Chromaticity of Gravitational Microlensing Events

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ABSTRACT

In this paper, we investigate the color changes of gravitational microlensing events caused by the two different mechanisms of differential amplification for a limb-darkened extended source and blending. From this investigation, we find that the color changes of limb-darkened extended source events (color curves) have dramatically different characteristics depending on whether the lens transits the source star or not. We show that for a source transit event, the lens proper motion can be determined by simply measuring the turning time of the color curve instead of fitting the overall color or light curves. We also find that even for a very small fraction of blended light, the color changes induced by the blending effect is equivalent to those caused by the limb-darkening effect, causing serious distortion in the observed color curve. Therefore, to obtain useful information about the lens and source star from the color curve of a limb-darkened extended source event, it will be essential to eliminate or correct for the blending effect. We discuss about the methods for the efficient correction of the blending effect.

Subject headings: gravitational lensing – stars: giants, main sequence – limb darkening

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1. Introduction

Searches for Galactic dark matter by monitoring light variations of stars caused by gravitational microlensing have been and are being carried out towards the Galactic bulge and the Magellanic Cloud fields by several groups (MACHO: Alcock et al. 1993; OGLE: Udalski et al. 1993; EROS: Aubourg et al. 1993; DUO: Alard & Guibert 1997). With their efforts, ∼ 500 events have been detected to date.

One of the important characteristics of gravitational microlensing amplification is that the color of the light curve does not change during the event. However, the achromaticity of the light curve is violated for some cases of gravitational microlensing events. The first mechanism that makes the lensing event light curve wavelength dependent is the differential amplification for a limb-darkened extended source (see § 2). By measuring the color changes induced by this mechanism (hereafter color curve), one can not only investigate the intensity profile of the source star but also determine the lens proper motion from which the mass and location of the lens can be significantly better determined.\(^1\) Another mechanism making the lensing light curve chromatic is blending. If the measured flux of the source star is affected by the blended light from unresolved stars with different colors, the color of the light curve is changed during the event (see § 3).

In this paper, we investigate the color changes of gravitational microlensing events caused by the two different mechanisms of differential amplification for a limb-darkened extended source and blending. From this investigation, we find that the color changes of limb-darkened extended source events have dramatically different characteristics depending on whether the lens transits the source star or not. We show that for a source transit event, the lens proper motion can be determined by simply measuring the turning time of the color curve instead of fitting the overall color or light curves. We also find that even for a very small fraction of blended light, the color changes induced by the blending effect is equivalent to those caused by the limb-darkening effect, causing serious distortion in the observed color curve. Therefore, to obtain useful information about the lens and source star from the color curve of a limb-darkened extended source event, it will be essential to eliminate or correct for the blending effect. We discuss about the methods for the efficient correction of the blending effect.

\(^1\)Investigation of the intensity profile of the source star and determination of the lens proper motion are also possible by analyzing a single band light curve of the event. This is because not only the color but also the amplification is affected by the extended source effect. However, if the lens does not transit the source star, the amplification change induced by the extended source effect simply masquerades as changes in lensing parameters, and thus it cannot be detected. By contrast, the color effects cannot be mimicked by the changes in lensing parameters, since point-source lensing event is achromatic. Gould & Welch (1996) showed that by detecting the color changes, one can double the chance to detect the extended source effect.
2. Color Change Caused by Limb Darkening

The light curve of an event with an isolated point source is represented by

\[ A_0 = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}, \tag{2.1} \]

where \( u \) is the lens-source separation in units of the angular Einstein ring radius \( \theta_E \). The Einstein ring represents the effective region of gravitational lensing amplification within which the source star amplification becomes greater than \( \sqrt{5}/3 \). The angular Einstein ring radius is related to the lens mass \( M \) and its location by

\[ \theta_E = \left( \frac{4GM}{c^2 D_{ls} D_{os}} \right)^{1/2}, \tag{2.2} \]

where \( D_{ol}, D_{ls}, \) and \( D_{os} \) are the separations between the observer, lens, and source star, respectively. The value of \( \theta_E \) for a Galactic bulge event caused by a solar mass lens located at the half way between the source star and observer (i.e. \( D_{ol}/D_{os} = 0.5 \)) is \( \theta_E \sim 0.3 \) milli-arcsec. On the other hand, typical main-sequence stars in the Galactic bulge have radii that extend only \( \theta_s \sim < 1 \mu\text{-arcsec} \). Therefore, equation (2.1) is a good approximation for most of Galactic microlensing events. The gravitational amplification for a point source event is characterized by a symmetric, non-repeating, and achromatic light curve.

However, for a very close lens-source impact event with a considerable source star size such as giants, the source star can no longer be approximated by a point source. For an extended source event, the light curve is given by the intensity-weighted amplification averaged over the surface of the source star, i.e.

\[ A_\nu = \frac{\int_0^{2\pi} \int_0^{r_s} I_\nu(r, \vartheta) A_0(|r - r_L|) r dr d\vartheta}{\int_0^{2\pi} \int_0^{r_s} I_\nu(r, \vartheta) r dr d\vartheta}, \tag{2.3} \]

where \( r_s \) is the radius of the source star, \( I_\nu(r, \vartheta) \) is the surface intensity distribution of the source star, and the vectors \( r_L \) and \( r = (r, \vartheta) \) represent the displacement vector of the center of source star with respect to the lens and the orientation vector of a point on the source star surface with respect to the center of the source star, respectively. For the radially symmetric distribution of the source star intensity, equation (2.3) is simplified into

\[ A_\nu = \frac{\int_0^{r_s} I_\nu(r) A_0(|r - r_L|) r dr}{\int_0^{r_s} I_\nu(r) r dr}. \tag{2.4} \]

The light curve of an extended source event can become chromatic due to limb darkening. Because observations at different wavelengths probe material at different depth in stellar atmosphere, the radial surface intensity profile of a star is wavelength dependent. In longer wavelength bands, which probe the cooler outer regions of the star, the stellar disk will appear less limb darkened. When the lens passes close to the source
star, different parts of the source star disk (with varying surface intensity and spectral energy distribution) are amplified by different amount due to the differences in distance to the lens: differential amplification (Gould 1994; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994). As a result, the microlensing event with a limb-darkened extended source star will have different amplifications depending on the observed wavelength bands. Let us define \( F_{\nu i} = 2 \pi \int_{r_0}^{r^*} I_{\nu i}(r) r dr; \ i=1, 2 \) and \( m_{\nu i} \) as the unamplified source star fluxes and the corresponding magnitudes measured in two different wavelength bands \( \nu 1 \) and \( \nu 2 \). Then the color changes during the lensing event caused by the limb darkening of the source star are computed by

\[
\Delta(m_{\nu 2} - m_{\nu 1}) = (m_{\nu 2} - m_{\nu 1}) - (m_{\nu 2} - m_{\nu 1})_0 = -2.5 \log \left( \frac{A_{\nu 2}}{A_{\nu 1}} \right),
\]

where \((m_{\nu 2} - m_{\nu 1})_0 = -2.5 \log(F_{\nu 2}/F_{\nu 1})\) and \(m_{\nu 2} - m_{\nu 1} = -2.5 \log(A_{\nu 2}F_{\nu 2}/A_{\nu 1}F_{\nu 1})\) represent the color differences before and during the gravitational amplification. From equation (2.5), one finds that the color change caused by the limb darkening effect depends on the ratio of the amplifications observed in two bands \( A_{\nu 2}/A_{\nu 1} \), but does not depends on the source star fluxes \( F_{\nu i} \).

2.1. Types of Color Curve Patterns

To see the pattern of the color curves, we compute the color changes of example events for a limb-darkened source star with an angular radius of \( \theta_\ast = 0.1 \theta_E \) and the resulting light curves are presented in Figure 1. On the left side, the trajectories of the lens with respect to the source star (the shaded circle) of the individual events are marked by straight lines with different line types. In the right panel, the resulting color curves \( \Delta(U - I) \) for the individual events are drawn by the same line types as those of the corresponding trajectories in the left panel. The unamplified color of the source star is \((U - I)_0 = 2.98\), which corresponds to that of a K-type giant (Allen 1973; Schmidt-Kaler 1982; Peletier 1989). For the surface intensity profile, we adopt a linear form of

\[
I_{\nu}(r) = 1 - C_{\nu} \left[ 1 - \sqrt{1 - (r/r_\ast)^2} \right],
\]

(2.1.1)

where the limb-darkening coefficients are \( C_{\nu} = 1.050 \) and 0.503 in \( U \) and \( I \) bands, respectively, which correspond to those of a K giant with \( T_{\text{eff}} = 4,750 \) K, \( \log g \sim 2.0 \), and a metallicity similar to the sun (Van Hamme 1993).

From Figure 1, one finds that the patterns of the color curve can be classifed broadly into two categories depending on whether the lens transits the source star disk or not. First, if the lens transits the source star (transit event), the resulting color curve is characterized by the turn of the color curve during the event. As the lens approaches the source star but before the transit, the color of the source star becomes redder because the closer (and thus more amplified) part of the source star to the lens is the cooler outer part. The source star
appears to be most reddish at the moment when the lens is located at the limb of the star, and thus the effect of differential amplification is maximized. As the lens further approaches and transits the source star, on the other hand, the closer part of the source star to the lens is the hotter inner region, causing a turn in the color curve. Second, if the lens passes close to the limb of the source star but does not actually transit (passing event), the color of the source star continues to become redder without any turn as the lens approaches the source star. In addition, the amount of the color changes for passing events are smaller than those of crossing events.

2.2. Source-Transit Event Rate

An interesting finding from the color curve of a transit event is that one can determine the lens proper motion by simply measuring the turning time \( t_\cap \) instead of fitting the overall color or light curves. The turn of the color change occurs when \( t_\cap = \pm \left[ \frac{(\theta_*/\theta_E)^2 - \beta^2}{2} t_E \right]^{1/2} \). Since the values of the lensing parameters \( \beta \) and \( t_E \) can be measured from the overall shape of the light curve, one can determine the angular Einstein ring radius by

\[
\theta_E = \frac{\theta_*}{\sqrt{\beta^2 + (|t_\cap|/t_E)^2}}.
\]  

Determining \( \theta_E \) is equivalent to the determination of the lens proper motion \( \mu \) relative to the observer-source line of sight because \( \mu = \theta_E/t_E \).

Then what will be the fraction of transit events out of total Galactic bulge events with giant source stars? Another question that should be answered is for what fraction of these source-transit events can be actually detected by the current lensing experiments. To answer these questions, we compute the event rate as a function of the Einstein time scale for Galactic bulge events with giant source stars by

\[
\Gamma(t_E) = \int_0^\infty dD_{os} \rho(D_{os}) \int_0^{D_{os}} dD_{ol} \rho(D_{ol}) \int_0^\infty \int_0^\infty dv_y dv_z v f(v_y, v_z) \\
\times \int_0^\infty dM \left( \frac{4GM}{c^2 D_{ol} D_{ls}} \right)^{1/2} \Phi(M) \delta \left[ t_E - \left( \frac{4GM}{c^2 v^2 D_{os}} \right)^{1/2} \right],
\]

where \( \rho(D_{os}) \) and \( \rho(D_{ol}) \) are the number densities of giant source stars and lenses along the line of sight towards the Galactic bulge, \((v_y, v_z)\) are the two components of the lens-source transverse velocity \( \mathbf{v} \), \( f(v_y, v_z) \) represents their distribution, and \( \Phi(M) \) is the mass function of lenses. For the matter distribution, we adopt a ‘revised COBE’ Galactic bulge model (Dwek et al. 1995) and a double-exponential Galactic disk model (Bahcall 1986). The transverse velocity distribution is modeled by a Gaussian. The detailed descriptions about the matter and transverse velocity distributions are found in Han & Gould (1996). The lens mass function is modeled by a power law with a mass cutoff, i.e. \( \Phi(M) \propto M^{-p} \Theta(m - m_{cut}) \), where the adopted values of the power and the cutoff mass are respectively \( p = 2.1 \) and...
$m_{\text{cut}} = 0.04 \, M_{\odot}$, following the determination of Han & Gould (1996). The relative event rate distribution for the total Galactic bulge events with giant source stars is presented in Figure 2 (dashed curve).

To become a transit event, the source star radius normalized by $\theta_E$ should be greater than the lens-source impact parameter of the event, i.e.

$$\beta_{\text{crit}} > \beta; \quad \beta_{\text{crit}} = \frac{\theta_s}{\theta_E}$$

(see Figure 3). With this definition of a source-transit event and the adopted source star radius of a K-giant, we determine the event rate distribution for source-transit events by using the same equation in (2.2.2) and the resulting distribution is presented also in Figure 2 (dot-dashed curve). We find that $\sim 16\%$ of the total Galactic bulge events with giant source stars will be transit events.

However, not all the source-transit events can be detected by the current lensing experiments. From Figure 2 one finds that source-transit events tend to have Einstein time scales substantially shorter than the average value of the total events. This implies that most source-transit events are caused by low-mass lenses. Then the most important restriction in detecting source-transit events is given by the short duration of source crossing. For a transit event with an impact parameter $\beta$, the duration of source crossing is computed by

$$t_{\text{cross}} = \frac{2(\beta_{\text{crit}}^2 - \beta^2)^{1/2}t_E}{\theta_E}.$$  

(2.2.4)

We, therefore, estimate the fraction of detectable source-transit events by assuming that only events with $t_{\text{cross}}$ longer than 2 days, which is the minimum duration for intensive monitoring by the followup observations\footnote{Since the color changes caused by the limb darkening effect will be small ($\sim 0.05$ mag), intensive monitoring of events with high precision followup observations will be essential for the detection of transit events. Currently several followup observation teams are under operation (GMAN: Pratt et al. 1996; PLANET: Albrow et al. 1998; MPS: Rhie et al. 1999). Since by observing giant source star events one can not only construct light curves with small uncertainties but also has better chance to detect the finite-source effect, most of these events are monitored by followup observations.}, can be detected. With this definition of detectable source-transit events, we compute the event rate distribution for these events and it is presented in Figure 3 (solid curve). We find that source-transit can be detected for $\sim 2\%$ of Galactic bulge events with giant source stars.

### 3. Color Change Caused by the Blending Effect

The light curve of a microlensing event can also become chromatic by blending. If one lets the blended fluxes in two different bands $B_{\nu_i}$, the colors of a point-source event...
before and during the event are \((m_{\nu 2} - m_{\nu 1})_0 = -2.5 \log((F_{\nu 2} + B_{\nu 2})/(F_{\nu 1} + B_{\nu 1}))\) and \(m_{\nu 2} - m_{\nu 1} = -2.5 \log((A_0 F_{\nu 2} + B_{\nu 2})/(A_0 F_{\nu 1} + B_{\nu 1}))\), respectively. Then, the color changes caused by the blending effect for a point-source event is computed by

\[
\Delta(m_{\nu 2} - m_{\nu 1}) = -2.5 \log \left( \frac{A_0 + f_{\nu 2}}{A_0 + f_{\nu 1}} \right) \left( \frac{1 + f_{\nu 2}}{1 + f_{\nu 1}} \right)^{-1},
\]

where \(f_{\nu i} = B_{\nu i}/F_{\nu i}\) represent the fractions of the blended light in the individual wavelength bands. Then, the change in color becomes 0 either when there is no blended light, i.e. \(f_{\nu i} = 0\), or when the source star is not gravitationally amplified, i.e. \(A_0 = 1.0\). While the color change induced by the limb darkening effect depends only on the amplification ratio measured in the two wavelength bands [see equation (2.5)], the color change caused by the blending effect depends on the amplification and the blended light fractions.

To see the pattern of the color changes caused by the blending effect, we compute the color changes expected when an example Galactic bulge event is affected by various fractions of blended light. For the direct comparison with the color changes caused by the limb-darkening effect analyzed in previous section, we assume the same impact parameter and the same lensed source star type of a K giant. For blended stars, we assume that they are K-type main-sequence stars, which are the most numerous type of stars in the Galactic bulge field. We determine the apparent magnitudes (i.e. \(U\) and \(I\)) of the lensed and blended stars by using the data of absolute luminosities and colors provided by Allen (1973), Schmidt-Kaler (1982), and Peletier (1989) along with the adopted distance to source stars of \(D_{os} = 8\) kpc: \(U = 17.03\) and \(I = 14.05\) for the lensed giant and \(U = 21.67\) and \(I = 19.01\) for each of the blended main-sequence star. Then, if a single main-sequence star is blended, the blended light fractions in the individual bands are \(f_U = 1.39\%\) and \(f_I = 1.04\%\), respectively. The individual curves in Figure 4 represent the color changes when the numbers of the blended main-sequence stars are 1, 3, 5, 10, and 20, respectively.

From Figure 1 and Figure 4, one finds that the pattern of the color changes caused by the blending effect is similar to the color changes caused by the limb-darkening effect for passing events; increasing the amount of color change with decreasing lens-source separation. In addition, one finds that although the amount of color change caused by the effect of blending depends on the fractions of the blended light, the induced color change is equivalent to that caused by the limb-darkening effect.

Since the densities of stars both in the Galactic bulge and the Magellanic Cloud fields are very high, it is very likely that most of events detected towards these directions are affected by blending. Then if the blending effect is not corrected for, the obtained color curve of an event will be seriously distorted by the color changes induced the blending effect, causing one to obtain wrong information both about the source star brightness profile and the lens proper motion. By considering both the effects of limb darkening and blending, the observed color change of an event is computed by

\[
\Delta(m_{\nu 2} - m_{\nu 1}) = -2.5 \log \left( \frac{A_\nu 2 + f_{\nu 2}}{A_\nu 1 + f_{\nu 1}} \right) \left( \frac{1 + f_{\nu 2}}{1 + f_{\nu 1}} \right)^{-1}.
\]

(3.2)
Note that due to the limb-darkening effect the amplification is no longer achromatic unlike the amplification for a point-source event [cf. equation (3.1)]. In Figure 5, we present the color curve of an event distorted by the blending effect (solid curve), in which the flux of a lensed K-type giant source star is affected by the fluxes from two blended main-sequence stars. One finds that even for a very small fraction ($\sim 2\%$) of the blended light the observed color curve significantly differs from the unblended color curve (dotted curve) due to the color changes caused by blending (dot-dashed curve).

Then, how often the color curves of passing events will be affected by the blending effect. To see this, we compute the average fraction of blended light for an event with a K-type giant source star by adopting the luminosity function of Holtzman et al. (1998) constructed from the observations by using the Hubble Space Telescope (HST). From this computation, we find that the average fraction of blended light, which comes mostly from main-sequence stars, is considerable ($\gtrsim 5\%$) even for a bright giant source star. Considering the fact that the blending effect on the observed light curve is significant even for $\sim 2\%$ of blended light, the color curves of most Galactic bulge events will be affected by the blending effect.

4. Blending Correction

In previous section, we demonstrated that since the color changes caused by the blending effect can be equivalent to those induced by the limb-darkening effect, the color curve of an event can be seriously distorted by the blending effect. Therefore, to obtain useful information both about the lens and the source star from the color curve of the event, it will be essential to eliminate or correct for the blending effect. There have been various methods proposed for the correction of the blending effect. In this section, we discuss the applicabilities of these methods to the precise construction of color curves which are free from the blending effect.

First, one can correct for the blending effect by detecting the shift of a source star image centroid toward the blended star: centroid shift method (Alard, Mao, & Guibert 1995; Alard 1996; Goldberg & Woźniak 1998; Han, Jeong, & Kim 1998). Noticeable color changes induced by the limb-darkening effect occurs for events with giant stars. For these bright source events, the fraction of blended light will be small. Then, since the expected centroid shift caused by a faint blended star will be too small to be detected, this method is not appropriate for the detection and correction for the blending effect caused by a small fraction of blended light.

Second, the effect of blending can also be corrected for by astrometrically observing microlensing events with a high precision interferometer such as the Space Interferometry Mission (SIM): astrometric method (Han & Kim 1999). When a source star is gravitationally amplified, it is split into two images, and their center of light moves along an elliptical trajectory (astrometric ellipse), which can be measured from the astrometric observations by using the SIM, during the event (Walker 1995; Høg, Novikov & Polnarev1995; Paczyński
1998; Boden, Shao & Van Buren 1998; Han & Chang 1999). If the event is affected by the blending effect, on the other hand, the observed trajectory of the source image centroid shift will deviate from the elliptical one. Since the deviation of the trajectory will be significant even for a small fraction of blended light, astrometric observations of microlensing events will be an efficient method to correct for the blending effect. The problem of this method is, however, that the mission is scheduled to be launched in 2005, and thus it is not immediately available.

Third, with the recently developed method of the difference image analysis method (DIA) one can measure light variations which are free from the effect of blending: DIA method (Alard 1999; Alard & Lupton 1998; Melchior et al. 1998, 1999; Alcock et al. 1999a, 1999b). The DIA method detects and measures the variations of source star flux $\Delta F_\nu$ by subtracting an observed image from a convolved reference image. Then, with the DIA method one can measure the true color of a point-source star by $\Delta(m_{\nu2} - m_{\nu1}) = -2.5 \log(\Delta F_{\nu2}/\Delta F_{\nu1})$ because the ratio between the variations of source star flux measured in two passbands by the DIA method is identical to the flux ratio of the source star fluxes, i.e. $\Delta F_{\nu2}/\Delta F_{\nu1} = F_{\nu2}/F_{\nu1}$. However, the color change induced by the limb-darkening effect cannot be determined by the DIA method. This is because the ratio of the light variation for the limb-darkened extended source event is $\Delta F_{\nu2}/\Delta F_{\nu1} = (F_{\nu2}/F_{\nu1})[(A_{\nu1} - 1)/(A_{\nu2} - 1)]$, while the value required for the color measurement is $A_{\nu2}F_{\nu2}/A_{\nu1}F_{\nu1}$.

Fourth, one can also correct for the blending effect from the high resolution space observations by using the HST: HST method (Han 1997). Blending corrections for all events by using this method will be difficult due to a large number of hours of HST time. However, if the HST observations are selectively conducted for relatively rare high amplification events with giant source stars, for which the probability to detect the limb-darkening effect is very high, the observations can be conducted with a reasonable amount of HST time. Actually, the MACHO groups already had HST observations of microlensing sources for a number of important events (Alcock et al. 1999a).

5. Summary

We analyze the color changes of gravitational microlensing events caused by the two different mechanisms of limb darkening and blending. The findings from these analyses are summarized as follows.

1. The color changes induced by the limb-darkening effect have dramatically different patterns depending on whether the lens transits the surface of the source star. The color curve of a source-transit event is characterized by the turn of the color change, while that of a passing event has no turn and relatively smaller amount of color change.
2. For a source-transit event, one can measure the lens proper motion by simply measuring the turning time of the color curve instead of fitting the overall light or color curves.

3. Among the total Galactic bulge events with giant source stars, $\sim 16\%$ of them are expected to be source-transit events and $\sim 2\%$ of events will have source-crossing intervals long enough to be detectable by the current followup observations.

4. Even for a very small fraction of blended light, the color change caused by the blending effect can become equivalent to those induced by the limb-darkening effect, causing serious distortion in the observed color curve of the event. Therefore, to obtain useful information both about the lens and source star, it will be essential to correct for the blending effect.

5. Among the methods proposed to correct for the blending effect, the best solution for the precise construction of the color curve is provided by the HST observations of events.

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Figure 1: The color changes of microlensing events with a limb-darkened extended source star. The source star has an angular radius of $\theta_* = 0.1 \theta_E$. On the left side, the trajectories of the lens with respect to the source star (the shaded circle) are marked by straight lines with various line types. In the right panel, the resulting color curves $\Delta(U-I)$ for the individual events are drawn by the same line types as those of the corresponding lens trajectories in the left panel. For the brightness profiles of the source star in the two wavelength bands, see the text.
Figure 2: The event rate distributions of Galactic bulge events as a function of Einstein time scale. While the dashed curve represents the distribution for the total events with giant source stars, dot-dashed curve is only for source-transit events. The solid curve represents the distribution for detectable source-transit events. The total event rate is arbitrarily normalized, but the individual distributions are relatively scaled. To better show the distribution for transit events, the region with short $t_E$ is expanded and presented in a separate box.
Figure 3: The lens system geometries for passing and transit events. The dotted circle on each side represents the Einstein ring and the lens ($L$) is located at the center of the Einstein ring. The surface of the source star ($S$) is represented by a shaded circle. The value $\beta_{\text{crit}}$ represents the source star radius normalized by $\theta_E$. One finds that to become a source-transit event $\beta_{\text{crit}}$ should be greater than the impact parameter ($\beta$) of the event.
Figure 4: The color changes of microlensing events affected by various fractions of blended light. The lensed star is a K-type giant with $U = 17.03$ and $I = 14.05$, while the individual blended stars are assumed to be main-sequence stars with $U = 21.67$ and $I = 19.01$. The individual curves represent the color changes when the numbers of main-sequence stars contributing to blended light are 1, 3, 5, 10, and 20, respectively. The blended light fractions by each blended star in the individual wavelength bands are $F_U = 1.39\%$ and $F_I = 1.04\%$. The marked number for each curve represents the number of blended main-sequence stars.
Figure 5: The color curve of an event affected by blending (solid curve). The dotted curve represents the color curve expected when the event is not affected by the blending effect, while the color changes caused purely by the blending effect is marked by a dot-dashed curve. The source star radius ($\theta_*/\theta_E$), the impact parameter of the event ($\beta$), and the fractions of blended light in the individual bands ($f_U$ and $f_I$) are also marked.