Remove Blur Image using Bi-Directional Akamatsu Transform and Discrete Wavelet Transform

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Abstract.

**Purpose:** Image is an imitation of everything that can be materialized, and digital images are taken using a machine. Although digital image capture uses machines, digital images are not free from interference. Image restoration is needed to restore the quality of the damaged image.

**Methods:** Bi-directional Akamatsu Transform is proven to have an effective performance in reducing blur in images. Meanwhile, Discrete Wavelet Transform has been widely used in digital image processing research. We had been investigated the image restoration method by combining Bi-directional Akamatsu Transform and Discrete Wavelet Transform. Bi-directional Akamatsu Transform applied in Low-Low (LL) sub-band is the Discrete Wavelet Transform decomposition image most similar to the original image before decomposing. In this study, there are still shortcomings, including the determination of the values of N, up_enh, and down_enh, which are still manual. Manually setting the three values makes the Bi-directional Akamatsu Transform method not get the best results. With the use of machine learning methods can get better restoration results. Further testing is also needed for a more diverse and robust blur. The image data has a resolution of 256x256, 512x512, and 1024x1024. The image will be directly converted to a grey-scale image. The converted image will be given an attack model: average blur, gaussian blur, and motion blur. The image that has been attacked will apply two restoration methods: the proposed method and the Bi-direction Akamatsu Transform. These two restoration images will then be compared using PSNR.

**Result:** The average PSNR value from the restoration of the proposed method is 0.1446 higher than the average PSNR value from the restoration of the Bi-directional Akamatsu Transform method. When we compare it with the average PSNR value of the Akamatsu Transform restoration method, the average PSNR of the proposed method is 0.2084.

**Value:** The combination of DWT and akamatsu transform results produce good PSNR values even though they have gone through the blurring method in image restoration.

**Keywords:** Digital Image, Restoration, Akamatsu Transform Bi-directional, Akamatsu Transform, Discrete Wavelet Transform

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INTRODUCTION

Image is an imitation, resemblance, or representation of people, places, objects, and everything that can be materialized [1]. The image is a painting since prehistoric times. But along with the development of human civilization, the form of an image is no longer just a painting [2]. The medium used for the image is starting to vary with the technology. The mediums used range from canvas and paper to digital. Digital image is an image taken using a machine based on sampling and quantization. Digital images are not free from interference, even if taken using a machine. Light, noise, blur, and others cause the image quality to decrease [3]. These things cause the information contained in the image to be reduced. Blur can be caused by a dirty camera lens [4]. The camera or a moving object can also cause it. Image restoration is needed to restore the quality of the damaged image. In the image restoration process, there are various methods, such as Lucy-Richardson [5], Wiener Filter [6], Discrete Wavelet Transform [7]–[9], Discrete Cosine Transform [10], Fourier Transform [11], Akamatsu Transform [6] and Bi-directional Akamatsu Transform [12].

Research using the Wiener algorithm proposed by [13] to restore blurry images. The image restoration process is carried out with two sub-processes. The first part degrades the image quality by adding noise and blur, and the second part removes noise and blur from the degraded image and restores the original image.

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This study uses the Non-Blind Restoration technique, which is a technique that assists in the reconstruction of the original image from the degraded image when the way the image is degraded is known. The method used in implementing the restoration technique is the iterative method. The modified iterative Wiener filter used in this study is an efficient method of restoring degraded medical images. The MSE value of the restored image decreases with an increasing number of iterations until the results converge. Most of the convergence occurs after ten iterations. In the end, the best result of the restored image is obtained when setting the parameter equal to one. In another study by [14], the efficiency of image restoration methods was compared for various blurry space images. The methods compared in this study are Kalman and Wiener filters. The image used in this study is assumed to have no noise. The image is applied motion blur, which has an average length equal to 5. Zero mean Gaussian noise, known as standard deviation, is also added. The following process is image restoration using wiener and Kalman filters on the tested image. From the results of the experiments, the Kalman filter is better than the Wiener filter in restoring blur images that are affected by space-variant blur.

Research [15] correcting or restoring images' blur using the Akamatsu Transform method. Akamatsu Transform is a technique Norio Akamatsu proposed to create differential and integral signals from the given original signals. The resulting integral from the Akamatsu Transform is similar to that of a low-pass filter. In contrast, the differential is assumed to be the high-frequency components of the original image. So that the characteristics of the original image can be seen in the differential. This method has a reasonably practical restoration result in restoring the image. But this method of restoring a stronger blur image requires further research to determine the appropriate downEnh and upEnh.

In a study [16], the blurred image restoration used the Bi-directional Akamatsu Transform method. This method is a research development of the Akamatsu Transform method [15]. The development of this method is that this method performs the Akamatsu Transform horizontally or in the direction of the X-axis and vertically or in the direction of the Y-axis. While the Akamatsu Transform method proposed in research [16] only performs horizontally or in the direction of the X-axis. The restoration method's results in this study reduced the blur effect on the image. Based on the experiments conducted in the study, the proposed method reduces the blur effect better than the method in the study [15]. However, in this method, the determination of downEnh and upEnh is still done manually.

Wavelet Transform has been widely used in digital image processing and [12] pattern recognition. Discrete Wavelet Transform is a form of the wavelet transform. DWT, in decomposing the image, the image will be divided into four sub-bands, namely Low-Low (LL), Low-High (LH), High-Low (HL), and High-High (HH). LL is a representation of the image and is the sub-band that is most similar to the original image before being decomposed. While the other three sub-bands, namely LH, HL, and HH, are variations of wavelets vertically, horizontally, and diagonally.

The proposed method is used to restore noise and blur images. The image will be decomposed using DWT. Then the results of the decomposition will be applied to the Akamatsu Transform. The LH sub-band uses the Akamatsu Transform vertically. The HL sub-band uses Akamatsu transform horizontally. And HH uses Akamatsu transform horizontally and vertically. The results of this restoration method do not change the characteristics of the original image significantly, but the noise in the image is reduced. For restoration results on blur, the image restoration method in [15] produces a more precise image than the restoration image of the proposed method. Bi-directional Akamatsu Transform is a technique developed based on the Akamatsu Transform [16] method. This method is proven to reduce blur in the image better than the Akamatsu Transform method.

This research will use two combined methods: Bi-directional Akamatsu Transform and Discrete Wavelet Transform. Bi-directional Akamatsu transform is used for the sub-band LL image, which is the DWT decomposition image most similar to the original image before decomposing. Testing the restoration results had been calculated by PSNR and compared with previous studies.

METHODS

Akamatsu Transform

Akamatsu transform is developed from integral and differential transformation [17]. Norio Akamatsu first developed and patented this transformation in 2006 [6]. This transformation is used in image processing to restore the overall blur image. Akamatsu transform can be applied to recognize sounds in vowels [15], [16], where the target signal is in the transformation. With the target signal, the result of the transformation will be divided into two parts, namely, the right side of the target signal and the left side. [18] The signal on the right of P(x,y) we define as VRS(x) and on the left as VLS(x). The target signal is used to determine the
result of the transformation. To get the target signal, it is necessary to calculate the integral and differential values. The integral value can be calculated by equation (1).

\[
A(x) = \frac{[AvrS_R(x) + AvrS_L(x)]}{2}
\]

Where,
- \(A(x)\) = Akamatsu transform integral value
- \(AvrS_R(x)\) = Average of the intensity values to the right of the target signal
- \(AvrS_L(x)\) = Average of the intensity values to the left of the target signal

**Discrete Wavelet Transform**

The wavelet-based method is used to obtain robustness that also maintains the two data types being transformed, frequency and spatial information [19]. Signal transformation is another form of signal representation that does not change the information contained in the signal. The wavelet transform has two development series: Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT). Discrete Wavelet Transform (DWT) algorithm [20] is an image-processing algorithm that uses the same mathematical calculations as DCT [21]. DWT has the characteristics of the image decomposition model into four parts (subbands), namely LL, LH, HL, and HH [22]. Frequency L is Low, and H is High. DWT is also an orthogonal transform with a fast computational process [23]. Figure 1 illustrates the DWT sub-band decomposition. Decomposition on DWT can be done using (2) and (3).

\[
\Phi_{j,a,b}(x,y) = 2^j \phi(2^j - a, 2^j y - b)
\]

\[
\psi_{i,j,a,b}(x,y) = 2^j(2^j - a, 2^j - a, 2^j y - b), i = \{H, V, D\}
\]

Where,
- \(i = \text{wavelet } \{\text{LL, LH, HL, HH}\}\)
- \(a, b = \text{image size}\)

![Figure 1. Sub-band decomposition on DWT](image)

Reconstruction is carried out by inverse DWT using the equation as shown in (4).

\[
f(x,y) = \sum_{a}^{\alpha} \sum_{b}^{\beta} w_{\phi}(j_0, a, b) \phi_{j_0,a,b}(x,y) + \sum_{i=H, V, D} \sum_{j=j_0}^{\alpha} \sum_{a}^{\alpha} \sum_{b}^{\beta} w_{\psi}(j_0, a, b) \psi_{i,j_0,a,b}(x,y)
\]

Where,
- \(i = \text{wavelet } \{\text{LL, LH, HL, HH}\}\)
- \(a, b = \text{image size}\)

**Preprocessing**

The image data collected cannot be directly used in the restoration process. The data collected will go through a preprocessing stage, including conversion and application of the attack model. After the data has gone through both stages, the data can be used for research. Uniformity of the type of image color map is required to eliminate the computational and processing burdens of RGB, binary, and grey-scale images. To simplify processing, the collected image data will be converted into a grey-scale image. The image used for research will be given two attack models before restoration. The images are given average blur, gaussian blur, and motion blur. After the images used for research are attacked, these images will be used in the proposed method’s restoration process.
Method
Figure 2 shows the proposed method of this study. In this study, the image is given blur attacks, namely average blur, motion blur, and gaussian blur. The image given a blurred attack will be decomposed using DWT. The steps of the restoration process are described as follows:

1) The image given a blurred attack will be decomposed using DWT to get the LL, LH, HL, and HH sub-bands.
2) The LL sub-band will be applied using the Bi-directional Akamatsu Transform method to obtain the LL sub-band as a result of restoration.
3) The Inverse DWT process was carried out using LL restoration results, LH, HL, and HH.
4) The image resulting from the Inverse DWT process is the image resulting from the restoration of the proposed method.

![Figure 2. Proposed Method](image-url)
Experiment

The restoration process was carried out using the MATLAB application. In this research, 20 images have been applied, where each image has three different pixel sizes, and among the selected images, there are color and grayscale images. The image data used has a resolution of 256x256, 512x512, and 1024x1024, as shown in Figure 3. Dataset taken from Irawan et al. [16], Karungaru et al. [15], Wellia [8], Eko Hari [24], and also https://sipi.usc.edu/database/. The image will be directly converted to a grey-scale image. The converted image will be given an attack model, namely average blur, gaussian blur, and motion blur with their respective intensities, on disc two or N=2. The image that has been attacked will apply two restoration methods: the proposed method and the Bi-direction Akatamatsu Transform. These two restoration images will then be compared using PSNR.

Figure 3. Original Images

Evaluation

The evaluation process will be carried out by looking at the PNSR value of the restoration. The PSNR value of the proposed method will be compared with the PSNR value of the restoration in studies [16] and [15].

RESULT AND DISCUSSION

Table 1 shows the average PSNR value of the proposed method has a higher value than the methods [15] and [16]. The average PSNR value of the proposed method is 0.0641 higher than the average value of the method [16]. When compared with the average value of the method [15], the average PSNR value of the proposed method is higher by 0.1576. The PSNR values of the three methods are derived from restoring the average blur filter image with a constant H of 1. All three restoration methods use the same N, which is 2.

| Image         | Resolution | Proposed Method | Irawan et al [16] | Karungaru et al [15] |
|---------------|------------|-----------------|-------------------|----------------------|
| 256_1.tiff    | 256x256    | 26.7420         | 26.6234           | 26.5104              |
| 256_1_reverse.tiff | 256x256    | 26.7420         | 26.6234           | 26.5104              |
| 256_1_90.tiff | 256x256    | 26.7420         | 26.6234           | 26.4276              |
| 256_1_180.tiff| 256x256    | 26.7420         | 26.6234           | 26.5104              |
| 256_1_270.tiff| 256x256    | 26.7420         | 26.6234           | 26.4276              |
| 256_2.tiff    | 256x256    | 25.0440         | 24.9643           | 24.8958              |
Based on Table 2, the average PSNR value for the restoration of the proposed method has a higher value than the average value for the methods [15] and [16]. The average value of the restoration results of the proposed method has a higher value of 0.1977 than the average PSNR value of the method [16]. When compared with the average PSNR value of the method [15], the average PSNR value of the proposed method is 0.2632. The PSNR values of the three methods are derived from the restoration of motion blur filter images with a len of 5 and theta of 90. The three restoration methods use the same N, namely, 2.

Table 2. PSNR value of restoration with N = 2 for motion blur

| Image         | Resolution | Proposed Method | Irawan et al [16] | Karungaru et al [15] |
|---------------|------------|-----------------|-------------------|----------------------|
| 256_2_reverse.tiff | 256x256 | 24.4816 | 24.2286 | 24.0711 |
| 256_2_90.tiff   | 256x256 | 24.4816 | 24.2286 | 24.0711 |
| 256_2_180.tiff  | 256x256 | 24.4816 | 24.2286 | 24.0711 |
| 256_2_270.tiff  | 256x256 | 24.2614 | 23.9265 | 23.6958 |
| 256_3_180.tiff  | 256x256 | 24.2614 | 23.9265 | 23.6958 |
| 256_3_270.tiff  | 256x256 | 24.2614 | 23.9265 | 23.6958 |
| 256_4_180.tiff  | 256x256 | 24.2614 | 23.9265 | 23.6958 |
| 256_4_270.tiff  | 256x256 | 24.2614 | 23.9265 | 23.6958 |
| 512_1_90.tiff   | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 512_1_180.tiff  | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 512_1_270.tiff  | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 512_2_90.tiff   | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 512_2_180.tiff  | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 512_2_270.tiff  | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 512_3_90.tiff   | 512x512 | 25.9039 | 25.8042 | 25.7608 |
| 512_3_180.tiff  | 512x512 | 25.9039 | 25.8042 | 25.7608 |
| 512_3_270.tiff  | 512x512 | 25.9039 | 25.8042 | 25.7608 |
| 512_4_90.tiff   | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 512_4_180.tiff  | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 512_4_270.tiff  | 512x512 | 22.8082 | 22.8247 | 22.7671 |
| 1024_1_90.tiff  | 1024x1024 | 27.7022 | 27.6545 | 27.5767 |
| 1024_1_180.tiff | 1024x1024 | 27.7022 | 27.6545 | 27.5767 |
| 1024_1_270.tiff | 1024x1024 | 27.7022 | 27.6545 | 27.5767 |
| 1024_2_90.tiff  | 1024x1024 | 28.9933 | 28.9975 | 28.8319 |
| 1024_2_180.tiff | 1024x1024 | 28.9933 | 28.9975 | 28.8319 |
| 1024_2_270.tiff | 1024x1024 | 28.9933 | 28.9975 | 28.8319 |
| Average        |          | 25.4573 | 25.3932 | 25.2997 |

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Based on Table 3, the average PSNR value for the restoration of the proposed method has a higher value than the average value for the methods [15] and [16]. The average value of the restoration results of the proposed method has a higher value of 0.1719 than the average PSNR value of the method [16]. Compared with the method’s average PSNR value [15], the average PSNR value of the proposed method is 0.2043 higher. The PSNR values of the three methods are derived from the restoration of the Gaussian blur filter image with a sigma constant of 1. All three restoration methods use the same N, which is 2.

Table 3. PSNR value of restoration with N = 2 for gaussian blur

| Image            | Resolution | PSNR     |
|------------------|------------|----------|
|                  |            | Proposed Method | Irawan et al [16] | Karungaru et al [15] |
| 256_1_270.tiff   | 256x256    | 26.0289   | 25.7490          | 25.7264              |
| 256_1_reverse.tiff | 256x256    | 26.0289   | 25.7490          | 25.7264              |
| 256_1_90.tiff    | 256x256    | 26.0289   | 25.7490          | 25.7264              |
| 1024_1.tif      | 1024x1024  | 26.0069   | 25.7956          | 25.7563              |
| 1024_1_reverse.tif | 1024x1024  | 26.0069   | 25.7956          | 25.7563              |
| 1024_1_90.tif   | 1024x1024  | 26.0069   | 25.7956          | 25.7563              |
| Average         |            | 23,4469   | 23,2492          | 23,1837              |
Based on Table 4, using average blur got the same blur reduction. However, the difference between the three restored images is not very visible. So from a visual comparison, all three methods have the same ability to reduce blur. Whereas using motion blur, the image restored by the three methods has strengthened image characteristics and reduced blur. If the three methods are compared visually, the three methods have the same capabilities. That's because the images from the restoration of the three methods look similar, from 256x256 to 1024 resolution. Using Gaussian blur, image restoration results from the three methods have stronger image characteristics and reduced blur.

| Average blur          | Motion blur          | Gaussian blur         |
|----------------------|----------------------|----------------------|
| Proposed By [16]     | Proposed By [15]     | Proposed By [16]     |
| 256_1_180.tiff       | 256x256              | 26.0289              |
| 256_1_270.tiff       | 256x256              | 25.7490              |
| 256_2.tiff           | 256x256              | 24.2879              |
| 256_2_reverse.tiff   | 256x256              | 23.9166              |
| 256_2_90.tiff        | 256x256              | 22.9166              |
| 256_2_180.tiff       | 256x256              | 22.8823              |
| 256_2_270.tiff       | 256x256              | 22.8823              |
| 256_3.tiff           | 256x256              | 23.0532              |
| 256_3_reverse.tiff   | 256x256              | 22.9166              |
| 256_3_90.tiff        | 256x256              | 22.9166              |
| 256_3_180.tiff       | 256x256              | 22.9166              |
| 256_3_270.tiff       | 256x256              | 22.9166              |
| 256_4.tiff           | 256x256              | 23.0532              |
| 256_4_reverse.tiff   | 256x256              | 22.9166              |
| 256_4_90.tiff        | 256x256              | 22.9166              |
| 256_4_180.tiff       | 256x256              | 22.9166              |
| 512_1.tiff           | 512x512              | 21.8145              |
| 512_1_reverse.tiff   | 512x512              | 21.7574              |
| 512_1_90.tiff        | 512x512              | 21.7574              |
| 512_1_180.tiff       | 512x512              | 21.7574              |
| 512_1_270.tiff       | 512x512              | 21.7574              |
| 512_2.tiff           | 512x512              | 22.4364              |
| 512_2_reverse.tiff   | 512x512              | 22.2755              |
| 512_2_90.tiff        | 512x512              | 22.2755              |
| 512_2_180.tiff       | 512x512              | 22.2755              |
| 512_2_270.tiff       | 512x512              | 22.2755              |
| 512_3.tiff           | 512x512              | 25.0924              |
| 512_3_reverse.tiff   | 512x512              | 24.8971              |
| 512_3_90.tiff        | 512x512              | 24.8971              |
| 512_3_180.tiff       | 512x512              | 24.8971              |
| 512_3_270.tiff       | 512x512              | 24.8971              |
| 512_4.tiff           | 512x512              | 21.7048              |
| 512_4_reverse.tiff   | 512x512              | 21.5638              |
| 512_4_90.tiff        | 512x512              | 21.5638              |
| 512_4_180.tiff       | 512x512              | 21.5638              |
| 512_4_270.tiff       | 512x512              | 21.5638              |
| 1024_1.tiff          | 1024x1024            | 26.4702              |
| 1024_1_reverse.tiff  | 1024x1024            | 26.2620              |
| 1024_1_90.tiff       | 1024x1024            | 26.4702              |
| 1024_1_180.tiff      | 1024x1024            | 26.4702              |
| 1024_1_270.tiff      | 1024x1024            | 26.4702              |
| 1024_2.tiff          | 1024x1024            | 26.3633              |
| 1024_2_reverse.tiff  | 1024x1024            | 26.1609              |
| 1024_2_90.tiff       | 1024x1024            | 26.1609              |
| 1024_2_180.tiff      | 1024x1024            | 26.1609              |
| 1024_2_270.tiff      | 1024x1024            | 26.1609              |

| Average             | 24.2262              | 24.0543              | 24.0219              |
The visual differences in the restoration images of the three restoration methods are not very visible to the naked eye, as shown in Table 5. Our proposed method yields the highest PSNR against blurring filters. By implementing attack blur, namely average blur, motion blur, and Gaussian blur where we equate the intensity according to Irawan’s research [16] with $N=2$ (disk 2) and Karungaru [15] with intensity $xy^2$ ($N=2$). Thus it can be proven that the PSNR value we produce is slightly higher than the two studies above.

| Table 5. The average PSNR value of the overall restoration |
|----------------------------------------------------------|
| Type of blur filter | Proposed | Irawan [16] | Karungaru [15] |
|---------------------|----------|-------------|----------------|
| Average             | 25,4573  | 25,3932     | 25,2997        |
| Motion              | 23,4469  | 23,2492     | 23,1837        |
| Gaussian            | 24,2262  | 24,0543     | 24,0219        |
| Average Result      | 24,3768  | 24,2322     | 24,1684        |

CONCLUSION
This research combines two transformation methods, Bi-directional Akamatsu Transform and Discrete Wavelet Transform, to eliminate blur in the image. The restoration results prove that the three methods can reduce blur in the image. Visually, the image restoration of the proposed method [16] and method [15] looks similar. Therefore, the images from the restoration of the three methods need to be compared by PSNR. From the comparison table, the average PSNR value from the restoration of the proposed method is 0.1446 higher than the average PSNR value from the restoration method [16]. When compared with the average PSNR value of the restoration method [15], the proposed method’s average PSNR value is 0.2084. The proposed method has a higher average PSNR value than the two methods because the Bi-directional Akamatsu Transform restoration method is not applied directly to the blurred image but to the LL sub-band, which is the result of decomposition of the blurred image using Discrete Wavelet Transform.

REFERENCES
[1] H. Xue and H. Cui, “Research on Image Restoration Algorithms Based on BP Neural Network,” J. Vis. Commun. Image Represent., vol. 59, no. January, pp. 204–209, Feb. 2019.
[2] K. Sakashita and S. Muramatsu, “Image Restoration by Group Sparsity with Union of Hierarchical DirLOTs,” 2020 Asia-Pacific Signal Inf. Process. Assoc. Annu. Summit Conf. APSIPA ASC 2020 - Proc., no. December, pp. 1182–1187, 2020.
[3] N. Wadhwa et al., “Synthetic Depth-Of-Field with a Single-Camera Mobile Phone,” ACM Trans. Graph., vol. 37, no. 4, 2018.
[4] C. Hu, B. B. Sapkota, J. A. Thomasson, and M. V. Bagavathiannan, “Influence of Image Quality and Light Consistency on the Performance of Convolutional Neural Networks for Weed Mapping,” Remote Sens., vol. 13, no. 11, 2021.
[5] S. Sharma and R. Mehra, “Image Restoration using Modified Lucy Richardson Algorithm in the Presence of Gaussian and Motion Blur,” Adv. Electron. Electr. Eng., vol. 3, no. 8, 2013.
[6] S. Yoshimori, Y. Mitsukura, M. Fukumi, and N. Akamatsu, “An Adaptive Graininess Suppression Method for Restoration of Color Degraded Images,” IEEJ Trans. Electron. Inf. Syst., vol. 127, no. 12, pp. 2093–2100, 2007.
[7] P. Megha, M. Swarna, V. Sowmya, and K. P. Soman, “Low Contrast Satellite Image Restoration Based on Adaptive Histogram Equalization and Discrete Wavelet Transform,” Int. Conf. Commun. Signal Process. ICCSP 2016, pp. 402–406, 2016.
[8] W. S. Sari, E. H. Rachmawanto, D. R. I. M. Setiadi, and C. A. Sari, “A Good Performance OTP
Encryption Image based on DCT-DWT Steganography,” *TELKOMNIKA*, vol. 15, no. 4, pp. 1987–1995, 2017.

[9] Y. Zhang, Z. Liu, M. Huang, Q. Zhu, and B. Yang, “Multi-Resolution Depth Image Restoration,” *Mach. Vis. Appl.*, vol. 32, no. 3, pp. 1–15, 2021.

[10] Y. Abe and Y. Iiguni, “Image Restoration From a Downsampled Image by Using the DCT,” *Signal Processing*, vol. 87, no. 10, pp. 2370–2380, 2007.

[11] T. Tahara, T. Akamatsu, Y. Arai, T. Shimobaba, T. Ito, and T. Kakue, “Algorithm for Extracting Multiple Object Waves Without Fourier Transform From a Single Image Recorded by Spatial Frequency-Division Multiplexing and Its Application to Digital Holography,” *Opt. Commun.*, vol. 402, no. June, pp. 462–467, Nov. 2017.

[12] J. M. Akbar and D. R. I. M. Setiadi, “Joint Method Using Akamatsu and Discrete Wavelet Transform for Image Restoration,” *Appl. Comput. Informatics*, pp. 0–7, 2019.

[13] A. H. Sheer and A. A. Al-Ani, “The Effect of Regularization Parameter Within Non-Blind Restoration Algorithm Using Modified Iterative Wiener Filter for Medical Image,” *Proc. - 2018 1st Annu. Int. Conf. Inf. Sci. AiCIS 2018*, pp. 77–81, 2019.

[14] A. Thakur, A. Kausar, and A. Iqbal, “Comparison Efficacy of Restoration Method for Space Variant Motion Blurred Images Using Kalman & Wiener Filter,” *Proc. 2016 6th Int. Conf. - Cloud Syst. Big Data Eng. Confl.*. 2016, pp. 508–512, 2016.

[15] S. Karungaru, M. Sugizaki, M. Fukumi, Y. Mitsukura, and N. Akamatsu, “Out-of-Focus Blur Image Restoration Using The Akamatsu Transform,” *IECON Proc. (Industrial Electron. Conf.*)*, pp. 4257–4261, 2009.

[16] C. Irawan, J. M. Akbar, D. R. Ignatius Moses Setiadi, E. H. Rachmawanto, C. A. Sari, and A. Winarno, “Image Deblurring Using Bi-directional Akamatsu Transform,” *Proc. - 2019 Int. Semin. Appl. Technol. Inf. Commun. Ind. 4.0 Retrospt. Prospect. Challenges, iSemantic 2019*, pp. 17–22, 2019.

[17] X. Liu, J. Zhu, Q. Zheng, Z. Li, R. Liu, and J. Wang, “Bidirectional Loss Function for Label Enhancement and Distribution Learning,” *Knowledge-Based Syst.*, vol. 213, no. 61573273, pp. 0–2, 2021.

[18] K. Tanaka, Y. Hirai, T. Suzuki, M. Akamatsu, K. Sakai, and H. Sakai, “Characterizing Water Behavior in A-Gel (A-Type Hydrated Crystal) Formed from Monohexadecyl Phosphate and L-Arginine,” *J. Oleo Sci.*, vol. 68, no. 3, pp. 225–231, 2019.

[19] M. Taufif, E. Khan, M. Hasan, and M. Reisslein, “SMFrWF: Segmented Modified Fractional Wavelet Transform: Fast Low-Memory Discrete Wavelet Transform (DWT),” *IEEE Access*, vol. 7, pp. 84448–84467, 2019.

[20] E. H. Rachmawanto, C. Atika Sari, and R. P. Pradana, “DWT-SVD Combination Method for Copyrights Protection,” *Sci. J. Informatics*, vol. 7, no. 1, pp. 113–124, 2020.

[21] E. Kartikadarma, E. Udayanti, … C. S.-S. J. of, and U. 2021, “A Comparison of Non Blind Image Watermarking Using Transformation Domain,” *Sci. J. Informatics*, vol. 8, no. 1, pp. 104–110, 2021.

[22] M. A. Alzain and J. F. Al-Amri, “Application of Data Steganographic Method in Video Sequences Using Histogram Shifting in the Discrete Wavelet Transform,” *Int. J. Appl. Eng. Res.*, vol. 13, no. 8, pp. 6380–6387, 2018.

[23] W. S. Sari and C. A. Sari, “A High Result in Wavelet Watermarking Using Singular Value Decomposition,” *Kinet. Game Technol. Inf. Syst. Comput. Network, Comput. Electron. Control.*, vol. 4, no. 3, pp. 269–276, Jul. 2019.

[24] E. H. Rachmawanto, D. R. I. M. Setiadi, C. A. Sari, and N. Riijati, “Imperceptible and secure image watermarking using DCT and random spread technique,” *Telecommunication Comput. Electron. Control.*, vol. 17, no. 4, 2019.