D-BRANE DYNAMICS

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ABSTRACT

I calculate the semiclassical phase shift ($\delta$), as function of impact parameter ($b$) and velocity ($v$), when one D-brane moves past another. From its low-velocity expansion I show that, for toroidal compactifications, the moduli space of two identical D-branes stays flat to all orders in $\alpha'$. For K3 compactifications, the calculation of the D-brane moduli-space metric can be mapped to a dual gauge-coupling renormalization problem. In the ultrarelativistic regime, the absorptive part of the phase shift grows as if the D-branes were black disks of area $\sim \alpha' \ln \frac{1}{v^2}$. The scattering of large fundamental strings shares all the above qualitative features. A side remark concerns the intriguing duality between limiting electric fields and the speed of light.

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Introduction. D(irichlet)-branes [1, 2, 3, 4] are dynamical extended defects, described by the fact that open strings have their end-points stuck on them. In an important development Polchinski has recently shown [1] that D-branes carry unit charge under the Ramond-Ramond gauge fields of closed type-II string theory, so that together with their bound states [5] they could provide the excitations required by various forms of string duality [6]. This bold conjecture makes such excitations amenable to study with the techniques of conformal field theory. Here I want to use such techniques to study certain aspects of the dynamics of D-branes. Of great help in the discussion will be the (perturbative) duality between brane motion and electromagnetic open-string backgrounds: obscure statements in one language become transparent in the other, and vice versa. I will therefore start by explaining briefly this duality.

D-brane motion and electromagnetic backgrounds. Consider a 0-brane or “point particle” describing some trajectory \( Y^j(X^0) \) where \( j \) is as usual a spatial index. The appropriate world-sheet action on the disk reads

\[
S = \frac{1}{4\pi \alpha'} \int d^2z \partial^{a} X^\mu \partial_a X_\mu + \frac{1}{2\pi \alpha'} \int d\tau Y^j(X^0) \partial_\sigma X_j , \tag{1}
\]

where \( z = \tau + i\sigma \), the bulk integral is over the upper half plane, and the coordinates \( X^0, X^j \) of the string obey Neumann, respectively fixed Dirichlet conditions. In writing this action we make use of the fact that the brane coordinates couple to the boundary vertex operator \( \partial_\sigma X^j/(2\pi\alpha') \) [2, 3]. The \( \beta \)-function equations for this coupling can thus be interpreted as the classical equations of motion of the 0-brane. Rather than do the calculation, let us map this into an identical mathematical problem:

\[
\tilde{S} = \frac{1}{4\pi \alpha'} \int d^2z \partial^{a} X^\mu \partial_a X_\mu + ie \int d\tau A^j(X^0) \partial_\tau X_j , \tag{2}
\]

where all string coordinates obey now Neumann conditions. This is the action on the disk in the presence of a time-varying, constant in space electric field \( E^j = \partial_\tau A^j \), coupling to a boundary that carries charge \( e \). To see that the
two problems are identical, notice that on the boundary

\[ < \partial_\sigma X^j(\tau) \partial_\sigma X^k(\tau') > |_{\text{Dirichlet}} = \]

\[ = - < \partial_\tau X^j(\tau) \partial_\tau X^k(\tau') > |_{\text{Neumann}} = \frac{2\alpha' \delta^{jk}}{(\tau - \tau')^2}, \quad (3) \]

so that modulo zero modes the loop expansions of (1) and (2) are the same. Now the dynamics of a slowly-varying electric field is known to be governed by the Born-Infeld action \([7]\). Under the exchange \(2\pi \alpha' e E^j \leftrightarrow v^j \equiv \partial_0 Y^j\) this becomes the action for a relativistic point particle:

\[ L_{\text{BI}} \propto \sqrt{1 - (2\pi \alpha' e E)^2} \leftrightarrow L_{\text{particle}} \propto \sqrt{1 - v^j v_j}. \quad (4) \]

What we see here is a manifestation, noticed previously by Leigh \([3]\), of the well-known perturbative duality between Neumann and Dirichlet conditions \([2, 8]\). Though technically trivial, the above identification is all the same startling: the Born-Infeld action is the result of resummation of all, stringy in nature, \(\alpha'\) corrections, and implies among other things the existence of a limiting electric field \(E_{\text{crit}} = (2\pi \alpha' e)^{-1}\). In the dual picture on the other hand all this is just a consequence of the laws of relativistic particle mechanics for the 0-brane, the limiting velocity being simply the speed of light! Put differently, the fact that the solitons of type-II string theory should behave as relativistic particles, fixes uniquely the form of the (abelian) Born-Infeld action.

The Regge slope \(\alpha'\) does not in fact completely disappear from the 0-brane dynamics. Indeed, derivatives of the electric field modify the Born-Infeld action \([10]\), so that by duality the acceleration measured in units of \((\alpha')^{-1/2}\) should also enter into the full 0-brane action. This looks at first sight paradoxical: aren’t straight world lines after all the only allowed motions of a point particle? The resolution of the puzzle has to be that \(Y^j(X^0)\) loses its meaning as a point-particle trajectory when one looks at it at scales shorter than \(\sqrt{\alpha'}\). I will come back to this point in the very end.

* It is nevertheless a curious coincidence that the leading two-derivative terms, \((\partial F)^2 F^n\), are absent in the supersymmetric case \([10]\).
The above discussion can be extended easily to the case where some spatial coordinates, \( X^1, \ldots, X^p \), enter on the same footing as \( X^0 \). From the gauge-field point of view this corresponds to having both electric and magnetic backgrounds. In the dual language, on the other hand, \( Y^M(X^\alpha) \) with \( \alpha = 0, 1, \ldots, p \) and \( M = p + 1, \ldots, d \) now describe the transverse motion of a p-brane. After some straightforward matrix algebra the Born-Infeld action for the above backgrounds takes the form,

\[
\mathcal{L}_{BI} \propto \sqrt{-\det(\eta_{\mu\nu} + 2\pi\alpha' e F_{\mu\nu})} \leftrightarrow \mathcal{L}_{p-brane} \propto \sqrt{-\det(\eta_{\alpha\beta} + \partial_\alpha Y^M \partial_\beta Y_M)} .
\]

Not surprisingly the right-hand side is the Nambu-Gotto action, in the gauge in which the first \( p + 1 \) (longitudinal) space-time coordinates are used to parametrize the world history of the brane. The two sides of eq. (5) can be combined more generally into the Dirac-Born-Infeld action [3], which describes electromagnetic fields on a fluctuating membrane. Higher-order terms would again modify this action at string scales. It is also interesting to comment on what happens when some of the coordinates are compactified. As one moves around a compact longitudinal coordinate, the total displacement in a compact transverse dimension must be an integer multiple of the period. This is a classical-geometric analog of Dirac’s quantization condition, with the role of magnetic charge being played by the transverse winding number.

Calculation of the phase shift. Uniform motion and orientation of a single D-brane cannot have any non-trivial consequences, since it depends on the choice of an inertial frame. Invariant meaning can however be attached to the relative motion and orientation of two branes, which can be sensed by open strings that stretch out between them. For instance relative disalignment breaks space-time supersymmetry exactly like a magnetic field does in the dual case [11]. Here I want to concentrate, instead, on relative motion of two parallel branes, and calculate the phase shift of their forward scattering amplitude. The boundary conditions for an open string with ends attached to two parallel moving p-branes read:

\[
\partial_\sigma X^1 \ldots p = X^{p+1} \ldots d-2 = 0 \quad \text{at} \quad \sigma = 0, \pi
\]

\[ (6a) \]
and

\[ X^{d-1} = X^d - v_1 X^0 = \partial_\sigma (v_1 X^d - X^0) = 0 \quad \text{at} \quad \sigma = 0 \]

\[ X^{d-1} - b = X^d - v_2 X^0 = \partial_\sigma (v_2 X^d - X^0) = 0 \quad \text{at} \quad \sigma = \pi . \quad (6b) \]

Here \( v_1 \) and \( v_2 \) are the brane velocities in the transverse \( X^d \) direction, and \( b \) is the impact parameter. Notice that the last conditions \((6b)\) are a consequence of the world-sheet boundary equations of motion. The minus sign is due to the Minkowski signature and plays a crucial role in what follows. Now the coordinates \( X^{1,...,d-1} \) are either pure Neumann or pure Dirichlet and have conventional mode expansions. The mode expansions of the remaining two coordinates can be worked out easily with the result:

\[ X^0 \pm X^d = i\sqrt{\alpha'} \sqrt{\frac{1 \pm v_1}{1 \mp v_1}} \times \]

\[ \times \sum_{n=-\infty}^{\infty} \left\{ \frac{a_n}{n + i\epsilon} e^{-i(n+i\epsilon)(\tau \pm \sigma)} + \frac{\tilde{a}_n}{n - i\epsilon} e^{-i(n-i\epsilon)(\tau \mp \sigma)} \right\} , \quad (7) \]

where

\[ \pi \epsilon \equiv \text{arctanh}(v) = \text{arctanh}(v_2) - \text{arctanh}(v_1) . \quad (8) \]

We will assume from now on that \( v_2 > v_1 \) so that \( \epsilon \) is positive. Reality imposes the conditions \( a_n^* = a_{-n} \) and \( \tilde{a}_n^* = \tilde{a}_{-n} \), while the canonical commutation relations imply

\[ [a_n, \tilde{a}_m] = (n + i\epsilon) \delta_{n+m,0} . \quad (9) \]

Finally the total world-sheet Hamiltonian reads

\[ L_0 = \frac{b^2}{4\pi^2\alpha'} + \sum_{n=1}^{\infty} a_n^* \tilde{a}_n + \sum_{n=0}^{\infty} \tilde{a}_n^* a_n + \frac{i\epsilon(1-i\epsilon)}{2} + \text{standard} , \quad (10) \]

where “standard” are the contributions of the pure Neumann or Dirichlet coordinates \( X^{1,...,d-1} \), other than the impact-parameter dependent piece which we explicitly exhibit. The net effect of the brane motion on stretched strings can be now summarized as follows: \((i)\) the frequencies of oscillation in the \((X^0, X^d)\) hyperplane are shifted by \( \pm i\epsilon \), and \((ii)\) there is an overall velocity-dependent energy subtraction. Technically it is as if the stretched strings belong to a twisted sector of an orbifold with imaginary twist angle.
All this is again analogous to the spectrum of free open strings in a constant electric-field background \([7, 8]\). Different velocities correspond to different end-point charges, while the expression (8) for the twist parameter, which has no obvious interpretation in the electric-field case, is here recognized as the relativistic composition of velocities of the two branes. Indeed, as dictated by Lorentz invariance, the spectrum only depends on the velocity \(v\) of one brane in the rest frame of the other. The annulus diagram, which gave the induced one-loop Lagrangian in the electric-field case, corresponds now to the phase shift for the forward scattering of two D-branes \([9]\). The extension of the analysis to superstrings, as well as the calculation of the annulus diagram are both straightforward, when one keeps the orbifold analogy in mind. They have been worked out in detail for the electric field in ref. \([8]\), so I will refrain from repeating the parallel steps in our case. The only significant difference is the absence of zero modes for \(X^0 \pm X^d\), which correctly accounts for the fact that the D-branes interact locally in transverse space and in time. The final expression for the phase shift in the supersymmetric case reads:\(^\ddagger\)

\[\delta \to -V^{(p)} \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-(p+1)/2} \times \]
\[\times \sum_{\alpha=2,3,4} \frac{1}{2} e_{\alpha} \Theta_\alpha^4(0) \frac{\eta}{\sqrt{2}} \eta^{-12} \left( \frac{it}{2} \right) \int_{-\infty}^\infty d\tau e^{-(\beta^2 + v^2 \tau^2)/t} \phi \].

Note also that if the branes wrap around a compact torus, one should replace \((2\pi^2 t)^{p/2}\) by a discrete momentum sum in the above expression.

\(^\dagger\) This is true for D-branes in type-II theory, since type-I D-branes may also exchange an open string corresponding to the disk topology.

\(^\ddagger\) The factor of 2 in front takes care of the fact that for oriented strings one can change the role of the two endpoints\([1]\). As a check of the normalizations note that in the limit of vanishing velocity, eq. (11) reduces formally to the result of the adiabatic approximation in which the branes are treated as quasi-stationary at a separation \(v\tau\) in the \(X^d\) direction:
\[
\delta(b,v) = -2 \times \frac{V(p)}{4\pi} \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-p/2} e^{-t^2/2} \frac{\Theta_1'(0,t)}{\Theta_1(0,t/2)} \times
\sum_{\alpha=2,3,4} e_\alpha \Theta_\alpha(\frac{ct}{2},\frac{it}{2}) \Theta_3^\alpha(0,\frac{it}{2}) \eta^{-12}(\frac{it}{2})
\]

(\ref{11})

where \(e_2 = -e_3 = e_4 = -1\), the volume \(V(p)\) of the p-branes should be simply dropped in the special case \(p = 0\), \(\Theta_\alpha\) and \(\eta\) are the usual Jacobi and Dedekind functions, and finally we have set \(2\alpha' = 1\). Expression (\ref{11}) is our basic formula. It was obtained by treating the D-branes as classical sources, and by neglecting higher world-sheet topologies: both the Compton wavelength and the Schwarzschild radius of the D-brane are thus taken to vanish. Our result is on the other hand exact as function of \(b, v\) and \(\alpha'\).

Flatness of moduli space. Consider the behaviour of the phase shift in the low-velocity limit, \(\pi \epsilon \approx v \ll 1\). The spin-structure sum in expression (\ref{11}) starts out in this limit at quartic order,

\[
\sum_{\alpha=2,3,4} e_\alpha \Theta_\alpha(\nu|\tau) \Theta_3^\alpha(0|\tau) \sim o(\nu^4)
\]

(\ref{12})

This follows from the well-known supersymmetry identity, and the fact that the \(\Theta\) functions solve the diffusion equation

\[
[\partial_\tau + \frac{i}{4\pi} \partial_\nu^2] \Theta_\alpha(\nu|\tau) = 0
\]

(\ref{13})

The absence of the zeroth-order term in this expansion is due to the cancellation of gravitational attraction and Ramond-Ramond repulsion for static D-branes \(\square\). The absence of the quadratic term, on the other hand, shows that the \(o(\nu^2)\) forces also vanish, i.e. that identical type-II D-branes do not scatter at very low velocities. Put differently, the moduli space of two D-branes is exactly flat to all orders in the \(\alpha'\) expansion. This statement

\(\square\)They are also even functions of their first argument, so that odd powers of \(v\) will automatically vanish.
will evidently stay true if any number of coordinates is torroidally compactified. In dual language it corresponds to the well-known fact that for maximally-supersymmetric theories the Maxwell term ($\sim F^2$) is not renormalized. Based on this duality we may in fact conjecture that the moduli space stays flat when higher string-loop corrections are taken into account. In open-string compactifications that break half of the space-time supersymmetries [12], on the other hand, the Maxwell term is generically renormalized at the one open-string-loop level. This implies by duality that in generic type-II compactifications on a K3 surface, the moduli space of D-branes wrapping around the entire K3 won't be flat, but that it may be determined at large scales entirely by a single closed-string exchange. Note that the potential infrared divergences of the open-string channel are cutoff by the impact parameter ($b$) in expression (11).

The flatness of moduli space seems to be a consequence of maximal supersymmetry. It has been established previously for fundamental type-II strings and for neutral fivebranes [13, 14], while fundamental and solitonic heterotic strings were shown to exhibit non-trivial scattering in the low-velocity limit [14]. Our conclusions for D-branes are compatible with these results, and are valid to all orders in the $\alpha'$ expansion.

Absorptive part. Let us turn now to the imaginary part of the phase shift, which arises from the zeroes of $\Theta_1$ at integer values of its argument: $\epsilon t/2 = k = 1, 2, 3...$ The corresponding poles must all be traversed on the same side, so as to allow a rotation of the $t$-integration axis in the complex plane. The absorptive part can therefore be obtained by summing the residues of the integrand on these poles. The result after some straightforward manipulations reads:

$$\text{Im}(\delta) = \frac{V^{(p)}}{2(2\pi)^p} \sum_{k=1}^{\infty} \frac{1}{k} \left(\frac{\epsilon}{k}\right)^p e^{-b^2 k/\pi \epsilon} \times \left\{ Z_{\text{ferm}}(\frac{ik}{\epsilon}) - (-i)^k Z_{\text{bos}}(\frac{ik}{\epsilon}) \right\}, \quad (14)$$

where $Z_{\text{ferm}} = \frac{1}{2} \Theta_2^4/\eta^{12}$ and $Z_{\text{bos}} = \frac{1}{2}(\Theta_3^4 - \Theta_4^4)/\eta^{12}$ are the usual bosonic and fermionic open-string partition functions. This dissipation rate is the dual counterpart of open-string pair production in a constant electric background.
Its physical interpretation in our case is as follows: a pair of open strings stretching between the two branes can nucleate out of the vacuum and slow down or stop the relative motion.

Now the energy cost for nucleating two strings must be gained back through stretching due to the brane motion, so we should expect this tunneling phenomenon to be exponentially suppressed at low velocity. Indeed for \( \pi \epsilon \simeq v \ll 1 \) only massless open-string states contribute to the partition functions in (14) and the dissipation rate reduces to

\[
\text{Im}(\delta) \simeq \frac{8V^{(p)}}{(2\pi)^p} \sum_{k=1}^{\infty} \frac{1}{k} \left( \frac{v}{\pi k} \right)^{\frac{p}{2}} e^{-b^2k/v}.
\]

This is vanishingly small all the way down to impact parameters \( b \sim \sqrt{\alpha' v} \), i.e. substantially shorter than the fundamental string scale. In the ultrarelativistic limit, on the other hand, we have

\[
\epsilon \simeq -\frac{1}{2\pi} \log \frac{1-v}{2} \simeq \frac{1}{\pi} \log \left( \frac{s}{M^2_p} \right) \gg 1
\]

with \( M_p \) the mass of the p-brane. The absorptive part is now dominated by the asymptotics of the open-string partition functions near the origin, and reads

\[
\text{Im}(\delta) \simeq \frac{V^{(p)}}{2(2\pi)^p} \frac{s}{M^2_p} \left( \log \frac{s}{M^2_p} \right)^{\frac{p}{2}-4} \times e^{-b^2/\log(s/M^2_p)}.
\]

This exhibits the characteristic behaviour of scattering of fundamental strings at very high energies [15]. The D-branes behave in this limit like black absorptive disks of logarithmically growing area, \( b_{cr}^2 \sim \alpha' \ln(s/M^2_p) \). As is the case for highly-energetic dilatons and gravitons [16], ultrafast p-branes cannot probe each other at sub-stringy scales.

**Outlook.** The above results can be considered as dynamic evidence that D-branes behave like fundamental strings. Several extensions of the analysis are being envisaged: an explicit calculation of the metric of moduli-space for D-branes in K3 compactifications, a study of polarization dependence through the insertion of spin-flip operators on the boundaries of the annulus,
and a study of the phase shift for a brane moving past an anti-brane.

Do our results shed any light on Shenker’s intriguing conjecture for a new dynamical scale in string theory, that involves the string coupling constant? The one thing we can safely conclude is that fast probes have no chance of detecting such a scale. The low-velocity scattering of toroidally-compactified D-branes, on the other hand, appears indeed to be trivial at distances much shorter than the string size. As our discussion of D-brane coordinates however showed, it is very questionable whether such a well-localized probe can be prepared as an initial state in the first place.

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