Close-coupled model of Feshbach resonances in ultracold $^3$He* and $^4$He* atomic collisions

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Helium atoms in the metastable $^2(^3)S_1$ state (He*) have unique advantages for ultracold atomic experiments. However, there is no known accessible Feshbach resonance in He*, which could be used to manipulate the scattering length and hence unlock several new experimental possibilities. Previous experimental and theoretical studies for He* have produced contradictory results. We aimed to resolve this discrepancy with a theoretical search for Feshbach resonances, using a new close-coupled model of He* collisions in the presence of an external magnetic field. Several resonances were detected and the existing literature discrepancy was resolved. Although none of the resonances identified are readily experimentally useable, an interesting non-Feshbach scattering length variation with magnetic field was observed in heteronuclear collisions, at field strengths that are experimentally accessible.

I. INTRODUCTION

Ultracold atomic gases are a unique experimental platform for the study of atomic and molecular physics and quantum mechanics. The simplicity, isolation and fewer degrees of freedom offered by such systems allow exciting experiments to be performed such as Bose-Einstein condensation [1] and the trapping of atoms in periodic optical lattices [2]. One feature of ultracold gases is that the collisions between atoms are accurately described by a single parameter—the s-wave scattering length. Feshbach resonances allow the tuning of the scattering length, typically via the application of an external magnetic field. Suitable resonances have facilitated experiments such as exploring quantum phase transitions [3], superfluidity regimes [4], realising negative temperature states [5], or ‘Bose-nova’ condensate collapse [6].

Metastable helium (He*) atoms possess unique advantages in ultracold experiments as the large internal energies of these atoms allow charged plate detectors to achieve individual atom detection with relatively high spatial and temporal resolution [7]. He* atoms are trapped in the $^2(^3)S_1$ excited state of helium, which has a lifetime of 2.2 hours [8], longer than the timescale of typical ultracold experiments, and an internal energy of 19.8 eV [9], the largest of any metastable atom. These properties have facilitated unique experiments such as studies of Hanbury Brown-Twiss style matter wave interference [10], production and detection of Bell correlated atomic pairs [11, 12] and ghost imaging [13]. Several extensions to these, as well as other proposed experiments for He*, would benefit from the use of a Feshbach resonance to tune the scattering length. These include controlling the rate of entanglement generating collisions [11, 14], adjusting the two-body interaction parameter in many-body lattice experiments [15] and increasing the quantum depleted fraction of a BEC [16, 17].

Unfortunately, He* experiments have been unable to use a Feshbach resonance, because it is unknown if an experimentally accessible Feshbach resonance exists in this state. There has been a single theoretical study of this problem [18] which used a perturbative treatment called the Asymptotic Bound State model. That study predicted multiple resonances over the different isotopic mixtures, albeit many were too narrow and/or required too high magnetic field strengths to be experimentally useful. However, an experimental search for the most promising of the predicted $^4$He* resonances failed to observe it [19], suggesting further investigation is required.

In this manuscript we develop a non-perturbative, ab-initio close coupled model of metastable helium atoms in the presence of a magnetic field. Our results resolve the discrepancy between the prior study and experiment: the resonance that was not seen in the experiment is broadened by ionisation effects that were discounted in the perturbative treatment. We identify several other resonances, as well as interesting non-Feshbach resonance magnetic field dependence.

II. METHODS

This approach mostly follows that of [20], extending the method to include the Zeeman interaction. Briefly, we write the scattering wavefunction of the colliding atoms in a basis of states called channels, and use that decomposition to solve the scattering Schrödinger equation. Fitting to the large-distance solution yields a scattering matrix containing information such as cross sections. The model was written in the JULIA programming language [21] using the DIFFERENTIALEQUATIONS package [22].

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A. Hamiltonian

The scattering of two metastable helium atoms in an external magnetic field is governed by the Hamiltonian:

$$\hat{H} = \hat{T} + \hat{H}_{\text{sd}} + \hat{H}_{\text{rot}} + \hat{H}_{\text{el}} + \hat{H}_{\text{hfs}},$$

where

$$\hat{T} = -\frac{\hbar^2}{2\mu} \frac{\partial}{\partial R} \left( R^2 \frac{\partial}{\partial R} \right)$$

is the kinetic term, and

$$\hat{H}_{\text{rot}} = \frac{\hat{J}^2}{2\mu R^2}$$

is the centripetal operator effective in spherical coordinates.

$$\hat{H}_{\text{el}} = \hat{H}_1 + \hat{H}_2 + \hat{H}_{12}$$

is the electronic Hamiltonian, $\hat{H}_i$ representing the kinetic term for the electrons in atom $i$ and $\hat{H}_{12}$ representing the Coulombic interaction between the two atoms.

$$\hat{H}_{\text{sd}} = -\frac{\xi}{\hbar^2 R^3} [3(\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{R}})(\hat{\mathbf{S}}_2 \cdot \hat{\mathbf{R}}) - \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2]$$

is the spin-dipole interaction between the magnetic dipole moments of the two atoms. In the above, $\hat{\mathbf{S}}_i$ is the spin operator for atom $i$, $\hat{\mathbf{R}}$ is the unit vector along the internuclear axis, and

$$\xi = \alpha^2 \left( \frac{\mu_e}{\mu_B} \right)^2 E_h a_0^3.$$  

(6)

Here $\alpha$ is the fine structure constant, $\mu_e$ and $\mu_B$ are respectively the electron magnetic moment and Bohr magneton, $1 E_h \approx 27.2 \text{ eV}$ is a Hartree, and $1 a_0 \approx 0.529 \text{ Å}$ is a Bohr radius.

$$\hat{H}_{\text{hfs}} = -\frac{g_s \mu_B}{\hbar} \mathbf{B} \cdot (\mathbf{S}_1 + \mathbf{S}_2)$$

is the Zeeman interaction of the atomic spins in the magnetic field, where $g_s \approx 2$ is the electronic spin g-factor.

Finally, $\hat{H}_{\text{hfs}}$ is the hyperfine structure term which is only for $^3\text{He}^*$ atoms. The form of $\hat{H}_{\text{hfs}}$ is given below in Section IID.

B. Coupled channels approach

We use the coupled channel formalism, which expands the scattering wavefunction $|\Psi\rangle$ into a basis of molecular states $|\alpha\rangle$ using radial functions $G_a(R)$:

$$|\Psi(R)\rangle = \sum_a \frac{1}{R} G_a(R) |\alpha\rangle.$$  

(8)

The molecular states, called channels, are eigenstates of the system Hamiltonian at asymptotically large separation distances. We label them open or closed if the eigenvalues of $\hat{H}_{\text{asymp}} = \hat{H}_1 + \hat{H}_2 + \hat{H}_{\text{sc}} + \hat{H}_{\text{hfs}}$ are lesser or greater, respectively, than the total energy $E$ (i.e. closed channels are energetically forbidden at large separation distances). We also invoke the Born-Oppenheimer approximation, assuming that the channels have only a parametric dependence on the internuclear separation $R$.

Using this approximation the time-independent Schrödinger equation,

$$\hat{H}|\Psi\rangle = E|\Psi\rangle,$$

(9)

can be manipulated to yield the multichannel equations for the radial wavefunctions:

$$\sum_a \left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} \delta a' - V_{a' a}(R) - E \delta_{a' a} \right] G_a(R) = 0,$$

(10)

where $V_{a' a}(R) = \langle a' | \hat{V}(R) | a \rangle$ and $\hat{V} = \hat{H} - \hat{T}$.

C. Basis

It is difficult to directly write down a basis that simultaneously diagonalises the Zeeman and Hyperfine interactions which are both present at large separation distances. Instead we perform the calculation in a basis whose elements can be easily listed and find the eigenstates by numerically diagonalising the long-distance Hamiltonian.

The easily enumerated basis we use is the hyperfine basis:

$$|\alpha\rangle \equiv |\alpha \beta \ell m_l\rangle \equiv |(S_\alpha f_\alpha m_{f_{\alpha}})(S_\beta f_\beta m_{f_{\beta}}) \ell m_l\rangle.$$  

(11)

Here $\alpha$ and $\beta$ list the magnetic spin numbers for atoms $\alpha$ and $\beta$, with the electronic and nuclear spins of each atom coupled as $S_j + i_j = f_j$. $l = 0, 1, 2, \ldots$ is the rotational angular momentum of the two-atom system.

In the case of homonuclear collisions these basis states must be symmetrised to obey the fermionic and bosonic symmetry of $^3\text{He}^*-^3\text{He}^*$ and $^4\text{He}^*-^4\text{He}^*$ respectively [23]. We form symmetrised states $|(\alpha \beta) \ell m_l\rangle$ as linear combinations [20]:

$$|(\alpha \beta) \ell m_l\rangle = \frac{|\alpha \beta\rangle + (-1)^{\ell + \ell_{a \beta} + m_l} |\beta \alpha\rangle}{\sqrt{2(1 + \delta_{a \beta})}} \otimes |\ell m_l\rangle.$$  

(12)

In the case of $^3\text{He}^*-^4\text{He}^*$ collisions we use the unsymmetrised $|(\alpha \beta) \ell m_l\rangle$ states.

D. Matrix elements

When integrating the multichannel equations (10) we must calculate the matrix elements $V_{a' a}(R)$. 
The rotational interaction has the simplest matrix element:

\[
\langle a' | \hat{H}_{\text{rot}} | a \rangle = \delta_{a,a'} \frac{l(l+1)\hbar^2}{2\mu R^2}.
\]  

(13)

The matrix elements for \( \hat{H}_{\text{el}} \), \( \hat{H}_{\text{eel}} \), and \( \hat{H}_{\text{ad}} \) are evaluated by changing into the basis associated with the coupling scheme \( S_\alpha + S_\beta = S \), using the expansion in terms of Clebsch-Gordan coefficients:

\[
|\alpha\beta\rangle = \sum_{m_\alpha} \sum_{m_\beta} \sum_{m_S} C_{m_\alpha, m_\beta}^{S} m_S \hat{S}_{m_\alpha}^{\beta} \hat{S}_{m_\beta}^{S} \langle i_{\alpha} m_{\alpha}^i | j_{\beta} m_{\beta}^j \rangle.
\]  

(14)

In this basis \( \hat{H}_{\text{el}} \) is diagonal:

\[
\hat{H}_{\text{el}} | S m_s \rangle = 2S+1 V_\text{el}(R) | S m_s \rangle.
\]  

(15)

The quintet Born-Oppenheimer \( ^5\Sigma_u^+ \) potential is taken as the analytic form described by Przybytek and Jeziorski [24]. The singlet \( ^3\Sigma_u^+ \) and triplet \( ^3\Sigma_u^- \) potentials are interpolated from the tabulated values of Müller et al. [25] within the \( 3a_0 < R < 14a_0 \) range where those values are given, and fitted to the \( ^3\Sigma_u^+ \) potential at larger distances. Following the method of [20], this fitting is done using an exponentially decaying exchange term:

\[
1.3 k V_\text{e}(R > 14a_0) = k V_\text{e}(R < A_1, 3) \exp(-\beta_1 R),
\]  

where \( A_1 = 5.9784, \beta_1 = 0.7367, A_3 = 1.7980, \) and \( \beta_3 = 0.6578. \)

We write the spin-dipole as the product of two second-rank irreducible tensors \( \hat{H}_{\text{ad}} = V_p(R) \hat{T}^2 \cdot \hat{C}^2 \), where \( V_p(R) = b/R^3 \) and \( b = -\sqrt{6} \xi \) [26]. This gives the matrix elements as

\[
\langle a' | \hat{H}_{\text{ad}} | a \rangle = V_p(R) D_{aa'}. \]

(16)

The coupling coefficient is

\[
D_{aa'} = \delta_{m_{aa'} + m_{a',m_{S,a} + m_{a',m_{S,a}}}} (-1)^{m_{S,a} - m_{S,a'}}
\]

\[
\times C^{S_{a'} m_{a'}^{S_{a'}}}_{m_{a'} m_{a'}^{S_{a'}}} C^{S_{a'} m_{a'}^{S_{a'}}}_{m_{a'} m_{a'}^{S_{a'}}} \times \langle S_{a'} S_{a'}' S' || \hat{T}^2 || S_{a} S_{a} S \rangle \langle \hat{T}' || \hat{C}^2 || \rangle.
\]  

(17)

The reduced matrix elements for the tensors \( \hat{T}^2 \) and \( \hat{C}^2 \) are

\[
\langle S_{a'} S_{a'}' S' || \hat{T}^2 || S_{a} S_{a} S \rangle = \delta_{S_{a}, S_{a'}} \delta_{S_{a}, S_{a'}} \delta_{S_{a}, S_{a'}} \times \frac{\sqrt{S_{a}(S_{a} + 1) S_{a} + 1}}{\sqrt{5(2S_{a} + 1)(2S_{a} + 1)}} \times \begin{vmatrix} S_{a} \ S_{a} \ \mathbf{1} \ \mathbf{1} \\ \mathbf{1} \ \mathbf{1} \ 2 \end{vmatrix},
\]

(18)

where the last factor is a Wigner 9-j coefficient, and

\[
\langle \hat{T}' || \hat{C}^2 || \rangle = \sqrt{\frac{2l + 1}{2l'} + 1} C^{l, l'}_{l'}.
\]

(19)

The matrix elements for the Zeeman interaction are simply

\[
\langle S' m_{s'} || \hat{H}_{\text{eel}} || S m_s \rangle = \delta_{S S'} \delta_{m_{s}, m_{s'}} \frac{g_a \mu_B \mu}{\hbar} B m_s.
\]  

(20)

The relative difference between the electron spin g-factor and the \( 2^3S_1 \) g-factor is of the order of \( 10^{-5} \) [27] and we therefore neglect it. We also neglect the contribution from nuclear spin because the nuclear magnetic moment is on the order of \( 10^9 \) smaller than the electronic magnetic moment [28].

\[
\hat{H}_{\text{eel}} \text{ is diagonal in this basis, with the matrix elements [29]:}
\]

\[
\langle a' | \hat{H}_{\text{eel}} | a \rangle = \delta_{a,a'} (E_{i_{a},s_{a}}^{\text{hfs}} + E_{i_{a},s_{a}}^{\text{hfs}})
\]  

(21)

where

\[
E_{i,s}^{\text{hfs}} = \begin{cases} 
1.519830 \times 10^{-7} E_h & i = \frac{1}{2}, f = \frac{3}{2}, \\
0 & \text{otherwise}
\end{cases}
\]  

(22)

To optimise the numerical integration routine we recognise that the non-diagonal interactions—\( \hat{H}_{\text{el}} \), \( \hat{H}_{\text{ad}} \) and \( \hat{H}_{\text{eel}} \)—can be written separately as products of R-independent matrices and a scalar functions of \( R \). For example in the case of the electronic term,

\[
\langle a' | \hat{H}_{\text{el}}(R) | a \rangle = \sum_{S=0}^{2} C^{2S+1}_{aa'} V^{2S+1}(R),
\]  

(23)

where each \( C^{2S+1}_{aa'} \) is a coupling coefficient. This separation of R-independent coupling terms can be written more generally as:

\[
V_{aa'}(R) = \sum_{k} V_{aa'}^{k} \otimes f_{k}(R),
\]  

(24)

where \( V_{aa'}^{k} \) would be a coupling coefficient and \( f_{k} \) a radial function. There are three different radial independent terms for \( \hat{H}_{\text{el}} \) (corresponding to the three values of \( S = 0, 1, 2 \)) and each for \( \hat{H}_{\text{ad}} \) and \( \hat{H}_{\text{eel}} \). We pre-calculate the radial-independent coupling coefficients for the non-diagonal interactions so that only the radial factors are repeatedly calculated.

E. Scattering Matrices and Cross sections

The scattering problem is constrained by inner and outer boundary conditions as \( R \to 0 \) and \( R \to \infty \) respectively. All radial wavefunctions must vanish at the inner boundary where the potential diverges; additionally, all closed channel wavefunctions must vanish at the outer boundary. We define linearly independent initial conditions representing both sets of boundary conditions, and integrate those conditions to a single middle point. We then use a QR decomposition to find linear combinations of the boundary conditions that produce matching
solutions at that point. These matched solutions satisfy both boundary conditions.

The matched solutions, evaluated at the outer boundary where the electronic and spin-dipole interactions can be safely neglected, are then fitted to spherical bessel functions, which represent the oscillatory asymptotic form of the scattering wave. Specifically, the solutions $F(R)$ are matched to the form \[ F(R) \xrightarrow{R \to \infty} J(R)A + N(R)A, \] where $J(R)$ and $N(R)$ are diagonal matrices with the entries \[ J_{aa}(R) = \sqrt{k_a R} j_{lm}(k_a R), \] \[ N_{aa}(R) = \sqrt{k_a R} n_{lm}(k_a R). \] (26)

The matrices have one entry for each open channel $a$.

The fitted matrices $A$ and $B$ are used to define the reactance matrix $K = BA^{-1}$, which then defines the scattering matrix:

\[ S = (I + jK)(I - jK)^{-1}. \] (27)

Cross sections for scattering from one channel into another (including elastic scattering where the incoming and outgoing channels are identical) are defined from the scattering matrix:

\[ \sigma(\gamma \rightarrow \gamma') = \frac{\pi}{k^2} \sum_{l'm_l'} |T_{\gamma'lm_l',\gamma lm_l}|^2, \] (28)

where the transition matrix $T = I - S$.

Similarly, the ionisation cross sections are found from the non-unitariness of the scattering matrix:

\[ \sigma(\gamma \rightarrow PI) = \sum_{l'm_l} \left[ 1 - \sum_{\gamma'lm_l'} |S_{\gamma'lm_l',\gamma lm_l}|^2 \right]. \] (29)

F. Resonance search

Elastic and ionisation cross sections are calculated at constant collisional energy over a grid of magnetic fields ranging from 0 G to $10^4$ G (0 T to 1 T). A maximum grid spacing of 400 G is used to observe the broad behaviour and identify resonances, with smaller grid spacing used around features of interest and where resonances were predicted in [18].

For each isotopic mixture we consider the two most experimentally relevant scattering channels, which are the two channels with lowest energy (under the hyperfine and Zeeman interactions). In bosonic He$^*$ the lowest energy channel corresponds to spin polarised atoms in the $m_f = +1$ state—this channel is the most experimentally accessible because the polarisation minimises Penning ionisation [31]. We do not expect resonances in the lowest energy fermionic channel as that channel involves identical fermions. However, we consider resonances in the second lowest energy fermionic channel, which can be experimentally accessed for example by using a radiofrequency sweep to transfer atoms from the lowest energy $|f, m_f\rangle = |3/2, 3/2\rangle$ state to the $|3/2, 1/2\rangle$ state before collision.

From the elastic scattering cross sections we calculate resonance locations and widths, by fitting Fano profiles to the data according to the equation:

\[ \sigma_{cl}^{\text{Fano}}(B) = \sigma_0 \left( \frac{q + \frac{\Delta}{2} + B - B_0}{\left(\frac{\Delta}{2}\right)^2 + (B - B_0)^2} \right)^2. \] (30)

Here $\sigma_0$ is the non-resonant background cross section, $B_0$ is the resonance location, $q$ is a shape parameter, and $\Delta$ is the resonance width.

III. RESULTS AND DISCUSSION

Seven resonances identified in this study are described in Table I. Three each are identified in the lowest energy scattering channels of heteronuclear and bosonic collisions. Another resonance is identified in the second-lowest energy scattering channel of fermionic collisions.
Figure 1. Elastic cross section of bosonic $^4$He$^*$$^4$He collisions in the lowest energy scattering channel. Blue shows cross sections with Penning ionisation accounted for; orange shows cross sections with Penning ionisation neglected. The orange curve lies under the blue curve except for the first resonance.

In this discussion we compare with the results of [18], which is the only existing theoretical study of He$^*$ Feshbach resonances. Two of the three resonances identified in homonuclear $^4$He$^*$ collisions (rightmost two resonances in Figure 1) roughly coincide with the previous theoretical study. The resonance at 345.0(7) G is located within the range of previously forecast locations, which was 99 G to 460 G [18]. This resonance has been investigated experimentally but not found [19]. However, upon switching off ionisation in our model (blue line in Figure 1), our results indicate that it is broadened massively by ionisation processes, which were neglected in the prior study. These results offer a complete explanation of the discrepancy between theory and experiment.

We believe the resonance seen at 4881(9) G in Figure 1 is the same as a previously predicted resonance at 5460 G [18]. Single channel calculations of bound state energies via counting of wavefunction nodes [32], performed using this model, matched the resonance seen here to the same associated bound state as in the previous prediction. The third resonance at $9.75(2) \times 10^3$ G was not previously predicted, however its appearance is unsurprising. The $4881(9)$ G resonance is caused by coupling between the $|S = 2, m_S = +2\rangle$ scattering channel and a bound state in the $|S = 2, m_S = 0\rangle$ closed channel; however, that scattering channel may also couple to a bound state in the $|S = 2, m_S = +1\rangle$ channel. Because the Zeeman potential is directly proportional to the spin projection, when the spin projection between the channels is halved, the resonance condition for the same bound state:

$$\frac{g_S \mu B}{h} B_0 (m_{s_{\text{scat}}} - m_{s_{\text{bound}}}) = E_{\text{bound}} - \frac{\hbar^2 k^2}{2\mu},$$

occurs at twice the magnetic field. Indeed, $9.75(2) \times 10^3$ G is within 0.1% of $2 \times 4881$ G. We in fact used this difference (between our model data and the precise factor of 2) to estimate the relative uncertainty in $B_0$, giving an uncertainty estimate which we propagated to all other resonances in our model. Note that this estimate does not reflect uncertainty from our use of the Born-Oppenheimer potentials, which themselves are uncertain—particularly the singlet and triplet potentials of [25].

The fermionic Feshbach resonance seen in Figure 2 was also predicted previously [18], with our calculated value at the upper bound of the previously predicted range of values. Those previous bounds were determined by variation in the $S = 1$ potential.

The heteronuclear $A$ channel resonance structure (Figure 3) is qualitatively very similar to the bosonic $A$ channel structure (as shown in Figure 1). This is likely due to the similarity between the homonuclear bosonic system and the reduced set of heteronuclear channel states that are directly coupled by the Zeeman interaction. Both of the larger two heteronuclear resonances were previ-
Figure 4. Elastic cross section of heteronuclear $^3\text{He}^*-^4\text{He}^*$ collisions in the second-lowest energy scattering channel. Inset: while no Fano profiles indicative of Feshbach resonances are identified, the cross section varies by over an order of magnitude across an experimentally accessible range of magnetic fields weaker than 100 G.

ously predicted to exist [18]. Interestingly, the resonance centred at $9.55(2) \times 10^3$ G was predicted to be narrower than the $4749(6)$ G resonance, but in these results resonance widths increase as $B_0$ increases. As with the small bosonic resonance, the ionisation process broadens the $130$ G resonance to the extent that it cannot be seen.

No Fano profiles were observed in the heteronuclear B channel (Figure 4), not even a very wide resonance previously predicted at $1214$ G. Three other resonances in this channel were previously predicted, however without resonance widths [18]. We thoroughly searched for a resonance predicted at $3618(6)$ G, scanning from $3598$ G to $3638$ G with a grid spacing of $1$ G, however no field dependence was identified in the cross section.

Interestingly, Figure 4 shows that in this scattering channel there are variations in the elastic cross section that are inconsistent with a Fano profile. Similar curves appear in the ionisation cross sections (not shown). The sharp spike in cross section between $0$ G and $100$ G—reminiscent of a shape resonance [33]—is particularly interesting, with the elastic cross section there varying by over an order of magnitude. This magnetic field range is experimentally accessible, so while not being a true Feshbach resonance (i.e. with a diverging scattering length), this behaviour could allow nontrivial manipulation of the scattering length of heteronuclear $\text{He}^*$ collisions, opening up several experimental possibilities. Experimental verification of this variation is an interesting and open question.

Unfortunately, our results do not identify a Feshbach resonance that appears experimentally useful. In the bosonic A channel ionisation processes broaden the $345$ G resonance to the point of insignificance. The resonance at $4881(9)$ G would likely pose difficulties in an experiment, as its location and width would demand a very powerful but stable electromagnet; these difficulties are magnified for the $9.55(2) \times 10^3$ resonance. This state of affairs also describes the heteronuclear A channel where the resonance structure is similar. The singular resonance in the fermionic B channel occurs at an even larger magnetic field and is therefore even less useful.

In contrast to the identified (unbroadened) Feshbach resonances, the non-Feshbach variation in the $3$-$4$ B channel cross section occurs over a very accessible range of magnetic fields. Because this variation is not a Feshbach resonance the scattering length may not diverge. However, with the elastic cross section varying by a factor of $\sim 30$, by $\sigma \approx 4\pi a^2$ that would imply the scattering length varies by a factor of $\sim 5$, which is significant. Ionisation would likely be a hindrance practically however, as our model calculates the ionisation cross section for heteronuclear B channel collisions to be $\sim 10^7$ times larger than for spin polarised bosonic helium.

IV. CONCLUSION

We developed a new close coupled model of ultracold $\text{He}^*$ atom collisions in the presence of a magnetic field and used it to search for Feshbach resonances. Several resonances were identified across bosonic, fermionic, and heteronuclear scattering channels, however none are likely to be experimentally accessible. We identified that a predicted Feshbach resonance that was not seen in experiment is actually broadened by ionisation processes. We also identified magnetic field dependence in the cross section of the second-lowest energy heteronuclear scattering channel, which although not a Feshbach resonance, occurs at low magnetic fields that may be experimentally accessible.

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