Diffraction Slopes for Elastic Proton-Proton and Proton-Antiproton Scattering

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Abstract. The diffraction slope parameter is investigated for elastic proton-proton and proton-antiproton scattering based on the all available experimental data at low momentum transfer values. Energy dependence of the elastic diffraction slopes is approximated by various analytic functions. The expanded "standard" logarithmic approximations allow to describe experimental slopes in all available energy range reasonably. Various approximations differ from each other both in the low energy and very high energy domains. Predictions for diffraction slope parameter are obtained for elastic proton-proton scattering at RHIC and LHC energies, for proton-antiproton elastic reaction in FAIR energy domain for various approximation functions.

Keywords: proton, antiproton, elastic scattering, slope
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INTRODUCTION

Elastic hadron-hadron scattering, the simplest type of hadronic collision process, remains one of the topical theoretical problems in fundamental interaction physics at present. In the case of $pp$ and $\bar{p}p$ elastic scattering, although many experiments have been made over an extended range of initial energies and momentum transfer, these reactions are still not well understood. The nuclear slope $B$ for elastic scattering is of interest in its own right. This quantity defined according to the following equation:

$$B(s) = \left[ \frac{\partial}{\partial t} \left( \ln \frac{\partial \sigma(s,t)}{\partial t} \right) \right]_{t=0},$$

is determined experimentally. On the other hand the study of $B$ parameter is important, in particular, for reconstruction procedure of full set of helicity amplitudes for elastic nucleon scattering [1]. The present status of slope for elastic $pp$ and $\bar{p}p$ scattering is discussed over the full energy domain.

SLOPE ENERGY DEPENDENCE

We have attempted to describe the energy behaviour of the elastic nuclear slopes for $pp$ and $\bar{p}p$ reactions. Specifically, we have focused on the low $|t|$ domain. The ensemble of experimental data on slopes for elastic nucleon collisions can be fitted successfully by many phenomenological models, at least for $\sqrt{s} > 5$ GeV. The new "expanded" logarithmic parameterizations have been suggested in [1] for description of the elastic slope at all available energies. Thus taking into account standard Regge parameterization
and quadratic function of logarithm from [2] the following analytic equations are under study here:

\[
\begin{align*}
B(s) &= B_0 + 2a_1 \ln(s/s_0), \\
B(s) &= B_0 + 2a_1 \ln(s/s_0) + a_2 [\ln(s/s_0)]^{a_3}, \\
B(s) &= B_0 + 2a_1 \ln(s/s_0) + a_2 (s/s_0)^{a_3}, \\
B(s) &= B_0 + 2a_1 \ln(s/s_0) + a_2 [\ln(s/s_0)]^2,
\end{align*}
\]

where \(s_0 = 1 \text{ GeV}^2\), in general case parameters \(B_0, a_i, i = 1, 2, 3\) depend on range of \(|t|\) which is used for approximation.

Experimental data are from [3, 4, 5, 6, 7, 8]. Total number of experimental points is equal 133 / 129 for \(pp/\bar{p}p\) scattering at low \(|t|\), respectively. Thus the experimental sample is significantly larger than that for some early investigations [1,2]. As known the measurements of nuclear slope, especially at intermediate \(|t|\) do not form a smooth set in energy, in contrast to the situation for global scattering parameters \(\rho\) and \(\sigma_{\text{tot}}\), where there is a good agreement between various group data [2]. Thus the data samples for approximations are somewhat smaller because of exclusion of points which differ significantly from the other experimental points at close energies. The maximum fraction of excluded points is equal 3.1% for low \(|t|\) domain.

The energy dependence for experimental slopes and corresponding fits by (2a)–(2d) are shown in Fig.1a and Fig.1b for \(pp\) and \(\bar{p}p\) correspondingly. The fitting parameter values are indicated in Table 1. The available systematic errors were added in quadrature to the statistical errors at calculation of resulting errors for fitted data points. One can see that the fitting functions (2a), (2d) describe the \(pp\) (Fig.1a) and \(\bar{p}p\) (Fig.1b) experimental data statistically acceptable only for \(\sqrt{s} \geq 5 \text{ GeV}\). The parameter \(a_1\) for function (2a) agree with estimate for Pomeron intercept \(\alpha'_{\rho} \approx 0.25 \text{ GeV}^{-2}\) for \(\bar{p}p\) data but this parameter is some larger than \(\alpha'_{\rho}\) for proton-proton data. One can see the \(a_2\) parameter is equal zero within errors for approximation of \(\bar{p}p\) data by (2d). Thus the function (2d) can be excluded from the set of approximation functions for \(\bar{p}p\) at low \(|t|\) values. The RHIC point for proton-proton collisions has a large error and can’t discriminates the approximations. The parameterizations (2b), (2c) allow to describe experimental data at all energies with reasonable fit quality for \(pp\). The functions (2a)–(2c) are close to each other at \(\sqrt{s} \geq 5 \text{ GeV}\), especially, the Regge model approximation and (2c) fit. It seems the ultra-high energy domain is suitable for separation of various parameterizations. The qualities of (2b), (2c) approximations for \(\bar{p}p\) elastic scattering data are much poorer because of very sharp behaviour of experimental data near the low energy boundary. But one can see that the functions (2b), (2c) agree with experimental points at qualitative level and (very) close to each other for all energy range.

One can get a predictions for nuclear slope parameter values for some facilities based on the results shown above. The \(B\) values at low \(|t|\) for different energies of FAIR, RHIC, and LHC are shown in the Table 2. According to the results above the function (2d) are not considered for \(\bar{p}p\) collisions. As expected the functions (2b) and (2c) predicted the very close slope parameter values for FAIR. The first approximation function (2a) can predicts for \(\sqrt{s} \geq 5 \text{ GeV}\) only. Functions (2a)–(2c) predict much smaller values for \(B\) in high-energy \(pp\) collisions than (2d) approximation. Perhaps, the future more precise RHIC results will agree better with predictions based on experimental data fits under study. It should be emphasized that the fits (2a)–(2c) of available experimental
FIGURE 1. Energy dependence of the elastic slope parameters for proton-proton (a) and proton-antiproton (b) scattering for low $|t|$ domain. The inner picture for (b) shows the experimental data and fits at the same scale as well as for (a). The curves correspond to the fitting functions as following: (2a) – dot, (2b) – thick solid, (2c) – dot-dashed, (2d) – thin.

TABLE 1. Fitting parameters for slope energy dependence in low $|t|$ domain

| Function          | Parameter | $B_0$, GeV$^{-2}$ | $a_1$, GeV$^{-2}$ | $a_2$, GeV$^{-2}$ | $a_3$  | $\chi^2$/n.d.f. |
|-------------------|-----------|------------------|------------------|------------------|-------|-----------------|
|                   |           |                  |                  |                  |       |                 |
| proton-proton      |           |                  |                  |                  |       |                 |
| scattering (2a)    |           | 8.43 ± 0.08      | 0.270 ± 0.007    | –                | –     | 162/87          |
| (2b)              |           | 8.77 ± 0.12      | 0.248 ± 0.009    | –27 ± 2          | –3.45 ± 0.14 | 329/125          |
| (2c)              |           | 8.33 ± 0.08      | 0.278 ± 0.007    | –181 ± 23        | –2.21 ± 0.09 | 340/125          |
| (2d)              |           | 9.3 ± 0.3        | 0.11 ± 0.06      | 0.03 ± 0.01      | –       | 155/86          |
| proton-antiproton  |           |                  |                  |                  |       |                 |
| scattering (2a)    |           | 9.4 ± 0.4        | 0.24 ± 0.02      | –                | –       | 29/22           |
| (2b)              |           | 11.98 ± 0.06     | 0.127 ± 0.006    | 489 ± 97         | –13.0 ± 0.5 | 1180/122        |
| (2c)              |           | 12.01 ± 0.06     | 0.125 ± 0.005    | (2.7 ± 1.4) · 10^6 | –9.3 ± 0.4 | 1260/122        |
| (2d)              |           | 9.4 ± 1.6        | 0.24 ± 0.16      | (2 ± 157) · 10^{-4} | –       | 29/21           |

TABLE 2. Predictions for nuclear slope based on the parameterizations (2a) – (2d) for low $|t|$ domain

| Fitting function | Facility energies, $\sqrt{s}$ |
|------------------|-------------------------------|
|                  | FAIR, GeV | RHIC, TeV | LHC, TeV |
|------------------|-----------|-----------|----------|
|                  | 3 | 5 | 6.5 | 14.7 | 0.2 | 0.5 | 14 | 28 | 42* |
| (2a)             | – | 10.95 | 11.20 | 11.98 | 14.15 | 15.14 | 18.74 | 19.49 | 19.93 |
| (2b)             | 12.56 | 12.80 | 12.93 | 13.35 | 14.02 | 14.93 | 18.24 | 18.93 | 19.33 |
| (2c)             | 12.56 | 12.81 | 12.95 | 13.35 | 14.22 | 15.24 | 18.95 | 19.72 | 20.17 |
| (2d)             | – | – | – | – | 15.00 | 16.67 | 24.44 | 26.39 | 27.58 |

* The ultimate energy upgrade of LHC project [9]
data predict the slower increasing of $B$ with energy than most of phenomenological models.[10] The $B$ values predicted for LHC at $\sqrt{s} = 14$ TeV by (2a) and (2c) are most close to the some model expectations [11, 12]. Moreover one needs to emphasize that the model estimates at $\sqrt{s} = 14$ TeV were obtained for $B(t = 0)$ and the $t$-dependence of slope shows the slight decreasing of $B$ at growth of momentum transfer up to $t \approx 0.1 − 0.2$ GeV$^2$ in particular for the models [11, 12]. Thus one can expect the some better agreement between model estimates and predicted values of $B$ from Table 2 for finite (non-zero) low $|t|$ values. Possibly the saturation regime, Black Dick Limit, will be reached at the LHC. The one of the models in which such effects appear, namely, DDM predicts the slope $B(t = 0) \approx 23$ GeV$^{-2}$ at $\sqrt{s} = 14$ TeV [13].

SUMMARY

The main results of this paper are the following. Most of all available experimental data for slope parameter in elastic nucleon collisions are approximated by different analytic functions. The suggested parameterizations allow to describe experimental nuclear slope at all available energies in low $|t|$ domain for $pp$ quite reasonably. The new approximations agree with experimental $\bar{p}p$ data at qualitative level but these fits are still statistically unacceptable because of very sharp behavior of $B$ near the low energy limit. Predictions for slope parameter are obtained for elastic proton-proton and proton-antiproton scattering in energy domains of some facilities.

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