On Some New Generalized Difference Sequence Space of Fuzzy Numbers and Statistical Convergence

Ganie AH1* and Syed MA2
1Department of Applied Science and Humanities, SSM College of Engineering and Technology, Kashmir, India
2Department of Mathematics, Faculty of Science Jazan university, Jazan, Saudi Arabia

Abstract

The main aim of the present paper is to introduce the concept of lacunary almost statistical convergence and strongly almost convergence of the generalized difference sequences of Fuzzy numbers. We also investigate their some basic topological properties.

Keywords: Almost convergence; Fuzzy set; Lacunary sequence; Generalized difference sequence spaces

2010 Mathematics subject classification: 40A05; 26A03; 11B05

Introduction

A sequence space is defined to be a linear space of real or complex sequences. Throughout the paper N, R and C denotes the set of non-negative integers, the set of real numbers and the set of complex numbers respectively. Let ω denote the space of all sequences (real or complex) and let and be Banach spaces of bounded and convergent sequences with supremum norm . Let T denote the shift operator on ω, that is, . A Banach limit L is defined on as a non-negative linear functional such that L is invariant i.e., . Let denote the set of almost convergent sequences by . This was further studied by many others [4,7-9,12-18]. Lorentz called a sequence almost convergent if all Banach limits of x, L(x), are same and this unique Banach limit is called . X(t0) = 1. A fuzzy real number is called convex if . Let denote the set of all closed and bounded intervals x, L(χ). In his paper, Lorentz proved the following criterion for almost convergence.

A sequence x = {xn} ∈ I∞ is almost convergent with F - limit of L(x) if and only if

\[
\lim_{n \to \infty} \limsup_{m \to \infty} t_n(x) = L(x)
\]

where, \( t_n(x) = \frac{1}{m} \sum_{j=1}^{m} f_x(T^j) \) uniformly in n ≥ 0.

We denote the set of all convergent sequences by f. This was further studied by Ganie [2], Nanda [3] and many others.

A complex number sequence x is said to be statistically convergent to the number L if for every \( \epsilon > 0 \),

\[
\lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \left| k \leq n \mid x_k - L \geq \epsilon \right| = 0,
\]

where the vertical bars indicate the number of elements in the enclosed set. In this case we write S - limit x = L or \( x_n \to L(S) \).

By a lacunary sequence we mean an increasing integer sequence \( \theta \) such that \( h_0 = 0 \) and \( h_k = k_h - k_{h-1} \) → ∞ as \( r \to \infty \). Throughout this paper, the intervals determined by \( \theta \) will be denoted by \( l_\theta = (k_h, k_{h+1}] \) and the ratio \( k_h / k_{h+1} \) will be abbreviated by \( q_\theta \).

Let \( \theta \) be a lacunary sequence; the number sequence \( x \) is \( S_\theta \) - convergent to \( L \) provided that for every \( \epsilon > 0 \),

\[
\lim_{n \to \infty} \frac{1}{l_\theta} \sum_{k \in I_\theta} \left| k \leq n \mid x_k - L \geq \epsilon \right| = 0.
\]

In this case we write \( S_\theta \) - limit \( x = L \) or \( x_n \to L(S_\theta) \), and we define \( S_\theta = \{ x : \text{for some } L, S_\theta - \lim x = L \} \).

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [10] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. Matloka [11] introduced bounded and convergent sequences of fuzzy numbers and studied their some properties and has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by many others [4,7-9,12-18].

Let \( D \) denote the set of all closed and bounded intervals \( X = [a_1, a_2] \) on the real line R. For \( X, Y \in D \) we define \( d(X : Y) = \max\{|a_1 - b_1|, (a_1 - b_2)\} \).

Where \( X = [a_1, a_2], Y = [b_1, b_2] \). It is known that \( (D, d) \) is a complete metric space.

Let \( I = [0,1] \). A fuzzy real number X is a fuzzy set on R and is a mapping \( X : R \to I \) associating each real number t with its grade of membership X(t). A fuzzy real number X is called convex if \( X(t) \geq X(s) \geq X(r) = \min\{X(s), X(r)\} \), where \( s < t < r \).

A fuzzy real number X is called if normal if there exists \( t_0 \in R \) such that \( X(t_0) = 1 \).

A fuzzy real number X is called upper semi-continuous if for each \( \epsilon > 0, X^{-1}([0, a + \epsilon]) \) for all a ∈ I and given \( \epsilon > 0, X^{-1}([0, a + \epsilon]) \) is open in

*Corresponding author: Ganie AH, Department of Applied Science and Humanities, SSM College of Engineering and Technology, Kashmir, India. Tel: 09906413186; E-mail: ashamidq@rediffmail.com

Received November 13, 2015; Accepted December 16, 2015; Published December 21, 2015

Citation: Ganie AH, Syed MA (2015) On Some New Generalized Difference Sequence Space of Fuzzy Numbers and Statistical Convergence. J Appl Comput Math 4: 273. doi:10.4172/2168-9679.1000273

Copyright: © 2015 Ganie AH, et al. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.
the usual topology of \( R \). The set of all upper semi-continuous, normal, convex fuzzy numbers is denoted by \( R(I) \). The \( a \) - level set of a fuzzy real number \( X \) for \( 0 < a \leq 1 \) is denoted by \( X_a \) and is defined by \( X_a = \{ t \in R : X(t) \geq a \} \). The \( 0 \) - level set is the closure of strong \( 0 \) - cut.

For each \( r \in R, \tau \in R(I) \) is defined by \( \tau = \{ r \}, \) if \( t = r, \) \( \{ 0 \}, \) if \( t \notin \tau. \)

The absolute value of \( |X| \) of \( X \in R(I) \) is defined by [19].

\[
|X(t)| = \begin{cases} \max(X(t), X(-t)), & \text{if } t \geq 0, \\ 0, & \text{if } t < 0. \end{cases}
\]

Let \( \delta : R(I) \times R(I) \to \mathbb{R} \) be defined by

\[
\delta(X, Y) = \sup_{x \in X} d(x, Y).
\]

Then \( \delta \) defines a metric on \( R(I) \) [19]. The additive identity and multiplicative identity in \( R(I) \) are denoted by \( 0 \) and \( 1 \), respectively.

A Fuzzy number is a function \( X \) from \( R^* \) to \( [0,1] \), which is normal, fuzzy convex, upper-semi continuous and the closure of \( \{ x \in R^* : X(x) > 0 \} \) is compact. These properties imply that for each \( 0 < a \leq 1 \), the \( a \) - level set \( X_a \) is a non-void compact convex subset of \( R^* \), with support \( X_a \) in \( R^* : X(t) > 0 \).

We denote by \( L(R^*) \) the set of all fuzzy number. The linear structure of \( L(R^*) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( R(I) \) of \( \lambda \) \( \in \) \( R \) of a level set in \( R^* \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in R \}, \quad X \lambda = \{ \lambda X(t) : t \in R \}.
\]

For each \( 0 \leq a \leq 1 \), let \( X(t) > 0 \) and \( \alpha \leq X(t) \geq a \) then

\[
d(X, Y)_\alpha = \delta(X, Y) \sup_{x \in X} d(x, Y).
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]

We denote by \( L(I) \) the set of all Fuzzy number. The linear structure of \( L(I) \) induces the addition \( X + Y \) and the scalar multiplication \( X \lambda \) \( \in \) \( I \) of \( \lambda \) \( \in \) \( I \), defined by

\[
X + Y = \{ X(t) + Y(t) : t \in I \}, \quad X \lambda = \{ \lambda X(t) : t \in I \}.
\]
Consequently, we have \( Q > 0 \) such that
\[
\frac{1}{\lambda} \sum_{n=1}^{\infty} d\left(\lambda_n^+ X_n\right) \leq Q.
\]

If, for \( \varepsilon > 0 \), we have
\[
\frac{1}{\lambda} \sum_{n=1}^{\infty} d\left(\lambda_n^+ X_n\right) \leq \varepsilon
\]

on taking limits as \( \lambda \to \infty \). Hence, we conclude that \( X \in \left[M^*_\sigma, \lambda_n^+(\sigma)\right] \).

Proof: For \( \varepsilon > 0 \), we have
\[
\left\| k \in I : d\left(\lambda_n^+ X_k\right) \geq \varepsilon \right\| \leq \frac{1}{\lambda} \sum_{n=1}^{\infty} d\left(\lambda_n^+ X_n\right) \geq \varepsilon
\]

This gives,
\[
\frac{1}{\lambda} \sum_{n=1}^{\infty} d\left(\lambda_n^+ X_n\right) \leq \varepsilon
\]

On taking limits as \( \lambda \to \infty \) and using the fact that \( \lim_{\lambda \to \infty} \left(\frac{1}{\lambda}\right) = 0 \), we conclude that \( X \) is \( \lambda_n^+(\sigma) \)-statistically convergent to \( X^* \).

Theorem 4.5: Let \( \theta = (k) \) be a lacunary sequence. If the sequence \( X = (X_n) \) is almost \( \lambda_n^+(\sigma) \)-statistically convergent to the Fuzzy number \( X^* \), and
\[
\liminf_{\lambda_n \to \infty} \left( \frac{1}{\lambda_n} \right) > 0,
\]

we have \( \lim_{\lambda_n \to \infty} \left( \frac{1}{\lambda_n} \right) > 0 \), then it is almost \( \lambda_n^+(\sigma) \)-statistically convergent to \( X^* \).

Proof: For \( \varepsilon > 0 \), we have
\[
\left\| k \in I : d\left(\lambda_n^+ X_k\right) \geq \varepsilon \right\| \leq \frac{1}{\lambda} \sum_{n=1}^{\infty} d\left(\lambda_n^+ X_n\right) \geq \varepsilon
\]

This gives,
\[
\frac{1}{\lambda} \sum_{n=1}^{\infty} d\left(\lambda_n^+ X_n\right) \leq \varepsilon
\]

On taking limits as \( \lambda \to \infty \) and using the fact that \( \lim_{\lambda \to \infty} \left(\frac{1}{\lambda}\right) = 0 \), we conclude that \( X \) is \( \lambda_n^+(\sigma) \)-statistically convergent to \( X^* \).

References

1. Lorenz GG (1948) A contribution to the theory of divergent series. Acta Math 80: 167-190.
2. Ganie AH, Sheik NA (2012) A note on almost convergent sequences and some matrix transformations. Int J Mod Math Sci 4: 126-132.
3. Nanda S (1979) Matrix transformations and almost boundedness. Glasnik Matematicki 14: 99-107.
4. Fast H (1951) Sur la convergence. Colloq Math 2: 241-244.
5. Siew C (1999) A topological and functional analytic approach to statistical convergence. Appl and Numerical Har Anal pp: 403-413.
6. Friday JA (1985) On statistical convergence. Analysis 5: 301-313.
7. Nuray F (1998) Lacunary Statistical convergence of sequences of Fuzzy numbers. Fuzzy Sets Syst 99: 353-356.
8. Salat T (1980) On statistically convergent sequences of real numbers. Math Slovaca 30: 139-150.
9. Tripathy BC, Sen M (2001) On generalised statistically convergent sequences. Indian J Pure and App Maths 32: 1689-1694.
10. Zadeh LA (1965) Fuzzy sets. Inform Control 8: 338-353.
11. Freedman AR, Sember JJ, Raphael M (1978) some Cesaro type summability spaces. Proc London Math Soc 37: 508-520.
12. Altin Y, Et M, Basarir M (2007) On some generalized difference sequences of fuzzy numbers. J Sci Engrg 34: 1-14.
13. Chaudhry AK, Das P (1993) Some results on fuzzy topology on fuzzy sets. Fuzzy Sets and Systems 56: 331-336.
14. Diamond P, Kloeden P (1990) Metric spaces of fuzzy sets. Fuzzy Sets Syst 35: 241-249.
15. Ganie AH, Sheikh NA, Sen M (2013) The difference sequence space defined by orlicz functions. Int J Mod Math Sci 6: 151-159.
16. Matloka M (1986) Sequences of fuzzy numbers. BUSEFAL 28: 28-37.
17. Nanda S (1989) On sequences of fuzzy numbers. Fuzzy Sets Syst 33: 123-126.
18. Savas E (2000) A note on sequence of fuzzy numbers. Inf Sci 124: 297-300.
19. Kelava O, Seikkala S (1984) On fuzzy metric spaces. Fuzzy Sets Syst 12: 215-229.
20. Kizmaz H (1981) On certain sequence spaces. Canad Math Bull 24: 169-175.
21. Tripathy BC, Esi A, Tripathy BK (2005) On a new type of generalized difference Cesaro sequence spaces. Sccochow J Math 31: 333-340.
22. Esi A, Hazarika B (2012) Some new generalized classes of sequences of fuzzy numbers defined by an Orlicz function. Annals Fuzzy Math Infor 4: 401-406.
23. Ganie AH, Sheikh NA (2013) Generalized difference sequences of fuzzy numbers. Int J Modern Mat Sci 4: 915-922.
24. Aytar S (2004) Statistical limit points of sequences of Fuzzy numbers. Inform Sci 165: 129-138.
25. Basarir M, Mursaleen M (2003) Some difference sequence spaces of fuzzy numbers. J Fuzzy Math 11: 1-7.
26. Çolak R, Altink H, Et M (2000) Generalized difference sequences of fuzzy numbers. Chaos, Solitons and Fractals 40: 1106-1117.
27. Esi A (2008) On some new classes of sequences of Fuzzy numbers. Int J Math Anal 2: 837-844.
28. Ganie AH, Sheikh NA (2013) Generalized difference sequences of fuzzy numbers. J Math, New York 19: 431-438.
29. Tripathy BC, Sarma B (2008) Sequence spaces of fuzzy real numbers defined by orlicz functions. Math Slovaca 56: 621-628.