Geometric linearization of theories for incompressible elastic materials and applications

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We derive geometrically linearized theories for incompressible materials from nonlinear elasticity theory in the small displacement regime. Our nonlinear stored energy densities may vary on the same (small) length scale as the typical displacements. This allows for applications to multiwell energies as, e.g., encountered in martensitic phases of shape memory alloys and models for nematic elastomers. Under natural assumptions on the asymptotic behavior of such densities we prove Gamma-convergence of the properly rescaled nonlinear energy functionals to the relaxation of an effective model. The resulting limiting theory is geometrically linearized in the sense that it acts on infinitesimal displacements rather than finite deformations, but will in general still have a limiting stored energy density that depends in a nonlinear way on the infinitesimal strains. Our results, in particular, establish a rigorous link of existing finite and infinitesimal theories for incompressible nematic elastomers.