Evaluation points of the complex frequency characteristics of a dynamic object using pulse testing impact

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Abstract. In the report we consider the problem of periodic identification of dynamic object is a single test pulse at a fixed interval of observation. The authors analyze the decomposition in Fourier series of the observed signal at the output of the object and the signal at the input to the observation interval. It is shown that the relationship of the amplitudes of the harmonics with the same number of output and input signals with certain accuracy can indicate the coordinates of corresponding points of complex frequency characteristics of the object. This precision depends on the observation interval and the inertial properties of the object. The advantage of this approach is its good noise immunity, and relatively low time for identification. Conditions imposed on the observed duration of the transition process at the output of the dynamic object with the given parameters of the testing pulse, which allows the required accuracy to estimate the coordinates of complex frequency characteristics of the industrial site and to adapt the controller settings.

1. Introduction
Identification of parameters of an industrial facility according to the results of the experiment is well designed, in practice, a technical procedure [1,2]. While most commonly used two kinds of testing signals. This is either a step signal, the response of the object to which the acceleration is subjected to further processing or a series of harmonic signals of different amplitude and frequency, which estimates separate points of complex frequency characteristics (CFC) object. The result of such identification is usually approximating (simplified) the transfer function of an object in a possible serial connection of the link delay, one, two or sometimes several aperiodic links.

Due to the fact that the real object is, in general, is not linear in the whole range of input signals, then the resulting transfer function reflects its properties only in the vicinity of a certain operating mode. The parameters of this mode can therefore change over time, and, accordingly, will change the parameters of the transfer function, towards those that were identified initially. This circumstance is taken into account in the algorithms of modern adaptive controllers [3,4,5], which periodically carry out adjustments of their parameters depending on change of the parameters of the transfer function of the object. Thus the estimation of changing parameters of the transfer function of the object is estimated by the adaptive controller to change the coordinates of several feature points of the object CFC. The coordinates of these points in accordance with the method [4,5] allow tuning of the controller parameters. In accordance with this technique, the adaptive controllers periodically adds to the control signal operating the closed system of a test harmonic signal of a certain frequency and amplitude and analyze the response of the object to these messages. The number of such signals, their amplitude and duration are determined by the parameters of the transfer function of the object, and the level and spectrum of noise in the signal response of the object.
A significant drawback of this method is identification of parameters employed in the system of the object is relatively large the time taken for the process of identification, due to the necessity of waiting the end of the transient into account the effect of the additive noise. In fact, such identification leads to additional noisy signal management, and, of course, affects the process flow. Use the overclocking features in practice estimation of changes in the parameters of the working object, in spite of the sufficient accuracy of the applied integrally modulation processing methods [6,7], is ill-suited for objects with no self-regulation and functioning in closed systems, especially in conditions of action of hindrances.

In this regard, of interest is the identification process, using pulse testing effect. In this case, the duration of the identification process associated with the duration of the testing pulse and duration of the end of the transitional process after removal of the test signal. The observed duration of the transition process also depends on the amplitude of the test pulse and the level of additive noise. If you continue to use the expansions in Fourier series of the observed signal at the output of the object and the signal at the input to the observation interval, the mapping parameters of the harmonics expansions with the same numbers with a certain precision may indicate the coordinates of corresponding points with a CFC object. This precision depends on the observation interval and the inertial properties of the object. The advantage of this approach is its good noise immunity, since the decomposition in Fourier series is an integral transform, and the relatively low time spent on identification.

All of the above issues relating to identification of dynamic object is a single test pulse at a fixed interval of observation are the subject of this work.

2. The decomposition in Fourier series of a single pulse at a predetermined monitoring interval

A single pulse at a predetermined monitoring interval $T$ for the decomposition in Fourier series can be replaced by a decomposition of the pulse sequence $f(t)$ with the repetition period of pulses is equal to the observation interval $T$. A sequence shown in figure 1. The function $f(t)$ can be represented as a Fourier series:

$$f(t) = b_0 + \sum_{n=1}^{\infty} (a_n \sin n\omega t) + \sum_{n=1}^{\infty} (b_n \cos n\omega t),$$

where the coefficients of the Fourier series $a_n, b_n, b_0$ are determined according to known proportions (the constant component $b_0$ further in the analysis does not participate):

$$a_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t) \, dt \quad ; \quad b_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t) \, dt \quad ; \quad b_0 = \frac{1}{T} \int_0^T f(t) \, dt \quad (1)$$

Figure 1. Analyze the sequence of pulses

The amplitude of the harmonics series by using (1) are defined as:
Note that in the decomposition (2) are numbers of harmonics of zero amplitude when \( ny = 1, 2, 3 \ldots \). This fact must be considered, varying the observation interval \( T \), if you need to explore the frequencies where these harmonics.

3. The decomposition in Fourier series of the response of a linear object to a single impulse testing and assessment of CFC points of the object.

We assume that a linear object is described by transfer function, the roots of the denominator of which are valid and negative. The zeros of the numerator are also valid, but their distribution on the real axis is not specified. It is easy to show using the theory of deduction, that this object can be represented by parallel connected and aperiodic real differentiating links of the first order. The number of such units will be equal to the number of roots of the denominator of the transfer function of the object. Thus, the reaction of the object on a single test pulse will consist of the amount of reactions on this momentum and aperiodic real differentiating links. In this regard, consider the decomposition in Fourier series of the response of these links on a single test pulse.

4. The decomposition in Fourier series of the response of an aperiodic link to a single test pulse and assessment points CFC link

Aperiodic link is described by the equation \( T_0 \frac{dy}{dt} + y = Kx \), where \( x \) – input, \( y \) – output, \( T_0 \) and \( K \) – are respectively time constant and the transfer coefficient of the link. CFC of such a link is determined by the amplitude - and phase-frequency characteristics:

\[
|W(j\omega)| = A(\omega) = \frac{K}{\sqrt{1 + \left(\omega T_0\right)^2}}; \quad \phi(\omega) = -\arctg \omega T_0; \quad (3)
\]

The reaction of an aperiodic link to a single test pulse is shown in figure 2 and is described by relations (4).

![Figure 2. The reaction of an aperiodic link to a single test pulse](image)

\[
f(t) = \begin{cases} 
XK(1 - e^{-\beta t}), & 0 \leq t \leq \tau \\
XK(1 - e^{-\beta \tau}) - XK(1 - e^{-\beta(t-\tau)}), & \tau \leq t \leq T
\end{cases} \quad (4)
\]

Using (2) and determine the expansion coefficients in Fourier series of the signal (4):

\[
a_n = \frac{XK}{\pi} \left(1 - \cos \frac{x_1}{1 + \alpha^2} \right) + \frac{1}{1 + \alpha^2} \left(\cos x_1 + \alpha \sin x_1 \right) - 1 - ke^{-\alpha x_2}, \quad (5)
\]

\[
b_n = \frac{XK}{\pi} \left(1 + \frac{1}{1 + \alpha^2} \left(\alpha \cos x_1 - \sin x_1 - \alpha - ak e^{-\alpha x_2}\right)\right), \quad (6)
\]

where \( \Omega = \frac{\beta}{\beta} = \alpha \). \( \Omega \tau = x_1; \Omega T = x_2, \ k = (e^{ax_1} - 1) \)
Note that in (5) and (6) the last terms depend on $x_1$, when $T \to \infty$ seeking to 0. In this case, the calculation module of the harmonics in the decomposition of the output signal (4) leads to the following relation:

$$c_{n, \text{out}} = \frac{2K}{\pi \sqrt{1+a^2}} \sin \left( \frac{x_1}{2} \right), \quad \text{where} \quad \frac{a}{\sqrt{1+a^2}} = \frac{1}{\sqrt{1+(\omega T)^2}}$$  \hspace{1cm} (7)

The relationship of the amplitudes of the harmonics with the same number of the output signal (7) to the input (2) give the exact value of the modulus CFC aperiodic link frequencies associated uniquely with the numbers of the harmonics. A similar result is obtained by evaluation of the phase-frequency characteristics of the link using (5) and (6).

Theoretically, when testing of an aperiodic link of the single pulse of the exponent of the output signal with increasing observation time $T$ only tend to zero, but the values do not reach. Therefore, when determining the points with a CFC using (5) and (6) inevitably, there will be errors in estimation of the magnitude and phase of vector CFC on the complex plane. This error make the last terms in (5) and (6). However, the absolute values of these errors depend only on the values of $\frac{T}{T_0}$. If this ratio is greater than 10, for example, when $\gamma = 0.1$, as the calculations demonstrate, for the first 30 harmonics of the error in the estimation of the harmonics does not exceed 0.3%. The exceptions are the harmonic numbers, which obey the equality $n\pi = 1, 2, 3 \ldots$ Figure 3 shows the change of error in the determination of the module of the transfer function aperiodic link with $T_0 = 0.1\ell; T = 1\ell; K = 1$ for the first thirty harmonics.

![The dependence of the error from the number of a harmonic](image)

The dependence of the error from the number of a harmonic

Error for 20 and 30 harmonics figure 3 is not shown, since they are two orders higher than the other. The errors for the 10 harmonics are also significantly larger than the error for the neighboring harmonics. In this example, harmonics of the input signal X are not calculated by the ratio of (2) and numerical methods (1) and therefore, the values of harmonics of the input signal for these frequencies are different from zero.

5. The decomposition in Fourier series of the response of real differentiating link for single pulse testing and assessment of CFC points of the link

The real differentiating link is described by the equation $T_0 \frac{dy}{dt} + y = K \frac{dx}{dt}$ where $x$ – input, $y$ – output, $T_0$ and $K$ – are respectively time constant and the transfer coefficient of the link. CFC of such a link is determined by the amplitude - and phase-frequency characteristics:
\[ W(j\omega) = A(\omega) = \frac{K\omega}{\sqrt{1 + (\omega T_0)^2}}; \quad \varphi(\omega) = \frac{\pi}{2} - \arctg \frac{\omega T_0}; \quad (8) \]

Analyzing the response of such a link on testing a single pulse and further including the decomposition in Fourier series, we obtain the components of the \(a_n\) and \(b_n\) similar to those which were obtained for the reaction at the output aperiodic link. Interestingly, the fact that these expressions in \(T \to \infty\) also allow to accurately assess CFC real differentiating link, and at finite values of \(T\) the error is determined exactly the same terms, dependent \(x_1\), that in expressions (5) and (6). Thus, all the results obtained above for aperiodic managers are fair and for the real differentiating link.

6. Evaluation of CFC points of the line feature

Given that a linear object with real and negative roots of the characteristic equation can be represented as a parallel connection of aperiodic real differentiating links, his response to a single test pulse contains the algebraic sum of the reactions of each link in the structure. While insertion errors in the CFC rating points increase, however, these errors in the result entirely determined by the degree of attenuation of the transition process at the output of the object after removing the test pulse at the end of the monitoring interval, a limited time you selected \(T\).

The presence of transport lag in the object structure does not prevent the assessment of points CFC using the relations (1).

7. The choice of parameters of a single test pulse and monitoring interval

The main parameters of a single test pulse are amplitude \(X\) and a duration \(\tau\). In addition to these two parameters must also be specified the size of the observation interval \(T\). These parameters must be "linked" with a preliminary (rough) assessment of the duration of the transition process at the output of the object from the action of the jump or pulse input, as well as the level of additive noise (RMS) output. These two parameters allow us to estimate the minimum time of observation. Pulse amplitude and duration are selected so that all processes remained in the linear zone static characteristics of the object.

8. Conclusion

The results of the research presented in the paper show the possibility of estimating the changes in the positions of certain points with a CFC of a dynamic object using a single test pulse for the purpose of correction settings adaptive controller system.

9. References

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