tilde{g}} \left[ (\nabla \Phi)^2 - 2V(\Phi) \right] - \frac{1}{8\pi G} \int d^3x \sqrt{h} \left( 1 - 8\pi G \xi \Phi^2 \right) \text{Tr} K, \]

where \( \xi \) accounts for the coupling between gravity and the scalar field, and all other symbols keep their conventional meaning. We shall use the metric

\[ ds^2 = \frac{2G}{3\pi} a(\eta)^2 \left( N^2 d\eta^2 + d\Omega_3^2 \right), \]

where \( \eta = \int dt/a \) is the conformal time, \( N \) is the lapse function and \( d\Omega_3^2 \) is the metric on the unit three-sphere. For conformal coupling \( \xi = 1/6 \) and flat geometry, the action integral (2.24) becomes

\[ S = -\frac{1}{2} \int d\eta N \left[ -\left( \frac{a'}{N} \right)^2 + \left( \frac{\chi'}{N} \right)^2 - \frac{8\pi G}{3} V(\Phi) a^4 \right] \]
\[ = -\frac{1}{2} \int d\eta N \left[ -\left( \frac{a'}{N} \right)^2 + \left( \frac{\chi'}{N} \right)^2 - \frac{8\pi G}{3} (1 - \omega) \rho_0 a^{1-3\omega} \right], \]

in which \( \chi = \sqrt{4\pi G/3a}\Phi \) is the conformal field, and we have used the definitions of the pressure and energy density \( p = \omega \rho = L = \Phi^2/2 - V(\Phi), \rho = \Phi^2/2 + V(\Phi) \) and Eq. (2.5). The equation of motion for \( \chi \) and the constraint \( (\delta/\delta N) \) are given by (in the gauge \( N = 1 \))

\[ \chi'' = 0 \]  

(2.27)
\[(a')^2 - (\chi')^2 - \frac{8\pi G(1 - \omega)\rho_0}{3} a^{1-3\omega} = 0, \quad (2.28)\]

with the equation of motion for \(a\) yielding also Eq. (2.28). From Eqs. (2.27) and (2.28) we obtain finally
\[(a')^2 - M^2 - \frac{8\pi G(1 - \omega)\rho_0}{3} a^{1-3\omega} = 0, \quad (2.29)\]

where \(M\) is an arbitrary integration constant. Now, if \(\omega = 1/3\), then \(a = (K_1 + K_2t)^{1/2}\), i.e. unless by a time re-scaling, essentially the same result as that we obtained from Eq. (2.3) for \(\omega = 1/3\) by assuming a minimal coupling \(\xi = 0\).

In terms of the conformal time \(\eta\) the scale factor (2.3) for \(\xi = 0\) is given by
\[a(\eta) = \left[\frac{3(1 + 3\omega)\eta}{2}\right]^{2/(1+3\omega)}, \quad (2.30)\]

which of course is not a solution of Eq. (2.29) for any value of \(\omega\) other than \(\omega = 1/3\). Thus, for \(\omega = -1\) we obtain from Eq. (2.29)
\[a(t) = \left(\frac{3M^2}{16\pi G\rho_0}\right)^{1/4} \sinh^{1/2} \left(\sqrt{\frac{64\pi G\rho_0}{3}} t\right). \quad (2.31)\]

Finally, at the onset of the accelerating regime \(\omega = -1/3\),
\[a(t) = \frac{9M^2}{32\pi G\rho_0} \left(\frac{32\pi G\rho_0 t^2}{9} - \frac{9M^2}{32\pi G\rho_0}\right)^{1/2}. \quad (2.32)\]

If we then assume that each particular value of \(\omega\) corresponds to a given radiation field, it follows that all of the resulting fields are associated with a minimal coupling between the scalar field and gravity, except when the state equation parameter takes on a value \(\omega = 1/3\), for which case the radiation field can be obtained by either conformally coupling or minimally coupling the field to gravity. It is in this sense that we shall consider in what follows that a universe dominated by dark energy is equivalent to a universe dominated by radiation.

### III. COSMOLOGICAL INFORMATION AND THE ANTHROPIC PRINCIPLE

The concepts of negative temperature and negative entropy are both meaningless within the realm of classical thermodynamics. The mathematical definition of entropy cannot in fact accommodate negative values both in statistical thermodynamics and in the Shannon theory of information, where the entropy (respectively denoted by \(S\) and \(H\)) is given by the conventional equivalent formulas
\[S = -k_B \log P, \quad H = -k \log_2 N, \quad (3.1)\]

with \(k_B\) the Boltzmann constant, \(k\) a given constant usually assumed to be unity, \(P\) the probability of any given microstate, and \(N\) the probability of the signal given the reference class of possible signals that could have been sent. The fact that \(P\) and \(N\) cannot exceed unity makes it impossible to have finite negative values for \(S\) and \(H\) in the classical theory. However, more recent developments have shown that quantum theory can allow the emergence of both negative temperatures and negative entropies.

Cerf and Adami studied [32] what happens to the Shannon theory when qubits of quantum information are accounted for. In doing that these authors extended classical probabilities to function describing also the quantum characteristics of the system, including both bits and qubits, within a consistent density-matrix description that otherwise parallels classical Shannon theory. A rather tantalizing implication from that generalization is that negative entropies crop up in the case of quantum systems having no classical analog. In particular, it was shown [33] that entropy becomes negative when two particles are quantum-mechanically entangled, even though that does not violate special relativity. On the other hand, the occurrence of negative absolute temperatures has already become a little bit more conventional [26]. These take also place in quantum systems or phenomena without classical analog, e.g. in quantum nuclear spin systems [34], and their existence has already been experimentally checked [34].
We shall now contend that at least in systems which admit a statistical thermodynamic description, negative temperature implies negative entropy, in the following sense. Given the specific properties of the "log" function (which actually relate characteristics of linear evolution with characteristics of nonlinear evolution), one can still introduce a tentative statistical definition of the entropy function which encompasses both positive and negative values of temperature and entropy simultaneously. Using the formula for Carnot efficiency
\[ E = \frac{T_2 - T_1}{T_2}, \] (3.2)
one can actually introduce a new entropy formula
\[ S' = k_B \log(E^{-1}) = k_B \log \left( \frac{T_2}{T_2 - T_1} \right). \] (3.3)
We notice that Eq. (3.3) connects the statistical content of entropy with its purely thermodynamic meaning, as it can now be checked by considering a small variation of \( T_1 \) which, if \( T_1 \gg T_2 \), by Eq. (3.3) induces a small decrease of entropy, such that \( \delta S = -k_B \delta T_1/T_1 \). In any event, if we assume that \( T_2 < 0 \), then the efficiency \( E > 1 \) and the entropy as defined by Eq. (3.3) becomes negative. It follows that a negative temperature would imply a negative entropy, except in the case that the system exchanges negative amounts of internal energy, for which case Eq. (3.3) no longer holds and must be replaced for
\[ S' = k_B \log(E) = -k_B \log \left( \frac{T_2}{T_2 - T_1} \right). \] (3.4)
Eq. (3.4) would in turn predict that a negative temperature \( T_2 < 0 \) must imply a positive entropy. That is precisely the case to which phantom energy belongs. Or more precisely, phantom energy must be regarded as a quantum entity with no classical analog, which is characterized by a negative temperature given by Eq. (2.17) and a positive entropy given by Eq. (2.19). If we would for a moment adhere to the alternate Alcaniz-Lima view [22], then the phantom energy would still be a quantum entity without classical analog, but characterized now by a positive temperature and a negative entropy. The difference with the interpretation provided in Ref. [22] is that in the present scenario having a phantom negative entropy is not necessarily physically meaningless due to the essential quantum nature of the phantom fluid.

We are going next to establish in exactly what sense the phantom fluid can be considered as a quantum entity. It is known [7] that in order to preserve weak energy condition, \( \rho > 0 \), the phantom scalar field should be Wick rotated (e.g., \( \phi \to i\Phi \)). As pointed out in Sec. II, we can readily check that such a rotation is equivalent to Wick rotating the time \( t \) itself (e.g., \( t \to -i\tau \)), while preserving the field unchanged. Thus, in the present general dark-energy scenario, the scalar field \( \phi \) is expressed in terms of time \( t \) as [7]
\[ \phi + \phi_0 = \frac{2}{3\sqrt{1 + \omega t_P}} \times \ln \left[ a_0^{3(1+\omega)/2} + \frac{3(1 + \omega)\sqrt{t_P t_0}}{2}(t - t_0) \right], \] (3.5)
where \( a_0, \phi_0 \) and \( \rho_0 \) are the initial values of the scale factor, the scalar field and the energy density, respectively, with \( t_P \) the Planck length. In the case that \( \omega < -1 \) it can be readily seen that Eq. (3.5) can be approximated to
\[ \phi + \phi_0 \propto i(t - t_0), \] (3.6)
along the entire cosmic evolution ending at the big rip. From Eq. (3.6) we can in fact deduce that Wick rotating \( \phi \) while keeping \( t \) unchanged is equivalent to Wick rotating time \( t \) while keeping \( \phi \) unchanged. Now, it is well known that a Euclideanized spacetime metric would describe a somehow quantized system [27]. It is in this sense that the cosmic phantom fluid is a quantized entity; that is, in a way which would parallel e.g. the procedure through which the quantum temperature and entropy of black holes can be derived by simply Wick rotating time in the maximally extended Kruskal metric [27].

Once we have made a preliminary discussion of some quantum properties of the cosmic phantom field, let us consider the simplest case in which the universe is initially filled only with phantom stuff. If we then add an elementary piece of ordinary matter to this observationally empty universe, according to our discussion before, the entropy would drop down, meaning that at least a bit (or qubit) of information necessarily about the added particle has been created and is available to potential observers (Note that essentially no direct information can be made available on the phantom fluid as this is not directly observable by definition). This is exactly the opposite to what would occur if such an
elementary piece of matter with positive energy is added to a universe filled with dark energy with $\omega > -1$, in which case the entropy of the universe increased. The creation of negative entropy in the case of a phantom fluid cannot be interpreted as being due to the creation of entangled correlations between the added particle and all "phantom quanta" as this implied knowledge about phantom being available to potential observers. Imagine now the onset of phantom energy domination in our real universe. The above reasoning would lead to the idea that a huge amount of bits and qubits of information about all observable pieces of matter encountered by the phantom field at the start of its domination would then be created and made available to potential observers. This is what can be dubbed as phantom-energy induced cosmological information. It actually calls for the presence of observers if one appeals to a general principle for natural economics. An important point should be mentioned at this point. It is that once phantom energy starts dominating in the universe, as its temperature is negative, and hence "hotter" than anything observable, it will gravitationally be ceaselessly accreted onto all kinds of observable matter steadily annihilating it, all the way until the big rip at which point all of the matter in the universe, including black holes, will completely disappear. Neither the above available cosmological information nor this process of all matter annihilation would nevertheless be present if instead of phantom energy the universe were filled with a dark energy vacuum component having $\omega > -1$.

Another standpoint from which one can look at the question of cosmic phantom energy is the very concept of life and its origin. In his rather controversial book "What is Life?", which collected a series of lectures delivered in Dublin, Schrödinger posed [36] a key conclusion that, a way or another, remains still inescapable today: what a living organism feeds upon is negative entropy, and this in its very statistical thermodynamic sense. That is currently interpreted by considering that living beings necessarily produce a positive supply of entropy to the universe during their vital activities which would compensate the needed stream of negative entropy upon themselves. However acceptable this may be for keeping the second law alive, it goes without solving the key question: for an essential element of life to make its first appearance in the universe, the site where it appeared ought to be prepared to provide the necessary and immediate supply of negative entropy for the incipient life element to continue, as in this case "egg would precede hen". Thus, since any living organisms are made up of positive energy, while original life elements could not be consolidated if the universe was then filled with any form of dark energy with $\omega \geq -1$, it can perfectly do so in a universe filled with phantom energy, as in this case the very act of the organism appearance does imply a supply of negative entropy. On the other hand, as discussed before, an universe filled with phantom energy is also prepared to be observed. It is in these senses that it could be thought that any form of anthropic principle [37] had to be formulated in terms of the emergence of an epoch when the universe started to be dominated by phantom energy.

IV. THE WIEN SPECTRUM

We shall apply in this and the next sections some aspects of the thermodynamic theory for $\gamma$-fluid developed by Lima and Maia [38] to the case of a general quintessential model of dark energy and, in particular, to the case of phantom energy. According to the discussion in Sec. II we can assume that the dark-energy field corresponds to a kind of radiation field with a Wien-type spectrum given by

$$\rho_T(\nu) = \alpha \nu^\beta \phi(\nu T^\lambda), \tag{4.1}$$

where $\alpha$ is a positive constant, $\nu$ is the frequency, $T$ is absolute temperature, and the parameters $\beta$ and $\lambda$ will be determined by imposing the two following constraints to be satisfied by Eq. (4.1)

$$\rho(T) = \int_0^\infty \rho_T(\nu)d\nu \propto T^{(1+\omega)/\omega}, \tag{4.2}$$

$$N(T) = \int_0^\infty \frac{\rho_T(\nu)d\nu}{\nu} \propto T^{1/\omega}, \tag{4.3}$$

which respectively define the energy density and the particle number density. Following Lima and Maia [38], let us then define a new variable $u = \nu T^\lambda$, so that the constraints (4.2) and (4.3) can be re-cast as

$$\rho(T) = \frac{\alpha}{T^{\lambda(1+\beta)}} \int_0^\infty u^\beta \phi(u)du \propto T^{(1+\omega)/\omega}, \tag{4.4}$$

$$N(T) = \frac{\alpha}{T^{\lambda\beta}} \int_0^\infty u^{\beta-1}\phi(u)du \propto T^{1/\omega}, \tag{4.5}$$
where we have used Eq. (4.1). From Eqs. (4.4) and (4.5) we can finally deduce that $\lambda = -1$ and $\beta = 1/\omega$ and, therefore, our generalized Wien law is given by

$$\rho_T(\nu) = \alpha \nu^{1/\omega} \phi \left( \frac{\nu}{T} \right),$$

(4.6)

which, as expected, reduces to the known Wien law for blackbody radiation if we set $\omega = 1/3$.

We note then that the temperature should not appear as simply $T$ but always in the combination $T/(1 + \omega)$ (see Sec. II), both in the energy density $\rho_T(0)$ and in the energy density $\rho(T)$. We can therefore write the Wien spectrum (4.1) for the phantom energy case as

$$\rho_T(\nu) = \alpha \nu^{1/\omega} \phi \left( \frac{\nu}{|T|} \right),$$

(4.7)

and hence using the same procedure as before, we recover the same Wien law, but referred to absolute value of temperature, also for a phantom radiation, i.e. for $\omega < -1$ and $T < 0$,

$$\rho_T(\nu) = \alpha \nu^{1/\omega} \phi \left( \frac{\nu}{|T|} \right).$$

(4.8)

This actually gives the most general expression for the Wien law and is valid for any positive and negative value of the parameter $\omega$.

V. SPONTANEOUS AND STIMULATED ABSORPTION

Let us consider a set of two level systems characterized by the resonant frequency $\nu$, immersed in phantom radiation at negative temperature $T < 0$. Assuming the simple dipole-moment approximation we label the energy levels of the system by $n$ and $m$ (see Fig. 1), with $E_m - E_n = h\nu$. Following a line of reasoning analogous to that which led Einstein to introduce his celebrated absorption and emission coefficients [39], we can now assume the occurrence of novel radiative processes if the radiation field is made up of a substance characterized by an equation of state with $\omega < -1$. In that framework the probability that a system is in the energy level $E_i$ ($i = n, m$) is given by

$$W_i = p_i e^{E_i/(k_B |T|)},$$

(5.1)

where $K_B$ is the Boltzmann constant and $p_i$ is the temperature-independent statistical weight of the $i$th state. Now, due to the presence of phantom energy which would only exchange discrete amounts of negative internal energy $(1 + \omega)\rho \Delta V$ with the level systems, there will be three new Einstein-like coefficients, here denoted by $B_n^m$, $B_m^n$ and $A_n^m$ (See Fig. 2). $A_n^m$ will correspond to a spontaneous absorption coefficient which would be evaluable from first principles using quantum mechanics and happens in the absence of any phantom radiation, but creates quanta of phantom energy; $B_n^m$ is a new induced emission coefficient and $B_m^n$ is a new stimulated absorption coefficient. The latter two coefficients would only occur in the presence of phantom radiation. These three coefficients will be related with the following rates of probability transition.

$$\dot{W}_{mn} = B_n^m \rho_T(\nu)$$

(5.2)

$$\dot{W}_{nm} = B_m^n \rho_T(\nu)$$

(5.3)

$$\dot{W}_{nm} = A_n^m$$

(5.4)

Thus, an equilibrium condition can be introduced in the considered system which reads

$$p_m e^{E_m/(k_B |T|)} B_m^n \rho_T = p_n e^{E_n/(k_B |T|)} (B_n^m \rho_T + A_n^m),$$

(5.5)

and if the approximation $|T| >> 0$ is applied, then $p_m B_m^n \approx p_n B_m^n$, and hence

$$\rho_T(\nu) = \frac{A_n^m}{B_m^n (e^{h\nu/(k_B |T|)} - 1)},$$

(5.6)
FIG. 1: Population of given energy levels in the cases that: (left) $\omega > -1$ and hence $T > 0$, and (right) $\omega < -1$ and hence $T < 0$. The area of the circles over the energy levels would qualitatively be proportional to the population.

$P = e^{-\text{energy}/T}$  \quad $P = e^{+\text{energy}/|T|}$
where we have used $E_n - E_m = h\nu$. Comparing with the generalized Wien law (4.8), we finally obtain a generalized Planck law, with [34]

$$A^m_n = \alpha B^m_n \nu^{1/\omega},$$ (5.7)

$$\phi(\nu/|T|) = \frac{1}{e^{h\nu/(k_B|T|)} - 1},$$ (5.8)

being the average occupation number, and finally

$$\rho_T(\nu) = \frac{\alpha \nu^{1/\omega}}{e^{h\nu/(k_B|T|)} \pm 1},$$ (5.9)

where the case of fermions has been also included. The law (5.9) is the most general Planck law and can be applied to radiation characterized by an equation of state $p = \omega \rho$, where $\omega$ can take on any positive or negative value.

As a test of consistency we finally calculate e.g. $\rho(T)$ from Eq. (5.9) for the fermion case. We in fact obtain

$$\rho(T) = \alpha \int_0^\infty \frac{\nu^{1/\omega} d\nu}{e^{h\nu/(k_B|T|)} + 1} = \alpha \left( \frac{k_B|T|}{\hbar} \right)^{(1+\omega)/\omega} \times \left( 1 - 2^{-1/\omega} \right) \Gamma \left( 1 + \frac{\omega}{1 + \omega} \right) \zeta \left( 1 + \frac{1 + \omega}{\omega} \right),$$ (5.10)

where $\Gamma$ and $\zeta$ are the Gamma and zeta functions [40], respectively. That is the result what was to be in fact expected.

Before closing up this section we would like to briefly comment on the effect that negative phantom temperatures may have on laser effect. In fact, it is well known that laser theory is based on the occurrence of stimulated emission which, together with population inversion, leads to the necessary coherent radiation amplification leading to the laser operation. In the case of negative phantom temperature, even though population inversion is guaranteed, there is anything like a stimulated emission and lasing can never be produced. What one could instead suppose is the possibility for a contrary or anti-lasing process. If in our negative temperature system one would restore a larger population in the lower energy level by the use of an appropriate procedure, then stimulated absorption would produce a damping, rather than amplification, of the coherent radiation. The construction of such an anti-laser, or more properly "lasar" (light attenuation by stimulated absorption of radiation), device could be of some usefulness in atomic and molecular physics. On the other hand, stimulated absorption processes might provide a clue for explaining the lack of phantom-energy radiative processes in cosmology.

VI. CONCLUSION AND FURTHER COMMENTS

There have been two recent important contributions to the question of the thermodynamics of dark energy. On one hand, Alcaniz and Lima [22] derived general dark energy thermodynamic expressions that, when extended to the phantom regime, implied positive values for its temperature and negative values for its entropy. By using two independent compelling arguments, here it is nevertheless shown that what becomes negative in the phantom regime is temperature, while entropy is kept definite positive along the entire range of negative state equation parameter. In any event, since classically having either a negative entropy or a negative temperature, or both, makes no physical meaning, the first obvious conclusion seems to be that phantom energy cannot exist. This situation is somehow reminiscent to the one which was posed in the mid-seventies when it was first realized that the black hole entropy had to be finite [41]. Hawking himself has many times stressed [42] that, at that time, it seemed that since black hole cannot emit anything classically, such a conclusion was meaningless, too. The solution to this paradoxical situation was, as it is now widely acknowledged, to appeal to the quantum nature of black holes which allowed them to radiate thermal particles. We argue in this paper that the solution to the above apparent phantom thermodynamic paradox is again by appealing to the essential quantum nature of the phantom stuff. In fact, if, in spite of by definition violating the dominant energy condition, i.e. $p + \rho < 0$, phantom energy is consistently assumed to satisfy the weak energy condition, $\rho > 0$, then it can be interpreted that the phantom spacetime becomes Euclideanized and this is known to provide the natural framework where quantum temperature and quantum entropy can be consistently defined [27]; i.e. the cosmic phantom fluid can be essentially regarded as a quantum system having not any classical analogs. Now, negative temperatures (or even entropies) can perfectly exist in the quantum realm. It is in this way that the above thermodynamic phantom paradox is solved in the present paper. Our conclusion thus is that there can
FIG. 2: Pictorial representation of the possible radiative processes induced by phantom radiation on a two-level system as expressed in terms of the Einstein-like coefficients: $A_{m}^{n}$ for spontaneous absorption, $B_{m}^{n}$ for induced absorption, and $B_{n}^{m}$ for stimulated absorption. No stimulated emission can take place in the presence of phantom energy with negative temperature.
exist a cosmic quantum field characterized by a temperature that is always negative and a positive definite entropy, and that, moreover, if the universe would happen to be currently dominated by such phantom energy, then this would introduce a universal element of cosmological information that would make every piece of observable matter to be really observable and allow the very existence of living organisms in the universe. On the other hand, when considering accretion of phantom energy onto black holes, Babichev, Dokuchaev and Eroshenko have showed [8] that the mass of the black holes undergo a gradual decrease and tend all to zero at the big rip. In this work we confirm and extend that result, consistently interpreting it in terms of the above conclusion on phantom thermodynamics. This realization is important in at least two respects. It firstly implies that once black holes are formed before or after phantom energy domination, all known evolution processes of existing black holes become inexorably dominated by phantom energy accretion and finally subject to what could be dubbed as a "democracy before death" principle by which such black holes are all equalized before disappearing all at once at the big rip, no matter their initial mass or the time when they were formed. The reason of such a domination of phantom energy accretion (which actually extends over any emission processes of any observable matter system) essentially resides on the fact that, however small its absolute value may be, a negative temperature is always hotter than any positive temperature, even if this is infinite. Secondly, it is also a conclusion of the present paper that, since the phantom-induced annihilation process prevails always over Hawking thermal emission, during the whole black hole evolution process leading from formation to final disappearance, quantum coherence is preserved, as in this case neither the black hole nor any thermal radiation emitted from it is left in the final state. This offers a rather comfortable solution to the so-called black-hole information paradox [27] and becomes still another positive consequence from the existence of phantom energy in the universe.

The formulation of some main theoretical basis for establishing the quantum statistic thermodynamics of phantom energy has been also aimed at in this work. By assuming that any form of dark energy can be taken to be a radiation field, we have thus considered a generalized Wien law and hence a generalized Planck radiation law. These laws turn out to be described by the same general expressions as for the usual dark energy case first derived by Lima and Maia [38], but referred to the absolute value of phantom temperature. This conclusion comes about as a consequence from the necessary introduction of novel Einstein coefficients and the different probability law for the case of phantom energy. The extra Einstein coefficients correspond to new radiative processes that include a stimulated absorption phenomenon, which would attenuate the intensity of the phantom radiation.

It will only be by collecting more cosmological data that we will be able to finally decide on whether or not phantom energy is the form of dark energy that currently operates in the universe to drive its observed accelerating expansion. However, even in the event that other kind of dark energy turned out to be the favoured stuff dominating the current universe, there could still be sufficient room for phantom energy to dominate over all other cosmic stuffs during other epochs along the universal evolution, including the primordial inflationary period, such as has been recently suggested [43]. Anyway, we hope that the contents of the present paper may help to convince cosmologists that, whatever the final conclusion on the above subjects may be, phantom energy is a physical concept with sufficient theoretical interest by itself as to pursue active research on it.

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