Nonlinear dynamics of the marine rotor-bearing system during the pitching motion

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Abstract—In order to study the motion law of the rotor-bearing system under ship pitching motion, the dynamic model of the marine rotor-bearing system is established. The dynamic characteristics of the system with different speeds are analysed by numerical method, such as Poincaré map, the frequency spectrum, and the largest Lyapunov exponents. The results show that with the increase of the rotational speed, there are abundant dynamic phenomena, such as single period, quasi-period and quasi-period of the upper and lower branches. Moreover, due to influence of the pitching motion of ship, the rotor tilts in the bearings and its procession occurs obviously, which affects the stability of the operation of rotor-bearing system. The findings of this study will contribute a comprehension of the nonlinear dynamic behaviours of the rotor-bearing system subjected to ship pitching motion.

1. INTRODUCTION

When a ship is sailing in a harsh marine environment, it performs some periodic oscillating motions near the equilibrium position due to the influence of stormy waves, including heaving, pitching, rolling, surging, swaying, and yawing\cite{1-3}. These oscillations affect the stability of ship power plant. In order to ensure the stable and safe navigation of ship in stormy waves, it is necessary to predict the influences of ship oscillations on the dynamics of rotor-bearing system in the design stage.

Many researchers made great efforts to study the dynamic response of the rotor system in consideration of the transport motion under maneuvering conditions. Yang et al. \cite{4} analysed the effects of aircraft hovering flight on the nonlinear dynamic response of a cracked rotor. Then, Ni et al. \cite{5} investigated the dynamic characteristics of the transverse bending of the helicopter tail axis during the condition of maneuver flight by Hamilton principle. Hou et al. \cite{6} established a dynamic model of cracked rotor under climbing-diving flight condition and discussed the dynamic characteristics of the system. Furthermore, the studies of dynamics of the rotor-bearing system considering basic motion are also drawing more attention. El-Saeidy et al. \cite{7} used Lagrange equation to develop the dynamic equation of the rotor system subjected to the fundamental excitation, and the vibration characteristics of the system were discussed by means of analytical solution and numerical solution respectively. Chen et al. \cite{8} derived a dynamic model of the flexible rotor system during time-varying basic excitation, and discussed the influence of the basic excitation on the dynamic response of the system by experimental
and numerical methods. Recently, Liu et al. [9] developed a modified oil film force model by introducing the basic motion speed, and studied the dynamic behaviours of the system by numerical method.

However, it is worth mentioning that those above works mainly focused on the dynamic characteristics of the rotor system under the conditions of maneuvering flight or basic motion, while few studies focused on the vibration law of the rotor system during ship motion. To investigate the nonlinear dynamic behaviours of the marine rotor-bearing system under pitching motion, the dynamic model of marine rotor-bearing system is established in this paper, and the nonlinear dynamic characteristics of the system are discussed by the numerical solution.

2. Nonlinear Dynamic Model

Figure 1 illustrates the ship pitching motion and the marine rotor-bearing system, in which $R_0(x_0, y_0, z_0)$ is the inertial reference frame, $R(x, y, z)$ is the non-inertial reference frame fixed to the ship, and $R_1(x_1, y_1, z_1)$ is the local one established in the disc. In this paper, the pitching motion of ship rotating around the $y_0$ axis in the $R_0(x_0, y_0, z_0)$ is simplified to $\theta = \theta_0 \sin(\Omega_0 t)$, where $\theta_0$ and $\Omega_0$ are the amplitude and angular frequency of the pitching motion, respectively.

![Schematic diagram of rotor-bearing system under pitching motion](image)

**Figure 1.** The schematic diagram of rotor-bearing system under pitching motion. (a) Ship pitching motion. (b) Rotor-bearing system.

2.1. Motion Equations

According to the lagrange equation, the motion equations of rotor-bearing system under ship pitching motion are obtained as follows

$$
\begin{align*}
mx & = F_{x1} + F_{x2} + mg \cos \theta + m\dot{\theta}^2 \cos \theta \cos \Omega t \\
my & = F_{y1} + F_{y2} + m\dot{\theta}^2 \cos \theta + m\Omega^2 \cos \Omega t, \\
ml & = F_{l1} + F_{l2} + me \Omega^2 \sin \Omega t, \\
J_d \dot{\theta}_d & = (F_{l1} - F_{l2})l - J_d \dot{\theta}_d - J_p \Omega \dot{\theta}, \\
J_p \dot{\theta}_p & = (F_{l1} - F_{l2})l + J_d \Omega \dot{\theta}_d - J_p \dot{\theta}.
\end{align*}
$$

(1)

where $m$ is the mass of the rotor, $e$ is the eccentricity, $l$ is the half length of the rotor, $J_d$ and $J_p$ are the diameter moment of inertia and pole moment of inertia of the rotor, respectively, $\Omega$ is the rotor speed, and $x$, $y$, $\theta$, and $\dot{\theta}$ denote the displacement and the angle of the rotor in the $R(x, y, z)$, respectively. $F_{x1}$, $F_{x2}$, $F_{y1}$ and $F_{y2}$ are the nonlinear oil film forces at both ends of rotor derived from the theory of short bearings[10], the nonlinear oil film forces can be expressed as...
2.2. Nondimensional Motion Equations
In order to make the research have universality and wide applicability, regard oil film gap c, the weight of rotor and time as the basis, and the dimensionless transformations of equation (1) is performed. The relevant dimensionless parameters are shown in Table 1. In which, $B$, $R$, and $\sigma$ are the bearing length, bearing radius and Sommerfeld number, respectively.

### TABLE 1. THE DIMENSIONLESS EXPRESSION OF PARAMETERS

| Parameter description | Expression               |
|-----------------------|--------------------------|
| Nondimensional eccentricity $\alpha$ | $\alpha$ |
| Nondimensional speed $\omega$ | $\omega$ |
| The ratio of length to diameter $\lambda$ | $\frac{B}{(2R)}$ |
| Nondimensional displacement of rotor $X$ | $\frac{X}{c}$ |
| Nondimensional displacement of rotor $Y$ | $\frac{Y}{c}$ |
| Nondimensional half of the rotor length $\xi$ | $\frac{\xi}{c}$ |
| Nondimensional diameter radius of gyration $\gamma$ | $\sqrt{J_x/mc^2}$ |
| Nondimensional pole radius of gyration $\eta$ | $\sqrt{J_y/mc^2}$ |
| Nondimensional frequency ratio $\nu$ | $\frac{\Omega_0}{\Omega}$ |
| Nondimensional time $\tau$ | $\frac{\tau}{\Omega}$ |
| Nondimensional Sommerfeld number $\sigma$ | $\frac{\sigma}{mc^2}$ |
| Nondimensional nonlinear oil film force $f_{xi}$ | $\frac{F_{xi}}{mg}$, $i=1,2$ |
| Nondimensional nonlinear oil film force $f_{yi}$ | $\frac{F_{yi}}{mg}$, $i=1,2$ |

Finally, the nondimensional formulation of motion can be obtained

$$\begin{align*}
X^* &= \cos(\theta_0\nu \sin \nu \tau)[1 + \alpha \Omega_0^2 \nu^2 \cos^2 \nu \tau + X \theta_0^2 \nu^2 \cos^2 \nu \tau] \\
& \quad + \frac{f_{x1}}{\omega^2} + \frac{f_{x2}}{\omega^2} + \frac{\xi}{2} \theta_0^2 \nu^2 \sin \nu \tau + \alpha \cos \tau, \\
Y^* &= \frac{f_{y1}}{\omega^2} + \frac{f_{y2}}{\omega^2} + \alpha \sin \tau, \\
\theta_x' &= \frac{(f_{x2} - f_{x1}) \xi}{2\gamma^2 \omega^2} + \frac{\eta^2}{\gamma^2} \theta_x' - \frac{\eta^2}{\gamma^2} \theta_0 \nu \cos \nu \tau, \\
\theta_y' &= \frac{(f_{y1} - f_{y2}) \xi}{2\gamma^2 \omega^2} + \frac{\eta^2}{\gamma^2} \theta_y' + \theta_0 \nu \sin \nu \tau.
\end{align*}$$

3. Numerical Results and Discussions
Equation (3) is four second-order nonlinear differential equations, it is difficult to obtain the analytical solution of the rotor system. Thus, the numerical solution of the Runge-Kutta method is used to get dynamic characteristics of the marine rotor-bearing system subjected to ship pitching motion, and the parameters of system are $\sigma=3$, $\alpha=0.09$, $\gamma=400$, $\eta=531.5$, $\lambda=0.2$, $\xi=900$, $\theta_0=\pi/36$, $\omega=[1.0,3.35]$, and $\nu=[0.0016,0.0053]$. 

Finally, the nondimensional formulation of motion can be obtained

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| Nondimensional half of the rotor length $\xi$ | $l/c$ |
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| Nondimensional frequency ratio $\nu$ | $\Omega_0/\Omega$ |
| Nondimensional time $\tau$ | $\Omega t$ |
| Nondimensional Sommerfeld number $\sigma$ | $\sigma/(mc^2\sqrt{gc})$ |
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& \quad + \frac{f_{x1}}{\omega^2} + \frac{f_{x2}}{\omega^2} + \frac{\xi}{2} \theta_0^2 \nu^2 \sin \nu \tau + \alpha \cos \tau, \\
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\end{align*}$$
3.1. Displacement bifurcation diagram and largest Lyapunov exponents

Figure 2 shows bifurcation diagram and largest Lyapunov exponents of the rotor-bearing system under pitching motion, in which the dimensionless speed $\omega$ varied from 1.0 to 3.35, so that the dynamic behaviours of the rotor system can be observed globally. Initially, the response of rotor system exhibits single periodic motion. However, when $\omega$ changes from 1.21 to 2.205, the system passes through the quasi-periodic bifurcation into the quasi-periodic state. Thereafter, as the speed continues to increase, the rotor system transitions to the quasi-periodic motion state of the upper and lower branches. Until after $\omega=2.74$, the rotor system returns to the quasi-periodic motion of single-branch. As the $\omega$ exceeds 3.12, the rotor amplitude starts to increase continuously, and the largest Lyapunov exponents curve in this stage is still not greater than 0, indicating that the rotor is still in the quasi-periodic motion state.

![Bifurcation diagram](image)

**Figure 2.** Bifurcation diagram and largest Lyapunov exponents of the rotor bearing system under pitching motion.

3.2. The steady-state response

![Steady-state response](image)

**Figure 3.** The steady-state response of rotor system under pitching motion when $\omega=2.25$, $v=0.0023$. (a) Poincaré map. (b) Displacement response. (c) Frequency spectrum. (d) Axial end orbit in left. (e) Disc orbit. (f) Axial end orbit in right.
Figure 3 depicts the dynamic steady-state response of the rotor system when the rotor speed $\omega=2.25$ and $v=0.0023$. The Poincaré map appears as two narrow closed curves; in the frequency spectrum, there are the frequency $f_0$ caused by the pitching motion, the power frequency $f$ due to mass eccentricity of rotor and the multiple frequency $2f, 3f, etc.$, moreover, the combinational frequency $(f+f_0)/2$ with large peak and its odd multiples are also obvious; the largest Lyapunov exponent of the system is -0.17892. In summary, it can be concluded that the rotor system performs quasi-periodic motion at this stage. Furthermore, the disc orbit of the rotor and the orbit of the two axial ends have obvious differences, which mean that the rotor tilt in the bearings and the precession phenomenon appears.

Figure 4 shows the dynamic characteristics of the rotor system at $\omega=3.25$ and $v=0.00163$. The Poincaré map consists of some complicated closed curves; some new combined frequencies $(2(f+f_0)/5, 7(f+f_0)/5, etc.)$ appears in the frequency spectrum; the disc orbit is limited in an ellipsoid region and is distinct from the two axial end orbits, this indicates that the rotor is precessing. The largest Lyapunov exponent corresponding to the rotor system is -0.0273. All of these features indicate that the rotor system is in a state of quasi-periodic motion.

![Figure 3](image.png)

![Figure 4](image.png)

**Figure 3.** The dynamic steady-state response of rotor system when $\omega=2.25$ and $v=0.0023$. The Poincaré map appears as two narrow closed curves; in the frequency spectrum, there are the frequency $f_0$ caused by the pitching motion, the power frequency $f$ due to mass eccentricity of rotor and the multiple frequency $2f, 3f, etc.$, moreover, the combinational frequency $(f+f_0)/2$ with large peak and its odd multiples are also obvious; the largest Lyapunov exponent of the system is -0.17892. In summary, it can be concluded that the rotor system performs quasi-periodic motion at this stage. Furthermore, the disc orbit of the rotor and the orbit of the two axial ends have obvious differences, which mean that the rotor tilt in the bearings and the precession phenomenon appears.

**Figure 4.** The steady-state response of rotor system under pitching motion when $\omega=3.25, v=0.00163$. (a) Poincaré map. (b) Displacement response. (c) Frequency spectrum. (d) Axial end orbit in left. (e) Disc orbit. (f) Axial end orbit in right.

**4. CONCLUSIONS**

According to the lagrange equation, the dynamic model of the rotor system during ship pitching motion has been developed, and the dynamic behaviours of the system have been discussed by numerical method. The results show that with the increase of the rotor speed, the system appears rich dynamic phenomena, such as single period, quasi-period and the quasi-period of the upper and lower branches. Affected by the pitching motion, the rotor tilts in the bearings and produces obvious precession, which affects the stability of the system operation. The findings of this study will contribute an comprehension of the nonlinear dynamic characteristics of the marine rotor-bearing system considering the pitching motion.
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