Neutrino Mixing: from the Broken $\mu$-$\tau$ Symmetry to the Broken Friedberg-Lee Symmetry \(^1\)

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Abstract

I argue that the observed flavor structures of leptons and quarks might imply the existence of certain flavor symmetries. The latter should be a good starting point to build realistic models towards deeper understanding of the fermion mass spectra and flavor mixing patterns. The $\mu$-$\tau$ permutation symmetry serves for such an example to interpret the almost maximal atmospheric neutrino mixing angle ($\theta_{23} \sim 45^\circ$) and the strongly suppressed CHOOZ neutrino mixing angle ($\theta_{13} < 10^\circ$). In this talk I like to highlight a new kind of flavor symmetry, the Friedberg-Lee symmetry, for the effective Majorana neutrino mass operator. Luo and I have shown that this symmetry can be broken in an oblique way, such that the lightest neutrino remains massless but an experimentally-favored neutrino mixing pattern is achievable. We get a novel prediction for $\theta_{13}$ in the CP-conserving case: $\sin \theta_{13} = \tan \theta_{12} |(1 - \tan \theta_{23})/(1 + \tan \theta_{23})|$. Our scenario can simply be generalized to accommodate CP violation and be combined with the seesaw mechanism. Finally I stress the importance of probing possible effects of $\mu$-$\tau$ symmetry breaking either in terrestrial neutrino oscillation experiments or with ultrahigh-energy cosmic neutrino telescopes.

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1 “Who ordered that?”

There are five categories of fundamental particles that we have known today: (a) charged leptons $e$, $\mu$ and $\tau$; (b) neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$; (c) up-type quarks $u$, $c$ and $t$; (d) down-type quarks $d$, $s$ and $b$; (e) gauge bosons $\gamma$, $W^\pm$, $Z^0$ and $g$. The particles belonging to category (e) are the carriers of electromagnetic ($\gamma$), weak ($W^\pm$ and $Z^0$) and strong ($g$) forces, respectively. In comparison, the particles belonging to categories (a) – (d) are the building blocks of the world of matter. But we have no idea about what constitutes the cold dark matter of the Universe.

The afore-mentioned fermions have three families: the first family includes $u$, $d$, $e$ and $\nu_e$; the second family contains $c$, $s$, $\mu$ and $\nu_\mu$; and the third family is composed of $t$, $b$, $\tau$ and $\nu_\tau$. The leptons or quarks of each category have the same gauge quantum numbers, but their masses are different from one another. What distinguishes different fermion families? In other words, “Who ordered that?” Although the definite answer to this fundamental question has been lacking, some people have speculated the possibility that there might exist certain hidden flavor quantum numbers or flavor symmetries which distinguish one family from another.

The mysterious Koide relation for the pole masses of three charged leptons can be taken as a good example to illustrate the puzzles of flavors [1]:

$$Q^\text{pole}_l \equiv \frac{m_e + m_\mu + m_\tau}{\left( \sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau} \right)^2} = \frac{2}{3}. \quad (1)$$

Given the experimental values [2]

$$\begin{align*}
m_e &= (0.510998918 \pm 0.000000044) \text{ MeV}, \\
m_\mu &= (105.6583692 \pm 0.0000094) \text{ MeV}, \\
m_\tau &= (1776.99^{+0.29}_{-0.26}) \text{ MeV},
\end{align*} \quad (2)$$

Eq. (1) holds up to the accuracy of $O(10^{-5})$; i.e., $-0.00001 \leq Q^\text{pole}_l - 2/3 \leq +0.00002$ [3]. This precision is so amazing that I cannot help to ask what the underlying physics is behind the Koide relation. As pointed out by Koide himself in this workshop [4], Eq. (1) might naturally stem from a kind of flavor symmetry, such as the discrete $S(3)$ symmetry. I stress that one has to distinguish between the concepts of running and pole masses of fermions when building models at certain energy scales, where certain flavor symmetries exactly exist.

Needless to say, flavor symmetries may serve as a promising guiding principle for model building and their spontaneous or explicit breaking schemes may help understand

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3Such a question was first asked by the famous American physicist I.I. Rabi, when he heard that the muon ($\mu$), a sister of the electron, was discovered from cosmic rays in 1936.
the observed mass spectra and flavor mixing patterns of leptons and quarks [5]. Here let me focus on neutrino mixing, which is described by the $3 \times 3$ Maki-Nakagawa-Sakata (MNS) matrix [6] at low energy scales. In September 1996, Fritzsch and I proposed the so-called “democratic” neutrino mixing pattern by starting from the $S(3)_L \times S(3)_R$ symmetry of the charged-lepton mass matrix and the $S(3)$ symmetry of the neutrino mass matrix [7]:

$$V_{FX} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{3}$$

A few years later, Harrison, Perkins and Scott [8] advocated the so-called “tri-bimaximal” neutrino mixing pattern,

$$V_{HPS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{4}$$

which can be derived from certain discrete flavor symmetries (such as the non-Abelian $A_4$ symmetry [9]). In comparison with $V_{FX}$, $V_{HPS}$ is much more compatible with today’s solar [10], atmospheric [11], reactor [12] and accelerator [13] neutrino oscillation data.\textsuperscript{4} In particular, the large-mixing-angle Mikheyev-Smirnov-Wolfenstein (MSW) solution [15] to the solar neutrino problem can simply be accommodated and the very special values of two mixing angles $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$ are closely related to the $\mu$-$\tau$ permutation symmetry.

2 \mu-\tau permutation symmetry

In the limit of $\mu$-$\tau$ permutation symmetry, the (effective) Majorana neutrino mass matrix $M_\nu$ takes the form

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix}. \tag{5}$$

Here and hereafter we work in the basis where the flavor eigenstates of charged leptons are identified with their mass eigenstates. It is easy to check that $M_\nu$ is invariant under the $\mu$-$\tau$ permutation, which is equivalent to the transformation $O_\nu M_\nu O_\nu^T$ with

$$O_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{6}$$

\textsuperscript{4}Radiative corrections to both $V_{FX}$ and $V_{HPS}$ have been analyzed by Luo, Mei and me [14] from a superhigh energy scale to the electroweak scale.
After a straightforward diagonalization of $M_\nu$, one may obtain $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$ for the MNS matrix $V$. The solar neutrino mixing angle $\theta_{12}$ is not fixed, nevertheless.

The breaking of $\mu$-$\tau$ symmetry will in general lead to $\theta_{13} \neq 0^\circ$, $\theta_{23} \neq 45^\circ$ and a non-trivial CP-violating phase $\delta$ [16, 17, 18, 19]. If the $\mu$-$\tau$ symmetry is softly broken, however, $\theta_{13} = 0^\circ$ or $\theta_{23} = 45^\circ$ might survive. For instance, it is possible to break the $\mu$-$\tau$ symmetry in a proper way such that the resultant MNS matrix takes one of the following two forms [17, 18, 19, 20]:

$$V = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & is \\ 0 & -is & c \end{pmatrix},$$

(7)

from which $\theta_{13} \neq 0^\circ$, $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ can be predicted. Here $c_{12} \equiv \cos \theta_{12}$, $s_{12} \equiv \sin \theta_{12}$, $c \equiv \cos \theta$ and $s \equiv \sin \theta$ are defined. The angle $\theta$ depends closely on the parameters of $\mu$-$\tau$ symmetry breaking. An example for the soft $\mu$-$\tau$ symmetry breaking will be given in section 5 of my talk.

A generic parametrization of the $3 \times 3$ Majorana neutrino mixing matrix needs three mixing angles ($\theta_{12}$, $\theta_{23}$, $\theta_{13}$) and three CP-violating phases ($\delta$, $\rho$, $\sigma$) [21]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & e^{i\rho}00 \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & 001 \end{pmatrix}. \tag{8}$$

One usually refers to $\delta$ as the “Dirac” phase, because it is also present in the $3 \times 3$ Dirac neutrino mixing matrix. In comparison, $\rho$ and $\sigma$ are commonly referred to as the “Majorana” phases, which do not manifest themselves in neutrino oscillations and can be rotated away if neutrinos are Dirac particles. A complex $\mu$-$\tau$ symmetry breaking scheme is possible to generate both non-trivial $\delta$ and non-trivial $\rho$ and $\sigma$ in a given neutrino mass model.

This talk will focus on a new kind of flavor symmetry, the so-called Friedberg-Lee (FL) symmetry [22], to discuss the neutrino mass matrix and neutrino mixing. One will see that the well-known $\mu$-$\tau$ symmetry can also manifest itself in the neutrino mass operator with the FL symmetry.
3 What is the FL symmetry?

The effective neutrino mass operator proposed recently by Friedberg and Lee [22] is of the form

\[
L_{\text{FL}} = a (\bar{\nu}_\tau - \bar{\nu}_\mu) (\nu_\tau - \nu_\mu) + b (\bar{\nu}_\mu - \bar{\nu}_e) (\nu_\mu - \nu_e) + c (\bar{\nu}_e - \bar{\nu}_\tau) (\nu_e - \nu_\tau) + m_0 (\bar{\nu}_e \nu_e + \bar{\nu}_\mu \nu_\mu + \bar{\nu}_\tau \nu_\tau),
\tag{9}
\]

where \(a, b, c\) and \(m_0\) are all assumed to be real. A salient feature of \(L_{\text{FL}}\) is its partial translational symmetry; i.e., its terms are invariant under the transformation \(\nu_\alpha \rightarrow \nu_\alpha + z\) (for \(\alpha = e, \mu, \tau\)) with \(z\) being a space-time independent constant element of the Grassmann algebra. Corresponding to Eq. (9), the neutrino mass matrix \(M_\nu\) reads

\[
M_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} b + c & -b & -c \\ -b & a + b & -a \\ -c & -a & a + c \end{pmatrix}.
\tag{10}
\]

Diagonalizing \(M_\nu\) by the transformation \(V_{\text{FL}}^\dagger M_\nu V_{\text{FL}}^* = \text{Diag}\{m_1, m_2, m_3\}\), in which \(m_i\) (for \(i = 1, 2, 3\)) stand for the neutrino masses, one may obtain the neutrino mixing matrix

\[
V_{\text{FL}} = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
\cos \frac{\theta}{2} & 0 & \sin \frac{\theta}{2} \\
0 & 1 & 0 \\
-\sin \frac{\theta}{2} & 0 & \cos \frac{\theta}{2}
\end{pmatrix},
\tag{11}
\]

where \(\theta\) is given by \(\tan \theta = \sqrt{3} (b - c) / [(b + c) - 2a]\). This interesting result leads us to the following observations [17]:

1. If \(\theta = 0^\circ\) holds, \(V_{\text{FL}}\) will reproduce the exact tri-bimaximal neutrino mixing pattern [8]. The latter, which can be understood as a geometric representation of the neutrino mixing matrix [23], is in good agreement with current experimental data. Non-vanishing but small \(\theta\) predicts \(\sin \theta_{13} = (2/\sqrt{6}) \sin (\theta/2)\), implying \(\theta \leq 24.6^\circ\) for \(\theta_{13} < 10^\circ\). On the other hand, \(\theta_{23}\) will mildly deviate from its best-fit value \(\theta_{23} = 45^\circ\) if \(\theta\) (or \(\theta_{13}\)) takes non-zero values.

2. The limit \(\theta = 0^\circ\) results from \(b = c\). When \(b = c\) holds, it is straightforward to see that the neutrino mass operator \(L_{\text{FL}}\) has the exact \(\mu-\tau\) symmetry (i.e., \(L_{\text{FL}}\) is invariant under the interchange of \(\mu\) and \(\tau\) indices). In other words, the tri-bimaximal neutrino mixing is a natural consequence of the \(\mu-\tau\) symmetry of \(M_\nu\) in the FL model. Then \(\theta_{13} \neq 0^\circ\) and \(\theta_{23} \neq 45^\circ\) measure the strength of \(\mu-\tau\) symmetry breaking, as many authors have discussed in other neutrino mass models.

Note that \(L_{\text{FL}}\) is only valid for Dirac neutrinos. Recently Zhang, Zhou and I have generalized \(L_{\text{FL}}\) to describe Majorana neutrinos [17]. In particular, we allow \(a, b, c\) and \(m_0\) to be complex so as to accommodate leptonic CP violation. Two special scenarios
have been discussed in detail. In scenario (A), we require that $a$ and $m_0$ be real and $b = c^*$ be complex. We find that the $\mu$-$\tau$ symmetry of $M_\nu$ is softly broken in this case, leading to the elegant predictions $\theta_{13} \neq 0^\circ$, $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$. Two Majorana CP-violating phases $\rho$ and $\sigma$ keep vanishing. In scenario (B), we assume that $a$, $b$ and $c$ are real but $m_0$ is complex. We find that the results of $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ obtained from $V_{\text{FL}}$ keep unchanged in this case, but some non-trivial values of the Majorana CP-violating phases $\rho$ and $\sigma$ can now be generated. The Dirac CP-violating phase $\delta$ remains vanishing.

To see the point of the FL symmetry more clearly, let me switch off the term proportional to $m_0$ and then rewrite $L_{\text{FL}}$ for Majorana neutrinos:

$$L_{\text{mass}} = \frac{1}{2} \left[ a(\overline{\nu}_{\tau L} - \overline{\nu}_{\mu L})(\nu_{\tau L}^c - \nu_{\mu L}^c) + b(\overline{\nu}_{\mu L} - \overline{\nu}_{e L})(\nu_{\mu L}^c - \nu_{e L}^c) + c(\overline{\nu}_{e L} - \overline{\nu}_{\tau L})(\nu_{e L}^c - \nu_{\tau L}^c) \right] + \text{h.c.}, \quad (12)$$

where $a$, $b$ and $c$ are in general complex, and $\nu_{\alpha L}^c \equiv C\nu_{\alpha L}^T$ (for $\alpha = e, \mu, \tau$). Corresponding to Eq. (12), the Majorana neutrino mass matrix $M_\nu$ is simply

$$M_\nu = \begin{pmatrix} b + c & -b & -c \\ -b & a + b & -a \\ -c & -a & a + c \end{pmatrix}. \quad (13)$$

The diagonalization of $M_\nu$ is straightforward: $V^\dagger M_\nu V^* = \overline{M}_\nu$, where $V$ is just the MNS matrix, and $\overline{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$. From Eq. (13) together with the parametrization of $V$ in Eq. (8), it is easy to verify

$$\text{Det}(M_\nu) = \text{Det} \left( V\overline{M}_\nu V^T \right) = \text{Det}(\overline{M}_\nu) [\text{Det}(V)]^2 = m_1 m_2 m_3 e^{2i(\rho + \sigma)} = 0. \quad (14)$$

This result, which is an immediate consequence of the FL symmetry in $L_{\text{mass}}$, implies that one of the three neutrinos must be massless. Because of $m_2 > m_1$ obtained from the solar neutrino oscillation data [10], the $m_2 = 0$ case has been ruled out. In Eq. (9) the FL symmetry is broken by $m_0 \neq 0$, hence all the three neutrinos are massive. One may wonder whether it is possible to break the FL symmetry but keep $m_1 = 0$ or $m_3 = 0$ unchanged. The answer to this question is actually affirmative, as one will see in the subsequent section.

4 Oblique FL symmetry breaking

Is it really possible to break the FL symmetry in $L_{\text{mass}}$ but keep $m_1 = 0$ or $m_3 = 0$ unchanged? Luo and I [19] have found that the simplest way to make this possibility realizable is to transform one of the neutrino fields $\nu_\alpha$ into $\kappa^* \nu_\alpha$ with $\kappa \neq 1$. Given
$\nu_e \to \kappa^* \nu_e$ as an example, the resultant Majorana neutrino mass operator reads

$$\mathcal{L}'_{\text{mass}} = \frac{1}{2} \left[ a(\nu_{eL} - \nu_{\mu L})(\nu_{eL} - \nu_{\mu L}^c) + b(\nu_{\mu L} - \kappa \nu_{eL})(\nu_{\mu L}^c - \kappa \nu_{eL}^c) \right.$$  
$$\left. + c(\kappa \nu_{eL} - \nu_{\tau L})(\kappa \nu_{eL}^c - \nu_{\tau L}) \right] + \text{h.c.} \quad (15)$$

Accordingly, the neutrino mass matrix is given by

$$M'_\nu = \begin{pmatrix} \kappa^2 (b + c) & -\kappa b & -\kappa c \\ -\kappa b & a + b & -a \\ -\kappa c & -a & a + c \end{pmatrix}. \quad (16)$$

We are then left with $\text{Det}(M'_\nu) = \kappa^2 \text{Det}(M_\nu) = 0$, which is independent of the magnitude and phase of $\kappa$. Thus we obtain either $m_1 = 0$ or $m_3 = 0$ from $M'_\nu$. The effect of $\kappa \neq 1$ is referred to as the “oblique” symmetry breaking, because it breaks the FL symmetry but keeps the lightest neutrino to be massless.

My next step is to show that a generic bi-large neutrino mixing pattern can be derived from $M'_\nu$. Let me focus on the $m_1 = 0$ case,\(^5\) in which $M'_\nu$ is diagonalized by the transformation $V^\dagger M'_\nu V^* = M_\nu$ with $M_\nu = \text{Diag}\{0, m_2, m_3\}$. As the best-fit values of the atmospheric and CHOOZ neutrino mixing angles are $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$ respectively [24], we may decompose $V$ into a product of two special unitary matrices: $V = UO$, where

$$U = \begin{pmatrix} 1 & \frac{\sqrt{2} \kappa}{\sqrt{2|\kappa|^2 + 1}} & 0 \\ \frac{\sqrt{2|\kappa|^2 + 1}}{\sqrt{2|\kappa|^2 + 1}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{2|\kappa|^2 + 1}}{\sqrt{2|\kappa|^2 + 1}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (17)$$

and $O$ depends on the details of $M'_\nu$. Note that the first column of $U$ is associated with $m_1 = 0$. $O$ contains a single $(2, 3)$-rotation angle, which can be obtained from the diagonalization of $U^\dagger M'_\nu U^*$.

For simplicity, Luo and I have assumed that $a$, $b$ and $c$ are all real [19]. In this case, $U^\dagger M'_\nu U^*$ is a real symmetric matrix and the calculation of $O$ is quite easy. We arrive at

$$V = \begin{pmatrix} \frac{1}{\sqrt{2|\kappa|^2 + 1}} & \frac{\sqrt{2} \kappa \cos \theta}{\sqrt{2|\kappa|^2 + 1}} & \frac{\sqrt{2} \kappa \sin \theta}{\sqrt{2|\kappa|^2 + 1}} \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2 + 1}} & \frac{\cos \theta}{\sqrt{2|\kappa|^2 + 1}} + \sin \theta & \frac{\sin \theta}{\sqrt{2|\kappa|^2 + 1}} \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2 + 1}} & \frac{\cos \theta}{\sqrt{2|\kappa|^2 + 1}} - \sin \theta & \frac{\sin \theta}{\sqrt{2|\kappa|^2 + 1}} \end{pmatrix}, \quad (18)$$

where $\theta$ is given by $\tan 2\theta = (b - c) \sqrt{2|\kappa|^2 + 1} / [(b + c) |\kappa|^2 - 2a]$. One can see that $b = c$, which is a clear reflection of the $\mu - \tau$ permutation symmetry in $\mathcal{L}'_{\text{mass}}$ or $M'_\nu$, leads to $\theta = 0^\circ$ or equivalently $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$. After rephasing the expression of $V$ with the transformations of charged-lepton and neutrino fields $e \to e^{i\phi_e} e$, $\mu \to -\mu$, $\nu_1 \to e^{i\phi_{\nu_1}} \nu_1$

\(^5\)The $m_3 = 0$ case is actually disfavored, if one intends to achieve $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$ from $M'_\nu$ in the leading-order approximation.
and $\nu_3 \rightarrow -\nu_3$, where $\phi_\kappa \equiv \text{arg}(\kappa)$, we may directly compare it with Eq. (8). A novel prediction turns out to be [19]

$$
\sin \theta_{13} = \frac{1 - \tan \theta_{23}}{1 + \tan \theta_{23}} \tan \theta_{12}.
$$

(19)

Since the mixing angles $\theta_{12}$ and $\theta_{23}$ are already known to a reasonable degree of accuracy, they can be used to determine the unknown mixing angle $\theta_{13}$. Eq. (19) indicates that the deviation of $\theta_{13}$ from zero is closely correlated with the deviation of $\theta_{23}$ from $45^\circ$. Such a correlation can easily be tested in the near future. On the other hand, Eq. (18) allows us to achieve

$$
|\kappa| = \frac{\sin \theta_{12}}{\sqrt{\cos 2\theta_{12} + \sin 2\theta_{23}}}.
$$

(20)

By using the 99% confidence-level ranges of $\theta_{12}$ and $\theta_{23}$ (i.e., $30^\circ \leq \theta_{12} \leq 38^\circ$ and $36^\circ \leq \theta_{23} \leq 54^\circ$) [24], we obtain $\theta_{13} \leq 7.1^\circ$ from Eq. (19) and $0.41 \leq |\kappa| \leq 0.56$ from Eq. (20). Note that $|\kappa| = 1/2$ is in particular favorable and it implies that $U$ takes the tri-bimaximal mixing pattern [8]. The numerical dependence of $\theta_{13}$ on $\theta_{12}$ and $\theta_{23}$ is illustrated in Fig. 1. The upper bound on $\theta_{13}$ extracted from Fig. 1 is certainly more stringent than $\theta_{13} < 10^\circ$ obtained from a global analysis of current neutrino oscillation data [24]. It will be interesting to see whether our prediction for the correlation between the unknown mixing angle $\theta_{13}$ and two known angles can survive the future measurements.

We have taken $\nu_e \rightarrow \kappa^* \nu_e$ to break the FL symmetry and achieve a realistic pattern of the neutrino mass matrix with $m_1 = 0$. A careful analysis shows that neither $\nu_\mu \rightarrow \kappa^* \nu_\mu$ nor $\nu_\tau \rightarrow \kappa^* \nu_\tau$ with $\kappa \neq 1$, which automatically breaks the $\mu-\tau$ permutation symmetry, is favored to reproduce the exactly or approximately tri-bimaximal neutrino mixing pattern.

### 5 CP violation and seesaw realization

Although the above discussions are based on the assumption of real $a$, $b$ and $c$, they can easily be extended to the case of complex $a$, $b$ and $c$ in order to accommodate CP violation. For simplicity of illustration, here we assume that $a$ remains real but $b = c^*$ is complex. We may then simplify the expression of $U^\dagger M'_\nu U$ by taking into account $b + c = 2\text{Re}(b)$ and $b - c = 2i\text{Im}(b)$. After an analogous calculation, we obtain the MNS matrix [19]

$$
V = \begin{pmatrix}
\frac{1}{\sqrt{2|\kappa|^2 + 1}} & \frac{i \sqrt{2} \kappa \cos \theta}{\sqrt{2|\kappa|^2 + 1}} & \frac{i \sqrt{2} \kappa \sin \theta}{\sqrt{2|\kappa|^2 + 1}} \\
\frac{-\kappa^*}{\sqrt{2|\kappa|^2 + 1}} & \frac{1}{\sqrt{2}} \left( i \frac{\cos \theta}{\sqrt{2|\kappa|^2 + 1}} + \sin \theta \right) & \frac{1}{\sqrt{2}} \left( \cos \theta - i \frac{\sin \theta}{\sqrt{2|\kappa|^2 + 1}} \right) \\
\frac{-\kappa}{\sqrt{2|\kappa|^2 + 1}} & \frac{1}{\sqrt{2}} \left( i \frac{\cos \theta}{\sqrt{2|\kappa|^2 + 1}} - \sin \theta \right) & \frac{1}{\sqrt{2}} \left( \cos \theta + i \frac{\sin \theta}{\sqrt{2|\kappa|^2 + 1}} \right)
\end{pmatrix},
$$

(21)
where $\theta$ is given by $\tan 2\theta = -\text{Im}(b)\sqrt{2|\kappa|^2 + 1}/[a + \text{Re}(b)(|\kappa|^2 + 1)]$. Two immediate but important observations are in order:

1. In this simple scenario $V$ contains two nontrivial CP-violating phases: $\delta = -90^\circ$ (when $\theta < 0^\circ$) or $\delta = +90^\circ$ (when $\theta > 0^\circ$) and $\sigma = -90^\circ$. Both of them are attributed to the purely imaginary term $b - c$. The Jarlskog rephasing invariant of leptonic CP violation\cite{25} reads $J = |\kappa|^2 |\sin 2\theta| / [2 (2|\kappa|^2 + 1)^{3/2}]$. A numerical analysis yields $0.41 \leq |\kappa| \leq 0.57$ and $|\theta| < 19.4^\circ$. Thus we arrive at $J \leq 0.041$.

2. $\tan \theta_{23} = 1$ or $\theta_{23} = 45^\circ$ can be achieved, although the effective Majorana neutrino mass operator $\mathcal{L}_{\text{mass}}'$ does not possess the exact $\mu$-$\tau$ permutation symmetry. The reason is simply that $|b| = |c|$ holds in our scenario. In other words, the phase difference between $b$ and $c$ signifies a kind of soft $\mu$-$\tau$ symmetry breaking which can keep $\theta_{23} = 45^\circ$ but cause $\theta_{13} \neq 0^\circ$.

Finally I like to point out that it is possible to derive the Majorana neutrino mass operator $\mathcal{L}_{\text{mass}}'$ from the minimal seesaw model (MSM) \cite{26}, a canonical extension of the standard model with only two heavy right-handed Majorana neutrinos. The neutrino mass term in the MSM can be written as

$$-\mathcal{L}_{\text{MSM}} = \frac{1}{2}(\nu_L, N_R)\begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu_L \\ N_R \end{pmatrix} + \text{h.c.}, \quad (22)$$

where $\nu_L$ and $N_R$ denote the column vectors of $(\nu_e, \nu_\mu, \nu_\tau)_L$ and $(N_1, N_2)_R$ fields, respectively. Provided the mass scale of $M_R$ is considerably higher than that of $M_D$, one
may obtain the effective (left-handed) Majorana neutrino mass matrix \( M'_{\nu} \) from Eq. (22) via the well-known seesaw mechanism [27]:

\[
M'_{\nu} = M_D M_R^{-1} M_D^T.
\]

As \( M_R \) is of rank 2, \( \text{Det}(M'_{\nu}) = 0 \) holds and \( m_1 = 0 \) (or \( m_3 = 0 \)) is guaranteed. We find that the expression of \( M'_{\nu} \) given in Eq. (16) can be reproduced from \( M_D \) and \( M_R \) if they take the following forms:

\[
M_D = \Lambda_D \begin{pmatrix}
\kappa & 0 \\
-1 & -1 \\
0 & 1
\end{pmatrix},
\]

\[
M_R = \frac{\Lambda_D^2}{ab + bc + ca} \begin{pmatrix}
a + c & c \\
c & b + c
\end{pmatrix},
\]

(23)

where \( \Lambda_D \) characterizes the mass scale of \( M_D \). Such textures are by no means unique, but they may serve as a good example to illustrate how the seesaw mechanism works to give rise to \( M'_{\nu} \) or \( L'_\text{mass} \) in the MSM.

6  Concluding remarks

I have argued that the observed flavor structures of leptons and quarks might imply the existence of certain flavor symmetries. A flavor symmetry itself is far away from a flavor theory, but it might be taken as a reasonable starting point to build a realistic flavor model towards deeper understanding of the fermion mass spectra and flavor mixing patterns. The \( \mu-\tau \) permutation symmetry has been taken as an example to interpret the almost maximal atmospheric neutrino mixing angle (\( \theta_{23} \sim 45^\circ \)) and the strongly suppressed CHOOZ neutrino mixing angle (\( \theta_{13} < 10^\circ \)). The effect of \( \mu-\tau \) symmetry breaking could even show up in the ultrahigh-energy neutrino telescopes [28].

Let us anticipate that IceCube [29] and other second-generation neutrino telescopes [30] are able to detect the fluxes of ultrahigh-energy (UHE) \( \nu_e (\bar{\nu}_e) \), \( \nu_\mu (\bar{\nu}_\mu) \) and \( \nu_\tau (\bar{\nu}_\tau) \) neutrinos generated from very distant astrophysical sources. For most of the currently-envisioned sources of UHE neutrinos [31], a general and canonical expectation is that the initial neutrino fluxes are produced via the decay of pions created from \( pp \) or \( p\gamma \) collisions and their flavor content can be expressed as

\[
\{ \phi_e, \phi_\mu, \phi_\tau \} = \left\{ \frac{1}{3}, \frac{2}{3}, 0 \right\} \phi_0,
\]

(24)

where \( \phi_\alpha \) (for \( \alpha = e, \mu, \tau \)) denotes the sum of \( \nu_\alpha \) and \( \bar{\nu}_\alpha \) fluxes, and \( \phi_0 = \phi_e + \phi_\mu + \phi_\tau \) is the total flux of neutrinos and antineutrinos of all flavors. Due to neutrino oscillations, the flavor composition of such cosmic neutrino fluxes to be measured at the detector of a neutrino telescope has been expected to be [32]

\[
\{ \phi^D_e, \phi^D_\mu, \phi^D_\tau \} = \left\{ \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\} \phi_0.
\]

(25)
However, it is worth remarking that this naive expectation is only true in the limit of $\mu$-$\tau$ symmetry (or equivalently, $\theta_{13} = 0$ and $\theta_{23} = \pi/4$). Starting from the hypothesis given in Eq. (24) and allowing for the slight breaking of $\mu$-$\tau$ symmetry, I have shown that [28]

$$
\phi_e^D : \phi_\mu^D : \phi_\tau^D = (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta)
$$

holds to an excellent degree of accuracy, where

$$
\Delta = \frac{1}{4} \left( 2\varepsilon \sin^2 \theta_{12} - \theta_{13} \sin 4\theta_{12} \cos \delta \right)
$$

with $\varepsilon \equiv \theta_{23} - \pi/4$ characterizes the effect of $\mu$-$\tau$ symmetry breaking (i.e., the combined effect of $\theta_{13} \neq 0$ and $\theta_{23} \neq \pi/4$). I obtain $-0.1 \leq \Delta \leq +0.1$ from the present neutrino oscillation data [24]. It would be fantastic if $\Delta$ could finally be measured at a km$^2$-scale neutrino telescope in the future.

The main body of this talk is actually to highlight the FL symmetry and its oblique breaking scheme. I have emphasized that the FL symmetry is a new kind of flavor symmetry applicable to the building of neutrino mass models. Imposing this symmetry on the effective Majorana neutrino mass operator, Luo and I have shown that it can be broken in such a novel way that the lightest neutrino remains massless but an experimentally-favored bi-large neutrino mixing pattern is achievable. This phenomenological scenario predicts a testable relationship between the unknown neutrino mixing angle $\theta_{13}$ and the known angles $\theta_{12}$ and $\theta_{23}$ in the CP-conserving case: $\sin \theta_{13} = \tan \theta_{12} |(1 - \tan \theta_{23})/(1 + \tan \theta_{23})|$. Such a result is suggestive and interesting because it directly correlates the deviation of $\theta_{13}$ from zero with the deviation of $\theta_{23}$ from $\pi/4$. We have discussed a simple but instructive possibility of introducing CP violation into the Majorana neutrino mass operator, in which the soft breaking of $\mu$-$\tau$ symmetry yields $\delta = \pi/2$ (or $\delta = -\pi/2$) but keeps $\theta_{23} = \pi/4$. We have also discussed the possibility of incorporating our scenario in the MSM.

In conclusion, the FL symmetry and its breaking mechanism may have a wealth of implications in neutrino phenomenology. The physics behind this new flavor symmetry remains unclear and deserves a further study.

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