Investigation of convective heat transfer enhancement for nanofluid flow over flat plate

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Abstract. In this paper, the laminar flow over flat plate of three different nanofluids is investigated. The fluid is considered as water with three different kind of solid particles Al₂O₃, TiO₂, and Fe₃O₄ of different volume fractions (1, 2, 3, and 4%). The values of viscosity, heat capacity, density, and thermal conductivity depending on volume fraction for the three nanofluids are evaluated. Numerical simulation has been conducted for analysis the impact of the nanoparticles on the heat transfer properties, on the temperature and velocity profiles, on the wall shear stress and on the skin friction coefficient. The results show that as the value of volume fraction increases the values of the wall shear stress, skin friction, and heat transfer increase, but the velocity decreases. A comparison between the three nanofluids presents the difference in the thermal enhancement, velocity profiles, and thermal boundary layer. With the volume fraction for all three nanofluids polynomial connections has been obtained to describe for the relation of the wall shear stress, skin friction coefficient and average heat transfer coefficient.

1. Introduction:
The traditional fluids (e.g., water, ethylene, oil etc) have limited heat transfer capabilities due to the poor thermal properties such as the thermal conductivity, which led researchers to try to overcome this barrier by improving the thermal conductivity of this fluids to have more efficient systems. The list of the industrial applications, where better generation of heat transfer fluids could be utilized, is endless for example hot rolling, drying of paper, biomedicine, food processing, nuclear reactors etc. Generally, the thermal conductivity of metal particles is higher than that of the base fluid. Therefore, many techniques have been implemented to enhance thermal performance of the traditional fluids. One of the early methods is via suspension of nanoparticles in base fluid. Adding particles of various materials that have higher thermal conductivity than the base fluid can enhance the thermal properties [1], [2]. This method was introduced by Cho [3] he coined the term nanofluid. Nanofluids have a bigger effective thermal conductivity due to the extremely large surface area of nanoparticles and led them to be potential candidate considering in the heat transfer media. This the thermal performance enhancement method has attracted considerable attention in wide range of industrial applications and academic field. The analysis of the thermal physical properties of nanofluids includes several parameters for instance: volume fraction, base fluid (water, oil ….), nanoparticles size and shape, and particles migration patterns, all these parameters play vital roles on the final nanofluid performance. The influence of the nanoparticles’ types on heat transfer has been studied by many researchers. Anuar studied the effect of Al₂O₃,Cu, and TiO₂ particles in the classical Blasius problem [4]. The volume fraction has been one of the weightiest parameters in nanofluid. Lee showed that the thermal conductivity has been increased linearly with the increase of volume fraction [5]. Khanafer developed a 2D model to study the heat transfer performance of nanofluids inside an enclosure [6]. In [7] Congedo investigated the natural convection flow for Al₂O₃-water nanofluid. The solution of the nanofluid problems has been investigated by different methods. In the literature, authors have used single phase
approach in various geometries such as flat plate, wedge, square channel, circular tube and flow over cylinder[2], [8], [9], [10]. These studies highlight the enhancement of the heat transfer in the presence of nanoparticles in the base fluid. In the present study we numerically investigate the characteristics of the thermal and hydraulic flow of nanofluid passed over flat plate for three types of nanoparticles and volume fractions (0.01, 0.02, 0.03, 0.04 vol %). CFD simulations are performed, the variation of the velocity and temperature is presented. The skin friction coefficient and the local Nusselt number in the nanofluids are calculated in comparison of the impact of the nanoparticles and particle concentrations.

2. Mathematical Model

Consider the nanofluid flow in two-dimension above a steady plate. The sheet is maintained at a constant temperature $T_w = 400$ K (see Figure 1). In our CFD simulation, single phase approach is employed for modelling the thermal properties of nanofluid. The flow is laminar and steady-state. Considering a Cartesian coordinate system $(x, y)$ for two-dimensional flow of a nanofluid over flat plate, where $x$ is the coordinate measured along the plate and $y$ is normal to it. The fluid flows on the top surface with constant velocity $U_{nf} = 0.01$ m/s and the temperature outside the thermal boundary layer is $T_{nf} = 300$ K.

2.1. Governing equations

The basic equations that describe the mass, momentum and energy in the fluid flow can be written in vectorial form as:

- conservation of mass:
  \[ \nabla \cdot V = 0, \]  
  (1)

- conservation of momentum:
  \[ (V \cdot \nabla)V = - \frac{1}{\rho_{nf}} \nabla p + \frac{\mu_{nf}}{\rho_{nf}} \nabla^2 V \]  
  (2)

- conservation of energy:
  \[ (V \cdot \nabla)T = \alpha_{nf} \nabla^2 T, \]  
  (3)

where the following notations are used:
- $V$: the velocity vector,
- $T$: the temperature of the nanofluid,
- $p$: the pressure of the nanofluid,
- $\mu_{nf}$: the dynamic viscosity of the nanofluid,
- $\rho_{nf}$: the density of the nanofluid,
- $\alpha_{nf}$: thermal diffusivity of the nanofluid.

2.2. Physical properties of the nanofluid

Here we the physical properties of the nanofluid are given with the dimensionless nanoparticle concentration $\phi$ as follows:

a) Viscosity

Here, $\mu_{nf}$ is the viscosity, $\mu_b$ is the viscosity of the base fluid (water), and $\phi$ denotes nanoparticle volume fraction as Brinkman [12], [13]:

\[ \mu_{nf} = \frac{\mu_b}{(1-\phi)^{2.5}}. \]  
(4)

b) Density and heat capacity

The effective density of the nanofluid is given by [14],
\[ \rho_{nf} = (1 - \phi)\rho_b + \phi\rho_p, \]  
(5)

where \(\rho_b\) and \(\rho_p\) denote the density base fluid, and nanoparticles, respectively, and the heat capacity of the nanofluid \(C_{pnf}\) is assumed as below [15], [14]:

\[ C_{pnf} = \frac{\phi(\rho C_p)_p + (1 - \phi)(\rho C_p)_b}{\rho_{nf}} \]  
(6)

c) Thermal conductivity

In our calculations, \(k_{nf}\) denotes the thermal conductivity of nanofluid, \(k_b\) the thermal conductivity of base fluid, \(k_p\) the thermal conductivity of the particles given as follows [16]:

\[ k_{nf} = k_b \frac{k_p + 2k_b - 2\phi(k_p - k_b)}{k_p + 2k_b + \phi(k_p - k_b)} \]  
(7)

3. The effect of the nanoparticles on the fluid properties

The effect of adding different nanoparticles on the physical properties are studied by calculating the numerical values of the physical parameters for \(\text{Al}_2\text{O}_3\), TiO\(_2\), and Fe\(_3\)O\(_4\). Moreover, we analyse the effect of the volume fraction from 1 to 4 % on the thermo-physical properties of the base fluid for all three nanofluids. Table 1 shows the thermo-physical properties for water, Al\(_2\)O\(_3\), Fe\(_3\)O\(_4\), and TiO\(_2\) particles [18].

| Parameter | Fe\(_3\)O\(_4\) | TiO\(_2\) | Al\(_2\)O\(_3\) | H\(_2\)O |
|-----------|----------------|-----------|----------------|---------|
| \(\rho [kg/m^3]\) | 5180 | 4250 | 3970 | 997.1 |
| \(C_p [J/kg\cdot K]\) | 670 | 686.2 | 765 | 4179 |
| \(k [w/m\cdot K]\) | 9.7 | 8.9538 | 40 | 0.613 |
| \(\mu [Pa\cdot s]\) | - | - | - | 0.001 |

The effect of volume fraction on density for \(\text{Al}_2\text{O}_3\), TiO\(_2\), and Fe\(_3\)O\(_4\) water nanofluids is depicted in Figure 2. When volume fraction increases, density of all three types of nanofluids increases and the increase is more for Fe\(_3\)O\(_4\)-water nanofluid than the TiO\(_2\)-water, and Al\(_2\)O\(_3\)-water nanofluids. The lowest nanofluid density was noted in Al\(_2\)O\(_3\)-water due to the low density of the alumina particles comparing to the other additives.

![Figure 2. The effect of nanoparticles on the density](image-url)

The result of increasing the quantity of nanoparticles in base fluid on the thermal capacity on the base of equation (6) is plotted in Figure 3. It shows a decrease in the thermal capacity with increasing the value of \(\phi\). The comparison between the three nanofluid mixtures showed that the fluid with the
highest thermal capacity is Al₂O₃ with base water. That variation of the thermal capacity is influenced by the value of the density and the thermal capacity of the nanoparticles.

Figure 3. The effect of nanoparticles on the thermal capacity

Figure 4 shows the impact of increasing \( \phi \) on the viscosity of the nanofluid. Applying the viscosity equation (4) (Brinkman formula), it can be seen that the viscosity is only influenced by parameter \( \phi \), and not sensitive to the type of nanoparticles. The three nanoparticles show the same effect on the viscosity, which is increasing when the concentration has increased. The influence of increasing the concentration of nanoparticles on the thermal conductivity of the nanofluid are plotted in Figure 5. The thermal conductivity (7) increases with increasing the value of \( \phi \). Al₂O₃-water nanofluid has the highest thermal conductivity followed by Fe₃O₄ and TiO₂ respectively. Figure 6 presents the effect of \( \phi \) on \((\rho C_p)\). It is noted that the highest values are obtained for Fe₃O₄.

Figure 4. The effect of nanoparticles on the viscosity

Figure 5. The effect of nanoparticles on the thermal conductivity
4. CFD procedure

4.1. Numerical procedure

The fluid flows above a flat plate with constant velocity 0.01 m/s at temperature 300 K, the horizontal plate is stationary with no slip condition and with fluid temperature 400 K. The equations (1)-(3) of continuity, momentum, and energy are discretized and solved using ANSYS 18. For the computational domain and mesh, the computational domain geometry was generated using Design Modeler and grid are generated using ANSYS Fluent mesh. The pre-processing module for the Fluent software is as in [19]. The CFD domain consists of inlet and outlet, which have been divided by the number of division type with 100 divisions, the behaviour is set to hard with bias factor 40 to increase the number of subdomains near to the plate and to increase the preciseness near the wall. The sides AD and BC are symmetry and wall, respectively. Both are divided using the same method with 200 divisions. The mesh generation sensitivity has been considered in the Grid independency part. The boundary conditions have been set up as shown in Table 2. Laminar model is used with pressure-velocity coupling. The relaxation factor is 1 for density; body force and energy, the thermophysical properties of nanofluids including; density, thermal conductivity, viscosity, and thermal capacity are calculated using single phase approach as it is extensively used in the literature [9], [20], [21]. This approach assumes that the mixture is homogenous, and the presence of the nanoparticles is present by modifying the physical properties of the mixture fluid. The following assumptions are applied: there is no difference between the velocity of the fluid and particles, and both of them in thermal equilibrium [8]. These assumptions have been applied in calculating the effect of adding nanoparticles to the water in the CFD simulation.

Table 2. The boundary conditions

|   | AB  | BC  | CD  | AD  |
|---|-----|-----|-----|-----|
|   | Velocity inlet | wall | outlet | symmetry |

4.2. Grid independency

The grid independence test is made to ensure the obtained solution is mesh independent because the accuracy of the finite volume methods is directly related to the quality of the discretization used. Therefore, a comprehensive mesh sensitivity study has been done to minimize the numerical influences introduced by the size of the meshes. The simulations are first performed for the different meshes with various number of cells (see Table 3) in process called mesh refinement where the mesh has been improved in each mesh refinement process. The analysis of the mesh sensitivity has been done for five meshes and the test has compared the average Nusselt number on the plate for each mesh and the results are presented in Table 3. It can be seen that the Nusselt number for the mesh 4 with 20,000 cells found to be satisfactory to ensure the accuracy of the solution as well as the independency of the grid.
5. Comparison of the analytical results with CFD results

The comparison of the CFD solution and analytical solution is used for validation, which distilled pure water as working fluid. To validate the accuracy and reliability of the present CFD simulation, the calculated results are compared with analytical solution [23], for local Nusselt number along the plate, skin friction, and average heat transfer coefficient. The comparison of the local skin friction coefficient between the analytical and CFD simulation is presented in Figure 7. The CFD result showed a good agreement with the analytical solution with difference 2.426% for the skin friction coefficient. The comparison of the average heat transfer given in Table 4 shows a maximum 0.838% difference between the two solutions. The comparison of the local Nusselt number are plotted in Figure 8 with maximal difference 7.5%.

Table 3. The grid independency examination

| Mesh | Number of cells | Average Nusselt number | Average heat transfer coefficient |
|------|----------------|------------------------|----------------------------------|
| 1    | 7000           | 121.162                | 74.2728                          |
| 2    | 9600           | 123.036                | 75.4210                          |
| 3    | 14400          | 124.041                | 76.0376                          |
| 4    | 20000          | 124.662                | 76.4165                          |
| 5    | 26400          | 124.580                | 76.3678                          |

Table 4. Average heat transfer coefficient comparison (water)

| Method | CFD            | Analytical     | Difference % |
|--------|----------------|----------------|--------------|
| $\dot{h}$ | 76.42085      | 77.0670        | 0.838        |

Figure 7. Skin friction comparison (water)

Figure 8. The local Nusselt numbers’ comparison for water
6. Results

6.1. The effect of nanoparticles on the wall shear stress (Al₂O₃-water)

Figure 9 is plotted for exhibiting the variation of the wall shear stress with the volume fraction for Al₂O₃-water nanofluid. As can be observed that the value of the wall shear stress has increased as the value of the nanoparticle’s concentration increased.

![Figure 9. The effect of Al₂O₃ nanoparticles on the wall shear stress](image1)

Figure 10 shows a polynomial increase in the wall shear stress for the three nanoparticles. This increase in the wall shear stress can be predicted using the correlations for each nanoparticle as in Table 5 for volume fractions ranging from 0 to 4 %. The highest increase in wall shear stress is observed for Fe₃O₄-water nanofluid, while the increase was slightly higher for TiO₂ than for Al₂O₃. The effect of the value ϕ showed the same effect on the skin friction coefficient, and the effect is presented in Fig. 11. The increasing tendency of the skin friction coefficient can be predicted due to the correlations in Table 6 for the nanofluids.

![Figure 10. The wall shear stress vs volume fraction at x=0](image2)

| Nanofluid      | Wall shear stress                  |
|----------------|-----------------------------------|
| Al₂O₃-water    | \( \tau_w = 0.0431 \phi^2 + 0.0367 \phi + 0.01378 \) |
| Fe₃O₄-water    | \( \tau_w = 0.0420 \phi^2 + 0.0423 \phi + 0.01378 \) |
| TiO₂-water     | \( \tau_w = 0.0511 \phi^2 + 0.03765 \phi + 0.01378 \) |
Figure 11. The effect of $\phi$ on the skin friction coefficient

Table 6. The connection of the skin friction coefficient with $\phi$ ($R^2=1$)

| Nanofluid     | Skin friction coefficient |
|---------------|---------------------------|
| Fe$_3$O$_4$-water | $C_f = 0.6836 \phi^3 + 0.6908 \phi + 0.225$ |
| TiO$_2$-water  | $C_f = 0.830 \phi^2 + 0.6150 \phi + 0.225$ |
| Al$_2$O$_3$-water | $C_f = 0.7114 \phi^2 + 0.5984 \phi + 0.225$ |

6.2. The effect of the volume concentration on the velocity profile (Al$_2$O$_3$-water)

Figure 12 presents the effect of the volume fraction on the velocity profile for Al$_2$O$_3$-water nanofluid at different values of $\phi$. It can be seen that the velocity increase and boundary layer thickness decrease with the increase of the volume fraction.

The comparison of the velocity profiles for the three different nanofluids with max.4 % volume fraction are plotted in the Figure 13. It can be noted that Fe$_3$O$_4$ has the thinnest boundary layer, and the boundary layer of the Al$_2$O$_3$ is the thickest among the nanofluids, while for the TiO$_2$ it was slightly thinner than the boundary layer of Al$_2$O$_3$. The decrease in velocity can be referred to the density of the nanoparticles, since the viscosity is influenced only by the value of $\phi$, and not by the type of the nanoparticles.
6.3. The effect of nanoparticles on the temperature distribution

Figure 14 presents the effect of nanoparticle volume fraction on the temperature profile for Al$_2$O$_3$-water. It can be concluded that the increase in volume fraction is to increase the temperature. The thermal boundary layer increases as the volume fraction increases due to higher resistance between the fluid and the wall and higher thermal conductivity of the mixture. Comparison of the temperature profiles for the three nanofluids at $\phi = 0.04$ shows that Al$_2$O$_3$-water has the thickest thermal boundary layer followed by TiO$_2$-water and Fe$_3$O$_4$-water, respectively. The result of comparing the thermal boundary layers for the three mixtures is presented in Figure 15.

Figure 13. The velocity profile for different nanofluids for $\phi = 0.04$

Figure 14. The temperature profiles for different value of $\phi$, (Al$_2$O$_3$ -water)
Figure 15. Comparison of the temperature for water based nanofluids with different nanoparticles ($\phi = 0.04$)

6.4. The effect of nanoparticles on the average heat transfer coefficient

Figure 16 illustrates the impact of adding nanoparticles on the average heat transfer of the mixture. It can be observed that the value of volume fraction has influenced the heat transfer rate positively, the value of the heat transfer coefficient increased with increasing the value of the volume fraction. A comparison among $\text{Al}_2\text{O}_3$, $\text{Fe}_3\text{O}_4$, and $\text{TiO}_2$ with different concentrations is presented in Table 7. Each nanoparticle has enhanced the heat transfer, while the higher enhancement is obtained for the $\text{Al}_2\text{O}_3$.

Table 7. The average heat transfer coefficient for different particles and different volume fraction

| $\phi$ | $\text{Al}_2\text{O}_3$-water | $\text{Fe}_3\text{O}_4$-water | $\text{TiO}_2$-water |
|-------|-----------------------------|-----------------------------|---------------------|
| 0     | 76.42085                    | 76.42085                    | 76.42085            |
| 0.01  | 77.85308                    | 77.81988                    | 77.66741            |
| 0.02  | 79.29031                    | 79.20593                    | 78.91704            |
| 0.03  | 80.73311                    | 80.58955                    | 80.15044            |
| 0.04  | 82.17327                    | 81.96828                    | 81.39145            |

Figure 16. The effect of adding nanoparticles on the average heat transfer coefficient

Table 8. Average heat transfer coefficient ($R^2=1$)

| Nanofluid   | Average heat transfer coefficient |
|-------------|----------------------------------|
| $\text{Al}_2\text{O}_3$-water | $\dot{h} = 15.307 \phi^2 + 143.24 \phi + 76.42$ |
| $\text{Fe}_3\text{O}_4$-water | $\dot{h} = -30.736 \phi^2 + 139.87 \phi + 76.42$ |
| $\text{TiO}_2$-water | $\dot{h} = -19.521 \phi^2 + 125.02 \phi + 76.42$ |
A polynomial best fit curve has been founded (see Table 8). The increase in the thermal conductivity has played an important role in the enhancement of the heat transfer between the working fluid, and the heated surface. The average heat transfer along the plate is presented in Figure 17. For Al\(_2\)O\(_3\)-water nanofluid with \(\phi=0.04\). The result shows that additional loading of nanoparticles enhances the heat transfer. The comparison of different nanofluids is plotted in Figure 18.

**Figure 17.** The average heat transfer along the plate (Al\(_2\)O\(_3\)-water case)

**Figure 18.** Comparison of average heat transfer along the plate for water based nanofluids with different nanoparticles (\(\phi=0.04\))

### 7. Conclusion
In this paper, the laminar flow above a steady flat surface for three different types of nanoparticles has been investigated. Water was the base fluid. The problem is modelled and solved using CFD method. The following main results are founded:
- The velocity of the Fe\(_3\)O\(_4\)-containing nanofluid for each \(\phi\) is greater than that of the other oxide-containing nanofluid. The increase of volume fraction causes an increase in velocity and decrease in the boundary layer thickness.
• Temperature of $\text{Al}_2\text{O}_3$-water is higher in the boundary layer than for $\text{TiO}_2$ or $\text{Fe}_3\text{O}_4$. Increasing the volume fraction shows an increase in the temperature profile, and the thermal boundary layer thickness increased as the volume fraction $\phi$ is increased.

• The wall shear stress and skin friction coefficient increase with increasing $\phi$. A polynomial connection can be given to predict the increase of these properties for all three nanofluids in volume concentration range 0 to 4 %.

• The type of the nanoparticle is a key factor for the heat transfer enhancement. Each nanoparticle shows different impact on the heat transfer enhancement, the average heat transfer coefficient is higher for $\text{Al}_2\text{O}_3$, then for $\text{Fe}_3\text{O}_4$, or $\text{TiO}_2$. This growth property was characterized by giving polynomial relations.

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