QCD critical point, baryon number fluctuations, and final state interactions

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Abstract. Baryon number cumulants are important quantities to diagnose the primordial stage of heavy ion collisions. In experiments, however, proton number cumulants have been measured as substitutes. In fact, proton number fluctuations are further modified in the hadron phase and different from those of baryon number. We explore the relation between proton and nucleon number fluctuations in the final state in relativistic heavy ion collisions.

1. Introduction
The order of the phase transition of quantum chromodynamics (QCD) at nonzero temperature ($T$) is believed to change from crossover [1] to first order as baryon chemical potential ($\mu_B$) increases. The existence of the QCD critical point is thus expected in the phase diagram on $T$-$\mu_B$ plane [2]. Experiments to explore the phase structure at nonzero $\mu_B$, especially the existence of the critical point, are now ongoing in the energy scan program at the Relativistic Heavy Ion Collider (RHIC) [3, 4, 5], and will also be performed in future facilities. The establishment of the QCD phase structure is an important issue to deepen our knowledge on the matter described by QCD.

Fluctuations, which are experimentally measured by event-by-event analyses in heavy ion collisions, are promising observables to probe properties of created fireballs [6], as their behaviors are sensitive to the state of the matter. Among the fluctuation observables, those of conserved charges can reflect fluctuations produced in earlier stages than non-conserved ones [7, 8, 9]. This is because the variation of a conserved charge in a volume is achieved only through diffusion, which makes the relaxation to equilibrium slower.

The dependences of proton number fluctuations, cumulants up to fourth order, on the beam energy $\sqrt{s}$ are recently measured by STAR collaboration at RHIC [3, 4, 5]. The result looks almost consistent with the prediction of the hadron resonance gas (HRG) model [10]. The proton number, however, is not a conserved quantity, and in fact we will later see that its fluctuations significantly evolve in the hadronic stage, which makes the experimentally-measured fluctuations close to those in the equilibrated hadronic matter. The agreement between the experiments and the HRG model in the proton number fluctuations [5] is in part due to these effects, and hence it does not immediately exclude the slow baryon number diffusion in the hadronic stage. Although the measurement of baryon number, which is a conserved charge, is desirable, its direct experimental measurement has been considered to be impossible because of the difficulty in the detection and identification of neutrons.
In the following, we show that the experimentally-measured proton number fluctuations are nevertheless directly related to baryon number fluctuations in earlier stages, and present concrete formulas that relate baryon number cumulants and these experimental observables. The key observation is that the distributions of (anti-)proton and (anti-)neutron numbers in the final state are well described by binomial distributions. As will be argued in detail later, this observation is justified well at least for RHIC energy, and is expected to hold for \( \sqrt{s} \geq 10\text{GeV} \). Throughout this article, we use \( N_X \) to represent the number of particles \( X \) leaving the system after each collision event, where \( X = p, n, \) and \( B \) represent proton, neutron, and baryon, respectively, and their anti-particles, \( \bar{p}, \bar{n}, \) and \( B \). The net and total numbers are denoted as \( N_X^{(\text{net})} = N_X - N_{\bar{X}} \) and \( N_X^{(\text{tot})} = N_X + N_{\bar{X}} \), respectively.

2. Formulation

Let us first briefly consider how the proton number fluctuations evolve in the hadronic stage. The most important process responsible for the variation of the proton number is the charge exchange reactions with thermal pions mediated by \( \Delta^{+}(1232) \) and \( \Delta^{0}(1232) \) resonances, \( p(n) + \pi \rightarrow \Delta^{+0} \rightarrow n(p) + \pi \). Because of a little energy required and the large cross sections, these reactions proceed even after chemical freeze-out. We note that these reactions do not alter the average abundances, \( \langle N_p \rangle \) and \( \langle N_{\bar{p}} \rangle \), if the isospin chemical potential vanishes, while they modify the fluctuations of \( N_p \) and \( N_{\bar{p}} \). Because chemical freeze-out is a concept to describe ratios between particle abundances such as \( \langle N_p \rangle / \langle N_{\bar{p}} \rangle \), their existence below the chemical freeze-out temperature, \( T_{\text{chem}} \), does not contradict the statistical model. The success of the model, on the other hand, indicates that creations and annihilations of (anti-)nucleons hardly occur below \( T_{\text{chem}} \).

The importance of the above reactions below \( T_{\text{chem}} \) is confirmed by evaluating the mean time of nucleons for these reactions. The mean time is evaluated to be \( 3 - 4 \text{fm} \) for \( T = 150 - 170 \text{MeV} \). One can also check that this mean time hardly changes even for moving protons in the range of momentum \( p \lesssim 3T \). On the other hand, dynamical models for RHIC energy predict that protons stay in the hadronic gas and continue to interact for several tens of fm on average at midrapidity [13]. From this result and the lifetime of \( \Delta \), it is concluded that nucleons in the fireball indeed undergo this reaction several times on average in the hadronic stage. The ratio of the probabilities that a proton in medium produces a \( \Delta^{+} \) or \( \Delta^{0} \), and then decays into \( p \) and \( n \) is 5 : 4, which is determined by the isospin symmetry. Whereas this probability is not even, after repeating the above processes several times in the hadronic stage nucleons completely forget their initial isospin.

The above observation shows that the evolution of proton number fluctuations in the hadronic stage is dominantly made via the exchanges of the two isospin states of nucleons. Now, we further assert that isospins of all nucleons in the final state are uncorrelated. This statement is well justified when the hadronic medium fulfills the following two conditions: (i) medium effects on the branching ratios and formation rates of \( \Delta \) are insensitive to the proton and neutron number densities, \( n_p \) and \( n_n \) (and the same holds for the anti-particle sector as well), and (ii) (anti-)nucleon-(anti-)nucleon interactions generating correlations between two nucleons hardly occur. As we will see later, these two conditions are well satisfied below \( T_{\text{chem}} \) except for low energy collisions. The probability distribution of finding \( N_p \) and \( N_n \) (\( N_{\bar{p}} \) and \( N_{\bar{n}} \)) particles in the final state in each event then becomes binomial. Under the isospin symmetry [10], this fact enables to factorize the probability distribution \( P(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) \) having \( N_p, N_n, N_{\bar{p}}, \) and \( N_{\bar{n}} \) particles in each event as

\[
P(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = F(N_B, N_{\bar{B}})B(N_{\bar{p}}; N_{\bar{B}})B(N_{\bar{n}}; N_{\bar{B}}),
\]

where \( B(k; N) = 2^{-N} N! / (k!(N - k)!) \) is the binomial distribution function with an equal
probability. On the right hand side (RHS) of Eq. (1) we have used $N_B$ and $N_B$ defined by $N_B = N_p + N_n$ and $N_B = N_p + N_n$. For the details of the derivation of Eq. (1), see Ref. [11, 12].

Equation (1) enables to represent baryon number cumulants by those of net and total proton numbers as

$$\langle N_B^{(\text{net})} \rangle = 2\langle N_p^{(\text{net})} \rangle,$$

$$\langle (\delta N_B^{(\text{net})})^2 \rangle = 4\langle (\delta N_p^{(\text{net})})^2 \rangle - 2\langle N_p^{(\text{tot})} \rangle,$$

$$\langle (\delta N_B^{(\text{net})})^3 \rangle = 8\langle (\delta N_p^{(\text{net})})^3 \rangle - 12\langle (\delta N_p^{(\text{net})})^2 \delta N_p^{(\text{tot})} \rangle + 6\langle N_p^{(\text{net})} \rangle,$$

$$\langle (\delta N_B^{(\text{net})})^4 \rangle = 16\langle (\delta N_p^{(\text{net})})^4 \rangle - 48\langle (\delta N_p^{(\text{net})})^2 \delta N_p^{(\text{tot})} \rangle + 48\langle (\delta N_p^{(\text{net})})^2 \rangle$$

$$+ 12\langle (\delta N_p^{(\text{tot})})^2 \rangle - 26\langle N_p^{(\text{tot})} \rangle,$$

where $\delta N_X = N_X - \langle N_X \rangle$. Since the RHSs of Eqs. (2) - (5) consist of only $N_p^{(\text{net})}$ and $N_p^{(\text{tot})}$, which are experimentally observable, these are formulas that express baryon number cumulants solely in terms of experimental observables. We remind that no specific form of $F(N_B, N_B)$ is assumed in deriving these results.

We remark that $N_B^{(\text{net})}$ ($N_B^{(\text{tot})}$) in Eqs. (2) - (5) are interpreted to be the sum of all net (total) baryon numbers entering a region in the phase space in the final state of each event. Since the diffusion of baryon number in the hadronic stage is slow [7, 8], the information of primordial fluctuations remains in $F(N_B, N_B)$ in Eq. (1) and, as a result, baryon number cumulants.

Next, let us inspect the validity of Eq. (1) in more detail. First, we consider the conditions (i) and (ii) introduced above Eq. (1). At RHIC energy, the Boltzmann approximation is well applied to nucleons below $T_{\text{chem}}$ since $T \ll m_N$ and $|\mu_B| \ll m_N$. Thus, the Pauli blocking effect can be almost ignored. The Bose factor, on the other hand, has a nonnegligible contribution since $m_\pi \simeq T_{\text{chem}}$. The density of pions, however, is more than one order larger than that of nucleons below $T_{\text{chem}}$. The Bose factor thus must be insensitive to $n_p$ and $n_n$, while it leads to the enhancement of the decay of $\Delta$ in medium, which acts in favor of the isospin randomization.

The large pion density also means that the mean time of a nucleon to form $\Delta$ is insensitive to $n_p$ and $n_n$. The condition (i) is thus well satisfied below $T_{\text{chem}}$ at RHIC energy. The validity of the condition (ii) is conjectured from the success of the statistical model as follows. The statistical model indicates that the pair annihilation of a $N$ and an $\bar{N}$ terminates at $T_{\text{chem}}$. $NN$ and $\bar{N}N$ reactions are then also expected to terminate there, because the elastic cross section of $N\bar{N}$ is significantly smaller than the inelastic one, and the total cross section of $NN$ behaves similarly as that of $N\bar{N}$ for $E_{\text{cm}} < 1\text{GeV}$ [14]. The condition (ii) thus should also be satisfied for $T < T_{\text{chem}}$. Intuitively speaking, in hot medium nucleons are so dilutely distributed that they do not feel other ones’ existence, while there are so many pions which can be regarded as the heat bath when the nucleon sector is concerned. The large abundance of pions would also enable to neglect the effects of nucleon isospin fluctuations possibly generated in the early stage. Second, strange baryons, on the other hand, decay via the weak or electromagnetic interaction outside the fireball. In particular, the $\Lambda$ and $\Sigma$ are important among them. The $\Lambda$ decays into $p$ and $n$ with the branching ratio $16 : 9$. Provided that the three isospin states of the $\Sigma$ are produced with an equal probability in medium, the ratio of probabilities that a $\Sigma^+$ decays into $p$ and $n$ is about $13 : 12$ [14], while a $\Sigma^-$ decays into $n\pi^-$. A $\Sigma^0$ decays into a $\Lambda$. If the $\Lambda$ and $\Sigma$ multiples are created with an equal probability, the production ration of form their decays is about $9 : 11$. Actually, because of the mass splitting between the $\Lambda$ and $\Sigma$ triplets, the production of the $\Sigma$ triplets are a bit suppressed compared to that of the $\Lambda$. This makes the ratio even closer to even. Thus the number of nucleons produced by these decays can be incorporated into $N_p$ and $N_n$ in Eq. (1). This promotes the nucleon numbers to those of baryons in Eq. (1). Inclusion of higher baryonic resonances and light nuclei such as deuterons will not affect our conclusions owing to their negligible abundances.
While the factorization Eq. (1) is fully established for RHIC energy, the binomiality will eventually break down as the beam energy is decreased. At very low beam energy, pions are not produced enough and nucleons will not undergo charge exchange reactions sufficiently below $T_{\text{chem}}$. We deduce that this happens when $T_{\text{chem}} \lesssim m_{\pi}$. When the reactions scarcely occur, the isospin correlations generated at the hadronization will remain until the final state. At low beam energy, also nucleon density becomes comparable with that of pions, and the latter can no longer be regarded as the heat bath to absorb isospin fluctuations of the former. From the $\sqrt{s}$ dependence of the chemical freeze-out line on the $T$-$\mu_B$ plane [15] and considering the validity of these two conditions we deduce that Eq. (1) is well applicable to $\sqrt{s} \gtrsim 10\text{GeV}$.

In the argument to derive Eqs. (2) - (5), we have implicitly assumed that the hadronic medium is isospin symmetric. While the effect of nonzero isospin density should be well suppressed for large $\sqrt{s}$ where large number of particles having nonzero isospin charges are produced, at lower energies this effect gives rise to nonnegligible modification to Eqs. (2) - (5). This effect is, as long as the conditions (i) and (ii) introduced above Eq. (1) hold, incorporated to our results by simply replacing the binomial function $B(N_p; N_B)$ in Eq. (1) with that having probability $k = \langle N_p \rangle / (N_p + N_n)$, and similar replacement to $B(N_{\bar{p}}; N_{\bar{B}})$ [12].

3. Conclusion
In the present article, we have derived relations between baryon and proton number cumulants, Eqs. (2) - (5), on the basis of the binomial nature of (anti-)nucleon isospin numbers in the final state. These results enable to immediately determine the baryon number cumulants with experimental results in heavy ion collisions, which will provide significant information on the QCD phase diagram. Though these results are obtained for the isosymmetric case, incorporation of nonzero isospin density is straightforward and is discussed in [12].

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