Letter

Fusion burn equilibria sensitive to the ratio between energy and helium transport

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Abstract

An analysis of the burn equilibria of fusion reactors of the tokamak family is presented. The global (zero-dimensional) analysis is self-consistent in that it takes into account the dependence of the energy confinement on the variables of the burning plasma, such as temperature and density. Universal burn contours are presented for a selection of commonly used scaling laws for energy confinement. It is shown that the output power of a fusion reactor is to good approximation inversely proportional to the particle confinement time, due to the choking effect of the accumulation of helium, the ash of the fusion reaction. It is further shown that, whereas a fusion reactor requires a minimum energy confinement time to ignite, the output power reaches a maximum for an energy confinement that lies about 30% above this minimum. Further improvement of confinement will lower the output, although in some cases the $\beta$ limit will be the limiting factor. Given that for maximum performance density the confinement and fuel mix are best chosen to be optimal, the particle confinement is proposed as an attractive parameter for burn control.

Keywords: nuclear fusion, burning plasma, alpha heating, ignition, confinement, DEMO, scaling law

(Some figures may appear in colour only in the online journal)

1. Introduction

In a fusion reactor of the tokamak type, a plasma of the hydrogen isotopes deuterium and tritium is kept at a temperature of hundreds of millions of kelvins, confined in a toroidal geometry by means of magnetic fields. To start the reactor, external heating is applied to bring the plasma to the ‘ignition’ point: a combination of sufficiently high temperature and density at which the heating power delivered by the fusion reactions balances the heat loss. Once the plasma is ignited its temperature—and thereby the fusion power—increases autonomously until the stable ‘burn temperature’ ($T_{\text{burn}}$) is reached. Above this temperature the heat losses increase faster than the fusion power. For a given $T_{\text{burn}}$, the electron density $n_e$ is therefore determined by the reaction rate [1] and heat loss rate, which is the sum of the radiation and conduction losses. The latter are conveniently expressed by the energy confinement time $\tau_E = W_P/\dot{W}_{\text{cond}}$, defined as the ratio of the kinetic energy content $W$ of the hot plasma and the conductive power losses $\dot{W}_{\text{cond}}$. As the heat loss is the result of complex turbulent processes, empirical scaling laws are used, which express $\tau_E$ as a function of operational parameters such as the geometry of the reactor, $n_e$ and heating power. There are only a few global parameters under operator control that influence $T_{\text{burn}}$ and might be used for burn control. Important ones are $n_e$, the mixing ratio of the two fuel components deuterium (D) and tritium (T) and the quality of confinement, expressed by the $H$-factor $H_{98} = \tau_E/\tau_{\text{IPB98}(y,21)}$, i.e. the value of $\tau_E$ compared to the scaling law prediction. A fourth and less obvious burn control parameter is the ratio of particle and energy transport

$$\rho = \frac{\tau_p}{\tau_E}$$

In tokamak reactors, particle confinement is much better than energy confinement, with $\rho$ typically between 5 and 10 [2], and values of 10–30 also reported [3]. The paradox of the fusion reactor is that, whereas good energy confinement is essential to reduce power losses, good particle confinement makes the reaction choke on its own ash. The effect of particle confinement on the burn equilibrium is evident from the contours in the $n_e \tau_E, T$-plane (assuming $T = T_e = T_i$) for which the fusion power heating balances the losses,
an analysis already performed by Reiter et al [4]. Note that, whereas the contours are open towards high energy confinement when the choking effect of particle confinement is neglected ($\rho = 0$), taking this effect into account closes and consticts the contours. For $\rho > 14.7$ no sustained burn is possible. To complicate matters, a further constraint comes from the fuel cycle, which requires $\rho$ to be sufficiently high, as was shown by Sawan et al [5].

For these reasons, we address the question how the $T_{\text{burn}}$ and $P_{\text{burn}}$ change under variation of $H_{98}$ and $\rho$, as well as $n_e$, while assuming that the fuel mix is 50–50. We first introduce the basic equations and definitions of the 0D model, following the work of Freidberg [7], Reiter et al [4] and Rebhan et al [8, 9]. Although inclusion of profile effects will quantitatively change the analysis, Reiter et al [10] showed that the qualitative properties of the system remain the same. We then present an expression for $n_e(T)$, construct universal burn contours and derive two new results for the influence of energy and particle confinement on the burn equilibrium.

2. Theory

The energy balance of a burning fusion plasma is approximated by

$$S_\rho = S_{\text{rad}} + S_{\text{o}}$$

(2)

with $S_\rho$ the alpha particle heating and $S_{\text{rad}}$ and $S_{\text{o}}$ the losses due to radiation and conduction, respectively. External and ohmic heating are neglected as they have a minor influence on the burn equilibrium. The alpha power density is given by $S_\rho = n_\rho \gamma_\rho (\langle \sigma \nu \rangle) E_{\nu}$, where $\langle \sigma \nu \rangle$ is the DT reactivity [1] and $E_{\nu} = 3.52 \text{ MeV}$ the energy of the alpha particle that is produced in the DT reaction, while $n_j$ denotes the number density of species $j$ in units of m$^{-3}$. The dominant radiation loss is due to the Bremsstrahlung, given by $S_{\text{rad}} = \sum_j Z_j n_j \mu_{\text{eff}} \langle \sigma \nu \rangle_{\text{e}} l_{\text{e,keV}}$ with $C_\text{b} = 5.35 \times 10^{-37} \text{ W m}^3$, $T_{\text{keV}}$ the electron temperature in keV, $Z$ the ion charge, $g_\text{e}$ the Gaunt factor, which we have approximated by $2 \sqrt{3}/\pi$, and the summation is over all ion species. To account for the helium density ($n_\rho$) resulting from the fusion reactions we write $n_\rho \gamma_\rho (\langle \sigma \nu \rangle) = n_u/n_e$, thereby assuming that the confinement of alpha particles is the same as that of other species. We further introduce the fuel dilution parameter $f_i = (n_D + n_T)/n_e$ and the alpha fraction $f_u = n_u/n_e$. Using these notations, [4] finds

$$n_e \tau_{E} = \frac{4 f_u}{\rho \langle \sigma \nu \rangle}$$

(3)

and by solving these equations arrives at burn contours, i.e., the contours in the $n_e \tau_{E}$, $T$-plane for which the alpha heating balances the losses.

3. Results

We have used the same formalism to produce the burn contours shown in figure 1. Note that $n_e \tau_{E}$ must exceed a critical value for burn to occur. For given $n_e \tau_{E}$ there are two solutions: the unstable ignition temperature (left-hand branch) and the stable burn temperature (right-hand branch), in agreement with the intuitive picture of ignition and burn.

In this calculation $\tau_{E}$ is an independent parameter, whereas in fact it depends on plasma parameters. Rebhan et al [9] proposed a self-consistent analysis by using a scaling law which expresses $\tilde{\tau}_E = W/(P_{\text{cond}} + P_{\text{rad}})$ as a function of plasma parameters. They used the ITER89 L-mode scaling [11] to find burn contours in the $n_e$, $T$-plane for this specific scaling, for a specific choice of reactor parameters. We follow this approach, using the now more relevant scaling for H-mode confinement, the IPB98(y, 2)-scaling [12], which is commonly used to predict the performance of future fusion devices. Since the radiation losses are not included in IPB98(y, 2), the method applied in [9] cannot be used. Instead, we inserted the expression for the alpha heating power to eliminate the confinement time and derived an expression for $n_e$ as a function of $T$ which is valid for all scaling laws of the form $\tau_{E} = K A^k n_e^{-m}$.

$$n_e = \left( \frac{4 f_u A^k}{\rho K} \right)^{1/m} \left[ \frac{1}{4} f_i \langle \sigma \nu \rangle_{\text{e}} l_{\text{e,keV}} E_{\nu}^m \right]^{1/m}$$

(5)

Here $K$ is a constant that depends on the engineering parameters of the reactor, $A$ is the average ion mass in amu and $P$ the power deposited in the plasma (by the alpha particle or external sources).

Since $K$ and the plasma volume $V$ are the only reactor specific parameters in this equation, $n_e (V^{-m} K^{1/(1-2m-n)})$ represents a normalized density that is the same for all fusion reactors that follow the same scaling law.

Figure 2 shows the POPCONs for ITER ID [13] and three conceptual reactor designs DEMO A–C, as described in the conceptual power plant study [14]. There is a large difference between the IPB98(y, 2) and 89L scaling for the
ITER ID reactor. The solid curves for ITER ID and DEMO A–C are isomorphic, which can be shown by applying the normalization described above.

It is important to note that, while the formalism using the scaling laws leads to burn equilibria at values of $n_e$ and $T$ that are far from the normal operating conditions of a fusion reactor, these are probably artefacts due to the mathematical form of the scaling laws. Reliable extrapolations can only be made in the parameter range where the database on which the scaling laws are based is well populated, i.e. with $n_e$ in the range $10^{19}$–$10^{21}$ m$^{-3}$.

To elucidate the role of particle confinement, while zooming in on the reactor relevant $n_e$ range, figure 3 shows the burn contours of DEMO A in the $n_e$, $T$-plane. The plot shows clearly how, at constant density, the reactor will move from ignition at a temperature of 5–8 keV to burn at a temperature around 30 keV, while the fusion power at the same time increases by an order of magnitude. The fusion power at ignition and burn depends quadratically on $n_e$, and therefore the Greenwald density limit $n_G = I_p/\pi a^2$ is of fundamental importance. For all but the lowest $\rho$-values this limit is more restrictive than the Troyon pressure limit, given by $\beta_{\text{max}} = g_T \frac{I_p}{\pi a^2}$ with $I_p$ in MA and the Troyon factor $g_T = 0.03$.

Calculating the $\beta$ limit requires knowledge about the composition of the plasma, and this is only available on the equilibrium contours, so plotting it is not straightforward. We have taken the following approach: for figures 2 and 3 we have calculated the $\beta$ limit for a pure DT plasma, which results in an underestimation of $n_e$ of at most 20%. For figures 4 and 5 we have taken the values of $f_a$ and $T$ at the equilibrium and used these to calculate the value of $n_e$ and subsequently the fusion power at the $\beta$ limit. This results in a $\beta$ limit that has two values at one value of $\rho$ and $H_{98}$ (because every point on an equilibrium contour is associated with a point on a $\beta$ limit contour).
In another projection of the parameter space, the influence of confinement is too low. For the stable burn branch (top half of the contour), an increase in $H_{98}$ beyond its minimum value initially results in ignition at lower temperature and power. The two branches meet again at the maximum $H_{98} = 6.38$ and 2.79 ($\rho = 5$ and 10), and beyond this there are no more burn equilibria, i.e. the fuel has become so diluted that the fusion power cannot balance the conduction and radiation losses.

In short, for a given reactor there is no gain to be expected from improvement of energy confinement. Either the plasma exceeds the $\beta$ limit, or the power output decreases. Rather, the reactor should be designed in such a way that its operating point is at $H_{\text{max}}$, provided it does not conflict with the $\beta$ limit. Of course, better confinement does allow one to reach ignition in a reactor with smaller dimensions and lower $P_{\text{burn}}$.

4. Conclusions

We have derived an analytical expression (equation 5 relating $T$ and $n_e$) in a fusion reactor with self-consistent treatment of fuel burn up and helium accumulation, using the IPB98(y,2) scaling law for confinement time. This expression is valid for all fusion reactors that obey the same energy confinement scaling if one takes into account a scale factor that depends on the reactor parameters. Using these results we have plotted the burn contours of the DEMO A design in the $n_e$, $T$-plane, including the curves of constant fusion power and the Greenwald and Troyon limits. The fusion power at these equilibria was found to be very sensitive to changes in $n_e$, $T$, and $H_{98}$. The fusion power scales quadratically with the density around the Greenwald density, although this will be different for reactors that have a minimum density for ignition that is close to this limit. The dependence on $\rho$ is especially strong for intermediate to high values of $\rho$, and, since the value of $\rho$ is to a large extent determined by the helium exhaust at the plasma edge, this offers possibilities for burn control using helium pumping [15, 16]. The effect of $H_{98}$ on the fusion power could have implications for advanced tokamak scenarios, where values of $H_{98}$ well above unity are expected.

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