Three-loop MSSM Higgs-Boson Mass Predictions and Regularization by Dimensional Reduction

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Abstract

The evaluation of three-loop contributions to the MSSM Higgs-boson mass is considered at the orders enhanced by the strong gauge coupling and top or bottom Yukawa couplings, i.e. at the orders $\mathcal{O}(\alpha_t\alpha_b^2, \alpha_t^2\alpha_s, \alpha_b^3)$. We prove that regularization by dimensional reduction preserves supersymmetry at the required level. Thus generating counterterms by multiplicative renormalization is correct. Technically, we extend a previous two-loop analysis to the three-loop level. The extension covers not only the genuine three-loop Higgs potential counterterms but also a large sector of two-loop counterterms, required for subrenormalization.

1 Introduction

After the discovery of the Higgs boson at the LHC [1], the Higgs boson mass $M_h$ has become a precision observable. Supersymmetric (SUSY) extensions of the Standard Model are special because they allow to predict the Higgs boson mass thanks to SUSY relations between the Higgs potential and known gauge couplings. Specifically in the MSSM, $M_h$ is smaller than the $Z$ boson mass at tree level, and calculable higher-order corrections can push it up to the measured value, resulting in important constraints on SUSY parameters.

Precision evaluations of these MSSM higher-order corrections have a long history. One-loop and leading 2-loop corrections have been evaluated long ago; we refer to the recent review [2] for details and references. The leading corrections are governed by Yukawa couplings $\alpha_{t,b}$ and the strong gauge coupling $\alpha_s$. They can be evaluated in the so-called gaugeless limit, where the electroweak gauge couplings and $M_{W,Z}$ go to zero with fixed ratio $M_W/M_Z$ and fixed Higgs vacuum expectation values. At the 3-loop level, the full corrections of order $\mathcal{O}(\alpha_t\alpha_s^2)$ have been evaluated in the gaugeless limit in Refs. [3] and further developed in
Refs. [4, 5]. Leading and subleading 3-loop large logarithms have been evaluated in Ref. [6]. Further recent progress includes 2-loop computations beyond the gaugeless limit and the resulting zero-momentum approximation [7, 9], resummation of large logarithms using EFT and renormalization group methods [10, 11] and the development of hybrid approaches [12, 13]. The remaining theory uncertainties of these current calculations depends on the details of the spectrum and on the calculational scheme, see e.g. the recent discussions in Refs. [11, 13]. However, the theoretical uncertainty is significantly larger than the experimental one — hence it is motivated to further improve the accuracy of theoretical calculations.

Here we consider the technically necessary step of regularization of higher-order loop contributions. The commonly used scheme is regularization by dimensional reduction (DRED) [14, 15] (see also the recent review [16] of regularization schemes). This scheme is consistent with SUSY for the 2-loop calculations in the gaugeless limit [17]. We ask whether the same is true at the 3-loop level. If this turned out not to be the case, additional SUSY-restoring counterterms would be required, which would affect the finite, non-logarithmic parts of a 3-loop calculation, i.e. the parts which cannot be obtained by renormalization-group methods.

In the remainder of this Introduction, we provide more details on potential non-symmetric counterterms and DRED.

In a 3-loop calculation of the Higgs boson masses in the gaugeless limit of the MSSM several types of counterterms are required:

- genuine 3-loop counterterms in the Higgs sector;
- 1-loop and 2-loop counterterms in the Higgs, Yukawa, and SUSY-QCD sectors.

Usually, the structure of the counterterm Lagrangian and Feynman rules is generated by a renormalization transformation respecting the symmetries of the theory. In this way the generated counterterms respect in particular SUSY. The present paper investigates whether this is correct and sufficient for calculations of MSSM Higgs boson masses at the 3-loop level in the gaugeless limit.

Cases in which a renormalization transformation respecting SUSY is not enough and additional, non-SUSY counterterms are needed are familiar in dimensional regularization (DREG). DREG breaks SUSY already at the 1-loop level, and non-SUSY counterterms have to be added e.g. to the gluino–squark–quark interaction. Such non-SUSY counterterms required in DREG have been evaluated and documented in Refs. [18–20].

For DRED, on the other hand, many studies have confirmed the compatibility with SUSY and the absence of non-SUSY counterterms, for overviews of earlier
results see [17,21,22]. In particular, several 2-loop cases of relevance for us have been studied: self energies and SUSY transformations of chiral multiplets [22] in general SUSY gauge theories, quartic Higgs coupling in the gaugeless limit of the MSSM [17], three-point interactions in the gauge/gaugino sector of SUSY Yang-Mills theories [23] and in the gauge/gaugino/quark/squark sector of SUSY QCD [24]. The existing studies do not cover the case of potential, finite non-SUSY counterterms to Higgs masses at the 3-loop level.

Also beyond the question of SUSY preservation, the status of DRED is similarly mature as the one of dimensional regularization: Both schemes can be formulated in a mathematically consistent way and the quantum action principle holds [22,25], and the two schemes are equivalent [26], i.e. one can always find local transition counterterms which translate between the two schemes. The infrared divergence structure of DRED regularized QCD amplitudes has been understood [27]. All these statements require to distinguish the formally $D$-dimensional parts of vector fields and the remaining $(4 - D)$-dimensional parts (the so-called $\epsilon$-scalars), and in particular to treat the renormalization and factorization of $\epsilon$-scalars as independent. The need for the independent renormalization has also been stressed in concrete multi-loop calculations [28].

The recent progress in understanding regularizations and the need for more precise Higgs boson mass computations motivates our study. The outline of the remainder of the paper is as follows. In section 2 we describe the setup and notation, and we discuss the different structure of SUSY and non-SUSY counterterms. We also explain the methods used to determine potential non-SUSY counterterms. Section 3 is devoted to the analysis of 2-loop counterterms, which are required for subrenormalization and shows that DRED is consistent with SUSY on this level. Section 4 finally discusses the proof that no SUSY-restoring counterterm is required for the MSSM Higgs boson mass calculations at the 3-loop level in the gaugeless limit.

2 Setup and SUSY versus non-SUSY counterterms

2.1 Setup

In this subsection we describe our setup, provide relevant notation and discuss the structure of the usual symmetric counterterms. We work in the MSSM with neglected electroweak gauge couplings, $g_{1,2} \to 0$, the so-called gaugeless limit. The relevant fundamental fields are the Higgs and Higgsino doublets $H_k^i$, $\tilde{H}_k^i$ (where $k = 1, 2$ and $i$ is the SU(2) index and where the vacuum expectation values have already been split off), the left-handed quark and squark doublets $q_L^i$
and $\bar{q}^L_i$, and the right-handed quark and squark singlets $u_R, d_R, \tilde{u}_R, \tilde{d}_R$, as well as the gluon and gluino fields $A^\mu, \tilde{g}$. All spinors are taken as 4-spinors; for the quarks we abbreviate $q_L = P_L q, u_R = P_R u$ etc; the gluino spinor is a Majorana spinor; the Higgsino spinors are taken as purely left-handed, $\tilde{H}^i_k \equiv \tilde{H}^{iL}_k = P_L \tilde{H}^{i}_k$. Gluon and gluino fields without SU(3) index are taken as contracted with Gell-Mann matrices, $\tilde{g} = \tilde{g}^a \frac{\lambda^a}{2}$, etc, and (s)quark colour indices are always suppressed.

The Lagrangian containing the dimension-4-terms relevant for our analysis can be decomposed as

$$L = L_{\text{kin}} + L_{\text{yu}} + L_{\text{gluino}} - V_{\text{quartic}}.$$  

(1) Here $L_{\text{kin}}$ contains the usual kinetic terms for all fields with the SU(3) gauge covariant derivative,

$$L_{\text{kin}} = \sum_{q=\text{u,d}} \bar{q}i \partial q + \sum_{k=1,2} |\partial^\mu H_k|^2 + \text{kinetic terms for } \tilde{q}, \tilde{H}, A^\mu, \tilde{g},$$

(2) $D^\mu = \partial^\mu + ig_s A^\mu$.  

(3) The remaining terms are the up-type Yukawa interactions, gluino interactions and the quartic Higgs potential:

$$L_{\text{yu}} = y_u \epsilon_{ij} \left( H_1^i \bar{u}^L q^j + \tilde{u}^i_R \tilde{H}^{jL}_2 + \bar{u} \tilde{H}^{i}_2 \tilde{q}^j_l \right) + \text{h.c.},$$

(4)  

$$L_{\text{gluino}} = -\sqrt{2}g_s \sum_{q=\text{u,d}} \left( \tilde{q}^i_L g \bar{q} q^j + \tilde{q}^i_R \tilde{q} q^j - \tilde{q}^i_R g_\tilde{q} q^j - g_\tilde{q} \bar{q} \tilde{q} \right),$$

(5)  

$$V_{\text{quartic}} = \frac{g_1^2 + g_2^2}{8} \left( |H_1|^2 - |H_2|^2 \right)^2 + \frac{g_2^2}{2} \left| H_1^1 H_2^2 \right|^2.$$

(6) In the last term the electroweak gauge couplings $g_1, g_2$ have been retained for reference. We have only considered up-type Yukawa couplings in $L_{\text{yu}}$ and suppressed corresponding down-type Yukawa coupling terms because they can be treated in the same way. We have also suppressed quartic terms involving squarks and Lagrangian terms of dimension $\leq 3$ and soft SUSY breaking terms since they are irrelevant for our goal of determining potential SUSY breaking in the dimension-4 sector of the Higgs potential.

The basic symmetries of the MSSM are Lorentz and gauge invariance and (softly broken) SUSY. The MSSM further respects a softly broken Peccei-Quinn symmetry, which forbids a dimension-4 mixing between the two Higgs doublets. The symmetries can be encoded in Slavnov-Taylor and Ward identities, allowing a full study of the renormalization of the MSSM [29] and providing the most general form of symmetric counterterms, i.e. the correct counterterm structure for the desirable case where the regularization preserves the symmetries of the theory. Specializing the results of Ref. [29] for the full MSSM to the case at
hand we obtain the following multiplicative renormalization transformation of
the parameters and fields:

\[ g_s \rightarrow Z_{g_s} g_s, \]  
\[ y_u \rightarrow Z_{y_u} y_u, \]  
\[ \varphi \rightarrow \sqrt{Z_{\varphi}} \varphi \text{ for all fields } \varphi = u_{L,R}, d_{L,R}, \tilde{u}_{L,R}, \tilde{d}_{L,R}, H_k, \tilde{H}_k, A^\mu, \tilde{g}, \]  
\[ Y_\varphi \rightarrow \sqrt{Z_{\varphi}^{-1}} Y_\varphi. \]  

In the last of these equations we have also specified the renormalization trans-
formation for sources of BRS transformations, which will appear later in the
Slavnov-Taylor identity.

Applying this symmetric renormalization transformation to the classical ac-
tion \( \Gamma_{cl} \) generates a bare action (i.e. the sum of classical and counterterm action)

\[ \Gamma_{cl} \rightarrow \Gamma_{cl+ct-sym}, \]  

which contains the usual symmetric counterterms.

For our purposes, the decisive features of the symmetry-preserving nature of
the resulting counterterms \( \Gamma_{ct-sym} \) are that the renormalization of all three terms
in \( L_{y_u} \) and the interaction terms involving gluons or gluinos are correlated, i.e.
that the SUSY-relations between interactions of particles and superpartners are
respected. Furthermore, the resulting counterterm for \( V_{quartic} \) vanishes in the
gaugeless limit. Later we will compare this form of the counterterms to a more
general one, which does not respect SUSY.

2.2 Method to determine potential non-SUSY counter-
terms

The basic idea to compute or check the absence of non-SUSY counterterms is
common to all mentioned studies in the literature. One needs to evaluate quan-
tities which depend on the potential non-SUSY counterterms and which satisfy
known relations. Relations employed in the literature include

- equality of on-shell amplitudes in the exact SUSY limit;
- equality of \( \beta \) functions at higher orders;
- validity of SUSY Slavnov-Taylor identities.

In particular Refs. [23, 24] used the second approach and computed 3-loop \( \beta \) functions for various gluon, gluino, and \( \epsilon \)-scalar couplings. These computations
depend on finite 2-loop counterterms, and it was shown that the SUSY relations between the β functions are satisfied if finite, non-SUSY counterterms are absent at the 2-loop level.

Now we give a brief overview of our approach, which is the one of Refs. [17, 30, 31] and based on Slavnov-Taylor identities. The first ingredient is the Slavnov-Taylor identity (STI) expressing SUSY and gauge invariance and the respective (anti-)commutation relations on the level of Green functions. It can be written as

\[ S(\Gamma) = 0, \]  

(9)

where \( \Gamma \) is the generating functional of one-particle irreducible (1PI) Green functions, and where

\[ S(\Gamma) = \int d^4x \left( \frac{\delta \Gamma}{\delta Y_{\varphi_i}(x)} \frac{\delta \Gamma}{\delta \varphi_i(x)} + s_{\varphi_i}(x) \frac{\delta \Gamma}{\delta \varphi'_i(x)} \right). \]  

(10)

Here \( \varphi_i \) runs over the MSSM quantum fields with nonlinear BRS transformations \( s_{\varphi_i} \), and \( \varphi'_i \) runs over the MSSM quantum fields with linear BRS transformations \( s_{\varphi'_i} \). The BRS transformations contain in particular the SUSY transformations, where the SUSY ghost \( \epsilon \) acts as the transformation parameter. The \( Y_{\varphi_i} \) are sources of the nonlinear BRS transformations, i.e. the classical Lagrangian contains a part \( \mathcal{L}_{\text{ext}} = Y_{\varphi_i} s_{\varphi_i} \). The explicit form of the MSSM STI identity can be found in Ref. [29], and adaptations to 4-spinor notation are given in Refs. [30, 31].

The STI has to hold after renormalization, i.e. after taking into account counterterms of all orders. Evaluating the STI at the \( n \)-loop level thus constrains the genuine \( n \)-loop counterterms in terms of regularized (and subrenormalized) \( \leq n \)-loop diagrams. In the case that the STI is already valid on the regularized/subrenormalized level, the non-SUSY counterterms are zero and the usual renormalization transformation Eq. (7) (the full form of which has been given in Ref. [29]) is sufficient.

Specific STIs for concrete sets of Green functions can be obtained by taking functional derivatives of the equation \( S(\Gamma) = 0 \) with respect to sets of fields. Such specific STIs can then be used to determine potential non-SUSY counterterms in specific sectors of the MSSM.

The other ingredient of our approach is the regularized quantum action principle in DRED [22]. It provides a direct way to check the validity of STIs on the regularized level and thus to check the absence of non-SUSY counterterms. Applied to the STI it can be written as

\[ S(\Gamma^{\text{DRED}}) = i[\Delta] \cdot \Gamma^{\text{DRED}}, \quad \Delta \equiv S(\Gamma_{\text{cl+ct}}) \]  

(11)

where all quantities are regularized in DRED and taken in \( D \) dimensions. The notation for such regularized quantities is as follows. \( \Gamma_{\text{cl+ct}} \) is the bare action
which is used to define the Feynman rules of the regularized (and partially or fully renormalized) theory. $\Gamma^{\text{DRED}}$ is the resulting generating functional for 1PI Green functions. $[\Delta] \cdot \Gamma^{\text{DRED}}$ denotes the insertion of the operator $\Delta$ into the 1PI Green functions; the operator is defined via the STI applied to the bare action (the sum of classical and counterterm action). The general result for the lowest-order term $\Delta_{cl} \equiv S(\Gamma_{cl})$ has been given in Ref. [22]. This operator is non-vanishing, corresponding to the fact that in $D$ dimensions even in DRED, Fierz identities are invalid and therefore the classical action cannot be shown to be supersymmetric.

In the following two sections we will use the STI and the quantum action principle to determine first the structure of 2-loop counterterms in all sectors and then the 3-loop counterterms for the Higgs mass calculation.

3 Lower-order results

In this section we determine the potential non-SUSY counterterms at 2-loop order in all relevant sectors of the MSSM in the gaugeless limit. These counterterms are important for two reasons. On the one hand they would appear in explicit computations of $M_h$ at the 3-loop level such as Refs. [3] or future extensions thereof; on the other hand they would appear in the computation of $\Delta$ in the quantum action principle Eq. (11) needed to evaluate the 3-loop STI below in section 4.

The required structure of 2-loop counterterms is already essentially known. Ref. [22] has considered self energies and SUSY transformations of chiral multiplets; Refs. [23, 24] have considered 3-point interactions between (s)quarks and gluons, gluinos and $\epsilon$-scalars using the method of higher-order $\beta$ functions. The result of all these references is that non-SUSY counterterms are not needed and the renormalization transformation respecting SUSY gives the correct counterterm structure.

Here we slightly extend these results, in order to illustrate and confirm the method of the STI and quantum action principle. We focus on counterterms involving interactions between Higgs/Higgsino and (s)quarks, and terms involving the BRS sources $Y_{\psi_i}$ and SUSY ghosts $\epsilon$. We remark that 2-loop counterterms to 4-point self interactions of squarks or $\epsilon$-scalars are not important for our purposes and not covered by either the analysis of Ref. [23, 24] or the following analysis.

We begin in subsection 3.1 with a detailed discussion of an exemplary STI; later in subsection 3.2 we will be briefer.
3.1 Exemplary case

Here we explain in detail the derivation of the 2-loop counterterm structure in the sector of the Yukawa interactions between Higgs/Higgsino and up-(s)quarks. We will show that the symmetric counterterms generated by Eq. (7) are correct in the context of DRED. To set the stage we first provide the form of the most general bare Lagrangian of this sector, which is not restricted by SUSY, only by manifest symmetries of DRED (such as Lorentz and gauge invariance and (softly broken) Peccei-Quinn symmetry). It reads

\[ L_{y_u}^{\text{ct, general}} = y_u \epsilon_{ij} \left( Z_{H_d q u} H_d^2 \bar{u}_L^j q_L^j + Z_{H_d q u} \bar{H}_d^2 q_L^j + Z_{H_d q u} \bar{u}_R^i H_d^2 \bar{q}_L^j \right) + \text{h.c.}. \]  

(12)

It contains the same terms as the tree-level Lagrangian \( L_{y_u} \), but each term appears multiplied with an independent coefficient \( Z_i \).

Our claim is that symmetric counterterms are correct in this sector. Equivalently we need to show that the following special choice for the \( Z_i \), corresponding to the symmetric renormalization transformation (7), is correct:

\[ Z_{H_d q u} = y_u \sqrt{Z_{H_d} Z_{q u} Z_{u_R}}, \]  

(13a)

\[ Z_{\bar{H}_d q u} = y_u \sqrt{Z_{\bar{H}_d} Z_{q u} Z_{\bar{u}_R}}, \]  

(13b)

\[ Z_{H_d \bar{q} u} = y_u \sqrt{Z_{H_d} Z_{\bar{q} u} Z_{u_R}}. \]  

(13c)

The first of these three equations can be assumed to hold by definition — it defines the quantity \( Z_{y_u} \); the other two equations are then nontrivial. The physical meaning is that there is only one fundamental Yukawa coupling parameter, which governs all three interactions.

In order to prove the second equation of (13), we consider the functional derivative of the STI

\[ 0 = \frac{\delta^4 S(\Gamma)}{\delta q_L^i \delta \bar{u}_R^i \delta H_d^2 \delta \bar{\epsilon}}. \]  

(14)

Here \( \epsilon \), the SUSY ghost, is a bosonic Majorana spinor. The equation thus corresponds to a SUSY relation connecting \( q_L, H_d, \bar{u}_R \). More details to the use, derivation and evaluation of such identities can be found in Refs. [30, 31]. Like in Ref. [30] we will use the identity only at leading order in the external momenta, so that contributions from soft supersymmetry breaking or electroweak symmetry breaking can be neglected. Evaluating Eq. (14) yields the following STI between 1PI Green functions,

\[ 0 = -\Gamma \bar{u}_R^i \epsilon_{y u} \Gamma q_L^i H_d^2 \bar{u}_R - \Gamma H_d^2 \bar{u}_R \epsilon_{y H} \Gamma \bar{q}_L^i u_R \Gamma H_d^2 \bar{u}_R \epsilon_{y u} \Gamma q_L^i \bar{q}_L^i - \Gamma H_d^2 \bar{u}_R \epsilon_{y H} \Gamma \bar{q}_L^i u_R \Gamma q_L^i \bar{q}_L^i + \Gamma q_L^i \epsilon_{y H} \Gamma \bar{q}_L^i u_R \Gamma q_L^i \bar{q}_L^i. \]  

(15)
Figure 1: Diagrams contributing to (17) at the two-loop level in the gauge-less limit. The insertion of the operator \( \Delta^{\leq 1L} \equiv S(\Gamma^{\leq 1L}_{cl+ct}) \) is marked by a cross. Quarks, gluons and Higgs bosons are denoted by \( q, g, H \); squarks and Higgsinos are denoted by \( \tilde{q} \) and \( \tilde{H} \).

Generically, the symbols \( Y_{\varphi_i} \) denote the sources of the BRS variations of the respective fields \( \varphi_i \), where lower-case notation \( y_{\varphi_i} \) is used for 4-spinor fields; \( \phi_l \) runs over all Higgs field components.

Each of the six terms is a product of one elementary 1PI Green function and one 1PI Green function with an insertion of a BRS transformation. The three terms in the second line can be ignored in the following: the last two terms in the second line involve Green functions which do not receive tree-level or counterterm contributions. The first term in the second line is subleading in the external momenta due to its Lorentz structure. Furthermore, we will now specialize to the case in which the quark field \( q^i_L \) carries no momentum, \( p_{q^i_L} = 0 \). Then also the last term in the first line can be neglected, and the identity simplifies to

\[
0 = -\Gamma^{\tilde{u}^\dagger_R H_2^\dagger} q^i_L H_2^\dagger \delta\epsilon + \ldots
\]

where the dots denote the neglected terms.

This is an identity between the first and second kind of the Yukawa interactions in \( \mathcal{L}_{y_u} \). The Green functions appearing as prefactors correspond to loop-corrected SUSY transformations of \( u_R \) into \( \tilde{u}_R \) and of \( H_2 \) into \( H_1 \), respectively.

As a first step we have to check whether the STI (14) is valid on the regularized 2-loop level with 1-loop subrenormalization. According to the quantum action principle the potential violation is given by

\[
\frac{\delta^4 S(\Gamma^{\text{DRED}})}{\delta q^i_L \delta \tilde{u}^\dagger_R \delta H_2^\dagger \delta \epsilon} = ([\Delta^{\leq 1L}] \cdot \Gamma^{\text{DRED}}) q^i_L \tilde{u}^\dagger_R H_2^\dagger \delta \epsilon
\]

i.e. by 1PI diagrams with external fields \( q^i_L \tilde{u}^\dagger_R H_2^\dagger \delta \epsilon \) and one insertion of the operator \( \Delta^{\leq 1L} \equiv S(\Gamma^{\leq 1L}_{cl+ct}) \). Here \( \Gamma^{\leq 1L}_{cl+ct} \) is the 1-loop bare action. At this level DRED is clearly SUSY-preserving, and \( \Gamma^{\leq 1L}_{cl+ct} \) is obtained from the classical action by the usual renormalization transformation. Hence also \( \Delta^{\leq 1L} \) at this order.
Table 1: Properties of the three topologies defined in Ref. [22] for the diagrams involving an insertion of the operator $\Delta \equiv S(\Gamma_{cl})$ attached to an open fermion line (A) and one fermion loop (B). The second column specifies whether the fermions in line A or B coupling to the insertion are gauginos (denoted by $\tilde{g}$) or fermions of a chiral SUSY multiplet ($\psi$). The last column specifies the maximum number of “allowed” $\gamma$-matrices in the $\gamma$-string B. If the respective bound is satisfied, the diagram vanishes [22].

| Topology       | Fermions at $A/B/B$ | # allowed $\gamma$-matrices |
|----------------|---------------------|-----------------------------|
| Topology (a)   | $\tilde{g}/\tilde{g}/\tilde{g}$ | $\leq 4$                    |
| Topology (b)   | $\tilde{g}/\psi/\psi$ | $\leq 2$                    |
| Topology (c)   | $\psi/\psi/\psi$ or $\psi/\psi/\tilde{g}$ | $\leq 3$                    |

is obtained from $\Delta_{cl} \equiv S(\Gamma_{cl})$ given in Ref. [22] simply by the renormalization transformation, and the structure of $\Delta^{\leq 1L}$ and its Feynman rules are the ones of Ref. [22].

Fig. 1 shows representative Feynman diagrams contributing to Eq. (17). All diagrams have a common structure with one open fermion line and one closed fermion loop. As in Refs. [17,22] we denote the chain of $\gamma$-matrices corresponding to the open fermion line as A, and the $\gamma$-chain corresponding to the closed fermion loop as B. In the terminology of those references the first three diagrams of Fig. 1 are of topology (c), and the last diagram is of topology (a) or (b), in case of gluinos/quarks in the fermion loop. According to the rules given in that reference, the diagrams vanish if the $\gamma$ chain B can be expressed as a product of up to 4, 2, 3 $\gamma$-matrices (for topology (a), (b), (c), respectively). These three topologies and their properties are summarized in Tab. 1.

After integrating over the fermion-loop momentum, the fermion loop can depend on up to two momenta (the second loop momentum and the single non-vanishing external momentum). Hence the fermion loop can be expressed as a product of up to two (three) gamma matrices in diagrams of Fig. 1 (a,b,d) (Fig. 1 (c)). According to the rule mentioned above, this is sufficient to know that all diagrams of Fig. 1 vanish. It is easy to see that the same is true for all other diagrams which could contribute to the breaking Eq. (17). Hence Eq. (17) vanishes, and the STI (16) is valid at the regularized 2-loop order with 1-loop subrenormalization.

The important consequence of this is that the genuine 2-loop counterterm contributions have to fulfill the STI (16) by themselves. This implies the following identities of renormalization constants (at the 2-loop level in the gaugeless limit):

$$Z_{\tilde{u}\tilde{e}\psi R} Z_{H_{2q}qu} = Z_{H_{2\tilde{e}\gamma}y_{2}} Z_{H_{2q}\tilde{u}} ,$$

where we have used the notation of the bare Lagrangian (12) and self-explanatory notation for the renormalization constants of the Green functions with BRS
sources. The latter Green functions are themselves constrained by Slavnov-Taylor identities relating the self energies of the scalar and fermionic components of chiral SUSY multiplets. The respective STIs have been shown to be valid on the regularized 2-loop level in Ref. [22], and as a result we know that the respective renormalization constants can be expressed in terms of elementary field renormalization constants as

\[
Z_{\tilde{\mu}y u_R} = \sqrt{Z_{\tilde{\mu}R}}/\sqrt{Z_u}, \quad (19a)
\]

\[
Z_{H_2\bar{\epsilon}y_2} = \sqrt{Z_{H_2}}/\sqrt{Z_{H_2}}, \quad (19b)
\]

Combining the previous two equations (19) with the result of the STI (18) and assuming Eq. (13a) to hold by definition then yields the desired result

\[
Z_{\tilde{H}_2q\tilde{\mu}} = Z_{y u} \sqrt{Z_{\tilde{H}_2}Z_{q_L}Z_{\tilde{\mu}u_R}}. \quad (20)
\]

To summarize: evaluating the functional derivative of the STI (14) leads to the specific STI (16), which is valid on the regularized 2-loop level in the gaugeless limit. As a result we can prove the second identity of our claim Eq. (13) in the desired order. All these steps can be repeated for the case in which the role of \(q_L\) and \(u_R\) are exchanged, i.e. for the functional derivative w.r.t. the set of fields \(\tilde{q}_L^I \tilde{u}_R^I H^j_2 \bar{\epsilon}\). Without going through the details it is clear that the result is then given by

\[
Z_{\tilde{H}_2\tilde{q}u} = Z_{y u} \sqrt{Z_{\tilde{H}_2}Z_{\tilde{q}L}Z_{u_R}}, \quad (21)
\]

the third and final identity of Eq. (13).

### 3.2 Other lower-order cases

We will now present a list of further STIs, which can be treated in an analogous way. In all cases we have verified that the identities are satisfied in the gaugeless limit at the 2-loop level with 1-loop subrenormalization, and we have derived the resulting identities between renormalization constants. The list is as follows:

- Starting from the same identity as (14),

\[
0 = \frac{\delta^4 S(\Gamma)}{\delta q^L_R \delta \tilde{\mu}^\dagger_R \delta H^j_2 \delta \bar{\epsilon}}, \quad (22)
\]

but evaluating it for the case in which the Higgs field carries no momentum, \(p_H = 0\) but \(p_{q_L} \neq 0\), leads to

\[
0 = -\Gamma_{\tilde{\mu}^\dagger_R \varepsilon y u_R} \Gamma_{q^L_R H^j_2 \tilde{u}^\dagger_R} - \Gamma_{H^j_2 \tilde{\mu}^\dagger_R \varepsilon \tilde{q}^L_R \tilde{q}^L_R} + \ldots . \quad (23)
\]
Here and in the subsequent equations the dots have the analogous meaning to Eq. (16). Using self-explanatory notation, this leads to the following identity between renormalization constants:

$$Z_{\tilde{u}\tilde{e}y_{uR}} Z_{H_2 q_u} = Z_{H_2 \tilde{u}\tilde{e}y_{qL}} Z_{q_L}.$$  (24)

This determines the renormalization of the BRS transformation of the quark field $q_L$ into an F-term given by the product $H_2 \tilde{u}_R^\dagger$:

$$Z_{H_2 \tilde{u}\tilde{e}y_{qL}} = Z_{yu} \sqrt{Z_{H_2} Z_{\tilde{u}_R}/Z_{q_L}}.$$  (25)

in agreement with the symmetric renormalization transformation (7).

• An analogous identity with $q_L$ and $u_R$ exchanged yields

$$Z_{\tilde{q}\tilde{e}y_{qL}} Z_{H_2 q_u} = Z_{H_2 \tilde{q}\tilde{e}y_{uR}} Z_{u_R}$$  (26)

and determines the renormalization of the BRS transformation of the quark field $u_R$ into the appropriate F-term:

$$Z_{H_2 \tilde{q}\tilde{e}y_{uR}} = Z_{yu} \sqrt{Z_{H_2} Z_{\tilde{q}_L}/Z_{u_R}}.$$  (27)

• Taking the functional derivative w.r.t. the set of fields $\tilde{q}_L^i \tilde{u}_R^\dagger H_2^j \tilde{H}_2^l C^\dagger \tilde{e}$ and evaluating it for the case with $p_{\tilde{q}_L} = 0$ but $p_{\tilde{H}_2} \neq 0$ leads to

$$0 = -\Gamma_{\tilde{u}_R^\dagger} \epsilon_{u_R} \Gamma_{\tilde{q}_L^i} H_2^j \tilde{u}_R^\dagger q_u - \Gamma_{\tilde{q}_L^i} \epsilon_{u_R} \Gamma_{\tilde{H}_2^l C^\dagger} \Gamma_{\tilde{u}_R^\dagger} H_2^j \tilde{H}_2^l \tilde{e} + \ldots.$$  (28)

The resulting identity between renormalization constants reads

$$Z_{\tilde{u}\tilde{e}y_{uR}} Z_{H_2 \tilde{q}_u} = Z_{\tilde{q}\tilde{e}y_{uR}} Z_{H_2}$$  (29)

and determines the renormalization of the BRS transformation of the Higgsino field $\tilde{H}_2$ into the appropriate F-term:

$$Z_{\tilde{q}\tilde{e}y_{\tilde{H}_2}} = Z_{yu} \sqrt{Z_{\tilde{q}_L} Z_{\tilde{u}_R}/Z_{\tilde{H}_2}}.$$  (30)

• Taking the functional derivative w.r.t. the set of fields $\tilde{u}_R \tilde{u}_R^\dagger H_2^j \tilde{H}_2^l C^\dagger \tilde{e}$ and evaluating it leads to

$$0 = -\Gamma_{H_2^j \tilde{u}_R^\dagger} \epsilon_{u_R} \Gamma_{\tilde{u}_R^\dagger} H_2^j \tilde{H}_2^l C \tilde{q}_L^i + \Gamma_{H_2^j C^\dagger} \epsilon_{\tilde{H}_2^l} \Gamma_{\tilde{u}_R^\dagger} H_2^j \tilde{u}_R^\dagger \tilde{H}_2^l \tilde{e} + \ldots.$$  (31)

The resulting identity between renormalization constants reads

$$Z_{H_2 \tilde{u}\tilde{e}y_{qL}} Z_{\tilde{H}_2 q_u} = Z_{\tilde{H}_2 e y_{\tilde{H}_2}} Z_{\tilde{u}\tilde{u} H_2^2 H_2}.$$  (32)

This identity determines the quartic interaction between two right-handed squarks $\tilde{u}_R$ and two Higgs bosons $H_2$ in terms of previously determined renormalization constants:

$$Z_{\tilde{u}\tilde{u} H_2^2 H_2} = Z_{yu}^2 Z_{H_2} Z_{\tilde{u}_R}.$$  (33)
- A similar identity which contains $\tilde{q}_L$ instead of $\tilde{u}_R$ determines the renormalization constant $Z_{\tilde{q}\tilde{q}H_2H_2}$ between two left-handed squarks $\tilde{q}_L$ and two Higgs bosons $H_2$:

$$Z_{\tilde{q}\tilde{q}H_2H_2} = Z_{y_u}^2 Z_{H_2} Z_{\tilde{q}_L}. \quad (34)$$

- Taking the functional derivative w.r.t. the set of fields $A'_aA''_bH_2\tilde{H}_2\bar{\epsilon}$ leads to the identity

$$0 = \Gamma_{\tilde{H}_2\bar{\epsilon}Y_{H_2}^\dagger} \Gamma_{A'_aA''_bH_2\tilde{H}_2} + \ldots. \quad (35)$$

This identity determines the interaction of two gluons and two Higgs bosons. Again, the identity is valid in DRED on the regularized 2-loop level. This is true in particular for the $D$-dimensional gauge fields but also for the so-called $\epsilon$-scalar part of the gluons, i.e. the $(4 - D)$ extra gluon components which appear on the regularized level and which behave like adjoint scalar fields and not like gauge fields. Following e.g. Ref. [22] we denote the $D$-dimensional gauge field gluons as $\hat{A}_a^\mu$ and the $\epsilon$-scalars as $\tilde{A}_a^\mu$. As a result of the identity, the renormalization constants for both the quartic $\hat{A}\hat{A}H_2H_2$ and the quartic $\tilde{A}\tilde{A}H_2H_2$ interactions are determined to vanish,

$$Z_{\hat{A}\hat{A}H_2H_2} = 0 \quad Z_{\tilde{A}\tilde{A}H_2H_2} = 0. \quad (36)$$

The first of these equations would also follow from gauge invariance, but the second would not. If the renormalization constant $Z_{\tilde{A}\tilde{A}H_2H_2}$ would not vanish at the 2-loop level, it could contribute to Higgs boson self energies at the 3-loop level.

- All previous identities involve the Higgs doublet $H_2$ and/or right-handed up-(s)quarks. Identities involving $H_1$ and/or right-handed down-(s)quarks can be derived in the same way, with analogous results.

The previous identities are summarized in Tab. 2. They lead to Eqs. (20, 21, 25, 30, 33, 34, 36) and thus determine a variety of renormalization constants. In all cases, the results agree with the symmetric counterterms generated by the renormalization transformation (7). The considered cases cover all Green functions with up to two Higgs/Higgsino fields and up to two coloured fields, up to the 2-loop level in the gaugeless limit. They include not only physical fields but also $\epsilon$-scalars, sources for BRS transformations and SUSY ghosts. In other words, the previous identities establish that SUSY-restoring counterterms are not required in this considered sector.

In addition, Refs. [23, 24] have shown the absence of SUSY-restoring counterterms in the pure-SUSY-QCD sector of triple interactions between quarks, squarks, gluons, gluinos and $\epsilon$-scalars. Quartic interactions involving four coloured fields are not covered by any of these analyses.
Table 2: Summary of the SUSY STIs used in sec. 3 for the 2-loop counterterms for subrenormalization.

| STI              | Relevant part | Expressed symmetry properties                                                                 |
|------------------|---------------|-------------------------------------------------------------------------------------------------|
| $\epsilon\tilde{q}_L^\dagger q_L$ | $\propto p_q^2$ | Connects the SUSY transformations of squark into quark and vice versa to field renormalizations (from Ref. [22]; similar for $\tilde{q}_L^\dagger u_R$) |
| $\epsilon\tilde{q}_L^\dagger Y_{\tilde{q}_L}$ | $\propto \tilde{p}$ | Connects the Yukawa coupling of the Higgs boson with quarks to the Yukawa coupling of quark/squark and Higgsino (similar for $\tilde{q}_L^\dagger u_R$) |
| $q_L\tilde{u}_R^\dagger H_2\epsilon$ | $p_{qL} = 0$ | Determines SUSY transformation of quark into an F-term involving Higgs and squark field |
| same             | $p_H = 0$     |                                                                                                 |
| $\tilde{q}_L\tilde{u}_R^\dagger \tilde{H}_2\epsilon$ | $p_{qL} = 0$ | Determines SUSY transformation of Higgsino into an F-term involving two squark fields |
| $\tilde{u}_R\tilde{u}_R^\dagger H_2\tilde{H}_2\epsilon$ | $p = 0$ | Determines quartic interactions between Higgs and squark fields (similar for $\tilde{q}_L$) |
| $A^\mu A^\nu H_2\tilde{H}_2\epsilon$ | $p = 0$ | Determines quartic coupling between Higgs bosons and gluons or $\epsilon$-scalars |

4 3-loop results

In this section we determine the potential non-SUSY counterterms which might enter the Higgs boson mass calculation at the 3-loop level in the gaugeless limit, i.e. at the orders $O(\alpha_t, \alpha_s^2, \alpha_t \alpha_s, \alpha_s^3)$. Ref. [17] has identified the relevant STI. It is obtained by taking

$$0 = \frac{\delta^5 S(\Gamma)}{\delta \phi_a \delta \phi_b \delta \phi_c \delta \tilde{H}_{kl}^i \delta \epsilon}, \quad (37)$$

where $\phi_i$ denote any components of the MSSM Higgs bosons $H^i$, $H^i_{1\dagger}$, and $\tilde{H}^i_k$ is the Higgsino partner of $H^i_k$. Evaluating the derivative without taking the gaugeless limit leads to the following identity:

$$0 = \sum_{\phi_i} \Gamma_{\tilde{H}_{kl}^i Y_{\phi_i}} \epsilon \Gamma_{\phi_a \phi_b \phi_c \phi_i} + \sum_{\lambda} \Gamma_{\phi_a \phi_b \phi_c \phi_i} \Gamma_{\tilde{H}_{kl}^i \phi_c \lambda} + \text{perm. + fin}. \quad (38)$$
Figure 2: Possible Feynman rules corresponding to $\Delta_{\text{extra}}^{2L}$ in Eq. (40). The first Feynman rule could appear if a non-SUSY 2-loop counterterm to an interaction of the type $\tilde{y}_g \tilde{q} \tilde{q}^\dagger$ is required, where $\tilde{y}_g$ is the source of the gluino BRS transformation. The second Feynman rule could appear if a non-SUSY 2-loop counterterm to a 4-squark interaction is required. Terms in $\Delta_{\text{extra}}^{2L}$ with less than three coloured fields are excluded by the discussion of Sec. 3.

Here the abbreviation “fin.” summarizes terms which vanish at tree-level and which at $n$-loop order don’t receive $n$-loop counterterm contributions; “perm” denotes terms corresponding to all possible permutations of $\phi_{a,b,c}$. The sums run over all Higgs field components $\phi_i$ and all electroweak gauginos $\lambda$. This identity describes the fundamental supersymmetry relation between the quartic Higgs-boson self-coupling and the electroweak gauge couplings, which is behind all Higgs-boson mass predictions in the MSSM. The gauge couplings are reflected in the second term of Eq. (38), which corresponds to the SUSY transformation of electroweak gauginos into the $D$-terms, which in turn contain products of gauge couplings and Higgs fields.

Although this identity is rather involved, it determines unambiguously the desired counterterm. From the arguments given in Ref. [17] we obtain immediately the following implication: “If Eq. (38) is valid on the regularized $n$-loop level in the gaugeless limit (with $(n-1)$-loop subrenormalization), then the potential non-SUSY counterterms to the Higgs sector are zero.”

Hence we will now study the identity at the 3-loop level with 2-loop subrenormalization. Using again the quantum action principle, we can write the potential breaking of the identity (38) at this level as

$$\left(\left[\Delta^{\leq 2L}\right] \cdot \Gamma^{\text{DRED}}\right)_{\phi_a \phi_b \phi_c \tilde{H}^L_{kL}}$$

where now $\Delta^{\leq 2L} \equiv S(\Gamma_{\text{ct}}^{\leq 2L})$ and where $\Gamma_{\text{ct}}^{\leq 2L}$ is the 2-loop bare action. We will evaluate the potential breaking at zero external momenta. This is sufficient since all Green functions appearing in the STI (58) are of dimension 4, and correspondingly the quartic Higgs counterterm to be determined is momentum-independent.

First we need to clarify the structure of the insertion $\Delta^{\leq 2L}$ and its Feynman rules. As discussed in the previous section the 2-loop counterterm structure is
Figure 3: First set of diagrams contributing the potential breaking of the Slavnov-Taylor identity Eq. (39) at the 3-loop level in the gaugeless limit. These diagrams contain one fermion loop, connected by up to one boson line to the external fermion line. The insertion of the operator $\Delta_{\text{ren.transf.}}$ is marked by a cross. Quarks, gluinos and Higgsinos are denoted by solid lines, gluons by circles lines; Higgs and squark lines are dashed. The lines corresponding to the external Higgs bosons $\phi_{a,b,c}$ can be attached in all possible ways.

Given essentially by the usual SUSY-preserving renormalization transformation, but the given proof does not exclude non-SUSY counterterms to quartic interactions of four coloured scalars (squarks or $\epsilon$-scalars). Hence we can write the insertion as

$$\Delta^{\leq 2L} \equiv S(\Gamma^{\leq 2L}_{\text{cl+ct}}) = \Delta_{\text{ren.transf.}} + \Delta^{2L}_{\text{extra}},$$

where the first term is obtained from the result given in Ref. [22] by the symmetric renormalization transformation (7). The second term might be present in the case that non-SUSY 2-loop counterterms are indeed necessary; non-SUSY 2-loop counterterms are constrained by the discussion of the previous section. Hence $\Delta^{2L}_{\text{extra}}$ would contain at least three powers of coloured fields and lead to Feynman rules like the ones shown in Fig. 2. It could contribute to Eq. (39) earliest if inserted into 2-loop diagrams but is itself of 2-loop order. Hence for the purpose of the 3-loop evaluation of Eq. (39) we can ignore $\Delta^{2L}_{\text{extra}}$. The relevant Feynman rules for $\Delta^{\leq 2L}$ are then the ones given in Ref. [22], up to the usual renormalization transformation.

Figs. 3, 4, 5 show representative 3-loop diagrams contributing to the potential breaking of the STI, i.e. to the Green function in Eq. (39). Similar to the 2-loop case of Fig. 1 each diagram contains one open fermion line and one fermion loop connected to the insertion $\Delta_{\text{ren.transf.}}$. We will again refer to the $\gamma$-string of the open fermion line as $A$ and the $\gamma$-string of the fermion loop as $B$. The diagrams can be classified into three classes, according to the number of boson lines connecting $A$ and $B$ and according to whether there is a second fermion loop. We now discuss each class in turn; a summary can be found in Tab. 3.

The diagrams shown in Fig. 3 contain up to one boson line connecting $A$ and $B$ and only a single fermion loop. All such diagrams are of Topology (c) (see
Figure 4: Second set of diagrams contributing the potential breaking of the Slavnov-Taylor identity Eq. (39) at the 3-loop level in the gaugeless limit. These diagrams contain one fermion loop, connected by two boson lines to the external fermion line. The notation is as in Fig. 3.

Figure 5: Third set of diagrams contributing the potential breaking of the Slavnov-Taylor identity Eq. (39) at the 3-loop level in the gaugeless limit. These diagrams contain two fermion loops. The notation is as in Fig. 3.

Tab. 1). The fermion attached to the insertion in line A is either a Higgsino or quark; the connecting boson line can only be a squark line. After integrating over all loop momenta in diagrams (A1) and (A2) and over the fermion and gluon loop momenta in diagram (A3), the resulting $\gamma$-string $B$ can at most depend on the single covariant $k$, where $k$ is the remaining loop momentum. As summarized in Tab. 1 the criteria given in Ref. 22 allow up to 3 $\gamma$ matrices. Hence all diagrams of this class are shown to vanish.

The diagrams shown in Fig. 4 involve two boson lines connecting the external fermion line and the fermion loop. One of the two boson lines can be a gluon — in this case the other one must be a squark, and the diagram must be of topology (c), as in diagram (B1). If both connecting bosons are squarks, the diagram can be of topology (b), as in diagram (B2). After integrating over the fermion loop momentum, the $\gamma$-string $B$ of these diagrams can depend on the two remaining loop momenta $k_1, k_2$. Thus in diagrams like (B1) the $\gamma$-string $B$ can be reduced to at most three $\gamma$-matrices $k_1 k_2 \gamma^\mu$, where $\gamma^\mu$ corresponds to the gluon vertex. In diagrams like (B2), the $\gamma$-string $B$ can be reduced to at most two $\gamma$-matrices $k_1 k_2$. Hence again in all cases the criteria of Tab. 1 show that the diagrams vanish.
Table 3: Summary of the properties of the classes of diagrams contributing to the breaking of the STI (39). Representative diagrams can be found in Figs. 3, 4, 5. “max. appearing γ’s” denotes the maximum number of different γ-matrices in the actual diagrams, see text. The column “allowed γ’s” is copied from the last column of Tab. 1.

| Class | # fermion-loops | # connecting bosons | Topology | max. appearing γ’s | allowed γ’s |
|-------|----------------|--------------------|----------|-------------------|------------|
| A1, A2 | 1 | 0 | (c) | 0 | 3 |
| A3    | 1 | 1 | (c) | 1 | 3 |
| B1    | 1 | 2 | (c) | 3 | 3 |
| B2    | 1 | 2 | (a,b,c) | 2 | 2 |
| C1    | 2 | 0 | (c) | 3 | 3 |
| C2    | 2 | 1 | (c) | 3 | 3 |

The diagrams shown in Fig. 5 contain a second fermion loop. The diagrams are of topology (c). After integrating over the two fermion-loop momenta, the γ-string B can be reduced to at most three covariants \( k_{\mu} \gamma_{\gamma}^{\nu} \), where \( k \) is the remaining loop momentum and where \( \gamma_{\mu,\nu} \) correspond to gluon vertices. Hence again the criteria for the diagrams to vanish are satisfied.

In summary (see Tab. 3), we have shown all diagrams contributing to potential breaking (39) to vanish. Using this result in the Slavnov-Taylor identity (38) proves that the Higgs potential counterterm vanishes,

\[
V_{\text{quartic, ct}} = 0
\]  

(41)

at the 3-loop level in the gaugeless limit. This corresponds to the result of the symmetric renormalization transformation — hence DRED preserves SUSY manifestly at this level and symmetric counterterms are sufficient.

5 Conclusions

We have investigated DRED for Higgs boson mass calculations in the MSSM. We have verified that DRED is consistent with SUSY at the 3-loop level in the gaugeless limit. As a result, the usual symmetric counterterms, generated by a symmetric renormalization transformation, are sufficient. Concrete calculations of the MSSM Higgs boson mass at the 3-loop order \( \mathcal{O}(\alpha_t, \alpha_s^2, \alpha_t^2, \alpha_s, \alpha_b^2) \) such as the existing calculation of the order \( \mathcal{O}(\alpha_t \alpha_s^2) \) [3, 5] and future extensions do not need SUSY-restoring counterterms.

The result has been obtained by extending the analysis of Ref. [17] to the 3-loop level. Slavnov-Taylor identities describing suitable SUSY relations were identified and shown to hold at the regularized level. In this way the potential
SUSY-restoring counterterms are shown to vanish. The result includes not only the genuine 3-loop counterterms in the Higgs sector but also — combined with Refs. [23,24] — all necessary 2-loop counterterms required for subrenormalization. The increased complexity of the 3-loop case required several additional technical steps compared to the 2-loop case. Counterterms required for subrenormalization were analyzed and the potential appearance of non-SUSY 2-loop counterterms in the quantum action principle was characterized. The Slavnov-Taylor identities were evaluated in specific limits for external momenta which are sensitive to the desired counterterms and which enabled the application of the quantum action principle.

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References

[1] G. Aad et al., ATLAS collaboration, Phys. Lett. B 176 (2012) 1-29.
S. Chatrchyan et al., CMS collaboration, Phys. Lett. B716 (2012) 30-61.

[2] P. Draper and H. Rzehak, Phys. Rept. 619 (2016) 1 [arXiv:1601.01890 [hep-ph]].

[3] R. V. Harlander, P. Kant, L. Mihaila and M. Steinhauser, Phys. Rev. Lett. 100 (2008) 191602 [Phys. Rev. Lett. 101 (2008) 039901] [arXiv:0803.0672 [hep-ph]]; JHEP 1008 (2010) 104 [arXiv:1005.5709 [hep-ph]].

[4] D. Kunz, L. Mihaila and N. Zerf, JHEP 1412 (2014) 136 [arXiv:1409.2297 [hep-ph]].

[5] R. V. Harlander, J. Klappert and A. Voigt, Eur. Phys. J. C 77 (2017) no.12, 814 [arXiv:1708.05720 [hep-ph]].

[6] S. P. Martin, Phys. Rev. D 75 (2007) 055005 [hep-ph/0701051].

[7] G. Degrassi, S. Di Vita and P. Slavich, Eur. Phys. J. C 75 (2015) no.2, 61 [arXiv:1410.3432 [hep-ph]].
[8] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich and W. Hollik, Eur. Phys. J. C 74 (2014) no.8, 2994 [arXiv:1404.7074 [hep-ph]]; Eur. Phys. J. C 75 (2015) no.9, 424 [arXiv:1505.03133 [hep-ph]].

[9] S. Borowka, S. Paßehr and G. Weiglein, [arXiv:1802.09886 [hep-ph]].

[10] J. Pardo Vega and G. Villadoro, JHEP 1507 (2015) 159 [arXiv:1504.05200 [hep-ph]]. P. Draper, G. Lee and C. E. M. Wagner, Phys. Rev. D 89 (2014) no.5, 055023 [arXiv:1312.5743 [hep-ph]]. E. Bagnaschi, G. F. Giudice, P. Slavich and A. Strumia, JHEP 1409 (2014) 092 [arXiv:1407.4081 [hep-ph]]. G. Lee and C. E. M. Wagner, Phys. Rev. D 92 (2015) no.7, 075032 [arXiv:1508.00576 [hep-ph]].

[11] E. Bagnaschi, J. Pardo Vega and P. Slavich, Eur. Phys. J. C 77 (2017) no.5, 334 [arXiv:1703.08166 [hep-ph]].

[12] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, Phys. Rev. Lett. 112 (2014) no.14, 141801 [arXiv:1312.4937 [hep-ph]]. H. Bahl and W. Hollik, Eur. Phys. J. C 76 (2016) no.9, 499 [arXiv:1608.01880 [hep-ph]]. H. Bahl, S. Heinemeyer, W. Hollik and G. Weiglein, Eur. Phys. J. C 78 (2018) no.1, 57 [arXiv:1706.00346 [hep-ph]].

[13] P. Athron, J. h. Park, T. Steudtner, D. Stöckinger and A. Voigt, JHEP 1701 (2017) 079 [arXiv:1609.00371 [hep-ph]].

[14] W. Siegel, Phys. Lett. B 84 (1979) 193.

[15] D. M. Capper, D. R. T. Jones and P. van Nieuwenhuizen, Nucl. Phys. B 167 (1980) 479.

[16] C. Gnendiger et al., Eur. Phys. J. C 77 (2017) no.7, 471 [arXiv:1705.01827 [hep-ph]].

[17] W. Hollik and D. Stöckinger, Phys. Lett. B 634, 63 (2006).

[18] S. P. Martin and M. T. Vaughn, Phys. Lett. B 318 (1993) 331.

[19] L. Mihaila, Phys. Lett. B 681 (2009) 52 [arXiv:0908.3403 [hep-ph]].

[20] D. Stöckinger and P. Varso, Comput. Phys. Commun. 183 (2012) 422 [arXiv:1109.6484 [hep-ph]].

[21] I. Jack and D. R. T. Jones, [arXiv:hep-ph/9707278].

[22] D. Stöckinger, JHEP 0503 (2005) 076, [hep-ph/0503129]

[23] R. V. Harlander, D. R. T. Jones, P. Kant, L. Mihaila and M. Steinhauser, JHEP 0612 (2006) 024 [hep-ph/0610206].
[24] R. V. Harlander, L. Mihaila and M. Steinhauser, Eur. Phys. J. C 63 (2009) 383 [arXiv:0905.4807 [hep-ph]].

[25] P. Breitenlohner and D. Maison, Commun. Math. Phys. 52 (1977) 11.

[26] I. Jack, D. R. T. Jones and K. L. Roberts, Z. Phys. C 63, 151 (1994), Z. Phys. C 62, 161 (1994).

[27] A. Signer and D. Stöckinger, Phys. Lett. B 626 (2005) 127; Nucl. Phys. B 808 (2009) 88 [arXiv:0807.4424 [hep-ph]]. C. Gnendiger, A. Signer and D. Stöckinger, Phys. Lett. B 733 (2014) 296 [arXiv:1404.2171 [hep-ph]]. A. Broggio, C. Gnendiger, A. Signer, D. Stöckinger and A. Visconti, Eur. Phys. J. C 75 (2015) no.9, 418 [arXiv:1503.09103 [hep-ph]]; JHEP 1601 (2016) 078 [arXiv:1506.05301 [hep-ph]]. W. Kilgore, Phys. Rev. D 86 (2012) 014019 [arXiv:1205.4015].

[28] R. Harlander, P. Kant, L. Mihaila and M. Steinhauser, JHEP 0609 (2006) 053 [hep-ph/0607240]. W. B. Kilgore, Phys. Rev. D83 (2011) 114005.

[29] W. Hollik, E. Kraus, M. Roth, C. Rupp, K. Sibold and D. Stöckinger, Nucl. Phys. B 639 (2002) 3, hep-ph/0204350.

[30] W. Hollik and D. Stöckinger, Eur. Phys. J. C 20 (2001) 105, hep-ph/0103009

[31] I. Fischer, W. Hollik, M. Roth and D. Stöckinger, Phys. Rev. D 69 (2004) 015004, hep-ph/0310191.