Method for modeling the process of wear of gear teeth

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Abstract. When modeling gears by means of roller analogues it is necessary to take into account that the degree of slippage is equal to zero in the meshing pole of the gears, in the contact zone of initial circles of the gears. The greatest slippage occurs between the tooth head of the master gear and the tooth foot of the driven gear, and between the tooth head of the driven gear and the tooth foot of the driven gear. Thus, the simulation of the wear process of gear wheels with roller counterparts is achieved by ensuring equality of geometric and kinematic parameters (radii of contact point curvature, degree of relative slippage of the surfaces, friction paths) and contact pressure.

1. Introduction
Modern mechanisms and machines contain many moving interfaces, a large proportion of which are gears. This type of mechanical interface is used in almost all areas of agricultural machinery: tractors, harvester combines [1-5], seeders [6], grain cleaning machines [7-9], choppers, grinders and others [10-14].

Also it is necessary to note that the time and material costs required for reliability and durability testing are high. In this regard, the use of gearing simulation is of great importance and can considerably accelerate, simplify and reduce the cost of the endurance tests. The cause of failure of gearing elements in most cases is wear caused by friction processes. For this reason, the modelling of the gearing must consider the friction surfaces in the gears and the resulting changes in the geometry of the wheel and gear. Mathematical evolutionary models of gearing are of great practical importance both in the theoretical study of material properties and technologies and in the practical application and production of gearing elements [15].

In addition, the experimental results obtained can only be used for gears with specific meshing modulus and kinematic parameters. The above-mentioned disadvantages associated with the testing of gears for wear resistance can be eliminated by simulating their wear process with roller counterparts, because on friction machines the samples made of gear materials have higher circumferential speeds compared to the gears in the unit.

2. Materials and research methods
It is known from the theory of gearing that the mating surfaces of gears must have a common normal passing through the engagement pole.

The outermost points of the working section of the meshing line K1 and K2 are at the intersection of the straight line BKBW with the circles of the drive and idler gears protrusions (Fig.1) [16].
The angle between the engagement line and the perpendicular to the centre line OkOwrestored at engagement pole P is called the engagement angle.

At the point of contact P, the peripheral velocity projections of the pinion and idler on the meshing line are equal.

![Figure 1. Schematic diagram for calculating the geometric and kinematic parameters of the gearing](image)

Figure 1. Schematic diagram for calculating the geometric and kinematic parameters of the gearing

The circumferential velocities of the drive and idler gears on a common tangent of the working surfaces of the teeth, perpendicular to the meshing line, are unequal. At point C, the velocity component of the driving gear VW, directed along the tangent to the profile, is greater than the similar velocity component of the driven gear Vk. Thus, during friction the surfaces of the mating teeth simultaneously roll and slide relative to each other.

In involute gearing, the circumferential speeds of the gears are identical on both, the starting and main circles.

At the start of engagement, the tooth head of the pinion slides on the tooth stalk of the idler up to the engagement pole. From the meshing pole, the tooth head of the idler slides on the tooth stalk of the drive pinion [17]. This causes the gear head and tooth stalk in the meshing area to wear much more than the rest of the tooth surface.

To simulate the profile of the gear teeth, the height of the head and foot of the teeth of the drive and driven gears were divided into several equal segments using roller analogues. The ratio of the length of each segment to the height of the tooth head or foot was called meshing coefficient in relation to the initial circles of the gears (k)[18]. The value of this coefficient depending on the location of their contact area varies from +1 at the head, to -1 at the foot of the gears, and at the point of contact of the starting circles of the gears, the value of the coefficient k is zero. According to the values of the coefficient k, the radii of curvature of the contact point of the teeth of the driven and driven gears are determined. The calculated values of curvature radii at tooth contact point, are also the radius of roller counterparts.
The radius of curvature of the tooth profile (roller analogue of the modelling tooth) of the driving gear is [19]:

\[ \rho_w = 0.5m \sqrt{z_w^2 \sin^2 \alpha + 4kz_w^2 + 4k^2}, \text{ m.} \] (1)

The radius of curvature of the tooth profile (the roller analogue of the modelling tooth) of the driven gear taking into account expression (2.38) is equal:

\[ \rho_k = 0.5m \left[ (i+1)\sin \alpha - \sqrt{z_w^2 \sin^2 \alpha + 4kz_w^2 + 4k^2} \right], \text{ m.} \] (2)

We consider the case where the engagement point of the pinion is inside the dividing circle at a distance km, in the direction of the tooth stalk.

The degree of slippage is then relative:

of the teeth (roller counterpart of the modelling tooth) of the drive pinion

\[ \xi_w = \frac{(i+1)\sqrt{z_w^2 \sin^2 \alpha + 4kz_w^2 + 4k^2} - z_w \sin \alpha}{i\sqrt{z_w^2 \sin^2 \alpha + 4kz_w^2 + 4k^2}}; \] (3)

teeth (roller counterpart of the modelling tooth) of the idler pinion

\[ \xi_k = \frac{(i+1)\sqrt{z_w^2 \sin^2 \alpha + 4kz_w^2 + 4k^2} - z_w \sin \alpha}{z_w (i+1)\sin \alpha - i\sqrt{z_w^2 \sin^2 \alpha + 4kz_w^2 + 4k^2}}. \] (4)

3. Research results and discussion

When meshing takes place between the tooth foot (modelling roller counterpart) of the driving gear and the tooth head (modelling roller counterpart) of the driven gear, conversely, the radius of curvature of the tooth foot (modelling roller counterpart) of the driving gear is reduced, the tooth head (modelling roller counterpart) of the driven gear is increased. With equal centre distance and gear ratio, as the number of teeth increases, the degree of slippage between the teeth (roller counterparts) of the gears (with the same values of k) decreases.

The sliding speed between the roller counterparts is determined by the difference in rolling speeds of the roller counterparts of the drive and idler gears at the point of contact:

\[ v_{ch} = \pi m_k \left[ z_w (i+1)\sqrt{z_w^2 \sin^2 \alpha + 4kz_w^2 + 4k^2} - z_w \sin \alpha \right], \text{ m/s} \] (5)

The cumulative rolling speed of the gears (simulation roller counterparts) is the sum of the rolling speeds of the teeth (simulation roller counterparts) of the driven and driven gears at their contact point:

\[ v_{cyw} = \pi m_k \left[ z_w (i+1)\sin \alpha (i-1)z_w^2 \sin^2 \alpha + kz_w^2 + 4k^2 \right], \text{ m/c.} \] (6)

The sliding path between the teeth (roller counterparts of simulation gears) of the gears depends on the degree of slippage and the number of teeth of the gear. The sliding path between the teeth (roller counterparts of the simulation gears) of the gears is calculated:

\[ S = \frac{\pi m(i+1)\sqrt{z_w^2 \sin^2 \alpha + kz_w^2 + 4k^2} - z_w \sin \alpha}{z_k}, \text{ m.} \] (7)

Depending on the location of the gearing, the ratio of the gearing relative to the pitch circle of the driving pinion is constantly changing.
Table 1. Variation of the degree of tooth slippage (roller analogs of modelling teeth) of spurred spur gears, from the value of A and $z_w$, at $i=2; \alpha=20^\circ$

| Number of drive gear teeth, $z_w$ | k  | 300 | 60 | 30 | 20 | 15 | 12 |
|----------------------------------|----|-----|----|----|----|----|----|
| **Tooth head (modelling tooth roller counterpart) of driving pinion and tooth foot (modelling tooth counterpart) of driven pinion** |    |     |    |    |    |    |    |
| 0.1                              |    | 0.0084 | 0.0409 | 0.0787 | 0.1138 | 0.1466 | 0.1772 |
| 0.2                              |    | 0.0167 | 0.0787 | 0.1466 | 0.2059 | 0.2584 | 0.3053 |
| 0.3                              |    | 0.0249 | 0.1138 | 0.2059 | 0.2825 | 0.3475 | 0.4036 |
| 0.4                              |    | 0.0329 | 0.1466 | 0.2584 | 0.3475 | 0.4207 | 0.4822 |
| 0.5                              |    | 0.0409 | 0.1772 | 0.3053 | 0.4036 | 0.4822 | 0.5468 |
| 0.6                              |    | 0.0487 | 0.2059 | 0.3475 | 0.4527 | 0.5348 | 0.6013 |
| 0.7                              |    | 0.0563 | 0.2330 | 0.3858 | 0.4961 | 0.5806 | 0.6481 |
| 0.8                              |    | 0.0639 | 0.2584 | 0.4207 | 0.5348 | 0.6209 | 0.6888 |
| 0.9                              |    | 0.0713 | 0.2825 | 0.4527 | 0.5697 | 0.6567 | 0.7247 |
| 1.0                              |    | 0.0787 | 0.3053 | 0.4822 | 0.6013 | 0.6888 | 0.7566 |
| **Tooth shank (roller counterpart of modelling tooth) of driving pinion and tooth head (roller counterpart of modelling tooth) of driven pinion** |    |     |    |    |    |    |    |
| 0.1                              |    | 0.0084 | 0.0407 | 0.0782 | 0.1128 | 0.14449 | 0.1748 |
| 0.2                              |    | 0.0167 | 0.0782 | 0.1449 | 0.2026 | 0.2533 | 0.2981 |
| 0.3                              |    | 0.0249 | 0.1128 | 0.2027 | 0.2763 | 0.3381 | 0.3909 |
| 0.4                              |    | 0.0329 | 0.1449 | 0.2533 | 0.3381 | 0.4069 | 0.4640 |
| 0.5                              |    | 0.0407 | 0.1748 | 0.2981 | 0.3909 | 0.4640 | 0.5233 |

Depending on the rotation speed of the gears (roller counterparts), the sliding path between the teeth changes. A gear tooth (roller counterpart) with higher angular velocity has a larger sliding path.

The duration of abrasion testing of roller counterparts of gears corresponding to one period of oil change in the unit, for gears and their roller counterparts, were determined according to the change in abrasive particle activity. The amount of oil poured into the tub of the friction machine, corresponding to the amount of oil in the unit was determined from the condition of equality of the ratio of the amount of oil poured into the tub of the friction machine to the amount of oil in the unit and the duration of wear resistance testing of roller counterparts on the friction machine to the period of oil replacement in the unit.
Table 2. Change of geometric and kinematic parameters of the gearing (roller counterpart) from m and Z_w, at a=0.45 m; k=0.5; n_k=0.5 m/s; i=2; α-20°

| m | Z_w | ρ_w | ρ_k | v_ck | S_w | S_k |
|---|-----|-----|-----|------|-----|-----|
| m | M   | m   | m   | m/s  | m   | m   |

**Tooth head (modelling tooth roller counterpart) of driving pinion and tooth foot (modelling tooth counterpart) of driven pinion**

|       |     |     |     |      |      |      |
|-------|-----|-----|-----|------|------|------|
| 0.001 | 300 | 0.0528 | 0.1012 | 1.0855 | 0.030*10^-3 | 0.015*10^-3 |
| 0.005 | 60  | 0.0582 | 0.0957 | 1.0684 | 0.712*10^-3 | 0.370*10^-3 |
| 0.010 | 30  | 0.0645 | 0.0894 | 1.0487 | 2.684*10^-3 | 1.444*10^-3 |
| 0.015 | 20  | 0.0703 | 0.0836 | 1.305  | 5.728*10^-3 | 3.190*10^-3 |
| 0.020 | 15  | 0.0757 | 0.0782 | 1.0135 | 9.707*10^-3 | 5.594*10^-3 |
| 0.025 | 12  | 0.0809 | 0.0731 | 0.9973 | 14.516*10^-3 | 8.648*10^-3 |

**Tooth shank (roller counterpart of modelling tooth) of driving pinion and tooth head (roller counterpart of modelling tooth) of driven pinion**

|       |     |     |     |      |      |      |
|-------|-----|-----|-----|------|------|------|
| 0.001 | 300 | 0.0529 | 0.1014 | 1.0855 | 0.030*10^-3 | 0.015*10^-3 |
| 0.005 | 60  | 0.582  | 0.0961 | 1.0687 | 0.719*10^-3 | 0.346*10^-3 |
| 0.010 | 30  | 0.0642 | 0.0901 | 1.0499 | 2.738*10^-3 | 1.267*10^-3 |
| 0.015 | 20  | 0.0696 | 0.0847 | 1.0331 | 5.912*10^-3 | 2.628*10^-3 |
| 0.020 | 15  | 0.0745 | 0.0798 | 1.0177 | 10.142*10^-3 | 4.323*10^-3 |
| 0.025 | 12  | 0.0790 | 0.0753 | 0.0030 | 15.361*10^-3 | 6.270*10^-3 |

4. Conclusion

Table 1 shows the results of calculating the degree of slip between the teeth (roller analogs) of spur gears obtained by expressions (3) and (4) depending on the value of k and Z_w, at α=2; α=20°. The degree of slip between the teeth (roller analogs of modeling teeth) of the gears increases with **increasing the value of k, because, when the gear tooth meshing between the tooth head (roller analog of modeling tooth) of the leading and the tooth foot (roller analog of modeling tooth) of the driven gears, the radius of curvature of the tooth head of the leading gear increases, the tooth foot of the driven gear decreases.**

From the data in Table 2, it can be deduced that in gears with increasing engagement modulus and with decreasing number of teeth, the curvature radius and sliding path between the teeth (roller analogues of model gears) of the gears increases, the sliding speed decreases.

The load transmitted by the gear and roller counterparts was determined according to the Hertz formula for contact surfaces having cylindrical shapes. The contact widths of the samples were chosen according to the contact pressures between the gear teeth.

References

[1] Matchanov R, Rizayev A, Astanakulov K, Tolibaev A, Karimov N IOP Conf. Series: Earth and Environmental Science 677(2021) 052021
[2] Astanakulov K, Shovazov K, Borotov A, Turdibekov A, Ibrokhimov S E3S Web of Conferences 227, 07001 (2021), doi.org/10.1051/e3sconf/202122707001
[3] Astanakulov K, Abdillaev T, Umirov A, Fozilov G, Hatamov B E3S Web of Conferences 227,
Ochihide O, Fozilov G, Achildiev Sh, Karimov M, Ashurov N. E3S Web of Conferences 227, 07002 (2021), doi.org/10.1051/e3sconf/202122707003

[5] Rizaev A, Malikov Z, Yuldashev A, Kuldoshev D. Materials Science and Engineering 1030, 012175 (2021), doi:10.1088/1757-899X/1030/1/012175

[6] Astanakulov K. IOP Conf. Series: Materials Science and Engineering 883(2020) 012137, doi:10.1088/1757-899X/883/1/012137

[7] Astanakulov K D, Karimov Y Z, Fozilov G G. Agricultural mechanization in Asia, Africa and Latin America. 42(4), 37-40 (2011)

[8] Astanakulov K. IOP Conf. Series: Materials Science and Engineering 883(2020) 012151, doi:10.1088/1757-899X/883/1/012151

[9] Astanakulov K D, Karimov M R, Khudaev I, Israilova D A and Muradimova F B. IOP Conf. Series: Earth and Environmental Science 614(2020) 012141, doi:10.1088/1755-1315/614/1/012141.

[10] Borotov A. IOP Conf. Series: Materials Science and Engineering 883(2020) 012160, doi:10.1088/1757-899X/883/1/012160

[11] Astanakulov K D, Fozilov G G, Kurbanov N M, Adashev B Sh and Boyturayev S A. IOP Conf. Series: Earth and Environmental Science 614(2020) 012129, doi:10.1088/1755-1315/614/1/012129

[12] Astanakulov K D, Fozilov G G, Kodirov B Kh, Khudaev I, Shermukhamedov Kh and Umarova F. IOP Conf. Series: Earth and Environmental Science 614(2020) 012130, doi:10.1088/1755-1315/614/1/012130

[13] Astanakulov K D, Gapparov Sh, Karshiev F, Makhsumkhonova A and Khudaynazarov D. IOP Conf. Series: Earth and Environmental Science 614(2020) 012158, doi:10.1088/1755-1315/614/1/012158

[14] Astanakulov K, Karshiev F, Gapparov Sh, Khudaynazarov D and Azizov Sh. E3S Web of Conferences 264, 04038

[15] Irgashev B A, Irgasheva A I. 2015 Journal of Friction and Wear. 36(5) 441-447

[16] Mirzayev Q Q, Irgasheva A. 2014 Journal of Friction and Wear. 35(5) 439-442

[17] Kragelsky I V et al. 1977 Mechanical Engineering 526

[18] Pulatov T R. 2021 Journal of Friction and Wear 1, 72-80

[19] Khaldoon F B, Dong Z, Fengshou Gu, Andrew D B. 2017 Mechanism and Machine Theory 117, 210-229