Possible Molecular Pentaquark States with Different Spin and Quark Configurations

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Abstract

We investigate three possible pentaquark candidates, one of which contains a single charm quark and the other two contain triple charm quarks in their substructure. To this end we apply QCD sum rule method and take into account both the positive and negative parity states corresponding to each possible pentaquark channel having spin 3/2 or 1/2. Insisting on the importance of identification of the members of pentaquark family we obtain their spectroscopic parameters such as masses and residues. These parameters are the main inputs in the searches for their electromagnetic, strong and weak interactions.

I. INTRODUCTION

The exotic hadrons with non-conventional quark substructures have been investigated for many years. Having such non-conventional configurations, different from the standard hadrons composed of tree quarks or a quark and an antiquark, make them interesting both theoretically and experimentally. Indeed, they have been searched for very long time in the experiment and their nature and probable internal structure have been theoretically investigated for many years. Finally the long sought result have been achieved and in 2003 X(3872) was observed by Belle Collaboration [1]. This triggered the subsequent experimental searches to identify those non-conventional hadrons, especially XYZ states, and measure their parameters. And finally the LHCb Collaboration [2] heralded the observation of another ones which are the pentaquark states, \( P^+_c(3872) \) and \( P^{++}_c(4450) \). These states were reported to have possibly \( J^P = (3/2^+, 5/2^+) \) quantum numbers, though not being well determined yet. These observations have triggered other investigations on such states and some other states were also interpreted as possible pentaquark states such as some of the newly observed \( \Omega_c \) states by LHCb [3] as stated in Refs. [4] and, the states \( N(1875) \) and \( N(2100) \) [5].

We have a lack of knowledge about the inner structure and properties of these pentaquark states. To identify their structure different models were suggested. Among these models are the diquark-diquark antiquark model [6–13], diquark-triquark model [6, 14, 15], topological soliton model [16, 17] and meson baryon molecular model [6, 18, 19, 20, 21, 22]. Beside the observed \( P^+_c(3872) \) and \( P^{++}_c(4450) \) states there are other possible candidates with possible five quark structure such as the ones studied in Ref [21] in which the masses of charmed-strange molecular pentaquark states as well as other hidden charmed molecular ones were predicted. In Refs. [20, 21, 22] along with the observed ones the pentaquark states containing \( b \) quark were also investigated.

The observation of pentaquark states by LHCb has brought some questions. One of them is about what possible internal structure these particles may have and whether they are tightly bound states or molecular ones. The other one is about the existence of the other possible stable pentaquark states. To shed light on these questions there have been an intense theoretical studies on these particles so far. However to understand them better, to identify their internal structure and their possible other candidates we need more investigations both on their spectroscopic properties and decay mechanism. The theoretical studies on these states may provide a deeper understanding on the nature and substructure of them and possible insights to the experimental researches as well as a deeper understanding on the strong interaction. With these motivations, in this work, we predict masses and residues of the three possible pentaquark states considering them in the meson-baryon molecular structure. For the investigation of the masses of these exotic particles we apply the QCD sum rules method [23, 24]. This method is among the effective nonperturbative methods which has been used widely in hadron physics giving reliable results consistent with the experimental observations.

In this work we firstly consider recent announcement of the LHCb Collaboration on the observation of five new \( \Omega_c \) states in \( \Xi_c^+ K^- \) channel [2]. In Refs. [25, 26] considering the closeness of their masses to a meson and a baryon threshold \( \Omega_c \) mesons were investigated with the possible molecular pentaquark assumption. Considering these interpretations we make a prediction on the mass of the possible molecular pentaquark states having single charm quark with spin parity \( J^P = \frac{3}{2}^\pm \). To this end, we chose a current in \( \Xi_c^+ K^- \) molecular form.

In addition to these states considering another observation of LHCb Collaboration on double-charm baryon \( \Xi_{cc}^{++} \) [47] we study the possible triple charmed pentaquark states and calculate the masses and residues of them for both positive and negative parity cases. The interpolating currents in the calculations are chosen in the \( \Xi_{cc}(3621)D^0 \) and \( \Xi_{cc}(3621)D^{*0} \) molecular form with spin parity quantum numbers \( J^P = \frac{1}{2}^- \) and \( J^P = \frac{3}{2}^- \), respectively. Such a molecular interpretation of the possi-
The triple charmed pentaquark state was also considered in Ref. [18] in which via one-boson-exchange model two possible molecular pentaquark states were predicted.

The outline of the article is as follows. In Section II we present the detailed QCD sum rules calculations for the single charmed molecular pentaquark and triple charmed pentaquark states. Section III is devoted to the numerical analysis of the results. Finally we summarize and discuss our results in section IV.

II. QCD SUM RULES CALCULATION

The details of the calculations for the considered possible three types of pentaquark states are presented in this section. In the calculation there are three steps to obtain QCD sum rules and these steps start from the correlation function. The mentioned correlation function is written in terms of the interpolating currents of the considered states and has a general form

$$\Pi_{\mu \nu}(p) = i \int d^4x e^{ipx} \langle 0 | T \{ J_{\mu}(x) \bar{J}_{\nu}(0) \} | 0 \rangle.$$  

(1)

In the first step the above correlation function is calculated in terms of hadronic degrees of freedom such as mass of the hadron, current coupling constant of the hadron etc. This side of calculation is represented as physical or phenomenological side. In the second step the same correlation function is calculated in terms of QCD degrees of freedom containing mass of quarks and quark gluon condensates and named as theoretical or QCD side. Final step requires a match between the result of mentioned two sides of calculations considering the coefficient of same Lorentz structure from both sides. For the improvement of the analysis Borel transformation is used to suppress the contribution coming from higher states and continuum together with quark hadron duality assumption.

A. Phenomenological Side

In this side we treat the interpolating currents as operators to annihilate or create the hadrons. To calculate the physical side, a complete set of hadronic states having the same quantum numbers with the considered interpolating current are inserted into the correlation function. Then the integration over $x$ is performed. The results appear in terms of masses and current coupling constant of the considered states, i.e. in terms of hadronic degrees of freedom.

The single charmed pentaquark states with $J = \frac{1}{2}$

To calculate the physical side of the single charmed pentaquark states we follow the above given steps and firstly calculate the correlation functions in terms of hadronic degrees of freedom. For that purpose we insert complete sets of hadronic state having the same quantum numbers with the considered interpolating current into correlation function. The integral over $x$ gives us the following result:

$$\Pi_{\mu \nu}^{\text{Phys}}(p) = \frac{\langle 0 | J_{\mu} \frac{3}{2}^+(p) \rangle \langle \frac{3}{2}^+(p) | J_{\nu} \rangle}{m_{\frac{3}{2}^+}^2 - p^2} + \frac{\langle 0 | J_{\mu} \frac{3}{2}^- (p) \rangle \langle \frac{3}{2}^- (p) | J_{\nu} \rangle}{m_{\frac{3}{2}^-}^2 - p^2} + \cdots,$$  

(2)

where $m_{\frac{3}{2}^+}$ and $m_{\frac{3}{2}^-}$ represent the masses of the positive and negative parity particles, respectively. The ellipsis corresponds to contributions of the higher states and continuum. Using the following matrix elements

$$\langle 0 | J_{\mu} \frac{3}{2}^+(p) \rangle = \lambda_{\frac{3}{2}^+}^+ \gamma_5 u_\mu^+(p),$$  

$$\langle 0 | J_{\mu} \frac{3}{2}^- (p) \rangle = \lambda_{\frac{3}{2}^-}^- u_\mu^-(p)$$  

(3)

parameterized in terms of the residues $\lambda_{\frac{3}{2}^+}^+$ and $\lambda_{\frac{3}{2}^-}^-$, and corresponding spinor, in Eq. (2) we obtain the Borel transformed correlation function as

$$B_{\mu \nu} \Pi_{\mu \nu}^{\text{Phys}}(p) = -\lambda_{\frac{3}{2}^+}^+ e^{-\frac{m_{\frac{3}{2}^+}^2}{M^2}} (-\gamma_5) (p + m_{\frac{3}{2}^+}) g_{\mu \nu} \gamma_5$$

$$-\lambda_{\frac{3}{2}^-}^- e^{-\frac{m_{\frac{3}{2}^-}^2}{M^2}} (p + m_{\frac{3}{2}^-}) g_{\mu \nu} + \cdots,$$  

(4)

where $M^2$ is the Borel mass squared.

The triple charmed pentaquark states with $J = \frac{1}{2}$ and $J = \frac{3}{2}$

Following similar steps as in single charmed case, we again start the calculation of the correlation functions in terms of hadronic degrees of freedom for triple charmed pentaquark states. Insertion of complete sets of hadronic state and integration over $x$ gives us the following result:

$$\Pi_{\mu \nu}^{\text{Phys}}(p) = \frac{\langle 0 | J_{\mu} \frac{1}{2}^+ (p) \rangle \langle \frac{1}{2}^+ (p) | J_{\nu} \rangle}{m_{\frac{1}{2}^+}^2 - p^2} + \frac{\langle 0 | J_{\mu} \frac{1}{2}^- (p) \rangle \langle \frac{1}{2}^- (p) | J_{\nu} \rangle}{m_{\frac{1}{2}^-}^2 - p^2} + \cdots,$$  

(5)

for spin−1/2 states, with masses $m_{\frac{1}{2}^+}$ and $m_{\frac{1}{2}^-}$ corresponding to the positive and negative parity particles, respectively. The ellipsis is again used for the representation of the contributions coming from the higher states.
and continuum. Using the following matrix elements

\[ \langle 0 | J_{1/2}^+ (p) | \rangle = \lambda_{1/2}^+ \gamma_5 u(p), \]
\[ \langle 0 | J_{1/2}^- (p) | \rangle = \lambda_{1/2}^- u(p) \]

(6)

in Eq. (5), the Borel transformed correlation function for this case is obtained as

\[ B_{p^2} \Pi^{\text{phys}} (p) = -\lambda_{1/2}^+ \frac{e^{-\frac{m^2_{1/2}}{m^2}}} (-\gamma_5) (\not{p} + m_{\frac{1}{2}} + \gamma_5) \]
\[ -\lambda_{1/2}^- e^{-\frac{m^2_{1/2}}{m^2}} (\not{p} + m_{\frac{1}{2}}) + \cdots. \]

(7)

As for the triple charmed states with spin−3/2 a similar procedure and similar steps as in single charmed pentaquark case are applied. Therefore we will skip the details for this calculation and remark that the results obtained here have the same forms as Eq. (2), Eq. (3) and Eq. (1).

Here we need to mention that for spin−3/2 parts, for both the single charmed and triple charmed pentaquark states, only the structures seen in Eq. (4) are given explicitly among the others. This is because of the fact that, these ones are the structures isolated from the spin−1/2 pollution and giving contributions to only spin−3/2 particles.

**B. Theoretical Side**

The second step in the QCD sum rule calculation requires the computation of the correlation function in terms of QCD degrees of freedom. In this part, the correlation function is reconsidered and it is calculated with explicit form of the interpolating currents of the interested states. In the calculations the quark fields present in interpolating currents are contracted via the Wick’s theorem which ends up with emergence of light and heavy quark propagators. These quark propagators are presented in [33] in coordinate space and are used in the calculations, following which we transform the calculations to the momentum space by means of Fourier transformation. As in physical side, for the suppression of contribution of higher states and continuum we apply Borel transformation to this side also. Taking the imaginary parts of the results of the specified structure to be used in analysis we achieve the spectral densities.

**The single charmed pentaquark states with J = \frac{1}{2}**

The interpolating current to be used in Eq. (1) for single charmed pentaquark states with spin−3/2 has the following form:

\[ J_\mu = [\epsilon^{abc} (q_u^T C \gamma_\mu s_b) c_c] [\bar{d} \gamma_5 s_d]. \]

(8)

In Eq. (8), the subscripts a, b, c and d are used to represent the color indices, C is charge conjugation operator and q represents u or d quark. This current does not only couple to the negative parity state but also to positive parity one. In the present analysis we consider both the negative and the positive parity cases. Following the mentioned procedure, usage of interpolating current of single charmed state in correlation function and application of Wick’s theorem results in

\[ \Pi^{\text{QCD}} (p) = i \int d^4 x e^{ixp} e^{abce} e^{ib'c'} \left\{ \text{Tr} \left[ \gamma_5 S_s^{dd'} (x) \gamma_\nu \right] \times \right. \]
\[ \left. C \gamma_\lambda S^{aa'}_u (x) C \gamma_\mu S^{bb'}_d (x) \gamma_5 S^{dd'}_d (x) \right\} \]
\[ \times C \gamma_\nu S^{dd'}_c (x) \right\}. \]

(9)

Then the propagators of light and heavy quarks are used in this equation and following straightforward mathematical calculations we obtain the results for this side. Imaginary parts of the results obtained for chosen Lorentz structures provide us with the spectral densities. To provide samples for the spectral densities obtained in this work, we present the results of this subsection in the Appendix.

**The triple charmed pentaquark states with J = \frac{3}{2} and J = \frac{\bar{3}}{2}**

The interpolating currents used for triple charm pentaquark states with spin J = \frac{3}{2} and J = \frac{3}{2} are as follows:

\[ J = [\epsilon^{abc} (q_u^T C \gamma_\mu c_b) c_c] [\bar{u} \gamma_5 c_d], \]
\[ J_\mu = [\epsilon^{abc} (q_u^T C \gamma_\mu c_b) c_c] [\bar{u} \gamma_5 c_d], \]

(10)

respectively. The results for the triple charmed states are obtained after the contraction as

\[ \Pi^{\text{QCD}} (p) = \mp i \int d^4 x e^{ipx} \epsilon^{abce} e^{ib'c'} \gamma_\mu \gamma_5 S^{cc'}_q (x) \gamma_5 \gamma_\nu \times \right. \]
\[ \text{Tr} \left[ \gamma_\nu C S^{T bb'}_e (x) C \gamma_\lambda S^{dd'}_u (x) \gamma_i S^{dd'}_d (x) \right] \]
\[ \times \gamma_j S^{da} (x) - \text{Tr} \left[ \gamma_\nu C S^{T ba'}_e (x) C \gamma_\mu S^{dd'}_u (x) \right] \]
\[ \gamma_j S^{dd'}_d (x) \right\} \gamma_\nu C S^{T ab'}_e (x) + \text{Tr} \left[ \gamma_\nu C S^{T ab'}_e (x) \gamma_i S^{dd'}_d (x) \right] \]
\[ \times C \gamma_\mu S^{dd'}_d (x) \gamma_i S^{dd'}_d (x) \gamma_j S^{da} (x) \right\} \gamma_\nu C S^{T ba'}_e (x) \gamma_i S^{dd'}_d (x) \right\} \]
\[ \times \gamma_j S^{dd'}_d (x) + \text{Tr} \left[ \gamma_\nu C S^{T bb'}_e (x) C \gamma_\mu \right] \]
\[ \times S^{aa'}_c (x) \right\} \text{Tr} \left[ \gamma_\nu S^{dd'}_u (x) \gamma_j S^{dd'}_d (x) \right] \]
\[ - \text{Tr} \left[ \gamma_\nu C S^{T ba'}_e (x) C \gamma_\mu S^{ab'}_c (x) \right] \times \gamma_j S^{dd'}_d (x) \right\}. \]

(11)
In Eq. (11) the − and + signs at the beginning of the equation are for spin−1/2 and spin−3/2 particles, respectively and the $g_i$ and $g_j$ is used for $g_i = g_j = g_5$ for spin−1/2 and $g_i = g_{13}$ and $g_j = g_{13}$ for spin−3/2 case, respectively.

C. QCD sum rules

After the calculations of both sides are completed we choose the same Lorentz structures from each side and we match the coefficients to obtain the QCD sum rules giving us the physical quantities that we seek for. From this procedures we obtain

\[
m_i + \lambda_i^p e^{-m_i^p/M^2} - m_i - \lambda_i^\rho e^{-m_i^\rho/M^2} = \Pi_i^m, \\
\lambda_i^p e^{-m_i^p/M^2} + \lambda_i^\rho e^{-m_i^\rho/M^2} = j\Pi_i^p,
\]

for single and triple charmed pentaquark states, where $j^\pm$ are used to represent the spin−1/2$^\pm$ and spin-3/2$^\pm$ states, $j$ is + for spin-1/2 and − for spin-3/2 cases. The $\Pi_i^m$ and $\Pi_i^p$ are the functions obtained in QCD sum rules from the coefficients of the structures 1 and $\rho$ for spin−1/2 and $g_{\mu\nu}$ and $\rho_{\mu\nu}$ for spin−3/2 cases and they are written as

\[
\Pi_i^{m(p)} = \int_0^{s'} ds \rho_i^{m(p)}(s)e^{-s/M^2},
\]

in terms of spectral densities, where $s' = (2m_i + m_c)^2$ for single charmed pentaquark and $s' = 9m_c^2$ for triple charmed ones. The spectral densities $\rho_i^{m(p)}$ contain both perturbative and nonperturbative parts and can be represented for each structure denoted by $m(p)$ as

\[
\rho_i^{m(p)}(s) = \rho_i^{m(p),\text{pert}}(s) + \sum_{k=3}^{6} \rho_{i,k}^{m(p)}(s),
\]

with $\rho_{i,k}^{m(p)}(s)$ part containing the nonperturbative contributions. In the Appendix we present the results of spectral densities obtained for the single charmed pentaquark state to provide an example.

To obtain the present four unknown physical quantities, namely $\lambda_i^p$, $\lambda_i^\rho$, $m_i^p$ and $m_i^\rho$ for each possible pentaquark states considered in this work, besides the two equations given in Eq. (12) we need two more equations. We obtain them taking the derivative of both sides of Eq. (12) with respect to $1/s$. Simultaneous solution of obtained four equations will result in the physical quantities that we are after.

III. NUMERICAL RESULTS

The expressions of physical parameters obtained in the last subsection contain QCD degrees of freedom, Borel parameter $M^2$ as well as continuum threshold $s_0$. These are all input parameters in the calculations to acquire the physical quantities of interest. Among these input parameters are the masses of light quarks $u$ and $d$ and they are taken as zero. Table I includes some of these input parameters.

The analyses in the work we have two auxiliary parameters: threshold parameter $s_0$ and Borel parameters $M^2$. To carry over the analysis they working intervals are needed. To determine these intervals one needs the criteria which brings some limitations on their values. For the Borel window these criteria are the convergence of the series of OPE and the adequate suppression of the contributions of higher states and continuum. The threshold parameter is not completely arbitrary and it is related to the energy of the first corresponding excited state. In its fixing we again consider the pole dominance and OPE convergence. The analyses done with these criteria result in the intervals given in Table II for these parameters:

Now, we would like to draw the graphs for masses and residues of positive and negative parity states pointing out the dependence of the results obtained for $\Xi_c^\pm K$ molecular pentaquark on Borel mass $M^2$ and threshold parameter $s_0$ in figures 1-4. These graphs depict weak dependency on the auxiliary parameters which is an expected result. The Borel parameter is not a physical parameter. Although there should be no dependence on it in reality, the weak dependence is acceptable in practice bringing some uncertainty to the calculation. This uncertainty manifest itself as errors in the results.
The working intervals and the other input parameters are used in the QCD sum rule results to obtain the phys-
and the corresponding masses and residues are given as in Ref. [51] in diquark-diquark-antiquark configuration. Such possible pentaquark states both the molecular and diquark-diquark-antiquark interpretations can be considered for their inner structures. Therefore to identify them we need more theoretical works not only on the spectroscopic properties of these type of particles but also on their possible interactions with other particles. On the other hand one can not look over the contribution of such theoretical studies for gaining deeper understanding in the nonperturbative realm of QCD.

### IV. SUMMARY AND OUTLOOK

In this work we consider some possible pentaquark states containing single or triple charm quark. We assign their structure in molecular form and find their masses and residues using QCD sum rules method. The calculations include both positive and negative parity states corresponding to each pentaquarks. The single charmed pentaquark state is considered as $\Xi^*_cK$ molecular state with $J^P = 3/2^+$ and the triple charmed pentaquarks are as $\Xi_{cc}(3621)D^0$ and $\Xi_{cc}(3621)D^{*0}$ molecular states with corresponding $J^P = 1/2^+$ and $J^P = 3/2^+$, respectively. The results obtained in this work are compared with the other present results for differently chosen quark configurations in literature. From this comparison it has been seen that the obtained results are in agreement. The results of present study may give an insight into the future experimental searches but it is clear that to distinguish the inner structure of prospective pentaquark states having such quark substructure these mass predictions, though necessary, may not be enough and it is needed to study other properties of them such as their possible decays. It can be concluded that, it is important to study such states theoretically in different respects not only to provide some insights into the future experiments but also to better understand the properties of these possible states. The theoretical studies on these states will also improve our knowledge on the present pentaquark states as well as on the nonperturbative nature of the QCD.
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APPENDIX: SPECTRAL DENSITIES

To exemplify the spectral density results, in this appendix, the perturbative and nonperturbative parts of the spectral densities for the single charmed pentaquark states are presented in terms of the Feynman parameters $x$ and $y$. These results are corresponding to the coefficients of the structures $g_{\mu\nu}$ and $\rho g_{\mu\nu}$.

For the structure $g_{\mu\nu}$:

\[
\begin{align*}
\rho_{g_{\mu\nu}}^{m,\text{pert.}}(x) &= \frac{1}{2^{20} \cdot 5^2 \cdot 3^2 \pi^8 r^5} \int_0^1 dx \frac{m_c x^4 (m_c^2 + sr)^4 \left(30m_r^2 r(-4 + r) - 11(m_c^2 + sr)x(-5 + r)\right)}{5 \cdot 3^3 \cdot 2^{19} \pi^6 r^4} \Theta[L], \\
\rho_{g_{\mu\nu}}^{m,\text{Dim3}}(x) &= \frac{1}{21^4 \cdot 3^2 \pi^6 r^2} \int_0^1 dx \frac{m_c m_s x^3 (m_c^2 + sr)^3 \left(10(\bar{d}d)(-3 + r) - 40(\bar{q}q) - 13(\bar{s}s)(-3 + r)\right)}{5 \cdot 3^3 \cdot 2^{19} \pi^6 r^4} \Theta[L], \\
\rho_{g_{\mu\nu}}^{m,\text{Dim4}}(x) &= -\frac{1}{21^4 \cdot 3^2 \pi^6 r^2} \int_0^1 dx \left(\frac{\alpha_s GG}{\pi} + \frac{x^2 m_c (m_c^2 + sr)^2 \left[5m_c^2 x(180 - 263x + 67x^2) + sr^2 \left(sx(900 - 1315x + 269x^2 + 11x^3) + 6m_s^2(30 - 5x^2 - 3x^3) + m_c^2 r(6m_s^2(30 - 15x^2 - x^3) + sx(1800 - 2630x + 604x^2 + 11x^3))\right)\right]}{5 \cdot 3^3 \cdot 2^{19} \pi^6 r^4} \Theta[L], \\
\rho_{g_{\mu\nu}}^{m,\text{Dim5}}(x) &= \frac{1}{21^4 \cdot 3^2 \pi^6 r^2} \int_0^1 dx \frac{m_c x^2 (m_c^2 + sr)^2 \left[45(\bar{q}q) - 15(\bar{d}d)(-2 + r) + 14(\bar{s}s)(-2 + r)\right]}{5 \cdot 3^3 \cdot 2^{19} \pi^6 r^4} \Theta[L], \\
\rho_{g_{\mu\nu}}^{m,\text{Dim6}}(x) &= \frac{1}{21^4 \cdot 3^2 \pi^6 r^2} \int_0^1 dx \left\{\frac{m_c x^2 (m_c^2 + sr)^2 \left[30(\bar{q}q) + (\bar{s}s)(-2 + r)\right] - (\bar{d}d)\left[3(\bar{q}q) + 10(\bar{s}s)(-2 + r)\right]}{5 \cdot 3^3 \cdot 2^{19} \pi^6 r^4} \right\} \Theta[L], \\
\end{align*}
\]

and for the structure $\rho g_{\mu\nu}$:

\[
\begin{align*}
\end{align*}
\]
\[ \rho_{p,\text{pert.}}^{3} = \frac{1}{2^{20} \cdot 3^{2} \cdot 5^{2} \pi^{4}} \int_{0}^{1} dx \frac{x^{4}(m_{c}^{2} + sr)^{4}(-30m_{c}^{2}r(-4 + r) + 11x(m_{c}^{2} + sr)(-5 + r))}{\Theta[L]}, \]

\[ \rho_{p,\text{Dim3}}^{3} = \frac{1}{2^{15} \cdot 3^{2} \pi^{2}r^{2}} \int_{0}^{1} dx \frac{m_{s}x^{3}(m_{c}^{2} + sr)^{3}(40\langle \bar{q}q \rangle - 10\langle \bar{d}d \rangle(-3 + r) + 13\langle \bar{s}q \rangle(-3 + r))}{\Theta[L]}, \]

\[ \rho_{p,\text{Dim4}}^{3} = \frac{1}{2^{19} \cdot 3^{2} \cdot 5^{2} \pi^{4}} \int_{0}^{1} dx \frac{x^{2}(m_{c}^{2} + sr)^{2}m_{0}^{4}x(-900 + 2215x - 1696x^{2} + 326x^{3}) - 5sr^{3}(sx(-180 + 263x - 63x^{2})}{\Theta[L]}, \]

\[ \rho_{p,\text{Dim5}}^{3} = \frac{1}{2^{24} \cdot 3^{2} \pi^{6}r} \int_{0}^{1} dx \frac{(m_{c}^{2} + sr)^{2}m_{0}^{2}(15\langle \bar{d}d \rangle(-2 + r) - 14\langle \bar{s}q \rangle(-2 + r) - 45\langle \bar{q}q \rangle)}{\Theta[L]}, \]

\[ \rho_{p,\text{Dim6}}^{3} = \frac{1}{2^{11} \cdot 3^{2} \pi^{4}r} \int_{0}^{1} dx \frac{x^{2}(m_{c}^{2} + sr)^{2}[3\langle \bar{q}q \rangle + 10\langle \bar{s}q \rangle(-2 + r) - \langle \bar{s}q \rangle(30\langle \bar{q}q \rangle + 3\langle \bar{s}q \rangle(-2 + r))]}{\Theta[L]}, \]

\[ + \frac{118^{2}((\bar{d}d)^{2} + \langle \bar{q}q \rangle^{2} + 2\langle \bar{s}q \rangle^{2})(m_{c}^{2} + sr)^{2}x^{2}(-2 + r)}{2^{15} \cdot 3^{2} \pi^{6}r} \]

where \( \Theta[L] \) is the step function and

\[ L = -m_{c}^{2}x + srx, \]

\[ r = -1 + x. \]
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