Modeling the Shape of the Roll Contact Curves in Two-Roll Modules

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Abstract. The study is devoted to mathematical modeling of the shape of the roll contact curves in two-roll modules. A two-roll module is considered, in which the rolls are positioned relative to the vertical with a tilt to the right, have unequal diameters and elastic coatings from the materials of different stiffness and friction coefficients, the material layer is fed with a tilt downward relative to the center line. The equations of the curves of roll contact in the considered two-roll module are derived. It is revealed that the obtained models are general in the sense that they are applicable for particular cases of interaction in a two-roll module.

1. Introduction
In two-roll modules, the forces applied to the roll axis of the rolls are transferred to the material being processed along the curves of the roll contact. One of the main tasks in the theory of contact interaction of two-roll modules is a mathematical modeling of the shape of these curves.

In two-roll modules with rolls having elastic coatings, mutual strain in the contacting bodies occurs. In this case, the contact curves have complex configurations. In this regard, the theoretical determination of contact curves is a rather complicated task. Therefore, in many cases, the curve shape of the roll contact is approximated by the arcs of a circle, an ellipse and a parabola.

The shape of the contact curves is judged by the change pattern in strain in the material layer in the process of interaction with the rolls. Some researchers believe that the strain in the material layer with the rolls at each point of time occurs in the radial direction to the axis of the rolls, others think that it occurs in the vertical direction to the layer. Based on this, studies on analytical description of the curve shape of the roll contact can be divided into two groups. In the first group of studies, the strain in the material layer with rolls at each point of time is considered in the radial direction to the axis of the rolls and the contact curves of the rolls are described by formulas of the form [1, 2, 3].

\[ r = a + \frac{b}{\cos \theta} \]

In the second group of studies, the strain in the material layer with the rolls at each point of time is considered in the vertical direction to the layer and the contact curves of the rolls are described by equations of the form [4, 5]

\[ r = \frac{a}{1 + b \cos \theta} \]

Contact interaction in two-roll modules with rolls having elastic coatings can be considered by analogy with the rolling of an elastic wheel over deformable soil. In the theory of wheel rolling, the...
analytical definition of the contact line is associated with the analysis of the ratio of the strain rates of interacting bodies [2]. In a number of publications, this ratio is considered constant [6, 7]. Then the ratio of wheel and soil strain is equal to the ratio of their strain rates [4].

The aim of the work is a mathematical modeling the curve shape of the roll contact in two-roll modules.

2. Analytical solution of the task
To systematize the studies, consider a generalized model of two-roll modules. In a two-roll module, the rolls are located relative to the vertical with a tilt to the right at an angle \( \beta \), have unequal diameters \( R_1 \neq R_2 \) and elastic coatings from materials of different stiffness and friction coefficients \( f_1 \neq f_2 \), the lower shaft is a driving one, the upper one is free. The material layer has a uniform thickness \( \delta_1 \) and is fed with a tilt downward relative to the line of centers at an angle \( \gamma_1 \) (figure 1).

![Figure 1. Interaction scheme in a two-roll module](image)

Study two cases of the curve shapes of the roll contact in two-roll modules. In the first case, assume that, in the process of interaction, the strain in the material layer occurs in the radial direction to the axis of the rolls, and in the second case - in the vertical direction.

First analyze the first case of the curve shapes of the roll contact.

Consider the interaction of the material layer with the lower roll. The shape of the contact curve is analyzed in polar coordinates. Curve (curve \( A_1A_2 \)) consists of two sections \( A_1K \) and \( KA_2 \) (figure 1). On the \( A_1K \) section, the interacting bodies are compressed, and on \( KA_2 \) the recovery occurs, where \( K \) is the point of the contact curve of the lower roll lying on the center line of the rolls. Any point of \( B_1 \) of section \( A_1K \) is determined by the polar coordinates \( r_{11} \) and \( \theta_{11} \), and point \( B_2 \) of the section \( KA_2 \) by \( r_{12} \) and \( \theta_{12} \). According to figure 1, \(- \varphi_1 \leq \theta_1 \leq 0 \), \( 0 \leq \theta_2 \leq \varphi_2 \), where \( \varphi_1, \varphi_2 \) - are the contact angles of the lower roll.

In the process of interaction with the rolls, the angle of inclination of the processed material changes: in section \( A_1K \) it decreases from \( \gamma_1 \) to zero, and on \( KA_2 \) it increases from zero to \( \gamma_2 = m\gamma_1 \), where \( m \) - is the proportionality coefficient.

Consider that
\begin{align*}
\gamma = \gamma(\theta_{11}) &= a_1\theta_{11} + b_1, \quad -\varphi_1 \leq \theta_{11} \leq 0, \\
\gamma = \gamma(\theta_{12}) &= a_2\theta_{12} + b_2, \quad 0 \leq \theta_{12} \leq \varphi_{12}.
\end{align*}

Coefficients \(a_{11}, b_{11}, a_{12}\) and \(b_{12}\) are found from initial and boundary conditions:

- at \(\theta_{11} = -\varphi_1, \quad \gamma = \gamma_{11};\)
- at \(\theta_{12} = \varphi_{12}, \quad \gamma = 0;\)
- at \(\theta_{12} = \varphi_{12}, \quad \gamma = m\gamma_1.\)

Then we have

\[ \gamma = \frac{\gamma_{11}\theta_{11}}{\varphi_{11}}, \quad -\varphi_1 \leq \theta_{11} \leq 0, \quad \gamma = \frac{m\gamma_{12}}{\varphi_{12}}, \quad 0 \leq \theta_{12} \leq \varphi_{12}. \tag{1} \]

Similar to the theory of rolling wheels, consider the surface layers strain ratio of the rolls and the layer of material equal to the ratio of their strain rates.

At each point of the roll contact curve, the strain of the contacting bodies occurs along the \(n - n\) line perpendicular to the roll contact curve. Then at point \(B_1\) of section \(A_1K\) we have (figure 1)

\[ \frac{D_1B_1}{B_1C_1} = \frac{dr_{11}}{dt} \quad \text{or} \quad \frac{D_1B_1}{B_1C_1} = \lambda_{11}, \tag{2} \]

where \(\lambda_{11} = \frac{dr_{11}}{dh}\) is the strain rates ratio of the surface layer of lower roll and the layer of material under compression; \(\psi\) - the angle between the radius \(r_{11}\) and the line \(n - n.\)

From figure 1 we find

\[ D_1B_1 = O_1D_1 - O_1B_1 = R_{11} - r_{11}, \quad B_1C_1 = O_1B_1 - O_1C_1 = r_{11} - R_{11} \frac{\cos(\phi_{11} + \gamma)}{\cos(\theta_{11} + \gamma)}. \tag{3} \]

With expressions (3), equalities (2) take the form

\[ \frac{R_{11} - r_{11}}{\cos(\phi_{11} + \gamma)} = \lambda_{11}. \tag{4} \]

Having solved equalities (4) with respect to \(r_{11}\), we find the equation of the contact curve of the section \(A_1K\)

\[ r_{11} = \frac{R_{11}}{1 + \lambda_{11}} \left(1 + \lambda_{11} \frac{\cos(\phi_{11} + \gamma)}{\cos(\theta_{11} + \gamma)} \right). \tag{5} \]

To simplify research, expression \(\cos(\phi_{11} + \gamma)\) in formula (5), where \(0 \leq \gamma \leq \gamma_1\), is replaced by expression \(\cos(\theta_{11} + \gamma)\).

Then we have

\[ r_{11} = \frac{R_{11}}{1 + \lambda_{11}} \left(1 + \lambda_{11} \frac{\cos(\phi_{11} + \gamma)}{\cos(\theta_{11} + \gamma)} \right), \quad \gamma = \frac{\gamma_{11}\theta_{11}}{\varphi_{11}}, \quad -\varphi_1 \leq \theta_{11} \leq 0. \tag{6} \]

Geometrical analysis shows that the transition from formula (5) to formula (6) means replacing the straight line \(A_1E_1\) with straight line \(A_1M_1\) (Fig. 1). Since the inclination angle of the material layer relative to the center line is usually small, such a replacement is admissible (in this case, the calculation error does not exceed 1.5% - 3.2%).

By analogy with formula (5), the equation of the contact curve of the section \(KA_2\) is written as:
\[
\gamma = \frac{m_{1}\gamma_{12}}{\varphi_{12}}, \quad 0 \leq \theta_{12} \leq \varphi_{12}.
\]

(7)

where \( \lambda_{42} = \frac{d\gamma_{12}}{dh_{2}} \) is the ratio of strain rates of the lower roll surface layer and the material layer at recovery.

Summarizing equations (6) and (7) and taking into account expressions (1), we find the equations of the lower roll contact curve:

\[
\begin{align*}
\gamma_{1} &= \frac{R_{1}}{1 + \lambda_{11}} \left( 1 + \lambda_{11} \frac{\cos(\varphi_{11} + \gamma_{1})}{\cos(\theta_{11} + \gamma)} \right), \\
\gamma &= \frac{\gamma_{1} \theta_{11}}{\varphi_{11}} - \varphi_{11} \leq \theta_{11} \leq 0,
\end{align*}
\]

(8)

By analogy of definition of the equation of the lower roll contact curve, we determine the equations of upper roll contact curve

\[
\begin{align*}
\gamma_{1} &= \frac{R_{2}}{1 + \lambda_{21}} \left( 1 + \lambda_{21} \frac{\cos(\varphi_{21} - \gamma_{2})}{\cos(\theta_{21} - \gamma)} \right), \\
\gamma &= \frac{\gamma_{1} \theta_{21}}{\varphi_{21}}, \quad -\varphi_{21} \leq \theta_{21} \leq 0,
\end{align*}
\]

(9)

where \( \lambda_{21} = \frac{d\gamma_{12}}{dh_{1}} \), \( \lambda_{22} = \frac{d\gamma_{21}}{dh_{2}} \) = are the ratios of the strain rates of the upper roll surface layer and the material layer at compression and recovery, respectively.

Now examine the curve shapes of the roll contact in the second case. Similar to the first case, at point \( B_{1} \) of section \( A_{1}K \) we have (figure 2)

\[
\frac{D_{1}B_{1}}{B_{1}C_{1}} = \lambda_{11},
\]

(10)

In a two-roll module, the angle \( \psi \) can range from zero to \( \theta_{11} + \gamma \) : \( \psi = 0 \), when the roll is stiff; \( \psi = \theta_{11} + \gamma \) when the layer of material is stiff. Therefore, we assume that \( \psi = C(\theta_{11} + \gamma) \), at \( 0 \leq C \leq 1 \).

Then equality (10) has the form:

\[
\frac{D_{1}B_{1}}{B_{1}C_{1}} \cdot \frac{\cos(1 - C)(\theta_{11} + \gamma)}{\cos(\theta_{11} + \gamma)} = \lambda_{11},
\]

or after transform

\[
\frac{D_{1}B_{1}}{B_{1}C_{1}} \cdot (\cos(\theta_{11} + \gamma) + \sin(\theta_{11} + \gamma)\tan C(\theta_{11} + \gamma)) = \lambda_{11}.
\]

We simplify this equality by accepting \( \sin(\theta_{11} + \gamma)\tan C(\theta_{11} + \gamma) \approx 0 \), since it has the values of the third power close to zero:

\[
\frac{D_{1}B_{1}}{B_{1}C_{1}} \cdot \cos(\theta_{11} + \gamma) = \lambda_{11}.
\]

(11)

From figure 2 it follows

\[
D_{1}B_{1} - O_{1}D_{1} - O_{1}B_{1} = R_{1} - r_{11}, \quad B_{1}C_{1} = r_{11}\cos(\theta_{11} + \gamma) - R_{1}\cos(\theta_{11} + \gamma).
\]

(12)

With expression (12) equality (11) has the form
\[
\frac{(R_i - r_{1i}) \cos(\theta_{1i} + \gamma)}{r_{1i} \cos(\theta_{1i} + \gamma) - R_i \cos(\phi_{1i} + \gamma)} = \lambda_{1i}.
\]  \hspace{1cm} (13)

Solving equality (13) relative to \( r_{1i} \), the equation of the contact curve of the section \( A_iK \) is obtained:

\[
r_{1i} = \frac{R_i}{1 + \lambda_{1i}} \left( 1 + \lambda_{1i} \frac{\cos(\phi_{1i} + \gamma)}{\cos(\theta_{1i} + \gamma)} \right).
\]

This equation coincides with equation (5). By analogy to the first case, we obtain the equations of the roll contact curves for the second case. They coincide with the systems of equations (8) and (9).

Thus, the equations of the roll contact curves in the two-roll modules in the first and second cases are described by systems (8) and (9). The effect of geometrical characteristics of one of the rolls on the contact curve shape of the second roll determines the contact angles, and the strain characteristics of the material layer and both rolls are completely included in the rate ratio. In the two-roll module under consideration, the upper roll is free. It can be a driving roll. In this case, the two-roll module has a transmission mechanism between the rolls. The gripping conditions in a two-roll module with one driving roll differ from the gripping conditions with two driving rolls. The gripping conditions in the two roll module determine the contact angles. Therefore, two-roll modules with one driving roll and with two driving rolls have different contact angles [8]. In this regard, we can say that the effect of the transmission mechanism on the curve shape of contact rolls is also determined by the contact angles.

All partial cases of the two-roll module under consideration and the corresponding partial types of the system of equations (8) and (9) are analyzed. It was revealed that the formulas obtained are general in the sense that they describe the cases of the interaction of a material layer with pairs of two-roll modules. So, they can be used in modeling the friction stresses.

3. Results

For the first time, formulas (systems of equations (8) and (9)) were found that describe the shape of the contact curves of the rolls in an asymmetric two-roll module.
4. Conclusions
1. For the first time, models of roll contact curves in a two-roll module were found, in which: the rolls are positioned relative to the vertical by tilting to the right, have unequal diameters and elastic coatings from materials with different stiffnesses and friction coefficients; lower roll drive, upper - free; the material layer is tilted downward relative to the center line.
2. It is revealed that the obtained models are general in the sense that they are applicable for particular cases of interaction in a two-roll module.

5. References
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