$\gamma^*\gamma^* \rightarrow \rho\rho$ at very high energy.

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The next generation of $e^+e^-$-colliders will offer a possibility of clean testing of QCD dynamics in the Regge limit. Recent progress in the theoretical description of exclusive processes permits for many of them a consistent use of the perturbative QCD methods. We advocate that the exclusive diffractive production of two $\rho$ mesons from virtual photons at very high energies should be measurable at the future linear collider (LC).

1. Introduction

The high energy limit of strong interaction has a very long story, which started much before the development of QCD \cite{QCD}. The Regge limit corresponds to the kinematical regime of large scattering energy square $s$ and small momentum transfer square $t$, $s \gg -t$. Soon after QCD was proposed as a theory for strong interactions, its Regge limit was studied by Balitsky, Fadin, Kuraev and Lipatov \cite{BFKL}. The evaluation of the elastic scattering amplitude of two infrared safe objects was performed, as an infinite series in $\alpha_s \ln s$. This so-called Leading Log Approximation (LLA), where small values of perturbative $\alpha_s$ are compensated by large values of $\ln s$, is expressed as an effective ladder with two reggeized gluons in $t$-channel (gluons dressed by interaction, resulting in appearance of Regge trajectories) interacting with $s$-channel gluonic rungs, through the effective Lipatov vertex which generalizes the usual triple Yang-Mills vertex. The net result for this hard Pomeron intercept is $\alpha_P(0) = 1 + c\alpha_s$, where $c$ is a strictly positive constant, which thus leads to a violation of the Froissart bound at perturbative level.

In order to test the hard Pomeron, it is not enough to study large $s$ experiments. It is also compulsory to select processes where a hard scale enables one to use perturbative QCD. In DIS, the virtuality of the photon naturally provides a hard scale. At the level of both total and diffractive cross-sections, it was possible to describe HERA data using models based on BFKL type of evolution, although the distinction with standard DGLAP evolution \cite{DGLAP} is not conclusive \cite{DGLAP}. Exclusive vector meson production was also proposed in order to see BFKL effects, selecting events with a large gap in rapidity between the vector meson and the outgoing proton (or its remnants). These approaches needed however some ansatz for the non-perturbative proton-Pomeron coupling.
2. $\gamma^*\gamma^*$ processes: the gold plated experiment

From the theoretical point of view, the best way for studying typical Regge behaviour in perturbative QCD is provided by the scattering of small transverse size objects. Such a reaction is naturally provided by photons of high virtuality as produced in $e^+e^-$ tagged collisions. This was investigated at the level of total $\gamma^*\gamma^*$ cross section by various groups [5]. Typical Pomeron enhancement can hardly be seen at LEP, but should be definitely measurable at LC. One of the key point in order to reveal this effect is that the detectors should be able to tag the outgoing particle with minimal tagging angle down to 20 mrad.

Another possibility is to select specific heavy bound states ($J/\Psi, \Upsilon, ...$) in the final state. This has been studied in the case of double diffractive photo production of $J/\Psi$ [5]. Several tens of thousand events are expected at LC, with an enhancement factor of the order of 50 with respect to the Born estimate.

We study the process of exclusive electroproduction of two $\rho$-mesons in $\gamma^*\gamma^*$ collisions. The measurable cross section in $e^+e^-$ collisions is related to the amplitude of this process through the usual photon flux factors:

$$
\frac{Q_1^2Q_2^2d\sigma(e^+e^- \rightarrow e^+e^-\rho\rho)}{dy_1dy_2dQ_1^2dQ_2^2} = \frac{\alpha}{2\pi}P_{\gamma/e}(y_1)P_{\gamma/e}(y_2)\sigma(\gamma^*\gamma^* \rightarrow \rho\rho),
$$

where $y_i$ are the longitudinal momentum fractions of the Bremstrahlung photons with respect to the respective leptons and with $P_{\gamma/e}(y) = 2(1-y)/y$ for longitudinally polarized photons. The virtualities $Q_1^2$ and $Q_2^2$ of the scattered photons play the role of the hard scales. This allows one to scan $Q_1^2$, $Q_2^2$, as well as $t$ to test the structure of the hard Pomeron. It is also possible to study various polarizations of both photons and mesons. As a first step in this direction we have calculated the Born order contribution to this process with longitudinally polarized photons and $\rho$-mesons, as illustrated in Fig.1. The

![Figure 1. Amplitude for the process $\gamma^*\gamma^* \rightarrow \rho\rho$ at Born order. The dots denote the effective coupling of t-channel gluons to the impact factors. Virtualities are defined by $Q_{1(2)}^2 = -q_{1(2)}^2$.](image)

choice of longitudinal polarizations of both the scattered photons and produced vector mesons is dictated by the fact that this configuration of the lowest twist-2 gives the dominant contribution in the powers of the hard scales $Q_{1,2}^2$. $Q_1^2$ and $Q_2^2$ should be taken of the same order so as to reduce phase space for conventional parton evolution.

We use the usual impact representation, where the meson vertex is treated in the collinear approximation which neglects in the hard part of the amplitude the relative transverse momentum of the quarks. This results in appearance of the Distribution Amplitude (DA): the meson wave function integrated over the relative momentum of quarks.
The amplitude for the process reads, defining $\bar{z} = 1 - z$,

$$M = -i s 2\pi \frac{N_c^2 - 1}{N_c^2} \alpha_s^2 \alpha_{em} f_\rho^2 Q_1 Q_2 \int_0^1 dz_1 dz_2 z_1 \bar{z}_1 \phi(z_1) z_2 \bar{z}_2 \phi(z_2) M(z_1, z_2). \quad (2)$$

Here $f_\rho$ is the $\rho$-meson coupling constant $\phi(z) = 6z\bar{z}$ is the asymptotic DA of the $\rho$ meson, $z$ ($\bar{z}$) being the light-cone quark (antiquark) $\rho$ momentum fraction, and

$$M(z_1, z_2) = \int \frac{d^2k}{k^2(k - \bar{k})^2} \left[ \frac{1}{z_1^2 \bar{z}_1^2 + \mu_1^2} + \frac{1}{\bar{z}_1 \bar{\bar{z}}_1 \bar{k}^2 + \mu_1^2} - \frac{1}{(z_1 \bar{z}_1 - \bar{k})^2 + \mu_1^2} \right]$$

$$\times \left[ \frac{1}{z_2^2 \bar{z}_2^2 + \mu_2^2} + \frac{1}{\bar{z}_2 \bar{\bar{z}}_2 \bar{k}^2 + \mu_2^2} - \frac{1}{(z_2 \bar{z}_2 - \bar{k})^2 + \mu_2^2} \right] \quad (3)$$

is the transverse momentum convolution of the impact factors with $2t$-channel gluon propagators. It can be expressed in terms of three kind of integrals, namely

$$I_2 = \int \frac{d^2k}{k^2(k - \bar{p})^2}, \quad I_{3m} = \int \frac{d^2k}{k^2(\bar{k} - \bar{p})^2((\bar{k} - \bar{q})^2 + m^2)}, \quad (4)$$

$$I_{4mm} = \int \frac{d^2k}{k^2(\bar{p} - \bar{q})^2((\bar{k} - \bar{q})^2 + m^2)((\bar{k} - \bar{b})^2 + m_1^2)}, \quad (5)$$

where we use the dimensional regularization $d = 2 + 2\epsilon$. These integrals were computed exactly using a generalized version of a technique used in coordinate space when evaluating diagrams of massless two dimensional conformal field theories.

The final result for $M(z_1, z_2)$ is too lengthy to be given here. It is regular in $z_1$ and $z_2$. After numerical integration over $z_1$ and $z_2$ and squaring, one obtains the differential cross-section $d\sigma^{\gamma^*\gamma^*\to\rho\rho}/dt$, shown in Fig.2a, for various values of $Q_1^2 = Q_2^2 = Q^2$. It is rapidly decreasing in $t$, and flat in $s$. Any BFKL type of resummation would give a rising shape in $s$. Integrating over $t$, one gets the $\sigma^{\gamma^*\gamma^*\to\rho\rho}$ cross-section, as shown in Fig.2b.
The expected number of events at LC, for a nominal luminosity of 100 fb$^{-1}$, is of the order of 1000 events per year. This is only a lower bound since the contribution of the transverse photon case has to be added. Moreover, we expect a net and visible enhancement of this cross section, because of resummation effects à la BFKL.

3. Conclusions

Double diffractive $\rho$ production in $e^+e^-$ collisions is a crucial test for QCD in Regge limit. The Born contribution for longitudinally polarized photon and meson gives a measurable cross-section. BFKL enhancement remains to be evaluated.

e$^+e^-$ collisions would be also a very good place to observe and test the Odderon. Such an object is the partner of the Pomeron, with opposite charge conjugation. We propose to study double diffractive $\pi^0$ production from two highly virtual photons, which should be dominated by the $t-$channel exchange of an Odderon. In QCD, such a state is constructed from at least 3 gluons, and resummation effects are expected in the Regge limit [7]. To test the existence of Odderon at the amplitude level, one may study interference effects between Odderon and Pomeron exchange in $\gamma^*\gamma^* \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)$ processes, using the fact that the C-parity is not fixed for such final states [8].

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