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Design and analysis of moving magnet synchronous surface motor with linear halbach array

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Abstract

To solve the difficulty of lower speed and precision of conventional stage, a moving magnet type synchronous surface motor (MMSSM) with slotless iron yoke and Halbach array is designed, where the stator and mover are composed of four electromagnetic coils and four Halbach arrays respectively. The magnetic field distributions are derived analytically in terms of magnetic scalar potential with the method of separation of variables. On the basis of the analytical field, the expressions of the flux linkage, back-electromotive force (EMF) and thrust force are acquired and examined, the chief design parameters, such as air gap length, pole number, magnet thickness and coil thickness, are optimized to maximize the thrust force of the MMSSM. A comparison between the results of the proposed method and finite element analysis verifies the feasibility and credibility, which facilitates the characterization of MMSSM and provides a basis for system dynamic modeling and simulation and servo control development.

1. Introduction

In many high technology fields, the requirement of two dimensional high precision positioning device is increasing, such as integrated-circuit photolithography (stepper and repeat positioning), material science (scanning tunneling microscopy), medicine and biology (cell biology research), and so on. Traditional two-dimensional positioning devices use two stacked linear motors as actuator of high precision positioning system. Thus, position precision and response speed of the positioning devices are limited. Moreover, it has disadvantage of high manufacturing cost, complicated structure of controller and complexity of the calibration test, etc. Therefore, the direct drive surface motor has been suggested to replace above method. Nowadays, many types of surface motors have been proposed [1] [2] [3] [4]. According to their working principles, most
of the planar motors can be classified into three types, i.e., variable reluctance planar motor, induction planar motor and synchronous permanent magnet planar motor. Among them, synchronous surface motor attracted more and more attention of the academic and industrial field. This is due to the fact that they have several advantages: high force density, excellent servo characteristics and high efficient [5].

However, current synchronous surface motors have some shortcomings. First, they have slot or without iron core, which produce an undesirable destabilizing tooth ripple cogging force [6] [7]. For the synchronous surface motor without iron core, the magnetic circuit of the magnet array is open to the air gap and therefore flux density in the air gap is decreased seriously [8]. Second, the moving parts consist of many electromagnet windings and the stationery part is composed of permanent magnets array [9]. Thus, the moving cable not only decreases the reliability, but also limits the mover’s movement.

In this paper, a moving magnet synchronous surface motor (MMSSM) with slotless stator iron core is proposed. Its driving characteristics are predicted according to design parameters such as magnet thickness, air gap length, coil thickness, and pole pairs. Thus, we can better understand the working principle of the MMSSM. Furthermore, it can be used to develop controller of the MMSSM.

2. Structure of the MMSSM

2.1 Structure

Fig.1 shows the fundamental structure of the MMSSM. And its driving method is the same as in the case of an ordinary three-phase synchronous permanent magnet rotary motor. The MMSSM contains machine frame, stator iron core, X and Y thrust force windings, Halbach array and stage. X and Y thrust force windings are concentrated full-pitch winding having three isolated phase sets. Two X thrust force windings are used for driving in the X-direction and two Y thrust force windings for Y-direction. Four linear Halbach array are attached under the stage. To prevent influences from friction, compressed air is poured from the holes of the bottom of the stator iron core to keep a gap floating the mover (stage and Halbach array). By switching off the current value in the three-phase windings appropriately, the mover can move quickly in the plane surface.

![Fig.1 MMSSM with slotless iron core. (a) MMSSM, (b) Halbach array and stage](image)

Compared to the conventional synchronous surface motor, the MMSSM has the following distinctive features.

Firstly, the stator core is made of iron material, and there is no slot on the iron core. Thus, it eliminates the tooth ripple cogging effect, and thereby improves the dynamic performance and servo characteristics.
Secondly, Halbach array is used in the surface motor, thus, magnetic flux density exposed to coil becomes denser than conventional magnetic array [10]. The more magnetic flux is exposed to the thrust force winding, the more thrust force is produced by Lorentz force.

Finally, the surface motor is a moving magnet type. In the case of the moving magnet surface motor, the moving part is the magnet and the stationary part is the windings. Consequently, the mover is free of the wire and can rotate itself in addition to performing motions in two directions on the plane by means of the appropriate excitation of the windings.

2.2 Permanent magnet material

The material of the permanent magnet is an important factor to the success of the proposed MMSSM. These permanent magnets, such as AlNiCo magnets have low coercivity, Ferrite magnets have low remanence, Samarium magnets are quite expensive at present. Advanced rare earth NdFeB magnets have high residual strength which enhance the magnetic forces, and strong coercive force which lower a demagnetizing effect and are very stable under higher working temperatures. Therefore, the NdFeB permanent magnet is deemed as the best choice. In the design analyzed here, the NdFeB permanent manget have the following characteristics: residual induction $B_r=1.24\sim1.26T$; coercive force $H_c=920\sim960 \text{KA}\cdot\text{m}^{-1}$; maximum energy product $BH_{\text{max}}=295\sim303 \text{KJ}\cdot\text{m}^{-3}$. These characteristics minimize the energy required and maximize the flux, and thus high ratio of thrust force to permanent magnet volume may be achieved.

3. Magnetic field analysis

In order to design the MMSSM precisely, it is necessary to analyze magnetic field in the whole analysis region including air gap and coil area. As one of the numerical methods in magnetic field analysis, the FEM is known to allow an accurate analysis of electrical machines and can consider geometric details and the nonlinearity of the magnetic material [19]. However, FEM requires long computation time particularly at the initial design stage. Therefore, the analytic technique that solved the magnetic scalar potentials is used for the calculation of magnetic field distribution.

3.1 Magnetic field by Halbach array

A: analysis model

Fig. 2 shows magnetic field analysis model by Halbach array. The motor is idealized as a 2-D structure. And followings are assumed in order to solve the magnetic scalar potential equations.

1) The motor has the periodicity in the $x$ direction.
2) The magnetization value of permanent magnet is constant.
3) The relative permeability of the iron core is infinity.

B: Magnetic Equations

The magnetic field distribution was obtained analytically by solving the equations of the magnetic scalar potential.

1) In the air gap

The magnetic scalar potential $\varphi_1$ satisfies the following Laplace equation

$$\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_1}{\partial z^2} = 0 \quad (1)$$
Let $H_i$ and $B_i$ denote the magnetic field strength and magnetic flux density in the air gap, respectively. Then, the following equations are satisfied

\begin{align}
H_i &= -\nabla \varphi_i \quad (2) \\
B_i &= \mu_0 H_i \quad (3)
\end{align}

2) In the permanent magnets

The magnetic scalar potential $\varphi_2$ in the permanent magnets satisfies the following equation

\[ \frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial z^2} = \frac{M}{\mu_w} \quad (4) \]

Where $\mu_w$ is the relative recoil permeability of the permanent magnets and $M$ is the remanent magnetization.

For a permanent magnet having a linear demagnetization characteristic, $\mu_w$ is constant and the magnetization $M$ is related to the remanence $B_r$ by $M = B_r/\mu_0$.

As $\nabla \times M = 0$ in the magnet, the equation of $\varphi_2$ can be deduced, i.e.,

\[ \frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial z^2} = 0 \quad (5) \]

Let $H_2$ and $B_2$ denote the magnet field strength and magnet flux density in the permanent magnet, respectively. Then, the following equations are satisfied

\begin{align}
H_2 &= -\nabla \varphi_2 \quad (6) \\
B_2 &= \mu_0 (\mu_r H_2 + M) \quad (7)
\end{align}

C: Boundary Condition

Equations (1) and (4) can be solved using the following boundary conditions.

\begin{align}
H_i(x,0) \big|_{z=0} &= 0 \\
H_2(x,d + g) \big|_{z=d + g} &= 0 \\
H_i(x, g) \big|_{z=g} &= H_i(x, g) \big|_{z=g} \\
B_i(x, g) &= B_2(x, g) \quad (8)
\end{align}

D: solving equations

The magnetization layer of the Halbach array is of thickness $d$, and within this layer the magnet array is represented by an infinite Fourier series in horizontal ($x$-directed) and vertical ($z$-directed) magnetization components through terms $M_z(x)$ and $M_x(x)$

\begin{align}
M_z(x) &= M \sum_{n=1}^{\infty} B_n \sin \left( \frac{2\pi n x}{\lambda} \right) = M \sum_{n=1}^{\infty} \frac{4}{\pi n} \sin \left( \frac{\pi n}{2} \right) \cos \left( \frac{\pi n}{4} \right) \cos (\pi n) \cos \left( \frac{2\pi n x}{\lambda} \right) \\
M_x(x) &= M \sum_{n=1}^{\infty} A_n \cos \left( \frac{2\pi n x}{\lambda} \right) = M \sum_{n=1}^{\infty} \frac{4}{\pi n} \sin \left( \frac{\pi n}{2} \right) \cos \left( \frac{\pi n}{4} \right) \cos (\pi n) \sin \left( \frac{2\pi n x}{\lambda} \right) \quad (9, 10)
\end{align}

Where $\lambda$ is the wavelength of the Halbach array. $A_n$ is the $n^{th}$ harmonic amplitude of $M_z(x)$, and $B_n$ is the $n^{th}$ harmonic amplitude of $M_x(x)$. From (9) and (10), it follows that $A_n = B_n = 0$ for $n = 0, 2\ell$, ...
\[ A_n = B_n \text{ for } n = 4i + 1. \]

By applying the above boundary conditions to the interfaces between different material regions, and then the magnetic field strength and magnetic flux density in the air gap are given by

\[ H_{1x} = \sum_{n=1,5,9,13,...(4i+1)...} MA_n \left( 1 - e^{-\frac{2\pi n d}{\lambda}} \right) e^{\frac{2\pi n}{\lambda}x} \sin \left( \frac{2\pi n}{\lambda}x \right) \] (11)

\[ H_{1z} = \sum_{n=1,5,9,13,...(4i+1)...} MA_n \left( 1 - e^{-\frac{2\pi n d}{\lambda}} \right) e^{\frac{2\pi n}{\lambda}z} \cos \left( \frac{2\pi n}{\lambda}x \right) \] (12)

\[ B_{1x} = \mu_0 H_{1x} = \sum_{n=1,5,9,13,...(4i+1)...} \mu_0 MA_n \left( 1 - e^{-\frac{2\pi n d}{\lambda}} \right) e^{\frac{2\pi n}{\lambda}x} \sin \left( \frac{2\pi n}{\lambda}x \right) \] (13)

\[ B_{1z} = \mu_0 H_{1z} = \sum_{n=1,5,9,13,...(4i+1)...} \mu_0 MA_n \left( 1 - e^{-\frac{2\pi n d}{\lambda}} \right) e^{\frac{2\pi n}{\lambda}z} \cos \left( \frac{2\pi n}{\lambda}x \right) \] (14)

Where subscript \( x \) and \( z \) denote \( x \)-direction and \( z \)-direction, respectively. \( z \) is the distance from winding surface to the Halbach array underside.

According to (14), the first harmonic of the magnetic flux density function \( B_x(x) \) is expressed as follows:

\[ B_x(x) = B_x A \left( 1 - e^{-\frac{2\pi x}{\lambda}} \right) e^{\frac{2\pi x}{\lambda}} \cos \left( \frac{2\pi x}{\lambda} \right) = B_m \cos \left( \frac{2\pi x}{\lambda} \right) \] (15)

Where \( B_m = B_x A \left( 2\frac{\sqrt{2}}{\pi} \right) \left( 1 - e^{-\frac{2\pi x}{\lambda}} \right) e^{\frac{2\pi x}{\lambda}} \) is the peak magnetic flux density.

According to the analysis as mentioned above, the magnetic field generated by Halbach array is obtained.

![Fig.2 Magnetic field analysis model by Halbach array](image)

3.2 Magnetic field by stator winding current

The magnetic field analysis model by stator winding current is shown in Fig. 3. In contrast with the mover, the stator is long enough to be assumed that it is infinitely long. Therefore, using the magnetic scalar potential equation (16), the magnetic fields can be solved in similar way

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = -\mu_0 J(x) \] (16)

Where \( J(x) \) is armature current density.

The magnetic field due to stator winding current is
Where $K$ is $\frac{\pi}{\tau}$.

As mentioned above, total magnetic field is superposition of the field due to Halbach array and stator winding current.

4. Back EMF and Thrust Force

The symbols used for calculation of the magnetic flux linkage are shown in Fig. 4. The reference frame $XYZ$ is fixed on the stator. Initially, one of coil of phase $A$ $x$-winding are fixed on the position $(x_c, y_c)$. The magnetic flux linkage of a winding can be derived by integrating three isolated areas of rectangle area $ABDEA$, arc area $BCDB$, and arc area $AEFA$. And the equation for calculating the flux linkage of phase $A$ $x$-winding is expressed as follows:

$$
\phi_{ax} = \phi_{AD} + \phi_{BD} + \phi_{AE} = NN_x \int B_z(x)ds = NN_x \int B_n \cos\left(\frac{2\pi x}{\lambda}\right)ds
$$

(18)

where $N$ is turns of each coil, $N_x$ is the number of series turns phase $A$ of $x$-winding. $\phi_{AD}$, $\phi_{BD}$, and $\phi_{AE}$ are the fluxes through the faces ABDEA, BCDB, and AEFA, respectively.

$$
\phi_{AD} = N \int_{x_c}^{x_c + \frac{l}{2}} \int_{y_c}^{y_c + \frac{h}{2}} B_n \cos\left(\frac{2\pi y}{\lambda}\right)dx dy = \frac{NB_n l_x}{\pi} \sin\left(\frac{\pi l_x}{2\lambda}\right) \cos\left(\frac{2\pi x}{\lambda}\right)
$$

(19)

$$
\phi_{BD} = N \int_{x_c}^{x_c + \frac{l}{2}} \int_{y_c}^{y_c + \frac{h}{2}} B_n \cos\left(\frac{2\pi y}{\lambda}\right)dx dy = \frac{NB_n l_x}{\pi} \sin\left(\frac{\pi l_x}{2\lambda}\right) \cos\left(\frac{2\pi x}{\lambda}\right) \left(\frac{1}{4}\right)
$$

(20)

$$
\phi_{AE} = \frac{\phi_{AD}}{2}
$$

(21)

Consequently, the total magnetic flux linkage of phase $A$ $x$-winding is expressed as follows

$$
\phi_{ax} = NN_x \left(\frac{B_n l_x}{\pi} \sin\left(\frac{\pi l_x}{2\lambda}\right) \cos\left(\frac{2\pi x}{\lambda}\right) + \frac{2B_n l_x}{\pi} \sin\left(\frac{2\pi l_x}{2\lambda}\right) \cos\left(\frac{2\pi x}{\lambda}\right) \left(\frac{1}{4}\right)\right)
$$

(22)

When the mover is moving along the $x$-direction, the magnetic field is moving whilst the stator is static; hence, the distribution of the air gap magnetic field varies with the position of mover. So, the flux coupled with each winding is variant with time. The back EMF of each phase can be written as
Thus, back EMF is given by the product of velocity \( v \) and the rate of change in flux linkage with respect to position. Hence, the back EMF of phase \( A \), which results from relative motion between the stator and mover, is obtained as

\[
E_a = -\frac{d\phi_{\mu_a}}{dt} = -\frac{d\phi_{\mu_a}}{dx}v = 2N_B a \sin \left( \frac{2\pi x_c}{2\lambda} \right) \cos \left( \frac{\pi t_a}{2\lambda} \right) + l_a \sin \left( \frac{2\pi}{\lambda} \left( x_c + \frac{L_a}{4} \right) \right) \cdot v
\]

(24)

where \( v \) is the motor velocity along the \( x \)-direction.

For the fully-pitch winding, the distribution of each phase of the three phase winding cover a third of the pole pitch \( \tau \). Then, the magnetic flux linkage of phase B and phase C are obtained from (22) substituting \( x_c \) with \( x_c - \frac{2\tau}{3} \) and \( x_c - \frac{4\tau}{3} \) respectively.

Consequently, the back EMF of phase B and C are obtained as

\[
E_b = E_a \left( x_c - \frac{2\tau}{3} \right)
\]

(25)

\[
E_c = E_a \left( x_c - \frac{4\tau}{3} \right)
\]

(26)

Where \( E_b \) and \( E_c \) are phase B and C back EMF, respectively.

The thrust force that is exerted on the mover in the surface motor, resulting from the interaction between the winding current and the permanent magnet field, is given by

\[
F = \left( E_a I_a + E_b I_b + E_c I_c \right) / v = \frac{E_a}{v} \left( I_a + I_b \left( x_c - \frac{2\tau}{3} \right) + I_c \left( x_c - \frac{4\tau}{3} \right) \right)
\]

(27)

Where \( I_a \), \( I_b \), and \( I_c \) are current vectors of phase A, phase B, and phase C, respectively.

Fig. 4 Winding shape and symbols for flux calculation of magnetic flux linkages

5. Thrust force analysis according to design parameters

A: Thrust force analysis according to air gap length
First, this paper examined the effect of air gap length on the thrust force for different pole numbers with the magnet array size fixed. Fig. 5 shows thrust force curves acting on the mover along the air gap length with pole numbers equal to 1, 2.5, 4 and 5.5. It can be seen that the more pole numbers increase, the more variation of thrust force is greatly affected by air gap length. In case of air gap<0.4mm, 1 poles is superior to the others for thrust characteristics according to air gap length, but if air gap length becomes short, it is difficult to maintain fixed air gap. In case of air gap>0.63mm, 5.5 poles is superior to the others for thrust characteristics according to air gap length, but air gap length is too long to produce high thrust. Therefore, these analyses make us choose 2.5 poles for values of various air gap lengths, under constraint condition fixed other parameters.

![Fig.5 Thrust force curves for different pole pairs](image)

**B: Thrust force analysis according to magnet thickness**

Next, this paper examined the effect of magnet array thickness on the thrust force with the air gap length fixed. Fig. 6(a) shows thrust force curves acting on the mover versus magnet array thickness for different pole numbers. It can be discovered that thrust force increases with increasing poles numbers. In case of pole numbers >2.5, the increasing rate slightly decreases. In additionally, It can also be observed that for the different pole numbers, thrust force variation according to magnet array thickness, as the value of magnet array thickness is 12mm or more, seldom change. So, for a fixed air gap length, magnet array thickness can be chosen as 12mm.

This paper continued this investigation of magnet array thickness on the thrust force for different pole pitch with the air gap length and pole numbers fixed. Fig. 6(b) shows thrust force variations acting on the mover versus magnet array thickness for different values of pole pitch. It can be seen that the more values of magnet array thickness increase, the more thrust force increases and the more values of pole pitch increase, the more difference of thrust force variations are slowed down. In particular, it can be observed that thrust variation according to magnet thickness in case of the value of pole pitch=8mm is almost twice as much as that in case of the value of pole pitch=4mm, whereas thrust force variation according to magnet array thickness in case of the value of pole pitch=8mm makes little difference that in
case of the value of pole pitch=16mm. So, for a fixed air gap length, proper magnet pole pitch 8mm can be determined from these results.

![Thrust force curves](image)

**Fig. 6 Thrust force curves. (a) for different pole pairs, (b) for different pole pitch**

C: Thrust force analysis according to winding thickness

Finally, this paper investigated the effect of the winding thickness on the thrust with the air gap length and magnet array size fixed. Fig. 7(a) shows the variation of the current density in the stator windings with the thickness of the windings under the constraint of constant ohmic power dissipation in the stator windings. Plotted in Fig. 7(b) is the thrust force per current density in the windings, which is the integral of the $z$-component of the no-load air gap magnetic flux density over the volume occupied by the stator windings. The product of the Fig. 7(a) and (b), shown in Fig. 7(c), indicated that there is an optimal thickness of stator windings (in this case 6mm) for highest motor thrust force.

6. Design results

The specification of the MMSSM from the design process is shown in Table I. And the result of thrust force computation corresponding to a series of load current values are shown in Fig. 8. It can be seen that the thrust force ripple every 60° is generated. This is because of the mismatching of the current and the magnetic field. And also the thrust force by FEM is somewhat less than that analytical method. The reason is that the nonlinear characteristic of permeability in iron core is considered in the FEM. Maximum thrust force of above 30N is got while the excited current is 2A.

7. Conclusions

A moving magnet synchronous surface motor (MMSSM) with slotless iron core has been proposed. It has higher force density compared to the coreless MMSSM. Two sets of windings for generating $x$-direction thrust force and $y$-direction thrust force is perpendicularly fixed in the $x$-direction and $y$-direction of the stator iron core, respectively.

For the MMSSM, magnetic field distribution has a very significant effect on the driving characteristic. By using the method of separation of variables, Analytical solutions of magnetic field distribution are derived, and the results are verified against the 2-D FEM.
The shape of coils and Halbach array has significant effect on the thrust force. This paper, therefore, investigated the effect under constant MMF. And thrust force ripple taking into account driving manner is also analyzed that is generated by mismatching the current and magnetic field. The design results are useful in developing the control system of the MMSSM.

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