Decoherence of a Macroscopic Quantum Superposition

Francesco De Martini\textsuperscript{1,2}, Fabio Sciarrino \textsuperscript{3,1}, and Nicolò Spagnolo\textsuperscript{1}

\textsuperscript{1}Dipartimento di Fisica dell’Università “La Sapienza” and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Roma, 00185 Italy
\textsuperscript{2} Accademia Nazionale dei Lincei, via della Lungara 10, I-00165 Roma, Italy
\textsuperscript{3}Centro di Studi e Ricerche “Enrico Fermi”, Via Panisperna 89/Â, Compendio del Viminale, Roma 00184, Italy

The high resilience to decoherence shown by a recently discovered Macroscopic Quantum Superposition (MQS) involving a number of photons in excess of $5 \times 10^4$ motivates the present theoretical and numerical investigation. The results are placed in close comparison with the properties of the well known MQS based on $|\alpha\rangle$ states. The very critical decoherence properties of the latter MQS are found to be fully accounted for, in a direct a simple way, by a unique "universal" function: indeed a new property of the quantum "coherent states".

PACS numbers:

Since the early decades of Quantum Mechanics, the counter-intuitive properties associated with the coherent superposition state of macroscopic objects was the object of an intense debate epitomized by the celebrated "Schrödinger Cat" paradox [1, 2]. In particular, the actual feasibility of such quantum object has always been tied to the alleged infinitely short persistence of its quantum coherence, i.e. of its overwhelmingly rapid "decoherence". In modern times the latter property, establishing a rapid merging of the quantum rules of microscopic systems into the classical dynamics, has been interpreted as a consequence of the entanglement between the macroscopic quantum system with the environment [3, 4]. By discarding the environmental variables in the final calculations the initially pure quantum state generally appears to decay irreversibly towards a probabilistic classical mixture [5]. Recently, the general interest for decoherence has received a renewed interest in the framework of quantum information theory where it plays a fundamental detrimental role since it conflicts with the experimental realization of the quantum computer or of any quantum device bearing any useful, relevant complexity [6]. In this respect a large experimental effort has been devoted recently to the implementation of Macroscopic (i.e. many-particle) Quantum Superpositions states (MQS), adopting photons, atoms and electrons in superconducting devices. Particular attention has been devoted to the realization of the MQS involving "coherent states" of light, which exhibits interesting and elegant Wigner function representations [7]. The most notable results of this experimental effort have been reached with atoms interacting with microwave fields trapped inside a cavity [3, 9] or for freely propagating fields [10]. However, in spite of the long lasting efforts spent in these endeavors, in these realizations the MQS has always proved to be so fragile that even the loss of a single particle was found to be able to spoil any possibility of a direct observation of its quantum properties. Precisely on the basis of these negative results in many scientific communities, and also within very influential editorial teams, grew the opinion that the "Schrödinger Cat" is indeed a ill defined, abstruse and then avoidable concept since it fundamentally lacks of any directly observable property [8].

However, very recently a new kind of MQS involving a number of particles $N$ in excess of $5 \times 10^4$ has been realized that allows the direct observation of entanglement between a microscopic and a macroscopic photonic state and shows a very high resilience to decoherence by coupling with environment [11]. Precisely, the MQS was generated by a quantum-injected optical parametric amplifier (QI-OPA) seeded by a single-photon belonging to an EPR entangled pair: indeed a high-gain phase-covariant Cloning Machine [12, 13, 14, 15, 17]. By this device, that includes an Orthogonality Filter (O-Filter, OF) for better state discrimination, the macroscopic-macroscopic state non-separability was successfully tested and the microscopic-macroscopic Violation of the Bell'inequalities for Spin-1 excitations was attained [11, 10]. It then appears that several prejudices about MQS should need a revision and that a careful analysis looks necessary in order to learn more about the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Schematicization of the decoherence model by a linear beam-splitter of transmittivity $T$. (a) Analysis of quantum superposition of coherent states $|\psi^{\pm}\rangle = \frac{1}{\sqrt{N}} (|\alpha\rangle \pm |-\alpha\rangle)$. (b) Analysis of amplified states of the collinear optical parametric amplifier injected by a single photon qubit generated in a type-II EPR source.}
\end{figure}
MQS dynamics and its decoherence. The present work is precisely intended to provide a first glimpse in this elusive albeit fundamental matter.

Criteria for macroscopic superposition: In order to distinguish between two different quantum states, let us introduce a definition of distance in the Hilbert space. An useful parameter to characterize quantitatively the overlap of two quantum states is the “fidelity” $\mathcal{F}$ between two generic density matrices $\hat{\rho}$ and $\hat{\sigma}$, defined as: $\mathcal{F}(\hat{\rho}, \hat{\sigma}) = \text{Tr} \left( \sqrt{\sqrt{\hat{\rho}} \hat{\sigma} \sqrt{\hat{\rho}}} \right)$ and $0 \leq \mathcal{F} \leq 1$, where $\mathcal{F} = 1$ for identical states, and $\mathcal{F} = 0$ for orthogonal states. This quantity is still not a metric in the quantum state space, but can be adopted to define different useful metrics. Amongst them we consider here the “Bures distance”:

$$D(\hat{\rho}, \hat{\sigma}) = \sqrt{1 - \mathcal{F}(\hat{\rho}, \hat{\sigma})}$$

In this paper we shall adopt this “distance” as it is connected to the probability of obtaining an inconclusive result with a suitable Positive Operator Valued Measurement (POVM) [21], which is $\mathcal{F}(|\phi\rangle, |\psi\rangle) = |\langle\psi|\phi\rangle|$ for pure states. When dealing with the distance amongst two MQS, we shall also refer to $D$ as the MQS “Visibility”.

Distinguishability, MQS Visibility, the lossy Channel.

Let us characterize two macroscopic states $|\phi_1\rangle$ and $|\phi_2\rangle$ and the corresponding MQS’s: $|\phi^\pm\rangle = \frac{N_{\pm}}{\sqrt{2}} (|\phi_1\rangle \pm |\phi_2\rangle)$ by adopting two criteria. I) The distinguishability between $|\phi_1\rangle$ and $|\phi_2\rangle$ can be quantified as $D(|\phi_1\rangle, |\phi_2\rangle)$. Indeed, the value of the MQS visibility depends exclusively on the relative phase of the component states $|\phi_1\rangle$ and $|\phi_2\rangle$. Assume two orthogonal superpositions $|\phi^\pm\rangle$: $D(|\phi^+\rangle, |\phi^-\rangle) = 1$. In presence of losses the relative phase between $|\phi_1\rangle$ and $|\phi_2\rangle$ progressively randomizes and the superpositions $|\phi^+\rangle$ and $|\phi^-\rangle$ approach an identical fully mixed state leading to: $D(|\phi^+\rangle, |\phi^-\rangle) = 0$. The aim of this paper is to study the evolution in a lossy channel of the phase decoherence acting on two macroscopic states $|\phi_1\rangle$ and $|\phi_2\rangle$ and on the corresponding superpositions $|\phi^\pm\rangle$ and the effect on the size of the corresponding $D(|\phi_1\rangle, |\phi_2\rangle)$ and $D(|\phi^+\rangle, |\phi^-\rangle)$.

The effects of losses are analyzed through the effect of a generic linear beam-splitter (BS) with transmittivity $T$ and reflectivity $R = 1 - T$ acting on a generic quantum state associated with a single mode beam: Fig.1 [22]. The procedure for the calculation of the output density matrix is the insertion of the BS unitary transformations as a function of the parameters $T$ or $R$, and the evaluation of the partial trace of the emerging field on the reflected mode (R-trace), i.e. on the loss variables. In this paper we shall study quantitatively how the distinguishability of macroscopic quantum states and the corresponding MQS Visibility is affected by the parameters $R$ or $T$ of the lossy channel. We are going to carry out this study for two classes of states that bear a particular relevance in the present context, as discussed above in the introductory comments.

Quantum superposition of coherent states. The first class of states we are going to analyze is the quantum superposition of coherent states $|\phi_1, \alpha\rangle = |\pm\alpha\rangle$, defined as: $|\psi^\pm\rangle = \frac{N_\pm}{\sqrt{2}} (|\alpha\rangle \pm |\alpha\rangle)$. As it is well known, this MQS exhibits several very interesting quantum properties, such as squeezing and sub-poissonian statistics and has been so far considered as the paradigmatic representation of the Schrödinger’s cat State.

Since the BS doesn’t affect the poissonian character on the field, the application of the losses model to the input component coherent states $|\pm\alpha\rangle$ of opposite phase leads again to an output coherent-state density matrix of the form $\rho_{\alpha}^T = |\sqrt{T}\alpha\rangle \langle \sqrt{T}\alpha|$.) Then the distance between the two states with opposite phase is: $D(|\sqrt{T}\alpha\rangle, |\sqrt{T}\alpha\rangle) = \sqrt{1 - e^{-2T|\alpha|^2}}$, a value close to 1 except for $T = 0$, i.e. for total loss of all particles. Hence the coherent states $|\pm\alpha\rangle$ keep their mutual distinguishability through the lossy channel.

Let’s now consider the MQS Visibility. Applying the previous losses procedure to the $|\psi^\pm\rangle$ states, the density matrices after losses have the general form:

FIG. 2: (a)-(d): Plot of the distribution of the number of photons in the $|\psi^\pm\rangle$ state for $\alpha = 4$, corresponding to an average number of photons $\langle n \rangle = 16$, for reflectivities $R = 0$ (fig.2-a), $R = 0.1$ (fig.2-b), $R = 0.5$ (fig.2-c) and $R = 0.8$ (fig.2-d). (e): Plot of the universal curve that describes the distance between $|\psi^+\rangle$ and $|\psi^-\rangle$ after losses as a function of $R\langle n \rangle \sin^2 \varphi$. 

![Graphs showing MQS Visibility](image-url)
where \( |\beta| = |\alpha \sqrt{T}| \). For the coherent state MQS with no losses \( (T = 1) \), the distribution in the Fock space exhibits only elements with an even number of photons for \( |\psi^+\rangle \) or an odd number of photons for \( |\psi^-\rangle \). This very peculiar comb structure is indeed very fragile under the effect of losses since the R-trace operation must be carried out in the space of the non-orthogonal coherent-states. This is shown in Fig 2(a)-(b). Furthermore, Fig 2(c)-(d) shows the effects of decoherence on the \( |\psi^\pm\rangle \) states for increasing particle loss. The MQS Visibility of these states has been evaluated analytically in closed form and is found extremely sensitive to decoherence: \( \bar{D} = \frac{1}{2} \left( |\beta| \langle \beta | + | - \beta \rangle \langle - \beta | \pm e^{-2R|\alpha|^2} (| - \beta \rangle \langle \beta | + | \beta \rangle \langle - \beta |) \right) \) \( (2) \)

with \( |\beta| = |\alpha \sqrt{T}| \). Note that the function \( D(x) \) depends only on the average number of lost photons \( x \equiv R|\alpha|^2 = R < n \), for any value of \( < n \). The average loss of 1 photon leads to the MQS Visibility value: \( D = 0.996 \), and then to the practical cancellation of any detectable interference effects involving \( \bar{D}_{\psi^\pm} \). This is fully consistent with the experimental observations \[8, 9\]. The previous calculations can be generalized to the general coherent state MQS: \( |\psi^+_\phi\rangle = \Sigma_{\phi} \left( |\alpha e^{i\phi}\rangle \pm |\alpha e^{-i\phi}\rangle \right) \) leading respectively to the distinguishability: \( D(|\alpha e^{i\phi}\rangle,|\alpha e^{-i\phi}\rangle) = \sqrt{1 - e^{-2T|\alpha|^2 \sin^2 \phi}} \) and the MQS Visibility: \( D(|\psi^+_\phi\rangle,|\psi^-\phi\rangle) = \sqrt{1 - e^{-4T|\alpha|^2 \sin^2 \phi}} \). We obtain the previous results by replacing in \( D(x) \) the coordinate \( x \) by the rescaled quantity: \( x = R|\alpha|^2 \sin^2 \phi \). We can summarize the theoretical results for the MQS Visibility by tracing the unique function: \( D(|\psi^+_\phi\rangle,|\psi^-\phi\rangle) = D(x) \), shown in Fig. 2 (c). We consider this "universal" function an additional important property of the "coherent states", not previously discovered to the best of our knowledge. Note that the function \( D(x) \) approaches its minimum value with zero slope: \( \text{SL} = \lim_{R \to 1} |dD(x)/dx| = 0 \).

Quantum superposition by phase-covariant quantum cloning. The lossy channel method has been applied to the amplified single photon qubits by a collinear Q1-OPA. The interaction Hamiltonian of this process is: \( \hat{H}_{\text{coll}} = i\hbar \hat{\alpha}_H^\dagger \hat{\alpha}_H + \text{h.c.} \) in the \{ \( \hat{\pi}_H, \hat{\pi}_V \) \} polarization basis, and \( \hat{H}_{\text{coll}} = i\hbar e^{-i\phi} \left( \hat{\pi}_H^2 - e^{2i\phi} \hat{\pi}_V^2 \right) \) for any "equatorial" basis \{ \( \hat{\pi}_\phi, \hat{\pi}_\phi \perp \) \} on the Poincaré sphere where: \( \hat{\pi}_\phi = \sqrt{\frac{1}{2}} (\hat{\pi}_H + e^{i\phi} \hat{\pi}_V) \). Two relevant equatorial basis are \{ \( \hat{\pi}_+, \hat{\pi}_- \) \} and \{ \( \hat{\pi}_R, \hat{\pi}_L \) \} corresponding respectively to \( \phi = 0 \) and \( \phi = \pi/2 \). We remind that the phase-covariant cloning process amplifies identically all "equatorial" qubits. The symbols H and V refer to horizontal and vertical field polarizations, i.e. the extreme "poles" of the Poincaré sphere. By direct calculation, the amplified states for an injected qubit \( \pi = \{ H, V \} \) is:

\[
|\Phi^\mp\rangle = \frac{1}{C^2} \sum_{i=0}^{\infty} \Gamma^{i} \sqrt{i+1} |(i+1)\pi, i\pi\perp\rangle
\]

While for an injected equatorial qubit the amplified state is:

\[
|\Phi^\phi\rangle = \sum_{i,j=0}^{\infty} \gamma_{ij} |(2i+1)\phi, (2j)\phi\perp\rangle
\]

where \( \gamma_{ij} = \frac{1}{C^2} \left( \frac{R}{2} \right)^i \left( \frac{\sqrt{2} \sin \phi}{2} \right)^j \left( \sqrt{(2i+1)} \Gamma \right) \left( \sqrt{(2j)} \right) \). In these expressions \( C = \cosh g \) and \( \Gamma = \tanh g \), where g is the non linear gain of the amplifier.

Let us consider the MQS of the macrostates \( |\Phi^\pm\rangle \) and \( |\Phi^-\rangle \): \( |\Phi^\pm\rangle = \frac{\sqrt{2}}{2} (|\Phi^+\rangle \pm i|\Phi^-\rangle) \). Due to the linearity of the amplification process \[14\], it can be easily found that \( |\psi^\pm\rangle = |\Phi^{R/L}\rangle \). The distribution in the Fock space \( P(n_\phi, n_{\phi\perp}) \) corresponding to each "equatorial" macrostate \( |\Phi^\phi\rangle \), evaluated by Eq. (3) exhibits a comb structure similar to the one shown in Fig 2 (a). Indeed only terms with a specific parity, in particular with odd number of photons for \( \hat{\pi}_\phi \) polarization and even number of photons for its orthogonal \( \hat{\pi}_\phi \perp \) are non vanishing. Furthermore, the amplified \{ \( |\Phi^{R/L}\rangle \) \} states are characterized by a diagonal distribution.

In Fig 3 the effects of losses in the distribution of both equatorial and \{ \( |\Phi^{R/L}\rangle \) \} amplified states are shown for different values of \( T \). The peculiar features of these Fock space distributions are progressively smoothed by the effect of losses.
As a following step, we have evaluated numerically the distinguishability of \{|\Phi^{+}m\rangle\} through the distance \(D(|\Phi^{+}m\rangle,|\Phi^{+}m\rangle)\) between the states: Fig.4-(a). It is found that this property of \{|\Phi^{+}m\rangle\} coincides with the MQS Visibility of \(|\Psi^{+}\rangle\), in virtue of the phase-covariance of the process: \(D(|\Psi^{+}\rangle,|\Phi^{+}\rangle) = D(|\Phi^{R}\rangle,|\Phi^{L}\rangle) = D(|\Phi^{+}\rangle,|\Phi^{-}\rangle)\). The visibility of the MQS \{\{\Psi^{+}m\rangle\}\} has been evaluated numerically analyzing \(D(x)\) as a function of the average lost photons: \(x \equiv R < n >\). The results for different values of the gain are reported in Fig.4-(a). Note that for small values of \(x\) the decay of \(D(x)\) is far slower than for the coherent state case shown in Fig. 2-(e). Furthermore, after a common inflexion point at \(D \sim 0.5\) the function \(D(x)\) drops to zero for \(R = 1\) and then for: \(< n > \sim N\), the total number of particles in the primary beam. Very important, for large \(< n >\), i.e. \(R \rightarrow 1\) the slope of the functions \(D(x)\) increase fast towards a very large value: \(R \rightarrow 1: SL = \lim_{R \rightarrow 1} |dD(x)/dx| \approx \infty\). All this means that the MQS Visibility can be large even if the average number \(x\) of lost particles is close to the total number \(N\), i.e. for \(R \sim 1\). As seen, this behavior is opposite to the case of coherent states where the function \(D(x)\) approaches zero value with zero slope: Fig.2(e). We believe that this lucky and quite unexpected behavior is at the core of the high resilience to decoherence of our QI-OPA MQS solution. Note that this behavior was responsible for the well resolved interference pattern with visibility: \(V \approx 20\%\) obtained in absence of O-Filter (OF) by: [13].

**Orthogonality Filter** The demonstration of microscopic-macroscopic entanglement by adopting the O-Filter was reported in [11]. The POVM like technique implied by this device locally selects the events for which the difference between the photon numbers associated with two orthogonal polarizations \(|m-n\rangle > k\), i.e. larger than an adjustable threshold, \(k[17]\). By this method a sharper discrimination between the output states \(|\Phi^{k}\rangle e |\Phi^{\perp}\rangle\) can be achieved. The action of the OF can be formalized through the measurement operator \(P_{OS} = \sum_{m,n} |m+n,m-n\rangle \langle m+n,m-n|\), where the sum over \(m,n\) extends over the terms for which the above inequality holds. In Fig.4 the results of a numerical analysis carried out for \(g = 0.8\) and different values of \(k\) are reported. Note the the increase of the value of \(D(x)\), i.e. of the MQS Visibility, by increasing \(k\) and, again: \(SL = \lim_{R \rightarrow 1} |dD(x)/dx| \approx \infty\). Of course here the high interference visibility is achieved at the cost of a lower success probability, as expected.

The present work was intended to give a firm theoretical basis to the very high resilience to decoherence demonstrated in recent experiments by our QI-OPA generated Macroscopic Quantum Superposition. This novel MQS system is expected to play a relevant role in the future investigations on the Foundational structure of Quantum Mechanics. We acknowledge useful discussions with Chiara Vitelli. Work supported by PRIN 2005 of MIUR and INNESCO 2006 of CNISM.

[1] A. Einstein, et al., *Phys. Rev.* 47, 777 (1935).
[2] E. Schroedinger, *Naturwissenschaften* 23, 807 – 812, 823 – 828, 844 – 849 (1935).
[3] M.A. Nielsen and I.L.Chuang, *Quantum Information and Quantum Computation* (Cambridge University Press, 2000)
[4] W. Zurek, Physics Today, October 1991, pag.36; *Rev. Mod. Phys.* 75, 715 (2003); *Progr. Math. Phys.* 48, 1 (2007)
[5] W. Dur, et al., *Phys. Rev. Lett.* 89, 210402 (2002) ; W. Dur, et al., *Phys. Rev. Lett.* 92, 180403 (2004)
[6] T. Gorin, et al., *Phys. Rev. Lett.* 99, 240405 (2007)
[7] W.P. Schleich, Quantum Optics in Phase Space (Wiley, New York, 2001), Chaps. 11 and 16.
[8] M. Brune, et al., *Phys. Rev. A* 45, 5193 (1992)
[9] J.M. Raimond,et al., *Rev. Mod. Phys.* 73, 565 (2001)
[10] A. Ourjoumtsev, et al., *Science* 382, 83 (2006); A. Ourjoumtsev, et al., *Nature* 448, 784 (2007)
[11] F. De Martini, et al., *Phys. Rev. Lett.* 100, 253601 (2008).
[12] F. De Martini, *Phys. Rev. Lett.* 81, 2842 (1998); *Phys. Lett. A* 250, 15 (1998).
[13] F. De Martini et al., *Prog. Quant. Elect.* 29, 165 (2005).
[14] F. De Martini, et al., *Phys. Rev. Lett.* 95, 240401 (2005).
[15] F. Sciarrino, et al., *Phys. Rev. A* 72, 062313 (2005)
[16] F. De Martini, et al., [quant-ph/0804034]v1.
[17] E. Nagali, et al., *Phys. Rev. A* 76, 042126 (2007).
[18] R. Jozsa, *J. Mod. Opt.* 41, 2315 (1994)
[19] D. Bures, *Trans. Am. Math. Soc.* 135, 190 (1969)
[20] M. Hubner, *Phys. Lett. A* 163, 239 (1992); *Phys. Lett. A* 179, 226 (1993)
[21] B. Huttrer, et al., *Phys. Rev. A* 54, 3783 (1996)
[22] R. Loudon, *The Quantum Theory of Light.*
[23] W. Schleich, et al., *Phys. Rev. A* 44, 2172 (1991)
[24] The closed-form calculations leading to Eq.3 as well as similar expression for \(D(x)\) will be reported in a forthcoming paper.
[25] A. Peres, *Quantum Theory: Methods and Concepts* (Kluwer Academic Publishers, Dordrecht, 1995).