Waves propagating in a randomly uneven medium with correlation length larger than the wavelength $\lambda$ can form filaments in a tree-branch-like manner [1]. This phenomenon, known as branched flow, has been observed in diverse systems and at many length scales [2–20]. For example, instead of smoothly diffusing or spreading, an electron beam passing through two-dimensional (2D) electron gases can form branching strads that become successively narrower [2–6], giant freak ocean waves are attributed to the random unevenness on the ocean floor [7–11], and branching of microwave radiation emitted by pulsars is attributed to the interstellar dust clouds [12–14]. Recently, branched flow of light (specifically continuous-wave laser) has been found on thin soap films [17, 18]. The light branching is attributed to variations of the film’s refractivity $\eta$, which bend and bundle the light rays at favorable locations and form caustics [17–20].

Despite its complexity, branched flow is usually a linear phenomenon and the distance $d_0$ from the source to the first branching point follows the scaling $d_0 \propto l_c v_0^{-2/3}$ [1, 15, 21–24], where $l_c$ is the correlation length of the medium’s unevenness, and $v_0$ is its strength parameter (to be defined later).

At present, lasers with intensity $I$ ranging from $10^{14}$ to $10^{20}$ W/cm$^2$ are readily available. The propagation of such intense laser light through matter sets off new phenomena [25]. When $I \gtrsim 10^{14}$ W/cm$^2$, the atomic Coulomb barrier is suppressed by the strong laser electric field, electrons are set free and the affected medium is ionized into plasma [26], whose optical properties then become dominated by electron dynamics. Moreover, at higher laser intensities $I \gtrsim 1.37 \times 10^{18}$ W/cm$^2$, namely, the laser intensity is above the relativistic threshold, in addition to photoionization, the laser ponderomotive force and relativistic plasma motion can significantly modify the original unevenness in the density as well as the local refractivity, thereby affecting the laser propagation [25]. Whether light branching can occur in such intense laser-matter interaction involving with complex nonlinear effects is a timely and critical question.

In this Letter, we present the first investigation of nonlinear branched flow of intense light in laser-matter interaction. Particle-in-cell (PIC) simulations show that laser branching can occur at moderate laser intensities ($I \approx 10^{14} – 10^{17}$ W/cm$^2$) in inhomogeneous plasma with randomly uneven density distribution. In contrast to linear light branching, in this regime the branching depends on the laser intensity. In particular, photoionization induced by the strong laser electric field raises the density unevenness along the laser paths and enhances the branching. However, relativistic lasers can suppress branching by smoothing the unevenness and thus the local refractivity of the plasma. An analysis of the branching process and the resulting properties consistent with the simulation results is also given.

Light branching is usually three dimensional. However, if the irregularity of the uneven background medium is isotropic, branching can be effectively 2D [1, 21, 28–30]. Accordingly, we shall conduct 2D PIC simulations of intense-laser branching using the EPOCH code [31].

In the simulations, the initial background medium (see Fig. 1(a), in blue color at the bottom) is weakly pre-ionized SiO$_2$ plasma (with Si$^{2+}$ and O$^+$ ions) with uneven density distribution located in $0 < x < 215 \mu m$, $-55 \mu m < y < 55 \mu m$. The density unevenness is of an isotropic correlation length $l_\mu = 4.8 \mu m$, as obtained from the autocorrelation function (ACF) [32] shown in Fig. 1(c). The average densities of Si$^{2+}$, O$^+$, and...
laser has not been reported before. A useful quantity for characterizing the branch pattern is the scintillation index \( \Sigma = (\langle I^2 \rangle / \langle I \rangle^2)^{1/2} - 1 \), which measures the relative strength of intensity fluctuations [11]. For statistical accuracy, here \( \Sigma \) is obtained from simulations using a reference planewave with otherwise identical interaction parameters. We see that \( \Sigma \) increases from 0 at the plasma front surface to 1.23 at \( x = 19.8 \) \( \mu m \). The dependence of \( \Sigma \) on \( x \) agrees well with the branch pattern of the laser intensity. Since optical turbulence is defined by \( \Sigma > 1 \) [39], we can consider that the laser pulse evolves from Gaussian to strongly fluctuating one within only 19.8 \( \mu m \), much shorter than that of typical filamentation instabilities [40–45]. We emphasize that the laser branching observed here is quite different from laser filamentation, where a sufficiently intense laser can break up into narrow parallel filaments, usually without further bifurcation as they propagate.

A feature of intense-laser branching is the nonlinear response of the background plasma medium, which changes the refractivity and the optical unevenness along the laser paths. In Fig. 2(a), we show the evolution of potential strength \( v_0 = \sqrt{(\langle n^2 - n_{eff}^2 \rangle^2)/2n^2} \), or unevenness, of the plasma by the action of laser with intensity ranging from \( 10^{14} \) to \( 10^{20} \) \( \text{W/cm}^2 \) as obtained from PIC simulations. Here, \( n^2 = \langle n_{eff}^2 \rangle \) is the mean-square value of the plasma refractivity \( n_{eff} \). For laser intensity below the relativistic threshold (\( \lesssim 10^{17} \) \( \text{W/cm}^2 \)), \( v_0 \) reaches a plateau after a sharp increase by photoionization. However, in the relativistic regime (\( I \gtrsim 10^{18} \) \( \text{W/cm}^2 \)), \( v_0 \) after the peaking decrease continuously to even smaller values than the initial one. The decrease is due to plasma homogenization by the laser interaction, at a time scale estimated to be \( \tau = t_c/2c_s \), where \( c_s = \sqrt{Z/Z_2 a_0^2 \omega^2} \) is the ion acoustic speed, \( Z \) and \( m_i \) are the charge number and rest mass of each ion species, \( T_e \sim (\gamma - 1)mc_e^2 \) is the bulk electron temperature [46], \( \gamma = (1 + a_0^2) \) for circular polarization, \( a_0 = e(\omega E)/m_e c \omega \) is the normalized laser electric field \( E \), \( m_e \) and \( -e \) are the electron rest mass and charge, \( \omega \) is the laser frequency, and \( c \) is the speed of light in vacuum. One can see in Fig. 2(b) that for non-relativistic lasers, \( \tau \) is much larger than the pulse duration. Therefore, the plasma unevenness \( v_0 \) can be considered as quasistatic after the rapid increase by photoionization, consistent with the plateaus shown in Fig. 2(a). In contrast, for relativistic lasers, \( \tau \) becomes comparable or smaller than the pulse duration. In this case, removal of the density unevenness by the laser interaction becomes dominant. The background plasma loses its original uneven quality along the laser path, resulting in the pronounced decrease of \( v_0 \).

For non-relativistic lasers, since the plasma homogenization during the laser interaction can be neglected and the refractivity of the uneven plasma can be considered as slow varying, propagation of the laser
FIG. 2. (a) Evolution of the potential strength $v_0$ for different laser intensities in the region $0 < x < 30 \, \mu m$, $-2 \, \mu m < y < 2 \, \mu m$ within the laser spot. (b) Characteristic time $\tau$ of plasma homogenization for different laser intensities. The black dotted line marks the pulse duration. The blue and red patches in (a) and (b) mark the non-relativistic and relativistic intensity regimes, respectively.

can be described by the Helmholtz equation [17, 30]

$$-\nabla^2 E + k_0^2 (\eta^2 - \eta_{\text{eff}}^2) E = k_0^2 \eta^2 E,$$

where $k_0 = 2\pi/\lambda_0$ is the wavenumber in vacuum, $\eta_{\text{eff}} = \sqrt{1 - n_e(I)/\gamma}$ is the effective refractivity, and $\eta^2 = \langle \eta_{\text{eff}}^2 \rangle$ with average taken over the laser spot area. Note that the electron number density $n_e$ is an explicit function of the instantaneous laser intensity $I$ since electrons are produced by photoionization. In this case, one obtains $v_0 = \sqrt{\delta(I)^2/2(n_e - n_e(I)/\gamma)}$, where $\delta(I) = n_e(I)/\gamma - n_e(I)/\gamma$ is the local fluctuating strength of the electron density, and $\gamma \approx 1$ for non-relativistic lasers. We see that since $v_0$ includes the effect of photoionization, it increases with $I$ due to the increased ionization rate, in agreement with the simulation results in Fig. 2(a). Since laser branching is directly related to $v_0$, the branching is also enhanced by photoionization.

Comparison with simulations where photoionization is switched off further confirms the above analysis. As shown Fig. 3(a), considerable increase of the electron density $n_e$ along the laser paths is observed if photoionization is included. At the laser intensity $I = 1 \times 10^{16} \, \text{W/cm}^2$, tunneling ionization of both the Si and O ions to the +4 state can occur [26, 30]. For our simulation parameters, the average $n_e$ around the laser axis is $\sim 0.24n_e$, three-fold of the initial value. In addition, since plasma homogenization can be ignored in the non-relativistic regime, the electrons have the initial disordered distribution of the ions [30]. Therefore, the ACF of the electron density at $t = 1095 \, \text{fs}$ remains almost the same as the initial one. Likewise, the local potential strength $v_0(x)$ keeps its initial correlations (indicated by the vertical streaks) after the rapid increase due to photoionization, as shown in Fig. 3(c). Note that the correlation length $l_c$ of 4.8 $\mu m$ is much smaller than the laser spotsize, i.e. the effect of Gaussian intensity profile across the beam on $l_c$ can be ignored. The increase of $v_0$ leads to stronger (but still relatively weak) scattering of the laser, resulting in the enhancement of the branch pattern, as shown in Fig. 3(b).
To further characterize the branching, we consider the angle dependence of the laser electric field in the Fourier space, defined by \( \tilde{E}_z(\phi) = | \int e^{-i k_0 x \cos \phi} e^{-i k_0 y \sin \phi} E_z(\phi) dx dy | \). Here, \( E_z \) is used instead of \( E_y \) to exclude the self-generated fields. As shown in Fig. 3(d), the dependence of \( \tilde{E}_z(\phi) \) on \( \phi \) at \( t = 35.3 \) fs is quite similar for both cases, indicating that most of the laser energy still flows in the \( x \) direction. However, at \( t = 1095 \) fs, as a result of many successive weak scatterings in the uneven plasma, a large amount of laser light is branched into other directions. The spread angle \( \Theta \) of the light branches after the first caustics can be defined as that when \( \tilde{E}_z \) drops to \( 1/4 \) of its maximum. We find \( \Theta \sim 2\pi/9 \) when photoionization is included, which is about two times larger than that without photoionization. This result further demonstrates that photoionization enhances the unevenness of the refractivity and thus the branching.

For \( I > 1.37 \times 10^{18} \) W/cm\(^2\) relativistic laser, most of the electrons on the outer shells of the ions are freed and they can be accelerated to light speed by the laser fields within a single cycle. In this case, further ionization becomes marginal and relativistic laser-plasma-interaction effects become significant. The plasma homogenization time \( \tau \sim 217 \) fs becomes much smaller than the pulse duration. The local refractivity along the laser path now changes simultaneously as the laser propagates, and Eq. (1) becomes inapplicable [30]. In fact, Figs. 4(a) and (b) show that the unevenness in the initial electron density distribution vanishes right behind the laser pulse front. Rapid plasma homogenization leads to the decrease of \( l_e \), and electron resonance in the laser fields causes longitudinal modification of the density distribution and the ACF, as can be seen in Fig. 4(b). In addition, the strong laser ponderomotive force expels the affected electrons, resulting in the formation of plasma channel behind the laser front [41–44], and further reduction of the density unevenness, as shown in Fig. 4(c). Figure 4(e) shows that the corresponding local potential strength \( v_0(x) \) decreases to much less than the initial one after the rapid increase caused by photoionization. The initial correlation of the unevenness also vanishes. As shown in Fig. 4(d), branching of the laser is suppressed and its spread angle \( \Theta \) remains small at \( 2\pi/67 \).

Figure 5(a) for the spread angle \( \Theta \) of the laser branches for different initial laser intensities shows that \( \Theta \) increases with \( I_0 \) until \( I_0 \lesssim 10^{17} \) W/cm\(^2\), then it decreases as \( I_0 \) increases further. This is in good agreement with the dependence of the potential strength \( v_0 \) on the laser intensity at \( t = 1095 \) fs shown in Fig. 2(a). Such dependence of \( \Theta \) on \( I_0 \) can be considered as evidence of nonlinear branched flow of intense laser light in experiments.

Another parameter for characterizing flow-branching is the distance \( d_0 \) from the boundary (where the flow enters) of the uneven medium to the first branching point. In the linear case, the flow has no influence on the medium, and a universal scaling law for \( d_0 \) is \( d_0 \propto l_e v_0^{-2/3} \). For non-relativistic picosecond lasers where plasma homogenization can be ignored, and \( v_0 = \)
The spread angle $\Theta$ (in radians) of the laser branches at $t = 1095$ fs for different initial laser intensities. The distance from the source to the first caustics $d_0$ (in $\mu$m) for different laser intensities. The red solid curve is obtained from simulations at the time when the first caustics appear. The blue patch shows the evolution of the flucuting strength given by $\langle \delta(I)^2 \rangle$, and increases with $\langle n_e - n_e(I)/\gamma \rangle$, indicating that the higher the effective plasma density $n_e/\gamma$ is, the earlier branching occurs. The quasilinear scaling agrees well with the branched flow of non-relativistic lasers, as shown in Fig. 5(b). However, for $I > 10^{18}$ W/cm$^2$ relativistic lasers, laser branching becomes suppressed due to plasma homogenization. The first caustics, instead of being a branching point, now mark the location where self-focusing starts. Figure 5(b) shows that $d_0$ now increases with the laser intensity. It is of interest to note that Eq. (2) still agrees fairly well with the simulation results, even though Eq. (1) no longer holds in this regime.

In conclusion, we have shown that an intense laser propagating through uneven plasma can form complex light branches. Photoionization can raise unevenness in the density, and thus enhance branch formation. However, relativistic effects of too-intense lasers can suppress branch formation by smoothing the plasma unevenness. These regimes can be potentially verified by experiments based on laser interaction with pre-ionized low-density fibrous or foamy materials, or gas clusters. Our work extends the existing studies of optical branching to the nonlinear regime. The results can be relevant to optical communications, nonlinear optics, strong field physics, as well as laser interaction with foam or turbulent plasma.

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