Relative yield of charged and neutral heavy meson pairs in $e^+e^-$ annihilation near threshold

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Abstract

The subject of the charged-to-neutral yield ratio for $B\bar{B}$ and $D\bar{D}$ pairs near their respective thresholds in $e^+e^-$ annihilation is revisited. As previously argued for the $B$ mesons, this ratio should exhibit a substantial variation across the $\Upsilon(4S)$ resonance due to interference of the resonance scattering phase with the Coulomb interaction between the charged mesons. A simple alternative derivation of the expression describing this effect is presented here, and the analysis is extended to include the $D$ meson production in the region of the $\psi(3770)$ resonance. The available data on Kaon production at the $\phi(1020)$ resonance are also discussed in connection with the expected variation of the charged-to-neutral yield ratio.
1 Introduction

The near-threshold resonances $\phi(1020)$, $\psi(3770)$, and $\Upsilon(4S)$, decaying respectively to pairs of pseudoscalar mesons $K\bar{K}$, $D\bar{D}$, and $B\bar{B}$ are well known ‘factories’ for production of these mesons in $e^+e^-$ annihilation. In a number of experimental studies it is important to know the relative yield of the pairs of charged and neutral mesons in the corresponding resonance region, i.e. the ratio

$$R_{c/n} = \frac{\sigma(e^+e^- \to P^+P^-)}{\sigma(e^+e^- \to P^0\bar{P}^0)},$$

where $P$ stands for the pseudoscalar meson, i.e. $K$, $D$, or $B$. The knowledge of this ratio is of particular importance for the studies of the $B$ mesons at the $B$-factories, and dedicated measurements have been done at the $\Upsilon(4S)$ resonance by CLEO$^{[1, 2]}$, BABAR$^{[3, 4]}$ and Belle$^{[5]}$. The central values of the ratio $R_{c/n}$ found in these measurements typically range from 1.01 to 1.10 with the latest result$^{[4]}$ being $1.006 \pm 0.036(\text{stat}) \pm 0.031(\text{sys})$. The same ratio for the production of $D$ meson pairs is likely to be studied in detail at the $\psi(3770)$ resonance by the imminent CLEO-c experiment$^{[6]}$. For the Kaon pair production the most detailed available scan of production of charged and neutral mesons in the $\phi$ resonance region has been done by the SND collaboration$^{[7]}$.

The theoretical treatment of the ratio $R_{c/n}$ at these three thresholds near the corresponding resonances bears considerable similarity, although specific factors contributing to the ratio in each case are slightly different. In a way, the simplest case is presented by the threshold $B$ pair production, where the deviation of $R_{c/n}$ from one is essentially entirely determined by the Coulomb interaction between the charged $B$ mesons, since the mass difference between the $B$ mesons is very small: $m_{B^0} - m_{B^+} = 0.33 \pm 0.28 \text{MeV}$. For the $D\bar{D}$ pair production at the $\psi(3770)$ peak the difference in the $P$-wave kinematical factor $p^3$ due to the substantial mass difference between the charged and neutral $D$ mesons is the main source of deviation of $R_{c/n}$ from one, and the Coulomb interaction effect is somewhat smaller, but is still of a measurable $O(10\%)$ magnitude. Finally, in the $\phi(1020)$ resonance region in addition to the mass difference and the Coulomb interaction effects, the amplitudes of production of $K^+K^-$ and $K_LK_S$ also differ due to a non-resonant isovector, $I = 1$, contribution coming from the electromagnetic current of the $u$ and $d$ quarks. Another technical difference between the heavy mesons and the Kaons is that the former can safely be considered nonrelativistic at energies in the region of the corresponding resonance (the velocities of the mesons at the corresponding resonances are $v(B)/c \approx 0.06$ and $v(D^+)/c \approx 0.13$), while
the velocity of the charged Kaons at the $\phi$ resonance is $v(K^+)/c \approx 0.25$, and the $O(v^2/c^2)$ relativistic effects can show up at some level of intended accuracy in $R_{c/n}$.

The kinematical effect of the mass difference, and of the non-resonant isovector amplitude for the case of Kaon production, lead to factors in the $R_{c/n}$, which are rather smoothly varying within the width of the corresponding resonance. The behavior of the Coulomb interaction effect is however quite different. Namely, as recently pointed out [9], with a proper treatment of the strong resonant interaction between the mesons, there is a phase interference between the Coulomb interaction and the resonance (Breit-Wigner) scattering phase. Since the latter phase changes by $\pi$ across the resonance, the effect of the Coulomb interaction in $R_{c/n}$ should exhibit a substantial variation with energy within the width of the resonance. Thus in particular a comparison between different measurements of the charged-to-neutral ratio at the $\Upsilon(4S)$ peak can only be meaningful with proper account for the differences in the specific experimental setups such as the beam energy spread, stability, calibration, etc. Needless to mention that the best approach would be a dedicated scan of the $\Upsilon(4S)$ peak. The details of this variation depend [9] on the specifics of the transition from the strong interaction region at short distances to the long-range Coulomb interaction and also on such details as the non-resonant scattering phase for the mesons. A dedicated scan of the charged-to-neutral ratio thus could possibly provide an information on rather fine properties of strong interaction between the heavy mesons, which are not likely to be accessible by other means. It may well be that a more experimentally feasible object for such study is provided by the $\psi(3770)$ resonance, and possibly can be explored in detail in the forthcoming CLEO-c experiment.

The main motivation of the present paper is to extend the analysis of Ref. [9] to the case of the $D\overline{D}$ production in the region of this resonance, and to estimate the magnitude of the expected effects, in the extent that it can be done within the present (rather approximate at best) understanding of the details of the strong interaction between heavy mesons.

The typical amplitude of the varying part of the ratio $R_{c/n}$ due to the Coulomb interaction is set by the Coulomb parameter $\alpha/v$ for the charged mesons, which tells us that for the $D$ mesons at $\psi(3770)$ the amplitude of the variation should be approximately twice smaller than for the $B$ mesons at $\Upsilon(4S)$ (and another factor of two smaller for the Kaons at $\phi$). In either case the Coulomb parameter is sufficiently small to justify limiting the consideration to only the linear term in the Coulomb interaction treated as perturbation. A simple derivation of the linear in the Coulomb interaction correction to production of a charged meson pair with the exact treatment of the strong interaction scattering is presented in Section 2.
derivation is slightly different from that given in Ref. [9], and hopefully, more transparently leads to the expression for the $O(\alpha/v)$ term found there. In Section 3 are presented specific estimates of the behavior of the ratio $R_{c/n}$ across the $\psi(3770)$ resonance, which take into account both the mass difference between the $D$ mesons and the effect of the Coulomb interaction.

It can be noticed that the very fact of the existence of the variation of $R_{c/n}$ within the resonance width is essentially model independent and is a direct consequence of a somewhat standard application of the quantum mechanical scattering theory. Nevertheless, there has been some scepticism expressed regarding the existence of the predicted rapid variation of $R_{c/n}$ at the resonance, most notably in the introductory part of the recent experimental paper [4]. Although the reported in Ref. [4] measurement has been done at just one point in the energy at the peak of the $\Upsilon(4S)$ resonance, the paper implicitly puts the prediction of the variation in doubt by arguing that “such rapid variation in the charged-to-neutral ratio has not been observed in scans across the $\phi(1020)$ resonance” (with a reference to the scans by the SND collaboration [7]). Thus it appears to be useful to point out the difference between the predictions for the $B$ production at $\Upsilon(4S)$ and the Kaon production at $\phi(1020)$ following from the discussed approach. The main difference is that the amplitude of the variation at the $\phi$ is expected to be suppressed by a factor of approximately 0.25 due to about four times larger velocity of the Kaons at the relevant energy, and additional differences in the details arise through different parameters (e.g. widths) of the $\phi$ and $\Upsilon(4S)$ resonances. A comparison of a ‘representative’ theoretical curve with the behavior of $R_{c/n}$ across the $\phi$ resonance extracted from the scan data [7] is presented in Section 4. Given the existing theoretical and experimental uncertainties, a meaningful comparison can only be done at a mostly qualitative level, and it is left entirely up to the reader to assess from the presented comparison whether the scan data indeed exclude a variation with expected amplitude of the charged-to-neutral ratio across the $\phi$ resonance.

2 Coulomb interaction correction to production of charged mesons

If the mesons were point particles devoid of strong interaction and produced by a point source, the Coulomb interaction between charged mesons would enhance their production.
cross section by the textbook factor

\[ F_c = 1 + \frac{\pi \alpha}{2v} + O\left(\alpha^2/v^2\right), \tag{2} \]

where \( v \) is the velocity of each of the mesons in the center of mass frame, which was the early prediction\[10\] for the ratio \( R^{c/n} \) at the \( \Upsilon(4S) \) resonance. It was however then realized that the effects of the meson electromagnetic form factor\[11\] and of the form factor in the decay vertex \( \Upsilon(4S) \to BB\tag{12, 13} \) tend to reduce the enhancement. In all these approaches the Coulomb rescattering of the produced mesons is considered, assuming that the propagation of the meson pair between the production vertex and the rescattering as well as after the Coulomb rescattering is described by a free propagator, and thus ignoring the fact that the strong interaction introduces the scattering phase factors \( \exp(i\delta) \) in these propagators. This would be a reasonable approximation if the strong phase \( \delta \) were small. In the region of a strong resonance however this assumption is definitely invalid, since the phase changes by \( \pi \) within the width of the resonance. The calculation described in Ref.\[9\] allows to completely take into account the strong scattering phase as well as the Coulomb interaction between the mesons. In the first order in the Coulomb rescattering the expression\[9\] for the Coulomb correction factor has a simple form

\[ F_c = 1 + \frac{1}{v} \text{Im} \left[ e^{2i\delta} \int_a^{\infty} e^{2ipr} \left(1 + \frac{i}{pr}\right)^2 V(r) \, dr \right], \tag{3} \]

where \( p \) is the meson momentum and \( V(r) \) is the potential for the rescattering interaction. Clearly, up to a possible form factor, \( V(r) = -\alpha/r \) for the Coulomb interaction. The short-distance cutoff parameter \( a \) in eq.(3) accounts for the fact that in the region of a strong interaction at short distances the mesons spatially overlap and in fact the system is a mixture of heavy and light quarks and gluons rather than a state of two individual mesons. Therefore at such distances there is no separation between the states with charged and neutral mesons and thus any difference in their Coulomb interaction disappears. If one introduces form factors in the Coulomb interaction in order to account for the structure of the mesons, the interaction potential \( V(r) \) would automatically vanish at short distances and the ultraviolet cutoff in the integral in eq.(3) would be provided by the form factors. If as in the earlier analyses, one sets the strong scattering phase \( \delta \) to zero, the expression in eq.(3) is finite even if an unmodified Coulomb potential is taken down to \( a = 0 \), in which limit it reproduces eq.(2). However at any finite \( \delta \) there is an essential dependence on the ultraviolet cutoff parameter \( a \) which cannot be removed.
In this section we present a slightly different from that in Ref.[9] derivation of eq.(3)
by considering the following problem. Two mesons, each with mass $m$ interact strongly at
distances $r < a$. The strong interaction in the $P$ wave at the total kinetic energy $E$ produces
the scattering phase $\delta$. At longer distances, $r > a$ the mesons interact via the potential
$V(r)$. The meson pair is produced by a source localized at distances shorter than $a$ (this
obviously corresponds to the production in the $e^+e^-$ annihilation). The problem is to find
the correction of the first order in $V$ to the production rate.

In order to solve the formulated problem we consider the radial part of the wave function
of a stationary $P$-wave scattering state, written in the form $R(r) = \chi(r)/r$. According to
the general scattering theory[14] the asymptotic form of $\chi(r)$ at $r \to \infty$ in the absence of
the long-range potential $V(r)$ is

$$\chi(r) = 2 \cos(pr + \delta) = e^{i\delta} e^{ipr} + e^{-i\delta} e^{-ipr}.$$  \hfill (4)

Furthermore, in the absence of the potential $V$ the function $\chi(r)$ satisfies the Schrödinger
equation for free motion in the $P$ wave:

$$\chi''(r) + \left(p^2 - \frac{2}{r^2}\right) \chi(r) = 0$$  \hfill (5)

at all distances $r$ outside the region of strong interaction, i.e. at $r > a$. Thus the solution
valid at all such distances, and having the asymptotic form (4) can be written explicitly:

$$\chi(r) = e^{i\delta} f(pr) + e^{-i\delta} f^*(pr) ,$$  \hfill (6)

in terms of the function

$$f(pr) = \left(1 + \frac{i}{pr}\right) e^{ipr} ,$$  \hfill (7)

and of its complex conjugate $f^*$. At distances $r < a$ the dynamics is unknown to the extent
that continuing description of the system in terms of the meson pair wave function in the
region of strong interaction does not make much sense, since most likely such description
should involve entirely different degrees of freedom. Nevertheless whatever complicated the
‘inner’ dynamics of the system may be, the wave function in eq.(5) at $r = a$ provides the
boundary condition for the ‘inner’ problem. In particular, it determines the normalization of
the state for the ‘inner’ problem. Thus the rate of production of the system, and consequently
of the meson pair, by a source localized at $r \ll a$ is proportional to $|\chi(a)|^2$.

When the long-range potential $V(r)$ is turned on and considered as a perturbation the
wave function at $r > a$ changes to $\chi(r) + \delta\chi(r)$. Once the corrected solution is normalized
at \( r \to \infty \) in the standard way, i.e. as the asymptotic wave in eq. (4), the normalization at \( r = a \) generally changes and hence also changes the production rate. In the first order in \( V \) the correction factor for the production rate is obviously given by

\[
F_c = 1 + 2 \frac{\delta \chi(a)}{\chi(a)} .
\]  

(8)

The perturbation \( \delta \chi \) of the scattering state wave function can be found in the standard way by writing it as a sum of outgoing and incoming waves:

\[
\delta \chi(r) = \delta \chi_+(r) + \delta \chi_-(r) ,
\]

similarly to eq. (6), and where \( \delta \chi_+(r) \) contains at \( r \to \infty \) only an outgoing wave (\( \exp(ipr) \)) while \( \delta \chi_- = \delta \chi_+^* \) contains at asymptotic \( r \) only an incoming wave. The function \( \delta \chi_+(r) \) is a perturbation of the outgoing wave part of the function \( \chi(r) \) in eq. (6) and is thus determined by the equation

\[
\delta \chi''_+(r) + \left( p^2 - \frac{2}{r^2} \right) \delta \chi_+(r) = m V(r) e^{i\delta} f(pr) .
\]  

(9)

The solution to this equation is found by the standard method using the Green’s function \( G_+(r, r') \) satisfying the equation

\[
\left( \frac{\partial^2}{\partial r^2} + p^2 - \frac{2}{r^2} \right) G_+(r, r') = \delta(r - r') ,
\]  

(10)

and the condition that \( G_+(r, r') \) contains only an outgoing wave when either of its arguments goes to infinity. The Green’s function is constructed from two solutions of the homogeneous equation, i.e. from the functions \( f(pr) \) and \( f^*(pr) \), as

\[
G_+(r, r') = \frac{1}{2i}\left[ f(pr) f^*(pr') \theta(r - r') + f(pr') f^*(pr) \theta(r' - r) \right] ,
\]  

(11)

where \( \theta \) is the standard unit step function. The solution to the equation (9) is then found as

\[
\delta \chi_+(r) = m e^{i\delta} \int_a^\infty G_+(r, r') V(r') f(pr') \, dr' .
\]  

(12)

It can be noted that at no point in this consideration a knowledge of the dynamics at \( r < a \) is required. In particular the integral in eq. (12) runs from the lower limit at \( a \), since by our assumptions the perturbation potential has support only at \( r > a \).

It is further important that adding the found perturbation \( \delta \chi \) to the wave function does not change the normalization of the wave function at \( r \to \infty \). Indeed in this limit one has \( r > r' \) in the Green’s function in eq. (12), so that the asymptotic behavior of \( \delta \chi_+(r) \) should be derived from the expression

\[
\delta \chi_+(r) \big|_{r \to \infty} = -\frac{i}{2\nu} e^{i\delta} f(pr) \int_a^\infty V(r') |f(pr')|^2 \, dr' ,
\]  

(13)
which gives the complex phase of $\delta \chi_+(r)$ at the asymptotic distances manifestly orthogonal to that of the outgoing-wave part of $\chi(r)$, since the integral is explicitly real\(^1\). In other words, at $r \to \infty$ the correction changes only the scattering phase by adding the ‘Coulomb’ phase to $\delta$.

At $r = a$ the perturbation $\delta \chi_+(a)$ is found from the same expression (12). In this case one has $r < r'$ in the entire integration region, so that

$$
\delta \chi_+(a) = -\frac{i}{2v} e^{i\delta} f^*(pa) \int_a^\infty V(r') \left[f(pr')\right]^2 \, dr'.
$$

(14)

Using this expression one readily finds the change in the normalization of the wave function at $r = a$ due to the potential $V$, and thus arrives at the expression (3).

As is clear from the presented derivation, the result in eq.(3) is valid at arbitrary $P$-wave scattering phase $\delta$, and also generally does not assume any condition for the value of the product $pa$. At energy $E$ near a strong resonance dominating the dynamics of the meson pair, the scattering phase is determined by the Breit-Wigner formula

$$
\delta = \delta_{BW} + \delta_1, \quad e^{2i\delta_{BW}} = \frac{\Delta - i\gamma}{\Delta + i\gamma},
$$

(15)

where $\Delta = E - E_0$ is the distance in energy to the nominal position $E_0$ of the resonance, and $\gamma$ is generally a function of energy such that its value at $E_0$ is related to the nominal width $\Gamma$ of the resonance as $\gamma(E_0) = \Gamma/2$. Finally, $\delta_1$ in eq.(15) is the non-resonant $P$-wave scattering phase.

The properties of the discussed resonances, $\Upsilon(4S)$ and $\psi(3770)$ are determined by the two meson-pair channels: $P^+P^-$ and $P^0\bar{P}^0$. The analytical properties of the scattering amplitude in the $P$ wave require that the contribution of each channel to $\gamma$ and $\delta_1$ starts at the corresponding meson pair threshold as cubic in the corresponding momentum. Thus at the energy close to both thresholds one can parametrize these quantities as

$$
\gamma(E) = c_1 \left(p_+^3 + p_0^3\right), \quad \delta_1(E) = c_2 \left(p_+^3 + p_0^3\right),
$$

(16)

where $p_+$ and $p_0$ are the momenta of respectively the charged and the neutral mesons at the energy $E$. Due to the very small mass difference between the $B$ mesons, the thresholds for

\(^1\)A minor technical point is that formally the integral is logarithmically divergent at $r' \to \infty$ for the Coulomb potential, which might put into question the applicability of the condition $r > r'$ in the entire integration region. This is the standard infrared divergence of the Coulomb scattering phase, and it can be easily dealt with by temporarily introducing a small photon mass $\lambda$, so that the potential is cut off by the factor $\exp(-\lambda r)$ and the integral is convergent. In the final result one can obviously take the limit $\lambda \to 0.$
the charged and neutral $B$ mesons practically coincide and so do the momenta $p_+$ and $p_0$. However for the $D\overline{D}$ pairs the $D^+D^-$ threshold is by 9.6 $MeV$ higher than that for $D^0\overline{D}^0$, which difference is substantial at the energy of the $\psi(3770)$ resonance.

It should be noticed that neglecting the higher terms of expansion in the momenta for the parameters $\gamma$ and $\delta_1$ is justified only inasmuch as the momenta are small in the scale of the size of the strong interaction region $a_s$. In the calculation of the Coulomb interaction effect it is essential that the distance parameter $a$ for the onset of the Coulomb interaction is not smaller than $a_s$: $a \geq a_s$. Otherwise no restriction on the value of the product $pa$ is implied. In Ref.[9] these parameters were reasonably assumed to be approximately equal. However it should be emphasized that generally the applicability of the formula (8) for the Coulomb correction is separate from the applicability of the first terms of the threshold expansion in eq.(16), and these two issues can be studied separately in experiments2.

In conclusion of this section we write the explicit formula for the correction described by eq.(3) in the resonance region in terms of real quantities[9]:

$$F_c = 1 + \frac{\alpha}{\nu} \left[ \frac{\Delta^2 - \gamma^2}{\Delta^2 + \gamma^2} (A \cos 2\delta_1 + B \sin 2\delta_1) - \frac{2\gamma}{\Delta^2 + \gamma^2} (B \cos 2\delta_1 - A \sin 2\delta_1) \right].$$  \hspace{1cm} (17)

The coefficients $A$ and $B$ are found as the imaginary and the real parts of the integral in eq.(3) with the Coulomb potential:

$$A = -\int_{pa}^{\infty} \left[ \left( 1 - \frac{1}{u^2} \right) \sin 2u + \frac{2 \cos 2u}{u} \right] \frac{du}{u} = \frac{\pi}{2} - \frac{\cos 2pa}{pa} + \frac{\sin 2pa}{2(pa)^2} - Si(2pa),$$

$$B = \int_{pa}^{\infty} \left[ \frac{2 \sin 2u}{u} - \left( 1 - \frac{1}{u^2} \right) \cos 2u \right] \frac{du}{u} = \frac{\cos 2pa}{2(pa)^2} + \frac{\sin 2pa}{pa} - Ci(2pa),$$  \hspace{1cm} (18)

where

$$Si(z) = \int_0^z \sin t \frac{dt}{t} \quad \text{and} \quad Ci(z) = -\int_z^\infty \cos t \frac{dt}{t}.$$

3 \ Estimated of the expected variation of $R_c/n$ across the $\psi(3770)$ resonance

Here we apply the formulas of the previous section, specifically the equations (17), (18), and (16), to an estimate of the scale of the variation in the ratio $R_c/n$ for the $D\overline{D}$ pair production2.

\footnote{In fact an attempt at detecting higher than $p^3$ terms in the width parameter for the $T(4S)$ resonance has been done by ARGUS collaboration[15]. However the deviation from the cubic behavior turned out to be too small within the experimental accuracy.}
at the $\psi(3770)$ resonance. In doing so it should be clearly understood that at present one can only guess (in some ‘reasonable’ range) the appropriate values of the scattering phase $\delta_1$ and the cutoff parameter $a$ for the Coulomb interaction (or more generally make a guess about an appropriate model for the onset at short distances of the Coulomb interaction between the charged $D$ mesons).

As different from the case of the $B$ meson production at the $\Upsilon(4S)$, in the charged-to-neutral ratio for the $D$ meson production at $\psi(3770)$ the Coulomb effect multiplies the ratio of the $p^3$ factors:

$$R_{c/n} = F_c \left(\frac{p_z}{p_0}\right)^3.$$  \hfill (19)

Also, according to the Particle Data Tables\cite{PDG}, the width of $\psi(3770)$ is somewhat larger than that of the $\Upsilon(4S)$: $\Gamma(\psi''') = 23.6 \pm 2.7 \text{MeV}$ as opposed to $14 \pm 5 \text{MeV}$, which tends to smoothen the variation of $R_{c/n}$ near the $\psi(3770)$ peak.

![Figure 1](image.png)

Figure 1: The energy dependence of the ratio $R_{c/n}$ for $D$ and $B$ meson pair production in the region of the respective resonance: $\psi(3770)$ and $\Upsilon(4S)$ (the center positions are assumed at $E_0 = 3.770 \text{GeV}$ and $E_0 = 10.580 \text{GeV}$ respectively) for some values of $a$ and $\delta_1(E_0)$: $a^{-1} = 200 \text{MeV}$, $\delta_1(E_0) = 30^0$ (solid), $a^{-1} = 200 \text{MeV}$, $\delta_1(E_0) = -30^0$ (dashed), $a^{-1} = 300 \text{MeV}$, $\delta_1(E_0) = 30^0$ (dashdot), and $a^{-1} = 300 \text{MeV}$, $\delta_1(E_0) = -30^0$ (dotted).

A sample expected behavior of the ratio $R_{c/n}$ for the $D$ pair production near the $\psi(3770)$ resonance is shown in Fig.1 for some ‘representative’ values of the parameters $a$ and $\delta_1(E_0)$.  

9
For comparison the same curves for the $B$ pairs in the vicinity of the $\Upsilon(4S)$ peak are also shown in a separate plot in Fig.1. It should be noted in connection with this comparison that generally it is not expected that the phase $\delta_1$ is the same in these two cases, although the parameter $a$ viewed as characterizing the electric charge structure in a heavy meson is likely to be quite similar for $D$ and $B$ mesons, if one considers both $c$ and $b$ quarks as asymptotically heavy. With all the present uncertainty in the knowledge of the strong interaction parameters, one can see from this comparison that the expected variation of the ratio $R^{c/n}$ is quite less prominent for $D$ mesons at the $\psi(3770)$ peak than for $B$ mesons at the $\Upsilon(4S)$. Hopefully, the amplitude of the variation of $R^{c/n}$ at the $\psi(3770)$ is still sufficient for a study in the upcoming CLEO-c experiment.

4 Discussion of the $\phi(1020)$ data and summary

So far the most detailed scan data for the production cross section of charged and neutral mesons are available only for the Kaons in the vicinity of the $\phi(1020)$ resonance[7], and it is quite natural to discuss whether any hint at the discussed variation of $R^{c/n}$ is indicated by those data. A consideration of this ratio at the $\phi$ peak however encounters certain peculiarities. As mentioned above, the Coulomb interaction effect is relatively smaller for production of $K$ mesons at the $\phi$ resonance, and also the relativistic effects can play a certain role. The analysis of the ratio $R^{c/n}$ is further complicated by the fact that the production amplitude also receives an isovector contribution, which has opposite sign for $K^+K^-$ and $K_LK_S$ and thus changes the charged-to-neutral ratio. (In the vector dominance model this contribution is considered as the ‘tail’ of the $\rho$ resonance.) Neglecting a smooth variation of the non-resonant $I = 1$ amplitude and also of the non-resonant part of the $I = 0$ amplitude (e.g. the ‘tail’ of the $\omega$ resonance) one can approximate the formula for $R^{c/n}$ in this case as

$$R^{c/n} = F_c \left(\frac{p_+}{p_0}\right)^3 \frac{|1 + (A_0 - A_1)(\Delta + i\gamma)|^2}{|1 + (A_0 + A_1)(\Delta + i\gamma)|^2},$$  (20)

where $A_1$ and $A_0$ stand for the appropriately normalized relative contribution of the non-resonant $I = 1$ and $I = 0$ amplitudes. The charged $K$ mesons are lighter than the neutral ones, thus the cube of the ratio of the momenta is larger than one and decreases towards one at energies far above the threshold. The data[7] however display a slight general increase of the ratio with energy, which shows that the effect of $A_1$ is not negligible. This makes any ‘absolute’ prediction of $R^{c/n}$ in this case rather troublesome, as well as of its general
variation with energy on a scale somewhat larger than the width of $\phi$. A real fit of the
involved parameters, including the discussed here rapid variation of the Coulomb factor $F_c$
requires knowledge of the raw data and of the correlation in the errors. For this reason
here in Fig.2 is shown a comparison of the data sets listed in Ref.[7] with a ‘representative’
thoretical curve.

Figure 2: The SND data[7] at the $\phi(1020)$ resonance represented as the charged-to-neutral
yield ratio for the two reported in [7] scans (PHI9801 and PHI9802) and for two methods
of identifying the $K_S$ mesons by their decay into neutral or into charged pions: PHI9801
neutral - stars, PHI9801 charged - diamonds, PHI9802 neutral - triangles, PHI9802 charged
- squares. The solid curve is explained in the text.

More specifically, in the paper [7] are listed (in the Table IX) the scan data for the
cross section for production of separately the $K^+K^-$ pairs and the $K_LK_S$ pairs acquired in
two different scans (called PHI9801 and PHI9802). Furthermore the detection of the $K_S$
mesons was done by two separate methods, i.e. by detecting their decay into charged pions,
and into neutral pions. Accordingly the paper [7] lists two separate sets of data for the
$K_LK_S$ production measured by each of these two methods. The systematic error for the
$K^+K^-$ cross section is listed as 7.1%, and for the neutral Kaons as 4% and 4.2% for the
two identification techniques used. The statistical errors are listed for each individual entry
for the cross section. The ‘data points’ in the plot of Fig.2 are the charged-to-neutral ratio
calculated from the data listed in Ref.[7] with the errors corresponding to the listed statistical
errors only. The ‘representative’ theoretical curve corresponds to $1.38F_c$. In other words the absolute normalization is set rather arbitrarily, given the described theoretical uncertainty in calculating the absolute normalization $R_{c/n}$ at the $\phi$ resonance and given the systematic uncertainty in the data. Also any overall variation of the kinematical and the amplitude factors in the shown energy interval is neglected. Changing the normalization of the curve and also including the overall energy dependence would result in a vertical shift and a slight tilt of the curve. The behavior of the Coulomb correction factor in the curve shown in Fig.2 is calculated assuming $E_0 = 1019.5\, MeV$, $\Gamma = 4.26\, MeV$, $a^{-1} = 200\, MeV$, and $\delta(E_0) = 40^0$. The only purpose of the ‘theoretical’ curve in Fig.2 is to illustrate the approximate magnitude of the expected effect of the variation of $R_{c/n}$ at the $\phi(1020)$ resonance, and by no means it should be considered as a detailed prediction.

Due to large experimental and theoretical uncertainties as explained in the text and can be seen from Fig.2, it is not entirely clear whether the available data\cite{7} exclude or rather suggest a variation with the expected magnitude of the ratio $R_{c/n}$ within the width of the $\phi$ resonance. Perhaps a detailed global fit of the raw data could provide a statistically significant evaluation of the amplitude of such variation.

In summary. The strong interaction phase significantly modifies the behavior of the cross section for production of pairs of charged pseudoscalar mesons near threshold in $e^+e^-$ annihilation. A simple calculation of this effect in the first order in the Coulomb interaction is presented in Section 2. The existence of a strong near-threshold resonance gives rise to a rapid variation of the charged-to-neutral yield ratio due to the interference of the resonance scattering phase with the Coulomb phase. The specific behavior of this variation is sensitive to details of the structure of the mesons and of their strong interaction. Thus measuring the discussed effect for $B$ mesons at the $\Upsilon(4S)$ resonance and for $D$ mesons at the $\psi(3770)$ resonance might provide an information on these details, which are not accessible by other means. The estimated effect for $D$ mesons in the region of the $\psi(3770)$ is less prominent than for $B$ mesons at the $\Upsilon(4S)$ but possibly is still measurable. The expected effect of the rapid variation of the charged-to-neutral ratio for the Kaon production at the $\phi(1020)$ resonance is the smallest, and, conservatively, the available data appear to be not conclusive enough to confirm or exclude such variation with a statistical significance.
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