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Theory of Weak Inclusive Decays and Lifetimes of Heavy Hadrons

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Abstract

The theory of preasymptotic effects in inclusive decays of heavy flavors is briefly reviewed.
1 Introduction

Heavy flavor hadrons $H_Q$ contain a heavy quark $Q$ plus a cloud built from light quarks (antiquarks) and gluons. The heavy quark $Q$ experiences a weak transition. The nature of this transition is of no concern to me here. It can be a radiative transition, like $b \rightarrow s\gamma$, semileptonic decay like $b \rightarrow c\ell\nu_\ell$ or a non-leptonic decay to lighter quarks, e.g. $b \rightarrow c\bar{u}d$. It is assumed that at short distances the amplitude is known from the electroweak theory. The task of the QCD-based theory is to calculate preasymptotic effects in the decay rate of the hadron $H_Q$ and other decay characteristics: the energy spectra, the average invariant mass of the hadronic state produced, etc. These effects are due to interactions with the soft degrees of freedom in the light cloud.

The foundation of the theory was laid in the eighties [1] when it was realized that the operator product expansion [2] could be used in application to the so-called transition operator of the type

$$\hat{T}(Q \rightarrow f \rightarrow Q) = i \int d^4x \{\mathcal{L}_W(x), \mathcal{L}_W^\dagger(0)\}_T,$$  

(1)

where $\mathcal{L}_W$ is the short-distance weak Lagrangian governing the transition $Q \rightarrow f$ under consideration. The momentum operator $P_\mu$ of the heavy quark $Q$ is written as a sum of two terms, $P_\mu = m_Q v_\mu + \pi_\mu$, where $v_\mu$ is the four-velocity of the heavy hadron $H_Q$, $\pi_\mu$ is the residual momentum operator, responsible for the interaction with the “background” gluon field in the light cloud. The large mechanical part $m_Q v_\mu$ in the heavy quark momentum guarantees that the transition operator (1) can be found as a sum of local operators (with some reservations to be discussed below). These operators are ordered according to their dimensions. At the level of $1/m_Q^2$ we have only two operators; extra four-fermion operators are added at the level of $1/m_Q^3$. A few of these were calculated 8 years ago [1].

At the next stage each term in the expansion must be averaged over the hadronic state $H_Q$. At this stage the bound state dynamics is accounted for.

After the initial excitement the OPE-based theory of the inclusive heavy flavor decays was in a rather dormant state until recently. The revival it experiences now is due to a combination of several factors. First, a very concise and convenient language was created, the heavy quark effective theory (HQET) [3]. Calculations of the non-perturbative effects were translated in this language and developed in Refs. [4, 5, 6]. Second, all relevant operators appearing at the level up to $1/m_Q^3$ were catalogued and our understanding of their matrix elements (I mean the numerical values) was significantly advanced. Finally, the issue of convergence of the non-perturbative series was clarified. In the beauty family one expects that the first two or three terms in the expansion ensure reasonable accuracy of the predicted lifetimes. As for the charmed quark we will see that it is, perhaps, too light for duality to set in. Since OPE is used in the Minkowski domain the validity of duality is crucial for the whole approach. Theoretically the onset of duality is correlated with the behavior of high-order terms in the non-perturbative series.
2 Master equation

The OPE-based approach is applicable in a very wide range of problems. Here we will concentrate on the total inclusive widths. Generically the $m_Q^{-1}$ expansion for the width has the form (for definiteness I will speak about the beauty family)

$$\Gamma(H_b \rightarrow f) = \frac{G_F^2 m_b^5}{192\pi^3} |\text{CKM}|^2 \times \left\{ c_3(f) \frac{\langle H_b | \bar{b}b | H_b \rangle}{2M_{H_B}} + \frac{c_5(f)}{m_b^2} \frac{\langle H_b | (\bar{b}/2i)\sigma G b | H_b \rangle}{2M_{H_B}} + \sum_i \frac{c_i^{(i)}(f)}{m_b^6} \frac{\langle H_b | (\bar{b}\Gamma_i q)(\bar{q}\Gamma_i b) | H_b \rangle}{2M_{H_B}} + \mathcal{O}(m_b^{-4}) \right\}. \quad (2)$$

The coefficient functions $c_i(f)$ depend on the particular inclusive transition considered and are calculable. They are determined by short-distance QCD provided that the energy release is large enough. On the other hand, the matrix elements on the rhs describe the response of the soft degrees of freedom on the instantaneous perturbation, the $b$-quark decay. These quantities are essentially non-perturbative. But they are universal, and, as seen from Eq. (2), there are only a few of them.

The matrix element of the chromomagnetic operator $\sigma G$ is expressible through spin splittings, say $M_{B^*} - M_B$. The four-fermion operators of dimension 6 can be evaluated, in the case of mesons, within factorization. For baryons a reliable calculation of the corresponding matrix elements is a problem essentially unsolved so far. As for the scalar density, $\bar{b}b$, it is this term that exactly reproduces the parton model (asymptotic) result, plus preasymptotic corrections. This operator also can be written as an expansion,

$$\bar{b}b = \bar{b}\gamma_0 b - \frac{1}{2m_Q^2} \bar{b}(\vec{\pi}^2 - (i/2)\sigma G)b + \frac{1}{4m_Q^2} g^2 \bar{b}\gamma_0 T^a b \sum_q \bar{q}\gamma_0 T^a q + \mathcal{O}(1/m_b^4) \quad (3)$$

where the sum runs over the light quarks. A new operator appearing here is $\bar{b}\vec{\pi}^2 b$, the square of the spatial momentum of the $b$-quark. The matrix element of this operator in the $B$ meson can be limited from below, for a detailed discussion see Ref. [4]. The average spatial momentum turns out to be surprisingly large, larger than 0.6 GeV! The QCD sum rule calculations [8] yield even a larger value, $\sim 0.7$ GeV. The expectation value of $\vec{\pi}^2$ in baryons is expected to be close to that in mesons.

Time/space limitations do not allow me to go into further details. Let me point out only the most remarkable features of the overall picture.

(i) The total rates do not contain non-perturbative corrections of order $1/m_b$, the so called CGG/BUV theorem [4, 5]. The corrections start at the level $1/m_b^2$. This
sets the scale of preasymptotic effects in the beauty family at the level of several per cent since \( \langle B|\bar{b}\sigma G b|B\rangle/2m_b^3 \approx 0.03 \). In particular, deviations of the lifetime ratios from unity are expected to be of this order of magnitude. At the level \( 1/m_b^2 \) all \( B \) mesons have the same lifetimes (disregarding some small \( SU(3)_f \) breaking effects).

(ii) The difference in the lifetimes of baryons and mesons is due to the fact that the expectation values of the operators in Eq. (2) are different for mesons and baryons. This difference arises at the level \( 1/m_b^2 \).

(iii) Four-fermion operators of dimension 6 produce effects formally scaling like \( 1/m_b^3 \), although numerically they seem to be enhanced since the corresponding coefficients have one loop less and, additionally, a key constant \( f_B \) turns out to be rather large. This enhancement may lead to the fact that dimension 5 and 6 operators are competitive in the beauty family. The four-fermion operators shift the lifetimes of mesons versus baryons and split the meson lifetimes from each other.

(iv) Situation with \( B_s \) is exceptional. The lifetime difference between \( B_s,\text{short} \) and \( B_s,\text{long} \) is due to a mechanism not exhibited in Eq. (2), namely \( B - \bar{B} \) oscillations. The corresponding estimates were done in Ref. [9].

3 Phenomenological implications

Assembling all theoretical elements discussed above (and those which are discussed in the original literature) we arrive at the following pattern. The lifetime of a charged \( B \) meson is predicted to exceed that of a neutral \( B \) meson,

\[
\frac{\tau(B^-)}{\tau(B_d)} - 1 \approx 0.05(f_B/200\text{MeV})^2 \sim 0.05.
\] (4)

At this level it is expected that \( \bar{\tau}(B_d) \approx \bar{\tau}(B_s) \) where \( \bar{\tau} \) denotes the average lifetime of the two mass eigenstates in the \( B^0 - \bar{B}^0 \) system. It is curious that \( B_s \) oscillations will seemingly produce the largest lifetime difference,

\[
\frac{\Delta\Gamma(B_s)}{\Gamma(B_s)} \approx 0.18(f_B/200\text{MeV})^2 \sim 0.18.
\] (5)

The baryon matrix elements are always most difficult for consistent analysis; therefore, the baryon-to-meson lifetime ratios should be taken with caution. Still, plausible estimates indicate that one can expect \( \tau(\Lambda_b)/\tau(B_d) \sim 0.9 \).

4 A grain of salt: \( Br_{sl}(B) \)

The theory of preasymptotic effects which I have just sketched, being applied to the problem of the semileptonic branching ratio in the \( B \) mesons, leads to a paradox. In this case the heavy quark expansion can be readily carried out up to terms of order \( 1/m_b^3 \). One obtains a formula very similar to Eq. (2), with the same
structure and the same operators \([10]\). The leading non-perturbative correction \(\mathcal{O}(m_b^{-2})\) tends to diminish the branching ratio while the term \(\mathcal{O}(m_b^{-3})\) tends to increase it. Both effects, however, are far too small to produce a noticeable impact on the branching ratio. At best they shift the prediction for the branching ratio by 0.5% or less. Thus we are forced to conclude that the prediction for \(\text{Br}_s(B)\) is controlled by perturbative QCD. People believe that perturbative QCD typically yields \(\text{Br}_s(B) \approx 13\%\); twisting arms allows one to go down to 12.5% \([11]\). At the same time experimentalists, both CLEO and ARGUS, seem to be firm in their conclusion that \(\text{Br}_s(B) < 11\%\). A natural question is what went wrong?

I leave aside the possibility that the experimental numbers are wrong. There is no visible loophole in the OPE-based theory of preasymptotic effects either. Then the remaining logical options are as follows: (i) something is missing in the perturbative analysis; (ii) new physics shows up in the \(B\) meson decays. Both options must be investigated. In a recent paper \([12]\) a new contribution in the perturbative calculation is identified, not included in the analysis of Ref. \([11]\), which seemingly works in the right direction – diminishes \(\text{Br}_s(B)_{\text{pert}}\) by \(\sim 0.5\%\). It remains to be seen whether the perturbative number can reach the 11% mark under realistic choice of relevant theoretical parameters (the quark masses, \(\alpha_s\), etc.). (Let me make a side remark: I do not believe that \(\alpha_s(M_Z)\) can be as large as 0.126, as is allegedly implied by the so called global fits at the \(Z\) peak at present. A wealth of low-energy data point to a significantly lower value of \(\alpha_s\), something like 0.114 or even lower. In terms of \(\Lambda_{\text{QCD}}\) the difference is drastic. I urge to take this discrepancy very seriously.)

5 The family of charm

It might seem to be a trivial exercise to substitute \(m_b\) by \(m_c\) in the master equation. Yes, technically this is easy, and formally all \(1/m_c^2\) and \(1/m_c^3\) corrections have been written down and classified. They are much larger, of course, than in the beauty family; typically of order of 0.5. I refer those interested to a very detailed recent update \([13]\). Qualitatively the pattern of the lifetimes in the charm family emerging in the heavy quark expansion agrees with experiment. Namely, those particles that live longer are predicted to live longer, etc. However, quantitatively the \(\mathcal{O}(m_c^{-2})\) and \(\mathcal{O}(m_c^{-3})\) preasymptotic terms are smaller than what one needs in order to reproduce, say, \(\tau(D^+)/\tau(\Xi_c^0)_{\text{exp}} \sim 12\). Since arithmetically the calculation is certainly correct one may start suspecting that something went wrong in the basics.

The operator product expansion, the foundation of the whole approach, is a well defined procedure in the Euclidean domain. A specific feature of the transition operator \([1]\) is its essentially Minkowski character. Therefore, in justifying the short-distance calculation of the coefficient functions one must always keep in mind a kind of analytic continuation, through a dispersion relation. Thus, strictly speaking, theoretical predictions for \(c_i(f)\) in Eq. \([2]\) refer to quantities integrated over energy in some energy range.
If the energy release is large enough so that duality is valid and the integrand is smooth, this smearing is unimportant; one can predict the coefficient functions for the given energy release, locally. It is always tacitly assumed that this is the case. The onset of duality is governed by exponential terms, not visible to any finite order in $1/m_Q$ expansion. Due to this reason the onset must be abrupt.

We are inclined to think that the $c$ quark is not heavy enough to warrant duality. The strongest argument comes from consideration of $\Gamma_{sl}(D)$. Indeed, with the reasonable value of $m_c (m_c(m_c) \sim 1.3 \text{ GeV})$ the parton-model prediction is close to the experimental number. The first perturbative correction is negative [1] and the second seems to be negative as well [4]. The non-perturbative corrections follow the same pattern. The leading $1/m_c^2$ term is known from Ref. [1] while the $1/m_c^3$ correction has been estimated recently [13], both are negative. The combined effect of the leading corrections amounts to reducing $\Gamma_{sl}(D)$ by 50%, and the next-to-leading terms only worsen the situation!

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