Kinematic and compliance analysis of active suspension system and Development of control algorithm to maximize ride comfort

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Abstract. It is important to understand the interrelationship between steering stability and NVH to adjust stiffness of the suspension to satisfy the aim of maximum ride comfort. This work aims to perform kinematic and compliance analysis on comprehensive quarter car model of McPherson suspension system to validate its reliability with respect to various road profiles and development of control algorithm to control its stiffness. Despite the wide usage of PID controllers, controllers based on Adaptive Neuro Fuzzy Inference System (ANFIS) are proved to give better results as they consider the non-linearity of active car suspension models. The modelling of the comprehensive non-linear quarter car active McPherson suspension system is done using MSC ADAMS and the control algorithm is implemented using MATLAB. The approach used is to perform parametric sensitivity analysis for prime parameters such as toe angle, camber angle and caster angle. The approach used to design a better robust control algorithm is achieved using µ-synthesis function of MATLAB.

1. Introduction

Developing a reliable vehicle suspension system often involve compromises in various incongruous parameters. Vehicle handling and ride comfort are two such incongruous measures which need certain compromises among them. Further designing the car only for its ride comfort demands compromise in the space requirements those including the boot space and leg room space requirements [1]. Conventional suspension models have been used in the evaluation of active and semi-active suspension systems [2]. The wide use of McPherson Suspension system due to its features such as light weight and low-cost characteristics makes it the perfect candidate to carry out to be used for vehicle dynamic and kinematic analysis [3].

In general, using the conventional suspension models involve linear models, which often neglect the non-linearity introduced due to incorrect consideration of tyre to be rigid element or often only the vertical movement of tyre is considered. This only compensates for the spring stiffness of the tyre. Vertical damping [4] and lateral deflection [5] both should be taken into account to make the suspension system truly non-linear. To perform a similar kind of Kinematic and Dynamic analysis on a double wishbone suspension quarter car model system a comprehensive model is created by incorporating...
wheel hop, vertical longitudinal damping and compliance of bushing at joints [5]. Deriving useful data from kinematic analysis of suspension system Stenson et al. proposed a dynamic model of McPherson suspension. Though this study included the non-linearity in some sense it ignored the dynamics due to vibration induced in chassis and those due to tyre [6]. Considering rotation of control arm and longitudinal displacement of unsprung mass as generalized co-ordinates, two different models have been worked out in the previous studies. Model by Hong et. al. consists of strut mounted on control arm [7], while model by Fallah et. al. consists of strut mounted on wheel spindle [8]. The study also included the disturbances caused due kingpin angles, caster and camber. Previous work also included working on the three-dimensional models to study the kinematics of McPherson suspension system, but none included the variations caused due to tyre hop, its longitudinal damping and lateral deviation caused during the actual events [9] [10]. Kim et al. performed a similar work on a three-dimensional model in ADAMS Software [11]. Anderson et al. based on the readings obtained from the experimental McPherson suspension setup in the laboratory concluded that both linear and non-linear models are comparable if correct parameters are identified. The very little difference in these two models is due to small variation in strut angle [12]. It is very important to create a reliable model of suspension system taking into account all its non-linearities to save the time and resources spend on an incorrect model and realizing at a further time when making the changes in the template prepared in vehicle dynamic simulation software like MSC ADAMS. It is very tedious work to make the desired changes in the template at later time.

This work proposes to develop a non-linear McPherson suspension system comprising of strut mounted firmly on wheel spindle while considering the effects due to tyre hop and lateral tyre displacement. Kinematics and Compliance Analysis tests are performed in MSC ADAMS to ensure that the virtual suspension system developed is in agreement with the real suspension system. Thus, the suspension system developed is more reliable and more realistic and is used for the second part of this work comprising of the development of control algorithm that can truly increase the ride comfort by actuating a correct signal that develops precise amount of counter force by force actuators to reduce the disturbances caused due to road unevenness and various road profiles. Figure 1 shows typical McPherson Suspension system.

Figure 1: Typical example of McPherson Suspension system

With the continuous development of technology, passenger safety and ride comfort has become a topic of prime importance for the automotive industries. To increase the ride comfort it is very crucial to optimize stiffness of spring \((k_t)\), various damping coefficient like tyre damping co-efficient and damping co-efficient related with sprung mass, unsprung mass spring stiffness of tyre. Sprung mass acceleration is inversely proportional to ride comfort. ISO 2631-1: 1997 defines ride comfort. Vibrations are caused due to various factors out of which vibrations due to road disturbances are predominant. Acceleration of sprung mass is the measure of Ride comfort.

Adverse effects of road excitations on the ride comfort has been studied by Griffin et. al. [12]. Added advantage of smooth transition of coefficient of damping of semi-active suspension makes it very effective in providing a better ride comfort when compared with passive suspension system [13]. Much research has been carried out on the semi-active suspension system that uses Magnetorheological...
damper. Although semi-active suspension system is the most commonly used suspension system, active suspension system turns to work out more effectively as it can perform better over wide range of frequencies [14]. Working space, ride comfort and handling have been determined as the objective function in the previous work involving 8 DOF model [15]. G. Verros et. al. presents the foundation to optimize parameters such as spring stiffness and damping co-efficient for non-linear suspensions models under random excitations [16].

Comparative study of Fuzzy Control system and Linear Quadratic Gaussian system has been carried out on Two Degree of Freedom suspension system [17]. It is seen that Adaptive-Network based Fuzzy Inference System (ANFIS) controllers performs better when optimising Active suspension system. The ANFIS controllers rely upon the data from FOPID control system to better compensate the non-linearity of suspension systems [18]. LQR control system has been suggested for Active suspension systems [19] [20]. This paper proposes a control algorithm with two approaches, H∞ approach and μ-synthesis approach implemented using MATLAB on the SIMULINK model prepared after verifying the results of Kinematics and Compliance Analysis.

The following paper is organised as follows. The development of two DOF non-linear model of McPherson System is developed and Kinematics and Compliance (Dynamic) Analysis is performed in section 2. In section 3 based on the SIMULINK model developed in section 2, control Algorithm is implemented. Section 4 comprises of conclusion of work.

2. Modelling of the suspension and Kinematic and Compliance Analysis:
The model is built with sheer consideration that the unsprung mass of the suspension system which basically consists of wheel assembly experiences vertical translational motion as well as rotational motion and the chassis (sprung mass) undergoes only vertical translational motion. Figure 2 shows a typical example of non-linear model of McPherson suspension system.

![Figure 2. McPherson suspension (a) Non-linear representation (b) kinematic representation](image)

The system mainly consists of 4 main elements: (1) Car Body (Chassis)- sprung mass, (2) Wheel Assembly – Unsprung mass, (3) Control Arm and (4) Strut. The displacement associated with the sprung mass i.e., chassis is denoted by $X_c$, the displacement related with the unsprung mass is denoted $X_u$ and the displacement due to road disturbances is denoted by $X_r$. The spring stiffness of the chassis is denoted by $K_c$ and the vertical spring stiffness of the wheel is denoted by $K_{wv}$ and the lateral spring stiffness of wheel (tyre) is denoted by $K_{wl}$, similarly the damping co-efficient associated with suspension is denoted as $B_s$ and the damping co-efficient of the wheel is denoted as $B_w$. Thus represents the non-linearity due to tire hop. Considering the lateral tyre deflection which causes changes in caster and camber angles, the Moment of inertia of the wheel assembly at centre C is denoted by $I_c$. The key features or assumptions of these models are (1) The control arm and the strut are of insignificant masses so they can be ignored. (2) The tyre behaves as rigidless element (3) Springs and Dampers behave linearly.
Consider \( Y_c \) and \( X_c \) to be the coordinates of the wheel centre \( C \) and \( Y_{c0} \) and \( X_{c0} \) as their initial values and \( \phi \) is the camber angle.

\[
\begin{align*}
Y_p - Y_c &= b\phi + a & (1) \\
X_F - a\phi &= X_{F0} + X_w & (2) \\
Y_L - Y_c &= d\phi + c & (3) \\
X_L + c\phi &= X_{L0} + X_w & (4) \\
Y_B - Y_c &= f\phi + e & (5) \\
X_P + e\phi &= X_{B0} + X_w & (6)
\end{align*}
\]

Here constants \( a, b, c, d, e \) and \( f \) are obtained from equilibrium conditions in initial conditions and their values are equal to: \( a = Y_{F0} - Y_{C0}, b = X_{F0} - X_{C0}, c = Y_{L0} - Y_{C0}, d = X_{L0} - X_{C0}, e = Y_{B0} - Y_{C0} \) and \( f = X_{B0} - X_{C0} \).

Equations (1)-(6) represent the kinematic behaviour of the unsprung mass and thus establish the relation between \( \phi \) and \( X_w \). Figure 3 shows motion of the system for the case of negative \( \phi \). Due to the motion of the system, there is lateral as well as vertical displacement of wheel. The resultant lateral displacement of the wheel is denoted \( \delta Y_{wl} \). From the geometry of the system shown in Figure 3, we get three more equations as:

\[
\begin{align*}
Y_{B0}Y_B - (2X_w - X_{B0})X_B &= p - X_{B0}X_w & (7) \\
Y_B &= -L\theta - q & (8) \\
X_P + q\theta &= L + X_w & (9)
\end{align*}
\]

where \( p = Y_{B0}^2 + X_{B0}^2; \) \( L = L_1\sin\theta_0; \) \( q = L_1\cos\theta_0. \)

Equations (1)-(8), represents linear system with nine variables. These equations can further be evaluated in terms of \( X_c \) and \( X_w \). We get series of expressions each of which is for different elements of suspension. Out of these expressions the three predominant key factors are \( Y_c, \theta \) and \( \phi \). These are required for Compliance analysis:

\[
\begin{align*}
Y_c &= -e - \frac{f(X_{B0} + X_w)}{e} + \sigma\left[ \left( L + \frac{f_q}{e}\right)(p + X_c) - X_{B0} + (q^2 + L^2 + LX_c) \left( \frac{Y_{B0}}{e} + 2X_c - X_{B0} \right) \right] & (10) \\
\phi &= \frac{X_{B0} + X_w}{e} - \frac{L}{G}Y_{B0}\frac{q^2}{e} + q\frac{(p - X_{B0}X_c)}{e} & (11) \\
\theta &= -\sigma\left[ p - X_{B0}X_c + Y_{B0}q - (L + X_c)(X_{B0} - 2X_c) \right] & (12)
\end{align*}
\]

where, \( \sigma = \frac{1}{LY_{B0} + qX_{B0} - 2qX_c} = \frac{1}{G} \) and \( G \) denotes denominator equivalent.

For further simplicity in solving the equations, following terms are established:

\[
\begin{align*}
d' &= p(L + \frac{f_q}{e})(q^2 + L^2)(\frac{Y_{B0}}{e} - X_{B0}) \\
e' &= X_{B0} \left( L + \frac{f_q}{e} \right) + L \left( \frac{Y_{B0}}{e} + X_{B0} \right) + 2(q^2 + L^2)
\end{align*}
\]
\[ c' = -\frac{f X_{B0}}{e} - e \]
\[ i' = -q X_{B0} - L Y_{B0} \]
\[ g' = \frac{X_{B0}}{e} \]
\[ h' = pq + (q^2 + L^2) Y_{B0} \]
\[ f' = 2L \]
\[ t' = d' + \frac{Rh'}{e} \]
\[ u' = e' + \frac{Rh'}{e} \]

Lateral deflection of tyre, \( \delta Y_{wl} \) is then calculated using Equations (10) and (11) as:
\[ \delta Y_{wl} = (Y_C - \phi R) - Y_{C0} = s' + \frac{f + R}{e} X_{wl} + \frac{1}{e} (t' + u' X_C + f' X_C^2) \tag{13} \]

For the dynamic part of analysis, if \( D \) denotes the dissipation energy, \( m_c \) and \( m_w \) represent the masses of chassis (sprung mass) and wheel assembly (unsprung mass) and \( \delta_{wv} \) represents the vertical displacement of spring and damper assembly of wheel, then value of \( \delta_{wv} \) is given by:
\[ \delta_{wv} = \sqrt{(L_{03})^2 + k \theta - L_{03}} \tag{14} \]
where, due to small angle:
\[ (L_{03})^2 + k \theta = (L_3)^2 \tag{15} \]
And constant \( k \) is determined by:
\[ k = -2L_1L_2 \sin (y_0) \]

The Lagrangian \( L \) to determine the dynamic response of the system is given by:
\[ L = 0.5 \times \left[ m_c X_C^2 + I_C \phi^2 + m_c (Y_C^2 + X_C^2) - K_w \delta_{wv}^2 - K_w (X_w - X_r)^2 - K_{wl} \delta_{wl}^2 \right] \tag{16} \]
and
\[ D = 0.5 [B_c \dot{\delta}^2 + B_w \dot{X}_C^2] \tag{17} \]
where,
\[ X_t = X_w + X_r \tag{18} \]

combining value of \( L \) with \( X_w \), we get:
\[ \frac{d}{dt} \left[ \frac{\partial L}{\partial X_w} \right] - \frac{\partial L}{\partial X_C} + \frac{\partial D}{\partial X_w} = 0 \tag{19} \]

Using values of equations (16) and (17) in equation (19), we get:
\[ m_w \ddot{X}_w + m_w \ddot{Y}_w + l_c \ddot{\phi} + B_w \dot{X}_w - \dot{X}_r + K_w \delta_{wv} \frac{\partial (\delta Y_{wl})}{\partial X_w} + K_w (X_w - X_r) \]
\[ \tag{20} \]

Equation (20) represents non-linear differential equation as a function of \( X_w \).

Figure 4: Non-linear McPherson suspension system developed in MSC ADAMS

With the help of above equations, a SIMULINK block model is derived for further use in MATLAB for developing control algorithm to optimize suspension. Figure 4 shows, equivalent quarter car model
of non-linear McPherson Suspension system into consideration developed using MSC ADAMS for Kinematics and Compliance Analysis.

The modelled is carefully taking into account certain design parameters as hard point geometry the suspension model is subjected under vertical parallel movement test and vertical oppose movement test, with prime objective of optimizing camber angle, caster angle and track width as these three parameters play major role in providing ride comfort in vehicles with active McPherson suspension system. The results of these test on MSC ADAMS model are compared with the results obtained from solving mathematical model and similar evaluation on experimental model in previous research work.

2.1 Results and Discussion of Kinematics and Dynamic Analysis

Previous research work shows that similar kind of experimental evaluation has been done on a real McPherson suspension system in lab environment. The experimental setup underwent similar tests i.e., vertical parallel movement test and vertical oppose movement test. To make a comparative evaluation between experimental and virtual suspension system, certain values like vertical displacement (bound and rebound) travel value and steps.

A total of 6 graphs were obtained from the test data set. Figure 5 and figure 6 shows the comparison between the three non-linear models – experimental model, virtual model created using MSC ADAMS and mathematical model. Figure 5 particularly shows the result of Kinematic & Compliance (dynamic) analysis (K & C analysis) when the models are subjected to vertical oppose movement test and similarly Figure 6 shows results obtained from vertical parallel movement test.

From Figure 6 (a) and 6 (b) we can see there is negative slope of in the graph which denotes decrease in value of toe change and caster change with increase in wheel travel with time. This is advantageous as it allows better handling and good stability during cornering [21] [22]. From figure 5 and figure 6, it is evident that the non-linear virtual McPherson suspension system created in MSC ADAMS is in great agreement with the results of real experimental suspension system. Therefore this virtual suspension
model can be worked with for developing control algorithm to control the input to force actuators that can counteract on the force disturbances that vehicle experience.

3. Development of Control Algorithm:
While in motion vehicle experiences vibration due to various force acting on it. Some of the forces include vibrations due to irregularities in road like bumps and potholes, vibrations due to aerodynamic forces acting on it. When a car hits a bump or a pothole, the tire absorbs some of the impact force and compresses a little bit and the force is then transfer to the car body causing it to bounce. The desirable function of the suspension system is to dampen these vibrations to provide ride comfort to occupants and better vehicle handling. Vehicle nowadays are equipped with active suspension systems. Ride experience can be augmented using Active Suspension Systems. The Vehicle chassis is fitted with numerous sensors also including those to measure body acceleration.

In general, the ideal suspension system is one which work in accordance with Skyhook theory. Active suspension system has force actuator in addition to the elements of the passive suspension systems. Body acceleration of the vehicle is converted into a suitable form to act as an input signal to the force actuators.

In the development of this control algorithm, H-infinity synthesis was used to create a nominal plant model that although guarantees performance but is not necessarily robust in the sense it does not operate over a large range of frequencies. Later an uncertain model is created using mu-synthesis to design a controller that is robust. Mu-synthesis of unregulated controllers works similar to h-infinity synthesis only difference being that mu synthesis takes into account the uncertainty of the system.

Based on the calculation of mathematical model, SIMULINK Model is created and is further used for developing this control algorithm. Two approaches are used to develop the control algorithm. One is using H∞ (H-infinity) approach and the second approach used is using µ-synthesis. The Robust control toolbox of MATLAB is used in the process.

The controller takes body acceleration \((a_b)\) and suspension deflection \((S_d)\) as inputs and produces hydraulic force \((f_h)\) as output. This hydraulic force act as a counter force to the forces causing undesired vibrations in system. Thus, the force created by hydraulic force try to dampen these undesired vibrations. Whenever the car hits a bump or a goes through a pothole, the disturbances (undesired vibrations) are introduced in the system. These disturbances are amplified in some ways which cause the car body to hop (\(a_h\)) and the suspension to compress (\(S_d\)). The performance of active suspension systems is determined by how well the control algorithm is developed and implemented that it produces certainly the right amount of actuator force to isolate the vibrations away from passengers and give them better ride experience. The feature of a best controller is that it minimizes this amplification of the disturbances in the least amount of actuator force possible. To manipulate the signals and treat them unequally within the model we add weighting factors. Using \(H_\infty\) controllers This enables us to create different modes like comfort mode, balanced mode and Handling mode (Performance mode). We generally see these different modes that can be adjusted from car’s dashboard on supercars. This way if we want a very smooth ride, we can choose comfort mode which decreases the weighting factor \((\beta)\) of actuating energy of force actuator which dampen the vibrations slowly or other way if we are concerned more about the life of suspension and want a better handling feature, we can select handling mode which dampens the vibrations quickly increasing the weighting factor of actuating energy. A balanced mode is trade-off between ride comfort and vehicle handling capabilities. The gist of the control algorithm is to determine which factors we have access to (in this case \(a_h\) and \(S_d\)) and which variables can be controlled \((f_h)\), then we need to choose the error signals and decide their cost functions by setting weight factors. The weighting functions \((\beta)\) can be frequency dependent, so high pass filter is set up on actuator signal to penalize high frequency controller commands than low frequency commands, which allows to limit the bandwidth of operational frequencies. The tire-hop frequency and rattlespace frequency were calculated to be 84.56 rad/s and 34.46 rad/s respectively.

Setting up the model for \(H_\infty\) controller is easier as we only need to create an open loop function and declare the variables that it has access to \((a_h, S_d)\) using the ‘hinfsyn’ command of Robust control
toolbox of MATLAB. ‘hinfsyn’ command solves the optimization problem to find a controller ‘K’ that minimizes the amplification of energy from external input signals ‘w’ (vibration due to forces acting on the body) to error signals (\(a_b, S_d, f_s\)) of virtual suspension model. The gain factors of the three desired modes (gain sets) i.e., comfort, balanced and performance is declared. ‘hinfsyn’ command is setup such that it calculated the optimized controller for each of the three gain sets. The simulation is carried out for time of 1 second and the road bump of 5cm.

From Figure 7 (a), we can observe that although the body travel for open loop suspension system i.e. in absence of force actuators and body travel in case of comfort mode and open loop response are same, the body returns to its original position slowly damping away the vibrations thus increasing the ride comfort for passengers. On the other hand, in handling mode the body quickly returns to its position enabling better handling of the vehicle. The green line represents the road bump. The blue line is representation of passive suspension system (open loop). As a result of high gain factor, \(\beta\) of performance mode the suspension creates a higher actuating force which quickly dampens the vibrations in less suspension deflection. The higher control force can be seen in figure 7 (b).

![Figure 7. Comparison of different modes using control algorithm (a) Body travel and Body acceleration (b) Suspension deflection and control force](image)

The only problem with using this controller designed using \(H_\infty\)-synthesis is it works on a nominal model and it fails to compensate for the noise (error) in the sensors value and dynamics of the actuator. Figure 8 shows comparision of simulations performed on Nominal \(H_\infty\) controller and \(\mu\)-synthesis controllers. The models are simulated with uncertain model simulated for 5 cm road bump and 100 random actuator variations for balanced mode.

![Figure 8. Simulation result of Nominal \(H_\infty\) controller vs \(\mu\)-synthesis controller for 100 random actuator variations](image)
It can be seen that out of the 100 actuator models, most of the actuators give a reasonably stable response. However some actuators produce unstable response. For a system with such high uncertainties, controllers developed using \( \mu \)-synthesis provides better results at it takes into account all the uncertainties from beginning of simulation. \( \mu \)-synthesis controllers work similar to \( H_\infty \) controllers except that it used decay iterative process to minimize the worst case scenario gain across range of uncertainties. It keeps calculating repeatedly the robustness of the nominal \( H_\infty \) controllers and scale each of them based on the uncertainty of the system. This process continues until the the robustness value (peak \( \mu \) value) of the suspension attains some constant value. The robustness value for the Nonlinear McPherson suspension system designed was calculated to be 0.946. The lesser this value, more robust is the suspension. It can be observed that \( \mu \)-synthesis controller (represented by red lines) produce stable response with less variability due to uncertainty. The wild acceleration is associated with the variations that deviate more from the nominal actuator. So we a suspension system is designed near the variation limit the system is stable but comes at the expense of rougher ride.

4. Conclusion:
The widely used McPherson suspension poses a challenging modelling problem of its nonlinear asymmetric behaviour. Certain important parameters are considered while designing the mathematical model of the suspension compensate the non-linearity in actual suspension system. The parameters include (1) rotation and translation for the unsprung mass (2) wheel mass and its inertia moment about the longitudinal axis and (3) tyre damping and lateral deflection due to the suspension mechanism. The proposed model is compared with ADAMS model and the SIMULINK model is prepared based on the mathematical model.

If actuator dynamics vary greatly, controllers developed using \( H_\infty \)-synthesis is not a great solution.

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