The influence of anti-chiral edge states on Andreev reflection in graphene-superconductor junction

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Using the tight binding model and the non-equilibrium Green function method, we study Andreev reflection in graphene-superconductor junction, where graphene has two nonequal Dirac Cones split in energy and therefore time reversal symmetry is broken. Due to the anti-chiral edge states of the current graphene model, an incident electron travelling along the edges makes distinct contribution to Andreev reflections. In a two-terminal device, because Andreev retro-reflection is not allowed for just the anti-chiral edges, in this case the mutual scattering between edge and bulk states is necessary, which leads that the coefficient of Andreev retro-reflection is always symmetrical about the incident energy. In a four-terminal junction, however, the edges are parallel to the interface of superconductor and graphene, so at the interface an incident electron travelling along the edges can be retro-reflected as a hole into bulk modes, or specularly reflected as a hole into anti-chiral edge states again. It is noted that, the coefficient of specular Andreev reflection keeps symmetric as to the incident energy of electron which is consistent with the reported results before, however the coefficient of Andreev retro-reflection shows an unexpected asymmetrical behavior due to the presence of anti-chiral edge states. Our results present some new ideas to study the anti-chiral edge modes and Andreev reflection for a graphene model with the broken time reversal symmetry.

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I. INTRODUCTION

Andreev reflection\cite{1} is related to a particle tunneling process on the interface of conductor and superconductor, where an incident electron of conductor is reflected as a hole into conductor and simultaneously a Cooper pair is formed in superconductor. When the bias between conductor and superconductor is small enough, the transport property of the conductor-superconductor junction is mainly determined by Andreev reflection. Therefore, Andreev reflection is always a widely studying issue in the field of condensed matter physics. In the conventional Andreev reflection, the incident electron and the reflected hole come from the same band (conduction band or valence band). This is why the reflected hole holds the opposite transport direction of the incident electron. In general, the conventional Andreev reflection is called as Andreev retro-reflection or intraband reflextion.

After successfully producing the graphene in experiment\cite{2}, the physical characteristic of graphene\cite{3,4} is one of the research focuses in the condensed matter physics. In the graphene-superconductor junction, besides Andreev retro-reflection there exists another kind of Andreev reflection, which is called as specular Andreev reflection, due to the gapless energy band of graphene. Unlike Andreev retro-reflection, the reflected hole travels along the specular path of the incident electron in specular Andreev reflection. Especially, the specular reflected hole and the incident electron belong to the different bands, which makes that specular Andreev reflection is also named as interband Andreev reflection. For example, if an incident electron comes from the conduction band, the reflected hole must enter into the valence band in the specular Andreev reflection. Ever since, many research groups have studied the electron transport properties in the graphene-superconductor hybrid system\cite{5,6}.

For the band structure of a graphene nanoribbon with zigzag edges, valence and conduction bands touch each other at two points (K and K’), which are called as Dirac points. Due to the time-reversal symmetry, the incident electron and the reflected hole come from the different valleys in Andreev reflection. But in some conditions the hole can be reflected into the same valley as the incident electron, so there are many works\cite{7,8} to study manipulating the valley index for designing the electronic device. As we know, for the pristine graphene ribbon there are zero-energy flat bands connecting the two Dirac points, which are dispersionless. In previous work\cite{9,10,11}, the edge modes behave as not only chiral states with the quantum Hall effect, but also helical states with the quantum spin Hall effect. When a second-neighbor hopping term is included along a certain direction in Hamiltonian\cite{12}, which is called as the split hopping term in this work, the two Dirac points are split in energy along opposite directions [see Figs. 1 (a)]. Because of this, both of the edge modes acquire the same dispersion, namely the same movement velocity, which therefore means the breakdown of the time reversal symmetry.

In this paper, we study Andreev reflection in the graphene-superconductor junction, where the Dirac points of graphene are split in opposite directions. Different from the cases reported in previous works\cite{13,14,15} in our model both edge states of the ribbon acquire the same velocity and dispersion. As is known, a chiral edge
and the formulas for calculating the Andreev reflection coefficients are derived. Our main results are shown and discussed in Sec.III. Finally, a brief conclusion is presented in Sec.IV.

II. MODEL AND FORMULA

The graphene-superconductor junction investigated here is related to a graphene ribbon with the split Dirac cones [see Figs. 1(a)] and superconductor terminals connected to the graphene ribbon [see Figs. 1(b) and 1(c)]. The total Hamiltonian of this junction can be represented as

\[
H = H_G + H_S + H_T,
\]

where \(H_G\), \(H_S\) and \(H_T\) are the Hamiltonians of the graphene ribbon, superconductor terminals and the coupling between the graphene ribbon and superconductor terminals, respectively.

In the tight-binding representation, \(H_G\) is given by

\[
H_G = \sum_{m,n} E_0 [a_{m,n}^\dagger a_{m,n} + b_{m,n}^\dagger b_{m,n}] - t [a_{m,n}^\dagger b_{m,n} + a_{m,n} b_{m,n} - t_1 e^{i\phi} (a_{m-1,n}^\dagger b_{m,n} + b_{m,n}^\dagger a_{m-1,n}) + H.c.],
\]

where \(a_{m,n}^\dagger\) (\(a_{m,n}\)) and \(b_{m,n}^\dagger\) (\(b_{m,n}\)) are the creation (annihilation) operators of the sublattices A and B at site \((m,n)\). \(E_0\) is the onsite energy, which corresponds to the position of Dirac points for pristine graphene. The second term in Eq. (2) stands for the nearest-neighbor hopping Hamiltonian. The third term is the splitting between the graphene ribbon and superconductor terminals. Described by a continuum model the superconductor terminal is represented by BCS Hamiltonian,

\[
H_S = \sum_{k,\sigma} \varepsilon_k C_{k\sigma}^\dagger C_{k\sigma} + \sum_k (\Delta C_{k\uparrow}^\dagger C_{-k\uparrow} + \Delta^* C_{-k\downarrow}^\dagger C_{k\downarrow}^\dagger),
\]

where \(\Delta = \Delta e^{i\theta}\). Here \(\Delta_s\) is the superconductor gap and \(\theta\) is the superconducting phase. The coupling between superconductor terminal and graphene is described by

\[
H_T = -\sum_{m,n,\sigma} t_c (a_{m,n,\sigma}^\dagger b_{m,n,\sigma}^\dagger) C_\sigma(x_i) + H.c.
\]

Here \(x_i\) and \((m,n)\) represent the positions of the coupling atoms on the interface of superconductor and graphene, and \(C_\sigma(x) = \sum_k e^{i k_x x} C_{k,\sigma}\) is the annihilation operator in real space. Note that \(\sigma\) represents the spin index and \(t_c\) is the coupling strength between graphene and superconductor terminals.

We now turn to analyze that an incident electron from the graphene terminal is reflected into a
hole with a Cooper pair emerging in the superconductor terminal. Using nonequilibrium Green function method, we can calculate the retarded and advanced Green function $G_r(E) = (E - H_C - \Sigma^{\alpha}_{\alpha})^{-1}$, where $H_C$ is the Hamiltonian of the center region in the Nambu representation and $I$ is the unit matrix with the same dimension as $H_C$. The center region is the rectangular region surrounded by the dashed line in Fig. 1 (b) and (c). $\Sigma^\alpha_{\alpha} = t_c g_t^\alpha(E)t_c$ is the retarded self-energy due to the coupling to the terminal $\alpha$, where $g_t^\alpha(E)$ is the surface Green function of the terminal $\alpha$. We can numerically calculate the surface Green function of the graphene terminals. For superconductor terminals, the surface Green function\textsuperscript{17,41–43} in real space is

$$g_{\alpha,i}^\alpha(E) = i\pi \rho \beta(E) J_0[k_f(x_i - x_j)] \left( \frac{1}{\Delta^s/E} \frac{\Delta/E}{1} \right),$$

where $\alpha = 2, 4$ and $\rho$ is the density of normal electron states. $J_0[k_f(x_i - x_j)]$ is the Bessel function of the first kind with the Fermi wavevector $k_f$. $\beta(E) = -iE\sqrt{\Delta^2_x - E^2}$ for $|E| < \Delta_s$ and $\beta(E) = |E|\sqrt{E^2 - \Delta^2_s}$ for $|E| > \Delta_s$.

The Andreev reflection coefficients for the incident electron coming from the graphene terminal 1 can be obtained\textsuperscript{17,49,95}

$$T_{A,11}(E) = \text{Tr}\{\Gamma_{1,\uparrow\uparrow}G_{1,\uparrow\downarrow}\Gamma_{1,\downarrow\downarrow}G_{1,\uparrow\uparrow}^\dagger\},$$

$$T_{A,13}(E) = \text{Tr}\{\Gamma_{1,\uparrow\downarrow}G_{1,\uparrow\uparrow}\Gamma_{1,\downarrow\uparrow}G_{1,\uparrow\downarrow}^\dagger\},$$

where the subscripts $\uparrow\uparrow$, $\downarrow\downarrow$, $\uparrow\downarrow$, and $\downarrow\uparrow$ represent the 11, 22, 12 and 21 matrix elements, respectively, in the Nambu representation. The linewidth function $\Gamma_\alpha$ is defined with the aid of self-energy as $\Gamma_\alpha = i[\Sigma^\alpha_{\alpha} - (\Sigma^\alpha_{\alpha})^\dagger]$. $T_{A,11}$ and $T_{A,13}$ represent the coefficients of Andreev retro-reflection and specular Andreev reflection, respectively. Because there is only one graphene terminal for the two-terminal junction in this paper, the coefficient of Andreev reflection $T_{A,11}$ is written as $T_A$ for simplicity.

### III. RESULTS AND DISCUSSION

In numerical calculations, we set the nearest-neighbour hopping energy $t = 2.75eV$. The length of the nearest-neighbor C-C bond is set to be $a_0 = 0.142$ nm as in a real graphene sample. The superconductor gap is set to be $\Delta_s = 0.02t$, the superconductor phase $\theta = 0$ and the Fermi wavevector $k_f = 10$ nm$^{-1}$. For the convenience of discussing the influence of the split hopping term, the Fermi energy is set to be zero and the hopping phase $\phi$ in the split hopping term takes the value $\pi/2$.

Fig. 2 shows the Andreev reflection coefficient $T_A$ as a function of the incident energy $E$ for the different split hopping strength $t_1$ in the two-terminal graphene-superconductor junction [see Fig. 1 (b)], where $E_0 = 0$. In the two-terminal junction, the reflected hole can only travel back into the left graphene terminal, no matter which kind of Andreev reflection occurs. Due to the upright incidence of electron to the interface, the momentum component parallel to the interface could be supposed to be zero. This supposition always keeps true, no matter the incident electron travels along the edges or within the bulk. So when Andreev reflections take place in the two-terminal junction, the momentum of the incident electron can only be changed in the direction perpendicular to the interface, which exactly corresponds to the case of Andreev retro-reflection. Namely, specular Andreev reflection is largely restrained in the two-terminal junction.

When $t_1 = 0$, the sample is exactly the pristine graphene ribbon with zigzag edges. Undoubtedly, from the perspective of band structure, only the interband reflection is permitted at $E_0 = 0$, which corresponds to the case of the specular Andreev reflection. In Fig. 2 it can be therefore obtained that $T_A$ is zero for $t_1 = 0$. In a word, both Andreev retro-reflection and specular Andreev reflection should be zero in the two-terminal pristine graphene-superconductor junction\textsuperscript{17,49}, due to the combined restrictions of the two-terminal junction and band structure.

The case above is upset immediately for a nonzero split hopping term $t_1$, where two Dirac cones are split in energy and thus time reversal symmetry is broken now. As shown in Fig. 2, $T_A$ shows nonzero values for nonzero $t_1$. When $t_1$ takes a small value $0.2\Delta_s$, the coefficient of $T_A$ still keeps zero for $|E| > \delta$, where $\delta = \sqrt{3t_1}$ represents the energy difference between the present Dirac cones and the pristine ones. It is easy to understand from the fact, specular Andreev reflection is forbidden in the two-terminal junction and thus the interband reflec-
tion for $|E| > \delta$ should be zero. Consequently, it can be concluded that the nonzero coefficient of $T_A$ for $|E| < \delta$ should be ascribed to the intraband reflection, namely Andreev retro-reflection.

When $t_1$ increases to $0.4\Delta_s$, the nonzero part of $T_A$ is mainly confined to the range of $|E| < \delta$, as discussed above. In addition, two symmetrical peaks appear at the boundaries of $T_A$. It is easy to verify that bulk states account for these peaks, which will be discussed in detail below. As $t_1$ rises further, the velocity of the two anti-chiral edge states becomes larger, which induces the reduction of its contribution to the density of states at a fix energy. Thus, the altitude of $T_A$ becomes smaller as $t_1$ continues changing from $0.6\Delta_s$ to $1.0\Delta_s$. When the split energy $\delta$ of Dirac cones is larger than the superconductor gap $\Delta_s$, the nonzero range of $T_A$ is mainly regulated by the superconductor gap. To sum up, for the two-terminal hybrid junction above there is mainly the intraband Andreev reflection, namely Andreev retro-reflection, and the anti-chiral edge states play a key role on Andreev reflection.

As is known, there are two kinds of Andreev reflection at interface between pristine graphene and superconductor, Andreev retro-reflection and specular Andreev reflection. Due to time-reversal symmetry, the incident electron and the reflected hole must come from different valleys in general. When the split hopping term is considered in our model, the low-energy effective Hamiltonian can be written as $H_A = \delta \tau_z \otimes \sigma_0 + k_x \tau_1 \otimes \sigma_x + k_y \tau_0 \otimes \sigma_y$ [10], where the Pauli matrices $\tau_k$ and $\sigma_k$ represent the valley and sublattice indices, respectively. It is easy to verify that due to the nonzero split hopping term the time-reversal symmetry is broken, $T H(k) T^{-1} \neq H(-k)$, where $T = \tau_x \otimes \sigma_0 \mathcal{C}$ is the time reversal operator [11] with the complex operator $\mathcal{C}$. Without the constraint of time reversal symmetry, an incident electron coming from the valley $K$ could be reflected as a hole into not only the other valley $K'$, but also the same valley $K$ [see Figs. 1(a)].

Besides, since the Dirac points of the adopted Hamiltonian are split in energy along the opposite directions, the two edge states of zigzag ribbon acquire the same dispersion and become anti-chiral. An electron traveling along the edge can be also reflected as a hole at the interface between graphene and superconductor. From Fig. 2, we can see that when the split hopping term is nonzero, the Andreev coefficient is nonzero for $|E| < \delta$ in the two-terminal junction. A question naturally arises as to what the role the anti-chiral edge states plays in Andreev reflection.

For the sake of simplification, in the rest of the paper the value of the split hopping strength is fixed to be $t_1 = 0.6\Delta_s$, which results in the split of two Dirac points, $\delta = \sqrt{3}t_1 \approx 1.0\Delta_s$. In order to get a rudimentary understanding on the system, we first plot the band structures for different values of $E_0$ in Fig. 3. As shown in Fig. 3(a) with $E_0 = 0$, the two Dirac points are split along the opposite direction, distributing symmetrically on the two sides of the Fermi energy $E_F = 0$. As increasing $E_0$ gradually, the band structure moves down in whole and the inversion symmetry about the Fermi energy is also ruined, as seen in Fig. 3(b) and (c). It can be seen that a new bulk state at the $K$ point intersects with the Fermi energy for $E_0 = 0.6\delta$ in Fig. 3(c). When $E_0$ continues increasing up to $E_0 = \delta$, the $K$ point here goes back again to the Fermi energy exactly. No new bulk state appears around the Fermi energy in this case. Obviously, the onsite energy $E_0$ can be used to tune the density of states near the Fermi energy.
Next we calculate Andreev reflection coefficients with the incident energy $E$ for different values of $E_0$ in the two-terminal junction. The curves are shown in Fig. 4. When $E_0 = 0.2\delta$, two maximum values of $T_A$ up to 0.9 can be observed within $|E| < 0.8\Delta_s$. In this case, $T_A$ decays fast to zero for $|E| > 0.8\Delta_s$. As $E_0$ increasing to 0.4$\delta$, the two symmetrical peaks of $T_A$ continue rising to 1.2, and $T_A$ also reduces to almost zero for $|E| > 0.6\Delta_s$. However, only one peak preserves at $E_0 = 0.6\delta$. In addition, $T_A$ with nonzero value is confined to the region of $|E| < 0.4\Delta_s$. As $E_0$ takes the value 0.8$\delta$, instead of increasing $T_A$ presents an obvious drop in strength. Also, it becomes almost zero for $|E| > 0.2\Delta_s$. Unexpectedly, $T_A$ drops near to zero for $E_0 = 1.0\delta$, even though the density of states is nonzero at Fermi energy. The similar situation can be observed at $E_0 = 1.25$.

Based on the detailed presentation and discussion above, here the main conclusions include four aspects: (1) In the case of $E_0 < \delta$, $T_A$ takes nonzero values within the region of $|E| < (\delta - E_0)$ and decays fast to zero for $|E| > (\delta - E_0)$. (2) The two symmetrical peaks of $T_A$ for small $E_0$ should be ascribed to the appearance of a few bulk states. (3) The anti-chiral edge modes due to the breaking of time reversal symmetry dominate the Andreev reflections near the energy of Dirac points, which leads to nonzero values of $T_A$ for small $E_0$ but almost zero value for $E_0 \geq \delta$. It is worth noting that the broken time reversal symmetry results in the anti-chiral edge modes with the same dispersion, the characteristics of $K$ and $K'$ Dirac points are mixed together for the anti-chiral edge modes, and in particular $T_A$ becomes almost zero when the anti-chiral edge modes are shift away from the Fermi energy. These observations exactly illustrate that the broken time reversal symmetry accounts for the nonzero coefficient of Andreev reflection $T_A$. (4) What should be particularly noted that $T_A$ quickly becomes almost zero for $E_0 \geq \delta$, even there are a few edge and bulk states yet. In this sense, the role of the residual edge and bulk states in Andreev reflection is fundamentally different to that of the anti-chiral edge states due to the broken time reversal symmetry.

Andreev reflection in the two-terminal junction with the anti-chiral edge states is studied above. It is shown that the incident electron travelling along the edges can be retro-reflected as a hole at the interface of the two-terminal junction. Due to only one graphene terminal connected with the superconductor terminal in the two-terminal junction, the reflected hole can only flow back into the same terminal as that of the incident electron. It is well clear that the anti-chiral edge states due the broken time reversal symmetry play a key role in Andreev reflections. However, it is still difficult to present a clear physical illustration about the distinct role of anti-chiral edge states.

In order to find out the answer, we choose a four-terminal junction, and study how the two kinds of Andreev reflections are influenced by the anti-chiral edge states. As shown in Fig. 1 (c), the terminals 1 and 3 are chosen to be graphene zigzag ribbon, and the terminals 2 and 4 are superconducting leads. Due to the anti-chiral edge states, the incident electron of the edge modes travels parallelly to the interface of the graphene ribbon and the superconductor terminals. Therefore, when an incident electron comes from the terminal 1, the retro-reflected and specular reflected hole should flow into the terminal 1 and 3, respectively.

Setting the split hopping $t_1 = 0.6\Delta_s$, we calculate the Andreev reflection coefficients in the four-terminal junction shown in Fig. 5 where the Andreev reflection coefficients change with the incident energy $E$. It can be seen that for the different $E_0$, both of $T_{A,11}$ and $T_{A,13}$ are quit large when $|E| < \Delta_s$ and show peaks at the gap edge $|E| = \Delta_s$, which is in good accord with the theory. On the whole, the value of $T_{A,13}$ is larger than that of $T_{A,11}$ for $E_0 = 0$, but it becomes smaller than $T_{A,11}$ with increasing $E_0$. Taking the case of $E_0 = 1.0\delta$ as an example, we can observe that the maximum of $T_{A,11}$ reaches up to 1.35 whereas $T_{A,13}$ only takes 0.35 for the same energy. In this sense, Andreev retro-reflection dominates when the onsite energy $E_0$ is shift up or down.

In Fig. 6 (a), the curves of $T_{A,11}$ and $T_{A,13}$ are symmetric about $E = 0$. In the other three figures, $T_{A,13}$ always keeps symmetric about $E = 0$, but $T_{A,11}$ is not symmetric about $E = 0$. As far as we know, the coefficients of Andreev reflection are generally symmetrical to the incident energy $E$, and the asymmetry of $T_{A,11}$ or $T_{A,13}$ is never reported before. Obviously, this asymmetry of $T_{A,11}$ should be ascribed to the broken time reversal symmetry or anti-chiral edge states. However, there are a few of issues worth clarification and discussion: one is how the anti-chiral edge states, which breaks time reversal symmetry, result in the asymmetry of $T_{A,11}$, and show peaks at the gap edge $|E| = \Delta_s$, which is in good accord with the theory. On the whole, the value of $T_{A,13}$ is larger than that of $T_{A,11}$ for $E_0 = 0$, but it becomes smaller than $T_{A,11}$ with increasing $E_0$. Taking the case of $E_0 = 1.0\delta$ as an example, we can observe that the maximum of $T_{A,11}$ reaches up to 1.35 whereas $T_{A,13}$ only takes 0.35 for the same energy. In this sense, Andreev retro-reflection dominates when the onsite energy $E_0$ is shift up or down.
the other is why $T_{A,13}$ always keeps symmetric and $T_{A,11}$ does not show asymmetrical characteristic for the two-terminal junction as shown in Figs. 2 and 4.

In order to uncover the underlying physical reason, we plot the band structures for both electron (black) and hole (red) at (a) $E_0 = 0$ and (b) $E_0 = 0.25$. (c) and (d) represent the distribution of the electron states along the sample cross section. (e) The schematic diagram of anti-chiral states in two-terminal and four-terminal junctions.

FIG. 6: (Color online) The band structures for both electron (black) and hole (red) at (a) $E_0 = 0$ and (b) $E_0 = 0.25$. (c) and (d) represent the distribution of the electron states along the sample cross section. (e) The schematic diagram of anti-chiral states in two-terminal and four-terminal junctions.

Therefore, we observe symmetrical features of both $T_{A,11}$ and $T_{A,13}$ in Fig. 5(a).

The symmetry is destroyed when a nonzero value of $E_0$ is taken. Noted that no matter the electron band (black) or the hole band (red) is no longer symmetrical to the incident energy $E = 0$ because of a nonzero value of $E_0$, as shown in Fig. 6(b). Due to electron-hole symmetry, an electron band (black) at the energy $E$ keeps the same to that of the corresponding hole state at the energy $-E$. For sake of concreteness, we restrict to the case of $E_0 = 0.25$. In Fig. 6(b), there are also four electron states for a small energy ($E = |\eta|$) above the Fermi energy, represented by $O$, $P$, $Q$, $M$, respectively. If an incident electron from the state $O$ is Andreev reflected as a hole of the state $O'$ or $P'$, it will contribute to the coefficient of $T_{A,13}$. While this incident electron is Andreev reflected as a hole of the state $Q'$ or $M'$, it will contribute to the coefficient of $T_{A,11}$. It is because both the initial electron state and the final hole state must move along the same direction for the cross Andreev reflection $T_{A,13}$, and the reflected hole must move oppositely as to the incident electron for the Andreev retro-reflection $T_{A,11}$.

Although in the cross Andreev reflection the initial electron state and the final hole state at the small positive energy $E = |\eta|$ are different from those at $E = -|\eta|$, they keeps always conjugated with each other, namely the initial electron state and the final hole state at $E = |\eta|$ equivalent to the final hole state and the initial electron state $E = -|\eta|$. It is not difficult to obtain that $T_{A,13}$ will maintain the symmetry about the Fermi energy even for a nonzero $E_0$. But the symmetrical feature is destroyed in the Andreev retro-reflection $T_{A,11}$. To sum up, the asymmetrical characteristic of $T_{A,11}$ in Fig. 5 (b-d) should be ascribed to the coexistence of anti-chiral states and asymmetrical band structure.

All curves in Fig. 6 become understandable by using the above-mentioned theory. In Fig. 5(a), $T_{A,13}$ shows a peak at small energy and decreases gradually with increasing the incident energy. However, $T_{A,11}$ keeps growing as the incident energy increases away from zero. It is because, the momentum separation between edge states and bulk states becomes more and more small with increasing the incident energy, which is conducive to $T_{A,11}$ but obstructive to $T_{A,13}$. For a nonzero value of $E_0$, the initial electron state is separated from the final hole state in momentum, as shown in Fig. 6(b), so $T_{A,13}$ becomes smaller and $T_{A,11}$ takes a nonzero value at $E = 0$. As $E_0$ takes a larger value, which leads to a large momentum separation between electron and hole anti-chiral edge states but a small one between electron edge states and hole bulk states, $T_{A,11}$ becomes dominated and $T_{A,13}$ is suppressed largely, as shown in Fig. 6(d). In a word, the variations of $T_{A,11}$ and $T_{A,13}$ in Fig. 6 are mainly influenced by the magnitude of the transferred momentum in Andreev reflections.

There is still a serious question, why $T_A$ (namely $T_{A,11}$) keeps symmetrical in a two-terminal junction, distinctly different from the case of four-terminal junction. We
can get hints from the schematic diagram of anti-chiral states in Fig. 6 (c) and the band structure in Fig. 6 (b). Obviously, for a small value of \( E_0 \), there are two edge states with positive velocity and two bulk states with negative velocity, and the movement directions of corresponding hole states are the same to those of electron states. Therefore, the mutual scattering between edge and bulk states is inevitable when incident electrons from the terminal 1 are Andreev reflected as holes of the terminal 1. Although the system has anti-chiral states and asymmetrical band structure with broken time reversal symmetry, the abundant and sufficient scattering between anti-chiral edge and bulk states have balanced their influences on the Andreev retro-reflection coefficient \( T_{A,11} \). So \( T_{A,11} \) manifests a symmetrical feature about the incident energy.

At last, Fig. 7 shows that the coefficients of Andreev reflection in the four-terminal junction as the function of the length \( L \). The incident energy is set to be small enough in order to be close to the Fermi energy. With \( L \) increasing, there are multiple Andreev reflections occurring between two interfaces. Therefore, both \( T_{A,11} \) and \( T_{A,13} \) oscillate intensely with \( L \) increasing no matter what the value \( E_0 \) takes. Comparing the curves of \( T_{A,11} \) and \( T_{A,13} \) in Fig. 7 (a), it can be seen that the amplitude of \( T_{A,13} \) is bigger than that of \( T_{A,11} \). Especially, there is one distinct difference between two curves in Fig. 7 (a). When \( L \) is small enough, \( T_{A,13} \) can reach up to 1.8, which is substantially larger than \( T_{A,11} \) in Fig. 7 (a). However, both \( T_{A,11} \) and \( T_{A,13} \) are close to zero for small \( L \) in Fig. 7 (b). It is easy to understand by analyzing the band structures at \( E_0 = 0 \) and \( E_0 = \delta \). In the case of \( E_0 = 0 \), there are anti-chiral edge states at \( E = 0.01\Delta_s \), so incident electron from the terminal 1 is apt to be specularly reflected to terminal 3. Whereas normal bulk states are predominant at \( E = 0.01\Delta_s \) in the case of \( E_0 = \delta \), so incident electron is probable to tunnel directly into terminal 3, which leads to very small values for both \( T_{A,13} \) and \( T_{A,11} \). As increasing the length \( L \) over 80, in Fig. 7 (b) \( T_{A,11} \) and \( T_{A,13} \) increase gradually due to the growing contribution of bulk states to Andreev reflection.

**IV. CONCLUSIONS**

We study the Andreev reflection in the zigzag graphene ribbon with the split Dirac cones. Due to the anti-chiral edge modes with the same velocity and dispersion, the time reversal symmetry in pristine graphene model is broken up. Different from the pristine graphene model, the incident electrons of the anti-chiral edge states can make an obvious contribution on Andreev reflection.

In a two-terminal graphene-superconductor junction, the Andreev reflection coefficient \( T_A \) takes nonzero value within the range of \( |E| < (\delta - E_0) \), where \( \delta = \sqrt{3}\Delta_t \) represents the energy difference between the present Dirac cones and the pristine ones. Especially, \( T_A \) maintains a symmetrical feature about the incident energy, as reported in previous papers. It is worth noting that the strength of \( T_A \) can be tuned by changing the onsite energy \( E_0 \).

Different from the case of the two-terminal junction, in a four-terminal junction the coefficient of Andreev reflection \( T_{A,11} \) shows a symmetrical feature about the incident energy but \( T_{A,11} \) manifests asymmetrical characteristic, which is never reported before. Through analysis, this distinct characteristic should be ascribed to the coexistence of anti-chiral states and asymmetrical band structure. Noted that, there should be an abundant and sufficient scattering between anti-chiral edge and bulk states for Andreev retro-reflections of a two-terminal junction, which helps \( T_{A,11} \) to get rid of the influence of broken time reversal symmetry and preserve the symmetrical characteristic. This is why the Andreev reflection coefficient \( T_{A,11} \) in the two-terminal junction keeps always symmetrical to the incident energy.

The results in this paper are very important to understand intervalley reflection, intravalley reflection, interband reflection, and intraband reflection and helpful to exploit the graphene-superconductor junction. In addition, this paper presents a clear physical picture about the behaviours of anti-chiral states in Andreev reflection. It could be important to find new materials and functional quantum devices.

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