A Mechanism for Ordinary-Sterile Neutrino Mixing

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Abstract

Efficient oscillations between ordinary (active) and sterile neutrinos can occur only if Dirac and Majorana mass terms exist which are both small and comparable. It is shown that this can occur naturally in a class of string models, in which higher-dimensional operators in the superpotential lead to an intermediate scale expectation value for a scalar field and to suppressed Dirac and Majorana fermion masses.
I. INTRODUCTION

There have been suggestions [1] that there may be mixing between ordinary neutrinos (with normal weak interactions) and sterile neutrinos (which have no ordinary interactions except by mixing) [2]. This is motivated by the various hints for neutrino masses or oscillations, including the MSW interpretation of the solar neutrino spectral anomaly [3,4], the anomalous $\mu/e$ ratio produced by atmospheric neutrinos [5], the candidate events for $\nu_\mu \rightarrow \nu_e$ (or $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) in the LSND experiment [6], and the possibility of a hot (neutrino) component to the dark matter (mixed dark matter) [7]. The first three of these suggest neutrino oscillations, most likely with different ranges for the neutrino mass-squared differences $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$, where $m_i$ is the mass of the $i^{th}$ mass eigenstate neutrino, around $10^{-5}$, $10^{-3} - 10^{-2}$, and $10^{-1} - 10$ eV$^2$, respectively. The mixed dark matter scenario suggests one or more mass eigenstate neutrinos $\nu_i$ in the several eV range. Given the constraint $N_\nu = 2.990 \pm 0.011$ from the $Z$ lineshape [10] on the number of light ordinary neutrinos, one cannot accomodate all of these possibilities unless there are additional sterile neutrinos (which don’t contribute significantly to $N_\nu$) [1]. Mixed dark matter also suggests the possibility of near degeneracies, so that $|\Delta m^2_{i,j}| \ll m^2_{i,j}$ [1].

Another implication is for big bang nucleosynthesis [11]. There are conflicting estimates of the abundance of primordial deuterium from QSO absorption lines, but the low $D$ observations, combined with canonical estimates of the primordial $^4He$ abundance, suggest only $N_{eff} = 1.9 \pm 0.3$ light neutrinos in equilibrium at the time of decoupling of the neutron to proton ratio [12]. The resolution may well be that the systematic uncertainties in the $^4He$ abundance have been underestimated (i.e., that there is more $^4He$, as predicted by $N_{eff} = 3$), or possibly that the high $D$ QSO observations are correct. However, it is possible that the number of effective neutrinos $N_{eff}$ is indeed smaller than three. Mechanisms for achieving this include a decaying $\tau$ neutrino, which for certain masses, lifetimes, and decay modes can lead to a lower energy density [13]. Another possibility is a significant $\nu_e - \bar{\nu}_e$ asymmetry [14]. A $\nu_e$ excess would deplete the $n/p$ ratio prior to the decoupling of the $\nu_e n \leftrightarrow e^- p$ reaction, leading to the production of less $^4He$, and thus $N_{eff} < 3$.

The mixing of ordinary and sterile neutrinos could affect $N_{eff}$ in two ways. The combination of ordinary-sterile oscillations with the rescattering of the active component (which destroys the phase coherence of the oscillating state and serves as a measurement) can lead to the production of sterile neutrinos. Ignoring effects of lepton asymmetries, a number of authors [15] have argued that for a wide range of ordinary-sterile neutrino parameters, the mixing could lead to the production of light sterile neutrinos in equilibrium numbers prior to nucleosynthesis, increasing the energy density and increasing $N_{eff}$ to four, aggravating the difficulty. However, Foot and Volkas have recently argued [16] that the asymmetric interaction of neutrinos and antineutrinos with matter could, for a significant parameter range, 

\footnote{Several authors [8] have suggested that it might be possible to accomodate the atmospheric neutrinos and LSND by a single $\Delta m^2 \sim 10^{-1}$ eV$^2$ in a three-neutrino mixing scheme. However, this range is strongly disfavored by the azimuthal and energy distributions in the most recent Superkamiokande data [8].}
lead to the generation of a large neutrino-antineutrino asymmetry prior to nucleosynthesis. This could both suppress the production of sterile neutrinos, and in some cases lead to a significant excess of $\nu_e$ with respect to $\bar{\nu}_e$, thus weakening the nucleosynthesis constraint on ordinary sterile mixing, or even accounting for $N_{\text{eff}} < 3$.

Yet another motivation for ordinary-sterile neutrino mixing comes from heavy element synthesis in supernova explosions. One promising site for the (neutron enriched) $r$-processes are in the ejecta of neutrino-heated supernova explosions [17]. Unfortunately, $\nu_e$ emissions would render the $r$-process impossible, due to the destruction of neutrons by $\nu_e n \rightarrow e^- p$. The problem would be even worse for $\nu_e \leftrightarrow \nu_\mu$ or $\nu_\tau$ conversions with $\Delta m^2 > 10^{-2} \text{ eV}^2$, which would create more energetic $\nu_e$'s [18]. However, it has recently been argued by Caldwell, Fuller, and Qian [19] that ordinary-sterile neutrino mixing would yield a robust solution to the difficulty, by allowing $\nu_\mu$ to convert to sterile neutrinos and escape from the supernova, followed by $\nu_e$ converting into (harmless) $\nu_\mu$.

Clearly, none of these motivations for ordinary-sterile neutrino mixing is compelling, but nevertheless they motivate an examination of the theoretical possibilities for significant ordinary-sterile neutrino mixing. In Section II I review the basic constraints on Dirac and Majorana neutrino masses needed to generate significant ordinary-sterile mixing. In particular, it will only occur in theories in which the Dirac and Majorana masses are both small and comparable to each other [2], which is difficult to achieve without fine-tuning in most models of neutrino mass [20]. In Section III, however, I discuss a framework in which it is quite plausible to have the necessary ingredients without fine-tuning. It was recently argued [21] that in perturbative superstring vacua realistic hierarchies of quark and charged lepton masses may be generated by higher-dimensional operators, in which the effective Higgs Yukawa couplings are suppressed by powers of the ratio of an intermediate scale vacuum expectation value to the string scale. It was also shown that in such models it is possible to have naturally small Dirac neutrino masses, as well as Majorana masses for the sterile neutrinos that can be either large or small depending on the dimensions of the relevant operators. These ideas are extended in Section III, in which it is shown that the Dirac and Majorana masses can indeed be small and comparable if two conditions are satisfied: (i) the scale $m_{\text{soft}}$ of supersymmetry breaking in the observable sector is comparable to the electroweak scale (i.e., no more than a TeV), as is necessary if supersymmetry is to be relevant to the stabilization of the electroweak scale and is predicted in models of radiative electroweak breaking; (ii) a simple relation is satisfied between the (integer) dimensions of the higher-dimensional operators responsible for the Dirac masses, the Majorana masses, and an operator associated with the intermediate scale vacuum expectation value. Although this relation will not be satisfied in all or even most string compactifications, it is sufficiently simple as to provide a plausible framework for ordinary-sterile mixing.

II. ORDINARY-STERILE NEUTRINO MIXING

An ordinary (active) neutrino occurs in an $SU(2)$ doublet with a charged lepton, and thus has standard charged and neutral current weak interactions. For example, the left-chiral $\nu_{eL}$ is the partner of the left-handed $e^-_L$. $\nu_{eL}$ is necessarily associated by CPT with the right-chiral antineutrino $\nu^c_{eR}$; the $SU(2)$ partner of the $e^+_R$. $\nu_{eL}$ and $\nu^c_{eR}$ together constitute
a Weyl two-component neutrino. A sterile neutrino is an $SU(2)$ singlet Weyl neutrino $N^c_L$ and its CPT partner $N^c_R$. It has no gauge interactions except by mixing. Most extensions of the standard model involve such sterile neutrinos. The only real issues are whether the sterile neutrinos are light, and whether there is significant mixing between the ordinary and sterile states of the same chirality.

In the presence of one ordinary and one sterile Weyl neutrino, the most general mass matrix is

$$L = \frac{1}{2} \left( \begin{array}{c} \nu_L \varepsilon^c \end{array} \right) \left( \begin{array}{ccc} m_T & m_D & m_M \\ m_T^T & m_D^T & m_M \\ m_M^T & m_D^T & m_M \end{array} \right) \left( \begin{array}{c} \nu_R \varepsilon^c \\ N_R \end{array} \right) + h.c.,$$

(1)

where $m_T$ and $m_M$ are Majorana mass terms for the ordinary and sterile neutrinos, respectively, and $m_D$ is a Dirac mass term, which connects two distinct Weyl neutrinos. $m_T$, $m_M$, and $m_D$ break weak isospin by 1, 0, and 1/2 units, respectively, and can be generated by the vacuum expectation values of Higgs triplets, singlets, and doublets. In many models, $m_T$ is absent. Diagonalizing (1), one obtains two Majorana mass eigenstate neutrinos, $\nu_1$ and $\nu_2$. The left chiral states are related by

$$\nu_L = \nu_{1L} \cos \theta + \nu_{2L} \sin \theta$$

$$N^c_L = -\nu_{1L} \sin \theta + \nu_{2L} \cos \theta,$$

(2)

where $\theta$ is the mixing angle. A similar relation holds for the right-chiral components. In the generalization to three families, one can interpret $\nu_L$, $N^c_L$, $\nu_R$, and $N_R$ as three-component vectors in family space, and $m_T$, $m_D$, and $m_M$ as 3 × 3 matrices ($m_D^T$ is the transpose of $m_D$; $m_T$ and $m_M$ are symmetric), yielding six Majorana mass eigenstates. The further generalization to models in which there are are a different number of sterile states is straightforward.

Most models of neutrino mass involve limiting cases of (1) in which there is little or no mixing between $\nu_L$ and $N^c_L$. For example, the pure Dirac case ($m_T = m_M = 0$) yields two degenerate Majorana eigenstates, which can be combined to form a four-component Dirac neutrino. One finds $\theta = \pi/4$, but can transform to an appropriate basis for the degenerate states in which it is manifest that there is a conserved lepton number and there are no $\nu_L \rightarrow N^c_L$ transitions. In the pure Majorana case $m_D = 0$, one has $\theta = 0$, and the sterile

\footnote{Which chiral state is referred to as the particle, and which as the antiparticle, is mainly a matter of convention, or is motivated if there is a conserved or approximate lepton number in the theory.}

\footnote{$m_M$ could in principle be a bare mass, but this is forbidden by additional symmetries in most extensions of the standard model.}

\footnote{For a general discussion, see \cite{20}.}

\footnote{The simplest model \cite{22} with $m_T \neq 0$ is excluded phenomenologically \cite{20}.}
neutrino decouples. Similarly, in the seesaw limit \[ m_M \gg m_D, \] there is one naturally light Majorana state \( \nu_1L \sim \nu_L \) with \( m_1 \sim m_D^2/m_M \), and one heavy state \( \nu_2L \sim N^c_L \) with \( m_2 \sim m_M \), with negligible mixing (\( \theta \sim m_D/m_M \)).

Significant ordinary-sterile mixing can only occur if \( m_D \) is of the same order of magnitude as \( m_M \) and/or \( m_T \). For definiteness, I will concentrate on \( m_M \). Because of the limits on neutrino mass, one needs not only that \( m_D \) and \( m_M \) are comparable, but both must be much smaller than the quark and charged lepton masses. If these conditions are satisfied, then ordinary-sterile (referred to as second class \( \nu_D \)) neutrino oscillations between \( \nu_L \) and \( N^c_L \) can occur, with the usual formula

\[
P(\nu_L \rightarrow N^c_L) = \sin^2 2\theta \sin^2 \left( \frac{1.27\Delta m^2 (eV^2)L(km)}{E(GeV)} \right),
\]

where \( \Delta m^2 = m_2^2 - m_1^2 \), \( L \) is the distance traveled, and \( E \) is the energy.

Similar statements apply to the \( 6 \times 6 \) generalization. In the pure Dirac, pure Majorana, or seesaw limits, there can be ordinary flavor (first class) oscillations between the three ordinary neutrinos, but no second class oscillations between the \( \nu_L \) and the \( N^c_L \) because of the absence of significant mixing between the two sectors (and because the sterile states are heavy in the seesaw model). However, if the Dirac and Majorana masses are small and comparable, then there can be significant mixing between the six left-chiral states, and both first and second class oscillations can occur.

Thus, significant ordinary-sterile mixing can occur, but it appears to require two miracles, i.e., small Dirac and Majorana masses. However, in the next Section I will describe a plausible framework in which this can occur naturally.

### III. Mass Generation by Higher-Dimensional Operators

Recently, a simple mechanism was found to generate small Dirac neutrino masses without invoking a seesaw, and also Majorana masses for the sterile neutrinos that could be either large or small \[ m_T \]. It utilized higher-dimensional operators \[ \mathcal{O} \] in the superpotentials of perturbative superstring compactifications based on the free fermionic construction \[ \mathcal{W} \] with additional non-anomalous \( U(1) \) gauge factors. The basic idea is that these operators involve additional fields which are singlets under the standard model gauge group. In some cases, these fields acquire intermediate scale vacuum expectation values, leading to effective Yukawa couplings that are suppressed by powers of the ratio of the intermediate scale to

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6One usually assumes \( m_T = 0 \) in the seesaw model, but similar conclusions apply if \( m_T \) is comparable to \( m_D^2/m_M \).

7Another possible mechanism is described in \[ [24] \].

8Other studies of the implications of higher-dimensional operators for neutrino masses may be found in \[ [25] \].
the string scale. Here, I will show that in some cases the Dirac and Majorana masses will naturally be small and comparable, although it is hard to make precise numerical predictions without a specific fully realistic string model. Some of the considerations could also occur in a more general class of models, but they are especially well motivated in the case of the superstring models with an extra $U(1)$.

In the typical supergravity model $[27]$ with radiative electroweak breaking, the mass-squares of the Higgs doublets $H_{1,2}$ at the Planck scale are positive and of order $m_{\text{soft}}^2$, where $m_{\text{soft}}$ is a typical soft supersymmetry breaking parameter. However, the large top-Yukawa drives one of the running Higgs mass-squares to a negative value of order $-m_{\text{soft}}^2$ at low energy, leading to electroweak breaking at a scale $\langle H_2 \rangle \sim m_{\text{soft}}$. Many superstring models involve an extra non-anomalous $U(1)$ in the observable sector. Recently, it was argued $[28]$ that this $U(1)$ would either: (i) be unbroken; (ii) be broken near the electroweak scale (i.e., below 1 TeV); or (iii) broken at a scale intermediate between the electroweak and string scales. This is because in most string models with supergravity mediated supersymmetry breaking all of the scalar mass-squares are positive and of the same order of magnitude, $m_{\text{soft}}^2$, at the string scale. (These string models do not allow supersymmetric mass terms.) Furthermore, Yukawa couplings at the string scale are typically either zero or simply related to a gauge coupling of order unity. Breaking can occur if there is a standard model singlet field $S$ that is charged under the extra $U(1)$ and which has a large Yukawa coupling to other fields. Then a radiative mechanism can occur analogous to the radiative electroweak breaking, i.e., the scalar would acquire an expectation value $\langle S \rangle \sim m_{\text{soft}}$, breaking the $U(1)$ near the electroweak scale $[9]$.

However, if there are two or more such fields with opposite signs for their $U(1)$ charges, then, depending on the details of the soft breaking, the minimum may occur along a $D$-flat direction. In many string models, these directions are also $F$-flat up to higher-dimensional terms. For example, for two fields $S_{1,2}$ with equal and opposite charges $Q$, the low energy potential is

$$V(S_1, S_2) = m_1^2 |S_1|^2 + m_2^2 |S_2|^2 + \frac{g'^2 Q^2}{2} (|S_1| - |S_2|)^2, \quad (4)$$

where $g'$ is the gauge coupling of the extra $U(1)$. For $m_1^2 < 0$ but $m_1^2 + m_2^2 > 0$, the minimum occurs for $\langle S_1 \rangle \sim m_{\text{soft}}$ and $\langle S_2 \rangle = 0$, similar to the single field case. However, for $m_1^2 + m_2^2 < 0$, the minimum occurs along the $D$-flat direction $\langle S_1 \rangle = \langle S_2 \rangle \equiv \langle S \rangle$, for which

$$V(S) = m^2 |S|^2, \quad (5)$$

where $m^2 = m_1^2 + m_2^2$. This potential appears to be unbounded from below. However, the potential will be stabilized by one or both of the following mechanisms: (i) higher-loop terms in the effective potential lead to the replacement of $m^2$ in (3) by the running

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9 Such models were more fully explored and shown to yield a natural solution to the $\mu$ problem in $[29]$.

10 A similar situation could occur for a scalar field $S$ not charged under any gauge group.
$m^2(S)$, evaluated at the scale $S$. This renormalization group improved potential has a minimum close to the point $\mu_{\text{RAD}}$ at which $m^2$ crosses zero \cite{21}. $\mu_{\text{RAD}}$ is sensitive to the soft parameters and Yukawas, and can occur anywhere between 1 TeV and the string scale. (ii) Higher-dimensional terms in the superpotential involving $S$, which are generally expected in string models, can also stabilize the potential. The latter mechanism usually dominates whenever $\mu_{\text{RAD}} \gtrsim 10^{12}$ GeV. The consequences of both mechanisms were explored in \cite{21}. In particular, it was shown that other higher-dimensional terms involving $S$ could also lead to a reasonable effective $\mu$ parameter, and to interesting hierarchies of quark, charged lepton, and neutrino masses.

For definiteness, I will consider the case in which the minimum of the potential occurs along a one-dimensional flat direction, characterized by an effective field $S = s/\sqrt{2}$. The potential for $s$ is \cite{21}

$$V(s) = \frac{1}{2} m^2 s^2 + \frac{1}{2(K+2)} \left(\frac{s^{2+K}}{\mathcal{M}^K}\right)^2,$$

where $\mathcal{M}$ is of the order of the string scale (e.g., $\mathcal{M} \sim 10^{17} - 10^{18}$ GeV), and $m^2 = \mathcal{O}(-m^2_{\text{soft}})$ is the running soft mass squared evaluated at $s$. The second term, with $K = 1, 2, \cdots$, is a higher-dimensional operator associated with the non-renormalizable term

$$W_{\text{NR}} = \left(\frac{\alpha_K}{M_{\text{Pl}}}\right)^K \hat{S}^{3+K},$$

in the superpotential. In (7), $\hat{S}$ is the superfield corresponding to $S$, $M_{\text{Pl}}$ is the Planck scale, and $\alpha_K$ is a calculable coefficient in a given string model. $\mathcal{M}$ and $M_{\text{Pl}}$ are related by $\mathcal{M} = C_K M_{\text{Pl}}/\alpha_K$, where $C_K = [2^{K+1}/((K+2)(K+3)^2)]^{1/(2K)}$. I will assume that the potential is stabilized by the higher-dimensional operator and that the running of $m^2$ can be neglected for the minimization. Then, the minimum occurs for

$$\langle s \rangle = \left[\sqrt{(-m^2)\mathcal{M}^K}\right]^{\frac{1}{K+1}} \sim (m_{\text{soft}} \mathcal{M}^K)^{\frac{1}{K+1}}.$$ 

As was discussed in \cite{21}, the electroweak scale in the same model can occur by the radiative mechanism at $\langle H_{1,2} \rangle = \mathcal{O}(m_{\text{soft}})$.

Dirac neutrino masses may be generated by the operators \cite{21}

$$W_D \sim \hat{H}_2 \hat{L}_L \hat{N}^c_L \left(\frac{\hat{S}}{\mathcal{M}}\right)^{P_D},$$

where $\hat{L}_L$ is the superfield corresponding to a left chiral doublet which includes an active $\nu_L$, and $\hat{N}^c_L$ is the superfield corresponding to a sterile neutrino. This yields an effective Dirac Yukawa coupling of order $(\langle S \rangle/\mathcal{M})^{P_D}$. Similar terms can lead to a reasonable hierarchy of quark and charged lepton masses. The integer powers $P_D$ can differ for quarks, charged leptons, and neutrinos, and can be generalized to include family indices and generational mixing. They are model dependent, but calculable in a specific string model. In some cases, the allowed powers for each type of term are restricted by the $U(1)$ and other symmetries of
the effective field theory. However, it is often the case that terms that would be allowed by the effective field theory symmetries are absent due to selection rules in the underlying string theory. It is reasonable that in some models the neutrino mass terms occur in higher orders than those for the quarks and charged leptons, leading to naturally small Dirac neutrino masses.

Majorana mass terms for both the ordinary and active neutrinos may be generated by the higher-dimensional operators

\[ W_T \sim \frac{\hat{H}_2 \hat{L}_L}{\mathcal{M}} \left( \frac{\hat{S}}{\mathcal{M}} \right)^{P_T} \]

and

\[ W_M \sim \hat{N}_L^c \hat{N}_L^c \hat{S} \left( \frac{\hat{S}}{\mathcal{M}} \right)^{P_M} \]

respectively.

Hence, one obtains neutrino masses

\[ m_D \sim \langle H_2 \rangle \left( \frac{\langle S \rangle}{\mathcal{M}} \right)^{P_D} \sim \langle H_2 \rangle \left( \frac{m_{\text{soft}}}{\mathcal{M}} \right)^{\frac{P_D}{K+1}}, \]

\[ m_T \sim \langle H_2 \rangle^2 \left( \frac{\langle S \rangle}{\mathcal{M}} \right)^{P_T} \sim \langle H_2 \rangle^2 \left( \frac{m_{\text{soft}}}{\mathcal{M}} \right)^{\frac{P_T}{K+1}}, \]

\[ m_M \sim \langle S \rangle \left( \frac{\langle S \rangle}{\mathcal{M}} \right)^{P_M} \sim m_{\text{soft}} \left( \frac{m_{\text{soft}}}{\mathcal{M}} \right)^{\frac{P_M - K}{K+1}}, \]

which depend on the powers \( K, P_D, P_T, \) and \( P_M \). The second expressions in (12) result from substituting (8) for \( \langle S \rangle \).

One expects \( m_{\text{soft}}/\mathcal{M} \sim 10^{-14} - 10^{-16} \) for \( m_{\text{soft}} \sim 100 - 1000 \) GeV and \( \mathcal{M} \sim 10^{17} - 10^{18} \) GeV. Since \( \langle H_2 \rangle = \mathcal{O}(100 \text{ GeV}) \), \( m_D \) can be in the interesting range of \( 10^{-3} - 10 \) eV for \( P_D/(K + 1) \lesssim 1 \). One has \( \langle H_2 \rangle^2/\mathcal{M} \sim 10^{-4} \) eV for \( \mathcal{M} \sim 10^{17} \) GeV, so \( m_T \) is too small to be phenomenologically interesting unless \( P_T = 0 \). Even then, \( m_T \) is too small to be relevant to the MSW conversions of solar neutrinos unless one stretches the parameters [3]. It could possibly be relevant to vacuum oscillations [31].

The sterile neutrino Majorana mass \( m_M \) may be large or small. For \( P_M - K < 0 \) one has \( m_M \gg m_{\text{soft}} \), allowing a conventional seesaw model. However, \( m_M \) is naturally small for \( P_M - K > 0 \). The special case \( P_D = P_M - K \) is particularly interesting, because it implies \( m_M/m_D \sim m_{\text{soft}}/\langle H_2 \rangle = \mathcal{O}(1) \), just the condition needed for significant ordinary-sterile neutrino mixing. This is quite encouraging for this class of models. However, it is difficult to be more quantitative because the precise relation between \( m_{\text{soft}} \) and \( \langle H_2 \rangle \), and the values of the effective mass \( \mathcal{M} \), which could vary somewhat from operator to operator, are model dependent.

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11For a recent discussion, see [30].
IV. CONCLUSIONS

There are a number of laboratory, astrophysical, and cosmological hints for neutrino masses and mixing. While it is too early to be certain of any or all of these, the pattern suggests the possibility of significant ordinary \((SU(2)\)-doublet\) -sterile \((SU(2)\)-singlet\) neutrino mixing. Such mixing could even play an important role in understanding heavy element synthesis in supernova explosions. It is therefore useful to seriously examine the theoretical possibilities for ordinary-sterile mixing.

Most extensions of the standard model predict the existence of sterile neutrinos. The only real questions are whether the ordinary and sterile neutrinos of the same chirality mix significantly with each other, and whether the mass eigenstate neutrinos are sufficiently light. When there are only Dirac masses, the ordinary and sterile states do not mix because of the conserved lepton number. Pure Majorana masses do not mix the ordinary and sterile sectors either. In the seesaw model the mixing is negligibly small, and the (mainly) sterile eigenstates are too heavy to be relevant to oscillations. The only way to have significant mixing and small mass eigenstates is for the Dirac and Majorana neutrino mass terms to be extremely small and to also be comparable to each other\(^\text{12}\). This appears to require two miracles in conventional models of neutrino mass.

However, it has been argued in this paper that there is a natural framework for achieving the necessary conditions. The essential ingredient is that the effective Yukawa couplings which induce the Dirac and Majorana masses are suppressed by integer powers of \(\langle S \rangle / M\) where \(S\) is a standard model singlet which can acquire an expectation value \(\langle S \rangle\) at a scale intermediate between the electroweak (or supersymmetry-breaking) scale and a large (e.g., string) scale \(M\). This situation is motivated by perturbative superstring compactifications, which often have additional non-anomalous \(U(1)\) gauge symmetries in the observable sector. These may lead to intermediate scale breaking if the minimum of the scalar potential occurs along a \(D\)-flat direction \(^\text{21}\). Furthermore, such theories involve higher-dimensional operators needed to stabilize the scalar potential, generate an effective electroweak \(\mu\) term, and generate effective Yukawa couplings for hierarchies of quark, charged lepton, and neutrino masses. The allowed higher-dimensional operators depend in general on both the symmetries of the effective four-dimensional field theory and on the underlying string dynamics.

For appropriate higher-dimensional operators, it is possible to achieve small Dirac neu-

\(^1\text{12}\)I am using the term Dirac to refer to a mass term connecting a left-chiral doublet with a right-chiral singlet, and Majorana for a term connecting either left and right-chiral singlets or left and right-chiral doublets. In generalizations involving two or more singlet Weyl neutrinos there are limiting cases in which pairs of Majorana mass eigenstates are degenerate and can be combined to form a four component (Dirac) neutrino with a conserved lepton number. Whether one chooses to refer to these masses as Dirac is a matter of convention. There is an analogous situation for pairs of doublet Weyl neutrinos (the Mahmoud-Konopinski model). Even in these cases the relevant singlet-singlet or doublet-doublet mass terms are still controlled by operators analogous to (11) and (10), respectively. The estimates in (12) and implications for ordinary-sterile mixing still apply. I am grateful to Y. Grossman for emphasizing this case to me.
trino masses (without invoking a seesaw), and Majorana masses which can be either large or small. When a specific condition, $P_D = P_M - K$, for the dimensions of the operators defined in (7), (9), and (11) is satisfied, then the Dirac and Majorana masses will indeed be comparable for a supersymmetry breaking scale less than around 1 TeV, leading to significant mixing. This condition is very specific and is not expected to hold in all or even most compactifications, so ordinary-sterile mixing cannot be considered to be a prediction of this class of models. It is nevertheless a simple relation between (presumably small) integers which could hold in many models. It is certainly not a case of fine-tuning.

It is difficult to be more precise than the order of magnitude estimates presented in this paper. The exact relation between the soft breaking scale and the expectation values of $S$ and the Higgs doublets depends on the details of the soft breaking and the running of the parameters to low energies. Also, the coefficients of the various higher-dimensional operators (the $\alpha_K$ in (7)) are model dependent. Detailed calculations would require a specific realistic string model and supersymmetry breaking mechanism. However, the basic ideas are expected to hold in a wide class of models.

The framework presented here may provide a viable mechanism for small Dirac and Majorana masses and significant ordinary-sterile mixing. Unfortunately, it will be difficult to establish such mixing unambiguously. Ordinary-sterile oscillations are probably only observable in disappearance experiments. In principle, the sterile neutrinos could be observed directly, e.g., by the production of the wrong sign lepton in appearance experiments, if they have (suppressed) interactions from new physics, but such effects are expected to be extremely small [32]. If ordinary-sterile mixing really occurs, it will be necessary to establish it by difficult indirect methods, such as careful measurements of the spectral distortions in solar neutrino experiments [33], or by angular distributions [34] or neutral current rates [35] for atmospheric neutrinos.

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