Evaluation of Void Ratio of Sands with Various Amount of Fines On the Basis of Shear Wave Velocity Measurement

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Abstract. The paper is focused on the possibility of evaluation of soil state of cohesionless soils containing fines on the basis of shear wave velocity. The paper presents laboratory test results obtained on the basis of large volume of triaxial tests data. Shear wave velocity was measured with piezoelectric transducers embodied in top and bottom platens of the triaxial cell. Three kinds of cohesionless soils containing various amount of fines (from 0 to 57%) were tested. In order to obtain vast ranges of void ratio, specimens were reconstituted by moist tamping technique with application of under compaction. For each kind of material, a series of tests was carried out in which shear wave velocity was measured. The data were categorized in several ranges of void ratio. In sandy material it was possible to distinguish a few subranges of void ratio, while in sands containing large amount of fines, the range of void ratio was significantly reduced. The tests delivered shear wave velocity values related to state of material represented by void ratio, and mean effective stress. Analysis of test results revealed that relationship between shear wave velocity and mean effective stress $p'$ can be approximated by logarithm function in distinguished void ratio ranges. This made possible to set a formula for calculating void ratio for a given state of stress on the basis of shear wave velocity measurement. Such formulae were obtained for all tested type of soil.

1. Introduction

Interest in seismic waves velocity measurement for geotechnical purposes has considerably increased during the last three decades. There are a few reasons for that. The first one results from the fact that the actual measurement refers to very small strain range (around 10⁻⁴ %). Such a small strain corresponds to true elastic response of material and therefore shear wave velocity can be used to calculate initial shear modulus $G_0$ which is one of fundamental parameter describing elastic behaviour of soil. The second reason of increased popularity of seismic waves velocity results from considerable progress which has been done in quality of measurement. It concerns field and laboratory techniques as well. Regarding shear wave velocity measurement in the laboratory, piezoelement of bender type introduced by [1] has improved ability to correct identification of arriving signal. Also much experimental work has been done concerning improvement of data interpretation [2]. It should be also pointed out that shear and longitudinal waves velocity can be measured at any stage of laboratory tests, thus delivering additional independent measurement which enhances interpretation capability. For instance, measurement of longitudinal wave velocity during saturation stage can be used as an indicator of degree of saturation. It correlates very well with Skempton's B parameter. Although both kinds of waves are useful, shear wave attracted much more attention during the last fifty years. This is
due to the fact, that shear waves are propagated through contact of grains and particles and thus reflects changes in soil fabric, effective stress and void ratio. The last two components constitute state of material and therefore shear wave velocity can reflect it quantitatively. Much research has been devoted to this issue [3], [4], [5], [6], [7]. Numerous formulae have been proposed in scientific papers describing relation between shear wave velocity and state. The majority of these formulae concerns cohesionless soils. This is due to the fact, that mechanism of compressibility in sandy soils is more complex than in case of cohesive soils. In cohesionless soils there is an infinite number of compressibility curves which positions depend on the initial (after deposition) void ratio. These curves are almost parallel with slight tendency to converge when the mean effective stress increases. Due to small compressibility of sand, monotonic drained loading can change void ratio of relatively small value. Shaking of material at small effective stress would cause bigger change of void ratio. If initial void ratio represents loose state a sandy material would have contractive response during shearing. If conditions are drained the material will be densified. In case of undrained conditions, the sand will be in unstable conditions. In order to determine the degree to which material is in unstable conditions, it is necessary to compare location of compressibility line with steady state line, which is created by set of points representing the state of material at the end of undrained shearing. If void ratio at a compressibility line is higher than void ratio at steady state at a given mean effective stress the material is in unstable equilibrium. Such a state with presence of shear stress resulting from gravitational forces (e.g. sloping ground) fulfil conditions to liquefaction of soil. Therefore, evaluation of state of soil represented by void ratio and mean effective stress is a crucial issue in evaluation of susceptibility of cohesionless soil to liquefaction [8], [9]. All aforementioned formulae are different with respect to various way of accounting void ratio, state of effective stress and component dependent on fabric. Void ratio and state of stress components are usually expressed by power function with various exponents. Soil fabric, in turn, is strictly related to method of deposition and stress history and therefore it is difficult to set representative function to reflect it. This is the reason why derived formulae cannot predict accurately all relevant components of the equation. This especially concerns void ratio which in case of problems associated with liquefaction is a key parameter.

The purpose of this paper is to present derivation of formulae to predict void ratio on the basis of soil kind and a given mean effective stress. In order to examine the proposed approach for more general conditions the carried out research concern three kinds of soils and thus can deliver meaningful information concerning range of applicability of the proposed approach.

2. Material and test procedure
As indicated, the tests were carried out on three kinds of materials, from pure sand to silt. The first material is a pure medium silica sand with no fines. The second material was also silica sand but with 8% of fines (understood as fraction finer than 0.063 mm). The third material is a sandy silt which from a granulometric point of view can be described as sand with 57% of fines. Grain size distribution of tested materials is shown in figure 1.

The tests were carried out in a triaxial apparatus with cell equipped with proximity transducers (as described in [10]). Triaxial tests were carried out on reconstituted specimens which were 50 mm in diameter and 100 mm in height. Specimen were prepared by moist tamping by under compaction method as described in [11]. Moist tamping was selected in order to enable preparation of samples of possible wide range of void ratio. When a triaxial cell was assembled a specimen was flushed with CO₂ and then with deaired water. The actual saturation was done with application of back pressure until Skempton's parameter B exceeded value of 0.96. After saturation specimens were consolidated with application of isotropic or anisotropic stress. In case of the latter, zero lateral strain method of stress application was used. When consolidation was terminated, shear wave velocity was measured. Piezoelectric transducer of bender type was used for sending and receiving the signals.
3. Algorithm of setting up formulae for void ratio calculation

As mentioned, the only method which allows to reconstitute a specimen to predetermined value of void ratio is most tamping by under compaction. It is even more important in case of silts, since such material prepared by other (dry or wet) methods would deliver very similar or unique compressibility line. This is very important, since the objective of the paper is to evaluate capability of shear wave velocity to describe state of granular soils with various amount of fines. With respect to this, for each kind of material, the data have been presented in the form of two charts:

- Void ratio against mean effective stress (\( e \sim p' \))
- Shear wave velocity against mean effective stress (\( V_s \sim p' \))

These data are presented in figures 2, 3 and 4 respectively for medium sand with no fines, sand with 8% of fines and silty material with 57%. As it can be deduced from the charts for each kind of material various ranges of void ratio were identified. In spite of the fact that all data for pure sand fall into relatively narrow range i.e. 0.78-0.92 the data are grouped in three subranges of void ratio. This certainly results from very flat compressibility lines for pure medium sand.

Much more subranges were identified for sand with 8% of fines. Range of void ratio 0.61-1.04 were divided into 8 subranges. For very loose samples (\( e > 0.9 \)) compressibility lines are visibly inclined. For void ratio higher than 1.0, which is not natural for this material, a 50kPa variation of mean effective stress changes void ratio from 1.04 to 1.00. For void ratio lower than 0.9, compressibility lines are very flat, similarly as for pure sand. For finest material, with 57% of fines, although the range of void ratio is from 0.51 to 0.72, only two subranges of void ratio were identified. It should be emphasized that the upper subrange (0.6-0.72) was possible to achieve only because of application of undercompaction method.

Another important issues which should be emphasized are assumption concerning function used for regression of shear wave velocity against effective stress. In the paper two assumption has been made with regards to this subject. The first one concerns the effective stress. It was assumed that mean effective stress \( p' \) will represent state of stress, no matter what kind of consolidation (isotropic or anisotropic) is considered. This kind of assumption is partly justified by the fact that in case of normally consolidated cohesionless soils, relation between horizontal and vertical effective stress ratio is strictly related to value of void ratio at the end of consolidation. The other assumption concerns regression of shear wave velocity against mean effective stress relations in the identified subranges of void ratio. The most common approximation which is used with respect to this relation is power
function. However, when one analyses various kind of soils having various stress history, and especially in case of cohesionless soils, when various initial fabric is considered, this approximation may not be optimal for regressions for relationship between shear wave velocity and mean effective stress. For this reason, it was assumed in the paper that these relationships can be approximated by logarithmic function. In the chart of shear wave velocity against logarithm of mean effective stress, the data for each soil and subranges of void ratio, can be approximated by linear regressions. The results of these approximations are shown in figure 5.

![Medium sand (no fines)](image)

**Figure 2.** Shear wave velocity data in various void ratio ranges of clean sand

As it can be deduced from the charts, the regressions reflects void ratio subranges. Another observation is that slope of regressions lines for subranges of void ratio increases with void ratio decreasing. These observations were the premises to derive an algorithm for setting up formulae for calculation of void ratio for each kind of soil. The flow chart of this algorithm is shown in figure 6.

The starting point for these considerations are linear regressions between shear wave velocity and logarithm of mean effective stress. $A$ and $B$ parameter of these linear regressions for each subrange of void ratio are determined. Then, average value of $B$ parameter is calculated.

It should be pointed out that slope of each linear regression reflects void ratio, therefore when $A$ parameter is established it is assumed to be a linear function of void in the form $A = \alpha e + \beta$. Then, $\alpha$ and $\beta$ parameters are determined on the bases of experimental data. As these parameters are known it is possible to derive a formula for void ratio calculation on the basis of measured shear wave velocity and determined constants $B$, $\alpha$ and $\beta$. 
Figure 3. Shear wave velocity data in various void ratio ranges of for cohesionless soils containing 8% of fines

Figure 4. Shear wave velocity data in various void ratio ranges of for silty material containing 57% of fines
Figure 5. Linear regressions for $V_s \sim \log p'$ relations for each subrange of void ratio of tested materials

$V_s = 44.8\ln(p') + 12$
$R^2 = 0.97$

$V_s = 45.8\ln(p') + 16$
$R^2 = 0.97$

$V_s = 52.1\ln(p') - 11$
$R^2 = 0.98$

$V_s = 63.72\ln(p') - 25.65$
$R^2 = 0.93$

$V_s = 63.82\ln(p') - 39.41$
$R^2 = 1.00$

$V_s = 61.50\ln(p') - 36.30$
$R^2 = 0.95$

$V_s = 53.06\ln(p') - 23.05$
$R^2 = 0.97$

$V_s = 52.85\ln(p') - 30.33$
$R^2 = 0.97$

$V_s = 64.63\ln(p') - 107.20$
$R^2 = 0.97$

$V_s = 44.72\ln(p') - 28.03$
$R^2 = 0.97$

$V_s = 40.29\ln(p') - 22.12$
$R^2 = 0.98$

$V_s = 58.16\ln(p') - 4.14$
$R^2 = 0.84$

$V_s = 48.38\ln(p') - 18.11$
$R^2 = 0.78$

$V_s = 59.16\ln(p') - 4.14$
$R^2 = 0.84$

void ratio range

$> 0.88$

$0.84 - 0.88$

$< 0.84$

Shear wave velocity $V_s$ [m/s]

Mean effective stress $p'$ [kPa]
4. Discussion of the results

The above procedure has been applied to all three tested kinds of soil. A chart comparing calculated against measured void ratio was assumed as a measure of effectiveness of the applied procedure. Such charts were prepared for each kind of material and shown in figures 7, 8 and 9. For the sake of better comparison, besides resulting regression of predicted and measured values of void ratio, equal values line was drawn on each chart.

Regarding the results obtained for medium sand with no fines (figure 7), it should be noticed that tests were done for a relatively narrow void ratio range (0.78-0.92). Although the regression and equal values line have similar slope, scatter of the data is large ($R^2=0.20$). The range of predicted values is much bigger than the measured ones. From these two observation can be summarized that accuracy is not bad (similar slope of lines) but resolution of the method is low (large scatter of data) for pure sand.

Much better situation is for sand with 8% of fines (figure 8). Here two lines have almost the same slope and scatter of the data is very small. Only prediction for very dense material are poor. In spite of this $R$-squared is 0.86. Without the data for dense state the final result would be much better. So it can be summarized that the accuracy of the method as well as its resolution is good. A conceivable explanation might result from compressibility. Wide range of void ratio (9 subranges) creates large potential for shear wave velocity change. In the presence of small percent of fines, changes of effective stress might change also number of grains contacts, what is resulting in shear wave velocity change. In less compressible material with no fines, increase of effective stress changes quality of contacts but does not change substantially their number.
Figure 7. Comparison of measured and calculated void ratio for clean sand

![Medium sand (no fines) comparison](image)

\[ e_{\text{calculated}} = 1.17e_{\text{measured}} - 0.14 \]
\[ R^2 = 0.20 \]

Figure 8. Comparison of measured and calculated void ratio for cohesionless soils containing 8% of fines

![Sand with 8% of fines comparison](image)

\[ e_{\text{calculated}} = 1.04e_{\text{measured}} - 0.03 \]
\[ R^2 = 0.86 \]

For silty material (figure 9) with 57% of fines, situation is different with respect to other materials. Here equal values line has slope twice bigger than the regression line for data. Also scatter of the data is considerably high \((R^2=0.29)\). Therefore, it can be stated that accuracy and resolution of this method in application to that soil are low. Most probably the reason for that is again compressibility. In soil with 57% of fines compressibility lines tend to converge and for wet preparation method it would be probably a unique line. Only moist tamping by undercompaction enabled creation of the second subrange of higher void ratio.
5. Conclusions
The experimental test programme carried out on the three kinds of soil i.e. medium sand with no fines, sand with 8 % of fines and silty material with 57% of fines, allowed formulation of the following conclusions:

- Large number of shear wave velocity measurements, carried out for various void ratio and state of stress, enable to create an algorithm for derivation of formulae allowing to calculate void ratio for a given kind of soil at mean effective stress in the range used in engineering applications.
- Logarithmic function can be used successfully to approximate relationship between shear wave velocity and mean effective stress for all tested kinds material.
- It is possible to use shear wave velocity to evaluate state of soil, however effectiveness of the method depends on the kind of soil.
- In medium sand with no fines accuracy of the method is acceptable, however resolution is not satisfactory. More data for verification of this statement is needed, especially for higher densities.
- In silty material, containing 57% of fines, accuracy as well as resolution of the applied method are not satisfactory.
- In sand with 8% of fines accuracy and resolution of the method is very good and therefore application of shear wave velocity for detection of state of soil in such material is highly recommended.

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