Cantilever-based electret energy harvesters

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Abstract
Integration of structures and functions has permitted the electricity consumption of sensors, actuators and electronic devices to be reduced. Therefore, it is now possible to imagine low-consumption devices able to harvest energy from their surrounding environment. One way to proceed is to develop converters able to turn mechanical energy, such as vibrations, into electricity: this paper focuses on electrostatic converters using electrets. We develop an accurate analytical model of a simple but efficient cantilever-based electret energy harvester. We prove that with vibrations of 0.1 g (∼1 m s⁻²), it is theoretically possible to harvest up to 30 μW per gram of mobile mass. This power corresponds to the maximum output power of a resonant energy harvester according to the model of William and Yates. Simulation results are validated by experimental measurements, raising at the same time the large impact of parasitic capacitances on the output power. Therefore, we ‘only’ managed to harvest 10 μW per gram of mobile mass, but according to our factor of merit, this is among the best results so far achieved.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Thanks to size reduction, micro-electro-mechanical-systems (MEMS) are consuming less and less energy, giving them the opportunity to harvest energy in their surrounding environment. This field of research called ‘energy harvesting’ consists of the development of converters able to turn ambient energy (light, air flux, variation of temperatures, vibrations) into electricity that is used to power the microsystem. Many principles of conversion have already been developed: photovoltaic, thermoelectric, biofuel cells, etc. As for mechanical energy from vibrations, it can be converted by three main different principles: piezoelectric, electromagnetic and electrostatic conversion. In this study, we focus on electrostatic converters which are based on a capacitive architecture (two charged electrodes spaced by an air gap) and connected to a load. Vibrations induce changes in the geometry of the capacitor and a circulation of charges between electrodes through the electrical load. The electronic circuit that manages power conversion of ‘standard’ electrostatic energy harvesters [1] is quite complicated and induces losses and therefore a decrease in efficiency. To limit the use of a managing electronic circuit, it is possible to use electrets (stable electrically charged dielectrics) that polarize the capacitance and allow us to harvest energy from vibrations without using cycles of charging and discharging.

Many electret-based energy harvesters have been developed, demonstrating the interest in such devices [2–22]. Most of these devices are in-plane structures where the variation of capacitance is obtained by a variation in the surface between patterned electrodes, while the gap is kept constant. These structures are generally hard to manufacture, using elaborate clean room processes and especially DRIE (deep reactive ion etching), but give the opportunity to harvest energy when the vibrations of the ambient environment are not constant because they avoid contacts between electrets and electrodes. In this paper, we have chosen to study a simpler structure: a ‘cantilever-based electret energy harvester’. This structure does not maximize the output power of the energy harvester if vibrations are not well defined but is particularly suitable when vibrations are stable in terms of frequency and amplitude in time. Moreover, this structure is quite easy to manufacture and therefore of low cost.

In section 2, we present the theory of vibration energy harvesting and more especially of energy harvesting using electrets. Then, we develop an accurate analytical model,
A vibration occurs modeled by an electrostatic force \( f_{\text{elec}} \) while the other part is converted into electricity, which is the displacement of the mobile mass \( x \) as piezoelectric, magnetic or piezoelectric, resonant energy harvesters can be modeled as a mobile mass (1):

\[
m\ddot{x} + kx + f_{\text{elec}} + f_{\text{mec}} = -m\ddot{y}.
\]

When forces can be modeled as viscous forces, \( f_{\text{elec}} = b_e\dot{x} \) and \( f_{\text{mec}} = b_m\dot{x} \), where \( b_e \) and \( b_m \) are, respectively, electrical and mechanical damping coefficients, William and Yates [23] have proven that the maximum output power of a resonant energy harvester subjected to an ambient vibration is reached when the natural angular frequency \( (\omega_n) \) of the mass–spring structure is tuned to the angular frequency of ambient vibrations \( (\omega) \) and when the damping rate \( \xi_e = b_e/(2m\omega_n) \) of the electrostatic force \( f_{\text{elec}} \) is equal to the damping rate \( \xi_m = b_m/(2m\omega_n) \) of the mechanical friction force \( f_{\text{mec}} \). This maximum output power \( P_{\text{W&Y}} \) can be simply expressed with (2), when \( \xi_e = \xi_m = \xi \):

\[
P_{\text{W&Y}} = \frac{mY^2\omega_n^2}{16\xi}.
\]  

As \( P_{\text{W&Y}} \) is a good approximation to know the output power of vibration energy harvesters when forces are modeled as viscous forces, comparing the output power \( (P) \) of a resonant energy harvester to \( P_{\text{W&Y}} \) gives a legitimate factor of merit \( \alpha_{\text{W&Y}} \):

\[
\alpha_{\text{W&Y}} = \frac{P}{P_{\text{W&Y}}}. \tag{3}
\]

Nevertheless, in many studies, the weight of the mobile mass is not given while the surface area of the electrodes \( (S) \) is often provided. Therefore, to compare systems, in a previous study we developed another factor of merit, normalized by the active surface \( S \) in place of the mass [24]:

\[
\chi = \frac{P}{Y^2\omega_n^3S}. \tag{4}
\]

These two factors of merit will be used in the next parts to compare our system with the state of the art.

2. Electret-based energy harvesters using cantilevers

Our electret-based energy harvester is a microsystem able to convert mechanical energy from vibrations into electricity. It is a type of vibration energy harvester, the general model of which is presented hereafter.

2.1. William and Yates’ general model for vibration energy harvesters

Regardless of the conversion principle (electrostatic, electromagnetic or piezoelectric), resonant energy harvesters can be modeled as a mobile mass \( (m) \) suspended to a support by a spring \( (k) \) and damped by forces \( (f_{\text{elec}} \text{ and } f_{\text{mec}}) \). When a vibration occurs \( y(t) = Y \sin(\omega t) \), it induces a relative displacement of the mobile mass \( x(t) = X \sin(\omega t + \phi) \) compared to the frame (figure 1). Part of the kinetic energy of the moving mass is lost due to mechanical damping \( (f_{\text{mec}}) \) while the other part is converted into electricity, which is modeled by an electrostatic force \( (f_{\text{elec}}) \) in electrostatic energy harvesters. Ambient vibrations are generally low in amplitude (typically \( Y = 25 \mu m \)) and the use of a mass–spring structure enables us to take advantage of a resonance phenomenon that amplifies the amplitude of vibrations perceived by the mobile mass and the harvested energy. Newton’s second law gives the differential equation that rules the movement of the mobile mass (1):

\[
m\ddot{x} + kx + f_{\text{elec}} + f_{\text{mec}} = -m\ddot{y}.
\]

Figure 1. The mechanical system.

Figure 2. An electret film.
mechanism results in the implantation of charges at the surface (figure 2), into the bulk or at the interfaces of the material. The grid is used to limit the surface voltage of the electret to a required final value. Nevertheless, dielectrics are not perfect insulators and implanted charges can move inside the material or can be compensated by other charges or environmental conditions, and finally disappear. A focal area of research on electrets concerns their stability [15, 27, 28]. Nowadays, many materials are known as good electrets able to keep their charges for years: for example, Teflon® and silicon dioxide (SiO₂) whose stability is estimated at more than 100 years [29–32].

A structure able to turn vibrations into electricity using electrets is introduced in figure 4: the system is composed of a counter-electrode and an electrode on which is deposited an electret, spaced by an air gap and connected by an electrical load (here a resistor). The electret has a constant charge \( Q_1 \), and, due to electrostatic induction and charge conservation, the sum of charges on the electrode and on the counter-electrode equals the charge on the electret: \( Q_1 = Q_1 + Q_2 \).

When a vibration occurs, it induces a change in the capacitor geometry (e.g. the counter-electrode moves away from the electret, changing the air gap and then, modifying the influences of the electret on the counter-electrode) and a reorganization of charges between the electrode and the counter-electrode through the load. This induces a current across the load \( R \) and part of the mechanical energy is then turned into electricity.

The converter introduced in figure 4 is integrated into a clamped–free beam mechanical structure (figure 5). The lower face of the beam is metallized and is used as the counter-electrode. The electret and the electrode are placed under the beam, separated by an air gap and electrically connected by a load (figure 5(a)). According to the formula of William and Yates (2), the output power is proportional to the mobile mass; consequently, to increase the output power, a proof mass \( m \) is added at the free end of the cantilever. Vibrations \( y(t) \) induce a relative displacement \( x(t) \) of the mobile mass compared to the electrode. Structure parameters are presented in figure 5(b) where \( h \) is the beam thickness, \( w \) is its width, \( L \) is the length between the clamping and the gravitational center of the mass, \( 2L_m \) is the mass length, \( t \) is the thickness of the air gap between the counter-electrode and the electret, \( g_0 \) is the thickness of this air gap without vibrations and without the weight effects \( W, V \) is the surface voltage of the electret, \( C_1 \) is the capacitance of the electret, \( C_2 \) is the capacitance of the air gap and finally \( λ \) is the electrode length.

To identify the main parameters of this kind of energy harvester and to maximize the output power for a given vibration \( y(t) = Y \sin(\omega t) \), it is necessary to find coupled mechanical and electrostatic equations that rule the energy harvester.

### 3. Analytical model of the cantilever-based electret energy harvester

To determine the output power of the energy harvester for a given vibration \( y(t) \), it is necessary to solve the equation of motion and to find the quantity of charge transferred to the output. Therefore, the goal of section 3 is to develop an analytical model for the cantilever-based electret energy harvester parameterized in figure 5 for mechanical and electrostatic parts.

#### 3.1. Model of the mechanical system

The clamped–free beam with a mass at the free end can be modeled as a damped mass–spring structure as presented in figure 1 and by adding the effect of weight \( W = mg \). The mechanical friction forces can be modeled as viscous forces \( f_m = b_m \dot{x} \) and the electrostatic force is the derivative of the electrostatic energy of the capacitor \( W_e \) with respect to the displacement \( x \). \( W_e \) is equal to the charge on the upper electrode \( Q_2 \) squared, divided by twice the capacitance as a function of time \( C(t) \). Thereby, the mechanical system is ruled by (5).

\[
m\ddot{x} + b_m \dot{x} + k x - \frac{d}{dx} \left( W_e \right) - mg = -m \ddot{y} \Rightarrow \dot{m} \ddot{x} + b_m \dot{x} + k x = -d \left( \frac{Q_2^2}{2C(t)} \right) - mg = -m \ddot{y}.
\]

To maximize the output power of the energy harvester, the natural angular frequency \( \omega_n = \sqrt{k/m} \) of the mass–spring structure has to be tuned to the angular frequency of the ambient vibrations \( \omega \). Moreover, according to equations from mechanical structure theory, the spring constant \( k \) can be deduced from the geometric parameters of the beam as follows:

\[
k = \frac{m \omega_n^2}{L} = \frac{3EI}{L^3} = \frac{Ew^3}{4L^3}
\]
where $E$ is the Young’s modulus and $I$ the quadratic moment of the beam.

Because of the mass, the behavior of the beam has to be studied in two parts. A drawing of the structure is presented in figure 6 and shows the deformation of the cantilever $\delta (z)$ as a function of the position on the cantilever $z$ for a forced deflection $x$ at $z = L$. The first part ($z \in [0, L_1 = L - L_m]$) does not have an additional mass: its behavior corresponds to that of a clamped–free beam whose deflection at the end ($x_1$) is imposed and given by $\delta (z) = L^2 / 2L_1^2 (3L_1 - z)$. The second part that has the additional mass ($z \in [L_1, L_2 = L + L_m]$) follows the deflection of part 1: the derivative of the deflection ($\delta (z)$) with respect to the position ($z$) for part 2 is constant and equal to the derivative of the deflection of part 1 at $z = L_1$ (7):

$$c = \left. \frac{d \delta (z)}{dz} \right|_{z = L_1} = \left. \frac{d \delta (z)}{dz} \right|_{z \in [L_1, L_2]} = \frac{3}{2} \frac{x_1}{L_1}$$

with $x_1 = x - cL_m$.

Therefore, for a given static deflection ($x$) on the position $L$ of the beam, the deformation of the beam can be simply expressed as a function of the parameters in both parts:

$$\delta (z) = \begin{cases} \frac{x_1}{2L_1^2} (3L_1 - z) & [\text{part 1}] \\ c(z - L) + x & [\text{part 2}] \end{cases}$$ (8)

Figure 7 presents the beam deformation resulting from equation (8) for a beam of $L = 30$ mm and $L_m = 2$ mm and for an imposed static displacement of $x = 300 \mu m$ compared to the deformation performed by FEM calculation (Comsol® Multiphysics). This proves that our calculations fit with FEM results.

Nevertheless, the problem we want to solve is not static but dynamic. Therefore, it is useful to verify that the beam deformation behavior is the same in dynamic and in static modes. We have verified this using FEM: it confirms that the deformation in dynamic and in static modes can be considered as equivalent. Thus, we can consider that the deflection in the dynamic mode can be simply expressed with (8) assuming that $x$ is the imposed deflection on the mass gravity point.

### 3.2. Modeling of the electrostatic system

The equivalent model of the energy harvester is presented in figure 8, where $Q_2$ is the charge on the counter-electrode. $V$ is the surface voltage of the electret and $C(t)$ the capacitance between the beam and the electrode. This capacitance corresponds to the serial capacitance formed by the capacitance $C_1$ of the electret dielectric material and the capacitance $C_2(t)$ of the air gap. Kirchhoff’s laws give the differential equation that governs the electrostatic system (9):

$$\frac{dQ_2}{dt} = \frac{V}{R} - \frac{Q_2}{R} \left[ \frac{1}{C(t)} \right] = \frac{V}{R} - \frac{Q_2}{R} \left[ \frac{1}{C_1} + \frac{1}{C_2(t)} \right].$$ (9)

Moreover, the electrostatic energy stored in the capacitor is

$$W_c = \frac{1}{2} \frac{Q_2^2(t)}{C(t)}.$$ (10)

To solve (9), it is necessary to know the capacitance of the electrostatic converter as a function of the imposed deflection ($x$). Knowing the cantilever deformation, and considering a
By integrating these expressions, the total capacitance between the imposed deflection. With our parameters (\(\varepsilon\), \(C\)) capacitor of infinitesimal length (dz) (figure 6), one can get the infinitesimal capacitance on both parts (dC_p1 and dC_p2) for a given x:

\[
dC_{p1}(x) = \frac{\varepsilon_0 w \, dz}{g_0 - \delta(z) + \frac{L}{\varepsilon_t}} \quad \text{with} \quad \delta(z) = \frac{x_1}{2L_1} + (3L_1 - z) \quad [\text{part 1}]
\]

\[
dC_{p2}(x) = \frac{\varepsilon_0 w \, dz}{g_0 - \delta(z) + \frac{L}{\varepsilon_t}} \quad \text{with} \quad \delta(z) = c(z - L) + x \quad [\text{part 2}].
\]

By integrating these expressions, the total capacitance between both electrodes is

\[
C(x) = C_{p1}(x) + C_{p2}(x) = \varepsilon_0 w \int_{L_1-L}^{L_1} \frac{dz}{g_0 - x + \frac{d(M-x)}{2L_1} + d} + \frac{\varepsilon_0 w}{\varepsilon} \ln\left(\frac{g_0 + \frac{x}{\varepsilon} + cL_m - x}{g_0 + \frac{x}{\varepsilon} - cL_m - x}\right).
\]

The integral defining \(C_{p1}(x)\) cannot be analytically calculated and will be numerically computed.

This capacitance expression has been compared to a FEM simulation and the curves presented in figure 9 show that the results are in excellent agreement. These results were also compared to the formula of a simple plane capacitor neglecting fringe effects \(C(x) = \varepsilon_0 S/(g_0 - x + d/\varepsilon_t)\), where S is the surface of the electrodes, \(g_0\) is the initial gap and x is the imposed deflection. With our parameters (\(L_m = 2 \, \text{mm}, \ L = 30 \, \text{mm}, \ g_0 = 505 \, \mu\text{m}, \ d = 100 \, \mu\text{m}, \ w = 12.33 \, \text{mm}, \ \varepsilon_t = 2, \ \lambda = 10 \, \text{mm}\)), we have found that the model of the simple plane capacitor overestimates (up to 35%) the maximal capacitance of the energy harvester.

This accurate value of the capacitance for a given deflection is then applied in the mechanical system introduced in section 3.1.

3.3. Complete analytical model

In order to get the output power of the energy harvester, mechanical and electrostatic systems have to be coupled.

\[
\text{Figure 8. Equivalent electric model of the energy harvester.}
\]

\[
\text{Figure 9. Capacitance between the electrodes (C) versus forced displacement (x) (L_m = 2 \, \text{mm}, \ L = 30 \, \text{mm}, \ g_0 = 505 \, \mu\text{m}, \ d = 100 \, \mu\text{m}, \ w = 12.33 \, \text{mm}, \ \varepsilon_t = 2, \ \lambda = 10 \, \text{mm}).}
\]

From (5) and (9), one can find that the system of equations that governs the energy harvester is (13):

\[
m\ddot{x} + b_m\dot{x} + k x - \frac{d}{2C(t)}\left(\frac{Q_2^2}{2\varepsilon V}\right) - mg = -m\ddot{y}
\]

\[
\frac{dQ_2}{dt} = \frac{V}{R} - \frac{Q_2}{C(t)R}.
\]

Nevertheless, it is not possible to get an analytical expression for \(x\) and \(Q_2\). Therefore, the system is numerically solved in Matlab/Simulink (figure 10).

The deflection (x) given by Matlab and the voltage across the resistor \(U_R = R(dQ_2/dt)\) versus time are presented in figures 11(a) and (b) for \(V = 1400 \, \text{V}, \ d = 127 \, \mu\text{m}, \ g_0 = 1 \, \text{mm}, \ \lambda = 20 \, \text{mm}, \ R = 300 \, \Omega\).

Figure 11(a) shows that the output voltage of cantilever-based electret energy harvesters can be higher than 200 V. This can greatly simplify rectification of the output voltage using diode bridges. Moreover, figure 11(a) shows a peculiarity of the output voltage of cantilever-based electret energy harvesters: the output voltage presents a discontinuity when it passes from its higher value (the capacitance is just before its maximum \(C_{\text{max}}\)) to its lower value (the capacitance is just after its maximum \(C_{\text{max}}^+\)) because the current changes direction when the capacitance crosses its maximum. The current also changes direction when the capacitance crosses its minimum \(C_{\text{min}}\). But, since the output voltage equals zero when the capacitance is minimum, no discontinuity appears on the output voltage.

In this section we have developed the complete analytical model of the energy harvester and its implementation in Simulink. Thanks to this model, the system can be optimized to give the maximum output power.

4. Output power and optimization

The goal of this section is to maximize the average output power of the energy harvester \(P\) for a given vibration \(y(t)\). Actually, the power can be simply computed from the derivative of \(Q_2\) given by the Simulink model presented in
We will determine in section 4.1 the parameters to optimize before optimizing them in section 4.2:

\[ P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} R \left( \frac{dQ_z}{dt} \right)^2 dt \]  

(14)

where \( t_1 \) and \( t_2 \) are times taken in the steady state.

### 4.1. Parameters to optimize

We have chosen to consider \( L, L_m, m, w, k, Y, \omega = 2\pi f, V, \varepsilon_s, d \) and \( \xi \) as given parameters and the load \( R \), the electrode length \( \lambda \) and the initial air gap \( g_0 \) as parameters to optimize. Figures 12(a)–(c) present the output power of the energy harvester when one parameter varies while the others are kept constant. Constant values are: \( Y = 10 \, \mu m, f = 50 \, Hz, m = 5 \, g, \xi = 1/150, V = 1400, w = 12.3 \, mm, d = 127 \, \mu m, \varepsilon_s = 2, L_m = 2 \, mm, L = 30 \, mm \) and \( R = 1 \, G\Omega, \lambda = 10 \, mm, g_0 = 2 \, mm \) when one of them varies.

It is obvious that those three parameters play an important role in the behavior of the energy harvester and it is also obvious that an optimum is present in each curve signifying that optimal parameters exist. As these parameters are not independent, they have to be optimized together in a single loop.

### 4.2. Optimization and maximum output power

The optimization process uses the \texttt{fminsearch} Matlab function to minimize the inverse of the output power \((1/P)\) considering \((R, \lambda, \text{and } g_0)\) as parameters. Table 1 gives the maximum output powers and the optimal parameters for different mobile masses for an ‘ambient’ vibration of \((Y, f) = (10 \, \mu m, 50 \, Hz) \approx 1 \, ms^{-2}\): this proves that the model of William and Yates gives a good approximation of the output power but is not rigorously exact in cantilever-based electret energy harvesters. Actually, if the system could be modeled by the William and Yates model, \(\alpha_{W\&Y}\) should be equal to 1 whatever the value of the mobile mass. Moreover, when the electrostatic force is sufficiently high, which is directly linked to the surface voltage of the electret, output power of the energy harvester is bigger than the output power determined by William and Yates’ model. When the surface voltage of the electret is not high enough to induce a sufficient electrostatic force that can absorb the kinetic energy of the mobile mass, the optimal energy cannot be obtained with the mobile mass (e.g. when \(m = 10 \, g\)). For \(m = 5 \, g\), a surface voltage of 1400 V should

![Figure 10. Simulink model of the energy harvester.](image)

![Figure 11. (a) Example of output voltage and (b) deflection versus time.](image)
Table 1. Output power \(P\) as a function of the mass \(m\) with \(V = 1400\) V.

| \(m\) (g) | \(R_{\text{opt}}\) (GΩ) | \(\lambda_{\text{opt}}\) (mm) | \(g_{\text{opt}}\) (μm) | \(P\) (μW) | \(P_{\text{WAY}}\) (μW) | \(P/m\) (μW g\(^{-1}\)) | \(\alpha_{\text{WAY}}\) |
|-----------|-----------------|-----------------|-----------------|--------|-----------------|-----------------|-----------|
| 1         | 10              | 6.4             | 700             | 36.59  | 29.07           | 36.59           | 1.26      |
| 2         | 7.1             | 5.9             | 602             | 71.7   | 58.14           | 35.85           | 1.23      |
| 3         | 4.36            | 7.4             | 600             | 104    | 87.21           | 34.67           | 1.19      |
| 5         | 2.18            | 9.6             | 593             | 160    | 145.34          | 32              | 1         |
| 10        | 0.8             | 14.30           | 901             | 173    | 290.68          | 17.3            | 0.6       |

Figure 13. Output power as a function of variations of \(\lambda\) and \(g_0\) around their optimal value.

Figure 14. Effect of the vibrations on the optimized system.

allow the harvesting of \(160\) μW which corresponds to a power density per mass unit of \(\sim 30\) μW g\(^{-1}\).

The results presented in table 1 are given with an accuracy of 1 μm for \(g_0\) and 10 μm for \(\lambda\). These precisions will not be easy to obtain. To see the effect of inaccuracies in \(\lambda\) and \(g_0\) on the response of the system, we have plotted the output power of the system when \(g_0\) and \(\lambda\) range from 100 μm to their optimal values (zero corresponds to the optimal value). Results presented in figure 13 prove that the output power does not vary much near the optimal values of \(g_0\) and \(\lambda\), but to avoid a contact between the counter-electrode and the electret, for the prototype, we will choose a value of \(g_0\) slightly higher than the optimal value. Therefore, even with inaccuracies on \(\lambda\) and \(g_0\) (\(\sim 50\) μm), the output power should be equal to at least 140 μW.

Similarly, the effect of the frequency and the amplitude of vibrations was evaluated for the optimal design. As the system is resonant and low damped, variations in \(f\) and \(Y\) induce a large change in output power (figure 14): these parameters are critical but can be adjusted with a good accuracy. Therefore this proves that, when the parameters of vibrations are constant, cantilever-based electret energy harvesters are good energy harvesters. But, if the amplitude of vibrations increases, it can lead to contact between the upper electrode and the electret that can damage the latter.

The resonant system has been optimized and theoretical results have proven that up to \(160\) μW could be reached with low vibrations (10 μm@50 Hz) \(\approx 1\) ms\(^{-2}\). These parameters are now tested on a prototype.

5. Experimental results and new model taking parasitic capacitances into account

The goal of the experimental results presented in section 5 is to validate theoretical results that have been obtained in the previous sections and to see the limits of our model. We conclude this section with a better analytical model of the energy harvester that takes parasitic capacitances into account.
5.1. Prototype and expected output power

To ensure a good flatness, the beam is made of silicon and attached to the frame at one end. The mobile mass attached to the other end is made of tungsten that has a high density \( d = 17 \) to limit its size. The electret is made from Teflon FEP (fluorinated ethylene propylene) that is well known for being a good electret [32]. In this study, the electret is charged to 1400 V. Our prototype design is presented in figure 15.

The natural frequency of the structure, computed with FEM, is actually 48.77 Hz. The difference from our model is due to the simplification in (6). To fit the vibrations imposed by the environment (50 Hz), the beam width is slightly changed to 13 mm. Therefore, the output power of the energy harvester should be 140 \( \mu \)W and the voltage across the resistance versus time should look like the one presented on figure 16(a) given by simulation (figure 16(b) is a zoom of figure 16(a)).

The prototype was made on a glass support (figure 17(a)) to limit parasitic capacitances. A mass of 5 g of tungsten was added at the free end of the cantilever. The electret is obtained by evaporating a 300 nm thick layer of aluminum on the rear face of a Teflon FEP film. It is glued on a sheet of copper to ensure the flatness of the electret during charging. The electret is charged using a standard corona discharge as presented in figure 3 with a point voltage \( V_p \) of 10 kV and a grid voltage \( V_g \) of 1400 V. It is placed in an oven at 175 °C and cooled to the ambient temperature while charging to improve stability. The long-term stability was not studied, but the short-term (some days) experiments showed low charge losses (−0.21% in 8 days) (figure 17(b)).

5.2. Output power, comparison to the theory and limits

We now present the experimental results we obtained on our prototype. The optimized parameters were applied to the prototype introduced in figure 17(a). The output power of our prototype is presented in figure 18(a). Experimental and theoretical curves do not fit and the output power is much lower than expected. These differences are due to parasitic capacitances that become important when using high-value loads. In those cases, the model given in figures 5(b) and 8 should be modified to take a parasitic capacitance \( C_{par} \) in parallel with the load into account.

To avoid these phenomena, the value of the load was limited to 300 M\( \Omega \) (chosen after some experimental measurements in order to limit the parasitic capacitances induced by the load) and the optimization process was restarted on \( g_0 \) and \( \lambda \). ‘New’ optimized values are presented in table 2.

The structure was tested again with the new load and its output voltage is presented in figure 18(b). Experimental and
Figure 18. Experimental output voltages (a) for $R = 2,2 \Omega$ and (b) for $R = 300 \Omega$.

Table 2. Parameters and values.

| Parameter Designation | Value |
|-----------------------|-------|
| $M_{\text{beam}}$    | Material of the beam Silicon |
| $E$                   | Young’s modulus of silicon 160 GPa |
| $L$                   | Distance between the clamping and the center of gravity of the mass 30 mm |
| $h$                   | Thickness of the beam 300 $\mu$m |
| $w$                   | Width of the beam/width of the electret 13 mm |
| $2L_m$                | Length of the mobile mass 4 mm |
| $m$                   | Mobile mass 5 g |
| $\omega_n$           | Natural angular frequency/angular frequency of vibrations $50 \times 2 \pi$ rad s$^{-1}$ |
| $Q_m$                 | Mechanical quality factor of the structure 75 |
| $M_{\text{rect}}$     | Material of the electret FEP |
| $\varepsilon_r$       | Dielectric constant of the electret 2 |
| $d$                   | Thickness of the electret 127 $\mu$m |
| $V$                   | Surface voltage of the electret 1400 V |
| $g_0$                 | Thickness of the initial air gap 700 $\mu$m |
| $\lambda$             | Length of the electrode 22.8 mm |
| $R$                   | Load 300 $\Omega$ |

Theoretical curves fit, except for negative voltages. This is once again due to parasitic capacitances that clip the signal in its negative part. The mean output power of the energy harvester is 50 $\mu$W when it is submitted to vibrations of 10 $\mu$m@50 Hz (1 ms$^{-2}$) (our simulation predicted 80 $\mu$W).

Our experimental results correspond to a factor of merit $\alpha_{\text{WAV}}$ equal to 34% and to a factor of merit $\chi$ equal to 38.75, putting our results among the best of the state of the art (table 3); however, our experimental results are quite different from the theoretical results.

5.3. Model taking parasitic capacitances into account

In order to explain the differences between our theoretical and experimental results, we have developed a new model that takes parasitic capacitances into account. The parasitic capacitance of the whole system is modeled as a capacitor $C_{\text{par}}$ in parallel with the energy harvester and the load, as presented in figure 19. $U$ is the voltage across the resistor, the parasitic capacitance and the electret energy harvester.

In order to model the behavior of the energy harvester taking parasitic capacitances into account, the equation that rules the electrostatic part is modified as following (15) (obtained using Kirchhoff’s laws) while the equation that rules the mechanical part is the same as in (5) and (13):

\[
\frac{dQ_2}{dr} = \frac{1}{1 + \frac{C_{\text{par}}}{C(t)}} \left( \frac{V}{R} - Q_2 \left( \frac{1}{RC(t)} - \frac{C_{\text{par}}}{C(t)^2} \frac{dC(t)}{dr} \right) \right).
\]

The instantaneous harvested power is given by (16):

\[
p(t) = \frac{U^2}{R} = \frac{1}{R} \left( V - \frac{Q_2}{C(t)} \right)^2.
\]
Therefore, to limit their effects, the load should be chosen so as not to exceed $Z_{par} = \frac{1}{C_{par} \omega}$, the impedance of the parasitic capacitances which is roughly equal to $Z_{par} = 500 \, \Omega$ in our case.

6. Conclusion and perspectives

We have developed an analytical model of a cantilever-based electret energy harvester that is in agreement with FEM results. The optimization process has shown that the power harvested by these structures is of the same magnitude as the theoretical output powers developed by William and Yates as soon as the surface voltage of the electret is sufficient to absorb the kinetic energy of the mobile mass. Finally, we validated our model with experimental results which reach up to $10 \, \mu W$ per gram of mobile mass for low ambient vibrations of $0.1 \, g$ ($1 \, m \, s^{-2}$), using a resonant system.

Cantilever-based energy harvester can be a good low-cost solution to harvest energy when vibrations are constant in frequency and amplitude. The output power meets the magnitude of powers reached by piezoelectric or electromagnetic solutions.

Table 3. Comparison with seven prototypes among the most recent state-of-the-art electret energy harvesters.

| Author       | References       | Vibrations                  | Active surface ($S$) | Electret potential ($V$) | Output power ($P$) | Figure of merit ($\chi$) |
|--------------|------------------|-----------------------------|----------------------|--------------------------|---------------------|--------------------------|
| Suzuki       | [14]             | 1 mm@37 Hz (54.0 ms$^{-2}$) | 2.33 cm$^2$          | 450 V                    | 0.28 $\mu W$        | 9.56 $\times 10^{-5}$    |
| Halvorsen    | [16]             | 2.8 $\mu m$@596 Hz (39.2 ms$^{-2}$) | 0.48 cm$^2$          | 25 V                     | 1 $\mu W$           | 5.06 $\times 10^{-2}$    |
| Kloub        | [17]             | 0.08 $\mu m$@1740 Hz (9.6 ms$^{-2}$) | 0.42 cm$^2$          | 12 $\mu W$              | 14.2                |
| Naruse       | [18]             | 25 mm@2 Hz (3.9 ms$^{-2}$)   | 9 cm$^2$             | 600 V                    | 3.58 $\times 10^{-2}$ |
| Edamoto      | [19]             | 500 $\mu m$@21 Hz (8.7 ms$^{-2}$) | 3 cm$^2$             | 120 V                    | 14.2                |
| Miki         | [20]             | 100 $\mu m$@63 Hz (15.7 ms$^{-2}$) | 3 cm$^2$             | 180 V                    | 1 $\mu W$           | 5.37 $\times 10^{-3}$    |
| Honzumi      | [21]             | 9.35 $\mu m$@500 Hz (92 ms$^{-2}$) | 0.01 cm$^2$          | 52 V                     | 90 $p W$           | 3.32 $\times 10^{-5}$    |
| This work (th.) |               | 10 $\mu m$@50 Hz (1.0ms$^{-2}$) | 4.16 cm$^2$          | 1400 V                   | 152 $\mu W$        | 117.84                   |
| This work (exp.) |             | 10 $\mu m$@50 Hz (1.0ms$^{-2}$) | 4.16 cm$^2$          | 1400 V                   | 50 $\mu W$        | 38.75                    |

Figure 20. Experimental output voltages (a) for $R = 2, 2 \, G\Omega$ and (b) for $R = 300 \, M\Omega$ and comparison to theory taking parasitic capacitances into account.

Our Simulink model was modified to take these changes into account. Our experimental results were then compared to our theoretical results taking parasitic capacitances into account (figures 20(a) and (b)) where parasitic capacitances with the 300 M$\Omega$ load are estimated to be 5 pF and to 10 pF with the 2.2 G$\Omega$ load.

Figures 20(a) and (b) show that theoretical and experimental results fit perfectly and validate our new model. Therefore, it appears that parasitic capacitances have a large impact on the behavior of the energy harvester, decreasing the harvested power, especially when using high-value resistors. Unfortunately, as parasitic capacitances greatly depend on the load, restarting an optimization process taking parasitic capacitances into account would be difficult. Moreover, it would be of limited interest since parasitic capacitances can change a lot with the use of managing electronic circuits. Therefore, to limit their effects, the load should be chosen so as not to exceed $Z_{par} = 1/C_{par} \omega$, the impedance of the parasitic capacitances which is roughly equal to $Z_{par} = 500 \, M\Omega$ in our case.

6. Conclusion and perspectives

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