Motion Simulation for Triangular-Shaped Autonomous Underwater Vehicle (TAUV) with Controllable Rudder

I Y Amran1, K Isa1*

1 Faculty of Electrical and Electronic Engineering, Universiti Tun Hussein Onn Malaysia, 86400 Parit Raja, Johor.

*halid@uthm.edu.my

Abstract. The dynamic model and motion simulation for a Triangular-Shaped Autonomous Underwater Vehicle (TAUV) with independently controlled rudders are described in this paper. The TAUV is designed for biofouling cleaning in aquaculture cage fishnet. It is buoyant underwater and moves by controlling two thrusters. Hence, in this research work, the authors designed a TAUV that is propelled by two thrusters and maneuvered by using an independently controllable rudder. This paper discussed the development of a mathematical model for the TAUV and its dynamic characteristics. The mathematical model was simulated by using Matlab and Simulink to analyze the TAUV’s motion based on open-loop control of different rudder angles. The position, linear and angular velocities, angle of attack, and underwater vehicle speed are all demonstrated in the findings.

1. Introduction

The Malaysian fisheries industry is a multibillion-ringgit industry, and the aquaculture industry is required to expand into large-scale commercial industries under the Third National Agricultural Policy (NAP3). However, biofouling is a problem in aquaculture, and it can cause low oxygen concentration in the fish cage, which can reduce fish production [1]. Cleaning the nets is important to maintain the flow of oxygen concentration in the fish cage. Consequently, an advanced underwater vehicle should be implemented in biofouling cleaning and prevention by introducing a bio-inspired control mechanism for cleaning the fish cages.

The Autonomous Underwater Vehicle (AUV) study is a widely discussed subject around the world. There are two types of underwater vehicles: Autonomous Underwater Vehicles (AUVs) and Remotely Operated Vehicles (ROVs) [2,3]. An AUV operates autonomously with the user command and returns after completing the task given. On the other hand, the ROV operates with wires attached to the body and transmits the data through the wires that convey power and enables the operator to monitor the ROV [4,5].

The model of AUV proposed in this paper is small, flattened, and triangular-shaped. The research work presented in this paper is a continuous work from [6,7] to gain a deeper understanding of the dynamics and motion of TAUV. Matlab and Simulink are used to carry out the motion simulations of the proposed TAUV in an open-loop control model. In the simulation, the motion characteristics of the TAUV were successfully established. These findings lay the basis for developing a biologically inspired control algorithm for a Triangular-Shaped Autonomous Underwater Vehicle.
1.1. Coordinate Frames
The body-fixed frame (b-frame) and inertial frame (i-frame) are two reference frames frequently applied in researching underwater vehicle motion. The b-frame is the moving coordinate frame that is conveniently fixed to the vehicle, and i-frame is the non-rotating frame. There is another frame used in studying underwater vehicles, which is known as the wind frame. The hydrodynamic forces and moments are expressed by using the frame shown in Figure 1. Figure 1 shows the description of the fixed body and inertial coordinate systems and the 6 DOF (Degree of Freedom) illustrations.

![Figure 1. The illustration of 6 DOF.](image)

1.2. Dynamic Model
A dynamic model is a mathematical description of how it will work when a physical system is subject to load-like forces and moments caused by actuators, atmosphere, and gravity. The mathematical expression would never accurately represent the actual system, meaning that all models are simplifications to varying degrees.

In principle, an AUV would be traveling in 6 DOF that is linear and rotational motion in three directions, respectively, as illustrated in Figure 1. According to [8,9], the underwater vehicle dynamic in 6 DOF can be defined by

\[ \eta = J(\eta)\nu \]  
\[ M\ddot{\nu} + C(\nu)\nu + D(\nu) + g(\eta) = \tau \]  

Where the vector \( \eta \) is the position and orientation of the AUV with relation to an inertial reference, and also the vector \( \nu \) is the body-fixed linear and angular velocities. The matrices \( M, C(\nu), \) and \( D(\nu) \) represent inertia, Coriolis, and damping, while the vector \( g(\eta) \) represents the forces of gravity and buoyancy. Finally, the vector \( \tau \) represents the force applied by its actuators and environmental disturbances on the AUV.

The notation by SNAME 1950 [10] is used in this study for linear and angular velocities, positions, forces, and Euler angles [11]. The notation possible to express the position of the vector and the velocity variables as:

\[ \eta = [\eta_1^T, \eta_2^T]^T, \eta_1 = [x, y, z]^T, \eta_2 = [\phi, \theta, \psi]^T \]  
\[ \nu = [\nu_1^T, \nu_2^T]^T, \nu_1 = [u, v, w]^T, \nu_2 = [p, q, r]^T \]  

The model formulation in equation (1) and equation (2) is derived in this paper. In order to reduce the complexity and number of parameters in the model, a number of assumptions and simplifications...
are made that mitigate the parameter estimation process. Simplified models are often sufficient for control design since unmodelled dynamics can be considered disturbances that the controller compensates for.

1.3. Kinematics

The kinematics relations of the body-fixed frame that transforms linear velocities into the inertial frame is given by equation (5) and (6), where $J_1(\eta_2)$ is the xyz convention rotation matrix, and $s \cdot$ and $c \cdot$ are $sin(\cdot)$ and $cos(\cdot)$.

$$\eta_1 = J_1(\eta_2)v_1$$

$$J_1(\eta_2) = \begin{bmatrix} c\psi c\theta & s\psi s\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\theta s\psi c\phi \\ -s\theta & -c\theta s\phi & c\theta c\phi \end{bmatrix}$$

Due to the orthogonality of the rotation matrices, the inverted transformation matrix is given.

$$J_1(\eta_2)^{-1} = J_1(\eta_2)^T$$

Transformation of angular velocities is given by transformation matrix $J_2(\eta_2)$, where $s(\cdot), c(\cdot),$ and $t(\cdot)$ are $sin(\cdot), cos(\cdot),$ and $tan(\cdot),$ respectively.

$$\eta_2 = J_2(\eta_2)v_2$$

$$J_2(\eta_2) = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\theta c\phi \end{bmatrix}$$

In comparison to the transformation matrix for linear velocities, the transformation matrix for angular velocities is not orthogonal, meaning is $J_1(\eta_2)^{-1} \neq J_1(\eta_2)^T$. Instead, it is given by

$$J_1(\eta_2)^{-1} = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & c\theta s\phi \\ 0 & -s\phi & c\theta c\phi \end{bmatrix}$$

Using Euler angles to describe the kinematic relationship, as shown in equation (9), is singularity for $\theta = \pm \pi/2$. This can be solved by instead using quaternion to express the angle. Since the AUV considered in this study is not intended to function in this singularity’s nearer, rather it is undesired, the quaternion representation is not appropriate.

The combination of linear and angular velocity transformation gives the kinematic relation in equation (1) as

$$\dot{\eta} = J(\eta)v = \begin{bmatrix} J_1(\eta_2) & 0_{3 \times 3} \\ 0_{3 \times 3} & J_2(\eta_2) \end{bmatrix}$$

1.4. Rigid Body Kinetics

Rigid body kinetics is used when subject to external loads to explain the motion of the AUV. The equations of motion can be derived using formation Newton-Euler are expressed in equation (12) and equation (13), where $M_{RB}$ is the matrix of mass and inertia, $C_{RB}(\pi)$ represents the Coriolis and centripetal matrix, and vector with external loads acting on the rigid body [12]. The matrix of mass and inertial is a matrix of $6 \times 6$ given by

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{xx} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{yy} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{zz} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{xy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{xz} \end{bmatrix}$$

$$C_{RB}(\pi) = \begin{bmatrix} 0 & -c(\phi) s(\theta) & c(\phi) c(\theta) & 0 & -s(\phi) s(\theta) & c(\phi) c(\theta) \\ c(\theta) s(\phi) & c(\phi) c(\theta) & -s(\phi) & 0 & -c(\theta) s(\phi) & -c(\phi) c(\theta) \\ c(\phi) & 0 & s(\phi) & 0 & c(\phi) & s(\phi) \\ 0 & -c(\phi) & s(\phi) c(\theta) & 0 & -s(\theta) & c(\phi) c(\theta) \\ s(\phi) & 0 & -c(\phi) & 0 & c(\phi) & s(\phi) \\ c(\phi) & 0 & -s(\phi) & 0 & -c(\phi) & c(\phi) \end{bmatrix}$$
\[ M_{RB} \ddot{v} + C_{RB}(v)v = \tau_{RB} \] (12)

\[ M_{RB} = \begin{bmatrix}
    ml_{3 \times 3} & -mS(r_g) \\
    mS(r_g) & I_b
\end{bmatrix} \] (13)

Where \( m \) is the mass of the AUV, \( I_b \) is the matrix of inertia defined as the center of origin (CO) and \( r_g \) is the vector from CO to the center of gravity \( (C_g) \). \( l_{3 \times 3} \) is a matrix 3 \times 3 identity. Assuming that the \( C_g \) is similar to the CO, the off-diagonal elements in equation (13) will become irrelevant, leading to

\[ M_{RB} = \begin{bmatrix}
    ml_{3 \times 3} & 0 \\
    0 & I_b
\end{bmatrix} \] (14)

Since the rigid body will rotate relative to the inertial frame, it will be subjected to Coriolis and centripetal forces as defined in equation (15) matrix \( C_{RB}(v) \) by [12]. Once again, assuming that \( C_g \) is close to CO, and also that the AUV is symmetrical on the three planes, Coriolis and centripetal forces make that resulting contribution.

\[ C_{RB}(v)v = \begin{bmatrix}
    mS(v_2) & -mS(v_2)S(r_g) \\
    S(r_g)S(v_2) & -S(I_bv_2)
\end{bmatrix} \begin{bmatrix}
    m(qw - rv) \\
    m(ru - pw) \\
    m(pv - qu) \\
    qr(I_{zz} - I_{yy}) \\
    pr(I_{xx} - I_{zz}) \\
    pq(I_{yy} - I_{xx})
\end{bmatrix} \] (15)

1.5. Hydrodynamics

As the AUV moves underwater, the forces and moments are induced by the surrounding fluid, called hydrodynamic forces. These forces and moments could define as

\[ \tau_{dyn} = -M_A \ddot{v} - C_A(v)v - D(v)v \] (16)

Where \( M_A \) represents added mass, \( C_A(v) \) represents Coriolis and centripetal forces and moments induced by added mass, and \( D(v) \) represents as damping by [12]. The research in [8] stated that when an AUV accelerates or decelerates, the immediate surrounding fluid also accelerates or decelerates. This effect can be interpreted as adding an extra mass to the AUV. Through integrating the assumption that the AUV has three planes of symmetry with the fact that the AUV travels at low speed, the additional mass contributions to the expressions can be simplified in equation (17) and equation (18), where \( X_u, Y_v, Z_w \) are the added mass parameter in the direction of each axis, and \( K_p, M_q, N_r \) are the added inertial [12].

\[ M_A = -diag\{X_u, Y_v, Z_w, K_p, M_q, N_r\} \] (17)

\[ C_A(v) = \begin{bmatrix}
    0 & 0 & 0 & 0 & -Z_wv & Y_vv \\
    0 & 0 & 0 & Z_wv & 0 & -X_uu \\
    0 & Z_wv & 0 & Y_vv & 0 & -N_rv \\
    -Z_wv & 0 & -X_uu & N_rv & 0 & -K_pp \\
    Y_vv & X_uu & 0 & -M qq & & \\
    -Y_vv & X_uu & 0 & -M q & &
\end{bmatrix} \] (18)
Several factors, such as drag friction, are responsible for hydrodynamic damping [12]. The specific contributions are classified into two separate concepts, one for linear damping, $D_l$ and the other for nonlinear damping, $D_{nl}$. The nonlinear concept at [12] consists of quadratic damping. Assuming that the AUV only conducts non-coupling motion, for example, only moves in 1 DOF simultaneously, a diagonal damping matrix $D$ is reasonable to assume from [12], where $X_{u|u|u}$, $Y_{v|v|v}$, $Z_{w|w|w}$, $K_{p|p}$, $M_{q|q}$, $N_{r|r}$ are linear damping coefficients, while $X_{u|u|u}$, $Y_{v|v|v}$, $Z_{w|w|w}$, $K_{p|p}$, $M_{q|q}$, $N_{r|r}$ are nonlinear damping coefficients.

$$D(\nu) = D_l + D_{nl}(\nu)$$
$$= (-\text{diag}\{X_{u|u|u}, Y_{v|v|v}, Z_{w|w|w}, K_{p|p}, M_{q|q}, N_{r|r}\})$$
$$+ (-\text{diag}\{X_{u|u|u}, Y_{v|v|v}, Z_{w|w|w}, K_{p|p}, M_{q|q}, N_{r|r}\})$$

Lastly, it should be noted that all hydrodynamic terms will depend on the relative velocity vector $\nu_r$ defined as

$$\nu_r = \nu - \nu_c$$

Where $\nu_r \epsilon R^6$ is the water current velocity referred to [12]. Nevertheless, it was presumed that the ocean currents were identically equal to zero for the simplified model in this research resulting in $\nu_r \equiv \nu$.

2. Dynamic Modelling Simulation Results

The simulation results of open-loop control TAUV are shown in this section. Table 1 shows the angle of the rudder utilized in the simulation. The rudder is predefined in the simulation. The value of rudder angle in Table 1 was appointed randomly to observe the maneuvering motion of TAUV while the rudder turned in different angles. The illustration of the angle rudder is shown in Figure 2.

| Simulation number | Rudder angle (°) |
|-------------------|-----------------|
| 1                 | 0°              |
| 2                 | 20°             |
| 3                 | -20°            |
| 4                 | 45°             |
| 5                 | -45°            |

Table 1. The angle of Triangle-Shaped Autonomous Underwater Vehicle’s rudder.

![Figure 2. Illustration of angle rudder.](image)
In order to characterize the TAUV dynamics, the simulation was constructed in Matlab and Simulink to produce 14 outputs in 100 seconds. The angle of attack and the underwater vehicle speed are among the outputs, as are twelve system states in generalized position and velocity. In the simulation, external disturbances such as oceans waves and currents were ignored.

![Figure 3. The Triangular-Shaped Autonomous Underwater Vehicle states when rudder at 0° angle.](image)

Figure 3 presents the result of simulation number 1. Based on the rotation roll, pitch, and yaw were caused by forces and moments acting on the TAUV body area and angle of attack, as shown in the graph. From the graph, the angle of attack value is around -0.2°. Moreover, the underwater vehicle’s speed is proximate to 1.01m/s with surge velocity, u is also proximately 1.01m/s and heave velocity, w is around -0.120m/s. The speed of the TAUV is calculated using the equation:

\[ U = \sqrt{u^2 + v^2 + w^2} \] (21)

Figure 4 shows the comparisons of the TAUV motion simulation of 4 different angle rudders: 20°, -20°, 45°, and -45°, and also compare with 7 graphs: roll, pitch, u (surge), v (sway), w (heave), attack of angle and speed of the underwater vehicle. These simulations revealed the TAUV motion in 4 different values of angle rudder. When angle rudder state at 20°, there is a slightly different value between figure 3 and figure 4, particularly roll and pitch. Turn out the moment coefficient of the roll increased, and the value of the pitch increased. The angle of attack value on the graph given is proximate -0.2°. Furthermore, the speed of the underwater vehicle proximately 1.02m/s with u is around 1.02m/s, and w is around -0.120m/s.

When the rudder angle was set to -20°, the angle of attack value tune to get more or less -0.1°; additionally, the underwater vehicle’s speed is around 1.12m/s with u tune to obtain more or less 1.125m/s, and w is almost -0.105m/s.

For simulation number 4, the rudder angle at states 45°, the roll’s moment coefficient is greater than the roll when states at 20°, and the value of the pitch kept increasing in multiple numbers. According to the graph, the value angle of attack is proximate -0.1°. Furthermore, the underwater vehicle’s speed proximately 1.11m/s with u is around 1.11m/s and w proximately -0.09m/s. The speed of maneuvering motion increases by about 1.3m/s when comparing the speed graphs at states 20° and 45°.

When angle rudder at states -45° in simulation number 5, the moment coefficient of the roll is lesser than the roll when states at -20°, and the value of the pitch kept increasing in multiple numbers. From the graph, the value angle of attack is more or less -0.1°. Furthermore, the speed of the underwater
vehicle is 1.12m/s with $u$ is almost 1.12m/s and $w$ approximately to -0.09m/s. When comparing the speed graphs at states -20° and -45°, the results showed that the speed increases almost to 1.3m/s during maneuvering motion of Triangular-Shaped Autonomous Underwater Vehicle.

Figure 4. The simulation of 4 different angle rudders.
3. Conclusions
This paper presents the Triangular-Shaped Autonomous Vehicle (TAUV) motion simulation to analyze the motion characteristics when the TAUV’s rudder is independently controllable. The goal of this research is to compute the TAUV’s open-loop motion control based on various rudder angles and then evaluate the TAUV’s position, Euler angles, velocities, speed, and angle of attack. Simulation results show that the speed of TAUV, surge velocity, and heave velocity kept increasing during maneuvering motion in each rudder angle from 0° to 20° or -20°, and from 20° or -20° to 45° or -45°. The sway velocity occurred due to the forces and moments present on the TAUV body area and angle of attack in every turn of angle rudder. The model’s controllability and observability are both full rankings, according to the simulation results. As a result, the simulation model is valid, and the open-loop control is reasonable. However, the TAUV is still in development, and the simulation findings cannot be compared to experimental results. The authors want to enhance the TAUV controller in the future by researching navigation and path planning and designing a biologically inspired control algorithm for multiagent TAUV with independently controlled rudder and thrusters; the model and open-loop control outcomes are required.

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