Maxwell-affine gauge theory of gravity

O. Cebeciöğlu\textsuperscript{1} and S. Kibaroğlu\textsuperscript{1}

\textsuperscript{1}Department of Physics, Kocaeli University, 41380 Kocaeli, Turkey

(Dated: April 1, 2015)

Abstract

Maxwell extension of affine algebra with additional tensorial generators is given. Using the methods of nonlinear realizations, we found the transformation rules for group parameters and corresponding generators. Gauging the Maxwell-affine algebra we presented two possible invariant actions for gravity: one is the first order and the other one is the second order in affine curvature. We noticed that equations of motion for the action, second order in affine curvature, lead to the generalized Bianchi identities on the choice of appropriate coefficients for a particular solution of the constraint equation.

PACS numbers: 02.20.Sv; 04.20.Fy; 11.15.-q; 02.40.-k

\textsuperscript{a} E-mail: ocebecioglu@kocaeli.edu.tr

\textsuperscript{b} E-mail: salihkibaroğlu@gmail.com
I. INTRODUCTION

Following the pioneering work of Bacry [1], the idea of Maxwell symmetry has been systematically studied by Schrader [2], and has received much attention, starting again with Soroka [3], over the past 10 years or so. The attention has focused mainly on two distinct directions one with the dynamical particle realization both relativistic and super symmetric [4–7] and the second direction is the localization of the Maxwell symmetry in order to get extended gravity theories [8, 9]. The formulation of different types of Maxwell gravities has already been studied in [10–14]. In these theories, Maxwell extension was applied to the Poincare, superPoincare and (A)dS groups. With this motivation, we have presented the Maxwell-affine gravity in a similar fashion by observing the result of [10–14]. We have already gone one more step beyond these groups by adding dilatation into the scene [15] and now we are pushing even more step beyond these taking into account the group of affine transformations. The aim of this paper is to establish the framework of the gauge theory of Maxwell-affine group, \( MA(4, R) \). This will be done by constructing the noncentral extension of affine group. As we shall see, the Maxwell-affine algebra is constructed from the translation generators \( P_a \) and the noncentral extension \( Z_{ab} \) together with the \( gl(4, R) \) generators \( L^a_b \).

The organization of the paper is as follows. In Sec. II, after reviewing some properties of the affine algebra, we study its extensions as described in [16, 17]. Applying the techniques of non-linear coset realization [18–21] to the Maxwell-affine group, the transformation rules for generalized coordinates are found and explicit expression for the generators of the Maxwell-affine algebra are given. In Sec.III, we present two types of action for gravity based on gauged Maxwell-affine algebra, one with first order and the other second order in affine curvature. We derive and discuss the field equations for both actions. Finally, Sec.IV concludes the paper.

II. AFFINE ALGEBRA AND ITS TENSOR EXTENSION

The affine group, \( A(4, R) \), is a group of all linear transformations in four dimensional space [22]:

\[
x^\alpha' = \Lambda^a_b x^b + e^a
\]

and it can be written as a semi direct product of its homogeneous and inhomogeneous parts namely the general linear group, \( GL(4, R) \), and the translational group \( T^4 \) parts respectively. The 20 generators of affine transformation can be decomposed into the 4 translations \( P_a \) and 16 general linear transformations \( L^a_b \). Its Lie algebra is defined by the commutation relations

\[
[L^a_b, L^c_d] = i(\delta^c_b L^a_d - \delta^a_d L^c_b)
\]

\[
[L^a_b, P_c] = -i\delta^a_c P_b
\]

\[
[P_a, P_b] = 0
\]

where the tensor indices \( a, b \) take the values \((0, 1, 2, 3)\). The elements of the affine group are here represented by exponentials according to

\[
g(x, \vec{\omega}) = e^{ix^a P_a} e^{i\vec{\omega} \cdot L^a_b}
\]

where \( e^{i\vec{\omega} \cdot L^a_b} \in GL(4, R) \) [23, 24].
The Maxwell extension of the affine algebra can be constructed in complete analogy to the Maxwell algebra obtained in \cite{16,17}. To construct Maxwell extension of affine group, we need to introduce Maurer-Cartan, MC, form defined by

\[ \Omega = -ig^{-1}dg \]

where \( g \) is a general element of affine group. This left invariant 1-form satisfies the Maurer-Cartan structure equation:

\[ d\Omega + \frac{i}{2}[\Omega, \Omega] = 0. \]

For the affine case, we have

\[ d\Omega^a_P + \Omega^a_{Lb} \wedge \Omega^b_P = 0 \]
\[ d\Omega^a_{Lb} + \Omega^a_{Le} \wedge \Omega^e_{Lb} = 0. \]

Freezing the GL(4,R) degrees of freedom \( (L^a_{b} \rightarrow 0) \), the most general closed invariant 2-form which cannot be written as the differential of an invariant 1-form is of the form

\[ \Omega_2 = f_{[ab]} \Omega^a_P \wedge \Omega^b_P \]

where the constant parameters \( f_{[ab]} \) is a second rank antisymmetric tensor. Therefore, we find that the non-trivial 2-forms an antisymmetric tensor representation of the \( GL(4,R) \) group. The 1-form potential associated to this 2-form is denoted by \( Z_{ab} \) and satisfies the MC structure equation

\[ d\Omega^{ab}Z - \frac{1}{2}\Omega^a_P \wedge \Omega^b_P = 0 \]

where the coefficient \((-\frac{1}{2})\) chosen for notational convenience. When the general linear transformation included, the extended set of MC 1-forms satisfies the equations

\[ d\Omega^a_P + \Omega^a_{Lb} \wedge \Omega^b_P = 0 \]
\[ d\Omega^a_{Lb} + \Omega^a_{Le} \wedge \Omega^e_{Lb} = 0 \]
\[ d\Omega^{ab}_Z + \Omega^a_{Le} \wedge \Omega^e_{Lb} - \frac{1}{2}\Omega^a_P \wedge \Omega^b_P = 0 \]

where second term in third equation implies that \( Z_{ab} \) generators transform as a tensor with respect to \( GL(4,R) \) transformations. Eqs.\((10)\) implies the 26 dimensional Maxwell-affine algebra with the following non-zero commutation rules:

\[ [L^a_{b}, L^c_{d}] = i(\delta^c_b L^a_{d} - \delta^a_d L^c_{b}) \]
\[ [L^a_{b}, P^c] = -i\delta^c_b P^a \]
\[ [P^a, P^b] = iZ_{ab} \]
\[ [L^a_{b}, Z_{cd}] = i(\delta^a_d Z_{bc} - \delta^a_c Z_{bd}) \]

To realize the group action on \( \mathcal{MA}(4,R)/GL(4,R) \), we make use of the formula

\[ g(a, \varepsilon, u)K(x, \theta) = K(x', \theta')h(\tilde{\omega}, g) \]

which defines the non-linear group action, choosing exponential parametrization for the coset

\[ K(x, \theta) = e^{ixp e^{i\theta Z}} \]
where the variables \( x^a, \theta^{ab} \) the coset parameters and

\[
h(\tilde{\omega}) = e^{i\tilde{\omega}^b_a L^a_b} \tag{14}\]

an element of stability subgroup \( GL(4, R) \). It can be easily evaluated through the use of the well known Baker-Hausdorff-Campell formula:

\[
e^{A+B} = e^A e^B e^{\frac{1}{2}[A,B]} \tag{15}\]

which holds when \([A,B]\) commutes with both \( A \) and \( B \). The transformation laws of the coset space parameters under the infinitesimal action of the \( \mathcal{MA}(4, R) \) are

\[
\begin{align*}
\delta x^a &= a^a + u^a_b x^b \\
\delta \theta^{ab} &= \varepsilon^{ab} - \frac{1}{4} a[x^b] + u^{[a} \theta^{cb]} \\
\tilde{\omega}^a_b &= u^a_b 
\end{align*} \tag{16}\]

where antisymmetrization is defined by \( x^{[a} y^{b]} = x^a y^b - x^b y^a \) and the corresponding generators are

\[
\begin{align*}
P_a &= i(\partial_a - \frac{1}{2} x^b \partial_{ab}) \\
Z_{ab} &= i \partial_{ab} \\
L^a_b &= i(x^a \partial_b + 2 \theta^{ac} \partial_{bc}) \tag{17}\end{align*}\]

where \( \partial_a = \frac{\partial}{\partial x^a} \), \( \partial_{ab} = \frac{\partial^2}{\partial x^a \partial x^b} \), and one can check that these generators fulfill the Maxwell-affine algebra and verify self-consistency of Jacobi identities.

### III. GAUGING THE MAXWELL-AFFINE ALGEBRA

The gauge theories of affine gravity treated in [25, 26]. The nonlinear gauge theories of gravity on the basis of the affine group \( A(4, R) \) as the principal group was used in [22–24]. More complete references on affine gauge theory and the metric affine gravity up to 1995 can be found in [27]. To gauge affine algebra we introduce the 1-form potential \( \mathcal{A} \) with the values in Lie algebra of the \( \mathcal{MA}(4, R) \) group, defined by

\[
\mathcal{A} = e^a P_a + B^{ab} Z_{ab} + \tilde{\omega}^b_a L^a_b \tag{18}\]

where \( e^a = e^a_\mu dx^\mu \), \( B^{ab} = B^{ab}_\mu dx^\mu \) and \( \tilde{\omega}^a_b = \tilde{\omega}^a_\mu dx^\mu \) are vector fields with respect to the space-time index \( \mu \). The variation of these fields under infinitesimal gauge transformations in tangent space is given by

\[
\delta \mathcal{A} = -d\zeta - i [\mathcal{A}, \zeta] \tag{19}\]

with the gauge generator

\[
\zeta(x) = y^a(x) P_a + \varphi^{ab}(x) Z_{ab} + \lambda^b_a(x) L^a_b \tag{20}\]
where \( y^a(x) \) are space-time translations, \( \varphi^{ab}(x) \) are translations in tensorial space, and \( \lambda^b_a(x) \) are the general linear transformation parameters respectively. The transformation properties of the 26 gauge fields under infinitesimal MA(4, R) are

\[
\begin{align*}
\delta e^a &= -dy^a - \tilde{\omega}^a_b y^b + \lambda^a_b e^b \\
\delta B^{ab} &= -d\varphi^{ab} - \tilde{\omega}^{[a}_c \varphi^{cb]} + \lambda^{[a}_c B^{cb]} + \frac{1}{2} e^{[a} y^{b]} \\
\delta \tilde{\omega}^a_{\ b} &= -d\lambda^a_{\ b} - \tilde{\omega}^a_{\ c} \lambda^c_{\ b} + \lambda^a_{\ c} \tilde{\omega}^c_{\ b}
\end{align*}
\]  

(21)

The curvature 2-form \( \tilde{\omega} \) is given by the structure equation

\[
\tilde{\omega} = \frac{i}{2} [A, A] \tag{22}
\]

whence writing

\[
\begin{align*}
\tilde{\omega} &= F_a^a P_a + F_{ab} Z_{ab} + \tilde{R}_{ab}^a L_{ab} \\
F_a^a &= de^a + \tilde{\omega}^a_{\ b} \wedge e^b \\
F_{ab} &= dB^{ab} + \tilde{\omega}^{[a}_{\ c} \wedge B^{cb]} - \frac{1}{2} e^{[a} \wedge e^{b]} \\
\tilde{R}_{ab} &= d\tilde{\omega}^a_{\ b} + \tilde{\omega}^a_{\ c} \wedge \tilde{\omega}^c_{\ b} \tag{23}
\end{align*}
\]

we find

\[
\begin{align*}
\delta F_a^a &= -\tilde{R}_{a}^b y^b + \lambda^a_{\ b} F^b \\
\delta F_{ab} &= -\tilde{R}^{[a}_{\ c} \varphi^{cb]} + \lambda^{[a}_{\ c} F^{cb]} - \frac{1}{2} y^{[a} F^{b]} \\
\delta \tilde{R}_{ab} &= \lambda^a_{\ c} \tilde{R}^c_{\ b} - \tilde{R}^a_{\ c} \lambda^c_{\ b}. \tag{24}
\end{align*}
\]

These are the general-affine torsion, a new curvature 2-form for tensor generator \( Z_{ab} \), and the general-affine curvature 2-form respectively. Under an infinitesimal gauge transformation with parameters \( \zeta \), the curvature 2-form \( \tilde{\omega} \) transform as

\[
\delta \tilde{\omega} = i [\zeta, \tilde{\omega}] \tag{25}
\]

and hence one gets

\[
\begin{align*}
\delta F_a^a &= -\tilde{R}_{a}^b y^b + \lambda^a_{\ b} F^b \\
\delta F_{ab} &= -\tilde{R}^{[a}_{\ c} \varphi^{cb]} + \lambda^{[a}_{\ c} F^{cb]} - \frac{1}{2} y^{[a} F^{b]} \\
\delta \tilde{R}_{ab} &= \lambda^a_{\ c} \tilde{R}^c_{\ b} - \tilde{R}^a_{\ c} \lambda^c_{\ b}. \tag{26}
\end{align*}
\]

Having found the transformation of the gauge fields and the curvatures, we are ready to look for invariant lagrangians under these transformations. We assume that the generalized gravitational Lagrangian 4-form to be gauge invariant under the homogeneous local \( GL(4, R) \) subgroup only. To construct gauge invariant Lagrangian 4-form first order in affine curvature, we start from the Euler density in four dimensions and substitute each curvature by a concircular curvature.[28]

\[
R^{ab} \rightarrow \tilde{R}^{ab}(\beta) = R^{ab} - \beta e^a \wedge e^b \tag{27}
\]

where the last term recalls the contribution to Lorentz curvature \( R^{ab} \) in (A)dS gravity, enters through the new gauge field strenght \( F_{ab} \). In the light of this substitution we combine the curvatures \( \tilde{R}^{ab} \) and \( F^{ab} \) into a single curvature as follows

\[
\tilde{R}^{ab} \rightarrow J^{ab}(\mu) = \tilde{R}^{ab} g^{eb} - \mu F^{ab} \tag{28}
\]
where we also introduce an extra field, the premetric symmetric tensor field $g^{ab}$, its infinitesimal transformation under local $GL(4, R)$ is

$$\delta g^{ab} = \lambda^a e^b + \lambda^b e^a.$$  \hfill (29)

The reason for introducing this premetric symmetric field stems from non-flatness of group space in ordinary affine gauge theory in contrast to the local Lorentz group, the space defined by this group is flat and characterized by flat Minkowski metric $^{29}$. After these preliminaries we start from the following gauge invariant Yang-Mills type action

$$S_1 = \frac{1}{2\kappa} \int J \wedge^* J = \frac{1}{4\kappa} \int \eta_{abcd} J^{ab} \wedge J^{cd}$$  \hfill (30)

where $\eta_{abcd} = \sqrt{-g} \varepsilon_{abcd}$. Writing out $J^{ab}$ in terms of the curvatures $\tilde{R}^a_{\ e} g^{eb}$ and $F^{ab}$, the action becomes

$$S_1 = \frac{1}{4\kappa} \int \eta_{abcd} \left( \tilde{R}^a_{\ e} g^{eb} \wedge \tilde{R}^c_{\ f} g^{fd} - 2\mu \tilde{R}^a_{\ e} g^{eb} \wedge F^{cd} + \mu^2 F^{ab} \wedge F^{cd} \right).$$  \hfill (31)

The second term of this Maxwell affine action contains the direct generalization of the Einstein Hilbert action to the affine case $^{30}$. One discovers that variation with respect to $\tilde{\omega}^b_{\ a}$ gives the generalized torsion

$$D (g^{ac*} J_{bc}) - 2\mu B^{ac} \wedge J_{cb} = 0$$  \hfill (32)

while variation with respect to the vierbein field $e^a$ gives the field equation

$$* J^{ab} \wedge e^b = 0.$$  \hfill (33)

By a variation with respect to premetric field $g^{ab}$, we get

$$\tilde{R}^{c}_{\ (a} \wedge^* J_{cb)} - \frac{1}{2} \eta_{abcd} J^{cd} \wedge^* J_{cd} = 0$$  \hfill (34)

where symmetrization is defined by $A_{(a} \wedge B_{b)} = A_a \wedge B_b + A_b \wedge B_a$. Finally, variation with respect to $B^{ab}$ leads to

$$D^* J_{ab} = 0$$  \hfill (35)

where $D = d + \tilde{\omega}$ is the general linear exterior covariant derivative. We conclude our discussion by emphasizing that Eq.$^{31}$ is the generalization of the equation given in $^{31}$. Remaining three equations of motion are local $GL(4, R)$ modified version of Azcarraga’s equations of motion $^8$. Thus, the field equations of Einstein-Cartan like theory are recovered.

To construct gauge invariant action whose Lagrangian 4-form, second order in curvatures, we start from the Pontryagin densities in 4 dimensions and substitute each product of two curvatures $^{28}$ according to

$$R^a_{\ b} \wedge R^b_{\ a} \to \tilde{R}^a_{\ b} (\beta) \wedge \tilde{R}^b_{\ a} (\beta) - \gamma F^a \wedge F_a$$  \hfill (36)

where $\beta$ and $\gamma$ are scale constants and $R^a_{\ b}$ is a Lorentz curvature. This time we chose our curvature as $\mathcal{R}^a_{\ b} = \tilde{R}^a_{\ b} - \mu F^a_{\ b}$ without any reference to premetric tensor field because in this form it transforms as a nice tensor under local $GL(4, R)$ which makes it possible to construct a gauge invariant action in the form of
\[ S_2 = \frac{1}{2\kappa} \int \mathcal{Y}^a_b \wedge \mathcal{Y}^b_a + \frac{1}{\rho} \int F^a \wedge F_a \]  

(37)

where \( \kappa \) and \( \rho = -\frac{2\kappa}{\gamma} \) are coupling constants. By construction the action Eq.(37) is manifestly diffeomorphism invariant and possess local \( GL(4, R) \) invariance. The field equations follow from the variation of the action. From the variation of Eq.(37) with respect to \( \tilde{\omega} \) we get the following equation for the generalized torsion tensor

\[ \mathcal{D}\mathcal{Y}^a_b - \mu [B, \mathcal{Y}]^a_b + \frac{\kappa}{\rho} [F, e]^a_b = 0. \]

(38)

where \([B, \mathcal{Y}]^a_b\) denotes taking the wedge product on the form part and the commutator on the Lie algebra part, i.e. \([B, \mathcal{Y}]^a_b = (B^a_c \wedge \mathcal{Y}^c_b - \mathcal{Y}^a_c \wedge B^c_b)\), similarly we have \([F, e]^a_b = (F^a \wedge e_b - e^a \wedge F_b)\). Furthermore, the \( e \) variation of the action gives following two equations

\[ \mathcal{Y}^a_b \wedge e^b - \left(\frac{2\kappa}{\mu \rho}\right) \mathcal{D}F^a = 0 \]

\[ e_b \wedge \mathcal{Y}^b_a + \left(\frac{2\kappa}{\mu \rho}\right) \mathcal{D}F_a = 0. \]

(39)

and finally

\[ \mathcal{D}\mathcal{Y}^a_b = 0 \]

(40)

obtained by varying with respect to the \( B \). Substituting Eq.(40) into Eq.(38) and taking exterior covariant derivative and making use of Eq.(39) leads the following constraint equation

\[ F^a_c \wedge \tilde{R}^c_b - \tilde{R}^a_c \wedge F^c_b = 0. \]

(41)

From special solution of Eq.(41)

\[ \tilde{R}^c_b = -\mu F^c_b \]

(42)

we get

\[ \mathcal{Y}^a_b = -2\mu F^a_b = 2\tilde{R}^a_b \]

(43)

If we insert this solution to the equations of motion, we arrived the result that equations of motion are the generalized Bianchi identities as the \( \left(\frac{\kappa}{\mu \rho}\right) \to 1 \) limit,

\[ \mathcal{D}F^{ab} = \tilde{R}^{[a}_c \wedge B^{b]} - \frac{1}{2} F^{[a} \wedge e^{b]} = 0 \]

\[ \mathcal{D}F^a = \tilde{R}^a_b \wedge e^b \]

\[ \mathcal{D}\tilde{R}^a_b = 0. \]

(44)

For completeness let us give the conservation laws which follow from the invariance of the action under the local \( \mathcal{MA}(4, R) \) symmetry. Under diffeomorphism, the variation of the Lagrangian is given by its Lie derivative \( l_\xi \mathcal{L} = d l_\xi \mathcal{L} + i_\xi d \mathcal{L} \) along \( \xi \). Now \( d \mathcal{L} = 0 \) because \( \mathcal{L} \) is a top form and the first term is a total divergence which can be ignored as a surface integral then we have

\[ \delta S_{diff} = \int l_\xi \mathcal{L} = 0. \]

(45)
In order to show the diffeomorphism invariance of the action explicitly, one substitutes the transformation rules Eq. (21) to the following action integral

\[
\delta S = \int \delta e^a \wedge E_a + \delta B^{ab} \wedge V_{ab} + \delta \tilde{\omega}^b_a \wedge C^a_b
\]

then one gets the following conservation rules

\[
\mathcal{D} E_a + V_{ab} \wedge e^b = 0 \\
\mathcal{D} V_{ab} = 0 \\
\mathcal{D} C^a_b + 2 B^{ac} \wedge V_{cb} + e^a \wedge E_b = 0
\]

where \( E_a, C^a_b, \) and \( V_{ab} \) are Einstein, Cartan and Maxwell 3-forms respectively. They can be found from actions Eq. (31) and Eq. (37) for the respective Lagrangians.

**IV. CONCLUSION**

In the present paper we enlarged the general affine algebra by using an antisymmetric tensor generator and constructed a non-linear realization of the Maxwell-affine group on its coset space with respect to the general linear group. We have shown two new set of field equations within the framework of the Maxwell affine gauge theory of gravity starting from Euler density, we formed an action which is linear in affine curvature leading to the generalized Einstein-Hilbert action and thus generalizing the results of [31], and we have also constructed an action second order in curvatures by making use of Pontryagin density which in turn leads to the generalized Bianchi identities.

**ACKNOWLEDGEMENTS**

The authors wish to thank Abdurrahman Andiç and Mustafa Erkovan for helpful discussions and useful remarks.

[1] H. Bacry, P. Combe, and J.L. Richard, Nuovo Cimento 67, 267-299 (1970).
[2] R. Schrader, Fortschritte der Physik 20, 701 (1972).
[3] D.V. Soroka and V.A. Soroka, Phys. Lett. B 607, 302-305 (2005).
[4] J. Gomis, K. Kamimura, and J. Lukierski JHEP 08, 39 (2009).
[5] S. Bonanos, J. Gomis, K. Kamimura, and J. Lukierski, Phys. Rev. Lett. 104, 090401 (2010).
[6] S. Bonanos, J. Gomis, K. Kamimura, and J. Lukierski J. Math. Phys. 51, 102301 (2010).
[7] S. Fedoruk and J. Lukierski, JHEP 02, 128 (2013).
[8] J.A. de Azcarraga, K. Kamimura, and J. Lukierski, Phys. Rev. D 83, 124036, (2011).
[9] D.V. Soroka and V.A. Soroka, Phys. Lett. B 707, 160-162 (2012).
[10] R. Durka, Kowalski-Glikman, and M. Szczachor, Mod. Phys. Lett. A 26, 2689-2696 (2011).
[11] R. Durka, Kowalski-Glikman, and M. Szczachor, Mod. Phys. Lett. A 27, 1250023 (2012).
[12] R. Durka and Kowalski-Glikman, arXiv:1110.6812v1 [hep-th].
[13] S. Hoseinzadeh and A. Rezaei-Aghdam, Phys. Rev. D 90, 084008 (2014).
[14] J. A. de Azcarraga and J. M. Izquierdo, Nucl. Phys. B 885, 34-45 (2014).
[15] O. Cebecioglu and S. Kibaroglu, Phys. Rev. D 90, 084053 (2014).
[16] S. Bonanos and J. Gomis, J. Phys. A: Math. Theor. 42, 145206 (2009).
[17] S. Bonanos and J. Gomis, J. Phys. A: Math. Theor. 43, 015201 (2010).
[18] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2239 (1969).
[19] C. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2247 (1969).
[20] A. Salam and J. Strathdee, Phys. Rev. 184, 1750 (1969).
[21] A. Salam and J. Strathdee, Phys. Rev. 184, 1760 (1969).
[22] A.B. Borisov and V.I. Ogievetskii, Theor. Math. Phys. 21, 1179-1188 (1974).
[23] A. Lopez-Pinto, A. Tiemblo, and R. Tresguerres, Class. Quant. Grav. 12, 1503-1516 (1995).
[24] R. Tresguerres and E.W. Mielke, Phys. Rev. D 62, 044004 (2000).
[25] F. W. Hehl, E. A. Lord, and Y. Ne’eman, Phys. Rev. D 17, 428-433 (1978).
[26] E. Lord, Phys. Lett. A 65, 1-4 (1978).
[27] F.W. Hehl, J.D. Mccrea, E.W. Mielke, and Y. Ne’eman, Phys. Rept. 258, 1-171 (1995).
[28] A. Mardones and J. Zanelli, Class. Quant. Grav. 8, 1545-1558 (1991).
[29] R. F. Sobreiro and V. J. Vasquez Otoya, J. Geom. Phys. 61, 137-150 (2011).
[30] M. Leclerc, Annals Phys. 321, 708-743 (2006).
[31] B. Julia and S. Silva, Class. Quant. Grav. 15, 2173-2215 (1998).