Theoretical Physics Institute
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TPI-MINN-97/06-T
UMN-TH-1532-97
hep-ph/9703437

Key Distributions for Charmless Semileptonic B Decay

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Abstract

We present theoretical predictions for a few phenomenologically interesting distributions in the semileptonic $b \to u$ decays which are affected by Fermi motion. The perturbative effects are incorporated at the one-loop level and appear to be very moderate. Our treatment of Fermi motion is based directly on QCD, being encoded in the universal distribution function $F(x)$. The decay distributions in the charged lepton energy, invariant mass of hadrons, hadron energy, and $q^2$ are given. We note that typically about 90% of all decay events are expected to have $M_X < M_D$; this feature can be exploited in determination of $|V_{ub}|$.

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1 Introduction

The measurement of heavy quark decay distributions is important in extracting key parameters of the standard model. Towards this end, theorists are faced with the tough task of separating strong binding effects, arising from strong interactions at large distances, from the relatively simple quark-lepton Lagrangian, known at short distances. Both perturbative corrections and little understood nonperturbative effects are very moderate in beauty decays if one is interested in sufficiently inclusive, integrated characteristics [1, 2]. In experimental studies of $b \to u\ell\nu$ decays, attempts to separate $b \to c$ transitions without explicitly identifying charmed particles, often suggests using a narrow slice of the overall decay kinematics reflecting the low invariant mass of final-state hadronic system accessible only in $b \to u$ decays. The strong interaction effects are magnified in this case. This fact was realized already in early papers where the QCD-based treatment of inclusive decay rates was elaborated. In particular, it was pointed out in [2] that the QCD-based OPE itself leads to the emergence of the phenomenon of “Fermi motion” of the heavy quark inside a hadron, an effect introduced ad hoc much earlier [3] for the description of heavy flavor decays in phenomenological models. For example, the impact of Fermi motion in simple quark models on the invariant mass of hadrons, $M_X$, was also addressed in [3]. The concrete manifestation of Fermi motion in QCD has some peculiarities [2, 5] making it somewhat different from the simple-minded Fermi motion of quark models. The QCD treatment was later described in detail in a number of publications [5, 6, 7, 8, 9]. Conceptually, Fermi motion is similar to the leading-twist nonperturbative effects in DIS. Further details and more extensive references can be found in [9].

In the present work we apply the methods of [9] to a few experimentally important distributions in the $b \to u$ semileptonic decays, namely, the distribution of the electron energy $d\Gamma/dE_\ell$, the invariant hadron mass $d\Gamma/dM_X$ and the hadron energy $d\Gamma/dE_h$; we also give a plot of $d\Gamma/dq^2$. The first three distributions are directly affected by the Fermi motion. Our treatment is in certain aspects simplified compared to [9]; we will mention these elements and justification later.

2 General Strategy – Including Fermi Motion in Heavy Quark Decay Distributions

Strong interactions have two faces in $b$-decays: one, associated with the hard modes of the gluon fields, is to produce perturbative corrections, the other, associated with the soft modes is to produce nonperturbative corrections, in particular Fermi motion. The way to separate the hard and soft modes is dictated by Wilson’s OPE: the introduction of the hard separation scale $\mu$. The ‘identity’ of the two regimes has no absolute sense and depends on the choice of $\mu$.

The hadronic part of the semileptonic decays is described by five structure func-
tions \( w_i(q_0, q^2) \), only three of which are relevant for \( \ell = e \) or \( \mu \) (for definitions, see \[10\]). The effect of Fermi motion is encoded in the heavy quark distribution function \( F(x) \); note that the primordial momentum is normalized to \( \Lambda = M_B - m_b \) so that we deal with the dimensionless parameter \( x \). Then the expression for the structure functions is written in the following way:

\[
w(q_0, q^2) = \int w_{\text{pert}} \left( q_0 - \frac{x\Lambda}{M_B} \sqrt{q_0^2 - q^2}, q^2 \right) F(x) \left( 1 - \frac{x\Lambda}{M_B} \frac{q_0}{\sqrt{q_0^2 - q^2}} \right) \, dx.
\]

Here \( w_{\text{pert}} \) is a parton structure function dressed with short-distance (perturbative) corrections. This expression coincides with those of Ref. \[5\] through leading-twist terms, summation of which yields the effect of the primordial momentum distribution of the heavy quark. The form of Eq. (1) is convenient since it closely follows the simple picture of the decay of the heavy quark boosted to the velocity \( x\Lambda/M_B \) along the direction of \( \vec{q} \), with \( F(x) \) determining the probability of the corresponding initial-state configuration.

The moments \( a_i \) of \( F(x) \) are given by the expectation values of local heavy quark operators. In the adopted normalization

\[
a_0 = 1, \quad a_1 = 0, \quad a_2 = \frac{\mu_x^2}{3\Lambda}, \quad a_3 = -\frac{\rho_D^3}{3\Lambda^3},
\]

with

\[
\mu_x^2 = \frac{1}{2M_B} \langle B| \bar{b}(i\vec{D})^2 b|B \rangle, \quad \rho_D^3 = \frac{1}{4M_B} \langle B| \bar{b}D_\mu G_{\mu0} b|B \rangle.
\]

One can chose a particular functional form for \( F(x) \) and adjust a few parameters to fit the phenomenologically deduced moments. This procedure was undertaken in \[9\] where the following ansatz was suggested:

\[
F(x) = \theta(1-x) e^{cx} (1-x)^a [a + b(1-x)^k].
\]

To construct the distributions we are interested in, we thus use the perturbatively corrected parton distributions \( w_{\text{pert}}(q_0, q^2) \), add the effect of the nonperturbative distribution via the convolution of Eq. (1), and calculate the decay distributions with the full \( w(q_0, q^2) \). This procedure, therefore, amounts to averaging the perturbative decay distributions over nonrelativistic primordial motion of the heavy quark governed by the distribution function \( F \).

This approach is simplified in one important aspect. In QCD, contrary to nonrelativistic models, \( F(x) \) depends essentially on \( q^2 \) and on the final state quark mass [3] even when perturbative effects are switched off. In particular, \( F(x) \) completely changes when \( m_b - \sqrt{q^2} \lesssim M_X \) where \( M_X \) is the invariant mass of the final hadronic system. For example, relations (2) are modified. In the presence of gluon bremsstrahlung \( M_X \) can become large, \( \gg \Lambda_{\text{QCD}} \), even for \( b \to u \) transitions. Certain
relations between the moments of $F$ at different $q^2$ still hold\footnote{The concrete form of Eq. (1) is chosen to preserve the lowest moments.} and we expect that the effects of changing $F(x)$ are insignificant in what concerns single-differential distributions.

Another complication of the QCD description is that $F(x)$ together with $\Lambda$ and local operators defining moments $a_i$ are normalization-point dependent when perturbative corrections to parton distributions appear. The perturbative structure functions are $\mu$-dependent as well; only the convolution (1) is $\mu$-independent. The most noticeable effect that arises is due to the running mass $m_b(\mu)$, and related variation of $\Lambda(\mu)$ \footnote{The concrete form of Eq. (1) is chosen to preserve the lowest moments.}.

3 Fermi motion in $d\Gamma/dE_\ell$, $d\Gamma/dM_X$, and $d\Gamma/dE_h$

With the structure functions (1) we arrive at the following lepton spectrum in $b \to u \ell \nu$ decays:

$$
\frac{d\Gamma}{dE_\ell} = \int \frac{d\Gamma^{\text{pert}}}{dE} \left( E_\ell - \frac{\Lambda E_\ell}{M_B} \right) F(x) \left( 1 - \frac{x\Lambda}{M_B} \right) dx . \tag{5}
$$

For the two other distributions, $d\Gamma/dM_X$ and $d\Gamma/dE_h$, we need to recall the kinematic definitions:

$$
M_X^2 = M_B^2 + q^2 - 2M_Bq_0 = [m_b^2 + q^2 - 2m_bq_0] + 2(m_b - q_0)\Lambda + \Lambda^2 \tag{6}
$$

and

$$
E_h = M_B - q_0 = m_b - q_0 + \Lambda . \tag{7}
$$

From this we get

$$
\frac{d\Gamma}{dM_X^2} = \frac{1}{2M_B} \int dq^2 \int dx F(x) \left( 1 - \frac{x\Lambda}{M_B} \right) \left( 1 - \frac{4q^2M_B^2}{(M_B^2 + q^2 - M_X^2)^2} \right)^{-1/2} \times
$$

$$
\frac{d^2\Gamma^{\text{pert}}}{dq_0 dq^2} \left( \frac{M_B^2 + q^2 - M_X^2}{2M_B} - \frac{x\Lambda}{M_B} \left( \frac{M_B^2 + q^2 - M_X^2}{2M_B} \right)^2 - q^2 , q^2 \right) \tag{8}
$$

and

$$
\frac{d\Gamma}{dE_h} = \int dq^2 \int dx F(x) \left( 1 - \frac{x\Lambda}{M_B} \right) \left( 1 - \frac{q^2}{(M_B - E_h)^2} \right)^{-1/2} \times
$$

$$
\frac{d^2\Gamma^{\text{pert}}}{dq_0 dq^2} \left( m_b - E_h + \Lambda \left( 1 - \frac{x\sqrt{(M_B - E_h)^2 - q^2}}{M_B} \right) , q^2 \right) . \tag{9}
$$
4 Perturbative corrections

Perturbative corrections are parametrically enhanced in the kinematics close to the free-quark decay. In particular, in $b \to u$ ($b \to s$) transitions double log effects appear. These enhanced corrections must be summed up in the decay $b \to s + \gamma$ where $q^2 = 0$, which was done in [9], including the effect of running $\alpha_s$. In semileptonic transitions the bremsstrahlung effects are much softer since the typical configuration corresponds to a significant invariant mass of the lepton pair. Therefore, we use instead the exact one-loop radiative corrections calculated in Ref. [11]; the running of $\alpha_s$ is thus discarded as well. It seems justified since even in the case of $b \to s + \gamma$ these effects – although very important in purely perturbative calculations including integration over low-momentum gluons, – are marginally seen when one proceeds with the Wilson’s OPE where the evolution of the coefficient functions and effective low-energy parameters is done on the same footing. We will demonstrate also that the results are fairly insensitive to variation of $\alpha_s$, which is another justification.

On the other hand, limiting ourselves to the one-loop fixed-$\alpha_s$ calculations, and keeping in mind the extremely weak numerical $\mu$-dependence of the obtained physical spectrum, we formally put $\mu = 0$ which allows using the expressions of Ref. [11] literally. This, however, necessitates using the perturbative one-loop value of the $b$-quark pole mass

$$\tilde{m}_b \simeq m_b(\mu) + \frac{16 \alpha_s}{9 \pi} \mu, \quad (10)$$

likewise a similar subtracted value of $\mu^2_\pi$

$$\tilde{\mu}^2_\pi \simeq \mu^2_\pi(\mu) - \frac{4 \alpha_s}{3 \pi} \mu^2, \quad (11)$$

etc. (for more details, see review [12]). We use $\alpha_s/\pi \simeq 0.1$ in the analysis; at $\mu \approx 1$ GeV it corresponds to $\tilde{m}_b \simeq 4.82$ GeV, and $\tilde{\mu}^2_\pi$ being about 0.1 GeV$^2$ smaller than $\mu^2_\pi$ normalized at the scale 0.5–1 GeV.

Although the perturbative distributions are known to one loop only, one can easily write them in exponentiated form (see, e.g. [9]). This is technically convenient since the elastic peak affected by virtual corrections disappears and one can then deal with smooth perturbative distributions. One relatively simple form of the exponentiated corrections for the double distribution was suggested in Eq. (17) of Ref. [13]. To simplify the calculations we used this form of radiative corrections for $d\Gamma/dM_X$ and $d\Gamma/dE_h$ where double-differential distributions must be used. The numerical difference of exponentiation is negligible after incorporating the effect of the primordial Fermi motion.

\footnote{The residual $\mu$-dependence always remains in theoretical calculations due to their approximate nature.}
5 Results and Discussion

The results of calculation of the distributions are shown in Figs. 1–3 \((d\Gamma/dE_\ell)\), Fig. 4 \((d\Gamma/dM_X)\) and Fig. 5 \((d\Gamma/dE_h)\). Our basic set of parameters is \(\tilde{m}_b = 4.82\) GeV, \(\tilde{\mu}_2 = 0.2\), 0.4 and 0.6 GeV\(^2\) which correspond to \([\alpha, a, b, c] = [1.5, 3, 16, -3.3], [0.5, 2, 0.76, -1.75]\) and \([0.1, 1.2, 0, -1.1]\), \(^{(12)}\) respectively, in the ansatz for \(F(x)\) (we use \(k = 1\)); the value of \(\alpha_s\) is set to 0.3.

To illustrate dependence on \(m_b\) we show also the plot for \(d\Gamma/dE_\ell\) for \(\tilde{m}_b = 4.73\) GeV (Fig. 2). Figure 3 shows the dependence of the lepton spectrum on \(\alpha_s\); it is clearly rather weak.

As in the case of \(b \to s + \gamma\), the effect of the Fermi motion – where present – is more pronounced than perturbative modifications. As expected, it shows up in a much softer way in \(d\Gamma/dE_\ell\) compared to \(d\Gamma/dE_\gamma\) in \(b \to s + \gamma\). The situation with \(b \to s + \gamma\) is in closer analogy with \(d\Gamma/dM_X\), but even here the effect is suppressed due to significant average invariant mass of the lepton pair, i.e. effectively smaller energy release and recoil momentum.

Apart from smearing by Fermi motion of non-smooth parts of the partonic distributions, the most significant nonperturbative effect in \(d\Gamma/dM_X\) and \(d\Gamma/dE_h\) is associated with the difference \(\Delta = M_B - m_b\) in the value of \(E_h = M_B - q_0\). This nonperturbative effect was first pointed out and analyzed in \(^{[14]}\). The predicted shape of the distributions depends to some extent on the adopted form of the distribution function \(F(x)\). We believe, nonetheless, that possible variations are not significant as long as the same underlying parameters, \(m_b(\mu)\) and \(\mu_2(\mu)\) are employed.

A more significant effect is possible from the next-to-leading twist operators, since the effective energy release is not very large. An example of such effects is the chromomagnetic interaction whose effect can be estimated \(^{[3, 9]}\), for instance, as due to the final-state mass difference between \(\pi\) and \(\rho\), or due to the initial-state mass difference for weak decays of \(B\) and \(B^*\) – these effects are neglected in the analysis. They can cause certain shifts of the distributions which are predicted if smearing over the corresponding interval is done. For example, in the distribution over \(M_X^2\) the necessary interval of smearing over \(M_X^2\) constitutes, probably, about \(\pm 0.2\) GeV\(^2\). In the regular parts of the distributions such a smearing is superfluous, of course.

For the same reason the very beginning of the distribution over \(M_X\) and \(E_h\) cannot be taken literally; in particular, one cannot deduce from it the exclusive decay rate into \(\pi\) or \(\rho\), or any particular resonance. In the limit \(m_b \to \infty\) the distance between the successive resonances in \(M_X\) would disappear in the scale of \(\langle M_X \rangle\); for the actual case of \(B\) it still produces a relatively coarse ‘grid’.

A related peculiarity of the presented plots is that they show a nonvanishing rate for \(M_X\) or \(E_h\) below pion mass, which, from the OPE viewpoint, is completely an artefact of neglecting higher-twist effects. To get rid of this unphysical feature one can, for example, consider \(ad hoc\) the given distribution as the one over \(M_X^2 - M_\pi^2\)
rather than over $M_X^2$ — distributions modified in such a way are formally equivalent to the order in $1/m_b$ expansion we work.

The decay distribution $d\Gamma/dq^2$ is not essentially affected by strong interactions. At maximal $q^2$ particularly close to $m_b^2$, nevertheless, these effects blow up. It is important to realize that it is not literally the effect of Fermi motion that shapes $d\Gamma/dq^2$. For example, the integral over the large-$q^2$ domain is governed by the flavor-dependent expectation value of the six-quark operator and thus can be completely different in decays of $B^\pm$ and $B^0$. The effects originating at $q^2 \to m_b^2$ were discussed in detail in [14]. Here we remind that studying the difference of the decay distributions for $B^\pm$ and $B^0$ near maximal $q^2$, at small $M_X$ or $E_h$ — or merely in the end-point electron spectrum — is a way to directly measure the four-fermion expectation values which are important for $B$ physics.

The decay distribution $d\Gamma/dq^2$ is shown in Fig. 6; except near the kinematic boundary, the deviation from the tree-level parton shape is very small. We obtained this distribution merely adding known nonperturbative $1/m^2$ effects [10] to the parton expressions calculated through order $\alpha_s$ [11].

An interesting observation we infer from the plots is that a significant fraction, $\approx 90\%$, of the decay events is expected to have $M_X < 1.87$ GeV, i.e. lie below the charmed states. It is in contrast with the case of semileptonic spectrum where only a small fraction of the decays proceed to the domain above the kinematical bound for $b \to c$ transitions, and even small measurement errors lead to sizeable bias. For the distribution in invariant mass, with a lower cutoff on $M_X$ between 1.5 and 1.6 GeV (to allow for a possible leaking of higher-mass states due to experimental uncertainties), the majority of decays appears in the low-$M_X$ region (80\% for $M_X < 1.6$ GeV and about 75\% for $M_X < 1.5$ GeV), and can be reliably calculated theoretically. On the other hand, this can possibly be determined in experiment. This would suggest a way for a trustworthy determination of $|V_{ub}|$ with a relatively good accuracy.

If a measurement of the distribution over $M_X^2$ is possible, it yields the possibility to determine independently the hadronic parameters. In analogy to the $b \to s + \gamma$ decays, the center of gravity of the distribution is sensitive to the $b$ quark mass and its width determines $\mu^2_\pi$. Figure 4 shows a reasonable sensitivity of the distribution over $M_X^2$ to $\mu^2_\pi$.

6 On a Possible Improvement of Theoretical Predictions

The presented analysis of the decay distributions is in many aspects simplified. While the $b$ quark mass is large enough for fully integrated characteristics, it is not the case when the detailed differential predictions are attempted. It is not therefore clear that further refinements based on more advanced treatment of perturbative corrections and nonperturbative effects can yield an essential and trustworthy improvement. The first candidate for the refinement is, obviously, inclusion of known
corrections which are not incorporated in the leading-twist effects summed up by the Fermi motion. The anticipated scale of these effects was mentioned above.

Another direction for refinements is improving the perturbative description. The impact of the perturbative corrections on the distributions even in $B$ decays appears to be smaller than those of nonperturbative effects. In particular, we expect a small effect from inclusion of higher-order $\alpha_s$-corrections. This applies, however, only when a proper treatment of the infrared domain is done – otherwise the running of $\alpha_s$, typically the dominant effect among the second-order corrections, apparently generates effects which blow up. This does not happen when one literally follows Wilson’s procedure of constructing the OPE. Experience shows that this procedure is necessary already when the second-order corrections in $b$ decays are addressed - the procedure enforces the safeguard from inconvenience of the miraculous compensation of significant effects coming from different sources [15, 16].

While introduction of the IR cutoff in calculating the perturbative coefficient functions is more or less straightforward in usual perturbative calculations, this is not so simple when exponentiation of soft/collinear effects is employed. A method applicable to these problems was described in [4]; it combines a few essential elements: reproducing exact one-loop and (all-order) BLM-improved answer and, simultaneously, proper exponentiation of singular double-log corrections. This feasible way is expected to yield more than enough accuracy in evaluating perturbative corrections.

While such a treatment is necessary when one intends to determine underlying parameters of the heavy quark expansion like $m_b$ and $\mu^2$ with the accuracy when their scale-dependence is essential, we anticipate a small overall impact of the higher-order perturbative corrections on various hadronic characteristics involved, for example, in measuring $|V_{ub}|$, well below the effect of the leading nonperturbative phenomena.

7 Outlook

One of the most important practical applications of the analysis of the $b \to u \ell\nu$ distributions is the extraction of $|V_{ub}|$. As was realized long ago [17], the theoretically cleanest way to determine it is from the total $b \to u \ell\nu$ width. In particular [17],

$$|V_{ub}| \approx 0.00415 \left( \frac{\text{BR}(B \to X_u \ell\nu)}{0.0016} \right)^{\frac{1}{2}} \left( \frac{1.55 \text{ ps}}{\tau_B} \right)^{\frac{1}{2}}$$

where theoretical uncertainties lie well below the level which is experimentally relevant now and in the near future. However, a completely model-independent experimental measurement of such a width is embryonic at present.

Another extreme way, suggested long ago, was to consider decay events with $E_\ell \gtrsim (M_B^2 - M_D^2)/2M_B = 2.31$ GeV, where charm decays cannot contribute. The decay rate in this too narrow slice of kinematics, on the other hand, depends on poorly known details of strong dynamics. Within our calculation this fraction of all
decay events does not vary too significantly; nevertheless, more work is needed to put this observation on a more firmly grounded quantitative basis.

There are many intermediate options whose advantages mainly depend on experiment. The larger the kinematic domain, the more it encompasses the parton-level kinematics, the better, in general, is the theoretical description of such an inclusive width. We found that the distribution over $M_X$ studied in experiment is rather promising in this respect. A similar suggestion was made to look at the distribution over $E_b$ [13]. It is clear that an accurate determination of $|V_{ub}|$ requires a detailed analysis of the kinematics of the decay events, which would ultimately lead to determination of just the double-differential distributions over $q_0$ and $q^2$. Which particular integral appears to be most suitable for extracting $|V_{ub}|$ will eventually depend on the available experimental technique.

ACKNOWLEDGMENTS: We are grateful to P. Henrard and his ALEPH collaborators for suggesting to us the extension of our $b \to s + \gamma$ considerations to the calculation of the semileptonic $b \to u$ distributions, useful discussions, critical comments on the results and incorporating them in the analysis of the ALEPH data [18]. We are thankful to I. Bigi and M. Shifman for stimulating comments and their encouraging interest. N.U. thanks the CERN Theory Division for kind hospitality during the period when the main calculations of this work were done. This work was supported in part by DOE under the grant number DE-FG02-94ER40823 and by NSF under the grant number PHY 92-13313.

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Figure 1: The electron energy distribution $d\Gamma_{sl}(b \rightarrow u)/dE_l$, arbitrary units. Long-dashed, solid, and short-dashed lines correspond to $\bar{\mu}^2 = 0.2, 0.4$ and 0.6 GeV$^2$. The $b$-quark mass $\bar{m}_b = 4.82$ GeV, $\alpha_s = 0.3$.

Figure 2: Dependence of $1/\Gamma_{sl}(b \rightarrow u) \ d\Gamma/dE_l$ on $m_b$. The solid line corresponds to $m_b = 4.82$ GeV, and the dashed line is for $m_b = 4.72$ GeV, while $\bar{\mu}^2 = 0.4$ GeV$^2$ is kept fixed.
Figure 3: Effect of radiative corrections on $d\Gamma/dE_l$: solid line shows $\alpha_s = 0.3$ and dashed line is $\alpha_s = 0$. Here $\bar{m}_b = 4.82$ GeV and $\bar{\mu}_\pi^2 = 0.4$ GeV$^2$.

Figure 4: The invariant mass distribution $d\Gamma/dM_x$; long-dashed, solid, and short-dashed lines correspond to $\bar{\mu}_\pi^2 = 0.2, 0.4$ and 0.6 GeV$^2$. The $b$-quark mass $\bar{m}_b = 4.82$ GeV, $\alpha_s = 0.3$. All distributions are normalized to the same total width.
Figure 5: The hadronic energy distribution $d\Gamma/dE_h$ in the same setting as in Figs. 1 and 4.

Figure 6: The $q^2$-distribution $d\Gamma/dq^2$ is made with the same conventions as for Figs. 1 and 4.