A study on stochastic term of calorimetric energy resolution

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Abstract. The energy deposited in the active medium of the crystal fluctuates event by event. This fluctuations in the lateral shower containment contributes to the stochastic term. The lateral shower shape determines the distribution of the energy deposition in a cluster of crystals around the impact point. The contribution to the stochastic term coming from fluctuations in the lateral shower containment of calorimeter prototypes of PbWO₄ crystals have been simulated by GEANT4 for incident electrons at different energies.

1. Introduction

The particle detection using crystal–avalanche photodiode system is an important technique in research, medicine and industry. Also, for some selected applications, mainly in high-energy physics experiments. As an example, the CMS collaboration has chosen the PbWO₄ crystal for the electromagnetic calorimeter (ECAL) system. Compared to the other scintillating crystals, PbWO₄ crystal is an attractive detector material because of its high density and fast response. The drawbacks of this crystal are a high sensitivity to a temperature variation and poor light yield. Use of the PbWO₄ crystal for the CMS ECAL was given in reference [1] extensively.

2. Energy Resolution in Calorimeters

The Electromagnetic Calorimeter (ECAL) performance is expressed in terms of the energy resolution which is conventionally parameterized by the following quadratic sum,

$$\frac{\sigma(E)}{E} = \sqrt{a^2 + b^2 + c^2}$$

where denotes the square root of the quadratic sum, a is the stochastic term, b is the constant term, c is the noise term and E is the incident particle energy in GeV.

The stochastic term of the energy resolution for crystal-photodide combination is composed of a contribution from event to event fluctuations in the lateral shower containment (a_{lateral}) and a contribution from photo-electron statistics (a_{pe}) given in Eq.2.

$$a = \sqrt{a_{lateral}^2 + a_{pe}^2}$$

The lateral shower shape determines the distribution of the energy deposition in a cluster of crystals around the impact point. The energy deposited in the active medium of the crystal fluctuates event by event. In order to compare the lateral shower development, it is possible to use the ratio of the energy contained into a single crystal (E1) over the energy contained into a 3x3 (E9) and a 5x5 (E25) crystal matrix centred around the hit crystal [2].
3. Simulation and Results

The contribution to the stochastic term coming from energy deposition fluctuations in the lateral shower containment of calorimeter prototypes of PbWO$_4$ crystals have been simulated by GEANT4 [3] for 0.2-100 GeV electrons. Electrons were injected into the central of the crystals and energy deposition and its fluctuation in the crystals has been obtained for the single crystal 1x1 (E1), 3x3 (E9) and 5x5 (E25) crystals matrices. The simulated size of the crystal is same size used in CMS ECAL. The crystal has a length of 23 cm (25.8X0) and a truncated-pyramidal shape, resulting in a cross section that varied from 2.2x2.2 cm$^2$ on front side to 2.6x2.6 cm$^2$ to the rear side.

A simulation of the EM shower has been made for a 3x3 PbWO$_4$ crystal calorimeter. In the simulation the electrons at different energies were assumed to be injected in the center of the central crystal of the matrix. The energy of the incident particle is deposited in active medium of the crystals. Figure 1 shows the energy deposition spectra, at 1 GeV for example, obtained in the nine individual PbWO$_4$ crystals.

![Energy deposits in 3x3 PbWO$_4$ crystals for 1 GeV electrons injected into the center of the central crystal.](image)

**Figure 1.** Energy deposits in 3x3 PbWO$_4$ crystals for 1 GeV electrons injected into the center of the central crystal.

As a result of that around 78% of the energy of the incident electron was deposited in the central crystal and that the total deposited energy in the nine PbWO$_4$ crystals was around 93%. Thus, the ratio of simulated energy deposition in the central crystal to that in all nine crystals was obtained around 84%. By summing up all the energies deposited in the nine crystal blocks, the energy spectra for the incident electrons at different energies were obtained as shown in figure 2. In order to compare GEANT4 results with the previous simulation results obtained by EGS4 [4], the simulation was performed in the order from 0.2 GeV to 1 GeV (figure 3).
Figure 2. Energy spectra obtained by summing up all energy deposits in the nine crystals.

Figure 3. Energy resolution of the 3x3 crystal calorimeter for incident electrons of 0.2, 0.4, 0.6, 0.8, and 1.0 GeV.

The simulation has been repeated for the calorimeter comprises 25 crystals arranged in a 5x5 matrix. In the simulation energy depositions and their fluctuations have been obtained up to 100 GeV incident electron energies. It turned out that the total deposited energy in the 25 PbWO₄ crystals was around 96%. Figure 4 shows the variation of energy deposition in the central crystal 1x1 (E1) and in the matrix of 3x3 (E9) and 5x5 (E25) crystals.
As can be seen from figure 5, the deposited energy fraction was fairly independent of the incident energy in the relevant energy region.

As a result of this work, intrinsic energy resolution ($\Delta_{\text{t}}$) for the 1x1, 3x3, and 5x5 crystal matrices have been obtained. Figure 6 shows intrinsic energy resolution for PbWO$_4$ crystal matrices as a function of incident electron energy.
As can be seen from the variations, intrinsic energy resolutions decrease up to 60 GeV and then the resolutions are constant for all crystal matrices. When a particle with energy $E$ creates a signal, then the relative precision of the calorimetric measurement of the energy $\sigma_E/E$ will be related by the relative fluctuation in the photo-detector signal $\sigma_S/S$. Decrease in the intrinsic energy resolution, $\sigma_E/E$, correspond to a decrease in $\sigma_S/S$.

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