The Law of Diminishing Returns in Determining the Environmental Cost of Mining in the Eastern Mediterranean Sea

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Abstract: Environmental standards, as independent/explanatory variables of C_{21} capital costing functions, may refer to parts of space that we want to control in order to protect subsystems that are particularly sensitive and/or importance. In these cases, we need a quantitative relationship that links the environmental characteristics of the source of pollution to those of the reference/control points. In this article we will identify the capital cost functions C_{21} = f (S_f), where S_f is a spatially distributed parameter (e.g. BOD), characteristic of an environmental model.

Key words: Law of diminishing returns, environmental cost, BOD, pollution.

1. Introduction

In the present work, two subsystems are examined, one-dimensional river-type flow and full three-dimensional artificial lake-type dispersion. In the first subsystem, the control station is located (AC) downstream of point A of the disposal of treated waste Q and BOD Se. Untreated waste has BOD S_0 while the actual capacity of the plant (expressed in secondary treatment reactor volume units), determined by minimizing the capital cost C_{21}, is V [1]. The biological material balance at point A is S_r Q_r + S_e Q (Q_r + Q), where S_r and SA are the BOD values of the river before and after mixing with the treated waste, respectively, while Q_r is the volumetric flow of the river before point A [2]. Taking into account that the one-dimensional flow in river simulates flow in PFR, we have

\[ S_e = S_0 \exp \left( -k \frac{Q_r + Q}{u} \right) \]

Where: Sc is the BOD value at point C, k is the constant rate of degradation/neutralization of waste in the conditions prevailing in the river and u is the average linear velocity of water mass transfer in it, the above balance is written as follows:

\[ S_r Q_r + S_e Q = S_f \exp \left[ k \frac{(AC)}{u} \right] (Q_r + Q) \] (1)

2. Implementation

The S_e = S_f commitment ensures that V = V_{\text{min}} to minimize the capital cost C_{21}: If V > V_{\text{min}} then the capital cost increases in order to increase the active capacity, resulting in S_e < S_f, i.e. excessive water purification, without this required by the application of the environmental standard [3, 4]. If V < V_{\text{min}} then results in S_e > S_f, i.e. insufficient water purification in relation to the predetermined and agreed (as provided either by the operating license of the industrial plant, to which the waste treatment plant belongs, or by the plant itself, if it has been authorized independently, because it serves many industrial/domestic users) BOD value resulting in the imposition of financial and administrative sanctions. From the models of decomposition/neutralization of waste in CFSTR and PFR type installations (real capacity V_c and V_p respectively), we obtain:

\[ S_e = S_0 / (1+k_0V_c/Q) \] and \[ S_e = S_0 \exp (-k_0V_p/Q) \] (2)
Where the \( k_0 \) is the rate of decomposition/neutralization rate of the waste under the conditions prevailing in the secondary treatment reactors. In the first case (CFSTR type installation the material balance (1) is written:

\[
S, Q_r + [S, / (1 + k, V, / Q)]Q = S, (Q, + Q) \exp[k(AC) / u] \\
V, = \frac{Q}{k_0} \frac{S, Q}{S, (Q, + Q) \exp[k(AC) / u] - S, Q_r}
\]

So the capital cost is given by the relation:

\[
C_{21} = B_1 + \frac{Q B_2}{k_0} \left[ \frac{S, Q}{S, (Q, + Q) \exp[k(AC) / u] - S, Q_r} \right]^{1 / Q'}
\]

While the verifying the validity of the Law of Diminishing Returns is a function of differential costs (as to the change of the environmental model \( S_f \)),

\[
\frac{dC_{21}}{dS_f} = \frac{B_2 Q}{k_0} \frac{S, Q (Q, + Q) \exp[k(AC) / u]}{[S, (Q, + Q) \exp[k(AC) / u] - S, Q_r]} < 0
\]

In the second case (PFR type installation) the material balance (1) is written:

\[
S, Q_r + S_0 Q \exp[k, V, / Q] = S, (Q, + Q) \exp[k(AC) / u] \\
V, = \frac{Q}{k_0} \ln \frac{S, Q}{S, (Q, + Q) \exp[k(AC) / u] - S, Q_r}
\]

So the capital cost is given by the relation:

\[
C_{21} = B_1 + \frac{Q B_2}{k_0} \frac{S, Q}{S, (Q, + Q) \exp[k(AC) / u] - S, Q_r}
\]

While the verifying the validity of the Law of Diminishing Returns is a function of differential costs (as to the change of the environmental model \( S_f \)) is:

\[
\frac{dC_{21}}{dS_f} = \frac{Q B_2}{k_0} \frac{(Q, + Q) \exp[k(AC) / u]}{[S, (Q, + Q) \exp[k(AC) / u] - S, Q_r]} < 0
\]

In the second subsystem, the control station is located inside the artificial lake, where the treated waste \( Q \) and BOD \( S_e \) are disposed of [5, 6]. As before, the untreated waste has a BOD \( S_0 \) while the actual capacity of the plant (expressed in units of the secondary treatment reactor volume), determined by minimizing the capital cost \( C_{21} \), is \( V \). Considering that the system is simulated with CFSTR of real capacity \( V_l \) in steady state conditions (with waste neutralization rate constant \( k \), flow outflow \( Q \) and BOD \( S_t = S_f \) to minimize the capacity \( V \) of the waste treatment plant, hence the corresponding capital cost), balance of biological material, gives the relation:

\[
\frac{S_f}{S_e} = 1/(1 + k V/Q)
\]

Fig. 1. Illustration of the two cases referring to the determination of the capacity \( (V_c \) or \( V_p) \) of the waste treatment plant (type CFSTR or PFR respectively) in the first subsystem. The minimization of \( V_c \) and \( V_p \), minimization of the capital cost, is achieved for \( S_e = S_0 \), where \( S_0 \), the maximum permissible value of \( S \) according to the valid environmental standard and/or the approved environmental study that accompanies the granted operating license of the industrial enterprise, which produces or simply treats (subcontracted) liquid waste with average volumetric flow \( Q \) and BOD index \( S_0 \).
The Law of Diminishing Returns in Determining the Environmental Cost of Mining in the Eastern Mediterranean Sea

3. Conclusion

Because models (2) are still valid, we again distinguish the respective two cases. In the first (installation of CFSTR type with \( V = V_c \)) the (3) is written:

\[
S_f = \frac{S_0}{(1 + kV_c/Q)(1 + kV_c/Q)} \Rightarrow V_c = \frac{Q}{k_o} \left( \frac{S_0}{S_f(1 + kV_c/Q)} - 1 \right)
\]

So the capital cost is given by the relation:

\[
C_{21} = C_2 + \frac{QB_2}{k_o} \left[ \frac{S_0}{S_f(1 + kV_c/Q)} - 1 \right] (t'Q')
\]

While the verifying the validity of the Law of Diminishing Returns as a function of differential cost (as to the change of the environmental model \( S_f \)) is:

\[
\frac{dC_21}{dS_f} = -\frac{QB_2}{k_o} \frac{S_0}{S_f(1 + kV_c/Q)} \frac{1}{(1 + kV_c/Q)} < 0
\]

In the second case (PFR type installation, with \( V = V_p \)), (3) is written:

\[
S_f = \frac{S_0 \exp(-kV_p/Q)}{1 + kV_p/Q} \Rightarrow V_p = \frac{Q}{k_o} \ln \left( \frac{S_0}{S_f(1 + kV_p/Q)} \right)
\]

So the capital cost is given by the relation:

\[
C_{21} = C_2 + \frac{QB_2}{k_o} \left[ \ln \left( \frac{S_0}{S_f(1 + kV_p/Q)} \right) \right] (t'Q')
\]

If \( S_f < S_f \) of the water stored in the artificial lake, it is possible to increase up to \( V = V_{max} \), in which case the stocks for industrial/agricultural use increase, while the required capacity of the biological treatment plant decreases, hence the corresponding capital cost. This can be done by zeroing the output of the lake, for a period of time \( t = (V_{max} - V_0)/Q \), where \( V = V_0 \), for \( t' = 0 \) and \( Q \), the volumetric supply of waste to and from the biological plant at which the input and output BOD value is \( S_0' \) and \( S_0 \), respectively, while the liquid phase volume is \( V_c \) or \( V_p \) with \( c, p \), denoting CFSTR or PFR reactors, respectively.

Considering that, in order to minimize the capital cost, we must have \( S = S_f \) (I) when CFSTR is used in the biological treatment plant of the domestic type of wastewater and industrial organic waste (so \( S_0/S_0' = [1 + kV_c/Q]^{-1} \)) where \( k_n \) the rate of degradation/neutralization of the organic load of the waste in the treatment plant:
The Law of Diminishing Returns in Determining the Environmental Cost of Mining in the Eastern Mediterranean Sea

\[ S_j = S_0 e^{-\beta_j} + \frac{S_0(1-e^{-\beta_j})}{1 + k_j V_o / Q (V_o / Q + t) k} \]

\[ V_o = \frac{Q}{k_j} \left( \frac{S_0(1-e^{-\beta_j})}{(S_j - S_0 e^{-\beta_j})(V_o / Q + t)k} - 1 \right) \]

So the capital cost is given by the relationship:

\[ C_{21} = \left( B_1 + \frac{B_2 Q}{k_j} \left( \frac{S_0'(1-e^{-\beta_j})}{(S_j - S_0 e^{-\beta_j})(V_o / Q + t)k} - 1 \right) \right) / (t'Q') \]

While the verifying the validity of the Law of Diminishing Returns is a function of differential costs (as to the change of the environmental model \( S_j \)) is:

\[ \frac{dC_{21}}{dS_j} = -\frac{QB_2}{k_j} \left( \frac{S_0'(1-e^{-\beta_j})}{(S_j - S_0 e^{-\beta_j})(V_o / Q + t)k} \right) (S_j - S_0 e^{-\beta_j}) < 0 \]

(II) When PFR (where \( S_j/S_0 = \exp[-k_j V_o/Q] \) where \( k_j \) is the rate of decomposition/neutralization of the organic load of the waste at the treatment plant is used in the biological treatment plant of domestic sewage and industrial organic waste).

\[ S_j = S_0 e^{-\beta_j} + \frac{S_0'(1-e^{-\beta_j})(e^{-\beta_j V_o / Q})}{(V_o / Q + t)k} \]

\[ V_o = \frac{Q}{k_j} \ln \left( \frac{S_0'(1-e^{-\beta_j})}{(S_j - S_0 e^{-\beta_j})(V_o / Q + t)k} \right) \]

So the capital cost is given by the relationship:

\[ C_{21} = \left( B_1 + \frac{B_2 Q}{k_j} \ln \left( \frac{S_0'(1-e^{-\beta_j})}{(S_j - S_0 e^{-\beta_j})(V_o / Q + t)k} \right) \right) / (t'Q') \]

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