Supersymmetric superheavy dark matter

V. Berezhinsky,1,2 M. Kachelrieß,3 and M. Aa. Solberg3

1INFN, Laboratori Nazionali del Gran Sasso, I–67010 Assergi (AQ), Italy
2Institute for Nuclear Research of the RAS, Moscow, Russia
3Institutt for fysikk, NTNU Trondheim, N–7491 Trondheim, Norway

(Dated: October 16, 2008)

We propose the lightest supersymmetric particle (LSP) as a well-suited candidate for superheavy dark matter (SHDM). Various production mechanisms at the end of inflation can produce SHDM with the correct abundance, \( \Omega_{\text{LSP}}h^2 \sim 0.1 \), if its mass is sufficiently high. In particular, gravitational production requires that the mass \( m_{\text{LSP}} \) of the LSP is above \( 3 \times 10^{13} \) GeV. Weak interactions remain perturbative despite the large mass hierarchy, \( m_{\text{LSP}} \gg m_Z \), because of the special decoupling properties of supersymmetry. As a result the model is predictive and we discuss the relevant cosmological processes for the case of a superheavy neutralino within this scheme.

PACS numbers: 95.35.+d, 12.60.Jv, 14.80.Ly

I. INTRODUCTION

Uncovering the nature of dark matter (DM) is one of the most pressing problems of current research in particle physics and cosmology. A wealth of observational data suggests that a viable DM candidate has to be non-baryonic and should be non-relativistic, at least from the time of matter-radiation equilibrium on \([1]\). The various baryonic and should be non-relativistic, at least from the data suggests that a viable DM candidate has to be non-thermal relics have either sufficiently small interactions or a high enough mass \( m_X \) to never be produced efficiently by processes like, e.g., \( e^-e^+ \rightarrow XX \).

The present relic abundance \( \Omega_X \) of a thermal relic scales approximately as \( \Omega_X \propto 1/\sigma_{\text{ann}} \) with its annihilation cross section \( \sigma_{\text{ann}} \). Moreover, unitarity of the \( S \) matrix restricts annihilations into the \( l \)-th partial-wave of particles with relative velocity \( v_{\text{rel}} \) as \( \sigma_{\text{ann}}^{(l)} \lesssim (2l+1)\pi/(v_{\text{rel}}M_X^2) \). Since for non-relativistic point-particles higher partial-waves are suppressed, the observed value \( \Omega_{\text{CDM}}h^2 = 0.11 \) of the DM abundance constrains the mass of any thermal relic as \( m \lesssim 100 \) TeV. On the other hand, the requirement that the DM is cold translates for thermal relics into a lower mass limit of the order 10 keV. Thus the mass of thermal relics should lie in the 10 keV – 100 TeV range.

Supersymmetry (SUSY) provides with the lightest neutralino one of the most attractive candidates for thermal DM, see Ref. \([4]\) for a review. Low-energy SUSY models are a natural extension of the standard model (SM), offering a solution to the hierarchy mass problem and a realization of electroweak symmetry breaking by radiative corrections \([5]\). The typical range of neutralino masses in these models extends from a few tens of GeV up to \( \sim 10 \) TeV.

Two notable non-thermal DM candidates are axions and superheavy DM (SHDM) particles. Axions were proposed as solution to the strong CP problem, but the still viable “axion window” includes the possibility that axions are the main contribution to the DM abundance. Depending on the inflationary scenario, axions may be non-thermally produced either by the misalignment mechanism or the decay of axionic topological defects \([6]\).

We will review the status of the SHDM model in the next section, before we discuss “superheavy supersymmetry” in Sec. III. Results for the relevant cosmological processes of a superheavy neutralino like elastic scattering on light fermions, self-scatterings and annihilations are presented in Sec. IV. After that, we review various possible production mechanisms of SHDM for the special case of a superheavy neutralino in Sec. V and summarize in Sec. VI.

II. SUPERHEAVY DARK MATTER

The first proposal of SHDM in Refs. \([7,8]\) was motivated by observations of ultrahigh energy cosmic rays (UHECR), which revealed not the expected suppression of the energy spectrum due to the interaction of extragalactic protons with cosmic microwave photons. Since CDM is gravitationally accumulated in the halo of our galaxy, the secondaries produced in decays or annihilations of SHDM do not suffer energy losses and their energy spectrum is characterized by a flat spectrum up to the kinematical cutoff \( m_X/2 \). In this scenario, the mass \( m_X \) of the SHDM particle should exceed \( 10^{12} \) GeV. Meanwhile, this original motivation for proposing SHDM has disappeared in the light of new UHECR data compatible with the expected flux suppression, for more details see Ref. \([9]\).

Superheavy dark matter particles with the density required by cosmological observations can be efficiently produced at inflation by many mechanisms including thermal production \([9,8,10]\). The most detailed description of this process within an inflationary framework is given in Ref. \([11]\).

A variety of different production mechanisms can pro-
vide a non-thermal distribution of superheavy particles in the expanding universe. Since the energy density of non-relativistic particles decreases slower than the one of radiation, their abundance increases by the factor $a(t_0)/a(t_\ast)$ with respect to radiation, where $a(t_0)$ and $a(t_\ast)$ are the scale factors of the universe today and at the epoch of particle generation, respectively. If particle production happens at the earliest relevant time, i.e. during inflation, this factor can become extremely large, $\sim 10^{22}$. Not surprisingly, such a small energy fraction can be transferred to SHDM particles by many different mechanisms, as thermal production at reheating [2, 10], the non-perturbative regime of a broad parametric resonance at preheating [12, 13], and production by topological defects [2, 14].

We discuss first the generation of superheavy particles by gravitational interactions from vacuum at the end of inflation [13, 16]. Since this production mechanism relies only on the gravitational coupling of the SHDM particle, it is unavoidably present in contrast to other, more model-dependent generation mechanisms. Neither inflation is needed for this production, it rather limits the gravitational production of the particles. Since this production is caused by the time variation of the Hubble parameter $H(t)$, only particles with masses $m_X \lesssim H(t)$ can be produced. In inflationary scenarios $H(t) \lesssim \dot{\phi}$, where $\dot{\phi}$ is the mass of the inflaton. It results in the limit on the mass of the produced particles, $m_X \lesssim 10^{13}$ GeV [15, 16].

The numerical calculations of Ref. [16] for the present abundance of fermionic SHDM can be approximated as

$$\Omega_X h^2 \approx \frac{T_R}{10^8 \text{GeV}} \begin{cases} (m_X/H_1)^2, & m_X \ll H_1 \\ \exp(-m_X/H_1), & m_X \gg H_1 \end{cases},$$

(1)

where $H_1 \approx 10^{13}$ GeV is the Hubble parameter during inflation, and $T_R$ is the reheating temperature.

Other generation mechanisms can occur additionally to gravitational production. If the SHDM particles couples directly or through an intermediate particle to the inflaton field, the time-dependence of the classical inflaton field induces particle production. SHDM particles may be also efficiently produced at preheating [12]. This stage, predecessor of reheating, is caused by oscillations of the inflaton field relative to the minimum of the potential after inflation. Such an oscillating field can produce non-perturbatively in the regime of a broad parametric resonance intermediate bosons which then decay to SHDM particles. The mass of the SHDM particles can be one or even two orders of magnitude larger than the inflaton mass.

Another mechanism is the so-called instant preheating [13]. It works only in specific models, where the mass of the intermediate boson $\chi$ is proportional to the inflaton field, $m_\chi = g\phi$. When the inflaton crosses the potential minimum $\phi = 0$, $\chi$ particles are massless and they are efficiently produced. When $|\phi|$ increases, $m_\chi$ increases, too, and can reach values up to the Planck mass.

While these additional production mechanisms can increase the abundance of SHDM particles relative to Eq. (1), entropy production as for instance in thermal inflation can reduce their abundance. Therefore there exists only a lower, not an upper limit for the mass of SHDM, arising from the condition that the SHDM particles do not reach chemical equilibrium.

What are the particle candidates for SHDM?

The first problem one meets is the particle life-time. Superheavy particles are expected to be very short-lived: Even gravitational interactions, e.g. described by dimension 5 operators suppressed by the Planck mass, result in lifetimes much shorter than the age of the universe $t_0$. Superheavy particles must be thus protected from fast decays by a symmetry which is respected even by gravity. Such symmetries are known: They are discrete gauge symmetries. These symmetries can be very weakly broken, e.g. by wormhole [3] or instanton effects [8], to provide a sufficiently long lifetime $\tau$, $\tau \gtrsim t_0$. A systematic analysis of broken discrete gauge symmetries is given in Ref. [17]. For instance, the lifetime of SHDM with mass $m_X \sim 10^{13} - 10^{14}$ GeV was found to be in the range $10^{11} - 10^{26}$ yr in the case of the symmetry group $Z_{10}$.

Various models that contain either absolutely stable or unstable particles with life-times larger than the age of the universe have been discussed [17, 18, 19]. Most of the suggested SHDM candidates belong to a new sector that has no tree-level interactions with SM particles. By contrast, we study in this work the possibility of having a SHDM particle with SM-like couplings to the weak gauge bosons. Since the longitudinal part of gauge bosons couples as $\propto gM_X/m_Z$ to a particle with mass $M_X$, weak interactions become generically strong for $M_X \gtrsim m_Z$ and thus the perturbative expansion fails. Using partial-wave unitarity, Chanowitz, Furman and Hinckliff [20] derived thereby an upper limit of $M_X \sim 10^5 - 10^6$ GeV for particles coupling with SM strength to the weak gauge bosons. An exception to this bound are supersymmetric theories, if only mass terms are added that break supersymmetry (SUSY) softly [21], and in particular the minimal supersymmetric extension of the SM (MSSM) [22]. Therefore we are led to suggest superheavy supersymmetry, i.e. the case where all masses of supersymmetric particles are of order $10^{11}$ GeV or larger, as a concrete model for SHDM with SM weak interactions. For definiteness, we choose the LSP as the lightest neutralino but we note that other possibilities as a sneutrino are also viable.

### III. SUPERHEAVY SUPERSYMMETRY

Our main motivation for the introduction of superheavy supersymmetry is the search for a particle candidate for superheavy dark matter. As discussed in the previous section, the particle candidates found so far [17, 18, 19] are in new particle sectors, such as e.g. the hidden sectors of supergravity or string models. In this section we discuss a more natural and more familiar
candidate, the lightest supersymmetric particle, in the case that all supersymmetric particles are superheavy. The longevity of this dark matter candidate is provided by a $Z_2$ discrete gauge symmetry (R-parity) and its production is guaranteed by gravitational interactions in any standard inflationary scenario. The scale of SUSY breaking may be determined by LHC experiments, and our model can be soon falsified by the discovery of low-scale SUSY at LHC.

Independent of the scale of symmetry breaking, supersymmetry remains an inevitable feature of any theory that wants to unify internal gauge symmetries such as SU(5) or SO(10) with the symmetry group of Minkowski space-time, the Poincaré group. Promoted to a local symmetry, gravity and gauge interactions are on a similar footing, with the gravitino as gauge field of gravity. Another motivation to consider (superheavy) SUSY is that it is a generic ingredient of consistent string theories. Thus various esthetical reasons suggest that SUSY may be realized in Nature.

Below we discuss the status of superheavy supersymmetry in comparison with low-scale symmetry breaking.

There are three pieces of evidence pointing towards "low-scale SUSY", i.e. a mass scale $M_{\text{SUSY}}$ of the supersymmetric partners of the SM particles below or around 1 TeV. First, the unification of coupling constants fails in the SM, while the three couplings meet in the MSSM assuming $M_{\text{SUSY}} \sim 1 \text{ TeV}$ [22]. Second, the fine-tuning problem of the SM Higgs is remedied in the MSSM, only if the mass splitting between the SM particles and their SUSY partners is small enough. Third, the SM contains no suitable DM candidate, while the lightest supersymmetric particle (LSP) of the MSSM, assuming that $R$ parity is conserved, is a promising CDM candidate. Assuming further that the LSP is a thermal relic requires again that at least part of the SUSY mass spectrum is below or close to the TeV scale.

These attractive properties of low-scale SUSY are overshadowed by several less appealing features: Low-scale SUSY predicts generically excessive flavor and CP violation as well as proton decay through dimension-5 operators that is close to or exceeds observational bounds. Moreover, the non-observation of SUSY particles with masses around 100 GeV re-introduces a "small" fine-tuning problem [24]: For instance, electroweak symmetry breaking requires that

$$\frac{m_{\tilde{Z}}^2}{2} = \frac{m_{\tilde{Z}}^2 - m_{\tilde{W}}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2,$$

where $m_i$ are the usual mass parameters of the Higgs potential that depend implicitly on the soft SUSY breaking masses. For $m_{\tilde{Z}}^2 \ll m_{\tilde{W}}^2 \mu^2$, a certain amount of cancellation between the terms on the RHS is required. Similarly, an upper limit on neutralino mass mass of 200 GeV arises if one limits accidental fine tuning to the level of 1% [25]. Moreover, most of the so-called “bulk region” in which the lightest neutralino has naturally the correct DM abundance is meanwhile excluded [26].

Recently, split SUSY has been proposed as a model avoiding the problems of low-scale SUSY while keeping gauge coupling unification [27]. In this model, the mass spectrum of SUSY particles is separated in two parts: Gauginos and gluinos are kept at the TeV scale providing with the lightest neutralino a suitable thermal DM candidate, while all scalars additional to the SM Higgs are heavy, with masses possibly close to the GUT scale. Motivated by the cosmological constant problem and the landscape picture [28] suggested by string theory, the naturalness principle is given up, keeping as guiding principles only experimental observations: The existence of DM, and the hint for GUT from gauge coupling unification.

In this work, we go one step further by abandoning also for the gaugino and gluino masses the weak scale. This becomes possible because we assume that the LSP is produced non-thermally at the end of inflation. As a result, the mass of the LSP should be generically above $\sim 3 \times 10^{11} \text{ GeV}$. The mass of the gluinos and of the SUSY scalars could be either close or, in a similar but not as extreme set-up as in split SUSY, much larger. In such a set-up, alternative approaches as modular [29] or conformal invariance may provide a solution to the hierarchy problem and to gauge coupling unification.

Experimental data from LHC will soon decide if the supersymmetric particles are at least partly close to the weak scale. If this is not the case, then both the MSSM and split supersymmetry are disfavoured. Our proposal that superheavy LSPs (SHLSP) are the DM particles may be then an interesting alternative connecting SUSY to the physical world. The prospects to detect DM in the form of stable superheavy neutralinos will be discussed in a subsequent work [31].

### A. Neutralino as LSP

We assume throughout that the lightest neutralino $\chi_1$ is the lightest of the supersymmetric particles. The neutralino mass matrix $M_\chi$ in the $(B, W^0, H^0_1, H^0_2)$ basis is given by [32]

$$
\begin{pmatrix}
M_1 & 0 & -c_\beta m_Z s_W & m_Z s_W s_\beta \\
0 & M_2 & c_W c_\beta m_Z & -c_W m_Z s_\beta \\
-c_\beta m_Z s_W & c_W c_\beta m_Z & 0 & -s_\beta \\
m_Z s_W s_\beta & -c_W m_Z s_\beta & -s_\beta & 0
\end{pmatrix}
$$

with $s_\beta = \sin \beta$, $c_\beta = \cos \beta$ where tan $\beta = v_1/v_2$ is the ratio of the two Higgs vevs, $s_W = \sin \vartheta_W$, $c_W = \cos \vartheta_W$ with $\vartheta_W$ as Weinberg angle, and $\mu$ as the Higgs mixing parameter.

The consequences of the limit $M_1, M_2, |\mu| \gg m_Z$ for the neutralino has been already extensively discussed for a neutralino as thermal relic [32]. Neglecting the terms of order $m_Z$, the four neutralino mass eigenstates become a pure bino, wino, and the symmetric and anti-symmetric
In order to decide which of the two higgsino combinations to the annihilation cross section into fermions with mass $m$, the annihilation cross section would be independent from $m$ and $\mu$. Assuming that there are no cancellations between the mass parameters, the sign of $\mu > 0$ in the following.

This case of a (partially) degeneration between the neutralino mass parameters $\mu$, $M_1$, and $M_2$ is the LSP for $|\mu| \ll M_1, M_2$. To be definite, we shall choose always $\mu > 0$ in the following.

If the mass difference between the two lighter of the three mass parameters $\mu$, $M_1$, and $M_2$ is not degenerate, the LSP for $|\mu| \ll M_1, M_2$. This case of a (partially) degeneration between the neutralino mass parameters $\mu$, $M_1$, and $M_2$ was dubbed well-tempered neutralino and discussed in Ref. [34].

**B. Unitarity for superheavy particles**

We illustrate with two explicit examples how superheavy SUSY avoids that longitudinal gauge bosons become strongly coupled to neutralinos for $m_\chi > m_Z$. The first case is the annihilation of neutralinos into fermion pairs in the limit of zero relative velocity $v$. Then the amplitude consists of sfermion, $Z$, and $A$ exchange, $\mathcal{M} = \mathcal{M}_Z + \mathcal{M}_A + \mathcal{M}_{s\ell}$. An inspection of the annihilation cross section given e.g. in Ref. [34] shows that the longitudinal component $Z_L$ contributes the term

$$\langle \sigma v \rangle_{Z_L} = \frac{\beta_f^3}{16\pi} \frac{g^4}{c_W} |O_{1L}''|^2 T_3^2 \frac{m_f^2}{m_Z^2}$$

(7)

to the annihilation cross section into fermions with mass $m_f$, isospin $T_3$, coupling $O_{1L}''$, and $\beta_f^2 = 1 - m_f^2/m_Z^2$. Assuming that there are no cancellations between $\mathcal{M}_Z$ and $\mathcal{M}_A + \mathcal{M}_{s\ell}$ as well as that the factor $O_{1L}''$ is of $O(1)$, the annihilation cross section would be independent from $m_\chi$ and thus violate perturbative unitarity for $m_\chi > m_Z$.

The resolution of this apparent problem lies partly in the specific form of the neutralino coupling and mass matrix. The interaction between $Z^0$ and the neutralinos are given in the unitary gauge by the Lagrangian [4]

$$\mathcal{L}_{Z\chi}\chi^0 = -\frac{g}{2c_W} Z_\mu \left[ \bar{\chi}_u^0 \gamma^\mu (O^L_{nm} P_L + O^R_{nm} P_R) \chi^0_{mn} \right],$$

(8)

where $n = m = 1$ yields the interaction between $Z^0$ and the LSP. Moreover, expressing $O_{ij}^{L,R}$ by the neutralino mixing matrix elements gives

$$O_{nm}^{\nu L} = -O_{mn}^{\nu R*} = \frac{1}{2} (-N_{3n} N_{3m} + N_{4n} N_{4m}).$$

Neglecting as always below possible CP violation, the coupling becomes simply $\propto (N_{13}^2 - N_{14}^2)$ for $m = n = 1$. For a completely (anti-) symmetric higgsino, this coupling vanishes because of $|N_{13}| = |N_{14}|$, while a bino and wino LSP have $|N_{13}| = |N_{14}| = 0$ in the limit $m_Z \to 0$. The leading contribution is thus suppressed by $(m_Z/M_{\mathrm{SUSY}})^2$ and given by

$$O_{11L}' = \begin{cases} \cos(2\beta) m_\chi^2 m_f^2, & \text{if } M_{1L} \ll M_2, \\ \frac{c_W^2 (\mu - \mu_2)}{2(\mu - \mu_2)} m_\chi^2, & \text{if } M_{1L} \ll M_2, \\ \frac{c_W^2 (\mu - \mu_2)}{2(\mu - \mu_2)} m_\chi^2, & \text{if } M_{1L} \ll M_2, \end{cases}$$

(10)

Thus this coupling vanishes in the limit $m_\chi/m_Z \to \infty$, because the longitudinal components of the gauge bosons couple only to the deviation from a completely (anti-) symmetric mixing of the higgsino components. Although the approximations are valid only for non-degenerate masses, this conclusion holds also for degenerate masses, because $|N_{13}| \approx |N_{14}|$ remains valid. Taking into account this suppression factor from the neutralino mixing matrix, already the single term from $Z_L$ exchange in the annihilation cross section is consistent with the unitary bound, $\langle \sigma v \rangle_{\mathrm{ann}} \propto 1/m_\chi^2$.

Another way to understand how the apparently dangerous terms $m_\chi/m_Z$ disappear in physical quantities is to compare the coupling of the $Z$ and its goldstone boson $G_Z$ in different gauges. For simplicity, we use for this comparison the unitary and the $R_c$-gauge restricted to $\xi = 1$. Then the $Z$ propagator becomes purely transversal in the $R_c$-gauge, and the interactions of neutralinos with the goldstone $G_Z$ have to agree with those with the longitudinal part of the $Z$ boson in unitary gauge.

We consider as example the annihilation of neutralinos into a $Z$ and the lightest CP even higgs boson in the limit of vanishing relative velocity $v$. Then the longitudinal part $Z_L$ gives for $\chi(p) + \chi(p') \to Z(k) + h(k')$ annihilation at rest [35]

$$\mathcal{M}(\chi\chi \to Z h)_{Z_L} = \frac{-ig^2 O_{11}'' m_Z^2}{2c_\omega^2} \chi \cdot \epsilon(k) \frac{q \cdot \epsilon(k)}{q^2 - m_Z^2},$$

(11)

where $q = p + p'$ denotes the momentum of the virtual $Z_L$ and $\epsilon$ the polarization vector of the real $Z$ boson. Although the neutral Goldstone boson is part of the $Z$
boson in the unitary gauge, its coupling to neutralinos differs and is given by

\[ C_{\chi n \chi m}^G = \frac{ig O_{nm}^G}{2 \cos \vartheta_W} \]  

(12)

with

\[ O_{nm}^G = (N_{n2} c_W - N_{n1} s_W) (c_\beta N_{m3} + s_\beta N_{m4}) + (n-m). \]  

(13)

For the lightest neutralino annihilation, \( n = m = 1 \), and since the coupling is imaginary, the Goldstone only couples to the axial part of the neutralino. The amplitude for the Goldstone exchange diagram is thus

\[ \mathcal{M}(\chi \chi \rightarrow Zh)_G = i \frac{g^2 O_{11}^G}{2 c_W^2} \chi_5 \chi \frac{q_\varphi (k)}{q^2 - m_Z^2}. \]  

(14)

Comparing (11) and (14), gauge independence requires as relation between the couplings

\[ O_{11}^Z \frac{m_Z}{m_Z} = - \frac{1}{2} O_{11}^G, \]  

(15)

or expressed in terms of the neutralino mixings and masses,

\[ (N_{14}^2 - N_{15}^2) \frac{m_Z}{m_Z} = \]

\[ = - (c_W N_{12} - s_W N_{11}) (s_\beta N_{14} + c_\beta N_{13}). \]  

(16)

The authors of Ref. \[35\] noted that this relation can be derived directly from the definition of the neutralino mixing matrix \( N \),

\[ (NM)_{nm} = m_n N_{nm}, \]  

(17)

since the identity

\[ (m_n + m_m) (NP N^{-1})_{nm} = (N (MP + PM) N^{-1})_{nm} \]  

(18)

holds for any matrix \( P \). Choosing for \( P \) the isospin operator that flips the first higgsino sign compared to the second one, \( P = \text{diag}(0, -\sigma_3) \), reproduces a generalization of Eq. \[16\]. It is the special structure of the neutralino mass matrix that makes \( PM + MP \) off-diagonal, and thereby leads to the vanishing of the left-hand side of \[18\] with \( m_Z \).

Finally, superheavy particles in a theory with chiral particles may lead to radiative effects that do not vanish for \( m_\chi \rightarrow \infty \). The authors of Ref. \[22\] discussed in a series of works, if the SM can be viewed as the low-energy limit of the MSSM in the sense of the Appelquist-Carazzone theorem \[36\]. They showed that all virtual effects of the SUSY particles are either suppressed by inverse powers of their mass or can be absorbed in the renormalization of SM parameters \[41\]. In conclusion, superheavy SUSY particles do neither lead to a violation of perturbative unitarity or to non-decoupling effects in virtual corrections.

### IV. RELEVANT PROCESSES AND CROSS SECTIONS

#### A. Elastic scattering on fermions and the energy relaxation time

Kinetic equilibrium of neutralinos in the late universe may be reached by scattering on light fermions like neutrinos and electrons. In the rest frame of the neutralino, the Mandelstam variables become

\[ s = 2 \omega m_\chi + m_\chi^2, \quad t = -2 \omega^2 (1 - \cos \vartheta), \]  

(19)

where \( m_\chi \) is the mass of the lightest neutralino, \( \omega \) is the initial energy of the lepton and \( \vartheta \) is the scattering angle. We consider here only the case of a broken electroweak symmetry, i.e. the case of temperatures \( T \) below the weak scale, when the following hierarchy holds

\[ \omega \ll m_Z \ll m_\chi. \]  

(20)

The assumption \( m_Z \ll m_\chi \) leads also to several simplifications in the Higgs sector of the MSSM that we shall employ below. Additionally, we require that the neutralino mass parameters are not too degenerate,

\[ |\mu - M_1|, |\mu - M_2|, |M_2 - M_1| \gg m_Z. \]  

(21)

We consider explicitly the case where the lightest neutralino is a bino or a higgsino and scatters on a neutrino. The case of a wino is almost identical to the one of the bino.

The Feynman amplitudes of the process \( \chi + \nu_e \rightarrow \chi + \nu_e \) consists of three contributions: Sneutrino exchange in the \( s \) and \( u \) channel, and \( t \) channel exchange of higgses and the \( Z \), \( |\mathcal{M}|^2 = |\mathcal{M}_s - \mathcal{M}_u + \mathcal{M}_t|^2 \). Since the neutralino is a Majorana particle, the amplitudes \( \mathcal{M}_s \) and \( \mathcal{M}_u \) can be obtained by interchanging the neutralino in the initial and final states and thus they should be subtracted.

1. The bino as the LSP

Using the approximations explained above, we obtain as the leading contribution to the total spin-averaged squared Feynman amplitude in the case of a bino \[37\]

\[ |\mathcal{M}_u|^2 = |\mathcal{M}_s|^2 = \frac{e^4 M_T^2 \omega^2}{2 c_W^4 (M_0^2 - M_1^2)^2}, \]  

(22)

\[ |\mathcal{M}_t|^2 = \frac{e^4 M_T^2 \omega^2 (3 - \cos \theta) \cos^2 (2\beta)}{2 c_W^4 (\mu^2 - M_1^2)^2}, \]  

(23)

\[ 2 \text{Re}(\mathcal{M}_s \mathcal{M}_u^*) = - \frac{e^4 M_T^2 \omega^2 \sin^2 (\theta/2)}{c_W^4 (M_0^2 - M_1^2)^2}, \]  

(24)
polarization and energy and the number density of relativistic fermions with one to the total squared Feynman amplitude

\[ 2\text{Re}(M_M^*) = \frac{e^4 \cos(2\beta) M_1^2 \omega^2 (3 - \cos(\vartheta))}{2e^4(M_1^2 - M_2^2)(M_1^2 - \mu^2)} \]  

(25)

Here, we used neutrinos as scattering target and denoted by \( M_\nu \) the sneutrino mass.

The energy relaxation time can be calculated as (see e.g. Ref. 20)

\[ \frac{1}{\tau_{\text{rel}}} = \frac{N_{\text{eff}}}{2E_x m_x} \int_0^\infty d\omega \int d\Omega n_0(\omega)(\delta p)^2 \left( \frac{d\sigma_{\text{el}}}{d\Omega} \right)_{\mu_\chi}, \]  

(26)

where \( E_k = (3/2)T \) is the mean kinetic energy of the neutralinos, \( \delta p \) the neutralino momentum obtained in one scattering,

\[ (\delta p)^2 = 2\omega^2 (1 - \cos(\vartheta)) \]  

(27)

and the number density of relativistic fermions with one polarization and energy \( \omega \) is

\[ n_0 = \frac{1}{2\pi^2} \frac{\omega^2}{e^{\omega/T} + 1} = \frac{1}{2\pi^2} \omega^2 e^{-\omega/T}. \]  

(28)

Finally, the factor \( N_{\text{eff}} \) counts the number of relevant relativistic degrees of freedom, weighted with the relative size of their cross-section compared to a neutrino. Combining the different contributions and performing the integrals gives

\[ \tau_{\text{rel}} = \frac{\pi^3 c_4^2 M_1 (M_2^2 - M_1^2)^2 (\mu^2 - M_1^2)^2}{25 N_{\text{eff}} e^4 T^6 \left[ \cos(2\beta) M_0^2 + \mu^2 - 2c_2^2 M_1^2 \right]}. \]  

(29)

2. The higgsino as the LSP

In the case of a higgsino as the LSP the contributions to the total squared Feynman amplitude \( |M|^2 \) are given by

\[ |M_1|^2 = \frac{2 e^4 \mu^2 \omega^2 m_2^4 (M_1^2 c_2^2 + M_2^2 s_2^2 - \mu^4) (c_\beta + s_\beta)^4}{s_2^4 (\mu - M_1)^4 (\mu - M_2)^4 (M_2^2 - \mu^2)^2}, \]  

(30)

\[ |M_2|^2 = \frac{2 e^4 \mu^2 \omega^2 m_2^4 (M_1^2 c_2^2 + M_2^2 s_2^2 - \mu^4) (c_\beta + s_\beta)^4}{(\mu - M_1)^4 (\mu - M_2)^4 (M_2^2 - \mu^2)^2}, \]  

(31)

\[ |M_4|^2 = \frac{2 e^4 \omega^2 c_\beta (3 - \cos \vartheta) (M_1^2 c_2^2 + M_2^2 s_2^2 - \mu^2)^2}{s_2^4 (\mu - M_1)^2 (\mu - M_2)^2}, \]  

(32)

\[ 2\text{Re}(M_4 M_4^*) = 2 e^4 (c_\beta + s_\beta/2) (3 - \cos \vartheta) \times \frac{\mu m_2^2 \omega^2 (M_1^2 c_2^2 + M_2^2 s_2^2 - \mu^4)}{s_2^4 (\mu - M_1)^4 (\mu - M_2)^4 (M_2^2 - \mu^2)^2}. \]  

(33)

\[ 2\text{Re}(M_6 M_6^*) = -4 e^4 (c_\beta + s_\beta)^4 \sin^2(\vartheta/2) \times \frac{\mu^2 \omega^2 m_2^4 (M_1^2 c_2^2 + M_2^2 s_2^2 - \mu^4)}{s_2^4 (\mu - M_1)^4 (\mu - M_2)^4 (M_2^2 - \mu^2)^2}. \]  

(34)

\[ 2\text{Re}(M_8 M_8^*) = -2 e^4 (c_\beta + s_\beta/2) (3 - \cos \vartheta) \times \frac{\mu \omega^2 m_2^4 (M_1^2 c_2^2 + M_2^2 s_2^2 - \mu^4)}{s_2^4 (\mu - M_1)^4 (\mu - M_2)^4 (M_2^2 - \mu^2)^2}. \]  

(35)

Analogously to the case of the bino, the energy relaxation time for a higgsino as lightest neutralinos follows as

\[ \tau_{\text{rel}} = \frac{\pi^3 \mu^3 (\mu - M_1)^2 (\mu - M_2)^2 s_2^4}{100 N_{\text{eff}} e^4 T^6 c_2^2 (M_1^2 c_2^2 + M_2^2 s_2^2 - \mu^4)}. \]  

(36)

B. Elastic neutralino-neutralino scattering

We use in this subsection again the assumptions (20), but denote now with \( \omega \) the kinetic energy of the colliding neutralinos in their center of mass frame. Then the Mandelstam variables are

\[ s = 4(M_2^2 + \omega^2), \quad t = -2\omega^2 (1 - \cos \vartheta). \]  

(37)

1. The Bino as the LSP

Neutralino-neutralino scattering can occur through \( s \) and \( t \) channel exchange of the \( Z \) and the three neutral Higgs bosons. We shall see that it is sufficient to consider only the squared matrix elements. They are given in the unitary gauge by

\[ |M_{Z-\text{exch}}|^2 = \frac{9 e^4 \cos^2(2\beta) m_2^4 M_1^4 \tan^2(\vartheta W)}{2 (\mu^2 - M_1^2)^4}, \]  

(38)

\[ |M_{Z-\text{ann}}|^2 = \frac{e^4 \cos^2(2\beta) m_2^4 M_1^4 \tan^2(\vartheta W)}{2 (\mu^2 - M_1^2)^4}. \]  

(39)

\[ |M_{\text{h-ann}}|^2 = \frac{e^4 \omega^2 m_2^4 (\mu \sin(2\beta) + M_1^4 \tan^2(\vartheta W)}{2 M_1^4 (\mu^2 - M_1^2)^4}. \]  

(40)

\[ |M_{\text{h-exch}}|^2 = \frac{8 e^4 m_2^4 M_1^4 (\mu \sin(2\beta) + M_1^4 \tan^4(\vartheta W)}{M_1^4 (\mu^2 - M_1^2)^4}. \]  

(41)

\[ |M_{\text{H-exch}}|^2 = \frac{8 e^4 \mu^4 \cos^2(2\beta) m_2^4 M_1^4 \tan^4(\vartheta W)}{M_1^4 (\mu^2 - M_1^2)^4}. \]  

(42)

\[ |M_{\text{H-ann}}|^2 = \frac{8 e^4 \mu^4 \omega^4 \cos^2(2\beta) m_2^4 M_1^4 \tan^4(\vartheta W)}{(M_2^4 - 4 M_1^4)^2 (\mu^2 - M_1^2)^4}. \]  

(43)

\[ |M_{\text{A}^0-\text{ann}}|^2 = \frac{8 e^4 m_2^4 M_1^4 (\mu + \sin(2\beta) M_1) \tan^4(\vartheta W)}{(M_2^4 - 4 M_1^4)^2 (\mu^2 - M_1^2)^4}. \]  

(44)
and
\[ |M_{A^0\text{-exch}}|^2 = e^4 \tan^4 (\vartheta_W) \frac{\omega^4 (\cos(2\vartheta) + 7) m_Z^4 (\mu + \sin(2\beta) M_1)^4}{M_{A^0}^4 (\mu^2 - M_1^2)^4}. \] (45)

We note first that we can neglect the squared amplitudes proportional to \( \omega^4 \). Because of the hierarchy in the Higgs masses,
\[ O(M_h) = O(m_Z) \ll O(M_H) = O(M_{A^0}), \] (46)
the \( h \)-exchange channel \( (11) \) that is of order \( O(M_{\text{SU}_Y}^0) \) compared to other channels of \( O(m_Z^4/M_{\text{SU}_Y}^0) \) dominates the self-scattering of superheavy neutralinos. With \( |M_{\chi^0\chi^0\to\chi^0\chi^0}|^2 = |M_{h\text{-exch}}|^2 \), the total cross section of neutralino-neutralino scattering follows as
\[ \sigma = \frac{e^4 m_Z^4 M_1^2 (\mu\sin(2\beta) + M_1)^4 \tan^4 (\vartheta_W)}{16 M_{A^0}^4 (\mu^2 - M_1^2)^4}. \] (47)

2. The Higgsino as the LSP

Analogously to the bino case, the leading contribution to higgsino-higgsino scattering is given by the exchange of the light, SM-like Higgs \( h \). With
\[ |M_{\chi^0\chi^0\to\chi^0\chi^0}|^2 = |M_{h\text{-exch}}|^2 = \frac{e^4 (\beta + s_\beta)^8 \mu^4 m_Z^4 (M_1 \cos^2 (\vartheta_W) - \mu + \sin^2 (\vartheta_W) M_2)^4}{M_{h^0}^4 (\mu - M_1)^4 (\mu - M_2)^4}, \] (48)
the total cross section of neutralino-neutralino scattering follows as
\[ \sigma = \frac{e^4 (\beta + s_\beta)^8 \mu^2 m_Z^2 (M_{h^0}^2 + M_2^2 - \mu)^4}{256 \pi e_{W}^4 s_W^4 M_{h^0}^4 (\mu - M_1)^4 (\mu - M_2)^4}. \] (49)

C. Annihilations

The annihilations of neutralinos have been studied in great detail. Annihilations of superheavy neutralinos are, in the bino case, dominated by the channels \(ZH, hA, AH, W^\pm H^\mp\), since all other channels are suppressed by powers of \( m_Z/M_{\text{SU}_Y} \). In the higgsino case all bosonic channels contribute at leading order except annihilation into \( Z^0 + A^0 \) and \( h + H \). Annihilation into fermions are always suppressed.

The fermionic annihilation channels do not give leading order contributions in any case.

1. The bino as the LSP

Assume that \( \{M_{\chi^1}, M_{\chi^2}, M_{\chi^3}, M_{\chi^4}\} \) correspond to \( \{M_1, M_2, \mu, -\mu\} \). Then the squared matrix-elements of these dominant channels are given by
\[ |M_{\chi^0\chi^0\to Z^0 H^0}|^2 = e^4 (M_{H^0}^2 - 4 M_1^2)^2 \times \frac{(-4 \mu M_1^2 + A + B + \mu^2 M_A^2 \beta_{23})^2}{8 c_W^4 (\mu^2 - M_1^2)^2 (M_A^2 - 4 M_1^2)^2 (\mu^2 - 2 M_1^2 + M_H^2)^2}, \] (50)
with
\[ A = (M_A^2 - 2 M_H^2) s_{23} M_1^2, \] (51)
\[ B = 2 \mu (2 \mu^2 + M_A^2 - M_H^2) M_1, \] (52)
\[ |M_{\chi^0\chi^0\to A^0 H^0}|^2 = e^4 (M_A^2 - M_H^2)^2 / (8 c_W^4 s_{23}) \times \frac{(-8 s_{23} M_1^4 - 4 \mu M_1^2 + C + D + E)^2}{(\mu^2 - M_1^2)^2 (M_A^2 - 4 M_1^2)^2 (\mu^2 - 2 M_1^2 + M_A^2 + M_H^2)^2}, \] (53)
with
\[ C = 2 (M_A^2 + M_H^2) s_{23} M_1^2, \] (54)
\[ D = 4 \mu (-2 \mu^2 + M_A^2 + M_H^2) M_1, \] (55)
\[ E = c_\beta M_A^2 (-4 \mu^2 + M_A^2 + M_H^2) s_{23}, \] (56)
\[ |M_{\chi^0\chi^0\to h A^0}|^2 = \frac{e^4 (\sin(2\beta) M_2^4 + 4 \mu M_1)^2}{8 c_W^4 (M_A^2 - 2 \mu^2 - 2 M_1^2)^2}. \] (57)
and
\[ |M_{\chi^0\chi^0\to h^0 H^\pm}|^2 = e^4 (M_{H^\pm}^2 - 4 M_1^2)^2 / 8 c_W^4 \times \frac{(-4 \mu M_1^2 + F + G + \mu^2 M_A^2 \beta_{23})^2}{(M_{H^\pm}^2 - 2 \mu^2 - 2 M_1^2)^2 (\mu^2 - M_1^2)^2 (M_A^2 - 4 M_1^2)^2}, \] (58)
where
\[ F = (M_A^2 - 2 M_{H^\pm}^2) s_{23} M_1^2, \] (59)
\[ G = 2 \mu (-M_{H^\pm}^2 + 2 \mu^2 + M_A^2) M_1. \] (60)
The annihilation cross section of the various channels follows then as
\[ \sigma v = \frac{\beta_{12}^2}{32 \pi m_{\chi}^2} \sum_i |M_i|^2. \] (61)

2. The higgsino as the LSP

Assume that \( \{M_{\chi^1}, M_{\chi^2}, M_{\chi^3}, M_{\chi^4}\} \) correspond to \( \{\mu, -\mu, M_1, M_2\} \). Then the squared matrix-elements of these dominant channels are given by
\[ |M_{\chi^0\chi^0\to Z^0 Z^0}|^2 = \frac{2 e^4}{\sin(2\vartheta_W)^4}, \] (62)
\[ |M_{\chi^0\chi^0\to W^+ W^-}|^2 = \frac{e^4}{4 \sin(2\vartheta_W)^4}, \] (63)
\[ |M_{\chi^0, \chi^0 \to Z^0 h}|^2 = \frac{2e^4 \mu^2 c_{2\beta}^2}{(\mu^2 + M_1^2)(\mu^2 + M_2^2)} \times (64) \]
\[ (\mu^2 M_2^2 c_{W}^2 + M_1^2 M_2^2 c_{W}^2 + M_1(\mu^2 + M_2^2) s_{W}^2)^2, \]
\[ |M_{A^0, \chi^0 \to A^0 h}|^2 = \frac{e^4(c_{\beta} - s_{\beta})^4}{2(-2\mu^2 - 2M_1^2 + M_2^2)^2 A^2 s_{W}^4} \times (4\mu M_1 s_{W}^2 A - 2c_{W} M_1^2 B + 4\mu c_{W} M_2 C + D)^2, (65) \]
where
\[ A = (-2\mu^2 - 2M_1^2 + M_2^2) \]
\[ B = (M_1^2 + 4\mu M_2) \]
\[ C = (M_2^2 - 2\mu^2) \]
\[ D = M_1^2 - 2\mu^2 M_1^2 - 2M_1^2 s_{W}^2 M_2, (69) \]
The annihilation into \( W^\pm, H^\mp \) are dominated by the chargino exchange and the \( A^0 \) annihilation channels. In order to obtain relatively compact expressions, we set \( M_{A^0} = M_H \) and assume that \( \mu \cos(\beta) + \sin(\beta) M_2 \neq 0 \). As a result we obtain
\[ |M_{\chi^0, \chi^0 \to W^\pm H^\mp}|^2 = \frac{e^4(s_{2\beta} M_1^2 + 4\mu M_2)^2}{8(-2\mu^2 - 2M_1^2 + M_2^2)^2 s_{W}^2}. (70) \]

To simplify the squared matrix elements both for annihilations into \( Z^0 + H \) and \( A^0 + H \), we set \( M_{A^0} = M_H \). The results are
\[ |M_{\chi^0, \chi^0 \to Z^0 H}|^2 = \frac{e^4(c_{\beta} - s_{\beta})^4}{H(-2\mu^2 - 2M_1^2 + M_2^2)^2} \times (M_1^4 - 2\mu^2 M_1^2 + E - F - G)^2, (71) \]
with
\[ E = 4\mu M_1(2\mu^2 + 2M_1^2 - M_2^2) s_{W}^2 \]
\[ F = 2c_{W} M_1^2 (M_1^2 - 4\mu M_2) \]
\[ G = 2M_2(2\mu(M_1^2 - 2\mu^2) c_{W}^2 + M_2 M_1^2 s_{W}^2) \]
\[ H = 2s_{2W}^2(-2\mu^2 - 2M_1^2 + M_2^2)^2, (75) \]
and
\[ |M_{\chi^0, \chi^0 \to A^0 H}|^2 = \frac{2e^4 \mu^2 c_{2\beta}}{(\mu^2 + M_1^2 - M_H^2)^2 I} \times (M_1^2 M_2^2 c_{W}^2 + M_2(\mu^2 - M_H^2) c_{W}^2 + J)^2, (76) \]
with
\[ I = (\mu^2 + M_2^2 - M_H^2)^2 s_{2W}^4 \]
\[ J = M_1(\mu^2 + M_2^2 - M_H^2) s_{W}^4. (78) \]
The annihilation channels \( Z^0 + A^0 \) and \( h + H \) do not give an \( O(1) \) contribution.

V. COSMOLOGICAL PRODUCTION

All the mechanisms for the production of SHDM particles described in section 11 apply also to SHLSPs. In particular, the abundance of neutralinos due to gravitational production is given by Eq. (1). In most works on SHDM particles the dependence on \( T_R \) and the two-fold degeneracy in Eq. (1) was fixed by choosing first for \( T_R \) the highest value allowed by the gravitino problem, \( T_R = 10^9 \text{GeV} \). Then the larger of the two possible masses was selected, \( m_\chi \sim 3 \times 10^{13} \text{GeV} \), so that secondaries of SHDM decays could explain the cosmic rays of the highest energies, \( E \sim 10^{20} \text{eV} \).

Both constraints can be relaxed in our scenario: Superheavy gravitinos decay before big-bang nucleosynthesis and we do not insist that SHDM decays explain UHE-CRs with energies, \( E \gtrsim 10^{20} \text{eV} \). As a result, the only constraint additionally to Eq. (1) is the requirement that neutralinos do not thermalize, \( T_R \ll m_\chi/30 \). Thus the choice \( T_R = 10^{10} \text{GeV} \) and \( m_\chi = 3 \times 10^{11} \text{GeV} \) gives the smallest possible neutralino mass. Finally we note that decays of heavier SUSY particles with mass \( \tilde{m} \), that are more effectively produced in the regime \( \tilde{m} \ll H_I \) than neutralinos, can change the relation (1) but not the smallest possible neutralino mass.

VI. SUMMARY

We have suggested the lightest supersymmetric particle as a well-suited candidate for superheavy dark matter. The requirement that SHDM is produced with the correct abundance by gravitational interactions at the end of inflation leads to a lower mass limit of \( 3 \times 10^{11} \text{GeV} \) for the masses of all SUSY particles. Since weak interactions remain perturbative despite the large mass hierarchy, \( m_\chi \gg m_Z \), and the mass scales are approximately fixed, we could reliably calculate the relevant processes for the special case of a superheavy neutralino.

Our proposal can be falsified in the near future by the discovery of low-scale/split SUSY at the LHC. If this is not the case, then SHLSPs as DM particles may be the unique opportunity to connect SUSY to the physical world. The observable consequences of meta-stable SHDM were discussed already in detail in Refs. [33, 40], while the prospects to detect DM in the form of stable superheavy neutralinos despite of their small number density and annihilation cross section will be discussed in a subsequent work [31].

Acknowledgments

We would like to thank T. Plehn for helpful discussions and especially A. Pukhov for advice on the use of CalcHEP.
