On the number of instabilities of cosmological solutions in an Einstein–Yang–Mills system

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Abstract

A detailed numerical stability analysis of the static, spherically symmetric globally regular solutions of the Einstein–Yang–Mills equations with a positive cosmological constant, \( \Lambda \), is carried out. It is found that the number of unstable modes in the even parity sector is \( n \) for solutions with \( n = 1, 2 \) nodes as \( \Lambda \) varies. The solution with \( n = 3 \) nodes exhibits a rather surprising behaviour in that the number of its unstable modes jumps from 3 to 1 as \( \Lambda \) crosses (from below) a critical value. In particular the topologically 3-sphere type solution with \( n = 3 \) nodes has only a single unstable mode.

As we have learned during the last few years, the coupling of nonabelian gauge fields to Einstein’s gravity leads to a rather large class of solutions of potential physical interest, such as “gravitating glueballs” of Bartnik–McKinnon [1], and “hairy black holes” [2–4]. These globally regular solutions are static, spherically symmetric, and they form a discrete family labelled by the number of nodes, \( n \), of the (single and purely “magnetic”) Yang–Mills amplitude. Furthermore, the above solutions have been generalized to a continuous family by allowing for a positive cosmological constant, \( \Lambda \) [5]. These “cosmological” solutions exist only for a limited range of the cosmological constant, \( 0 < \Lambda \leq \Lambda_{reg}(n) \), \( n \) being the node number of the Yang–Mills amplitude.

The important problem of stability has been studied mostly in linear perturbation theory, with the conclusion that all of these solutions are unstable [6–10]. The spectrum of small fluctuations can be divided with respect to parity into even (“gravitational”) and odd (“sphaleronic”) sectors. Within the most general spherically symmetric ansatz, mostly numerical results give convincing evidence that a gravitating glueball or black hole with \( n \) nodes possesses exactly \( n \) unstable modes in the even parity sector [6,7], and in Refs. [8,9] analytic arguments have been presented that there are also precisely \( n \) unstable modes in the odd parity sector.

The aim of this Letter is to point out that in the even parity sector the number of unstable modes of the cosmological solutions shows an unexpected dependence on the cosmological constant, \( \Lambda \), contrary to a previous claim [9]. Since the geometry of the cosmological solutions changes substantially for a cer-
tain value of \( \Lambda = \Lambda_{\text{crit}}(n) \), as then an equator develops, i.e., the surface area, \( 4\pi r^2 \), of the invariant 2-spheres becomes stationary for a certain value of \( r \) and then it starts to decrease. A careful numerical analysis reveals that the number of unstable modes in the even parity sector is 1 respectively 2 for solutions with \( n = 1 \) respectively \( n = 2 \) nodes independently of \( \Lambda \) as it varies from 0 to \( \Lambda_{\text{reg}}(n) \). The number of unstable modes of the \( n = 3 \) nodes solution exhibits, however, a rather surprising behaviour. As \( \Lambda \) tends to a certain value, \( \Lambda_{\text{merg}} \approx 0.2658 \), two eigenvalues of the unstable modes approach each other, actually merging at \( \Lambda_{\text{merg}} \) and then they simply disappear from the spectrum. In particular the globally regular, spatially compact and topologically 3-sphere-type solution (where \( \Lambda = \Lambda_{\text{reg}}(n) \)) with \( n = 3 \) zeros has only a single unstable mode. This is a somewhat unexpected phenomenon, since the linearized perturbation equations can be transformed to a standard 1-dimensional Schrödinger eigenvalue problem for the perturbation, \( \delta W \), of the Yang–Mills background for a particular choice of the coordinates (Schwarzschild gauge). The unstable modes are then nothing but bound states of the corresponding Schrödinger equation. Now for such a Schrödinger problem one does not expect that the values of bound-state energy levels could even coincide as a single parameter \( (\Lambda) \) is varied, let alone disappear from the spectrum without crossing the zero energy threshold. The Schwarzschild gauge is, however, singular when an equator is present, which is always the case for \( \Lambda \in [\Lambda_{\text{crit}}(n), \Lambda_{\text{reg}}(n)] \). In that case the potential in the associated Schrödinger equation becomes so singular that the standard theory of self adjoint operators is no longer applicable. It is quite remarkable that the number of unstable modes can actually change as the cosmological constant varies and it would be clearly desirable to achieve a better understanding of this phenomenon.

The most general spherically symmetric line element can be written as

\[
d s^2 = e^{2\nu(R,t)} dR^2 - e^{2\lambda(R,t)} dR^2 - r^2 (R,t) d\Omega^2, \tag{1}
\]

and the minimal spherically symmetric ansatz for the YM field is given by

\[
\Lambda = (T^1 W(R,t) d\theta + T^2 W(R,t) \sin \theta d\phi + T^3 \cos \theta d\phi), \tag{2}
\]

where the \( T^i \) \( (i = 1, 2, 3) \) denote an orthonormal set of Lie algebra generators of SU(2). For the above ansätze the (suitably rescaled) Einstein equations take the form:

\[
1 - U - 2V - e^{-2\lambda} \nu'(r^2) + e^{-\nu}(r^2) e^{-\nu} = 0, \tag{3a}
\]

\[
1 + U - 2V - e^{-\lambda} (e^{-\lambda}(r^2))' + e^{-2\nu} \lambda(r^2) = 0, \tag{3b}
\]

\[
e^{-\nu}(r'e^{-\nu})_r - e^{-\lambda} (r'e^{-\lambda})_r = -e^{-\nu-\lambda} \frac{\partial V}{\partial r}, \tag{3c}
\]

\[
r'_r + r' \lambda_r - r_t v' + \frac{2W_r W_t}{r} = 0, \tag{3d}
\]

with the Yang–Mills field equation for the amplitude \( W \):

\[
(e^{-\nu-\lambda} W')' - (e^{-\nu} W_t)_t - e^{-\nu+1} \frac{W(W^2 - 1)}{r^2} = 0, \tag{4}
\]

where

\[
V = \frac{(1 - W^2)^2}{2r^2} + \frac{\Lambda r^2}{2}, \quad U = e^{-2\nu} \left( r^2 - 2W^2 \right) + e^{-2\lambda} \left( r^2 - 2W^2 \right),
\]

and \( r_z := \partial r/\partial \tilde{t} \), while prime denotes \( \partial/\partial R \).

Following Ref. [11] the static field equations are conveniently written as

\[
\dot{\nu} = N, \tag{5a}
\]

\[
\dot{N} = (\kappa - N) \frac{N}{r} - \frac{2\tilde{W}^2}{r}, \tag{5b}
\]

\[
\dot{\kappa} = 1 - \kappa^2 + 2W^2 - 2\Lambda r^2 p^{-1}, \tag{5c}
\]

\[
\dot{\tilde{W}} = \frac{W^2 - 1}{r^2} - \frac{\kappa - N}{r} \frac{\tilde{W}}{r}, \tag{5d}
\]

where from now on \( \dot{f} := df/d\sigma := e^{-\lambda} f' \), and \( N, \kappa \) are defined as \( N := \dot{r}, \kappa := N + r \dot{v} \). The \((RR)\) component of the Einstein equations \((3a)\) yields the constraint:

\[
1 + N^2 + 2\tilde{W}^2 - 2V = 2\kappa N. \tag{6}
\]
Solutions of Eqs. (5) with a regular origin have the following power series expansion

\[ r = \sigma - \frac{1}{3}\left(2b^2 + \frac{A}{6}\right)\sigma^3 + O(\sigma^5). \]

\[ \kappa = 1 + \left(2b^2 - \frac{A}{2}\right)\sigma^2 + O(\sigma^4), \]

\[ v = \left(2b^2 - \frac{A}{6}\right)\sigma + O(\sigma^3), \]

\[ N = 1 - \left(2b^2 + \frac{A}{6}\right)\sigma^2 + O(\sigma^4), \]

\[ W = 1 - b\sigma^2 + O(\sigma^4), \quad (7) \]

where \( b \) is a free parameter. We consider solutions with a cosmological horizon, characterized in our notations by the vanishing of \( N(\sigma_h) \) and a simple pole of \( \kappa(\sigma) \) at \( \sigma = \sigma_h \). The corresponding series expansions of the functions near the cosmological horizon are given as:

\[ r = r_h + r_h^{(2)} x^2 + O(x^4), \]

\[ \kappa = -\frac{r_h}{x} + k_h^{(1)} x + O(x^3), \]

\[ v = v_h + \ln(x) + O(x^2), \]

\[ N = N_h^{(1)} x + O(x^2), \]

\[ W = W_h + W_h^{(2)} x^2 + O(x^4), \quad (8) \]

with \( x = \sigma_h - \sigma \), and

\[ r_h^{(2)} = -\frac{N_h^{(1)}}{2}, \]

\[ k_h^{(1)} = \frac{1}{3}\left(2Ar_h - \frac{1}{r_h} + \frac{N_h^{(1)}}{2}\right), \]

\[ N_h^{(1)} = \frac{1}{2r_h}\left(\frac{1 - W_h^2}{r_h^2} + Ar_h^2 - 1\right), \]

\[ W_h^{(2)} = -\frac{1}{4}W_h - \frac{1}{4}W_h^2, \quad (9) \]

where \( W_h, v_h \) and \( r_h \) are free parameters.

As found in Ref. [5] the cosmological solutions naturally fall into three qualitatively different classes as the cosmological constant varies from \( \Lambda = 0 \) to \( \Lambda = \Lambda_{reg}(n) \). All of these solutions may be indexed by the number of zeros, \( n \), of the Yang–Mills amplitude, \( W \).

For \( \Lambda = 0 \) the spacetime corresponding to the globally regular Bartnik–McKinnon solutions is asymptotically flat. As soon as \( \Lambda > 0 \) the globally regular cosmological solutions correspond to asymptotically de Sitter spacetimes (with a cosmological horizon), as long as the cosmological constant does not exceed a critical value, i.e., \( 0 < \Lambda < \Lambda_{crit}(n) \), constituting the first class. The geometry of the solutions changes when \( \Lambda \) exceeds \( \Lambda_{crit}(n) \), since then an equator develops outside of the cosmological horizon, i.e., the radius of the invariant 2-spheres, \( r(\sigma) \), reaches a maximum and then it decreases down to zero where a geometrical singularity develops. The equator is located outside of the cosmological horizon for \( \Lambda < \Lambda_{nreg}(n) \). As \( \Lambda \) further increases, \( \Lambda_{reg}(n) < \Lambda < \Lambda_{reg}(n) \), the position of the equator moves inside the horizon. Such solutions with an equator correspond to a singular geometry of the “bag of gold” type. Finally for a special value \( \Lambda = \Lambda_{reg}(n) \) the position of the horizon coincides with the singularity and then remarkably, there exists a globally regular solution corresponding to a spacetime whose spatial sections are compact. No regular solution seems to exist outside this range of the cosmological constant. For \( n = 1 \), this solution is analytically known [12]:

\[ \kappa = N = W = \cos(\sigma/\sqrt{2}), \]

\[ r = \sqrt{2}\sin(\sigma/\sqrt{2}), \quad \Lambda_{reg}(1) = 3/4, \quad (10) \]

corresponding to a static Einstein universe whose spatial section are 3-spheres, with a constant energy density due to the Yang–Mills fields.

To study the linear stability problem of the above solutions, one introduces time dependent perturbations of the form

\[ f(\sigma, t) = f(\sigma) + \delta f(\sigma) e^{i\omega t}, \quad (11) \]

defined where now \( f \) stands for any of the functions \( [r, N, \kappa, W] \), and one linearizes the field equations (3a–d) around a static background, \( f(\sigma) \). In this way one obtains the following set of linear differential equations for the perturbations:

\[ \delta \dot{r} = \frac{\kappa - N}{r} \frac{\delta r}{\delta r} - \frac{2\dot{W}}{r} \delta W + N \delta \kappa, \quad (12a) \]

\[ \delta \dot{\kappa} = \left(\dot{\kappa} - e^{-2\nu} \alpha^2 \right) \delta \lambda - 2 \frac{\kappa - N}{r} \delta \kappa + 4 \frac{\dot{W}}{r} \delta W \]

\[ - \left(4\Lambda + \frac{3\kappa}{r} + e^{-2\nu} \alpha^2 \right) \delta r, \quad (12b) \]

where \( N, W, \kappa, V, r \) are perturbed quantities with \( \delta N, \delta W, \delta \kappa, \delta V, \delta r \) being the corresponding perturbations.
δW = −κ − N \frac{\delta W}{r} + \left( \frac{3W^2 - 1}{r^2} - \frac{2W^2}{r^2} - e^{-2\nu} \omega^2 \right) \delta W

− 2 \frac{\dot{W}}{r} \delta r − \frac{\dot{W}}{r} \delta \kappa

+ \left( 2 \dot{W} + \frac{\kappa - N}{r} \dot{W} \right) \delta \lambda + \ddot{W} \delta \dot{\lambda}, \quad (12c)

with the linearized constraint (3a):

N\delta \kappa = \left( \dot{\kappa} - e^{-2\nu} \omega^2 r \right) \delta r

+ 2 \dot{W} \delta W - 2 \ddot{W} \delta W + 2 \dot{W}^2 \delta \lambda = 0. \quad (13)

We remark that Eq. (12a) is nothing but the (t, R) component of the linearized Einstein equations. The system (12) is invariant with respect to the following gauge (diffeomorphism) transformations:

δλ → δλ + \dot{F},

δκ → δκ + (δ − e^{-2\nu} \omega^2 r) F,

δr → δr + NF, \quad \delta W → δW + \dot{W} F, \quad (14)

where F is a function of the variable σ. As already mentioned, due to the presence of an equator the stability analysis of the cosmological solutions is more complicated than that of asymptotically flat ones. When there is no equator there exists a particular regular gauge—the Schwarzchild gauge—defined by δσ = 0, in which the gravitational degrees of freedom can be eliminated from the equation of the Yang–Mills perturbation amplitude in Eq. (12c). Indeed, in this gauge one obtains just a standard Schrödinger equation:

−e^\nu (e^\nu \dot{\delta W}) + V_S \delta W = \omega^2 \delta W, \quad (15)

where the potential, given by

V_S = e^{2\nu} \frac{3W^2 - 1}{r^2} + 4e^\nu \left( e^{\frac{\dot{W}^2}{r N}} \right),

becomes singular when the solution has an equator, V_S \propto \frac{1}{(\sigma - \sigma_{eq})^2}, due to the vanishing of N. The unstable mode of the analytically known 3-sphere background (10) corresponds to the following solution of the Schrödinger equation (15):

δW = \sin(\sigma/\sqrt{2}) \tan(\sigma/\sqrt{2}), \quad \omega^2 = -1. \quad (17)

Note that the solution (17) is not even square integrable because of its divergence at the equator \sigma_{eq} = \pi/\sqrt{2}. According to the classical theory of self-adjoint operators one should impose boundary conditions at the equator which would, however, just exclude the solution (17) (see Ref. [9] for a discussion on this point), so it seems that the correct interpretation of the singular Schrödinger equation (15) in the present context is beyond the standard theory. It is clear for physical reasons that one cannot impose boundary conditions for the perturbations at the equator, since the singularity there is an artifact due to the gauge choice.

On the other hand, one can simply choose a globally regular gauge where the equator is a regular point of Eqs. (12). The only disadvantage of such regular gauges is that one does not obtain a Schrödinger-type perturbation equation, but a somewhat unusual eigenvalue problem. It is then not easy to interpret the eigenfunctions and give a meaning to the number of nodes as in the case of a Schrödinger equation. Nevertheless we find it both conceptually clearer and also better suited for our numerical procedure to work in a globally regular gauge, which we have chosen to be δσ = 0.

The solutions of Eqs. (12) corresponding to locally regular perturbations admit the following power series expansions at the origin:

δr(σ) = \frac{4b}{3} \delta W_2 \sigma^3 + O(\sigma^5),

δκ(σ) = -4b \delta W_2 \sigma^2 + O(\sigma^4),

δW(σ) = \delta W_2 \sigma^2 + O(\sigma^4), \quad (18)

and near the horizon

δr(x) = \delta r_h x^a (x^2 + O(x^4)),

δκ(x) = \delta \kappa_h x^a (x + O(x^3)),

δW(x) = \delta W_h x^a (1 + O(x^2)), \quad (19)

with the following definitions

a^2 = -e^{-2\nu} \omega^2,

\delta r_h = - \frac{4W_h^{(2)}}{r_h(a + 1)} \delta W_h,

\delta \kappa_h = - \frac{(a - 1)^2}{(a + 3)} \delta r_h. \quad (20)
where $\delta W_2$, $\omega^2$ and $\delta W_h$ are free parameters. For the background (10) the unstable mode can be written in the $\delta \lambda = 0$ gauge as:

$$
\delta W = \delta \kappa = \langle \sigma / \sqrt{2} \rangle \sin(\sigma / \sqrt{2}),
$$

$$
\delta r = \sqrt{2} \sin(\sigma / \sqrt{2}) - \sigma \cos(\sigma / \sqrt{2}).
$$

We have numerically integrated Eqs. (12) by a fifth order adaptive step-size Runge–Kutta method from both the origin and the horizon subject to the boundary conditions (18) and (19) in a given static background solution, and by varying the free parameters we have matched the solution at an intermediate point (shooting to a fitting point).

Our main numerical results are presented on Fig. 1 ($n = 1, 2, 3$) and in Tables 1–4. The $A$-dependence of the unstable mode of the $n = 1$ background solution

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**Fig. 1.** The eigenvalues of the unstable modes plotted as functions of $A$ for the $n = 1, 2, 3$ solutions.

| $n$ | $1$ | $2$ | $3$ |
|-----|-----|-----|-----|
| $\omega^2$ | $-3.284026$ | $-3.283974$ | $-3.287678$ |
| $r_h$ | $\infty$ | $-0.983515$ | $-0.947872$ |
| $W_h$ | $10.0$ | $16.431260$ | $-0.904637$ |
| $b$ | $0.45371627$ | $53.924708$ | $-0.859362$ |
| $A$ | $0.001$ | $16.431260$ | $-0.947872$ |
| $\delta \lambda$ | $0.4538471$ | $-0.983515$ | $-0.947872$ |
| $\sigma$ | $16.431260$ | $-0.947872$ | $-0.947872$ |
| $\delta W_2$ | $0.4538471$ | $-0.983515$ | $-0.947872$ |
| $\delta W_h$ | $-0.983515$ | $-0.947872$ | $-0.947872$ |
| $\delta W_2$ | $-0.983515$ | $-0.947872$ | $-0.947872$ |
| $\delta W_h$ | $-0.983515$ | $-0.947872$ | $-0.947872$ |
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| $A$ | $0.001$ | $16.431260$ | $-0.904637$ |
| $\delta \lambda$ | $0.4538471$ | $-0.983515$ | $-0.904637$ |
| $\sigma$ | $16.431260$ | $-0.947872$ | $-1.649222$ |
| $\delta W_2$ | $0.4538471$ | $-0.983515$ | $-1.649222$ |
| $\delta W_h$ | $-0.983515$ | $-0.947872$ | $-1.649222$ |
| $\omega^2$ | $-3.284026$ | $-3.283974$ | $-3.287678$ |
| $r_h$ | $\infty$ | $-0.983515$ | $-0.947872$ |
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| $A$ | $0.001$ | $16.431260$ | $-0.904637$ |
| $\delta \lambda$ | $0.4538471$ | $-0.983515$ | $-0.904637$ |
| $\sigma$ | $16.431260$ | $-0.947872$ | $-1.649222$ |
| $\delta W_2$ | $0.4538471$ | $-0.983515$ | $-1.649222$ |
| $\delta W_h$ | $-0.983515$ | $-0.947872$ | $-1.649222$ |
| $\omega^2$ | $-3.284026$ | $-3.283974$ | $-3.287678$ |
| $r_h$ | $\infty$ | $-0.983515$ | $-0.947872$ |
| $W_h$ | $10.0$ | $16.431260$ | $-0.904637$ |
| $b$ | $0.45371627$ | $53.924708$ | $-0.859362$ |
| $A$ | $0.001$ | $16.431260$ | $-0.904637$ |
Schrödinger description—the highest energy bound state wave function in the $n$ eigenvalue unstable mode with Ref. [9]. The main new behaviour of the unstable displayed on Fig. 1 (n = 1) agrees with that of Ref. [9]. The main new behaviour of the unstable modes is exhibited on Fig. 1 (n = 3): the largest eigenvalue unstable mode with two nodes of the $n = 3$ node background solution—corresponding to the highest energy bound state wave function in the Schrödinger description—looses one of its nodes at $A_{\text{merg}} \approx 0.2657847$ where its eigenvalue coincides with that of the single node bound state. At $A_{\text{merg}}$ the coinciding eigenvalues are $\omega_2^2 = \omega_1^2 \approx -69$. For $A > A_{\text{merg}}$ these modes with nodes completely disappear from the spectrum, so only a single unstable mode without nodes (the lowest lying bound state) survives. It is worth noting that the lowest lying eigenvalues of the bound states for the $n = 2$ and in particular for the $n = 3$ node background solution exhibit quite large variations in a rather small $A$ interval close to $A = 0$. For example, the energy of the lowest lying bound state in the $n = 3$ background drops from $\approx -300$ to $\approx -190$ as $A$ varies from 0 to $\approx 0.02$. For backgrounds with $n > 3$ this phenomenon is getting more and more pronounced, making numerical integration quite difficult.

Preliminary investigations [13] suggest that in fact for all $n > 3$ node solutions there is such a $A_{\text{merg}}(n)$ where precisely two negative eigenvalues collide, leading to a diminution of the unstable modes by two. Therefore, it is natural to conjecture that the number of instabilities of the $n \geq 3$ node topologically 3-sphere-type solution in the even parity sector is in fact $n - 2$.

Finally for completeness we have also computed the eigenvalues of the unstable modes in the odd parity sector, in a regular gauge for solutions with $A = 0$ and for the topologically 3-sphere solutions. Omitting all computational details here, we just state that our results show that the number of unstable modes for both $A = 0$ and $A = A_{\text{reg}}(n)$ is $n$ (up to $n = 3$), in agreement with the analytical arguments of Ref. [9]. The numerical eigenvalues are provided in Table 4.

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Table 2
Numerical values of the parameters for some $n = 2$ solutions

| $A$   | $b$     | $r_h$ | $W_h$ | $\omega_1^2$ | $\omega_2^2$ |
|-------|---------|-------|-------|--------------|--------------|
| 0.0   | 0.65172553 | $\infty$ | 1.0   | -17.7898     | -93.9115     |
| 0.001 | 0.65357281 | 53.775678 | 0.837449 | -20.0993     | -100.6350    |
| 0.01  | 0.66188220 | 16.271892 | 0.560250 | -39.3607     | -153.0908    |
| 0.1   | 0.63599416 | 4.224264  | 0.332182 | -76.8804     | -222.8341    |
| 0.3   | 0.49283156 | 1.244954  | 0.647999 | -13.2317     | -17.9967     |
| 0.35  | 0.44326287 | 0.419923  | 0.938400 | -5.7734      | -9.5080      |
| 0.36424423 | 0.42959769 | 0          | 1.0   | -4.7046      | -8.0863      |

Table 3
Numerical values of the parameters for some $n = 3$ solutions

| $A$   | $b$     | $r_h$ | $W_h$ | $\omega_1^2$ | $\omega_2^2$ | $\omega_3^2$ |
|-------|---------|-------|-------|--------------|--------------|--------------|
| 0.0   | 0.69704005 | $\infty$ | -1.0  | -53.3132     | -390.5425    | -2946.8374   |
| 0.001 | 0.70174503 | 53.754661 | -0.321763 | -370.5305    | -1235.9470   | -1440.84     |
| 0.01  | 0.70073671 | 16.257493 | -0.119067 | -764.4160    | -9097.367    | -94777.96    |
| 0.1   | 0.66137928 | 4.206730  | -0.075888 | -493.7848    | -9938.45     | -89162.65    |
| 0.26578472 | 0.34488550 | 1.351120 | -0.444434 | -69.0027     | -69.0678     | -153.7337    |
| 0.29  | 0.51312783 | 0.347933  | -0.947303 | -           | -           | -35.7286     |
| 0.29321764 | 0.50882906 | 0      | -1.0  | -           | -           | -30.7787     |

Table 4
Eigenvalues of the unstable modes in the odd parity sector

| $n$ | $A = 0$ | $A = A_{\text{reg}}(n)$ |
|-----|---------|------------------------|
| 1   | -3.872828 | -1.0                   |
| 2   | -24.083467 | -3.665216              |
| 3   | -82.480704 | -4.619788              |
|     | -73.653692 | -7.938386              |
|     | -325.34471 | -9.190614              |
|     | -3005.4050 | -33.599200             |
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