Finite volume corrections to forward Compton scattering off the nucleon

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In collaboration with: Agadjanov, Gegelia, Meißner and Rusetsky

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Nucleon Forward Compton Scattering

The doubly-virtual forward Compton scattering.

Spin-averaged Compton tensor:

\[ T^{\mu\nu} = (q^\mu q^\nu - g^{\mu\nu} q^2) T_1(\nu, q^2) \]
\[ + \frac{1}{m^2} \left\{ (p^\mu q^\nu + p^\nu q^\mu) p \cdot q - g^{\mu\nu} (p \cdot q)^2 - p^\mu p^\nu q^2 \right\} T_2(\nu, q^2) \]

Two scalar amplitudes involved: \( T_1(\nu, q^2) \) and \( T_2(\nu, q^2) \), \( \nu \equiv p \cdot q/m \).
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- the proton-neutron mass difference
- Lamb shift in the muonic hydrogen
Lamb Shift: Polarizability Contribution

Two-photon exchange of lepton-nucleon scattering.

Higher-order proton structure corrections to the Lamb shift in muonic hydrogen: Carlson, Vanderhaeghen 2011

\[ \Delta E_{nS} = \frac{\alpha_{\text{em}} \phi_n^2}{4\pi^3 m_l} \frac{1}{i} \int d^3q \int_0^\infty d\nu \frac{(q^2 - 2\nu^2) T_1(\nu, q^2) - (q^2 + \nu^2) T_2(\nu, q^2)}{q^4[(q^2/4m_l^2) - \nu^2]} . \]

Muonic Lamb-shift \( \rightarrow T_1, T_2 \ (q^2 < 0) \)
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\]

Muonic Lamb-shift $\rightarrow T_1, T_2 (q^2 < 0)$

The $T_1(\nu, q^2)$ can be evaluated using the once-subtracted dispersion integral

A problem: $S_1(q^2) \equiv T_1(0, q^2)$ is not fixed by experiments.
Determination of $S_1$

Reggeon dominance hypothesis. Gasser et al. 2015.

- Chiral effective field theories J. Alarcón et al. 2014
- Phenomenological approaches Walker-Loud et al. 2012 Erben et al. 2014
- Reggeon dominance Gasser et al. 2015
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- Lattice QCD: model-independent
External Field Method

- Using **lattice QCD**, the **Compton tensor** can be studied
- Study the two-point function in an **external** em. field

\[ \text{Detmold et al. 2006} \]

- **Uniform electromagnetic field** → **polarizabilities**  
  \[ \text{Detmold et al. 2006} \]
Nucleon in a Periodic Magnetic Field

- **Static** magnetic field $\rightarrow$ stable energy levels.
- The energy shift of a nucleon on the lattice, using the external field method, is

  Agadjanov, Rusetsky and Meißner 2017, Schierholz et al. 2017

  $$
  \delta E = \frac{e^2 B^2}{4m} S_1(-\omega^2) + O(B^3).
  $$

- \( B = (0, 0, -eB \cos(\omega nx)) \), \( n = (0, 1, 0) \) and \( \omega = \frac{2\pi n}{L} \)
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More recently, this result was rederived in a finite volume

$\delta E = -\frac{1}{4m} \left( \frac{eB}{\omega} \right)^2 T^{11}(p, q) + O(B^3)$.

Kinematics: $p = (m, 0)$, $q = (0, 0, \omega, 0)$. 
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Work left to do: Subtract finite-volume correction to \( T^{11} \rightarrow S_1 \).
Finite Volume Artifacts

- Lattice simulations are done in a finite-volume → finite-volume effects.

- Two kinds of FVE:
  - Type 1: Polarization Effects: exponential
  - Type 2: Multi-hadron intermediate states: power law

In this work, we deal with FVE of the first type.
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- How to estimate them? → baryon ChPT in a finite-volume

- The Lagrangians are the same in the infinite and in a finite volume

- The 3-momentum integrals changed by sums:
  \[
  \int \frac{d^3k}{(2\pi)^3} \rightarrow \frac{1}{L^3} \sum_k, \quad k = \frac{2\pi}{L} n, \quad n \in \mathbb{Z}^3
  \]
Results in the infinite-volume

Model A: $S_{\text{inel}}(0) = (0.8 \pm 2.7) \text{GeV}^{-2} \rightarrow \text{Purely experimental}$

Model B: $S_{\text{inel}}(0) = (-0.7 \pm 1.0) \text{GeV}^{-2} \rightarrow \text{Experimental + Reggeon}$

Model C: $S_{\text{inel}}(0) = (-1.2 \pm 0.5) \text{GeV}^{-2} \rightarrow \text{Experimental + Lattice}$
Results in the infinite-volume

\[ O(p^4) \]

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\end{align*}
$Q^2 = M^2_{\pi}, \text{ neutron}$

$$\Delta \equiv \frac{T_{L}^{11}(p,q) - T_{11}^{11}(p,q)}{T_{11}^{11}(p,q)}$$
\[ Q^2 = 0.1M^2_\pi, \text{ neutron} \]

\[ \Delta \equiv \frac{T^{11}_L(p,q) - T^{11}(p,q)}{T^{11}(p,q)} \]
$Q^2 = 0.01M_{\pi}^2$, neutron

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$Q^2 = M^2_{\pi}$, proton

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\[ Q^2 = 0.1 M_{\pi}^2, \text{ proton} \]

\[ \Delta = \frac{T_{L}^{11}(p,q) - T_{11}^{11}(p,q)}{T_{11}^{11}(p,q)} \]
$Q^2 = 0.01 M^2_{\pi}$, proton

$\Delta \equiv \frac{T_{L}^{11}(p,q) - T^{11}(p,q)}{T^{11}(p,q)}$

Graph showing the behavior of $\Delta$ as a function of $M_{\pi} L$. The graph includes curves for $O(p^3)$ and $O(p^4)$. The graph's $x$-axis represents $M_{\pi} L$ values ranging from 3 to 6, and the $y$-axis represents $\Delta$ values ranging from 0 to 0.05.
A finite-volume calculation of the forward Compton scattering tensor was performed in the framework of ChPT to order $O(p^4)$. For the physical pion mass $\pi^L \simeq 4$, $\Delta$ is not bigger than 3% for both proton and neutron. Low energy constants at $O(p^4)$, although not accurately known, do not pose a problem in the convergence of our results. The extraction of the infinite-volume $S_1$ with good accuracy is possible for reasonable large lattices.
Summary and Conclusion

- A finite-volume calculation of the forward Compton scattering tensor was performed in the framework of ChPT to order $O(p^4)$.
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