RESONANCES OF MULTIPLE EXOPLANETS AND IMPLICATIONS FOR THEIR FORMATION

XIAOJIA ZHANG, HUI LI, SHENGTAI LI, AND DOUGLAS N. C. LIN

1 Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064, USA; xzhang47@ucsc.edu
2 Los Alamos National Laboratory, Los Alamos, NM 87545, USA
3 Institute for Advanced Studies, Tsinghua University, Beijing, China

Received 2014 January 28; accepted 2014 May 29; published 2014 June 19

ABSTRACT

Among \(\sim 160\) of the multiple exoplanetary systems confirmed, about 30\% of them have neighboring pairs with a period ratio \(\leq 2\). A significant fraction of these pairs are around mean motion resonance (MMR), and, more interestingly, peak around 2:1 and 3:2, with a clear absence of more closely packed MMRs with period ratios less than 4:3, regardless of planet masses. Here, we report numerical simulations demonstrating that such MMR behavior places important constraints on the disk evolution stage out of which the observed planets formed. Multiple massive planets (with mass \(\geq 0.8 \, M_{\text{Jup}}\)) tend to end up with a 2:1 MMR mostly independent of the disk masses, but low-mass planets (with mass \(\leq 30 \, M_{\oplus}\)) can have MMRs larger than 4:3 only when the disk mass is quite small, suggesting that the observed dynamical architecture of most low-mass-planet pairs was established late in the disk evolution stage, just before it was dispersed completely.

Key words: planet-disk interactions – planetary systems – protoplanetary disks

1. INTRODUCTION

Many multiple planetary candidates were discovered by Kepler’s transit search. It has been pervasively suggested that the majority of them are indeed genuine multiple-planet systems (Lissauer et al. 2012; Batalha et al. 2013). A significant fraction of adjacent pairs of planets are locked in or near first- or second-order mean motion resonances (MMRs) with period ratios around 2:1, 3:2, 5:3, etc. (Lissauer et al. 2011b), although members of most multiple systems do not have nearly commensurable orbits (Mayor et al. 2009).

The MMR of exoplanet systems has been well studied under various perspectives (Goldreich 1965; Wisdom 1980; Henrard 1982; Weidenschilling & Davis 1985; Ogihara & Ida 2009; Ogihara et al. 2010; Lee et al. 2013). A widely adopted scenario is that resonant pairs captured each other on their mutual MMRs through convergent migration (Bryden et al. 2000; Kley 2000; Lee & Peale 2002b). Migration mechanisms include tidal interaction between very short period planets and their host stars (Schlaufman et al. 2010) as well as protoplanet interaction with their natal disks (Lin et al. 1996). For planets with periods longer than a few days, MMRs provide supporting evidence of the disk migration scenario.

Gas-giant planets with sufficient mass to open gaps in their natal disks undergo Type II migration (Lin & Papaloizou 1986). Most Kepler candidates have much lower masses and undergo Type I migration (Goldreich & Tremaine 1980; Ward 1997; Paardekooper et al. 2011; Kretke & Lin 2012). In either case, orbital convergence would occur if the inward migration of a planet catches up with that of its siblings close to the host star or if a planet is trapped at some special disk region and its siblings migrate toward it. Convergent migration leads to the possibility of resonant capture (Lee & Peale 2002a; Kley et al. 2004).

Many multiple planet systems are indeed locked in or are close to MMRs. The current database on The Extrasolar Planets Encyclopaedia Web site (http://exoplanet.eu) contains 157 confirmed multiple planet systems with \(\sim 225\) neighboring pairs. This sample includes 118 systems of 2 planets, 22 systems of 3 planets, 9 systems of 4 planets, 4 systems of 5 planets, and 4 systems of 6 planets. Figure 1 shows the distribution of these adjacent pairs as a function of their period ratios, focusing on systems with a period ratio around and less than 4. Overall, \(\sim 70\) pairs (i.e., \(\sim 1/3\) of all pairs) have a period ratio around or less than 2. The subsets of pairs with both planets’ masses \(\geq 0.8 \, M_{\text{Jupiter}}\) and below \(\sim 30 \, M_{\oplus}\) (designated as gas-giant and low-mass-planet systems, respectively) are also shown in Figure 1. Most gas giants are discovered by the radial velocity method. Although their mass determinations are lower limits, this observational bias does not affect their statistics. Most multiple low-mass-planet systems are Kepler candidates. Their mass has been estimated by using an empirical formula to convert a planet’s radius to mass (Lissauer et al. 2011a, 2011b).

Figure 1 confirms the previously known results that there are statistically clusters of systems in or near the 2:1 and 3:2 MMRs (Lissauer et al. 2011b; Fabrycky & Kepler Science Team 2012). Since the confirmed Kepler multi-exoplanetary systems measured with transit timing variations are biased the samples toward planets near resonances, the peaks may be overrepresented, but one can still find this statistical clustering feature by including all pairs of Kepler candidates (Goldreich & Schlichting 2014). The excess near the 3:2 MMR is mostly associated with low-mass-planet systems, whereas that near the 2:1 MMR contains both low-mass-planet and gas-giant systems. If these systems formed through resonant capture during planetary migration, it appears that their asymptotic MMR is a sensitive function of planet mass, orbital characteristics, and relative migration rates (Ogihara & Kobayashi 2013). Gas-giant pairs such as GJ876 (Laughlin et al. 2001; Rivera & Lissauer 2001; Laughlin et al. 2005) and HD 82943 (Lee et al. 2006; Tan et al. 2013) cluster mostly around the 2:1 MMR because the Type II migration rate is relatively slow compared with the libration timescale for the 2:1 MMR.

However, Type I migration is considerably faster than Type II migration (Lin & Papaloizou 1986). This allows migrating pairs to bypass wider resonances, with slower libration timescales (Murray & Dermott 1999), and end up in more tightly packed configurations. It is noteworthy that there is a deficit of both low-mass-planet and gas-giant pairs with period ratios smaller than 4:3. Most Kepler planetary candidates have orbital periods within a few months, and the possibility of not detecting a
transiting companion with a similar period is small. Thus, the observed deficit of closely packed resonant pairs and the preferential concentration of lower-mass pairs near the 3:2 MMR relative to the 2:1 MMR are statistically significant.

2. METHOD

In order to reproduce these observations in terms of the resonant-capture scenario, we use a two-dimensional (2D) module of the LA-COMASS package developed at Los Alamos to investigate the orbital evolution of multiple planets in a protostellar disk. This 2D hydrodynamical polar grid code solves the continuity and isothermal Navier–Stokes equations for gas inside a quasi-Keplerian disk subject to the gravity of a one-solar-mass central star. The motion of multiple planets is calculated with a fourth-order Runge–Kutta solver. The evanescent boundary condition has been implemented in the code to provide wave killing zones at each edge of the disk (de Val-Borro et al. 2006). This code has been used extensively for planet–disk interaction studies (e.g., Li et al. 2009).

In our simulations, the 2D disk is modeled within the radial range of \([0.3 \, R_0, 3.3 \, R_0]\), where \(R_0\) is the distance unit in the code. We investigate two main configurations: a disk with multiple gas giants and a disk with multiple 10 \(M_\oplus\) planets. For the migration of gas giants, since their Type II migration does not sensitively depend on the disk structure (Lin & Papaloizou 1986), we adopt models with smooth density, temperature, and viscosity profiles. However, the pace and direction of the Type I migration does depend sensitively on the disk properties and they may be trapped in a disk near the magnetospheric truncation radius, the inner edge of “dead zone,” or other locations such as the boundary between the viscous heating and irradiation heating regions (Kretke & Lin 2012).

For the migration of low-mass planets, we adopt a two-zone disk model based on the \(\alpha\) prescription for viscosity. In order to approximate a disk structure in which the magnetohydrodynamic turbulence is prevalent throughout the inner region and the mostly neutral outer regions with a “dead” midplane zone, we adopt a high value of \(\alpha_{\text{vis}} = 0.004\) for disk radius \(a < a_{\text{crit}} = 0.6 \, R_0\) and \(\alpha_{\text{vis}} = 0.001\) for \(a > a_{\text{crit}}\). Typical values of \(R_0\) may range from a few stellar radii (for the inner disk boundary or the inner boundary of the dead zone) to a few AU (for the interface between the viscously heated and irradiated disk regions). In a steady state (Kretke et al. 2009), this
evolution timescale of the disk, the gas giants can hardly pass through the 2:1 MMR location unless they are in a disk with an extremely high accretion rate.

We also simulated the evolution of initially compact gas-giant pairs (with a period ratio less than 2). Population synthesis models (Ida et al. 2013) indicate that multiple gas-giant systems can form with relatively compact orbits because their progenitor embryos are separated by ~10 Hill’s radii before they acquire sufficient mass to accrete gas efficiently. Our extensive simulations show that the orbits of these compact gas-giant pairs are not stable against their intense gravitational perturbation on each other (Zhou et al. 2007). In all cases, one of the two gas giants is scattered to large distances from its initial location. In the inner regions, the scattered planets may resume their migration. If they are able to catch up with their companions, they would eventually settle into the 2:1 resonance. The gas-giant resonance “barrier” at the 2:1 MMR is confirmed by these simulations.

Typically the Type I migration timescale of a planet on a circular orbit is $t_0 \sim h^2 q^{-1} M_*/(\Sigma_0 R_p^2 \Omega_p^{-1})$, where $h$ is the aspect ratio of the disk, $q$ is the mass ratio of planet to star, $R_p$ is the location of planets, and $\Sigma_0$ is the local surface density at $R_p$.

The migration rate is also sensitively dependent on the gradient of the surface density and temperature of the disk (Paardekooper et al. 2010, 2011). In this Letter, we only control the migration rate by applying different surface densities. Our disk model leads to the convergent evolution between adjacent pairs of two, three, or four low-mass-planet systems. In these simulations, the mass of each planet is assumed to be $10 M_\oplus$ and the gas mass within the computational domain is $2 \times 10^{-3}$ that of the host star. The asymptotic period ratio of all neighboring pairs is around 5:4 or smaller. This ability to closely pack multiple low-mass-planet systems is due to their relatively rapid converging speed.

Such compact systems are rarely found among the Kepler planetary candidates. This apparent discrepancy between observations and simulations can be reduced considerably with much slower converging Type I migration rates. In order to illustrate this conjecture, we have performed a large set of simulations of multiple low-mass systems for a broad range of disk surface densities.

In these simulations, we have chosen the central star mass $M_* = 1 M_\odot$, $h/r = 0.05$, and $a_{\text{crit}} = 0.004/0.001$ for $a$ interior/exterior to $a_{\text{crit}}$. In a steady state, the disk accretion rate is given by $\dot{M} = 1.5 \times 10^{-4} \cdot f \cdot (1AU/R_0)^{-1.5} M_\odot yr^{-1}$, where $f = (\Sigma_0 R_G^2/M_*)$ is the disk-to-star mass ratio and $\Sigma_0$ is the disk surface density (in $g/cm^2$) at $R_0$. The transition radius $a_{\text{crit}} = 0.6 R_0$ is also dependent on the disk accretion rate (Kretke et al. 2009) as $a_{\text{crit}} \propto M^{0.5}$. Both the disk surface density normalization and the trapping location (associated with $R_0$) decrease as the disk accretion rate diminishes during disk depletion.

From these simulations, we find that the asymptotic period ratios between adjacent pairs decrease toward unity as the accretion rate increases. Figure 3 shows a general trend that lower disk accretion rates lead to relatively wider spacing for low-mass-planet systems. These results suggest that the accretion rate must be $<2 \times 10^{-8} M_\odot yr^{-1}$ to reproduce the observed paucity. If this paucity is a signature of some alternative
Panels from top to bottom show the final period ratio of adjacent pairs in multiple low-mass-planet systems with two, three, and four planets of $10 \, M_\oplus$ each. The horizontal lines indicate the first-order MMRs at 3:2, 4:3, 5:4, 6:5, and 7:6. The vertical line marks the observed disk accretion rate for a typical young solar mass system with an age $\sim 1$ Myr (Natta et al. 2006).

Pairs of low-mass planets undergo Type I migration and capture each other within $10^3$ orbits, as indicated by the left panel. Continued simulations up to $10^4$ orbits of such systems demonstrate that the pair remains around the 5:4 MMR. A gradual decrease of the disk surface density is implemented, as shown in the insert. If a dynamical process, like instability, then we can constrain the forming stage of the planetary systems from the accretion rate corresponding to the final period ratio larger than the 4:3 MMR. The corresponding disk-to-star mass ratio in this region is $< 6.7 \times 10^{-4}$. The reproduction of the observed enhancement of adjacent low-mass-planet pairs with 3:2 MMR requires even lower disk accretion rates ($\dot{M} < 10^{-8} \, M_\odot \, yr^{-1}$), especially in systems with more than two planets. Note that the inferred accretion rates around classical T Tauri stars in the Taurus and Ophiuchus complex (Natta et al. 2006) range mostly between $10^{-8}-10^{-7} \, M_\odot \, yr^{-1}$.

In order to extract constraints on the disk accretion rate from Figure 1, we need to verify that the stability of compact systems may be preserved (Gladman 1993). Systematic studies (Zhou et al. 2007) show that in a gas-free environment, multiple equal-mass low-mass-planet systems with initially circular orbits separated by $k_0 = \Delta a / R_H < 10$ (where $R_H = (2M_p/3M_*)^{1/3} a$) become dynamically unstable over the Gyr main-sequence lifespan of the host stars. For non-resonant $M_p = 10 \, M_\oplus$ low-mass-planet systems, this stability criterion requires the period ratio to be larger than $\sim 1.43$, which could account for the paucity of multiple systems with a period ratio between adjacent planets smaller than that of the 3:2 MMR. Multiple gas-giant systems with a period ratio between adjacent planets smaller than that of the 2:1 MMR are also unstable on a Gyr timescale.

However, the stability of systems with near MMRs may be better preserved against long-term dynamical instability. To illustrate, we select a pair of low-mass planets that are captured into each other’s 5:4 MMR in a disk initially with $f = 0.002$. Such a system corresponds to $k_0 = 6$. If it is out of resonance, it would become dynamically unstable in $< 10^3$ orbits in the absence of gas and $< 10^4$ orbits if it is embedded in a minimum mass solar nebula. We extend our simulation for an additional $10^4$ orbits, while the disk surface density is prescribed to decrease exponentially over that timescale. The results in Figure 4 indicate that once the low-mass-planet pairs are captured into a tight MMR, they tend to remain in these MMRs in the absence of major perturbations before the depletion of the disk. However, the later long-term orbital evolution of planets with an absence of gas may break the system. Although we cannot exclude the instability criterion as an contribution to the paucity of a period ratio smaller than the 4:3 MMR, we can take this as a clue to the formation stage. If instability is responsible for the paucity, it might indicate that
the low-mass-planet pairs with a period ratio larger than 4:3 formed in a somewhat late stage of the disk.

Based on the results from Figures 3 and 4, we infer that low-mass-planet systems can capture each other into 4:3 and/or 3:2 MMRs, provided they undergo Type I migration when the accretion rate ($\dot{M}$) or the mass of their natal disks is relatively low. The results in Figure 3 indicate that in order for a pair of $10 M_\oplus$ low-mass planets to attain asymptotic period ratios of 4:3 and 3:2, the upper limits of the disk-to-star mass ratios are at $f = 6.7 \times 10^{-4}$ and $f = 4 \times 10^{-4}$, respectively. Since the Type I migration rate is linearly proportional to the product of $q$ and $f$, low-mass-planet pairs would approach each other’s 4:3 (or 3:2) MMRs with the same critical relative speed if their $q \cdot f \sim 2 \times 10^{-8}$ (or $\sim 1.2 \times 10^{-8}$). The condition for MMR capture requires that the migration timescale through the characteristic width ($t_{\text{ mig}} \sim \Delta a/\dot{a}$) is longer than the libration timescale ($t_{\text{ lib}}$). If we take into account that the libration time of the lowest-order MMR is $\propto q^{-1/2}$ and $\Delta a \propto q^{1/2}$ in the first order of the expansion (Murray & Dermott 1999), the critical condition for the lowest order of MMR capture would be roughly independent of the planet-to-star mass ratio.

In order to quantify the stage of the disk according to the accretion rate, we use the observations of a relatively young ($t_a \sim 1$ Myr) star-forming region, ρ Oph (Natta et al. 2006), where the accretion rate around $\sim 1 M_\odot$ star is about $4 \times 10^{-8} M_\odot$ yr$^{-1}$. For $R_0 = 4.1$AU, the corresponding value $f = 0.002$. The accretion rate is also observed to decline over a timescale of 3–5 Myr. We introduce an approximation for the disk accretion rate $\dot{M} = 4 \times 10^{-8} \cdot e^{-t/\tau_{\text{D}}} M_\odot$ yr$^{-1}$ for $t \geq t_a$, where $t$ indicates the age of the disk, $t_a \sim 1$ Myr, and $\tau_{\text{D}}$ is the disk lifetime. Figure 5 shows the dependency of final period ratios on the planet masses and the MMRs formation times. It confirms that a relatively large period ratio (such as 3:2) requires the low-mass-planet pairs to have migrated and captured each other late in the disk evolution stage when the disk surface density was sufficiently depleted.

4. SUMMARY

These simulation results for multiple gas-giant and low-mass-planet systems place important constraints on the planet formation stage with respect to the disk evolution. According to the core accretion scenario, the formation of gas giants must be preceded by the emergence of sufficiently massive ($>10 M_\oplus$) protostellar embryos in a gas-rich environment, presumably during the early stage of disk evolution. These cores, if retained in a dense disk, would either congregate and effectively merge near some trapping radius to become cores of proto-gas-giant planets or be mostly scattered into or far from their host stars. A significant “left-over” population would have produced compact pairs with small period ratios that are not consistent with observations. Around stars that only bear relatively low-mass planets, their dynamical configuration may be established during the advanced stages of disk evolution when the disk gas is severely depleted. It is also possible that these low-mass planets were assembled over several millions of years.

We thank the referee, Frederic Rasio, for helpful comments that improved the manuscript. We acknowledge support from the LDRD program and IGPP of Los Alamos National Laboratory. H.L. and D.N.C.L. also acknowledge support from the UC fee program of University of California. Simulations were carried out using the Institutional Computing resources at LANL.

REFERENCES

Batalha, N. M., Rowe, J. F., Bryson, S. T., et al. 2013, ApJS, 204, 24
Bryden, G., Różycka, M., Lin, D. N. C., & Bodenheimer, P. 2000, ApJ, 150, 1091
de Val-Borro, M., Edgar, R. G., Artymowicz, P., et al. 2006, MNRAS, 370, 529
Fabrycky, D. C., & Kepler Science Team. 2012, in AAS/Division of Dynamical Astronomy Meeting, Vol. 43, AAS/Division of Dynamical Astronomy Meeting (Mount Hood, OR: AAS/DADA), 01.03
Gladman, B. 1993, Icar, 106, 247
Goldreich, P. 1965, MNRAS, 130, 159
Goldreich, P., & Schlichting, H. E. 2014, AJ, 147, 32
Goldreich, P., & Tremaine, S. 1980, ApJ, 241, 425
Henrand, J. 1982, CeMec, 27, 3
Ida, S., Lin, D. N. C., & Nagasawa, M. 2013, ApJ, 775, 42
Kley, W. 2000, MNRAS, 313, L47
Kley, W., Peitz, J., & Bryden, G. 2004, A&A, 414, 735
Kretke, K. A., & Lin, D. N. C. 2012, ApJ, 755, 74
Kretke, K. A., Lin, D. N. C., Garaud, P., & Turner, J. N. 2009, ApJ, 690, 407
Laughlin, G., Butler, R. P., Fischer, D. A., et al. 2005, ApJ, 622, 1182
Laughlin, G., Chambers, J., & Fischer, D. 2001, BAAS, 33, 1304
Lee, M. H., Butler, R. P., Fischer, D. A., Marcy, G. W., & Vogt, S. S. 2006, ApJ, 641, 1178
Lee, M. H., Fabrycky, D., & Lin, D. N. C. 2013, ApJ, 774, 52
Lee, M. H., & Peale, S. J. 2002a, ApJ, 567, 596
Lee, M. H., & Peale, S. J. 2002b, arXiv:astro-ph/0209176
Li, H., Liboz, S. H., Li, S., & Lin, D. N. C. 2009, ApJ, 690, L52
Lin, D. N. C., Bodenheimer, P., & Richardson, D. C. 1996, Natur, 380, 606
Lin, D. N. C., & Papaloizou, J. 1986, ApJ, 309, 846
Lissauer, J. J., Fabrycky, D. C., Ford, E. B., et al. 2011a, Natur, 470, 53
Lissauer, J. J., Marcy, G. W., Rowe, J. F., et al. 2012, ApJ, 750, 112
Lissauer, J. J., Ragozzine, D., Fabrycky, D. C., et al. 2011b, ApJS, 197, 8
Mayor, M., Udry, S., Lovis, C., et al. 2009, A&A, 493, 639
Murray, C. D., & Dermott, S. F. 1999, Solar System Dynamics (Cambridge: Cambridge Univ. Press)
Natta, A., Testi, L., & Randich, S. 2006, A&A, 452, 245
Ogilvie, M., Duncan, M. J., & Ida, S. 2010, ApJ, 721, 1184
Ogilvie, M., & Ida, S. 2009, in AAS/Division for Planetary Sciences Meeting Abstracts, Vol. 41, AAS/Division for Planetary Sciences Meeting Abstracts (Fajardo: AAS/DPS), 40.05
Ogilvie, M., & Kobayashi, H. 2013, ApJ, 775, 34
Paardekooper, S.-J., Baruteau, C., Crand, A., & Kley, W. 2010, MNRAS, 410, 1950
Paardekooper, S.-J., Baruteau, C., & Kley, W. 2011, MNRAS, 410, 293
Rivera, E. J., & Lissauer, J. J. 2001, ApJ, 558, 392
Schlaufman, K. C., Lin, D. N. C., & Ida, S. 2010, ApJ, 724, L53
Tan, X.-P., Payne, M. J., Lee, M. H., et al. 2013, ApJ, 777, 101
Ward, W. R. 1997, ApJ, 482, L211
Weidenschilling, S. J., & Davis, D. R. 1985, Icar, 62, 16
Wisdom, J. 1980, AJ, 85, 1122
Zhou, J.-L., Lin, D. N. C., & Sun, Y.-S. 2007, ApJ, 666, 423