Single Isospin Decay Amplitude and CP Violation

N.G. Deshpande, Xiao-Gang He, and Sandip Pakvasa

1 Institute of Theoretical Science, University of Oregon
Eugene, OR 97403-5203, USA

2 School of Physics, University of Melbourne
Parkville, Vic. 3052, Australia

and

3 Department of Physics and Astronomy, University of Hawaii
Honolulu, HI 90822

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Abstract

We study partial rate asymmetry in decays with single isospin final state. For K meson or hyperon decays, the partial rate asymmetries are always zero if the final states are single isospin states. In B decays the situation is dramatically different and partial rate asymmetries can be non-zero even if the final states are single isospin states. We calculated partial rate asymmetries for several B decays with single isospin amplitude in the final states using factorization approximation. We find that in some cases the asymmetries can be large.
CP violation is one of the few remaining unresolved mysteries in particle physics. The explanation in the Standard Model (SM) based on Cabibbo-Kobayashi-Maskawa (CKM) matrix \([1]\) is still not established, although there is no conflict between the observation of CP violation in the neutral K-system and theory \([2]\). It is important to carry out more experiments to test CP violation in the SM. The study of CP violation in the B system is very important which may provide crucial information about CP violation \([3]\). The B system offers several final states that provide a rich source for the study of this phenomena. In many cases CP violation in B decays occurs in a quite different form from K or hyperon decays. In this paper we study CP violating partial rate asymmetries in B decays. We clarify some subtleties for CP violation in partial rate asymmetry in relation to isospin analysis.

The decay amplitudes for Kaon or hyperon are customarily parametrized according to isospin decomposition in the final states because isospin states are eigenstates of strong interaction. For example the amplitude for \(K^0 \to \pi^+\pi^-\) decay can be parametrized as

\[
A = A_0 e^{i\delta_0^w + \delta_0^s} + A_2 e^{i\delta_2^w + \delta_2^s},
\]

where superscript 0 and 2 indicate the isospin of the final states. \(\delta_i^w\) and \(\delta_i^s\) are the CP violating weak and CP conserving strong phases, respectively.

The partial rate asymmetry \(A_{asy}\) is given by

\[
A_{asy} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = -\frac{2A_0A_2\sin(\delta_0^w - \delta_2^w)\sin(\delta_0^s - \delta_2^s)}{A_0^2 + A_2^2 + 2A_0A_2\sin(\delta_0^w - \delta_2^w)\sin(\delta_0^s - \delta_2^s)}. \tag{2}
\]

The conditions for non-zero partial rate asymmetry in this case are: there must exist at least two different isospin decay amplitudes with different weak, and strong rescattering phases. It is clear that for final states with single isospin, for example \(K^- \to \pi^-\pi^0\), the partial rate asymmetry vanishes. The same equation could be used for \(B \to \pi\pi\) decays. But the argument is now incorrect. We wish to consider the difference in some detail.

We now consider the same process as above from the quark level. In the SM the effective Hamiltonian responsible for \(K \to \pi^+\pi^-\) at the quark level can be parametrized as \([4]\),

\[
H_{eff} = \sum_i [V_{us}V_{ud}^*c_i + V_{cs}V_{cd}^*c_i + V_{ts}V_{td}^*c_i] O_i , \tag{3}
\]
where $c_i^f$ are Wilson Coefficients (WC) of the corresponding quark operators $O_i$. At the one loop level $c_i^u$ contain the tree and $u$ internal quark contributions, $c_i^{c,t}$ contain internal $c$- and $t$- quark contributions. Since the $u$, $\bar{u}$ pair is lighter than $s$-quark, at the one loop level with $u$ quark in the loop, absorptive amplitudes will be generated, and $c_i^u$ has a strong rescattering phase \[\text{[5]}\]. On the other hand no absorptive parts exist for $c_i^{c,t}$ \[\text{[5]}\]. Naively, one would obtain partial rate asymmetry for $K^0 \to \pi^+\pi^-$ which seems to have nothing to do with strong phase shifts in different isospin amplitudes. However, this argument turns out to be wrong. It has been pointed out by Gerard and Hou \[\text{[6]}\] that CPT theorem is violated if one is not careful to include all diagrams of the same order \[\text{[7]}\]. The interference term responsible for rate asymmetry due to $c_i^u$ is an interference between penguin amplitudes of order $\alpha_s^2$. To this order there is also another contribution which is the interference between the tree amplitude and the higher order penguin amplitudes with absorptive part developed in the vacuum polarization of the virtual gluon. This contribution cancels the previous contribution. In practical calculation, $c_i^u$ must be treated as real in this case. The rule is that for the phase of any one of the penguin WC, if there is a tree amplitude with the same CKM factor, the phase in that penguin WC must be removed when the final states are the same as the tree amplitude. In a more general formulation of the problem from CPT theorem and unitarity considerations, Wolfenstein showed that any diagonal strong phase (the phase due to rescattering of the states which are the same as the final states) do not contribute to partial rate asymmetry \[\text{[8]}\]. The phases in $c_i^u$ are diagonal phases in the present case \[\text{[8]}\].

At the hadron level, the decay amplitude is given by

$$A(K^0 \to \pi^+\pi^-) = \sum_i <\pi^+\pi^-| [V_{ud}^*V_{us}c_i^u + V_{cd}^*V_{cs}c_i^c + V_{td}^*V_{ts}t_i^c]O_i|K>. \quad (4)$$

The strong rescattering phases are generated by rescattering the two pions in the final states. This is because in this case, only two pion final states are opened in this kinematic region with the right parity. Three pions in the intermediate states are allowed kinematically, but three pions would not rescatter into two pions because of G parity conservation. The part
responsible for absorptive amplitude is given by

\[ A(K^0 \to \pi^+\pi^-) = \sum_i \sum_I \langle \pi^+\pi^- | [V_{ud} V_{us}^* c_i^u + V_{cd}^* V_{cs} c_i^c + V_{td}^* V_{ts} c_i^t] | (\pi\pi)_I \rangle \times < (\pi\pi)_I | O_i | K > , \]

(5)

where I is summed over isospin eigenstates.

Since isospin symmetry is respected by strong interaction, we can generally parametrize the hadronic matrix elements as, after the rescattering strong phase shifts are included,

\[ \sum_I < \pi^+\pi^- | [V_{ud} V_{us}^* c_i^u + V_{cd}^* V_{cs} c_i^c + V_{td}^* V_{ts} c_i^t] | (\pi\pi)_I \rangle < (\pi\pi)_I | O_i | K > = [V_{ud} V_{us}^* c_i^u + V_{cd}^* V_{cs} c_i^c + V_{td}^* V_{ts} c_i^t] [x_i^0 a_0 e^{-i\delta_s^0} + x_i^2 a_2 e^{-i\delta_s^2}] , \]

(6)

where \( a_{0,2} \) and \( \delta_s^{0,2} \) are the isospin eigen-amplitudes and strong rescattering phases from all contributions, and \( x_i^{0,2} \) are the Clebsch-Gordan coefficients for each operator. Knowing that the absorptive parts in \( c_i^u,c^c \) are canceled by the other effects, we should take all \( c_i^{u,c} \) in the above equation to be real. We have for the isospin amplitudes \( A_{0,2} \)

\[ A_0 = \sum_i [V_{ud} V_{us}^* c_i^u + V_{cd}^* V_{cs} c_i^c + V_{td}^* V_{ts} c_i^t] x_i^0 a_0 e^{-i\delta_s^0} , \]

\[ A_2 = \sum_i [V_{ud} V_{us}^* c_i^u + V_{cd}^* V_{cs} c_i^c + V_{td}^* V_{ts} c_i^t] x_i^2 a_2 e^{-i\delta_s^2} . \]

(7)

Since in general \( x_i^0 \) is not equal or proportional to \( x_i^2 \), \( A_0 \) and \( A_2 \) do not have the same weak phases

\[ \delta_0^u = Arg(\sum_i [V_{ud} V_{us}^* c_i^u + V_{cd}^* V_{cs} c_i^c + V_{td}^* V_{ts} c_i^t] x_i^0) , \]

\[ \delta_2^u = Arg(\sum_i [V_{ud} V_{us}^* c_i^u + V_{cd}^* V_{cs} c_i^c + V_{td}^* V_{ts} c_i^t] x_i^2) . \]

(8)

Thus the decay amplitude for \( K^0 \to \pi^+\pi^- \) can be parametrized in the form in eq.(6). The discussion can be easily generalized to hyperon decays.

For \( K^- \to \pi^-\pi^0 \), the final state has only \( I = 2 \) amplitude, the decay amplitude is of the form,

\[ \tilde{A}_2 = \sum_i [V_{ud} V_{us}^* c_i^u + V_{cd}^* V_{cs} c_i^c + V_{td}^* V_{ts} c_i^t] x_i^2 \tilde{a}_2 e^{i\delta_s^2} . \]

(9)
It is clear that the particle-antiparticle rate asymmetry vanishes.

For $B \to \pi\pi$ the situation is, however, very different. We will use the same notation for the effective Hamiltonian. Of course we should keep in mind that now the operators $O_i$ contain a $b$ quark. Now because the $b$ quark is heavier than a $u$, $\bar{u}$ pair, and also a $c$, $\bar{c}$ pair, both $c_i^u$, $c_i^c$ have strong rescattering phases at the one loop level. At the hadron level, the part responsible for absorptive amplitude is given by

$$A(B^0 \to \pi^+\pi^-) = \sum_i \sum_l <\pi^+\pi^-| [V_{ud}^* V_{ub} c_i^u + V_{cd}^* V_{cb} c_i^c + V_{td}^* V_{tb} c_i^t] |I> \langle I|O_l|B^0 >$$

$$+ \sum_i \sum_l <\pi^+\pi^-| [V_{cd}^* V_{cb} c_i^c] |I_c> \langle I_c|O_l|B^0 > ,$$

(10)

where $I$ is summed over non-charmed on-shell intermediate states like, $\pi^+\pi^-$, $\pi^0\pi^0$, ..., and $I_c$ is summed over charmed on-shell particle intermediate states like, $D\bar{D}$ etc. In this case we can remove the phases in $c_i^u$ because in this decay there is a tree amplitude with the same CKM factor. However, now there is another class of phase shift due to $<\pi^+\pi^-|c_i^c|I_c>$ which can not be removed. The phase shift of this type to the lowest order is due to the absorptive part in $c_i^c$. Because these new phases are generated by charmed particles in the intermediate state, these phases will only appear in the term proportional to $V_{cb}V_{cd}^*$. We obtain the $I=0,2$ decay amplitudes $A'_{0,2}$ for $B \to \pi\pi$

$$A'_0 = \sum_i [V_{ud}^* V_{ub} c_i^u + V_{cd}^* V_{cb} c_i^c + V_{td}^* V_{tb} c_i^t] x_i^0 a_0 e^{i\delta_i^0}$$

$$A'_2 = \sum_i [V_{ud}^* V_{ub} c_i^u + V_{cd}^* V_{cb} c_i^c + V_{td}^* V_{tb} c_i^t] x_i^2 a_2 e^{i\delta_i^2} .$$

(11)

Now we should treat $c_i^{u,t}$ to be real and only $c_i^c$ to be complex (non-zero rescattering phases) up to order $\alpha_s^2$ in the asymmetry.

Let us now compare these amplitudes with the amplitudes for $K \to \pi\pi$. First we note that because in the case for $B \to \pi\pi$ more on-shell intermediate states are allowed, i.e. $\pi\pi$, $\pi\pi\pi\pi$ etc, the rescattering phases $\delta_i^{0,2}$ include inelastic channels unlike the elastic phase shifts $\delta_i^{0,2}$. Second we note that for $B \to \pi\pi$ there are additional strong rescattering phases due to on-shell charmed intermediate states. The strong rescattering phases in $c_i^c$ are not
canceled by any other contributions. This has a very important consequence that particle-antiparticle rate asymmetry can occur in a single isospin amplitude. In fact this happens quite often in $B$ decays. In the following we study four different processes representing different types of $B$ decays, $b \rightarrow \psi s$, $b \rightarrow \phi s$, $B^{-} \rightarrow \eta \pi^{-}$, and $B^{-} \rightarrow \eta K^{-}$.

When considering partial rate asymmetry for single isospin final state in $B$ decays, we do not need to know the values for the overall strong rescattering phases. We, however, need to know the relative strong rescattering phase shifts between amplitudes with different CKM factors by calculating various on-shell rescattering processes. This calculation is very difficult to carry out. However, we believe that the WC’s and the phases calculated at the quark level could be good indications of the sizes and the signs of the strong rescattering phases by appealing to duality. The absorptive parts of hadronic processes are given quite accurately by considering the corresponding quark loops as in the calculation of $R$ in $e^+e^-$ scattering. In our later calculation, we will use this approximation.

In the SM the amplitudes for $B$ decays are generated by the following effective Hamiltonian:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [V_{tb}V_{tb}^* (c_{1f} + c_{2f}) - \sum_{i=3}^{10} (|V_{ub}|^2 c_{i} + |V_{cb}|^2 c_{i} + |V_{tb}|^2 c_{i}) O_{i}] + H.C., \quad (12)$$

where the superscripts $u$, $c$, $t$ indicate the internal quarks, $f$ can be $u$ or $c$ quark. $q$ can be $d$ or $s$ quark depending on if the decay is a $\Delta S = 0$ or $\Delta S = -1$ process. The operators $O_{i}$ are defined as

$$O_{1} = \bar{q}_\alpha \gamma_\mu L u_\beta \bar{u}_\gamma \gamma^\mu L b_\alpha, \quad O_{2} = \bar{q}_\alpha \gamma_\mu L u_\beta \bar{u}_\gamma L b_\alpha,$$

$$O_{3,5} = \bar{q}_\alpha \gamma_\mu L b_\beta \bar{q}_\gamma L (R) q_\alpha, \quad O_{4,6} = \bar{q}_\alpha \gamma_\mu L b_\beta \bar{q}_\gamma \gamma_\mu L (R) q_\alpha,$$

$$O_{7,9} = \frac{3}{2} \bar{q}_\alpha \gamma_\mu L b_\beta e_\gamma q' \gamma^\mu R(L) q_\alpha, \quad O_{8,10} = \frac{3}{2} \bar{q}_\alpha \gamma_\mu L b_\beta e_\gamma q'_\gamma \gamma_\mu R(L) q_\alpha,$$

where $R(L) = 1 + (-)\gamma_5$, and $q'$ is summed over $u$, $d$, and $s$. $O_{1}$ are the tree level and QCD corrected operators. $O_{3-6}$ are the strong gluon induced penguin operators, and operators $O_{7-10}$ are due to $\gamma$ and $Z$ exchange, and “box” diagrams at loop level. The WC’s $c_{i}^f$ are defined at the scale of $\mu \approx m_b$ which have been evaluated to the next-to-leading order in
QCD. We give the non-zero coefficients below for $m_t = 176$ GeV, $\alpha_s(m_Z) = 0.117$, and $\mu = m_b = 5$ GeV,

$$
c_1 = -0.307, \quad c_2 = 1.147, \quad c_3 = 0.017, \quad c_4 = -0.037, \quad c_5 = 0.010, \quad c_6 = -0.045, \quad c_7 = -1.24 \times 10^{-5}, \quad c_8 = 3.77 \times 10^{-4}, \quad c_9 = -0.010, \quad c_{10} = 2.06 \times 10^{-3},
$$

$$
c_{u,c}^{3,5} = -c_{u,c}^{4,6}/N = P_s^c/N, \quad c_{u,c}^{7,9} = P_e^c, \quad c_{u,c}^{8,10} = 0
$$

where $N$ is the number of color, $c_i^t$ are the regularization scheme independent WC’s obtained in Ref. [10]. The leading contributions to $P^{s,e}_i$ are given by:

$$
P^{s}_i = (\alpha_s/8\pi)c_2(10/9 + G(m_i, \mu, q^2)) \quad \text{and} \quad P^{e}_i = (\alpha_{em}/9\pi)(Nc_1 + c_2)(10/9 + G(m_i, \mu, q^2)).
$$

The function $G(m, \mu, q^2)$ is given by

$$
G(m, \mu, q^2) = 4 \int_0^1 x(1-x)d\ln \frac{m^2 - x(1-x)q^2}{\mu^2}.
$$

All the above coefficients are obtained up to one loop order in electroweak interactions. When $q^2 > 4m^2$, $G(m, \mu, q^2)$ becomes imaginary. In our calculation, we will use $m_u = 5$ MeV, $m_d = 10$ MeV, $m_s = 175$ MeV, $m_c = 1.35$ GeV, and the averaged value $m_b^2/2$ for $q^2$.

We must be careful in using absorptive parts of the above WC’s. We should always remove phase shift discussed previously, otherwise one would obtain results violating CPT theorem.

To obtain exclusive decay amplitudes, we need to calculate relevant hadronic matrix elements. Since no reliable calculational tool exists for two body modes, we shall use factorization approximation to get an idea of the size of asymmetry $A_{asy}$. The numerical numbers obtained should be viewed as an order of magnitude estimates. The important message is that CP violating partial rate asymmetry in some single isospin channel decays are indeed non-zero and can reach significant magnitude.

**Partial rate asymmetry in $b \to \psi s$**

Since $\psi$ carries no isospin, the final state is a single isospin state in this decay. At the hadronic level, this decay includes $B \to \psi K$, $B \to \psi K^*$, and etc. This decay is particularly interesting because it has a large branching ratio. CP violating partial rate asymmetry in
this type of decay was first studied in the early 80’s by Brown, Pakvasa and Tuan [11]. Let
us now analyze this asymmetry using factorization approximation. In this approximation,
the partial rate asymmetries for $b \to \psi s$ and $B^- \to \psi K^-$ are the same. We have

$$A(b \to \psi s) = \frac{G_F}{\sqrt{2}} \bar{s} \gamma_\mu (1 - \gamma_5) b \{ < \psi | \bar{u} \gamma_\mu u | 0 > V_{ub} V_{us}^* (c_1 + \frac{c_2}{N})$$

$$+ < \psi | \bar{c} \gamma_\mu c | 0 > [ V_{cb} V_{cs}^* (C_1 + \frac{C_2}{N}) - \sum_i V_{ib} V_{is}^* (c_i^1 + \frac{c_i^4}{N} + c_i^5 + \frac{c_i^6}{N} + c_i^7 + \frac{c_i^8}{N} + c_i^9 + \frac{c_i^{10}}{N})] \}.$$  \hspace{1cm} (16)

Here the first term corresponds to an annihilation contribution which is usually small. If
this term is neglected, we have

$$A(b \to \psi s) = \frac{G_F}{\sqrt{2}} \bar{s} \gamma_\mu (1 - \gamma_5) b < \psi | \bar{c} \gamma_\mu c | 0 > \times [ V_{ub} V_{us}^* (C - c_1^u - c_2^u) - V_{cb} V_{cs}^* (C + c_1 + \frac{c_2}{N} - c_7 - c_8) ] ;$$  \hspace{1cm} (17)

where $C = c_1^u + c_5 + c_7 + c_9 + (c_1 + c_6 + c_8 + c_{10})/N$. Any phase shift in $c_i^\mu$ should be removed
from our previous discussions. Only the absorptive part in $c_i^u$ generate effective strong
rescattering phases. The interference which cause the partial rate difference in this case is
of order $\alpha_{em}$ instead naively expected $\alpha_s$ because the strong penguin generated absorptive
amplitude cancel in $c_i^u + c_4 + c_{10}/N$. In our numerical calculations we will use $N = 2$ favored
by experimental data, and $|V_{us}| = 0.2205$, $|V_{ub}| = 0.04$, and $|V_{ub}/V_{ub}| = 0.08$. We find that
the partial rate asymmetry is less than $10^{-4}$. The result is shown in Figure 1. One can
easily obtain the asymmetry for $b \to \psi d$ by scaling the asymmetry by a factor of $|V_{ub}/V_{ub}|^2$.
The asymmetry in this case is much large but the branching ratio is much smaller. In Ref.
[12] using absorptive amplitudes generated by rescattering color octet state, asymmetry was
estimated for $b \to \psi d$. Using the same calculation for $b \to \psi s$, we find that the partial
rate asymmetry is about the same order of magnitude obtained here. This asymmetry is
an order of magnitude smaller than that obtained in Ref. [11,13]. Note that here we have
neglected the annihilation contribution estimated in Ref. [11,13]. However, it is found that
when current knowledge about CKM parameters is used, the caculation of Ref. [11,13] also
yields an asymmetry below $10^{-4}$. Hence including the annihilation diagram is not going to
change our result significantly.
Partial rate asymmetry in $b \to \phi s$

The final state is a single isospin state because $\phi$ is isospin singlet. This decay is induced by pure penguin interaction. CP violation in this process was first evaluated in Ref. [14]. This process is not affected by the previously mentioned effect. The partial rate asymmetry is generated by interference of different penguin amplitude which is of order $\alpha_s^2$. We have

$$A(b \to \phi s) = \frac{G_F}{2\sqrt{2}} \bar{s}\gamma_\mu(1-\gamma_5)b < \phi|\bar{s}\gamma_\mu s|0>$$

$$\times [V_{ub}V^\ast_{ud}(C' - 2(c_4^u + \frac{c_4^d}{N}) + c_4^s(1 + \frac{1}{N}) + c_5^u)] + V_{cb}V^\ast_{cs}(C' - 2(c_4^u + \frac{c_3^d}{N} + c_5^s(1 + \frac{1}{N}) + c_5^u)]\ ,$$

where $C' = (2(c_3^d + c_4^d + c_5^d) + 2(c_3^d + c_4^d + c_5^s)/N - (c_7^d + c_9^d + c_1^d + (c_4^s + c_5^s + c_1^s)/N)$. In this case we find the partial rate asymmetry is of order $O(10^{-3})$. The result is shown in Figure 2.

Partial rate asymmetry in $B^- \to \eta\pi^-$

This is an exclusive decay. Because $\eta$ is an $I = 0$ particle, the final state is a single isospin state with $I = 1$. In the factorization approximation, we obtain

$$A(B^- \to \eta\pi^-) = \frac{G_F}{\sqrt{2}} [V_{ub}V^\ast_{ud}(T^{\eta}_{B\pi}C^{ut} + T^{\pi}_{B\eta}D^{ut})] + V_{cb}V^\ast_{cs}(T^{\eta}_{B\pi}C^{ct} + T^{\pi}_{B\eta}D^{ct})\ ,$$

where

$$C^{ut} = c_1 + \frac{c_2}{N} - \frac{c_3^d}{3} - c_4^u - + \frac{3}{2}(c_7^u + \frac{c_8^u}{N}) + \frac{1}{2}(c_9^u + c_{10}^u)$$

$$+ X_d(-2\frac{c_3^u}{N} - 2c_6^u + c_7^u + \frac{c_8^u}{N} - c_9^u - \frac{c_{10}^u}{N}) + \{c_i^u \to -c_i\} \ ,$$

$$D^{ut} = \frac{c_1}{N} + c_2 - (\frac{c_3^u}{N} + c_4^u + \frac{c_9^u}{N} + c_{10}^u + 2X_\pi(c_5^u + c_6^u + \frac{c_7^u}{N} + c_8^u)) + \{c_i^u \to -c_i\} \ ,$$

where $X_d = m_{\eta}^2/(2md(m_b - m_d))$ and $X_\pi = m_\pi^2/(m_u + m_d)(m_b - m_d)$. $C^{ct}$ and $D^{ct}$ are obtained by setting $c_{1,2}$ to be zero and replacing the superscript $u$ by $c$. $T^{\eta}_{B\pi}$ and $T^{\pi}_{B\eta}$ are defined as,

$$T^{\eta}_{B\pi} = <\pi^-|\bar{d}\gamma_\mu(1-\gamma_5)b|B^- > <\eta|\bar{u}\gamma_\mu(1-\gamma_5)u|0>$$
where \( F_0^B(q^2) \) are the transition form factors between \( B \) meson and \( p \) meson (where \( p \) could be \( \pi \) or \( \eta \)) defined in Ref. [13,16], \( f_\pi = 93\text{MeV} \), and we will use \( f_\pi \approx f_\eta \). This time any strong phase in \( c_1^u \) must be removed. The interference causing partial rate difference is of order \( \alpha_s \). The result is shown in Figure 3. In the numerical calculation, we have included the \( \eta-\eta' \) mixing effect with the mixing angle \( \theta = -20^\circ \). The figures are plotted with \( f_{\eta_1} = f_{\eta_8} = f_\pi \). The results are not sensitive to the mixing effect. The asymmetry can be quite large. The branching ratio is 3 to 4 times larger (\( O(10^{-5}) \)) if the form factors in Ref. [15] are used than the one obtained using the form factors in Ref. [14].

### Partial rate asymmetry in \( B^- \rightarrow \eta K^- \)

The final state is a pure \( I = 1/2 \) state. We have

\[
A(B^- \rightarrow \eta K^-) = \frac{G_F}{\sqrt{2}} [V_{ub}V_{us}^*(T_{BK}^\eta \tilde{C}^{ut} + T_{B\eta}^\eta \tilde{D}^{ut})
+ V_{cb}V_{cs}^*(T_{BK}^\eta \tilde{C}^{ct} + T_{B\eta}^\eta \tilde{D}^{ct})],
\]

\[
\tilde{C}^{ut} = c_1 + \frac{c_2}{N} + \left(2\frac{c_3^u}{N} + 2c_4^u - c_9^u - c_1^c\right) + \frac{3}{2}(c_7^u + \frac{c_8^u}{N} - c_9^u - \frac{c_10^u}{N})
+ 2X_s\left(\frac{c_5^u}{N} + c_6^u - \frac{c_7^u}{N} - c_8^u\right) + \{c_i^u \rightarrow -c_i^t\}
\]

\[
\tilde{D}^{ut} = \frac{c_1}{N} + c_2 - \frac{c_3^u}{N} - c_4^u - 2X_s\left(\frac{c_5^u}{N} + c_6^u + \frac{c_7^u}{N} + c_8^u\right)
- \frac{c_9^u}{N} - c_10^u + \{c_i^u \rightarrow -c_i^t\},
\]

where \( X_K = m_K^2/(m_s + m_u)(m_b - m_u) \), \( X_s = m_s^2/(2m_s(m_b - m_s)) \), \( T_{BK}^\eta = i(f_\eta/\sqrt{3})F_0^{BK}(m_\eta^2)(m_B^2 - m_\eta^2) \), \( T_{B\eta}^\eta = i(f_\eta/\sqrt{3})F_0^{B\eta}(m_\eta^2)(m_B^2 - m_\eta^2) \). Similarly the coefficients \( \tilde{C}^{ct} \) and \( \tilde{D}^{ct} \) are obtained by setting \( c_{1,2} \) to be zero, and replacing the superscript \( u \) by \( c \) in \( C_{ut} \) and \( D_{ut} \). This time the interference term causing partial rate difference is, again, of order \( \alpha_s \). The result is shown in Figure 4. In this case the results are sensitive to the mixing effect. Within the allowed ranges for \( f_{\pi,\eta_1,\eta_8} \), the branching ratio can change by a factor of 5. The branching ratio can be as large as \( 4 \times 10^{-6} \) using the form factors in Ref.
and it is smaller by a factor of 3 to 4 using the form factors in Ref. [16]. The mixing effect on the asymmetry is less sensitive. Again, the asymmetry can be quite large. For the same values of form factors, we agree with the results obtained by Du and Guo in Ref. [9]. If the electroweak penguin effect is neglected, we also agree with Kramer, Palmer and Simma in Ref. [3] when the same form factors are used.

To conclude, we have shown that partial rate asymmetry in decays of particles containing a strange quark with single isospin final states, is always zero because the allowed intermediate on-shell states are limited. However, the situation in B decays is dramatically different. In the latter case, more intermediate on-shell states with different CKM factors are allowed, CP violating partial rate asymmetries need not to be zero even if the final state contains only a single isospin state.

We have carried out detailed analyses for four types of B decays. The partial rate asymmetry in $b \to \psi s$ is small because the interference causing rate deference in particle and anti-particle decay rates is of order $\alpha_{em}$ due to cancellation. The partial rate asymmetry in $b \to \phi s$ is also small ($O(10^{-3})$. The asymmetry in exclusive decays considered here are larger. The partial rate asymmetry in $B^- \to \eta\pi^-$ and $B^- \to \eta K^-$ can be quite large.

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FIGURES

FIG. 1. The partial rate asymmetry for $b \rightarrow \psi s$. The vertical axis is the asymmetry and the horizontal axis is the value in degree for the phase angle $\gamma$ in the Wolfenstein parametrization.

FIG. 2. The partial rate asymmetry for $b \rightarrow \phi s$. 
FIG. 3. The partial rate asymmetry for $B^- \rightarrow \eta\pi^-$. 
FIG. 4. The partial rate asymmetry for $B^- \rightarrow \eta K^-$. 