Transforming a time-domain electromagnetic signal to a frequency-domain electromagnetic response using Gauss elimination method

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Abstract. Time-domain electromagnetic method (TEM) is an electromagnetic prospecting method that has developed rapidly and widely used in recent years. The inversion of this method uses frequency domain electromagnetic method, so the key is how to transform electromagnetic signals from time domain to frequency domain. Aiming at this problem, this paper proposes an inversion algorithm for transforming a time-domain electromagnetic signal to a frequency-domain electromagnetic response. Assuming that the time domain electromagnetic signal is transformed by cosine or sine and fitted by polynomial under the condition of turn-off current, the linear equations composed of these time signals are solved by Gauss elimination method, and then the frequency domain response can be obtained. When this algorithm is applied to the synthesis of the perpendicular magnetic field of the magnetic dipole, the relative error between the inversion frequency domain response and the theoretical result is less than 1%. Thus, this method can maintain a relatively high accuracy when transform the time domain response to the frequency domain response.

1. Introduction

TEM inversion is very important in the practice of many fields, so improving the accuracy of inversion has always been a research hotspot. In order to solve the non-uniqueness problems and 3D imaging problem in large-scale inversion, different data reweighting schemes are proposed. Further, a hybrid model parametrization approach is presented, where traditional cell-based model parameters are used simultaneously within a parametric inversion in order to allow for a more focusing inversion. In this paper, we first transform the time domain signal to get the frequency domain response. If the sampling frequency Fs does not satisfy the sampling theorem, it will cause spectral aliasing near Fs/2, causing analytical error. Generally, as long as the analog signal has discontinuity point, its frequency function is not sharp-cut, so we need to use various window functions to cut off the analog signal. However, the frequency bandwidth will produce two kinds of error phenomena that cause errors: the fence effect and the spectrum leakage, which makes it impossible to perform inversion directly. Therefore, how to avoid these two phenomena is particularly important for our inversion and analysis. For the solution to the fence effect, improving the accuracy need to reduce the length of the polyline as much as possible [1]. So increasing the number of polylines so that the polyline can be as close as possible to the true curve we need. When the sampling frequency is greater than twice the maximum frequency of the signal contained in the signal, the sampled data can describe the signal without distortion. When the sampling frequency does not meet this condition, frequency foldback and frequency repetition will occur. Therefore, in order to ensure that the sampled data can be as close as possible to the real situation and to ensure the effect of later inversion, it is necessary to unify and take
a large number of samples of the original signal. So this paper chooses to sample at regular intervals in
the natural logarithmic domain.

According to the sampling theorem, Nyquist frequency is determined by the sampling interval
which is the reciprocal of the sampling interval by two times. If high frequency is desired, the
sampling frequency must be increased to shorten the sampling interval. In the time range of this paper,
RAICHE's "flow through" method is used to select \( t \) for evenly spaced sampling within the natural
logarithmic domain.

In the frequency range of the magnetic dipole, it is found that with the increase of
the resistivity, the cosine transform accuracy is more dependen
t on the frequency band \([1]\); The coil
radius and separation distance increase. In either case, the optimal frequency range is basically to
\([10^3, 10^4]\) Hz, and when the bandwidth is constant \((\log f_H - \log f_L \geq 12)\), the calculation accuracy of the
magnetic dipole can be guaranteed. Therefore, the effective frequency selection range is
\([10^3, 10^4]\).

Different from Weng Aihua's optimal regularization, this paper
uses gauss elimination to solve
linear equations. As a kind of direct solution m
ethod, Gauss elimination method can know the sum of
calculation in advance, which is very important for our field data workload prediction, design
algorithm and so on.

2. Algorithm

2.1. Time-frequency domain conversion method

With a cosine transform

\[
f(t) = \int_{-\infty}^{\infty} F(\omega) \cos(\omega t) d\omega \tag{1}
\]

For the purpose of numerical calculation, the curve \( F(\omega) \) is divided into segments of \( F(\omega) \) certain
length (which can be determined according to the needs of actual operation), and then the polyline
formed by each segment is used to approach \( F(\omega) \). The length of segment mainly depends on the
smoothness of \( F(\omega) \) curve. As long as \( F(\omega) \) curve is smooth enough and segments are fine enough, its
approximation error can be small enough.

Now let each segment of the polyline be \( S_k(\omega) \), and its sum approximation integral is

\[
f(t) = \sum_{k=0}^{N-1} \int_{\omega_k}^{\omega_{k+1}} F_k(\omega) \cos(\omega t) d\omega \approx \sum_{k=0}^{N-1} \int_{\omega_k}^{\omega_{k+1}} S_k(\omega) \cos(\omega t) d\omega \tag{2}
\]

where

\[
F_k(\omega) \approx S_k(\omega) = \frac{F(\omega_{k+1}) - F(\omega_k)}{\omega_{k+1} - \omega_k} \left( \omega_k - \omega t \right) + F(\omega_k) \tag{3}
\]

And

\[
\int_{\omega_k}^{\omega_{k+1}} S_k(\omega) \cos(\omega t) d\omega = \frac{F(\omega_{k+1}) - F(\omega_k)}{\omega_{k+1} - \omega_k} \left( \cos(\omega_{k+1} t) - \cos(\omega_k t) \right)
+ \frac{F(\omega_{k+1})}{t} \sin(\omega_{k+1} t) - \frac{F(\omega_k)}{t} \sin(\omega_k t) \tag{4}
\]

Substitute equation (4) into equation (2) to obtain

\[
f(t) = \int_{-\infty}^{\infty} F(\omega) \cos(\omega t) d\omega \]
Equation (5) is the numerical formula of equation (1) by using broken line approximation. If the step response of the vertical magnetic field of the electric dipole source is calculated, the transient response can be written as:

\[
H_z(t) = \frac{2}{\pi} \int_0^\infty \frac{\text{Im}H_\omega(\omega)}{\omega} \cos \omega d\omega \\
\approx \frac{2}{\pi} \sum_{k=0}^{N-1} \left[ \frac{\text{Im}H(\omega_{k+1})}{\omega_{k+1}} - \frac{\text{Im}H(\omega_k)}{\omega_k} \right] \frac{\cos(\omega_{k+1}t) - \cos(\omega_k t)}{t^2} \\
+ \frac{1}{t} \left[ \frac{\text{Im}H(b)}{b} \sin(bt) - \frac{\text{Im}H(a)}{a} \sin(at) \right]
\]

Equation (5) tends to infinity, in equation (6) is too small and can be ignored.

When calculating equation (6), it is necessary to control the step length so as to approach the original function well and speed up the calculation. We use the error control step size to make the calculation step size both regular and accurately reflect the nature of the original function.

2.2. Gauss elimination method for solving equations

With a few simple operations, equation 5 can be transformed into the following matrix form:

\[
d_i = \sum_{j=1}^N L_{ij} D_j
\]

(7)

Here \(d_i\) is the time domain signal on the time channel, \(L_{ij}\) is the coefficient matrix, and \(D_j\) is the frequency domain response on the frequency channel. The coefficient matrix can be expressed as follows

\[
L_{ij} = \begin{cases} 
0 - w_{ij}, & j = 1; \\
\omega_i, & i = j; \\
w_{i,j-1} - w_{ij}, & \text{others}; \\
w_{i,N} - 0, & j = N, \\
\omega_i, & j = N;
\end{cases}
\]

(8)

Where

\[
w_{ij} = -\frac{2}{\pi i^2} \frac{\cos(\omega_j t_j) - \cos(\omega_i t_j)}{\omega_{j+1} - \omega_j}
\]

(9)

For the sake of the neatness of the formula, equation 10 above can be expressed in a compact format

\[
LD = d
\]

(10)

The basic idea of Gauss elimination method is to reduce the number of unknowns in the equation by elimination [2], so that each equation contains only one summation number, thus obtaining the required solution. However, due to the errors in the actual calculation process, the results of the
elimination method are often not accurate, and there is a problem of numerical stability. Therefore, we add the condition of smoothing limit in the frequency domain response, and select the pivot in the algorithm to ensure the stability of the value. At the same time, in order to avoid the instability of numerical calculation in gaussian elimination method, it is generally necessary to add a pivot element selection process before each normalization, and exchange elements with the largest or larger modulus to the position of the pivot element. In this algorithm, we exchange the largest module element. Obviously, you cannot exchange only one pair of elements, but you must exchange the whole row or the whole column at the same time. Row exchange does not change the element position of solution vector, while column vector changes the element position of solution vector, so it is necessary to restore the elements of solution vector according to the history of column exchange.

To sum up the above process, Gauss elimination method is actually to transform the original linear simultaneous equations into a linear simultaneous equations in the form of lower triangle or upper triangle through Gauss elimination method, and then forward replacement algorithm is used to solve the problem

\[
\begin{align*}
    a_1x_1 + a_{12}x_2 + \Lambda + a_{1m}x_m &= b_1 \\
    a_{21}x_1 + a_{22}x_2 + \Lambda + a_{2m}x_m &= b_2 \\
    & \quad \vdots \ \\
    a_{m1}x_1 + a_{m2}x_2 + \Lambda + a_{mm}x_m &= b_m
\end{align*}
\]

Equation (11) is a form that assumes the original linear simultaneous equations

\[
\begin{align*}
    l_{11}x_1 &= b_1 \\
    l_{21}x_1 + l_{22}x_2 &= b_2 \\
    & \quad \vdots \\
    l_{m,1}x_1 + l_{m,2}x_2 + \cdots + l_{m,m}x_m &= b_m
\end{align*}
\]

Equation (12) is a linear simultaneous equation of the lower triangle form after Gaussian elimination

\[
\begin{align*}
    x_1 &= \frac{b_1}{l_{11}}, \\
    x_2 &= \frac{b_2 - l_{21}x_1}{l_{22}}, \\
    & \quad \vdots \\
    x_m &= \frac{b_m - \sum_{i=1}^{m-1} l_{m,i}x_i}{l_{m,m}}
\end{align*}
\]

Equation (13) is solved by forward substitution algorithm.

2.3. Numerical results presentation

The analytical expression of the field can be obtained only under quasi-static approximation conditions and when the source and receiver are placed on the homogeneous surface of the earth. The model environment used in this paper is a vertical magnetic dipole source on the semi-space surface of the homogeneous earth. Of course, this method also applies to the conversion of frequency domain to time domain of other sources.

The calculation formula of frequency domain response is

\[
H_z = \frac{m}{2\pi k^2 \rho^3} \left[ 9 - (9 + 9i k \rho - 4 k^2 \rho^2 - i k^3 \rho^3) e^{-i k \rho} \right]
\]

In the above formula \( k^2 = -i \mu \omega \sigma \), \( \rho \) is the transmit and receive distance.
The calculation formula of the response in the time domain is

\[
h(t) = -\frac{m}{4\pi^3} \left[ \frac{9}{2\theta^2 \rho^5} \text{erf}(\theta \rho) + \text{erf}(\theta \rho) - \frac{1}{\pi^{1/2}} \left( \frac{9}{\theta \rho} + 4\theta \rho \right) e^{-\theta^2 \rho^2} \right]
\]

In the above formula, \( \theta = \sqrt{\frac{\mu_0 \sigma}{4t}} \), \( \sigma = \frac{1}{\rho} \), \( \mu_0 = 8.854 \times 10^{-12} \text{ F/m} \), and other meanings are the same as above.

Figure 1. The time domain and frequency domain of vertical electromagnetic field are reproduced by inversion method

The magnetic field shown in figure 1 is generated by excitation of a ground magnetic dipole source. (a) is the frequency domain electromagnetic signal, (b) is the time domain comparison between theory and inversion, and (c) is the relative error between theory and inversion data in the time domain. The parameters of the device are: magnetic moment of magnetic couple source 1 Am\(^2\), receiving and transmitting distance 100m, uniform half space resistivity 100 \( \Omega \text{•m} \).

The black and red lines in Figure a represent the absolute values of the real and imaginary parts of the theoretical frequency domain response calculated in Equation 14 in double logarithmic coordinates, respectively.

The black line in Figure b represents the theoretical time domain response value calculated by Equation 15, and the red line represents the theoretical calculated value after the inversion method.

In Figure c, we can see that in the medium term, most of the time domain errors are less than 0.1%. In the early stage, although there is a large error, it is still less than 1%.
The high-precision method at low frequencies is to transform the frequency domain response at low frequencies into a monotone function. However, in most of the low and high frequency phases of exploration there is an oscillatory decline in the electromagnetic field. Therefore, in the early and late period time, the time domain response is extremely unstable, so that the time domain response of inversion deviates from the actual response, resulting in a poor accuracy.

3. Conclusions
Assuming a step current, the time domain electromagnetic signal can evaluate the frequency domain response by matrix equation LD = D. In this paper, the matrix equation LD = D is solved directly by Gauss elimination, and the response in frequency domain is obtained by inversion of time domain signal under the limitation of smoothing.

This theoretical study reveals that the second derivative of the electromagnetic field spectrum is close to zero in the middle period, which proves the basis and feasibility of physical constraints. This is a very important study and it's different from other methods of doing the Fourier transform directly.

It is very difficult to measure the response in the time domain in the early stage, and high sensitivity electromagnetic tools are needed to meet the requirements of accurate time domain electromagnetic methods. Using the inversion method, we can obtain the weak signal with high precision in the frequency domain from the time domain signal with high sensitivity.

Although the data of the transient electromagnetic method used in this article comes from the transient response of terrestrial magnetic dipole sources, due to its similarity, this method can be applied to other transient electromagnetic methods.

References
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