Van Hiele Levels: Errors in Solving Geometry Problems from Mathematical Disposition

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Abstract. The purpose of this study is to analyze student errors based on van Hiele's level of understanding in solving geometric problems in terms of mathematical dispositions. This research is considered important as it can reveal mistakes often made by students based on van hiele levels. There are 5 levels contained in van hiele levels, namely level I-visualization, level II-analysis, level III-ordering, level IV-deduction, and level V-rigor. This research is a qualitative descriptive study. Subjects in this study were selected based on high, medium, low mathematical disposition abilities. This research shows that students with high mathematical disposition in solving geometric problems reach level IV-deduction. Students with moderate dispositions are able to reach level II-analysis and make mistakes at level III-ordering. Furthermore, students with low mathematical disposition only reach level I-visualization and make errors at level II-analysis.

1. Introduction

Mathematics is closely related to problem solving. The Regulation of the Minister of Education and Culture of the Republic of Indonesia Number 36 of 2018 states that the main focus in learning mathematics is problem solving skills. Mathematical problem solving is closely related to difficulties and mistakes. Mathematical errors are common and there is a need for further analysis of the errors that are made and how to solve them (Gagatsis & Kyriakides, 2000). Errors in solving mathematical problems often occur. Bray (2013) and Clements (1982) mention the importance of analyzing mathematical errors made by students. With this error analysis, students can learn from their mistakes and reduce the possibility of making the same mistakes in the future. Errors made when solving problems are generally due to errors in understanding concepts, procedures or principles (Roselizawati Hj Sarwadi & Shahrill, 2014). Teachers must know what mistakes are made by students in solving problems to provide them with strategies to reduce their tendency to make mistakes so that learning becomes effective (Kingsdorf & Krawec, 2014).

There are many types of errors that often occur in solving math problems. These are misunderstanding the problem/concept and transformation errors (Fatahillah et al., 2017; Nur Afifah et al., 2018; Layn & Kahar, 2017; Jana, 2018). In line with the results of research from Nurhikmah & Febrian (2016) which states that mistakes are mistakes that are usually made in solving problems are errors in the procedures used. Marshall (1983) states that the most common mathematical errors are calculation errors and errors in translating tables, graphs, or geometric drawings.
One of the courses where students often make mistakes in solving their problems is the geometry course. Students still have difficulty solving geometry problems (Dosinaeng et al., 2019) and only high-ability students can solve geometric problems correctly. Meanwhile, Bahr et al and Basarear (Luneta, 2015) stated that in the school learning curriculum, students must be able to analyze the properties, characteristics and relationships of flat and spatial geometric shapes.

There are various levels of understanding in mathematics, one of which is Van Hiele's level of understanding. Van Hiele's level of thought was developed by Pierre Van Hiele and Diba van Hiele-Geldof in 1957 and 1986 and focuses on geometry. There are 5 levels of thinking in geometry based on Van Hiele, namely level I-visualization, level II-analysis, level III-ordering, level IV-deduction and level V-rigor (Halat, 2006; Erdogan, 2020). Level I-visualization mainly identified elements and properties of geometric shapes, if there is no image visualization, students can visualize based on the problem given. Level II-analysis is when students have the ability to analyze the relationship between concepts and known geometric elements (Crowley, 1987; Mason, 1998; Mayberry, 1983). Meanwhile, level III-ordering is the ability to relate definitions, properties and known elements with informal arguments (Mason, 1998). Level IV-deduction is the ability to use postulates, axioms, and theorems in deductive proof (Crowley, 1987; Vojkuvkova, 2012). Lastly, level V-rigor is the ability to explain theorems and postulates used in different systems and compare them (Mason, 1998; Perdikaris, 1994).

Research results from Hendroanto et al. (2019) which is based on Van Hiele's level of thinking, discover that students have difficulty in solving geometric problems, especially in understanding the questions and determining the strategies used. Meanwhile, Luneta (2015); and Razak & Sutrisno (2017) state that most students are still at level 2-analysis at Van Hiele’s thinking level. Rizki et al. (2018) in their research also stated that in solving geometry problems, most students had not reached level III-ordering, level IV-deduction and level V-rigor. Yudianto et al., (2018) state that most students in solving geometry problems only reach level I-visualization. Some of the research above demonstrates students' low ability to solve geometry problems. There needs to be further research on how to solve geometric problems by prospective teachers so that when they become teachers, they can help their students to be able to improve their students' problem-solving abilities.

Students' thinking ability is also influenced by affective aspects. One of them is mathematical disposition. Sumarmo (Fitrianna et al., 2018) states that mathematical disposition is a strong willingness, awareness and interest in learning mathematics. It is important to know the ability of mathematical dispositions to enable educators to find out the individual condition of the students and how they are interested in mathematics (Purwashi & Bernad, 2018). Maxwell (Rahmadhani, 2018; Cooke, 2015) also writes that mathematical disposition needs particular attention because it is a major factor in determining the success of learning mathematics. In addition, Feldhaus (2014) and Clark et al. (2014) also emphasize the importance of recognising the mathematical disposition of prospective teachers, particularly at what level their disposition towards mathematics (Cooke, 2015).

Based on the explanation above about the importance of knowing the mathematical errors made in solving geometric problems and the importance of knowing the mathematical disposition, an analysis of students' mathematical errors is carried out in solving geometric problems based on Van Hiele's theory in terms of mathematical dispositions. Van Hiele's level of understanding that is used in this research is level I-visualization, level II-analysis, level III-ordering, level IV-deduction and level V-rigor.

2. Methodology
This study aims to analyze student errors based on Van Hiele's level of thinking in solving geometric problems in terms of mathematical disposition. This research is a qualitative descriptive study. The research subjects were students of FKIP UHAMKA who were given a mathematical disposition questionnaire. The scores were classified into high, medium, and low dispositions. The selection of research subjects was done through purposive sampling technique. At least one student is selected as a research subject from each category of mathematical disposition. The selected subjects have the same initial ability and are of the same gender. The selected subjects are given a geometry problem solving test (TPM). The validity of the data were tested using time triangulation.
The problem solving test instrument was used to reveal student errors in solving geometric problems based on the van hiele level. Another instrument used was a mathematical disposition questionnaire to determine the disposition ability of the subject in the high, medium, and low categories. The indicators of mathematical disposition used were self-confidence, curiosity, persistence, flexibility and reflection.

Data analysis used the constant comparative method (Moleong, 2013: 287). The steps of the analysis were (1) data reduction and categorization; selecting the required data and removing unnecessary data; (2) data presentation; the reduced data is presented for the analysis of errors in solving geometric problems according to Van Hiele's level of understanding in terms of mathematical disposition; and (3) drawing conclusions and verification; the data that has been presented were then compared and analyzed for errors based on Van Hiele's level of understanding, adjusted for each category of mathematical disposition.

3. Result and Discussion
As the basis for determining the research subject, the subject was given a mathematical disposition questionnaire. The subjects were then categorized based on high, medium, and low mathematical disposition abilities. Apart from being based on the mathematical disposition category, the research subjects were selected based on the initial value and the same gender. After being categorized, 3 research subjects were selected with one subject in each category of mathematical disposition. From the categorization results, there were 10 subjects with a high category mathematical disposition, 7 people with a medium category disposition, and 12 subjects with a low category disposition. The determination of the three subjects was of the same gender and had an initial value of 73. The research subjects were coded with S1 (high mathematical disposition), S2 (moderate mathematical disposition), and S3 (low mathematical disposition).

Data from this research were in the form of data on the results of geometry problem solving tests and interview data. Interview data on subjects in each category of mathematical disposition were transcribed to be analyzed for error rates based on Van Hiele's level of understanding of geometry. The interviewer was denoted by "P" and the voting subject was denoted by "S". The geometry problem solving test consisted of 3 questions. The following section described the results of the geometry problem solving tests and interviews in each category of mathematical disposition.

High Mathematical Disposition (S1 Subject)
The research subject was asked a question: “It is known that the ABCD.EFGH cube has a rib length of 10 cm. Determine the distance of point H to the diagonal of the AC side! ”. The results of the geometry problem solving test and interview excerpts for the S1 subject are as follows.

Figure 1. The results of the geometry problem-solving test for S1 subjects

P1.11: question number 3 asks us to determine the distance from point H to the diagonal of the AC side. S1.11: number 3 uses the concept of the Pythagorean theorem. The first is the shortest distance is the perpendicular line. It was then pulled from point H to the diagonal AC, making a right triangle. As the
length is 10, the diagonal length of the side is $10\sqrt{2}$. After that, AO is half of AC, so AO is $5\sqrt{2}$. After that, from the HO line, a right triangle AOH is made again using the Pythagorean theorem to get HO is $5\sqrt{10}$.

Q1.12: is there any other way for knowing distances apart from using the Pythagorean theorem?

S1.12: As far as I know this is the only way.

The geometry problem-solving test and the interview above show that the S1 subject did not make a mistake at the visualization level. S1 subjects were able to represent problems in geometric shapes accurately and clearly. At the analysis level, the S1 subject was able to connect geometric elements, namely the diagonal side and the concept of the Pythagorean theorem. This can be seen from the results of interviews and visualization of images that have been made, so that the S1 subject did not make mistakes at this level. Furthermore, at the ordering level, the S1 subject was able to connect the side diagonals of the cube and the Pythagorean theorem as a first step in solving the given problem. At the deduction level, S1 subjects were able to use theorems in solving problems, but there were slight misconceptions when calculating. In the last step, the S1 subject used the Pythagorean theorem but made a mistake in the calculation, which should use subtraction, but the S1 subject uses addition. At the same time, the rigor step was not carried out by S1 subjects.

Medium Mathematical Disposition (S2 Subject)

The research subject was asked a question: “It is known that the ABCD.EFGH cube has a rib length of 10 cm. Determine the distance of point H to the diagonal of the AC side!”. The results of the geometry problem solving test and interview excerpts for the S2 subject are as follows.

![Diagram of a cube with a triangle labeled ABC and a diagonal AC.](image)

**Figure 2.** The results of the S2 subject geometry problem-solving test

Q2.8: Explain the solution to question number 3?

S2.8: The cube has the same edges so that the length is 10. As for the distance H to AC, AC is the diagonal of the sides. I took triangle ABC and found the value of AC with $AB^2 + BC^2 = 10\sqrt{2}$. Since the distance of point H is equal to the distance to the point on the diagonal of the AC side, the distance is the same, which is $10\sqrt{2}$.

The results of the geometry problem solving test and the interview above demonstrate that at the visualization level there are no errors and the question was done properly by the S2 subject. Furthermore, at the level of analysis, the S2 subject recognised that the side diagonal element and the Pythagorean theorem was used in problem solving. Whereas in the ordering step, the S2 subject made a mistake in determining the distance from the point to the line precisely. The S2 subject used the length of the diagonal of the side as the length of the distance referred to in the question. This is in line with the results of Jana (2018) which also states that mistakes that are often made are understanding the concept. The deduction and rigor levels were not carried out by the S2 subject.
Low Mathematical Disposition (S3 Subject)

The research subject was asked a question: "It is known that the ABCD.EFGH cube has a rib length of 10 cm. Determine the distance of point H to the diagonal of the AC side!". The results of the geometry problem-solving test and interview excerpts for the S3 Subject are as follows.

\[ \text{Figure 3. The results of the S3 subject geometry problem-solving test} \]

**P3.8:** number 3 asks us to find the distance of a point to the diagonal of the side

**S3.8:** the length of the edge of the cube is 10, then the distance from point H to the diagonal of the side AC is \( HO \). \( AC = 10\sqrt{2} \). Then \( \frac{1}{2} AC = 5\sqrt{2} \). To find \( HO \),

\[ HO^2 = (10\sqrt{2})^2 - (5\sqrt{2})^2 = \sqrt{10} \text{ cm} \]

The results of the geometry problem solving test and interview above showcase that at the visualization level there was no error and can be answered properly by the S3 subject. Furthermore, at the level of analysis, the S3 subject understood the side diagonal element and the Pythagorean theorem was used in problem solving. In the ordering step, the S3 subject made an error in determining the distance from point to line precisely. Subject S3 used the length of the diagonal of the sides as the length of the distance referred to in the problem. The level of deduction and rigor were not carried out by S3 subjects.

In 2 other geometry problem solving test questions, the S1 subject only reached the deduction level. The S2 subject only attained the level of analysis and part of the level of ordering, although remain misusing the concept. Meanwhile, in the other 2 questions, the S3 subject was only able to reach the level of visualization. The S3 subject made a mistake at the level of analysis. Based on these results, most students still had difficulty solving geometry problems (Dosinaeng et al., 2019). Only highly skilled students could solve geometry problems correctly. Subjects with moderate mathematical disposition only arrived at the analysis level and made mistakes at the deduction level. Errors that occurred in solving problems were misunderstanding the problem/concept and transformational errors (Fatahilla et al., 2017; Nur Afifah et al., 2018; Layn & Kahar 2017). This is in accordance with the research by Nurhikmah & Febrian (2016) which states that mistakes that are often made in solving problems are errors in the procedures used.

Based on the explanation on each of the mathematical disposition categories above, it was found that the subject's error in completing the geometric problem-solving test was based on Van Hiele's level of understanding as follows.
Table 3. Summary of Geometry Problem Solving Test Results

| Van Hiele Levels   | Higher Mathematical Disposition (S1) | Medium Mathematical Disposition (S2) | Low Mathematical Disposition (S3) |
|-------------------|--------------------------------------|--------------------------------------|----------------------------------|
| Level I – Visualization | √                                   | √                                   | √                                |
| Level II – Analysis | √                                   | √                                   | ×                                |
| Level III – Ordering | √                                   | ×                                   | ×                                |
| Level IV – Deduction | √                                   | ×                                   | ×                                |
| Level V – Rigor    | ×                                   | ×                                   | ×                                |

In table 3 above, it can be seen that subjects with a high mathematical disposition made it to the deduction level but made a few mistakes at that level. Meanwhile, subjects with moderate mathematical disposition only arrived at the analysis level and made mistakes at the ordering level. Subjects with moderate mathematical disposition only reached the level of visualization; the S3 subject had misunderstanding of the concept at the analysis level. This is in line with the research by Yudianto et al., (2018) which states that most students do not make mistakes up to the level of visualization alone. At moderate mathematical disposition, students were able to reach the level of analysis. A research by Rizki et al., (2018) also stated that most students had not yet reached the ordering level. Luneta (2015) and Razak & Sutrisno (2017) stated that most students are still at level 2 (analysis) at the Van Hiele’s level of understanding.

From the results above, it can be seen that the subjects with medium and low dispositions only reached the analysis and visualization levels. This is the reason why mathematical dispositions are so important. In solving the results of the geometric problem solving test, the subjects must have a high mathematical disposition. This is in line with Maxwell Rahmadhani, (2018) and Cooke (2015) who state that mathematical disposition needs attention because it is a major factor in determining the success of learning mathematics.

4. Conclusion

The purpose of this study was to determine the level of student error in solving geometry problems based on Van Hiele’s level of thinking when viewed from a mathematical disposition. The results and discussion above demonstrate that students with high mathematical disposition in solving geometry problems can arrive at the IV-ordering level, but experience a little error at that level. Meanwhile, students with moderate disposition were able to reach level II - analysis. Students with mathematical disposition were experiencing errors when entering the deduction level. Furthermore, students with low mathematical dispositions only reached level I - visualization only and they made mistakes when entering the analysis level. These errors are in identifying the elements and theorems that should be used in problem solving.

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