Can the Tajmar effect be explained using a modification of inertia?

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Abstract – The Tajmar effect is an unexplained acceleration observed by accelerometers and laser gyroscopes close to rotating supercooled rings. The observed ratio between the gyroscope and ring accelerations was $3 \pm 1.2 \times 10^{-8}$. Here, a new model for inertia which has been tested quite successfully on the Pioneer and flyby anomalies is applied to this problem. The model assumes that the inertia of the gyroscope is caused by Unruh radiation that appears as the ring and the fixed stars accelerate relative to it, and that this radiation is subject to a Hubble-scale Casimir effect. The model predicts that the sudden acceleration of the nearby ring causes a slight increase in the inertial mass of the gyroscope, and, to conserve momentum in the reference frame of the spinning Earth, the gyroscope rotates clockwise with an acceleration ratio of $1.78 \pm 0.25 \times 10^{-8}$ in agreement with the observed ratio. However, this model does not explain the parity violation seen in some of the gyroscope data. To test these ideas the Tajmar experiment (setup B) could be exactly reproduced in the Southern Hemisphere, since the model predicts that the anomalous acceleration should then be anticlockwise.

Introduction. – The Tajmar effect is a small unexplained acceleration observed in accelerometers and laser gyroscopes close to supercooled rotationally accelerated rings of niobium, aluminium, stainless steel and other materials [1–3]. The effect is similar to the Lense-Thirring effect (frame-dragging) predicted by General Relativity, but is 20 orders of magnitude larger. The effect has not yet been reproduced in another laboratory.

The authors of ref. [4] proposed an explanation for the anomaly that relies on a Higgs mechanism that causes the graviton to gain mass. This theory was called the gravitometric London effect, but it seems to have been discredited because the inception of the Tajmar effect does not coincide with the superconducting transition temperature, only with very low temperatures.

The model suggested here as an explanation for the effect was proposed by [5]. It assumes that the inertial mass of an object with respect to an attracting body is caused by Unruh radiation which is generated by the relative acceleration of the two bodies, and that this radiation is subject to a Hubble-scale Casimir effect (in which longer waves are increasingly disallowed). The model could be called Modified Inertia due to a Hubble-scale Casimir effect (MHiSCE) or perhaps Quantised Inertia, and was tested by [5] on the Pioneer anomaly (observed by [6]). In [5] the inertial mass ($m_I$) was defined as

$$m_I = m_g \left(1 - \frac{\beta n^2 c^2}{|a| \Theta}\right),$$

where $m_g$ is the gravitational mass, $\beta = 0.2$ (empirically derived by Wien as part of Wien’s law), $c$ is the speed of light, $\Theta$ is the Hubble diameter ($2.7 \times 10^{26}$ m, derived from [7]) and the magnitude of the acceleration ($a$) in this case was the acceleration of the Pioneer craft relative to their main attractor: the Sun. This model predicted a small loss of inertial mass for the Pioneer spacecraft that increased their Sunward acceleration by an amount close to the observed Pioneer anomaly (when the spacecraft were unbound beyond 10 AU from the Sun).

Reference [8] applied MHiSCE to the unexplained velocity jumps observed in Earth flybys of interplanetary probes (the flyby anomalies observed by [9]) and found

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that these anomalies could be reproduced quite well if the acceleration in eq. (1) was taken to be that of the spacecraft relative to all the particles of matter in the spinning Earth. Using MiHsC and the conservation of momentum the predicted anomalous jump for spacecraft passing by the spinning Earth was

\[ dv' = \frac{\beta \pi^2 v^2}{\Theta} \left( \frac{v_2}{a_2} - \frac{v_1}{a_1} \right), \]

where \( a_1 \) and \( a_2 \) were the average accelerations of all the matter in the spinning Earth seen from the point of view of the incoming \( (a_1) \) and outgoing \( (a_2) \) craft. This formula predicted the observed flyby anomalies quite well and was similar to the empirical formula suggested by [9].

The Tajmar effect is similar to the flyby anomalies, although instead of being an anomalous acceleration of a spacecraft close to a spinning planet, it is an anomalous acceleration observed in laser gyroscopes close to a super-cooled spinning ring. Therefore, in this paper MiHsC is applied to the Tajmar effect, but also, following Mach’s principle, in this paper the relative accelerations of the fixed stars are also considered.

**Method.** – The assumed experimental set up is that of [3] and their setup configuration B, and is shown in fig. 1, with a rotating super-cooled ring of radius \( r \). A laser gyroscope of mass \( m \) is located symmetrically above the ring. The fixed stars are also shown schematically. They have a huge combined mass, but are very far away. Assuming the conservation of the angular momentum of the gyroscope (subscript \( G \)) from one time to another (subscripts 1 and 2) in the reference frame of the fixed stars (subscript \( s \)) we have

\[ I_{G1}w_{Gs1} = I_{G2}w_{Gs2}, \]

where \( I \) is the moment of inertia \( (I = \Sigma mr^2) \) and \( w \) is the angular velocity \( (w = v/r) \). For a segment of the circular laser gyroscope (see “\( g \)” on fig. 1) this simplifies to the conservation of momentum parallel to the ring

\[ m_{g1}v_{gs1} = m_{g2}v_{gs2}. \]

Replacing the inertial masses with the modified inertia of [5] leaves

\[ v_{gs1} \left( 1 - \frac{\beta \pi^2 v^2}{a_{g1}} \right) = v_{gs2} \left( 1 - \frac{\beta \pi^2 v^2}{a_{g2}} \right) \]

and rearranging

\[ \frac{v_{gs2} - v_{gs1}}{v_{gs1}} = \frac{\beta \pi^2 v^2}{a_{g2}} \left( \frac{v_{gs2}}{a_{g2}} - \frac{v_{gs1}}{a_{g1}} \right). \]

Of course, this is similar to eq. (2), which was derived from MiHsC for the flyby anomalies. For this new case, the initial and final accelerations \((a_{g1} \) and \( a_{g2} \)) of the gyroscope with respect to all the surrounding masses now need to be defined. First we assume that because of cooling the temperature-dependent acceleration of nearby atoms is small. We can say that the acceleration relative to the fixed stars \((a_s) \) of an object fixed to the Earth at the latitude of Seibersdorf in Austria where the experiment was performed \((48^\circ N)\) is the same as the Coriolis acceleration: \(fv\), where \( f \approx 0.0001 \text{s}^{-1} \) in mid-latitudes, and \( v \), the spin velocity of the Earth at this latitude is 311 m/s, so \( a_s = 0.0311 \cdot v^2/2 \). To this should be added the acceleration due to the Earth’s orbit around the Sun \((so we have: a_s = (0.0311 + 0.006) \text{m/s}^2)\). The acceleration due to the Sun’s orbit around the Galaxy is far smaller and can be neglected. So in the above formula

\[ a_{g1} = 0.0371 \text{m/s}^2. \]

The sudden acceleration of the Tajmar ring causes an acceleration of \( a_R = 33 \text{rad/s}^2 = 2.5 \text{m/s}^2 \) (since the radial position of the gyroscope was 0.075 m). Therefore \( a_{g2} = fR(a_s, a_R) \). However, to find the average acceleration we have to consider the relative importance of the fixed stars and the ring for determining the inertia of the gyroscope. We will assume here that the importance of an object for the inertia of another one is equivalent to its gravitational importance, which is proportional to its mass over the distance squared. This details of this assumption do not affect the final result as we will see. Therefore \( a_{g2} \) is

\[ a_{g2} = \frac{m_R/a_s + m_R/a_R}{m_R/R^2}, \]

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where \( m_s \) is the mass of all the fixed stars, and \( r_s \) is their mean distance away and \( m_R \) is the mass of the ring and \( r_R \) is its distance away. Using eq. (7) in eq. (6) gives

\[
v_{gs2} - v_{gs1} = \frac{\beta \pi^2 c^2}{\Theta} \left( \frac{v_{gs2}}{0.96 a_s + 2200 a_R} - \frac{v_{gs1}}{a_s} \right)
\]

These values were approximated as a total stellar mass of \( m_s \sim 2.4 \times 10^{52} \) kg from [10], at a distance of \( r_s \sim 2.7 \times 10^{20} \) m \( (r_s = 2c/H) \), derived from the Hubble constant, \( H \), from [7]), and a ring mass of \( m_R \sim 0.336 \) kg (stainless steel has a density of about 8000 kg/m\(^3\), and the ring had a circumference of \( 2 \pi \times 0.075 \), a height of 0.015 m and a width of 0.006 m) and a ring distance of \( r_R \sim 0.039 \) m (the distance from the above-ring gyroscope to the nearest part of the ring). Using these values we have

\[
v_{gs2} - v_{gs1} = \frac{\beta \pi^2 c^2}{\Theta} \left( \frac{v_{gs2}}{0.39 a_s + 2200 a_R} - \frac{v_{gs1}}{a_s} \right).
\]

We can therefore neglect \( a_s \) in the denominator of the first term in brackets (this simplification similarly applies to the aluminium and niobium rings) to give

\[
v_{gs2} - v_{gs1} \sim \frac{\beta \pi^2 c^2}{\Theta} \frac{v_{gs2}}{a_R} - \frac{v_{gs1}}{a_s}.
\]

Since \( v_{gs2} \sim v_{gs1} \) and \( a_R \) is 2 orders of magnitude greater than \( a_s \) we can say

\[
dv' \sim \frac{\beta \pi^2 c^2}{\Theta} \frac{v_{gs1}}{a_s}.
\]

Differentiating to find the resulting acceleration

\[
d\alpha' \sim \frac{\beta \pi^2 c^2}{\Theta} \frac{a_{gs}}{a_s} \sim \frac{-6.6 \times 10^{-10}}{0.0371} a_{gs} \\
\sim -1.78 \pm 0.25 \times 10^{-8} a_{gs}.
\]

Since \( a_{gs} \) (the rotational acceleration of the gyroscope with respect to the fixed stars) is anticlockwise in the Northern Hemisphere because of the Earth’s spin, eq. (12) implies that the anomalous rotational acceleration of the gyroscope will be clockwise, as observed by [3] for setup B. The error of 0.25 was derived by assuming a 9% error in the Hubble constant (and therefore the estimated Hubble diameter) following [7]. The predicted acceleration is close to the ratio of \( da'/a_R \) that was observed by [3]. Using niobium, aluminium and stainless steel rings they observed a velocity coupling between the rings and the gyroscope (equal to the acceleration coupling factor since acceleration here is angular velocity times a constant radius) which increased from zero above the transition temperature at 25 K, to reach values of about \(-3 \pm 1.2 \times 10^{-8} \) at 5 K (see [3], figs. 3 and 6). This ratio agrees within error bars with the ratio predicted by eq. (12), but the parity violation ([3], fig. 2), only observed for their setup A, is not predicted. Rings of YBCO and Teflon produced similar results.

**Discussion.** – In the model used here (MiHsC) the inertial mass of the gyroscope is assumed to be determined by Unruh radiation that appears as it accelerates with respect to every other mass in the Universe. The Unruh radiation is also subject to a Hubble-scale Casimir effect. Before its surroundings are cooled the gyroscope sees large accelerations due to the vibration of nearby atoms, it is surrounded by Unruh radiation of short wavelengths, MiHsC has only a small effect, and the inertial mass of the gyroscope is close to its gravitational mass. The nearby atomic accelerations reduce when the surroundings (the cryostat) are cooled, so that the inertia of the gyroscope is more sensitive to the accelerations of the fixed stars (it is on the rotating Earth). This is a small acceleration, so the Unruh waves it sees are long and a greater proportion are disallowed by MiHsC, so the gyroscope’s inertial mass (from eq. (1)) falls very slightly below its gravitational mass. In this case it loses \( 2 \times 10^{-10} \) kg for every kilogram of mass. However, when the nearby ring accelerates, the gyroscope suddenly sees the higher accelerations of the ring, the Unruh waves shorten, fewer are disallowed, and its inertial mass increases again following eq. (1). The important point is that to conserve momentum the gyroscope must accelerate in the opposite direction to the Earth’s spin: clockwise.

This can be summarised by saying that if an object A changes its inertial mass by MiHsC because of changes in the relative acceleration of an object B, then A must accelerate to conserve the momentum of the joint system (A + B) in the reference frame of the fixed stars. A more accurate calculation of the predicted acceleration ratio using eq. (8) gives \(-1.8 \times 10^{-8} \) and only a slow decay with distance \( r_R \). If the velocity jump derived from eq. (8) is calculated for gyroscopes at various distances from the ring \( (r_R) \) starting at \( r_R = 0.016 \) m and increasing, then the new MiHsC effect decreases to half its original size at \( r_R = 56 \) m, i.e. 56 m away. However, this slow decrease only applies to a uniformly cold environment such as space, and does not take account of the warmer environment further away from the ring whose thermal accelerations would hide the effect of MiHsC. As discussed above (eq. (7)) the importance of the cold ring for the inertia of the accelerometer decreases inversely as the square of the distance, so, for a gyroscope, say, four times as far from the cold environment surrounding the ring as the above-ring gyroscope, the MiHsC effect due to the cold ring will be 16 times smaller, and far less detectable. It is interesting in this respect, that while the gyroscope was exposed on the outside to the cold liquid helium the anomalous signal did not decay with distance ([3], setup A) whereas when it was moved outside the cryostat the signal did decay quickly ([3], setup C).

One criticism of MiHsC is that it makes no mention of the accelerations of the atoms in spinning stars, for example. Why should the acceleration of the accelerometer with respect to the fixed stars be only 0.0371 m/s\(^2\) as assumed here, when the fixed stars are rotating with larger...
accelerations than this and contain atoms with very large accelerations?

**Tests.** – One way to test these ideas would be to reproduce setup B of the Tajmar experiment (see [3]) in the Southern Hemisphere, since MiHsC predicts that then the anomalous acceleration of the gyroscope should be anticlockwise instead of clockwise. This hemisphere-dependence may already have been observed by [2], who mentioned that the Earth’s rotation may therefore be somehow involved.

The size of \( a_s \) in eq. (11) could be reduced by performing the Tajmar experiment in a frame that rotates to follow the fixed stars, to counter the Earth’s rotation. A partial example of such a system would be a Foucault pendulum. The anomalous jump seen should then be larger. This test would be easier to achieve at the Earth’s poles.

The Tajmar experiment could be repeated, slowing the spin of the ring so that the ring’s acceleration relative to the gyroscope is close to that of the fixed stars (0.036 m/s\(^2\)) then according to eq. (10) the anomalous jump should disappear. For even lower accelerations (values of \( a_R \)) the anomalous effect should reverse. Equation (9) implies that for very small values of \( a_R \) the anomalous acceleration is predicted to be a maximum of about 229 times greater than the anomaly predicted by eq. (11). However, at these lower accelerations the acceleration ratio could be undetectable given the precision of the gyroscopes.

Since the relative acceleration of the fixed stars has a different value depending on latitude, the inertial mass of objects, especially supercooled objects, should vary slightly with latitude. This should be apparent as a change in weight when objects are moved latitudinally. A move towards the equator would decrease the apparent acceleration of the fixed stars, so according to MiHsC the inertial mass should decrease, the object would respond more actively to the Earth’s gravity (i.e.: it would not follow a straight line through space so dutifully) and so its gravitational mass would appear to increase. In a move towards the pole, the weight would seem to decrease.

**Conclusions.** – The anomalous clockwise accelerations observed by laser gyroscopes close to rotating supercooled rings (see [3], for setup B) can be predicted by a theory (MiHsC) that assumes that the inertial mass of the gyroscope is caused by Unruh radiation that appears because of its mutual acceleration with the fixed stars and the spinning ring, and that this radiation is subject to a Hubble-scale Casimir effect (MiHsC has already been shown to offer a reasonably successful explanation for the Pioneer and flyby anomalies.)

MiHsC does not explain the parity violations seen in the results of [3] using their setup A, in which the gyroscopes were positioned asymmetrically with respect to the spinning ring.

This explanation for the Tajmar effect could be tested by exactly reproducing the experiment of [3] (their setup B) in the Southern Hemisphere, since MiHsC predicts that then the anomalous acceleration of the gyroscopes should be anti-clockwise.

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