The rare top quark decays $t \to cV$ in the topcolor-assisted technicolor model

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Abstract

We consider the rare top quark decays in the framework of topcolor-assisted technicolor (TC2) model. We find that the contributions of top-pions and top-Higgs predicted by the TC2 model can enhance the SM branching ratios by as much as 6-9 orders of magnitude. i.e., in the most case, the orders of magnitude of branching ratios are $Br(t \to cg) \sim 10^{-5}$, $Br(t \to cZ) \sim 10^{-5}$, $Br(t \to c\gamma) \sim 10^{-7}$. With the reasonable values of the parameters in TC2 model, such rare top quark decays may be testable in the future experiments. So, rare top quark decays provide us a unique way to test TC2 model.

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I Introduction

It is widely believed that the top quark, which with a mass of the order of the electroweak scale, plays an important role in particle physics. Its unusually large mass makes it more sensitive to certain types of flavor-changing (FC) interactions.

In the standard model (SM), due to the GIM mechanism, the rare top quark decays $t \rightarrow cV$ ($V = Z, \gamma, g$) are very small[1], far below the feasible experimental possibilities at the future colliders (LHC or LC)[2]. In some new physics models beyond the standard model (SM), the decay widths of the rare top quark decays $t \rightarrow cV$ may be significantly enhanced because of the appearance of large flavor changing couplings at the tree-level. Various rare top quark decays have been extensively studied in the SM[1], the multi Higgs doublets models (MHDM)[3][4][5], the technicolor models[6][11], the MSSM models[7][8][9], and other new physics models. They have shown that, with reasonable values for the parameters, the branching ratios $Br(t \rightarrow cV)$ could be within the observable threshold of future experiments.

The topcolor-assisted technicolor (TC2) model[10] connects the top quark with the electroweak symmetry breaking (EWSB). In this model, the topcolor interactions make small contributions to the EWSB, and give rise to the main part of the top quark mass $(1 - \epsilon)m_t$ with a model dependent parameter $0.03 \leq \epsilon \leq 0.1$. The technicolor (TC) interactions play a main role in the breaking of the electroweak gauge symmetry. The extend technicolor (ETC) interactions give rise to the masses of the ordinary fermions including a very small portion of the top quark mass $\epsilon m_t$. This kind of model predicts three top-pions ($\Pi^0_t, \Pi^\pm_t$) and one top-Higgs ($h_t$) with large Yukawa couplings to the third generation. These new particles can be regarded as a typical feature of the TC2 model. Thus, studying the possible signature of these particles and their contributions to some processes at high energy colliders is a good method of testing the TC2 model. There have been many publications related to this field[11][12][13]. Another feature of the TC2 model is the existence of large flavor-changing couplings. For TC2 models, topcolor interactions are non-universal and therefore does not posses a GIM mechanism, which results in a
new flavor-changing coupling vertices when one writes the interactions in the quark mass eigen-basis. Thus, the top-pions and top-Higgs predicted by this kind of models have large Yukawa couplings to the third generation and can induce the new flavor-changing couplings. Such flavor-changing couplings would give contributions to the rare decays $t \to cV$. Because the rare top quark decays $t \to cV$ can hardly be detected in the SM, any observation of rare top quark decays would be an unambiguous signal of new physics. So, the study of the rare top quark decays within the framework of the TC2 model would be a feasible method to test the TC2 model. Ref.[11] has considered the contributions of these particles to the rare top quark decay $t \to cg$. However Ref.[11] only considered the contributions of neutral top-pion $\Pi^0_t$ and did not consider the contributions of the charged top-pions $\Pi^{\pm}_t$. In this paper, we systematically calculate the contributions of the top-pions ($\Pi^0_t, \Pi^\pm_t$) and top-Higgs ($h_t$) to the rare top quark decays $t \to cV$ in the TC2 model, and find that the TC2 model can significantly enhanced the rare top quark decays $t \to cV$, and may approach the detectability threshold of the future experiments.

II The rare top quark decays $t \to cV$ in the TC2 model

The TC2 model predicts the existence of the top-pions $\Pi^0_t, \Pi^\pm_t$, top-pions would give the new flavor changing couplings at tree-level. The relevant interactions of these top-pions with the $b, t$ and $c$ quarks can be written as [10][12]:

$$
\frac{m_t}{\sqrt{2}F_t} \left[ \frac{\sqrt{v^2_\omega - F_t^2}}{v_\omega} \left[ iK^{tt\bar{u}}_URU_L^{tt}t_R^\dagger \Pi^0_t + \sqrt{2}K^{tt\bar{b}}_URU_L^{tt}b_R^\dagger \Pi^{0\pm}_t + iK^{tc\bar{u}}_URU_L^{tt}c_R^\dagger \Pi^0_t + \sqrt{2}K^{tc\bar{b}}_URU_R^{tt}b_L^\dagger \Pi^{0\pm}_t + h.c. \right] \right] (1)
$$

where $v_\omega = v/\sqrt{2} \approx 174 GeV$, $F_t$ is the decay constant of the top-pions. $K_{UL}^{ij}$ are the matrix elements of the unitary matrix $K_{UL}$ from which the Cabibbo-Kobayashi-Maskawa (CKM) matrix can be derived as $V = K_{UL}^{-1}K_{DL}$, and $K_{UR}^{ij}$ are the matrix elements of the right-handed rotation matrix $K_{UR}$. Their values can be written as:

$$
K^{tt}_UR = K^{bb}_DL = 1, \quad K^{tt}_UR = 1 - \epsilon, \quad K^{tc}_UR \leq \sqrt{2\epsilon - \epsilon^2} (2)
$$

In the following calculation, we take $K^{tc}_UR = \sqrt{2\epsilon - \epsilon^2}$ and take $\epsilon$ as a free parameter.
The TC2 model also predicts a CP-even scalar $h_t$, called top-Higgs \cite{12}, which is a $t\bar{t}$ bound and analogous to the $\sigma$ particle in low energy QCD. Its couplings to quarks are similar to that of the neutral top-pion except that the neutral top-pion is CP-odd. All the Feynman rules of top-pions and top-Higgs relevant to $t \to cV$ are shown in Appendix One.

The above large Yukawa couplings will effect the rare top quark decays $t \to cV$. The relevant Feynman diagrams for the contributions of the top-pions and top-Higgs to the rare top quark decays $t \to cV$ are shown in Fig.1. Using Eq.[1] and other relevant Feynman rules, we obtain the relative amplitudes of the rare top quark decays $t \to cV$:

$$M_V = \overline{\psi}_c L (F_{V1} \gamma^\mu + F_{V2} p_t^\mu + F_{V3} p_c^\mu) u_t \varepsilon_\mu(\lambda)$$

(3)

where $L = (1 - \gamma_5)/2$ is the left-handed projector, the expressions of $F_{Vi}(V = Z, \gamma, g, i = 1, 2, 3)$ in Eq.[3] can be obtained by a straightforward calculations of the diagrams shown in Fig.1. Because of $m_t >> m_c(m_b)$, for the sake of simplicity, we have neglected the terms proportional to $m_c, m_b$ in Eq.[3]. It can be seen that each diagram actually contain ultraviolet divergences. Because there are no corresponding tree-level terms to absorb these divergences, all the ultraviolet divergences cancel in the effective vertex. Then, the widths of the rare top quark decays contributed by top-pions and top-Higgs can be written as:

$$\Gamma(t \to cZ) = \frac{1}{16\pi m_t} \left[1 - \frac{M_Z^2}{m_t^2}\right] \frac{1}{8M_Z^2} \left[F_{Z1}^2(4m_t^2M_Z^2 - 8M_Z^4 + 4m_t^4)ight.
+F_{Z2}^2(-3m_t^4M_Z^2 + m_t^6 + 3m_t^2M_Z^4 - M_Z^6) + F_{Z3}^2(m_t^2 - M_Z^2)^3
+(F_{Z1} \cdot F_{Z2}^* + F_{Z2} \cdot F_{Z1}^*)(-4m_t^3M_Z^2 + 2m_t^5 + 2M_Z^4m_t)
+(F_{Z2} \cdot F_{Z3}^* + F_{Z3} \cdot F_{Z2}^*)(-3m_t^4M_Z^2 + m_t^6 + 3m_t^2M_Z^4 - M_Z^6)
+2(F_{Z1} \cdot F_{Z3}^* + F_{Z3} \cdot F_{Z1}^*)(m_t^2 - M_Z^2)^2m_t]$$

(4)

$$\Gamma(t \to c\gamma) = \frac{1}{16\pi m_t} \left[F_{\gamma1}^2m_t^2 - \frac{1}{2}F_{\gamma2}^2m_t^4 - \frac{1}{2}(F_{\gamma1}F_{\gamma2}^* + F_{\gamma2}F_{\gamma1}^*)m_t^3
- \frac{1}{4}(F_{\gamma2}F_{\gamma3}^* + F_{\gamma3}F_{\gamma2}^*)m_t^4\right]$$

(5)
Figure 1: The Feynman diagrams for the contributions of the top-pions ($\Pi_0^t$, $\Pi_\pm^t$) and top-Higgs ($h_t$) to the rare top quark decays $t \rightarrow cV$. 
\[ \Gamma(t \rightarrow cg) = \frac{1}{16\pi m_t} \left[ F_{g1}^2 m_t^2 - \frac{1}{2} F_{g2}^2 m_t^4 - \frac{1}{2} (F_{g1} F_{g2}^* + F_{g2} F_{g1}^*) m_t^3 \right] 
- \frac{1}{4} (F_{g2} F_{g3}^* + F_{g3} F_{g2}^*) m_t^4 \] (6)

where \( m_t \) and \( M_Z \) denote the masses of top quark and \( Z \) boson, respectively. The explicit expressions of the form factors \( F_{g\gamma}, F_{Z\gamma}, F_{gi} \) are given in Appendix Two.

III The numerical results and conclusions

According to the above calculations, we can give the numerical results of the branching ratio of \( t \rightarrow cV \) contributed by \( \Pi_t \) and \( h_t^0 \). In this paper, we adopt the branching ratios \( Br(t \rightarrow cV) \) defined as\(^{[1]}\):

\[ Br(t \rightarrow cV) = \frac{\Gamma(t \rightarrow cV)}{\Gamma(t \rightarrow W^+b)} \] (7)

Before numerical calculations, we need to specify the parameters involved. We take \( m_t = 175 \) GeV, \( M_Z = 91.18 \) GeV, \( s_W^2 = \sin^2 \theta_W = 0.23, \alpha_e = 1/128.9 \) and \( \alpha_s = 0.118 \). Now, there are still four free parameters: \( \epsilon, m_{\Pi_0^t}, m_{\Pi_\pm t}, m_{h_t} \). \( \epsilon \) is a model dependent parameter and we take it in the range of \( 0.03 \sim 0.1 \), \( m_{\Pi_0^t}, m_{\Pi_\pm t}, m_{h_t} \) denote the masses of neural top-pion \( \Pi_0^t \), charged top-pion \( \Pi_\pm t \) and top-Higgs \( (h_t) \), respectively. Due to the split of the \( m_{\Pi_0^t} \) and \( m_{\Pi_\pm t} \) only come from the electroweak interactions, the different of \( m_{\Pi_0^t} \) and \( m_{\Pi_\pm t} \) is very small and can be ignored. Here, we take \( m_{\Pi_0^t} = m_{\Pi_\pm t} = m_{\Pi_t} \). Ref.\(^{[10]}\) have estimated the mass of top-pions, the results show that the \( m_{\Pi_t} \) is allowed to be in the region of a few hundred GeV depending on the models. Estimating the contributions of top-pions to the rare top quark decays \( t \rightarrow cV \), we take the mass of top-pion to vary in the range of 200 GeV \( \sim 500 \) GeV in this paper. The mass of \( h_t \) can be estimated in the Nambu-Jona-Lasinio (NJL) model in the large \( N_c \) approximation and is found to be of the order of \( m_{h_t} \approx 2m_t \)\(^{[12]}\). This estimation is rather crude and the masses well below the \( t\bar{t} \) threshold are quite possible and occur in a variety of cases \(^{[15]}\). As the branching ratios are proportional to \( (2\epsilon - \epsilon^2)(1 - \epsilon)^2 \), to cancel the influence of \( \epsilon \) on the branching ratio, we summarized the final numerical results of \( \frac{Br(t \rightarrow cV)}{(2\epsilon - \epsilon^2)(1 - \epsilon)^2} \) in Figs. 2-4.

Fig.2-4 are the plots of the \( \frac{Br(t \rightarrow cV)}{(2\epsilon - \epsilon^2)(1 - \epsilon)^2} \) versus \( m_{\Pi_0^t} \) (200 GeV \( \sim 500 \) GeV) for \( m_{h_t} = 200 \) GeV, 250 GeV, 300 GeV, respectively. We can see that, the branching ratio of \( t \rightarrow c\gamma \)
Figure 2: The branching ratio $\frac{Br(t \rightarrow c\gamma)}{(2\epsilon - c^2)(1 - \epsilon)^2}$ as a function of top-pion mass $m_{\Pi_t}$ for the mass of top-Higgs $m_{ht} = 200$ GeV (solid line), $m_{ht} = 250$ GeV (dashed line), $m_{ht} = 300$ GeV (dotted line), respectively.

Figure 3: The same as Fig. 2 but for the process of $t \rightarrow cZ$
is two order smaller than that of $t \rightarrow cZ$ and $t \rightarrow cg$. The $Br(t \rightarrow c\gamma)$ decreases as $m_{H_t}$ increase and $m_{h_t}$ decrease for small $m_{H_t}$, but for large $m_{H_t}$, it increases with $m_{H_t}$ increasing and $m_{h_t}$ decreasing. The $Br(t \rightarrow cZ)$ are very sensitive to top-pions mass and decreases with $m_{H_t}$ and $m_{h_t}$ increasing. But for very large $m_{H_t}$, the branching ratio of $t \rightarrow cZ$ hardly changes with the $m_{h_t}$. As for $t \rightarrow cg$, the branching ratio decreases very sharply as $m_{H_t}$ increase for small $m_{H_t}$. In the most case, the orders of magnitude of branching ratios are $Br(t \rightarrow cg) \sim 10^{-5}$, $Br(t \rightarrow cZ) \sim 10^{-5}$, $Br(t \rightarrow c\gamma) \sim 10^{-7}$.

Comparing with the theoretical predictions in the other models, we list the maximum levels of $Br(t \rightarrow cV)$ predicted by the SM [1], the MSSM [5] and the TC2 model as follows:

|                      | SM       | MSSM     | TC2     |
|----------------------|----------|----------|---------|
| $Br(t \rightarrow cZ)$ | $O(10^{-13})$ | $O(10^{-7})$ | $O(10^{-4})$ |
| $Br(t \rightarrow c\gamma)$ | $O(10^{-13})$ | $O(10^{-7})$ | $O(10^{-6})$ |
| $Br(t \rightarrow cg)$    | $O(10^{-11})$ | $O(10^{-4})$ | $O(10^{-4})$ |

Table 1: Theoretical predictions for branching ratios of the rare top quark decays $t \rightarrow cV$. 

Figure 4: The same as Fig.2 but for the process of $t \rightarrow cg$. 

It is shown that the branching ratios of $t \to cV$ in TC2 model are significant large than that in SM and MSSM. The contributions of $\Pi_t$ and $h_t$ can enhance the SM branching ratios by as much as 6-9 orders of magnitude. On the other hand, $Br(t \to cZ)$ predicted by TC2 model is about 3 orders of magnitude larger than that predicted by MSSM. So, the mode of $t \to cZ$ is especially important for us to distinguish TC2 from MSSM.

To assess the discovery reach of the rare top quark decays in the future high energy colliders, Ref.[16] has roughly estimated the following sensitivities for $100 fb^{-1}$ of integrated luminosity:

$$LHC : Br(t \to cV) \geq 5 \times 10^{-5},$$

$$LC : Br(t \to cV) \geq 5 \times 10^{-4},$$

$$TEV33 : Br(t \to cV) \geq 5 \times 10^{-3}.$$ (10)

Comparing the theoretical predictions in TC2 model with the sensitivities of future high luminosity colliders(LHC,LC,TEV33), we can conclude that TC2 model can enhance the branching ratios $Br(t \to cV)$ to be within the observable threshold of future experiments, especially for $t \to cZ$. LHC seems to be the most suitable collider where to test rare top quark decays. The LC is limited by statistics but in compensation every collected event is clear-cut. So, this machine could eventually be of much help, especially for high luminosity.

In conclusion, we have calculated the rare top quark decays $t \to cV$ in the TC2 model. We find that the contributions arising from $\Pi_t$ and $h_t$ predicted by the TC2 model indeed significantly enhance the branching ratios of the rare top quark decays. The channels $t \to cZ$ and $t \to cg$ are found to have the larger branching ratios, which can reach $10^{-4}$ for the favorable parameter values and may be detectable in the future high energy colliders. Therefore, the rare top quark decays provide us a unique way to test TC2. Otherwise, $Br(t \to cZ)$ predicted by TC2 model is much larger than that predicted by MSSM. So, we can distinguish TC2 from MSSM via $t \to cZ$ mode.
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Appendix One: The feynman rules needed in the calculations

Based on the effective Yukawa couplings to ordinary fermions of the top-pions and top-Higgs in the TC2 model, we can write down the relevant Feynman rules used in this paper:

\[
\Pi_{t}^{0}t \rightarrow t : \quad -\frac{m_{t}}{\sqrt{2}F_{t}} \frac{\sqrt{v_{\omega}^{2} - F_{t}^{2}}}{v_{\omega}} (1 - \epsilon) \gamma_{5}
\]

\[
\Pi_{t}^{0}t \rightarrow c : \quad \frac{m_{t}}{\sqrt{2}F_{t}} \frac{\sqrt{v_{\omega}^{2} - F_{t}^{2}}}{v_{\omega}} \frac{1 - \gamma_{5}}{2} \sqrt{2\epsilon - \epsilon^{2}}
\]

\[
\Pi_{t}^{+}b \rightarrow b : \quad i\sqrt{2} \frac{m_{t}}{\sqrt{2}F_{t}} \frac{\sqrt{v_{\omega}^{2} - F_{t}^{2}}}{v_{\omega}} \frac{1 + \gamma_{5}}{2} (1 - \epsilon)
\]

\[
\Pi_{t}^{0}b \rightarrow c : \quad i\sqrt{2} \frac{m_{t}}{\sqrt{2}F_{t}} \frac{\sqrt{v_{\omega}^{2} - F_{t}^{2}}}{v_{\omega}} \frac{1 - \gamma_{5}}{2} \sqrt{2\epsilon - \epsilon^{2}}
\]

\[
h_{t}^{0}t \rightarrow t : \quad im_{t} \frac{\sqrt{v_{\omega}^{2} - F_{t}^{2}}}{v_{\omega}} (1 - \epsilon)
\]

\[
h_{t}^{0}t \rightarrow c : \quad \frac{im_{t}}{\sqrt{2F_{t}}} \frac{\sqrt{v_{\omega}^{2} - F_{t}^{2}}}{v_{\omega}} \frac{1 - \gamma_{5}}{2} \sqrt{2\epsilon - \epsilon^{2}}
\]

\[
Zh_{t}^{0} \rightarrow \Pi_{t}^{0} : \quad \frac{g}{2e_{W}} (p_{1} - p_{2})_{\mu}
\]

\[
ZZh_{t}^{0} : \quad i \frac{F_{t} gM_{Z}}{v_{\omega} c_{W}} g_{\mu\nu}
\]

\[g = \frac{e}{2e_{W}}, \quad c_{W} = \cos \theta_{W}\] is the Weinberg angle.

Appendix Two: The explicit expressions of the form factors: \( F_{V i} \)

The explicit expressions of the form factors \( F_{V i} \) used in (3)-(6) can be written as:

\[
F_{Z_i} = k_{Z} \sum_{\alpha=a}^{i} F_{Zi}^{\alpha} + k'_{Z} \sum_{\beta=j}^{k} F_{Zi}^{\beta}; \quad F_{\gamma i} = k_{\gamma} \sum_{\alpha=a}^{i} F_{\gamma i}^{\alpha}; \quad F_{g i} = k_{g} \sum_{\alpha=a}^{i} F_{g i}^{\alpha}
\]

Here, \( \alpha = a, b, c, d, e, f, g, h, i \) and \( \beta = j, k \) denote each Feynman diagrams in Fig.1. \( F_{V i}^{\gamma}(V = \gamma, Z, g i = 1, 2, 3) \) are the contributions arising from corresponding Feynman
\[ F_{Z1}^b = \frac{8}{3} s_W^2 (B_0^b + B_1^b) \]  
\[ F_{Z1}^c = 2\left(1 - \frac{2}{3} s_W^2\right) [-m_t^2 (C_{11}^c - C_{12}^c + C_{21}^c + C_{22}^c - 2C_{23}^c) + (m_t^2 - M_Z^2) (C_{22}^c - C_{23}^c) - 2C_{24}^c + \frac{1}{2}] \]  
\[ F_{Z2}^c = -4\left(1 - \frac{2}{3} s_W^2\right) m_t (C_{22}^c - C_{23}^c) \]  
\[ F_{Z3}^c = 4\left(1 - \frac{2}{3} s_W^2\right) m_t (C_{11}^c - C_{12}^c + C_{21}^c + C_{22}^c - 2C_{23}^c) \]  
\[ F_{Z1}^d = 4\left(1 - 2 s_W^2\right) C_{24}^d \]  
\[ F_{Z2}^d = -2\left(1 - 2 s_W^2\right) m_t (4C_{23}^d - 2C_{22}^d - 2C_{21}^d + C_{12}^d - C_{11}^d) \]  
\[ F_{Z3}^d = -2\left(1 - 2 s_W^2\right) m_t (2C_{22}^d - 2C_{23}^d + C_{12}^d - C_{11}^d) \]  
\[ F_{Z1}^e = \frac{4}{3} s_W^2 (B_0^e - B_0^{*e}) \]  
\[ F_{Z1}^f = \frac{4}{3} s_W^2 (B_1^f + B_1^{*f} + 2B_0^{*f}) \]  
\[ F_{Z1}^g = (1 - \frac{4}{3} s_W^2) [m_t^2 (C_{22}^g - 2C_{23}^g + C_{21}^g + C_{22}^g - 2C_{23}^g + C_{24}^g) + (2C_{24}^g + 2C_{24}^g - 1)] - \frac{4}{3} s_W^2 m_t^2 (C_{11}^g - C_{12}^g + C_{11}^g - C_{12}^g) \]  
\[ - (1 - \frac{4}{3} s_W^2) (m_t^2 - M_Z^2) (C_{22}^g - C_{23}^g + C_{22}^g - C_{23}^g) + \frac{8}{3} s_W^2 m_t^2 C_0^g \]  
\[ F_{Z2}^g = -2\left(1 - \frac{4}{3} s_W^2\right) m_t (C_{21}^g + C_{22}^g - 2C_{23}^g + C_{21}^g + C_{22}^g - 2C_{23}^g - 2C_{12}^g - 2C_{11}^g) \]  
\[ F_{Z3}^g = -2\left(1 - \frac{4}{3} s_W^2\right) m_t (C_{23}^g - C_{22}^g + C_{23}^g - C_{22}^g) - \frac{8}{3} s_W^2 m_t (C_{12}^g - C_{12}^g) \]  
\[ F_{Z1}^h = -2C_{24}^h \]  
\[ F_{Z2}^h = m_t (4C_{23}^h - 2C_{22}^h - 2C_{21}^h + C_{0}^h - C_{12}^h + C_{11}^h) \]  
\[ F_{Z3}^h = m_t (2C_{22}^h - 2C_{23}^h + 3C_{12}^h - C_{11}^h + C_{0}^h) \]  
\[ F_{Z1}^i = -2C_{24}^i \]  
\[ F_{Z2}^i = m_t (4C_{23}^i - 2C_{22}^i - 2C_{21}^i - C_{0}^i + 3C_{12}^i - 3C_{11}^i) \]  
\[ F_{Z3}^i = m_t (2C_{22}^i - 2C_{23}^i - C_{12}^i - C_{11}^i - C_{0}^i) \]  
\[ F_{Z1}^j = \frac{4}{3} s_W^2 m_t (C_{12}^j - C_{11}^j) \]  
\[ F_{Z3}^j = -\frac{8}{3} s_W^2 C_{12}^j \]
\[ F_{Z1}^k = m_t [1 - \frac{4}{3}s_W^2](C_{12}^k - C_{11}^k) + \frac{4}{3}s_W C_0^k \] (22)

\[ F_{Z2}^k = -2(1 - \frac{4}{3}s_W^2)(C_{12}^k - C_{11}^k) \] (23)

\[ F_{\gamma 1}^b = -\frac{4}{3}(B_0^b + B_1^b) \] (24)

\[ F_{\gamma 1}^c = \frac{2}{3}[-m_t^2(C_{11}^c - C_{12}^c + C_{21}^c - C_{23}^c) - 2C_{24}^c + \frac{1}{2}] \] (25)

\[ F_{\gamma 2}^c = \frac{4}{3}m_t(C_{11}^c - C_{12}^c + C_{21}^c + C_{22}^c - 2C_{23}^c) \] (26)

\[ F_{\gamma 3}^c = -\frac{4}{3}m_t(C_{22}^c - C_{23}^c) \] (27)

\[ F_{\gamma 1}^d = 4C_{24}^d \] (28)

\[ F_{\gamma 2}^d = -2m_t(4C_{23}^d - 2C_{22}^d - 2C_{21}^d + C_{12}^d - C_{11}^d) \] (29)

\[ F_{\gamma 3}^d = -2m_t(2C_{22}^d + 2C_{23}^d + C_{12}^d - C_{11}^d) \] (30)

\[ F_{\gamma 1}^e = -\frac{2}{3}(B_0^e - B_0^{*e}) \] (31)

\[ F_{\gamma 1}^f = -\frac{2}{3}(B_1^f + B_1^{*f} + 2B_0^{*f}) \] (32)

\[ F_{\gamma 1}^g = -\frac{2}{3}[-m_t^2(C_{21}^g - C_{23}^g + C_{11}^g - C_{12}^g + C_{21}^{*g} - C_{23}^{*g} + C_{11}^{*g} - C_{12}^{*g}) \]

\[ + (-2C_{24}^g - 2C_{24}^{*g} + 1)] - \frac{4}{3}m_t^2C_{0}^{*g} \] (33)

\[ F_{\gamma 2}^g = -\frac{4}{3}m_t(C_{21}^g + C_{22}^g - 2C_{23}^g + 2C_{11}^{*g} + C_{22}^{*g} - 2C_{23}^{*g} + C_{21}^{*g} - 2C_{12}^{*g}) \] (34)

\[ F_{\gamma 3}^g = -\frac{4}{3}m_t[C_{23}^g - C_{22}^g + C_{21}^{*g} - C_{22}^{*g} + C_{12}^{*g}] \] (35)

\[ F_{g1}^b = -2(B_0^b + B_1^b) \] (36)

\[ F_{g1}^c = -2[-m_t^2(C_{11}^c - C_{12}^c + C_{21}^c - C_{23}^c) - 2C_{24}^c + \frac{1}{2}] \] (37)

\[ F_{g2}^c = -4m_t(C_{11}^c - C_{12}^c + C_{21}^c + C_{22}^c - 2C_{23}^c) \] (38)

\[ F_{g3}^c = 4m_t(C_{22}^c - C_{23}^c) \] (39)

\[ F_{g1}^e = -(B_0^e - B_0^{*e}) \] (40)

\[ F_{g1}^f = -(B_1^f + B_1^{*f} + 2B_0^{*f}) \] (41)

\[ F_{g1}^g = m_t^2(C_{21}^g - C_{23}^g + C_{11}^g - C_{12}^g + C_{21}^{*g} - C_{23}^{*g} + C_{11}^{*g} - C_{12}^{*g}) \]

\[ -2C_{24}^g - 2C_{24}^{*g} + 1 - 2m_t^2C_{0}^{*g} \] (42)

\[ F_{g2}^g = -2m_t(C_{21}^g + C_{22}^g - 2C_{23}^g + 2C_{11}^{*g} + C_{22}^{*g} - 2C_{23}^{*g} + C_{21}^{*g} - 2C_{12}^{*g}) \] (43)
\[ F_{g3}^g = -2m_t [C_{23}^g - C_{22}^g + C_{22}^{ag} - C_{22}^{ag} + C_{12}^g - C_{12}^{ag}] \]  

\[
\begin{align*}
k_Z &= - \frac{i}{16\pi^2} \frac{m_t^2}{2F_t^2} \frac{v_\omega^2 - F_t^2}{v_\omega^2} \sqrt{2\epsilon - \epsilon^2} (1 - \epsilon) \frac{g}{2c_W} \\
k'_Z &= - \frac{i}{32\pi^2} \frac{m_t M_Z}{\sqrt{2} v_\omega} \frac{\sqrt{v_\omega^2 - F_t^2}}{v_\omega} \frac{g^2}{c_W} \\
k_\gamma &= - \frac{i}{16\pi^2} \frac{m_t^2}{2F_t^2} \frac{v_\omega^2 - F_t^2}{v_\omega^2} \sqrt{2\epsilon - \epsilon^2} (1 - \epsilon) \frac{e}{g_s} T^a \\
k_g &= - \frac{i}{16\pi^2} \frac{m_t^2}{2F_t^2} \frac{v_\omega^2 - F_t^2}{v_\omega^2} \sqrt{2\epsilon - \epsilon^2} (1 - \epsilon) g_s T^a 
\end{align*}
\]

Here \( g_s = \sqrt{4\pi\alpha_s} \), \( T^a \) are the Gell-Mann SU(3)\(_c\) matrices. \( B_i \), \( C_{ij} \) are two-point and three-point scalar integrals. \( p_V \) represents the momenta of \( Z, \gamma, g \), respectively.

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