A Blind Separation Method of PCMA Signals Based on MS-Gibbs Algorithm

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Abstract. A blind separation method of PCMA signals with different symbol rates based on MS-Gibbs (multiple states Gibbs) algorithm is proposed. The prior probability of input symbol pair is calculated based on channel state and two input signal components, which is used to guide the update of symbol sequence. Simulation results show that the performance of the proposed algorithm is similar to that of DG-PSP algorithm, but the complexity of separation is greatly reduced through the proposed algorithm. Compared with no iteration in the 10⁻² order of magnitude of BER (bit error rate), the proposed algorithm can obtain nearly 2dB SNR (signal-noise ratio) gain after 2 iterations. If the number of iterations is 4, the proposed algorithm can obtain nearly 4dB SNR (signal-noise ratio) gain.

1. Introduction
In recent years, with the wide application of radio communication technology, communication environment becomes more and more complex. It is inevitable that the receiver receives two or more signals at the same time [1]. Taking satellite communication as an example, Paired Carrier Multiple Access (PCMA) technology is used to improve the capacity of satellite communication, which has been widely used nowadays [2]. According to the characteristic of the PCMA signals, the blind separation technology for PCMA signals can only be realized through single-channel reception [3-5]. Furthermore, the PCMA signals with different symbol rates are appearing more and more frequently. The update of input symbols in two signal components are not synchronization, which makes the blind separation of PCMA signals with different symbol rates become very difficult, so the study of PCMA signals with different symbol rates has a very important epochal significance.

The traditional single-channel blind separation algorithm of PCMA signals mainly includes particle filtering algorithm [6] and per-survivor processing algorithm [7]. In order to improve the performance of algorithm, some people unite PSP algorithm and SOVA (soft output viterbi algorithm) to obtain the soft information of symbol sequence. The decoding algorithm is also used to correct the soft information, which improves the performance of demodulation [8-9]. In terms of the complexity of algorithm, Yang Yong reduced the complexity of separation by Gibbs separation algorithm, and the performance of it is similar to PSP algorithm [10]. However, the signal model of the above algorithm is established for the PCMA signals with the same symbol rate. It is not applicable for PCMA signals with different symbol rates. There are few literatures about the blind separation of PCMA signals with different symbol rates, so the separation of PCMA signals with different symbol rates is a technical problem nowadays [11].

For the limited applicable range of traditional algorithms, this paper focuses on the blind separation of PCMA signals with different symbol rates under non-cooperative reception. On the basis of channel
status and two input signal components, we calculate the prior probability of each input symbol pair, and update input symbol pairs according to the prior probability.

2. Signal model

In the system of non-cooperative PCMA communication, the receiver receives the mixed signal of two MPSK- or QAM-modulated signal components. \( T(i=1 \text{ or } 2) \) denotes the period of symbol. \( P \) denotes oversampling multiplier. The received signal is sampled at sampling frequency \( \frac{1}{T} \), and the discrete-time form of it can be written as follows

\[
\begin{align*}
    z_k &= h_1 e^{j2\pi x f_1 k T} x_{1,k} + h_2 e^{j2\pi x f_2 k T} x_{2,k} + v_k \\
    &= h_1 e^{j2\pi x f_1 k T} x_{1,k} + h_2 e^{j2\pi x f_2 k T} x_{2,k} + v_k \\
\end{align*}
\]

Where \( v_k \) \((k=1,2,\ldots)\) is the sampled sequence of complex additive white Gaussian noise with zero mean and variance \( N_0 \). \( h_1, \Delta f_1, \varphi_i \) are the amplitudes, frequency offsets, and initial phase offsets, respectively. \( x_{1,k} \) and \( x_{2,k} \) denote the digital baseband signal of useful component and interference component, respectively. According to the independence of \( x_{1,k} \) and \( x_{2,k} \), \( x_{i,k} \) \((i=1 \text{ or } 2)\) can be expressed as follows

\[
\begin{align*}
    x_{1,k} &= \sum_{n=-L}^{L} s_{1,n}^1 g_1(K_{1}^1 T_1 - m T_1 + K_{1}^2 T_1 + \tau_i) \\
    x_{2,k} &= \sum_{n=-L}^{L} s_{2,n}^2 g_2(K_{2}^3 T_2 - m T_2 + K_{2}^4 T_2 + \tau_2) \\
\end{align*}
\]

Where \( L \) \((L = 2L_c + 1)\) is defined as the memory length of the channel filter. \( K_{1}^1 = \mod(k,P) \), \( K_{1}^2 = k - PK_{1}^1 \), \( K_{2}^3 = \mod(k,\frac{T_2}{T}) \), \( K_{2}^4 = k - \frac{T_2}{T} K_{2}^3 \). \( \mod(\cdot) \) denotes modular arithmetic. \( s_{1,k} \) and \( s_{2,k} \) denote the transmit symbols of two signal component, respectively. \( g_i(\cdot) \) is the pulse response of the channel filter.

\[
\begin{align*}
    s_{1,k} &= \left\{ s_{1,n}^1 \right\} \\
    s_{2,k} &= \left\{ s_{2,n}^2 \right\} \\
    g_{1,k} &= h_1 e^{j2\pi x f_1 k T} \left[ g_1(K_{1}^1 T_1 - L T_1 + K_{1}^2 T_1 + \tau_i) : \\
    &\quad K_{1}^1 T_1 + L T_1 + K_{1}^2 T_1 + \tau_i) \right] \\
    g_{2,k} &= h_2 e^{j2\pi x f_2 k T} \left[ g_2(K_{2}^3 T_2 - L T_2 + K_{2}^4 T_2 + \tau_2) : \\
    &\quad K_{2}^3 T_2 + L T_2 + K_{2}^4 T_2 + \tau_2) \right] \\
\end{align*}
\]

The received signal can be written as follows

\[
z_k = g_1^T s_{1,k} + g_2^T s_{2,k} + v_k \\
\]

The blind separation for PCMA signals is to estimate the sequence \( \{s_{1,k}, s_{2,k}, k=0,1,\ldots\} \) according to the received sequence \( \{z_k, k=0,1,\ldots\} \).

3. The blind separation of PCMA signals with different symbol rates

3.1. Blind separation algorithm

If the symbol rate of the two signal components is different, the input symbols of two signal components are not updated synchronously in channel. However, these input symbol pairs are updated regularly. The symbol rate of the two signal components are defined as \( f_{d_1} \) and \( f_{d_2} \), respectively.
Where \( fd_i > fd_d \). In this paper, the sampling frequency is \( f_s = 2fd_s \). \( y_k \) is received signal, where \( y_k = [y_k, y_{k+\frac{1}{2}}] \). There are three possible states of input symbol pair at time \( k \) will be analyzed as follows.

**Station 1** The input symbol pair is \((s_k^{(i)}, s_k^{-2})\) at time \( k - 1 \). According to the sampling point \((y_k, y_{k+\frac{1}{2}})\) in Figure 1, the state of input symbol pair is updated into \((s_k^{(i)}, s_k^{-2})\) at time \( k \). In this station, we construct \((2L_c) \times 1\) matrix \( s_k^{(i)} \) and \((2L_c) \times 1\) matrix \( s_k^{-2} \). \( s_k^{(i)} \) and \( s_k^{-2} \) are expressed as follows

\[
\begin{align*}
\bar{s}_k^{(i)} &= [s_{k-L_c}^{(i)}, \ldots, s_{k-1}^{(i)}, s_k^{(i)}, \ldots, s_{k+L_c}^{(i)}]^	op \\
\bar{s}_k^{-2} &= [s_{k-L_c}^{-2}, \ldots, s_{k-1}^{-2}, s_k^{-2}, \ldots, s_{k+L_c}^{-2}]^	op
\end{align*}
\]  

(6)

In order to express formula expeditiously, we construct the distance function \( Q_{k,k_i} \) as follows

\[
Q_{k,k_i} = \frac{1}{N_0} \sum_{j=k-L_c}^{k+L_c} \left| y_j - g_i \bar{s}_j^{(i)} - g_i^\top \bar{s}_j^{-2} \right|^2
\]  

(7)

The conditional probability \( \beta_{k,k_i} \) of the input symbol pair at time \( k \) can be written as follows.

\[
\beta_{k,k_i} = p(s_k^{(i)} = a_1, s_k^{-2} = a_2 \mid y, \bar{s}_k^{(i)}, \bar{s}_k^{-2})
\]  

\[
= \frac{p(y \mid \bar{s}_k^{(i)}, \bar{s}_k^{-2})}{p(y \mid \bar{s}_k^{(i)}, \bar{s}_k^{-2})} p(s_k^{(i)} = a_1, s_k^{-2} = a_2)
\]  

\[
= \frac{\exp(\bar{Q}_{k,k_i}) p(s_k^{(i)} = a_1, s_k^{-2} = a_2)}{\sum_{a_1, a_2 \in S} \{ \exp(Q_{k,k_i}) p(s_k^{(i)} = a_1, s_k^{-2} = a_2) \}}
\]  

(8)

The \( l-th \) bit of the \( k-th \) symbol in the \( i-th \) signal component is defined as \( c_{i,l}^{(i)} \) (\( l = 0, \ldots, Q-1 \)). The posterior probability distribution of \( c_{i,l}^{(i)} \) can be expressed as follows

\[
L(c_{i,l}^{(i)} \mid y) = \frac{\sum_{\forall a_1, a_2 \in [0,1]} p(s_k^{(i)} = a_1, s_k^{-2} = a_2 \mid y)}{\sum_{\forall a_1, a_2 \in [0,1]} p(s_k^{(i)} = a_1, s_k^{-2} = a_2 \mid y)}
\]  

(9)

Therefore, the extrinsic information of \( c_{i,l}^{(i)} \) can be written as follows

\[
\lambda_{i,l}^{(i)} = L(c_{i,l}^{(i)} \mid y) - \ln \frac{p(c_{i,l}^{(i)} = 1)}{p(c_{i,l}^{(i)} = 0)}
\]  

(10)

The core task of the proposed algorithm is to calculate the prior probability distribution of input symbol pairs, so it is necessary to convert the extrinsic information of bit sequence into the prior probability of input symbol pair \([10]\).
Where $p(c_i^{(0)}) = 1/(1 + e^{q_{i}^{(0)}})$, $p(c_i^{(1)}) = e^{q_{i}^{(1)}}/(1 + e^{q_{i}^{(1)}})$.

**Station 2** The state of input symbol is updated into $s_{k+1}^{(1)}$ at time $k$. The prior probability of input symbol pair in this state can be expressed as follows

$$p(s_{k+1}^{(1)} = a_i) = \sum_{j=0}^{2^M-1} p(c_{i,j}^{(1)}) \quad (12)$$

**Station 3** The $\beta_{a_{1},a_{2}}$ in this state can be expressed as follows

$$\beta_{a_{1},a_{2}} = p(s_{k+1}^{(1)} = a_i | y_s^{(0)}, y_s^{(1)}) \times p(s_{k+2}^{(2)} = a_2 | y_s^{(0)}, y_s^{(1)}), \quad (13)$$

The prior probability of input symbol pair in this state can be expressed as follows

$$p(s_{k+2}^{(2)} = a_{1}, s_{k+2}^{(2)} = a_{2}) = \sum_{i=0}^{2^M-1} \sum_{j=0}^{2^M-1} p(c_{i,j}^{(1)}) \quad (14)$$

3.2. **The steps of the proposed algorithm**

**Step 1:** The initial prior probability of the input symbol pair is $1/M^2$, where $M$ is the modulation order and the random symbol sequence are generated according to the initial probability $(s_{0,0}^{(1)}, s_{1,0}^{(1)}, s_{1,0}^{(2)}, \ldots, s_{K-1,0}^{(1)}, s_{K-1,0}^{(2)}), \quad (s_{K-1,0}^{(1)}, s_{K-1,0}^{(2)})$, where $s_{i,n}$ is defined as the symbol of $i$-th signal in $n$-th iteration at time $k$. $g_{1,0}$ and $g_{2,0}$ are initialized according to the estimation of channel parameters, where the parameters include amplitude, frequency offset, initial phase offset and time delay.
Step2: The conditional probability distribution of the input symbol at time $k$ is calculated according to equation (7) (11) (13).

Step3: The prior probability of input symbol pair at time $k$ is calculated according to equation (10) (12) (14), and the input symbol pairs are updated based on the prior probability.

Step4: If $n \geq N_g/2$, the channel response $g_{1,n}^r$ and $g_{2,n}^r$ are updated through LMS tracking algorithm.

Step5: If $n = N_g$, we have achieved the estimation of symbol sequence in this experiment.

4. Simulation

4.1. Experiment 1

The BER and FER of the proposed algorithm decreases obviously with the increase of SNR in Figure 4 and 5. Moreover, the performance of the algorithm is close to the joint performance bound. Compared with no iteration in the $10^{-2}$ order of magnitude of BER and FER, the algorithm obtains nearly 2 dB SNR gain after 2 iterations. When the number of iterations is 4, the algorithm obtains nearly 4 dB SNR gain. However, after 4 iterations, the curve of BER decreases no longer obviously with the increase of iterations. With the increasing of iterations, the higher the signal-to-noise ratio, the more obvious the improvement of algorithm’s performance. When the signal-to-noise ratio is 8dB, compared with no iteration, the performance of algorithm is improved one and two orders of magnitude respectively after 2 iterations. When the SNR is 0, the number of iterations has little effect on the performance of algorithm.

![Figure 4. Effect of $N$ on BER](image)

![Figure 5. Effect of $N$ on FER](image)
4.2. Experiment 2

Figure 6 show the performance curves of the algorithm versus signal-to-noise ratio when the amplitude ratio is 1:0.8, 1:0.9 and 1:1. If the number of iterations is 6, and the signal-to-noise ratio is 8, when the amplitude ratio increases from 1:1 to 1:0.9, the BER of the algorithm will reduce from $10^{-3}$ to $10^{-4}$ order of magnitude. When the amplitude ratio increases from 1:0.9 to 1:0.8, the BER of the algorithm will reduce from $10^{-3}$ to $10^{-5}$ order of magnitude. We can also see that the greater the difference of two signal components’ amplitude, the better the separation performance of the algorithm. These results can be explained as follows. If the difference of two signal components’ amplitude is great, the correlation of the two signal components is weak, which improves the performance of separation, but when the amplitude ratio is 1:1, and the signal-to-noise ratio is 8, the algorithm can also reach $10^{-3}$ order of magnitude, so the algorithm is also suitable for PCMA signals with little difference of amplitude.

![Figure 6. Effect of the amplitude ratio on BER](image)

5. Conclusion

In this study, the single-channel blind separation of PCMA signals with different symbol rates is realized through MS-Gibbs algorithm. Currently, most of traditional algorithms cannot be used to solve such problem. Compared with the DG - PSP algorithm, the complexity of separation is greatly reduced. The proposed algorithm combines decoding module and demodulation module, which provides a new idea for the efficient separation of PCMA signals with different symbol rates.

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