A precise sum rule among four $B \rightarrow K\pi$ CP asymmetries 1

Michael Gronau

Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

and

Physics Department, Technion
Haifa 32000, Israel

A sum rule relation is proposed for direct CP asymmetries in $B \rightarrow K\pi$ decays. Leading terms are identical in the isospin symmetry limit, while subleading terms are equal in the flavor SU(3) and heavy quark limits. The sum rule predicts $A_{\text{CP}}(B^0 \rightarrow K^0\pi^0) = -0.17 \pm 0.06$ using current asymmetry measurements for the other three $B \rightarrow K\pi$ decays. A violation of the sum rule would be evidence for New Physics in $b \rightarrow s\bar{q}q$ transitions.

PACS codes: 12.15.Hh, 12.15.Ji, 13.25.Hw, 14.40.Nd

CP asymmetry measurements in neutral $B$ decays involving an interference between $B^0$–$\bar{B}^0$ mixing and $b \rightarrow \bar{c}\bar{c}s$ or $b \rightarrow \bar{u}\bar{u}d$ transitions improve our knowledge of the Cabibbo-Kobayashi-Maskawa (CKM) phases $\beta \equiv \text{Arg}(-V^*_{cb}V_{cd}/V^*_{tb}V_{td})$ and $\alpha \equiv \text{Arg}(-V^*_{tb}V_{td}/V^*_{ub}V_{ud})$ beyond information obtained from all other CKM constraints [1]. While time-dependent asymmetries in $b \rightarrow s\bar{q}q$ transitions ($q = u, d, s$) indicate a potential deviation from $\sin 2\beta$ [2], the current statistical significance of the discrepancy is insufficient for claiming a serious anomaly.

Extraction of the weak phase $\gamma \equiv \text{Arg}(-V^*_{ub}V_{ud}/V^*_{cb}V_{cd})$ from the direct CP asymmetry measured recently in $B^0 \rightarrow K^+\pi^-$ [3] [4] is obstructed by large theoretical uncertainties in strong interaction phases. Direct CP asymmetries can provide evidence for New Physics in $B^+ \rightarrow \pi^+\pi^0$, where the Standard Model predicts a vanishing asymmetry, including tiny electroweak penguin contributions [5]. Other tests based on direct asymmetries, which require studying carefully U-spin symmetry breaking effects, are provided by pairs of processes, e.g. $B^+ \rightarrow K^0\pi^+$ and $B^+ \rightarrow \bar{K}^0K^+$ or $B^0 \rightarrow K^+\pi^-$ and $B_s \rightarrow \pi^+K^-$, in which CP asymmetries are related by U-spin symmetry interchanging $d$ and $s$ quarks [6]. Precision tests would be provided by CP asymmetry relations, in which isospin relates dominant terms in the asymmetries while flavor SU(3) relates subdominant terms. The motivation for this study is proposing such a relation among $B \rightarrow K\pi$ asymmetries, a violation of which could serve as an alternative clue for physics beyond the Standard Model in $b \rightarrow s\bar{q}q$ transitions.

1To be published in Physics Letters B.
Table I: CP asymmetries $A_{CP}$ for $B \to K\pi$ decays. In parentheses are corresponding branching ratios in units of $10^{-6}$.

| Decay mode | Babar \cite{7} | Belle \cite{8} | Average |
|------------|----------------|----------------|---------|
| $B^0 \to K^+\pi^-$ | $-0.133 \pm 0.030 \pm 0.009$ | $-0.113 \pm 0.022 \pm 0.008$ | $-0.120 \pm 0.019$ |
| | $(19.2 \pm 0.6 \pm 0.6)$ | $(18.5 \pm 1.0 \pm 0.7)$ | $(18.9 \pm 0.7)$ |
| $B^+ \to K^+\pi^0$ | $0.06 \pm 0.06 \pm 0.01$ | $0.04 \pm 0.04 \pm 0.02$ | $0.05 \pm 0.04$ |
| | $(12.0 \pm 0.7 \pm 0.6)$ | $(12.0 \pm 1.3_{-0.9}^{+1.3})$ | $(12.1 \pm 0.8)$ |
| $B^0 \to K^0\pi^0$ | $-0.06 \pm 0.18 \pm 0.03$ | $0.11 \pm 0.18 \pm 0.08$ | $0.02 \pm 0.13$ |
| | $(11.4 \pm 0.9 \pm 0.6)$ | $(11.7 \pm 2.3_{-1.3}^{+1.3})$ | $(11.5 \pm 1.0$ |
| $B^+ \to K^0\pi^+$ | $-0.09 \pm 0.05 \pm 0.01$ | $0.05 \pm 0.05 \pm 0.01$ | $-0.02 \pm 0.04$ |
| | $(26.0 \pm 1.3 \pm 1.0)$ | $(22.0 \pm 1.9 \pm 1.1)$ | $(24.1 \pm 1.3)$ |

Direct CP asymmetries in all four $B \to K\pi$ decay processes, measured by the Babar \cite{7} and Belle \cite{8} collaborations, are quoted in Table I together with their averages. (We do not quote earlier CLEO measurements which involve considerably larger errors.) One defines by convention

$$A_{CP}(B \to f) \equiv \frac{\Gamma(B \to \bar{f}) - \Gamma(B \to f)}{\Gamma(B \to f) + \Gamma(B \to f)} .$$

A nonzero asymmetry was measured in $B^0 \to K^+\pi^-$, $A_{CP} = (-12.0 \pm 1.9)\%$, where the experimental error is smallest among the four $B \to K\pi$ decays. The other three asymmetries, of which that in $B \to K^0\pi^0$ involves the largest experimental error, are consistent with zero. Table I quotes also for later use corresponding CP-averaged branching ratios in units of $10^{-6}$ \cite{9}, including a very recent Babar measurement of $B(B^0 \to K^+\pi^-)$ \cite{10}.

The purpose of this Letter is to prove a sum rule among the four $B \to K\pi$ CP rate differences,

$$\Delta(K^+\pi^-) + \Delta(K^0\pi^+) \approx 2\Delta(K^+\pi^0) + 2\Delta(K^0\pi^0) ,$$

where we define

$$\Delta(B \to f) \equiv \Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f) .$$

This sum rule, reminiscent of a similar sum rule among partial decay rates \cite{11,12}, is expected to hold within an accuracy of several percent. Using the approximation (see branching ratios in Table I and the discussion in the paragraph below Eq. \cite{24}),

$$\Gamma(K^+\pi^-) \approx \Gamma(K^0\pi^+) \approx 2\Gamma(K^+\pi^0) \approx 2\Gamma(K^0\pi^0) ,$$

this implies at a somewhat lower precision a sum rule among CP asymmetries,

$$A_{CP}(K^+\pi^-) + A_{CP}(K^0\pi^+) \approx A_{CP}(K^+\pi^0) + A_{CP}(K^0\pi^0) .$$

The equality of leading terms in the sum rule \cite{24} will be shown to follow from isospin symmetry, while subleading terms are equal in the flavor SU(3) and heavy quark limits.
For the most part, we will make no assumption relating $B \to K\pi$ decays to $B \to \pi\pi$ decays.

A somewhat less precise relation excluding $A_{CP}(K^0\pi^+)$ in \[\text{13}\], which holds under more restricted conditions, was proposed recently in a broader context \[\text{13}\]. An over-simplified but too crude relation,

$$A_{CP}(K^+\pi^-) \sim A_{CP}(K^+\pi^0),$$

was suggested several years ago \[\text{11}\] making too strong an assumption about color-suppressed tree amplitudes. The latter relation, which is quite far from what is being measured (see Table I), has recently provoked discussions about an anomalously large color-suppressed amplitude, an enhanced electroweak penguin amplitude and possible New Physics effects \[\text{14}\].

Let us recapitulate for completeness the structure of hadronic amplitudes in charmless $B$ decays, specifying carefully our assumptions. The effective Hamiltonian governing $B \to K\pi$ decays is given by \[\text{15}\]

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left[ \sum_{U=u,c} \lambda_U (c_1 O^U_1 + c_2 O^U_2) - \lambda_t \sum_{i=3}^{10} c_i O_i \right].$$

where $\lambda_U = V^*_{Ub} V_{Us}$, $\lambda_t \equiv V^*_{tb} V_{ts}$. The dozen operators $O^U_j$ and $O_i$ are four-quark operators, with given flavor and chiral structure, including current-current operator $O^U_{1,2}$, QCD-penguin operators $O_i$, $i = 3 - 6$, and electroweak penguin (EWP) operators $O_i$, $i = 7 - 10$. The real Wilson coefficients, which were calculated beyond the leading logarithmic approximation, are $c_1 \approx 1.10$, $c_2 \approx -0.20$, $c_{3-6} \sim \text{few} \times 10^{-2}$, $c_{7,8} \sim \text{few} \times 10^{-4}$, $c_9 \approx -0.010$ and $c_{10} \approx 0.0020$. Contributions of $O_7$ and $O_8$ can be safely neglected, as one does not expect a huge enhancement of their hadronic matrix elements relative to those of $O_9$ and $O_{10}$. The latter operators, involving larger Wilson coefficients, have a $(V-A)(V-A)$ structure similar to the current-current operators.

All four quark operators can be written as a sum of SU(3) representations, \[\overline{15}, 6\] and \[3\], into which the product $\overline{3} \otimes \overline{3} \otimes 3$ can be decomposed \[\text{16, 17}\]. Current-current and EWP operators which involve the same $(V-A)(V-A)$ structure consist of identical SU(3) operators. Thus one finds simple proportionality relations between current-current (here denoted by a subscript $T$ for “tree”) and EWP operators belonging to \[\overline{15}\] and \[6\] representations \[\text{5, 18, 19}\],

$$H_{\text{EW P}}(\overline{15}) = -\frac{3c_9 + c_{10}}{2} \frac{\lambda_t}{\lambda_u} H_T(\overline{15}),$$

$$H_{\text{EW P}}(6) = \frac{3c_9 - c_{10}}{2} \frac{\lambda_t}{\lambda_u} H_T(6).$$

These operator relations have useful consequences in $B \to K\pi$ decay amplitudes. The first relation was applied in \[\text{18}\] and both relations were used in \[\text{5}\]. In the following discussion we will apply SU(3) to the subdominant EWP amplitudes by using Eqs. (8)–(9). Dominant terms in $B \to K\pi$ asymmetries will be shown to be related by isospin symmetry alone.
The effective Hamiltonian (7) permits a general decomposition of the four $B \to K\pi$ amplitudes into terms of distinct topologies representing hadronic matrix elements of corresponding operators in (7). Using the unitarity of the CKM matrix, $\lambda_u + \lambda_c + \lambda_t = 0$, and defining $P_{tc} \equiv P_t - P_c, P_{uc} \equiv P_u - P_c$, one has (10):

$$- A(K^+\pi^-) = \lambda_u (P_{uc} + T) + \lambda_t (P_{tc} + 2/3 P_{EW}^C),$$
$$-\sqrt{2}A(K^+\pi^0) = \lambda_u (P_{uc} + T + C + A) + \lambda_t (P_{tc} + P_{EW} + 2/3 P_{EW}^C),$$
$$\sqrt{2}A(K^0\pi^0) = \lambda_u (P_{uc} - C) + \lambda_t (P_{tc} - P_{EW} - 1/3 P_{EW}^C),$$
$$A(K^0\pi^+) = \lambda_u (P_{uc} + A) + \lambda_t (P_{tc} - 1/3 P_{EW}^C).$$

The amplitudes $P_u, T, C, A$ and $P_c$ are contributions from the first sum in (7), corresponding to $U = u$ and $U = c$, respectively, while $P_t, P_{EW}$ and $P_{EW}^c$ originate from the second sum. The terms $P, T, C$ and $A$ represent penguin, color-allowed tree, color-suppressed tree and annihilation topologies, respectively. Specific EW contributions were expressed in terms of color-allowed and color suppressed amplitudes, $P_{EW}$ and $P_{EW}^c$, using a simple substitution (10):

$$\lambda_u C \to \lambda_u C + \lambda_t P_{EW}, \quad \lambda_u T \to \lambda_u T + \lambda_t P_{EW}^C, \quad \lambda_u P_{uc} \to \lambda_u P_{uc} - \frac{1}{3} \lambda_t P_{EW}^C. \quad (14)$$

The four physical amplitudes can also be decomposed into three isospin amplitudes (20), a contribution $B_{1/2}$ with $I(K\pi) = 1/2$ from the isosinglet part of $H_{\text{eff}}$, and two amplitudes $A_{1/2,3/2}$ with $I(K\pi) = 1/2, 3/2$ from the isotriplet part of $H_{\text{eff}}$:

$$- A(K^+\pi^-) = B_{1/2} - A_{1/2} - A_{3/2},$$
$$-\sqrt{2}A(K^+\pi^0) = B_{1/2} + A_{1/2} - 2A_{3/2},$$
$$\sqrt{2}A(K^0\pi^0) = B_{1/2} - A_{1/2} + 2A_{3/2},$$
$$A(K^0\pi^+) = B_{1/2} + A_{1/2} + A_{3/2}. \quad (18)$$

One has

$$B_{1/2} = \lambda_u \left[ P_{uc} + \frac{1}{2} (T + A) \right] + \lambda_t \left[ P_{tc} + \frac{1}{6} P_{EW}^C \right], \quad (19)$$
$$A_{1/2} = -\lambda_u \frac{1}{6} [T - 2C - 3A] + \lambda_t \frac{1}{3} \left[ P_{EW} - \frac{1}{2} P_{EW}^C \right], \quad (20)$$
$$A_{3/2} = -\lambda_u \frac{1}{3} [T + C] - \lambda_t \frac{1}{3} \left[ P_{EW} + P_{EW}^C \right]. \quad (21)$$

Eqs. (10)-(13) are quite general, providing a common basis for QCD calculations of $B \to K\pi$ amplitudes (21,22). The terms in parentheses involve magnitudes of hadronic amplitudes and strong interaction phases, which are hard to calculate without making further assumptions. For instance, the term $P_c$ may involve sizable long distance “charming penguin” contributions which must be fitted to the data (23). Our following
arguments will be independent of specific hadronic calculations, relying mainly on isospin and flavor SU(3) symmetry properties of certain terms. SU(3) breaking corrections will be estimated using generalized factorization [24].

We will use Eqs. (8) and (9), which imply approximate SU(3) relations between 
P_{EW}, P_{EW}^C, on the one hand, and \( T \) and \( C \), on the other [5][15].

\[ P_{EW} + P_{EW}^C \approx -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} (T + C) \],  
\[ P_{EW}^C \approx -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} C \].  

(22)

(23)

In the second equation we used \((c_9 - c_{10})/(c_1 - c_2) \approx (c_9 + c_{10})/(c_1 + c_2) \) [15], neglecting a small exchange contribution [25] which vanishes at leading order in \(1/m_b\) and \(\alpha_s\) [28]. SU(3) breaking effects on \((22)\), calculated by using generalized factorization [24], were found to be about 10% in the magnitude of ratio \((P_{EW} + P_{EW}^C)/(T + C)\) and less than 5° in its phase. Similar effects will be assumed in \((23)\), as estimated by similar considerations.

The terms in the amplitudes \([10]-[13] \) multiplying \(\lambda_i\) dominate the decay amplitudes because \(|\lambda_u/\lambda_t| \approx 0.02\). The penguin amplitude \(P_{tc}\) is pure isosinglet, thus contributing equally to the two decay amplitudes involving a charged pion and contributing a term smaller by factor \(\sqrt{2}\) to the two amplitudes involving \(\pi^0\). Dominance by \(P_{tc}\) is exhibited clearly by the four \(K\pi\) branching ratios in Table I which obey Eq. (11) to a reasonable approximation. (The effect of a lifetime difference between \(B^+\) and \(B^0\) will be discussed later.) All other terms in \([10]-[13] \) are smaller than \(\lambda_iP_{tc}\) and may be considered subdominant.

Using Eq. (22) and noting that \(T + C\) dominates the amplitude of \(B^+ \to \pi^+\pi^0\) [16], the measured ratio of branching ratios \(B(\pi^+\pi^0)/B(K^0\pi^+)\) shows that the higher order electroweak amplitude \(P_{EW} + P_{EW}^C\) is indeed much smaller than \(P_{tc}\) [18][29],

\[ \left| \frac{P_{EW} + P_{EW}^C}{P_{tc}} \right| \approx \frac{3}{2} \frac{|c_9 + c_{10}|}{|c_1 + c_2|} \frac{\sqrt{f_\pi}}{f_K} \frac{|V_{cb}|}{|V_{ub}|} \frac{B(\pi^+\pi^0)}{B(K^0\pi^+)} \approx 0.11 \],  

(24)

where \(f_\pi\) and \(f_K\) are meson decay constants, and a value \(B(\pi^+\pi^0) = (5.5 \pm 0.6) \times 10^{-6}\) [9] was used.

We will not assume color suppression for \(C\) and \(P_{EW}^C\), nor will we assume that \(P_{uc}\) is smaller than \(T\) or \(C\). That is, the triplet of amplitudes \((T, C, P_{uc})\) and the doublet \((P_{EW}, P_{EW}^C)\) could each consist of amplitudes with comparable magnitudes. Several questions have been raised recently concerning these relative magnitudes [30][31][32][33][34] in view of an apparent disagreement with a hierarchy assumption [16] \(|C| \sim 0.2|T|\), \(|P_{EW}^C| \sim 0.2|P_{EW}|\) and with calculations in QCD [21][22]. We will make use of the fact that the amplitude \(A\) and the strong phase of \(C/T\) vanish to leading order in \(1/m_b\) and \(\alpha_s\) [28]. We note that a small value of Arg\((C/T)\) is not favored by a global SU(3) fit to all \(B\) meson decays into two charmless pseudoscalars [32], although the error on the output value of this phase is still very large. While the fit assumes common magnitudes and strong phases for SU(3) amplitudes in \(B \to K\pi\) and \(B \to \pi\pi\) decays, our assumption about SU(3) in Eqs. (22)-(23) is restricted to \(B \to K\pi\). As
mentioned, these SU(3) breaking effects have been calculated to be very small implying $|\text{Arg}[(P_{EW} + P_{EW}^C)/(T + C)]| < 5^\circ$.

Direct CP asymmetries in $B \to K\pi$ processes occur through the interference of two terms in the amplitudes involving different CKM factors, $\lambda_t$ and $\lambda_u$, and different strong phases. Using the definition (3) we find

$$\Delta(K^+\pi^-) = \text{Im} \left[ (P_{tc} + \frac{2}{3}P_{EW}^C)(T + P_{uc})^* \right] I ,$$  
(25)

$$2\Delta(K^0\pi^0) = \text{Im} \left[ (P_{tc} + 2P_{EW} + \frac{2}{3}P_{EW}^C)(T + C + A + P_{uc})^* \right] I ,$$  
(26)

$$2\Delta(K^0\pi^0) = \text{Im} \left[ (P_{tc} - P_{EW} - \frac{1}{3}P_{EW}^C(-C + P_{uc})^* \right] I ,$$  
(27)

$$\Delta(K^0\pi^+) = \text{Im} \left[ (P_{tc} - \frac{1}{3}P_{EW}^C(A + P_{uc})^* \right] I ,$$  
(28)

where $I = 4\text{Im}(\lambda_t\lambda_u^*)$ is a common CKM factor.

Combining the four CP rate differences by defining a difference $\delta_{K\pi}$ between pairs involving charged and neutral pions,

$$\delta_{K\pi} \equiv \Delta(K^+\pi^-) + \Delta(K^0\pi^+) - 2\Delta(K^+\pi^0) - 2\Delta(K^0\pi^0) ,$$  
(29)

we find

$$\delta_{K\pi} = -\text{Im} \left[ (P_{EW} + P_{EW}^C(T + C)^* + (P_{EW}C^* - P_{EW}^C T^*) + (P_{EW} + P_{EW}^C)A)^* \right] I .$$  
(30)

All terms involving the dominant $P_{tc}$ term cancel in $\delta_{K\pi}$. This follows from isospin symmetry [35], as these terms describe an interference of $P_{tc}$ with a term multiplying $\lambda_u$ in a combination which vanishes by Eqs. (15)-[13],

$$- A(K^+\pi^-) + A(K^0\pi^0) + \sqrt{2}A(K^+\pi^0) - \sqrt{2}A(K^0\pi^0) = 0 .$$  
(31)

All the terms on the right-hand-side of (30) involve EWP amplitudes and are thus suppressed relative to corresponding terms involving $P_{tc}$ by about an order of magnitude. The first term vanishes in the SU(3) limit because it involves an interference of two contributions which carry a common strong phase by [22]. A potential SU(3) breaking strong phase difference, $|\text{Arg}[(P_{EW} + P_{EW}^C)/(T + C)]| < 5^\circ$ [24], suppresses this term by at least an order of magnitude. The second term vanishes in the SU(3) limit at leading order in $1/m_b$ and $\alpha_s$, as can be seen by using [22]-[23] and $\text{Arg}(C/T) \approx 0$. This implies a suppression of this term either by an order of magnitude from SU(3) breaking, or by $1/m_b$ and $\alpha_s$. The last term on the right-hand-side involves an interference between two subdominant amplitudes, $P_{EW} + P_{EW}^C$ and $A$, each of which is suppressed relative to corresponding leading terms, $P_{tc}$ and $T$, respectively. Since all three terms on the right-hand-side of (30) are doubly suppressed relative to $\Delta(K^+\pi^-)$ by two factors each about an order of magnitude, we expect the ratio $\delta_{K\pi}/\Delta(K^+\pi^-)$ to be at most several percent. Therefore, one may safely take $\delta_{K\pi} = 0$ which proves [2].

The proposed sum rule may be written in terms of CP asymmetries, taking into account differences among the four $B \to K\pi$ CP-averaged branching ratios and the $B^+$
to $B^0$ lifetime ratio $\tau_+/\tau_0 = 1.076 \pm 0.008$ [9]:

$$A_{\text{CP}}(K^+\pi^-) + A_{\text{CP}}(K^0\pi^+)\frac{B(K^0\pi^+)}{B(K^+\pi^-)}\frac{\tau_0}{\tau_+} = A_{\text{CP}}(K^+\pi^0)2\frac{B(K^+\pi^0)}{B(K^+\pi^-)}\frac{\tau_0}{\tau_+} + A_{\text{CP}}(K^0\pi^0)2\frac{B(K^0\pi^0)}{B(K^+\pi^-)}.$$  (32)

Using branching ratios from Table I, we predict a negative CP asymmetry in $B^0 \to K^0\pi^0$ in terms of the other asymmetries,

$$A_{\text{CP}}(K^0\pi^0) = -0.17 \pm 0.06.$$  (33)

This value is not inconsistent with the average measured value in Table I. Alternatively, the approximate sum rule [5] among CP asymmetries reads in terms of corresponding current measurements,

$$(-0.120 \pm 0.019) + (-0.02 \pm 0.04) \approx (0.05 \pm 0.04) + (0.02 \pm 0.13).$$  (34)

While central values on the two sides have opposite signs, errors in the asymmetries (in particular that in $B^0 \to K^0\pi^0$) must be reduced before claiming a discrepancy.

The proposed sum rule [2] makes no assumption about the smallness of the amplitudes $P_{uc}$ and $C$ relative to $T$, or about their strong phases relative to that of the dominant $P_{tc}$ amplitude. The contribution of $P_{uc}$ to the asymmetries has been neglected in a sum rule suggested recently when studying $b \to s$ penguin amplitudes in $B$ meson decays into two pseudoscalars [13]. A $P_{uc}$ contribution comparable to $T$ would be observed by a nonzero $A_{\text{CP}}(K^0\pi^+)$, unless the strong phase of $P_{uc}$ relative to $P_{tc}$ is very small. A sizable $P_{uc}$ comparable to $T$ is an output of a global SU(3) fit to $B \to K\pi$ and $B \to \pi\pi$ decays [32]. Bounds on $A_{\text{CP}}(K^0\pi^+)$ derived from Table I favor a small relative phase between $P_{tc}$ and a sizable $P_{uc}$. Another output of the fit, a large amplitude $C$ comparable to $T$, also obtained in separate analyses of $B \to K\pi$ [33] and $B \to \pi\pi$ [34], provides a simple interpretation for the failure of the oversimplified relation [6] which had assumed $C$ to be color-suppressed.

To conclude, we have shown that direct CP asymmetries in the four $B \to K\pi$ decay processes obey the sum rule [2] within several percent, or the sum rule [5] in the approximation of equal rates in [1]. Isospin and flavor SU(3) symmetries have been used to relate leading QCD penguin and subleading electroweak penguin terms in the sum rule, respectively. While we assumed a suppression of an annihilation amplitude relative to a color-allowed tree amplitude and a suppression of $\text{Arg}(C/T)$, no assumption was made about the magnitudes of color-suppressed tree, electroweak penguin amplitudes and a term $P_{uc}$ associated with intermediate $u$ and $c$ quarks. A violation of the sum rule would provide evidence for New Physics in $b \to s\bar{q}q$ transitions. The most likely interpretation of the origin of a potential violation would be an anomalous $\Delta I = 1$ operator in the effective Hamiltonian. A generalization of our argument using [31] implies that in the isospin symmetry limit contributions to CP asymmetries from any $\Delta I = 0$ operator cancel in the sum rule.
I wish to thank the SLAC theory group for its very kind hospitality. I am grateful to Helen Quinn, Jonathan Rosner and Denis Suprun for very useful discussions. This work was supported in part by the Department of Energy contract DE-AC02-76SF00515, by the Israel Science Foundation founded by the Israel Academy of Science and Humanities, Grant No. 1052/04, and by the German-Israeli Foundation for Scientific Research and Development, Grant No. I-781-55.14/2003.

References

[1] For two very recent reviews, see M. Gronau, invited talk presented at the Tenth International Conference on B Physics at Hadron Machines (Beauty 2005), 20–24 June 2005, Assisi, Perugia, Italy, [http://www.pg.infn.it/beauty2005](http://www.pg.infn.it/beauty2005). Y. Sakai, invited talk presented at the 25th International Symposium on Physics in Collision (PIC 2005), 6–9 July 2005, Prague, Czech Republic, [http://www.particle.cz/conferences/pic2005](http://www.particle.cz/conferences/pic2005).

[2] For two very recent reviews, see K. Abe, invited talk presented at the 22nd International Symposium on Lepton-Photon Interactions at High Energy (LP 2005), 30 June – 7 July 2005, Uppsala, Sweden, [http://lp2005.tsl.uu.se/~lp2005](http://lp2005.tsl.uu.se/~lp2005). R. Bartoldus, invited talk presented at the 25th International Symposium on Physics in Collision (PIC 2005), 6–9 July 2005, Prague, Czech Republic, [http://www.particle.cz/conferences/pic2005](http://www.particle.cz/conferences/pic2005).

[3] Babar Collaboration, B. Aubert et al., Phys. Rev. Lett. 93 (2004) 131801.

[4] Belle Collaboration, Y. Chao et al., Phys. Rev. Lett. 93 (2004) 191802; K. Abe et al., [hep-ex/0507045](http://arxiv.org/abs/hep-ex/0507045).

[5] M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D 60 (1999) 034021.

[6] M. Gronau, Phys. Lett. B 492 (2000) 297.

[7] Babar Collaboration, B. Aubert et al., Ref. [3]; Phys. Rev. Lett. 94 (2005) 181802; [hep-ex/0503011](http://arxiv.org/abs/hep-ex/0503011) [hep-ex/0507023](http://arxiv.org/abs/hep-ex/0507023).

[8] Belle Collaboration, Ref. [4]; Y. Chao et al., Phys. Rev. D 71 (2005) 031502; K. Abe et al., [hep-ex/0507037](http://arxiv.org/abs/hep-ex/0507037).

[9] J. Alexander et al., Heavy Flavor Averaging Group, [hep-ex/0412073](http://arxiv.org/abs/hep-ex/0412073). Updated results and references are tabulated periodically by this group: [http://www.slac.stanford.edu/xorg/hfag/rare](http://www.slac.stanford.edu/xorg/hfag/rare).

[10] Babar Collaboration, reported by W. T. Ford at the 25th International Symposium on Physics in Collision (PIC 05), 6–9 July 2005, Prague, Czech Republic, [http://www.particle.cz/conferences/pic2005](http://www.particle.cz/conferences/pic2005).

[11] M. Gronau and J. L. Rosner, Phys. Rev. D 59 (1999) 113002.
[12] H. J. Lipkin, Phys. Lett. B 445 (1999) 403.

[13] M. Gronau and J. L. Rosner, Phys. Rev. D 71 (2005) 074019.

[14] M. Neubert, talk presented at the Third Workshop on the Unitarity Triangle: CKM 2005, 15–18 March 2005, San Diego, CA, http://ckm2005.ucsd.edu/; W. S. Hou, M. Nagashima and A. Soddu, hep-ph/050307; M. Beneke, talk presented at the International Conference on QCD and Hadronic Physics, 16–20 June 2005, Beijing, China, http://www.phy.pku.edu.cn/qcd/.

[15] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125.

[16] M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D 50 (1994) 4529; Phys. Rev. D 52 (1995) 6374.

[17] B. Grinstein and R. F. Lebed, Phys. Rev. D 53 (1996) 6344.

[18] M. Neubert and J. L. Rosner, Phys. Lett. B 441 (1998) 403.

[19] M. Gronau, Phys. Rev. D 62 (2000) 014031.

[20] H. J. Lipkin, Y. Nir, H. R. Quinn and A. Snyder, Phys. Rev. D 44 (1991) 1454; M. Gronau, Phys. Lett. B 265 (1991) 389.

[21] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606 (2001) 245; M. Beneke and M. Neubert, Nucl. Phys. B 675 (2003) 333.

[22] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Lett. B 504 (2001) 6; Y. Y. Keum and A. I. Sanda, Phys. Rev. D 67 (2003) 054009.

[23] M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. B 501 (1997) 271; M. Ciuchini, R. Contino, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. B 512 (1998) 3; [Erratum-ibid. B 531 (1998) 656].

[24] M. Neubert, JHEP 9902 (1999) 014.

[25] Evidence for suppression by an order of magnitude of an exchange amplitude relative to a tree amplitude exists in $B^0 \rightarrow D_s^- K^+$ [26]. The measured suppression factor in the charmless decay process $B^0 \rightarrow K^+ K^-$, for which a strict upper limit has been recently obtained [10] [27], has not yet reached this level.

[26] S. Eidelman et al. (Particle Data Group Collaboration), Phys. Lett. B 592 (2004) 1.

[27] Belle Collaboration, K. Abe et al., hep-ex/0506080.

[28] C. W. Bauer and D. Pirjol, Phys. Lett. B 604 (2004) 183; C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 70 (2004) 054015.

[29] M. Gronau, J. L. Rosner and D. London, Phys. Rev. Lett. 73 (1994) 21.
[30] T. Yoshikawa, Phys. Rev. D 68 (2003) 054023.

[31] M Gronau and J. L. Rosner, Phys. Lett. B 572 (2003) 43.

[32] C. W. Chiang, M. Gronau, J. L. Rosner and D. Suprun, Phys. Rev. D 70 (2004) 034020. This study uses instead of $P_{uc}$ a quantity $P_{tu} = V_{ub}^* V_{ud} (P_{tc} - P_{uc})$.

[33] S. Baek, P. Hamel, D. London, A. Datta and D. Suprun, Phys. Rev. D 71 (2005) 057502.

[34] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Phys. Rev. Lett. 92 (2004) 101804; Nucl. Phys. B 697 (2004) 133; Acta Phys. Polonica 36 (2005) 2015.

[35] D. Atwood and A. Soni, Phys. Rev. D 58 (1998) 036005. This paper mentioned a similar sum rule among CP differences of branching ratios rather than partial decay rates, using isospin symmetry and neglecting EWP contributions. I thank David Atwood and Amarjit Soni for pointing this out.