s-Vector Control of Battery Energy Storage System: Definition and Application

Zhao Yuan, Member, IEEE

Abstract—We give the definition of s-Vector control by using the complex power vector in the p-q plane to quantify the feasible operational region of battery energy storage system (BESS) and to find the optimal power set-point solution. The s-Vector control features in the avoidance of using costly optimization solvers to solve optimization models online which is also an execution time bottleneck in real-time control. The advantages of s-Vector control are fast and stable computational efficiency. We validate a s-Vector based real-time controller of BESS to provide frequency response and voltage support as ancillary services for the power grid. The objective is to maximize the utility of the BESS asset. We formulate the dynamic capability curve of the DC-AC converter and the security requirements of the battery cells as constrains of the control system. The initial power set-points are obtained based on the traditional droop control approach. Based on s-Vector control, a fast optimization solution algorithm is proposed to find the optimal power set-point and guarantee the security. According to the experimental validation in our 720 kVA / 560 kWh BESS on EPFL campus, we achieve 100 ms of refreshing the real-time control loop. As a benchmark, using optimization solver requires 200 ms to update the real-time control loop.

Index Terms—s-Vector Control, Battery Energy Storage System, Real-time Control, Optimization, Ancillary Service.

NOMENCLATURE

Parameters:

- $\Delta t$: Time step interval.
- $P_{AC,i}^0$: Initial active power set-point.
- $Q_{AC,i}^0$: Initial reactive power set-point.
- $K_{max}$: Maximum number of iterations.
- $V_{DC}^t$: Voltage on the DC-bus.
- $f_t$: Grid frequency measurement.
- $V_{AC,nominal}^t$: Nominal grid voltage.
- $\Delta f$: Deviation of grid frequency measurement.
- $\Delta V_{AC}^t$: Deviation of grid voltage measurement.
- $\alpha_0$: Grid frequency droop coefficient.
- $\beta_0$: Grid voltage droop coefficient.
- $SOC_{min}^t$: Lower bound of $SOC$.
- $SOC_{max}^t$: Upper bounds of $SOC$.
- $C_{max}$: Maximum storage capacity of the battery.
- $\eta$: Efficiency of the DC-AC converter.
- $p_{DC,\text{min}}^t$: Lower bound of $p_{DC}^t$.
- $p_{DC,\text{max}}^t$: Upper bound of $p_{DC}^t$.
- $q_{AC,i}^0$: Initial reactive power set-point.
- $\theta_{AC,i}$: Complex power at angle $\theta$.
- $\theta_{0}$: Angle of the initial complex power set-point.
- $\theta_{0,\text{min}}$: Lower bound of $\theta_0$.
- $\theta_{0,\text{max}}$: Upper bound of $\theta_0$.

Variables:

- $t$: Time step.
- $\theta$: Angle.
- $i$: Index of equality constraints.
- $j$: Index of inequality constraints.
- $k$: Iteration.

Set:

- $\mathcal{R}$: Feasible operational region.
- $\partial \mathcal{R}$: Boundary of $\mathcal{R}$.
- $\Omega$: Decision variables.
- $\Theta$: Angles.

Index:

- $t$: Time step.
- $\theta$: Angle.

Zhao Yuan is with the Distributed Electrical Systems Laboratory (DESIL), École Polytechnique Fédérale de Lausanne, Switzerland (EPFL), e-mail: zhaoyuan@epfl.ch.

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Recent decades witness a dramatic advancement of battery and grid converter technology [1]–[6]. Both are key components of the BESS for power system applications. For the battery, the pioneering works by Nobel laureates M. Stanley Whittingham, John B. Goodenough and Akira Yoshino revolutionize the lithium-ion battery technology in terms of energy density, safety, voltage level and lifetime [1]–[3]. Since the year of 2010, the price of lithium-ion battery has dropped over 70% [7]. Although these developments are mostly driven by portable electronics [8], the applications of BESS in power system are expanding rapidly [9]–[13]. For the grid converter, the increasing power capacity and switching frequency of solid-state switching devices, circuit topology as well as control technology are extensively improving the power quality, power rating and applicability of BESS in the power system [4]–[6]. The ultra-fast response from BESS is a feasible solution to balance the fluctuation of renewable energy resource through real-time regulation, frequency response and voltage support [13]–[17]. Moreover, BESS plays an important role in solving the challenges of the growing distributed energy resources (DERs) in power distribution networks [18], [19].
The real-time control of BESS is generally formulated in an optimization model and then solved by available optimization solvers or iterative approaches [19]–[23]. [19] uses distributed optimization algorithm i.e. the alternating direction method of multipliers (ADMM) within a model predictive control (MPC) framework to operate the microgrid with energy storage as one type of the considered DERs. The distributed optimization algorithm requires only local information of the DERs to coordinate with each other and converge to the optimal solution very close to the solution using centralized optimization [19]. Authors in [21] model the real-time control of a hybrid energy storage system including battery and super capacitor in a convex optimization problem. To solve the optimization problem in [21], the optimization solver SeDumi is deployed [20]. The daily scheduling of BESS is optimized in [22] using generic algorithm (GA) to minimize operation cost considering day-ahead market prices. An adaptive dynamic programming method is proposed and validated in [23] to coordinate the charging of multiple batteries. Under mild assumptions about the state space and some parameters, the authors in [23] prove the convergence of the method and the bound of the used approximations by fuzzy systems. By using the functional analogy between the data storage in computer system and energy storage in power system, [24] designs a cache control to coordinate the operation of multiple levels of energy storage in the power system. This cache control supports plug-and-play of energy storage system as a module integrated in the power system [24]. [25] proposes a two-stage framework to schedule the energy storage in the power system with renewable energy resources. The schedule of energy storage is obtained from the solution of the first-stage stochastic unit commitment model and then limit the look-ahead horizon in the second-stage of the proposed framework [25]. An energy-conservation method is proposed in [26] to integrate the two controllers of BESS for photovoltaic capacity firming and energy time shift into one controller. The proposed method in [26] also relies on solving an optimization routine. Real-time control of BESS without relying on optimization solvers has not been well addressed in the literature.

The contributions of this paper are three-fold: (1) definition of s-Vector control; (2) design of the real-time controller of BESS based on s-Vector control; (3) numerical and experimental validations. The rest of this paper is organized as follows. Section II gives the definition of s-Vector control. Section III presents the optimization model of the real-time control of BESS to provide grid ancillary services. Section IV derives the real-time control algorithm based on s-Vector control for the BESS. Section V shows the numerical results. Section VI shows the experiments and analysis. Section VII concludes.

II. DEFINITION OF s-VECTOR CONTROL

For the mathematical symbols, we denote known parameters or measurements in upper case (except the frequency measurement and nominal value) and unknown variables in lower case. To distinguish with scalars, we denote vectors in bold.

Definition 1. Feasible Operational Region. The feasible operational region \( \mathcal{R} \) of an energy resource is defined as the set of all the feasible power set-points \((p_{AC}^t, q_{AC}^t)\) satisfying all the operational constraints of the energy resource.

If the initial power set-point from the operator of the energy resource (e.g. BESS) is \((P_{AC}^t, Q_{AC}^t)\), the optimal power set-point can be found by solving following optimization problem (1a)-(1b):

\[
\begin{align*}
& \text{Minimize} \quad (p_{AC}^t - P_{AC}^t)^2 + (q_{AC}^t - Q_{AC}^t)^2 \\
& \text{subject to} \quad (p_{AC}^t, q_{AC}^t) \in \mathcal{R}
\end{align*}
\]

Where \( \Omega = \{p_{AC}^t, q_{AC}^t\} \) is the set of decision variables. The optimal solution of the power set-point \((p_{AC}^t, q_{AC}^t)\) is the power set-point closest to the initial power set-point \((P_{AC}^t, Q_{AC}^t)\) inside the feasible operational region of the energy resource. If the initial power set-point \((P_{AC}^t, Q_{AC}^t)\) is already inside the feasible operational region of the energy resource, the optimal solution of power set-point \((p_{AC}^t, q_{AC}^t)\) is equal to \((P_{AC}^t, Q_{AC}^t)\). Otherwise, if the initial power set-point \((P_{AC}^t, Q_{AC}^t)\) is not feasible for the energy resource, the optimal solution of power set-point \((p_{AC}^t, q_{AC}^t)\) is a projection of \((P_{AC}^t, Q_{AC}^t)\) to the boundary of the feasible operational region of the energy resource. If \( \mathcal{R} \) is convex, the optimization model (1a)-(1b) is convex since we are minimizing a convex objective function over a convex feasible region.

Definition 2. s-Vector. s-Vector is defined as the complex power vector \( s_{AC}^t = (p_{AC}^t, q_{AC}^t) \) in the p-q (active power - reactive power) plane.

Definition 3. s-Vector Control. s-Vector control is defined as the control of an energy resource using the s-Vector \( s_{AC}^t \) to quantify the feasible operational region of the energy resource.

Screening the s-Vector over the feasible operation region \( \mathcal{R} \) can quantify the size of \( \mathcal{R} \) which is very useful in the real-time control of the energy resource. If \((0, 0) \in \mathcal{R} \) and \( \mathcal{R} \) is convex and bounded, the length of s-Vector at \( \tan(\theta) = \frac{q_{AC}^t}{p_{AC}^t} \) can be found by solving following optimization problem [(1b), (1c)-(1i)]. Note here \( \theta \in (0^\circ, 360^\circ) \) is given as a parameter.

\[
\begin{align*}
& \text{Maximize} \quad (p_{AC}^t)^2 + (q_{AC}^t)^2 \\
& \text{subject to} \quad (1b)
\end{align*}
\]
Theorem 1. If (1) \( \mathcal{R} = \{(p_t^{AC}, q_t^{AC}) | (p_t^{AC}, q_t^{AC}) \in [s_{i-1}^{AC}, q_t^{AC}] = 0, g_{j-1}(p_t^{AC}, q_t^{AC}) \leq 0\} \) is convex and bounded; (2) \( \mathcal{R} = \{(p_t^{AC}, q_t^{AC}) | (p_t^{AC}, q_t^{AC}) \in [s_t^{AC} = 0, g_{j-1}(p_t^{AC}, q_t^{AC}) = 0\} \) is the boundary of \( \mathcal{R} \) and \( 0 \notin \mathcal{R} \), the optimal solution of optimization problem [(1b), (1c)-(1i)] is equal to the solution of [(1b), (1d)-(1i), \( \mathcal{R} \)]. In other words, solving the optimization problem [(1b), (1c)-(1i)] is equivalent to solving the equations [(1b), (1d)-(1i)] where there is no objective function.

Proof. We prove theorem 1 by contradiction. Since \( 0 \in \mathcal{R} \) and \( \mathcal{R} \) is convex and bounded, the optimization problem [(1b), (1c)-(1i)] and equations [(1b), (1c)-(1i), \( \mathcal{R} \)] are always feasible or solvable. Moreover, because \( \mathcal{R} \) is convex, the solution of equations [(1b), (1c)-(1i)] is unique (otherwise \( \mathcal{R} \) is concave). We denote the optimal solution of optimization problem [(1b), (1c)-(1i)] as \( (p_t^{AC}, q_t^{AC}) \). The associated s-Vector is \( s_t^{AC} \). The solution of equations [(1b), (1c)-(1i), \( \mathcal{R} \)] is denoted as \( (p_t^{AC}, q_t^{AC}) \). The associated s-Vector is \( s_t^{AC} \). Since \( (p_t^{AC}, q_t^{AC}) \) gives the maximal objective function value for the optimization problem [(1b), (1c)-(1i)] and \( (p_t^{AC}, q_t^{AC}) \) is unique, we know \( |s_t^{AC}| > |s_t^{AC}| \). Assume \( (p_t^{AC}, q_t^{AC}) \neq (p_t^{AC}, q_t^{AC}) \), because \( \mathcal{R} \) is convex and \( 0 \in \mathcal{R} \), there exists \( 0 < \lambda < 1 \) such that:

\[
|s_t^{AC}| = \lambda s_t^{AC} + (1 - \lambda)0 \tag{1j}
\]

Or:

\[
(p_t^{AC}, q_t^{AC}) = \lambda(p_t^{AC}, q_t^{AC}) + (1 - \lambda)(0, 0) \tag{1k}
\]

This contradicts that \( (p_t^{AC}, q_t^{AC}) \) is the solution of equations [(1b), (1c)-(1i), \( \mathcal{R} \)] which means it is on the boundary of \( \mathcal{R} \). So \( (p_t^{AC}, q_t^{AC}) \neq (p_t^{AC}, q_t^{AC}) \) cannot hold.

Theorem 1 shows that we can find the maximum length of the s-Vector by solving equations [(1b), (1d)-(1i), \( \mathcal{R} \)]. This can be done by using either Newton–Raphson or Gauss-Seidel method. We propose following heuristic solution algorithm 1 which can also avoid the usage of optimization solvers.

Algorithm 1: Find the Maximum Length of s-Vector \( s_t^{AC} \)

**Result:** Maximum Length of S-Vector \( s_t^{AC} \)

**Initialization:**
1. Set \( s_t^{AC} = (p_t^{AC}, q_t^{AC}) = (0, 0) \); 
2. Set \( \Delta s_t^{AC} = (\Delta p_t^{AC}, \Delta q_t^{AC}) \); 
3. Set \( k = 0 \); 
4. while Constraints [(1b), (1d)-(1i), \( \mathcal{R} \)] are valid and \( k < k_{max} \) do:
   1. \( s_t^{AC} = s_t^{AC} + \Delta s_t^{AC} \); 
   2. \( k = k + 1 \); 
   3. \( \Delta s_t^{AC} = \frac{\Delta s_t^{AC}}{k} \); 
end

Where \( \Delta s_t^{AC} > 0 \) is the increment step of \( s_t^{AC} \). \( \Delta p_t^{AC}, \Delta q_t^{AC} \) is the increment step of active power and reactive power. \( k \) is the index of iteration. \( k_{max} \) is the maximum allowed number of iterations. These are the parameters of the algorithm 1 which are set a priori. As we have mentioned before, \( \tan(\theta) = \frac{\Delta q_t^{AC}}{\Delta p_t^{AC}} \). The feasibility check of constraints [(1b), (1d)-(1i), \( \mathcal{R} \)] can be done by substituting the value of \( (p_t^{AC}, q_t^{AC}) \) in these constraints. The increment step \( \Delta s_t^{AC} \) can be set to a large value at the beginning of the iteration and reduce it in later iterations as we have shown in algorithm 1 in \( \Delta s_t^{AC} = \frac{\Delta s_t^{AC}}{k} \). Other approaches to reduce \( \Delta s_t^{AC} \) are also applicable. The solution algorithm 1 is executed offline and when all the maximum lengths of s-Vector for all considered \( \theta \) are obtained, we can use these values in the real-time control of the energy resource to guarantee the feasibility of all the operational constraints. In this way, we avoid the usage of any optimization solver in real-time control which stabilizes the computational performance. It is worth to mentioning that algorithm 1 can only be used if the feasible operational region \( \mathcal{R} \) is convex and \( 0 \notin \mathcal{R} \). We show the application of s-Vector control in the next sections of this paper.

III. OPTIMIZATION MODEL OF THE REAL-TIME CONTROL FOR BESS

We use lower case to denote variables and upper case to denote parameters. The real-time control of BESS is formulated as the following optimization problem (2a)-(2f):

\[
\begin{align*}
& \text{Minimize} \quad \hat{f}(\hat{p}_t^{AC} - P_0^{AC})^2 + (\hat{q}_t^{AC} - Q_0^{AC})^2 \quad \text{(2a)} \\
& \text{subject to} \quad \frac{\hat{q}_t^{AC}}{\hat{p}_t^{AC}} = \frac{Q_0^{AC}}{P_0^{AC}} \quad \text{or} \quad \hat{q}_t^{AC} P_0^{AC} = \hat{p}_t^{AC} Q_0^{AC} \quad \text{(2b)} \\
& \quad C_{AC}(\hat{p}_t^{AC}, \hat{q}_t^{AC}, V_t^{DC}, V_t^{AC}) \leq 0 \quad \text{(2c)} \\
& \quad \text{soc}_{t+1} = \text{soc}_t + \int_{t}^{t+1} \frac{\hat{p}_t^{DC}}{C_{max}} dt \quad \text{(2d)} \\
& \quad \text{soc}_{t_{min}} \leq \text{soc}_t \leq \text{soc}_{t_{max}} \quad \text{(2e)} \\
& \quad \hat{p}_t^{DC} = \begin{cases} \eta P_0^{AC}, & P_0^{AC} > 0 \\ \frac{P_0^{AC}}{\eta}, & P_0^{AC} \leq 0 \end{cases} \quad \text{(2f)}
\end{align*}
\]

Where \( \Omega = \{\hat{p}_t^{AC}, \hat{q}_t^{AC}\} \) is the set of decision variables i.e. the active power and reactive power set-points. \( \Phi = \{(P_0^{AC}, Q_0^{AC}, V_t^{DC}, V_t^{AC}, \eta, \text{soc}_t)\} \) is the set of parameters from a priori calculations or measurements. \( (P_0^{AC}, Q_0^{AC}) \) are the initial active power and reactive power set-points. Constraint (2b) maintains the power factor of the power set-point. Constraint (2c) is the dynamic capability of the DC-AC converter connecting the BESS through a 300 V / 20 kV transformer to the EPFL campus power grid. One of the dynamic capability curves is shown in Fig. 1. We use a series of linear and quadratic functions to depict the actual capability of the DC-AC converter (the original measurement data-set is from the converter provider and we interpolate the data-set). \( V_t^{DC} \) is the voltage measurement on the DC-bus. \( V_t^{AC} \) is the voltage measurement at the AC side. Constraint (2d) represents the dynamics of the battery state of charge \( \text{soc}_t \) which is bounded by constraint (2e). \( \text{soc}_t \) is the battery state of charge. \( \text{soc}_{t_{min}}, \text{soc}_{t_{max}} \) are the lower and upper bounds of \( \text{soc}_t \). \( i_{t_{DC}}^{max} \) is the current on the DC-bus. \( C_{max}^{max} \) is the maximum storage capacity of the battery in ampere-hour or Ah. Constraint (2f) relates the active power on the DC-bus \( \hat{p}_t^{DC} \) to the active power at the AC side \( p_t^{AC} \). \( \eta = 97\% \)
is the efficiency of the DC-AC converter. The initial power set-points \( \left( P_{0,t}^{AC}, Q_{0,t}^{AC} \right) \) are from the command of the BESS operator. Typically in the droop control of frequency response and voltage support, equations (2g)-(2h) are used to determine \( \left( P_{0,t}^{AC}, Q_{0,t}^{AC} \right) \). The parameters \( \alpha_0 = 9 \text{ MW/Hz}, \beta_0 = 8.39 \text{ kVar/V} \) are the initial droop coefficients (see [27] for the estimation of these parameters). \( \Delta f_t = f_t - f_{nominal} \) is the power grid frequency deviation from the nominal frequency \( f_{nominal} = 50 \text{ Hz} \). \( \Delta_{\min} f_t \) is the dead-band of frequency deviation. \( \Delta V_{AC} = V_{AC} - V_{AC,nominal} \) is the power grid AC voltage magnitude deviation from the nominal voltage \( V_{AC,nominal} = 300 \text{ V} \) (line voltage). \( \Delta_{\min} V_{AC} = 10 \text{ mV} \) is the dead-band of grid voltage deviation.

\[
P_{0,t}^{AC} = \begin{cases} \alpha_0 \Delta f_t, & \forall |\Delta f_t| > \Delta_{\min} f_t \\ 0, & \forall |\Delta f_t| \leq \Delta_{\min} f_t \end{cases} \quad (2g)
\]

\[
Q_{0,t}^{AC} = \begin{cases} \beta_0 \Delta V_{AC}, & \forall |\Delta V_{AC}| > \Delta_{\min} V_{AC} \\ 0, & \forall |\Delta V_{AC}| \leq \Delta_{\min} V_{AC} \end{cases} \quad (2h)
\]

This optimization model is convex because: (1) the objective is to minimize a convex function; (2) constraint (2c) is a convex set; (3) constraint (2d) is linear (note we are using the final approximation expression); (4) constraint (2e) is linear; (5) constraint (2f) is formulated according to the initial power set-points \( \left( P_{0,t}^{AC}, Q_{0,t}^{AC} \right) \) and it is linear.

IV. S-VectoR CONTROL OF BESS TO PROVIDE Ancillary Services

Although solving optimization problem (2a)-(2f) directly by using convex optimization solvers is fast within 65 ms computational time for the most cases during the operation, the computational time can be over longer (rarely happens according to our experiments explained in the next section) which burdens the real-time control system. We propose a new solution algorithm based on s-Vector control. This new algorithm gives faster and more stable computational efficiency performance. Another motivation of proposing the new solution algorithm is due to the availability of convex optimization solvers. These solvers may be costly for some operators and thus cannot be used. Firstly, we consider the state of charge constraints of the battery and reformulate constraints (2d)-(2e) equivalently to constraints (3a)-(3b).

\[
p_t^{DC} \geq \frac{(SOC_{min}^{t} - SOC_{t})C_{max}^{t}}{\Delta t} = p_t^{DC, min} \quad (3a)
\]

\[
p_t^{DC} \leq \frac{(SOC_{max}^{t} - SOC_{t})C_{max}^{t}}{\Delta t} = p_t^{DC, max} \quad (3b)
\]

Note \( p_t^{DC, min} \leq 0 \) since \( SOC_{min} - SOC_{t} \leq 0 \), \( p_t^{DC, max} \geq 0 \) since \( SOC_{t} - SOC_{min} \geq 0 \). These relationships are important in deriving our improved optimization solution algorithm 2 explained later.

Secondly, we consider the converter capability curve constraint by investigating the maximum length (Euclidean norm) of the s-Vector i.e. \( |s_{t}^{AC}| = \left| \left[ p_{t}^{AC}, q_{t}^{AC} \right] \right| \). We define \( \theta = \arctan \left( \frac{q_{t}^{AC}}{p_{t}^{AC}} \right) \) as an index to discretize the s-Vector \( |s_{t}^{AC}| \) and calculate the maximum length of \( |s_{t}^{AC}| \) at a resolution of \( \Delta \theta = 1^\circ \) or 0.017 rad off-line. So in total the number of \( |s_{t}^{AC}| \) values that we can get is 360. Since the converter capability curve is convex, \( |S_{t,0}^{AC}| \) can be calculated by solving the following optimization problem off-line:

\[
\text{Maximize} \quad (p_t^{AC})^2 + (q_t^{AC})^2 \quad (3c)
\]

subject to \( (1d) - (1i), (2c) \) \( (3d) \)

Where \( \theta_{min} = |\theta_0| \in \Theta \) is the lower bound of \( \theta_0 \). \( \theta_{max} \) is the floor function returning the maximum integer smaller than \* \( \theta_0 \in \{ 1^\circ, 2^\circ \ldots 360^\circ \} \) (rad) is given as a known parameter in this optimization model. Note that smaller resolution of \( \Delta \theta \) can also be used to give more precise results. Constraint (1i) considers the valid argument domain of the tangent function \( \tan(\theta) \). As we have mentioned in Section II, this optimization problem can be equivalently solved by using algorithm 1. In order to make sure the power set-point \( \left( p_t^{AC}, q_t^{AC} \right) \) satisfies the converter capability curve, we can compare the length of the initial s-Vector \( |S_{t,0}^{AC}| = \left| \left[ P_{0,t}^{AC}, Q_{0,t}^{AC} \right] \right| \) with the corresponding maximum s-Vector \( S_{t,01}^{AC} \left| \left[ P_{t,01}^{AC}, Q_{t,01}^{AC} \right] \right| \) at \( \theta 
\]

\[
|s_t^{AC}| \leq |S_{t,01}^{AC}| \quad (3e)
\]

Otherwise, the constraint is:

\[
|s_t^{AC}| \leq \max \left\{ |S_{t,0}^{AC}|, |S_{t,0max}^{AC}| \right\} \quad (3f)
\]

Note that \( p_t^{DC} \leq |S_{t,0}^{AC}| \) and \( q_t^{DC} \leq |S_{t,0max}^{AC}| \), this approach is illustrated in Fig. 1 where we use one of the converter dynamic capability curve as an example (\( V_{DC}^{AC} = 300 \text{ V}, V_{AC}^{AC} = 600 \text{ V} \)). There are in total six different fitted functions distinguished by different colours for different sections in this capability curve. Note in Fig. 1 we exaggerate the space between \( s_t^{AC} \) and \( S_{t,0min}^{AC} \) for ease of illustrating the approach. The actual space between them is very small since we are using very small resolution of \( \Delta \theta \). A more conservative approach compared with constraint
is to use constraint (3g). This will guarantee the secure operation of BESS but sacrifices a bit more the utility of the BESS asset compared with the approach in (3f).

\[
|s^AC_t| \leq \min \left\{ |S^AC_{t,\theta^AC_{min}}|, |S^AC_{t,\theta^AC_{max}}| \right\} \quad (3g)
\]

Jointly considering the reformulated battery cell security constraints (3a)-(3b) and the discretized dynamic converter capability constraints (3e)-(3e), We propose algorithm 2 which gives stable performance of computational time within 35 ms. As we have explained earlier, since \( P_{DC,min}^t \leq 0 \) and \( P_{DC,max}^t \geq 0 \), updating \( p_{DC}^t \) in the way described in algorithm 2 always decreases the length (Euclidean norm) \( |S^AC_t| \) which contributes to the feasibility of the BESS constraints described in (2b)-(2f). This algorithm converges normally within two iterations. The converged solution is very close to the optimal solution of optimization problem (2a)-(2f). We show the performance of both algorithms in the next sections of this paper. Because we are using the complex power vector \( s^AC_t \) in the \((p^AC_t, q^AC_t)\) plane to quantify the feasible operational region of BESS and find the optimal power set-point solution, we denote our real-time controller as \( s\)-Vector controller.

**Algorithm 2: s-Vector Control of BESS**

**Result:** Optimal Power Set-Points \( p^AC_t, q^AC_t \)

**Initialization:**
Set \((p^AC_0^t, q^AC_0^t) = (P^AC_0^t, Q^AC_0^t)\) Based on Equations (2g)-(2h);
Set \( p_{DC}^t \) Based on Equation (2f);
Set \( \theta_0 = \arctan \left( \frac{Q^AC_0^t}{P^AC_0^t} \right) \);
Set \( \theta^AC_{min} = \lfloor \theta_0 \rfloor \);
Set \( \theta^AC_{max} = \lceil \theta_0 \rceil \);

while (3a)-(3b) or (3e)-(3f) are not valid do
  if (3a)-(3b) are not valid then
    Set \( S^AC_t = \max \left\{ S^AC_{t,\theta^AC_{min}}, S^AC_{t,\theta^AC_{max}} \right\} \);
    \( p_{DC}^t \) Based on Equation (2f);
  end
  if (3a)-(3b) are not valid then
    if \( p_{DC}^t < P_{DC,min}^t \) then
      Set \( p_{DC}^t = P_{DC,min}^t \);
    else
      \( p_{DC}^t = P_{DC,max}^t \);
    end
    \( p_{AC}^t \) Based on Equation (2f);
  end
end

V. NUMERICAL VALIDATION

We validate the performance of algorithm 2 in this section. Firstly, we compare the optimal solutions of algorithm 2 with the solutions from the built-in optimization solver FMINCON in MATLAB. We investigate the optimal solution for the initial power set-point \((P^AC_0^t, Q^AC_0^t)\) from \((-1000 \text{kW}, -1000 \text{kVar})\)
to (1000 kW, 1000 kVar). The results are shown in Fig. 2- Fig. 4. Fig. 2 shows the optimal solutions of active power set-points $p_t^{AC}$. For the solution accuracy, all the optimal solutions from our proposed algorithm 2 are very close to the optimal solutions from FMINCON. The absolute differences are less than 2 kW. The relative differences are less than 0.6%. Fig. 3 shows the optimal solutions of reactive power set-points $q_t^{AC}$. For the solution accuracy, all the optimal solutions from our proposed algorithm 2 are very close to the optimal solutions from FMINCON. The absolute differences are less than 2 kVar. The relative differences are less than 0.6%. The computational efficiency comparison between algorithm 2 and FMINCON is shown in Fig. 4. Our proposed algorithm 2 is at least 10 times faster than FMINCON according to the required CPU time to find the optimal solutions. Moreover, the computational efficiency of algorithm 2 is more stable than FMINCON according to the statistical analysis shown in Fig. 5. Both the mean value $\mu$ and standard deviation $\sigma$ parameters of the fitted normal distribution curve of the CPU time from our proposed algorithm 2 are much smaller than CPU time from FMINCON.

Labview is used in the real-time control. We compare the real-time control performance of algorithm 2 with the performance of solving the optimization problem (2a)-(2f) by using the built-in MATLAB optimization solver FMINCON. Using our proposed algorithm 2, we achieved 100 ms refresh rate of updating the control loop (monitoring all the equipment's status and sending the power set-point command to the DC-AC converter every 100 ms). As a comparison, if we use FMINCON, we achieved 200 ms refresh rate of updating the control loop. In all the results of this Section, positive power set-point means discharging for the BESS. Negative power set-point means charging for the BESS.

Fig. 5. Comparison of the Probability Distributions of CPU Time

VI. EXPERIMENT

We conduct the experiment on the 720 kVA / 560 kWh BESS installed on EPFL campus in Lausanne, Switzerland. The battery technology is Lithium-Titanate (LTO). A 4-quadrant DC-AC converter is used to connect the DC-bus of the battery through a 300 V / 20 kV transformer to the campus power grid. More details of the BESS can be found from [27]. The real-time control system of BESS shown in Fig. 6 is developed using Labview. The optimization model is coded by using YALMIP toolbox [28]. The MATLAB run-time engine for

Fig. 6. Real-time Control System of BESS
A. Performance of the FMINCON Solver

The frequency response and voltage support results (by using the FMINCON solver to solve the optimization model) are shown in Fig. 7 and Fig. 8. We can see there are some optimal power set-points which are different from the initial power set-points. This is because the initial power set-points are very large due to large frequency deviation at these moments. These initial power set-points are outside the feasible operational region of the BESS and the FMINCON solver projects these power set-points to the boundary of the feasible operational region. The statistic analysis of execution time is shown in Fig. 9. In average it takes around 28 ms to solve the optimization model. The maximum execution time is less than 65 ms. Considering all the monitoring and controlling process for the real-time controller, in this experiment, we achieve 200 ms to update the control loop.

B. Performance of the s-Vector Control

The frequency response and voltage support results (by using our proposed algorithm 2) are shown in Fig. 10 and Fig. 11.

A. Performance of the FMNCON Solver

The frequency response and voltage support results (by using the FMINCON solver to solve the optimization model) are shown in Fig. 7 and Fig. 8. We can see there are some optimal power set-points which are different from the initial power set-points. This is because the initial power set-points are very large due to large frequency deviation at these moments. These initial power set-points are outside the feasible operational region of the BESS and the FMINCON solver projects these power set-points to the boundary of the feasible operational region. The statistic analysis of execution time is shown in Fig. 9. In average it takes around 28 ms to solve the optimization model. The maximum execution time is less than 65 ms. Considering all the monitoring and controlling process for the real-time controller, in this experiment, we achieve 200 ms to update the control loop.

B. Performance of the s-Vector Control

The frequency response and voltage support results (by using our proposed algorithm 2) are shown in Fig. 10 and Fig. 11.
Note this experiment is conducted at another hour during the same day of the previous experiment using FMINCON. So the power grid frequency and voltage profiles are different. Again, we can see there are some optimal power set-points which are different from the initial power set-points. This is because the initial power set-points are outside the feasible operational region of the BESS and the s-Vector control projects these power set-points to the boundary of the feasible operational region. The statistic analysis of execution time is shown in Fig. 12. In average it takes around 8 ms to solve the optimization model using algorithm 2. The maximum execution time is less than 35 ms. Considering all the monitoring and controlling process for the real-time controller, in this experiment, we achieve 100 ms to update the control loop. In other words, s-Vector control reduces the required execution time from 200 ms to 100 ms.

VII. CONCLUSION

We define the concept of s-Vector control and show its application in the real-time control of BESS to provide ancillary services to the power grid. The key idea of s-Vector control is to quantify or approximate the feasible operational region of BESS discretely. Without relying on optimization solvers, s-Vector control can guarantee the feasibility and optimality of the power set-point for the BESS. The computational efficiency of s-Vector control over optimization solvers is validated numerically and experimentally. s-Vector control can avoid the usage of optimization solvers which can be costly for the BESS operators. Numerical results and statistical analysis show that s-Vector control is at least 10 times faster and more stable than the optimization solver FMINCON. These advantages are very preferred in real-time control because of the determinant execution time requirement. Frequency response and voltage support experiments in our utility-scale 720 kVA / 560 kWh BESS show that we can achieve 100 ms of time resolution to update the whole control loop. Using the FMINCON solver in MATLAB as a benchmark, the control loop takes 200 ms to update in the experiment. This fast response performance is highly valuable for the future power system to balance the fast fluctuations of renewable energy resources. We believe the proposed s-Vector control can overcome the traditional optimization routine which is one bottleneck in the real-time control of BESS.

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