Research Article

Design of Robust Adaptive Fuzzy Controller for a Class of Single-Input Single-Output (SISO) Uncertain Nonlinear Systems

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1. Introduction

Adaptive control technology provides a very effective mean to solve the uncertainty in the system [1–3]. Since the 1970s, adaptive control has attracted great attention in the field of control theory [2, 4, 5]. In actual engineering control, the model is always nonlinear and contains some uncertainty. Uncertain nonlinear systems are susceptible to two types of uncertainties: known structural uncertainties, i.e., parameter uncertainties [6] and uncertainties in structural unknowns, i.e., modeling errors and external disturbances [7]. Considering nonlinear systems with known uncertainties in structure, in the past research, significant results have been achieved in adaptive techniques for feedback linearization of nonlinear systems [8–10]. For nonlinear systems with structurally unknown uncertainties, robust control strategies can be applied. Since some controlled systems cannot know the bounds of the unknown uncertainty of their structure, adaptive control and robust control methods cannot be used for controller design. However, for such problems, neural networks and fuzzy control can approximate uncertain continuous functions of the unknown structure in the system [11, 12]. Ge et al. researched a robust adaptive neural network control method for a perturbed strictly feedback nonlinear system, which can guarantee the final boundedness in the case of unknown structural uncertainty [13]; Cao et al. proposed a new system control strategy using the global approximation property of the fuzzy system to approximate the unknown function of the designed system and ensure the stability of the whole system [14].

However, in the system described above, it is satisfied that the unknown nonlinearity and the control input appear in the same state space model equation. In order to overcome the limitations of such systems, relevant research scholars have proposed adaptive post-push technology [4, 6, 15, 16], furthermore successfully solved the constraint...
of the matching conditions imposed on the nonlinearity of the system. However, adaptive post-pushing technology has some disadvantages such as overparameterization. In order to solve this shortcoming, Yang et al. designed the controller with adaptive post-pushing technology and robust control technology, which made the closed-loop system globally consistent and bounded under the influence of unknown nonlinearity and parameter uncertainty factors [17]. In addition, robust adaptive neural networks and fuzzy controllers are studied for systems with matching conditions imposed by the abovementioned nonlinearities [18, 19]. Especially for high-order systems and multivariable systems, when neural network basis functions or fuzzy rule numbers are used to approximate uncertain nonlinear functions, many parameters need to be adjusted, resulting in a “dimension disaster” problem. As a result, the computational burden becomes too large and the “calculation expansion” problem arises at the same time, which is not conducive to engineering applications [19, 20]. Yang et al. investigated an adaptive control technique based on the T-S fuzzy system approximator, which adjusted the adaptive parameters of each system to two and solved the problem of “dimensionality disaster” [21]. Hedrick et al. proposed a “Dynamic Surface Control” (DSC) method that overcame the “computational expansion” problem by introducing a first-order low-pass filter at each step of the pushback technique [22].

2.1.1. Fuzzy Control Theory. Fuzzy Logic Control, referred to as Fuzzy Control, is a computer digital control technology based on fuzzy set theory, fuzzy linguistic variables, and fuzzy logic reasoning. Professor L. A. Zadeh of the University of California, United States, published the famous “Fuzzy Sets” paper [27] and proposed the concepts of fuzzy sets and fuzzy algorithms.

The fuzzy control system is mainly composed of four parts: fuzzification, rule base, fuzzy reasoning machine, and defuzzification:

(1) Fuzzification: the main function is to select the input amount of the fuzzy controller and convert it into a fuzzy amount that can be recognized by the system. It consists of three steps. First, the input volume is processed to meet the needs of fuzzy control. Second, the input amount is scaled. Third, the fuzzy language value of each input and the corresponding membership function are determined.

(2) Rule base: it consists of a fuzzy “If-Then” rule set. The fuzzy rule base contains many control rules and is a key step in the transition from actual control experience to fuzzy controllers.

(3) Fuzzy reasoning machine: based on combined reasoning or independent reasoning for fuzzy rule bases, a variety of fuzzy reasoning machines are proposed, which mainly implement knowledge-based reasoning decisions.

In this paper, a robust adaptive fuzzy control technique based on “Dynamic Surface Control” (DSC) method is proposed for generalized single-input single-output (SISO) uncertain nonlinear systems. The T-S fuzzy logic system is used to approximate the uncertain nonlinear function with an unknown structure in the system. The main contributions of the paper include the following:

(1) In the case of unknown structural uncertainty, the resulting closed-loop system is ultimately bounded.

(2) Only one function needs to have a T-S fuzzy logic system approximation and each subsystem needs only one parameter to be adjusted, which overcomes the problem of “dimensionality disaster.” In turn, the calculation amount of the control algorithm is greatly reduced, and the problem of “calculation expansion” is solved.

The chapters of this paper are distributed as follows. Section 1 mainly introduces the expressions and assumptions of the questions and provides some preliminary knowledge. A design of robust adaptive fuzzy controller based on “Dynamic Surface Control” (DSC) method and Lyapunov stability analysis are described in Section 2. Section 3 uses MATLAB to carry out simulation research to prove the effectiveness of the proposed method. Finally, the conclusion is reflected in Section 4.

2. Problem Formulation and Preparation

2.1. Problem Preparation

2.1.1. Fuzzy Control Theory. Fuzzy Logic Control, referred to as Fuzzy Control, is a computer digital control technology based on fuzzy set theory, fuzzy linguistic variables, and fuzzy logic reasoning. Professor L. A. Zadeh of the University of California, United States, published the famous “Fuzzy Sets” paper [27] and proposed the concepts of fuzzy sets and fuzzy algorithms.

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There are two types of fuzzy control systems: Mamdani and Takagi–Sugeno–Takagi (T-S).

Consider a fuzzy system approximating a continuous multidimensional function \( y = f(x) \), where \( x = (x_1, x_2, \ldots, x_p)^T \in U \) is the input vector and \( y \in V \subset R \) is the output vector. The fuzzy logic system constitutes a mapping from subspace \( U \) to subspace \( V \), as shown in Figure 1. The system usually consists of the following \( P > 1 \) fuzzy rules:

If \( x_1 = A_{i1}^0, x_2 = A_{i2}^0, \ldots, x_n = A_{in}^0 \), then \( y = B_{i1}^0 \), \( i = 1, 2, \ldots, P \), where \( A_{ij}, B_{ij} \) are the input fuzzy set and the output fuzzy set, respectively, represent the input fuzzy set and the output fuzzy set.

Backstepping method has obvious advantages in implementing robust control or adaptive control of uncertain nonlinear systems. However, the backstepping method itself does not have a good solution to the term expansion caused by the derivation of virtual control and the problem caused by the term expansion. This disadvantage is particularly prominent in higher-order systems. It is to overcome the computational complexity of the traditional inverse design. The main advantages of the dynamic surface control method are as follows: (1) it can eliminate the expansion of differential terms and make the controller and parameter design simple; (2) it can reduce the number of neural network and fuzzy system input variables used for modeling; and (3) there is no need to approximate the bounded error in the stability analysis, thereby avoiding the cyclical argument.

The following briefly introduces the design of dynamic surface control methods.

Take a second-order nonlinear system as an example:

\[
\begin{align*}
\dot{x}_1 &= x_2 + af_1(x_1), \\
\dot{x}_2 &= u + bf_2(x_2), \\
y &= x_1,
\end{align*}
\]

where \( x_2 = [x_1, x_2]^T \) represents the system state variable and \( u \) represents the system input.

Define the first error surface \( S_1 = x_1 - y_r \), where \( y_r \) represents the system reference input signal, and derive the error surface \( S_1 \) to obtain the following formula:

\[
\dot{S}_1 = x_2 + x_{2d} - x_{2d} + af_1(x_1) - y_r,
\]

where \( x_{2d} \) represents the intermediate virtual control rate.

In order to overcome the expansion of the differential term, a new low-pass filter is introduced to obtain a new variable \( z_2 \):

\[
\tau_z \dot{z}_2 + z_2 = x_{2d},
\]

where \( \tau_z \) represents the time constant of the first-order low-pass filter.

By introducing a first-order filter in a nonlinear system, the problem of term expansion caused by the derivation of virtual control in high-order systems is avoided. The design and stability analysis of the dynamic surface control method proves that the literature [31] gives specific steps.
Define the tracking error \( z \), track a given reference signal \( y_r(t) \), and assume that the given reference signal \( y_r(t) \) is bounded:

\[
z = y(t) - y_r(t).
\]

(9)

Make assumptions about the abovementioned system.

**Assumption 1.** The unknown smooth virtual control non-linear gain function \( g_i(\cdot) \) satisfies \( 0 \leq a_{\text{min}} \leq g_i(x, t) \leq b_{\text{max}} \), where \( a_{\text{min}} \) and \( b_{\text{max}} \) represent the upper and lower bounds of the parameter and are some unknown constant.

**Assumption 2.** It is uncertain that the external disturbance \( d_i(x, t) (1 \leq i \leq n) \) is bounded, and \( |d_i(x, t)| \leq d_i \), \( d_i \) is an unknown normal number.

**Assumption 3.** The weight of the fuzzy system \( \theta_i^r \) and the approximation error \( \varepsilon_i \) are bounded.

2.2. Design of Robust Adaptive Fuzzy Controller

2.2.1. Controller Design. Consider the first subsystem of the system shown in equation (8), defining the first tracking error \( z_i = x_i - y_r \); then,

\[
\dot{z}_i = g_i(x_i, t)z_i + f_i(x_i, t) + d_i - y_r.
\]

(10)

Using a T-S-type fuzzy system to approximate \( f_i(x_i, \omega) \), then

\[
f_i(x_i, \omega) = \xi_i(x_i)A_i x_i + \varepsilon_i = \xi_i(x_i)A_i z_i + \xi_i(x_i)A_i y_r + \varepsilon_i
\]

\[
= q_{0i} \xi_i \omega_i + \xi_i(x_i)A_i y_r + \varepsilon_i
\]

(11)

where \( \varepsilon_i \) represents the approximation error, \( q_{0i} \) is an unknown constant, and \( q_{0i} = \| A_i \| \), \( A_i = q_{0i} A_{1i}^m \), and \( \omega_i = A_i z_i \); then,

\[
\dot{z}_i = g_i(x_i, t)z_i + q_{0i} \xi_i(x_i)\omega_i + l_i - y_r,
\]

(12)

where \( l_i = \xi_i(x_i)A_i y_r + \varepsilon_i + d_i \), \( \| l_i \| \leq \| \xi_i(x_i)A_i y_r + \varepsilon_i + d_i \| \leq a_{\text{min}} \theta_1 \varepsilon_i \| (x_i) \), \( \theta_1 = a_{\text{min}} \max (\| A_i y_r \|, \| \varepsilon_i + d_i \|) \)

and \( \varepsilon_i(x_i) = 1 + \| \xi_i \| \).

Therefore, the intermediate stabilization function \( \psi_2 \) and the parameter adaptation rate are

\[
\psi_2 = -k_i z_i + \dot{y}_r - \lambda_i \rho_i(x_i)z_i,
\]

(13)

\[
\dot{\lambda}_i = \mu_i [\rho_i(x_i)z_i - \sigma_i(\lambda_i - \lambda_i^0)],
\]

(14)

\[
\rho_i(x_i) = \frac{1}{4\epsilon_i^2} x_i^T x_i + \frac{1}{4\epsilon_i^2} \xi_i^T(\theta_1^0 + \theta_2^0),
\]

(15)

where \( k_i, \mu_i, \lambda_i \), and \( \epsilon_i \) are normal numbers; \( \lambda_i^0 \) is an estimated value of \( \lambda_i \); \( \lambda_i = a_{\text{min}} \max (\| A_{1i}^m \|) \); and \( \lambda_i^0 \) is the initial value.

At this point, the estimated value \( s_2 \) of \( \psi_2 \) is obtained:

\[
s_2 = \psi_2 - T \eta \hat{s}_2,
\]

(16)

where \( \tau \) is the time constant.

Considering the \( i \)-th \((2 \leq i \leq n - 1)\) subsystems of the system shown in equation (8) and defining the \( i \)-th tracking error \( z_i = x_i - y_r \); then,

\[
\dot{z}_i = g_i(x_i, t)z_i + f_i(x_i, t) + d_i - \hat{s}_i.
\]

(17)

Using the T-S fuzzy system to approximate the uncertain function \( f_i(x_i, t) \), then

\[
f_i(x_i, t) = \xi_i(x_i)A_i x_i^m + \varepsilon_i = \xi_i(x_i)A_i y_r + \varepsilon_i
\]

\[
= q_{0i} \xi_i \omega_i + d_i,
\]

(18)

where \( d_i = \xi_i A_i^m y_r + \xi_i \sum_{j=2}^i A_j^m s_j + \varepsilon_i \), \( q_{0i} = \| A_i^m \|, A_i^m = q_{0i} A_{1i}^m \), and \( \omega_i = A_i^m z_i \); then,

\[
\dot{z}_i = g_i(x_i, t)z_i + q_{0i} \xi_i(x_i)\omega_i + l_i - \hat{s}_i,
\]

(19)

where \( l_i = d_i + d_i \), \( \| l_i \| \leq \| d_i + \xi_i A_i^m y_r + \xi_i \sum_{j=2}^i A_j^m + \varepsilon_i \| \leq a_{\text{min}} \theta_i \varepsilon_i \| (x_i) \), \( \theta_i = a_{\text{min}} \max (\| A_i^m y_r \|, \| \varepsilon_i + d_i \|) \), and \( \varepsilon_i(x_i) = 1 + \| \xi_i \| \).
Therefore, the intermediate stabilization function $\psi_{i+1}$ and the parameter adaptation rate are

$$\psi_{i+1} = -k_i \xi_i + \hat{\lambda}_i \rho_i (\xi_i) \xi_i,$$

$$\hat{\lambda}_i = \mu_i [\rho_i (\xi_i) \xi_i^2 - \sigma_i (\hat{\lambda}_i - \lambda_i^0)],$$

$$\rho_i (\xi_i) = \frac{1}{4t_i} \xi_i^2 + \frac{1}{4k_i^2} \varphi_i^2,$$

where $k_i$, $\mu_i$, $t_i$, and $\xi_i$ are normal numbers; $\hat{\lambda}_i$ is an estimated value of $\lambda_i$; $\lambda_i = a_{\min} \max (\varphi_i^2, \theta_i^2)$; and $\lambda_i^0$ is the initial value.

At this point, the estimated value $s_{i+1}$ of $\psi_{i+1}$ is obtained:

$$s_{i+1} = \psi_{i+1} - t_{i+1} \dot{s}_{i+1}.$$

Define the $n$-th tracking error $e_n = x_n - s_n$; similarly, there is

$$\dot{e}_n = \dot{g}_n (x, t) u + f_n (x, t) + d_n - \dot{s}_n,$$

$$f_i (x, t) = \xi_i (x, t) A_i (x, t + n) + e_n = \xi_i (x, t) A_n \left[ \begin{array}{c} z_1 + y_r \\
\vdots \\
z_n + s_n \end{array} \right] + e_n,$$

$$d_n = q_{\text{fin}} A_n (x, t) + d_n,$$

where $d_n$, $e_n$, and $g_n$ are normal numbers; $\lambda_n$ is an estimated value of $\lambda_i$; $\lambda_i = a_{\min} \max (\varphi_i^2, \theta_i^2)$; and $\lambda_i^0$ is the initial value.

2.2.2. Lyapunov Stability Analysis. Define a new tracking error:

$$y_{i+1} = s_{i+1} - \psi_{i+1}, i = 1, 2, \ldots, n-1.$$

From equation (23), $\dot{s}_i = ((s_i + \psi_i)/\tau_i) = ((y_i)/\tau_i)$; then,

$$\dot{y}_2 = \dot{s}_2 - \psi_2 = \frac{y_2}{\tau_2} + \left( \frac{\partial \psi_2}{\partial x_1} \xi_1 - \frac{\partial \psi_2}{\partial \theta_1} \hat{\theta}_1 - \frac{\partial \psi_2}{\partial \lambda_1} \hat{\lambda}_1 + \dot{y}_r \right) = \frac{y_2}{\tau_2} + O_2 (z_1, z_2, y_2, \hat{\theta}_1, \hat{\lambda}_1, y_r, \dot{y}_r, \dot{y}_r),$$

where $O_2 (\cdot)$ is the remainder. Similarly, there is

$$\dot{y}_{i+1} = \dot{s}_{i+1} - \psi_{i+1} = \frac{y_{i+1}}{\tau_{i+1}} + O_{i+1} (z_1, \ldots, z_{i+1}, y_2, \ldots, y_i, \hat{\theta}_1, \hat{\theta}_1, \hat{\lambda}_1, y_r, \dot{y}_r, \dot{y}_r).$$

According to the $i$-th tracking error equation and equation (29), $x_{i+1} = z_{i+1} + s_{i+1}$ and $s_{i+1} = y_{i+1} + \psi_{i+1}$, and the tracking error can be expressed as follows:

Then, bring equations (13)~(15), (19)~(21), and (26)~(28) into the abovementioned equation and obtain the following expression:
Define the closed-loop system Lyapunov function as

\[
V = \frac{1}{2} \sum_{i=1}^{n} (\zeta_i^T + \lambda_i^T a_{\min}^{-1} \lambda_i) + \frac{1}{2} \sum_{i=1}^{n-1} \gamma_i^T, 
\]

(34)

where \( \lambda_i = \lambda_i - \tilde{\lambda}_i \).

In the following, it will be proved that the given arbitrary \( \xi > 0 \), the existence of \( l_i \), and \( t_i, \kappa_i, \sigma_i, \) and \( \mu_i \) make the solution of the closed-loop system consistent and bounded.

The derivative of the closed-loop system Lyapunov function with respect to time is

\[
\dot{V} = \sum_{i=1}^{n} \left( g_i z_i + g_i \gamma_i z_i - a_{\min} k_i z_i^2 - a_{\min} \lambda_i^T a_{\min}^{-1} \lambda_i \right) + \sum_{i=1}^{n-1} \gamma_i^T. 
\]

(35)

Deduced analysis can be drawn

\[
\dot{V} \leq \sum_{i=1}^{n} \left( g_i z_i + g_i \gamma_i z_i - a_{\min} k_i z_i^2 - a_{\min} \lambda_i^T a_{\min}^{-1} \lambda_i \right) + \sum_{i=1}^{n-1} \gamma_i^T. 
\]

(36)

However,

\[
q_{\min} k_i z_i = q_{\min} k_i z_i - \tilde{\lambda}_i^T a_{\min}^{-1} \lambda_i + \sum_{i=1}^{n-1} \gamma_i^T. 
\]

(37)

At the same time, it can be known from \( \hat{\dot{z}}_i = ((-s_i + \psi_i))/r_i \):

\[
g_i z_i + g_i \gamma_i z_i \leq \frac{1 + b_{\max}}{r_i} z_i^2 + \frac{1 + b_{\max}}{4} \omega_i. 
\]

(38)

Then, equation (36) can be changed to

\[
\dot{V} \leq \sum_{i=1}^{n} \left( g_i z_i + g_i \gamma_i z_i - a_{\min} k_i z_i^2 - a_{\min} \lambda_i^T a_{\min}^{-1} \lambda_i \right) + \sum_{i=1}^{n-1} \gamma_i^T. 
\]

(39)

Since \( g_i \bar{\theta}_i \psi_i(X) ||z|| + a_{\min} \bar{\theta}_i \psi_i(X) ||z|| \leq g_i \tilde{\lambda}_i (1/4k_i^2) \)

\[
\bar{\theta}_i \psi_i(X) ||z||. 
\]

(40)
where
\( \delta_i = (1 + b_{\text{max}})O_i^2 + b_{\text{max}}k_i + (\sigma_i/2)|\lambda_i^* - \lambda_i^0|^2 \),
\( i = (1^2 + 2^2 + \ldots + n^2)^{1/2} \), and \( \omega = \{\omega_1, \omega_2, \ldots, \omega_n\}^T \).

Since \( \omega = A^{m\pi}_{m} \), \( A^{m\pi}_{m} \leq 1 \); however,
\[
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_n
\end{bmatrix} = 
\begin{bmatrix}
A_1^m & 0 & \cdots & 0 \\
A_2^m & A_2^m & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
A_n^m & A_n^m & \cdots & A_n^m
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
\vdots \\
z_n
\end{bmatrix} = Az.
\]

This can be obtained as
\[
\|\omega\| \leq \|A\|\|z\| \leq \|z\|. \tag{42}
\]

First inspection the properties of equation (8):

Let \( |O_i| \) be the maximum value of \( M_{i,1} \) on the set \( \Omega \), and let
\( (1/r_{i,1}) = ((3 - b_{\text{max}})/4r_{i,1})^{-1}((b_{\text{max}}/4) + (M_{i,2}/2\psi) + \psi_0) \), and since \( |y_{i,1}/O_i| \leq (y_{i,1}M_{i,1}^2/2\psi) + (\psi/2) \), then
\[
\frac{b_{\text{max}}}{4}y_{i,1}^2 - \frac{3}{4r_{i,1}}y_{i,1} + y_{i,1}O_i \leq \left(-\frac{b_{\text{max}}}{4} + \frac{M_{i,1}^2}{2\psi} + \psi_0\right)y_{i,1}^2 + \frac{b_{\text{max}}}{4}y_{i,1}^2 + \frac{M_{i,1}^2O_i}{2\psi} + \psi_2
\]
\[
= -\psi_0y_{i,1}^2 \left(1 - \frac{\Omega_i^2}{M_{i,1}^2}\right)y_{i,1}^2 + \psi\frac{\psi_2}{2}\tag{43}
\]

Let be \( (\sigma_i/2b_{\text{max}}a_{\text{min}}) = \psi_0 \), \( k_i = b_{\text{max}}/4 + (1 + b_{\text{max}}/4)/\psi_0 \), \( m_i = b_{\text{max}}/4 + (1 + b_{\text{max}}/4)/\psi_0 \), \( k_n = b_{\text{max}}/4 + (1 + b_{\text{max}}/4)/\psi_0 \), and \( \psi_0 \) is a normal number. Then, equation (36) becomes
\[
V \leq -\psi_0 \sum_{i=1}^{n} z_i^2 - \psi_0 \sum_{i=1}^{n} \left(\delta_i + \frac{1}{\psi_0}\right) - \psi_0 \sum_{i=1}^{n} \frac{z_i^2}{\psi_0}
\]
\[
+ \sum_{i=1}^{n} \left(\delta_i \omega_i^T + \chi \leq -2\psi_0V + \chi \right)
\]

where \( z = [z_1, z_2, \ldots, z_n]^T \), \( \chi = \sum_{i=1}^{n} (\delta_i) + \sum_{i=1}^{n} (\psi/2) \), then
\[
V(t) \leq \frac{\chi}{2\psi_0} + V(t_0)e^{-2\psi_0(t-t_0)}, \quad \forall t \geq t_0 \geq 0. \tag{45}
\]

Obviously, for any \( \eta > 0 \), exist \( T > 0 \) so that for all \( t \geq t_0 + T, \|z\| \leq \zeta \) is established. Then, all solutions of the closed-loop system are consistent and bounded, that is, by adjusting the controller parameters so that the tracking error \( z_i = y(t) - y_r(t) \) is as small as possible to achieve the desired tracking accuracy.

### 3. Simulation Studies of Three-Dimensional Elliptical Vibration Cutting System

#### 3.1. System Transformation

Make the following assumptions for system (8).

**Assumption 4.** System (8) has a strong relative \( r \).

Under the condition of hypothesis 3.1, there is a differential homeomorphic map \( (u, v) = T(x) \) so that equation (8) becomes

\[
\begin{cases}
\dot{y} = a_1(t) + [b_1(t) + g_1(u, v)]u + f_1(u, v) + d(u, v), \\
\dot{v} = q(u, v),
\end{cases}
\]

where \( L_0^h(x) = f_1(u, v), L_0L_0^{-1}h(x) = g_1(u, v), \dot{v} = q(u, v) \) denotes zero dynamics of the system, \( a_1(t) \) and \( b_1(t) \) are known functions that are continuously bounded, and \( a_1(t) = b_1(t) = 0 \) is assumed.

**Assumption 5.** System (1) is of the minimum phase and pairs and the variable \( u \) satisfies the Lipschitz condition:
\[
[q(u, v) - q(0, v)] \leq L|u|, \tag{47}
\]

where \( L \) is a constant.

Under the assumption of 5, if \( u \) is bounded, then \( v \) must be bounded.

Suppose the system has no zero dynamics, i.e., \( v = 0 \).

Let \( f(x) = f_1(u, 0), g(x) = g_1(u, 0), \) and \( u = T^{-1}(x) \); then, equation (8) can be changed to
\[
y^{(r)} = g(x)u + f(x, t) + d(x, t). \tag{48}
\]

For the three-dimensional elliptical vibration assisted cutting system of the previous research [21], considering this type of single-input single-output (SISO) nonlinear Wiener system, it can be expressed as follows:
\[
y^{(3)} = g(x, t)u(t) + f(x, t) + d(t), \tag{49}
\]

where \( g(x, t) \) and \( f(x, t) \) are nonlinear functions, \( g(x, t) > 0 \), and \( d(t) \) is external interference.

Based on the LabVIEW simulation analysis, five fuzzy sets are defined for each variable in the system model to be fuzzy: [NL, NM, ZE, PM, PL]. These fuzzy sets are described by the following membership functions:

\[
\begin{align*}
\Gamma_{NL} & = \exp[-(x + 1)^2], \\
\Gamma_{NM} & = \exp[-(x + 0.5)^2], \\
\Gamma_{ZE} & = \exp[-x^2], \\
\Gamma_{PM} & = \exp[-(x - 0.5)^2], \\
\Gamma_{PL} & = \exp[-(x - 1)^2].
\end{align*}
\]

According to the control algorithm proposed in Section 2, the control rate, intermediate stabilization function, and parameter adaptation rate are selected as follows:

\[
\begin{align*}
\psi_2 & = -5z_1 + y_d, \\
\psi_3 & = -0.05z_2 + z_2 - \lambda_3z_2, \\
u & = -0.02z_2 + z_3 - \lambda_3z_2, \\
\dot{\lambda}_2 & = 0.2[\rho_2z_2^2 - 0.5\lambda_2], \\
\dot{\lambda}_3 & = 0.1[\rho_3z_3^2 - 1.5\lambda_3],
\end{align*}
\]

\[
\rho_i(t) = \frac{1}{4\lambda_i^2}k_i^iT + \frac{1}{4k_i^i}, \quad i = 2, 3.
\]

\[\]

\[\]
3.2. Description of Three-Dimensional Elliptical Vibration Cutting System. The three-dimensional elliptical vibration assisted cutting device (shown in Figure 2) is driven mainly by two parallel, vertically placed piezoelectric stacks in a non-resonant manner. The three piezoelectric stacks are, respectively, distributed on the upper flexible hinge and the lower flexible hinge, and each of the piezoelectric stacks is placed in parallel with a displacement sensor. The 3D EVC system can be viewed as three single-input single-output (SISO) systems using a nonlinear Wiener model to describe the 3D EVC system. The system identification results are detailed in [32].

3.3. Simulation Studies. In order to further illustrate the effectiveness of the proposed method, the three-dimensional elliptical vibration cutting device developed by our group was used to design the nonlinear Wiener control system as an example. The robust adaptive fuzzy control design and simulation research were carried out. A type of single-input and single-output nonlinear control system that can express its system equations is as follows:

\[
\begin{align*}
\dot{y}^{(3)} &= f_1(x,t) + g_1(x,t)u(t) + d_1(t), \\
x &= y,
\end{align*}
\]  

(52)

Figure 2: Structure of the 3D EVC system.

Figure 3: Membership function graph.
where \( f_1(x, t) \) and \( g_1(x, t) \) represent nonlinear functions, \( g_1(x, t) > 0; \) and \( d_1(t) \) are unknown external disturbances.

The robust adaptive fuzzy control simulation is carried out by taking the system identification result of the \( X^+ \) direction subsystem of the designed three-dimensional elliptical vibration cutting device as an example. In the simulation, the same membership function as in equation (50) is taken, and use MATLAB software to generate the membership function graph shown in Figure 3. The external interference is taken as \( d_1(t) = \sin(t); \) the initial values of the system variables \( x_1, x_2, \) and \( x_3 \) are set to \([0.2, 2\pi, 0]\); and the reference tracking signal is \( y_{d} = \sin(\pi t) \). The simulation results of position tracking and speed tracking are shown in Figure 4.

The tracking error curve of robust adaptive fuzzy control for a three-dimensional elliptical vibratory cutting system is shown in Figure 4. It can be seen from Figures 4 and 5 that the nonlinear model identified by the three-dimensional elliptical vibration cutting device can effectively suppress the jitter problem by using the robust adaptive fuzzy controller. The control object has a little jitter at the beginning, but it can be quickly stabilized and smoothed to move along the ideal displacement and velocity signals. Figures 6(a) and 6(b) are simulation results of the estimated parameters \( \hat{\lambda}_2 \) and \( \hat{\lambda}_3 \), respectively. The parameter estimation simulation curve shows that the proposed controller has strong robust adaptability and can freely adjust its own parameters according to the needs of the controlled system.

The robust adaptive fuzzy control of the \( X^+ \) directional subsystem of the abovementioned three-dimensional elliptical vibration cutting system can make the system tend to be...
stable in a short time, is not affected by external disturbances and has strong robust adaptability.

4. Conclusions

In this paper, the design of a robust adaptive fuzzy controller for a class of single-input single-output (SISO) uncertain nonlinear systems is proposed, and the corresponding design steps are given. The conclusions reached are as follows:

(1) The most notable features of the proposed control algorithm are as follows: in the case of unknown structural uncertainty, the resulting closed-loop system is ultimately bounded. Only one function needs to have a T-S fuzzy logic system approximation, and each subsystem needs only one parameter to be adjusted, which overcomes the problem of “dimensionality disaster.” Thereby, the calculation amount of the control algorithm is greatly reduced and the problem of “calculation expansion” is solved.

(2) The simulation research on the nonlinear control system of the three-dimensional elliptical vibration cutting device developed by our group is carried out. It is verified that the robust adaptive fuzzy control can make the system tend to be stable in a short time, is not affected by external disturbances, and has strong robust adaptability.

Future research will focus on extending the parameter optimization problem of the analytical control system to prove that the proposed solution has more effective engineering application value.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] M. M. Polycarpou and P. A. Ioannou, “A robust adaptive nonlinear control design,” in Proceedings of the 1993 American Control Conference, pp. 1365–1369, San Francisco, CA, USA, June 1993.

[2] Y. Huang, D. Wang, and D. Liu, “Bounded robust control design for uncertain nonlinear systems using single-network adaptive dynamic programming,” Neurocomputing, vol. 266, pp. 128–140, 2017.

[3] S. N. Tiwari and R. Padhi, “Optimal and robust control of a class of nonlinear systems using dynamically re-optimised single network adaptive critic design,” International Journal of Systems Science, vol. 49, no. 2, pp. 246–263, 2018.

[4] K. Nam and A. Araposthathis, “A model reference adaptive control scheme for pure-feedback nonlinear systems,” IEEE Transactions on Automatic Control, vol. 33, no. 9, pp. 803–811, 1988.

[5] M. Hou, Z. Deng, and G. Duan, “Adaptive control of uncertain pure-feedback nonlinear systems,” International Journal of Systems Science, vol. 48, no. 10, pp. 2137–2145, 2017.

[6] A. Teel, R. Kadiyala, P. Kokotovic, and S. Sastry, “Indirect techniques for adaptive input-output linearization of nonlinear systems,” International Journal of Control, vol. 53, no. 1, pp. 193–222, 1991.

[7] Z. P. Jiang and D. J. Hill, “A robust adaptive backstepping scheme for nonlinear systems with unmodeled dynamics,” IEEE Transactions on Automatic Control, vol. 44, no. 9, pp. 1705–1711, 1999.

[8] D. Seto, “Adaptive control of a class of nonlinear system with a triangular structure,” IEEE Transactions, pp. 1666–1682, 1992.

[9] L. Huang, Y. Li, and S. Tong, “Fuzzy adaptive output feedback control for a class of switched non-triangular structure nonlinear systems with time-varying delays,” International Journal of Systems Science, vol. 49, no. 1, pp. 132–146, 2018.

[10] H. Wang, P. X. Liu, S. Li et al., “Adaptive neural output-feedback control for a class of nonlower triangular nonlinear systems with unmodeled dynamics,” IEEE Transactions on Neural Networks and Learning Systems, vol. 29, no. 8, pp. 3658–3668, 2018.

[11] C. Mu, Y. Zhang, Z. Gao et al., “ADP-based robust tracking control for a class of nonlinear systems with unmatchd uncertainties,” IEEE Transactions on Systems, Man, and Cybernetics: Systems.

[12] C. Mu and Y. Zhang, “Learning-based robust tracking control of quadrotor with time-varying and coupling uncertainties,” IEEE Transactions on Neural Networks and Learning Systems, vol. 31, no. 1, pp. 259–273, 2020.

[13] S. S. Ge and J. Jing Wang, “Robust adaptive neural control for a class of perturbed strict feedback nonlinear systems,” IEEE Transactions on Neural Networks, vol. 13, no. 6, pp. 1409–1419, 2002.

[14] S. G. Cao, N. W. Rees, and G. Feng, “Universal fuzzy controllers for a class of nonlinear systems,” Fuzzy Sets and Systems, vol. 122, no. 1, pp. 117–123, 2001.

[15] D. G. Taylor, P. V. Kokotovic, R. Marino et al., “Adaptive regulation of nonlinear systems with unmodeled dynamics,” in Proceedings of the 1988 American Control Conference, pp. 360–365, Atlanta, GA, USA, June 1988.

[16] N. Wang, T. Zhang, Y. Yi, and Q. Wang, “Adaptive control of output feedback nonlinear systems with unmodeled dynamics and output constraint,” Journal of the Franklin Institute, vol. 354, no. 13, pp. 5176–5200, 2017.

[17] Z.-J. Yang, T. Nagai, S. Kanae, and K. Wada, “Dynamic surface control approach to adaptive robust control of nonlinear systems in semi-strict feedback form,” International Journal of Systems Science, vol. 38, no. 9, pp. 709–724, 2007.

[18] S. S. Ge, P. Hong, and T. H. Lee, “Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients,” IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics), vol. 34, no. 1, pp. 499–516, 2004.

[19] K. Fischle and D. Schroder, “An improved stable adaptive fuzzy control method,” IEEE Transactions on Fuzzy Systems, vol. 7, no. 1, pp. 27–40, 1999.

[20] X.-l. Li, X.-f. Zhang, C. Jia, and D.-x. Liu, “Multi-model adaptive control based on fuzzy neural networks,” Journal of Intelligent & Fuzzy Systems, vol. 27, no. 2, pp. 965–975, 2014.

[21] Y. Yang, G. Feng, and J. Ren, “A combined backstepping and small-gain approach to robust adaptive fuzzy control for strict-feedback nonlinear systems,” IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans, vol. 34, no. 3, pp. 406–420, 2004.

[22] D. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, “Dynamic surface control for a class of nonlinear systems,” IEEE Transactions on Automatic Control, vol. 45, no. 10, pp. 1893–1899, 2000.

[23] D. Swaroop, J. C. Gerdes, P. P. Yip et al., “Dynamic surface control for a class of nonlinear systems,” IEEE Transactions on Fuzzy Systems, vol. 27, no. 1, pp. 172–184, 2019.

[24] C. Mu, W. Xu, and C. Sun, “On switching manifold design for terminal sliding mode control,” Journal of the Franklin Institute, vol. 353, no. 7, pp. 1553–1572, 2016.

[25] S. Sui, C. L. P. Chen, and S. Tong, “Fuzzy adaptive finite-time control design for nontriangular stochastic nonlinear systems,” IEEE Transactions on Fuzzy Systems, vol. 27, no. 1, pp. 172–184, 2019.

[26] C. Mu, W. Xu, and C. Sun, “On switching manifold design for terminal sliding mode control,” Journal of the Franklin Institute, vol. 353, no. 7, pp. 1553–1572, 2016.

[27] S. Sui, C. L. P. Chen, and S. Tong, “Neural network filtering control design for nontriangular structure switched nonlinear systems in finite time,” IEEE Transactions on Neural Networks and Learning Systems, vol. 30, no. 7, pp. 2153–2162, 2019.

[28] S. Sui, S. Tong, and C. L. P. Chen, “Finite-time filter decentralized control for nonstrict-feedback nonlinear large-scale systems,” IEEE Transactions on Fuzzy Systems, vol. 26, no. 6, pp. 3289–3300, 2018.

[29] L. A. Zadeh, “Fuzzy sets,” Information and Control, vol. 8, no. 3, pp. 338–353, 1965.

[30] T. Takagi and M. Sugeno, “Fuzzy identification of systems and its applications to modeling and control,” IEEE Transactions on Systems, Man, and Cybernetics, vol. SMC-15, no. 1, pp. 116–132, 1985.

[31] S. G. Cao, N. W. Rees, and G. Feng, “Analysis and design for a class of complex control systems Part I: fuzzy modelling and identification,” Automatica, vol. 33, no. 6, pp. 1017–1028, 1997.

[32] S. G. Cao, N. W. Rees, and G. Feng, “Universal fuzzy controllers for a class of nonlinear systems,” Fuzzy Sets and Systems, vol. 122, no. 1, pp. 117–123, 2001.

[33] S. G. Cao, N. W. Rees, and G. Feng, “Dynamic surface control of nonlinear systems,” in Proceedings of the American Control Conference, vol. 5, pp. 3028–3034, Albuquerque, NM, USA, June 1997.

[34] M. Lu, H. Wang, J. Lin et al., “A nonlinear Wiener system identification based on improved adaptive step-size glow-worm swarm optimization algorithm for three-dimensional elliptical vibration cutting,” The International Journal of Advanced Manufacturing Technology, vol. 103, no. 5–8, pp. 2865–2877, 2019.