The research of a movement dynamics two-legged robot with spring-loaded weight

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Abstract. The movement of a two-legged robot which moves due to oscillations of a load with a given mass fixed on the robot body with the help of the springs, is considered in this research. By changing parameters, such as mass of the load, stiffness of the springs, size of the robot's body and the initial speed of movement, the various trajectories of movement are executed. In this article mathematical model and calculation results have been presented.

Introduction
Mobile robots have vital differences associated not only with the use of a concrete construction of mover and a schematic structure, but also with the correlation of numbers of propulsion devices and degrees of freedom of a mobile robot as a mechanical system. [1, 2, 3].

For robotic transport platforms, a structural diagram and movers are widely used to ensure static stability. In this case, the number of propulsion devices is equal to or exceeds the number of degrees of freedom of the robot (for example, «Vosminog», «Ortonog») [4, 5].

When moving in aggressive environments, the construction of the robot may undergo changes, because the use of standard types of mover is difficult. The movable mass serves as a mover, being inside the robot and changing its position relative to the body. Such robots are applicable, for example, for movement in pipes [6].

The research of a two-legged robot movement which moves due to a movable mass fixed on springs is considered. The movement is carried out as follows: one of the supports is in continuous interaction with the surface, while the other is in the transfer phase, allowing the rod to rotate around the fixed support, then they change, and the robot changes the direction of rotation, thereby moving. This principle is similar to the movement of the "Celtic stone": if you put it on a horizontal plane and twist it around a vertical axis, then after a certain period of time it will stop rotating, vibrations will occur around other axes, after which the stone will begin to rotate around the vertical axis, but in the opposite direction [7].

Computational model
The movement of a two-legged robot is considered.

Design schedule (figure 1) presents a mechanical system consisting of two weightless supports A and B (1), connected by a weightless rod 2, moving under the influence of vibrations of a load 3 of
mass \( m \), fixed on springs 4 by stiffness \( c/2 \). The x-axis in the design diagram is aligned with the weightless bar \( AB \), with the origin at point \( C \). At the initial moment of time, the speed is set \( V_0 \).

It is assumed that when the two-legged robot under consideration moves, one of the supports at point \( A \) or \( B \) does not move relative to the surface \( Oyz \), allowing the rod \( AB \) to rotate around this support. The condition for immobility of one of the supports is the position \( M \) of the load 3 relative to the middle of the rod \( AB \) (figure 2 (a), (b)). Point \( C \) is in the middle of the road \( AB \) and is the origin of the \( Cx \) coordinate system. When changing supports, the calculation schemes of the robot's movement change (figure 2 (a), (b)):

\[
\begin{align*}
&x \leq 0 \\
&\begin{cases} 
  y_a = \text{const}, \\
  z_a = \text{const}; 
\end{cases} \\
&x > 0 \\
&\begin{cases} 
  y'_a = \text{const}, \\
  z'_a = \text{const}. 
\end{cases}
\]

The effect of frictional forces between the surface and the supports is not taken into account. Lagrange equations of the second kind are used as a solution method:
\[
\begin{aligned}
&\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}} \right) - \frac{\partial T}{\partial x} = -\frac{\partial P}{\partial x}, \\
&\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\varphi}} \right) - \frac{\partial T}{\partial \varphi} = -\frac{\partial P}{\partial \varphi}.
\end{aligned}
\] (1)

Two cases are considered: the first – support \( A \) is motionless, \( B \) – in the transfer phase (figure 2 (a)), the second - support \( B \) is motionless, \( A \) - in the transfer phase (figure 2 (b)). In the first case, expressions for kinetic and potential energies are:

\[
\begin{aligned}
T &= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{\varphi}^2 (a + x)^2, \\
P &= \frac{1}{2} c x^2;
\end{aligned}
\] (2)

where \( x \) – the movement of the load relative to the center \( O \), \( \varphi \) is the angle of transfer, and \( a \) is half the length of the robot. Then from (1, 2) the equations follow:

\[
\begin{aligned}
\dot{m} \dot{x} - m \dot{\varphi}^2 (a + x) &= -c x, \\
\ddot{m}(a + x)^2 + 2m \ddot{\varphi}(a + x) &= 0.
\end{aligned}
\] (3)

In the second case:

\[
\begin{aligned}
T &= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{\varphi}^2 (a - x)^2, \\
P &= \frac{1}{2} c x^2;
\end{aligned}
\] (4)

And instead of (4), the following equations take place:

\[
\begin{aligned}
\dot{m} \dot{x} + m \dot{\varphi}^2 (a - x) &= -c x, \\
\ddot{m}(a - x)^2 - 2m \ddot{\varphi}(a - x) &= 0.
\end{aligned}
\] (5)

The coordinates of the point \( C \) of the support \( AB \) of the robot (figure 3) are determined by the dependencies:

\[
\begin{aligned}
y_c &= y_a + a \cos \varphi = y_b - a \cos \varphi, \\
z_c &= z_a + a \sin \varphi = z_b - a \sin \varphi.
\end{aligned}
\] (6)

When calculating the movement, the following parameters of the mechanical system are set: \( a, m, c, \dot{\varphi}_0 \); initial conditions: \( x_0, \varphi_0, \dot{x}_0 \), as well as the coordinates of the support points \( A \) and \( B \) at the initial moment of time.

Then, at the first stage of calculating the robot's movement, system (6) takes the form:

\[
\begin{aligned}
y_c &= y_{a_0} + a \cos \varphi, \\
z_c &= z_{a_0} + a \sin \varphi;
\end{aligned}
\] (7)
At $x = 0$, at the end of the first stage, the values of the angular velocity $\dot{\phi}$ and angle $\phi_k$, as well as the coordinates of point $B$, are determined:

$$\begin{align*}
y_{B_1} &= y_{A_1} + 2a \cos \phi_k, \\
z_{B_1} &= z_{A_1} + 2a \sin \phi_k.
\end{align*}$$

(8)

At the second stage:

$$\begin{align*}
y_c &= y_{B_1} - a \cos \phi, \\
z_c &= z_{B_1} - a \sin \phi,
\end{align*}$$

(9)

then:

$$\begin{align*}
y_{A_2} &= y_{B_1} - 2a \cos \phi_k, \\
z_{A_2} &= z_{B_1} - 2a \sin \phi_k.
\end{align*}$$

(10)

For subsequent stages, calculations are carried out in a similar way.

**Results of solving the model problem**

Initial conditions: $x_0 = -0.3 \ m$; $\phi_0 = 0 \ rad$; $\dot{x}_0 = 0 \ m \cdot s^{-1}$; $\dot{\phi}_0 = 5 \ rad \cdot s^{-1}$. The parameters of the considered mechanical system: $m = 5 \ kg$, $c = 10 \ N \cdot m^{-1}$, $a = 0.5 \ m$. 

**Figure 3.** Determining the coordinates of the robot center and supports.
Figure 4. Dependence of the movement of the load $M$ relative to the middle of the rod $C$ on time: I – rotation around point $A$ (figure 2 (a)); II – rotation around point $B$ (figure 2 (b)).

Figure 5. Dependence of the change in the angle of rotation of the robot on time: I – rotation around point $A$ (figure 2 (a)); II – rotation around point $B$ (figure 2 (b)).
Figure 6. The movement trajectory of the robot center (point C) if: 1) \(x_0 = -0.3 \text{ m}; \ \phi_0 = 0 \text{ rad}; \ \dot{x}_0 = 0 \text{ m} \cdot \text{s}^{-1}; \ \dot{\phi}_0 = 5 \text{ rad} \cdot \text{s}^{-1}, m = 5 \text{ kg}, c = 10 \text{ N} \cdot \text{m}^{-1}, a = 0.5 \text{ m.} \ 2) x_0 = -0.3 \text{ m}; \ \phi_0 = 0 \text{ rad}; \ \dot{x}_0 = 0 \text{ m} \cdot \text{s}^{-1}; \ \dot{\phi}_0 = 5 \text{ rad} \cdot \text{s}^{-1}, m = 5 \text{ kg}, c = 100 \text{ N} \cdot \text{m}^{-1}, a = 0.5 \text{ m.} \ 3) x_0 = -0.3 \text{ m}; \ \phi_0 = 0 \text{ rad}; \ \dot{x}_0 = 2 \text{ m} \cdot \text{s}^{-1}; \ \dot{\phi}_0 = 5 \text{ rad} \cdot \text{s}^{-1}, m = 5 \text{ kg}, c = 10 \text{ N} \cdot \text{m}^{-1}, a = 0.5 \text{ m.} \ 4) x_0 = -0.4 \text{ m}; \ \phi_0 = 0 \text{ rad}; \ \dot{x}_0 = -0.5 \text{ m} \cdot \text{s}^{-1}; \ \dot{\phi}_0 = 5 \text{ rad} \cdot \text{s}^{-1}, m = 25 \text{ kg}, c = 10 \text{ N} \cdot \text{m}^{-1}, a = 0.5 \text{ m.}

Conclusion
Due to the movement of a load with mass \(m\) and giving an initial velocity \(V_0\) to the center of the body, occurs the robot’s motion, the pattern of which depends on the specified parameters and initial conditions. The movement is undulating. The trajectory and direction depends on the initial conditions.

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