On the Entity Hardening Problem in Multi-layered Interdependent Networks

Joydeep Banerjee, Arun Das, Chenyang Zhou, and Anisha Mazumder

School of Computing, Informatics and Decision Systems Engineering
Arizona State University
Tempe, Arizona, USA

Arunabha (Arun) Sen

ENGINE
Wroclaw University of Technology
(on sabbatical from Arizona State University)
Wroclaw, Poland

This project has received funding from the European Union’s Seventh Framework Programme for research, technological development and demonstration under grant agreement no 316097 [ENGINE].
ENGINE - European research centre of Network intelligence for Innovation Enhancement

This project has received funding from the European Union’s Seventh Framework Programme for research, technological development and demonstration under grant agreement no 316097 [ENGINE].
This project is partially funded by the European Commission under the **7th Framework Programme**, Coordination and Support Action in the **REGPOT (Research Potential)** program.

The **main goal** of the project is to **enhance the research potential** of the ENGINE Centre, an integral part of the Wrocław University of Technology.

Web page: [http://www.engine.pwr.edu.pl/en/](http://www.engine.pwr.edu.pl/en/)
• Staff training and development
• Organizing courses, training days, seminars, workshops and conferences
• Training and development of the staff through specialized trainings and through bilateral exchange with leading European research centres
• Recruitment of experienced researchers
• Updating and improving the research equipment and organization in the ENGINE Centre
• Increasing participation in the EU collaborative research projects resulting in the joint grant proposals for national and international research projects, especially FP7 and in future Horizon 2020
Recruitment of an experienced researcher (visiting professor) for the total duration of **6-9 months**. Salary: **7000 EURO per month (before tax)**

Recruitment of a post-doc for the total duration of **6-9 months**. Salary: **4000 EURO per month (before tax)**

Field: **Computer Network Optimization**

Location: Wroclaw University of Technology

**Required:**

- experience in conducting various types of research related to network optimization and content-oriented networks
- experience in preparing projects’ proposals, especially in the European Framework Programs
- significant record of cited publications in the area of computer network optimization
On the Entity Hardening Problem in Multi-layered Interdependent Networks

(Research supported in part by the RIPS Program of the National Science Foundation)
Agenda

Introduction & Background

The Implicative Interdependency Model (IIM)

Problem Formulation – Entity Hardening Problem

Computational Complexity

Solution Techniques – Optimal & Heuristic

Experimental Results
In the last few years there has been an increased awareness that the critical infrastructure networks are highly interdependent. This is particularly true for two vitally important critical infrastructures - the Power Grid and Communication Networks.

Communication networks need electric power and electric power generation, transmission, and distribution is impossible without communication networks as they are controlled by Supervisory Control and Data Acquisition (SCADA) system that requires communication with critical delay bounds.

A number of models have been proposed in the last few years to capture the interdependency between power and communication networks.
Multilayered Interdependent Network

- Power Network
- Communication Network
- Transportation Network
Interdependent Infrastructure Networks

Interdependent Power and Communication Infrastructure System
Efforts in Modeling of Interdependent Infrastructure Networks

Limitations of Existing Models & Proposed New Model
Figure 1  The graph corresponding to the Italian high-voltage (380 kV) transmission grid resulting from the available data.

Figure 4  The high-bandwidth backbone of the internet network dedicated to linking Italian universities and research institutions (GARR).
Rosato Model (2008)

Early effort in realistic modeling of Power Network (PN) and Communication Network (CN)

Effect of perturbation of PN on CN is analyzed based on a coupling parameter

The impact of CN on PN is not analyzed

The coupling parameter is not validated and is assumed
A component (a set of nodes) in a composite graph is connected if any two nodes have at least one blue path and one green path connecting them.
Buldyrev Model (2010)

Fault propagation with both intra link connection and inter link interdependencies in consideration

Network robustness --- maximum number of node removal from one network to get at least one giant connected cluster (percolation threshold)

Nodes in PN are representation of generators, substations or load are not stated and similarly nodes in CN not identified as routers or fiber optic lines or base stations

Actual working of SCADA system in CN needs to be considered in modeling interdependency
Peeta Model (2011)

Fig. 1. Multilayer infrastructure network (MIN) framework.
Precursor Effect

Kill Effect

Figure 3. General representation of an Interdependent Multi-Layer Network.
Interdependency in space based networks with shared subsystems

Two different type of interdependency among different subsystems --- kill effect and precursor effect

A failure propagation algorithm is developed

The cases considered in experiments though realistic are not validated
Liu Model (2012)
Liu Model (2012)

Effect of cyber intrusions in SCADA and EMS system on PN is analyzed through realistic test beds.

The experiments are confined to small domain.

Large cascades of failure owing to this effect is not analyzed.
Multiplicity of Models

WSCC 9 Bus System

Node weighted graph model with generator and load weights. Each Bus is represented as a node and two nodes have an edge between them if the corresponding buses are connected by a transmission line.

Graph Model of WSCC 9 Bus System

Edge weighted graph model with power flow on the transmission lines.
Modiano Model (2013)

Fig. 1. Cyber-Physical Interdependency Model - dotted lines represent power lines and solid lines represent communication lines.

Fig. 3. Graph structure under different interdependency models
(a) Unidirectional Interdependency  (b) Bidirectional Interdependency
The Modiano Model (2013)

Both networks have star topologies.

All of the substations in the power grid are directly connected to the generator;

No substation’s failure can disconnect the other substations from the generator.

All of the routers in the CCN are directly connected to the control center, and

No router’s failure can disconnect the other routers from the control center.

The failure of all nodes in both networks is known as Total Failure.

Q: What is the minimum number of nodes whose removals will lead to total failure?
Dependencies that exist between the entities of the Power and Communication networks are often complex, involving a combination of conjunctive and disjunctive terms representing the entities of these two types of networks.

Most of the proposed interdependency models are unequipped to capture such complex interdependencies.
Basic Structure of the Electric System

**Color Key:**
- Blue: Transmission
- Green: Distribution
- Black: Generation

**Transmission Lines**
- 500, 345, 230, and 138 kV

**Generating Station**
- Generator Step Up Transformer

**Transmission Customer**
- 138kV or 230kV

**Substation Step-Down Transformer**
- Subtransmission Customer
  - 26kV and 69kV

**Primary Customer**
- 13kV and 4 kV

**Secondary Customer**
- 120V and 240V
Battery backup

Generator Step Up Transformer

Transmission Lines
500, 345, 230 and 138 kV

Substation Step Down Transformer

Secondary Customer
120V and 240V

Generating Station

Color Key:
Blue: Transmission
Green: Distribution
Black: Generation

\[ a_1 \rightarrow a_2 \rightarrow a_3 \rightarrow a_5 \rightarrow a_6 \rightarrow a_7 \rightarrow a_8 \rightarrow a_9 \rightarrow a_{10} \rightarrow a_{11} \rightarrow a_{12} \rightarrow a_{13} \]
Limitations of Current Models

An example of limitation of the current models:

Power Network Entity $\text{PNE}_a$ (say, a generator, substation, transmission line, load) is “alive” if communication network entities (CNE)

- $\text{CNE}_b$ and $\text{CNE}_c$ and $\text{CNE}_d$ are alive, OR
- $\text{CNE}_e$ and $\text{CNE}_f$ are alive, OR
- $\text{CNE}_g$ is alive

Examples of communication network entities may include routers, cell towers, fiber optic lines, optical signal amplifiers.

Existing models cannot effectively model such complex combination of conjunctive and disjunctive interdependencies.
The Implicative Interdependency Model (IIM)* overcomes these limitations and models interdependency using Boolean Logic.

We express the dependency relation in the example in the previous slide in the following way:

This dependency relation is a necessary but not sufficient condition for $\text{PNE}_a$ to be “alive”.

* “Identification of K most vulnerable nodes in multi-layered network using a new model of interdependency”, A. Sen, A. Mazumder, J. Banerjee, A. Das, and R. Compton, Presented at NetSciCom 2014, 6th International Workshop on Network Science for Communication Networks held in conjunction with INFOCOM 2014.
Failures in a multi-layered network can cascade from layer to layer

Failures propagate in time steps

Cascading failures reach a steady state after $K$ time steps
Steady state in a multilayer network can be viewed as a “fixed point” in the system.

Interdependent multi-layer network can be viewed as a closed loop control system.
Cascading Failure Propagation in IIM

An example

**Power Network:**
\[ a_1 \leftarrow b_1 + b_2 \]
\[ a_2 \leftarrow b_1 b_3 + b_2 \]
\[ a_3 \leftarrow b_3 b_1 b_2 \]
\[ a_4 \leftarrow b_1 + b_2 + b_3 \]

**Communication Network:**
\[ b_1 \leftarrow a_1 + a_2 a_3 \]
\[ b_2 \leftarrow a_1 + a_3 \]
\[ b_3 \leftarrow a_1 a_2 \]

| Entities | \( t_1 \) | \( t_2 \) | \( t_3 \) | \( t_4 \) | \( t_5 \) | \( t_6 \) | \( t_7 \) |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \( a_1 \)  | 1         | 1         | 1         | 1         | 1         | 1         | 1         |
| \( a_2 \)  | 0         | 0         | 0         | 0         | 1         | 1         | 1         |
| \( a_3 \)  | 0         | 0         | 1         | 1         | 1         | 1         | 1         |
| \( a_4 \)  | 0         | 0         | 0         | 0         | 1         | 1         | 1         |
| \( b_1 \)  | 0         | 0         | 0         | 1         | 1         | 1         | 1         |
| \( b_2 \)  | 0         | 0         | 0         | 1         | 1         | 1         | 1         |
| \( b_3 \)  | 0         | 1         | 1         | 1         | 1         | 1         | 1         |
Problem: Identification of $K$ most vulnerable entities in a multi-layered network.

Definition: A set of entities in a multi-layered network is said to be the “most vulnerable” if failure of the $K$ entities induces failure of the largest number of other entities in the multi-layered network.

“Identification of K most vulnerable nodes in multi-layered network using a new model of interdependency”, A. Sen, A. Mazumder, J. Banerjee, A. Das, and R. Compton. Presented at NetSciCom 2014, 6th International Workshop on Network Science for Communication Networks held in conjunction with INFOCOM 2014.
If defender takes no action then the adversary will destroy $K$ most vulnerable entities in the system that will cause maximum damage.

Entity Defense: Defender takes some action so that the attacker cannot destroy the entity.

If the defender has the resources to defend $K$ entities then the attacker cannot inflict any damage to the system.

If the defender does not have resources to defend $K$ entities, but say $K'$ entities, where $K' \leq K$, the defender has to decide which $K'$ entities should be defended so that the impact of attack is minimized.
Entity Hardening Problem

Given

- An interdependent network with entity sets $A$ and $B$
- The set of $K$ vulnerable entities of the system $A' \cup B'$ where $A' \subseteq A$ and $B' \subseteq B$
- Two positive integers $k$, $k < K$ and $E_F$

Question: Is there a set of entities $H = A'' \cup B''$, $A'' \subseteq A$, $B'' \subseteq B$, $|H| \leq k$, such that hardening this set of entities ensures failure of no more than $E_F$ entities at the end of the cascading failure process, if the entities $(A' - A'') \cup (B' - B'')$ fail at time step $t = 0$. 
Dependency Relations:
Most general form of the dependency relation

\[ a_i \leftarrow b_j b_k b_l + b_m b_n + b_p \]

In the general form
- No. of Min-terms are arbitrary
- Size of Min-terms are arbitrary

We consider four special cases:

| Case     | No. of Min-terms | Size of Min-terms |
|----------|------------------|-------------------|
| Case 1   | 1                | 1                 |
| Case 2   | Arbitrary        | 1                 |
| Case 3   | 1                | Arbitrary         |
| Case 4   | Arbitrary        | Arbitrary         |
| (General)|                 |                   |
In case I, we consider ‘live equations’ of the form:

\[ a_1 \leftarrow b_1 \]
\[ a_2 \leftarrow b_2 \]
\[ b_1 \leftarrow a_3 \]
\[ b_3 \leftarrow a_3 \]

This problem is polynomial time solvable.

**Algorithm 1**

1. We construct a directed graph \( G=(V,E) \), where
   \( V \) is the set of entities, i.e., \( V=A \cup B \) and \( E \) is
   the set of implications

2. For each node \( x_i \in V \), construct a transitive closure set \( C_{x_i} \). Let \( x_i \) be called the “seed
   entity” for the set \( C_{x_i} \)

3. Remove all transitive closure sets which are proper subsets of some other transitive
   closure set

4. Sort the remaining transitive closure sets \( C_{x_i} \) in non increasing order of their cardinalities

5. The set of the seed entities of the top \( \kappa \) transitive closure sets is the required solution
   (select arbitrary entities if there are not enough transitive closure sets)
Time complexity of Algorithm 1 is \( O((n + m)^3) \), where \( n \) is the number of \( A \) type entities and \( m \) is the number of \( B \) type entities.

For each pair of transitive closure sets \( C_{x_i} \) and \( C_{x_j} \) produced in step 2 of Algorithm 1, either
\[ C_{x_i} \cap C_{x_j} = \emptyset \text{ or } C_{x_i} \cap C_{x_j} = C_{x_i} \text{ or } C_{x_i} \cap C_{x_j} = C_{x_j} \]
where \( x_i \neq x_j \)

Algorithm 1 gives an optimal solution for our problem when all the IDRIs belong to Case 1.
In case II, we consider Interdependency Implications of the form:

\[ a_1 \leftarrow b_1 b_2 \]

In this case, the problem of identification of \( K \) most vulnerable nodes is an NP-complete problem

The Densest \( k \)-subhypergraph problem can be reduced to this problem
Densest $p$-subgraph Problem: Given a graph $G$ and a parameter $p$, the densest $p$-subgraph problem is to find a set of $p$ vertices with maximum number of induced edges.

Densest $p$-subhypergraph (DpSH) Problem: Given a hypergraph $G = (V; E)$ and a parameter $p$, find a set of $p$ vertices with maximum number of hyperedges in the subgraph induced by this set. (M: parameter representing the maximum number of hyperedges in the decision version of the problem)

Construction of an instance of an ENH Problem from an instance of the DpSH Problem

1. $A = E$, $B = V$
2. $F(A, B)$: If $e_i = \{v_{i,1}, v_{i,2}, \ldots, v_{i,q}\}$ then $a_i \leftarrow b_{i,1} b_{i,2} \ldots b_{i,q}$
3. $A' = A$, $B' = B$
4. $k = p$, $E_F = |V| + |E| - (p + M)$
The Entity Hardening (ENH) problem

INSTANCE: Given:
(i) An interdependent network system \( T(A, B, \mathcal{F}(A, B)) \), where the sets \( A \) and \( B \) represent the entities of the two networks, and \( \mathcal{F}(A, B) \) is the set of IDR.
(ii) The set of \( \mathcal{K} \) most vulnerable entities of the system \( A' \subseteq A \) and \( B' \subseteq B \)
(iii) Two positive integers \( k, k < \mathcal{K} \) and \( E_F \).

QUESTION: Is there a set of entities \( \mathcal{H} = A'' \cup B'', A'' \subseteq A, B'' \subseteq B, |\mathcal{H}| \leq k \), such that hardening \( \mathcal{H} \) entities results in no more than \( E_F \) entities to fail after entities \( A' \cup B' \) fail at time step \( t = 0 \).

Construction of in instance of an ENH Problem from an instance of the DpSH Problem

1. \( A = E, B = V \)
2. \( F(A, B): \) If \( e_i = \{v_{i,1}, v_{i,2}, \ldots, v_{i,q}\} \) then \( a_i \leftarrow b_{i,1} b_{i,2} \ldots b_{i,q} \)
3. \( A' = A, B' = B \)
4. \( k = p, E_F = |V| + |E| - (p + M) \)

**DpSH \rightarrow ENH**

If there exists a subgraph of \( p \) vertices in \( G = (V, E) \) that completely contains \( M \) hyperedges, then in the instance of ENH, the corresponding \( p, B \) type entities will harden additional \( M, A \) type entities. As such the maximum number of nodes that can fail, \( E_F \), can at most be \( |V| + |E| - (p + M) \).

**ENH \rightarrow DpSH**

If there exists \( p \) entities, whose hardening ensures hardening of additional \( M \) entities, then these \( p \) entities must be of type \( B \). Then the corresponding nodes in \( G = (V, E) \), must be Inducing a subgraph of \( G \) where \( M \) hyperedges are completely contained.
Consider the instances of ENH where only A type entities appear on the LHS of the Implicative Dependency Rules. This set of instances constitute a restricted version of ENH (RENH). The restricted version of ENH is equivalent to the DpSH. As DpSH is hard to estimate, so is RENH and hence ENH.
In case III, IDR are of the form:

\[
\begin{align*}
    a_1 & \leftarrow b_1 + b_2 \\
    a_2 & \leftarrow b_2 + b_3 \\
    b_1 & \leftarrow a_3 + a_4 \\
    b_3 & \leftarrow a_1 + a_2
\end{align*}
\]

In this case, the problem of Entity Hardening Problem is an NP complete.

Transformation from the Set Cover Problem
Set Cover (SC) Problem

Instance: A set \( S = \{s_1, s_2, ..., s_n\} \), a set \( S \) of \( m \) subsets of \( S \), i.e., \( S = \{S_1, ..., S_m\} \), where \( S_i \subseteq S \), \( \forall \ i, \ 1 \leq i \leq m \), integer \( p \).

Question: Is there a \( p \) element subset \( S' \) of \( S \) that completely covers of the set \( S \)?

The Entity Hardening (ENH) problem

INSTANCE: Given:
(i) An interdependent network system \( \mathcal{I}(A, B, \mathcal{F}(A, B)) \), where the sets \( A \) and \( B \) represent the entities of the two networks, and \( \mathcal{F}(A, B) \) is the set of IDRMs.
(ii) The set of \( \mathcal{K} \) most vulnerable entities of the system \( A' \cup B' \), where \( A' \subseteq A \) and \( B' \subseteq B \)
(iii) Two positive integers \( k, k < \mathcal{K} \) and \( E_F \).

QUESTION: Is there a set of entities \( \mathcal{H} = A'' \cup B'' \subseteq A, B'' \subseteq B, |\mathcal{H}| \leq k \), such that hardening \( \mathcal{H} \) entities results in no more than \( E_F \) entities to fail after entities \( A' \cup B' \) fail at time step \( t = 0 \).

Construction of an instance of an ENH Problem from an instance of the SC Problem

1. \( A = S, B = S' \)
2. \( F(A, B): \) If \( s_i \in \{S_{i1}, ..., S_{iq}\} \) then
   \( s_i \leftarrow S_{i1} + ... + S_{iq} \)
3. \( A' = \emptyset, B' = B \)
4. \( k = p, E_F = m - p \)
Set Cover (SC) Problem

Instance: A set $S=\{s_1, s_2, ..., s_n\}$, a set $\mathcal{S}$ of $m$ subsets of $S$, i.e., $\mathcal{S} = \{S_1, ..., S_m\}$, where $S_i \subseteq S$, $\forall$ $i$, $1 \leq i \leq m$, integer $p$.

Question: Is there a $p$ element subset $S'$ of $\mathcal{S}$ that completely covers of the set $S$?

Max Cover Problem: Set Cover Problem with a budget, i.e., try to cover as many elements of $S$ as possible, subject to a budget constraint.

Protection set – The protection set of an entity $x_i \in \{A \cup B\}$ is the set difference between the set of entities that would eventually fail if the entities $\{A \cup B\}$ fail at time $t = 0$, and the set of entities that would eventually fail if the entities $\{\{A \cup B\} - \{x_i\}\}$ fail at time $t = 0$. 
The Entity Hardening (ENH) problem

INSTANCE: Given:
(i) An interdependent network system $\mathcal{I}(A, B, \mathcal{F}(A, B))$, where the sets $A$ and $B$ represent the entities of the two networks, and $\mathcal{F}(A, B)$ is the set of IDR.s.
(ii) The set of $K$ most vulnerable entities of the system $A' \cup B'$, where $A' \subseteq A$ and $B' \subseteq B$
(iii) Two positive integers $k, k < K$ and $E_F$

QUESTION: Is there a set of entities $\mathcal{H} = A'' \cup B'', A'' \subseteq A, B'' \subseteq B, |\mathcal{H}| \leq k$, such that hardening $\mathcal{H}$ entities results in no more than $E_F$ entities to fail after entities $A' \cup B'$ fail at time step $t = 0$.

Theorem 5. There exists an $1 - \frac{1}{e}$ approximation algorithm that approximates the ENH problem for Case III.

Utilizes the Approximation Algorithm for the Max Cover Problem
In Case IV, we consider IDR\(s\) of the form:
\[
a_1 \leftarrow b_1 b_2 + b_3
\]

Since ENH Problem Case II and Case III are subsets of ENH IV, and ENH II and ENH III are NP-complete, ENH IV must be NP-complete.

We provide an optimal solution using Integer Linear Programming.

We also provide a heuristic solution and compare its performance against the optimal solution.
Entity Hardening – Optimal Solution

- Solved using Integer Linear Programming (ILP)
- Consider entity set A having m entities and entity set B having n entities
- Objective of ILP: Minimize the number of entities dead at \( t = m + n - 1 \) (final time step) to get the desired set of k entities hardened at \( t = 0 \)

\[
\min \sum_{i \in A} + \sum_{j \in B} \\
\sum_{i \in A} \min_{q \in A} \sum_{j \in B} 1_{q \in A \text{ leading time}} \\
\min_{j \in B} \sum_{i \in A} 1_{q \in B \text{ leading time}} \\
\]

\( x_{ij} = \begin{cases} 
1 & q \in A \text{ leading time} \\
0 & \text{otherwise} 
\end{cases} \)

\( y_{ij} = \begin{cases} 
1 & q \in B \text{ leading time} \\
0 & \text{otherwise} 
\end{cases} \)
• **Constraints of the ILP**
  
  - **Constraint Set 1**: The number of entities that is to be hardened is fixed to $k$
  
  - **Constraint Set 2**: Only if an entity is not defended then the entity can fail at the initial time step
  
  - **Constraint Set 3**: If entity $a_i$ ($b_i$) fails at a time step, it should continue to remain failed for all remaining time steps
  
  - **Constraint Set 4**: Failure propagation modeling for Case 4 type dependencies (3-Step procedure taking into account the hardening process). Also takes into account that if an entity is hardened at initial time step it does not fail at any subsequent time step
Protection set – The protection set of an entity \( x_i \in \{A \cup B\} \) is the set difference between the set of entities that would eventually fail if the entities \( \{A \cup B\} \) fail initially, and the set of entities that would eventually fail if the entities \( \{\{A \cup B\} - \{x_i\}\} \) fail initially. This is represented as \( P(x_i \mid A' \cup B') \).

Minterm Coverage Number - For an entity \( x_i \in A \cup B \) the Minterm Coverage Number is defined as the number of minterms that can be removed from the IDR without affecting the cascading process by hardening the entity \( x_i \) when all entities in \( A' \cup B' \) fails initially. This is represented as \( M(x_i \mid A' \cup B') \).
Entity Hardening – Heuristic Solution

Heuristic Solution

Data: An interdependent network $\mathcal{I}(S, \mathcal{F}(A, B))$ (with $S = A \cup B$), set of entities $A' \cup B'$ failed initially causing maximum failure in the interdependent network with $|A'| + |B'| = \mathcal{K}$ and hardening budget $k$.

Result: Set of hardened entities $\mathcal{H} = A'' \cup B''$.

begin
  \begin{algorithm}
  Initialize $S' \leftarrow A' \cup B'$;
  \begin{algorithm}
  \begin{algorithm}
  while ($k$ entities are not hardened) do
  \begin{algorithm}
  For each entity $x_i \in S$ compute the Protection Set $P(x_i|S')$;
  \begin{algorithm}
  Choose the entity $x_d$ with highest cardinality of the set $|P(x_d|S')|$;
  if (more than one entity has the same highest cardinality value) then
  \begin{algorithm}
  For each such entity $x_j$ compute the Minterm Coverage Number $M(x_j|S')$;
  \begin{algorithm}
  Choose the entity $x_d$ with highest Minterm Coverage Number;
  In case of a tie choose arbitrarily;
  \end{algorithm}
  Update $S \leftarrow S - P(x_d|S')$;
  Update $\mathcal{F}(A, B)$ by removing (i) IDR's corresponding to all entities in $P(x_d|S')$, and (ii) occurrence of these entities in IDR's of entities not in $P(x_d|S')$;
  if ($x_d \in S'$) then
  \begin{algorithm}
  Update $S' \leftarrow S' - \{x_d\}$;
  \end{algorithm}
  \end{algorithm}
  \end{algorithm}
  \end{algorithm}
  \end{algorithm}
  \end{algorithm}
  \end{algorithm}
  \end{algorithm}
  \end{algorithm}
Experimental Results

Data Collection for Maricopa County

- **Communication Network Data:** Data of Cell Towers, Fiber Lit Buildings and Fiber Routes collected from Geotel (www.geotel.com) for Maricopa County.

- **Power Network Data:** Data of Power Plants and Transmission Lines collected from Platts (www.platts.com) for Maricopa County.
Data from Maricopa County

Power Network (PNE):
- Power plants: 70, Transmission Lines: 470

Communication Network (CNE):
- Cell Towers: 2960, Fiber-lit building: 7100,
- Fiber Links: 42,723

We wanted to evaluate if certain “regions” of Maricopa county are more vulnerable than others. A “region” is made up of one or more zip codes.
Sample Problem Instance

Data from a region of Maricopa County (Zip Code 85354)

Power Network (PNE):
Generators: 5, Transmission Lines: 24

Communication Network (CNE):
Cell Towers: 10, Routers: 5, Fiber Links: 4

Dependency Relations:
\[
\begin{align*}
a_0 & \leftarrow b_3 + b_{10}b_{15} \\
a_1 & \leftarrow b_5 \\
a_2 & \leftarrow b_4 + b_{11}b_{16} \\
a_3 & \leftarrow b_4 + b_{14}b_{17} \\
a_4 & \leftarrow b_9 + b_{13}b_{18} \\
b_0 & \leftarrow a_0a_{28} + a_1a_5 \\
b_1 & \leftarrow a_4a_6 \\
b_2 & \leftarrow a_0a_7 + a_4a_8 \\
b_3 & \leftarrow a_0a_9 \\
b_4 & \leftarrow a_2a_{10} + a_3a_{11} \\
\end{align*}
\]

\[
\begin{align*}
b_5 & \leftarrow a_1a_{12} + a_2a_{13} \\
b_6 & \leftarrow a_1a_{14} \\
b_7 & \leftarrow a_2a_{15} + a_4a_{16} \\
b_8 & \leftarrow a_1a_{17} + a_4a_{18} \\
b_9 & \leftarrow a_4a_{19} \\
b_{10} & \leftarrow a_2a_{20} + a_4a_{21} \\
b_{11} & \leftarrow a_0a_{22} + a_4a_{23} \\
b_{12} & \leftarrow a_0a_{24} + a_3a_{25} \\
b_{13} & \leftarrow a_2a_{26} \\
b_{14} & \leftarrow a_4a_{27}
\end{align*}
\]
Experimental Results

Comparison of Optimal (ILP) and Heuristic solutions for 5 Maricopa County regions
Experimental Results (Continued ..)

Region 3

Region 4

Region 5
Thank you!