1. Introduction

In the portfolio selection problem, given a set of available securities or assets, we want to find out the optimum way of investing a particular amount of money in these assets. Each way of the different ways to diversify this money between the several assets is called a portfolio. For solving this portfolio problem, Markowitz \cite{10, 11} has set up a quantitative framework. Markowitz’s model which is called Mean-Variance model assumes that the return on a portfolio of assets can be completely described by the expected return and the variance of returns (risk) between these assets. For a particular universe of assets, the set of portfolios of assets that offer the minimum risk for a given level of return is the set of efficient portfolios. These portfolios can be found by convex quadratic programs (QP). But the Markowitz’s standard model, does not contain some practical constraints. For example, the standard Mean-Variance model has not got any bounding constraints limiting the amount of money to be invested in each asset neither prevents very small amounts of investments in each asset. This kind of constraints is very useful in practice and is called buy-in threshold constraints \cite{11}. In order to overcome these inconveniences, the standard model can be generalized to include these constraints.

In this paper we focus on solving the problem of portfolio selection under buy-in threshold constraints. We investigate a local deterministic approach based on DC (Difference of Convex functions) programming and DCA (DC Algorithms) that were introduced by Pham Dinh Tao in their preliminary form in 1985. They have been extensively developed since 1994 by Le Thi Hoai An and Pham Dinh Tao and become now classic and more and more popular (see e.g. \cite{4, 6, 8, 12, 13, 15} and references therein). DCA has been successfully applied to many large-scale (smooth or nonsmooth) nonconvex programs in various domains of applied sciences, for which it provided quite often a global solution and proved to be more robust and efficient than standard methods (see e.g. \cite{4, 6, 8, 12, 13, 15} and reference therein).

The existence of buy-in threshold constraints makes the corresponding portfolio selection problem nonconvex and so very difficult to solve by existing algorithms. By introducing the binary variables, we first express the buy-in threshold constraints as mixed zero-one linear constraints; then, using an exact penalty result, we reformulate the last problem in terms of a DC program. A so-called DC program is that of minimizing a DC function over a convex set. We then suggested using DC programming approach and DCA to solve this portfolio selection problem. For testing the efficiency of DCA we compare it with a Branch-and-Bound algorithm.

The paper is organized as follows. After the introduction, we present in section 2 the model of the portfolio selection problem under buy-in threshold constraints, and the reformulation in term of a DC program. Section 3 deals with DC programming and a special realization of DCA to the underlying portfolio problem. Section 4 is devoted to preliminary experimental results and some conclusions are reported in section 5.

2. Portfolio selection problem under buy-in threshold constraints

2.1. Problem formulation

First of all, as we introduce the notations that we are going to use in this paper, let us remind the well known Markowitz’s Mean-Variance model for the portfolio selection problem. Let $n$ be the number of available assets, $r_i$ be the mean return of asset $i$, $Q$ be an $n \times n$ Variance-Covariance (positive semidefinite) matrix such that its $(i, j)$-th element, that is $\sigma_{ij}$ is the covariance between returns of assets $i$ and $j$ and its value is calculated by using the following formula:

$$
\sigma_{ij} = (1/m) \sum_{k=1}^{m} (r_{ik} - \bar{r}_i)(r_{jk} - \bar{r}_j).
$$

(1)

Here $r_{ik}$ is the $(i, k)$-th historical data and $m$ is the number of periods that we have considered. Let $R$ be the desired expected return and the decision variables $y_i$ represent the
proportion \((0 \leq y_i \leq 1)\) of capital to be invested in asset \(i\) and \(y^T = (y_1, \ldots, y_n)\). Using this notation, the standard Markowitz’s Mean-Variance model is (11)

\[
\min V(y) := y^T Q y \tag{2}
\]

s.t.: \(\left\{ \sum_{i=1}^{n} r_i y_i = R, \sum_{i=1}^{n} y_i = 1, y_i \geq 0, i = 1, \ldots, n \right\}.\)

By solving this problem, one minimizes the total variance (risk) associated with the portfolio by ensuring that the portfolio has an expected return \(R\). In this paper no short-sale is allowed.

This formulation is a simple convex quadratic program for which efficient algorithms are available. By resolving the above QP for varying values of \(R\), we can trace out the efficient frontier, a smooth non-decreasing curve that gives the best possible tradeoff of risk against return.

For generalizing the standard Markowitz model with the inclusion of buy-in threshold constraints, we will use some additional notations. Let \(a_i\) and \(b_i\) be, respectively, the lower and upper bounds for the proportion of capital to be invested in asset \(i\), with \(0 < a_i \leq b_i \leq 1\). The generalized Mean-Variance model for the portfolio selection problem under buy-in threshold constraints can be written as

\[
\min V(y) := y^T Q y \tag{3}
\]

s.t.

\[
\left\{ \sum_{i=1}^{n} r_i y_i = R, \sum_{i=1}^{n} y_i = 1, y_i \in \{0\} \cup [a_i, b_i], i = 1, \ldots, n \right\}.
\]

Due to the last constraints \(y_i \in \{0\} \cup [a_i, b_i]\), this is a hard problem for which efficient algorithms are not available.

2.2. Reformulation

The later problem can be reformulated as a mixed integer quadratic problem by introducing the additional variables \(z_i\) such that

\[
z_i = 1 \text{ iff } y_i \in [a_i, b_i], 0 \text{ otherwise}.
\]

The new mixed integer quadratic programming formulation of the problem is

\[
\min V(y) := y^T Q y \tag{4}
\]

s.t.

\[
\left\{ \sum_{i=1}^{n} r_i y_i = R, \sum_{i=1}^{n} y_i = 1, a_i z_i \leq y_i \leq b_i z_i, z_i \in \{0, 1\}, i = 1, \ldots, n \right\}.
\]

Using the exact penalty result presented in [9], we will formulate (4) in the form of a convex-concave minimization problem with linear constraints which is consequently a DC program. Let

\[
A := \{(y, z) \in \mathbb{R}^n \times [0, 1]^n : \sum_{i=1}^{n} r_i y_i = R, \sum_{i=1}^{n} y_i = 1, a_i z_i \leq y_i \leq b_i z_i, i = 1, \ldots, n \}.
\]

Define the function

\[
p(y, z) := \sum_{i=1}^{n} z_i (1 - z_i).
\]

Clearly, \(p\) is a concave function with nonnegative values on \(A\) and the feasible region of (4) can be written as

\[
\left\{(y, z) \in A : z_i \in \{0, 1\} \right\} = \{(y, z) \in A : p(y, z) = 0\} = \{(y, z) \in A : p(y, z) \leq 0\}.
\]

So, (4) can be expressed as

\[
\min\{V(y) := y^T Q y : (y, z) \in A, \ p(y, z) \leq 0\}. \tag{5}
\]

Since the objective function \(V\) is convex and \(A\) is a bounded polyhedral convex set, according to [9], there is \(t_0 \geq 0\) such that for any \(t > t_0\), the program (5) is equivalent to

\[
\min\{F(y, z) := y^T Q y + tp(y, z) : (y, z) \in A\}. \tag{6}
\]

The function \(F\) is convex in variable \(y\) and concave in variable \(z\). Consequently it is a DC function. A natural DC formulation of the problem (5) is

\[
\min\{g(y, z) - h(y, z) : (y, z) \in \mathbb{R}^n \times \mathbb{R}^n\}, \tag{8}
\]

where

\[
g(y, z) := y^T Q y + \chi_A(y, z),
\]

and

\[
h(y, z) := t \sum_{i=1}^{n} z_i (z_i - 1).
\]

Here \(\chi_A\) is the indicator function on \(A\), i.e. \(\chi_A(y, z) = 0\) if \((y, z) \in A\) and \(+\infty\) otherwise.

3. Solution method via DC programming and DCA

3.1. DCA for general DC programs

Let \(\Gamma_0(\mathbb{R}^n)\) denote the convex cone of all lower semicontinuous proper convex functions on \(\mathbb{R}^n\), and consider the general DC program

\[
(P_{dc}) \quad \alpha = \operatorname{inf}\{f(x) := g(x) - h(x) : x \in \mathbb{R}^n\} \tag{7}
\]

where \(g, h \in \Gamma_0(\mathbb{R}^n)\). Such a function \(f\) is called DC function, and \(-h\), DC decomposition of \(f\) while the convex functions \(g\) and \(h\) are DC components of \(f\).

Let \(C\) be a convex set. The problem

\[
\inf\{f(x) := k - h(x) : x \in C\} \tag{8}
\]

can be transformed to an unconstrained DC program by using the indicator function on \(C\), i.e.,

\[
\inf\{f(x) := g(x) - h(x) : x \in \mathbb{R}^n\} \tag{9}
\]

where \(g := k + \chi_C\).

Let \(g^*(y) := \sup\{(x, y) - g(x) : x \in \mathbb{R}^n\}\) be the conjugate function of \(g\). Then, the following program is called the dual program of (P_{dc}):

\[
(D_{dc}) \quad \alpha_D = \inf\{h^*(y) - g^*(y) : y \in \mathbb{R}^n\}. \tag{10}
\]

Under the natural convention in DC programming that is \(+\infty - (+\infty) = +\infty\), and by using the fact that every function \(h \in \Gamma_0(\mathbb{R}^n)\) is characterized as a pointwise supremum of a collection of affine functions, say

\[
h(x) := \sup\{(x, y) - h^*(y) : y \in \mathbb{R}^n\},
\]
one can prove that \( \alpha = \alpha_D \). We observe the perfect symmetry between primal and dual DC programs: the dual to \((P_{dc})\) is exactly \((P_d)\).

Recall that, for \( \theta \in I_0(\mathbb{R}^n) \) and \( x_0 \in \text{dom} \theta := \{x \in \mathbb{R}^n : \theta(x) < +\infty\} \), \( \partial \theta(x_0) \) denotes the subdifferential of \( \theta \) at \( x_0 \), i.e.,

\[
\partial \theta(x_0) := \{ y \in \mathbb{R}^n : \theta(x) \geq \theta(x_0) + \langle x-x_0, y \rangle, \forall x \in \mathbb{R}^n \}. \tag{11}
\]

The subdifferential \( \partial \theta(x_0) \) is a closed convex set in \( \mathbb{R}^n \). It generalizes the derivative in the sense that \( \theta \) is differentiable at \( x_0 \) if and only if \( \partial \theta(x_0) \) is reduced to a singleton which is exactly \( \{ \theta'(x_0) \} \). The necessary local optimality condition for the primal DC program \((P_{dc})\) is:

\[
\partial g(x^*) \supset \partial h(x^*). \tag{12}
\]

A point \( x^* \) verifies the condition \( \partial h(x^*) \cap \partial g(x^*) \neq \emptyset \) is called a critical point of \( g - h \). The condition (12) is also sufficient for many important classes of DC programs, for example, in case of the function \( f \) is locally convex at \( x^* \) (\([7,8,12]\)).

The transportation of global solutions between \((P_{dc})\) and \((P_d)\) is expressed by:

\[
[\cup_{y \in \mathbb{R}^n} \partial g^*(y^*)] \subset \mathcal{P}, \quad [\cup_{x \in \mathbb{R}^n} \partial h(x^*)] \subset \mathcal{D} \tag{13}
\]

where \( \mathcal{P} \) and \( \mathcal{D} \) denote the solution sets of \((P_{dc})\) and \((P_d)\) respectively. Under technical conditions, this transportation holds also for local solutions of \((P_{dc})\) and \((P_d)\) (\([6,8,12,13]\)).

Based on local optimality conditions and duality in DC programming, the DCA consists in the construction of two sequences \( \{x^k\} \) and \( \{y^k\} \), candidates to be optimal solutions of primal and dual programs respectively, such that the sequences \( \{g(x^k) - h(x^k)\} \) and \( \{h^*(y^k) - g^*(y^k)\} \) are decreasing, and \( \{x^k\} \) (resp. \( \{y^k\} \)) converges to a primal feasible solution \( \bar{x} \) (resp. a dual feasible solution \( \bar{y} \)) verifying local optimality conditions and

\[
\bar{x} \in \partial g^*(\bar{y}), \quad \bar{y} \in \partial h(\bar{x}). \tag{14}
\]

The DCA then yields the next scheme:

\[
y^k \in \partial h(x^k); \quad x^{k+1} \in \partial g^*(y^k). \tag{15}
\]

In other words, these two sequences \( \{x^k\} \) and \( \{y^k\} \) are determined in the way that \( x^{k+1} \) (resp. \( y^k \)) is a solution to the convex program \((P_k)\) (resp. \( (D_k) \)) defined by

\[
\begin{align*}
\inf\{g(x) - h(x^k) - \langle x-x^k, y^k \rangle : x \in \mathbb{R}^n\}, \quad (P_k) \\
\inf\{h^*(y) - g^*(y^k+k) - \langle y-y^k, x^k \rangle : y \in \mathbb{R}^n\}, \quad (D_k).
\end{align*}
\]

In fact, at each iteration one replaces in the primal DC program \((P_{dc})\) the second component \( h(x) \) by its affine minorization \( h_k(x) := h(x^k) + \langle x-x^k, y^k \rangle \) at a neighbourhood of \( x^k \) to give birth to the convex program \((P_k)\) whose the solution set is nothing but \( \partial g^*(y^k) \). Likewise, the second DC component \( g^* \) of the dual DC program \((P_{dc})\) is replaced by its affine minorization \( g^*_k(y) := g^*(y^k) + \langle y-y^k, x^k \rangle \) at a neighbourhood of \( y^k \) to obtain the convex program \((D_k)\) whose \( \partial h(x^k+1) \) is the solution set. DCA performs so a double linearization with the help of the subgradients of \( h^* \) and \( g^* \).

It is worth noting that \([6,8,12,13]\) DCA works with the convex DC components \( g \) and \( h \) but not the DC function \( f \) itself. Moreover, a DC function \( f \) has infinitely many DC decompositions which have crucial impacts on the qualities (speed of convergence, robustness, efficiency, globality of computed solutions,...) of DCA.

Convergence properties of DCA and its theoretical basis can be found in \([6,8,12]\), for instance it is important to mention that:

- DCA is a descent method (the sequences \( \{g(x^k) - h(x^k)\} \) and \( \{h^*(y^k) - g^*(y^k)\} \) are decreasing) without line-search;
- If the optimal value \( \alpha \) of problem \((P_{dc})\) is finite and the infinite sequences \( \{x^k\} \) and \( \{y^k\} \) are bounded then every limit point \( z \) (resp. \( y \)) of the sequence \( \{x^k\} \) (resp. \( \{y^k\} \)) is a critical point of \( g - h \) (resp. \( h^* - g^* \)).
- DCA has a linear convergence for general DC programs.

### 3.2. DCA for solving \((6)\)

According to the general framework of DCA, we first need computing a sub-gradient of the function \( h \) defined by

\[
h(y, z) := t \sum_{i=1}^{n} z_i (1 - z_i). \tag{16}
\]

From the definition of \( h \) we have

\[
(u^k, v^k) \in \partial h(y^k, z^k) \Leftrightarrow u^k_i = 0, v^k_i = t(2z^k_i - 1), \quad i, j = 1, \ldots, n. \tag{17}
\]

Secondly, we have to compute an optimal solution of the following convex quadratic program

\[
\min \{ y^T Q y - \langle (y, z), (u^k, v^k) \rangle : (y, z) \in A \} \tag{18}
\]

that will be \( (y^{k+1}, z^{k+1}) \). To sum up, the DCA applied to \((6)\) can be described as follows.

**Algorithm DCA**

1. **Initialization**: Let \( \varepsilon \) be a sufficiently small positive number, let \( (y^0, z^0) \in \mathbb{R}^n \times [0,1]^n \), and set \( k = 0 \).

2. **Iteration**: \( k = 0, 1, 2, \ldots \)

   - set \( u^k_i = 0 \) and \( v^k_i = t(2z^k_i - 1) \) for \( i = 1, \ldots, n \).

   - Solve the following quadratic program

     \[
     \min \{ y^T Q y - \langle (y, z), (u^k, v^k) \rangle : (y, z) \in A \}
     \]

     to obtain \( (y^{k+1}, z^{k+1}) \).

3. If \( \| y^{k+1} - y^k \| + \| z^{k+1} - z^k \| \leq \varepsilon \), then stop.

   - \( (y^{k+1}, z^{k+1}) \) is a solution, otherwise set \( k = k + 1 \) and go to step 2.

For evaluating the quality of solutions computed by DCA and by the way their globality, we solve the problem by a classical Branch-and-Bound algorithm for mixed zero-one programming \([4]\). More precisely, the lower bound is computed by solving the classical relaxed problem of \((4)\) (the binary constraints \( z_i \in \{0,1\} \) are replaced by \( 0 \leq z_i \leq 1 \)) which is a convex quadratic program, and the upper bound is updated when a better feasible solution to \((4)\) is discovered. The subdivision is performed in the way that \( z_i = 0 \) or \( z_i = 1 \).
4. Computational experiments

We have tested the algorithms on two sets of data that have been already used in [2][3][5]. These data correspond to weekly prices from March 1992 to September 1997 and they come from the indices: Dax 100 in Germany and Nikkei 225 in Japan. The number \( n \) of different assets considered for each one of the test problems is 85 and 225, respectively. The mean returns and covariances between these returns have been calculated for the data. All the results presented here have been computed using the values \( a_i = 0.05 \) and \( b_i = 1.0 \) in (4). We have tested DCA and the classical Branch-and-Bound algorithm for different values of desired expected return \( R \). The parameter \( t \) is taken the value 0.01 for the first set of data and 0.02 for the second one. The tolerance \( \varepsilon \) is equal to \( 10^{-7} \).

The algorithms are coded in C++ and run on a Pentium 1.600GHz of 512 DDRAM.

Finding a good initial point for DCA.

In fact, one of the key questions in DCA is how to find a good initial solution for it. The question is still open. In this work, in order to find a good initial solution we first solve the relaxed problem of (4). In general the obtained solution is not necessarily integer and thus we have to modify it to get a feasible solution to (4). This new solution is taken as the initial point for DCA. The procedure can be summarized as follows:

1. **Solution of the relaxed problem**
   Solve the relaxed problem of (4) to obtain the optimal solution \((\hat{y}, \hat{z})\).

2. **Finding an integer solution**
   obtain an integer solution \(\tilde{z}\) by rounding each nonzero value \(\hat{z}_i\) to one.

The new solution \((\hat{y}, \tilde{z})\) may not be feasible to (6). We need just one iteration of DCA to obtain a feasible solution of (6), and all the other iterations of DCA will improve the solution.

We have tested DCA from different initial points:

- The point obtained by the above procedure;
- The optimal solution of the relaxed problem of (4);
- The optimal solution of the next problem

\[
\min \left\{ p(y, z) := \sum_{i=1}^{n} z_i (1 - z_i) : (y, z) \in \mathcal{A} \right\}.
\]

In our experiments the initial point of DCA given by the first procedure is the best.

In Tables 1, 2, 3, and 4, we give the results for two considered data sets. In these tables, the number of iterations (iter), the computer time in seconds (CPU), and the solutions obtained by each of the algorithms are shown.

The computational results show that DCA gives a good approximation of the optimal solution within a very short time. The running time is less than 2 seconds and the number of iterations is at most 4 for computing each solution.

Tab. 1: Numerical results of Branch-and-Bound algorithm for the first set of data

| R     | Optimal value | iter | CPU  |
|-------|---------------|------|------|
| 0.00001 | 0.000305      | 12   | 10.953 |
| 0.00002 | 0.000305      | 13   | 10.656 |
| 0.00003 | 0.000305      | 12   | 10.516 |
| 0.00004 | 0.000305      | 12   | 11.703 |
| 0.00005 | 0.000305      | 13   | 11.610 |
| 0.00006 | 0.000305      | 13   | 11.547 |
| 0.00007 | 0.000305      | 13   | 11.672 |
| 0.00008 | 0.000305      | 13   | 11.813 |
| 0.00009 | 0.000305      | 13   | 11.813 |
| 0.0001  | 0.000305      | 13   | 11.890 |
| 0.0002  | 0.000305      | 14   | 13.110 |
| 0.0003  | 0.000305      | 14   | 13.110 |
| 0.0004  | 0.000308      | 16   | 15.407 |
| 0.0005  | 0.000310      | 24   | 22.844 |
| 0.0006  | 0.000312      | 15   | 14.250 |
| 0.0007  | 0.000315      | 15   | 14.328 |
| 0.0008  | 0.000319      | 32   | 30.563 |
| 0.0009  | 0.000322      | 32   | 30.563 |
| 0.001   | 0.000326      | 30   | 29.265 |
| 0.002   | 0.000390      | 12   | 12.140 |
| 0.003   | 0.000517      | 11   | 11.657 |

Tab. 2: Numerical results of DCA for the first set of data

| R     | Optimal value | iter | CPU  |
|-------|---------------|------|------|
| 0.00001 | 0.000306      | 2    | 1.594 |
| 0.00002 | 0.000306      | 2    | 1.609 |
| 0.00003 | 0.000306      | 2    | 1.594 |
| 0.00004 | 0.000306      | 2    | 1.625 |
| 0.00005 | 0.000306      | 2    | 1.609 |
| 0.00006 | 0.000306      | 2    | 1.578 |
| 0.00007 | 0.000306      | 2    | 1.610 |
| 0.00008 | 0.000306      | 2    | 1.594 |
| 0.00009 | 0.000306      | 2    | 1.609 |
| 0.0001  | 0.000306      | 2    | 1.703 |
| 0.0002  | 0.000305      | 2    | 1.750 |
| 0.0003  | 0.000307      | 2    | 1.719 |
| 0.0004  | 0.000310      | 2    | 1.781 |
| 0.0005  | 0.000311      | 2    | 1.735 |
| 0.0006  | 0.000314      | 2    | 1.719 |
| 0.0007  | 0.000316      | 2    | 1.719 |
| 0.0008  | 0.000322      | 2    | 1.781 |
| 0.0009  | 0.000324      | 2    | 1.687 |
| 0.001   | 0.000328      | 2    | 1.781 |
| 0.002   | 0.000391      | 2    | 1.828 |
| 0.003   | 0.000519      | 2    | 1.953 |

5. Conclusions

In this paper we present a new approach for solving the portfolio selection problem. Instead of the standard Markowitz mean-variance model, we have used an extension including buy-in threshold and bounding constraints. These constraints make the corresponding portfolio selec-
Tab. 3: Numerical results of Branch-and-Bound algorithm for the second set of data

| R       | Optimal value | iter | CPU    |
|---------|---------------|------|--------|
| 0.0001  | 0.000174      | 1348 | 147.953|
| 0.0002  | 0.000170      | 718  | 77.343 |
| 0.0003  | 0.000167      | 549  | 59.313 |
| 0.0004  | 0.000162      | 671  | 72.625 |
| 0.0005  | 0.000159      | 788  | 86.500 |
| 0.0006  | 0.000158      | 1475 | 158.54 |
| 0.0007  | 0.000156      | 1980 | 209.610|
| 0.0008  | 0.000153      | 204  | 22.062 |
| 0.0009  | 0.000141      | 140  | 15.875 |
| 0.001   | 0.000134      | 129  | 14.406 |

Tab. 4: Numerical results of DCA for the second set of data

| R       | Optimal value | iter | CPU    |
|---------|---------------|------|--------|
| 0.0001  | 0.000186      | 2    | 0.235  |
| 0.0002  | 0.000189      | 2    | 0.234  |
| 0.0003  | 0.000193      | 2    | 0.218  |
| 0.0004  | 0.000182      | 3    | 0.266  |
| 0.0005  | 0.000174      | 3    | 0.266  |
| 0.0006  | 0.000173      | 4    | 0.312  |
| 0.0007  | 0.000170      | 4    | 0.313  |
| 0.0008  | 0.000167      | 3    | 0.266  |
| 0.0009  | 0.000167      | 4    | 0.313  |
| 0.001   | 0.000167      | 4    | 0.312  |
| 0.002   | 0.000156      | 2    | 0.219  |
| 0.003   | 0.000159      | 2    | 0.234  |
| 0.004   | 0.000207      | 2    | 0.203  |

The problem nonconvex and so very difficult to solve by existing algorithms. We have transformed this problem into a mixed integer quadratic program and developed a deterministic approach based on DC programming and DCA. Preliminary numerical simulations show the efficiency of DCA, its inexpensiveness and its superiority with respect to standard branch-and-bound techniques. They suggest to us extending the numerical experiments in higher dimension, and combining DCA and Branch and Bound algorithms for globally solving the problem of portfolio selection. Work in these directions is currently in progress.

REFERENCES

[1] M. Bartholomew-Biggs, “Nonlinear Optimization with Financial Applications”, Kluwer Academic Publishers, First edition, 2005.

[2] Chang T.J., N. Meade, J.E. Beasley and, Y.M. Sharaiha, “Heuristics for cardinality constrained portfolio optimization”, Computers and Operations Research, Vol. 27, pp1271-1302, 2000.

[3] Fernandez A., Y.M. Gomez, “Portfolio selection using neural networks”, Computers & Operations Research, To appear.

[4] Harrington J.E., B.F. Hobbs, J.S. Pang, A. Liu, G. Roch, “Collusive game solutions via optimisation”, Math. Program. Ser. B, Vol. 104, No. 1-2, pp407-435, 2005.

[5] Jobst N., M. Horniman, C. Lucas, G. Mitra, “Computational aspects of alternative portfolio selection models in the presence of discrete asset choice constraints”, Quantitative Finance, Vol. 1, pp1-31, 2001.

[6] Le Thi, H.A., “Contribution à l’optimisation non convexe et l’optimisation globale: Théorie, Algorithmes et Applications”, Habilitation à Diriger des Recherches, Université de Rouen, 1997.

[7] Le Thi H.A. and T. Pham Dinh, “A continuous approach for globally solving linearly constrained quadratic zero-one programming problems”, Optimization, Vol. 50, No. 1-2, pp93-120, 2001.

[8] Le Thi H.A. and T. Pham Dinh, “The DC (difference of convex functions) Programming and DCA revisited with DC models of real world non convex optimization problems”, Annals of Operations Research, Vol. 133, pp23-46, 2005.

[9] Le Thi H.A., T. Pham Dinh, V.N. Huynh, “Exact Penalty Techniques in DC Programming”, Research Report, LMI, National Institute for Applied Sciences - Rouen, France, July, 2005.

[10] Markowitz, Harry M. “Portfolio Selection”, Journal of Finance, Vol. 7, No. 1, pp77-91, 1952.

[11] Harry M. Markowitz, “Portfolio Selection”, John Wiley and Sons, New York, First Edition, 1959.

[12] Pham Dinh T. and H.A. Le Thi, “Convex analysis approach to d.c. programming: Theory, Algorithms and Applications”, Acta Mathematica Vietnamica, dedicated to Professor Hoang Tuy on the occasion of his 70th birthday, Vol. 22, No. 1, pp289-355, 1997.

[13] Pham Dinh T. and H.A. Le Thi, “DC optimization algorithms for solving the trust region subproblem”, SIAM J. Optimization, Vol. 8, pp476-505, 1998.

[14] R.T. Rockafellar, “Convex Analysis”, Princeton University Press, Princeton, First edition, 1970.

[15] Stefan Weber, Christoph Schnörr, Thomas Schüle, Joachim Hornegger, “Binary Tomography by Iterating Linear Programs”, R. Klette, R. Kozera, L. Noakes and J. Weickert (Eds.), “Computational Imaging and Vision - Geometric Properties from Incomplete Data”, Kluwer Academic Publishers, 2005.