Signatures of gluon saturation from structure-function measurements  arXiv:2203.05846

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Motivation

- Color Glass Condensate (CGC) framework describes non-linear effects (gluon saturation)
  - Bjorken-\( x \) dependence from Balitsky-Kovchegov (BK) evolution equation
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- In collinear factorization framework the \(Q^2\) evolution comes from Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations

- To see saturation effects on experimental data we have to distinguish the genuine difference between DGLAP and BK dynamics

- Both frameworks require input which are fitted to the same experimental data
  - The results do not deviate dramatically and the distinguishing DGLAP/BK evolution is difficult
Our method to see difference in DGLAP/BK

We want to be as independent as possible of initial condition parametrization.
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2. We "force" collinear factorization and CGC $F_{2,L}$ to agree in a line in $(x, Q^2)$ plane

Matching line in $(x, Q^2)$ plane
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4. With differences we can approximate the accuracy of $F_{2,L}$ saturation measurements in EIC and LHeC/FCC-he
**Collinear factorization:**
- Collinear factorization $F_{2,L}$ using APFEL [1] and LHAPDF [2] libraries
- NNPDF31_nlo_as_0118_1000 as proton PDF set
- nNNPDF20_nlo_as_0118_Au197 as nuclear PDF set
- Both PDF sets have 1000 Monte Carlo replicas

**Color Glass Condensate (CGC):**
- Dipole picture $F_{2,L}$ fitted to HERA data
- Leading order total photon-nucleus cross sections
- Running coupling BK evolution

We match collinear factorization $F_{2,L}$ to corresponding CGC structure functions in a line in $(x, Q^2)$ plane

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1 T. Lappi and H. Mäntysaari. “Single inclusive particle production at high energy from HERA data to proton-nucleus collisions”. In: *Phys. Rev. D* 88 (2013), p. 114020. arXiv: 1309.6963 [hep-ph]
PDF matching

Bayesian reweighting method [4, 5]:

For each PDF replica \( f_k \) we define

\[
\chi_k^2 = \sum_{i=1}^{N_{\text{data}}} \frac{(O_i - O_i[f_k])^2}{(\delta_{\text{BK}} O_i)^2}
\]

and so called Giele-Keller weights \([6]\)

\[
\omega_k = e^{-\frac{1}{2} \chi_k^2} \frac{1}{N_{\text{rep}} \sum_{k=1}^{N_{\text{rep}}} e^{-\frac{1}{2} \chi_k^2}}
\]

which always sum up to unity,

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\frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \omega_k = 1
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Giele-Keller weights favor replicas with \( \chi_k^2 \approx 0 \).

Then we define reweighted observables as

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O_{\text{Rew}} = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \omega_k O[f_k]
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We also construct a PDF set matched to BK in \((x, Q^2)\) line (Back up)
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We also construct a PDF set matched to BK in $(x, Q^2)$ line (Back up)
Fixing matching parameters

- We want to match the reweighted values to BK values as closely as possible
  - Finite number of replicas (1000) prevent the absolute match
- Effective number of replicas \([4, 7]\)

\[
N_{\text{eff}} = \exp \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \omega_k \ln \left( \frac{N_{\text{rep}}}{\omega_k} \right)
\]

gives an approximation on how many PDF replicas have significant weight

- We adjust \(\delta_{\text{BK}}\) in \(\chi_k^2\) in order to fix \(N_{\text{eff}} \approx 10\)

\[
\begin{align*}
\chi_k^2 &= \sum_{i=1}^{N_{\text{data}}} \frac{(y_i - y_i[f_k])^2}{(\delta_{\text{BK}} y_i)^2} \\
\omega_k &= e^{-\frac{1}{2} \chi_k^2} \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} e^{-\frac{1}{2} \chi_k^2} \\
O^{\text{Rew}} &= \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \omega_k O[f_k]
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Choosing the matching line

We want to do the matching in a common region of validity for both frameworks:
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  - With enough small \( \alpha_s \log(Q^2) \) so that DGLAP evolution dynamics is reliable
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$\rightarrow$ We choose to do the matching on points $Q^2(x) \approx 10 \times Q_s^2(x)$
Proton matching

(a) $F_2$
(b) $F_L$

The structure functions for proton as a function of $x$ at $Q^2 \approx 10Q_s^2(x)$

- Separate matching for proton $F_2$ and $F_L$ are both almost perfect
Relative difference of proton $F_2^{\text{Rew}}$ to $F_2^{\text{BK}}$

For proton $F_2$ the relative difference is only a few percent

Generically slower $x$ dependence in BK evolution
Relative difference of proton $F_L^{\text{Rew}}$ to $F_L^{\text{BK}}$

For proton $F_L$ the relative difference is:

- $\lesssim 10\%$ for $x = 10^{-3} \ldots 5.6 \times 10^{-3}$ (EIC)
- $\lesssim 40\%$ for $x = 10^{-5} \ldots 10^{-4}$ (LHeC/FCC-he)

$F_L$ is much more sensitive to saturation than $F_2$
The structure functions for $^{197}$Au as a function of $x$ at $Q^2 \approx 10Q_s^2(x)$.

- Nuclear reweight is not as successful as for proton since there are not enough Monte Carlo replicas to get a precise match.
Relative difference of nuclear $F_2$ to $F_2^{BK}$

For nuclear $F_2$ the relative difference is $\lesssim 10\%$

The relative difference is much larger than in the proton case
  - It is expected since saturation effects are stronger in nuclei
Relative difference of nuclear $F_L^{\text{Rew}}$ to $F_L^{\text{BK}}$

The relative difference $(F_L^{\text{BK}} - F_L^{\text{Rew}})/F_L^{\text{BK}}$.

For nuclear $F_L$ the relative difference is:

- $\lesssim 15\%$ for $x = 10^{-3} \ldots 10^{-2}$ (EIC)
- $\lesssim 60\%$ for $x = 10^{-5} \ldots 10^{-4}$ (LHeC/FCC-he)
With Bayesian reweighting we match proton/nuclear structure functions to corresponding BK values in a line $Q^2 \approx 10 \times Q_s^2(x)$
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The deviation outside the matching line describes signatures of saturation.
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In order to see saturation in protons in EIC:

- $F_L$ the measurements have to be $\mathcal{O}(10\%)$.
- $F_2$ the measurements have to be $\mathcal{O}(1\%)$.
Summary

- With Bayesian reweighting we match proton/nuclear structure functions to corresponding BK values in a line $Q^2 \approx 10 \times Q_s^2(x)$

- The deviation outside the matching line describes signatures of saturation

- In order to see saturation in protons in EIC
  - $F_L$ the measurements have to be $\mathcal{O}(10\%)$
  - $F_2$ the measurements have to be $\mathcal{O}(1\%)$

- In LHeC/FCC-he the differences are a few times larger
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Saturation is stronger in heavy nuclei than in proton.
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- $F_L$ the measurements have to be $O(10\%)$
- $F_2$ the measurements have to be $O(1\%)$

In LHeC/FCC-he the differences are a few times larger

Saturation is stronger in heavy nuclei than in proton

$F_L$ is more sensitive to saturation than $F_2$
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Reweighting has slightly stronger effect on gluon distribution than on up quark.

Moderate effects expected since NNPDF3.1 PDFs are fitted to same HERA data as BK boundary conditions.
Nuclear PDFs are affected more than proton PDFs

Reweighting has stronger effect on gluon distribution than on up quark
Back up: Reweight with smaller $x$ region

![Diagram of $197\text{Au} F_2$ and $197\text{Au} F_L$ with $Q^2$ versus $x$](image)

(a) $F_2$

(b) $F_L$

Nuclear reweight in region $x = 10^{-4} \ldots 10^{-2}$
Back up: Reweight with smaller $x$ region

(a) $F_2$

The relative difference $(F_2^{BK} - F_2^{Rew})/F_2^{BK}$ with nuclear reweight in region $x = 10^{-4} \ldots 10^{-2}$.

(b) $F_2$

The relative difference $(F_2^{BK} - F_2^{Rew})/F_2^{BK}$ with nuclear reweight in region $x = 10^{-4} \ldots 10^{-2}$.

(c) $F_L$

The relative difference $(F_L^{BK} - F_L^{Rew})/F_L^{BK}$ with nuclear reweight in region $x = 10^{-4} \ldots 10^{-2}$.
Back up: Reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$

\begin{align*}
\text{(a) } F_2 \\
\text{Nuclear reweight in line } Q^2(x) &\approx 27 \times Q_s^2(x).
\end{align*}
Back up: Reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$

The relative difference $(F_2^{\text{BK}} - F_2^{\text{Rew}})/F_2^{\text{BK}}$ with nuclear reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$.

(a) $F_2$

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The relative difference $(F_L^{\text{BK}} - F_L^{\text{Rew}})/F_L^{\text{BK}}$ with nuclear reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$.

(a) $F_L$

(b) $F_L$
Giele-Keller weights which favor replicas with $\chi^2/N_{\text{data}} \approx 0$

$$\omega_k = \frac{e^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} e^{-\frac{1}{2}\chi_k^2}}$$

Weights used with experimental data favor replicas with $\chi^2/N_{\text{data}} \approx 1$

$$\omega_k = \frac{(\chi_k^2)(N_{\text{data}}-1)/2 e^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} (\chi_k^2)(N_{\text{data}}-1)/2 e^{-\frac{1}{2}\chi_k^2}}$$