P-wave bottom baryons of the SU(3) flavor $6_F$

Hui-Min Yang$^1$ and Hua-Xing Chen$^{1,2,*}$

$^1$School of Physics, Beihang University, Beijing 100191, China
$^2$School of Physics, Southeast University, Nanjing 210094, China

We investigate P-wave bottom baryons of the SU(3) flavor $6_F$, and systematically study their D-wave decays into ground-state bottom baryons and pseudoscalar mesons. Together with Refs. [1–4], a rather complete study is performed on both mass spectra and decay properties of P-wave bottom baryons, using the method of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory. Among all the possibilities, we find four $\Sigma_b$, four $\Xi_b$, and six $\Omega_b$ baryons, with limited widths and so capable of being observed. Their masses, mass splittings within the same multiplets, and decay properties are extracted (summarized in Table VI) for future experimental searchings.

Keywords: heavy baryon, bottom baryon, heavy quark effective theory, QCD sum rules, light-cone sum rules

I. INTRODUCTION

The strong interaction holds quarks and gluons together inside a single hadron. It is similar to the electromagnetic interaction in some aspects, which holds electrons and protons together inside a single atom. The latter leads to the well-known fine structure of line spectra, and it is interesting to investigate whether the former also leads to some fine structure of hadron spectra [5–8]. An ideal platform to study this is the singly bottom baryon system [9–12]: light quarks together with gluons circle around the nearly static bottom quark, and the whole system behaves as the QCD analogue of the hydrogen.

In recent years important experimental progresses have been made in the field of singly bottom baryons. Until three years ago there were only two excited bottom baryons, $\Lambda_b(5912)^0$ and $\Lambda_b(5920)^0$, which were observed by LHCb and CDF in 2012 [13, 14]. However, in the past three years the LHCb and CMS Collaborations discovered as many as nine excited bottom baryons:

- In 2018 the LHCb Collaboration observed the $\Sigma_b(6097)^\pm$ in the $\Lambda_b^0\pi^\pm$ invariant mass spectrum, and the $\Xi_b(6227)^-$ in the $\Lambda_b^0K^-$ and $\Xi_b^0\pi^-$ invariant mass spectra [15, 16]:

  \[
  \Sigma_b(6227)^- : M = 6226.9 \pm 2.0 \pm 0.3 \pm 0.2 \text{ MeV},
  \Gamma = 18.1 \pm 5.4 \pm 1.8 \text{ MeV},
  \]

  \[
  \Sigma_b(6097)^+ : M = 6095.8 \pm 1.7 \pm 0.4 \text{ MeV},
  \Gamma = 31 \pm 5.5 \pm 0.7 \text{ MeV},
  \]

- In 2019 the LHCb Collaboration observed the $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$, and $\Omega_b(6350)^-$ in the $\Xi_b^0K^-$ invariant mass spectrum [17]:

  \[
  \Omega_b(6316)^- : M = 6315.64 \pm 0.31 \pm 0.07 \pm 0.50 \text{ MeV},
  \Gamma < 2.8 \text{ MeV},
  \]

  \[
  \Omega_b(6330)^- : M = 6330.30 \pm 0.28 \pm 0.07 \pm 0.50 \text{ MeV},
  \Gamma < 3.1 \text{ MeV},
  \]

  \[
  \Omega_b(6340)^- : M = 6339.71 \pm 0.26 \pm 0.05 \pm 0.50 \text{ MeV},
  \Gamma < 1.5 \text{ MeV},
  \]

  \[
  \Omega_b(6350)^- : M = 6349.88 \pm 0.35 \pm 0.05 \pm 0.50 \text{ MeV},
  \Gamma = 1.4^{+1.0}_{-0.9} \pm 0.1 \text{ MeV}.
  \]

- In 2020 the LHCb Collaboration observed the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ in the $\Lambda_b^0\pi^+\pi^-$ invariant mass distribution [18]. Later in 2020 the CMS Collaboration confirmed them, and further observed a broad excess of events in the $\Lambda_b^0\pi^+\pi^-$ mass distribution in the region of 6040–6100 MeV [19], whose mass and width were later measured by LHCb to be [20]:

  \[
  \Lambda_b(6072)^0 : M = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2 \text{ MeV},
  \Gamma = 72 \pm 11 \pm 2 \text{ MeV}.
  \]

Various theoretical methods and models have been applied to study singly bottom baryons in the past thirty years, such as various quark models [21–30], various molecular models [31–42], the quark pair creation model [43–46], the chiral perturbation theory [47–49], QCD sum rules [50–52], and Lattice QCD [53–55], etc. More theoretical studies can be found in Refs. [56–60], and we refer to recent reviews for detailed discussions [5, 9–12, 61, 62].

Especially, the $\Lambda_b(5912)^0$ and $\Lambda_b(5920)^0$ were studied by Capstick and Igisur in 1986 as P-wave bottom baryons [63], and their predicted masses are in very good agreement with the LHCb and CDF measurements [13, 14]. The $\Sigma_b(6097)^\pm$ and $\Xi_b(6227)^-$ are also good candidates of P-wave bottom baryons [27, 43–45, 50, 51, 58–60], while there exists the molecular interpretation for the $\Xi_b(6227)^-$ [41, 42]. Besides, the four excited $\Omega_b$ baryons, $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$, and

*Electronic address: hxchen@buaa.edu.cn
\( \Omega_b(6350)^- \), are all good candidates of \( P \)-wave bottom baryons [30, 46, 52, 64]. Other than this, the \( \Lambda_b(6146)^0 \) and \( \Lambda_b(6152)^0 \) can be well interpreted as \( D \)-wave bottom baryons [65–71].

In this paper we shall investigate \( P \)-wave bottom baryons of the \( SU(3) \) flavor \( 6_F \), and systematically study their \( D \)-wave decays into ground-state bottom baryons and pseudoscalar mesons. Previously in Refs. [1–4], we have systematically studied their mass spectra and \( S \)-wave decay properties using the method of QCD sum rules [72, 73] and light-cone sum rules [74–78] within the heavy quark effective theory (HQET) [79–81]. These results will be reanalysed in the present study, so that a rather complete study can be performed on both mass spectra and decay properties of \( P \)-wave bottom baryons. Similar methods applied to investigate singly heavy mesons and baryons can be found in Refs. [82–96].

This paper is organized as follows. In Sec. II, we briefly introduce our notations for \( P \)-wave bottom baryons of the \( SU(3) \) flavor \( 6_F \), and categorize them into four bottom baryon multiplets, \([6_F, 1, 0, \rho], [6_F, 0, 1, \lambda], [6_F, 1, 1, \lambda], \) and \([6_F, 2, 1, \lambda] \). Then in Sec. III we study their \( D \)-wave decays into ground-state bottom baryons and pseudoscalar mesons (\( \pi \) or \( K \)), separately for these four multiplets. In Sec. IV we discuss the results and conclude this paper.

II. \( P \)-WAVE BOTTOM BARYONS

At the beginning we briefly introduce our notations. A singly bottom baryon is composed by one bottom quark and two light up/down/strange quarks, with the following internal structures:

- According to the Pauli principle, the total symmetry of the two light quarks is antisymmetric.

- The color structure of the two light quarks is antisymmetric (\( 3_F \)).

- The flavor structure of the two light quarks is either symmetric (\( SU(3) \) flavor \( 6_F \)) or antisymmetric (\( SU(3) \) flavor \( \bar{3}_F \)).

- The spin structure of the two light quarks is either symmetric (\( s_l \equiv s_{\gamma l} = 1 \)) or antisymmetric (\( s_l = 0 \)).

- The orbital structure of the two light quarks is either symmetric (\( s_{\gamma l} = 0 \) and \( l_\lambda = 1 \), meaning that the orbital excitation is between the bottom quark and the two-light-quark system) or antisymmetric (\( s_{\gamma l} = 1 \) and \( l_\lambda = 0 \), meaning that the orbital excitation is between the two light quarks).

Accordingly, we categorize \( P \)-wave bottom baryons into eight baryon multiplets, four of which belong to the \( SU(3) \) flavor \( 6_F \) representation, as shown in Fig. 1. We use \([F(\text{flavor}), j_l, s_l, \rho/\lambda] \) to denote them, where \( j_l \) is the total angular momentum of the light components, satisfying \( j_l = l_\lambda \otimes l_\rho \otimes s_l \). Each multiplet contains one or two bottom baryons with the total angular momenta \( j = j_l \otimes s_b = |j_l \pm 1/2| \), which have similar masses according to the heavy quark effective theory.

In Refs. [1, 2] we have systematically studied the mass spectrum of \( P \)-wave bottom baryons, and the results are reanalysed in the present study, as summarized in Table I. Some of them are used as input parameters when studying decay properties of \( P \)-wave bottom baryons. Especially, we use the following mass values when calculating their decay widths:

- In Ref. [97] we found that the \( \Omega_b(6316)^- \) can be explained as a \( P \)-wave \( \Omega_b \) baryon of either \( J^P = 1/2^- \) or \( 3/2^- \), belonging to the \([6_F, 1, 0, \rho] \) doublet. Hence, we use the following mass values for this doublet, taken from the LHCb experiment [17] as well as their mass sum rules:

\[
\begin{align*}
M_{[\Omega_b(1/2^-),1,0,\rho]} &= 6.05 \text{ GeV}, \\
M_{[\Omega_b(3/2^-),1,0,\rho]} &= 6.05 \text{ GeV}, \\
M_{[\Xi_b(1/2^-),1,0,\rho]} &= 6.18 \text{ GeV}, \\
M_{[\Xi_b(3/2^-),1,0,\rho]} &= 6.19 \text{ GeV}, \\
M_{[\Omega_b(1/2^-),1,0,\rho]} &= 6315.64 \text{ MeV}, \\
M_{[\Omega_b(3/2^-),1,0,\rho]} &= 6315.64 \text{ MeV}.
\end{align*}
\]

- For the \([6_F, 0, 1, \lambda] \) singlet, we use the following mass values taken from their mass sum rules:

\[
\begin{align*}
M_{[\Omega_b(1/2^-),0,1,\lambda]} &= 6.05 \text{ GeV}, \\
M_{[\Xi_b(1/2^-),0,1,\lambda]} &= 6.20 \text{ GeV}, \\
M_{[\Omega_b(1/2^-),0,1,\lambda]} &= 6.34 \text{ GeV}.
\end{align*}
\]

- In Ref. [97] we found that the \( \Omega_b(6330)^- \) and \( \Omega_b(6340)^- \) can be explained as \( P \)-wave \( \Omega_b \) baryons of \( J^P = 1/2^- \) and \( 3/2^- \) respectively, belonging to the \([6_F, 1, 1, \lambda] \) doublet. Hence, we use the following mass values for this doublet, taken from the LHCb experiment [17] as well as their mass sum rules:

\[
\begin{align*}
M_{[\Omega_b(1/2^-),1,1,\lambda]} &= 6.06 \text{ GeV}, \\
M_{[\Omega_b(3/2^-),1,1,\lambda]} &= 6.07 \text{ GeV}, \\
M_{[\Xi_b(1/2^-),1,1,\lambda]} &= 6.21 \text{ GeV}, \\
M_{[\Xi_b(3/2^-),1,1,\lambda]} &= 6.22 \text{ GeV}, \\
M_{[\Omega_b(1/2^-),1,1,\lambda]} &= 6330.30 \text{ MeV}, \\
M_{[\Omega_b(3/2^-),1,1,\lambda]} &= 6339.71 \text{ MeV}.
\end{align*}
\]

- In Refs. [97, 98] we found that the \( \Sigma_b(6097)^\pm \), \( \Xi_b(6227)^- \), and \( \Omega_b(6350)^- \) can be explained as \( P \)-wave bottom baryons of \( J^P = 3/2^- \), belonging to the \([6_F, 2, 1, \lambda] \) doublet. Hence, we use the following mass values for this doublet, taken from the
TABLE I: Parameters of the $P$-wave bottom baryons belonging to the $SU(3)$ flavor $6_F$ representation. See Refs. [1, 2] for detailed discussions. In the last column we list the decay constant $f$, where isospin factors are explicitly taken into account, satisfying $f_{\Sigma^+_b} = f_{\Sigma^-_b} = \sqrt{2} f_{\Xi^0_b}$ and $f_{\Xi^0_b} = f_{\Xi^0_b}^{-}$.

| Multiplets | B | $\omega_c$ (GeV) | Working region (GeV) | $\Lambda$ (GeV) | Baryons $(j^P)$ | Mass $(\text{MeV})$ | Difference $(\text{MeV})$ | $f$ (GeV$^4$) |
|------------|---|------------------|----------------------|----------------|----------------|----------------|-------------------|-------------|
| $[6_F, 1, 0, \rho]$ | $\Sigma_b$ | 1.93 | 0.27 < $T$ < 0.34 | 1.31 ± 0.11 | $\Sigma_b(1/2^-)$ | 6.05 ± 0.12 | 3 ± 1 | 0.079 ± 0.019 (\Sigma^+_b (1/2^-)) |
| | | | | | $\Sigma_b(3/2^-)$ | 6.05 ± 0.12 | 0.037 ± 0.009 (\Sigma^-_b (3/2^-)) |
| | $\Xi'_b$ | 1.98 | 0.26 < $T$ < 0.36 | 1.45 ± 0.11 | $\Xi'_b(1/2^-)$ | 6.18 ± 0.12 | 3 ± 1 | 0.072 ± 0.016 (\Xi^-_b (1/2^-)) |
| | | | | | $\Xi'_b(3/2^-)$ | 6.19 ± 0.11 | 0.034 ± 0.008 (\Xi^-_b (3/2^-)) |
| | $\Omega_b$ | 2.13 | 0.26 < $T$ < 0.37 | 1.58 ± 0.09 | $\Omega_b(1/2^-)$ | 6.32 ± 0.11 | 2 ± 1 | 0.133 ± 0.028 (\Omega^+_b (1/2^-)) |
| | | | | | $\Omega_b(3/2^-)$ | 6.32 ± 0.11 | 0.063 ± 0.013 (\Omega^-_b (3/2^-)) |
| $[6_F, 0, 1, \lambda]$ | $\Sigma_b$ | 1.70 | 0.26 < $T$ < 0.32 | 1.25 ± 0.10 | $\Sigma_b(1/2^-)$ | 6.05 ± 0.11 | - | - |
| | | | | | $\Xi'_b(1/2^-)$ | 6.20 ± 0.11 | - | - |
| | $\Omega_b$ | 2.00 | 0.27 < $T$ < 0.34 | 1.54 ± 0.09 | $\Omega_b(1/2^-)$ | 6.34 ± 0.11 | - | 0.127 ± 0.028 (\Omega^-_b (1/2^-)) |
| $[6_F, 1, 1, \lambda]$ | $\Sigma_b$ | 1.94 | 0.29 < $T$ < 0.36 | 1.25 ± 0.11 | $\Sigma_b(1/2^-)$ | 6.06 ± 0.13 | 6 ± 3 | 0.075 ± 0.016 (\Sigma^+_b (1/2^-)) |
| | | | | | $\Sigma_b(3/2^-)$ | 6.07 ± 0.13 | 0.035 ± 0.008 (\Sigma^-_b (3/2^-)) |
| | $\Xi'_b$ | 1.97 | 0.35 < $T$ < 0.38 | 1.38 ± 0.09 | $\Xi'_b(1/2^-)$ | 6.21 ± 0.11 | 7 ± 2 | 0.069 ± 0.012 (\Xi^-_b (1/2^-)) |
| | | | | | $\Xi'_b(3/2^-)$ | 6.22 ± 0.11 | 0.032 ± 0.006 (\Xi^-_b (3/2^-)) |
| | $\Omega_b$ | 2.00 | 0.38 < $T$ < 0.39 | 1.48 ± 0.07 | $\Omega_b(1/2^-)$ | 6.34 ± 0.10 | 6 ± 2 | 0.122 ± 0.019 (\Omega^-_b (1/2^-)) |
| | | | | | $\Omega_b(3/2^-)$ | 6.34 ± 0.09 | 0.058 ± 0.009 (\Omega^-_b (3/2^-)) |
| $[6_F, 2, 1, \lambda]$ | $\Sigma_b$ | 1.84 | 0.27 < $T$ < 0.34 | 1.30 ± 0.13 | $\Sigma_b(3/2^-)$ | 6.11 ± 0.16 | 12 ± 5 | 0.102 ± 0.028 (\Sigma^-_b (3/2^-)) |
| | | | | | $\Sigma_b(5/2^-)$ | 6.12 ± 0.15 | 0.043 ± 0.012 (\Sigma^-_b (5/2^-)) |
| | $\Xi'_b$ | 1.96 | 0.26 < $T$ < 0.35 | 1.41 ± 0.12 | $\Xi'_b(3/2^-)$ | 6.23 ± 0.15 | 11 ± 5 | 0.091 ± 0.023 (\Xi^-_b (3/2^-)) |
| | | | | | $\Xi'_b(5/2^-)$ | 6.24 ± 0.14 | 0.038 ± 0.010 (\Xi^-_b (5/2^-)) |
| | $\Omega_b$ | 2.08 | 0.26 < $T$ < 0.37 | 1.53 ± 0.10 | $\Omega_b(3/2^-)$ | 6.35 ± 0.13 | 10 ± 4 | 0.162 ± 0.035 (\Omega^-_b (3/2^-)) |
| | | | | | $\Omega_b(5/2^-)$ | 6.36 ± 0.12 | 0.069 ± 0.015 (\Omega^-_b (5/2^-)) |

LHCb experiments [15–17] as well as their mass sum rules:

\[
M_{[\Sigma_b(3/2^-), 2, 1, \lambda]} = 6096.9 \text{ MeV},
M_{[\Sigma_b(3/2^-), 2, 1, \lambda]} - M_{[\Sigma_b(3/2^-), 2, 1, \lambda]} = 12 \text{ MeV},
M_{[\Xi'_b(3/2^-), 2, 1, \lambda]} = 6226.9 \text{ MeV},
M_{[\Xi'_b(3/2^-), 2, 1, \lambda]} - M_{[\Xi'_b(3/2^-), 2, 1, \lambda]} = 11 \text{ MeV},
M_{[\Omega_b(5/2^-), 2, 1, \lambda]} = 6349.88 \text{ MeV},
M_{[\Omega_b(5/2^-), 2, 1, \lambda]} - M_{[\Omega_b(3/2^-), 2, 1, \lambda]} = 10 \text{ MeV}. \quad (12)
\]
Note that the above interpretations are just possible assignments, and there exist many other possibilities for the $\Sigma_b(6097)^+\to\Xi_b(6227)^-, \Omega_b(6316)^-, \Omega_b(6330)^-, \Omega_b(6340)^-$, and $\Omega_b(6350)^-$.

We use the following parameters for ground-state bottom baryons, pseudoscalar and vector mesons [8]:

$$\Lambda_b(1/2^+): m = 5619.60 \text{ MeV},$$
$$\Xi_b(1/2^+): m = 5793.20 \text{ MeV},$$
$$\Sigma_b(1/2^+): m = 5813.4 \text{ MeV},$$
$$\Sigma_b^* (3/2^+): m = 5833.6 \text{ MeV},$$
$$\Xi_b^*(1/2^+): m = 5935.02 \text{ MeV},$$
$$\Xi_b^*(3/2^+): m = 5952.6 \text{ MeV},$$
$$\Omega_b(1/2^+): m = 6046.1 \text{ MeV},$$
$$\Omega_b^*(3/2^+): m = 6063 \text{ MeV},$$
$$\pi(0^-): m = 138.04 \text{ MeV},$$
$$K(0^-): m = 495.65 \text{ MeV},$$
$$\rho(1^-): m = 775.21 \text{ MeV},$$
$$\Gamma = 148.2 \text{ MeV}, g_{\rho\pi}\pi = 5.94,$$
$$K^*(1^-): m = 893.57 \text{ MeV}.$$  

\[ \Gamma = 49.1 \text{ MeV}, g_{K^*K\pi} = 3.20, \]

with the following Lagrangians:

\[ \mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} \times \left( \rho_0^\mu \pi^\nu \partial^\mu \pi^- - \rho_0^\mu \pi^- \partial^\mu \pi^\nu \right) + \cdots, \]
\[ \mathcal{L}_{K^*K\pi} = g_{K^*K\pi} \times (K^- \partial^\mu \pi^0 - \partial^\mu K^- \pi^0) + \cdots. \quad (14) \]

### III. D-WAVE DECAY PROPERTIES

In the present study we shall investigate $P$-wave bottom baryons of the $SU(3)$ flavor group, and study their $D$-wave decays into ground-state bottom baryons together with pseudoscalar mesons ($\pi$ or $K$). Their $S$-wave decay properties have been systematically studied in Refs. [3, 4], and we shall use the same method of light-cone sum rules within the heavy quark effective theory to investigate the following decay channels (the coefficients at right hand sides are isospin factors):

\[(a1) \quad \Gamma \left[ \Sigma_b[1/2^-] \rightarrow \Sigma_b^0 + \pi \right] = 2 \times \Gamma \left[ \Sigma_b^{-}[1/2^-] \rightarrow \Sigma_b^{0} + \pi^- \right], \quad (15) \]
\[(b1) \quad \Gamma \left[ \Sigma_b[3/2^-] \rightarrow \Lambda_b + \pi \right] = \Gamma \left[ \Sigma_b^{-}[3/2^-] \rightarrow \Lambda_b^0 + \pi^- \right], \quad (16) \]
\[(b2) \quad \Gamma \left[ \Sigma_b[3/2^-] \rightarrow \Sigma_b^* + \pi \right] = 2 \times \Gamma \left[ \Sigma_b^{-}[3/2^-] \rightarrow \Sigma_b^{0*} + \pi^- \right], \quad (17) \]
\[(b3) \quad \Gamma \left[ \Sigma_b[3/2^-] \rightarrow \Sigma_b^* + \pi \right] = 2 \times \Gamma \left[ \Sigma_b^{-}[3/2^-] \rightarrow \Sigma_b^{0*} + \pi^- \right], \quad (18) \]
\[(c1) \quad \Gamma \left[ \Sigma_b[5/2^-] \rightarrow \Lambda_b + \pi \right] = \Gamma \left[ \Sigma_b^{-}[5/2^-] \rightarrow \Lambda_b^0 + \pi^- \right], \quad (19) \]
\[(c2) \quad \Gamma \left[ \Sigma_b[5/2^-] \rightarrow \Sigma_b^* + \pi \right] = 2 \times \Gamma \left[ \Sigma_b^{-}[5/2^-] \rightarrow \Sigma_b^{0*} + \pi^- \right], \quad (20) \]
\[(c3) \quad \Gamma \left[ \Sigma_b[5/2^-] \rightarrow \Sigma_b^* + \pi \right] = 2 \times \Gamma \left[ \Sigma_b^{-}[5/2^-] \rightarrow \Sigma_b^{0*} + \pi^- \right], \quad (21) \]
\[(d1) \quad \Gamma \left[ \Xi_b^{-}[1/2^-] \rightarrow \Xi_b^0 + \pi \right] = \frac{3}{2} \Gamma \left[ \Xi_b^{-}[1/2^-] \rightarrow \Xi_b^{0} + \pi^- \right], \quad (22) \]
\[(d2) \quad \Gamma \left[ \Xi_b^{-}[1/2^-] \rightarrow \Sigma_b^0 + K \right] = 3 \Gamma \left[ \Xi_b^{-}[1/2^-] \rightarrow \Xi_b^{0} + K^- \right], \quad (23) \]
\[(e1) \quad \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Sigma_b^0 + \pi \right] = \frac{3}{2} \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Xi_b^{0} + \pi^- \right], \quad (24) \]
\[(e2) \quad \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Lambda_b + K \right] = \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Lambda_b^0 + K^- \right], \quad (25) \]
\[(e3) \quad \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Xi_b^* + \pi \right] = \frac{3}{2} \Gamma \left[ \Xi_b^{-}[1/2^-] \rightarrow \Xi_b^{0} + \pi^- \right], \quad (26) \]
\[(e4) \quad \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Sigma_b^0 + K \right] = 3 \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Xi_b^{0} + K^- \right], \quad (27) \]
\[(e5) \quad \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Xi_b^* + \pi \right] = \frac{3}{2} \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Xi_b^{0} + \pi^- \right], \quad (28) \]
\[(e6) \quad \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Sigma_b^0 + K \right] = 3 \Gamma \left[ \Xi_b^{-}[3/2^-] \rightarrow \Xi_b^{0} + K^- \right], \quad (29) \]
\[(f1) \quad \Gamma \left[ \Xi_b^{-}[5/2^-] \rightarrow \Sigma_b^0 + \pi \right] = \frac{3}{2} \Gamma \left[ \Xi_b^{-}[5/2^-] \rightarrow \Xi_b^{0} + \pi^- \right], \quad (30) \]
We shall calculate their decay widths through:

\[\mathcal{L}_{X_b(1/2^-)\rightarrow Y_b(3/2^+)} = g\bar{X}_b(1/2^-)\gamma_\mu\gamma_5 Y_{b\nu}(3/2^+)\partial^\mu\partial^\nu P,\]

\[\mathcal{L}_{X_b(3/2^-)\rightarrow Y_b(3/2^+)} = g\bar{X}_b(3/2^-)\gamma_\mu\gamma_5 Y_{b\nu}(1/2^+)\partial^\mu\partial^\nu P,\]

\[\mathcal{L}_{X_b(3/2^-)\rightarrow Y_b(3/2^+)} = g\bar{X}_b(3/2^-) Y_{b\nu}(3/2^+)\partial^\mu\partial^\nu P,\]

\[\mathcal{L}_{X_b(5/2^-)\rightarrow Y_b(3/2^+)} = g\bar{X}_b(5/2^-) Y_{b\nu}(1/2^+)\partial^\mu\partial^\nu P,\]

\[\mathcal{L}_{X_b(5/2^-)\rightarrow Y_b(3/2^+)} = g\bar{X}_{b\mu\nu}(5/2^-) Y_{b\nu}(5/2^+)\partial^\mu\partial^\rho P,\]

where \(X^{(\mu\nu)}_b\), \(Y^{(\mu\nu)}_b\), and \(P\) denote the \(P\)-wave bottom baryon, ground-state bottom baryon, and pseudoscalar meson, respectively.

As an example, we shall study the \(D\)-wave decay of \(\Omega^-_b(3/2^-)\) belonging to \([6_F, 2, 1, \lambda]\) into \(\Xi^{(1/2^+)}_b(3/2^+)\) and \(K^- (0^-)\). To do this we need to calculate the three-point correlation function

\[\Pi^\alpha (\omega, \omega') = \int d^4x e^{-i (\omega - \omega') x} \langle 0| J^\alpha_{3/2^- - \Omega^-_b, 2, 1, \lambda}(0) J_{\Xi^{(1/2^+)}_b}(x)| K^- (q) \rangle = \frac{1 + \frac{1}{(2\pi)^2} C_{\Omega^-_b(3/2^-)} G_{\Xi^{(1/2^+)}_b}}{2},\]

at both hadron and quark-gluon levels. In this expression \(k' = k + q\), \(\omega = v \cdot k\), and \(\omega' = v \cdot k'\). The two interpolating fields \(J^\alpha_{3/2^- - \Omega^-_b, 2, 1, \lambda}\) and \(J_{\Xi^{(1/2^+)}_b}\) have been constructed in Refs. [1, 94]:

\[J^\alpha_{3/2^- - \Omega^-_b, 2, 1, \lambda}\]

\[i\epsilon_{abc} \left( [D^\mu_1 s^T] C\gamma^\nu s^b + s^a T C\gamma^\nu [D^\mu_1 s^b] \right) \times \left( g_1^{\alpha\nu} \gamma_5 + g_2^{\alpha\nu} \gamma_5 - \frac{2}{3} g_3^{\alpha\nu} \gamma^5 \right) h^c,\]

\[J_{\Xi^{(1/2^+)}_b} = \epsilon_{abc} [s^T C\gamma^5 s^b] h^c,\]

which couple to \(\Omega^-_b(3/2^-)\) and \(\Xi^{(1/2^+)}_b(1/2^+),\) respectively.
At the hadron level, we write $G_{\Omega_b^{-}\to \Xi_b^- K^-}^{\alpha}$ as:

$$G_{\Omega_b^{-}\to \Xi_b^- K^-}^{\alpha}(\omega, \omega') = g_{\Omega_b^{-}\to \Xi_b^- K^-}^{\alpha} \times \frac{f_{\Omega_b^{-}\to \Xi_b^-}}{(\Lambda_{\Omega_b^-} - \omega')(\Lambda_{\Xi_b^-} - \omega)} \times \gamma \cdot q \gamma_5 q^\alpha + \cdots,$$  \hspace{1cm} (51)

where $\cdots$ denotes other possible decay amplitudes.

At the quark-gluon level, we calculate $G_{\Omega_b^{-}\to \Xi_b^- K^-}^{\alpha}$ using the method of operator product expansion (OPE):

$$G_{\Omega_b^{-}\to \Xi_b^- K^-}^{\alpha}(\omega, \omega') = \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega t} e^{iu\omega t} \times \left( \frac{f_{Km_3}m_3}{4\pi^2 f_2}\phi_{2,K}^3(u) + \frac{f_{Km_3}m_3}{12(m_u + m_s)\pi^2 f_2}\phi_{3,K}^5(u) \right)$$

$$+ \frac{f_{Km_3}m_3}{48(m_u + m_s)\pi^2 f_2}\phi_{3,K}^5(u) + \frac{f_{Km_3}m_3}{288(m_u + m_s)}(\bar{s}s)\phi_{3,K}^5(u) + \frac{f_{Km_3}m_3}{3072}(g_s\bar{s}G_s)\phi_{3,K}^5(u) \times \gamma \cdot q \gamma_5 q^\alpha + \cdots.$$  \hspace{1cm} (52)

Then we perform double Borel transformations to both Eqs. (51) and (52), and obtain:

$$g_{\Omega_b^{-}\to \Xi_b^-}^{\alpha} - f_{\Omega_b^{-}\to \Xi_b^-}^{\alpha} = \frac{\alpha}{\pi^2} \delta_{\Xi_b^-} - \frac{\alpha}{\pi^2} \delta_{\Xi_b^-}$$  \hspace{1cm} (53)

In the above expression, $\omega$ and $\omega'$ have been transformed to $T_1$ and $T_2$; we work at the symmetric point $T_1 = T_2 = 2T$ so that $u_0 = \frac{T_1 + T_2}{2} = 1$; $f_n(x) \equiv 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}$. We refer to Refs. [77, 78, 99–104] for explicit forms of the light-cone distribution amplitudes contained in the above sum rule equations, and more examples can be found in Appendix A.

In the present study we use the following values for various quark and gluon parameters at the renormalization scale 2 GeV [8, 105–112]:

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV}^3),$$

$$\langle s\bar{s} \rangle = (0.8 \pm 0.1) \times \langle \bar{q}q \rangle,$$

$$\langle g_s\bar{s}G_s \rangle = M_0^2 \times \langle \bar{q}q \rangle,$$

$$\langle g_s\bar{s}G_s \rangle = M_0^3 \times \langle \bar{q}q \rangle,$$

$$M_0^2 = 0.8 \text{ GeV}^2,$$

$$\langle g_s^2GG \rangle = (0.48 \pm 0.14) \text{ GeV}^4.$$  \hspace{1cm} (54)

After fixing $\omega_c = 1.665$ GeV to be the average of the threshold values of the $\Omega_c^- (3/2^-)$ and $\Xi_b^- (0^-)$ mass sum rules, we calculate the coupling constant $g_{\Omega_b^{-}\to \Xi_b^-}$ from Eq. (53) to be:

$$g_{\Omega_b^{-}\to \Xi_b^-}^{\alpha} = \frac{M_0^2}{\omega_c} = \frac{27}{1.665} + 1.01 + 1.22 + 2.34 + 1.96 \text{ GeV}^{-2}$$

$$= 27.23^{+3.92}_{-2.83} \text{ GeV}^{-2}.$$  \hspace{1cm} (55)

Here the uncertainties are due to the Borel mass, the parameters of $\Xi_b^0$, the parameters of $\Omega_c^- (3/2^-)$, and various quark and gluon parameters listed in Eq. (54), respectively. Some of these parameters can be found in Sec. II.
The variation of $g_\Omega^{-}(-\frac{3}{2}^{-}) \rightarrow \Xi_{c} K^{-}$ is shown in Fig. 4(d) as a function of the Borel mass $T$, where we find that its Borel mass dependence is moderate and acceptable.

The $D$-wave decay of $\Omega_{c}^{-}(3/2^{-})$ into $\Xi_{c}^{0} K^{-}$ is kinematically allowed, and its amplitude is

$$\mathcal{M}(0 \rightarrow 1 + 2) = \mathcal{M}\left(\Omega_{c}^{-}(3/2^{-}) \rightarrow \Xi_{c}^{0} + K^{-}\right)$$

$$= g_{0 \rightarrow 1 + 2} \bar{u}_{0}^{c} \gamma^{\nu} \gamma_{5} u_{1} \gamma_{\mu} p_{2, \rho} p_{2, \rho} \sigma,$$

where $0, 1, 2$ denote the initial state $\Omega_{c}^{-}(3/2^{-})$, the final state $\Xi_{c}^{0}$, and the final state $K^{-}$, respectively. This amplitude can be further calculated its width through

$$\Gamma\left(0 \rightarrow 1 + 2\right) \equiv \Gamma\left(\Omega_{c}^{-}(3/2^{-}) \rightarrow \Xi_{c}^{0} + K^{-}\right)$$

$$= \left|\frac{\bar{p}_{0}}{32\pi^{2} m_{0}^{2}} \times \frac{g_{0}^{2}}{2 \mu_{2, \rho} p_{2, \rho} p_{2, \rho} \sigma} \times \text{Tr}\left[\gamma^{\nu} \gamma_{5} (p_{1} + m_{1}) \gamma^{\sigma} \gamma_{5}\right]
\left(\frac{g^{2\nu - \sigma}}{3} - \frac{p_{0}^{\nu} p_{0}^{\sigma}}{3m_{0}^{2}}\right)\right|,$$

from which we obtain

$$\Gamma_{\Omega_{c}^{-}(3/2^{-}) \rightarrow \Xi_{c}^{0} K^{-}} = 4.6^{-3.3}_{+1.9} \text{ MeV}. \quad (58)$$

In the following subsections we shall follow the same procedures to separately investigate the four bottom baryon multiplets, $[6_F, 1, 0, \rho]$, $[6_F, 0, 1, \lambda]$, $[6_F, 1, 1, \lambda]$, and $[6_F, 2, 1, \lambda]$.

### B. The $[6_F, 1, 0, \rho]$ doublet

There are six bottom baryons contained in the $[6_F, 1, 0, \rho]$ doublet, that are $\Sigma_{b}(\frac{1}{2}^{-} / \frac{3}{2}^{-})$, $\Xi_{b}^{0}(\frac{1}{2}^{-} / \frac{3}{2}^{-})$, and $\Omega_{b}(\frac{1}{2}^{-} / \frac{3}{2}^{-})$. We study their $D$-wave decays into ground-state bottom baryons and pseudoscalar mesons, and find twelve non-zero coupling constants:

(a1) $g_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 4.80 \text{ GeV}^{-2}$,  
(b2) $g_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 5.79 \text{ GeV}^{-2}$,  
(b3) $g_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 3.37 \text{ GeV}^{-2}$,  
(d1) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 3.10 \text{ GeV}^{-2}$,  
(d2) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 3.63 \text{ GeV}^{-2}$,  
(c3) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 3.79 \text{ GeV}^{-2}$,  
(e4) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 4.45 \text{ GeV}^{-2}$,  
(e5) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 2.18 \text{ GeV}^{-2}$,  
(e6) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 2.55 \text{ GeV}^{-2}$,  
(g1) $g_{\Omega_{b}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 4.54 \text{ GeV}^{-2}$,  
(h2) $g_{\Omega_{b}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 5.56 \text{ GeV}^{-2}$,  
(h3) $g_{\Omega_{b}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 3.19 \text{ GeV}^{-2}$.

We show some of these coupling constants as functions of the Borel mass $T$ in Fig. 2. Based on them, we further find six $D$-wave decay channels that are kinematically allowed:

(a1) $\Gamma_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 0.62 \text{ MeV}$,  
(b2) $\Gamma_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 0.84 \text{ MeV}$,  
(b3) $\Gamma_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 0.098 \text{ MeV}$,  
(d1) $\Gamma_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 0.29 \text{ MeV}$,  
(e3) $\Gamma_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 0.47 \text{ MeV}$,  
(e5) $\Gamma_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 0.064 \text{ MeV}$.

We summarize these $D$-wave decay widths in Table II, where possible experimental candidates are given for comparisons. In Refs. [1, 2] we have studied the mass spectrum of $P$-wave bottom baryons, and the results are reanalysed and summarized in this table. In Refs. [3, 4] we have studied $S$-wave decay properties of $P$-wave bottom baryons into ground-state bottom baryons together with pseudoscalar mesons or vector mesons, and the results are also reanalysed and summarized in this table.

### C. The $[6_F, 0, 1, \lambda]$ singlet

There are three bottom baryons contained in the $[6_F, 0, 1, \lambda]$ singlet, that are $\Sigma_{b}(\frac{1}{2}^{-})$, $\Xi_{b}(\frac{1}{2}^{-})$, and $\Omega_{b}(\frac{1}{2}^{-})$. We study their $D$-wave decays into ground-state bottom baryons and pseudoscalar mesons, but find all the coupling constants to be zero. For completeness, we summarize these results in Table III, together with their mass spectrum, $S$-wave decay properties, and possible experimental candidates.

### D. The $[6_F, 1, 1, \lambda]$ doublet

There are six bottom baryons contained in the $[6_F, 1, 1, \lambda]$ doublet, that are $\Sigma_{b}(\frac{1}{2}^{-} / \frac{3}{2}^{-})$, $\Xi_{b}(\frac{1}{2}^{-} / \frac{3}{2}^{-})$, and $\Omega_{b}(\frac{1}{2}^{-} / \frac{3}{2}^{-})$. We study their $D$-wave decays into ground-state bottom baryons and pseudoscalar mesons, and find twelve non-zero coupling constants:

(a1) $g_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 1.40^{+0.09}_{-0.140} \text{ GeV}^{-2}$,  
(b2) $g_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 3.48^{+1.89}_{-1.47} \text{ GeV}^{-2}$,  
(b3) $g_{\Sigma_{b}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 2.01^{+1.08}_{-0.85} \text{ GeV}^{-2}$,  
(d1) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 1.44^{+0.66}_{-0.61} \text{ GeV}^{-2}$,  
(d2) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 0.78 \text{ GeV}^{-2}$,  
(e3) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 2.22^{+0.96}_{-0.80} \text{ GeV}^{-2}$,  
(e4) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Sigma_{b}^{+}\pi} = 1.74 \text{ GeV}^{-2}$,  
(e5) $g_{\Xi_{b}^{0}(\frac{1}{2}^{-}) \rightarrow \Xi_{b}^{+}\pi} = 1.28^{+0.58}_{-0.46} \text{ GeV}^{-2}$.
We show some of these coupling constants as functions of the Borel mass $T$: (a) $g_{\Sigma_b^{*}(3/2^-) \rightarrow \Sigma_b^{*}(1/2^+)}$, (b) $g_{\Xi_b^{*}(3/2^-) \rightarrow \Xi_b^{*}(1/2^+)}$, (c) $g_{\Xi_b^{*}(3/2^-) \rightarrow \Sigma_b^{*}(1/2^+)}$, (d) $g_{\Xi_b^{*}(3/2^-) \rightarrow \Xi_b^{*}(1/2^+)}$, (e) $g_{\Sigma_b^{*}(3/2^-) \rightarrow \Sigma_b^{*}(1/2^+)}$, and (f) $g_{\Xi_b^{*}(3/2^-) \rightarrow \Xi_b^{*}(1/2^+)}$. Here the bottom baryon doublet $[6_F, 1, 0]$ is investigated.

We show some of these coupling constants as functions of the Borel mass $T$ in Fig. 3. Based on them, we further find six $D$-wave decay channels that are kinematically allowed:

(a1) $\Gamma_{\Sigma_b^{*}(3/2^-) \rightarrow \Sigma_b^{*}(1/2^+)} = 0.076^{+0.144}_{-0.070}$ MeV,

(b2) $\Gamma_{\Sigma_b^{*}(3/2^-) \rightarrow \Sigma_b^{*}(1/2^+)} = 0.55^{+0.74}_{-0.36}$ MeV,

(b3) $\Gamma_{\Sigma_b^{*}(3/2^-) \rightarrow \Sigma_b^{*}(1/2^+)} = 0.070^{+0.096}_{-0.047}$ MeV,

(d1) $\Gamma_{\Xi_b^{*}(3/2^-) \rightarrow \Xi_b^{*}(1/2^+)} = 0.16^{+0.18}_{-0.10}$ MeV,  

(e3) $\Gamma_{\Xi_b^{*}(3/2^-) \rightarrow \Xi_b^{*}(1/2^+)} = 0.34^{+0.35}_{-0.26}$ MeV,  

(e5) $\Gamma_{\Xi_b^{*}(3/2^-) \rightarrow \Xi_b^{*}(1/2^+)} = 0.051^{+0.057}_{-0.036}$ MeV.

We summarize these $D$-wave decay widths in Table IV, together with their mass spectrum, $S$-wave decay properties, and possible experimental candidates.

E. The $[6_F, 2, 1, \lambda]$ doublet

There are six bottom baryons contained in the $[6_F, 2, 1, \lambda]$ doublet, that are $\Sigma_b(3/2^-) / (1/2^-)$, $\Xi_b(3/2^-) / (5/2^-)$, and $\Omega_b(3/2^-) / (5/2^-)$. We study their $D$-wave decays into
and find twelve non-zero coupling constants:

\[ g_{\Sigma_{b}^{-} K_{b}^{0}} \rightarrow \Lambda_{b}^{0} \pi^{0}, \]
\[ g_{\Sigma_{b}^{-} K_{b}^{0}} \rightarrow \Sigma_{b}^{0} \pi^{-}, \]
\[ g_{\Sigma_{b}^{-} K_{b}^{0}} \rightarrow \Xi_{b}^{0} \pi^{-}, \]
\[ g_{\Xi_{b}^{-} K_{b}^{0}} \rightarrow \Sigma_{b}^{0} \pi^{-}, \]
\[ g_{\Xi_{b}^{-} K_{b}^{0}} \rightarrow \Xi_{b}^{0} \pi^{-}, \] and (f) \[ g_{\Xi_{b}^{-} K_{b}^{0}} \rightarrow \Xi_{b}^{0} \pi^{-}. \]

Here the bottom baryon doublet \([6_F, 1, 1, \lambda]\) is investigated.

We show some of these coupling constants as functions of the Borel mass \( T \) in Fig. 4. Based on them, we further find eight \( D \)-wave decay channels that are kinematically allowed:

\[ (b1) \Gamma_{\Sigma_{b}^{-} K_{b}^{0}} \rightarrow \Lambda_{b}^{0} \pi^{0} = 49.6 +^{76.4}_{-32.9} \text{MeV}, \]
\[ (b2) \Gamma_{\Sigma_{b}^{-} K_{b}^{0}} \rightarrow \Sigma_{b}^{0} \pi^{-} = 1.6 +^{3.2}_{-1.1} \text{MeV}, \]

FIG. 3: The coupling constants as functions of the Borel mass \( T \): (a) \( g_{\Sigma_{b}^{-} K_{b}^{0}} \rightarrow \Lambda_{b}^{0} \pi^{0} \), (b) \( g_{\Sigma_{b}^{-} K_{b}^{0}} \rightarrow \Sigma_{b}^{0} \pi^{-} \), (c) \( g_{\Sigma_{b}^{-} K_{b}^{0}} \rightarrow \Xi_{b}^{0} \pi^{-} \), (d) \( g_{\Xi_{b}^{-} K_{b}^{0}} \rightarrow \Sigma_{b}^{0} \pi^{-} \), (e) \( g_{\Xi_{b}^{-} K_{b}^{0}} \rightarrow \Xi_{b}^{0} \pi^{-} \), and (f) \( g_{\Xi_{b}^{-} K_{b}^{0}} \rightarrow \Xi_{b}^{0} \pi^{-} \).
FIG. 4: The coupling constants as functions of the Borel mass $T$: (a) $g_{\Sigma_b - [3/2] - \rightarrow \Lambda_0^b \pi}$, (b) $g_{\Xi'_b - [3/2] - \rightarrow \Xi_0^b \pi}$, (c) $g_{\Xi'_b - [3/2] - \rightarrow \Lambda_0^b K}$, (d) $g_{\Omega_b - [3/2] - \rightarrow \Xi_0^b K}$, (e) $g_{\Sigma'_b - [3/2] - \rightarrow \Sigma_0^b \pi}$, (f) $g_{\Xi'_b - [3/2] - \rightarrow \Xi'_0 \pi}$, (g) $g_{\Sigma'_b - [3/2] - \rightarrow \Sigma^*_0 \pi}$, and (h) $g_{\Xi'_b - [3/2] - \rightarrow \Xi^*_0 \pi}$. Here the bottom baryon doublet $[6_F, 2, 1, \lambda]$ is investigated.
We summarize these $D$-wave decay widths in Table V, together with their mass spectrum, $S$-wave decay properties, and possible experimental candidates.

IV. SUMMARY AND DISCUSSIONS

To summarize this paper, we have investigated the $P$-wave bottom baryons belonging to the $SU(3)$ flavor $6_F$ representation, and studied their $D$-wave decays into ground-state bottom baryons and pseudoscalar mesons. Together with Refs. [1–4], we have performed a rather complete study on both mass spectra and decay properties of $P$-wave bottom baryons using the method of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory.

Accordingly to the heavy quark effective theory, we categorize the $P$-wave bottom baryons of the $SU(3)$ flavor $6_F$ into four multiplets: $[6_F, 1, 0, \rho]$, $[6_F, 0, 1, \rho]$, $[6_F, 1, 1, \lambda]$, and $[6_F, 2, 1, \lambda]$. In this paper we have studied their $D$-wave decay properties, and the results are separately summarized in Tables II/III/IV/V. Besides, in Refs. [1, 2] we have studied the mass spectrum of $P$-wave bottom baryons, and the results are reanalyzed and summarized in these tables; in Refs. [3, 4] we have studied $S$-wave decay properties of $P$-wave bottom baryons into ground-state bottom baryons together with pseudoscalar mesons or vector mesons, and the results are also reanalyzed and summarized in these tables.

Before drawing our conclusions, we note that there are considerable (theoretical) uncertainties in our results for absolute values of the bottom baryon masses due to their significant dependence on the bottom quark mass [1, 2]; however, their mass splittings within the same doublets do not depend much on this, so they are produced quite well with much less (theoretical) uncertainties and give more useful information; moreover, we can extract even (much) more useful information from $S$- and $D$-wave strong decay properties of $P$-wave bottom baryons. Based on the results summarized in Tables II/III/IV/V, we can well understand $P$-wave bottom baryons as a whole:

- The $[6_F, 0, 1, \rho]$ doublet contains six bottom baryons: $\Sigma_b(\frac{1}{2}, \frac{1}{2})$, $\Xi_b(\frac{1}{2}, \frac{1}{2})$, and $\Omega_b(\frac{1}{2}, \frac{1}{2})$. Their total widths are all calculated to be very large, preventing them to be observed in any experiment.

- The $[6_F, 1, 0, \rho]$ doublet contains six bottom baryons: $\Sigma_b(\frac{3}{2}, \frac{3}{2})$, $\Xi_b(\frac{3}{2}, \frac{3}{2})$, and $\Omega_b(\frac{3}{2}, \frac{3}{2})$. The total widths of $\Sigma_b(\frac{3}{2}, \frac{3}{2})$ and $\Xi_b(\frac{3}{2}, \frac{3}{2})$ are all calculated to be very large, while the total widths of $\Omega_b(\frac{3}{2}, \frac{3}{2})$ are both extracted to be zero.

- The $[6_F, 1, 1, \lambda]$ doublet contains six bottom baryons: $\Sigma_b(\frac{1}{2}, \frac{1}{2})$, $\Xi_b(\frac{1}{2}, \frac{1}{2})$, and $\Omega_b(\frac{1}{2}, \frac{1}{2})$. Their total widths are all calculated to be less than 100 MeV.

- The $[6_F, 2, 1, \lambda]$ doublet contains six bottom baryons: $\Sigma_b(\frac{3}{2}, \frac{3}{2})$, $\Xi_b(\frac{3}{2}, \frac{3}{2})$, and $\Omega_b(\frac{3}{2}, \frac{3}{2})$. Their total widths are all calculated to be less than 100 MeV.

Hence, among all the possible $P$-wave bottom baryons of the flavor $6_F$, we find altogether four $\Sigma_b$, four $\Xi_b$, and six $\Omega_b$ baryons, with limited widths (< 100 MeV) and so capable of being observed. Their masses, mass splittings within the same multiplets, and decay properties are summarized in Table VI. Their possible experimental candidates are also given in this table for comparisons. We suggest the LHCb and CMS Collaborations to search for these excited bottom baryons, but note that it still depends on the production rates whether these baryons can be observed or not. Especially, it is interesting to further investigate the $A_b(6072)^0$, i.e., the broad excess of events in the $A_b^0\pi^+\pi^-$ mass distribution in the region of 6040–6100 MeV [19, 20].

In the present study the $\rho$-mode doublet $[6_F, 1, 0, \rho]$ is found to be lower than the two $\lambda$-mode doublets $[6_F, 1, 1, \lambda]$ and $[6_F, 2, 1, \lambda]$, which behaviour is consistent with our previous results for their corresponding doublets of the $SU(3)$ flavor $3_F$ [1, 2], but in contrast to the quark model expectation [6, 25]. However, this may be simply because that the mass differences between different multiplets have considerable uncertainties in our framework, similar to the absolute values of baryon masses, but unlike the mass differences within the same multiplet. We propose to verify whether the $\rho$-mode doublet $[6_F, 1, 0, \rho]$ exist or not by investigating: a) the spin-parity quantum number of the $\Omega_b(6316)^-$, b) whether it can be separated into two states almost degenerate, and c) whether its $\Sigma_b$ and $\Xi_b'$ partners states can be observed.

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TABLE II: Decay properties of the $P$-wave bottom baryons belonging to the $[6_F, 1, 0, \rho]$ doublet. The sixth column “$D$-wave width” is newly obtained in the present study. Besides, in Refs. [1, 2] we studied the mass spectrum of $P$-wave bottom baryons, and the results are reanalysed and summarized in the second and third columns; in Refs. [3, 4] we studied the $S$-wave decay properties of $P$-wave bottom baryons into ground-state bottom baryons together with pseudoscalar mesons or vector mesons, and the results are reanalysed and summarised in the fifth column. Possible experimental candidates are given in the last column for comparisons.

| Baryon $(j^P)$ | Mass (GeV) | Difference (MeV) | Decay channels | $S$-wave width (MeV) | $D$-wave width (MeV) | Total width (MeV) | Candidate |
|----------------|-----------|------------------|----------------|----------------------|---------------------|------------------|-----------|
| $\Sigma_b(\frac{1}{2}^-)$ | 6.05 ± 0.12 | 3 ± 1 | $\Sigma_b(\frac{1}{2}^-) \rightarrow \Sigma_b\pi$ | 710 | – | 710 | – |
| | | | $\Sigma_b(\frac{1}{2}^-) \rightarrow \Sigma_b^0\pi$ | – | 0.62 | – | – |
| | | | $\Sigma_b(\frac{1}{2}^-) \rightarrow \Lambda_b\rho \rightarrow \Lambda_b\pi\pi$ | 4.3 × 10^{-3} | – | 4.3 × 10^{-3} | – |
| $\Sigma_b(\frac{3}{2}^-)$ | 6.05 ± 0.12 | 3 ± 1 | $\Sigma_b(\frac{3}{2}^-) \rightarrow \Sigma_b\pi$ | – | 0.84 | – | – |
| | | | $\Sigma_b(\frac{3}{2}^-) \rightarrow \Sigma_b^0\pi$ | 410 | 0.098 | 410 | – |
| | | | $\Sigma_b(\frac{3}{2}^-) \rightarrow \Lambda_b\rho \rightarrow \Lambda_b\pi\pi$ | 5.1 × 10^{-3} | – | 5.1 × 10^{-3} | – |
| $\Xi_b(\frac{1}{2}^-)$ | 6.18 ± 0.12 | 3 ± 1 | $\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b^0\pi$ | 250 | – | 250 | – |
| | | | $\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b\pi$ | – | 0.29 | – | – |
| | | | $\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b^0\pi$ | 1.2 × 10^{-5} | – | 1.2 × 10^{-5} | – |
| $\Xi_b(\frac{3}{2}^-)$ | 6.19 ± 0.11 | 2 ± 1 | $\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b\pi$ | 160 | 0.064 | 160 | – |
| | | | $\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b^0\pi$ | 8.0 × 10^{-5} | – | 8.0 × 10^{-5} | – |
| $\Omega_b(\frac{1}{2}^-)$ | 6.32 ± 0.11 | 2 ± 1 | – | – | – | – | – |
| $\Omega_b(\frac{3}{2}^-)$ | 6.32 ± 0.11 | 2 ± 1 | – | – | – | – | – |

TABLE III: Decay properties of the $P$-wave bottom baryons belonging to the $[6_F, 0, 1, \lambda]$ singlet. See the caption of Table II for detailed explanations.

| Baryon $(j^P)$ | Mass (GeV) | Difference (MeV) | Decay channels | $S$-wave width (MeV) | $D$-wave width (MeV) | Total width (MeV) | Candidate |
|----------------|-----------|------------------|----------------|----------------------|---------------------|------------------|-----------|
| $\Sigma_b(\frac{1}{2}^-)$ | 6.05 ± 0.11 | – | $\Sigma_b(\frac{1}{2}^-) \rightarrow \Lambda_b\pi$ | 1300 | – | 1300 | – |
| $\Xi_b(\frac{1}{2}^-)$ | 6.20 ± 0.11 | – | $\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b^0\pi$ | 990 | – | 990 | – |
| | | | $\Xi_b(\frac{1}{2}^-) \rightarrow \Lambda_bK$ | 910 | – | 910 | – |
| $\Omega_b(\frac{1}{2}^-)$ | 6.34 ± 0.11 | – | $\Omega_b(\frac{1}{2}^-) \rightarrow \Xi_bK$ | 2700 | – | 2700 | – |

Appendix A: Sum rule equations

In this appendix we give several examples of sum rule equations, which are used to study $D$-wave decays of $P$-wave bottom baryons into ground-state bottom baryons and pseudoscalar mesons.

The sum rule equation for the $\Xi_b^{*-} \frac{1}{2}^-$ belonging to $[6_F, 1, 0, \rho]$ is

$$G_{\Xi_b^{*-}\frac{1}{2}^-\rightarrow \Xi_b^0\pi}(\omega, \omega') = g_{\Xi_b^{*-}\frac{1}{2}^-\rightarrow \Xi_b^0\pi} \times \frac{f_{\Xi_b} \bar{f}_{\Xi_b^0}}{(\xi_{\Xi_b^{*-}\frac{1}{2}^-} - \omega')(\xi_{\Xi_b^0} - \omega)}$$

$$= \int_0^\infty dt \int_0^1 du e^{i(u-1)\omega'} \bar{\epsilon} e^{iu\omega} \times \left( \frac{f_{\Xi_b} m_{\Xi_b}^2 m_{\Xi_b^0}^2 u}{96(m_u + m_d)\pi^2} \phi_{3,\pi}(u) + \frac{f_{\Xi_b} m_{\Xi_b^0} u}{128\pi^2} \phi_{4,\pi}(u) \right)$$

(A1)
TABLE IV: Decay properties of the $P$-wave bottom baryons belonging to the $[6_F, 1, 1, \Lambda]$ doublet. See the caption of Table II for detailed explanations.

| Baryon $(j^P)$ | Mass (GeV) | Difference (MeV) | Decay channels | $S$-wave width (MeV) | $D$-wave width (MeV) | Total width (MeV) | Candidate |
|----------------|------------|------------------|----------------|---------------------|---------------------|-------------------|-----------|
| $\Sigma_b(\frac{3}{2}^-)$ | 6.06 ± 0.13 | 6 ± 3 | $\Sigma_b(\frac{1}{2}^-) \to \Sigma_b\pi$ | 14.1 ± 21.2 | – | 14.3 ± 21.2 | – |
| | | | $\Sigma_b(\frac{1}{2}^-) \to \Sigma_b\pi$ | – | 0.076 ± 0.144 | – | 0.144 – 0.076 | – |
| | | | $\Sigma_b(\frac{3}{2}^-) \to \Lambda_b\rho \to \Lambda_b\pi\pi$ | 0.087 ± 0.244 | – | 0.244 – 0.087 | – |
| $\Xi_b(\frac{1}{2}^-)$ | 6.07 ± 0.13 | 6 ± 3 | $\Sigma_b(\frac{3}{2}^-) \to \Sigma_b\pi$ | 3.9 ± 5.8 | – | 5.8 ± 2.9 | – |
| | | | $\Sigma_b(\frac{1}{2}^-) \to \Lambda_b\rho \to \Lambda_b\pi\pi$ | 0.23 ± 0.45 | – | 0.45 – 0.20 | – |
| $\Xi_b(\frac{1}{2}^-)$ | 6.21 ± 0.11 | 7 ± 2 | $\Xi_b(\frac{1}{2}^-) \to \Xi_b\pi$ | 4.5 ± 5.8 | – | 5.8 ± 3.3 | – |
| | | | $\Xi_b(\frac{3}{2}^-) \to \Xi_b\pi$ | – | 0.16 ± 0.18 | – | 0.18 – 0.10 | – |
| $\Xi_b(\frac{1}{2}^-)$ | 6.22 ± 0.11 | 6 ± 2 | $\Xi_b(\frac{3}{2}^-) \to \Xi_b\pi$ | 0.43 ± 0.079 | – | 0.079 – 0.038 | – |
| | | | $\Xi_b(\frac{1}{2}^-) \to \Xi_b\rho \to \Xi_b\pi\pi$ | 0.34 ± 0.35 | – | 0.35 – 0.20 | – |
| $\Omega_b(\frac{1}{2}^-)$ | 6.34 ± 0.10 | 6 ± 2 | $\Xi_b(\frac{1}{2}^-) \to \Xi_b\rho \to \Xi_b\pi\pi$ | 0.078 ± 0.147 | – | 0.147 + 0.068 | – |
| $\Omega_b(\frac{3}{2}^-)$ | 6.34 ± 0.09 | 6 ± 2 | $\Xi_b(\frac{3}{2}^-) \to \Xi_b\rho \to \Xi_b\pi\pi$ | 1.8 ± 1.07 | – | 1.07 – 0.92 | – |

The sum rule equation for the $\Omega_b^-(\frac{3}{2}^-)$ belonging to $[6_F, 0, 1, \Lambda]$ is

$$G_{\Omega_b^-}(\frac{3}{2}^-) \to \Xi_b^o \kappa^- (\omega, \omega') = g_{\Omega_b^-}(\frac{3}{2}^-) \to \Xi_b^o \kappa^- \times \frac{f_{\Omega_b^-}(\frac{3}{2}^-) f_{\Xi_b^o}}{(\Lambda_{\Omega_b^-}(\frac{3}{2}^-) - \omega')(\Lambda_{\Xi_b^o} - \omega)} = 0. \quad (A2)$$

The sum rule equation for the $\Sigma_b^-(\frac{3}{2}^-)$ belonging to $[6_F, 1, 1, \Lambda]$ is

$$G_{\Sigma_b^-}(\frac{3}{2}^-) \to \Sigma_b^{0 \pi^-}(\omega, \omega') = g_{\Sigma_b^-}(\frac{3}{2}^-) \to \Sigma_b^{0 \pi^-} \times \frac{f_{\Sigma_b^-}(\frac{3}{2}^-) f_{\Sigma_b^{0 \pi^-}}}{(\Lambda_{\Sigma_b^-}(\frac{3}{2}^-) - \omega')(\Lambda_{\Sigma_b^{0 \pi^-}} - \omega)} = 0. \quad (A3)$$
TABLE V: Decay properties of the $P$-wave bottom baryons belonging to the $[6_F, 2, 1, \lambda]$ doublet. See the caption of Table II for detailed explanations.

| Baryon $(j^P)$ | Mass (GeV) | Difference (MeV) | Decay channels | $S$-wave width (MeV) | $D$-wave width (MeV) | Total width (MeV) | Candidate |
|----------------|------------|------------------|----------------|----------------------|----------------------|-------------------|-----------|
| $\Sigma_b(\frac{3}{2}^+)$ | 6.11 ± 0.16 | 12 ± 5 | $\Sigma_b(\frac{3}{2}^+) \rightarrow \Lambda_b \pi$ | – | 49.6 $^{+76.4}_{-32.9}$ | 51.4 $^{+76.5}_{-32.9}$ | $\Sigma_b(6097)^+$ [16] |
| $\Sigma_b(\frac{5}{2}^+)$ | 6.12 ± 0.15 | – | $\Sigma_b(\frac{5}{2}^+) \rightarrow \Sigma_b \pi$ | – | 1.6 $^{+3.2}_{-1.1}$ | – | – |
| $\Xi_b(\frac{1}{2}^-)$ | 6.23 ± 0.15 | 11 ± 5 | $\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b \pi$ | – | 19.0 $^{+29.3}_{-13.3}$ | – | – |
| $\Omega_b(\frac{3}{2}^-)$ | 6.35 ± 0.13 | 10 ± 4 | $\Omega_b(\frac{3}{2}^-) \rightarrow \Omega_b K$ | – | 7.4 $^{+11.0}_{-4.8}$ | – | – |
| $\Omega_b(\frac{5}{2}^-)$ | 6.36 ± 0.12 | – | $\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b \pi$ | – | 0.79 $^{+1.2}_{-0.79}$ | 27.3 $^{+28.5}_{-14.0}$ | $\Xi_b(6227)^-$ [15] |
| $\Omega_b(\frac{5}{2}^-)$ | 6.44 ± 0.14 | – | $\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b \pi$ | – | 0.007 $^{+0.223}_{-0.007}$ | 0.12 $^{+0.17}_{-0.08}$ | – |

The sum rule equation for the $\Sigma_b(\frac{3}{2}^-)$ belonging to $[6_F, 2, 1, \lambda]$ is

\begin{equation}
G_{\Sigma_b(\frac{3}{2}^-)}^{(2+)} \propto \frac{f_{S_{\Sigma_b(\frac{3}{2}^-)}}}{f_{A_{\Sigma_b(\frac{3}{2}^-)}}} \propto \frac{f_{S_{\Sigma_b(\frac{3}{2}^-)}}}{f_{A_{\Sigma_b(\frac{3}{2}^-)}}} (\Lambda_{\Sigma_b(\frac{3}{2}^-)} - \omega).
\end{equation}

[1] H. X. Chen, W. Chen, Q. Mao, A. Hosaka, X. Liu and S. L. Zhu, $P$-wave charmed baryons from QCD sum rules, Phys. Rev. D 91, 054034 (2015).

[2] Q. Mao, H. X. Chen, W. Chen, A. Hosaka, X. Liu and S. L. Zhu, QCD sum rule calculation for $P$-wave bottom baryons, Phys. Rev. D 92, 114007 (2015).

[3] H. X. Chen, Q. Mao, W. Chen, A. Hosaka, X. Liu and S. L. Zhu, Decay properties of $P$-wave
TABLE VI: Among all the possible $P$-wave bottom baryons of the flavor $6_f$, we find altogether four $\Sigma_b$, four $\Xi'_b$, and six $\Omega_b$ baryons, with limited widths ($< 100$ MeV) and so capable of being observed. Their masses, mass splittings within the same multiplets, and decay properties are extracted for future experimental searchings. We note that there are considerable uncertainties in our results for absolute values of the bottom baryon masses due to their significant dependence on the bottom quark mass [1, 2]; however, their mass splittings within the same doublets do not depend much on this, so they are produced quite well with much less uncertainties and give more useful information; moreover, we can extract even more useful information from $S$- and $D$-wave strong decay properties of $P$-wave bottom baryons.

| B   | Multiplet | Baryon $(j^P)$ | Mass (GeV) | Difference (MeV) | Decay channel | Total width (MeV) | Candidate |
|-----|-----------|----------------|------------|------------------|---------------|-------------------|-----------|
| $\Sigma_b$ | $[6_f, 1, 1, \lambda]$ | $\Sigma_b(\frac{1}{2}^-$) | 6.06 ± 0.13 | 6 ± 3 | $\Gamma\left( \Sigma_b(\frac{1}{2}^-) \rightarrow \Sigma_b^0 \pi \right) = 14.1 \pm 21.2 \text{ MeV}$ | 14.3 ± 12.2 MeV | – |
|       |           |                |            |                  | $\Gamma\left( \Sigma_b(\frac{1}{2}^-) \rightarrow \Sigma_b^0 \pi \right) = 0.076 \pm 0.076 \text{ MeV}$ |                     |           |
|       |           |                |            |                  | $\Gamma\left( \Sigma_b(\frac{1}{2}^-) \rightarrow \Lambda_b \rho \rightarrow \Lambda_b \pi \pi \right) = 0.087 \text{ MeV}$ |                     |           |
| $\Xi'_b$ | $[6_f, 1, 1, \lambda]$ | $\Xi'_b(\frac{1}{2}^-)$ | 6.07 ± 0.13 | 7 ± 2 | $\Gamma\left( \Xi'_b(\frac{1}{2}^-) \rightarrow \Xi'_b^0 \pi \right) = 4.5 \pm 5.8 \text{ MeV}$ | 4.7 ± 5.8 MeV | – |
|       |           |                |            |                  | $\Gamma\left( \Xi'_b(\frac{1}{2}^-) \rightarrow \Xi'_b^0 \pi \right) = 0.16 \pm 0.16 \text{ MeV}$ |                     |           |
|       |           |                |            |                  | $\Gamma\left( \Xi'_b(\frac{1}{2}^-) \rightarrow \Xi_b^0 \pi \pi \right) = 0.043 \text{ MeV}$ |                     |           |
| $\Xi_b$ | $[6_f, 2, 1, \lambda]$ | $\Xi_b(\frac{1}{2}^-)$ | 6.21 ± 0.11 | 11 ± 5 | $\Gamma\left( \Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b^0 \pi \right) = 10.3 \pm 22.3 \text{ MeV}$ | 12.1 ± 22.3 MeV | 11.5 ± 4 MeV |
|       |           |                |            |                  | $\Gamma\left( \Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b^0 \pi \right) = 10.3 \pm 22.3 \text{ MeV}$ |                     |           |
|       |           |                |            |                  | $\Gamma\left( \Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b^0 \pi \right) = 0.13 \pm 0.06 \text{ MeV}$ |                     |           |
| $\Omega_b$ | $[6_f, 1, 1, \lambda]$ | $\Omega_b(\frac{3}{2}^-)$ | 6.34 ± 0.10 | 6 ± 2 | $\Gamma\left( \Omega_b(\frac{3}{2}^-) \rightarrow \Xi_b^0 \pi \right) = 10.3 \pm 22.3 \text{ MeV}$ | 12.1 ± 22.3 MeV | 11.5 ± 4 MeV |
|       |           |                |            |                  | $\Gamma\left( \Omega_b(\frac{3}{2}^-) \rightarrow \Xi_b^0 \pi \right) = 0.13 \pm 0.06 \text{ MeV}$ |                     |           |
|       |           |                |            |                  | $\Gamma\left( \Omega_b(\frac{3}{2}^-) \rightarrow \Xi_b^0 \pi \right) = 0.13 \pm 0.06 \text{ MeV}$ |                     |           |
| $\Omega_b$ | $[6_f, 2, 1, \lambda]$ | $\Omega_b(\frac{3}{2}^-)$ | 6.34 ± 0.09 | 10 ± 4 | $\Gamma\left( \Omega_b(\frac{3}{2}^-) \rightarrow \Xi_b^0 K \right) = 10.3 \pm 22.3 \text{ MeV}$ | 12.1 ± 22.3 MeV | 11.5 ± 4 MeV |
|       |           |                |            |                  | $\Gamma\left( \Omega_b(\frac{3}{2}^-) \rightarrow \Xi_b^0 \pi \right) = 10.3 \pm 22.3 \text{ MeV}$ |                     |           |
charmed baryons from light-cone QCD sum rules, Phys. Rev. D 95, 094008 (2017).

[4] H. M. Yang, H. X. Chen, E. L. Cui, A. Hosaka and Q. Mao, Decay properties of P-wave heavy baryons accompanied by vector mesons within light-cone sum rules, Eur. Phys. J. C 80, 80 (2020).

[5] H. X. Chen, W. Chen, X. Liu, Y. R. Liu and S. L. Zhu, A review of the open charm and open bottom systems, Rept. Prog. Phys. 80, 076201 (2017).

[6] L. A. Copley, N. Isgur and G. Karl, Charmed Baryons in a Quark Model with Hyperfine Interactions, Phys. Rev. D 20, 768 (1979), Erratum: [Phys. Rev. D 23, 817 (1981)].

[7] M. Karliner, B. Keren-Zur, H. J. Lipkin and J. L. Rosner, The Quark Model and b Baryons, Annals Phys. 324, 2 (2009).

[8] M. Tanabashi et al. [Particle Data Group], Review of Particle Physics, Phys. Rev. D 98, 030001 (2018).

[9] J. G. Körner, M. Kramer and D. Pirjol, Heavy baryons, Prog. Part. Nucl. Phys. 33, 787 (1994).

[10] A. V. Manohar and M. B. Wise, Heavy quark physics, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 10, 1 (2000).

[11] S. Bianco, F. L. Fabbri, D. Benson and L. Bigi, A Cicerone for the physics of charm, Riv. Nuovo Cim. 26N7, 1 (2003).

[12] E. Klempt and J. M. Richard, Baryon spectroscopy, Rev. Mod. Phys. 82, 1095 (2010).

[13] R. Aaij et al. [LHCb Collaboration], Observation of excited \( \Lambda_b^0 \) baryons, Phys. Rev. Lett. 109, 172003 (2012).

[14] T. A. Aaltonen et al. [CDF Collaboration], Evidence for a Bottom Baryon Resonance \( \Lambda_b^{0*} \) in CDF Data, Phys. Rev. D 88, 071101 (2013).

[15] R. Aaij et al. [LHCb Collaboration], Observation of a new \( \Xi_b^- \) resonance, Phys. Rev. Lett. 121, 072002 (2018).

[16] R. Aaij et al. [LHCb Collaboration], Observation of two resonances in the \( \Lambda_b^{0*} \pi^\pm \) systems and precise measurement of \( \Sigma_b^e \) and \( \Sigma_b^{e\pm} \) properties, Phys. Rev. Lett. 122, 012001 (2019).

[17] R. Aaij et al. [LHCb Collaboration], First observation of excited \( \Xi_b^- \) states, arXiv:2001.00851 [hep-ex].

[18] R. Aaij et al. [LHCb Collaboration], Observation of New Resonances in the \( \Lambda_b^{0*} \pi^+\pi^- \) System, Phys. Rev. Lett. 123, 152001 (2019).

[19] A. M. Sirunyan et al. [CMS Collaboration], Study of excited \( \Lambda_b^{0*} \) states decaying to \( \Lambda_b^{0*} \pi^+\pi^- \) in proton-proton collisions at \( \sqrt{s} = 13 \text{ TeV} \), arXiv:2001.06533 [hep-ex].

[20] R. Aaij et al. [LHCb Collaboration], Observation of a new baryon state in the \( \Lambda_b^{0*} \pi^+\pi^- \) mass spectrum, arXiv:2002.05112 [hep-ex].

[21] H. Garcia-Recio, J. Vijnade and A. Valcarce, Faddeev study of heavy baryon spectroscopy, J. Phys. G 34, 961 (2007).

[22] D. Ebert, R. N. Faustov and V. O. Galkin, Masses of excited heavy baryons in the relativistic quark model, Phys. Lett. B 659, 612 (2008).

[23] W. Roberts and M. Pervin, Heavy baryons in a quark model, Int. J. Mod. Phys. A 23, 2817 (2008).

[24] P. G. Ortega, D. R. Entem and F. Fernandez, Quark model description of the \( \Lambda_b^* \) (2940) \(^+\) as a molecular \( D^*N \) state and the possible existence of the \( \Lambda_b^*(6248) \), Phys. Lett. B 718, 1381 (2013).

[25] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka and K. Sadato, Spectrum of heavy baryons in the quark model, Phys. Rev. D 92, 114029 (2015).

[26] H. Nagahiro, S. Yasui, A. Hosaka, M. Oka and H. Nouni, Structure of charmed baryons studied by pionic decays, Phys. Rev. D 95, 014023 (2017).

[27] K. L. Wang, Q. F. Li and X. H. Zhong, Interpretation of the newly observed \( \Sigma_b(6097)^e \) and \( \Xi_b(6227)^{-} \) states as the P-wave bottom baryons, Phys. Rev. D 99, 014011 (2019).

[28] L. X. Gutierrez-Guerrero, A. Bashir, M. A. Bedolla and E. Santopinto, Masses of Light and Heavy Mesons and Baryons: A Unified Picture, Phys. Rev. D 100, 114032 (2019).

[29] Y. Kawakami and M. Harada, Singly heavy baryons with chiral partner structure in a three-flavor chiral model, Phys. Rev. D 99, 094016 (2019).

[30] L. Y. Xiao, K. L. Wang, M. S. Liu and X. H. Zhong, Possible interpretation of the newly observed \( \Omega_b \) states, arXiv:2001.05110 [hep-ph].

[31] C. Garcia-Recio, J. Nieves, O. Romanets, L. S. Salcedo and L. Tolos, Odd parity bottom-flavored baryon resonances, Phys. Rev. D 87, 034032 (2013).

[32] W. H. Liang, C. W. Xiao and E. Oset, Baryon states with open beauty in the extended local hidden gauge approach, Phys. Rev. D 89, 054023 (2014).

[33] C. S. An and H. Chen, Observed \( \Omega_b^e \) resonances as pentaquark states, Phys. Rev. D 96, 034012 (2017).

[34] G. Montana, A. Pei and A. Ramos, A meson-baryon molecular interpretation for some \( \Omega_b^e \) excited states, Eur. Phys. J. A 54, 64 (2018).

[35] V. R. Debastiani, J. M. Dias, W. H. Liang and E. Oset, Molecular \( \Omega_b^e \) states generated from coupled meson-baryon channels, Phys. Rev. D 97, 094035 (2018).

[36] R. Chen, A. Hosaka and X. Liu, Searching for possible \( \Omega_b^e \)-like molecular states from meson-baryon interaction, Phys. Rev. D 97, 036016 (2018).

[37] J. Nieves, R. Pavao and L. Tolos, \( \Omega_b^e \) excited states within a SU(6)$_{lsf}$ ×HQSS model, Eur. Phys. J. C 78, 114 (2018).

[38] Y. Huang, C. J. Xiao, Q. F. Li, R. Wang, J. He and L. Geng, Strong and radiative decays of \( D_s \) molecule states and newly observed \( \Omega_b^e \) states, Phys. Rev. D 97, 094013 (2018).

[39] J. Nieves, R. Pavao and L. Tolos, \( \Xi_b^- \) and \( \Xi_b^0 \) excited states within a SU(6)$_{lsf}$ ×HQSS model, Eur. Phys. J. C 80, 22 (2020).

[40] W. H. Liang and E. Oset, The observed \( \Omega_b^e \) spectrum and meson-baryon molecular states, arXiv:2001.02929 [hep-ph].

[41] Q. X. Yu, R. Pavao, V. R. Debastiani and E. Oset, Description of the \( \Xi_b^- \) and \( \Xi_b^0 \) states as molecular states, Eur. Phys. J. C 79, 167 (2019).

[42] Y. Huang, C. J. Xiao, L. S. Geng and J. He, Strong decays of the \( \Sigma_b(6227) \) as a \( \Sigma_b K \) molecule, Phys. Rev. D 99, 014008 (2019).

[43] B. Chen, K. W. Wei, X. Liu and A. Zhang, Role of newly discovered \( \Sigma_b(6227)^{-} \) for constructing excited bottom baryon family, Phys. Rev. D 98, 031502 (2018).

[44] B. Chen and X. Liu, Assigning the newly reported \( \Sigma_b(6097) \) as a P-wave excited state and predicting its partners, Phys. Rev. D 98, 074032 (2018).

[45] P. Yang, J. J. Guo and A. Zhang, Identification of the newly observed \( \Sigma_b(6097)^e \) baryons from their strong decays, Phys. Rev. D 99, 034018 (2019).

[46] W. Liang and Q. F. Liu, Strong decays of the newly ob-
served narrow $\Omega_b$ structures, arXiv:2001.02221 [hep-ph].

[47] J. X. Lu, Y. Zhou, H. X. Chen, J. J. Xie and L. S. Geng, *Dynamically generated $J^P = 1/2^+ (3/2^+)$ singly charmed and bottom heavy baryons*, Phys. Rev. D 92, 014036 (2015).

[48] H. Y. Cheng and C. K. Chua, *Strong Decays of Charmed Baryons in Heavy Hadron Chiral Perturbation Theory*, Phys. Rev. D 75, 014006 (2007).

[49] H. Y. Cheng and C. K. Chua, *Strong Decays of Charmed Baryons in Heavy Hadron Chiral Perturbation Theory: An Update*, Phys. Rev. D 92, 074014 (2015).

[50] T. M. Aliev, K. Azizi, Y. Sarac and H. Sundu, *Determination of the quantum numbers of $\Omega_b (6097)^+$ via their strong decays*, Phys. Rev. D 99, 094003 (2019).

[51] T. M. Aliev, K. Azizi, Y. Sarac and H. Sundu, *Structure of the $\Omega_b (6227)^+$ resonance*, Phys. Rev. D 98, 094014 (2018).

[52] Z. G. Wang, *Analysis of the $\Omega_b (6316)$, $\Omega_b (6330)$, $\Omega_b (6340)$ and $\Omega_b (6350)$ with QCD sum rules*, arXiv:2001.02961 [hep-ph].

[53] M. Padmanath, R. G. Edwards, N. Mathur and M. Peardon, *Excited-state spectroscopy of singly, doubly and triply-charmed baryons from lattice QCD*, arXiv:1311.4806 [hep-lat].

[54] B. Chen, K. W. Wei and A. Zhang, *Assignments of $\Lambda_b$ and $\Sigma_b$ baryons in the heavy quark-light diquark picture*, Eur. Phys. J. A 51, 82 (2015).

[55] M. Karliner and J. L. Rosner, *Prospects for observing the lowest-lying odd-parity $\Sigma_c$ and $\Sigma_b$ baryons*, Phys. Rev. D 92, 074026 (2015).

[56] C. K. Chua, *Color-allowed bottom baryon to charmed baryon nonleptonic decays*, Phys. Rev. D 99, 014023 (2019).

[57] M. Karliner and J. L. Rosner, *Scaling of $P$-wave excitation energies in heavy-quark systems*, Phys. Rev. D 98, 074026 (2018).

[58] D. Jia, W. N. Liu and A. Hosaka, *Regge Behaviors in Low-lying Singly Charmed and Bottom Baryons*, Phys. Rev. D 101, 034016 (2020).

[59] V. Crede and W. Roberts, *Progress towards understanding baryon resonances*, Rept. Prog. Phys. 76, 076301 (2013).

[60] H. Y. Cheng, *Charmed baryons circa 2015*, Front. Phys. (Beijing) 10, 101406 (2015).

[61] S. Capstick and N. Isgur, *Baryons in a Relativized Quark Model with Chromodynamics*, Phys. Rev. D 34, 2809 (1986).

[62] H. Mutuk, arXiv:2002.03695 [hep-ph].

[63] K. L. Wang, Q. F. Liu and X. H. Zhong, *Interpretation of the newly observed $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ states in a chiral quark model*, Phys. Rev. D 100, 114035 (2019).

[64] W. Liang, Q. F. Liu and X. H. Zhong, *Canonical interpretation of the newly observed $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ via strong decay behaviors*, Phys. Rev. D 100, 054013 (2019).

[65] B. Chen, S. Q. Luo, X. Liu and T. Matsuki, *Interpretation of the observed $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ states as 1D bottom baryons*, Phys. Rev. D 100, 094032 (2019).

[66] K. Azizi, Y. Sarac and H. Sundu, *The $\Lambda_b(6146)^0$ state newly observed by LHCb*, arXiv:2001.04953 [hep-ph].

[67] H. Y. Cheng, Q. Mao, A. Hosaka, X. Liu and S. L. Zhu, *D-wave charmed and bottomed baryons from QCD sum rules*, Phys. Rev. D 94, 114016 (2016).

[68] Q. Mao, H. X. Chen, A. Hosaka, X. Liu and S. L. Zhu, *D-wave heavy baryons of the SU(3) flavor 6p*, Phys. Rev. D 96, no. 7, 074021 (2017).

[69] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *QCD and Resonance Physics. Theoretical Foundations*, Nucl. Phys. B 147, 385 (1979).

[70] L. J. Reinders, H. Rubinstein and S. Yazaki, *Hadron Properties from QCD Sum Rules*, Phys. Rept. 127, 1 (1985).

[71] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, *Nucl. Phys. B 312, 509 (1989).*

[72] V. M. Braun and I. E. Filyanov, Z. Phys. C 44, 157 (1989) [Sov. J. Nucl. Phys. 50, 511 (1989) ] [Yad. Fiz. 50, 818 (1989)].

[73] V. L. Chernyak and I. R. Zhitnitsky, *Nucl. Phys. B 345, 137 (1990).*

[74] P. Ball, *Theoretical update of pseudoscalar meson distribution amplitudes of higher twist: The Nonsinglet case*, JHEP 9901, 010 (1999).

[75] P. Ball, V. M. Braun and A. Lenz, *Higher-twist distribution amplitudes of the K meson in QCD*, JHEP 0605, 004 (2006).

[76] B. Grinstein, *The Static Quark Effective Theory*, Nucl. Phys. B 339, 253 (1990).

[77] E. Eichten and B. R. Hill, *An Effective Field Theory for the Calculation of Matrix Elements Involving Heavy Quarks*, Phys. Lett. B 234, 511 (1990).

[78] A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, *Heavy Meson Form-factors From QCD*, Nucl. Phys. B 343, 1 (1990).

[79] E. Bagan, P. Ball, V. M. Braun and H. G. Dosch, *QCD sum rules in the effective heavy quark theory*, Phys. Lett. B 278, 457 (1992).

[80] M. Neubert, *Heavy meson form-factors from QCD sum rules*, Phys. Rev. D 45, 2451 (1992).

[81] D. J. Broadhurst and A. G. Grozin, *Operator product expansion in static quark effective field theory: Large perturbative correction*, Phys. Lett. B 274, 421 (1992).

[82] T. Huang and C. W. Luo, *Light quark dependence of the Isgur-Wise function from QCD sum rules*, Phys. Rev. D 50, 5775 (1994).

[83] Y. B. Dai, C. S. Huang, M. Q. Huang and C. Liu, *QCD sum rules for masses of excited heavy mesons*, Phys. Lett. B 390, 350 (1997).

[84] P. Colangelo, F. De Fazio and N. Paver, *Universal tau(1/2)(y) Isgur-Wise function at the next-to-leading order in QCD sum rules*, Phys. Rev. D 58, 116005 (1998).

[85] S. Groote, J. G. Korner and O. I. Yakovlev, *QCD sum rules for heavy baryons at next-to-leading order in alphases*, Phys. Rev. D 55, 3016 (1997).

[86] S. L. Zhu, *Strong and electromagnetic decays of p wave heavy baryons $\Lambda_b^+$, $\Lambda_b^+$*, Phys. Rev. D 61, 114019 (2000).

[87] J. P. Lee, C. Liu and H. S. Song, *QCD sum rule analysis of excited Lambda(c) mass parameter*, Phys. Lett. B 476, 303 (2000).

[88] C. S. Huang, A. L. Zhang and S. L. Zhu, *
cited heavy baryon masses in HQET QCD sum rules, Phys. Lett. B 492, 288 (2000).

[92] D. W. Wang and M. Q. Huang, Excited heavy baryon masses to order $\Lambda_{QCD}/m_Q$ from QCD sum rules, Phys. Rev. D 68, 034019 (2003).

[93] F. O. Duraes and M. Nielsen, QCD sum rules study of $\Xi_c$ and $\Xi_b$ baryons, Phys. Lett. B 658, 40 (2007).

[94] X. Liu, H. X. Chen, Y. R. Liu, A. Hosaka and S. L. Zhu, Bottom baryons, Phys. Rev. D 68, 034019 (2003).

[95] D. Zhou, E. L. Cui, H. X. Chen, L. S. Geng, X. Liu and S. L. Zhu, D-wave heavy-light mesons from QCD sum rules, Phys. Rev. D 90, 114035 (2014).

[96] D. Zhou, H. X. Chen, L. S. Geng, X. Liu and S. L. Zhu, F-wave heavy-light meson spectroscopy in QCD sum rules and heavy quark effective theory, Phys. Rev. D 77, 014031 (2008).

[97] H. X. Chen, E. L. Cui, A. Hosaka, Q. Mao and H. M. Yang, Excited $\Omega_b$ baryons and fine structure of strong interaction, arXiv:2001.02147 [hep-ph].

[98] E. L. Cui, H. M. Yang, H. X. Chen and A. Hosaka, Identifying the $\Xi_b(6227)$ and $\Xi_b(6097)$ as P-wave bottom baryons of $J^P = 3/2^-$, Phys. Rev. D 92, 114015 (2015).

[99] P. Ball and R. Zwicky, $B_{d,s} \to \rho, \omega, K^*, \phi$ decay form-factors from light-cone sum rules revisited, Phys. Rev. D 71, 014029 (2005).

[100] P. Ball and V. M. Braun, Exclusive semileptonic and rare $B$ meson decays in QCD, Phys. Rev. D 58, 094016 (1998).

[101] P. Ball, V. M. Braun, Y. Koike and K. Tanaka, Higher twist distribution amplitudes of vector mesons in QCD: Formalism and twist - three distributions, Nucl. Phys. B 529, 323 (1998).

[102] P. Ball and V. M. Braun, Higher twist distribution amplitudes of vector mesons in QCD: Twist - 4 distributions and meson mass corrections, Nucl. Phys. B 543, 201 (1999).

[103] P. Ball and G. W. Jones, Twist-3 distribution amplitudes of $K^*$ and phi mesons, JHEP 0703, 069 (2007).

[104] P. Ball, V. M. Braun and A. Lenz, Twist-4 distribution amplitudes of the $K^*$ and phi mesons in QCD, JHEP 0708, 090 (2007).

[105] K. C. Yang, W. Y. P. Hwang, E. M. Henley and L. S. Kisslinger, QCD sum rules and neutron proton mass difference, Phys. Rev. D 47, 3001 (1993).

[106] W. Y. P. Hwang and K. C. Yang, QCD sum rules: $\Delta - N$ and $\Sigma_0 - \Lambda$ mass splittings, Phys. Rev. D 49, 460 (1994).

[107] S. Narison, Withdrawn: QCD as a theory of hadrons from partons to confinement, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 17, 1 (2002), arXiv:hep-ph/0205006.

[108] V. Gimenez, V. Lubicz, F. Mescia, V. Porretti and J. Reyes, Operator product expansion and quark condensate from lattice QCD in coordinate space, Eur. Phys. J. C 41, 535 (2005).

[109] M. Jamin, Flavor symmetry breaking of the quark condensate and chiral corrections to the Gell-Mann-Oakes-Renner relation, Phys. Lett. B 538, 71 (2002).

[110] B. Ioffe and K. N. Zyablyuk, Gluon condensate in charmonium sum rules with three loop corrections, Eur. Phys. J. C 27, 229 (2003).

[111] A. A. Ovchinnikov and A. A. Pivovarov, QCD Sum Rule Calculation Of The Quark Gluon Condensate, Sov. J. Nucl. Phys. 48, 721 (1988) [Yad. Fiz. 48, 1135 (1988)].

[112] P. Colangelo and A. Khodjamirian, At the Frontier of Particle Physics/Handbook of QCD (World Scientific, Singapore, 2001), Volume 3, 1495.