Adding impurities or defects destroys crystalline order. Occasionally, however, extraordinary behaviour emerges that cannot be explained by perturbing the ordered state. One example is the Kondo effect, where magnetic impurities in metals drastically alter the temperature dependence of resistivity. In Type-II superconductors, disorder generally works to pin vortices, giving zero resistivity below a critical current $j_c$. However, peaks have been observed in the temperature and field dependences of $j_c$. This peak effect is difficult to explain in terms of an ordered Abrikosov vortex lattice. Here we test the widespread paradigm that an order-disorder transition of the vortex ensemble drives the peak effect. Using neutron scattering to probe the vortex order in superconducting vanadium, we uncover an order-disorder transition from a quasi-long-range-ordered phase to a vortex glass. The peak effect, however, is found to lie at higher fields and temperatures, in a region where thermal fluctuations of individual vortices become significant.
It has taken many decades to unravel the effect of weak disorder on the vortex lattice. In the 1970s, investigations initiated by the famous Russian theorist Anatoly Larkin found that any random disorder, no matter how weak, would destroy long-range order\(^1\). In Larkin’s perturbative approach\(^1,2\), vortex displacements grow linearly with vortex separation \(r\), as measured by the displacement correlation function \(b(r) = \left\langle \frac{u_j - u_i}{r} \right\rangle^2\), where \(u_j\) is the displacement of the \(j\)th vortex. This leads to an absence of divergent Bragg peaks. Yet resolution-limited Bragg peaks from vortex ensembles are clearly observed in scattering experiments\(^3,6\). It turns out the perturbative approach becomes invalid in in-plane spacings \(r = r_c\), where displacements become larger than \(\xi\), the characteristic scale of the disorder potential. The behaviour at spacings \(r < r_c\) is accordingly described as belonging to the Larkin or random-force regime. At larger scales, vortices compete for minima in the disorder potential, slowing the algebraic growth of displacements to \(b(r) \propto r^{-\zeta}\), where the roughness exponent \(\zeta < \frac{1}{2}\). In this so-called random manifold regime, theoretical values for \(\zeta\) can be derived from the elastic Hamiltonian by scaling estimates\(^7,8\), renormalisation group analysis\(^9\) or variational replica-symmetry breaking techniques\(^10,11\). A summary of theoretical values for \(\zeta\) is reproduced in Table 1.

At larger scales \(r > r_c\), where vortex displacements become of order \(a_0 = (2\Phi_0/\sqrt{3}b)^{\frac{1}{2}}\), the lattice spacing set by flux quantisation, the periodicity of the system becomes crucial\(^12\). Here the random manifold gives way to a Bragg glass regime. This asymptotic regime at \(r > r_c\) is characterised by a slower, logarithmic growth\(^13,14\) \(b(r) \propto \ln r\) and a translational order correlation function \(c_g(r) = \left\langle e^{\mu(u_i - u_j/r)} \right\rangle\) that decays algebraically with an exponent \(\eta_b\) (Table 1)\(^9,11\). The result is quasi-long-range order with algebraically diverging Bragg peaks and so the resulting vortex phase is referred to as the Bragg glass. Note that this term is used to describe both the asymptotic regime and, synergodochically, all three regimes in length scale. Where clarity is called for, we refer to the former as the Bragg glass regime and the latter as the Bragg glass picture.

The Bragg glass picture is expected to break down when distortions become important. Upon increasing field or disorder strength, a transition to a short-range ordered vortex glass phase is expected as the pinning energy exceeds the plastic deformation energy\(^14,15\). Notionally, disorder affects translational order more than orientational order, so the vortex glass is likely hexatic\(^17\). The orientational order is characterised by the correlation function \(g_\theta(r) = \left\langle e^{i\theta(u_i - u_j/r)} \right\rangle\), where \(\theta\) is the nearest neighbour bond angle\(^17,18\). Experimentally \(g_\theta(r)\) is observed to decay algebraically \(\propto r^{-\delta}\) in both the Bragg glass and vortex glass phases. A previous SANS study\(^19\) of the Bragg glass regime in niobium reported \(\eta_b = 0.07\). In the vortex glass, images of disordered vortex ensembles on the surfaces of \(\text{NbSe}_2\) and \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8\)\(^20,21\) yield \(\eta_b = 0.06-0.35\). These images also show a much faster, exponential decay of translational order, consistent with a hexatic vortex glass.

At temperatures close to the upper critical field \(B_{c2}(T)\) at which bulk superconductivity disappears, thermal fluctuations become increasingly important. They drive a proliferation of dislocations and a thermodynamic melting of the vortex lattice. The relative role of thermal fluctuations is quantified by the Ginzburg number\(^8\) \(G_i \approx (\mu_i k_B T)^{\frac{1}{2}}/2\xi^2 B_{c2}^2\). The thermal melting line \(B_m(T)\) of the vortex lattice can be estimated from the phenomenological Lindemann criterion \((\delta^2) = c_i^2(\delta_0^2)\) whereby melting occurs when displacements become a fraction \(c_i\) of the lattice spacing \(\delta_0\). Typically \(c_i = 0.1-0.2\). This gives\(^22-24\)

\[
(B_c(T) - B_m(T))/B_{c2}(T) \approx 0.43 c_i^{-\frac{1}{2}} G_i^{\frac{1}{2}}(1 - t^2)^{\frac{1}{2}}
\]

where the reduced temperature \(t = T/T_{c2}\). Since \(G_i \propto \xi^{-6}\), the position of the melting line is strongly dependent on the coherence length \(\xi\). For cuprate superconductors, \(\xi\) is a few nanometres, so \(G_i \approx 10^{-2}-10^{-3}\) and the melting line is expected 0.1–10 K (depending on the field) below \(B_{c2}(T)\). The expected position of the melting line is confirmed by experiments on the cuprates\(^25-27\). On the other hand, in low-\(\kappa\) superconductors the melting line is much more difficult to resolve\(^5,28\) due in part to the smallness of \(G_i\). For our vanadium sample \(c_i = 1.3\), \(\xi = 26\) nm (see Methods) and \(G_i \approx 6 \times 10^{-10}\), so \(B_m(T)\) is anticipated to lie very close to \(B_{c2}(T)\). For example, at 0.3 T and with \(c_i = 0.2\), \(B_m(T)\) is within 8 mK of \(B_{c2}(T)\).

Equation (1) is obtained with consideration of thermal fluctuations only. A Lindemann-like approach can also be used to predict the field and temperature dependence of the order-disorder transition line \(B_{du}(T)\) separating the Bragg glass and the vortex glass phases\(^23,29,30\). To do this, disorder-induced fluctuations must be considered. At \(B_{du}(T)\) the topologically ordered Bragg glass phase becomes unstable to the formation of

### Table 1 Properties of vortex-glass correlations in superconducting vanadium

| Sample conditions | Lattice constant | Correlation function | Orientational | Translational | Random manifold | Displacement crossover length scale |
|-------------------|------------------|----------------------|---------------|--------------|-----------------|-----------------------------------|
| \(T(K)\) | \(B(T)\) | \(a_0 (\text{nm})\) | \(g_c(r) \propto r^{-\eta_b}\) | \(c_g(r) \propto r^{-\eta_b}\) | \(c_g(r) \propto e^{-c_i \delta^2/(\delta_0^2)}\) | \(b(r) \propto r^{-\delta}\) | \(r_{\Delta}\) |
| 1.6 | 0.13 | 135 | 0.13 | 0.78 | 0.44 | 0.31 | 1600 |
| 1.6 | 0.18 | 115 | 0.20 | 0.93 | 0.60 | 0.20 | 1900 |
| 1.6 | 0.23 | 102 | 0.22 | 0.96 | 0.56 | 0.22 | 1800 |
| 2.7 | 0.17 | 119 | 0.07 | - | 0.42 | 0.20 | - |

*The topologically pristine Bragg glass phase is predicted by elastic theory with weak underlying disorder\(^11\). It has quasi-long-range positional order comprising of logarithmically growing displacements \(\propto \ln r\) and algebraically diverging translational correlations \(c_g(r) \propto r^{-\eta_b}\) at large length scales \(r \gg \delta_0\). At smaller scales \(r \approx \delta_0\) correlations show the behaviour of a random manifold with \(b(r) \propto r^{-\delta}\) and \(c_g(r) \propto e^{-c_i \delta^2/(\delta_0^2)}\). Our observed exponents on the Bragg glass phase are compared with predicted values from elastic theory\(^29\). A slight dependence of these exponents on the elastic modulus is expected\(^23\) and here we list values appropriate for our vanadium sample. The Bragg glass is expected to become unstable at higher fields or temperatures. As shown by the data at 2.7 K and 0.17 T, the asymptotic regime is the first to be suppressed as the vertex ensemble disorders.*
dislocations. The dislocation network is anticipated\textsuperscript{16} to appear at scales $\approx r_a$, where displacements are on the order of $a_0$. This can be shown to be equivalent to a generalised Lindemann criterion of the form $u^2(a_0,0) = c_l^2a_0^2$ where displacements $u(r, l)$ are now parameterised by the separations in and out of the vortex plane\textsuperscript{29,30}. Note that the order-disorder transition can be driven entirely by disorder-induced displacements. Consequently as thermal fluctuations vanish for $T \rightarrow 0$, $\Delta_j(T)$ remains distinct from and less than $B_{c2}(0)$ (Fig. 1), in contrast to the fluctuation-induced melting line $B_{m}(T)$, which curves up to meet $B_{c2}(0)$.

Here we use small-angle neutron scattering (SANS) to probe the long-range correlations of vortex ensembles in a vanadium single crystal. The SANS technique provided the first experimental evidence for the Bragg glass picture: a dependence of the diffracted peak intensity upon magnetic field $B$ that could not be explained by perfect crystalline order\textsuperscript{3}. We also find evidence from the field dependence of the diffracted intensity for a Bragg glass picture at intermediate fields in the phase diagram (Fig. 1). Furthermore we demonstrate the presence of a Bragg glass regime by characterising the shape of the diffraction peak in a high-resolution experimental set-up (Fig. 2) and using reverse Monte Carlo refinement to extract correlation functions from our data\textsuperscript{19}. Our manuscript proceeds as follows: first we characterise the underlying disorder and the peak effect using magnetometry data on our vanadium single crystal. We subsequently examine our SANS data collected over several experiments and using different experimental setups. These SANS experiments allow the order-disorder transition to be located. Comparing the order-disorder transition line $B_{\text{dis}}(T)$ determined from SANS with the critical current density $j_c$ determined by magnetometry, we discern no jump in $j_c$ around $B_{\text{dis}}$ in our sample. Instead, we observe a nascent peak effect at fields and temperatures close to $B_{c2}(T)$. This dissimilarity is at odds to the commonly held notion that the peak effect is underpinned by the order-disorder transition from Bragg glass to vortex glass.

\section*{Results}

\subsection*{Critical current from magnetometry.}

Our sample is a vanadium single crystal of cylindrical shape, with length $10$ mm and diameter $2R = 5$ mm. The [111] cubic crystal direction is coincident with the cylindrical axis. We measured the isothermal magnetisation $M$ as a function of magnetic field $H$ applied parallel to [111], collecting $M(H)$ curves at several temperatures. Figure 3a shows a typical $M(H)$ loop, measured at $T = 1.6$ K. Extracting the upper critical field from these loops, we find $B_{c2}(T)$ is well described by the empirical relationship\textsuperscript{31} $B_{c2}(T) = B_{c2}(0)/(1 - t^3/(1 + 1/4 t^4))$ with $B_{c2}(0) = 0.440$ T. Compared to pure vanadium where $T_c = 5.47$ K\textsuperscript{32}, the small increase in $B_{c2}(T)$ and the small suppression of $T_c = 5.31$ K measured for our sample indicate that the underlying disorder is weak. We obtain a mean free path $l = 48$ nm (see Methods). From the $M(H)$ loops, the critical current density $j_c = 3.5aM/2R$ is calculated using the critical-state model of Bean\textsuperscript{33}. $j_c$ is seen to be small $<10^7$ A m\textsuperscript{-2} (c.f. Fig. 3b) compared to the depairing current density $j_{nd} \approx H_c/\lambda \approx B_{c2}/(\sqrt{2}\kappa\lambda) \approx 4 \times 10^{12}$ A m\textsuperscript{-2}, consistent with weak pinning.

Underlying disorder constrains vortex lattices with a pinning force density $F_p$ of magnitude $j_B$. Close to $B_{c2}(T)$, thermal fluctuations enable vortices to ride over the pinning potential, and $F_p$ is accordingly expected to decrease at high fields and temperatures. It follows that there must be a maximum in $F_p$ as a function of $B$. Many models have been proposed to describe the field dependences of $F_p$ and $j_c$. Motivated by data on alloy superconductors with strong pinning, the first models estimated the average pinning force per vortex, accounting semi-empirically for the type and geometry of the pins\textsuperscript{34-37}. For isotropic superconductors like vanadium, these models\textsuperscript{37,38} place the maximum in $F_p(B)$ at fields $\approx 0.2R_B$. For our vanadium sample we find $F_p(B)$ does scale with $B_{c2}(T)$, but the maximum is located at 0.13$B_{c2}$, i.e. at lower fields than expected and the observed $j_c(B)$ dependence is not accurately reproduced by these semi-empirical models\textsuperscript{34-37}.

More recent models for $j_c(B)$ consider the contribution to pinning at different length scales\textsuperscript{8} and the dynamic effects of flux creep\textsuperscript{39}. These models give various $j_c$ behaviours depending on the regime of field and temperature. For example, the large bundle pinning regime is entered when pinning is weak and vortex length scales are large compared to the penetration depth $\lambda$. Here non-locality of the elastic constants can be neglected. In our low-$\kappa$ superconductor, $\lambda = 35$ nm is short. The minimum vortex lattice spacing $a_0 = 74$ nm, the value at $B_{c2}(0)$, so vortex lattice length scales generally exceed $\lambda$ and we expect to be in the large bundle regime in most of the mixed state. In this regime, $j_c$ is theoretically predicted to follow\textsuperscript{3}

$$j_c \approx j_0 e^{-6(\lambda a_0/l_c^2)}$$

where $l_c$ is the scale delimiting the Larkin regime parallel to the vortices. The $a_0^6$ dependence yields\textsuperscript{5,39} an algebraic decay of $j_c(B) \propto B^{-3}$. As shown in Fig. 3b, we do find $j_c(B)$ can be described by an algebraic decay for $0.1 T < B < 0.33 T$. With values of $j_0$ and $l_c$ for our sample, Eq. (2) gives $l_c \approx 10a_0$ over this field.
Fig. 2 Neutron scattering geometries. Small-angle neutron scattering (SANS) instruments are usually pinhole cameras with small apertures to tightly collimate the incoming neutron beam. The magnetic field profile presented by vortices in the sample diffracts neutrons. Rocking curves are collected by rotating the sample, field and vortices together through the Bragg condition. Two experimental geometries may be used, exploiting the significantly better instrument resolution in the direction probed by the rocking curve: (a) The applied magnetic field and vortices are orientated roughly parallel to the neutron beam. Here the rocking curve is most sensitive to correlations along the vortices. A typical image of the 2D SANS multidetector at the peak of the rocking curve of the right Bragg spot is shown. The rotation axis for this rocking curve is indicated by $\omega$. The unscattered neutron beam in the centre of the image at $q = 0$ is blocked by a beamstop. (b) The rocking curve probes correlations in the plane of the vortices when the applied magnetic field and vortices are orientated perpendicular to the neutron beam. The detector image shows the peak of the rocking curve of the right spot at 0.13 T.
The magnetisation loop. The sample has low fluctuations start depinning vortices. The dashed line is a fit to an algebraic decay \( j \propto B^{-1.7} \). Inset is a magnification of the region around \( B_{pp} \).

**Fig. 3** Magnetometry at 1.6 K. **a** M(H) loop for our vanadium single-crystal with \( H \parallel [111] \). **b** Critical current density \( j_c \) calculated from the width \( \Delta M \) of the magnetisation loop. \( j_c(B) \) decreases monotonically except for a small peak, labelled \( B_{pp} \), observed just before the sharp downturn where thermal fluctuations start depinning vortices. The dashed line is a fit to an algebraic decay \( j_c \propto B^\alpha \). Fitted value of \( \alpha = -1.7 \). Inset is a magnification of the region around \( B_{pp} \).

However the observed algebraic decay is slower than the \( B^{-3} \) predicted, with a reduced exponent \( j_c(B) \propto B^{1-\nu} \).

Perhaps it is not too surprising that no literature model describes our \( j_c \) data precisely. All of the theoretical and empirical models described focus on high-\( \kappa \) superconductors. Our sample has low \( \kappa = 1.3 \) and a large part of the superconducting phase diagram is occupied by a Meissner regime (Fig. 1). In what follows we concentrate our analysis at high \( B \), away from the Meissner regime.

For most of the mixed state, i.e. from \( B = 0.10 \) to 0.33 T at 1.6 K (Fig. 3b), the sample supports a finite critical current, indicating that vortices are pinned and frozen. At \( B = 0.34 \) T there is a sharp downturn in \( j_c \) and it falls rapidly up to \( B_{c2} \). This indicates vortices are depinning as thermal fluctuations take hold just below \( B_{c2} \). Just below the depinning region at \( B = 0.335 \) T, a nascent peak in \( j_c \) can be seen (Fig. 3b inset). This peak effect is less pronounced than in other reports, which is to be expected given the weak underlying disorder in our sample. The location of the peak effect is consistent with previous transport measurements and magnetometry on niobium and vanadium under neutron irradiation and with transport measurements on niobium and Nb-Ta alloys under plastic deformation. This systematic studies all show the same development of the peak effect: with increasing irradiation or deformation, the peak effect emerges first at high fields close to \( B_{c2} \) before developing into a larger peak that pushes to lower fields as the density of pinning centres increases. The reverse effect, where the peak effect is reduced and pushed back up to high fields, is also observed when samples are annealed following neutron irradiation.

**Order–disorder line from small-angle neutron scattering.** Following the seminal study by Larkin and Ovchinnikov on the collective pinning of vortex ensembles, we might expect the jump in \( j_c \) to be associated with a loss of vortex order. Indeed, peak effects have commonly been linked with order-disorder transitions of vortex lattices in the literature. To test this paradigm, we directly probe the vortex lattice order in our sample using SANS (Fig. 2).

Neutron scattering is sensitive to correlations between pairs of vortices via the structure factor \( S = \sum_{i<j} e^{-\lambda_i r_{ij}} \) where \( r_{ij} \) is the position of the \( j \)th vortex and \( q \) is the scattering vector. The measured scattered intensity as a function of \( q \) is the convolution of \( |h(q)|^2 S \) with the instrument resolution function, where \( h(q) \) is the form factor of a single vortex. For a perfect vortex lattice, the structure factor consists of \( \delta \)-function Bragg peaks at reciprocal lattice vectors \( \mathbf{G} \) with the first such vector appearing at \( g = 2\pi/\sqrt{3} \) in the case of a triangular lattice. Experimentally the Bragg peak intensity is quantified by integrating the measured scattering over three directions in reciprocal space. This is achieved by measuring rocking curves (Fig. 4a), where the sample, field and vortex lattice are rotated together through the Bragg condition. An area on the 2D SANS multidetector encompassing the Bragg spot is then summed and these summed counts, plotted versus rocking angle (Fig. 4a), are fitted to a Gaussian. The resulting integrated intensity \( I(B) \) of the first order Bragg peak is shown in Fig. 4b, for \( B = 1.6 \) K after field-cooling. In the vortex glass, rapidly decaying translational order \( \epsilon_q \) is expected that does not support Bragg peaks. Combined with the rapid fall-off of the form factor, this means little, if any, neutron scattering will be discernible from the vortex glass in our SANS experiments.

We see in Fig. 4b that at \( B = 1.6 \) K the integrated intensity \( I(B) \) begins a downturn before quickly becoming indiscernible from the background at 0.29 T. We may therefore locate the vortex order–disorder line \( B_{dis}(T) \) at 1.6 K, 0.29 T. Similarly we locate \( B_{dis}(T) \) from the field dependence of \( I(B) \) at two other temperatures (Fig. 5). All three \( B_{dis}(T) \) points identified (crosses in circles in Fig. 1) lie well below \( B_{c2}(T) \). \( B_{dis}(T) \) may also be determined by measuring the scattered intensity as a function of temperature at constant field. In Fig. 6 we show the scattering measured at the peak of the rocking curve upon warming at 0.13 T, starting from a well-ordered vortex ensemble at 2 K. These data also demonstrate the vanishing of neutron intensity well below the upper critical field and confirm that \( B_{dis}(T) \) lies deep in the mixed state.

**Bragg glass picture from integrated neutron intensity.** For a perfect crystalline lattice, integrating the structure factor \( S \) yields a factor \( 1/g^2 \) at small scattering angles, so the field dependence of \( I \) is described by \( |h(g)|^2/g \). For the Bragg glass picture, on the other hand, the factor \( 1/g^2 \) from integrating \( S \) depends on whether the SANS instrument resolution \( s \) is larger or smaller than the crossover scale between random manifold and Bragg glass regimes, at which displacements are of the order \( \sigma_0 \). For SANS measurements with the field and vortices aligned roughly parallel to the neutron beam, the width of the rocking curve probes correlations along the direction of the vortices (see Methods). Then if \( s < \lambda_{c} \) where \( \lambda_{c} \) is the crossover scale parallel to the vortices, integrating the structure factor in the Bragg glass picture...
yields a $1/g$, i.e. a $B^{-1/2}$ dependence similar to the perfect crystalline case. If $s > l_A$, the Bragg glass picture produces an additional factor $B^{-3/2}$. This hypothesis was born out by the $I(B)$ measured on (K,Ba)BiO$_3$ in the first experimental report of the Bragg glass\textsuperscript{3}. Similar $I(B)$ Bragg glass dependences have been reported in the electron-doped cuprate Nd$_{1.85}$Ce$_{0.15}$CuO$_4$ and in underdoped La$_{2-x}$Sr$_x$CuO$_4$ ($x < 0.15$)\textsuperscript{49}.

In Fig. 4b we compare the integrated intensity $I(B)$ measured at 1.6 K with the field dependence expected for a perfect crystalline lattice and that expected in the Bragg glass picture. For these measurements, the instrument resolution $s = 240a_0$ along the direction of the vortices (see Methods). Up to $B = 0.26$ T, i.e. before $I$ falls off more sharply as $B_{B0}$ is approached, it can be seen that the field dependence in the Bragg glass picture describes the data rather better than for the perfect crystalline case. We deduce that $l_A < s = 240a_0$ from our experiments. In elastic theory\textsuperscript{9}, length scales along the vortices and length scales in the vortex plane are coupled through the elastic moduli via $\sqrt{c_{44}/c_{66}}$. Approximate expressions for the tilt modulus $c_{44} \approx BH$ and for the shear modulus are\textsuperscript{50}

$$c_{66} \approx \frac{B\Phi_0}{16\pi^4\mu_0} \left(1 - \frac{B}{B_{c2}}\right)^2 \left(1 - \frac{1}{2k^2}\right)$$

(3)

The shear modulus $c_{66}$ softens close to the upper critical field line as vortex cores start to overlap. Deeper in the mixed state, e.g. for fields $0.1 < B < 0.33$ T, we find the calculated aspect ratio $\sqrt{c_{44}/c_{66}}$ lies in the range from 2 to 30. An upper bound for the crossover scale $r_A$ in the plane of the vortices can thus be determined e.g. at 0.23 T, this upper bound is $s/\sqrt{c_{44}/c_{66}} \approx 30a_0$.

In the next section, we see $r_A$ can be precisely quantified using a high-resolution SANS setup.

**Bragg glass regime from reverse Monte Carlo refinement.** To gain more detailed information about the Bragg glass and vortex glass phases in the sample, we employ a second experiment geometry in our SANS measurements. The SANS data in Fig. 4 are collected in the first experiment geometry where applied field and vortices are orientated roughly parallel to the incoming neutron beam (Fig. 2a). In the second experiment geometry, the applied field and vortices are orientated perpendicular to the beam (Fig. 2b). In this geometry, a high instrument resolution in the plane of the vortices is achieved (see Methods) and rocking curves, collected by rotating the sample, field and vortex ensemble together through the Bragg condition, probe correlations within the vortex plane, transverse to the reciprocal lattice vector (Fig. 7).

Such data (Fig. 7e) are seen to exhibit an algebraic decay with rotation angle $\omega$. This algebraic decay is characteristic of the Bragg glass regime\textsuperscript{11,19}. The observed exponent $\eta_r \approx 3$ predicted\textsuperscript{11} for the Bragg glass regime where $c_r(\omega)$ decays algebraically $\propto \omega^{-\eta_r}$, with $\eta_r \approx 1$ (Table 1).

To uncover more information about the vortex correlations, we use the reverse Monte Carlo (RMC) technique to refine the peak shapes of rocking curves collected in the perpendicular field geometry\textsuperscript{19}. Up to 220,000 vortices are simulated on a computer.

**Fig. 4** Neutron scattering in parallel field geometry at 1.6 K. a Rocking curves are collected by rotating the sample, field and vortices together through the Bragg condition. Neutron counts are summed over an area on the 2D multidetector encompassing the Bragg spot (red sector in Fig. 2a). Error bars are determined by Poisson statistics. Solid line is a Gaussian fit, with $W_0$ denoting the measured rocking curve width. b Field dependence, at $T = 1.6$ K, of the scattered neutron intensity $I$ integrated over the rocking curve. Vortex ensembles are prepared by field-cooling, i.e. by cooling in the desired field from the normal state. Dotted line is a guide to the eye illustrating the form expected if vortices were arranged in a perfect 2D lattice. A steeper descent arises from the quasi-long-range order of the Bragg glass picture and scales to the data well (solid line) up to 0.26 T, whereupon $I$ decays rapidly. Note that $I$ vanishes much before the peak effect $B_{pp}$, c $W_\parallel$ probes correlations along the vortices in the parallel field geometry (Fig. 2a). The dominant contribution to the instrument resolution is the angular spread $\alpha$ of the incoming beam, illustrated by the dashed line. $\alpha = 0.077^\circ$ corresponds to a maximum resolvable correlation length $s = 2.35/(\alpha a_0) = 240a_0$. d Radial width $W_r$ of the Bragg spot on the 2D neutron multidetector at the peak of the rocking curve (Fig. 2a) is sensitive to correlations along the vortices and to translational correlations in the vortex plane. Solid line depicts the calculated radial width under the assumption of infinite translational correlation length $1/\gamma$, i.e. it represents the contribution to $W_r$ from instrument resolution and from finite correlations along the vortices. The rapid departure from this line at 0.27 T indicates collapsing translational order as the order-disorder transition is approached.

\[|DO: 10.1038/s41467-018-03267-z | www.nature.com/naturecommunications\]
times are needed for the tails where the scattering is weak. We measured such datasets at four selected fields and temperatures in the phase diagram (Fig. 7).

Once the simulated rocking curve numerically reproduces the experimentally measured rocking curve (Fig. 7), correlation functions may be calculated from the simulated vortex positions. A simulated vortex ensemble is not unique: there are many possible simulated microstates that can reproduce the experimental peak shape. However, similar correlation functions are observed to result from such microstates. In Fig. 8 we show typical correlation functions extracted using RMC at each of the four points measured in the phase diagram. Looking first at the three datasets collected at \( T = 1.6 \) K and \( B = 0.13, 0.18 \) and 0.23 T, we see these show the same qualitative features, namely two distinct regimes in length scale. At small \( r \), the displacement correlation function \( b(r) \) is seen to increase algebraically while \( c_b(r) \) is seen to decay as a stretched exponential. This clearly represents a random manifold regime and accordingly here we fit the data to \( b(r) \propto r^{\beta} \) and \( c_b(r) \propto \exp(-r/\Lambda_0)^{\eta} \). The fitted values of roughness exponent \( g \) and exponential decay exponent \( \beta \) and the effective correlation length \( \Lambda_0 \) agree well with theoretical predictions (Table 1). The absence of an observable Larkin regime at small \( r \) is consistent with the weak disorder of our sample. Using the above estimate of \( l_c \approx 10a_0 \) from the critical current, the Larkin regime would be expected at in-plane length scales below \( r_c \approx l_c/\sqrt{c_6/c_{66}} \approx 2a_0 \), a value which is too small to be resolvable from the \( r \)-dependence of correlation functions.

At large \( r \), we identify a Bragg glass regime (Fig. 8). In this asymptotic regime, \( b(r) \propto \ln r \) and \( c_b(r) \propto r^{-\eta} \), with the fitted \( \eta \approx 1 \) (Table 1). The orientational order \( g(r) \) follows an algebraic decay \( \propto r^{-\eta} \) through both the random manifold and the Bragg glass regimes. The fitted exponent \( \eta \approx 2 \) (Table 1) is rather less than the translational exponent \( \eta_0 \) reflecting the longer range of bond orientational order compared to translational order. We conclude that a Bragg glass phase is present at fields and temperatures below the order-disorder transition \( B_{ds}(T) \).

Fracturing of the vortex lattice. We turn now to the fourth dataset collected with long counting times in the perpendicular geometry at 0.17 T and 2.7 K, a point which lies close to the...
order-disorder transition $B_{d0}(T)$ (c.f. Fig. 1). Here the rocking curve exhibits a distinctly different shape to the other three RMC datasets measured deeper in the Bragg glass phase. Its brow appears broader (Fig. 7d) and its tails seem less algebraic (Fig. 7e). The correlation functions extracted using RMC show a concomitant suppression of the asymptotic Bragg glass regime: the algebraic decay at large $r$ in $c_6(r)$ is no longer visible (Fig. 8c). Only a stretched exponential decay remains, with fitted exponents indicative of the random manifold regime (Table 1). $b(r)$ also shows the sublinear growth expected in this regime.

There is more intriguing behaviour at large $r$. Here, instead of the slow logarithmic growth that would signal a Bragg glass, $b(r)$ is seen to saturate at $\approx 30a_0$ (Fig. 8a). This suggests the vortex ensemble is fracturing into domains, with reasonable positional order being maintained within each domain and large jumps in displacement, with possible locally amorphous regions, at the domain walls. Indeed, a finite crystalline domain of dimension $\approx 30a_0$ would produce a Bragg peak of finite width $\approx 1.6^\circ$, which is roughly consistent with the broadening observed in the rocking curve (Fig. 7d). The orientational order $g_6(r)$ for this RMC dataset is reduced compared to the other three RMC datasets, but it continues to decay slowly, with a fitted algebraic exponent $\eta_6 = 0.07$. Moreover, $g_6(r)$ persists beyond the domain length scale $\approx 30a_0$, indicating that domains share roughly similar nearest neighbour directions. The fracturing observed of the Bragg glass is evocative of the prediction of a multidomain glass phase, that should separate the Bragg glass and vortex liquid phases. There are differences between prediction and experiment, however. The predicted multidomain glass also has both orientational and translational order decaying rapidly beyond the domain length scale, but here we see the orientational order $g_6(r)$ survives to larger length scales even in the fractured vortex ensemble. Interestingly, qualitatively similar correlation functions were measured by decoration experiments on the high-$T_c$ layered cuprate $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ (BSCCO). The formation of domains

Fig. 7 Neutron scattering in perpendicular field geometry. In the perpendicular field geometry (Fig. 2b), rocking curves probe correlations in the plane of the vortices. a-d Rocking curves of the first order diffraction peak at four selected points in the phase diagram, collected with long counting times to afford high quality data for reverse Monte Carlo refinement. Backgrounds measured at zero field have been subtracted. Ordinate axes are normalised for comparison. In reverse Monte Carlo refinement, simulated vortices are displaced using a Monte Carlo method until the experimental rocking curve (circles) is reproduced (red lines). Black lines indicate instrument resolution. e Rocking curves of a and d on logarithmic axes. Dashed line is a guide showing an algebraic decay. Algebraic tails are expected from the Bragg glass picture. Gaussian fits (blue lines) are drawn for comparison.
was proposed to be a nonequilibrium effect due to finite cooling rates\textsuperscript{51}. We have initiated further SANS explorations to clarify this.

**Discussion**

The peak effect and other features in magnetometry and transport data are frequently assumed to be underpinned by changes in the positional order of the vortex lattice. Under this assumption, conclusions are sometimes drawn as to the nature of vortex phases without directly probing them. Several features can be identified in magnetometry or transport data, including the irreversibility line ($H_{irr}$), the peak field ($H_{pp}$) and the onset of the peak effect regime ($H_{on}$). In the high-$T_c$ cuprates, a broad peak in $\Delta M$ may also be present at low temperatures and fields deep in the mixed state and is accordingly designated the fishtail effect or second magnetisation peak\textsuperscript{52}. There are reports of both a sharp peak effect close to $B_{c2}$ and a broad second magnetisation peak well below $B_{c2}$ being observed in the same sample\textsuperscript{53,54} indicating that these two effects are distinct. Accordingly, we should add to our list of features the onset field ($H_{on}$), the fishtail peak field ($H_{fp}$) and the field at which the magnetisation shows a kink ($H_{kk}$).

On the other hand, the route from ordered vortex ensemble to vortex liquid is envisaged to occur in only one or two sharp steps, i.e. a direct melting from Bragg glass to vortex liquid, or an order-disorder transition to a vortex glass followed by melting to the liquid. Recent scanning tunnelling microscope (STM) experiments on Co-doped NbSe\textsubscript{2} indicate that there may be two vortex glass phases—a hexatic vortex glass and an amorphous vortex glass\textsuperscript{55}—though it is not clear whether the two phases identified are truly distinct. Nonetheless, there are far fewer disordering transitions than there are features in $\Delta M$ and $f_c$. It is an open question as to which of these features, if any, should be tied to an underlying order-disorder transition in the vortex ensemble. The irreversibility line may reasonably be associated with the melting transition\textsuperscript{25}, since a vortex liquid phase cannot ordinarily support a finite critical current. To which feature, on the other hand, should we associate the Bragg glass to vortex glass transition?

To date only a handful of studies have directly probed the order of the vortex ensemble and simultaneously pinpointed features in magnetisation or critical current. The first such study found the SANS intensity disappeared at the irreversibility line in BSCCO\textsuperscript{25}. This occurred at temperatures consistent with estimates of the melting line using the Ginzburg number and the Lindemann criterion, signalling a link between vortex lattice melting and $H_{irr}$. In addition, the diffracted neutron intensity was found to vanish abruptly with increasing fields at temperatures well below the irreversibility line, with no reported corresponding features in the bulk magnetisation\textsuperscript{25}. A subsequent muon spin rotation study indicated that this vanishing was due to a 3D to 2D crossover, where the pancake vortices in BSCCO, which arise due to its highly anisotropic layered structure, become decoupled between the superconducting layers\textsuperscript{56}.

Subsequent studies have focussed on the quasi-two-dimensional layered 2H-NbSe\textsubscript{2} system. Here, due to coexisting charge order, the vortex cores are strongly anisotropic in the plane, adopting a sixfold star shape\textsuperscript{57,58}. Magnetic impurities in 2H-NbSe\textsubscript{2} are adorned by bound states with the same sixfold star shape\textsuperscript{59} which, we suggest, may drive the additional transition from hexatic glass to an amorphous vortex glass observed recently in Co-doped NbSe\textsubscript{2} samples\textsuperscript{55}. Interestingly this transition appears to coincide with the peak field $H_{pp}(T)$ in these samples\textsuperscript{55}, indicating that the Bragg glass to hexatic glass transition should lie at lower fields and temperatures, e.g. perhaps at the onset $H_{on}(T)$. This scenario is consistent with a recent SANS study\textsuperscript{60} on clean NbSe\textsubscript{2} where the SANS intensity all but disappears at $H_{on}(T)$.

Unfortunately the experimental picture is neither so simple nor general. In Fe-doped NbSe\textsubscript{2}, a well-ordered vortex lattice can be observed by SANS at fields much higher than $H_{pp}$ leading to the conclusion that the peak effect is unrelated to a bulk order–disorder transition\textsuperscript{60}. This conclusion is shared by an earlier decoration study on clean and Fe-doped NbSe\textsubscript{2}\textsuperscript{61}. The picture is muddied further by studies on isotropic conventional superconductors, like niobium, vanadium and V\textsubscript{3}Si, where vortices have the usual line structure. In V\textsubscript{3}Si\textsuperscript{62,63}, the diffracted SANS intensity disappears at $H_{pp}(T)$ but in Nb there are conflicting reports: in one SANS study $H_{pp}(T)$ appears to coincide with the loss of in-plane positional order\textsuperscript{63} but in another study a clear SANS signal indicating good vortex order is observed above the peak effect\textsuperscript{4}. Together with our observation of a Bragg glass to vortex glass transition lying at much lower fields and temperatures than $H_{pp}(T)$, we are drawn to the inevitable conclusion that

**Fig. 8** In-plane correlations educed by reverse Monte Carlo refinement. Correlation functions calculated from simulated vortex ensembles (open symbols) reproduce the neutron scattering data in Fig. 7. For clarity curves are offset with a multiplication factor $Y$. (a) Displacement correlator $b(r) = \langle \mathbf{u} \cdot \mathbf{u} \rangle^2$. (b) Hexatic orientational order $g_0(r) = \langle e^{i \mathbf{b} \cdot \mathbf{r}} \rangle$ is fitted with a single algebraic decay $\propto r^{-\alpha}$. (c) Translational order $c_2(r) = \langle e^{i \theta} \rangle$ and $b(r)$ comprise three regimes in the Bragg glass picture: the Larkin, random manifold and Bragg glass regimes (see text). Here the Larkin regime lies at indiscernibly small length scales and two-part functions fit $b(r)$ and $c_2(r)$ (solid lines) for the three datasets at 1.6 K. For the 0.17 T, 2.7 K dataset, which lies close to the order-disorder line, a single function fits $c_2(r)$ corresponding to the random manifold regime.
the origin of the order–disorder transition and the origin of the peak effect are not a priori the same.

Recent experimental investigations also show that metastability of the vortex configuration and the associated dependences on history affect the vortex order and consequently the perceived position of the order–disorder transition \[T_{\text{fi}} \rightarrow T_{\text{fl}}\]. Bulk characteristics from transport \[\sigma_0(B)\] and magnetic susceptibility \[\chi(B)\] are also similarly affected. Here we note that our data point for \(B_{\text{diw}}(T)\) at 0.13(T) and 3.74 K (Fig. 6) is obtained by warming the sample at constant field, so it is possible that the vortex configuration is superheated at the perceived order–disorder transition temperature of 3.74 K. If this were the case however, the equilibrium order–disorder transition would lie at even lower temperatures than we report here. Our conclusion, that the order–disorder transition and the peak effect are not necessarily related, would abide.

What are the origins of the order–disorder transition and of the peak effect? One possibility is a change in the nature of the underlying pinning such as a crossover from weak collective to strong pinning \[\lambda = \text{strong}\] for weak pinning, the pinning force density underlying pinning such as a crossover from weak collective to strong pinning \[\lambda = \text{strong}\]. This instability, localised to the cores, is not significant of 3.74 K. If this were the case however, the equilibrium order–disorder transition would lie at even lower temperatures than we report here. Our conclusion, that the order–disorder transition and the peak effect are not necessarily related, would abide.

For weak pinning, the pinning force density \(F_p = j_B = \int [F_p(\lambda) / V] dV / V\), where \(F_p\) is the elementary pinning force and \(N_p = 1/2\ell^2\) is the density of pins, \(V_c = l_r^2 \ell\) is the correlation volume over which displacements reach the superconducting coherence length \(\xi\) following the random force model. Strong and/or single pinning is identified when \(F_p\) overcomes the Labusch force \[\xi_l \approx \Delta_{\text{fl}} / 4\pi\mu_0 \lambda^2 \approx 10^{-11} \text{ N}\] for our sample. Since our data do not provide a direct measure of \(l_r\) or \(\xi\), we make an overestimate of \(V_c \approx \sqrt{\xi_{\text{fl}} / \xi_{\text{fl}} r_c^2}\) to yield an upper bound for \(F_p\) which is maximal at low inductions, reaching \(\sim 10^{-13} \text{ N}\). Thus throughout the mixed state \(F_p \ll \xi_l\) and weak collective pinning is effective. There is no change in pinning regime at \(B_{\text{diw}}(T)\) or at the peak effect in our sample.

We return to the effect of thermal fluctuations. These allow the vortices to ride over the pinning potential and result in a collapse of the critical current. The onset of fluctuations in our sample is marked by the sharp downturn in \(j_c\) close to the upper critical field. At \(T = 1.6\) K, this depinning of vortices occurs at \(B = 0.34\) T (Fig. 3b). At 0.3 T, the depinning line lies \(\approx 140\) mK below \(B_{\text{fi}}(T)\), substantially below the thermally driven vortex lattice melting field. We infer that continuous translational symmetry is broken at the Bragg glass to vortex glass transition. It is still a matter of debate, however, as to which symmetry—if any—is broken between the depinned vortex liquid and pinned vortex glass phases. We expect an exciting era in vortex matter physics, where intriguing possibilities such as the vortex glass being no more than a pinned, hexatic liquid are experimentally explored using increasingly available high-quality STM apparatus.

Overall, our data show that the peak effect and similar features in magnetometry or transport data may not a priori be due to a vortex order–disorder transition. SANS provides a direct probe of vortex order and reveals the order–disorder transition \(B_{\text{diw}}(T)\) in our sample. It is mediated only by the weak underlying disorder, lying deep in the mixed state, far from the regime dominated by thermal fluctuations. A jump in \(j_c\) around \(B_{\text{diw}}\) might be expected following the theory of Larkin and Ovchinnikov, \[\xi_{\text{fl}}\] cannot be detected in our \(j_c\) data derived from magnetometry (Fig. 3b). In contrast, these data show a nascent peak effect at high temperatures and fields, where thermal fluctuations become apparent.

Methods

Laboratory characterisation. Bulk magnetic measurements were carried out using a high-field cryogen free measurement system (CFMS) at the DTU Risø Campus. The field was applied parallel to the [111] crystal axis. We used the Goodman–Gor’kov relations \[\xi_{\text{fl}}\] to compare the upper critical field \(B_{\text{ci}}(T)\) and superconducting critical temperature \(T_c\) of our sample to the values reported by ref.\[32\] for varying sample purity. We calculate a mean free path \(l = 48\) nm, impurity parameter \(a = 0.84\), superconducting coherence length \(\xi_{\text{fl}} = 26\) nm and London penetration depth \(\lambda(0) = 35\) nm implying \(\chi = 1.3\) for our sample.

Neutron scattering. SANS experiments were performed on three instruments: D22 at the Institut Laue-Langevin (Fig. 4), NG7 at the NIST Center for Neutron Research (Fig. 6) and, SANS-II at the Swiss Spallation Neutron Source (Figs. 5, 7 and 8). In a typical setup on D22, neutrons of wavelength \(\lambda = 0.9\) nm with spread \(\Delta\lambda / \lambda = 0.01\) were collimated over a 18 m distance, providing a beam of angular spread \(\Delta\alpha = 0.077\). Scattered neutrons were detected using a 2D multidetector placed 18 m behind the sample (Fig. 2). NG7 and D22 are at reactor sources where the neutron flux at the sample position is constant over experiment timescale. SANS-II is at a continuous spallation source with varying neutron flux, so in these experiments multidetector count rates are normalised with a monitor detector situated upstream of the sample. Rather than plotting detector counts/monitor counts, in Fig. 5a and in Fig. 7e the ordinate are scaled to a typical monitor value that was used for measurements containing low or zero vortex signal. When the spallation source is stable, this corresponds to a measurement time of 5 min per rocking angle.
The [111] axis of our vanadium crystal was aligned to within 0.2° of the applied magnetic field direction. Unless otherwise noted, vortex ensembles were prepared by cooling in the desired magnetic field from the normal state, i.e. field-cooled. Any misalignment between the applied field and the vortex directions due to crystalline anisotropy would have been much smaller than the observed peak widths. In niobium, which has a large crystalline anisotropy than vanadium, 0.2° of misalignment from [111] would distort the vortex direction by 0.007°\(^7\). The edges of the sample were masked from the incident neutrons in view of possible demagnetisation effects.

Two experimental geometries were used in our SANS measurements: (a) the rocking curve measured in the perpendicular geometry, (b) the rocking curve measured in-plane to the beam (Fig. 2b). The parallel geometry was used with a = 0.077° on D22 to survey the field dependence (Fig. 4). This geometry also confirmed the absence of transitions in vortex lattice shape away from the hexagonal symmetry expected for fields along [111]. The perpendicular geometry was used on NG7 with vertical applied field (Fig. 2b) for the temperature dependence (Fig. 5c). It was also used on SANS-II with horizontal applied field to further map the phase diagram (Fig. 5) and to collect rocking curves with high resolution (a = 0.035°) (Fig. 7) suitable for evaluating in-plane correlations by reverse Monte Carlo refinement. To describe the field dependences of the intensity I integrated over the rocking curve (Figs. 4b and 5c), we used the form factor assuming in-plane positions of vortices are simulated on a computer. These positions are generated in each RMC run that all reproduce the shape of the measured rocking curve width.

As an initial approximation, the instrument resolution and mosaic spread of the specimen were masked from the incident neutrons in view of possible misalignment between the applied field and the vortex directions due to crystalline anisotropy. The perpendicular geometry was used on SANS-II with horizontal applied field to further map the phase diagram (Fig. 5) and to collect rocking curves with high resolution (a = 0.035°) (Fig. 7) suitable for evaluating in-plane correlations by reverse Monte Carlo refinement. To describe the field dependences of the intensity I integrated over the rocking curve (Figs. 4b and 5c), we used the form factor calculated for a hexagonal vortex lattice.

An ensemble of this study are available from the corresponding author upon request.

Received: 21 August 2017 Accepted: 1 February 2018
Published online: 02 March 2018

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