Drinfeld Yangian of the queer Lie superalgebra $sq_1$

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Abstract. Drinfeld Yangian of a queer Lie superalgebra $sq_1$ is defined as the quantization of a Lie bisuperalgebra of twisted polynomial currents. An analogue of the new system of generators of Drinfeld is being constructed. It is proved for the case Lie superalgebra $sq_1$ that this so defined Yangian and the Yangian, introduced earlier by M. Nazarov using the Faddeev-Reshetikhin-Takhtadzhian approach, are isomorphic.

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1. Introduction
The Yangians of simple Lie superalgebras were defined by Drinfeld in the mid-1980s (see, for example, [3], [4], [5], [6], [1]), as quantizations (or non-commutative deformations in the Hopf algebra class) of Lie polynomial Lie bisuperalgebras with a bracket, determined by a rational $r$-matrix (the Yang matrix). But the Yangian of general linear algebra appeared earlier in the works of the mathematical physicists of the Leningrad school L.D. Faddeev. In the second case, it was determined using the defining relations given by the Yang quantum $R$-matrix. V.Drinfeld established the equivalence of these two different definitions of the Yangian for a special linear algebra, but he did not publish proof of this fact. The first published evidence appeared much later (first for $sl_2$-case and later, almost 20 years after the first works of Drinfeld and for the case $sl_n$ (see [10])).

Later, in the early 90s of the last century, L.D. Faddeev, N.Yu. Reshetikhin and L. Takhtadzhian published a detailed description of the approach to the definition of quantum groups based on the use of the quantum $R$-matrix for defining a system of defining relations ([10]). Such an approach was naturally related to the quantum method of the inverse scattering problem developed by them, which is now more often called the algebraic Bethe ansatz. Because, it is natural to call their name the traditional definition for the mathematical physicists of the Yangian, in contrast to V. Drinfeld’s definition. In the 90s of the last century, the Yangians of Lie superalgebras began to be investigated (see the works [12], [18]). Moreover, in the first paper the definition of the Yangian of the general linear Lie superalgebra was given by the Faddeev-Reshetikhin-Takhtadzhian approach (the FRT approach), and in the second work, the Yangian of the special linear superalgebra was defined in accordance with Drinfeld’s approach. The Yangian thus defined we call the Drinfeld Yangian. We note that the Yangians of Lie superalgebras have now found numerous applications in quantum field theory, in particular, in...
the quantum theory of superstrings ([2], [17]).

For definition Yangian in accordance with the Drinfeld approach it is necessary to have a nondegenerate invariant bilinear form on the original Lie algebra (or Lie superalgebra), which is used in the definition of the Young matrix or for definition cobracket in Lie bisuperalgebra. But such a form is absent in the queer Lie superalgebra, which prevents the extension of the Drinfeld definition of Yangian on this case. Nevertheless, as will be seen below, instead of deforming the superalgebra of polynomial currents, one can consider the deformation of a twisted superalgebra of currents. The Yangians associated with the queer Lie superalgebra (see [9], [7]) were first constructed by M. Nazarov ([13]), using approach Faddeev-Reshetikhin-Takhdazhjan. Later in the paper by M. Nazarov and A. Sergeev ([14]) was given definition of Yangian of the queer Lie superalgebra using a general approach based on the centralizer construction of G. Olshansky. This definition was later used in [15] to establish the connection between the Yangian of queer Lie superalgebra and W-algebras, introduced in [16].

In the paper [23], the Yangians of the queer Lie superalgebra was introduced, following the Drinfeld approach as a quantization of the Lie bisuperalgebra of twisted currents. There was also obtained his description in terms of the current system of generators and defining relations (a new system of generators in the terminology of V. Drinfel’d, [5]). It should be noted that the Yangian of a queer Lie superalgebra is defined as the deformation of a Lie bisuperalgebra of twisted polynomial currents in which the structure of a Lie superalgebra is given by a rational r-matrix (see [20]).

We note that this approach can not be fully extended to quantizations of other twisted superalgebras of currents, since, generally speaking, they can not be endowed with the structure of a Lie bisuperalgebra (see [24]). Nevertheless, with a small modification, it can be used for definition of twisted Yangians, which, generally speaking, are not Hopf superalgebras.

In this paper, which is the first part of the work devoted to the study of the Yangian of the queer Lie superalgebra (see also [29], [28]), we define the Yangian of the queer Lie superalgebra based on the Drinfeld approach, and establish a connection with the Yangian Nazarov of the queer Lie superalgebra in the particular case of the Lie superalgebra \( q_1 \). A few words about the organization of this work. In the second section we recall the definitions of the queer Lie superalgebra and the twisted current Lie bisuperalgebra. We describe the deformation of the twisted current Lie bisuperalgebra and introduce the main actor, the Yangian of the queer Lie superalgebra. In the section 3 we recall the definition of the Yangian of the queer Lie superalgebras given by M. Nazarov and introduce the current system of generators for Drinfeld Yangian in section 4. We prove in the section 5 the theorem on the isomorphism of the Drinfeld Yangian of the queer Lie superalgebra and the Yangian introduced by M. Nazarov for the partial case \( Q_1 \). Our proof is based on well-known ideas and essentially uses the triangular decomposition of the transfer matrix used in definition of the defining relations in the Yangian introduced by M. Nazarov and the construction of the quasi-determinants that was introduced by I.M. Gelfand and V.S. Retakh (see [8]).

In this paper we give the sketch of idea of the proof of general case using quasi-determinant theory (see [8], [11]) and uses some features of the root system of the queer Lie superalgebra (see also [29]).

We note that simplifying the proof of the theorem on isomorphism, we somewhat changed the current system of generators in this paper in comparison with the papers [22], [23], [25], [28] which led to a somewhat simpler and more natural form of defining relations. Let’s note that exist isomorphism between super Yangian and quantum loop superalgebra for special linear superalgebra, which is twin of queer Lie superalgebra ([26], [27]). To construct such isomorphism for Yangian of queer Lie superalgebra is an open problem.
2. Queer Lie superalgebra $q_1$, graded by involution Lie superalgebra $gl(1,1)$ and Yangian

In this case we are dealing not with the basic Lie superalgebra. Nevertheless, the general construction of quantization considered in the previous subsection can also be used in this particular case. We recall the definition of the Lie superalgebra $q_1$. This superalgebra is generated by the generators $h_0, h_1$, and $p(h_0) = 0, p(h_1) = 1$, which satisfy the following relations

$$[h_0, h_0] = 0, \quad [h_0, h_1] = 0, \quad [h_1, h_1] = 2h_0. \quad (1)$$

The relations for a twisted superalgebra of currents with values in $gl(1,1)$ have the following form.

$$[h_0, h_0^1] = [h_0^1, h_0] = 0, \quad [h_0^1, h_1^1] = -[h_1^1, h_0^1] = \frac{1}{2}h_1, \quad (2)$$

$$[h_1^1, h_1^1] = -\frac{1}{2}h_0, \quad [h_0^1, h_1] = 2h_1^1, \quad [h_1^1, h_1] = 2h_1^1. \quad (3)$$

The Casimir element corresponding to the invariant scalar product in $gl(1,1)$ is defined by the formula:

$$t_0 = h_1^1 \otimes h_1 + h_0^1 \otimes h_0 - h_1 \otimes h_1^1 + h_0 \otimes h_0^1. \quad (4)$$

We note that in this case the structure of the bisuperalgebra on the twisted current algebra $gl(1,1)^{tw}[t]$ is defined by a bracket

$$\delta : gl(1,1)^{tw}[t] \rightarrow gl(1,1)^{tw}[t] \otimes gl(1,1)^{tw}[t],$$

which is given by formula:

$$\delta : a(u) \rightarrow [a(u) \otimes 1 + 1 \otimes a(v), r_{\sigma}(u,v)], \quad r_{\sigma}(u,v) = \frac{1}{2} \frac{t_0 + t_1}{u - v} + \frac{1}{2} \frac{t_0 - t_1}{u + v}, \quad (5)$$

where

$$t_0 = h_0 \otimes h_1^1 - h_1 \otimes h_1^1, \quad t_1 = h_0^1 \otimes h_0 - h_1^1 \otimes h_1.$$

Next we will be interested in the Yangian of the Lie superalgebra $sq_1$, that is, $Y(sq_1)$, and also Yangian $Y(Q_1)$.

**Definition 2.1.** Yangian $Y(sq_1)$ is an associative superalgebra generated by generators $h_{0,0}, h_{1,0}, h_{1,1}, h_{1,1}$ which satisfy the following defining relations:

$$[h_{0,i}, h_{0,j}] = 0, \quad i, j \in \{0,1\}, \quad [h_{1,0}, h_{1,0}] = 2h_{0,0}, \quad (6)$$

$$[h_{0,0}, h_{i,k}] = 0, \quad i, j \in \{0,1\}, \quad [h_{0,1}, h_{1,0}] = 2h_{1,1}, \quad (7)$$

$$[h_{1,0}, h_{1,1}] = 0. \quad (8)$$

Yangian $Y(sq_1)$ is a Hopf superalgebra with comultiplication on generators which is given by the following formulas:

$$\Delta(h_{i,0}) = h_{i,0} \otimes 1 + 1 \otimes h_{i,0}, \quad i = 0,1, \quad (9)$$

$$\Delta(h_{0,1}) = h_{0,1} \otimes 1 + 1 \otimes h_{0,1} + h_{1,0} \otimes h_{1,0}, \quad (10)$$

$$\Delta(h_{1,1}) = h_{1,1} \otimes 1 + 1 \otimes h_{1,1} - h_{1,0} \otimes h_{0,0} + h_{0,0} \otimes h_{1,0}. \quad (11)$$

We obtain system generators and defining relations for $Y(q_1)$ as in papers [25], [28]. Further we need in some modification of system generators, namely in so-called new (or current) system generators (see section 4).
3. Nazarov Yangian $Y_N(q_1)$

Recall that Yangian $Y_N(q_1)$ introduced by M. Nazarov ([13], [14]) is the associative unital superalgebra over $C$ with countable set of generators

$$t_{i,j}^m, \quad i, j = \pm 1, \quad m = 1, 2, \ldots.$$  

The $Z_2$ grading of the $Y_N(q_1)$ is defined as follows $p(t_{i,1}^m) = p(t_{-i,-1}^m) = 0$, $p(t_{i,-1}^m) = p(t_{-i,1}^m) = 1$.

To write down the defining relations for these generators we employ the formal series in $Y_N(q_1)[[u^{-1}]]$:

$$t_{i,j}(u) = \delta_{i,j} \cdot 1 + t_{i,j}^1 u^{-1} + t_{i,j}^2 u^{-2} + \ldots.$$  

Then for all possible indices $i, j, k, l$ we have the relations:

$$
(u^2 - v^2)[[t_{i,j}(u), t_{k,l}(v)]] \cdot (-1)^p(i)(p(k) + (i)p(l) + (k)p(l)) = \\
(u + v)(t_{k,j}(u)t_{l,i}(v) - t_{k,j}(v)t_{l,i}(u)) - (u - v)(t_{-k,j}(u)t_{-l,i}(v) - t_{k,-j}(v)t_{-l,-i}(u))(-1)^p(k) + p(l),
$$

(12)

where $v$ is a formal parameter independent for $u$, and we have an equality in the algebra of formal Laurent series in $u^{-1}, v^{-1}$ with coefficients in $Y(q_1)$. Also we have the relations

$$
t_{i,j}(-u) = t_{-i,-j}(u).  
$$

(13)

Note, that the relations (12) and (13) are equivalent to the following defining relations:

$$
([t_{i,j}(u), t_{k,l}(v)] - [t_{i,j}^m, t_{k,l}^{m+1}]) \cdot (-1)^p(i)(p(k) + (i)p(l) + (k)p(l)) = \\
t_{k,j}^m t_{l,i}^{m+1} - t_{k,j}^{m+1} t_{l,i}^m - t_{k,j}^{m+1} t_{l,i}^m + \\
(-1)^p(k) + p(l)(-t_{-k,j}^m t_{-l,i}^m + t_{-k,j}^m t_{-l,i}^m + t_{k,-j}^m t_{-i,-l}^m - t_{k,-j}^m t_{i,-l}^m),
$$

(14)

and as above

$$
t_{-i,j}^m = (-1)^m t_{i,j}^m.  
$$

(15)

where $m, r = 1, 2, \ldots$ and $t_{i,j}^0 = \delta_{i,j}$.

Recall that $Y_N(q_1)$ is a Hopf algebra with comultiplication given by the formula

$$
\Delta(t_{i,j}^m) = \sum_{s=0}^{m} \sum_k (-1)^{(i) + p(k)}(p(j) + (k)p(l)) t_{i,k}^s \otimes t_{j,k}^{m-s}.  
$$

(16)

Let’s note that this definition can be rewrite using generating functions as follows. Let, $I = \{1, 2, \ldots, n\}$, $I_1 = I \cup (-I) = \{\pm 1, \pm 2, \ldots, \pm n\}$ and as above

$$
J = \sum_{i \in I_1} E_{i,-i}(-1)^{(i)}
$$

$$
P = \sum_{i,j \in I_1} E_{i,j} \otimes E_{j,i}(-1)^{(j)}, \quad P_1 = P \otimes 1, \quad P_2 = 1 \otimes P,
$$

$$
T(u) = \sum_{i,j \in I_1} E_{i,j} \otimes t_{i,j}(u), \quad T_1(u) = T(u) \otimes 1, \quad T_2(u) = 1 \otimes T(u).
$$

We define quantum R-matrix by formula:

$$
R(u, v) = 1 - \frac{P}{u - v} + \frac{P J_1 J_2}{u + v}.
$$

(17)
Then defining relations can be presented in the following form

\[(R(u, v) \otimes 1)T_1(u)T_2(v) = T_2(v)T_1(u)(R(u, v) \otimes 1).\]  \hspace{1cm} (18)

We write explicitly the relations for the Yangian \(Y_N(q_1)\):

\[\{t^m_{-1,1}, t^k_{1,1}\} = \sum_{r=1}^{m-1} \left( t^{k+r-1}_{-1,1} t^m_{-1,1} - t^{m-r}_{-1,1} t^{k+r-1}_{-1,1} \right) + \sum_{r=1}^{m-1} (-1)^r \left( (-1)^{k+m-k+r} t^m_{1,1} - t^m_{-1,1} t^{k+r-1}_{1,1} \right).\]  \hspace{1cm} (19)

Similarly, we obtain, that

\[\{t^m_{1,1}, t^k_{1,1}\} = - \sum_{r=1}^{m-1} \left( t^{k+r-1}_{1,1} t^m_{1,1} - t^{m-r}_{1,1} t^{k+r-1}_{1,1} \right) + \sum_{r=1}^{m-1} (-1)^r \left( (-1)^{k+m-k+r} t^m_{1,1} - t^m_{1,1} t^{k+r-1}_{1,1} \right).\]  \hspace{1cm} (20)

Last relation has the following form:

\[\{t^m_{1,1}, t^k_{-1,1}\} = \sum_{r=1}^{m-1} \left( t^{k+r-1}_{1,1} t^m_{-1,1} - t^{m-r}_{1,1} t^{k+r-1}_{1,1} \right) + \sum_{r=1}^{m-1} (-1)^r \left( (-1)^{k+m-k+r} t^m_{1,1} - t^m_{1,1} t^{k+r-1}_{1,1} \right).\]  \hspace{1cm} (21)

4. Yangian \(Y(sq_1)\). Current generators.

Let

\[\{a, b\} := a \cdot b + (-1)^{p(a)p(b)} b \cdot a\]

be an anticommutator.

**Definition 4.1.** Yangian \(\tilde{Y}(sq_1)\) is the associative unital superalgebra over \(C\) with countable set of generators \(\tilde{h}_{0,k}, \tilde{h}_{1,k}, k \in \mathbb{Z}_{k \geq 0}\), which satisfy the following system of defining relations.

\[
\begin{align*}
[\tilde{h}_{0,2k}, \tilde{h}_{0,2r}] &= [\tilde{h}_{2k+1}, \tilde{h}_{2r+1}] = 0, \quad [\tilde{h}_{0,0}, \tilde{h}_{0,k}] = 0, \\
[\tilde{h}_{0,1}, \tilde{h}_{1,k}] &= 2\tilde{h}_{1,k} + \frac{1}{2}(\tilde{h}_{0,0}\tilde{h}_{1,k} + \tilde{h}_{1,k}\tilde{h}_{0,0}), \\
[\tilde{h}_{1,0}, \tilde{h}_{1,k}] &= \tilde{h}_{0,k}, \\
[\tilde{h}_{0,k+1}, \tilde{h}_{1,r}] &= [\tilde{h}_{0,k}, \tilde{h}_{1,r+1}] + \{\tilde{h}_{0,k}, [\tilde{h}_{1,k}, \tilde{h}_{1,r}]\} + \{\tilde{h}_{1,k}, [\tilde{h}_{0,k}, \tilde{h}_{1,r}]\}, \\
[\tilde{h}_{1,k+1}, \tilde{h}_{1,r}] - [\tilde{h}_{1,k}, \tilde{h}_{1,r+1}] &= [\tilde{h}_{1,k}, [\tilde{h}_{1,r}, \tilde{h}_{1,r}] + \{\tilde{h}_{1,k}, \tilde{h}_{1,r}\} + \{\tilde{h}_{1,k}, \tilde{h}_{0,r}\},
\end{align*}
\]

**Theorem 4.1.** ([25], [28], [29]) Yangian \(Y(sq_1)\) is isomorphic as an associative algebra to \(\tilde{Y}(sq_1)\).
Let’s define the new generators in \( Y(sq_1) \) by the following formulas:

\[
  h_{1,k+1} := [h_{0,1}, h_{1,k}], \quad h_{0,2k} := [h_{1,0}, h_{1,2k}].
\]

Easy to check that the following relations are satisfied:

\[
  [h_{1,k+2}, h_{0,2r}] = [h_{1,k}, h_{0,2(r+1)}] + \ldots
\]

We define the system of generators and defining relations of Drinfeld Yangian of the Lie superalgebra \( sq_1 \).

**Theorem 4.2.** Yangian \( Y_D(sq_1) \) is an associative Hopf superalgebra over \( C \) generated by generators \( h_{0,2k}, h_1, k, k \in \mathbb{Z}_{\geq 0} \), which satisfy the following defining relations:

\[
  [h_{2k}, h_{0,2r}] = 0, \\
  [h_{0,2k+2}, h_{1,l}] = [h_{1,2k+1}, h_{0,l+1}] + [h_{0,2k}, h_{1,l+2}] + \{h_{0,2k}, h_{1,2k}\} + (-1)^l\{h_{1,2k}, h_{0,l}\}, \\
  [h_{1,k+2}, h_{1,l}] = [h_{1,k}, h_{1,l+2}] = [h_{1,k+1}, h_{1,l}] + [h_{0,k}, h_{0,l+1}].
\]

5. **Isomorphism between Nazarov Yangian \( Y_N(q_1) \) and Drinfeld Yangian \( Y(q_1) \)**

I recall the triangular decomposition construction for Yangian \( Y_N(q_1) \). Let \( T(u) = (t_{i,j}(u))_{i,j = \pm 1} \).

Then we have the following decomposition

\[
( t_{1,1}(u) \quad t_{1,-1}(u) \quad t_{-1,1}(u) \quad t_{-1,-1}(u) ) = \left( \begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & h_{-1}(u) & 0 & 0 \\
0 & 0 & d_2(u) & d_1(u) \\
0 & 0 & d_2(u) & h_{-1}(u)
\end{array} \right) \cdot \left( \begin{array}{c}
h_1(u) \\
0 \\
0 \\
0
\end{array} \right).
\]

So, we have the following equality

\[
( t_{1,1}(u) \quad t_{1,-1}(u) \quad t_{-1,1}(u) \quad t_{-1,-1}(u) ) = \left( \begin{array}{cccc}
d_1(u) & 0 & 0 & d_1(u) \\
0 & h_{-1}(u) \cdot d_1(u) & d_2(u) + h_{-1}(u) d_1(u) h_1(u) & 0 \\
0 & 0 & d_2(u) & h_{-1}(u)
\end{array} \right).
\]

We can formulate the main result of this subsection.

**Theorem 5.1.** The correspondence

\[
t_{1,1}(u) \rightarrow h_0(u), \quad t_{1,1}(u)^{-1} t_{-1,1}(u) \rightarrow h_1(u)
\]

define the epimorphism

\[
F : Y_N(q_1) \rightarrow Y_D(sq_1),
\]

where \( KerF = (h_{0,3}, h_{0,5}, \ldots) \) the ideal generated by elements \( h_{0,2k+1} \) for \( k \geq 1 \).

**Proof.** Let’s check that map \( F \) respects the defining relations of \( Y_N(q_1) \) and \( Y_D(sq_1) \). We have that

\[
t_{1,-1} = d_1(u) \cdot h_1(u).
\]

Therefore

\[
t_{1,-1} = \sum_{k=1}^{\infty} t_{1,-1} u^{-k} = \left( 1 + \sum_{k=0}^{\infty} h_{0,k} u^{-k-1} \right) \left( \sum_{r=0}^{\infty} h_{1,r} u^{-r-1} \right) = \sum_{n=0}^{\infty} \left( \sum_{k=0}^{n-1} h_{0,k} \cdot h_{n-k} \right) u^{-n-1}.
\]

Here \( h_{0,-1} = 1 \). Therefore

\[
t_{1,-1}^{n+1} = \sum_{k=-1}^{n-1} h_{0,k} \cdot h_{n-k} = -h_{1,n} + h_{0,0} h_{1,n-1} + \ldots + h_{0,n-1} h_{0,0}.
\]
Also, we obtain
\[ t_{1,1} = 1 + \sum_{n=1}^{\infty} t_{1,1}^n u^{-n} = d_1(u) = h_0(u) = 1 + \sum_{n=0}^{\infty} h_{0,n} u^{-n-1}. \]

Therefore
\[ t_{1,1}^n = h_{0,n-1}. \]

As a corollary of proved theorem we obtain a definition of Yangian
\[ Y_N(q_1) := Y_N(q_1)/\text{Ker} F. \]

In Nazarov Yangian \( Y_N(q_1) \) we’ll use only even generators \( t_{11}^{2k} \), because only this generators corresponds to generators \( h_{0,2k} \) in Drinfel’d Yangian \( Y_D(q_1) \). We have the following lemma is hold.

**Lemma 5.1.** We have the following relation
\[ [t_{11}^{2k}, t_{11}^{2r}] = 0. \quad (28) \]

for all \( r, k > 0 \).

**Proof.** We give a sketch of proof. For \( r, k = 0,1 \) the formula (28) is evidently satisfied. Easy to check also that \([t_{11}^0, t_{11}^{2r}] = 0\) and \([t_{11}^0, t_{11}^{2k}] = 0\) for all \( r \geq 0 \).

We suggest that \([t_{11}^{2k}, t_{11}^{2r}] = 0\) and let’s prove that \([t_{11}^{2(k+1)}, t_{11}^{2(r+1)}] = 0\) is also true. \( \square \)

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