We report time-of-flight measurements on electrons travelling in quantum-Hall edge states. Hot-electron wave packets are emitted one per cycle into edge states formed along a depleted sample boundary. The electron arrival time is detected by driving a detector barrier with a square wave that acts as a shutter. By adding an extra path using a deflection barrier, we measure a delay in the arrival time, from which the edge-state velocity \( v \) is deduced. We find that \( v \) follows a \( 1/B \) dependence, in good agreement with the \( \vec{E} \times \vec{B} \) drift. The edge potential is estimated from the energy-dependence of \( v \) using a harmonic approximation.

Electronic analogues of fundamental photonic quantum-optics experiments, so-called “electron quantum optics”, can be performed using the beams of single-electron wave packets. The demonstration of entanglement and multi-particle interference with such wave packets would set the stage for quantum-technology applications such as quantum information processing [1]. Various theoretical proposals [2–7] and experimental realisations [8–17] employ quantum-Hall edge states [18] as electron waveguides. The group velocity and dispersion relation of edge states are important parameters for understanding and controlling electron wave-packet propagation. For edge-magnetoplasmons, the velocity can be deduced. We find that \( v \) is inversely proportional to the magnetic field \( B \).

In this Letter, we demonstrate an experimental method for probing the bare edge-state velocity of electrons travelling in a depleted edge of a two-dimensional system. The electrons are emitted from a tunable-barrier single-electron pump [25–27] as hot single-electron wave packets (\( \sim 100 \) meV above the Fermi energy) [13, 28]. These electrons are injected into an edge where the background two-dimensional electron gas (2DEG) is depleted to avoid the influence of electron-electron interactions. The arrival time of these wave packets is detected by an energy-selective detector barrier with a picosecond resolution [13, 16]. The travel length between the source and detector is switched by a deflection barrier. The time of flight of the extra path is measured as a delay in the arrival time at the detector [29]. The edge-state velocity is calculated from the length of the extra path and the time of flight. We find that the edge-state velocity is inversely proportional to the magnetic field applied perpendicular to the plane of the 2DEG in a good agreement with the \( \vec{E} \times \vec{B} \) drift velocity, where \( \vec{E} \) is the electric field and \( \vec{B} \) is the magnetic field. We probe the dispersion of the edge states by controlling the electron emission energy. From the energy dependence of the velocity, we deduce the edge potential profile and obtain the information on the spatial positions of the edge states.

The measurements presented in this work are performed on...
two samples, Sample A and Sample B, with slightly different device parameters. Figure 1(a) shows a scanning electron micrograph of a device with the same gate design as Sample B. Both samples are made from GaAs/AlGaAs heterostructures with a 2DEG 90 nm below the surface, but the 2DEG carrier density is slightly different (1.8 × 10^{15} m^{-2} for Sample A and 1.6 × 10^{15} m^{-2} for Sample B). The active part of the device is defined by shallow chemical etching and Ti/Au metal deposition using electron-beam lithography. The device comprises 5 surface gates: the pump entrance gate (G1), pump exit gate (G2), detector gate (G3), deflection gate (G4), and depletion gate (G5). L_1 is the path length along the deflection gate and is the same for both samples (1.5 µm). L_2 is the path length along the loop section defined by shallow etching, and is 2 µm for Sample A and 5 µm for Sample B. The measurements are performed in a cryostat with the base temperature at 300 mK.

Figure 1(a) also shows the measurement circuit. The rf sine signal V_{G2}^{RF} (with a peak-to-peak amplitude ~ 1 V) applied to G1 pumps electrons over the barrier formed by the dc voltage V_{G2} applied on G2 [32]. The rf signal is repeated periodically at a frequency f = 240 MHz, producing the pump current I_p. When the device pumps exactly one electron per cycle, I_p = e f ≈ 38 pA, where e is the elementary charge. In a magnetic field B applied perpendicular to the plane of the 2DEG [in the direction indicated in Fig. 1(a)], the electrons emitted from the pump follow the sample boundary and enter the region where the background 2DEG is depleted by the negative voltage V_{G5} on G5 (we apply ~0.45 V for Sample A and ~0.3 V for Sample B, well in excess of the typical depletion voltage of ~ −0.2 V). The bottom of the lowest Landau level is raised above the Fermi energy E_F but is kept lower than the electron emission energy as shown in Fig. 1(b). The electrons travel along the edge approximately 500 nm (roughly equal to the extent that G5 covers from the edge defined by shallow etching) away from the nearest 2DEG [as indicated by the red dot in Fig. 1(b)].

Depending on the voltage V_{G4} applied to the deflection gate G4, the electron wave packets reach the detector (G3) either through the shorter route [solid red line in Fig. 1(a)] or the longer route (dashed line). In both cases, the majority of electrons reach the detector without measurable energy loss for the magnetic field considered here. Electrons that lose energy through LO-phonon emission [13, 30] are reflected by the detector barrier and do not contribute to the detector current.

The longer route adds an extra length 2L_1 + L_2 to the electron path, causing a delay in the arrival time at the detector. The electron arrival time at the detector is detected using a time-dependent signal on the detector [13, 16]. A square wave V_{G2}^{RF} with a peak-to-peak amplitude of ~ 20 mV is applied to G3 [16]. The detector current I_D is monitored as V_{DC} is swept and the relative delay time t_d between V_{G2}^{RF} and V_{G3}^{RF} is varied with a picosecond resolution [see Fig. 1(c)] [16].

Figures 2(a)-(c) show the behaviour of the detector current for three values of V_{G4} taken at B = 14 T with Sample A. Here, dI_D/dV_{G3}^{DC} is plotted in colour scale as a function of V_{G3}^{DC} and t_d. The pump current is set at the quantised value for one electron emission per cycle (i.e. I_p ≈ e f) with V_{G1}^{DC} = −0.46 V and V_{G2} = −0.56 V. When the detector barrier is sufficiently low (i.e. V_{G3}^{DC} is less negative), all emitted electrons that do not suffer energy loss during the travel from the pump enter the detector contact and contribute to I_D. Therefore, I_D ≈ I_p, as the LO phonon emission is negligible at B = 14 T in these samples. When the detector barrier is sufficiently high, all electrons are blocked, and I_D = 0. When the detector barrier is matched to the energy of incoming electrons, a peak in dI_D/dV_{G3}^{DC} appears [13, 16]. The peak position (or the detector threshold) in V_{G3}^{DC} depends on t_d because a square wave is applied to the detector gate and the sum V_{DC}^{G3} + V_{RF}^{G3} determines the detector barrier height. When t_d is small (large), electrons arrive when the square wave is negative (positive), and hence it shifts the detector threshold to more positive (negative) in V_{G3}^{DC}. The transition of the detector threshold in V_{G3}^{DC} from more positive to more negative occurs at t_d where the square-wave transition coincides with the electron arrival at the detector. As V_{G4} is made more negative, the detector transition shows splitting [Fig 2(b)] and finally settles to larger t_d [Figs. 2(c)]. The splitting happens as G4 splits the wave packets into the shorter and longer routes, and hence two sets of electron wave packets arrive at the detector with a time delay. The shift of the detector transition to larger t_d occurs because the longer route causes a delay in the
arrival time.

In Fig. (d), \( \frac{dI_d}{d\tau_d} \) is plotted as \( \tau_d \) is swept through the centre point of the detector transition marked by red crosses for the cases of \( V_{G4} = -0.48 \) V (solid line) in Fig. (a) and -0.51 V (dashed line) in Fig. (c). These two curves represent the arrival-time distributions for the shorter and longer routes, and hence the time difference \( \tau_d \) between the two peaks is the time of flight of the extra path \((2L_1 + L_2)\) taken by the longer route. The edge-state velocity in the extra path can be calculated as \( v = (2L_1 + L_2)/\tau_d \). In this example, \( v = 5 \mu m/95 \) ps = \( 5 \times 10^4 \) m/s.

The uncertainty in the velocity measurement arises from the uncertainties in \( 2L_1 + L_2 \) and \( \tau_d \). The value of \( 2L_1 + L_2 \) is likely to be accurate only to \( \pm 10\% \) as we can only estimate it from the device geometry. This gives the same systematic error to all velocity estimates within the same sample, and hence it does not affect the discussions in the later sections qualitatively. The uncertainty in \( \tau_d \) is more problematic. This is because the arrival time does not just switch between two values as the edge-state path is switched. As plotted in Fig. (e), the arrival time initially changes slowly towards larger \( \tau_d \) as \( V_{G4} \) is made more negative, then it starts to move through a series of changes for \( \tau_d \) as the edge-state path is switched. As plotted in Fig. (e), the rapid change in the velocity along \( G4 \), and is the main source of a large uncertainty due to a shorter time of flight. The velocity along the path \( L_1+2L_2 \) is plotted as \( v = (2L_1 + L_2)/\tau_d \) in Fig. (a) and \( v^2 \) as a function of relative emission energy \( \Delta E \). Solid lines are fits to a linear relation.

A typical uncertainty in \( \tau_d \) estimate by this method is \( \pm 5 \) ps.

A more rigorous velocity estimate can be introduced by excluding the contribution from the electron paths along \( G4 \). Figure (f) shows the time-of-flight data taken from Sample B plotted in the same manner to Fig. (e). With Sample B, the arrival time changes more rapidly as \( V_{G4} \) is made more negative after the electron path is switched to the longer route. As the case with Sample A, this is considered to result from a rapid change in the velocity along \( G4 \), and is the main source of the uncertainty in velocity estimates. In order to reduce the uncertainty, we break up the time of flight into two parts, \( \tau_{d1} \) along \( G4 \) (length \( 2L_1 \)) and \( \tau_{d2} \) along the loop (length \( L_2 \)), i.e., \( \tau_d = \tau_{d1} + \tau_{d2} = 2L_1/v_1 + L_2/v_2 \), where \( v_1 \) is the velocity along the path \( L_1+2L_2 \). From this, one can see \( \tau_d \rightarrow \tau_{d2} = L_2/v_2 \) in the limit \( v_1/v_2 \gg 2L_1/L_2 \). Once the electron path is deflected, \( v_1 \) increases as \( V_{G4} \) is made more negative, whereas \( v_2 \) is unaffected. Therefore, in the limit of large negative \( V_{G4} \), the time of flight settles to \( \tau_{d2} \), the time of flight around the loop section. It is not trivial to know exactly how \( v_1 \) changes with \( V_{G4} \), but a linear relation \( (v_1 \propto -V_{G4}) \) fits well to the experimental data [solid line in Fig. (f)]. As shown in Fig. (f), \( \tau_{d2} \) can be estimated as the difference between the saturated values of arrival time at the positive and negative ends of \( V_{G4} \). The velocity around the loop section is calculated as \( v_2 = L_2/\tau_{d2} \). We find that the uncertainty is reduced approximately by a factor of three using this method. We note that we cannot apply this method to Sample A as \( L_2 \) is too small to observe the saturation in the arrival time at the negative end in \( V_{G4} \).

Now, we investigate the magnetic-field and emission-energy dependence of the velocity to see if the studied edge-state system is consistent with a simple, interaction-free, quantum-Hall edge-state model. Figure (a) shows the magnetic field dependence of the measured edge-state velocity for both samples \( v \) (the velocity along the whole extra path, \( 2L_1 + L_2 \)) is plotted for Sample A and \( v_2 \) (the velocity along the loop section only, \( L_2 \)) for Sample B]. Clear 1/B dependence is observed for both samples down to \( B = 5 \) T. This is in good agreement with the \( \vec{E} \times \vec{B} \) drift velocity, where \( v = |\vec{E} \times \vec{B}|/B^2 \propto 1/B \), and \( \vec{E} \) is the electric field due to the edge potential. For Sample A, the data point at \( B = 5 \) T deviates from the 1/B dependence. This may be a sign of crossover from the skipping orbits to the \( \vec{E} \times \vec{B} \) drift observed by McClure et al. (23), but this could also be simply because of a large uncertainty due to a shorter time of flight.

In order to estimate the edge confinement potential and the spatial position of edge states, we consider the dispersion rela-
The electron emission energy decreases in this particular dataset) and assuming the LO phonon energy point used in this work set at zero] is made by calibrating [shown on the right vertical axis in Fig. 3(b) with the highest energy point used in this work set at zero] is made by calibrating V_{G2} against LO-phonon emission peaks [13] (not visible in this particular dataset) and assuming the LO phonon energy of 36 meV [33]. The electron emission energy decreases linearly as V_{G2} is made more positive. When V_{G2} is swept further positive, multiple (N) electrons are pumped per cycle. Even then, we can still resolve the emission energy of the last electron emitted, as the emission energies of other electrons are well separated at lower values and can be blocked by the detector.

Figure 3(c) plots v^2 measured as a function of relative emission energy ΔE at B = 14 T for both samples. As expected, they fit well to straight lines. From this fit, we deduce the edge-confinement energy Ω_{y} = 2.7 meV and 1.8 meV, and the bottom of the confinement potential at ΔE ≈ −47 meV and −61 meV, for Samples A and B, respectively. From these, we can reconstruct the edge-confinement potential $\phi = \frac{1}{2} m^* \omega_y^2 y^2 / 2\epsilon$ as shown in Fig. 3(d). Here we set the potential at the bottom of the parabola as zero. From each data point in Fig. 3(c), we can deduce the potential energy $−\epsilon \phi$ at the position of the guiding centre, averaged over the length of the path, by subtracting the kinetic energy $\frac{1}{2} m^* v^2$ from the total (relative) energy ΔE. We can then visualise the spatial position of the edge states as plotted in Fig. 3(d).

In summary, we have shown the measurements of the time of flight of electron wave packets travelling through edge states. The electrons travel in the region where the background 2DEG is depleted and electron-electron interaction is minimised. We find that the electron velocity is in good agreement with the expected $\vec{E} \times \vec{B}$ drift. From the energy dependence, we deduce the edge confinement potential. Our technique provides a way of characterising the edge-state transport of single-electron wave packets with picosecond resolutions. The method that we have developed to transport electron wave packets in depleted edges could provide an ideal electron waveguide system where decoherence due to interactions can be avoided for electron quantum optics experiments.

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[29] One may argue that time-of-flight measurements could be performed by using two detector gates on the electron path, and detecting the difference in the arrival time between the two detectors. However, in this method, synchronising two detector signals with a picosecond resolution would be challenging. Our method circumvents such a timing issue by using a single detector.

[30] We find that the depletion gate can be used to suppress the energy relaxation due to LO-phonon emission [13]. With a negative voltage on G5 enough to deplete the 2DEG underneath, LO-phonon scattering becomes negligible down to ($B \sim 7 - 8$ T). Even at lower fields where LO-phonon scattering is enhanced, we can still perform velocity measurements as an enough number of electrons arrive at the detector with the original emission energy. We believe that the suppression of LO-phonon emission is caused by the decreased overlap between the initial and final states due to shallower slope at the bottom of parabolic potential. The details of this effect will be published elsewhere [34].

[31] Although, in this approximation, the channel is confined by a harmonic potential whereas in our device the confinement potential is only on one side (the other edge is far away), it is still appropriate to use this approximation in high $B$ limit as the wave function becomes localised on one edge only.

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