Generalized STAR (1;1) Model with Outlier - Case Study of Begal in Medan, North Sumatera

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Abstract. This study was aimed to compare generalized space autoregressive (GSTAR) (1;1) model with its modification. The modification of GSTAR (1;1) model was done by adding the outlier factor which gained by outlier detection procedure. A case study was done towards the amount of monthly crime activity-begal at seven police sectors in Medan, North Sumatera. By using three steps of Box-Jenkins, Autoregressive (1) model was applied to each location and resulted West Medan as fifth location with additive outlier (AO) in the third data of time which weighted -57.5247. The comparison of model was done by calculating the root mean square of error (RMSE) in each model. Since GSTAR (1;1) model has larger RMSE (8.1728) rather than its modification (7.9700), then GSTAR (1;1) model with outlier is the best model in this case study. This invention shows that the model of space time with outlier model is usable to find precise result rather than space time model only.

Keyword. GSTAR (1;1), Additive Outlier (AO), space time, root mean square error (RMSE).

1. Introduction

Many events which occured in everyday life were not only related to the events in previous time but also with the locations or surrounding area. At first, time series analysis and spatial analysis studies were discussed separately. If multiple locations are fixed, then time series analysis is used. Conversely, if multiple locations are not fixed but time is fixed, then spatial analysis is used. However, along with the development of science and technology, space-time analysis was emerged.

The name of Generalized Space Time Autoregressive (GSTAR) model was introduced by Ruchjana in 2002. Previously, [1] in 1995 adopted the name of GSTAR in a different context--i.e. the STAR model with spatial correlation which occur at same time and same parameters in each location. On the contrary, Ruchjana refered GSTAR as STAR model in heterogeneous locations with different parameter values at each location [2]. Avoiding different interpretations, the definition of GSTAR in this paper refers to Ruchjana’s.

Many researchers were using GSTAR model to be applied in space-time data. Nurhayati et al. (2012) applied GSTAR model on gross domestic product (GDP) in West European countries [3]. In recent time, GSTAR model is developed and resulting some modifications. For example, Ilmi et al. applied GSTAR model with time correlated errors to red chili weekly prices at some traditional markets in Bandung. This time correlated error assumption produced better forecasting rather than GSTAR model only [4]. Meanwhile, Masteriana et al. tried to develop spatial and time correlated error assumption and produced better result by doing monte carlo simulation [5]. Considering the possibility of better performance by doing some modifications of model, this paper will discuss about adding outlier factor to GSTAR model. Outlier is problem which often encountered and producing bias model. Therefore, an idea arose in this paper to apply outlier detecting procedure such that outlier may be covered and resulting better model.
A case study in this paper was done toward the amount of monthly crime activity which named as begal at seven police sectors in Medan, North Sumatera. According to The Great Dictionary of The Indonesian Language of The Language Center (KBBI), begal is defined as process and way of act of stamping or robbery in the street [6]. Usually, this activity is done towards motorcycle drivers in a sadistic way. The criminals may pull and force the victims, until they take the valuable things i.e. money or the motorcycle. No kidding at all, the criminals even using gun, stiletto, or knife to force the victims and kill the victim if he resists. Begal is one of extraordinary crime in Medan, North Sumatera which strengthen by the fact that one leader of begal may act this crime up to 32 times [7]. Since begal is kind of crime which is extraordinary, modelling and predicting this case become an interesting thing to be executed. In order to simplify the analysis, GSTAR (1;1) is choosen in this study. Moreover, since there is a location which detected has additive outlier (AO), a comparison between GSTAR (1;1) model and its modification with outlier will be done. The best result of this case study is determined by finding the smaller root mean square error (RMSE). Result of this case study furthermore may be used especially by the police in North Sumatera to handle and prevent begal happened.

2. Generalized Space Time Autoregressive (GSTAR) Model

Given process \( Y(t) = (Y_1(t), Y_2(t), \ldots, Y_N(t))^T \) at location \( i = 1, 2, \ldots, N \) and time \( t \). \( Y_i(t) \) is following GSTAR model with time order \( p \) and spatial lag \( \lambda_1, \ldots, \lambda_p \), or written as GSTAR \( (p; \lambda_1, \ldots, \lambda_p) \) if satisfy

\[
Y_i(t) = \sum_{k=1}^{p} \phi_{k0} Y(t - k) + \sum_{s=1}^{\lambda_k} \phi_{ks} W^{(s)} Y(t - k) + \varepsilon_i(t) \tag{1}
\]

where \( \lambda_k \) representing spatial order for the \( k \)-th time lag and \( \phi_{ks} \) representing regression parameter of \( k \)-th time order and \( s \)-th spatial order. Moreover, weight matrix \( W^{(s)} \) representing the weight of \( s \)-th order spatial with size \( N \times N \) and \( \varepsilon_i(t) \) is error vector in time \( t \) with dimension \( N \) [8]. Equation (1) in matrix notation is written as follow.

\[
Y(t) = \sum_{k=1}^{p} \sum_{s=1}^{\lambda_k} \phi_{ks} W^{(s)} Y(t - k) + \varepsilon(t) \tag{2}
\]

where

\[
Y(t) = \begin{bmatrix} Y_1(t) \\ Y_2(t) \\ \vdots \\ Y_N(t) \end{bmatrix}, \varepsilon(t) = \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \vdots \\ \varepsilon_N(t) \end{bmatrix}, \Psi = \begin{bmatrix} 0 & w_{12} & \cdots & w_{1N} \\ w_{21} & 0 & \cdots & w_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N1} & w_{N2} & \cdots & 0 \end{bmatrix}
\]

for every \( s \)-th spatial order which satisfy \( \sum_{j=1}^{N} w_{ij} = 1 \), and \( \Phi_{ks} = diag(\phi_{k1}, \phi_{k2}, \ldots, \phi_{kN}), k = 1, 2, \ldots, p \) for every \( s \)-th spatial order.

Consider \( Y_i(t) \) following GSTAR (1;1) model which given by

\[
Y(t) = (\Phi_0 + \Phi_1 W) Y(t - 1) + \varepsilon(t). \tag{3}
\]

Since the process of \( Y(t) \) is assumed to be centered, then \( E[Y(t)] = 0 \) for every \( t \). Consequently, STAR (1;1) model is special case of GSTAR(1;1) which satisfies \( \Phi_0 = \phi_{10}I \) and \( \Phi_1 = \phi_{11}I \). Furthermore, the stationarity of GSTAR (1;1) is gained if \( \phi_{10}^{(i)} \) and \( \phi_{11}^{(i)} \) satisfy \( |\phi_{10}^{(i)} + \phi_{11}^{(i)}| \leq 1 \) for every \( i = 1, \ldots, N \) [9].

3. Spatial Weight Matrix

Spatial weight matrix is main key in most models which need spatial structure representation [10]. The construction of spatial weight matrix is built upon spatial structure principal which involving the consideration of each affected unit and not. Let \( W \) be spatial weight matrix with \( N \times N \) size and \( w_{ij} \) is matrix element at location \( i, j \) in \( n \) location. According to the convention, the main diagonal element \( (w_{ii}) \) must be zero and total of each row is one. Meanwhile, the spatial configuration between location is represented by the value of matrix element except the main diagonal. In this paper, we use the kind of uniform weight matrix. This spatial weight matrix is affected by amount of observed locations in such order. If location \( i \) has \( n \) nearest location, then weight between location \( i \) and \( j \) is determined by:
\[ w_{ij}^{(s)} = \begin{cases} \frac{1}{n_i}, & j \text{ is neighbor } i \text{ in } s\text{-th order} \\ 0, & \text{others.} \end{cases} \]

Consequently, amount of weight between close location in one order is same (uniform).

4. Three Stages Iteration of Box-Jenkins

Three stages iteration of Box-Jenkins is adopted to analyze the model of GSTAR.

4.1. Identification Model of GSTAR

In space time model, identification step is executed by analyzing Space-Time Autocorrelation Function (STACF) and Space-Time Partial Autocorrelation Function (STPACF). Similar with time series analysis, determining model is done by observing cut-off or tail-off pattern from STACF and STPACF which theoretically given by Table 1. below [9].

| Model          | STACF               | STPACF                  |
|----------------|---------------------|-------------------------|
| STAR \( (p; \lambda_1, ..., \lambda_p) \) | tail off            | cut off after time lag \( p\text{-th} \) and spatial lag \( \lambda_p\text{-th} \) |
| STMA \( (q; m_1, ..., m_p) \)           | cut off after time lag \( q\text{-th} \) and spatial lag \( m_q\text{-th} \) | tail off                  |
| STARMA \( (p; \lambda_1, ..., \lambda_p, q; m_1, ..., m_p) \) | tail off            | tail off                  |

4.2. Parameter Estimation of GSTAR

Parameter estimation is done by using least square method. Assume error of model is mutually independent. Equation (2) for time \( t = 1, 2, ..., T \) and location \( i = 1, 2, ..., N \), is stated in linear model

\[ Y = X\phi + \epsilon \quad (4) \]

where:

\[
Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{N_{(NT\times1)}} \end{bmatrix} \quad \text{with} \quad Y_i = \begin{bmatrix} Y_1(t) \\ Y_2(t) \\ \vdots \\ Y_{(T \times 1)} \end{bmatrix} \quad \text{and} \quad V_i = \text{diag}(X_1, ..., X_N)_{(NT\times2N)} \quad \text{with} \quad X_i = \begin{bmatrix} Y_1(0) \\ Y_2(1) \\ \vdots \\ Y_{(T-1)}(T-1) \end{bmatrix}_{(T\times2N)}
\]

\[
\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix}_{(N_{(2\times N)})} \quad \text{with} \quad \phi_i = \begin{bmatrix} \phi_{0i} \\ \phi_{1i} \\ \vdots \\ \phi_{Ni} \end{bmatrix}_{(N_{(T\times1)})}, \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{bmatrix}_{(N_{(T\times1)})} \quad \text{with} \quad \epsilon_i = \begin{bmatrix} \epsilon_{i1} \\ \epsilon_{i2} \\ \vdots \\ \epsilon_{iN} \end{bmatrix}_{(T\times1)}
\]

The parameter of \( \phi \) are able to be estimated by \( \hat{\phi}_T = (X'X)^{-1}X'Y \) if \( X'X \) is non-singular matrix [8].

4.3. Diagnostic Checking

Diagnostic checking is executed to prove whether the assumption of mutual independent error is satisfied or not. Consider residual is difference between real data and estimated model, then diagnostic
checking is done by plotting the Autocorrelation Function (ACF) of model and doing normality test. If plot of residual’s ACF is cut-off in lag 0, then there is no any correlation of error. It means past errors do not affect the future errors. Moreover, the result of normality test may strengthen the assumption.

5. Time Series Outlier
Sometimes, extreme values are appeared in time series data. The data with these extreme values often becomes the outliers of those observations. If the time and cause of the outlier is known, an intervention model can be used to analyze it. However, the occurrence of outlier is sometimes unknown. Therefore, time series outlier model is needed [11]. There are two kinds of outlier as follow.

5.1. Additive Outlier (AO)
Additive outlier is a kind of outlier which affect the time series model in time $T$ only. Let $Z_t$ is observed series and $X_t$ is series of the outlier. AO is given in equation below.

$$Z_t = \begin{cases} X_t, & t \neq T \\ X_t + \omega, & t = T \end{cases} = X_t + \omega I^T_t = \frac{\theta(B)}{\phi(B)} a_t + \omega I^T_t \tag{5}$$

with

$$I^T_t = \begin{cases} 1, & t \neq T \\ 0, & t = T. \end{cases}$$

5.2. Innovational Outlier (IO)
Innovational outlier is a kind of outlier which affect the time series model in time $T$ until $T + m$, with $m$ is time when the outlier is still affected. Let $Z_t$ is observed series and $X_t$ is series of the outlier. IO is given in equation below.

$$Z_t = X_t + \frac{\theta(B)}{\phi(B)} I^T_t = \frac{\theta(B)}{\phi(B)} (a_t + \omega I^T_t). \tag{6}$$

Generally, time series with $k$ outliers which consisted of AO and IO is written as

$$Z_t = \sum_{j=1}^{k} \omega_j v_j(B) I^T_t + X_t \tag{7}$$

with $X_t = \frac{\theta(B)}{\phi(B)} a_t$; $v_j(B) = 1$ for AO model and $v_j(B) = \frac{\theta(B)}{\phi(B)}$ for IO model in time $t = T_j$.

6. Procedure of Analysis
Procedure of analysis is given by flowchart in Figure 1. below.

![Flowchart of analysis procedure.](image-url)
The selection of Autoregressive (AR) (1) model to whole locations is done since we are using GSTAR (1;1) model. Detecting outlier procedure which done by using this model was explained by [12]. Result of this procedure is maximum absolute value of lambda which shows that the detected outlier will be chosen and inserted to the model. In this case study, additive outlier (AO) is found in fifth location, West Medan with weight of -57.5024 in the third data of time. Moreover, in GSTAR (1;1) model with outlier, the weight of outlier is added behind the Equation (2). Furthermore, three stages iteration of Box-Jenkins are used in both model. Finally, the best model is determined by seeing the smaller root mean square error (RMSE) which produced.

7. Result and Discussion

7.1. Descriptive Statistics

Case study in this paper was done towards monthly amount of begal in Medan, North Sumatera along October 2013 – July 2017. This means there are 46 data of time in each location. Seven locations of sector police in Medan are Criminal Detective (Reskrim), Medan Area, Medan City, East Medan, West Medan, Medan Baru and Percut Sei Tuan (Ps. Tuan) as given in Figure 2. Consider that descriptive statistics which given in Table 2, show that the maximum amount of begal is reached by location Ps.Tuan. Meanwhile, the maximum summation of data is gained by Reskrim. It means both of these locations contribute higher amount of begal in Medan. Other interesting fact which gained from this statistics is the average of begal action which recorded by the police was about 56.8913 in Reskrim and 56.4130 in Ps.Tuan. This means almost two person in a day is being victim of begal in that location.

Consider the Pearson correlation between data in seven locations which given in Table 3. The result shows positive sign with different value. It means if amount of begal happened in a location increases, then other location also increases. The value indicates how strong the influence that appear. Noted that the strong correlation is reached by M.Barat and Reskrim with correlation 0.757. It means when begal is happened in M.Barat, then begal will be happened in Reskrim too with high probability.

| Table 2. Descriptive statistics. |
|----------------------------------|
|        | N  | Min. | Max. | Sum  | Mean | Std. Dev |
| Reskrim | 46 | 25.00| 107.00| 2670.00| 56.8913| 17.34232 |
| M.Area  | 46 | 16.00| 51.00 | 1301.00| 28.2826| 7.90896  |
| Medan City | 46 | 12.00| 56.00 | 1227.00| 26.6739| 10.05773 |
| East Medan | 46 | 8.00 | 51.00 | 1223.00| 26.6322| 8.68335  |
| West Medan | 46 | 3.00 | 79.00 | 787.00 | 17.1087| 13.34038 |
| M.Baru  | 46 | 12.00| 60.00 | 1384.00| 30.8070| 9.69494  |
| Ps.Tuan | 46 | 30.00| 144.00| 2959.00| 56.4130| 23.52453 |

| Table 3. Correlation of data in seven locations. |
|-----------------------------------------------|
|        | Reskrim | M.Area | M.Kota | M.Timur | M.Barat | M.Baru | Ps.Tuan |
| N      | 46      | 46     | 46     | 46       | 46       | 46     | 46      |
| Reskrim | 1       | .259   | .618   | .755    | .755     | .325   | .249    |
| M.Area  | .259    | 1      | .542   | .702    | .64       | .009   | .46      |
| M.Kota  | .618    | .542   | 1      | .725    | .757      | .669   | .595    |
| M.Timur | .755    | .702   | .725   | 1       | .626      | .454   | .416    |
| M.Barat | .325    | .64    | .64    | .626    | 1         | .56    | .503    |
| M.Baru  | .009    | .009   | .009   | .009    | .56       | 1      | .328    |
| Ps.Tuan | .249    | .46    | .46    | .46     | .56       | .328   | 1       |

* Correlation is significant at the 0.05 level (2-tailed)
** Correlation is significant at the 0.01 level (2-tailed)
7.2. Spatial Weight Matrix

Before determining the spatial weight matrix, we need to determine the spatial lag. In this study, the amount of spatial lag is determined by using the formula of Sturge which is usually used for amount determination of class. By using the formula, we have four spatial lag. By calculating the Euclidean distance between two locations in whole locations, the distance matrix may be gained as given as Table 4. below. Note that the spatial lag division is done by calculating the difference of minimum and maximum distance and dividing it by four (since amount of spatial lag is four). Moreover, the uniform spatial weight matrix is constructed by following Table 4 and Section (3).

Table 4. Distance matrix and spatial lag division. This value is needed to construct the weight matrix.

|       | Reskrim | M.Area  | M.Kota  | M.Timur | M.Barat | M.Baru  | Ps.Tuan |
|-------|---------|---------|---------|---------|---------|---------|---------|
| Reskrim | 0       | 0.04019 | 0.05448 | 0.06916 | 0.00887 | 0.1742  | 0.10676 |
| M.Area  | 0.04019 | 0       | 0.02155 | 0.10463 | 0.04555 | 0.02867 | 0.07178 |
| M.Kota  | 0.05448 | 0.02155 | 0       | 0.12257 | 0.05584 | 0.03713 | 0.07700 |
| M.Timur | 0.06916 | 0.10463 | 0.12257 | 0       | 0.07139 | 0.59539 | 0.15441 |
| M.Barat | 0.00887 | 0.04455 | 0.05584 | 0.07139 | 0       | 0.02069 | 0.11349 |
| M.Baru  | 0.01742 | 0.02386 | 0.03713 | 0.08578 | 0.02069 | 0       | 0.09391 |
| Ps.Tuan | 0.10676 | 0.07178 | 0.07700 | 0.15441 | 0.11349 | 0.09391 | 0       |

7.3. Identification Model

Consider the space time autocorrelation function (STACF) which given in Figure 3. Since the pattern is tail-off, then this graph shows the pattern of STAR theoretically. Moreover, ensuring STAR model and determination of order are executed by analyzing space time partial autocorrelation function (STPACF) which given in Figure 4. Since the pattern is cut-off, then this graph shows the pattern of STAR theoretically. Note that, there are another possible model is GSTAR (1;3). But, indication of GSTAR (1;1) is also found (red marked in Figure 4). Since higher order will spent more calculation and sometimes it is not always fit the data, then we limit the model to be GSTAR (1;1). So, the analysis of model in this paper is done for GSTAR (1;1) model only.

Figure 3. Graph of STACF. This graph gives readers information about pattern of data which suitable to such model. Pattern of tail-off in STACF below indicates AR characteristic.
Figure 4. Graph of STPACF. This graph helps readers deciding suitable order of GSTAR model. Since the graph shown cut-off pattern in spatial lag 1, then AR model is indicated.

7.4. Parameter Estimation
Consider GSTAR (1:1) model which given in equation (3). By using least square method, the linear model in (4) is solvable and produces parameter \( \hat{\Phi}_{10} = \text{diag} (-0.5034, -0.4963, -0.4454, -0.5156, -0.1077, -0.4812, -0.3351) \) and \( \hat{\Phi}_{11} = \text{diag} (0.4398, -0.1845, 0.1765, -0.0290, -0.1231, 0.1500, 0.3464) \). Using those same value, the modification of GSTAR (1;1) model which inserting its outlier factor is given as

\[
Y_t = (\Phi_0 + \Phi_1 W)Y_{t-1} + e_t + X
\]

where \( X \) is outlier matrix which given by:

\[
X = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \ddots & -57.5024 & \vdots & \vdots \\
0 & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

7.5. Diagnostic Checking
Consider the normality plot of residual which given below. Most of the points lie in normal line, except in fifth location. The normality plot of residual in fifth location is more precise and close to normal line in GSTAR (1;1) model with outlier. This results show that assumption of normal error is satisfied.

Figure 5. Normality plot of residual. The plot which given by GSTAR (1:1) model with outlier, especially in Police Sector 5 show closeness to normal line. (Explanation: x axis:= theoritical quartiles, meanwhile y axis:= sample quartiles)
Consider the autocorrelation function (ACF) of residual which given in Figure 6. below. Note that, most plots are under the significance level of 0.3 and cut-off in lag 0. This means the assumption of uncorrelated and mutual independent error is satisfy in both models.

![Figure 6. Autocorrelation Function (ACF) plot of residual.](image)

(a) GSTAR (1;1)  
(b) GSTAR (1;1) with outlier

7.6. The Best Model
The best model in this case study is determined by using the formula of root mean square error (RMSE). The RMSE for GSTAR (1;1) model is 8.1728 meanwhile for its modification is 7.900. Since the smaller value of RMSE is given by modification of GSTAR (1;1), the best model is GSTAR (1;1) with outlier.

![Figure 7. Result of applied model.](image)

(a) GSTAR (1;1)  
(b) GSTAR (1;1) with outlier

The blue line describes real data meanwhile The red line describes the model. It is clearly seen the best approximation reached by GSTAR (1;1) model with outlier.
Another interesting fact is given in Figure 7. Although we only have small difference of RMSE between GSTAR (1;1) model and its modification, but how the model may capture the data in Police Sector 5 in the third data is good. For the next research, it is a need to check the characteristics of GSTAR model with outlier. In this case study we do not execute forecasting stage since Additive Outlier (AO) will not give impact in the future time. This becomes something interesting to be checked later.

8. Conclusion
Based on the result and discussion about begal in Medan, North Sumatera above, the smaller value of RMSE is given by GSTAR (1;1) model with outlier with value 8.1728, meanwhile GSTAR (1;1) has RMSE 7.900. So, the best model in this case study is GSTAR (1;1) with outlier. This invention shows that space-time model with outlier is able to get precise result rather than use space time model only.

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