Non-Singular Method of Fundamental Solutions for Two-dimensional Thermoelasticity

QG Liu¹, B Šarler¹,²
¹University of Nova Gorica, Nova Gorica, Slovenia
²Institute of Metals and Technology, Ljubljana, Slovenia

E-mail: qingguo.liu@ung.si, bozidar.sarler@ung.si

Abstract. A framework for simulation of thermomechanical processing of microstructures by the method of fundamental solutions, free of artificial boundary, is shown. The formulation of the method for two-dimensional thermal and mechanical models is presented. In particular, the formulation for elastic and thermo-elastic problems is discussed. In the thermal and mechanical models, the concentrated point sources are replaced by the distributed sources over the sphere around the singularity to regularize the singularities. The balance of heat fluxes and forces is used to calculate some of the otherwise singular diagonal coefficients. This procedure enables also to solve a broad spectra of thermomechanical problems. The novel boundary meshless method has been assessed by comparison with the method of fundamental solutions, and analytical solution. The method is easy to code, accurate, and efficient.

1. Introduction
When an elastic solid exhibits volume changes due to temperature, it is subject to thermoelastic loadings. The present papers deals with a novel variant of the Method of Fundamental Solutions (MFS) for solving two-dimensional thermoelastic problems. The MFS is a numerical technique that belongs to the class of methods generally called boundary methods. It has been successfully applied to a large variety of physical problems, an account of which may be found in the survey papers [1,2]. The MFS was in conjunction with the method of particular solutions and the dual reciprocity method already applied [3,4] to the numerical solution of direct problems in three-dimensional isotropic linear thermoelasticity. Afterwards, the two-dimensional linear thermoelasticity [5,6] problems were solved by MFS. Some recent applications of MFS to thermoelastic problems can be found in [7,8,9].

In the traditional MFS, the determination of the proper distance between the real boundary and the fictitious boundary is troublesome. In recent years, various efforts have been made, aiming to remove this drawback of the MFS, so that the source points can be placed on the real boundary directly [10,11]. A new boundary meshfree approach named Non-singular MFS (NMFS) for isotropic [12] and anisotropic [13] elasticity problems has been recently presented, based on the Boundary Distributed Source (BDS) method [14]. The NMFS has been recently extended to solve porous media problems with moving boundaries [15] and Stokes flow problems [16]. The NMFS has been recently upgraded for solving the multi-body elastic problems [17]. The NMFS does not involve fictitious boundaries and singularities. In NMFS, the concentrated point sources are replaced with the area-distributed sources covering the source points for 2D problems. These area-distributed sources represent analytical integration of the original singular fundamental solution, so that they preserve the
2. Governing Equation
Consider a two-dimensional ideally isotropic thermoelastic solid confined to domain $\Omega$ with boundary $\Gamma$. Let us introduce a two-dimensional Cartesian coordinate system with orthonormal base vectors $\mathbf{i}_x$ and $\mathbf{i}_y$, and coordinates $p_x$ and $p_y$ of point $P$ with position vector $\mathbf{p} = p_x \mathbf{i}_x + p_y \mathbf{i}_y$. The governing thermoelastic equations for a two-dimensional steady-state heat conduction and displacement in an isotropic homogeneous medium are

$$
\frac{\partial^2 T}{\partial p_x^2} + \frac{\partial^2 T}{\partial p_y^2} = 0,
$$

$$
\frac{2\mu(1-\nu)}{(1-2\nu)} \frac{\partial^2 u_x}{\partial p_x^2} + \frac{\partial^2 u_x}{\partial p_y^2} + \frac{\mu}{(1-2\nu)} \frac{\partial^2 u_x}{\partial p_x \partial p_y} - \frac{Eh}{(1-2\nu)} \frac{\partial T}{\partial p_x} = 0,
$$

$$
\frac{2\mu(1-\nu)}{(1-2\nu)} \frac{\partial^2 u_y}{\partial p_x^2} + \frac{\partial^2 u_y}{\partial p_y^2} + \frac{\mu}{(1-2\nu)} \frac{\partial^2 u_y}{\partial p_x \partial p_y} - \frac{Eh}{(1-2\nu)} \frac{\partial T}{\partial p_y} = 0,
$$

where $T$ is temperature, $\nu$ is Poisson’s ratio, $E$ is Young’s modulus, $h$ is the coefficient of linear thermal expansion, and $\mu = E / 2(1+\nu)$. The boundary is divided into two not necessarily connected parts $\Gamma^T = \Gamma^T + \Gamma^d$ for the thermal problem and $\Gamma^{me} = \Gamma^d + \Gamma^e$ for the mechanical problem. On the part $\Gamma^T$ the Dirichlet boundary conditions are given, and on the part $\Gamma^d$ the Neumann boundary conditions are given. On the part $\Gamma^d$ the displacement boundary conditions are given, and on the part $\Gamma^e$ the traction boundary conditions are given

$$
T(p) = \bar{T}(p); \quad p \in \Gamma^T, \quad q(p) = \bar{q}(p); \quad p \in \Gamma^d, \quad u_\zeta(p) = \bar{u}_\zeta(p); \quad p \in \Gamma^d, \quad t_\zeta(p) = \bar{t}_\zeta(p); \quad p \in \Gamma^e, \quad \zeta = x, y,
$$

where $\bar{T}$, $\bar{q}$, $\bar{u}_\zeta$ and $\bar{t}_\zeta$ represent known functions. The normal heat flux $q$ on the boundary is related to the temperature gradients by

$$
q(p) = -\kappa \left( \frac{\partial T(p)}{\partial p_x} n_x + \frac{\partial T(p)}{\partial p_y} n_y \right),
$$

where $\kappa$ is the thermal conductivity, $n_x$ and $n_y$ denote the coordinates of the outward normal $\mathbf{n}$ at the boundary point $p$.

In the framework of isotropic linear thermoelasticity, the strain tensor $\varepsilon$ is related to the stress tensor $\sigma$ by means of the constitutive law of thermoelasticity, namely

$$
\varepsilon_{\xi \zeta} = \frac{1 + \nu}{E} \sigma_{\xi \zeta} - \frac{\nu}{E} \left( \sigma_{\xi x} + \sigma_{\xi y} \right) \delta_{\xi \zeta} + hT \delta_{\xi \zeta}, \quad \xi, \zeta = x, y.
$$
The constitutive law of thermoelasticity Eq. (6) can be conveniently expressed in terms of the stresses as

\[
\sigma_{\xi\eta} = 2\mu \left[ e_{\xi\eta} + \frac{v}{1-2v} (e_{xx} + e_{yy}) \delta_{\xi\eta} \right] - \frac{2\mu(1+v)h}{1-2v} T \delta_{\xi\eta}, \quad \xi, \eta = x, y, \tag{7}
\]

where \( v' = v / (1 + v) \), \( h' = h(1 + v) / (1 - 2v) \) for plane stress, and \( \delta_{\xi\eta} \) is the Kronecker delta

\[
\delta_{\xi\eta} = \begin{cases} 
1, & \xi = \eta, \\
0, & \xi \neq \eta, 
\end{cases} \quad \xi, \eta = x, y. \tag{8}
\]

The tractions \( t_x \) and \( t_y \) are defined in terms of the stresses as

\[
t_x = \sigma_x n_x + \sigma_y n_y, \quad \xi = x, y. \tag{9}
\]

3. Solution procedure

3.1. Fundamental solution

The fundamental solution \( T^* (p, s) \) of the heat balance equation (1) for the two-dimensional steady-state heat conduction in an isotropic homogeneous medium is

\[
T^* (p, s) = -\frac{1}{2\pi} \log \frac{r}{r_0}, \tag{10}
\]

where \( p(p_x, p_y) \) is a collocation point, \( s(s_x, s_y) \) is a source point, \( r = \left[ (p_x - s_x)^2 + (p_y - s_y)^2 \right]^{1/2} \), and \( r_0 \) stands for a constant. The corresponding fundamental normal heat flux is

\[
q^* (p, s) = -\kappa \left( \frac{\partial T^* (p, s)}{\partial p_x} n_x + \frac{\partial T^* (p, s)}{\partial p_y} n_y \right). \tag{11}
\]

The displacement can be divided into two parts as follows \( u = u'' + u''', \) where \( u'' \) is the fundamental solution of 2D elasticity problem and \( u''' \) is a particular solution of the non-homogeneous equilibrium equations (2). The corresponding strain and stress tensors are

\[
e_{\xi\eta}'' = \frac{1}{2} \left( \frac{\partial u_x''}{\partial p_x} + \frac{\partial u_y''}{\partial p_y} \right), \quad \Phi = P, H, \tag{12}
\]

\[
\sigma_{\xi\eta}'' = 2\mu \left[ e_{\xi\eta}'' + \frac{v}{1-2v} (e_{xx}'' + e_{yy}'') \delta_{\xi\eta}'' \right], \quad \Phi = P, H. \tag{13}
\]

The corresponding traction is

\[
t = t'' + t''' = \frac{2\mu(1+v)h}{1-2v} T n. \tag{14}
\]

In case of Cauchy-Navier system, associated with the two-dimensional isotropic linear elasticity, the fundamental solution for the displacement vector \( U'' \) is given as

\[
U''_{\xi\eta} (p, s) = \frac{1}{8\pi\mu(1-v)} \left[ -(3-4v) \log \frac{r}{r_0} \delta_{\xi\eta} + r_x r_y \right]. \tag{15}
\]
The fundamental solution for the traction vector \( T^H \) in case of two-dimensional isotropic linear elasticity is obtained as [12]

\[
T^H_{\psi\varphi}(\mathbf{p}, \mathbf{s}) = \frac{2\mu}{1-2\nu} \left[ (1-\nu) \frac{\partial U^H_\varphi(\mathbf{p}, \mathbf{s})}{\partial \hat{p}_\psi} + \nu \frac{\partial U^H_\psi(\mathbf{p}, \mathbf{s})}{\partial \hat{p}_\varphi} \right] n_\varphi + \mu \left[ \frac{\partial U^H_\varphi(\mathbf{p}, \mathbf{s})}{\partial \hat{p}_\psi} + \frac{\partial U^H_\psi(\mathbf{p}, \mathbf{s})}{\partial \hat{p}_\varphi} \right] n_\psi, \zeta \neq \xi. \tag{16}
\]

The particular solution for the displacement vector \( U^p \) is given as

\[
U^p_\psi(\mathbf{p}, \mathbf{s}) = -\frac{h(1+\nu)}{4\pi(1-\nu)(p_\xi - s_\xi)} \log \frac{r}{r_0}. \tag{17}
\]

The corresponding particular traction vector \( T^p \) is given by [5]

\[
T^p_\psi(\mathbf{p}, \mathbf{s}) = \frac{\mu}{1-2\nu} \left[ (1-\nu) \frac{\partial U^p_\varphi(\mathbf{p}, \mathbf{s})}{\partial \hat{p}_\psi} + \nu \frac{\partial U^p_\psi(\mathbf{p}, \mathbf{s})}{\partial \hat{p}_\varphi} \right] n_\varphi + \mu \left[ \frac{\partial U^p_\varphi(\mathbf{p}, \mathbf{s})}{\partial \hat{p}_\psi} + \frac{\partial U^p_\psi(\mathbf{p}, \mathbf{s})}{\partial \hat{p}_\varphi} \right] n_\psi, \zeta \neq \xi. \tag{18}
\]

It can be shown that the following \( T, u_\xi \) and \( u_\eta \) satisfy the governing Eqs. (1) and (2)

\[
T(\mathbf{p}) = \gamma T^H(\mathbf{p}, \mathbf{s}),
\]

\[
u_\xi(\mathbf{p}) = u^H_\xi(\mathbf{p}) + u^p_\xi(\mathbf{p}) = U^H_\theta(\mathbf{p}, \mathbf{s}) - \alpha + U^H_\xi(\mathbf{p}, \mathbf{s}) - \beta + U^p_\xi(\mathbf{p}, \mathbf{s}) - \gamma, \zeta = x, y, \tag{20}
\]

where \( \gamma, \alpha, \beta \) and represent arbitrary constants. The expressions \( U^H_\psi(\mathbf{p}, \mathbf{s}), U^p_\psi(\mathbf{p}, \mathbf{s}) \) and \( T^p(\mathbf{p}, \mathbf{s}) \) are singular when \( \mathbf{p} = \mathbf{s} \). We have in the past [12,13] used the desingularization technique, proposed by [14], for evaluating the singular values. We modify the approach [14] in the sense of preserving the original fundamental solution in all points except near the singularity, and by scaling the singularity with the area of the circle over which the desingularization integration is performed. This allows us to treat the MFS and the NMFS in a formally the same way. The desingularization is performed as

\[
T^\star(\mathbf{p}, \mathbf{p}_\nu) = \begin{cases} 
T^H(\mathbf{p}, \mathbf{p}_\nu), & r > R, \\
\frac{1}{\pi R^2} \int_{p \in A(p^n, R)} T^H(\mathbf{p}, \mathbf{p}_\nu) \, dA, & r \leq R,
\end{cases} \tag{21}
\]

\[
U^H_\psi(\mathbf{p}, \mathbf{p}_\nu) = \begin{cases} 
U^H_\psi(\mathbf{p}, \mathbf{p}_\nu), & r > R, \\
\frac{1}{\pi R^2} \int_{p \in A(p^n, R)} U^H_\psi(\mathbf{p}, \mathbf{p}_\nu) \, dA, & r \leq R,
\end{cases} \tag{22}
\]

\[
U^p_\psi(\mathbf{p}, \mathbf{p}_\nu) = \begin{cases} 
U^p_\psi(\mathbf{p}, \mathbf{p}_\nu), & r > R, \\
\frac{1}{\pi R^2} \int_{p \in A(p^n, R)} U^p_\psi(\mathbf{p}, \mathbf{p}_\nu) \, dA, & r \leq R,
\end{cases} \tag{23}
\]

where \( A(p^n, R) \) (see Fig. 1) represents a circle with radius \( R \), centred around \( \mathbf{p}_\nu \). And

\[
U^H_{xx}(\mathbf{p}, \mathbf{p}_\nu) = U^H_{yy}(\mathbf{p}, \mathbf{p}_\nu) = \frac{1}{8\pi\mu(1-\nu)} \left[ (3-4\nu) \log \left( \frac{R}{r_0^2} \right) + \frac{1}{2} \right], \quad U^H_{xy}(\mathbf{p}, \mathbf{p}_\nu) = U^H_{yx}(\mathbf{p}, \mathbf{p}_\nu) = 0, \tag{24}
\]
\[ \tilde{U}_x(p, n, p_n) = \tilde{U}_y(p, n, p_n) = 0, \]  
\[ \tilde{T}^r(p, n, p_n) = -\frac{1}{2\pi} \left( \log \frac{R - r_0}{r_0/2} \right). \]  

When \( p \neq p_n \), are the corresponding expressions are

\[ q(p) = \gamma_n \tilde{q}^r(p, p_n), \]

\[ t_v(p) = t_v^H(p, n) + t_v^F(p, n) - \frac{2\mu(1 + \nu)h}{1 - 2\nu} T(p)n_v \]

\[ = \tilde{T}_{\xi x}^H(p, p_n)\alpha_n + \tilde{T}_{\xi y}^H(p, p_n)\beta_n + \left( \tilde{T}_{\xi x}^F(p, p_n) - \frac{2\mu(1 + \nu)h}{1 - 2\nu} \tilde{T}^r(p, p_n)n_v \right)\gamma_n, \]

where

\[ \tilde{q}^r(p, p_n) = q^r(p, p_n), \quad \tilde{T}_{\xi x}^H(p, p_n) = T_{\xi x}^H(p, p_n), \quad \tilde{T}_{\xi y}^H(p, p_n) = T_{\xi y}^H(p, p_n). \]

3.2. Discretisation

The solution of the problem is sought in the form

\[ u_v(p) = \sum_{n=1}^{N} \tilde{U}_{\xi x}^H(p, p_n)\alpha_n + \sum_{n=1}^{N} \tilde{U}_{\xi y}^H(p, p_n)\beta_n + \sum_{n=1}^{N} \tilde{U}_{\xi}^F(p, p_n)\gamma_n, \]

\[ T(p) = \sum_{n=1}^{N} \tilde{T}_{\xi}^r(p, p_n)\gamma_n, \quad p \notin \bigcup_{n=1}^{N} A(p_n, R). \]

The heat flux and the tractions can be expressed as

\[ q(p) = \sum_{n=1}^{N} \tilde{q}^r(p, p_n)\gamma_n, \quad p \notin \bigcup_{n=1}^{N} A(p_n, R). \]

\[ t_v(p) = \sum_{n=1}^{N} \tilde{T}_{\xi x}^H(p, p_n)\alpha_n + \sum_{n=1}^{N} \tilde{T}_{\xi y}^H(p, p_n)\beta_n + \sum_{n=1}^{N} \tilde{T}_{\xi}^F(p, p_n) - \frac{2\mu(1 + \nu)h}{(1 - 2\nu)} \tilde{T}^r(p, p_n)n_v \right)\gamma_n, \]

The coefficients \( \alpha_n, \beta_n \) and \( \gamma_n \) are calculated from a system of \( 3N \) algebraic equations

\[ Ax = b, \]
where $A$ stands for a $3N \times 3N$ matrix, $x$ is a $3N \times 1$ vector, and $b$ is a $3N \times 1$ vector. The diagonal terms $\tilde{T}_{xx}^H(p_m, p_m)$, $\tilde{T}_{xx}^P(p_m, p_m)$ and $\tilde{q}'(p_m, p_m)$, $\varsigma, \xi = x, y, m = 1, ..., N$ in Eqs. (33) and (32) are determined indirectly by invoking the fact that the boundary integration of the forces and the normal gradient of the potential on the body should vanish in mechanical equilibrium. We obtain the diagonal terms by solving the following integral equations

$$
\int_{\Gamma} t_\varsigma(p)d\Gamma = \sum_{n=1}^{N} \int_{\Gamma} \tilde{T}_{xx}^H(p_m, p_n)d\Gamma \alpha_n + \sum_{n=1}^{N} \int_{\Gamma} \tilde{T}_{xx}^H(p_m, p_n)d\Gamma \beta_n + \sum_{n=1}^{N} \int_{\Gamma} \tilde{T}_{xx}^P(p_m, p_n)d\Gamma \gamma_n = 0, \tag{35}
$$

$$
\int_{\Gamma} \tilde{q}(p)d\Gamma = \sum_{n=1}^{N} \int_{\Gamma} \tilde{q}'(p_m, p_n)d\Gamma \gamma_n = 0. \tag{36}
$$

Since Eqs.(35) and (36) should be satisfied for arbitrary conditions or source density distribution, we have

$$
\int_{\Gamma} \tilde{T}_{xx}^H(p_m, p_n)d\Gamma = \int_{\Gamma} \tilde{T}_{xx}^P(p_m, p_n)d\Gamma = \int_{\Gamma} \left( \tilde{T}_{xx}^P(p_m, p_n) - \frac{2\mu h(1+v)}{(1-2v)} T_{xx}(p_m, p_n) n_{m,\varsigma} \right) d\Gamma = 0, \tag{37}
$$

$$
\int_{\Gamma} \tilde{q}'(p_m, p_n)d\Gamma = 0 \tag{38}
$$

and $\tilde{T}_{xx}^H(p_m, p_m)$, $\tilde{T}_{xx}^P(p_m, p_m)$ and $\tilde{q}'(p_m, p_m)$, $\varsigma, \xi = x, y$, can be evaluated as

$$
\tilde{T}_{xx}^H(p_m, p_m) = -\frac{1}{l_m} \sum_{n=1 \text{ or } m}^{N} \tilde{T}_{xx}^H(p_n, p_m) l_m, \tag{39}
$$

$$
\tilde{T}_{xx}^P(p_m, p_m) = -\frac{1}{l_m} \left( \sum_{n=1 \text{ or } m}^{N} \tilde{T}_{xx}^P(p_n, p_m) - \frac{2\mu h(1+v)}{(1-2v)} T_{xx}(p_m, p_m) n_{m,\varsigma} \right) l_m + \frac{2\mu h(1+v)}{(1-2v)} T_{xx}(p_m, p_m) n_{m,\varsigma}, \tag{40}
$$

$$
\tilde{q}'(p_m, p_m) = -\frac{1}{l_m} \sum_{n=1 \text{ or } m}^{N} \tilde{q}'(p_n, p_m) l_m , \tag{41}
$$

by approximating the integrals by assuming a constant value of the fundamental tractions on the boundary segments $l_m$. The Eqs. (39) - (41) represent the essence of the NMFS. By knowing all the elements of $A$ and $b$ of the system (34), we can determine the values of $x$ (i.e. $\alpha_s$, $\beta_s$ and $\gamma_s$). Afterwards, we can calculate the solution of the governing equation from

$$
\begin{align*}
\mu_s(p) &= \sum_{n=1}^{N} U_{\varsigma}^{H}(p, p_m)\alpha_n + \sum_{n=1}^{N} U_{\varsigma}^{H}(p, p_m)\beta_n + \sum_{n=1}^{N} U_{\varsigma}^{P}(p, p_m)\gamma_n, \quad \varsigma = x, y, \\
T(p) &= \sum_{n=1}^{N} T_{\varsigma}(p, p_m)\gamma_n, 
\end{align*}
\tag{42}
$$

where $p$ is any point inside the domain or on the boundary.

4. Numerical example

Due to the limited space, we present only one numerical example with Dirichlet boundary conditions. The material constants are taken to be as follows: $\mu = 4.8 \times 10^{10}$ N/m$^2$, $v = 0.34$, $\kappa = 4.01$ W/m$^2$K$^{-1}$.
Figure 2. Comparison of numerically calculated temperatures and displacements at the selected 12 points with the analytical solution. (analytical solution: –. MFS result: • N = 150. NMFS result: ◊ N = 60, ▷ N = 90, ● N = 120, ◯ N = 150).
\( h = 16.5 \times 10^{-6} \ \text{C}^{-1} \), and \( r_0 = 2 \). We consider an example (as in [7]) on the unit disk with exact solution \( T(p) = 100 \log \|p-p_0\| \), \( u(p) = \left[h(1+\nu)/2(1-\nu)\right] (p-p_0) T(p) \). The Dirichlet boundary condition is defined on the whole boundary. The number of boundary nodes used is 60, 90, 120, and 150. The radius of the circular disk for the distributed area source covering each node is set as \( R = d/4 \), where \( d \) is the smallest distance between the two nodes on the boundary. A total number of \( M = 12 \) field points is selected inside the circle along the line \( r = 0.5 \textrm{m} \) and with \( 0 \leq \theta \leq 360 \), and the solution is computed and compared with the analytical solution in Fig. 2. A good agreement of the NMFS results with the analytical solution and MFS results is observed in Fig. 2. The solution improves with the increased number of the boundary nodes. The solution of MFS and NMFS almost coincides for \( N = 150 \).

5. Conclusion

In the present paper, a novel NMFS for two-dimensional steady-state thermoelasticity problems is developed. The fundamental solution is integrated over small disks to avoid the singularity in this approach. The fact that the boundary integration of the forces and the normal gradient of the potential on the body should vanish in mechanical equilibrium is used to determine the respective desingularized fundamental tractions and flux. The NMFS gives similar results as the MFS without the artificial boundary. The NMFS is very general and can be adapted and extended to handle many related problems, such as three-dimensional thermoelasticity problems, large multi-body problems, and thermoplasticity problems. The listed problems represent the directions of our future work.

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