Bilayer graphene in strong ultrafast laser fields

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Abstract
We theoretically investigate the interaction of an ultrastrong femtosecond-long linearly polarized optical pulse with AB-stacked bilayer graphene (BLG). The pulse excite electrons from the valence into the conduction band (CB), resulting in finite CB population. Such a redistribution of electrons results in the generation of current which can be manipulated by the angle of incidence of the pulse. For the normal incidence, the current along a direction transverse to the polarization of the optical pulse is zero. However, the interlayer symmetry is broken up by a finite angle of incidence due to which BLG possesses a single axis of symmetry. Thus, for an oblique incidence, if the pulse is polarized normal to the symmetry axis then there is an induction of electric current in the direction perpendicular to the polarization of the pulse. We show that the magnitude and the direction of such a current as well as charge transfer along this direction can be manipulated by tuning the angle of incidence of the laser pulse. Further, the symmetry of the system prohibits the generation of transverse current if the pulse is polarized along the axis of symmetry of BLG.

Keywords: bilayer graphene, ultrafast optical pulse, oblique incidence, current, charge

(Some figures may appear in colour only in the online journal)

1. Introduction
Graphene, a two dimensional (2D) material with honeycomb lattice structure, is a promising candidate for future technology [1]. The uniqueness of graphene is due to its remarkable physical properties [2]: the valence band (VB) and the conduction band (CB) are touching each other at the Dirac points thereby making it a gapless semiconductor [3]. It is chiral and is characterized by the Berry phase of $\pm \pi$ at the Dirac points [4–6]. Interestingly, the electron dynamics in graphene is governed by a massless Dirac equation [7]. This unusual electronic behavior gives rise to various unprecedented phenomena, such as half-integer quantum Hall effect [8–10], ballistic transport [11], and Klein tunneling due to the absence of back scattering [12].

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Interestingly, multilayer graphene systems come to demand for their increased electrical or thermal properties [13, 14] and optical signatures [15]. The simplest and thinnest intercalated structure of interest is the bilayer graphene (BLG) [16, 17]. Despite of having many properties similar to monolayer graphene [1], BLG is potentially different in terms of underlying features: (i) in BLG the dispersion near the Dirac points is parabolic [18] unlike the monolayer graphene where it is linear; (ii) the charge carriers in BLG are massive chiral quasiparticles [19] rather than massless ones in monolayer graphene; (iii) the Berry phase in bilayer [20] is $2\pi$ while in monolayer it is $\pi$; (iv) each individual layer in BLG can be tuned separately by doping [21] or gating [22, 23]. Moreover, the perpendicular electric field removes the interlayer symmetry [24] by introducing the difference in the on-site energies, which opens a band gap at the $K$ and $K'$ points. The band gap can be tuned by the magnitude of transverse electric field. This distinctive
Such a transverse current also results in the finite charge transfer. However, symmetry consideration of the system forbids any such transverse current for \( y \)-polarization of the pulse. We also show that the ultrafast electron dynamics in BLG remains nonadiabatic and irreversible, likewise in graphene, even when the interlayer symmetry is broken by the normal component of the applied pulse.

The paper is organized as follows. In section 2, we describe the model of BLG and introduce the main equations. In section 3, we present and discuss our main results. Finally, the concluding remarks are given in section 4.

2. Model

Monolayer of graphene has honeycomb crystal structure whose unit cell contains two non-equivalent carbon atoms A and B. The BLG, on the other hand, contains two coupled monolayers each of which has hexagonal lattice structure. The unit cell of BLG consists of four carbon atoms: with the contribution of two atoms each from the bottom and top layers. The BLG can have three configurations [17]: AA stacking, AB stacking (Bernal stacking), and twisted bilayer. The AA stacking arises when each carbon atom in the top layer is exactly over the corresponding atom of the bottom layer. However, in AB-stacking configuration one of the carbon atom \( B_b \) from the bottom layer is placed exactly below the carbon atom \( A_t \) of the top layer, as shown in figure 1(a). In the third configuration, the top graphene layer is rotated at some angle with respect to the bottom layer [35]. In this paper, we consider Bernal configuration of the BLG as shown in figure 1(a). The primitive reciprocal lattice vectors of the BLG are \( \mathbf{b}_1 = 2\pi/a (1,1,3) \), and \( \mathbf{b}_2 = 2\pi/a (1,-1,3) \). The first Brillouin zone of the BLG is also a hexagon whose vertices are the Dirac points at \( K = 2\pi/a (−1/3,1/\sqrt{3}) \) and \( K' = 2\pi/a (1/3,1/\sqrt{3}) \), where \( a = 2.46 \) Å is the lattice constant [figure 1(b)]. In AB-stacked BLG, the pair of sites (atom \( B_b \) from the bottom layer and \( A_t \) from the top layer), which exactly overlap, are referred as ‘dimers’. The interlayer coupling between the dimer sites is relatively strong because the orbitals of the dimer sites strongly overlap with each other. As a consequence of this, the hopping \( \gamma_1 \) between the dimer sites is the strongest. Thus, in ultrafast electric currents have been observed experimentally [31].

Contrary to linearly polarized pulses, the chiral optical pulses selectively populate the \( K \) and \( K' \) valleys [32], thereby generating valley polarization in gapped Dirac materials [33, 34]. Moreover, these effects are attributed to the existence of topological resonance which arises due to competition between dynamic and topological phase [34].

In this paper, we study the interaction of AB-stacked BLG with an ultrafast optical pulse. The laser pulse is applied at an oblique incidence and the normal component of it results in the interlayer asymmetry which opens a dynamical band gap at the \( K \) and \( K' \) points. Unlike monolayer, BLG in AB-stacking is axially symmetric only about \( y \)-axis and not along \( x \)-axis. We show that if the pulse is polarized along \( x \)-direction then it leads to the generation of a transverse electric current which can be maneuvered by means of the angle of incidence. Such a transverse current also results in the finite charge transfer. However, symmetry consideration of the system forbids any such transverse current for \( y \)-polarization of the pulse.

The feature of BLG makes it a potential candidate for electronic applications, which in monolayer graphene are limited due to its semimetal nature.

Recent advances in ultrafast laser technology have provided a versatile platform to explore the coherent control of electron dynamics at the sub-femtosecond time scale [25]. In particular, the interaction of strong ultrafast laser pulses with 2D materials opens a pathway to probe their extremely nonlinear behavior [26, 27]. Such optical pulses produce dramatic changes in the electron dynamics of graphene [28]. For instance, linearly polarized pulses produce interference fringes in the CB population distribution in the reciprocal space of graphene. This effect is due to quantum interference caused by a double passage by an electron of the Dirac points [29] during the pulse. Consequently, in graphene there exist a current along the direction of the polarization of the pulse. However, the interaction of linearly polarized pulse with gapped Dirac materials leads to the generation of an ultrafast current in the direction transverse to the plane of polarization of the pulse [30]. Such
The Brillouin zone is shown by black line and the application of a perpendicular field of amplitude $F_0 = 0.5 \text{ V \AA}^{-1}$ opens up a band-gap [16] of 0.371 eV between $v_3$ and $c_1$ at $K$ and $K'$. Such approximation we consider the following tight-binding Hamiltonian for BLG with the nearest neighbor hopping [17]

$$H_0 = \begin{pmatrix}
0 & -\gamma_0 f(k) & 0 & 0 \\
-\gamma_0 f^*(k) & 0 & \gamma_1 & 0 \\
0 & \gamma_1 & 0 & -\gamma_0 f(k) \\
0 & 0 & -\gamma_0 f^*(k) & 0
\end{pmatrix},$$

where $\gamma_0 = 3.16$ eV is the hopping integral, $\gamma_1 = 0.381$ eV represents the coupling between the orbitals on the dimer sites and $f(k) = \sum \delta_{ij} e^{i\mathbf{k}\cdot\delta_j}$ $(\delta_j, j = 1, 2, 3)$ are the vectors connecting the nearest neighbor intralayer sites describes the intralayer nearest-neighbor hopping between the orbitals $A_i$ and $B_i (i = b, t)$ and is given by the following expression

$$f(k) = \exp \left( \frac{iak_y}{\sqrt{3}} \right) + 2 \exp \left( -\frac{ia k_y}{2\sqrt{3}} \right) \cos \left( \frac{ak_x}{2} \right).$$

The energy spectrum of BLG can be found from equation (1). It consists of four bands: two CBs and two VBs with the following energy dispersion

$$E_{v_1} = -E_{v_2} = -(\gamma_1 - F)/2,$$
$$E_{v_2} = -E_{v_1} = (\gamma_1 + F)/2,$$

where $F = \sqrt{4\gamma_0^2 |f(k)|^2 + \gamma_1^2}$. The energy spectrum is shown in figure 2. The splitting between bands is determined by the interlayer coupling $\gamma_1$, as illustrated in figure 2(a). At the Dirac points, two bands $c_1$ (CB) and $v_2$ (VB) are degenerate. The band structure of the BLG can be modified by applying a perpendicular electric field. This breaks the interlayer symmetry and as a result the low-energy bands shows a ‘Mexican hat’ shape with a band gap.

Now, we consider a p-polarized ultrashort optical pulse of a single oscillation that is incident on BLG as shown in figure 1(a). The direction of propagation of the pulse is characterized by angle $\theta$, while the in-plane orientation of the pulse is determined by angle $\phi$ measured with respect to the $x$-axis [see figure 1(a)]. In the presence of the optical pulse, the Hamiltonian of the system takes the following form

$$H(t) = H_0 - eF_0(t)r - eL_zF_z(t)/2,$$

where $e$ is the electron charge, $F_0(t) = (F_x(t), F_y(t), F_z(t)) = F(t)\cos \theta (\cos \phi, \sin \phi)$ is the in-plane component of the electric field of the pulse, $L_z = 3.35$ Å is the interlayer spacing, and $F_z(t) = F(t)\sin \theta$ represents the normal component. Note that here $F_x(t)$ creates the interlayer asymmetry by introducing difference in the on-site energies of the two layers and consequently it opens up a dynamic band gap at the $K$ and $K'$ points. The applied ultrafast pulse which contains no dc Fourier component in the electric field is parameterized by the following equation

$$F = F_0(1 - 2u^2)e^{-u^2},$$

where $F_0$ is the amplitude of the optical pulse, and $u = t/\tau$, where $\tau$ is the pulse duration, which we consider to be 1 fs. The profile of the pulse in equation (6) is displayed in figure 3 and has zero area, $\int_{-\infty}^{\infty} F(t)dt = 0$. Note that the pulse waveform in equation (6) has been chosen in order to keep the electron in the laser focusing region during the whole pulse and to avoid the electron motion after the end of the pulse [36]. Further, there is no definite frequency for the ultrafast pulse and we use following relation to obtain its mean frequency [37]

$$\omega = \frac{\int |\mathbf{F}(\omega)|^2d\omega}{\int |\mathbf{F}(\omega)|^2d\omega},$$

where $\mathbf{F}(\omega)$ is the Fourier transform of $\mathbf{F}(t)$. For $\tau = 1$ fs, the mean frequency is 1.5 eV $\hbar^{-1}$.

We assume that the electron dynamics in the presence of the external electric field of the optical pulse is coherent. This is because the duration of the pulse in equation (6) is shorter than the characteristic electron scattering time of 10–100 fs [38–40]. Such a coherent electron dynamics is described by the time-dependent Schrödinger equation

$$i\hbar \frac{d\Psi}{dt} = H(t)\Psi.$$
The applied strong electric field generates both interband and intraband electron dynamics. The intraband dynamics is governed by the Bloch acceleration theorem [41], which provides following time-dependent crystal momentum, \( k(q, t) \)

\[
k(q, t) = q + \frac{e}{\hbar} \int_{-\infty}^{t} F_p(t') dt',
\]

(9)

where \( q \) is the initial crystal momentum. For intraband electron dynamics the corresponding wave functions, which are the solutions of equation (8) within a single band, i.e., without interband coupling, are well known Houston functions [42]

\[
\phi_{\alpha}^{(B)}(r, t) = \Psi_{kq,\alpha}^{(a)}(r) \exp \left[ -i \left( \phi_{\alpha}^{(D)}(t) + \phi_{\alpha}^{(B)}(t) \right) \right],
\]

(10)

where \( \alpha = v_1, v_2, e_1, e_2 \) for VB and CB, respectively, and \( \Psi_{k}^{(a)} \) are the periodic Bloch-band eigenfunctions in the absence of the optical pulse. Here \( \phi_{\alpha}^{(D)}(t) \) is the geometric (Berry) phase in the band \( \alpha \) and is given by,

\[
\phi_{\alpha}^{(D)}(t) = \frac{e}{\hbar} \int_{-\infty}^{t} \mathcal{A}^{\alpha}(k(q, t')) F_p(t') dt',
\]

(11)

where \( \mathcal{A}^{\alpha}(q) = (\Psi_{q}^{(a)}, \frac{\partial}{\partial q} | \Psi_{q}^{(a)} \rangle \) is the non-Abelian Berry connection [43]. For the definition of dynamic phase, \( \phi_{\alpha}^{(D)}(t) \), see [30]. Using the Houston functions (10) as the basis, we express the solution of equation (8) in the following form

\[
\Psi_{q}(r,t) = \sum_{\alpha = v_1, v_2, e_1, e_2} \beta_{\alpha q}(t) \phi_{\alpha}^{(B)}(r, t),
\]

(12)

where \( \beta_{\alpha q}(t) \) are the expansion coefficients which satisfy following set of equations

\[
i\hbar \frac{\partial B_q(t)}{\partial t} = H'(q, t) B_q(t),
\]

(13)

where \( B_q(t) \), and Hamiltonian \( H'(q, t) \) are defined as

\[
B_q(t) = \begin{bmatrix}
\beta_{v_1 q}(t) \\
\beta_{v_2 q}(t) \\
\beta_{e_1 q}(t) \\
\beta_{e_2 q}(t)
\end{bmatrix},
\]

(14)

\[
H'(q, t) = -eF(t) \mathbf{A}_{3d}(q, t),
\]

(15)

\[
\mathbf{A}_{3d}(q, t) = \begin{bmatrix}
D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) \\
D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) \\
D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) \\
D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t) & D_{3d}^{\alpha\alpha}(q, t)
\end{bmatrix},
\]

(16)

where \( F(t) = F_p(t) + F_z(t) \), and \( \mathbf{A}_{3d}^{\alpha\alpha} = (\mathbf{A}_{3d}^{\alpha\alpha})^* \). Also we have introduced following quantities

\[
\mathbf{D}_{3d}^{\alpha\alpha}(q, t) = \mathbf{D}_{p}^{\alpha\alpha}(q, t) + \mathbf{D}_{z}^{\alpha\alpha}(q, t)
\]

\[
= \left( \mathbf{A}_{\alpha\alpha}^{\alpha\alpha} [k(q, t)] + \mathbf{A}_{\alpha\alpha}^{\alpha\alpha} [k(q, t)] \right) \times \exp \left[ i \left( \phi_{\alpha}^{(D)}(q, t) + \phi_{\alpha}^{(B)}(q, t) \right) \right],
\]

(17)

\[
\phi_{\alpha}(q, t) = \phi_{\alpha}^{(D)}(q, t) - \phi_{\alpha}^{(D)}(q, t),
\]

(18)

\[
\phi_{\alpha}(q, t) = \phi_{\alpha}^{(B)}(q, t) - \phi_{\alpha}^{(B)}(q, t).
\]

(19)

Here \( \mathbf{D}_{p}^{\alpha\alpha}(q, t) \) and \( \mathbf{D}_{z}^{\alpha\alpha}(q, t) \) represents the respective in-plane and normal component of the non-Abelian Berry connection. Specifically, \( \mathbf{D}_{p}^{\alpha\alpha}(q, t) \) represents the interband coupling introduced by the normal component of the electric field of the applied laser pulse. The analytic expressions of \( \mathbf{A}_{\alpha\alpha}^{\alpha\alpha} \), and \( \mathbf{A}_{\alpha\alpha}^{\alpha\alpha} \) are provided in the appendix A.

The time-dependent electric field of the optical pulse causes the polarization of the system which generates an electric current. Both intraband \( \mathbf{J}^{\text{intra}}(t) \) and interband \( \mathbf{J}^{\text{inter}}(t) \) currents contribute to the total current, \( \mathbf{J}(t) = \mathbf{J}^{\text{intra}}(t) + \mathbf{J}^{\text{inter}}(t) \), in the system. Here, intraband and interband currents can be expressed in the following form:

\[
\mathbf{J}^{\text{intra}}(t) = 2e \int dq \sum_{\alpha = v_1, v_2, e_1, e_2} \beta_{\alpha q}^* V_j^\alpha \beta_{\alpha q} (j = x, y),
\]

(20)

\[
\mathbf{J}^{\text{inter}}(t) = 2e \int dq \sum_{\alpha, \alpha' = v_1, v_2, e_1, e_2} \beta_{\alpha q}^* V_j^{\alpha\alpha'} \beta_{\alpha' q} (\alpha \neq \alpha'),
\]

(21)

where \( V_j^\alpha \) is the intraband velocity (group velocity), \( V_j^{\alpha\alpha'} \) is the interband velocity [30], and factor of 2 is due to spin degeneracy. The matrix elements of the intraband velocity operator are presented in appendix B.

3. Results

By using the formalism described in section 2, we numerically solve the system of equation (13) under the initial conditions of initially occupied VBs 1 and 2 i.e. \( \beta_{v_1 q}, \beta_{v_2 q}, \beta_{e_1 q}, \beta_{e_2 q} = (0, 0, 1, 0) \) and \( \beta_{v_1 q}, \beta_{v_2 q}, \beta_{e_1 q}, \beta_{e_2 q} = (0, 0, 1, 0) \). From the solution of equation (13), we calculate the CB populations after the pulse which is known as residual CB populations, \( \mathbf{N}_{\alpha}(q) = |\beta_{\alpha q}(t = \infty)|^2 \) and \( \mathbf{N}^{\text{intra}}_{\alpha}(q) = |\beta_{\alpha q}(t = \infty)|^2 \). To further characterize the interband dynamics we define the time-dependent total population of the CBs

\[
\mathbf{N}_{\text{CB,}\alpha}(t) = \sum_q |\beta_{\alpha q}(t)|^2,
\]

(22)
Figure 4. Residual CB population (a) and (b) $N_{c1}^{\text{res}}(k)$ and (c) and (d) $N_{c2}^{\text{res}}(k)$ of two CBs for BLG in the reciprocal space as a function of the wave vector ($k$) for different angles of incidence ($\theta$). The boundary of the first Brillouin zone is shown by white lines. The amplitude of the pulse is fixed at $F_0 = 0.5$ V Å$^{-1}$, while the angle of incidence is (a) and (c) $\theta = 0^\circ$, and (b) and (d) $\theta = 60^\circ$. Here $\phi = 0^\circ$, and rest of the parameters are same as in figure 3.

where $\alpha = c_1, c_2$ and $i = 1, 2$ corresponds to initial conditions described above.

3.1. $x$-polarized pulse

The electron dynamics in BLG depends on the angle of incidence and the intensity of the applied pulse. To explore this effect, we first consider the case of $\phi = 0^\circ$, i.e., when the pulse is polarized along the $x$-direction. Also, to probe the influence of the oblique incidence, we fix the incidence angle dependence in the plane of BLG. Thus, the in-plane field amplitude of the pulse remains fixed. In figure 4, we show the distribution of the residual CB population in the first Brillouin zone at the end of the linearly polarized pulse for different angles of incidence and for fixed amplitude of the pulse, 0.5 V Å$^{-1}$. For the normally incident pulse, see figures 4(a) and (c), the CB population distributions is symmetric with respect to the $x$-axis and have hot spots with large CB population separated by dark regions with small CB population. These distributions are similar to the one that was observed in monolayer graphene [29] and are due to interference that is caused by the double passage by electrons of the Dirac points during one cycle of the optical pulse.

At a finite angle of incidence, the normal component of the optical field opens up a band gap between VB 2 and CB 1. This will cause an asymmetry in CB population distribution with respect to the $x$-axis, which is clearly visible in figures 4(b) and (d). Further, the CB population distribution can be reversed for the negative angle of incidence.

Redistribution of electrons between the valence and CBs results in the generation of electric currents during and after the pulse, governed by equations (20) and (21). Here we explore the dependence of the ultrafast currents the angle of incidence and the amplitude of the linearly polarized pulse.

Since the CB population distribution is symmetric with respect to the $x$-axis for $\theta = 0$, then the corresponding generated electric current is longitudinal i.e. it has only $x$-component, see figure 5, while the transverse component is zero. Both interband and intraband electron dynamics produce

Figure 5. (a) Electric current density along the longitudinal direction of the polarization of the pulse as a function of time. Here $\theta = 0^\circ$, $\phi = 0^\circ$ and $F_0 = 0.5$ V Å$^{-1}$. The other parameters are same as in figure 4.
Figure 6. (a) Electric current density along the direction transverse to the polarization of the optical pulse for different angles of incidence. (b) The same as (a) except for the negative angles of incidence. The parameters are same as in figure 5.

Figure 7. Electric current density along the direction (a) longitudinal and (b) transverse to the polarization of the optical pulse for different field amplitudes. Here $\theta = 60^\circ$ and rest of the parameters are same as in figure 5.

Figure 8. The transferred charge density in the direction parallel to the $x$-polarized optical pulse as a function of time. The other parameters are same as in figure 5.

The electron redistribution between CBs and VBs also depends on the amplitude of the pulse. This influences the longitudinal as well as transverse current and the magnitude of the both increases with increase in field amplitude, as shown in figure 7. However, the longitudinal current switches sign if the direction of the field maximum is reversed whereas the transverse current remains unaffected.

The current generated in the system results in the charge transfer through the system, which can be calculated from the following expression

$$ Q_j = \int_{-\infty}^{+\infty} dt J_j(t) e^{-t/\tau}, $$

(23)
Figure 9. (a) The charge density transferred along a direction transverse to the plane of incidence of the optical pulse as a function of the field amplitude for different angles of incidence. (b) The same as (a) except for negative angles of incidence. The parameters are same as in figure 5.

where \( j = ||, \perp \). Note that in equation (23), we introduce the exponential decay in current with a relaxation time \( \tau_r \). This is done to relax the oscillations in the current with increase in the integration time. Practically it is relevant since increase in integration time results in the introduction of relaxation in the electron dynamics and consequently decay in the current. Here for numerical calculations, we set \( \tau_r = 10 \text{ fs} \).

The charge transferred along the longitudinal direction is shown in figure 8 as a function of the field amplitude. Two main features of the charge transfer are to be noted from figure 8: (i) the magnitude of the charge transfer increases with increase in the field amplitude. (ii) The direction of charge transfer is positive which means that it follows the direction of field maximum.

In figure 9, we show the charge transferred along the transverse direction i.e. along the \( y \)-direction for different angles of incidence. The transferred charge increases with the field amplitude and with the angle of incidence. The direction of the charge transfer depends on the sign of the \( z \) component of the field maximum. Namely, for positive angle \( \theta \), i.e., for the \( z \)-component of the field maximum is positive, the charge is transferred in the negative direction of the \( z \)-axis, while for negative angle \( \theta \), which corresponds to the negative \( z \)-component of the field maximum, the transferred charge is positive. However, at \( F_0 = 1 \text{ V Å}^{-1} \) and \( \theta > \pm 44° \), the charge transfer switches sign both for positive and negative angles of incidences, as shown in figure 9.

The dependence of the charge transferred in the transverse direction, \( Q_{\perp} \), on the angle of incidence is shown in figure 10. The transferred charge has almost linear dependence on angle \( \theta \). It is positive for negative \( \theta \) and odd function of the angle.

3.2. \( y \)-polarized pulse

In the preceding section, the pulse was polarized in the \( x-z \) plane and the electric current was generated along the \( y \)-direction due to the breaking of the interlayer symmetry by the normal component of the applied pulse. Now we fix \( \phi = 90° \) i.e. the polarization of pulse is along \( y \)-axis which is also the axis of symmetry of BLG. For this case, no symmetry breaking occurs and the CB distribution along \( y \)-axis remains partially symmetric i.e. the red fringes around \( y \)-axis are partially symmetric even for non-zero angle of incidence as shown in figure 11. For close up residual CB population \( N_{(res)}^{(y)} \), see figure 12.

The current produced from such a distribution is shown in figure 13 for an angle of incidence of 60° and field amplitude of 0.5 V Å\(^{-1}\). There is a longitudinal current i.e. the current along the \( x \)-axis remains zero during the pulse. This is attributed to the fact that here the pulse is applied along the axis of symmetry which forbids the generation of transverse current.

3.3. CB population dynamics

One of the characteristics of the interband electron dynamics is the temporal evolution of the total CB population given by equation (22). Such time dependence of the total population of the first and the second CBs are shown in figure 14 for different values of the angle of incidence. The data show that the electron dynamics is irreversible and the residual CB populations for both bands are comparable to the maximum populations during the pulse. The dependence of the
Figure 11. Residual CB population (a) and (b) $N_{C_1}^{\text{res}}(k)$ and (c) and (d) $N_{C_2}^{\text{res}}(k)$ of two CBs in BLG for $\gamma$-polarized pulse in the reciprocal space as a function of the wave vector ($k$) for different angles of incidence ($\theta$). The boundary of the first Brillouin zone is shown by white lines. The amplitude of the pulse is fixed at $F_0 = 0.5 \text{ V Å}^{-1}$, while the angle of incidence is (a) and (b) $\theta = 0^\circ$, and (c) and (d) $\theta = 60^\circ$. Here $\phi = 90^\circ$, and rest of the parameters are same as in figure 3.

CB populations on the angle of incidence is different for two bands, while the CB population of the first CB decreases with $\theta$, the CB population of the second CB increases with $\theta$, see figure 14.

The residual CB populations of the first and the second CBs monotonically increase with the field amplitude, see figure 15(a). At the field amplitude of $F_0 = 1 \text{ V Å}^{-1}$, the CB population of the first CB reaches 12%, while the CB population of the second CB is about 9%. As expected, the CB population of the second CB is always less than the CB population of the first CB. This difference, which can be characterized as a ratio $N_{C_1}^{\text{res}} / N_{C_2}^{\text{res}}$ is shown in figure 15(b). At small field amplitude this ratio is about 2.3. With increasing the field amplitude the CBs become more equally populated, which is due to the fact that electrons are excited from the VBs over the energy range that is proportional to the field amplitude.

Figure 12. Close up of residual CB population $N_{C_2}^{\text{res}}(k)$ around (a) $K'$ and (b) $K$ points for $\theta = 60^\circ$. Rest of the parameters are same as in figure 11.

Figure 13. Electric current density parallel (solid black line) and perpendicular (dashed line) to the $\gamma$-polarization of the optical pulse as a function of time. Here $\theta = 60^\circ$ and rest of the parameters are same as in figure 5.
Figure 14. Population of (a) the first CB and (b) the second CB as a function of time for different angles of incidence of the optical pulse. The other parameters are the same as in figure 5.

Figure 15. (a) Residual population of the first, $N_{C1}^{(res)}$, and the second, $N_{C2}^{(res)}$, CB as a function of the pulse amplitude, $F_0$ and for different angles of incidence. Here solid and dashed lines correspond to $N_{C1}^{(res)}$ and $N_{C2}^{(res)}$, respectively. (b) Ratio of the residual CB populations, $N_{C1}^{(res)}/N_{C2}^{(res)}$ as a function of the field amplitude for different angles of incidence.

4. Conclusions

The electron dynamics in AB-stacked BLG is sensitive to the angle of incidence of the optical pulse. Such sensitivity is due to the fact that the normal component of the electric field of the pulse breaks the interlayer symmetry and, correspondingly, the inversion symmetry of BLG. As a result, for BLG in such an optical field, only the y-axis is the axis of symmetry but not the x-axis. Breaking of inversion symmetry in BLG opens a dynamic band gap. When the plane of incidence of the pulse is x–z plane, such band gap results in nontrivial topological phase that behaves differently above and below the $K$ ($K'$) point. Here the topological phase is defined as the sum of the geometric (Berry) phase and the phase of the interband dipole matrix element. Competition between the dynamic phase and the nontrivial topological phase results in a topological resonance, which has different strength above and below the $K$ ($K'$) point. This behavior results in two effects: (i) the CB population distribution in the reciprocal space is asymmetric with respect to the plane of incidence of the pulse, x–z plane, (ii) there is a nonzero transverse electric current generated during the pulse in the direction perpendicular to the plane of incidence, i.e., in y direction. These features occur only for BLG with the bandgap, which, in our case, is the dynamic band gap generated by the normal component of the field. Thus, for the normally incident optical pulse, the CB population distribution is symmetric with respect to the plane of polarization of the pulse and the transverse electric current is exactly zero. Furthermore, if the plane of incidence of the pulse is the y–z plane, which is still the plane of symmetry of BLG even in the optical field, no transverse current is generated in the system during the pulse, even for oblique incidence.

Interestingly, unlike the linearly polarized pulse, a completely different electron dynamics of BLG occurs under the influence of oblique incidence of a circular pulse. This is due to the fact that apart from the inversion symmetry breaking by the normal component, the in-plane component of the circular pulse removes the time-reversal symmetry thereby creating a selective population in $K$ and $K'$-valleys and produces valley polarization [44], an effect that cannot be realized with linearly polarized pulse. Further, such a valley polarization can be manipulated by employing a linearly polarized optical pulse which incident normally and follows the obliquely incident chiral optical pulse. We will explore the interaction of BLG with chiral pulses elsewhere.
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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A. Matrix elements of the non-Abelian Berry connection

The matrix elements of the in-plane component of the non-Abelian Berry connection has the following form

\[ A_{y}^{(v_1)} = A_1^{v_2} = A_1^{v_2} = A_1^{v_1} = - \frac{\alpha l_I}{\sqrt{3}|f(k)|^2}. \] (A.10)

where

\[ U_1 = \sin \left( \frac{ak_y}{2} \right) \sin \left( \frac{\sqrt{3}ak_x}{2} \right), \] (A.11)

\[ U_2 = \sin(ak_y) + \sin \left( \frac{ak_x}{2} \right) \cos \left( \frac{\sqrt{3}ak_x}{2} \right), \] (A.12)

\[ U_3 = \cos(ak_y) - \cos \left( \frac{ak_x}{2} \right) \cos \left( \frac{\sqrt{3}ak_x}{2} \right), \] (A.13)

\[ U_4 = \cos \left( \frac{ak_x}{2} \right) \sin \left( \frac{\sqrt{3}ak_x}{2} \right), \] (A.14)

\[ U_{v_1} = U_{v_1} = \frac{\gamma_0|f(k)|}{\sqrt{2(E_{c_1}^2 + \gamma_0^2|f(k)|^2)}}, \] (A.15)

\[ U_{v_2} = U_{v_2} = \frac{\gamma_0|f(k)|}{\sqrt{2(E_{c_2}^2 + \gamma_0^2|f(k)|^2)}}, \] (A.16)

and \( F = \sqrt{4\gamma_0^2|f(k)|^2 + \gamma_0^2}. \)

Also, the matrix elements of the normal component of the non-Abelian Berry connection can be written as

\[ A_1^{v_1} = 2L_z N_{v_1} N_{v_1}, \] (A.17)

\[ A_1^{v_2} = 2L_z N_{v_2} N_{v_1}, \] (A.18)

\[ A_2^{v_1} = L_z N_{v_2} N_{v_1} \left( 1 - \frac{E_{c_2}^2}{\gamma_0^2|f(k)|^2} \right), \] (A.19)

\[ A_2^{v_2} = L_z N_{v_1} N_{v_2} \left( 1 - \frac{E_{c_1}^2}{\gamma_0^2|f(k)|^2} \right), \] (A.20)

and \( A_2^{v_2} = A_1^{v_1} = 0. \)

Appendix B. Matrix elements of the intraband velocity

From the known eigenstates, the matrix elements of the intraband velocities can be expressed in the following form:

\[ V_x^{v_2} = -V_x^{v_1} = -\frac{2\alpha\gamma_0}{\hbar F} \left( E_{c_2}^2 \right), \] (B.1)

\[ V_y^{v_2} = -V_y^{v_1} = -\frac{2\sqrt{3}\alpha\gamma_0}{\hbar F} \left( E_{c_1}^2 \right). \] (B.2)

The other intraband velocities can be expressed as \( V_x^{v_1} = V_x^{v_1} = V_x^{v_2}, \) and \( V_y^{v_1} = V_y^{v_2} = V_y^{v_2}. \)

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