Parameterized LMI Based Diagonal Dominance Compensator Study for Polynomial Linear Parameter Varying System

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Abstract. For dynamic decoupling of polynomial linear parameter varying (PLPV) system, a robust dominance pre-compensator design method is given. The parameterized pre-compensator design problem is converted into an optimal problem constrained with parameterized linear matrix inequalities (PLMI) by using the conception of parameterized Lyapunov function (PLF). To solve the PLMI constrained optimal problem, the pre-compensator design problem is reduced into a normal convex optimization problem with normal linear matrix inequalities (LMI) constraints on a new constructed convex polyhedron. Moreover, a parameter scheduling pre-compensator is achieved, which satisfies robust performance and decoupling performances. Finally, the feasibility and validity of the robust diagonal dominance pre-compensator design method are verified by the numerical simulation on a turbofan engine PLPV model.

1. Introduction
For MIMO control problem, the decoupling performances are important because coupling interaction among different control loops can significantly wors the control characteristics. An promising methodology for multi-input multi-output (MIMO) control systems are convert the control system into single-input single-output (SISO) control system by using decoupling precompensation, then design controllers for every decoupled control loop[1,2]. In diagonal dominance precompensation study for decoupling purpose, Hawkins has proposed a pseudo-diagonalization method which minimize the off-diagonal effects at certain frequencies[3]. Neil Munro obtains optimal reduction in the off-diagonal elements based on Perron-Frobenius theory and the conception of generalised diagonal dominance[4]. Nobakhti uses genetic algorithms to achieve diagonal dominance without choosing a design frequency in previous methods[5]. In[6] Chughtai etc. give a new method for determining a constant precompensator for reducing the interactions in MIMO system by solving LMI constrained optimization problem. B. Labibi presents a methodology for diagonal dominance of large-scale systems via Eigen structure assignment[7]. Based on the theory of generalized diagonal dominance, S. Jamebozorg gives a design method of static precompensator for the reduction of interaction in linear multivariable systems[8]. For LPV systems, the decoupling compensation is important and difficult in control system design with the change of scheduling parameters[9]. In [10], A LPV system output regulation is achieved by nonlinear compensation approach based on PLF. J. Mohammadpour proposes a method using parameter-dependent static inversion or SVD decomposition of the system to minimize the effects of the off-diagonal terms in the MIMO system[11,12]. Based on minimization of the $H_2$ norm, Amin Nobakhti proposes a LMI method for the design of these precompensators which improve the efficiency in speed of computation and design conservatism[13]. In this study a LMI
based decoupling compensator is designed to obtain the diagonal dominance of LPV system using parameterized Lyapunov function, then the global solutions are achieved by solving LMI restrained optimization problems on constructed convex polyhedron vertex set. Finally, the given decoupling compensator design method is verified by numerical simulations.

2. Problem formulation
For LPV decoupling compensator design problem, consider the following PLPV system (1), which depends on the parameter $\sigma$ with bounded parameter variation rate.

$$\Sigma p \begin{cases} \dot{x}_p = A_p(\sigma)x_p + B_p(\sigma)u \\ y_p = C_p(\sigma)x_p + D_p(\sigma)u \end{cases}$$ (1)

Where $x_p \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^n$ is the input vector, $y_p \in \mathbb{R}^n$ is the output vector. The coefficient matrices of system (1) have the same form, which is defined as follows:

$$\ell(\sigma) = \sum_{i=0}^{\hat{\sigma}} \ell_i, \quad \sigma \in [\sigma_{\min}, \sigma_{\max}], \quad |\sigma| < \Delta_{\max}$$ (2)

Where $\ell_i$ are known real constant matrices with appropriate dimensions. Moreover, we consider the following form of PLF needed in system analysis and synthesis.

$$V(x, \sigma) = x^T P(\sigma)x, \quad P(\sigma) = \sum_{i=0}^{m} \sigma^i P_i$$ (3)

Where, $P(\sigma) \in \mathbb{R}^{m \times m}$ is a symmetric positive definite polynomial Lyapunov matrix of degree $m$, $P_i$ are symmetric real constant matrices with appropriate dimensions. The LPV decoupling compensator design purposes are design compensator $K_p(\sigma)$ which can decoupling the interaction between different control loops under the varying of scheduling parameter $\sigma$, then the MIMO control problem can be converted into several SISO control problems easily.

3. LMI based decoupling compensator design
The decoupling compensator design purpose is to find a static compensator that optimize diagonal dominance condition of compensated system under certain constrains[6]. Firstly, the following parameterized regular splitting definition is given based on the conceptions of parameterized regular splitting and parameterized fundamental dominance which extended from the basic conceptions in[6]:

**Parameterized regular splitting**: Let $A(s,\sigma) \in \mathbb{C}^{m \times m}$ be a parameterized complex matrix, then, $A(s,\sigma)$ is said to have a regular splitting if

$$A(s,\sigma) = B(s,\sigma) + D(s,\sigma)$$ (4)

such that $B(s,\sigma)$ is non singular. Then the fundamental dominance is defined as:

**Parameterized Fundamental dominance**: Let $A(s,\sigma) \in \mathbb{C}^{m \times m}$ be a complex matrix, and let $A(s,\sigma) = B(s,\sigma) + D(s,\sigma)$ be a regular splitting, where $B(s,\sigma) \in \mathbb{C}^{m \times m}$ and $D(s,\sigma) \in \mathbb{C}^{m \times m}$. Then, the term $B(s,\sigma)$ is said to be the dominant term of $A(s,\sigma)$ if and only if

$$\rho(B(s,\sigma)^{-1}D(s,\sigma)) < 1$$ (5)

where $\rho(.)$ is the spectral radius and is used to scale the magnitude of the maximum eigenvalue.

If any square parameterized transfer function matrix (PTFM) $G(s,\sigma)$ has a regular splitting of the form

$$G(s,\sigma) = X(s,\sigma) + Y(s,\sigma)$$ (6)

and the parameterized precompensator $K_p(\sigma) = X(\sigma)^{-1}$; then the compensated system would be

$$G(s,\sigma)K_p(\sigma) = I + Y(s,\sigma)K_p(\sigma)$$ (7)
Thus, if $X(\sigma)$ is the dominant term of $G(s,\sigma)$, then the identity will be the dominant term of the compensated system. Since all eigenvalues of a matrix are bounded above by its maximum singular value or $\rho(.)<\sigma(.)$ and $\|\|_{\sup} \sigma(.)$; the inequality (5) could be presented as
\[ \|B(s,\sigma)'^TD(s,\sigma)\| \leq 1 \] (8)

Then, for the system having the regular splitting as in (6) the problem of achieving diagonal dominance can be presented as an optimization problem: Find a compensator $K_p(\sigma)$ can minimize.
\[ \|Y(s,\sigma)K_p(\sigma)\|_\infty = \|G(s,\sigma)K_p(\sigma)-\| \leq \gamma \] (9)

The minimization of $\gamma$ can be presented in the form of LMIs by using the Bounded Real Lemma (BRL)[6]. According to BRL, the state space system (1) corresponding PTFM form is $G(s,\sigma)=C_p(\sigma)(sI-A_p(\sigma))^{-1}B_p(\sigma)+D_p(\sigma)$, then $A_p$ is stable and $\|G(s,\sigma)\|_\infty < \gamma$, if and only if there exists a positive defined polynomial parameterized symmetric matrix $P(\sigma)$ satisfied LMI constrains (10).
\[
\begin{bmatrix}
A_p(\sigma)P(\sigma)+P(\sigma)A_p(\sigma)^T + B_p(\sigma)P(\sigma)C_p(\sigma)^T \\
* & -\gamma I \\
* & * & -\gamma I
\end{bmatrix} < 0
\] (10)

Let $G(s,\sigma)$ is the plant without compensation and define $\tilde{G}(s,\sigma)=G(s,\sigma)K_p(\sigma)-I$ as the compensated plant. Based on (10) the diagonal dominant constant precompensator $K_p(\sigma)$ will be the solution of optimization problem (11) restrained by LMIs.
\[
\min_{\sigma} \gamma \quad \text{s.t.} \\
\begin{bmatrix}
A_p(\sigma)P(\sigma)+P(\sigma)A_p(\sigma)^T + B_p(\sigma)K_p(\sigma)P(\sigma)C_p(\sigma)^T \\
* & -\gamma I \\
* & * & -\gamma I
\end{bmatrix} < 0
\] (11)

4. Convex polyhedron construction based compensator design
For LPV model, the PLF reduces the control system analysis and synthesis problem into an optimization problem with the LMI constraints. To guarantee the global characters of compensator design, we convert it into a normal convex optimization problem with LMI constraints based on a methodology of convex polyhedron construction[14].

Suppose $\Phi_p=(P_1,\ldots,P_n)^T$, considering (2) and (3), then the constraints (11) can be written as:
\[
F(\sigma)=\sum_{i=0}^{n} \sigma^i F_i(\Phi_p,\gamma) < 0 \] (12)

Where $F_i(\Phi_p,\gamma)$ is a symmetric matrix function that affinely depends on $\Phi_p$ and $\gamma$. For (12), we have the following theorem:

**Theorem 1[15]:** In $m$ dimensional linear space, assume that there exists a convex polyhedron $H_0$ including curve $\Gamma$ which is defined as:
\[
\Gamma=\{(\sigma,\ldots,\sigma^n)^T \mid \sigma \in [\sigma_{\min},\sigma_{\max}]\}
\] (13)

The $p' \in R^n$ and the vertex set of $H_0$ is $V_0=\{p_j \mid j=1,\ldots,q\}$. If there exist $\Phi_p$ and $\gamma$ that satisfy (14):
\[
F(p_j)=\sum_{i=0}^{n} p_{ij} F_i(\Phi_p,\gamma) < 0, \forall p_j \in V_0
\] (14)

Where $p_{ij}$ is the $j$th dimensional coordinate of $p_j$, then (12) will be satisfied on $\Gamma$.

Based on theorem 1, the global compensator design can be achieved by satisfying (14) on the constructed convex polyhedron vertex set. Then the convex polyhedron construction method in [14] is used and the construction method is not given in detail because the scope of this article. For LPV system (1), if the constructed convex polyhedron vertex set is $V=\{v_j \mid j=1,\ldots,q\}$, then the optimization problem (11) can be formulated as (15).
\[ \min_{\gamma} \gamma \quad \text{s.t.} \quad \sum_{q=0}^{k_1} q_{q}^{2} P_{q} > 0, \]

\[ = \sum_{q=0}^{k_1} q_{q}^{2} P_{q} + \sum_{q=0}^{k_2} q_{q}^{2} A_{q}^{T} P_{q} + \sum_{q=0}^{k_3} q_{q}^{2} P_{q} A_{q} + \sum_{q=0}^{k_4} q_{q}^{2} B_{q}^{T} K_{q} + \sum_{q=0}^{k_5} q_{q}^{2} B_{q}^{T} P_{q} B_{q} + \sum_{q=0}^{k_6} q_{q}^{2} B_{q}^{T} C_{p} B_{q} - \gamma I \]

Where, \( q_{q} \) is the \( q \) th dimensional coordinate of vertex \( v_{j} \). Then the compensator design problem for PLPV system with constraints(11) can be reduced into a normal LMI constrained convex optimization problem.

5. Numerical simulation

To verify the LMI based decoupling compensation method, the following PLPV model (16) is considered and the coefficient matrices satisfied the definition (1) and (2) with degree 2[14].

\[
\begin{bmatrix}
\dot{x}(t) \\
y(t)
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix} + \begin{bmatrix}
u \end{bmatrix}
\] (16)

Where the coefficient matrices \([A_{1}, A_{2}, A_{3}], [B_{0}, B_{1}, B_{2}], [C] \) and \([D] \) are:

\[
[A_{1}][A_{2}] = \begin{bmatrix}
-4.365 & -0.672 & -0.336 \\
0.581 & 0.855 & 0.589 \\
-0.670 & -1.375 & -0.991
\end{bmatrix},
[B_{0}] = \begin{bmatrix}
2.374 & 0.749 & 0.160 & -0.352 & 0.156 & 0.131
\end{bmatrix},
[C] = \begin{bmatrix}
0 & 0 & 0
\end{bmatrix}
\] (17)

The scheduling parameter \( \sigma \in [0, 1] \) and \( D_{\max} = 10 \).

In simulation, the pseudo-diagonalization is used as a comparative method with the given LMI based compensator. The coefficient matrices of designed compensator \( K_{\sigma} \) is given in (18).

\[
K_{\sigma} = \begin{bmatrix}
16.67 & 4.80 & -12.25 & -5.72 & -3.30 & 9.69 \\
1.56 & -1.61 & -2.07 & 1.30 & 0.03 & 0.12
\end{bmatrix}
\] (18)

In figure 1 and figure 2, the dominance ratio and Inverse Nyquist Array (INA) with row Gershgorin discs of \( \Sigma_{0} \) are given respectively where the \( \sigma = 0.1 \). Figure 3 and figure 4 show the dominance ratio and INA with row Gershgorin discs of pseudo-diagonalization compensated system \( \Sigma_{1} \) respectively where the key compensation frequency is 0Hz and \( \sigma = 0.1 \). Figure 5 and figure 6 show the dominance ratio and inverse Nyquist array (INA) with row Gershgorin discs of LMI based compensated system \( \Sigma_{2} \) respectively where the parameter \( \sigma = 0.1 \).
In figure 1 and figure 2, the first row of $\Sigma_i$ is almost non-diagonal dominance with large scale of Gershgorin discs size and the second row shows better diagonal dominance condition. Figure 3 and figure 4 shows that the first row of $\Sigma_i$ is almost non-diagonal dominance, the second row is non-diagonal dominance and both rows with large scale of Gershgorin discs size. As shown in figure 5 and figure 6, the LMI based compensation model show good diagonal dominance characteristics in working frequency domain and get satisfying decoupling performance with small scale of Gershgorin discs size. For LPV model, the LMI based compensation method also have global characteristics of diagonal dominance compensation without specifying certain frequency domain which the classical method can not satisfy. The more simulations on $\sigma \in [0,1]$ also show satisfying diagonal dominance performance results of compensated models.

6. Conclusion
In MIMO LPV control system analysis and synthesis, the interaction between different control loops can dramatically influence the control performance which changed with the varying schedule parameters. For a PLPV system a global robust dominance pre-compensator design method is proposed. Based on parameterized Lyapunov function, this parameterized compensator design is converted into an optimal problem constrained with parameterized linear matrix inequalities; To get the global solution, the design problem was reduced into a normal convex optimization problem with normal LMI constraints on constructed convex polyhedron. Finally, the feasibility and validity of the parameterized robust diagonal dominance pre-compensator design method are verified by the numerical simulation on a PLPV model.
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