How Well Can Cosmological Parameters Be Estimated from CMB Observations?

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Abstract

The CMB anisotropy depends sensitively upon the slope and amplitude of primordial density and gravitational wave fluctuations, the baryon density, the Hubble constant, the cosmological constant, the ionization history, etc. We report on recent work showing how well multi-scale measurements of the anisotropy power spectrum can resolve these factors. We identify a hyper-surface in cosmic parameter space that can be accurately localized by observations, but along which the likelihood will hardly vary. Other cosmic observations will be needed to break this degeneracy.

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6 FIGURES, no tables
In this paper, we discuss the degree to which the CMB anisotropy observations can determine cosmological parameters such as the slope of the initial power spectrum, the age of the universe and the cosmological constant. These proceedings are a summary of and expansion upon a recent series of studies [Bond et al. 1994, Crittenden et al. 1993a,b]. Our central conclusion is that CMB anisotropy measurements alone cannot fix the parameters individually; however, a non-trivial combination of them can be determined. More concretely, for models based on the generation of gaussian, adiabatic fluctuations by inflation, we have identified a new variable $\tilde{n}_s$, a function of the basic parameters that can be fixed to great precision by CMB anisotropy observations. Distinct models with nearly the same value of $\tilde{n}_s$ cannot be discriminated by CMB data alone. However, combined with other cosmological observations, determining $\tilde{n}_s$ is a powerful tool for testing models and measuring fundamental parameters.

We imagine parameterizing the space of cosmological models by

$$\left( C_{S,T}^{(S,T,Is,...)}, n_{S,T,Is,...}, h, \Omega_B, \Omega_\Lambda, \Omega_{CDM}, \Omega_{HDM}, \ldots \right),$$

where $H_0 = 100$ h km sec$^{-1}$Mpc$^{-1}$ is the Hubble parameter, and $\Omega_{B,A,CDM,HDM,...}$ are the energy densities associated with baryons, cosmological constant ($\Lambda$), cold and hot dark matter, etc., divided by the critical density. We use the CMB quadrupole moments $C_{S,T}^{(S,T,Is,...)}$ to parameterize the overall amplitudes of energy density (scalar metric), gravitational wave (tensor metric), isocurvature scalar and other primordial fluctuations predicted by the model. We parameterize the shape of the initial (e.g., post-inflation) fluctuation spectra in wavenumber $k$ by power law indices $n_{S,T,Is,...}$; defined at time $t_i$ by $k^3 \langle (\delta \rho/\rho)(k,t_i)^2 \rangle \propto k^{ns+3}$ and $k^3 \langle \tilde{h}_{+,-}(k,t_i)^2 \rangle \propto k^{nr}$, where $\delta \rho/\rho$ and $h_{+,-}$ are the amplitudes of the energy density and gravitational wave metric fluctuations (for two polarizations), respectively.

We shall restrict ourselves to subdomains of this large space consistent with inflation models of fluctuation generation. Inflation produces a flat universe, $\Omega_{total} \approx 1$. We also take $\Omega_{HDM} = 0$, but note that, for angular scales $\gtrsim 10'$, the anisotropy for mixed dark matter models with $\Omega_{CDM} + \Omega_{HDM} \approx 1$ is quite similar to the anisotropy if all of the dark matter is cold. Given $\Omega_B$, we impose the nucleosynthesis estimate [Walker et al. 1991], $\Omega_B h^2 = 0.0125$, to determine $h$; we also satisfy globular cluster and other age bounds [Kolb and Turner 1990], and gravitational lens limits [Maoz and Rix 1993]; we range from $h \lessapprox 0.65$ for $\Omega_\Lambda = 0$ to $h \lessapprox 0.88$ for $\Omega_\Lambda \lessapprox 0.6$.

Inflation produces adiabatic scalar [Bardeen, Steinhardt and Turner 1983, Guth and Pi 1982, Starobinskii 1982, Hawking 1982] and tensor [Rubakov et al. 1982, Starobinskii 1985, Abbott and Wise 1984] Gaussian fluctuations. The COBE quadrupole fixes $C_{S,T}^{(T)} + C_{S}^{(S)}$, but the tensor-to-scalar quadrupole ratio $r \equiv C_{S,T}^{(T)}/C_{S}^{(S)}$ is undetermined (e.g., see Fig.1 in Davis et al. 1992). The indices $n_s$ and $n_t$ are determined by power-law best-fits to the theoretical prediction over the scales probed by the CMB. For generic models of inflation, including new, chaotic, and extended models, inflation gives [Davis et al. 1992, Lucchin et al. 1992, Salopek 1992, Liddle and Lyth 1992, Sahni and Souradeep 1992, Lidsey and Coles 1992, Adams et al. 1993, Crittenden et al. 1993a]

$$n_t \approx n_s - 1 \quad \text{and} \quad r \equiv C_{S,T}^{(T)}/C_{S}^{(S)} \approx 7(1-n_s). \quad (1)$$

Measuring $r$ and $n_s$ to determine whether they respect Eq. (1) is a critical test for inflation. Exceptions to Eq. (1) require additional fine-tuning of parameters or initial conditions,
The band-powers are placed at \( \langle C_\ell \rangle \), \( \ell = 1 \). Using this expression, the anisotropy predictions for the exceptional models which violate Eq. (4) can be extrapolated from the results shown in the figures. With our set of assumptions including Eq. (4), we have reduced the parameter-space to three-dimensions, \( (r|n_s, \Omega) \) (where \( \Omega_B = 0.0125h^{-2} \) and \( \Omega_{CDM} = 1 - \Omega_B - \Omega_{\Lambda} \)). We explicitly display both \( r \) and \( n_s \) but with a “|” as a reminder that \( r \) is determined by Eq. (4) given \( n_s \); we have also assumed \( n_t = n_s - 1 \).

Our results are based on numerical integration of the general relativistic Boltzmann, Einstein, and hydrodynamic equations for both scalar [Bond and Efstathiou 1984, Bond 1988, Efstathiou 1990] and tensor metric fluctuations using methods reported elsewhere [Crittenden et al. 1993a]. Included in the dynamical evolution are all the relevant components: baryons, photons, dark matter, and massless neutrinos. The temperature anisotropy, \( \Delta T/T (\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi) \), is computed in terms of scalar and tensor multipole components, \( a_{\ell m}^{(S)} \) and \( a_{\ell m}^{(T)} \), respectively. For inflation, each multipole for the two modes is predicted to be statistically independent and Gaussian-distributed, fully specified by angular power spectra, \( C_{\ell}^{(S)} = \ell(\ell + 1) \langle |a_{\ell m}^{(S)}|^2 \rangle/(2\pi) \) and \( C_{\ell}^{(T)} = \ell(\ell + 1) \langle |a_{\ell m}^{(T)}|^2 \rangle/(2\pi) \). Fig. 1

Figures 1-4 show sample spectra, normalized to match the COBE detection. The points on the curves are weighted averages \( \langle C_{\ell} \rangle_{W_{th}} \) of the \( C_{\ell}'s \), \( \text{wrt weight functions } W_{\ell} \):

\[
\langle C_{\ell} \rangle_{W_{th}} = \langle f_{\ell} \rangle, \quad \text{where } \langle f_{\ell} \rangle = \sum_{\ell} \frac{(\ell + \frac{3}{2})}{\ell(\ell + 1)} f_{\ell} \tag{2}
\]

defines the “logarithmic integral” \( \langle f_{\ell} \rangle \) of a function \( f_{\ell} \). We choose the \( W_{\ell} \) to be filter functions for a set of existing anisotropy experiments spanning the range 10° to 2°, some of which report detections in these Proceedings: \( dmr \) (Smoot et al. 1992), \( firs \) (Ganga et al. 1993), \( ten \) (Watson 1993), \( sp91 \) (Gaier et al. 1992), \( bp \) (Wollack et al. 1993), \( pyth \) (Dragovan et al. 1993), \( msam2 \), \( msam3 \) (Cheng et al. 1993), \( max \) (Meinhold et al. 1993, Gunderson et al. 1993), \( wd2 \) (Tucker et al. 1993), \( ov7 \) (Readhead et al. 1989 - \( ov22 \) is a new OVRO experiment).

\( \langle C_{\ell} \rangle_{W_{th}} \) characterizes the broad-band power that the experiment is sensitive to. It is simply a renormalization by the factor \( \langle f_{\ell} \rangle \) (typically \( \sim 1 \)) of the \( \text{rms fluctuations } (\Delta T/T)_{rms}^2 \). The band-powers are placed at \( \langle \ell \rangle \), and the horizontal bars (when present) delineate the range that \( W_{\ell} \) covers. Errors in the estimation of the band-power \( \langle C_{\ell} \rangle \) arise from experimental noise and the theoretical cosmic variance. The error bars shown represent the limiting resolution achievable with CMB experiments, if there were full-sky coverage and errors from cosmic variance alone,

\[
\langle C_{\ell} \rangle = \langle C_{\ell} \rangle_{W_{th}} \pm \sqrt{\frac{\langle \langle C_{\ell}^{2} \rangle/W_{\ell}^2 \rangle}{\langle \ell(\ell + 1) \rangle}} \tag{3}
\]

The fractional error \( \sim \langle \ell \rangle^{-1} \) is so tiny for intermediate and small angle experiments that it would appear that even extremely subtle differences in the spectra could be determined, in
spite of the ~ 10% unavoidable error from COBE-type experiments. Thus in the figures the error bars are much smaller than the size of the points for $\ell \gtrsim 50$ and for $\ell \gtrsim 200$ they basically merge. For more realistic error bars, consider a detection obtained from measurements $\langle \Delta T/T \rangle_i \pm \sigma_D$ (where $\sigma_D$ represents detector noise) at $i = 1, \ldots, N_D$ experimental patches sufficiently isolated from each other to be largely uncorrelated. For large $N_D$, the likelihood function falls by $e^{-\frac{1}{2}}$ from a maximum at $\langle \mathcal{C}_\ell \rangle_{W,maxL}$ when

$$\langle \mathcal{C}_\ell \rangle_W = \langle \mathcal{C}_\ell \rangle_{W,maxL} \pm \sqrt{2/N_D} \left[ \langle \mathcal{C}_\ell \rangle_{W,maxL} + \frac{\sigma_D^2}{I[\mathcal{W}_\ell]} \right].$$

An experimental noise $\sigma_D$ below $10^{-5}$ is standard now, and a few times $10^{-6}$ is soon achievable; hence, if systematic errors and unwanted signals can be eliminated, the 1-sigma relative uncertainty in $\langle \mathcal{C}_\ell \rangle_W$ will be from cosmic-variance alone, $\sqrt{2/N_D}$, falling below 20% for $N_D > 50$. The observed likelihood maximum is itself centered around $\langle \mathcal{C}_\ell \rangle_{W,h}$ with a relative error of $1/\sqrt{N_D}$, but still lies within the error bar, which in fact includes this effect. The optimal variances shown in the figures roughly correspond to filling the sky with patches separated by $2\theta_{\text{fwhm}}$.

Fig. 1 shows how the small-angular signal is increasingly suppressed as $r$ increases and $n_s$ decreases [Davis et al. 1992, Crittenden et al. 1993a]. For large maps, cosmic variance is significant for large-angle experiments [White et al. 1993] but shrinks to insignificant levels at smaller scales. It appears that $r|n_s$ could be resolved if $\Lambda$, $h$ and ionization history were known.

Fig. 2 shows the effects of varying $\Omega_\Lambda$ or $H_0$ compared to our baseline (solid line) spectrum ($r = 0|n_s = 1, h = 0.5, \Omega_\Lambda = 0$). Increasing $\Omega_\Lambda$ (or decreasing $h$) enhances small-angular scale anisotropy by reducing the red shift $z_{eq}$ at which radiation-matter equality occurs. Increasing $\Omega_\Lambda$ also changes slightly the spectral slope for $\ell \lesssim 10$ due to $\Lambda$-supression of the growth of scalar fluctuations [Kofman and Starobinskii 1986]. The band-powers show that either $r|n_s$, $\Omega_\Lambda$, or $h$ can be resolved if the other two parameters are known.

A degree of “cosmic confusion” arises, though, if $r|n_s$, $\Omega_\Lambda$ and $h$ vary simultaneously. Fig. 3 shows spectra for models lying in a two-dimensional surface of $(r|n_s, h, \Omega_\Lambda)$ which produce nearly identical spectra. In one case, $r|n_s$ is fixed, and increasing $\Omega_\Lambda$ is nearly compensated by increasing $h$. In the second case, $h$ is fixed, but increasing $\Omega_\Lambda$ is nearly compensated by decreasing $n_s$ [Kofman et al. 1992].

Further cosmic confusion arises if we consider ionization history. Let $z_R$ be the red shift at which we suppose sudden, total reionization of the intergalactic medium. Fig. 4 compares spectra with standard recombination (SR), no recombination (NR) and late reionization (LR) at $z_R = 50$, where $h = 0.5$ and $\Omega_\Lambda = 0$. NR represents the behavior if reionization occurs early ($z_R >> 200$). The spectrum is substantially suppressed for $\ell \gtrsim 200$ compared to any SR models. Experiments at $\lesssim 0.5^\circ$ scale can clearly identify NR or early reionization ($z_R \gtrsim 150$ gives qualitatively similar results to NR). Reionization for $20 \lesssim z_R \lesssim 150$ results in modest suppression at $\ell \approx 200$, which can be confused with a decrease in $n_s$ (see figure). Inflation-inspired models, e.g., cold dark matter models, are likely to have negligibly small $z_R$ [Bond and Efstathiou 1984, Bond 1988, Efstathiou 1990], the large $z_R$ examples shown here suggest the small angular-scale suppression characteristic of models which require large $z_R$, such as cosmic string and texture models.

The results can be epitomized by some simple rules-of-thumb: Over the $30^\prime - 2^\circ$ range,
\langle C_\ell \rangle_W \text{ is roughly proportional to the maximum of } C_\ell \text{ (the first Doppler peak). Since the maximum (corresponding to } \sim 5^\circ \text{ scales) is normalized to COBE’s DMR band-power, } \langle C_\ell \rangle_{dmr} \text{ (with } W_\ell \text{ the dmr beam, corresponding to } \sim 8^\circ \text{ scales), its value is exponentially sensitive to } n_s. \text{ Since scalar fluctuations account for the maximum, the maximum decreases as } r \text{ increases. The maximum is also sensitive to the red shift at matter-radiation equality (or, equivalently, } (1 - \Omega_\Lambda)h^2), \text{ and to the optical depth at last scattering for late-reionization models, } \sim z_R^{3/2}. \text{ These observations are the basis of an empirical formula (accurate to } \lesssim 15\%):}

\frac{C_\ell}{\langle C_\ell \rangle_{dmr}} \bigg|_{\text{max}} \approx A e^{B \tilde{n}_s},

(5)

with } A = 0.15, B = 3.56, \text{ and}

\tilde{n}_s \approx n_s - 0.28 \log(1 + 0.8r) - 0.52[(1 - \Omega_\Lambda)h^2]^{1/2} - 0.00036 z_R^{3/2} + 0.26,

(6)

where } r \text{ and } n_s \text{ are related by Eq. (4) for generic inflation models, and } z_R \lesssim 150 \text{ is needed to have a local maximum. } (\tilde{n}_s \text{ has been defined such that } \tilde{n}_s = n_s \text{ for } r = 0, h = 0.5, \Omega_\Lambda = 0, \text{ and } z_R = 0.) \text{ [In a forthcoming paper, we show how increasing } \Omega_B h^2 \text{ increases the Doppler peak and changes the spectral shape.] Hence, the predicted anisotropy for any experiment in the range } 10^\prime \text{ and larger is not separately dependent on } n_s, r, \Omega_\Lambda, \text{ etc.; rather, it is function of the combination } \tilde{n}_s. \text{ Eq. (5) shows explicitly the separate dependence on } n_s \text{ and } r, \text{ and so can be applied to exceptional inflationary models which violate Eq. (4).}

Figure 5 shows how one might use this result, in conjunction with other astrophysical observations, to determine cosmic parameters. Eq. (4) implies that the CMB anisotropy measurements are exponentially sensitive to } \tilde{n}_s. \text{ Hence, we envisage that } \tilde{n}_s \text{ will be accurately determined in the foreseeable future. We suppose, for the purposes of illustration, that experiments indicate a value } \tilde{n}_s = 0.85. \text{ Then, Eq. (6) implies that the values of the cosmological parameters are constrained to the surface illustrated in Fig. 5. (For simplicity, we have assumed } z_R \lesssim 20, \text{ as is anticipated for standard cold dark matter models.) Cosmological models corresponding to any point on this surface yield indistinguishable CMB anisotropy power spectra. To determine which point on the surface corresponds to our universe requires other astrophysical measurements. For example, limits on the age of the universe from globular clusters, on } h \text{ from Tully-Fisher techniques, on } n_s \text{ from galaxy and cluster counts, and on } \Lambda \text{ from gravitational lenses all reduce the range of viable parameter space. It is by this combination of measurements that the CMB power spectrum can develop into a high precision test of cosmological models.}

In the discussion thus far, we have focused on what can be learned from the CMB anisotropy measurements based on the power spectrum only. The power spectrum represents only the two-point temperature correlation function. From a CMB anisotropy map, one can hope to measure three- and higher-point correlation functions, } e.g., \text{ to test for non-gaussianity of the primordial spectrum. Another conceivable test is the CMB polarization. Our calculations, though, suggest that the polarization is unlikely to be detected or to provide particularly useful tests of cosmological parameters [Crittenden et al. 1993b]. For example, it had been hoped that large-scale (small } \ell \text{) polarization measurements would be useful for discriminating scalar and tensor contributions to the CMB anisotropy [Polnarev}
1985, Ng and Ng 1993, see also M.V. Sazhin in these Proceedings], thereby measuring \( r \).

In the upper panel of Fig. 6, we show the percentage polarization (in \( \Delta T/T \)) for scalar and tensor modes for a model with \( r = 1 \) and \( n_s = .85 \), an example where there are equal tensor and scalar contributions to the quadrupole moment. The figure shows that, indeed, there is a dramatically different polarization expected for scalar versus tensor modes for small \( \ell \). However, the magnitude of the polarization is less than 0.1\%, probably too small to be detected in the foreseeable future. On scales less than one degree (\( \ell > 100 \)), the total polarization rises and approaches 10\%, a more plausible target for detection. However, the tensor contribution on these angular scales is negligible, so detection does not permit us to distinguish tensor and scalar modes. In fact, the predictions are relatively insensitive to the cosmological model, a notable exception being the reionization history. The lower panel of Figure 6 illustrates the prediction for a model with no recombination. The overall level of polarization is increased. The tensor contribution is suppressed relative to scalar, so polarization remains a poor method of measuring \( r \). However, the polarization at angular scales of a few degrees (\( \ell \approx 50 \)) rises to nearly 5\%, perhaps sufficient for detection. An observation of polarization at these angular scales would be consistent with a non-standard reionization history.

We thank R. Crittenden and G. Efstathiou, who collaborated in the research from which this summary is drawn. This research was supported by the DOE at Penn (DOE-EY-76-C-02-3071), NSERC at Toronto, and the Canadian Institute for Advanced Research.

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FIGURE CAPTIONS

**Figure 1.** Power spectra as a function of multipole moment $\ell$ for $(r=0|n_s=1)$, $(r=0.7|n_s=0.9)$ and $(r=1.4|n_s=0.8)$ where $h = 0.5$ and $\Omega _\Lambda = 0$ for all models. The spectra in all figures are normalized so that the COBE band-power is $10^{-10}$. It is observed to be $(1.0 \pm 0.2) \times 10^{-10}$. The (elongated) vertical bars on the band-powers $\langle C_\ell \rangle_W$ are 1-sigma cosmic variance error bars assuming full-sky coverage for the 11 experiments shown (see also Eq. [4]). The band-powers are placed at the average $\langle \ell \rangle_W$ of $\ell$ over the filter. The horizontal error bars on the $n_s=1$ case give the ranges where the filters fall by half an e-folding from their maxima.

**Figure 2.** Power spectra as a function of $\ell$ for scale-invariant models, with $r = 0|n_s = 1$. The middle curve shows $h = 0.5$ and $\Omega _\Lambda = 0$. In the upper curve, $\Omega _\Lambda$ is increased to 0.4 while keeping $h = 0.5$. In the lower curve, $\Omega _\Lambda = 0$ but $h$ is increased from 0.5 to 0.65 (hence $\Omega _B$ drops from 0.5 to 0.3). The spectra are insensitive to changes in $h$ for fixed $\Omega _B$. Increasing $\Omega _\Lambda$ or $\Omega _B$ increases the power at $\ell \sim 200$.

**Figure 3.** Examples of different cosmologies with nearly identical spectra of multipole moments and values of the band-powers. The solid curve is $(r = 0|n_s = 1, h = 0.5, \Omega _\Lambda = 0)$. The other two curves explore degeneracies in the $(r = 0|n_s = 1, h, \Omega _\Lambda)$ and $(r|n_s, h = 0.5, \Omega _\Lambda)$ planes. In the dashed curve, increasing $\Omega _\Lambda$ is almost exactly compensated by increasing $h$. In the dot-dashed curve, the effect of changing to $r = 0.42|n_s = 0.94$ is nearly compensated by increasing $\Omega _\Lambda$ to 0.6.

**Figure 4.** Power spectra for models with standard recombination (SR), no recombination (NR), and ‘late’ reionization (LR) at $z = 50$. In all models, $h = 0.5$ and $\Omega _\Lambda = 0$. NR or reionization at $z \geq 150$ results in substantial suppression at $\ell \geq 100$. Models with reionization at $20 \leq z \leq 150$ give moderate suppression that can mimic decreasing $n_s$ or increasing $h$; e.g., compare the $n_s = 0.95$ spectrum with SR (thin, dot-dashed) to the $n_s = 1$ spectrum with reionization at $z = 50$ (thick, dot-dashed).

**Figure 5.** The surface in the parameter-space $(n_s, h, \Omega _\Lambda)$ corresponding to $\tilde{n}_s = 0.85$, as determined by Eq. (3). The grey-scale varies with the height of the surface, or, equivalently, the value of the spectral tilt $n_s$, the darkest strip corresponds to $n_s \approx 0.85$. Note that $\tilde{n}_s = 0.85$ is also consistent with values of $n_s$ quite different from 0.85. Choices of cosmological parameters corresponding each point along the surface yield virtually indistinguishable CMB anisotropy.

**Figure 6.** The percentage polarization in $\Delta T/T$ versus multipole moment $\ell$ predicted for an inflationary model with $n_s = 0.85$, $h = 0.5$, cold dark matter, and standard recombination.
(For this value of $n_s$, inflation predicts equal scalar and tensor contributions to the unpolarized quadrupole.) The upper panel represents the prediction for standard recombination and the lower panel is for a model with no recombination.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9402041v1