Spin Seebeck coefficient in the 2d Hubbard model

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We investigate the spin Seebeck coefficient \( S_s \) in the square lattice Hubbard model at high temperatures of relevance to cold-atom measurements. We solve the model with the finite-temperature Lanczos and with the dynamical mean-field theory methods and find they give similar results in the considered regime. \( S_s \) exceeds the atomic 'Heikes' estimates and the Kelvin entropic estimates drastically. We analyze the behavior in terms of a mapping onto the problem of a doped attractive model and derive an approximate expression that allows relating the enhancement of \( S_s \) to distinct scattering of the spin-majority and spin-minority excitations. Our analysis reveals the limitations of entropic interpretations of Seebeck coefficient even in the high-temperature regime. Large values of \( S_s \) could be observed on optical lattices. We also calculated the full diffusion matrix. We investigated to what extent the thermoelectric effects change the spin-diffusion and discuss the results in the context of recent measurements of the spin-diffusion constant in cold atoms.

I. INTRODUCTION

Cold-atom systems on optical lattices provide a novel lens on poorly understood transport regimes of correlated electrons [1, 2]. The cold atoms realize the Hubbard model – the standard model of correlated electrons that interact with an onsite repulsion \( U \) and move on the lattice (hopping \( t \)) – without the real world complications, such as lattice vibrations and disorder. Hence one can directly and quantitatively compare the outcome of the experiment to those of the numerical solutions of the Hubbard model [1, 2]. Such a crossverification turned very successful in the measurements of the charge diffusivity [1] and was besides an important mutual benchmark and motivation for further quantification of the vertex corrections [3]. Intriguingly, related measurements of spin diffusivity revealed a disagreement between the numerical methods and the experiment [2]. An independent numerical investigation confirmed the results of the theory [4] but disagrees with the experiment.

In this setting, quantitative disagreement between the experimentally reported spin-conductivity is surprising and calls for a close inspection of the underlying assumptions of the experimental analysis. One of the assumptions is that the spin-thermolectric effect is unimportant. This holds strictly at vanishing magnetization, but in the actual experiment this condition was only approximately met as some spin imbalance \( m_z = (n_\uparrow - n_\downarrow)/2 \) is seen in the measurements: \( |m_z| \lesssim 0.05 \) [2]. Spin-thermopower is a quantity that is relevant for spintronics applications but was not investigated broadly for the Hubbard model, and it is not clear how large it is compared to \( k_B \), a natural unit for the spin-Seebeck coefficient.

Some intuition could be expected from considerations that relate the Seebeck coefficient to thermodynamic quantities, such as the high-temperature Heikes limit \( S = \mu/T \) or the Kelvin formula \( S = d\mu/dT \) that relate the Seebeck coefficient to the temperature dependence of the chemical potential [5–8]. For ordinary Seebeck effect it was demonstrated that often at high temperatures the Kelvin formula describes the Seebeck coefficient well [7] (but some exceptions to this were also noted [9]).

To use this intuition for the case of spin-Seebeck coefficient it is convenient to take advantage of a mapping that relates the magnetized repulsive Hubbard model to a doped attractive Hubbard model [10–17]. This mapping proceeds via a particle-hole transformation on particles of only one, e.g., \( \downarrow \) spin with \( c_{i,\downarrow} \rightarrow (-1)^i c_{i,\downarrow}^{\dagger} \) and results in an interchange of spin and charge degrees of freedom, explicitly \( n_\uparrow - n_\downarrow \rightarrow n_\uparrow + n_\downarrow \) and \( n_\uparrow n_\downarrow \rightarrow -n_\uparrow n_\downarrow \). Similarly Hubbard repulsion goes to attraction, \( U n_\uparrow n_\downarrow \rightarrow -U n_\uparrow n_\downarrow \). Via this mapping, one can relate the spin Heikes estimate for spin Seebeck coefficient \( S_s^H = B/T|_{m_z=\text{const}} \) (or Kelvin estimate \( S_s^K = dB/dT|_{m_z=\text{const}} \)) to the corresponding charge Heikes and Kelvin estimates for a model with opposite sign of repulsion (that is, attractive model for the case of interest here). Actually, exploiting the mapping one can use the results from the literature [18] and obtain \( S_s = 8k_Bm_z \) at a high temperature \( (T > U) \) and \( S_s = 4k_Bm_z \) at a lower temperature \( (T < U) \). From these estimates – that one is inclined to trust, especially in the high temperature regime \( T > t \) pertinent to cold atom measurements – one expects only small values of the spin Seebeck coefficient \( S_s \lesssim k_B \), since \( m_z \ll 1 \).

In this paper we show that this reasoning is incorrect. We calculate the spin-Seebeck coefficient \( S_s \) for a square lattice Hubbard model at high-\( T \) using finite temperature Lanczos and the dynamical mean-field
theory (DMFT) methods and find that it strongly exceeds the bounds just discussed. \( S_s \) violates the thermodynamic expectations even in the high temperature regime, an unexpected finding based on what was previously known for the ordinary thermoelectric effect. Large values of \( S_s \) call for a reexamination of the possible spin-thermoelectric effects in cold atom measurements of spin-diffusion. In order to theoretically evaluate those, we calculated the full diffusion matrix. The deviations of the eigenvalues from the value corresponding to pure spin-diffusion are given in terms of deviation of \( S_s \) from values given by the Kelvin formula. We discuss why in spite of these being sizeable, the final influence on the measured spin-diffusion is at moderate values of magnetization unimportant.

We note that in the DMFT large values of Seebeck coefficient for the attractive model were earlier found [17] but were not compared to the thermodynamic estimates and the importance of those results for the spin-thermoelectric response was not discussed.

II. MODEL AND METHOD.

We study the square lattice Hubbard model,

\[
H = -t \sum_{\langle i,j \rangle, s=\uparrow,\downarrow} c^\dagger_{i,s} c_{j,s} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}
\]

with \( t \) being the hopping between the nearest neighbors. We take \( \hbar = k_B = e = g\mu_B = 1 \). We likewise take lattice spacing \( a = 1 \). We use \( t \) as the energy unit.

We solve the Hamiltonian with the finite-temperature Lanczos method (FTLM) [19–21] on a \( N = 4 \times 4 \) cluster and we compare the results also to the dynamical mean-field theory (DMFT) [22] results obtained using numerical-renormalization group (NRG) [23] in the NRG-Ljubljana implementation [24] as the impurity solver. We focus on a half filled case (electron doping \( \delta = n - 1 = 0 \)) with finite magnetization \( m_z \).

III. RESULTS

Fig. 1(a) displays \( S_s \) as a function of temperature for \( U = 10 \) evaluated with the FTLM (full, thick) and DMFT (symbols). At highest temperatures, \( S_s \) approaches the high temperature Heikes value, \( 2 \log(1 + 2m_z)/(1 - 2m_z) \approx 8m_z \), which is the expected behavior. The Kelvin estimate from \( dB/dT \) evaluated using FTLM (thin; DMFT gives similar results) agree with the Kubo evaluation in this regime. On lowering the temperature, surprisingly, instead of diminishing in amplitude as suggested by the Heikes value corresponding to \( T < U \), \( \log[(1 + 2m_z)/(1 - 2m_z)] \approx 4m_z \) [18], \( S_s \) increases and reaches a maximum value well above the Heikes estimate, and only drops consequently at lower \( T \). This behavior with a substantial increase of the spin-thermoelectric coefficient above the high-temperature and thermodynamic estimates becomes even more pronounced for larger \( U \), as displayed in Fig. 1(b).

The magnitude of the peak diminishes with decreasing magnetization but increases with increasing \( U \). It is, as we show below, proportional to \( m_z U/T \). The fact that the spin-Seebeck coefficient reaches values of or-
der \( k_B \) at weak magnetizations means that in measurements on optical lattices one should control the magnetization very accurately to properly probe the transport free from the thermoelectric effects.

Throughout the considered regime, the temperatures are high \( (T > t) \) and one cannot attribute the deviations from the thermodynamic estimates to coherent transport or to a proximity to a magnetically ordered regime. The observed deviations are in stark contrast to the behavior of the charge thermopower that at temperatures \( T > t \) does follow the thermodynamic estimates [7].

### IV. DMFT DESCRIPTION OF TRANSPORT

In order to understand this behavior it is convenient to discuss the transport properties within the DMFT approach. The DMFT expresses the transport coefficients in terms of the transport function [25]

\[
\Phi(\omega) = \sum_k v_k^2 A^2_{k\sigma}(\omega)
\]

where \( v_k \) is the band velocity: \( v_k = \partial \epsilon_k / \partial k_x \) with \( \epsilon_k \) the band energy. \( A_{k\sigma} \) is the spectral function at momentum \( k \) and spin \( \sigma \). The charge \( S_c \) and spin \( S_s \) Seebeck coefficient are, respectively,

\[
S_{(c,s)} = (1,2) \frac{\int (\Phi_\uparrow(\omega) \pm \Phi_\downarrow(\omega)) (-\omega/T)(-df/d\omega)d\omega}{\int (\Phi_\uparrow(\omega) + \Phi_\downarrow(\omega)) (-df/d\omega)d\omega}. \tag{3}
\]

As seen in Fig. 1(a), the DMFT results are similar, even though not identical to the FTLM ones. To what extent the differences are technical (it is quite challenging to converge the DMFT calculations in this regime as discussed in Ref. [17]) or physical, such as emanating in non-local fluctuations and/or the vertex corrections neglected in DMFT is a question that goes beyond the scope of the present paper. It is likely that at low \( T \) the vertex corrections become more important as the DMFT calculation gives a spin gapped insulator in the otherwise spin conductive (Heisenberg) regime. For our purpose we will ignore these differences between the two methods and exploit the more transparent DMFT formulation of transport to interpret the FTLM results.

It is convenient to analyze the results in terms of the mapping of the spin to the charge degrees of freedom for an attractive model, that is \( S_s(U) = 2S_c(-U) \), with the spin polarization \( 2m_z \to \delta \) becoming the charge doping with factors of 2 occurring due to the spin definition.

Fig 2(a) presents the spectral function of the doped attractive model at \( T > 4t \), for two values of attraction \(-U = 10, 20\). As discussed in earlier studies of the attractive model [12, 13, 17, 26], the spectral function consists of two peaks, which are as for the repulsive Hubbard model centered at \(-\mu\) and \(-\mu + U\), with \( \mu \sim U/2 \). The crucial distinction between the doped attractive and doped repulsive model is in the behavior of the chemical potential with temperature that can at high-\( T \) be most simply obtained from a grand-canonical treatment of the atomic problem. There, average electron occupancy can be evaluated from

\[
n = \frac{2 \exp(\beta \mu) + 2 \exp(-\beta(U - 2\mu))}{1 + 2 \exp(\beta\mu) + \exp(-\beta(U - 2\mu))} \tag{4}
\]

In the repulsive model, one can simply ignore the terms that include \( \exp(-\beta U) \), whereas in the attractive model these grow at low temperatures and should be retained. One obtains

\[
\mu = U/2 + \delta \mu = U/2 + T \log((1 + \delta)/(1 - \delta)) \tag{5}
\]

The fact that \( \mu \sim U/2 \) leads to a crucial difference between the attractive and the repulsive case. In the attractive one, \( \mu \) tends towards \( U/2 \) and the spectral function is gapped, that is, it consists of two peaks displaced by \( \delta \mu \propto T \) from the \( \pm U/2 \), which is shown on Fig 2(a) for two values of attraction \( U \). Because at large \( U \) the gap is well developed, the lower and upper Hubbard bands must have unequal spectral weight to yield a finite doping. Conversely, for doped repulsive model, \( \mu \sim 0 \) or \( \mu \sim U \) for hole and electron doping respectively; in each case there is no gap in the single-particle spectral function.

Fig. 2(d) presents the transport spectral function. One sees that this exhibits two Hubbard bands and is overall similar to the density-of-states. There is however an important difference: because \( \Phi \) contains \( A^2_{k\sigma} \), the weights of the upper and the lower Hubbard band parts are affected by the amplitude of scattering. When spectral function is a sharply peaked function, \( A^2_{k\sigma} \sim A_k/(2\pi \Gamma_k) \) with \( \Gamma_k = \text{Im}\Sigma(\omega_k) \), namely the value of the self-energy at the peak frequency \( \omega_k \). This is, e.g., frequently used to connect the bubble expression to the Boltzmann calculation.

### V. PHENOMENOLOGICAL ANALYSIS

We will now use advantage of the known general shape of the transport function to obtain approximate simple expression for the Seebeck coefficient. The temperatures of our simulations (and of the cold atom experiments, that motivate our investigation) are large. In Ref. [2] the estimated entropies are 1.1, which pertains to \( T \sim 0.3U \), comparable to the bandwidth. Hence the derivative of the Fermi function \( df/d\omega \) does not change substantially
over each of the Hubbard bands and hence in the transport integrals, Eq. 3, one can approximate the transport function by the two \( \delta \)-peaks as

\[
\Phi(\omega) = \phi_- \delta(\omega - U/2 + \delta \mu) + \phi_+ \delta(\omega + U/2 + \delta \mu),
\]

with different weights \( \phi_- \) and \( \phi_+ \) for the negative and positive frequency peaks, respectively. Note that \( U < 0 \) in this case. One can evaluate the Seebeck coefficient using this ansatz transport function. Taking also into account that \( \delta \mu/T \) is small, one can expand the derivative of the Fermi function, that is

\[
-\frac{df}{d\omega}(\pm U/2 - \delta \mu) = -\frac{df}{d\omega}(U/2)(1 \pm t_0 \delta \mu/T)
\]

where we define \( t_0 = \tanh(U/4T) \). Introducing effective weights (modified from \( \phi \) due to \( df/d\omega \)) \( \phi_{\pm} = \phi_z(1 \pm t_0 \delta \mu/T) \), one obtains a simple expression for the Seebeck coefficient

\[
S = \frac{-U}{2T} \frac{\phi_- - \phi_+}{\phi_- + \phi_+} + \frac{\delta \mu}{T} = S_1 + \frac{\delta \mu}{T}
\]

The high-\( T \) Seebeck coefficient for the attractive model thus has a Heikes’ term (second term of this expression, predicted by Chaikin and Beni [18]), but, crucially also the first term \( S_1 \), that is proportional to \( U/2T \) and the difference between the effective weights of the peaks of the transport function. Whenever this difference (that, as we discuss next, relates to a different scattering of electrons and holes) does not vanish, the Seebeck coefficient cannot be interpreted in terms of the entropic considerations alone. This explains large values of spin-Seebeck coefficient seen in numerical results of Fig. 1.

Is scattering really important? Would not the effective weights differ already due to the different weights of the corresponding Hubbard band weights in the density of states? If this were the case, one would have \( \phi_{\pm} \propto (1 \mp \delta) \) (simply from the considerations of occupancy). Incidentally, the influence of \( \delta \mu \) just cancels at small \( T \). Namely, for small doping \( \delta \mu/T \approx \delta \). As \( T/|U| \) becomes small, \( t_0 \rightarrow -1 \). Hence one has, \( \phi_{\pm} \approx \phi_{\pm}(1 \pm \delta) \), and hence \( \phi_+ = \phi_- \). In this limit \( S_1 \) would vanish. One needs the to take the scattering into account to understand the occurrence of deviation from entropic estimates.

We plot the weight \( \phi_- \), obtained from the integral of the transport function over negative frequencies normalized such that the total integral is 2 on Fig. 3(b). One sees that in most of the studied temperature range \( \phi_- \) is close to the value expected from the dependence \( (1 + \delta)^2 \). Only at smaller temperatures the weight \( \phi_- \) decreases and actually approaches a smaller value \( (1 + \delta) \).

On Fig. 3(a) we compare numerical results for \( S_z \) to the result of Eq. 7 using the estimate for the weights \( \phi_{\pm} \propto (1 \mp \delta)^2 \) with \( z = 1 \) (blue) and with \( z = 2 \) (green). At small temperatures these lead to a behavior \( S_1 = -U/2T(z - 1)\delta \) (where corrections of order \( \delta^2 \) and higher are ignored). As seen from this expression, using \( z = 1 \), which corresponds to taking into account just the different number of carriers, does not reproduce the behavior of numerical simulations (except at lower \( T \)), one needs to take into account their different scattering (as embodied in \( z = 2 \)), too. It can further be shown, that ignoring the difference of scattering between positive and negative frequencies and fixing the spectral weight of the lower peak to \( 1 + \delta \) (and accordingly of the upper peak) and then taking into account the proper shift of chemical potential with \( T \) would result in a Seebeck coefficient given by the Kelvin estimate. This again emphasizes the importance of scattering effects.

![FIG. 3. (a) Spin-Seebeck coefficient for \( U = 10 \) and spin polarization \( m_z = 0.05 \) compared to simple analytical expressions based on the assumption of the weights of the spectral function \( \phi_{\pm} = (1 \mp \delta)^2 \) for \( z = 1, 2 \). (b) \( \phi_- \) for two values of \( U \). The phenomenological estimate \( \phi_- = (1 + \delta)^2 \) with \( z = 2 \) is also indicated (dashed), see text.](image)

To understand the implications of this discussion for the repulsive model one can exploit the relation \( A_{k\uparrow}(\omega) = A_{k\downarrow}(\omega) \) which holds for the repulsive model at particle-hole symmetry. The mapping Fig. 2 show the \( s_z = \uparrow \) components. The \( s_z = \downarrow \) components can be obtained from \( \omega \rightarrow -\omega \). \( \phi_- \) weight thus corresponds to electron contribution to conductivity for the \( s_z = \uparrow \) channel and hole contribution for the \( s_z = \downarrow \) channel. The different scattering of states in the upper and lower Hubbard band of the attractive model thus maps to a different scattering of the spin-minority and majority carriers.

### VI. DIFFUSION MATRIX, DIFFUSION EIGENVALUES, AND RELEVANCE FOR EXPERIMENT

Large values of spin-Seebeck coefficient at large \( T \) and the increase of the peak value with increasing \( U \) (or the corresponding analogue in the attractive case) should be
easily measured in future cold-atom experiments. In the
introduction we also raised a possibility that the exist-
ing measurements of spin-diffusion would be affected by
spin-thermoelectric effects, which could account for va-
values of the spin-diffusion and the spin-conductivity that
were found to be larger that theoretically expected.

Let us try to estimate the influence of thermoelectric
effects on spin-conductivity using a hand-waving argu-
ment. At finite magnetization, gradients of mag-
netic field are accompanied by a gradient of energy,
and, assuming thermalization, a gradient of temper-

ature, \( \nabla T \sim \nabla E/c = m_z \nabla B/c \), with \( c \) the specific
heat. Temperature gradients drive the spin current via
spin-thermoelectric effect. Writing spin-current as \( j_s =
-L_{ss} \nabla B - L_{sz} \nabla T \), one has \( j_s = L_{ss}(1 - S_s m_z/c) \nabla B \).
At \( T \approx 3 \) (relevant to experiment [2]), the values of
specific heat are of the order of \( 0.3k_B \), hence the cor-
rection of the estimated spin-conductivity due to spin-
thermoelectricity for magnetization 0.05 where \( S_s \approx 0.8 \)
would be at the 15% level. Importantly, because \( S_s \) and
\( m_z \) are of equal sign, this estimate actually antici-
ates the spin-conductivity is reduced compared to the case
where spin-thermoelectricity is neglected.

In order to make this discussion more precise one must
consider a generalization of the Nerst-Einstein relation
to a matrix formulation allowing for mixed response
[27], where the off-diagonal entries involve the mixed
transport coefficient \( L_{sq} \) and thermoelectric suscepti-
ability \( \xi = -\partial^2 f/\partial B \partial T \). The diffusion constant in matrix
form reads \( D = -L A^{-1} \), where \( \{j_q, j_s\} = L \{\nabla T, \nabla B\} \)
defines the conductivity matrix \( L \) and susceptibility ma-
trix \( A \) is defined by \( \{\nabla q, \nabla m_z\} = A \{\nabla T, \nabla B\} \).
The diffusion eigen-modes that involve in general non-
vanishing spin and heat components are obtained by
diagonalizing the matrix \( D \). We denote \( D_- \) \( (D_+ \) to be
the diffusion eigenvalue whose mode contains pre-
dominantly spin \( (\text{heat}) \) component, respectively. These
are shown in Fig. 4(a) as a function of temperature
for \( U = 10, m_z = 0.05 \) and are compared to the bare
spin diffusion constant \( D_s = \sigma_s / \chi_s \) and bare heat dif-
fusion constant \( D_q = \kappa / c \). One sees that, consistent with
the hand-waving discussion above, the influence of the
spin-thermoelectric effects is only moderate, the dif-
sion eigenvalues are close to the values obtained when
there is no mixing. The behaviour of \( D_\pm \) being smaller
(larger) than the corresponding bare diffusion without
spin-thermoelectricity has similar origin as a standard
level repulsion.

The relatively small admixing occurs because the
geometric mean of the offdiagonals \( \sqrt{D_{sq}D_{qs}} \) is sig-
nificantly smaller \( (\lesssim 10\%) \) than \( D_{qq} - D_{ss} \). When any of
the offdiagonals is zero, the spin-thermoelectric effect
on diffusion vanishes. We inspect more closely \( D_{sq} \).
It can be rewritten as \( D_{sq} = -D_s \left( \frac{s + S_{sq}}{s} \right) \). This expres-
sion explicitly expresses \textit{thermal spin diffusion}, i.e. spin-
thermoelectric influence on the spin-diffusion. Note that
\( S_s \) is multiplied by \( \chi_s / c \), which becomes large in value
at \( T = 0 \). This hints at a rich behaviour at low tem-
peratures and should be explored in future work. Using
the Kelvin formula \( S^K_s = dB/dT |_{m_z} \), one can further
rewrite: \( D_{sq} = -D_s \frac{\chi}{c} (S_s - S^K_s) \). Interestingly, when
the spin thermoelectricity or spin Seebeck coefficient is
given by the Kelvin estimate, there is no admixing be-
tween the spin and the thermal diffusion.

In experiment, an initial gradient of magnetization is
imposed. Could larger values of the experimentally in-
ferred spin-diffusion constant occur because there is a
significant contribution of the heat eigen-mode in the
initial state that decays faster? The components of
eigen-vectors \( \vec{v}_\pm \) are shown on Fig. 4(b). At \( T =
3.1 \) we find the spin dominated eigen-vector \( \vec{v}_- =
\{-0.2t, -0.98\} \), i.e. it contains a significant component
of heat current that only increases with temperature.
(In this expression we reintroduced \( t \) to indicate that the
t two components of \( \vec{v} \) are in different units; because
\( t \) is a natural unit for the energy it is meaningful to com-
pare the numerical values of the two components setting
\( t = 1 \).) At the same temperature, the heat dominated
\( \vec{v}_+ = \{-0.999t, 0.02\} \) is mostly single component.
One can first assume that the initial state is given by pure
magnetization profile. Expanding this profile in terms of
\( \vec{v}_\pm \) we find only small part of magnetization is contained
in \( \vec{v}_+ \). To be specific, at \( T = 3.1 \), \( \vec{v}_+ \) contains \( \lesssim 0.5\% \)
of the initial spin modulation (the relative weight of \( \vec{v}_+ \)
is sizeable, but it carries only a small magnetization).
Hence, the faster decay of the \( \vec{v}_+ \) cannot importantly
affect the evolution of the magnetization. What if the

\[ \begin{align*}
D & \text{ spin-diffusion constant} \\
D_q & \text{ bare heat diffusion constant} \\
D_{sq} & \text{ and the two eigenvalues of } D \\
T & \text{ for } U = 10 \\
m_z & = 0.05 \\
S & \text{ spin-thermoelectric effect. Writing spin-current as} \\
\n\end{align*} \]
heat gradient component is also initially present? Using the estimate $\delta q \sim -\frac{3}{2} \Delta \delta m_z$ (describing the situation where the magnetic field responsible for $\delta m_z$ is switched off and excess energy is instantly converted into heat), we find that $\bar{v}_+^m$ is more prominent in the initial state, but still accounts for $\lesssim 2\%$ of the initial magnetization modulation. In both cases, the majority of spin diffusion is therefore governed by $D_- \sim D_+^m$.

VII. CONCLUSIONS

In summary, we calculated the spin-Seebeck coefficient in the Hubbard model and discovered a rich behavior with temperature. The spin-Seebeck coefficient exceeds significantly the entropic estimate. This occurs due to the unequal scattering of spin minority and spin-majority carriers which gives rise to an $\propto m_z U/T$ dependence that adds up to the Heikes’ estimate. This is a striking demonstration of the breakdown of the entropic interpretation of the thermopower in a high-temperature regime where a priori one would trust it the most. Our predictions for large spin-thermoelectric effect could be tested on optical lattices. We calculated the diffusion matrix eigenvalues and estimated the effect could be tested on optical lattices. We calculated the most. Our predictions for large spin-thermoelectric temperature regime where a priori one would trust it to be moderate and insufficient to explain the discrepancy between the experiment and the theory. Possible directions for future research include simulating explicitly the time-dependence in such experiments and the study of possible nonlinear effects.

ACKNOWLEDGEMENTS

We acknowledge helpful discussions with Rok Žitko and Antoine Georges. This work was supported by the Slovenian Research Agency (ARRS) under Program No. P1-0044 and Project No. J1-2458, N1-0088, and J1-2455-1.

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