ECCENTRIC BLACK-HOLE–NEUTRON-STAR Mergers

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Received 2011 May 16; accepted 2011 June 29; published 2011 July 15

ABSTRACT

Within the next few years gravitational waves (GWs) from merging black holes (BHs) and neutron stars (NSs) may be directly detected, making a thorough theoretical understanding of these systems a high priority. As an additional motivation, these systems may represent a subset of short-duration gamma-ray burst progenitors. BH–NS mergers are expected to result from primordial, quasi-circular inspiral as well as dynamically formed capture binaries. The latter channel allows mergers with high eccentricity, resulting in a richer variety of outcomes. We perform general relativistic simulations of BH–NS interactions with a range of impact parameters, and find significant variation in the properties of these events that have potentially observable consequences, namely, the GW signature, remnant accretion disk mass, and amount of unbound material.

Key words: black hole physics – gamma-ray burst: general – gravitation – gravitational waves – stars: neutron

Online-only material: color figures

1. INTRODUCTION

Merging binaries consisting of black holes (BHs) and neutron stars (NSs) are prime targets for observation by ground-based gravitational wave (GW) detectors (such as LIGO; Abramovici et al. 1992) and may be the progenitors of some short-hard gamma-ray bursts (sGRBs; Narayan et al. 1992). The great diversity of sGRB characteristics and the potential variation in the corresponding GW signals motivates a thorough investigation of the possible outcomes of binary compact object (BCO) mergers. BCOs may form through evolution of primordial binaries or through dynamical processes in star clusters (O’Leary et al. 2009; Lee et al. 2010). The latter population motivates this study of BH–NS interactions for systems that are initially marginally unbound.

Star clusters at the centers of galaxies undergo mass segregation, resulting in heavier objects concentrated toward the center (see, e.g., Bahcall & Wolf 1977). Recent Fokker–Plank models suggest that the Galactic nuclear cluster (NC) should have \(\sim 1800\) BHs and \(\sim 400\) NSs in the central 0.1 pc (Hopman & Alexander 2006). Such clusters are thus promising sites for BH–NS close encounters. Using models of galactic NCs, O’Leary et al. (2009) calculate the rate of binary BH formation through GW emission in close encounters. They find corresponding Advanced LIGO detection rates between 5 and 2700 per year, and estimate that the BH–NS rate could be around 1% of this. These capture binaries form with relatively small periapsis separations, \(r_p\); in particular, \(\sim 30\%\) form with \(r_p \lesssim 10\) M, where \(M\) is the total mass of the system (Figure 4 of O’Leary et al. 2009). (Unless otherwise stated we employ geometric units with \(G = c = 1\).) This is due in part to the large velocity dispersion in the cluster core (\(\sim 1000\) km s\(^{-1}\)), but also to gravitational focusing, which may be understood as follows. The total rate is proportional to the cross-section: \(\Gamma \propto \pi b^2\), where \(b\) is the impact parameter. However, for Newtonian hyperbolic orbits with relative velocity \(w\) at infinity, \(r_p = b^2 w^2 / 2M + O(w^4)\). Thus, the rate is linearly proportional to \(r_p\).

Globular clusters (GCs) that have undergone core collapse may also host BCO close encounters due to the high density of compact objects in their cores (Fabian et al. 1975; Grindlay et al. 2006). For example, models of M15 calibrated to the observational velocity dispersion yield an NS fraction of \(\sim 55\%\) in the inner 0.2 pc (Dull et al. 1997). Lee et al. (2010) calculate the expected rate of BCO interactions inside M15 as a function of time and then scale these results for GCs with a distribution of half-mass relaxation times. Depending upon the GC evolution model, they find that the global rate for BH–NS collisions (i.e., events for which \(r_p \lesssim R_{\text{NS}} + R_{\text{BH}}\)) peaks at \(\sim 8–25\) yr\(^{-1}\) Gpc\(^{-3}\) at redshifts between \(z = 0.36\) and \(z = 0.97\), and slowly declines to between \(50\%\) and \(85\%\) of peak by \(z = 0\). (We obtained the BH–NS collision rate by re-scaling their NS–NS results according to the factors in Table 3 of Lee et al. 2010.) For the fiducial BH–NS system considered by Lee et al. (2010), collisions occur at \(r_p \lesssim 2.7\) M. Since the rate scales linearly with \(r_p\), this implies an interaction rate \(\sim 30–100\) yr\(^{-1}\) Gpc\(^{-3}\) with (for example) \(r_p \lesssim 10\) M.

Population synthesis models (Belczynski et al. 2010) find comparable rates for primordial BH–NS mergers: from \(\sim 0.1\) yr\(^{-1}\) Gpc\(^{-3}\) (pessimistic) to \(\sim 120\) yr\(^{-1}\) Gpc\(^{-3}\) (optimistic). However, primordial BH–NS binaries will enter the LIGO band with essentially zero eccentricity (Kowalska et al. 2011). Thus, GW signals from BH–NS close encounters should be readily distinguishable due to their significant eccentricities. We further note that the sGRB rate, \(8–30\) yr\(^{-1}\) Gpc\(^{-3}\) (Guetta & Piran 2006), is comparable to the primordial BH–NS rate, and somewhat less than the expected NS–NS merger rate (30–400 yr\(^{-1}\) Gpc\(^{-3}\); Belczynski et al. 2010). The estimates of Lee et al. (2010) thus suggest that close encounters in clusters could contribute significantly to the sGRB progenitor population, especially if their emission is less tightly beamed than that of primordial mergers (Grindlay et al. 2006).

BH–NS mergers in clusters with \(r_p \lesssim 10\) M will exhibit complicated behaviors probing the strong field regime of general relativity (GR). For stellar mass BH companions, one cannot treat the NS as a perturbation of the BH spacetime. Furthermore, the nonlinear nature of GR will most strongly manifest during a close encounter of the BH–NS. Numerical simulations within full GR are thus the preferred tool for exploring these systems. To date, such simulations have been performed by several groups (Etienne et al. 2009; Kyutoku et al. 2010; Duez et al. 2009).
Our three-dimensional numerical code solves the Einstein field equations using finite difference (FD) techniques with Berger and Oliger style adaptive mesh refinement (AMR; Berger & Oliger 1984). The numerical scheme for evolving the spacetime metric is substantively the same as the generalized harmonic method described by Pretorius (2005a, 2005b), except that the FD scheme is fourth-order accurate and uses fourth-order Runge–Kutta time integration, and enforce strict conservation at AMR boundaries using the flux correction method of Berger & Colella (1989). We have implemented several methods for calculating inter-cell fluxes (HLL, Harten et al. 1983; the Roe field equations using finite difference (FD) techniques with & Colella (1989). We have implemented several methods for at AMR boundaries using the flux correction method of Berger & Colella (1989), except that the FD scheme is fourth-order accurate and uses fourth-order Runge–Kutta time integration, and enforce strict conservation

We have performed three-dimensional, high-resolution simulations of the two-body problem for initial encounters with error bars (where appropriate) computed from convergence calculations. Our simulations employ compactified coordinates such that the outer boundaries extend to spatial infinity. Thus, the global (ADM) $M$ and $J$ should be conserved. In practice, however, we must evaluate these quantities at a finite distance, making them subject to gauge artifacts, some propagating outward from the central BH/NS region from $t = 0$. For $t < 200 M$, an extraction sphere of 300 $M$ is free of propagating artifacts, whence $M (J)$ is conserved to better than 0.3 (2.0)% for all cases at medium resolution.

Based on the above of runs, some guidance from perturbative results (Peters & Mathews 1963; Turner 1977; Berry & Gair 2010), and a zoom-whirl geodesic analog of the two-body problem (Pretorius & Khurana 2007), we conjecture the following qualitative behavior as a function of $r_p$ for initial encounters resulting in a bound system.

Consider $n$, the non-negative, integer number of periapsis passages before disruption/plunge and $r_p^{(n)}$, the periapsis distance on the $n$th encounter for $1 \leq i \leq n$. Define $r_p^{(n+1)}$ to be an effective periapsis distance for the final close encounter (i.e., the corresponding $r_p$ before the disruption/plunge part of the final encounter). Group (1) above has $n = 0$, group (2) $n = 1$, and group (3) $n \geq 1$. The behavior of a close encounter will depend sensitively on the distance $\delta r_p = r_p^{(n)} - r_p$ between $r_p^{(n)}$ and a radius $r_c$ of an effective unstable circular orbit with the same energy and angular momentum. If $\delta r_p$ is sufficiently small (relative to $M$), the orbit will exhibit a whirl phase, where it asymptotes to a nearly circular orbit. The smaller $\delta r_p$, the longer the duration of the whirl, with a maximum when $\delta r_p = 0$ that equals the time required for the binary to lose its excess orbital kinetic energy, also adequately describes such events in GCs. We superimpose initial data for the BH and NS at an initial separation of 50 $M$ (498 km) with initial velocities according to a Newtonian orbit with the desired $r_p$. Though these superimposed initial data do not strictly satisfy the constraint equations except at infinite separation, tests performed at various initial separations indicate that 50 $M$ is sufficiently large that the constraint violation does not appreciably affect the system (relative to truncation error).

3. RESULTS AND DISCUSSION

We consider a range of periapsis separations from $r_p/M = 5.0$ to 15 (i.e., 50 to 150 km). (Henceforth, we will consider $r_p$ to be normalized by $M$.) In all of these cases, sufficient energy is carried away by GWs to result in a bound system. Our simulations exhibit three types of behavior: (1) a direct plunge ($r_p = 5.0, 5.83, 6.67, 6.81$), (2) following the initial periapsis passage, a single elliptical orbit and then a plunge ($r_p = 6.95, 7.22, 7.5$), and (3) following the initial periapsis passage, a long-period elliptical orbit ($r_p = 8.75, 10.0, 12.5, 15.0$). For the latter group (and the high resolution $r_p = 7.5$ run), the entire orbit is prohibitively long to simulate, and we focus on the burst of GWs associated with the first periapsis passage. For one case in each class ($r_p = 5.0, 7.5, 10.0$), we ran three simulations with different characteristic mesh spacings (but always with seven refinement levels) for convergence studies. At $t = 0$, the low (medium, high) resolution run had finest meshes covering the BH and NS of roughly $80^3 (100^3, 150^3)$ cells, resolving the NS diameter with $\sim 40 (50, 75)$ cells and the BH horizon diameter with $\sim 70 (85, 130)$ cells. (We note that the level structure is set by truncation error estimates and is adjusted with time.)

All other simulations were run at medium resolution. Unless otherwise noted, results will be reported for medium resolution, with error bars (where appropriate) computed from convergence calculations. Our simulations employ compactified coordinates such that the outer boundaries extend to spatial infinity. Thus, the global (ADM) $M$ and $J$ should be conserved. In practice, however, we must evaluate these quantities at a finite distance, making them subject to gauge artifacts, some propagating outward from the central BH/NS region from $t = 0$. For $t < 200 M$, an extraction sphere of 300 $M$ is free of propagating artifacts, whence $M (J)$ is conserved to better than 0.3 (2.0)% for all cases at medium resolution.

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either via GW emission or tidal transfer of energy to the NS material. The excess energy is the difference between the total energy of the binary entering the whirl phase and a putative binary on a quasi-circular inspiral at \( r_p \).

Because of the requirement that \( \delta r_p^{\dot{r}_p} \) be small, we only expect the possibility of significant whirling near the ultimate or penultimate encounter. If \( \delta r_p^{\dot{r}_p} \) is negative, the whirl will directly transition to a plunge. If positive and small, there will be a separation following the whirl; however, the effective \( r_c \) for the next encounter increases while \( r_p^{\dot{r}_p} \) decreases due to GW emission, and since this is quite sizeable for our 4:1 mass ratio system, \( \delta r_p^{\dot{r}_p} \) will likely be negative, resulting in a subsequent plunge. Furthermore, the separation where the NS starts to be tidally disrupted is within the radii of unstable whirl orbits, hence the NS will not survive any prolonged whirl phase. Prior encounters (for larger \( n \) cases) will not exhibit significant whirling, and while \( \delta r_p^{\dot{r}_p} \) is large the orbital evolution could better be described as a series of precessing ellipses with decreasing eccentricity and semimajor axis.

Considering the number of orbits \( n \) as a function of the initial \( r_p, n(r_p) \) is monotonically increasing, and the values of \( r_p \) where one could see a notable final whirl would be near the steps in \( n(r_p) \). The most pronounced whirl behavior will occur for small \( n(r_p) \); as \( n(r_p) \) increases, the amount of excess kinetic energy left over once the final orbit is reached decreases, and for sufficiently large \( n \) the late stages will essentially follow a quasi-circular inspiral.

Figure 1 illustrates some of the varied phenomena encountered near the transition between \( n(r_p) = 0 \) and \( n(r_p) = 1 \), occurring at \( r_p \approx 6.88 \pm 0.08 \). Most striking is the amount of rest mass remaining after the merger as a function of \( r_p \) (Table 1). For cases in which the NS plunges without significant disruption, such as \( r_p = 5 \) or \( r_p = 7.5 \), less than 1% of the initial mass is available to form an accretion disk. However, for \( r_p \) closer to the first transition \( (r_p = 6.67 \) and 6.81, see Figure 1), the NS is stretched into a long tidal tail, and a sizeable amount of bound material is left to form an accretion disk—12% of the initial NS rest mass for \( r_p = 6.81 \).

Figure 2 shows the approximate rate of fallback as a function of time for \( r_p = 6.81, 6.95, 7.22, \) and 7.5. This is the rate at which material on elliptical orbits is expected to return to the accretion disk (see Rosswog 2007). (These accretion rates are likely upper limits since they do not account for nuclear burning; see Metzger et al. 2010.) The fallback rate is larger for cases with larger disk masses, such as \( r_p = 6.81 \), but all cases exhibit an approximate \( t^{-5/3} \) falloff. This time dependence was predicted for stellar disruptions around supermassive BHs by Rees (1988).

It appears unlikely that BH–NS mergers with fallback rates as in Figure 2 will be able to explain sGRBs with extended emission (see, e.g., Norris & Bonnell 2006) if this emission is due to feeding of the accretion disk at late times. For example, by \( t \approx 100 \) s, the luminosity for the \( r_p = 6.81 \) case would be only \( L \sim \eta M c^2 \sim 2 \times 10^{52} \) erg s\(^{-1}\) (assuming an efficiency \( \eta = 0.1 \)).

Table 1 also shows the total energy and angular momentum lost to GWs for \( r_p \) between 5 and 12.5. For the cases that we followed through merger, we find 0.7%-1.7% of the total mass lost to GW energy and estimate the final spins of the BHs to be \((0.49 \pm 0.01, 0.45, 0.37, 0.47, 0.50, 0.50)\) for \( r_p = (5.00, 6.67, 6.81, 6.95, 7.22, 7.50) \), respectively. The
energy loss is largest for the transitional case \((r_p = 6.95)\), which has a large pulse from the whirling first passage and a second burst from the merger (Figure 3). Table 1 also shows the GW losses for the initial encounter in cases where the NS survives the periapsis passage (Columns 5 and 6). These fly-by pulses can be compared (Figure 3, lower panels, and Figure 4) with the prediction of Turner (1977), who used the Newtonian orbit together with quadrupole physics for the GW emission, which we will call the Newtonian Quadrupole Approximation (NQA). Our waveforms show roughly the same pulse shape as the NQA prediction but have larger amplitudes for the smaller \(r_p\) cases. At \(r_p = 15\) we find the gravitational waveform from the initial fly-by to be indistinguishable from the NQA prediction at our resolution (\(\pm 10\%\)).

The enhancement in GW energy losses for close encounters may be due (in part) to zoom-whirl-like behavior. Figure 4

![Figure 3. Upper panel: the Newman–Penrose scalar \(\Psi_4\) on the z-axis (orthogonal to the orbit) for \(r_p = 6.95\). The first pulse is from the initial close encounter, the second from the merger-ringdown. Between the pulses there is an oscillation due to the rotating, distorted neutron star, which is significantly torqued during the first encounter. Here, \(t = 0\) corresponds to the start of the simulation. Lower panels: the real and imaginary components (black diamonds and red squares) of the \(C_{22}\) for \(r_p = 7.5\) (left), and \(r_p = 10\) (right). For comparison the NQA analytical results are shown multiplied by an overall factor so that the magnitude and phase match at peak (\(t = 0\)). (A color version of this figure is available in the online journal.)](image-url)
shows the GW energy loss as a function of $r_p$, along with the NQA prediction and a fit consistent with zoom-whirl dynamics (Pretorius & Khurana 2007) in the regime ($r_p \lesssim 10$) where we start to see significant departures from the NQA approximation.

4. CONCLUSIONS

An interesting result of this general relativistic study that is qualitatively consistent with previous Newtonian studies (e.g., Lee et al. 2010) is the great variability of the outcome as a function of impact parameter. For example, the remnant disk and conclusions with brief preliminary comments. These signals may be difficult to detect with instruments such as LIGO since they lack a long inspiral phase and most of the power is at high frequencies (1500–2000 Hz for the masses considered here). Using the broadband AdLIGO noise curve, we find sky-averaged signal-to-noise ratio (S/N) of 3–9 for $r_p = 5–10$, assuming a distance of 100 Mpc. Scaling the GW emission up to ($M_{\text{NS}}, M_{\text{BH}} = (2, 8) M_\odot$) and assuming optimal orientation gives S/N of 8 out to 340 Mpc for $r_p = 7.5$. Encounters with larger $r_p$ could be easier to detect since they will have a number of fly-by GW bursts before the merger. Given that some BH–NS capture binaries emit the majority of their GW power at the high frequency end of the LIGO noise curve and could be relatively weak (both in terms of GW emission and extrapolated electromagnetic emission based on disk mass), it may be a worthwhile exercise to revisit the analysis of GRB 070201 (Abbott et al. 2008) with burst templates adapted to capture driven BH–NS encounters.

We thank Adam Burrows, John Friedmann, Roman Gold, Benjamin Lackey, and Richard O’Shaughnessy for useful conversations. This research was supported by the NSF through TeraGrid resources provided by NICS under grant TG-PHY100053, the Bradley Program fellowship (B.C.S.), the NSF Graduate Research Fellowship under grant DGE-0646086 (W.E.), NSF grants PHY-0745779 (F.P.) and PHY-1001515 (B.C.S.), and the Alfred P. Sloan Foundation (F.P.). Simulations were also run on the Woodhen cluster at Princeton University.

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