Hiding neutrinoless double beta decay in split seesaw model with 2+1 right-handed neutrinos

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Abstract

In the paper by Asaka, Ishida and Tanaka [Phys. Rev. D 103 (2021) 015014, arXiv:2012.12564], they proposed a novel possibility that the neutrinoless double beta decay can be hidden in the minimal seesaw model with two right-handed neutrinos which have a hierarchical mass structure. The heavier right-handed neutrino is sufficiently heavy to decouple from the neutrinoless double beta decay while the lighter one is lighter enough than the typical Fermi-momentum scale of nuclei. They show that, under some specific condition on the neutrino Yukawa couplings, the neutrinoless double beta decay can be hidden. In this paper, we perform a further study of such an interesting possibility in the complete seesaw model with three right-handed neutrinos. Our framework is same as theirs except that there are two heavier right-handed neutrinos so that the baryon-antibaryon asymmetry of the Universe can be explained via the leptogenesis mechanism. We first give the condition on the neutrino Yukawa couplings for hiding the $0\nu\beta\beta$ decay, discuss its realization by employing an Abelian flavor symmetry, and study its implications for the mixing of the lighter right-handed neutrino with three left-handed neutrinos. We then successively study the implications for leptogenesis of the interesting scenarios where $M_D$ is a triangular matrix or respects the $\mu$-$\tau$ reflection symmetry.

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1 Introduction

As we know, the phenomena of neutrino oscillations indicate that neutrinos are massive and the lepton flavors are mixed [1]. On the one hand, the type-I seesaw mechanism [2] where the Standard Model is extended with the right-handed neutrinos \( N_I \) (for \( I = 1, 2, 3 \)) is the most attractive and natural way to generate the observed small neutrino masses. These new particles not only have the Yukawa couplings with the left-handed neutrinos \( \nu_\alpha \) (for \( \alpha = e, \mu, \tau \)) and the Higgs field \( H \) in the form of \( (Y_\nu)_\alpha H N_I \) with \( (Y_\nu)_\alpha \) being dimensionless coefficients, which lead to the Dirac neutrino mass terms \( (M_D)_{\alpha I} = (Y_\nu)_\alpha v \) with \( v \equiv \langle H \rangle = 174 \text{ GeV} \) being the vacuum expectation value of \( H \), and have their own Majorana masses. In this paper, without loss of generality, we will work in the basis \( \alpha \) \( I \) where the mass eigenstates of three charged leptons are identical (for \( I = 1, 2, 3 \)) and the lightest neutrino mass, nor any constraint on the Majorana CP phases. Their values can only be inferred from some non-oscillatory processes (e.g., the neutrinoless double beta decay [5]) or cosmological observations. Unfortunately, so far there has not been any lower constraint on the lightest neutrino mass, nor any constraint on the Majorana CP phases.

As neutral particles, neutrinos are the unique candidate for Majorana fermions in the Standard Model. Furthermore, they would really be Majorana particles if their masses were generated via the seesaw mechanism. Therefore, testing the nature (Majorana or Dirac) of neutrinos will not only tell us if there exist Majorana fermions in the nature but also be helpful to revealing the origin of neutrino masses. At present, the most promising way to tackle this issue is to find the phenomenon of the neutrinoless double beta \((0\nu\beta\beta)\) decay [5]. The decay can be mediated by massive Majorana neutrinos and its rate is controlled by the so-called effective Majorana neutrino mass \( m_{\beta\beta} \). When the right-handed neutrinos are much heavier than the electroweak scale, their direct contributions to \( m_{\beta\beta} \)
are negligibly small. But they can contribute to $m_{\beta\beta}$ indirectly through their contributions to the light neutrino mass term $(M_\nu)_{ee}$. When a certain right-handed neutrino is lighter than the typical scale of Fermi momentum of a nucleus $\Lambda_\beta \sim O(100)\text{ MeV}$, due to the intrinsic property of the seesaw mechanism, its direct contribution to $m_{\beta\beta}$ and indirect contribution to $m_{\beta\beta}$ through the contribution to $(M_\nu)_{ee}$ exactly cancel out each other [6].

In Ref. [7], T. Asaka, H. Ishida and K. Tanaka propose a novel possibility that the 0$\nu\beta\beta$ decay can be hidden in the minimal seesaw model with two right-handed neutrinos which have a hierarchical mass structure [8]. The heavier right-handed neutrino is sufficiently heavy to decouple from the 0$\nu\beta\beta$ decay while the lighter one is lighter enough than $\Lambda_\beta$. Under some specific condition on the neutrino Yukawa couplings, the 0$\nu\beta\beta$ decay can be hidden. In this paper, we perform a further study of such an interesting possibility in the complete seesaw model with three right-handed neutrinos. Our setup is same as in Ref. [7] except that there are two heavier right-handed neutrinos so that the baryon-antibaryon asymmetry of the Universe can be explained via the leptogenesis mechanism. The rest part of this paper is organized as follows. In section 2, we give the condition on the neutrino Yukawa couplings for hiding the 0$\nu\beta\beta$ decay, discuss its realization by employing an Abelian flavor symmetry, and study its implications for the mixings of the lighter right-handed neutrino with three left-handed neutrinos. In sections 3 and 4, we respectively study the implications for leptogenesis of the interesting scenarios of $M_D$ being a triangular matrix and respecting the $\mu$-$\tau$ reflection symmetry. In section 5, we summarize our main results.

## 2 Condition on neutrino Yukawa couplings for hiding 0$\nu\beta\beta$ decay

In our framework, the right-handed neutrino mass spectrum is taken to be $M_1 \ll \Lambda_\beta \ll M_2 < M_3$. In this case, $N_2$ and $N_3$ are sufficiently heavy to decouple from the 0$\nu\beta\beta$ decay and will be responsible for leptogenesis, while $N_1$ is lighter enough than $\Lambda_\beta$ so that its direct contribution to $m_{\beta\beta}$ and indirect contribution to $m_{\beta\beta}$ through the contribution to $(M_\nu)_{ee}$ exactly cancel out each other. Obviously, for the following form of $M_D$ [i.e., $(Y_\nu)_{e2} = (Y_\nu)_{e3} = 0$]

$$M_D = \begin{pmatrix}
a_1 \sqrt{M_1} & 0 & 0 \\
a_2 \sqrt{M_1} & b_2 \sqrt{M_2} & c_2 \sqrt{M_3} \\
a_3 \sqrt{M_1} & b_3 \sqrt{M_2} & c_3 \sqrt{M_3}
\end{pmatrix},$$

Table 1: The best-fit values, 1$\sigma$ errors and 3$\sigma$ ranges of six neutrino oscillation parameters extracted from a global analysis of the existing neutrino oscillation data [3].

| Parameter                        | Normal Ordering | Inverted Ordering |
|----------------------------------|-----------------|-------------------|
| $\sin^2 \theta_{12}$            | $0.318^{+0.016}_{-0.016}$ | $0.318^{+0.016}_{-0.016}$ |
| $\sin^2 \theta_{23}$            | $0.566^{+0.016}_{-0.022}$ | $0.566^{+0.018}_{-0.023}$ |
| $\sin^2 \theta_{13}$            | $0.0222^{+0.00055}_{-0.00078}$ | $0.02250^{+0.00056}_{-0.00076}$ |
| $\delta/\pi$                    | $1.20^{+0.23}_{-0.14}$ | $1.54^{+0.13}_{-0.13}$ |
| $\Delta m_{21}^2/(10^{-5}\text{ eV}^2)$ | $7.50^{+0.22}_{-0.20}$ | $7.50^{+0.22}_{-0.20}$ |
| $|\Delta m_{31}^2|/(10^{-3}\text{ eV}^2)$ | $2.56^{+0.03}_{-0.04}$ | $2.46^{+0.03}_{-0.03}$ |
the $0\nu\beta\beta$ decay will be hidden. This can be easily understood as follows: the seesaw formula gives

$$M_\nu \simeq - \begin{pmatrix} a_1^2 & a_1 a_2 & 0 \\ a_1 a_2 & a_2^2 + b_2^2 + c_2^2 & a_2 a_3 + b_2 b_3 + c_2 c_3 \\ a_1 a_3 & a_2 a_3 + b_2 b_3 + c_2 c_3 & a_3^2 + b_3^2 + c_3^2 \end{pmatrix}.$$  \hfill (4)

In this case $(M_\nu)_{ee}$ only receives a contribution from $N_1$, so its contribution to $m_{\beta\beta}$ will be exactly cancelled out by the direct contribution of $N_1$ to $m_{\beta\beta}$.

As we will see, the texture zeros of $M_D$ in Eq. \[3\] can be realized by employing the $Z_{12}$ symmetry \[9\], under which the left-handed lepton doublets $L_\alpha$, the right-handed charged leptons $\alpha$ and the right-handed neutrinos have the following transformation properties

$$
\begin{align*}
L_e &\to \omega L_e, & e &\to \omega e, & N_1 &\to \omega N_1, \\
L_\mu &\to \omega^3 L_\mu, & \mu &\to \omega^2 \mu, & N_2 &\to \omega^2 N_2, \\
L_\tau &\to \omega^4 L_\tau, & \tau &\to \omega^5 \tau, & N_3 &\to \omega^5 N_3,
\end{align*}
$$

with $\omega = \exp(i\pi/6)$. Then, the bilinears $L_{\alpha \beta}$ and $L_{\alpha N_I}$, respectively responsible for the charged lepton mass terms $(M_l)_{\alpha \beta}$ and the Dirac neutrino mass terms $(M_D)_{\alpha I}$, transform according to the following matrix

$$
\begin{pmatrix}
\omega^2 & \omega^3 & \omega^6 \\
\omega^4 & \omega^5 & \omega^8 \\
\omega^9 & \omega^{10} & \omega
\end{pmatrix},
$$

while the bilinears $N_I^T N_J$, responsible for the right-handed neutrino mass terms $(M_R)_{IJ}$, transform according to the following matrix

$$
\begin{pmatrix}
\omega^2 & \omega^3 & \omega^6 \\
\omega^3 & \omega^4 & \omega^7 \\
\omega^6 & \omega^7 & \omega^{10}
\end{pmatrix}.
$$

In order to render a certain element of $M_D$ [e.g., $(M_D)_{e1}$] non-zero, one may introduce some singlet that has an appropriate transformation property under $Z_{12}$ [e.g., $\phi_{e1} \to \omega^{10} \phi_{e1}$] and will acquire a non-zero vacuum expectation value. If the singlets $\phi_{e2}$ and $\phi_{e3}$ that have the transformation properties $\phi_{e2} \to \omega^9 \phi_{e2}$ and $\phi_{e3} \to \omega^6 \phi_{e2}$ under $Z_{12}$ are absent, then the vanishing of $(M_D)_{e2}$ and $(M_D)_{e3}$ will be protected by the $Z_{12}$ symmetry. On the other hand, $M_I$ (and similarly for $M_R$) will be of the diagonal form as desired if there only exist $\phi_{ee}$, $\phi_{\mu\mu}$ and $\phi_{\tau\tau}$ that have the transformation properties $\phi_{ee} \to \omega^{10} \phi_{ee}$, $\phi_{\mu\mu} \to \omega^7 \phi_{\mu\mu}$ and $\phi_{\tau\tau} \to \omega^{11} \phi_{\tau\tau}$ under $Z_{12}$. Besides, an auxiliary $Z_3$ symmetry is also needed in order to distinguish the singlets $\phi_{\alpha\beta}$, $\phi_{\alpha I}$ and $\phi_{IJ}$ (responsible for the generations of the non-zero elements of $M_I$, $M_D$ and $M_R$, respectively), under which the relevant fields have the following transformation properties

$$
\begin{align*}
L_\alpha &\to \omega' L_\alpha, & \alpha &\to \omega' \alpha, & N_I &\to N_I, \\
\phi_{\alpha\beta} &\to \omega' \phi_{\alpha\beta}, & \phi_{\alpha I} &\to \omega'^2 \phi_{\alpha I}, & \phi_{IJ} &\to \phi_{IJ},
\end{align*}
$$

with $\omega' = \exp(2i\pi/3)$.

On the other hand, the smallness of $M_I$ compared to $M_2$ and $M_3$ can be explained by means of the $U(1)$ Froggatt-Nielsen (FN) symmetry \[10\] \[10\]. One may assign an FN number $n$ for $N_1$ and introduce

\[1\] In Ref. \[11\], the splitting among the right-handed neutrino masses is realized by invoking the extra dimensional theory.
a singlet $\phi$ with an FN number $-1$. Then, the mass term of $N_1$ will be subject to the suppression factor $(\langle \phi \rangle / \Lambda)^{2n}$ compared to those of $N_2$ and $N_3$, with $\Lambda$ being the cutoff scale of the FN mechanism. And the Yukawa couplings of $N_1$ are subject to the suppression factor $(\langle \phi \rangle / \Lambda)^n$ compared to those of $N_2$ and $N_3$. In this way the contribution of $N_1$ to the light neutrino masses is comparable to those of $N_2$ and $N_3$ \[12\]. This is because in the seesaw formula the contributions of a right-handed neutrino to the light neutrino masses are inversely proportional to its mass but quadratically proportional to its Yukawa couplings.

Then, we reconstruct the model parameters $a_i$, $b_i$ and $c_i$ in Eq. \[3\] in terms of the low-energy neutrino observables (i.e., the lepton flavor mixing parameters and neutrino masses). This can be done by making a direct comparison between $M_\nu$ in Eq. \[4\] and that obtained via the reconstruction relation $M_\nu = U D_\nu U^T$ [see Eq. \[1\]]:

\[
\begin{align*}
(M_\nu)_{ee} &= m_1 e^{2i\rho} c_{12}^2 c_{13}^2 + m_2 e^{2i\sigma} s_{12}^2 c_{13}^2 + m_3 s_{13}^2 e^{-2i\delta}, \\
(M_\nu)_{\mu\mu} &= m_1 e^{2i\rho} (s_{12} c_{23} + c_{12} s_{23} s_{13} e^i)^2 + m_2 e^{2i\sigma} (c_{12} c_{23} - s_{12} s_{23} s_{13} e^i)^2 + m_3 c_{13}^2 s_{23}, \\
(M_\nu)_{\tau\tau} &= m_1 e^{2i\rho} (s_{12} s_{23} - c_{12} c_{23} s_{13} e^i)^2 + m_2 e^{2i\sigma} (c_{12} s_{23} + s_{12} c_{23} s_{13} e^i)^2 + m_3 c_{13}^2 c_{23}, \\
(M_\nu)_{e\mu} &= -m_1 e^{2i\rho} c_{12} c_{13} (s_{12} c_{23} + c_{12} s_{23} s_{13} e^i) + m_2 e^{2i\sigma} s_{12} c_{13} (c_{12} c_{23} - s_{12} s_{23} s_{13} e^i) + m_3 c_{13} s_{23} s_{13} e^{-i\delta}, \\
(M_\nu)_{e\tau} &= m_1 e^{2i\rho} c_{12} c_{13} (s_{12} s_{23} - c_{12} c_{23} s_{13} e^i) - m_2 e^{2i\sigma} s_{12} c_{13} (c_{12} s_{23} + s_{12} c_{23} s_{13} e^i) + m_3 c_{13} s_{23} s_{13} e^{-i\delta}, \\
(M_\nu)_{\mu\tau} &= -m_1 e^{2i\rho} (s_{12} s_{23} - c_{12} c_{23} s_{13} e^i) (s_{12} c_{23} + c_{12} s_{23} s_{13} e^i) - m_2 e^{2i\sigma} (c_{12} c_{23} - s_{12} s_{23} s_{13} e^i) (c_{12} s_{23} + s_{12} c_{23} s_{13} e^i) + m_3 c_{13}^2 c_{23} s_{23}. \tag{9}
\end{align*}
\]

Note that there are totally 7 model parameters but $M_\nu$ only contains 6 independent elements. Hence the model parameters can not be completely reconstructed in terms of the low-energy neutrino observables and we will be left with one free parameter. It is useful to note that $M_\nu$ in Eq. \[4\] keeps invariant with respect to the following transformation

\[
\begin{pmatrix}
 b_2' \\
 c_2'
\end{pmatrix} = \begin{pmatrix}
 \cos z & -\sin z \\
 \sin z & \cos z
\end{pmatrix}
\begin{pmatrix}
 b_2 \\
 c_2
\end{pmatrix}, \quad
\begin{pmatrix}
 b_3' \\
 c_3'
\end{pmatrix} = \begin{pmatrix}
 \cos z & -\sin z \\
 \sin z & \cos z
\end{pmatrix}
\begin{pmatrix}
 b_3 \\
 c_3
\end{pmatrix}, \tag{10}
\]

with $z$ being a complex parameter. With the help of this observation, the reconstruction result can be obtained in a way as follows: we first derive it in the particular scenario of $c_2 = 0$ (where $M_D$ is a triangular matrix as will be studied in the next section). A direct calculation gives

\[
\begin{align*}
a_1 &= i \eta_a \sqrt{ (M_{\nu})_{ee} }, \quad
a_2 &= i \eta_a \frac{(M_{\nu})_{e\mu}}{\sqrt{(M_{\nu})_{ee}} }, \quad
a_3 &= i \eta_a \frac{(M_{\nu})_{e\tau}}{\sqrt{(M_{\nu})_{ee}} }, \\
b_2 &= \eta_b \frac{(M_{\nu})_{e\mu}^2 - (M_{\nu})_{ee}(M_{\nu})_{\mu\mu}}{(M_{\nu})_{ee} }, \\
b_3 &= \eta_b \frac{(M_{\nu})_{e\mu}(M_{\nu})_{e\tau} - (M_{\nu})_{ee}(M_{\nu})_{\mu\tau}}{(M_{\nu})_{ee}(M_{\nu})_{e\mu} - (M_{\nu})_{ee}(M_{\nu})_{\mu\mu} } , \\
c_3 &= \eta_c \frac{(M_{\nu})_{e\tau}^2 - (M_{\nu})_{ee}(M_{\nu})_{\tau\tau} - [(M_{\nu})_{e\mu}(M_{\nu})_{e\tau} - (M_{\nu})_{ee}(M_{\nu})_{\mu\tau}]^2}{(M_{\nu})_{ee}(M_{\nu})_{e\mu} - (M_{\nu})_{ee}(M_{\nu})_{\mu\mu} } , \tag{11}
\end{align*}
\]
with $\eta_{a_1}, \eta_{a_2}, \eta_{a_3} = \pm 1$. Then, the reconstruction result in the generic scenario of $c_2 \neq 0$ can be expressed as

$$b_2 = \eta_{b_2} \sqrt{\frac{(M_\nu)^2_{e\mu} - (M_\nu)_{ee}(M_\nu)_{\mu\mu}}{(M_\nu)_{ee}}} \cos z,$$

$$c_2 = \eta_{b_2} \sqrt{\frac{(M_\nu)^2_{e\mu} - (M_\nu)_{ee}(M_\nu)_{\mu\mu}}{(M_\nu)_{ee}}} \sin z,$$

$$b_3 = \eta_{b_2} \frac{M_\nu_{\mu\mu}(M_\nu)_{e\mu} - (M_\nu)_{ee}(M_\nu)_{\mu\mu}}{(M_\nu)_{ee}(M_\nu)_{e\mu} - (M_\nu)_{ee}(M_\nu)_{\mu\mu}} \cos z,$$

$$-\eta_{c_3} \sqrt{\frac{(M_\nu)^2_{e\tau} - (M_\nu)_{ee}(M_\nu)_{\tau\tau}}{(M_\nu)_{ee}}} \sin z,$$

$$c_3 = \eta_{b_2} \frac{M_\nu_{\mu\mu}(M_\nu)_{e\mu} - (M_\nu)_{ee}(M_\nu)_{\mu\mu}}{(M_\nu)_{ee}(M_\nu)_{e\mu} - (M_\nu)_{ee}(M_\nu)_{\mu\mu}} \cos z,$$

$$+\eta_{c_3} \sqrt{\frac{(M_\nu)^2_{e\tau} - (M_\nu)_{ee}(M_\nu)_{\tau\tau}}{(M_\nu)_{ee}}} \sin z,$$

while the expressions of $a_i$ are same as in Eq. (11). It is obvious that $z$ acts as the above-mentioned remaining free parameter.

Finally, we study the implications of $(Y_\nu)_{e2} = (Y_\nu)_{e3} = 0$ for the mixings of $N_1$ with three left-handed neutrinos. The magnitudes of these mixings directly determine the discovery prospects of $N_1$ in relevant experiments [13]. And the relative sizes of these mixings for three lepton flavors directly determine which flavor-specific channel will be the most promising one for the discovery of $N_1$. From the above reconstruction result, the mixing strengths $|\Theta_{\alpha 1}|^2 \equiv |(M_D)_{\alpha 1}|^2/M_1^2$ of $N_1$ with $\nu_\alpha$ are directly obtained as

$$|\Theta_{e1}|^2 = \frac{|a_1|^2}{M_1}, \quad |\Theta_{\mu 1}|^2 = \frac{|a_2|^2}{M_1}, \quad |\Theta_{\tau 1}|^2 = \frac{|a_3|^2}{M_1} = \frac{|(M_\nu)_{e\tau}|}{M_1}. \quad (13)$$

It is interesting that the magnitudes of $|\Theta_{\alpha 1}|^2$ can be completely determined from the low-energy neutrino observables (plus $M_1$). In Fig. 1 we show the maximally and minimally allowed values of $|\Theta_{\alpha 1}|^2$ as functions of the lightest neutrino mass in the NO and IO cases. These results are obtained by taking the best-fit values of the lepton flavor mixing angles and neutrino mass squared differences as typical inputs (same as below), and allowing the CP phases to vary in their whole ranges. And $M_1 = 10$ MeV has been taken as a benchmark value. Given that $|\Theta_{\alpha 1}|^2$ are inversely proportional to $M_1$, the results of the former for other values of the latter can be obtained by rescaling the lines in Fig. 1 proportionally. From Fig. 1 one can make the following observations. We first note that the behaviours of $|\Theta_{\mu 1}|^2$ and $|\Theta_{\tau 1}|^2$ are quite similar to each other. This fact is due to the approximate $\mu$-$\tau$ flavor symmetry in the neutrino sector [14] [15] which will be the subject of section 4. In the NO case, $|\Theta_{\mu 1}|^2$ and $|\Theta_{\tau 1}|^2$ can be vanishingly small for sufficiently large $m_1$, while $|\Theta_{e1}|^2$ can be so only for $2.4 \lesssim m_1/\text{meV} \lesssim 7.1$. The maximally allowed values of $|\Theta_{\mu 1}|^2$ and $|\Theta_{\tau 1}|^2$ are about twice as large as (are approximately equal to) those of $|\Theta_{e1}|^2$ for sufficiently small (large) $m_1$. In the IO case, $|\Theta_{\mu 1}|^2$ and $|\Theta_{\tau 1}|^2$ can be vanishingly small for the whole range of $m_3$, while the contrary is true for $|\Theta_{e1}|^2$. The maximally allowed values of $|\Theta_{\mu 1}|^2$ and $|\Theta_{\tau 1}|^2$ are approximately equal to those of $|\Theta_{e1}|^2$ for the whole range of $m_3$. And the maximally allowed values of $|\Theta_{\alpha 1}|^2$ are much larger than in the NO case when the lightest neutrino mass is sufficiently small.
3 Scenario of $M_D$ being a triangular matrix

In this section, we consider the interesting scenario of $M_D$ being a triangular matrix obtained by taking $c_2 = 0$ in Eq. (3). As discussed in section 2, for this scenario, the model parameters can be completely reconstructed from the low-energy neutrino observables as in Eq. (11). Then, we study the implications of this scenario for leptogenesis.

As we know, the type-I seesaw model also provides an attractive explanation (i.e., the leptogenesis mechanism) for the baryon-antibaryon asymmetry of the Universe \[ Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} = (8.67 \pm 0.15) \times 10^{-11}, \]
with $n_B$ ($n_{\bar{B}}$) being the baryon (antibaryon) number density and $s$ the entropy density. The leptogenesis mechanism works in a way as follows [17, 18]: a lepton-antilepton asymmetry $Y_L \equiv (n_L - n_{\bar{L}})/s$ is first generated from the CP-violating and out-of-equilibrium decays of the right-handed neutrinos and then partially transported to the baryon asymmetry: $Y_B \simeq c Y_L$ with $c = 28/79$. In the unflavored regime which holds for $M_I > 10^{12}$ GeV, the three lepton flavors are indistinguishable. In this regime, the final baryon asymmetry generated from the decays of $N_I$ can be expressed as follows \[ Y_{IB} = -c r \varepsilon_I \kappa (\bar{m}_I). \]
Here $r \approx 3.9 \times 10^{-3}$ measures the ratio of the number density of $N_I$ to the entropy density. $\varepsilon_I$ is the total CP asymmetry for the decays of $N_I$:

\[ \varepsilon_I = \sum_{\alpha} \varepsilon_{I\alpha} = \sum_{\alpha} \frac{\Gamma(N_I \rightarrow L_{\alpha} + H) - \Gamma(N_I \rightarrow \bar{L}_{\alpha} + \bar{H})}{\Gamma(N_I \rightarrow L_{\alpha} + H) + \Gamma(N_I \rightarrow \bar{L}_{\alpha} + \bar{H})} \]

\[ = \frac{1}{8\pi (M_D^\dagger M_D)_{II}} \sum_{j \neq I} \text{Im} \left[ (M_D^\dagger M_D)^2_{IJ} \right] \mathcal{F} \left( \frac{M_J^2}{M_I^2} \right), \]

while the flavored CP asymmetries are given by \[ \varepsilon_{I\alpha} = \frac{1}{8\pi (M_D^\dagger M_D)_{II}} \sum_{j \neq I} \text{Re} \left[ (M_D^\dagger M_D)_{\alpha j} (M_D^\dagger M_D)_{Ij} \right] \mathcal{F} \left( \frac{M_J^2}{M_I^2} \right) \]

\[ + \text{Im} \left[ (M_D^\dagger M_D)_{\alpha j} (M_D^\dagger M_D)_{Ij} \right] \mathcal{G} \left( \frac{M_J^2}{M_I^2} \right), \]

\[ (17) \]

In this section, we consider the interesting scenario of $M_D$ being a triangular matrix obtained by taking $c_2 = 0$ in Eq. (3). As discussed in section 2, for this scenario, the model parameters can be completely reconstructed from the low-energy neutrino observables as in Eq. (11). Then, we study the implications of this scenario for leptogenesis.

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Here $r \approx 3.9 \times 10^{-3}$ measures the ratio of the number density of $N_I$ to the entropy density. $\varepsilon_I$ is the total CP asymmetry for the decays of $N_I$:

\[ \varepsilon_I = \sum_{\alpha} \varepsilon_{I\alpha} = \sum_{\alpha} \frac{\Gamma(N_I \rightarrow L_{\alpha} + H) - \Gamma(N_I \rightarrow \bar{L}_{\alpha} + \bar{H})}{\Gamma(N_I \rightarrow L_{\alpha} + H) + \Gamma(N_I \rightarrow \bar{L}_{\alpha} + \bar{H})} \]

\[ = \frac{1}{8\pi (M_D^\dagger M_D)_{II}} \sum_{j \neq I} \text{Im} \left[ (M_D^\dagger M_D)^2_{IJ} \right] \mathcal{F} \left( \frac{M_J^2}{M_I^2} \right), \]

while the flavored CP asymmetries are given by \[ \varepsilon_{I\alpha} = \frac{1}{8\pi (M_D^\dagger M_D)_{II}} \sum_{j \neq I} \text{Re} \left[ (M_D^\dagger M_D)_{\alpha j} (M_D^\dagger M_D)_{Ij} \right] \mathcal{F} \left( \frac{M_J^2}{M_I^2} \right) \]

\[ + \text{Im} \left[ (M_D^\dagger M_D)_{\alpha j} (M_D^\dagger M_D)_{Ij} \right] \mathcal{G} \left( \frac{M_J^2}{M_I^2} \right), \]
with $\mathcal{F}(x) = \sqrt{x\{(2-x)/(1-x) + (1+x)\ln|x/(1+x)|\}}$ and $\mathcal{G}(x) = 1/(1-x)$. And $\kappa(m_f) < 1$ is the efficiency factor taking account of the effects of the processes (e.g., the inverse decays) that can washout the generated baryon asymmetry. Its value is determined by the so-called washout mass parameter

$$\tilde{m}_f = \sum_{\alpha} \tilde{m}_{f\alpha} = \sum_{\alpha} \frac{|(M^\dagger_D)_{\alpha f}|^2}{M^\dagger_D}.$$  

(18)

A detailed study shows that the dependence of the efficiency factor $\kappa(\tilde{m})$ on a certain washout mass parameter $\tilde{m}$ can be well described by the following analytical fits [19]

$$\frac{1}{\kappa(\tilde{m})} \simeq \frac{3.3 \times 10^{-3} \text{ eV}}{\tilde{m}} + \left(\frac{\tilde{m}}{5.5 \times 10^{-4} \text{ eV}}\right)^{1.16}.$$  

(19)

In the two-flavor regime which holds for $10^{12} > M_f/\text{GeV} > 10^9$, the $\tau$ flavor becomes distinguishable from the other two flavors which remain indistinguishable. In this regime, the final baryon asymmetry generated from the decays of $N_f$ can be expressed as follows [20]

$$Y_{1B} = -cr \left[ \varepsilon_{1\tau} \kappa \left( \frac{390}{589} \tilde{m}_{1\tau} \right) + \varepsilon_{1e} \kappa \left( \frac{417}{589} \tilde{m}_{1e} \right) \right],$$  

(20)

with $\varepsilon_{1\tau} = \varepsilon_{1e} + \varepsilon_{1\mu}$ and $\tilde{m}_{1\tau} = \tilde{m}_{1e} + \tilde{m}_{1\mu}$. In the temperature range below $10^9$ GeV, all the three flavors are distinguishable and should be treated separately. But the requirement for leptogenesis to be viable would place a lower bound $\sim 10^9$ GeV for the right-handed neutrino masses [21], hence we will just consider the unflavored and two-flavor regimes in the following discussions.

As mentioned above, in our framework it is $N_2$ and $N_3$ that will be responsible for leptogenesis. Depending on the relative sizes of $M_{2,3}$ and $10^{12}$ GeV (i.e., the boundary between the unflavored and two-flavor regimes), there are the following three possible leptogenesis scenarios. **Scenario I:** for $M_3 > M_2 > 10^{12}$ GeV, the flavor effects have not come into play. After the $N_3$-leptogenesis phase, the baryon asymmetry $Y_{3B}^0$ produced from the decays of $N_3$ can be calculated according to Eq. (15) with

$$\varepsilon_3 = \frac{M_3}{8\pi v^2 |c_3|^2} \text{Im} (b_3^2 c_3^2) \mathcal{F} \left( \frac{M_2^2}{M_3^2} \right), \quad \tilde{m}_3 = |c_3|^2.$$  

(21)

Due to $M_3 > M_2$, $Y_{3B}^0$ is subject to the washout effects of $N_2$. After the $N_2$-leptogenesis phase, the surviving amount of $Y_{3B}^0$ is given by

$$Y_{3B} = \left[ p_{32} \exp \left( -\frac{3\pi \tilde{m}_2}{8m_*} \right) + 1 - p_{32} \right] Y_{3B}^0,$$  

(22)

with $m_* \simeq 1.1 \times 10^{-3}$ eV and

$$p_{32} = \frac{|(M_D^\dagger_M_{D})_{32}|^2}{(M_D^\dagger_M_{D})_{22}(M_D^\dagger_M_{D})_{33}} = \frac{|b_3|^2}{|b_2|^2 + |b_3|^2}.$$  

(23)

Taking account of the baryon asymmetry $Y_{2B}^0$ produced from the decays of $N_2$, which can be calculated according to Eq. (15) with

$$\varepsilon_2 = \frac{M_3}{8\pi v^2(|b_2|^2 + |b_3|^2)} \text{Im} (b_3^2 c_3^2) \mathcal{F} \left( \frac{M_2^2}{M_3^2} \right), \quad \tilde{m}_2 = |b_2|^2 + |b_3|^2,$$  

(24)

the final baryon asymmetry is given by $Y_B = Y_{2B} + Y_{3B}$. 

8
For the purpose of illustration, we only study the cases where only one of $\delta$, $\rho$ and $\sigma$ acts as the source for CP violation while the other two of them simply take trivial values (i.e., 0 or $\pi$ for $\delta$, 0 or 0.5$\pi$ for $\rho$ and $\sigma$). In Fig. 2, we plot the lower bounds of $M_2$ as functions of the lightest neutrino mass ($m_1$ in the NO case, $m_3$ in the IO case), imposed by requiring the leptogenesis scenario under consideration to be successful. These results are obtained by allowing one of $\delta$, $\rho$ and $\sigma$ to vary in the whole range, for the various trivial-value combinations of the other two of them. Note that when the right-handed neutrinos are heavier than $\sim 10^{14}$ GeV, their contributions to leptogenesis would be exponentially suppressed \[12\], so we have just shown the results for $M_2 < 10^{14}$ GeV. From these results one can make the following observations. In the NO case, when $\delta$ is the only source for CP violation, for $[\rho, \sigma] = [0, 0], [0, 5, 0] \pi$ and $[0.5, 0.5] \pi$, there exist no constraints on $M_2$ for small values of $m_1$ but the lower bounds of $M_2$ increase rapidly for large values of $m_1$. For $[\rho, \sigma] = [0, 0.5] \pi$, there almost exist no constraints on $M_2$ in the whole $m_1$ range. When $\rho$ or $\sigma$ is the only source for CP violation, only in some small ranges of $m_1$ there exist relatively small lower bounds for $M_2$. These results indicate that leptogenesis is more difficult to be successful when $\delta$ is the only source for CP violation. This can be easily understood from that the effects of $\delta$ are always suppressed by $s_{13}$. In the IO case, the results are similar. When $\delta$ is the only source for CP violation, the ranges of $M_2$ for leptogenesis to be successful are very small. But when $\rho$ or $\sigma$ is the only source for CP violation, there almost exist no constraints on $M_2$. Furthermore, to see more clearly the constraints on the CP phases from leptogenesis, for some benchmark values of $M_2$ (between the lower bounds of $M_2$ obtained above and $10^{14}$ GeV), in Figs. 3 and 4 (for the NO and IO cases, respectively) we plot the values of the CP phases versus the lightest neutrino mass for leptogenesis to be successful. We see that when $\delta$ is the only source for CP violation, the results for $[\rho, \sigma] = [0, 0]$ (or $[0, 0.5] \pi$) are similar to those for $[\rho, \sigma] = [0.5, 0.5] \pi$ (or $[0.5, 0] \pi$). When $\rho$ or $\sigma$ is the only source for CP violation, the results for $\delta = 0$ are similar to those for $\delta = \pi$. This is also due to the suppression effect of $s_{13}$ for the effects of $\delta$. Furthermore, in the IO case, the results for the case of $\rho$ being the only source of CP violation are similar to those for the case of $\sigma$ being the only source of CP violation. This is simply because one has $m_1 \approx m_2$ in the IO case.

**Scenario II:** for $M_3 > 10^{12}$ GeV > $M_2$, after the $N_3$-leptogenesis phase, the baryon asymmetry $Y_{3B}^0$ produced from the decays of $N_3$ can be calculated in a same way as in **Scenario I**. After the $N_2$-leptogenesis phase, the surviving amount of $Y_{3B}^0$ is given by

$$Y_{3B} = Y_{3B}^0 \exp \left( \frac{-3\pi m_{2\tau}}{8m_*} \right),$$

(25)

with $m_{2\tau} = |b_3|^2$. And the baryon asymmetry produced from the decays of $N_2$ can be calculated according to Eq. (20) with $\varepsilon_{2\tau} = \varepsilon_2$ and $\varepsilon_{2\gamma} = 0$. For this scenario, we have performed a similar analysis as for the former one. In Fig. 5, we plot the lower bounds of $M_2$ as functions of the lightest neutrino mass, imposed by requiring the leptogenesis scenario under consideration to be successful. From these results one can make the following observations. In the NO case, when $\delta$ is the only source for CP violation, leptogenesis has no (or very small) chance to be successful for $[\rho, \sigma] = [0.5, 0.5] \pi$ (or $[0, 0]$). For $[\rho, \sigma] = [0, 0.5] \pi$ and $[0.5, 0] \pi$, leptogenesis has chance to be successful for $0.001 \lesssim m_1/eV \lesssim 0.1$ and $M_2 \gtrsim 10^{10}$ GeV. When $\rho$ or $\sigma$ is the only source for CP violation, leptogenesis has chance to be successful for $m_1 \gtrsim 0.001$ eV and $M_2 \gtrsim 10^{10}$ GeV. In the IO case, when $\delta$ is the only source for CP violation, there exists no parameter space for leptogenesis to be successful at all. When $\rho$ or $\sigma$ is the only source for CP violation, leptogenesis has chance to be successful for $m_1 \gtrsim 0.001$ eV and $M_2 \gtrsim 10^{11}$ GeV. Similarly, for some benchmark values of $M_2$ (between the lower bounds of $M_2$ obtained above and $10^{12}$ GeV), in Figs. 6 and 7 (for the NO and IO cases, respectively) we plot the
reflection symmetry and study its implications for leptogenesis. In this section, we consider the interesting scenario of \( M_D \) being the triangular matrix, for the various trivial-value combinations of \( [\rho, \sigma] \), the lower bounds of \( M_2 \) as functions of the lightest neutrino mass \( (m_1 \text{ in the NO case}, m_3 \text{ in the IO case}) \), imposed by requiring the **Scenario I** leptogenesis to be successful. These results are obtained by allowing \( \delta \) to vary in the whole range. Second (third) column: same as the first column, except that the roles of \( \delta \) and \( \rho \) (\( \sigma \)) are interchanged.

values of the CP phases versus the lightest neutrino mass for leptogenesis to be successful. One can see that the observations made at the end of last paragraph basically still hold.

**Scenario III:** for \( 10^{12} \text{ GeV} > M_3 > M_2 \), after the \( N_3 \)-leptogenesis phase, the baryon asymmetry \( Y_{3B}^0 \) produced from the decays of \( N_3 \) can be calculated according to Eq. (26) with

\[
\varepsilon_{3\tau} = \varepsilon_3, \quad \varepsilon_{3\gamma} = 0, \quad \bar{m}_{3\gamma} = |c_3|^2, \quad \bar{m}_{3\gamma} = 0.
\]

After the \( N_2 \)-leptogenesis phase, the surviving amount of \( Y_{3B}^0 \) and the baryon asymmetry produced from the decays of \( N_2 \) can be calculated in a same way as in the former scenario. The numerical results for this scenario are very similar to those for the former one, so we will not explicitly show them.

### 4 Scenario of \( M_D \) respecting \( \mu-\tau \) reflection symmetry

In this section, we consider the interesting scenario of \( M_D \) in Eq. (3) also respecting the so-called \( \mu-\tau \) reflection symmetry and study its implications for leptogenesis.

Due to the closeness of the lepton flavor mixing angles to some special values (e.g., \( \sin^2 \theta_{23} \simeq 1/2 \) and \( \sin^2 \theta_{12} \simeq 1/3 \)), it is believed by many people that there may exist some flavor symmetry in the lepton sector. And many flavor symmetry models have been proposed to explain the particular values of the lepton flavor mixing parameters [22]. One popular and attractive candidate of them is just the \( \mu-\tau \) reflection symmetry [23]. This symmetry is defined in a way as follows: the neutrino mass matrix should keep invariant with respect to the following transformations of three left-handed neutrino fields

\[
\nu_e \leftrightarrow \nu_e^c, \quad \nu_\mu \leftrightarrow \nu_\mu^c, \quad \nu_\tau \leftrightarrow \nu_\tau^c,
\]
where the superscript $c$ denotes the charge conjugation of relevant neutrino fields. Under this symmetry, the elements of $M_\nu$ satisfy the following relations

$$(M_\nu)_{e\mu} = (M_\nu)^*_{e\tau}, \quad (M_\nu)_{\mu\mu} = (M_\nu)^*_{\tau\tau}, \quad (M_\nu)_{ee} = (M_\nu)^*_{ee}, \quad (M_\nu)_{\mu\tau} = (M_\nu)^*_{\mu\tau},$$

which lead to the following interesting predictions for the lepton flavor mixing parameters

$$\theta_{23} = \frac{\pi}{4}, \quad \delta = \pm \frac{\pi}{2}, \quad \rho, \sigma = 0 \text{ or } \frac{\pi}{2}. \quad (29)$$

Taking account of the results in Eq. (29), the expressions in Eq. (9) become

$$(M_\nu)_{ee} = m_1 \eta_\rho c_1^2 c_{13}^2 + m_2 \eta_\sigma s_1^2 c_{13}^2 - m_3 s_{13}^2,$$

$$(M_\nu)_{e\mu} = -\frac{1}{\sqrt{2}} m_1 \eta_\rho c_1^2 c_{13} (s_{12} \pm i c_{12}s_{13}) + \frac{1}{\sqrt{2}} m_2 \eta_\sigma s_1^2 c_{13} (c_{12} \mp i s_{12}s_{13}) \mp \frac{1}{\sqrt{2}} i m_3 c_{13} s_{13},$$

$$(M_\nu)_{\mu\mu} = \frac{1}{2} m_1 \eta_\rho (s_{12} \pm i c_{12}s_{13})^2 + \frac{1}{2} m_2 \eta_\sigma (c_{12} \mp i s_{12}s_{13})^2 \mp \frac{1}{2} m_3 c_{13}^2,$$

$$(M_\nu)_{\mu\tau} = -\frac{1}{2} m_1 \eta_\rho (s_{12}^2 + c_{12}^2 s_{13}^2) - \frac{1}{2} m_2 \eta_\sigma (c_{12}^2 + s_{12}^2 s_{13}^2) + \frac{1}{2} m_3 c_{13}^2,$$

$$(M_\nu)_{e\tau} = -(M_\nu)^*_{e\mu}, \quad (M_\nu)^*_{\tau\tau} = (M_\nu)^*_{e\tau},$$

where $\eta_\rho = 1$ or $-1$ for $\rho = 0$ or $\pi/2$ (and similarly for $\eta_\sigma$), and the upper and lower signs of $\pm$ and $\mp$ respectively correspond to $\delta = \pi/2$ and $-\pi/2$. Note that the sign difference for $(M_\nu)_{e\mu}/(M_\nu)_{e\tau}$ in Eqs. (28, 30) is due to the unphysical phases.
Figure 4: Same as Fig. 3 except that the results are for the IO case.

Under the \( \mu-\tau \) reflection symmetry, \( M_\nu \) in Eq. (32) can be reexpressed in the following form

\[
M_D = \begin{pmatrix}
\sqrt{\eta_1} a_1 \sqrt{M_1} & 0 & 0 \\
\sqrt{\eta_1} a_2 \sqrt{M_1} & \sqrt{\eta_2} b_2 \sqrt{M_2} & \sqrt{\eta_3} c_2 \sqrt{M_3} \\
\sqrt{\eta_1} a_2^* \sqrt{M_1} & \sqrt{\eta_2} b_2^* \sqrt{M_2} & \sqrt{\eta_3} c_2^* \sqrt{M_3}
\end{pmatrix},
\]

with \( a_1 \) being real now and \( \eta_I = \pm 1 \). It is direct to verify that the resulting \( M_\nu \) from such an \( M_D \)

\[
M_\nu \simeq - \begin{pmatrix}
\eta_1 a_1^2 & \eta_1 a_1 a_2 & \eta_1 a_1 a_2^* \\
\eta_1 a_1 a_2 & \eta_1 a_2^2 + \eta_2 b_2^2 + \eta_3 c_2^2 & \eta_1 |a_2|^2 + \eta_2 |b_2|^2 + \eta_3 |c_2|^2 \\
\eta_1 a_1 a_2^* & \eta_1 |a_2|^2 + \eta_2 |b_2|^2 + \eta_3 |c_2|^2 & \eta_1 a_2^* + \eta_2 b_2^* + \eta_3 c_2^* 
\end{pmatrix},
\]

does obey the relations in Eq. (28). As before, one can reconstruct the model parameters \( a_i, b_i \) and \( c_i \)
in Eq. (31) in terms of the low-energy neutrino observables by making a direct comparison between \( M_\nu \) in Eq. (32) and that described by Eq. (30). Note that there are totally 7 degrees of freedom among the model parameters but \( M_\nu \) only contains 6 degrees of freedom. Hence the model parameters can not be completely reconstructed in terms of the low-energy neutrino observables and we will be left with one degree of freedom. Taking account of the effect of the unphysical phases as mentioned below.
Figure 5: Same as Fig. 2 except that the results are for the Scenario II leptogenesis.

Eq. (30), we obtain the following reconstruction result

\[ \sqrt{\eta_1} a_1 = i \eta_{a_1} \sqrt{(M_e)_{ee}}, \quad \sqrt{\eta_1} a_2 = i \eta_{a_1} \frac{(M_\nu)_{e\mu}}{(M_\nu)_{ee}} \]

\[ \sqrt{\eta_2} b_2 = \eta_{b_2} \sqrt{\frac{(M_\nu)_{e\mu} - (M_\nu)_{ee}(M_\nu)_{e\mu}}{(M_\nu)_{ee}}} \cos z, \]

\[ \sqrt{\eta_3} c_2 = \eta_{b_2} \sqrt{\frac{(M_\nu)_{e\mu} - (M_\nu)_{ee}(M_\nu)_{e\mu}}{(M_\nu)_{ee}}} \sin z, \]

\[ \eta_2 |\cos^2 z| + \eta_3 |\sin^2 z| = \left| \frac{(M_\nu)_{ee}}{(M_\nu)_{e\mu} - (M_\nu)_{ee}(M_\nu)_{e\mu}} \left[ (M_\nu)_{e\mu} - \eta_1 \frac{(M_\nu)_{e\mu}}{(M_\nu)_{ee}} \right] \right|. \quad (33) \]

If \( z \) is parameterized to be \( x + iy \) with \( x \) and \( y \) being real parameters, then one has

\[ \eta_2 |\cos^2 x| + \eta_3 |\sin^2 x| = \pm \frac{1}{2} (e^{2y} + e^{-2y}) \quad \text{or} \quad \pm \cos 2x, \quad (34) \]

for \( \eta_2 = \eta_3 = \pm 1 \) or \( \eta_2 = -\eta_3 = \pm 1 \). These results show that we can always determine one of \( x \) and \( y \) from the low-energy neutrino observables, while the other one of them acts as the remaining degree of freedom.

Then, let us study the implications of the present scenario for leptogenesis. Substituting \( M_D \) in Eq. (31) into the expressions of \( \varepsilon_1 \) and \( \varepsilon_{1a} \) in Eqs. (16, 17), we arrive at \( \varepsilon_1 = \varepsilon_{1e} = 0 \) and

\[ \varepsilon_{2\mu} = -\varepsilon_{2\tau} = \frac{M_3}{8\pi v^2 b_2 c_2} \Re(b_2^* c_2) \Im(b_2 c_2) \left[ \eta_2 \eta_3 \mathcal{F} \left( \frac{M_3^2}{M_2^2} \right) + \mathcal{G} \left( \frac{M_3^2}{M_2^2} \right) \right], \]

\[ \varepsilon_{3\mu} = -\varepsilon_{3\tau} = \frac{M_2}{8\pi v^2 c_2^2} \Re(b_2 c_2^* b_2^* c_2) \left[ \eta_2 \eta_3 \mathcal{F} \left( \frac{M_2^2}{M_3^2} \right) + \mathcal{G} \left( \frac{M_2^2}{M_3^2} \right) \right]. \quad (35) \]

As a result of \( \varepsilon_1 = 0 \), in the unflavored regime, the baryon asymmetry produced from the decays of \( N_2 \) and \( N_3 \) is vanishing, making leptogenesis impossible. On the other hand, in the two-flavor regime,
Figure 6: Same as Fig. 3 except that the results are for the Scenario II leptogenesis.

the baryon asymmetry produced from the decays of $N_I$ (for $I = 2, 3$) is given by

$$Y_{IB} = -c_r \varepsilon_I \mu \left[ \kappa \left( \frac{417}{589} \tilde{m}_{I\mu} \right) - \kappa \left( \frac{390}{589} \tilde{m}_{I\mu} \right) \right],$$

which is obtained from Eq. (36) by taking account of $\varepsilon_{Ie} = \tilde{m}_{Ie} = 0$, $\varepsilon_{I\mu} = -\varepsilon_{I\tau}$ and $\tilde{m}_{I\mu} = \tilde{m}_{I\tau}$. Thanks to the difference between the coefficients $417/589$ and $390/589$, a successful leptogenesis becomes possible.

Depending on the relative sizes of $M_{2,3}$ and $10^{12}$ GeV, there are the following two possible leptogenesis scenarios. Scenario I: for $M_3 > 10^{12}$ GeV > $M_2$, the baryon asymmetry produced from the decays of $N_3$ is vanishing, while that produced from the decays of $N_2$ can be calculated according to Eq. (36) with $\varepsilon_{2\mu}$ as in Eq. (35) and $\tilde{m}_{2\mu} = |b_2|^2$. In Fig. 8 for the various combinations of $(\eta_1, \eta_2, \eta_3)$ and $[\rho, \sigma]$ that can accommodate a successful leptogenesis, we plot the lower bounds of $M_2$ as functions of $m_1$ in the NO case. These results are obtained by allowing the unconstrained one of $x$ and $y$ to vary in the whole range. Note that these results are same for $\delta = \pi/2$ and $-\pi/2$. We see that the requirement of leptogenesis being successful can help us exclude many possible combinations of $(\eta_1, \eta_2, \eta_3)$ and $[\rho, \sigma]$. For the viable combinations of them, in some cases $M_2$ needs to be larger than $10^{11}$ GeV or even close to $10^{12}$ GeV in order for leptogenesis to be successful, while in some other cases leptogenesis has chance to be successful for $M_2 \sim 10^{10}$ GeV and certain values of $m_1$. In Fig. 9 we give the results for the IO case. As in the NO case, only for certain combinations of $(\eta_1, \eta_2, \eta_3)$ and $[\rho, \sigma]$ can leptogenesis have chance to be successful. For these cases, $M_2$ needs to be larger than $10^{11}$ GeV in order for leptogenesis to be successful.

Scenario II: for $10^{12}$ GeV > $M_3 > M_2$, after the $N_3$-leptogenesis phase, the baryon asymmetry $Y_{3B}^0$ produced from the decays of $N_3$ can also be calculated according to Eq. (36) with $\varepsilon_{3\mu}$ as in Eq. (35) and $\tilde{m}_{3\mu} = |c_2|^2$. After the $N_2$-leptogenesis phase, the surviving amount of $Y_{3B}^0$ can be calculated according to Eq. (25) with $\tilde{m}_{2\tau} = |b_2|^2$. Taking account of the baryon asymmetry $Y_{2B}^0$
produced from the decays of $N_2$, which can be calculated can be calculated in a same way as in the former scenario, the final baryon asymmetry is simply given by $Y_B = Y_{2B} + Y_{3B}$. Given that the contribution of $N_3$ to leptogenesis suffers the washout effects of $N_2$, the numerical results for this scenario are similar to those for the former one, so we will not explicitly show them.

5 Summary

In summary, in this paper we have performed a further study of an interesting possibility proposed by Asaka, Ishida and Tanaka: the $0\nu\beta\beta$ decay can be hidden in the minimal seesaw model with two right-handed neutrinos which have a hierarchical mass structure. The heavier right-handed neutrino is sufficiently heavy to decouple from the $0\nu\beta\beta$ decay while the lighter one is lighter enough than the typical Fermi-momentum scale of nuclei. Under some specific condition on the neutrino Yukawa couplings, the $0\nu\beta\beta$ decay can be hidden. Our framework is same as theirs except that there are two heavier right-handed neutrinos so that the baryon-antibaryon asymmetry of the Universe can be explained via the leptogenesis mechanism.

We have first given the condition on the neutrino Yukawa couplings [i.e., $(Y_\nu)_{e2} = (Y_\nu)_{e3} = 0$] for hiding the $0\nu\beta\beta$ decay [see Eq. [3]], discussed its realization by employing an Abelian flavor symmetry, and studied its implications for the mixing strengths $|\Theta_{\alpha 1}|$ of the lighter right-handed neutrino with three left-handed neutrinos. It is interesting that $|\Theta_{\alpha 1}|$ can be completely determined from the low-energy neutrino observables (plus $M_1$). We have then successively studied the implications for leptogenesis of the interesting scenarios where $M_\Delta$ is a triangular matrix or respects the $\mu-\tau$ reflection symmetry.

In the scenario of $M_\Delta$ being a triangular matrix, the model parameters can be completely reconstructed from the low-energy neutrino observables [see Eq. [11]]. In this case, the low-energy neutrino observables will be subject to the constraints from leptogenesis. For the purpose of illustration, we have only studied the cases where only one of $\delta$, $\rho$ and $\sigma$ acts as the source for CP violation while the other two of them simply take trivial values. For all the possible cases we have given the lower...
Figure 8: In the scenario of $M_D$ respecting the $\mu-\tau$ reflection symmetry, for the various combinations of $(\eta_1, \eta_2, \eta_3)$ and $[\rho, \sigma]$, the lower bounds of $M_2$ as functions of $m_1$ in the NO case, imposed by requiring the **Scenario I** leptogenesis to be successful. These results are same for $\delta = \pi/2$ and $-\pi/2$.

bounds of $M_2$ as functions of the lightest neutrino mass that are obtained by requiring leptogenesis to be successful. It is found that leptogenesis is more difficult to be successful when $\delta$ is the only source for CP violation. In particular, in the IO case, when $\delta$ is the only source for CP violation, the ranges of $M_2$ for leptogenesis to be successful are very small. This can be easily understood from that the effects of $\delta$ are always suppressed by $s_{13}$. Furthermore, to see more clearly the constraints on the CP phases from leptogenesis, for some benchmark values of $M_2$ we have plotted the values of the CP phases versus the lightest neutrino mass for leptogenesis to be successful.

In the scenario of $M_D$ respecting the $\mu-\tau$ reflection symmetry [see Eq. (31)], the lepton flavor mixing parameters are predicted to take some special values as given in Eq. (29), and the model parameters can be reconstructed from the low-energy neutrino observables in combination with the unconstrained one of $x$ and $y$ [see Eqs. (33) and (34)]. Because of the $\mu-\tau$ reflection symmetry, only in the two-flavor regime can leptogenesis have chance to be successful. For the various combinations of $(\eta_1, \eta_2, \eta_3)$ and $[\rho, \sigma]$, we have given the lower bounds of $M_2$ as functions of the lightest neutrino mass that are obtained by requiring leptogenesis to be successful.

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**References**

[1] Z. Z. Xing, Phys. Rep. **854**, 1 (2020).
Figure 9: Same as Fig. 8 except these results are for the IO case.

[2] P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, edited by P. van Nieuwenhuizen and D. Freedman, (North-Holland, 1979), p. 315; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979), p. 95; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980).

[3] P. F. de Salas, D. V. Forero, S. Gariazzo, P. Martinez-Mirave, O. Mena, M. Tortola and J. W. F. Valle, JHEP 02, 071 (2021).

[4] F. Capozzi, E. Lisi, A. Marrone and A. Palazzo, Prog. Part. Nucl. Phys. 102, 48 (2018); I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou, JHEP 09, 178 (2020).

[5] For some reviews, see W. Rodejohann, Int. J. Mod. Phys. E 20, 1833 (2011); S. M. Bilenky and C. Giunti, Int. J. Mod. Phys. A 30, 0001 (2015); S. Dell’Oro, S. Marcocci, M. Viel and F. Vissani, Adv. High Energy Phys. 2016, 2162659 (2016); J. D. Vergados, H. Ejiri and F. Simkovic, Int. J. Mod. Phys. E 25, 1630007 (2016).

[6] M. Blennow, E. Fernandez-Martinez, J. Lopez-Pavon and J. Menendez, JHEP 07, 096 (2010).

[7] T. Asaka, H. Ishida and K. Tanaka, Phys. Rev. D 103, 015014 (2021).

[8] See also T. Asaka, H. Ishida and K. Tanaka, PTEP 2021, 063B01 (2021); arXiv:2101.12498; D. L. Fang, Y. F. Li and Y. Y. Zhang, arXiv:2112.12779.

[9] W. Grimus, A. S. Joshipura, L. Lavoura and M. Tanimoto, Eur. Phys. J. C 36, 227 (2004).

[10] C. D. Froggatt, H. B. Nielsen, Nucl. Phys. B 147 (1979) 277.

[11] A. Kusenko, F. Takahashi and T. T. Yanagida, Phys. Lett. B 693, 144 (2010).
[12] J. Barry, W. Rodejohann and H. Zhang, JCAP 01, 052 (2012).
[13] M. Drewes and B. Garbrecht, Nucl. Phys. B 921, 250 (2017).
[14] T. Fukuyama and H. Nishiura, arXiv:hep-ph/9702253; E. Ma and M. Raidal, Phys. Rev. Lett. 87, 011802 (2001); C. S. Lam, Phys. Lett. B 507, 214 (2001); K. R. S. Balaji, W. Grimus and T. Schwetz, Phys. Lett. B 508, 301 (2001).
[15] For a review, see Z. Z. Xing and Z. H. Zhao, Rept. Prog. Phys. 79, 076201 (2016).
[16] P. A. R. Ade et al. (Planck Collaboration), Astron. Astrophys. A 16, 571 (2014).
[17] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).
[18] For some reviews, see W. Buchmuller, R. D. Peccei and T. Yanagida, Ann. Rev. Nucl. Part. Sci. 55, 311 (2005); W. Buchmuller, P. Di Bari and M. Plumacher, Annals Phys. 315, 305 (2005); S. Davidson, E. Nardi and Y. Nir, Phys. Rept. 466, 105 (2008).
[19] G. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685, 89 (2004).
[20] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP 0604, 004 (2006); E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP 0601, 164 (2006).
[21] S. Davidson and A. Ibarra, Phys. Lett. B 535, 25 (2002).
[22] S. F. King and C. Luhn, Rept. Prog. Phys. 76, 056201 (2013); F. Feruglio and A. Romanino, Rev. Mod. Phys. 93, 015007 (2021).
[23] P. H. Harrison and W. G. Scott, Phys. Lett. B 547, 219 (2002).