Z-map based cutting force prediction for elliptical ultrasonic vibration-assisted milling process

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Abstract
Elliptical ultrasonic vibration-assisted milling (EUVAM) adds high-frequency vibration to conventional milling (CM) to realize high-frequency intermittent milling. It has broad application prospects in the processing of difficult-to-cut materials such as titanium alloys, superalloys, and hard and brittle materials. To reveal the mechanism of the highly intermittent cutting nature in EUVAM, according to the motion relationship between the cutting edge and the workpiece and the Z-map representation of the workpiece, a method and its algorithm for calculating the undeformed cutting thickness and thus the cutting force in EUVAM are proposed. The simulation results show that EUVAM can improve the actual cutting speed when compared with CM, and the proportion of idle cutting time will directly determine the intermittent degree of the milling process. The experiment of EUVAM is performed to verify the correctness of the proposed cutting force model, and the impact of spindle speed on the cutting force in EUVAM is also analyzed. It is shown that UEVAM can reduce the cutting force by up to 50% under appropriate cutting conditions.

Keywords
Elliptical ultrasonic vibration-assisted milling · Cutting force prediction · Separating characteristics · Z-map model

1 Introduction
Elliptical ultrasonic vibration-assisted cutting technology (EUVAC) is a new machining method that is based on the combination of traditional cutting methods and ultrasonic vibration machining technology; that is, by improving the structure and function of traditional machine tools, ultrasonic vibration is applied to tools or workpieces, and the ultrasonic vibration energy is added to the machining process to further enhance its machining performance [1]. On the basis of one-dimensional ultrasonic vibration assisted cutting, Shamoto and Moriwaki [2] first proposed the concept of EUVAC and revealed its principle. This is a cutting process in which the tool moves along an elliptical path generated by adding the cyclic vibrations on the cutting tool in the cutting direction and chip flow direction.

Milling is an intermittent cutting process since each cutting tooth touches the workpiece periodically within the range of entry and the exit angle at the tooth-passing frequency. When an additional elliptical ultrasonic vibration is applied to the conventional milling process, it becomes the elliptical ultrasonic vibration-assisted milling (EUVAM). The externally applied elliptical ultrasonic vibrations cause additional high-frequency tool-workpiece contact and separation, which change the kinematics and dynamics of milling process. It has been demonstrated that EUVAM can reduce cutting forces and heat generation, which improves the surface finish and the cutting tool life. As a result, the EUVAM is especially suitable for precision machining of hardened steel molds with mirror surface finish [2], carbon fiber-reinforced plastic (CFRP) parts in the aviation industry to avoid delamination [3], and titanium alloys to achieve good surface quality and high machining efficiency [4, 5].

In order to achieve high-efficiency chatter-free EUVAM, the dynamics of the vibration-assisted milling process should be modeled accurately. Although the stability of conventional milling operations has been studied extensively,

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in the literatures [6, 7], the stability of milling under ultrasonic vibrations has not been investigated sufficiently. Xiao et al. [8] numerically and experimentally studied the chatter stability of a 1-DOF vibration-assisted turning process and it was said that greater axial depth of cut can be achieved, but they did not predict stability lobes of the process. Ma et al. [9] excited elliptical ultrasonic vibration on a turning tool and analyzed the chatter stability in the frequency domain using the Nyquist criteria. They considered the periodic contact time of the actuator by Fourier series and used the dynamic orthogonal cutting model in the stability solution. Wan et al. [10] reported that the first analytical chatter stability of the vibration-assisted milling process using the semi-discrete method. In his study, the vibration was excited along the feed or normal direction from the workpiece side, and the high-frequency tool-workpiece separations changed the time delay dynamically and hence shifted the stability lobes. Gao and Altintas [11] proposed a dynamic model of the synchronized elliptical ultrasonic vibration-assisted milling, solved it using the semi-discrete time-domain method, and validated the predicted stability diagram experimentally.

Cutting force is a significant factor affecting the quality and efficiency of cutting process, especially when machining thin-walled parts of nickel-base alloy and titanium alloy with complex structure and shape. In order to reveal the influence of EUVAM on cutting force and surface integrity, systematic researches have been conducted using numerical simulation or experimental methods. Using the 3D finite element method, Muhammad et al. [12] compared the cutting force between ultrasonic vibration cutting and conventional cutting and considered that the vibration cutting can reduce cutting force due to the increase of cutting speed and decrease of tool-workpiece engagement. Tong and Wei [13] using both finite element method and experimental method to investigate the transient cutting processes of titanium alloy with EUVAC method compared the main cutting force of EUVAC with that of conventional cutting (CC) and discovered that under EUVAC, the reduction rate of cutting forces decreases with the increase in cutting depth and increases with the increase in cutting speed. Based on the kinematic characteristics of tool-workpiece and the separation criteria of intermittent cutting condition during high-speed EUVAM, Liu et al. [14] established a chip thickness model and the tool dynamic cutting relief angle model and investigated the effects of vibration amplitude on cutting force and surface integrity. Abootorabi et al. [15] experimentally studied the effects of cutting speed and workpiece vibration amplitude on the cutting force. Results showed that as the cutting speed increases, the effect of ultrasonic vibration on the milling process decreases and the cutting force in the EUVAM and CM processes become closer to each other. Han and Zhang [16] introduced a velocity coefficient $K$ to describe the separating characteristic of the EUVAM and theoretically analyzed the separating type and non-separating type cutting using the EUVAM method. Theoretical analysis and experimental results show that compared with non-separating EUVAM, the separating one can bring more reduction of the average radial cutting force.

The Z-map model was first proposed by Lee and Ko [17], in which an entity is projected to the XY plane, and then, it is meshed according to the accuracy requirements. From the perspective of data structure, as the Z-map model uses a two-dimensional array to record the Z-direction height of each grid, the model is particularly suitable for 2.5-axis milling process simulation. Based on the Z-map model of workpiece, Li and Liu [18] proposed a model to predict the machined surface topography during peripheral milling, in which the regenerative effect is considered.

As mentioned above, although many studies involve the cutting force of EUVAM, most of them only focus on the characteristics of EUVAM, lack of mechanism analysis on cutting force, and cannot reveal the effect of cutting conditions on the undeformed cutting thickness and even the cutting force. Even though some studies try to predict the undeformed cutting thickness and the cutting force in EUVAM by analytical method [19], due to the complex nature of EUVAM, the cutting edges may lose contact with the workpiece many times rather than just once during single tooth-passing period. If the undeformed cutting thickness is simply expressed by the distance between the current tooth $j$ position ($p_j(t)$) at time $t$ and the previous in-cut tooth $q$ in the radial direction with $r$ delay ($p_q(t-r)$, it cannot reflect the real situation of EUVAM. Therefore, the cutting thickness and cutting force cannot be accurately predicted. In order to solve this problem, this paper uses the Z-map model to represent the workpiece, establishes the motion equations of the cutting edges, calculates the actual undeformed cutting depth according to intersection between the cutting edge and the workpiece at each discrete time, and constantly updates the Z-map model of the workpiece, so as to realize the accurate prediction of cutting force in EUVAM.

The paper is organized into six sections. In Sect. 2, the cutting edge trajectory equation of EUVAM is established, and the influence of spindle speed on its cutting speed is analyzed and compared with that of CM. The methodology and algorithm for calculating the undeformed cutting thickness and updating the workpiece Z-map model in each discrete time are given in detailed in Sect. 3 based on the Z-map representation of the workpiece. In Sect. 4, the cutting force of EUVAM is predicted by applying the instantaneous rigid force model to each cutting edge element. Section 5 introduces the verification test of EUVAM, compares the predicted cutting force with the measured one, and analyzes the effect of spindle speed on cutting force. Section 6 gives the conclusion of the paper.
2 Kinematic modeling of EUVAM

2.1 Displacement equation of the cutting edge

Elliptical vibration is formed when two vibrations with a certain frequency and amplitude are applied to the tool holder in two directions perpendicular to each other in a plane perpendicular to the tool axis. As shown in Fig. 1, due to the existence of elliptical ultrasonic vibration, the movement of the cutting edge becomes very complex. Its movement includes the rotation of the tool around the spindle, the elliptical vibration around the tool axis, and the tool feed movement of the cutting tool.

The feed motion of the cutting tool can be expressed as follows:

\[
\begin{align*}
x_f &= 0 \\
y_f &= v_ft \\
v_f &= f_tN/60
\end{align*}
\]

(1)

where \(v_f\) is the feed rate, \(f_t\) is the feed rate per tooth, \(n\) is the spindle speed, \(N\) is the teeth number, and \(t\) is the time. The rotation of the cutting tool around the spindle axis can be expressed as follows:

\[
\begin{align*}
x_r &= R\cos(\alpha t) \\
y_r &= R\sin(\alpha t) \\
\alpha &= \omega t \quad \text{where } \alpha = \omega t, \text{the direction of the long axis of the ellipse is consistent with the direction of the } U\text{-axis in the } Ouv \text{ coordinate system. The motion equation of elliptical vibration can be expressed as follows:}
\end{align*}
\]

\[
\begin{align*}
u &= 0.5a\cos(2\pi ft) \\
v &= 0.5b\sin(2\pi ft)
\end{align*}
\]

(3)

where \(a\) is the amplitude in the feed direction, \(b\) is the amplitude perpendicular to the feed direction, and \(f\) is the vibration frequency.

When the ellipse equations in Ouv are transformed into the trajectory equations in Oxy, the coordinate transformation can be expressed as follows:

\[
\begin{align*}
x_e &= uc\cos(\alpha t) - vsin(\alpha t) \\
y_e &= us\sin(\alpha t) + vc\cos(\alpha t)
\end{align*}
\]

(4)

Considering the above three parts of motion, the motion equation of the cutting edge can be obtained as follows:

\[
\begin{align*}
x &= R\cos(\alpha t) + uc\cos(\alpha t) - vsin(\alpha t) \\
y &= v_ft + R\sin(\alpha t) + us\sin(\alpha t) + vc\cos(\alpha t)
\end{align*}
\]

(5)

According to Eq. (5), the cutting edge trajectory in EUVAM can be as shown in Fig. 3. The figure shows that except for the elliptical vibration locus in detail, the cutting edge locus in EUVAM is similar to that in CM, which is approximately an arc. Different cutting conditions may lead to two completely different cutting situations. Figure 3b shows the case when the rotation of the cutting tool is dominant. Although the elliptical vibration makes the cutting edge trajectory no longer an approximate arc, the cutting teeth only disengages from the workpiece only once within one tooth-passing period. Figure 3c shows the case when the elliptical vibration is dominant, the elliptical vibration not only makes the cutting edge trajectory no longer an approximate arc but also makes the cutting edge separate from the workpiece many times within one tooth-passing period. When the elliptical vibration is dominant, points A, B, C, D, and E correspond to an ultrasonic vibration period. The cycle can be divided into 4 stages: A-B is the empty cutting stage, the tooth moves towards the workpiece at a high speed, and the cutting force is 0; B-C is the cutting-in stage,
and the cutting thickness increases gradually; C-D is the cutting-out stage, and the cutting thickness decreases gradually; D-E is the empty cutting stage. The tooth cuts out of the workpiece from point D, separates from the workpiece, and the cutting force remains 0.

2.2 Velocity equation of the cutting edge

To find the derivative of Eq. (5), the velocity equation of the cutting edge can be obtained as follows:

\[
\begin{align*}
    v_x &= -R \sin(\omega t) + u' \cos(\omega t) - w \sin(\omega t) - v' \sin(\omega t) - w \cos(\omega t) \\
    v_y &= v_f + R \cos(\omega t) + u' \sin(\omega t) + w \cos(\omega t) + v' \cos(\omega t) - v \sin(\omega t)
\end{align*}
\]

(6)

The synthesis velocity \(v\) and its direction \(\beta\) are as follows:

\[
\begin{align*}
    v &= \sqrt{v_x^2 + v_y^2} \\
    \beta &= \tan^{-1} v_y / v_x
\end{align*}
\]

(7)

The tangential and radial cutting velocities are as follows:

\[
\begin{align*}
    v_r &= -v_x \sin(\omega t) + v_y \cos(\omega t) \\
    v_t &= v_x \cos(\omega t) + v_y \sin(\omega t)
\end{align*}
\]

(8)

According to Eqs. (6)–(8), the tangential cutting velocity is predicted by changing the spindle speed; the comparison of tangential cutting speed between EUVAM and CM at different spindle speed is shown in Fig. 5. The figure shows that due to the participation of elliptical ultrasonic vibration, the tangential cutting speed in EUVAM fluctuates around that in CM; when the spindle speed is high, the minimum tangential cutting speed is always greater than 0; that is, during one tooth-passing period, the cutting edge keeps cutting the workpiece until it exceeds the exit angle. Under these circumstances, although the tangential cutting speed fluctuates, the cutting edge is still performing the conventional milling in essence, which can be supported by the cutting edge trajectory shown in Fig. 3b. As the spindle speed decreases, the minimum tangential speed crosses the zero-line downward; that is, the cutting edge no longer cuts the workpiece forward, but retreats backward. The phenomenon is bound to cause the cutting edge to disengage with the workpiece, resulting in a temporary interruption of the milling process and brief cooling of the cutting zone, which may also the purpose and advantage of EUVAM. The minimum tangential cutting speeds in Fig. 4a–c are greater than 0, approximately 0, and less than 0, respectively. In Fig. 4c, d, the idle cutting time, that is, the time when the tangential cutting speed is negative, accounts for 1/4 and 1/3 of the whole time period, respectively. It can be seen that with the decrease of spindle speed, that is, the proportion of elliptical ultrasonic vibration increases, the idle cutting time will increase significantly. However, the optimal proportion of idle cutting time needs to be further studied.

Figure 5 shows the change of the cutting edge cutting speed in two typical EUVAM. As shown in the figure, regardless of the spindle speed, the maximum cutting speed in EUVAM is significantly higher than that in CM. It can also be seen from Figs. 3, 4, and 5, during one elliptical vibration cycle, B-C-D is the cutting section. When the cutting edge moves to point C, the ultrasonic vibration speed direction is consistent with the rotation speed direction of the cutting tool, and its synthesis cutting speed reaches the maximal. When the cutting edge moves to point D, the synthesis cutting speed reaches the minimal. As the spindle speed \(n\) is 600 r/min, the maximum cutting speed is 49.48 m/min; the minimum cutting speed is 15.10 m/min, and both are greater than the cutting speed in CM (15.09 m/min). As the spindle speed \(n\) is 300 r/min, the maximum cutting speed is 46.20 m/min; the minimum cutting speed is 22.60 m/min, and both are greater than the cutting speed in CM (7.54 m/min).

From the above analysis, it can be seen that the cutting speed in EUVAM is much higher than that in CM even at the same spindle speed due to the existence of elliptical ultrasonic vibration. The cutting speed reaches the maximum value at point C, which has reached the cutting speed required for high-speed milling of titanium alloy and...
Fig. 4 Comparison of tangential cutting speed between EUVAM and CM under different spindle speed (a = 0.012 mm, b = 0.008 mm, \( f = 20,000 \) Hz, \( R = 4 \) mm, \( f_z = 0.1 \) mm/tooth, \( N = 2 \))

Fig. 5 Cutting speed curve in EUVAM under different spindle speed (a = 0.012 mm, \( b = 0.008 \) mm, \( f = 20,000 \) Hz, \( R = 4 \) mm, \( f_z = 0.1 \) mm/tooth, \( N = 2 \))
superalloy. The time period the cutting speed near the maximal value occupies 1/5 of the whole cutting period. It can be seen that even under the condition of low spindle speed, it is possible to realize high-speed milling to a certain extent by using EUVAM. If the frequency and amplitude of elliptical ultrasonic vibration can be further optimized, the effect of high-speed cutting under the condition of low spindle speed may be more significant.

3 Calculation of undeformed cutting thickness

3.1 Z-map model of the workpiece

So far, the most widely used approach for calculating the undeformed cutting thickness was proposed by Tlusty et al., in which the trajectory of the cutting tool is approximately regarded as an arc. Suppose the feed rate per tooth is \( f_r \), when the rotation angle of the cutting tool is \( \phi \), the undeformed cutting thickness can be approximately expressed as follows:

\[
h = f_r \sin \theta
\]  

(9)

However, during the process of EUVAM, due to the comprehensive action of tool rotation and elliptical ultrasonic vibration, the path of cutting tooth becomes very complex (Fig. 3), which makes the above equation no longer applicable to the elliptical ultrasonic milling process. The existence of elliptical ultrasonic vibration leads to the cutting edge cutting into and out of the workpiece at high frequency during single tooth-passing period, which leads to the inconsistency between the workpiece contour and the cutting edge trajectory. The value obtained from the difference between the current tooth position and the previous tooth position in the radial direction with \( \tau \) delay is not always the undeformed cutting thickness. As shown in Fig. 6, \( O_p(t) \) is the origin of the workpiece coordinate system, \( O_f(t) \) is the tool center position which is affected
by the feed rate, the elliptical vibration amplitude, and frequency, \( P(t) \) and \( Q(t) \) are the current tooth position and the intersection point of the machined surface with the cutting tool in the radial direction of the cutting tooth, respectively, and \( h(t) \) is the undeformed cutting thickness in the radial direction. Therefore, it is necessary to find other methodologies to calculate the undeformed cutting thickness.

In the following, a Z-map model is used to represent the surface of the workpiece to obtain the undeformed cutting thickness.

The Z-map representation of the workpiece for EUVAM is shown in Fig. 7, \( O_i \) and \( P_i \) are the tool center and the tooth position at time \( t \), \( O_{i+1} \) and \( P_{i+1} \) are the tool center and the tooth position at time \( t + dt \), \( R \) is the radius of the cutting tool, \( dx \) and \( dy \) are the grid size in the \( x \) and \( y \) direction, symbol “●” represents the grid in the unmachined surface, symbol “○” represents the grid in the surface to be machined during period \( dt \), and the blank indicates the grid in the machined surface.

### 3.2 Methodology and algorithm for obtaining the undeformed cutting thickness

The methodology and algorithm used for calculating the undeformed cutting thickness based on the Z-map model are shown in Fig. 8, and the basic procedures are as follows:

1. Set the appropriate grid size \( dx \) and \( dy \) to ensure the accuracy of the Z-map model, and make \( Z(1: x_{num}, 1: y_{num}) = a_p \) to initialize the Z-map model of the workpiece.
2. Set the appropriate time increment \( dt \) to realize the discretization of the elliptical ultrasonic vibration milling process.
3. For each discrete time \( t \), calculate the coordinates of the tool center position \( O_{i+1} \) and the current tooth point \( P_{i+1} \) according to Eqs. (4)-(5).
4. Using the coordinates of \( O_i, O_{i+1}, P_i, \) and \( P_{i+1} \) to obtain the rectangle which is represented by \( (x_{min}, y_{min}, x_{max}, y_{max}) \) (see the dashed line in the figure), and mesh this rectangle with \( dx \) and \( dy \).
5. Use the binary search method to obtain the intersection point \( Q_{i+1} \), between the cutting edge and the workpiece located at the straight line \( O_{i+1}P_{i+1} \), and then obtain the instantaneous undeformed cutting thickness.
6. For each grid within the rectangle, use the library function \textit{inpolygon} in Matlab to judge whether a specified point is inside the quadrilateral \( O_iP_iP_{i+1}O_{i+1} \), if \( Z(j, k) = a_p \) and if the grid is inside the quadrilateral, update the grid by setting \( Z(j, k) = 0 \).
7. Repeat step 3 to step 6 until the end of the simulation.

### 3.3 Simulation and analysis of undeformed cutting thickness in EUVAM

According to the above-mentioned approaches, the undeformed cutting thickness of each cutting edge at any time in EUVAM can be obtained. The simulation results under different cutting conditions are shown in Fig. 9. It is not difficult to understand from the figure that the undeformed cutting thickness in EUVAM is no longer regular as that in CM, but an irregular curve varying with the frequency and amplitude of the elliptical vibration and the spindle speed. When the spindle speed is high, as shown in Fig. 9a, b, as the rotation of the cutting tool is dominant, although the elliptical ultrasonic vibration will make the undeformed cutting thickness change sharply, it will not cause the cutting edge to leave the cutting zone only when the cutting tool starts to

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**Flow chart for calculating the undeformed cutting thickness based on Z-map model**

**Fig. 8** Flow chart for calculating the undeformed cutting thickness based on Z-map model
cut into the workpiece, will there be a short period of high intermittent cutting process. When the spindle speed is low, as shown in Fig. 9c, d, as the elliptical vibration is dominant, it will cause the cutting edge to leave the cutting zone at a high frequency and make the deformed cutting thickness 0. At this time, the cutting edge of the current tooth is separated from the workpiece and the idle cutting occurs. With the further decrease of spindle speed, that is, elliptical ultrasonic vibration is more dominant, and the proportion of idle cutting time increases. Therefore, it is not difficult to infer that the intermittent degree of elliptical ultrasonic milling process is more significant at this time. The high-frequency engagement and disengagement between the cutting edge and the workpiece will help the coolant enter the cutting zone to fully cool the cutting teeth, so as to improve the tool life and the surface quality of the workpiece.

4 Cutting force modeling for EUVAM

In order to reveal the detailed process of cutting force modeling for the elliptical ultrasonic vibration milling process, this paper focuses on cylindrical end mill. The modeling parameters include tool parameters: radius $R$, number of teeth $N$, helix angle $\beta$, pitch angle $\phi_p$; the process parameters: spindle speed $n$, federate per tooth $f_r$, axial depth of cut $a_p$, radial depth of cut $a_e$; vibration parameters: ultrasonic frequency $f$, ultrasonic amplitude $a$ and $b$.

It is found that there is no essential difference between EUVAM and CM in the cutting mechanism and cutting force distribution along the cutting edge, except that the former adds additional small-amplitude high-frequency elliptical vibration. The workpiece material is extruded on the rake face, resulting in elastic–plastic deformation and

![Comparison of undeformed cutting thickness between EUVAM and CM under different spindle speed](image)

Fig. 9 Comparison of undeformed cutting thickness between EUVAM and CM under different spindle speed ($a=0.012$ mm, $b=0.008$ mm, $f=20,000$ Hz, $R=4$ mm, $f_r=0.2$ mm/tooth, $N=2$)

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shear slip, and the cutting layer metal is separated from the matrix to form chips; the flank of the cutting tool scrapes on the machined surface, resulting in tissue fibrosis and work hardening. Therefore, the classical instantaneous rigid force model is also suitable for EUVAM. Refer to the mechanistic cutting force model for CM, the cutting edge is discretized into \( m \) disks with thickness \( dz \) along its axial direction, and the cutting force elements acting on tooth \( j \)th, cutting edge element \( l \)th in the tangential, radial, and axial direction are expressed as follows:

\[
\begin{align*}
\Delta F_{tjl} &= g(\phi_j)(K_{tc}h(\phi_j) + K_{ae})dz \\
\Delta F_{rjl} &= g(\phi_j)(K_{rc}h(\phi_j) + K_{ae})dz \\
\Delta F_{ajl} &= g(\phi_j)(K_{ac}h(\phi_j) + K_{ae})dz
\end{align*}
\]

(10)

where \( h \) is the instantaneous undeformed cutting thickness of the cutting edge element, which can be obtained from the method proposed in the above section; \( K_{tc}, K_{rc}, K_{ac} \) are the tangential, radial, and axial specific cutting force; \( K_{tc}, K_{rc}, K_{ae} \) are the tangential, radial, and axial edge force coefficients. \( g(\phi_j) \) is a step function used to indicate whether the current cutting edge element participates in cutting or not. Different from the cutting force model of CM, when calculating the undeformed cutting thickness of EUVAM in the above section, whether the cutting tool is separated from the workpiece has been considered. Therefore, we can make \( g(\phi_j) = 1 \).

As shown in Fig. 10, due to the existence of a helix angle, a point on the cutting edge will lag behind a point on the tooltip.

The lag angle of a point at \( z \) height in the axial direction can be expressed as follows:

\[
\psi = z\tan\beta/R
\]

(11)

For a cutting tool with uniformly distributed cutting teeth, the pitch angle \( \phi_p = 2\pi/N \). Suppose the rotation angle at the tooltip of the first tooth is \( \phi_{10} \), then the rotation angle of the \( l \)th cutting edge element located on the \( j \)th tooth can be expressed as follows:

\[
\phi_{jl} = \phi_{10} + (j - 1)\phi_p + ldz\tan\beta/R
\]

(12)

Through coordinate transformation, the cutting force component in the machine tool coordinate system (Cartesian coordinate system) can be obtained as follows:

\[
\begin{align*}
\Delta F_{xjl} &= -\Delta F_{yjl}\cos(\phi_j) - \Delta F_{yjl}\sin(\phi_j) \\
\Delta F_{yjl} &= \Delta F_{xjl}\sin(\phi_j) - \Delta F_{yjl}\cos(\phi_j) \\
\Delta F_{zjl} &= \Delta F_{zjl}
\end{align*}
\]

Integrate the cutting force acting on the cutting edge element along the axial direction, and sum the cutting force for each tooth to yield the three-dimensional cutting forces as follows:

\[
\begin{align*}
F_x &= \sum_{j=1}^{N} \sum_{l=1}^{n} \Delta F_{xjl} \\
F_y &= \sum_{j=1}^{N} \sum_{l=1}^{n} \Delta F_{yjl} \\
F_z &= \sum_{j=1}^{N} \sum_{l=1}^{n} \Delta F_{zjl}
\end{align*}
\]

(14)
Experimental verification and discussion

Cutting force verification tests of EUVAM are carried out on a 5-axis vertical machining center Rambaudi Ramatic-1201 g, and the experimental devices are shown in Fig. 11 [20]. The tool holder used in EUVAM is composed of a tool handle, an elliptical piezoelectric transducer and a 12-mm diameter four-tooth cemented carbide end mill. The device can produce two-dimensional high-frequency vibrations in two directions perpendicular to each other, and its vibration frequency is about 17,880 Hz. The material of the test piece is titanium alloy Ti-6Al-4 V, and its size is 200 mm × 120 mm × 60 mm. The cutting force measuring devices adopts a three-way dynamometer Kistler9257b, the charge amplifier is Kistler5017a, and the cutting force measuring software is DynoWare signal analyzer, and the sampling frequency is set to 10 kHz.

The cutting force coefficient of titanium alloy Ti-6Al-4 V obtained through the cutting force coefficient identification experiment is shown in Table 1. The cutting parameters used in the verification tests are spindle speed \( n = 600 \text{ r/min} \), feed per tooth \( f_t = 0.20 \text{ mm} \), radial depth of cut \( a_e = 0.2 \text{ mm} \) and axial cutting depth \( a_p = 5.0 \text{ mm} \); the elliptical vibration parameters are the amplitude in feed direction \( a = 0.012 \text{ mm} \), the amplitude normal to feed direction \( b = 0.008 \text{ mm} \), and the vibration frequency \( f = 17,880 \text{ Hz} \).

### Table 1 Cutting force coefficients for material Ti-6Al-4 V

| \( K_{tc} \text{ (MPa)} \) | \( K_{rc} \text{ (MPa)} \) | \( K_{ac} \text{ (MPa)} \) | \( K_{te} \text{ (MPa)} \) | \( K_{re} \text{ (MPa)} \) | \( K_{ae} \text{ (MPa)} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 2315            | 383.2           | -522.78         | 32.72           | 55.87           | 10.17           |

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Fig. 12 Comparison of measured cutting force and predicted one in EUVAM \((n=600 \text{ r/min}, f_t=0.20 \text{ mm}, a_e=0.2 \text{ mm}, a_p=5.0 \text{ mm}, a=0.012 \text{ mm}, b=0.008 \text{ mm}, f=17,880 \text{ Hz})\)
Fig. 13 Comparison of predicted cutting forces between EUVAM and CM ($f_0=0.10$ mm, $a_c=12.0$ mm, $a_p=2.0$ mm, $a=0.012$ mm, $b=0.008$ mm, $f=17,880$ Hz)
The comparison between the measured and predicted cutting forces in X and Y directions is shown in Fig. 12. The figure shows that the predicted cutting force in X direction is in good agreement with the measured one both in amplitude and shape, and the predicted cutting force in Y direction is very consistent with the measured one in shape, but there are some differences in amplitude, and the maximum relative error is 20%. Due to the small radial cutting depth used in the test, the cutting force in Y direction is very small, less than 1/10 of that in X direction. Although the relative error is considerable, the absolute value is not large, only a few Newtons. This error may come from the manufacturing error and installation error of cutter teeth, or from other unknown factors. In general, the cutting force prediction model proposed in this paper for EUVAM has significant prediction accuracy.

In order to further reveal the effect of the proportion of empty cutting time in EUVAM, that is, the spindle speed on cutting force, the cutting forces in EUVAM, and CM under different spindle speeds are predicted, and the comparison is shown in Fig. 13. The figure shows that with the decrease of spindle speed, that is, the elliptical vibration speed is dominant and the proportion of empty cutting time increases; the cutting force of EUVAM in both X direction and Y direction shows a significant decrease. As the spindle speed $n = 900 \text{ r/min}$, there is little difference in the cutting force between EUVAM and CM; however, as the spindle speed $n = 300 \text{ r/min}$, the cutting force in EUVAM decreases by nearly 50% compared with CM in both X direction and Y direction.

Figure 14 shows the comparison of the measured cutting force in X and Y directions between EUVAM and CM when the spindle speed $n = 600 \text{ r/min}$. The figure shows that when the spindle speed is 600, EUVAM can reduce the cutting force by 30% compared with CM, which is very consistent with the simulation results.

6 Conclusions

In this paper, the motion equations of the cutting edge in EUVAM are first established. On the basis, the undeformed cutting thickness is obtained and the cutting force prediction model is established through Z-map representation of the workpiece. The proposed model is verified by cutting tests, and the influence of cutting conditions on both the undeformed cutting thickness and cutting force is discussed.

1. Due to the action of elliptical ultrasonic vibration, the actual cutting speed in EUVAM is much higher than that in CM under the same cutting parameters. Therefore, the adoption of EUVAM can realize high-speed milling to some extent at a relatively low spindle speed.

2. The adoption of the Z-map model can greatly simplify the calculation complexity of undeformed cutting thickness in EUVAM. The dominant degree of elliptical ultrasonic vibration relative to rotary motion, that is, the proportion of empty cutting time, determines the intermittent degree of EUVAM. An appropriate proportion of idle cutting time is helpful to improve the tool life and the surface quality of the workpiece.

3. According to the comparison between the predicted cutting force and the experimental one, the cutting force prediction method proposed in this paper can accurately predict the cutting force in EUVAM. Simulation results show that, with the increase of the proportion of idle cutting time, the cutting force of EUVAM in both X direction and Y direction shows a significant decrease, and compared with the conventional milling, the maximum cutting force can be reduced by up to 50%.

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Declarations

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