CP-ODD PHASES IN SLEPTON PAIR PRODUCTION

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The effects of CP-odd supersymmetric phases on slepton pair production are considered. It is shown that CP-even observables in $e^+e^-$ and $e^-e^-$ collisions, such as the total selectron cross section, can depend on CP-odd supersymmetric phases through interference between different tree level amplitudes. Left handed selectron pair production in $e^-e^-$ collisions is particularly sensitive to the relative phase between the bino and wino masses. This sensitivity is not limited to any kinematic regime and extends over all of neutralino parameter space. The relative phase between the bino and wino masses is a renormalization group invariant at one-loop, and as such provides a clean probe for operators which violate gaugino universality at the messenger scale.

1. Introduction

If nature is supersymmetric at the weak scale, a plethora of superpartners are waiting to be discovered at future colliders. The spectrum and couplings of the superpartners provide an indirect window to the messenger scale for supersymmetry breaking. Precision measurements of the superpartners could therefore provide indirect information about physics at scales well beyond those directly accessible to colliders. In this paper the possibility of measuring CP-odd supersymmetric phases in selectron pair production is considered.

The CP-violating phases of the minimal supersymmetric standard model (MSSM) are reviewed in the context of slepton pair production in the next section. The two basis independent combinations of phases in the neutralino mass matrix are identified. The possibility and advantages of measuring CP-odd phases with CP-even observables through interference between different tree level amplitudes is discussed in section 3. It is shown that selectron pair production in $e^+e^-$ or $e^-e^-$ collisions can depend on the CP-odd phases in the neutralino mass matrix. These processes are interesting in that effects of CP-odd phases arise from interference between amplitudes in the same kinematic channel, and so are not limited to any particular

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region of phase space. The relative sensitivity in different helicity modes to these phases in the gaugino or Higgsino limit is explained in section 3.1 in terms of the chiral properties of the tree level amplitudes. In section 3.2 the production of left handed sleptons in $e^-e^-$ collisions is shown to be particularly sensitive to the relative phase between the bino and wino masses. This process is unique in that the phase sensitivity extends over all of supersymmetry parameter space. Other modes are suppressed outside the mixed gaugino-neutralino region of parameter space.

The relative phase between the bino and wino masses is a renormalization group invariant at one-loop, and therefore provides a clean probe for violations of gaugino universality at the messenger scale. Precision measurements of left handed slepton production in the $e^-e^-$ mode of the Next Linear Collider (NLC) provide an interesting window to the messenger scale through this phase. This mode also provides probably the best opportunity to measure any supersymmetric CP-odd phase at the NLC, and is complimentary to low energy electric dipole measurements which are not directly sensitive to the relative phase between the bino and wino masses.

2. CP-Violating Phases in the MSSM

The CP-violating phases which arise in the MSSM beyond those of the standard model appear in Lagrangian mass parameters. The first appears in the superpotential Dirac mass parameter $\mu$, 

$$W = \mu H_u H_d$$

Assuming squark and slepton universality, the remaining phases appear in the soft SUSY breaking mass parameters $m_i$, $A$, and $m^2_{ud}$,

$$\mathcal{L} = -\frac{1}{2}m_i\lambda_i\lambda_i - A (h_uQH_u\bar{u} - h_dQH_d\bar{d} - h_eLH_d\bar{e}) - m^2_{ud}H_uH_d + h.c.$$  

where $\lambda_i$, $i = 1, 2, 3$ are the gauginos, and $h_i$ are the Yukawa coupling matrices. Only a subset of all the phases in the Lagrangian parameters and represent basis independent combinations of physical CP-violating phases. The simplest way to determine these basis independent combinations is to notice that for $\{\mu, m_i, A, m^2_{ud}\} \rightarrow 0$ the MSSM possesses additional $U(1)_{PQ}$ and $U(1)_R$ global symmetries. The mass parameters can therefore be treated as background spurious which spontaneously break the global symmetries. A particular assignment of background charges to the mass parameters and fields is listed in Table 1. Physical amplitudes must be invariant under the background symmetries. The invariant combinations of mass parameters which can have a non-trivial phase and appear in physical amplitudes are

$$A^*m_i\quad A\mu(m^2_{ud})^*$$

$$m_i\mu(m^2_{ud})^*\quad m^*_im_j$$  

For selectron production the gluino mass does not appear through two loops, and effects suppressed by the electron Yukawa coupling are irrelevant. Observable effects
of non-zero phases can therefore only appear in the parameters $\{\theta^i\}$ with $i = 1,2$. Among these there are two linear combinations of phases which may be taken to be

$$\text{Arg}\left(m_1 \mu (m_2^2)\right) \quad \text{Arg}\left(m_1^* m_2\right) \quad (5)$$

At tree level these phases can affect selectron production only through the neutralino mass matrix.

In the basis $(-i\tilde{B}, -i\tilde{W}, \tilde{H}_d, \tilde{H}_u)$ the neutralino mass matrix is

$$\mathcal{L} = -\frac{1}{2} \lambda M \lambda + \text{h.c.}$$

$$M = \begin{pmatrix}
  m_1 & 0 & -\left(\frac{g'}{\sqrt{2}}\right) H^0_u & \left(\frac{g'}{\sqrt{2}}\right) H^0_d \\
  0 & m_2 & \left(\frac{g}{\sqrt{2}}\right) H^0_u & -\left(\frac{g}{\sqrt{2}}\right) H^0_d \\
  -\left(\frac{g'}{\sqrt{2}}\right) H^0_u & \left(\frac{g}{\sqrt{2}}\right) H^0_d & 0 & -\mu \\
  \left(\frac{g'}{\sqrt{2}}\right) H^0_d & -\left(\frac{g}{\sqrt{2}}\right) H^0_u & -\mu & 0
\end{pmatrix} \quad (6)$$

where $H^0_u$ and $H^0_d$ are understood to be expectation values. In a general basis all terms in the mass matrix are complex. The off diagonal gauge interaction terms which mix the gauginos and Higgsinos clearly depend on the phase of the Higgs condensates. The relative phase of the two Higgs condensates is not arbitrary but determined dynamically by the Higgs potential. The only tree level potential term which depends on the relative phase is

$$V \supset -m^2_{ud} H^0_u H^0_d + \text{h.c.}$$

$$= -|m^2_{ud} H^0_u H^0_d| \cos \left[\text{Arg}(m^2_{ud}) + \text{Arg}(H_u H_d)\right] \quad (7)$$

In the ground state, with broken electroweak symmetry, the phases of the Higgs condensates dynamically adjust to $\text{Arg}(m^2_{ud}) = -\text{Arg}(H_u H_d)$. This corresponds to vanishing expectation value for the pseudo-scalar Higgs boson $A^0$, and is generally not modified by quantum corrections. With this alignment, the complex phases
which appear in the mass matrix are those of $m_1, m_2, \mu$, and $m_{ud}^2$. From the discussion of the basis independent combinations of phases given above it is clear that a diagonalization of (6) can only depend on the two phases (5). It is these two phases which can have an effect on slepton pair production as discussed in the next section. The rotation between mass and interaction eigenstates, $\lambda = V\chi$, is in general complex for non-zero phases (5). It is always possible to work in a basis in which the mass eigenvalues are real, although it is sometimes convenient to leave the eigenvalues complex, as is done in the next section. It is however not possible in general to absorb all the phase dependence in (6) onto the eigenvalues.

The phase $\text{Arg}(m_1\mu(m_{ud}^2)^*)$ only appears implicitly in the neutralino mass matrix in the off diagonal mixing terms through the phase of the Higgs condensates relative to the other parameters. Its effects are therefore unsuppressed only in the mixed region of parameter space, $|\mu|^2 - |m_1|^2 \lesssim m_Z^2$, in which gaugino-Higgsino mixings are important. In contrast, the effects of the phase $\text{Arg}(m_1^2 m_2)$ do not require mixing and are unsuppressed over all of the neutralino parameter space.

The phases $\text{Arg}(A^*m_1)$ and $\text{Arg}(m_1\mu(m_{ud}^2)^*)$ are bounded by electric dipole moment measurements. Over most of the SUSY parameter space these phases are typically bounded to be less than $10^{-3}$. Electric dipole measurements are however not directly sensitive to $\text{Arg}(m_1^2 m_2)$. Mixing effects in the neutralino mass matrix do allow for non-vanishing contributions to electric dipole moments, but these are suppressed in the gaugino or Higgsino limit. This is in contrast to left handed slepton pair production in $e^-e^-$ collisions discussed in section 3.2, which has an unsuppressed sensitivity to $\text{Arg}(m_1^2 m_2)$ in these regions of parameter space. Measurements in this mode of slepton pair production are therefore complimentary to electric dipole moment measurements.

The expectations for the magnitude of these CP-violating phases of course depends on the model for the messenger and supersymmetry breaking sectors. Under the assumption of gaugino universality, such as would occur in dilaton dominated supersymmetry breaking, $\text{Arg}(m_1^2 m_2)$ would be expected to vanish. However, even in this class of theories violations of universality can in general induce non-zero $\text{Arg}(m_1^2 m_2)$. For example, in the dilaton dominated ansatz, Planck scale slop can induce a small relative phase between the bino and wino masses. Since the gaugino mass renormalization group equations are homogeneous at one-loop, the relative phases are preserved at this order under renormalization group evolution. Two loop renormalization group modifications of $\text{Arg}(m_1^2 m_2)$ from mixing with $\text{Arg}(A^*m_3)$ and $\text{Arg}(m_1^2 m_3)$ typically amount to only a fraction of a percent even for high scale supersymmetry breaking. The sensitivity of slepton pair production to $\text{Arg}(m_1^2 m_2)$ discussed below therefore provides a clean probe for operators which violate gaugino universality at the messenger scale.

In the CP-conserving case the phases (5) reduce to two sign ambiguities in the neutralino mass matrix

$$\text{sgn}(m_1\mu m_{ud}^2) = \pm \quad \text{sgn}(m_1 m_2) = \pm$$ (8)
The first of these is often referred to in the literature as \( \text{sgn}(\mu) \) with some particular choice of basis. The second sign ambiguity is often ignored, and \( m_1 \) and \( m_2 \) are tacitly assumed to have the same sign. Even in the CP-conserving case, the signs (\( \mu \)) can have very large effects on selectron pair production as discussed below.

3. Slepton Pair Production

Any physical process in general receives contributions from multiple quantum amplitudes. The probabilities then depend on both the magnitudes and relative phases of the amplitudes. Physical observables can depend on CP-odd phases directly through the interference of relative phases of the amplitudes. This is true even for CP-even observables. In this case, since a CP-odd phase changes sign under CP or T, \( \varphi \to -\varphi \), the observable must depend on the CP or T even quantity \( \cos \varphi \). Because of this, a CP-even measurements can only determine a CP-odd phase up to a \( Z_2 \) ambiguity.\(^\dagger\) If near destructive interference occurs between two amplitudes for some values of CP-odd phases, then certain CP-even observables can in fact be quite sensitive to these phase. This is in fact the case for left handed selectron production in \( e^- e^+ \) collisions as discussed in section 3.2.

The general scheme for determining CP-odd phases outlined here differs significantly from the standard treatment. Almost all discussions of the effects of CP-violating phases in the literature rely on the fact that amplitudes which depend on CP-odd phases are conjugated under CP or T. However, CP-conserving final state rescatterings give an imaginary contribution to the amplitude which does not change sign under CP or T. A CP-odd observable then receives a contribution from interference between the CP-violating and final state amplitudes, proportional to \( \sin \varphi \). While such CP-odd observables provide a direct probe for CP-violation, in the absence finite width enhancements for nearly degenerate states, they are generally unobservably small for supersymmetric phenomena at colliders since final state rescatterings only occur at one-loop. In contrast, the CP-even observables described below are sensitive to CP-odd phases at tree level.

Specializing to the case of charged slepton pair production, it is interesting to determine which channels are sensitive to supersymmetric CP-violating phases through interference between different amplitudes. As discussed in the previous section, since Yukawa coupling effects are generally irrelevant to production, the only possible phase dependence arises in the neutralino mass matrix. Slepton production at hadron colliders proceeds through s-channel \( \gamma^* \) and \( Z^* \) exchange, and so is not sensitive to neutralino phases. The same applies to smuon and stau final states at \( e^+ e^- \) colliders. Selectron final states at \( e^+ e^- \) and \( e^- e^- \) colliders do in general, however, have contributions from \( t^- \) and \( u^- \)-channel neutralino amplitudes.

\( ^\dagger \)This \( Z_2 \) sign ambiguity differs from the sign ambiguities of the neutralino mass matrix in the CP-conserving case. The latter sign ambiguities can be determined by CP-even measurements as discussed below.
Table 2. Selectron production modes which are sensitive to neutralino phases through interference. Summary of the chiral structure of the neutralino propagator, overall magnitude of the cross section, and sensitivity of the cross section to neutralino phases in the gaugino or Higgsino limit.

| Mode | Neutralino Propagator | Gaugino/Higgsino Limit | Overall Magnitude | Phase Sensitivity |
|------|------------------------|------------------------|------------------|------------------|
| $e^+_R e^-_R \rightarrow \tilde{e}^+_R \tilde{e}^-_R$ | Chirally Violating | Suppressed | Unsuppressed |
| $e^+_L e^-_L \rightarrow \tilde{e}^+_R \tilde{e}^-_L$ | Chirally Violating | Suppressed | Unsuppressed |
| $e^-_R e^-_R \rightarrow \tilde{e}^+_R \tilde{e}^-_R$ | Chirally Violating | Unsuppressed | Suppressed |
| $e^-_L e^-_R \rightarrow \tilde{e}^+_L \tilde{e}^-_R$ | Chirally Conserving | Suppressed | Unsuppressed |
| $e^-_L e^-_L \rightarrow \tilde{e}^+_L \tilde{e}^-_L$ | Chirally Violating | Unsuppressed | Unsuppressed |

3.1. The Neutralino Functions

In order to discuss selectron production at $e^+ e^-$ and $e^- e^-$ colliders it is useful to work in the helicity or equivalently chiral basis. In this basis the right chiral initial states couple only to neutralinos through the bino component, while left chiral initial states couple through both bino and wino components. The $t$- and $u$-channel neutralino propagators with these chiral couplings can be written in compact form in terms of the neutralino functions introduced by Peskin. The couplings of the $i$-th neutralino mass eigenstate, $\chi_i$, to left and right handed chiral states are

$$\sqrt{2} e V_{Ri} = \sqrt{2} e \left( \frac{1}{\cos \theta_w} V_{1i} \right)$$

$$\sqrt{2} e V_{Li} = \sqrt{2} e \left( \frac{1}{2 \cos \theta_w} V_{1i} + \frac{1}{2 \sin \theta_w} V_{2i} \right)$$

where $\lambda = V \chi$, and the diagonal mass matrix is $V^t MV$. The neutralino functions are then defined to be proportional to the sum over mass eigenstates of the neutralino propagators weighted by the chiral couplings (9) and (10)

$$N_{ab}(t) = \sum_i V^*_{ai} \frac{1}{|m_i|^2 - t} V_{bi}$$

$$M_{ab}(t) = \sum_i V^*_{ai} \frac{m_i}{|m_i|^2 - t} V_{bi}$$

for $a,b = L,R$. The functions $N_{ab}(t)$ arise from chirally conserving neutralino propagators, while $M_{ab}(t)$ are from the chirally violating propagators. The contributions of the four mass eigenstates to the neutralino functions in general have some non-trivial interference. Peskin’s dimensionless neutralino functions, $N_{ab}(t)$ and $M_{ab}(t)$, are related to these by $N_{ab}(t) = |m_1|^2 N_{ab}(t)$ and $M_{ab}(t) = |m_1| M_{ab}(t)$.

For non-zero phases in the neutralino mass matrix, the chirally violating propagator functions $M_{ab}(t)$ are complex in general. The differential cross section for selectron production with $e^+ e^-$ and $e^- e^-$ initial states with a net chirality
therefore in general depends on the neutralino phases via interference between the neutralino mass eigenstates. The chirally conserving propagator function $N_{LR}(t)$ also has non-trivial interference in general. However $\text{Im} N_{aa}(t) = 0$ as a result of hermiticity of the chirally conserving bino-bino and wino-wino propagators. This has the consequence that for $e^+e^-$ collisions, modes with pairs of right handed selectrons or pairs of left handed selectrons in the final state do not depend on the phases through interference between different amplitudes. The remaining modes which are sensitive to neutralino phases through interference are listed in Table 2.

The differential cross sections for these modes are

$$
\frac{d\sigma}{dt}(e^+_Re^-_R \to \tilde{e}^+_L\tilde{e}^-_R) = 3R |M_{LR}(t)|^2
$$

(13)

$$
\frac{d\sigma}{dt}(e^+_Le^-_L \to \tilde{e}^+_R\tilde{e}^-_L) = 3R |M_{LR}(t)|^2
$$

(14)

$$
\frac{d\sigma}{dt}(e^+_Re^-_R \to \tilde{e}^+_R\tilde{e}^-_R) = \frac{3}{2}R |M_{RR}(t) + M_{RR}(u)|^2
$$

(15)

$$
\frac{d\sigma}{dt}(e^+_Le^-_L \to \tilde{e}^+_L\tilde{e}^-_L) = \frac{3}{2}R |M_{LL}(t) + M_{LL}(u)|^2
$$

(16)

$$
\frac{d\sigma}{dt}(e^+_Le^-_R \to \tilde{e}^+_L\tilde{e}^-_R) = 3R \left[ \frac{(t - m_{\tilde{\epsilon}}^2)(m_{\tilde{\epsilon}}^2 - t)}{s} - t \right] |N_{LR}(t)|^2
$$

(17)

where $R = \sigma(e^+e^- \to \mu^+\mu^-) = 4\pi\alpha^2/3s$, and the angular integrations are over $-1 \leq \cos \theta \leq 1$. The first four of these are s-wave near threshold, while the last one is p-wave.

An important feature of the modes listed above is that the phase dependent interference takes place between different neutralino mass eigenstates in the same channel. Because the interference is between amplitudes in the same kinematic channel, the effects are not limited to a particular kinematic region of phase space, and are not suppressed for production well above threshold. This is in contrast to analogous chargino and neutralino processes mentioned in the conclusions.

The magnitudes and phases of the neutralino eigenstate contributions to the neutralino functions are determined by diagonalization of the neutralino mass matrix (6). In order to understand the physical content of this diagonalization it is instructive to consider the mostly gaugino or mostly Higgsino limit. In this limit the physical mass eigenstates are mostly the gaugino and Higgsino eigenstates with small admixtures of the other states induced by the off diagonal mixing terms in (6). This limit is reached if the level splitting between the gauginos and Higgsinos is large compared with the off diagonal mixing terms, $||\mu||^2 - |m_1|^2 \gtrsim m_2^2$. This limit holds regardless of whether the lightest neutralino is gaugino or Higgsino like, subject to the small mixing criterion above.

In the gaugino or Higgsino limit the bino-wino propagators only arise through
mixing with intermediate Higgsino states. The bino-wino propagators projected onto the physical mass eigenstates are therefore suppressed by $\mathcal{O}(m_2^2/(\mu^2 - m_1^2))$ as compared with the bino-bino or wino-wino propagators. This has the effect that all the $LR$ neutralino functions are suppressed in magnitude by a similar amount compared with the $RR$ or $LL$ functions. These functions, although reduced in magnitude, are sensitive to the phases in the neutralino mass matrix.

The chirally violating neutralino function $\mathcal{M}_{LL}(t)$ receives contributions from both bino-bino and wino-wino propagators. In the gaugino or Higgsino limit these propagators projected onto the physical mass eigenstates are dominated by the mostly bino and mostly wino states. Interference between these two amplitudes is therefore sensitive in this limit to the relative phase between the bino and wino masses, $\text{Arg}(m_1^* m_2)$. However, the chirally violating neutralino function $\mathcal{N}_{RR}(t)$ is dominated only by the mostly bino state and can not have a large interference with the other states in this limit. So while the magnitude of this function is not suppressed in this limit, its sensitivity to phases is suppressed.

The (non)suppressions of the overall rate and phase sensitivity in the gaugino or Higgsino limit for the $e^+e^-$ and $e^-e^-$ modes discussed above are summarized in Table 2. The relative suppression of some of the phase sensitive modes is best illustrated by considering the pure gaugino or Higgsino limit, in which case the relevant neutralino functions reduce to

\begin{align*}
\mathcal{M}_{RR}(t) & = \frac{1}{\cos^2 \theta_w} \frac{m_1}{|m_1|^2 - t} \\
\mathcal{M}_{LL}(t) & = \frac{1}{4 \sin^2 \theta_w} \left( \frac{m_1 \tan^2 \theta_w}{|m_1|^2 - t} + \frac{m_2}{|m_2|^2 - t} \right) \\
\mathcal{M}_{LR}(t) & = 0 \\
\mathcal{N}_{LR}(t) & = 0
\end{align*}

Since there is no bino-wino mixing in this limit, the $LR$ functions vanish. The chirally violating function $\mathcal{M}_{LL}(t)$ is given by pure bino and wino exchange in this limit, and as such the interference term in $|\mathcal{M}_{LL}(t)|^2$ depends on $\text{Arg}(m_1^* m_2)$. The chirally violating function $\mathcal{M}_{LL}(t)$ is given by pure bino exchange in this limit, and $|\mathcal{M}_{RR}(t)|^2$ therefore does not depend on either phase in the neutralino mass matrix.

In the mixed region of neutralino parameter space the differential cross sections for all the modes listed in Table 2 depend in a non-trivial way on the phases (5). However, it is important to note that even in the gaugino or Higgsino limit the suppressed modes still have non-trivial dependence on the phases. It is in fact these suppressed modes which would help to determine the $\mu$ parameter in the gaugino limit if the heavier neutralino states are kinematically inaccessible. This makes clear that any precision fit of neutralino parameters to data must include the phases (5).
Even if CP-conservation is assumed the sign ambiguities must be included a fit.

### 3.2. Phase Dependence in $e^-e^-$ Collisions

The only mode of selectron pair production in which both the rate and phase sensitivities are unsuppressed in the gaugino or Higgsino limit is $e^-e^L \rightarrow \tilde{e}^-e^L$. It is therefore worthwhile to consider this process in the pure gaugino or Higgsino limit. The magnitude squared of the neutralino function for this case is

$$|M_{LL}(t)|^2 = \frac{1}{16 \sin^4 \theta_w} \left( \frac{|m_1|^2 \tan^4 \theta_w}{(|m_1|^2 - t)^2} + \frac{2|m_1||m_2| \tan^2 \theta_w \cos (\text{Arg}(m_1^* m_2))}{(|m_1|^2 - t)(|m_2|^2 - t)} \right)$$

The bino-wino interference term depends on $\cos (\text{Arg}(m_1^* m_2))$. Note that maximal constructive (destructive) interference is obtained for $\text{Arg}(m_1^* m_2) = 0, \pi$ or in the CP-conserving case $\text{sgn}(m_1 m_2) = +(-)$. The total cross sections $\sigma(e^- e^L \rightarrow \tilde{e}^- e^L)$ and $\sigma(e^- e^R \rightarrow \tilde{e}^- e^R)$ are shown in Fig. 1 as a function of $\text{Arg}(m_1^* m_2)$ with a typical set of parameters at the NLC. The differential cross sections for the same set of parameters are shown in Fig. 2. For constructive interference between bino and wino exchange in $e^- e^L \rightarrow \tilde{e}^- e^L$ the differential cross section is a monotonic function of $\cos \theta$ in the forward hemisphere. Any non-monotonic deviation implies $\text{Arg}(m_1^* m_2) \neq 0$. The sensitivity to $\text{Arg}(m_1^* m_2)$ is very pronounced in the forward direction.
The large forward peak in both $\ell^- R \ell^- R \rightarrow \tilde{\ell}^- R \tilde{\ell}^- R$ and $\ell^- L \ell^- L \rightarrow \tilde{\ell}^- L \tilde{\ell}^- L$ is less pronounced for scattering near threshold, and with heavier neutralinos. The differential cross sections for slightly more massive states are shown in Fig. 3. For this set of parameters the $\ell^- L \ell^- L \rightarrow \tilde{\ell}^- L \tilde{\ell}^- L$ distribution happens to be nearly flat in $\cos \theta$ for destructive interference.

The process $\ell^- L \ell^- L \rightarrow \tilde{\ell}^- L \tilde{\ell}^- L$ is apparently quite sensitive to the phase $\text{Arg}(m_1^* m_2)$. This fortuitous sensitivity is the result of a numerical coincidence which allows near complete constructive or destructive interference over much of the kinematic phase space. Well above threshold the total cross section is dominated by small $t$ corresponding to scattering in the nearly forward direction. For $t = 0$

$$|M_{LL}(0)|^2 = \frac{1}{16 \sin^2 \theta_w} \left( \tan^4 \theta_w + \frac{2|m_1|}{|m_2|} \tan^2 \theta_w \cos(\text{Arg}(m_1^* m_2)) + \frac{|m_1|^2}{|m_2|^2} \right)$$

The interference between bino and wino exchange can be nearly completely destructive for $\text{Arg}(m_1^* m_2) = \pi$ if $\tan^2 \theta_w \sim |m_1/m_2|$. This is in fact the case for the gaugino unification value of $|m_1/m_2| \approx \frac{1}{2}$. In the pure gaugino limit, assuming gaugino unification, $|M_{LL}(0)|^2 \approx 0.82(0.04)$ for $\text{Arg}(m_1^* m_2) = 0(\pi)$.

Away from the pure gaugino or pure Higgsino limit these results are modified slightly, but the qualitative features are not changed at lowest order. The overall rates for $\ell^- R \ell^- R \rightarrow \tilde{\ell}^- R \tilde{\ell}^- R$ and $\ell^- L \ell^- L \rightarrow \tilde{\ell}^- L \tilde{\ell}^- L$ are modified at $\mathcal{O}(m_Z^2/(\mu^2 - m_1^2))$ by mixing with the Higgsino states. The phase $\text{Arg}(m_1^* m_2)$, interpreted as the relative phase of the masses of the mostly bino and mostly wino states, is also renormalized at $\mathcal{O}(m_Z^2/(\mu^2 - m_1^2))$ by $\text{Arg}(m_1 \mu (m_{ud}^2)*)$ through mixing with the Higgsino states. An upper limit on this modification can however be obtained from the electron
Fig. 3. Differential cross sections for $e_R^- e_R^- \rightarrow \tilde{e}_R^- \tilde{e}_R^-$ and $e_L^- e_L^- \rightarrow \tilde{e}_L^- \tilde{e}_L^-$ in the pure gaugino or Higgsino limit for $\text{Arg}(m_1^* m_2) = 0, \pi$. The parameters are $\sqrt{s} = 500$ GeV, $|m_1| = 150$ GeV, $|m_2| = 300$ GeV, $m_{\tilde{e}_R} = 170$ GeV, and $m_{\tilde{e}_L} = 210$ GeV.

electric dipole moment experimental bound. Finally, the rate for $e_L^- e_R^- \rightarrow \tilde{e}_L^- \tilde{e}_R^-$ is down by $O(m_4^2/Z^2)$ compared with the unsuppressed modes.

A precision measurement of $\text{Arg}(m_1^* m_2)$ at the NLC in the $e^- e^-$ mode must contend with uncertainties in the kinematic masses, beam polarization, supersymmetry parameters responsible for mixing with the Higgsino states, and detector monte carlo uncertainties in both signal and background efficiencies with cuts. For typical parameters at the NLC it is estimated that even from a total cross section measurement, $\text{Arg}(m_1^* m_2)$ could be determined to a precision of a few percent in a year of running. A slightly better measurement could be obtained from observables which are optimized to take account of angular dependences. As discussed in section 2 this phase is a clean probe of gaugino universality violations at the messenger scale. So the mode $e_L^- e_L^- \rightarrow \tilde{e}_L^- \tilde{e}_L^-$ at the NLC provides an interesting window to the messenger scale for supersymmetry breaking.

4. Conclusions

CP-odd phases in the neutralino mass matrix have an effect on CP-even observables associated with selectron production in $e^+ e^-$ and $e^- e^-$ collisions through interference between different tree level amplitudes in the same kinematic channel. In the gaugino or Higgsino limit, left handed slepton production in $e^- e^-$ collisions is very sensitive to the relative phase between the bino and wino masses. Since this phase is not modified at a significant level by renormalization group evolution, this process provides a clean probe for gaugino non-universality at the messenger scale. This is in contrast to small violations of gaugino universality in the magnitudes
of the gaugino masses which can receive significant modifications from two-loop
renormalization group evolution.

In addition to the differential cross section for longitudinally polarized beams,
it is possible to form other CP-even observables which are sensitive to interference
between different amplitudes with transversely polarized beams. These are however
suppressed outside the mixed gaugino-Higgsino regions of parameter space, and
require interference between different kinematic channels. Other possibilities for
observing CP-odd phases in CP-even observables from interference effects are for
$\chi^0_i \chi^0_j, i \neq j$ and $\chi^\pm_i \chi^\mp_j$ final states. These however are suppressed outside the mixed
gaugino-Higgsino regions of parameter space and require a mass insertion in the final
state so are additionally suppressed well above threshold. Left handed selectron
production in $e^- e^-$ collisions therefore provides probably the best opportunity to
measure a CP-odd phase at the NLC, and is complimentary to low energy electric
dipole measurements.

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