Spin Transfer from the point of view of the ferromagnetic degrees of freedom

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Abstract

Spintronics is the generic term that describes magnetic systems coupled to an electric generator, taking into account the spin attached to the charge carriers. For this topical review of Spin Caloritronics, we focus our attention on the study of irreversible processes occurring in spintronic devices, that involve both the spins of the conduction electrons and the ferromagnetic degrees of freedom. The aim of this report is to clarify the nature of the different kinds of power dissipated in metallic ferromagnets contacted to an electric generator, and to exploit it in the framework of the theory of mesoscopic non-equilibrium thermodynamics. The expression of the internal power (i.e. the internal entropy production multiplied by the temperature) dissipated by a generic system connected to different reservoirs, allows the corresponding kinetic equations to be derived with the introduction of the relevant phenomenological kinetic coefficients. After derivation of the kinetic equations for the ferromagnetic degrees of freedom (i.e. the Landau-Lifshitz equation) and the derivation of the kinetic equations for the spin-accumulation effects (within a two channel model), the kinetic equations describing spin-transfer are obtained. Both spin-dependent relaxation (usual spin-accumulation) and spin-precession in quasi-ballistic regime (transverse spin-accumulation) are taken into account. The generalization of the Landau-Lifshitz equation to spin-accumulation is then performed with the introduction of two potential energy terms, that are experimentally accessible.

PACS numbers: 75.40.Gb,72.25.Hg,75.47.De
I. SUMMARY

The approach of spintronics adopted here is that of Non-Equilibrium Thermodynamics [1–5] applied at the Mesoscopic scale [6, 7] (MNET). The analysis is based on the expression of the entropy production, i.e. on the expression of the power dissipated through the different relaxation mechanisms that characterize the system. The theory is adapted to the description of a plurality of out-of-equilibrium sub-systems exchanging energy, in which the role of environmental microscopic degrees of freedom are reduced to transport coefficients (damping, diffusion coefficients, conductivity, thermoelectric power, gyromagnetic ratio, and other Onsager coefficients) for the collective variables under consideration. The description holds at the mesoscopic scales, under the hypothesis of local equilibrium extended to internal degrees of freedom. The aim of this report is to propose an application of MNET to metallic spintronic devices that includes uniform ferromagnetic degrees of freedom explicitly.

Before presenting the detailed derivation of the kinetic equations in the forthcoming sections II, III and IV, let us first summerize the general scheme of the report. In this introductory section, the usual thermoelectric effect is first presented, and extended to the case of a bi-valuated internal variables: this is the two-channel model. An analogous approach is then performed in the space of the ferromagnetic degrees of freedom $\Sigma$. The coupling between the $\Sigma$ space and the internal degree of freedom of the electronic sub-system leads us to the spin-transfer kinetic equations. The detailed treatment of the ferromagnetic transport is given in section II, the detailed treatment of the two channel model of electric transport is performed in section III, and the coupling between both subsystems, i.e. the spin-transfer [8, 9], is presented in the last section.

A. Thermoelectric effects

The internal power dissipated by a system is given by the internal entropy production $\frac{dS^i}{dt}$ (where $S$ is the entropy of the system) multiplied by the temperature $T$. In the case of electric charges moving in one dimension $z$ in a wire of section unity (i.e. the wire is contacted to two reservoirs of electric charge) and maintained at uniform temperature, the power dissipated inside the system is given by the Joule heating, i.e. the product of the electric current by the electric field $\mathcal{E}$:
\[
\frac{dS_i}{dt} = J^e \mathcal{E} = -J^e e \frac{1}{e} \frac{\partial \mu^e}{\partial z} 
\]

(1)

where \( J^e \) is the electric current, \( e \) the absolute value of the electric charge, and \( \mu^e \) is the electrochemical potential. The electric field is given by \( \mathcal{E} = -\frac{1}{e} \frac{\partial \mu^e}{\partial z} \).

The application of the second law of thermodynamics \( \frac{dS_i}{dt} \geq 0 \) leads us to define a first positive Onsager coefficient \( \sigma \) (which is a function of the state variables) in order to build a positive quadratic form. The electric current writes:

\[
J^e = -\sigma \frac{\partial \mu^e}{\partial z} 
\]

(2)

which is Ohm’s law and \( \sigma \) is the electric conductivity.

On the other hand, the power dissipated inside a wire contacted to two heat reservoirs is the product of the heat flow \( J^Q \) and the conjugated force \( \frac{\partial}{\partial z} \left( \frac{1}{T} \right) \) multiplied by \( T \):

\[
T \frac{dS_i}{dt} = T J^Q \frac{\partial}{\partial z} \left( \frac{1}{T} \right) 
\]

(3)

The Fourier’s law is deduced from the second law of thermodynamics after introducing a positive Onsager coefficient \( \kappa \):

\[
J^Q = -\kappa \frac{\partial T}{\partial z} 
\]

(4)

where \( \kappa \) is the thermal conductivity.

If both electric and heat reservoirs are contacted to the same wire, the internal power dissipated is:

\[
T \frac{dS_i}{dt} = -J^e e \frac{1}{e} \frac{\partial \mu^e}{\partial z} + T J^Q \frac{\partial}{\partial z} \left( \frac{1}{T} \right) 
\]

(5)

Now, beyond Ohm’s law and Fourier’s law, the two currents are coupled through the relevant Onsager thermoelectric cross-coefficients,

\[
\begin{cases}
J^e = -\frac{\sigma}{e} \frac{\partial \mu^e}{\partial z} + S \sigma \frac{\partial T}{\partial z} \\
J^Q = \frac{S T \sigma}{e} \frac{\partial \mu^e}{\partial z} - (\kappa + T S^2 \sigma) \frac{\partial T}{\partial z}
\end{cases} 
\]

(6)

where the Onsager cross-coefficients are expressed with the help of the Seebeck coefficient \( S \), defined at zero electric current by the relation \( \mathcal{E} = S \frac{\partial T}{\partial z} \) and the conductivities \( \sigma \).
and $\kappa$. Note that according to the Onsager reciprocity relations, the Peltier coefficient $\Pi$, defined without electric field by the relation $J_Q = \Pi J^e$, verify to the relation $\Pi = T S$. The existence of the cross-coefficients is justified by the fact that the charge carriers are also contributing to the transport of heat, or inversely, the transport of heat is contributing to the transport of electric charges. If a detailed microscopic theory is possible in the case of the Ohm’s law and the Fourier’s law, this is no longer the case in general for thermoelectric effects or other cross-effects. However, the knowlege of the detailed mechanisms of heat transport due to electric carriers is not necessary in order to derive Eq. (6). This justifies the interest of a thermokinetic phenomenological appoach applied to spin caloritronics, for which the underlaying relaxation mechanisms are also not well known [10, 11].

B. Two-channel relaxation and spin-accumulation

Let us assume that the ensemble of electric charges is composed of two different populations. The difference is introduced through an internal degree of freedom, restricted here to a bi-valuated variable that takes the value $\alpha$ and $\gamma$. In the context of semiconductor physics, the two channel model was introduced in order to describes the transport of both electrons and holes [4, 5]. In the context of the usual spin-accumulation effect due to spin-flip relaxation, the two channels account for the spin up or spin down attached to the conduction electrons: $\alpha = \uparrow$ and $\gamma = \downarrow$ [12–15]. However, from the point of view adopted here, this scheme should be generalized to a band structure in order to account for the $s - d$ relaxation mechanisms, that are responsible for the coupling between the spin of the conduction electron (mainly $s$ electron band) and the ferromagnetic degrees of freedom (related to $d$ electron band) [16]. In this case, electronic transport is also spin-dependent because the $d$ band is full for majority spins (e.g. $\uparrow$) in usual 3d metallic ferromagnets [17].

However, without entering into the complexity of the spin-dependent relaxation mechanisms, it is easy to generalize Ohm’s law with adding the parameters $\alpha$ and $\gamma$ to the transport coefficients. A third kinetic equation should be introduced in order to take into account the power dissipated by the $\alpha \rightarrow \gamma$ relaxation (spin-flip relaxation or $s - d$ relaxation). This relaxation is formally equivalent to a chemical reaction, driven by the chemical affinity $\Delta \mu = \mu_\alpha - \mu_\gamma$ [15]. Indeed, the power dissipated by the system reads then:
\[ T \frac{dS^e}{dt} = -J_\alpha^e \frac{1}{e} \frac{\partial \mu^e_\alpha}{\partial z} - J_\gamma^e \frac{1}{e} \frac{\partial \mu^e_\gamma}{\partial z} + \dot{\Psi} \Delta \mu \]  

(7)

where the flux of particles relaxing from one channel to the other (relaxation occurring in the space of the internal degrees of freedom) is given by \( \dot{\Psi} \). The corresponding kinetic equations are deduced, after introducing a supplementary Onsager coefficient \( L \).

\[
\begin{cases}
J_\alpha^e = -\frac{\sigma_\alpha}{e} \frac{\partial \mu^e_\alpha}{\partial z} \\
J_\gamma^e = -\frac{\sigma_\gamma}{e} \frac{\partial \mu^e_\gamma}{\partial z} \\
\dot{\Psi} = L \Delta \mu
\end{cases}
\]  

(8)

The set of equations Eqs (8) is sufficient and necessary in order to describe, in the stationary regime, spin-accumulation effects or any non-equilibrium contribution to the resistance due to \( \Delta \mu \) occurring at an interface. The corresponding effects (spin accumulation, giant magnetoresistance, etc) will be discussed in Section III. Onsager cross-coefficients may also be added at this stage of the analysis [18].

It is important to note the introduction of the parameter \( \Delta \mu \) in the irreversible processes described in Eq. (7): in spintronics, \( \Delta \mu \) is called "spin accumulation" and it was first introduced in this context by Van Kempen et al. [13]. This parameter plays the role of the pumping force that is responsible for the out-of-equilibrium relaxation occurring from one channel to the other. It will be also responsible for spin-transfer as described at the end of this report.

Accordingly, it is convenient to rewrite Eq. (8) as a function of the variable \( \Delta \mu \). Let us define the conductivity asymmetry by the parameter \( \beta \) such that \( \beta = \frac{\sigma_\alpha - \sigma_\gamma}{\sigma_0} \) and the mean conductivity \( 2\sigma_0 = \sigma_\alpha + \sigma_\gamma \). On the other hand, the spin-polarized electric current is \( \delta J^e = J_\alpha^e - J_\gamma^e \) and the spin-independent current is \( J_0^e = J_\alpha^e + J_\gamma^e \). In this new system of equations, the Onsager matrix re-writes:

\[
\begin{pmatrix}
\delta J^e \\
J_0^e \\
\dot{\Psi}
\end{pmatrix} =
\begin{pmatrix}
\sigma_0 & \beta \sigma_0 & 0 \\
\beta \sigma_0 & \sigma_0 & 0 \\
0 & 0 & L
\end{pmatrix}
\begin{pmatrix}
-\frac{1}{e} \frac{\partial \Delta \mu^e}{\partial z} \\
-\frac{1}{e} \frac{\partial \nu^e_0}{\partial z} \\
\Delta \mu
\end{pmatrix}
\]  

(9)

where \( \mu_0 = \mu_\alpha + \mu_\gamma \).
The generalization of the thermoelectric effect to the two channel case is straightforward, assuming that the electrons are thermalized (i.e. the temperature is the same for each channel):

\[
\begin{align*}
J^e_\alpha &= -\sigma_\alpha \frac{\partial \mu^e_\alpha}{\partial z} + S_\alpha \sigma_\alpha \frac{\partial T}{\partial z} \\
J^Q_\alpha &= -S_\alpha T \sigma_\alpha \frac{\partial \mu^e_\alpha}{\partial z} - (\kappa_\alpha + T S^2_\alpha \sigma_\alpha) \frac{\partial T}{\partial z} \\
J^e_\gamma &= -\sigma_\gamma \frac{\partial \mu^e_\gamma}{\partial z} + S_\gamma \sigma_\gamma \frac{\partial T}{\partial z} \\
J^Q_\gamma &= -S_\gamma T \sigma_\gamma \frac{\partial \mu^e_\gamma}{\partial z} - (\kappa_\gamma + T S^2_\gamma \sigma_\gamma) \frac{\partial T}{\partial z}
\end{align*}
\]

(10)

where \( S_i \) and \( \kappa_i \), \( i = \{\alpha, \gamma\} \), are respectively the Seebeck and Fourier coefficient of each channel. However, in this report, we will not further investigate this set of equations. Some consequences in relation with experiments have been investigated and reported [19–28]. The set of equations (10), or other theoretical descriptions beyond the two channel model [29–31] investigated the caloritronic properties of spintronic systems. In contrast, the goal of this report is to describe the transport properties of the ferromagnetic degrees of freedom and the consequences of its interaction with spin-dependent electric sub-systems.

C. Introduction of the ferromagnetic degrees of freedom

Let us introduce the ferromagnetic degrees of freedom. This observable is defined in a physical space that is not the usual space \( \mathbb{R} \) considered above, but a space of magnetic moments \( \Sigma \). This space should be considered as a space of internal degrees of freedom, in the same sense as the bi-valuated variable \( \{\alpha, \gamma\} \) of the two channel model (and in the same sens as the spin space in quantum mechanics). The power dissipated by the magnetic system is then given by the flux \( \vec{J}^F_0 \) of magnetic moments (defined in the corresponding vectorial space \( \Sigma \)) multiplied by the magnetic force:

\[
T \frac{dS^F}{dt} = \vec{J}^F_0 \cdot \vec{\nabla}_\Sigma \mu^F_0
\]

(11)

where \( \mu^F_0 \) is the ferromagnetic chemical potential and \( \vec{\nabla}_\Sigma \) is the gradient defined in the space \( \Sigma \) (see Section III). The application of the second law of thermodynamics leads us to introduce the positive Onsager matrix \( \tilde{L}_0 \) such that
\[ \vec{J}_0^F = \mathcal{L}_0 \vec{\nabla}_\Sigma \mu_0^F \] (12)

This kinetic equation for the ferromagnetic degrees of freedom is actually the simplest formulation of the Landau-Lifshitz equation for the magnetization \( \vec{M} \) (see Section II below).

By analogy with the thermoelectric effect, it is tempting to formally introduce a gradient of temperature in the corresponding configuration space. We expect then the existence of a supplementary force acting on the magnetization:

\[
\begin{pmatrix}
\vec{J}_F \\
\vec{J}_F^Q
\end{pmatrix} =
\begin{pmatrix}
\mathcal{L} & \bar{S}^F \\
\bar{\Pi}^F & \bar{\lambda}
\end{pmatrix}
\begin{pmatrix}
\vec{\nabla}_\Sigma \mu^F \\
\vec{\nabla}_\Sigma T
\end{pmatrix}
\] (13)

where \( \bar{S}^F \), \( \bar{\lambda} \) and \( \bar{\Pi}^F \) are arbitrary Onsager matrices that formally generalises the Seebeck coefficient, the thermal conductivity, and the Peltier coefficient in the \( \Sigma \) space. The question is to understand the physical meaning of a temperature gradient in the configuration space of the magnetization. This situation is analogous but very different from the case of two thermostats of different temperatures localized in two places (in the \( \Re \) space). The question about the physical signification of a quantity like \( \vec{\nabla}_\Sigma T \) is not trivial.

However, the situation described in Eq. (13) is rather similar to the result obtained at the end of the report, providing that the effective temperature gradient \( \vec{\nabla}_\Sigma T \) is replaced by the voltage drop due to spin-accumulation: \( \Delta \mu \approx \int \frac{\partial \Delta \mu}{dz} dz \). A supplementary force is acting on the ferromagnet.

Indeed, let us consider now the system with both spin-dependent electric and ferromagnetic dissipation. The ferromagnetic system is not closed, since spins are transfered from the electric subsystem to the ferromagnetic subsystem. However, the total system is closed. The total internal entropy production allows to access to the kinetic equations of the coupled system. The power dissipated is now:

\[
T \frac{dS_i}{dt} = \vec{J}_0^F \cdot \vec{\nabla}_\Sigma \mu^F - \delta \vec{J}_e \cdot \frac{\partial \Delta \mu^e}{\partial \mu^e} - J_0^e \frac{\partial \mu^e}{\partial \mu^e} + \dot{\Psi} \Delta \mu^e
\] (14)

where \( \vec{J}_0^F \) is the total ferromagnetic flux that includes spin transfer and where the vectorial form of the pumping force \( \vec{\Delta \mu}^e \) is introduced in order to take into account the transverse spin-accumulation mechanisms discussed in the litterature related to microscopic theories of spin-transfer-torque.
Ignoring the electric dissipation (i.e. the two last terms in Eq. (14)), the following form is obtained for the ferromagnetic system (after some crude simplifications see Section IV):

$$
\begin{pmatrix}
\vec{J}_{tot}^{F} \\
\vec{J}_{Q}^{F}
\end{pmatrix} =
\begin{pmatrix}
\vec{L} & \vec{l} \\
\vec{\bar{l}} & \vec{\bar{\sigma}}
\end{pmatrix}
\begin{pmatrix}
\vec{\nabla} \Sigma \mu^{F} \\
\Delta \mu^{c}
\end{pmatrix}
$$

(15)

where the matrices \(\vec{L}, \vec{l}, \vec{\bar{l}}\) and \(\vec{\bar{\sigma}}\) are related to measurable experimental parameters. In the same way as for the thermoelectric power in Eq. (6), the presence of the cross-coefficients is justified by the fact that the diffusion of the spin carriers at the interface (e.g. \(s – d\) relaxation) is contributing to the transport of ferromagnetic moments.

The consequences of the suplementary term in the expression of the current \(\vec{J}_{tot}^{F}\) in Eq. (15) are investigated in terms of a generalized Landau-Lifshitz equation that includes drift and diffusion contributions due to spin-transfer [32] (Section IV). The rough arguments presented in this introductory section will be developed and detailed in the following sections.

II. DERIVATION OF LANDAU-LIFSHITZ EQUATION FROM THE CORRESPONDING POWER DISSIPATION

In order to treat statistically the time dependence of a unique uniform ferromagnetic moment \(\vec{M} = M_s \vec{u}_r\) (with radial unit vector \(\vec{u}_r\)) of a fluctuating magnetic nanostructure, the ergodic property is used. It allows work with a statistical ensemble of a large number of ferromagnetic moments \(\vec{m}\) oriented in the direction \(\{\theta \pm d\theta, \varphi \pm d\varphi\}\) of a sphere \(\Sigma\) of radius \(M_s\). The density \(\rho_F^{\theta, \varphi}\), defined on the surface of the sphere, is then identified with the statistical distribution of ferromagnetic moments. The introduction of the density is justified by the nanoscopic size of the magnetic single domain, for which the fluctuations play a major role. To that point of view, the system is mesoscopic. Accordingly [47], the chemical potential \(\mu_0^{F}\) takes the general form :

$$
\mu_0^{F} = kT ln(\rho_0^{F}) + V^{F}
$$

(16)

in which the ferromagnetic potential is for instance \(V^{F}(\vec{H}, \theta) = K \sin(\theta) - M_s H \cos(\theta - \phi)\) in the case of a single domain with uniaxial anisotropy of constant \(K\) and with an external magnetic field \(\vec{H}\) applied at an angle \(\phi\) from the anisotropy axis.
The subscript 0 stands for a closed ferromagnetic system (no source of magnetic moments). The corresponding current of magnetic moments $\vec{J}_0^F$ is related to the density by the conservation law:

$$\frac{d\rho_0^F}{dt} = -\text{div}_\Sigma \vec{J}_0^F$$

(17)

where $\text{div}_\Sigma$ is the divergence operator defined on the surface of the sphere $\Sigma$.

The power dissipated by the ferromagnetic system is given by the corresponding internal entropy production $\frac{dS_i^F}{dt}$, and is given by the product of the generalized flux by the generalized force. Assuming a uniform temperature $T$ we have:

$$T \frac{dS_i^F}{dt} = \vec{J}_0^F \cdot \vec{\nabla}_\Sigma \mu_0^F$$

(18)

The application of the second law of thermodynamics $dS_i^F/\frac{dt}{dt} \geq 0$ allows the transport equation to be deduced by writing the relation that links the generalized flux (the current $\vec{J}_0^F$) of the extensive variables under consideration and to the generalized force defined in the corresponding space $\Sigma$. Both quantities, flux and forces, are related by the Onsager matrix of the transport coefficients $\bar{L}_0$:

$$\vec{J}_0^F = \bar{L}_0 \vec{\nabla}_\Sigma \mu_0^F$$

(19)

The problem is solved as soon as the Onsager matrix is known. In the present case, we started from the hypothesis that the magnetic domain was uniform: the modulus of the magnetization is conserved. The trajectory of the magnetization (in the configuration space) is then confined on the surface of a sphere of radius $M_s$, and the flow is a two component vector defined with the unit vectors $\{\vec{u}_\phi, \vec{u}_\theta\}$ of $\Sigma$. Accordingly, the Onsager matrix is a 2 by 2 matrix defined by four transport coefficients $\{L_{\theta\theta}, L_{\theta\phi}, L_{\phi\theta}, L_{\phi\phi}\}$. Furthermore, the Onsager reciprocity relations impose that $L_{\theta\phi} = -L_{\phi\theta}$.

However, the magnetization is defined by a given axis (unit vector $\{\vec{u}_r\}$) in 3D space. The choice of the two other vectors is arbitrary, so that $L_{\theta\theta} = L_{\phi\phi}$. Let us now introduce a dimensionless supplementary coefficient $\alpha$, which is the ratio of the off diagonal to the diagonal coefficients: $\alpha = L_{\theta\phi}/L_{\theta\theta}$. In conclusion, the ferromagnetic kinetic equation is defined by two ferromagnetic transport coefficients $L_{\theta\phi} = \rho_0^F L_F$ and $\alpha$:
On the other hand, the generalized force \( \vec{\nabla} \Sigma \mu^F_0 \), thermodynamically conjugated to the magnetization, defines a "generalized" effective magnetic field

\[
\vec{H}_{\text{eff}} \equiv -\vec{\nabla} \Sigma \mu^F_0
\]

It is a generalization in the sense that this effective field includes the diffusive term \([33]\) that has first been introduced by Brown in the rotational Fokker-Planck equation \([34]\).

The equation Eq. (19) is the well known phenomenological Landau-Lifshitz (LL) equation:

\[
\vec{J}^F_0 = -\rho^F_0 L_F \left( \begin{array}{cc} \alpha & 1 \\ -1 & \alpha \end{array} \right) \vec{H}_{\text{eff}}
\]

Actually, it could be rather surprising to claim that Eq. (22), that takes the form of the Fick’s law or thermoelectric laws (with the cross-coefficients), is the "well-known LL equation" because the LL equation is the dynamical equation of the ferromagnetic variable \( \vec{M} = M_s \vec{u}_r \). However, it is sufficient to rewrite Eq. (22) in 3D space with re-introducing the radial unit vector \( \vec{u}_r = (1, 0, 0) \) of the reference frame \( \{ \vec{u}_r, \vec{u}_\theta, \vec{u}_\phi \} \), and recalling that the current is the density multiplied by the velocity \( \vec{J}^F_0 = \rho^F_0 d\vec{u}_r/dt \), in order to recover the traditional LL equation from (22):

\[
\frac{d\vec{u}_r}{dt} = L_F \left\{ \vec{u}_r \times \vec{H}_{\text{eff}} - \alpha \vec{u}_r \times \left( \vec{u}_r \times \vec{H}_{\text{eff}} \right) \right\}
\]

Furthermore, it is well-known that LL equation is equivalent to the following Gilbert \([35]\) equation, that allows the damping coefficient \( \eta \) to be defined:

\[
\frac{d\vec{u}_r}{dt} = \vec{u}_r \times \Gamma \left( \vec{H}_{\text{eff}} - \eta M_s \frac{d\vec{u}_r}{dt} \right)
\]

where \( \Gamma \) is the gyromagnetic ratio. The equivalence between the two equations defines the coefficients \( \alpha \) and \( L_F \) has a function of the coefficients \( \eta \) and \( \Gamma \). \( \alpha \) is the dimensionless damping coefficient:
\[ \alpha = \eta \Gamma M_s \]  
(25)

and \( L_F \) is defined by the relation

\[ L_F = \frac{\Gamma}{M_s (1 + \alpha^2)} \]  
(26)

The corresponding Fokker-Planck stochastic equation first derived by Brown \[34\] is obtained directly by inserting Eq. (22) into Eq. (17).

\[ \frac{d\rho_F^0}{dt} = -\vec{\nabla}_\Sigma \vec{L}_0 \vec{\nabla}_\Sigma \mu_F^0 \]  
(27)

Using Eq. (16), Eq. (25), Eq. (26), and the explicit expression of the Laplacian \( \nabla^2_\Sigma \) in spherical coordinates, the Fokker-Planck equation reads:

\[ \frac{\partial \rho_F^0}{\partial t} = \frac{L_F}{\sin \theta \partial \theta} \left\{ \sin \theta \left[ \left( \frac{\alpha \partial V^F}{\partial \theta} - \frac{1}{\sin \theta} \frac{\partial V^F}{\partial \phi} \right) \rho_F^0 + kT \alpha \frac{\partial \rho_F^0}{\partial \theta} \right] \right\} \]
\[ + \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} \left\{ \left( \frac{\partial V^F}{\partial \theta} + \frac{\alpha}{\sin \theta} \frac{\partial V}{\partial \phi} \right) \rho_F^0 + kT \frac{\alpha}{\sin \theta} \frac{\partial \rho_F^0}{\partial \phi} \right\} \]  
(28)

The driving force responsible for the magnetization dynamics is distributed between drift and diffusion terms for the probability distribution. The equilibrium solution of the equation is the Boltzmann distribution, as it was assumed in expression Eq. (16) for the definition the chemical potential.

Since the equation depends on the determinist potential \( V^F \) (that contains the energy due to the external magnetic field, the magnetocrystalline anisotropy, dipolar energy, etc), it is non linear. Only a few simple configurations can find an analytical solution for the non-equilibrium statistical distribution \( \rho_F^0 (\theta, \varphi, t) \) \[36\]. This is typically the case for linear expansions near equilibrium states in the context of ferromagnetic resonance, or for the Néel-Brown activation process \[34, 36\] at long time scales. Eq. (28) will be extended to spin-transfer contributions introduced in Section III, after the study of spin-accumulation effects below.
III. DERIVATION OF SPIN-ACCUMULATION FROM THE CORRESPONDING POWER DISSIPATION

In this section, we focus on the electric transport only (we forget the role of the ferromagnetic variable and the existence of the $\Sigma$ space). The corresponding electric wire is defined along the $z$ axis, with a section unity: the relevant configuration space is the one dimensional real space $\mathbb{R}$. However, in order to take into account the spin-dependent electric current, the two channel model is introduced. Beyond the diffusive two channel model approach, transverse spin accumulation parameters are also introduced, in order to take into account the spin precession in quasi-ballistic regime near the interface.

A. two channel relaxation and spin-diffusion

The system is composed of two populations of conduction electrons with a relaxation process that allows the electrons of one population relax into the other. The difference between the two populations is introduced through an internal degree of freedom, and the relaxation process occurs within the space defined by this internal degree of freedom. Inside the bulk, the relaxation from one channel to the other is compensated by the opposite relaxation: the electronic populations are maintained at equilibrium. However, the presence of an interface with inhomogeneous transport parameters puts out-of-equilibrium the electronic populations. At steady state, a diffusion process (of the spin-density) in the $\mathbb{R}$ space occurs that compensates the forced relaxation defined in the internal space. This diffusion process is called spin-accumulation in the case of spin-dependent transport. Formally, the two-channel model consisted of defining a bi-valuated internal variable for the transport parameters, that takes the values $\alpha$ or $\gamma$. Typically, the values of the internal variables are $\alpha = \uparrow$ and $\gamma = \downarrow$ for spin-flip scattering, or $\alpha = s (\uparrow \downarrow)$ and $\gamma = d (\downarrow)$ for spin-dependent $s-d$ scattering [16, 26] (the three channel model in which the internal variable takes the values $\{s \uparrow, s \downarrow, d \downarrow\}$ is developed in reference [16]). Accordingly, the local electrochemical potentials are defined by $\mu^\alpha_e$ and $\mu^\gamma_e$, and the electric currents generated in each channel is noted $\{J^{\alpha}_e, J^{\gamma}_e\}$

The conservation laws write:
\[
\begin{align*}
\frac{dn_\alpha}{dt} &= -\frac{\partial J^e_\alpha}{\partial z} - \dot{\Psi} \\
\frac{dn_\gamma}{dt} &= -\frac{\partial J^e_\gamma}{\partial z} + \dot{\Psi}
\end{align*}
\] (29)

where \(n_\alpha\) and \(n_\gamma\) are the densities of charge carriers in the channels \(\{\alpha, \gamma\}\), and the spin-dependent relaxation is taken into account by the flux \(\dot{\Psi}\). This is the velocity of the reaction (or relaxation of the spin-dependent internal variable) that transforms a conduction electron \(\alpha\) into the conduction electron \(\gamma\). This generalized flux defines a "spin current" (density times velocity) in the configuration space of the internal variable (somehow related to \(\Sigma\): see next section). Note however that in the literature the term "spin current" is devoted to the spin-polarized electric current \(\delta J^e = J^e_\alpha - J^e_\gamma\) defined in the real space \(\Re\).

The power dissipated by the electric system is given by the corresponding internal entropy variation, i.e. by the product of the currents by the electric fields:

\[
T \frac{dS_i^e}{dt} = -J^e_\alpha \frac{\partial \mu^e_\alpha}{e \partial z} - J^e_\gamma \frac{\partial \mu^e_\gamma}{e \partial z} + \dot{\Psi} \Delta \mu^e
\] (30)

where we introduced the difference of the chemical potentials \(\Delta \mu^e = \mu^e_\alpha - \mu^e_\gamma\).[13, 14]

the application of the second law of thermodynamics leads to the kinetic equations, after introducing the transport coefficients: the conductivities \(\sigma_\alpha, \sigma_\gamma\), and the Onsager coefficient \(L\), such that:

\[
\begin{align*}
J^e_\alpha &= -\frac{\sigma_\alpha}{e} \frac{\partial \mu^e_\alpha}{\partial z} \\
J^e_\gamma &= -\frac{\sigma_\gamma}{e} \frac{\partial \mu^e_\gamma}{\partial z} \\
\dot{\Psi} &= L \Delta \mu^e
\end{align*}
\] (31)

where the two first equations are Ohm’s law applied to each channels. The effect of the electric charge relaxation is described in reference [16]. The Onsager coefficient \(L\) is shown to be inversely proportional to the electronic relaxation times \(\tau_{\alpha+\gamma}\). The total electric current is spin-independent:

\[
J^e_0 = J^e_\alpha + J^e_\gamma = -\frac{1}{e} \frac{\partial}{\partial z} \left( \sigma_\alpha \mu^e_\alpha + \sigma_\gamma \mu^e_\gamma \right)
\] (32)

However, it is not possible to measure separately the different conduction channels, since any realistic electric contact short cuts the two channels. What is measured is necessarily the usual Ohm’s law, \(J^e_0 = -2\sigma_0 \frac{\partial \zeta}{\partial z}\), that imposes the reference electric potential \(\zeta\) to be introduced, together with the mean conductivity \(\sigma_0 = (\sigma_\alpha + \sigma_\gamma)/2\). The potential \(\zeta\) is hence:
\[ e\zeta = \frac{2}{\sigma_0}(\sigma_\alpha \mu_\alpha + \sigma_\gamma \mu_\gamma) \tag{33} \]

The reference configuration is defined by the two channels collapsing to a unique conduction channel (e.g. parallel magnetization of a junction of two identical ferromagnetic layers: \( \Delta \mu_e^{eq}(0) = 0 \)). The non-equilibrium (\( \Delta \mu_e(0) \neq 0 \)) contribution of the junction to the resistance, \( R^{ne} \), is calculated through the relation:

\[ J_0^e R^{ne} = \int_A^B \frac{\partial}{\partial z}(\mu_\alpha^e - e\zeta(z))dz = \int_A^B \frac{\partial}{\partial z}(\mu_\gamma^e - e\zeta(z))dz \tag{34} \]

so that

\[ R^{ne} = -\frac{1}{J_0^e} \int_A^B \frac{\sigma_\alpha - \sigma_\gamma}{\sigma_0} \frac{\partial \Delta \mu_e}{\partial z}dz \tag{35} \]

where the measurement points \( A \) and \( B \) are located far enough in each side of the interface (inside the bulk) so that \( \Delta \mu_e(A) = \Delta \mu_e(B) = 0 \). The integral in Eqs. (34) is performed over the regular part of the function only (across the interface \( \zeta \) and \( \sigma_i \) are discontinuous at this scale): this resistance is proportional to the discontinuity at the interface. It is convenient to describe the conductivity asymmetry by the parameter \( \beta \) such that \( \sigma_\alpha = \sigma_0(1 + \beta) \) and \( \sigma_\gamma = \sigma_0(1 - \beta) \). On the other hand, the spin-polarized electric current is \( \delta J^e = J^e_\alpha - J^e_\gamma \).

With these new variables, Eq. (35) rewrites:

\[ R^{ne} = -\frac{2\beta}{J_0^e} \int_A^B \frac{\partial \Delta \mu_e}{\partial z}dz \tag{36} \]

and the Onsager matrix reads:

\[
\begin{pmatrix}
\delta J^e \\
J_0^e \\
\dot{\Psi}
\end{pmatrix} = \begin{pmatrix}
\sigma_0 & \beta \sigma_0 & 0 \\
\beta \sigma_0 & \sigma_0 & 0 \\
0 & 0 & L
\end{pmatrix} \begin{pmatrix}
-\frac{1}{e} \frac{\partial \Delta \mu_e}{\partial z} \\
-\frac{1}{e} \frac{\partial \mu_0^e}{\partial z} \\
\Delta \mu
\end{pmatrix} \tag{37}
\]

The system of equations Eq. (37) allows the diffusion equation for \( \Delta \mu(z) \) to be derived for the stationary conditions \( \partial \tilde{J}_0^e / \partial z = 0 \) and \( \partial \delta \tilde{J}^e / \partial z = -2\dot{\Psi} \):

\[ \frac{\partial^2 \Delta \mu}{\partial z^2} = \frac{\Delta \mu}{l_{diff}^2} \tag{38} \]

where \( l_{diff}^2 = \frac{2eL}{\mu_0 \zeta^2} \). The resistance \( R^{ne} \) can then be calculated for each specific device configurations [16, 26].
The two channel approximation with the internal degree of freedom that takes the value \{\uparrow, \downarrow\} describes the consequences of the spin-flip scattering. However, the model is also sufficient for the description of spin-dependent \( s - d \) relaxation [16, 26], where the \( d \) band is full for the majority spins \( \uparrow \). Indeed, the \( s - d \) relaxation with spin-flip (from \( s \uparrow \) to \( d \downarrow \)) has a very small probability to occur, compared to the relaxation without spin-flip (from \( s \downarrow \) to \( d \downarrow \)). As a consequence, the two channel model can also be used, and it also leads to a redistribution of the spin populations at the interfaces, i.e. to spin-accumulation. In both cases, the spin-accumulation is described by the function \( \Delta \mu(z) \), which is solution of is the the diffusive equation Eq. (38).

### B. Quasi-ballistic effect and transverse spin-accumulation

However, the description proposed above with a spin-dependent internal variable that takes the two values \{\( \alpha, \gamma \)\} is not able to take into account the precession of the spins occurring in a magnetic field. If the precession contribution is not relevant in the case of the processes that lead to giant magnetoresistance (because the mean values are averaged out over the spin-diffusion length) this is no longer the case in a quasi ballistic regime near the interface.

In order to take into account quasi-ballistic effects near the interface (i.e. sub-nanometric scales in metallic devices), the two-channel model has been recently generalized to transverse spin-accumulation in the context of spin-transfer-torque investigations [37–42]. The transverse spin-accumulation is introduced with the corresponding current \( \delta J^\perp \) and the corresponding chemical potential \( \Delta \mu^\perp \). Transverse means here that the spin density is considered in the plan perpendicular to the quantification axis \( \uparrow \) that defines the spin up and spin down in the two channel-model.

The coefficient \( \sigma^\perp \) can also be defined through the corresponding diffusion coefficient \( D^\perp = \frac{\sigma^\perp kT}{n^\perp} \) where \( n^\perp \) is the density of transverse spins [43]. It is then also possible to define a ”pseudo” spin-diffusion process in the case of spin-decoherence. Note however that the two potentials \( \Delta \mu^e \) and \( \Delta \mu^\perp \) are defined at very different length scales and it is necessary to refer to quantum approaches in order to understand the physical signification of the transverse parameters [44, 45]. The corresponding contribution to the power dissipated is
\[ T \frac{dS^e}{dt} = -\delta J^e \frac{\partial \Delta \mu^e}{e \partial z} \]  

(39)

Putting all together we have the following Onsager relations for the electric system:

\[
\begin{pmatrix}
\delta J^e \\
J_0^e \\
\delta J_\perp^e \\
\hat{\Psi}
\end{pmatrix} =
\begin{pmatrix}
\sigma_0 & \beta \sigma_0 & 0 & 0 \\
\beta \sigma_0 & \sigma_0 & 0 & 0 \\
0 & 0 & \sigma_\perp & 0 \\
0 & 0 & 0 & L
\end{pmatrix}
\begin{pmatrix}
\frac{-1}{e} \frac{\partial \Delta \mu^e}{\partial z} \\
\frac{-1}{e} \frac{\partial \mu^e}{\partial z} \\
\frac{-1}{e} \frac{\partial \Delta \mu^e}{\partial z} \\
\Delta \mu
\end{pmatrix}
\]  

(40)

IV. DERIVATION OF SPIN-TRANSFER DUE TO LONGITUDINAL AND TRANSVERSE SPIN-ACCUMULATION

In usual experimental configurations for spin-transfer, an electric current is injected in a circuit that includes a ferromagnet (in series or in non-local configuration [48]) and the magnetoresistance, i.e. the potential drop (of the form \(-\frac{\beta}{2} \int_A^B \frac{\partial \Delta \mu}{e \partial z} dz\) Eq. (36)) allows the magnetization states to be measured. The effect of strong electric currents on the magnetization states can then be observed. In such a configuration, the two sub-systems described in the sections above exchange magnetic moments at the junctions and both are open systems.

In order to describe the dynamics of the ferromagnetic degrees of freedom (following step by step the method presented in Section II), we have to deal with a closed system. The system of interest is now the ferromagnetic system that includes spin-accumulation effects at the junctions. This total ferromagnetic system is such that the density of ferromagnetic moments \(\rho^F_{tot}\) and the total ferromagnetic flux \(\vec{J}^F_{tot}\) are related by the conservation law:

\[ d\rho^F_{tot}/dt = -\text{div}_\Sigma \vec{J}^F_{tot}. \]

The initial configuration space of magnetic moments is then extended to 1D real space parametrized by the internal variable \(\Sigma \otimes \mathcal{R}_{\alpha\gamma}\). The important point here is that the internal variable is spin-dependent, and related to the ferromagnetic space \(\Sigma\) (e.g. through s – d relaxation and the corresponding spin-accumulation). This accounts for the coupling, i.e. the transfer, of magnetic moment between the two sub-systems.

The dissipation is given by the internal power dissipated in the total system \(T dS_i/dt\) :
\[ T \frac{dS_i}{dt} = J^F_i \cdot \nabla_F \mu^F - \delta J^e \cdot \frac{\partial \Delta \mu^e}{e \partial z} - \delta J^e \cdot \frac{\partial \Delta \mu^e}{e \partial z} - J^0_0 \frac{\partial \mu^e_0}{e \partial z} + \dot{\Psi} \Delta \mu^e \] (41)

Where the first term in the right hand side is the power dissipated by the total ferromagnetic sub-system (including the ferromagnetic contribution due to spin-transfer), the two following terms are the power dissipated by spin-dependent electric transport, and the fourth term is the spin-independent Joule heating. The last term is the power dissipated by spin-flip or \( s - d \) relaxation.

In Eq. (41), the vectors are defined on the sphere \( \Sigma \) with the help of two angles \( \theta \) and \( \varphi \). The total ferromagnetic current \( \vec{J}^F_{\text{tot}} = J^F_{\text{tot}} \vec{u}_\theta + J^F_{\text{tot}} \vec{u}_\varphi \) includes the contribution due to spin-accumulation mechanisms. The chemical potential \( \mu^F \) accounts for the energy of a ferromagnetic layer. On the other hand, the system is contacted to electric reservoirs with the electric currents and the corresponding chemical potentials. Applying the second law of thermodynamics, we obtain the general Onsager relations:

\[
\begin{pmatrix}
J^F_{\varphi} \\
J^F_{\theta} \\
\delta J^\perp_e \\
\delta J^e \\
J^0_e \\
\dot{\Psi}
\end{pmatrix} = 
\begin{pmatrix}
\alpha \rho_0 L_F & \rho_0 L_F & l_{\varphi \varphi} & l_{\varphi \theta} & 0 & 0 \\
-\rho_0 L_F & \alpha \rho_0 L_F & l_{\theta \varphi} & l_{\theta \theta} & 0 & 0 \\
l_{\varphi \varphi} & l_{\varphi \theta} & \sigma_\perp & 0 & 0 & 0 \\
l_{\theta \varphi} & l_{\theta \theta} & 0 & \sigma_0 & 0 & \beta \sigma_0 \\
0 & 0 & 0 & \beta \sigma_0 & \sigma_0 & 0 \\
0 & 0 & 0 & 0 & 0 & L
\end{pmatrix} 
\begin{pmatrix}
\frac{1}{\sin(\theta)} \frac{\partial \mu^e}{\partial \varphi} \\
\frac{\partial \mu^e}{\partial \theta} \\
\frac{1}{e} \frac{\partial \Delta \mu^e}{\partial z} \\
\frac{1}{e} \frac{\partial \Delta \mu^e}{\partial z} \\
\frac{1}{e} \frac{\partial \mu^e_0}{\partial z} \\
\Delta \mu
\end{pmatrix}
\] (42)

All coefficients were defined in the previous sections, except the new cross-coefficients \( \{l_{\varphi \varphi}, l_{\varphi \theta}, l_{\theta \theta}, l_{\theta \varphi}\} \), introduced in this model as spin-transfer coefficients, related to the experimental parameters. The coefficients \( \{l_i\}_{i=\{\theta, \varphi\}} \) are deduced from the coefficients \( \{l_i\} \) through the Onsager reciprocity relations.

The total ferromagnetic current can be written after integrating over the volume \( v \) of the ferromagnetic layer of section unity and the spin accumulation zone. This volume is such that \( v = \int_A^B dz \), where \( z = A \) and \( z = B \) are two sections close to the interface but far enough with respect to the diffusion length \( l_{\text{diff}} \). We assume here that \( l_{\text{diff}} \) is much smaller than the width of the ferromagnetic layer in order to simplify the calculation: the volume of the ferromagnet is identified as \( v \). Let us define \( \vec{X} \) as the correction due to the spin-transfer deduced from the two first equations of the matrix equation Eq. (42), after integrating over the volume \( v \) :
\[ v \vec{J}^F_{\text{tot}} = v \mathcal{L}_0 \vec{\nabla} \mu^F_t + \vec{X} \]  

(43)

where \( \mathcal{L}_0 \) is the matrix defined in Eq. (20).

The assumption of constant modulus of the magnetization imposes that \( \vec{X} \) is confined on the surface of the sphere \( \Sigma \). The Helmoltz decomposition theorem can then be applied: the vector \( \vec{X} \) can be decomposed in a unique way with the introduction of the two potentials \( \chi \) and \( \Phi \) (i.e. a potential vector) such that:

\[ \vec{X} = \vec{u}_r \times \vec{\nabla}_\Sigma \Phi + \vec{\nabla}_\Sigma \chi \]  

(44)

where the first term is divergenceless and the second term is curlless (i.e. non conservative). The method used here is hence not equivalent to that of adding a spin-transfer source term in the conservation equation of the time variation of \( \rho^F_{\text{tot}} \). The two potentials will be described in more details below.

The total correction to the Landau-Lifshitz-Gilbert equation writes:

\[ v \vec{J}^F_{\text{tot}} = v \mathcal{L}_0 \vec{\nabla}_\Sigma \mu_t + \vec{u}_r \times \vec{\nabla}_\Sigma \Phi \]  

(45)

where the electro-spin chemical potential that describes the ferromagnetic system with the addition of the spin-transfer contribution writes

\[ \mu_t = kT \ln(\rho^F_{\text{tot}}) + V^F + \frac{\chi}{L_F} \]  

(46)

However, in Eq. (46), the density \( \rho^F_0 \) is no longer relevant because the ferromagnetic system alone is an open system, and only the total density \( \rho^F_{\text{tot}} \) is defined. The canonical form of the chemical potential of the total system that contains the total density \( \rho_{\text{tot}} \) is:

\[ \mu^F_t = kT \ln(\rho^F_{\text{tot}}) + V^F. \]  

The total density is deduced with identifying with Eq. (46) \( \rho^F_{\text{tot}} = \rho^F_0 e^{\chi L_F} \).

The generalized LLG takes the form:

\[ \frac{d\vec{u}_r}{dt} = \frac{\gamma}{M_s(1 + \vec{\alpha}^2)} \left\{ \vec{u}_r \times \left( \vec{H}_{\text{eff}} + \frac{\vec{\nabla}_\Sigma \Phi}{v \rho^F_0 L_F} \right) - \alpha \vec{u}_r \times \left[ \vec{u}_r \times \left( \vec{H}_{\text{eff}} - \frac{\vec{\nabla}_\Sigma \chi}{v \rho^F_0 L_F \alpha} \right) \right] \right\} \]  

(47)

This is the main result of this work.
The corresponding Fokker-Planck equation is obtained by inserting the expression of $\vec{J}_{tot}$ into the conservation equation: $d\rho_{tot}/dt = -\text{div}_\Sigma \vec{J}_{tot}^F$. The study of the resulting stochastic equation is however beyond the aim of this paper.

In order to give an expression of the two potential-energy terms \{\chi, \Phi\}, we will make the following assumption. According to previous discussions \[26, 32, 46\] based on the separation of the typical relaxation time scales involved during the ferromagnetic processes, the usual spin-accumulation due to spin-dependent relaxation $\Delta \mu^e$ is coupled to the relaxation of the magnetization $j_{tot}^{\theta F}$ only (because $l_{\text{diff}}$ defines a mesoscopic variable that scales with the magnetization). On the other hand, we assume that the transverse spin accumulation is coupled to the precession only (i.e. acting at subnanosecond time-scale).

In this case and after integrating over the volume $v$ the ferromagnetic current writes:

\[
\begin{align*}
  v J^{F\theta}_{tot} &= v J^{F\theta}_0 + \frac{1}{\sin \theta} \frac{\partial \Phi}{\partial \phi} \\
  v J^{F\phi}_{tot} &= v J^{F\phi}_0 + \frac{\partial \Phi}{\partial \theta}
\end{align*}
\]

(48)

On one hand the potential $\chi$ is directly associated to the voltage drop due to spin-dependent relaxation (Eq. 36):

\[
\frac{\partial \chi}{\partial \theta} = -l_{\theta \theta} e \int_A^B \frac{\partial \Delta \mu^e}{\partial z} dz = -\frac{J^e_0 l_{\theta \theta}}{2\beta} R_{\text{GMR}}
\]

(49)

and this expression can be generalized to specific device configurations. The potential energy $\chi$, function of the magnetic coordinates, can be measured in the context of two-level-fluctuation experiments performed on individual magnetic nanostructures \[32, 46\].

On the other hand, the potential $\Phi$ is associated to the discontinuity of the transverse spin accumulation and is responsible for the spin-transfer torque:

\[
\begin{align*}
  \frac{\partial \Phi}{\partial \phi} &= \frac{l_{\theta \phi} \sin \theta}{e} \int_A^B \frac{\partial \Delta \mu^e}{\partial z} dz \\
  \frac{\partial \Phi}{\partial \theta} &= -\frac{l_{\theta \phi}}{e} \int_A^B \frac{\partial \Delta \mu^e}{\partial z} dz
\end{align*}
\]

(50)

In these expressions, and in analogy with spin-accumulation due to the spin-dependent relaxation, the voltage drop $\int_A^B \frac{\partial \Delta \mu^e}{\partial z} dz = J^e_0 R_{\text{eff}}^{\text{trans}}$ is also able to define a non-equilibrium interface magnetoresistance $R_{\text{eff}}^{\text{trans}}$ for transverse spin-accumulation. Experimentally, the potential $\Phi$ is identified to the so called Slonczewski term used in the context of resonance experiments (FMR) performed in the GHz range.
V. CONCLUSION

In order to describe spin-transfer effect, spin-accumulation has been taken into account explicitly in the dynamical equation of the macroscopic ferromagnetic degrees of freedom. This dissipative coupling was described in terms of Onsager cross-coefficients $l_i$ appearing in the Onsager matrix in Eq. (42).

All the other terms appearing in the equation Eq. (42) (the other transport coefficients of the matrix, the generalized flux, and the conjugated generalized forces) have first been defined independently in the two preceding sections. In the case of the well-known dynamics of the ferromagnetic order parameter (supposed uniform), the approach proposed allows us to define the transport coefficients, the ferromagnetic current and the ferromagnetic generalized force from the expression of the entropy production and the conservation equations. The dynamics of the magnetization is summarized in the Onsager equation Eq. (22), which is the simplest form of the well-known Landau-Lifshitz equation. On the other, the spin-dependent electronic relaxations (spin-dependent $s-d$ relaxation or spin-flip relaxation) were treated on an equal footing in the context of the two channel model of electric conductivity. The resulting kinetic equations are also summarized by a Onsager equation Eq(37) with the relevant flux and forces, which are also a simple form of well known kinetic equations (e.g. that derived from the Valet-Fert model). The corresponding transport coefficients and forces ($\beta$, $\sigma_0$, $\delta J^e$, $\Delta \mu$) can be measured through the giant magnetoresistance and related effects. Due to quasi-ballistic precession of the spins of the conduction electrons, it is furthermore necessary to generalize the spin-accumulation effect to ”transverse spin-accumulation” (according to recent reports on spin-transfer torque).

Due to the two forms of spin-accumulation mechanisms, there are also two forms of coupling, namely spin-accumulation coupling (due to spin-dependent relaxations) and transverse spin-transfer-torque coupling due to quasi-ballistic spin precession. The model proposes a method able to formalize this coupling, and to deduce the consequences in terms of Landau-Lifshitz equation. In both cases, the coupling between the spin of the conduction electrons and the ferromagnetic parameters are introduced through the four phenomenological Onsager cross-coefficients $l_{ij}$. The generalization of the Landau-Lifshitz equation to these contributions is performed with two measurable potentials $\{\Psi, \chi\}$ (functions of the magnetic coordinates) defined in Eqs. (45) and (46). The potential $\chi$ is associated to
the spin-accumulation generated by the spin-dependent relaxation, and the potential $\Phi$ is associated to the conservation of the transverse moments (spin-transfer-torque). The two functions $\{\Psi, \chi\}$ are experimentally accessible.

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