Proposed neutron interferometry test of Berry’s phase for a circulating planar spin

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The energy eigenstates of a spin $-\frac{1}{2}$ particle in a magnetic field confined to a plane, define a planar spin. If the particle moves adiabatically around a loop in this plane, it picks up a topological Berry phase that can only be an integer multiple of $\pi$. We propose a neutron interferometry test of the Berry phase for a circulating planar spin induced by a magnetic field caused by a very long current-carrying straight wire perpendicular to the plane. This Berry phase causes destructive interference in the direction of the incoming beam of thermal neutrons moving through a triple-Laue interferometer.

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I. INTRODUCTION

Neutron interferometry has been used to demonstrate experimentally various quantum interference effects. These include gravity-induced phase shifts [1], the Aharonov-Casher effect [2], the scalar Aharonov-Bohm phase shift [3], as well as the Levy-Leblond confinement phase [4]. While the neutron spin plays no role or a passive role in these experiments, phase effects induced by the evolution of the neutron spin have been demonstrated in terms of the geometric phase [5–7].

The Berry phase is the geometric phase arising in cyclic adiabatic evolution of general quantum systems [8]. In the case of a spin, this phase is proportional to the enclosed solid angle $\Omega$ of the slowly rotating magnetic field. Specifically, if the spin projection is $m_s$, the Berry phase is $-m_s\Omega$ for a positively oriented path. In the case of a neutron, $m_s = \pm \frac{1}{2}$ and the Berry phase can take the values $\mp \frac{\pi}{2} \Omega$.

If the neutron spin is confined to a plane, forming a planar spin, the enclosed solid angle is restricted to an integer multiple of $2\pi$. This implies that the Berry phase picked up when the neutron moves around a loop has a topological character as it can only be an integer multiple of $\pi$. For an odd multiple, this phase shift results in complete destructive interference of certain outcomes. This effect has been predicted in scattering of ultrasonic neutrons [9], and in time evolution of other two-level quantum systems, such as $E \otimes \epsilon$ Jahn-Teller molecules [10], cavity QED systems [11], and degenerate pairs of dark states in cold atoms [12].

The setup proposed in Ref. [9] consists of ultraslow neutrons that scatter on a very long current-carrying straight wire. The required low speed of the neutrons (typically in the order of cm/s) in combination with high angular resolution and precise centering of the beam is experimentally challenging. In order to deal with this, we develop here an interferometry-based experimentally simpler version of the scattering setup. We demonstrate how the Berry phase of the neutron spin associated with the magnetic field circling around the wire can be tested for feasible electrical currents and existing interferometry techniques for thermal neutrons in a triple-Laue setup [13].

II. ADIABATIC SPIN DYNAMICS

Consider a narrow beam of neutrons moving at velocity $\mathbf{v}$ with an impact parameter $b$ on a very long straight wire carrying an electrical current $I_w$, as shown in Fig. 1a. The magnetic field is given by Biot-Savart’s law

$$\mathbf{B} = \frac{\mu_0 I_w}{2\pi r}\mathbf{e}_\theta,$$

(1)

where we have assumed that the wire points along the $z$-axis. Here, $\theta$ is the polar angle in cylindrical coordinates with corresponding unit basis vector $\mathbf{e}_\theta$, $r$ is the distance from the wire, and $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am) is permeability of vacuum. The magnetic field induces a local energy splitting of the neutron spin states, as described by the Zeeman Hamiltonian

$$\mathcal{H} = -\mathbf{\mu} \cdot \mathbf{B} = -\frac{1}{2} \mathbf{\sigma} \cdot \mathbf{B}$$

(2)

with $\mu = -9.65 \cdot 10^{-27}$ J/T the neutron magnetic moment and $\mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ the standard Pauli operators representing the neutron spin. The eigenvalues of the Hamiltonian are

$$V_\pm(r) = \pm \frac{|\mu|\mu_0 I_w}{4\pi r} \equiv \pm C_0 I_w \frac{\mu_0 I_w}{r^2}.$$ 

(3)

Numerically, we find $C_0 = 9.65 \cdot 10^{-34}$ JmA$^{-1}$. The local spin eigenvectors $|\pm; r\rangle$ are eigenvectors of $\sigma_\theta = \mathbf{e}_\theta \cdot \mathbf{\sigma}$. Each eigenvector defines a local spin $|\pm; \sigma \rangle = |\pm; r\rangle$ confined to the $xy$ plane with winding number $+1$, as shown in Fig. 1b. A Berry phase factor of $-1$ when the neutron circles adiabatically around the wire once is a manifestation of the nontrivial topology of this circulating planar spin.

In a semi-classical approach, the spatial motion of the neutrons is treated classically, while their spin is a quantum-mechanical degree of freedom. The orbits associated with the two spin eigenstates are solutions of

$$m_n \mathbf{v} = -\nabla_r V_\pm(r) = \pm C_0 I_w \frac{\mu_0 I_w}{r^2} \mathbf{e}_r,$$

(4)
are constants of the motion, we find the solution evolution as governed by the semi-classical Hamiltonian of the quantum-mechanical spin interaction with the magnetic field. The local spin is confined to the $xy$ plane with winding number $+1$. For sufficiently strong electrical current and sufficiently slow neutrons, the spin of the neutron follows adiabatically the circular magnetic field $B$. This makes it possible to measure the Berry phase manifesting the nontrivial winding number associated with the circulating local spin.

where $m_n$ is the mass of the neutron. By using that the mechanical energy

$$E = \frac{m_n v^2}{2} + \frac{L^2}{2m_n r^2} \pm \frac{C_0 I_w}{r}$$

and the angular momentum along the $z$ axis

$$L = m_n v^2 \dot{\theta}$$

are constants of the motion, we find the solution

$$r = \frac{L^2}{m_n C_0 I_w} \left(1 + \sqrt{1 + \frac{2EL^2}{m_n C_0^2 I_w^2}} \sin \theta \right)^{-1}.$$  

(7)

Here, we have assumed that the neutron initially moves horizontally from the left with the origin at the center of the wire, as shown in Fig. 1. In the interferometer setup, $E > 0$ so that the eccentricity

$$\varepsilon = \sqrt{1 + \frac{2EL^2}{m_n C_0^2 I_w^2}} > 1.$$  

(8)

Thus, the trajectory is hyperbolic.

Next, we analyze the adiabatic condition for the Zeeman Hamiltonian of the quantum-mechanical spin interacting with the magnetic field along the orbit of the neutron. In the adiabatic regime, the local spin is parallel or anti-parallel to the magnetic field at the location of the neutron. We examine the adiabatic condition for the spin evolution as governed by the semi-classical Hamiltonian

$$\mathcal{H}_{sc} = -\mu \frac{1}{2} \sigma \cdot B(r_t).$$  

(9)

In other words, for the local spin eigenstates $|\pm; r_t\rangle$ we check whether or not the adiabatic condition

$$h \left| \frac{\langle +; r_t | \hat{\mathcal{H}}_{sc}(r_t) | -; r_t \rangle}{(E_+ - E_-)^2} \right| \ll 1$$

(10)

is satisfied. We find:

$$\hat{\mathcal{H}}_{sc}(r_t) = -\frac{C_0 I_w}{r^2} (v_r \sigma_y + v_\theta \sigma_r),$$  

(11)

where $v_r = \dot{r}$ ($v_\theta = r \dot{\theta}$) is the radial (angular) velocity and $\sigma_r = e_r \cdot \sigma$, $e_r$ being a unit basis vector in the radial direction. Since $|\pm; r_t\rangle$ are eigenvectors of $\sigma_\theta$, it follows that $\langle +; r_t | \sigma_\theta | -; r_t \rangle = 0$ and

$$\langle +; r_t | \hat{\mathcal{H}}_{sc}(r_t) | -; r_t \rangle = -\frac{C_0 I_w}{r^2} v_\theta (\langle +; r_t | \sigma_r | -; r_t \rangle = i \frac{C_0 I_w}{r^2} v_\theta.$$  

(12)

Furthermore,

$$(E_+ - E_-)^2 = \frac{4C_0^2 I_w^2}{r^2}.$$  

(13)

By inserting Eqs. (12) and (13) into Eq. (10), we finally obtain the adiabatic condition

$$h \frac{C_0 I_w}{r^2} |v_\theta| \frac{r^2}{4C_0^2 I_w^2} = \frac{h}{4C_0 I_w} |v_\theta| \ll 1$$

$$\Rightarrow I_w \gg \frac{h}{4C_0} |v_\theta|.$$  

(14)

III. INTERFEROMETER SETUP

Thermal neutrons moving at speed $v \sim 2000$ m/s have been used extensively for interferometry tests [13]. We
demonstrate that such neutrons can be used to observed the above described Berry phase of the circulating planar spin. To this end, we first show that there exists a wide electrical current range for which the neutron moves along a straight line, and its spin evolution at the same time satisfies the adiabatic condition in Eq. (14).

We use that \( E = \frac{1}{2} m_\text{n} v^2 \) and \( L = m_\text{n} v b \), \( b \) being the impact parameter (see Fig. 1a), are constants of the motion, to rewrite the eccentricity as

\[
\varepsilon = \sqrt{1 + \frac{m_\text{n}^2 v^2 b^2}{C_0 I_w^2}}. \tag{15}\n\]

The neutron moves along a straight line provided \( \varepsilon \gg 1 \), which combined with Eq. (14) yields

\[
\frac{\hbar}{4C_0} |v_0| \ll I_w \ll \frac{m_\text{n} v^2 b}{C_0}. \tag{16}\n\]

For an impact parameter in the order of \( 10^{-1} \) m and \( \max(|v_0|) \sim v \sim 2000 \) m/s, we find

\[
50 \text{A} \ll I_w \ll 7 \cdot 10^{11} \text{A}. \tag{17}\n\]

This confirms that the straight line assumption can be combined with adiabatic spin evolution. By taking a reasonable upper limit for current density of 500 A/cm², a Copper wire of radius 0.5 cm would allow for a sufficiently large current in the order of 400 A.

The triple-Laue interferometer setup designed to measure the Berry phase is shown in Fig. 2. A beam of thermal neutrons is coherently superposed at the first crystal plate \( P \) acting as a beam-splitter in front of the wire. A second crystal plate \( M \) acting as mirrors recombine the beam-pair at the third crystal plate \( Q \). The intensity at the two output detectors \( D_1 \) and \( D_2 \) is finally measured. At \( Q \), the probability amplitude reads

\[
\psi_Q = \frac{1}{\sqrt{2}} \psi_{\text{up}} e^{i \int_{\text{up}, P}^Q A_\pm(r) \cdot dr} + \frac{1}{\sqrt{2}} \psi_{\text{down}} e^{i \int_{\text{down}, P}^Q A_\pm(r) \cdot dr}, \tag{18}\n\]

where

\[
A_\pm(r) = i \langle \pm; r | \nabla_r | \pm; r \rangle = - \frac{1}{2r} e_\theta. \tag{19}\n\]

are the Berry vector potentials of the two spin eigenstates and \( \psi_{\text{up}} (\psi_{\text{down}}) \) is the probability amplitude for the upper (lower) beam. The vector potential is of Aharonov-Bohm type and corresponds to half a flux unit at the wire.

The beam-splitter at \( Q \) induces the transformations \( \psi_{\text{up}} \mapsto \psi_{D_1} + \psi_{D_2} \) and \( \psi_{\text{down}} \mapsto \psi_{D_1} - \psi_{D_2} \) yielding the output

\[
\psi_{\text{out}} = \frac{1}{2} \psi_{D_1} e^{i \int_{\text{up}, P}^Q A_\pm(r) \cdot dr} \left( 1 + e^{i \oint_{\text{up}} A_\pm(r) \cdot dr} \right) + \frac{1}{2} \psi_{D_2} e^{i \int_{\text{up}, P}^Q A_\pm(r) \cdot dr} \left( 1 - e^{i \oint_{\text{up}} A_\pm(r) \cdot dr} \right), \tag{20}\n\]

where \( C \) is the interferometer loop in the counterclockwise direction and \( \psi_{D_1} (\psi_{D_2}) \) is the amplitude for the upper (lower) output beam. The gauge independent \( e^{i \oint_{\text{up}} A_\pm(r) \cdot dr} \) is the Berry phase factor associated with the planar local spin of the neutron. By using Eq. (19), we find \( e^{i \oint_{\text{up}} A_\pm(r) \cdot dr} = e^{i \pi} = -1 \), yielding

\[
\psi_{\text{out}} = \psi_{D_2} e^{i \int_{\text{up}, P}^Q A_\pm(r) \cdot dr}. \tag{21}\n\]
This shows that the Berry phase of the planar spin gives rise to a destructive interference effect that suppresses the probability amplitude in the direction of the incoming beam, leading to that all neutrons are detected at $D_2$. This is the interferometer analog of the destructive interference effect that creates a nodal line in the forward direction seen in the scattering setup proposed in Ref. [9].

In general, a dynamical phase is accompanying the Berry phase. This phase may result in a dynamical contribution that would be sensitive to fluctuations in the neutron speed and thereby potentially destabilize the interference effect. This can be resolved by putting the wire on the symmetry line of the interferometer connecting the two beam crossing points at $P$ and $Q$. In this way, the dynamical phase contributions along the two beams are identical and therefore cancel so that only the Berry phase influences the interference effect measured at the two detectors $D_1$ and $D_2$.

A typical neutron source produces unpolarized neutrons, as characterized by a spin density operator $\rho = \frac{1}{2} I$. The interference effect is determined by the time evolution operator

$$U(Q; P) = e^{i \oint C A_+ (r) \cdot dr} |+\rangle \langle +| + e^{i \oint C A_- (r) \cdot dr} |-\rangle \langle -| = -I$$

of the spin, yielding the interference effect [15]

$$I_{D_1} = I_{in} - I_{D_2} \propto 1 + |\text{Tr}(\rho U)| \cos \text{arg} \text{Tr}(\rho U) = 0.$$  \hspace{1cm} (23)

Thus, unpolarized neutrons can be used to demonstrate the Berry phase induced destructive interference effect. This considerably simplifies the experimental realization of the setup as the neutrons can be used directly from the source without undergoing any intensity-reducing spin filtering.

IV. CONCLUSIONS

A very long straight wire carrying an electrical current defines a circulating planar local spin of a neutron. We have shown that there exists an electrical current range for which sufficiently slow neutrons can be used to test the Berry phase accompanying adiabatic evolution of the local spin around the interferometer loop. A complete cancelation of the probability amplitude in the direction of the incoming neutron beam provides a clear signature of the topological Berry phase of the circulating planar spin. The adiabatic condition is satisfied for electrical currents in the order of a few hundreds of $\text{A}$ in the case of thermal neutrons in a triple-Laue setup.

The Aharonov-Casher effect [16] occurs when an electrical neutral particle moves around a charged straight wire. The resulting phase shift is topological in that it only depends on the winding number of the particles path around the wire. The effect has been observed by using thermal neutrons [2]. Our proposed experiment can be viewed as an analog to this effect where the static charges are replaced by moving charges that constitute the electrical current through the uncharged wire. We note that, while the Aharonov-Casher phase shift is tiny (it is essentially a relativistic effect), the proposed Berry phase shift is large (integer multiple of $\pi$) and can therefore easily be observed provided the adiabatic condition is satisfied and the dynamical phases can be made to vanish by aligning the interferometer setup.

Our proposed experiment is built upon existing well-proven techniques and requires achievable experimental parameter values. On the other hand, the corresponding scattering experiment, put forward in Ref. [9] and designed to test the Berry phase of the planar spin, requires ultraslow neutrons moving at a speed which is four to five orders of magnitude lower than for thermal neutrons. In combination with the high angular resolution and precise centering of the neutron beam, the need for such slow neutrons would be highly challenging. The present neutron interferometry setup avoids these problems and opens up for a simpler verification of the Berry phase of the circulating planar spin.

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