Estimation of local heat flux for turbulent flow in a helical coil tube by conjugate gradient method

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Abstract. Estimation of local heat flux is challenging in a helical coil tube heat exchanger due to the complex flow field developed by tube curvature. The heat flux has uneven distribution in the angular direction of the tube cross-section. The current research aims to estimate the local heat flux at the fluid-solid interface for the turbulent flow of water in a helical coil tube by solving the inverse heat conduction problem (IHCP). Conjugate gradient method (CGM) with an adjoint problem is used as an inverse algorithm. First, the commercial CFD software ANSYS FLUENT is used for solving the governing equations of continuity, momentum, and energy for turbulent flow to obtain the heat flux at the fluid-solid interface. This heat flux is used to determine the temperature distribution at the outer surface of the tube. The heat flux is then considered unknown and it is estimated by CGM algorithm with the developed in-house code in MATLAB. The result shows that the estimation of heat flux by CGM is very accurate.

1. Introduction
Helical coil heat exchangers are most widely used in process industries, food industries, refrigeration systems, power plants, etc. [1]. Figure 1(a) shows a typical helical coil tube with its terminology and Figure 1(b) shows the inner and outer tube wall for fluid-solid interface and corresponding angular position. The curvature of the tube leads to centrifugal force while the helix angle of the coil causes the torsional effect on fluid, which generates a secondary flow pattern in the fluid. The fluid near the outer side of the tube moves faster than the inner side of the tube and this uneven velocity field induces uneven heat transfer at the internal wall of the tube [3]. Dean (1963) studied the effect of curvature on flow field inside the coiled and show that the flow field is governed by the Dean number given by,

\[ \text{De} = \frac{\text{Re}\delta^{1/2}}{\mu} \]  

where \( \delta \) is called curvature ratio which is the ratio of tube diameter (2r) to coil diameter (2Rc). Re is Reynolds number given by

\[ \text{Re} = \frac{\rho u_{av}}{\mu} \]  

where \( \mu \) is fluid viscosity, \( u_{av} \) is average fluid velocity and \( \rho \) is the fluid density. Many researchers have estimated the local heat transfer coefficient in coil tube experimentally [3] [4] [5] and numerically for laminar as well as turbulent flow [6] [7] [8]. The literature survey indicates that the estimation of local heat transfer coefficient or heat flux is difficult. Among all methods, the inverse methods are found to be feasible and accurate enough. In inverse heat transfer problems (IHTP), the unknown quantity is estimated by temperature data through accessible distance [9] [10] [11].
Conjugate gradient method (CGM) with adjoint problem is the most widely used inverse method [9]. The current study includes the estimation of local heat flux inside the helical coil tube for turbulent flow of water at different Re by using CGM.

Figure 1. (a) Helical tube terminology (b) Angular position of inner and outer tube wall at fluid-surface interface (c) Computational mesh for numerical analysis (d) The variation of $\frac{Nu_\theta}{Nu_{max}}$ with angular position ($\theta$) for validation with experimental work

2. Methodology

The methodology includes two steps:

1. Solve continuity, momentum, and energy equations for turbulent flow heat transfer through helical coil tube by ANSYS FLUENT and obtain the peripheral distribution of local heat flux.
2. Formulate IHCP for two-dimensional heat conduction through the cross-section of the tube and estimate the heat flux with temperature data at the outer surface of the tube.

2.1. CFD software analysis of heat transfer through the helical coil.

The main aim of this analysis is to resolve the flow field induced by the curvature effect of the helical coil tube and get the local distribution of heat flux at the fluid-solid interface. The inner diameter of the coil tube ($2r$) is taken as 11 mm with 8 turns and a pitch of 60 mm. The coil dimensions are chosen in such a way that the flow becomes thermally fully developed flow at the outlet [5]. Finite volume mesh is generated with inflation mesh to resolve the boundary layer near the wall as shown in Fig. 1(c). The first layer thickness is chosen in such a way that non-dimensional wall distance $y+$ is less than 1 for SST- $k$-$\omega$ turbulence model [6]. The water ($Pr = 3.56$) is considered a working fluid. About 4202212 elements were found to be optimum after the grid independence test. The velocity inlet boundary condition is given at the inlet; the pressure outlet is given at the outlet. Figure 1(d) shows the variation of normalized Nusselt number ($\frac{Nu_\theta}{Nu_{max}}$) for present numerical work and experimental work by Bozzoli et al. [5] for Re = 10000. Both works have the same trend shown in the figure.

2.2. Estimation of heat flux by CGM with adjoint problem.

Once the distribution of local heat flux over circumference is obtained by CFD from the above step, it can be used to determine the temperature distribution on the outside surface of the tube wall. However, in actual experimentation, this temperature distribution can be measured by infrared thermography. Bozzoli et al. [5] used a well-calibrated infrared thermographic system in experimental work for measuring the temperature of known emissivity of coil surface. The temperature data at different locations were recorded first and then overall temperature distribution was constructed. The temperature distribution is used in the inverse algorithm for estimating the heat flux at the fluid-surface interface. The computational domain for IHCP is shown in Fig. 2(a). The governing equation for the direct problem is steady-state two-dimensional heat conduction with polar coordinates given by
with boundary conditions

\[ k \frac{\partial T}{\partial r} \left( \frac{r \partial T}{\partial r} \right) + k \frac{\partial^2 T}{r^2 \partial \theta^2} = 0 \]  

(3)

where \( T_{\text{env}} \) is ambient air temperature and \( h \) is overall heat transfer coefficient for tube outer wall. The heat flux boundary condition at inner wall (\( W_{\text{int}} \)) given by:

\[ -k \frac{\partial T}{\partial r} = q(\theta) \]

(3b)

Steel (\( k = 16.27 \text{ W/mK} \)) is chosen as a material with a tube thickness of 2 mm and the inner diameter is equivalent to the exit diameter of the coil (11 mm). A finite volume structured grid is used where the total number of divisions in radial directions (\( N_r \)) and angular directions (\( N_\theta \)) are 31. Both the direct problem as well as inverse problems are formulated and solved in MATLAB R2016b. The CGM is used for solving IHCP. It starts with an initial guess of unknown flux \( q(\theta) \) and then converges toward a solution by minimizing the least square-based objective function given by

\[ J(q(\theta)) = \int_0^{2\pi} \sum_{m=1}^{M} [T_{\text{est}}(\theta, r = r_{\text{ext}}) - T_m(\theta, r = r_{\text{ext}})]^2 d\theta \]  

(4)

where, \( T_{\text{est}} \) is estimated temperatures and \( T_m \) are measured temperatures at the external wall (\( W_{\text{ext}} \)) of tube and M are the number of sensors located at the outer circumference of the tube. For solving current IHCP by CGM, different auxiliary problems are required to be solved [9] as shown in Figure 2(b). The algorithm terminates as the objective function reaches stopping criteria.

3. Results

The heat flux distribution at the fluid-solid interface with the angular position is estimated for three different Reynolds numbers viz \( Re = 10000, 7000 \) and \( 14200 \) and \( Pr = 3.56 \). Figure 3(a) shows the convergence of an objective function \( J(q(\theta)) \) with iterations (G). The CGM will converge in the first
2-3 iterations very rapidly. Figure 3(b) shows the profiles of the actual (q) heat flux distribution obtained by CFD software analysis and heat flux estimated (qe) by CGM. The accuracy of prediction is measured by root mean square (RMS) error is given by,

$$\text{RMS} = \sqrt{\frac{1}{N_\theta} \sum_{n=1}^{N_\theta} [q(\theta) - qe(\theta)]^2}$$

where $N_\theta$ are the number of space steps in the angular direction. The CGM is predicting perfectly the flux profile for $Re = 10000$ (RMS = 0.1190 W/m$^2$) and $Re = 7000$ (RMS = 0.1184 W/m$^2$). For $Re = 14200$ the RMS error is 3.085 W/m$^2$ which is higher than the previous two cases but still, it is a good estimate.

4. Conclusions

The distribution of local heat flux at the exit of the helical coil is estimated by CGM with an adjoint problem. The continuity, momentum, and energy equation of fluid are solved by commercial CFD software ANSYS FLUENT and distribution of heat flux at coil exit is obtained which is used as an unknown quantity for IHCP. Water is chosen as a working fluid and estimation is done for three Reynolds numbers. Following are the main conclusions:

- CGM with an adjoint method is an accurate algorithm for the estimation of heat flux for turbulent flow in a helical coil tube.
- The distribution of local heat flux is uneven on the circumference of the tube. The value is highest at the outer wall (at $\theta = 0$ and 6.28 radians) and lowest at the inner wall (at $\theta = 3.14$ radians).

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