Spectral distortions to the Cosmic Microwave Background from the recombination of hydrogen and helium

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ABSTRACT
The recombination of hydrogen and helium at \( z \sim 1000–7000 \) gives unavoidable distortions to the Cosmic Microwave Background (CMB) spectrum. We present a detailed calculation of the line intensities arising from the Ly\( \alpha \) (2p–1s) and two-photon (2s–1s) transitions for the recombination of hydrogen, as well as the corresponding lines from helium. We give an approximate formula for the strength of the main recombination line distortion on the CMB in different cosmologies, this peak occurring at about 170 \( \mu \text{m} \). We also find a previously undescribed long wavelength peak (which we call the pre-recombination peak) from the lines of the 2p–1s transitions, which are formed before significant recombination of the corresponding atoms occurred. Detailed calculations of the two-photon emission line shapes are presented here for the first time. The frequencies of the photons emitted from the two-photon transition have a wide spectrum and this causes the location of the peak of the two-photon line of hydrogen to be located almost at the same wavelength as the main Ly\( \alpha \) peak. The helium lines also give distortions at similar wavelengths, so that the combined distortion has a complex shape. The detection of this distortion would provide direct supporting evidence that the Universe was indeed once a plasma. Moreover, the distortions are a sensitive probe of physics during the time of recombination. Although the spectral distortion is overwhelmed by dust emission from the Galaxy, and is maximum at wavelengths roughly where the cosmic far-infrared background peaks, it may be able to tailor an experiment to detect its non-trivial shape.

Key words: lines: formation – cosmology: cosmic microwave background – cosmology: early universe – cosmology: theory – atomic processes – infrared: general.

1 INTRODUCTION
Physical processes in the plasma of the hot early Universe thermalize the radiation content, and this redshifts to become the observed Cosmic Microwave Background (CMB; see Scott & Smoot 2004, and references therein). Besides the photons from the radiation background, there were some extra photons produced from the transitions when the electrons cascaded down to the ground state after they recombined with the ionized atoms. The transition from a plasma to mainly neutral gas occurred because as the Universe expanded the background temperature dropped, allowing the ions to hold onto their electrons. The photons created in this process give a distortion to the nearly perfect blackbody CMB spectrum. Since recombination happens at redshift \( z \sim 1000 \), then Ly\( \alpha \) is observed at \( \sim 100\mu\text{m} \) today. There is approximately one of these photons per baryon, which should be compared with the \( \sim 10^9 \) photons per baryon in the entire CMB. However, the recombination photons are superimposed on the Wien part of the CMB spectrum, and so make a potentially measurable distortion.

From the Far-Infrared Absolute Spectrophotometer (FIRAS) measurements, Fixsen et al. (1996) and Mather et al. (1999) showed that the CMB is well modelled by a 2.725±0.001 K blackbody, and that any deviations from this spectrum around the peak are less than 50 parts per million of the peak brightness. Constraints on smooth functions, such as \( \mu\)- or \( y\)-distortions are similarly very stringent. However, there are much weaker constraints on narrower features in the CMB spectrum. Moreover, within the last decade it has been discovered Puget et al. (1996) that there is a Cosmic Infrared Background (CIB; see Hauser & Dwek 2001, and references therein), which peaks at 100–200\( \mu\text{m} \) and is mainly composed of luminous infrared galaxies at moderate
redshifts. The existence of this background makes it more challenging to measure the recombination distortions than would have been the case if one imagined them only as being distortions to Wien tail of the CMB. However, as we shall see, the shape of the recombination line distortion is expected to be much narrower than that of the CIB, and hence the signal may be detectable in a future experiment designed to measure the CIB spectrum in detail.

The first published calculations of the line distortions occur in the seminal papers on the cosmological recombination process by Peebles (1968) and Zel’dovich, Kurt & Sunyaev (1968). One of the main motivations for studying the recombination process was to answer the question: ‘Where are the Lyα line photons from the recombination in the Universe?’ (as reported in Rubino-Martin, Hernandez-Monteagudo & Sunyaev 2005). In fact these studies found that for hydrogen recombination (in a cosmology which is somewhat different than the model favoured today) there are more photons created through the two-photon 2s–1s transition than from the Lyα transition. Both Peebles (1968) and Zel’dovich et al (1968) plot the distortion of the CMB tail caused by these line photons, but give no detail about the line shapes. Other authors have included some calculation or discussion of the line distortions as part of other recombination related studies, e.g. Boschan & Biltzinger (1998), and most recently Switzer & Hirata (2003). However, the explicit line shapes have never before been presented, and the helium lines have also been neglected so far. The only numerical study to show the hydrogen lines in any detail is a short conference report by Dell’Antonio & Rybicki (1993), meant as a preliminary version of a more full study which never appeared. Although their calculation appears to have been substantially correct, unfortunately in the one plot they show of the distortions (their fig. 2) it is difficult to tell precisely which effects are real and which might be numerical.

Some of the recombination line distortions from higher energy levels, n > 2, have also been calculated (Dubrovich 1972, Lyubarskii & Sunyaev 1983, Fahr & Loch 1991, Burdyuzha & Chekmezov 1994, Dell’Antonio & Rybicki 1995, Dubrovich & Stolatoro 1997, Burcin 2003, Khojipenko, Ivanikh & Varshalovich 2003). However, these high n lines are extremely weak compared with the CMB (below the 10⁻⁶ level), while the Lyα line is well above the CMB in the Wien region of the spectrum. As trumpeted by many authors, we are now entering into the era of precision cosmology. Hence one might imagine that future delicate experiments may be able to measure these line distortions. Since the lines are formed by the photons emitted in each transitions of the electrons, they are strongly dependent on the rate of recombination of the atoms. The distortion lines may thus be a more sensitive probe of recombination era physics than the ionization fraction x_e, and the related visibility function which affects the CMB anisotropies. This is because a lot of energy must be injected in order for any physical process to change x_e substantially (e.g. Peebles, Seager & Hu 2000). In general that energy will go into spectral distortions, including boosting the recombination lines.

This also means that a detailed understanding of the physics of recombination is crucial for calculating the distortion. The basic physical picture for cosmological recombination has not changed since the early work of Peebles (1968) and Zel’dovich et al (1968). However, there have been several refinements introduced since then, motivated by the increased emphasis on obtaining an accurate recombination history as part of the calculation of CMB anisotropies. Seager, Sasselov & Scott (1999,2000) presented a detailed calculation of the whole recombination process, with no assumption of equilibrium among the energy levels. This multi-level computation involves 300 levels for both hydrogen and helium, and gives us the currently most accurate picture of the recombination history. In the context of the Seager et al (2000) recombination calculation, and with the well-developed set of cosmological parameters provided by Wilkinson Microwave Anisotropy Probe (WMAP; Spergel et al 2003) and other CMB experiments, it seems an appropriate time to calculate the distortion lines to higher accuracy in order to investigate whether they could be detected and whether their detection might be cosmologically useful.

The aim of this paper is to calculate the line distortions on the CMB from the 2p–1s and 2s–1s transitions of H and the corresponding lines of He (i.e. the 2s–1p–1s transitions of HeI, and the 2p–1s and 2s–1s transitions of HeII) during recombination, using the standard cosmological parameters and recombination history. In Section 2 we will describe the model we used in the numerical calculation and give the equations used to calculate the spectral lines. An approximate formula for the magnitude of the distortion in different cosmologies will also be given. Other possible modifications of the spectral lines and their potential detectability will be discussed in Section 3. And finally, we will present our conclusions in the last section.

2 BASIC THEORY

2.1 Model

Instead of adopting a full multi-level code, we use a simple 3-level model atom here. For single-electron atoms (i.e. H and HeI), we consider only the ground state, the first excited state and the continuum. For the 2-electron atom (HeII), we consider the corresponding levels among singlet states. In general, the upper level states are considered to be in thermal equilibrium with the first excited state. Case B recombination is adopted here, which means that we ignore recombinations and photo-ionizations directly to ground state. This is because the photons emitted from direct recombinations to the ground state will almost immediately reionize a nearby neutral H atom (Peebles 1968; Seager et al 2000). We also include the two-photon rate from 2s to the ground state for all three atoms, with rates: A_{2s-1s}^I = 8.229063 s⁻¹ (Goldman 1983), Santos, Parente & Indelicato 1998; A_{2s-1s}^He = 51.02 s⁻¹ (Derevianko & Johnson 1997), although it makes no noticeable difference to the calculation if one uses the older value of 51.3 s⁻¹ from Drake, Victor & Daliganerro (1969); and A_{2s-1s}^HeII = 526.532 s⁻¹ (Lipke, Novick & Tolk 1963; Goldman 1983). This 3-level atom model is similar to the one used in the program recomb, with the main difference being that here we do not assume that the rate of change of the first excited state n₂ is zero.
The rate equations for the 3 atoms are similar, and so we will just state the hydrogen case as an example:

\[
(1+z) \frac{dn_i^H(z)}{dz} = -\frac{1}{H(z)} \left[ \Delta R^H_{2p-1s} + \Delta R^H_{2s-1s} \right] + 3n_i^H; \quad (1)
\]

\[
(1+z) \frac{dn_j^H(z)}{dz} = -\frac{1}{H(z)} \left[ n_n a H n_i^H H \right] \quad (\text{for } i \neq j, 1 \text{-- } 12) \quad (2)
\]

\[
(1+z) \frac{dn_i^H(z)}{dz} = -\frac{1}{H(z)} \left[ 3n_i^H \right] + 3n_i^H; \quad (3)
\]

\[
(1+z) \frac{dn_p^H(z)}{dz} = -\frac{1}{H(z)} \left[ 3n_p^H \right] + 3n_p^H. \quad (4)
\]

Here the values of \(n_i\) are the number density of the \(i\)th state, where \(n_n\) and \(n_p\) are the number densities of electrons and protons respectively. \(\Delta R^H_{i,j}\) is the net bound-bound rate between state \(i\) and \(j\) and the detailed form of \(\Delta R^H_{2p-1s}\) and \(\Delta R^H_{2s-1s}\) will be discussed in the next subsection. \(H(z) \equiv \dot{a}/a\) is the Hubble factor.

\[
H(z)^2 = \frac{\Omega_M (1+z)^3 + \Omega_K (1+z)^2 + \Omega_L}{1+z_{eq}}, \quad (5)
\]

where \(\Omega\) represents the fraction of the critical density in matter, curvature or cosmological constant, and the Hubble parameter today \(H_0 = 100h\) km s\(^{-1}\) Mpc\(^{-1}\). Finally \(\Omega_M\) is the Case B recombination coefficient from \(\text{Hunmer (1994)}\),

\[
\alpha H = 10^{-19} \frac{a t^b}{1+ct^d} \text{ m}^3 \text{s}^{-1}, \quad (6)
\]

which is fitted by \(\text{Pequignot, Pettitiec & (1991)}\), with \(a = 4.309, b = -0.6166, c = 0.6703, d = 0.5300\) and \(t = T_M/10^4\) K, while \(\beta H\) is the photo-ionization coefficient:

\[
\beta H = \alpha H \left( \frac{2 \pi \nu_{2s-c} \nu_T M}{h^2} \right) \exp \left\{ -\frac{h \nu_{2s-c}}{k_B T_M} \right\}, \quad (7)
\]

where \(k_B\) is Boltzmann’s constant, \(h\) is Planck’s constant, \(M\) is the mass of electron, \(T_M\) is the matter temperature and \(\nu_{2s-c}\) is the frequency of the energy difference between state \(2s\) and the continuum. For the rate of change of \(n_i^H\), we include only the Compton and adiabatic cooling terms \(\text{Seager et al. (2000)},\) i.e.

\[
(1+z) \frac{dn_i^H(z)}{dz} = \frac{8 \pi g \nu U}{3 H(z) m_e c n_e + n_H + n_e (T_M - T_R) + 2 T_M}, \quad (8)
\]

where \(T_R\) is the radiation temperature, \(c\) is the speed of light, \(U = a_H T_R^4\), \(a_H\) is the radiation constant and \(\sigma_T\) is the Thompson scattering cross-section.

We use the Bader-Deuflhard semi-implicit numerical integration scheme (see Section 16.6 in \(\text{Press et al. (1992)}\) to solve the above rate equations. All the numerical results are made using the \(\Lambda\)CDM model with parameters: \(\Omega_B = 0.046; \quad \Omega_M = 0.3; \quad \Omega_L = 0.7; \quad \Omega_K = 0; \quad Y_e = 0.24; \quad T_0 = 2725 \text{ K and } h = 0.7\) (see e.g. \(\text{Spergel et al. (2003)}\)). Here \(Y_e\) is the primordial He abundance and \(T_0\) the present background temperature.

### 2.2 Spectral distortions

We want to calculate the specific line intensity \(I_\nu(z = 0)\) (i.e. energy per unit time per unit area per unit frequency per unit solid angle, measured in W m\(^{-2}\) Hz\(^{-1}\) sr\(^{-1}\)) observed at the present epoch, \(z = 0\). The detailed calculation of \(I_\nu(z = 0)\) for the Ly \(\alpha\) transition and the two-photon transition in hydrogen are presented as examples (the notation follows Section 2.5 in Padmanabhan (1993)). A similar derivation holds for the corresponding transitions in helium. To perform these calculations we first consider the emissivity \(j_\nu(z)\) (energy per unit time per unit volume per unit frequency, measured in W m\(^{-3}\) Hz\(^{-1}\)) of photons due to the transition of electrons between the 2p and 1s states at redshift \(z\):

\[
j_\nu(z) = h \nu \Delta R^H_{2p-1s}(z) \phi(\nu), \quad (9)
\]

where \(\phi(\nu)\) is the frequency distribution of the emitted photons from the emission process and \(\Delta R^H_{2p-1s}\) is the net rate of photon production between the 2p and 1s levels, i.e.

\[
\Delta R^H_{2p-1s} = p_{12} \left( n_{2p}^H R_{21} - n_{1s}^H R_{12} \right). \quad (10)
\]

Here \(n_i^H\) is the number density of hydrogen atoms having electrons in state \(i\), the upward and downward transition rates are

\[
R_{12} = B_{12} J, \quad (11)
\]

\[
R_{21} = \left( A_{21} + B_{21} J \right). \quad (12)
\]

with \(A_{21}, B_{12}\) and \(B_{21}\) being the Einstein coefficients and \(p_{12}\) the Sobolev escape probability (see \(\text{Seager et al. (2000)}\)), which accounts for the redshifting of the Ly \(\alpha\) photons due to the expansion of the Universe. As \(n_1^H \gg n_2^H\), \(p_{12}\) can be expressed in the following form:

\[
p_{12} = 1 - e^{-\tau_\nu}, \quad (13)
\]

\[
\tau_\nu = A_{21} \lambda^2_{2p-1s} (g_{2p}/g_{1s}) n_1. \quad (14)
\]

We approximate the background radiation field \(J\) as a perfect blackbody spectrum by ignoring the line profile of the emission (see \(\text{Seager et al. (2000)}\)). We also neglect secondary distortions to the radiation field (but see the discussion in Section 11). These secondary distortions come from photons emitted earlier in time, during recombination of H or He, primarily the line transitions described in this paper. Assuming a blackbody we have

\[
J(T_M) = \frac{2 h \nu^3}{c^2} \left[ \exp \left( \frac{h \nu_\alpha}{k_B T_M} \right) - 1 \right]^{-1}, \quad (15)
\]

where \(\nu_\alpha = c/121.5682 \text{ nm} = 2.466 \times 10^{15} \text{ Hz}\) and corresponds to the energy difference between states 2p and 1s, while the frequency of the emitted photons is equal to \(\nu_\alpha\). Therefore, we can set \(\phi(\nu) = \delta(\nu - \nu_\alpha)\), i.e. a delta function centred on \(\nu_\alpha\), so that

\[
j^\nu L^\nu/\alpha(z) = h \nu \Delta R^H_{2p-1s}(z) \delta(\nu - \nu_\alpha). (16)
\]

The increment to the intensity coming from time interval \(dt\) at redshift \(z\) is

\[
dI_\nu(z) = \frac{\nu}{4 \pi} j_\nu(z) dt, \quad (17)
\]

which redshifts to give
\[ dI_{\nu_0}(z = 0) = \frac{c}{4\pi} \frac{j_\nu}{(1 + z)^3} dt. \]

We assume that the emitted photons propagate freely until the present time. Integration over frequency then gives

\[ I_{\nu_0}^{\nu_3}(z = 0) = \frac{c}{4\pi} \int \frac{j_\nu}{(1 + z)^3} dt = \frac{\hbar c \Delta R_{2p-1s}(z_0)}{4\pi H(z_0)(1 + z_0)^3}, \]

with

\[ 1 + z_0 = \frac{\nu_0}{\nu}, \]

using

\[ \nu(z) = \nu_0(1 + z) \quad \text{and} \quad \frac{dt}{dz} = -\frac{1}{H(z)(1 + z)}. \]

Equation (19) is the basic equation for determining the Ly \( \alpha \) line distortion, using \( \Delta R_{2p-1s}(z) \) from the 3-level atom calculation.

For the two-photon emission between the 2s and 1s levels, the emissivity at each redshift is

\[ j_\nu = h_\nu \nu \Delta R_{2s-1s}(z) \phi(\nu(z)), \]

and the calculation is slightly more complicated, since for \( \phi(\nu) \) we need the frequency spectrum of the emission photons of the 2s–1s transition of H \( \phi(\nu) \) to integrate equation (22) numerically from \( z = 0 \) to the time when \( \Delta R \) is sufficiently small that the integrand can be neglected.

3 RESULTS

Each of the line distortions are shown separately in Fig. 2 and summed for each species in Fig. 3. The shape of the lines from H, HeI and HeII are fairly similar. There are two distinct peaks to the 2p–1s emission lines. We refer to the one located at longer wavelength as the ‘pre-recombination peak’, since the corresponding atoms had hardly started to recombine during that time. The physics of the formation of this peak will be discussed in detail in section 3.3.1. The second (shorter wavelength) peak is the main recombination peak, which was formed when the atoms recombined. While the longer wavelength peak actually contains almost an order of magnitude more flux, it makes a much lower relative distortion to the CMB. The ratio of the total distortion to the CMB is intensity is shown in Fig. 4. It is approximately one for the main recombination peak, but \( \sim 10^{-4} \) for the pre-recombination peak.

In Fig. 3 we plot the lines from H and HeI together with the CMB and an estimate of the CIB. We can see that the lines which make the most significant distortion to the CMB are the Ly \( \alpha \) line and the \( 2p^1s^1 \) line of HeI, and that these lines form a non-trivial shape for the overall distortion. The sum of all the spectral lines and the CMB is shown in Fig. 3. Note that these lines will also exist in the presence of the CIB – but the shape of this background is currently quite poorly determined (Fixsen et al. 1998; Hansen et al. 1998).

We now discuss details of the physics behind the shapes of each of the main recombination lines.
3.1 Lines from the recombination of hydrogen

During recombination, the Lyman lines are optically thick, which means that photons emitted from the transition to $n = 1$ are instantly reabsorbed. However, some of the emitted photons redshift out of the line due to the expansion of the Universe and this makes the Ly$\alpha$ transition one of the possible ways for electrons to cascade down to the ground state. The other path for electrons going from $n = 2$ to $n = 1$ is the two-photon transition between $2s$ and $1s$. Fig. 5 shows the net photon emission rate of the Ly$\alpha$ and two-photon transitions as a function of redshift for the standard $\Lambda$CDM model. The two-photon rate dominates at low redshift, where the bulk of the recombinations occur. This means that there are more photons emitted through the two-photon emission process (54% of the total number of photons created during recombination of H) than through the Ly$\alpha$ redshifting process. This conclusion agrees with Zeldovich et al. (1968) – although of course the balance depends on the cosmological parameters (see Seager et al. 2004) and for today’s best fit cosmology the two processes are almost equal. Despite this fact, the overall strength of the two-photon emission lines are weaker because the photons are not produced with a single frequency, but with a wide spectrum ranging from 0 to $r_\alpha$. The location of the two-photon peak (see Fig. 2) is also somewhat unexpected, since it is almost at the same wavelength as the Ly$\alpha$ recombination peak, rather than at twice the wavelength. The reason for this will be discussed in section 3.1.2.

We should also note that the tiny dip in our curves for the long-wavelength tail of the pre-recombination peak (see Fig. 2) is due to a numerical error, when the number density of the ground state is very small. This can also be seen in the pre-recombination peak for HeII.

### 3.1.1 The pre-recombination emission peak

The highest Ly$\alpha$ peak (shown in Fig. 2) is formed before the recombinations of H has already started, approximately at $z > 2000$. During that time the emission of Ly$\alpha$ photons is controlled by the bound-bound Ly$\alpha$ rate from $n = 2$ (i.e. the $n_2 R_{21}$ term in equation (10)) and the photo-ionization rate ($n_{20\alpha}$). From Fig. 6 we can see that at early times the bound-bound Ly$\alpha$ rate is larger than the photo-ionization rate. This indicates that when an electron recombines to the $n = 2$ state, it is more likely to go down to the ground state by emission of a Ly$\alpha$ photon than to get ionized. The excess Ly$\alpha$ alpha photons are not reabsorbed by ground state H, but are redshifted out of the absorption frequency due to the expansion of the Universe; they escape freely and form the pre-recombination emission line. Note that there is very little net recombination of H, since the huge reservoir...
of $> 13.6$ eV CMB photons keeps photo-ionizing the ground state H atoms (see Fig. 12).

We now turn to a more detailed explanation of the pre-recombination emission peak. The bound-bound Lyα rate from $n = 2$ is initially approximately constant, as it is dominated by the spontaneous de-excitation rate (the $A_{21}$ term in equation (21)). At the same time the photo-ionization rate is always decreasing as redshift decreases, since the number of high energy photons keeps decreasing with the expansion of the Universe. Therefore, with a constant bound-bound Lyα rate and the decreasing photo-ionization rate, the emission of Lyα photons rises. The peak of this pre-recombination line of H occurs at around $z = 3000$, by which time only a very tiny amount of ground state H atoms have formed ($n_1/n_{1H} < 10^{-7}$, see Fig. 6). These ground state H atoms build up until they can reabsorb the Lyα photons and this lowers the bound-bound Lyα rate. The decrease of the bound-bound Lyα rate is represented in the Sobolev escape probability $p_{12}$ in equation (13). At high redshift, $p_{12}$ is 1 and there is no trapping of Lyα photons. When H starts to recombine, the optical depth $\tau_n$ increases and the Lyα photons can be reabsorbed by even very small amounts of neutral H. For $\tau_n \gg 1$, we can approximate $p_{12} \simeq 1/\tau_n$ and $p_{12} \propto H(z)/n_1$. Because of the increase in the number density of the ground state and the decrease in $H(z)$, the pre-recombination line decreases. One can therefore think of the 'pre-recombination peak' as arising from direct Lyα transitions, before enough neutral H has built up to make the Universe optically thick for Lyman photons. This process occurs because the spontaneous emission rate ($A_{21}$ term) is faster than the photoionization rate for $n = 2$; it increases as the Universe expands, due to the weakening CMB blackbody radiation, and is quenched as the fraction of atoms in the $n = 1$ level grows. The shorter wavelength peak, on the other hand, comes from the process of redshifting out of the Lyα line during the bulk of the recombination epoch.

By using the RECFAST program (Seager et al. 1999), we can generate the main Lyα recombination peak and also the two-photon emission spectrum, by simply adding a few lines into the code. However, the pre-recombination peak cannot be generated from RECFAST, since there the rate of change of the number density of the first excited state $n_2$ is assumed to be negligible and is related to $n_1$ via thermal equilibrium. Moreover, in the effective 3-level formalism, the Lyα line is assumed to be optically thick throughout the whole recombination process of H (in order to reduce the calculation into a single ODE), which is not valid at the beginning of the recombination process. Hence, one needs to follow the rate equations of both states (i.e. $n = 1$ and $n = 2$) to generate the full Lyα emission spectrum. The pre-recombination peak of H was mentioned and plotted in the earlier work of Dell’Antonio & Rybicki (1993) as well, although they did not describe it in any detail.

Another way to understand the line formation mechanism is to ask how many photons are made in each process per atom. We find that for the main Lyα peak there are approximately 0.47 photons per hydrogen atom (in the standard cosmology). During the recombination epoch, net photons for the $n = 2$ to $n = 1$ transitions are only made when atoms terminate at the ground state. Hence we expect exactly one $n = 2$ to $n = 1$ photon for each atom, split between the Lyα redshifting and 2-photon processes (and the latter splits the energy into two photons, so there are 1.06 of these photons per atom). For the 'pre-recombination peak', on the other hand, the atoms give a Lyα photon when they reach $n = 1$, but they then absorb a CMB continuum photon to get back to higher n or become ionized. The number of times an atom cycles through this process depends on the ratio of the relevant rates. If we take the rate per unit volume from Fig. 8 and divide by the number density of hydrogen atoms at $z \simeq 3000$ then we get a rate which is about an order of magnitude larger than the Hubble parameter at that time. Hence we expect about 10 'pre-recombination peak' photons per hydrogen atom. A numerical calculation gives the more precise value of 8.11.

### 3.1.2 The two-photon emission lines

Surprisingly, the location of the peak of the line intensity of the 2s–1s transition is almost the same as that of the Lyα transition, as shown in Fig. 2 while one might have expected it to differ by a factor of 2. In order to understand this effect, we rewrite the equation (22) in the following way:

$$I_{\nu_0}^\delta(z = 0) = \int_0^\infty \phi'(z')I_{\nu_0}^\delta(z = 0; z') dz', \quad (23)$$

where $\phi(z') = \nu_0 \phi(\nu')$, and

$$I_{\nu_0}^\delta(z = 0; z') = I_{\nu_0}^\delta(z = 0; z'(\nu')) = \frac{c\nu_0}{4\pi} \frac{R_{2\gamma}(z')}{H(z')(1+z')^3}, \quad (24)$$

with $1 + z' = \frac{\nu'}{\nu_0}$.

Equation (24) gives the redshifted flux (measured now at $z = 0$) of a single frequency $\nu'$ coming from redshift $z'$ and corresponding to the redshifted frequency $\nu_0$.

We first calculate the line intensity of the two-photon emission with a simple approximation: a delta function spectrum $\delta(\nu - \nu_0/2)$, where $\nu_0/2$ is the frequency corresponding to the peak of the two-photon emission spectrum $\phi(\nu)$. Fig 7 shows the intensity spectrum of two-photon emission using...
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Figure 7. The line intensity of the 2s–1s transition (two-photon emission) \( I_{\nu_0}(z = 0) \) as a function of redshifted frequency \( \nu_0 \) for three different assumptions: the correct frequency spectrum of two-photon emission (solid); the delta function approximation \( \delta(\nu - \nu_0/2) \) (dashed); and the flat spectrum approximation (dotted).

Figure 8. The top panel shows the redshifted flux from single emission frequency \( I_{\nu_0}^R(z = 0; z') \) plotted against the redshift of emission, \( 1 + z' \). The bottom panel shows the frequency spectrum of two-photon emission \( \phi(\nu(z')) \) plotted against \( z' \) for three redshifted frequencies: \( \nu_0 = 10^{12} \text{ Hz}; 1.6 \times 10^{12} \text{ Hz}; \) and \( 5 \times 10^{12} \text{ Hz}. \)

a delta frequency spectrum \( \delta(\nu - \nu_0/2) \) compared with the two-photon emission using the correct spectrum \( \phi(\nu) \). We can see that there is a significant shift in the line centre compared with the \( \delta \)-function case. Where does this shift come from?

We know that the frequencies of emitted photons are within the range of \( 0 \) to \( \nu_0 \) at the time of emission. For a fixed redshifted frequency \( \nu_0 \) now, we can calculate the range of emission redshifts contributing to \( \nu_0 \) (referred to as the ‘contribution period’ from now on), which is represented by \( \phi(z') \) or \( \phi(\nu') \). In Fig. \[\text{S}\] we show the spectral distribution \( \phi(\nu(z')) \) as a function of redshift \( z' \) for specific values of \( \nu_0 \). For example, if we take \( \nu_0 = 5 \times 10^{12} \text{ Hz}, \) then photons emitted between \( 1 + z = 1 \) (i.e. \( \nu = \nu_0 \)) and \( \sim 500 (\nu = \nu_0) \) will give contributions to \( \nu_0 \). The smaller the redshifted frequency \( \nu_0 \), the wider the contribution period.

We might expect that the line intensity of this two-photon emission will be larger if the contribution period is longer, as there are more redshifted photons propagating from earlier times. However, this is not the case, because the rate of two-photon emission \( R_{2\gamma} \) also varies with time, and is sharply peaked at \( z \simeq 1300 - 1400. \) Hence \( I_{\nu_0}^R(z = 0; z') \) is also sharply peaked at \( z \simeq 1300 - 1400. \) In Fig. \[\text{S}\] the redshifted flux integral \( I_{\nu_0}^R(z = 0, z) \) and the emission spectrum \( \phi(\nu(z)) \) are plotted on the same redshift scale. For \( \nu_0 = 5 \times 10^{12} \text{ Hz} \) (lowest panel), we can see that the contribution period covers a redshift range when \( I_{\nu_0}^R(z = 0, z) \) and \( R_{2\gamma} \) are small in value. The contribution period widens with decreasing \( \nu_0 \) and covers more of the redshift range when two-photon emission is high. Therefore, the flux \( I_{\nu_0}^R(z = 0) \) is expected to increase with decreasing \( \nu_0 \) until the contribution period extends to the redshifts at which the two-photon emission peaks. As \( \nu_0 \) gets even smaller (e.g. \( \nu_0 = 10^{12} \text{ Hz} \)), then the contribution period becomes larger than the redshift range for two-photon emission and hence only lower energy photons can be redshifted to that redshifted frequency. As a result, the flux peaks at \( \nu_0 \approx 10^{12} \text{ Hz} \) when we use the \( \delta \)-function approximation. However, from Fig. \[\text{S}\] we can see that the contribution period for \( \nu_0 \approx 10^{12} \text{ Hz} \) is much greater than that of the two-photon emission period, and therefore this is not the location of peak. Based on the argument presented above, we expect the peak to be at around \( 1.6 \times 10^{12} \text{ Hz}, \) or \( 200 \mu\text{m}. \)

The basic mathematical point is that \( \phi(y) \) is extremely poorly represented by a \( \delta \)-function. Since the spectrum \( \phi(\nu) \) is quite broad, it can be better approximated as a uniform distribution than as a \( \delta \)-function. Another crude approximation would be to assume a flat spectrum for \( \phi(\nu) \) in Fig. \[\text{S}\]. (Fig. \[\text{S}\] compares the intensity \( I_{\nu_0}^R(z = 0) \) found using the correct form for \( \phi(\nu) \) with the \( \delta \)-function and flat spectrum approximations. This shows that the flat spectrum gives qualitatively the same results as the correct form of the spectrum, and that the peak occurs fairly close to that of Ly\(\alpha, \) but is much broader. The same general arguments apply to the two-photon lines of He\(\alpha \) and He\(\alpha \) (as we discuss in Section 3.2).

3.1.3 Dependence of \( \Omega_M \) and \( \Omega_B \)

The largest distortion on the CMB is from the shorter wavelength recombination peak of the hydrogen Ly\(\alpha \) line (see Fig. \[\text{S}\]). It may therefore be useful to estimate the peak of this line’s intensity as a function of the cosmological parameters. The relevant parameters are the matter density \( (\propto \Omega_M h^2) \) and the baryon density \( (\propto \Omega_B h^2). \) This is because \( \Omega_M h^2 \) affects the expansion rate, while \( \Omega_B h^2 \) is related to the number density of hydrogen. No other combinations of cosmological parameters have a significant impact on the physics of recombination.

We can crudely understand the scalings of these parameters through the following argument. Regardless of the escape probability \( p_{12}, \) the remaining part of the rate \( (n_{12}^R R_{21} - n_{12}^R R_{12}) \) is roughly proportional to \( n_{12}^R \propto \Omega_M h^2(1 - x_e). \) The escape probability \( p_{12} \) can be approximated as 1 at the beginning of recombination (\( \tau_e \ll 1 \)) and \( 1/\tau_e \) during the bulk of the recombination process (with \( \tau_e \gg 1 \)). Note that \( \tau_e \propto H(z)/n_{12}^R \propto (\Omega_M h^2)^{1/2}/(\Omega_B h^2(1 - x_e))^{-1}. \) Therefore,
\[ \Delta R_{2p-1s} \propto \begin{cases} (\Omega M h^2) (\Omega B h^2 (1 - x_e)) & \text{for } \tau_s \ll 1 \\ (\Omega M h^2)^{1/2} (\Omega B h^2 (1 - x_e))^2 & \text{for } \tau_s \gg 1 \end{cases} \] 

and thus

\[ I_{\lambda_0} \propto \frac{\Delta R}{H(z)} \propto \begin{cases} (\Omega M h^2)^{-1/2} (\Omega B h^2 (1 - x_e)) & \text{for } \tau_s \ll 1 \\ (\Omega M h^2)^{5/2} (\Omega B h^2 (1 - x_e))^2 & \text{for } \tau_s \gg 1 \end{cases}. \]

From this rough scaling argument, we may expect that the \( \Omega M \) dependence of the peak of the Ly \( \alpha \) line is an approximate power law with index between \(-1/2\) and 0, while for \( \Omega B \) the corresponding power-law index is expected to lie between 0 and 1. The dependence of \( \Omega M \) is actually more complicated when one allows for a wider range of values (see Dell’Antonio & Rybicki 1993). The above estimation just gives a rough physical idea of the power of the dependence.

A more complete numerical estimate of the peak of the recombination Ly \( \alpha \) distortion is:

\[ (\lambda_0 I_{\lambda_0})_{\text{peak}} \simeq 8.5 \times 10^{-15} \left( \frac{\Omega M h^2}{0.0224} \right)^{0.57} \left( \frac{\Omega B h^2}{0.147} \right)^{0.15} \text{Wm}^{-2}\text{sr}^{-1}, \]

where we have normalized to the parameters of the currently favoured cosmological model. The peak occurs at

\[ \lambda_0 \simeq 170 \mu\text{m} \]

for all reasonable variants of the standard cosmology.

### 3.2 Lines from the recombination of helium (HeI and HeII)

We compute the recombination of HeI and HeII in the same way as for hydrogen. For the two-electron atom HeI, we ignore all the forbidden transitions between singlet and triplet states due to the low population of the triplet states (see Seager et al. 1999, 2000). The \( 2^1p-1^1s \) transitions of HeI are optically thick, the same situation as for H. This makes the electrons take longer to reach the ground state and causes the recombination of HeI to be slower than Saha equilibrium. However, unlike for H, and despite the optically thick \( 2^1p-1^1s \) transition line, the \( 2^1p-1^1s \) rate dominates, as shown in Fig. 9. For HeII, due to the fast two-photon transition rate (see Fig. 10), there is no ‘bottleneck’ at the \( n = 2 \) level in the recombination process. Hence HeII recombination can be well approximated by using the Saha equilibrium formula (Seager et al. 2000).

We can see the effect of the above differences in recombination history on the lines: the width of the recombination peak of both H and HeI is larger than that of HeII. Overall, the spectral lines of HeI are of much lower amplitude than those of H (see Fig. 2) with the distortion to the CMB about an order of magnitude smaller.

The peaks of the line distortions from H and HeI are located at nearly the same wavelengths. For hydrogenic ions the 1s–2p energy (and all the others) scales as \( Z^2 \), where \( Z \) is the atomic number. Hence for HeI recombination takes place at \( z \approx 6000 \) rather than the \( z \approx 1500 \) for hydrogen. Hence the line distortion from the 2p–1s transition of HeII redshifts down to about 200 \( \mu\text{m} \), just like Ly \( \alpha \).

The two-photon frequency spectrum of HeII is the same as for H, since they both are single-electron atoms (Tung et al. 1984). However, it is complicated to calculate the two-photon frequency spectrum of HeI very accurately, since there is no exact wave-function for the state of the atom. Drake et al. (1963) used a variational method to calculate the two-photon frequency spectrum of HeI with values given up to 3 significant figures. Drake (1984) presented another calculation, giving one more digit of precision, and making the spectrum smoother, as shown in Fig. 11. These two calculations differ by only about 1%, which makes negligible change to the two-photon HeI spectral line.

All of the H and He lines (for \( n = 2 \) to \( n = 1 \)) are presented in Fig. 2 and the sum is shown as a fractional distortion to the CMB spectrum in Fig. 3. We find that in the standard cosmological model, for HeI recombination, there are about 0.67 photons created per helium atom in the ‘main’ \( 2^1p-1^1s \) peak, 0.70 per helium atom in the ‘pre-recombination peak’, and 0.66 in the two-photon process. The numbers for HeII recombination are 0.62, 0.76 and 6.85 for these three processes, respectively.

### 4 DISCUSSION

#### 4.1 Modifications in the recombination calculation

There are several possible improvements that we could make to the line distortion calculation. However, as we will discuss below, we do not believe that any of them will make a substantial difference to the amplitudes of the lines.

In order to calculate the distortion lines to higher accuracy, we should use the multi-level model without any thermal equilibrium assumption among the bound states. And we also need to take into account the secondary spectral distortion in the radiation field, i.e., we cannot approximate the background radiation field \( J \) as a perfect blackbody spectrum. This means, for example, that the extra photons from the recombination of HeI may redshift into an energy range that can photoionize \( H(n = 1) \) (Dell’Antonio & Rybicki 1993; Seager et al. 2000). We can assess how significant
Figure 10. Comparison of the net 2p–1s (solid) and 2s–1s two-photon (dashed) transition rates of HeII as a function of redshift. The two-photon process is greater through most of the recombination epoch, so that most of the cosmological HeIII → HeII process happens through the two-photon transition.

Figure 11. The normalized emission spectrum for the two-photon emission process \((2^1s-1^1s)\) in HeI. Here \(y = \nu/\nu_{2s-1s}\), where \(\nu_{2s-1s} = 4.9849 \times 10^{15} \text{ Hz}\). The crosses are the calculated points from Drake et al. (1969) and Drake (1986), while the line is a cubic spline fit.

This effect might be by considering the ratio of the number of CMB background photons with energy larger than \(E_\gamma\), \(n_\gamma(>E_\gamma)\), to the number of baryons, \(n_B\), at different redshifts (see Fig. 12).

Roughly speaking, the recombination of \(H\) occurs at the redshift when \(n_\gamma(>h\nu_\alpha)/n_B\) is about equal to 1. This is because at lower redshifts there are not enough high energy background photons to photo-ionize or excite electrons from the ground state to the upper states (even to \(n = 2\)), while at higher redshift, when such transitions are possible, there are huge numbers of photons able to ionize the \(n = 2\) level. The solid line in Fig. 12 shows the effect of the helium line distortions on the number of high energy photons (above Ly\(\alpha\)) per baryon. The amount of extra distortion photons with redshifted energy larger than \(h\nu_\alpha\) coming from the recombination of HeI is only about 1 per cent of the number of hydrogen atoms. Their effect is therefore expected to be negligibly small for \(x_e\). We neglect the effect of the helium recombination photons on the hydrogen line distortion, since it is clearly going to make a small correction (at much less than the 10 per cent level).

As well as this particular approximation, there have been some other recent studies which have suggested that it may be necessary to make minor modifications to the recombination calculations presented in Seager et al. (1999, 2000). Although these proposed modifications would give only small changes to the recombination calculation, it is possible that they could have more significant effects on the line amplitudes and shapes. Recent papers have described 3 separate potential effects.

In the effective three-level model, Leung, Chan & Chu (2004) argued that the adiabatic index of the matter should change during the recombination process, as the ionized gas becomes neutral, giving slight differences in the recombination history. Dubrovich & Grachev (2004) have claimed that the two-photon rate between the lowest triplet state and the ground state and that between the upper singlet states and the ground state should not be ignored in the recombination of HeI. And Chluba & Sunyaev (2003) suggested that one should also include stimulated emission from the 2s state of H, due to the low frequency photons in the CMB blackbody spectrum. Even if all of these effects are entirely completely correct, we find that the change to the amplitude of the main spectral distortion is much less than 10 per cent. We therefore leave the detailed discussion of these and other possible modifications to a future paper.

4.2 Possibility of detection

There is no avoiding the fact that detecting these CMB spectral distortions will be difficult. There are 3 main challenges to overcome: (1) achieving the required raw sensitivity; (2)
removing the Galactic foreground emission; and (3) distinguishing the signal from the CIB.

Let us start with the first point. We can estimate the raw sensitivity achievable in existing or planned experiments (even although these instruments have not been designed for measuring the line distortion). Since the relevant wavelength range is essentially impossible to observe from the ground, it will be necessary to go into space, or at least to a balloon-based mission. One existing experiment with sensitivity at relevant wavelengths is BLAST (Devin et al. 2004), which has an array of bolometers operating at 250 µm on a balloon payload. The estimated sensitivity is 236 mJy in 1 s, for a 30 arcsec FWHM beam, which corresponds to $\lambda I_\lambda = 1.2 \times 10^{-7}$ W m$^{-2}$ sr$^{-1}$. Comparing with equation (24) for the peak intensity, it would take $\sim 10^5$ such detectors running for a year to detect the line distortion. The SPIRE instrument on Herschel will have a similar bolometer array, but with better beamsize. The estimated sensitivity of 2.5 mJy at 5ø in 1 hour for a 17.4 arcsec FWHM beamsize (Griffin, Swinyard & Vigroux 2001) corresponds to $\lambda I_\lambda = 4.4 \times 10^{-8}$ W m$^{-2}$ sr$^{-1}$ per detector for the 1σ sensitivity in 1 second. So detection of the line would still require $\sim 10^6$ such detectors operating for a year.

These experiments are limited by thermal emission from the instrument itself, and so a significant advance would come from cooling the telescope. This is one of the main design goals of the proposed SAFIR (Leisawitz 2004) and SPICA (Nakagawa et al. 2004) missions. One can imagine improvements of a factor $\sim 100$ for far-IR observations with a cooled mirror. This would put us in the regime where arrays of $\sim 10^4$ detectors (of a size currently being manufactured for sub-mm instruments) could achieve the desired sensitivity.

One could imagine an experiment designed to have enough spectroscopic resolution to track the shape of the expected line distortion. The minimum requirement here is rather modest, with only $\lambda/\Delta \lambda \sim 10$ in at least 3 bands. An important issue will be calibration among the different wavelengths, so that the non-thermal shape can be confidently measured. To overcome this, one might consider the use of direct spectroscopic techniques rather than filtered or frequency-sensitive bolometers.

Another way of quoting the required sensitivity is to say that any experiment which measures the recombination line distortion would have to measure the CIB spectrum with a precision of about 1 part in $10^5$, which is obviously a significant improvement over what can be currently achieved. A detection of the line distortion might therefore naturally come out of an extremely precise measurement of the CIB spectrum, which would also constrain other high frequency distortions to the CMB spectrum.

Some of the design issues involved in such an experiment are discussed by Fixsen & Mather 2002. They describe a future experiment for measuring deviations of the CMB spectrum from a perfect blackbody form, with an accuracy and precision of 1 part in 10$^5$. This could provide upper limits on Bose-Einstein distortion $\mu$ and Compton distortion $y$ parameters at the $\sim 10^{-7}$ level (the current upper limits for $y$ and $\mu$ are $15 \times 10^{-6}$ and $9 \times 10^{-5}$, respectively; Fixsen et al. 1996). The wavelength coverage they discuss is 2–120 cm$^{-1}$ (about 80–5,000 µm), which extends to much longer wavelengths than necessary for measuring the line distortion. The beam-size would be large, similar to FIRAS, but the sensitivity achieved could easily be 100 times better. An experiment meant for detecting the line distortion would have to be another couple of orders of magnitude more sensitive still.

Turning to the second of the major challenges, it will be necessary to detect this line in the presence of the strong emission from our Galaxy. At 100 µm the Galactic Plane can be as bright as $\sim 10^3$ MJy sr$^{-1}$ which is about a billion times brighter than the signal we are looking for! Of course the brightness falls dramatically as one moves away from the Plane, but the only way to confidently avoid the Galactic foreground is to measure it and remove it. So any experiment designed to detect the line distortion will need to cover some significant part of the sky, so that it will be possible to extrapolate to the cosmological background signal. The spectrum of the foreground emission is likely to be smoother than that of the line distortion, and it may be possible to use this fact to effectively remove it. However, it seems reasonable to imagine that the most efficient separation of the signals will involve a mixture of spatial and spectral information, as is done for CMB data (see e.g. Patanchon et al. 2004).

In the language of spherical harmonics, the signal we are searching for is a monopole, with a dipole at the $\sim 10^{-3}$ level and smaller angular scale fluctuations of even lower amplitude. Hence we would expect to be extrapolating the Galactic foreground signals so that we can measure the overall DC level of the sky. This is made much more difficult by the presence of the CIB, which is also basically a monopole signal. Hence spatial information cannot be used to separate the line distortion from the CIB. The measurement of the line distortion is therefore made much more difficult by the unfortunate fact that the CIB is several orders of magnitude brighter – this is the third of the challenges in measuring the recombination lines.

The shape of the CIB spectrum is currently not very well characterised. It was detected using data from the DIRBE and FIRAS experiments on the COBE satellite. Estimates for the background ($\lambda I_\lambda$) are: 9 nW m$^{-2}$ sr$^{-1}$ at 60 µm (Miville-Deschênes, Lagache & Puget 2002); 23 nW m$^{-2}$ sr$^{-1}$ at 100 µm (Lagache, Haffner & Reynolds 2000); 15 nW m$^{-2}$ sr$^{-1}$ at 140 µm (Lagache et al. 1999; Hauser et al. 1998); and 11 nW m$^{-2}$ sr$^{-1}$ at 240 µm (Lagache et al. 1999; Hauser et al. 1998). In each case the detections are only at the 3–5σ level, and the precise values vary between different prescriptions for data analysis (see also Schlegel, Finkbeiner & Davis 1998; Finkbeiner, Davis & Schlegel 2000; Hauser & Dwek 2001; Wright 2004). The short wavelength distortion of the CMB, interpreted as a measurement of the CIB (Puget et al. 1996) can be fit with a modified blackbody with temperature 18 ± 5 K and emissivity index 0.64 (although there is degeneracy between these parameters), which we plotted in Fig. 2.

The CIB is thus believed to peak somewhere around 100 µm, which is just about where we are expecting the recombination line distortion. The accuracy with which the CIB spectrum is known will have to improve by about 5 orders of magnitude before the distortion will be detectable. Fortunately the spectral shape is expected to be significantly narrower than that of the CIB – the line widths are similar to the $\delta z/z \sim 0.1$ for the last scattering surface thickness, as
opposed to $\delta \lambda / \lambda \sim 1$ for a modified blackbody shape (potentially even wider than this, given that the sources of the CIB come from a range of redshift $\Delta z \sim 1$).

One issue, however, is how smooth the CIB will be at the level of detail with which it will need to be probed. It may be that emission lines, absorption features, etc. could result in sufficiently narrow structure to obscure the recombination features. We are saved by 2 effects here: firstly the CIB averaged over a large solid angle patch is the sum of countless galaxies, and hence the individual spectral features will be smeared out; and secondly, the far-IR spectral energy distributions of known galaxies do not seem to contain strong features of the sort which might mimic the recombination distortion (see e.g. Lagache, Puget & Dole 2005). As we learn more about the detailed far-IR spectra of individual galaxies we will have a better idea of whether this places a fundamental limit on our ability to detect the recombination lines.

Overall it would appear that the line distortion should be detectable in principle, but will be quite challenging in practice.

5 CONCLUSIONS

We have studied the spectral distortion to the CMB due to the Ly$\alpha$ and 2$s$–1$s$ two-photon transition of H and the corresponding lines of HeI and HeII. Together these lines give a quite non-trivial shape to the overall distortion. The strength and shape of the line distortions are very sensitive to the details of the recombination processes in the atoms. Although the amplitude of the spectral line is much smaller than the Cosmic Infrared Background, the raw precision required is within the grasp of current technology, and one can imagine designing an experiment to measure the non-trivial line shape which we have calculated. The basic detection of the existence of this spectral distortion would provide incontrovertible proof that the Universe was once a hot plasma and its amplitude would give direct constraints on physics at the recombination epoch.

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