The Nature of Massive Neutrinos

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Abstract

The compelling experimental evidences for oscillations of solar, reactor, atmospheric and accelerator neutrinos imply the existence of 3-neutrino mixing in the weak charged lepton current. The current data on the 3-neutrino mixing parameters are summarised and the phenomenology of 3-ν mixing is reviewed. The properties of massive Majorana neutrinos and of their various possible couplings are discussed in detail. Two models of neutrino mass generation with massive Majorana neutrinos - the type I see-saw and the Higgs triplet model, are briefly reviewed. The problem of determining the nature - Dirac or Majorana, of massive neutrinos is considered. The predictions for the effective Majorana mass $\langle m \rangle$ in neutrinoless double beta $(\beta\beta)_{0\nu}$ decay in the case of 3-neutrino mixing and massive Majorana neutrinos are summarised. The physics potential of the experiments, searching for $(\beta\beta)_{0\nu}$ decay for providing information on the type of the neutrino mass spectrum, on the absolute scale of neutrino masses and on the Majorana CP-violation phases in the PMNS neutrino mixing matrix, is also briefly discussed. The opened questions and the main goals of future research in the field of neutrino physics are outlined.

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1.1 Introduction: The Three Neutrino Mixing (an Overview)

It is a well-established experimental fact that the neutrinos and antineutrinos which take part in the standard charged current (CC) and neutral current (NC) weak interaction are of three varieties (types) or flavours: electron, $\nu_e$ and $\bar{\nu}_e$, muon, $\nu_\mu$ and $\bar{\nu}_\mu$, and tauon, $\nu_\tau$ and $\bar{\nu}_\tau$. The notion of neutrino type or flavour is dynamical: $\nu_e$ is the neutrino which is produced with $e^+$, or produces an $e^-$ in CC weak interaction processes; $\nu_\mu$ is the neutrino which is produced with $\mu^+$, or produces $\mu^-$, etc. The flavour of a given neutrino is Lorentz invariant. Among the three different flavour neutrinos and antineutrinos, no two are identical. Correspondingly, the states which describe different flavour neutrinos must be orthogonal (within the precision of the current data): $\langle \nu_l | \bar{\nu}_l \rangle = \delta_{ll}$, $\langle \bar{\nu}_l | \bar{\nu}_l \rangle = \delta_{ll}$, $\langle \bar{\nu}_l | \nu_l \rangle = 0$.

It is also well-known from the existing data (all neutrino experiments were done so far with relativistic neutrinos or antineutrinos), that the flavour neutrinos $\nu_l$ (antineutrinos $\bar{\nu}_l$), are always produced in weak interaction processes in a state that is predominantly left-handed (LH) (right-handed (RH)). To account for this fact, $\nu_l$ and $\bar{\nu}_l$ are described in the Standard Model (SM) by a chiral LH flavour neutrino field $\nu_{LL}(x)$, $l = e, \mu, \tau$. For massless $\nu_l$, the state of $\nu_l$ ($\bar{\nu}_l$), which the field $\nu_{LL}(x)$ annihilates (creates), is with helicity (-1/2) (helicity +1/2). If $\nu_l$ has a non-zero mass $m(\nu_l)$, the state of $\nu_l$ ($\bar{\nu}_l$) is a linear superposition of the helicity (-1/2) and (+1/2) states, but the helicity +1/2 state (helicity (-1/2) state) enters into the superposition with a coefficient $\propto m(\nu_l)/E$, $E$ being the neutrino energy, and thus is strongly suppressed. Together with the LH charged lepton field $l_L(x)$, $\nu_{LL}(x)$ forms an $SU(2)_L$ doublet in the Standard Model. In the absence of neutrino mixing and zero neutrino masses, $\nu_{LL}(x)$ and $l_L(x)$ can be assigned one unit of the additive lepton charge $L_l$ and the three charges $L_l$, $l = e, \mu, \tau$, are conserved by the weak interaction.

At present there is no compelling evidence for the existence of states of relativistic neutrinos (antineutrinos), which are predominantly right-handed, $\nu_R$ (left-handed, $\bar{\nu}_R$). If RH neutrinos and LH antineutrinos exist, their interaction with matter should be much weaker than the weak interaction of the flavour LH neutrinos $\nu_l$ and RH antineutrinos $\bar{\nu}_l$, i.e., $\nu_R$ ($\bar{\nu}_R$) should be “sterile” or “inert” neutrinos (antineutrinos) [1]. In the formalism of the Standard Model, the sterile $\nu_R$ and $\bar{\nu}_R$ can be described by $SU(2)_L$ singlet RH neutrino fields $\nu_R(x)$. In this case, $\nu_R$ and $\bar{\nu}_R$ will have no gauge interactions, i.e., will not couple to the weak $W^\pm$ and $Z^0$ bosons. If present in an extension of the Standard Model (even in the minimal one), the RH neutrinos can play a crucial role i) in the generation of neutrino masses and mixing, ii) in understanding the remarkable disparity between the magnitudes of neutrino masses and the masses of the charged leptons and quarks, and iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via the leptogenesis mechanism [2], see also, e.g., [3]). In this scenario which is based on the see-saw theory [4], there is a link between the generation of neutrino masses and the generation of the baryon asymmetry of the Universe. The simplest hypothesis (based on symmetry considerations) is that to each LH flavour neutrino field $\nu_{LL}(x)$ there corresponds a RH neutrino field $\nu_R(x)$, $l = e, \mu, \tau$, although schemes with less (more) than three RH neutrinos are also being considered (see, e.g., [5]).

The experiments with solar, atmospheric, reactor and accelerator neutrinos (see [6] and the references quoted therein) have provided compelling evidences for flavour neutrino oscillations [1 7 8] - transitions in flight between the different flavour neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$ (antineutrinos $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$), caused by nonzero neutrino masses and neutrino mixing. As a consequence of the results of these experiments the existence of oscillations of the solar $\nu_e$, atmospheric $\nu_\mu$ and $\nu_\tau$, accelerator $\nu_\mu$ (at $L \sim 250$ km, $L \sim 730$ km and $L \sim 295$ km, $L$ being the distance traveled by the neutrinos) and reactor $\bar{\nu}_e$ (at $L \sim 180$ km and $L \sim 1$ km), was firmly established. The data imply the presence of neutrino mixing
in the weak charged lepton current:

\[ \mathcal{L}_{CC} = - \frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{L}_l(x) \gamma_\alpha \nu_L(x) W^{\alpha\dagger}(x) + \text{h.c.}, \quad \nu_L(x) = \sum_{j=1}^{n} U_{ij} \nu_{jL}(x), \]  

where \( \nu_{jL}(x) \) are the flavour neutrino fields, \( \nu_{jL}(x) \) is the left-handed (LH) component of the field of the neutrino \( \nu_j \) having a mass \( m_{ij} \), and \( U \) is a unitary matrix - the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix \[11 \ 7 \ 8\], \( U \equiv U_{PMNS} \). All compelling neutrino oscillation data can be described assuming 3-neutrino mixing in vacuum, \( n = 3 \). The number of massive neutrinos \( n \) can, in general, be bigger than 3 if, e.g. there exist right-handed (RH) sterile neutrinos and they mix with the LH flavour neutrinos. It follows from the current data that at least 3 of the neutrinos \( \nu_j \), say \( \nu_1, \nu_2, \nu_3 \), must be light, \( m_{1,2,3} \lesssim 1 \text{ eV} \), and must have different masses, \( m_1 \neq m_2 \neq m_3 \). At present there are no compelling experimental evidence for the existence of more than 3 light neutrinos. Certain neutrino oscillation data exhibit anomalies that could be interpreted as being due to the existence of one or two additional (sterile) neutrinos with mass in the eV range, which have a relatively small mixing \( \sim 0.1 \) with the active flavour neutrinos (see, e.g., \[9\] and the references quoted therein.)

In the case of 3 light neutrinos we will concentrate on in this Section, the neutrino mixing matrix \( U \) can be parametrised by 3 angles and, depending on whether the massive neutrinos \( \nu_j \) are Dirac or Majorana \[10\] particles, by 1 or 3 CP violation (CPV) phases \[11 \ 12 \ 13\]:

\[ U = VP, \quad P = \text{diag}(1, e^{i \frac{\alpha_{21}}{2}}, e^{i \frac{\alpha_{31}}{2}}), \]  

where \( \alpha_{21} \) and \( \alpha_{31} \) are the two Majorana CPV phases and \( V \) is a CKM-like matrix containing the Dirac CPV phase \( \delta \):

\[ V = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & 0 \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{12}c_{13} \\
    s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{13}
\end{pmatrix}. \]

In eq. (1.3), \( c_{ij} = \cos \theta_{ij}, \ s_{ij} = \sin \theta_{ij} \), the angles \( \theta_{ij} = [0, \pi/2], \ \delta = [0, 2\pi] \) and, in general, \( 0 \leq \alpha_{j1}/2 \leq 2\pi, \ j = 2, 3 \ [14]. \) If CP invariance holds, we have \( \delta = 0, \pi, \) and \( [15 \ 13], \alpha_{21(31)} = k^{(1)} \pi, \) \( k^{(1)} = 0, 1, 2, 3, 4. \)

Thus, in the case of massive Dirac neutrinos, the neutrino mixing matrix \( U \) is similar, in what concerns the number of mixing angles and CPV phases, to the CKM quark mixing matrix. The presence of two additional physical CPV phases in \( U \) if \( \nu_j \) are Majorana particles is a consequence of the special properties of the latter (see, e.g. refs. \[11 \ 16\]). On the basis of the existing neutrino data it is impossible to determine whether the massive neutrinos are Dirac or Majorana fermions.

The neutrino oscillation probabilities depend, in general, on the neutrino energy, \( E \), the source-detector distance \( L \), on the elements of \( U \) and, for relativistic neutrinos used in all neutrino experiments performed so far, on the neutrino mass squared differences \( \Delta m^2_{ij} \equiv (m_i^2 - m_j^2), \ i \neq j \) (see, e.g., ref. \[6 \ 16\]). In the case of 3-neutrino mixing there are only two independent neutrino mass squared differences, say \( \Delta m^2_{21} \neq 0 \) and \( \Delta m^2_{31} \neq 0 \). The numbering of massive neutrinos \( \nu_j \) is arbitrary. We will employ here the widely used convention of numbering of \( \nu_j \) which allows to associate \( \theta_{13} \) with the smallest mixing angle in the PMNS matrix \( U \), and \( \theta_{12}, \Delta m^2_{21} > 0 \), and \( \theta_{23}, \Delta m^2_{31(32)} \), with the parameters which drive, respectively, the solar \( \nu_e \) oscillations at \( L \sim 180 \text{ km} \) (see, e.g., \[6\]).

\[ \text{Under the assumption of CPT invariance, which we will suppose to hold throughout this article, } \theta_{12} \text{ and } \Delta m^2_{21} \text{ drive also the reactor } \nu_e \text{ oscillations at } L \sim 180 \text{ km} \text{ (see, e.g., } [6]). \]
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(and accelerator $\nu_i$) oscillations. In this convention $m_1 < m_2$, $0 < \Delta m^2_{21} < |\Delta m^2_{31}|$, and, depending on $\text{sgn}(\Delta m^2_{31})$, we have either $m_3 < m_1$ or $m_3 > m_2$ (see further). In the case of $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$), the neutrino mass squared difference $\Delta m^2_{21}$, as it follows from the data to be discussed below, is much smaller than $|\Delta m^2_{31(32)}|$, $\Delta m^2_{21} \ll |\Delta m^2_{31(32)}|$. This implies that in each of the two cases $m_1 < m_2 < m_3$ and $m_3 < m_1 < m_2$ we have $|\Delta m^2_{31} - \Delta m^2_{32}| = \Delta m^2_{21} \ll |\Delta m^2_{31, 32}|$.

The angles $\theta_{12}$ and $\theta_{23}$ are sometimes called “solar” and “atmospheric” neutrino mixing angles, and are often denoted as $\theta_{12} = \theta_{\odot}$ and $\theta_{23} = \theta_{\text{atm}}$, while $\Delta m^2_{21}$ and $\Delta m^2_{31(32)}$ are sometimes referred to as the “solar” and “atmospheric” neutrino mass squared differences and are sometimes denoted as $\Delta m^2_{\odot}$ and $\Delta m^2_{\text{atm}}$.

The neutrino oscillation data, accumulated over many years, allowed to determine the parameters which drive the solar, reactor, atmospheric and accelerator neutrino oscillations, $\Delta m^2_{21}$, $\theta_{12}$, $|\Delta m^2_{31(32)}|$ and $\theta_{23}$, with a rather high precision (see, e.g., [17]). Furthermore, there were spectacular developments in the period since June 2011 in what concerns the angle $\theta_{13}$ (see, e.g., [6]). They culminated in March of 2012 in a high precision determination of $\sin^2 2\theta_{13}$ in the Daya Bay experiment with reactor $\bar{\nu}_e$ [18):

$$\sin^2 2\theta_{13} = 0.098 \pm 0.0001 \pm 0.005.$$

Subsequently [7], the RENO [19], Double Chooz, and T2K experiments [20] reported, respectively, 4.9$\sigma$, 2.9$\sigma$ and 3.2$\sigma$ evidences for a non-zero value of $\theta_{13}$, compatible with the Daya Bay result.

A global analysis of the latest neutrino oscillation data presented at the Neutrino 2012 International Conference [17], was performed in [21]. We give below the best fit values of $\Delta m^2_{21}$, $\sin^2 \theta_{12}$, $|\Delta m^2_{31(32)}|$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$, obtained in [21]:

$$\Delta m^2_{21} = 7.54 \times 10^{-5} \text{ eV}^2, \quad |\Delta m^2_{31(32)}| = 2.47 (2.46) \times 10^{-3} \text{ eV}^2,$$

$$\sin^2 \theta_{12} = 0.307, \quad \sin^2 \theta_{23} = 0.39, \quad \sin^2 \theta_{13} = 0.0241 (0.0244),$$

where the values (the values in brackets) correspond to $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$). The 1$\sigma$ uncertainties and the the 3$\sigma$ ranges of the neutrino oscillation parameters found in [21] are given in Table 1.1.

A few comments are in order. We have $\Delta m^2_{21}/|\Delta m^2_{31(32)}| \approx 0.031 \ll 1$, as was indicated earlier. The existing data do not allow to determine the sign of $\Delta m^2_{31(32)}$. As we will discuss further, the two possible signs correspond to two different basic types of neutrino mass spectrum. Maximal solar neutrino mixing, i.e. $\theta_{12} = \pi/4$, is ruled out at more than 6$\sigma$ by the data. Correspondingly, one has $\cos 2\theta_{12} \geq 0.28$ at 3$\sigma$. The results quoted in eq. (1.5) imply that $\theta_{23} \cong \pi/4$, $\theta_{12} \cong \pi/5.4$ and that $\theta_{13} \cong \pi/20$. Thus, the pattern of neutrino mixing is drastically different from the pattern of quark mixing.

As we have noticed earlier, the neutrino oscillations experiments are sensitive only to neutrino mass squared differences $\Delta m^2_{ij} \equiv (m^2_i - m^2_j)$, $i \neq j$, and cannot give information on the absolute values of the neutrino masses, i.e., on the absolute neutrino mass scale. They are insensitive also to the nature - Dirac or Majorana, of massive neutrinos $\nu_j$ and, correspondingly, to the Majorana CPV phases present in the PMNS matrix $U$ [11, 22].

\footnote{The preceding part of the text of the present article follows closely parts of the text of the review article [6].}

\footnote{We have reported in eq. (1.4) the latest result of the Daya Bay experiment, published in the second article quoted in [13].}

\footnote{Note that we have quoted the value of $|\Delta m^2_{31(32)}|$ in eq. (1.5), while the mass squared difference determined in [21] is $|\Delta m^2_A| = |\Delta m^2_{31} - \Delta m^2_{21}/2| (|\Delta m^2_A| = |\Delta m^2_{32} + \Delta m^2_{21}/2|).
Table 1.1: The best-fit values and 3σ allowed ranges of the 3-neutrino oscillation parameters, derived in [21] from a global fit of the current neutrino oscillation data. The values (values in brackets) correspond to \( m_1 < m_2 < m_3 \) \( (m_3 < m_1 < m_2) \). The definition of \( \Delta m^2_{31} \) used: \( \Delta m^2_A = m^2_3 - (m^2_2 + m^2_1)/2 \). Thus, \( \Delta m^2_A = \Delta m^2_{31} = \Delta m^2_{32} + \Delta m^2_{21}/2 \) for \( m_3 < m_1 < m_2 \).

| Parameter | best-fit \((\pm 1\sigma)\) | 3σ |
|-----------|-----------------|-----|
| \( \Delta m^2_{31} \) \(10^{-5} \text{ eV}^2\) | 7.54 \(\pm 0.26\) | 6.99 - 8.18 |
| \( |\Delta m^2_A| \) \(10^{-3} \text{ eV}^2\) | 2.43 \(\pm 0.06\) \((2.42 \pm 0.07)\) & 2.19(2.17) - 2.62(2.61) |
| \( \sin^2 \theta_{12} \) | 0.307 \(\pm 0.018\) | 0.259 - 0.359 |
| \( \sin^2 \theta_{23} \) | 0.386 \(\pm 0.024\) \((0.392 \pm 0.023)\) & 0.331(0.335) - 0.637(0.663) |
| \( \sin^2 \theta_{13} \) | 0.0241 \(\pm 0.0025\) \((0.0244 \pm 0.0023)\) & 0.0169(0.0171) - 0.0313(0.0315) |

After the successful measurement of \( \theta_{13} \), the determination of the absolute neutrino mass scale, of the type of the neutrino mass spectrum, of the nature - Dirac or Majorana, of massive neutrinos, as well as getting information about the status of CP violation in the lepton sector, are the most pressing well as getting information about the status of CP violation in the lepton sector, are the most pressing and challenging problems and the highest priority goals of the research in the field of neutrino physics (see, e.g., [6]).

As was already indicated above, the presently available data do not permit to determine the sign of \( \Delta m^2_{31(2)} \). In the case of 3-neutrino mixing, the two possible signs of \( \Delta m^2_{31(2)} \) correspond to two types of neutrino mass spectrum. In the widely used convention of numbering the neutrinos with definite mass employed by us, the two spectra read:

i) **spectrum with normal ordering (NO):** \( m_1 < m_2 < m_3 \), \( \Delta m^2_{atm} = \Delta m^2_{31} > 0 \), \( \Delta m^2_{\odot} \equiv \Delta m^2_{21} > 0 \), \( m_3(3) = (m^2_3 + \Delta m^2_{32(31)})^{1/2} \);

ii) **spectrum with inverted ordering (IO):** \( m_3 < m_1 < m_2 \), \( \Delta m^2_{atm} = \Delta m^2_{32} < 0 \), \( \Delta m^2_{\odot} \equiv \Delta m^2_{21} > 0 \), \( m_2 = (m^2_2 + \Delta m^2_{32} - \Delta m^2_{23})^{1/2} \).

Depending on the value of the lightest neutrino mass, \( m(j) \), the neutrino mass spectrum can be:

a) **Normal Hierarchical (NH):** \( m_1 \ll m_2 < m_3 \), \( m_2 \cong (\Delta m^2_{32})^{1/2} \approx 8.68 \times 10^{-3} \text{ eV} \), \( m_3 \cong |\Delta m^2_{atm}|^{1/2} \approx 4.97 \times 10^{-2} \text{ eV} \); or

b) **Inverted Hierarchical (IH):** \( m_3 \ll m_1 < m_2 \), with \( m_{1,2} \cong |\Delta m^2_{atm}|^{1/2} \approx 4.97 \times 10^{-2} \text{ eV} \); or

c) **Quasi-Degenerate (QD):** \( m_1 \cong m_2 \cong m_3 \cong m_0 \), \( m^2_j \gg |\Delta m^2_{atm}|, m_0 \gtrsim 0.10 \text{ eV} \).

The type of neutrino mass spectrum (hierarchy), i.e., the sign of \( \Delta m^2_{31(2)} \), can be determined i) using data from neutrino oscillation experiments at accelerators (NO\( \nu A, T2K \), etc.) (see, e.g., [23]), ii) in the experiments studying the oscillations of atmospheric neutrinos (see, e.g., [24]), as well as iii) in experiments with reactor antineutrinos [25]. The relatively large value of \( \theta_{13} \) is a favorable factor for the \( \text{sgn}(\Delta m^2_{31(2)}) \) determination in these experiments. If neutrinos with definite mass are Majorana particles, information about the \( \text{sgn}(\Delta m^2_{31(2)}) \) can be obtained also by measuring the effective neutrino Majorana mass in neutrinoless double \( \beta \)-decay experiments [26].

More specifically, in the cases i) and ii) the \( \text{sgn}(\Delta m^2_{31(2)}) \) can be determined by studying oscillations of neutrinos and antineutrinos, say, \( \nu_\mu \leftrightarrow \nu_e \) and \( \bar{\nu}_\mu \leftrightarrow \bar{\nu}_e \), in which matter effects are sufficiently large. This can be done in long base-line \( \nu \)-oscillation experiments (see, e.g., [23]). For \( \sin^2 \theta_{13} \approx 0.05 \) and \( \sin^2 \theta_{23} \gtrsim 0.50 \), information on \( \text{sgn}(\Delta m^2_{31(2)}) \) might be obtained in atmospheric neutrino exper-
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iments by investigating the effects of the sub-dominant transitions $\nu_{\mu}(e) \rightarrow \nu_{e}(\mu)$ and $\bar{\nu}_{\mu}(e) \rightarrow \bar{\nu}_{e}(\mu)$ of atmospheric neutrinos which traverse the Earth (for a detailed discussion see, e.g., [24]). For $\nu_{\mu}(e)$ (or $\bar{\nu}_{\mu}(e)$) crossing the Earth core, new type of resonance-like enhancement of the indicated transitions takes place due to the (Earth) mantle-core constructive interference effect (neutrino oscillation length resonance (NOLR)) [27] (see also [28]). As a consequence of this effect the corresponding $\nu_{\mu}(e)$ (or $\bar{\nu}_{\mu}(e)$) transition probabilities can be maximal [29] (for the precise conditions of the mantle-core (NOLR) enhancement see [27, 29]). For $\Delta m_{31(32)}^{2} > 0$, the neutrino transitions $\nu_{\mu}(e) \rightarrow \nu_{e}(\mu)$ are enhanced, while for $\Delta m_{31(32)}^{2} < 0$ the enhancement of antineutrino transitions $\bar{\nu}_{\mu}(e) \rightarrow \bar{\nu}_{e}(\mu)$ takes place [27] (see also [28, 29, 31]), which might allow to determine $\text{sgn}(\Delta m_{31(32)}^{2})$. Determining the type of neutrino mass spectrum is crucial for understanding the origin of neutrino masses and mixing as well.

All possible types of neutrino mass spectrum we have discussed above are compatible with the existing constraints on the absolute scale of neutrino masses $m_{j}$. Information about the absolute neutrino mass scale can be obtained by measuring the spectrum of electrons near the end point in $^{3}$H $\beta$-decay experiments [32] and from cosmological and astrophysical data (see, e.g., [33]). The most stringent upper bound on the $\bar{\nu}_{e}$ mass was obtained in the Troitzk [34] experiment (see also [35]):

$$m_{\bar{\nu}_{e}} < 2.05 \text{ eV} \quad \text{at 95\% C.L.}$$  \hspace{1cm} (1.7)

We have $m_{\bar{\nu}_{e}} \approx m_{1,2,3} > 0.1$ eV in the case of quasi-degenerate (QD) spectrum. The KATRIN experiment [35], which is under preparation, is planned to reach sensitivity of $m_{\bar{\nu}_{e}} \sim 0.20$ eV, i.e., it will probe the region of the QD spectrum [3]. The Cosmic Microwave Background (CMB) data of the WMAP experiment, combined with supernovae data and data on galaxy clustering can be used to derive an upper limit on the sum of neutrinos masses (see, e.g., [33]). Depending on the model complexity and the input data used one obtains [37]: $\sum_{j} m_{j} \lesssim (0.3 - 1.3)$ eV, 95\% C.L. Data on weak lensing of galaxies, combined with data from the WMAP and PLANCK experiments, may allow $\sum_{j} m_{j}$ to be determined with an uncertainty of $\sigma(\sum_{j} m_{j}) = (0.04 - 0.07)$ eV [38].

Thus, the data on the absolute scale of neutrino masses imply that neutrino masses are much smaller than the masses of the charged leptons and quarks. If we take as an indicative upper limit $m_{j} \lesssim 0.5$ eV, $j = 1, 2, 3$, we have

$$\frac{m_{j}}{m_{l,q}} \lesssim 10^{-6}, \quad l = e, \mu, \tau, \quad q = d, s, b, u, c, t.$$  \hspace{1cm} (1.8)

It is natural to suppose that the remarkable smallness of neutrino masses is related to the existence of a new fundamental mass scale in particle physics, and thus to new physics beyond that predicted

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6 We note that the Earth mantle-core (NOLR) enhancement of neutrino transitions differs [27] from the MSW one. It also differs [27, 29] from the mechanisms of enhancement discussed, e.g., in the articles [30]: the conditions of enhancement considered in [30] cannot be realised for the $\nu_{\mu}(e) \rightarrow \nu_{e}(\mu)$ or $\bar{\nu}_{\mu}(e) \rightarrow \bar{\nu}_{e}(\mu)$ transitions of the Earth core crossing neutrinos.

7 The so-called “neutrino oscillograms of the Earth”, showing in the case of $\Delta m_{31(32)}^{2} > 0$, for instance, the dependence of, e.g., the $\nu_{\mu}(e) \rightarrow \nu_{e}(\mu)$ transition probability on the nadir angle of the neutrino trajectory through the Earth for given values of the neutrino energy $E$ (or $\Delta m_{31(32)}^{2}/E$) and of the relevant neutrino mixing angle, and discussed in detail in [31] and in the talks given by the authors of [31], first appeared in the publications quoted in [28].

8 Information on the type of neutrino mass spectrum can also be obtained in $\beta$-decay experiments having a sensitivity to neutrino masses $\sim \sqrt{\Delta m_{31(32)}^{2}} \approx 5 \times 10^{-2}$ eV [30] (i.e., by a factor of ~4 better sensitivity than that of the KATRIN experiment [35]). Reaching the indicated sensitivity in electromagnetic spectrometer $\beta$-decay experiments of the type of KATRIN does not seem feasible at present.
by the Standard Model. A comprehensive theory of the neutrino masses and mixing should be able to explain the indicated enormous disparity between the neutrino masses and the masses of the charged leptons and quarks.

At present no experimental information on the Dirac and Majorana CPV phases in the neutrino mixing matrix is available. Therefore the status of the CP symmetry in the lepton sector is unknown. The importance of getting information about the Dirac and Majorana CPV phases in the neutrino mixing matrix stems, in particular, from the possibility that these phases play a fundamental role in the generation of the observed baryon asymmetry of the Universe. More specifically, the CP violation necessary for the generation of the baryon asymmetry within the “flavoured” leptogenesis scenario \(^{39,40}\) can be due exclusively to the Dirac and/or Majorana CPV phases in the PMNS matrix \(^{41}\), and thus can be directly related to the low energy CP-violation in the lepton sector. If the requisite CP violation is due to the Dirac phases \(\delta\), a necessary condition for a successful (“flavoured”) leptogenesis is that \(\sin \theta_{13} \gtrsim 0.09\) \(^{41}\), which is comfortably compatible with the Daya Bay result, eq. (1.4).

With \(\theta_{13} \neq 0\), the Dirac phase \(\delta\) can generate CP violating effects in neutrino oscillations \(^{42}\) (see also \(^{43}\)), i.e., a difference between the probabilities of \(\nu_l \to \nu_{l'}\) and \(\bar{\nu}_l \to \bar{\nu}_{l'}\) oscillations in vacuum: \(P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'}), l \neq l' = e, \mu, \tau\). The magnitude of the CP violating effects of interest is determined \(^{44}\) by the rephasing invariant \(J_{CP}\) associated with the Dirac CPV phase \(\delta\) in \(U\). It is analogous to the rephasing invariant associated with the Dirac CPV phase in the CKM quark mixing matrix \(^{45}\). In the “standard” parametrisation of the PMNS neutrino mixing matrix, eqs. (1.2) - (1.3), we have:

\[
J_{CP} \equiv \text{Im}(U_{\mu 3}^* U_{e 3} E_{\mu 2} U_{\mu 1}^*) = \frac{1}{8} \cos \theta_{13} \sin 2 \theta_{13} \sin \delta. \tag{1.9}
\]

Thus, given the fact that \(\sin \theta_{12}, \sin \theta_{23}\) and \(\sin \theta_{13}\) have been determined experimentally with a relatively good precision, the size of CP violation effects in neutrino oscillations depends essentially only on the magnitude of the currently unknown value of the Dirac phase \(\delta\). The current data imply \(|J_{CP}| \lesssim 0.039\), where we have used eq. (1.9) and the 3σ ranges of \(\sin^2 \theta_{12}, \sin^2 \theta_{23}\) and \(\sin^2 \theta_{13}\) given in Table 1.1. Data on the Dirac phase \(\delta\) will be obtained in the long baseline neutrino oscillation experiments T2K, NO\(\nu\)A, etc. (see, e.g., refs. \(^{6}\)). Testing the possibility of Dirac CP violation in the lepton sector is one of the major goals of the next generation of neutrino oscillation experiments (see, e.g., \(^{23,46}\)). Measuring the magnitude of CP violation effects in neutrino oscillations is at present also the only known feasible method of determining the value of the phase \(\delta\) (see, e.g., \(^{17}\)).

If \(\nu_j\) are Majorana fermions, getting experimental information about the Majorana CPV phases in the neutrino mixing matrix \(U\) will be remarkably difficult \(^{45,49,50,51,52,53,54}\). As we will discuss further, the Majorana phases of the PMNS matrix play important role in the phenomenology of neutrinoless double beta ((\(\beta\beta\))0ν- decay - the process whose existence is related to the Majorana nature of massive neutrinos \(^{55}\): \(A, Z \rightarrow (A, Z + 2) + e^- + e^-\). The phases \(\alpha_{21,31}\) can affect significantly the predictions for the rates of the (LFV) decays \(\mu \rightarrow e + \gamma, \tau \rightarrow \mu + \gamma, \) etc. in a large class of supersymmetric theories incorporating the see-saw mechanism \(^{56}\). As was mentioned earlier, the Majorana phase(s) in the PMNS matrix can be the leptogenesis CP violating parameter(s) at the origin of the baryon asymmetry of the Universe. \(^{41,14}\).

Establishing whether the neutrinos with definite mass \(\nu_j\) are Dirac fermions possessing distinct antiparticles, or Majorana fermions, i.e., spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for understanding the origin of \(\nu\)-masses and mixing and the underlying symmetries of particle interactions. Let us recall that the neutrinos \(\nu_j\) with definite mass \(m_j\) will be Dirac fermions if particle interactions conserve some additive lepton number, e.g., the total lepton
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charge \( L = L_e + L_\mu + L_\tau \). If no lepton charge is conserved, the neutrinos \( \nu_j \) will be Majorana fermions (see, e.g. [16]). The massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of neutrino mass generation [4], which also provides an attractive explanation of the smallness of neutrino masses and, through the leptogenesis theory [2], of the observed baryon asymmetry of the Universe. The observed patterns of neutrino mixing and of neutrino mass squared differences driving the solar and the dominant atmospheric neutrino oscillations, can be related to Majorana massive neutrinos and the existence of an approximate symmetry in the lepton sector corresponding to the conservation of the non-standard lepton charge \( L' = L_e - L_\mu - L_\tau \) [57]. They can also be associated with the existence of approximate discrete symmetry (or symmetries) of the particle interactions (see, e.g., [58]). Determining the nature (Dirac or Majorana) of massive neutrinos is one of the fundamental and most challenging problems in the future studies of neutrino mixing [6].

1.2 The Nature of Massive Neutrinos

1.2.1 Majorana versus Dirac Massive Neutrinos (Particles)

The properties of Majorana particles (fields) are very different from those of Dirac particles (fields). A massive Majorana neutrino \( \chi_j \) (or Majorana spin 1/2 particle) with mass \( m_j > 0 \) can be described in local quantum field theory which is used to construct, e.g., the Standard Model, by 4-component complex spin 1/2 field \( \chi_j(x) \) which satisfies the Dirac equation and the Majorana condition:

\[
C \left( \bar{\chi}_j(x) \right)^T = \xi_j \chi_j(x), \quad |\xi_j|^2 = 1,
\]

where \( C \) is the charge conjugation matrix, \( C^{-1} \gamma_\alpha C = - (\gamma_\alpha)^T \) \( (C^T = - C, C^{-1} = C^\dagger) \), and \( \xi_j \) is, in general, an unphysical phase. The Majorana condition is invariant under proper Lorentz transformations. It reduces by a factor of 2 the number of independent components in \( \chi_j(x) \).

The condition (1.10) is invariant with respect to \( U(1) \) global gauge transformations of the field \( \chi_j(x) \) carrying a \( U(1) \) charge \( Q, \chi_j(x) \rightarrow e^{i\alpha Q} \chi_j(x), \) only if \( Q = 0 \). As a result, i) \( \chi_j \) cannot carry nonzero additive quantum numbers (lepton charge, etc.), and ii) the field \( \chi_j(x) \) cannot “absorb” phases. Thus, \( \chi_j(x) \) describes 2 spin states of a spin 1/2, absolutely neutral particle, which is identical with its antiparticle, \( \chi_j \equiv \bar{\chi}_j \). As is well known, spin 1/2 Dirac particles can carry nonzero \( U(1) \) charges: the charged leptons and quarks, for instance, carry nonzero electric charges.

Owing to the fact that the Majorana (neutrino) fields cannot absorb phases, the neutrino mixing matrix \( U \) contains in the general case of \( n \) charged leptons and mixing of \( n \) massive Majorana neutrinos \( \nu_j \equiv \chi_j \), altogether

\[
n_{\text{CPV}}^{(M)} = \frac{n(n-1)}{2}, \quad \text{Majorana } \nu_j,
\]

CPV phases [11]. In the case of mixing of \( n \) massive Dirac neutrinos, the number of CPV phases in \( U \), as is well known, is

\[
n_{\text{CPV}}^{(D)} = \frac{(n-1)(n-2)}{2}, \quad \text{Dirac } \nu_j.
\]

Thus, if \( \nu_j \) are Majorana particles, \( U \) contains the following number of additional Majorana CP violation phases: \( n_{\text{MCPV}}^{(M)} \equiv n_{\text{CPV}}^{(M)} - n_{\text{CPV}}^{(D)} = (n-1) \). In the case of \( n \) charged leptons and \( n \) massive Majorana neutrinos, the PMNS matrix \( U \) can be cast in the form [11]

\[
U = V P,
\]
where the matrix $V$ contains the $(n-1)(n-2)/2$ Dirac CP violation phases, while $P$ is a diagonal matrix with the additional $(n-1)$ Majorana CP violation phases $\alpha_{21}, \alpha_{31}, \ldots, \alpha_{n1}$,

$$P = \text{diag} \left( 1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}, \ldots, e^{i\frac{\alpha_{n1}}{2}} \right).$$

As will be discussed further, the Majorana phases will conserve CP if $\alpha_{21} = \pi q_j$, $q_j = 0, 1, 2, j = 2, 3, \ldots, n$. In this case $\exp(i\alpha_{21}) = \pm 1$ and $\exp[i(\alpha_{21} - \alpha_{k1})] = \pm 1$ have a simple physical interpretation: these are the relative CP-parities of the Majorana neutrinos $\nu_j$ and $\nu_l$ and of $\nu_j$ and $\nu_k$, respectively.

It follows from the preceding discussion that the mixing of massive Majorana neutrinos differs, in what concerns the number of CPV phases, from the mixing of massive Dirac neutrinos. For $n = 3$ of interest, we have one Dirac and two Majorana CPV phases in $U$, which is consistent with the expression of $U$ given in eq. (1.2). If $n = 2$, there is one Majorana CP phase and no Dirac CPV phases in $U$. Correspondingly, in contrast to the Dirac case, there can exist CP violating effects even in the system of two mixed massive Majorana neutrinos (particles).

The Majorana phases do not enter into the expressions of the probabilities of oscillations involving the flavour neutrinos and antineutrinos [11][22], $\nu_l \to \nu_p$ and $\bar{\nu}_l \to \bar{\nu}_p$. Indeed, the probability to find neutrino $\nu_p$ (antineutrino $\bar{\nu}_p$) at time $t$ if a neutrino $\nu_l$ (antineutrino $\bar{\nu}_l$) has been produced at time $t_0$ and it had traveled a distance $L \equiv t$ in vacuum, is given by (see, e.g., [16][4]):

$$P(\nu_l \to \nu_p) = \left| \sum_j U_{lj} \exp\left(-i(E_j t - p_j L)\right) U_{lj}^\dagger \right|^2,$$

(1.15)

$$P(\bar{\nu}_l \to \bar{\nu}_p) = \left| \sum_j U_{lj} \exp\left(-i(E_j t - p_j L)\right) U_{lj}^\dagger \right|^2,$$

(1.16)

where $E_j$ and $p_j$ are the energy and momentum of the neutrino $\nu_j$. It is easy to show, using the expression for $U$ in eq. (1.13) that $P(\nu_l \to \nu_p)$ and $P(\bar{\nu}_l \to \bar{\nu}_p)$ do not depend on the Majorana phases present in $U$ since

$$\sum_j (VP)_{pj} \exp(-i(E_j t - p_j L)) (VP)^\dagger = \sum_j V_{pj} \exp(-i(E_j t - p_j L)) V_{pj}^\dagger.$$  

(1.17)

The same result holds when the neutrino oscillations take place in matter [22].

If $CP$-invariance holds, Majorana neutrinos (particles) have definite $CP$-parity $\eta_{CP}(\chi_j) = \pm i$:

$$U_{CP} \chi_j(x) U^{-1}_{CP} = \eta_{CP}(\chi_j) \gamma_0 \chi_j(x_p), \quad \eta_{CP}(\chi_j) \equiv i \rho_j = \pm i,$$

(1.18)

where $x = (x_0, \mathbf{x})$, $x_p = (x_0, -\mathbf{x})$ and $U_{CP}$ is the unitary $CP$-transformation operator. In contrast, Dirac particles do not have a definite $CP$-parity — a Dirac field $f(x)$ transforms as follows under the $CP$-symmetry operation:

$$U_{CP} f(x) U^{-1}_{CP} = \eta f \gamma_0 C(\bar{f}(x_p))^T, \quad |\eta|^2 = 1,$$

(1.19)

$\eta$ being an unphysical phase factor. In the case of $CP$ invariance, the $CP$-parities of massive Majorana fermions (neutrinos) can play important role in processes involving real of virtual Majorana particles (see, e.g., [16][60][61]).
Using eqs. (1.19) and (1.18) and the transformation of the $W^\pm$ boson field under the CP-symmetry operation,

$$U_{CP}W_\alpha(x)U_{CP}^\dagger = \eta_W \kappa_\alpha(W_\alpha(x_p))^{\dagger}, \quad |\eta_W|^2 = 1, \quad \kappa_0 = -1, \quad \kappa_{1,2,3} = +1.$$  \hspace{1cm} (1.20)

where $\eta_W$ is an unphysical phase, one can derive the constraints on the neutrino mixing matrix $U$ following from the requirement of CP-invariance of the leptonic CC weak interaction Lagrangian, eq. (1.1). In the case of massive Dirac neutrinos we obtain:

$$\eta_l^* \eta_W U_{ij}^* = U_{ij}, \quad l = e, \mu, \tau, \quad j = 1, 2, 3.$$  \hspace{1cm} (1.21)

In the case of massive Majorana neutrinos we obtain using eqs. (1.10), (1.18) (1.19) and (1.20):

$$\xi_j^* (i\rho_j) \eta_l^* \eta_W U_{ij} = U_{ij}^*.$$  \hspace{1cm} (1.22)

Thus, in the convention used the elements of the PMNS matrix can be either real or purely imaginary by us) convention we get (16):

$$CP \text{ invariance: } \quad U_{ij}^* = U_{ij}, \quad l = e, \mu, \tau, \quad j = 1, 2, 3, \quad \text{Dirac } \nu_j.$$  \hspace{1cm} (1.21)

$$CP \text{ invariance: } \quad U_{ij}^* = \rho_j U_{ij}, \quad \rho_j = +1 \text{ or } (-1), \quad l = e, \mu, \tau, \quad j = 1, 2, 3, \quad \text{Majorana } \nu_j.$$  \hspace{1cm} (1.22)

In eqs. (1.19) and (1.18) and the transformation of the $W^+$, one can derive the constraints on the neutrino mixing matrix $U$ following from the requirement of CP-invariance of the leptonic CC weak interaction Lagrangian, eq. (1.1). In the case of massive Dirac neutrinos we obtain:

$$\eta_l^* \eta_W U_{ij}^* = U_{ij}, \quad l = e, \mu, \tau, \quad j = 1, 2, 3.$$  \hspace{1cm} (1.21)

In the case of massive Majorana neutrinos we obtain using eqs. (1.10), (1.18) (1.19) and (1.20):

$$\xi_j^* (i\rho_j) \eta_l^* \eta_W U_{ij} = U_{ij}^*.$$  \hspace{1cm} (1.22)

One can obtain in a similar way the CP-invariance constraint on the matrix of neutrino Yukawa couplings, $\lambda_{kl}$, which plays a fundamental role in the leptogenesis scenario of baryon asymmetry generation, based on the (type I) see-saw mechanism of generation of neutrino masses [2, 3, 47]:

$$\mathcal{L}_Y(x) = -\lambda_{kl} \overline{N_{kR}}(x) H^\dagger(x) \psi_{lL}(x) + h.c., \quad \hspace{1cm} (1.23)$$

$$\mathcal{L}^N_M(x) = -\frac{1}{2} M_k \overline{N_{k}}(x) N_k(x). \quad \hspace{1cm} (1.24)$$

Here $N_{kR}(x)$ is the field of a heavy right-handed (RH) sterile Majorana neutrino with mass $M_k > 0$, $\psi_{lL}$ denotes the Standard Model left-handed (LH) lepton doublet field of flavour $l = e, \mu, \tau$, $\psi_{lL}^\dagger = (\nu_{1L}^\dagger, l_1^L)$, and $H$ is the Standard Model Higgs doublet field whose neutral component has a vacuum expectation value $v = 174$ GeV. The term $\mathcal{L}_Y(x) + \mathcal{L}^N_M(x)$ includes all the necessary ingredients of the see-saw mechanism. Assuming the existence of two heavy Majorana neutrinos, i.e., taking $k = 1, 2$ in eqs. (1.23) and (1.24), and adding the term $\mathcal{L}_Y(x) + \mathcal{L}^N_M(x)$ to the Standard Model Lagrangian, we obtain the minimal extension of the Standard Model in which the neutrinos have masses and mix and the leptogenesis can be realised. In the leptogenesis formalism it is often convenient to use the so called “orthogonal parametrisation“ of the matrix of neutrino Yukawa couplings [59]:

$$\lambda_{kl} = \frac{1}{v} \sqrt{M_k} R_{kj} \sqrt{m_j} (U^\dagger)_{ji}, \quad \hspace{1cm} (1.25)$$

where $R$ is, in general, a complex orthogonal matrix, $R R^T = R^T R = 1$. The CP violation necessary for the generation of the baryon asymmetry of the Universe is provided in the leptogenesis scenario of interest by the matrix of neutrino Yukawa couplings $\lambda$ (see, e.g., [3, 47]). It follows from eq. (1.25) that it can be provided either by the neutrino mixing matrix $U$, or by the matrix $R$, or else by both the matrices $U$ and $R$. It is therefore important to derive the conditions under which $\lambda$, $R$ and $U$ respect the CP symmetry. For the PMNS matrix $U$ these conditions are given in eq. (1.22). For the
matrices $\lambda$ and $R$ in the convention in which i) $N_k(x)$ satisfy the Majorana condition with a phase equal to 1 (i.e., $\xi_k = 1$), ii) $\eta^H = i$ and $\eta^L = 1$, $\eta^I$ and $\eta^D$ being the unphysical phase factors which appear in the CP-transformations of the LH lepton doublet and Higgs doublet fields $\psi^L(x)$ and $H(x)$, respectively, they read \[41\]:

$$\lambda^*_{kl} = \lambda_{kl} \rho^N_k, \quad \rho^N_k = \pm 1, \quad j = 1, 2, 3, \quad l = e, \mu, \tau, \quad (1.26)$$

$$R^*_{kj} = R_{kj} \rho^N_k \rho_j, \quad j, k = 1, 2, 3. \quad (1.27)$$

where $i\rho^N_k \equiv \eta_{CP}(N_k) = \pm i$ is the CP-parity of $N_k$. Thus, in the case of CP invariance also the elements of $\lambda$ and $R$ can be real or purely imaginary. Note that, as it follows from eqs. \[1.22\], \[1.26\] and \[1.27\], given which elements are real and which are purely imaginary of any two of the three matrices $U$, $\lambda$ and $R$, determines in the convention we are using and if CP invariance holds, which elements are real or purely imaginary in the third matrix. If, for instance, $U_{e2}$ is purely imaginary ($\rho_2 = -1$) and $\lambda_{l\mu}$ is real ($\rho^l_1 = 1$), then $R_{12}$ must be purely imaginary. Thus, in the example we are considering, a real $R_{12}$ would signal that the CP symmetry is broken \[41\].

The currents formed by Majorana fields have special properties, which make them also quite different from the currents formed by Dirac fields. In particular, it follows from the Majorana condition that the following currents of the Majorana field $\chi_k(x)$ are identically equal to zero (see, e.g., \[16\]):

$$\bar{\chi}_k(x) \gamma_5 \gamma_\alpha \chi_k(x) \equiv 0, \quad (1.28)$$

$$\bar{\chi}_k(x) \sigma_{\alpha\beta} \chi_k(x) \equiv 0, \quad (1.29)$$

$$\bar{\chi}_k(x) \sigma_{\alpha\beta} \gamma_5 \chi_k(x) \equiv 0. \quad (1.30)$$

Equations \[1.28\], \[1.29\] and \[1.30\] imply that Majorana fermions (neutrinos) cannot have nonzero $U(1)$ charges and intrinsic magnetic and electric dipole moments, respectively. A Dirac spin 1/2 particle can have non-trivial $U(1)$ charges, as we have already discussed, and nonzero intrinsic magnetic moment (the electron and the muon, for instance, have it). If CP invariance does not hold, Dirac fermions can have also a nonzero electric dipole moments. Equations \[1.29\] and \[1.30\] imply also that the Majorana particles (neutrinos) cannot couple to a real photon. The axial current of a Majorana fermion, $\bar{\chi}_k(x) \gamma_\alpha \gamma_5 \chi_k(x) \neq 0$. Correspondingly, $\chi_k(x)$ can have an effective coupling to a virtual photon via the anapole momentum term, which has the following form in momentum space:

$$\left(g_{\alpha\beta} q^2 - q_\alpha q_\beta \right) \gamma_5 \gamma_\beta F_a^{(k)}(q^2), \quad (1.31)$$

where $q$ is the momentum of the virtual photon and $F_a^{(k)}(q^2)$ is the anapole formfactor of $\chi_k$. The fact that the vector current of $\chi_k$ is zero while the axial current is nonzero has important implications in the calculations of the relic density of the lightest and stable neutralino, which is a Majorana particle and the dark matter candidate in many supersymmetric (SUSY) extensions of the Standard Model \[62\].

In certain cases (e.g., in theories with a keV mass Majorana neutrino (see, e.g., \[9\]), in the TeV scale type I see-saw model (see, e.g., \[62\], in SUSY extensions of the Standard Model) one can have effective interactions involving two different massive Majorana fermions (neutrinos), say $\chi_1$ and $\chi_2$.\[9\] This convention is similar to, and consistent with, the convention about the unphysical phases we have used to derive the CP-invariance constraints on the elements of the PMNS matrix $U$.\[11\]
The Nature of Massive Neutrinos

We will consider two examples. The first is an effective interaction with the photon field, which can be written as:

\[ \mathcal{L}_{\text{eff}}^{(A)}(x) = \bar{\chi}_1(x) \sigma_{\alpha \beta} (\mu_{12} - d_{12} \gamma_5) \chi_2(x) F^{\alpha \beta}(x) + \text{h.c.}, \tag{1.32} \]

where \( \mu_{12} \) and \( d_{12} \) are, in general, complex constants, \( F^{\alpha \beta}(x) = \partial^\alpha A^\beta(x) - \partial^\beta A^\alpha(x) \), \( A^\mu(x) \) being the 4-vector potential of the photon field. Using the Majorana conditions for \( \chi_1(x) \) and \( \chi_2(x) \) in the convention in which the phases \( \xi_1 = \xi_2 = 1 \), it is not difficult to show that the constants \( \mu_{12} \) and \( d_{12} \) enter into the expression for \( \mathcal{L}_{\text{eff}}^{(A)}(x) \) in the form: \( (\mu_{12} - d_{12}) = 2\text{Im}(\mu_{12}) \equiv \tilde{\mu}_{12}, \) \( (d_{12} + d_{12}') = 2\text{Re}(d_{12}) \equiv \tilde{d}_{12} \), i.e., \( \tilde{\mu}_{12} \) is purely imaginary and \( \tilde{d}_{12} \) is real\(^{10} \). In the case of CP invariance of \( \mathcal{L}_{\text{eff}}^{(A)}(x) \), the constants \( \mu_{12} \) (\( \tilde{\mu}_{12} \)) and \( d_{12} \) (\( \tilde{d}_{12} \)) should satisfy:

\[ \text{CP invariance : } \mu_{12} = -\rho_1 \rho_2 \mu_{12}, \quad d_{12} = +\rho_1 \rho_2 d_{12}. \tag{1.33} \]

Thus, if \( \rho_1 = \rho_2 \), i.e., if \( \chi_1(x) \) and \( \chi_2(x) \) posses the same CP-parity, \( \mu_{12} = 0 \) and \( d_{12} \) (and \( \tilde{d}_{12} \)) can be different from zero. If \( \rho_1 = -\rho_2 \), i.e., if \( \chi_1(x) \) and \( \chi_2(x) \) have opposite CP-parities, \( d_{12} = 0 \) and \( \mu_{12} \) (and \( \tilde{\mu}_{12} \)) can be different from zero. If CP invariance does not hold, we can have both \( \mu_{12} \neq 0 \) and \( d_{12} \neq 0 \) (\( \tilde{\mu}_{12} \neq 0 \) and \( \tilde{d}_{12} \neq 0 \)).

As a second example we will consider effective interaction of \( \chi_1 \) and \( \chi_2 \) with a vector field (current), which for concreteness will be assumed to be the \( Z^0 \)-boson field of the Standard Model:

\[ \mathcal{L}_{\text{eff}}^{(Z)}(x) = \bar{\chi}_1(x) \gamma_\alpha (v_{12} - a_{12} \gamma_5) \chi_2(x) Z^\alpha(x) + \text{h.c.}, \tag{1.34} \]

Here \( v_{12} \) and \( a_{12} \) are, in general, complex constants. Using the Majorana conditions for \( \chi_1(x) \) and \( \chi_2(x) \) with \( \xi_1 = \xi_2 = 1 \), one can easily show that \( v_{12} \) has to be purely imaginary, while \( a_{12} \) has to be real\(^{11} \). The requirement of CP invariance of \( \mathcal{L}_{\text{eff}}^{(Z)}(x) \), as can be shown, leads to \( (\xi_1 = \xi_2 = 1) \):

\[ \text{CP invariance : } v_{12} = -\rho_1 \rho_2 v_{12}, \quad a_{12} = +\rho_1 \rho_2 a_{12}. \tag{1.35} \]

Thus, we find, similarly to the case considered above, that if \( \chi_1(x) \) and \( \chi_2(x) \) posses the same CP-parity \((\rho_1 = \rho_2)\), \( v_{12} = 0 \) and \( a_{12} \) can be different from zero; if \( \chi_1(x) \) and \( \chi_2(x) \) have opposite CP-parities \((\rho_1 = -\rho_2)\), \( a_{12} = 0 \) while \( v_{12} \) can be different from zero. If CP invariance does not hold, we can have both \( v_{12} \neq 0 \) and \( a_{12} \neq 0 \).

These results have important implications, in particular, for the phenomenology of the heavy Majorana neutrinos \( N_k \) in the TeV scale (type I) see-saw models, for the neutralino phenomenology in SUSY extensions of the Standard Model, in which the neutralinos are Majorana particles, and more specifically for the processes \( e^- + e^+ \rightarrow \chi_1 + \chi_2 \), \( \chi_2 \rightarrow \chi_1 + l^+ + l^- \) \((m(\chi_2) > m(\chi_1))\), \( l = e, \mu, \tau \), where \( \chi_1 \) and \( \chi_2 \) are, for example, two neutralinos of, e.g., the minimal SUSY extension of the Standard Model (see, e.g., \(^{60, 61} \)).

Finally, if \( \Psi(x) \) is a Dirac field and we define the standard propagator of \( \Psi(x) \) as

\[ < 0| T(\Psi_\alpha(x)\Psi_\beta(y)) |0> = S^F_{\alpha \beta}(x - y), \tag{1.36} \]

\(^{10} \text{In the case of } \chi_1(x) \equiv \chi_2(x) = \chi(x), \text{ the current } \bar{\chi}(x)\sigma_{\alpha \beta}(\mu_{12} - d_{12} \gamma_5)\chi(x) \text{ has to be hermitian, which implies that } \tilde{\mu}_{12} \text{ should be real while } \tilde{d}_{12} \text{ should be purely imaginary. Combined with constraints on } \tilde{\mu}_{12} \text{ and } \tilde{d}_{12} \text{ we just obtained, leads to } \tilde{\mu}_{12} = \tilde{d}_{12} = 0, \text{ which is consistent with eqs. (1.29) and (1.30).} \]

\(^{11} \text{In the case of } \chi_1(x) \equiv \chi_2(x) = \chi(x), \text{ the hermiticity of the current } \bar{\chi}(x)\gamma_\alpha(v_{12} - a_{12} \gamma_5)\chi(x) \text{ implies that both } v_{12} \text{ and } a_{12} \text{ have to be real. This, together with constraints on } v_{12} \text{ and } a_{12} \text{ just derived, leads to } v_{12} = 0, \text{ which is consistent with the result given in eq. (1.28).} \]
one has
\[ <0|T(\Psi_\alpha(x)\Psi_\beta(y))|0> = 0 , \quad <0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0> = 0 . \]  
(1.37)
In contrast, a Majorana neutrino field \( \chi_k(x) \) has, in addition to the standard propagator
\[ <0|T(\chi_{k\alpha}(x)\bar{\chi}_{k\beta}(y))|0> = S^{FK}_{\alpha\beta}(x - y) , \]  
(1.38)
two non-trivial non-standard (Majorana) propagators
\[ <0|T(\chi_{k\alpha}(x)\chi_{k\beta}(y))|0> = -\xi_k S_{\alpha\delta}^{FK}(x - y) C_{\delta\beta} , \]  
(1.39)
\[ <0|T(\bar{\chi}_{k\alpha}(x)\bar{\chi}_{k\beta}(y))|0> = \xi_k C^{-1}_{\alpha\delta} S_{\delta\beta}^{FK}(x - y) . \]  
(1.40)
This result implies that if \( \nu_j(x) \) in eq. (1.1) are massive Majorana neutrinos, \((\beta\beta)^{0\nu}_{\alpha\nu}\)-decay can proceed by exchange of virtual neutrinos \( \nu_j \) since \( <0|T(\nu_{\nu_{\alpha}}(x)\nu_{\nu_{j\beta}}(y))|0> \neq 0 \). The Majorana propagators play a crucial role in the calculation of the baryon asymmetry of the Universe in the leptogenesis scenario of the asymmetry generation (see, e.g., [3, 47]).

### 1.2.2 Generating Dirac and Majorana Massive Neutrinos

The type of massive neutrinos in a given theory is determined by the type of the (effective) mass term \( \mathcal{L}_m' \) neutrinos have, more precisely, by the symmetries of \( \mathcal{L}_m' \) and of the total Lagrangian \( \mathcal{L}(x) \) of the theory. A fermion mass term is any bi-linear in the fermion fields which is invariant under the proper Lorentz transformations.

Massive Dirac neutrinos arise in theories in which the neutrino mass term conserves some additive quantum number that could be, e.g., the (total) lepton charge \( L = L_e + L_\mu + L_\tau \), which is conserved also by the total Lagrangian \( \mathcal{L}(x) \) of the theory. A well known example is the Dirac mass term, which can arise in the minimally extended Standard Model to include three RH neutrino fields \( \nu_R \), \( l = e, \mu, \tau \), as \( SU(2)_L \) singlets:
\[ \mathcal{L}'_D(x) = -\bar{\nu}_{R\alpha}(x) M_D \nu_{L\alpha}(x) + h.c. , \]  
(1.41)
where \( M_D \) is a \( 3 \times 3 \), in general complex, matrix. The term \( \mathcal{L}'_D(x) \) can be generated after the spontaneous breaking of the Standard Model gauge symmetry by an \( SU(2)_L \times U(1)_{Y_w} \) invariant Yukawa coupling of the lepton doublet, Higgs doublet and the RH neutrino fields [63]:
\[ \mathcal{L}_Y(x) = -Y_{w\nu}^{\alpha\beta} \bar{\nu}_{R\alpha}(x) H^\dagger(x) \psi_{L\beta}(x) + h.c. , \]  
(1.42)
\[ M_D = v Y_w. \]  
(1.43)
If the nondiagonal elements of \( M_D \) are different from zero, \( M_D' \) is not, \( l \neq l' = e, \mu, \tau \), will be conserved. Nevertheless, the total lepton charge \( L \) is conserved by \( \mathcal{L}'_D(x) \). As in the case of the charged lepton and quark mass matrices generated via the spontaneous electroweak symmetry breaking by Yukawa type terms in the SM Lagrangian, \( M_D \) is diagonalised by a bi-unitary transformation: \( M_D = U^{lep}_R \chi_{lep}_D(U^{lep}_L)^\dagger \), where \( U^{lep}_R \) and \( U^{lep}_L \) are \( 3 \times 3 \) unitary matrices. If the mass term in eq. (1.41) is written in the basis in which the charged lepton mass matrix is diagonal, \( U^{lep}_L \) coincides with the PMNS matrix, \( U^{lep}_L \equiv \chi_{PMNS} \). The neutrinos \( \nu_j \) with definite mass \( m_j > 0 \) are Dirac particles: their fields \( \nu_j(x) = (U^{lep}_L)^\dagger_{ji} \nu_{jL}(x) + (U^{lep}_R)^\dagger_{ji} \nu_{jR}(x) \) do not satisfy the Majorana condition, \( \chi_j(x) = (\nu_j(x))^T \neq \xi_j \nu_j(x) \). Although the scheme we are considering is phenomenologically
viable\footnote{\textsuperscript{12}It does not contain a candidate for a dark matter particle.} it does not provide an insight of why the neutrino masses are much smaller than the charged fermion masses. The only observable “new physics” is that related to the neutrino masses and mixing: apart from the neutrino masses and mixing themselves, this is the phenomenon of neutrino oscillations\footnote{\textsuperscript{64}}.

Indeed, given the fact that the lepton charges \( L_l, l = e, \mu, \tau, \) are not conserved, processes like \( \mu^+ \to e^+ + \gamma \) decay, \( \mu^- \to e^- + e^+ + e^- \) decay, \( \tau^- \to e^- + \gamma \) decay, etc. are allowed. However, the rates of these processes are suppressed by the factor \[64\] [\( |U_{\nu j}U_{\nu j}^\ast m_j^2/M_W^2|\)^2, \( l' \neq l, M_W \cong 80 \text{ GeV} \)] being the \( W^\pm\)-mass and \( l = \mu, l' = e \) for the \( \mu^\pm \to e^\pm + \gamma \) decay, etc., and are unobservably small. For instance, for the \( \mu \to e + \gamma \) decay branching ratio we have \[64\] (see also \[65\]):

\[
BR(\mu \to e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{\nu e}U_{\mu j}^\ast \frac{m_j^2}{M_W^2} \right|^2 \cong (2.5 - 3.9) \times 10^{-55},
\]

where we have used the best fit values of the neutrino oscillation parameters given in eqs.(1.5) and (1.6) and the two values correspond to \( \delta = \pi \) and 0. The current experimental upper limit reads \[66\]:

\[
BR(\mu^+ \to e^+ + \gamma) < 5.7 \times 10^{-13}.
\]

Thus, although the predicted branching ratio \( BR(\mu^+ \to e^+ + \gamma) \neq 0 \), its value is roughly by 43 orders of magnitude smaller than the sensitivity reached in the experiments searching for the \( \mu \to e + \gamma \) decay, which renders it unobservable in practice.

As was emphasised already, massive Majorana neutrinos appear in theories with no conserved additive quantum number, and more specifically, in which the total lepton charge \( L \) is not conserved and changes by two units. In the absence of RH singlet neutrino fields in the theory, the flavour neutrinos and antineutrinos \( \nu_l \) and \( \bar{\nu}_l \), \( l = e, \mu, \tau \), can have a mass term of the so-called Majorana type:

\[
\mathcal{L}_M^\nu(x) = -\frac{1}{2} \bar{\nu}_R(x) M_{l,l'} \nu_L(x) + \text{h.c.} , \nu^c_{\nu R} \equiv C (\nu_{lL}(x))^T,
\]

where \( M \) is a \( 3 \times 3 \), in general complex, matrix. In the case when all elements of \( M \) are nonzero, \( M_{l,l'} \neq 0 \), \( l, l' = e, \mu, \tau \), neither the individual lepton charges \( L_l \) nor the total lepton charge \( L \) is conserved: \( L_l \neq \text{const.}, L \neq \text{const.} \). As it is possible to show, owing to the fact that \( \nu_{lL}(x) \) are fermion (anti-commuting) fields, the matrix \( M \) has to be symmetric (see, e.g., \[16\]): \( M = M^T \). A complex symmetric matrix is diagonalised by the congruent transformation:

\[
M^{\text{diag}} = UMU^T , \ U - \text{unitary},
\]

where \( U \) is a \( 3 \times 3 \) unitary matrix. If \( \mathcal{L}_M^\nu(x) \) is written in the basis in which the charged lepton mass matrix is diagonal, \( U \) coincides with the PMNS matrix: \( U \equiv U_{\text{PMNS}} \). The fields of neutrinos \( \nu_j \) with definite mass \( m_j \) are expressed in terms of \( \nu_{lL}(x) \) and \( \nu^c_{\nu R} \):

\[
\mathcal{L}_M^\nu(x) = \frac{1}{2} \bar{\nu}_R(x) m_j \nu_j(x),
\]

\[
\nu_j(x) = U_{j,l}^\dagger \nu_{lL}(x) + U_{j,l}^T \nu^c_{\nu R} = C (\bar{\nu}_{jL}(x))^T, \ j = 1, 2, 3.
\]

They satisfy the Majorana condition with \( \xi_j = 1 \), as eq. (1.48) shows.

The Majorana mass term (1.45) for the LH flavour neutrino fields \( \nu_{lL} \) can be generated i) effectively after the electroweak symmetry (EWS) breaking in the type I see-saw models \[4\], ii) effectively after the EWS breaking in the type III see-saw models \[67\].
iii) directly as a result of the EWS breaking by an $SU(2)_L$ triplet Higgs field which carries two units of the weak hypercharge $Y_W$ and couples in an $SU(2)_L \times U_Y$ invariant manner to two lepton doublets \[68\] (the Higgs Triplet Model (HTM) sometimes called also “type II see-saw model”), iv) as a one loop correction to a Lagrangian which does not contain a neutrino mass term \[69\] (see also \[70\]), iv) as a two loop correction in a theory where the neutrino masses are zero at tree and one loop levels \[71\] (see also \[70\]), v) as a three loop correction in a theory in which the neutrino masses are zero at tree, one loop and two loop levels \[70\].

In all three types of see-saw models, for instance, the neutrino masses can be generated at the EWS breaking scale and in this case the models predict rich beyond the Standard Model physics at the TeV scale, some of which can be probed at the LHC (see, e.g., \[72\] and further). We will consider briefly below the neutrino mass generation in the type I see-saw and the Higgs triplet models.

In a theory in which the $SU(2)_L$ singlet RH neutrino fields $\nu_{R\ell}$, $\ell = e, \mu, \tau$, are present \[10\], the most general neutrino mass Lagrangian contains the Dirac mass term (1.41), the Majorana mass term for the LH flavour neutrino fields (1.45) and a Majorana mass term for the RH neutrino fields $\nu_{R\ell}(x)$ \[74\]:

\[ \mathcal{L}_{D+M}^\nu(x) = - \bar{\nu}_{R\ell}(x) M_{D\ell\ell} \nu_{L\ell}(x) - \frac{1}{2} \bar{\nu}_{R\ell}^c(x) M_{L\ell\ell} \nu_{L\ell}(x) - \frac{1}{2} \bar{\nu}_{R\ell}^c(x) M_{R\ell\ell} \nu_{R\ell}(x) + \text{h.c.} , \] (1.49)

where $\nu_{L\ell}^c \equiv C (\bar{\nu}_{R\ell}(x))^T$ and $M_D$, $M_L$ and $M_R$ are $3 \times 3$, in general complex, matrices. By a simple rearrangement of the neutrino fields this mass term can be cast in the form of a Majorana mass term which is then diagonalised with the help of the congruent transformation \[16\]. In this case there are six Majorana mass eigenstate neutrinos, i.e., the flavour neutrino fields $\nu_{L\ell}(x)$ are linear combinations of the fields of six Majorana neutrinos with definite mass. The neutrino mixing matrix in eq. (1.1) is a $3 \times 6$ block of a $6 \times 6$ unitary matrix.

The Dirac-Majorana mass term is at the basis of the type I see-saw mechanism of generation of the neutrino masses and appears in many Grand Unified Theories (GUTs) (see, e.g., \[16\] for further details). In the see-saw models, some of the six massive Majorana neutrinos typically are too heavy to be produced in the weak processes in which the initial states of the flavour neutrinos and antineutrinos $\nu_{\ell}$ and $\bar{\nu}_{\ell}$, used in the neutrino oscillation experiments, are being formed. As a consequence, the states of $\nu_{\ell}$ and $\bar{\nu}_{\ell}$ will be coherent superpositions only of the states of the light massive neutrinos $\nu_j$, and the elements of the neutrino mixing matrix $U_{\text{PMNS}}$, which are determined in experiments studying the oscillations of $\nu_{\ell}$ and $\bar{\nu}_{\ell}$, will exhibit deviations from unitarity. These deviations can be relatively large and can have observable effects in the TeV scale see-saw models, in which the heavy Majorana neutrinos have masses in the $\sim (100 - 1000)$ GeV range (see, e.g., \[75\]).

If after the diagonalisation of $\mathcal{L}_{D+M}^\nu(x)$ more than three neutrinos will turn out to be light, i.e., to have masses $\sim 1$ eV or smaller, active-sterile neutrino oscillations can take place (see, e.g., \[9\] \[9\]): a LH (RH) flavour neutrino $\nu_{L\ell}$ (antineutrino $\bar{\nu}_{R\ell}$) can undergo transitions into LH sterile antineutrino(s) $\bar{\nu}_{\ell}(x)$ (RH sterile neutrino(s) $\nu_{R\ell}(x)$). As a consequence of this type of oscillations, one would observe a “disappearance” of, e.g., $\nu_e$ and/or $\nu_\mu$ ($\bar{\nu}_e$ and/or $\bar{\nu}_\mu$) on the way from the source to the detector.

We would like to discuss next the implications of CP invariance for the neutrino Majorana mass matrix, eq. (1.45). In the convention we have used to derive eqs. (1.26) and (1.27), in which the
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unphysical phase factor in the CP transformation of the lepton doublet field $\Psi_{lL}(x)$, and thus of $\nu_{lL}(x)$, $\eta^l = i$, the requirement of CP invariance leads to the reality condition for $M$:

$$\text{CP} - \text{invariance} : \quad M^* = M . \quad (1.50)$$

Thus, $M$ is real and symmetric and therefore is diagonalised by an orthogonal transformation, i.e., if CP invariance holds, the matrix $U$ in eq. (1.46) is an orthogonal matrix. The nonzero eigenvalues of a real symmetric matrix can be positive or negative \(^{13}\). Consequently, $M^{\text{diag}}$ in eq. (1.46) in general has the form:

$$M^{\text{diag}} = (m'_1, m'_2, m'_3), \quad m'_j = \rho_j m_j, \quad m_j > 0, \quad \rho_j = \pm 1 . \quad (1.51)$$

Let us denote the neutrino field which has a mass $m'_j \neq 0$ by $\nu'_j(x)$. According to eq. (1.48), the field $\nu'_j(x)$ satisfies the Majorana condition: $\nu'_j(x) = C (\nu'_j(x))^T$. One can work with the fields $\nu'_j(x)$ remembering that some of them have a negative mass. It is not difficult to show that the CP-parity of the fields $\nu'_j(x)$ is $\eta_{\text{CP}}(\nu'_j) = i, j = 1, 2, 3$. The physical meaning of the signs of the masses $m'_j \neq 0$ of the Majorana neutrinos becomes clear if we change to a “basis” of neutrino fields $\nu_j(x)$ which have positive masses $m_j > 0$. This can be done, e.g., by introducing the fields \(^{16}\):

$$\nu'_j(x) = (-\gamma_5)^{1/2} (1 - \rho_j) \nu_j(x) : \quad \nu'_j(x) = \nu_j(x) \quad \text{if} \quad \rho_j = 1 ; \quad \nu'_j(x) = -\gamma_5 \nu_j(x) \quad \text{if} \quad \rho_j = -1 . \quad (1.52)$$

As it is not difficult to show, if $\nu'_j(x)$ has a mass $m'_j < 0$, CP-parity $\eta_{\text{CP}}(\nu'_j) = i$ and satisfies the Majorana condition $C (\nu'_j(x))^T = \rho_j \nu'_j(x)$, the field $\nu_j(x)$ possess a mass $m_j > 0$, CP-parity $\eta_{\text{CP}}(\nu_j) = i \rho_j$, and satisfy the Majorana condition $C (\nu_j(x))^T = \rho_j \nu_j(x)$:

$$\nu_j : \quad m_j > 0, \quad \eta_{\text{CP}}(\nu_j) = i \rho_j, \quad C (\nu_j(x))^T = \rho_j \nu_j(x) . \quad (1.53)$$

Thus, in the case of CP invariance, the signs of the nonzero eigenvalues of the neutrino Majorana mass matrix determine the CP-parities of the corresponding positive mass Majorana (mass eigenstate) neutrinos \(^{15}\).

1.2.3 A Brief Historical Detour

It is interesting to note that B. Pontecorvo in his second seminal article on neutrino oscillations \(^{7}\), which was published in 1958 when only one type of neutrino and antineutrino was known, assumed that the state of the neutrino $\nu$, emitted in weak interaction processes is a linear superposition of the states of two Majorana neutrinos $\nu^M_1$ and $\nu^M_2$ which have different masses, $m_1 \neq m_2$, opposite CP-parities, $\eta_{\text{CP}}(\nu^M_1) = -\eta_{\text{CP}}(\nu^M_2)$ and are maximally mixed, while the state of the corresponding antineutrino $\bar{\nu}$ is just the orthogonal superposition of the states of $\nu^M_1$ and $\nu^M_2$:

$$|\nu > = |\nu^M_1 > + |\nu^M_2 > \sqrt{2} , \quad (1.54)$$

$$|\bar{\nu} > = |\nu^M_1 > - |\nu^M_2 > \sqrt{2} . \quad (1.55)$$

\(^{13}\)The absolute value of the difference between the number of positive and number of negative eigenvalues of a real symmetric matrix $A$ is an invariant of the matrix with respect to transformations $A' = PAP^T$, where $P$ is a real matrix which has an inverse.

\(^{15}\) For further discussion of the properties of massive Majorana neutrinos (fermions) and their couplings see, e.g., \(^{16}\).
Thus, the oscillations are between the neutrino $\nu$ and and the antineutrino $\bar{\nu}$, in full analogy with the $K^0 - \bar{K}^0$ oscillations. From contemporary point of view, B. Pontecorvo proposed active - sterile neutrino oscillations with maximal mixing and massive Majorana neutrinos. To our knowledge, the 1958 article quoted in [7] was also the first in which fermion mixing in the weak interaction Lagrangian was introduced.

The article of Z. Maki, M. Nakagawa and S. Sakata [8] was inspired, in part, by the discovery of the second type of neutrino - the muon neutrino, in 1962 at Brookhaven. These authors considered a composite model of elementary particles in which the electron and muon neutrino states are superpositions of the states of composite Dirac neutrinos $\nu_1^D$ and $\nu_2^D$ which have different masses, $m_1^D \neq m_2^D$:

\begin{align}
|\nu_e> &= |\nu_1^D> \cos \theta_c + |\nu_2^D> \sin \theta_c, \quad (1.56) \\
|\nu_\mu> &= -|\nu_1^D> \sin \theta_c + |\nu_2^D> \cos \theta_c, \quad (1.57)
\end{align}

where $\theta_c$ is the neutrino mixing angle. The model proposed in [8] has lepton-hadron symmetry built in and as consequence of this symmetry the neutrino mixing angle coincides with what we call today the Cabibbo angle $\theta_C \approx 0.22$. The authors of [8] discuss the possibility of $\nu_\mu - \nu_e$ oscillations, which they called “virtual transmutations”.

In an article [78] by Y. Katayama, K. Matsumoto, S. Tanaka and E. Yamada, published in 1962 somewhat earlier than [8], the authors also introduce two-neutrino mixing. However, this is done purely for model construction purposes and does not have any physical consequences since the neutrinos in the model constructed in [78] are massless particles.

In 1967 B. Pontecorvo independently considered the possibility of $\nu_e \leftrightarrow \nu_\mu$ oscillations in the article [79], in which the notion of a “sterile” or “inert” neutrino was introduced. Later in 1969, V. Gribov and B. Pontecorvo [79] introduced for the first time a Majorana mass term for the LH flavour neutrinos $\nu_e$ and $\nu_\mu$, the diagonalisation of which lead to two Majorana neutrinos $\nu_1^M$ and $\nu_2^M$ with definite but different masses $m_{1,2}$, $m_1 \neq m_2$, and two-neutrino mixing with an arbitrary mixing angle $\theta$:

\begin{align}
|\nu_e> &= |\nu_1^M> \cos \theta + |\nu_2^M> \sin \theta, \quad (1.58) \\
|\nu_\mu> &= -|\nu_1^M> \sin \theta + |\nu_2^M> \cos \theta. \quad (1.59)
\end{align}

This was the first modern treatment of the problem of neutrino mixing which anticipated the way this problem is addressed in gauge theories of electroweak interactions and in Grand Unified Theories (GUT’s). In the same article for the first time the analytic expression for the probability of $\nu_e \leftrightarrow \nu_\mu$ oscillations was also derived.

1.2.4 Models of Neutrino Mass Generation: Two Examples

Type I See-Saw Model. A natural explanation of the smallness of neutrino masses is provided by the type I see-saw mechanism of neutrino mass generation [80]. An integral part of this rather simple mechanism are the RH neutrinos $\nu_{1R}$ (RH neutrino fields $\nu_{1R}(x)$). The latter are assumed to possess a Majorana mass term $\mathcal{L}_M^N(x)$ as well as Yukawa type coupling $\mathcal{L}_Y(x)$ with the Standard Model lepton and Higgs doublets, $\psi_{1L}(x)$ and $H(x)$, given in eq. (1.42). In the basis in which the Majorana mass matrix of RH neutrinos is diagonal, we have:

\begin{align}
\mathcal{L}_{Y,M}(x) \equiv \mathcal{L}_Y(x) + \mathcal{L}_M^N(x) = -\left(\lambda_{k} \overline{N_k(x)} H(x) \psi_{1L}(x) + \text{h.c.}\right) - \frac{1}{2} M_k \overline{N_k(x)} N_k(x), \quad (1.60)
\end{align}

The article by Z. Maki, M. Nakagawa and S. Sakata [8] appeared before the article by N. Cabibbo [76] in which the “Cabibbo angle” $\theta_C$ was introduced and the hadron phenomenology related to this angle was discussed, but after the article by M. Gell-Mann and M. Levy [77] in which $\theta_C$ was also introduced (by the way, in a footnote).
where we have combined the expressions given in eqs. (1.23) and (1.24). When the electroweak symmetry is broken spontaneously, the neutrino Yukawa coupling generates a Dirac mass term: 

\[ m_{\nu}^D = g_{\nu} L R(x) V L(x) + h.c., \]

with \( m^D = v \lambda, v = 174 \text{ GeV} \) being the Higgs doublet v.e.v. In the case when the elements of \( m^D \) are much smaller than \( M_k, |m_{\nu}^D| \ll M_k, i, k = 1, 2, 3, l = e, \mu, \tau, \) the interplay between the Dirac mass term and the mass term of the heavy (RH) Majorana neutrinos \( N_k \) generates an effective Majorana mass (term) for the LH flavour neutrinos [4]:

\[
M_{\nu l} \equiv - (m_{\nu}^D)^T_{l k} M_k^{-1} m_{\nu}^D = - v^2 (\lambda)^T_{l k} M_k^{-1} \lambda_{kl}. \tag{1.61}
\]

In grand unified theories, \( m^D \) is typically of the order of the charged fermion masses. In \( SO(10) \) theories, for instance, \( m^D \) coincides with the up-quark mass matrix. Taking indicatively \( M \sim 0.1 \text{ eV}, m^D \sim 100 \text{ GeV}, \) one obtains \( M_k \sim M_N \sim 10^{14} \text{ GeV} \), which is close to the scale of unification of the electroweak and strong interactions, \( M_{\text{GUT}} \equiv 2 \times 10^{16} \text{ GeV} \). In GUT theories with RH neutrinos one finds that indeed the heavy Majorana neutrinos \( N_k \) naturally obtain masses which are by few to several orders of magnitude smaller than \( M_{\text{GUT}} \) (see, e.g., the second and third articles quoted in ref. [4]). Thus, the enormous disparity between the neutrino and charged fermion masses is explained in this approach by the huge difference between effectively the electroweak symmetry breaking scale and \( M_{\text{GUT}} \).

An additional attractive feature of the see-saw scenario under discussion is that the generation and smallness of neutrino masses is related via the leptogenesis mechanism [2] (see also, e.g., [3]) to the generation of the baryon asymmetry of the Universe. Indeed, the Yukawa coupling in [1,60], in general, is not CP conserving. Due to this CP-nonconserving coupling the heavy Majorana neutrinos undergo, e.g., the decays \( N_j \rightarrow l^+ + H^(-), N_j \rightarrow l^- + H^(+), \) which have different rates: \( \Gamma(N_j \rightarrow l^+ + H^(-)) \neq \Gamma(N_j \rightarrow l^- + H^(+)) \). When these decays occur in the Early Universe at temperatures somewhat below the mass of, say, \( N_1 \), so that the latter are out of equilibrium with the rest of the particles present at that epoch, CP violating asymmetries in the individual lepton charges \( L_l \) and in the total lepton charge \( L \) of the Universe are generated. These lepton asymmetries are converted into a baryon asymmetry by \( (B - L) \) conserving, but \( (B + L) \) violating, sphaleron processes, which exist in the Standard Model and are effective at temperatures \( T \sim (100 - 10^{12}) \text{ GeV} \) [80]. If the heavy neutrinos \( N_j \) have hierarchical spectrum, \( M_1 \ll M_2 \ll M_3, \) the observed baryon asymmetry can be reproduced provided the mass of the lightest one satisfies \( M_1 \gtrsim 10^9 \text{ GeV} \) [81]. Thus, in this scenario, the neutrino masses and mixing and the baryon asymmetry have the same origin - the neutrino Yukawa couplings and the existence of (at least two) heavy Majorana neutrinos. Moreover, quantitative studies based on advances in leptogenesis theory [39,40], in which the importance of the flavour effects in the generation of the baryon asymmetry was understood, have shown that the Dirac and/or Majorana phases in the neutrino mixing matrix \( U \) can provide the CP violation, necessary in leptogenesis for the generation of the observed baryon asymmetry of the Universe [41]. This implies, in particular, that if the CP symmetry is established not to hold in the lepton sector due to the PMNS matrix \( U \), at least some fraction (if not all) of the observed baryon asymmetry might be due to the Dirac and/or Majorana CP violation present in the neutrino mixing.

In the see-saw scenario considered, the scale at which the new physics manifests itself, which is set by the scale of masses of the RH neutrinos, can, in principle, have an arbitrary large value, up to the GUT scale of \( 2 \times 10^{16} \text{ GeV} \) and even beyond, up to the Planck mass. An interesting possibility, which can also be theoretically well motivated (see, e.g., [84]), is to have the new physics at the TeV scale, i.e., \( M_k \sim (100 - 1000) \text{ GeV} \). Low scale see-saw scenarios usually predict a rich phenomenology.

\[17\] In specific type I see-saw models this bound can be lower by a few orders of magnitude, see, e.g., [82].
at the TeV scale and are constrained by different sets of data, such as, e.g., the data on neutrino oscillations, from EW precision tests and on the lepton flavour violating (LFV) processes $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu^- - e^- \rightarrow$ conversion in nuclei. In the case of the TeV scale type I see-saw scenario of interest, the flavour structure of the couplings of the heavy Majorana neutrinos $N_k$ to the charged leptons and the $W^\pm$ bosons, and to the LH flavour neutrinos $\nu_L$ and the $Z^0$ boson, are essentially determined by the requirement of reproducing the data on the neutrino oscillation parameters [63]. All present experimental constraints on this scenario still allow i) for the predicted rates of the $\mu \rightarrow e + \gamma$ decay, $\mu \rightarrow 3e$ decay and $\mu - e$ conversion in the nuclei to be [84] within the sensitivity range of the currently running MEG experiment on $\mu \rightarrow e + \gamma$ decay [?] planned to probe values of $\text{BR}(\mu^+ \rightarrow e^+ + \gamma) \gtrsim 10^{-13}$, and of the future planned experiments on $\mu \rightarrow 3e$ decay and $\mu - e$ conversion [85,86,87,88,89], ii) for an enhancement of the rate of neutrinoless double beta ($((\beta\beta)_{0\nu})$) decay [63], which thus can be in the range of sensitivity of the $((\beta\beta)_{0\nu})$-decay experiments which are taking data or are under preparation (see, e.g., [90]) even when the light Majorana neutrinos possess a normal hierarchical mass spectrum (see further), and iii) for the possibility of an exotic Higgs decay channel into a light neutrino and a heavy Majorana neutrino with a sizable branching ratio, which can lead to observable effects at the LHC [72] (for further details concerning the low energy phenomenology of the TeV scale type I see-saw model see, e.g., [63,83,84]).

Let us add that the role of the experiments searching for lepton flavour violation to test and possibly constrain low scale see-saw models, and more generally, extensions of the Standard Model predicting “new” (lepton flavour violating) physics at the TeV scale, will be significantly strengthened in the next years. Searches for $\mu - e$ conversion at the planned COMET experiment at KEK [86] and Mu2e experiment at Fermilab [87] aim to reach sensitivity to conversion rates CR($\mu$) within the sensitivity range of the currently running MEG experiment on $\mu \rightarrow e + \gamma$ decay [?] planned to probe values of $\text{BR}(\mu^+ \rightarrow e^+ + \gamma) \gtrsim 10^{-13}$, and of the future planned experiments on $\mu \rightarrow 3e$ decay and $\mu - e$ conversion [85,86,87,88,89], ii) for an enhancement of the rate of neutrinoless double beta ($((\beta\beta)_{0\nu})$) decay [63], which thus can be in the range of sensitivity of the $((\beta\beta)_{0\nu})$-decay experiments which are taking data or are under preparation (see, e.g., [90]) even when the light Majorana neutrinos possess a normal hierarchical mass spectrum (see further), and iii) for the possibility of an exotic Higgs decay channel into a light neutrino and a heavy Majorana neutrino with a sizable branching ratio, which can lead to observable effects at the LHC [72] (for further details concerning the low energy phenomenology of the TeV scale type I see-saw model see, e.g., [63,83,84]).

The Higgs Triplet Model (HTM). In its minimal formulation this model includes one additional $SU(2)_L$ triplet Higgs field $\Delta$, which has weak hypercharge $Y_W = 2$ [85]:

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} \\ \Delta^0 \\ -\Delta^+\sqrt{2} \end{pmatrix}.$$ (1.62)

The Lagrangian of the Higgs triplet model which is sometimes called also the “type II see-saw model”, reads [85]

$$\mathcal{L}_{\text{HTM}} = -M_{\Delta}^2 \text{Tr} \left( \Delta^\dagger \Delta \right) - \left( h_{\ell\ell'} \bar{\psi}_L \gamma^\mu \ell_2^\mu \Delta \psi_{L\ell'} + \mu_\Delta H^\dagger \Delta^\dagger i\tau_2 H^* + \text{h.c.} \right),$$ (1.63)

where $\bar{\psi}_{\ell L}(L) \equiv (\nu_{\ell L}^C C^{-1} - \ell_L^C C^{-1})$, $C$ being the charge conjugation matrix, $H$ is the SM Higgs doublet and $\mu_\Delta$ a real parameter characterising the soft explicit breaking of the total lepton charge conservation. We will discuss briefly the low energy version of HTM, where the new physics scale

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18We do not give here, for simplicity, all the quadratic and quartic terms present in the scalar potential (see, e.g., [92]).
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$M_\Delta$ associated with the mass of $\Delta$ takes values $100 \text{ GeV} \lesssim M_\Delta \lesssim 1 \text{ TeV}$, which, in principle, can be probed by LHC (see [94, 95] and references quoted therein).

The flavour structure of the Yukawa coupling matrix $h$ and the size of the lepton charge soft breaking parameter $\mu_\Delta$ are related to the light neutrino Majorana mass matrix $M_\nu$, which is generated when the neutral component of $\Delta$ develops a “small” vev $v_\Delta \propto \mu_\Delta$. Indeed, setting $\Delta^0 = v_\Delta$ and $H^T = (v \ 0)^T$ with $v \simeq 174$ GeV, from Lagrangian (1.63) one obtains:

$$M_\nu^\ell \approx 2 h_\ell \rho \ v_\Delta.$$  

(1.64)

The matrix of Yukawa couplings $h_\ell \rho$ is directly related to the PMNS neutrino mixing matrix $U_{\text{PMNS}} \equiv U$, which is unitary in this case:

$$h_\ell \rho \equiv \frac{1}{2v_\Delta} \left( U^* \text{diag}(m_1,m_2,m_3) U^\dagger \right)_\ell \rho, \quad \rho_j \geq 0.$$  

(1.65)

An upper limit on $v_\Delta$ can be obtained from considering its effect on the parameter $\rho = M_\ell^2 / M_\pi^2 \cos^2 \theta_W$. In the SM, $\rho = 1$ at tree-level, while in the HTM one has

$$\rho = 1 + \delta \rho = \frac{1 + 2x^2}{1 + 4x^2}, \quad x \equiv v_\Delta / v.$$  

(1.66)

The measurement $\rho \approx 1$ leads to the bound $v_\Delta / v \lesssim 0.03$, or $v_\Delta < 5$ GeV (see, e.g., [95]).

For $M_\Delta \sim (100 - 1000)$ GeV, the model predicts a plethora of beyond the SM physics phenomena (see, e.g., [95, 96, 97]), most of which can be probed at the LHC and in the experiments on charged lepton flavour violation, if the Higgs triplet vacuum expectation value $v_\Delta$ is relatively small, roughly $v_\Delta \sim (1 - 100)$ eV. As can be shown (see, e.g., [95]), the parameters $v_\Delta$ and $\mu_\Delta$ are related: for $M_\Delta \sim v = 174$ GeV we have $v_\Delta \approx \mu_\Delta$, while if $M_\Delta^2 >> v^2$, then $v_\Delta \approx \mu_\Delta v^2 / (2M_\Delta^2)$. Thus, a relatively small value of $v_\Delta$ in the TeV scale HTM implies that $\mu_\Delta$ has also to be small, and vice versa. A nonzero but relatively small value of $\mu_\Delta$ can be generated, e.g., at higher orders in perturbation theory [98]. The smallness of the neutrino masses is therefore related to the smallness of the vacuum expectation value $v_\Delta$, which in turn is related to the smallness of the parameter $\mu_\Delta$.

Under the conditions specified above one can have testable predictions of the model in low energy experiments, and in particular, in the ongoing MEG and the planned future experiments on the lepton flavour violating processes $\mu \to e\gamma$, $\mu \to 3e$ and $\mu + N \to e + N'$ (see, e.g., [84]). The HTM has also an extended Higgs sector including neutral, singly charged and doubly charged Higgs particles. The physical singly-charged Higgs scalar field (particle) practically coincides with the triplet scalar field $\Delta^+$, the admixture of the doublet charged scalar field being suppressed by the factor $v_\Delta / v$. The singly- and doubly-charged Higgs scalars $\Delta^+$ and $\Delta^{++}$ have, in general, different masses [98]: $m_{\Delta^+} \neq m_{\Delta^{++}}$. Both cases $m_{\Delta^+} > m_{\Delta^{++}}$ and $m_{\Delta^+} < m_{\Delta^{++}}$ are possible. The TeV scale HTM predicts the existence of rich new physics at LHC as well, associated with the presence of the singly and doubly charged Higgs particles $\Delta^+$ and $\Delta^{++}$ in the theory (see, e.g., [95, 97]).

1.3 Determining the Nature of Massive Neutrinos

The Majorana nature of massive neutrinos typically manifests itself in the existence of processes in which the total lepton charge $L$ changes by two units: $K^+ \to \pi^- + \mu^+ + \mu^+$, $\mu^- + (A, Z) \to \mu^+ + (A, Z - 2)$, etc. Extensive studies have shown that the only feasible experiments having the
potential of establishing the Majorana nature of massive neutrinos at present are the \((\beta\beta)_{0\nu}\)-decay experiments searching for the process \((A, Z) \rightarrow (A, Z + 2) + e^- + e^-\) (for reviews see, e.g., [16, 90, 99]).

The observation of \((\beta\beta)_{0\nu}\)-decay and the measurement of the corresponding half-life with sufficient accuracy, would not only be a proof that the total lepton charge is not conserved, but might provide also information i) on the type of neutrino mass spectrum [26], and ii) on the absolute scale of neutrino masses (see, e.g., [49]).

The observation of \((\beta\beta)_{0\nu}\)-decay and the measurement of the corresponding half-life with sufficient accuracy, combined with data on the absolute neutrino mass scale might provide also information on the Majorana phases in \(U\) [100, 48, 49, 50, 51]. If the neutrino mass spectrum is inverted hierarchical or quasi-degenerate, for instance, it would be possible to get information about the phase \(\alpha_{21}\). However, establishing even in this case that \(\alpha_{21}\) has a CP violating value would be a remarkably challenging problem [51] (see also [52]). Determining experimentally the values of both the Majorana phases \(\alpha_{21}\) and \(\alpha_{31}\) is an exceptionally difficult problem. It requires the knowledge of the type of neutrino mass spectrum and high precision determination of both the absolute neutrino mass scale and of the \((\beta\beta)_{0\nu}\)-decay effective Majorana mass, \(|\langle m \rangle|\) (see, e.g., [48, 50, 51]).

### 1.3.1 Majorana Neutrinos and \((\beta\beta)_{0\nu}\)-Decay

Under the assumptions of 3-\(\nu\) mixing, for which we have compelling evidence, of massive neutrinos \(\nu_j\) being Majorana particles, and of \((\beta\beta)_{0\nu}\)-decay generated only by the \((V-A)\) charged current weak interaction via the exchange of the three Majorana neutrinos \(\nu_j\) having masses \(m_j \lesssim \text{few MeV}\), the \((\beta\beta)_{0\nu}\)-decay amplitude of interest has the form (see, e.g. [16, 48, 99]): \(A(\beta\beta)_{0\nu} \equiv m < m > M\), where \(M\) is the corresponding nuclear matrix element (NME) which does not depend on the neutrino mixing parameters, and

\[
|\langle m \rangle| = \left|m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i(\alpha_{31} - 2\delta)}\right|, \tag{1.67}
\]

is the effective Majorana mass in \((\beta\beta)_{0\nu}\)-decay, \(|U_{e1}| = c_{12} c_{13}, |U_{e2}| = s_{12} c_{13}, |U_{e3}| = s_{13}\). In the case of CP-invariance one has \(2\delta = 0\) or \(2\pi\) and [15, 13],

\[
\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \tag{1.68}
\]

\(\eta_{21(31)}\) being the relative CP-parity of the Majorana neutrinos \(\nu_{2(3)}\) and \(\nu_1\).

It proves convenient to express [101] the three neutrino masses in terms of \(\Delta m_{31}^2\) and \(\Delta m_{31(32)}^2\); measured in neutrino oscillation experiments, and the absolute neutrino mass scale determined by \(\text{min}(m_j)\) [19]. In both cases of neutrino mass spectrum with normal and inverted ordering one has (in the convention we use): \(\Delta m_{21}^2 > 0, m_2 = (m_1^2 + \Delta m_{21}^2)^{\frac{1}{2}}\). For normal ordering, \(\Delta m_{21}^2 > 0\) and \(m_3 = (m_1^2 + \Delta m_{31}^2)^{\frac{1}{2}}\), while if the spectrum is with inverted ordering, \(\text{min}(m_j) = m_3, \Delta m_{32}^2 < 0\) and \(m_1 = (m_2^2 + \Delta m_{32}^2 - \Delta m_{21}^2)^{\frac{1}{2}}\). Thus, given \(\Delta m_{21}^2, \Delta m_{31(32)}^2\), \(\theta_{12}\) and \(\theta_{13}\), \(|\langle m \rangle|\) depends on \(\text{min}(m_j)\), Majorana phases \(\alpha_{21}, \alpha_{31}\) and the type of \(\nu\)-mass spectrum.

The problem of obtaining the allowed values of \(|\langle m \rangle|\) given the constraints on the parameters following from \(\nu\)-oscillation data, and more generally of the physics potential of \((\beta\beta)_{0\nu}\)-decay experiments, was first studied in [101] and subsequently in a large number of papers [29] (see, e.g., [48, 51, 102, 103, 104]). The results of this analysis are illustrated in Fig. 1. The main features of the

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\(^{19}\)For a detailed discussion of the relevant formalism see, e.g. [16, 48, 99].

\(^{20}\)Extensive list of references on the subject is given in [99].
predictions for $|\langle m \rangle|$ in the cases of the NH, IH and QD spectra are summarised below.

i) NH spectrum:

$$|\langle m \rangle| \cong |(\Delta m_{21}^2)^{\frac{1}{2}} s_{12}^2 + (\Delta m_{31}^2)^{\frac{1}{2}} s_{13}^2 e^{-i(\alpha_{21}-\alpha_{31}+2\delta)}|.$$  (1.69)

Using the $3\sigma$ allowed ranges of the relevant neutrino oscillation parameters we get:

$$4.7 \times 10^{-4} \text{ eV} \lesssim |\langle m \rangle| \lesssim 4.8 \times 10^{-3} \text{ eV}, \quad \text{NH}.$$  (1.70)

ii) IH spectrum:

$$|\langle m \rangle| \cong (|\Delta m_{32}^2|)^{\frac{1}{2}} (1 - \sin^2 2\theta_{21} \sin^2 \frac{\alpha_{21}}{2})^{\frac{1}{2}},$$  (1.71)

$$|\Delta m_{32}^2| \lesssim |\langle m \rangle| \lesssim (|\Delta m_{32}^2|)^{\frac{1}{2}}.$$  (1.72)

Numerically one finds:

$$0.014 \text{ eV} \lesssim |\langle m \rangle| \lesssim 0.050 \text{ eV}, \quad \text{IH},$$  (1.73)

the upper and the lower bounds corresponding to the CP-conserving values of $\alpha_{21} = 0; \pi$.

iii) QD spectrum:

$$|\langle m \rangle| \cong m_0 (1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{21}}{2})^{\frac{1}{2}},$$  (1.74)

$$m_0 \gtrsim |\langle m \rangle| \gtrsim m_0 \cos 2\theta_{12} \gtrsim 0.028 \text{ eV},$$  (1.75)

with $m_0 \geq 0.1 \text{ eV}$, $m_0 < 2.05 \text{ eV}$ [34] (see also [35]), or $m_0 \leq (0.3 - 1.3) \text{ eV}$ [37] (see eq. (1.7) and the discussion following after it).

For the IH (QD) spectrum we have also [100, 48]:

$$\sin^2 \left( \frac{\alpha_{21}}{2} \right) \cong \left( 1 - \frac{|\langle m \rangle|^2}{m_0^2} \right) \frac{1}{\sin^2 2\theta_{12}}, \quad m_0^2 = |\Delta m_{32}^2|^2 (m_0^2), \quad \text{IH (QD)}.$$  (1.76)

Thus, a measurement of $|\langle m \rangle|$ and $m_0 (|\Delta m_{32}^2|^2)$ for QD (IH) spectrum can allow to determine $\alpha_{21}$.

The experimental searches for $(\beta\beta)_{0v}$-decay have a long history (see, e.g., [105]). The most stringent upper limits on $|\langle m \rangle|$ were set by the IGEX [106], CUORICINO [107], NEMO3 [108] and EXO-200 [109] experiments with $^{76}\text{Ge}$, $^{130}\text{Te}$, $^{100}\text{Mo}$ and $^{136}\text{Xe}$, respectively [21]. The IGEX collaboration has obtained for the half-life of $^{76}\text{Ge}$, $T_{1/2}^\beta > 1.57 \times 10^{25} \text{ yr} \ (90\% \ C.L.)$, from which the limit $|\langle m \rangle| < (0.33 - 1.35) \text{ eV}$ was derived [106]. Using the recent more advanced calculations of the corresponding nuclear matrix elements (including the relevant uncertainties) [110] one finds: $|\langle m \rangle| < (0.22 - 0.35) \text{ eV}$. The NEMO3 and CUORICINO experiments, designed to reach a sensitivity to $|\langle m \rangle| \sim (0.2 - 0.3) \text{ eV}$, set the limits: $|\langle m \rangle| < (0.61 - 1.26) \text{ eV}$ [108] and $|\langle m \rangle| < (0.19 - 0.68) \text{ eV}$ [107] (90\% C.L.), where estimated uncertainties in the NME are accounted for. The two upper limits were derived from the experimental lower limits on the half-lives of $^{100}\text{Mo}$ and $^{130}\text{Te}$, $T_{1/2}^\beta > 5.8 \times 10^{23} \text{ yr} \ (90\% \text{C.L.})$ [108] and $T_{1/2}^\beta > 3.0 \times 10^{24} \text{ yr} \ (90\%\text{C.L.})$ [107]. With the NMEs and their uncertainties

\footnote{The NEMO3 collaboration has searched for $(\beta\beta)_{0v}$-decay of $^{82}\text{Se}$ and other isotopes as well.}
calculated in [110], the NEMO3 and CUORICINO upper limits read, respectively: $|\langle m \rangle| < (0.50 - 0.96)$ eV and $|\langle m \rangle| < (0.25 - 0.43)$ eV. A very impressive lower limit on the half-life of $^{136}\text{Xe}$ was obtained recently in the EXO-200 experiment [109]: $T_{1/2}^{0\nu}(^{136}\text{Xe}) > 1.6 \times 10^{25}$ yr (90% C.L.).

The best lower limit on the half-life of $^{76}\text{Ge}$, $T_{1/2}^{0\nu} > 1.9 \times 10^{25}$ yr (90% C.L.), was found in the Heidelberg-Moscow $^{76}\text{Ge}$ experiment [111]. It corresponds to the upper limit [110] $|\langle m \rangle| < (0.20 - 0.35)$ eV. A positive $(\beta\beta)_{0\nu}$-decay signal at $> 3\sigma$, corresponding to $T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}$ yr (99.73% C.L.) and implying $|\langle m \rangle| = (0.1 - 0.9)$ eV, is claimed to have been observed in [112], while a later analysis reports evidence for $(\beta\beta)_{0\nu}$-decay at $6\sigma$ corresponding to $|\langle m \rangle| = 0.32 \pm 0.03$ eV [113].

Most importantly, a large number of projects aim at a sensitivity to $|\langle m \rangle| \sim (0.01 - 0.05)$ eV [90]: CUORE ($^{130}\text{Te}$), GERDA ($^{76}\text{Ge}$), SuperNEMO, EXO ($^{136}\text{Xe}$), MAJORANA ($^{76}\text{Ge}$), MOON ($^{100}\text{Mo}$), COBRA ($^{116}\text{Cd}$), XMASS ($^{136}\text{Xe}$), CANDLES ($^{48}\text{Ca}$), KamLAND-Zen ($^{136}\text{Xe}$), SNO+ ($^{150}\text{Nd}$), etc. These experiments, in particular, will test the positive result claimed in [113].

The existence of significant lower bounds on $|\langle m \rangle|$ in the cases of IH and QD spectra [26], which lie either partially (IH spectrum) or completely (QD spectrum) within the range of sensitivity of the next generation of $(\beta\beta)_{0\nu}$-decay experiments, is one of the most important features of the predictions of $|\langle m \rangle|$. These minimal values are given, up to small corrections, by $\Delta m^2_{32} \cos 2\theta_{12}$ and $m_0 \sin 2\theta_{12}$. According to the combined analysis of the solar and reactor neutrino data [21], i) the possibility of $\cos 2\theta_{12} = 0$ is excluded at $\sim 6\sigma$, ii) the best fit value of $\cos 2\theta_{12}$ is $\cos 2\theta_{12} \cong 0.39$, and iii) at 99.73% C.L. one has $\cos 2\theta_{12} \cong 0.28$. The quoted results on $\cos 2\theta_{12}$ together with the range of possible values of $\Delta m^2_{32}$ and $m_0$ lead to the conclusion about the existence of significant and robust lower bounds on $|\langle m \rangle|$ in the cases of IH and QD spectrum. At the same time one can always have $|\langle m \rangle| \ll 10^{-3}$ eV in the case of spectrum with normal ordering [49]. As Fig. 1 indicates, $|\langle m \rangle|$ cannot exceed $\sim 5$ meV for NH neutrino mass spectrum. This implies that $\max(|\langle m \rangle|)$ in the case of NH spectrum is considerably smaller than $\min(|\langle m \rangle|)$ for the IH and QD spectrum. This opens the possibility of obtaining information about the type of $\nu$-mass spectrum from a measurement of $|\langle m \rangle| \neq 0$ [26].

In particular, a positive result in the future $(\beta\beta)_{0\nu}$-decay experiments with $|\langle m \rangle| > 0.01$ eV would imply that the NH spectrum is strongly disfavored (if not excluded). For $\Delta m^2_{31(32)} > 0$, such a result would mean that the neutrino mass spectrum is with normal ordering, but is not hierarchical. If $\Delta m^2_{31(32)} < 0$, the neutrino mass spectrum would be either IH or QD. Prospective experimental errors in the values of oscillation parameters in $|\langle m \rangle|$ and the sum of neutrino masses, and the uncertainty in the relevant NME, can weaken but do not invalidate these results [27, 116, 51, 54].

As Fig. 1 indicates, a measurement of $|\langle m \rangle| \gtrsim 0.01$ eV would either [49] i) determine a relatively narrow interval of possible values of the lightest neutrino mass $\min(m_j)$, or ii) would establish an upper limit on $\min(m_j)$. If an upper limit on $|\langle m \rangle|$ is experimentally obtained below 0.01 eV, this would lead to a significant upper limit on $\min(m_j)$.

The possibility of establishing CP-violation in the lepton sector due to Majorana CPV phases has been studied in [49, 52] and in much greater detail in [50, 51]. It was found that it is very challenging: it requires quite accurate measurements of $|\langle m \rangle|$ (and of $m_0$ for QD spectrum), and holds only for a limited range of values of the relevant parameters. More specifically [50, 51], establishing at $2\sigma$ CP-violation associated with Majorana neutrinos in the case of QD spectrum requires for $\sin^2 \theta_\odot = 0.31$, in particular, a relative experimental error on the measured value of $|\langle m \rangle|$ and $m_0$ smaller than 15%, a “theoretical uncertainty” $F \lesssim 1.5$ in the value of $|\langle m \rangle|$ due to an imprecise knowledge of the

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27Encouraging results, in what regards the problem of calculation of the NME, were reported at the MEDEX’11 Workshop on Matrix Elements for the Double-beta decay Experiments [114]. For the bounds on $|\langle m \rangle|$ obtained using the current results on the NME see, e.g., [54].
corresponding NME, and value of the relevant Majorana CPV phase $\alpha_{21}$ typically within the ranges
of $\sim (\pi/4 - 3\pi/4)$ and $\sim (5\pi/4 - 7\pi/4)$.

The knowledge of NME with sufficiently small uncertainty is crucial for obtaining quantitative
information on the $\nu$-mixing parameters from a measurement of $(\beta\beta)_{0\nu}$-decay half-life. The observation
of a $(\beta\beta)_{0\nu}$-decay of one nucleus is likely to lead to the searches and eventually to observation of the
decay of other nuclei. One can expect that such a progress, in particular, will help to solve completely
the problem of the sufficiently precise calculation of the nuclear matrix elements for the $(\beta\beta)_{0\nu}$-decay.

If the future $(\beta\beta)_{0\nu}$-decay experiments show that $|\langle m \rangle| < 0.01$ eV, both the IH and the QD
spectrum will be ruled out for massive Majorana neutrinos. If in addition it is established in neutrino
oscillation experiments that the neutrino mass spectrum is with inverted ordering, i.e. that $\Delta m^2_{31(32)} < 0$,
one would be led to conclude that either the massive neutrinos $\nu_j$ are Dirac fermions, or that $\nu_j$
are Majorana particles but there are additional contributions to the $(\beta\beta)_{0\nu}$-decay amplitude which interfere destructively with that due to the exchange of light massive Majorana neutrinos. The case of more than one mechanism generating the $(\beta\beta)_{0\nu}$-decay was discussed recently in, e.g., [117, 118], where
the possibility to identify the mechanisms inducing the decay was also analysed. If, however, $\Delta m^2_{31(32)}$ is determined to be positive in neutrino oscillation experiments, the upper limit $|\langle m \rangle| < 0.01$ eV
would be perfectly compatible with massive Majorana neutrinos possessing NH mass spectrum, or
mass spectrum with normal ordering but partial hierarchy, and the quest for $|\langle m \rangle|$ would still be
open.

If indeed in the next generation of $(\beta\beta)_{0\nu}$-decay experiments it is found that $|\langle m \rangle| < 0.01$ eV,
while the neutrino oscillation experiments show that $\Delta m^2_{31(32)}> 0$, the next frontier in the searches for
$\beta\beta$-decays would most probably correspond to values of $|\langle m \rangle| \sim 0.001$ eV. Taking $|\langle m \rangle| = 0.001$
eV as a reference value, the conditions under which $|\langle m \rangle|$ in the case of neutrino mass spectrum with
normal ordering would be guaranteed to satisfy $|\langle m \rangle| \gtrsim 0.001$ eV, were investigate in [102]. In the
analysis performed in [102], the specific case of normal hierarchical neutrino mass spectrum, and the
general case of spectrum with normal ordering, partial hierarchy and values of $\theta_{13}$, including the value
measured in the Daya Bay, RENO, Double Chooz and T2K experiments, eq. (1.4), were considered.
The ranges of the lightest neutrino mass $m_1$ and/or of $\sin^2 \theta_{13}$, for which $|\langle m \rangle| \gtrsim 0.001$ eV were
derived as well, and the phenomenological implications of such scenarios were discussed.

1.4 Outlook

The last 14 years or so witnessed a spectacular experimental progress in the studies of the properties
of neutrinos. In this period the existence of neutrino oscillations, caused by nonzero neutrino masses
and neutrino mixing, was established and the parameters which drive the oscillations, were determined
with a relatively high precision. In spite of these remarkable achievements one has to admit that we
are still completely ignorant about some of the fundamental aspects of neutrino mixing: the nature -
Dirac or Majorana, of massive neutrinos, the type of spectrum the neutrino masses obey, the absolute
scale of neutrino masses, the status of CP symmetry in the lepton sector. Finding out these aspects
and understanding the origins of the neutrino masses and mixing and the patterns they and possibly
leptonic CP violation exhibit, requires an extensive and challenging program of research. The main
goals of such a research program include:

\[\footnote{A possible test of the NME calculations is suggested in [49] and is discussed in greater detail in [115].}\]
• Determining the nature - Dirac or Majorana, of massive neutrinos $\nu_j$. This is of fundamental importance for making progress in our understanding of the origin of neutrino masses and mixing and of the symmetries governing the lepton sector of particle interactions.

• Determination of the sign of $\Delta m_{31}^2$ ($\Delta m_{31}^2$) and of the type of neutrino mass spectrum.

• Determining or obtaining significant constraints on the absolute neutrino mass scale.

• Determining the status of CP symmetry in the lepton sector.

• Understanding at a fundamental level the mechanism giving rise to neutrino masses and mixing and to $L_d$—non-conservation. This includes understanding the origin of the patterns of neutrino mixing and neutrino masses, suggested by the data. Are the observed patterns of $\nu$-mixing and of $\Delta m_{21,31}^2$ related to the existence of a new fundamental symmetry of particle interactions? Is there any relation between quark mixing and neutrino (lepton) mixing? What is the physical origin of CP violation phases in the neutrino mixing matrix $U$? Is there any relation (correlation) between the (values of) CP violation phases and mixing angles in $U$? Progress in the theory of neutrino mixing might also lead to a better understanding of the mechanism of generation of baryon asymmetry of the Universe.

The successful realization of this research program would be a formidable task and would require many years. It already began with the high precision measurement of $\theta_{13}$ in the Daya Bay and RENO experiments, which showed that $\sin^2 2\theta_{13}$ has a relatively large value, eq. (1.4). The Double Chooz and T2K experiments also found values of $\sin^2 2\theta_{13}$, which are different from zero respectively at $3.1\sigma$ and $3.2\sigma$ and are compatible with those obtained in the Daya Bay and RENO experiments. These results on $\theta_{13}$ have far reaching implications. As we have already mentioned or discussed, the measured relatively large value of $\theta_{13}$ opens up the possibilities, in particular,

i) for searching for CP violation effects in neutrino oscillation experiments with high intensity accelerator neutrino beams, like T2K and NO$\nu$A

ii) for determining the sign of $\Delta m_{32}^2$, and thus the type of neutrino mass spectrum, in neutrino oscillation experiments with sufficiently long baselines (see, e.g., [23, 25]).

A value of $\sin \theta_{13} > 0.09$ is a necessary condition for a successful “flavoured” leptogenesis with hierarchical heavy Majorana neutrinos when the CP violation required for the generation of the matter-antimatter asymmetry of the Universe is provided entirely by the Dirac CP violating phase in the neutrino mixing matrix [41].

With the measurement of $\theta_{13}$, the first steps on the long “road” leading to a comprehensive understanding of the patterns of neutrino masses and mixing, of their origin and implications, were made. The future of neutrino physics is bright.

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$^{24}$The sensitivities of T2K and NO$\nu$A on CP violation in neutrino oscillations are discussed, e.g., in [46].
Figure 1.1: The effective Majorana mass $|\langle m \rangle|$ (including a $2\sigma$ uncertainty), as a function of $\min(m_j)$ for $\sin^2 \theta_{13} = 0.0236 \pm 0.0042$ \cite{ref} and $\delta = 0$. The figure is obtained using also the best fit values and $1\sigma$ errors of $\Delta m^2_{21}$, $\sin^2 \theta_{12}$, and $|\Delta m^2_{31(32)}|$ given in Table 1.7 in ref. \cite{ref}. The phases $\alpha_{21,31}$ are varied in the interval $[0,\pi]$. The predictions for the NH, IH and QD spectra are indicated. The red regions correspond to at least one of the phases $\alpha_{21,31}$ and $(\alpha_{31} - \alpha_{21})$ having a CP violating value, while the blue and green areas correspond to $\alpha_{21,31}$ possessing CP conserving values. (From ref. \cite{ref}.)
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