A new topological model is proposed in three dimensions as an extension of the BF-model. It is a three-dimensional counterpart of the two-dimensional model introduced by Chamseddine and Wyler ten years ago. The BFK-model, as we shall call it, shows to be quantum scale invariant at all orders in perturbation theory. The proof of its full finiteness is given in the framework of algebraic renormalization.

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II. THE BFK-MODEL IN $D = 3$ AND ITS SYMMETRIES

A. The classical action

The classical action of the BFK-model in $D = 3$ is given by

$$\Sigma_{\text{BFK}}^{\text{inv}} = \frac{1}{2} \text{Tr} \int d^3x \, \epsilon^{\mu \nu \rho} \{B_\mu F_{\nu \rho} + K_{\mu \nu} D_\rho \phi\} , \quad (1)$$

where $B_\mu$ is a vector field, $K_{\mu \nu}$ is a rank-2 antisymmetric tensor and $\phi$ is a scalar. The second piece of the action (1) can be seen as a topological matter coupling, where the matter fields $K_{\mu \nu}$ and $\phi$ lie in the adjoint representation of the gauge group. In two dimensions similar topological matter term was proposed by Chamseddine and Wyler \[3\]. Any field, $\varphi$, is to be assumed as Lie algebra valued, in such a way that

$$\varphi \equiv \varphi^a \tau_a , \quad (2)$$

where the matrices $\tau$ are the generators of the gauge group and obey

$$[\tau_a, \tau_b] = f_{abc} \tau_c \quad \text{and} \quad \text{Tr}(\tau_a \tau_b) = \frac{1}{2} \delta_{ab} . \quad (3)$$

The field strength, $F_{\mu \nu}$, is defined as

$$F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] , \quad (4)$$

and the covariant derivative reads

$$D_\mu \phi = \partial_\mu \phi + [A_\mu, \phi] . \quad (5)$$

B. Gauge symmetries

The action (1) possesses two symmetries:

1. The standard gauge symmetry

$$\delta_\alpha A_\mu = -D_\mu \alpha \equiv -(\partial_\mu \alpha + [A_\mu, \alpha]) , \quad \delta_\alpha B_\mu = [\alpha, B_\mu] , \quad \delta_\alpha K_{\mu \nu} = [\alpha, K_{\mu \nu}] \quad \text{and} \quad \delta_\alpha \phi = [\alpha, \phi] . \quad (6)$$

2. The topological symmetry

$$\delta_\beta A_\mu = 0 , \quad \delta_\beta B_\mu = -(D_\mu \beta + [\beta_\mu, \phi]) , \quad \delta_\beta K_{\mu \nu} = -(D_\mu \beta_\nu - D_\nu \beta_\mu) \quad \text{and} \quad \delta_\beta \phi = 0 . \quad (7)$$

C. BRS symmetry

The corresponding BRS transformations of the fields, $A_\mu$, $B_\mu$, $K_{\mu \nu}$ and $\phi$, stemming from the symmetries (6) and (7), are given by

---

1 The gauge group is considered as a general compact one.
2 The commutators are assumed to be graded, namely, $[\varphi_{g1}^{a1}, \varphi_{g2}^{a2}] \equiv \varphi_{g1}^{a1} \varphi_{g2}^{a2} - (-1)^{g1\cdot g2} \varphi_{g2}^{a2} \varphi_{g1}^{a1}$, where the upper indices, $g1$ and $g2$, are the Faddeev-Popov charges (ΦΠ) carried by $\varphi_{g1}^{a1}$ and $\varphi_{g2}^{a2}$, respectively.
\[
\begin{align*}
sA_\mu &= -D_\mu c \equiv -(\partial_\mu c + [A_\mu, c]) , \\
sB_\mu &= -D_\mu B^1 + [\phi, B^1_\mu] + [c, B_\mu] , \\
sK_{\mu\nu} &= -(D_\mu B^1_\nu - D_\nu B^1_\mu) + [c, K_{\mu\nu}] , \\
\phi &= [c, \phi] \quad \text{and} \quad sc = c^2 ,
\end{align*}
\]

where \( c \) and \( B^1 \) are scalar ghosts, and \( B^1_\mu \) is a vector ghost, all of them are anticommuting fields with Faddeev-Popov charge (ghost number) one. Bearing in mind that we have a residual degree of freedom from \( sK_{\mu\nu} \) caused by a zero mode \( (B^1_\mu = D_\mu \varphi^2) \), i.e., it is a reducible symmetry, has to be fixed, therefore, yielding a ghost \( B^2 \) for the ghost \( B^1_\mu \).

Now, fixing the zero mode by introducing the ghost for ghost, \( B^3 \),\footnote{The dimension (d) and the ghost number (\( \Phi \Pi \)) of all fields are displayed in TABLE I.} where the multiplier fields belong to the following BRS-doublets:

\[
sB^3 = [\phi, B^2] + [c, B^1] , \quad sB^1_\mu = D_\mu B^2 + [c, B^1_\mu] \quad \text{and} \quad sB^2 = [c, B^2] .
\]

It should be noticed that the BRS operator \( s \) is nilpotent up to the field equations for \( B_\mu \) and \( K_{\mu\nu} \), since

\[
s^2 B_\mu = \frac{1}{2} \left[ B_\mu^2, \varepsilon_{\mu\nu\rho} \delta \Sigma_{\text{inv}}^{\text{BFK}} \right] \quad \text{and} \quad s^2 K_{\mu\nu} = \left[ B^2_{\mu\nu}, \varepsilon_{\mu\nu\rho} \delta \Sigma_{\text{inv}}^{\text{BFK}} \right] ,
\]

called on-shell nilpotency.

D. Gauge-fixing

The gauge-fixing we are considering here is of the Landau-type. Since we are dealing with a more complex model than the Yang-Mills one, face its symmetries, some subtleties arise as their consequence. In order to implement the gauge-fixing we couple the Lagrange multiplier fields \( b, \pi^0, \pi^{0\mu} \) and \( \pi^{-1} \) to

\[
\begin{align*}
\text{Tr} \ b \partial^\mu A_\mu , \quad \text{Tr} \ \pi^0 \partial^\mu B_\mu , \quad \text{Tr} \ \pi^{0\mu} (\partial^\mu K_{\mu\nu} + \partial_\nu \rho^0) \quad \text{and} \quad \text{Tr} \ \pi^{-1} (\partial^\mu B^1_\mu + \lambda^1) ,
\end{align*}
\]

where the multiplier fields belong to the following BRS-doublets:

\[
\begin{align*}
s\bar{c} = b , \quad sb = 0 ; \\
s\bar{c}^{-1} = \pi^0 , \quad s\pi^0 = 0 ; \\
s\bar{c}^{-1}_\mu = \pi^{0\mu} , \quad s\pi^{0\mu} = 0 ; \\
s\bar{c}^{-2} = \pi^{-1} , \quad s\pi^{-1} = 0 .
\end{align*}
\]

We stress here that for the fields, \( K_{\mu\nu} \) and \( B^1_\mu \), inhomogeneous gauge conditions have been chosen. In which concerns the field \( K_{\mu\nu} \), a gauge condition of the type \( \pi^{0\mu} \partial^\nu K_{\mu\nu} \) would not fix completely the gauge, since a residual degree of freedom is present by \( \pi^{0\nu} \rightarrow \pi^{0\nu} + \partial^\nu \rho^0 \), therefore, due to this fact an inhomogeneous gauge-fixing condition has to be adopted. Since it is an Abelian transformation does not demand further ghost fields. Besides the condition on \( K_{\mu\nu} \) has introduced a new field \( \rho^0 \), it yields also the consideration of another inhomogeneous gauge condition associated to \( B^1_\mu \) by putting into the game the field \( \lambda^1 \). In fact, the introduction of \( \lambda^1 \) is due to cohomological arguments, so in order to protect the independence of BRS-cohomology in those fields introduced by hand, we force them to belong to a BRS-doublet

\[
s\rho^0 = \lambda^1 , \quad s\lambda^1 = 0 .
\]

Bearing in mind that a neutral Faddeev-Popov charge action is desired, the fields \( \rho^0 \) and \( \lambda^1 \) have already had their charges fixed\footnote{The dimension (d) and the ghost number (\( \Phi \Pi \)) of all fields are displayed in TABLE I.}, 0 and 1, respectively.

Now, to introduce a BRS-trivial gauge-fixing compatible with the gauge conditions, we add to the action the following four pieces:

\[
\begin{align*}
\Sigma_{\text{gl}}^1 &= s \text{Tr} \int d^3 x \ \bar{c} \partial^\mu A_\mu , \quad \Sigma_{\text{gl}}^2 = s \text{Tr} \int d^3 x \ \bar{c}^{-1} \partial^\mu B_\mu , \\
\Sigma_{\text{gl}}^3 &= s \text{Tr} \int d^3 x \ \bar{c}^{-1} \nu (\partial^\nu K_{\mu\nu} + \partial_\nu \rho^0) \quad \text{and} \quad \Sigma_{\text{gl}}^4 = s \text{Tr} \int d^3 x \ \bar{c}^{-2} (\partial^\mu B^1_\mu + \lambda^1) .
\end{align*}
\]
Moreover, since those four pieces of the gauge-fixing shall break the on-shell nilpotency of the BRS operator, further modifications of the BRS transformations are necessary, however, in order to restore the BRS invariance an additional term has to be added to the gauge-fixing sector as well. The full modified gauge-fixing action,

\[ \Sigma_{gf} = \Sigma_{gf}^1 + \Sigma_{gf}^2 + \Sigma_{gf}^3 + \Sigma_{gf} + \Sigma_{mod}, \]

with

\[ \Sigma_{mod} = \text{Tr} \int d^3 x \varepsilon^{\mu \nu \rho} \partial_\mu \bar{c}_\nu^{-1} \partial_\rho \bar{c}^{-1}, B^2, \]

reads

\[ \Sigma_{gf} = \text{Tr} \int d^3 x \left\{ b \delta \rho^\mu A_\mu + \pi^0 \delta \rho^\mu B_\mu + \pi^{0 \nu} (\partial_\mu K_{\mu \nu} + \partial_\nu \rho^0) + \pi^{-1} (\partial_\mu B^1_\mu + \lambda^1) + \right. \]
\[ + \bar{c} \delta \rho^\mu D_\mu c + \bar{c}^{-1} \partial_\mu (D_\mu B^1 - [\phi, B^1_\mu] - [c, B_\mu] - \varepsilon_{\mu \nu \rho} (\partial_\nu \bar{c}^{-1} \rho, B^2)) + \]
\[ + \bar{c}^{-1 \nu} [\partial_\mu (D_\mu B^1_\nu - D_\nu B^1_\mu) - [c, K_{\mu \nu}] - \varepsilon_{\mu \nu \rho} (\partial_\rho \bar{c}^{-1}, B^2)] - \partial_\nu \lambda^1 + \]
\[ + \bar{c}^{-2} \partial_\mu (D_\mu B^2 + [c, B^1_\mu]) + \varepsilon_{\mu \nu \rho} \partial_\mu \bar{c}_\nu^{-1} \partial_\rho \bar{c}^{-1}, B^2 \right\}, \]

where the enlarged BRS transformations associated to \( B_\mu \) and \( K_{\mu \nu} \) are given by

\[ sB_\mu = -D_\mu B^1 + [\phi, B^1_\mu] + [c, B_\mu] + \varepsilon_{\mu \nu \rho} (\partial_\nu \bar{c}^{-1} \rho, B^2), \]
\[ sK_{\mu \nu} = -(D_\mu B^1_\nu - D_\nu B^1_\mu) + [c, K_{\mu \nu}] + \varepsilon_{\mu \nu \rho} (\partial_\rho \bar{c}^{-1}, B^2). \]

Let us now introduce the action in which the nonlinear BRS transformations are coupled to the antifields (BRSt invariant external fields), so as to control the renormalization of those transformations:

\[ \Sigma_{ext} = \text{Tr} \int d^3 x \left\{ B^*_\mu sB^\mu + \frac{1}{2} K_{\mu \nu}^* sK^{\mu \nu} + A_\mu^* sA^\mu + \phi^* s\phi + e^* sc + B^2 s B^2 + B^1 s B^1 + B^{1*} s B^{1*} + \right. \]
\[ + \frac{1}{2} \bar{c} \varepsilon_{\mu \nu \rho} B^*_\mu [K_{\nu \rho}, B^2] \right\}. \]

The total classical action for the BFK-model, \( \Gamma^{(0)} \):

\[ \Gamma^{(0)} = \Sigma_{inv} + \Sigma_{gf} + \Sigma_{ext}, \]

is invariant under the following BRS transformations

\[ sB_\mu = -D_\mu B^1 + [\phi, B^1_\mu] + [c, B_\mu] + \varepsilon_{\mu \nu \rho} (\partial_\nu \bar{c}^{-1} \rho, B^2), \]
\[ sK_{\mu \nu} = -(D_\mu B^1_\nu - D_\nu B^1_\mu) + [c, K_{\mu \nu}] + \varepsilon_{\mu \nu \rho} (\partial_\rho \bar{c}^{-1}, B^2), \]
\[ sA_\mu = -D_\mu c, \, s\phi = [c, \phi], \, sc = c^2, \]
\[ sB^1 = [\phi, B^2] + [c, B^1], \, sB^1_\mu = D_\mu B^2 + [c, B^1_\mu], \, sB^2 = [c, B^2], \]
\[ s\bar{c} = b, \, sb = 0; \, s\bar{c}^{-1} = r^0, \, s\bar{r}^0 = 0; \]
\[ s\bar{c}^{-1} = \bar{r}^- = 0, \, s\bar{r}^- = 0, \]
\[ s\rho^0 = \lambda^1, \, s\lambda^1 = 0. \]

The BRS invariance of the action \( \Sigma^{(0)} \) is expressed through the Slavnov-Taylor identity

\[ \mathcal{S}(\Gamma^{(0)}) = \text{Tr} \int d^3 x \left\{ \frac{\delta \Gamma^{(0)}}{\delta B^*_\mu} \frac{\delta \Gamma^{(0)}}{\delta B^\mu} + \frac{1}{2} \frac{\delta K^*_{\mu \nu}}{\delta K^{\mu \nu}} + \frac{\delta \Gamma^{(0)}}{\delta A^\mu} \frac{\delta \Gamma^{(0)}}{\delta A^\mu} + \frac{\delta \Gamma^{(0)}}{\delta \phi^*} \frac{\delta \Gamma^{(0)}}{\delta \phi} + \frac{\delta \Gamma^{(0)}}{\delta \rho^0} \frac{\delta \Gamma^{(0)}}{\delta \rho^0} + \frac{\delta \Gamma^{(0)}}{\delta B^{1*}} \frac{\delta \Gamma^{(0)}}{\delta B^1} + \right. \]
\[ + \frac{\delta \Gamma^{(0)}}{\delta B^{1*}} \frac{\delta \Gamma^{(0)}}{\delta B^1} + \frac{\delta \Gamma^{(0)}}{\delta B^{1*}} \frac{\delta \Gamma^{(0)}}{\delta B^1} - \frac{\partial}{\partial \bar{c}} \left( \frac{\delta \Gamma^{(0)}}{\delta \bar{c}^{-1}} + \frac{\delta \Gamma^{(0)}}{\delta \bar{c}^{-1}} + \frac{\delta \Gamma^{(0)}}{\delta \bar{c}^{-1}} + \frac{\delta \Gamma^{(0)}}{\delta \bar{c}^{-1}} + \lambda^1 \frac{\delta \Gamma^{(0)}}{\delta \rho^0} \right) = 0, \]
as = ∫ d³x \left\{ -\frac{1}{2} \varepsilon_{\mu \nu \rho} (\partial^\mu \bar{c} + A^\mu) \frac{\delta}{\delta B^\nu} - \frac{1}{2} \varepsilon_{\mu \nu \rho} (\partial^\mu \bar{c}^{-1}) + B^{*\nu} \right\} \frac{\delta}{\delta \phi} + A_\mu \frac{\delta}{\delta c} + B^1_\nu \frac{\delta}{\delta B^1_\nu} - B^{*\nu} \right\} + \frac{1}{2} \varepsilon_{\mu \nu \rho} (\partial^\mu \bar{c}^{-1}) + B^{*\nu} \right\} \frac{\delta}{\delta \phi} + A_\mu \frac{\delta}{\delta c} + B^1_\nu \frac{\delta}{\delta B^1_\nu} - B^{*\nu} \right\} .

By similar reasons to the existence of a vector-supersymmetry in the BFK-model, a scalar-supersymmetry can be found:

\[ L \Gamma^{(0)} = \Delta L, \]

where its Ward operator reads

\[ L = \int d³x \left\{ \frac{1}{2} \varepsilon_{\mu \nu \rho} (\partial^\mu \bar{c} + A^\mu) \frac{\delta}{\delta K_{\mu \rho}^{(0)}} - \frac{1}{2} \varepsilon_{\mu \nu \rho} (\partial^\mu \bar{c}^{-1}) + B^{*\nu} \right\} \frac{\delta}{\delta \phi} + A_\mu \frac{\delta}{\delta c} + B^1_\nu \frac{\delta}{\delta B^1_\nu} - B^{*\nu} \right\} .

and its linear breaking in the quantum fields, \( \Delta L \), is given by

\[ \Delta L = \int d³x \left\{ \frac{1}{2} \varepsilon_{\mu \nu \rho} \left( K^{*\mu \nu \rho} b - A^{\mu \nu} \right) \right\} .

E. Gauge conditions, ghost and antighost equations

This Subsection is devoted to establish the gauge conditions, ghost and antighost equations, and two others quite important symmetries in the proof of the exact quantum scale invariance of the BFK-model.

The gauge conditions read

\[ \frac{\delta \Gamma^{(0)}}{\delta b} = \partial^\mu A_\mu, \quad \frac{\delta \Gamma^{(0)}}{\delta \pi^0} = \partial^\mu B_\mu, \quad \frac{\delta \Gamma^{(0)}}{\delta \pi^0} = \partial^\mu K_{\mu \rho} + \partial_\rho \right\} \quad \text{and} \quad \frac{\delta \Gamma^{(0)}}{\delta \pi^0} = \partial^\mu B^1_\mu + \lambda^1, \]

therefore, the conditions fulfilled by the BRS-doublet auxiliary fields, \( \rho^0 \) and \( \lambda^1 \), are given by
\[
\frac{\delta \Gamma^{(0)}}{\delta \rho^0} = -\partial^\mu \pi^0_\mu \quad \text{and} \quad \frac{\delta \Gamma^{(0)}}{\delta \lambda^1} = -\partial^\mu \bar{c}_{\mu}^{-1} - \pi^{-1} .
\] (31)

The ghost equations:
\[
\begin{align*}
G(\Gamma^{(0)}) = & \frac{\delta \Gamma^{(0)}}{\delta c} + \partial^\mu \frac{\delta \Gamma^{(0)}}{\delta A^*_\mu} = 0 , \\
G^1(\Gamma^{(0)}) = & \frac{\delta \Gamma^{(0)}}{\delta \bar{c} \nu} + \partial^\mu \frac{\delta \Gamma^{(0)}}{\delta B^\nu_\mu} = 0 , \\
G^1_\mu(\Gamma^{(0)}) = & \frac{\delta \Gamma^{(0)}}{\delta c_{\mu}^{-1}} + \partial^\nu \frac{\delta \Gamma^{(0)}}{\delta K^{\nu}_{\mu}} = -\partial_{\mu} \bar{c}^1 \quad \text{and} \quad G^2(\Gamma^{(0)}) = \frac{\delta \Gamma^{(0)}}{\delta \bar{c}^{-2}} - \partial^\mu \frac{\delta \Gamma^{(0)}}{\delta B^1_{\mu}} = 0 ,
\end{align*}
\] (32)

mean that \( \Gamma^{(0)} \) depends on the antighosts, \( \bar{c}, \bar{c}^{-1}, \bar{c}^{-2} \), and the antifields, \( A^*_\mu, B^*_\mu, K^{\nu}_{\mu} \) and \( B^1_{\mu} \), through the combinations
\[
\begin{align*}
\bar{A}^*_\mu = & \bar{A}^*_\mu + \partial_{\mu} \bar{c} , \\
\bar{B}^*_\mu = & B^*_\mu + \partial_{\mu} \bar{c}^{-1} , \\
\bar{K}^{\nu}_{\mu} = & K^{\nu}_{\mu} + \partial_{[\mu} \bar{c}^{-1}_{\nu]} \quad \text{and} \quad \bar{B}^1_{\mu} = B^1_{\mu} - \partial_{\mu} \bar{c}^{-2} .
\end{align*}
\] (33)

In the BFK-model there are two antighost equations, they are listed as below:
\[
\begin{align*}
\overline{G}(\Gamma^{(0)}) = & \int d^3x \left\{ \frac{\delta \Gamma^{(0)}}{\delta B^2} + \left[ \bar{c}^{-1}, \frac{\delta \Gamma^{(0)}}{\delta b} \right] \right\} = \Delta \overline{G} , \quad \text{where} \\
\Delta \overline{G} = & \int d^3x \left\{ [A^*_\mu, B^*_\mu] + [c, B^1] \right\} , \\
\mathcal{E}(\Gamma^{(0)}) = & \int d^3x \left\{ \frac{\delta \Gamma^{(0)}}{\delta B^2} - \left[ \bar{c}^{-2}, \frac{\delta \Gamma^{(0)}}{\delta b} \right] \right\} = \Delta \mathcal{E} , \quad \text{where} \\
\Delta \mathcal{E} = & \int d^3x \left\{ \left. \frac{1}{2} \bar{c} \nu \mu \rho \left( [B^{\nu}_{\mu}, (\partial [\nu \bar{c}^{-1}]^{\nu} + K^{\nu}_{\mu}] + [K^{\nu}_{\mu}, \partial^{\rho} \bar{c}^{-1}] \right) + [B^2, c] + [B^{1*}, \phi] + [B^{1}, A^*] \right\} .
\end{align*}
\] (34)

From those operators, \( \overline{G} \) and \( \mathcal{E} \), two others can be found by grading commutations with the Slavnov-Taylor operator (see the operatorial algebra in the next Subsection):
\[
\overline{F}(\Gamma^{(0)}) = \int d^3x \left\{ A^*_\mu, \frac{\delta \Gamma^{(0)}}{\delta B^2} \right\} - \left[ B^*_\mu, \frac{\delta \Gamma^{(0)}}{\delta A^*_{\mu}} \right] + \left[ c, \frac{\delta \Gamma^{(0)}}{\delta B^1} \right] + \left[ B^{1*}, \frac{\delta \Gamma^{(0)}}{\delta c^*} \right] + \left[ \bar{c}^{-1}, \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} \right] + \\
+ \left[ \pi^0, \frac{\delta \Gamma^{(0)}}{\delta b} \right] = 0 ,
\] (36)

\[
\Omega(\Gamma^{(0)}) = \int d^3x \left\{ \frac{1}{2} \bar{c} \nu \mu \rho \left\{ \left( \partial [\nu \bar{c}^{-1}]^{\nu} + K^{\nu}_{\mu} \right), \frac{\delta \Gamma^{(0)}}{\delta B^2} \right\} + \left( \partial \nu \bar{c}^{-1} + B^{\nu}_{\mu} \right), \frac{\delta \Gamma^{(0)}}{\delta K^{\nu}_{\mu}} \right\} - \left[ c, \frac{\delta \Gamma^{(0)}}{\delta B^1} \right] - \left[ \pi^0, \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} \right] + \\
- \left[ A^*_\mu, \frac{\delta \Gamma^{(0)}}{\delta B^2} \right] - \left[ \bar{c}^{-2}, \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} \right] + \left[ \bar{c}^{-1}, \frac{\delta \Gamma^{(0)}}{\delta b} \right] + \left[ B^{1*}, \frac{\delta \Gamma^{(0)}}{\delta A^*_{\mu}} \right] + \left[ B^1, \frac{\delta \Gamma^{(0)}}{\delta \phi^*} \right] - \left[ B^2, \frac{\delta \Gamma^{(0)}}{\delta c^*} \right] = \Delta \Omega
\]

where \( \Delta \Omega = \int d^3x \left\{ \frac{1}{2} \bar{c} \nu \mu \rho \left( \left[ \pi^0, \partial^\mu K^{\nu*}_{\rho} \right] - \left[ \pi^0, \partial^\mu B^{2*}_{\rho} \right] \right) \right\} .
\] (37)

It should be pointed out that the breakings, \( \Delta \overline{G}, \Delta \mathcal{E} \) and \( \Delta \Omega \), being linear in the quantum fields are not subjected to renormalization.

**F. Operatorial algebra**

All operators introduced previously satisfy the following off-shell algebra for any functional \( \mathcal{K} \) with even Faddeev-Popov charge:

1. Slavnov-Taylor operator identities
\[
\begin{align*}
S_K S(\mathcal{K}) = & 0 \quad \forall \mathcal{K} , \\
S_K S_K = & 0 \quad \text{if} \ S(\mathcal{K}) = 0 , \\
\mathcal{W}_\mu S(\mathcal{K}) + S_K (\mathcal{W}_\mu S(\mathcal{K}) - \Delta W^\nu_\mu) = & \mathcal{P}_\mu (\mathcal{K}) , \\
\mathcal{L} S(\mathcal{K}) + S_K (\mathcal{L} S(\mathcal{K}) - \Delta L) = & 0 , \\
\frac{\delta S(\mathcal{K})}{\delta b} - S_K \left( \frac{\delta K}{\delta b} - \partial^\mu A_\mu \right) = & \mathcal{G}(\mathcal{K}) , \\
\frac{\delta S(\mathcal{K})}{\delta \pi^0} - S_K \left( \frac{\delta K}{\delta \pi^0} - \partial^\mu B_\mu \right) = & \mathcal{G}^1(\mathcal{K}) , \\
\end{align*}
\]
\[
\frac{\delta S(K)}{\delta \pi^{0 \nu}} - S_K \left( \frac{\delta K}{\delta \pi^{0 \nu}} - \partial^\mu K_{\mu \nu} - \partial^\mu \rho^\nu \right) = G_\nu(K) + \partial_{\nu} \lambda^1, \quad \frac{\delta S(K)}{\delta \pi^{-1}} + S_K \left( \frac{\delta K}{\delta \pi^{-1}} - \partial^\mu B^1_{\mu} - \lambda^1 \right) = G^2(K),
\]
\[
\mathcal{G} S(K) + S_K \mathcal{G}(K) = 0, \quad \mathcal{G}^1 S(K) + S_K \mathcal{G}^1(K) = 0,
\]
\[
\frac{\delta S(K)}{\delta \pi^0} - S_K \left( \frac{\delta K}{\delta \pi^0} + \partial^\mu \pi^0_{\mu} \right) = 0, \quad \frac{\delta S(K)}{\delta \pi^1} + S_K \left( \frac{\delta K}{\delta \pi^1} + \partial^\mu \pi^1_{\mu} + \pi^1 \right) = \frac{\delta K}{\delta \pi^0} + \partial^\mu \pi^0_{\mu},
\]
\[
\mathcal{G} S(K) + S_K \mathcal{G}(K) - \Delta \mathcal{G} = \mathcal{F}(K), \quad \mathcal{F} S(K) - S_K \mathcal{F}(K) = 0,
\]
\[
\mathcal{A} S(K) - S_K \left( \mathcal{A}(K) - \Delta \mathcal{A} \right) = \mathcal{O}(K) - \Delta \mathcal{O}, \quad \mathcal{O} S(K) + S_K \left( \mathcal{O}(K) - \Delta \mathcal{O} \right) = 0;
\]

(38)

2. Other identities

\[
\{ \mathcal{W}_\nu, \mathcal{W}_\nu \} = 0, \quad \{ \mathcal{W}_\mu, L \} = 0, \quad \{ L, L \} = 0,
\]
\[
\mathcal{W}_\mu \mathcal{G}(K) - \Delta \mathcal{G} + \mathcal{G}(\mathcal{W}_\mu(K) - \Delta \mathcal{W}_\mu) = 0, \quad L(\mathcal{G}(K) - \Delta \mathcal{G}) + \mathcal{G}(L(K) - \Delta L) = \mathcal{A}(K) - \Delta \mathcal{A},
\]
\[
\mathcal{W}_\mu \mathcal{F}(K) - \mathcal{F}(\mathcal{W}_\mu(K) - \Delta \mathcal{W}_\mu) = 0, \quad L(\mathcal{F}(K) - \Delta \mathcal{F}) + \mathcal{F}(L(K) - \Delta L) = \mathcal{O}(K) - \Delta \mathcal{O},
\]
\[
\mathcal{W}_\mu (\mathcal{E}(K) - \Delta \mathcal{E}) - \mathcal{E}(\mathcal{W}_\mu(K) - \Delta \mathcal{W}_\mu) = 0, \quad L(\mathcal{E}(K) - \Delta \mathcal{E}) - \mathcal{E}(L(K) - \Delta L) = 0,
\]
\[
\mathcal{W}_\mu (\mathcal{O}(K) - \Delta \mathcal{O}) + \mathcal{O}(\mathcal{W}_\mu(K) - \Delta \mathcal{W}_\mu) = 0, \quad L(\mathcal{O}(K) - \Delta \mathcal{O}) + \mathcal{O}(L(K) - \Delta L) = 0,
\]
\[
\mathcal{G}(\mathcal{G}(K) - \Delta \mathcal{G}) + \mathcal{G}(\mathcal{G}(K) - \Delta \mathcal{G}) = 0, \quad \mathcal{G}(\mathcal{G}(K) - \Delta \mathcal{G}) + \mathcal{G}(\mathcal{G}(K) - \Delta \mathcal{G}) = 0,
\]
\[
\mathcal{A}(\mathcal{A}(K) - \Delta \mathcal{A}) + \mathcal{A}(\mathcal{A}(K) - \Delta \mathcal{A}) = 0, \quad \mathcal{A}(\mathcal{A}(K) - \Delta \mathcal{A}) + \mathcal{A}(\mathcal{A}(K) - \Delta \mathcal{A}) = 0,
\]
\[
\mathcal{O}(\mathcal{O}(K) - \Delta \mathcal{O}) + \mathcal{O}(\mathcal{O}(K) - \Delta \mathcal{O}) = 0, \quad \mathcal{O}(\mathcal{O}(K) - \Delta \mathcal{O}) + \mathcal{O}(\mathcal{O}(K) - \Delta \mathcal{O}) = 0,
\]
\[
\mathcal{P}_\mu, \Theta = 0 \quad \forall \Theta \in \{ S_K, \mathcal{W}_\mu, L, \mathcal{G}, \mathcal{G}^1, \Delta \mathcal{W}_\mu, \mathcal{G}^2, \mathcal{F}, \mathcal{A}, \mathcal{O}, \mathcal{P}_\mu \},
\]

(39)

where \( \mathcal{P}_\mu \) is the Ward operator associated to translations:

\[
\mathcal{P}_\mu = \sum_\phi \text{Tr} \int d^3x \, \partial_\mu \phi \frac{\delta}{\delta \phi}.
\]

(40)

The first group of identities involving the Slavnov-Taylor operator given by \( \mathcal{G} \) are those which yield the conditions (the well-known Wess-Zumino consistency condition is one of them) to be satisfied by the quantum breaking of the Slavnov-Taylor identity \( \mathcal{G} \) allowed by the Quantum Action Principle \( \mathcal{I} \).

### III. FINITENESS

In this Section is summarized the results on the study of stability of the classical action under radiative corrections, the implementation of the Slavnov-Taylor identity at the quantum level and the conclusion on the finiteness of the BFK-model at all orders in perturbation theory.

#### A. Stability

In order to check whether the action in the tree-approximation is stable under radiative corrections or not, we perturb it by an arbitrary integrated local functional \( \Sigma^c \), such that

\[
\hat{\Gamma}^{(0)} = \Gamma^{(0)} + \varepsilon \Sigma^c,
\]

(41)
where $\varepsilon$ is an infinitesimal parameter. The functional $\Sigma^c$ has the same quantum numbers as the classical action, $\Gamma^{(0)}$, and the deformed action, $\tilde{\Gamma}^{(0)}$, must obey in the same way all the identities $\Gamma^{(0)}$ does, leading therefore, to the following homogeneous conditions to be fulfilled by the counterterm, $\Sigma^c$:

$$S_{\Gamma^{(0)}} \Sigma^c = W_{\mu} \Sigma^c = L \Sigma^c = \frac{\delta \Sigma^c}{\delta \theta^0} = \frac{\delta \Sigma^c}{\delta \pi^0} = \frac{\delta \Sigma^c}{\delta \pi^1} = \frac{\delta \Sigma^c}{\delta \rho^0} = \frac{\delta \Sigma^c}{\delta \lambda^1} = 0,$$

$$G \Sigma^c = G^1 \Sigma^c = G^2 \Sigma^c = \bar{G} \Sigma^c = \bar{A} \Sigma^c = \bar{F} \Sigma^c = \delta \Sigma^c = 0.$$  \hspace{1cm} (42)

In searching for the most general counterterm satisfying all the constraints listed above, it can be shown that there is no integrated local polynomial in the fields which survives those requirements, then

$$\Sigma^c = 0,$$ \hspace{1cm} (43)

meaning that the usual ambiguities due to the renormalization procedure do not appear in the BFK-model. It remains now to prove the absence of anomalies in order to conclude about the perturbative finiteness of the model.

### B. Anomaly and finiteness

At the quantum level the vertex functional, $\Gamma$, which coincides with the classical action $\Gamma^{(0)}$, at order 0 in $\hbar$,

$$\Gamma = \Gamma^{(0)} + O(\hbar),$$  \hspace{1cm} (44)

has to satisfy the same constraints as the classical action does. However, according to the Quantum Action Principle \cite{6} the Slavnov-Taylor identity \cite{22} gets a quantum breaking

$$S(\Gamma) = \Delta \cdot \Gamma = \Delta + O(\hbar \Delta),$$ \hspace{1cm} (45)

where $\Delta$ is an integrated local functional with ghost number 1 and dimension 3. The absence of anomalies amounts to show that the Slavnov-Taylor identity can be implemented at the quantum level at the expenses of a BRS-trivial breaking $\Delta = S_{\Gamma} \hat{\Delta}^{(0)}$, called noninvariant counterterm.

The nilpotency identity, $S_{\Gamma} S(\Gamma) = 0$, together with

$$S_{\Gamma} = S_{\Gamma^{(0)}} + O(\hbar),$$ \hspace{1cm} (46)

implies the Wess-Zumino consistency condition for the breaking $\Delta$:

$$S_{\Gamma^{(0)}} \Delta = 0,$$ \hspace{1cm} (47)

beyond that, through the algebra \cite{18}, $\Delta$ satisfies:

$$W_{\mu} \Delta = L \Delta = \delta \Delta = \frac{\delta \Delta}{\delta \theta^0} = \frac{\delta \Delta}{\delta \pi^0} = \frac{\delta \Delta}{\delta \pi^1} = \frac{\delta \Delta}{\delta \rho^0} = \frac{\delta \Delta}{\delta \lambda^1} = 0,$$

$$G \Delta = G^1 \Delta = G^2 \Delta = \bar{G} \Delta = \bar{A} \Delta = \bar{F} \Delta = \delta \Delta = 0.$$  \hspace{1cm} (48)

The Wess-Zumino consistency condition \cite{17} constitutes a cohomology problem in the sector of ghost number one. Its solution can always be written as a sum of a trivial cocycle $S_{\Gamma^{(0)}} \hat{\Delta}^{(0)}$, where $\hat{\Delta}^{(0)}$ has ghost number 0, and of nontrivial elements belonging to the cohomology of $S_{\Gamma^{(0)}}$ \cite{23} in the sector of ghost number one, $A^{(1)}$:

$$\Delta = A^{(1)} + S_{\Gamma^{(0)}} \hat{\Delta}^{(0)}.$$  \hspace{1cm} (49)

Although the constraints imposed to $\Delta$ in \cite{17} and \cite{18} show that $A^{(1)} = 0$ \cite{17}, it can be proved quite generally that in three-dimensions there is no anomaly, since the cohomology in the sector of ghost number 1 is empty up to possible terms in the Abelian ghosts \cite{19}. However, through the arguments of \cite{24} we conclude that the $U(1)$-ghosts do not contribute to the anomaly due to their freedom or soft coupling, then the Slavnov-Taylor identity is implemented at the quantum level.

In conclusion, the absence of counterterms in the study of stability, as presented in Subsection III A, together with the result of Subsection III B concerning the absence of anomaly lead to a proof on the finiteness of the BFK-model at all orders in perturbation theory.

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\(^4\)See TABLE for the dimension and the ghost number of all fields and anti-fields.
TABLE I. Dimension $d$ and ghost number $\Phi_{\Pi}$.

| $d$ | $\Phi_{\Pi}$ | $\pi_\mu$ | $\bar{c}^{-1}$ | $\rho$ | $\lambda$ | $B_\mu$ | $K_{\mu\nu}$ | $A_{\mu}$ | $\phi$ | $c$ | $B_1^\mu$ | $\bar{c}$ | $b$ | $\bar{c}^{-1}$ | $\pi''$ | $c_{\mu}''$ |
|-----|--------------|-----------|--------------|-------|---------|--------|-----------|--------|-------|----|--------|---------|----|---------|--------|------------|
| 1   | 0            | 1         | 1            | 0     | -1      | 0      | 0         | 1      | 1     | 1  | 1      | 0       | 1  | -1      | 0      | 1          |
| 2   | 1            | 2         | 2            | 1     | 1       | 2      | -1       | 0      | -1    | 0  | -1     | -2      | -3 | -2      | -3     | -2         |

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