Non-Supersymmetric Attractors in BI black holes

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Abstract

We study attractor mechanism in extremal black holes of Einstein-Born-Infeld theories in four dimensions. We look for solutions which are regular near the horizon and show that they exist and enjoy the attractor behavior. The attractor point is determined by extremization of the effective potential at the horizon. This analysis includes the backreaction and supports the validity of non-supersymmetric attractors in the presence of higher derivative interactions in the gauge field part.
1 Introduction

The study of attractor mechanism [1]-[4] in extremal black holes of general theories of gravity and string theory has drawn a lot of attention recently [5]-[19]. This, in part, is due to the realization that the concept of attractor mechanism is rather general and goes beyond the original motivation, where supersymmetry was the key ingredient. This mechanism can be used to study the properties of extremal black holes in supersymmetric theories, which do not respect any supersymmetry, or those in non-supersymmetric theories. In all these cases, the statement of attractor mechanism is that, in a generic situation, the near horizon geometry and the black hole entropy turn out to be completely independent of the asymptotic behavior of scalar fields of the theory and depend only on certain conserved quantities, like mass, charge and angular momentum. In this context, the entropy function formalism [7] has proved to be a very useful tool for calculating the entropy of extremal black holes in a general theory of gravity, with any set of higher derivative terms and in higher dimensions. This formalism is based on the fact that, knowing the near horizon symmetries of the black hole is enough to generate the entropy through the use of Wald’s entropy formula, and attractor equations are essentially some linear combinations of the equations of motion of all the fields of the theory.

The concept of attractor mechanism is extremely useful in calculating the entropy of extremal black holes. It also turns out to be helpful in seeking to comprehend the structure of higher derivative terms in a general theory of gravity [20]-[26]. String theory at low energies gives rise to a variety of higher derivative terms. Understanding the structure of these higher derivative terms is of paramount importance, not just for the case of black holes, but also because they hold a lot of information about the unitarity and renormalizability properties of the theory in question. With the use of the tools provided by attractor mechanism and entropy function formalism, interesting aspects of Lovelock terms, Chern-Simons terms, Born-Infeld terms etc., can be studied [27]-[30]. Although, most of the issues have been studied keeping in mind that the black holes cannot have any hair, it is also important to study the structure of higher derivative terms in the case when the black hole has hair. One of the key reasons is that, in non-supersymmetric theories, the assumption of a flat potential for the scalar fields can be erroneous, in which case one is left with scalar hair. This is also important from the point of view of the need to introduce a small amount of non-extremality, in certain situations involving higher derivative terms [31].

Although considerable progress has been made in understanding the physics of attractor mechanism, there are a number of issues which need to be addressed, especially, in the absence of supersymmetry. For instance, in the absence of supersymmetry, though non-supersymmetric attractor mechanism can be used to calculate the entropy of black holes, the existence of a full black hole solution interpolating between the Bertotti-Robinson geometry and the asymptotically flat space is not
guaranteed. This issue becomes more difficult to deal with when there are higher
derivative terms in addition. Thus, it becomes important to formulate and study
non-supersymmetric attractor mechanism when there are different kinds of higher
derivative terms following from the low energy limit of string theory, in the Einstein
action. Another issue is the stability of solutions of non-supersymmetric attractor
equations. One of the methods to address the problem of stability of attractor points
is to follow [4,8] and check on a case by case basis.

In this note, we study attractor mechanism in a general Einstein-Born-Infeld
type of gravity coupled to moduli fields. Born-Infeld terms are known to arise in
the low energy limit of a configuration where gauge fields are coupled to open bosonic
or superstrings. In fact, the low energy theory on the world-volume of a D-brane
is governed by a Born-Infeld action. The importance of Born-Infeld terms in the
context of extremal black holes and their connection with elementary string states
was stressed in [51]. It was argued that virtual black holes going around closed loops
can give rise to Born-Infeld type corrections to extremal black hole configurations
with non-trivial dilaton profiles. On the other hand, Einstein-Born-Infeld black holes
in presence of string generated low energy fields have been studied, for example
in [33]-[38]. Thus, it is important to study if the attractor mechanism works in
the case of extremal black holes in Einstein-Born-Infeld theory. Furthermore, if the
mechanism works, then, the entropy function formalism can be used to calculate the
entropy in this case [29].

The rest of this paper is organized as follows. In section 2, we start by recollecting
relevant features of attractor mechanism needed for our purposes in the case of
Einstein-Maxwell theory coupled to scalar field and discuss the possible attractor
solutions in a general theory of gravity coupled to gauge fields and scalars. Section 3
is devoted to studying attractor mechanism in Einstein-Born-Infeld theory coupled
to a scalar field. In section 4 we present a general perturbative analysis to show the
presence of attractor mechanism in this theory. Our conclusions are summarized in
section 5.

2 Non-Supersymmetric Attractors: General features

Let us start with a few relevant aspects of non-supersymmetric attractors which are
needed for our purposes. We consider the class of following gravity theories coupled
to $U(1)$ gauge fields and scalar fields as in [8]:

$$S = \frac{1}{\kappa^2} \int d^4 x \sqrt{-g} \left( R - 2(\partial \phi_i)^2 - f_{ab}(\phi_i) F_{\mu\nu}^a F^{b\mu\nu} - \frac{1}{2\sqrt{-g}} \tilde{f}_{ab}(\phi_i) F_{\mu\nu}^a F_{\rho\sigma}^b \epsilon^{\mu\nu\rho\sigma} \right),$$

(1)
where \( F_{\mu \nu}^a, a = 0, \ldots N \) are gauge fields and \( \phi^i, i = 1, \ldots n \) are scalar fields. The scalar-dependent couplings of gauge fields are motivated from analogy with the supersymmetric theories. Any additional potential term for the scalar fields will lead to a breakdown of attractor mechanism in asymptotically flat spaces. Rest of the notations are as in \[8\].

A static spherically symmetric ansatz is:

\[
\begin{align*}
\text{d}s^2 &= -\alpha(r)^2 \text{d}t^2 + \alpha(r)^{-2} \text{d}r^2 + \beta(r)^2 \text{d} \Omega^2.
\end{align*}
\]  

(2)

On the other hand, the Bianchi identity and equations of motion of gauge fields can be solved by taking the gauge field strengths to be of the form:

\[
F^a = f^{ab}(\phi_i)(Q_{eb} - \tilde{f}_{bc}Q^c_m) \frac{1}{\beta^2} \text{d}t \wedge \text{d}r + Q^a_m \sin \theta \text{d}\theta \wedge \text{d}\varphi,
\]  

(3)

where \( Q^a_m \) and \( Q_{ea} \) are constants that determine the magnetic and electric charges carried by the gauge fields \( F^a \), and \( f^{ab} \) is inverse of \( f_{ab} \).

For a brief recap of non-supersymmetric attractor mechanism, it is enough to concentrate only on the equations of motion of scalar fields:

\[
\partial_r (2\alpha^2 \beta^2 \partial_r \phi_i) = \frac{1}{\beta^2} \partial_r V_{\text{eff}},
\]  

(4)

with the effective potential given by:

\[
V_{\text{eff}}(\phi_i) = f^{ab}(Q_{ea} - \tilde{f}_{ac}Q^c_m)(Q_{eb} - \tilde{f}_{bd}Q^d_m) + f_{ab}Q^a_m Q^b_m.
\]  

(5)

Non-supersymmetric attractor equations can be derived from \( \partial_r V_{\text{eff}}(\phi_0) = 0 \), which also determines the attractor values of scalar fields in terms of the fixed charges of the extremal black hole. This effective potential can in fact be shown to be equivalent to Sen’s entropy function prescription, as discussed in \cite{cite0601016}.

For matching the microscopic and macroscopic results for the entropy, it is important to consider the impact of higher derivative terms. For a particular set of curvature squared terms in \( N = 2 \) supergravity, the corrections to entropy can be calculated \cite{21}. For non-supersymmetric extremal black holes, the effect of higher derivative corrections can also be calculated as in \cite{22}. As discussed in \cite{23}, using the set up described in this section, non-supersymmetric attractor mechanism can be shown to be present when one includes a certain set of higher derivative terms coming from the gravity side in the Einstein-Maxwell action. It was also argued in \cite{23} that, in the presence of general \( R^2 \) terms in the action, the effective potential gets modified by additional terms, and was in fact called as \( W_{\text{eff}} \). The scalar field equation of motion remains as in \cite{21}, with \( V_{\text{eff}} \) replaced by \( W_{\text{eff}} \).

Before proceeding, it should be mentioned that \( W_{\text{eff}} \) will in general depend on \( r \). However, near the horizon all the quantities are independent of \( r \). In this special
situation, the \( r \) dependence in \( W_{\text{eff}} \) drops out. As a result, the horizon radius computed from \( W_{\text{eff}} \) will also be a constant, but modified by higher derivative terms. For instance, let us note down the general form of the scalar field equation near the horizon, in the presence of Gauss-Bonnet terms:

\[
(\alpha^2 \beta^2 \phi')' = \frac{1}{2 \beta^2} \frac{dW_{\text{eff}}}{d\phi},
\]

where,

\[
W_{\text{eff}}(\phi) = V_{\text{eff}}(\phi) + 4 a G(\phi),
\]

and there is no \( r \) dependence. The additional term \( 4 a G(\phi) \) also modify the entropy of the black hole via Wald’s entropy formula. This is in parallel to the analysis in Sen’s entropy function formalism, where the addition of Gauss-Bonnet term gives rise to a finite area to the horizon and hence to the entropy of small black holes in heterotic string theory.

In the following sections, we show that the above analysis of [8, 23] can be extended to include higher derivative interactions in the gauge field part, in particular to the case of Born-Infeld terms. Black hole solutions in Einstein-Born-Infeld theories have been studied quite a lot in literature. It is known that one can have particle-like and BIon solutions in these theories. However, finding explicit black holes solutions in the presence of scalar couplings in the Einstein-Born-Infeld action is non-trivial. In four dimensions, when looking for asymptotically flat solutions in these theories, it is reasonable to assume that the near horizon geometry of these black holes preserve the symmetries of \( \text{AdS}_2 \times S^2 \). In order to understand the effect of higher order Born-Infeld corrections to the entropy of extremal black holes, an entropy function analysis of small black holes in heterotic string theory was presented in [29]. However, it is important to check if the attractor mechanism works when considering the full black hole solution. As in [8, 23], in this work, we carry out a perturbative analysis to show that the moduli fields take fixed values as they reach the horizon and that a double horizon Einstein-Born-Infeld black hole continues to exist. We show that the attractor mechanism works in the case of Born-Infeld black holes. In effect, we show that, once one obtains critical values of the effective potential and ensures that \( \partial_i \partial_j V_{\text{eff}}(\phi) > 0 \), the perturbative analysis signifies that there is always a solution of equations of motion where the scalar fields get attracted to fixed points, which remain stable.

### 3 Non-Supersymmetric Attractors in Einstein-Born-Infeld theories

Non-supersymmetric attractor mechanism in Einstein-Born-Infeld theories can be studied using the entropy function formalism [29]. However, to see that the moduli indeed get attracted to fixed points near the horizon, one has to use the formalism for
non-supersymmetric attractor mechanism reviewed in the previous section, which makes explicit use of the general solutions and equations of motion for two \[8\] and higher derivative \[23\] gravities.

In this section, we follow the analysis outlined in the previous section. Using a perturbative approach to study the corrections to the scalar fields and taking the backreaction corrections into the metric, it is possible to show that the scalar fields are indeed drawn to their fixed values at the horizon. Here, the requirements are the existence of a *double degenerate horizon solution*. We now concentrate on the analysis using equations of motion explicitly and study the attractor mechanism in the case of Einstein-Born-Infeld black holes coupled to moduli fields.

### 3.1 Born-Infeld theory

Born-Infeld theory is one of the most important generalizations to non-linear electrodynamics. It was proposed to obtain a finite self-energy of the electron in arbitrary dimensions as follows:

\[
\mathcal{L}_{BI} = 4b \left\{ \sqrt{-\text{det} \eta_{\mu\nu} - \sqrt{-\text{det} (\eta_{\mu\nu} + \frac{1}{\sqrt{b}} F_{\mu\nu})} \right\}, \tag{8}
\]

where \( \eta_{\mu\nu} \) and \( F_{\mu\nu} \) represent the Minkowski metric and the electromagnetic field strength tensor, respectively, and \( b \) is a parameter characteristic of the Born-Infeld dynamics, and measures the nonlinearity of the theory. Born-Infeld theory has received attention again for it plays a significant role in string theory. It arises naturally in open superstrings and in D-branes \[39\]-\[48\]. In open superstring theory, loop calculations lead to the above Lagrangian with \( b = (2\pi\alpha')^{-2} \). It has also been observed that the Born-Infeld action arises as an effective action governing the dynamics of vector fields on D-branes.

In four-dimensional spacetime, the Lagrangian can be expanded out to be:

\[
\mathcal{L}_{BI} = 4b \left\{ 1 - \left[ 1 + \frac{1}{2b} F_{\mu\nu} F_{\mu\nu} - \frac{1}{16b^2} (F_{\mu\nu} \star F_{\mu\nu})^2 \right]^\frac{1}{2} \right\}, \tag{9}
\]

where \( \star F_{\mu\nu} \) denotes the dual tensor, \( \star F_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} \). The above Lagrangian reduces to the usual Maxwell one in the weak field limit. In the open superstring theory where the dilaton field is expressed by \( \phi \), the Lagrangian is modified to (See \[50\] \[51\] \[53\]):

\[
\mathcal{L}_{BI} = 4b e^{2\phi} \left\{ \sqrt{-\text{det} g_{\mu\nu} - \sqrt{-\text{det} (g_{\mu\nu} + \frac{e^{-2\phi}}{\sqrt{b}} F_{\mu\nu})} \right\}, \tag{10}
\]
3.2 BI Attractor Equations

For the purpose of studying non-supersymmetric attractor mechanism in Einstein-Born-Infeld black holes coupled to moduli fields, we start from the following action with general couplings:

\[
S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( R_g - 2(\partial\phi)^2 + \mathcal{L}_{BI} \right),
\]

where,

\[
\mathcal{L}_{BI} = 4bf(\phi) \left\{ 1 - \left[ 1 + \frac{f^{-2}(\phi)}{2b}F^2 - 2f^{-4}(\phi) - \frac{16}{b^2}(F\times F)^2 \right]^{\frac{1}{2}} \right\}.
\]

where \( F_{\mu\nu} \) denotes the field strength and \( f(\phi) \) stands for a dilaton like coupling of the scalar field \( \phi \). Various interesting BI solutions ensuing from the action were studied in \[35, 38\]. We have further tried to ensure that the action in eqn. (11), reduces to the action considered in \[8\] in the limit \( b \to \infty \). The case considered in \[8\] is more general and from (10), in the following, we specialize to the case \( f(\phi) = e^{2\gamma\phi} \) where \( \gamma \) characterizes the strength of dilaton field. It is one for string theory. We keep this parameter arbitrary, so that even more general theories of gravity, apart from the ones descending from string theory could also be considered. In the present case, we restrict ourselves to the case of a single gauge field (excepting the example in section 3.3) and scalar field, and with the black hole carrying dyonic charges. It is important not to have a potential for the scalar fields, so as to allow for the existence of a moduli space to vary. In the absence of any moduli fields, Einstein-Born-Infeld black holes have been constructed in \[33\]. In what follows, we shall be interested in asymptotically flat spacetime solutions, although, the generalization to include a cosmological constant should also be possible. In fact, it might be interesting to include a cosmological constant \[49\] in view of the results in \[29\].

Now, one makes an ansatz for a static spherically symmetric metric which must satisfy the field equations following from the Einstein-Born-Infeld action in eqn. (11). It should be mentioned, that although we are working with a system of gauge fields coupled to scalar fields, to lowest order, we are looking for the solutions of the equations of motion only for constant values of the moduli fields. The Birkhoff’s theorem holds in this case and we may assume the solution to be static and spherically symmetric, to be of the form:

\[
ds^2 = -\alpha(r)^2 dt^2 + \frac{dr^2}{\alpha(r)^2} + \beta(r)^2 d\Omega_2^2,
\]

\[
F = F_{tr} dt \wedge dr + F_{\theta\phi} d\theta \wedge d\phi.
\]

The induction tensor \( G_{\mu\nu} \) is defined by

\[
G^\mu_{\nu} = \frac{1}{2} \frac{\partial L}{\partial F_{\mu\nu}}.
\]
The Maxwell equations and Bianchi identity are
\[ \frac{dG}{dG} = 0, \quad \frac{dF}{dF} = 0. \] (15)

The above two equations give us the following solutions:
\[ F_{tr} = \frac{Qe^{2\gamma\phi}}{\beta^2 \sqrt{1 + \frac{Q^2 + Q_m^2 e^{-4\gamma\phi}}{b^2 \beta^4}}}, \quad F_{\theta\phi} = Q_m \sin \theta. \] (16)

Although, for simplicity, we consider the case of a single scalar field and a gauge field, the generalization to many scalar fields is straightforward. The equations of motion and the Hamiltonian constraint derived from the action \( S \) with the above solution for gauge fields and the metric ansatz turn out to be:
\[ -1 + \alpha^2 \beta^2 + \frac{\alpha'^2 \beta^2}{2} - \alpha^2 \beta^2 (\partial_r \phi)^2 + \frac{1}{\beta^2} V_{\text{eff}} = 0, \] (17)
\[ \alpha^2 + \alpha'' + \frac{2\alpha \alpha' \beta'}{\beta} - 2be^{2\gamma\phi} \left( 1 - \frac{1}{\sqrt{1 + \frac{Q^2 + Q_m^2 e^{-4\gamma\phi}}{b^2 \beta^4}}} \right) = 0, \] (18)
\[ \partial_r (2\alpha^2 \beta^2 \partial_r \phi) - \frac{\partial_r V_{\text{eff}}}{\beta^2} = 0, \] (19)
\[ (\partial_r \phi)^2 + \frac{\beta''}{\beta} = 0, \] (20)

where \( V_{\text{eff}} \) plays the role of an ‘effective potential’ for the scalar fields. A difference with [8] is that \( V_{\text{eff}} \) in this case, is a function of \( r \), as seen below:
\[ V_{\text{eff}} = 2b \beta^4 e^{2\gamma\phi} \left( \sqrt{1 + \frac{Q^2 + Q_m^2 e^{-4\gamma\phi}}{b^2 \beta^4}} - 1 \right). \] (21)

However, as discussed in [11], it is possible to treat \( r \) as just a parameter near the horizon. Extremizing the effective potential gives the fixed values taken by the moduli at the horizon.

### 3.3 Exact Solution

Let us study some exact solutions, which in the \( b \to \infty \) limit give the solutions considered in [54, 8]. For the time being, let us consider only the electrically charged case, i.e., \( Q_m = 0 \). For instance, if we have two gauge fields and a single scalar field, then, the effective potential turns out to be:
\[ V_{\text{eff}} = 2b \beta^4 S^{-\gamma_1} \left( \sqrt{1 + \frac{Q_1^2}{b^2 \beta^4}} - 1 \right) + 2b \beta^4 S^{-\gamma_2} \left( \sqrt{1 + \frac{Q_2^2}{b^2 \beta^4}} - 1 \right), \] (22)
where \( S = e^{-2\phi_0} \) is the notation near the horizon. Extremizing the effective potential, the near horizon value of the scalar field gets fixed at:

\[
S = \left( \frac{\gamma_2 \left( \sqrt{1 + \frac{Q_2^4}{b^4}} - 1 \right)}{\gamma_1 \left( 1 - \sqrt{1 + \frac{Q_1^4}{b^4}} \right)} \right)^{1/(\gamma_2 - \gamma_1)} \tag{23}
\]

The second derivative of the effective potential is,

\[
\partial_\phi^2 V_{\text{eff}} = S^{\gamma_2 - 2} \left( \sqrt{1 + \frac{Q_2^4}{b^4}} - 1 \right) \left( \frac{\gamma_1}{\gamma_2} \right) - \left( \frac{\gamma_2}{\gamma_1} \right) \tag{24}
\]

which is positive if \( \gamma_1 \) and \( \gamma_2 \) are of opposite sign. The critical value of the scalar field in eqn. (23) is also independent of the asymptotic value of the moduli field.

Area of the event horizon is,

\[
\text{Area} = 4\pi \beta_H^2 = 4\pi V_{\text{eff}}(\phi_0),
\]

\[
= 8\pi b r_H^4 \eta \left( \sqrt{1 + \frac{Q_2^4}{b^4 H}} - 1 \right) - \left( \sqrt{1 + \frac{Q_1^4}{b^4 H}} - 1 \right) \tag{25}
\]

where, \( r_H \) is the radius of the horizon and,

\[
\eta = \left( -\frac{\gamma_2}{\gamma_1} \right)^{(\gamma_1/(\gamma_1 - \gamma_2))} + \left( -\frac{\gamma_2}{\gamma_1} \right)^{(\gamma_2/(\gamma_2 - \gamma_1))}. \tag{26}
\]

In the case, \( \gamma_1 = -\gamma_2 \), we have,

\[
\frac{1}{4} \text{Area} = 4\pi b r_H^4 \left( \sqrt{1 + \frac{Q_2^4}{b^4 H}} - 1 \right)^{1/2} \left( \sqrt{1 + \frac{Q_1^4}{b^4 H}} - 1 \right)^{1/2}. \tag{27}
\]

The radius of the horizon can be found as follows. Assuming a double horizon solution to the equations of motion (18)-(20) as,

\[
\alpha_0(r) = \alpha_H \left( 1 - \frac{r_H}{r} \right), \quad \beta_0(r) = r, \tag{28}
\]

one can use the hamiltonian constraint in eqn. (17) to obtain,

\[
\beta_H^2 = V_{\text{eff}}(\phi_0), \tag{29}
\]

where \( \phi_0 \) is the critical point of the effective potential. Solving this equation for the special case of \( \gamma_2 = -\gamma_1 \) and \( Q_1 = Q_2 = Q \), one gets:

\[
r_H^2 = \beta_H^2 = 2 \left( Q^2 - \frac{1}{16b} \right). \tag{30}
\]

This result for the radius of the horizon is similar to [51], after some redefinitions, although in [51], there were no moduli fields. Thus, we have,

\[
\frac{1}{4} \text{Area} = 2\pi \left( Q^2 - \frac{1}{16b} \right). \tag{31}
\]
4 Perturbative Analysis

It is well known that the equations of motion (17)-(20), admit $AdS_2 \times S^2$ as a solution in the case of constant moduli. However we wish to address the attractor behavior considering double horizon black hole solutions, which are asymptotically flat. Thus, we start with an extremal black hole solution in this theory, obtained by setting the scalar fields at their critical values of the effective potential. Then, as one varies the values of scalar fields at asymptotic infinity, we show that the double horizon nature of black holes continues to exist. Further, the critical values of the scalar fields remain stable, as the asymptotic values of these moduli fields are somewhat different from attractor values.

In view of the fact that the four equations governing $(\alpha(r), \beta(r), \phi(r))$ are a set of highly complicated coupled differential equations of order four, we follow the Frobenius method to solve these equations as in [23]. We call these four sets of equations of motion $EqA, EqB, Eq\Phi, EqC$. As a variable of expansion we define $x \equiv (1 - \frac{r_H}{r})$, ranging from 0 to 1 to cover $r \geq r_H$ completely. Requiring that the solution: (a) be extremal: meaning that we have a double degenerate horizon as, $\alpha^2(r) = (r - r_H)^2 \tilde{\alpha}^2(r)$, with $\tilde{\alpha}^2(r)$ being analytic at the horizon $r = r_H$, (b) be asymptotically flat: meaning that the black hole geometry tends to be flat and moduli fields take arbitrary values at asymptotic infinity and (c) be regular at the horizon the most general Frobenius expansions of $\alpha(r)$, $\beta(r)$ and $\phi(r)$ take the form:

\[
\begin{align*}
\alpha^2(r) &= \alpha_H^2 x^2 (1 + \sum_{n=1}^{\infty} a_n x^{\lambda_1 n}), \\
\beta(r) &= r (1 + \sum_{n=1}^{\infty} b_n x^{\lambda_2 n}), \\
\phi(r) &= \phi_0 + \sum_{n=1}^{\infty} \phi_n x^{\lambda_3 n},
\end{align*}
\]

with $\lambda_i > 0$.

When $V(\phi)$ of (21) is of pure magnetic (electric) type, the case given in (21) does not have an extremum for any finite value of $\phi$. To have an extremum with electric or magnetic fields and not both, one needs at least two gauge fields. Here we consider a dyonic case whence both electric and magnetic charges are non-zero.

**Zeroth order results**

\footnote{Just like the cases studied in references [8, 23] there is a solution where the scalar blows up at the horizon. In the supersymmetric case, the well behaved solution is automatically chosen.}
At zeroth order perturbation we start with a double horizon black hole solution as follows,

\[ \phi(r) = \phi_0, \quad \beta(r) = r, \quad \alpha(r) = \alpha_H \left( 1 - \frac{r_H}{r} \right), \quad (35) \]

where, for given electric and magnetic charges, \( \phi_0, \alpha_H \) and \( r_H \), can be found from the following equations in terms of these charges,

\[ e^{4\gamma\phi_0} = \frac{Q_m^2}{Q_e^2} - \frac{1}{4bQ_e^2}, \quad (36) \]

\[ r_H^4 = 4Q_e^2Q_m^2 - \frac{Q_e^2}{b}, \quad (37) \]

\[ \alpha_H^2 = 1 - \frac{1}{4bQ_m^2}. \quad (38) \]

We should mention, that from the above equations we find a lower bound for the value of magnetic charge to be \( 4bQ_m^2 \geq 1 \). This bound relaxes in the limit \( b \to \infty \), where, the Born-Infeld theory reduces to the Maxwell theory. In this limit \( \phi_0 \) and \( r_H \) approach values that one can find in Einstein-Maxwell-Dilaton theory [8]. In this case, a Reissner-Nordstrom black hole with constant scalars, is an exact solution of the equations of motion.

Notice that the equations (35-38), together, determine both the attractor value of the moduli field and the horizon radius in terms of the charges and the parameters of the action. In fact, both the above results are quite useful. Due to (34), the Bekenstein-Hawking entropy of the solution is given by the value of the \( V_{\text{eff}}(\phi_0) \), up to a numerical prefactor.

This, in fact fixes \( \phi_0 \) at its extremum point. From (34), \( \phi_0 = \phi(r_H) \) and so the value of the moduli field is fixed at the horizon, regardless of any other information. To complete the proof of the attractor behavior, one has to be able to show that the four sets of equations of motion, denoting a coupled system of differential equations, admit the expansions (32), (33) and (34). Also, one should see that there are solutions to all orders in the expansion parameter \( x \), with arbitrary values taken by scalar fields at asymptotic infinity, where their value at the horizon is fixed to be \( \phi_0 \). The existence of a complete set of solutions with desired boundary conditions (considering the fact that we have coupled non-linear differential equations) in the present case is very interesting. Moreover, it is easy to show that, in our theory, there is no asymptotically flat solution with everywhere constant moduli.

**First order results**

To start with the first order perturbation theory, we write,

\[ \delta \phi \equiv \phi - \phi_0, \quad (39) \]
where, we keep $\delta \phi$ as a small parameter in perturbation theory. From the equation of motion of the scalar field, we find,

$$\delta \phi = \phi_1 \left(1 - \frac{r_H}{r}\right)^k.$$  \hspace{1cm}(40)

where,

$$k = \frac{1}{2} \left(-1 + \sqrt{1 + 8 \gamma^2}\right).$$ \hspace{1cm}(41)

Here, $\phi_1$ is an undetermined constant of integration. Since, we are considering the case where $k > 0$, in the asymptotic region $r \to \infty$, we have $\delta \phi \to \phi_1$, which means that, there is a moduli space where the scalar fields can take arbitrary values, since, $\phi_1$ can take arbitrary values. However, near the horizon, $\delta \phi$ vanishes, as seen from eq. (40), i.e., the value of the scalar field remains fixed at $\phi_0$ regardless of its asymptotic value. This shows that the attractor mechanism works to first order in perturbation theory.

In comparison to the Einstein-Maxwell theory where a Reissner-Nordstrom black hole case was considered in \cite{3}, here we have a correction to the components of the metric at the first order in perturbation theory. At this order $\beta(r)$ does not get any correction, while $\alpha(r)$ receives corrections as follows:

$$\alpha_1(r) = \alpha_H^2 a_1 \left(1 - \frac{r_H}{r}\right)^{\frac{1}{2} \left(3 + \sqrt{1 + 8 \gamma^2}\right)\phi_1},$$ \hspace{1cm}(42)

where,

$$a_1 = \frac{4\gamma}{bQ_m^2 \sqrt{(1 + \sqrt{1 + 8 \gamma^2})(3 + \sqrt{1 + 8 \gamma^2})}} \phi_1.$$ \hspace{1cm}(43)

This correction however, vanishes at the horizon faster than $\left(1 - \frac{r_H}{r}\right)$. Thus to this order, the solution continues to be a double horizon black hole with vanishing surface gravity. Asymptotically this correction runs to a constant so the solution continues to be asymptotically flat to this order.

**Second order results**

At second order in perturbation theory the non-constant value of the scalar field we found at first order, plays the role of a source terms, resulting in corrections to the components of the metric. We should also consider boundary conditions as follows. Since we are interested in extremal black hole solutions with vanishing surface gravity, we should have a horizon where $\beta(r)$ is finite and $\alpha^2(r)$ has a “double horizon”. In other words, $\alpha(r) = (r - r_H)\tilde{\alpha}(r)$ where $\tilde{\alpha}(r)$ is finite and non-zero at the horizon. It is useful to note, that by an appropriate gauge choice, we can always take the horizon to be at $r = r_H$. Plugging \cite{32,33} into equations \cite{17,20}, the solutions for $\alpha$ and $\beta$ corresponding to the above boundary conditions are:

$$\alpha_2(r) = \alpha_H^2 a_2 \left(1 - \frac{r_H}{r}\right)^{\frac{1}{1 + \sqrt{1 + 8 \gamma^2}}},$$ \hspace{1cm}(44)
\[ \beta_2(r) = b_2 r (1 - \frac{r H}{r})^{-1 + \sqrt{1 + 8\gamma^2}} , \] (45)

where, the constants \( a_2 \) and \( b_2 \) are

\[
a_2 = \frac{\gamma}{bQ_m^2 \sqrt{1 + 8\gamma^2} (1 + \sqrt{1 + 8\gamma^2})} \phi_2 ,
\]
\[
+ \frac{1}{\sqrt{1 + 8\gamma^2} (1 + \sqrt{1 + 8\gamma^2})} \left( 2(1 - \sqrt{1 + 8\gamma^2})(2 + \sqrt{1 + 8\gamma^2}) \right.
\]
\[
+ \frac{1}{bQ_m^2} - 4(1 - \frac{1}{4bQ_m^2})^2 \bigg) b_2 , \quad (46)
\]

\[
b_2 = -\frac{1}{4} \left( \frac{1 - \sqrt{1 + 8\gamma^2}}{2 - \sqrt{1 + 8\gamma^2}} \right) \phi_1^2 , \quad (47)
\]

\[
\phi_2 = \frac{1}{4bQ_m^2} \left[ \frac{4\sqrt{1 + 8\gamma^2}(\sqrt{1 + 8\gamma^2} - 1)}{\gamma(\sqrt{1 + 8\gamma^2} + 1)(\sqrt{1 + 8\gamma^2} + 3)} + \frac{\sqrt{1 + 8\gamma^2} - 1}{4(2 - \sqrt{1 + 8\gamma^2})} \right] \phi_1^2 . \quad (48)
\]

These solutions, however, vanish at the horizon. Since \( \beta_1(r) \) vanishes, area of the horizon also does not change to second order in perturbation theory and is therefore independent of the asymptotic value of dilaton. Further, \( \alpha_2(r) \) also vanishes at the horizon faster than \( \alpha_1(r) \), thus to second order in perturbation theory, the solution continues to be a double horizon black hole with vanishing surface gravity. Asymptotic behavior of the metric components as \( r \to \infty \) is, \( \alpha_2(r) \sim a_2 \) and \( \beta_2(r) \sim b_2 r \). Therefore, the solution continues to be asymptotically flat to this order.

The scalar field also gets a correction to the second order in perturbation theory. This can be calculated in a way similar to the above analysis. We discuss this correction along with higher order corrections.

**Higher order results**

We solve the system of equations \((EqA, EqB, Eq\Phi, EqC)\) order by order in the \( x \)-expansion. To first order, we find that one variable, say \( \phi_1 \), can not be fixed by the equations. Let us denote the value of \( \phi_1 \) as \( K \). We thus find \( a_1 \) and \( b_1 \) as functions of \( K \). One can check that at any order \( n \geq 2 \), one can substitute the resulting values of \( (a_m, b_m, c_m) \), for all \( m \leq n \) from the previous orders. Then \((EqB, Eq\Phi, EqC)\) of the current order together with \( EqA \) of order \((n - 1)\), consistently give,

\[
b_n = b_n(K) ; \quad a_n = a_n(K) ; \quad \phi_n = \phi_n(K) . \quad (49)
\]

as polynomials of order \( n \) in terms of \( K \).

\( K \) remains a free parameter to all orders in the \( x \)-expansion. From (32), (33) and (34), the asymptotic values of \( (a(r), b(r), \phi(r)) \) are given by a sum of all the coefficients in the \( x \)-expansion of the corresponding function. As a consequence of
one notices that \((a_\infty, b_\infty, \phi_\infty)\) are free to take different values, given different choices for \(K\). The convergence of the series is not addressed in detail, but it would be the case for small enough values for \(|K|\).

The value of \(\phi\) remains arbitrary at infinity, \(\phi = \phi_\infty\), while its value at the horizon is fixed to be \(\phi_0\). This signifies the presence of attractor mechanism in this theory.

5 Conclusions

In this paper, we studied non-supersymmetric attractor mechanism in a theory of gravity coupled to gauge fields and scalar fields, with Born-Infeld corrections in the action. By investigating solutions of the equations of motion, we observed the attractor behavior explicitly. We looked for all possible solutions which admit the criteria of being regular at the horizon and free in the asymptotic region. We used the perturbative approach of \[8\] to study the corrections to the scalar fields and took these backreaction corrections into the metric, to show that the scalar fields are indeed drawn to their fixed values at the horizon.

It is useful to make a few comparisons with the case of \[8\]. In the case of Reissner-Nordstrom black holes \[8\], there were no corrections to the metric components to first order in perturbation theory. In the present case, \(\beta(r)\) does not receive any correction to first order, so, the horizon area does not change to this order. However, \(\alpha(r)\) receives corrections. This, in particular means that the mass of the black hole starts changing from first order. Since, \(\beta(r) = r\) to this order, from the \(1/r\) piece of the \(g_{rr}\) component of the metric, it is possible to extract the first order correction to mass of the BI black hole by a redefinition of the metric as (in the notation of \[8\]):

\[
M = r_H + \frac{\epsilon a_1 k r H}{2(1 + \epsilon a_1)}
\]

where \(a_1\) is defined in eqn. \[13\] and \(k\) is defined in eqn. \[11\] and \(\epsilon\) is a perturbation parameter. Notice that this first order correction is positive, indicating that the lowest mass black hole is the extremal black hole found at zeroth order. Furthermore, this correction vanishes in the limit where the BI parameter \(b \to \infty\). This is consistent with the results in \[8\], where the mass of the RN black holes starts receiving correction only at second order in perturbation theory and this should continue to hold at higher orders as well. To calculate the mass corrections at second and higher orders, one has to look at the \(1/y\) term of the \(g_{yy}\) component of the metric, where \(y = \beta(r)\) as in \[8\]. It is in general difficult to calculate it in the present case, as it is not possible to find a general solution to the equation of motion in \[18\] due to its non-linear nature.

Thus, following the analysis in \[8\] for the Reissner-Nordstrom black holes, our analysis of section-4 shows that the non-supersymmetric attractor behavior continues to hold in the case of Born-Infeld black holes as well, as long as the effective
potential given in eqn. (21) has a critical point $\phi_0$ and the second derivative $\partial^2 V_{\text{eff}}$ evaluated at the critical point, has positive eigen values i.e., $\gamma_i > 0$. These conditions are enough to ensure that a double zero horizon BI black hole continues to exist to all orders in perturbation theory and thus, the attractor mechanism works to all orders in perturbation theory. In is worth mentioning that, as in [8], our perturbative analysis blows up when $k = 1/2$. This can be explicitly seen from eqn. (47). Thus, it is expected that this feature continues to hold whenever $k = 1/n$ for an integer $n$. The function $\beta(r)$ does not receive any corrections to first order and further corrections as seen in section-4 vanishes at the horizon, starting from second order eqn. (15). So, the entropy also remains uncorrected in perturbation theory as for the Reisnner-Nordstrom black hole [8].

In the case of [8], a general eight charge black hole of heterotic string theory [22] can be obtained by an appropriate choice of the scalar couplings of the gauge fields. However, in this work, we only considered the case of a single scalar and gauge field with the black hole carrying dyonic charges. In a more general case, there can be further scalar couplings of the type $h_a(\phi_i) L_{BI}^a$, in the action in eqn. (11). With such couplings and many scalar fields, it should be possible to study BI black holes carrying further electric and/or magnetic charges and there could in general be multiple basins of attractions. In the present case, we explicitly showed that there are different black hole solutions characterized by different values taken by the scalar fields of the theory at asymptotic infinity. Near the horizon, the scalar fields however get fixed to critical values determined by the effective potential. It should be interesting to generalize this analysis to the case of AdS black holes. Furthermore, since the Born-Infeld terms start contributing at order $\alpha'$, together with Gauss-Bonnet terms in the string effective action, it may also be interesting to check whether the attractor mechanism works when both sets of terms are included.

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