Sensitivity of modal parameters for damage detection in corroded beam elements of the Pescara benchmark

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Abstract. Sensitivity and identifiability problems of the modal parameters in the presence of corrosion damage are studied. The first concerns the rate of change of the modal parameters against the damage increase. The second concerns the uncertainty intervals overlapping of the modal parameters between the sound and damaged states. To this end, different testing methods, different identification methods (time, frequency and time-frequency methods), different corrosion levels and different thermal condition (summer-winter) are considered. Prestressed concrete beams of identical geometry endowed with low to high reinforcement ratios are dynamically tested. The free decaying vibrations are used to identify the modal parameters: frequency, modal shapes and damping. The prestressing force is found to be not a variable of the problem. The damage levels range from very low to moderate. It is found that reliable damage identification is possible only for moderate damage. In spite of higher scatter, damping reveals a more stable and sensitive indicator than frequency. Modal shapes shows regular changes, but within the range of the uncertainty intervals.

1. Introduction
Experimental modal analysis is often used in health monitoring strategies to assess the system state. Robustness of the results depend on the sensitivity of the modal parameters to damage. This problem is still an open question for civil structures prone to corrosion. In [1] the result of dynamic tests on a p.c. beam are studied. Damage due to corrosion was simulated by the progressive cut of the prestressing strands. Changes in natural frequencies were detected when the reinforcement reduction was over 30%. The first bending frequency suffered the highest decay. In [2] three r.c. beams subjected to cycles of accelerated corrosion in the compressive zone are tested. One beam was kept undamaged for reference. Neither the corrosion entity nor its effect on the concrete are quantified. It is reported that the first and third frequencies decay respectively of about 4% and 3%. In [3] two r.c. beams were corroded up to 7% loss of steel area in the tensile zone and one was kept sound as control beam. Concrete cracking ranged from very slight spalling to moderate cracking. The drop of the first seven frequencies was quite different between the two beams: 4% to 11% and 1 to 4%. The higher frequencies suffered the higher decay. Changes of the modal damping ratios were also measured for the first three modes. However the changes were not consistent with those observed for the frequencies. In [4] 26 r.c. beams, with the same geometry and reinforcement scheme, were subjected to different chemical attacks (sulfate, carbonation, chloride). All the beams were subjected to a preliminary loading process to induce cracks and favour the chemical attack. After one year of
chemical attack variations in natural frequencies were not clearly defined. Actually, a slight increment of the stiffness was observed. In [5] the dependence of the first three natural frequencies on the level of reinforcement corrosion is examined by the help of a non linear f.e. model calibrated on experimental results. Corrosion damage up to 20% of steel area loss is considered. It is found that higher order frequencies are more sensitive to corrosion damage.

The literature results do not lead to clear and univocal conclusions. The paper aims at evaluating the modal parameters sensitivity of corrosion damaged beams. To this end the free decaying dynamic response relevant to a large set of p.c. beams subjected to different corrosion levels is investigated. The modal parameters are identified and compared using time domain, frequency domain and joint time-frequency domain techniques. The performed tests allow for some statistics so that mean values and standard deviations can be estimated. Successful damage assessment is achieved only when the uncertainty intervals of the modal parameters between the sound and damaged states are well spaced and do not overlap.

2. Prestress force effects on the beam dynamics
When studying the dynamic behaviour of p.c. beams one should ask whether the prestress force \( P_f \) has any influence on the beam response. First considerations were given in [6] where \( P_f \) was treated as an external force. It was found that frequencies are lowered by \( P_f \). In [7] hypotheses similar to [6] were used to compare theoretical and laboratory results. A mismatch between the two was observed. It was found that the experimental frequencies decrease as \( P_f \) decreases. Subsequent works faced the problem, but remained defective in the theoretical assumptions [8], [9] and [10]. They did not account for the coupling between the compressed beam and the tensioned strands. Following [7] a debate was opened. Various authors arrived at similar conclusions: in a linear model the influence of \( P_f \) is negligible [11], the influence of \( P_f \) does not modify the beam frequencies in any case [12], \( P_f \) cannot cause changes of stiffness since it should be considered an internal force [13]. A general and rigorous demonstration was provided in [14]. The coupling between the compressed concrete beam and the tensioned steel strands was imposed using proper equilibrium and compatibility conditions. It was found that \( P_f \) does not affect the natural frequencies of a p.c. beam. In conclusion, as concerns the present work, this parameter will not be further considered in the interpretation of results.

3. Experimental tests
The detailed description of the experimental tests carried out is reported in a separate companion paper. In the following only the main features are summarised and specialized to the purpose of the paper. In order to study the effects of the corrosion damage on concrete, the beams were chosen with identical dimensions and differ only by the amount of prestress steel, figure 1a. Three different groups of beams derived from mass production were considered: T1, T4 and T7 composed by 5 joists each. One beam per group was preserved sound, the others corroded. The nominal characteristics of the beams are given in figure 1b. Note that the number and position of the steel strands has been appropriately chosen in order to favour spalling or delamination. In view of the geometric identity and since \( P_f \) does not affect the dynamic response, it is expected that all the beams share the same modal parameters in the sound state. This is not the case actually. Dimensional measurements showed that the beams are similar, but not identical and that changes in the dynamic parameters are of the order of the damage induced changes. As a consequence, a single sound beam cannot be taken as the reference beam of the group, but each beam should be self referenced and tested in the sound conditions prior the corrosion takes place. Finally, it is worth to note that the beam bending stiffness is not significantly altered by the steel amount, therefore damage depends essentially on concrete cracking caused by the expansive character of the corrosion products.

The beams were corroded only in the central part \( L_c = 0.6L \) with \( L = 320 \) cm. Both ends were not corroded to prevent loss of bond. The corrosion entity is expressed by the loss (%) of the effective steel area since this parameter provides also for a direct measure of the beam strength. The corrosion levels \( C_{ni} \) listed in table 1 were considered. To help understand the table the sample beam T14, i.e. the
fourth beam of group T1, is discussed. This beam was first corroded to $C_\% = 6\%$ and tested, then it was corroded to $C_\% = 15\%$ and tested again. The same applies for the other beams.

| Table 1. Corrosion levels (steel area loss) |
|------------------------------------------|
| Corrosion [\%] | 0  | 3  | 5  | 6  | 10 | 12 | 15 | 30  |
|----------------|----|----|----|----|----|----|----|-----|
| Group T1       | - T12 |    |    | T11, T14 |    |    | T11, T14, T15 | T13 |
| Group T4       | T41 | T42 |    | T45 | T43, T44 | T42, T45 |    |    |
| Group T7       | T76 |    | T71, T73, T74 |    | T74 |    | T71, T73 |    |

The corrosion process was artificial and accelerated. The central part of the beams was immersed in a water solution at 5% NaCl and the electric current provided and kept constant by a dc power supply. The Faraday’s law was used to have a theoretical relationship between the time over which the impressed current was allowed to flow and the extent of corrosion. Current densities comparable to maximum literature values were used. At the end of each corrosion step a map of the concrete cracking was taken. The cracking maps of group T7 are shown in figure 2. Some remarks are helpful to better appreciate the results of identification described in the next paragraph. The T7 beams were subjected to two corrosion steps: 5\% for step 1 and 10-15\% for step 2. Cracking at step 1 is quite different among the three beams either for crack distribution and length and for crack opening. At step 2 the cracking of beam T73 remained almost unchanged, whereas beam T74 increased only the crack number and length and beam T71 turned out to be highly damaged. In conclusion, even if the beams experienced the same corrosion process the damage suffered (cracking) revealed quite different.

The beams were tested in the simply supported configuration with the supports provided by heavy steel blocks. The span length was $L = 300$ cm. Three different impulse tests were considered: (M) hammer, (R) sudden release of a point load, (C) sudden settlement at one of the supports. The impulse was not measured. Different impulse positions and different impulse intensities were considered. Each test was repeated at least twice. The full set of tests was carried out once for the sound beams and then repeated for all the corrosion steps. The dynamic response was recorded using 9 accelerometers with $f_{\text{sampl}} = 1000-2000$ Hz. The sensors were not equally spaced along the beam length. Their position corresponds to nodes or peaks of the first five modal shapes and therefore hampers or cancels the modal contribution at a specific point.

![Figure 1. Beam section and reinforcement (a); beam characteristics (b)](image1)

![Figure 2. Step 1: T71 5\% (a), T73 5\% (b), T74 5\% (c); Step 2: T71 15\% (a), T73 15\% (b), T74 10\% (c)](image2)
4. Identification methods

The impulse tests have been processed using three different identification methods according to as many analysis domain: frequency, time and joint time-frequency domain. The three methods are all of output-only type and in this sense the operational modal parameters are identified. The frequency domain method has been considered as a first level processing extended to all measurements. For that reason the peak picking method based on the Fourier transform (FT) has been used. It is in essence a sdof type method that is repeatedly applied to all the interested frequencies. All the modal parameters can be identified yet with some approximation. To get a more robust and accurate identification of the frequency and damping the time domain Hilbert transform method (HT) has been applied to a selected subset of tests. This is also a sdof type method and has been used to process only the fundamental vibration mode. The HT has been used also to qualify the type of damping and possible non linearity of the beam response. The same subset of tests has been analysed using the Gabor transform method (GT). This is a considerably general method of mdof type with intermediate accuracy between FT and HT. The major features of the three methods are given in the following to appreciate the results of the identification. The linear free vibrations of the beam at each measurement station \( h = 1...M \) are given by the superimposed damped contributions of \( N \) vibration modes:

\[
x^h(t) = \sum_{k=1}^{N} a_k(0)\phi_k^h e^{-\omega_k\xi_t} \sin \left[ \sqrt{1 - \xi^2_k} \omega_k t + \theta_k(0) \right]
\]

(1)

4.1. Frequency domain identification

The FT of (1), bounded in the neighbourhood of the \( k \)-th resonance frequency \( \omega_k \), is approximated by the dominant contribution of mode \( k \) [15], the others being neglected because of the well spaced frequencies of the beam under consideration:

\[
X^h(\omega) = \sum_{k=1}^{N} \frac{a_k(0)\phi_k^h}{\omega_k^2 - \omega^2 + i2\omega_k\xi_k} \approx \frac{a_k(0)\phi_k^h}{\omega_k^2 - \omega^2 + i2\omega_k\xi_k} \text{ for } \omega \in I_{\omega_k}
\]

(2)

Each identified resonance frequency \( \omega_k \) corresponds to one of the \( N \) peaks of the FT amplitude function \( |X^h(\omega)| \). The \( h \) component \( |\phi_k^h| \) of the \( k \) mode shape is proportional to \( |X^h(\omega_k)| \) and the sign is derived from the phase angle \( \angle X^h(\omega_k) \), here simplified because of the a priori knowledge of the modal shapes. The damping is estimated through the half power bandwidth method:

\[
\xi_k = \frac{\omega_h - \omega_a}{2\omega_k}, \text{ where: } |X^h(\omega_a)|^2 = |X^h(\omega_h)|^2 = \frac{1}{2} |X^h(\omega_k)| \text{ with } \omega_a < \omega_k < \omega_h
\]

(3)

Since the FT method provides for \( M \) independent estimates of frequencies and damping, more stable results are obtained by averaging on the estimated values.

4.2. Time domain identification

The HT of the time signal (1) is a new time signal \( H[x^h(t)] \) that is in quadrature with \( x^h(t) \). When the spectral properties of \( x^h(t) \) met some requirements [16], the HT action can be thought as a filter action that leaves unchanged the magnitude of the signal \( |X^h(\omega)| \), but shifts the phase \( \angle X^h(\omega) \) of each frequency component by \( \pm \pi/2 \) depending whether they are negative or positive frequencies (modulation property). For monochromatic signals \( x^h_k(t) = a_k^h(t)\cos \theta_k(t) \) the modulation property states that \( H[x^h_k(t)] = -a_k^h(t)\sin \theta_k(t) \) and allows to refer to the analytic signal defined by:

\[
z^h_k(t) = x^h_k(t) + iH[x^h_k(t)]
\]

(4)
In the complex plane, $z^h(t)$ can be split into two new easy computed time signals: the instantaneous envelope $a^h_k(t)$ and the instantaneous phase $\theta^h_k(t)$:

$$a^h_k(t) = \left| \frac{z^h_k(t)}{e^{\theta^h_k(t)}} \right|$$

$$\theta^h_k(t) = \arg \left( \frac{z^h_k(t)}{e^{\theta^h_k(t)}} \right)$$

The envelope and phase $a^h_k(t)$ and $\theta^h_k(t)$ account for the decay and the oscillatory properties of the dynamic response and are used to identify damping and frequency through the following theoretical relations:

$$a^h_k(t) = a_k(0) \phi^h_k e^{-\omega_k \xi t}$$

$$\omega_{k,\text{damp}}(t) = \frac{d\theta^h_k(t)}{dt}$$

Both functions (6) are time functions, exponentially decaying the first, constant the second for linear viscously damped systems. The damping and the frequency are easily identified by fitting (6) to the experimentally derived curves. The deviation of the experimental curves from the theoretical behaviour allows to assess whether different sources of energy dissipation exist or if some kind of non linearity is involved during vibrations or if coupled mechanisms are present.

4.3. Joint time-frequency domain identification

The method is more general than herein needed and allows to identify the so called evolutionary modal model EMM of weakly nonlinear structures with multiple degrees of freedom [17]. The EMM collapses to constant time values in the present case of linear vibrating beams. According to (1), the EMM is constituted by the following quantities:

$$a_k(t) = \left\{ \sum_{h=1}^{M} \left[ a^h_k(t) \right]^2 \right\} \quad \phi^h_k(t) = \frac{a^h_k(t)}{a_k(t)} e^{i \phi^h_k(t)} \quad \omega_{k,\text{damp}}(t) = \frac{d\theta^h_k(t)}{dt}$$

where $a_k(t)$, $\phi^h_k(t)$ and $\omega_k(t)$, are respectively the $k$-th (instantaneous) amplitude, modal shape component and natural frequency. These quantities are identified through a least squares surface fitting between the experimental computed and the theoretical expression of GT.

The GT is a linear time-frequency transform [18] that belongs to the Short Time Fourier Transforms family when the moving time window $w(t)$ is given by the gaussian function:

$$G_{\text{exp}}(t, \omega) = \int_{-\infty}^{\infty} w(\tau - t) x^h(\tau) e^{i\omega \tau} d\tau \quad w(\tau) = \frac{1}{\sqrt{\pi \Delta_t}} e^{-\left( \frac{\tau}{\Delta_t} \right)^2}$$

the capabilities of GT are strictly related to the width (duration) $D_t$ of $w(t)$ in consideration of the uncertainty principle for which one cannot be simultaneously tight in time and in frequency. Consequently, the optimal $D_t$ should be tuned case by case. The theoretical expression of GT in terms of the EMM quantities reads [19]:

5
where $R_{k}^h(\omega)$ is a term depending on the nonstationarity and nonlinearity of the dynamics response. The surface fitting is repeated at each measurement station $h$. Finally it is underlined that when the structure vibrates linearly the modal quantities are time constant and therefore, in both cases of HT and GT, the standard deviations can be computed and used to qualify the consistency of the data and the reliability of the estimates.

Illustrative outcomes of the adopted identification methods are reported in the figures below. In figure 3 the free decaying vibrations (1) of midspan accelerometer of beam T73 are shown. The windowed portion highlights the quick decay of a modal component (3rd mode) with respect to the fundamental one (1st mode). In figure 4 the FT, HT and GT functions of the above signal are plotted. The HT shows the strong linearity of the beam response ($\omega_1 = \text{const}$) according to the first dominant mode. The FT shows that the signal is mainly composed by two frequencies (1st and 3rd modes). The GT shows that mode 1 is sustained in time, whereas mode 3 damps down quickly.

![Figure 3. Beam T73-sound, midspan sensor: overall signal (a), windowed signal (b)](image)

![Figure 4. Beam T73-sound; midspan sensor. Maps of the transformed signal: (a) Fourier transform (amplitude), (b) Hilbert transform (frequency function), (c) Gabor transform (amplitude)](image)
5. Results
The dynamic tests were preceded by static characterization of the beams both in the sound and damaged state. The static tests highlighted a linear behaviour for static displacement fields comparable to the deformed dynamic configurations. The static stiffness $\beta = EI$ was computed by means of linear regressions of the load deflection curves. Reference numerical results were obtained using a f.e. Euler beam model (E-model) with stiffness $\beta$ and fixed supports. For such a model the first five frequencies are in the ratios: 1, 4, 9, 16, 25. The effects of the homogenization, H-model, are practically negligible for groups T1 and T4 (low to medium steel to concrete ratios $A_s/A_c$), but should be accounted for group T7 ($A_s/A_c > 1\%$) for which $\beta_H/\beta = 4.4\%$. In this latter case the frequencies increase uniformly of about 2%. The inclusion of shear deformation, T-model, involves an opposite effect. However, in this case the frequency decrease is not uniform and is negligible for the first three natural frequencies. The E-model was used to validate the numerical results against the experimental ones. The results are given in table 2 for beam T71. In the sound case, when the static stiffness $\beta$ is used a satisfactory correlation between measured and computed frequencies is observed. The frequency matching becomes excellent, at least for the first three frequencies and mode shapes (not shown), if some flexibility of the supports is allowed. If the same comparison is performed in the damaged state ($C_{\%} = 15\%$), a less satisfactory correlation is observed, mainly due to some dispersion of the experimental results, particularly in the even order frequencies. In conclusion, it is observed that a corrosion level $C_{\%} = 15\%$ induces a damage, loss of beam stiffness, of about $(1-\beta_{\text{dam}}/\beta_{\text{sound}})\times 100 = 5\%$.

| Table 2. Beam T71: numerical vs. experimental frequencies |
|----------------------------------------------------------|
| $\beta$ | $f_1$ [Hz] | $f_2$ [Hz] | $f_3$ [Hz] | $f_4$ [Hz] | $f_5$ [Hz] |
| Exp | 228.4 | 19.24 | 74.82 | 161.17 | 278.27 | 432.53 |
| Num | 228.4 | 19.34 | 77.04 | 172.18 | 303.29 | 468.39 |
| | | | | | | |
| Damaged Exp | - | 18.79 | 78.03 | 155.08 | 284.85 | 441.55 |
| Num | 217.2 | 18.79 | 73.85 | 160.00 | 261.70 | 345.63 |

1: fixed supports; 2: flexible supports (stiffness: $K = 2.5E4$ kN/m

The figure 5a was constructed assuming the above found relation between corrosion, stiffness and frequency up to a maximum $C_{\%} = 30\%$. It is observed that higher order modes are less sensitive than lower modes. It is also observed that for a corrosion level of 10% the change of frequency of the third mode is about 1%. This value can be considered a limit for damage identification in view of the experimental uncertainties. Although the identification process can detect stiffness reduction, safety checks require to evaluate the corresponding loss of strength of the beam. The moment curvature relations for sample beams of groups T1 and T7 are shown in figure 5b. The residual beam bending capacity is evaluated in the range 5-30% of the corrosion attack. Apart a very different qualitative behaviour related to the steel amount in T1 (1.0%) and T7 (4.4%), the capacity loss is not as much different between the two groups. For $C_{\%} = 15\%$ the loss of strength is 11% and 7% respectively for T1 and T7 and reaches 22% and 15% when corrosion rises to 30%. It is also interesting to compare the numerical and experimental mode shapes. The comparison is made quantitative by means of the MAC index. Three cases are compared, figure 6a: numerical vs. experimental in both sound and damaged states and experimental sound vs. experimental damaged state. In any case the MAC values are very high > 0.95. When such values are encountered it is generally agreed to have a good correlation between the modal shapes. In view of the above, one should conclude that the MAC index is hardly effective to detect differences induced by corrosion. However, it should also be considered that the present case is not the best suited to display modal shape changes since the damage is almost uniformly distributed and diffused on a long span of the beam. It should also be observed that even in this limit condition the MAC values show a regular trend and that the higher modes show the higher
sensitivity. In field testing the climatic conditions can destroy the identification if not properly accounted for. To investigate this aspect, some beams have been tested both in summer and winter with a thermal jump of about 30°C. In figure 6b the frequencies (avg. values and std. intervals) relevant to the first vibration mode of the T4 beams in different corrosion states are reported. The summer and winter values are well spaced and no superposition exist between the uncertainty intervals. Standard deviations are generally higher in winter than summer and the distances between summer and winter values are greater for corroded beams than the sound ones, even if no definite trends can be observed. The features observed for the first mode do not maintain for higher modes. For such modes the width of the uncertainty intervals tends to increase and superimpose with the mode order. In this instance possible differences between the climatic conditions are lost.

Figure 5. Loss of frequency vs. corrosion (a); Moment-curvature curves: sound vs. corroded

Figure 6. MAC num. vs. exp. (a); summer-winter freq. and uncertainty intervals (b)

Figure 6. Frequency (a) and damping (b) vs. acceleration intensity for tests M, C and R.
A further evaluation concerns the stability of the results against the type of test performed. A typical result is given in figure 6 for beam T11 where both the first frequency and the associated damping are plotted against the peak acceleration. Three impulse tests are compared: M, C and R. The more stable and less dispersed values appear those for test type C and, to a less extent, M; whereas test type R shows large scatter both in frequency and damping.

The main results for beams T12, T14, and T15 of group T1 are summarised in figure 7a. Similar results were obtained for T7 whereas some inconsistencies were found for T4. In the figure, the frequency ratios $R_f$ between the corroded and sound states are shown for the first five natural frequencies. The uncertainty intervals in the sound state are also reported. For the first two frequencies $R_f$ is correctly below 1 regardless the corrosion level. Only for corrosion of order 15% all the five $R_f$ are systematic lower than 1. Moreover, only the first and third frequencies are outside the uncertainty intervals and allow for robust damage identification. In the literature it is warned that the higher order frequencies are more sensitive to corrosion damage. This feature is not shared by the $R_f$ index. In figure 7b the identified damping is plotted against the peak acceleration of the different tests performed. The results for beam T41, T42 and T43 in the sound state are shown. The damping values confirm the tendency to be more dispersive than frequency. In spite of that, if one compares the damping values between the sound and the corroded state a systematic damping increase is observed. Further, this change is greater than the uncertainty intervals and allows for safe identification. This is better revealed if the average values among different test are taken in place of a single outcome as shown by the figure 7c where damping for T1 and T7 is reported according to increasing corrosion. In conclusion one can state the even if frequencies are better identified they appear less sensitive than damping to detect corrosion damage.

6. Conclusions
The sensitivity of modal quantities to damage in corroded prestressed concrete beams was studied. The effects of several parameters were considered. It was found that changes of the prestressing force due to corrosion do not affect the natural frequencies. On the contrary, impulse tests of different type show some influence on the identified frequencies and scatter. These differences are intrinsic to the
test performed and do not depend on the identification method adopted. In any case damping shows more sensitive than frequency even if endowed with higher dispersion. Mode shapes are not presently a proper indicator in view of the uniform damage configuration. The dependence on the thermal conditions appears hampered by corrosion and should be accounted for. Safe damage detection is obtained when the modal parameters are systematically outside the uncertainty intervals. This happens for corrosion levels greater than 10% to which corresponds roughly a 4% loss of beam bending stiffness and a residual beam capacity of 0.92-0.95 depending on the amount of the reinforcement.

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