Pre-big bang model has Planck problem

D.H. Coule

School of Mathematical Sciences
University of Portsmouth, Mercantile House
Hampshire Terrace, Portsmouth PO1 2EG

Abstract

The pre-big bang’s kinetic driven inflationary mechanism is not an adequate form of inflation: the Planck length grows more rapidly than the scale factor. In order to explain our large universe, the resulting post-big bang universe requires the same unnatural constants (Planck problem) as those of any other non-inflationary big bang model.

PACS numbers: 04.20. Ex, 02.30. Hq, 98.80
1 Introduction

The pre-big bang scenario, inspired by superstring theory, is an alternative inflationary universe model to that of the usual scalar potential driven one - for reviews see [1]. This expansion can now start at time \( t \to -\infty \) and is of the form \( a \sim (-t)^p \) with \( p < 0 \). This kinetic or pole-law inflation is a more general phenomena that occurs in various alternative gravity theories such as induced gravity [2].

Most interest has developed in string theory where such an expansion is driven by the dilaton’s kinetic energy; alternatively the model can be recast as a Brans-Dicke theory with parameter \( \omega = -1 \).

Most of the studies of this model have concentrated on the singularity that occurs as \( t \to 0^- \) and how one can undergo a branch change to get to a less expansionary post-big bang region, now with expansion rate \( a \sim t^p \), \( 0 < p \leq 1 \). This branch change has proven difficult to implement [3] but one hopes that additional terms in the string action could be responsible [4] or else appeal to quantum cosmological notions such as tunneling [5]. We will not be concerned with the actual mechanism of the branch change but assume it can be correctly accounted for.

The simple model we consider is that from the low-energy string theory with action [6]

\[
S = \int d^4x \sqrt{-g} \exp(-\phi) \left( R - \omega (\partial_\mu \phi)^2 \right). \tag{1}
\]

We have only included the dilaton \( \phi \) term as this is the fundamental component that can possibly drive an expansion. Using a field redefinition \( \Phi = \exp(-\phi) \) the action can be rewritten in the more usual Brans-Dicke form

\[
S = \int d^4x \sqrt{-g} \left( \Phi R - \frac{\omega}{\Phi} (\partial_\mu \Phi)^2 \right). \tag{2}
\]

For stability in Lorentzian space one requires \( \omega > -3/2 \). Duality symmetry of string theory requires \( \omega = -1 \) [7] but we keep this \( \omega \) term general for now.

We will only be interested in the FRW ansatz

\[
d s^2 = -d t^2 + a^2(t) d \Omega_3^2, \tag{3}
\]

and will use Planck units throughout. We should stress that although Newton’s constant \( G \) and its corresponding Planck length \( (l_p \equiv G^{1/2}) \) are time
dependant in Brans-Dicke theory, we take Planck units to be their present values.

The Brans-Dicke model is known to slow down the expansion rate of a potential driven inflationary model: the field $\Phi$ can gain kinetic energy which drains energy from that being used for expansion -see eg.[8]. For $\omega < 1/2$ the conventional route of inflation becomes impossible to implement: any scalar potential becomes too steep to have inflation [9]. If instead the kinetic energy is coupled to the gravitational field it can possibly drive the expansion. In the pre-big bang scenario there are two types of solution starting at time $t = -\infty$, one which starts at infinite size and collapses $a \sim (-t)^{1/\sqrt{3}}$ and another which has the required inflationary expansion $a \sim (-t)^{-1/\sqrt{3}}$.

Brans-Dicke theory can be understood in either the Jordan frame or conformally transformed to alternative frames such as the so-called Einstein frame [10]. In this Einstein frame it simply takes the form of a massless scalar field which is not considered inflationary. We will see that both frames are equivalent for understanding the pre-big bang phase and that a deficiency of kinetic driven inflation is apparent in either frame.

2 Conformal transformation to the Einstein frame

By means of a conformal transformation of the form -see eg. [8,10]

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

where $\Omega^2 = \Phi$, we can find an equivalent action to expression (2), which can be expressed as

$$S = \int d^4x \sqrt{-\tilde{g}} \left( R(\tilde{g}) - 1/2(\tilde{\nabla}\sigma)^2 \right) ,$$

where the scalar field $\sigma$ can be defined from [8,10]

$$\Phi = \exp(\beta\sigma) ,$$

and $\beta^2 = 1/(2\omega + 3)$. This action is simply that of a massless scalar field whose field equations with a FRW metric are,

$$H^2 + \frac{k}{a^2} = \dot{\sigma}^2$$

$$\ddot{\sigma} + 3\frac{\dot{a}}{a}\dot{\sigma} = 0 \Rightarrow \dot{\sigma}^2 = \frac{\dot{A}}{a^6}$$
The solution of this equation ignoring the curvature is simply $a = A^{1/3}|t|^{1/3}$, with $t = 0$ being the initial singularity. We can now highlight a serious problem that occurs with a big bang model that uses this matter source. If such a model is to account for our present universe then the constant $A$ has to be extremely large $\sim 10^{120}$ in Planck units. This is true for any big bang model with a matter source that obeys the strong energy condition (e.g. dust or radiation). For a perfect fluid with equation of state $p = (\gamma - 1)\rho$, the expansion is $a \sim t^{2/\gamma}$, so $\gamma = 2$ corresponds to the stiff fluid or kinetic driven phase we have obtained $a \sim t^{1/3}$.

Consider a universe created with Planck radius ($\sim 10^{-33}$ cm) and Planck density ($\sim 10^{93}$gcm$^{-3}$). If such a universe expands to its present size greater than $\sim 10^{28}$cm then the density would be of order

$$\rho \sim 10^{93} \left(10^{28}/10^{-33}\right)^{-6} \simeq 10^{-273}$gcm$^{-3}$ \tag{9}$$

This should be compared to the present energy density $\sim 10^{-30}$gcm$^{-3}$. Even if the kinetic energy density was immediately converted into dust the resulting energy density would be $\sim 10^{-90}$gcm$^{-3}$. To account for this discrepancy we require the constant $A$ to be so large that the energy density is vastly greater than the Planck value for when the universe is $\sim$ Planck size or equivalently the size of the universe is already much bigger than Planck size for time $\sim$ Planck time ($t_p$). This problem we have outlined we will call the Planck density problem \cite{11} which is in many ways the most fundamental problem we first need to try and solve with a cosmological model. It is present in flat $k = 0$ universe as the natural size of a radiation dominated universe with scale factor $a \sim t^{1/2}$, with todays lifetime $10^{60}t_p$ is only $\sim 10^{-33}$cm $\ast 10^{30} \sim 10^{-3}$cm!

The closely related flatness problem is solved by having an exceedingly big value of $\dot{a}$ at the Planck time. This sets the density parameter $\Omega$, where

$$\Omega = 1 + \frac{k}{\dot{a}^2} \tag{10},$$

extremely close to unity so that even today at time $\sim 10^{60}t_p$ it still has not departed significantly from unity. Again the large value of the constant $A$ can achieve this.

So far we have not included any inflationary early stage: indeed if inflation was actually required the big-bang model would have been rejected
long ago! However, with inflation the Planck density problem is helped by having a huge expansion while the energy density remains roughly constant. This obviates the need for arbitrary constants that usually set parameters, particularly $\dot{a}$, vastly post-Planckian where quantum gravity is utterly dominant. With inflation, the constant “$A$” is automatically forced large without requiring unnatural initial values -see eg.[18].

There is a further problem of having a kinetic, or equivalently stiff equation of state $\gamma = 2$, driven early stage. There is over production of gravitational waves in the resulting universe [12]. This was taken to rule out this equation of state and and more recent studies show eg.[13] that the scalar density fluctuation spectrum is “blue shifted” - too much power on small scales for galaxy formation.

3 Jordan or string frame description
The field equations from the Brans-Dicke action (2) are well established and have solutions [1]

$$a \propto |t|^q \quad; \quad \Phi = |t|^{1-3q}$$ \hspace{1cm} (11)

where,

$$q = \frac{1 + \omega \pm \sqrt{1 + 2\omega / 3}}{4 + 3\omega}$$ \hspace{1cm} (12)

There are two different sets of solutions separated by a curvature singularity at time $t_0$ which without loss of generality can be taken to occur at time $t_0 = 0$. Note that there is an arbitrary constant that can be introduced but we initially assume it to be unity.

For the string theory case with $\omega = -1$ these solutions simplify for times $t < 0$

$$a \propto (-t)^{\mp \frac{1}{\sqrt{3}}} \quad; \quad \Phi = (-t)^{1 \pm \sqrt{3}}$$ \hspace{1cm} (13)

The upper signs correspond to the inflationary behaviour while the lower signs correspond to collapse -see Fig.(1). Incidentally this inflationary solution seems fragile in that the presence of any additional matter fields would tend to destabilize it towards the collapsing solution. But we ignore this weakness and proceed with the pure vacuum dilaton model. The post-big bang solutions are simply got from equation (13) by substitution of $t \to -t$. The required pre-big bang solution is

$$a \sim (-t)^{-\frac{1}{\sqrt{3}}} \quad; \quad \Phi \sim (-t)^{1 + \sqrt{3}} \Rightarrow l_p = (-t)^{-(1 + \sqrt{3})/2}$$ \hspace{1cm} (14)
while the expanding post-big bang solution is

\[ a \sim (t)^{\frac{1}{\sqrt{3}}} ; \Phi \sim (t)^{1+\sqrt{3}} . \]  

This post-big bang phase is chosen at it can most easily be joined to a conventional FRW expansion \( a \sim t^{2/3} \), particulary radiation \( a \sim t^{1/2} \). There is also a collapsing post-big bang branch which is presently ignored. However the field \( \Phi \) has to fixed in the post-big bang phase as there are strong constraints eg.[14] on any time evolution of \( G \equiv \Phi^{-1} \) or equivalently the Planck length \( l_p \equiv G^{1/2} \). Note that initially at \( t \to -\infty \), \( G \to 0 \) and it grows towards infinity as \( t \to 0_- \), this incidentally is the cause of the fragility as any matter will eventually dominate and cause collapse.

In Fig.(1) we show the initial expanding pre-big bang solution undergoing a branch change so that it can match to a radiation dominated phase. We assume that a branch change occurs when \( G \) takes its present value and remains constant in the subsequent post-big bang phase. Although this is somewhat ad-hoc “quenching ” of the model it would seem to give it the best chance of working as the “wanted” post-big bang solution unrealistically sets \( G = 0 \) again for \( t = 0_+ \). The other collapsing post-big bang solution would have set \( G = \infty \) at \( t = 0_+ \). This does rather constrain how the branch change should occur but there appear deficiencies in the model while it is still in the “weak coupling” domain.

Note that the scale factor and the Planck length do not have the same time dependance \( a \sim (-t)^{-0.6} \) while \( l_p \sim (-t)^{-1.4} \) -see Fig.(2). At early times the scale factor starts larger than the Planck length \( a/l_p \sim (-t) \). Even so the scale factor is infinitesimally small while the Planck length starts even smaller and grows for increasing time: we leave aside how it is consistent for strings to be actually present. As \( t \to 0_- \) the Planck length starts catching up with the scale factor \( a \) and overtakes it for \( |t| < 1 \) - see Fig.(2). This is not what one wants with inflation because we are going to require \( a >> l_p \) at the Planck time in the post-big bang phase in order to set the constant “\( A \)” there large and so avoid any Planck problem.

Kinetic inflation is actually detrimental and is taking the initial state towards a quantum gravitational region and away from what we require. As the time approaches the singularity the Planck length together with \( G \) increases until it presumably reaches its present value. Recall these values then remain constant during and after the branch change. But at this branch
change it is unclear why the scale factor is still much larger than \( l_p \) unless we again introduce an arbitrary constant \( A^{\natural} \) on the pre-big bang side, where now \( a = A^{\natural}(-t)^{-1/\sqrt{3}} \). Otherwise to keep \( a >> l_p \) the branch change has to occur long before the singularity is reached measured in the Planck time units of the post-big bang phase. This is still the ubiquitous mismatch of scales of the usual big bang model and entirely analogous to choosing the constant \( A \) in the massless scalar field model. We suspect this is true in all the alternative gravity theories, and certainly Brans-Dicke with \( \omega > -3/2 \), that display kinetic inflation as they all are conformally equivalent to a massless scalar field [8,10].

The pre-big bang model is just equivalent to the massless scalar field, no better or worse: it does not give an extra mechanism for resolving the Planck density problem. Further, the initial state at \( t \to -\infty \) seems rather contrived in that while not within the “quantum foam” it is still infinitesimally small. The kinetic inflation takes this state and drives it towards a reversed state where the Planck length is bigger than the scale factor: quantum gravitational dominance. Before this can happen some other scale is intervening and causing the natural \( a \sim l_p \) scale at the branch change to be broken to \( a >> l_p \). One might try to make the branch change account for this or have other inflationary mechanisms, but this would negate any advantage the pre-big bang phase might have over other models.

It might be objected that the scale factor has no intrinsic meaning as a FRW metric is conformally invariant with scale invariance \( a \to \lambda a \). But our actual universe does not display this symmetry simply once masses are introduced: eg. Planck mass in our case. Our fixing of \( \Phi \) in the post-big bang phase breaks the underlying duality symmetries, but these symmetries eg. \( a \to 1/a \), are now obviously broken in the present universe. It is now crucial that this actual value of the scale factor is large.

Another length scale is introduced when curvature is present and results in the flatness problem of why this length scale is so large. Usually this is resolved in a similar fashion to the Planck density problem but that cannot be assumed here. Indeed it has already been shown that the pre-big bang model is vulnerable to curvature dominating [15]. We can see this roughly since \( \dot{a} \sim (-t)^{-(1/\sqrt{3})-1} \) tends to zero as \( t \to -\infty \) cf. eq.(10), and any curvature present will dominate (recall \( G \to 0 \) in this domain so that LHS of Einstein’s equations are isolated). Depending on whether \( k = \pm 1 \) the
scale factor will either be infinite or contract to a singularity as $t \to -\infty$ [15]. Either way other mechanisms would be required to explain these initial conditions and which relegates the question of whether there is subsequent inflation. We note in passing that the initial conditions based on a canonical classical measure do suggest an infinite scale factor [16]- although there are a number of caveats with such arguments[17].

4 Quantum fluctuations as $t \to 0$.

In scalar potential driven inflationary models there is the possibility of the fluctuations being so large that eternal inflation results, where many separate universes are created. In the potential driven case this is possible for energy densities below Planck values[18]. While trying to find the analogous thing for kinetic driven inflation we were first led to doubts about this type of inflation. In kinetic driven inflation the only scale for fluctuations to dominate is now for Hubble parameter $H = (-t)^{-1} \geq 1$ but the scale factor is only $\sim 1$ at this time. Already by this stage the quantum fluctuations are dominating over the classical solution and easily cause a shift to the collapsing solution [19]. This is consistent with our criticism that the universe is not adequately being inflated to be large and classical as it can so easily be totally switched to the collapsing phase.

The fluctuation spectrum for scalar density perturbations has the same worrisome “blue spectrum” as that of a massless scalar field-see eg.[13]. It also has gravitational waves [20] which earlier were taken to rule out the massless scalar field or stiff equation of state model [12]. It seems ironic that this is being made a virtue, or at least a major point of interest[1].

5 Quantum cosmology and branch changes

It has been suggested that quantum cosmology could explain the branch change in this or related models that might avoid the problems we have outlined [5]. This is very suspect as a quantum description should not be relevant after an inflationary phase when the universe is driven large and classical. It is certainly not tidy to require quantum cosmology twice, once for the initial condition at $t \to -\infty$ and then also to avoid a singularity at time $t = 0$. In general, quantum gravitational phenomena should only occur over small volume regions and then they would be expected to cause large perturbations that probably need an inflationary mechanism to remove them. Indeed, quantum cosmology is a source of the Planck density problem, it requires inflation to expand the initial $\sim$ Planck size region it usually predicts see eg.[18].
6 Outlook and conclusions

What can be learnt from this model? Most of the misunderstanding seems to have come from taking inflation to be defined as a change in the comoving Hubble length \((Ha)^{-1} \equiv \dot{a}^{-1}\), which suggests near infinite expansion of \(a\) is possible. This has not picked up the inadequacy of kinetic inflation for solving the Planck density problem. Indeed using this definition has led one to conclude that the contracting phase of a massless scalar field in the Einstein frame is an adequate inflationary model [21] - an obvious inadequacy of any realistic model. Most previous work does not seem to have adequately made contact with our actual universe which also no longer displays duality symmetries, but rather has a fixed value of Newton’s constant. An exception is ref.[22] where similar doubts have been expressed about reconciling the pre-big bang scenario with the actual universe.

What about using the contracting phase in these model, such a phase does display a mechanism for describing fluctuations as they are “left outside” the contracting Hubble radius [1]. Such a contraction might be incorporated into a cosmological model along the lines of refs.[23,24]. Of course such contractions have nothing per se to do with inflation, recall pure radiation Fig.(1). But they do beg the question of why the universe is large in the first place and how does one avoid the inevitable approaching singularity? One needs a “bounce” that requires violation of the strong-energy condition [23,24]. In ref.[24] one uses the two post-big bang solutions and creates a bounce between them, although this uses other coupled matter fields to violate the strong-energy condition. If one is anyway going to violate the strong-energy condition one might as well use inflation as this can also explains the Planck problem together with a mechanism for creating fluctuations see eg. [18].

In conclusion, the pre-big bang model is only equivalent to an ordinary (strong-energy satisfying) big bang model. If they are to explain our present universe, both suffer, from requiring constants that exceed natural Planck values drastically. This is unlike potential driven inflation, although we leave aside arguments that that itself suffers from other problems of fine tuning.

The kinetic driven inflation is defective as the Planck length grows faster than the scale factor so sending any universe towards quantum gravitational dominance. kinetic inflation is a mirage: it goes away in the Einstein frame, and cannot inflate to give our large universe.

Acknowledgement
I should like to thank Janna Levin, David Wands and Andrei Linde for helpful discussions.

Figure Captions

Figure 1. The pre-big bang scenario. The two solutions to eq. (11) are plotted for \( \omega = -1 \) -solid lines. The required sequence of solutions are indicated by the solid arrows. The wanted expanding solution \( a \sim (-t)^{-1/\sqrt{3}} \) starting at time \( t = -\infty \) undergoes a branch change when \( t \approx 0 \) to the expanding post-big bang solution. It can then connect to the radiation dominated universe (dotted line). Note that the two unwanted collapsing branches (open arrows) of eq. (11) are conveniently ignored in this scenario. Pure radiation has a collapsing phase for negative time.

Figure 2. The Planck problem of the pre-big bang model. The scale factor (solid lines) \( a = (-t)^{-1/\sqrt{3}} \) for \( t < 0 \); and that of radiation \( a \sim t^{1/2} \) for \( t > 0 \). The Planck length (dotted lines) during the pre-big bang phase grows more rapidly than scale factor: in the post-big bang region it is fixed to its present value. Without introducing arbitrary large constants it is not clear why \( a \gg l_p \) at \( t \approx t_p = 1 \) after the branch change occurs. The resulting universe suffers a Planck problem identical to that of conventional big bang model without inflation being present.
References

1. M. Gasperini and G. Veneziano, Astropart. Phys. 1 (1993) p.317. M. Gasperini, “Birth of the Universe in String cosmology”, preprint gr-qc/9706037. S.J. Rey, “Recent progress in string inflationary cosmology”, preprint hep-th/9609115. J.J. Levin, “Gravity driven inflation”, preprint gr-qc/9506017. Available at pre-big bang web site: http://www.to.infn.it/teorici/gasperini/

2. M.D. Pollock and D. Shahdev, Phys. Lett. B 222 (1989) p.12.

3. R. Brustein and G. Veneziano, Phys. Lett. B 329 (1994) p. 429.

4. S.J. Rey, Phys. Rev. Lett. 77 91996) p.1929.

5. M. Gasperini, J. Maharana and G. Veneziano, Nucl. Phys. B 472 (1996) p.349. M. Gasperini and G. Veneziano, Gen. Rel. Grav. 28 (1996) p. 1301. J.E. Lidsey, Phys. Rev. D 55 (1997) p.3303.

6. E.S. Fradkin and A.A. Tseytlin, Phys. Lett. B 158 (1985) p.316. C.G. Callan, D. Friedan, E.J. Martinec and M.J. Perry, Nucl. Phys. B 262 (1985) p.593. C. Lovelace, Nucl. Phys. B 273 (1985) p.135.

7. G. Veneziano, Phys. Lett. B 263 (1991) p. 287.

8. S. Kalara, N. Kaloper and K.A. Olive, Nucl. Phys. B 341 (1990) p.252.

9. R. Brustein and P.J. Steinhardt, Phys. Lett. B 302 (1993) p.196.

10. G. Magnano and L.M. Sokolowski, Phys. Rev. D 50 (1994) p. 5039. L.M. Sokolowski, preprint gr-qc/9511073.

11. Y.B. Zeldovich, “My Universe: selected reviews”, Harwood Academic Press (1992)
12. Y.B. Zeldovich and I.D. Novikov, “The structure and evolution of the universe: relativistic astrophysics vol. 2”, Chicago University Press (1983) p.666.

13. J. Hwang and H. Noh, Phys. Rev. D 54 (1996) p. 1460. “Density spectrums from kinetic inflations”, preprint gr-qc/9612065

14. J.D. Barrow and P. Parsons, Phys. Rev. D 55 (1997) p.1906.

15. M.S. Turner and E.J. Weinberg, Phys. Rev. D 56 (1997) p.4604.

16. S.W. Hawking and D.N. Page, Nucl. Phys. B 298 (1988) p. 789.

17. D.H. Coule, Class. Quant. Grav. 12 (1995) p.455.

18. A.D. Linde, “Particle Physics and Inflationary cosmology”, Harwood press: Switzerland (1990).

19. Z. Lalak and R. Poppe,“Scalar field fluctuations in pre-big bang cosmologies”, preprint gr-qc/9704083.

20. J. Hwang, “Gravitational wave spectrums from kinetic inflations”, preprint gr-qc/9710061.

21. M. Gasperini and G. Veneziano, Mod. Phys. Lett. A 8 (1993) p. 3701.

22. J.J. Levin, Phys. Rev. D 51 (1995) p. 462. ibid p.1536.

23. R. Durrer and J. Laukenmann, Class. Quant. Grav. 13 (1996) p. 1069.

24. F.G. Alvarenga and J.C. Fabris, Class. Quant. Grav. 12 (1995) p. L69.