Nucleation of squat cracks in rail, calculation of crack initiation angles in three dimensions

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Abstract. A numerical model of wheel-track system is developed for nucleation of squat-type fatigue cracks in rail material. The model is used for estimating the angles of squat cracks in three dimensions. Contact mechanics and multi-axial fatigue analysis are combined to study the crack initiation mechanism in rails. Nonlinear material properties, actual wheel-rail geometries and realistic loading conditions are considered in the modelling process. Using a 3D explicit finite element analysis the transient rolling contact behaviour of wheel on rail is simulated. Employing the critical plane concept, the material points with the largest possibility of crack initiation are determined; based on which, the 3D orientations/angles of the possible squat cracks are estimated. Numerical estimations are compared with sample results of experimental observations on a rail specimen with squat from the site. The findings suggest a proper agreement between results of modelling and experiment. It is observed that squat cracks initiate at an in-plane angle around 13°-22° relative to the rail surface. The initiation angle seen on surface plane is calculated around 29°-48°, while the crack tend to initiate in angles around 25°-31° in the rail cross-section.

1. Introduction
Squat is a type of rolling contact fatigue (RFC) that occurs in the railhead and can develop into the rail fracture, if it is not detected and treated with maintenance measures. Squats are one of the most prevalent forms of RCF defects in the Netherlands. These defects usually appear along with surface and subsurface cracks in the railhead. The squat cracks after initiation, continue to grow during the railway service. A large and growing body of literature has investigated different stages of squat crack life in rails; see for example [1, 2]. Among various stages of a squat crack development, the initial phase is technically a crucial issue that has considerable importance in determining the life time of rail materials. Some researchers [3-5] concentrated on microstructural features related to fatigue crack initiation in rail materials. In this category of works, squat initiation is treated with various laboratory techniques and field studies, mainly to provide information at the micron level and study of the material microstructure. A number of numerical models [6-8] tried to estimate the stress-strain levels inside rail material and to predict RCF initiation, mainly based on finite element method (FEM) and wheel-rail contact mechanics. In spite of a considerable amount of work on crack initiation studies in rails, there has been little discussion about the angles of squat cracks in the initiation phase. Crack initiation angles in rails under rolling contact were predicted in number of studies using 2D models [9-11], which did not take into account the real geometry and loading conditions of the wheel-rail system. In addition, there has been few examples of 3D models [12, 13] for RCF crack initiation; however,
they were not dedicated to rail squats, as they predicted cracks in gauge corner (GC) rolling contact conditions (an indication for another sort of RCF cracks called as head checks). The present study seeks to remedy this lack of knowledge, by developing a 3D model of wheel-track contact using the actual geometries, material properties and contact loading conditions. The nucleation orientations of the squat crack in rail are presented using the results of numerical simulations. Moreover, some experimental observations, carried out on an example of rail with squat are presented to assess the effectiveness of the numerical predictions.

Figure 1 gives various phases (A to F) of development of a squat crack in the railhead based on the observations on the majority of the squat cracks [14, 15]. Using this figure, stage A (shown with the dashed-line rectangle) is considered as the objective phase of crack initiation in the present study.

**Figure 1.** Searching for the critical plane with the maximum $FP$ in the first critical point.

2. **Numerical modelling of squat crack initiation**

A wide range of FE modelling approaches is available for wheel-rail contact problem. Considering the effect of vehicle-track dynamic interaction on RCF phenomenon, a 3D model is developed in this research to study the initiation process of squat cracks. As shown in Figure 2, one loading cycle of a single wheel component, running at limited length of a straight railway track is modelled. The general methodology of the FE simulation and the process of wheel-rail contact are taken from [16, 17].

**Figure 2.** (a) The 3D FE model of the wheel-track system, (b) magnification on rail surface in the solution zone with the detailed meshing style.

2.1. **Multi-axial fatigue criteria of crack initiation**

A multi-axial fatigue model of the rail material under rolling contact is considered in the present study based on [18]. The fatigue parameter in this model is expressed by:
\[ FP_{\text{max}} = \left( \langle \sigma_{\text{max}} \rangle \frac{\Delta \varepsilon}{2} + J \Delta \tau \Delta \gamma \right)_{\text{max}} \]  

where in each rolling cycle, \( \Delta \varepsilon \) is the normal strain range, \( \sigma_{\text{max}} \) is the maximum normal stress, \( \Delta \tau \) is the shear strain range, \( \Delta \gamma \) is the shear stress range, \( J \) is a material-dependent constant and \( \langle \rangle \) denotes the McCauley bracket \( \langle x \rangle = (|x| + x)/2 \). The constant \( J \) is obtained from tension/torsion tests. This model considers the influence of normal and shear loadings on damage occurrence. The first term of (1) considers the mean stress effect in normal direction, while the second term incorporates the shear stress-strain effects. The proposed multi-axial fatigue model has the capability of capturing the synergism between the shear and normal stress components. Stress and strain components are the integral parts of the multi-axial fatigue analysis. The histories of loads are obtained from the results of FE analysis. This fatigue criterion has been recently used in [19, 20], where the crack initiation life of rail material was governed by the cyclic fatigue, rather than the ratchetting failure. Therefore, the ratchetting-based criterion (see for instance [21]) is not used in this research.

2.2. Critical plane approach

All the stress and strain quantities of (1) need to be obtained at the critical plane where the largest fatigue parameter \( (FP_{\text{max}}) \) occurs. Critical plane approach incorporates the effects of all stress components in three dimensions to find out the angles situation that produces the maximum possibility of fatigue damage. Through a tensor rotation for the stress and strain, the maximum fatigue parameter and the critical plane angles are determined by surveying all the possible planes at a material point. In tensor analysis, various mathematical formulations exist to express the stress components in three dimensions, when the coordinate system is subjected to rotations. In the present research, the spatial rotations of the stress matrices are carried out by the application of Euler angles and mathematical transformations. The 3D transformation matrix can be viewed as a series of three successive rotations about coordinate axes. As shown in Figure 3(a), the first rotation occurs with the angle \( \alpha \) about the z-axis and then the second rotation of angle \( \beta \) takes place about the new x-axis (which has already rotated itself due to the first rotation). Finally, the last rotation of \( \gamma \) happens about the new-tilted y-axis to generate the new-rotated coordinate system \( x'y'z' \).

Figure 3. (a) Original and rotated coordinate systems, demonstration of the Euler angles, (b) transforms of stress components.

When a Cartesian coordinate system \( xyz \) (the original state, in which the stresses are obtained) is subjected to three sequential rotations demonstrated in Figure 3, the transformation matrix of the stress tensor can be related to the Euler angles \( \alpha, \beta, \gamma \):
\[\sigma = \begin{pmatrix} \tau_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \tau_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \tau_z \end{pmatrix}, \quad \sigma' = \begin{pmatrix} \tau_{x'} & \tau'_{xy} & \tau'_{xz} \\ \tau'_{xy} & \tau'_{y'} & \tau'_{yz} \\ \tau'_{xz} & \tau'_{yz} & \tau'_z \end{pmatrix}\]  
\[\sigma' = Q \cdot \sigma \cdot Q^T, \quad \sigma'_{im} = \lambda_{m - \sigma} \cdot \sigma_g, \quad \lambda = \cos(x', x)\]  
\[\sigma' = \begin{pmatrix} \cos(x', x) & \cos(x', y) & \cos(x', z) \\ \cos(y', x) & \cos(y', y) & \cos(y', z) \\ \cos(z', x) & \cos(z', y) & \cos(z', z) \end{pmatrix} \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix} \begin{pmatrix} \cos(x', x) & \cos(y', x) & \cos(z', x) \\ \cos(x', y) & \cos(y', y) & \cos(z', y) \\ \cos(x', z) & \cos(y', z) & \cos(z', z) \end{pmatrix}\]  
\[Q = Q_x Q_y Q_z = \begin{pmatrix} \cos \beta \cos \alpha & \cos \gamma \cos \alpha + \sin \beta \cos \gamma \sin \alpha & \sin \gamma \sin \alpha - \cos \gamma \sin \beta \cos \alpha \\ \sin \gamma \cos \alpha - \sin \beta \sin \gamma \sin \alpha & \cos \gamma \cos \alpha & \sin \gamma \sin \beta \cos \alpha \\ -\sin \gamma \cos \beta & -\sin \gamma \sin \beta \cos \alpha & \cos \beta \cos \alpha \end{pmatrix}\]

2.3. Combining fatigue analysis and critical plane approach

Based on the prescribed formulations, there must be dozens of variations with respect to the spatial angles, since any combination of axes can be considered to perform stress transformation. To determine the location and angles of crack in rail, the material points that are experiencing the largest von Mises stress (criterion for determining the critical damage point) under wheel passage are determined. Then the critical plane approach is employed to determine the angles in which the maximum fatigue parameter is achieved (multi-axial fatigue criterion). These angles provide the situation by which the rail is highly susceptible to crack initiation. Finally, the critical angles are represented by in-plane angles of crack initiation for rail material. As shown in Figure 4(a), there are two forms of demonstrations available for crack plane angles. The in-plane angles \(\theta, \phi, \psi\) are used in the present study to show the orientations of crack initiation. This angles are given in Figure 4(b) with details of traffic direction, gauge corner side and axes definitions.

![Figure 4](image-url)  
**Figure 4.** (a) Two forms of angle definitions for a crack plane, (b) definition of angles in rail for the present research.

2.4. Material and loading parameters

The consecutive model of the rail material in the contact interface are defined by the combined nonlinear isotropic and kinematic hardening model, which is developed by Lemaitre and Chaboche [22, 23]. This model can be used for simulation of cyclic plasticity and ratchetting material response with decaying ratchetting rate [11]. The yield stress in this model is defined by a Von Mises yield surface, where the yield function \(\phi\) is expressed by:
\[ \phi(\sigma, X, K) = \sqrt{\frac{3}{2}} |\tau_{\text{dev}}| - K - \sigma_y \]
\[ \tau_{\text{dev}} = \sigma_{\text{dev}} - X, \quad |\tau_{\text{dev}}| = \sqrt{\tau_{\text{dev}} : \tau_{\text{dev}}} \]  
(6)

Where \( \sigma \) is the Cauchy stress tensor, \( X \) is the backstress tensor, \( K \) is the drag stress, \( \sigma_y \) is the yield stress, \( \tau \) is the deviator stress tensor, \( \sigma_{\text{dev}} \) is the deviator stress tensor of \( \sigma \), and the operator \( " : " \) defines the contraction \( a : b = a_{ij} b_{ij} \). The material is elastic when \( \phi(\sigma, X, K) < 0 \) and plastic, when \( \phi(\sigma, X, K) = 0 \). The isotropic hardening law \( (K) \) and \( X \) tensors are defined by:

\[ \dot{K} = \dot{\mathcal{B}}\left(1 - \frac{K}{K_\infty}\right), \quad \dot{X} = \dot{\mathcal{K}} \mathcal{C} \left(\frac{2}{3}n_{\text{dev}} - \gamma X\right), \quad n_{\text{dev}} = \frac{|\tau_{\text{dev}}|}{\tau_{\text{dev}}} \]

(7)

where \( \dot{b} \) is the initial rate of the isotropic hardening, \( \dot{\mathcal{B}} \) is the plastic multiplier, \( K \) is the drag stress, and \( K_\infty \) is the saturated drag stress due to isotropic hardening. The kinematic hardening law follows the nonlinear Armstrong and Frederick law, where \( \mathcal{C} \) and \( \gamma \) are material parameters. The rail steel that is generally being used in the Netherlands is the typical pearlitic steel. The material constants that are used in the failure models can be obtained by normal fatigue tests like tension-compression and torsion fatigue tests. In the present study, however, these constants are taken from [24]. The material and load dependent constant \( J \) was set to 0.2, based on results in the literature [18]. The plasticity parameters of the material model used in the current study are extracted from [11]. The parameters of the FE model in the present research are listed in Table 1. This table also gives the properties of the pearlitic rail steel employed in this work.

**Table 1: Mechanical/loading parameters of the wheel-track system and rail material**

| Parameters of the model and linear materials | Values | Pearlitic rail steel (nonlinear) | Values |
|---------------------------------------------|--------|---------------------------------|--------|
| Static wheel load, \( P_0 \) (kN)          | 120    | \( \sigma_y \) (MPa)           | 456    |
| Wheel weight (kg)                           | 900    | \( k_e \) (MPa)                | 234    |
| Sleeper mass \( M_s \) (kg)                | 280    | \( E \) (GPa)                 | 210    |
| Stiffness of ballast, \( K_b \) (kN/m)     | 45000  | \( \nu \)                      | 0.3    |
| Damping of ballast, \( C_b \) (N.s/m)      | 32000  | \( \varepsilon' \) (%)         | 10.3   |
| Stiffness of rail pad, \( K_p \) (kN/m)    | 130000 | \( \sigma_f' \) (MPa)          | 936    |
| Damping of rail pad, \( C_p \) (N.s/m)     | 45000  | \( \gamma_f' \) (%)           | 15.45  |
| Stiffness of primary suspension, \( K_{ps} \) (kN/m) | 800 | \( \tau_f' \) (MPa) | 468 |
| Damping of primary suspension, \( C_{ps} \) (N.s/m) | 4000 | \( b \) (MPa) | 10.7 |
| Young’s modulus of wheel-rail, \( E_r \) (GP) | 210 | \( c \) (GPa) | 6.49 |
| Poisson’s ratio of wheel-rail material, \( \nu_r \) | 0.3 | \( \frac{2}{\nu_f} \) | 14.4 |
| Density of wheel-rail material, \( \rho_w \) (kg/m3) | 7800 | \( K_w \) (MPa) | 22.8 |
| Young’s modulus of concrete, \( E_c \) (GP) | 38.4 | \( \varepsilon_c \) (%) | 11.5 |
| Poisson’s ratio of concrete material, \( \nu_c \) | 0.2 | \( b \) | -0.089 |
| Density of sleeper material, \( \rho_s \) (kg/m3) | 2520 | \( c \) | -0.559 |
| Rolling speed (km/h)                        | 140    |                                 |        |
| Friction coefficient of wheel-rail contact  | 0.6    |                                 |        |
| Traction coefficient (ratio of tangential load) | 0.25 |                                 |        |

3. Results of numerical investigations

The histories of stresses and strains under rolling contact were computed from FE analysis. The rail material points with the largest von Mises stresses were determined as the critical damage points. The fatigue parameters (FP) of these points were calculated in a range of 3D transformations. The results
of FP calculations for six critical points (the first six points with largest von Mises stress) are given in Figure 5,6, with respect to angles variations. The angles in these figures are the ones defined in Figure 4(b). The dimensionless FP values were obtained by normalizing fatigue parameters over the peak FP in the entire loading cycle. It should be noted that for the search of the critical plane, only two of three angles need to be calculated. The third angle is dependent open the first two angles by spherical law of cosines.

![Figure 5](image5.png)

**Figure 5.** Searching for the critical plane with the maximum FP in material points P1-P3.

![Figure 6](image6.png)

**Figure 6.** Searching for the critical plane with the maximum FP in material points P4-P6.

Based on these results, angles of crack initiation were estimated and listed in Table 2. Three approximate limits are given in the last column of the table, as tentative ranges for the angles. Note that, from the proposed numerical calculation it is not possible to expect exact values for the angles; however suggesting the obtained ranges seem to be rational.

| Critical point | P1 | P2 | P3 | P4 | P5 | P6 | Range     |
|---------------|----|----|----|----|----|----|-----------|
| Angle θ       | 19 | 20 | 18 | 22 | 15 | 13 | 13-22     |
| Angle ϕ       | 31 | 29 | 48 | 42 | 36 | 34 | 29-48     |
| Angle ψ       | 25 | 31 | 30 | 25 | 29 | 28 | 25-31     |

### 4. Experimental observations on a sample squat

So far, orientations of the initiated cracks were numerically studied. To assess the effectiveness of the proposed model, a rail sample with relatively severe squat (squat class B according to [25]) is taken from the site. The overall view of the rail sample with squat is demonstrated in Figure 7(a), representing the local plastic deformation of the surface and a typical lung-shape pattern. Traffic direction was from left to right and the rail was taken from the left side of a straight track between Meppel and Steenwijk in the Netherlands. The chosen rail is an R260Mn type Dutch rail made of steel with a chemical composition of 0.64%C, 1.51%Mn, 0.25%Si, 0.02%P and 0.02%S (mass %). The rail was manufactured in 1989 and was loaded with an annual gross tonnage of 4.1 million tons. Experimental observations were made to assess the locations and orientations of the squat cracks for the chosen rail. Owing to this, a cubic sample was cut from the rail top in the area by which the squat cracks can be relatively captured. Figure 7(b, c) gives the location of the selected cube on rail surface.
The sectioning pattern and dimensions of the cutting samples are also shown in the figure. The size of the cube was 14 mm × 15 mm in the longitudinal and lateral directions, respectively. The cube was 10 mm deep in the vertical direction. According to Figure 7, the cube was divided into seven thin samples with 15*10*2 mm dimensions. This was done to reproduce the shape/geometry of the cracks with serial sectioning. Using this method, the actual geometry of cracks was observed. As demonstrated in Figure 7, direction of cutting was perpendicular to the traffic direction and the interval length between sections was 2 mm. Therefore, seven slices were obtained in total, namely S1-S7.

Figure 7. (a) The rail sample with relatively severe squat, (b) rail sectioning pattern, dimensions of cutting samples, (c) sliced samples after the serial cutting process.

All rail slices were cut and the faces of the cuts were ground and roughly polished. After sample preparation, the microstructure of the rail was observed with optical microscopes in transverse directions. The geometries of squat cracks in all sequential sections were traced. The microstructures of the sections with cracks were mapped with 2D images including details of crack tip locations, angles, sizes and depth of propagation. Figure 8 shows the microstructure of the cracked rail in sections S1, S3, S5 as an example of such observations.

Figure 8. Microstructure of the cracked rail in sections S1, S3, S5, ruler indentations are in mm.

Geometries of the cracks in 2D images were incorporated to generate the 3D configurations of the squat cracks. Figure 9 gives the 3D representation of the cracks, constructed using the prescribed method. This figure shows the full morphology of the cracks in 3 dimensions.

A survey was done about locations and geometries of squat cracks on the rail surface. Based on the 3D geometry of cracks in Figure 8, the intersection lines (boundary curves) of the crack with the rail surface were determined as shown in Figure 10. This figure shows the geometric shape of squat cracks from the top view. Note that two different views of the same sample are shown in Figure 10(a, b). According to these observations, there were two separate lines (crack mouths) on rail surface. Each crack mouth had a turning point somewhere in the middle of the curve. The turning points (marked
with 1 and 2 in Figure 10) can be possible points of crack initiation at rail surface. By the way, the experimental observations cannot confirm that the cracks are initiated at such turning points.

![Figure 9. 3D views of the cracked rail; vectors at cube corners show the rolling direction.](image)

Based on results of numerical simulations in this paper, the critical damage points were located at the rail surface. Therefore, it can be hypothesized that the crack initiation point is located somewhere in the surface (along the cracks boundaries in Figure 10, but not necessarily on turning points). Based on this hypothesis, four sides of crack faces (C1-C4) are defined in Figure 10 as the possible faces for crack initiation. Any of two crack mouths on rail surface was divided into two individual curves to generate such faces. Now, it can be hypothesized that the squat is initiated from each of the C1-C4 faces, but still at the surface.
Considering C1-C4 as the crack faces carrying the crack initiation point, it is possible to determine the angles of the initiated crack. By mapping each of these faces on different planes of the Cartesian coordinate system, the angles of crack initiations were calculated. The results of angle measurements are shown in Figure 11, in which the orientation angles of each individual face were separately demonstrated. The corresponding angles of crack initiation for C1-C4 crack faces are listed in Table 3. Note that all angles were measured from 0 to 180° with respect to the original axes in Figure 3.

Comparing the results in tables 2 and 3, the crack angles from numerical studies relatively matches with the results of C1 crack face in experimental investigation. This means that among the four crack faces, only one face offers initiation angles, having proper agreement with the numerical predictions. This evidence suggests that the C1 face is the possible face, in which the squat crack has initiated in the rail sample. Although the squat in this sample was a kind of severe squat with well-developed cracks, only situations of cracks at locations very close to the surface was studied. Therefore, it is possible to consider such results for the squat at its initiation phase. By the way, it was assumed that the initial situation of crack angles has not been changed during rail service (after crack initiation). Looking at results of $\theta$, $\varphi$, $\psi$ in table 3, it can be seen that the values of $\theta$ and $\psi$ are in the range that is suggested in Table 2 ($\theta$: 14°-23° and $\psi$: 29°-52°). However, experimental observation on the chosen squat reported a value of 25.7° for $\varphi$, which is not in the range 29°-52° suggested by the numerical work (the experimental value of $\varphi$ was slightly lower than numerical one). This is mainly due to the

Figure 10. Intersection lines of cracks with rail surface and crack meeting points on rail, (a,b) 3D views, (c) surface view with nomination of crack-surface meeting points and cube dimensions.
fact that, many sorts of uncertainties may play a role in the complex orientation of the cracks that cannot be seen in the numerical models. Apart from this discrepancy, the findings of experimental observations appropriately supported the numerical predictions.

Figure 11. Crack initiation angles in different 2D views for C1-C4 crack faces

| Crack mouth | $\theta$   | $\phi$   | $\psi$   |
|-------------|------------|----------|----------|
| C1          | 19.20      | 25.70    | 19.40    |
| C2          | 159.30     | 139.80   | 22.90    |
| C3          | 16.90      | 21.10    | 161.10   |
| C4          | 169.50     | 157.20   | 168.90   |

5. Conclusions
This paper presented the results of crack initiation angles in rail squats using both numerical investigations and experimental observations. The 3D finite element model of the wheel-track system was built, capable of simulating rolling contact conditions. Applying the multi-axial fatigue analysis and critical plane approach, the material points and angles that are mostly susceptible to fatigue damage were determined. A rail specimen with squat taken from the site was cut with serial sectioning and the shape/geometry of the cracks was reproduced. Comparing the results of experimental work with numerical predictions, a positive correlation was observed. Based on results of analyses, it can be concluded that squat cracks initiate at an angle $13^\circ$-$22^\circ$, relative to the rail surface ($\theta$). The initiation angle seen on surface plane ($\phi$) was around $29^\circ$-$48^\circ$. Looking at rail cross-section, the angle of crack initiation toward rail surface ($\psi$) was in the range of $25^\circ$-$31^\circ$. Although the chosen squat in the experimental work was relatively severe with well-developed cracks, it was possible to trace the initiation point and angles of the squat crack, using the prescribed method. As an attempt to quantify the angles of initiated fatigue crack in rails, this study contributed to ascertain the mechanism by which rail squats are generated.

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Corrigendum: Nucleation of squat cracks in rail, calculation of crack initiation angles in three dimensions

The following are the corrected versions of equation 7 and figures 9 and 10:

\[
\dot{K} = \dot{\lambda} h \left(1 - \frac{K}{K_x}\right), \quad \dot{X} = \dot{\lambda} c \left(\sqrt{\frac{2}{3}} n_{\text{dev}} - \frac{2}{3} X\right), \quad n_{\text{dev}} = \frac{\tau_{\text{dev}}}{|\tau_{\text{dev}}|}
\]  

(7)

Figure 9. 3D views of the cracked rail; vectors at cube corners show the rolling direction.
Figure 10. Intersection lines of cracks with rail surface and crack meeting points on rail. (a,b) 3D views, (c) surface view with nomination of crack-surface meeting points and cube dimensions.