Magnetization processes in quantum spin chains with regularly alternating intersite interactions

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Abstract

We consider the dependence magnetization vs. field at zero temperature for the spin–\(\frac{1}{2}\) chains in which the intersite interactions regularly vary from site to site with a period \(p\). In the limiting case when the smallest value of the intersite interactions tends to zero the chain splits into noninteracting identical fragments of \(p\) sites and the dependence magnetization vs. field can be examined rigorously. We demonstrate explicitly the appearance of plateaus in such dependence and discuss a presence of the magnetization values \(m\) predicted by the condition \(p \left( \frac{1}{2} - m \right) = \text{integer} \). We comment on the influence of the anisotropy in the interspin interaction on the magnetization profiles. Finally, we show how the case of nonzero smallest value of the intersite interactions can be considered.

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The theoretical study of (quantum) spin chains attracts much attention during last years. On the one hand, a number of quasi–one–dimensional magnetic compounds, the properties of which can be reasonably described by the one–dimensional quantum spin models, becomes available. On the other hand, quantum spin chains should exhibit various interesting properties the examining of which is of great importance from the academic point of view. Thus, the analysis of the magnetization processes at low temperatures may yield a step–like dependence magnetization vs. field. The latter problem has received a lot of interest in a numerous theoretical, numerical and experimental papers concerning a variety of spin chains and ladders [1, 2, 3].

In what follows we discuss one mechanism which generates a step–like dependence of the magnetization vs. field, that is a regular alternation of the intersite interactions. Namely, we consider a chain of $N \to \infty$ spins governed by the Hamiltonian

$$H = -h \sum_{n=1}^{N} s_n^z + \sum_{n=1}^{N} \left( J_n^x s_n^x s_{n+1}^x + J_n^y s_n^y s_{n+1}^y + J_n^z s_n^z s_{n+1}^z \right)$$

(1)

assuming that the intersite (antiferromagnetic) interactions $J_n^\alpha (\geq 0)$ vary regularly from site to site with a period $p$, i.e. a sequence of parameters is \{ $J_1^x, J_1^y, J_1^z, \ldots, J_p^x, J_p^y, J_p^z, J_1^x, J_1^y, J_1^z, \ldots$ \}. If $J_n^x = J_n^y = J_n^z = J_n$ Eq. (1) is the Hamiltonian of the isotropic Heisenberg ($XXX$) chain, if $J_n^x, J_n^y \neq 0, J_n^z = 0$ Eq. (1) corresponds to the anisotropic XY chain. For the latter chain one can differ two limiting cases, namely, i) $J_n^x = J_n^y = J_n$ — the isotropic XY ($XX$) chain and ii) $J_n^x = J_n, J_n^y = 0$ — the Ising chain. Besides, the Hamiltonian (1) contains a uniform external field $h$ directed along $z$ axis (that is called the transverse field for XY chains).

We shall be interested in the magnetization per site $m = \frac{1}{N} \sum_{n=1}^{N} \langle s_n^z \rangle$ (the angle brackets denote the thermodynamical canonical average) or more precisely in the dependence $m$ vs. $h$ at zero temperature. Oshikawa et al. [1] using the Lieb–Schultz–Mattis theorem and the bosonization techniques for a general quantum spin $s$ chain with axial symmetry argue that the magnetization obeys $p(s - m) = \text{integer}$, i.e. in the case $s = \frac{1}{2}$ the possible values of magnetization are $m = \frac{p - 2k}{2p}$, $k = 0, 1, 2, \ldots, p$. A lot of work has been done to check the above mentioned criteria using various approximate analytical approaches and numerical techniques. On the other hand, for the XX chain in a transverse field the magnetization $m$ can be calculated rigorously. The result at zero temperature reads

$$m = \frac{1}{2} \int_{-\infty}^{\infty} dE \rho(E)(2\theta(E) - 1),$$

(2)

where $\rho(E)$ is the density of states of the Jordan–Wigner fermions which is known explicitly for any finite period $p$ [4] and $\theta(E)$ is the Heaviside step function. Regular nonuniformity leads to a splitting of the fermion band of the uniform chain into several subbands that, in its turn, immediately leads to plateaus in
the dependence \( m \) vs. \( h \) as follows from the above formula (2) for \( m \). The values of the characteristic fields at which plateaus start and end up are the solutions of two algebraic equations of \( p \)th order, whereas the possible values of \( m \) are connected with the possible differences in the numbers of subbands at \( E < 0 \) and at \( E > 0 \) (see (2)). One more related work concerning the regularly alternating transverse Ising chain has been reported in Ref. [5]. Contrary to the transverse XX chain, the regular alternation of exchange interactions for the transverse Ising chain does not lead to plateaus in the dependence \( m \) vs. \( h \).

In what follows we consider in some detail a particular case of the regularly alternating spin–\( \frac{1}{2} \) chain (1) when the smallest value of the intersite interactions equals to zero. Without a loss of generality we may put \( J_0^\alpha = 0 \). In such limiting case a simple picture for explanation the zero temperature magnetization profiles emerges. Really, in this limit the chain consists of the noninteracting clusters every one of which contains \( p \) sites. The magnetization of the chain per site \( m \) follows from the magnetization of the cluster \( M_p \) after dividing by \( p \), and \( M_p = \langle \text{GS}|S_p|\text{GS} \rangle \), where \( S_p = s^z_1 + \ldots + s^z_p \) and \( |\text{GS}\rangle \) is the ground state eigenvector of the cluster Hamiltonian. The appearance of plateaus arises due to the change of the ground state with varying of the field. Such a viewpoint is known as the strong–coupling approach. It was exploited in a number of papers devoted to the spin chains with a periodic modulation of the intersite interactions and the spin ladders [3]. We discuss the strong–coupling limit to get a better understanding of the obtained earlier rigorous results for the transverse XX and transverse Ising chains by means of the continued fraction approach [4, 5] as well as to discuss the influence of the anisotropy in spin interaction on the zero temperature magnetization profiles. On the other hand, demonstrating how does the strong–coupling approach work in the exactly solvable case we can reveal a region of validity of this approximate method.

We start with the regularly alternating XX chain with \( p = 2 \). Assuming \( J_2 = 0 \) one splits the chain into noninteracting clusters containing two sites. The Hamiltonian of the cluster reads

\[
H_2 = -h (s^z_1 + s^z_2) + J_1 (s^x_1 s^x_2 + s^y_1 s^y_2). \tag{3}
\]

The eigenvalues of (3) are

\[
E_1 = -\frac{1}{2} J_1, \quad E_2 = -h, \quad E_3 = h, \quad E_4 = \frac{1}{2} J_1; \tag{4}
\]

the corresponding eigenvectors are

\[
|1\rangle = \frac{1}{\sqrt{2}} (|\uparrow_1 \downarrow_2\rangle - |\downarrow_1 \uparrow_2\rangle), \quad |2\rangle = |\uparrow_1 \uparrow_2\rangle, \quad |3\rangle = |\downarrow_1 \downarrow_2\rangle, \quad |4\rangle = \frac{1}{\sqrt{2}} (|\uparrow_1 \downarrow_2\rangle + |\downarrow_1 \uparrow_2\rangle). \tag{5}
\]

Moreover, as it follows from (4), \( \langle 1|S_2|1\rangle = 0, \langle 2|S_2|2\rangle = 1, \langle 3|S_2|3\rangle = -1, \langle 4|S_2|4\rangle = 0 \). For \( 0 \leq h < \frac{1}{2} J_1 \) one concludes from (4) that \( |\text{GS}\rangle = |1\rangle \) and therefore \( M_2 = 0 \). For \( \frac{1}{2} J_1 < h \) one finds that \( |\text{GS}\rangle = |2\rangle \) and therefore \( M_2 = 1 \). Similarly the case \( h \leq 0 \) can be considered. As a result one deduces that the magnetization...
curve $m$ vs. $h$ should exhibit a plateau at $m = 0$ (if $-\frac{1}{2}J_1 < h < \frac{1}{2}J_1$) and at $m = \pm \frac{1}{2}$ (if $h > \frac{1}{2}J_1$ and $h < -\frac{1}{2}J_1$).

Let us consider further the case $p = 3$. The cluster Hamiltonian is as follows

$$H_3 = -h (s_1^x s_2^x + s_3^x) + J_1 (s_1^y s_2^y + s_3^y) + J_2 (s_1^z s_2^z + s_2^z s_3^z).$$

The eigenvalues of the Hamiltonian (6) are as follows

$$\langle \psi \rangle = \langle \psi | H | \psi \rangle$$

Moreover, $\langle 3 | S_3 | 3 \rangle = \frac{3}{2}, \langle 1 | S_3 | 1 \rangle = \langle 4 | S_3 | 4 \rangle = \langle 7 | S_3 | 7 \rangle = \frac{7}{2}, \langle 2 | S_3 | 2 \rangle = \langle 5 | S_3 | 5 \rangle = \langle 8 | S_3 | 8 \rangle = -\frac{1}{2}, \langle 6 | S_3 | 6 \rangle = -\frac{3}{2}$. Therefore, for $0 < h < \frac{1}{2} \sqrt{J_1^2 + J_2^2}$ (since $|GS| = |1\rangle$ (see (6))) one finds $M_3 = \frac{3}{2}$ and $m = \frac{1}{2}$, whereas for $\frac{1}{2} \sqrt{J_1^2 + J_2^2} < h$ (since $|GS| = |3\rangle$) one finds $M_3 = \frac{3}{2}$ and $m = \frac{1}{2}$. As a result one concludes that the dependence $m$ vs. $h$ exhibits plateaus at $m = \frac{1}{2}$ (if $\frac{1}{2} \sqrt{J_1^2 + J_2^2} < h$), at $m = \frac{1}{2}$ (if $0 < h < \frac{1}{2} \sqrt{J_1^2 + J_2^2}$), at $m = -\frac{1}{6}$ (if $-\frac{1}{2} \sqrt{J_1^2 + J_2^2} < h < 0$), and at $m = -\frac{1}{2}$ (if $h < -\frac{1}{2} \sqrt{J_1^2 + J_2^2}$).

It is interesting to note that the relevant for zero temperature magnetization low-lying levels of the Hamiltonians (3) and (6) follow from the following Hamiltonians

$$\mathcal{H}_2 = -h S^z + \frac{1}{2} J_1 \left( (S^z)^2 - 1 \right), \quad S^z = \{ \pm 1, 0 \}$$

and

$$\mathcal{H}_3 = -h S^z + \frac{1}{4} \sqrt{J_1^2 + J_2^2} \left( (S^z)^2 - \frac{9}{4} \right), \quad S^z = \left\{ \pm \frac{3}{2}, \pm \frac{1}{2} \right\},$$

respectively. The appearance of plateaus becomes evident from (3) (or (6)) since, e.g., at small $h > 0$ spins are fixed to $S^z = 0$ (or to $S^z = \frac{1}{2}$) and for large $h > 0$ they should be in the $S^z = \frac{5}{2}$ (or $S^z = 1$) state.

We pass to the case $p = 4$. The eigenvalues of the cluster Hamiltonian are as follows

$$E_1 = -\frac{1}{2} \sqrt{J_2^2 + (J_1 + J_3)^2} = -E_{16},$$

$$E_2 = -h - \frac{1}{2 \sqrt{2}} \sqrt{J_1^2 + J_2^2 + J_3^2 + \sqrt{J_1^4 + J_2^4 + J_3^4 - 2J_1^2 J_2^2 + 2J_1^2 J_3^2 + 2J_2^2 J_3^2}} = -E_{15},$$

$$E_3 = h - \frac{1}{2 \sqrt{2}} \sqrt{J_1^2 + J_2^2 + J_3^2 + \sqrt{J_1^4 + J_2^4 + J_3^4 - 2J_1^2 J_2^2 + 2J_1^2 J_3^2 + 2J_2^2 J_3^2}} = -E_{14},$$

$$E_4 = -\frac{1}{2} \sqrt{J_2^2 + (J_1 - J_3)^2} = -E_{13},$$

$$E_5 = -h - \frac{1}{2 \sqrt{2}} \sqrt{J_1^2 + J_2^2 + J_3^2 - \sqrt{J_1^4 + J_2^4 + J_3^4 - 2J_1^2 J_2^2 + 2J_1^2 J_3^2 + 2J_2^2 J_3^2}} = -E_{12},$$

$$E_6 = h - \frac{1}{2 \sqrt{2}} \sqrt{J_1^2 + J_2^2 + J_3^2 - \sqrt{J_1^4 + J_2^4 + J_3^4 - 2J_1^2 J_2^2 + 2J_1^2 J_3^2 + 2J_2^2 J_3^2}} = -E_{11},$$

$$E_7 = -2h = -E_{10}, \quad E_8 = 0 = E_9.$$

(10)
Usually, a chain with \( p = 4 \) (e.g., if \( J_1 = J_2 = J_3 = J \)) exhibits plateaus at \( m = 0 \) for \(-h_1 < h < h_1\) when \( |GS⟩ = |1⟩\), \( ⟨S_4|1⟩ = 0\), at \( m = 1/4\) \((m = -1/4)\) for \( h_1 < h < h_2\) \((-h_2 < h < -h_1)\) when \( |GS⟩ = |2⟩\), \( 2|S_4⟩ = 1 \(|GS⟩ = |3⟩\), \( 3|S_4⟩ = -1\), and at \( m = 1/2 \((m = -1/2)\) for \( h_2 < h \) \((h < -h_2)\) when \( |GS⟩ = |7⟩\), \( 7|S_4⟩ = 2 \(|GS⟩ = |10⟩\), \( 10|S_4⟩ = -2\). However, for special values of parameters \( J_1, J_2, J_3 \) not all possible values of \( m \), i.e. \( 0, \pm1/4, \pm1/2 \), are observed. For example, if \( E_1 = E_2 \) in \( ⟨1⟩ \) (this occurs when \( J_1 = J_2 = J, J_3 = 0 \)) the plateau at \( m = 0 \) disappears. In terms of the Jordan–Wigner fermions in such a case one has \( ρ(E) = 1/4δ(E + h + \sqrt{2}J) + 1/2δ(E + h) + 1/4δ(E + h - √2J). \) Therefore, in accordance with \( K \) \( m = 0 \) occurs exactly when \( h = 0 \) since any small positive (negative) \( h \) immediately yields \( m = 1/2 \) \((m = -1/2)\).

Let us turn to the transverse Ising chain. For \( p = 2 \) the cluster Hamiltonian eigenvalues are

\[
E_1 = -\sqrt{h^2 + 1/16J^2} = -E_4, \quad E_2 = -1/4J_1 = -E_3.
\]

Note, that the ground state is \( |1⟩ \) for any \( h \). Moreover, \( ⟨1|S_2|1⟩ = 0 \) if \( h = 0 \) and \( ⟨1|S_2|1⟩ → 1 \) if \( h → ∞ \). Thus, the considered chain does not exhibit plateaus that is in agreement with the result for a general (without restriction \( J_2 = 0 \)) transverse Ising chain with \( p = 2 \) reported in \( [3] \). In fact, the absence of plateaus in magnetization curve is not surprising and is conditioned by the different symmetry of the transverse Ising chain. Thus, \( ∑n s_n^2 \) for this model with arbitrary \( J_2 \) does not commute with the Hamiltonian (in contrast to the case of the transverse \( XX \) chain) and hence \( ⟨GS|S_p|GS⟩ \) should vary continuously with changing \( h \).

One can convinced himself that the formulae \( [4], [7], [10], [11] \) are valid for ferromagnetic sign of all or a part of interactions and hence the described picture is not restricted to the case \( J_n ≥ 0 \). For the isotropic Heisenberg chain with \( p = 2 \) with the antiferromagnetic interaction \( J_1 > 0 \) the cluster Hamiltonian eigenvalues are \( E_1 = -3/4J_1, E_2 = -h + 1/4J_1, E_3 = 1/4J_1, E_4 = h + 1/4J_1 \), and therefore one concludes that \( m = 0 \) for \(-J_1 < h < J_1\) and \( m = 1/4 \) \((m = -1/4)\) for \( J_1 < h \) \((h < -J_1)\). However, for the ferromagnetic interaction \( J_1 < 0 \) one finds instead \( E_1 = -h - 1/4|J_1|, E_2 = -1/4|J_1|, E_3 = h - 1/4|J_1|, E_4 = -3/4|J_1| \), and hence the plateau at \( m = 0 \) does not appear.

Evidently, in the considered limit \( J_p^o = 0 \) an arbitrary type of the intersite interaction (the anisotropic \( XY \) chain, the Heisenberg–Ising (\( XXZ \) chain etc.) and the value of spin \( s \) can be examined easily. In addition to the total magnetization the on–site magnetization can be calculated in a similar way. Besides, the treatment in that limit can be applied to the spin systems of higher dimensions.

Finally, let us discuss briefly how the obtained results can be used for the analysis beyond the limit \( J_p = 0 \). The transverse \( XX \) chain or the transverse Ising chain can be studied rigorously \( [4],[3] \) in contrast to the Heisenberg or more complicated chains. Different ways to take into account a nonzero value of the smallest interaction perturbatively at certain value of \( m \) have been elaborated \( [3] \). Let us demonstrate how it can be done considering for concreteness the regularly alternating \( XX \) chain of period 2 in which now
$J_1 \gg J_2 \neq 0$. The Hamiltonian of that system naturally splits into two parts

$$H = H_0 + V,$$

$$H_0 = \ldots - h (s_{101}^z + s_{102}^z) + \frac{1}{2} J_1 \left(s_{101}^+ s_{102}^- + s_{101}^- s_{102}^+\right) - \ldots,$$

$$V = \ldots + \frac{1}{2} J_2 \left(s_{102}^+ s_{103}^- + s_{102}^- s_{103}^+\right) + \ldots.$$ 

Only the two lowest levels of the two-site cluster discussed above will be taken into account. We assume $h \geq 0$; the relevant states are $|1\rangle_{51} = \frac{1}{\sqrt{2}} (|\uparrow_{101}\downarrow_{102}\rangle - |\downarrow_{101}\uparrow_{102}\rangle)$ and $|2\rangle_{51} = |\uparrow_{101}\uparrow_{102}\rangle$. Let us introduce new spin $\frac{1}{2}$ operators $\sigma_i^\alpha$ which act in a following way

$$\ldots, \quad \sigma_{51}^z |1\rangle_{51} = -\frac{1}{2} |1\rangle_{51}, \quad \sigma_{51}^+ |1\rangle_{51} = |2\rangle_{51}, \quad \sigma_{51}^- |1\rangle_{51} = 0,$$

$$\sigma_{51}^z |2\rangle_{51} = \frac{1}{2} |2\rangle_{51}, \quad \sigma_{51}^+ |2\rangle_{51} = 0, \quad \sigma_{51}^- |2\rangle_{51} = |1\rangle_{51}, \quad \ldots.$$ 

Then $H_0$ can be written approximately (only the lowest levels are reproduced) as

$$H_0 = \sum_{i=1}^{L} \left( -\frac{1}{2} J_1 \left( \frac{1}{2} - \sigma_i^z \right) - h \left( \frac{1}{2} + \sigma_i^z \right) \right) = -\frac{1}{2} \left( h + \frac{1}{2} J_1 \right) L - \left( h - \frac{1}{2} J_1 \right) \sum_{i=1}^{L} \sigma_i^z$$

with $L = \frac{1}{2} N$. Really,

$$\left( -\frac{1}{2} \left( h + \frac{1}{2} J_1 \right) - \left( h - \frac{1}{2} J_1 \right) \sigma_i^z \right) |1\rangle_{l} = -\frac{1}{2} J_1 |1\rangle_{l},$$

$$\left( -\frac{1}{2} \left( h + \frac{1}{2} J_1 \right) - \left( h - \frac{1}{2} J_1 \right) \sigma_i^z \right) |2\rangle_{l} = -h |2\rangle_{l},$$

that are just the two lowest levels of $l$th cluster. To write the intercluster interaction, e.g., $V_{51,52} = \frac{1}{2} J_2 \left(s_{102}^+ s_{103}^- + s_{102}^- s_{103}^+\right)$ in terms of the operators $\sigma_i^\alpha$ let us consider the following equations

$$V_{51,52}|1\rangle_{51} = \frac{1}{2} J_2 s_{103}^- \frac{1}{\sqrt{2}} |2\rangle_{51} = \frac{1}{2 \sqrt{2}} J_2 s_{103}^- \sigma_{51}^+ |1\rangle_{51},$$

$$V_{51,52}|2\rangle_{51} = \frac{1}{2} J_2 s_{103}^- \frac{1}{\sqrt{2}} |1\rangle_{51} = \frac{1}{2 \sqrt{2}} J_2 s_{103}^+ \sigma_{51}^- |2\rangle_{51}$$

(note, that although $|\uparrow_{101}\downarrow_{102}\rangle = \frac{1}{\sqrt{2}} (|1\rangle_{51} + |4\rangle_{51})$, we put $|\uparrow_{101}\downarrow_{102}\rangle = \frac{1}{\sqrt{2}} |1\rangle_{51}$ since we take into account only $|1\rangle_{51}$ and $|2\rangle_{51}$). Hence

$$V_{51,52} = \frac{1}{2 \sqrt{2}} J_2 \left(s_{103}^+ \sigma_{51}^- + s_{103}^- \sigma_{51}^+\right).$$

Further,

$$V_{51,52}|1\rangle_{52} = \frac{1}{2 \sqrt{2}} J_2 \left(\sigma_{51}^+ \frac{1}{\sqrt{2}} |3\rangle_{52} + \sigma_{51}^- \frac{1}{\sqrt{2}} (-|2\rangle_{52})\right) = -\frac{1}{4} J_2 \sigma_{51}^- \sigma_{52}^+ |1\rangle_{52},$$

$$V_{51,52}|2\rangle_{52} = \frac{1}{2 \sqrt{2}} J_2 \sigma_{51}^+ |1\rangle_{52} = -\frac{1}{4} J_2 \sigma_{51}^+ \sigma_{52}^- |2\rangle_{52}.$$ 

As a result one concludes that

$$V_{51,52} = -\frac{1}{4} J_2 \left(\sigma_{51}^+ \sigma_{52}^- + \sigma_{51}^- \sigma_{52}^+\right).$$
and therefore

\[ V = -\frac{1}{2} J_2 \sum_{l=1}^{L} (\sigma_l^x \sigma_{l+1}^x + \sigma_l^y \sigma_{l+1}^y). \]  

(20)

The Hamiltonian (12), (14), (20) describes the uniform spin-\(\frac{1}{2}\) transverse XX chain of L spins

\[ H = -\frac{1}{2} \left( h + \frac{1}{2} J_1 \right) L - \left( h - \frac{1}{2} J_1 \right) \sum_{l=1}^{L} \sigma_l^z \sum_{l=1}^{L} \left( \sigma_l^x \sigma_{l+1}^x + \sigma_l^y \sigma_{l+1}^y \right). \]  

(21)

Finally, acting like while deriving (17), (19) one finds the relation between the operators \(s_{101}^\alpha\) and \(\sigma_{101}^\alpha\)

\[ \cdots, \quad s_{101}^+ = -\frac{1}{\sqrt{2}} \sigma_{51}^-, \quad s_{101}^- = -\frac{1}{\sqrt{2}} \sigma_{51}^+, \quad s_{101}^z = \frac{1}{2} \left( \frac{1}{2} + \sigma_{51}^z \right), \]

\[ s_{102}^+ = \frac{1}{\sqrt{2}} \sigma_{51}^-, \quad s_{102}^- = \frac{1}{\sqrt{2}} \sigma_{51}^+, \quad s_{102}^z = \frac{1}{2} \left( \frac{1}{2} + \sigma_{51}^z \right), \quad \cdots. \]  

(22)

Consider now the magnetization which owing to (22) can be written as

\[ m = \frac{1}{2} \left( \frac{1}{2} + \frac{1}{L} \sum_{l=1}^{L} \langle \sigma_l^z \rangle \right). \]

Using the well-known results for the uniform spin-\(\frac{1}{2}\) transverse XX chain (see, e.g. [4]) one concludes that \(m = 0\) at \(0 \leq h \leq \frac{1}{2} (J_1 - J_2)\) and \(m = \frac{1}{2}\) at \(\frac{1}{2} (J_1 + J_2) \leq h\). If \(h\) increases from \(\frac{1}{2} (J_1 - J_2)\) to \(\frac{1}{2} (J_1 + J_2)\) the magnetization increases from 0 to \(\frac{1}{2}\) as \(\frac{1}{2} - \frac{1}{2\pi} \arcsin \sqrt{1 - \left(\frac{2h - J_1 J_2}{J_1 J_2}\right)^2}\).

In Fig. 1 we plotted the zero temperature magnetization profiles for the spin-\(\frac{1}{2}\) transverse XX chain of period 2 with \(J_1 = 1 + \delta, J_2 = 1 - \delta\) for \(\delta = 1, 0.8, 0.6, 0.4, 0.2, 0\) as they follow from 1) the exact formula (2) [4] (solid curves) and 2) the approximate Hamiltonian (21) (dotted curves). These plots demonstrate how does the strong-coupling approach work as \(\delta\) deviates from 1.

To summarize, we have reconsidered the zero temperature magnetization processes in the regularly alternating spin-\(\frac{1}{2}\) XY chains within the frames of the strong-coupling approach. Besides discussing the magnetization plateaus in terms of the spins rather than in terms of the Jordan-Wigner fermions we have demonstrated to what extent the strong-coupling approximation can reproduce the exact magnetization profiles.

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Figure caption

Fig. 1. \( m \) vs. \( h \) for the spin–\( \frac{1}{2} \) transverse \( XX \) chain of period 2 with \( J_1 = 1 + \delta \), \( J_2 = 1 - \delta \), \( \delta = 1 \) (a),
\( \delta = 0.8 \) (b), \( \delta = 0.6 \) (c), \( \delta = 0.4 \) (d), \( \delta = 0.2 \) (e), \( \delta = 0 \) (f) at zero temperature. Solid curves correspond
to the exact results following from [2], [3], dotted curves correspond to the results obtained with the help of
the approximate Hamiltonian [3].
