Doppler Tomograms from Hydrodynamical Models of Dwarf Nova Disks

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Abstract

We present three-dimensional models of accretion disks in U Gem -like systems and calculate their Doppler tomograms. The tomograms are based on two different assumptions concerning the origin of line emission from the disk. The assumption of lines originating due to irradiation of the surface layer of the disk by the central source leads to a better agreement with observations. We argue that fully three-dimensional modelling is necessary to properly interpret the observed tomograms.

1 Introduction

With their typical dimensions of less than $10^{-4}$ arcseconds, Dwarf Nova (DN) disks are far too small to be resolved directly. However, an indirect observational insight into their structure became possible already in mid-eighties, when a powerful technique of Doppler tomography was introduced (for a recent review see Marsh 2001). Nowadays tomographic observations are a standard, but the interpretation of Doppler tomograms in terms of theoretical models of DN disks is often problematic. This is particularly well visible in the case of spiral waves, which have been suggested as one of the agents responsible for the angular momentum transfer through the disk (for a recent review see Boffin 2001). The excellent discussion of the subject can be found in a recent paper by Smak (2001), and there is no need to repeat it here.

The main problem associated with the interpretation of Doppler tomograms is related to the nature of the data derived from the theory. Model calculations yield spiral patterns in the distribution of disk surface density, while patterns in observational tomograms are related to the distribution of the emissivity in specific spectral lines. While comparing these two sets of data one usually makes an implicit assumption that emissivity is proportional to surface density, which certainly is an oversimplification.

A more sophisticated approach was presented by Steeghs and Stehle (1999), who based their Doppler tomograms on emission line profiles calculated from 2D disk models. For the origin of the lines they adopted a purely thermal model with local Planckian source function. Such model was criticized by Smak (2001), who pointed out that it requires factor of 100 overabundance of helium in order to reproduce the observed intensities, while the relative brightness of the features it produces in the tomograms is incompatible with observations.

Smak himself proposed to base the theoretical tomograms on the distribution of velocity divergence. He argued that at a given radial distance
from the disk center the compression regions with $\nabla \vec{v} < 0$ would be distinguished by a higher-than-average disk thickness (because their density and temperature would be higher). As a result, the surface layer of the disk would be better exposed to the irradiating flux from the white dwarf and the boundary layer, and a local enhancement in line emission would be observed. Based on the three-body model of gas flow in a close binary he identified regions of maximum compression in the orbital plane and showed that they well reproduced shape, location and relative intensities of the arch-like structures observed in Doppler tomograms of DN disks.

Motivated by his paper, we obtained two- and three-dimensional hydrodynamical models of DN disks and calculated their tomograms. The details of the modelling procedure are given in Section 2. The results are presented in Section 3 and discussed in Section 4.

2 Numerical methods, input physics and initial conditions

All models presented here were obtained with the help of the ZEUS-3D code (Clarke & Norman 1994, Clarke 1996). The original code was modified to include conservative angular momentum transport (Kley 1998). Spherical coordinates $(r, \theta, \phi)$ centered on the primary and corotating with the system were used. The grid was extending from 0 to $2\pi$ in $\phi$, from 0 to $0.2\pi$ in $\theta$, and from $r_{in} = 0.1a$ to $r_{out} = 0.5a$ in $r$, with $a$ standing for the orbital separation. Grid spacing was uniform in $(\theta, \phi)$ and logarithmic in $r$, resulting in zones of identical shape. After a few experiments we decided to limit the resolution to $100 \times 20 \times 64$ zones in $r$, $\theta$, $\phi$ directions, respectively (test runs, with resolution increased by a factor of 2 in $r$, $\theta$, $\phi$ consecutively, did not introduce any significant changes into the models).

A standard periodic boundary condition was imposed at $\phi = 2\pi$, and symmetry with respect to the orbital plane was assumed, implying a reflecting boundary condition at $\theta = 0$. A free outflow was allowed for at $r_{in}$ and $r_{out}$. In the four innermost radial zones the radial velocity was reduced by 5% at every time-step, preventing the reflection of waves from the inner boundary of the grid. To check whether this damping procedure did not influence the structure of the disk or the shape of the spiral pattern, we calculated model B2 with $r_{in}$ moved to 0.045a (see Table 1). It was found that shifting the inner grid boundary toward the white dwarf did not introduce any significant changes in the model.

The simulations did not include explicit viscosity (the von Neumann & Richtmyer and scalar linear artificial viscosities originally implemented in ZEUS were only used with coefficients $C_1 = 0.5$ and $C_2 = 2.0$, where $C_1$ is responsible for the magnitude of the artificial viscous pressure, and $C_2$ is a shock-spreading parameter, see the definitions in Stone and Norman (1992)). The energy equation was not solved; a polytropic equation of state ($p = \kappa \rho^\gamma$) with $\gamma = 5/3$ was employed instead. In a system of units in which gravitational constant, orbital separation, and primary’s mass are all equal to 1, the value of $\kappa$ was set to 6500. To mimic U Gem - like systems, all models had the same mass ratio, $q = 0.5$. The stream flowing from the secondary through the $L_1$ point was not included.

Every simulation consisted of three phases (relaxation, switch-on, and proper). The mass ratio was set to 0 in the relaxation phase and to 0.5 in the proper phase, while in the switch-on phase it was linearly increasing in time. At the beginning of each simulation ($t = 0$) the grid was initialized.
with an exponential density distribution

\[ \rho(r, \theta) = \max \left( \rho_0 e^{-\alpha r^2 \sin^2 \theta}, \rho_{\min} \right), \]  

(1)

where

\[ \alpha = \frac{G M_1}{2 r^3 c_{s,0}^2}, \]  

(2)

and

\[ c_{s,0}^2 = \frac{\partial p}{\partial \rho} \bigg|_{\theta=0} = \gamma \kappa \rho_0^{\gamma-1}. \]  

(3)

In our system of units the value of the midplane density, \( \rho_0 \), was set to \( 10^{-8} \), corresponding to \( \sim 2 \cdot 10^{-8} \) g cm\(^{-3} \) in U Gem (with U Gem parameters taken from Groot 2001). The azimuthal velocity of the disk was given a purely Keplerian pattern, and the remaining two velocity components were set to 0. Since the content of the grid was not in hydrostatic equilibrium, we allowed it to relax for \( \sim 3.9 \) orbital periods (\( P_{\text{orb}} \)). Throughout the relaxation phase the model was strictly axisymmetric, so that it was possible to speed the computations up by reducing the number of angular grid points to 2.

Because of extremely steep vertical gradients of density at the surface of the disk it was necessary to introduce a density limit. Every time step the grid was scanned for cells with \( \rho < \rho_{\min} \), and whenever such a cell was found \( \rho \) was reset to \( \rho_{\min} \). We wanted \( \rho_{\min} \) to be small enough to minimize side-effects caused by the newly-added matter falling onto the disk, and, simultaneously, large enough to avoid excessive computational slowdowns due to formation of strong shocks in the rarefied medium above the disk. After a few experiments \( \rho_{\min} \) was set to \( 10^{-13} \) for all models. At the end of the relaxation phase the midplane density of the disk increased, reaching up to \( 4 \cdot 10^{-8} \) (corresponding to \( \sim 8 \cdot 10^{-8} \) g cm\(^{-3} \) in U Gem).

At the beginning of the switch-on phase the relaxed model was mapped onto the standard grid, and the secondary’s gravity was “switched on”. The final value of \( q \) was achieved at \( t = 5.8 P_{\text{orb}} \). At the end of the switch on phase the total mass contained in the grid, and scaled to U Gem, \( M_{\text{disk}} \), was equal \( \sim 10^{24} \) g. The proper phase with \( q = 0.5 \) lasted for another \( \sim 3.9 P_{\text{orb}} \) so that the simulation was ended at \( t \approx 9.7 P_{\text{orb}} \). By that time shape and location of the disk edge stabilized, and a stationary spiral pattern developed in the disk.

## 3 Results

The tidal forces affect the relaxed disk in two ways. First, some material is stripped from its outer edge and driven out of the grid through the outer grid boundary. Second, angular momentum is removed from the
remaining material and transferred into the orbital momentum of the binary, causing the disk to shrink. In the three-body approximation, the radius of the disk cannot be larger than the radius of the largest non-intersecting orbit, $r_{\text{max}}$. U Gem-like systems with $q = 0.5$ have $r_{\text{max}} \approx 0.3$, and in fact at the end of the simulation the disk barely extends beyond $r \approx 0.3$. The rest of the gas originally located at $0.3 < r < 0.5$ now resides in a ring-like density enhancement between $r \approx 0.15$ and $r \approx 0.3$.

The ring is markedly elliptical, but, when averaged over the azimuthal angle, it shows a well-defined density maximum at $r \approx 0.19$, i.e. slightly beyond the circularization radius ($r_{\text{circ}} = 0.16$ for $q = 0.5$). The maximum density is factor of $\sim 2.5$ higher than the midplane density of the inner disk ($r < 0.15$). The ratio $h/r$, where $h$ is the half-thickness of the disk, varies from $\sim 0.1$ in the inner disk to $\sim 0.2$ in the ring. The overall structure of the final model is reminiscent of the one expected at the early phase of the outburst, when the outer radius of the disk just begins to increase. However, because of the polytropic equation of state we employ, the interior of the disk is unrealistically hot (for a disk composed of pure hydrogen $T$ grows from $\sim 10^4 \text{ K}$ at the surface of the disk to $\sim 6.7 \cdot 10^5 \text{ K}$ at the density maximum). We find that both 2-D and 3-D calculations produce disks of nearly the same shape and extent (Fig. 1). Below we shall argue, however, that fully three-dimensional models are needed to properly interpret the Doppler tomograms.

Both 2-D and 3-D models develop spiral shocks shown in Figs. 1 and 2. At a first glance, location and inclination of the shocks do not significantly depend on the number of dimensions of the model. Significant differences become visible when compression regions ($\nabla \vec{v} < 0$) are compared: while model A has a clear two-armed pattern, three arms are present in models B1 and B2. We speculate that the third arm may originate due to tidal forcing of the disk matter in the direction perpendicular to the orbital plane (an effect entirely absent in 2D). Thus, to the long dispute on whether spiral shocks can exist in three dimensions, or rather their existence is limited to the two-dimensional world, we add a vote in favor of the first possibility. In 3D the shocks are definitely there, but their pattern is different than in 2D. Obviously, the validity of this conclusion

Figure 1: Left panel: distribution of disk density, $\rho$, in model A. Right panel: distribution of disk surface density, $\Sigma = \int \rho(z)dz$, in model B1. The white loop marks the Roche lobe.
Figure 2: Left panel: distribution of the compressional power per unit volume, \(-\min(p\nabla \vec{v}, 0)\), in model A. Right panel: distribution of the compressional power per unit surface, \(-\int \min(p\nabla \vec{v}, 0)dz\), in model B1.

is limited to hot polytropic disks with \(0.1 \leq h/r \leq 0.2\).

Following the approach of Smak (2001), we obtained Doppler tomograms of the compressional power due to tidal forces, \(pdV\). The distribution of brightness on the \((v_x, v_y)\) plane was calculated from the integral

\[
L_{pdV}(v_x, v_y) = -\int_V \min(p\nabla \vec{u}, 0)B(u_x, v_x, \delta v_x)B(u_y, v_y, \delta v_y)dV, \quad (4)
\]

where \((u_x, u_y)\) are velocity components of the volume element \(dV\), and the so-called boxcar function \(B\) is defined as

\[
B(u, v, \delta) = \begin{cases} 
1 & |u - v| < \frac{\delta}{2} \\
0 & \text{otherwise}
\end{cases} \quad (5)
\]

The resolution parameter \(\delta\), related to the resolution of images \((400 \times 400\) pixels), was set to \(6v_{\text{orb}}/400\).

In the three-body approximation of Smak (2001), the inner disk is essentially Keplerian, and its Doppler tomogram cannot show any non-axisymmetric structures in the high-velocity range. However, in a more realistic polytropic disk the spiral shocks excited at its outer edge propagate deeply into the high-velocity regions. As a result, our tomograms (Fig. 3) show extended spirals instead of two crescent-shaped maxima reported by Smak (2001). Such spirals are not observed in real disks (Groot 2001). To account for the limited resolution of the observational data we blurred the tomograms to such a degree that the width of the brightest segments of the spiral became comparable to the width of the observed features (\(\sim 400 \div 500\) km s\(^{-1}\); see Groot 2001). However, the spiral pattern extending up to velocities of \(\sim 1000\) km s\(^{-1}\) was still clearly visible. Moreover, both location and relative intensity of the brightest segments of the spiral did not agree with those observed in U Gem.

Should this disagreement be regarded as an argument against the presence of spiral waves in CV disks? Certainly not. As we already indicated in the Introduction, the observed tomograms refer to the distribution of the emissivity in specific spectral lines rather than to the distribution of physical parameters directly obtainable from hydrodynamical simulations. The presently available hydrocodes are not sophisticated enough
Figure 3: Doppler tomograms of compressional power distributions shown in Fig. 2. Top row: raw data. Bottom row: the same data blurred by the convolution with a Gaussian: $\exp\left[-\left(\frac{v_x - v_{x0}}{1000}\right)^2 - \left(\frac{v_y - v_{y0}}{1000}\right)^2\right]$.

to predict the detailed spectrum of the disk, and the correspondence between those two sets of data is by no means clear. However, the models can yield data much more closely related to line emissivity than simple physical parameters or their combinations.

To obtain such data, we assume the line emission to originate mainly due to the irradiation of the surface layer of the disk by the central white dwarf and/or boundary layer (cf. Robinson et al. 1993, Smak 1991). Since our models are not detailed enough to resolve the surface layer, we assume that the lower boundary of the layer coincides with a constant density surface $S_l$ at which $\rho = \rho_l = 10^{-8}$. Typical distance between $S_l$ and the midplane of the disk, $h_l$ was such, that $h_l/r = 0.08$ and $h_l/r = 0.2$ in the inner disk and in the outer ring, respectively. Further, we assume that the line flux from each element of the layer is proportional to the mass contained within that element, $\rho dV$, multiplied by the irradiating flux. For simplicity, we also assume that all irradiating photons are emitted from a point source located at the centre of the white dwarf, so that the irradiating flux is given by $L/r^2$, where $r$ is the radial coordinate of the volume element $dV$, and $L = L_{wd} + L_{bl}$ is the combined luminosity of the white dwarf and the boundary layer. The distribution of brightness on
the \((v_x, v_y)\) plane is now given by the integral
\[
L_{irr}(v_x, v_y) = \int V \rho r^2 S(r, \theta, \phi) B(u_x, v_x, \delta v_x) B(u_y, v_y, \delta v_y) dV. \tag{6}
\]

The function \(S(r, \theta, \phi)\) describes the shadow cast by \(S_l\), and it is given by
\[
S(r, \theta, \phi) = 1 - \mathcal{H} \left( \int_0^r \mathcal{H}(\rho(r', \theta, \phi) - \rho_l) dr' \right), \tag{7}
\]
where
\[
\mathcal{H}(x) = \begin{cases} 
1 & x > 0 \\
0 & \text{otherwise}
\end{cases}
\tag{8}
\]
is the Heaviside step function. The third velocity component, \(v_z\), was neglected in \(6\) because nearly everywhere in the disk its value was smaller than \(\sim 5\%\) of the local azimuthal velocity, \(v_\phi\). Formally, the integral in Eq. 6 subtends the whole space above \(S_l\). Practically, due to steeply falling density, only volume elements closest to \(S_l\) contribute to it significantly. The boundary layer above \(S_l\) has a mass of \(\simeq 0.15 M_{\text{disk}}\), but only about 25\% of its volume is directly illuminated.

Location and relative intensity of the brightest areas on tomograms resulting from the irradiation approach (Fig. 4) agree rather well with those observed by Groot (2001) at an advanced outburst phase of U Gem (his Fig. 2, Episode 2). The major discrepancy is the bright ring visible in
our tomograms at $\sqrt{u_2^2 + v_2^2} \simeq 1000 \text{ km s}^{-1}$. In both models the ring is too bright compared to the observational data, but relative to the maximum intensity obtained in the model it is weaker in B2 where the disk extends down to $r = 0.045$. We conclude that in B1 the ring is enhanced by a spurious contribution from the inner edge of the disk at $r = 0.1$. The ring in B2 would be still weaker if absorption of the irradiating flux by the gas in the surface layer was taken into account in equation (6). Unfortunately, the present models are too crude for such an operation to be reliable. It is clear, however, that the effect of absorption should be particularly strong for $r \lesssim 0.1$, where $h/r$ is nearly constant (see Fig. 4), and the irradiating quanta propagate nearly parallel to $S_l$. Further reduction in ring intensity could probably be achieved if the inner boundary of the grid was moved even closer to the white dwarf, as the shadow cast by $S_l$ at $r < 0.045$ might partly screen the region at $r \sim 0.1$ where the ring originates. On the other hand, in some phases of the activity cycle the intensity of the brightest areas in our irradiation tomograms is underestimated relative to the ring. This is because substantial $p_dV$ work is done by tidal forces directly on the gas in the surface layer of the outer disk, where the low-velocity emission originates.

The subsurface maximum of $p_dV/\rho$ in Fig. 5 originates mainly due to tidal forcing in the direction perpendicular to the orbit. It also contains a contribution from the ambient gas falling onto the disk; however the “rainfall” heating is much less efficient than the tidal one. We checked this by re-running simulation B1 with $\rho_{\text{min}}$ reduced to $3 \cdot 10^{-14}$: at $t \approx 8.0 P_{\text{orb}}$ virtually no changes were seen in the distribution of $p_dV/\rho$.

### 4 Discussion

As discussed in Section 3, we find that spiral waves efficiently propagate from excitation regions at the outer edge of the disk toward the white dwarf, reaching to at least $\sim 0.05a$. This conclusion concerns both two-dimensional and three-dimensional models; it is however limited to hot polytropic disks presented in this paper. The main spiral features seen in the density distribution (two-dimensional case) and in the surface density distribution (three-dimensional case) are hard to distinguish. On the other hand, clear differences are visible in the distributions of the tidal heating rate, $p_dV$: in 3D the two main spiral arms are less tightly wound than in 2D, and a weaker third arm is excited. We suggest that the third arm may originate from tidal forcing in the direction perpendicular to the orbital plane. The effects of this forcing seem to be responsible for the origin of the clear maximum of tidal heating rate per unit mass, $p_dV/\rho$, which is
located away from the midplane in subsurface layers of the outer disk.

Doppler tomograms of tidal heating rate derived from both 2D and 3D models correlate rather poorly with observed tomograms of U Gem (Groot 2001). A better agreement (but still not entirely satisfactory) is obtained for tomograms of the irradiation flux from the white dwarf through the surface layer of the disk. The brightest areas of such tomograms coincide with arches observed in U Gem at an advanced stage of the outburst. The irradiation tomograms can be derived from 3D models only, which indicates that fully three-dimensional modelling is needed for a reliable interpretation of the observational data on DN disks.

According to our results the arches originate in the outer part of the disk, fairly high above the midplane ($h > 0.1r$). For this to happen, the outer disk would have to be substantially bulged. The bulging phenomenon may be explained within the following (not entirely new) scenario: prior to the outburst the gas transferred from the secondary mainly collects in a ring at the circularization radius, and only partly accretes through the disk onto the white dwarf. The ring expands as the gas flows in, but it remains cool until heating from tidal forcing in the orbital plane grows so strong that it cannot be balanced by radiative cooling. The ring begins to expand even more rapidly, and within it the gas located away from the midplane begins to receive additional internal energy from tidal forcing in the direction perpendicular to the orbital plane. Eventually, strong spiral shocks develop, and a dynamical instability of the kind described by Różycki & Spruit (1993) sets in.

Obviously, such scenario cannot be the whole story, as it is not linked to the thermal instability believed to be at least partly responsible for eruptive phenomena in DN and other classes of cataclysmic variables. Nevertheless, it seems to indicate a promising direction of further research.

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