An algorithm for the optimal management of air-source heat pumps and PV systems

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Abstract. The progressive decentralization and decarbonisation of the energy system calls for an increased flexibility of the demand side. Model predictive control techniques can help users consume more efficiently by shifting their energy demand toward most convenient time windows. The present work proposes an algorithm to optimally shift the heating demand of a residential building equipped with an air-source heat pump and PV panels by exploiting the inertia provided by its thermal mass. Results show that the proposed look-ahead control strategy is able to achieve significant cost savings for the users without affecting the thermal comfort in the indoor environment.

1. Introduction

The power industry has experienced major changes over the last years. Traditionally, large generators were scheduled to follow a rigid energy demand, but this paradigm is shifting toward the need for flexible power consumption, due to the increasing penetration of stochastic and non-dispatchable energy sources, such as wind and solar power. These changes are also slowly affecting small consumers. For instance, several pricing schemes incentive energy consumption during hours with low system’s demand or high production from renewable energy sources. For the control and management of the future power systems, several decentralized concepts based on Multi-Agent System architectures have been proposed, pursuing either local or system-level objectives that are generally different from one another [1]. Both the thermal inertia of building structures and thermal energy storage systems offer a huge potential for shifting heating and cooling loads as a source of flexibility [2]. According to the IEA EBC Annex 67, the energy flexibility of buildings can be defined as their ability to manage its demand and generation according to local climate conditions, user needs and grid requirements [3]. According to Arteconi et al. [2], higher penetration rates of responsive buildings reduces the operational costs at system level, but on the other hand decreases the savings per participant since less load shifting per dwelling is necessary. Liu and Heiselberg [4] showed that a weather predictive control can reduce costs for space heating and cooling by 26% compared to a simple rule-based control based on setpoint changes on a NZEB. Yin et al. [5] showed that aggressive setpoint changes are not effective when the temperature difference between the indoors and the outdoors drops below a certain threshold. Pau et al. [6] proposed a smart control strategy formulated as a Mixed Integer Linear Programming (MILP) problem to reduce the peak load of a pool of heat pumps using hot water tanks as thermal buffers.
The present work proposes an algorithm to optimally shift the heat loads of buildings equipped with heat pumps and PV solar panels using only the thermal inertia of the building structures, i.e. no additional thermal energy storage systems are required. The algorithm is designed as a model predictive controller aimed at minimizing the heating cost associated to the heat pumps. At this step of the work, the effectiveness of the algorithm is demonstrated only at local level, whereas in the future the same concept will be applied for achieving system-level objectives.

2. Models
All models are developed using PYTHON according to the object-oriented modeling paradigm.

2.1. Building model
The thermal behaviour of the building is simulated using the lumped-capacitance model described in the German Standard VDI 6007 \[7\], and implemented in PYTHON. Lumped-capacitance models assume that the distributed thermal mass of the dwelling is lumped into a discrete number of thermal capacitances. The aforementioned model, shown in Figure 1, has seven thermal resistances and two thermal capacitances and will be referred to as \(7R2C\) model.

![Figure 1. Lumped-capitance building model of German Standard VDI 6007.](image)

This model distinguishes between adiabatic and non-adiabatic building components, and assigns an equivalent thermal capacity to each of the two groups. Further details on the model, including a discussion on its accuracy, can be found in Vivian et al. \[8\]. The equations of the building model are not shown here for the sake of brevity.

2.2. PV model
The PV system is modeled using the open-source PYTHON package \textit{pvlib} \[9\]. This tool provides a set of functions and classes for simulating the performance of photovoltaic energy systems. \textit{pvlib} performs classical solar processing calculations to find the incident solar radiation on PV modules for given tilt angle and orientation throughout the year. Then, the performance of the PV modules is computed using the air temperature and the wind speed as additional inputs. The PV modules and the inverters parameters can be manually inserted or loaded from an existing library available in the \textit{pvlib} documentation.

2.3. Heat pump model
A simplified model of the heat pump is given by

\[
\text{COP}_t = 0.3935 + 0.00802 \theta_e^t
\]  (1)
where the COP is a linear function of the outdoor temperature $\theta_i$, as in [6].

3. Case-study

The algorithm was tested on a typical Italian apartment from the 1990s equipped with a 3 kW photovoltaic system on the roof. The external walls have thermal transmittance of 0.67 W/(m² K) and face the East and the West. The floor area is 94.4 m² and the ratio S/V is approximately 0.75. The windows have a thermal transmittance of 2.75 W/(m² K) and an area of 4.5 m² towards the West and 12.4 m² towards the East. Most of the thermal mass of the building comes from adiabatic building components. Indeed, floor, ceiling, internal walls and divisory walls together reach an internal heat capacity $C_{1,\text{int}} = 5.70 \cdot 10^7$ J/K while the internal heat capacity of the external walls is $C_{1,\text{aw}} = 7.95 \cdot 10^6$ J/K. The overall internal heat capacity of the building is $C_m = 6.55 \cdot 10^7$ J/K.

An air-to-water heat pump supplies heat to the building via low temperature radiators. A case-study the Test Reference Year of Venice (Italy) [10]. The latter schedules the operation of the heat pump solving the MILP optimization problem described in Section 4.1. The simulation time-step is set to 15 minutes.

4. Methods

The benchmark used to evaluate the effectiveness of the proposed controller is a conventional thermostat that shares the same set-point of the controller.

The latter schedules the operation of the heat pump solving the MILP optimization problem described in Section 4.1. The simulation time-step is set to 15 minutes.

4.1. Optimal control policy

The model predictive control aims at minimizing the following objective function:

$$\min \sum_{t \in K} \left( \lambda_t^b w_t^b - \lambda_t^i w_t^i \right) + \sum_{t \in K} \gamma \left( \delta_t^f + \delta_t^i \right)$$

(2)

where $\lambda_t^b / \lambda_t^i$ is the electricity buy/sell price during interval $t$, $w_t^b / w_t^i$ is the amount of electricity bought/sold at $t$, $\delta_t^f / \delta_t^i$ is the amount of degrees exceeding the maximum/minimum temperature comfort level, and $\gamma$ is the penalty associated with a violation of the internal temperature comfort level. The objective function (2) is associated with a set of constraints, that can be divided into three main blocks. The first block models the evolution in time of the internal temperature $\theta_t^i$ according to the well-known Standard ISO 13790 [11]. However, here the parameters of the equivalent electrical circuit are not calculated by means of the procedure described in the Standard, but are instead determined through a parameter identification procedure that minimizes the distance between the measured indoor temperature and the one calculated by the simplified model. The first block of constraints is

$$H^{\text{re}} (\theta_t^\text{ma} - \theta_t^i) + H^{\text{ir,iso}} (\theta_t^i - \theta_t^e) + \phi_t^\text{lc} + \phi_t^\text{ia} = 0, \quad \forall t \in K$$

(3a)

$$H^{\text{ir,iso}} (\theta_t^i - \theta_t^e) + H^{\text{ir,w}} (\theta_t^e - \theta_t^i) + H^{\text{ir,ms}} (\theta_t^\text{ma} - \theta_t^i) + \delta_t^i = 0, \quad \forall t \in K$$

(3b)

$$H^{\text{ir,ms}} (\theta_t^i - \theta_t^\text{ma}) + H^{\text{ir,em}} (\theta_t^e - \theta_t^\text{ma}) + \phi_t^\text{ma} = C_m (\theta_t^\text{ma} - \theta_{t-1}^\text{ma}), \quad \forall t \in K$$

(3c)

where $\theta_t^\text{ma}$ is the temperature of the air supplied to the building through the ventilation system, that coincides with the external air temperature $\theta_t^e$ in naturally ventilated buildings. Temperatures $\theta_t^i$ and $\theta_t^\text{ma}$ can be interpreted as the average temperatures of the internal surfaces and of the thermal mass of the building, respectively. The thermal load of the HVAC system is given by $\phi_t^\text{lc}$, and the internal and solar heat gains are split between the air node ($\phi_t^\text{ia}$), the
surface node \((\phi^s_t)\) and the thermal mass node \((\phi^m_t)\), according to coefficients proposed by the Standard. The remaining coefficients are parameters that depend on the thermal properties and size of the building components. They are lumped into five thermal transmittances, i.e., \(H^{ve}\), \(H^{tr, is}\), \(H^{tr, w}\), \(H^{tr, ms}\), \(H^{tr, em}\), and one thermal capacitance \(C^m\).

The second block of constraints imposes the electric energy balance between the household and the power grid and the thermal comfort in the indoor environment as

\[
\begin{align*}
w_b + w_{pv} &= w_s + w_{hp}, \quad \forall t \in \mathcal{K} \\
\theta_i^l - \delta^l_t &\leq \theta^l_i \leq \theta^u_i + \delta^u_t, \quad \forall t \in \mathcal{K} \\
w_b \geq 0, \quad w_s \geq 0, \quad \delta^l_t \geq 0, \quad \delta^u_t \geq 0, \quad \forall t \in \mathcal{K}
\end{align*}
\]  

where \(w_{pv}\) is the energy generated by the PV panels during interval \(t\), \(w_{hp}\) is the energy consumed by the heat-pump at \(t\), \(\theta^l_i\) and \(\theta^u_i\) are the minimum and maximum indoor comfort temperature levels. The third and last group of equations is related to the heat pump performance

\[
\begin{align*}
w^{hc}_t &= F_{hp} \phi^{hc}_t, \quad \forall t \in \mathcal{K} \\
\phi^{hc}_t &\leq Q^{hp}, \quad \forall t \in \mathcal{K} \\
F_{hp} &= K_0 + K_1 \theta^l_t, \quad \forall t \in \mathcal{K}
\end{align*}
\]

where \(Q^{hp}\) is the maximum energy consumption of the heat-pump, \(F_{hp}\) is the inverse of the COP, \(K_0\) and \(K_1\) are coefficients associated with the heat-pump coherently with Eq. (1). The set of decision variables of (2) is \(\Xi = \{w^b_t, w^s_t, w^{hp}_t, \phi^{hp}_t, \theta^l_i, \delta^l_t, \delta^u_t\}\). The optimization has a time horizon of 12 hours, which means that at each time step the optimization calculates the value of the decision variables for the next 48 time steps.

4.2. Thermostat control

The behaviour of the heat pump is then reproduced using a thermostat in the indoor environment. The thermostat sets a benchmark for assessing the improvement brought by the optimal control policy described in Section 4.1. The thermostat dead-band \(\delta \theta_t\) is coherent with the values adopted in Equation (4b). Therefore, the following holds true:

\[
\delta \theta_t = \frac{\theta^u_t - \theta^l_t}{2}
\]

5. Results

For ease of reading, the control system proposed in Section 4.1 will be addressed from here on under the acronym \textit{BEMS}, abbreviation for Building Energy Management System. A first analysis of the results shows that the \textit{BEMS} tries to maintain the internal temperature at the minimum allowed temperature in order to minimize costs. This is possible because we assume to have a variable-speed compressor in the heat pump that is able to modulate the thermal output of the heat pump between 0 and 100%. As a result, a considerable difference emerged between the thermal energy supplied by the heat pump in case the latter was controlled by the \textit{BEMS} compared to the benchmark case with thermostat control (thermostat). Therefore, we assume this case as an \textit{economic-oriented} use of the \textit{BEMS} and we named this operational mode \textit{BEMS}_eco. A second operational mode is then set out to have the same minimum thermal comfort conditions guaranteed by the thermostat. This was obtained by resetting the thermal comfort band between the set-point temperature and the upper bound temperature, \(\overline{\theta}_t\). This \textit{comfort-oriented} use of the \textit{BEMS} is referred to as \textit{BEMS}_comf.
δθ = 1°C

|                  | BEMS _eco | BEMS _comf | thermo | BEMS _eco | BEMS _comf | thermo |
|------------------|-----------|------------|--------|-----------|------------|--------|
| Q<sub>hp</sub>  | 7433      | 8225       | 8202   | 6631      | 8243       | 8173   |
| W<sub>hp</sub>   | 2279      | 2511       | 2527   | 2046      | 2513       | 2572   |
| W<sub>pv</sub>   | 665       | 665        | 665    | 665       | 665        | 665    |
| W<sub>b</sub>    | 1917      | 2143       | 2381   | 1712      | 2126       | 2469   |
| W<sub>s</sub>    | 302       | 296        | 518    | 330       | 277        | 561    |
| W<sub>self</sub> | 362       | 369        | 146    | 335       | 387        | 103    |
| C<sub>tot</sub>  | 368.3     | 413.7      | 450.3  | 325.8     | 411.3      | 465.7  |
| R<sub>tot</sub>  | 82.1      | 36.6       | 0.0    | 139.8     | 54.4       | 0.0    |

|                  | BEMS _eco | BEMS _comf | thermo | BEMS _eco | BEMS _comf | thermo |
|------------------|-----------|------------|--------|-----------|------------|--------|
| Q<sub>hp</sub>  | 7433      | 8225       | 8202   | 6631      | 8243       | 8173   |
| W<sub>hp</sub>   | 2279      | 2511       | 2527   | 2046      | 2513       | 2572   |
| W<sub>pv</sub>   | 665       | 665        | 665    | 665       | 665        | 665    |
| W<sub>b</sub>    | 1917      | 2143       | 2381   | 1712      | 2126       | 2469   |
| W<sub>s</sub>    | 302       | 296        | 518    | 330       | 277        | 561    |
| W<sub>self</sub> | 362       | 369        | 146    | 335       | 387        | 103    |
| C<sub>tot</sub>  | 368.3     | 413.7      | 450.3  | 325.8     | 411.3      | 465.7  |
| R<sub>tot</sub>  | 82.1      | 36.6       | 0.0    | 139.8     | 54.4       | 0.0    |

Table 1: Summary of simulation results.

The simulation results are summarized in Table 1. The heat demand of the building with the thermostat control is 8202 kWh, i.e., approximately 87 kWh/(m² yr). For the reason mentioned above, the BEMS _eco operational mode brings a variation of the heat demand of -9.4% with δθ<sub>t</sub> = 1°C and of -18.9% with δθ<sub>t</sub> = 2°C, and the average indoor temperature in the two cases is 20.2°C and 19.2°C, respectively. On the other hand, the BEMS _comf operation shows an almost negligible increase in the heat demand of the building, i.e., +0.3% with δθ<sub>t</sub> = 1°C and +0.9% with δθ<sub>t</sub> = 2°C, and an average annual internal temperature close to the set-point one (21.1°C for both δθ<sub>t</sub> values), whereas with the thermostat control the average indoor temperature coincides with the set-point temperature. The cost for the electricity consumption is significantly reduced by using the BEMS in both operational modes. Due to the difference between the price for electricity bought from the grid (as high as 0.20 €/kWh) and the price at which the electricity is sold to the grid (as low as 0.05 €/kWh), it is more convenient to self-consume the electricity produced by the PV panels rather than selling it to the grid. Following this logic, the BEMS exploits the thermal mass of the building to shift the electricity consumption towards the most convenient periods, i.e., during the daytime hours when the PV module is generating electricity. At the same time, the penalty added to the objective function avoids the temperature of the indoor environment exceeding the comfort limits.

Figure 2 shows the behavior of the indoor temperature with BEMS _eco (red continuous line), BEMS _comf (blue continuous line), and thermostat (green continuous line). It can be seen that with BEMS _eco and BEMS _comf the increase in the indoor temperature always occurs when the PV generates electricity (black dashed line). Note that there is a natural correlation between PV solar production and indoor temperature due to the solar gains during the hours in which the PV is generating electricity. Accordingly, also the thermostat control seems to follow the PV production pattern, even though it simply ensures that the temperature lies between its minimum and maximum comfort limit. Depending on the δθ<sub>t</sub> adopted, the share of self-produced electricity increases from 4.0-5.8% to 14.7-15.4% with the BEMS _comf operation mode and up to 15.9-16.4% with the BEMS _eco operation mode. The latter, due to the combination of lower energy needs and increased self-production from the PV system, allows to drastically reduce the energy bill for space heating from 450 to 368 €/year with δθ<sub>t</sub> = 1°C (-18%) and from 466 to 326 €/year with δθ<sub>t</sub> = 2°C (-30%). The BEMS _comf mode allows to reduce costs without affecting the indoor environmental quality. Therefore, the cost reduction is lower but still significant: from -8% to -12% depending on the temperature flexibility allowed around the set-point.
6. Conclusions
A model predictive control algorithm is developed with the aim of minimizing the heating cost in a residential dwelling equipped with an air-source heat pump and a PV solar system. Its performance is compared to that of a conventional thermostat controller. The proposed control policy is able to significantly reduce the user’s cost for electricity with respect to the naive control policy thanks to an increased share of self-produced electricity, approximately from 5% to 16%. The energy bill for the final user is then reduced from from 8% to 12% without affecting the thermal comfort in the indoor environment. Note that the controller exploits the thermal mass of the building to shift the heat load pattern with no need of additional thermal energy storage systems. Future research will address the uncertainty of weather forecast and the stability of the controller.

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