

Fluctuation-induced quantum Zeno effect

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An isolated quantum gas with a localized loss features a non-monotonic behavior of the particle loss rate as an incarnation of the quantum Zeno effect, as recently shown in experiments with cold atomic gases. While this effect can be understood in terms of local, microscopic physics, we show that novel many-body effects emerge when non-linear gapless quantum fluctuations become important. To this end, we investigate the effect of a local dissipative impurity on an one-dimensional gas of interacting fermions. We show that the escape probability for modes close to the Fermi energy vanishes for an arbitrary strength of the dissipation. In addition, transport properties across the impurity are qualitatively modified, similarly to the Kane-Fisher barrier problem. We substantiate these findings using both a microscopic model of spinless fermions and a Luttinger liquid description.

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Introduction — The quantum Zeno effect (QZE) entails that, perhaps surprisingly, a frequent measurement of a microscopic quantum system does not allow it to evolve unitarily [1]. Recently, experiments with ultracold atoms have revealed the QZE in many-body systems. Here, different loss processes play the role of a continuous measurement. For example, it has been demonstrated that strong two-body losses give rise to an effective two-body hardcore constraint, in this way turning losses into a tool to create strong correlations [2,3]. For strong three-body losses, this effect has also been predicted theoretically to give rise to intriguing many-body phenomena such as dimer superfluids and -solids [4], or the fractional quantum Hall effect [5]. Another paradigmatic setup was introduced in Refs. [6–8], where a local dissipative impurity is considered to study strong three-body losses, this effect has also been predicted theoretically to give rise to intriguing many-body phenomena such as dimer superfluids and -solids [4], or the fractional quantum Hall effect [5]. Another paradigmatic setup was introduced in Refs. [6–8], where a local dissipative impurity is considered to study strong three-body losses, this effect has also been predicted theoretically to give rise to intriguing many-body phenomena such as dimer superfluids and -solids [4], or the fractional quantum Hall effect [5]. Another paradigmatic setup was introduced in Refs. [6–8], where a local dissipative impurity is considered to study strong three-body losses, this effect has also been predicted theoretically to give rise to intriguing many-body phenomena such as dimer superfluids and -solids [4], or the fractional quantum Hall effect [5].

An isolated quantum gas with a localized loss features a non-monotonic behavior of the particle loss rate as an incarnation of the quantum Zeno effect, as recently shown in experiments with cold atomic gases. While this effect can be understood in terms of local, microscopic physics, we show that novel many-body effects emerge when non-linear gapless quantum fluctuations become important. To this end, we investigate the effect of a local dissipative impurity on an one-dimensional gas of interacting fermions. We show that the escape probability for modes close to the Fermi energy vanishes for an arbitrary strength of the dissipation. In addition, transport properties across the impurity are qualitatively modified, similarly to the Kane-Fisher barrier problem. We substantiate these findings using both a microscopic model of spinless fermions and a Luttinger liquid description.

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We show that fluctuations strongly renormalize the loss barrier in the vicinity of the Fermi momentum $k_F$, even if it is weak on the microscopic scale. For repulsive interactions, the loss barrier is indefinitely enhanced at $k_F$ (cf. Fig. 1 right panel). A fluctuation-induced QZE then manifests itself by the loss barrier becoming fully opaque for momenta $\sim k_F$. The opposite behavior with a renormalization group (RG) flow towards a transparent fixed point is observed for attractive interactions. This leads to a fluctuation-induced transparency.

Although the defining feature of the localized dissipation is the absence of a unitarity constraint for scattering off it, unitarity is thus emerging exactly at the Fermi level. In fact, the fixed points are analogous to the ones of Kane and Fisher [9]. However, the approach to the fixed points, i.e. the physics in the vicinity of the Fermi surface, strongly deviates from the Kane-Fisher scenario due to the open-system nature. This is highlighted for attractive interactions: here, observables scale logarithmically with, e.g., temperature, instead of the more common algebraic behavior.

The open-system nature of the setup provides an opportunity to probe the system via its output and thus calls for suitable new observables without closed system counterpart. We show that the momentum or energy resolved escape probability turns out to be a realistic measure to detect the fluctuation-induced QZE in future experiments with ultracold atoms.

In the following, we substantiate these findings in two complementary approaches: first, within a minimalistic bosonization approach, providing a simple qualitative picture. Second, via a microscopic calculation, taking into account the dynamical nature of the open system problem and elucidating the physical mechanism behind our results.

Microscopic model — We consider a wire of spinless fermions with mass $m$, interacting through a short-range potential $V(x)$, thus obeying the Hamiltonian

$$H = -\int_x \psi^\dagger(x) \frac{\nabla^2}{2m} \psi(x) + \int_{x,y} V(x-y)n(x)n(y),$$  (1)
with \( \psi \), \( \bar{\psi} \) fermionic operators and \( n(x) = \psi^\dagger(x)\psi(x) \).

We assume the wire to be infinitely long, so that \( \int L = \int dz \), and to be prepared at \( T = 0 \). At time \( t = 0 \), a localized loss is switched on at \( x = 0 \): this will generate particle currents, thus driving the system out of equilibrium. This setup is locally equivalent to a wire with a local loss coupled to two reservoirs at \( T = 0 \) at its far ends, feeding the particle currents (see Fig. 1). We model the loss as a localized coupling to an empty Markovian bath, thus describing the irreversible loss of atoms from the wire. Its dynamics will be then conveniently described by the quantum master equation [12]:

\[
\partial_t \rho = -i[H, \rho] + \int_x \Gamma(x) \left[ L\rho L^\dagger - \frac{1}{2}(L^\dagger L, \rho) \right],
\]

with \( L(x) = \psi(x) \), \( \Gamma(x) = \gamma \delta(x) \), \( \gamma \) being the dissipation strength.

**Luttinger liquid description** — To obtain a first simple picture, we consider a long-wavelength description of Hamiltonian [1] in terms of a Luttinger liquid [13]:

\[
H = \frac{v}{2\pi} \int_x \left[ g(\partial_x \phi)^2 + g^{-1}(\partial_x \theta)^2 \right]
\]

with \( v \) the speed of sound, \( g \) a parameter encoding the effect of interactions, and \( \theta \) and \( \phi \) bosonic fields related to density and phase fluctuations, respectively. Given the non-equilibrium nature of the system, the usual equilibrium techniques are inadequate to treat the problem, and therefore it is convenient to resort to a Keldysh description [14–16]. In order to include the local loss in the bosonization language, we map the master equation [2] onto a Keldysh action, and then bosonize the fields [15–19]. Analogously to the case of a potential barrier [9–10], this yields a local backscattering term

\[
S_{\text{back}} = -2i \gamma \int_{x,t} \delta(x) \left( \psi^\dagger \phi q - \cos \theta q \right) \cos \theta c,
\]

where the labels \( c,q \) denote the classical and quantum fields, respectively [14]. Notice that both \( \theta \) and \( \phi \) appear, differently from the case of a potential barrier, where only \( \theta \) is involved: the field \( \phi \) accounts for the currents flowing towards the impurity. Following Refs. [9–10], we study the renormalization of the barrier in the limiting case of weak dissipation \( \gamma \to 0 \). The renormalization of \( \gamma \) at long wavelengths is then determined by integrating out degrees of freedom within a momentum shell \( k \in [\Lambda e^{-\gamma}, \Lambda] \), with \( \Lambda \) an ultraviolet cutoff, and by rescaling momenta and frequencies in order to restore the original value of the cut-off [16]. This produces the flow equation

\[
\frac{d\gamma}{dt} = (1 - g)\gamma.
\]

This entails that the particle loss is expected to be suppressed for slow modes. For attractive interaction, \( \gamma \to 0 \) and, thus, the flow suppresses the dissipation strength. In contrast, for repulsive interactions \( g < 1 \) the strength of the localized loss flows to infinity and its effect becomes suppressed by the QZE. Eq. (3) is remarkably similar to the one obtained for the renormalization of a potential barrier [9–10], despite the fact that the present system is subject to dissipation and is out of equilibrium.

In order to certify the domain of validity of Eq. (3) during the time evolution of the system, and its effect on the observables, we will analyze directly the microscopic model in Eq. (1). Moreover, while the previous analysis is perturbative in \( \gamma \), the following, complementary approach is exact in \( \gamma \) and perturbative in the microscopic interaction.

**Dynamical regimes** — As the dynamics following the quench of the dissipative impurity is remarkably complex [20–23], we clarify its different stages by first solving numerically the non-interacting model on a lattice with periodic boundary conditions, described by the Hamiltonian \( H = -\sum_j \psi^\dagger_j \psi_j + h.c. \), with \( L \) the size of the system and \( \psi_j, \psi_j^\dagger \) the fermionic operators on site \( j \). The characterizing parameters are the system length \( L \), the initial density \( n_0 \), and the dissipation strength \( \gamma \). Fig. 2 (upper panel) shows the particle loss rate as a function of time, from which one can identify three regimes: i) for \( t < t_1 \sim \gamma^{-1} \) an initial, fast depletion of particles occurs close to the site of loss impurity. ii) for \( t_1 < t < t_{11} \), a constant current of particles flowing towards the loss site is established: in this regime, regions at a distance \( l > vt \) (with \( v \) the speed of sound) from the impurity are still unaffected and act as particle reservoirs. This regime, which we will focus on, lasts up to a *macroscopic* time \( t_{11} \sim L \). For \( t > t_{11} \), the entire system experiences the effect of the dissipation and the particle loss rate slows down, until the system is eventually depleted. In Fig. 2 (lower panel) we show the density profile during the second regime \( t_1 < t < t_{11} \). At the impurity site the density is strongly depleted, while the density in the surrounding region exhibits a less pronounced depletion, which heals back to the initial value of the density on a distance \( \sim vt \). Crucially, the density around the impurity
By taking the limit evaluated exactly by solving the corresponding Dyson equation \( G_\gamma \) indicates the regime of constant loss. For all curves \( L = 501, \gamma = 3 \) and \( N(0)/L = 0.25 \).

**FIG. 2.** (Color online). Upper panel: Particle loss rate \( \ell \) as a function of time elapsed from the quench, for different system sizes \( L \). The dashed line indicates the regime of constant loss. For all curves \( \gamma = 6 \) and \( N(0)/L = 0.5 \). Lower panel, main plot: Density profile from the lattice model, for different times elapsed from the quench. Inset: Friedel oscillations around the loss site. For all curves \( L = 501, \gamma = 3 \) and \( N(0)/L = 0.25 \).

By using the Green’s functions method \([16]\), we derive the time-dependent density profile \( n(x,t) = \int_k |G_R(x,−k,t)|^2 n_0(k), \) with \( G_R(x,k,t) \) the single-particle retarded Green’s function in a mixed real- and momentum-space representation, and \( n_0(k) = \delta(k_0^2−k^2) \) the momentum distribution in the initial state. The single-particle retarded Green’s function \( G_R \) is then evaluated exactly by solving the corresponding Dyson equation \([14]\), and its explicit form is reported in \([16]\). By taking the limit \( t \to \infty \), we obtain a stationary value for \( n(x) \) corresponding to the second regime discussed above: in fact, by having already taken the thermodynamic limit \( L \to \infty \) we implicitly assumed \( t_1 \to +\infty \), so that the system is “frozen” in the second regime.

We then find \([16]\) \( n(x) \sim n_{\text{ness}} \propto \sin(2k_Fx)/x \), which holds for \( x \gg k_F^{-1} \), with \( n_{\text{ness}} \) the uniform background of the stationary state. These density modulations will generate, in an interacting system, an additional barrier renormalizing the original one. In fact, for momenta close to \( k_F \), the virtual scattering processes between the two barriers add up to an effective impenetrable one (for repulsive interactions) or a vanishing one (for attractive interactions) \([24][25]\). In the following, we show that this mechanism also applies to the present non-equilibrium, dissipative case.

**Transport properties —** To gain further insight on Eq. (5), we consider the transport properties. The transmission and reflection probabilities \( T(k) \) and \( R(k) \), respectively, for a particle with momentum \( k > 0 \) impinging upon the dissipative impurity can be read off the retarded Green’s function \([16]\). The loss of unitarity related to the scattering off the loss barrier is then quantified by

\[
\eta(k) = 1 − T(k) − R(k).
\]

The escape probability \( \eta(k) \) describes the probability that a particle with momentum \( k \) is absorbed into the bath. \( \eta(k) \), which is related to the Fourier transform of \( \langle \psi(t,x = 0)\psi(t',x = 0) \rangle \) \([16]\), is the key quantity of the present analysis, for three reasons: i) it bears signatures of the QZE, ii) as a momentum-resolved quantity, it is sensitive to the renormalization of long-wavelength modes, and iii) it can be directly related to experimentally measurable quantities.

For the interactionless case \( \eta_0(k) = 2\gamma v_k/(\gamma + v_k)^2 \), with \( v_k = |k|/m \) the group velocity, showing that losses are suppressed for both \( v_k/\gamma \to 0 \) and \( v_k/\gamma \to \infty \). The perturbative corrections to \( T, R \) due to the interaction potential \( V(x) \) can be computed in analogy to equilibrium \([24][26]\) and it yields \([16]\)

\[
\begin{align*}
\delta T &= 2\alpha \frac{\eta_0}{\langle R \rangle} \log |d(k−k_F)|, \\
\delta R &= \alpha \frac{\eta_0}{\langle R \rangle} \log |d(k−k_F)|, \\
\end{align*}
\]

with \( \eta_0, \langle R \rangle \) the bare values, \( d \) a length scale to be chosen as the largest between the spatial extent of the interaction \( V(x) \) and the Fermi wavelength, and \( \alpha = (\hat{V}(0)−\hat{V}(2k_F))/(2\pi v_F) \). \( \hat{V}(k) \) being the Fourier transform of \( V(x) \) and \( v_F \equiv |k_F|/m \) the Fermi velocity. The two contributions \( \hat{V}(0) \) and \( \hat{V}(2k_F) \) derive from the exchange and Hartree part of the interaction, respectively; \( \alpha > 0 \) corresponds to repulsive interactions, and \( \alpha < 0 \) to attractive ones. While the perturbative corrections \([7]\) are in principle controlled by an expansion in \( \alpha \ll 1 \), they actually diverge logarithmically for \( k \to k_F \). These divergences can be resummed by an RG treatment \([10][24][25]\), leading to the RG flow equations

\[
\frac{dT}{d\ell} = -2\alpha \frac{T}{R}, \quad \frac{dR}{d\ell} = -\alpha \frac{R}{(R + T - 1)},
\]

with the flow to be stopped at \( \ell = -\log |d(k−k_F)| \). Eqs. \([8]\) have one stable fixed point: \( T^* = 0, \ R^* = 1 \) for \( \alpha > 0 \) and \( T^* = 1, \ R^* = 0 \) for \( \alpha < 0 \). Physically, this entails that tunneling through the dissipative impurity is suppressed at \( k = k_F \) for repulsive interactions, while it is maximally enhanced for attractive interactions, similarly to the case of a potential barrier \([24][25]\). However, two
novel remarkable features, compared to the potential barrier problem, emerge from the solutions of Eqs. (8). First, for both attractive and repulsive interactions, $\eta^* = 0$, implying that particles with $k = k_F$ are not emitted into the bath, but are actually “trapped” inside the wire, signalling emergent unitarity at the Fermi level. Second, $\eta(k)$ approaches its fixed point value in qualitatively different ways, depending on the sign of $\alpha$:

$$\eta(k) \sim \begin{cases} |k - k_F|^\alpha & \text{for } \alpha > 0, \\ -1/ \log |d(k - k_F)| & \text{for } \alpha < 0. \end{cases} \quad (9)$$

This asymmetry, also visible in the behaviour of $T(k)$ and $R(k)$ for $k \rightarrow k_F$, does not occur for the case of a potential barrier, where the fixed-point values are approached algebraically in both cases [24][25]. Eq. (9) is the key result of this work: the escape probability at the Fermi momentum is strongly renormalized by fluctuations, which suppress it. For $\alpha > 0$, this happens as if $\gamma \rightarrow +\infty$, thus producing a QZE; for $\alpha < 0$, instead, this happens as if $\gamma \rightarrow 0$, the impurity thus becoming transparent.

Fig. 3 (upper panels) shows the RG flow of $T$, $R$, and $\eta$, for both repulsive and attractive interactions. The flow of $\eta$ may be non-monotonic depending on the sign of $\alpha$ and on the bare value $\eta_0$. To make contact with the Luttinger formulation and the right panel in Fig. 1, it is possible to reparametrize $T(k)$, $R(k)$, and $\eta(k)$ in terms of a single function $\tilde{\gamma}(k)$ [10]. For the escape probability one finds $\eta(k) = 2\tilde{\gamma}(k)/[1 + \tilde{\gamma}(k)]^2$. The RG flow of $\tilde{\gamma}$ can be determined from Eqs. (8) as

$$\frac{d\tilde{\gamma}}{dt} = \alpha \tilde{\gamma}^2 / (1 + \tilde{\gamma}). \quad (10)$$

The fixed points of Eq. (5) translate then to $\tilde{\gamma}^* = \infty$ for $\alpha > 0$ and $\tilde{\gamma}^* = 0$ for $\alpha < 0$. $\tilde{\gamma}$ can therefore be interpreted as the effective strength of the localized dissipation, thus bridging the Luttinger result [5] with the one obtained from the microscopic model. In fact, for a Luttinger parameter $g \simeq 1$, one has $1 - g \simeq g^{-1} - 1 \simeq \alpha$ [27], and therefore Eqs. (5) and (10) coincide for $\tilde{\gamma} \gg 1$, upon the identifications $\tilde{\gamma} \equiv \gamma$. The discrepancy for $\tilde{\gamma} \ll 1$, i.e., for the weak loss limit, can be understood as the limits $\gamma \rightarrow 0$ and $\alpha \rightarrow 0$ do not commute for the nonequilibrium stationary state.

The behaviour of $\eta(k)$ under RG can be therefore rationalized in terms of the flow of $\eta(\ell)$ (see Fig. 1 right panel): $\eta$ reaches its fixed point $\eta^* = 0$ either for $\gamma \rightarrow \infty$ or for $\gamma \rightarrow 0$, in the former case thus resulting in a fluctuation-induced QZE, and in the latter one a fluctuation-induced transparency. In Fig. 3 (lower panels) we show the value of $\eta(k)$ as a function of $k$ reconstructed from the RG flow: its value drops to zero at $k_F$ for both attractive and repulsive interactions. As a consequence of the non-monotonicity of the RG flow, $\eta(k)$ may increase for momenta in the vicinity of $k_F$ (right panel).

**Observables** — The fluctuation-induced QZE can be naturally detected by harnessing the energy or momentum resolved flow of particles leaving the wire, without destructive measurements. For illustrative purposes, we consider the following model, inspired by the input-output formalism of quantum optics [12]. The local dissipation originates from the wire being coupled to a continuum of fermionic modes outside the wire. For definiteness, we assume particles to exit the wire by expanding isotropically in the surrounding vacuum, which could e.g. be realized in setup Ref. [11] by a local transfer to an untrapped internal state. The particles could be described by operators $c_q, c_q^\dagger$, with $q$ a three-dimensional momentum. These modes have energy $\epsilon_q$ and to be coupled to the wire at $x = 0$ through the Hamiltonian $H_{\text{int}} = \sum_q (\epsilon_q c_q^\dagger \psi_0 + \text{h.c.})$, with $g_q$ the coupling of the $q$-mode to the wire and $\psi_0 \equiv \psi(x = 0)$. In the second regime, the constant rate of particles with momentum $q$ leaving the wire is then given by [10]:

$$\frac{d(c_q^\dagger c_q)}{dt} = \theta(E_F - \epsilon_q) \frac{|g_q|^2}{\gamma} \eta(\epsilon_q), \quad (11)$$

with $E_F = k_F^2/2m$, thus providing a connection between $\eta$ and an experimentally accessible quantity. Notice that the bare Fermi distribution enters Eq. (11) as the approach is perturbative in the interaction: it is expected to be smeared out by stronger interactions [13].

**Finite temperature and size** — The unavoidable presence of a finite temperature $T$ and system size $L$ in realistic systems can be accounted for by our RG analysis, and their variation actually leveraged to disclose the novel
collective behaviours described above. In fact, a finite $T$ (resp. $L$) cuts off the RG flow a scale $\ell_T = -\log(dT)$ (resp. $\ell_L = \log(L/d)$). As a consequence $\eta(k_F)$ can exhibit a non-monotonic behaviour as a function of the considered length-scale (cf. Fig. 5 upper panels). For instance, the value of $\eta(k_F)$ increases up to $\sim 100\%$ by reducing the temperature of a gas from $T_{\text{Fermi}}$ to $0.1T_{\text{Fermi}}$, thus suggesting that the effects above discussed are observable within the current experimental setups [11] [28].

Conclusions — We have shown that a one dimensional ultracold gas of fermions displays novel many-body effects in presence of a localized loss. Loss is suppressed close to the Fermi energy, effectively restoring unitarity. Moreover, transport properties are modified similarly to the case of a potential barrier. These effects would be experimentally accessible by analyzing the ejected flow of particles energy or momentum resolved, without further destructive measurements. Our results call moreover for a development of observables based on a microscopic approach: in particular, a Landau-Büttiker theory for experimentally accessible by analyzing the ejected flow of particles energy or momentum resolved, without further destructive measurements. Moreover, transport properties are modified similarly to the case of a potential barrier. These effects would be experimentally accessible by analyzing the ejected flow of particles energy or momentum resolved, without further destructive measurements. Our results call moreover for a development of observables based on a microscopic approach: in particular, a Landau-Büttiker theory for transport in driven-dissipative quantum systems appears to be crucial to guide future studies.

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SUPPLEMENTAL MATERIAL

I. EXACT RESULTS FOR THE NON-INTERACTING CONTINUUM MODEL WITH LOCALIZED LOSS

Retarded Green’s function

The quantum master equation (see Eq. (2) in the main text) can be mapped onto a Keldysh action \( S = S_0 + S_{\text{loss}} \) \([S1, S2]\), with

\[
S_0 = \int_{x,t} \left[ i\psi_+^* \dot{\psi}_+ - H(\psi_+^*, \psi_+) - i\psi_-^* \dot{\psi}_- + H(\psi_-^*, \psi_-) \right], \tag{S1}
\]

and

\[
S_{\text{loss}} = -i \int_{x,t} \Gamma(x) \left[ L_+^* L_+ - \frac{1}{2} (L_+^* L_+ + L_-^* L_-) \right], \tag{S2}
\]

where \( \Gamma(x) = \gamma \delta(x) \) and \( L_\pm(x) = \psi_\pm(x) \). The retarded Green’s function \( G(x, x', \omega) \) can be then computed conveniently by using the Dyson’s equation \([S2]\):

\[
G(x, x', \omega) = G_0(x, x', \omega) + \int_y G_0(x, y, \omega) \Sigma(y) G(y, x', \omega), \tag{S3}
\]

where

\[
G_0(x, x', \omega) = \frac{1}{2i\sqrt{\omega}} e^{i\sqrt{\omega} |x-x'|} \tag{S4}
\]

is the (translation invariant) retarded Green’s function of the system in absence of localized loss, while \( \Sigma(y) \) is the self-energy associated with the localized loss. Since the loss appear as a quadratic operator in the Keldysh action, the self-energy is a field-independent function which reads \( \Sigma(y) = -i\gamma \delta(y) \), so that Eq. \([S3]\) can be rewritten as

\[
G(x, x', \omega) = G_0(x, x', \omega) - i\gamma G_0(x, 0, \omega) G(0, x', \omega), \tag{S5}
\]

which can readily be solved, yielding

\[
G(x, x', \omega) = G_0(x, x', \omega) - i\gamma \frac{G_0(x, 0, \omega) G_0(0, x', \omega)}{1 + i\gamma G_0(0, 0, \omega)} = \frac{1}{2i\sqrt{\omega}} \left[ e^{i\sqrt{\omega} |x-x'|} + r(\omega) e^{i\sqrt{\omega} (|x|+|x'|)} \right], \tag{S6}
\]

where

\[
r(\omega) = \frac{-\gamma}{2\sqrt{\omega} + \gamma} \tag{S7}
\]

is a function to be discussed below. We report, for later convenience, also the expression for the Green’s function in a mixed momentum-real space representation \( G(x, q, \omega) \):

\[
G(x, q, \omega) = \frac{1}{\omega + i\epsilon - q^2} \left[ e^{iqx} + r(\omega) e^{i\sqrt{\omega} |x|} \right], \tag{S8}
\]

with \( \epsilon \) an infinitesimal dissipation needed in order to ensure causality.

Transmission and reflection coefficients

The transmission and reflection coefficients for the scattering of a particle off the lossy barrier can be inferred from Eqs. \([S6, S8]\). For instance, Eq. \([S8]\) can be interpreted as the response to an external field \( h(x,t) = h \exp(iqx - i\omega t) \) corresponding to a plane wave incoming from left. From the on-shell condition \( \omega \simeq q^2 \) one finds

\[
G(x, q, \omega \simeq q^2) \propto \begin{cases} 
  e^{iqx} + r(q^2) e^{-iqx} & x < 0 \\
  t(q^2) e^{iqx} & x > 0,
\end{cases} \tag{S9}
\]

where \( r(q^2) \) is defined in Eq. \([S7]\) and corresponds to the reflection coefficient, while \( t(q^2) = 1 + r(q^2) \) corresponds to the transmission coefficient. Using Eq. \([S7]\) one finds

\[
t(q^2) = \frac{2|q|}{2|q| + \gamma}, \quad r(q^2) = -\frac{\gamma}{2|q| + \gamma}. \tag{S10}
\]
One-particle correlation function

We study the evolution of the one-particle correlation function \( C(x, y, t, t') = \langle \psi(x, t) \psi(y, t') \rangle \) after the quench of the impurity at time \( t_0 \). Since the wire is coupled to an empty bath which absorbs irreversibly particles from it, \( C(x, y, t, t') \) can be written as a propagation of the correlation at time \( t_0 \), by using the retarded Green’s function previously computed:

\[
C(x, y, t, t') = \int_{x', y'} G^*(x, x', t - t_0) G(y, y', t' - t_0) C(x', y', t_0, t_0).
\]

(S11)

With the initial condition \( C(x, y, t_0, t_0) = \int_q e^{iq(y-x)} n_F(q) \), where \( n_F(q) \) is the Fermi distribution at \( T = 0 \), Eq. (S11) becomes

\[
C(x, y, t, t') = \int_{-k_F}^{k_F} \frac{dq}{2\pi} G^*(x, -q, t) G(y, -q, t').
\]

(S12)

We now compute the stationary value of \( C(x, y, t, t') \) by taking the limit \( t_0 \to -\infty \). For the sake of clarity, we consider the case \( x = y \) and \( t = t' \), but the reasoning can be straightforwardly generalized to the case \( x \neq y \) and \( t \neq t' \).

\[
|G(x, -q, t - t_0)|^2 = \left| \int_0^{\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t_0)}}{\omega + i - q^2} \left[ e^{-iqx} + r(\omega)e^{i\sqrt{\omega}|x|} \right]^2 \right|_0^t \times \left| \int_0^{\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t_0)}}{\omega + i - q^2} \left[ e^{-iqx} + r(\omega)e^{i\sqrt{\omega}|x|} \right] \left( \frac{d\omega}{2\pi} \delta(\omega - q^2) \right) \right|^2 = \left| e^{-iqx} + r(q^2)e^{i|q||x|} \right|^2,
\]

(S13)

where in the third step we took the limit \( t_0 \to -\infty \). The local density of particles at time \( t \) is given by \( n(x, t) = C(x, x, t, t) \). By making use of the previous equation we obtain

\[
n(x) = \frac{1}{\pi} \int_0^{k_F} dq \left[ 1 + r(q^2)^2 + r(q^2) + r(q^2) \cos(2q|x|) \right] \equiv n_{ess} + \delta n(x),
\]

(S14)

where \( n_{ess} \) is the homogeneous background and \( \delta n(x) \) the density modulations, which exhibits for \( |x| \gg k_F^{-1} \) Friedel oscillations

\[
\delta n(x) = r(k_F) \frac{\sin(2k_F|x|)}{2\pi|x|}.
\]

(S15)

II. BOSONIZATION OF A DISSIPATIVE IMPURITY

The low-energy properties of a system interacting one-dimensional fermions is captured by the Luttinger Hamiltonian described in the main text (see Eq. (3)). The mapping between fermions and bosons is done via the transformation (considering the two leading harmonics) \([S3][S5] \):

\[
\psi \sim \sqrt{P_0} e^{ik_F x} e^{i(\phi + \theta)} + e^{-ik_F x} e^{i(\phi - \theta)},
\]

(S16)

where \( \phi \) and \( \theta \) are bosonic fields describing phase fluctuations and density fluctuations, respectively. The associated Keldysh action \( S_0 \) can be written as \([S0][S8] \):

\[
S_0 = \int_{k, \omega} \chi^\dagger(k, \omega) G^{-1}(k, \omega) \chi(k, \omega)
\]

(S17)

where \( \int_{k, \omega} \equiv \int d\omega/(2\pi)^2 \), \( \chi \equiv (\phi_c, \theta_c, \phi_q, \theta_q)^T \),

\[
G^{-1}(k, \omega) = \begin{pmatrix}
0 & P_A(k, \omega) \\
P_R(k, \omega) & P_K(k, \omega)
\end{pmatrix},
\]

(S18)
and

\[ P_R = P_A^1 = \frac{1}{2\pi} \begin{pmatrix} v g k^2 + i\omega & -k\omega \\ -k\omega & v g k^2 + i\omega \end{pmatrix}, \quad P_K = \frac{1}{2\pi} \begin{pmatrix} 2i\epsilon\omega \coth \frac{\omega}{T} & 0 \\ 0 & 2i\epsilon\omega \coth \frac{\omega}{T} \end{pmatrix}. \tag{S19} \]

The retarded and Keldysh Green’s functions, \( G_R \) and \( G_K \), respectively, can be obtained as \( G_R = P_R^{-1} \) and \( G_K = P_K^{-1} \). By using the mapping \([S2]\) and the bosonization formula \([S16]\), the backward scattering due to the impurity reads:

\[ S_{\text{back}} = -i 2\gamma \int_{x,t} \delta(x) \left[ \cos \theta_c (e^{i\phi_q} - \cos \theta_q) \right], \tag{S20} \]

where \( \theta_{c,q} = \theta_+ \pm \theta_- \) and \( \phi_{c,q} = \phi_+ \pm \phi_- \).

A renormalization group analysis is then performed by integrating out fast modes lying in the momentum-shells \( k \in [\Lambda e^{-\ell}, \Lambda] \) and subsequently rescaling as \( (x, t) \to (e^\ell x, e^\ell t) \). In the weak coupling limit \( (\gamma \to 0) \) we consider the leading term in the expansion in \( \gamma \):

\[ \langle e^{iS_{\text{back}}} \rangle_{\text{fast}} \simeq \langle 1 + i S_{\text{back}} \rangle_{\text{fast}} \simeq e^{iS_{\text{back}}} \langle 1 \rangle_{\text{fast}}. \tag{S21} \]

By making use of the Gaussian identity \( \langle e^{ix} \rangle = \exp[-(x^2)/2] \) and the correlation functions from Eq. \([S19]\), we obtain from Eq. \([S20]\)

\[ \langle S_{\text{loss}}[\phi_{\text{fast}}, \theta_{\text{fast}} + \theta_{\text{slow}}] \rangle_{\text{fast}} = S_{\text{loss}}[\phi_{\text{slow}}, \theta_{\text{slow}}] e^{-\langle \theta^2(x,t) \rangle_{\text{fast}}}. \tag{S22} \]

This first order corrections can thus be calculated from \([S19]\), and for \( T = 0 \) it reads

\[ \langle \theta^2_c(x,t) \rangle_{\text{fast}} = g \int_{\Lambda e^{-\ell}} \frac{dk}{k} = g\ell. \tag{S23} \]

Finally, by noticing that the canonical scaling dimension of the dissipation strength \( \gamma \) is \( [\gamma] = 1 \), we obtain the flow equation \((5)\) in the main text.

### III. Renormalization of Scattering Amplitudes

We proceed by computing the corrections of the scattering probabilities \( \mathcal{T} \) and \( \mathcal{R} \) due to interactions \( V \) (see Eq. \((1)\) in the main text) to first order in perturbation theory in the interactions. To this end, its convenient to focus on the transmission and reflection amplitudes, which relate to the corresponding probabilities via

\[ 0 = (1) \text{ in the main text) to first order in perturbation theory in the interactions.} \]

To this end, its convenient to focus on the transmission and reflection amplitudes, which relate to the corresponding probabilities via

\[ \langle 1 \rangle_{\text{fast}} = e^{iS_{\text{back}}} \langle 1 \rangle_{\text{fast}}. \tag{S21} \]

By making use of the Gaussian identity \( \langle e^{ix} \rangle = \exp[-(x^2)/2] \) and the correlation functions from Eq. \([S19]\), we obtain from Eq. \([S20]\)

\[ \langle S_{\text{loss}}[\phi_{\text{fast}} + \phi_{\text{slow}}, \theta_{\text{fast}} + \theta_{\text{slow}}] \rangle_{\text{fast}} = S_{\text{loss}}[\phi_{\text{slow}}, \theta_{\text{slow}}] e^{-\langle \theta^2(x,t) \rangle_{\text{fast}}}. \tag{S22} \]

This first order corrections can thus be calculated from \([S19]\), and for \( T = 0 \) it reads

\[ \langle \theta^2_c(x,t) \rangle_{\text{fast}} = g \int_{\Lambda e^{-\ell}} \frac{dk}{k} = g\ell. \tag{S23} \]

Finally, by noticing that the canonical scaling dimension of the dissipation strength \( \gamma \) is \( [\gamma] = 1 \), we obtain the flow equation \((5)\) in the main text.

\[ \delta G(x, y, \omega) = \int_{x', y'} G_0(x, x', \omega) [V_H(x', y') + V_{ex}(x', y')] G_0(y', y, \omega), \tag{S24} \]

where the Hartree and exchange potentials \( V_H \) and \( V_{ex} \) are given by

\[ V_H(x, y) = \delta(x - y) \int_{x', t} V(x - x') C(x', t, t), \quad V_{ex}(x, y) = -V(x - y) C(x, y, t, t), \tag{S25} \]

with \( C(x, y, t, t) \) evaluated in the stationary limit. From Eq. \([S24]\) one obtains the following corrections (see Ref. \([S9]\),

\[ t = t_0 + \alpha t_0 r_0^2 \log |d(k - k_F)|, \tag{S26a} \]

\[ r = r_0 + \frac{\alpha}{2} r_0 (r_0^2 + t_0^2 - 1) \log |d(k - k_F)|, \tag{S26b} \]

with \( t, r \equiv t(k), r(k) \) for \( k \sim k_F \), \( d \) is the typical length scale of \( V(x) \), and \( \alpha = [\langle V(2k_F) - V(0) \rangle/(2\pi v_F)] \). The logarithmic divergences of the perturbation theory remain at higher orders and are cured by a proper resummation, achieved by a real space or a frequency RG \([S9], [S10]\) leading to

\[ \frac{dt}{d\ell} = -\alpha t^2, \tag{S27a} \]

\[ \frac{dr}{d\ell} = -\frac{\alpha}{2} r \left(t^2 + r^2 - 1 \right). \tag{S27b} \]
From here it is straightforward to derive the flow equations of the scattering probabilities (Eqs. (8) in the main text). Since both, the continuity relation \( t = 1 + r \) is preserved and \( t, r \) remain real-valued along the flow, it is possible to parametrize the flow of \( r \) and \( t \) by a single function \( \tilde{\gamma} \), such that
\[
r = -\frac{\tilde{\gamma}}{1 + \tilde{\gamma}}, \quad t = \frac{1}{1 + \tilde{\gamma}}.
\] (S28)

The flow equation for \( \tilde{\gamma} \) is then easily derived from Eq. (S27) and reads
\[
\frac{d\tilde{\gamma}}{d\ell} = \alpha \frac{\tilde{\gamma}^2}{1 + \tilde{\gamma}}.
\] (S29)

### IV. INPUT-OUTPUT FORMALISM

We investigate the loss of particles from the system at \( x = 0 \). To this end, we study \( G(x = 0, p, \omega) \), describing the response at \( x = 0 \) to a plane wave perturbation, as discussed in Sec. I (cf. Eq. (S8)):
\[
G(x = 0, p, \omega) = t_\eta(\omega)G_0(p, \omega).
\] (S30)

Here, we introduced the amplitude \( t_\eta \), related to \( \eta \) through
\[
\frac{\gamma}{\ell} t_\eta^2 = 1 - T - \mathcal{R} \equiv \eta.
\] (S31)

Eqs. [S30] and [S31] can then be regarded as an operative definition to evaluate \( \eta \) directly from the response function. We then introduce \( g(t_2 - t_1) = \langle \psi(t_2) | \psi(t_1) \rangle \) and obtain as a generalization of the calculation in Sec. I
\[
g(t_2 - t_1) = \int_{-k_F}^{k_F} \frac{dp}{2\pi} G(x = 0, -p, t_1 - t_0)G(x = 0, -p, t_2 - t_0) = \int_{-k_F}^{k_F} \frac{dp}{2\pi} e^{-p^2(t_2 - t_1)}|t_\eta(p^2)|^2.
\] (S32)

As discussed in the main text, we consider the wire being coupled to a continuum of free fermionic modes, described by \( H_{\text{int}} = \sum_q (g_q c_q^\dagger \psi_0 + \text{h.c.}) \). The momentum resolved loss rate is then related to \( t_\eta \) by
\[
\frac{d\langle c_q^\dagger c_q \rangle}{dt} = |g_q|^2 g(\omega = \epsilon_q) = |g_q|^2 \frac{\theta(E_F - \epsilon_q)}{\sqrt{\epsilon_q}} |t_\eta(\omega = \epsilon_q)|^2,
\] (S33)

with \( E_F = k_F^2/2m \).

The renormalization procedure from Sec. III can be adapted to obtain the renormalization of \( t_\eta \). As the starting point, the perturbative correction of \( G(x = 0, p, \omega) \) is given by
\[
\delta G(x = 0, p, \omega) = \int_{x', y'} \langle x', y' | V_H(x', y') + V_{ee}(x', y') \rangle |G(y', p, \omega)|^2.
\] (S34)

Crucially, we observe that \( t_\eta \) enters the expression via \( G(x = 0, x', \omega) \), while the logarithmic divergencies are generated by the asymptotic forms of \( G(y', y, \omega), V_H(x', y') \) and \( V_{ee}(x', y') \) in which accordingly \( t \) and \( r \) appear.

The flow equation of \( t_\eta \equiv t_\eta(k) \) for \( k \sim k_F \) is found based on these considerations as
\[
\frac{dt_\eta}{d\ell} = -\frac{\alpha}{2} t_\eta (r^2 + tr).
\] (S35)

One can readily check that the flow equations of \( \eta = 1 - t_\eta^2 - r^2 \) and \( t_\eta^2 \) coincide, in agreement with Eq. (S31). According to Eqs. (S33) and (S31), we can relate the momentum resolved loss rate to the escape probability \( \eta \) as the key loss indicator of the system and arrive at the central result given in Eq. (11) of the main text.

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