Numerical solution of optimal departure frequency of Taipei TMS

Lih-jier Young\textsuperscript{1} and Chin-Hsin Chiu\textsuperscript{2}

\textsuperscript{1} Professor, Chung Hua University, Taiwan, R.O.C.
\textsuperscript{2} Ph.D. Student, Chung Hua University, Taiwan, R.O.C.
E-mail: young@chu.edu.tw

Abstract. Route Number 5 (Bannan Line) of Taipei Mass Rapid Transit (MRT) is the most popular line in the Taipei Metro System especially during rush hours periods. It has been estimated there are more than 8,000 passengers on the ticket platform during 18:00~19:00 at Taipei main station. The purpose of this research is to predict a specific departure frequency of passengers per train. Monte Carlo Simulation will be used to optimize departure frequency according to the passenger information provided by 22 stations, i.e., 22 random variables of route number 5. It is worth mentioning that we used 30,000 iterations to get the different samples of the optimization departure frequency, i.e., 10 trains/hr which matches the practical situation.

1. Introduction

The mass rapid transit (MRT) system is the most important, efficient, convenient transportation between modern metropolitan and urban areas. Because the rail is always underground or elevated, MRT offers the fastest way from downtown to any other place \cite{1}. MRT is therefore the most important mode of modern city transportation. It can carry more than 1,000 passengers on each train to reach their destination get elsewhere in even less than 10 minutes. However, during the 18:00~19:00 rush hour more than 8,000 passengers crowd into the platform of Route Number 5 (Bannan Line) of Taipei main station \cite{2}. Maintaining optimization, i.e., get the optimization departure frequency to have these passengers on their way quickly and of course maintain the most benefit of the Taipei Metro System is the main goal of this research \cite{3}.

2. Monte Carlo Simulation

Due to the reasons of nonparametric statistics, full valuation, probabilistic iteration and easy operating, the Monte Carlo Simulation is chosen to model the complex situation of the train Taipei MRT system during the rush hours. Monte Carlo
Simulation is a very popular method of estimating a large amount of calculation. It is used in many research fields, such as statistical engineering, bio-medicine, psychology, atmospheric science, and management, but this application is specific to mass rapid transit transfer system. A lot of papers outline the risk estimate by MCS such as [4-7]. Probability estimate is based on the assumption of normal distribution. Gross [8], estimate the probability by MCS. Remaining life assessment of low pressure turbine rotor has been recommended based on MCS [9]. With the advent of computers, MCS has become a very common numerical method. Mark Demaria [10] used MCS to estimate Tropical Cyclone Wind Speed Probabilities. It is a random event process, similar to the scientific experimental process [11]. It is also used to model error estimation in order to estimate social risk [12].

Table 1 Numbers of Departure passenger of each station of Bannan Line during 18:00~19:00 per weekday.

| Station       | Departure Passengers |
|---------------|----------------------|
| Yongning      | 1607                 |
| Tucheng       | 860                  |
| Haishian      | 2951                 |
| Far East Hospital | 2080            |
| Futhong       | 4271                 |
| Banqiao       | 4956                 |
| Xinpu         | 6074                 |
| Jiang zihui   | 3863                 |
| Longshan Temple | 3244             |
| Ximen         | 6858                 |
| Taipei Main Station | 8746            |
| Shandao Temple | 2541                |
| Zhongxiao Xinsheng | 2377          |
| Fuxing        | 4784                 |
| Zhongxiao Dunhua | 4701             |
| S.Y.S. Memorial Hall | 2603         |
| Taipei City Hall | 5015             |
| Yongchun      | 2449                 |
| Houshanpi     | 2537                 |
| Kunyang       | 1502                 |
| Nangang       | 1103                 |
| Taipei Nangang Exhibition Center | 1977          |
To analyze the optimization frequency of the Bannan Line of Taipei MTR from inputting each of the 22 independent parameters from each correspondence station the final income of each train plays a important role of this research. Table 1 shows the arrival of passengers for 22 stations of the Bannan Line during the time 18:00~19:00 of one weekday. In this paper, the Random Number Generator is made by Linear Congruential Generator (LCG) the equation is

\[ Y_{i+1} = [(AY_i + C) \mod M] \quad i = 0, 1, 2, \ldots \]

where \( Y_0 \) is the seed value, \( A \) is the constant multiplier, \( C \) is the increment, and \( M \) is the modulus. The value of \( Y_0 \) is preset. The sequence of all other \( Y_i \)'s will be obtained according to eq. (1) and the normalized value (Pseudo-Random Number) \( R_0, R_1, R_2, \ldots, R_i \) will be assigned from eq. (2)

\[ R_i = \frac{Y_i}{M} \]

Performing each Monte Carlo iteration, a random number is generated for each random variable of those 22 stations as described above. The 22 simulations are processed deterministically for a few chosen standard deviations of the above mentioned 22 random variables. It is worth mentioning we repeated the iteration 30,000 times to get the sample of the profile of the Bannan Line with different departure frequencies.

3. Simulation the income of each train

To simulate the income of each train we have to set up a model to get the beginning and the end point of each passenger journey. It is reasonable to assume the alighting rate can be obtained as per the equations below:

\[ GOR_s = \left( \frac{DP_s - \frac{DP_s}{n}}{\sum_{s=1}^{22} DP_s} \right) \times 100\% \quad s = 1, 2, 3, \ldots, 11 \quad \text{(before station k)} \]  

\[ GOR_s = \left( \frac{DP_s}{\sum_{s=1}^{22} DP_s} \right) \times 100\% \quad s = 12, 13, 14, \ldots, 22 \quad \text{(after station k)} \]

where \( GOR_s \) is the alighting rate of each station,
\( DP_s \) is the number of the departing passengers from each station,
\( n \) is a specific number corresponding to each station and described in Table 2.
### Table 2 Values of $n$ at each station where the alphabet corresponds to Table 1.

|   | a   | b  | c  | d  | e  | f  | g  | h  | i  | j  | k  | l  | m  | n  | o  | p  | q  | r  | s  | t  | u  | v  |
|---|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| $n$| 1   | 1.1| 1.2| 1.3| 1.4| 1.5| 1.6| 1.7| 1.8| 1.9| 2  | 1.9| 1.8| 1.7| 1.6| 1.5| 1.4| 1.3| 1.2| 1.1| 1.1| 1  |

It is worthy of mention that the value of $n$ for each station is assumed to have linearly increase from station $a$ to $k$ and then to decrease linearly from station $k$ to $v$. It is reasonable that $n = 2$ at station $k$ because station $k$ is the central station of the line, i.e., the number of passenger for each journey is equal.

### Table 3 Ticket Price of Bannan Line where the alphabet corresponds to Table 1.

| Station | a   | b  | c  | d  | e  | f  | g  | h  | i  | j  | k  | l  | m  | n  | o  | p  | q  | r  | s  | t  | u  | v  |
|---------|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| $a$     | 0   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $b$     | 20  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $c$     | 20  | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $d$     | 20  | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $e$     | 25  | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $f$     | 25  | 25 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $g$     | 25  | 25 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $h$     | 30  | 25 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $i$     | 35  | 30 | 25 | 25 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $j$     | 35  | 35 | 30 | 30 | 25 | 25 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $k$     | 35  | 35 | 35 | 30 | 30 | 25 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |    |
| $l$     | 40  | 35 | 35 | 30 | 30 | 25 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |    |
| $m$     | 40  | 40 | 35 | 35 | 30 | 30 | 25 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |    |
| $n$     | 40  | 40 | 40 | 35 | 35 | 30 | 30 | 25 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |
| $o$     | 45  | 40 | 40 | 35 | 35 | 30 | 30 | 25 | 20 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |    |
| $p$     | 45  | 45 | 40 | 35 | 35 | 30 | 30 | 25 | 25 | 20 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |    |
| $q$     | 45  | 45 | 40 | 40 | 35 | 35 | 30 | 25 | 25 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |    |
| $r$     | 50  | 45 | 45 | 40 | 40 | 35 | 35 | 30 | 25 | 25 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |    |    |
| $s$     | 50  | 45 | 45 | 40 | 40 | 35 | 35 | 30 | 25 | 25 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |    |    |    |    |    |
| $t$     | 50  | 50 | 45 | 45 | 40 | 40 | 40 | 35 | 35 | 30 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 20 | 20 | 20 | 20 | 20 |
| $u$     | 55  | 50 | 50 | 45 | 45 | 45 | 40 | 35 | 35 | 30 | 30 | 25 | 25 | 25 | 25 | 25 | 25 | 20 | 20 | 20 | 20 | 20 |
| $v$     | 55  | 55 | 50 | 50 | 45 | 45 | 45 | 40 | 35 | 35 | 30 | 30 | 25 | 25 | 25 | 25 | 25 | 20 | 20 | 20 | 20 | 20 |
After we have GORs of each station and consult the transportation fee listed in, we can than calculate the total income of each train by summation all incomes of each 22 stations. The ticket price of Bannan Line is shown on Table 3.

4. Results

The results of five different departure frequencies, i.e., 1, 2, 3, 10 and 20 trains each hour are shown in Tables 4 to 8. It is obvious that the income of each train can be calculated by the equations (5) and (6) below.

\[ I = \sum_{i=1}^{21} (\text{Inc}) \]  \hspace{1cm} (5)

\[ \text{Inc} = \sum_{i=1}^{\text{Num}} \text{Pri} \]  \hspace{1cm} (6)

I: Total income of the train,
Inc: Income of each station,
Num: Number of passengers who board the train at on a specific station,
Pri: Ticket price that of each passenger needs to pay.

It is obvious from Table 4 where the departure frequency is 1 train/hr that some passengers cannot board the train at stations f, j, k, n, o, and q for the reason that the train’s upper loading limit is 1,936 passengers (highlighted with yellow background). Therefore, the total income is NT$584,972 which is less than others. However, if we double the frequency, it can be seen from Table 5 that only if the passengers board the train at stations other than station k, then the total income will increase to NT$677,840 (338920×2). If the frequency increase three times, as the original Table 6 shows, the passengers of every station can get on the train and the total income increases to NT$691,308 (230436×3). Even if everyone can board the train, it will likely be crowded, especially at stations j and k. For the sake of passenger comfort level, we increase the frequency ten times from the original. It is very clear that even if the income is NT$691,330 (69133×10) which maintains the same income as Table 7, the comfort level is increased along with the willingness of passengers to take the line. Table 8 shows the result of 20 trains/hr with the income NT$681,040 (34052×20).
Table 4 Passengers of each station and total income of each train with departure frequency 1 train/hr where the alphabet corresponds to Table 1.

| Station | Passengers | Ticket Price | Total Income |
|---------|------------|--------------|--------------|
| a       | 65         | 20           | 1230         |
| b       | 107        | 20           | 2130         |
| c       | 223        | 25           | 2525         |
| d       | 791        | 25           | 2025         |
| e       | 1223       | 25           | 3057         |
| f       | 2807       | 30           | 8421         |
| g       | 2232       | 30           | 6696         |
| h       | 1308       | 30           | 3924         |
| i       | 7521       | 30           | 22563        |
| j       | 2633       | 30           | 7900         |
| k       | 2633       | 30           | 7900         |
| l       | 2989       | 30           | 8967         |
| m       | 5176       | 30           | 15528        |
| n       | 5176       | 30           | 15528        |
| o       | 5176       | 30           | 15528        |
| p       | 4832       | 30           | 14502        |
| q       | 4550       | 30           | 13650        |
| r       | 29         | 30           | 870          |
| s       | 193        | 30           | 5790         |
| t       | 300        | 30           | 9000         |
| u       | 300        | 30           | 9000         |
| v       | 300        | 30           | 9000         |

Note: 1936 is the upper limit of passengers each train.
Table 5 Passengers of each station and total income of each train with departure frequency 2 trains/hr where the alphabet corresponds to Table 1.

|      | a  | b  | c  | d  | e  | f  | g  | h  | i  | j  | k  | l  | m  | n  | o  | p  | q  | r  | s  | t  | u  | v  |
|------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| GOR  | 0  | 0.2| 1.3| 1.2| 3.1| 4.2| 5.8| 4  | 3.7| 9.2| 11.1| 3.4| 3.4| 7.2| 7.5| 4.4| 9.1| 4.7| 0  | 3.5| 2.6|
| Ticket Price | 20 | 20 | 20 | 25 | 25 | 25 | 30 | 35 | 35 | 35 | 40 | 40 | 40 | 45 | 45 | 45 | 50 | 50 | 50 | 55 | 55 |
| Passengers | 810| 205| 465| 510| 630| 1020| 930| 505| 615| 1770| 1936| 955| 890| 1030| 1100| 500| 1200| 285| 165| 105| 60 |
| a  | 32 | 211| 194| 628| 851| 1175| 972| 1049| 2608| 3147| 1102| 1102| 2333| 2734| 1604| 3317| 1904| 41 | 1418| 1158|
| b  | 53 | 49 | 159| 215| 297| 246| 265| 660| 796| 279| 279| 590| 692| 406| 839| 482| 10 | 359| 293 |
| c  | 112| 360| 448| 674| 558| 602| 1407| 1807| 632| 632| 1339| 1569| 921| 1094| 1093| 23 | 814 | 665 |
| d  | 395| 536| 740| 612| 660| 1642| 1981| 694| 694| 1469| 1721| 1010| 2088| 1199| 26 | 893 | 729 |
| e  | 662| 914| 756| 816| 2029| 2448| 857| 857| 1814| 2126| 1247| 2580| 1481| 32 | 1103| 901 |
| f  | 1479| 1224| 1321| 3284| 3963| 1387| 1387| 2938| 3443| 2020| 4177| 2397| 5 | 1785| 1459 |
| g  | 1116| 204| 2955| 3613| 1265| 1265| 2678| 5139| 1841| 3808| 2186| 47 | 1628| 1330 |
| h  | 654| 1626| 1962| 687| 687| 1454| 1704| 1000| 2068| 1187| 25 | 884 | 722 |
| i  | 1980| 2389| 836| 836| 1771| 2076| 1278| 2518| 1445| 51 | 1076| 879 |
| j  | 1876| 2407| 2407| 5908| 5974| 3505| 7248| 4160| 89 | 8308| 2531 |
| k  | 2623| 2623| 5576| 6534| 3833| 7928| 4559| 57 | 3388| 2768 |
| l  | 1295| 2759| 3223| 1891| 3911| 2244| 48 | 1671| 1366 |
| m  | 2563| 3004| 1762| 3645| 2092| 45 | 1558| 1273 |
| n  | 3476| 2039| 4218| 2421| 52 | 1803| 1473 |
| o  | 2178| 4505| 2585| 55 | 1025| 1573 |
| p  | 2146| 933| 20 | 693 | 566 |
| q  | 2867| 61 | 2135| 1745 |
| r  | 14 | 499 | 408 | 0 |
| s  | 193 | 157 | 0 |
| t  | 150 | 0 |
| u  | 0 |
| v  | 0 |

Note: 1936 is the upper limit of passengers each train.

Total Income: 333920
Table 6 Passengers of each station and total income of each train with departure frequency 3 trains/hr where the alphabet corresponds to Table 1.

| Passengers | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $j$ | $k$ | $l$ | $m$ | $n$ | $o$ | $p$ | $q$ | $r$ | $s$ | $t$ | $u$ |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| GOR        | 0   | 0.2 | 1.3 | 1.2 | 3.1 | 4.2 | 5.8 | 4   | 3.7 | 9.2 | 11.1 | 3.4 | 3.4 | 7.2 | 7.5 | 4.4 | 9.1 | 4.7 | 0.1 | 3.5 | 2.6 | 0   |
| Ticket Prix | 0   | 20  | 20  | 25  | 25  | 25  | 30  | 35  | 35  | 35  | 40  | 40  | 40  | 40  | 40  | 40  | 45  | 45  | 45  | 50  | 50  | 55  |
| $a$         | 22  | 140 | 129 | 417 | 505 | 780 | 646 | 697 | 1732| 2090| 732  | 732 | 1509| 1816| 1065| 2203| 1294| 27  | 942 | 769 | 0   |
| $b$         | 35  | 33  | 105 | 143 | 197 | 163 | 176 | 438 | 528 | 185 | 185  | 392 | 459 | 269 | 557 | 320 | 7   | 238 | 194 | 0   |
| $c$         | 76  | 246 | 334 | 461 | 382 | 412 | 1024| 1235| 432  | 432 | 916  | 1073| 630 | 1302| 747 | 16  | 557 | 455 | 0   |
| $d$         | 260 | 353 | 487 | 403 | 435 | 1082| 1305| 457 | 457  | 968 | 1134| 665 | 1376| 790 | 17  | 588 | 480 | 0   |
| $e$         | 442 | 610 | 505 | 545 | 1356| 1636| 573 | 573 | 1212 | 1421| 834 | 1724| 989 | 21  | 737 | 602 | 0   |
| $f$         | 986 | 816 | 881 | 2190| 2642| 925 | 925 | 958 | 2295| 1346| 2785| 1598| 34  | 1190| 972 | 0   |
| $g$         | 754 | 813 | 2022| 2440| 854 | 854 | 1809| 2120| 1243| 2572| 1476| 31  | 1099| 898 | 0   |
| $h$         | 448 | 1114| 1344| 471 | 471 | 996 | 1168| 685 | 1417| 813 | 17   | 606 | 495 | 0   |
| $i$         | 1317| 1589| 556 | 556 | 1178| 1380| 810 | 1675| 961  | 20  | 716  | 585 | 0   |
| $j$         | 4619| 1617| 1617| 3424| 4013| 2354| 4869| 2794| 59   | 2081| 1700 | 0   |
| $k$         | 1939| 1939| 4107| 4813| 2823| 5839| 3551| 71  | 2966| 2039 | 0   |
| $l$         | 866 | 1835| 2150| 1261| 2609| 1497| 32  | 1115| 911  | 0   |
| $m$         | 1714| 2068| 1178| 2437| 1398| 30  | 1041| 851  | 0   |
| $n$         | 2325| 1364| 2821| 1619| 34  | 1206| 985  | 0   |
| $o$         | 1445| 2989| 1716| 37  | 1278| 1044| 0   |
| $p$         | 1622| 931  | 20  | 693 | 566  | 0   |
| $q$         | 1915| 4142 | 1165| 0   |
| $r$         | 10  | 348  | 285  | 0 |
| $s$         | 193  | 157  | 79  |
| $t$         | 79  |
| $u$         | 0   | 0   |
Table 7 Passengers of each station and total income of each train with departure frequency 10 trains/hr where the alphabet corresponds to Table 1.

| 10 trains/hr, passengers and income | Rev |  |
|---|---|---|
| a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v |
| GOR | 0 | 0.2 | 1.8 | 1.2 | 3.1 | 4.2 | 5.8 | 4 | 3.7 | 9.2 | 11 | 3.4 | 3.4 | 7.2 | 7.5 | 4.4 | 9.1 | 5 | 0 | 3.5 | 2.6 | 0 |
| Ticket Price | 0 | 20 | 20 | 20 | 25 | 25 | 30 | 35 | 35 | 40 | 40 | 45 | 45 | 50 | 50 | 50 | 55 | 55 |
| Passengers | 162 | 41 | 93 | 102 | 126 | 204 | 186 | 101 | 173 | 354 | 477 | 191 | 178 | 706 | 720 | 118 | 244 | 57 | 53 | 21 | 17 | 0 |
| a | 6 | 42 | 39 | 126 | 170 | 235 | 194 | 210 | 522 | 629 | 220 | 220 | 467 | 547 | 321 | 663 | 381 | 8 | 284 | 232 | 0 | 0 |
| b | 11 | 10 | 32 | 43 | 59 | 49 | 53 | 132 | 159 | 56 | 56 | 118 | 138 | 81 | 168 | 96 | 2 | 72 | 59 | 0 | 0 |
| c | 22 | 72 | 98 | 135 | 112 | 120 | 299 | 361 | 126 | 126 | 268 | 314 | 184 | 381 | 219 | 5 | 163 | 133 | 0 | 0 |
| d | 79 | 107 | 148 | 122 | 132 | 328 | 396 | 139 | 139 | 294 | 344 | 202 | 418 | 240 | 5 | 179 | 146 | 0 | 0 |
| e | 132 | 183 | 151 | 163 | 406 | 490 | 171 | 171 | 363 | 428 | 249 | 516 | 516 | 622 | 180 | 0 | 0 |
| f | 296 | 245 | 264 | 657 | 793 | 277 | 277 | 588 | 689 | 404 | 835 | 479 | 10 | 357 | 292 | 0 | 0 |
| g | 223 | 241 | 599 | 723 | 253 | 253 | 536 | 628 | 368 | 762 | 437 | 9 | 526 | 266 | 0 | 0 |
| h | 131 | 325 | 392 | 137 | 137 | 291 | 341 | 200 | 414 | 237 | 5 | 177 | 144 | 0 | 0 |
| i | 396 | 478 | 167 | 167 | 354 | 415 | 244 | 504 | 289 | 6 | 215 | 176 | 0 | 0 |
| j | 1375 | 483 | 481 | 1020 | 1195 | 701 | 1450 | 832 | 18 | 620 | 506 | 0 | 0 |
| k | 581 | 581 | 1230 | 1441 | 845 | 1749 | 1003 | 21 | 747 | 611 | 0 | 0 |
| l | 260 | 550 | 645 | 378 | 782 | 449 | 10 | 334 | 273 | 0 | 0 |
| m | 513 | 601 | 352 | 729 | 418 | 9 | 312 | 312 | 0 | 0 |
| n | 695 | 408 | 844 | 484 | 10 | 361 | 295 | 0 | 0 |
| o | 436 | 901 | 517 | 11 | 385 | 315 | 0 | 0 |
| p | 483 | 277 | 6 | 207 | 169 | 0 | 0 |
| q | 573 | 12 | 427 | 349 | 0 | 0 |
| r | 3 | 100 | 82 | 0 | 0 |
| s | 58 | 47 | 0 | 0 |
| t | 30 | 0 | 0 |
| u | 0 | 0 |
| v | 0 | 0 |

Total Income= 69133
Table 8 Passengers of each station and total income of each train with departure frequency 20 trains/hr where the alphabet corresponds to Table 1.

| Passengers and Income | a   | b   | c   | d   | e   | f   | g   | h   | i   | j   | k   | l   | m   | n   | o   | p   | q   | r   | s   | t   | u   | v   |
|-----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| GOR                   | 0   | 0   | 1   | 1   | 3   | 4   | 2   | 5   | 8   | 4   | 3.7 | 9.2 | 11.1| 3.4 | 3.4 | 7.2 | 7.5 | 4.4 | 9.1 | 5   | 0.1 | 3.5 | 3   | 0   |
| Ticket Price          | 20  | 20  | 20  | 25  | 25  | 25  | 30  | 35  | 35  | 35  | 40  | 40  | 40  | 45  | 45  | 45  | 50  | 50  | 50  | 55  | 55  | 55  |
| Total Income          | 80  | 81  | 46  | 61  | 102 | 63  | 93  | 50  | 61  | 177 | 213 | 95  | 89  | 103 | 110 | 110 | 110 | 123 | 123 | 160 | 332 | 190 | 142 | 116 |
| Rev                   | 2758| 680 | 1552| 1709| 2062| 3231| 2811| 1452| 1692| 4339| 4394| 1831| 1594| 1548| 1282| 281 | 681 | 90  | 51  | 14  | 0   | 0   |

Total Income = 34052
It can be seen from Fig. 1 that the total income almost the same after the frequency increases up to 3 trains/hr and there is a maximum income value at the frequency of 10 trains/hr.

Figs 2-6 show the distribution and statistical data of 30,000 iterations of each different frequency. Each of them is normal standard distribution. Table 9 gives statistical data of the different departure frequency of each train.

According to the results shown above, it is wise to choose the departure frequency 10 trains/hr. The reason why we did not pick up 20 trains/hr is that there is still an electricity cost of NTD$50,000 for each train. After consulting
the departure time table for the Bannan Line we found that the departure frequency is 11 trains/hr which matches the results of our research.
Figure 5 Distribution of 30,000 iterations of departing frequency 10 trains/hr.

Figure 6 Distribution of 30,000 iterations of departing frequency 20 trains/hr.
Table 9: Statistical data of different departure frequencies.

| Statistical Data | 1     | 2     | 3     | 4     | 5     |
|------------------|-------|-------|-------|-------|-------|
| departure frequency | 584972 | 338923 | 230435 | 69134 | 34052 |
| mean             | 0.13  | 0.129 | 0.13  | 0.13  | 0.13  |
| standard error   | 584972 | 338923 | 230435 | 69134 | 34052 |
| median           | 584977 | 338926 | 230434 | 69133 | 34017 |
| mode             | 584977 | 338926 | 230434 | 69133 | 34017 |
| standard deviation | 22.58  | 22.32  | 22    | 22.51 | 22    |
| variance         | 509.8  | 498.34 | 500   | 507   | 496   |
| kurtosis         | 0.018  | 0.0411 | -0.02 | 0.05  | 0.001 |
| skewness         | -0.015 | -0.013 | -0.01 | -0.02 | -0.011|
| range            | 183    | 193    | 173   | 197   | 197   |
| minimum          | 584880 | 338828 | 230347 | 69026 | 33941 |
| maximum          | 585063 | 339021 | 230520 | 69223 | 34138 |
| numbers of iteration | 30000  | 30000  | 30000 | 30000 | 30000 |

5. Conclusions

This research focuses on the MRT of the Bannan line, which is the most popular line of the 5 lines of the MRT system. Based on the total entrance of passengers into each station during the time of 18:00~19:00 on one weekday, we set up a procedure to obtain the optimization of departure frequency of trains by using Monte Carlo Simulation for 60,000 iterations. The frequency is 10 trains/hr which is almost the same as the real situation, i.e., 11 trains/hr. We can apply this research to other lines of the MRT and even other cities. However, there are still a few points that need to be discussed.

a. The given data varies according to the number assigned to the variable n. It is just an assumption for the specific number of each station n to calculate the number of passenger of different journeys. The results will be varied according to the different ways this number.

b. The number of passenger alighting the train is another assumption in this research. It is based on the GORn number which is the alighting rate calculated from the number of the departing passengers from each station. The results will be different if this assumption is changed.

c. Cost is an important factor of the total profit of the Bannan Line. We made no mention of any cost of this research owing to the reason that the comfort level is another factor of passengers’ willingness to take the MRT.
6. References

[1] Lo, S. C., & Chang, W. J. (2012). Design of real-time fuzzy bus holding system for the mass rapid transit transfer system. Expert Systems with Applications, 39, 1718-1724.

[2] Caprara, A., Kroon, L., Monaci, M., Peeters, M., Toth, P. (2006). Passenger Railway Optimization. ARRIVAL-TR-0035.

[3] Ke, B. R., Lin, C. L., & Lai, C. W. (2011). Optimization of train-speed trajectory and control for mass rapid transit systems. Control engineering Practice, 19, 675-687.

[4] Kahneman, D., Slovic, P., & Tversky, A. (1982). Judgment under Uncertainty: Heuristics and Biases, Cambridge University Press.

[5] Schumacher, E. H., Seymour, T. L., Glass, J. M., Fencik, D. E., Lauber, E. J., Kieras, D. E., & Meyer, D. E. (2001). Virtually Perfect Time Shearing in Dual-Task Performance: Unlocking the Central Cognitive Bottleneck. Psychological Science, 12, 101-108.

[6] Rahman, A., Hosokawa, S., Oono, Y., Amakawa, T. Goto, N., & Tsurumi, S. (2002). Auxin and Ethylene Response Interactions during Arabidopsis Root Hair Development Dissected by Auxin Influx Modulators. Plant Physiology, 130, 1-10.

[7] Knaff, J. A., Cram, T. A., Schumacher, A. B., Kossin, J. A. P., & Demaria, M. (2007). Objective Identification of Annular Hurricanes. American Meteorological Society, 23, 17-28.

[8] Gross, B. O., Pgunwuyi, F., Moshary, S., Ahmed, & Cairns, B. (2004). Aerosol Retrieval Over Urban Areas Using Spatial Regression Between V/NIR and MIR Hyperion Channels, San Diego, Cal., Proc. SPIE, vol. 5547, 111, doi:10.1117/12.560097.

[9] Young, L. J., Yeh, B. H., & Young, P. (2014). Remaining Life Assessment of Low Pressure Turbine Rotor Using Monte Carlo Simulation. America Society of Testing Materials, 42, 3, 1-9.

[10] Mark Demaria, John A. Knaff, Richard Knabb, Charles R. Sampson, Robert T. Demaria, 2009, A New Method for Estimating Tropical Cyclone Wind Speed Probabilities. American Meteorological Society, 24, 1573-1591.

[11] Vořechovský, M., (2012). Correlation Control in Small Sample Monte Carlo Type Simulation II: Analysis of Estimation Formulas, Random Correlation and Perfect Uncorrelatedness. Probabilistic Engineering Mechanics, 29, 105-120.

[12] David F., Andersen, J. A. M., Vennix, G. Richardson, P., & Rouwette, E. A. J. A. (2007). Group Model Building: Problem Structuring, Policy Simulation and Decision Support. The Journal of the Operational Research Society, 58(5), 691-694.