Gauge Mediated SUSY Breaking via Seesaw

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We present a simple scenario for gauge mediated supersymmetry breaking where the messengers are also the fields that generate neutrino masses. We show that the simplest such scenario corresponds to the case where neutrino masses are generated through the Type I and Type III seesaw mechanisms. The entire supersymmetric spectrum and Higgs masses are calculable from only four input parameters. Since the electroweak symmetry is broken through a doubly radiative mechanism, meaning a nearly zero $B$-term at the messenger scale which runs down to acceptable values, one obtains quite a constrained spectrum for the supersymmetric particles whose properties we describe. We refer to this mechanism as “$\nu$-GMSB”.

I. INTRODUCTION

The Minimal Supersymmetric Standard Model (MSSM) is one of the most appealing extensions of the Standard Model with a mechanism to protect the Higgs mass from radiative corrections, realizable high scale gauge coupling unification [1–4], and a candidate for the cold dark matter of the Universe, even when the so-called discrete $R$-parity symmetry is broken. Furthermore, the mechanism of electroweak baryogenesis can be employed to explain the matter-antimatter asymmetry in the Universe and one has the appealing mechanism for radiative electroweak symmetry breaking (EWSB) [5].

One of the open issues in the MSSM is the origin of supersymmetry (SUSY) breaking (see Ref. [6] for a review on supersymmetry breaking). Gauge Mediation [7] is one of the most appealing mechanisms to address this issue. Superpartner masses are predicted assuming the existence of a SUSY breaking hidden sector. This breaking is then transmitted to the MSSM sector through gauge interactions via messenger fields. This process generates masses for all superpartner masses and avoids the so-called flavor problem in SUSY theories since mixings between the sfermions of different families are not generated.

In this paper we present a simple scenario for gauge mediated supersymmetry breaking where the messengers are also the fields that generate neutrino masses. We refer to this scenario as “$\nu$-GMSB”. We build
up to this scenario by discussing previous implementations of gauge mediation through the already existing particle content of the simplest $SU(5)$ supersymmetric grand unified theories (GUT). Since it is expected that in such models we will also generate neutrino masses in a consistent way, we then consider seesaw extensions of $SU(5)$. The so called seesaw fields can also mediate SUSY breaking and since the seesaw scale, $M_{\text{Seesaw}} \lesssim 10^{11-14}$ GeV, is much smaller than the GUT scale, $M_{\text{GUT}} \approx 10^{16-17}$ GeV, the seesaw contributions will dominate the SUSY breaking masses. This idea was first discussed in Refs. [8–10].

We investigate this hypothesis discussing all possible scenarios for gauge mediation in the context of $SU(5)$ theories and find that neutrino mass generation through both the Type I and Type III seesaw mechanisms provides the simplest framework for gauge mediation via seesaw fields. We then pursue this idea in detail, finding that the spectrum depends on four parameters and that while the bilinear Higgs term is very small at the messenger scale, it can run to acceptable values for electroweak symmetry breaking (EWSB) at the SUSY scale. We obtain a constrained SUSY spectrum whose phenomenological aspects are then discussed.

In Section II we discuss the different implementation of the gauge mediation mechanism for supersymmetry breaking in the context of $SU(5)$ grand unified theories. In Section III we discuss the predictions for superpartner masses in the case where the messengers are the fields responsible for the Type III seesaw mechanism. In Section IV we discuss the constraints from gauge coupling unification and proton decay, while in Section V we summarize our findings.

II. SUPERSYMMETRIC SU(5) UNIFICATION AND GAUGE MEDIATED SUSY BREAKING

In the minimal supersymmetric $SU(5)$ [11] the MSSM matter fields of one family are unified in $\hat{5} = (\hat{D}C, \hat{L})$, and $\hat{10} = (\hat{U}C, \hat{Q}, \hat{E}C)$, while the Higgs sector is composed of $\hat{5}_H = (\hat{T}, \hat{H})$, $\hat{24}_H = (\hat{T}, \hat{H})$, and $\hat{24}_H = (\hat{T}, \hat{H})$. The relevant interactions for breaking $SU(5)$ to the SM gauge group are:

\[ W_2 = m_\Sigma \text{Tr} \hat{24}_H^2 + \lambda_\Sigma \text{Tr} \hat{24}_H^3 + \mathcal{O}(24_H/M_{Pl}), \]

and the interactions between the different Higgs chiral superfields are:

\[ W_3 = m_H \hat{5}_H \hat{5}_H + \lambda_H \hat{5}_H \hat{24}_H \hat{5}_H + \mathcal{O}(24_H/M_{Pl}). \]

Here we assume the existence of higher-dimensional operators for consistent fermion masses. The relevant interactions for breaking $SU(5)$ to the SM gauge group are:

\[ W_2 = m_\Sigma \text{Tr} \hat{24}_H^2 + \lambda_\Sigma \text{Tr} \hat{24}_H^3 + \mathcal{O}(24_H/M_{Pl}), \]
See Refs. [12–14] and references therein for the status of this model.

At first sight, the most appealing way to proceed is to break both the GUT symmetry and SUSY with the same field, $\hat{24}_H$ [15, 16] assuming, $\langle \hat{24}_H \rangle = v_{24} + \theta^2 F_{24}$. This would lead to contributions to the soft terms from the following four sets of would-be messengers: the Higgses—$H$ and $\bar{H}$ [17]; the $SU(3)$ color triplets—$T$ and $\bar{T}$; the components of the $\hat{24}_H$—$\Sigma_3$ and $\Sigma_8$; and the $SU(5)$ heavy gauge bosons—$X$ and $Y$ [18]. The largest contribution by far is the first one, via the Higgses, since they are the lightest of these fields. SUSY breaking in this case is transmitted once we generate the term $H\bar{H}F_{24}$ using the scalar interactions from Eq. (3). Unfortunately, this possibility is ruled out due to negative leading order contributions to the sfermion masses, which produce a tachyonic stop [19]. While the coupling of the Higgses to the $\hat{24}_H$ must exist in order to achieve double-triplet splitting, it is important to note that the $H\bar{H}F_{24}$ term can be eliminated by invoking extra fine-tuning on top of the fine-tuning needed for doublet-triplet splitting. We find this possibility unappealing and will not discuss it further.

Since the fields present in the theory cannot be used to transmit SUSY breaking, the simplest approach is to introduce a new singlet field, $\hat{S}$, which couples both to the $SU(5)$ visible sector and to the hidden sector. We assume this singlet does not couple to $H$ and $\bar{H}$ to avoid the tachyonic stop issue mentioned above. Once this field gets a VEV, $\langle \hat{S} \rangle = m_S + \theta^2 F_S$, superpartner masses can be generated. Assuming that SUSY breaking is transmitted through the mass term for the field used to break $SU(5)$ to the SM, we replace $m_{\Sigma} \text{Tr} 24_H^2$ by $\lambda_S \langle \hat{S} \rangle \text{Tr} 24_H^2$. In this case SUSY breaking is mediated through $\Sigma_8$ and $\Sigma_3$, but since these fields do not carry hypercharge, the bino and right-handed charged sleptons, $\tilde{e}_c^i$, remain massless (at the two-loop level) at the messenger scale while running effects would drive the latter mass to tachyonic values. A realistic SUSY spectrum then requires transcending the minimal model by introducing extra representations which can be used as messengers.

We proceed by appealing to neutrino masses for guidance. Explicitly, we compare the possible mechanisms for neutrino masses and their role in gauge mediation. Neutrino masses can be generated through the Type I [21–25], Type II [26–30] or Type III [31–36] seesaw mechanism ($R$-parity violating interactions can also be used to generate neutrino masses but do not provide messenger candidates and we continue by assuming $R$-parity conservation). Now, since the right-handed neutrinos needed for Type I seesaw are SM singlets they cannot generate a realistic superpartner spectrum, leaving Type II and Type III seesaw as the only viable options. Type II seesaw necessitates the introduction of two chiral superfields, $\hat{15}_H$ and $\hat{\bar{15}}_H$, and was shown to produce a realistic spectrum in Ref. [9]. On the other hand Type III seesaw requires

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1. One could avoid this problem adding several 24 representations [20].
2. For the study of flavour and CP violation in non-renormalizable $SU(5)$ models see for example Ref. [37].
introducing only one chiral superfield, a $\hat{24}$, making it more minimal and worthy of study.

Recently, several groups have investigated the Type III seesaw mechanism in the context of $SU(5)$ grand unified theories \cite{32-35}. In order to realize this mechanism one has to introduce a new matter superfield in the adjoint representation:

$$24 = (\hat{\rho}_8, \hat{\rho}_3, \hat{\rho}_{(3,2)}, \hat{\rho}_0) = (8, 1, 0) \bigoplus (1, 3, 0) \bigoplus (3, 2, -5/6) \bigoplus (3, 2, 5/6) \bigoplus (1, 1, 0).$$ \hspace{1cm} (4)

In the context of non-SUSY $SU(5)$, this idea was pursued in a non-renormalizable model \cite{33} and then in a fully renormalizable scenario \cite{34}. This mechanism was studied for the first time in the context of a supersymmetric grand unified theory in Ref. \cite{35}.

In our case the relevant superpotential, which is used to generate neutrino masses through both the Type III and Type I seesaw mechanisms, is given by

$$W_4 = h_1 \hat{\bar{\rho}}_5 \hat{24} \hat{\rho}_5 H + \hat{\bar{\rho}}_5 (h_2 \hat{24} \hat{24} H + h_3 \hat{24} H \hat{24} + h_4 \text{Tr}(\hat{24} \hat{24} H)) \hat{\bar{\rho}}_5 H / M_{Pl}. \hspace{1cm} (5)$$

The mass of the seesaw fields, $\rho_0$ and $\rho_3$, responsible for Type I and Type III seesaw respectively, are computed using the following superpotential

$$W_5 = m \text{Tr} \hat{24}^2 + \lambda \text{Tr} \hat{24}^2 \hat{24} H + O(24 H / M_{Pl}). \hspace{1cm} (6)$$

Once $SU(5)$ is broken, $\langle 24 H \rangle = \text{diag}(2, 2, 2, -3, -3) v_{\Sigma}/\sqrt{30}$, the above superpotential can be used to compute the masses for the fields in the $24$ multiplet:

$$M_{\rho_0} = m - A, \hspace{1cm} (7)$$

$$M_{\rho_3} = m - 3 A, \hspace{1cm} (8)$$

$$M_{\rho_8} = m + 2 A, \hspace{1cm} (9)$$

$$M_{\rho_{(3,2)}} = m - \frac{1}{2} A. \hspace{1cm} (10)$$

Where $A = \lambda v_{\Sigma}/\sqrt{30}$ and we neglect the effect of higher-dimensional operators for simplicity. The neutrino mass matrix is given by

$$M_{ij}^\nu = \frac{c_i c_j}{M_{\rho_0}} + \frac{b_i b_j}{M_{\rho_3}}, \hspace{1cm} (11)$$

where $c_i$ and $b_i$ are a linear combination of the couplings $h_1 - h_4$ in Eq. (5).
and typically, it is assumed that $\rho_3$ and $\rho_0$ are around the “seesaw scale”, $M_{\rho_0}, M_{\rho_3} \approx 10^{11-14}$ GeV. Notice that in this case the neutrino masses are generated through the Type I and Type III seesaw mechanisms and one neutrino remains massless. The spectrum can then have a Normal Hierarchy with $m_1 = 0$ or an Inverted Hierarchy with $m_3 = 0$.

In order to transmit SUSY breaking in this scenario one needs to replace the mass term for the $\hat{\mathbf{24}}$ field in Eq. (6):

$$m \text{ Tr } \hat{\mathbf{24}}^2 \to \lambda_S \hat{S} \text{ Tr } \hat{\mathbf{24}}^2,$$

(14)

so that both the fermion and scalar components of $\hat{\mathbf{24}}$ get a squared mass contribution of $\lambda_S^2 m_S^2$ but the scalars get a further mass squared mixing term, $\lambda_S F_S \hat{\mathbf{24}} \hat{\mathbf{24}}$. The upshot of this is that a SUSY breaking mass difference exists between the scalars and fermions of the $\hat{\mathbf{24}}$:

$$|m_{\hat{\mathbf{24}}}^2 - m_{\hat{\mathbf{24}}}^2| = |\lambda_S F_S|.$$

(15)

This difference is communicated to the visible sector by the messengers through the mass parameter $\Lambda \equiv \lambda F_s/M_{\text{Mess}}$.

We again stress that in this scenario, “$\nu$–GMSB”, by adding only one extra chiral superfield, $\hat{\mathbf{24}}$, we are able to generate neutrino masses in agreement with experiments and have a consistent mechanism for gauge mediation since the components of the $\hat{\mathbf{24}}$ have color, weak and hypercharge charges. Since Type I seesaw cannot generate masses for the superpartners and Type II seesaw needs two chiral superfields $^3$, Type III seesaw provides the simplest framework for gauge mediation in $SU(5)$ grand unified theory via seesaw fields. It is important to emphasize the differences between this scenario and that studied in Ref. [10], where the authors: studied a more involved case with several copies of the 24 field; neglected the very relevant interaction—$\text{Tr } \hat{\mathbf{24}}^2 \hat{\mathbf{24}} H$, which tells us that the seesaw scale is large; did not discuss that neutrino masses are generated through both the Type I and Type III seesaw mechanisms; and did not consider radiative $B$-term generation. In our opinion, these are crucial features of our scenario which deserve attention and we investigate their effects in detail.

$^3$ Notice that the authors in Ref. [9] have more representations since they need $\hat{\mathbf{15}}_H$ and $\hat{\overline{\mathbf{15}}}_H$, and in general their superpotential contains the following terms:

$$W^{II}_4 = Y_{\nu} \hat{\mathbf{5}} \hat{\mathbf{15}}_H \hat{\mathbf{5}} + \eta \hat{X} \text{ Tr } \hat{\mathbf{15}}_H \hat{\mathbf{15}}_H + \mu_1 \hat{\mathbf{5}} \hat{\mathbf{15}}_H \hat{\mathbf{5}} + \mu_2 \hat{\mathbf{5}} \hat{\mathbf{15}}_H \hat{\mathbf{5}},$$

$$W^{II}_5 = \lambda_3 \text{ Tr } \hat{\mathbf{15}}_H \hat{\mathbf{24}}_H \hat{\mathbf{15}}_H + O(24_H/M_{Pl}).$$

(16)

Then, one could say that they have less parameters only when some of the interactions above, which in general are relevant, are neglected. For example, the term $\text{Tr } \hat{\mathbf{15}}_H \hat{\mathbf{24}}_H \hat{\mathbf{15}}_H$ gives a mass splitting between the messengers after $SU(5)$ is broken.
III. $\nu$-GMSB PREDICTIONS

In gauge mediation scenarios it is typically assumed that the messengers are degenerate and therefore all associated with the same contributions, $\Lambda$, to the soft masses. Being in a specific GUT model allows us the advantage of seeing that this not necessarily true. Here the messengers, the $\hat{2}_4$, attain mass splittings from Eq. (6) due their couplings to the $SU(5)$ breaking $\hat{2}_4_{1H}$. The masses are given in Eqs. (7-10), where we take $m \rightarrow \lambda_S m_S$ to allow for gauge mediation. Since these masses differ from each other, each messenger field will have a different $\Lambda$ parameter associated with it: $\Lambda_i$ where $i = (\rho_0, \rho_3, \rho_8, \rho_{(3,2)})$. In this section, we will for convenience reparameterize the mass relations in terms of $M_{\rho_3}$ and $\hat{m} = M_{\rho_8} / M_{\rho_3}$. Also, any phase in Eqs. (7-10) can always be rotated away to yield positive values for each of the masses. Then assuming no relevant phase between $m$ and $A$ leads to three possible cases for this reparameterization:

- **Case I:** $m < A/2$; $M_{\rho_{(3,2)}} = \frac{1}{2} M_{\rho_3} (1 - \hat{m})$ where $0 < \hat{m} < 1$,
- **Case II:** $A/2 < m < 3A$; $M_{\rho_{(3,2)}} = \frac{1}{2} M_{\rho_3} (\hat{m} - 1)$ where $\hat{m} > 1$,
- **Case III:** $m > 3A$; $M_{\rho_{(3,2)}} = \frac{1}{2} M_{\rho_3} (\hat{m} + 1)$ where $\hat{m} > 0$.

For the remainder of the paper we will focus on case III since the range of $\hat{m}$ is the union of cases I and II. Specifically we will consider $0.1 < \hat{m} < 10$ to reduce the fine tuning between the components of the $\hat{2}_4$.

In general, at each seesaw field threshold, the gaugino masses will receive a one-loop contribution, which must be evolved down to the next threshold, modified by the new contribution and evolved again. However, the effect from running between these thresholds is small since the messengers are never separated by more than an order of magnitude. Therefore, one can simply state the gaugino masses as a boundary condition at $M_{\text{Mess}} \equiv M_{\rho_3}$. Computing the gaugino masses at one-loop yields the following results at the messenger scale:

\[
M_3(M_{\text{Mess}}) = a_3 \left( 3\Lambda_{\rho_8} + 2\Lambda_{\rho_{(3,2)}} \right), \tag{18}
\]

\[
M_2(M_{\text{Mess}}) = a_2 \left( 2\Lambda_{\rho_3} + 3\Lambda_{\rho_{(3,2)}} \right), \tag{19}
\]

\[
M_1(M_{\text{Mess}}) = a_1 \frac{5}{4} \Lambda_{\rho_{(3,2)}}, \tag{20}
\]

where $a_i = \alpha_i / 4\pi$ and $\Lambda_i = \lambda_S F_S / M_i$.

Scalar masses are generated at two-loops and can be calculated using the same philosophy discussed for the gauginos. In general, scalar masses will also receive contributions from their Yukawa couplings to the messengers. However, once these become sizable, at higher messenger scales, they lead to low energy lepton number violation. We postpone a study of this effect to a future paper and continue with the assumption that
$M_{\text{Mess}} \ll 10^{14-15}$ GeV. We will use $M_{\text{Mess}} = 10^{11}$ GeV to illustrate the numerical results. This also means that as in minimal models of GMSB, the trilinear $a$-terms and bilinear $B$-term will be zero at the messenger scale but non-zero at the SUSY scale due to RGE effects. As a result, the boundary conditions for the scalar parameters are:

\begin{align*}
    m_\tilde{Q}^2(M_{\text{Mess}}) & = 8 a_3^2 \Lambda_{\rho_8}^2 + 3 a_2^2 \Lambda_{\rho_3}^2 + \left( \frac{16}{3} a_3^2 + \frac{9}{2} a_2^2 + \frac{1}{6} a_1^2 \right) \Lambda_{\rho_{(3,2)}}^2, \\
    m_\tilde{U}^2(M_{\text{Mess}}) & = 8 a_3^2 \Lambda_{\rho_8}^2 + \left( \frac{16}{3} a_3^2 + \frac{8}{3} a_1^2 \right) \Lambda_{\rho_{(3,2)}}^2, \\
    m_\tilde{D}^2(M_{\text{Mess}}) & = 8 a_3^2 \Lambda_{\rho_8}^2 + \left( \frac{16}{3} a_3^2 + \frac{2}{3} a_1^2 \right) \Lambda_{\rho_{(3,2)}}^2, \\
    m_\tilde{L}^2(M_{\text{Mess}}) & = m_{\tilde{H}_u}^2 = m_{\tilde{H}_d}^2 = 3 a_2^2 \Lambda_{\rho_3}^2 + \left( \frac{9}{2} a_2^2 + \frac{3}{2} a_1^2 \right) \Lambda_{\rho_{(3,2)}}^2, \\
    m_\tilde{e}^2(M_{\text{Mess}}) & = 6 a_1^2 \Lambda_{\rho_{(3,2)}}^2, \\
    a_i(M_{\text{Mess}}) & = 0; \quad i = u,d,e, \\
    B(M_{\text{Mess}}) & = 0.
\end{align*}

See the Appendix for our notation. It is well-known that in gauge mediation, the masses of all the generations of a given sfermion type are degenerate since they have the same charges, i.e. $m_{\tilde{Q}_1} = m_{\tilde{Q}_2} = m_{\tilde{Q}_3}$, while Yukawa effects in the running will push the third generation mass below the degenerate masses of the first and second generation. The right-handed components have different masses than the left-handed ones because of their different charges.

Armed with this information we are ready to study the predictions of this model, focusing on case III. Calculations are done by inputting the gauge couplings and fermion masses at the $Z$ mass scale with a guess for $\tan \beta$ and then evolving up to the messenger scale using one-loop renormalization group equations (RGEs). At that scale, the boundary conditions for the soft terms are calculated and those values are then evolved to the SUSY scale using one-loop RGEs. The electroweak symmetry breaking (EWSB) constraints are then used to solve for $\mu$ and $B$. If the $B$ value from the EWSB conditions does not match the one given from the RGEs, a new guess for $\tan \beta$ is used and the process repeats until it converges on a value of $\tan \beta$.

It is important to keep in mind that there are only four input parameters:

\[ \hat{m}, \quad \Lambda \equiv \Lambda_{\rho_3}, \quad M_{\text{Mess}} \equiv M_{\rho_3} \quad \text{and} \quad \text{sign}(\mu), \]

so that validation of this scenario could possibly begin once three superpartner masses are known and the rest of the spectrum can be calculated.
A. Doubly Radiative Electroweak Symmetry Breaking

Before diving into the spectrum, it would be useful to contemplate EWSB and the $\mu/B\mu$ problem. The latter arises in GMSB when generation of the $\mu$ term is linked to SUSY breaking, which usually results in the untenable situation $B \gg \mu$. However, in our approach, $\mu$ is a parameter of the superpotential that arises from doublet-triplet splitting and we do not attempt to link it to SUSY breaking.

It is common in the literature to assume a value for $\tan \beta$ and then use the EWSB equations to solve for $B$, hence indicating an ignorance of the mechanism which generates this term. We see no reason to adopt this approach since $B$ is radiatively generated in these SUSY breaking scenarios (we refer to this as doubly radiative EWSB). Therefore, even though $B$ is very close to zero at the messenger scale, an appropriate value is generated by RGE running from the messenger to the SUSY scale. The EWSB equations can then be used to determine the appropriate value of $\tan \beta$ and $\mu$:

$$B \mu = \frac{\tan \beta}{1 + \tan^2 \beta} \left( 2|\mu|^2 + m_{Hu}^2 + m_{Hd}^2 \right),$$

(28)

$$|\mu|^2 = -\frac{1}{2} M_Z^2 + \frac{m_{Hu}^2 \tan^2 \beta - m_{Hd}^2}{1 - \tan^2 \beta}.$$  

(29)

Satisfying these equations automatically allows for a nontrivial vacuum and guarantees that the potential is bounded from below:

$$(B \mu)^2 > \left( m_{Hd}^2 + |\mu|^2 \right) \left( m_{Hu}^2 + |\mu|^2 \right),$$

(30)

$$2 \ B \mu < m_{Hu}^2 + m_{Hd}^2 + 2|\mu|^2,$$

(31)

respectively.

In typical models of GMSB, where $B(M_{\text{Mess}}) = 0$, such as in Ref. [38], the value of $\tan \beta$ turns out to be large. This is because $\tan \beta$ is inversely related to $B$, which does not run very large. This is not necessarily the case here as can be seen in Fig. 1 which plots $B$ and $\mu$ as a function of $\hat{m}$ for $\Lambda = 50$ TeV and $M_{\text{Mess}} = 10^{11}$ GeV. Solid dashed lines in the upper part of the figure indicates values of constant $\tan \beta$. Solutions to the right (above) of the dots on the $\mu > 0$ and $B > 0$ ($\mu < 0$ and $B < 0$) curves are ruled out by the constraint on the stau mass.

The behavior of $\tan \beta$ is displayed in Fig. 2 for the same values of the input parameters. The wide range of possible $\tan \beta$ values is due to the $\hat{m}$ parameter, which reflects the hierarchy between the colored and non-colored superpartners: as $\hat{m}$ increases, this hierarchy decreases. Typical gauge mediation models have

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4 For a recent phenomenological analysis in this type of scenarios see Ref. [39].
FIG. 1: Values of $B$ and $\mu$ at the SUSY scale versus $\hat{m}$ for $\Lambda = 50$ TeV and $M_{\text{Mess}} = 10^{11}$ GeV. Note that $\mu$ and $B$ are both positive (negative) above the $\hat{m}$ axis (below the $\hat{m}$ axis). The dashed lines represent lines of constant $\tan \beta$ above the $\hat{m}$ axis only. Below this axis, one can compare with Fig. [2] the top (bottom) of the $B < 0$ ellipsoid shape corresponds to $\tan \beta \sim 38$ ($\tan \beta \sim 22$). The $\mu < 0$ curve also forms an ellipsoid but a much thinner one, which appears as a line on this plot. Solutions to the right of the dots on the $\mu > 0$ and $B > 0$ curves are ruled out by the constraint on the stau mass, while the same is true of solutions above the two dots on $\mu < 0$ and $B < 0$ curves.

the same $\Lambda$ for both the colored and non-colored sectors and so correspond to $\hat{m} = 1$ or $\tan \beta \sim 20$ (from Fig [2]). To understand the wider range of $\tan \beta$ values here it is useful to investigate the largest contributions to the $B$-term beta function:

$$
\beta_B \sim 6y_t a_t + 6y_b a_b + \frac{6}{5}g_1^2 M_1 + 6 g_2^2 M_2,
$$

(32)

where the $a$-terms run negative tending to cancel the effects of the gaugino masses, thereby prohibiting $B$ from running too large. However, since the $a$-terms are mostly driven by the gluino mass parameter, decreasing $\hat{m}$ increases the gluino mass and allows the $a$-terms to dominate over the electroweak gaugino masses leading to larger positive $B$ values. This in turn allows for smaller $\tan \beta$ values for small $\hat{m}$ (LEP2 experiments constrain $\tan \beta > 2.4$ [40]).

Increasing $\hat{m}$ allows for two options. The first, if $\mu > 0$, which also implies $B > 0$, is that $\tan \beta$ continues to increase with $\hat{m}$, as one would naively expect. The second is for $\mu, B < 0$. It is due to gaugino
\( \mu > 0 \) and \( \mu < 0 \). The latter is unique in GMSB with doubly radiative EWSB and exists in a small part of the parameter space. When \( \mu > 0 \) the solutions above the dot are ruled out by the lower bound on the stau mass. In the case \( \mu < 0 \) one finds consistent solutions below the two points in the curve.

masses dominating in \( \beta_B \) thereby running \( B \) negative but only when \( \tan \beta \lesssim 40 \), since larger values would increase \( a_b \) in magnitude and allow the \( a \)-terms to dominate once more. The region \( \mu < 0 \) is unusual for models of gauge mediation where the \( B \)-term is generated radiatively. It is interesting to note that the region of \( \mu < 0 \) is allowed only for a small part of the parameter space: \( 3.2 \lesssim \hat{m} \lesssim 4.4 \) and \( 22 \lesssim \tan \beta \lesssim 38 \).

For the sake of brevity, we will focus most of the remaining paper on the \( \mu > 0 \) region noting here that the major difference between these two regions is the value of \( \tan \beta \) which will result in heavier masses for the lightest stau and sbottom in the \( \mu < 0 \) region.

As a final note, notice that EWSB solutions exist only when \( \hat{m} \leq 4.8 \) indicating a deep relationship between high scale physics—the mass splittings in the \( 2\Delta \), \( \hat{m} \)—and the low scale physics—EWSB.
**B. Superpartner Spectrum**

As in any gauge mediation mechanism the lightest supersymmetric particle (LSP) is the gravitino since its mass is given by, \( m_{3/2} \approx F/M_{Pl} \approx 10^2 - 10^4 \) keV, where in our case if we take as messenger scale, \( M_{Mess} \approx 10^{11} \) GeV, \( \sqrt{F} \approx 10^7 - 10^8 \) GeV is the SUSY breaking scale and \( M_{Pl} \sim 10^{18} \) GeV, is the reduced Planck scale. The rest of the spectrum has some distinctive features from the typical gauge mediation due to \( \hat{m} \).

We begin by examining the gaugino mass parameters versus \( \hat{m} \) at the SUSY scale for \( \Lambda = 50 \) TeV and \( M_{Mess} = 10^{11} \) GeV, Fig. 3. This plot reflects the fact that as \( \hat{m} \) increases, the hierarchy between the colored and non-colored sectors decreases thus reducing the ratio \( M_3 : M_2 : M_1 \) from \( 20 : 2 : 1 \) at \( \hat{m} = 0.1 \) to \( 4 : 3 : 1 \) at \( \hat{m} = 4.5 \).

![Gaugino Mass Parameters](image.png)

**FIG. 3:** Gaugino mass parameters at the SUSY scale versus \( \hat{m} \) for \( \Lambda = 50 \) TeV and \( M_{Mess} = 10^{11} \) GeV.

This effect of \( \hat{m} \) is also reflected in Fig. 4 where we see the squark masses drawing closer to the slepton masses as \( \hat{m} \) increases. We see two other interesting features as \( \hat{m} \) increases: \( m_{H_u}^2 \) becomes less negative so that eventually EWSB would not be possible (as was seen in Fig. 1) and that the stau mass parameter eventually becomes tachyonic. To understand the former behavior, examine the largest contributions to the
FIG. 4: Sfermion mass parameters versus $\hat{m}$ for $\Lambda = 50$ TeV, $M_{M_{\text{mess}}} = 10^{11}$ GeV and $\mu > 0$. The actual values plotted are $\text{sign}(m^2_\phi)\sqrt{|m^2_\phi|}$ so that negative values indicate negative mass squared values. The dashed lines correspond to constant values of $\tan \beta$.

$m^2_{H_u}$ beta function:

$$\beta m^2_{H_u} \sim 6|y_t|^2(m^2_{H_u} + m^2_{\tilde{Q}_3} + m^2_{\tilde{t}_c}) - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2. \quad (33)$$

Typically, $m^2_{H_u}$ runs negative due to the product of the large top Yukawa coupling with the stop masses. However, as $\hat{m}$ increases, this product decreases compared to the gaugino masses, eventually $m^2_{H_u}$ does not run negative enough thus spoiling radiative EWSB. In fact Fig. 4 cuts-off when EWSB is no longer possible, at around $\hat{m} = 4.8$ for $\Lambda = 50$ TeV. Again, this result is interesting because it specifies that there cannot be too much splitting in the $\tilde{24}$ multiplet due to the constraints of EWSB. The right-handed stau parameter becomes tachyonic for large $\hat{m}$ because of the large $\tan \beta$ values, which increase the value of $y_\tau$ running $m^2_{\tilde{\tau}}$ negative. This latter feature is more constraining on the parameter space and places the upper bound $\hat{m} \lesssim 2.4$ (for stau masses consistent with LEP 2 bounds, $m_{\tilde{\tau}} > 100$ GeV).

A lower bound on $\hat{m}$ can also be derived if one wishes to limit the amount of fine tuning necessary to satisfy EWSB, Eq. (29). This can be most clearly seen by examining this equation in the limit $\tan \beta \gg 1$

$$|\mu|^2 = -\frac{1}{2}M_Z^2 - m^2_{H_u}. \quad (34)$$
The amount of cancellation needed between $\mu^2$ and $m_{H_u}^2$ to produce $\frac{1}{2}M_Z^2$ is a measure of the necessary fine-tuning and increases with the magnitude of $|m_{H_u}|$ and decreasing $\hat{m}$. In the interest of fine tuning, we restrict $|m_{H_u}| \sim |\mu| < 500$ GeV which when combined with the stau bounds lead us to study the range:

$$1.8 < \hat{m} < 2.4,$$

(35)

![Graph showing physical neutralino, chargino, and gluino masses versus $\hat{m}$ for $\Lambda = 50$ TeV, $M_{\text{mess}} = 10^{11}$ GeV and $\mu > 0$. In this plot we focus on the region of reduced fine-tuning, $1.8 \leq \hat{m} \leq 2.4$.](image)

The physical spectrum for the gauginos plotted versus $\hat{m}$ is shown in Fig. 5. To understand the composition of the neutralinos and charginos first consider Eq. (34) which further reduces to $\mu = |m_{H_u}|$ for $|m_{H_u}|^2 \gg M_Z^2$, typically a good assumption. Since the Higgsino masses are proportional to $\mu$, they are also proportional to $|m_{H_u}|$. Consulting with Figs. 3 and 4 indicates that the neutralinos, from lightest to heaviest are mostly: bino, wino-Higgsino mix, Higgsino and wino-Higgsino mix while the charginos are both wino-Higgsino mixes. The gluino is the heaviest gaugino for the the value of $\hat{m}$ shown.

The physical sfermion spectrum is shown in Fig. 6 with dashed lines of constant $\tan \beta$. In this region of minimal fine-tuning, the mass ratio of squarks to left-handed sleptons—$m_{\tilde{q}} : m_{\tilde{l}_2} \sim 2 : 1$. Furthermore, the Higgs mass is above the LEP 2 lower bound of 114.4 GeV for this range of $\hat{m}$ and the most serious constraint comes from the mass of the lightest stau.
Because gaugino masses go as the dynkin index of the messengers while the sfermion masses are proportional to the square root of the dynkin index, large messengers representations or many copies of messengers lead to gaugino masses larger than the corresponding sfermion masses. This applies in our case where the gluino is heavier than the squarks, the wino heavier than the sleptons and the lightest stau is the next to lightest supersymmetric particle (NLSP). The fact that $\tan \beta$ is large at large $\hat{m}$ is a further contribution making the stau the NLSP. Since the coupling of TeV particles to the LSP gravitino is highly suppressed, the NLSP plays an important role in collider physics. Depending on the lifetime of the stau NLSP, it will produce charged tracks or displayed vertices, both of which would be spectacular signals at the Large Hadron Collider (LHC) [41].

![Graph showing physical sfermion masses versus $\hat{m}$ for $\Lambda = 50$ TeV, $M_{\text{Mess}} = 10^{11}$ GeV and $\mu > 0$ with dashed lines of constant of $\tan \beta$. In this plot we focus on the region of reduced fine-tuning, $1.8 \leq \hat{m} \leq 2.4$.]

FIG. 6: Physical sfermion masses versus $\hat{m}$ for $\Lambda = 50$ TeV, $M_{\text{Mess}} = 10^{11}$ GeV and $\mu > 0$ with dashed lines of constant of $\tan \beta$. In this plot we focus on the region of reduced fine-tuning, $1.8 \leq \hat{m} \leq 2.4$. As a final attempt to familiarize the reader with the features of the spectrum, we list the masses for $\hat{m} = 2$ in Table I, which reflects the features noted thus far. We also give a similar table for $\mu < 0$ and $\hat{m} = 4$, Table II to get a feeling for this part of the parameter space. Since this region has smaller values of $\tan \beta$, the bounds on the lightest stau are satisfied even with $\hat{m}$ larger than the range discussed above and also leads to a less hierarchical spectrum. We also note that our results for $\tan \beta$ are consistent with the constraints coming from $Y_b = Y_t$ unification. See for example Ref. [42].
FIG. 7: The dots indicate the allowed parameter space in the $\hat{m} - \Lambda$ plane given collider constraints on the supersymmetric masses and EWSB for $\mu > 0$ and $M_{\text{Mess}} = 10^{11}$ GeV.

Finally, in order to understand the predictions in the full parameter space we show the allowed range in the $\hat{m} - \Lambda$ plane given collider constraints on the supersymmetric masses and EWSB for $\mu > 0$ and $M_{\text{Mess}} = 10^{11}$ GeV in Fig. 7. The stau is the NLSP in the entire parameter space.

IV. CONSTRAINTS FROM GAUGE COUPLING UNIFICATION AND PROTON DECAY

We study in this section the possible constraints obtained by requiring unification of the gauge couplings when the gaugino and squark masses are determined by gauge mediated SUSY breaking mechanism proposed in this paper. For the general constraints in minimal SUSY $SU(5)$ see Ref. [43]. Let us analyze the case where $M_T = M_V = M_{\text{GUT}}$, and leave $M_{\Sigma_3}$ and $M_{\Sigma_8}$ as free parameters. Solving the RGE’s in Eqs. (43)-(45), we find

$$M_{\text{GUT}} = M_Z \left[ \hat{m}^{-6} \left( \frac{1 + \hat{m}}{2} \right)^{12} M_Z^0 \frac{m_Q^0 m_W^0 m_{\Sigma_3}^0}{M_{\Sigma_3}^0 M_{\Sigma_8}^0} \exp \left[ 2\pi \left( 5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1} \right) (M_Z) \right] \right]^{1/24} \quad (36)$$
TABLE I: Particles mass spectrum at the SUSY scale for \( \tan \beta = 34 \), \( \Lambda = 50 \) TeV, \( M_{\text{Mess}} = 10^{11} \) GeV, \( \hat{m} = 2 \) and \( \mu > 0 \). First and second generation masses are degenerate.

| Particle, Symbol | Mass (GeV) |
|-----------------|------------|
| stop \( \tilde{t}_1, \tilde{t}_2 \) | 718, 953 |
| sbottom \( \tilde{b}_1, \tilde{b}_2 \) | 834, 920 |
| up squarks \( \tilde{u}_1, \tilde{u}_2 \) | 906, 996 |
| down squarks \( \tilde{d}_1, \tilde{d}_2 \) | 900, 999 |
| stau, tau sneutrino \( \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\nu}_\tau \) | 135, 450, 433 |
| selectron, electron sneutrino \( \tilde{e}_1, \tilde{e}_2, \tilde{\nu}_e \) | 209, 450, 443 |
| neutralinos \( \tilde{N}_1, \tilde{N}_2, \tilde{N}_3, \tilde{N}_4 \) | 228, 418, 451, 552 |
| charginos \( \tilde{C}_1, \tilde{C}_2 \) | 416, 552 |
| gluino \( \tilde{g} \) | 1123 |
| Higgses \( m_{A^0}, m_{H^\pm}, m_{H^0}, m_{h^0} \) | 506, 512, 506, 127 |

TABLE II: Particles mass spectrum at the SUSY scale for \( \tan \beta = 22 \), \( \Lambda = 50 \) TeV, \( M_{\text{Mess}} = 10^{11} \) GeV, \( \hat{m} = 4 \) and \( \mu < 0 \). First and second generation masses are degenerate.

| Particle, Symbol | Mass (GeV) |
|-----------------|------------|
| stop \( \tilde{t}_1, \tilde{t}_2 \) | 368, 612 |
| sbottom \( \tilde{b}_1, \tilde{b}_2 \) | 478, 564 |
| up squarks \( \tilde{u}_1, \tilde{u}_2 \) | 498, 610 |
| down squarks \( \tilde{d}_1, \tilde{d}_2 \) | 494, 615 |
| stau, tau sneutrino \( \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\nu}_\tau \) | 106, 369, 359 |
| selectron, electron sneutrino \( \tilde{e}_1, \tilde{e}_2, \tilde{\nu}_e \) | 130, 371, 363 |
| neutralinos \( \tilde{N}_1, \tilde{N}_2, \tilde{N}_3, \tilde{N}_4 \) | 111, 155, 167, 429 |
| charginos \( \tilde{C}_1, \tilde{C}_2 \) | 141, 429 |
| gluino \( \tilde{g} \) | 629 |
| Higgses \( m_{A^0}, m_{H^\pm}, m_{H^0}, m_{h^0} \) | 357, 366, 357, 118 |

Notice that the unification scale does not depend explicitly on the absolute value of the masses of the \( \rho \) multiplet, but on their mass splitting \( \hat{m} = M_{\rho_8}/M_{\rho_3} \). Remember that if \( \hat{m} > 1 \), the \( \rho_3 \) field is the lightest partner of the \( \hat{24} \) representation, and \( \rho_8 \) is the heaviest, while the opposite is true when \( \hat{m} < 1 \). The corresponding gauge coupling at the unification scale is given by...
\[
\alpha_{\text{GUT}}^{-1}(M_{\text{GUT}}) = \frac{1}{24} \left\{ -25\alpha_1^{-1} + 15\alpha_2^{-1} + 34\alpha_3^{-1} \right\}(M_Z)
\]

\[
+ \frac{1}{\pi} \log \left[ \hat{m}^{51} \left( \frac{1 + \hat{m}}{2} \right)^{-6} \frac{M_{\rho_3}^{60} m_Q^{27} m_{\tilde{W}}^6 M_{W}^{10} M_{g}^{34} M_{\Sigma_3}^{15} M_{\Sigma_8}^{51}}{M_{Z}^{194} m_{\tilde{u}_{1c}}^{3/2} m_{\tilde{e}_{1c}}^{15/2}} \right] \right\},
\]

(37)

and contrary to the unification scale it depends on the absolute value of the masses of the $\rho$ multiplet. The unification scale and the gauge coupling at the unification scale are both independent of the Higgsino masses.

**FIG. 8:** Parameter $\Lambda_{\rho_3}$ as a function of the mass splitting $\hat{m}$, and different values of the masses of the $\Sigma_3$ and $\Sigma_8$ fields.

**FIG. 9:** Unification scale as a function of the mass splitting $\hat{m}$, and different values of the masses of the $\Sigma_3$ and $\Sigma_8$ fields.
\( m_{\tilde{H}_u} \) and \( m_{\tilde{H}_d} \). The product of their masses is, however, constrained by unification:

\[
m_{\tilde{H}_u} m_{\tilde{H}_d} = M_Z^2 \left[ \frac{m_{\tilde{\nu}}^2}{2} \right]^{12} \frac{M_Z^{28} m_{\tilde{\nu}}^9 m_{\tilde{e}}^{21} m_{\tilde{u}}^{12} M_\Sigma^{36} \Sigma_\nu}{m_Q^9 m_L^{12} M_W^4 M_{\tilde{H}_u}^4 M_{\tilde{H}_d}^4} \exp \left[ 6\pi \left( 5\alpha_1^{-1} - 11\alpha_2^{-1} + 6\alpha_3^{-1} \right) (M_Z) \right] \right]^{1/8}.
\]

By imposing a lower bound on the product of the Higgsino masses, the latter condition sets, as a function of the \( \rho \) mass splitting \( \hat{m} \), an upper limit on the parameter \( \Lambda_{\rho_3} \). Furthermore, since the lightest sfermion at the messenger scale is \( \tilde{e}^c \), and by using Eq. (27), a lower bound on \( \Lambda_{\rho_3} \) can be obtained from a given value of \( m_{\tilde{e}^c} \), neglecting the running of its mass.

In Fig. 8 we show the allowed values of \( \Lambda_{\rho_3} \) which are compatible with the limits \( m_{\tilde{e}^c} > 100 \text{ GeV} \), with \( M_{\text{MSSM}} = 10^{11} \text{ GeV} \), and \( (m_{\tilde{H}_u} m_{\tilde{H}_d})^{1/2} > 100 \text{ GeV} \). Under these conditions, and for \( M_{\Sigma_3} = M_{\Sigma_8} = M_{\text{GUT}} \), the parameter \( \Lambda_{\rho_3} \) is constrained and can take values only in the range

\[
25 \text{ TeV} < \Lambda_{\rho_3} < 1000 \text{ TeV} ,
\]

which is fairly independent of the messenger scale because the gauge couplings run rather slowly at very high energy scales. The corresponding \( \rho \) mass splitting is constrained to be in the range

\[
1.3 < \hat{m} < 50 .
\]

These limits, however, can be relaxed if the \( \Sigma_3 \) and \( \Sigma_8 \) fields are lighter than the unification scale. Indeed, for \( M_{\Sigma_3} = M_{\Sigma_8} = M_{\text{GUT}} \) the unification scale is of the order of \( 10^{16.1} \text{ GeV} \), which might be in conflict with proton decay if we do not suppress the couplings of the colored triplets mediating proton decay to matter. The unification scale becomes larger, and compatible with proton decay, if these two fields become lighter. This is illustrated in Fig. 9 showing the allowed values of the unification scale as a function of \( \hat{m} \), assuming \( M_{\Sigma_3} = M_{\Sigma_8} \) at or below \( M_{\text{GUT}} \). As it has been discussed in detail in Ref. [12], the lower bound on the mass of the colored triplet mediating proton decay is basically \( M_T > 10^{17} \text{ GeV} \) if no additional suppression mechanism is used. Notice that this is perhaps the simplest solution to suppress proton decay since in this case one does not have mixings between the squarks of the different families. However, since in Eq. (1) we assume the existence the higher-dimensional operators, one can always suppress the dimension five contributions to proton decay using the fact that the couplings of the colored triplets to matter are free parameters in general.

The gauge coupling at the unification scale is represented in Figs. 10(a) and 10(b) for different values of the \( \rho_3 \) mass, and different choices of \( M_{\Sigma_3} \) and \( M_{\Sigma_8} \). It is worth mentioning that GMSB together with unification requires the Higgsino masses and \( m_{\tilde{e}^c} \) to be relatively light.
Here we have seen that suppressing proton decay by pushing up the GUT scale requires the $\Sigma_3$ and $\Sigma_8$ fields to be below the GUT scale, in particular only when these fields are below $10^{14}$ GeV one can achieve unification at $10^{17}$ GeV. Notice that using these results one can find a lower bound on the messenger scale which is $M_{\rho_3} > 10$ TeV, see Fig. 10 However, such low-scale gauge mediation in this context one requires fine-tuning the messenger masses because of the $\text{Tr} \hat{2}_4^2 \hat{2}_4 H$ term, which tells us that the masses of the seesaw fields should be very large.

\[
\begin{align*}
M_{\Sigma_3} &= M_{\Sigma_8} = M_{\text{GUT}}, \\
M_{\rho_3} &= 10^{14} \text{ GeV}
\end{align*}
\]

FIG. 10: Gauge coupling at the unification scale as a function of the mass splitting $\hat{m}$, and of the mass of the $\rho_3$ field for (a) $M_{\Sigma_3} = M_{\Sigma_8} = M_{\text{GUT}}$, and (b) $M_{\Sigma_3} = M_{\Sigma_8} = 10^{14}$ GeV.

V. SUMMARY AND OUTLOOK

We have presented a simple scenario for gauge mediated supersymmetry breaking where the messengers are the fields that generate neutrino masses. We refer to this mechanism as “$\nu$-GMSB”. In this scenario the neutrino masses are generated through the Type I and Type III seesaw mechanisms and in the simplest case where the contributions of Yukawa couplings to soft masses are not considered we find:

- Sparticle and Higgs masses are predicted from only four free parameters: $\hat{m}$ defining the splitting in the $\hat{2}_4$ representation, the messenger scale $M_{\text{Mess}}$, $\lambda = \lambda_S F_S / M_{\text{Mess}}$ and $\text{sign}(\mu)$.

- EWSB is achieved through a doubly radiative mechanism, where the $B$-term is very small at the messenger scale and radiatively generated at the SUSY scale. resulting in a constrained spectrum.
• Imposing “minimal” fine-tuning, $100 \text{ GeV} \leq |\mu|, |m_{H_u}|(M_Z) \leq 500 \text{ GeV}$, EWSB conditions and collider constraints, and $M_{\text{Mess}} \approx 10^{11} \text{ GeV}$, we find small mass splitting between the messengers and the mostly right-handed stau is always the NLSP.

• For $\mu < 0$, $\hat{m}$ and $\tan \beta$ are in a small range leading to a very constrained spectrum. For example, when $\Lambda = 50 \text{ TeV}$ and $M_{\text{Mess}} = 10^{11} \text{ GeV}$, $3.2 \lesssim \hat{m} \lesssim 4.4$ and $22 \lesssim \tan \beta \lesssim 38$.

• LHC signatures include charged tracks as is typical in for GMSB with a stau NLSP. It is well-known, that this allows for the reconstruction of the gaugino and squark masses to determinate the spectrum.

• The lower bound on the messenger scale from the constraint $\alpha_{\text{GUT}} < 1$ is $M_{\rho_3} > 10 \text{ TeV}$.

• In a future publication we plan to study in this model the predictions and/or constrains from rare decays, the baryogenesis and leptogenesis mechanism, and the analysis of the Yukawa coupling contributions to the soft masses.

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APPENDIX: NOTATION AND RGES

In order to set our notation we include the superpotential of the MSSM:

$$W_{\text{MSSM}} = \hat{u}^c y_u \hat{Q} \hat{H}_u + \hat{d}^c y_d \hat{Q} \hat{H}_d + \hat{e}^c y_e \hat{L} \hat{H}_d + \mu \hat{H}_u \hat{H}_d,$$

(41)

where $y_{u,d,e}$ are matrices in family space, and the soft SUSY-breaking Lagrangian is given by

$$L_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} ^\dagger \tilde{W} + M_1 \tilde{B} ^\dagger \tilde{B} + \text{h.c.} \right)$$

$$- \left( \tilde{u}^c a_u \tilde{Q} H_u + \tilde{d}^c a_d \tilde{Q} H_d + \tilde{e}^c a_e \tilde{L} H_d + B \mu H_u H_d + \text{h.c.} \right)$$

$$- \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{u}^c m_{u^c}^2 \tilde{u}^c - \tilde{d}^c m_{d^c}^2 \tilde{d}^c - \tilde{e}^c m_{e^c}^2 \tilde{e}^c$$

$$- m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d,$$

(42)
where $M_3$, $M_2$, and $M_1$ are the gluino, wino, and bino mass terms. The second line in Eq. (42) contains the (scalar) supersymmetry and the scale of unification, and $a_{u,d,e}$ are complex $3 \times 3$ matrices in family space. Here again $m^2_{\tilde{Q}}$, $m^2_{\tilde{L}}$, $m^2_{\tilde{e}}$, $m^2_{\tilde{d}}$, and $m^2_{\tilde{c}}$ are $3 \times 3$ matrices in family space.

The renormalization group of equations (RGEs) for the gauge couplings in this model are given by

$$
\alpha^{-1}_1(M_Z) = \alpha^{-1}_{\text{GUT}} + \frac{1}{2\pi} \left( 4 \ln \frac{M_{\text{GUT}}}{M_Z} + \frac{3}{10} \ln \frac{M_{\text{GUT}}}{m_L} + \frac{3}{5} \ln \frac{M_{\text{GUT}}}{m_{\tilde{e}}} + \frac{1}{10} \ln \frac{M_{\text{GUT}}}{m_{\tilde{Q}}} + \frac{4}{5} \ln \frac{M_{\text{GUT}}}{m_{\tilde{d}}} \\
+ \frac{1}{5} \ln \frac{M_{\text{GUT}}}{m_{\tilde{d}}} + \frac{1}{10} \ln \frac{M_{\text{GUT}}}{m_{H_u}} + \frac{1}{10} \ln \frac{M_{\text{GUT}}}{m_{H_d}} + \frac{1}{5} \ln \frac{M_{\text{GUT}}}{m_{H_d}} \\
+ \frac{1}{5} \ln \frac{M_{\text{GUT}}}{m_{H_u}} - 10 \ln \frac{M_{\text{GUT}}}{M_V} + \frac{2}{5} \ln \frac{M_{\text{GUT}}}{M_T} + 5 \ln \frac{M_{\text{GUT}}}{M_{\rho(3,2)}} \right),
$$

(43)

$$
\alpha^{-1}_2(M_Z) = \alpha^{-1}_{\text{GUT}} + \frac{1}{2\pi} \left( -\frac{20}{6} \ln \frac{M_{\text{GUT}}}{M_Z} + \frac{1}{2} \ln \frac{M_{\text{GUT}}}{m_L} + \frac{3}{2} \ln \frac{M_{\text{GUT}}}{m_{\tilde{Q}}} + \frac{1}{6} \ln \frac{M_{\text{GUT}}}{m_{H_u}} + \frac{1}{6} \ln \frac{M_{\text{GUT}}}{m_{H_d}} \\
+ \frac{1}{3} \ln \frac{M_{\text{GUT}}}{m_{H_d}} + \frac{1}{3} \ln \frac{M_{\text{GUT}}}{m_{H_u}} + \frac{4}{3} \ln \frac{M_{\text{GUT}}}{M_W} - 6 \ln \frac{M_{\text{GUT}}}{M_V} \\
+ 2 \ln \frac{M_{\text{GUT}}}{M_{\Sigma_3}} + 2 \ln \frac{M_{\text{GUT}}}{M_{\rho_3}} + 3 \ln \frac{M_{\text{GUT}}}{M_{\rho(3,2)}} \right),
$$

(44)

$$
\alpha^{-1}_3(M_Z) = \alpha^{-1}_{\text{GUT}} + \frac{1}{2\pi} \left( -7 \ln \frac{M_{\text{GUT}}}{M_Z} + \ln \frac{M_{\text{GUT}}}{m_{\tilde{Q}}} + \frac{1}{2} \ln \frac{M_{\text{GUT}}}{m_{\tilde{e}}} + \frac{1}{2} \ln \frac{M_{\text{GUT}}}{m_{\tilde{d}}} \\
+ 2 \ln \frac{M_{\text{GUT}}}{M_{\tilde{g}}} - 4 \ln \frac{M_{\text{GUT}}}{M_V} + \ln \frac{M_{\text{GUT}}}{M_T} + 3 \ln \frac{M_{\text{GUT}}}{M_{\Sigma_8}} \\
+ 2 \ln \frac{M_{\text{GUT}}}{M_{\rho(3,2)}} + 3 \ln \frac{M_{\text{GUT}}}{M_{\rho_8}} \right).
$$

(45)

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