Influence of the initial state of a distributed system on the optimal control of quartz optical fiber drawing

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Abstract. An optimal control problem for a distributed system with compromise control is considered out of this work. The solvability is shown and an optimization system is obtained for the optimal control issue with boundary monitoring and boundary supervision described by equations of melts continuity and movement. An algorithm for implementing the optimal control problem is suggested and the outcome is analyzed. The issue is resolved for a few types of initial conditions used with the radius deviation function from its programmed determination.

1. Introduction
Optimal control theoretics of distributed systems is a relatively young scientific discipline. Despite this fact, it is very rich in results and applications. The theory of optimal control which has been effectively developed causes real objects to refer to various nature (physical, chemical, economical, etc) are apportioned in space [1, 2]. In this paper, an object of control is the process of optical fiber drawing which is portrayed through partial differential equations.

Fresh manufacture of optical fibers is a complex high-accuracy technological operation in which many physical and chemical phenomena are realized. The constancy of the characteristics of the resulted fiber along its length is one of the main indicators of fiber quality. However, it is impossible to realize the continuous monitoring of most indicators in the process of fiber production. In this situation, the fiber diameter is unceasingly measured solely at the latter stage of manufacture – fiber drawing. Based on experience there is a fine correlation between the constancy of the diameter of the followed fiber and the constancy of its different specifications along the length. Consequently, the available control systems for the fiber drawing are rooted in this base [3].

Nowadays, the quality of the optical fiber is evaluated only after the drawing process, i.e. the optical characteristics of the resulted fiber are estimated. In this work, the authors propose a qualitatively different approach to this problem. It is adjusting the fiber diameter by controlling its winding speed. This approach will improve the quality of the fiber even at the drawing stage.

Taking everything mentioned above into consideration, the main purpose of this study is to develop the theoretical foundations for the optimal control of the fiber drawing as the control of a distributed system.

2. Materials and methods
In an ideal case, the permanence of the final fiber diameter, over the isothermal conditions, is possible to acquire and are in the steady-state drawing mode, i.e. with permanent feeding and drawing velocities.
The motion of the system at certain values of the speeds will be named the programmed movement, as well as the appropriate control is programmed control.

However, the actual (factual) movement of the system always differs from the programmed one for a number of reasons: a) incorrect realization of programmed control; b) incorrect implementation of original and boundary conditions; c) incorrect data regarding the physical and mechanical characteristics of the material; d) imperfect data regarding external impacts affecting the system, etc. Thus the actual values of the radius of the fiber \( R(t,z) \), the speed of the material \( V(t,z) \) and control \( u(t,z) \) are described by the following functions

\[
\begin{align*}
R(t,z) &= R_{st}(z)(1 + \tilde{R}(t,z)), \\
V(t,z) &= V_{st}(z)(1 + \tilde{V}(t,z)), \\
u(t,z) &= u_{st}(z)(1 + \tilde{u}(t,z)),
\end{align*}
\]

where \( t \) is time, \( z \) is the longitudinal coordinate, \( R_{st}(z) \) and \( V_{st}(z) \) are programmed (stationary) movement, \( u_{st}(z) \) is the programmed control appropriate to the stationary state, \( \tilde{R}(t,z) \) and \( \tilde{V}(t,z) \) are deviations (disturbance) of the factual motion from the programmed movement, \( \tilde{u}(t,z) \) is the deviation of the practical control from the programmed control.

Taking into account the remarks stated above, the goal of optimal control is to reduce the deviations of the actual fiber radius values of the programmed ones: \( R_{st}(z)\tilde{R}(t,z) \rightarrow 0 \). That kind of monitoring is called the optimal stabilizing control.

In order to realize the assigned problem, it is necessary to obtain the perturbed movement equations describing the deviations of the practical motion from the one that is programmed.

For the investigated process, the system describing the perturbation of the steady-state drawing mode (isothermal case) has the given form [3]:

\[
\begin{align*}
-\frac{\partial \tilde{R}}{\partial t} &= V_{st} \frac{\partial \tilde{R}}{\partial z} + \frac{V_{st}}{2} \frac{\partial \tilde{V}}{\partial z}, \\
\frac{\partial \tilde{V}}{\partial t} &= 3\mu \frac{\partial^2 \tilde{V}}{\partial z^2} + \beta_1(z) \frac{\partial \tilde{V}}{\partial z} + \beta_2(z) \tilde{V} + \alpha_1(z) \frac{\partial \tilde{R}}{\partial z} + \alpha_2(z) \tilde{R}.
\end{align*}
\]

Where coefficients \( \beta_1(z) \), \( \beta_2(z) \), \( \alpha_1(z) \), \( \alpha_2(z) \) have the given form

\[
\begin{align*}
\beta_1(z) &= -V_{st} + \frac{3\mu}{\rho V_{st}} \frac{\partial V_{st}}{\partial z} + \frac{3\mu}{\rho R_{st}^2 V_{st}} \left( R_{st}^2 \frac{\partial V_{st}}{\partial z} \right), \\
\beta_2(z) &= -2 \frac{\partial V_{st}}{\partial z} + \frac{3\mu}{\rho R_{st}^2 V_{st}} \frac{\partial}{\partial z} \left( R_{st}^2 \frac{\partial V_{st}}{\partial z} \right), \\
\alpha_1(z) &= \frac{6\mu}{\rho V_{st}} \frac{\partial V_{st}}{\partial z} + \frac{\sigma}{2\rho R_{st} V_{st}}, \\
\alpha_2(z) &= -2 \frac{\partial V_{st}}{\partial z} + \frac{6\mu}{\rho R_{st}^2 V_{st}} \frac{\partial}{\partial z} \left( R_{st}^2 \frac{\partial V_{st}}{\partial z} \right) + \frac{2g}{V_{st}} + \frac{\sigma}{2\rho R_{st}^2 V_{st}} \frac{\partial R_{st}}{\partial z}.
\end{align*}
\]

Here \( \rho \) and \( \mu \) are density and viscosity of the melt, \( \sigma \) is the coefficient of surface tension, \( g \) is gravitational acceleration. \( \beta_1(z) \), \( \beta_2(z) \), \( \alpha_1(z) \), \( \alpha_2(z) \) coefficients are calculated through the functions \( R_{st}(z) \) and \( V_{st}(z) \), which are the solutions of the system of equations for the steady-state [3].

The authors formulate the problem of boundary control and boundary observation of a parabolic system that is joined by original conditions and the first kind boundary conditions for the disturbed motion.
The functional (4) is protuberant, lower semicontinuous, and coercive [4]; therefore, taking into the account the theorem of the optimal element existence [5], there is the function $\tilde{u}_0(t)$ that works out an inferior limit to the functional:

$$F(\tilde{u}_0) = \inf_{\tilde{u} \in L^2(\Omega)} F(\tilde{u}(t)).$$

(5)

Considering the criterion of optimality [4, 6], the value of Gateaux differential on the optimal element $\tilde{u}_0$ is zero:

$$\frac{1}{2} \langle F'(\tilde{u}_0), w - \tilde{u}_0 \rangle + \int_0^\tau (R_{st}(L)\tilde{R}(t, L))^2 \tilde{\tau} dt = 0 ,$$

(6)

where $\tilde{R} = \tilde{R}_{t=0}$ is the operator of weak differentiation, $\delta \tilde{u}_0 = w - \tilde{u}_0$ is the variation of the optimal control. Take into the account that the differential issue in the formulation of (3) is linear concerning the control function $\tilde{u}(t)$, consequently, the single of its solutions, the function $\tilde{R}(t, z)$, can be treated as an effective outcome of some linear operator $\Lambda$ to the function $\tilde{u}(t)$:

$$\tilde{R}(t, z) = \Lambda(\tilde{u}(t)) = \Lambda(\tilde{u}).$$

(7)

The linearity conditions (distributivity, associativity) are appropriate for the operator $\Lambda$. Then the equality (6) could be done in this way:

$$\int_0^\tau \Lambda \delta \tilde{u}_0 \Lambda(w - \tilde{u}_0) dt + \alpha \int_0^\tau \tilde{u}_0 (w - \tilde{u}_0) dt = 0 .$$

(8)

Further, using the method described in the work [7], the authors obtain the optimality system in this way:
\[\begin{aligned}
&-\frac{\partial \tilde{R}}{\partial t} = V_{st} \frac{\partial \tilde{R}}{\partial z} + \frac{V_{st}}{2} \frac{\partial \tilde{V}}{\partial z}, \\
&\tilde{R}\big|_{t=0} = R_{f}(z), \quad \tilde{V}\big|_{z=0} = \tilde{R}_{0}(t), \\
&\frac{\partial \tilde{V}}{\partial t} = \frac{3\mu}{\rho} \frac{\partial^{2} \tilde{V}}{\partial z^{2}} + \beta_{1}(z) \frac{\partial \tilde{V}}{\partial z} + \beta_{2}(z) \tilde{V} + \alpha_{1}(z) \frac{\partial \tilde{R}}{\partial z} + \alpha_{2}(z) \tilde{R}, \\
&\tilde{V}\big|_{t=0} = V_{f}(z), \quad \tilde{V}\big|_{z=0} = V_{0}(t), \quad \tilde{V}\big|_{z=L} = \frac{1}{\alpha} \left( \frac{R_{st}^{2} \tilde{R}}{2V_{st}^{2}} - \frac{3 \mu R_{st}}{\rho} \frac{\partial \tilde{p}}{\partial z} \right).
\end{aligned}\]

(9)

where \( q(t, z), p(t, z) \in L_{2}(\Omega) \) are given auxiliary functions and the optimal control function
\[\tilde{u}_{0}(t) = \frac{1}{\alpha} \left( \frac{R_{st}^{2} \tilde{R}}{2V_{st}^{2}} - \frac{3 \mu R_{st}}{\rho} \frac{\partial \tilde{p}}{\partial z} \right) \big|_{z=L} .\]

3. Results and discussion

The numeric realization of the optimality system solution (9) was fulfilled with the finite-element method in multiphysics simulation software (Comsol Multiphysics). The solving operation can be partitioned into the separate phases:

- looking for a stationary solution (functions \( R_{f}(z) \) and \( V_{f}(z) \)). The functions were detected with the solution of the system, which describes the stationary state of the drawing process [3];
- finding functions \( \alpha_{1}(z), \alpha_{2}(z), \beta_{1}(z), \beta_{2}(z) \), relying on stationary states;
- finding the solution to the optimality system (9) and revealing the optimal control function \( \tilde{u}_{0}(t) \).

- examination of the acquired outcome.

We would like to characterize the implementation process.

Firstly, stationary solutions \( R_{st}(z) \) and \( V_{st}(z) \) were found numerically (Figure 1).

**Figure 1.** (a) Radius of the quartz melt flow, (b) Velocity of the quartz melt flow
The optimality system was solved for these input parameters: \( t = 160 \text{ s} \), \( \rho = 2200 \text{ kg/m}^3 \), \( \mu = 10000 \text{ Pa s} \), \( \sigma = 0.3 \text{ N/m} \), \( L = 0.3 \text{ m} \). The deviation of the fiber radius from its stationary solution was presented by the convex function seen in Figure 2(a). The upper limit deviation of the fiber radius from its stationary (programmed) solution was 10%.

![Figure 2. Radius drawing deviation (as a fraction of the stationary solution)](image)

Significant awareness has to be given to the quantity of the control price \( \alpha \). The parameter \( \alpha \) must be considered beforehand or picked by resolving test issues. In this example, its value is 0.5.

In Figure 3(a) the ensued control function \( V(t, L) \) is presented.

![Figure 3. Optimal control function](image)

The authors get rid of the identical issue on the speculation that the function \( R(z) \) describing the original deviation of the fiber radius is now ascertained by the function in the given way (Figure 2(b)). The upper limit deviation of the fiber radius from its stationary (programmed) solution in this example is also 10%. After that, resolving the optimality system (9) for the identical values of the entry parameters, the function \( V(t, L) \) is acquired in the given way (Figure 3(b)).

Researching the outcome of resolving the optimality system (9) containing two various original conditions for the equation of state, its possible to notice that the values of the control function (winding velocity regulation values) vary from -15% to +16%, which match to the proficiency of practical manufacturing.

The objective functional (4) was figured for different values of the control function. Notably, accompanied by an invariable winding velocity of the manufactured fiber (\( \tilde{u} = 0 \)), the value of the functional surpass the value acquired, when applying the optimal state \( \tilde{u}_0(t) = \frac{1}{\alpha} \left( \frac{R_{tt}^2 \tilde{R}}{2V_{tt}^2} + \frac{3\mu R_{tt}}{\rho V_{tt}^3} \frac{\partial p}{\partial z} \right) \bigg|_{z=L} \). Figure 4 shows the deviations of the finished fiber radii for the two different drawing modes, which are described above, and for two different initial conditions (Figures 2(a) and 2(b)).
The solid lines correspond to the values of the deviation of the radius at a constant drawing speed, and the dash-dotted line corresponds to the values of the deviation of the radius in the optimal control mode. As can be seen, the process tends to stabilize in both cases in the optimal control mode already from the first seconds. Radius deviations are reduced.

4. Conclusion
Thereby, the issue dedicated to optimal stabilizing control of quartz optical fiber manufacturing was put together, explained and resolved as a result of this work. The issue was figured out in a one-dimensional formulation. The control function is the outcome of fiber winding velocity. The linearization was done and the optimality system was acquired in a strong way for the control issue, i.e. for deviations of the radius and velocity of drawing, likewise for their linked states in the kind of boundary value issue in partial derivatives. The issue is solved for two various original conditions that indicate the deviation of the radius with regard to its stationary (programmed) state. In the first and second occurrence, the acquired solutions of the optimality systems were delivered, as good as optimal control functions in addition.

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