Sensitivity analysis in stability evaluation of earthen embankments

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ABSTRACT

This study has investigated sensitivities of input parameters in stability analysis of embankments to safety factor \( F_s \) based on experimental design. In addition to the geotechnical parameters, parameters about geometry such as height, width, and slope gradient of embankments were also considered in the analysis. This study applied regression analysis based on orthogonal design to the sensitivity analysis. The geotechnical parameters required for the stability analysis were used in the sensitivity analysis, and the sensitivity of these parameters to \( F_s \) was investigated. The results of the sensitivity analysis showed that the parameters about geometry of embankments such as height and slope gradient are more sensitive to \( F_s \), although the parameters about hydraulic characteristics of embankment material are less sensitive to the \( F_s \).

Keywords: sensitivity analysis, slope stability, experimental design

1 INTRODUCTION

When stability of embankments is evaluated, factor of safety (\( F_s \)) has been commonly used in practice, and the methods calculating \( F_s \) has been proposed by many researchers so far (Ugai 1989; Ugai et al. 1998; Wakai et al. 2000). Particularly, Ugai (1989) proposed a sophisticated method for evaluating slope stability by coupling elasto-plastic shear strength reduction finite element method, called SSRFEM, with finite element saturated-unsaturated seepage analysis and verified its effectiveness through some application examples.

Although it is widely known that the usefulness of finite element based stability analysis, the methods usually need many geotechnical parameters. If the van Genuchten model (VG model, van Genuchten 1980) is used for representing hydraulic characteristics of an embankment material, at least nine parameters are necessary to conduct the stability analysis. In fact, most of the parameters about hydraulic characteristics are usually not obtained in geotechnical practices for economical reasons.

In geotechnical practices, more influential parameters on stability of embankment should be obtained with high accuracy. On the other hand, less influential parameters on the stability do not need high accuracy and are allowed to determine empirically. Therefore, identifying the parameters which are high sensitive to stability (\( F_s \)), can lead to an efficient and economic design of embankments.

The objective of this study is to identify more influential parameters on stability analysis of embankments based on the concept of sensitivity analysis (Saltteli et al. 1993). Although methods of sensitivity analysis include direct methods (Dickson and Gelinus 1976), Fourier methods (Cukier et al. 1978), and Monte Carlo methods (Sobol’ 2001), this study employs a regression analysis based on experimental design which has been developed in quality engineering. This is because the methods based on experimental design are more efficient and easier to conduct than other methods. In addition to geotechnical parameters, this study focuses on parameters about geometry of embankments such as height, width, and slope gradient, and those sensitivity to embankment stability \( F_s \) are analyzed.

The structure of this paper is as follows. Section 2 reviews some methods of sensitivity analysis which have been proposed so far, and an overview of the regression method used in this study is provided in this section. Section 3 shows an application example of the method described in Section 2, and its usefulness is discussed. The conclusion and summary are given in Section 4.

2 SENSITIVITY ANALYSIS

2.1 Review of techniques

Sensitivity analysis is a methodology which investigates the relationship between the simulation
models, usually implemented in a computer program, and its parameters. In other words, sensitivity analysis tries to quantitatively identify how variable the output is due to change of input parameters. The methodology has been particularly developed in computer experiments, and there are many applications.

In general, sensitivity analysis is classified into two categories, local sensitivity analysis and global sensitivity analysis. The local sensitivity analysis only addresses sensitivity relative to the point estimates, and they include direct methods and variational methods. On the other hand, global sensitivity analysis is the how the uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input and includes the Fourier amplitude sensitivity test (FAST) and the Sobol' method.

### 2.2 Regression analysis based on experimental design

Regression analysis is one of the methodologies of the global sensitivity analysis and has been developed in quality engineering. Regression analysis is to fit regression models to the output and to assess the "sensitivity" of input parameters by their regression coefficients. These methodologies are most effective when the design is orthogonal.

In this method, standardization of input parameters are introduced. A standardized value of \( x'_i \) is given by the following equation:

\[
  x'_i = \frac{x_i - \overline{x}_i}{\sigma_i}
\]

where, \( x_i \) denotes input parameters \( x_1, x_2, \ldots, x_n \), \( \overline{x}_i \) is the mean value of \( x_i \), and \( \sigma_i \) is the standard deviation of \( x_i \). Using standardized values places all parameter on a common scale. Having standardized all parameter, fit the first-order regression model to the standardized data. The regression coefficients are called the standardized regression coefficients (SRCs). Because all parameters have been placed on a common scale, and the relative magnitudes of the SRCs indicate the relative sensitivity of the output to each input. The validity of this implementation depends on the overall fit of the regression model, either as indicated by measures such as the coefficient of determination \( R^2 \), or by predicted sums of squares. If the overall fit is poor, the regression model does not adequately describe the relation between the output and input, and the SRCs do not reflect the effect of the inputs on the output.

As mentioned before, regression analysis is most effective when the design is orthogonal. Therefore, this study uses orthogonal designs (array), which is one of the fractional factorial designs, to meet the orthogonality of the computer experiments about slope stability.

### 3 APPLICATION EXAMPLE

#### 3.1 Slope model

This application focuses on stability analysis of existing embankments of irrigation tank, and the sensitivity of the input parameters to its \( F_s \), is quantitatively investigated. A conceptual illustration of an embankment of irrigation tank to be analyzed is shown in Figure 1.

![Conceptual illustration of an embankment of irrigation tank](image)

#### 3.2 Slope stability analysis

In this study, stability analysis of embankments was performed thorough the finite element based software called GUSLOPE (Geotechnical Engineering Laboratory, Gunma University 2006). GUSLOPE can consistently conduct saturated-unsaturated seepage analysis and slope stability analysis. The application examples of the method to geotechnical problems were shown in some literatures (Ugai 1989; Ugai et al. 1998; Wakai et al. 2000).

#### 3.3 Input parameters

Input parameters shown in Table 1 have been used in the sensitivity analysis. The parameters which evidently have high sensitivity to \( F_s \) such as cohesion \( c \) and internal friction angle \( \phi \) were not focused on in the analysis. As mentioned before, parameters about geometry of embankments were also used in the analysis in addition to geotechnical parameters.

In order to standardize the parameters shown in Table 1, we have collected numerous data from some literatures (Takeshita and Kohno 1993; Lee et al. 2009) and estimated their mean values and standard deviations. The obtained statistical values of the parameters and their standardized parameters are summarized in Table 2. Other parameters which are not shown in Table 1 were set as fixed parameters, and they are summarized in Table 3. In the table, \( E \) and \( v \) mean elastic modulus and Poisson's ratio respectively and are also set as fixed parameters. This is because Ugai (1989) reported that \( E \) and \( v \) are much less sensitivity to factor of safety.
Table 1. Input parameters used in sensitivity analysis.

| Parameter                  | Unit weight of the embankment material $\gamma_t$ (kN/m$^3$) | Saturated hydraulic conductivity $K_s$ (m/d) | Empirical parameter-1 $\alpha$ | Empirical parameter-2 $n$ | Residual volumetric moisture content $\theta_r$ | Saturated volumetric moisture content $\theta_s$ | Height of embankment (downstream) $H$ (m) | Difference in height of the foundation $dH$ (m) | Slope gradient (upstream) $\alpha_1$ | Slope gradient (downstream) $\alpha_2$ | Width of the crest $B$ |
|----------------------------|---------------------------------------------------------------|---------------------------------------------|-------------------------------|--------------------------|-----------------------------------------------|-----------------------------------------------|---------------------------------------------|-------------------------------------------|------------------------------------------|------------------------------------------|
| x1                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x2                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x3                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x4                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x5                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x6                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x7                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x8                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x9                         |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x10                        |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |
| x11                        |                                                               |                                              |                               |                          |                                               |                                               |                                             |                                           |                                         |                                          |

Table 2. Mean values, standard deviations, and standardized parameters.

| Parameter | $\mu$ (Level 0) | $\sigma$ | Level | 1 | -1 |
|-----------|-----------------|----------|-------|---|----|
| x1 $\gamma_t$ | 17.5          | 1.4      | 18.9  | 16.1 |    |
| x2 $\log K_s$ | -3.604        | 1.195   | -2.409 | -4.799 |   |
| x3 $\log a$ | -1.710         | 0.541    | -1.169 | -2.251 |   |
| x4 $n$ | 2.009           | 1.257    | 3.266  | 0.752  |   |
| x5 $\theta_r$ | 0.104         | 0.070    | 0.174  | 0.034  |   |
| x6 $\theta_s$ | 0.434         | 0.080    | 0.514  | 0.354  |   |
| x7 $H$ | 8.24            | 2.65     | 10.89  | 5.59   |   |
| x8 $dH$ | 1.33            | 1.19     | 2.52   | 0.14   |   |
| x9 $\alpha_1$ | 0.58          | 0.11     | 0.69   | 0.47   |   |
| x10 $\alpha_2$ | 0.58         | 0.09     | 0.67   | 0.49   |   |
| x11 $B$ | 3.46            | 0.59     | 4.05   | 2.87   |   |

Table 3. Fixed parameters in the sensitivity analysis.

| Parameter | $c$ (kN/m$^3$) | $\phi$ (degree) | $S_i$ (kN/m$^2$) | $E$ (kN/m$^2$) | $\nu$ |
|-----------|----------------|-----------------|------------------|---------------|------|
| x1        | 2.0            | 30.0            | 0.0              | 20,000        | 0.30 |

3.4 Orthogonal design

Table 4 shows orthogonal array used in the analysis. The orthogonal array is denoted as $L_a(b^c)$ where $L$ means Latin square, $a$ denotes the number of test trials, $b$ denotes the number of levels for each column, and $c$ denotes the number of columns in the array. The highlighted area in the table shows unused rows. The No.17 column is added to the original table in order to confirm if the results show convex or concave shape. The results of stability analysis ($F_s$) obtained from the orthogonal experiment is also shown in Table 4.

3.5 Sensitivity analysis

Sensitivity of the parameters to slope stability $F_s$ was studied by using a multiple linear regression model given by the following equation:

$$y = X\beta + \varepsilon$$  \hspace{1cm} (2)$$

where $y$ is the $n \times 1$ vector of the $F_s$, $X$ is the $n \times p$ matrix consisting of the levels of the independent parameters (shown in Table 1), $\beta$ is the $p \times 1$ vector of the regression coefficients (SRCs), and $\varepsilon$ is the vector of random errors. Note that the equation (2) can analyze only main effects of each input parameters. In other words, it cannot analyze interaction effects between input parameters.

The method of least squares was used to estimate the regression coefficients in the equation (2), and the least squares estimator of $\beta$, that is shown as $\hat{\beta}$, is given by:

$$\hat{\beta} = (X^TX)^{-1}X^Ty$$  \hspace{1cm} (3)$$

where the superscript "T" means "transpose".

The least squares fit $\hat{y}$ with the regression coefficients reported to three decimal place is:

$$\hat{y} = (0.067)x1 - (0.010)x2 + (0.005)x3 + (0.007)x4 - (0.005)x5 + (0.014)x6 - (0.115)x7 + (0.013)x8 - (0.006)x9 - (0.120)x10 - (0.007)x11 + 0.450.$$

(4)

Figure 2 shows the results of the orthogonal computer experiment and its regression model (Equation (4)). In this figure, plots (red color) show the results of $F_s$, and the surfaces (green color) shows the regression model, which is also called "Response Surface (RS)" in design experiment. Since a set of the $F_s$ show neither convex nor concave shape, the results obtained from the orthogonal design can be analyzed using multiple linear regression models. In addition, the determination coefficient $R^2$ of the above model is 0.997. Therefore, Equation (4) adequately describes the relation between the $F_s$ and input parameters, and the SRCs reflect the effect of the input parameters on the $F_s$. 

| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | $F_s$ |
|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|-----|
| 1   | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 0.297|
| 2   | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1  | -1  | 0.530|
| 3   | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 0.518|
| 4   | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1  | 1  | 0.707|
| 5   | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1  | 1  | 0.754|
| 6   | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1  | 0.498|
| 7   | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 0.530|
| 8   | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1  | 1  | 0.308|
| 9   | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 0.404|
| 10  | -1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 0.617|
| 11  | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1  | 0.119|
| 12  | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1  | 0.339|
| 13  | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 0.418|
| 14  | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1  | 0.145|
| 15  | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1  | 0.671|
| 16  | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1  | 0.362|
| 17  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0.425|

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Figure 3 compares each coefficient of input parameters. The vertical and the horizontal axis indicate the coefficients, which implies sensitivity in this case, and input parameters, respectively. The most sensitive parameter is the height of embankment $H$, and the second is slope gradient (downstream) $\alpha_2$. The input parameters about geometry are more sensitive than geotechnical parameters. The most sensitive parameters in geotechnical parameters is unit weight, and the other parameters cannot be sensitive to $F_s$.

4 CONCLUSIONS

This study has presented sensitivity analysis in stability analysis for earthen embankments of existing irrigation tanks to quantitatively investigate sensitivity of the input parameters to $F_s$ of the embankments. In addition to the geotechnical parameters, parameters about geometry of embankments were also considered in the analysis. The results of the sensitivity analysis showed that although the parameters about hydraulic characteristics of embankment materials are less sensitive to the $F_s$, the parameters about geometry of embankments such as height and slope gradient are more sensitive to $F_s$. Conclusions summarize the most important propositions concisely derived from the study.

REFERENCES

1) Cai, F., Ugai, K., Wakai, A. and Li, Q. (1998): Effects of horizontal drains on slope stability under rainfall by three dimensional finite element analysis, Computers and Geotechnics, 23, 255-275.
2) Cukier, R. I., Levine, H. B. and Shuler, K. E. (1978): Nonlinear sensitivity analysis of multiparameter model systems, Journal of Computational Physics, 26, 1-42.
3) Dickson, R. P. and Gelinas, R. J. (1976): Sensitivity analysis of ordinary differential equation systems - A direct method, Journal of Computational Physics, 21, 123-143.
4) Geotechnical Engineering laboratory, Gunma Univ.: (2006).
5) Lee, K., Koyama, T., Ohnish i, Y., Furukawa, H. and Kobayashi, T. (2009): Coupled stress-flow simulations for the river levee considering overflow, Japanese Geotechnical Journal, 4(1), 1-9 (in Japanese).
6) Saltelli, A., Andres, T. H. and Homma, T. (1993): Sensitivity analysis of model, Computational Statistics & Data Analysis, 15, 211-238.
7) Sobol', I. M. (2001): Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates, Mathematics and Computers in Simulation, 55, 271-280.
8) Takeshita, Y. and Kohno, I. (1993): A method to predict hydraulic properties for unsaturated soils and its application to observed data, Jiban-to-Kensetsu, 11(1), 95-104 (in Japanese).
9) Ugai, K. (1989): A method of calculation of global safety factor of slopes by elasto-plastic FEM, Soils and Foundations 29(2), 190-195 (in Japanese).
10) Ugai, K., Cai, F., Sakajo, S. and Wakai, A. (1998): Evaluation of global safety factor of slopes during rainfall, Journal of Japan Landslide Society, 35(1), 19-23 (in Japanese).
11) van Genuchten, M. T. (1980): A closed-form equation for predicting the hydraulic properties of unsaturated soils, Soil Science Society of America Journal, 44(5), 892-898.
12) Wakai, A., Cai, F. and Ugai, K. (2000): Evaluation of groundwater level and slope stability based on numerical analyses of saturated-unsaturated seepage insoil, Journal of Japan Landslide Society, 36(4), 8-13 (in Japanese).