Non-equilibrium steady state of the superfluid mixtures of helium isotopes at classical temperatures

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Abstract

The size effect in the gas of impuritons of the superfluid mixtures of helium isotopes is investigated by taking into consideration the contribution of thermal excitations. The solution is obtained for the set of kinetic equations describing a non-equilibrium state of the phonon-impuriton system of a superfluid mixture situated in the volume filled with the macroparticles. It allows to find the condition describing a steady, thermodynamically non-equilibrium state of $^3He - ^4He$ mixture in confined geometry. The Knudsen effect in the gas of impuritons of a superfluid mixture is investigated by taking into account the contribution of phonons. A model for the collision operator has been proposed to analyze the exact results in the context of the concrete physical situations. An experiment for the investigation of the Knudsen effect in a superfluid mixture of helium isotopes is proposed.
I. INTRODUCTION

There is a row of the unique features of a size effect in the superfluid mixtures of helium isotopes. The first evident difference between the latter one and it’s classical analogue is concerned with a non-ideality of the gases of quasiparticles of a superfluid mixture. It can be observed explicitly in the degeneracy region where the contribution of impuriton interaction to all the thermodynamic quantities of the mixture reaches the significant values [1]. The approximation based on the classical Knudsen relation [2] can not be used in the ultra low temperature region where the gas of impuritons shows it’s quantum properties [3].

A very different situation takes place in a region of classical temperatures. Here, the contribution of quasiparticle interaction is very small [1] and can not be considered as a reason for a deviation from the well-known Knudsen relation. On the other hand, the latter one, evidently, can not be used to describe the steady state of superfluid mixture even at classical temperatures. Indeed it should be expected that a significant contribution of thermal excitations to the pressure at the temperatures $T \geq 0.6K$ [4] results in the change of the classical relation between the gradients of temperature and pressure describing the steady state of a superfluid mixture.

The relation which describes the steady, thermodynamically non-equilibrium state of a superfluid mixture in confined geometry is presented in this work. We use the kinetic method to investigate the steady non-equilibrium state of a superfluid mixture situated in the volume filled with a gas of macroparticles. Such an approach can be considered as a development of the well-known ”dust laden” gas model [5,6,7] describing the process of diffusion of the ”light” particles in a gas of ”heavy” immobile macrons. It should be noted that the requirement of immobility of the ”heavy” particles in their collisions with the ”light” ones should be considered as an idealization. Indeed, it corresponds to the zeroth order approximation with respect to the ratio of the particle mass to the macron mass [2]. The model which is considered in this paper is valid for a broad spectrum of practical applications. It can be used to describe the process of particle diffusion in very
different physical systems such as a volume filled with the crocus [19] and a gas of the crystal formations associated with the positive ions in a superfluid mixture [8].

The paper is organized as follows: Section II presents the mathematical solution of the above described problem in the framework of the exact kinetic approach. In Section III a convenient model for the inter-particle collision operator is proposed to investigate the general solution of the kinetic equation and to obtain explicitly the mass velocities of the quasiparticle gases. The conditions of a steady state formation are obtained and analyzed in Section IV for different relations between the frequencies of the inter-quasiparticles and quasiparticles-macrons collisions. In this Section we also describe an experiment which would make it possible to check the obtained theoretical results. A brief summary is given in Section V.

II. SOLUTION OF THE KINETIC EQUATIONS

The approach based on the quasiparticle description [2] makes it possible to use the kinetic method for investigation of a slightly non-equilibrium state of a superfluid mixture of helium isotopes. The specificity of the $^3$He $- ^4$He superfluid mixture externalizes in the fact that one has to consider a non-equilibrium state of a three-component system of the gases of quasiparticles (phonons, rotons and impuritons) with non-quadratic energy-momentum relations. If temperature is less than 0.9 K the contribution of rotons to all the thermodynamic quantities can be neglected and one can restrict oneself to the analysis of the two-component system of the impuritons and phonons. In order to formulate exactly the kinetic problem it is necessary to consider the Hilbert spaces $\mathcal{H}_i$ of the momentum functions $g(\vec{p}_i)$ with the scalar product

$$
(h(\vec{p}_i), g(\vec{p}_i)) = - \int f^{(i)}_0 \left( \frac{\varepsilon_i}{T} \right) h^*(\vec{p}_i) g(\vec{p}_i) d\Gamma_i
$$

(1)

Here $\vec{p}_i$ is the quasiparticle momentum in s-system (the coordinate system in which a superfluid component is at rest), $d\Gamma_i$ is the corresponding volume element of the momentum
phase space ($i = 3$ corresponds to impurities, $i = 4$ to phonons), $\mu_3$ is the chemical potential of the impurities, $f^{(3)}_0(x) = \frac{1}{\exp(x - \frac{\mu_3}{T}) + 1}$, $f^{(4)}_0(x) = \frac{1}{\exp(x) - 1}$, $\varepsilon_3 = \frac{p^2}{2m^*}$, $\varepsilon_4 = cp_4$, $c$ is the sound velocity, $p_i$ are the moduli of corresponding vectors, the prime denotes differentiation with respect to the argument.

We introduce the Hilbert space $\mathbb{R}$ of vectors $|\phi(p^3_1) ; \varphi(p^4_1)\rangle^T$ with the components $\phi(p^3_1)$, $\varphi(p^4_1)$ belonging to the Hilbert spaces $\mathbb{R}_3$, $\mathbb{R}_4$ respectively. The scalar product in the space $\mathbb{R}$ is defined by the equality:

$$\langle \phi_1(p^3_1) ; \varphi_1(p^4_1) | \phi_2(p^3_1) ; \varphi_2(p^4_1) \rangle = (\phi_1(p^3_1), \phi_2(p^3_1))_3 + (\varphi_1(p^4_1), \varphi_2(p^4_1))_4$$

(2)

The set of kinetic equations describing a non-equilibrium state of the two-component system of impurities and phonons can be represented in the operator form:

$$I_{33} |g\rangle + I_{44} |g\rangle + I_{34} |g\rangle + L |g\rangle = |V\rangle$$

(3)

Here $|g\rangle = |g_3 ; g_4\rangle^T$, $|V\rangle = |v^3_3 \nabla_3^3 ; v^4_3 \nabla_4^3\rangle^T$, $\nabla_3^3 = \frac{\partial (H_3 - \mu_3)}{\partial T}$, $\nabla_4^3 = \frac{\partial (H_4 - \mu_4)}{\partial T}$, $v^k_i$ are the velocities of quasiparticles of species $i$ in the s-system, $f_i$ are the quasiparticle distribution functions, $H_3 = \varepsilon_0 + \varepsilon_3 + p^3_3 v^3_s$, $H_4 = \varepsilon_4 + p^4_4 v^4_s$ are the Hamiltonians of the impurities and phonons respectively, $v^k_s$ is the velocity of the superfluid component, $g_i = \left(f^{(i)}_0 \left(\frac{H_i}{T}\right)\right)^{-1} (f_i - f^{(i)}_0)$ are the small corrections to the equilibrium distribution functions $f^{(i)}_0$. We defined the collision operators in equation (3) as:

$$I_{33} = \begin{pmatrix} \tilde{I}_{33} & 0 \\ 0 & 0 \end{pmatrix}; \quad I_{44} = \begin{pmatrix} 0 & 0 \\ 0 & \tilde{I}_{44} \end{pmatrix}; \quad I_{34} = \begin{pmatrix} \tilde{I}_{34}^{(3)} & \tilde{I}_{34}^{(4)} \\ \tilde{I}_{43}^{(3)} & \tilde{I}_{43}^{(4)} \end{pmatrix}; \quad L = \begin{pmatrix} \tilde{L}_3 & 0 \\ 0 & \tilde{L}_4 \end{pmatrix}$$

$$\tilde{I}_{33} g_i = \int W_{i_2}(p^3_1, p^4_1 | p^3_{i_2}, p^4_{i_2}) \left(1 \pm f^{(i)}_0(p_{i_2})\right)^{-1} \left(1 \pm f^{(i)}_0(p_{i_3})\right) f^{(i)}_0(p_{i_2}) f^{(i)}_0(p_{i_3}) \left[ g_i(p^3_{i_2}) + g_i(p^4_{i_2}) - g_i(p^3_{i_3}) - g_i(p^4_{i_3}) \right] d\Gamma_{i_2} d\Gamma_{i_3} d\Gamma_{i_3}$$

(4)

$$\tilde{I}_{34}^{(i)} g_i = \int W_{i_2}(p^3_1, p^4_1 | p^3_{i_2}, p^4_{i_3}) \left(1 \pm f^{(i)}_0(p_{i_2})\right)^{-1} \left(1 \pm f^{(i)}_0(p_{i_3})\right) f^{(i)}_0(p_{i_2}) f^{(i)}_0(p_{i_3}) \left[ g_i(p^3_{i_2}) - g_i(p^4_{i_2}) - g_i(p^3_{i_3}) + g_i(p^4_{i_3}) \right] d\Gamma_{i_2} d\Gamma_{i_3} d\Gamma_{i_3}$$

(5)

$$\tilde{I}_{44}^{(i)} g_i = \int W_{i_2}(p^3_1, p^4_1 | p^3_{i_2}, p^4_{i_3}) \left(1 \pm f^{(i)}_0(p_{i_2})\right)^{-1} \left(1 \pm f^{(i)}_0(p_{i_3})\right) f^{(i)}_0(p_{i_2}) f^{(i)}_0(p_{i_3}) \left[ g_i(p^3_{i_2}) - g_i(p^4_{i_2}) - g_i(p^3_{i_3}) + g_i(p^4_{i_3}) \right] d\Gamma_{i_2} d\Gamma_{i_3} d\Gamma_{i_3}$$

(6)
Here $\hat{I}_{ij}$ are the linearized impuriton-impuriton ($i = 3$) and phonon-phonon ($i = 4$) collision operators \[1\, 2\], $\hat{I}^{(i)}_{ij}$ and $\hat{I}^{(j)}_{ij}$ are the components of the linearized operators of cross impuriton-phonon collisions which act in the spaces $\mathbb{R}_i, \mathbb{R}_j$ respectively, $L$ is the operator describing the quasiparticle-macron collisions, sign “+” corresponds to phonons, “-” to impuritons.

The procedure of solving of kinetic equation (3) is simplified considerably if one uses the Lorentz approximation \[2\] for the operator $L$ describing the scattering of the ”light” quasiparticles by the ”heavy” macrons. The components of this operator describing the impuriton-macron ($i = 3$) and phonon-macron ($i = 4$) collisions can be presented in the form:

$$\hat{L}_i g_i = N \int W (\overrightarrow{p}_i | \overrightarrow{p}_{i_1}) (g (\overrightarrow{p}_{i_1}^\alpha) - g (\overrightarrow{p}_i^\alpha)) \delta (\varepsilon_i (p_i) - \varepsilon_i (p_{i_1}) + \overrightarrow{v}_s (\overrightarrow{p}_i - \overrightarrow{p}_{i_1})) d\Gamma_{i_1}$$

(7)

where $N$ is the macron number density.

The ”light” particles diffusing in a ”heavy” gas change only the direction of their motion due to the collisions so that their energy remains unchanged in the system where the scatterers are at rest \[4\]. It is similar to a gas of long-wavelength phonons which are scattered by macrons \[4, 5\]. Thus, every scattering of a quasiparticle by the macron leads to a change in the quasiparticle energy which is equal to $\overrightarrow{v}_s (\overrightarrow{p}_i - \overrightarrow{p}_{i_1})$ in the s-system (see (7)). It should be noted that the corresponding change of energy in the process of the quasiparticles collisions with one another goes to zero in the s-system due to momentum conservation law. The energy conservation law in the collisions of quasiparticles with macrons is taken into account by the $\delta$ -functional factor in collision integral (7). One can neglect the term proportional to $\overrightarrow{v}_s^2$ in the argument of $\delta$ - function to the first order in the thermodynamic gradients. Thus, the collision operators $\hat{L}_i$ act only on the polar and azimuthal components of the function of the corresponding quasiparticle momentum.

In order to inverse the collision operator and solve operator equation (3) we introduce the orthonormal basis in the Hilbert space $\mathbb{R}$, according to the definitions :
\[
|\psi_{100}\rangle = \frac{1}{\sqrt{4\pi}} \begin{pmatrix}
1 \\
1 \\
1 \\
1 \\
1
\end{pmatrix};
\]
\[
|\psi_{200}\rangle = \frac{1}{\sqrt{4\pi}} \begin{pmatrix}
\tau B_1 \\
F \frac{1}{2} \\
F \frac{1}{2} \\
F \frac{1}{2} \\
F \frac{1}{2}
\end{pmatrix};
\]
\[
|\psi_{300}\rangle = \frac{1}{\sqrt{4\pi}} \begin{pmatrix}
|p^2\rangle \\
q
\end{pmatrix};
\]
\[
|\psi_{400}\rangle = \frac{1}{\sqrt{4\pi}} \begin{pmatrix}
4\tau \pi^2 B_2 p^2 \\
-\frac{5}{2} F \frac{1}{2} q
\end{pmatrix};
\]
\[
|\psi_{110}\rangle = -i\sqrt{\frac{4\pi}{\tau}} \begin{pmatrix}
Y_{10}(\theta) p \\
Y_{10}(\theta) q
\end{pmatrix};
\]
\[
|\psi_{210}\rangle = -i\sqrt{\frac{4\pi}{\tau}} \begin{pmatrix}
4\tau \pi^2 B_2 Y_{10}(\theta) p \\
-\frac{1}{2} F \frac{1}{2} Y_{10}(\theta) q
\end{pmatrix};
\]
\[
|\psi_{111}\rangle = \sqrt{\frac{2\pi}{3}} \begin{pmatrix}
(Y_{11}(\theta, \varphi) \pm Y_{1-1}(\theta, \varphi)) p \\
(Y_{11}(\theta, \phi) \pm Y_{1-1}(\theta, \phi)) q
\end{pmatrix};
\]
\[
|\psi_{211}\rangle = i\sqrt{\frac{2\pi}{3}} \begin{pmatrix}
\frac{4}{3} \tau \pi^2 B_2 \left( Y_{11}(\theta, \varphi) \pm Y_{1-1}(\theta, \varphi) \right) p \\
-\frac{1}{2} F \frac{1}{2} \left( Y_{11}(\theta, \phi) \pm Y_{1-1}(\theta, \phi) \right) q
\end{pmatrix}
\]
and
\[
|\psi_{2klm}\rangle = \begin{pmatrix}
0 \\
Y_{lm}(\theta, \phi) q^{k-1-\delta_l^k \delta_m^l \delta_l^k}
\end{pmatrix};
\]
\[
|\psi_{2k-1lm}\rangle = \begin{pmatrix}
Y_{lm}(\theta, \varphi) p^{k-1-\delta_l^k \delta_m^l \delta_l^k} \\
0
\end{pmatrix}
\]
where \(k = 3, 4, 5...\) for \(l = 0, k = 2, 3, 4...\) for \(l = 1, k = 1, 2, 3...\) for \(l \geq 2\).

Here \(Y_{lm}(\theta, \varphi)\) are the spherical functions \([10]\), \(\vec{p} = \frac{\vec{p}}{\sqrt{2m+1}}\) and \(\vec{\tau} = \frac{\tau}{\hbar}\) are the dimensionless values of the momentum of impuriton and phonon respectively, \(\tau = \left(\frac{T}{\hbar}\right)^3\), \(B_n\) are Bernoulli’s numbers, \(F_\nu = \left(\frac{\sqrt{2m+1}}{\hbar}\right)^3 \frac{1}{2\pi^2} \int_0^\infty \frac{x^\nu dx}{\exp(x/\hbar)+1}\) is the Fermi-function, \(\delta^l_m\) is the Kronecker symbol. Let us note also that the vectors introduced above satisfy the conditions of orthonormality:
\[
\langle \psi_{2k-1lm} | \psi_{2klm} \rangle = 0 \quad k=1,2,3...;l=0,1,2...
\] (8)

Using the set of vectors, introduced above, one can represent the conservation laws for impuriton-phonon collisions in the form:
\[
I_{34} |\psi_{nm\ell}\rangle = 0
\] (9)
where the set of indices \((n, m, \ell)\) corresponds to:

- \((1, 0, 0)\) - number density conservation law;
- \((3, 0, 0)\) - energy conservation law;
- \((1, 1, 0)\) - \(z\) - component of momentum conservation law;
- \((1, 1, 1)\) - \(x\) - component of momentum conservation law;
(1, 1, −1) - y- component of momentum conservation law.

A similar formula describes the conservation laws in the collisions of quasiparticles of same species with one another:

\[
I \psi_{nml} = 0 \tag{10}
\]

with the operator \( I = I_{33} + I_{44} \) and the sets of indices to be added to the above mentioned, as follows: \((2, 0, 0), (4, 0, 0), (2, 1, 0), (2, 1, 1), (2, 1, -1)\).

It should be noted that according to formulae (9), (10) the operators \( I_{34} \) and \( I \) have kernels which represent subspaces of different dimensions. It is concerned with the fact that the operator \( I \) has ten independent collision invariants: five for impuritons collisions with one another and five for phonons ones. The operator \( I_{34} \) has only five invariants corresponding to number density, energy and momentum conservation laws in impuriton-phonon collisions.

The above complete set of the vectors can be orthonormalized by means of the recurrent formulae:

\[
|\varphi_{0lm}\rangle = \frac{|\psi_{0lm}\rangle}{\sqrt{\langle \psi_{0lm} | \psi_{0lm} \rangle}} \tag{11}
\]

\[
|\varphi_{n+1lm}\rangle = \frac{\left( E - P_{0}^{(n,l,m)} \{ |\varphi_{klm}\rangle_{k=1}^{n} \} \right) |\psi_{n+1lm}\rangle}{\sqrt{\langle \psi_{n+1lm} | \psi_{n+1lm} \rangle - ||\psi_{n+1lm}||_{n}^{2}}} \tag{12}
\]

where

\[
P_{0}^{(n,l,m)} \{ |\varphi_{klm}\rangle_{k=1}^{n} \} = \sum_{k=1}^{n} |\varphi_{klm}\rangle \langle \varphi_{klm}| \tag{13}
\]

is the set of projectors produced by the orthonormal basis \( \{ |\varphi_{klm}\rangle \} \),

\[
||\psi_{n+1lm}||_{n} = \sqrt{\sum_{k=1}^{n} \langle \varphi_{klm} | \psi_{n+1lm} \rangle^{2}} \tag{14}
\]

is the norm of a projection of the vector \( |\psi_{n+1lm}\rangle \) to the subspace with the basis \( \{ \varphi_{klm}\rangle_{k=1}^{n} \).

The operators

\[
P_{0}^{[3,4]} = P_{0}^{(2,0,0)} \{ |\varphi_{2k-100}\rangle_{k=1}^{2} \} + \sum_{m=-1}^{1} P_{0}^{(1,1,m)} |\varphi_{11m}\rangle \tag{15}
\]
\[ P_0 = P_0^{(4,0,0)} \left[ \{ \varphi_{k00} \}^{4}_{k=1} \right] + \sum_{m=-1}^{1} P_0^{(2,1,m)} \left[ \{ \varphi_{k1m} \}^{2}_{k=1} \right] \] (16)

represent the projectors of Hilbert space \( \mathcal{H} \) to the kernels of operators \( I_{34} \) and \( I \), respectively. It is convenient to rewrite operator (16) in the form of a multiplication:

\[ P_0 = P_0^{[3,3]} P_0^{[4,4]} \] (17)

where

\[ P_0^{[3,3]} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} P_{0}^{(\infty,l,m)} \left[ \{ \varphi_{2k-100} \}^{2}_{k=1} , \{ \varphi_{11m} \}^{1}_{m=-1} , \{ \varphi_{2klm} \}^{\infty}_{k=1} \right] \] (18)

\[ P_0^{[4,4]} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} P_{0}^{(\infty,l,m)} \left[ \{ \varphi_{2k00} \}^{2}_{k=1} , \{ \varphi_{21m} \}^{1}_{m=-1} , \{ \varphi_{2k-1lm} \}^{\infty}_{k=1} \right] \] (19)

are the projectors to the kernels of operators \( I_{33} \) , \( I_{44} \) describing the mutual collisions of impurities and phonons, respectively. Besides the operators (16), (18), (19) we introduce the projector \( P_L \) to the kernel of operator \( L \). According to the definition of the operator \( L \) and all the notes to formula (7) concerning it’s kernel one can obtain:

\[ P_L = P^{(\infty,0,0)} \] (20)

The above introduced sets of the orthonormal vectors and the derived projectors make it possible to obtain the general solution of operator equation (3). For this purpose let us project the latter to the ranges of values of operators \( P_0^{[1]} = P_0^{[3,4]} P_L, P_0^{[2]} = (P_0 - P_0^{[3,4]}) P_L, P_0^{[3]} = P_0^{[3,4]} - P_0^{[1]} \) and to their orthogonal complement. As a consequence one derives the set of equations which can be written as follows:

\[ \left( (E - P_0^{[3,3]}) I_{33} (E - P_0^{[3,3]}) + (E - P_0^{[4,4]}) I_{44} (E - P_0^{[4,4]}) + (E - P_0^{[3,4]}) I_{34} (E - P_0^{[3,4]}) + (E - P_L) L (E - P_L) \right) |g\rangle = (E - P_0^{[1]}) |V\rangle \] (21)

\[ P_0^{[3]} L (E - P_L) |g\rangle = P_0^{[3]} |V\rangle \] (22)

\[ P_0^{[2]} I_{34} (E - P_0^{[3,4]}) |g\rangle = P_0^{[2]} |V\rangle \] (23)
\[ P_0^{[1]} |V\rangle = 0 \]  \hspace{1cm} (24)

Here \( E \) is the identity operator.

Because \( P_L |V\rangle = 0 \), equality (24) is fulfilled trivially and can be omitted. Equality (23) means that the vector \( I_{34} |g\rangle \) has to belong to kernel of the projector \( P_0^{[2]} \).

It should be noted that the operators \((E - P_0^{[i,j]}) I_{ij} (E - P_0^{[i,j]}) \) \((i,j = 3,4)\) in equation (21) represent the restrictions of the collision operators \( I_{ij} \) with respect to the orthogonal complements of their kernels. Because all the collision operators are non-positive ones the sum in the left-hand side of equation (21) can be inverted one-to-one. The latter condition makes it possible to solve the set of operator equations (21) - (24) for the components of a non-equilibrium term \( |g\rangle \) to be represented in the form:

\[ |g\rangle = |g_0\rangle + |g'\rangle \]  \hspace{1cm} (25)

where \( |g_0\rangle \) is an arbitrary vector which belongs to the range of values of operator \( P_0^{[1]} \) (i.e. the solution of homogeneous operator equation corresponding to kinetic equation (3)), and

\[ |g'\rangle = \left( (E - P_0^{[3,3]}) I_{33}(E - P_0^{[3,3]}) + (E - P_0^{[4,4]}) I_{44}(E - P_0^{[4,4]}) + (E - P_0^{[3,4]}) I_{34}(E - P_0^{[3,4]}) + (E - P_L) L (E - P_L)^{-1} (E - P_0^{[1]}) \right) |V\rangle \]  \hspace{1cm} (26)

The obtained solution of kinetic problem (25)-(26) is exact and can be used for the investigation of a steady non-equilibrium state of the two-component impuriton-phonon system in the presence of macrons. It takes into account that the collision operator represents the sum of the operators with kernels of different dimensions. It can be elucidated in the limiting cases when the contribution of a certain type of collisions can be neglected. In these cases solution (26) leads to the correct limiting expressions under the condition that one omits the corresponding collisions operator in it. If one can neglect the contribution of phonons formula (26) gives the result presented in [17].

If the contribution of mutual collisions of both species of quasiparticles should be taken into account solution (26) can be represented in the form:
\[ |g'\rangle = ((E - P_0) I (E - P_0) + (E - P_0^{[3,4]}) I_{34} (E - P_0^{[3,4]}) + (E - P_L) L (E - P_L)^{-1} (E - P_0^{[1]}) |V\rangle \] (27)

It should be noted that solution of the kinetic equation (27) has to fulfill conditions (22) and (23). To illustrate it let us consider the hydrodynamic limit when the contribution of quasiparticle - macron collisions can be neglected. To obtain the correct limiting transition in the set of equations (21) - (24) one should use the equality \( P_L = E \). As a result equation (22) reduces to the form:

\[ P_0^{[3,4]} |V\rangle = 0 \] (28)

Equality (28) yields the condition of solvability for equation (3) describing a steady non-equilibrium state of the two-component phonon-impuriton system. Using the explicit expressions for projector \( P_0^{[3,4]} \) (9) and vector \( |V\rangle \) one can rewrite the condition (28) as follows:

\[ \frac{\partial P_3}{\partial T} + S_{ph} \frac{\partial T}{\partial T} = 0 \] (29)

where \( P_3 = \frac{2}{3} TF_3 \) is the osmotic pressure of the gas of impuritons; \( S_{ph} (T) \) is the entropy of phonons per unit volume and \( S_{ph} \frac{\partial T}{\partial T} \) is the gradient of fountain pressure caused by phonons [1,4]. Relation (29) represents a conventional condition of a mechanical equilibrium in a closed system: the total pressure must be constant in entire volume of the mixture [1].

The remarkable fact is that one can even avoid to use solution (25) of the kinetic equation to obtain the condition describing a steady non-equilibrium state of the mixture in the hydrodynamic limit. For this purpose it is enough to analyze the condition of it’s solvability (22) which reflects the momentum conservation law in impuriton - phonon collisions.

In the presence of the macrons the phonon-impuriton system is unclosed. The operator \( L \) is not equal to zero and relation (23) is not fulfilled. So, in order to investigate the non-equilibrium state of the mixture one has to analyze solution (25) of the kinetic equation.
III. FURTHER ANALYSIS OF THE SOLUTION OF A KINETIC EQUATION BY USING THE MODEL REPRESENTATION OF A COLLISION OPERATOR

The exact solution of the kinetic equation (27) allows us to construct the approximations which take certain properties of true collision operator into account. Here we restrict ourselves to the model for a two-component system collision operator which should be considered as a generalization of the approach developed in \[15,13\] for the case of the one-component gas.

The essential features of the true collision operators which is reflected in the proposed model are as follows:

1) the model collision operators have to satisfy conditions \(\text{(9), (10)}\) expressing the conservation laws in the collisions;

2) linearized collision operators must be self-adjoint;

3) the H-theorem \[2\] must be satisfied.

According to the first two conditions the collision operators must satisfy the relations:

\[
I |g\rangle = (E - P_0) I (E - P_0) |g\rangle \quad (30)
\]

\[
I_{34} |g\rangle = \left( E - P_0^{[3,4]} \right) I_{34} \left( E - P_0^{[3,4]} \right) |g\rangle \quad (31)
\]

Developing the idea of the Bhatnagar-Gross-Krook approximation \[15\] for the one-component gas it is natural to approximate the true collision operators \(I\) and \(I_{34}\) by the multiplication operators. Such an approach results in the model:

\[
I + I_{34} = (E - P_0) \tilde{\lambda} \left( E - P_0 \right) + \left( E - P_0^{[3,4]} \right) \tilde{\lambda}_{34} \left( E - P_0^{[3,4]} \right) \quad (32)
\]

where \(\tilde{\lambda} = \text{diag} \{-\nu_{33}, -\nu_{44}\}\), \(\tilde{\lambda}_{34} = \text{diag} \{-\nu_{34}, -\nu_{43}\}\) are the diagonal matrices with real elements. It should be noted that the parameters \(\nu_{ij} (i, j = 3, 4)\) must be positive to satisfy the conditions of H-theorem. To prove it let us consider the entropy of the non-equilibrium phonon-impuriton system:
\[ S = -\int (f_3 \ln f_3 + (1 - f_3) \ln (1 - f_3)) d\Gamma_3 - \int (f_4 \ln f_4 - (1 + f_4) \ln (1 + f_4)) d\Gamma_4 \tag{33} \]

Taking the time derivative of \( S \) and linearizing the result with respect to the non-equilibrium term \(|g|\) we represent the condition of H-theorem in the form:

\[ \langle g | I + I_{34} | g \rangle \leq 0 \tag{34} \]

Here, equality holds if (and only if) the vector \(|g|\) belongs to the kernel of operator \( I + I_{34} \).

According to condition (34) the model operators must be negative and, consequently, the above mentioned parameters must satisfy \( \nu_{ij} > 0 \).

The idea behind the representation of the model collision operator in the form (32) is that the average effect of the collisions is to change the distribution functions by amounts proportional to the \(|g_i|\), i.e. by the deviations of \( f_i \) from their equilibrium values. The parameters \( \nu_{ij} \) play a role of coefficients at the non-equilibrium terms \(|g_i|\) which should be associated with the collision frequencies of an according type.

Model (32) satisfies the conditions 1)-3) for arbitrary positive values \( \nu_{ij} \). But the above explained physical meaning implies that the frequencies of impuriton-phonon (\( \nu_{34} \)) and phonon-impuriton (\( \nu_{43} \)) collisions are connected with one another. In order to find this relation between the frequencies let us consider the corresponding true collision operators in details. The collision term can be represented in the form:

\[ \hat{I}_{ii} g_i + \hat{I}_{ij}^{(i)} g_i + \hat{I}_{ij}^{(j)} g_j = \sum_{j=3}^{4} \overline{w}_{ij} (\overline{p}_i) (M_{ij} - g_i) \quad (i = 3, 4) \tag{35} \]

where

\[ \overline{w}_{ij} (\overline{p}_i) = \int W_{ij} (\overline{p}_i, \overline{p}_{j_1}, \overline{p}_{j_2}, \overline{p}_{j_3}) \left( 1 \pm f_{0}^{(i)} (p_i) \right)^{-1} \left( 1 \pm f_{0}^{(i)} (p_{j_1}) \right) f_{0}^{(j)} (p_{j_1}) f_{0}^{(j)} (p_{j_1}) \] \[ \left( 1 \pm f_{0}^{(j)} (p_{j_3}) \right) d\Gamma_{i_0} d\Gamma_{j_1} d\Gamma_{j_3} \tag{36} \]

\[ M_{ij} (\overline{p}_i) = \int W_{ij} (\overline{p}_i, \overline{p}_{j_1}, \overline{p}_{j_2}, \overline{p}_{j_3}) \left( 1 \pm f_{0}^{(i)} (p_i) \right)^{-1} \left( 1 \pm f_{0}^{(i)} (p_{j_2}) \right) f_{0}^{(j)} (p_{j_1}) f_{0}^{(j)} (p_{j_3}) d\Gamma_{i_0} d\Gamma_{j_1} d\Gamma_{j_3} \tag{37} \]
Here, sign "+" corresponds to phonons, "-" to impurities.

Hence, according to definition \((36)\) the frequencies \(\omega_{34}, \omega_{43}\) are connected with one another by the relation:

\[
\int \omega_{34} f_0^{(3)'} d\Gamma_3 = \int \omega_{43} f_0^{(4)'} d\Gamma_4 \tag{38}
\]

Expression \((35)\) has the same structure as model collision term \((32)\). Indeed, the latter can be represented in the form:

\[
\sum_{j=3}^{4} \nu_{ij} \left( f_0^{(i)} \left( \frac{\varepsilon_i - p_{iz} V_{ij}}{T} \right) - f_i \right) \quad (i = 3, 4) \tag{39}
\]

where

\[
V_{ii} = -T \frac{(p_{ix}, g_{ix})_i}{(g_{ix}, p_{ix})_i} \quad (i = 3, 4) \tag{40}
\]

are the moduli of the mass velocities of the quasiparticles of species \(i\). The index \(z\) denotes the projection of corresponding vector on the direction of gradients. The values \(V_{34}, V_{43}\) are determined one-to-one by a comparison of formulae \((39)\) and \((32)\). Thus, the complicated emission terms in the true collision integrals are replaced by the multiplication of a Maxwellian by the corresponding collision frequency in model presentation \((32)\). It should be noted that such an approximation looks appropriate for the conditions where the individual distribution functions are not far away from their equilibrium values. If the dependencies of \(\omega_{ij}\) with respect to the momentum are smooth enough it can be approximated by their mean values, i.e. the effective collision frequencies \(\nu_{ij}\) in model \((32)\). These frequencies may be conveniently introduced as follows. Let us apply the second mean value theorem to the integrals in equality \((38)\). It gives:

\[
\nu_{34} \int f_0^{(3)'} d\Gamma_3 = \nu_{43} \int f_0^{(4)'} d\Gamma_4 \tag{41}
\]

where the mean values are defined by:

\[
\nu_{ij} = \frac{(1, \omega_{ij})_i}{(1, 1)_i} \tag{42}
\]
Equality (41) reduces to well-known relation between the cross collision frequencies and the number densities of gases in their two-component mixture [13] by substituting the classical Maxwell distribution functions into it. Substituting the classical distribution function of impuritons and the Bose-distribution function of phonons into equality (41) we obtain:

\[ n_3 \nu_{34} = 1.37 n_{ph} \nu_{43} \]  

(43)

where \( n_{ph} \) is the number density of phonons and \( n_3 \) is the number density of impuritons.

We investigate now the operator \( L \) describing the quasiparticle-macron collisions. Let us consider the non-equilibrium state of a mixture which is caused by a non-uniformity of the thermodynamic values along the z-axis. Thus, the non-equilibrium term \( |g'\rangle \) cannot be a function of the polar angles in the momentum spaces of both impuritons and phonons. The result of the action of the collision operator on the non-equilibrium term \( |g\rangle \) can be written in the form:

\[ L |g\rangle = - \sum_{n,l} \hat{\nu}_{Kn}^l |\varphi_{nl0}\rangle \langle \varphi_{nl0}|g\rangle \]  

(44)

where

\[ \hat{\nu}_{Kn}^l = \text{diag} \left\{ \nu_{3Kn}^l, \nu_{4Kn}^l \right\} \]  

(45)

\[ \nu_{iKn}^l = 2\pi N v_T^{(i)} \int (1 - P_l(\cos \alpha)) \sigma_i(p_i, \alpha) \sin \alpha d\alpha \]  

(46)

are the partial frequencies of the collisions of quasiparticles of species \( i \) with the macrons, \( v_T^{(3)} = \left( \frac{2\pi}{m} \right)^{\frac{3}{2}} p, v_T^{(4)} = c, \sigma_i(p_i, \alpha) \sin \alpha d\alpha \) are the differential scattering cross-section for the quasiparticles of species \( i \); \( P_l(\cos \alpha) \) is the Legendre polynomial of the \( l \)th order.

When substituting expressions (32) and (44) into the kinetic equation (3) one has to take into account that the model operators \( I \) and \( I_{34} \) present the operators of multiplication. It means that the sum of operators \( I + I_{34} + L \) maps into itself the subspace of vectors (12) corresponding to the \((l,0)\)th harmonics. The latter makes it possible to look for a solution
of equation \( (3) \) in the subspace corresponding to the same harmonics as the vector in the left-hand side of kinetic equation \( (3) \) belongs to.

Inserting \( (32) \) and \( (44) \) into \( (3) \) we get:

\[
\left( \hat{\lambda} P_n + (P_n + P_c) \hat{\lambda}_{34} (P_n + P_c) + \hat{\nu}^1_{Kn} \right) |g\rangle = |V\rangle
\]

(47)

with:

\[
P_c = |\varphi_{210}\rangle \langle \varphi_{210}|
\]

(48)

\[
P_n = E - P_c - |\varphi_{110}\rangle \langle \varphi_{110}|
\]

(49)

Analyzing the solution of the kinetic equation \( (47) \) we restrict ourselves to the case when the macrons can be regarded as rigid spheres of radius \( a \). Thus the phonon-macron collision frequency \( \nu^1_{4Kn} \) which describes the Rayleigh scattering of the long-wavelength phonons by rigid spherical macron in equation \( (47) \), is represented in the form \( (50) \):

\[
\nu^1_{4Kn} = cN \left( \frac{4\pi}{3\hbar^2} a^3 \left( 1 - \frac{\rho_k}{\rho} \right) \right)^2 q^4
\]

(50)

where \( \rho \) is the density of a mixture, \( \rho_k \) is the average density of macron. The frequency of collisions of "light" impuriton with the "heavy" macron can be written as:

\[
\nu^1_{3Kn} = p\nu^1_{3Kn}
\]

(51)

where \( \nu_{3Kn} = \pi Na^2 \left( \frac{2T}{m^*} \right)^{1/2} \).

In the case when frequency of phonon-macron collisions is much less than the sum of frequencies of phonon-impuriton and phonon-phonon ones the contribution of thermal excitation to a size-effect in the gas of impuritons should be expected to be maximal. Such a limit can be realized practically by choosing the appropriate parameters in formula \( (50) \). In this limiting case the procedure of inversion of the collision operator and solution of kinetic equation \( (47) \) is simplified significantly. In zeroth order with respect to the parameter \( \frac{\nu^1_{4Kn}}{\nu_{44} + \nu_{43}} \) the expression for the non-equilibrium terms \( g_i \) can be written as follows:
\[
g_3 = -\frac{\nu_{3z}}{\nu_3 + \nu_{3K_n}^1} \left( \nabla_3 - \frac{S_{ph} \nabla T}{n_3 \mathbb{T}} \right) - \frac{\gamma}{p + \gamma} \frac{p_{3z} V_{33}}{\mathbb{T}} \tag{52}
\]

\[
g_4 = -\frac{1}{\nu_4} \left( v_{4z} \nabla_4 + \frac{p_{4z} c^2 \nabla T}{T} \right) - \frac{p_{4z} V_{44}}{T} \tag{53}
\]

where \( \gamma(n_3, T) = \frac{\nu_3}{\nu_{3K_n}} \), \( \nu_3 = \nu_{33} + \nu_{34} \), \( \nu_4 = \nu_{44} + \nu_{43} \).

Using definitions (40) one can obtain the explicit expressions for the mass velocities of impuritons and phonons

\[
\vec{V}_{33} = -\frac{1}{n_3 m^* \nu_{3K_n}^1} \frac{G_0(\gamma)}{G_1(\gamma)} (T \nabla n_3 + n_3 \eta(\gamma) \nabla T + S_{ph} \nabla T) \tag{54}
\]

\[
\vec{V}_{44} = -\frac{1}{n_3 m^* \nu_{3K_n}^1} \frac{G_0(\gamma)}{G_1(\gamma)} (T \nabla n_3 + n_3 \eta(\gamma) \nabla T) - \left( \frac{G_0(\gamma)}{\nu_{3K_n} G_1(\gamma)} - \frac{(1 + \delta)^2}{\nu_{34} + \nu_{43} \delta} \right) \frac{S_{ph} \nabla T}{n_3 m^*} \tag{55}
\]

respectively.

Here, \( G_n(\gamma) = \frac{4}{3 \sqrt{\pi}} \int_0^\infty \frac{e^{-\gamma y} - \frac{\nu_{3n}}{\gamma} \rho_{nph}}{\gamma + \sqrt{y}} dy \), \( \delta = \frac{n_{nn} m^*}{\rho_{nph}} \), \( \rho_{nph} \) is the normal density of phonons and

\[
\eta(\gamma) = \frac{G_2(\gamma)}{G_0(\gamma)} - \frac{3}{2} \tag{56}
\]

The model proposed in this section makes it possible to obtain the explicit results without accounting a large amount of details of the inter-quasiparticles interaction, assuming that it is not likely to influence the experimentally measured values significantly. The results (52) - (55) which were obtained from model (32) are used to analyze the steady non-equilibrium state of a superfluid mixture in confined geometry.

**IV. STEADY NON-EQUILIBRIUM STATE OF THE SUPERFLUID MIXTURE OF HELIUM ISOTOPES.**

There are two different stages in the process of the formation of the steady non-equilibrium state of the superfluid mixture in confined geometry. During the first stage the superfluid component which does not encounter any resistance, overflows rapidly to ensure the condition under which the chemical potential of \(^4\text{He}\) is constant all over the mixture.
At this stage the pressure gradient in the mixture is equal to the sum of the gradients of the osmotic and the fountain pressures.

The second prolonged stage corresponds to the steady state formation in the quasiparticles system of a superfluid mixture. During this stage the slow impuriton flow induced by the unbalanced gradients of temperature and concentration emerges in the system.

The actual steady state sets in only as a result of a second stage, when the concentration gradient balances the temperature gradient so that the mass velocities of both the impuriton gas and the mixture are equal to zero. Evidently, the duration of the second stage depends essentially on the amount of the quasiparticle-macron collision contribution to the process of a steady state formation.

To analyze the described processes quantitatively let us write the above mentioned conditions of vanishing of impuritons and mixture mass flow in the form

\[ -V_{33} = 0 \]  \hspace{1cm} (57)

\[ \rho_s \vec{v}_s + \rho_{nph} \vec{V}_{44} + n_3 m^* \vec{V}_{33} = 0 \]  \hspace{1cm} (58)

respectively. Here \( \rho_s \) is the density of the superfluid component of the mixture.

The explicit form of condition (57) represents the relation between the temperature gradient and the concentration gradient ensuring the steady state of a mixture. This relation can be investigated with any degree of accuracy by using definition (40) and the general solution of the kinetic equation (26). Relation (58) defines the velocity of a superfluid motion which compensates the normal one to ensure the steady state of a mixture.

In the region of degeneracy the terms of the sum in expression (54) corresponding to the contribution of thermal excitations can be neglected. So, in this case relation (57) transforms into the result presented in [17], describing the steady non-equilibrium state of a superfluid mixture in the ultra-low temperature region.

Thus, to study the contribution of phonons to the process of the steady state formation in the superfluid mixture one can restrict oneself to the case of a classical mixture. Proceeding
from the explicit expression (54) for the mass velocity of the impuritons and the condition of stationarity (57) we get:

\[ n_3 \eta(\gamma) \nabla T + T \nabla n_3 + S_{ph} \nabla T = 0 \]  

(59)

For further analysis relation (59) can be conveniently expressed in a coordinate independent-form as a function \( \Psi \) of the impuriton number density and the temperature:

\[ \nabla \Psi (n_3, T) = 0 \]  

(60)

Integrating the equality (59) one can obtain the explicit expression for the function \( \Psi \):

\[ \Psi = n_3 \theta^{n(\theta, \gamma_\lambda)} - \int_{\theta}^{1} S_{ph} (tT_\lambda) t^n(t, \gamma_\lambda) \frac{dt}{t} \]  

(61)

where \( \theta = \frac{T}{T_\lambda} \) is the dimensionless temperature of a mixture, \( T_\lambda \) is the temperature of the \( \lambda \)-transition, \( \gamma_\lambda = \gamma(n_3, T_\lambda) \) and

\[ n(\theta, \gamma_\lambda) = -\frac{1}{6.5 \ln \theta} \int_{\theta \theta_\lambda}^{\frac{1}{\theta_\lambda}} \eta(\gamma_\lambda t) \frac{dt}{t} \]  

(62)

It should also be noted that the function \( \Psi \) is defined up to an arbitrary constant.

Relation (53), the actual dependence of the phonon-impuriton collision frequency on the temperature \( \nu_{34} \sim T^7 \) [18] and the fact that in classical temperature region the frequency of impuriton-impuriton collisions is much less than the frequency of impuriton-phonon ones [18] have been taken into account when deriving formula (61).

Results (59) and (60) describe the steady, thermodynamically non-equilibrium state of the phonon-impuriton system for arbitrary relations between the frequencies of impuriton-impuriton, impuriton-phonon and impuriton-macron collisions. They allow to investigate all cases which can be realized in experiments. In the hydrodynamic limit (\( \gamma \rightarrow \infty \)), when the contribution of impuriton-macron collisions can be neglected in comparison with both impuriton-impuriton and impuriton-phonon collisions, result (59) transforms to relation (29). In the opposite, Knudsen limiting case (\( \gamma = 0 \)) relation (60) gives:

\[ \nabla \left( n_3 \sqrt{T} + \frac{2}{7} S_{ph} \sqrt{T} \right) = 0 \]  

(63)
Relation (63) describes the Knudsen effect in the gas of impuritons of a superfluid mixture of helium isotopes by taking the contribution of phonons into consideration. The existence of a second term in the sum in the parentheses in formula (63) makes the difference with famous Knudsen relation [2]. The relative contributions of the ”impuriton” and ”phonon” terms to the sum in (63) are determined by the relation between the normal densities of impuritons and phonons. Hence, it should be expected that relation (63) deviates significantly from the classical Knudsen one for dilute mixtures at classical temperatures.

Result (63) can be checked experimentally by the conventional setup which is used for osmotic pressure measurements [19]. It represents the system of two containers filled with the superfluid mixture and connected through the supergap containing a porous material. The containers are maintained at constant but different temperatures so that the unbalanced gradients of temperature and concentration lead to an overflow of the mixture through the supergap. The experiments [19] focus on the difference in pressures of the mixture in containers which is formed as a result of the rapid overflow of the superfluid component. An increase in the expectation time in the experiments similar to those described in [19] must result in a realization of the above-mentioned second prolonged stage of a steady state formation. An experiment like this would make it possible to analyze the Knudsen effect in the superfluid mixture described by formula (63).

To investigate the condition of stationarity (57) in an intermediate region one has to use relation (60) with function Ψ (61). The parameter \( n(\frac{1}{2} \leq n \leq 1) \) introduced in (61) should be considered as an index characterizing the regime which was formed in the impuritons system of a superfluid mixture. The value \( n = \frac{1}{2} \) corresponds to Knudsen limit, \( n = 1 \) to the hydrodynamic one. The intermediate region with arbitrary relations between inter-quasiparticles and impuriton-macron collisions frequencies was investigated numerically. The result for \( T = 0.7K \) is shown on the Fig. [1]. As it can be seen the hydrodynamic regime is established in the system exponentially fast as parameter \( \gamma_A \) increases.

If one can neglect the temperature dependence of the parameter \( n \) a simple approximation for the function \( \Psi \) can be derived from (61) to rewrite the condition of stationarity (60):
\[ \nabla \left( n_3 T^n + \frac{S_{ph} T^n}{n + 3} \right) = 0 \] (64)

Formula (64) describes explicitly the transition from the Knudsen regime \( n = \frac{1}{2} \) to the hydrodynamic one \( n = 1 \).

There is an explicit physical explanation behind the deviation of conditions (59) and (63) describing the steady non-equilibrium state of a superfluid mixture from the classical Knudsen relation. The requirement of vanishing flow of an impuriton gas in form (57) describes the situation when the diffusion processes caused by the temperature gradient and the concentration gradient are equilibrated. According to (54) the flow of phonons caused by the gradient of temperature contributes to the coefficient of thermal diffusion of impuritons due to phonon-impuriton collisions. Hence, the existence of the phonon gas flow must lead to an effective increase of the concentration gradient to equilibrate the process of thermal diffusion of impuritons while satisfying condition (57). Thus, the gradient of impuriton concentration must be larger in the presence of phonons than in the case of the classical Knudsen effect at the same conditions.

V. CONCLUSIONS

The size effect in a superfluid mixture of helium isotopes was investigated by taking the contribution of thermal excitations into consideration. The exact solution (25), (26) of set of the kinetic equations (3) describing the steady, thermodynamically non-equilibrium state of a phonon-impuriton system has been obtained. The problem of the steady flow of the superfluid mixture caused by the thermodynamic gradients through the volume filled with the macrons was solved. Conditions (57) and (58) ensuring the steady state of a superfluid mixture in the volume filled with macrons were obtained and analyzed explicitly.

A model (32) reflecting the significant features of the true collision operator was proposed. It made it possible to simplify considerably the main results obtained within the considered approximation. In particular, the explicit results for the mass velocities (54) and (55) of quasiparticle gas flow were obtained and used in conditions (57) and (58).
which describe the steady non-equilibrium state of a superfluid mixture in the volume filled with macrons. As a result, the conditions of stationarity (57) and (58) were presented in the form of relation between the thermodynamic gradients ensuring the steady state of a mixture. The relation obtained was investigated in the region of classical temperatures where the contribution of phonons is maximal. All the results are available for arbitrary relations between the frequencies of collisions of quasiparticles with one another and with macrons. The steady state condition (59) was studied for arbitrary relations between the frequencies of collisions of different types.

The hydrodynamic \( \nu_3 \gg \nu_{3Kn} \) (29) and Knudsen \( \nu_3 \ll \nu_{3Kn} \) (63) limits of the conditions ensuring a steady state of a superfluid mixture were investigated in details by taking the contribution of phonons into account. In particular, the correction to the classical Knudsen condition caused by the presence of phonons was obtained. A convenient interpolation formula (64) was derived from the exact result to observe explicitly the steady state condition (60) in the intermediate region \( \nu_3 \sim \nu_{3Kn} \).

The results were applied to the investigations of a steady non-equilibrium state of a superfluid mixture situated in the volume filled with the macroparticles. Two different stages in the process of a steady state formation were found and studied. The experiment for an investigation of the Knudsen effect in a superfluid mixture was proposed.

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FIG. 1. The transition from the Knudsen regime to hydrodynamical one in relation (61) describing the sufficient condition ensuring a steady state of the superfluid mixture ($T = 0.7K$).