Dissipative Transport of a Bose-Einstein Condensate

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We investigate the effects of impurities, either correlated disorder or a single Gaussian defect, on the collective dipole motion of a Bose-Einstein condensate of $^7$Li in an optical trap. We find that this motion is damped at a rate dependent on the impurity strength, condensate-center-of-mass velocity, and interatomic interactions. Damping in the Thomas-Fermi regime depends universally on the disordered potential strength scaled to the condensate chemical potential and the condensate velocity scaled to the speed of sound. The damping rate is comparatively small in the weakly interacting regime, and in this case, is accompanied by strong condensate fragmentation. In situ and time-of-flight images of the atomic cloud provide evidence that this fragmentation is driven by dark soliton formation.

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I. INTRODUCTION

The creation of Bose-Einstein condensates (BECs) of ultracold atomic gases \cite{1--3} has enabled investigations of some of the most fundamental concepts of condensed matter physics \cite{4}. One of the most fruitful avenues of research has involved the use of BECs to probe the nature of superfluidity itself. Early studies led to observations of the critical velocity for the onset of dissipation \cite{5--7} and quantized vortices \cite{8, 11}.

Recently, there has been much interest in using BECs to emulate disordered superfluids (c.f. \cite{12, 13}). Results from such experiments have wide ranging implications, from the transport of superfluid He in porous media \cite{14} to the motion of atomic BECs in microchip traps or matter waveguides \cite{15--18}. Of particular interest is how disorder can disrupt, or even destroy, superfluidity. Due to their exquisite controllability, atomic BECs are ideal physical systems with which to systematically study the interplay between superfluidity, disorder, and interatomic interactions.

In this paper, we report measurements of the dissipation of the superfluid flow of an elongated BEC subject to either a disordered potential or a single Gaussian defect. We characterize the superfluid nature of the harmonically trapped cloud through detailed measurements of the velocity dependent damping of the collective dipole mode. We use a BEC of $^7$Li in the $|F = 1, m_F = 1\rangle$ internal state, where the interactions may be tuned via a wide Feshbach resonance located at 737 G \cite{19--21}. This resonance includes a shallow zero-crossing that enables the Feshbach resonance located at 737 G \cite{19--21}. This resonance includes a shallow zero-crossing that enables the

\[ v_L = \min \frac{\epsilon(p)}{p}, \] (1)

where $\epsilon(p)$ is the energy of an elementary excitation of momentum $p$. For the case of a weakly interacting BEC with uniform density $n$, Bogoliubov theory gives the excitation energy as \cite{20}

\[ \epsilon(p) = \sqrt{\left( \frac{p^2}{2m} \right)^2 + c^2 p^2}, \] (2)

where $m$ is the atomic mass and $c$ is the bulk speed of sound. For small $p$, this spectrum reduces to the well known relation $\epsilon(p) = cp$ describing phonon excitations with

\[ c = \sqrt{nU/m}, \] (3)

where $U = 4\pi\hbar^2a/m$. Application of Eq. \[ 1 \] gives $v_L = c$, implying that only supersonic flow can dissipate energy through the creation of elementary excitations; conversely, if the flow is subsonic, excitations are energetically forbidden, and the flow is superfluid. Application of Eq. \[ 1 \] to the case of a non-interacting condensate implies
that $v_L = 0$, suggesting that superfluidity cannot exist in an ideal gas.

The dynamics of highly elongated BECs can be accurately modeled using an effective one-dimensional (1D) nonlinear Schrödinger equation (NLSE) \[23\]. In such a treatment, one starts from the 3D Gross-Pitaevskii equation (GPE), and integrates out the radial dimension. The effect of this integration is a reduction in $c$ relative to Eq. \(5\) due to the average over the nonuniform radial density. For the case of a harmonically trapped BEC in the Thomas-Fermi regime, the bulk density $n$ is replaced with the average density $n_0/2$. Therefore, the speed of sound becomes

$$c_0 = \sqrt{\frac{n_0 U}{2m}}$$ \(4\)

A theoretical description of an elongated BEC beyond the standard 1D NLSE leads to a reduction (on the order of 10%) to the speed of sound relative to Eq. \(9\) \[31\]. In addition, the spectrum of axially propagating excitations in a cylindrical BEC can differ dramatically from Eq. \(2\) when $\mu \gg \hbar \omega_r$, where $\mu$ is the chemical potential, leading to an additional reduction in $v_L$ \[32\]. The highest $\mu$ condensates created in our system have $\mu/\hbar \omega_r \sim 13$, resulting in a predicted 20% reduction \[32\].

When attempting to explain the onset of dissipation in any particular experimental situation, care must be taken to apply Landau’s criterion locally, by using the local density $n(r = 0, z)$ instead of $n_0 \equiv n(r = 0, z = 0)$ in Eq. \(9\) \[33\]. For arbitrary trapping potentials, excitations will be nucleated first in regions of low density where the local speed of sound is small, and the critical velocity is reduced relative to the bulk. As a consequence of this effect, experimentally observed critical velocities are often much lower than the bulk speed of sound \[34\].

The remainder of this paper is organized as follows: In Sec. II we describe our experimental methods for creating a BEC in either a disordered harmonic potential or a harmonic potential with a single Gaussian defect; in Secs. III and IV we discuss our results for the induced dissipation for these two scenarios, where both the 3D Thomas-Fermi and the quasi-1D weakly interacting regimes are discussed for each case. We conclude in Sec. V with a discussion relating the similarities and differences between dissipation in the two types of potentials, and directions for future studies.

II. EXPERIMENTAL METHOD

We create a BEC of $^7\text{Li}$ in a highly elongated, cylindrically symmetric, hybrid magnetic-optical dipole trap \[21\], [34] with radial and axial trapping frequencies in the ranges of $\omega_r/(2\pi) \sim 220-460\text{ Hz}$ and $\omega_z/(2\pi) \sim 4-5.5\text{ Hz}$, respectively. The radial confinement is dominated by the optical trapping potential formed by a single focused laser beam with wavelength 1030 nm and a $1/e^2$ Gaussian radius of 33 $\mu$m, while the axial confinement is dominated by an adjustable, harmonically confining magnetic field. A set of Helmholtz coils provides a uniform bias field along the long ($z$) axis of the trap, allowing for the tuning of $a$ via a Feshbach resonance at 737 G \[19\]–[21]. The BEC is created at a field of 717 G where $a$ is positive and large enough (\approx 200 a_0) to allow for efficient evaporative cooling in the optical trap, but small enough to avoid substantial three-body losses. At this field, the trap lifetime is limited to $\sim 10\text{ s}$ due to three-body recombination with a loss coefficient of $L_3 \sim 10^{-26}\text{ cm}^6/\text{s}$. After evaporation, the BEC has no discernible thermal component from which we estimate that the temperature $T < 0.5 T_C$, where $T_C$ is the BEC transition temperature. The bias field is then ramped over a timescale on the order of seconds to achieve the desired value for $a$.

We excite the collective dipole mode of the condensate by pulsing on an axially oriented magnetic gradient, thereby abruptly shifting the center of the harmonic trap. After 1/4 of an oscillation period, the condensate is at the peak of an oscillation and we abruptly switch on either a disordered potential with an extent exceeding the oscillation amplitude of the condensate, or a single, narrow Gaussian defect located near the trap center. By varying the duration of the gradient pulse we precisely vary the amplitude $A$ of the oscillation, and therefore the initial peak velocity $v_0$ of the condensate center of mass, where $v_0 = A \omega_z$. At various times thereafter we image the cloud to track the center of mass location as well as the shape of the density distribution. We investigate the dependence of the damped dipole motion on $v_0$, the strength of the disordered potential or single Gaussian defect, and on the value of $a$.

The disordered potential is an optical speckle pattern created by passing a laser beam through a diffuser plate in a manner similar to previous studies \[34\]–[36]. This beam is directed perpendicular to the trap $z$-axis. Figure 1 shows a characteristic intensity slice of the disorder. The disorder speckle size $\sigma_D$ is defined to be the $1/e^2$ radius of a Gaussian fit to the autocorrelation of the in-
tensity pattern and is measured to be $\sigma_D = 5.5 \mu m$. The beam has been cylindrically focused such that in the radial direction the speckle size is much larger than the radial Thomas-Fermi radius $\sim 10 \mu m$, making the disorder effectively 1D. We have verified that the intensity distribution of the disorder follows a decaying exponential $P(I) = (I/I_0)^{-1} e^{-I/I_0}$, as expected for fully developed speckle \[30\]. The average value of the speckle intensity $\langle I \rangle$ determines the disorder strength through the relation $V_D = \hbar \Gamma^2 \langle I \rangle / (4I_{sat}\Delta)$, where the transition linewidth $\Gamma = (2\pi) 5.9$ MHz and the saturation intensity $I_{sat} = 5.1$ mW/cm$^2$. The detuning from the $^7$Li $2S \rightarrow 2P$ transition $\Delta = (2\pi) 300$ GHz, producing a repulsive disorder potential. For the strongest disorder used in these studies, off-resonant scattering from the disorder occurs at a rate of $\sim 0.1 s^{-1}$. The statistical properties of the speckle pattern are measured by direct imaging with a CCD camera before the optical system is installed onto the experimental apparatus.

A cylindrically focused laser beam is used for the studies involving a single Gaussian defect. This beam has a Gaussian intensity distribution $I(z,r) = I_0 e^{-2(z^2/w_z^2+r^2/w_r^2)}$, with beam waists $w_r = 5 \text{mm}$ and $w_z = 12 \mu m$. The radial size of the defect $w_r$ is much larger than $R_{TF}$, ensuring that flow around the defect is suppressed. We conduct experiments using both a repulsive (blue detuned) and an attractive (red detuned) defect with $|\Delta| = 300$ GHz.

We adjust the healing length through an approximate range $0.5 \mu m < \xi < 20 \mu m$ by tuning $a$. Thus, a wide range of values are achievable for the relevant dimensionless quantities, $0.1 < \xi/\sigma_D < 3.6$ and $0.04 < \xi/w_z < 1.7$.

### III. DISORDER INDUCED DISSIPATION

#### A. Thomas-Fermi Regime

Figure 2 shows the position of the center of a condensate at various times during a dipole oscillation in a disordered potential. The dipole oscillation is initiated by a kick that produces an initial peak velocity of $v_0 = 20 \text{mm/s}$ when the condensate passes through the center of the trap. For this data, the condensate begins its motion well into the supersonic regime with $v_0 \sim 4 \mu m/s$. The resulting oscillation is characterized by a time-dependent damping, suggesting that the damping depends on $v_0$. The damping rate is initially small, goes through a maximum after about 3.8 s, and then diminishes at later times. We fit 4-period sections of the data in Fig. 2 to the form of a damped harmonic oscillator:

$$z(t) = A e^{-\beta t} \cos (\omega t + \phi), \quad (5)$$

where $\omega = (\omega_z^2 - \beta^2)^{1/2}$. The peak velocity $v_0$ is then computed from the fitted $A$ for each data subset to obtain the damping coefficient $\beta$ as a function of $v_0$, with the results shown in Fig. 3. The damping monotonically

![Figure 2](image-url) (Color online) Damping of a condensate initially traveling supersonically through a disordered potential with $V_D/\hbar = 280$ Hz. The center of the BEC (circles) is extracted from a Thomas-Fermi fit to the radially integrated column density (the “axial density”). The thick lines tracing the amplitude are phenomenological guides to the eye. The initial amplitude is $A = 0.6 \text{mm}$ yielding an initial peak velocity of $v_0 = 20 \text{mm/s}$. For this data, $\omega_x = (2\pi) 5.5$ Hz, $\omega_y = (2\pi) 260$ Hz, $a = 25 \sigma_0$, and $\mu = 7/2 m \omega_z^2 R_{TF}^2 = \hbar (1.1 \text{kHz})$, where $R_{TF}$ is the axial Thomas-Fermi radius. In addition, $c_0 = 5.6 \text{mm/s}$, $\xi = 0.8 \mu m$, and $\xi/\sigma_D = 0.2$. The insets show details of the oscillation at early and late times.

![Figure 3](image-url) (Color online) Velocity dependent damping. Results of fitting the data of Fig. 2 to Eq. 5 using a traveling 4-period window. The peak velocity $v_0$ is obtained from $v_0 = A \omega_z$. The solid line is a square-root function convolved with an exponential decay and is meant as a guide to the eye. The inset shows the same data on a semi-log plot, emphasizing the nearly exponential decay of $\beta/\omega_z$ for large $v_0/c_0$. Vertical error bars correspond to the range in $\beta$ for which $\Delta \chi^2 = 1$ for the fit to Eq. 5 while simultaneously adjusting $A$, $\beta$, and $\phi$ to minimize $\chi^2$. Horizontal error bars are determined using an identical process for $A$ in Eq. 5 and are typically $\sim 15\%$. The effects of systematic uncertainty in imaging magnification and variations in $N$ are $\sim 10\%$ in the horizontal axis and $\sim 5\%$ in the vertical axis, these are not included in the displayed error bars.
increases for small \(v_0\), peaking near \(v_0 \sim 1.1c_0\), followed by a nearly exponential decay of \(\beta\) for \(v_0 > c_0\).

A perturbative theoretical treatment has produced a closed form solution for the velocity-dependent damping, resulting in good quantitative agreement with our measurements \([40]\). For weak disorder the qualitative behavior shown in Fig. 3 can be understood through a local Landau critical velocity argument. At low velocities, Bogoliubov quasiparticles are only created within a thin shell at the surface of the condensate, where the low density leads to a low local speed of sound, and therefore a low local \(v_L\). As the velocity of the condensate increases, a larger condensate volume can support excitations because a larger fraction of the atoms violate the local Landau criterion. The maximum damping occurs near the point where the velocity of the BEC reaches the peak speed of sound \(c_0\) in the condensate. At even larger velocities the excitation volume cannot increase further, but the Bogoliubov density of states decreases, resulting in the observed exponential decrease of the damping.

Except for the absence of a critical velocity, the qualitative behavior of the velocity dependent damping shown in Fig. 3 is remarkably similar to that predicted by 1D NLSE simulations of a uniform, repulsive BEC in the presence of an oscillating Gaussian obstacle \([41, 42]\). In these simulations, above a certain impurity strength-dependent critical velocity, the impurity moving at a velocity \(v\) deposits energy into the BEC in the form of density fluctuations. The average rate of condensate energy growth \(\langle dE/dt \rangle\) increases nearly linearly with \(v\), to a peak at \(v \sim c\) as the defect excites dark solitons and linear sound waves. As the velocity of the defect is increased further, the density fluctuations decrease significantly, accompanied by an exponential decrease of \(\langle dE/dt \rangle\), similar to our experimental observations. In contrast to a single impurity in a uniform condensate, a defect is always present in a low density region of a condensate in a disordered harmonic trap. Consequently, \(v_0\) is always greater than the local speed of sound at the edge of the condensate and excitations are always present. Previous experimental \([44, 45, 46]\) and numerical \([47]\) studies of the damping of collective modes and the damping of Bloch oscillations in a disordered lattice potential \([48, 46]\) have found qualitatively similar results.

Figure 4 shows \(\text{in situ}\) polarization phase-contrast images of the data shown in Fig. 2 at various times. The images are nearly equally spaced in time between the time labels.

**FIG. 4.** Characteristic \(\text{in situ}\) polarization phase-contrast images of the data shown in Fig. 2 at various times. The images are nearly equally spaced in time between the time labels.

**FIG. 5.** Generation of a non-condensed component. (a) Squares show the center of the Thomas-Fermi (condensed) component and circles show the center of the Gaussian (non-condensed) component. The Gaussian center trails behind the Thomas-Fermi center and has a smaller amplitude of oscillation. Within experimental uncertainty, \(\omega_s = (2\pi)\, 5.1(2)\, \text{Hz}\) for both components. For this data, \(a = 200\, a_0\), \(N = 3 \times 10^5\), \(\mu/h = 1.8\, \text{kHz}\), \(V_D/\mu = 0.22\), \(v_0 = 28\, \text{mm/s}\), \(c_0 = 7.2\, \text{mm/s}\), and \(\omega_r = (2\pi)\, 220\, \text{Hz}\). (b–d) Axial density distributions with bimodal fits (solid lines) and a single component Thomas-Fermi fit (dashed lines) at various times during the oscillation: (b) 28 ms, (c) 100 ms, (d) 190 ms. The condensates in (b) and (d) are traveling in the positive direction whereas the condensate in (c) is traveling in the negative direction.
temperature was high enough that damping occurred due to the interaction between a BEC and a thermal component. In contrast to those results, we observe that the dipole oscillation is undamped in the absence of the disordered potential. Furthermore, there is no observable heating due to the quick switch on of the disorder. In our experiment, therefore, the presence of the non-condensed component seems to be linked to the motion of the BEC in the disordered potential. A recent numerical simulation using a truncated Wigner method predicts the emission of incoherent atoms from a BEC moving supersonically through a disordered potential \[ \text{[14]}, \] consistent with our observations.

We have investigated this effect in further detail using in situ absorption imaging, which allows for determination of the low density non-condensed wings of the distributions. Figure 6 shows that by fitting the cloud to a bimodal Thomas-Fermi plus Gaussian profile, a phase difference of \( \Delta \phi = 0.23 \) between the condensed and non-condensed cloud centers is found. Note that the interaction strength is different for this data than that shown in Figs. 2–4.

We have systematically investigated the dependence of \( \beta \) on the disorder strength \( V_D/\mu \). Figure 7 shows the normalized damping parameter \( \beta/\omega_z \) plotted against the normalized disorder strength \( V_D/\mu \), where \( \mu \) is the chemical potential of the condensate prior to the kick and before the disorder is switched on. We find the data fits well to a power law

\[
\frac{\beta}{\omega_z} \propto \left( \frac{V_D}{\mu} \right)^p,
\]

for all measured velocities. The precise value of \( p \), however, depends weakly on \( v_0/c_0 \) across the range of velocities \( 0 < v_0/c_0 < 5 \), with a mean value of \( p = 2.1(5) \) (see Fig. 8 inset).

Figure 8 presents the measured values of \( \beta \) as a function of both \( V_D \) and \( v_0 \). As expected, a vertical trace through this plot shows a qualitative similarity to Fig. 7. We observe two distinct regimes of reduced damping: one where \( v_0/c_0 \ll 1 \) and the other when \( v_0/c_0 \gg 1 \), with the damping reaching a maximum at \( v_0 \sim c_0 \). A numerical simulation using an effective 1D NLSE has produced qualitatively similar results [14].

### B. Variation with Interaction Strength

We observe nearly universal behavior for \( \beta \) as a function of both \( V_D/\mu \), and \( v_0/c_0 \) for BECs in the Thomas-Fermi regime. As already shown in Fig. 6 \( \beta \propto (V_D/\mu)^2 \) for condensates with \( \mu \) differing by a factor of 3. Shown
Approaching the strongly damped regime near \(v/c\), consequently, the system displays a strong dependence of \(\beta\) from the 3D to the quasi-1D regime where one can gain a qualitative understanding of this behavior going from high to low \(\beta\) vs. \(v/c\), with \(\beta\) when \(\mu\) and \(v_0/c_0\) differ nearly a factor of 5, showing that while the general shape of the damping curve is universal, the magnitude of the damping is not.

An investigation of the effect of interatomic interactions on the peak damping \((v_0/c_0 \approx 1)\) at fixed \(V_D/\mu\) is shown in Fig. 8. We find that \(\beta\) scales linearly with \(a\), going to zero with decreasing interactions, consistent with the disappearance of the low energy phonon portion of the excitation spectrum as \(U \rightarrow 0\).

The elongated confinement geometry in our system facilitates the investigation of the dimensional crossover from the 3D to the quasi-1D regime where \(\mu \ll \hbar \omega_r\), as shown in Fig. 9. Shown in Fig. 10 are measurements of \(\beta\) vs. \(\omega_r\) at constant \(V_D\) and \(v_0\). When \(\mu > \hbar \omega_r\) (to the right of the vertical dashed line) and \(v_0\) is comparable to, or less than \(c_0\) (as is the case for the data shown as open and closed circles), we find \(\beta \propto \mu^{-1.4}\). By reference to Fig. 4 one can gains a qualitative understanding of this behavior going from high to low \(\mu\): starting subsonically (open and filled circles), the system travels along a path from the weakly damped regime (lower left corner of Fig. 7) towards the regime of strong damping (middle right region). As \(\mu\) decreases, the quantities \(V_D/\mu\) and \(v_0/c_0\) increase correspondingly, and the system follows a path which crosses several contours of constant \(\beta\) while approaching the strongly damped regime near \(v/c \approx 1\). Consequently, the system displays a strong dependence of \(\beta\) on \(\mu\). Blue squares depict a different situation where the system is supersonic for all \(\mu\) investigated. For large \(\mu\) the system occupies a point in Fig. 4 with \(v_0/c_0 > 1\) and \(V_D/\mu < 1\). As \(\mu\) decreases, the system follows a diagonal path, roughly tracing a contour line of constant \(\beta\), moving into the regime of \(v_0/c_0 \gg 1\) and \(V_D/\mu \gg 1\).
the collective dipole mode brought on by extensive fragmentation. Because $V_D > \mu$, it is perhaps not surprising that the condensate quickly fragments. While the center of mass of the cloud damps after about 5 oscillation periods, examination of shot-to-shot differences in the damped density distributions reveal that the position of the fragments are highly non-repeatable, suggesting that some fragments remain in motion. This residual motion is consistent with the long thermalization time expected from weak two-body interactions. It is interesting to note that the maximum single particle kinetic energy, $E_K = \frac{1}{2}m\omega_z^2A^2 = h(295 \text{ Hz})$, is 2.8 times larger than the average height of the disordered potential. The observed dephasing is therefore consistent with the expected behavior of a gas of non-interacting particles interacting with a disordered potential where the disorder strength is smaller than the kinetic energy of the individual particles.

IV. DISSIPATION INDUCED BY A SINGLE GAUSSIAN DEFECT

A. Thomas-Fermi Regime

In an effort to better understand the mechanisms responsible for the damping by disorder, we have investigated the dissipation induced by a single Gaussian defect. The defect potential is described by $V(z) = V_D e^{-z^2/w_z^2}$, where $w_z = 12 \mu m$. The static effect of either an attractive or repulsive defect on a repulsively interacting BEC in the Thomas-Fermi regime is shown in Fig. 12.

As expected, the attractive defect leads to an increase of the density in the region of the defect, accompanied by a small decrease of the density in the wings of the distribution, while the opposite is true for a repulsive defect.

The dynamical distributions can differ dramatically from the static case, as shown in Fig. 13 where in situ axial densities are displayed for various times throughout the dipole oscillation. In the following discussion we refer to the upstream side of the condensate as the portion of the BEC that reaches the barrier after the leading or downstream portion. The interaction of the BEC with the repulsive defect, shown in Fig. 13(b), produces a deep downstream density rarefaction as well as a large upstream density compression bearing a qualitative similarity to a shock wave. Similar structures have been predicted in effective 1D theoretical treatments and interpreted as upstream and downstream dispersive shocks. In contrast, the interaction of the BEC with the attractive defect, shown in Fig. 13(a), produces no such shock waves. However, the cloud is slightly compressed near the defect simply due to the attractive defect potential. Because $v_0 > c_0$, phonon excitations cannot be emitted in the upstream direction as they would have to propagate faster than the speed of sound. Close inspection of Fig. 13(a) reveals minimal density modulation of

FIG. 11. (Color online) Damping of a nearly non-interacting gas. For this data $a = 0.4 \, a_0$, $N = 2 \times 10^5$, $\mu/h = 26 \text{ Hz}$, $c = 1.2 \, \text{mm/s}$, $V_D = 4 \mu$, $\omega_z = (2\pi) \, 240 \text{ Hz}$, and $\omega_m = (2\pi) \, 5.3 \text{ Hz}$. (a) Center of mass position (circles) and radius (squares) of the condensate as a function of time. (b) Axial density traces at various times in the oscillation: (b) 40 ms (c) 100 ms, (d) 960 ms. After two full oscillations, the cloud has fragmented and spread to a size comparable with the initial oscillation amplitude.
which is consistent with the density fluctuations visible downstream propagating dark solitons in addition to an upstream side, while more modulation is evident on the downstream side.

Several 1D theoretical studies predict the formation of downstream propagating dark solitons in addition to an upstream dispersive shock as a repulsive defect is supersonically swept through a condensate [30, 42, 44, 52–56], which is consistent with the density fluctuations visible in Fig. 13. However, the size of the dark solitons will be on the order of the healing length, $\xi = 0.5 \mu m$ for these condensates, which is a factor of 6 smaller than our imaging resolution.

Figure 14 shows time-of-flight images of the BEC oscillating in the presence of a single repulsive defect. In contrast to the in situ images of Fig. 13(b), after time-of-flight additional structures emerge which were not previously visible. These structures are consistent with dark solitons that form from short length scale in situ phase fluctuations that map onto larger scale density modulations after time-of-flight. However, in situ phase fluctuations may also arise from thermal excitations in highly elongated BECs, and these can also manifest as density fluctuations after time-of-flight [57]. Close inspection of Fig. 14 reveals that deep density modulations are present only in the downstream portion of the BEC (after the first pass through the defect), consistent with the dark soliton interpretation. Similar density fluctuations have also been interpreted as dark solitons in an experiment using a moving defect and a stationary BEC [58].

We have measured $\beta$ as a function of both $V_D$ and $v_0$, with characteristic results presented in Fig. 14. Contrary to what was observed for a disordered potential, we observe a critical velocity $v_c$ below which the motion is undamped, for both the attractive and repulsive defects. We find that the peak damping for an attractive defect is significantly weaker than for a repulsive one. Figure 16 presents measurement results of $\beta$ as a function of both $V_D$ and $v_0$. For an attractive defect, we find that $v_c/c_0 \approx 0.7$ with $v_c$ depending only weakly on $V_D$. However, for a moderately strong repulsive defect, $v_c/c_0$ occurs significantly below 1 and depends strongly on $V_D$. For both attractive and repulsive defects $v_c$ tends to $c_0$ as $|V_D/\mu|$ is reduced to zero.

Once again, a model based on a local Landau criterion is sufficient to explain the dependence of $v_c$ on $V_D$. For simplicity, consider a uniform density flow impinging on
Note the difference in scale between damping induced by an attractive impurity at low velocities as well as undamped motion at large velocities. Both types of impurities show critical behavior. Coordinates of the black squares are the values of $\mu/h = 2 \text{kHz}$. (b) Repulsive defect with (squares) $V_D/\mu = 0.2$ and other parameters as stated in Fig. 13(a) except with $N = 8 \times 10^5$ and $\mu/h = 2 \text{kHz}$. (b) Repulsive defect with (squares) $V_D/\mu = 0.4$ or (circles) $V_D/\mu = 0.2$ and other parameters as stated in Fig. 13(b).

We quantify this picture, in the case of a repulsive defect, by applying the local Landau criterion at the instant the center of the BEC crosses the defect. In Ref. [44] the authors used an effective 1D NLSE in the high density regime to determine the locus of points where the local condensate velocity $v(z)$ is equal to the local speed of sound $c(z)$; this defines the curve

$$\frac{v_c}{\omega_b} = \left( 1 - \frac{V_D}{\mu} \right)^{5/2}, V_D > 0,$$

where $V_D/\mu = \delta n_0/n_0$ is the fractional change in the peak density at the peak of the repulsive defect. When $v_0/\omega_b < 1$, we can ignore effects of the axial Thomas-Fermi profile of the condensate because $A < R_{TF}$ for our trap, where $R_{TF}$ is the axial Thomas-Fermi radius. Equation (7) is plotted in Fig. 16 when $V_D > 0$ and is found to agree with the measured $v_c$ for the range of $V_D$ explored experimentally. Therefore, the observed reduction of the critical velocity below $c_0$ is consistent with the local Landau critical velocity without invoking more exotic mechanisms, such as vortex nucleation. This is in contrast with several experiments involving BECs in less elongated configurations $[6, 7]$, as well as in superfluid $^4$He where nucleation of vortex lines and rings can result in $v_c < v_L [23]$.

In the case of the attractive defect, the density, and therefore $c(z)$, is enhanced at the location of the defect and reduced only slightly elsewhere. We find that the reduction in density in the bulk due to the enhancement at the defect is less than 1% for the strongest barriers used, leading to an essentially unperturbed speed of sound in the bulk. The ratio of the local fluid velocity to the local speed of sound can then be found by considering only the bare Thomas-Fermi profile, and is given by

$$\frac{\psi(z)}{c(z)} = \frac{\omega_b}{\omega_c} \left( 1 - \frac{z^2}{A^2} \right)^{1/2},$$

where, using Eq. (4) $v_0/\omega_b = 2A/R_{TF}$. If $2A < R_{TF}$ then $v_0/\omega_b = 1$ and the local Landau criterion is satisfied everywhere inside the condensate:

$$\frac{v_c}{\omega_b} = 1, V_D < 0,$$

implying that $v_c$ is independent of $V_D$. Our measurements, however, show that $v_c$ depends weakly on $V_D$ with

\begin{figure}[h]
\centering
\includegraphics{fig15.png}
\caption{(Color online) Velocity dependence of $\beta$ induced by a single Gaussian defect. (a) Attractive defect with (squares) $V_D/\mu = -0.8$ or (circles) $V_D/\mu = -0.3$ and other parameters as stated in Fig. 13(a) except with $N = 8 \times 10^5$ and $\mu/h = 2 \text{kHz}$. (b) Repulsive defect with (squares) $V_D/\mu = 0.4$ or (circles) $V_D/\mu = 0.2$ and other parameters as stated in Fig. 13(b). Both types of impurities show critical behavior at low velocities as well as undamped motion at large $v_0/\omega_b$. Note the difference in scale between damping induced by an attractive versus a repulsive impurity. Vertical and horizontal error bars are as described in Fig. 14.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics{fig16.png}
\caption{(Color) Transport regimes of a BEC traveling through a harmonic potential with a central Gaussian defect. Coordinates of the black squares are the values of $V_D/\mu$ and $v_0/\omega_b$ for the data sets used to extract $\beta/\omega_b$ from a fit to Eq. 4. The color map (and contour lines) for $\beta/\omega_b$ is derived from an interpolation using the measured results. Dashed white lines show the local Landau critical velocity as given by Eqs. 7 and 9. The attractive and repulsive cases are qualitatively similar: superfluidity for $v_0/\omega_b \ll 1$, increased damping as $v_0/\omega_b \rightarrow 1$, and reduced damping for $v_0/\omega_b \gg 1$. Damping induced by an attractive impurity is an order of magnitude weaker than for a repulsive one. Data with $V_D < 0$ and $V_D > 0$ correspond to parameters in Figs. 13(a) and (b), respectively.}
\end{figure}
\(v_c/c_0 \to 1\) only in the weak impurity limit. Our experimental results are consistent with numerical simulations using a 1D NLSE \[44\] for which the local Landau criterion accurately describes the repulsive impurity case, but slightly overestimates \(v_c\) in the attractive case.

Figures \[13\] and \[14\] demonstrate that damping is significantly suppressed deep into the supersonic regime. We observe undamped motion when \(v_0\) is greater than a \(V_D\)-dependent upper critical velocity \(v_c^+\). Numerical simulations \[12, 44, 59\] have shown that for “wide and smooth” barriers (\(\xi \ll w_z\)) the emission of radiation from the defect in the form of phonons and solitons can be very small for supersonic velocities. In fact, it has been shown analytically that the radiation emission rate resulting from a defect moving supersonically through a condensate decreases exponentially with the ratio \(\xi/w_z\) \[60\]. Without emission of radiation, energy dissipation is inhibited and the flow persists, even though Landau’s criterion is violated. For the data presented in Figs. \[12\]–\[17\] \(\xi/w_z \sim 0.04\), well within the regime where supersonic non-dissipative flow is predicted. Experiments similar to ours have also shown a reduction in soliton emission from a barrier moving through a condensate in the supersonic regime \[58\].

We therefore observe three distinct regimes of flow in the single defect system: subsonic superfluid (\(v_0/c_0 < 1\)), dissipative (\(v_0/c_0 \sim 1\)), and supersonic non-dissipative (\(v_0/c_0 > v_c\)). Figure \[16\] displays axial densities from \textit{in situ} polarization phase-contrast images at the instant the defect passes through the peak of the condensate for the three different velocity regimes. As expected, for the superfluid flow regime the axial density profiles look very much like the equilibrium profiles of Fig. \[12\] there is an increase (decrease) in the density at the location of the attractive (repulsive) defect. In the dissipative flow regime, on the other hand, the flow patterns for \(V_D > 0\) show significant distortion, while for \(V_D < 0\) there is little distortion, as discussed in detail above. Finally, in the supersonic non-dissipative flow regime, we observe a counter-intuitive density inversion with respect to the superfluid regime, where the attractive defect produces a density depression while the repulsive defect causes a density peak.

The physical origin of this counter-intuitive density inversion can be understood by considering the behavior of the gas at large \(v_0\). In this regime, as in the disordered case, the Bogoliubov excitation spectrum, given by Eq. \[2\] is dominated by the \(p^2/2m\) term and therefore, dominated by plane waves with wavenumber \(k = p/\hbar\) rather than phonons. For this “quasi-ideal” gas, the drag should be determined by the scattering of these plane waves off of the defect \[54\]. At high velocities, scattering of these waves from the defect is greatly suppressed, leading to low dissipation. If we extend this argument further and consider the atoms to be classical particles, one expects the atoms to slow down in the presence of the repulsive defect, resulting in a density increase near the defect, while the opposite is expected for an attractive defect.

Density inversions similar to the ones presented here have also been discussed in the context of dissipationless stationary states at supersonic velocities \[53, 61, 62\] as well as sonic black holes \[63\]. Under our experimental conditions, when \(v_0/c_0 \sim 1\) the edge of the barrier can serve as a sonic event horizon. Such systems have been proposed as possible candidates with which to study “table top” astrophysics, where exotic effects, such as Hawking radiation, should be observable. Interestingly, in this system the experimenter plays the role of the so-called super-observer, having access to the regions both outside and inside the event horizon \[63\].

### B. Weakly Interacting Regime

Figure \[18\] shows results of measurements of a weakly interacting condensate (\(a = 0.6 a_0\)) oscillating in the presence of a repulsive defect. Under these conditions the condensate is in the quasi-1D regime with \(\mu/\hbar \omega_r = 0.15\). We find that the axial density profile of the condensate becomes increasingly modulated during the damped oscillation, consistent with theory \[42, 53\]. We compute the
The dipole mode is caused primarily by the creation of interaction energy. We therefore conclude that the damping of density modulation qualitatively matches the loss of oscillation energy. The time-dependent increase in the density traces and Gaussian fits at various oscillation times: (c)–(e) In situ axial density traces and Gaussian fits at various oscillation times: (c) 0 ms; (d) 140 ms, at the second crossing of the defect; and (e) 1260 ms, after several crossings of the defect. At large times we find that the large density modulations are accompanied by only a slight increase of the axial size of the condensate.

The root-mean-square deviation of the axial density distribution from a Gaussian fit \( n_{\text{fit}} \) as a proxy for the increased internal energy of the condensate due to the density modulation

\[
\Delta = \sqrt{\frac{1}{L} \int_L \left[ \frac{n(z) - n_{\text{fit}}(z)}{n_{\text{fit}}(z)} \right]^2 dz},
\]

where we take the integration length \( L \) to be over the central 70% of the condensate to minimize edge effects. Figure 18(b) shows that \( \Delta \) initially increases in time and then saturates. The time-dependent increase in the density modulation qualitatively matches the loss of oscillation energy. We therefore conclude that the damping of the dipole mode is caused primarily by the creation of in situ density modulations in the cloud.

Results of measurements of the velocity-dependence of the damping by a repulsive defect with \( a = 0.6 a_0 \) are shown in Fig. 19. As was the case with disordered potentials, we find that the timescale for damping in the quasi-1D regime with a strong impurity strength is much longer than that observed in the Thomas-Fermi regime with a weak impurity strength.

C. Dark Soliton Production in the Weakly Interacting Regime

Of particular interest in the quasi-1D regime is the ability to create and observe long-lived dark solitons. These nonlinear excitations have been previously created in BECs with repulsive interatomic interactions through a variety of means, including direct phase imprinting \( [67, 68] \), spatially selective microwave transfer \( [69] \), slow light \( [70] \), two condensate interference \( [71, 72] \), and, similar to the work presented here, as a result of a BEC crossing a semi-permeable defect \( [55] \).

In general, the decay of dark solitons occurs as a result of dynamical instability or as a result of dissipative dynamics associated with the interaction of the soliton with quasiparticle excitations of the BEC. However, it is known that dark solitons can have very long lifetimes in the quasi-1D regime \( [23] \). For the most weakly interacting BECs presented here, \( \mu/\hbar \omega_c = 0.13 \), making our system ideally suited to study long-lived dark solitons.

We have studied the formation of deep in situ density modulations in BECs for different values of \( a \), with the results shown in Fig. 20. Dipole motion is initiated af-
after the field is slowly ramped to a desired value near the scattering length zero-crossing at 544 G. Panels (a–e) of Fig. 20 show the cloud after 3/4 of a complete oscillation. The defect is switched off after the first pass of the field is slowly ramped to a desired value near the trapping frequencies for this data are \( \omega_r = 5 \text{ Hz}, \omega_z = 77 \text{ Hz}, \xi = 3.06 \mu m, \xi_s = 2.8(4) \mu m \); (d), (h): \( a = 5.4 a_0, N = 2.2 \times 10^5, \mu/h = 144 \text{ Hz}, \xi = 2.24 \mu m, \xi_s = 2.5(3) \mu m \). The trap frequencies for this data are \( \omega_r = (2\pi) 240 \text{ Hz} \) and \( \omega_z = (2\pi) 4.75 \text{ Hz} \). The dashed lines show fits to Eq. (11). We omit the fit in (d) for clarity. For comparison, the thin dashed line in (a) is only the Gaussian portion of the fit. Error bars for \( \xi_s \) are given by the standard deviation of a collection of images.

V. SUMMARY AND FUTURE DIRECTIONS

We have conducted comprehensive measurements of the dissipation of superfluid flow in an elongated BEC subject to either a disordered potential or a single Gaussian defect. By measuring the velocity and disorder strength-dependent damping parameter, we have characterized the breakdown of superfluidity of a harmonically trapped cloud in both the 3D Thomas-Fermi and the quasi-1D weakly interacting regimes.

Our data largely support the validity of the Landau criterion for a critical velocity above which the superfluid motion is damped, as long as the criterion is applied locally. The local criterion accounts for the inherent inhomogeneity of trapped gases, as well as density modifications produced by large defects. The only exception is for attractive defects of relatively large strength, where we find that \( v_c \) decreases to \( v_c \approx 0.7 \alpha_0 \) for \( V_D/\mu < -0.5 \). Dissipation is also found to diminish for velocities greater than \( v_c \), which we associate with reduced excitation of dark solitons and phonons.

Throughout the 3D Thomas-Fermi regime, the damping is found to be well described by a universal relation depending on the dimensionless defect strength \( V_D/\mu \) and velocity \( v_0/c_0 \). The universal damping peaks at \( v_0/c_0 \sim 1 \) for any \( V_D/\mu \) and scales as \( (V_D/\mu)^2 \) for all \( \mu \). As \( \mu \) decreases, the peak damping rate decreases as well, consistent with the disappearance of the phonon portion of the excitation spectrum as \( c_0 \rightarrow 0 \). Damping in the quasi-1D regime is qualitatively different. In this case, we find for fixed absolute \( V_D \) and \( v_0 \) that \( \beta \) is independent of \( \mu \). In this regime, damping is accompanied by fragmentation and spreading of the cloud, with the damping monotonically increasing with \( V_D/E_K \), where \( E_K \) is the maximum single particle kinetic energy.

A particularly intriguing possibility for the future is to explore the transport properties of a weakly attractive gas. In the case of a disordered potential there exists the opportunity to study the transport properties of bright matter-wave solitons with the prospect of observing Anderson localization in such systems. For a single defect, there is also a possibility for the creation of coherently split solitons or solitonic Schrödinger’s cat states.

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