Numerical thermal featuring in $\gamma\mathrm{Al}_2\mathrm{O}_3$-$\mathrm{C}_2\mathrm{H}_6\mathrm{O}_2$ nanofluid under the influence of thermal radiation and convective heat condition by inducing novel effects of effective Prandtl number model (EPNM)

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Abstract
The investigation of thermal transport in the nanofluid attained much interest of the researchers due to their extensive applications in automobile, mechanical engineering, radiators, aerodynamics, and many other industries. Therefore, a nanofluid model is developed for $\gamma\mathrm{Al}_2\mathrm{O}_3$-$\mathrm{C}_2\mathrm{H}_6\mathrm{O}_2$ by incorporating the novel effects of Effective Prandtl Number Model (EPNM), thermal radiations, and convective heat condition. The model discussed numerically and furnished the results against the governing flow quantities. It is examined that the nanofluid velocity alters significantly due to combined convection and stretching parameter. Induction of thermal radiation in the model significantly contributed in the temperature of nanofluids and high temperature is observed by strengthen thermal radiation ($R_d$) parameter. Further, convection from the surface (convective heat condition) provided extra energy to the fluid particles which boosts the temperature of $\gamma\mathrm{Al}_2\mathrm{O}_3$-$\mathrm{C}_2\mathrm{H}_6\mathrm{O}_2$. The comparison of nanofluid ($\gamma\mathrm{Al}_2\mathrm{O}_3$-$\mathrm{C}_2\mathrm{H}_6\mathrm{O}_2$) temperature with base fluid ($\mathrm{C}_2\mathrm{H}_6\mathrm{O}_2$) revealed that $\gamma\mathrm{Al}_2\mathrm{O}_3$-$\mathrm{C}_2\mathrm{H}_6\mathrm{O}_2$ has high temperature and would be fruitful for future industrial applications. Moreover, the study is validated with previously reported literature and found reliability of the study.

Keywords
$\gamma\mathrm{Al}_2\mathrm{O}_3$ nanoparticles, thermal radiation, thermal performance, EPNM

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Introduction
The investigation of thermal performance in the nanofluids attained much fame among the researcher’s community from the last few decades. The nanofluids are newly introduced fluids that have much ability to transfer heat in comparison with conventional liquids. The regular liquids have not enough thermal transport characteristics, therefore, these fluids are limited to industrial applications. Usually, huge amount of heat is acquired to accomplish the cycle of industrial products and the conventional liquids have not enough ability to produce such amount of heat. Therefore, the nanofluids are then extensively applicable in various industries and engineering side as well.

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Primarily, nanofluids are the mixture of nano-sized nanoparticles in the host liquids which dissolve stably and thermally in equilibrium. An intermixture of various metals and oxides composed by the metals significantly alter the heat transport characteristics of the nanofluids. Due to their superior heat transport capability, the nanofluids strengthen their roots in every aspect of life. Their applications include from small level, that is, kitchen appliances to the peak of nanotechnology, that is, aerodynamics, manufacturing of air bus parts, parts of super computers, paint industries, detection of cancer cells in human body (using Ag and TiO$_2$ nanoparticles), civil engineering, biomedical engineering and many more. Due to the rising applications of the nanofluids, researchers focused to explore the heat transport characteristics from various aspects of fluid dynamics.

The dynamics of nanoliquid past a vertically placed stretchable sheet reported by Thammanna et al.$^1$ They examined heat transfer, shear stresses, and the local heat transport mechanism under the influences of imposed Lorentz forces, thermal radiation, and mixed convection. The dimensionless model was obtained by inducing the similarity transforms with appropriate differentiation w.r.t to space coordinates. They found that the convectively heated surface significantly alters heat transport mechanism in the nanoliquids. Recently, Shahzad et al.$^2$ explored the dynamics of nanofluid saturated by magnetite nanomaterial. Further, they considered the porosity and mixed convection effects in the constitutive model and discussed the results against them. Reduction in the heat transfer due to rotational parameter is reported in the analysis.

The inspection of heat transfer on Oldroyd B fluid past a stretched surface addressed in.$^3$ They impinged influences of nonlinear thermal radiation and internal heat source to explore fascinating results for heat transfer of the fluid. The flow model was obtained by adopting boundary layer approximation theory and then solved the corresponding model numerically. Augmentation in the local thermal performance of the fluid is reported due to nonlinear thermal radiation impinged in the energy equation. The study of magnetized 3D flow past a stretchable surface by taking nonlinear thermal radiations into account is described in.$^4$ They revealed that by altering the volumetric fraction of the nanoparticle, heat transport in the nanofluid rises over the desired domain. Further, the role of convective surface parameter on the heat transfer is examined in the study and found that convective parameter boosts the temperature and is better for thermal enhancement in the nanofluid.

Recently, Sreedevi et al.$^5$ addressed unsteady flow of hybrid nanoliquid by incorporating the influences of Lorentz forces. They extended the existing work for heat and mass transport in addition to chemical reaction and slip effects over a radiated stretchable surface. The hybrid mixture was made by adding hybrid nanomaterial composed by carbon nanotubes and silver particles in the host liquid. Augmentations in the temperature of hybrid nanoliquid were examined against both steady and unsteady flows. Moreover, numerical computation for local heat transport mechanism is provided by varying the flow quantities. Another significant analysis of the heat transfer over a porous stretching cylinder is described in.$^6$ The work was carried out for Cu-H$_2$O nanofluid and solved the colloid flow model via Keller box technique. The thermal behavior of the fluid examined graphically against the parameters involved in the model. Further, they concluded that the temperature improved due to elevating Reynolds number in the presence of internal heat source.

In 2019, Elgazery$^7$ performed the thermal analysis in the nanofluids by considering permeability effects of the surface. They analyzed the behavior of thermal transport in the nanoliquids composed by different tiny particles namely TiO$_2$, Cu, Ag, and Al$_2$O$_3$ with water as a host liquid. To enhance the novelty of the study, slanted magnetic field and non-uniform internal heat source are plugged in the constitutive model. The reduction in the fluid velocity and augmentation in the shear stresses are examined due to imposed slanted magnetic field. The heat transfer over a melting curved stretched surface by taking first order chemical reaction is addressed in Ref.$^8$ The study revealed that the hybrid nanoliquid has high thermal performance as compared to that of conventional nanoliquid over a curved surface.

The significant recent studies for potential class of fluids for microbial analysis of spinning microbes in blood, heat transport mechanism in Williamson nanofluid, entropy optimization and thermal performance in ZnO-SAE50, MHD influences in the presence of internal heat generation/absorption over perpendicular porous surface, Darcy Forchheimer flow (DFF) over inclined disk, multiphase behavior of the nanofluid, comparative temperature variations in nano and hybrid nanofluids under mixed convection influences and MHD flow with chemical species over an inclined surface were reported in.$^9$-$^{16}$ respectively.

Recently, Patel$^{17}$ examined internal heat source and thermal radiation effects on electrical conducting fluid in porous media. Thereafter, heat and mass transport mechanism in MHD Casson fluid over oscillatory surface with ramped surface temperature, thermal performance of two different nanofluids, cross diffusion gradients and combined convection effects on MHD Carreau fluid, dynamics of micropolar fluid in vertical walls, influences of molecular diameter and freezing temperature on thermal behavior of the nanofluid, multiple shape effects of nanoparticles on the
temperature of nanofluids, soret, and heat generation phenomena in porous media and combined convection effects on the heat and mass transport were reported in\textsuperscript{18–26}.

From the above cited literature, it is noted that no one made attempted to analyze thermal enhancement in the nanofluids by considering EPNM in the constitutive model. The novelty and research question that will be addressed in conducted study are as follows:

- How EPNM contributes in temperature of $\gamma$-$\text{Al}_2\text{O}_3$/$\text{C}_2\text{H}_6\text{O}_2$ nanofluid.
- In which type of nanofluid ($\gamma$-$\text{Al}_2\text{O}_3$/$\text{C}_2\text{H}_6\text{O}_2$) heat transport mechanism is rapid.
- Role of thermal radiations in the temperature enhancement of $\gamma$-$\text{Al}_2\text{O}_3$/$\text{C}_2\text{H}_6\text{O}_2$ nanofluid.
- How the temperature can be improved in $\gamma$-$\text{Al}_2\text{O}_3$/$\text{C}_2\text{H}_6\text{O}_2$ nanofluids by engaging convective heat condition.
- Among $\gamma$-$\text{Al}_2\text{O}_3$/$\text{C}_2\text{H}_6\text{O}_2$ nanofluids, which one has high thermal performance under above mentioned physical effects (thermal radiations and convective heat condition).

### Mathematical modeling

#### Problem statement and geometry

The flow of $\text{C}_2\text{H}_6\text{O}_2$ saturated by $\gamma$-nanomaterial of aluminum oxide is considered over a convectively heated vertically situated surface. The surface is radiated and has ability to stretching along the coordinate axes. The velocity components $\tilde{u}_w = \tilde{a}(x + y)^n$ and $\tilde{v}_w = b(x + y)^n$ are taken along x and y-axes, respectively. It is assumed that the $\gamma$-nanomaterial and water are thermally in equilibrium and no slip exists between them. Further, to improve the temperature enhancement, EPNM is ingrained in the energy equation. The flow configuration of said nanofluid is elaborated in Figure 1.

### Thermophysical correlations

In order to enhance thermal performance of the nanofluid, the following thermophysical correlations and effective Prandtl number model (EPNM) for $\gamma$-$\text{Al}_2\text{O}_3$-$\text{C}_2\text{H}_6\text{O}_2$ are used\textsuperscript{27–31}:

$$
\left( \frac{\rho C_p}{\rho} \right)_{nf} = \left( \frac{\rho C_p}{\rho} \right)_{f} \left[ (1 - \phi) + \frac{\phi \left( \frac{\rho C_p}{\rho} \right)_{p}}{\left( \frac{\rho C_p}{\rho} \right)_{f}} \right],
$$

$$
\bar{\rho}_{nf} = \left[ (1 - \phi) + \frac{\phi \left( \rho C_p \right)_{p}}{\left( \rho C_p \right)_{f}} \right] \bar{\rho}_{f},
$$

$$
\left( \frac{\rho \beta}{\rho} \right)_{nf} = \left( \frac{\rho \beta}{\rho} \right)_{f} \left[ (1 - \phi) + \frac{\phi \left( \rho \beta \right)_{p}}{\left( \rho \beta \right)_{f}} \right],
$$

$$
\bar{\mu}_{nf} = \bar{\mu}_{f}(306\phi^2 - 0.19\phi + 1).
$$

$$
\tilde{k}_{nf} = \tilde{k}_{f}(28.905\phi^2 + 2.8273\phi + 1)
$$

$$
\tilde{Pr}_{nf} = \tilde{Pr}_{f}(254.3\phi^2 - 3.0\phi + 1)
$$

Table 1 describing thermophysical values for the nanofluid:

#### Development of the flow model

By following the flow configuration of the nanofluid, we have the below dimensional version by

![Figure 1. The flow sketch of $\gamma$-$\text{Al}_2\text{O}_3$-$\text{C}_2\text{H}_6\text{O}_2$ nanofluid.](image)
incorporating the influences of mixed convection and thermal radiations in which equation (1) is the continuity equation for incompressible fluid, equations (8–10) obtained in the view of Prandtl Boundary Layer Approximation Theory (PBLAT), equation (11) is the energy equation and thermal radiation effects calculated by using Roseland approximation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (7)
\]

\[
\tilde{u} = \tilde{u}(x+y)^n \tilde{F}', \quad \tilde{v} = \tilde{u}(x+y)^n \tilde{G}'
\]

\[
\tilde{w} = - (\partial \tilde{v})^{0.5} (x+y)^{0.5(n-1)} [0.5(n+1)(F + G) + 0.5(n-1)\eta(F' + G')] \frac{\partial \tilde{w}}{\partial z} = 0, \quad (12)
\]

The conditions at convectively heated surface and far from it are given in the following version, respectively:

\[
\tilde{u} = \tilde{u}_w + \sigma_v (2 - \sigma_v) \lambda_0 \frac{\partial u}{\partial z},
\]

\[
\tilde{v} = \tilde{v}_w + \sigma_v (2 - \sigma_v) \lambda_0 \frac{\partial v}{\partial z}, \quad \tilde{w} = 0,
\]

\[
\tilde{k}_n \frac{\partial \tilde{T}}{\partial z} = \tilde{h}_f (\tilde{T} - \tilde{T}_w)
\]

and

\[
\tilde{u} \rightarrow 0, \tilde{v} \rightarrow 0, \tilde{T} \rightarrow \tilde{T}_w.
\]

In which \(\tilde{u}_w\) and \(\tilde{v}_w\), are the stretching velocities along the coordinate axes, respectively. Furthermore, \(\tilde{h}_f\), and \(\lambda_0\) representation the thermal transport and the velocity coefficients, respectively. These coefficients are taken of variable type and mathematically described by the following formulas:

\[
\lambda_0 = \lambda_0(x+y)^{0.5(1-n)} \quad \text{and} \quad \tilde{h}_f = \tilde{c}(x^{-n})^{-0.5},
\]

respectively and the terms \(\lambda_0\) and \(\tilde{c}\) are constants.

**Invertible transformations**

The following suitable invertible transformations are defined to carry out the nondimensionalization process of the model:

\[
\frac{(306\phi^2 - 0.19\phi + 1)}{(1 - \phi + \phi \frac{\phi}{\beta})} F'' - n(F' + G')F' + \frac{(n + 1)}{2} (F + G)F'' + \frac{(1 - \phi) + \phi (\frac{\phi}{\beta})}{(1 - \phi + \phi \frac{\phi}{\beta})} \lambda \beta = 0 \quad (13)
\]

\[
\frac{(306\phi^2 - 0.19\phi + 1)}{(1 - \phi + \phi \frac{\phi}{\beta})} G'' - n(F' + G')G' + 0.5(n+1)(F + G)G'' = 0 \quad (14)
\]

\[
\left(1 + \frac{Rd}{28.905\phi^2 + 2.8273\phi + 1}\right) \beta'' = \frac{(254.3\phi^2 - 3\phi + 1)}{(1 - \phi + \phi \frac{\phi}{\beta})} \left(\frac{\phi}{\beta}\right) \quad (15)
\]

The supporting dimensionless form of the flow conditions over the surface (\(\eta = 0\)) and far from the surface (\(\eta \rightarrow \infty\)) are transformed in the following version:

**Table 1.** Thermophysical values of the nanofluid.

| Properties               | \(\dot{\rho}(kg/m^3)\) | \(\beta\) (1/k) | \(\dot{c}_p(J/Kg K)\) | \(\dot{k}(W/mK)\) |
|--------------------------|------------------------|----------------|------------------------|----------------|
| Ethylene Glycol (C\(_2\)H\(_6\)O\(_2\)) | 1116.6                  | 65\times10\(^{-5}\) | 2382                  | 0.249         |
| Al\(_2\)O\(_3\)            | 3970                   | 0.85\times10\(^{-5}\) | 765                  | 40            |

\(\eta = \rho c_{p} v^{2} / D\) is the kinematic viscosity, where \(D\) is the dissipative coefficient.
\[ F'(\eta_{-0}) = 1 + SF''(\eta_{-0}), \quad G'(\eta_{-0}) = c + SG'(\eta_{-0}), \quad F(\eta_{-0}) = 0, \]
\[ G(\eta_{-0}) = 0, \quad \frac{\kappa_n}{\kappa_f} \beta'(\eta_{-0}) = B_1(\beta(\eta_{-0}) - 1) \]
\[ F'(\eta_{-\infty}) \rightarrow 0, \quad G'(\eta_{-\infty}) \rightarrow 0, \quad \beta(\eta_{-\infty}) \rightarrow 0 \]

The governing flow quantities appeared in the model are thermal radiation parameter \( (Rd = \frac{16\nu_T^2}{3\lambda_k^2}) \), stretching ratio number \( (c = \frac{\delta}{\alpha}) \), velocity slip \( \left( S = \frac{2\nu_T}{\sigma_v} \lambda_0 \left( \frac{\partial \nu}{\partial \nu} \right)^{0.5} \right) \) and Biot parameter \( \left( B_1 = \frac{c}{\kappa_f} \left( \frac{\nu}{\nu} \right)^{0.5} \right) \) due to convectively heated surface.

It is imperative to organize the formula to investigate the behavior of the shear stresses and the local thermal performance in the nanofluid past a vertical convectively heated surface. Therefore, the following formulas are developed to inspect aforementioned quantities of engineering interest: The behavior of shear stresses and local thermal transport rate at the surface is described by the following equations:

\[ \tilde{C}_F = \frac{\mu_n}{\mu_n} \left( \frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} \right) \]
\[ \tilde{C}_F = \frac{\mu_n}{\mu_n} \left( \frac{\partial \tilde{w}}{\partial \tilde{x}} + \frac{\partial \tilde{w}}{\partial \tilde{y}} \right) \]
\[ Nu = \left( \frac{x+y}{k_f(T_f-T_{\infty})} \left( 1-k_n \left( \frac{\partial \tilde{T}}{\partial \tilde{x}} \right) \right)_{\tilde{z}=0} + \tilde{q}_{rw} \right) \]

By imposing the feasible differentiation and effective values of the nanofluid, the following version is obtained:

\[ \tilde{C}_F \tilde{R}^{0.5} = (306\phi^2 - 0.19\phi + 1)F''(\eta_{-0}) \]
\[ \tilde{C}_F \tilde{R}^{0.5} = (306\phi^2 - 0.19\phi + 1)G''(\eta_{-0}) \]
\[ Nu \tilde{R}^{0.5} = -28905\phi^2 + 28273\phi + 1 \]

The local Reynolds number in the above mathematical expressions are defined as \( \tilde{R}_{x}^{0.5} = \frac{u_{x}(x+y)}{\nu} \) and \( \tilde{R}_{y}^{0.5} = \frac{u_{y}(x+y)}{\nu} \), respectively.

**Mathematical analysis of \( \gamma Al_2O_3-C_2H_6O_2 \) model**

Many physical phenomena arising in engineering and fluid mechanics are modeled via nonlinear coupled system of differential equations which does not possess closed or exact form of the solution. Therefore, solution of the model is a key tool to observe and analyze the characteristics of the model. In such scenario we prefer numerical techniques rather than the exact mathematical techniques that do not work significantly over an infinite region and for highly coupled nonlinear models. Thus, a numerical algorithm based on shooting technique is adopted to handle the model. Primarily, this method works for a system of first order initial value problem. Therefore, we need to transform our colloidal model into the desired system firstly. The whole mathematical procedure described in Figure 2. The flow chart of mathematical analysis is given below.

For this, the following transformations are adjusted:

\[ F(\eta) = \tilde{A}_1, \quad F'(\eta) = \tilde{A}_2, \quad F''(\eta) = \tilde{A}_3, \]
\[ F''(\eta) = \tilde{A}_4, \quad G(\eta) = \tilde{B}_4, \]
\[ G'(\eta) = \tilde{B}_3, \quad G''(\eta) = \tilde{B}_6, \quad G'''(\eta) = \tilde{B}_6', \]
\[ \beta(\eta) = \tilde{C}_7, \quad \beta'(\eta) = \tilde{C}_5, \quad \beta''(\eta) = \tilde{C}_6. \]

By plugging these transformations in the model, the version is obtained:

\[ \frac{(306\phi^2 - 0.19\phi + 1)}{\left( 1 - \phi + \frac{\phi \tilde{q}_{rw}}{\nu} \right)} \tilde{A}_1 - n(\tilde{A}_2 + \tilde{B}_5) \tilde{A}_2 \]
\[ + \frac{(n+1)}{2} (\tilde{A}_1 + \tilde{B}_4) \tilde{A}_3 = 0 \]

\[ \frac{(306\phi^2 - 0.19\phi + 1)}{\left( 1 - \phi + \frac{\phi \tilde{q}_{uw}}{\nu} \right)} \tilde{B}_6' - n(\tilde{A}_2 + \tilde{B}_5) \tilde{B}_6 \]
\[ + 0.5(n+1)(\tilde{A}_1 + \tilde{B}_4) \tilde{B}_6 = 0 \]

\[ \tilde{C}_9 \]

The code is run in Mathematica 10.0 and plotted the results against various flow quantities embedded in the model.

**Graphical results with discussion**

**\( \gamma Al_2O_3-C_2H_6O_2 \) axial and transverse motion**

The governing flow parameters play significant role in the nanofluid motion. This subsection deals with the axial and transverse motion of the nanofluid by altering the velocity slip parameter \( S \) and the surface stretching parameter \( c \). The effects of these parameters are elaborated in Figure 3(a) and (b), respectively.

The fluid motion pattern against the velocity slip effects is decorated in Figure 3(a). It is noted that the
axial velocity drops by increasing the strength of slip parameter. Near the surface, axial motion declines abruptly and far from the sheet, the fluid motion becomes stable and finally vanishes asymptotically. Further, most rapid decrement is observed against $n = 0.5$.

The transverse velocity of the nanofluid due to stretching parameter $c$ is highlighted in Figure 3(b). The transverse fluid motion abruptly rises against the stronger stretching parameter. Physically, stretching surface provides larger flowing area over the surface due to which the fluid particles move faster. Because of this reason, the fluid transverse velocity is maximum near the surface. As for as we move away from the surface, the stretching effects drops and increment in the fluid motion becomes slow-down in uniform manner.
\( \gamma \text{Al}_2\text{O}_3$-$\text{C}_2\text{H}_6\text{O}_2 \) thermal distribution

Figure 4 elaborates the temperature distribution in nanofluid against thermal radiations, stretching, and Biot parameters, respectively. From Figure 4(a), it is analyzed that addition of thermal radiations in the constitutive model is better for thermal enhancement in the nanofluid. Physically, internal energy of the nanofluid rises due to imposed thermal radiations (\( \text{Rd} \)) effects which lead to increment in the temperature \( \beta(\eta) \). At the surface, these effects are very prominent due to stronger thermal radiations influences. On the other side, Figure 4(b) portrayed that by increasing the strength of stretching parameter \( c \), the nanofluid thermal performance drops over the region of interest.

The temperature behavior due to convectively heated surface (Biot effects) is captured in Figure 4(c). As, Biot number is the ratio of convective heat transfer to conductive heat transfer which involves thermal conductivity of the geometry (\( k_\text{s} \)). The energy transfer in the surface due to conduction mode and then it transfers energy to the particles adjacent to the surface. As a result, the temperature of the particles upsurges and density of the particles become lighter due to which the particles escape away from the surface. The maximum temperature at the surface is due to the stronger convection and it decays asymptotically. Figure 4(d) depicts the comparative temperature profile for nanofluid and base fluid. It is observed that for nanofluid, the temperature is optimum than base fluid.

Quantities of engineering concern

Figures 5 and 6 representing the trends in shear stresses along \( x \) and \( y \)-axes, respectively. The behavior of shear stresses for both nanofluid as well conventional fluid is decorated against varying \( n \) and the slip parameter \( S \). From Figure 5, it is analyzed that the shear stresses at the surface declines for both fluids. However, maximum decreasing trends observed for nanofluid. By increasing the strength of slip parameter, declines in the shear stresses becomes slow. On the other hand, the shear stresses along \( y \)-direction varies oppositely for conventional and nanofluid, respectively. These trends captured in Figure 5(b) in which \( S \) varies horizontally.

Figure 6(a) highlights that the shear stresses for varying the velocity slip effects are rises for both sort of fluids. However, maximum increasing trends are observed by altering \( n \) horizontally. The velocity slip
parameter opposes the shear stresses along y-direction and for smaller values of n and S, the decrement is slow. These trends are captured in Figure 6(b).

The impacts of thermal radiations and Biot number on the local thermal performance (local Nusselt number) of the nanofluid are elucidated in Figure 7(a) and (b), respectively. As, Nusselt number is described the relation between convective heat transfer to conductive heat transfer by considering thermal conductivity of the nanofluid. Therefore, dominant convection and high thermal conductivity of the nanofluid play significant role in Nusselt number variations. From the captured results it is explore that the local heat transport rises by impinging thermal radiation in the energy model in the presence of convectively heated surface. Thermal radiations and convective surface transfer the energy to the fluid particles which boosts the local heat transfer rate in the nanofluid.

Table 2 presenting the numerical computation against different parameters for $F'(0)$ and $G''(0)$ over the desired domain region.

| n     | S  | $\lambda$ | $F'(0)$  | $G''(0)$ |
|-------|----|------------|----------|----------|
| 0.1   | 0.5| 0.5        | 0.318249 | 0.152139 |
| 0.3   | 0.5| 0.5        | 0.359998 | 0.164490 |
| 0.5   | 0.5| 0.5        | 0.396567 | 0.174985 |
| 0.7   | 0.5| 0.5        | 0.429025 | 0.183934 |
| 0.9   | 0.5| 0.5        | 0.458133 | 0.191575 |
| 1.1   | 0.5| 0.5        | 0.484458 | 0.198099 |
| 0.3   | 0.1| 0.5        | 0.436935 | 0.203008 |
| 0.3   | 0.3| 0.5        | 0.394549 | 0.181811 |
| 0.5   | 0.3| 0.5        | 0.359998 | 0.164490 |
| 0.7   | 0.3| 0.5        | 0.331228 | 0.150061 |
| 0.9   | 0.3| 0.5        | 0.306861 | 0.137853 |
| 1.1   | 0.3| 0.5        | 0.285935 | 0.127389 |
| 0.5   | 0.1| 0.5        | 0.384409 | 0.159960 |
| 0.3   | 0.3| 0.5        | 0.372116 | 0.162249 |
| 0.5   | 0.3| 0.5        | 0.359998 | 0.164490 |
| 0.7   | 0.3| 0.5        | 0.348046 | 0.166683 |
| 0.9   | 0.3| 0.5        | 0.335254 | 0.168833 |
| 1.1   | 0.3| 0.5        | 0.324615 | 0.170940 |
**Validation of the study**

It is important to authenticate the study with previously published literature for replication of the study. Therefore, the study is compared with literature results under certain assumptions. The comparative results are given in Table 3 and it is revealed that the results showing an excellent agreement with the results of Thammanna et al.\(^1\) which validate the study. Further, assumptions for validation highlighted in Table 3.

**Table 3.** Validation of the study with previously published literature.

| Assumptions | \( S = 0, \lambda = 0, \phi = 0 \) and \( c \) varies | \( F'(\eta = 0) \) Present result | \( F'(\eta = 0) \) Thammanna et al.\(^1\) results | \( G'(\eta = 0) \) Present result | \( G'(\eta = 0) \) Thammanna et al.\(^1\) results |
|-------------|-----------------------------------------------------|-------------------------------|-----------------------------|-----------------------------|-------------------------------|
| 0.0         | -1.0                                                 | -1.0                          | 0.0                         | 0.0                         |
| 0.5         | -1.22474                                             | -1.22474                      | -0.612373                   | -0.612373                   |
| 1.0         | -1.41421                                             | -1.41421                      | -1.41421                    | -1.41421                    |

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**Appendix**

**Nomenclature**

\( \bar{u}, \bar{v}, \bar{w} \) (x, y, z) velocity components [m/s]

\( \bar{T} \) temperature [Kelvin]

\( \bar{T}_\infty \) ambient temperature [Kelvin]

\( \bar{T}_w \) surface temperature [Kelvin]

\( \bar{C}_p \) (nf) effective heat capacitance [J/K]

\( \bar{C}_p \) (f) EG heat capacity [J/K]

\( \bar{C}_p \) (np) Al2O3 heat capacity [J/K]

\( \phi \) Al2O3 volume fraction

\( \bar{\rho} \) (nf) effective density of the nanofluid [kg/m³]

\( \bar{\rho} \) (np) Al2O3 density [kg/m³]

\( \bar{\rho} \) (f) EG density [kg/m³]

\( \bar{\mu} \) (nf) effective dynamic viscosity [kg/ms]

\( \bar{\mu} \) (f) EG dynamic viscosity [kg/ms]

\( \bar{k} \) (nf) nanofluid thermal conductivity [W/mK]

\( \bar{k} \) (f) EG thermal conductivity [W/mK]

\( \bar{k} \) (np) Al2O3 thermal conductivity [W/mK]

\( \bar{\rho} \) Pr(nf) effective Prandtl number

\( \bar{\rho} \) Prf fluid Prandtl number

\( \lambda \) mixed convection number

Rd Thermal radiation parameter

S velocity slip number

\( \bar{c} \) stretching parameter

\( \bar{B}_i \) Biot number

\( \bar{F} \) (η) dimensionless velocity

\( \bar{G} \) (η) dimensionless velocity

\( \bar{\beta} \) (η) dimensionless temperature