Exact Results in SO(11) SUSY Gauge Theories with Spinor and Vector Matter

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Abstract

We investigate the confining phase vacuum structure of supersymmetric SO(11) gauge theories with one spinor matter field and \( N_f \leq 6 \) vectors. We describe several useful tricks and tools that facilitate the analysis of these chiral models and many other theories of similar type. The forms of the \( N_f = 5 \) and \( N_f = 6 \) quantum moduli spaces are deduced by requiring that they reproduce known results for \( SU(5) \) SUSY QCD along the spinor flat direction. After adding mass terms for vector fields and integrating out heavy degrees of freedom, we also determine the dynamically generated superpotentials in the \( N_f \leq 4 \) quantum theories. We close with some remarks regarding magnetic duals to the \( N_f \geq 7 \) electric \( SO(11) \) theories.
1. Introduction

During the past few years, remarkable progress has been made in understanding non-perturbative aspects of \( \mathcal{N} = 1 \) supersymmetric gauge theories. Pioneering work in this area by Seiberg and collaborators has shed light upon such interesting strong interaction phenomena as phase transitions, confinement, and chiral symmetry breaking \[1\]. Their studies have also opened up several new directions for model building which potentially have important phenomenological applications. Supersymmetric model investigations have thus yielded valuable insights into several basic issues in quantum field theory and particle physics.

Many of the recent key advances were developed within the context of SUSY QCD which represents the prototype \( \mathcal{N} = 1 \) gauge theory \[2,3\]. Unfortunately, it has often proven difficult to extend the new ideas beyond this relatively simple model to more complicated theories. For example, finding weakly coupled duals to strongly coupled models with no simplifying tree level superpotentials remains an outstanding challenge despite significant theoretical efforts to uncover patterns among known dual pairs. Confining phase analyses are generally more tractable than those which focus upon questions related to free magnetic and nonabelian Coulomb phases in various theories. But even addressing confinement issues in models with more complicated matter contents than those like SUSY QCD with fields in only fundamental representations frequently requires one to overcome nontrivial technical problems.

In this note, we investigate the confining phase vacuum structure of a supersymmetric theory based upon an \( SO(11) \) gauge group with one spinor field and \( N_f \leq 6 \) vectors. Our motivations for studying and presenting results on this particular model are threefold. Firstly, we wish to describe a number of useful tools that greatly facilitate the analysis of this nontrivial theory’s confining phase. These tricks can be applied to the study of many other supersymmetric theories’ low energy dynamics. While most of the simple methods which we employ have been known to nonperturbative SUSY model experts for some time \[4,5\], we believe it is worthwhile to discuss these previously undocumented techniques so as to make them accessible to a larger community. Secondly, the 32-dimensional spinor irrep of \( SO(11) \) is pseudoreal. Since no mass term for it can be written down, our model is chiral. It may therefore have interesting applications for dynamical supersymmetry breaking. Finally and most importantly, understanding the confining phases of \( \mathcal{N} = 1 \) theories is invaluable in searching for duals. As we shall see, a dual to this \( SO(11) \) model
would act as a generator for several other magnetic descriptions of various electric theories. While we have not yet constructed such a dual, our present analysis restricts its possible form.

Our article is organized as follows. In section 2, we discuss the low energy description of the microscopic \( SO(11) \) model and identify gauge invariant operators which label its flat directions. We demonstrate that this theory confines when it contains \( N_f \leq 6 \) vector fields. We then proceed in section 3 to analyze the quantum moduli spaces in the \( N_f = 5 \) and \( N_f = 6 \) theories. After adding mass terms for vector fields and systematically integrating them out, we also deduce the dynamically generated superpotentials in the \( N_f \leq 4 \) quantum theories. Finally, we close in section 4 with some remarks and speculations regarding duals to the \( SO(11) \) models with \( 7 \leq N_f \leq 22 \) vector fields.

2. The \( SO(11) \) model

We begin our study of the \( SO(11) \) model by listing its full symmetry group

\[
G = SO(11)_{\text{local}} \times [SU(N_f) \times U(1)_V \times U(1)_Q \times U(1)_R]_{\text{global}},
\]

superfield matter content

\[
V^{\mu i} \sim (11; N_f; 1, 0, 0)
\]

\[
Q^A \sim (32; 1; 0, 1, 0) \tag{2.2}
\]

\[
\Lambda^{b_0} \sim (1; 1; 2N_f; 8, 10 - 2N_f)
\]

and one-loop Wilsonian beta function coefficient

\[
b_0 = \frac{1}{2}[3K(\text{Adj}) - \sum_{\text{matter reps } \rho} K(\rho)] = 23 - N_f. \tag{2.3}
\]

In the absence of any tree-level superpotential, the classical theory remains invariant under an arbitrary \( G \) transformation. But in the quantum theory, each of the \( U(1) \) factors in eqn. (2.1) is anomalous. The theta parameter in the \( SO(11) \) Lagrangian undergoes a shift

\[
\theta_{SO(11)} \rightarrow \theta_{SO(11)} + C\alpha \tag{2.4}
\]

\footnote{We adopt the \( SO(11) \) irrep index values \( K(11) = 2, K(55) = 18 \) and \( K(32) = 8. \)}
when an anomalous $U(1)$ rotation through angle $\alpha$ is performed. As a result, the spurion field

$$\left(\frac{\Lambda}{\mu}\right)^{b_0} \equiv \exp\left[\frac{-8\pi^2}{g(\mu)^2} + i \theta_{SO(11)}\right]$$

(2.5)

acquires a $U(1)$ charge equal to the anomaly coefficient $C$. The charge assignments for $\Lambda^{b_0}$ in (2.2) therefore simply equal the group theory coefficients of the $SO(11)^2U(1)_V$, $SO(11)^2U(1)_Q$ and $SO(11)^2U(1)_R$ anomalies [6].

$G$ invariance restricts the possible form of any dynamically generated superpotential $W_{\text{dyn}}$ which can arise within a low energy description of the $SO(11)$ model. For example, $W_{\text{dyn}}$’s dependence upon $\Lambda^{b_0}$ is completely fixed since it is the only field that carries nonvanishing R-charge. The net numbers of spinor and vector fields appearing within the nonperturbative superpotential are also determined by $U(1)_V$ and $U(1)_Q$ invariance. We thus easily find that $W_{\text{dyn}}$ must assume the schematic forms listed in Table 1 as a function of $N_f$. The results displayed in the table suggest that the $N_f = 5$ model is analogous to $N_f = N_c$ SUSY QCD inasmuch as the R-charge assignment for $\Lambda^{b_0}$ vanishes in this case. As a result, no superpotential may be dynamically generated. But nonperturbative effects can alter the Kähler potential and quantum mechanically constrain the matter fields. The form such a constraint would have to take multiplied by a Lagrange multiplier field $X$ is shown in Table 1. The $N_f = 6$ $SO(11)$ model is similarly analogous to $N_f = N_c + 1$ SUSY QCD.
Table 1: Schematic forms for dynamically generated superpotentials

| $N_f$ | $R(\Lambda^{b_0})$ | $W_{\text{dyn}}$ |
|-------|------------------|------------------|
| 0     | 10               | $[\Lambda^{23}/Q^8]^{\frac{1}{5}}$ |
| 1     | 8                | $[\Lambda^{22}/V^2 Q^8]^{\frac{1}{4}}$ |
| 2     | 6                | $[\Lambda^{21}/V^4 Q^8]^{\frac{1}{3}}$ |
| 3     | 4                | $[\Lambda^{20}/V^6 Q^8]^{\frac{1}{2}}$ |
| 4     | 2                | $\Lambda^{19}/V^8 Q^8$ |
| 5     | 0                | $X[V^{10}Q^8 - \Lambda^{18}]$ |
| 6     | -2               | $V^{12}Q^8/\Lambda^{17}$ |

We next need to find $SO(11)$ invariant combinations of vectors and spinors that act as moduli space coordinates in the low energy effective theory. Equivalently, we need to determine the D-flat directions of the scalar potential in the microscopic theory. Identifying independent classical solutions to D-flatness conditions is generally a difficult task. However, we can avoid this complicated group theory exercise if we know instead the gauge symmetry breaking pattern realized at generic points in moduli space. The solution to this latter mathematical problem was worked out years ago in ref. [7] for a large class of theories including our particular $SO(11)$ model:

$$SO(11) \xrightarrow{32} SU(5) \xrightarrow{11} SU(4) \xrightarrow{11} SU(3) \xrightarrow{11} SU(2) \xrightarrow{11} 1.$$  \hspace{1cm} (2.6)

Given this information, it is straightforward to count the number of $SO(11)$ singlet operators which enter into the low energy effective theory. In Table 2, we display the number of parton level matter degrees of freedom as well as the generic unbroken color subgroup as a function of $N_f$. We also list the number of chiral superfields eaten by the superHiggs mechanism. The number of independent color-singlet hadrons in the low energy theory...
then simply equals the difference between the initial and eaten matter field degrees of freedom.

| $N_f$ | Parton DOF | Unbroken Subgroup | Eaten DOF | Hadrons |
|-------|------------|--------------------|-----------|---------|
| 0     | 32         | $SU(5)$            | $55 - 24 = 31$ | 1       |
| 1     | 43         | $SU(4)$            | $55 - 15 = 40$ | 3       |
| 2     | 54         | $SU(3)$            | $55 - 8 = 47$ | 7       |
| 3     | 65         | $SU(2)$            | $55 - 3 = 52$ | 13      |
| 4     | 76         | 1                  | 55        | 21      |
| 5     | 87         | 1                  | 55        | 32      |
| 6     | 98         | 1                  | 55        | 43      |

Table 2: Number of independent hadron operators

In order to figure out how to explicitly combine vector and spinor partons into gauge invariant hadrons, it is useful to recall some basic elements of $SO(11)$ group theory [8–10]. The tensor product of two 32-dimensional spinor fields decomposes into irreducible $SO(11)$ representations as follows:

$$2^5 \times 2^5 = [0]_A + [1]_S + [2]_S + [3]_A + [4]_A + [5]_S.$$  \hspace{1cm} (2.7)

Here $[n]$ denotes a tensor irrep with $n$ antisymmetric vector indices, and its “S” or “A” subscript indicates symmetry or antisymmetry under spinor field exchange. Since our model contains just one spinor flavor, all hadrons can only involve spinor products belonging to the symmetric $^{(11)}_{11} = 11$, $^{(11)}_{12} = 55$ or $^{(11)}_{55} = 462$ dimensional irreps. We contract vector
fields into these spinor combinations using the $SO(11)$ Gamma matrices

\begin{align*}
\Gamma_1 &= \sigma_2 \times \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3 \\
\Gamma_2 &= -\sigma_1 \times \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3 \\
\Gamma_3 &= 1 \times \sigma_2 \times \sigma_3 \times \sigma_3 \times \sigma_3 \\
\Gamma_4 &= -1 \times \sigma_1 \times \sigma_3 \times \sigma_3 \times \sigma_3 \\
\Gamma_5 &= 1 \times 1 \times \sigma_2 \times \sigma_3 \times \sigma_3 \\
\Gamma_6 &= -1 \times 1 \times \sigma_1 \times \sigma_3 \times \sigma_3 \\
\Gamma_7 &= 1 \times 1 \times 1 \times \sigma_2 \times \sigma_3 \\
\Gamma_8 &= -1 \times 1 \times 1 \times \sigma_1 \times \sigma_3 \\
\Gamma_9 &= 1 \times 1 \times 1 \times 1 \times \sigma_2 \\
\Gamma_{10} &= -1 \times 1 \times 1 \times 1 \times \sigma_1 \\
\Gamma_{11} &= \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3
\end{align*}

(2.8)

and charge conjugation matrix $C = \sigma_2 \times \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3$. We thus form the hadrons

\begin{align*}
L &= (Q^T \Gamma_\mu CQ) (Q^T \Gamma_\mu CQ) \\
M^{ij} &= (V^T)_i \nu^{\mu j} \\
N^i &= Q^T \nu^i CQ \\
O^{[ij]} &= \frac{1}{2!} Q^T \nu^{[i} \nu^{j]} CQ \\
P^{ijklm} &= \frac{1}{5!} Q^T \nu^{[i} \nu^{j} \nu^{k} \nu^{l} \nu^{m]} CQ \\
R^{ijklm} &= \frac{1}{5!} \epsilon^{\mu_1 \cdots \mu_{11}} (Q^T \Gamma_{\mu_1} \Gamma_{\mu_2} \Gamma_{\mu_3} \Gamma_{\mu_4} \Gamma_{\mu_5} CQ) (Q^T \Gamma_{\mu_6} CQ) V^{[i} \nu^{j} \nu^{k} \nu^{l} \nu^{m]} \nu^{\mu_7} \nu^{\mu_8} \nu^{\mu_9} \nu^{\mu_{10}} CQ \\
T^{ijklmn} &= \frac{1}{6!} Q^T \nu^{[i} \nu^{j} \nu^{k} \nu^{l} \nu^{m} \nu^{n]} CQ.
\end{align*}

(2.9)

where Greek and Latin letters respectively denote color and flavor indices, square brackets indicate antisymmetrization and $\nu^i = V^i_\mu \Gamma^\mu$. Expectation values of these operators act as coordinates on the microscopic $SO(11)$ theory’s moduli space of independent flat directions.

We can now check that the composite fields in eqn. (2.9) account for all independent hadronic degrees of freedom within the $SO(11)$ model’s confining phase. In Table 3, we list the number of nonvanishing hadrons as a function of flavor number. For $N_f \leq 4$, the $L$, $M$, $N$ and $O$ degrees of freedom sum up to the number of color-singlet composites entering into the low energy effective theory which we previously found in Table 2. This counting works in large part due to the antisymmetric flavor structure of the various hadrons in (2.9). When $N_f = 5$, the number of nonvanishing hadronic degrees of freedom exceeds the required number of composites by one. So a single constraint must exist among $L$, $M$, $N$, $O$, $P$ and $R$. This conclusion is consistent with our earlier finding that a relation among these fields is compatible with $R$-charge considerations in the $N_f = 5$ quantum theory.
Similarly, a larger number of independent constraints must exist in the $N_f = 6$ case in order for the simple counting arguments to hold. By analogy with $N_f = N_c + 1$ SUSY QCD, we expect these classical relations to persist in the quantum theory.

| $N_f$ | Hadrons | $L$ | $M$ | $N$ | $O$ | $P$ | $R$ | $T$ | constraints |
|-------|---------|-----|-----|-----|-----|-----|-----|-----|-------------|
| 0     | 1       | 1   |     |     |     |     |     |     |             |
| 1     | 3       | 1   | 1   | 1   |     |     |     |     |             |
| 2     | 7       | 1   | 3   | 2   | 1   |     |     |     |             |
| 3     | 13      | 1   | 6   | 3   | 3   |     |     |     |             |
| 4     | 21      | 1   | 10  | 4   | 6   |     |     |     |             |
| 5     | 32      | 1   | 15  | 5   | 10  | 1   | 1   | -1  |             |
| 6     | 43      | 1   | 21  | 6   | 15  | 6   | 6   | 1   | -13         |

Table 3: Hadron degree of freedom count

The simple tools which we have so far utilized to analyze the $SO(11)$ theory restrict the possible matter content of its low energy description. But since these heuristic methods are clearly not rigorous, we need further cross checks on our conclusions regarding the $SO(11)$ model’s vacuum structure. We therefore examine massless parton and hadron contributions to global ’t Hooft anomalies. To begin, we abandon the anomalous global symmetry in eqn. (2.1) and work instead with the nonanomalous group

$$G_{\text{new}} = SO(11)_{\text{local}} \times [SU(N_f) \times U(1)_Y \times U(1)_R]_{\text{global}}.$$  \hspace{1cm} (2.10)

The generators of the new hypercharge and R-charge abelian factors are linear combinations of the three old $U(1)$ generators. After assigning the partonic matter fields the nonanomalous charges

$$V^{\mu i} \sim (11; N_f; -4, 1)$$

$$Q^A \sim (32; 1; N_f, -5/4),$$  \hspace{1cm} (2.11)
we can readily compute the quantum numbers under $G_{\text{new}}$ of all the composite operators in eqn. (2.9).

We next compare the $SU(N_f)^3$, $SU(N_f)^2U(1)_Y$, $SU(N_f)^2U(1)_R$, $U(1)_Y$, $U(1)^3_Y$, $U(1)_R$, $U(1)^3_R$, $U(1)^2_U(1)_R$, and $U(1)^2_R U(1)_Y$ global anomalies at the parton and hadron levels as a function of $N_f$. We find they precisely match when $N_f = 6$. \(^2\) This nontrivial agreement is consistent with our expectation that the $SO(11)$ model with six vectors and one spinor confines at the origin of moduli space like $N_f = N_c + 1$ SUSY QCD. It strongly suggests that the low energy effective theory contains only the composite fields in (2.9) and no additional colored or colorless massless degrees of freedom. \(^3\) We further observe that all global anomalies match when $N_f = 5$ provided we include a field $X \sim (1; 1; 0, 2)$ into the low energy spectrum. In the quantum theory, $X$ is naturally interpreted as a Lagrange multiplier which enforces a single constraint. Agreement between parton and hadron level anomalies occurs in other similar constrained $SO(N_c)$ theories so long as Lagrange multiplier contributions are taken into account. It is also important to note that global anomalies do not match when $N_f \geq 7$. The disagreement cannot be eliminated via inclusion of additional color-singlet composites into the low energy theory beyond those already listed in (2.9). So as in $N_f = N_c + 2$ SUSY QCD \(^4\), we interpret the anomaly mismatch as signaling the end of the $SO(11)$ model’s confining regime and the beginning of a new dual phase.

3. **Low energy superpotentials**

Having established an overall picture of the $SO(11)$ theory’s confining phase, we are now ready to investigate its low energy structure in detail. We seek to determine how non-perturbative effects in the quantum theory modify the classical moduli space of degenerate vacua. One way to proceed is by starting with the $N_f = 0$ model and postulating that a superpotential consistent with the requirements of Table 1 is dynamically generated. We can then try to systematically “integrate in” vector flavors and construct superpotentials in the effective theories corresponding to larger values of $N_f$ \(^3\). \(^6\). This bottom-up approach

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\(^2\) This anomaly agreement has recently also been noted in ref. [14].

\(^3\) Anomaly matching does not shed any light upon the presence or absence of hadrons in the effective theory that transform as singlets under the total global symmetry group. But as we shall see in the following section, such fields do not enter into the low energy $N_f = 6$ superpotential. Counting arguments then rule out the existence of such unconstrained singlets.
unfortunately becomes intractable for $N_f \geq 2$. Alternatively, we can follow a top-down procedure in which we first deduce the form of the nonperturbative superpotential for $N_f = 6$ flavors. Once $W_{\text{dyn}}$ is known in this case, it is straightforward to methodically integrate out vector flavors and uncover the vacuum structure of the $SO(11)$ model for smaller values of $N_f$. We will adopt this latter approach.

Actually, it is technically easier to first determine the quantum constraint in the $N_f = 5$ theory. Recall that $SO(11)$ breaks down to $SU(5)$ when the spinor field develops a nonvanishing vacuum expectation value. Along the spinor flat direction, the $SO(11)$ constraint must reduce to the well-known $N_f = N_c = 5$ relation $\det m - b\mathbf{F} = \Lambda_{SU(5)}^{10}$. This requirement fixes the exact form of the $SO(11)$ constraint.

In order to embed $SU(5)$ inside $SO(11)$, we first choose a set of 24 fundamental irrep $SU(5)$ generators $t_a$ whose Cartan subalgebra members look like

\begin{align}
  t_3 &= \frac{1}{\sqrt{2}} \text{Diag}(1, -1, 0, 0, 0) \\
  t_8 &= \frac{1}{\sqrt{6}} \text{Diag}(1, 1, -2, 0, 0) \\
  t_{15} &= \frac{1}{\sqrt{12}} \text{Diag}(1, 1, 1, -3, 0) \\
  t_{24} &= \frac{1}{\sqrt{20}} \text{Diag}(1, 1, 1, 1, -4).
\end{align}

We next form annihilation and creation operators

\begin{align}
  A_j &= \frac{1}{2} (\Gamma_{2j-1} - i\Gamma_{2j}) \\
  A_j^\dagger &= \frac{1}{2} (\Gamma_{2j-1} + i\Gamma_{2j})
\end{align}

which satisfy the anticommutation relations $\{A_j, A_k\} = \{A_j^\dagger, A_k^\dagger\} = 0$ and $\{A_j, A_k^\dagger\} = \delta_{jk}$. The $32 \times 32$ matrices $T_a = A_j^\dagger (t_a)_{jk} A_k$ then generate the $SU(5)$ subgroup of $SO(11)$ in the spinor irrep $[8]$.

$SO(11)$ vectors and spinors break apart into $1 + 5 + \overline{5}$ and $1 + 1 + 5 + \overline{5} + 10 + \overline{10}$ under $SU(5)$. We can explicitly see how the vector decomposes by inverting the relationship in eqn. (3.2) between the operators $A_j$ and $A_j^\dagger$ which respectively transform as 5 and $\overline{5}$ under
$SU(5)$ and the Gamma matrices which transform as 11 under $SO(11)$:

$$V^\mu = \begin{pmatrix} q_1 + \bar{q}_1 \\ i(q_1 - \bar{q}_1) \\ q_2 + \bar{q}_2 \\ i(q_2 - \bar{q}_2) \\ q_3 + \bar{q}_3 \\ i(q_3 - \bar{q}_3) \\ q_4 + \bar{q}_4 \\ i(q_4 - \bar{q}_4) \\ q_5 + \bar{q}_5 \\ i(q_5 - \bar{q}_5) \\ \phi \end{pmatrix}. \quad (3.3)$$

Similarly, the $SU(5)$ irreps to which each of the spinor field’s 32 elements belong are readily identified by acting upon $Q$ with the four Cartan subalgebra generators:

$$Q^T = \left[ \nu^c_R, \nu_L, e^+_L, d^c_R, d_R, u_R, u_L, d^c_L, d_L, u_R, u_L, d^c_R, d_R, u_L, d^c_L, d_L, e^+_R, e^+_L, \nu_R, \nu^c_L \right]. \quad (3.4)$$

The names for the components of this row vector have been intentionally chosen to mimic familiar Standard Model and $SU(5)$ GUT nomenclature.

Equal nonvanishing expectation values for the first and last entries in the spinor field break $SO(11) \to SU(5)$ while preserving D-flatness:

$$\langle Q \rangle^T = [a, 0, 0, \cdots, 0, 0, a]. \quad (3.5)$$

This vev’s dependence upon a single parameter $a$ is consistent with the counting argument conclusion that the $SO(11)$ model has one independent spinor flat direction. Once the gauge symmetry is broken, we find that the $SO(11)$ hadrons decompose into the following combinations of $SU(5)$ mesons $m^{ij}$, baryons $b$ and $\bar{b}$, and singlets $\phi^i$: 

$$L \to -4a^4$$

$$M^{(ij)} \to 2(m^{ij} + m^{ji}) + \phi^i\phi^j$$

$$N^i \to 2ia^2\phi^i$$

$$O^{[ij]} \to -4ia^2(m^{ij} - m^{ji})$$

$$P \to -32a^2(b + \bar{b}) + 4ia^2\epsilon_{ijklm}m^{ij}m^{kl}\phi^m$$

$$R \to 64a^4(b - \bar{b}). \quad (3.6)$$
With this information in hand, we can assemble the hadrons into flavor singlet combinations and adjust their coefficients so as to recover the \( SU(5) \) relation among mesons and baryons. After a lengthy computation, we thus deduce the quantum constraint in the \( N_f = 5 \) \( SO(11) \) theory:

\[
W_{N_f=5} = X \left\{ L^2 \det M - \frac{1}{4!} \epsilon_{i_1 i_2 i_3 i_4 i_5} \epsilon_{j_1 j_2 j_3 j_4 j_5} \left[ L M^{i_1 j_1} M^{i_2 j_2} M^{i_3 j_3} M^{i_4 j_4} N^{i_5} N^{j_5} 
- 2 L M^{i_1 j_1} M^{i_2 j_2} M^{i_3 j_3} O^{i_4 j_4} O^{i_5 j_5} + 6 M^{i_1 j_1} M^{i_2 j_2} N^{i_3} N^{j_3} O^{i_4 j_4} O^{i_5 j_5} 
- M^{i_1 j_1} O^{i_2 j_2} O^{i_3 j_3} O^{i_4 j_4} O^{i_5 j_5} \right] + \frac{1}{4} \epsilon_{ijklm} N^i O^{jk} O^{lm} P + LP^2 + R^2 - A_5^{18} \right\}.
\]  

(3.7)

Working in a similar fashion, we can determine the exact superpotential in the low energy \( N_f = 6 \) sigma model. Its form is tightly restricted by requiring that it be smooth everywhere on the moduli space and that its equations of motion yield valid classical relations among spinor and vector fields. Moreover, we must recover the \( N_f = 5 \) quantum constraint after giving mass to one of the vector flavors. The unique superpotential which satisfies these criteria is displayed below:

\[
W_{N_f=6} = \frac{1}{\Lambda_6^{17}} \left\{ -L^2 \det M + \frac{1}{5!} \epsilon_{i_1 i_2 i_3 i_4 i_5 i_6} \epsilon_{j_1 j_2 j_3 j_4 j_5 j_6} \left[ L M^{i_1 j_1} M^{i_2 j_2} M^{i_3 j_3} M^{i_4 j_4} M^{i_5 j_5} N^{i_6} N^{j_6} 
- \frac{5}{2} L M^{i_1 j_1} M^{i_2 j_2} M^{i_3 j_3} M^{i_4 j_4} O^{i_5 j_5} O^{i_6 j_6} + 10 M^{i_1 j_1} M^{i_2 j_2} M^{i_3 j_3} M^{i_4 j_4} N^{i_5} N^{j_5} O^{i_6 j_6} 
- \frac{5}{2} M^{i_1 j_1} M^{i_2 j_2} O^{i_3 j_3} O^{i_4 j_4} O^{i_5 j_5} O^{i_6 j_6} \right] - R_i M^{ij} R_j - LP_i M^{ij} P_j 
+ 2 i P_i O^{ij} R_j + (N^{i} P_i)^2 + \frac{1}{4} P_i M^{ij} \epsilon_{ijklmn} N^k O^{lm} O^{no} 
+ LT^2 + 2 i N^i R_i T - 2 (\text{Pf } O) T \right\}.
\]

(3.8)

We note that all combinations of \( SO(11) \) hadrons consistent with symmetry and smoothness considerations enter into \( W_{N_f=6} \) with nonvanishing coefficients. Although this result may seem natural, other theories analogous to \( N_f = N_c + 1 \) SUSY QCD are known to have zero coefficients for some \( a \text{ priori} \) allowed terms in their superpotentials \([12,13]\). So while it is relatively easy to figure out the basic polynomial form of the numerator in (3.8) as it is in many similar models \([14]\), determining the numerical values for each term’s coefficient requires a detailed computation.

Once the ground state structures of the \( N_f = 5 \) and \( N_f = 6 \) \( SO(11) \) theories are known, it is straightforward to flow down to models with fewer vector fields. We simply add a tree level mass term \( W_{\text{tree}} = \mu_{ij} M^{ij} \) to eqn. (3.7), rotate the meson field \( M^{ij} \) into
diagonal form via a flavor transformation and then integrate out heavy vector flavors. Since the modified quantum theory with the meson mass term contains no sources which transform under the $SU(N_f)$ flavor group like $N^i$ or $O^{ij}$, the expectation values of those fields containing heavy vectors must vanish. After systematically removing heavy vector flavors, we find the following tower of dynamically generated $SO(11)$ superpotentials:

$$W_{(N_f=4)} = \frac{\Lambda_{19}^4}{L^2 M^4 - 4LM^3N^2 + 6LM^2O^2 - 12MN^2O^2 + O^4}$$

$$W_{(N_f=3)} = 2 \left[ \frac{\Lambda_{20}^3}{L^2 M^3 - 3LM^2N^2 + 3LMO^2 - 3N^2O^2} \right]^\frac{1}{2}$$

$$W_{(N_f=2)} = 3 \left[ \frac{\Lambda_{21}^2}{L^2 M^2 - 2LMN^2 + LO^2} \right]^\frac{1}{2}$$

$$W_{(N_f=1)} = 4 \left[ \frac{\Lambda_{22}^1}{L^2 M - LN^2} \right]^\frac{1}{2}$$

$$W_{(N_f=0)} = 5 \left[ \frac{\Lambda_{23}^0}{L^2} \right]^\frac{1}{2}.$$

The strong interaction scales are related by requiring gauge coupling continuity across heavy vector thresholds:

$$\Lambda_{0}^{23} = \mu_{11} \Lambda_{1}^{22} = \mu_{11} \mu_{22} \Lambda_{2}^{21} = \mu_{11} \mu_{22} \mu_{33} \Lambda_{3}^{20} = \mu_{11} \mu_{22} \mu_{33} \mu_{44} \Lambda_{4}^{19} = \mu_{11} \mu_{22} \mu_{33} \mu_{44} \mu_{55} \Lambda_{5}^{18}$$

$$(3.10)$$

As a check, one can verify that these $N_f < 5$ $SO(11)$ expressions properly reduce to their $SU(5)$ descendants

$$W_{N_f < 5} = (N_c - N_f) \left[ \frac{\Lambda_{SU(5)}^{15-N_f}}{\det m} \right]^{\frac{1}{5-N_f}}$$

$$(3.11)$$

along the spinor flat direction.

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4 All flavor indices on $M^{ij}$, $N^i$ and $O^{ij}$ in $W_{(N_f=4)}$, $W_{(N_f=3)}$ and $W_{(N_f=2)}$ are implicitly contracted with $SU(N_f)$ epsilon symbols so as to form flavor singlet combinations.
The nonperturbative superpotentials in (3.9) lift the classical vacuum degeneracy and generate runaway scalar potentials. The $N_f \leq 4$ $SO(11)$ quantum theories can be stabilized by adding tree level superpotentials that increase along all D-flat directions. If $W_{\text{tree}}$ preserves some global symmetry which is spontaneously broken in the true ground state, the chiral $SO(11)$ model should dynamically break supersymmetry [20]. This condition on $W_{\text{tree}}$ cannot be satisfied in the $N_f = 0$ theory, for all polynomial functions of the spinor field $Q$ break the global $U(1)_R$ symmetry. Witten index arguments then suggest that SUSY remains unbroken in models with additional vector fields. Nevertheless, supersymmetry may be broken via other schemes such as coupling singlets to all the hadrons in the $N_f = 5$ quantum constraint [15]. So whether SUSY can be dynamically broken in this $SO(11)$ theory remains an interesting open question.

4. Conclusion

In this paper, we have investigated the low energy behavior of $SO(11)$ models containing $N_f \leq 6$ vectors and one spinor matter field. The tricks and tools which we used to analyze this particular theory can be profitably applied to the study of many other $\mathcal{N} = 1$ models. Knowing the pattern of gauge symmetry breaking at generic points in moduli space is especially valuable. Indeed, the confining phase structure of the $SO(11)$ theory is essentially fixed by $N_c = 5$ SUSY QCD along its spinor flat direction. Other $SO(N_c)$ models of similar type are likewise restricted [16–19].

It would be highly desirable to extend our understanding of the strongly interacting $SO(11)$ model into its dual phase. Our present confining phase results provide helpful clues in the search for a weakly coupled dual. In particular, the superpotential in eqn. (3.8) must be recovered from any magnetic dual to the $SO(11)$ electric theory when the number of vector flavors is reduced down to six. Our primary incentive for explicitly calculating $W_{N_f=6}$ was in fact to determine which nonvanishing terms must be reproduced by a magnetic theory. The complex superpotential expression in (3.8) suggests the dual is not simple.

Given that the spinor flat direction played a central role in our analysis of the $SO(11)$ theory’s confining phase, we naturally want to exploit it for studying the nonabelian Coulomb phase as well. Motivated by Seiberg’s dual to $N_c = 5$ SUSY QCD [3], we speculate that the magnetic gauge group $\tilde{G}$ contains $SU(N_f - 5)$ as a subgroup. Of course,
other duals beside Seiberg’s could exist for $SU(5)$ gauge theory with $N_f \geq 7$ quark flavors. So $\tilde{G}$ need not resemble $SU(N_f - 5)$ at all.

In closing, we mention that a weakly coupled magnetic description of the electric $SO(11)$ theory would yield several other dual pairs as interesting by-products. For example after Higgsing the $SO(11)$ gauge group, one should find duals to $SO(6) \simeq SU(4)$ and $SO(5) \simeq Sp(4)$ models with $N_f = 4$ quark flavors and various numbers of antisymmetric fields. These special cases might shed light upon more general $SU(2N_c)$ and $Sp(2N_c)$ models with fundamental and antisymmetric matter. Finding weakly coupled duals to these theories with zero tree level superpotential remains an unsolved and challenging problem.

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