Retraction

Retraction: A Shape Optimization Technique for Medical Images (J. Phys.: Conf. Ser. 1916 012233)

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This article (and all articles in the proceedings volume relating to the same conference) has been retracted by IOP Publishing following an extensive investigation in line with the COPE guidelines. This investigation has uncovered evidence of systematic manipulation of the publication process and considerable citation manipulation.

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IOP Publishing regrets that our usual quality checks did not identify these issues before publication, and have since put additional measures in place to try to prevent these issues from reoccurring. IOP Publishing wishes to credit anonymous whistleblowers and the Problematic Paper Screener [1] for bringing some of the above issues to our attention, prompting us to investigate further.

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A Shape Optimization Technique for Medical Images

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Abstract: Segmentation and restoration are among the primary objectives in the field of medical image processing. Among many methods available today, derivative method specifically domain based derivative method helps immensely in this field of medical imaging. There was a time-based improvement in the aspects of establishing results in a medical image with the aid of topological derivative. Advantage of this method lies in establishing a link between that of the perturbed domain and that of the original domain and solving for the required metrics in the problem. Treating the original domain as the original image and the diseased image to be the perturbed domain, topological derivative helps to solve the problem. Asymptotic analysis also helps in handling both large and small amount of data based on the big-O notation and small-o notation. It also establishes the supremacy of topological derived image metrics to that of other metrics. Here attempts are made for solving problems in medical images with the help of topological derivative method by adapting minimization procedure. Also, the end results are arrived in an outstanding way with help of the receiver operating characteristic curve.

1. Introduction

Irregularly shaped objects, human organs for example and the study about the geometrical domain of the same is an arduous process. Decomposition, an inherent property of wavelet is used in this paper with the aid of topological derivative. Asymptotic analysis plays an important role and determining the input size is the principle aspect of it. Running time of an algorithm dependent on the input size. Deciding factor for these are the types of algorithms that decides the speed of the running time. Simple cases are logarithmic based algorithms that help in getting results faster compared to that of the linear based algorithms even when the program is run on an efficient machine having faster speed. The algorithm is the main factor over here and not the efficacy of the machine. Asymptotic analysis handles the input size and determines the algorithm capability of picking up the number of inputs and matches for better performance of algorithm by taking into account, the speed factor. How fast the program is run by the type of algorithm selected, is decided with the aid of Asymptotic analysis. In other words, asymptotic analysis makes the best choice for faster execution time.
Landau symbol $\Theta$ is adopted when dealing with large amount of data. Asymptotic analysis decides on the function depending upon the big-O notation. Figure 1 illustrates the idea of order of growth. As seen, one is having linear growth and the other is having logarithmic growth. Using asymptotic analysis, the best choice for using either of the two functions above is determined. Faster computational time for $n$ iterated loops and $N$ number of inputs is determined. It is evident from the graph that log function gives best result for computational time because it has a higher rate of growth.

Asymptotic notation on the other hand is adopted for calculations involving only the constant terms and drops out the coefficients and other significant terms. There are three forms of asymptotic notation which are big- theta notation, big-O notation and bit-omega notation.

Mathematically we have big-O notation and that of small-o notations denoted as $O$ and $o$. $O$ stands for bigger order of growth if the input size is larger and $o$ stands for smaller order of growth if the input size is smaller. Asymptotic behavior can be applied to a large number of functions as

\begin{align}
  y &\to 0^+, y \to y_0, y \to \infty \\
  y &\to 0^-, y \to y_0, y \to -\infty \\
  x &\to 0^+, x \to x_0, x \to \infty \\
  x &\to 0^-, x \to x_0, x \to -\infty
\end{align}

Equations (1) and Equation (2) gives a general case of asymptotic situations for both $x$ and $y$ axes, as functions belonging to different classes have their asymptotes along either the $x$ or $y$ directions. Consider a class of real functions.
For functions representing $O$ and $o$, we have set of functions defined as $a$ and $b$, such that $a, b: \mathbb{R} \to \mathbb{R} \setminus \{0\}$, with constants $C$ and $d > 0$. Under such conditions for large number of inputs, the function is defined as $a = O(b)$, the normed vector space can be represented as

$$|a(y) \leq Cb(y)| \quad 0 < |y| < d$$

Equation (3) holds good only for the big $O$ notation. For moderate number of inputs, with constants $C$ and $d > 0$, the function is defined as $a = o(b)$ but with a different condition we have for every value of variable $\alpha > 0$ corresponding there is a value of $d > 0$, then

$$|a(y) \leq \alpha b(y)| \quad 0 < |y| < d$$

Equation (4) holds good only for small $o$ notation. Concept of topological derivative under asymptotic conditions is best realized when the equation is summoned for punctured neighborhood of $0^+$. This situation is best realized when

$$a(y) = \varepsilon_1 b(y) \quad y \to 0, b \neq 0$$

Equation (5) is valid if $a/b$ is a punctured neighborhood of $0^+$. Representing the punctured neighborhood of $0^+$ as $0^+ < |y - x_{yi}| < n$ equation (5), can be written as

$$a(y) = \varepsilon_1 b(y) \quad y \to 0, b \neq 0 \leftrightarrow y: |y - x_{yi}| < n$$

In Equation (6) $|y - x_{yi}| < n$ is a representation for the neighborhood of a point in the $x$-$y$ graph. $x_{yi}$ is the point under consideration which is present at a distance from $y$. Besides $a \times b^{-1}$ is bounded in the neighborhood

$$0^+ \to y: |y - x_{yi}| < n.$$ Also $y: |y - x_{yi}| < n$ represents $(x_{yi} + n)$ or $(x_{yi} - n)$.

The same in the case of small $o$ notation is represented as

$$a(y) = \varepsilon_2 b(y) \quad y \to 0, b \neq 0$$

Equation (7) can further be represented as

$$a(y) = \varepsilon_2 b(y) \quad y \to 0, b \neq 0 \leftrightarrow \begin{cases} a \times b^{-1} \quad \text{when} \quad y \to 0^+ \\ y: |y - x_{yi}| < n \end{cases}$$

Asymptotic condition is laid out as

$$a_{xy} \times b_{xy}^{-1} \to 1$$

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Equation (9) gives the firm condition for the existence of asymptotic equations. Interesting among the cases of trigonometric functions that are asymptotic have the form as

\[
\sin(x) \approx x \quad \text{and} \quad \sin(1/x) = O(1) \quad \text{at} \quad x \to 1
\]

(10)

Other functions of interest in the case of asymptotic functions are

\[
x = o(\log x) \quad \text{and} \quad e^{-1/x} = o(x^n) \quad \text{at} \quad x \to 0^+
\]

(11)

Equations (5) through (8) gives various conditions put down for both the big-O and small-o notations. Equation (9) put forth a condition for the asymptotic existence and equations (10) through (12) gives few possible examples in terms of exponential, logarithmic and trigonometric where asymptotic behavior can be realized.

Consider a sequence of functions \( \rho_n : \mathbb{R} \to \mathbb{R} \setminus \{0\} \) for \( y \to y_0^+ \), asymptotic sequence is defined as

\[
\rho_{n+1}(y) = \rho_n(y) \quad y \to y_0^+
\]

(13)

where \( n \) takes the values of 0, 1, 2, 3, etc. In Equation (13) \( \rho_0 \) is called as gauge function since its inception in 1886.

For \( \{\rho_n\} \) to be an asymptotic sequence and a function \( a \) defined as \( a : \mathbb{R} \to \mathbb{R} \setminus \{0\} \), we can have

\[
a(y) \sim \sum_{n=0}^{\infty} \alpha_n \rho_n(y) \quad y \to 0^+
\]

(14)

Equation (14) insists the condition of asymptotic. The expansion that results from the above equation is termed as Asymptotic expansion. Precisely Equation (14) is an asymptotic expansion of \( a(y) \) with respect to the asymptotic sequence \( \{\rho_n\} \) when \( y \to 0^+ \). Also, for each and every value of \( n \) designated as \( N = 0, 1, 2, 3 \ldots \), and so on, the above equation can be rewritten as

\[
a(y) = \sum_{n=0}^{N} \alpha_n \rho_n(y) = o(\rho) \quad y \to 0^+
\]

(15)

Equation (15) provides an interpretation of asymptotic expansion for different values of \( N = 0, 1, 2, 3 \ldots \)

The problem in this paper revolves around the punctured neighborhood. Also, the function \( \rho_n(y) \) must not vanish in the neighborhood of \( y \to y_0^+ \). Then, as a follow up from that of the asymptotic expansion, we can write

\[
\alpha_N + 1 = \lim_{n \to 0^+} a(y) - \sum_{n=0}^{N} \alpha_n \rho_n(y)
\]

(16)
Equation (16) implies that if a function exists as a series of expansion of gauge functions, then the expansion is unique. Best example of asymptotic series is \( \log(\sin(x)) \) which has \( \{ \log(x), x^2, x^4, x^6 \ldots \} \) as asymptotic sequences as \( x \to 0^+ \). The sequence is expanded as powers of \( x \) with a constant coefficient associated to all \( x \). Such kind of expansions pave way for asymptotic series which is expressed as

\[
a(y) \sim \sum_{n=0}^{\infty} a_n y^n \quad y \to 0^+ \tag{17}
\]

Generally, it takes the form \( f \sim \sum_{n=0}^{\infty} a_n x^n \). In physics one example of the usage of asymptotic series is Navier-Stokes equation. Taking \( \lambda_{mf} \cdot M_L^{-1} \ll 1 \), where \( \lambda_{mf} \) is the mean free path for molecules in the fluid and \( M_L \) is the macroscopic length scale for that of the fluid flow. Navier-Stokes equation associates a small parameter to this \( \lambda_{mf} \cdot M_L^{-1} \) which is comparable to that of the punctured neighborhood as discussed above in the previous paragraphs. Assigning \( \varepsilon \) parameter to this \( \lambda_{mf} \cdot M_L^{-1} \), we have \( \varepsilon = \lambda_{mf} \cdot M_L^{-1} \). Here in this Navier-Stokes problem \( \varepsilon \) is the viscosity of the fluid and \( \varepsilon \ll 1 \). This \( \varepsilon \) is the leading term here and the asymptotic sequence is built on the order of \( \varepsilon \) as,

\( \{ \varepsilon^2, \varepsilon^4, \varepsilon^6, \varepsilon^8, \varepsilon^{10}, \ldots \} \) or \( \{ \varepsilon, \varepsilon^3, \varepsilon^5, \varepsilon^7, \varepsilon^9, \ldots \} \). Under circumstances all the order of \( \varepsilon \), has its play in building the asymptotic expansion as

\[
f \sim ye^1 + ye^2 + ye^3 + ye^4 + ye^5 + ye^6 + ye^7 + ye^8 + ye^9 + \ldots \tag{18}
\]

Equation (18) paves the way for having \( \varepsilon \) as the perturbation factor in this paper. Then the asymptotic expansion is written as an asymptotic sequence in terms of \( \varepsilon \), which is associated with the neighborhood of \( y \to 0^+ \). Equation (5) of this paper calls in for the presence of punctured neighborhood. Equation (18) gives an example of asymptotic expansion as a follow up of the fluid dynamics problem. Physical situation having a domain associated with it and a punctured neighborhood within the domain, naturally calls in perturbed domain. Naming the original domain as \( \Omega \) and the perturbed domain as \( \Omega_{\varepsilon} \) will have an asymptotic expansion equated with the domain as an sequence of the order of magnitude \( \varepsilon \).

This concept comes in handy for problems in medical images. Treating the original domain \( \Omega \) as disease free human organ and the perturbed domain \( \Omega_{\varepsilon} \) as the human organ affected with disease in the medical image, attempt is made to solve the problem with the help of topological derivative. Topological derivative helps to relate the perturbed domain \( \Omega_{\varepsilon} \) with that of the original domain \( \Omega \). Solving for factors of \( \varepsilon \) helps to establish a solution in this paper. Establishing the supremacy of topological derivative derived results when compared with that of the other result obtained by solving for the respective metrics in the medical image, forms the basis of this paper.

Medical images are grey scale in most of the image modalities that are in existence. Pivotal point that lies in the grey scale image is by the difference in the intensity of the light and how this shows the way for detecting the disease in the medical image.
Figure 2. Morphological Grey Scale Image

Figure 2 represents an illustrative example of a variation of grey scale in terms of intensity on morphological scale fitted in three-dimensional rectangle. Similar variations throw light in detecting changes in intensity which account for the diseased nature of the organ. Variation in the hue factor of the grey scale image can bring out details in medical images.

2. Topological Derivative

Let $\Omega \subset \mathbb{R}^3$ be a domain of connected sets and $\gamma (\Omega)$ is the solution of the partial differential equation defined on $\Omega$. Cost function associated with this problem is $\Psi (\Omega) = \Psi (\gamma (\Omega_e))$. Values of $c$ to be solved which will be a perturbed domain and is represented as $\Omega_e$, where $\varepsilon$ is the perturbation parameter. Perturbed domain defines a ball of radius $\varepsilon$ and is represented by

$$\Omega_e = \Omega B_e^{-1} \varepsilon \to 0$$  \hspace{1cm} (19)

where, $B_e$ is a sphere of radius $\varepsilon$ and $\Omega_e$ is the perturbed domain. $B_e$ is defined as

$$B_e = x_0 + \varepsilon \mathbf{x} \quad x_0 \in \Omega$$  \hspace{1cm} (20)

Topology sensitivity problem calls in for asymptotic expansion of $\Psi (\Omega)$ as $\varepsilon \to 0^+$. Consequently, we have

$$\Psi (\Omega_e) - \Psi (\Omega) = f (\varepsilon) D_T (x_0) + \mathbb{R}(f (\varepsilon))$$  \hspace{1cm} (21)

where, $\Psi (\Omega_e)$ is the cost function of the perturbed domain, $\Psi (\Omega)$ is the cost function of the original domain, $f (\varepsilon)$ is the monotonic function, $D_T (x_0)$ is the topological derivative at point $x_0$, $\mathbb{R}(f (\varepsilon))$ is the remaining terms in the equation.
From Equation (21) the definition for the topological derivative can be written as,

$$D_T(x_0) = \lim_{\varepsilon \to 0^+} \frac{\psi(\varepsilon) - \psi(0)}{f(0)}$$  \hspace{1cm} (22)$$

It is evident from Equation (22) the formalism of topological derivative helps to establish the relation between the original domain and the perturbed domain. This formalism can be used for optimization process. Minimization of the cost function is applied to this problem. It is achieved by introducing perturbations where the function is negative. Advantage of this function is that the solution for the problem can be achieved quickly. It happens as a consequence of lesser number of iteration process or in other fast algorithm methods. Images are defined as piecewise smooth functions.

Figure 3 gives a graphical illustration of the image in terms of the piecewise smooth functions and edges which forms an important part of study are identifies as singularities. Distributional derivative helps in acquiring splayed out results in medical imaging. Piezoelectric transducers play an important role in ultrasound machinery which in turn helps in getting ultrasound medical images. Topological derivative method can also be used to get results in the electrical impedance tomography. Diffusion method is one of the methods that established supremacy for the detection of edges in medical images. Frequency region can also throw light in medical images. Edges in medical images can be studies in finer scale of details. Segmentation of various chambers of the heart can be achieved with the help of topological derivative method. Level set method can be used for studying the boundaries in a medical image. Big-O notation is used for comparison of running time also [1]. Growth of functions can be studied with the help of big-O notation [2]. The big-O notation for that of the multiple variables can be studied [3].

3. Methodology

Given medical image is treated as functions of both regular and irregular attributes as illustrated in Figure 3. The kind of functions varies between cases. Depending upon the complexity of the image the asymptotic analysis helps in assigning big-O notation or small-o notation. Thereafter condition for iteration takes its play for making the problem to run in the smoother way. Along with this big-o notation helps for the running the loop in a fast manner [4-8].

Topological derivative helps in finding the differences between the normal medical image and the diseased medical image with the aid of the perturbation parameter $\varepsilon$. Solving for the cost function will throw out the
results of required parameter. Minimization helps in getting results in a faster and efficient way. Step by step procedure of all these will give out segmented portion of the diseased organ with superficial results particularly concerning with that of the edges in the images [9-10].

4. Conclusion and Discussion

Topological derivative having its application in various fields of engineering and sciences also aids in the field of medical image processing. With the big-O notation helping in running iterated loops for scrutinizing the various portions in the medical image, topological derivative method helps in detecting edges which are the main contributing factor in the recognition of various types of diseases in the medical images. While the big-O notation helps in handling large number of inputs with in shorter period of time, minimization procedure helps in faster execution of the program in getting efficient results for an enhanced medical image. By using other metrics for getting the same results in medical images, the method of topological derivative achieves the same in a faster executing time.

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