Multiple gaps in quantum Hall effect including review of prl 1998-2001 papers

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Abstract

The fractions of constant conductivity, \( i \), where the conductivity is \( ie^2/h \) are interpreted to arise from the summation of two frequencies, \( \omega_1 + \omega_2 \), type of processes so that the quantum Hall effect becomes a problem of spectroscopic energy levels. We describe the conversion of resistivity to the frequency spectrum. An effort is made to look at the recent articles from the Phys.Rev.Lett. 1998-2001 to see if there is any data which does not agree with our theory. The theoretical wave functions which claim to provide the theoretical base are found to be irrelevant to the experimental data.
1 Introduction.

Recently, Eisenstein et al [1] have observed the quantum Hall conductivity quantized at $\nu = 7/2$ and $5/2$. On high field side of 7.2 the fraction $10/3$ is cleanly visible. Similarly, $11/3$ is seen at low field side. As we have explained $10/3$ and $11/3$ are particle-hole Kramers conjugates [2]. The fractions $16/5$ and $19/5$ are also clearly visible which require a suitable explanation. It has been noted that there is a spin branching. We make an effort to understand these fractions and the spin branching. Some of the data is obtained by illuminating the sample with a red light emitting diode. After a finite time, the diode is switched off and then the resistivity is recorded in the Hall geometry. The effect of light is therefore to populate some of the excited states which affect the resistivity.

In this paper, we interpret the fractions $7/2$, $10/3$, $11/3$, $16/5$ and $19/5$ and their particle-hole symmetry and hence the spin branching. We also look at 1998-2001 issues of the Phys. Rev. Lett. to see if there are any contractions between our theory and the data. It is found that in all of the cases our theory is in agreement with the data.

2 Summation process.

In our theory the conductivity is quantized in fractions determined by two series [3]. One of the series is the same as half of the Lande’s $g$ value and the other series is obtained by reversing the sign of the spin in the Lande’s formula. Therefore, the theory is sufficiently simple, free of errors and exact. The resistivity is obtained from the spectrum. Therefore, the plateau width is related to the line width in the spectrum which arises from the many-body interactions. The half of the Lande’s $g$-value gives one series,

$$\nu_+ = \frac{l + 1}{2l + 1}$$  

and the other series obtained by using negative sign for the spin is

$$\nu_- = \frac{l}{2l + 1}.$$  

Usually for conduction electrons only $l = 0$ is taken. However we have to take many finite values of $l$ so that,
predicts all of the fractions. As the fractions give energies, \( n\nu_\pm \) are also possible, where \( n \) is an integer. We can identify \( n \) as the Landau level quantum number, \( n \). The values \( \nu_+ \) and \( \nu_- \) are Kramer’s conjugate states. Multiplying the above numbers by \( n \) gives all of the integers when we multiply 1 by \( n \) which is the case for \( \nu_+ \) with \( l = 0 \). This is of course the usual conduction band which gives the quantum Hall step at 1, for which the resistivity is \( \hbar/e^2 \). The \( l \to \infty \) limit of the two series is 1/2. When we multiply this value by \( n \) we get \( n/2 \). This value is symmetric in the sense that one series is on the high-field side of \( n/2 \) and the other series is on the left. Thus for any given value of \( n \) we have both the series and the patterns of a center with left and right sides is repeated for all other values of \( n \). All these predicted features are the same as found experimentally in Fig. 18 of Stormer [4]. Therefore, it is believed that the theory is correct. In fact Willett et al [5] have used our series but did not give reference to our paper. We will now explain how the above two series can be added to derive the experimentally observed values. We consider \( \omega_1 + \omega_2 \) type summation process so that \( \omega_1 + n\omega_2 \) type frequencies can also be observed. In order to generate 2 in the denominator we make use of \( n/2 \). Now we take \( \lambda \to \infty \) limit and multiply it by 7 which gives 7/2. The fraction 1/3 has negative spin from above. Therefore, we obtain, \( 3+(1/3)=10/3 \) which is observed on the right hand side of 7/2. The positive spin gives 2/3 for \( l = 1 \) in the \( \nu_+ \) series and hence we predict, \( 3+(2/3)=11/3 \) which is observed on the left hand side of 7/2. Therefore, 10/3 and 11/3 are the Kramers particle-hole conjugate states with spin reversed. This provides the explanation for 11/3, 7/2 and 10/3 in terms of values as well as the symmetries. The difference between 11/3 and 7/2 and that between 7/2 and 10/3 is exactly equal so that 7/2 is in the centre of 10/3 and 11/3. The fraction 10/3 uses 1/3 which has negative spin while 11/3 uses 2/3 which has positive spin. Therefore, we are able to identify the spin branch correctly as displayed in Fig. 1.
Let us now look at 16/5 which may be written as 3+(1/5). For \( l = 2 \) for \( \nu_+ \) we have 3/5 and for \( \nu_- \) we have 2/5. The difference between 3/5 and 2/5 is 1/5. For \( \nu = 3 \) we obtain \( \omega_3 \), for 3/5 we have \( \omega_{3/5} \) and for 2/5 we have \( \omega_{2/5} \). The difference \( \omega_{3/5} - \omega_{2/5} \) generates \( \omega_{1/5} \) and hence \( 3 + \frac{1}{5} \) is generated by \( \omega_3 + (\omega_{3/5} - \omega_{2/5}) \). In this case the spin is mixed and it is not generated from 7/2.

We consider the next fraction 19/5 at which there is a plateau in the transverse resistivity. This fraction may be written by taking \( \nu_- = 2/5 \) so that the spin is negative. For \( l = 2 \), we obtain this value which we multiply by \( n = 2 \) which means that we have a level at \( n = 2, l = 2 \) with spin negative so that we get \( n \times 2/5 = 4/5 \). We consider the frequency due to \( \nu = 3 \) and add \( \omega_3 + \omega_{4/5} \) to obtain \( 3 + 4/5 = 19/5 \). In this way we are able to assign the spin to 19/5 and also understand it in terms of a summation process. Thus our theory explains 16/5 as well as 19/5.

### 3. Angular momentum.

The theory given in ref. 3 thus gives the same values for the fractions as those experimentally observed by Stormer [4] and Willett et. al [5]. It gives the spin value and hence couples the spin with charge by use of Bohr magneton. It also describes the Kramers particle-hole symmetry. We have compared this theory with a lot of experimental data and found that in all cases the fractions tabulated in ref. 3 are the same as found in the experimental data [6]. The high Landau levels are easily explained by this theory [7]. In fact, the theory of ref. 3 works so well in terms of getting agreement with the experimental data that many of its features are described in a book [8]. The interpretation of high Landau levels 5/2, 7/3 and 8/3 has been obtained and 7/3 and 8/3 are found to be particle-hole conjugate states [9]. The relative change in resistivity is given by [10],

\[
\frac{\delta R_{xx}}{R_{xx}} = 4\pi \chi'' Q
\]

where \( \chi'' \) is the imaginary part of the susceptibility and \( Q \) is the quality factor. The imaginary part of the susceptibility resonates with the relaxation rate \( T_2 \) and resonance
frequency $\omega_o$ as

\[
\chi'' = \chi' - i\chi'' \quad \text{ (4)}
\]

\[
\chi'' = \frac{\chi_o}{2\omega_o T_2} \frac{1}{1 + (\omega - \omega_o)^2 T_2^2}. \quad \text{ (5)}
\]

As one goes from one level to another, the peaks of $\chi''$ appear as peaks in the resistivity. The frequency $\omega$ is obviously proportional to the magnetic field. In fact, the table of $\nu_\pm$ given above predicts a soft mode of zero frequency so that we expect a Goldstone mode to emerge when the magnetic length is equal to the separation between two layers of a semiconductor [11]. When the sample are first illuminated by a red diode and the resistivity is recorded after switching off the light, considerable changes occur in the populations of the excited states which affect the resistivity data. In some cases, this type of experiment results into improved resolution [12]. The theoretical work predicts that there is a small correction to the value of the Bohr magneton [13]. The polarization of the half-filled Landau level is also well explained [14].

4 Discussion and the list of 1998-2001 prl papers.

We have to consider if our theory agrees with Laughlin’s theory of fractional charge. Unfortunately, the problem depends on the flux quantization so that the field multiplied by an area is a constant,

\[
B.A = \frac{hc}{e},
\]

(6)

replacing $e$ by $(1/3)e$ is exactly equivalent to replacing area $A$ by $(1/3)A$. Therefore, it is not necessary to change $e$ to $(1/3)e$. This problem is hidden in Laughlin’s theory of an area operator. Since the product $eA$ occurs, Laughlin could have chosen the $(1/3)$ of the area rather than $(1/3)$ of the charge. Hence, Laughlin’s theory is of no relevance to the experimental data of quantum Hall effect [15].

There are non-Abelian theories [16-18] and topology [19] has been applied. In the Maxwell equations apparently it is possible to make a correction to the vector potential if a correction is also made to the scalar potential. These corrections are called Chern-Simon fields which shift the magnetic field [20-21]. Many problems related to charging have been
reported [22-31]. The scaling theory have been discussed [32-40]. There are discussions of spin-orbit interaction [41-42]. As the resistivity oscillates between low and high values, the high values are associated with the terminology of an insulator and metal-insulator transition has been discussed [43-53]. Detailed work on conductivity [34-61] with specific fractions, integers [62-65] as well as (1/2)-fractions [66-73] is available. A large amount of theoretical work has been published to search for the correct interpretation of data but there is not much success. In one case theory of fractals [74], is applied to the curves of the resistivity but this theory neither agrees with the data nor is relevant in any way but the effort goes on. In any case quantum Hall effect is not represented by fractals. In the composite fermion (CF) theory [75-96], “even number” of fluxes are attached to the electron, i.e., two flux quanta are attached to one electron to explain the plateau in the Hall resistivity at $\nu = 1/3$. The magnetic field becomes $B^* = B - 2\rho \phi_0$ where $\phi_0 = hc/e$ and $\rho$ is the charge density. The correct way to quantize the field $B$ is $BA = n\phi_0$ and not by $B^*$, where $A$ is an area and $n$ an integer. When we make a coil with a current passing through the coil, a magnetic field is produced. This is called the Biot and Shavart’s law in classical electrodynamics as shown in Fig. 2a. If a flux is attached to the electron it will affect the field and the Biot and Shavart’s law will be modified as shown in Fig. 2b. We also show the even number of fluxes attached to one electron in Fig. 2c. The idea of attaching the flux tubes to electrons was considered by Wilczek [97]. The experimental data does not show the “even number” quantization. As far as the statistics is concerned, we find [98] that the composite fermions actually have a component of bosons whereas the theory of “composite fermions” demands that there quasiparticles be fermions. It is clear that composite fermions do not obey the Biot and Shavart’s law. Some experimentalists have actually claimed to find agreement between the composite fermions and the data. These experimental claims are obviously deceptive or incorrect. The composite fermion theory is not a physically observable theory and it is of no relevance to the experimental data on quantum Hall effect. It should be made clear that fluxes are not attached to the electrons which is another way of saying that the composite fermions are not observed.

Several workers have measured [99-106] the nuclear magnetic resonance near the mag-
nentic field value where plateaus occur in the quantum Hall effect. These experiments are able to find the polarization from the Knight shift [14]. It is clear that the temperature at which the paramagnetic electrons become ferromagnetic is zero. Therefore, the ferromagnetic phase is not found. There are several theories [107-115] of ferromagnetic phase in the quantum Hall effect but these theoretical claims of finding ferromagnetic phase in the quantum Hall effect are not correct because the Curie-Weiss critical temperature is zero. There is a considerable effort to understand edges [116-128] and stripes [129-193]. A Goldstone mode has been found [134-135] experimentally at half-filled Landau level for which several theorists have constructed [136-150] their models but due to lack of good foundation their ideas are fragmentary and hence not applicable to the data. The correct theory of the Goldstone mode in accordance with the experimental data, is given by Shrivastava [11]. The half-filled Landau level presents some analogy with the Bose-Einstein condensation [151-158] and levitating states have been reported [154].

The spin and charge fluctuations in the fluid are subject to exchange interaction which can lead to a ferromagnetic state. There is a phase transition when the Lande’s $g$ value approaches zero. The quasiparticles in this limit are called “skyrmions” [155,156]. An effort has been made to suggest that “skyrmions” are important for the quantum Hall effect [157-163]. However, the experimental evidence is not in favour of exchange interaction which gives a Curie temperature. The effect of spin has been considered but the theories are not in accord with the data [164-172]. Some authors [173-177] are arguing in favour of spin-charge decoupling in which the fermions and bosons are exchanged so that the statistics is mixed in one dimension but there is no experimental evidence in favour of such a phenomenon. The low field high integer Landau levels have been discussed and considerable effort has been made to find the interpretation [178-181]. In this case the angular momentum theory is in agreement with the experimental data [7]. The problem of dimensionality has been discussed. Although the electron gas is two dimensional, the quantum Hall effect appears to require three dimensions [182-184]. Our theory [3] also does not use the dimensionality explicitly. The emission of phonons has been studied in the quantum Hall effect [185-186] and the relaxation is the result
of phonon emission and absorption. Light scattering experiments [187-188] gave the usual results and evidence was found to support fractionally charged quasiparticles. The experimental measurements of mass and $g$ values [189-190] show that some of the masses are equal. We have explained that this equality arises from the particle-hole symmetry [2]. There is emphasis on the anisotropy of the currents [191-192] but the polynomials which determine the anisotropy have not yet been determined. The tunneling current [193-194] magnetization [195-196] and the drag effects have been discussed [197-198]. There are some indications of rotations [199-200]. An accurate measurement of the Planck’s constant has been reported [201].

5 Conclusions.

There are a large variety of experimental measurements in the quantum Hall effect. In all the cases our theory of angular momentum first published in ref. 3 gives the correct interpretation of the experimental data. The apparent fractional charge is determined in terms of spin and orbital angular momenta. There is no need of a wave function of a fractionally charged quasiparticle as it is clear that fractionally charged quasiparticles do not occur in the quantum Hall effect.

6 References.

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Figure Captions

**Fig.1:** The transverse resistivity data as a function of magnetic field showing $7/2$ with $10/3$ with negative spin and $11/3$ with positive spin.

**Fig.2:** (a) The correct Biot and Shavert’s law, (b) one flux attached to one electron and (c) two fluxes attached to one electron. The cases (b) and (c) are the modifications called the “composite fermions” (CF). It is obvious and (b) and (c) are not consistent with (a).
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