Efficient charge pumping in graphene

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Abstract
We investigate a graphene quantum pump, adiabatically driven by two thin potential barriers vibrating around their equilibrium positions. For the highly doped leads, the pumped current per mode diverges at the Dirac point due to the more efficient contribution of the evanescent modes in the pumping process. The pumped current shows an oscillatory behavior with an increasing amplitude as a function of the carrier concentration. This effect is in contrast to the decreasing oscillatory behavior of a similar normal pump (i.e. a pump based on a two-dimensional electron gas). The graphene pump driven by two vibrating thin barriers operates more efficiently than the graphene pump driven by two oscillating thin barriers.

Keywords: graphene, adiabatic processes, transport phenomena, quantum pumping

(Some figures may appear in colour only in the online journal)

1. Introduction
Quantum pumping is a coherent transport mechanism for producing a DC charge current in the absence of an external bias voltage by an appropriate periodical variation of the system parameters. The idea of quantum pumping was introduced by Thouless, who proposed a quantum pump driven by a moving potential for DC current generation \[1\]. Then, Brouwer developed an elegant formula for adiabatic quantum pumping in an open quantum system based on the scattering matrix approach \[2\]. Finally, Moskalets and Büttiker generalized the scattering matrix approach for AC transport and they also derived general expressions for the pumped current, heat flow and shot noise for an adiabatically driven quantum pump in the weak pumping limit \[3\]. It has been shown that the pumping current is related to the geometric (Berry) phases \[4\] and quantum interference effects \[5\]. Several proposals have been made for the application of quantum pumping: as a potential means to generate a dissipationless charge current in nanoscale devices \[6\]; as a promising method for generating a dynamically controlled flow of spin-entangled electrons \[7\]; as a way of producing spin polarized current which is important in spintronics \[8\]; and as a method to transfer charge in a quantized fashion \[9\]. Quantum pumping has been realized experimentally in different nanoscale systems such as quantum dots with Coulomb blockade \[10–12\] and Josephson junctions \[13, 14\].

Recently, experimental realization of graphene, a mono-layer of carbon atoms with hexagonal lattice structure, has introduced a new type of two dimensional material with unique properties \[15–17\]. Electrons in graphene behave identically to two-dimensional massless Dirac fermions due to its gapless semiconducting electronic band structure with a linear dispersion relation at low energies \[18, 19\]. Such a peculiar quasiparticle spectrum accompanied by the unique feature of chirality has led to anomalous behaviors in several transport phenomena including Klein tunneling \[20\], a conductivity minimum \[21\], the integer quantum Hall effect \[22\] and the Josephson effect \[23\].

Several graphene quantum pumps have been proposed, and different aspects of them have been investigated. Prada \textit{et al} proposed a graphene pump driven by two oscillating square potential barriers. Their study revealed that the evanescent modes in graphene make a dominant contribution to the pumped current which gives rise to a universal dimensionless pumping efficiency at the Dirac point \[24\]. It has also been shown that adding stationary magnetic barriers in the graphene pump leads to a valley-polarized and pure valley-pumped currents \[25\]. Zhu and Chen studied a quantum pump device composed of ballistic graphene coupled to the reservoirs via two oscillating tunnel barriers \[26\]. Bercioux \textit{et al} showed that the presence of a gate tunable spin–orbit interaction can generate a spin-polarized pumped current \[27\]. It has been
shown that the combination of high frequency vibrations and metallic transport in graphene makes it extremely suitable for charge pumping due to the sensitivity of its transport coefficients to perturbations in electrostatic potential and mechanical deformations [28]. Tiwari and Blaauboer found that the combination of a perpendicular magnetic field in the central pumping region with two oscillating electrical voltages in the leads causes both charge and spin pumped currents through traveling modes [29]. It has been shown that the presence of a superconducting lead enhances the pumped current per mode by a factor of four in the resonance condition [30]. Kundu et al showed that a graphene superconducting double barrier structure supports large values of pumped charge when the pumping contour encloses a resonance point [31].

The effect of the interlayer coupling on the pumped current in a bilayer graphene pump was investigated by Wakker et al [32]. A large pumped current around the Dirac point has been demonstrated in the bipolar regime by a single-parameter graphene pump invoking graphene’s intrinsic features of chirality and bipolarity [33].

In this paper we introduce a mechanism for efficient charge pumping in graphene. We propose a graphene quantum pump driven by two thin potential barriers vibrating around their equilibrium positions. We analyze the pumped current in the two different cases of highly doped and undoped leads. We compare the results with the more familiar case of charge pumping by oscillating thin barriers. The results of the latter case are very similar to the graphene pump considered in [24]. We find very efficient pumping by the vibrating thin barriers in comparison to the oscillating ones. In the case of highly doped leads and vibrating thin barriers the pumped current per mode diverges at the Dirac point, whereas for the oscillating thin barriers it tends to a limited value. An interesting and distinguishing feature of the pumped current generated by the vibrating thin barriers is its increasing oscillatory behavior as a function of the carrier concentration. The decreasing oscillatory behavior of the similar normal pump reveals that this is a unique feature of a Dirac fermion pump and can be attributed to the linear dispersion relation of the electrons in graphene. This feature is in contrast to the tendency of the pumped current generated by the oscillating thin barriers to a limited value at high carrier concentration.

The outline of the paper is as follows. In section 2 we introduce the proposed pump and basic equations which are used for calculation of the pumped current. Section 3 is devoted to studying the pumped current in the case of highly doped leads. In section 4 we give the results and discussions for the pump with undoped leads. Section 5 is devoted to a discussion about the main features of the pump and its experimental implementation. Finally, we summarize our results in section 6.

2. Model and basic equations

We consider a quantum pump composed of a graphene sheet of length $2L$ and width $W$ connected to two leads kept at zero bias voltage. It is driven out of equilibrium by two thin potential barriers as shown in figure 1. These thin barriers can be realized by electric fields or thin gates under the graphene. We study the two cases of highly doped and undoped leads and compare their results. For highly doped leads evanescent modes are induced in the pumping region. This is in contrast to the case of undoped leads for which all of the modes in the pumping region are propagated. This feature allows us to investigate the contribution of the evanescent modes in the pumped current [24]. In order to adiabatically pump a charge current between two leads kept at zero bias voltage, the scattering properties of the system should undergo a slow and periodic variation. This is achieved by cyclically changing some parameters of the system usually referred to as pumping parameters. The slow variation is attained when the pumping parameters vary more slowly than the dwell time of the carriers in the pump region. In this work we consider two different methods for driving the pump. These methods are realized by two oscillating or vibrating thin barriers. In the first case, the pump is driven by two thin barriers located at the fixed positions $X_1$ and $X_2$ with oscillating magnitudes of the potentials

\begin{equation}
U_1(t) = U_{1,0} + \delta U_1 \cos(\omega t + \phi),
U_2(t) = U_{2,0} + \delta U_2 \cos(\omega t + \phi),
\end{equation}

where $U_{1,0}$ and $U_{2,0}$ are static potentials, $\delta U_1$ and $\delta U_2$ are amplitudes of the oscillations and $\phi$ is their phase difference. This driving method is very similar to that in a previous work [24]. The second method is a mechanism usually referred to as the ‘snow plow’ mechanism [34]. The pump is driven by two thin barriers with fixed magnitudes of the potentials and periodic variation of their positions, as follows:

\begin{equation}
X_1(t) = X_{1,0} + \delta X_1 \cos(\omega t + \phi),
X_2(t) = X_{2,0} + \delta X_2 \cos(\omega t + \phi),
\end{equation}

Figure 1. (a) A graphene quantum pump driven by two thin barriers imposed via two thin gates. The periodic variations of the magnitudes or positions of the thin barriers are considered as pumping parameters. (b) Electrostatic potential through the system in the case of highly doped leads and (c) undoped leads. The solid lines show the instantaneous potential profile during the pumping cycle.

where $X_{1,0}$ and $X_{2,0}$ are the equilibrium positions of the thin barriers, $\delta X_1$ and $\delta X_2$ are amplitudes of the vibrations and $\phi$ is their phase difference. If we denote the two pumping parameters by $\eta_1$ and $\eta_2$, the adiabatic pumped current in the
bilinear response regime, where \( \delta \eta_1 \ll \eta_1 \) and \( \delta \eta_2 \ll \eta_2 \), is given by \([2, 24]\)

\[
I_p = I_0 \sum_{\alpha = L, R} \text{Im} \left( \frac{\partial S_{\alpha} \delta \partial S_{\alpha}^*}{\partial \eta_1 \partial \eta_2} \right),
\]

where \( I_0 = (\omega/2\pi)e^\delta \eta_1 \delta \eta_2 \sin \varphi \) and \( S_{L, R} \) is an element of the scattering matrix. In the above equation summation goes over the transverse modes in the left and right leads. The pumped charge depends only on the area spanned by the pumping cycle in the parameter space and not on its details. This equation shows that pumped current in the adiabatic limit is proportional to the variation frequency and vanishes at zero phase difference, when the area enclosed in the parameter space is zero during a cycle.

To calculate the pumping current we need to obtain the scattering matrix of the pump. The low energy excitations in the graphene are described by the two-dimensional Dirac equation

\[
\left[ i\hbar \mathbf{p} \cdot \hat{\sigma} + U(x) \right] \Psi = E \Psi,
\]

where \( \mathbf{p} \) is the momentum operator relative to the Dirac point, \( \hat{\sigma} = (\sigma_x, \sigma_y) \) is the vector of the Pauli matrices and \( U(x) \) is the potential energy across the system. We model the thin barriers by symmetric delta function potentials in our calculations. Thus, in the pumping region \( U(x) = U_1 \delta(x - x_1) + U_2 \delta(x - x_2) \) and in the leads \( U_{L,R} \rightarrow -\infty \) for highly doped leads and \( U_{L,R} = 0 \) in the case of undoped leads. In fact, the highly doped leads model normal metal leads connected to graphene. Figures 1(b) and (c) show the potential profiles through the system in the cases of highly doped and undoped leads, respectively. \( \Psi \) in equation (4) is a two-component spinor in the pseudospin space which refers to the two sublattices in the two-dimensional honeycomb lattice. We solve equation (4) in different regions of the pump for highly doped and undoped leads in the following sections. Due to the conservation of transverse momentum through the system, mode matching gives us the elements of the scattering matrix.

3. Highly doped leads

As it has been shown in the figure 1(b), the system has five regions in the case of highly doped leads. In the leads where \( U_{L,R} \rightarrow -\infty \), carrier densities are very large in contrast to the pumping region. This situation is realized by metallic leads. The highly doped leads induce evanescent modes in the pumping region, leading to the contribution of the evanescent modes in the pumped current. The wavefunctions in the left \((x < -L)\) and right \((x > L)\) leads are

\[
\Psi_L = e^{i(K_{L}x + qy)} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + re^{i(-K_{L}x + qy)} \begin{pmatrix} 1 \\ -1 \end{pmatrix},
\]

\[
\Psi_R = re^{i(K_{R}x + qy)} \begin{pmatrix} 1 \\ 1 \end{pmatrix},
\]

where \( r \) and \( t \) are reflection and transmission coefficients, respectively. In the above relations \( q \) is the transverse momentum and \( K_{L,R} = \sqrt{(E-U_{L,R}/\hbar^2)^2 - q^2} \) are the wavevectors in the leads. The wavefunctions in the pumping region, the region between the left lead and the first delta potential \(-L < x < X_1\), the region between two delta potentials \(X_1 < x < X_2\) and the region between the second delta potential and the right lead \(X_2 < x < L\), which are denoted by \( j = 1, 2, 3 \) respectively, read

\[
\Psi_j = a_j e^{i(k_j x + qy)} \begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix} + b_j e^{-i(k_j x + qy)} \begin{pmatrix} 1 \\ -e^{-i\phi} \end{pmatrix},
\]

where \( k = \sqrt{(E/\hbar^2)^2 - q^2} \) is the wavevector in the pumping region and \( \phi = \tan^{-1}(\frac{X_1}{X_2}) \) is the incident angle. The boundary conditions for these wavefunctions are the continuity at the \( x = \pm L \) and satisfying the following conditions at the positions of the delta potentials:

\[
\Psi_{2|x=x_1} = T_1 \Psi_{1|x=x_1}, \quad \Psi_{3|x=x_2} = T_2 \Psi_{2|x=x_2},
\]

where

\[
T_{1,2} = 1 + i \sigma_x U_{1,2} / 2 \hbar \nu \quad \Rightarrow e^{2i\sigma_x \tan^{-1}(U_{1,2}/\nu)}.\]

This boundary condition is obtained by integrating the Dirac equation through a symmetric delta function potential \([35]\). We obtain the transmission and reflection coefficients by solving these equations. The obtained expressions are too long to be given here. In the following we calculate the pumped current in the case of the highly doped leads for oscillating and vibrating thin barriers.

3.1. Driving by the oscillating thin barriers

Let us focus on the pumped current generated by variations of the magnitudes of two thin potential barriers given by equation (1). In this case the pumped current is obtained by the following relation:

\[
I_p = I_0 \sum_n \text{Im} \left\{ \frac{\partial r \partial r^*}{\partial U_1 \partial U_2} + \frac{\partial t \partial t^*}{\partial U_1 \partial U_2} \right\},
\]

where \( I_0 = (\omega/2\pi)e^\delta \eta_1 \delta \eta_2 \sin \varphi \) and summation is over the transverse modes denoted by \( n \). For short and wide graphene \((W \gg \nu)\) we can change the summation over \( n \) to integration over the continuous transverse momentum, \( \sum_q \rightarrow (W/2\pi) \frac{dq}{q} \). Thus, the pumping current reads

\[
I_p = N_m I_0 \int_{-\infty}^{+\infty} \frac{dq}{k_F} \text{Im} \left\{ \frac{\partial r \partial r^*}{\partial U_1 \partial U_2} + \frac{\partial t \partial t^*}{\partial U_1 \partial U_2} \right\},
\]

where \( N_m = 4k_F W/\pi \) is the number of propagating modes in the pump and \( k_F \) is the Fermi momentum. The coefficient 4 is due to the degeneracy including two valleys and two spin states in graphene. In figure 2 we show the normalized pumped current \( I_p/\pi N_m I_0 \), as a function of \( k_F L \) (characterizing the carrier concentration) for different configurations of the potential barriers. These plots reveal some important features. First, the pumped current changes sign around \( k_F L \sim 1/2 \).
This happens due to the generation of the pumped current by the evanescent modes \(|q| > k_F\) in the opposite direction to the pumped current generated by the extended modes \(|q| \leq k_F\) around the Dirac point. To explain this we show the kernel or the momentum distribution of the normalized pumped current in figure 3. This shows that around the Dirac point oscillating thin barriers drive electrons occupying extended and evanescent modes in different directions. At a specific value of \(k_FL\) these opposite contributions cancel each other and the pumped current vanishes. This feature can help to distinguish between the pumped and rectified currents [36]. Second, the minimum of the pumped current, i.e. the pumped current at the Dirac point, depends on the configuration of the potential barriers and, in contrast to [24], it does not have a universal value. Third, there is a weak oscillatory behavior in the curves which is caused by quantum interference due to the reflections from two thin potential barriers. Fourth, all of the plots tend to the same value \(I_P/\pi N_m I_0 = 0.25\) in the limit of large \(k_FL\). This happens irrespective of the pump configuration and is identical to the result of [24].

### 3.2. Driving by the vibrating thin barriers

In this section we consider the pumped current generated by the vibration of two thin potential barriers around their equilibrium positions, given by equation (2). For this case the pumped current reads

\[
I_P = N_m I_0 \int_{-\infty}^{+\infty} \frac{dq}{k_F} \Im \left\{ \frac{\partial r}{\partial X_1} \frac{\partial r^*}{\partial X_2} + \frac{\partial t}{\partial X_1} \frac{\partial t^*}{\partial X_2} \right\} , \tag{12}
\]

where \(I_0 = (\omega/2\pi) e \delta X_1 \delta X_2 \sin \varphi\). In the calculations we consider that \(U_1 = U_2 = h\varphi\). Figure 4 shows the normalized pumped current as a function of \(k_FL\). As is apparent from the figure, there are major differences between the pumped current generated by the position and variations in magnitude of the potential barriers. In the case of driving by position variation, the normalized pumped current (the pumped current per mode normalized by \(I_0\)) diverges at the Dirac point. This indicates the nonzero value of the pumped current at the vanishingly small density of states around the Dirac point. The pumped current shows asymmetric oscillations around zero as a function of the carrier concentration. For vibrating thin barriers the pumped current maintains an increasing oscillatory behavior by increasing \(k_FL\), as shown in the inset of figure 4. This is in contrast to the oscillating thin barriers for which the pumped current tends to a limited value at large \(k_FL\).

To clarify the obtained results, we compare the momentum distribution of the pumped current for oscillating and vibrating thin barriers. The momentum distribution is a symmetric

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**Figure 2.** The normalized pumped current as a function of the carrier concentration, \(k_FL\), for different configurations of the oscillating thin barriers (where we have considered \(U_{1,0} = U_{2,0} = 0\)).

**Figure 3.** Momentum distribution of the normalized pumped current as a function of the transverse momentum \(q\) and the carrier concentration \(k_FL\) in the case of oscillating thin barriers for \(U_{1,0} = U_{2,0} = 0\) and \(X_2 = -X_1 = 0.5L\). The white lines indicate the points with \(|q| = k_F\). Around the Dirac point, the evanescent modes \(|q| > k_F\) contribute to the pumped current in the opposite direction to the extended modes \(|q| \leq k_F\).

**Figure 4.** Normalized pumped current as a function of the carrier concentration \(k_FL\), for different configurations of vibrating thin barriers \((U_1 = U_2 = h\varphi\) is considered). Inset: normalized pumped current as a function of the carrier concentration in the wide range of \(k_FL\) for \(X_2 = -X_1 = 0.5L\).
function of the transverse momentum $q$. Thus, this lets us plot the momentum distributions for two different driving methods in one figure. The left half of figure 5 shows the momentum distribution for the oscillating thin barriers and the right half shows the vibrating thin barriers for $U_{1,0} = U_{2,0} = \hbar v_F$ and $X_{2,0} = -X_{1,0} = 0.5L$. As is apparent in the figure, in both cases the contribution of the normal incident Dirac fermions ($q = 0$) to the pumped current is zero due to Klein tunneling. The contribution of the evanescent modes in the pumped current around the Dirac point is considerable in both cases. Despite these similarities there is a major difference between two methods of driving. In the case of the vibrating thin barriers the contribution of the extended modes increases with increasing carrier concentration, whereas it decreases in the case of the oscillating thin barriers. This feature makes pumping by vibrating thin barriers more effective than with oscillating ones.

4. Undoped leads

There are three different regions in the system with undoped leads. The region on the left of the first delta potential, $x < X_1$, the region between two delta potentials, $X_1 < x < X_2$, and the region on the right of the second delta potential, $X_2 < x$. The wavefunctions in the left and right leads are as follows:

$$\Psi_L = e^{i(kx+qx)} \left( \frac{1}{e^{i\phi}} + re^{-i(kx+qx)} \right),$$

$$\Psi_R = te^{i(kx+qx)} \left( \frac{1}{e^{i\phi}} \right).$$

In the pump region the wavefunction is

$$\Psi_p = ae^{i(kx+qx)} \left( \frac{1}{e^{i\phi}} \right) + be^{-i(kx+qx)} \left( \frac{1}{e^{-i\phi}} \right).$$

where $k = \sqrt{\left( \frac{E}{\hbar v_F} \right)^2 - q^2}$ is the wavevector in the pumping region as well as the leads and $\phi = \tan^{-1}(\frac{q}{k})$ is the incident angle.

Figure 5. Comparison of the momentum distributions of the normalized pumped currents as a function of the transverse momentum $q$ and the carrier concentration $k_F L$ for oscillating (left half) and vibrating (right half) thin barriers. Here we have taken $U_{1,0} = U_{2,0} = \hbar v_F$ and $X_{2,0} = -X_{1,0} = 0.5L$. The white lines indicate the points with $|q| = k_F$.

These wavefunctions should satisfy the following boundary conditions:

$$\Psi_p|_{x=X_1} = T_1 \Psi_L|_{x=X_1}, \quad \Psi_R|_{x=X_2} = T_2 \Psi_p|_{x=X_2}. \quad (16)$$

$T_{1,2}$ are given by equation (9). Solution of these equations yields the reflection and transmission coefficients. Since the obtained expressions are lengthy we will not give them here.

4.1. Oscillating thin barriers

In this section we present the results for oscillating thin barriers in the case of undoped leads. The pumped current is given by equation (11) and we obtain a simple analytical equation for it:

$$I_p = N_m I_0 \int_{-\infty}^{+\infty} \frac{dq}{k_F} q^2 \sin(2k(X_2 - X_1)).$$

Figure 6. Normalized pumped current as a function of the carrier concentration, $k_F L$, for different configurations of oscillating thin barriers. Here $U_{1,0} = U_{2,0} = 0$. As has been shown in figure 6, the normalized pumped current vanishes at the Dirac point due to the absence of extended and evanescent modes. It has an oscillatory behavior as a function of the carrier concentration arising from quantum interference. At large values of $k_F L$ the normalized pumped current tends to a limited value $I_p/\pi N_m I_0 = 0.5$, as also indicated in [24].

4.2. Vibrating thin barriers

For vibrating thin barriers the pumped current is given by equation (12). The normalized pumped current is shown in figure 7 for different configurations of the pump and $U_1 = U_2 = \hbar v_F$. As is apparent from figure 7, the pumped current vanishes around the Dirac point. At the Dirac point all of the modes in the pump are unpopulated and there is no nonzero contribution in the pumped current. This is due to the absence of extended modes at the Dirac point and the evanescent modes in the graphene connected to the undoped leads. As in the case of highly doped leads, there is an increasing oscillatory behavior in the normalized pumped current as a function of $k_F L$. 

Figure 6. Normalized pumped current as a function of the carrier concentration, $k_F L$, for different configurations of oscillating thin barriers. Here $U_{1,0} = U_{2,0} = 0$. 

4. Undoped leads
5. Unique features

To emphasize the unique feature of pumping by vibrating thin barriers in graphene we compare it with a similar normal pump (i.e. a similar pump based on a two-dimensional electron gas). We notice that the pumped current in graphene shows similar behaviors for large values of carrier concentration for both highly doped and undoped leads. Thus, we just compare the results for undoped leads. For the normal pump we should solve the Schrödinger equation in the presence of the two delta function potentials and then mode matching will give us the reflection and the transmission coefficients. As with graphene, the pumped current for the normal pump shows an oscillatory behavior as a function of \( k_F L \). But unlike graphene its amplitude decreases with increasing carrier concentration. To clarify this we compare the momentum distributions of the normal and graphene pumps. In figure 8, the left half shows the momentum distribution for the normal pump and the right half for the graphene pump. The oscillatory behavior in both cases is clearly apparent in the figure. However, the contribution of the extended modes in the pumped current goes in opposite directions with increasing carrier concentration: it increases in the graphene pump, whereas it decreases in the normal pump. Thus, we can conclude that the increasing contribution of the extended modes in the pumped current is a unique feature for driving the Dirac fermions by the vibrating potential barriers.

Let us discuss practical situations. For highly doped leads the normalized pumped current \( I_p = I_p / \pi N_m I_0 \) generated by the oscillating thin barriers tends to a limited value at the Dirac point. This means that at \( k_F L \rightarrow 0 \), where the number of extended modes \( N_m \) vanishes, the pumped current \( I_p \) should also vanish. On the other hand, the normalized pumped current generated by the vibrating thin barriers diverges at the Dirac point. But when \( k_F L \rightarrow 0 \) and \( I_p \rightarrow \infty \), \( k_F L_i p \sim 1 \) the pumped current tends to a nonzero value at the Dirac point. We can estimate the magnitude of the pumped current by considering an adiabatic pumping frequency in the terahertz range, \( \omega/2\pi \sim 1.0 \) THz [24]. Assuming typical values in the experiments for the other parameters, \( W/L \sim 10–100 \), \( \delta X_{1,2}/L \sim 0.01–0.1 \) and \( \varphi = \pi/2 \), we have a pumped current around \( I_p \sim 0.5–500 \) nA. This is due to the efficient contribution of the evanescent modes in the pumped current generated by the vibrating thin barriers. This value for the pumped current at the Dirac point is easily accessible in the experiment.

6. Conclusion

In conclusion we have investigated a new mechanism for driving a graphene pump. In this graphene pump two thin potential barriers are employed to drive the system out of equilibrium using two different methods of magnitude oscillation and position vibration. The case of driving by oscillating thin barriers is similar to the graphene pump considered in [24]. As expected, for large carrier concentrations our results tend to their results. This is due to the fact that at large carrier concentrations the exact configuration of the pump has an insignificant effect on the pumped current. However, there are important differences in vicinity of the Dirac point. The minimum of the pumped current, arising due to the contribution of the evanescent modes, depends on the pump configuration, and does not have a universal value despite the results of [24]. Also, the pumped current changes sign around the Dirac point due to the opposite contributions of the evanescent and extended modes. On the other hand, new features appear in the case of vibrating thin barriers. The pumped current has an increasing oscillatory behavior around zero as a function of the carrier concentration. This is due to the increasing contribution of the extended modes in the pumped current with increasing carrier concentration. Comparison with a similar normal pump indicates that this is a unique feature of
Dirac fermions. Also, the normalized pumped current diverges at the Dirac point due to the more effective contribution of the evanescent modes when the thin barriers have nonzero magnitudes. Due to these facts, we can conclude that driving by oscillating and vibrating thin barriers are very effective methods for generating a pumped current in graphene. Thus, despite the practical difficulties for experimental realization of the considered pump we believe that it has a good chance of being confirmed by experiment.

We have considered the thin potential barriers as delta function potentials in our calculations. This is a limited situation, and in practice we expect a determinate width for the thin potential barriers. This results in a complex dependence of the pumped current on the width and height of the potential barriers. We will consider this situation in subsequent works.

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