Disruption of Dark Matter Minihalos in the Milky Way Environment: Implications for Axion Miniclusters and Early Matter Domination

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Abstract

Many theories of dark matter beyond the weakly interacting massive particles paradigm feature an enhanced matter power spectrum on subparsec scales, leading to the formation of dense dark matter minihalos. Future local observations are promising to search for and constrain such substructures. The survival probability of these dense minihalos in the Milky Way environment is crucial for interpreting local observations. In this work, we investigate two environmental effects: stellar disruption and (smooth) tidal disruption. These two mechanisms are studied using semianalytic models and idealized N-body simulations. For stellar disruption, we perform a series of N-body simulations of isolated minihalo–star encounters to test and calibrate analytic models of stellar encounters before applying the model to the realistic Milky Way disk environment. For tidal disruption, we perform N-body simulations to confirm the effectiveness of the analytic treatment. Finally, we propose a framework to combine the hierarchical assembly and infall of minihalos to the Milky Way with the late-time disruption mechanisms. We make predictions for the mass functions of minihalos in the Milky Way. The mass survival fraction (at $M_{mäh} \geq 10^{-12} M_\odot$) of dense dark matter minihalos, e.g., for axion miniclusters and minihalos from early matter domination, is ~60% with the relatively low-mass, compact population surviving. The survival fraction is insensitive to the detailed model parameters. We discuss various implications of the framework and future direct detection prospects.

Unified Astronomy Thesaurus concepts: Dark matter (353); Cosmology (343); N-body simulations (1083); Solar neighborhood (1509); Gravitational disruption (664)

1. Introduction

The gravitational clustering of dark matter has been well measured on galactic scales and supergalactic scales and is consistent with a nearly scale-invariant spectrum of primordial fluctuations (e.g., Aghanim et al. 2020). However, the matter power spectrum on extremely small scales ($k \gtrsim \text{pc}^{-1}$), which corresponds to sub-planetary-mass structures, is still weakly constrained and is sensitive to both the nature of dark matter and the thermal history of the early Universe. There have been proposals to detect small-scale structures in the mass range $10^{-13} - 10^{-2} M_\odot$ in the future with pulsar timing arrays (PTAs; e.g., Siegel et al. 2007; Bagram et al. 2011; Dror et al. 2019; Ramani et al. 2020; Lee et al. 2021a, 2021b) and lensing effects (e.g., Kolb & Tkachev 1996; Metcalf & Madau 2001; Diaz Rivero et al. 2018; Fairbairn et al. 2018; Katz et al. 2018; Van Tilburg et al. 2018; Dai & Miralda-Escudé 2020).

Many well-motivated dark matter theories can leave unique fingerprints on the primordial perturbations at small scales ($k \gtrsim \text{pc}^{-1}$), such as the quantum chromodynamics (QCD) axion/axion-like particles (ALPs) with the Peccei–Quinn (PQ) symmetry (Peccei & Quinn 1977) broken after inflation (e.g., Hogan & Rees 1988; Kolb & Tkachev 1993, 1994; Zurek et al. 2007), early matter domination (EMD; e.g., Eriksen & Sigurdson 2011; Fan et al. 2014), and vector dark matter produced during inflation (e.g., Nelson & Scholtz 2011; Graham et al. 2016). Therefore, small dark matter substructures provide unique insights into the microphysics of dark matter. The model space of interest here differs from that in more common weakly interacting massive particles (WIMP)-like collisionless cold dark matter (CDM) models. In those models, adiabatic fluctuations produced at the end of inflation can also seed small CDM subhalos down to the kinetic decoupling and free-streaming limit ($k \sim O(\text{pc}^{-1})$), roughly corresponding to the Earth mass ($\sim 10^{-6} M_\odot$; e.g., Hofmann et al. 2001; Berezinsky et al. 2003; Green et al. 2005; Loeb & Zaldarriaga 2005). The evolution of these subhalos in the Milky Way environment has been studied in the past (e.g., Angus & Zhao 2007; Goertd et al. 2007; Green & Goodwin 2007; Zhao et al. 2007; Schneider et al. 2010; Berezinsky et al. 2014; Delos 2019; Facchinetti et al. 2022). However, WIMP-like CDM formed its first nonlinear structures at relatively late times with correspondingly low density (e.g., $\rho \sim 60 < \rho_{eq}$ as shown in Green et al. 2005), and the minihalos are thus subject to significant disruption due to tidal stripping and disk shocking after falling onto their host halos (e.g., Ostriker et al. 1972; Gnedin et al. 1999; Goertd et al. 2007; Zhao et al. 2007; Schneider et al. 2010). The typical minihalos of WIMP-like CDM are out of reach for PTAs and other observations we discuss here (e.g., Lee et al. 2021a).

On the other hand, dark matter minihalos in the theories we consider here formed in the early Universe. Therefore, they are much denser and less likely to be disrupted by tidal forces than

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5 We note that the “small scale” here is fundamentally different from the small-scale problem of CDM (at kiloparsec scale) discussed in astrophysical studies (see review by Bullock & Boylan-Kolchin 2017).
WIMP-like CDM subhalos. For instance, a (pseudo) scalar field (e.g., the QCD axion) with the PQ symmetry broken after inflation (e.g., Hogan & Rees 1988; Zurek et al. 2007, hereafter called the “post-inflationary axion”) can induce order-unity isocurvature fluctuations on the horizon scale during the symmetry breaking. Regions with order-unity overdensities tend to collapse gravitationally very early, even before matter–radiation equality ($z_{\text{eq}} \sim 3000$), into small axion miniclusters (AMCs). The miniclusters underwent subsequent hierarchical clustering until the large-scale adiabatic perturbations intervened. As another example, EMD models can introduce a nonstandard thermal history that is not constrained by any current data, but adiabatic fluctuations within the horizon can grow during this early period of matter domination. The characteristic mass of dark matter minihalos formed in EMD models is determined by the reheating temperature of this period. In vector dark matter models, the longitudinal modes of the vector DM produced at the end of inflation give rise to a peak in the matter power spectrum on small scales, with the scale directly determined by the dark matter particle mass (Graham et al. 2016; Lee et al. 2021a). Those models have interesting dynamics in the early Universe, which are difficult to probe directly. However, the remnants of those early Universe dynamics, dark matter minihalos, may be detectable in local observations (e.g., Dror et al. 2019; Ramani et al. 2020; Lee et al. 2021a).

While the evolution of (some versions of) these dark matter minihalos in the nonlinear regime has been studied both semianalytically with the Press–Schechter model (Zurek et al. 2007; Enander et al. 2017; Fairbairn et al. 2018; Blinov et al. 2020; Lee et al. 2021a; Blinov et al. 2021) and numerically with $N$-body simulations (Zurek et al. 2007; Eggemeier et al. 2020; Buschmann et al. 2020; Xiao et al. 2021), the gravitational interactions between dark matter minihalos and the large-scale dark matter structures or baryonic structures are not well studied, due to the large dynamic range and nonlinear behaviors involved. Dark matter minihalos formed from nonstandard early universe dynamics can be as light as $\sim 10^{-12} M_\odot$, while the Milky Way has a halo mass of $\sim 10^{12} M_\odot$. Therefore, it is challenging to resolve these small structures while simultaneously simulating the dynamics of the largest structures.

However, it is critical to study the survival probability of minihalos in the Milky Way and to determine the prospects of detecting such structures in the local environment. This aspect is actively studied in the literature. For example, Kavanagh et al. (2021) made an analytic estimate of the minihalo mass loss through stellar disruption and the survival fraction of AMCs in the Milky Way environment. Dandoy et al. (2022) proposed a quantum mechanical description of AMCs and studied the impact of stellar encounters using standard perturbation theories. However, these studies neglect some important nonlinear effects, such as the realistic minihalo concentration distribution, multiple mechanisms of disruption, and their interplay. In this paper, we improve these estimates with detailed numerical simulations and semianalytic treatment to combine all the expected dominant disruption terms. To deal with the enormous dynamic range involved, our strategy is to numerically simulate the dynamics of the dark matter minihalos in individual encounters (varying, e.g., halo parameters and impact parameters), using these to build detailed semianalytic models, which can be used to treat the large-scale behavior. This allows us to capture the key nonlinear physics on small scales while making realistic predictions for the overall behavior of dark matter minihalos in the Milky Way (in ways that can be generalized, in principle, to a broad class of minihalo-like models). Broadly speaking, the disruption of dark matter minihalos in the Milky Way can be divided into two parts: stellar disruption and tidal disruption. The stellar disruption term is sensitive to close encounters with individual stars, while the tidal term depends on the gradient of the collective gravitational potential on large scales. We study these separately using $N$-body simulations and then combine them using semianalytic models.

This paper is organized as follows. In Section 2, we discuss the minihalo mass function, mass–concentration relation, and the analytic models for stellar disruptions due to individual stellar encounters and tidal disruptions. In Section 3, we present our numerical results from a series of idealized $N$-body simulations for isolated minihalos encountering a star and simulations of tidal stripping. In Section 4, we apply the semianalytic model calibrated using idealized simulations to a realistic Milky Way environment and combine both stellar disruption and tidal disruption. In Section 5, we study the survival fraction of dark matter minihalos by applying the framework to different physical models.

We assume a ΛCDM cosmology with parameters given as $h = 0.697$, $\Omega_m = 0.2814$, and $\Omega_{\Lambda} = 0.7186$, and adopt scalar spectral index $n_s = 0.9667$. These are consistent with the recent Planck results (Aghanim et al. 2020), and our conclusions are relatively insensitive to variations in these parameters (compared to the much larger uncertainties in, e.g., minihalo properties from different physical models).

2. Analytic Model

2.1. Initial Mass Functions and Concentrations of Small-scale Structures

The two models discussed in the introduction that lead to the formation of dense minihalos in the early Universe are as follows: i. (pseudo) scalars (e.g., the QCD axion) with symmetry broken after inflation (Hogan & Rees 1988; Zurek et al. 2007), and ii. the EMD model (Erckcek & Sigurdson 2011). These two models are physically well-motivated and are also representatives of models enhancing the matter power spectrum at small scales. Although the cosmological perturbations in those models have different origins, they share common features. They are generated at extremely small scales before matter–radiation equality, forming dark matter substructures as light as $10^{-12} M_\odot$ ubiquitously, decoupled from the usual adiabatic fluctuations. The model parameters are the axion mass $m_a$ for the AMC model and the reheating temperature $T_{rh}$ for the EMD model. We vary the axion mass from 1.25 to 500 $\mu$eV, which is roughly the mass window that can produce the correct dark matter relic abundance, given uncertainties introduced by the axion emission from strings as shown in Gorghetto et al. (2021), Buschmann et al. (2022). On the contrary, the reheating temperature is loosely constrained although $T_{rh}$ cannot be below a few MeV. Otherwise, Big Bang nucleosynthesis will be spoiled (see e.g., Kawasaki et al. 1999). We study the value from 15 to 120 MeV. We choose the fiducial values $m_a = 25 \mu$eV, and $T_{rh} = 30$ MeV, such that the minihalo mass function in both models peaks at $\sim 10^{-8} - 10^{-7} M_\odot$. 

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The formation of dense substructures from these small-scale perturbations occurs at early times (see Lee et al. 2021a for an analytic study; and Xiao et al. 2021 for simulations of axion miniclusters). The formation and hierarchical mergers of minihalos are described by the redshift-dependent mass function $dn_i/DM(z_i)$, calibrated by simulations in Xiao et al. (2021). Meanwhile, the minihalos can fall onto CDM substructures formed from adiabatic perturbations. The redshift of infall, $z_i$, which occurs after the minihalo formation, defines the time when they stop merging or accretion, owing to the high virial velocities in the normal CDM halos, and their properties (mass, concentration) remain unchanged afterward in the absence of disruption mechanisms.

Therefore, the final mass function, including minihalos falling onto the CDM structures, can be expressed as

$$\frac{dn_i}{dM} = \int_{z_i}^{\infty} \frac{df_{\text{col}}^{CDM}(z_i)}{dz_i} \frac{dn_i}{dM(z_i)}$$

where $f_{\text{col}}^{CDM}(z_i)$ is the collapse fraction of normal CDM halos formed from adiabatic fluctuations. The probability of infall at $z = z_i$ is proportional to $dn_{\text{sim}}(z_i)/dz_i$, where we have assumed there is no dynamic decoupling between collapsed minihalos and the overall dark matter content.

We can use the Press–Schechter model (Press & Schechter 1974) to compute the collapse fraction

$$f_{\text{col}}^{CDM}(z_i) = \text{erfc} \left( \frac{\delta_c}{\sqrt{2} \sigma_z} \right),$$

where $\delta_c = 1.686$ is the critical overdensity for spherical collapse, $D(z_i)$ is the growth function, and $\sigma_z^2(M)$ is the variance of the adiabatic fluctuations calculated using the code for anisotropies in the microwave background $^6$ (CAMB; Lewis et al. 2000; Howlett et al. 2012), and $M_{\text{min}}$ is the smallest CDM structure we consider formed from adiabatic fluctuations. We take $M_{\text{min}}$ to be $10^{-2} M_{\odot}$, corresponding to a scale where the CDM power spectrum dominates while the enhanced small-scale perturbations become subdominant. The result is insensitive to $M_{\text{min}}$ as $\sigma(M)$ depends logarithmically on $M$ at small scales. There could be even smaller CDM halos that can form, which will increase the collapse fraction $f_{\text{col}}^{CDM}(z_i)$, especially at high redshifts. However, below a certain mass, the normal CDM halos may have comparable masses to the enhanced substructures, and our assumptions will no longer hold. A detailed study of this infall process will ultimately be required for more detailed predictions, and we leave it for future study.

The method described above allows us to compute the collapse fraction at various redshifts analytically and obtain the final mass function for the enhanced substructures. This will also give us the distribution of the infall redshift $z_i$, $P(z_i)$, which determines the time when the structural changes of minihalos halt and thus the average density of minihalos. Including minihalos falling onto CDM halos, we obtain the initial (pre-disruption) minihalo mass function at $z = 0$ from different models, as shown in Figure 1. In Figure 2, we show the mass distribution of minihalos on the $z_i$ versus $M_{\text{min}}$ plane for AMC with $n_a = 25 \mu eV$.

We assume that the inner structure of the minihalo is described by the Navarro–Frenk–White (NFW) profile (Navarro et al. 1996, 1997). The concentration parameter $c$ is defined as the ratio between minihalo virial radius and scale radius $r_v$. Regarding the concentration of minihalos, we adopt the model in Lee et al. (2021a) to calculate the concentration from the power spectra of different models. For each mass, it evaluates the redshift when the corresponding primordial fluctuation mode collapses $(z_c)$ and assumes $c(M) \propto (1 + z_c(M))$, which is similar to the CDM results (e.g., Bullock et al. 2001). The only difference here is that we assume the merger or smooth accretion of minihalos stops at $z = z_c$ instead of $z = 0$. Therefore, effectively, we have $c(M) \propto (1 + z_c(M))/(1 + z_i)$.

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6 CAMB documentation: https://camb.info/

7 Integrated over the entire mass range, $P(z_i) \propto df_{\text{col}}^{CDM}(z_i)/dz_i$ is purely determined by CDM cosmology. However, if looking at minihalos at $z = 0$ in a certain mass range, the conditional probability distribution of $z_i$ will be mass-dependent. In the hierarchical formation of minihalos, more massive ones form at later times. Therefore, more massive minihalos found at $z = 0$ tend to have lower $z_i$. 

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Figure 1. Top: Initial (pre-disruption) mass function of minihalos in different physics models ($f$ is the mass fraction with respect to the total dark matter mass in the Universe). Here we show the mass functions for the AMC models with axion mass $n_a = 1.25$, 25, 500 $\mu eV$ and the EMD models with reheating temperature $T_0 = 15$, 30, 60, 120 MeV. The model parameter variations manifest as constant horizontal shifts of the mass function (see Appendix A for details). The EMD models exhibit sharper peaks than the AMC models. Bottom: Mass-concentration relation of minihalos in different models shown in the top panel. The mass-concentration relation in the EMD model is peaked due to its peaked matter power spectrum. In general, the halo concentration hits a floor at the massive end as the adiabatic CDM power spectrum takes over. At the low-mass end, since one would not expect minihalos to form before matter-radiation equality, a cap to the concentration appears at around $c(1 + z_i) = 10^7$. 

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and one should reproduce the mass–concentration relation of normal CDM halos. There is a cap for concentration at the low-mass end since one will not expect minihalo formation before matter–radiation equality. The axion mass for the AMC models or the reheating temperature of the EMD models only creates constant mass shifts in the mass function and the mass–concentration relation. For comparison, the typical concentration of WIMP-like CDM halos studied in, e.g., Green & Goodwin (2007), Delos (2019) is \( c \sim 1 - 20 \) at the redshift when minihalos are initialized (usually \( z \sim 60 – 100 \) in these studies). In terms of the central density, they are comparable to the relatively massive minihalos studied here.

2.2. Disruption in Late-time Evolution

Owing to the ultra-compact structure of the minihalos, they are largely immune to external perturbations through their evolutionary history after decoupling from the Hubble flow. However, after falling into a massive host system like the Milky Way, nonlinear gravitational interactions with the host halo and the dense baryonic structures in the host halo could lead to significant disruption of minihalos. The two leading disruption mechanisms are tidal disruption from the host halo (and the baryonic disk) and close encounters with stars (referred to as stellar disruption). The relevant spatial and time scales on which these two mechanisms operate are drastically different. In this section, we will review the analytic model developed in the literature.

We also note that the term disruption used in this paper describes the mass-loss of minihalo from external perturbations at various levels and is not restricted to the case where a minihalo is completely destroyed as in some literature.

2.2.1. Stellar Disruption

First, we consider the consequence of the encounter between a minihalo and a star. The virial radius of minihalos with a mass of interest (\( \sim 10^{10} M_\odot \)) at \( z_i = 0 \) is of the order of 0.01 pc \( \sim 2000 \) au, which is still much larger than the radius of main-sequence stars even though we are considering minihalos. Therefore, for simplicity, stars can be treated as point-like objects during encounters. In addition, after a stellar encounter, the structure of the minihalo cannot immediately react to the energy imparted during the encounter. It takes roughly a dynamical time for the minihalos to relax to the final state after disruption, which is given by

\[
t_{\text{dyn}} = \left( \frac{3 \pi}{16 G \rho_{\text{mh}}^*} \right)^{1/2} \approx 0.3 \, \text{Gyr} \left( \frac{1 + z_i}{1 + 5} \right)^{-3/2},
\]

where \( \rho_{\text{mh}}^* \) is the average density of the minihalo (assumed to be \( \Delta \) times the critical density of the Universe at \( z_i \); see the discussion after Equation (11)). The dynamical time \( t_{\text{dyn}} \) is comparable to the Hubble timescale for \( z_i = 0 \), but is much shorter than that if the minihalos fall into CDM structures at a high redshift. The duration of the star–minihalo encounter, however, is several orders of magnitude shorter than \( t_{\text{dyn}} \). Therefore, the impulse approximation holds, and the encounter can be treated as an instantaneous interaction (Spitzer 1958). In the distant-tide approximation (when the impact parameter is much larger than minihalo size), the imparted energy from a single star encounter can be expressed as (Spitzer 1958)

\[
\Delta E \approx \frac{4 G^2 m_s^2 M_{\text{mh}} (r^2)}{3 v_s^4 b^4} = \frac{4 \alpha^2 G^2 m_s^2 M_{\text{mh}} R_{\text{mh}}^2}{3 v_s^4 b^4},
\]

where \( \Delta E \) is the increase of internal energy of the minihalo, \( M_{\text{mh}} \) is the virial mass (radius) of the minihalo, and \( (r^2) \) represents the mean-squared radius of particles with respect to the center of the minihalo. \( m_s, v_s, b \) are the mass, impact parameter, and relative velocity of the stellar object, respectively. The mean-squared radius can be parameterized as \( (r^2) = \alpha^2 R_{\text{mh}}^2 \), where \( \alpha \) is a dimensionless parameter determined by the density profile \( \rho(r) \) of the dark matter minihalo

\[
\alpha^2 = \frac{(r^2)}{R_{\text{mh}}^2} = \frac{1}{M_{\text{mh}} R_{\text{mh}}^2} \int_0^{R_{\text{mh}}} d^3 r r^2 \rho(r).
\]

Assuming that the minihalo has the NFW profile, one obtains

\[
\alpha^2(c) = \frac{c(-3 - 3c/2 + c^2/2) + 3(1 + c)\ln(1 + c)}{c^2(-c + (1 + c)\ln(1 + c))},
\]

where \( c \) is the concentration number of the minihalo.

However, when the impact parameter becomes comparable to the size of the minihalo, the distant-tide approximation made by Equation (4) breaks down. The strong \( b^{-4} \) dependence of \( \Delta E \) will disappear once the star passes through the minihalo, and the disruption effect is suppressed (e.g., Gerhard & Fall 1983; Moore 1993; Carr & Sakellariadou 1999; Green & Goodwin 2007). For a single encounter, Green & Goodwin (2007) proposed a more general treatment of the imparted energy calibrated using simulations

\[
\Delta E = \begin{cases} 
\frac{4 \alpha^2(c) G^2 m_s^2 M_{\text{mh}} R_{\text{mh}}^2}{3 v_s^4 b^4} & (b > b_s), \\
\frac{4 \alpha^2(c) G^2 m_s^2 M_{\text{mh}} R_{\text{mh}}^2}{3 v_s^4 b_s^4} & (b \leq b_s), 
\end{cases}
\]

where \( b_s = f_s(2\alpha/3\beta)^{1/2} R_{\text{mh}} \) is the transition radius, which is close to the physical size of the minihalo up to a factor determined by structure parameters, and \( f_s \) is an order-unity correction factor we introduce to be determined by our

8 The original formula proposed in Green & Goodwin (2007), who were studying low concentration halos, does not have the correction term (equivalently \( f_s = 1 \)). However, minihalos with higher concentrations are studied in this work, and we find the correction term is necessary empirically to fit the simulation results.
simulations, which will be discussed in Section 3. β is another structural parameter defined as

\[ \beta^2 = (r^{-2}) R_{\text{min}}^2 = \frac{R_{\text{min}}^2}{\int_{r_{\text{c}}}^{R_{\text{vir}}} d\tau r^{-2} \rho(r)} \]

\[ \simeq \frac{c^2 \ln(r_*/r_c) + c^2/2 - 1/2}{\ln(1 + c) - c/(1 + c)} , \]  

(8)

where the NFW profile is assumed, and \( r_c \) is the smallest radius that the profile extends to, which we assume to be 0.01 \( r_c \). In principle, an axion star formed in the center of a minihalo can provide a natural physical scale for \( r_c \). However, the size of an axion star sensitively depends on particle physics parameters as well as the growth rate of axion stars (e.g., Helfer et al. 2017; Visinelli et al. 2018; Chen et al. 2021). A typical axion star radius is of \( \sim 10^{-3} \) au (e.g., Schive et al. 2014; Kavanagh et al. 2021; H. Xiao et al. 2024, in preparation), which is about 2 orders of magnitude smaller than the scale radius of AMCs with mass \( 10^{-10} M_\odot \) and \( c \sim 10^7 \). We note that the choice of \( r_c \) has weak effects since (1) it only appears in the logarithm; (2) the uncertainties of \( r_c \) and \( \beta \) have been effectively captured by the free parameter \( f_b \); (3) \( \beta \) in fact only matters for rare close encounters. Compared to full analytic calculations under the impulse and distant-tide approximations, Equation (7) gives better agreement with the simulations in the transitional regime.

Roughly speaking, disruption of a minihalo is expected to occur when the increase in internal energy of the minihalo given by Equation (4) exceeds the binding energy of the minihalo

\[ E_b = \gamma G M_{\text{min}}^2 / R_{\text{min}} . \]  

(9)

Here, \( \gamma \) is a dimensionless parameter again determined by the mass profile of the dark matter halo. For the NFW profile, it takes the form (Mo et al. 1998)

\[ \gamma(c) = \frac{c + 1 - 1/(1 + c)^2 - 2 \ln(1 + c)/(1 + c)}{c/(1 + c) - \ln(1 + c)^2} . \]  

(10)

Utilizing Equations (7) and (9), we obtain the normalized energy input to the minihalo as

\[ \frac{\Delta E}{E_b} = \left\{ \begin{array}{ll} \frac{4 \alpha^2(c) G M_{\text{sun}}^2}{3 \gamma(c) v_\gamma^2 b^4 M_{\text{min}}} & \frac{1}{\pi \gamma(c) v_\gamma^2 b^4 \tilde{\rho}_{\text{min}}} \quad (b > b_s) \\ \frac{4 \alpha^2(c) G M_{\text{sun}}^2}{3 \gamma(c) v_\gamma^2 b^4 M_{\text{min}}} & \frac{1}{f_b^4 \gamma(c) v_\gamma^2 M_{\text{min}} R_{\text{min}}} \quad (b \leq b_s) \end{array} \right. \]  

(11)

where \( \tilde{\rho}_{\text{min}} \equiv M_{\text{min}} / (4\pi R_{\text{min}}^3/3) \) is the average density of the minihalo. Assuming the minihalos are in virial equilibrium with respect to the background density at redshift \( z_i \) (the infall redshift), we obtain \( \tilde{\rho}_{\text{min}} = \Delta_c \tilde{\rho}_{\text{crit}}(z_i) \), where \( \tilde{\rho}_{\text{crit}}(z_i) \) is the critical density of the Universe at \( z_i \), and \( \Delta_c = 200 \) is the critical overdensity of collapsed objects (neglecting corrections from, e.g., \( \Omega_m \), \( \rho_{\text{crit}}(z_i) \sim (1 + z_i)^3 \)). Since \( \tilde{\rho}_{\text{min}} \) is independent of minihalo mass, the energy input in the large-\( b \) case in Equation (11) will be independent of minihalo mass while having a strong dependence on the impact parameter. The only free parameter that is left to be determined by simulations in Equation (11) is the correction factor \( f_b \).

From Equation (11) (assuming the \( b > b_s \) case), we can compute a characteristic impact parameter when \( \Delta E/E_b = 1 \):

\[ b_{\text{min}} = \frac{m_e}{\sqrt{v_\gamma} \left( \frac{\alpha^2(c) G}{\pi \gamma(c) \tilde{\rho}_{\text{min}}} \right)^{1/4}} \approx 0.07 \text{ pc} \left( \frac{\alpha^2(c) G}{\gamma(c)} \right)^{1/4} \times \frac{\left( M_\odot / 10^4 \text{ M}_\odot \right)^{1/2}}{\text{ (200 km s}^{-1} \text{)}^{-1/2} \left( 1 + z_i \right)^{-3/4}} , \]  

(12)

which gives a crude estimate of the condition for the destruction of minihalos. It is worth noting that the average density of a virialized halo is much larger when formed at higher redshifts, given as \( \tilde{\rho}_{\text{min}} \propto (1 + z_i)^3 \). Minihalos that collapsed and fell into the host halo earlier in cosmic time should be less vulnerable to stellar disruptions due to higher central densities. Although \( b_{\text{min}} \) serves as an indicator for significant disruption of the minihalo, the actual mass loss of the minihalo (as a function of \( \Delta E/E_b \) after a single encounter should be calibrated by numerical simulations presented in the following section).

2.2.2. Tidal Disruption

Another important disruption mechanism for minihalos is tidal disruption. After the minihalos fall into the Milky Way halo, they experience tidal forces from the Milky Way dark matter halo and the Galactic disk. In contrast to stellar disruption, which sensitively depends on close encounters with stars, the tidal forces are determined by gravity at galactic scales, dominated by the collective effects of the smooth gravitational potential rather than the fluctuating component from individual objects. Moreover, rather than an impulsive event, tidal disruption is secular and usually treated differently from stellar disruptions.

In a sufficiently strong and smooth external tidal field, the outskirts of the minihalo will be stripped away where the tidal force exceeds the self-gravity of the minihalo. This tidal radius is given by (e.g., King 1962; Taylor & Babul 2001; Zentner & Bullock 2003)

\[ r_t = R \left( \frac{M_{\text{min}}(r < r_t)/M_{\text{MW}}(r < R)}{2 - \frac{d \ln M_{\text{MW}}}{d \ln R} [r + V(R)/V(R)]} \right)^{1/3} , \]  

(13)

where \( R \) is the galactocentric distance of the minihalo, \( M_{\text{MW}}(r < R) \) is the mass of the Milky Way (including both dark and baryonic matter) enclosed within radius \( R \), and \( M_{\text{min}}(r < r_t) \) is the minihalo mass enclosed within \( r_t \). \( V(R) \) is the instantaneous tangential speed of the minihalo, which equals the circular velocity \( V(R) \) when the minihalo is on a circular orbit. The minihalo mass outside the tidal radius will be stripped away roughly over a dynamical timescale of the host, and the instantaneous mass-loss rate is often expressed as (e.g., Zentner & Bullock 2003; Taffoni et al. 2003; Oguri & Lee 2004; Pullen et al. 2014)

\[ M_{\text{min}} = - \frac{M_{\text{min}}(r > r_t)}{t_\odot(R)} . \]  

(14)
where \( t_{\text{ts}} \) is the characteristic timescale of tidal stripping, which is proportional to the dynamical timescale of the host halo (not the minihalo)

\[
t_{\text{ts}}(R) = A \frac{t^*_{\text{dyn}}(R)}{\rho_{\text{host}}(R)}
\]

\[
\approx 90 \text{ Myr} \left( \frac{A}{1} \right) \left( \frac{1.6 \times 10^5 \text{M}_\odot \text{kpc}^{-3}}{\rho_{\text{host}}(R)} \right)^{-1/2}
\]

(15)

where \( \rho_{\text{host}}(R) \) is the averaged density of the host halo within radius \( R \), and \( A \) is a constant that factor of order unity that is found to be \( \sim 0.5 \) – 3 in several previous studies (e.g., Zentner & Bullock 2003; Zentner et al. 2005; Pullen et al. 2014; van den Bosch et al. 2018; Green et al. 2021; Errani et al. 2023). If the tidal stripping occurs in the solar neighborhood, the timescale will be much shorter than the lifetime of minihalo being gravitationally bound to the Milky Way. As a simple estimation, the characteristic formation time (defined as when \( d \log M / d \log a \) falls below a threshold as proposed in Wechsler et al. 2002) of a Milky Way–mass halo is about \( a = 0.3 \) (corresponding to \( z \sim 2.5 \) and lookback time \( \gtrsim 10 \text{ Gyr} \). If the minihalos are dynamically coupled to the smooth dark matter content accreted by the Milky Way halo, the typical lifetime of minihalos in the Milky Way will be of the same order. Meanwhile, since \( t_{\text{ts}} \) is approximately the orbital period of the minihalo at the pericenter, a reasonable assumption is that the minihalo outskirts will be tidally disrupted immediately in the first pericenter passage and before any form of stellar disruptions take place.

3. Idealized Simulations for Stellar Encounters and Tidal Stripping

In this section, we use \( N \)-body simulations to systematically study the stellar disruption and tidal stripping effects for isolated minihalos initialized with the NFW profile. The goal is to test the analytic models described in Section 2 and calibrate them against various minihalo parameters.

3.1. Stellar Encounters

We perform a suite of \( N \)-body simulations for minihalos initialized with the NFW profile by varying minihalo concentrations, masses, and impact parameters of the encountering stars. The simulations adopt the code GIZMO\(^9\) (Hopkins 2015), which has been widely used in cosmological \( N \)-body or hydrodynamical simulations (e.g., Hopkins et al. 2014, 2018; Feldmann et al. 2023). The simulations aim to test and calibrate analytic predictions of the minihalo mass fraction disrupted in stellar encounters. The relaxation process of minihalos and the evolution of density profiles are also studied, which provides important insights into modeling the disruption effects under multiple encounters.

In these simulations, an isolated minihalo (composed of collisionless \( N \)-body dark matter particles) is initialized at \( z = 0\)\(^{10}\) with a star (represented by a single point-mass particle) having mass \( m_\star = 1 \text{M}_\odot \) at a large distance (10 pc) moving toward the minihalo with a relative velocity of 200 km s\(^{-1}\). Since the code is Lagrangian, it makes no difference whether the star or the minihalo is moving and in which frame we solve the dynamics equations. In Figure 3, the evolution of a minihalo during disruption is visualized. The encounter with a star will impart a certain amount of energy into the minihalo, disrupting the halo outskirts at roughly a minihalo dynamical time. Our default simulations resolve the minihalo with \( 10^6 \) dark matter particles initialized in an equilibrium NFW halo following the method in Springel (2005). We generate the initial condition using the package PYICS\(^11\) (Herpich et al. 2017). To systematically study stellar disruption effects, we vary the impact parameter of the star, the minihalo mass, and the minihalo concentration (or equivalently scale radius). The gravitational softening length of the dark matter particles is taken to be \( 10^{-9} \text{kpc} \), which is small enough to resolve the dense core of a minihalo with mass \( 10^{-10} \text{M}_\odot \), concentration \( c = 100 \), and scale radius \( r_s = 9.6 \times 10^{-8} \text{kpc} \). The time stepping in the simulation requires more careful consideration since most disruption occurs when the distance between the star and the minihalo is around its minimum. We choose the maximum size of the timestep to be \( 10^{-8} \text{Gyr} \) even when the star is still at a large distance (10 pc) from the minihalo and study the subsequent disruption as the star moves closer toward the minihalo. It is worth noting that this timestep is about an order of magnitude smaller than the crossing time of the star, \( \sim R_{\text{mh}} / v_\star \), so that the trajectory of the star around the minihalo can be well resolved. After the close encounter, we can relax the upper limit on the timestep to integrate the relaxation of the minihalo after the stellar disruption to arbitrarily long times (at a fairly low computational cost). A detailed discussion of numerical convergence is given in Appendix B. Experimenting with the maximum timestep and/or the gravitational softening length shows that smaller values produce essentially identical results (with larger computational costs). However, order-of-magnitude larger values of softening length can risk allowing the simulation to “over smooth” gravity or take excessively large timesteps for some particles, which “overshoot” the very brief duration of the encounter (for detailed numerical tests, see Hopkins et al. 2018; and Grudić & Hopkins 2020).

3.1.1. Disruption under Different Impact Parameters and Calibration of the Response Function

We first run three sets of simulations for encounters with fixed minihalo mass and concentration but different impact parameters. In all these simulations, we set a fixed minihalo mass \( M_{\text{mh}} = 10^{-10} \text{M}_\odot \), the star mass \( m_\star = 1 \text{M}_\odot \), and the initial star–minihalo relative velocity \( v_\star = 200 \text{ km s}^{-1} \). The halo concentration has been fixed to be \( c = 10, 30, 100 \) for each set of simulations, respectively. The goal is to characterize the relation between the mass loss of the minihalo after a stellar encounter and the normalized energy input. The imparted energy can be related to the impact parameter with Equation (11). After the minihalo becomes fully relaxed after the stellar encounter (at \( t = 1 \text{ Gyr} > t_{\text{dyn}}(z = 0) \)), dark matter particles, with kinetic energy (in the center-of-momentum frame) larger than the absolute value of their gravitational potential energy, are identified as unbound and disrupted. The remaining mass of the minihalo is measured as the fraction of

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9 GIZMO documentation: [www.tapir.caltech.edu/~phopkins/Site/GIZMO_files/gizmo_documentation.html](http://www.tapir.caltech.edu/~phopkins/Site/GIZMO_files/gizmo_documentation.html)

10 The simulations are not cosmological, but the redshift value is required for initializing the NFW halo. The response of minihalos to encounters we test in this section is not sensitive to the density normalization or the redshift we set up for the minihalo.

11 PYICS documentation: [jakobherpich.github.io/pyICs/](https://jakobherpich.github.io/pyICs/)
bound dark matter particles after minihalo relaxation, and we have verified that the remaining minihalo mass has converged at the time of measurement.

In Figure 4, we show the minihalo mass loss as a function of the normalized imparted energy for different choices of halo concentrations. The imparted energy is calculated using Equation (11). The mass loss is negligible when $\Delta E/E_b \ll 1$, and quickly increases in a power-law fashion with respect to $\Delta E/E_b$. In this regime, minihalos with low concentrations are more vulnerable to stellar encounters with steeper power-law slopes. In the following, the mass loss as a function of the imparted energy and halo concentration will be referred to as the response curve, $\mathcal{F}(\Delta E/E_b, c)$. We propose the following functional form to fit the response curve:

$$
\frac{M_{\text{mh}} - \Delta M_{\text{mh}}}{M_{\text{mh}}} \equiv \mathcal{F}(\Delta E/E_b, c) = \frac{2}{1 + \left(1 + \frac{\Delta E/E_b}{p(c)}\right)^{k(c)}},
$$

where $\Delta E/E_b$ should be evaluated using Equation (11) (the free order-unity parameter $f_1$ is yet to be determined, but the calibration here is done in the $b > b_0$ regime, so $f_1$ does not have any real impact). After exploring the response curve of each choice of minihalo concentration, we propose the following fitting formula for the parameters $p(c)$ and $k(c)$:

$$
\log p(c) = a_1 (\log c - \eta) + a_2 (\log c - \eta)^2 + a_3 (\log c - \eta)^3; \\
\log k(c) = b_0 + b_1 (\log c - 2).
$$

Then, we perform the least-square fits jointly on the results of all sets of simulations. The best-fit parameters are $\eta = 0.987$, $a_1 = -0.8$ (fixed), $a_2 = -0.586$, $a_3 = -0.034$, $b_0 = -0.583$, $b_1 = -0.559$. The best-fit model is also shown in Figure 4 and is in good agreement with the simulation results of minihalos with various concentrations. This best-fit model of the response curve will be the foundation we use to understand the disruption of minihalos with different masses or concentrations. For comparison, we also show the analytical model in Kavanagh et al. (2021) (where $c = 100$ is fixed) and the semianalytical model developed in Delos (2019; which is calibrated for large $\Delta E/E_b$ values). Our model agrees reasonably well with their results for shared dynamical ranges and minihalo parameters.

### 3.1.2. Disruption under Different Halo Concentrations

Given the calibrated response function, we next run an additional set of simulations with a fixed impact parameter $b = 0.05$ pc but for minihalos with different concentrations. The goal is to further validate this semianalytic model (the analytic calculation of imparted energy plus simulation calibrated response function) for minihalos with various compactness. The minihalo mass is still fixed to $M_{\text{mh}} = 10^{-10} M_\odot$, and the properties of the encountering star are the same as in Section 3.1.1.

The halo concentration will affect the mass loss from stellar encounters in two ways. First, the structural parameters $\alpha$, $\beta$, and $\gamma$ all have an explicit dependence on minihalo concentration, which will propagate to the calculation of energy imparted (i.e., $\Delta E/E_b \propto \alpha^2(c)/\gamma(c) \sim \ln(c)/c$ when $b \gg b_0$ and $c \gg 1$). Second, the response function also has a strong dependence on concentration (see Equation (16)), especially when $\Delta E/E_b \gg 1$. Less concentrated minihalos will become increasingly vulnerable to disruptive stellar encounters. Therefore, it is nontrivial for the semianalytic model to correctly capture the concentration dependence of minihalo mass loss. In Figure 5, we show the mass loss of minihalo versus minihalo concentration. The best-fit semianalytic model agrees with the simulation results over a wide range of concentrations, even extrapolating into the regimes not covered in our original calibration step above.

### 3.1.3. Disruption under Different Minihalo Masses

One remaining ingredient of the semianalytic model that needs to be calibrated is the disruption behavior in the $b \lesssim b_0$ regime. According to Equation (11), the imparted energy will stop rising as $b$ decreases when a characteristic scale $b_0$ is
reached. The free, order-unity correction factor should be calibrated by simulations. To fulfill that, we run an additional set of simulations for minihalos of different masses ranging from $10^{-10}$ to $10^{-3} \, M_{\odot}$ but fixing the impact parameter $b = 0.05 \, \text{pc}$, and the minihalo concentration $c = 100$. In low-mass minihalos, $b_s$ is small enough that the encounter is in the $b > b_s$ regime, where mass loss is independent of minihalo mass. As minihalo mass increases (and $b_s$ increases), the test cases with massive minihalos enter the $b < b_s$ regime, where stellar disruptions are suppressed. In Figure 6, we show the mass loss as a function of minihalo mass. A transition of the response of the minihalo occurs, and we use the mass of this transition to calibrate the free parameter to be $b_s \approx 6$. The predicted mass loss using the best-fit response curve for $c = 100$ halos is shown with the solid line in the figure. In general, the semianalytic model gives the correct location and shape of the transition at $b \sim b_s$. The mass loss remains a
constant at low minihalo masses as indicated by Equation (11) in the $b > b_s$ regime. A sharp transition occurs at $b \simeq b_s$ such that the mass-loss rate reduces to almost zero at high minihalo masses.

### 3.1.4. Density Profiles

The density profile will not immediately change after the close encounter with stars, but will gradually relax to the final minihalo profile within a few minihalo dynamical times (e.g., Delos 2019). This is mainly because the close encounter with a star occurs on a timescale much shorter than the minihalo dynamical time.

In Figure 7, we show the evolution of minihalo density profiles for minihalos with mass $10^{-10} M_\odot$ and concentration $c = 100$. In the top (bottom) panel, we show the case with impact parameter $b = 2 \times 10^{-5}$ pc ($5 \times 10^{-5}$ pc). The density profile is shown as $\rho r^3$, which is proportional to $\Delta M / \Delta \log r$, the contribution to total mass per unit logarithmic interval of radius. The vertical shaded region shows the convergence radius for collisionless particles based on the Power et al. (2003) criterion (this is roughly the radius interior to which the numerical two-body relaxation time drops below the Hubble time, a conservative indicator of where N-body integration error could be significant). In the $b = 5 \times 10^{-5}$ pc case, the outskirts of the halo are predominantly disrupted while the core of the halo remains relatively unperturbed, leaving approximately 60% of the minihalo mass disrupted. Although the central density exhibits a small decrease, its contribution to the total mass loss is negligible, and the scale of this decrease is close to the convergence radius of dark matter properties, which makes it hard to distinguish the decrease from a numerical artifact. At the outskirts of the minihalo, the density profile turns up, corresponding to a shell of unbound particles (heated by the encounter) propagating outward. For the $b = 2 \times 10^{-5}$ pc case, due to higher imparted energy, the disruption is more significant with over 80% of the minihalo mass disrupted. However, the behavior of the final density profile is rather similar to the previous case.

It is worth noting the remaining minihalos never relax to a new NFW profile. Instead, the final density profile (after the shell of unbound particles escapes) can be well described by a broken power law of the form

$$\rho(r) = \frac{\rho_0}{r^\eta(1 + \frac{r}{a})^k},$$

with the best-fit asymptotic slope $k = 3.2$, for $b = 2 \times 10^{-5}$ kpc, and $k = 3.3$, for $b = 5 \times 10^{-5}$ kpc. This slope implies that the density profile is close to a Hernquist (1990) profile ($k = 3$), or in a more general sense, the $\eta$-profile family (Dehnen 1993; Tremaine et al. 1994) with an asymptotic slope of $-4$. Unfortunately, given the scale-free physics involved, this is similar to well-studied simulations of impulsive high-mass ratio galaxy–galaxy encounters (e.g., Hernquist & Quinn 1988; Barnes & Hernquist 1992; Boylan-Kolchin et al. 2005; Hopkins et al. 2008, 2009), particularly the structure of shell galaxies (Hernquist & Quinn 1987; Hernquist & Spergel 1992) and the closely related slopes of the marginally bound layer of particles at apocenter in cosmological simulations defined as the halo “splashback” radius (e.g., Diemer & Kravtsov 2014; More et al. 2015). Generically, this slope arises from relaxation after dynamical mass ejection events, which (by definition) excite some material to a broad distribution of energies crossing the specific binding energy $E = 0$, as the outer slope $k + 1 = 4$ corresponds to the only finite-mass asymptotic power-law distribution function, which is continuous through $E = 0$ (Hernquist et al. 1993).

### 3.2. Numerical Test of Tidal Stripping

To validate the semianalytic description of tidal disruptions of minihalos in Section 2.2.2, we perform an idealized simulation of a minihalo traveling through the analytic gravitational potential of a Milky Way–mass host halo ($M_{\text{vir}} = 10^{12} M_\odot$). The host halo profile is modeled as an NFW profile with concentration $c = 12$ (e.g., Klypin et al. 2002; McMillan 2011; Deason et al. 2012; Bland-Hawthorn & Gerhard 2016). The
minihalo is assumed on a circular orbit with $R = R_\odot \approx 8$ kpc. In Figure 8, we show the evolution of the enclosed mass profile and density profile of the minihalo. The tidal radius calculated using Equation (13) is shown as the red vertical dashed line. After evolving for about 0.5 Gyr (for reference, the dynamical timescale of the host halo at $R = 8$ kpc is $t_{\text{dyn}}^{\text{host}} \approx 0.1$ Gyr; see Equation (15)), the enclosed mass profile starts flattening and eventually plateaus outside the analytically evaluated tidal radius. In Figure 9, we show the mass evolutionary history of this minihalo, and specifically the mass inside and outside the tidal radius. The mass within the tidal radius is almost immune to tidal stripping, with 75% of the mass remaining after 0.5 Gyr. On the other hand, the mass outside the tidal radius exhibits an exponential decay after about $t_{\text{dyn}}^{\text{host}}$. The mass loss of the minihalo can be well represented by Equation (14), implying $M(r > r_t) \sim e^{-t/t_{\text{dyn}}^{\text{host}}}$ with a fudge factor $A$ close to 1.8 for the minihalo tested, in broad agreement with previous numerical studies of more massive CDM subhalos (e.g., Zentner & Bullock 2003; Pullen et al. 2014; van den Bosch et al. 2018).

The simulation presented in this section demonstrates that the semianalytic treatment of tidal disruption works reasonably well in predicting the mass loss and the tidal stripping timescale. Given the order-unity fudge factor $A$ found here, the tidal stripping timescale $t_\text{ts}$ in Equation (15) is at least an order of magnitude smaller than the Hubble time. Therefore, tidal stripping can be treated as an instantaneous process in modeling the cosmological evolution of minihalos.

4. Disruptions in the Realistic Milky Way Environment

In the sections above, we described the physical process and consequence of stellar encounters and the effects of tidal fields. In the following, we will apply the calibrated model to study the evolution of minihalos under the influence of the real Milky Way halo and stellar disk.

4.1. Multiple Encounters in the Milky Way Disk

Disk stars are the dominant component of the stellar populations in the Milky Way, and thus will be the main contributor to the disruption of dark matter substructures. When a minihalo passes through the stellar disk, it will encounter a slab of stars within a short timescale

$$t_{\text{disk}}^s \approx 2 \text{ Myr} \left( \frac{H_d}{400 \text{ pc}} \right) \left( \frac{v_{\text{disk}}^\text{perp}}{200 \text{ km s}^{-1}} \right)^{-1},$$

(19)

where $H_d$ is the scale height of the Milky Way thin disk, and $v_{\text{disk}}^\text{perp}$ is the relative velocity of the minihalo perpendicular to the disk plane. Since this disk passage time is much shorter than the dynamical time of dark matter minihalos, the internal structure of the minihalos will not have enough time to relax between successive stellar encounters during a single passage. For the realistic velocity distribution of stars, a series of encounters with disk stars can be effectively considered as one encounter with the injected energy accumulated over the passage (see Appendix C).

A more complicated scenario arises when the minihalo has enough time to fully relax before the next encounter, such as multiple disk crossings with an orbital period comparable to or longer than the relaxation time. In the following, we will first show a simplified model, which integrates the disruption optical
with this simplification, the survival of the minihalo is far from negligible when the energy injection $\Delta E/E_b \sim 1$. The back-of-the-envelope model certainly breaks. This fact is shown in our calibrated response curve in Figure 6 and has been highlighted in studies on CDM substructures (e.g., van den Bosch et al. 2018).
characteristic mass that depends on the PDMF of stars in the Milky Way,
\[ m_c \rho_0 = \int dm \ n_m \ m^2. \]  

A simple approximation that gives a good fit to the Milky Way data is to take the Kroupa initial mass function (Kroupa 2001) as the PDMF at \( m_\ast < 1 M_\odot \) (since the evolutionary effects are small at low masses) with a power-law cutoff \( n_{m_\ast} \sim m_\ast^{-4.5} \) at \( m_\ast > 1 M_\odot \) (e.g., Scalo 1986; Kroupa 1993; Sollima 2019). After the integration, we obtain \( m_c \simeq 0.6 M_\odot \), which is insensitive to the minimum or maximum star mass assumed. \( m_c \) can be viewed as the characteristic mass of the most effective disruptor, which can vary in different environments. For example, a clumpy medium may have significantly higher \( m_c \) and thus stronger disruption effects. The integration in Equation (24) is carried out to infinite distances, where in principle the localized quantities we define in a disk patch no longer apply. This will not affect the results significantly since the contribution from distant stars is suppressed by the \( 1/b^4 \) dependence of energy imparted.

It is, however, still important to note that Equation (24) will no longer be valid if the stellar surface density is small enough that shot noise becomes important, i.e., when \( \sqrt{2 \Sigma_*/m_c} \sim 1 \).

If we take \( m_c = 0.6 M_\odot \) as estimated above, we obtain the cutoff impact parameter as
\[ b_c = 0.044 \text{ pc} \left( \frac{m_c}{0.6 M_\odot} \right)^{1/2} \left( \frac{\Sigma_*}{100 M_\odot \text{ pc}^{-2}} \right)^{-1/2}. \] (26)

If the shot noise becomes dominant, \( b_c \gg b_s \), the total energy injection becomes\(^{14}\)
\[ \frac{\Delta E_{\text{tot}}}{E_0} = \frac{Gm_c \Sigma_*}{\sigma_*^2 + v_{\text{m}}^2} \frac{\alpha^2(c)}{\gamma(\gamma)c} \frac{\rho_{\text{m}} b_c^2}{c^2}. \] (27)

Connecting the behavior at \( b_c \ll b_s \), the general solution of the accumulated energy injection during one disk passage can be written as
\[ \frac{\Delta E_{\text{tot}}}{E_0} = \frac{Gm_c \Sigma_*}{\sigma_*^2 + v_{\text{m}}^2} \frac{\alpha^2(c)}{\gamma(\gamma)c} \frac{2}{\gamma(\gamma)c} \frac{\rho_{\text{m}} b_c^2}{c^2} \approx 2 \times 10^{-3} \left( \frac{c}{100} \right)^{-1} \left( \frac{m_c}{0.6 M_\odot} \right) \left( \frac{\sqrt{\sigma_*^2 + v_{\text{m}}^2}}{250 \text{ km s}^{-1}} \right)^{-2} \times \left( \frac{\Sigma_*}{100 M_\odot \text{ pc}^{-2}} \right)^{-2} \left( \frac{b_c}{0.1 \text{ pc}} \right)^{-3} \left( \frac{1 + z_1}{1 + 5} \right)^{-3}, \] (28)

where we use the same asymptotic approximation of \( \alpha^2(c)/\gamma(c) \) as in Equation (22). Typically, one has \( b_s \gg b_c \) for massive minihalos with low concentrations and \( b_s \ll b_c \) for low-mass minihalos with high concentrations. The typical minihalo mass above which \( b_s \) is larger than \( \sqrt{2 \Sigma_*} b_c \) is \( \sim 10^{-6} (10^{-2}) M_\odot \) for minihalos before (after) tidal disruption. It is also worth noting that \( m_c \) will cancel out when \( b_s \gg b_c \), implying that the clumpiness of the medium only matters for minihalos with physical sizes comparable to or larger than the typical spacing of the disruptors.

When the Galactic disk is dominated by stars, our treatment by integrating the cumulative perturbations from disk stars to large distances (rather than considering only close encounters like in the back-of-the-envelope model) should be physically equivalent to the disk shocking effect (e.g., Ostriker et al. 1972; Binney & Tremaine 1987; Gnedin et al. 1999; Stref & Lavalle 2017) studied in the depletion of substructures in the Milky Way (e.g., D’Onghia et al. 2010; Stref & Lavalle 2017; Facchinetti et al. 2022). The disk shocking calculation implies (e.g., Binney & Tremaine 1987; Stref & Lavalle 2017)
\[ \frac{\Delta E_{\text{tot}}}{E_0} \propto \frac{R_{\text{disk}}^2}{\sigma_{\text{m}}^2} \frac{\Sigma_\ast}{v_{\text{m}}^3} \propto \frac{M_{\text{m}}}{M_{\text{disk}}} \frac{R_{\text{disk}}}{R_{\text{m}}} \frac{\Sigma_\ast}{v_{\text{m}}^3} \frac{v_{\text{m}}^3}{\Sigma_\ast}, \] (29)

where \( g_{\text{z, disk}} \Sigma_\ast \) is the gravitational acceleration at disk vicinity, \( \sigma_{\text{m}} \) is the internal velocity dispersion of the minihalo, which should scale as \( \sqrt{G M_{\text{m}}/R_{\text{m}}} \). However, taking our Equation (28) to a smooth disk limit (\( m_c \) being infinitesimal), we do not get the disk shocking limit naturally. The key difference is that we assume that all individual stellar encounter events are independent. We evaluate the energy injection in each independent event first before summing up, while the disk shocking evaluates the tidal field of the entire baryonic disk simultaneously before estimating the momentum and energy injection. This independence assumption we made is supported by the fact that the disk scale height, \( H \sim \mathcal{O}(100) \text{ pc} \), is at least 2 orders of magnitude larger than the characteristic impact parameter \( b_s \sim R_{\text{m}} \) of the minihalo, so the individual encounters should operate locally in an independent fashion. If we abandon the independence assumption, the break in energy injection at \( b_s \) in Equation (7) (which comes from resolving individual encounters) will disappear. We can integrate the \( 1/b^4 \) law until reaching \( b_s \) and would obtain
\[ \frac{\Delta E_{\text{tot}}}{E_0} \propto \frac{m_c \Sigma_*}{v_{\text{m}}^3} \frac{\rho_{\text{m}} b_c^2}{c^2} \propto \frac{\Sigma_*}{v_{\text{m}}^3} \frac{v_{\text{m}}^3}{\Sigma_*}, \] (30)

which is totally consistent with the disk shocking calculation.

Depending on the orbit of the minihalo, it can pass through the stellar disk multiple times after falling into the host. The total number of passages and stellar surface densities at the encounter point should take an ensemble average of all possible stellar orbits passing the location of the detector. In Appendix D, we calculate the relevant correction factors from the ensemble average of all possible orbits with a simplified model. The accumulated injected energy will ultimately be fed to the response function \( \mathcal{F}(\Sigma_* \Delta E_{\text{tot}}/E_0) \) to calculate the mass loss.

### 4.2. Semianalytic Model to Combine Stellar and Tidal Disruptions

As the minihalo moves closer to the Galactic center, both \( r_s \) (Equation (13)) and \( t_s(R) \) (Equation (15)) will decrease sharply. Therefore, the total mass loss of an infalling minihalo is dominated by its pericenter passages. For the minihalos of interest for detection (e.g., in the solar neighborhood \( R_s \sim 8 \text{ kpc} \)), the tidal-stripping timescale during a pericenter passage at \( R_p \) is \( O(100) \text{ Myr} \), which is of the same order as the minihalo orbital time and much shorter than the lifetime of the
minihalo in the host (∼$T_{\text{Hubble}}$). We assume that the mass of an infalling minihalo outside the tidal radius will be quickly stripped away during the first few pericenter passages before the impact of stellar disruptions starts to accumulate.

For simplicity, we evaluate the tidal radius at the target radius $r_{\text{obs}}$ of observation, assuming a circular orbit

$$r_i = r_{\text{obs}} \left[ \frac{M_{\text{min}}(r < r_i)/M_{\text{MW}}(r < r_{\text{obs}})}{3 - \frac{d \ln M_{\text{MW}}}{d \ln r} |_{r_{\text{obs}}}} \right]^{1/3}. \quad (31)$$

where the Milky Way halo mass distribution is modeled as a NFW profile for dark matter plus a Hernquist profile (Hernquist 1990) for the stellar content

$$M_{\text{MW}}(r < x) = M_{\text{dm}} \frac{f_{\text{nw}}(x/r_r)}{f_{\text{nw}}(c)} + M_b \frac{r^2}{r^2 + a^2}, \quad (32)$$

where $f_{\text{nw}}(x) \equiv \ln(1 + x) - x/(1 + x)$; the host halo parameters are $c = 12$, and $M_{\text{dm}} = 10^{12} M_\odot$ (e.g., Klypin et al. 2002; McMillan 2011; Deason et al. 2012; Bland-Hawthorn & Gerhard 2016). For baryon properties, abundance-matching studies have shown that a Milky Way mass system typically has a stellar-to-total-mass ratio of $0.414 \pm 0.01$ (Hernquist 1990).

The stripped minihalo forms the initial condition for the following stellar disruptions. Therefore, for a minihalo observed at $r_{\text{obs}}$, the energy accumulated from stellar encounters can be written as (following Equation (28) but considering multiple passages through the disk during the lifetime of the minihalo)

$$\frac{\Delta E_{\text{tot}}(r_{\text{obs}})}{E_b} = \left\{ N_p \left[ \frac{G M_{\text{c}, \Sigma_\text{a}}}{\sigma_\text{a}^2 + v_{\text{t}, \text{min}}^2} \right] \frac{\alpha^2(c_{\text{eff}})}{\gamma(c_{\text{eff}}) \Delta \beta_{\text{c}, r_{\text{obs}}, \Sigma_\text{a}}(\xi)} \right\}_{\sigma} \frac{2}{b_{\Sigma_{\text{c}}}^2(c_{\text{eff}}, R_{\text{eff}} + 2b_{\Sigma_{\text{c}}}^2(\Sigma_\text{a})) \Delta \beta_{\text{c}, r_{\text{obs}}, \Sigma_\text{a}}(\xi)} \frac{1}{1 + e^{-k \log(\Sigma_{\text{c}}/h_k)}}. \quad (36)$$

$$= \frac{2}{N_p} \frac{G M_{\text{c}, \Sigma_\text{a}}}{\sigma_\text{a}^2 + v_{\text{t}, \text{min}}^2} \frac{\alpha^2(c_{\text{eff}})}{\gamma(c_{\text{eff}}) \Delta \beta_{\text{c}, r_{\text{obs}}, \Sigma_\text{a}}(\xi)} \frac{2}{b_{\Sigma_{\text{c}}}^2(c_{\text{eff}}, R_{\text{eff}} + 2b_{\Sigma_{\text{c}}}^2(\Sigma_\text{a}))} \frac{1}{1 + e^{-k \log(\Sigma_{\text{c}}/h_k)}}. \quad (35)$$

where $N_p$ is the number of passages through the stellar disk; $\langle \rangle_\sigma$ denotes averaging over an ensemble of minihalos observed at $r_{\text{obs}}$ with all possible orbits. Therefore, $N_p$ represents the averaged number of passages over all possible orbits.

$N_p$ is the number of passages assuming the minihalo is on a circular orbit with radius $r_{\text{obs}}$ calculated based on the Hubble time $T_{\text{Hubble}}$ and the circular orbit period $T_{\text{c}, r_{\text{obs}}}$. $f_{\text{c}, \Sigma_\text{a}}$ characterizes the deviation of $\Delta \beta_{\text{c}, r_{\text{obs}}, \Sigma_\text{a}}$ from this circular orbit estimation. In Equation (35), $\Sigma_\text{a}$ is the stellar surface density where the minihalo crossed the disk (the surface density profile $\Sigma_\text{a}(r)$ is given below Equation (23)). $\langle \rangle_\sigma$ denotes averaging over all past disk crossings given the orbit of the minihalo. The correction factor $f_{\text{c}, \Sigma_\text{a}}$ characterizes the deviation of the averaged $\Sigma_\text{a}$ at all past encounter locations for all possible orbits from $\Sigma_\text{a}(r_{\text{obs}})$. $f_\text{c}$ accounts for the increased effective stellar surface density when the minihalo is not passing perpendicularly to the disk (see Appendix D for details). In Appendix D, $f_{\text{c}, \Sigma_\text{a}}$ and $f_{\text{c}, \Sigma_\text{a}}$ are estimated based on the orbital model of an isothermal halo. The combined effect of $f_{\text{c}, \Sigma_\text{a}}$ and $f_{\text{c}, \Sigma_\text{a}}$ on $\Delta E_{\text{tot}}/E_b$ is $O(10)$ at the solar neighborhood. The velocity term $\sigma_{\text{c}}^2 + v_{\text{t}, \text{min}}^2$ has a weak dependence on $r_{\text{obs}}$ and minihalo orbits, so it is assumed to be the constant value (250 km s$^{-1}$)$^2$ for simplicity.

We note that $b_{\Sigma_{\text{c}}}$ has an implicit dependence on the surface density at the encounter. When $b_{\Sigma_{\text{c}}} \gg b_s$ (when $\Sigma_\text{a}$ is large or $M_{\text{min}}$ is small), $\Delta E_{\text{tot}}/E_b$ will be proportional to $\Sigma_{\text{c}}$, and the correction factor $f_{\text{c}, \Sigma_\text{a}}$ should be replaced with $f_{\Sigma_{\text{c}}}$ (see the calculation in Appendix D). To properly account for this, we model the transition from $f_{\text{c}, \Sigma_\text{a}}$ to $f_{\Sigma_{\text{c}}}$ empirically as

$$f = f_{\Sigma_{\text{c}}} + (f_{\Sigma_{\text{c}}} - f_{\text{c}, \Sigma_\text{a}}) \frac{1}{1 + e^{-k \log(\Sigma_{\text{c}}/h_k)}}; \quad (36)$$

where $k = 3$ is assumed. We note that the value of $k$ or the detailed form of the transition does not affect the post-disruption mass function in any significant way.

### 4.3. Monte Carlo Sampling of the Minihalos

We are ready to implement all the physics of disruption discussed above to a sample of minihalos and track their mass loss. We model the evolution of the minihalo population in the Milky Way halo following the steps below:

1. We initialized a Monte Carlo sample of minihalos. First, we construct a grid of infall redshifts from $z_{\text{min}} = 0$, to $z_{\text{max}} = 150$ (uniform in $\log(1 + z)$), and compute the infall probability at each redshift point as $\Delta f_{\text{col}}^{CDM}(z_i)$ (using the matter power spectrum from adiabatic CDM fluctuations on small scales; see the discussion in Section 2.1). Then, at each redshift point, the minihalo masses are sampled uniformly over the dynamical range $10^{-14}$–$10^{-3} M_\odot$. The number densities of these minihalos are calculated following the redshift-dependent pre-infall mass function given in Section 2.1 and Appendix A. The weight of each individually sampled minihalo is proportional to the product of the number density and the infall probability at $z_i$. The minihalo concentrations are calculated following the mass–concentration relation given in Section 2.1. These sampled physical properties represent the initial status of the minihalos upon falling into the Milky Way host.

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15 A model variant that evaluates tidal disruption at smaller pericenter distances for eccentric orbits is considered in Appendix E. It leads to almost identical minihalo mass function when stellar disruption is also included.
2. Tidal stripping and structural corrections are applied to the sampled minihalos as described in Equations (33) and (34). Since the $r_i$ solved from Equation (31) after normalizing over $r_e$ is independent of minihalo mass, we calculate $r_i/r_e$ on a grid of $c$ and $z_i$ for several different choices of $r_{\text{obs}}$ and prepare them as lookup tables for efficient interpolation of the tidal radius.

3. After the tidal stripping and the implementation of relevant structural corrections, we apply the stellar disruption with the mass loss given by $\mathcal{F}(\Delta E_{\text{tot}}/E_b, c)$. The cumulative energy injection from stellar encounters, $\Delta E_{\text{tot}}/E_b$, is evaluated with Equation (35) using the structural parameters corrected after tidal disruptions. We include the orbit corrections derived in Appendix D.

All the results demonstrated below have passed the convergence tests over hyperparameters in the sampling approach above, including the maximum sample redshift $z_i^{\text{max}}$, the mass range of sampling, the number of redshift grid points, the number of samples at each redshift, and the resolution of tidal correction grid.

5. Results

5.1. Post-disruption Mass Functions

In the top panel of Figure 10, we show the mass function of minihalos (from AMC with the fiducial $m_a = 25$ µeV as an example) at the solar neighborhood ($r_{\text{obs}} = 8$ kpc). The mass functions are presented as the matter mass fraction (with respect to the total dark matter mass) in minihalos per unit logarithm interval (dex) of minihalo mass. We have assumed the axion gives the correct dark matter relic abundance. We present the initial (pre-disruption) mass function, the post-tidal/stellar disruption mass function, the mass function considering the combined effect of tidal and stellar disruptions, and the mass function aggregating tidal and stellar disruptions linearly. With both tidal and stellar disruption, the peak of the mass function is reduced by about 30% accompanied by roughly half an order of magnitude mass shift of the peak. The low-mass end of minihalos is less disrupted than the massive end because lighter minihalos generically form earlier in these models and are more concentrated. Comparing the two disruption mechanisms, the stellar disruption dominates over the entire mass range, which agrees with the conclusion of previous studies of AMC (e.g., Kavanagh et al. 2021; Lee et al. 2021a). It is worth noting that tidal disruption can alter the structural parameters and enhance the average density of minihalos, which could lead to nonlinear effects on the following stellar disruption. However, the effect is relatively weak on mass functions in our experiments. We note that a spike shows up in the post-disruption mass function at $M_{\text{nh}} \sim 10^{-7} M_\odot$ when considering only the stellar disruption. The spike is related to a turnover in the distribution of the minihalo initial concentrations and will be discussed in the following section.

In the bottom panels of Figure 10, we show the integrated fraction of disrupted minihalos in number and total mass, respectively. The survival fraction quoted hereafter is defined as the fraction of the integrated mass/number of minihalos above a certain mass threshold retained after disruptions. Both the mass and number survival fractions show a plateau as the mass threshold becomes sufficiently low compared to the peak of the initial mass function. Thus, we pick $M_{\text{nh}}^{\text{lim}} = 10^{-12} M_\odot$ as the limit to measure the overall survival fraction, which is insensitive to $M_{\text{nh}}^{\text{lim}}$ whatsoever. The overall survival fraction of minihalos with $M_{\text{nh}} \geq 10^{-12} M_\odot$ is about 83% in terms of number and about 58% in terms of mass. The survival fraction will quickly diminish to $\lesssim 30\%$ at $M_{\text{nh}} > 10^{-7} M_\odot$. Again, the dominance of the stellar disruption is manifest.

In Figure 11, we show the post-disruption mass function of minihalos at different galactocentric distances, $r_{\text{obs}} = 4, 8, 16$ kpc, for the $m_a = 25$ µeV AMC model. We find similar behavior of the post-disruption mass functions at different target radii. In all cases, the massive end is more severely disrupted, and the stellar disruption dominates. The disruption at smaller galactocentric distances is stronger primarily due to enhanced stellar surface densities, resulting in the peak of the mass function shifting toward lower masses. The geometric assumptions of our model will break at the central bulge ($r_{\text{obs}} \lesssim 1$ kpc) of the galaxy. The effective stellar density can be much higher than implied by the disk model, and the minihalos will spend most of their lifetime in dense stellar environments. According to the back-of-the-envelope estimation given in Equation (22), the survival probability in this scenario should diminish to zero with moderately high stellar densities.

5.2. Impacts of Different Disruption Mechanisms

To better illustrate the impact of tidal disruption, in Figure 12, we show the mass distribution of minihalos on the plane of $c$ versus $M_{\text{nh}}$ (left column) and $r_{\text{nh}}$ versus $M_{\text{nh}}$ (right column) before and after the tidal disruption. The initial concentrations and average densities of minihalos have a big scatter driven by the distribution of $z_i$. At the massive end, the scatter approaches zero since all of the minihalos there have $z_i \sim 0$. The tidal disruptions (along with corrections on minihalo structural parameters, see Equation (33)) effectively lower the concentration of minihalos, especially at the massive end. The average densities of minihalos are enhanced in the meantime, so the net effects over following stellar disruptions will depend on the trade-off between lowered concentrations and enhanced average densities (see Equation (35)). The population of minihalos with large $z_i$ (thus higher average densities and lower concentrations) are almost immune to tidal disruption, since the tidal radii $r_i$ of these minihalos become larger than the virial radius.

In Figure 13, we show the median mass loss versus the initial mass of minihalo decomposed by the disruption mechanism. The stellar disruption is the dominant mechanism over the entire mass range of interest. However, the tidal disruptions do have nonlinear effects in modifying the structural parameters of minihalos prior to stellar encounters. The net effect over stellar disruptions is a trade-off between lowered concentrations and effectively enhanced average densities (i.e., higher $\Delta\rho_{\text{eff}}$ of minihalos. It suppresses (promotes) the stellar disruption at $M_{\text{nh}} < 10^{-6} M_\odot$ ($M_{\text{nh}} > 10^{-6} M_\odot$). Nevertheless, the nonlinear effects do not affect much the mass functions or the overall survival probability of minihalos shown in Figure 10 since most of the differences show up at the decreasing edge of the mass function. Another interesting feature to note is the peak of the mass-loss curve due to stellar disruptions at around $M_{\text{nh}} \sim 10^{-5} M_\odot$ (10^{-6} if tidal disruption is not introduced). The peak is related to the turnover feature (at the same mass scale) of the mass–concentration distribution shown in the upper panel of Figure 12. The peak in the mass-loss curve corresponds to the spike in the minihalo mass function (stellar-
5.3. Disruption for Different Physics Models

Here, we explore the disruption of minihalos in different physics models summarized in Section 2.1. These models feature different initial mass functions and mass–concentration relations.

In the top panel of Figure 14, we present the pre- and post-disruption mass functions of minihalos in the AMC model with axion mass $m_a = 25 \mu$eV. For the EMD case, we show the model with reheating temperature $T_{rh} = 15, 30, 60$ MeV. The numerical sampling experiments are all conducted at $r_{obs} = 8$ kpc. Since both the initial mass functions and mass–concentration relations are almost self-similar with a horizontal mass shift (and the relative mass change only depends on $c$ and $r_{obs}$), the post-disruption mass functions are also similar albeit with a horizontal shift. The minihalo abundance reduction from disruptions has similar patterns between the AMC and EMD models. In the bottom panel of Figure 14, only case) shown in Figure 10. This is an example that the convolution of the mass–concentration relation and mass function gives rise to nonlinear features.

Figure 10. Top: Mass function of minihalos (from axion miniclusters) at the solar neighborhood ($r_{obs} \approx 8$ kpc). We assume the AMC model with $m_a = 25 \mu$eV. The mass function before disruption is shown as the gray dashed line. The mass function after processing only tidal (or stellar) disruption is shown as the blue (red) solid line. The mass function post-disruption, combining both tidal and stellar disruption, is shown as the solid black line. The mass function post-disruption, combining tidal and stellar disruption linearly, is shown as the dashed black line. In general, the disruptions taken together induce approximately a 30% decrease in the peak value of the mass function and shift the mass of the peak by roughly half an order of magnitude. The massive end is more strongly affected by disruption than the low-mass end. Bottom: We show the integrated number (left) and mass (right) of minihalos before and after the disruption. The typical survival fraction of minihalos with $M_{nh} \geq 10^{-12} M_\odot$ is 83% in terms of number and about 58% in terms of mass. Stellar disruption is the dominant disruption mechanism through the entire mass range of interest.

Figure 11. Mass function of minihalos (from AMC with $m_a = 25 \mu$eV) observed at three galactocentric distances ($r_{obs} = 4, 8, 16$ kpc). A similar reduction pattern in the mass density of minihalos is found at each distance, with stronger disruption at smaller radii and the massive end of the mass function. The peak of the post-disruption mass function is slightly shifted toward lower masses at smaller radii. Stellar disruption is the dominant disruption mechanism at all radii.
we show the mass survival fraction of minihalos. Regardless of the model choice, the overall survival fraction is about 60% for minihalos with \(M_{\text{mh}} > 10^{-12} M_\odot\). It is visible that the AMC (EMD) model with higher \(m_\alpha\) \((T_{\text{rh}})\) suffers from a stronger disruption at the massive end. This is due to the lower minihalo concentration at the same mass in these models (as shown in Figure 1). The EMD models exhibit sharper decreases in the survival fraction at the massive end. The mass function of EMD models features a steeper decline at the massive end, and therefore, at a fixed mass, the same level of a horizontal shift of mass function will give rise to a larger decrease in minihalo abundance.

From the comparisons shown in this section, we can conclude that the model variations have little impact on the mass function and survival fraction of minihalos up to the mass shift noted previously. The estimated mass survival fraction of minihalos in the solar neighborhood is about 60%.

In Figure 15, we show the survival (mass) fraction of minihalos after stellar and tidal disruption as a function of the radius of observation. We choose the fiducial AMC and EMD models (with \(m_\alpha = 25\mu eV\), and \(T_{\text{rh}} = 30\ MeV\)) for comparison here. The number of surviving minihalos is evaluated by integrating the mass function in three mass bins: \(\log(M_{\text{mh}}/M_\odot) \in [-12, -10], (-10, -8], (-8, -6] \). For both models, the survival fraction of minihalos significantly drops with increasing minihalo mass and decreasing galactocentric distance. Quantitatively, more than 70% of minihalos in the mass range \(\log(M_{\text{mh}}/M_\odot) \leq -8\) survive at any radius of observation, as opposed to \(\leq 50\% \) (\(\leq 30\%\)) survival fraction of minihalos with \(\log(M_{\text{mh}}/M_\odot) \leq -12\) at \(r_{\text{obs}} \leq 8\ kpc\) (4 kpc). Low-mass minihalos are less vulnerable to disruption with more concentrated structures, although the survival fraction still decreases sharply at small galactocentric radii due to
2021a destroys the outer shells of the minihalo. The density profile disruption has a limited impact on the central core but primarily for these concentrated remnants et al. 2015; Due et al. 2018. Thermostatistics can potentially probe a mass fraction of minihalos in different models before (dashed) and after (solid) disruption. Two different models are studied: post-inflationary AMCs and EMD. The reheating temperature for the EMD era is \( T_{\text{rh}} = 15, 30, 60 \text{ GeV} \) while the axion mass in the post-inflationary AMC scenario is chosen as \( m_a = 1.25, 25, 500 \mu \text{eV} \). These parameters are purely chosen for illustrative purposes, and one can use other parameters, which will shift the mass range, but the shape of the mass function will remain the same. Bottom: The integrated mass survival fraction of minihalos for the models shown in the top panel. Regardless of the model choices, the overall survival fraction of minihalos with \( M_{\text{min}} > 10^{-11} M_\odot \) is stably around 60%.

Figure 14. Top: Mass function of minihalos in different models before and after disruption. Two different models are studied: post-inflationary AMCs and EMD. The reheating temperature for the EMD era is \( T_{\text{rh}} = 15, 30, 60 \text{ GeV} \) while the axion mass in the post-inflationary AMC scenario is chosen as \( m_a = 1.25, 25, 500 \mu \text{eV} \). These parameters are purely chosen for illustrative purposes, and one can use other parameters, which will shift the mass range, but the shape of the mass function will remain the same. Bottom: The integrated mass survival fraction of minihalos for the models shown in the top panel. Regardless of the model choices, the overall survival fraction of minihalos with \( M_{\text{min}} > 10^{-11} M_\odot \) is stably around 60%.

Figure 15. Mass survival fraction of minihalos as a function of galactocentric distance. In addition to the overall survival fraction after stellar and tidal disruptions, we also show the survival (mass) fraction of minihalos in three mass bins: \( \log(M_{\text{min}}/M_\odot) \in [-12, -10], [-10, -8], [-8, -6] \). The survival fraction has a strong dependence on minihalo mass. Massive minihalos are more severely disrupted, especially at small galactocentric radii. Less than a half of the minihalos in the \([-8, -6]\) bin can survive at \( r_{\text{obs}} \gtrsim 8 \text{kpc} \). On the other hand, low-mass minihalos in the \([-12, -10]\) bin have \( \gtrsim 70\% \) survival fraction at any radius.

Encouraging for the prospects of axion direct detection in the post-inflationary scenario as well as minihalos formed from EMD. We leave the calculations of astrophysical signals of dark matter minihalos in the Milky Way for a follow-up study.

5.4. Limitations of the Model

Our fiducial model assumes a static Milky Way halo and stellar disk when evaluating stellar disruption. This fails to account for the natural mass and size evolution of both the halo and the disk in a fully cosmological setup. There are mainly two factors yet to be captured. The first is the orbit evolution of minihalos during the growth of the halo, due to dynamical friction, two-body interactions between minihalos, and adiabatic evolution as the halo potential changes during its growth. However, for the minihalo mass and number density of interest, both the dynamical friction times scale and the two-body relaxation time are much larger than the Hubble time (e.g., Binney & Tremaine 2008). Meanwhile, as found in studies of halo growth and structure (e.g., Wechsler et al. 2002; Zhao et al. 2003, 2009; Prada et al. 2012; Ludlow et al. 2014), the central halo has limited evolution at late stages when most of the mass growth is at the outskirts. Therefore, we consider the orbit evolution of minihalos subdominant. The second and dominant factor is the evolution of stellar disk surface density and host halo potential. It has been shown in cosmological simulations of Milky Way–like galaxies that the stellar disk mass can change by order of magnitude from \( z \approx 2 \) to \( z = 0 \) (e.g., Garrison-Kimmel et al. 2017). In Appendix E, we test a model variation where we assume a dynamically evolving stellar disk with redshift-dependent surface density taken from simulations.

Furthermore, due to the growth of the host halo, minihalos falling earlier can have lower orbital energy and angular momentum, ending up in more tightly bound orbits. In Appendix E, we test a model variation where the mass growth history of the Milky Way halo is considered, and the orbital corrections of minihalos depend on their infall redshifts.
Nevertheless, in this test, we have not considered the potential segregation of minihalos with different infall redshifts in the host halo. For example, less massive minihalos with earlier infall times (see Figure 2) have more tightly bound orbits and therefore are more likely to be found at smaller galactocentric radii. This implies more complicated $\zeta$ distribution and minihalo mass function that depend on the radius of observation even when no disruption mechanisms are considered, which are not captured by our model. Fully capturing the dynamical evolution of the halo and disk is beyond the scope of this work. They can be studied self-consistently with an on-the-fly evaluation of accumulated imparted energy, given the local stellar density and velocity dispersion field in cosmological hydrodynamical simulations.

Our semianalytic model of tidal disruption assumes that the density structure of tidally stripped minihalos is described as an NFW profile truncated at the tidal radius. However, some previous studies (e.g., Peñarrubia et al. 2010; Green & van den Bosch 2019) have shown that the central density of subhalos can be reduced through tidal stripping as well, especially in violent mass-loss cases. To evaluate the uncertainties arising from the treatment of violent tidal disruption cases, we consider alternative approaches of calculating the tidal disruption mass loss when $r_t$ becomes comparable to $r_c$ in Appendix E and find the results invariant when stellar disruption has been included. In Appendix E, we also test varying the galactocentric radius; we evaluate the strength of tidal disruption, and the results are invariant to the choices as well.

In this paper, we assume minihalos have the NFW density profile. Meanwhile, there are studies of initial inner structures of minihalos featuring more cuspy density profiles (e.g., Bertschinger 1985; Zurek et al. 2007; Ricotti & Gould 2009; Ishiyama et al. 2010; Fairbairn et al. 2018; Delos et al. 2019), which are suitable for the lightest minihalos not yet undergone mergers. However, the minihalos considered in this paper typically collapse gravitationally very early (e.g., at $z_{eq}$ for post-inflationary AMCs, Hogan & Rees 1988; Kolb & Tkachev 1993) and have undergone substantial amounts of mergers (e.g., Eggemeier et al. 2020; Xiao et al. 2021). Evaluating the cuspy density profile scenario self-consistently is beyond the scope of this paper, since it requires a completely different minihalo synthesis model and recalibration of the stellar disruption. In Kavanagh et al. (2021), a cuspy power-law density profile has been considered for AMCs, and it yields a significantly higher survival fraction than minihalos with the NFW profile due to more concentrated mass distribution and resistance to stellar disruption. Therefore, even when considering alternative assumptions about minihalo inner structures, our findings remain a conservative estimate of minihalo survivability.

6. Conclusions

In this paper, we systematically studied the environmental effects on dark matter minihalos (as light as $10^{-12} M_\odot$) after infall to the Milky Way. Due to the large dynamic range, it is impossible to simultaneously track the evolution of minihalos in a standard cosmological simulation of the Milky Way–mass galaxy. Therefore, we developed a framework to combine small-scale, idealized $N$-body simulations with an analytic model of the Milky Way to make these sorts of predictions tractable.

Stellar disruption and tidal disruption are the most critical environmental effects. For stellar disruption, the analytic expressions in the literature are inadequate to accurately capture the minihalo mass loss after stellar encounters. To address this, we developed a semianalytic model calibrated by a suite of $N$-body simulations of idealized encounters between a star and a minihalo, varying impact parameters, minihalo masses, and concentrations. On the other hand, for tidal disruption, our $N$-body simulations show that the disruption effects can be predicted accurately by relatively simple analytic models. To connect to galactic scales, we apply a simplified orbital model of minihalos and derive an orbital-averaged treatment of stellar and tidal disruptions. We perform Monte Carlo simulations to model the mass evolution of minihalos with various masses, concentrations, and infall histories. The framework allows us to make predictions for the survival fraction of minihalos in the Milky Way as a function of galactocentric radius and halo parameters. This paper focuses on the miniclusters of post-inflationary axions and minihalos in the EMD. However, the same framework can be easily applied to other types of dark matter substructure models.

Our major findings can be summarized below:

1. When the energy imparted ($\Delta E$) during a stellar encounter is much smaller than the binding energy of the minihalo ($E_b$), the mass loss is relatively insensitive to minihalo concentration. However, when $\Delta E/E_b$ exceeds unity and continues to increase, the minihalos with lower concentration will experience significantly larger mass loss in stellar encounters. This aspect has not been properly considered in many previous studies of dark matter substructures. We propose a simple and intuitive way to model successive encounters by aggregating the imparted energy from individual encounters and applying the same response curve calibrated for single encounters. This method works reasonably well when the imparted energy from individual encounters $\Delta E/E_b \ll 1$ (weak-disruption limit).

2. For tidal disruption, we confirm the existence of a tidal radius outside which minihalos are largely stripped. The analytical formula works remarkably well even for extremely light minihalos. On the other hand, the stellar disruption results in a shell of marginally bound particles propagating outwards, leaving a characteristic density profile similar to the Hernquist profile at large radii. The stellar disruption can also slightly reduce the minihalo central densities.

3. We show that there could be nontrivial nonlinear interactions/combined effects between these disruption processes. The first tidal or stellar disruption will leave dense cores of the original minihalos, which are typically less vulnerable to future disruptions. Therefore, the assumptions on the structures of minihalos need to be revised. Our analytic method includes analytical corrections on minihalo structural parameters after tidal disruptions.

4. Applying our methods to well-motivated models like post-inflationary AMC and EMD minihalos, we find a relatively stable mass (number) survival fraction of $\sim 60\%$ ($\sim 80\%$) at the solar neighborhood. The numbers are insensitive to the limiting minihalo mass we define as survival. The number is also insensitive to model choices. The survival fraction shows a strong dependence on the galactocentric radii, especially for massive minihalos. For
example, the mass survival fraction of minihalos above $10^{-8} M_\odot$ can drop to $\lesssim 50\%$ ($\lesssim 30\%$) at $r_{\text{obs}} < 8$ kpc (4 kpc). Over the entire mass range of interest, stellar disruption is the dominant disruption mechanism. The remaining minihalos are abundant and dense enough to give direct detection signals in, e.g., upcoming PTA observations.

The follow-up paper will focus on the direct detection signals of minihalos. In the future, the framework built in this paper can be applied to a spectrum of interesting topics in particle astrophysics, such as the dark matter annihilation rate in minihalos and axion minicluster–neutron star encounters.

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Appendix A

Minihalo Mass Function and Concentration in Different Models

The minihalo mass function arising from an enhanced matter power spectrum at small scales can be considered separately from the adiabatic power spectrum. We use the Press–Schechter model to compute the minihalo mass function (Press & Schechter 1974)

$$\frac{d^2n}{dM d\ln M} = \rho f(\nu) \frac{d\nu}{\nu},$$

where $\rho$ is the comoving density of dark matter, and $\nu$ is a dimensionless parameter that defines the rareness of the halo. $f(\nu)$ and $\nu$ are defined as

$$f(\nu) = \frac{\nu}{\sqrt{2\pi}} \exp(-\nu^2/2),$$

$$\nu = \frac{\delta_c^2(z)}{\sigma^2(M)},$$

where $\delta_c = 1.686$ is the critical density required for the formation of collapsed halos in spherical collapse models. $\sigma^2(M)$ is the variance of the initial perturbations smoothed with a top-hat filter of scale $R = (3M/4\pi\rho)^{1/3}$, which can be determined as

$$\sigma^2(M) = \int \frac{dk}{k} \frac{k^3P(k)}{2\pi^2} D^2_T(z)|W(kR)|^2,$$

where $W(x) = (3/x^3)[\sin(x) - x \cos(x)]$ is the spherical top-hat window function, $D_T(z)$ is the growth function normalized in the radiation era, and $P(k)$ is the primordial matter power spectrum introduced by new physics, such as axions. The variance of the white-noise power spectrum from the axion in the post-inflationary scenario can be expressed as

$$\sigma(M) = D_1(z) \sqrt{\frac{3A_{\text{osc}} M_0}{2\pi^2 M}}.$$}

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Given the value of $M_{\text{th}}$, one can obtain the minihalo mass function using the Press–Schechter model if the adiabatic fluctuations are neglected. It is worth noting that the variance is a function of $M/M_0$, such that the shape of the mass function will remain the same when we change the model parameters, but the characteristic mass will shift accordingly.

In the EMD scenario, the minihalo mass function can be calculated with a similar method. In this scenario, the primordial power spectrum remains the same as the adiabatic fluctuations, but their growth is modified. Effectively, this enhances the primordial power spectrum at small scales. The reheating temperature $T_{\text{rh}}$ is the only relevant physical parameter that determines the characteristic scale of the matter power spectrum, which corresponds to a characteristic mass scale $M_{\text{th}}$. The variance is enhanced at small scales due to EMD, in a manner that scales as

$$\sigma(M < M_{\text{th}}) \propto D_1(z) M/M_{\text{th}}^{(n+3)/6},$$

where $n = 0.963$ is the scalar spectral index (Erickcek & Sigurdson 2011), and $M_{\text{th}} \approx 9.06 \times 10^{-5} M_{\odot}$ (10 MeV/$T_{\text{rh}}$)³. Similar to the axion minicluster scenario, the variance is only a function of $M/M_{\text{th}}$, and the shape does not change with the reheating temperature. Thus, we can compute the minihalo mass function based on the variance using Press–Schechter. We can further compute the mass function including the minihalos that have fallen into the massive CDM halos by including the effect of adiabatic fluctuations, as shown in Figure 14.

Appendix B

Convergence Testing on the Simulation of Stellar Disruptions

Here, briefly, we discuss numerical tests of the simulations of stellar disruptions. The fiducial simulations presented in the main text employed the constant gravitation softening length $10^{-9}$ kpc and dark matter particle mass resolution $10^{-16} M_\odot$ for minihalos with a mass of $10^{-10} M_\odot$. We cap the timestep at $10^{-8}$ Gyr during the stellar encounter to resolve the star
trajectory since the stellar disruption is most relevant during the crossing. We justify those choices in more detail here.

B.1. Gravitational Softening

The gravitational softening length must be chosen appropriately in the simulation so that we can resolve the relevant physical scales. In our idealized simulations of stellar disruption, the minimum halo mass is $10^{-10} M_{\odot}$, which corresponds to a scale radius of $r_s = 9.6 \times 10^{-8}$ kpc, at $z = 0$. With the fiducial particle mass resolution, the convergence radius of dark matter calculated using the Power et al. (2003) criterion is $\sim 10^{-8}$ kpc. Therefore, the fiducial gravitational softening length, $10^{-9}$ kpc, and particle mass resolution should be sufficient to resolve the core profile of the halo. An additional run was performed with a gravitational softening length $2 \times 10^{-7}$ kpc, and the results of the disruption fraction and mass profile are robust. We have also verified that the results are robust to the particle number at our fiducial resolution.

B.2. Time Stepping

Since the crossing time of a stellar encounter is orders of magnitude shorter than the internal dynamical time of the minihalo, we need sufficiently small timesteps to resolve the trajectory of the star in the vicinity of the minihalo. The time stepping parameter in our fiducial simulations is capped at $10^{-8}$ Gyr, which is roughly $1/5 R_{\text{mb}}/v_s$, where $R_{\text{mb}}$ is the minihalo radius, and the corresponding minihalo mass is $10^{-10} M_{\odot}$. In an additional run, the time stepping parameter is changed to $2 \times 10^{-7}$ Gyr while other parameters including the minihalo parameters are exactly the same. We obtain the same disruption fraction, mass profile, and energy change after the halo is fully relaxed.

Appendix C
Modeling Multiple Encounters

In this section, we present the supplemented analysis and discussion for modeling the cumulative effects of multiple stellar encounters. They support the approach we proposed in the main text.

C.1. Multiple Encounters within a Dynamical Time

We first examine the amalgamation of encounters occurring within the dynamical time of the minihalo. A relevant illustration of such an event would be a minihalo traversing the Milky Way disk. The typical crossing time (given by Equation (19)) is significantly shorter than the dynamical time of the minihalo (given by Equation (3)). The minihalo does not have enough time to fully relax and realize the mass loss before the subsequent encounter occurs.

For a single encounter, the change in the velocity of a particle within a minihalo of size $R$ at position $r$ (relative to the center of the minihalo) due to an encounter with a perturber of mass $m$ moving with relative velocity $u$ at an impact parameter $b$ (when $b \gg R$) can be expressed as (Spitzer 1958; Green & Goodwin 2007)

$$\delta v \approx \frac{2 G m}{u b^2} [(r \cdot e_b) e_b - (r \cdot e_z) e_z]$$

(C1)

where $e_b$ is the unit vector in the direction of $b$, and $e_z$ is the unit vector perpendicular to $b$ and $u$ ($e_z = e_b \times e_u$). For a particle in the minihalo moving with velocity $v$, the change in energy is given by

$$\delta E = v \cdot \delta v + (\delta v)^2 / 2.$$  

(C2)

The ensemble average of $\delta E$ over all particles in the minihalo leads to the cancellation of the first term, under the assumption of isotropic particle velocities. This ultimately results in the expression for $\Delta E$ given in Equation (4).

When multiple encounters occur in a short enough time compared to the minihalo dynamical time, the aggregation of encounters is simply $\delta v = \sum_{i} \delta v_i$. According to Equation (C2), the aggregated $\delta E$ will have two types of terms: $\delta v_i \cdot \delta v_i$ and $\delta v_i \cdot \delta v_j$, where the former is simply energy injection from each individual encounters while the latter involves two different encounters

$$\delta v_i \cdot \delta v_j \propto \mathcal{K}(e_b^i, e_b^j) + \mathcal{K}(e_z^i, e_z^j) - \mathcal{K}(e_b^i, e_b^j) - \mathcal{K}(e_z^i, e_z^j),$$

$$\mathcal{K}(x, y) \equiv (r \cdot x)(r \cdot y)(x \cdot y).$$

(C3)

When the number of encounters aggregated is large, we can take the ensemble average over encounter parameters of the $i$th and $j$th events, of which the result depends on the local velocity distribution of stars with respect to the minihalo. Two typical scenarios are considered in the following.

(1) An isotropic velocity field. Both $e_b$ and $e_z$ are isotropic. Assuming the $i$th and $j$th encounters are independent, the ensemble averages (over the orientation of incident stars) of the four terms in Equation (C3) are the same and exactly cancel each other.

(2) A stream of stars. All the stars are moving in the same direction. However, the impact parameter $b$ is still isotropic in the cross section of the stream. Therefore, $e_b$ and $e_z$ are isotropic in this two-dimensional plane, leading to the cancellation of the four terms in Equation (C3) after the ensemble average. In a practical scenario, consider a minihalo moving through a cloud of stars with an isotropic velocity distribution; the relative velocity field of the stars with respect to the minihalo can be represented as a linear combination of the two previously discussed cases. Due to the linear nature of the ensemble average, the $\delta v_i \cdot \delta v_j$ terms can be ignored in this scenario as well. Consequently, only the $\delta v_i \cdot \delta v_i$ terms remain, and the total change in energy, $\delta E_{\text{tot}}$, can be expressed as the sum of $\delta E$ from individual encounters.

C.2. Multiple Encounters after Full Relaxation

A more complicated scenario arises when the minihalo has enough time to fully relax before the next encounter, such as multiple disk crossings with an orbital period comparable to or longer than the relaxation time. In such cases, the disruption model calibrated on single encounters needs to be revised because the internal structure of the minihalo can differ significantly from the initial condition. For example, in Section 3, we demonstrated that the density profile post-disruption is no longer NFW-like and features steeper slopes at the outskirts.

One approach often used in the literature is to integrate the effective optical depth. This relies on the approximation that minihalos are completely destroyed when the energy imparted exceeds a certain threshold. The number fraction of minihalos
survived scales exponentially with the optical depth of encounters (see the definition in Equation (22)). This approach has been used in, e.g., Schneider et al. (2010) for CDM minihalos. In Section 4, we have used it as a back-of-the-envelope estimate.

The second approach is to resolve individual encounters and process them sequentially, relying on a model to describe the internal states of minihalos after the encounter(s). Delos (2019) proposed a self-similar profile for post-encounter minihalos and calibrated disruption models accordingly. This method works reasonably well in the large $d\Delta E/E_0$ regime, as shown in the comparison in Figure 4. However, it is unknown whether it is accurate in the limit we are interested in. The typical $d\Delta E/E_0$ for a single stellar disk crossing is of the order $10^{-2}$ (see Equation (28)), which is outside the calibration range of this model.16

In Section 4, we have proposed an alternative and intuitive way to treat multiple encounters, which directly builds upon the knowledge about single encounters. This method requires no assumptions on parameterizing the density profiles of post-disruption minihalos and works reasonably well in a weak-disruption limit. Similar to what we derived for multiple encounters within a dynamical time, we accumulate the injected energy $\Delta E$ for successive encounters and use the response function $F(\Delta E_{\text{cal}}/E_0)$ (calibrated for single encounters) to obtain the total mass disruption fraction. The validity of this treatment is supported by the following observations. (1) The state of a minihalo after disruption could be well described by a single parameter, which can be the total energy, the gravitational binding energy, some characteristic density, etc. An example of such a description is given in Delos (2019), who found a self-similar density profile for minihalos after disruption described effectively by one parameter. Assuming virial equilibrium, the total energy of the minihalo is uniquely related to the density profile and thus this effective parameter. Inversely speaking, the state of a minihalo can be determined just by its total energy. (2) For the typical $d\Delta E/E_0 \sim 10^{-2}$ of a single stellar disk crossing, each encounter only causes a marginal disruption to the least bound particles. The shell of unbound particles that eventually escape the minihalo carries negligible energy at infinity, which is supported by both our simulations and the analytical calculations shown in Kavanagh et al. (2021). 17 Thus, the change in the total energy of the minihalo simply equals the cumulative energy imparted by the encounters, irrespective of the specific order in which these encounters occur. Combining this with the first point, the final state of the minihalo (determined by the final energy of the minihalo) is only related to the total imparted energy $\Delta E_{\text{cal}}$ and has no dependence on the exact history of encounters.

To validate this treatment, we perform a series of numerical tests. We take a minihalo with $M_{\text{halo}} = 10^{-10} M_\odot$, $c = 100$ and set the mass and velocity of the star as $1 M_\odot$ and $200 \text{ km s}^{-1}$.

16 It should be noted that the definition of $d\Delta E/E_0$ in Delos (2019) differs from the one used in this paper, as they normalize $d\Delta E$ by the binding energy of the central dark matter core.

17 In Kavanagh et al. (2021), during a stellar encounter, the change in the total energy of the minihalo is $E_i - E_f = (1 - f_0) \Delta E - E_{\text{bound}}^i$, where $E_i (E_f)$ is the initial (final) energy of the minihalo, $f_0$ is the fraction of imparted energy ends up in unbound particles, and $E_{\text{bound}}^i$ is initial energy of the subset of particles that are going to be unbound. When $\Delta E/E_0$ is $\sim 10^{-2}$, the corresponding $f_0$ is $\sim 10^{-2}$, and $E_{\text{bound}}^i/E_i$ is $\sim 10^{-4}$. Neglecting all second-order terms, we have $E_f - E_i \approx \Delta E$. The shell of unbound particles that eventually escape the minihalo has negligible energy at infinity, $(f_0 \Delta E + E_{\text{bound}}^i)/E_i \rightarrow 0$.

**Figure 16.** Mass loss of the minihalo from single/multiple encounters. The blue line and open circles show the response curve of $c = 100$ minihalos we calibrated in Section 3. The green circles show the minihalo mass loss as a function of accumulated imparted energy from successive encounters (with $\Delta E/E_0 = 0.1$ each). The red circles show the same test but with typical imparted energy $\Delta E/E_0 = 10$. Our treatment of accumulating imparted energy from independent encounters works reasonably well in the weak-disruption limit ($\Delta E/E_0 \ll 1$) but will severely underpredict the mass loss when $\Delta E/E_0 \gg 1$.

The impact parameter is left free so that we can simulate encounters of different $\Delta E/E_0$. In the first test, we simulate eight repetitive encounters with $\Delta E/E_0 = 0.1, 0.1, 0.1, 0.1, 0.1, 0.2, 0.2$. The minihalo is allowed to fully relax between encounters. In Figure 16, we compare the mass-loss curve of successive encounters with the response curve we calibrated for single encounters. The difference between the two is at $\lesssim 0.05 \text{ dex}$ level, and there is no sign of error accumulation as the number of encounters grows. In our real application, the maximum $\Delta E/E_0$ from individual disk crossing will be $\lesssim 10^{-2}$ (an estimate is given in Equation (28), and we have explicitly checked this in our numerical sampling of minihalos). Therefore, the approach we propose can reasonably accurately approximate the disruption fraction from multiple disk crossings. In the second test, we pick an impact parameter such that the injected energy from a single event is $\Delta E/E_0 \sim 10$ and repeat the encounter five times. This is a limit where all our assumptions above should fail. In this case, the response curve calibrated underpredicts the amount of mass loss, and the prediction error clearly inflates as the number of encounters increases.

**Appendix D**

**Orbital Model of Minihalos**

The analytic calculation of accumulated energy in Equation (28) comes from only one encounter with the minihalo velocities perpendicular to the plane of the Galactic disk. In reality, the minihalos after infall to the Milky Way halo will typically cross the stellar disks multiple times at various locations. To measure the accumulated energy input from a series of disk crossings, we need to evaluate the total number of passages through the disk, the stellar surface density where the encounter occurs, and the angle of incidence, with an ensemble average over all possible orbits for the minihalos eventually found at the observed radius.

For simplicity, we adopt a singular isothermal sphere model (a model variation assuming a Hernquist profile is considered in Appendix E) following the method in van den Bosch et al. (1999)
to estimate the uncertainties related to the orbits of minihalos. The density and potential of the system are given by

\[ \rho(r) = \frac{V_c^2}{4\pi Gr^2}, \quad \Phi(r) = V_c^2 \ln(r/r_0), \]  

where \( V_c \) is the constant circular velocity assumed to be 200 km s\(^{-1}\), and \( r_0 \) is the zero-potential reference point chosen to be 10 kpc. For a bound test particle in this potential, its halo-centric distance \( r \) will oscillate between the peri and apocenter with the period

\[ T = 2 \int_{r_1}^{r_2} \frac{dr}{\sqrt{2[E - \Phi(r)] - L^2/r^2}}, \]  

where \( E \) and \( L \) are the energy and angular momentum of the test particle, and \( r_1 \) and \( r_2 \) are the peri and apocenter of its orbit, which can be determined by solving the equation

\[ \frac{1}{r^2} + \frac{2(\Phi(r) - E)}{L^2} = 0. \]  

Due to the scale-free nature of the singular isothermal profile, the equations can be simplified by defining the maximum angular momentum \( L_c(E) = r_c(E)V_c \), where \( r_c(E) \) is the radius of the circular orbit, with energy \( E \) given by

\[ r_c(E) = r_0 \exp\left(\frac{E}{V_c^2} - \frac{1}{2}\right). \]  

Based on this, the circularity parameter is defined as \( \eta = L/L_c(E) \), and Equation (D3) is reduced to

\[ \frac{1}{x^2} + \frac{2 \ln(x) - 1}{\eta^2} = 0, \]  

where \( x \equiv r/r_c(E) \), and the two solutions correspond to the peri and apocenter distances, which are independent of the energy of the test particle if normalized by \( r_c(E) \). Similarly, if normalized by \( r_c(E)/V_c \), \( T \) also becomes independent of \( E \). The look-up tables of \( T, r_1, \) and \( r_2 \) with respect to \( \eta \) are computed numerically. Here, we do not consider the scattering and mergers of minihalos within the parent halo and treat their orbits as unperturbed from various relaxation mechanisms.

Assuming spherical symmetry, the ergodic phase-space distribution function \( f(\epsilon) \) can be derived through the Eddington inversion method

\[ f(\epsilon) = \frac{1}{\sqrt{2\pi} \epsilon^2} \frac{d}{d\epsilon} \int_0^{\epsilon} \frac{d\psi}{\sqrt{\epsilon - \psi}} \frac{d\rho}{d\psi}, \]  

where \( \psi \equiv -\Phi \), and \( \epsilon \equiv -E \). For the singular isothermal profile, the solution is simply the Boltzmann distribution

\[ g(E) = K \exp(-2E/V_c^2), \]  

where \( K \) is a constant normalization factor. Given a target radius for observation \( r_{\text{obs}} \), the normalized isotropic phase-space density function of dark matter particles localized around \( r_{\text{obs}} \) is

\[ f(E, L)|_{\text{obs}} = \frac{4\pi}{r_{\text{obs}}^2 \rho(r_{\text{obs}})} \frac{g(E)L}{\sqrt{2(E - \Phi(r_{\text{obs}})) - L^2/r_{\text{obs}}}}, \]  

where \( E \geq \Phi(r_{\text{obs}}) \) and \( L \leq 2r_{\text{obs}} \sqrt{E - \Phi(r_{\text{obs}})} \) are required. Replacing \( L \) as \( \eta L_c(E) \), we obtain

\[ f(E, \eta)|_{\text{obs}} = \frac{4\pi}{r_{\text{obs}}^2 \rho(r_{\text{obs}})} g(E) L_c(E) \frac{\eta}{\eta_{\text{max}}^2 - \eta^2}, \]  

where \( \eta \leq \eta_{\text{max}} = r_{\text{obs}} \sqrt{2(E - \Phi(r_{\text{obs}}))} / L_c(E) \). Following van den Bosch et al. (1999), we perform a Monte Carlo sampling of particles in the phase space based on this distribution function. For each sample particle, we compute its orbital period \( T \), pericenter and apocenter distances \( r_1 \) and \( r_2 \) based on the look-up table created earlier. We note that, because of the self-similar nature of the isothermal sphere, the value of \( r_1 \) and \( r_2 \) with respect to \( r_{\text{obs}} \) is independent of \( r_{\text{obs}} \), similarly for \( T/T_{\text{circ}}(r_{\text{obs}}) \). In Figure 17, we show the probability distribution function of eccentricities, pericenter, and apocenter distances, and orbital periods of sampled minihalo orbits. The distribution should be self-similar for any target radius of observation.

Assuming spherical symmetry, the orbit of a test particle will be confined in a plane, and the precession of the orbit will eventually lead to a rosette-like pattern. The phase of the precession when the orbit crosses the disk plane is random. Considering a large ensemble of dark matter particles, the distribution of the radial location where the encounter with the stellar disk takes place will be the same as the probability of the presence of the particle at that distance, and thus equivalent to the time-averaged radial distance of the test particle. Therefore, we have the averaged surface density given the orbit parameter \( E, L \) as

\[ \langle \Sigma_a \rangle_k(E, L) = \frac{2}{T} \int_{r_1}^{r_2} \frac{\Sigma_a(r)dr}{\sqrt{2(E - \Phi(r)) - L^2/r^2}}, \]  

where \( \langle \rangle_k \) denotes averaging over all possible minihalo orbits. The stellar surface density profile \( \Sigma_a(r) \) is given below Equation (23) following the measurements in McMillan (2011, 2017). The average of the second-order term \( \langle \Sigma_a^2 \rangle_k \) can be obtained similarly. In the top panel of Figure 18, we show the distribution of \( \langle \Sigma_a \rangle_k/\Sigma_a(r_{\text{obs}}) \) at the target radius 8 kpc, and the correction factor \( f_{\Sigma_a}(r_{\text{obs}}) \equiv \langle \Sigma_a \rangle_k/\Sigma_a(r_{\text{obs}}) \) (averaging over all possible orbits) is about 1.16. We compute the value of \( f_{\Sigma_a} \) and \( f_{\Sigma_a^2} \) at several different \( r_{\text{obs}} \) from 2 to 16 kpc and find that both \( f_{\Sigma_a}(r_{\text{obs}}) \) and \( f_{\Sigma_a^2}(r_{\text{obs}}) \) can be fitted by the functional form \( A e^{B/r_{\text{obs}}}/r_{\text{obs}}^C \). The best-fit parameters are \( A = 0.106, B = 2.03, r_1 = 12.961, \alpha = 2.048 \) for \( f_{\Sigma_a} \); and \( A = 0.318, B = 0.781, r_1 = 5.740, \alpha = 1.628 \) for \( f_{\Sigma_a^2} \).

An additional correction comes from the enhanced surface density when the minihalo trajectory is not perpendicular to the disk plane. To the leading order, the effective surface density along the trajectory of the incident minihalo (as well as the time duration the minihalo stays in the disk) should scale as \( 1/\cos \theta \),
where $\theta$ is the angle between the velocity vector of the mini-cluster and the normal vector of the disk. Assuming the velocities of dark matter particles are isotropic, the correction factor is

$$f_0 = \left( \frac{1}{\cos \theta} \right) = \int_{r_d/H_d}^{1} \frac{d \cos \theta}{\cos \theta} = \ln(r_d/H_d) \simeq 2,$$  \hspace{1cm} (D11)

where we have imposed a cutoff at $\cos \theta = H_d/r_d$ with $H_d$ and $r_d$ the scale height and length of the disk, assuming to be 400 pc and 3 kpc, respectively. Particles with even smaller incidence angles stay in the disk and will become completely disrupted (see Equation (22)), which will have no impact on the averaged energy imparted. Combining the two effects above, we obtain the correction factor for the effective stellar surface density.

**Appendix E**

**Test of Model Variations**

In this section, we will test a few variations of the fiducial model presented in the main paper and show how the minihalo mass function is affected by various assumptions we made. These variations include the following:

1. **A time-dependent stellar disk model following the fitting results of cosmological hydrodynamical simulations.** We assume that the surface density of the disk follows the scaling of disk mass in Garrison-Kimmel et al. (2017) while the scale radius of the disk is kept constant. At $z = 0$, it returns to the surface density distribution of the fiducial model. When calculating the stellar disruption of a minihalo, we use the time-averaged disk surface density since its infall to the Milky Way.

2. **A time-dependent model for halo potential.** The main progenitor mass of a Milky Way–mass halo is described as

$$M_{\text{vir}}(z) = \frac{M_{\text{vir}}(0)}{1 + \frac{(1 - \alpha z)}{2}} \exp(-\alpha z),$$

where $\alpha \simeq 0.6$ (Wechsler et al. 2002). To implement this time-dependent halo potential, in the orbital energy distribution of minihalos (Equation (D7) in Appendix D), we modify the circular velocity so that it scales with the virial velocity $V_{\text{vir}}$ of the halo at infall redshift $z_i$

$$V(z_i) = \frac{V_{\text{vir}}(z_i)}{V_{\text{vir}}(0)} = \frac{V_{\text{vir}}(0)}{M_{\text{vir}}(0)} \left( \frac{\rho_{\text{cen}}(z_i)}{\rho_{\text{cen}}(0)} \right)^{1/6}. \hspace{1cm} (E1)$$

However, when evaluating the phase–space distribution (Equation (D9)), the potential at $r_{\text{obs}}$ is kept the same as the $z = 0$ value. The $\eta$ distribution is also kept unchanged (which means that we keep the isotropic assumption about the velocity field) while $\eta_{\text{max}}$ will be reduced due to the systematically lower minihalo energy sampled. This accounts for the fact that minihalos falling earlier will be more tightly bound to the Milky Way halo, have smaller
or, orbital pericenter distances, and could suffer from stronger disruption.

3. Variation of the orbit model. We redo the calculation in Appendix D but assume the halo features a Hernquist density profile (Hernquist 1990); with halo mass $10^{12} M_\odot$ and scale radius 20 kpc. It represents a more realistic host halo density profile with an analytical formula of phase space density. At the central part ($r \lesssim 10$ kpc) of the halo, it yields an almost identical density profile as the NFW profile.

4. Different radii for the evaluation of tidal disruption. In the main paper, we choose $r_{\text{obs}}$ as the radius to evaluate tidal disruption while a minihalo could have closer pericenter passages. Here, we test evaluating tidal disruption at $0.5 \times r_{\text{obs}}$, which agrees with the median pericenter distance of minihalos (see Figure 17) in the fiducial orbit model.

5. Different treatment of minihalos suffering strong tidal disruption. In the fiducial model, we cut off the minihalo density profile at $r_t$. However, some previous studies (e.g., Peñarrubia et al. 2010; Green & van den Bosch 2019) have shown that the central density of subhalos can be reduced through tidal stripping as well, especially in violent mass-loss cases. To evaluate the uncertainties related to the treatment of strong tidal disruption events, we test two extreme scenarios. (1) When $r_t$ becomes smaller than the scale radius of the minihalo $r_s$, we simply drop the minihalo and consider it completely disrupted. (2) We also test keeping the minihalo unperturbed in such cases.

In Figure 19, we show the post-disruption mass function of minihalos (AMC, $m_a = 25 \mu$eV) at $r_{\text{obs}} = 8$ kpc with various model variations listed above. Assuming a time-dependent stellar disk can enhance the mass function at $M_{\text{halo}} \gtrsim 10^{-8} M_\odot$ and increase the survival fraction to $\gtrsim 70\%$. Assuming a time-dependent halo potential to correct minihalo orbits can decrease the mass function at $M_{\text{halo}} \gtrsim 10^{-8} M_\odot$ and decrease the survival fraction to $\sim 40\%$. These two model variations both consider some time-dependence of the stellar/halo properties of the host halo and have opposite impacts on the survival fraction at a similar magnitude. All other model variations result in almost identical mass functions and mass survival fractions to the fiducial model.

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Figure 19. Minihalo mass function in different model variations compared to the fiducial model. In the bottom panel, we show the survival mass fraction above a certain mass limit. These variations are described in the text, and most of them have marginal effects on the post-disruption mass function.
