Do wealth distributions follow power laws?
Evidence from “rich lists”

Michal Brzezinski
Faculty of Economic Sciences, University of Warsaw, Długa 44/50, 00-241, Warsaw, Poland

Abstract

We use data on wealth of the richest persons taken from the “rich lists” provided by business magazines like Forbes to verify if upper tails of wealth distributions follow, as often claimed, a power-law behaviour. The data sets used cover the world’s richest persons over 1996–2012, the richest Americans over 1988–2012, the richest Chinese over 2006–2012 and the richest Russians over 2004–2011. Using a recently introduced comprehensive empirical methodology for detecting power laws, which allows for testing goodness of fit as well as for comparing the power-law model with rival distributions, we find that a power-law model is consistent with data only in 35% of the analysed data sets. Moreover, even if wealth data are consistent with the power-law model, usually they are also consistent with some rivals like the log-normal or stretched exponential distributions.

Keywords: power-law model, wealth distribution, goodness of fit, model selection, Pareto model

1. Introduction

The search for universal regularities in income and wealth distributions has started over one hundred years ago with the famous work of Pareto [1]. His work suggested that the upper tails of income and wealth distributions follow a power law, which for a quantity $x$ is...
defined as a probability distribution $p(x)$ proportional to $x^{-\alpha}$, with $\alpha > 0$ being a positive shape parameter known as the Pareto (or power-law) exponent. Pareto’s claim has been extensively tested empirically as well as studied theoretically [2–6]. The emerging consensus in the empirical econophysics literature is that the bulk of income and wealth distributions seems to follow the log-normal or gamma distributions, while the upper tail follows power-law distribution. Recent empirical studies found a power-law behavior in the distribution of income in Australia [7, 8], Germany [9], India [10], Italy [7, 9, 11], Japan [12, 13], the UK [9, 14, 15], and the USA [9, 14, 16]. Another group of studies discovered a power-law structure of the upper tail of modern wealth distributions in China [17], France [18], India [10, 19], Sweden [20], the UK [14, 18, 20, 21], and the USA [18, 20, 22, 23]. Surprisingly, analogous result were obtained for wealth distribution of aristocratic families in medieval Hungary [24] and for the distribution of house areas in ancient Egypt [25].

However, as shown recently by Clauset et al. [26] detecting power laws in empirical data may be a difficult task. Most of the existing empirical studies exploit the fact that the power-law distribution follows a straight line on a log-log plot with the power-law exponent equal to the absolute slope of the fitted line. The existence of power-law behaviour is often confirmed visually using such a plot, while the exponent is estimated using linear regression. Such approach suffers, however, from several drawbacks [26, 27]. First, the estimates of the slope of the regression line may be very biased. Second, the standard $R^2$ statistic for the fitted regression line cannot be treated as a reliable goodness of fit test for the power-law behaviour. Third, even if traditional methods succeed in verifying that a power-law model is a good fit to a given data set, it is still possible that some alternative model fits the data better. A complete empirical analysis would therefore require conducting a statistical comparison of the power-law model with some other candidate distributions.

Using a more refined methodology for measuring power-law behaviour, Clauset et al. [26] have shown that the distribution of wealth among the richest Americans in 2003 as compiled in Forbes’ annual US “rich list” is not fitted well by a power-law model. Recently, Ogwang [28] has tested formally for a power-law behaviour in Forbes’ data on the wealth of the world’s billionaires for the years from 2000 to 2009. He has found that the Kolmogorov-Smirnov,
Anderson-Darling and $\chi^2$ goodness of fit tests all reject power-law behaviour for each of the data sets he used. However, Ogwang [28] has tested if the whole range of observations in his data sets follow a power-law behaviour, while in fact this may apply only to a subset of the very largest observations. A more appropriate methodology for detecting a power-law distribution would have, therefore, include a procedure for estimating a lower bound on the power-law behaviour.

The present paper uses a complete empirical methodology for detecting power laws introduced by Clauset et al. [26] to verify if upper tails of wealth distributions obey the power-law model or if some alternative model fits the data better. We estimate both the power-law exponent and the lower bound on the power-law behaviour. We also use goodness of fit tests and compare power-law fits with fits of alternative models. We analyze a large number of data sets on wealth distributions published annually by Forbes and other business magazines concerning wealth of 1) the richest persons in the world, 2) the richest Americans, 3) the richest Chinese, and 4) the richest Russians.

The paper is organized as follows. Section 2 presents the statistical framework used for measuring and analyzing power-law behavior in empirical data introduced by Clauset et al. [26]. Section 3 shortly describes our data sets drawn from the lists of the richest persons published by Forbes and other sources, while Section 4 provides the empirical analysis. Section 5 concludes.

2. Statistical methods

In order to detect a power-law behaviour in wealth distributions we use a toolbox proposed by Clauset et al. [26]. A density of continuous power-law model is given by

$$ p(x) = \frac{\alpha - 1}{x_{\text{min}}} \left(\frac{x}{x_{\text{min}}} \right)^{-\alpha}. $$

(1)

The maximum likelihood estimator (MLE) of the power-law exponent, $\alpha$, is

$$ \hat{\alpha} = 1 + n \left[ \sum_{i=1}^{n} \ln \frac{x_i}{x_{\text{min}}} \right], $$

(2)
where $x_i, i = 1, \ldots, n$ are independent observations such that $x_i \geq x_{\text{min}}$. The lower bound on the power-law behaviour, $x_{\text{min}}$, will be estimated using the following procedure. For each $x_i \geq x_{\text{min}}$, we estimate the exponent using the MLE and then we compute the well-known Kolmogorov-Smirnov (KS) statistic for the data and the fitted model. The estimate $\hat{x}_{\text{min}}$ is then chosen as a value of $x_i$ for which the KS statistic is the smallest.\footnote{The Kolmogorov-Smirnov statistic was also proposed by Goldstein et al. \cite{27} as a goodness of fit test for the discrete power-law model assuming, however, that the lower bound on power-law behaviour is known.} The standard errors for estimated parameters are computed with standard bootstrap methods with 10,000 replications.

The next step in measuring power laws involves testing goodness of fit. A positive result of such a test allows to conclude that a power-law model is consistent with a given data set. Following Clauset et al. \cite{26} again, we use a test based on a semi-parametric bootstrap approach. The procedure starts with fitting a power-law model to data using the MLE for $\alpha$ and the KS-based estimator for $x_{\text{min}}$ and calculating a KS statistic for this fit, $k_s$. Next, a large number of bootstrap data sets is generated that follow the originally fitted power-law model above the estimated $x_{\text{min}}$ and have the same non-power-law distribution as the original data set below $\hat{x}_{\text{min}}$. Then, power-law models are fitted to each of the generated data sets using the same methods as for the original data set and the KS statistics are calculated. The fraction of data sets for which their own KS statistic is larger than $k_s$ is the $p$-value of the test. The power-law hypothesis is rejected if this $p$-value is smaller than some chosen threshold. Following Clauset et al. \cite{26}, we rule out the power-law model if the estimated $p$-value for this test is smaller than 0.1. In our computations, we use 4,999 generated data sets.

If the goodness of fit test rejects the power-law hypothesis, we may conclude that the power law has not been found. However, if a data set is well fit by a power law, the question remains if there is other alternative distribution, which is equally good or better fit to this data set. We need, therefore, to fit some rival distributions and evaluate which distribution gives a better fit. To this end, Clauset et al. \cite{26} use the likelihood ratio test proposed by Vuong \cite{29}. The test computes the logarithm of the ratio of the likelihoods of the data under
two competing distributions, $LR$, which is negative or positive depending on which model fits data better. Vuong [29] showed that in the case of non-nested models the normalized log-likelihood ratio $NLR = n^{-1/2}LR/\sigma$, where $\sigma$ is the estimated standard deviation of $R$, has a limit standard normal distribution. This result can be used to compute a $p$-value for the test discriminating between the competing models. In case of nested models, Vuong [29] shows that $2LR$ has a limit a chi-squared distribution.

Each of the estimators and tests described above has been tested with good effects by Clauset et al. [26] using Monte Carlo simulations.²

3. Wealth data from the “rich lists”

In several countries business magazines publish annual lists of the richest individuals. The oldest and the most famous one is the Forbes 400 Richest Americans list, which started in 1982. Other “rich lists” published by Forbes include the World’s Billionaires and the 400 Richest Chinese. These lists provide rankings of rich individuals according to their net worth defined as a sum of their assets minus their debts. We use annual data from the Forbes 400 Richest Americans list for the period 1988–2012, from the Forbes World’s Billionaires list for the period 1996–2012 and from the Forbes 400 Richest Chinese list for 2006–2012. In addition, we use 2004–2011 data from the list of top Russian billionaires published by the Russian magazine Finans (www.finansmag.ru). Descriptive statistics for our data sets are presented using beanplots [30] in Appendix A.

4. Results

Power-law fits to our data sets are shown in Figure 1. The values of the power-law exponent are rather stable over time for all four groups of data sets studied. However, except for Russia, the estimated exponents are substantially higher than usually found in

²The Stata software implementing all methods described in this section is available from the author upon request. The original power-law-testing Matlab and R software written by Aaron Clauset and Cosma R. Shalizi can be obtained from http://tuvalu.santafe.edu/~aaronc/powerlaws/.
the previous literature on power-law behaviour of wealth distributions. In particular, the average estimate of the exponent for the world’s richest persons is 2.5, while the averages for the US, China and Russia are, respectively, 2.4, 2.7 and 1.9. This result is a consequence of the fact that previous studies have rarely attempted to estimate $x_{min}$ and instead often fitted power-law models to all available observations. However, estimating $x_{min}$ using KS-based approach as described in Section 2 leads to a substantially smaller range of observations that may potentially follow the power-law behaviour. According to our estimates, on average only about 306 observations (44% of all available observations) are above $\hat{x}_{min}$ in case of data sets covering the world’s richest persons. The average number of observations above $\hat{x}_{min}$ is 268 (60%), 261 (57%) and 220 (55%) for the US, Russia and China, respectively. The

\[3\] Richmond et al. [15] found that the estimated values of the power-law exponent range from 0.5 to 1.5 for wealth distributions and from about 1.5 to 3 for income distributions.
most striking conclusion from Figure 1 is that the majority of the data sets for the world’s richest persons, the richest Americans, and the richest Russians are not fitted well by the power-law model according to the goodness of fit test used. For Russia only 25% of data sets seem to follow a power-law behaviour, while for the richest persons in the world and for the richest Americans the number is in the range from 28 to 29%. Only for China in all but one case wealth distribution seems to follow a power-law model, but the period under study for this country is the shortest. These results suggest that, at least for the data sets drawn from the “rich lists”, wealth distributions often do not follow the power-law model and that testing goodness of fit should always precede a declaration that a power-law behaviour of a wealth distribution was found. Figure 2 shows typical examples when a power law is not a good fit for our data sets (left panel, goodness of fit test $p$-value = 0.02) and for the case when it seems to be a good fit (right panel, $p$-value = 0.64).

Figure 2: The complementary cumulative distribution functions and their power-law fits

Our results are inconsistent with those of Ogwang [28], who found that the power-law
behaviour of the wealth of the world’s richest persons according to Forbes’ data in every year between 2000 and 2009 is ruled out by conventional goodness of fit tests. On the contrary, we find that the wealth of the world’s billionaires is fitted well by a power-law model in 2000, 2001 and 2003. This inconsistency can be explained by noticing that Ogwang [28] has not estimated the lower bound on the power-law behaviour, $x_{\text{min}}$, but fitted power-law models to the whole range of Forbes’ observations. However, fixing $x_{\text{min}}$ at the minimum wealth level in Forbes’ data seems to be statistically unjustified.

The results of the likelihood ratio tests for all 20 data sets that passed the goodness of fit test (with $p$-value $> 0.1$) are given in Table 1. We have followed Clauset et al. [26] in choosing the following alternative distributions: log-normal, exponential, stretched exponential and power-law with exponential cut-off. Positive (negative) values of $NLR$ or $LR$ mean that the power-law model gives a better (worse) fit compared to a given alternative. If the $p$-value for the likelihood ratio test is small, then we may reject the model which gives a worse fit to data. If the $p$-value is larger than the chosen level, which is set to 0.1 in our analysis, then we are not able to choose between the compared models.

Results from Table 1 show there is no data set for which we may conclude that it is fitted well by a power-law model and that it is there is no plausible alternative model for it. There are only two data sets (the world’s richest persons in 1999 and the richest Chinese in 2011) for which the sign of the $NLR$ statistic suggests that the power law is better than the log-normal, exponential and stretched exponential distributions, but $p$-values for the statistic are in most cases so large that the tests are inconclusive. More generally, the exponential distribution can be ruled out as a plausible model for our wealth data in all but three cases (the world’s richest persons in 1999 and the richest Chinese in 2006 and 2007). On the other hand, the log-normal distribution appears to be empirically indistinguishable from the power law in our data – the $p$-values for the relevant tests are always larger than 0.11. Similar conclusion can be drawn for a comparison between the stretched exponential

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4See Clauset et al. [26] for definitions of these distributions. The pure power-law model is a nested within the power-law with exponential cut-off model and for this reason the latter always provides a fit at least as good as the former. The $LR$ statistic for comparing these models will therefore be always negative or zero.
Table 1: Power-law vs. other models for the upper tail of wealth distributions

| Data set | Power law | Log-normal | Exponential | Stretched exponential | Power law with cut-off | Support for power law |
|----------|-----------|------------|-------------|-----------------------|-----------------------|----------------------|
|          | $p$       | $NLR$ $p$  | $NLR$ $p$   |                       | $NLR$ $p$             | $LR$ $p$             |
| World    |           |            |             |                       |                       |                      |
| 1998     | 0.981     | 0.026      | 0.979       | 2.043                 | 0.041                 | 0.043 0.966         | 0.068 0.713          | moderate             |
| 1999     | 0.977     | 0.483      | 0.629       | 1.447                 | 0.148                 | 0.146 0.884         | 0.000 1.000          | moderate             |
| 2000     | 0.824     | -0.144     | 0.886       | 2.313                 | 0.021                 | -0.158 0.874       | -0.141 0.595         | moderate             |
| 2001     | 0.490     | -0.544     | 0.586       | 2.587                 | 0.010                 | -0.554 0.580       | -0.777 0.212         | moderate             |
| 2003     | 0.154     | -0.929     | 0.353       | 2.623                 | 0.009                 | -0.955 0.340       | -1.715 0.064         | with cut-off         |
| US       |           |            |             |                       |                       |                      |                      |
| 1993     | 0.297     | -0.680     | 0.496       | 3.146                 | 0.002                 | -0.694 0.488       | -0.957 0.167         | moderate             |
| 1999     | 0.506     | -0.116     | 0.908       | 2.325                 | 0.020                 | -0.137 0.891       | -0.141 0.595         | moderate             |
| 2000     | 0.189     | -0.361     | 0.718       | 3.630                 | 0.000                 | -0.361 0.718       | -0.660 0.251         | moderate             |
| 2004     | 0.315     | -0.431     | 0.666       | 2.429                 | 0.015                 | -0.441 0.659       | -0.637 0.259         | moderate             |
| 2008     | 0.268     | -0.879     | 0.379       | 2.171                 | 0.030                 | -0.902 0.367       | -1.533 0.080         | with cut-off         |
| 2011     | 0.381     | -0.307     | 0.759       | 4.615                 | 0.000                 | -0.316 0.752       | -0.721 0.230         | moderate             |
| 2012     | 0.386     | -0.791     | 0.429       | 2.831                 | 0.005                 | -0.813 0.416       | -1.432 0.091         | with cut-off         |
| China    |           |            |             |                       |                       |                      |                      |
| 2006     | 0.377     | -0.817     | 0.414       | 1.561                 | 0.119                 | -0.850 0.396       | -1.090 0.140         | moderate             |
| 2007     | 0.244     | -0.699     | 0.483       | 1.394                 | 0.163                 | -0.705 0.481       | -0.800 0.206         | moderate             |
| 2009     | 0.295     | -0.940     | 0.347       | 2.883                 | 0.004                 | -0.975 0.330       | -1.658 0.069         | with cut-off         |
| 2010     | 0.168     | -0.469     | 0.639       | 2.739                 | 0.006                 | -0.479 0.632       | -0.656 0.252         | moderate             |
| 2011     | 0.636     | 0.365      | 0.715       | 3.820                 | 0.000                 | 0.084 0.933        | -0.119 0.626         | moderate             |
| 2012     | 0.676     | -0.407     | 0.684       | 3.633                 | 0.000                 | -0.412 0.681       | -0.497 0.319         | moderate             |
| Russia   |           |            |             |                       |                       |                      |                      |
| 2005     | 0.101     | -1.596     | 0.110       | 3.200                 | 0.001                 | -1.664 0.096       | nc                   | moderate             |
| 2011     | 0.661     | -0.860     | 0.390       | 6.270                 | 0.000                 | -0.886 0.375       | nc                   | moderate             |

The first column gives $p$-values for the goodness of fit test for the power-law behaviour. For each non-nested alternative distribution we give the normalized log-likelihood ratio ($NLR$), while for the power law with exponential cut-off we give the log-likelihood ratio ($LR$). "nc" denotes a non-convergence of the MLE for a distribution. We also present $p$-values ($p$) for the significance of $NLR$ or $LR$. The last column presents the final judgement using the terminology of Clauset et al. [26]: "moderate" means that power law is a good fit but so are some plausible alternatives; "with cut-off" means that the power law with exponential cut-off is favoured over the pure power law, but there are also other plausible models. None of our data sets can be labelled "good", which means that the power law is a good fit and that none of the alternatives considered is plausible.
and the power law with the possible exception of data for Russia in 2005, for which the stretched exponential distribution seems to be marginally favored over the power law. The power law with an exponential cut-off seems to fit the data better than the pure power law in four cases (the world’s richest persons in 2003, the richest Americans in 2008 and 2012 and the richest Chinese in 2009). However, in these four cases the log-normal and stretched exponential distributions are also plausible fits to data.

In overall, none of the 57 data sets on wealth distribution analysed in this paper can be reliably described as fitted best by a power-law model. Only about 35% of data sets can be plausible considered to follow a power-law distribution, but even among these data sets the power law is empirically indistinguishable from the log-normal and stretched exponential distributions.

These results suggest that the hypothesis that upper tails of wealth distributions, at least when measured using data from “rich lists”, follow a power-law behavior is statistically doubtful. It seems obvious that this hypothesis should no longer be assumed before conducting an empirical analysis of a given data set using tools similar to those of Clauset et al. [26]. The existence of popular software implementing such empirical methods should make this task easier. The results of this paper seem also to cast some doubt on the theoretical literature in econophysics and economics that provides a theoretical structure for power-law behaviour of top wealth values. Theoretical models that make room also for some other distributions (especially the log-normal and stretched exponential distributions) describing upper tail of wealth distribution may be equally worth consideration.

5. Conclusions

In this paper, we have used a large number of data sets on wealth distribution taken from the lists of the richest persons published annually by business magazines like Forbes. Using recently developed empirical methodology for detecting power-law behaviour introduced by Clauset et al. [26], we have found that top wealth values follow the power-law behaviour only in 35% of analysed cases. Moreover, even if the data do not rule out the power-law
model usually the evidence in its favour is not conclusive – some rivals, most notably the log-normal and stretched exponential distributions, are also plausible fits to wealth data.

Acknowledgements

I would like to acknowledge gratefully the Matlab and R software written by Aaron Clauset and Cosma R. Shalizi, which implements methods described in Section 2. The software can be obtained from http://tuvalu.santafe.edu/~aaronc/powerlaws/. I also thank Moshe Levy for sharing data from Forbes 400 Richest Americans lists. I also appreciate the helpful comments of the participants of the 32nd International Association for Research in Income and Wealth conference, Boston, USA, August 5-11, 2012. All remaining errors are my own. This work was supported by Polish National Science Centre grant no. 2011/01/B/HS4/02809.

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Appendix A. Descriptive statistics for wealth data from the “rich lists” (may be put in the supplementary materials online)

Figure A.1: Beanplots for wealth of world billionaires, Forbes data, 1996–2012

The beanplot for a given year shows on a log scale individual wealth values as short vertical lines with the estimated density shown in gray. The vertical solid black lines show mean net worth for a given year, while the overall vertical dotted line shows the grand mean.
Figure A.2: Beanplots for wealth of the US billionaires, Forbes data, 1988–2012

a See note to Fig. A.1.
Figure A.3: Beanplots for wealth of the richest Chinese, Forbes data, 2006–2012

*a* See note to Fig. A.1.
Figure A.4: Beanplots for wealth of the richest Russians, Finans magazine data, 2004–2011

\(^a\) see note to Fig. A.1.