Selected Aspects of Modelling and Design Calculations of Roller Mills

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Abstract: An analysis of construction was conducted in this study, and the guidelines for designing roller mills used commonly in the agri-food, chemical, power and construction industries were presented. Quick designing of the machines required existence of adequate mathematical models. Within the framework of the study’s realization, a mathematical model was developed and we presented the selected results of calculations of the milling rollers loading from the assembly’s mill bowl. The simulation calculations conducted with the mathematical model unambiguously show that the roller’s rotation around a fixed point considerably influences the total loading of the mill’s bowl, and passing over that phenomenon is a mistake.

Keywords: shredding of grain material; milling; roller mills; design calculations; mathematical modelling; loading of a mill’s working assembly

1. Introduction

The shredding of material, similar to crushing and squeezing out, is one of the main technological operations performed in the agri-food, chemical, power and construction industries. These processes consist of splitting material into smaller parts with the use of working elements of a machine, which overcome the cohesion forces of the material’s particles. The effect of these processes is obtaining material shredded in the crushing and squeezing out processes, and oils and cold-pressed pulp in the process of kneading [1–5]. Material obtained in such a manner can directly, or after processing, be used for consumption purposes, for fodder for animals, as an energetic material during its combustion, as an additive to fossil fuels or in the construction industry [6–13].

The processes of shredding and crushing are the most popular. When selecting the method of shredding, one should take into consideration the mechanical properties of the material to be shredded, choosing a manner of the working elements’ interaction to obtain the required degree of grain material shredding at the lowest possible stress levels to avoid damage. The roller mills, in particular the roller-bowl mills, have found their extended application in the agri-food industry. They are used for the milling and crushing of cereal grain, fertilizers and mineral raw materials, among the others.

The essence of the milling process in the roller mills consists of a mutual interaction of mechanical forces exerted on two surfaces, where one of them rolls on the other one, and between them there is the milled and shredded material. The feed forces are exerted from the mill’s grinding media by the gravity forces, springs or a hydraulic clamp. The bodies of revolution in the form of rolls, toruses, tapers or pebbles act as grinding media. Grind- ing media always roll along the bearing surface, which is made of bowls of flat or shaped materials. In the known designs of mills for brands such as Loesche, Pfeiffer, Polysius and others, the grinding media have the diameter from 200 to 2500 mm [14–16]. They guarantee the material’s crushing effectiveness on the level from 2 to 500 t·h⁻¹.
For the purposes of selecting the design features and parameters of the mills of that type, the guidelines for their design are provided in the available literature [14,16]:

- The roll’s suspension system should be equipped with connecting members which fulfill the perpendicularity requirement with reference to the normal operation of centrifugal force,
- The rolls should be in the shape of a truncated cone, full torus or divided torus,
- The relationship of the width \( s \) of the roll to its diameter \( d \) should amount to between 0.27 to 0.32, depending on the mill’s type,
- There exist the limit values of bowls or rolls’ angular velocities, depending on their geometrical features, where these speeds increase together with the increase of the mill’s dimensions. In the case of bigger mills, the milled material has to overcome the frictional resistance along the longer bowl surface.

The maximum pressure of rolls, having the diameter \( d \) on a bowl, may be determined based on the Börner’s dependence [14]:

- for the mills of the Pfeiffer type:
  \[
  P_{\text{max}} = \left( 12.75 \ d^{2.24} - 0.2 \right) \times 10^4 \ N \tag{1}
  \]
- for the mills of the Loesche type:
  \[
  P_{\text{max}} = \left( 15.7 \ d^{2.60} + 0.4 \right) \times 10^4 \ N \tag{2}
  \]

In the opinion of the study’s authors, there is a failure to consider the maximum pressure of rollers and their angle speeds for the roller mill design. This is of special importance in the reversed system, when the bowl is motionless and the rollers move along the bowl in the rolling motion on a constant radius.

That is why the development of the mathematical model for the loading of the bowl from the roller assembly in the roller mill and the selected simulation calculations are the basic purposes of this study.

2. Materials and Methods

On the stage of development of the mill bowl’s mathematical loading, there have been two assumed premises, from which the following have resulted:

- Considering the description of all the important factors from the point of the view of the internal bowl’s loading, features and design parameters of the milling roller and the axis it is mounted on;
- Application of a simple mathematical recording, accurately mapping the real conditions of the roller mill’s operation.

Exemplary schemes of design solutions of the milling rollers are presented in Figure 1. Figure 2 presents the scheme of the bowl’s loading with the roller assembly and its axis, where it has been assumed that the point of tangency \( A \) of the roller with the background is in a given moment motionless, and the straight line connecting the points \( O \) and \( A \) is the temporary axis of the roller’s rotation.

The value of the reaction force \( P_{\text{max}} \) may be calculated in two ways. The first consists of the fact that we disregard the rotation of a fixed point of a milling roller, and then we have:

\[
P_{\text{max}} = G + R = mg + q(2b - a) \tag{3}
\]

where:

\( G \)–weight of the roller,
\( R \)–resultant loading of the roller’s axis,
\( m \)–mass of the roller
\( g \)–gravitational acceleration,
\( q \) – constant loading of the roller’s axis.

**Figure 1.** Schemes of design solutions for the milling rollers.

**Figure 2.** Scheme of the mill bowl’s loading.

In the second manner of the reaction force’s \( P_{\text{max}} \) calculation we consider in calculations the roller’s rotation on a fixed point. It is particularly important when designing mills equipped with rollers with big dimensions, the diameter of which have even been 2500 mm. According to the Figure 1, between \( \omega_1 \) and \( \omega_2 \), there occurs a dependence:

\[
\omega_2 = \omega_1 \cot \alpha = \frac{\omega_1}{r} b
\]

where:
- \( \omega_1 \) – the roller’s angular velocity in relation to the axis \( z \),
- \( \omega_2 \) – the roller’s angular velocity in relation to its own axis of revolution, which is axis \( y \).

The following dependence may be applied to determine the force \( P_{\text{max}} \):

\[
\frac{dK}{dt} = \overrightarrow{M_0}
\]

According to which, the differential coefficient in relation to the time of angular momentum of the material point system in relation to any optional motionless pole equals the geometrical sum of all the external forces’ moments in relation to the same pole. However, the components of the angular momentum \( K_0 \) may be calculated by determining the components \( K_x, K_y, \) and \( K_z \) from the dependence:

\[
K_{x,y,z} = I_{x,y,z} \omega_{x,y,z}
\]
where:

$I_{x,y,z}$—moments of inertia of a body making the rotational movement in relation to the axes $x$, $y$ and $z$,

$\omega_{x,y,z}$—projections of the angular speeds $\omega$ of vectors on the axes $x$, $y$, $z$.

According to Figure 2, we have:

$\omega_x = 0$; $\omega_y = -\omega_z$; $\omega_z = \omega_1$ that is respectively:

$K_x = 0$; $K_y = -I_y \omega_2$; $K_z = I_z \omega_1$.

Accordingly, we receive the equation:

$$\vec{K}_0 = I_y \vec{\omega}_2 + I_z \vec{\omega}_1$$  \hspace{1cm} (7)

Due to the fact that in the milling process assumes $\omega = \text{const}$, the value of the angular momentum also has not changed, because $K_0 = \text{const}$.

Therefore:

$$\vec{\omega}_1 \cdot \vec{K}_0 = \vec{M}_0$$  \hspace{1cm} (8)

Substituting for the Equation (7) instead for $K_0$ the equation from formula (8), we receive:

$$I_y (\vec{\omega}_1 \cdot \vec{\omega}_2) + I_z (\vec{\omega}_1 \cdot \vec{\omega}_1) = \vec{M}_0$$  \hspace{1cm} (9)

Therefore:

$$I_y \omega_1 \omega_2 + I_z \omega_1^2 = M_0$$  \hspace{1cm} (10)

The second constituent of the Equation (10) equals zero as the product of two parallel vectors and of $|\vec{\omega}_1 \vec{\omega}_2| = \omega_1 \omega_2$, which is why, finally:

$$I_y \omega_1 \omega_2 = M_0$$  \hspace{1cm} (11)

For the mill’s milling roller there are three external forces with reference to the point $0$:

- gravity force of the milling roller ($m \cdot g$),
- reaction force of the mill’s bowl ($P_{\text{max}}$),
- reaction force of the milling roller’s loading from the driving axle ($R = q \left(\frac{2b-a}{2}\right)$).

It has been assumed in the discussions that the axle driving the roller is supported in two points: point $0$ and in the middle of the roller’s width. That is why:

$$M_0 = P_{\text{max}} b - Gb - Rb$$  \hspace{1cm} (12)

Substituting the expression (12) to (11), we find:

$$I_y \omega_1 \omega_2 = P_{\text{max}} b - Gb - Rb$$  \hspace{1cm} (13)

Therefore:

$$P_{\text{max}} = I_y \frac{\omega_1 \omega_2}{b} + mg + q \left(\frac{2b-a}{2}\right)$$  \hspace{1cm} (14)

Assuming that the milling roller has the shape of a cylinder ($I_y = \frac{mr^2}{2}$) and $\omega_2 = \omega_1 \frac{b}{r}$, we find:

$$P_{\text{max}} = \frac{mr \omega_1^2}{2} + mg + q \left(\frac{2b-a}{2}\right)$$  \hspace{1cm} (15)

The expression $\kappa = \frac{mr \omega_1^2}{2}$ constitutes an additional value of a dynamic reaction because of the roller’s rotation around a fixed point. It may be assumed that the values have an impact on the total loading of the mill’s bowl, which is why, in the opinion of the authors of the study, at this stage of the mills’ design the dynamic reaction should not be omitted in case the working elements rotate around a fixed point.
3. Results

The selected simulation calculations aimed at determining the dynamic reaction \( \kappa \) were conducted for different design features and parameters of the milling rollers made of carbon steel of the mass density of \( \rho_{st} = 7850 \text{ kg} \cdot \text{m}^{-3} \) and of cast iron of the mass density of \( \rho_z = 7500 \text{ kg} \cdot \text{m}^{-3} \). For calculation purposes it has been assumed that the diameters of the rollers \( d \) in the mill were within the scope from 270 to 320 mm at their width \( s = 100 \text{ mm} \) (Table 1). The assumed system of values of \( d \) and \( s \) meet the guidelines concerning roller mills’ design, according to which \( s/d = 0.27–0.32 \). Moreover, it has been assumed that the angular speed of the roller \( \omega_1 \) with reference to the axle falls into the range of values from 0.7 to 1.6 rad\( \cdot s^{-1} \) and that the dimensions \( a \) and \( b \) (in accordance with the Figure 2) respectively assume the values \( a = 500 \text{ mm} \) and \( b = 550 \text{ mm} \).

![Table 1](image)

| Marking of a Roller | Diameter of a Roller (mm) | Width of a Roller (mm) | Dimensions | Weight of a Roller (kg) |
|---------------------|---------------------------|------------------------|------------|------------------------|
| A                   | 270                       | 100                    | 500        | 550                    | N9E | EN-GJS-700-2 |
| B                   | 280                       |                        |            |                        | 44.95 | 42.94 |
| C                   | 290                       |                        |            |                        | 48.34 | 46.18 |
| D                   | 300                       |                        |            |                        | 51.85 | 49.54 |
| E                   | 310                       |                        |            |                        | 55.49 | 53.01 |
| F                   | 320                       |                        |            |                        | 59.25 | 56.61 |

Detailed values of simulation calculations of the reaction force’s value \( P_{max} \), of additional dynamic reaction \( \kappa \) and its percentage share in the reaction force’s value \( P_{max} \) for individual design features and parameters are presented in the Tables 1–3. In the mentioned tables, letters A–F mark the milling rollers made of carbon steel N9E and of cast iron EN-GJS-700-2 in sequence from the lowest to the highest diameter \( d \).

![Table 2](image)

| Roller       | \( \omega_1 \) (rad s\(^{-1} \)) |
|--------------|-------------------------------|
|              | 0.7                           |
|              | 1.0                           |
|              | 1.3                           |
|              | 1.6                           |
| N9E          | EN-GJS-700-2                  |
| A \( P_{max} \) (N) | 7188.74 7074.05 8098.98 7943.59 |
| \( \kappa \) (N)  | 2123.88 2028.91 2043.12 2084.45 |
| B \( P_{max} \) (N) | 7466.77 7339.74 8481.91 8309.52 |
| \( \kappa \) (N)  | 2366.66 2262.82 2383.80 2323.60 |
| C \( P_{max} \) (N) | 7763.93 7624.04 8891.67 8701.53 |
| \( \kappa \) (N)  | 2631.38 2514.15 2759.12 2391.65 |
| D \( P_{max} \) (N) | 8081.48 7926.95 9330.01 9119.67 |
| \( \kappa \) (N)  | 2913.22 2783.02 3416.75 3795.75 |
| E \( P_{max} \) (N) | 8419.45 8250.33 9797.01 9566.51 |
| \( \kappa \) (N)  | 3214.31 3071.09 4591.87 4387.27 |
| F \( P_{max} \) (N) | 8778.48 8593.55 10293.61 10041.24 |
| \( \kappa \) (N)  | 3535.28 3377.92 5050.40 4823.60 |

\( \omega_1 \)—the roller’s angular velocity in relation to the axis \( z \).

In Figure 3, the impact of the milling roller’s mass \( m \) made of steel on the value of the dynamic reaction \( \kappa \) for the assumed angular speeds \( \omega_1 \) is presented.

In Figure 4, the impact of the value of the milling roller’s mass \( m \) on the reaction value \( \kappa \) when the roller is made of cast iron is presented.
Table 3. Results of calculations of percentage shares of dynamic reaction \( \kappa \) in the mill’s bowl loading.

| Roller | \( \omega_1 \) (rad \cdot s\(^{-1}\)) | \( \omega_1 \) (rad \cdot s\(^{-1}\)) | \( \omega_1 \) (rad \cdot s\(^{-1}\)) | \( \omega_1 \) (rad \cdot s\(^{-1}\)) |
|--------|-----------------|-----------------|-----------------|-----------------|
|        | N9E EN-GJS-700-2 | N9E EN-GJS-700-2 | N9E EN-GJS-700-2 | N9E EN-GJS-700-2 |
| A      | 29.54           | 28.68           | 37.46           | 36.49           |
| B      | 31.72           | 30.83           | 39.89           | 38.90           |
| C      | 33.89           | 32.98           | 42.28           | 41.28           |
| D      | 36.05           | 35.11           | 44.61           | 43.60           |
| E      | 38.18           | 37.22           | 46.87           | 45.86           |
| F      | 40.27           | 39.31           | 49.06           | 48.06           |

Figure 3. Impact of the roller’s weight \( m \) made of steel N9E and its angular speed \( \omega_1 \) on the dynamic reaction \( \kappa \) value.

Figure 4. Impact of the roller’s mass \( m \) made of cast iron and its angular speed \( \omega_1 \) on the dynamic reaction’s value \( \kappa \).

From the obtained simulation calculation results (Figures 3 and 4), it unambiguously shows that together with the increase of the roller’s weight \( m \) and its angular speed \( \omega_1 \), the reaction value \( \kappa \) increases approximately in the quadratic function, where the highest values of \( \kappa = 8080.64\) N for the roller made of steel and for \( \kappa = 7720.96\) N for the roller made of cast iron were received for the roller’s angular speed \( \omega_1 = 1.6\) rad \cdot s\(^{-1}\).

In Figure 5, the impact of the milling roller’s weight \( m \) made of steel on the value of reaction \( P_{\text{max}} \) for angular speeds \( \omega_1 \) is presented.
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4. Discussion

From the analysis of the results presented in Figures 5 and 6, it unambiguously shows that together with the increase of the roller’s weight $m$ and its angular speed $\omega_1$, the reaction value $P_{\text{max}}$ increases approximately in the quadratic function. The highest values $P_{\text{max}}$ assumes for the angular speed of the roller $\omega_1 = 1.6 \text{ rad s}^{-1}$, respectively, and $P_{\text{max}} = 13,323.85 \text{ N}$ for a roller made of steel and $P_{\text{max}} = 12,936.60 \text{ N}$ for a roller made of cast iron.

In Figure 7, the chart of percentage shares of dynamic reaction $\kappa$ in the complete value of reaction $P_{\text{max}}$, respectively, for both the roller materials is presented.
From the analysis of the chart in Figure 7, it unambiguously shows that the share of $\kappa$ is very big and for the variable system at the roller’s angular speed $\omega_1 = 1.3 \text{ rad} \cdot \text{s}^{-1}$ and the angular speed $\omega_1 = 1.6 \text{ rad} \cdot \text{s}^{-1}$, it may even exceed 50%.

In order to compare the obtained research and calculation results with those available in the literature, the authors state that others have not yet analyzed the effect of the additional loading of mill working elements due to the occurrence of the spherical motion effect. Only general mathematical models in this field have been presented in the literature [16–20].

The authors’ calculation model presented in this paper takes into account the gyroscopic phenomenon taking place during the crushing process. This results in an increased loading of the mill bowl with an additional dynamic reaction from the working unit, which may account for 60.65% of the total load. Therefore, using the calculation model proposed in the article, one gets a full picture of the actual loads on the working unit and the mill bowl, which should translate into a proper selection of their design parameters.

5. Conclusions

The processing of agri-food, chemical, power and construction industries’ raw materials products are being conducted, among the others, in gravitational, mixing, impact, vibration or roller mills. Over the last few years their big development has been observed, which has been aimed at the increase of their output and limitation of the unit energy consumption while maintaining the correct shredding degree of the milled material. Quick design of new mill constructions, in particular for roller ones, requires the development of adequate mathematical models. Within the frames of the study, a mathematical model of loading of the bowl from the working assembly of the roller for a roller mill was developed. It comprised an additional member of dynamic reaction $\kappa$, which has been so far omitted in the professional literature at the stage of design calculations of roller mills, and its share in the bowl’s calculations is considerable. The conducted simulation calculations on the developed mathematical model have shown that, for example, for a steel roller N9E of the weight of $m = 63.13$ kg and its angular speed $\omega_1 = 1.6 \text{ rad} \cdot \text{s}^{-1}$, its share amounts to 60.65%. However, for a roller made of cast iron EN-GJS-700-2 of the weight of $m = 60.32$ kg and an angular speed $\omega_1 = 1.6 \text{ rad} \cdot \text{s}^{-1}$, it amounts to 59.68%.

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