Using Full Field Data to Produce a Single Indentation Test for Fully Characterising the Mooney Rivlin Material Model

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Abstract. A theoretical testing method for fully characterising the Mooney-Rivlin hyper-elastic material model is proposed by capturing full-field data, namely displacement field and indentation force data. A finite element model with known parameters will act as the experimental model against which all data will be referenced. This paper proposes a method of inverse finite element analysis operating under the assumption of equally objective function optimal planes or "hyper-planes". The paper concludes that the Mooney-Rivlin material model can theoretically be fully characterised in a single indentation test by applying methods discussed in the paper when using full-field data operating under the assumption of hyper-planes.

1 Introduction

Hyper-elastic materials have found themselves in numerous applications from simple water tight seals to primary components in soft-robotics [11], simulating the behaviour of these materials for improved design requires the use of a hyper-elastic material model. A method of performing inverse finite element analysis (FE analysis) on an indentation test may improve the process of isolating the correct material parameters since this loading method induces complex load cases in the deformed material. Inverse FE analysis also has the advantage in that it may reduce the engineering time for obtaining a suitable material model for FE analysis [10].

This paper will demonstrate that when operating under the assumption that there exists linear planes in the material parameter design space wherein the parameters have a fixed relationship that results in the objective functions for each data type are equally optimal [1],[7]. The Mooney-Rivlin three-parameter model can be fully characterised from full-field data acquired from a single indentation test as when evaluating the material parameters along the intersection line between the two planes for the same indentation test at different depth the parameters are convergent.

2 Methods and Materials

2.1 Analysis Method

Understanding how well the inverse FE analysis method performs under ideal circumstances, as in the case of FE model data, provides valuable insight on which factors are the most

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relevant before attempting to do the same analysis on physical test sample data. Once the proposed method has demonstrated its effectiveness on accurately isolating the correct material parameters for a given material model based on simulated digital image correlation (DIC) data, as will be demonstrated in this paper. The proposed method can then be applied to physical test data. The evaluated material model in this paper will be the Mooney-Rivlin three-parameter model as seen in equation 3 which applies strain energy density to assess material behaviour [2].

\[ W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) + C_{20}(I_1 - 3)^2 \]  

The analysis is broken up into three distinct phases operating under the core assumption that there exists regions or planes in the design space wherein the objective function is equally optimal as mentioned in the introductory section. This means that on these planes the displacement field or indentation force would be identical even if the design variables are not correct, these planes will be referred to as hyperplanes throughout the rest of the paper. Evidence supporting the assumption can be shown from [7] who identified that simulated indentation force had non-unique sets of material parameters since they would result in identical indentation force being witnessed. [4] and [1] both witnessed that optimising the objective function for a single dataset would not always provide a good fit for another dataset or loading case. With [1] showing that the absolute values of the material parameters not being strongly correlated but their product and intern their ratio having a strong correlation.

Phase one would be locating the low objective function regions where the hyperplane can be found. This must be done for both the displacement field and indentation force data under a single variate optimisation scheme using DOT [8] as the optimiser. Once this optimization has converged this point will be assumed to lie on the hyperplane, these points will be identified for both datasets.

Phase two is where the equations of the hyperplanes are identified, this method incorporates a central composite design as recommended by [9] for choosing appropriate sample points for evaluating the response surface, this ensures the second-order model (Equation 2) has a valid dataset to perform the least-squares regression. Evaluating the principle plane using stationary point analysis on the second-order model’s parameters yields a linear surface equation for the hyperplane.

\[ f(x_1, x_2, x_3) = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 \]  

Phase three is where the intersection between the hyperplanes from the displacement field data is evaluated resulting in a single line in hyperspace which can be seen as one dimensional since all parameters can be coupled to a single variable \( \alpha \), the hyperplanes can be seen in Figures 3a and 3b. The optimisation algorithm now used is the method of Golden Sections analysing the displacement field for the same experimental model at a different indentation depth, this results in a third hyperplane that rotates about the "ideal" point of the data set making the error profile along the one-dimensional line is strictly uni-model.

### 2.2 Experimental Model

The geometry under investigation was a 20x20x5 mm cuboid with a cylindrical indenter placed lengthwise along the top surface’s centreline with a length of 30 mm with a radius of 3 mm. Because this geometry is symmetric about its centre planes it can be taken advantage of as only half the geometry needs to be simulated, significantly reducing the computational
cost. Applying symmetry conditions results in a simulated sample with dimensions 10x10x5 mm and the indenter being placed along the top surface’s edge as seen in Figure 2.

As in the physical test the indentation depth will be physically controlled therefore to replicate this in the FE model displacement control for an overall indentation depth of 3 mm over 1 second to induce material deformation revealing the material behaviour. The choice behind this geometry stems from the research of [4], the reasoning behind this geometry was for situations in which small biological tissue samples would need to be evaluated. These samples would not only be in short supply but due to their limited size, it would not be feasible to clamp the samples and perform uni-axial or biaxial tensile tests which opened the question as to whether or not the Mooney-Rivlin material model could be fully characterized with a single sample of simple geometry and relatively simplistic loading application.

The model also simulates the situation in which digital image correlation (DIC) would be used for extracting the displacement field data which is achieved by only evaluating the surface nodes and elements on a single face of the sample as seen in Figure 2.

In-order to generate the theoretical experimental data the same model used in the inverse FE analysis was run once with significantly denser mesh and only surface data on a single face was extracted mimicking the type of data that would result from the use of DIC.

\[ W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) + C_{20}(I_1 - 3)^2 \]  

(3)

### 2.3 Numerical Pipeline

The program used to perform the FE analysis in this investigation was MSC Marc Mentat 2021, this software was favourable since it can be controlled in either batch mode which allows for a procedure script that updates the model to be passed onto the software or direct control through python during operation using the PyMentat modules [5]. Marc Mentat also has a strong python-support library for post-processing of the results files with their built in library. In the numerical pipeline of this investigation, the batch mode method of control was used to updated the material parameters and run the simulation.

The initial optimisation phase was handled by the software package DOT (Design Optimization Tools) of which the sequential quadratic programming (SQP) algorithm was used which is a gradient-based optimisation scheme. This phase was performed separately on both the displacement field data and indentation force data until the "hyperplanes" or regions where the apparent non-unique minimum objective values are for either data set were identified.
2.4 Objective Functions

Inverse FE analysis at its core uses some method of optimisation to determine the material parameters of an evaluated sample. This is achieved by minimising an objective function based off error data generated from the experimental and simulation models. Since displacement field and indentation force are very different in terms of their data structure two objective functions were needed.

2.4.1 Displacement Field Objective Function

Root mean square (RMS) was used to determine the error between the simulated and measured displacement fields. This error is represented by a single value for each FE simulation, one for each coordinate direction resulting in three RMS errors as represented by equations 4,5, and 6. Directly summing up these equations will result in a single error value that describes how similar the two displacement fields are, however, this now introduces bias into the objective function favouring the larger displacement errors. The simulation has an indenter that moves in the Y-direction and therefore it is expected that Y-displacements will always be larger than displacements in the X and Z-directions. An attempt to remove this bias was to normalize the displacements with their respective maximum displacements in that direction, this is represented by equation 7. [10] used the same set of equations when characterising a non-linear orthotropic by means of bubble inflation tests.

\[
    e_x (m) = \sqrt{\frac{\sum_{i=1}^{N} (d_{exp_x}(m) - d_{sim_x}(m))^2}{N}} \tag{4}
\]

\[
    e_y (m) = \sqrt{\frac{\sum_{i=1}^{N} (d_{exp_y}(m) - d_{sim_y}(m))^2}{N}} \tag{5}
\]

\[
    e_z (m) = \sqrt{\frac{\sum_{i=1}^{N} (d_{exp_z}(m) - d_{sim_z}(m))^2}{N}} \tag{6}
\]
\[ e_{\text{disp}} = \frac{1}{M} \sum_{i=1}^{N} \frac{e_x(i)}{\max(|d_{\text{exp},x}(i)|)} + \frac{e_y(i)}{\max(|d_{\text{exp},y}(i)|)} + \frac{e_z(i)}{\max(|d_{\text{exp},z}(i)|)} \]  \hspace{1cm} (7)

### 2.4.2 Indentation Force Objective Function

The objective function for the indentation force was simpler than the displacement field in that there is only one point of measurement, in one direction for each indentation depth and not a "field". It was chosen that any force between the simulation and experimental would increase with the absolute difference between the two measurements. This means that the lower limit for the force objective function would be zero when the measured forces are equal. The force objective function is as follows:

\[ e_{\text{force}} = \sum_{i=1}^{m} |f_{\text{exp}}(i) - f_{\text{sim}}(i)| \]  \hspace{1cm} (8)

### 3 Results

#### 3.1 Phase One Convergence Points Results

After the SQP algorithm in DOT identified many convergence points for each respective dataset it was found, as expected, that none of the starting points converged to the same point in design space across different datasets. This was also true within each dataset itself that it would be extremely unlikely that two starting points in the displacement field or indentation force data would converge to the same point as seen in Figures 3a and 3b. However, these points did display the core behaviour behind this paper that the assumption of a hyperplane, where any point on the plane would result in identical material behaviour for the given data type, is valid since it was possible to fit the first-order surface through the converged points with almost all of these points being on that plane. The correlation coefficients for the linear fits on the two hyperplanes were \( R_{\text{force}} = 0.999 \) and \( R_{\text{disp}} = 1.000 \) which is a near-perfect correlation score providing further evidence supporting the assumption of hyper-planes being present in the data sets. It is also worth noting that none of the starting points did converge to correct material parameters as prescribed in the high density mesh finite element model which was used to generate the synthetic experimental data in this research paper. These findings agree with what was witnessed in the literature.

#### 3.2 Phase Two Central Composite Design Results

The results of phase two were that a second-order model based on equation 2 was successfully fit with the central composite design points for both the displacement field and indentation force datasets. The respective R-squared value for a central composite design centred at an arbitrarily chosen convergence point for each fit was \( R_{\text{disp}} = 0.981 \) and \( R_{\text{force}} = 0.978 \) and adjusted R-squared values of \( \text{Adj } R_{\text{disp}} = 0.948 \) and \( \text{Adj } R_{\text{force}} = 0.938 \) which all indicate a strong correlation with the second-order model. Although these values are dependent on where in design space the central composite design is chosen, the fit values do not differ significantly from the above values, they can be seen as a representative of what can be encountered when performing this analysis.
3.3 Phase Three Golden Section Results

The results for phase three were that the method of golden sections was successfully able to find the optimal point that fit both the displacement field and indentation force data simultaneously when following the one-dimensional line that is generated at the intersection point between the two linear surfaces, also referred to as hyperplanes. This final convergent point from the golden section phase also represents the correct material parameters for the Mooney-Rivlin model with a total magnitude of 2.32% error from the prescribed values and was absolutely convergent at a single point along the intersection line as seen in Figure 4b. This error can be further reduced by repeating phases two to three by implementing a central composite design having a smaller radius centred at the converged material point.
Golden section optimisation along intersection line between material upper and lower bounds of search space

Figure 4: Golden section optimisation on intersection line between the two hyperplanes

4 Conclusion

In this paper, a method of performing indentation tests for fully characterising the three-parameter Mooney-Rivlin model. This method is based on the assumption that within the data sets there are hyper-planes with fixed relationships between the material parameters wherein the objective function is equally optimal. It was demonstrated for both the force and displacement field data that this assumption is applicable since the linear plane was able to be fit through the converged points for random starting positions in the design space with near-perfect correlation coefficients of $R_{\text{force}} = 0.999$ and $R_{\text{disp}} = 1.000$.

When optimising parameters along the intersection line of the two hyper-planes it was demonstrated in section 3.3 that evaluating the indentation force for a different depth results in the objective function being convergent to a single point. The results, therefore, demonstrate that operating under the assumption of hyperplanes in the data sets the Mooney-Rivlin Material model can be fully characterised with a low level of error of only 2.32%.

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