DEPENDENCE OF ECS ON TEMPORAL CORRELATION STRUCTURE

DEPENDENCE OF INFERRED CLIMATE SENSITIVITY ON THE DISCREPANCY MODEL

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Abstract—We consider the effect of different temporal error structures on the inference of equilibrium climate sensitivity (ECS), in the context of an energy balance model (EBM) that is commonly employed in analyzing earth system models (ESM) and observations. We consider error structures ranging from uncorrelated (IID normal) to AR(1) to Gaussian correlation (Gaussian Process GP) to analyze the abrupt 4xCO₂ CMIP5 experiment in twenty-one different ESMs. For seven of the ESMs, the posterior distribution of ECS is seen to depend rather weakly on the discrepancy model used suggesting that the discrepancies were largely uncorrelated. However, large differences for four, and moderate differences for the rest of the ESMs, leads us to suggest that AR(1) is an appropriate discrepancy correlation structure to use in situations such as the one considered in this article.

Other significant findings include: (a) When estimates of ECS (mode) were different, estimates using IID were higher (b) For four of the ESMs, uncertainty in the inference of ECS was higher with the IID discrepancy structure than with the other correlated structures, and (c) Uncertainty in the estimation of GP parameters were much higher than with the estimation of IID or AR(1) parameters, possibly due to identifiability issues. They need to be investigated further.

I. INTRODUCTION

On the one hand, Earth System Models (ESMs) that comprise of atmosphere-ocean general circulation models (AOGCMs) coupled to other earth system components such as ice sheets, land surface, terrestrial biosphere, and glaciers are central to developing our understanding of the workings of the climate system, and are proving to be the most comprehensive tool available to study climate change and develop climate projections [1]. Concomitantly, simple concepts such as climate sensitivities—metrics used to characterise the response of the global climate system to a given forcing—are central not only to climate modeling, but also to discussions of the ongoing global warming (e.g., see [2], [3]). Nevertheless, the immense computational infrastructure required and the cost incurred in running ESMs precludes the direct evaluation of such metrics.

Simple climate models (SCMs) on the other hand consider only integral balances of important quantities such as mass and/or energy, are computationally cheap and can be used in myriad different ways (unlike ESMs that are typically run only in the forward mode). It is for these reasons that the use of SCMs to estimate climate sensitivities, both in the context of ESMs and actual observations, is now well established (e.g., see [4], [5] and others).

II. METHODOLOGY AND RESULTS

A particular form of an SCM that has been popular in summarizing integral thermal properties of AOGCMs and/or ESMs is the anomaly-based upwelling-diffusion (UD) energy-balance model (EBM) [4]. To briefly describe such a model, consider a horizontally-integrated model of the climate system that is partitioned into two active layers in the vertical. An upper (surface) layer that comprises the oceanic mixed layer, atmosphere and land surface and a bottom layer that comprises the ocean beneath the mixed layer. Evolution of the upper surface heat content anomaly per unit area is given by

\[ C_u \frac{dT_u}{dt} = F - \lambda T_u - \gamma (T_u - T_d) \]  

(1)

where \( \lambda \) represents. Exchange of heat between the surface layer and the ocean beneath is parameterized by the difference in temperature between the two layers. Similarly, evolution of the subsurface ocean heat content anomaly (again per unit area ) is given by

\[ C_d \frac{dT_d}{dt} = -\gamma (T_u - T_d) \]  

(2)

Such two layer models have been used extensively to obtain point estimates of ECS of AOGCMs/ESMs (e.g., see [5], [6], and others). However, and to the best of our knowledge, the dependence of estimates of ECS on the assumed temporal structure of the discrepancy between ESM representation of the surface air temperature (SAT) and the above EBM’s (Eqs. 1 & 2) representation of it has not been investigated:

\[ T_u^{ESM}(t) = T_u^{EBM}(t) + \epsilon_t \]  

(3)
Fig. 1. Inferred ECS for the 21 models considered using CMIP5 experiment abrupt4xCO$_2$. 
The equation above arises from the fact that the anomaly-based EBM considered has no representation of climate variability, unlike the more comprehensive ESM that it is used to analyze. We consider three correlation structures for $\epsilon_t$:

1. **IID**: $\Sigma(t-s) = \sigma^2 \delta(t-s)$
2. **AR(1)**: $\Sigma(t-s) = \sigma^2 \rho^{t-s}$
3. **AR(1)+GP**: $\Sigma(t-s) = \sigma^2 \exp(-\frac{(t-s)^2}{\lambda^2})$

where structure AR(1)+GP uses the sum of the covariance matrices indicated in items 2 and 3 above.

In the context of globally-averaged SAT (of which sea surface temperature or SST is a large component), we know, e.g., following the work of [7], that the mixed layer integrates (high-frequency) weather noise. Thus, SST (and therefore SAT) is expected to be auto-correlated in time, although the correlations themselves are highly spatiotemporally variable (e.g., winter SST anomalies are more persistent than summer SST anomalies, the tropical Pacific may display larger persistence than the tropical Atlantic, etc...). However, such correlations rarely exceed about six months and we are considering annual-averaged SAT. Physically, correlations on the interannual time scales are related to internal climate dynamics (phenomena such as the re-emergence of the winter mixed layer, delay-oscillations and others). While a casual inspection of some actual climate time series may suggest the unlikeliness of IID variability, it is not the case for the globally-averaged SAT time series in the abrupt4xCO2 CMIP5 experiment that we analyze, and as we will see later. However, it should also be noted that if the discrepancies are actually correlated, then an inference of EBM parameters using the IID discrepancy structure will result in estimates of uncertainty that are smaller than actual, and again as we will see later.

Figure 1 shows the posterior distribution of ECS with the three error structures (indicated in the legend) for the 21 ESMs. The prior is shown in the bottom-left panel. In this figure it is seen that

- For a substantial number of the ESMs, the three error structures lead to similar estimates of ECS (CNRMCM5, MIROC5, MPIESML/MR, MPIESMP, MRICGCM3, NorESM1M).
- When the estimates of ECS are different, estimates using IID tend to be higher (CCSM4, FGOALSs2, GFDLESML2G/M, HadGEM2ES, in-mcm4).
- Differences in ECS estimates from that between AR(1) and AR(1)+GP tend to be smaller than that between either and IID.
- In a majority of the ESMs considered, IID leads to smaller estimates of uncertainty in ECS suggesting that the discrepancies in those models are temporally correlated. The exceptions (CCSM4, GOALSs2, GFDLESML2G/M), are therefore surprising and need to be investigated further.

We also note that there was far more uncertainty in the estimation of GP parameters as opposed to estimation of either IID and AR(1) parameters. This is likely due not only to the shortness of the ESM runs considered (150 years) from the point of low-frequency variability that the GP component was intended to capture, but may involve issues of identifiability and needs to be investigated further. However, when such problems occur, the parameters involved act more as nuisance parameters and do not prevent reasonable inference of ECS and other EBM parameters.

### III. Discussion

Simple climate models play a valuable role in helping interpret both observations and the responses of comprehensive ESMs. As such, we used a simple and popular EBM in a Bayesian framework to interpret the abrupt4xCO2 CMIP5 experiment in 21 ESMs, in terms of their SAT response. We used three different statistical models to represent the discrepancy in the SAT response of the ESMs and SCMs. This discrepancy is largely due to natural variability—an aspect of climate that represented in the ESMs, but not in the SCMs. For seven of these models, the posterior distribution of ECS depended only very weakly on the discrepancy model used suggesting that the discrepancies were largely uncorrelated. For four of the models, the differences were large and for the rest of the models, the differences were moderate. Significant differences in a majority of the models, therefore, indicate the existence of temporal correlations in the discrepancies and the importance of accounting for them in a Bayesian inference framework.

Next, the differences in estimated ECSs were much smaller for inferences using AR(1) and AR(1)+GP as compared to differences between inferences using either of these models and IID. This coupled with the fact that the uncertainty in the estimation of GP parameters was much larger than that in the estimation of AR or IID parameters, leads us to conclude that AR(1) is a good choice in situations such as the one considered in this article.

A number of other issues need to be investigated further: the higher estimates of ECS when using the IID structure for some of the ESMs, the higher uncertainty in the estimation of ECS when using the IID structure for some of the ESMs, and the increased uncertainty in the estimation of GP parameters as compared to that in the estimation of IID or AR(1) parameters.

Additionally, the existence of an analytic inverse for the covariance of an AR(1) process makes it faster to compute with as compared to with a GP.
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