Spontaneous Symmetry Breaking and Quantum Hall Effect in Graphene

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In this article we briefly review recent experimental and theoretical work on quantum Hall effect in graphene, and argue that some of the quantum Hall states exhibit spontaneous symmetry breaking that is driven by electron-electron interaction. We will also discuss how to experimentally determine the actual manner in which symmetry breaking occurs, and detect the collective charge and neutral excitations associated with symmetry breaking. Other issues will also be briefly mentioned.

I. INTRODUCTION

Recent experimental work has established graphene as a new two-dimensional (2D) system with linear Dirac-like energy-band dispersion. Shortly after the original work performed at zero magnetic field, experimentalist applied a strong magnetic field (around 10T) perpendicular to the graphene sheet, and observed a set of integer quantum Hall plateaus on which the Hall conductance takes quantized values

\[
\sigma_{xy} = \nu e^2/h,
\]

with

\[
\nu = 4(n + 1/2),
\]

where \(n\) is an integer. This result, which was anticipated theoretically, may be understood in the following manner: the prefactor 4 reflects the two-fold spin and two-fold valley degeneracy in the graphene band structure, while the “shift” of 1/2 originates from the Berry phase due to the pseudospin (or valley) precession when a massless (and thus chiral) Dirac particle experiences cyclotron motion. This results thus provide direct evidence for the Dirac or relativistic nature of the charge carriers in graphene; it can also be understood from the Landau level structure of Dirac fermions that includes the quantum Hall effect, was considered by Semenoff and Haldane, and considerations of quantum Hall effect of relativistic particles dates back even further. We also note that such integer quantum Hall plateaus were found to persist to room temperature, paving the way for their application some day.

When a even stronger magnetic field (up to 45 T) is applied, more quantum Hall (QH) plateaus not in the sequence of Eq. (2) have been observed, these include \(\nu = 0, \pm 1, \) and \(\pm 4\). These new plateaus cannot be understood from Landau quantization alone, and their origin is currently under extensive theoretical study and considerable debate. In the next section we review and compare the existing theoretical proposals for these states, and discuss how to distinguish them experimentally. We will also briefly mention theoretical work on edge states and possible fractional quantum Hall state in graphene (which has not been observed yet), and some other related issues, in section III.

II. SPONTANEOUS SYMMETRY BREAKING AND THE NEW QUANTUM HALL STATES

As discussed earlier and in Appendix A, fully occupied Landau levels (LLs) of Dirac fermions that includes the spin and valley degeneracy lead to integer QH states that belong to the sequence Eq. (2). In the following for the sake of ease of discussion we introduce a reduced filling factor \(\nu_n\) for the highest LL (or valence LL, with index \(n\)) that is occupied by electrons:

\[
\nu_n = \nu - 4(n - 1/2) \leq 4.
\]

Obviously for the sequence (2), \(\nu_n = 4\) which reflects the fact that the highest occupied LL is fully occupied, while for the new QH states \(\nu_n = 1, 2\) or 3, reflecting partial occupation. In these new QH states the incompressibility or charge gap cannot be due to LL spacing. The origin of the gap in these cases is the subject of extensive theoretical work recently. With few exceptions, in these theories the gap has its origin in some spontaneously broken symmetry of the system, and in most cases the symmetry breaking is driven by electron-electron interactions. In the following we briefly review these theoretical works.

A. Spontaneous breaking of SU(4) symmetry and quantum Hall ferromagnetism

In this class of theories, SU(4) symmetry that corresponds to the invariance of electron-electron interaction under a rotation within the 4-fold spin/valley internal space, is spontaneously broken. In the symmetry breaking ground state, the system spontaneously picks \(\nu_n\) orthogonal states in the SU(4) space, and fill them with electrons in the \(n\)th LL:

\[
|\Psi_0\rangle = \prod_{1 \leq \sigma \leq \nu_n} \prod_k c_{\sigma}^\dagger \left| 0 \right\rangle.
\]
Here $c^\dagger$ is the electron creation operator, $|0\rangle$ is the vacuum state, $\sigma$ is the index of the internal state that runs from 1 to 4, $k$ is an intra LL orbital index; for example in the Landau gauge it is the wave vector along the plane wave direction, while in the symmetric gauge it is the angular momentum quantum number. Such a state can be shown to be an exact eigenstate of the electron-electron interaction Hamiltonian in the presence of full SU(4) symmetry, and very effectively gains exchange energy. For a broad class of repulsive interactions, we expect $\langle \Psi_0 \rangle$ to be the exact ground state of the system; this can be proven rigorously for the case $\nu_n = 1$ and short-range repulsions. Any additional electron(s) added to the system will then have to occupy the empty internal states, and lose exchange energy; this results in additional energy cost, leading to incompressibility of the system. The situation here is quite similar to the ordinary single layer QH states in GaAs at odd integer fillings, where electrons in the valence LL spontaneously magnetize to lower exchange energy even in the absence of Zeeman splitting; the spontaneously broken symmetry there is SU(2). Such broken symmetry states are collectively known as quantum Hall ferromagnets (QHF). In a recent numerical study, QHF ground states with large charge gaps have been observed for the graphene SU(4) case.

In general symmetry breaking leads to low-energy collective modes. Since SU(4) is a continuous symmetry, when broken spontaneously the systems must support gapless collective modes. These neutral, spin-wave like modes have been studied recently, and their full spectra have been determined exactly. Due to the larger symmetry, we find that number of such modes is $\nu_n (4 - \nu_n)$, larger than the corresponding SU(2) case, and is tied to the actual manner of symmetry breaking.

Perhaps more interesting is the appearance of topological solitons in the order parameter called skyrmions as low-energy charge excitations of the systems. Again due to the larger symmetry, skyrmions come in more species in the SU(4) QHF compared to the SU(2) QHF studied earlier. There is also an important quantitative difference due to the Dirac nature of the charge carriers in graphene. In a magnetic field, the eigen wave functions of the two Dirac components are very different; they actually each represent different LL way functions of the single component wave functions of non-relativistic electrons (see also Appendix A). As a result the form factor and thus effective electron-electron interaction in a given LL is quite different for Dirac fermions. One of the consequences of this is skyrmions are lowest energy charge excitations not only in the lowest LL (which is the only case this is true for of non-relativistic electrons), but also in some of the higher LL; more specifically they are lowest energy charge excitations for $|n| < 3$. These high LL skyrmions have been observed in a recent numerical work.

Small perturbations that explicitly break the SU(4) symmetry do exist in graphene; they may fix the direction of the SU(4) order parameter and open small gap(s) in the collective mode spectra. In the following we discuss different types of such perturbations.

**Zeeman Splitting.** This is probably the most obvious one, which lifts the spin degeneracy without affecting the valley degeneracy; formally it reduces the SU(4) symmetry to a (valley) SU(2) symmetry. Its effect is particularly important when $\nu_n = 2$, in which case it uniquely selects out a single ground state which is a valley singlet but spin fully polarized in the valence LL, with or without electron-electron interaction. In the absence of interaction the gap of this state is precisely the Zeeman splitting, and in some theories (to be discussed later) this is thought to be the sole source of gap for $\nu = \pm 4$. Quantitatively, the Zeeman splitting is of order 10 K, while electron-electron interaction scale $e^2/\epsilon \ell \sim 100 - 1000K$ ($\ell = \sqrt{\hbar c/eB}$ is the magnetic length); it thus qualifies as a small symmetry breaking perturbation.

**Lattice Effect.** The SU(4) symmetry is exact in the continuum limit $a \ll \ell$, where $a$ is lattice spacing. The lattice effect breaks the SU(4) symmetry, or more precisely, the SU(2) symmetry associated with valley symmetry. The strength of the symmetry breaking is proportional to $a/\ell$. For the lowest LL ($n = 0$), this introduces an Ising anisotropy in the valley SU(2) space, while for $|n| > 0$ an easy-plane (or XY) anisotropy is introduced. Disorder. Even if the disorder potential did not break any symmetry, in general it lifts LL degeneracy and favors SU(4) singlet ground states. In Ref. where the notion of QHF was first introduced in the graphene context, a Stoner-like criterion was derived for the development of QHF against disorder. The situation was further analysed in Ref. where the authors came to the conclusion that the existing samples may not be clean enough for QHF, and the system may actually be SU(4) paramagnets; they argue that the combination of Zeeman and symmetry breaking interactions can give rise the new QH states in the paramagnetic regime.

On the other hand Abanin et al. pointed out an interesting symmetry breaking effect of disorder potential, which may favor ordering: the random potential due to strain in the graphene lattice locally breaks the valley degeneracy in a random manner; it thus acts like a random field along $\hat{z}$ direction in the valley subspace. This forces the valley SU(2) order parameter into the XY plane, and allows for algebraic long-range order at finite temperature and a Kosterlitz-Thouless transition.

Before closing this subsection, we point out that many issues similar to those discussed here have been studied in the context of QHF in silicon, and bilayer QH systems. In particular the SU(4) symmetry was used by Arovas et al. and Ezawa and co-workers to organize the internal degrees of freedom in silicon and bilayer systems respectively. It should be noted however that graphene is a much better realization of the SU(4) symmetry due to the small parameters that control the symmetry-breaking perturbations. Such small parame-
B. Spontaneous mass generation and symmetry breaking

In another set of theories, it was argued that the system spontaneously generates a mass for the Dirac particles by breaking the translational symmetry, and the symmetry breaking is driven by electron-electron interaction. Such spontaneous mass generation was considered even before the actual experimental realization of monolayer graphene. In Ref. 20, it was argued that the ordering that generates the mass could be either charge density wave (CDW) or antiferromagnetism (AFM), depending on the relative strength of on-site and nearest neighbor electron-electron repulsion. The main selling point of these theories is the fact that a Dirac mass would lift the valley degeneracy in the $n = 0$ LL only, but not for $|n| > 1$ (see Appendix A for a discussion of this point). Combining with Zeeman splitting, these theories would predict that in the $n = 0$ LL one would get QH states for all integer $n$, but for $|n| > 1$ only even $n$ (and thus $\nu$) support QH states. This is consistent with existing experimental results that the odd integer QH states observed thus far, $\nu = \pm 1$, are in the $n = 0$ LL.

C. Electron-phonon interaction and spontaneously broken inversion symmetry

Fuchs and Lederer argued that in the presence of a magnetic field, electron-phonon interaction can induce a spontaneous lattice distortion similar to the Peierls effect, which breaks the inversion symmetry of graphene. This spontaneously broken inversion symmetry also generates a Dirac mass, although electron-electron interaction only plays a minor role here. The consequences of this mass generation are the same as the theories discussed in section IIB.

D. Comparison of competing theories and experimental distinctions

It should be obvious from the discussions above that the existing theories all share a common theme that spontaneous symmetry breaking plays a central role in stabilizing the new QH states observed in Ref. 12. They are not entirely orthogonal in details either. For examples, theories of subsections B and C both rely on spontaneous mass generation: in the QHF theories described in subsection A, the valley QHF for the $n = 0$ LL is a CDW due to the fact that wave functions of different valleys live in opposite sublattices. In the following we focus on their differences and how to distinguish among them experimentally.

Perhaps the first question to address is whether the origin of these new QH states lies in electron-electron interaction or other mechanism, like electron-phonon interaction. Experimentally, this question can be settled by measuring the magnetic field dependence of the gap at fixed filling factors, in sufficiently clean samples. If electron-electron interaction dominates the gap, we expect $\Delta \propto e^2/\ell \propto \sqrt{B}$; such a behavior would be strong indication of the electron-electron interaction mechanism. Also the size of $\Delta$ is an indicator too; $e^2/\ell$ is by far the largest energy scale in the system other than LL spacing.

The biggest difference between the QHF theory and other theories discussed in subsections B and C is it allows for QH states at all integer fillings, including odd integers in $|n| \geq 1$ LLs, in sufficiently clean samples. It would thus be highly desirable to have higher quality samples and study if more QH states can be found, including odd integers other than $\nu = \pm 1$; if so this would lend strong support to QHF. In addition, due to the breaking of a continuous symmetry in QHF, they support highly collective neutral and charge excitations, which are not shared in other theories. The neutral, spin-wave like modes may be detectable in inelastic light scattering or optical absorption experiments while charged skyrmions may be detected in transport and other experiments in particular, valley skyrmions have been seen in silicon samples. Some of these experiments performed in semiconductors to detect skyrmions can also be done in graphene. Obviously evidence for these neutral and charged collective excitations would also lend strong support to the QHF theory.

It is clear from the discussion above that higher quality (or mobility) samples in which disorder effects are minimal are crucial to further progress on the issues discussed in this section.

III. EDGE STATES, FRACTIONAL QUANTUM HALL STATES, AND OTHER ISSUES

Edge States. Edge states play a key role in QH transport. Due to the special lattice structure of graphene and Dirac nature of the carriers, the edge states here have certain features not shared elsewhere. Among many peculiar properties, the edge states of the $\nu = 0$ QH state (which clearly has no counterpart elsewhere) are particularly interesting – they are argued to carry no charge current but finite spin current in equilibrium. There appear to be some experimental evidence in support of this. The possibility of edge reconstruction has also been discussed.

Possible Fractional Quantum Hall States. Perhaps the most eagerly anticipated next experimental breakthrough is the observation of fractional Quantum Hall (FQH) States. Possible FQH states have been dis-
cussed in a number of papers and some of them may be QHF that break SU(4) symmetry as well. Possible compressible, composite fermion Fermi sea states have also been discussed.

**Lattice Effects, Disorder, Localization, and Integer Quantum Hall Transition.** Most of the papers above use the continuum description of Dirac fermions, which properly describe low-energy electronic state when the magnetic field strength is not too strong. When the field gets so strong that $a/\ell$ is no longer small, or when the Fermi energy becomes comparable to band width, the continuum description breaks down and the lattice effect must be taken into account. This issue has been addressed by studying the QH physics on a honeycomb lattice directly. The lattice model also allows for a straightforward numerical study of effects of disorder, and the associated localization and integer QH transition for non-interacting electrons problem. In particular, it was shown that disorder can split the critical states carrying non-zero Chern numbers associated with the two degenerate valleys in the absence of disorder; this offers yet another mechanism for the new QH states discussed earlier that does not involve any interaction. Most of the lattice work relies heavily on the calculation of the Chern number, which have been used extensively before in the study of both integer and fractional quantum Hall effects.

**IV. CONCLUDING REMARKS**

We hope the discussions above have made it clear that progress made in the field of quantum Hall effect in graphene thus far has been exciting, but the whole business is in its very early stage; many important issues remain unresolved. As usual experimentalists hold the key to further progress, and higher sample quality is most crucial to such progress.

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**APPENDIX A: LANDAU LEVELS OF DIRAC PARTICLES**

In this Appendix we discuss the Landau levels (LLs) of Dirac particles. Instead of repeating the theoretical treatment well documented in literature, here we provide a heuristic discussion of the LL energies and degeneracies, without actually writing down the Dirac equation.

**Landau Level Energies.** The dispersion relation of relativistic particles with mass $m$ is

$$E = \sqrt{m^2v_F^2 + v_F^2p^2}. \quad (A1)$$

In the graphene context Fermi velocity $v_F$ plays the role a speed of light, and the mass $m$ is either zero or extremely small. In the presence of a magnetic field $B$, classically the particle exercises cyclotron motion, and the semi-classical Bohr-Sommerfeld rule leads to the quantization condition for the radii of such cyclotron orbits:

$$\pi R_n^2 = \hbar c/eB, \quad (A2)$$

i.e., they enclose an integer number of flux quanta. This dictates the mechanical momentum of the particle to be

$$p_n = \sqrt{2n\hbar}/\ell, \quad (A3)$$

where $\ell = \sqrt{\hbar c/eB}$ is the magnetic length. Substituting into (A1), we obtain the Landau level energies:

$$E_n = \pm \sqrt{m^2v_F^2 + 2n\hbar v_F^2|eB|/c}, \quad (A4)$$

where the negative energy states correspond to the Dirac hole states, or the valence band states in the graphene context. Amazingly, this is the exact result! Normally energy levels obtained from the Bohr-Sommerfeld quantization condition misses the zero point energy; here the effect of zero point motion is apparently compensated for by the Berry phase shift discussed in section I.

**Landau Level Degeneracy.** Let us start by ignoring the physical spin, which contributes a trivial factor of two. We do explicitly include the two-fold valley degeneracy which is important for the following discussion. We know that two major triumphs of the Dirac theory are (i) the particles must have “spin”-1/2 to be consistent with Lorentz invariance; and (ii) the magnetic $g$-factor is exactly 2 for the “spin” in the non-relativistic limit. In the graphene context, this “spin” is actually the 2-fold sublattice degree of freedom associated with the hexagonal lattice, or a pseudospin; combining with the two-fold valley degeneracy we get a four component wave function for usual Dirac particles. We also know that $g = 2$ has a remarkable consequence in the non-relativistic limit: the Zeeman splitting is $g\mu_B B = \hbar \omega_c$, exactly the same as the Landau level spacing! This leads to a degeneracy of pseudospin-up states in the $n$th LL with the pseudospin-down states in the $n - 1$th LL; as a consequence of this each energy level has degeneracy $2N_\Phi$ (where $N_\Phi$ is the number of flux quanta in the system), and the wave functions of each degenerate level mix the $n$th and $n - 1$th LL wave functions. This is true except for the level corresponding to $n = 0$th LL with pseudospin up, whose degeneracy remains $N_\Phi$. Now putting back the factor...
of two associated with physical spin, one immediately obtains the quantization condition \( m = 0 \) for the Hall conductance of filled LLs. Remarkably, the degeneracy discussed above, and thus the condition \( m = 0 \), persist even in the extreme relativistic limit \( m = 0 \), to be discussed below.

**The Zero Mass Limit.** In the limit \( m \to 0 \), nothing significant happens to the \( n \geq 1 \) LLs. However the two \( n = 0 \) LLs with energies \( \pm m v_F^2 \) merge together to form a single level, now with degeneracy \( 4N_0 \) just like the other LLs (including physical spin). Since half of the states of this level come from the valence band, it is half-filled at zero doping; as a consequence the \( \nu = 0 \) state corresponds to a partially filled LL!

Another way to phrase the discussion above is a nonzero Dirac mass \( m \), either intrinsic or dynamically generated, lifts the valley degeneracy of the \( n = 0 \) LL, but does not affect the degeneracy of other LLs. This fact plays a crucial role in the theories discussed in sections IIB and IIC.

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1. K. S. Novoselov et al., Science 306, 666 (2004); Y. Zhang et al., Phys. Rev. Lett. 94, 176803 (2005); C. Berger et al., J. Phys. Chem. B 108, 19912 (2004).
2. K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos and A. A. Firsov, Nature 438, 197 (2005).
3. Yuanbo Zhang, Yan-Wen Tan, Horst L. Stormer and Philip Kim, Nature 438, 201 (2005).
4. Y. Zheng and T. Ando, Phys. Rev. B 65, 245420 (2002).
5. V. P. Gusynin and S. G. Sharapov, Phys. Rev. Lett. 95, 146801 (2005).
6. N. M. R. Peres, F. Guinea, and A. H. Castro Neto, Phys. Rev. B 73, 125411 (2006).
7. G. W. Semenoff, Phys. Rev. Lett. 53, 2015 (1988).
8. F. D. M. Haldane, Phys. Rev. Lett. 61, 2449 (1984).
9. A. H. MacDonald, Phys. Rev. B 28, 2235 (1983).
10. A. M. J. Schakel, Phys. Rev. D 95, 085002 (2017).
11. Tapash Chakraborty and Pekka Pietilainen, cond-mat/0703536.
12. S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B 47, 16419 (1993).
13. X.-G. Wu and S.L. Sondhi, Phys. Rev. B 51, 14725 (1995).
14. Csaba Toke, Paul E. Lammert, Jainendra K. Jain, and Vincent H. Crespi, Phys. Rev. B 74, 235417 (2006).
15. D. P. Arovas, A. Karlhede and D. Lilliehook, Phys. Rev. B 59, 13147 (1999).
16. K. Yang, K. Moon, L. Zheng, A. H. MacDonald, S. M. Girvin, D. Yoshioka, and Shou-Cheng Zhang, Phys. Rev. Lett. 72, 732 (1994); K. Moon, H. Morii, K. Yang, S. M. Girvin, A. H. MacDonald, L. Zheng, D. Yoshioka and S.-C. Zhang, Phys. Rev. B 51, 5138 (1995); K. Yang, K. Moon, L. Belkhir, H. Morii, S. M. Girvin, A. H. MacDonald, L. Zheng and D. Yoshioka, Phys. Rev. B 54, 11644 (1996).
17. K. Hasebe and Z. F. Ezawa, Phys. Rev. B 66, 155318 (2002); Z. F. Ezawa, G. Tsitsishvili, and K. Hasebe, Phys. Rev. B 67, 125314 (2003); Z. F. Ezawa and G. Tsitsishvili, Phys. Rev. D 72, 085002 (2005); G. Tsitsishvili and Z. F. Ezawa, Phys. Rev. B 72, 115306 (2005).
18. A. Iyengar, Jianhui Wang, H. A. Fertig, and L. Brey, cond-mat/0608364.
19. We note in passing that cyclotron resonance has been observed in graphene recently using optical probes: Z. Jiang, E.A. Henriksen, L.C. Tung, Y.-J. Wang, M.E. Schwartz, M.Y. Han, P. Kim, and H.L. Stormer, cond-mat/0703822; R.S. Deacon, K-C. Chung, R. JNicholas, K.S. Novoselov, and A.K. Geim, cond-mat/0704041.
20. S. E. Barrett, G. Dabbagh, L. N. Pfeiffer, K. W. West, and R. Tycko Phys. Rev. Lett. 74, 5112 (1995); A. Schmeller, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 75, 4290 (1995); E.H. Aifer, B.B. Goldberg, D.A. Broideo, Phys. Rev. Lett. 76, 680 (1996); V. Bayot, E. Grivei, S. Melinte, M.B. Santos, and M. Shayegan, Phys. Rev. Lett. 76, 4584 (1996); V. Bayot, E. Grivei, J.-M. Beuken, S. Melinte, and M. Shayegan, Phys. Rev. Lett. 79, 1718 (1997); D.R. Leadley, R.J. Nicholas, D.K. Maude, A.N. Utjuzh, J.C. Portal, J.J. Harris, and C.T. Foxon, Phys. Rev. Lett. 79, 4246 (1997); J.L. Osborne, A.J. Shields, M.Y. Simmons, N.R. Cooper, D.A. Ritchie, and M. Pepper, Phys. Rev. B 58, R4227 (1998); S. Melinte, E. Grivei, V. Bayot, and M. Shayegan, Phys. Rev. Lett. 82, 2764 (1999); J.H. Smet, R.A. Deutschmann, F. Ertl, W. der Wegschei, G. Abstreiter and K. von Klitzing, Phys. Rev. Lett. 92, 086802 (2004); P.G. Gervais, H.L. Stormer, D.C. Tsui, P.L. Kuhns, W.G. Moulton, A.P. Reyes, L.N.
Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. 94, 196803 (2005).

37 Y.P. Shkolnikov, S. Misra, N.C. Bishop, E.P. De Poortere, and M. Shayegan, Phys. Rev. Lett. 95, 066809 (2005).

38 For a review, see C. L. Kane and M. P. A. Fisher, in Ref. 25.

39 A. H. Castro Neto, F. Guinea, and N. M. R. Peres, Phys. Rev. B 73, 205408 (2006).

40 Dmitry A. Abanin, Patrick A. Lee, and Leonid S. Levitov, Phys. Rev. Lett. 96, 176803 (2006).

41 L. Brey and H.A. Fertig, Phys. Rev. B 73, 235411 (2006).

42 H.A. Fertig and Luis Brey, Phys. Rev. Lett. 97, 116805 (2006).

43 Y. Hatsugai, T. Fukui, and H. Aoki, Phys. Rev. B 74, 205414 (2006).

44 Dmitry A. Abanin, Kostya S. Novoselov, Uli Zeitler, Patrick A. Lee, Andre K. Geim, and Leonid S. Levitov, cond-mat/0702125.

45 A. H. MacDonald, S. R. E. Yang, and M. D. Johnson, Aust. J. Phys. 46, 345 (1993); C. de C. Chamon and X.-G. Wen, ibid 49, 8227 (1994); X. Wan, K. Yang, and E. H. Rezayi, Phys. Rev. Lett. 88, 056802 (2002); X. Wan, E. H. Rezayi, and K. Yang, Phys. Rev. B 68, 125307 (2003); K. Yang, Phys. Rev. Lett. 91, 036802 (2003).

46 V. M. Apalkov and T. Chakraborty, Phys. Rev. Lett. 97, 126801 (2006).

47 Csaba Toke and Jainendra K. Jain, cond-mat/0701026.

48 M. O. Goerbig and N. Regnault, cond-mat/0701661.

49 D. V. Khveshchenko, Phys. Rev. B 75, 153405 (2007).

50 G. Baskaran, cond-mat/0702420.

51 D. N. Sheng, L. Sheng, and Z. Y. Weng, Phys. Rev. B 73, 233406 (2006).

52 Yasumasa Hasegawa and Mahito Kohmoto, Phys. Rev. B 74, 155415 (2006).

53 B. Andrei Bernevig, Taylor L. Hughes, Han-Dong Chen, Congjun Wu, and Shou-Cheng Zhang, Int. J. of Mod. Phys. B 20, 3257 (2006).

54 C. P. Burgess and B. P. Dolan, cond-mat/0612269.

55 D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. 49, 405 (1982); Q. Niu, D. J. Thouless and Y. S. Wu, Phys. Rev. B 31, 3372 (1985); D. P. Arovas et. al., Phys. Rev. Lett. 60, 619 (1988); Y. Huo, and R. N. Bhatt, Phys. Rev. Lett. 68, 1375 (1992); D. N. Sheng and Z. Y. Weng, Phys. Rev. Lett. 75 2388 (1995); Phys. Rev. B54, R11070 (1996); K. Yang and R. N. Bhatt, Phys. Rev. Lett. 76, 1316 (1996); Phys. Rev. B 59, 8144 (1999); Phys. Rev. B 55, R1922 (1997); R. N. Bhatt and X. Wan, Pramana-J. Phys. 58, 271 (2002); Qinghong Cui, Xin Wan, and Kun Yang, Phys. Rev. B 70, 094506 (2004).

56 D. N. Sheng, Xin Wan, E. H. Rezayi, Kun Yang, R. N. Bhatt, and F. D. M. Haldane, Phys. Rev. Lett. 90, 256802 (2003); D. N. Sheng, Leon Balents, and Ziqiang Wang, Phys. Rev. Lett. 91, 116802 (2003); Xin Wan, D.N. Sheng, E.H. Rezayi, Kun Yang, R.N. Bhatt, and F.D.M. Haldane, Phys. Rev. B 72, 075325 (2005).