EXPLORING THE SPECTRUM OF QCD USING A SPACE-TIME LATTICE

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Some past and ongoing explorations of the spectrum of QCD using Monte Carlo simulations on a space-time lattice are described. Glueball masses in the pure-gauge theory are reviewed, and the energies of gluonic excitations in the presence of a static quark-antiquark pair are discussed. Current efforts to compute the baryon spectrum using extended three-quark operators are also presented, emphasizing the need to use irreducible representations of the cubic point group to identify spin quantum numbers in the continuum limit.

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1. Introduction

Spectroscopy is a powerful tool for distilling the key degrees of freedom in a given system. At the present time, the best way of extracting the spectrum of states from the QCD Lagrangian is Monte Carlo computer calculations using a space-time lattice. A spectrum determination requires the extraction of many excited-state energies, so a brief discussion on how excited-state energies can be determined from Monte Carlo estimates of correlation functions in Euclidean field theory is warranted. In this talk, several past explorations of the QCD spectrum are outlined, in particular, glueball masses in the pure-gauge theory and the energies of gluonic excitations of the so-called static quark-antiquark potential. Also, ongoing efforts by the Lattice Hadron Physics Collaboration (LHPC) to determine the baryon spectrum using extended three-quark operators are described.

2. Extracting Excited-State Energies and Resonances

The Monte Carlo method can be applied to obtain estimates of the path integrals which yield a Hermitian matrix of correlation functions $C_{ij}(t) = \langle 0 | O_i(t) \overline{O}_j(0) | 0 \rangle$, where $\overline{O}_j(0)$ creates the states of interest at time $t = 0$ and $O_i(t)$ annihilates such states at a later time $t$. The procedure for extracting the lowest energies $E_0, E_1, E_2, \ldots$ from this matrix is well known. Let $\lambda_n(t, t_0)$ denote the eigenvalues of the hermitian matrix $C(t_0)^{-1/2} C(t) C(t_0)^{-1/2}$, where $t_0$ is some fixed
reference time (typically small) and the eigenvalues, also known as the principal correlation functions, are ordered such that \( \lambda_0 \geq \lambda_1 \geq \cdots \) as \( t \) becomes large. Then one can show that

\[
\lim_{t \to \infty} \lambda_n(t, t_0) = e^{-E_n(t-t_0)} \left(1 + O(e^{-\Delta_n(t-t_0)})\right),
\]

\[
\Delta_n = \min_{k \neq n} |E_k - E_n|.
\]

Determinations of the principal correlators \( \lambda_n(t, t_0) \) for large temporal separations \( t \) yield the desired energies \( E_n \). Since statistical fluctuations grow with increasing \( t \), it is crucial that contributions from higher-lying states be diminished so that the desired lowest-lying energies dominate the principal correlators well before the signal-to-noise ratio falls. Judiciously chosen quark-field and gluon-field smearings is another important ingredient for reducing couplings to the short wavelength modes of the theory. The use of large sets of extended operators is another key ingredient.

Our Monte Carlo calculations are carried out in a finite-sized box with periodic boundary conditions. Given the finite volume and the discrete nature of the allowed momenta in the box, the masses and widths of resonances (unstable hadrons) cannot be calculated directly, but must be deduced from the discrete spectrum of finite-volume stationary states for a range of box sizes. Such applications require prohibitive computational resources, but our goal in the baryon project is to obtain a first exploratory scan of the spectrum, so simply obtaining the finite-volume spectrum for a few judiciously-chosen volumes should suffice for ferreting out the hadron resonances from the less interesting scattering states.

3. Glueballs and Gluonic Excitations in Presence of Static \( \bar{Q}Q \) Pair

Glueball masses in the pure-gauge theory, shown in Fig. 1, were the first exploration of the QCD spectrum in which I was involved. The spectrum can be fairly well described by a bag model description of the gluons, whereas string models seem to fare less well. Inclusion of light-quark effects is an ongoing challenge.

The issue of string formation of the gauge-field in the presence of a static quark-antiquark pair was my next exploration. For quark-antiquark separations \( R \) greater than 2 fm, the spectrum of gluonic excitations agreed without exception with that expected from an effective string theory description of the gauge field (see Fig. 1), and a fine structure provided tantalizing clues to the nature of such an effective string theory. A dramatic level reordering was observed as \( R \) became smaller, suggesting a bag model picture or multipole expansion may be more relevant at such scales. These energies were used as a starting point in a Born-Oppenheimer treatment of heavy-quark mesons, both conventional and hybrid (bound by an excited gluon field). Agreement of level splittings from direct Monte Carlo calculations with those in the leading Born-Oppenheimer approximation validated such a treatment and provided a compelling physical picture of both conventional and hybrid heavy-quark mesons.
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4. Baryons

I am currently a member of the Lattice Hadron Physics Collaboration (LHPC). One of our goals is to compute the spectrum of baryon resonances, with an eye towards later determining the meson spectrum. We have designed large sets of extended, gauge-invariant three-quark operators to facilitate this task. The usual operator construction which mimics the approach one would take in continuous space-time is very cumbersome, especially when tackling an entire spectrum. Our approach combines the physical characteristics of baryons with the symmetries of the lattice regularization of QCD used in simulations. For baryons at rest, our operators are formed using group-theoretical projections onto the irreducible representations (irreps) of the \( O_h \) symmetry group of a three-dimensional cubic lattice. There are four two-dimensional irreps \( G_1^g, G_1^u, G_2^g, G_2^u \) and two four-dimensional representations \( H^g \) and \( H^u \). The continuum-limit spins \( J \) of our states can be deduced by examining degeneracy patterns across the different \( O_h \) irreps.

![Fig. 1. The glueball mass spectrum in the pure-gauge theory (left). The spectrum of gluonic excitations in the presence of a static quark-antiquark pair separated by distance \( R \) (right).](image)

![Fig. 2. The spatial arrangements of the extended three-quark baryon operators used. Smeared quark fields are shown by solid circles, line segments indicate gauge-covariant displacements, and each hollow circle indicates the location of a Levi-Civita color coupling. For simplicity, all displacements have the same length in an operator.](image)
Baryons are expected to be rather large objects, and hence, local operators will not suffice. Our approach to constructing extended operators is to use covariant displacements of the quark fields. Displacements in different directions are used to build up the appropriate orbital structure, and displacements of different lengths can build up the needed radial structure. There are six different spatial orientations that we use, shown in Fig. 2. The singly-displaced operators are meant to mock up a diquark-quark coupling, and the doubly-displaced and triply-displaced operators are chosen since they favor the Δ-flux and Y-flux configurations, respectively.

We have now finished optimizing the quark-field and gauge-field smearing parameters, and have begun low-statistics runs to prune out the ineffectual and overly-noisy operators. Some sample effective mass plots are shown in Fig. 3. Our goal is to find the smallest set of operators out of the several hundred we have constructed which are useful for extracting some number of lowest-lying states, and to push the technology to maximize the number of excited-state energies which can be reliably determined. We shall report our findings in the near future. This work was supported by the U.S. National Science Foundation through grant PHY-0354982.

References

1. C. Michael, *Nucl. Phys. B* 259, 58 (1985).
2. M. Lüscher and U. Wolff, *Nucl. Phys. B* 339, 222 (1990).
3. B. DeWitt, *Phys. Rev.* 103, 1565 (1956).
4. U. Wiese, *Nucl. Phys. B (Proc. Suppl.)* 9, 609 (1989).
5. M. Lüscher, *Nucl. Phys. B* 364, 237 (1991).
6. K. Rummukainen and S. Gottlieb, *Nucl. Phys. B* 450, 397 (1995).
7. C. Morningstar and M. Peardon, *Phys. Rev. D* 60, 034509 (1999).
8. K. J. Juge, J. Kuti, and C. Morningstar, *Phys. Rev. Lett.* 90, 161601 (2003).
9. K.J. Juge, J. Kuti, and C.J. Morningstar, *Phys. Rev. Lett.* 82, 4400 (1999).
10. S. Basak, R. Edwards, G.T. Fleming, U.M. Heller, C. Morningstar, D. Richards, I. Sato, and S. Wallace, unpublished (hep-lat/0506029).