Electromagnetic Transitions with Related Quantities for $^{80}$Se and $^{82}$Kr Nuclei

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Abstract. The Hamiltonian is determined in the Interacting Boson Model-1 (IBM-1) framework for $^{80}$Se and $^{82}$Kr nuclei. Fit values of parameters are used to construct the Hamiltonian, energy levels, electromagnetic transitions (B(E2) and B(M1)) and multipole mixing ratios (E2/M1) for some even-even Se and Kr nuclei. These properties are calculated. The potential energy surfaces (PESs) to the IBM Hamiltonian have been obtained by using the intrinsic coherent state.

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1. Introduction

The concept of collectivity is one of the most fundamental findings in history of nuclear physics. Various nuclear models have been applied to describe this collective behavior of atomic nuclei like the geometrical models depicting the nucleus as a liquid drop with a given nuclear shape and algebraic models; take into account the pairs of proton and/or neutron only. Initially, Arima and Iachello had been introduced the Interacting Boson Model (IBM) which rather successful to described the collective properties of several medium and heavy nuclei [1-5]. The ingredients of the IBM were s monopole and d quadrupole bosons and it correspond to collective nucleon-pairs with angular momenta $J^\pi = 0^+$ and $2^+$ [6, 7], respectively. The IBM Hamiltonian was exact solutions in three dynamical symmetry limits U(5), SU(3) and O(6). It was geometrically analogous to the harmonic vibrator, axial rotor and $\gamma$-unstable rotor, respectively [8-10]. The another phenomenological study indicated that nuclei might have an intermediate structure of the U(5)$-$SU(3), U(5)$-$O(6) and SU(3)$-$O(6) limits [11-13].

In neutron number $N = 46$, $^{78}$Se and $^{80}$Kr nuclei have atomic number $Z = 34$ and 36, respectively. These nuclei with protons ($Z$) > 28, and neutrons ($n$) < 50 are allowed to occupy $g_{9/2}$, $p_{1/2}$, $p_{3/2}$ and $f_{5/2}$ orbitals. In the past few years, low-lying energy states and electromagnetic transition properties in several isotopes like Se and Kr have been measured for the first time or re-measured with higher precision in the A $\approx$ 82 mass region [14-21]. The aim of this work is to calculate the energy levels and electromagnetic transitions probabilities of B(E2) and B(M1), multipole mixing ratios and potential energy surface in the
80Se and 82Kr nuclei using the IBM-1. These calculations will be compared with the experimental data.

2 Method of Calculations
The IBM-1 model in its group theoretical formulation exhibits three dynamical symmetries and each it is corresponding to a particular way of breaking the degeneracy of the parent U(6) group.

\[ U(6) \supset \begin{cases} U(5) \supset O(5) \\ SU(3) \\ O(6) \supset O(5) \end{cases} \supset O(3) \]  

The IBM-1 Hamiltonian can be expressed as [6, 22, 23]:

\[ H = \varepsilon_s (s^\dagger s) + \varepsilon_d (d^\dagger d) + \sum_{L=0,2,4} \frac{1}{2} \left[ \frac{1}{2} c_L \left[ (d^\dagger x d^\dagger)^{(L)} \times (d \times d)^{(L)} \right]^{(0)} + \frac{1}{2} v_L \left[ (d^\dagger x s^\dagger)^{(2)} \times (d \times d)^{(2)} \right]^{(0)} 
+ \frac{1}{2} u_L \left[ (s^\dagger x s^\dagger)^{(2)} \times (d \times d)^{(2)} \right]^{(0)} \right] \]  

where \( \varepsilon_s \) and \( \varepsilon_d \) are the single-boson energies (in one body terms) and \( c_L \) (L = 0, 2 and 4), \( v_L \) (L = 0, 2), \( u_L \) (L = 0, 2) (in the two-body terms). However, \( N_b = n_s + n_d \), is the total number of boson conserved [23].

According to the Hamiltonian [6, 23], it can be discussed the calculated results separately, and plotting the potential energy surface \((N_b, \beta, \gamma)\) gives a final shape to the nucleus.

\[ E(N_b, \beta, \gamma) = a_2 N_b (N_b - 1) \left[ (1 + 3/4 \beta^4 - \sqrt{2} \beta^3 \cos 3\gamma) / (1 + \beta^2) \right], \ldots SU(3) \]  

\[ E(N_b, \beta, \gamma) = a_0 N_b (N_b - 1) \left[ (1 - \beta^2) / (1 + \beta^2) \right]^2, \ldots O(6), \]  

where \( \beta \) and \( \gamma \) are the intrinsic deformation parameters which determine the geometrical shape of the nucleus. These expression give (for large \( N_b \)) \( \beta_{\text{min}} = 0, \sqrt{2}, \) and 1 for U(5), SU(3), and O(6), respectively.

3 Results and discussion
In the framework of the IBM-1, the Selenium (Z=34) and the Krypton (Z=36) nuclei with neutron number \( N = 46 \). The Z values are near mid shell and the n value is near closed shell that would suggest structure the O(6) symmetry and O(6) – U(5) transition for 80Se and 82Kr, respectively. These nuclei have 5 and 6 bosons, respectively.

The energy ratio \( R_{4/2} = E_{4^+} / E_{2^+} \) used as a starting point to express the degree and type of collectivity. It is a good indicator of the shape deformation of the nucleus. Its value is 10/3 for the well-deformed nuclei SU(3), 2.5 for O(6) or \( \gamma \)-unstable nuclei and 2 for vibrational U(5), 2.2 for the analytically solvable symmetry E(5) on the U(5) - O(6) path and 2.9 for the approximate X(5) symmetry on the U(5)-SU(3) path[23, 26-30].

In Table 1, the experimental values of \( R = E_{4^+} / E_{2^+} \) of low–lying energy levels of 80Se and 82Kr nuclei are presented. From this Table, the \( R_{4/2} \) attains of the O(6) value is ~ 2.5 for the 80Se and for 82Kr lie close to the E(5) symmetry.
Table 1: The ratio \( \frac{R_{4/2}}{E_{2+}} = \frac{E_{41} + E_{21}}{\gamma} \) for \(^{80}\)Se and \(^{82}\)Kr nuclei [31-33].

| Nucleus | \(^{80}\)Se | \(^{82}\)Kr |
|---------|---------|---------|
| \( \frac{R_{4/2}}{E_{2+}} \) | 2.55 | 2.34 |

In IBM-1, the calculations have been performed with PHINT code [34] and the number of bosons which calculated from the sum of the proton bosons and the neutron bosons (no distinction made between neutron and proton bosons) of the close shells (28 and 50). The Hamiltonian in equation (2) can be written in general form as [23, 35]:

\[
\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{P} + a_1 \hat{L} + a_2 \hat{Q} + a_3 \hat{T}_3 + a_4 \hat{T}_4
\]

where \( \varepsilon \) is the boson energy, and the operators are:

\[
\hat{n}_d = \left( d^\dagger \cdot d \right)
\]

\[
\hat{P} = \frac{1}{2} \left( \left( d^\dagger \cdot d \right) - \left( s^\dagger \cdot s \right) \right)
\]

\[
\hat{L} = \sqrt{\lambda} \left( d^\dagger \times d \right)
\]

\[
\hat{Q} = \left[ d^\dagger \times \left( s^\dagger \times d \right) \right]^{(2)} + \chi \left( d^\dagger \times d \right)^{(2)}
\]

\[
\hat{T}_r = \left[ d^\dagger \times d \right]^{(r)}
\]

Here, \( \hat{n}_d \) is the total number of d-boson operator, \( \hat{P}, \hat{L}, \hat{T}_3, \hat{T}_4 \) represent pairing, angular momentum, octupole and hexadecapole interactions between the bosons, respectively. \( \hat{Q} \) is the quadrupole operator (\( \chi \) is the quadrupole structure parameter and take the values 0 and \( \pm \sqrt{7} \) [9, 36, 37]) and \( a_0, a_1, a_2, a_3 \) and \( a_4 \) represent the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecapole interaction between the bosons, respectively.

This Hamiltonian is able to give symmetries with the following parameters:

U(5): \( a_0 = 0, \chi = 0 \), SU(3): \( \varepsilon = 0, \chi = \pm \frac{\sqrt{7}}{2} \) and O(6): \( \varepsilon = 0, \chi = 0 \) [23].

The eigenvalues for these three limits are given by [23]:

\[
E = \varepsilon \left( n + \beta \right) + 2 \gamma L(L + 1) + 2 \delta L(L + 1) \quad \text{U(5)}
\]

\[
E = \frac{a_2}{2} \lambda^2 + \lambda \mu + \mu \nu + 3(\lambda + \mu) + \left( a_1 - \frac{3a_2}{8} \right) L(L + 1) \quad \text{SU(3)}
\]

\[
E = a_0/4 \left( N_b - \sigma \right) (N + \sigma + 4) + a_3/2 \tau (\tau + 3) + \left( a_1 - a_3/10 \right) \left( L(L + 1) \right) \quad \text{O(6)}
\]

The parameters of the IBM-1 Hamiltonian (eq.6) are given the best fitting between theoretical and experimental energy levels [31-33] of the above nuclei that presented in the Table 2 and \( a_2 \) and \( a_4 \) are equal to the zero.

Table 2: Adopted values of the parameters used for IBM-1 calculations. All parameters are given in MeV, excepted \( N_b \) for Se and Kr nuclei.

| Isotopes | \( N_b \) | \( \varepsilon \) | PAIR | ELL | OCT |
|---------|---------|---------|------|-----|-----|
| \(^{80}\)Se | 5 | --- | 0.123 | 0.091 | 0.055 |
| \(^{82}\)Kr | 6 | 0.25 | 0.106 | 0.112 | 0.062 |

(PAIR = \( a_2/2 \), ELL = \( 2a_1 \) and OCT = \( a_3/5 \)) [23].

For even-even Se and Kr nuclei, the calculated and experimental values of ground (gr), \( \beta^- \) and \( \gamma^- \) bands are plotted and shown in the Figure 1. In the Figure 1, the calculated energy levels are in good agreement with the experimental [31-33] ones for Se and Kr nuclei. The increase of the calculated values of the excitation energies with the angular momentum is too fast compare to the experimental data. Splitting of the \( 3^- -- 4^+ \) states in the gamma-band is too small compare to the experimental values. Some levels have been confirmed and other levels have been predicted. This leads to the prediction of electromagnetic transmissions within these levels. From our view, we are believed that it is opening the way for a new study of the practical measurements in the future.
Levels with "( )" in gr, γ and β states correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally.

Figure 1: (Color online) comparisons between the calculated IBM-1 and the experimental data [31-33] for $^{80}$Se and $^{82}$Kr with $N=46$.

Besides excitation energy spectra, the electromagnetic transitions are important factors within the collective nuclear structure and can also be analyzed in the framework of the IBM-1. The most general E2 transition operator can be written as: [1, 23, 38]

$$T(E2) = \alpha_2 [d^s s^s d^{d^s} + \beta_2 [d^d d^d]](2) = e_B \bar{Q}$$ (9),

where $(s^s, d^d)$ and $(s, d)$ are creation and annihilation operators for $s$ and $d$ bosons, respectively, while $\alpha_2$ and $\beta_2$ are two parameters, and $(\beta_2 = \chi \alpha_2, \alpha_2 = e_B (effective \ charge \ of \ boson))$. The electric transition probabilities ($B(E2)$ values) are defined in terms of reduced matrix elements as [23, 39]:

$$B(E2; L_i \rightarrow L_f) = \frac{1}{2L_i+1} |\langle L_f | T(E2) | L_i \rangle|^2$$ (10)

The values of effective charge $e_B = \alpha_2$ are estimated from the experimental $B(E2)$ value of the transition $(2^+_1 \rightarrow 0^+_1)$ and presented in Table 3. It is used to calculate the reduced probability for E2 transitions of the IBM-1 model. The $B(E2)$ in the low lying states for Se and Kr nuclei with the neutron number $N=46$ and compared with the experimental data [31-33] are presented in Table 4.

Table 3: Parameter (in eb) used to reproduce $B(E2)$ values for $^{80}$Se and $^{82}$Kr nuclei.

|       | A  | N_b | e_B  |
|-------|----|-----|------|
| $^{80}$Se | 5  | 0.074 |
| $^{82}$Kr | 6  | 0.061 |

The reduced transition probabilities $B(E2; 2^+_1 \rightarrow 0^+_1)$ and $B(E2; 4^+_2 \rightarrow 2^+_1)$ values decrease as proton number increases toward the middle of the shell in the Se and Kr nuclei. Description of $B(E2; 2^+_3 \rightarrow 2^+_1)$ is bad. Their experimental values are smaller than $B(E2; 2^+_1 \rightarrow 0^+_1)$, however, the results of calculations demonstrate an opposite tendency. The calculated values of $B(E2; 2^+_3 \rightarrow 0^+_2)$ are much larger than the experimental ones. The experimental data indicate on a possibility of the shape coexistence in these nuclei connected to the $0^2_2$ state. All most of the calculated results in IBM-1 are reasonably consistent with the available experimental data, except for few cases that deviate from the experimental data [31-33]. The $M1$ transition operator can be written as [6, 40]:

$$T(M1) = (g_B + A1N_b) \hat{L} + B[\hat{T}(E2) \times \hat{L}] + C\hat{n}_d\hat{L}$$ (11),

where $g_B$ (atomic number (Z)/mass number (A)) is the effective boson g factor [40, 41], $N_b$ is the number of bosons, $\hat{L}$ is the angular momentum operator, $\hat{T}(E2)$ is matrix elements of the
E2 operator, \( \hat{n}_d \) is d-boson number operator. The fitting of E2 matrix elements is essentials for the calculation of the M1 matrix elements, because their share with the \( \delta(E2/M1) \).

Table 4: The IBM-1 and Experimental [31-33] values of the B(E2) values for \(^{80}\text{Se}\) and \(^{82}\text{Kr}\) nuclei (in \( e^2\text{b}^2 \)).

| \( L_i \rightarrow L_f \) | \(^{80}\text{Se}\) EXP. | IBM EXP. | \(^{82}\text{Kr}\) EXP. | IBM |
|--------------------------|--------|--------|----------------|--------|
| \( 2^+_1 \rightarrow 0^+_1 \) | 0.0503 | 0.0504 | 0.0449 | 0.0440 |
| \( 2^+_2 \rightarrow 2^+_1 \) | 0.0377 | 0.0639 | 0.0116 | 0.0579 |
| \( 2^+_2 \rightarrow 2^+_2 \) | -- | 0.0000 | -- | 0.0000 |
| \( 2^+_2 \rightarrow 0^+_2 \) | 0.0001 | 0.0235 | 0.0001 | 0.0229 |
| \( 3^+_1 \rightarrow 2^+_1 \) | -- | 0.0061 | -- | 0.0040 |
| \( 3^+_1 \rightarrow 2^+_2 \) | -- | 0.0440 | -- | 0.0423 |
| \( 3^+_1 \rightarrow 4^+_1 \) | -- | 0.0137 | -- | 0.0132 |
| \( 4^+_1 \rightarrow 2^+_1 \) | 0.0718 | 0.0639 | 0.0675 | 0.0579 |
| \( 4^+_1 \rightarrow 4^+_1 \) | 0.0571 | 0.0293 | 0.0633 | 0.0282 |
| \( 5^+_1 \rightarrow 4^+_1 \) | -- | 0.0033 | -- | 0.0024 |

The parameters \( g_B, A1, B \) and \( C \) are used in the present work to calculate the \( T(M1) \) transitions and it is presented in Table 5. The calculated reduced probability for \( M1 \) transitions and the experimental data [34–36] are given in Table 6 for nuclei under study.

Table 5: The parameters of \( T(M1) \) used in the present work. All parameters are given in (\( \mu_N \)), except \( N_b \).

| Nucleus | \( N_b \) | \( g_B \) | \( A1 \) | \( B \) | \( C \) |
|---------|--------|--------|--------|--------|--------|
| \(^{80}\text{Se}\) | 5 | 0.425 | -0.083 | -0.29\times10^{-4} | -0.095 |
| \(^{82}\text{Kr}\) | 6 | 0.439 | -0.071 | -0.44\times10^{-4} | -0.192 |

Table 6: The IBM-1 and Experimental [31-33] values of the B(M1) for \(^{80}\text{Se}\) and \(^{82}\text{Kr}\) nuclei.

| \( L_i \rightarrow L_f \) | \(^{80}\text{Se}\) EXP. | IBM EXP. | \(^{82}\text{Kr}\) EXP. | IBM |
|--------------------------|--------|--------|----------------|--------|
| \( 2^+_2 \rightarrow 2^+_1 \) | 0.0007 | 0.0004 | 0.0008 | 0.0015 |
| \( 4^+_2 \rightarrow 4^+_1 \) | 0.0214 | 0.0007 | 0.0116 | 0.0027 |
| \( 8^+_3 \rightarrow 7^+_1 \) | -- | -- | 0.0179 | 0.0018 |

Instead of evaluate the E2 and M1 matrix elements for the Se and Kr nuclei under study which are essential in the theoretical mixing ratio \( \delta(E2/M1) \) calculations. It is possible to determine these ratios in an analytical form. The ratio \( \Delta(E2/M1) \) is defined as: the ratio of the reduced E2 matrix element to the reduced M1 matrix element and can be written as [23, 40]:

\[
\Delta(E2/M1) = \frac{\langle L_i || T(E2) || L_f \rangle}{\langle L_i || T(M1) || L_f \rangle}
\]

The quantity of this ratio is related to the usual \( \delta \)-mixing ratio given by [23, 42],

\[
\delta(E2/M1) = 0.835 [E_{\gamma}/(1\text{MeV})] \Delta(E2/M1)
\]

where \( E_{\gamma} \) is in MeV and \( \Delta(E2/M1) \) is in e\( ^2\text{b} \). By using Eq. (13), the \( \delta \) (E2/M1) multipole mixing ratios of the electromagnetic transitions between the energy states were calculated for Se and Kr nuclei. The calculated values are given in Table 7.

Table 7 The IBM-1 and the experimental values of \( \delta(E2/M1) \) multipole mixing ratios for Selenium and Krypton nuclei.

| \( I_i \rightarrow I_f \) | EXP. | IBM-1 | \(^{80}\text{Se}\) | \(^{82}\text{Kr}\) |
|--------------------------|--------|--------|--------|--------|
| \( J_i \rightarrow J_f \) | \(^{80}\text{Se}\) | \(^{82}\text{Kr}\) |
The potential energy surfaces $E(N_{\text{b}}, \beta, \gamma)$ are plotted in the Figure 2. This figure shows that $^{80}\text{Se}$ is a deformed and has $\gamma$-unstable-like characters ($\gamma \approx \frac{\pi}{6}$) and the shape phase transition from $\gamma$-unstable $O(6)$ to vibrational $U(5)$ symmetry for $^{82}\text{Kr}$.

\begin{table}
\begin{tabular}{|c|c|c|c|c|}
\hline
      & $E_\gamma$(MeV) & $\delta$(E2/M1) & $E_\gamma$(MeV) & $\delta$(E2/M1) \\
\hline
$^{80}\text{Se}$ & & & & \\
$2_2^+ \rightarrow 2_1^+$ & 0.783 & 4.827 & 0.837 & 8.736 \\
$4_2^+ \rightarrow 4_1^+$ & 0.793 & 1.079 & 1.116 & 6.127 \\
\hline
$^{82}\text{Kr}$ & & & & \\
$2_2^+ \rightarrow 2_1^+$ & 0.698 & 2.164 & 1.067 & 5.579 \\
$4_2^+ \rightarrow 4_1^+$ & 0.606 & 0.335 & 1.417 & 3.873 \\
$8_2^+ \rightarrow 7_1^+$ & 0.416 & 1.102 & 0.134 & 0.231 \\
\hline
\end{tabular}
\end{table}

4 Conclusions

The energy levels are calculated using IBM-1 for $^{80}\text{Se}$ and $^{82}\text{Kr}$ nuclei with neutron number $N=46$. The result shows good agreement with published experimental data. The increase of the calculated values of the excitation energies with the angular momentum is too fast compared to the experimental data. Splitting of the $3^+ \rightarrow 4^+$ states in the gamma-band is too small compared to the experimental values. The reduced transition probabilities $B(E2)$, $B(M1)$ and $\delta(E2/M1)$ values have been calculated using Interacting Boson Model (IBM). The reduced transition probabilities $B(E2; 2_1^+ \rightarrow 0_1^+)$ and $B(E2; 4_1^+ \rightarrow 2_1^+)$ values decrease as proton number increases toward the middle of the shell in the Se and Kr nuclei. Description of $B(E2; 2_2^+ \rightarrow 2_1^+)$ is bad. Their experimental values are smaller than $B(E2; 2_1^+ \rightarrow 0_1^+)$, however, the results of calculations demonstrate an opposite tendency. The calculated values of $B(E2; 2_2^+ \rightarrow 0_2^+)$ are much larger than the experimental ones. The experimental data indicate on a possibility of the shape coexistence in these nuclei connected to the $0_2^+$ state. The contour plot of PES shown that the $^{80}\text{Se}$ is a deformed and has $\gamma$-unstable-like characters, while $^{82}\text{Kr}$ lie close to the $E(5)$ symmetry.

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References

[1] A. Arima and F. Iachello, Advances in Nuclear Physics, (Plenum Press, New York) 13, 139 (1984).
[2] O. Scholten, Interacting Boson in Nuclear Physics, edited by F. Iachello (Plenum Press, New York), 17 (1979).
[3] G. Puddu, O. Scholten and T. Otsuka, Nucl. Phys. A 348, 109(1980).
[4] R. Bijker, A. E. L. Dieperink, O. Scholten and R. Spano, Nucl. Phys. A 344, 207(1980).
[5] I. M. Ahmed, Hewa Y. Abdullah, Mudhaffer M. Ameen, Huda H. Kassim and Fadhil I. Sharrad, Physics of Atomic Nuclei, 81, 695 (2018).
[6] K. A. Hussain, M. K. Mohsin and F. I. Sharrad, Ukrainian Journal of Physics 62, 653(2017).
[7] A. Bohr and B. R. Mottelson, Benjamin, Reading, Massachussets, Vol.11, (1975).
[8] A. Arima, F. Iachello, Ann. Phys. (N.Y.) 99, 253 (1976).
[9] A. Arima, F. Iachello, Ann. Phys. (N.Y.) 111, 201 (1978).
[10] A. Arima, F. Iachello, Ann. Phys. (N.Y.) 123, 468 (1979).
[11] F. Iachello, Phys. Rev. Lett. 85, 3580 (2000).
[12] P. Cejnar, J. Jolie, R.F. Casten, Rev. Mod. Phys. 82, 2155 (2010).
[13] R.F. Casten, E.A. McCutchan, J. Phys. G 34, R285 (2007).
[14] F. I. Sharrad, I. Hossain, I. M. Ahmed, H. Y. Abdullah, S. T. Ahmad and A. S. Ahmed Braz J Phys 45, 340 (2015).
[15] F. I. Sharrad, H. Y. Abdullah, N. Al-Dahan, N. M. Umran, A. A. Okhunov and H. Abu-Kassim, Chinese Physics C 37, 034101 (2013).
[16] I. Hossain, I. M. Ahmed, F. I. Sharrad, H. Y. Abdullah, A. D. Salman and N. Al-Dahan, Chiang Mai J. Sci. 42, 996 (2015).
[17] A. Shelley, I. Hossain, Fadhil I Sharrad, Hewa Y Abdullah and M A Saeed Prob. Atom. Sci. & Tech. 64 38 (2015).
[18] I. Hossain, H. H. Kassim, F. I. Sharrad and A. S. Ahmed, ScienceAsia 42, 22 (2016).
[19] M. A. Al-Jubbory, H. H. Kassim, F. I. Sharrad and I. Hossain, Nucl. Phys. A 955, 101 (2016).
[20] H. H. Khudher, A. K. Hasan and F. I. Sharrad, Ukrainian Journal of Physics 62, 152 (2017).
[21] M. O. Waheed and F. I. Sharrad, Ukrainian Journal of Physics 62, 757 (2017).
[22] K. Abrahams, K. Allaart and A. E. L. Dieperink, Nuclear Structure, Plenum press, New York and London (1981).
[23] R. F. Casten and D. D. Warner, Rev. Mod. Phys. 60, 389 (1988).
[24] A.E.L. Dieperink, O. Scholten, F. Iachello, Phys. Rev. Lett. 44, 1747 (1980).
[25] I. M. Ahmed, G. N. Flaiyh, H. H. Kassim, H. Y. Abdullah, I. Hossain and F. I. Sharrad, Eur. Phys. J. Plus 132, 84 (2017).
[26] F. Iachello, Phys. Rev. Lett. 85, 3580 (2000).
[27] F. Iachello, Phys. Rev. Lett. 87, 052502 (2001).
[28] R. F. Casten and N. V. Zamfir, Phys. Rev. Lett. 87, 30 (2001).
[29] H. H. Kassim and F. I. Sharrad, Nucl. Phys. A 933, 1 (2015).
[30] M. A. Al-Jubbory, F. Sh. Radhi, A. A. Ibrahim, S. A. Abdullah Albakrid, H. H. Kassim and F. I. Sharrad, Nuclear Physics A 971, 35 (2018).
[31] http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp.
[32] B. Singh, Nuclear Data Sheets, 105, 223 (2005).
[33] J. K. Tuli, Nuclear Data Sheets 98, 209 (2003).
[34] O. Scholten, Computer code PHINT, KVI; Groningen, Holland, (1980).
[35] M. A. Al-Jubbory, H. H. Kassim, F. I. Sharrad, A. Attarzadeh and I. Hossain, Nuclear Physics A 970, 438 (2018).
[36] A. Okhunov, F. I. Sharrad, A. A. Al-Sammareea and M. U. Khandaker, Chinese Physics C 39, 084101 (2015).
[37] H. H. Kassim, A. A. Mohammed-Ali, M. Abed Al-Jubbory, F. I. Sharrad, A.S. Ahmed and I. Hossain, J. Natn. Sci. Foundation Sri Lanka 46, 3 (2018).
[38] M. A. Al-Jubbory, K. A. Al-Miityu, K. I. Saeed and F. I. Sharrad, Chinese Physics C 41, 084103 (2017).
[39] I. Hossain, F. I. Sharrad, M. A. Saeed, H. Y. Abdullah and S. A. Mansour, Maejo Int. J. Sci. Technol. 10, 95 (2016).
[40] H. H. Kassim and F. I. Sharrad, International Journal of Modern Physics E 23, (2014) 1450070.
[41] M. O. Waheed and F. I. Sharrad, NUCLEAR PHYSICS AND ATOMIC ENERGY 18, 313(2017).
[42] H. H. Kassim, A. A. Mohammed-Ali, F.I. Sharrad, I. Hossain and K. S. Jassim, Iran J Sci Technol Trans Sci 42, 993 (2018).