Modelling Negotiations of Construction Subcontract based on a Game Theory – Results of an Experiment

Hubert Anysz

1 Warsaw University of Technology, Civil Engineering Department, I. Armii Ludowej 16, 00-637 Warsaw, Poland

h.anysz@il.pw.edu.pl

Abstract. The process of negotiation can be analyzed considering some scientific issues (e.g. psychological, sociological). When a construction subcontract is negotiated, as it is business activity, the rational choices should be considered more than other factors influencing the decisions of the negotiating parties. A game theory supports rational decision making based on calculations, expected values and strategic choices. An experiment was carried with the participation of the students of Civil Engineering Faculty. Five different games were prepared. The students started playing with the “nature”. Then the probabilities of the “nature” strategies were introduced. Finally, they played three one-on-one sets of repetitive games. The results prove that knowing the needs of the opposite player (knowing his/her possible strategies) leads to a win-win result where the score of both players is maximized. The experiment and its result are analyzed in the paper. The multiple negotiations, and then multiple cooperation of a general contractor with the same subcontractor on construction projects is an often case. These parties discuss the rules of their cooperation formalized in subcontracts several times. The typical image of a general contractor’s need is a low price given by a subcontractor. In fact, it is only one of the needs. Moreover, it is not a rare case where flexibility of engagement on a building site or a high number of equipment units or favorable terms of payment are desired by a general contractor more than the low price given by a subcontractor. A subcontractor’s standing can be different too. They can search for profitable cooperation or they have to provide work for some brigades not to have them unengaged. They can afford long term payment or they have to be paid in advance. The experiment proves that disclosure of the needs (strategies) by negotiating parties leads to achievement of higher gain for both of them (when the game is repetitive). The paper sets the basis for modelling negotiations of a construction subcontract based on a game theory.

1. Introduction

Scientific approaches to the negotiation processes (psychological, sociological) and a theory of rational choices – a game theory – were opposed to proposed way of solving conflicts presented in the book [1] gave inspiration to this article. The experiment based on [2], [3], [4] (set of the games and descriptions to them) was prepared to illustrate the students – the experiment participants – that the information about possible strategies of opposite player (negotiating party) can improve a lot the score achieved by both parties. In order to achieve top result in a final game, players were required to choose also the strategies where in a simple attempt they lost. Review of modern application of a game theory has shown the wide spectrum of its possible use [5] supporting decision making. Models based on the game theory were
applied for renewal of construction objects [6], analysis of bidding procedure [7] and profit distribution in construction projects [8]. The need to support the negotiation processes in a construction industry was recognized too [9]. The example of an application of the game theory to managing conflict situation between a client and a general contractor is presented in [10]. The model presented hereinafter is appropriate for negotiations between a general contractor and a subcontractor. The process of negotiations is less formalized (if a client is not a party) and there is a possibility of repeating cooperation of the same two parties. The above mentioned and an interesting results of the experiment were the reasons for creating mathematical model based on the game theory, of a subcontract negotiating.

2. The experiment

The experiment was run by the author as a part of a subject “Organisation and Steering of Construction Site” presented on the master course of Civil Engineering Faculty at Warsaw University of Technology for students who have chosen “Engineering of Construction Processes” specialisation. It is aimed at modelling of business situation where the conflict of interest usually exists and it proves to students that the clear presentation of the needs and cooperation of negotiating parties can make both parties winners. The students’ groups participated in the experiment during four semesters (spring 2017 to autumn 2018). The complete set i.e. 5 games was played in 1,5 hours. The lecture was interspersed with games and instructions to them. That’s why the engagement of the players was usually increasing up to final cooperative game.

2.1. Game 1 – against the nature

Following pay-off matrix was given to participants:

| Player’s Strategies | NATURE |
|---------------------|--------|
|                     | A  | B  | C  | D  |
| P                   | 1  | 0  | -1 | -2 |
| Q                   | 4  | -8 | 7  | -7 |
| R                   | -5 | -1 | -9 | 10 |
| S                   | -1 | 3  | -2 | -1 |

**Figure 1.** The player’s pay-offs in Game 1

The random character of nature’s choices was explained, as well as, the players’ aim of the game – to achieve the highest total result summarised from 16 attempts. The students were writing down their choices (from P, Q, R, S strategies), then the nature announced its chosen strategy (from A, B, C, D) in a single attempt. Then, they wrote down their scores summarized in table 1.

| Results          | Score achieved |
|------------------|----------------|
| Minimum          | -73            |
| Maximum          | 35             |
| Mean Average     | -6,16          |
| Standard deviation | 22,39         |

**Table 1.** Result achieved by participants in Game 1.

There were 180 fully fulfilled form sheets that students had been given empty before the start of the game. They were told that the nature randomly chooses its strategy, but in fact each strategy was chosen 4 times (with random sequence). Then, they were told about this fact and considering it, the expected value for each students’ strategy was calculated and explained.
2.2. Game 2 – against the nature with the distribution of its strategy known

The pay-off matrix was exactly the same like in Game 1. The result is shown in table 2.

Table 2. Result achieved by participants in Game 2.

| Results         | Score achieved |
|-----------------|----------------|
| Minimum         | -56            |
| Maximum         | 51             |
| Mean Average    | 6.87           |
| Standard deviation | 21.58       |

The game consisted – as before – of 16 attempts. All results (minimum, maximum and mean average) were improved in comparison to Game 1. Standard deviation remained similar to the previous game. It has to be noticed that the highest (but still negative) expected value was for S strategy and equal to 0.25. If any student plays only S strategy in all 16 attempts, the result would be -4. The mean average achieved in Game 2 was higher, equal to +6,87.

2.3. Game 3 – one-to-one, repetitive game with hidden strategies

It was one-to-one game, where students played in pairs. From 180 participants only 78 pairs of them fulfilled completely the sheet forms given to them. Result from 12 pairs of players couldn’t be considered. Each player received his/her own pay-off matrix and was asked not to disclose it to the opposite player.

![Figure 2. The pay-off matrixes in Game 3 a) given to the player on the left b) given to the player on the right side](image)

This time 10 attempts created full game. To provide simultaneous voting for strategies of both players, playing cards were used to vote. When both players declared their strategies by laying down chosen card (strategy) on the table reverse side up, the averse of cards were disclosed and each player could read the score on his/her own pay-off matrix. The game was created to destroy the players’ beliefs in expected value universal usefulness. When both players chose the strategies with the highest expected values (left player: diamonds, right player: hearts) the score was negative for both of them. The result achieved by 78 pairs of players is summarized in table 3.

Table 3. Result achieved by participants in Game 3.

| Results         | Score of L player | Score of P player | Sum of L and P |
|-----------------|-------------------|-------------------|----------------|
| Minimum         | -25               | -44               | -48            |
| Maximum         | 14                | 21                | 18             |
| Mean Average    | -3.56             | -12.10            | -15.67         |
| Standard deviation | 10.78            | 15.26             | 14.85          |
2.4. Game 4 – one-to-one, non-repetitive game with open strategies

It was only one attempt to the game. Each pair of players received pay-off matrixes as shown in figure 3. The scores of 80 pairs were recorded and summarized in table 4.

| Results      | Score of L player | Score of P player | Sum of L and P |
|--------------|-------------------|-------------------|----------------|
| Minimum      | -4                | -2                | 0              |
| Maximum      | 9                 | 9                 | 8              |
| Mean Average | 2.28              | 2.85              | 5.25           |
| Standard deviation | 1.76            | 1.80              | 1.12           |

As the strategies were opened for opposite players, they could discuss them, and the choice done was their joint decision.

2.5. Game 5 – one-to-one, repetitive game with open strategies

The final game in the experiment was designed to utilize the experience gained in the former four games by its participants. The pay-off matrix was identical like in Game 4. The number of attempts was designed as 10.

| Player on the right | A | B | C | D |
|---------------------|---|---|---|---|
| P                   | 2 \-2 | -10 \10 | 0 \8 | 1 \-1 |
| Q                   | 2 \3 | 3 \-3 | -1 \1 | -3 \3 |
| R                   | -2 \2 | 8 \-4 | 2 \-2 | 10 \-10 |
| S                   | 9 \-2 | 1 \-1 | -4 \4 | 3 \2 |

Figure 3. The pay-off matrix in Game 4 and Game 5 (the right number in a cell is the score of the player on the right side)

Only 76 pairs of players played all 10 attempts and wrote down the scores achieved. The results are shown in table 5. Players could discuss which pair of strategies they chose in every attempt.

| Results      | Score of L player | Score of P player | Sum of L and P |
|--------------|-------------------|-------------------|----------------|
| Minimum      | -25               | -20               | 3              |
| Maximum      | 45                | 40                | 76             |
| Mean Average | 28.95             | 26.63             | 55.58          |
| Standard deviation | 14.61           | 15.03             | 25.01          |

2.6. Discussion

The groups of students were not bigger than 24 people. The experiment lasting not more than 1,5 hour in each group was designed and conducted in the way, that the students’ knowledge about game theory, strategies, as well as experience in playing were increasing step by step. Players were told before each of the game that the aim was to achieve the highest score by a single player. They were also told that it wasn’t allowed to count the total score (from both players) and then to assign half of it to each player. Game 1 was an introductory one, but an effect of understanding the distribution of “nature’s” strategies is clearly visible. The minimum, maximum and mean average scores have increased (became
more favourable for the players). The standard deviations were similar in both games as it was random character of the “nature’s” choices with the same distribution (only the players didn’t realize that during Game 1). Introducing the concept of calculating the expected value of a player’s strategy (when the distribution of random choices of an opposite player is known) introduced a confusion in Game 3. But in that game, the choices of strategy done by the opposite player were not random. That pushed the students’ attention to cooperation. During the Game 3, when many negative score for both players appeared in initial attempts, they started asking, if their pay-off matrixes were identical or not. They were not provided with this information (pay-off matrixes were different). Students were disappointed with the Game 3 (average score for left and right player were negative – see table 3) and became more willing to cooperate, discuss (in pairs) their strategies before the choices were done. Having two sets of strategies (A-Q and D-S; see figure 3) which provided positive score for both players, 87.5 % (10 out of 80) of pairs chose one of these win-win pair of strategies in Game 4. The cooperation developed in Game 5, where in 39.5 % (30 out of 76) of cases, the sum of scores for the left and the right players was maximum or near the maximum i.e. 76 or 75 points. The summarized results for the left and the right player were then (36, 40) and (40, 35) respectively. The really good cooperation is clearly visible in figure 4, where median values and quartiles are marked.

The good cooperation is clearly shown by the results achieved in Game 3, where possible strategies were hidden for opposite players. The similar to figure 4c chart is shown in figure 5 (for the summarised total score of both players). In Game 3, 75 % of summarised scores were lower than -5 points. The highest achievement was 18 points. To provide the complete view how poor were the results in Game 3 – where cooperation was not allowed – table 6 was prepared. Some possible sets of successful strategies are shown there (based on pay-off matrixes shown in figure 3). None of the playing pairs achieved such good results.
Figure 5. Summarized total score in Game 3 for both players.

Moreover, there were only two pairs (among 78 played Game 3) with the positive scores of both players (left 5 and right 3 in both cases) after 10 attempts played. It means that in 97,4 % cases at least one player didn’t achieve a positive score. In fact, in 41,0 % cases both players scored below 0. These proportions of losers and long distance of real achievements to the highest possible scores in Game 3, compared to the results in Game 5 prove of the power of cooperation.

| Strategies | Score of L player | Score of P player | Sum of L and P |
|------------|-------------------|-------------------|----------------|
| (L spades; R clubs) 5 times, (L clubs; R hearts) 5 times | 35 | 10 | 45 |
| (L clubs; R diamonds) 10 times | 20 | 50 | 70 |
| (L hearts; R spades) 10 times | 20 | 30 | 50 |
| (L spades; R clubs) 4 times, (L clubs, R hearts) 6 times | 22 | 24 | 46 |

3. Modelling negotiations of construction subcontract

The character of construction subcontracts differs, as usual, from the contracts signed between a client and a general contractor. The potency of parties (measured by their yearly sales or employment level) differs more in subcontracts. A subcontractor – who doesn’t satisfy the needs of a general contractor during a project execution – can be changed much easier for another one than a general contractor changed by a client. The production means being at a subcontractor’s disposal are usually allocated to the smaller number of construction sites than a production means of a general contractor. So, the result achieved by a subcontractor on a single construction site influences the result of the subcontractor’s company more than the company of a general contractor. For all those reasons the negotiating power belongs much more to the buyer – a general contractor (GC) than to its prospective subcontractor (SC).
When the scope of works to execute is clear for both these parties, they meet to agree the price for it. They come to a negotiation table and present their initial prices keeping secret the limits. In case when limit prices overlap – as it is shown in figure 6 b – the details of the work execution are discussed and contract clauses are negotiated. Then parties, in the process of negotiation, come to the conclusion that the agreement becomes more and more possible. Finally the price is agreed from the range limited by the overlap of the price limits. When the limits do not overlap, the agreement is not possible – there is a gap between the highest possible GC’s price and the lowest possible SC’s price (see figure 6 a). After long negotiation process (due to not disclosing the limits explicitly), the parties start to realise that the achievement of the agreement is not possible. The origin of the price limit should be analysed to check the possibility of shifting the price limits (to create an overlap). A subcontractor price limit can be explained as the sum of direct costs of the works to be executed, planned profit to achieve and additional costs to be spent during the contract execution (most of them can be classified as indirect, but there are some other specific, necessary expenses).

\[
P^{(SC)} = Dc + P^{(SC)} + C
\]

where:

- \( P^{(SC)} \) subcontractor’s price limit
- \( Dc \) subcontractor’s planned direct costs
- \( P^{(SC)} \) subcontractor’s planned profit to achieve on a building site
- \( C \) additional subcontractor’s expenses

The examples of aforesaid specific factors affecting additional contractor’s expenses \( C \) are as follows:
- terms of payment,
- variability of employment level on a construction site,
- planned duration of works,
- accessibility of social containers on site,
- staff accommodation

and many more. Making the aforementioned factors favourable for a subcontractor can make SC willing to lower the price limit. For instance, it can be expensive (much over the market value) – for SC – to hire and secure the social containers, and store-container in a town were SC does not usually operate.
Relatively low cost of providing these services by GC (below the market level; e.g. some containers – owed by GC – were emptied by another subcontractor who had completed works). A general contractor’s price limit origin is different. For subcontractor’s works made for GC, it is calculated as the difference between planned value of that work (planned by GC in the offer placed and accepted by a client) and the GS’ planned profit. The GS’s price limit can be calculated as a difference between budgeted value and the profit (assumed to achieve on SC’s works).

\[ P^{(GC)} = B - P^{(GC)} \]  

(2)

where:

\( P^{(GC)} \) general contractor’s price limit

\( B \) general contractor’s budget (given as an offer to a client for subcontracted scope of works)

\( P^{(GC)} \) general contractor’s planned profit to achieve on a given subcontract

As the agreement is possible only when a subcontractor’s price limit is lower than a general contractor price limit (see figure 6 b), based on (1) and (2) the inequation (3) can be written.

\[ P^{(GC)} + P^{(SC)} + C < B - Dc \]  

(3)

The GC’s budget for subcontractor’s work and contractor’s direct cost of work cannot be lowered. The gap (presented in figure 6 a) exists when inequation (3) is not met. To bring the agreement possible components on left side of inequation (3) should be lowered. If the parties concentrate on the price negotiations and lowering their planned profits does not create the price limit overlap, the negotiations fail. While, according to [1], if the real needs of the opposite party are dug during the talks (by the opposite player), it can occur that what can be done by one party for a low cost (not disturbing the profit much), that can be of a really high value for the opposite party. There are specific, not rare situations, that solution of this kind can be found. Some examples are presented in table 7.

| Case No. | GC position | SC position | Agreed action | Value for GC | Value for SC |
|----------|-------------|-------------|---------------|--------------|--------------|
| 1        | Delayed, tight schedule; continuous front of SC’s works cannot be provided | Several workers on forced, partly paid leaves; lack of engagement for them | Flexible weekly engagement of SC’s forces on site | High        | Low          |
| 2        | Delayed schedule; increased risk of high penalties charged by a client; Free beds at hostel hired by GC; social containers – owed by GC – emptied by another SC after their works completion | Lack of possibilities of engaging external workers (to meet GC’s requirement of high speed work) acc. to low financial liquidity | Advance payment | Low         | High         |
| 3        | A high cost of accommodation of workers and social container on constr. site located in another town | Letting SC to use the hostel and container | Low          | High        |

The lack of going in-depth to the details of the opposite side during the negotiations process is like playing the Game 3. Parties try to find the cross section of their strategies to maximise the profits, but the reasons why they can’t agree are hidden (both sides may think, that the profit level of the opposite side is too high). When they concentrate only on the price to be agreed (not discussing unimportant matters e.g. 10 beds in a hostel for 2 moths), the gap between parties’ price limits cannot be cancelled i.e. agreement remains impossible to achieve. Discussion on the contract price only simultaneously...
shifts profits expected by the parties but the sum of their profit remains constant (4), so it doesn’t affect desirable meeting of the inequation (3).

\[ p^{(GC)} + p^{(SC)} = \text{const} \]  

(4)

Analysing case 1 (in table 7) and considering inequation (3), it can be found that the SC’s price limit can be shifted down, as the use of SC’s staff on forced leave does not add any cost. In fact, it lowers SC costs. Moreover, it brings a high value for GC. Some critical points in the schedule can be deleted and the problem of continuous providing the work front for SC is overcome. As a result of unexpected value given by SC, GC will be more willing to increase a bit the price limit. Hiring a subcontractor who keeps on site full staff ready to handle a peak requirement for work, would be much more costly for GC. The case 2, where covering small cost (beds in a hostel) by GC makes SC’s cost much lower (the reason why SC’s price limit can be shifted down), can be modelled by Game 5. There, players agreed in alternate attempts small loss and high gain, what result in high reward for both of them.

Let each of \( n \) strategies \( S_n \) be represented by set of values of \( m \) attributes \( Q_m \). Each attribute \( Q_m \) can have \( k_m \) different values \( q_{im}^{(i_m)} \) for \( 1 < i_m < k_m \). Then the strategy \( S_n \) can be written down as

\[ S_n = [q_1^{(i_1)}, q_2^{(i_2)}, q_3^{(i_3)}, \ldots, q_m^{(i_m)}] \]  

(5)

As the sequence of attributes’ values has no meaning, there is \( n \) possible strategies where:

\[ n = k_1 * k_2 * k_3 * \ldots * k_m \]  

(6)

During the negotiation of a subcontract, where the gap created by parties’ limit prices is recognized, the role of negotiators is to find out which attributes are important to the opposite party, as well as, to explaining which are important for themselves. Having found the attribute, a solution which causes low cost for one party but is a high gain for the other should be searched (as the examples presented in table 7). Once the important attributes are chosen (and agreed by both parties) and their value is also agreed (e.g. number of beds allowed, the sum of advance payment) but still the value of a given attribute have a different monetary value for each party. The GC’s pay-off \( V_n^{(GC)} \) can be calculated separately for each of \( n \) strategies according to the formula (7).

\[ V_n^{(GC)} = \sum_{i=1}^{p} v_i^{(GC)} (q_i^{(i_p)}) \]  

(7)

where:

\( p \) number of attributes considered in strategies (the same for each strategy)

\( v_i^{(GC)} \) function transforming the \( i \)-attribute value \( q_i^{(i_p)} \) to monetary value for GC

The subcontractor’s pay-off \( V_n^{(SC)} \) can be calculated in the similar way but the transforming functions will be different. At least one party pay-off is positive (what means savings) i.e. an ability to shift the price limit in the direction “to the other party” price limit (if the gap still exists). The pay-off of the opposite party can be a bit negative but so close to 0, that it does not require shifting the price limit.
When the sequence of strategies is the same in a column and a row of the pay-off matrix, the possible choices of SG and GC strategies are on the diagonal (as presented in figure 7). It is not the typical game as GC and SC strategies (at this stage) have to be the same (e.g. $S_3$ for both of them). The set of attributes is the same, their values too – parties agreed it previously. A given strategy – even if based on the same set of attributes and their values – provide different pay-offs to the parties. In Game 5 also, players didn’t consider a set of strategies providing negative scores for both players. Here, the strategies other than loose-loose are left on the diagonal (in figure 7). The aim of the negotiation is to find the strategy which, if applied, can shift parties price limits, thus creating the overlap in price limits and finally the conclusion of the agreement between a general contractor and a subcontractor.

4. Conclusions

Even the gap in question is overcome and there is a space to find the subcontract price it is recommended to analyse the full set of strategies as this may lead to a discovery of an even better strategy. Then the profit of a general contractor and a subcontractor can increase. An apparent opposition of theory of rational choices – game theory – and presented in [1] ways of solving conflicts, in fact, are not different approaches to negotiations. With the use of the experiment described and the model presented, both approaches support each other. The success in construction subcontract negotiations is a subject of one very important matter – openness of parties willing to sign the agreement. An openness does not mean disclosing commercial secrets of the company. Thanks to the game theory, the model could be created in a way that neither price limits, nor direct costs, nor value of the general contractor’s offer (comprising subcontractor’s scope of works) haven’t been disclosed. Draining the real needs of the opposite party during the negotiations process have to accompanied by openness. Then – if possible – solutions can be found. Experienced, successful negotiators (companies’ owners) are successful thanks to their intuition – they implicitly search for win-win solutions. However, if the process of getting to the agreement is structured and mathematically described, it can be optimised – that is the author’s plan for future exploration.

Acknowledgment

Author wishes to acknowledge the Warsaw University of Technology students’ for the participation in the experiment.

References

[1] R. Fisher, W. Ury, B. Patton, “Getting to Yes: Negotiating agreement without giving in”, Penguin Books, ISBN 978-1844131464, 2011
[2] P.D. Straffin, “Teoria Gier”, Wydawnictwo Naukowe SCHOLAR, Warsaw 2001
[3] K. Binmore, “Teoria Gier”, Wydawnictwo Uniwersytetu Łódzkiego, Łódź 2017
[4] A.K. Dixit, B.J. Nalebuff, “Sztuka strategii. Teoria gier w biznesie i życiu prywatnym”, MT Biznes, Warsaw 2016
[5] O. Kapliński, J. Tamozaitienė, „Game theory applications in construction engineering and
management”, Technological and Economic Development of Economy, Baltic Journal of Sustainability 16(2): 348-363, DOI: 10.3846/tede.2010.22

[6] J. Antuchevičienė, Z. Turskis, E. K. Zawadskas, „Modelling renewal of construction objects applying methods of the game theory”, Technological and Economic Development of Economy, Vol XII, No. 4, pp. 263-268, 2006

[7] M. W. Kembłowski, B. Grzyl, A. Siemaszko, „Game Theory Analysis of Bidding for A Construction Contract”, WMCAUS 2017, IOP Publishing, Materials Science Engineering 245 (2017) 062047, DOI:10.1088/1757-899X/245/6/062047

[8] Y. Teng, X. Li, P. Wu, X. Wang, “Using cooperative game theory to determine profit distribution in IPD projects”, International Journal of Construction Management, 2017 DOI: 10.1080/15623599.2017.1358075

[9] J. R. San Cristobal, “The use of Game Theory to solve conflicts in the project management and construction industry”, International Journal of Information Systems and Project Management, Vol. 3, No. 2, pp.43-58, 2015, DOI: 10.12821/ijispm030203

[10] B. Grzyl, M. Apollo, A. Kristowski, “Application of Game Theory to Conflict Management in a Construction Contract”, preprint, 2019, DOI:10.20944/preprints201901.0004.v2