Detection is truncation: studying source populations with truncated marginal neural ratio estimation

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Abstract

Statistical inference of population parameters of astrophysical sources is challenging. It requires accounting for selection effects, which stem from the artificial separation between bright detected and dim undetected sources that is introduced by the analysis pipeline itself. We show that these effects can be modeled self-consistently in the context of sequential simulation-based inference. Our approach couples source detection and catalog-based inference in a principled framework that derives from the truncated marginal neural ratio estimation (TMNRE) algorithm. It relies on the realization that detection can be interpreted as prior truncation. We outline the algorithm, and show first promising results.

1 Introduction

Point sources detection is crucial for astronomical surveys, and is the cornerstone for the compilation of source catalogues. Those source catalogues are then typically the basis for the inference of physical parameters that describe the sources at the population level. Upcoming astronomical facilities, such as the Square Kilometer Array (SKA) [1] and the Cherenkov Telescope Array (CTA) [2] will deliver large and complex datasets. In order to leverage their full potential, it is urgent to develop robust and automated source detection and source population parameters inference algorithms.

Recent developments in deep learning and more generally automatic differentiation frameworks [3] are increasingly used for tackling difficult astronomical data analysis challenges. The capability of deep learning techniques of point sources detection and population characterization has been demonstrated across different wavelengths surveys, e.g. in γ-ray data [4–7], radio data [8–11] and cosmic microwave background data [12]. In particular simulation-based machine learning approaches can be highly flexible, allowing to tailor developed pipelines to specific telescopes and science cases. A range of simulation-based inference (SBI) algorithms have been proposed in the literature (see Cranmer et al. [13] for a review). An appealing feature is that they generally allow to directly estimate marginal posteriors for parameters of interest [14]. Furthermore, sequential SBI approaches [15–17] have been shown to be particularly simulation efficient. Among those, truncated marginal neural ratio estimation (TMNRE) [14, 18] is a sequential SBI approach based on neural ratio estimation (NRE) [19], which particularly well composes with marginalization.

Here, we present a strategy for how to use TMNRE [1] to simultaneously perform source detection and population-level parameters inference. This enables to self-consistently combine information from both detected and sub-threshold sources, without being affected by detection biases. The key idea is to recast the traditional concept of source detection in terms of prior truncation. This will allow us to distill information of bright sources directly into the simulation model. Our proposed method is highly interpretable since it resembles components of traditional survey analysis workflows.

1 We use swyft TMNRE implementation that can be found at https://github.com/undark-lab/swyft

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2 Methodology

Background TMNRE (and NRE in general) performs posterior estimation by solving a binary classification problem. Given a model \( p(x, z) = p(x|z)p(z) \), where \( x \) is data, and \( z \) a set of parameters of interest, one trains a network to distinguish joined samples \( x, z \sim p(x)p(z) \) from marginal samples \( x, z \sim p(x)p(z) \). The networks learn to estimate the likelihood-to-evidence ratio \( r(z; x) = p(x|z)/p(x) \) which we can use to obtain weighted samples from the posterior \( p(z|x) = r(z; x)p(z) \). In order to improve the network’s learning and maximise the simulator efficiency, TMNRE concentrates in stages the regions in parameter space from which training examples are drawn based on a target observation. Hence, this truncation scheme restricts the prior distribution’s support without modifying its shape, as opposed to other sequential methods that employ a posterior estimate as proposal distribution for generating simulations for the next round \[15\].

Simulation model We consider here a simple Bayesian hierarchical source model,

\[
p(x, \bar{s}, \vartheta) = p(x|\bar{s})p(\vartheta) \prod_{i=1}^{N} p(s_i|\vartheta),
\]

where \( x \) is the observed sky map, \( s_i \equiv (F_i, \Omega_i) \) denotes the flux \( F_i \) and position \( \Omega_i \) of point source \( i \), and \( \vartheta \equiv \{N, \Sigma, h\} \) collects source population parameters, namely the number of sources \( N \), and the parameters \( \Sigma \) and \( h \) that control the flux and spatial distributions respectively. Finally, we add instrumental effects to simulated maps. We give more details on the model in appendix A. We show examples of our simulated maps in the top row of the right panel of fig. 2.

Source detection For source detection we consider the following likelihood-to-evidence ratio

\[
r_1(\Omega, F_{th}; x) = \frac{p(\Omega|x(F \geq F_{th}) = 1, \Omega|x)}{p(\Omega|x(F \geq F_{th}) = 0, \Omega|x)}.
\]

Here, the denominator corresponds to the prior probability of having a source at position \( \Omega \) with a flux \( F \geq F_{th} \) that exceeds some threshold flux \( F_{th} \). The numerator is the corresponding posterior. We model the source detection ratio estimator in eq. (2) as an image-to-image neural network that solves a binary classification problem in each image pixel. For simulated data, we call a simulated source \( s_i \) ‘detected’ when there is a corresponding compact region as function of \( \Omega \) where the detection significance is above threshold, \( r_1(\Omega, F_{th}; x) > 5 \). This effectively leads to a split between sources that are clearly identifiable and ‘sub-threshold’ sources that are difficult to detect a individual instances. Below, we assign the detection label \( d_i = 1 \) (\( d_i = 0 \)) to detected (undetected) sources.

In order to characterize the split between detected and sub-threshold sources, we introduce a source-sensitivity function, \( S(F, \Omega) \), which provides the probability that a source with flux \( F \) and at position \( \Omega \) would be detected by the ratio estimator in eq. (2). This function can be estimated by training the ratio estimator

\[
r_2(d; F, \Omega, x) \equiv \frac{p(d|F, \Omega, x)}{p(d)}
\]

which is marginalised over all other sources and source parameters. By omitting the dependence on the map, \( x \), which is then effectively marginalized, the ratio estimator can then simply be modeled as a \( \mathbb{R}^3 \rightarrow \mathbb{R} \) multi-layer perceptron. The source-sensitivity function can then be estimated as

\[
S(F, \Omega) = \sigma \left( \log \left( \frac{p(d = 1|F, \Omega)}{p(d = 0|F, \Omega)} \right) \right),
\]

where we have introduced the sigmoid function \( \sigma(y) = 1/(1 + e^{-y}) \).

We can make the concept of source detection part of our model as follows. In a random realization, each source \( i \) will be either detected, \( d_i = 1 \) (with probability \( S(F_i, \Omega_i) \)), or not detected, \( d_i = 0 \) (with probability \( 1 - S(F_i, \Omega_i) \)). To keep notation simple, we omit \( d_i \) and instead group detected

\[1\] In the next section we will use the notation \( r(a; b|c) = \frac{p(a,b|c)}{p(a|c)p(b|c)} \), where with |\( a \)’ we refer to conditioning on specific variables for all factors in the ratio definition. If necessary, multiple variables are comma separated, for example \( r(a, b; c|d) = \frac{p(a,b,c|d)}{p(a,b|d)p(c|d)} \). Training conditional ratios is a straightforward extension of NRE.
while sub-threshold sources vary freely. Two inference networks are trained to capture information from sub-threshold sources ($r_3$) and detected sources ($r_4$).

The second step in eq. (8) corresponds to the truncation approximation. It is exact (in the sense of leaving $p(\theta|x_o)$ unaffected) in the limit where $p(x_o|\tilde{s}_{det}) = 0 \rightarrow 0$. Training that ratio estimator directly with TMNRE would be challenging since $\mathbb{I}_{x_o}(\tilde{s}_{det}) = 1$ has very small support in the training data. Instead, we split the ratio into two computationally feasible ratios (this is in spirit similar to the telescoping ratio estimation approach presented in Rhodes et al. [20]).
We apply the proposed methodology to the target observation $x_o$, because detected sources are assumed to be fixed in the parameter space where $r$. The first ratio, $r_3(\hat{\theta}; x) \|_{\Omega_o} = 1$, can be estimated by training a peak-count network on targeted data, that is truncated to $I_{\Omega_o}(\hat{s}_{det}) = 1$. The second ratio can be estimated as

$$
r(\hat{\theta}; I_{\Omega_o}(\hat{s}_{det}) = 1) = \frac{p(I_{\Omega_o}(\hat{s}_{det}) = 1 | \hat{\theta})}{p(I_{\Omega_o}(\hat{s}_{det}) = 1)}
$$

so it can be estimated by training a network on detected sources lists on un-truncated training data. In practice, we generate weighted samples from the full posterior $p(\hat{\theta}| x)$ by sampling $\hat{\theta}, \hat{s}_{det} \sim p(\hat{\theta})p(\hat{s}_{det}|I_{\Omega_o}(\hat{s}_{det}) = 1)$ with weights $w = r_3(\hat{\theta}; x) \|_{\Omega_o}(\hat{s}_{det}) = 1 \cdot r(\hat{\theta}; I_{\Omega_o}(\hat{s}_{det}) = 1)$.

In the process, we trained four ratio estimation networks that are directly connected with traditional source analysis pipeline components: $r_1(\Omega, F; x)$ performs source detection; $r_2(d; F, \Omega, x)$ is the source sensitivity function; $r_3(\hat{\theta}; x) \|_{\Omega_o}(\hat{s}_{det}) = 1$ constraints $\hat{\theta}$ based on sub-threshold sources (because detected sources are assumed to be fixed in the parameter space where $I_{\Omega_o}(\hat{s}_{det}) = 1$); and $r_4(\hat{s}_{det}; \hat{\theta})$ constraints $\hat{\theta}$ using the detected sources catalog. We show a schematic overview of the inference framework used in this work in fig. [1].

### 3 Results

We apply the proposed methodology to the target observation $x_o$, shown in fig. [2]. We first train the source detection ratio estimator in eq. (2) on data simulated from the full model shown in eq. (1), and then apply it to $x_o$ to obtain a detection map. From the detection map we derive a catalog of detected sources $\hat{s}_{det}$, and define a truncated parameter space of interest, where $I_{\Omega_o}(\hat{s}_{det}) = 1$. In order to make source detection part of our model, we train the sensitivity ratio estimator in eq. (3) to estimate the sensitivity function $S(F, \Omega)$. We then generate targeted training data from our truncated simulation model in eq. (7). We show samples from the full model and the truncated one in fig. [2].

Finally, we train two inference networks to capture information regarding population parameters from sub-threshold and detected sources, as explained in section [2]. We show the constraints on population parameters $\vartheta$ from sub-threshold sources, detected ones, and their combination in fig. [3]. The different posteriors are consistent with each other, indicating that the proposed inference framework automatically accounts for detection biases. We see that different constraints are dominated by different neural networks, e.g. the number $N$ of point source is better constrained by sub-threshold ones, whereas the spatial distribution parameter $h$ by detected ones. The weaker constraint on the flux parameter $\Sigma$ inferred from detected sources is due to the fact that in eq. (9) we average over different detected sources realisations, always re-sampling the parameter $\Sigma$ (see appendix [A] for the hierarchical model details).

The four neural networks were trained on a NVIDIA GeForce RTX 3080 Ti GPU, the total computational time cost to obtain the results shown in fig. [3] is $\sim 2$ hours. We intend on describing in details the networks used for each task and choice of hyperparameters in future work.
4 Conclusions

We have introduced a novel method to self-consistently perform point sources detection and source population parameters inference using TMNRE. With this approach, we can exploit information of detected as well as sub-threshold sources for population-level parameter inference. Detection biases are automatically accounted for in our approach. Exemplary results of our approach are shown in fig. 3, where we show inference results on source population parameters from both detected point sources and sub-threshold sources separately, as well as their combination. Since the proposed method is essentially a specific implementation of TMNRE, we expect that it inherits its positive properties in terms of simulation-efficiency and scalability [18]. A possible shortcoming of this approach is that multiple neural networks need to be trained self-consistently, which on the other hand have a clear interpretation in terms of traditional analysis pipeline components. A potential application beyond those directly intended is to detectable and sub-threshold substructures in strong gravitational lenses.

Broader Impact This work is focusing on the analysis of astronomical surveys that contain point sources population via TMNRE. Variants of the presented approach could find application in other areas of the physical sciences. We do not expect any negative societal impact of the presented methods. However, we recommend the usual caution in inferring scientific conclusions based on a complex methodology.

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Table 1: Point source simulation model parameters and priors.

| Parameter                  | Prior          |
|----------------------------|----------------|
| Population parameters      |                |
| number of point sources    | $N$ $\sim \mathcal{U}(10, 500)$ |
| flux distribution parameter| $\Sigma$ $\sim \mathcal{U}(1, 3)$ |
| spatial distribution parameter | $h$ $\sim \mathcal{U}(1, 20)$ |
| Point source parameters    |                |
| flux                       | $F$ $\sim \log \mathcal{N}(1, \Sigma)$ |
| position                   | $\Omega \equiv (l, b)$ $\sim (\mathcal{N}(0, 20), \mathcal{N}(0, h))$ |

A Appendix: Simulation model

We describe in more detail the Bayesian hierarchical point source model adopted in this work. To generate an observation, first, we sample point sources population parameters $\vartheta \equiv \{N, \Sigma, h\}$ from their priors, given in table 1. Then, for each point source, we draw its flux $F$ and position on the map $\Omega \equiv (l, b)$ from the priors given in table 1. We then generate a $128 \times 128$ pixels map. To model instrumental effects, we add a point-spread function (PSF) with Gaussian kernel standard deviation $\epsilon = 1.5$ and Poisson noise, obtaining the final simulated map $x$. 