Time-jerk optimal trajectory planning of hydraulic robotic excavator

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Abstract
Due to the fact that intelligent algorithms such as Particle Swarm Optimization (PSO) and Differential Evolution (DE) are susceptible to local optima and the efficiency of solving an optimal solution is low when solving the optimal trajectory, this paper uses the Sequential Quadratic Programming (SQP) algorithm for the optimal trajectory planning of a hydraulic robotic excavator. To achieve high efficiency and stationarity during the operation of the hydraulic robotic excavator, the trade-off between the time and jerk is considered. Cubic splines were used to interpolate in joint space, and the optimal time-jerk trajectory was obtained using the SQP with joint angular velocity, angular acceleration, and jerk as constraints. The optimal angle curves of each joint were obtained, and the optimal time-jerk trajectory planning of the excavator was realized. Experimental results show that the SQP method under the same weight is more efficient in solving the optimal solution and the optimal excavating trajectory is smoother, and each joint can reach the target point with smaller angular velocity, and acceleration change, which avoids the impact of each joint during operation and conserves working time. Finally, the excavator autonomous operation becomes more stable and efficient.

Keywords
Trajectory planning, hydraulic robotic excavator, splines, jerk, execution time, optimization

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Introduction
Hydraulic excavators are widely used in extremely harsh environments for mining, transportation, and civil engineering. However, the following problems persist regarding the process of work: (1) the high intensity of labor required from the operators, (2) a dangerous working environment, and (3) the large amounts of money, material resources, and time required for an operator to develop the requisite skill. Thus, unmanned excavators have gradually replaced manually operated ones. Intelligent excavators can be used for submarine operations and earthquake relief work as well as at nuclear power plants. Improving the operating efficiency and reliability of the excavator is an important research subject. Trajectory planning is the premise and foundation for realizing control of the excavation trajectory. An appropriately planned trajectory enables quick and smooth completion of the excavation task. In particular, a large jerk value for each joint during operation results in unsmooth motion, which has a significant negative impact on the hydraulic cylinder. This also reduces the service life of the machine and damages the equipment.

To ensure the speed and stability of excavator in independent operation, the planning of time-jerk optimal excavation trajectory has become a research hot-
spot. In the trajectory planning method with optimal
time or smoothing as the goal, Liu et al.9 applied inter-
polation in joint space using B-splines and also
employed SQP for optimizing the minimum time tra-
jectory. However, they did not consider the adverse effects
of larger jerk on the work equipment. Boryga and
Grabos10 used a high-order polynomial with only one
unknown parameter to achieve time-optimal trajectory
planning but presupposed that the maximum acceler-
ation value of the terminal endpoint moving along the
path must be given. Elnagar and Hussein11 used the
numerical iterative method to obtain the optimal trajec-
tory of a robot in energy consumption in a 3D environ-
ment with certain boundary conditions. Lin et al.12
projected the minimum time trajectory of an industrial
robot by using a polyhedron search. By imposing con-
straints on velocity, acceleration, and jerk, cubic splines
have been used to interpolate optimal trajectory in the
joint space. Piazza and Visioli13 proposed interval anal-
ysis, which requires presetting the total time of the tra-
jectory in order to obtain the optimal trajectory.
Gasparetto and Zanotto,14 Gasparetto et al.,15 and
Zanotto et al.16 used cubic splines to plan a multi-
objective optimal trajectory and verified it for a robot
through experimentation. Wang et al.17 uses the Beetle
Swarm algorithm to optimize the minimum time and
joint rotation angle optimal trajectory. Zhang et al.18
solved the problem of non-convex global optimal tra-
jectory planning and produced an efficient and contin-
uous trajectory. For the point-to-point optimal
trajectory planning problem, Wang et al.19 obtained
the point-to-point time optimal trajectory based on the
expression of interpolation in the form of multiple
roots. However, each joint obtained by the solution
does not synchronously reach the limits of their allow-
able angular velocity, acceleration, and jerk. Qian
et al.20 improved the B-spline interpolation method and
optimal the time-jerk trajectory. Fang et al.21,22 use the
sigmoid piecewise function and the improved sinusoidal
function to achieve interpolation. According to the
given angular velocity, acceleration, and jerk con-
straints of each joint, they acquired the synchronized
motion curve. Zhao et al.23 solve the optimal trajectory
planning problem considering the hybrid objective of optimizing time, energy, and
jerk.24,25 During this process, the robots efficiency, sta-
bility, and energy consumption of the robot during the
operation were comprehensively considered. Kim
et al.26 regarded velocity and acceleration as given val-
es to optimize time-torque trajectory planning of the
hydraulic excavator. Xiao et al.27 used cubic splines to
fit joint angles in joint space to conduct online form
time-optimal trajectory planning for industrial robots.

In this paper, the Lipai PC1012 hydraulic robotic
excavator is taken as the research object. To ensure the
efficiency and stability of the excavator in the working
process, this paper takes the angular velocity, angular
acceleration, and jerks as the constraints, and uses the
SQP, PSO28 and DE29 algorithms to optimize the time-
jerk optimal cubic spline interpolation trajectory. The
experimental results show that the mining trajectory
obtained by the SQP algorithm is more efficient and
smooth, and the SQP algorithm more efficient to obtain
the optimal solution.

The remainder of this paper is organized as follows.
Section 2 briefly introduces joint trajectory using cubic
spline parameterization. Section 3 establishes the opti-
mization model of the multi-objective function and the
process of using SQP algorithm optimization is intro-
duced. Section 4 simulates the trajectory planning
problem using different weight coefficients and also
analyzed the optimal trajectory obtained through SQP,
DE, and PSO optimization. Finally, Section 5 sum-
marizes the conclusions.

Parameterized trajectory using cubic
splines

Cubic spline interpolation is a universal method that
can ensure the succession of acceleration in trajectory
planning, and the obtained trajectory is smoother and
continuous. Compared with the NURBS interpolation
method,30 the calculation amount and complexity of the
cubic spline are small. And it can also help avoid
Runge’s phenomenon that results in excessive oscilla-
tions and collisions in the case of high-order poly-
nomial interpolation. In this paper, cubic splines are used
for optimal trajectory planning in joint space.

Taking the \(j-th\) joint as an example, \(s_m\) indicates a
series of the given joint angles and every two adjacent
joint angle values are connected by cubic splines. \(t_1, t_2, \ldots, t_n\) are the time required for passing through
each interpolation point \((n = s_m + 2)\). The time interval
between continuous interpolation points is \(h_i = t_{i+1} - t_i\). The expressions of joint acceleration on the
interval are linear functions of \(t\).

\[
\ddot{Q}_{j,i}(t) = \frac{t_{i+1} - t}{h_i} \ddot{Q}_{j,i}(t_i) + \frac{t - t_i}{h_i} \ddot{Q}_{j,i}(t_{i+1}) \quad (1)
\]

By integrating equation (1) twice with respect to the
given boundary conditions \(\dot{Q}_{j,i}(t_i) = \dot{q}_{j,i}\) and
\(\ddot{Q}_{j,i}(t_{i+1}) = \ddot{q}_{j,i+1}\). The expression for joint displace-
ment \(Q_{j,i}(t)\) can be obtained as follows.

\[
Q_{j,i}(t) = \int_{t_i}^{t_{i+1}} \ddot{Q}_{j,i}(t) dt + \int_{t_i}^{t} \dot{Q}_{j,i}(t) dt + Q_{j,i}(t_i)
\]
The expressions for joint velocity were obtained by calculating the first derivative of equation (2). Based on the conditions of continuous joint velocity through each interpolation point, the relation among the interpolation time interval, joint displacement, and acceleration can be obtained as follows.

\[
Q_j(i) = \frac{\hat{Q}_j(t_i)}{6h_i}(t_{i+1} - t_i)^3 + \frac{\hat{Q}_j(t_{i+1})}{6h_i}(t - t_i)^3 + \left( \frac{q_{i+1}}{h_i} - \frac{\hat{Q}_j(t_{i+1})}{6h_i} \right)(t - t_i) + \left( \frac{q_{i}}{h_i} - \frac{\hat{Q}_j(t_{i})}{6h_i} \right)(t_{i+1} - t)
\]

\[i = 1, 2, \ldots n - 1\]

The expressions for joint velocity were obtained by calculating the first derivative of equation (2). Based on the conditions of continuous joint velocity through each interpolation point, the relation among the interpolation time interval, joint displacement, and acceleration can be obtained as follows.

\[
Aa_j = O_j \quad \forall j = 1, 2, 3, 4
\]

\[
A = \begin{bmatrix}
           c_{11} & c_{12} & 0 & 0 & 0 & 0 \\
           c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\
           0 & c_{32} & c_{33} & c_{34} & 0 & 0 \\
           0 & 0 & c_{43} & c_{44} & c_{45} & 0 \\
           0 & 0 & 0 & c_{54} & c_{55} & c_{56} \\
           0 & 0 & 0 & 0 & c_{65} & c_{66}
\end{bmatrix}
\]

\[
c_{11} = 3h_1 + 2h_2 + h_1^2/h_2, c_{12} = h_2, c_{21} = h_2 - h_1^2/h_2, c_{22} = 2(h_2 + h_3), c_{23} = h_3, c_{32} = h_3, c_{33} = 2(h_3 + h_4), c_{34} = h_4, c_{43} = h_4, c_{44} = 2(h_4 + h_5), c_{45} = h_5, c_{54} = h_5, c_{55} = 2(h_5 + h_6), c_{56} = h_6^2 - h_7^2/h_6, c_{65} = h_6, c_{66} = 3h_7 + 2h_6 + h_7^2/h_6.
\]

\[
a_j = \begin{bmatrix}
          \hat{Q}_j(t_2) \\
          \hat{Q}_j(t_3) \\
          \hat{Q}_j(t_4) \\
          \hat{Q}_j(t_5) \\
          \hat{Q}_j(t_6) \\
          \hat{Q}_j(t_7)
\end{bmatrix}
\]

\[
O_j = \begin{bmatrix}
          6\left( \frac{q_{j,3}}{h_2} + \frac{q_{j,1}}{h_1} \right) - 6\left( \frac{q_{j,1}}{h_2} + \frac{q_{j,1}}{h_1} \right) \\
          6\left( \frac{q_{j,1}}{h_2} + 6q_{j,4} \right) - 6\left( \frac{q_{j,3}}{h_2} + \frac{q_{j,3}}{h_1} \right) \\
          6\left( \frac{q_{j,5} - q_{j,4}}{h_3} - \frac{q_{j,4} - q_{j,3}}{h_1} \right) \\
          6\left( \frac{q_{j,6} - q_{j,5}}{h_5} - \frac{q_{j,5} - q_{j,4}}{h_4} \right) \\
          6\left( \frac{q_{j,8}}{h_7} + \frac{q_{j,6}}{h_6} \right) - 6\left( \frac{q_{j,6}}{h_7} + \frac{q_{j,6}}{h_6} \right)
\end{bmatrix}
\]

where \(A\) is nonsingular and diagonal about the interpolation time interval, \(a_j\) is the acceleration at each interpolation point, and \(O_j\) is a vector of the joint displacement and interpolation time interval.

### Modeling and trajectory optimization

To improve the efficiency of completing a given task, Wang et al.\(^3\) took time as the optimization goal and conducts time-optimal trajectory planning and to improve the smoothness of the trajectory, Lu et al.\(^1\) took jerk as the performance index to obtain a smooth optimal trajectory. While Huang et al.\(^4\) comprehensively considers multiple factors and establishes a multi-objective trajectory planning. So in this paper, to ensure the efficient and smooth motion of each joint, the problems of time and jerk were considered simultaneously for determining the optimal trajectory. The optimal objective function is as follows.

\[
\min f(h) = w_1 \sum_{i=1}^{n-1} h_i + (1 - w_1) \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \left[ \frac{(q_{j,i+1} - q_{j,i})^2}{h_i} \right] \\
\text{s.t.} \quad |\dot{q}_j(h)| - WC_j \leq 0 \\
|\ddot{q}_j(h)| - JC_j \leq 0 \\
|\dddot{q}_j(h)| - JC_j \leq 0 \\
j = 1, 2, 3, 4
\]

In equation (7), the first item is the total time needed to complete the task, which is related to efficiency. The second item is the total jerk at each joint, which is related to the problem of smoothness. The \(w_1\) is the weight coefficient. When \(w_1\) is small, the optimal trajectory focuses on the influence of jerk on the equipment, where \(WC_j, JC_j\) are related to the constraints on the velocity, acceleration, and jerk, respectively, of the revolute, boom, arm, and bucket joints.

It can be seen from equations (3)–(7) that the parameters to be solved in the expression are closely related to the interpolation time of each segment of the excavation path when calculating the cubic spline expression. The SQP is used to find the optimal value of each interpolation time. When each optimal interpolation time can be optimized, the expression of the joint angle curve can be obtained. Through time-jerk optimal trajectory planning, the excavator can reach the target point with a small range of angle changes in the actual moving process, so as to realize the completion of the excavation task in a short time and ensure the smooth operation of each joint in the working process to the servo control of the control system. By limiting the angular velocity, acceleration, and jerk of each joint in the process of moving, the negative effect on the hydraulic cylinder can be avoided in the process of operation.

The time intervals \(h_i\) used as optimization variables, which are solved by SQP, must have appropriate initial
values. Further, each value of $h_i$ has a lower bound that should be satisfied by equation (8).

$$h_{lb} \geq \max_{j=1,2,3,4} \left\{ \frac{|q_{j,i+1} - q_{j,i}|}{VC_j} \right\} > 0, \quad i = 1, 2, \ldots, n - 1$$  \tag{8}

In addition to considering the lower bound, the values of $k_1, k_2, k_3$ are calculated using equation (9).

$$
\begin{align*}
  k_1 &= \max_{j=1,2,3,4} \max_{t=[t_i, t_{i+1}], i=1,2,\ldots,n-1} \left\{ \frac{|Q_{j,i}(t_i)|}{VC_j} \right\} \\
  k_2 &= \max_{j=1,2,3,4} \max_{t=[t_i, t_{i+1}], i=1,2,\ldots,n-1} \left\{ \frac{|Q_{j,i}(t_i)|}{WC_j} \right\} \\
  k_3 &= \max_{j=1,2,3,4} \max_{t=[t_i, t_{i+1}], i=1,2,\ldots,n-1} \left\{ \frac{|Q_{j,i}(t_i)|}{JC_j} \right\}
\end{align*}
$$  \tag{9}

Based on the values of $k_1, k_2, k_3$, the suitable initial values $h^0$ can be obtained as follows.

$$h^0 = h_{lb} \cdot \max \left\{ 1 \quad k_1^{1/2} \quad k_2^{1/2} \quad k_3^{1/2} \right\}$$  \tag{10}

The trajectory planning problem is a nonlinear optimization problem. This problem is transformed into the Quadratic Programming (QP) problem to solve it using SQP. Through Taylor expansion, the objective function can be reduced to a quadratic function at the iteration point $h^k$ and the constraint functions can be reduced to linear functions. Further, the QP problem becomes a problem with respect to variable $H$ as follows.

$$
\min f(h) = \frac{1}{2} H^T XH + Y^T H
$$

$$
\begin{align*}
  \text{s.t.} \quad & \nabla \dot{q}(h^k)H + \ddot{q}(h^k) - VC_j \leq 0 \\
  & \nabla \dot{q}(h^k)H + \ddot{q}(h^k) - WC_j \leq 0 \\
  & \nabla \dot{q}(h^k)H + \ddot{q}(h^k) - JC_j \leq 0 \\
  & j = 1, 2, 3, 4
\end{align*}
$$  \tag{11}

where $H = h - h^0$, $X = \nabla^2 f(h^0)$, and $Y = \nabla f(h^0)$.

The specific steps for solving the optimization problems are as follows and the corresponding flowchart is shown in Figure 1.

1. Solve $h^0$ using equations (8)–(10), set $\varepsilon = 0, k = 0$, and calculate the Hessian matrix $X_0$.
2. Simplify point $h^0$ to a QP problem.
3. Solve the QP problem and consider $H^k = H^*$.
4. Execute the constrained one dimensional search of the objective function along the direction $H^k$ and output point $h^k + 1$.

Figure 1. Optimization flowchart of SQP algorithm.
between the coordinate systems. In Figure 3, the transformation relationship between the coordinate systems by deriving the homogeneous transformation matrix listed in Table 1. This method expresses the transformation relationship between the coordinate systems by the working device of the excavator. The mining path planned according to the corresponding angle values of each joint of the excavator moves at the same time.

Simulation and analysis of results

The PC1012 hydraulic robotic excavator is used as a research object, shown in Figure 2. According to the D-H coordinate system method, the excavator working device model is established, as shown in Figure 3. The relevant D-H parameters are listed in Table 1. This method expresses the transformation relationship between the coordinate systems by deriving the homogeneous transformation matrix between the coordinate systems. In Figure 3, the transformation matrix between the link joints is

\[ X_{k+1} = X_k + q_k q_i^T s_k - X_k^T s_k X_k s_k \]  

(12)

where \[ s_k = h_k + 1 - h_k, \quad q_k = \left[ \nabla f(h_k + 1) + \sum_{i=1}^9 \gamma_i \nabla^2 f_i(x_k) \right], \quad k = k + 1; \] then, return to Step 2.

Table 1. D-H parameters of the PC1012 excavator.

| Joint | \( d_i/m \) | \( a_i/m \) | \( \alpha_i/° \) | \( \theta_i/° \) |
|-------|-------------|-------------|----------------|----------------|
| 1     | 0.665       | 0.322       | 90             | -180 to 180    |
| 2     | 0           | 1.35        | 0              | -53.83 to 54.62|
| 3     | 0           | 0.7         | 0              | -156.61 to -32.2|
| 4     | 0           | 0.423       | 0              | -165.4 to 14.6 |

\[ ^0A_4 = \begin{bmatrix} c_1 s_{234} & -c_1 s_{234} & s_1 & c_1(a_4 c_{234} + a_3 c_{23} + a_2 c_2 + a_1) \\ s_1 s_{234} & -s_1 s_{234} & c_1 & s_1(a_4 c_{234} + a_3 c_{23} + a_2 c_2 + a_1) \\ s_{234} & c_{234} & 0 & a_4 s_{234} + a_3 s_{23} + a_2 s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  

(13)

where \[ c_i = \cos \theta_i, \quad s_i = \sin \theta_i, \quad c_ijk = \cos (\theta_i + \theta_j), \quad s_ijk = \sin (\theta_i + \theta_j) \], and \( a_1, a_2, a_3, a_4 \) are the joint lengths of the revolute, boom, arm, and bucket. The coordinates of the end position relative to the base coordinate system are

\[ ^0P = ^0A_4 P \]  

(14)

where \[ ^0A_4 = ^0A_1 ^1A_2 ^2A_3 ^3A_4, ^0P = [0, 0, 0, 1]^T \].

Set the position and posture coordinate point of the tip of the bucket as \([x, y, z, \xi]^T\). According to equation (14), the expression of the bucket tooth tip relative to the basal coordinate is shown below.

\[
\begin{align*}
  x &= c_1(a_4 c_{234} + a_3 c_{23} + a_2 c_2 + a_1) \\
  y &= s_1(a_4 c_{234} + a_3 c_{23} + a_2 c_2 + a_1) \\
  z &= a_4 s_{234} + a_3 s_{23} + a_2 s_2 + d_1 \\
  \xi &= \theta_2 + \theta_3 + \theta_4
\end{align*}
\]  

(15)

where \( \xi \) is the attitude angle of bucket tooth tip.

In Table 1, joint 1 represents the rotary platform, joint 2 is the boom, joint 3 is the arm, and joint 4 is the bucket.

Due to the limited range of the driving mechanism, according to the limited range of the angles of each joint in Table 1, without considering rotation \((y = 0)\), each joint of the excavator moves at the same time. The allowable working space at the end of the bucket tooth tip is shown in Figure 4.

This paper takes digging as an example and selects the excavation path point in the working range allowed by the working device of the excavator. The mining path points are directly given by the coordinate points at the end of the bucket tooth tip, while the trajectory is planned according to the corresponding angle values of each joint. Therefore, the coordinate value of the path point must be converted into the angle value of each joint corresponding to the coordinate point. This process realizes the conversion from the coordinate space to the
joint space through the inverse kinematics solution. According to equation (16), the analytical method is used to obtain the joint angle values of the boom, arm, and bucket corresponding to the excavation path point. And the corresponding results are shown in Table 2.

\[
\begin{align*}
\tan \theta_1 &= \frac{y}{x} \Rightarrow \theta_1 = \arctan \left( \frac{y}{x} \right) \\
\theta_2 &= \arctan \left( \frac{z - d_1 - a_4x_{234}}{\frac{1}{c_1} - a_1 - a_4x_{234}} \right) \\
\cos \theta_3 &= \frac{\left( \frac{1}{c_1} - a_1 - a_4x_{234} \right)^2 + (z - d_1 - a_4x_{234})^2 - a_2^2 - a_3^2}{2a_2a_3} \\
\sin \theta_3 &= \pm \sqrt{1 - \cos^2 \theta_3} \\
\theta_4 &= \xi - \theta_2 - \theta_3
\end{align*}
\]

There are two extra points in Table 2. They are not given and merely applied to satisfy the continuous condition of velocity and acceleration. Its expressions are as follows.

\[
q_{j,2} = q_{j,1} + h_1v_{j,1} + \frac{h_1^2}{3}a_{j,1} + \frac{h_1^2}{6}\ddot{q}_{j,2}(t_2)
\]

\(j = 1, 2, 3, 4\)

Table 2. Conversion of control point from Cartesian space into joint space.

| Points (m) | Rotation angle (°) | Boom angle (°) | Arm angle (°) | Bucket angle (°) |
|-----------|--------------------|---------------|---------------|-----------------|
| (2, 0, 0) | 0                  | 9.8099        | −55.1056      | −67.7043        |
| Extra point | (1.75, 0, −0.5)   | 0             | −10.612       | −67.7956        |
| (1.5, 0, −1) | 0             | −41.3316      | −47.1904      | −94.2349        |
| (1, 0, −1)  | 0                | −47.113       | −27.7956      | −105.0914       |
| (0.75, 0, −0.5) | 0          | −34.0584      | −83.5227      | −92.4189        |
| Extra point | (0.5, 0, 0)      | 0             | −38.1773      | −125.4859       |

Figure 4. Working space of the PC1012 excavator.

Table 3. Physical constraints of the PC1012 excavator.

| Constraints | Boom | Arm | Bucket |
|-------------|------|-----|--------|
| Velocity (°) | 28   | 35  | 30     |
| Acceleration (°) | 10   | 15  | 10     |
| Jerk (°)   | 10   | 10  | 10     |

\[
q_{j,n-1} = q_{j,n} - h_{n-1}v_{j,n} + \frac{h_{n-1}^2}{3}a_{j,n} + \frac{h_{n-1}^2}{6}\ddot{q}_{j,n-1}(t_{n-1})
\]

\(j = 1, 2, 3, 4\)

In the process of obtaining the optimal solution, the constraint conditions of joint angular velocity, acceleration, and jerk are shown in Table 3.

Under the same constraint conditions, SQP, PSO, and DE algorithms are used successively to optimize the best trajectory of time-jerk under different weight coefficients. The optimization results are shown in Table 4. In the same configuration conditions, Table 5 shows the time taken to find the best trajectory of the time-jerk of the cubic spline curve using these three algorithms.

When \(w_1 = 1\), the optimal joint angle, velocity, acceleration, and jerk curves of the boom, arm, and bucket solved by the SQP are shown in Figure 5.

Substituting the optimal solution corresponding to \(w_1 = 0.6\) into the equations (3)–(6), the expression of the joint angle is obtained. The joint angle expressions of the boom, arm, and bucket obtained by SQP, PSO, and DE algorithms as follow.

The expressions of the boom \(\theta_1\), arm \(\theta_2\), and bucket \(\theta_3\) joint obtained by the SQP algorithm are as follows equations (19)–(21). The expressions of the boom \(\theta_1\), arm \(\theta_2\), and bucket \(\theta_3\) joint obtained by the PSO algorithm are as follows equations (22)–(24). The expressions of the boom \(\theta_1\), arm \(\theta_2\), and bucket \(\theta_3\) joint obtained by the DE algorithm are as follows equations (25)–(27), respectively.
Table 4. Optimal value results.

\[
\begin{array}{cccccccccc}
  \text{Algorithm} & \text{w} \_1 = 1 & \text{w} \_1 = 0.6 & \text{w} \_1 = 0.5 & \text{w} \_1 = 0.4 & \text{w} \_1 = 0.3 \\
\hline
  & \text{h} \_1 & \text{h} \_2 & \text{h} \_3 & \text{h} \_4 & \text{h} \_5 & \text{h} \_6 & \text{h} \_7 & \sum \text{h} \_1 \\
\hline
  \text{SQP} & 2.4657 & 1.0826 & 3.2551 & 4.0899 & 2.1888 & 1.6012 & 2.0587 & 16.741 \\
  \text{PSO} & 7.4076 & 2.7014 & 2.4369 & 3.7895 & 3.0458 & 3.2611 & 4.4183 & 27.0606 \\
  \text{DE} & 2.9729 & 2.4868 & 4.8702 & 3.863 & 4.7782 & 3.0537 & 4.242 & 26.2668 \\
\hline
  \text{SQP} & 1.76 & 3.2279 & 3.5106 & 8 & 4.1536 & 6.5641 & 1.4041 & 28.6203 \\
  \text{PSO} & 3.4315 & 3.2231 & 7.5331 & 4.7774 & 6.3995 & 5.3864 & 3.7482 & 34.5082 \\
  \text{DE} & 4.7304 & 3.9718 & 6.3623 & 5.8292 & 6.277 & 5.189 & 3.6174 & 35.9771 \\
\hline
  \text{SQP} & 7.0269 & 8 & 2.2112 & 8 & 1.5529 & 2.2438 & 0.8332 & 29.868 \\
  \text{PSO} & 3.2083 & 4.2824 & 6.9677 & 6.0728 & 5.9249 & 5.5713 & 5.1746 & 37.202 \\
  \text{DE} & 4.8104 & 3.3844 & 7.1242 & 7.1163 & 6.2531 & 5.2408 & 3.3134 & 37.2426 \\
\hline
  \text{SQP} & 1 & 1.7994 & 3.3627 & 8 & 4.5797 & 7.3863 & 1.5757 & 27.7038 \\
  \text{PSO} & 5.1534 & 6.506 & 6.7648 & 6.4339 & 5.5619 & 5.359 & 4.2936 & 40.0726 \\
  \text{DE} & 5.345 & 2.9341 & 7.3894 & 5.054 & 6.4771 & 6.4594 & 4.2918 & 37.9508 \\
\hline
  \text{SQP} & 6.2946 & 7.9891 & 7.9912 & 8 & 7.8954 & 8 & 30.2922 \\
  \text{PSO} & 6.3903 & 7.7179 & 7.8636 & 7.8796 & 7.6522 & 7.4878 & 7.3369 & 52.3283 \\
\end{array}
\]

Table 5. The time of obtaining the optimal solution.

| Algorithm | The computation time of optimal solution (s) |
|-----------|---------------------------------------------|
| SQP       | 1.0272                                      |
| PSO       | 6.5325                                      |
| DE        | 8.3256                                      |

\[
\theta_{t1}(t) = 9.8099 - 0.2549097t^3 \quad (0 \leq t \leq 1.76) \\
\theta_{t2}(t) = 0.13898854t - 4.9879t^3 - 6.71333t - 0.0605594(t - 1.76) - 24.910196 \quad (1.76 \leq t \leq 4.9879) \\
\theta_{t3}(t) = 0.09966(t - 4.9879)^3 + 0.0556827(t - 8.4985)^3 - 10.665054t + 44.99337 \quad (4.9879 \leq t \leq 8.4985) \\
\theta_{t4}(t) = 1.4168t - 0.043734(t - 16.4985)^3 + 0.010305(t - 8.4985)^3 - 75.764388 \quad (8.4985 \leq t \leq 16.4985) \\
\theta_{t5}(t) = 4.4226849t - 0.054328(t - 16.4985)^3 - 0.019848(t - 29.6521)^3 - 121.502979 \quad (16.4985 \leq t \leq 20.6521) \\
\theta_{t6}(t) = 0.0343776(t - 27.2162)^3 - 2.83294t + 0.0180543(t - 20.6521)^3 + 34.170816 \quad (20.6521 \leq t \leq 27.2162) \\
\theta_{t7}(t) = -0.084403(t - 28.6203)^3 - 38.0584 \quad (27.2162 \leq t \leq 28.6203) \\
\]

\[
\theta_{t8}(t) = -0.022114t^3 - 55.1056 \quad (0 \leq t \leq 1.76) \\
\theta_{t22}(t) = -0.582398t + 0.282766(t - 1.76)^3 + 0.0120576(t - 4.9879)^3 - 53.795612 \quad (1.76 \leq t \leq 4.9879) \\
\theta_{t23}(t) = 17.869113t - 0.25899(t - 8.4985)^3 - 0.20167(t - 4.9879)^3 - 147.568654 \quad (4.9879 \leq t \leq 8.4985) \\
\theta_{t24}(t) = 0.0884989(t - 16.4985)^3 - 6.579104t - 0.033129(t - 8.4985)^3 + 96.790465 \quad (8.4985 \leq t \leq 16.4985) \\
\theta_{t25}(t) = 0.073026(t - 16.4985)^3 - 15.7175t + 0.0603419(t - 20.6521)^3 + 235.84366 \quad (16.4985 \leq t \leq 20.6521) \\
\theta_{t26}(t) = 0.03801(t - 20.6521)^3 - 0.0462093(t - 27.2162)^3 - 5.964745t + 26.5924456 \quad (20.6521 \leq t \leq 27.2162) \\
\theta_{t27}(t) = -0.17771(t - 28.6203)^3 - 125.4859 \quad (27.2162 \leq t \leq 28.6203) \\
\]
Figure 5. The $w_1 = 1$ optimal joint angle, velocity, acceleration, and jerk curves of the boom, arm, and bucket solved by the SQP: (a) the optimal joint angle of boom, arm, and bucket, (b) the optimal joint velocity of boom, arm, and bucket, (c) the optimal joint acceleration of boom, arm, and bucket, and (d) the optimal joint jerk of boom, arm, and bucket.

\[
\begin{align*}
\theta_{11}(t) &= -0.0008274t^3 - 67.7043 & (0 \leq t \leq 1.76) \\
\theta_{12}(t) &= 0.0004511(t - 4.9879)^3 - 0.1311825(t - 1.76)^3 - 0.02179t - 67.65528 & (1.76 \leq t \leq 4.9879) \\
\theta_{13}(t) &= 0.1214097(t - 8.4985)^3 - 8.623865t + 0.0691867(t - 4.9879)^3 - 23.938397 & (4.9879 \leq t \leq 8.4985) \\
\theta_{14}(t) &= -0.236541t - 0.0128257(t - 8.4985)^3 - 0.030361(t - 16.4985)^3 + 107.76941 & (8.4985 \leq t \leq 16.4985) \\
\theta_{21}(t) &= 3.5124243t - 0.001992(t - 16.4985)^3 - 0.024754(t - 20.6521)^3 - 164.815 & (16.4985 \leq t \leq 20.6521) \\
\theta_{22}(t) &= 3.246321t + 0.0012608(t - 27.2162)^3 - 0.020688(t - 20.6521)^3 - 159.10564 & (20.6521 \leq t \leq 27.2162) \\
\theta_{23}(t) &= 0.096719(t - 28.6203)^3 - 76.3368 & (27.2162 \leq t \leq 28.6203)
\end{align*}
\]
Figure 6. The $\theta_1 = 0.6$ optimal joint angle, velocity, acceleration, and jerk curves of the boom, arm, and bucket solved by the SQP: (a) the optimal joint angle of boom, arm, and bucket, (b) the optimal joint velocity of boom, arm, and bucket, (c) the optimal joint acceleration of boom, arm, and bucket, and (d) the optimal joint jerk of boom, arm, and bucket.

\[
\begin{align*}
\theta_{11}(t) &= -0.016286t^3 - 67.7043 \quad (0 \leq t \leq 3.4315) \\
\theta_{12}(t) &= 0.017291(t - 6.6636)^3 - 0.02393(t - 3.4315)^3 - 1.11722t - 63.9448 \quad (3.4315 \leq t \leq 6.6636) \\
\theta_{13}(t) &= 0.010268(t - 14.1967)^3 - 3.61538t + 0.00189(t - 6.6636)^3 - 43.71652 \quad (6.6636 \leq t \leq 14.1967) \\
\theta_{14}(t) &= 0.038776(t - 14.1967)^3 - 3.0894t - 0.0298(t - 18.9741)^3 - 50.70019 \quad (14.1967 \leq t \leq 18.9741) \\
\theta_{15}(t) &= 3.12201t - 0.02895(t - 25.3736)^3 + 0.001067(t - 18.9741)^3 - 171.91553 \quad (18.9741 \leq t \leq 25.3736) \\
\theta_{16}(t) &= 3.363446t - 0.001268(t - 30.76)^3 - 0.022786(t - 25.3736)^3 - 177.95975 \quad (25.3736 \leq t \leq 30.76) \\
\theta_{17}(t) &= 0.032745(t - 34.5082)^3 - 76.3368 \quad (30.76 \leq t \leq 34.5082)
\end{align*}
\]

\[
\begin{align*}
\theta_{21}(t) &= 9.8099 - 0.0379t^3 \quad (0 \leq t \leq 4.7304) \\
\theta_{22}(t) &= 0.04515(t - 8.7022)^3 - 0.01027(t - 4.7304)^3 - 4.68171t + 30.7724 \quad (4.7304 \leq t \leq 8.7022) \\
\theta_{23}(t) &= 0.006409(t - 15.0645)^3 - 5.94586t + 0.02119(t - 8.7022)^3 + 42.78054 \quad (8.7022 \leq t \leq 15.0645) \\
\theta_{24}(t) &= 0.023768(t - 15.0645)^3 - 1.013279t - 0.023135(t - 20.8937)^3 - 30.64967 \quad (15.0645 \leq t \leq 20.8937) \\
\theta_{25}(t) &= 4.0186t - 0.02207(t - 27.1707)^3 - 0.027136(t - 20.8937)^3 - 136.53545 \quad (20.8937 \leq t \leq 27.1707) \\
\theta_{26}(t) &= 0.0134258(t - 27.1707)^3 - 1.8405397t + 0.032825(t - 32.3597)^3 + 20.53671 \quad (27.1707 \leq t \leq 32.3597) \\
\theta_{27}(t) &= -0.0192587(t - 35.9771)^3 - 38.05839 \quad (32.3597 \leq t \leq 35.9771)
\end{align*}
\]

\[
\begin{align*}
\theta_{21}(t) &= 0.003349t^3 - 55.1056 \quad (0 \leq t \leq 4.7304) \\
\theta_{22}(t) &= 0.41359t + 0.09844(t - 4.7304)^3 - 0.39887(t - 8.7022)^3 - 56.957643 \quad (4.7304 \leq t \leq 8.7022) \\
\theta_{23}(t) &= 12.5351325t - 0.061454(t - 15.0645)^3 - 0.0821958(t - 8.7022)^3 - 172.1004 \quad (8.7022 \leq t \leq 15.0645) \\
\theta_{24}(t) &= 0.089713(t - 20.8937)^3 + 0.01367(t - 15.0645)^3 - 6.59157t + 112.693646 \quad (15.0645 \leq t \leq 20.8937) \\
\theta_{25}(t) &= 0.012695(t - 27.1707)^3 - 9.48577t + 0.00273(t - 20.8937)^3 + 173.53704 \quad (20.8937 \leq t \leq 27.1707) \\
\theta_{26}(t) &= 0.064893(t - 27.1707)^3 - 8.896133t - 0.003303(t - 32.3597)^3 + 157.7299 \quad (27.1707 \leq t \leq 32.3597) \\
\theta_{27}(t) &= -0.093086(t - 35.9771)^3 - 125.4859 \quad (32.3597 \leq t \leq 35.9771)
\end{align*}
\]
The first and second derivatives of the equations (19)–(27) are acquired to obtain the change curves of the angle joint, velocity, acceleration, and jerk of each joint, as shown in Figures 6 to 8.

It can be seen from Figures 6 to 8 that the cubic spline interpolation function uses SQP, PSO, and DE to find the optimal interpolation time that satisfies the constraints and obtains the smooth and continuous curves of the joint angle, velocity, acceleration, and jerk which velocity, acceleration, and jerk are within the constrained range. This shows that these three algorithms can be used to optimize the trajectory of the excavator under certain constraints. Secondly, it can be seen from Figure 5 that when only the efficiency problem is considered, although the time to complete the task is short, the jerk value of each joint is large compared to Figures 6 to 8.

For the time-jerk optimal joint angle curve obtained by the optimal solution, the corresponding joint angle value can be obtained from the corresponding time. Therefore, in the optimal joint angle curve obtained by the above optimization solution, under the premise that the time is known, substitute the optimal joint angle value of \( w_1 = 0.6 \) into the simulation model of the excavator working device, and the digging trajectory of the tip of the bucket tooth is shown in the Figure 9's mining path. It can be seen from Figure 9 that the second, third, and fifth parts of the excavation trajectory

\[
\begin{align*}
\theta_{31}(t) &= -0.004513t^3 - 67.7043 \quad (0 \leq t \leq 4.7304) \\
\theta_{32}(t) &= 0.005375(t - 8.7022)^3 - 0.03413(t - 4.7304)^3 - 0.553764t - 65.20868 \quad (4.7304 \leq t \leq 8.7022) \\
\theta_{33}(t) &= 0.02313(t - 15.0645)^3 - 4.760304t + 0.019723(t - 8.7022)^3 - 35.284813 \quad (8.7022 \leq t \leq 15.0645) \\
\theta_{34}(t) &= 0.0233549(t - 15.0645)^3 - 2.26514t - 0.3117(t - 20.8937)^3 - 62.429756 \quad (15.0645 \leq t \leq 20.8937) \\
\theta_{35}(t) &= 2.7163t - 0.02187(t - 27.1707)^3 + 0.004038(t - 20.8937)^3 - 167.36632 \quad (20.8937 \leq t \leq 27.1707) \\
\theta_{36}(t) &= 3.59607t - 0.026213(t - 27.1707)^3 - 0.0048526(t - 32.3597)^3 - 190.742279 \quad (27.1707 \leq t \leq 32.3597) \\
\theta_{37}(t) &= 0.0376(t - 35.9771)^3 - 76.3368 \quad (32.3597 \leq t \leq 35.9771)
\end{align*}
\]

\[w_1 = 0.6\]
generally overlap, but from the trajectory of the first and fourth segments shown in the Figure 10, it can be clearly seen that the best trajectory of the time-jerk obtained by the SQP method is relatively smooth and each joint has completed the given excavation task with the smallest joint angle change, which has helped reduce the wear of hydraulic cylinders and protects the equipment.

Conclusions and Summary

In this paper, the joint trajectory planning problem of hydraulic robotics excavators is studied. Firstly, the performance indexes considering both jerk and time are established, and the optimal cubic spline interpolation trajectory concerning the problems of time and jerk are obtained by using the three algorithms of the SQP, PSO, and DE. The results of the time-jerk optimal trajectory solution and the efficiency of obtaining the optimal solution show that the optimal trajectory of time obtained by the SQP under the same weight coefficient is the shortest, and the jerk value is relatively small. The results of this experiment show that it can

Figure 8. The $w_1 = 0.6$ optimal joint angle, velocity, acceleration, and jerk curves of the boom, arm, and bucket solved by the DE: (a) the optimal joint angle of boom, arm, and bucket, (b) the optimal joint velocity of boom, arm, and bucket, (c) the optimal joint acceleration of boom, arm, and bucket, and (d) the optimal joint jerk of boom, arm, and bucket.

Figure 9. Comparison diagram of mining path.
beneficial to reduce the impact and vibration in the process of motion, ensure the smooth continuity of the trajectory and the stability of the movement, improve the tracking accuracy of the trajectory, but also can protect the mechanical structure, reduce mechanical wear, prolong the service life of the equipment.

Secondly, the experimental results show that the greater the weight of time is, the greater the jerk value of each joint will be in the optimization process. Conversely, the greater the weight of the jerk, the longer the time it takes to complete a given task. Therefore, in engineering applications, the problems of time and jerk should both be considered, according to the given circumstances.

In further research, the existence of obstacles must be considered in the process of trajectory planning, perfect the actual working environment of hydraulic excavators, and improve the equipment’s ability to work autonomously in complex environments.

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