Field-Shift Aging Protocol on the 3D Ising Spin-Glass Model: Dynamical Crossover between the Spin-Glass and Paramagnetic States

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Spin-glass (SG) states of the 3-dimensional Ising Edwards-Anderson model under a static magnetic field $h$ are examined by means of the standard Monte Carlo simulation on the field-shift aging protocol at temperature $T$. For each process with $(T; t_w, h)$, $t_w$ being the waiting time before the field is switched on, we extract the dynamical crossover time, $t_{cr}(T; t_w, h)$.

We have found a nice scaling relation between the two characteristic length scales which are properly determined from $t_{cr}$ and $t_w$ and then are normalized by the static field crossover length introduced in the SG droplet theory. This scaling behavior implies the instability of the SG phase in the equilibrium limit even under an infinitesimal $h$. In comparison with this numerical result the field effect on real spin glasses is also discussed.

KEYWORDS: spin glass, aging phenomena, droplet picture, Ising EA model, Monte Carlo simulation

In spite of extensive studies in more than a decade the nature of ordering in spin glasses have remained unsettled. A most notable problem is on the stability of the equilibrium spin-glass (SG) phase in a static field $h$. The mean-field theory predicts its stability up to a certain critical magnitude of $h$, while by the droplet theory the SG phase is unstable even in an infinitesimal $h$. In our opinion, a fundamental difficulty of the SG studies lies on the following fact: spin dynamics governed by thermally activated processes is so slow that the time window of experiments on real spin glasses, not to mention that of numerical simulations, is insufficient for us to distinguish whether a SG-state we are observing is in an asymptotic regime close to equilibrium or far from it. In such circumstances, a possible strategy is a literal numerical experiment on nonequilibrium dynamics of model spin glasses without referring to their equilibrium properties which have been extensively looked for by some ingenious, but artificial methods. Following this strategy, we have been extensively studying aging phenomena in the three-dimensional (3D) Ising Edwards-Anderson (EA) model including the comparison of the results with those measured in real spin glasses.

In the present work we address the problem of the SG-phase stability in a field by the strategy mentioned above, concentrating on the following field-shift ($h$-shift) aging protocol. A system is instantaneously quenched at $t = 0$ from $T = \infty$ to a temperature $T$ below the transition temperature $T_c$, it is aged in zero field until $t = t_w$ when $h$ is switched on, and the induced magnetization, called zero-field-cooled (ZFC) magnetization and denoted as $M_{ZFC}(t')$, is measured as a function of $t' = t - t_w$. At $t < t_w$ isothermal and isobaric aging under $h = 0$ proceeds and the SG short-range order, or the mean size of SG domains, $R_T(t)$, grows as,

$$R_T(t) = b_T t^{1/z(T)},$$

with $b_T$ and $z(T)$ being constants. The characteristic time scale after the $h$-shift most frequently examined is the one where the logarithmic time derivative of $M_{ZFC}(t')$, i.e., $S(t') = \partial M_{ZFC}(t')/\partial \ln t'$ exhibits a peak. This behavior of $S(t')$ is interpreted to represent a crossover from the isobaric aging state under $h = 0$ to that under $h > 0$. The peak position of $S(t')$ is called here the crossover time and denoted as $t_{cr}$, though it has been frequently called the effective waiting time. It is a function of $T$, $t_w$ and $h$. We then expect that in the time range $t' \lesssim t_{cr}$, which we call the transient regime of $h$-shift processes, SG subdomains in local equilibrium with respect to $h > 0$ grow within each SG domains which were in local equilibrium with respect to $h = 0$ at $t = t_w$. Furthermore we intuitively introduce the growth law of mean size of the subdomains in the transient regime as

$$R_{T_0}(t') = b_T t'^{1/z(T)} \exp(-a_T h^2) \quad (t' \lesssim t_{cr}).$$

Compared with the growth law of Eq.(1), the second term with $a_T$ is added in the exponent.

The last step of our analysis is to relate the two characteristic length scales above-introduced, $R_w \equiv R_T(t = t_w)$ and $R_{cr} \equiv R_T(t' = t_{cr})$, to the field crossover length, $L_h$ introduced in the droplet theory. The latter separates characteristic behavior of droplet excitations in equilibrium by their size $L$, such that it is dominated by the Zeeman energy ($\sim hL^{d/2}$) for $L > L_h$ and by the SG free energy gap ($\sim \gamma L^d$) for $L < L_h$. Here $d$ is the spatial dimension, $\gamma$ the stiffness constant of domain walls, and $\theta$ the gap exponent. Explicitly $L_h$ is written as

$$L_h = L T h^{-\delta},$$

where $\delta = (d/2 - \theta)^{-1}$ and $T$ is a constant. Surprisingly it is found that, with a proper choice of the values of $a_T$, the scaled plot $R_{cr}/L_h$ vs. $R_w/L_h$ of all the data obtained for different $T$, $t_w$ and $h$ lie on a single curve. The consequence of this scaling is clear; the $h$-shift aging process is nothing but a dynamical crossover from the SG state in $h = 0$ to the paramagnetic state in $h > 0$, or, the SG phase is unstable under $h > 0$ in the equilibrium...
limit. Furthermore, when the simulated results are simply extended to the time window of the experiment on a real Ising spin glass Fe0.5Mn0.5TiO3 our picture of the dynamical crossover turns out to be consistent with the experimental data which were interpreted as an evidence of the AT phase transition predicted by the mean-field theory.\textsuperscript{16}

We carry out standard heat-bath Monte Carlo (MC) simulations on the 3D Ising EA model with interactions obeying a Gaussian distribution with mean zero and variance $J$ which plays a role of units of energy and temperature (with $k_B = 1$). The strength of field $h$ is represented by its associated Zeeman energy. The time is measured in unit of the MC steps per spin (mcs). In the present work we use the values of parameters $b_T$ and $z(T)$ in Eq.(1) obtained previously;\textsuperscript{9} $(T; b_T, z(T)) = (0.4; 0.82, 14.8), (0.5; 0.80, 11.8), (0.6; 0.78, 9.8), (0.7; 0.78, 8.7)$ and $(0.8; 0.76, 7.9)$. Note that $T_c \simeq 0.95\textsuperscript{17,18}$ for this SG model. In the temperature range $T \leq 0.8$ and in the time range $t \leq 10^5$ where we carry out simulations, $R_T(t)$ is less than a few lattice distances.\textsuperscript{9} Therefore we examine systems with a linear size of 24, but an average over at least 6400 samples with different sets of interactions is taken for evaluating physical quantities in each $h$-shift process discussed below.

Let us begin our discussions with the typical results of $M_{ZFC}(t')$ and $M_{FC}(t')$, the field-cooled (FC) magnetization defined by $M_{ZFC}(t')$ with $t_w = 0$, at $T = 0.6$ shown in Fig. 1. Both exhibit a jump at $t' = 1$, which is naturally attributed to one-spin flips of individual spins whose internal field at $t' = 0$ is smaller than and opposite to $h$. The number of such spins decreases in the ZFC process at $t \leq t_w$ and so $M_{ZFC}(1) > M_{ZFC}(1)$. After the jump the system ages to an isobaric aging state under $h$ gradually and irreversibly. Associated with this, subdomains in local equilibrium with respect to $h$ are expected to grow obeying Eq.(2), though we have not succeeded to observe them directly in simulation yet. Also shown in the figure is $M(t')$ which is observed in $h = 0.75$ after shift-down from the FC process with $h = 1.0$ of a period $t_w$. One can see that the magnetizations under $h = 0.75 (1.0)$ at $t' \geq 10^6 (10^4)$ are independent of the history of the system at $t' \leq 0$, i.e., the system reaches the equilibrium paramagnetic state at this time range. For a small $h$ such as $h = 0.1$, on the other hand, we cannot see even a precursor of saturation in $M_{ZFC}(t')$. Our interest is therefore in behavior of $M_{ZFC}(t')$ for $0.1 \leq h \leq 0.75$.

In Fig. 2 we demonstrate a set of $S(t')$ observed in the $h$-shift processes at $T = 0.6$ for various $h$ but with a fixed $t_w$. Except for $h = 2.0$, $S(t')$ exhibits a peak at $t' = t_{cr}$. All $t_{cr}$ obtained for various $h$ and $t_w$ are shown in the inset of Fig. 3 below. For smaller fields, $h \leq 0.1$, we obtain $t_{cr} \simeq t_w$, as has been observed since the very beginning of the SG aging study.\textsuperscript{19} For larger $h$, however, $t_{cr}$ becomes significantly smaller than $t_w$.

As noted at the beginning of this letter, a key quantity in the droplet theory on the SG aging in a field is the field crossover length, $L_h$, and its role on the dynamical crossover of present interest becomes apparent if we compare it with the characteristic length scales, $R_{cr}$ and $R_w$, of each $h$-shift process. As already defined, the latter are a function of $T$, $t_w$ and $h$. In processes with such a large $h$ or/and $t_w$ that $R_w \gg L_h$ holds, the aging dynamics after the $h$-shift is dominated by the Zeeman energy and so $R_{cr} \simeq L_h$ is expected irrespectively of $t_w$. In opposite processes with $R_w \ll L_h$, we expect $R_{cr} \simeq R_w$. These qualitative features among $R_{cr}$, $R_w$, and $L_h$ are made more transparent by the scaling plots of $R_{cr}/L_h$ vs $R_w/L_h$. Even if we simply put $R_{cr}(t') = R_T(t')$,\textsuperscript{20} i.e., $a_T = 0$ in Eq.(2), we obtain the scaling plot, from which we can see the tendency of saturation of $R_{cr}/L_h$ at large $R_w/L_h$.

More interestingly, as shown in Fig. 3, all the sets of $(R_{cr}/L_h, R_w/L_h)$ obtained in the $h$-shift processes at different temperatures, $T = 0.4, 0.5, 0.6, 0.7, 0.8$, and $0.9$, with different sets of $t_w$ and $h$, lie on a single curve, if the values of $a_T$ and $l_T$ are properly adjusted as $(T; a_T, l_T) = (0.4; 0.06, 0.85), (0.5; 0.075, 0.83), (0.6; 0.09, 0.81), (0.7; 0.115, 0.78)$ and $(0.8; 0.125, 0.75)$. We emphasize here that the values of the parameters other than $a_T$ and $l_T$ are those previously determined independently of the

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**Fig. 1.** $M_{FC}(t)$ ($t' = t$), $M_{ZFC}(t')$ with $t_w = 4096$, and $M(t')$ under $h = 0.75$ after the shift from $h = 1.0$ ($t_w = 4096$).

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**Fig. 2.** $S(t')$ for the $h$-shift processes at $T = 0.6$ with $t_w = 4096$ plotted vs $\ln t'$. The data are for $h = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.75, 1.0$ and $2.0$ from right to left.
present analysis; $\theta = 0.20, 0.21$ and so $\delta \sim 0.77$ in Eq. (3) with $d = 3$, and $(b_T, 1/z(T))$ already listed above. The adjustment of $a_T$ is essential to obtain a unique scaling curve for the data set at each temperature. The proper choice of $l_T$, which commonly scales both axes at each temperature, makes the scaling curves at different temperatures to lie on top of each other. Physically, the main $T$-dependence of $l_T$ can be attributed to that of the stiffness constant $\Upsilon$ which is expected to vanish at $T_c$.

The scaling behavior we have found implies that, for $h$ of any strength, if a system is aged under $h = 0$ up to $t_w$ for which $R_w \sim L_h$ is satisfied, the aging dynamics after the $h$-shift is governed dominantly by the Zeeman energy, i.e., the system exhibits paramagnetic behavior at $t' > t_{cr}$. If $t_w$ is smaller than such a value, say $t_w = 0$, it is faster for the system to reach the paramagnetic state as demonstrated in Fig. 1.

To reach the nice scaling shown in Fig. 3, the term intuitively added in the exponent in Eq.(2) plays an essential role, though the scaling behavior is not so sensitive on the precise value of $a_T$, i.e., even if it is changed by $\pm 0.01$ from the value listed above, the quality of the scaling does not become significantly worse. A positive $a_T$ implies that the subdomains in the transient regime grow more slowly than the domains do in the isobaric aging, i.e., $R_w(t = t'') < R_T(t)$. We consider it quite plausible that the domain growth in $h$ starting from a state already aged in $h' \neq h$ (including $h' = 0$) is slower than that starting from a state not aged at all. In this context it is worth noting that Joh et al. explained their experimental result $t_{cr} < t_w$ for relatively large $h$ in terms of the reduction of the barrier energy for excitations due to the Zeeman energy. Numerically we have also observed that $t_{cr} < t_w$, but our interpretation for it is quite different from theirs.

Our MC simulation alone cannot exclude a possible existence of the SG phase under sufficiently small fields ($h \lesssim 0.1$). However, the deviation of $R_{cr}$ from $R_w$ is clearly recognized even at small $x \equiv R_w/L_h$. It originates from the fact that the excitation gaps of droplets with a fixed size distribute continuously and a finite weight exists even at zero energy. This point has been recently emphasized by Jönsson et al.23 who have called the consequence of these excitations the weak-chaos effect. Our data of $R_{cr}/L_h$ vs. $R_w/L_h$ at small, though not sufficiently small, $x$ are consistent with their argument. We may better call this the weak-paramagnetic effect.

One more comment is in order on the out-of-phase component of the ac susceptibilities, $\chi''(\omega; t)$, measured in the $h$-shift aging protocol. It is evaluated via the fluctuation-dissipation theorem and its typical behavior is shown in Fig. 4. In the isobaric regime at $t \leq t_w$, $\chi''(\omega; t)$ is rather insensitive to $h$ as seen in the figure. Just after the $h$-shift, however, it exhibits a sharp jump-up, and subsequently it tends to merge to the one observed in the isobaric aging under $h$ from $t = 0$, which we call the reference curve. Qualitatively similar behavior has been observed experimentally.24 According to the droplet theory, the aging part of $\chi''(\omega; t)$ in the isobaric regime is given by a function of $L_\omega/R_T(t)$, where $L_\omega$ is a mean size of droplets which can respond to the ac field of frequency $\omega$. A naive extension of this idea is that $\chi''(\omega; t)$ after the $h$-shift is dominantly governed by $R_{cr}(t')$, the mean size of subdomains which is smaller than $R_w$. In fact, as demonstrated in Fig. 4, the time scale that the $\chi''(\omega; t)$ merges to the reference curve well coincides with $t_{cr}$, the peak position of $S(t')$. Thus the following picture on the aging dynamics around $t' \sim t_{cr}$ is extracted. At $t' \lesssim t_{cr}$ subdomains of mean size $R_{cr}(t')$ grow up obeying Eq.(2), and at $t' \gtrsim t_{cr}$ the subdomains become main domains in the system. For a process with $R_{cr} \lesssim L_h$ the system is regarded to be already in a paramagnetic state and SG domains do not grow any more. For a process with $R_{cr} < L_h$, on the other hand, the
mean size of SG domains grows following Eq.(1) with a shift of the time origin by a certain amount. This growth is observed through physical quantities, such as $\chi''(\omega; t)$ in Fig. 4, which associate a length scale smaller than the mean size of domains at $t'$. The value of $M_{ZFC}(t')$ at $t' \sim t_{cr}$ is, however, still significantly smaller than $M_{FC}(t')$, and it needs further long time for $M_{ZFC}(t')$ to catch up $M_{FC}(t')$, or the mean size of SG domains reaches $L_h$.

So far described are the results of our simulation on the EA Ising SG model. In order to look for their relevance to phenomena in real Ising spin glasses, let us introduce the dynamical crossover condition defined by $R_w(t_{cr}')/L_h = x_c$ with $x_c$ being a constant nearly equal to unity. It is rewritten as

$$h_{cr} = c_{cr}(t_{cr}'/t_0)^{-1/\delta z(T_{cr})},$$

with $c_T = (x_c T/b_T)^{1/\delta}$ and $t_0$ being the microscopic time unit. This equation gives us an estimate of the value $h_{cr}$, $T_{cr}$ or $t_{cr}'$ when the values of the other two parameters are specified, and when $h \gtrsim (\lesssim) h_{cr}$, for example, the system is regarded as in a paramagnetic (SG) state.

Katori and Ito [16] determined the characteristic temperature, $T_{XT}$, of Ising glass Fe$_0.5$Mn$_0.5$TiO$_3$ under $h$ as the one where the $M_{ZFC}$ starts to deviate from $M_{FC}$ within the observation time of $t_{ob} \simeq 10^2$ s at each temperature. Regarding this as the dynamical crossover with $t_{cr}' = t_{ob}$ and $T_{XT} = T_{cr}$ ($= T$), we evaluate $h_{cr}$ by Eq.(4) with $T_c = 21.5$ K and $t_0 = 10^{-12}$ s. This means that we simply extend the result extracted from our simulation to the time range of $10^{14}$ mcs as we did with success in our previous work. [13] Surprisingly, as shown in Fig. 5, the $h_{cr} - T$ plot thus evaluated with $x_c \simeq 1.5$ coincides very well with the $h \sim T_{XT}$ plot that Ito regard the AT critical line. [2] Similarly, the freezing temperatures $T_f(\omega, h)$ extracted by Mattsson et al. [25] through the ac susceptibility measurement are also satisfactorily reproduced by our analysis with $T_{cr} = T_f$, $t_{w}' = 2\pi/\omega$ and $h_{cr} = h$. These agreements, whose details will be reported elsewhere, are quite promising for us to consider that the field effect also on real spin glasses is well described by our dynamical crossover picture. [20] For $h$ of the magnitude less than 1 Oe, however, $t_{w}'$ becomes astronomically large, which explains, we consider, why $M_{ZFC}/h$ measured in laboratory-time deviate from $M_{FC}/h$, and $M_{FC}/h$ itself does so from the paramagnetic susceptibility at $T \simeq T_c$ for small $h$. [26]

To conclude we have demonstrated that the characteristic length scales associated with $h$-shift aging protocols in the 3D EA Ising SG model exhibit unique scaling behavior. Because of the latter, we have succeeded to reach one of the most important equilibrium properties of Ising spin glasses via simulations on nonequilibrium (aging) dynamics: their SG phase is unstable under a finite field in equilibrium.

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1) See the papers in Spin glasses and random fields, ed., A.P. Young, (World Scientific, Singapore, 1997).
2) J.R.L.de Almada and D.J. Thouless: J. Phys. A 11, (1978) 983.
3) D.S. Fisher and D.A. Huse: Phys. Rev. B 38 (1988) 373.
4) D.S. Fisher and D.A. Huse: Phys. Rev. B 38 (1988) 386.
5) A.J. Bray and M.A. Moore: Phys. Rev. Lett. 58 (1987) 57.
6) K. Hoshakala, J. Houdayer, O.C. Martin and G. Parisi: Phys. Rev. Lett. 87 (2001) 197204.
7) K. Katori and Ito: J. Phys. Soc. Jpn. 68 (1999) 3387.
8) T. Komori, H. Yoshino and H. Takayama: J. Phys. Soc. Jpn. 69 (2000) 1192.
9) T. Komori, H. Yoshino and H. Takayama: J. Phys. Soc. Jpn. 69 Suppl. A (2000) 355.
10) H. Takayama and K. Hoshakala: J. Phys. Soc. Jpn. 71 (2002) 3003.
11) L.W. Bernardi, H. Yoshino, K. Hoshakala, H. Takayama, A. Tobo and A. Ito: Phys. Rev. Lett. 86 (2001) 720.
12) J. Kisker, L. Santen, M. Schreckenberg and H. Rieger: Phys. Rev. B 53 (1996) 6418.
13) E. Marinari, G. Parisi, F. Ricci-Tersenghi and J.J. Ruiz-Lorenzo: J. Phys. A 31 (1998) 2611.
14) H. Aruga Katori and A. Ito: J. Phys. Soc. Jpn. 63 (1994) 3122.
15) E. Marinari, G. Parisi and J.J. Ruiz-Lorenzo: Phys. Rev. B 58 (1998) 14852.
16) P.O. Mari and I.A. Campbell: Phys. Rev. E 59, (1999) 2653.
17) L. Lundgren, P. Svedlindh, P. Nordblad and O. Beckman: Phys. Rev. Lett. 51, (1983) 911.
18) H. Takayama: to appear in J. Magn. Magn. Mater.