Dual superconductivity. Variations on a theme.

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Abstract

It is pointed out that the low energy effective theory that describes the low lying glueballs of the pure Yang Mills theory sustains static classical stringlike solutions. We suggest that these objects can be identified with the QCD flux tubes and their energy per unit length with the string tension.
It is a great pleasure for me to write this paper in honor of Larry Horowitz. Larry has a rare gift of being able to inspire and encourage people around him even when their work has little overlap with his main line of research. I for one, was very lucky to experience the invigorating atmosphere of long and frequent discussions with him, and much of my work is directly or indirectly influenced by it. One of the many research areas of Larry’s is the study of topological objects in field theory, and in particular in QCD. This perhaps is where my own research is closest to his. I have decided therefore to devote this paper to a subject which belongs to this area - the dual superconductor model of confinement in QCD.

Confinement is widely believed to be a property of strong interactions. It is certainly one of the most striking properties of the strong interactions. It is therefore somewhat ironic that it has also proved to be the most difficult one to understand theoretically. So much so that it is not even clear what objects exactly are confined by strong interactions.

A seemingly straightforward answer to this question is that those are perturbative quarks and gluons. A little reflection, however shows that this answer is totally unsatisfactory. Quarks and gluons are the main objects of perturbative QCD, and as such are very helpful concepts in the perturbative context. However in the strict sense quarks and gluons can not even in principle exist as asymptotic states. This is due to the fact that they carry nontrivial color charges. The color group is not a bona fide physical symmetry of the strong interactions, but is a gauge symmetry. Which means that it is not a symmetry of physical states but rather a redundancy of our way of description of strong interaction physics in terms of the standard QCD Lagrangian. Physical states are by construction color singlets and therefore can not contain a single quark or a gluon. As opposed to abelian gauge theories, where the global part of the gauge group is indeed a symmetry of physical states, in nonabelian theories not even the global color transformation is physical. So perturbative partons are confined by construction, and
this confinement does not seem to depend on the dynamics of the theory. For example, to the best of my understanding this “kinematical” confinement will be also present in the $SU(3)$ theory with more than 16 flavors, which is infrared free perturbatively, and which is not expected to be confining in the usual colloquial sense. What we mean by confinement must be some other genuinely dynamical effect.

Even though in non Abelian theories confinement is a somewhat undefined concept, it is intuitively clear that there is a dynamical phenomenon to be understood here. Many models of confinement have been put forth. Perhaps the most widely accepted one is the dual superconductor model due to t’Hooft and Mandelstam [1]. In this note I want to discuss one particular objection to this model if understood literally, and suggest a related but in certain aspects very different model which also exhibits the property of confinement and is motivated by the low energy spectrum of a pure Yang Mills theory.

Let me first briefly review the idea of dual superconductor. The idea itself is very simple and as such certainly very appealing. Consider a superconductor. It is described by the Landau - Ginzburg model of the complex order parameter field $H$ (the Higgs field) coupled to vector potential $A_i$. The free energy, or Lagrangian of this Abelian Higgs model (AHM) in the field theoretical context, for static field configurations is

$$- L = |D_i H|^2 + \frac{1}{2} B^2 + \lambda (H^* H - v^2)^2 \quad (1)$$

where

$$D_i = \partial_i - ig A_i \quad (2)$$

The superconducting phase is characterized by a nonvanishing condensate of the Higgs field

$$< H > = v \quad (3)$$

In this phase it is convenient to work in the unitary gauge, that is to rewrite Lagrangian
in terms of the field
\[ a_i = A_i - \frac{1}{g} \partial_i \phi \] (4)
where
\[ H = \rho \exp\{i\phi\} \] (5)
The Lagrangian then becomes
\[ -L = (\partial_i \rho)^2 + g^2 \rho^2 a_i^2 + \frac{1}{2} B^2 + \lambda (\rho^2 - v^2)^2 \] (6)

As is well known, the only way the magnetic field can penetrate a superconductor is in the form of the flux tubes with the core size of the penetration depth \( l = (gv)^{-1} \). We will recap the derivation of this property here, even though it is very well known since we will need to refer to some of its features later.

Let us consider a superconductor of the linear dimension \( L \), and try to find the configuration with minimal energy subject to condition that it sustains magnetic flux \( \Phi \) in the direction of the \( z \) axis. We also assume of course that \( L >> l \). In the absence of the condensate the vector field is massless. The magnetic field inside the medium therefore is basically the field of a dipole of the size \( L \), and for \( r_{\text{perp}} = \sqrt{x^2 + y^2} << L \) behaves as \( B_z \propto \frac{r_{\text{perp}}^2}{L^4} \). The vector potential is \( a_i \propto \epsilon_{ij} r_j r^2 / L^4 \), and the energy of such a configuration is finite in the limit of large \( L \). Now consider the superconducting state. The vector field is massive. The quadratic part of the Lagrangian becomes
\[ -L = (\partial_i \sigma)^2 + g^2 v^2 a_i^2 + \frac{1}{2} B^2 + m^2 \sigma^2 \] (7)
with \( \sigma = \rho - v \). The vector field \( a \) therefore can not possibly have such long range form in a state with minimal energy as in the nonsuperconducting state. Instead, it should vanish exponentially as \( \exp\{-gvr_{\text{perp}}\} \) for \( r_{\text{perp}} > l \). At first sight this seems impossible, since a vector potential can vanish so fast only in a state with vanishing magnetic flux.
\[ \Phi_z = \int_S d^2r_{\text{perp}} \epsilon_{ij} \partial_i a_j = \int_{C=ds} dr_i \epsilon_{ij} a_j = 0 \] (8)
Consider however a vector potential which far enough from the origin \((r_{\text{perp}} = 0)\) vanishes everywhere except on a half plain (let’s say \(x = 0, y > 0\)):

\[
a_i = \Phi_z \delta_{11} \delta(x) \theta(y)
\]  

(9)

It certainly describes a finite magnetic flux \(\Phi_z\). The singularity in \(a_i\) also does not contribute to the energy, if the singularity is quantized. The reason for that bizarre sounding statement is that the vector field mass term is strictly speaking not \(g^2 v^2 a_i^2\) as it is written in eq.7. Remember that it really is the remnant of the covariant derivative term \(|D_i H|^2\). This expression when written in terms of the phase \(\phi\) is \(g^2 \rho^2 (A_i - \frac{1}{g} \partial_i \phi \mod 2\pi)^2\). It does not feel jumps in the phase \(\phi\) which are integer multiples of 2\(\pi\). Since our field \(a_i\) was defined shifting \(A_i\) by \(\partial_i \phi\) the mass term in the Lagrangian eq.7 is actually

\[
g^2 v^2 (a_i^{\mod 2\pi \Delta})^2
\]

(10)

where \(\Delta\) is the ultraviolet regulator - lattice spacing. For smooth fields \(a_i\) there is no difference between the two mass terms. However eq.() admits singularities of the type of eq.(9). The singularity of this type therefore does not cost energy provided

\[
\Phi_z = \frac{2\pi n}{g}
\]

(11)

with integer \(n\). For these values of the magnetic flux the lowest energy configuration has the form \(a_i = \Phi_z \delta_{11} \delta(x) \theta(y) + b_i(x)\) with \(b_i\) a smooth, exponentially decreasing function. It represents a tube of magnetic flux along the \(z\) axis, or Abrikosov-Nielsen-Olesen (ANO) vortex. Properties of this solution are discussed, for example in [2].

So far we were discussing an ordinary superconductor. Suppose now, that in addition to the charged field \(\Phi\) our model also has very heavy magnetic monopoles. Since they are very heavy, they do not affect the dynamics of the order parameter. But the structure of the vacuum does affect strongly the interaction between the monopoles themselves. A monopole and an antimonopole in the superconducting vacuum feel linear confining
potential. Since a monopole is a source of magnetic flux, this flux in the superconducting vacuum will form a ANO flux tube, which terminates on an antimonopole. The energy of the flux tube is proportional to its length, hence the linear potential.

So, here is a theory in which certain objects are confined by a linear potential. The dual superconductor hypothesis asserts that the Yang Mills theory is basically an “upside down” version of the Abelian Higgs model, as far as the confinement mechanism is concerned. That is, if we use the following dictionary, the preceding discussion describes confinement in any non Abelian gauge theory, which is in the confining phase. The magnetic field of the Higgs model should be called the “color electric field”, the magnetic monopoles should be called the “color charges”, and by analogy the electrically charged field $H$ should turn into “color magnetic monopole”.

There are of course many questions that can be immediately asked. The first thing is who are those mysterious monopoles, which do not even appear in the QCD Lagrangian. But perhaps those could appear as a result of a duality transformation. One can certainly draw some encouragement from the simplest known confining theory - Abelian compact $U(1)$ model. This theory on the lattice is known to have a phase transition at some finite value of the coupling constant. In the strongly coupled phase it is confining. Note that in the Abelian, as opposed to non Abelian case, the global part of the gauge group is a physical symmetry. Charged states do exist in the Hilbert space a priori, and confinement is a purely dynamical effect.

Now, confinement in this theory does have a direct interpretation in terms of dual superconductivity. In fact, in the lattice theory one can perform a duality transformation which transforms the compact $U(1)$ pure gauge theory into a noncompact $U(1)$ theory with charged fields, i.e. Abelian Higgs model[d]. The “Higgs” field in this model actually describes excitations carrying magnetic charge - magnetic monopoles. Those are condensed in the confining phase and dual superconductivity captures perfectly the
physics of confinement in this theory.

The example of the $U(1)$ theory encouraging as it is, still leaves many questions unanswered. In non Abelian theories there is no gauge invariant definition of a monopole, or for that matter even of the flux that it carries. Since even the definition of monopoles depends on the gauge, it is unavoidable that the correlation between the quantitative properties of these monopoles (e.g. monopole density) and confinement (the value of the string tension) will also vary from gauge to gauge. This is indeed known to be the case. In the so called maximal Abelian gauge these correlations are indeed very strong\[5\]. This gauge is designed to make the gauge fixed Yang Mills Lagrangian to be as similar as possible to the compact $U(1)$ theory by attempting to maximally supress the fluctuations of all fields except the colour components of the gauge potential which belong to the Cartan subalgebra. In particular it seems that the monopoles contribute a large share of the string tension \[5\]. Monopole condensate disappears above the critical temperature in the deconfined phase. An interesting recent work attempts to calculate the “effective potential” for the monopole field, and the results are consistent with the “Mexican hat” type potential below the critical temperature and a single well above $T_c$ \[6\].

In other gauges correlations between the monopole properties and the confining properties of the theory are much weaker, and in some cases are completely absent\[4\]. Even in the maximal Abelian gauge it is not clear that the monopole dominance is not a lattice artifact. One expects that if the monopoles are indeed physical objects in the continuum limit, their size would be of order of the only dimensional parameter in the theory $\Lambda_{QCD}^{-1}$. The monopoles which appear in the context of lattice calculations on the other hand are all pointlike objects.

These are worrysome questions, but unfortunately I do not have anything new to add to these points. The point that will concern me in this note is a different one.
One can formulate yet another objection to the dual superconductor picture. The dual superconductor hypothesis in effect states that the effective low energy theory of the pure Yang Mills theory when written in terms of appropriate field variables is an Abelian Higgs model. If so, the low lying spectrum of pure Yang Mills should be the same as that of an Abelian Higgs model.

A statement about the form of the effective Lagrangian is not easily verified, since it necessitates an appropriate choice of variables. On the other hand a statement about the spectrum does not depend on the choice of variables and is in this sense universal and easily verifiable. The spectrum of the Abelian Higgs model is well known. The two lowest mass excitations are the massive vector particle and a massive scalar particle. In fact, the role of the vector particle was crucial in the discussion of confinement in the beginning of this paper. The spectrum of the Yang Mills theory is not known from analytical calculations. However, in recent years a rather clear picture of it emerged from the lattice QCD simulations [7]. The lowest lying particle in the spectrum is a scalar glueball with the mass $1.5 - 1.7$ Gev. The second excitation is a spin 2 tensor glueball with a mass of around 2.2 Gev. Vector (and pseudovector) glueballs are conspicuously missing in the lowest lying part of the spectrum. The simulations indicate that they are relatively heavy with masses above 2.8 Gev [7]. The pattern of the Yang Mills spectrum therefore seems to be rather different than the one suggested by the Abelian Higgs model. Moreover, since the vector glueball is so much heavier than the scalar and tensor ones, it seems very unlikely that it plays so prominent a role in the confinement mechanism as the one played by the massive photon in a superconductor.

This of course can not be completely ruled out. It could happen that for some exotic reason, it is the high energy part of the spectrum that has closer relation to confinement than the lowest lying excitations. However it seems more reasonable to assume that the low energy spectrum of the YM theory should serve as a guide to the confinement
mechanism. In that case it is almost inevitable that the spin 2 glueball should play a very important role. In fact, it is possible that the role of the spin 1 massive photon of the Abelian Higgs model is played by the spin 2 glueball in the real life YM. The purpose of this note is precisely to present this possibility and to explore it on a very simplistic level. In what follows we will write down an effective theory, which should on very general grounds describe the dynamics of the spin 0 and spin 2 glueballs. We will then show that in this theory certain external objects that carry an analog of the magnetic flux are indeed confined. The tensor field that describes the spin 2 glueball causes this confinement in a way very similar to the massive photon field in AHM. We will also show that this model has other similarities with AHM, so that even though its field content is very different, in a certain sense it can be considered as a variant or extension of a dual superconductor model.

We start therefore by writing down a theory which contains a scalar field $\sigma$ and a massive symmetric tensor field $G_{\mu\nu} = G_{\nu\mu}$ with a simple interaction.

$$L = \frac{1}{4}G_{\lambda\sigma}D^{\lambda\sigma\rho\omega}G_{\rho\omega} + \partial_\mu \sigma \partial^\mu \sigma - 2g^2 v \sigma G^{\mu\nu} G_{\mu\nu} - g^2 \sigma^2 G^{\mu\nu} G_{\mu\nu} - V(\sigma) \quad (12)$$

The operator $D$ which appears in the kinetic term of the tensor field is

$$D^{\lambda\sigma\rho\omega} = (g^{\lambda\rho} g^{\sigma\omega} + g^{\sigma\rho} g^{\lambda\omega})(\partial^2 + m^2) - 2g^{\lambda\sigma} g^{\rho\omega}(\partial^2 + M^2) \quad (13)$$

Several comments are in order here. A general symmetric tensor field has ten components. A massless spin two particle has only two degrees of freedom. The tensor structure of the kinetic term for the massless tensor particles is therefore determined so that it should project out two components out of $G_{\mu\nu}$. In fact it is easy to check that the kinetic term in eq.(14) in the massless case ($m^2 = M^2 = 0$) is invariant under the four parameter local gauge transformation

$$\delta G^{\mu\nu} = \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu \quad (14)$$
These four gauge invariances together with four corresponding gauge fixing conditions indeed eliminate eight components of $G^{\mu\nu}$. This gauge invariance is broken by the mass. However the equations of motion even in the massive case lead to four constraints

$$\partial_\mu G^{\mu\nu} = 0$$  \hspace{1cm} (15)

This eliminates four degrees of freedom out of ten\footnote{In the interacting theory, eq.12, the constraint equations are slightly modified, but they still provide four conditions on the fields.}. In addition the scalar field $G^\mu_\mu$ decouples from the rest of the dynamics and can be neglected. In fact we have added a parameter $M$ whose only purpose is to make $G^\mu_\mu$ arbitrarily heavy. All in all therefore we are left with five independent propagating degrees of freedom in $G^{\mu\nu}$, which is the correct number to describe a massive spin two particle.

The scalar glueball self interaction potential $V(\sigma)$ is fairly general and the following discussion will not depend on it. The natural choice to keep in mind is a mass term augmented with the standard triple and quartic self interaction, although one could also consider more complicated logarithmic potential as is becoming a dilaton field.

Our first observation is that the Lagrangian eq.12 allows for static classical solutions which are very reminiscent of the Abrikosov - Nielsen - Olesen vortices. Consider a static field configuration of the form

$$G^{ij} = G^{00} = 0$$
$$G^{0i} = G^{i0} \equiv a^i(\vec{x})$$
$$\sigma \equiv \rho(\vec{x}) - v$$  \hspace{1cm} (16)

It is easy to see that this configuration is a solution of equations of motion, provided $a^i$ and $\rho$ solve precisely the same equations as in the AHM with the only modification that the scalar potential is given by $V(\sigma)$. We will call this configuration the tensor flux.
tube (TFT). In fact it does carry a conserved tensor flux. To see this let us consider the following operator

\[ \tilde{F}^{\mu\nu\lambda} = \epsilon^{\mu\nu\rho\sigma} \partial_{\rho} G_{\sigma\lambda} \]  

(17)

For single valued field \( G_{\mu\nu} \) it satisfies the conservation equation

\[ \partial_{\mu} \tilde{F}^{\mu\nu\lambda} = 0 \]  

(18)

This is the analog of the homogeneous Maxwell equation of the AHM without monopoles

\[ \partial_{\mu} \tilde{F}^{\mu\nu} = 0 \]  

(19)

For our static TFT solution we have

\[ \tilde{F}_{i00} \equiv b_{i} = \epsilon_{ijk} \partial_{j} a_{k} \]  

(20)

and the flux through the plane perpendicular to the symmetry axis of TFT is

\[ \Phi \equiv \int dS_{i} b_{i} = 2\pi/g \]  

(21)

The energy per unit length of the TFT is directly related to the masses of the two glueballs and to the interglueball coupling constants \( g^2 \) and \( \nu \). It is natural to identify this quantity with the string tension of the pure Yang-Mills theory.

One important qualification to the above statements is, that we have assumed that just like in the AHM the mass term (and the interaction terms) of the tensor field allows quantized discontinuities of the form \( 2\pi\Delta \). This is not at all unnatural in the present framework. Note that our effective theory can be thought of as a gauge theory with the “spontaneously broken” gauge group of eq.14. The analog of the phase field \( \phi \) of eq.8 in our model is played by the gauge parameter \( \Lambda_{\mu} \) of eq.14. All that is needed for the discontinuities to be allowed is that \( \Lambda_{\mu} \) (or at least \( \Lambda_{0} \)) be a phase, or in other words for the gauge group to have a \( U(1) \) subgroup. This does not seem to be an extremely unnatural requirement, although to determine whether this is indeed true one
would need to have some information about dynamics on distance scales shorter than the inverse glueball mass.

Continuing the same line of thought, we can identify the objects which should play in our model the same role as magnetic monopoles in the dual superconductor. The fields $G$ and $\sigma$ can couple to much heavier objects, which from the low energy point of view are basically pointlike. At these short distances eq.18 should be modified, and we can consider the current

$$J^{\nu \lambda} = \partial_\mu \tilde{F}^{\mu \nu \lambda}$$

Again I stress that the current $J^{\nu \lambda}$ must have only very high momentum components $k >> m$, otherwise eq.12 will not describe faithfully the low energy sector of the theory. Of course, this situation is very similar to AHM with heavy magnetic monopoles where the homogeneous Maxwell equation is also violated on the distance scales of the order of the monopole size. Now, since $\tilde{F}^{\mu \nu \lambda}$ is antisymmetric under the interchange of $\mu$ and $\nu$, our newly born current is conserved

$$\partial_{\nu} J^{\nu \lambda} = 0$$

The components of the current which are relevant to our TFT configuration are $J^{\mu 0}$. Imagine that the underlying theory does indeed contain objects that carry the charge $Q = \int d^3 x J^{00}$ Then the argument for confinement of these objects is identical to the argument for confinement of magnetic monopoles in AHM. If these objects are identified with heavy quarks this becomes the picture of the QCD confinement from the low energy point of view. Of course, these “quarks” have very little to do with perturbative quarks. But here I appeal to the sentiment from the introductory part of this paper, namely that we really don’t know what should be the relation of the perturbative quarks to the physical confined objects.

\*\*\*At present we do not understand how to reconcile the tensorial nature of the current with the Coleman - Mandula theorem.\*\*\*
This discussion is very far removed from the underlying QCD Lagrangian, and for this reason is not immediately helpful in understanding confinement in terms of the ultraviolet degrees of freedom. This is also not my purpose here. The purpose of this note is to point out that the low energy effective theory that describes the low lying glueballs contains classical stringlike objects which are naturally interpreted as the QCD flux tubes. To verify this interpretation further study is necessary. I hope, however that this discussion can teach us two main lessons. First, that the dual superconductor model should not be thought of narrowly as the Abelian Higgs model, but rather as a much larger class of models that admit classical stringlike solutions.

Second, that the effective low energy theory of YM theory belongs to this class. In this sense our discussion lends support to the extended dual superconductor hypothesis.

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