A new method to characterize the curled wake shape under yaw misalignment

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Abstract. Wake Redirection Control due to intentional yaw misalignment is a promising method to enhance power yield at wind farm level. A turbine misaligned with the inflow wind produces a curled wake shape, which is currently not accounted for in wake tracking algorithms. This study proposes a new 2D wake description specifically designed to account for the non-elliptic shape of the redirected wake. The performance of this new method is evaluated by employing a Large Eddy Simulation model at different atmospheric stratifications. A comparison with traditionally used approaches indicates an improvement in describing wake shape and center position, and consequently a significantly higher accuracy in the power estimation of a virtual downstream turbine. A brief outlook suggests that this wake tracking algorithm is suited to study the effect of the most influential atmospheric and operational parameters on wake propagation under yaw misalignment and the development of a physically based empirical wake parameterization.

1. Introduction

As the wind energy industry is maturing, awareness of the negative effects of turbine wakes on power production in wind farms is increasing. Lately, focus is shifting from optimal control of individual turbines to optimization on a farm level. Wake Redirection Control (WRC) by intentional yaw misalignment [1, 2] is currently regarded as one of the most promising methods to enhance power yield at a farm level. By misaligning the turbine with the incoming flow, a crosswise thrust force component is introduced, redirecting the wake away from a downstream turbine. In recent years, the potential of WRC has been demonstrated by numerical simulations [3, 4, 5, 6, 7], wind tunnel experiments [1, 8, 9] and free-field measurements [10, 11, 12, 13] with reported power yield increases of up to 15% depending on studied inflow conditions, used yaw controller and turbine spacing. Additionally, [13, 14] reported a potential increase in Annual Energy Production (AEP) on farm level.

The magnitude of the crosswise thrust force component introduced by yaw misalignment is not uniform over the rotor area, but varies in the vertical with a maximum near hub height. As first reported by [15], this results in asymmetric curling of the wake to a kidney-like shape. This has implications for determining the wake position downstream of the turbine, as assuming a self-similar Gaussian distribution of the wake deficit is inaccurate. This in turn affects the quality of power and load assessments.

In turbine wake studies, including those applying WRC by intentional yaw misalignment, it is common practice to consider the wake center trajectory to be representative for the propagation
of the wake downstream of the turbine. However, there is no consensus in the literature how to define this wake center. The location of the maximum wind speed deficit and the Center of Mass (CoM) are two of the simplest, yet most frequently used methods in this respect. Both are, however, sensitive to turbulence, which might be undesirable when studying steady state situations. In [4], the wake center is determined using three methods: (1) by fitting a simple one-dimensional Gaussian at hub height, (2) by fitting a more convoluted two-dimensional bivariate Gaussian to the wind field and (3) by determining the location of the minimal potential available power of a virtual downstream turbine. Methods 2 and 3 are currently considered state-of-the-art methods to identify the wake center. They found dissimilarities between the three methods, especially under large turbine yaw angles, as none of the methods is able to accurately account for the curled wake shape.

To contribute to the growing interest in WRC by intentional yaw misalignment, this study proposes a new method to identify frequently studied wake characteristics, specifically designed to account for the non-elliptic shape of the redirected wake. Next to a new definition for the wake center, metrics to determine wake area and curliness are proposed. The performance of the proposed method is evaluated in comparison with traditional methods, in its ability to describe the location of the wake center and the wake as a whole. Potential applications include studying the dependency of wake characteristics on atmospheric and operational parameters and the development of a physically based empirical wake parameterization or wake meandering model.

2. Methodology

An overview of the LES model PALM and the simulation set-up is described in the following section, followed by an overview of traditional and the proposed wake tracking algorithms.

2.1. Large-Eddy Simulations

General

The performance of the discussed methods is evaluated by employing the PArallelized Large-eddy simulation Model (PALM, [16]), which uses a non-hydrostatic incompressible Boussinesq approximation of the Navier-Stokes equations on a regularly spaced grid using right-handed Cartesian coordinates. Information exchange between the surface and the lowest grid cell is achieved by applying the Monin-Obukhov Similarity Theory. In this study, model revision 3455 with default numerical schemes is used. All simulations have a spatial resolution of $\Delta = 5$ m in the boundary layer, while the vertical resolution above the boundary layer height increases with 6% per cell to save computational resources. The Coriolis parameter corresponds to $55^\circ$N and the surface roughness is constant at $z_0 = 0.1$ m, representing low crops. The simulation chain includes a precursor without and a main simulation with one turbine.

Precursor simulations

Precursor simulations generate realistic turbulent inflow conditions by adding random perturbations to an initially laminar flow. Two inflow conditions representing a Neutral Boundary Layer (NBL) and a Stable Boundary Layer (SBL), respectively, are generated, both having approximately the same mean wind speed and direction at hub height to allow for a fair comparison of the downstream wind field. A Convective Boundary Layer is omitted in this study, as WRC is found not to be beneficial under this condition [4]. The simulations use cyclic horizontal boundary conditions and the total simulation time is determined empirically until convergence to a stationary state occurs. The NBL does not prescribe a thermal forcing at the surface, while the SBL simulation specifies a constant cooling rate of $\partial\Theta/\partial t = 0.25$ K/h, following [17]. A cooling rate was described rather than a negative surface heat flux, as recommended by [18]. The details of the precursor simulations are summarized in Table 1.

Main simulations

The main simulations subsequently use the information generated in the precursor simulations.
by utilizing a turbulence recycle method, which adds a turbulent signal to a fixed mean inflow, as upstream boundary conditions. The downstream boundary condition ensures an undisturbed outflow by utilizing a radiation boundary condition. Table 1 summarizes the used simulation parameters. Total simulation time includes 20 minutes of spin-up time and a subsequent 60 minutes used for analysis. The size of the domain is only extended in streamwise direction and the recycle area has the same size as the precursor domain. One turbine is simulated, located in the center of the domain in crossstream direction and 6 rotor diameters downstream of the recycling area in streamwise direction. A 5MW NREL turbine, with a hub height of 90 m and a rotor diameter $D$ of 126 m [19], is simulated with an Actuator Disc Model with Rotation (ADMR) [20]. Turbine yaw angles ($\phi$) of $-30^\circ$, $0^\circ$, and $30^\circ$ are simulated for both inflow conditions, resulting in six simulations in total. A positive yaw angle is here defined as a clockwise rotation of the turbine, looking from above. Main simulations will hereafter be referred to as an abbreviation of its stability and yaw angle, e.g. NBL00; SBL30. The most relevant inflow parameters are displayed in Table 2, showing comparable wind speed for all simulations and dissimilar atmospheric conditions related to a Neutral and Stable Boundary Layer. The small spread of the parameters between the three simulations in the same boundary layer, indicated by the standard deviation, can be neglected.

For the downstream wind field, the following will solely study the wake deficit, simply defined as $U_{\text{def}} = U_{\text{wake}} - U_{\infty}$, where $U_{\infty}$ represents the undisturbed inflow and $U_{\text{wake}}$ the observed wind speed in the wake. To this end, frozen turbulence is assumed, meaning the advection velocity is assumed constant in streamwise direction. To reduce the impact of small-scale turbulence, the analyzed wind fields comprise of 60-minute averages.

### Table 1. Summary of simulation parameters for precursor and main simulations. The length ($t$) and size ($L_x$, $L_y$, $L_z$) of the precursor simulations (normalized by the rotor diameter ($D = 126$ m)) is determined empirically until convergence to a stationary state occurs. The size of the domain of the main simulations is extended only in streamwise direction. The geostrophic wind ($u_g$, $v_g$) is constant in precursor and corresponding main simulations.

|       | $t$ [h] | $L_x$ [D] | $L_y$ [D] | $L_z$ [D] | $u_g$ [m/s] | $v_g$ [m/s] |
|-------|---------|-----------|-----------|-----------|-------------|-------------|
| Precursor |         |           |           |           |             |             |
| NBL   | 28      | 40.6      | 20.3      | 6.3       | 10.115      | -3.969      |
| SBL   | 20      | 11.4      | 7.6       | 3.8       | 9.500       | -5.170      |
| Main  |         |           |           |           |             |             |
| NBL   | 1       | 61.0      | 20.3      | 6.3       | 10.115      | -3.969      |
| SBL   | 1       | 30.5      | 7.6       | 3.8       | 9.500       | -5.170      |

### Table 2. Summary of the most relevant inflow parameters, given as mean and standard deviation over the three main simulations. Results consist of rotor effective wind speed ($U_{\text{eff}}$), turbulence intensity at hub height ($TI_h$), wind shear ($\alpha$) and veer ($\delta\alpha$) over the rotor area and the Obukhov Length ($L$).

|       | $U_{\text{eff}}$ [m/s] | $TI_h$ [%] | $\alpha$ [-] | $\delta\alpha$ [°] | $L$ [m] |
|-------|------------------------|------------|--------------|---------------------|--------|
| NBL   | 8.28±0.05              | 10.30±0.17 | 0.166±0.002  | 2.03±0.27           | ∞      |
| SBL   | 8.11±0.02              | 5.68±0.01  | 0.322±0.001  | 9.61±0.04           | 145.0±0.2 |
2.2. Wake Tracking Algorithms

Traditional methods

As a reference, this study adopts the three wake center definitions used in [4]. Two of these utilize the principle that a wake deficit can be described by a Gaussian distribution. The simple one-dimensional Gaussian \( f_{1D} \) only considers a horizontal slice of the wind field at hub height, whereas the bivariate Gaussian \( f_{2D} \) assumes an elliptically shaped wake, thereby including information in two dimensions. Both identify the wake center as the location of the curve's minimum, found with a least squares fitting procedure. Additionally, the 95% confidence interval \( (1.96\sigma) \) can be used as a measure for the wake width, as proposed in [21]. The pragmatic Available Power method \( f_{AP} \) defines the wake center as the location of the minimal available power produced by a virtual turbine, making it dependent on the defined rotor area. This approach does not provide a metric for the wake width, but could be combined with other definitions such as 99% of the incoming wind speed as proposed in [22]. This concept is however very sensitive to turbulence and will therefore not be considered in this study.

Multiple 1D Gaussian method

The proposed method also applies the fundamental idea of describing a wake with a Gaussian distribution. It seeks to combine the strengths of \( f_{1D} \) (simple and robust) and \( f_{2D} \) (two dimensional). The Multiple 1D Gaussian \( f_{M1D} \) method fits a simple one-dimensional Gaussian distribution in crosswise direction at every vertical level where information is available, essentially identifying a center, width and magnitude at each altitude. This is illustrated in Figure 1 for hub height and upper tip height. To avoid including turbulent cells in the wake, a limit is introduced where the minimum of every fit needs to be larger than the deficit of the wake edge \((1.96\sigma)\) at hub height. This is particularly relevant when the wake deficit is small (e.g., far downstream and at higher altitudes).

The black crosses in Figure 2a illustrate the wake center positions as identified by the set of one-dimensional Gaussian distributions fitted in crosswise direction. By fitting another one-dimensional Gaussian through the magnitudes of this set of distributions (Figure 2b), one obtains the vertical position of the wake center. The center positions of this set of distributions can in turn be used to express the curliness and tilt of the wake (Figure 2c). A simple second-degree polynomial \( y = az^2 + bz + c \) is fitted to these positions between lower and upper tip height, where \( a \) represents the curliness and \( \arctan b \) the tilt of the wake. Comparing the dashed gray lines in Figures 2b and c, one can see that the maximum deficit and maximum curl do not necessarily occur at the same altitude. The wake area can simply be deduced from the 95% confidence intervals of the set of one-dimensional Gaussian distributions.

Figure 1. (a) Exemplary figure (SBL30, 5 rotor diameters downstream) illustrating the concept of the \( f_{M1D} \) method, where the thick black lines illustrate two horizontal cross-sections. (b) and (c) demonstrate the fitting procedure at these two heights.
3. Results & Discussion

This section will first present an evaluation of the proposed method, followed by a short overview of its future applications.

3.1. Evaluation of the proposed Multiple 1D Gaussian method

Figure 3a presents the vertical cross-section of the wake deficit five rotor diameters downstream of a simulated turbine in SBL00. Distances are normalized by the rotor diameter and centered around the turbine’s hub. The wake is elliptically shaped and stretched in crosswise direction due to strong wind veer in the SBL (Table 2). This strong veer also introduces a small wake deflection even in the absence of yaw misalignment. All four methods identify the wake center at very similar positions, where \( f_{1D} \) by definition excludes any vertical movement. A simplified wind field can be reconstructed from the Gaussian-based methods, using information from the fitted distributions. For this purpose, \( f_{1D} \) uses the same distribution in horizontal and vertical direction, describing a circular wake. These reconstructed wind fields are shown in Figures 3b-d. The circular shape described by \( f_{1D} \) severely underestimates the maximum wake deficit. Both \( f_{2D} \) and \( f_{M1D} \) are able to capture the elliptical shape, where the former describes the wake as a perfect ellipse and the latter allows slight deviations and therefore includes more detail. \( f_{2D} \) identifies the wake center as the location with the maximum deficit, which is not necessarily true for \( f_{M1D} \). Although not designed for an elliptically shaped wake, \( f_{M1D} \) presents the most accurate description of the wind field.

Figure 4 presents these results for a redirected wake (SBL30). Figure 4a shows that the \( f_{2D} \) and \( f_{AP} \) methods identify a wake center on the inner side of the curled wake, identifying practically no crosswise displacement. This is due to these methods’ assumptions of a circularly or elliptically shaped wake, which is not satisfied in this example. The \( f_{1D} \) method better captures the lateral displacement, but excludes any vertical movement. The \( f_{M1D} \) method does not have this limitation and locates the wake center closest to the location of the maximum deficit. The reconstructed wind fields in Figures 4b-d clearly show that the curled shape is only captured in \( f_{M1D} \), demonstrating a superiority of the proposed method. Interesting to note is that \( f_{1D} \) largely underestimates the total wake area, since it does not account for any vertical expansion. The LES and reconstructed wind fields can be used to estimate the available power of a virtual
Figure 3. Vertical cross-section (looking downstream) of the SBL00 LES wake deficit wind field 5 rotor diameters downstream of the simulated turbine and the wake center positions identified by all methods (a). Reconstructed wind fields based on $f_{1D}$ (b), $f_{2D}$ (c) and $f_{M1D}$ (d).

Figure 4. Same as Figure 3, but for SBL30.
turbine located downstream of the wake-producing turbine in mean wind direction. The available power is estimated by the cubed rotor effective wind speed. Figure 5 presents the percentage error of the available power computed with the reconstructed wind field relative to when computed with the LES wind field. The boxes include all six simulations described in Section 2.1. One observes that all methods perform better in the far wake, since there the wake deficit can be more accurately described by a Gaussian distribution. By contrast, the near wake region shows a high momentum zone in the central part of the near wake, introduced by a low thrust force around the turbine’s hub. Further, \( f_{2D} \) holds more accurate results than \( f_{1D} \), but \( f_{M1D} \) consistently outperforms both methods. Especially in the far wake (\( x/D \geq 4 \)), only \( f_{M1D} \) has no systematic bias and it has the smallest spread of all methods. This also indicates that \( f_{M1D} \) is competitive in estimating the available power, the main strength of the \( f_{AP} \) method. This result stresses the improvement of \( f_{M1D} \) over the other methods.

Figure 6 shows the horizontal wake center trajectories for the NBL (a) and SBL (b) at three yaw angles as determined by the four discussed methods. A 0° yaw angle results in no lateral displacement in the NBL and a small displacement in the SBL, as already observed in Figure 3. The differences between the four methods are rather small. There are more noticeable differences between the methods when there is a yaw misalignment. \( f_{2D} \) and \( f_{AP} \) typically show the smallest lateral deflection, whereas \( f_{M1D} \) is very comparable to \( f_{1D} \) and generally finds the largest lateral deflection. The wake displacement is asymmetric, which is due to wind veer and therefore best visible in the SBL. This corresponds to the findings of [23], who reported on more effective wake deflection under negative yaw angles compared to positive ones. It should be noted that the deviating trajectory of \( f_{2D} \) in SBL-30 is due to the detachment of the downstream wake in two separate cells as mentioned in [4], which is currently not accounted for in any of the wake center definitions.

Figure 6c-d shows the vertical wake center trajectories in the NBL (c) and SBL(d). It should be noted that the vertical displacement is about an order of magnitude smaller than the horizontal displacement. By definition, \( f_{1D} \) excludes any vertical displacement. An initial vertical displacement in the near wake is observed for all simulations. Interestingly, the wake center moves back to hub height in the far wake for non-yawed turbines, but not in the case of yaw misalignment. This suggests that yaw misalignment unintentionally also results in a slight vertical deflection. For this reason, to accurately describe the characteristics of the redirected wake, one should take the vertical displacement into account. In general, \( f_{M1D} \) is most similar to \( f_{2D} \), which is currently the standard to study vertical wake center displacement. These results demonstrate that \( f_{M1D} \) indeed combines the strengths of \( f_{1D} \) (best in horizontal) and \( f_{2D} \) (best in vertical).
3.2. Future applications

A potential application of the newly proposed wake tracking algorithm is to study the wake curliness as determined by the method described in Section 2.2. Figure 7 shows that a non-yawed turbine does not produce a curled wake and therefore has a curliness parameter around zero. The misaligned turbines indicate that the curl is mainly generated in the near wake and remains relatively constant further downstream. Exception to this is SBL-30, showing an ever increasing curliness parameter. This is again due to the detachment of the wake in two separate cells, which is currently not accounted for. These preliminary results suggest that atmospheric conditions affect the wake curliness as the curl parameter is larger in a SBL than in a NBL, for instance due to less mixing.

Figure 6. Wake center trajectories for NBL (a,c) and SBL (b,d) representing horizontal (a,b) and vertical (c,d) deflection for yaw angles of 0 (black), 30 (blue) and -30 (red) degrees.

Figure 7. Evolution of the wake curliness downstream of the turbine for two stratifications and three yaw angles.
Figure 8. Preliminary result showing the potential of the development of a physically based empirical wake deflection parameterization. Contours indicate the SBL30 LES wind field at hub height, the black line indicate the wake center trajectory and the red line the wake center trajectory predicted with a simple statistical model.

Figure 8 shows a first result of the development of a physically based empirical wake deflection parameterization. The black line indicates the wake center trajectory as determined directly from the LES using $f_{MID}$, while the red line indicates a predicted trajectory based on simple inflow and operational parameters (shear, veer, torque and yaw angle) by employing a simple Ordinary Least Squares fitting model. The testing data set merely encompasses SBL30, whereas the training data set includes five different inflow conditions (not SBL) combined with five yaw angles. These preliminary results suggest that it is possible to determine the wake trajectory of a redirected wake solely based on atmospheric and operational parameters.

In the future, this work will be elaborated on by including vertical wake center displacement and wake shape parameters, as well as the magnitude and distribution of the wake deficit. Additionally, the aspired wake parameterization should be applicable to a wide range of locations and atmospheric conditions, and will be validated with field measurements.

4. Conclusions
This study proposes a new 2D wake description specifically designed to account for shape deformations introduced by intentional yaw misalignment. This parameterized wake tracking algorithm describes frequently studied wake characteristics, such as the wake shape and center position.

Utilizing a Large-Eddy Simulation model at different atmospheric stratifications, a comparison with traditionally used approaches demonstrates a significantly more accurate wake description of the newly proposed method. This does not only include a better estimation of the wake center position, but also a better description of the shape of the redirected wake by introducing a curliness parameter. Consequently, it provides a significantly higher accuracy in the power estimation of a virtual downstream turbine compared to traditional approaches.

A drawback of the newly proposed method is its sensitivity to turbulence, especially when the wake deficit is small (e.g. far downstream and at higher altitudes).

A brief outlook suggests that this wake tracking algorithm is suited to study the effect of the most influential atmospheric and operational parameters on wake propagation under yaw misalignment and the development of a physically based empirical wake parameterization.
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