Robust Geometric Quantum Computation with Time-Optimal Control

Tao Chen and Zheng-Yuan Xu

Guangdong Provincial Key Laboratory of Quantum Engineering and Quantum Materials,
GPETR Center for Quantum Precision Measurement, and School of Physics and Telecommunication Engineering,
South China Normal University, Guangzhou 510006, China

(Dated: January 17, 2020)

Geometric phase is a promising element to induce high-fidelity and robust quantum operations due to its built-in noise-resilience feature. However, there are two main difficulties hinder the practical applications of quantum gates induced by geometric phases. The first one is that the implementation of geometric quantum computation needs complex interactions among multiple qubits. The second one is that a geometric gate is usually slower than its dynamical counterpart, and thus decoherence will result in more gate errors. Here, we propose to implement fast nonadiabatic geometric quantum computation on a scalable two-dimensional superconducting-qubit lattice, with experimental accessible two-body interaction. Meanwhile, we combine the time-optimal control technique to accelerate the implemented quantum gates, which decreases the needed gate time. In addition, the numerical results show that, comparing with dynamical gates, our implemented geometric gates have the higher gate fidelities and stronger robustness. Therefore, our scheme provides a promising method towards scalable fault-tolerant solid-state quantum computation.

I. INTRODUCTION

Based on the fundamental quantum mechanical principles, quantum computation (QC) can effectively deal with certain complex tasks that are hard for classical computers, e.g., the striking example of factoring large numbers \[1\]. This is because that QC can take the advantage of the intrinsic quantum parallelism. Therefore, various physical systems have been suggested for the implementation of QC, e.g., trapped ions \[2\], cavity QED \[3\], neutral atoms in optical lattices \[4, 5\], etc. However, considering the requirements of large scale integrability and flexibility, a promising candidate is the Josephson junctions based superconducting qubit \[6, 9\] due to the ability to compatible with the modern ultrafast optoelectronics as well as nanostructure fabrication and characterization.

The recent experimental advances in controlling the coherent evolution of the quantum states \[10, 13\] in large qubit lattice make superconducting nanocircuit a interesting candidate to implement scalable QC, which requires at least a two-dimensional (2D) square lattice of coupled qubits. Currently, experimental control induced unwanted cross-talk (frequency drift) among adjacent qubits in large qubit lattice is the main error source for quantum gate implementation. Meanwhile, due to the inevitable interaction between a quantum system and its surrounding environment, the coherence of this quantum system is very fragile. Therefore, how to suppress the effect from quantum operational imperfections and decoherence are the main challenge in realizing scalable QC.

To suppress the quantum operational imperfections, quantum gates induced by the geometric phases are promising \[14\], due to the built-in noise-resilience features of geometric phases. Explicitly, geometric quantum computation (GQC) \[14-16\] has been proposed by using adiabatic geometric phases. However, due to long gate-operation time required by the adiabatic condition, decoherence effect will cause considerable gate error. To break such limitation, nonadiabatic GQC has been proposed to achieve high-fidelity quantum gates based on Abelian \[17-20\] and non-Abelian geometric phases \[21-23\]. Therefore, the experimental demonstration of GQC has been made on various systems \[24-31\]. However, due to the need of additional auxiliary energy levels beyond qubit states, and/or additional auxiliary coupling elements, the implementation of the high-fidelity geometric quantum gates are experimentally difficult. Meanwhile, as to the effect of decoherence, the needed time for geometric quantum gates are usually longer than that of the gates from the dynamical evolution, leading to more decoherence-induced gate errors in GQC, and thus being the other main drawback of GQC.

Here, we propose to implement nonadiabatic GQC on a 2D square superconducting circuits, which can remove the two above-mentioned disadvantages of GQC. Our proposal is scalable as it is based on a general 2D qubit lattice, where the adjacent superconducting transmon qubits are capacitively coupled, without the increase of circuit complexity by adding coupling elements. Meanwhile, our scheme only utilize experimental accessible two-body interaction, by parametrically tunable coupling \[11, 12, 32\]. In addition, our proposal can be further combined with the time-optimal control (TOC) technique \[33, 34\], which can decrease the gate errors induced by decoherence, as TOC can select out a fastest evolution path for a target quantum gate. Finally, the numerical results show that, comparing with dynamical gates, our implemented geometric gates can perform with the higher gate fidelities and stronger robustness. Therefore, our proposal removes the two main drawbacks of GQC, making it to be a more promising quantum computation strategy.

II. RESULTS

A. General framework

We first illustrate how to implement nonadiabatic evolution for a general two-state system. Assuming $h = 1$ hereafter, the
FIG. 1. Comparison between the TOC and conventional GQC.
(a) The geometric illustration of the evolution paths of TOC (red line) and conventional (blue line) geometric gates in a Bloch sphere. (b) The X, Y, and Z-axis rotation gate time of TOC and conventional GQC as function of rotation angles, with a same time-dependent pulse shape of \( \Omega(t) = \Omega_m \sin(\pi t/\tau) \).

The general Hamiltonian of a driven two-level system is

\[
\mathcal{H}(t) = \frac{1}{2} \begin{pmatrix} -\Delta(t) & \Omega(t) e^{-i\phi(t)} \\ \Omega(t) e^{i\phi(t)} & \Delta(t) \end{pmatrix},
\]

where the basis is consisted of a ground state \(|0\rangle = (1, 0)\rangle\) and an excited state \(|1\rangle = (0, 1)\rangle\); \(\Omega(t)\) and \(\phi(t)\) are the amplitude and phase of the driving microwave field, respectively; \(\Delta(t)\) is the time-dependent detuning between the qubit transition frequency and the frequency of the microwave field. For the Hamiltonian \(\mathcal{H}(t)\) in Eq. (1), the corresponding dynamic Lewis-Riesenfeld invariant \(I(t)\) is

\[
I(t) = \mu \begin{pmatrix} \cos \chi(t) & \sin \chi(t) e^{-i\xi(t)} \\ \sin \chi(t) e^{i\xi(t)} & -\cos \chi(t) \end{pmatrix},
\]

where \(\mu\) is an arbitrary constant. Selecting its eigenvectors

\[
\begin{pmatrix} |\psi_+(t)\rangle \\ |\psi_-(t)\rangle \end{pmatrix} = \begin{pmatrix} \cos \frac{\chi(t)}{2} e^{-i\xi(t)/2}|0\rangle + \sin \frac{\chi(t)}{2} e^{i\xi(t)/2}|1\rangle \\ \sin \frac{\chi(t)}{2} e^{i\xi(t)/2}|0\rangle - \cos \frac{\chi(t)}{2} e^{-i\xi(t)/2}|1\rangle \end{pmatrix},
\]

as a set of two-dimensional orthogonal dressed states. By letting \(I(t)\) satisfies \(\partial I(t)/\partial t + i[\mathcal{H}(t), I(t)] = 0\), the evolution paths of these dressed states can be determined by the parameters of \(\mathcal{H}(t)\) as \(\xi(t) = -\Delta - \Omega(t) \cot \chi(t) \cos[\phi(t) - \xi(t)]\) and \(\chi(t) = \Omega(t) \sin[\phi(t) - \xi(t)]\). Therefore, at the final time \(\tau\), the evolution operator reads

\[
U(\tau) = e^{i\gamma} |\psi_+(\tau)\rangle \langle \psi_+(0)| + e^{-i\gamma} |\psi_-(\tau)\rangle \langle \psi_-(0)|,
\]

where \(\gamma = \gamma_d + \gamma_g = \int_0^\tau \langle \psi_+ (t) | (i \partial / \partial t - \mathcal{H}(t)) | \psi_+ (t) \rangle dt\) is the Lewis-Riesenfeld phase, including the dynamical phase \(\gamma_d = \frac{1}{2} \int_0^\tau \frac{\xi \sin^2 \chi + \Delta}{\cos \chi} dt\) and geometric phase \(\gamma_g = \frac{1}{2} \int_0^\tau \xi \cos \chi dt\). Our purpose is to make this phase \(\gamma\) an unconventional geometric phase [19] [25], satisfying the form of \(\gamma = \alpha_g + (1 + 2\gamma)\xi\), with \(\xi\) as a proportional constant and \(\alpha_g\) being a coefficient dependent only on the geometric feature of the quantum evolution path during the gate operation. Thus, we set \(\chi\) as a constant, then the constraints for other parameters reduce to

\[
\xi(t) = \phi(t), \quad \tan^{-1}[-\Omega(t)/(\xi(t) + \Delta)] = \chi,
\]

and the resulting evolution operator from Eq. (4) is

\[
U(\tau) = \begin{pmatrix} (c_\gamma + i \sigma c_\chi) e^{-i\xi/2} & i s_\gamma s_\chi e^{-i\xi/2} \\ i s_\gamma s_\chi e^{i\xi/2} & (c_\gamma - i s_\gamma c_\chi) e^{i\xi/2} \end{pmatrix},
\]

where \(c_i = \cos i, s_i = \sin i\) and \(\xi = \xi(t) + \xi(0)\). In this way, the target control of the X, Y, and Z-axis rotation operations for arbitrary angles \([0, \pi]\) can all be done by determining \(\gamma = \alpha_g = \frac{\pi}{2}, \frac{\pi}{2}\), and \(\pi\) with the same \(\ell \equiv -1\), where \(\alpha_g\) and \(\xi\) are obviously independent of the parameters of the qubit system. Note that, \(\xi(t)\) can be in arbitrary shape, providing its boundary values \(\xi(0)\) and \(\xi(\tau)\) to be fixed to realize different rotation operations, i.e., the shape of \(\phi(t)\) of the driving Hamiltonian \(\mathcal{H}(t)\) in Eq. (1) can be used as a degree of freedom to adopt the TOC.

In the construction of the quantum gate, to pursue the higher gate fidelity, we also need to minimize the gate time, to reduce the gate error induced by the decoherence effect. Therefore, we further combine the TOC technique with the above general framework of GQC, by engineering the shape of \(\phi(t)\), to accelerate the geometric gate, and thus strengthen its decoherence protection. As to the quantum dynamics under the driving Hamiltonian \(\mathcal{H}(t)\), the different selection of \(\Omega(t)\) and \(\phi(t)\) makes the quantum system evolve along different paths. The motivation of the TOC is to find the path with the shortest time, see Methods for details. Considering the restrictions on the realistic physical implementation, and then solving the quantum brachistochrone equation, we obtain the parameters’ restriction as

\[
\phi(t) = \phi_0 + \phi_1(t), \quad \phi_1(t) = \int_0^t |C_0 \Omega(t') - \Delta| dt',
\]

where \(\Omega(t)\) can be an arbitrary pulse shape, and the coefficient
$C_0$ is a constant that depends only on the type of target gates. Therefore, the time-optimal form of the driving Hamiltonian $\mathcal{H}(t)$ in Eq. (1) can be determined to realize universal GQC.

B. Single-qubit geometric gates with TOC

We now proceed to implement high-fidelity universal nonadiabatic GQC with TOC technique on a 2D square superconducting transmon-qubit lattice, as shown in Fig. 2(a). Starting from a single transmon qubit, where the computational subspace $\{|0\rangle, |1\rangle\}$ consists of the ground and excited states of the transmon. Conventionally, as shown in Fig. 2(b), arbitrary control over the transmon qubit can be realized by applying a microwave field driving with the time-dependent amplitude $\Omega(t)$ and phase $\phi(t)$ on its two lowest levels with a detuning $\Delta$. This driving coupling Hamiltonian can be written as $\mathcal{H}(t)$ in Eq. (1). To implement universal single-qubit geometric gates with TOC, we set the parameters’ restriction in Eq. (7) by defining $C_0 = \cot(\theta/2)$. In this way, the geometric X, Y, and Z-axis rotation operations with TOC, denoted as $R^x_0(\theta_x), R^y_0(\theta_y)$ and $R^z_0(\theta_z)$, for arbitrary angles $\theta_{x,y,z} \in [0, \pi]$ can all be realized by setting

$$A_0 = \frac{\pi}{2}, \quad \phi_1(\tau_x) = \pi, \quad \theta = \theta_x, \quad \phi_0 = -\frac{\pi}{2};$$

$$A_0 = \frac{\pi}{2}, \quad \phi_1(\tau_y) = \pi, \quad \theta = \theta_y, \quad \phi_0 = 0;$$

$$A_0 = \pi, \quad \phi_1(\tau_z) = \theta_z - 2\pi,$$

with the same $\phi_1(0) = 0$ and the minimum pulse area

$$\frac{1}{2} \int_0^{\tau_{x,y,z}} \Omega(t) dt = A_0/\sqrt{1 + \cot^2 \theta/2},$$

which are all less than $\pi$ required for the conventional geometric operations \cite{20, 21}. In addition, it is worth emphasizing that $\theta$ can be the arbitrary value in the construction of the geometric Z-axis rotation operations, thus the detuning $\Delta$ can be used as an additional degree of freedom to further accelerate the geometric Z-axis rotations. However, when constructing the geometric X and Y-axis rotations, one needs to set $\Delta$ to realize a target rotation angle. As an explicit demonstration, we take a simple pulse shape $\Omega(t) = \Omega_m \sin(\pi \tau/\tau)$ as an example, the time acceleration results are shown in Fig. 2(a), where for convenience we fix the detuning $\Delta = 0$ in the construction of the geometric Z-axis rotations.

However, in the realistic physical implementation, due to the weak anharmonicity of the target transmon, a target driving on the qubit states will still accommodate the sequential transitions among the higher excited states, in a dispersive way. Targeting such obstacle, we also apply the recent theoretical exploration of derivative removal via adiabatic gate (DRAG) \cite{16, 17} to suppress this leakage error to obtain the precise qubit manipulation. We only consider the influence of the third energy level, which is the main leakage source of our qubit states \cite{16, 17}. To end, the Hamiltonian describing a single-qubit system can be written as

$$\mathcal{H}_1(t) = \frac{1}{2} \mathbf{B}(t) \cdot \mathbf{S} - \alpha_1 |2\rangle\langle 2|,$$

where $\alpha_1$ is the intrinsic anharmonicity of the target transmon, $\mathbf{B}(t) = \mathbf{B}_0(t) + \mathbf{B}_d(t)$ is the vector of total microwave field including the original and additional DRAG correcting microwave fields, i.e.,

$$\mathbf{B}_0(t) = (B_x, B_y, B_z) = (\Omega(t) \cos(\phi_0 + \phi_1(t)), \Omega(t) \sin(\phi_0 + \phi_1(t)), -\Delta),$$

$$\mathbf{B}_d(t) = (B_{d,x}, B_{d,y}, B_{d,z}) = -\frac{1}{2\alpha_1}(-\dot{B}_y + B_z B_x, \dot{B}_x + B_z B_y, 0),$$

respectively, and the operator vector $\mathbf{S}$ is given by

$$S_x = \sum_{m=0}^{\infty} \sqrt{m+1} \langle m+1 \vert m \rangle + \langle m \vert m+1 \rangle,$$

$$S_y = \sum_{m=0}^{\infty} \sqrt{m+1} (i \langle m+1 \vert m \rangle - i \langle m \vert m+1 \rangle),$$

$$S_z = \sum_{m=0,1,2} (1 - 2m) \langle m \vert m \rangle.$$

To further analyze the performance of the single-qubit time-optimal geometric gates, we take the Hadamard $U(\tau_H)$, Phase $U(\tau_\pi)$ and $\pi/8$ gates $U(\tau_{\pi/8})$ as typical examples, which can be constituted a universal set for the arbitrary single-qubit geometric gates. Considering the effects of decoherence and the
high-order oscillating terms, the quantum dynamics of $H_t(t)$ can be simulated by the Lindblad master equation, see Method for details. In our simulation, from the state-of-art experiment [10], we choose the relaxation and dephasing rates of the transmon to be identical as $\kappa = \kappa_1 = \kappa_2 = 2\pi \times 4$ kHz, the anharmonicity $\omega_1 = 2\pi \times 220$ MHz. Here, to fully evaluate the implemented geometric gates, for a general initial state of $|\psi_1\rangle = \cos \theta_1 |0\rangle + \sin \theta_1 |1\rangle$ with $|\psi_{f_{\text{J-A}}}(t)\rangle = U(t_0)|\psi_1\rangle$ being the ideal final state, we define the single-qubit gate fidelity as $F^G_k = \frac{1}{2\pi} \int_0^{2\pi} \langle \psi_{f_k} | \rho_1 | \psi_{f_k} \rangle d\theta_1$ [39], where the integration is numerically done for 1001 input states with $\theta_1$ being uniformly distributed within $[0, 2\pi]$, and $\rho_1$ is a numerically simulated density matrix of the qubit system.

In Figs. 4a and 4b, we plot the fidelities of the Hadamard and Phase quantum gates as functions of the tunable parameters, where we find that, when $\Omega_\text{m} = 2\pi \times 30$ MHz with the corresponding restricted detuning parameter $\Delta \approx -2\pi \times 35$ MHz for the Hadamard gate, and $\Omega_\text{m} = 2\pi \times 20$ MHz with $\Delta \in 2\pi \times (35 \pm 2)$ MHz for the Phase gate, the gate fidelities of these two gates can both exceed 99.97%, which are higher than the best performance of the reported experiments for the dynamical gates of a single transmon qubit [10]. Furthermore, comparing with the corresponding dynamical gates, see Method for details, the robustness of the implemented geometric gates possess the better noise-resilient features for both the qubit frequency drift $\delta$ and the deviation $\epsilon$ of the driving amplitude in the form of $\Delta + \delta$ and $(1 + \epsilon)\Omega(t)$, as shown in Figs. 4c–4f. Note that the performance of the $\pi/8$ gate is very similar to that of the Phase gate, thus not present here.

C. Nontrivial two-qubit geometric gate with TOC

We next work on implementing the nontrivial two-qubit geometric gate with TOC technique on the 2D square superconducting transmon-quantum lattice in Fig. 2a. Nevertheless, for the capacitive coupled qubit lattice, the coupling strength of two adjacent transmons, e.g., $T_1$ and $T_2$ in the same row (or $T_1$ and $T_2$ in the same column), are fixed. Meanwhile, the frequency difference of two adjacent transmons, $\Delta_1 = \omega_2 - \omega_1$, is also generally set to be fixed, so that resonant coupling and/or off-resonant coupling are difficult to meet without changing a qubit frequency to deviate from its optimal working point.

To deal with these difficulties, we here introduce an additional qubit-frequency driving for the transmon $T_1$ which can be experimentally realized by a longitudinal driving field, in the form of $\epsilon(t) = F(t)$ [22], where $F(t) = \beta \sin[\nu t + \varphi(t)]$, with $\nu$ and $\varphi(t)$ indicating the frequency and phase of the longitudinal field, respectively, the circuit details are shown in Fig. 2c). Moving into the interaction picture, the coupling Hamiltonian reads

$$\begin{align*}
\mathcal{H}_{12}(t) &= g_{12} \left\{ |01\rangle_{12} \langle 10| \epsilon^{\Delta_1 t} + \sqrt{2} |11\rangle_{12} \langle 20| \epsilon^{i(\Delta_1 + \omega_1)} \right. \\
+ \sqrt{2} |02\rangle_{12} \langle 11| e^{i(\Delta_1 - \omega_2 t)} e^{-i\beta \sin[\nu t + \varphi(t)]} + \text{H.c.,}
\end{align*}$$

where $g_{12}$ is the coupling strength between transmons $T_1$ and $T_2$, $\omega_i$ is the intrinsic anharmonicity of the transmon $T_i$. Utilizing the Jacobi-Anger identity, and then neglecting the high-order oscillating terms, we find that the parametrically tunable coupling in the single-excitation subspace $\{|01\rangle_{12}, \{10\rangle_{12}\}$ and two-excitation subspace $\{|02\rangle_{12}, \{11\rangle_{12}, \{20\rangle_{12}\}$ can all be selectively addressed by adjusting the frequency $\nu$ of the longitudinal field. The corresponding energy spectrum of these two capacitively coupled transmons $T_1$ and $T_2$ is shown in Fig. 2d.

For example, by setting the frequency $\nu$ to satisfy $\Delta_1 = \nu - \Delta_1 + \omega_2$ with $|\Delta_1| \ll \{\Delta_1 - \omega_2, \nu\}$, and applying the unitary transformation, we can obtain the effective Hamiltonian in the two-excitation subspace $\{|02\rangle_{12}, \{11\rangle_{12}\}$ as

$$\mathcal{H}_t(t) = \frac{1}{2} \left( \begin{array}{cc} -\Delta_1 & g'_{12} e^{-i\varphi(t)} \\
-\Delta_2 & -\Delta_1 \\
g'_{12} e^{-i\varphi(t)} & \Delta_1 \end{array} \right),$$

where $g'_{12} = 2\sqrt{2} J_1(\beta) g_{12}$ with $J_1(\beta)$ being Bessel function of the first kind. In the following, to achieve the integration with the TOC, we further require that the parameter $\varphi(t) = \eta t$ with $\eta$ being a constant that depends only on the type of the target gates, see Method for details. Thus, within the two-qubit subspace $\{|00\rangle_{12}, \{01\rangle_{12}, \{10\rangle_{12}, \{11\rangle_{12}\}$, the evolution operator with TOC is $U_2(T_{\text{cp}}) = \text{diag}\{1, 1, 1, e^{i\zeta}\}$ with minimal time cost of $T_{\text{cp}} = T_0 \sqrt{1 - (1 + 4\Delta_1)\eta^2(1 - \zeta^2)^2}$. At this point, the two-qubit time-optimal Control-Phase geometric gate can be obtained. Obviously, gate time $T_{\text{cp}}$ is also faster than the corresponding conventional geometric operation, and the gate time of which is $T_0 = 2\pi / g'_{12}$ [20, 21].

We next take the two-qubit geometric gate with $\zeta = \frac{\pi}{2}$ as an typical example to fully evaluate its gate performance. Re-
alistically, we choose the coupling strength of the two adjacent transmons as $g_{12} = 2\pi \times 10$ MHz, the anharmonicity of the second transmon as $\Delta_2 = 2\pi \times 180$ MHz, and the relaxation and dephasing rates of the transmon to be identical as $\kappa = \kappa_1 = \kappa_2 = \kappa'_1 = \kappa'_2 = 2\pi \times 4$ kHz. For the general initial state of the two qubits as $|\psi_2\rangle = (\cos \vartheta_1 |0\rangle + \sin \vartheta_1 |1\rangle) \otimes (\cos \vartheta_2 |0\rangle + \sin \vartheta_2 |1\rangle)$ with $|\psi_{fp}\rangle = U_2(T_{cp}) |\psi_2\rangle$ being the ideal final state, we can define the two-qubit gate fidelity as $F_{cp} = \left\langle \psi_{fp}|U_2(T_{cp})|\psi_{fp}\right\rangle$. Numerically done for 10001 input states with $\vartheta_1, \vartheta_2$, parameter range in which high-fidelity two-qubit geometric gate fidelity higher than 99.80% can be realized. In particularly, the numerical regime, within which two-qubit gate with fidelity higher than $99.80\%$ can be realized. In this sense, we choose the coupling strength of the two adjacent transmons as $g_{12} = 2\pi \times 10$ MHz, the anharmonicity of the second transmon as $\Delta_2 = 2\pi \times 180$ MHz, and the relaxation and dephasing rates of the transmon to be identical as $\kappa = \kappa_1 = \kappa_2 = \kappa'_1 = \kappa'_2 = 2\pi \times 4$ kHz.

Thus, to avoid this type of leakage error as much as possible, it is necessary to optimize the qubit parameters to obtain a parameter range in which high-fidelity two-qubit geometric gate can be achieved. As shown in Eq. (12), the driving amplitude of the microwave field cannot be infinite, i.e., $f_1(\mathcal{H}_c) = \frac{1}{2} \text{Tr}(\mathcal{H}_c^2) - \frac{1}{2} \Omega^2(\mathcal{t}) = 0$.

In the realistic physical implementation, the considered interaction Hamiltonian $\mathcal{H}_c = \frac{1}{2} \Omega(\mathcal{t})[\cos \phi(\mathcal{t}) \sigma_x + \sin \phi(\mathcal{t}) \sigma_y]$ needs to satisfy certain constraints:

(i) the driving amplitude of the microwave field cannot be infinite, i.e., $f_1(\mathcal{H}_c) = \frac{1}{2} \text{Tr}(\mathcal{H}_c^2) - \frac{1}{2} \Omega^2(\mathcal{t}) = 0$.

(ii) the form of interaction Hamiltonian $\mathcal{H}_c$ is not arbitrary, i.e., $f_2(\mathcal{H}_c) = \text{Tr}(\mathcal{H}_c \sigma_z) = 0$, where the above $\sigma_{x,y,z}$ are the Pauli operators for the computational subspace $\{0\}, \{1\}$. And then, by solving quantum brachistochrone equation (39)

$$\frac{\partial F}{\partial t} = -i [\mathcal{H}(\mathcal{t}), F],$$

where $F = \partial (\sum_{j=1,2} \lambda_j f_j(\mathcal{H}_c)) / \partial H = \lambda_1 H_c + \lambda_2 \sigma_z$ with $\lambda_j$ being the Lagrange multiplier, we can determine that the restricted parameter as $\mathcal{F}(\mathcal{t})$ by defining $\lambda_1 = 1 / \Omega(\mathcal{t})$ and $\lambda_2 = -C_0 / 2$, with $\Omega(\mathcal{t})$ being an arbitrary pulse shape. In particular, when $\Omega(\mathcal{t})$ is a square pulse, the restricted parameter reduces to $\dot{\phi}(\mathcal{t}) = \eta$ with $\eta$ being a constant that depends only on the type of target gate. Then, the time-optimal form of the driving Hamiltonian $\mathcal{H}(\mathcal{t})$ in Eq. (1) can be characterized.

### III. DISCUSSION

To sum up, we have proposed a practical implementation of high-fidelity universal geometric quantum gates for QC. In a simple experimental setup, we can optimize accessible two-body interaction and avoid the introduction of additional auxiliary energy levels beyond qubit states and additional auxiliary coupling elements. Meanwhile, our scheme is robust against the main errors and less affected by the decoherence, and can further suppress the effects of higher excited states transitions by applying the correcting scenario and optimizing the qubit parameters. As the needed error correction is $\mathcal{F}(\mathcal{t})$ is the same as that of the single-excitation subspace of the exchange coupled spin systems. Our scheme can be readily extended to these systems, e.g., quantum dots, cavity QED, trapped ions, etc. Therefore, our implementation uses only the existing experimental technologies to remedy the main drawbacks of GQC, making it a promising strategy towards robust and scalable solid-state QC.

### IV. MATERIALS AND METHODS

#### A. Time-Optimal Control

As stated in the general framework, the different selection of $\Omega(\mathcal{t})$ and $\phi(\mathcal{t})$ makes the quantum system evolve along different paths under the driving Hamiltonian $\mathcal{H}(\mathcal{t})$. Next, based on the method of the TOC technique, we are targeted to find the path with the shortest gate time by setting constrains on the Hamiltonian parameters $\Omega(\mathcal{t})$ and $\phi(\mathcal{t})$. In the realistic physical implementation, the considered interaction Hamiltonian $\mathcal{H}_c = \frac{1}{2} \Omega(\mathcal{t})[\cos \phi(\mathcal{t}) \sigma_x + \sin \phi(\mathcal{t}) \sigma_y]$ needs to satisfy certain constraints:

(i) the driving amplitude of the microwave field cannot be infinite, i.e., $f_1(\mathcal{H}_c) = \frac{1}{2} \text{Tr}(\mathcal{H}_c^2) - \frac{1}{2} \Omega^2(\mathcal{t}) = 0$.

(ii) the form of interaction Hamiltonian $\mathcal{H}_c$ is not arbitrary, i.e., $f_2(\mathcal{H}_c) = \text{Tr}(\mathcal{H}_c \sigma_z) = 0$, where the above $\sigma_{x,y,z}$ are the Pauli operators for the computational subspace $\{0\}, \{1\}$. And then, by solving quantum brachistochrone equation (39)

$$\frac{\partial F}{\partial t} = -i [\mathcal{H}(\mathcal{t}), F],$$

where $F = \partial (\sum_{j=1,2} \lambda_j f_j(\mathcal{H}_c)) / \partial H = \lambda_1 H_c + \lambda_2 \sigma_z$ with $\lambda_j$ being the Lagrange multiplier, we can determine that the restricted parameter as $\mathcal{F}(\mathcal{t})$ by defining $\lambda_1 = 1 / \Omega(\mathcal{t})$ and $\lambda_2 = -C_0 / 2$, with $\Omega(\mathcal{t})$ being an arbitrary pulse shape. In particular, when $\Omega(\mathcal{t})$ is a square pulse, the restricted parameter reduces to $\dot{\phi}(\mathcal{t}) = \eta$ with $\eta$ being a constant that depends only on the type of target gate. Then, the time-optimal form of the driving Hamiltonian $\mathcal{H}(\mathcal{t})$ in Eq. (1) can be characterized.

### B. The Lindblad Master Equation

In the realistic physical implementation, the performance of the implemented time-optimal geometric gates are inevitably affected by the decoherence effect of the quantum system. Therefore, we next consider the effects of decoherence and the high-order oscillating terms, the quantum dynamics of the driving Hamiltonian $\mathcal{H}_d(\mathcal{t})$ can be simulated by the Lindblad master equation (38) of

$$\dot{\rho}_n = -i [\mathcal{H}_d(\mathcal{t}), \rho_n] + \sum_{i=1}^n \left\{ \frac{\kappa^i}{2} \mathcal{L}\left(|0\rangle_i \langle 1| + \sqrt{2} |1\rangle_i \langle 2| \right) 
+ \frac{\kappa^i}{2} \mathcal{L}\left(|1\rangle_i \langle 1| + 2 |2\rangle_i \langle 2| \right) \right\},$$

where $\rho_n$ is the density matrix of the considered quantum system, $\mathcal{L}(\mathcal{A}) = 2 A \rho_n, A - A^\dagger A \rho_n - \rho_n A^\dagger A$ is the Lindblad operator for operator $A$, and $\kappa^i$, $\kappa'^i$ are the relaxation and dephasing rates of the $i$th transmon, respectively. For the cases of a single qubit and two coupled qubits, the form of the driving Hamiltonian are expressed as $\mathcal{H}_d(\mathcal{t}) = \mathcal{H}_3(\mathcal{t})$ with $n = 1$ and $\mathcal{H}_d(\mathcal{t}) = \mathcal{H}_{12}(\mathcal{t})$ with $n = 2$, respectively. In addition, through the numerical simulation, we can find that, for all the implemented time-optimal geometric gates, their infidelities caused by the level leakage to the third energy level are all less than 0.01%, which is almost negligible, thus confirming that it is feasible to consider only the level leakage to the third energy level in our simulation.
C. Numerical comparison of the geometric and dynamical quantum gates

During the comparison of the robustness of quantum gates, universal dynamical quantum gates [40] based on purely dynamical evolution can be achieved by resonant interaction. However, different from our geometric gate implementation, the dynamical evolution can be achieved by resonant interaction. The resulting evolution operator can be obtained as

\[ U_d(\theta_d, \phi) = \begin{pmatrix} \cos \theta_d & -i \sin \theta_d e^{-i\phi} \\ -i \sin \theta_d e^{i\phi} & \cos \theta_d \end{pmatrix}, \] (15)

where \( \theta_d = \left[ \int_0^{\tau_d} \Omega(t) dt \right] / 2 \). In this way, a universal set for the arbitrary single-qubit gates, i.e., the Hadamard, Phase and \( \pi/8 \) gates, can be all realized by \( U(\pi/4, -\pi/4)U(\pi/2, 0), U(\pi/4, \pi/4)U(\pi/2, 0) \) and \( U(\pi/4, \pi/4)U(\pi/2, 0) \), respectively. In particular, to ensure the fairness of our robustness comparison, we also define the pulse shape of the amplitude to be \( \Omega(t) = \Omega_m \sin(\pi t / \tau_d) \), which is the same as that of the geometric gates.

ACKNOWLEDGMENTS

We thank B.-J. Liu for helpful discussion. This work was supported by the Key-Area Research and Development Program of GuangDong Province (Grant No. 2018B030326001), the National Natural Science Foundation of China (Grant No. 11874156), the National Key R&D Program of China (Grant No. 2016YFA0301803), and the Innovation Project of Graduate School of South China Normal University (Grant No. 2019LXK0006).

[1] P. W. Shor, in Proceedings of the 35th Annual Symposium on Foundation of Computer Science (IEEE Computer Society, Los Alamitos, CA, 1994), p. 124.
[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. 74, 4091 (1995).
[3] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble, Phys. Rev. Lett. 75, 4710 (1995).
[4] G. K. Brennen, C. M. Caves, P. S. Jessen, and I. H. Deutsch, Phys. Rev. Lett. 82, 1060 (1999).
[5] D. Jaksch, H.-J. Briegel, J. I. Cirac, C.W. Gardiner, and P. Zoller, Phys. Rev. Lett. 82, 1975 (1999).
[6] Y. Makhlin, G. Schön, and A. Shnirman, Nature (London) 398, 305 (1999).
[7] L. B. Ioffe, V. B. Geshkenbein, M.V. Feigelman, A. L. Faucher, and G. Blatter, Nature (London) 398, 679 (1999).
[8] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) 398, 786 (1999).
[9] J. E. Mooij, T. P. Orlando, L. Tian, C. van der Wal, L. Levitov, S. Lloyd, and J. J. Miao, Science 285, 1036 (1999).
[10] R. Barends et al., Nature (London) 508, 300 (2014).
[11] C.Neill et al., Science 360, 195 (2018).
[12] M. Reagor et al., Sci. Adv. 4, eaao3603 (2018).
[13] F. Arute et al., Nature (London) 574, 505 (2019).
[14] P. Zanardi and M. Rasetti, Phys. Lett. A 264, 94 (1999).
[15] J. Pachos, P. Zanardi, and M. Rasetti, Phys. Rev. A 61, 010305 (1999).
[16] L.-M. Duan, J. I. Cirac, and P. Zoller, Science 292, 1695 (2001).
[17] W. Xiang-Bin and M. Keiji, Phys. Rev. Lett. 87, 097901 (2001).
[18] S.-L. Zhu and Z. D. Wang, Phys. Rev. Lett. 89, 097902 (2002).
[19] S. L. Zhu and Z. D. Wang, Phys. Rev. Lett. 91, 187902 (2003).
[20] F. Z. Zhao, X. D. Cui, G. F. Xu, E. Sjöqvist, and D. M. Tong, Phys. Rev. A 96, 052316 (2017).
[21] E. Sjöqvist, D. M. Tong, L. M. Andersson, B. Hessmo, M. Johansson, and K. Singh, New J. Phys. 14, 103035 (2012).
[22] G. F. Xu, J. Zhang, D. M. Tong, E. Sjöqvist, and L. C. Kwek, Phys. Rev. Lett. 109, 170501 (2012).
[23] B.-J. Liu, X.-K. Song, Z.-Y. Xue, X. Wang, and M.-H. Yung, Phys. Rev. Lett. 123, 100501 (2019).
[24] D. Leibfried, B. DeMarco, V. Meyer, D. Lucas, M. Barrett, J. Britton, W. M. Itano, B. Jelenković, C. Langer, T. Rosenband, and D. J. Wineland, Nature (London) 422, 412 (2003).
[25] J. Du, P. Zou, and Z. D. Wang, Phys. Rev. A 74, 020302(R) (2006).
[26] A. A. Abdumalikov, J. M. Fink, K. Jullusson, M. Pechal, S. Berger, A. Wallraff, and S. Filipp, Nature (London) 496, 482 (2013).
[27] G. Feng, G. Xu, and G. Long, Phys. Rev. Lett. 110, 190501 (2013).
[28] C. Zu, W.-B. Wang, L. He, W.-G. Zhang, C.-Y. Dai, F. Wang, and L.-M. Duan, Nature (London) 514, 72 (2014).
[29] Y. Xu, W. Cai, Y. Ma, X. Mu, L. Hu, T. Chen, H. Wang, Y. P. Song, Z.-Y. Xue, Z.-Q. Yin, and L. Sun, Phys. Rev. Lett. 121, 110501 (2018).
[30] T. Yan, B.-J. Liu, K. Xu, C. Song, S. Liu, Z. Zhang, H. Deng, Z. Yan, H. Rong, K. Huang, M.-H. Yung, Y. Chen, and D. Yu, Phys. Rev. Lett. 122, 080501 (2019).
[31] Z. Zhu, T. Chen, X. Yang, J. Bian, Z.-Y. Xue, and X. Peng, Phys. Rev. Appl. 12, 024024 (2019).
[32] J. Chu et al., arXiv:1906.02992.
[33] X. Wang, M. Allegro, G. Jacobs, S. Lloyd, C. Lupo, and M. Mohseni, Phys. Rev. Lett. 114, 170501 (2015).
[34] J. Geng, Y. Wu, X. Wang, K. Xu, F. Shi, Y. Xie, X. Rong, and J. Du, Phys. Rev. Lett. 117, 170501 (2016).
[35] H. R. Lewis and W. B. Riesenfeld, J. Math. Phys. 10, 1458 (1969).
[36] F. Motzoi, J. M. Gambetta, P. Rebentrost, and F. K. Wilhelm, Phys. Rev. Lett. 103, 110501 (2009).
[37] T. Wang, Z. Zhang, L. Xiang, Z. Jia, P. Duan, W. Cai, Z. Gong, Z. Zong, M. Wu, J. Wu, L. Sun, Y. Yin, and G. Guo, New J. Phys. 20, 065003 (2018).
[38] J. F. Poyatos, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 78, 390 (1997).
[39] A. Carlini, A. Hosoya, T. Koike, and Y. Okudaira, Phys. Rev. A 75, 042308 (2007).
[40] S.-B. Zheng, C.-P. Yang, and F. Nori, Phys. Rev. A 93, 032313 (2016).