Interaction mechanics of the wheel with the drum

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Abstract. Currently, the evaluation of tire rolling resistance as well as determination of the lateral withdrawal resistance rate is often performed on drum stands. In addition, test procedures of vehicles using drum stands of various design features (with a different number of drums, their location and drive type) are becoming more and more widespread. However, there are no works considering mechanics of interaction of an elastic wheel with a drum and determining kinematic and power characteristics. The aim of this research is to eliminate this gap. This work considers a general idea of rolling mechanics of the wheel on the drum, taking into the consideration frictional interaction of the "wheel-drum" pair, including determination of rolling resistance due to hysteresis losses inside the wheel’s material, tangential force, acting in the contact point, the moment on the wheel, the power of friction losses in the contact point, areas coordinate of adhesion and sliding in the contact point of the wheel with the drum, the relative speed loss (slippage) of the wheel, the interaction of the wheel with two drums.

1. Introduction
At present, issues related to the mechanics of rolling the wheel on a flat surface are considered quite widely. However, the wheel rolling process on the drum remains poorly studied. In foreign and domestic literature there is no analysis of wheel rolling mechanics on the drum, although this requires close study, as both tire tests and wheel breaking and research tests of cars on drum stands are widely used today. This necessitates obtaining dependencies that determine the forces at the wheel's contact with the drum, friction loss of the contact power, wheel slip, and so on.

2. Research of the mechanics of interaction of the wheel with the drum
The mechanics of elastic wheel rolling on a drum is the same as wheel rolling on a flat rigid support surface.

When the driven wheel, loaded only with normal load force, is rolling due to imperfect material elasticity, internal friction losses occur in wheel material (hysteresis) that creates the moment of resistance \( M_t \) and the appearance of rolling resistance force \( F_t \) - longitudinal tangential force acting in the opposite direction of the wheel in its contact point with the base. A similar rolling resistance force arises also in the brake wheel loaded with additional braking torque \( M_b \), in comparison with the driven wheel. The presence of this force leads to the slippage of its treadmill elements relative to the base in the contact point and to the angular velocity loss of the wheel.

During the rolling action of the drive wheel, movement of which occurs after the action of the torque \( M_k \), in the contact pint, a driving (traction) force directed along the wheel is occurred. As in the previous
case, this force causes slippage of treadmill wheel elements in the base contact zone and the loss of linear speed of wheel axis.

The mechanism of sliding elements occurrence of wheel surface relative to the base is thoroughly described in [1-5]. Using the scheme of an inverted mechanism named “elastic wheel – is rigid base” and based on the theory of preliminary displacement, it was shown that during stable rolling condition, wheel elements, entering the contact zone, that were not yet “prepared” for the perception of tangential force. At the same time pressed towards the base by normal force, begin to move without sliding. Thus, tangential shift is obtained (with the opposite direction to the rolling of the brake and driving wheel, and in the rolling direction for the drive wheel). As coupled wheel elements and the base move in the reversed mechanism, their tangential displacements increase in the contact zone. Therefore, the tangential friction force between coupled elements also increases. In the contact point, where the increased friction force reaches the ultimate adhesion value, a breakdown occurs in the whole contact area located beyond the failure point (coordinate $x_f$, Figure 1). Glide happens regardless the location of the failure point (in the zone of decreasing or increasing normal pressures).

During the increase of wheel speed loss and corresponding increase of tangential force acting in the contact point, the slip zone increases, as well as the power of friction loss in the contact point, that characterizes the wear intensity of treadmill and partially the rolling resistance of the wheel.

Figure 1. Normal $q_n$ and tangential $q_t$ stress in the contact point of the wheel with the drum; $x_f$ is the coordinate of boundaries of adhesion and slip areas, $a$ is the half-length of the contact area.

The rolling resistance of the driven wheel, as it was already noted, is determined by rolling resistance moment caused by hysteresis loss in the wheel material. The most convenient way to determine the value of this moment [6] using the value of hysteresis power losses is $M_f = P_f / \omega_k$.

To determine the hysteresis power loss in the material of an elastic wheel rolling along a rigid drum, we will take into account only the normal wheel deformation that may be represented as a sum of items (Figure 2): $W = W' + W''$.

As $W' = (a^2 - x^2) / 2r$, $W'' = (a^2 - x^2) / 2r_\delta$, that $W = (a^2 - x^2) / 2 + (1/r + 1/r_\delta)$

(1)

The hysteresis power loss may be determined using the following method:

$P_f = \beta_f \int_0^a q_n \frac{dW}{dt} dx 2b$

(2)

where $\beta_f$ is hysteresis loss coefficient; $dW/dt$ - wheel warping speed:

$dW/dt = dW/dx \times dV_{esp}/dx = dW/dx \times \omega_k r$

(3)
Here, the \( x \) coordinate is located on OX axis (Figure 2): \( \frac{dx}{dt} = V_{\text{kip}} = \omega_k r \) - as the change of normal deformation value \( dW/dt \) as the wheel treadmill moves inward into the contact, occurs at a speed that is equal to peripheral wheel speed.

![Figure 2. Normal stress value \( q_n \) and deflection \( w \) while the wheel is pressed towards the drum.](image)

Considering (1):

\[
\frac{dW}{dt} = -x(\frac{1}{r} + \frac{1}{r_\delta})\omega_k r
\]  

(4)

As a result:

\[
P_r = \frac{3}{16} \beta_r F_a \omega_k r (\frac{1}{r} + \frac{1}{r_\delta})
\]  

(5)

Taking into the consideration the \( P_r \) dependence, the moment of hysteresis in tire material can be presented as follows:

\[
M_r = \frac{P_r}{\omega_k} = \frac{3}{16} \beta_r a F_a (1 + \frac{r}{r_\delta})
\]  

(6)

Then the shift shoulder of normal drum reaction will be as follows:

\[
h_0 = \frac{M_r}{F_a} = \frac{3}{16} \beta_r a (1 + \frac{r}{r_\delta})
\]  

(7)

According to [4,5] \( 3\beta_r a^{\alpha_3}/16 = f_0 r_k^c \approx f_0 r \).

Then

\[
h_0 = f_0 r (1 + \frac{r}{r_\delta}) \frac{a}{a^{\alpha_3}}
\]  

(8)

Here the \( a^{\alpha_3} \) is a half-length of a wheel contact area with a flat rigid supporting surface with the same load \( F_a \).

Knowing the shoulder value \( h_0 \), it is possible to find the dependence for tangential force (it is also known as rolling resistance force of a driven wheel) due to the hysteresis:

\[
F_{\tau_s} = F_a f_0 (1 + \frac{r}{r_\delta}) \frac{a}{a^{\alpha_3}}
\]  

(9)
Since the ratio of rolling resistance force to normal force $F_f / F_n = f_0^\delta$ is a wheel rolling resistance coefficient, then for the considered case of an elastic wheel rolling along a rigid drum

$$f_0^\delta = f_0 \left( \frac{r}{r_0} + 1 \right) \frac{a}{a_{nr}}$$

(10)

Since $r_0 \to \infty$, dependences (6), (9), (10) lead to formulas obtained for the case of rolling an elastic wheel on a flat rigid supporting surface. The comparison of these formulas with the above mentioned dependences leads to the conclusion that the hysteresis moment, force and rolling resistance coefficient of a driven elastic wheel along a rigid drum, caused by hysteresis increase by $a(1+r/r_0)/a_{pl}$ times in comparison with the rolling of the same wheel on a flat firm surface.

An increase of rolling resistance on a drum leads to a difference in lateral drag coefficients determined on the drum and during the wheel’s movement on a flat supporting surface.

While using lateral drag and rolling resistance coefficients, obtained experimentally on a drum stand, for the case of a wheel moving on a flat supporting surface, appropriate correction factors should be performed.

Tangential displacements of treadmill points in the contact zone caused by the realization of tangential contact force are determined by [1-5] dependence

$$U = (a - x) \left( \frac{\omega_k r^c_k}{\omega_\delta r_\delta} - 1 \right)$$

(11).

Applying to wheel rolling on a rigid drum, $V=V_\delta=\omega_\delta r_\delta$, where $\omega_\delta$ and $r_\delta$ are the angular velocity and radius of the drum. As a result, tangential displacements of surface points of an elastic wheel due to the implementation of tangential force in the contact point with the drum in the grip area can be represented as follows:

$$U = (a - x) \left( \frac{\omega_k r^c_k}{\omega_\delta r_\delta} - 1 \right) = \xi(a - x)$$

(12)

where

$$\xi = \frac{\omega_k r^c_k}{\omega_\delta r_\delta} - 1$$

(13)

is relative speed difference.

With the known value of $\xi$, ratios of angular wheel and drum velocities will be as follows:

$$\frac{\omega_k}{\omega_\delta} = (1 + \xi) \frac{r_\delta}{r^c_k}$$

(14)

Proceeding from the proportionality of tangential stresses (specific tangential forces) to tangential displacements, we can estimate that tangential stresses caused by the realization of tangential force in the contact point are as follows:

$$q_t = \lambda U = \lambda \xi (a-x),$$

(15)

where $\lambda$ is the coefficient of tangential wheel rigidity, defined [1] as follows:

$$\lambda = \lambda_k \frac{r_\delta}{r_\delta + r} = \frac{1.5qr}{a^3} \frac{1}{1 + r / r_\delta}$$

(16)

Taking into consideration the parabolic law of normal pressures distribution along the length of the contact area, the boundary coordinate of adhesion and slip areas (Figure 1), determined from the equality $q_t = \mu q_n$, can be represented by the dependence [1-3]:

$$x_f = a \pm \frac{\lambda \xi}{\mu q_n}$$

(17)

The moment on the drum, due to the action of tangential force, is as follows:
\[ M_r = 2b \int_{-a}^{a} r \phi_i \, dx + 2b \int_{-a}^{a} s \phi_i \, dx = \left[ 2b \int_{-a}^{a} \phi_i \, dx + 2b \int_{-a}^{a} \phi_i \, dx \right] r = F_r r_\delta \] (18)

The value in square brackets, equal to the algebraic sum of all specific tangential forces in the contact point, we will call as the circumferential traction force:

\[ F_r = 2b \left[ \frac{\lambda \xi}{2} \left( a - x_r \right) ^2 \pm \frac{1}{3} \mu q_n \left( 2a^3 + 3a^2 x_r - x_r ^3 \right) \right] \] (19)

Substituting in (19) \( \lambda \xi = x_\delta \mu q_n (a + x_r) \) formula, obtained from (17), after transformations, we get an equation the solution of which gives the dependence law for finding the boundary coordinate of adhesion and slip areas:

\[ x_r = a \left( 1 - 2\sqrt{1 - \frac{F_r}{\mu F_z}} \right) \] (20)

As a result,

\[ \xi = \pm \frac{1}{\lambda} 2\mu q_n \left( 1 - \frac{1}{3} \sqrt{1 - \frac{F_r}{\mu F_z}} \right) \] (21)

or, taking into consideration formulas for \( q_n = 1 \ldots 5 \), (12) and (16)

\[ \xi = \pm \frac{\mu a}{s} \left( \frac{1}{r} - \frac{1}{r_\delta} \right) \left( 1 - \frac{1}{3} \sqrt{1 - \frac{F_r}{\mu F_z}} \right) \] (22)

If we do not take into account the saturation factor of the treadmill pattern \( s \) (for a wheel without a treadmill pattern \( s = 1 \)), the last formula coincides with a similar formula obtained by G. Fromm [9, 10] (the difference is only in the degree of the radical: G. Fromm - square root) and then R.V. Virabov [1] for friction transmission consisting of two cylinders.

With the known dependence for \( \xi \), the ratio of angular velocities of the elastic wheel and rigid drum as a function of traction force \( F_r \) and normal load in the contact point, according to formulas (4), (12) and (13), can be represented as follows:

\[ \frac{\omega_k}{\omega_\delta} = \frac{r_\delta}{r} \left[ 1 \pm \frac{2\mu q_n}{\lambda} \left( 1 - \frac{1}{3} \sqrt{1 - \frac{F_r}{\mu F_z}} \right) \right] = \]  

\[ \frac{r_\delta}{r} \left[ 1 \pm \frac{\mu a}{s} \left( \frac{1}{r} - \frac{1}{r_\delta} \right) \left( 1 - \frac{1}{3} \sqrt{1 - \frac{F_r}{\mu F_z}} \right) \right] \] (23)

For low value of tangential forces (compared with coupling limit value \( F_r^{\text{upper}} = \mu F_z \)), last formulas can be simplified if formula \( \sqrt{1 - \frac{F_r}{\mu F_z}} \) is expanded in a grade range, rejecting values of the second order of smallness:

\[ x_r = a \left( 1 - \frac{2}{3} \frac{F_r}{\mu F_z} \right) \] (24)

\[ \xi = \frac{a}{3s} \left( \frac{1}{r} + \frac{1}{r_\delta} \right) \frac{F_r}{F_z} \] (25)

\[ \frac{\omega_k}{\omega_\delta} = \frac{r_\delta}{r} \left[ 1 - \frac{a}{3s} \left( \frac{1}{r} + \frac{1}{r_\delta} \right) \frac{F_r}{F_z} \right] \] (26)

In last formulas, \( F_r \) force is positive for the drive wheel and negative for the driven and brake wheels.
Power loss in the contact point due to friction of an elastic wheel with a rigid drum, caused by the realization of traction force at the contact point, is determined with the same formula as for wheel rolling on a flat supporting surface:

\[ P_{mp} = F_r \dot{z} V = F_r \dot{z} \omega_\delta r_\delta \]  

(27)

The point of application of resulting tangential force in the contact point (Figure 3) can be determined from the dependence [9],

\[ x_{F_r} = a(1 - \frac{\mu F_z}{F_r})(1 - \frac{1}{3} \left[ \frac{F_r}{\mu F_z} \right] ) \]  

(28)

that with low tangential forces (compared with the ultimate coupling value) can be represented in a simplified way, if decomposed into a grade range:

\[ x_{F_r} = a(1 - \frac{\mu F_z}{F_r}) - \frac{F_r}{3 \mu F_z} = a \left( \frac{F_r}{\mu F_z} - 1 \right) \]  

(29)

The action shoulder of tangential force relative to the wheel center is determined using geometrical considerations (Figure 3):

\[ h_\tau = O_\delta O_h \cos \alpha_\tau - r_\delta = (r_0 + r_\delta) \cos \alpha_\tau - r_\delta \]  

(30)

Where the angle is

\[ \alpha_\tau = x_{F_r} / r_\delta \]  

(31)

After conversions, we get

\[ h_\tau = r_0 \left( 1 - x_{F_r}^2 / 2 r_\delta r_\delta \right) \]  

(32)

As in the case of the elastic wheel rolling on a flat supporting surface, in the situation of the wheel interaction with a rigid drum, the normal pressures redistribute along the length of the contact area, and, as a result, there is an offset of the action point of the normal reaction of the drum relative to the wheel center (Figure 3).

The normal reaction shoulder of the drum relative to the center of the wheel is as follows:

\[ h_n = (r \cdot h_\tau) F_r / F_n \]  

(33)

Let us consider the case [10], when the elastic wheel rolls along two support drums (Figure 4). This is how, in particular, vehicle wheels are installed during running tests on drum stands.

Denote \( e' \) and \( e'' \) distance between the wheel center and centers of left and right drums, relatively:

\[ e' = r_\delta + r'_\delta \quad e'' = r''_\delta + r''_\delta, \]

where \( r'_\delta \) and \( r''_\delta \) - the shortest distance from the wheel axis of left and right drums.

Under the action of \( M_\tau \) moment that is applied to the wheel, the normal drum reactions are shifted by the value of \( h' \) and \( h'' \):

\[ h' = h'_n + h'_0 \quad h'' = h''_n + h''_0, \]  

(34)

where \( h'_n \) and \( h''_n \) are the displacements of normal reactions caused by the realization of tangential forces \( F'_r \) and \( F''_r \) in the contact point; \( h'_0 \) and \( h''_0 \) are the displacements of normal reactions due to hysteresis in tire material, which can be defined as follows:

\[ h'_0 = f_0 r (1 + \frac{r}{r_\delta}) \frac{a''(r)}{a''(r)} \]  

(35)

where \( a' \) and \( a'' \) are the length value of the contact area of a wheel with drums under the action of normal forces \( F'_n \) and \( F''_n \); \( a''(r) \) - a half of the length of the contact area on the surface under the action of forces \( F'_z = F'_n \) and \( F''_z = F''_n \).

Application points of tangential forces are determined using formulas (28) (for low tangential forces, using (29)).
Figure 3. Forces shoulders acting on the wheel.

Figure 4. Forces acting on the wheel while rolling on two drums.

γ angles that determine the displacement of the application point of normal reactions of drums, can be determined using geometrical considerations:

\[ \gamma' = \arcsin \frac{h'_n + h'_0}{e'} \quad \gamma'' = \arcsin \frac{h''_n + h''_0}{e''} \] (36)

Under the action of \( M_k \) moment, the wheel center is located asymmetrically relative to the drums axes; herewith:
\[
\alpha' = \arccos \left( \frac{(e')^2 + c^2 - (e'')^2}{2e'e''} \right)
\]
\[
\alpha'' = \arcsin \left( \frac{e' \sin \alpha'}{e''} \right)
\]

3. Conclusion

1. The formulas were obtained that allow to calculate the power and kinematic parameters of a wheel when it is rolling on one or two rough drums.

2. The rolling resistance coefficient of a driven wheel along a rough drum increases by \(a(1+r/r_{\delta})/a^{\alpha_{\delta}}\) times compared to rolling on a flat firm surface.

3. An increase of rolling resistance on a drum leads to a difference in lateral drag coefficients determined on a drum and during wheel movement on a flat supporting surface.

4. While using lateral drag and rolling resistance coefficients obtained experimentally on a drum stand, for the case of wheel movement on a flat support surface, appropriate correction factors should be performed.

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