Seismic Travel Time Tomography in Modeling Low Velocity Anomalies between the Boreholes

A Octova\textsuperscript{1*}, R Sule\textsuperscript{2}

\textsuperscript{1}Mining Engineering Department, Faculty of Engineering, Universitas Negeri Padang, Indonesia
\textsuperscript{2}Geophysical Engineering Department, Faculty of Mining and Petroleum Engineering, Institut Teknologi Bandung, Indonesia

*adree.octova@gmail.com

Abstract. Travel time cross-hole seismic tomography is applied to describing the structure of the subsurface. The sources are placed at one borehole and some receivers are placed in the others. First arrival travel time data that received by each receiver is used as the input data in seismic tomography method. This research is divided into three steps. The first step is reconstructing the synthetic model based on field parameters. Field parameters are divided into 24 receivers and 45 receivers. The second step is applying inversion process for the field data that consists of five pairs bore holes. The last step is testing quality of tomogram with resolution test. Data processing using FAST software produces an explicit shape and resemble the initial model reconstruction of synthetic model with 45 receivers. The tomography processing in field data indicates cavities in several place between the bore holes. Cavities are identified on BH2A-BH1, BH4A-BH2A and BH4A-BH5 with elongated and rounded structure. In resolution tests using a checker-board, anomalies still can be identified up to 2 meter x 2 meter size. Travel time cross-hole seismic tomography analysis proves this method is very good to describing subsurface structure and boundary layer. Size and anomalies position can be recognized and interpreted easily.

1. Introduction
Geophysics is one of exploration method that can be used to knowing subsurface imaging with a relatively short time and low cost. One of the geophysical methods that provides high resolution picture is seismic tomography between the boreholes. Crosshole Seismic tomography uses characteristic of artificial wave propagation to imaging the anomalies through it. The source of these artificial waves can be dynamite or modified sources for various purposes. The waves propagating from the source placed on one of the borehole will be received by the receivers at the other borehole. Travel time seismic tomography has several advantages. The results of the analyzed data are only the first wave travel time received by the receivers (first break). The other waves do not need to be analyzed further.

Geophysical modeling consists of two types that are forward modeling and inverse modeling. In seismic tomography, forward modeling is a way of seeing the movement of waves in a geological model. This modeling begins with initial modeling based on the parameter model. By doing mathematical analysis of physical phenomena, we will be known the movement of the wave of each stage of time. The Output from forward modeling is the travel time calculation. The result of this
forward modeling is using to testing the algorithm for a geological model. This process is known as inverse modeling.

The medium that through the ray of wave is divided over grids that have a certain speed. Eikonal equation can be used to calculate travel time to get positions, directions and time. Various ways can be applied from the Eikonal equation. Simplicity of Eikonal equation formulation is related to speed of calculation.

In this research will be presented a suitability method of crosshole seismic tomography in providing an overview between the boreholes. The first step is reconstructing the synthetic model. The second step is applying inversion process for the field data. The last step is testing quality of tomogram.

2. Seismic Tomography Modeling

Projection and reconstruction play important role in seismic tomography [1,2]. The Projection is a observation of object responses that given disturbance. The object is subsurface structures, while the disturbance is seismic waves through to the object. The Projection process is usually called as forward modeling. The opposite of the projection is the reconstruction process. This process is object development based on projected results. By doing it, we will know the object structure. The reconstruction process is usually called as inverse modeling.

Forward modeling can describe seismic patterns from geological model. This patterns include ray path which produces travel time of waves emitted from source to receivers. The medium which through out of the ray is divided into grids that have certain velocities. Travel time calculation is performed by resolve the Eikonal equation to obtain the position, direction and time propagation. Various ways can be applied to obtain the approach of Eikonal equation. Simplicity Eikonal equation formulation is associated with computing speed. Eikonal equation is expressed by :

\[
|\nabla t|^2 = \frac{1}{v^2} \text{ or } |\nabla t|^2 = s^2, \quad (1)
\]

\[
\frac{\partial T}{\partial x_i} \frac{\partial T}{\partial x_i} = \frac{1}{c^2} \quad (2)
\]

\[
\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = \frac{1}{c^2} \quad (3)
\]

s is slowness parameter

Eikonal equation is a solution of high frequency wave equation that can calculate the shortest travel time between source and receiver. This equation helps in the reconstruction of wave propagation in the medium in its path. Eikonal equations can be solved in several ways, one of them is finite difference methods [3-5] whose have developed FAST program.

The solution of the eikonal equation using finite difference method was proposed by [6]. The main advantage of this method is that data directly calculated to the point of the grid, no interpolation is needed next. Travel time functions are restricted to single-valued, thus this technique is suitable for first arrival time modeling.

Figure 1 shows a grid element, counter-clockwise numbering nodes, starting from the left bottom node. 0th node is a known travel time (t0). This travel time is used to determine time propagation in 1st, 2nd, and 3rd nodes (t1, t2, t3).
Inverse modeling is a process to get model from data. The input is delay time of wave propagating from source to receivers. This delay time is difference between observed and calculated wave travel time at specific velocity model of the earth. Travel time is determined from the observed wave arrival time reading on the receiver. The output of inverse modeling is subsurface geological model. Travel time tomography is a nonlinear inverse problem that can be solved by linearization and iteration. Travel time of seismic wave propagation through the slowness is obtained from:

\[ T_i = \int_{L_i} s(\vec{x}(l)) \, dl \]  \hspace{1cm} (4)

where \( dl \) is the segment of the ray length along the ray path \( L_i \) from source to receiver. Linearization of the Eikonal equation from the slowness is resulting from the changes of travel time to parts of slowness perturbation (\( \Delta s \)). It can be written be:

\[ T_i = \int_{L_i} [s_0(\vec{x}(l)) + \Delta s(\vec{x}(l))] \, dl = \int_{L_i} s_0(\vec{x}(l)) \, dl + \int_{L_i} \Delta s(\vec{x}(l)) \, dl \]  \hspace{1cm} (5)

Based on Fermat's principle which states that no changes of ray path length, while the rays to be divided (\( L_i \approx L_i^0 \)), so that the time delay can be written as:

\[ \delta t_i = T_i - T_i^0 \approx T_i^0 = \int_{L_i^0} \Delta s(\vec{x}(l)) \, dl \]  \hspace{1cm} (6)

or

\[ \delta t_i = \sum_{j=1}^{M} \delta s_j \partial L_i^0 \]  \hspace{1cm} (7)

where \( M \) is the number of cells, \( \partial L_i^0 \) is the length of the ray path segment of \( i^{th} \) ray in \( j^{th} \) cell of the reference model, as shown in Figure 2.

Inversion problem is usually solved by least-squares minimization method. This method estimate solution by finding model parameters that minimize of length measurement from the modeled data. Solution of linear least-squares inversion is calculating the error (\( E \)) that obtained on the model parameters (\( m \)) that produces a zero value. So it can be written:

\[ G^T G m - G^T d = 0 \]  \hspace{1cm} (8)

This equation is a quadratic matrix equation for unknown model parameters. Meanwhile, for known model parameters, the solution of least-squares method is:

\[ m = (G^T G)^{-1} G^T d \]  \hspace{1cm} (9)
The damped least-squares inversion will cause problems if there are a lot of "a priori" information that applied in the model parameters. This information will control the model parameters which produces misinterpretation final result. These results will only explain the influence of "a priori" dominant rather than observational data. Regularization is an approach by addition of the inversion data to omit the under-determined part. The regularization scheme of [5] minimizes an objective function that includes norms that measure model roughness and data misfit. The advantage of this scheme is its flexibility to combine the smallest, flattest, and smoothest perturbation constraints in the inversion scheme [7]. The objective function \( \Phi \) minimized at each iteration is:

\[
\Phi(m) = \delta t^T C_d^{-1} \delta t + \lambda [m^T C_h^{-1} m + \omega m^T C_v^{-1} m]
\]

where \( \delta t = t - G(m) \) is the data residual vector; \( m \) is the model vector; \( C_d \) is the data covariance matrix which is a diagonal matrix and consists of picking uncertainty information; \( C_h \) and \( C_v \) are the roughness matrices in horizontal and vertical direction, respectively; and \( \lambda \) is the trade-off parameter. For 2D tomographic, \( C_h \) and \( C_v \) are band-diagonal matrices, which non-zero elements represent the second-order Finite Difference scheme of the Laplace operator. The parameter \( \omega \) is the parameter that controls the inversion scheme which can be written by:

\[
\omega = \alpha s_z + (1 - \alpha) s_p
\]

where \( \alpha \) is the parameter that controls the relative weighting of fitting the smallest perturbation (sp) constraint equations versus smoothnest/flattest constraint equation. Meanwhile \( s_z \) is smoothness parameter between vertical and horizontal models.

The minimization of the objective function \( \Phi \) is related to the results of the model vector \( m \) that contain in equation:

\[
\begin{bmatrix}
C_d^{-1} L

\lambda C_h

\omega C_v
\end{bmatrix}

\delta m =
\begin{bmatrix}
C_d^{-1} \delta t

-\lambda C_h m_0

-s_z \lambda C_v m_0
\end{bmatrix}
\]

where \( L \) is the kernel data matrix containing the ray segment in each cell; \( m_0 \) is the current model; \( \delta m \) is the model perturbation using the updated model which is given by \( m = m_0 + \delta m \).

Generally, seismic tomography process can be shown in the flow chart in Figure 3 [8]. The most crucial processes in tomography are picking travel time observation and determinating initial model. Tomography gives updated model of the earth at each iteration in order to produce very small difference travel time observation data. The smallest difference can be used as the final model.
3. Synthetic Model

Simple models are made in order to travel time tomography test. Initial velocity model (background velocity) is set to 2000 m/s whole the medium. Positive and negative anomalies are positioned in the center of the medium with 4 m x 4 m dimensions. Positive anomaly has 2500 m/s velocity (Figure 4 top left) whereas negative anomaly has 1500 m/s velocity (Figure 4 bottom left).

The inversion results can be shown in perturbation velocity cross section, i.e. absolute velocity changes of absolute average overall value of the tomogram. In Figure 4 right, top and bottom can be seen clearly that the structure of the positive and negative anomalies are corresponding to the initial model. The structure and dimension of the anomaly are similarly in size and position. It has been proven the reconstruction of synthetic data in positive and negative anomalies with this method gives good result.
4. Application In Real Data
Seismic travel time tomography has been applied in synthetic data with good reconstruction of positive and negative anomalies. Then it is used in real data to see structures between boreholes in Kuala Lumpur Malaysia. There are five boreholes were applied in this area. Position of these boreholes are shown in Figure 5.

Figure 5. Boreholes position in Istana Island
Initial velocity model is the first step to choose velocity background as in situ velocity. It is obtained from travel time average against the distance of source to receiver. In Kuala Lumpur Malaysia the initial velocity model that chosen is 4521 m/s whole the model.

The model is divided into small cells in x, y, and z direction. The size distribution of this model affects for the quality of tomogram result and computing time. The smaller grid gives better tomogram result, but with a longer computing time. In Kuala Lumpur Malaysia, the grid that chosen is 0.1 m x 0.1 m.

After inversion process with FAST program, there are five sections tomogram that produced in Istana Island Malaysia. The rays move from sources to receivers not in straight line. They are moved away and assembled in several sections. These rays can be seen in Figure 6 left, Figure 7 left, Figure 8 left, Figure 9 left and Figure 10 left. There are only three sections indicate cavities presence in Figure 6 left, Figure 8 left, and Figure 10 left. The gathering area become a place with high velocity anomalies and the moved away area are low velocity anomalies as predicted as cavities. The perturbation sections as shown in Figure 6 middle, Figure 7 middle, Figure 8 middle, Figure 9 middle and Figure 10 middle give clear structures and clear cavities anomalies (Figure 6 middle, Figure 8 middle, and Figure 10 middle). Cavities are predicted in rounded line. The dashed line are the limit section with no data.

The absolute velocity sections are generally composed loose soil and limestone. Weaker (fractured and more porous) limestone or perhaps related to water infilled cavities are identified between the solid limestone. These cavities give dramatically velocity changes inside the limestone layer as shown in Figure 6 right, Figure 8 right, and Figure 10 right. The absolute of the cavities have range from 3800 m/s to 4800 m/s.

The combination of five pairs boreholes produce 3D section as shown in Figure 11. This section gives excellent layer boundaries between layers of soil and marble. There is interconnected layers boundary between one drill hole to another. This proves that seismic tomographic methods between the boreholes can provide a good overview of the quality results that can be interpreted easily.

5. Checker-Board Test
To make sure quality of the tomographic result, it needs to be done of resolution test [9]. The models are made in such a way using the synthetic data. FAST program is used to reconstruct a model of the inversion process. This resolution test is using the checker-board with the different size of anomalies.

The resolution test is done by making several positive and negative anomalies inside the models. There are three types of schemes. These are 4 m x 4 m, 3 m x 3 m, and 2 m x 2 m anomalies. This process is carried out to see the accuracy of the tomographic inversion process to reconstruct the size of the anomalies. Background velocity is set to the value of 2000 m/s. Positive anomalies are set to the value of 2500 m/s and the negative anomalies are set to value of 1500 m/s. These initial model can be seen in Figure 12 top.

The shots position are placed in the left side of the boreholes from 6 m to 50 m depth. Meanwhile the receivers are positioned in the right borehole from 5 m to 50 depth. Generally after doing the numbers of iterations, anomalies are still reconstructed with 2 m x 2m anomalies with same shape and position. But then, in the top of section, anomalies couldn’t give same shape as well as initial model. It caused of no data in 0 m to 5 m depth (Figure 12 bottom).
**Figure 6.** Inversion result of BH2A-BH1. left to right: ray tracing, perturbation section, absolute velocity section.

**Figure 7.** Inversion result of BH2A-BH3. left to right: ray tracing, perturbation section, absolute velocity section.
**Figure 8.** Inversion result of BH4A-BH2A. left to right: ray tracing, perturbation section, absolute velocity section.

**Figure 9.** Inversion result of BH4A-BH3. left to right: ray tracing, perturbation section, absolute velocity section.
**Figure 10.** Inversion result of BH4A-BH5. left to right: ray tracing, perturbation section, absolute velocity section.

**Figure 11.** 3D section
Figure 12. Checker-board test. Top: initial models. Bottom: reconstruction results. Left to right: 4 m x 4 m anomalies, 3 m x 3 m anomalies, and 2 m x 2 m anomalies.
6. Conclusions
Based on the tomogram obtained, crosshole seismic travel time tomography method can give good and clear subsurface structure. It proves from anomalies detected between the boreholes. Synthetic model with square model in the middle area could reconstructed similarly. Meanwhile for real data in Malaysia, could detected the cavities in certain of depth. The accuracy of tomogram resolution, is effected on the number of rays. More ray that passes through the medium will give better and more detail tomogram result. Checker-board test gives that anomalies still can be identified up to 2 m x 2 m size.

Acknowledgments
We thank the Department of Irrigation and Drainage - Malaysia for the permission in using the data.

Reference
[1]. Stewart, R.R 1991 Exploration Seismic Tomography : Fundamentals (Course Notes Series, Volume 3. Society of Exploration Geophysicists)
[2]. Widiyantoro, S 2000 Diktat Tomografi Geofisika Edisi ke-1 (Program Studi Geofisika Jurusan Geofisika dan Meteorologi Fakultas Ilmu kebumian dan Teknologi Mineral Institut Teknologi Bandung)
[3]. Aldridge, D.F., D.W.Oldenburg 1993 Two-Dimensional Tomographic Inversion with Finite-Difference Traveltimes (Journal of Seismic Exploration 2) p 257-274
[4]. Eaton, D.W.S 1990 Finite-difference Solution of The Eikonal Equation for Transversely Isotropic Media
[5]. Zelt, C.A., P.J.Barton 1998 Program Package for First Arrival Seismic Tomography (Rice Univercity, Houston)
[6]. Vidale, J 1988 Finite-difference Calculation of Traveltimes (Bull. Seism.Soc.Am) p 78:2062-2076
[7]. Sule, M.R 2004 Seismic Travel Time Tomography and Elastic Waveform Modeling Aplication to Ore-Dyke Characterizations (Dissertation, Logos Verlag Berlin)
[8]. Grandis, H 2009 Pengantar Pemodelan Inversi Geofisika (Himpunan Ahli Geofisika Indonesia)
[9]. Octova, A 2014 Rekonstruksi Model Sintetis 2D Seismik Tomografi Waktu Tempuh Antar Lubang Bor dengan Metode Beda Hingga (Jurnal Invotek ISSN 1411-3414 Vol. XIV, No. 1 - Februari 2014 pp 2875-2886