Outline

1 The beginning
2 LSZ – 1rst highlight
3 Intermediate years
4 Renormalization theory – 2nd highlight
   - Finite diagrams, equations of motion, symmetries
   - Operator product expansion
5 Reduction of couplings – 3rd highlight
6 The man behind the scientist
7 Summary
childhood, study

born February 17, 1928 in Freiburg im Breisgau (Germany)
father: medical doctor
older sister: theater, “Giganisch”
1946: entering university in Freiburg, study of mathematics & physics.
lectures/seminars:
“Either they were too fast or too slow for me. Either I had to think about
the new content – then I was too slow. Or I understood it instantly, then
the lecture was boring.”
measure: in 1950 doctoral degree in mathematics
thesis devoted to topology
earlier dissertation: but abandoned, because he found out that the
main result could be proven in a much simpler way, hence considered
this work as inadequate for a doctoral degree
a further article on topology (1952)
papers written in style and spirit of BOURBAKI
his comment: “I can read BOURBAKI like the newspaper.”
general remark

1952 WZ: research associate (group of Werner Heisenberg) Max-Planck-Institut f. Physik in Göttingen
first physics paper (1952): on thermodynamics of a Fermi gas
first QFT paper (1953): on the bound state problem in field theory with co-author Vladimir Glaser
entry ticket to “der Feldverein”
LSZ papers

truly famous: three papers (1955, 1955, 1957) with Harry Lehmann and Kurt Symanzik the “LSZ formalism” of quantum field theory

principles: Lorentz covariance, unitarity, causality realized on Green functions and S-matrix first axiomatic formulation of quantum field theory

conversely: Lehmann, Glaser and Zimmermann (1957) sufficient conditions on functions → a field theory

LSZ does not refer to perturbative expansions however: greatly successful in perturbative realization extremely powerful in practice until the present day the most efficient description of scattering amplitudes in particle physics.
asymptotic condition, reduction formula

key idea: in remote past and future scattering experiment deals with free particles
interaction only in a finite region of spacetime respective fields related by asymptotic condition:

\[
\phi(x) \xrightarrow{x^0 \to \pm \infty} \sqrt{z} \phi_{out}^{in}(x),
\]

\(z\) a number, \(\phi_{out}^{in}\) free fields

\[
\left(\Box + m^2\right) \phi_{out}^{in}(x) = 0,
\]

\(\phi(x)\) is an interacting field
limit: in the weak sense, i.e. it is valid only for matrix elements
Scattering experiment: $n_i$ particles in initial state into $n_f$ particles in final state. Transition by $S$-operator, matrix elements $S_{fi}$: LSZ-reduction formula

$$S_{fi} = \langle f | i \rangle = \langle p_1 \ldots p_{n_f} | q_1 \ldots q_{n_i} \rangle$$

$$= \left( \frac{-1}{\sqrt{z}} \right)^{n_f+n_i} \lim_{k,j} \prod_{k,j} (p_k^2 - m^2)(q_j^2 - m^2) \tilde{G}(-p_1, \ldots, -p_n, q_1, \ldots, q_l)$$

(with $\lim: p_k^2 \to m^2, q_j^2 \to m^2, p_k^0 > 0, q_k^0 > 0$)

Here $\tilde{G}$ denotes the FT of the Green functions

$$G(y_1, \ldots, y_{n_f}, x_1, \ldots, x_{n_i}) = \langle T \phi(y_1) \ldots \phi(x_{n_i}) \rangle,$$

vacuum expectation value of time ordered product of field operators determined by equations of motion
Historical remark:
Another axiomatic formulation of QFT has been initiated by Wightman (1956).
The relation of the LSZ-scattering theory to those axioms and clarification of the role of fundamental fields have been given by Haag (1958, 1959) and in particular by Ruelle (1962).
perturbative treatment

for Green functions, S-matrix: Feynman diagrams
for every elementary interaction: vertex
for particles propagating in spacetime: line

scattering process: vertices linked by lines
mathematical prescription for vertices, lines: “Feynman rules”
ordering of diagrams: by numbers of vertices
perturbation series: power series of coupling constants
consistent algorithm required
S-matrix: *Dyson*

\[ S = \langle Te^i \int \mathcal{L}_{\text{int}} \rangle \]

Green functions: *Gell’man-Low*

\[ G(x_1, ..., x_n) = \langle T(\phi(x_1)...\phi(x_n)) \rangle \]

\[ = \frac{\langle T(\phi^{(0)}(x_1)...\phi^{(0)}(x_n)e^i \int \mathcal{L}^{(0)}_{\text{int}}) \rangle}{\langle e^i \int \mathcal{L}^{(0)}_{\text{int}} \rangle} \]

evaluation: Wick’s theorem with \( \langle T(\phi(x_1)^{(0)}\phi(x_2)^{(0)}) = \Delta_c(x_1 - x_2) \)

fundamental axioms:
Lorentz covariance, unitarity, causality: satisfied
observe: propagator

\[ \tilde{\Delta}_c(p) = \frac{i}{p^2 - m^2 + i\varepsilon} \]

distribution, not a function

calculate: \( \Delta_c(x - y)\Delta_c(x - y) \)

find: infinite! meaningless!

many diagrams with closed loops not well-defined

problem: give mathematical meaning to such expressions

do not violate the fundamental axioms

Schwinger, Dyson

Bogoliubov & Parasiuk, Hepp (BPH) first satisfactory solution

Wolfhart Zimmermann (BPHZ), (s.b.)
various problems

1957 WZ leaves Göttingen
positions in: Instit. for Advanced Study in Princeton, Univ. of Hamburg
visitor at: Physics Dep. of UCB (Berkeley), CERN, Univ. of Vienna
problems studied: bound states, one-particle singularities of Green’s
functions, analyticity structure of scattering amplitudes

1962 appointed professor of physics at New York University
visitor at: Enrico Fermi Institute (Chicago)
IHES (Bures-sur-Yvette, France)
noteworthy: contribution to “relativistic” $SU(6)$-symmetry
(in hindsight: prepares the way to supersymmetry, anticommutators →
Jordan algebras (Hironari Miyazawa, (1967))
general remark

next absolute landmark work: renormalization theory
Bogoliubov & Parasiuk, Hepp (BPH): finite diagrams via recursive prescription
WZ: first step explicit solution of recursion – “forest formula”
    second step: subtractions in momentum space
    ∴ integrals absolutely convergent (BPH: conditional convergence)
“BPHZ renormalization scheme” (1968,1969)
→ S-matrix elements
→ Green functions involving arbitrary composite operators
→ equations of motions, currents, symmetries
→ precise notion of anomalies
    → link to mathematics
    → truely QFT effects
pivotal tool: “Zimmermann identities” between different normal products
(meaning even beyond perturbation theory)
finite diagrams

propagator decreases in $p$-space for large $p$ like $1/p^2$; implication

\[
\sim \lambda^2 \int d^4k \frac{1}{(p-k)^2 - m^2 + i\varepsilon} \frac{1}{k^2 - m^2 + i\varepsilon}
\]

integrand $\sim (k)^0$ \hspace{1cm} integral $\simeq (\ln(\frac{\Lambda}{m}))$

vertex correction: logarithmically divergent integral
subtract first Taylor term at $p = 0$
introduce Zimmermann’s $\varepsilon_Z = \varepsilon(m^2 + p^2)$
integral is absolutely convergent
limit $\varepsilon \to 0$: integral Lorentz covariant function.
no series problem for *non-overlapping* diagrams like

Here one can remove the divergences by subsequently removing in an analogous way first those of the subdiagrams and thereafter that of the entire diagram. The result does in particular not depend upon in which order the subdiagrams have been subtracted.
However, in diagrams like

\[ \gamma \]

removal of divergences in a subdiagram $\lambda$ interferes with those of the others and the removal of the overall divergence (i.e. of $\gamma$): the diagram $\gamma$ contains “overlapping divergences”
WZ: “forest formula”
- explicit solution of the recursion problem involved
- deals properly with the overlaps

\[ R_\Gamma(p, k) = \sum_{U \in \mathcal{F}_\Gamma} S_\Gamma \prod_{\gamma \in U} (-t_{p,\gamma}^{d(\gamma)} S_\gamma) I_\Gamma(U) \]

sum: over all families of non-overlapping diagrams (“forests”) in \( \Gamma \)

\( t \): Taylor subtractions at \( p = 0 \)
\( S \): relabels the momentum variables appropriately.

forest formula: \( \int (I_\Gamma - \cdots \text{subtractions}) \)

theorem: the integral over the internal momenta of the closed loops is absolutely convergent and yields in the limit \( \varepsilon \to 0 \) a Lorentz covariant vertex function or (for general Green functions) a Lorentz covariant distribution.
normal products, Zimmermann identity

“obvious” extension: standard vertices $\rightarrow$ composite operator via Green functions with composite operator as a special vertex and use of the respective reduction formula

$$
\langle T (Q(x)\varphi(y_1)...\varphi(y_k)) \rangle = \left\langle T \left( N_d [Q^{(0)}(x)] \varphi^{(0)}(y_1)...\varphi^{(0)}(y_k) e^{i \int L^{(0)}_{\text{int}}} \right) \right\rangle^{(0)}
$$

$d$: naive dimension of $Q$.
find: $\delta = d + c$, $c \in \mathbb{N}$ possible
result: Zimmermann identity

$$
N_\delta [Q] \cdot \Gamma = N_\varphi [Q] \cdot \Gamma + \sum_i u^{(Q)}_i N_\varphi [Q_i] \cdot \Gamma
$$

with $\varphi > \delta \geq \text{dim}(Q)$
harbours all fundamental deviations of quantum field theory from classical field theory
action principle, equation of motion

define functional differential operators which represent field transformations $\delta^X$
on $\Gamma$

$$W^X\Gamma \equiv i \int d^4 x \, \delta^X \varphi(x) \frac{\delta}{\delta \varphi(x)} \Gamma$$

(1)

for a massive scalar field with

$$\Gamma_{\text{eff}} = \int d^4 x \left( \frac{1}{2} (\partial \varphi \partial \varphi - m^2 \varphi^2) - \frac{\lambda}{4!} \varphi^4 \right) + \Gamma_{\text{counter}}$$

(2)

the action principle reads

$$\delta^X \varphi(x) \frac{\delta}{\delta \varphi(x)} \Gamma = \left[ \delta^X \varphi(x) \frac{\delta}{\delta \varphi(x)} \Gamma_{\text{eff}} \right] \cdot \Gamma \equiv \left[ Q^X(x) \right] \cdot \Gamma$$

(3)

(non-integrated transformation)
replace $\delta^X \varphi$ by 1:
$\rightarrow$ well-defined operator field equation via LSZ-reduction
symmetries, anomalies

suppose: variations $\delta^X$ satisfy an algebra

$$[W^X, W^Y] = iW^Z,$$  \hspace{1cm} (4)

implies algebraic restrictions on the insertions $Q^X$

if $Q^X(x) = \text{variation}$

modify $\Gamma_{\text{eff}}$: symmetry can be implemented

if not: anomaly

Note: method is \textit{constructive}; insertion $Q^X(x)$ in action principle is
determined uniquely, can be characterized by covariance and power counting;
extremely powerful tool
operator product expansion

arrive at normal products by merging external lines
isolate singularities, capture them as coefficients of operators
find: the operator product expansion (as introduced by K. Wilson)
provides existence proof for OPE in perturbation theory (1973)

study limit $\xi \to 0$ for $x = (x_1 + x_2)/2$ and $\xi = (x_1 - x_2)/2$. 
for a bilinear product of a scalar field $A$

$$\mathcal{T} A(x + \xi) A(x - \xi) = E_0(\xi) 1 + E_1(\xi) A(x) - i E_2^\mu(\xi) \partial_\mu A(x) + \frac{1}{2} E_3(\xi) N[A(x)^2] + R(x, \xi)$$

directional dependence of composite operators understood (1971)

$\rightarrow$ lightlike and spacelike operator product expansions

application:
strong sector (QCD) of standard model of particle physics
deep inelastic scattering of $\nu$’s and $e$’s off hadrons
composite structure of hadrons confirmed
general remark

• 1974 WZ scientific member of the Max-Planck Society
director at MPP, Munich, Germany
• 1977 honorary professor at Technical University of Munich
• visitor at:
Centre de Investigación y de Estudios Avanzados del IPN, Mexico City, Mexico
Purdue University West Lafayette, IN, USA.
• prime subject of his group: formulation of gauge and supersymmetric
  models to all orders, possible only with BPHZ
• WZ & Reinhard Oehme study asymptotically free theories like
  QCD and analyze the Renormalization Group in models with several
effective couplings (1984)
generalization of symmetry

starting point: a perturbatively renormalizable model has a primary coupling $g$ and $n$ secondary couplings $\lambda_i, i = 1, \ldots, n$. Effective couplings satisfy the renormalization group equations

$$
\frac{d}{dt} \bar{g}(t) = \beta_g(\bar{g}, \bar{\lambda}_i) \quad \frac{d}{dt} \bar{\lambda}_i(t) = \beta_{\lambda_i}(\bar{g}, \bar{\lambda}_i) \quad (5)
$$

eliminate scale parameter $t$, find

$$
\beta_g(\bar{g}, \bar{\lambda}(\bar{g})) \frac{d}{d\bar{g}} \bar{\lambda}_i(\bar{g}) = \beta_{\bar{\lambda}_i}(\bar{g}, \bar{\lambda}(\bar{g})). \quad (6)
$$

ode’s, singular at vanishing couplings, case by case study
power series solutions $\rightarrow$ initial value condition, no free parameter
general solution: $n$ free parameters, say, they replace $\lambda_i$
possible symmetries: solutions in the reduced model
simple examples

Simple examples (1984):
(1) massless theory, pseudo-scalar field $B$, spinor field $\psi$, interaction

$$ig\bar{\psi}\gamma_5 B\psi - \frac{\lambda}{4!} B^4$$

for $\lambda$ positive, $g$ sufficiently small

$$\exists_1 \text{ power series } \lambda = \frac{1}{3}(1 + \sqrt{145})g^2 + o(g^4)$$

embedded into general solution with $d_{11}g^{\frac{2}{5}\sqrt{145+2}} + \text{higher orders}$

$d_{11}$ arbitrary.

(2) massless Wess-Zumino model, couplings $g$, $\lambda$, interaction

$$g\bar{\psi}(A + i\gamma_5 B)\psi - \frac{\lambda}{2}(A^2 + B^2)^2$$

supersymmetric solution $\lambda = g^2$ embedded into a non-supersymmetric general solution $\lambda = g^2 + \rho_3g^8 + \sum \rho_j g^{2j+2}$, $\rho_3$ arbitrary

third solution $\lambda = -\frac{4}{5}g^2 + \sum \rho_j g^{2j+2}$ not related to supersymmetry
further notable examples

- non-supersymmetric embeddings of models which can have $N = 2, 4$ supersymmetry

- non-abelian gauge symmetry as unique solution, if embedding theory has rigid invariance

- “finite” models exist
  - finite: $\beta$-functions vanish to all orders
  - superconformal symmetry is realized as in the classical theory (mainly of theoretical interest)
phenomenological implications

reduction in the standard model?
problem:
coupling of abelian subgroup asymptotically free in the infrared

... of non-abelian subgroups asymptotically free in the ultraviolet
→ generalize reduction principle
find: bounds on Higgs and top mass (1991)
including two-loop corrections

\[ m_t = 89.6 \pm 9.2 \text{ GEV} , \quad m_h = 64.5 \pm 1.5 \text{ GEV} \]

values already overruled by precision experiments
model must be extended
within supersymmetric extensions of the standard model (2008):

\[ m_h = 121 \ldots 126 \text{ GeV} \text{ (uncertainty: 3 GeV)} \]

Wolfhart Zimmermann was pleased by this
1991 WZ was awarded the Max-Planck-Medal
1996 retirement; WZ kept ties to the institute until his end
up to now: the scientist and his work
the man:

enjoyed eating and drinking well
loved having company for dinner in his house
his wife graceful & competent host
generous towards members & guests of the institute
cared very much about his three daughters
loved music, theater, the flowers in his terrace garden

Let’s have another look at the person via anecdotes.
Why did WZ never do refereeing work for journals?

answer (comes close to comment at lectures & seminars):

“If the problem addressed in the paper is interesting I am attracted to solve it myself. If I don’t find it interesting I can not press myself to read it further and just do nothing but criticising. In any case it distracts me too long from my own work.”
He simply hated committee meetings, in particular those of the directorate of the institute. There was just too much of trouble and strife and bad behaviour for him. In the breaks of directorate meetings he used to come to my office to discuss physics as a kind of recreation. At some time there was a “chance” that he had to become executive director (Geschäftsführender Direktor). His comment: “Ach wissen Sie, ich habe einen Zettel in meiner Jackentasche. Darauf steht: mir kann ja nichts passieren.” Indeed, nothing happened to him; a colleague of his was very eager to get this job.
WZ as “boss”?
two remarks

first: he quoted a well-known mathematician: “Mr. X at university Y said once in public: ‘Ich bin ein Bonze und möchte als solcher behandelt werden.’ I would never say this.”

second: he himself filled in and kept the list of vacation days for the members of the theory group and not the administration of the institute. reason: a scientist is most effectively controlled by his work ad not by administrative measures like presence in the institute.

no abuse of this freedom in the theory group, people there quite well understood that their rank is being fixed by their scientific reputation.

It is obvious which sort of atmosphere is being created on such a background.
In the same spirit he supervised the guest program of the theory group. The only relevant criterion for admission was the expected scientific outcome and its quality. Mainstream arguments were not considered to be sufficient. And, of course, the program was international. No arguments like “Germany first” have ever been heard. This was seemingly trivial at that time. But it has to be outspoken today.
When looking at the highlights a clear pattern emerges:

• LSZ clarify basic notions in their fundamental papers. Those have been used over and over again and have become textbook knowledge.
• WZ improves the basis of renormalization theory. A wealth of papers tackles successfully the structure of models: equations of motion, symmetries, anomalies.
• WZ proves operator product expansion in Minkowski space. Measurable quantities in QCD become available; they confirm the theory.
• WZ formulates the principle of reduction of couplings. Within supersymmetric extensions of the standard model the Higgs mass can be predicted to quite some level of accuracy.

“Wenn Könige bauen, haben die Kärrner zu tun!”
(F. Schiller in den “Xenien” (1798) über Kant)
Wolfhart Zimmermann has ended a journey in which he not only devoted his gifts to mathematics and physics but above all of this to his family, his friends and his collaborators. We will miss him.
Thank you for your attention