Crossing the phantom divide barrier with scalar tensor theories

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Abstract. There is accumulating observational evidence (based on SnIa data) that the dark energy equation of state parameter \( w \) may be evolving with time and crossing the phantom divide barrier \( w = -1 \) at recent times. The confirmation of these indications by future data would indicate that minimally coupled quintessence cannot reproduce the observed expansion rate \( H(z) \) for any scalar field potential. Here we explicitly demonstrate that scalar tensor theories of gravity (extended quintessence) can predict crossing of the phantom divide barrier. We reconstruct phenomenologically viable scalar–tensor potentials \( F(\Phi) \) and \( U(\Phi) \) that can reproduce a polynomial best fit expansion rate \( H(z) \) and the corresponding dark energy equation of state parameter \( w(z) \) crossing the \( w = -1 \) line. The form of the reconstructed scalar tensor potentials is severely constrained but is not uniquely determined. This is due to observational uncertainties in the form of \( H(z) \) and also because a single observed function \( H(z) \) does not suffice for the reconstruction of the two potential functions \( F(\Phi) \) and \( U(\Phi) \).

Keywords: dark energy theory, gravity, cosmology of theories beyond the SM

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Detailed observations of the relation between luminosity distance and redshift for extragalactic Type Ia Supernovae (SnIa) indicates \cite{1,2} that the universe has entered a phase of accelerating expansion (the scale factor obeys \( \ddot{a} > 0 \)). In addition, a diverse set of other cosmological data which includes large scale redshift surveys \cite{3} and measurements of the cosmic microwave background (CMB) temperature fluctuations spectrum \cite{4} has shown that the spatial geometry of the universe is flat but there is not enough matter density to justify this flatness. Thus there is an additional cosmological component required to justify the observed flatness. This component should have repulsive gravitational properties to justify the observed present accelerated expansion. Such properties can either be due to a modified theory of gravity \cite{5} or, in the context of standard general relativity, to the existence of a smooth energy component with negative pressure termed ‘dark energy’ \cite{6,7}. This component is usually described by an equation of state parameter \( w \equiv \frac{p}{\rho} \) (the ratio of the homogeneous dark energy pressure \( p \) over the energy density \( \rho \)). For cosmic acceleration, a value of \( w < -\frac{1}{3} \) is required as indicated by the Friedmann equation

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p).
\] (1.1)

The cosmological constant \( (w = -1) \) is the simplest viable candidate for dark energy. It predicts an expansion history of the universe which is described by a Hubble parameter \( H(z) \) as a function of the redshift \( z \) given by

\[
H^2(z; \Omega_{0m}) = \left( \frac{\dot{a}}{a} \right)^2 = H_0^2[\Omega_{0m}(1 + z)^3 + (1 - \Omega_{0m})] \]

where flatness has been imposed and \( \Omega_{0m} \equiv \rho_0/\rho_c \) is the single free parameter of this simplest data consistent parameterization (LCDM). Such a model has two important disadvantages.

- It requires extreme fine tuning of the value of the cosmological constant \( \Lambda \) for the accelerating expansion to start at around the present cosmological time (coincidence problem).
• It does not provide the best possible fit to recent SnIa data. In particular, recent analyses [8]–[11] of the latest and most reliable SnIa dataset (the Gold dataset [2]) have indicated that even though LCDM cannot be ruled out, significantly better fits compared to LCDM are obtained by allowing for a redshift-dependent equation of state parameter. Extensive analyses of such parameterizations of \( H(z) \) have shown that the parameterizations that allow crossing of the \( w = -1 \) line (known also as the Phantom Divide Line: PDL) provide significantly better fits to the data (the parameter values corresponding to LCDM are in the range of 1\( \sigma \) to 2\( \sigma \) away from the best fit [9]–[11]). It is therefore important to construct physical models that allow for a redshift-dependent \( w \) that crosses the PDL. It has been shown however that this is a highly non-trivial task [12]–[14].

The simplest approach towards constructing a physical model for dark energy is to associate it with a homogeneous minimally coupled scalar field \( \phi \) with negative pressure whose dynamics is determined by a potential properly chosen so that the energy density of \( \phi \) comes to dominate the universe at the present time. Such models are described by Lagrangians of the form

\[
\mathcal{L} = \pm \frac{1}{2} \dot{\phi}^2 - V(\phi) \tag{1.3}
\]

where the upper (lower) sign corresponds to a quintessence [15] (phantom [16]) field. It should be noted however that phantom fields [16], with negative kinetic term \( (w < -1) \), violate the strong energy condition, the null energy condition \( (\rho + p \geq 0) \) and may be physically unstable. They also lead to a future ‘Big Rip’ singularity where all bound systems get dissociated due to the increasing repulsive gravitational force of phantom energy [17]. The phantom instability however can be cured in extended gravity theories [18]. The equation of state parameter corresponding to (1.3) is

\[
w = \frac{p}{\rho} = \frac{\pm(1/2)\dot{\phi}^2 - V(\phi)}{\pm(1/2)\dot{\phi}^2 + V(\phi)}. \tag{1.4}
\]

For quintessence (phantom) models with \( V(\phi) > 0 \) (\( V(\phi) < 0 \)) the parameter \( w \) remains in the range \(-1 < w < 1 \). For an arbitrary sign of \( V(\phi) \) the above restriction does not apply but it is still impossible for \( w \) to cross the PDL \( w = -1 \) in a continuous manner. The reason is that for \( w = -1 \) a zero kinetic term \( \pm \dot{\phi}^2 \) is required and the continuous transition from \( w < -1 \) to \( w > -1 \) (or vice versa) would require a change of sign of the kinetic term. The sign of this term however is fixed in both quintessence and phantom models. This difficulty in crossing the PDL \( w = -1 \) could play an important role in identifying the correct model for dark energy in view of the fact that data favour \( w \approx -1 \) and furthermore parameterizations of \( w(z) \) where the PDL is crossed appear to be favoured over the cosmological constant \( w = -1 \). Even for generalized \( k \)-essence Lagrangians [19, 20] of a minimally coupled scalar field, e.g.

\[
\mathcal{L} = \frac{1}{2} f(\phi) \dot{\phi}^2 - V(\phi) \tag{1.5}
\]

it has been shown [12] to be impossible to obtain crossing of the PDL. Multiple field Lagrangians (combinations of phantom with quintessence fields [21]–[24]) have been shown in principle to achieve PDL crossing, but such models are complicated and without clear physical motivation (but see [25] for an interesting physically motivated model).
The obvious class of theories that could lead to a solution of the above described problem is the non-minimally coupled scalar fields. Such theories are realized in a universe where gravity is described by a scalar–tensor theory and their study is well motivated for two reasons.

1. A scalar–tensor theory of gravity is predicted by all fundamental quantum theories that involve extra dimensions. Such are all known theories that attempt to unify gravity with the other interactions (e.g. supergravity (SUGRA), M-theory, etc).

2. As shown in the following sections, quintessence scalar fields emerging from scalar–tensor theories (extended quintessence) can predict an expansion rate \( H(z) \) that violates the inequality

\[
\frac{d(H(z)^2/H_0^2)}{dz} \geq 3\Omega_{0m}(1+z)^2.
\]

(1.6)

It is easy to show that the inequality (1.6) is equivalent to

\[
w(z) = \frac{\rho_{DE}(z)}{\rho(z)} = \frac{(2/3)(1+z)\ln H_0}{1 - (H_0/H)^2\Omega_{0m}(1+z)^3} \geq -1
\]

(1.7)

(see e.g. [10] for a derivation of \( w(z) \) in the form of equation (1.7)). Thus, violation of the inequality (1.6) is equivalent to crossing the PDL \( w = -1 \) and is favoured by the Gold SnIa dataset [11], [26]–[30]. The inequality (1.6) cannot be violated in minimally coupled quintessence theories (see equation (2.12)).

The usual approach [13], [30]–[34] in comparing quintessence models with observations is to start with a well-defined theoretical model Lagrangian, identify the predicted form of \( H(z) \) and compare with observational data to identify the consistency and the quality of fit of the model.

This approach is not particularly efficient in view of the infinite number of possible model Lagrangians that may be considered. An alternative, more efficient approach is to start from the best fit parameterization \( H(z) \) obtained directly from data and use this \( H(z) \) to reconstruct the corresponding theoretical model Lagrangian. This later approach was pioneered in [35]–[37] and has been further developed for the cases of both minimally coupled quintessence [38]–[40] and scalar tensor theories [41,42] (extended quintessence [43]–[46]). Extensions of this approach have recently also been applied to \( f(R) \) generalized gravity theories [47]. However, despite the high quality of the data of the Gold dataset that allows a fairly reliable determination of \( H(z) \) (especially at redshifts up to \( z \approx 1 \)) no attempt has been made to reconstruct quintessence or extended quintessence Lagrangians from the \( H(z) \) that best fits the Gold dataset. This task is undertaken in the present study.

We consider a simple polynomial parameterization of \( H(z) \) and fit it to the Gold dataset following [11,27,29]. The best fit form of \( H(z) \) is found to violate the inequality (1.6), i.e. to cross the PDL. Thus it is not consistent with minimally coupled quintessence. We thus use it to reconstruct a class of extended quintessence scalar–tensor Lagrangian which involves two scalar functions \( F(\Phi) \) and \( U(\Phi) \).

The structure of the paper is the following. In section 2 we present in some detail the reconstruction method described above for the case of extended quintessence and derive the main equations that relate the scalar–tensor potentials \( F(\Phi) \) and \( U(\Phi) \) with
the observed Hubble parameter $H(z)$ and other experimental constraints. In section 3 we apply this technique to reconstruct a class of $F(\Phi)$ and $U(\Phi)$ from the actual best fit polynomial $H(z)$ obtained from the Gold dataset. The reconstructed potentials are consistent with other observational constraints (solar system tests and Cavendish-type experiments). We show that in contrast to minimally coupled quintessence, this reconstruction is possible for extended quintessence. However, even though the reconstructed potentials $F(\Phi)$ and $U(\Phi)$ are severely constrained by the form of $H(z)$ and other consistency requirements, they are not uniquely determined. Finally, in section 4 we review the prospects for a more constrained reconstruction of scalar–tensor theories which may significantly contribute towards the identification of the fundamental theory realized in Nature.

2. Technique for reconstructing the scalar–tensor Lagrangian

The scalar–tensor Lagrangian is a generalization of the general relativistic Lagrangian of the form

$$L = \frac{F(\Phi)}{2} R - \frac{Z(\Phi)}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) + \mathcal{L}_m[\psi_m; g_{\mu\nu}]$$

(2.1)

where we have set $8\pi G = 1$ ($F_0 = 1$) and $\mathcal{L}_m$ represents the matter fields and does not depend on $\Phi$ so that the weak equivalence principle is satisfied. One of the degrees of freedom $F(\Phi)$ and $Z(\Phi)$ can be absorbed by a redefinition of $\Phi$. The following parameterizations are commonly used.

- $F(\Phi) \to \Phi, Z(\Phi) \to \omega(\Phi)/\Phi$ is the Brans–Dicke (BD) parameterization, where $\omega(\Phi) = F(\Phi)/(dF/d\Phi)^2$ is the BD parameter.
- $Z(\Phi) \to \pm 1$. The case $Z \to -1$ corresponds [41] to a BD parameter $-3/2 < \omega_0 < 0$ which contradicts solar system measurements [48] for any $U(\Phi)$ allowing cosmological evolution of $\Phi$.
- $g_{\mu\nu} \to g^*_\mu\nu \equiv F(\Phi) g_{\mu\nu}$ and $\Phi \to \varphi$ : $(d\varphi/d\Phi)^2 \equiv \frac{3}{2} (d\ln F(\Phi)/d\Phi)^2 + Z(\Phi)/2F(\Phi)$. This transformation corresponds to the Einstein frame as opposed to the original Jordan frame of equation (2.1). In the Einstein frame the kinetic terms of the graviton and the scalar field are diagonalized and the mathematical analysis of the theory is simplified at the expense of introducing an explicit coupling of the scalar field with matter. An advantage of the original Jordan frame is that the various physical quantities are those measured in experiments.

In what follows we choose the Jordan frame and the parameterization $Z \to 1$. Under this assumption, $F(\Phi)$ should satisfy the following constraints.

- $F(\Phi) > 0$ so that gravitons carry positive energy [41].
- $\omega_0^{-1} = (dF/d\Phi)_0^2 < 4 \times 10^{-4}$ from solar system measurements [48] (the subscript 0 denotes the present time).

Assuming a homogeneous $\Phi$ and varying the action corresponding to (2.1) in background of a flat FRW metric

$$ds^2 = -dt^2 + a^2(t) \left( dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right)$$

(2.2)
we find the coupled system of equations [42]

\[ 3F \cdot H^2 = \rho + \frac{1}{2} \dot{\Phi}^2 - 3H \cdot \dot{F} + U \]  
\[ (-2F \cdot \dot{H}) = (\rho + p) + \dot{\Phi}^2 + \dot{F} - H \cdot \dot{F} \]

where we have assumed the presence of a perfect fluid \((\rho, p)\). Eliminating \(\dot{\Phi}^2\) from (2.4), setting

\[ q(z) \equiv H(z)^2 / H_0^2 \]

and rescaling \(U \rightarrow U \cdot H_0^2\) while expressing in terms of redshift \(z\), we obtain

\[ F'' + \left[ \frac{q'}{2q} - \frac{4}{1 + z} \right] F' + \left[ \frac{6}{(1 + z)^2} - \frac{2}{(1 + z)2q} \right] F = \frac{2U}{(1 + z)^2q^2} + 3\frac{1 + z}{q^2} \Omega_{0m} \] 
\[ \Phi^2 = -\frac{6F'}{1 + z} + \frac{6F}{(1 + z)^2} - \frac{2U}{(1 + z)^2q^2} - \frac{6}{q^2} \Omega_{0m} \]

where the prime ' denotes differentiation with respect to redshift \((d/dz)\) and we have assigned properties of matter \((p = 0, \Omega_{0m} = 3\rho_{0m}/H_0^2)\) to the perfect fluid.

Given the form of \(H(z)\) from observations, equations (2.6) and (2.7) may be used with boundary conditions \(F(0) = 1, F'(0) = 0\) and \(\Phi(0) = 0\) to reconstruct sets of \(F(z), U(z), \Phi(z)\) which predict the given form of \(H(z)\) and are consistent with solar system tests. The system (2.6) and (2.7) may take a more convenient form by setting

\[ F(z) = f(z)(1 + z)^2. \]  

It then takes the form

\[ U(z) = \frac{1}{2}(1 + z)^4 q f'' + \frac{1}{4} q'(1 + z)^4 f' - \frac{3}{2}(1 + z)^3 \Omega_{0m} \] 
\[ \Phi'(z)^2 = -6f - 6(1 + z)f' - \frac{q'}{2q}(1 + z)^2 f' - (1 + z)^2 f'' - 3\Omega_{0m} \frac{1 + z}{q} \]

with boundary conditions \(\Phi(0) = 0, f(0) = 1\) and \(f'(0) = -2\).

The case of minimally coupled \(\Phi\) is regained for \(F(z) = 1\) \((f = 1/(1 + z)^2)\). Indeed, setting \(f(z) = 1/(1 + z)^2\) in (2.9) and (2.10) we find

\[ U(z) = 3q - \frac{q'}{2}(1 + z) - \frac{3}{2}(1 + z)^3 \Omega_{0m} \]
\[ \Phi^2(z) = \frac{1}{q(1 + z)}(q' - 3\Omega_{0m}(1 + z)^2) \]

which are in agreement with previous studies [35,38] reconstructing minimally coupled Lagrangians.

The reconstruction of \(U(\Phi)\) may proceed in the minimally coupled case by finding \(\Phi(z)\) from (2.12) \((\text{setting for example } \Phi(0) = 0)\), inverting for \(z(\Phi)\) and substituting in (2.11) to find \(U(\Phi)\). The inequality (1.6) \((\text{preventing PDL crossing})\), valid for the minimally coupled case, is recovered from (2.12) since \(\Phi^2 > 0\).

As a simple application we may consider the \(H(z)\) induced by a cosmological constant which implies

\[ q(z) = \Omega_\Lambda + \Omega_{0m}(1 + z)^3. \]
In this case (2.11) and (2.12) reduce to
\[ U(z) = 3\Omega_\Lambda = \rho_\Lambda \]  
(2.14)
\[ \Phi'(z) = 0 \]  
(2.15)
as expected. Thus, the reconstruction leads to a uniquely defined potential in the minimally coupled case if \( q' > 3\Omega_{\text{om}}(1 + z)^2 \).

If on the other hand there is a redshift range where \( q' < 3\Omega_{\text{om}}(1 + z)^2 \), an \( F(z) \neq 1 \) \( (f \neq 1/(1 + z)^2) \) is required to keep \( \Phi'^2 > 0 \) in (2.10). Since \( q' < 3\Omega_{\text{om}}(1 + z)^2 \) implies superacceleration which cannot be supported merely by the repulsive gravity of the potential \( U \), a modified strength of gravity will be required \( (F(z) < 1) \). Such an \( F(z) \) is also constrained to obey \( F(z) > 0 \) for all redshifts and \( F(z = 0) \approx 0 \). It is therefore possible that no such \( F(z) \) exists so that for a given \( q(z) \) the three requirements
\[ \Phi'(z)^2 \geq 0 \]  
(2.16)
\[ F(z) > 0 \]  
(2.17)
\[ F'(z = 0) = 0 \]  
(2.18)
are fulfilled. The last requirement is imposed in order to satisfy the observational constraint
\[ \omega_0^{-1} = (dF/d\Phi)^2 < 4 \times 10^{-4} \]  
\[ (dF/dz = (dF/d\Phi_0)\Phi_0'). \]  
In fact, for \( q(z) \) corresponding to a cosmological constant (equation (2.13)) and \( U(z) = 0 \) it has been shown that no acceptable function \( F(z) \) exists [42] because \( F(z) \) becomes negative at relatively low \( z \). If an acceptable \( F(z) \) is found to exist then it will not be uniquely defined because a perturbed \( F(z) \) will also lead to a positive \( \Phi'^2 \), leading to another acceptable solution.

The best case scenario for the existence of an acceptable \( F(z) \) is the case when \( \Phi'(z) = 0 \), i.e. when \( F(z) \) does not have to balance the attractive gravity of \( \Phi'^2 \). If no acceptable \( F(z) \) exists for \( \Phi' = 0 \) (i.e. if \( F(z) < 0 \) for \( z > z_c \)) then there will be no acceptable \( F(z) \), for any \( \Phi'^2 > 0 \) \( (F(z) \) will get even more negative to balance the larger attractive gravity of \( \Phi'^2 \)). Thus, in order to see if an acceptable scalar tensor theory exists for a given \( q(z) \) we simply have to set \( \Phi' = 0 \) in equation (2.10), solve for \( f(z) \) with \( f(0) = 1, f'(0) = -2 \) and see if the solution obeys \( f(z) > 0 \). If \( f(z) < 0 \) for some redshift range then it will remain so for any \( \Phi'^2 > 0 \), and therefore no acceptable scalar tensor Lagrangian exists predicting the given \( q(z) \). If on the other hand \( f(z) > 0 \) for all redshifts of interest, the solution is acceptable and we may start increasing \( \Phi'^2 \) (trying different functional forms) until \( f(z) < 0 \) for some redshift range. Thus we may identify a set of observationally acceptable scalar tensor Lagrangians that predict the given form of \( q(z) \). The case \( \Phi'(z) = 0 \) itself is not a physically interesting case as it cannot be inverted to give \( z(\Phi) \) and the \( \Phi \) dependence of \( F \). However, it does serve as a mathematically interesting starting point to construct realistic scalar–tensor Lagrangians that lead to crossing of the PDL. This procedure will be applied in the next section to identify scalar–tensor Lagrangians that predict the particular form of \( q(z) \) shown in figure 1 (see next section). This is the best fit form of \( q(z) \) in the context of a quadratic polynomial parametrization and the corresponding \( w(z) \) crosses the PDL line.
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Figure 1. The effective equation of state parameter $w(z)$ corresponding to the parameterization (3.1) along with the corresponding form of $H(z) = H_{P2}(z)$ at best fit. The form of $H(z) = H_{LCDM}(z)$ corresponding to a cosmological constant with the same prior is also shown for comparison (dashed line).

3. From the Gold dataset to the fundamental theory

A wide range of expansion history parameterizations $H(z)$ with a flat prior [8]–[11] has been fitted to the Gold and earlier SnIa datasets. The best fits among those parameterizations share the following common features.

- The inequality (1.6) is violated for $z \lesssim 0.3$, i.e. the effective equation of state is $w(z) < -1$ for $z \lesssim 0.3$.
- The inequality (1.6) is respected for $z \gtrsim 0.3$, i.e. the PDL is crossed at $z \gtrsim 0.2$–0.3.

A representative easy to handle parameterization that shares these common features at best fit and has been one of the first introduced in the literature is the polynomial parameterization [9]

$$q(z) = \Omega_{0m}(1 + z)^3 + a_2(1 + z)^2 + a_1(1 + z) + a_0$$  \hspace{1cm} (3.1)

where $a_0 \equiv 1 - a_2 - a_1 - \Omega_{0m}$ due to flatness. For a prior of $\Omega_{0m} = 0.3$ the parameter values that best fit the Gold dataset are [11] $a_1 = -4.16 \pm 2.53$, $a_2 = 1.67 \pm 1.03$. The effective equation of state parameter $w(z)$ corresponding to the parameterization (3.1) is shown in figure 1 along with the corresponding form of $H(z)$ at best fit. The form of $H(z)$ corresponding to a cosmological constant with the same prior is also shown for comparison.

In the redshift range $0 < z < 0.25$ where $w(z) < -1$ the inequality (1.6) is temporarily violated. The PDL is clearly crossed and therefore no minimally coupled scalar can account for the accelerating expansion crossing $w(z) = -1$ (see equation (2.12)). We will thus examine if a non-minimally coupled scalar can account for this type of expansion.

The parameterization (3.1) will be used for the reconstruction of the scalar tensor potentials along the lines described in the previous section. No attempt will be made to reconstruct the unique scalar tensor theory obtained from data. Instead we only examine
if it is in principle possible to construct extended quintessence Lagrangians that lead to a crossing of the PDL in the way favoured by the Gold dataset. We thus use arbitrary forms of $\Phi'(z)$ and first try the ‘best case scenario’ for the existence of $F(z)$ setting $\Phi'(z) = 0$ in (2.10) while using boundary conditions $f(0) = 1$, $f'(0) = -2$. The solution of (2.10) for $F(z) = (1 + z)^2 f(z)$ is easily found (see figure 2) to fulfil the requirement (2.17) ((2.18) is imposed by the boundary conditions) for all redshifts $z$ and is therefore observationally acceptable.

We next ask the following question: Can the requirements (2.16)–(2.18) continue to hold if we use a small positive $\Phi'(z)^2$? By numerically solving equation (2.10) for a few trial functions $\Phi'(z)^2$ and boundary conditions $f(0) = 1$, $f'(0) = -2$ it is easy to see that only small, decreasing with $z$ functions $\Phi'(z)$ can accommodate $F(z) > 0$ for all redshifts $z$. Examples of such functions include

$$\Phi'(z)^2 = a(1 + z)^{-n} \quad (3.2)$$

$$\Phi'(z)^2 = ae^{-z} \quad (3.3)$$

with $a \lesssim 0.35$ and $n \gtrsim 1$. All such functions lead to qualitatively similar forms for $f(z)$.

In order to reconstruct the scalar potentials $U(\Phi)$, $F(\Phi)$ of the scalar tensor theory for the best fit $q(z)$ of (3.1) we use the following steps.

(1) Pick a function $\Phi'(z) = g(z)$ leading to $F(z) > 0$ in the redshift range of interest. Such functions are given for example by equations (3.2) and (3.3). Use this function and the best fit $q(z)$ of equation (3.1) to solve equation (2.10) numerically.

(2) Use the resulting $f(z)$ along with the best fit $q(z)$ in equation (2.9) to find $U(z)$.

(3) To convert $U(z)$ to $U(\Phi)$ solve the differential equation $\Phi'(z) = g(z)$ using the selected function $g(z)$ with boundary condition $\Phi(0) = 0$ (set for example $\Phi$ to 0 at the present time). Use the solution $\Phi(z)$ to invert and find $z(\Phi)$.

Figure 2. The function $F(z)$ can fulfil the requirements (2.17) and (2.18) for a constant $\Phi(z)$ (e.g. $\Phi = 0$).

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(4) Substitute $z(\Phi)$ in the $U(z)$ found in step 2 and in $F(z) = (1+z)^2 f(z)$ found in step 1 to find $U(z(\Phi)) = U(\Phi)$ and $F(z(\Phi)) = F(\Phi)$.

Notice that the inversion $z(\Phi)$ cannot be made for all values of $\Phi$ but only for those values of $\Phi$ appearing in the $\Phi(z)$ plot. This is a limitation because we have found that $\Phi(z)$ tends to a constant at large $z$ (see figure 3) because of large cosmic friction at early times.

The above steps have been implemented in a Mathematica code. For definiteness we have assumed a $\Phi'(z)$ of an exponentially decreasing form given in (3.3) with $a = 0.35$. The resulting functions $\Phi(z)$, $U(z)$ and $F(z)$ are shown in figure 3 in the redshift range $0 < z < 2$. The potential $U(z)$ is found to be a slowly increasing function of $z$ while $F(z)$ has a shallow minimum at $z \simeq 1$ and increases monotonically beyond that redshift. The minimum of $F(z)$ in figure 3 (see also figure 5), however, is deeper compared to that of figure 2 corresponding to $\Phi'(z) = 0$ because $F(z)$ has to balance the attractive gravity of $\Phi'(z) \neq 0$, and therefore has to diverge more from $F(z) = 1$. It is easily shown that the resulting potentials remain qualitatively similar for other acceptable forms of $\Phi'(z)$. By inverting $\Phi(z)$ along the lines of step 3 the potentials $U(\Phi)$ and $F(\Phi)$ may be found and are shown in figures 4 and 5 respectively. In figure 4 the numerically obtained potential corresponds to the thick dotted line. The existence of such potentials is our main result and not their particular form which is not unique given the form of $H(z)$. Nevertheless for illustration purposes we attempt to fit the numerically obtained potentials with simple analytic functions. The continuous lines in figure 4 correspond to attempts to fit the numerically obtained potential with simple analytical functions. We have tried several simple analytical functions. A very good fit was provided by an exponential of the form

$$U(\Phi) = A e^{\lambda \Phi^2} \quad (3.4)$$

Figure 3. The reconstructed functions $\Phi(z)$, $U(z)$ and $F(z)$ in the redshift range $0 < z < 2$. 

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2 The Mathematica [56] file including the numerical analysis and the production of the figures of the paper can be downloaded along with the source files of the paper from the astro-ph archive or sent by e-mail upon request.
with $\lambda \simeq 3$. The fit was worse for other powers of $\Phi$ in the exponential (the next best approximation $U(\Phi) \sim e^{\lambda \Phi^2} (\lambda \simeq 5)$ is also shown in figure 4 and the fit is clearly not as good as with (3.4)).

It is straightforward to test our results, by substituting the reconstructed functions $U(z)$ and $f(z)$ back in equation (2.9) to solve for $q(z)$ numerically (with boundary condition $q(0) = 1$) and confirm that we get back the best fit form (3.1) with the appropriate parameter values. This test revealed that the numerically obtained functions $U$ and $F$ of figures 3–5 indeed lead to a prediction of exactly the best fit form of $q(z)$.

Alternative best fit forms of $q(z)$ that violate the inequality (1.6) crossing the PDL can be used to reconstruct potentials $F(\Phi), U(\Phi)$. For example [11] has indicated a best fit $q(z)$ parameterization that crosses the PDL repeatedly, showing evidence for oscillations in redshift space. This parameterization was shown to provide a better fit to the Gold dataset than any other parameterization tested in [11]. It is straightforward to reconstruct potentials $F$ and $U$ that reproduce the best fit oscillating $H(z)$ parameterization and show that the expansion history oscillations are inherited by the reconstructed potentials (see footnote 2). Thus the $U \sim e^{\lambda \Phi^2}$ form of the reconstructed potential is not a particularly robust prediction of the Gold dataset.

### 4. Conclusion

We have demonstrated that it is possible to reconstruct experimentally viable scalar–tensor potentials which predict exactly the best fit parameterizations obtained from the recent Gold SnIa dataset. These parameterizations share the common feature of temporally violating the inequality (1.6) or equivalently correspond to a dark energy equation of state that crosses the PDL. This feature is impossible to reproduce in the context of single field quintessence or even phantom models.
Figure 5. The reconstructed function $F(\Phi)$ has a local minimum at a small value of $\Phi$ which is deeper than that of figure 2. This minimum appears to be a generic feature found using other best fit parameterizations of $H(z)$ as well.

As an application we have used a quadratic polynomial parameterization for $H(z)$ at its best fit with respect to the Gold dataset to reconstruct the scalar–tensor potentials $F(\Phi)$ and $U(\Phi)$ taking into account consistency with solar system and Cavendish-type experiments. The reconstructed scalar–tensor potential $U$ was found to be well fitted by an analytical function of the form $U(\Phi) \sim e^{3\Phi^2}$ which may be thought of as physically motivated on the basis of SUGRA theories [49, 50].

The derived potentials however are not uniquely determined for two reasons.

• We used one observationally determined input function ($H(z)$) in the context of two coupled equations to construct three output functions ($\Phi(z)$, $F(z)$, $U(z)$). The additional solar system test experimental constraints were not enough to lead to a unique determination of the three output functions.

• The best fit form of $H(z)$ depends on the type of parameterization considered. The polynomial parameterization considered for $H(z)$ gives a very good quality of fit to the Gold dataset but is not unique. For example an oscillating parameterization gives a somewhat better fit, yet the reconstructed potentials are quantitatively different from those obtained from the polynomial parameterization at best fit.

Thus our main result is that in contrast to minimally coupled quintessence, scalar–tensor theories can reproduce the main features of the best fit Hubble expansion history obtained from the Gold dataset. However, the precise determination of the scalar–tensor theory potentials requires more accurate SNIa data and additional observational input [51]–[55] which could come for example from weak lensing or large scale structure surveys providing the structure formation evolution history $(\delta \rho/\rho)(z) \equiv \delta_m(z)$. Once $\delta_m(z)$ is known from observations its time evolution equation

$$\ddot{\delta}_m + 2H \dot{\delta}_m - 4\pi G_{\text{eff}}(t) \rho \delta_m \approx 0$$

(4.1)
may be used in addition to the system (2.9) and (2.10) to fully determine the scalar–
tensor Lagrangian. The unknown functions $F(z), U(z)$ and $\Phi(z)$ enter in equation (4.1)
though the effective Newton’s constant $G_{\text{eff}}(z)$ measured in Cavendish-type experiments
(Force $= G_{\text{eff}}(m_1m_2/r^2)$) which is connected with the Newton’s constant $G$ entering in
the scalar–tensor Lagrangian (2.1) by

$$G_{\text{eff}} \equiv G \left( \frac{2ZF + 4(dF/d\Phi)^2}{2ZF + 3(dF/d\Phi)^2} \right).$$

(4.2)

In terms of the redshift $z$, equation (4.1) takes the form

$$H^2 \delta''_m + \left( \frac{(q^2)'}{2} - \frac{q^2}{1+z} \right) \delta'_m \approx \frac{3}{2}(1+z)\frac{G_{\text{eff}}(z)}{G} \Omega_{\text{dm}} \delta_m.$$  

(4.3)

The use of equation (4.3) to supplement the system (2.9) and (2.10) and uniquely
determine the functions $F(z), U(z)$ and $\Phi(z)$ subject only to the observational
uncertainties of the observed input functions $H(z)$ and $\delta_m(z)$ is a particularly interesting
future prospect.

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