Constraints on features in the inflationary potential from future Euclid data

Ivan Debono,1⋆ Dhiraj Kumar Hazra,2,3,4† Arman Shafieloo,5,6 George F. Smoot,1,7,8 Alexei A. Starobinsky9

1Paris Centre for Cosmological Physics, Université de Paris, CNRS, Astroparticule et Cosmologie, F-75013 Paris, France
3The Institute of Mathematical Sciences, HBNI, CIT Campus, Chennai 600113, India
4Osservatorio di Astrofisica e Scienza dello Spazio di Bologna/Instituto Nazionale di Astrofisica, via Gobetti 101, I-40129 Bologna, Italy
5Istituto Nazionale Di Fisica Nucleare, Sezione di Bologna, Viale Berti Pichat, 6/2, I-40127 Bologna, Italy
6Korea Astronomy and Space Science Institute, Daejeon 34055, Korea
7University of Science and Technology, Daejeon 34113, Korea
8Physics Department and Lawrence Berkeley National Laboratory, University of California, Berkeley, 94720 CA, USA
9Landau Institute for Theoretical Physics RAS, Moscow, 119334, Russia

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
With Planck CMB observations, we established the spectral amplitude and tilt of the primordial power spectrum. Evidence of a red spectral tilt ($n_s = 0.96$) at 8σ provides strong support for the inflationary mechanism, especially the slow-roll of the effective scalar field in its nearly flat potential as the generator of scalar primordial perturbations. With the next generation of Large Scale Structure surveys, we expect to probe primordial physics beyond the overall shape and amplitude of the main, smooth and slowly-changing part of the inflaton potential. Using the specifications for the upcoming Euclid survey, we investigate to what extent we can constrain the inflation potential beyond its established slow-roll behaviour. We provide robust forecasts with Euclid and Planck mock data from 9 fiducial power spectra that contain suppression and wiggles at different cosmological scales, using the Wiggly Whipped Inflation framework to generate these features in the primordial spectrum. We include both Euclid cosmic shear and galaxy clustering, with a conservative cut-off for non-linear scales. Using MCMC simulations, we obtain an improvement in constraints on the WWI potential, as well an improvement for the background cosmology parameters. We find that apart from improving the constraints on the overall scale of the inflationary potential by 40-50%, we can also identify oscillations in the primordial spectrum that are present within intermediate to small scales ($k \sim 0.01 - 0.2 \text{Mpc}^{-1}$).

Key words: cosmology: inflation – gravitational lensing: weak – cosmology: cosmic background radiation

1 INTRODUCTION
Cosmology is at a point in its history where observations have caught up with theories, and physics at the largest cosmological scales is probed in full-sky surveys. Data from various observations allow us to measure the parameters in our cosmological model with increasing precision. These data include Cosmic Microwave Background (CMB) measurements such as WMAP (Hinshaw et al. 2013) and Planck (Planck Collaboration et al. 2014), supernovae compilations (e.g. Goldhaber 2009, SCP), large-scale structure maps (e.g. Ahn et al. 2014, SDSS), and weak-lensing observations (e.g. Parker et al. 2007; Schrabback et al. 2010). The next generation of observations, such as Euclid (Laureijs et al. 2011; Amendola et al. 2018), the Square Kilometre Array (Blake et al. 2004; Santos et al. 2015, Maartens et al. 2015, SKA), and the Large Synoptic Survey Telescope (Ivezić et al. 2019, LSST) are expected to provide order-of-magnitude improvements in precision, and in the ability to constrain different cosmological processes.

The fundamental questions facing modern-day cosmologists are not simply about parameter estimation in a known model, but about the possibility of new physics. They are
questions about model selection. In addition to estimating the values of the parameters in the model, this involves decisions on which parameters to include or exclude. In some cases, the inclusion of parameters is possible only by invoking new physical models.

The Λ Cold Dark Matter (ΛCDM) concordance model can fit different astrophysical datasets with only six parameters describing the mass-energy content of the Universe (baryons, CDM and a cosmological constant or constant dark energy) and the initial conditions. Any deviations from ΛCDM are too small compared to the current observational uncertainties to be inferred from cosmological data alone. However, it does not mean that additional parameters are ruled out.

There are several open questions in modern cosmology. Most of the matter in the Universe is dark matter, whose nature is not known. Another open question is the nature of the component causing the accelerated expansion of the Universe. The data are compatible with a cosmological constant, but do not exclude dynamical dark energy. Finally, there is the question of the conditions in the very early Universe.

In this paper we focus on the physics of the primordial Universe, and examine the ability of Euclid to provide information about features in the primordial power spectrum beyond that which Planck has provided. In Huang et al. (2012) and Ballardini et al. (2016), it was found that for models with features, a large-scale structure survey like Euclid will be essential to detect and measure these features. With CMB probes, we measure the angular power spectrum of the anisotropies in the two-dimensional multipole space. This is a projection of the power spectrum in the three-dimensional momentum space. This projection smooths out narrow features in momentum-space at large multipoles. This does not occur with large-scale structure surveys like Euclid.

This work is motivated by the above consideration. Planck and Euclid show substantial overlap in their ability to probe cosmological scales. With Planck, we have not been able to find strong evidence for the existence of features in the primordial power spectrum. Certain types of features in the intermediate and small scales persist in all data releases in Planck. Their existence, if detected, will directly identify the fine shape of the inflaton potential, its transitions, and its nature.

In the last four decades, various types of features in the primordial power spectrum generated by local or non-local modification in the potential have been proposed (e.g. Starobinsky 1992; Bennett et al. 1996; Peiris et al. 2003; Gariazzo et al. 2017; Beutler et al. 2019. See also Martin et al. 2014). In this work we aim to forecast parameter constraints for different types of primordial features appearing at different cosmological scales. Should the inflation potential really result in a primordial power spectrum with features, to what extent can Euclid observations probe these features?

In order to explore the features we use Wiggly Whipped Inflation (WWI) (Hazra et al. 2014a), which can generate a wide variety of primordial power spectra with features at different cosmological scales which otherwise can be obtained using different potentials. Due to its generic nature, the WWI framework, being confronted with Planck temperature and polarization data, was capable of offering a family of primordial power spectra that provided better fit to the combined data compared to the nearly scale-invariant spectrum (Hazra et al. 2016). By using Wiggly Whipped Inflation, we allow for a broad range of primordial power spectrum features in a single framework. So far, three types of features are known to provide improvement of fit to the data compared to power law primordial spectrum, namely, large-scale suppression or dip, intermediate scale oscillations near first acoustic peak and near ℓ = 600 – 800 and certain high-frequency oscillations that continue towards small scales.

In this paper, we use the best-fitting Wiggly Whipped Inflation models obtained using Planck to create fiducial cosmologies and data for Planck and Euclid. We use Markov chain Monte Carlo (MCMC) methods to forecast the ability of Euclid observations to add information and the possibility of identifying different features in the primordial power spectrum.

This paper is organized as follows. In section 2, we describe the different models for the primordial power spectrum considered in our work, and provide details of the Wiggly Whipped Inflation potential. We also explain how WWI is supported by the data. In section 3, we describe the methods we use to generate mock data for Planck CMB, Euclid cosmic shear and Euclid galaxy clustering, together the survey specifications and the theoretical error modelling. Our cosmological models and the various software codes used in this work are described in this section. We present our results in section 4. Finally, we provide some concluding remarks and perspectives for future work in section 5.

## 2 Primordial Physics

The large-scale structure that we observe today in the Universe is seeded by primordial quantum perturbations. These quantum fluctuations originated and evolved during the inflationary epoch. The form of the primordial power spectrum describing these perturbations depends on the inflation potential. Here we focus on the physics of the primordial Universe, and examine the ability of Euclid to provide information about features in the primordial power spectrum.

### 2.1 Power law primordial spectrum in the Concordance Model

The simplest primordial power spectrum, which is the one used in the six-parameter Concordance Model of cosmology, is a power law with the following phenomenological form:

\[ P^\text{Plaw}_S(k) = A_s \left( \frac{k}{k_0} \right)^{n_s - 1}, \tag{1} \]

where \( A_s \) is the amplitude and \( n_s \) is the tilt of the spectrum of primordial perturbations (see e.g. Kosowsky & Turner 1995; Bridle et al. 2003). In a scale-invariant power spectrum, \( n_s = 1 \). However, by now this spectrum is firmly excluded by observations.

Based on this model, Planck-2018 analysis reports \( n_s = 0.9649 \pm 0.0042 \). This 8σ statistical difference between the data and the scale-independent primordial power spectrum model is among the most remarkable results of the Planck mission (Planck Collaboration et al. 2014, 2016, 2018a,b).
The scale-invariant power spectrum is featureless. Broad features in the power spectrum can be described by logarithmic derivatives of the tilt (running and running-of-running), or by local and non-local wiggles in the power spectrum. Any variation beyond the tilt in the spectrum has not been established to date with any statistical significance.

However, since the initial release of WMAP (Spergel et al. 2003), through to different releases of Planck data, the ΛCDM model has shown certain outliers. In different model-dependent and independent reconstructions (Hannestad 2001; Tegmark & Zaldarriaga 2002; Brügge et al. 2003; Mukherjee & Wang 2003; Tocchini-Valentini et al. 2006; Shafieloo & Sroudadpe 2004; Kogo et al. 2005; Leach 2006; Shafieloo & Sroudadpe 2008; Paykari & Jaffe 2010; Nicola & Contaldi 2009; Gauthier & Bucher 2012; Hlozek et al. 2012; Vázquez et al. 2012; Hazra et al. 2013b; Hunt & Sarkar 2014; Dorn et al. 2014; Hazra et al. 2014a,d) primordial features at particular scales have been found to address these outliers. While the statistical significance of these outliers is rather low (3σ), they interestingly persisted in all releases of these two full-sky surveys and at the same angular scales (Hazra et al. 2014d). If we do not attribute the occurrence of these two full-sky surveys and at the same angular scales Euclid: inflation features

While the statistical significance of these outliers can be explained by detectable features in the spectrum. Projected constraints on broad features that are described by running and running-of-running of spectral index in the context of Euclid were studied in Debono et al. (2010) using Fisher analysis. In this work, we use the MCMC method to forecast the constraints on possible oscillations in the primordial spectrum. Instead of just imposing some parametric modification in the spectrum we model the existence of such features from inflation theory and therefore we project constraints in the shape of the inflationary potential.

2.2 Wiggly Whipped Inflation

The aforementioned form of power-law primordial power spectrum is a prediction of inflation where the scalar field (inflaton) slowly rolls down to the bottom of the flat inflationary potential. With the constraints on the tilt and an upper bound on the amplitude of tensor perturbation with respect to scalar perturbation, various surveys have ruled out a wide class of models. However, fundamental questions such as the energy scale of inflation and the detailed shape of the potential remain unanswered. Note that any changes in the nearly flat potential will eventually lead to certain features in the spectrum. Local glitches in the potential including rapid change of its amplitude, or the break in its first or second derivatives (Starobinsky 1992; Starobinsky 1998; Adams et al. 2001; Covi et al. 2006; Joy et al. 2008, 2009; Hazra et al. 2010; Miranda et al. 2012; Benetti 2013; Chluba et al. 2015; Cadavid & Romano 2015), false vacuum decay (leading to open inflation, in particular) (Linde 1999; Linde et al. 1999; Bousso et al. 2014), or an inflection point in the potential (Allahverdi & Mazumdar 2006; Jain et al. 2009), or oscillations in the potential (Aashooroon & Krause 2006; Biswas et al. 2010; Flauger et al. 2010; Palud et al. 2009; Aich et al. 2013; Hazra 2013; Peiris et al. 2013; Meierburg & Spergel 2014; Easther & Flauger 2014; Motohashi & Hui 2015; Miranda et al. 2016) all lead to local and non-local oscillations in the spectrum. Direct reconstruction of the primordial spectrum from the Planck data Hazra et al. (2014d) hints at large-scale oscillations, an intermediate-scale burst of oscillations, and persistent high-frequency oscillations within intermediate to small scales. While these types of features can be obtained by different potentials, in this work we will be using the Wiggly Whipped Inflation, which is known to provide these local and non-local features in a unified framework.

Wiggly Whipped Inflation was first proposed in Hazra et al. (2014a) as an extension of the Whipped Inflation model introduced in Hazra et al. (2014b). Both belong to the class of models with a large field inflaton potential. In Whipped Inflation, the inflaton starts with a power-law potential. After an initial period of fast roll that lasts until after a few e-folds inside the horizon, it transits to the attractor of the slow-roll part of the potential with a lower power. The initial motivation for Wiggly Whipped Inflation was the BICEP-2 result announced in BICEP2 Collaboration et al. (2014) and Ade et al. (2014) where the suppression of scalar power at large scales with appropriate tensor power spectrum amplitude ruled out the simplest power law form of the spectrum in Planck-BICEP2 joint analysis. The data was subsequently re-interpreted in BICEP2/Keck Collaboration et al. (2015), where dust polarization amplitude could consistently describe the observed B-modes at large scales. This re-interpretation reduced the statistical significance of WWI associated with large field models. Thereafter, in Hazra et al. (2016) the authors redesigned the potential in the light of new data. With Planck temperature and polarization data, using only two potentials in the WWI framework, the authors identified five types of spectra that provided improvement in fit to the Planck data compared to power-law spectrum by a Δχ2/ν = 12 − 14 with 2-4 extra parameters.

2.2.1 The inflationary potential

In the WWI framework, the two potentials we consider are Wiggly Whipped Inflation (WWI potential hereafter) and Wiggly Whipped Inflation Prime (WWIP potential hereafter).

The WWI potential is defined by the equation:

\[
V'(\phi) = V_i \left(1 - \left(\frac{\phi}{\mu} \right)^p \right) + \Theta(\phi_T - \phi) V_i \left(\frac{\gamma}{\phi_T - \phi} \right)^q + \phi_i^q, 
\]

where we note that \(V_S(\phi) = V_i \left(1 - \left(\frac{\phi}{\mu} \right)^p \right)\) has two parameters, \(V_i\) and \(\mu\). The parameter \(\mu\) and the index \(p\) determine the spectral tilt \(n_s\) and the tensor-to-scalar ratio \(r\).

We choose the values \(p = 4\) and \(\mu = 15 M_P\), where \(M_P = 1\) is the reduced Planck mass, such that \(n_s \approx 0.96\) and \(r \approx O(10^{-2})\) (as in Efstathiou & Chongchitnan 2006). The transition and discontinuity happen at the field value \(\phi_T\). In this case, a featureless primordial power spectrum is obtained if \(\gamma \approx 0\) and \(\phi_0 = 0\). The Heaviside Theta function \(\Theta(\phi_T - \phi)\) is modelled numerically as usual by a Tanh step \((\frac{1}{2} [1 + \tanh((\phi - \phi_T)/\delta)]\) and thereby introduces a new extra parameter \(\delta\).
The WWIP potential is defined by:

\[
V(\phi) = \Theta(\phi - \phi_0)V_1(1 - \exp[-\alpha(\phi - \phi_0)]) \\
+ \Theta(\phi - \phi_T)V_T(1 - \exp[-\alpha(\phi - \phi_0)])
\]  

(3)

This potential is same as that used in Hazra et al. (2016). It is composed of \(\alpha\)-attractor potentials (Kallosh & Linde 2013), which include the Einstein frame effective potential of the Starobinsky \(R + R^2\) inflationary model (Starobinsky 1980) as a particular case for \(\alpha = \sqrt{273} \approx 0.816\), where \(R\) is the Ricci scalar, with different slopes appearing in the exponent, allowing a discontinuity in the derivative. Since in this case the potential is continuous, \(V\), can be derived from \(V_1\). In the WWIP models considered in this paper, we set \(\alpha = \sqrt{273}\). In our convention, \(\kappa^2 = 8\pi G\) is equal to 1. The parameter \(G\) is the gravitational constant.

3 METHOD

Our forecasts use the MCMC technique, with mock data from fiducial cosmological models. We begin by giving brief details of the simulated Euclid and Planck data sets used in our forecasts. The Euclid likelihoods used in this paper are described exhaustively in Sprenger et al. (2019).

3.1 The simulated data

Data from the Euclid mission are not yet available, so we compute mock data from a fiducial cosmology following the method defined in Sprenger et al. (2019). Since our aim is to quantify the improvement in constraints from future Euclid data, we carry out two MCMC forecasts for each cosmological model: first with simulated Planck CMB data alone, then with joint Euclid galaxy clustering and cosmic shear, and Planck CMB data.

3.1.1 Cosmic microwave background

For Planck, we run our forecasts with mock data from a fiducial cosmology using the MontePython package, which models the full Planck mission. It is based on the fake realistic Planck likelihood provided with the publicly-available MontePython package, which models the full Planck mission. It is composed of \(\alpha\)-attractor potentials (Kallosh & Linde 2013), which include the Einstein frame effective potential of the Starobinsky \(R + R^2\) inflationary model (Starobinsky 1980) as a particular case for \(\alpha = \sqrt{273} \approx 0.816\), where \(R\) is the Ricci scalar, with different slopes appearing in the exponent, allowing a discontinuity in the derivative. Since in this case the potential is continuous, \(V\), can be derived from \(V_1\). In the WWIP models considered in this paper, we set \(\alpha = \sqrt{273}\). In our convention, \(\kappa^2 = 8\pi G\) is equal to 1. The parameter \(G\) is the gravitational constant.

3.1.2 Cosmic shear

In this section we give a brief description of the relevant quantities in our cosmic shear likelihood. Further details are found in Sprenger et al. (2019), and references therein. This code, and the code for the galaxy clustering likelihood are both publicly-available in the MontePython package.

Cosmic shear surveys map the alignments in the distortion of galaxies caused by weak gravitational lensing as a result of density inhomogeneities along the line of sight. It provides an effective way to map dark matter, is therefore a powerful probe of large-scale structure. Cosmological information is extracted from auto-correlations and cross-correlations of alignment maps at different redshifts (see e.g. Bartelmann 2010).

The matter power spectrum is defined as:

\[
\langle \delta(k)\delta'(k') \rangle = (2\pi)^3 \delta^3_D(k - k')P(k).
\]  

(4)

The three-dimensional matter power spectrum is projected onto a two-dimensional lensing correlation function for redshift bins \(i\) and \(j\) at multipole \(\ell\):

\[
C^{ij}_\ell = \frac{9}{16}\Omega^2_m H_0^2 \int_0^\infty \frac{dr}{r^2} g_r(r)g_i(r)P\left(k = \frac{\ell}{r}, z(r)\right).
\]  

(5)

The functions \(g_i(r)\) depend on the radial distribution of galaxies in the redshift bin \(i\).

A noise term \(N_\ell\) is added to the theoretical \(C^{ij}_\ell\) due to the intrinsic alignment of galaxies. The noise spectrum is:

\[
N_\ell^{ij} = \delta_{ij} \sigma_{\text{shear}}^2 n^{-1}_i,
\]  

(6)

where \(\sigma_{\text{shear}}\) is the root mean square of the galaxy intrinsic shear. We set this to 0.3. The term \(n_i\) is the number of galaxies per steradian in redshift bin \(i\). We divide the redshift range into 10 redshift bins, with an equal number of galaxies in each. Therefore, for every redshift bin we have:

\[
n_i = \frac{n_{\text{gal}}}{10} \times 3600 \left(\frac{180}{\pi}\right)^2,
\]  

(7)

where the number of observed galaxies \(n_{\text{gal}} = 30\) arcmin\(^{-2}\).

3.1.3 Cosmic shear likelihood

To calculate the cosmic shear likelihood, we use the method described in Sprenger et al. (2019), which in turn is taken from Audren et al. (2013). This method defines the likelihood as:

\[
-2\ln L \equiv \sum_\ell (2\ell + 1) f_{\text{sky}} \left(\frac{d^\text{mix}_\ell}{d^\text{th}_\ell} + \ln \frac{d^\text{th}_\ell}{d^\text{obs}_\ell} - N\right),
\]  

(8)

where ‘obs’ and ‘th’ denote observed and theoretical quantities, respectively. The term \(N\) is the number of redshift bins. Each \(C_\ell\) matrix has dimension \(N\), and the matrix determinants are denoted by \(d\). We have three kinds of determinant: the determinant of the theoretical angular power:

\[
d^\text{th}_\ell = \det\left(C^{ij}_\ell + N^{ij}_\ell\right),
\]  

(9)

that observed angular power spectrum:

\[
d^\text{obs}_\ell = \det\left(C^{\text{fiducial}ij}_\ell + N^{ij}_\ell\right),
\]  

(10)
and a mixed determinant:

\[ d_{\ell}^{\text{mis}} = \sum_k \det \left( N^{ij}_{\ell} + \begin{cases} C^{\text{th ij}}_{\ell}, & j \neq k \\ C^{\text{fiducial ij}}_{\ell}, & j = k \end{cases} \right) \]  

(11)

Note that the observed and theoretical spectra include a noise term \( N^{ij}_{\ell} \).

In our MCMC simulations, the sampled points in parameter space act as our observed power spectra, while the theoretical power spectrum is produced using the fiducial model.

### 3.1.4 Euclid cosmic shear specifications

We use the number density of galaxies with the corresponding redshift errors taken from Audren et al. (2013) and used in Sprenger et al. (2019), where the unnormalized redshift number density distribution is defined by:

\[ \frac{d n_{\text{gal}}}{dz} = z^3 \exp \left[-\left(\frac{z}{\alpha z_m}\right)^\gamma\right]. \]  

(12)

We set the values \( \alpha = \sqrt{2}, \beta = 2, \text{ and } \gamma = 1.5. \) In this equation, \( z \) is the redshift, while \( z_m = 0.9 \) is the median redshift of the sources.

The redshift uncertainty is parametrized by a Gaussian error which depends on the redshift \( z \). For redshifts up to \( z_{\text{photo-max}} = 4.0 \), we assign a large error \( \sigma_{\text{photo-max}} = 0.05. \) Beyond \( z_{\text{photo-max}} \), we use a small error \( \sigma_{\text{no-max}} = 0.3. \) We use a value of \( z_{\text{photo-max}} = 4.0 \). The details of the redshift uncertainty parametrization are found in Harrison et al. (2016) and Sprenger et al. (2019).

The sky coverage for Euclid \( f_{\text{sky}} = 0.3636 \), and we use the same value for galaxy clustering.

### 3.1.5 Galaxy clustering

Galaxies are not randomly distributed in space, but tend to be found in clusters. The galaxy power spectrum is defined as a function of a continuous density field, which represents the probability density \( p_g \) of finding a galaxy at some position \( r \). The galaxy density perturbation \( \delta_g \) is therefore a perturbation of this probability density:

\[ p_g(r) = \bar{n}(r)(1 + \delta_g(r)). \]  

(13)

The quantity \( \bar{n}(r) \) is the expected number density of galaxies on a homogeneous background, calculated as the mean density over a sufficiently large volume. In our galaxy clustering calculation, this will be the volume corresponding to one redshift bin. The spatial distribution of galaxies represents a biased tracer of the underlying dark matter distribution, so the conversion from the matter to the galaxy power spectrum must take into account various effects. We use the method developed in Sprenger et al. (2019).

The observed galaxy power spectrum \( P_g \) is related to the matter power spectrum \( P_m \) by:

\[ P_g(k, \mu, z) = f_{\text{AP}}(z) \times f_{\text{res}}(k, \mu, z) \times f_{\text{RSD}}(k, \mu, z) \times b^2(z) \times P_m(k, z). \]  

(14)

This relation assumes a flat-sky approximation (Lemos et al. 2017; Asgari et al. 2018), which allows us to define the angle between the Fourier modes \( k \) and the line-of-sight distance vector \( r \). Thus, in Equation 14,

\[ k = |k|, \]  

(15)

and

\[ \mu = \frac{k \cdot r}{|kr|}. \]  

(16)

The parallel part of a mode is given by \( k_c = \mu k \) and the perpendicular one by \( k_s = k \sqrt{1 - \mu^2}. \)

The first term in Equation 14 arises from the Alcock-Paczynski effect due to the relation between the Fourier modes of real space and those in the fiducial space. If we denote the values as \( H \) and \( D \) as the quantities in the real or true cosmology, and the values \( H_f \) and \( D_f \) as corresponding to the fiducial cosmology, we obtain:

\[ f_{\text{AP}}(z) = \frac{D_z^2 H}{D_z^2 H_f}. \]  

(17)

The second term in Equation 14 is due to the limited resolution of any telescope, which means that the observed small-scale perturbations are suppressed. Assuming Gaussian errors \( \sigma_g(z) \) and \( \sigma_\perp(z) \) on coordinates parallel and perpendicular to the line of sight at redshift \( z \), respectively, the suppression factor is:

\[ f_{\text{res}}(k, \mu, z) = \exp(-k^2 \left[ \mu^2 \left( \sigma_g^2(z) + \sigma_\perp^2(z) \right) \right]). \]  

(18)

The suppression factor is independent of the fiducial cosmology.

In any galaxy observation, there are additional sources of redshift alongside the cosmological redshift. The classical Doppler effect due to the velocity of galaxies produces an apparent anisotropy in the redshift-space power spectrum. The redshift-space distortion effects are parametrized by the third term in Equation 14:

\[ f_{\text{RSD}}(k, \mu, z) = \left(1 + \beta(\hat{k}, \hat{z}) \hat{\mu}^2\right)^2 e^{-k^2 \mu^2 \sigma_e^2}. \]  

(19)

The first term in brackets corresponds to the Kaiser formula (Kaiser 1987). The term \( \beta \) is the growth rate \( f(k, z) \) corrected by the galaxy bias \( b(z) \):

\[ \beta(\hat{k}, \hat{z}) \equiv \frac{f(\hat{k}, \hat{z})}{b(z)} \equiv \frac{1}{b(z)} \frac{d \ln \left( \frac{P_m(\hat{k}, \hat{z})}{P_m(\hat{k}, \hat{z})} \right)}{d \ln a} = \frac{1}{2b(z)} \frac{d \ln P_m(\hat{k}, \hat{z})}{dz}. \]  

(20)

The bias relates the density perturbations in the galaxy field \( \delta_g \) to the dark matter density perturbations \( \delta_m \). We assume a linear approximation where the bias is scale-independent, so that:

\[ \delta_g = b(z) \delta_m. \]  

(21)

The exponential term in Equation 19 accounts for the elongation in redshift-space maps along the line of sight within overdense regions, known as the ‘Fingers of God’ effect. We include the term \( \sigma_{e\perp} \) as a nuisance parameter in our forecasts, with a fiducial value of 7 Mpc, and a prior range from 4 to 10 Mpc.
Our galaxy clustering survey is divided into redshift bins of width \( \Delta z = 0.1 \) with mean redshift \( \bar{z} \). The experimental shot noise in each redshift bin is parametrized by:

\[
P_N(z) = \frac{1}{n(z)} = \frac{V_r(z)}{N(z)},
\]

where \( N(z) \) is the number of galaxies in the bin, \( V_r(z) \) the volume of the bin and \( n(z) \) the galaxy number density. The volume of each redshift bin is:

\[
V_r(z) = 4\pi f_{\text{sky}} \int_{\Delta z(z)} r^2 \, dr = \frac{4\pi}{3} f_{\text{sky}} \left[ \frac{r^3}{2} \left( \bar{z} + \frac{\Delta z}{2} \right) - r^3 \left( \bar{z} - \frac{\Delta z}{2} \right) \right],
\]

where \( f_{\text{sky}} \) is the fraction of the sky covered by the survey. We include massive neutrinos in our cosmological models. In such models, the clustering of halos is determined by cold dark matter and baryons only, and not by massive neutrinos. The calculation of the observed galaxy power spectrum in Sprenger et al. (2019) used here accounts for this by ignoring the contribution of light massive neutrinos with a free-streaming length larger than the typical size of a galaxy. It therefore includes only the cold dark matter and baryon field \( P_{\text{cb}} \), rather than the full matter field \( P_{\text{m}} \) (i.e., cold dark matter, baryons, and massive neutrinos) in Equation 14, so that the equation is modified to:

\[
P(g) = P(k, \mu, z) = f_{\text{AP}}(z) \times f_{\text{cb}}(k, \mu, z) \times f_{\text{ESD}}(\bar{k}, \bar{\mu}, \bar{z}) \times P_{\text{cb}}(\bar{k}, \bar{z}).
\]

The \( \beta \) term of the Kaiser formula in Equation 20 is then modified to:

\[
\beta(\bar{k}, \bar{z}) = \frac{1 + \bar{z}}{2b_{h}(\bar{k}, \bar{z})} \frac{d \ln P_{\text{cb}}(\bar{k}, \bar{z})}{d \bar{z}},
\]

with the bias now defined as \( \delta_b = b(z) \times \delta_{h} \).

Putting all this together, we finally obtain the observed galaxy clustering power spectrum in each bin:

\[
P_{\text{obs}}(k, \mu, \bar{z}) = P(g) = P(k, \mu, \bar{z}) + N(z).
\]

### 3.1.6 Galaxy clustering likelihood

Since we are dealing with simulated data, \( P_{\text{obs}} \) is either produced by our fiducial cosmology, or by the points sampled in parameter space. If we denote each quantity by the label \( f \) (fiducial cosmology), or \( s \) (sample cosmology), the galaxy clustering likelihood can be written as:

\[
\chi^2 = \sum \int f(k^2) \, dk^2 \sum_{\mu} \mu f(\mu) \frac{\sqrt{f}}{2 \ln(2\pi)} \times \left[ \frac{\partial f}{\partial A_{\text{cb}}} + \frac{H}{D_A} P_{\text{cb}}(k^2, \mu^2) + \frac{H^2}{D_A^2} P_{\text{cb}}(k^2, \mu^2) \right]^2,
\]

where \( \chi^2 = -2 \ln L \).

### 3.1.7 Euclid galaxy clustering specifications

We use a redshift range from 0.7 to 2.0, which is the approximate range accessible to Euclid. For the bin centres \( \bar{z} \), we use a minimum redshift \( z_{\text{min}} = 0.75 \) and a maximum redshift \( z_{\text{max}} = 1.95 \), with the entire redshift range divided into 13 bins.

The error on spectroscopic redshift measurements is assumed to be \( \sigma_z = 0.001(1 + \bar{z}) \). The effect of angular resolution is neglected, so that \( \sigma_\ell \) is set to 0. The radial distance error is a function of the redshift error, and is cosmology-dependent:

\[
\sigma_r = \frac{c}{H_0} \sigma_z.
\]

The galaxy number count distribution \( dN(z)/dz \) per deg\(^2\) assumes a limiting flux of \( 3 \times 10^{-16} \text{erg s}^{-1} \text{cm}^{-2} \), and is taken from Pozzetti et al. (2016).

We use a sky fraction of \( f_{\text{sky}} = 0.3636 \). The total number of detected galaxies in a given redshift bin is:

\[
N(z) = 41253 f_{\text{sky}} \text{deg}^2 \int_{\frac{z - \Delta z}{2}}^{\frac{z + \Delta z}{2}} \frac{dN(z)}{dz} \, dz.
\]

The bias factor corresponding to galaxies detected by Euclid is assumed to be close to the simple relation:

\[
b(z) = \sqrt{1 + z},
\]

which is also used in Audren et al. (2013).

Two parameters introduced in Sprenger et al. (2019) account for inaccuracies in this relation:

\[
b(z) = b_0^\text{Euclid}(1 + z)^{0.5} b_1^\text{Euclid}.
\]

We include \( b_0^\text{Euclid} \) and \( b_1^\text{Euclid} \) in our MCMC simulations as nuisance parameters. We assign a mean value of 1 to both, with an unbounded prior range.

### 3.2 The non-linear theoretical uncertainty

With its wide sky coverage and its catalogue of billions of stars and galaxies, Euclid will be a great leap forward in observational cosmology. However, this will significantly increase the size of the sampling variance and shot noise compared to current surveys. The leading source of error on small scales, which then impacts parameter extraction and model selection, will be theoretical errors.

In Sprenger et al. (2019), a novel method was introduced for dealing with the theoretical uncertainties, which we use here. This strategy defines a cutoff \( k_{\text{NL}} \). All theoretical uncertainties up to this wavenumber are ignored, while all the information above it is discarded. The redshift dependence of non-linear effects is parametrized as:

\[
k_{\text{NL}}(z) = k_{\text{NL}}(0)(1 + z)^{2(2 + n_s)}. \]

Our use of this parametrization with cosmological models where the tilt \( n_s \) is not explicitly present is justified by two considerations. First, the envelope of the wiggles on the Wiggly Whipped primordial power spectrum corresponds to the tilt on the power-law spectrum. Our parameter choices ensure that the resulting spectrum is compatible with the current best data from Planck. Secondly, the wiggles die down well before the non-linear part of the spectrum (see Figure 2).

In Sprenger et al. (2019) two frameworks for modelling the theoretical error are defined. The first is a ‘realistic’ case where the parametrization of the error is trusted up to large
wavenumbers. The information from small scales is gradually suppressed by a growing relative error function. The second is a ‘conservative’ case where the same error function is used with a sharp cut-off.

The parameters for our galaxy clustering forecast correspond to the ‘conservative’ setup. We adopt a cut-off on large wavelengths at $k_{\text{min}} = 0.02 \, \text{Mpc}^{-1}$. This eliminates scales which are bigger than the bin width or which violate the small-angle approximation. On small wavelengths, we use a theoretical uncertainty with $k_{\text{NL}}(0) = 0.2 \, h \, \text{Mpc}^{-1}$.

Similarly, we adopt the ‘conservative’ setup for the cosmic shear forecast. We include multipoles from $\ell_{\text{min}} = 5$ up to a bin-dependent non-linear cut-off determined by $k_{\text{NL}}(0) = 0.5 \, h \, \text{Mpc}^{-1}$.

### 3.3 Cosmology

In this paper, we work within the framework of Friedmann-Robertson-Walker cosmology and we assume a flat spatial geometry for all our models. We consider two main classes of cosmological models. The first is ΛCDM with a power-law primordial power spectrum, or the Concordance Model of Cosmology. The second class consists of Wiggly Whipped Inflation models. These two classes are distinguished by the shape of the primordial power spectrum. For the former, they are featureless. For the latter, they contain features.

The background ΛCDM cosmology for all the models contains baryonic and cold dark matter, massive neutrinos, and a cosmological constant or constant dark energy. This is parameterized by: the baryon density $\omega_b = \Omega_b h^2$, the cold dark matter density $\omega_{\text{cdm}} = \Omega_{\text{cdm}} h^2$, the Hubble parameter via the peak scale parameter $100\theta_s$, and the optical depth to reionization $\tau_{\text{reio}}$. We use the following fiducial values for the background cosmology to generate the mock data for all our models: $\omega_b = 0.221 \times 10^{-2}$, $\omega_{\text{cdm}} = 0.12$, $100\theta_s = 1.0411$, and $\tau_{\text{reio}} = 0.09$. We assume 3 neutrino species, with the total neutrino mass split according to a normal hierarchy. We therefore have 2 massless and 1 massive neutrino. We keep all the values of the neutrino parameters fixed as follows: the sum of the neutrino masses $M_{\text{total}} = 0.06 \, eV$; and the number of effective neutrino species in the early Universe $N_{\text{eff}} = 3.046$. Our choice of neutrino fiducial values is motivated by the latest data from neutrino oscillation experiments (de Salas et al. 2018), which show strong statistical support for a normal hierarchy. Our Concordance Model is parametrized by two additional parameters for the power-law primordial power spectrum: the scalar amplitude $A_s$ and the scalar spectral index $n_s$. We use the following fiducial values: $\ln(10^{10} A_s) = 3.0447$, and $n_s = 0.9659$, with the pivot scale $k_0$ fixed at $0.05 \, h \, \text{Mpc}^{-1}$.

The second class of models contain features in the primordial power spectrum with ΛCDM as background cosmology. Here, instead of using power law spectrum we use the numerical solution to the Klein-Gordon and Mukhanov-Sasaki equations for background scalar field evolution and cosmological perturbations respectively. Therefore in addition to four parameter describing the ΛCDM background we have the inflationary potential parameters. The fiducial values for inflation potential parameters used to produce the mock data are shown in Table 1. For our MCMC simulations, the sampled data is parametrized by five free inflation parameters for WWI (ln$\left(10^{10} V_0\right)$, $\phi_0$, $\gamma$, $\varphi_T$, ln$\delta$), and three free inflation parameters for WWIP (ln$(10^{10} V_0)$, $\phi_0$, $\varphi_T$. We consider five fiducial primordial spectra for the WWI potential and three for the WWIP potential. For the WWI potential, four of the fiducial power spectra contain different types of features at different cosmological scales which represent local and global best fits to the Planck data. We call these WWI-[A, B, C, D] Hazra et al. (2016). For the WWIP potential, we use the Planck global best-fitting Hazra et al. (2016) spectrum (hereafter, WWIP:Planck-best-fit) and another spectrum found within the 95% confidence limits of Planck data (hereafter, WWIP:Small-scale-feature). This has wiggles extending to smaller scales where the overlap with upcoming Euclid data is better. We also consider two spectra without features for both WWI and WWIP. These two fiducial spectra are obtained by fixing $\phi_0 = 0$, $\gamma = 0$ for WWI, and $\phi_0 = 0$ for WWIP, respectively. These two spectra are used as a null test. The primordial power spectra produced by the feature models are shown in Figure 1. Corresponding matter power spectra for these fiducial potential parameters are provided in Figure 2 and the CMB temperature and polarization angular power spectra are provided in Figure 3. Since these fiducials represent features at different scales and amplitude, large-scale-structure data will have different constraining power when combined with CMB from Planck. Using the sensitivity of the cosmic-variance-limited proposed CMB polarization survey CORE Di Valentino et al. (2018); Finelli et al. (2018), forecasts on these types of features were performed in Hazra et al. (2018). It was shown that while the largest scale features ($\ell < 50$) cannot be detected with next-generation CMB surveys beyond 95% C.L., intermediate and small-scale oscillations can be discovered with high statistical significance. Since the proposed CORE mission was not approved we expect that a joint combination of Euclid and Planck can identify certain types of features if they represent the true model of the Universe and fall within the good signal-to-noise region of both the surveys.

In order to compute the primordial power spectrum from the inflation models, we use the BINGO package (Hazra et al. 2013a). Since the existence of fast-roll limits the use of analytical approximations in obtaining the power spectrum, BINGO is a necessary tool. We solve both the cosmological background and perturbation equations during inflation using adaptive stepsize, and adapt BINGO to output the primordial power spectrum directly as an input for CLASS via an external command. We evaluate the sensitivity to cosmological parameters of Euclid combined with Planck by performing MCMC forecasts with the joint data, and compare these against Planck-only constraints. We generate spectra for the fiducial models, which we then use as our mock data. This method has two advantages over the Fisher matrix formalism. First, it avoids the problem of numerical instabilities, particularly those linked to the choice of step size for the numerical derivatives. Secondly, it allows us to work with non-Gaussian errors, especially those which may arise with non-standard cosmologies such as ours.

---

1 Using N-body simulations, the effects of some of these features have been tested in L’Huillier et al. (2018) and a few other features have been tested in Ballardini et al. (2019).

2 BI-spectra and Non-Gaussianity Operator. Available on https://github.com/dkhaz/bingo
Table 1. Fiducial values: Inflationary potential parameters used to obtain the fiducial primordial power spectrum. We have used two types of potential in this framework: WWI (see Equation 2) and WWIP (see Equation 3). For both inflation models, we include a featureless case (labelled WWI:Featureless and WWIP:Featureless, respectively). WWI–A, B, C and D and WWIP:Planck-best-fit represents the best fit potential parameters to the combined Planck temperature and polarization data. WWIP:Small-scale-feature corresponds to a particular power spectrum that has features at small scales (within Planck 95% confidence limits), which ensures better overlap with cosmological scales to be probed by Euclid.

Figure 1. The scalar primordial power spectrum for the fiducial models considered in this paper. The left and right panels show WWI and WWIP, respectively. The inset bottom panel in each plot shows the amplitude of the features relative to the featureless spectrum $P_0$ (i.e. $\Delta P(k) = P_0(k) - P(k)/P_0(k)$).

Figure 2. The full non-linear matter power spectrum at $z = 0$ for the fiducial models considered in this paper. The left and right panels show WWI and WWIP, respectively. The inset plots show the range from $k = 0.001$ to 0.1.
We use the MCMC sampler MontePython (Brinckmann & Lesgourgues 2019) with the Boltzmann solver CLASS (Blas et al. 2011) to generate MCMC samples using a Metropolis-Hasting algorithm. Since we do not yet have any data from Euclid, we use data generated from a fiducial cosmological model. We include both cosmic shear and galaxy clustering, using the likelihoods from Sprenger et al. (2019). We include the non-linear part of the power spectrum in our forecasts. The non-linear contribution is calculated within CLASS using the HALOFIT Takahashi et al. (2012); Bird et al. (2012) semi-analytical formula.

4 RESULTS

In this section, we present our results. We fit the theoretical sampled angular power spectra and matter power spectra to the fiducial mock data of the corresponding models. Using the MCMC technique, we obtained Planck-only (labelled ‘Planck’) and joint Euclid+Planck constraints, which are presented in this section. Euclid here includes galaxy clustering and cosmic shear. For convenience, we just use the label ‘Euclid’. For both Planck-only and Euclid+Planck, we keep the same mock data in order to have a consistent comparison. The MCMC chains were analysed using GetDist.

In Table 2 we provide the projected constraints on the ΛCDM model with power law primordial spectrum (i.e. the Concordance Model) with both dataset combinations. Note that first six parameters in the table are the parameters used for MCMC analysis. We include four derived parameters: the dark energy density ΩΛ, the matter density Ωm, the Hubble constant H0, and the power spectrum normalization parameter σ8, defined as the root-mean-square amplitude of the density contrast inside an 8h−1 mpc sphere. We also plot the one-dimensional posteriors on 4 parameters (Ωm, τreio, σ8 and H0) and their marginalized contours in Figure 4. The table reflects improvements in all the parameter constraints when Euclid mock likelihood is combined with CMB. While the baryon density experiences a marginal improvement, constraints on CDM density becomes 4 times tighter which is reflected in the posterior of matter density. Constraints on H0 are improved through its degeneracies with other parameters, especially Ωm. The amplitude (ln[1010 A_s]) and tilt (n_s) of the primordial power spectrum are expected to be constrained 30-40% better with Euclid compared to the present bounds. Since Euclid will probe small scales comparatively better than Planck and the weak lensing will probe order-of-magnitude smaller scales beyond the Planck CMB probed scales, the long lever arm on the

![Figure 3. The TT (top) and EE (bottom) CMB angular power spectrum for the fiducial models considered in this paper. The left and right panels show WWI and WWIP, respectively.](image-url)
two-point correlations at small scales will be able to improve the constraints. The $\sigma_8$ being the integral of the matter power spectrum (which, in turn, is defined by the primordial spectrum amplitude and tilt and the transfer function) it is also expected to be constrained two-fold tighter than present constraints from Planck. The optical depth is not directly associated with the physical processes probed by Euclid. However, since the amplitude of the primordial spectrum is completely degenerate with optical depth, improvement in the constraints on the amplitude also improves the constraints on the $\tau_{reio}$. Note that here the mean value of $\tau_{reio}$ is higher than the recently released Planck value that is obtained simply as an artefact of using higher $\tau_{reio}$ in the fiducial cosmology. Use of higher $\tau_{reio}$ does not affect our analysis as Euclid cannot directly constrain optical depth and in the forecast we are concerned only about the bounds on the parameters, and not their mean value.

Compared to the 2-parameter from of power law primordial power spectrum, the WWI framework has 5 and 3 parameters that defines the potential of inflation. Since the feature induced by these parameters are scale-dependent, the power of Euclid in constraining these parameters will be different in different fiducials. To begin with, we test the WWI-featureless fiducial obtained by keeping $\phi_0 = \gamma = 0$. This fiducial represents a nearly scale-invariant primordial spectrum with a spectral tilt of 0.96. When the WWI potential is compared against this fiducial using the combined Planck + Euclid mock likelihood, we can address to what degree we can rule out non-zero $\phi_0, \gamma$, if a featureless power spectrum represents the true model. The 68% constraints are provided in Table 3 and constraints on inflation potential parameters are plotted in Figure 5. While the constraints on the background parameters experience similar improvements with Euclid as the power law CDM model, we find that apart from $V_0$, we do not have any improvements with respect to Planck. In other words, if the primordial power spectrum does indeed follow power law, Euclid is not going to be able to rule out any large-scale power suppression (induced by $\gamma$) or oscillations (induced by $\phi_0$) with higher statistical significance than Planck has already done. It is expected as Planck being cosmic variance limited in temperature, provides the best constraints at the large scales ($l < 50$, $k < 5 \times 10^{-3}\text{Mpc}^{-1}$). At the same time, apart from high frequency oscillations Planck already rules out wigglers at small scales. Therefore a featureless fiducial representing the mock data is not expected to rule out potentials that are already ruled out by Planck. For WWI-B and WWI-D we will consider high frequency wigglers in the primordial spectra as fiducials. With the feature models, we add 1 (for WWIP) or 4 (for WWI) free parameters to our

| Parameter | Planck | Euclid+Planck |
|-----------|--------|---------------|
| $10^{-2}\omega_b$ | 2.210 ± 0.015 | 2.209 ± 0.011 |
| $\omega_{cdm}$ | 0.11997 ± 0.00091 | 0.11999 ± 0.00025 |
| $1000\theta$ | 1.04111 ± 0.00033 | 1.04111 ± 0.00031 |
| $\tau_{reio}$ | 0.0906 ± 0.0047 | 0.0902 ± 0.0029 |
| $\ln(10^{10}V_0)$ | 1.17313 ± 0.0098 | 1.17304 ± 0.0055 |
| $\phi_0$ | < 0.0195 | < 0.0207 |
| $\gamma$ | < 0.0953 | unbonded |
| $\phi_T$ | < 7.78 | < 7.78 |
| $\ln \delta$ | -4.51 ± 1.7 | -4.54 ± 1.7 |
| $\Omega_{\Lambda}$ | 0.6805 ± 0.0058 | 0.6804 ± 0.0013 |
| $\Omega_m$ | 0.3194 ± 0.0058 | 0.3195 ± 0.0013 |
| $H_0$ | 66.85 ± 0.42 | 66.83 ± 0.13 |
| $\sigma_8$ | 0.8299 ± 0.0049 | 0.8297 ± 0.0018 |

| Parameter | Planck | Euclid+Planck |
|-----------|--------|---------------|
| $10^{-2}\omega_b$ | 2.210 ± 0.014 | 2.210 ± 0.011 |
| $\omega_{cdm}$ | 0.11998 ± 0.00095 | 0.12005 ± 0.00027 |
| $1000\theta$ | 1.04111 ± 0.00033 | 1.04111 ± 0.00031 |
| $\tau_{reio}$ | 0.08841 ± 0.0045 | 0.0891 ± 0.0031 |
| $\ln(10^{10}V_0)$ | 1.73211 ± 0.0089 | 1.7331 ± 0.0058 |
| $\phi_0$ | < 0.0197 | < 0.0197 |
| $\gamma$ | unbonded | unbonded |
| $\phi_T$ | < 7.81 | < 7.80 |
| $\ln \delta$ | -4.28 ± 0.93 | -4.40 ± 1.6 |
| $\Omega_{\Lambda}$ | 0.6804 ± 0.0060 | 0.6801 ± 0.0013 |
| $\Omega_m$ | 0.3195 ± 0.0060 | 0.3198 ± 0.0013 |
| $H_0$ | 66.84 ± 0.44 | 66.82 ± 0.13 |
| $\sigma_8$ | 0.8304 ± 0.0051 | 0.8313 ± 0.0018 |

| Parameter | Planck | Euclid+Planck |
|-----------|--------|---------------|
| $10^{-2}\omega_b$ | 2.210 ± 0.014 | 2.210 ± 0.011 |
| $\omega_{cdm}$ | 0.11998 ± 0.00095 | 0.12005 ± 0.00027 |
| $1000\theta$ | 1.04111 ± 0.00033 | 1.04111 ± 0.00031 |
| $\tau_{reio}$ | 0.08841 ± 0.0045 | 0.0891 ± 0.0031 |
| $\ln(10^{10}V_0)$ | 1.73211 ± 0.0089 | 1.7331 ± 0.0058 |
| $\phi_0$ | < 0.0197 | < 0.0197 |
| $\gamma$ | unbonded | unbonded |
| $\phi_T$ | < 7.81 | < 7.80 |
| $\ln \delta$ | -4.28 ± 0.93 | -4.40 ± 1.6 |
| $\Omega_{\Lambda}$ | 0.6804 ± 0.0060 | 0.6801 ± 0.0013 |
| $\Omega_m$ | 0.3195 ± 0.0060 | 0.3198 ± 0.0013 |
| $H_0$ | 66.84 ± 0.44 | 66.82 ± 0.13 |
| $\sigma_8$ | 0.8304 ± 0.0051 | 0.8313 ± 0.0018 |
MCMC parameter space with respect to the Concordance Model, which should induce weaker parameter constraints in this kind of fiducial-based forecast. The reason is simple: more free parameters means greater statistical uncertainty. However, we note that there is no significant degradation in the constraints on the background parameters in the feature models. Indeed, we note an improvement in constraints on the CDM density compared to Concordance Model in both Planck and Planck+Euclid. In the potential when we fix \( \mu \) in Equation 2, the spectral tilt generated by the inflation gets fixed. Therefore in the MCMC runs, variation in the tilt is not allowed and that reduces the degeneracies with the background parameter resulting in marginally improved constraints.

In Table 4 we present the constraints on the WWI potential when WWI-A is used as fiducial cosmology and in Figure 6 we plot the constraints only on the inflation potential parameters. The table reflects improvement in constraints on the background parameters with Euclid similar to the Concordance Model. However, we do not find any improvement in the potential parameters except \( V_0 \) which represents the amplitude of the primordial spectrum. Compared to the power law, WWI-A specifically improves the fit to the Planck data at low multipoles (\( \ell < 20 \)) with the large-scale suppression, and the dip at near \( k \sim 2 \times 10^{-3} \) MPc\(^{-1}\).
fits the $\ell \sim 22$ dip in the angular power spectrum. The power spectrum at these largest scales probed by Planck can not be constrained better with Euclid data as these scales are dominated by cosmic variance, and we can only expect an improvement with cosmic-variance-limited polarization surveys Hazra et al. (2018). The Euclid measurement error at the largest scales is dominated by statistical uncertainties due to cosmic variance. Cosmic-variance errors on the dark energy equation of state (Valkenburg et al. 2013) and the Hubble parameter (Marra et al. 2013) are particularly important. Here we assume a cosmological constant, so we are not concerned by the former. But the latter effect may degrade the constraints on our other parameter through their degeneracies with $H_0$. These limitations can be reduced by using multiple tracers with different biases. We should note that both Planck and Planck+Euclid reject the high amplitude (high $\phi_0$) sharp oscillations (low ln$\delta$) as we can note from their correlations. A transition at higher field value ($\phi_T$) implies the occurrence of features at small scales as the small-scale modes leave the Hubble radius at a higher $\phi_T$ values. We have strong constraints in the increasing direction of both $\phi_0 - \phi_T$ and $\gamma - \phi_T$. Therefore, we also find high amplitude oscillations and suppression are only allowed at large scales and up to certain intermediate scales. Euclid can marginally tighten the constraint on the frequency of the oscillation by constraining ln$\delta$.

The WWI-B, C and D fiducial models represent wiggles in the primordial spectrum within intermediate to smaller
scales ($\sim 0.1\text{Mpc}^{-1}$) as in the inset of Figure 2. Note that these spectra fall within the high signal-to-noise region of both Planck and Euclid. Constraints on WWI model when WWI-B, WWI-C and WWI-D are used as fiducials are tabulated in Table 5, Table 6, Table 7 respectively and corresponding posteriors and marginalized contours are plotted in Figure 7, Figure 8, Figure 9. Out of these three cases we note a remarkable improvement in constraints for the WWI-B case when Euclid is combined with Planck. We obtain significant detection of $\phi_0$ and 1-2$\sigma$ preference of a suppression ($\gamma$). Therefore, if WWI-B represents the true model of our Universe, Euclid will certainly establish this with high statistical significance when Planck CMB data is used in combination. Since out of the four WWI fiducials, WWI-B has the maximum large-scale suppression, using this fiducial leads to a marginal preference for $\gamma$. However, due to cosmic variance, it is not possible to get more than 2$\sigma$ preference with power spectrum. A detection of $\phi_0$ represents a detection of the wiggles in the primordial power spectrum and therefore we obtain the position of the potential transition ($\phi_T$) and the sharpness of the transition ($\ln\delta$) determined with high statistical significance as well. WWI-C has wiggles in the intermediate scales but these oscillations decay at the smaller scales ($k \sim 10^{-2}\text{Mpc}^{-1}$). This limited overlap with Euclid-probed cosmological scales reduces the chances of a detection of these features. When the WWI-D fiducial is used as mock data, Euclid improves the constraints on the inflationary parameters compared to Planck-only results.
We find that the constraints on the location and sharpness of the feature can experience a slight improvement. However, unlike WWI-B we will cannot expect any detection of features. In this case, although WWI-D has the best overlap between Planck and Euclid-probed cosmological scales, the amplitude of the oscillations is the lowest among the WWI models and therefore is less likely to be detected by Euclid. Note that with CORE-like surveys it was found that the WWI-D spectrum has the best chance of being detected Hazra et al. (2018). However, in this case Euclid is not expected to resolve the high frequency oscillations as they will be binned and averaged out in the observed power spectrum.

The WWIP potential has 3 parameters describing the primordial physics. Apart from the amplitude, determined by $V_0$, two other parameters $\phi_0$ and $\phi_T$ are responsible for the transition in the potential and therefore for features. Note that this model produces both suppression and wiggles. We have used three fiducial primordial spectra to generate the mock spectra for Euclid and Planck. Similar to WWI potential, we use a featureless fiducial generated with $\phi_0 = 0$. For the second fiducial we use the best fit to Planck temperature and polarization data (Hazra et al. 2016). Another point in the potential parameter space that is allowed within Planck 95% C.L. and generates features extended towards smaller scales ($k \sim 0.2$ in $\text{hMPc}^{-1}$) in the primordial spectrum (compared to WWIP:Planck-best-fit), is used here as mock data termed as WWIP:small-scale-feature. The con-

Figure 7. One-dimensional posteriors and marginalized contours for the inflation parameters in the WWI-B model. The improvement in constraints for all inflation parameters with the addition of Euclid data is evident in this plot.
Figure 8. One-dimensional posteriors and marginalized contours for the inflation parameters in the WWI–C model. Planck alone provides no constraints on $\gamma$. The addition of Euclid data results in an upper bound.

constraints on the background and inflationary parameters for these three cases are presented in Table 8, Table 9, Table 10 respectively. The one-dimensional posteriors and marginalized contours for the inflationary potential parameters are plotted in Figure 10, Figure 11 and Figure 12 respectively. Similar to the results obtained so far, we notice 40% improvement in the constraints on $V_0$ (in logscale). In the featureless case we notice 10% improvement by Euclid with respect to Planck on the upper bound on $\phi_0$ (the amplitude of the wiggles). In Table 8, we note that adding Euclid information shifts the mean value of $\phi_T$ closer to the upper edge of the prior boundary, so the binning method for the analysis of the MCMC chains is unable to find two-tailed marginalized limits, hence the single (lower-only) bound for $\phi_T$ obtained with Euclid+Planck. In the case where the WWIP:Planck-best-fit represents the true model of the Universe, we find only marginal improvement in $\phi_0$ as can be seen in Figure 11. The current Planck best fit for WWIP has oscillations in the large to intermediate scales ($k \sim 10^{-3} - 10^{-2}\text{MPc}^{-1}$). These range of scales are already well-probed by Planck and from Euclid we are only expected to see marginal improvement. However, if WWIP:small-scale-feature represents a true model of the Universe, we can expect 40% improvement in the constraints on $\phi_0$ leading to a detection of features with Euclid +Planck (Note that with Planck mock also we are able to rule out featureless spectrum with more than 4$\sigma$ C.L.). Since WWIP:small-scale-feature has oscillations with higher magnitude and they extend to smaller scales with
better overlap with *Euclid*-probed scales, this improvement is expected.

Analysis with these fiducials clearly establishes the contribution of *Euclid* data in constraining inflationary parameters, but the improvement in constraints varies. Projected constraints depend on the model and on the fiducial cosmology, due to the non-Gaussian nature of the posteriors. Since inflation features appear at particular scales, the overlap of these features with scale probed by *Euclid* determines the improvements in constraints with respect to *Planck* CMB data. In the cases where wiggles in the primordial spectrum are located in the intermediate to small scales \((k \sim 10^{-3} - 10^{-1} \text{Mpc}^{-1})\), *Euclid* can play a significant role in detection along with *Planck* CMB.

A consistent feature of the results for all models is the improvement in constraints on the amplitude of the matter power spectrum \((A_s\) in the Concordance Model, or through \(V_0\) in WWI). *Euclid* spectroscopy provides better redshift resolution, which results in a better measurement of the redshift-space distortion signal. It therefore breaks the degeneracy between the bias parameter and the amplitude of the power spectrum, leading to better constraints on the amplitude.

5 CONCLUSIONS

For the first time, we present accurate and realistic forecasts for *Euclid* cosmic shear and galaxy clustering based on MCMC simulations for features in the primordial power
spectrum. We use a ΛCDM background cosmology including massive neutrinos. The features in the primordial power spectra that we consider are both local and non-local in nature and we use the Wiggly Whipped Inflation framework, which can produce different kinds of features relevant to the CMB data observed by Planck. With a discontinuity either in the potential or in its derivative, WWI provides large-scale suppression, localized and non-local oscillations. Our results compliment those in Huang et al. (2012) and Ballardini et al. (2016), where an MCMC-based forecast was carried out for galaxy clustering data from different probes including Euclid. Ballardini et al. (2018) report similar results based on Fisher analysis. We use up-to-date specifications for the Euclid survey, and recently-published likelihoods for cosmic shear and galaxy clustering, with a conservative model for the theoretical error in the nonlinear spectrum, with a redshift-dependent cut-off at $0.2\,h\text{Mpc}^{-1}$. By using this realistic error model with MCMC simulations, we are free from the assumption of Gaussianity which is the basis of Fisher analysis. For this reason, Fisher analysis tends to underestimate the error bounds.

Our results are in broad agreement with other studies such as (Audren et al. 2013; Sprenger et al. 2019). We show that the addition of Euclid data tightens the cosmological parameter constraints obtained by Planck alone, even with a ‘conservative’ setup for the non-linear uncertainties. The results strengthen the scientific case for Euclid and the use of multiple probes to exploit synergies and break parameter degeneracies.

Our conservative forecast establishes that MCMC is
necessary tool in the detection of features in the primordial power spectrum, as well as in obtaining accurate forecasts. Simulated data is a single realisation of the true Universe, and we find a significant change in the inflation parameter constraints depending on the amplitude and occurrence of features at different cosmological scales. This hints at a strong parameter dependence of the joint Planck-Euclid covariance matrix in the inflation sector, which means that the Fisher matrix approach may not be sufficiently accurate. This parameter dependence merits further study (see e.g. Heavens et al. 2017; Kodwani et al. 2019; and, for the Euclid dark energy Figure-of-Merit, Debono 2014; Schäfer & Reischke 2016).

Our main findings are:

The Euclid cosmic shear and galaxy clustering likelihoods and error modelling by (Sprenger et al. 2019) perform well with cosmological models containing features in the primordial power spectrum.

We find significant improvement in the constraints on the background parameters when Euclid is used with Planck compared to Planck alone. The improvement for \( \Omega_b h^2 \) is marginal, since it is already well-constrained by Planck.

Using cosmic shear with galaxy clustering, Euclid is expected to improve the bounds on the primordial spectrum amplitude and tilt by 30%-40% compared to Planck when the power-law form is used for the spectrum. Due to these tighter constraints, we also find indirect tighter bounds on the reionization optical depth.

When WWI models of inflation are used for the forecast,
Euclid: inflation features

Figure 12. One-dimensional posteriors and marginalized contours for the inflation parameters in the WWIP:Small-scale-feature model. We obtain closed contours for all the inflation parameters, with a significant improvement with Euclid data are added.

the scale of the potential, $V_0$ gets tightly constrained in all the cases, as the amplitude of the perturbation spectrum is directly dependent on $V_0$. The constraints on the background cosmology parameters are not significantly affected by the presence of features in the primordial power spectrum.

Features that are present at larger scales compared to our conservative large scale cutoff for Euclid ($0.02\, \text{Mpc}^{-1}$), cannot be constrained better than Planck with ‘conservative’ theoretical error for Euclid. However we notice marginal improvement in cases where due to better constraints on background and amplitude parameters that helps to reduce certain residual degeneracies.

Oscillations that are present at intermediate and small scales ($k \sim 0.02 - 0.2\, \text{Mpc}^{-1}$) in the power spectrum with $\sim 2\%$ amplitude with respect to the featureless spectrum, have a high probability of being detected with high statistical significance with combined Euclid and Planck data, if they represent the true model of the Universe.

The contribution of Euclid data to the detection of small-scale, high-frequency features is limited.

Our work validates the scientific potential of Euclid by contributing three main results.

First, our forecasts show that Euclid improves constraints in the background cosmology sector, even in the presence of features in the inflation potential.

Second, we show that Euclid data improves constraints in the overall scale of the slow-roll potential. This widens the
Table 6. 1σ confidence intervals for cosmological parameters with WWI–C as the fiducial cosmology.

| Parameter          | Planck   | Euclid+Planck |
|--------------------|----------|---------------|
| $10^{-2} \omega_b$ | 2.210 ± 0.014 | 2.212 ± 0.011 |
| $\omega_{cdm}$     | 0.12013 ± 0.00089 | 0.12004 ± 0.00027 |
| $1000h$            | 1.04111 ± 0.00032 | 1.04112 ± 0.00031 |
| $\tau_{rela}$      | 0.0889 ± 0.0046 | 0.0894 ± 0.0031 |
| $\ln(10^{10}V_0)$ | 1.7176 ± 0.0919 | 1.7186 ± 0.059 |
| $\phi_0$           | < 0.00844 | < 0.00868 |
| $\gamma$           | < 0.0919 |          |
| $\phi_T$           | > -7.63  | > -7.00 |
| $\Omega_{\Lambda}$ | 0.6795 ± 0.0057 | 0.6802 ± 0.0013 |
| $\Omega_m$         | 0.302 ± 0.0057 | 0.3019 ± 0.0013 |
| $H_0$              | 66.79 ± 0.42 | 66.83 ± 0.13 |
| $\sigma_8$         | 0.8249 ± 0.0051 | 0.8248 ± 0.0019 |

Table 7. 1σ confidence intervals for cosmological parameters with WWI–D as the fiducial cosmology.

| Parameter          | Planck   | Euclid+Planck |
|--------------------|----------|---------------|
| $10^{-2} \omega_b$ | 2.204 ± 0.015 | 2.207 ± 0.011 |
| $\omega_{cdm}$     | 0.12060 ± 0.00093 | 0.12037 ± 0.00028 |
| $1000h$            | 1.04095 ± 0.00032 | 1.04095 ± 0.00031 |
| $\tau_{rela}$      | 0.0868 ± 0.0056 | 0.0884 ± 0.0033 |
| $\ln(10^{10}V_0)$ | 1.751 ± 0.011 | 1.7537 ± 0.0060 |
| $\phi_0$           | < 0.00756 | < 0.00815 |
| $\gamma$           | 0.071± -0.034 | 0.072 ± 0.051 |
| $\phi_T$           | < 7.89   | 7.8481 ± 2.1 |
| $\ln \delta$      | > -6.66  | > -5.49 ± 0.36 |
| $\Omega_{\Lambda}$ | 0.6760 ± 0.0060 | 0.6776 ± 0.0014 |
| $\Omega_m$         | 0.3229 ± 0.0060 | 0.3223 ± 0.0014 |
| $H_0$              | 66.52 ± 0.45 | 66.63 ± 0.13 |
| $\sigma_8$         | 0.8402 ± 0.0057 | 0.8406 ± 0.0019 |

Table 8. 1σ confidence intervals for the inflation parameters in the WWIP–Featureless model.

| Parameter          | Planck   | Euclid+Planck |
|--------------------|----------|---------------|
| $10^{-2} \omega_b$ | 2.211 ± 0.014 | 2.212 ± 0.011 |
| $\omega_{cdm}$     | 0.12012 ± 0.00089 | 0.11992 ± 0.00027 |
| $1000h$            | 1.04115 ± 0.00033 | 1.04116 ± 0.00031 |
| $\tau_{rela}$      | 0.0918 ± 0.0045 | 0.0918 ± 0.0028 |
| $\ln(10^{10}V_0)$ | 0.2857 ± 0.0051 | 0.2839 ± 0.0057 |
| $\phi_0$           | < 0.450   | < 0.416 |
| $\phi_T$           | 4.590 ± 0.044 | > 4.57 |
| $\Omega_{\Lambda}$ | 0.6798 ± 0.0057 | 0.6811 ± 0.0013 |
| $\Omega_m$         | 0.3201 ± 0.0057 | 0.3188 ± 0.0013 |
| $H_0$              | 66.81 ± 0.42 | 66.90 ± 0.13 |
| $\sigma_8$         | 0.8352 ± 0.0049 | 0.8337 ± 0.0018 |

Table 9. 1σ confidence intervals for cosmological parameters with WWIP–Planck-best-fit as the fiducial cosmology.

| Parameter          | Planck   | Euclid+Planck |
|--------------------|----------|---------------|
| $10^{-2} \omega_b$ | 2.209 ± 0.014 | 2.209 ± 0.012 |
| $\omega_{cdm}$     | 0.12010 ± 0.00089 | 0.12009 ± 0.00030 |
| $1000h$            | 1.04109 ± 0.00032 | 1.04112 ± 0.00030 |
| $\tau_{rela}$      | 0.0860 ± 0.0044 | 0.0883 ± 0.0030 |
| $\ln(10^{10}V_0)$ | 0.2743 ± 0.0087 | 0.2790 ± 0.0056 |
| $\phi_0$           | < 0.435   | < 0.424 |
| $\phi_T$           | > 4.57    | > 4.57 |
| $\Omega_{\Lambda}$ | 0.6797 ± 0.0057 | 0.6798 ± 0.0013 |
| $\Omega_m$         | 0.3203 ± 0.0057 | 0.3201 ± 0.0013 |
| $H_0$              | 66.79 ± 0.41 | 66.80 ± 0.12 |
| $\sigma_8$         | 0.8307 ± 0.0050 | 0.8327 ± 0.0018 |

Table 10. 1σ confidence intervals for cosmological parameters with WWIP–Small-scale-feature as the fiducial cosmology.

| Parameter          | Planck   | Planck+Euclid |
|--------------------|----------|---------------|
| $10^{-2} \omega_b$ | 2.210 ± 0.014 | 2.210 ± 0.010 |
| $\omega_{cdm}$     | 0.11999 ± 0.00090 | 0.12001 ± 0.00026 |
| $1000h$            | 1.04111 ± 0.00033 | 1.04111 ± 0.00030 |
| $\tau_{rela}$      | 0.0903 ± 0.0054 | 0.0899 ± 0.0029 |
| $\ln(10^{10}V_0)$ | 0.301 ± 0.011 | 0.3000 ± 0.0054 |
| $\phi_0$           | 0.177 ± 0.037 | 0.178 ± 0.023 |
| $\phi_T$           | 4.50246 ± 0.00043 | 4.50245 ± 0.00017 |
| $\Omega_{\Lambda}$ | 0.6804 ± 0.0057 | 0.6803 ± 0.0012 |
| $\Omega_m$         | 0.3196 ± 0.00057 | 0.3196 ± 0.00012 |
| $H_0$              | 66.84 ± 0.42 | 66.84 ± 0.12 |
| $\sigma_8$         | 0.8413 ± 0.00053 | 0.8408 ± 0.00018 |

scientific scope of Euclid beyond the original dark energy and neutrino sectors.

Third, we provide the first Euclid forecasts using MCMC for the fine shape of the inflation potential in the presence of features. As the next generation of probes are activated, and the synergies between probes exploited, this is likely to become an important area of research. Planck pushed the boundaries of CMB observations to the point where future experiments cannot do better by using the same data, so new experiment must turn to new observables (CMB polarization, CMB lensing, and other secondary effects). Euclid’s ability to probe the primordial Universe though large-scale structure is a major milestone in observational cosmology. This work underlines the need for multiple probes in order to explore all cosmological scales as fully as possible.

As Euclid likelihood modelling is improved closer to the launch date, it will be straightforward to obtain more accurate forecasts using our pipeline. Further improvements include, but are not limited to: better modelling of the non-linear theoretical error, especially in the presence of features in the primordial power spectrum; more accurate galaxy number counts; improved modelling of massive neutrino effects. The availability of simulated data from Wiggly Whipped Model would open up new avenues of research. Additional constraining power can be provided by independent probes such as SKA 21-cm intensity mapping from reionization, from priors on the Hubble parameter through supernovae, and also from cross-correlations between galaxy clustering and cosmic shear. The calculation of power spectra from the inflation potential provides new opportunities for testing competing models, and for Bayesian model selection with Euclid.

MNARAS 000, 1–22 (2020)
