DECAY OF TURBULENCE IN FLUIDS WITH POLYTROPIC EQUATIONS OF STATE

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Abstract: We present numerical simulations of decaying hydrodynamic turbulence initially driven by solenoidal (divergence-free) and compressive (curl-free) driving. Most previous numerical studies for decaying turbulence assume an isothermal equation of state (EOS). Here we use a polytropic EOS, $P \propto \rho^\gamma$, with polytropic $\gamma$ ranging from 0.7 to 5/3. We mainly aim at determining the effects of polytropic $\gamma$ and driving schemes on the decay law of turbulence energy, $E \propto t^{-\alpha}$. We additionally study probability density function (PDF) of gas density and skewness of the distribution in polyturbulence driven by compressive driving. Our findings are as follows. First of all, we find that even if polytropic $\gamma$ does not strongly change scaling relation of the decay law, the driving schemes weakly change the relation; in our all simulations, turbulence decays with $\alpha \approx 1$, but compressive driving yields smaller $\alpha$ than solenoidal driving at the same sonic Mach number. Second, we calculate compressive and solenoidal velocity components separately and compare their decay rates in turbulence initially driven by compressive driving. We find that the former decays much faster so that it ends up having a smaller fraction than the latter. Third, the density PDF of compressively driven turbulence with polytropic $\gamma > 1$ deviates from log-normal distribution: it has a power-law tail at low density as in the case of solenoidally driven turbulence. However, as it decays, the density PDF becomes approximately log-normal. We discuss why decay rates of compressive and solenoidal velocity components are different in compressively driven turbulence and astrophysical implication of our findings.

Key words: ISM; general - hydrodynamics - turbulence

1. INTRODUCTION

Supersonic turbulence in the interstellar medium (ISM) is a well-known phenomenon and plays an essential role in star formation processes (Larson 1981; Padoan & Nordlund 2002; Mac Low & Klessen 2004). Given that driving mechanisms of astrophysical turbulence are usually intermittent in both space and time, it is natural for turbulence to decay. Earlier studies showed that non-driven turbulence decays quickly in approximately one large-eddy turnover time (for hydrodynamic turbulence, see e.g., Lesieur 2008; for magnetohydrodynamic turbulence, see Mac Low et al. 1998; Stone et al. 1998), which is consistent with the fact that energy cascade occurs within one large-scale eddy turnover time even in the case of strongly magnetized turbulence (Goldreich & Sridhar 1995). It has been analytically suggested that turbulence energy decays with a power-law form of $E \propto t^{-\alpha}$ (see e.g., chap.7 of Lesieur 2008). Results from previous numerical studies of turbulence have converged that the value of $\alpha$ is approximately unity, and it does not strongly depend on the degree of magnetization and compressibility (Mac Low et al. 1998; Stone et al. 1998; Biskamp & Müller 1999; Ostriker et al. 2001; Cho et al. 2002).

Even if the consensus that turbulence quickly decays has been numerically established for the last two decades, the previous numerical results depend heavily on isothermal condition. However, as long as various density and temperature phases in the ISM (Ferrière 2001) are concerned, the use of polytropic equation of state (EOS)

$$P = K \rho^\gamma,$$

where $P$ is the pressure, $\rho$ is the density, and both $K$ and $\gamma$ are constants, is a valid approach (see Vázquez-Semadeni et al. 1996 and reference therein). The polytropic EOS has been used for many astrophysical problems, such as complex chemical processes (Spaans & Silk 2004; Glover & Mac Low 2007b), or turbulence (Scalo et al. 1998; Li et al. 2003; Glover & Mac Low 2007b; Federrath & Banerjee 2015).

Besides a variety of density and temperature phases, a wide range of driving agents of turbulence also characterizes interstellar turbulence (see Federrath et al. 2017 for a review). Based on its compressibility, we may consider two extreme types of driving: solenoidal (divergence-free) and compressive driving (curl-free). Until recently, solenoidal driving had been mainly used for turbulence studies. However, Federrath et al. (2010) showed that compressive driving and solenoidal driving can have different statistics. For
example, they showed that “the former yields stronger compression at the same RMS Mach number than the latter, resulting in a three times larger standard deviation of volumetric and column density probability distributions.” To our best knowledge, scaling relations of decay of polytropic turbulence driven by compressive driving have not been studied yet.

The main goal of this paper is to examine whether decay exponent $\alpha$ depends on the value of the polytropic $\gamma$. Here we concentrate on decay of polytropic turbulence driven by either solenoidal or compressive driving in both transonic and supersonic regimes. Hence, we expect to demonstrate what impacts polytropic EOS and types of driving have on decaying turbulence. In addition, we also investigate probability density function (PDF) of gas density and skewness of the PDF in decaying polytropic turbulence initially driven by compressive driving.

The paper is organized as follows. We explain our motivation and numerical method in Section 2 and present the results from our numerical simulations in Section 3. We discuss our finding and its astrophysical implication and give summary in Section 4.

### 2. Motivation and Numerical Method

#### 2.1. Motivation

As we described earlier, decay of solenoidally driven isothermal turbulence follows $E \propto t^{-\alpha}$ with $\alpha \approx 1$. The type of driving or the polytropic $\gamma$ may affect this scaling relation.

First, if we use compressive driving, it yields more compressions at the same Mach number. Therefore, while decaying, compressed regions could generate additional kinetic energy via expansion, which could affect the rate of turbulence decay.

Second, regarding the effects of polytropic $\gamma$ on decaying turbulence, only limited parameter study is available. Mac Low et al. (1998) found that supersonic turbulence with $\gamma = 1.4$ decays with $\alpha \sim 1.2$. For isothermal cases (i.e., $\gamma = 1$), they found that $\alpha$ is nearly unity. This suggests that the scaling exponent $\alpha$ in $E \propto t^{-\alpha}$ only weakly depends on polytropic $\gamma$ as assumed by Davidovits & Fisch (2017). However, Mac Low et al. (1998) used random initial velocity perturbation, which follows a power-law, and a constant initial density. Therefore, it is necessary to test the decay law using initial velocity and density data cubes from actual turbulence simulations with both soft EOS (i.e., polytropic $\gamma < 1$) and stiff EOS (i.e., polytropic $\gamma > 1$).

Third, the effects of polytropic $\gamma$ and the type of driving on density PDF of turbulence have also been addressed in several previous studies. For example, Federrath et al. (2010) showed that solenoidal driving and compressive driving can produce different statistics of isothermal turbulence as mentioned earlier. In addition, Federrath & Banerjee (2013) found that density PDF of solenoidally driven turbulence with polytropic $\gamma = 5/3$ has a clear power-law tail at low density, which is not observed in isothermal turbulence. However, earlier studies have not addressed turbulence with polytropic

| Run          | Driving$^a$ | $\gamma^b$ | Resolution | $k^c_{\text{c}}$ | $M^d_{\text{s}}$ |
|--------------|-------------|-------------|-------------|-----------------|-----------------|
| SMS1-0.7     | Solenoidal  | 0.7         | 512$^3$     | 8.0             | $\sim 1$       |
| SMS1-1.0     | Solenoidal  | 1.0         | 512$^3$     | 8.0             | $\sim 1$       |
| SMS1-5/3     | Solenoidal  | 5/3         | 512$^3$     | 8.0             | $\sim 1$       |
| SMS3-0.7     | Solenoidal  | 0.7         | 512$^3$     | 8.0             | $\sim 3$       |
| SMS3-1.0     | Solenoidal  | 1.0         | 512$^3$     | 8.0             | $\sim 3$       |
| SMS3-5/3     | Solenoidal  | 5/3         | 512$^3$     | 8.0             | $\sim 3$       |
| SMS5-0.7     | Solenoidal  | 0.7         | 512$^3$     | 8.0             | $\sim 5$       |
| SMS5-1.0     | Solenoidal  | 1.0         | 512$^3$     | 8.0             | $\sim 5$       |
| SMS5-1.5     | Solenoidal  | 1.5         | 512$^3$     | 8.0             | $\sim 5$       |
| CMS1-0.7     | Compressive | 0.7         | 512$^3$     | 8.0             | $\sim 1$       |
| CMS1-1.0     | Compressive | 1.0         | 512$^3$     | 8.0             | $\sim 1$       |
| CMS1-5/3     | Compressive | 5/3         | 512$^3$     | 8.0             | $\sim 1$       |
| CMS3-0.7     | Compressive | 0.7         | 512$^3$     | 8.0             | $\sim 3$       |
| CMS3-1.0     | Compressive | 1.0         | 512$^3$     | 8.0             | $\sim 3$       |
| CMS3-1.5     | Compressive | 1.5         | 512$^3$     | 6.0             | $\sim 3$       |
| CMS5-0.7     | Compressive | 0.7         | 512$^3$     | 8.0             | $\sim 5$       |
| CMS5-1.0     | Compressive | 1.0         | 512$^3$     | 8.0             | $\sim 5$       |

$^a$ Driving schemes - either solenoidal or compressive driving.

$^b$ Polytropic exponent.

$^c$ The driving wavenumber at which the energy injection rate peaks.

$^d$ The sonic Mach number which is defined in Equation (3).
We use 512 grid points in our periodic computational box. The peak of energy injection occurs at \( k \approx 6 \) or 8, where \( k \) is the wavenumber. We drive turbulence in Fourier space and use solenoidal \((\nabla \cdot \mathbf{f} = 0)\) and compressive \((\nabla \times \mathbf{f} = 0)\) driving. In both drivings, the driving vectors continuously change with a correlation time comparable to the large-eddy turnover time. We also adopt polytropic EOS:

\[
P = P_0 \left( \frac{\rho}{\rho_0} \right)^\gamma = \left( \frac{\gamma c_s^2 \rho_0}{\gamma} \right) \left( \frac{\rho}{\rho_0} \right)^\gamma
\]

where \( P \) is the normalized pressure, and \( P_0, c_0 \), and \( \rho_0 \) are the initial pressure, sound speed, and density, respectively. The sonic Mach number \( M_s \) is defined by

\[
M_s = \frac{v_{rms}}{c_0},
\]

where \( v_{rms} \) is the rms velocity. We vary the polytropic \( \gamma \) and the sonic Mach number \( M_s \) to consider both soft and stiff EOS in transonic and supersonic regimes.

Table 1 lists our simulation models. We use the notation XMS\(Y\), where \( X = S \) or \( C \) refers to solenoidal or compressive driving; \( Y = 1, 3, \) or 5 refers to the sonic Mach number \( M_s \); \( Z = 0.7, 1.0, 1.5, \) or \( 5/3 \) refers to the value of polytropic \( \gamma \). We keep driving turbulence until the system reaches saturation stage, after which the driving is turned off to let turbulence freely decay. In decaying simulations, time is normalized by \( t = t_{code}/t_{ed} \). Here, \( t_{code} \) is the time in code unit, and \( t_{ed} \) is the large-eddy turnover time, where \( L = 2\pi \) is the size of the simulation box, \( k_{ed} = 6 \) or 8 is the driving wavenumber at which the energy injection rate peaks, and \( v_0 \) is the velocity at the moment turbulence starts decaying.
results imply that a similar argument holds true for $\gamma \neq 1$. Now, let us deal with decay of polytropic turbulence initially driven by compressive driving. Figures 3 and 4 show decay of $< v^2 >$ and $\sigma_{p/\rho_0}$, respectively. As in the case of solenoidal turbulence\(^1\), we use different values of $M_s$ (from left to right panel) and polytropic $\gamma$ (curves with different colors).

Similar to the result from solenoidal turbulence, both $< v^2 >$ and $\sigma_{p/\rho_0}$ in compressively driven turbulence also exhibit power-law decay, and polytropic $\gamma$ hardly affects the decay rate. According to Figure 3, the power-law exponent $\alpha$ is $\sim 1.0$ for $M_s \sim 1$, and $\sim 0.8$ for $M_s > 1$, which means that decay of compressively driven turbulence is slower than that of solenoidal turbulence at the same $M_s$. As can be seen from Figure 4, the power-law exponent $\alpha$ for the decay of $\sigma_{p/\rho_0}$ is nearly half of that for $< v^2 >$, which is consistent with the result from the previous section.

Note that, unlike the case of solenoidal turbulence, polytropic $\gamma$ slightly affects the decay of compressively driven turbulence. First, kinetic energy density in compressively driven turbulence shows bump-like features (indicated by the black arrow in each panel in Figure 3). This slight increase of kinetic energy density is most pronounced in the case of $M_s \sim 1$. Second, dip-like features (indicated by the black arrow in each panel in Figure 3) are clearly shown in the evolution of $\sigma_{p/\rho_0}$ at the nearly same time when the bump-like features in $< v^2 >$ occur. Third, we can see from Figure 4 that decay of $\sigma_{p/\rho_0}$ for $\gamma = 0.7$ is faster than that obtained for $\gamma > 0.7$ regardless of $M_s$.

3. RESULTS

3.1. Decay of Hydrodynamic Turbulence with Polytropic EOS

3.1.1. Decay of Turbulence Driven by Solenoidal Driving

In this subsection, we consider decaying polytropic turbulence initially driven by solenoidal driving. Figures 1 and 2 show decay of kinetic energy density $< v^2 >$ and standard deviation of density fluctuation $\sigma_{p/\rho_0}$, respectively. From left to right panel, the sonic Mach number $M_s$ is $\sim 1$, $\sim 3$, and $\sim 5$, respectively. Blue, red, cyan, green curves in each panel correspond to polytropic $\gamma = 0.7, 1.0, 1.5$, and $5/3$, respectively.

First of all, we can clearly see that decay of kinetic energy density follows a power-law form of $< v^2 > \propto t^{-\alpha}$, and $\alpha$ is almost same at the same $M_s$ regardless of the value of polytropic $\gamma$. The energy decay is steeper in the case of $M_s \sim 1$ ($\sim 1.2$) than supersonic cases ($\sim 1.0$). Second, similar to the case of $< v^2 >$, the decay of $\sigma_{p/\rho_0}$ is hardly affected by $\gamma$. In addition, the power-law exponent for $\sigma_{p/\rho_0}$ is nearly half of that for $< v^2 >$ at the same $M_s$. For the cases of $\gamma = 1$ (i.e., isothermal cases), this result is consistent with the fact that standard deviation of density fluctuation is approximately linear with the sonic Mach number (e.g., Padoan et al. 1997, Passot & Vázquez-Semadeni 1998), which implies $\sigma_{p/\rho_0} \propto M_s \propto \sqrt{< v^2 >} \propto t^{-\alpha/2}$. Our results imply that a similar argument holds true for $\gamma \neq 1$.

3.1.2. Decay of Turbulence Driven by Compressive Driving

Now, let us deal with decay of polytropic turbulence initially driven by compressive driving. Figures 3 and 4 show decay of $< v^2 >$ and $\sigma_{p/\rho_0}$, respectively. As in the case of solenoidal turbulence\(^1\), we use different values of $M_s$ (from left to right panel) and polytropic $\gamma$ (curves with different colors).

![Figure 3](image1.png)

Figure 3. The similar as Figure 1 but for compressively driven turbulence. Note that unlike Figure 1, a small amount of kinetic energy densities is generated, and this is most apparent in the case of $M_s \sim 1$.

![Figure 4](image2.png)

Figure 4. The similar as Figure 2 but for compressively driven turbulence.

\(^1\)We mean solenoidal turbulence by turbulence initially driven by solenoidal driving.
3.2. Decay of Solenoidal and Compressive Velocity Components in Compressively Driven Turbulence

In next two subsections, we only consider compressively driven turbulence. In this subsection, we first deal with how differently solenoidal and compressive modes decay. In order to address the issue, we decompose 3D velocity field of the compressively driven turbulence into solenoidal and compressive components. Figure 5 illustrates the decay of compressive ratio $\langle v_{\text{comp}}^2 \rangle / \langle v_{\text{tot}}^2 \rangle$ (upper panels) and solenoidal ratio $\langle v_{\text{spl}}^2 \rangle / \langle v_{\text{tot}}^2 \rangle$ (bottom panels). Here, $v_{\text{tot}}^2 = v_{\text{spl}}^2 + v_{\text{comp}}^2$, and $v_{\text{spl}}$ and $v_{\text{comp}}$ are solenoidal and compressive kinetic energy density, respectively. Blue, red, cyan, and green curves indicate polytropic $\gamma = 0.7, 1.0, 1.5$, and $5/3$, respectively.

First, Figure 5 shows that the compressive ratio decreases as turbulence decays. When $M_s \sim 1$ or $3$, the smaller the polytropic $\gamma$ is, the larger the compressive ratio is. However, when $M_s \sim 5$, we do not see strong dependence of the compressive ratio on $\gamma$. Second, and more importantly, the solenoidal ratio increases as turbulence decays and eventually becomes higher than the compressive ratio irrespective of $\gamma$ and $M_s$, which means that compressive energy density decays faster. Figure 5 clearly shows this in the case of isothermal turbulence initially driven by compressive driving, in which we plot the decay of solenoidal and compressive energy density separately. We can clearly see from the reference lines (see dotted lines with different colors in each panel) that compressive energy density (magenta curves) decays more quickly than solenoidal energy density (cyan curves), with this resulting in higher solenoidal energy density at the late stages of decay irrespective of $M_s$. 

Figure 6 illustrates the decay of compressive ratio in compressively driven turbulence. Only isothermal turbulence (i.e., polytropic $\gamma = 1.0$) is presented in this figure. Left panel: $M_s \sim 1$. Middle panel: $M_s \sim 3$. Right panel: $M_s \sim 5$. Black, cyan, and magenta curves represent total, solenoidal, and compressive energy density, respectively. The dotted lines with different colors are reference lines for different power-law forms. Note that compressive kinetic energy density decays much faster than solenoidal one.

Figure 5. Decay of ratio of both compressive (upper panels) and solenoidal (bottom panels) energy density in compressively driven turbulence. Left panels: $M_s \sim 1$. Middle panels: $M_s \sim 3$. Right panels: $M_s \sim 5$. Blue, red, cyan, and green curves denote polytropic $\gamma = 0.7, 1.0, 1.5$, and $5/3$, respectively.

Figure 6. Decay of both solenoidal and compressive energy density in compressively driven turbulence. Only isothermal turbulence (i.e., polytropic $\gamma = 1.0$) is presented in this figure. Left panel: $M_s \sim 1$. Middle panel: $M_s \sim 3$. Right panel: $M_s \sim 5$. Black, cyan, and magenta curves represent total, solenoidal, and compressive energy density, respectively. The dotted lines with different colors are reference lines for different power-law forms. Note that compressive kinetic energy density decays much faster than solenoidal one.
3.3. Density PDF and Skewness

In this subsection, we investigate density PDF and its skewness of compressively driven turbulence with polytropic EOS in driven and decay regime. We define skewness of the density PDF as follow:

\[
\text{Skew}(s) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{s_i - \langle s \rangle}{\sigma_s} \right)^3
\]

where \( N \) is the total number of data points, \( s \equiv \ln(\rho/\rho_0) \) is the natural logarithm of the density fluctuation, and \( \langle s \rangle \) denotes the spatial average value. Skewness measures asymmetry of a probability distribution. When the distribution is left (right)-skewed, skewness has a negative (positive) value.

Figures 7 and 8 show the density PDF of polytropic turbulence with \( M_s \sim 1 \) and \( \sim 3 \), respectively. Each solid curve with different colors in each panel cor-
Decay of Polytropic Turbulence

4. Discussion and Summary

The purpose of this study is to investigate the effects of EOS (i.e., value of polytropic $\gamma$) and driving schemes (i.e., solenoidal and compressive driving) on decaying turbulence and its statistics. In this paper, it is proved that the scaling relation of the decay law ($<v^2>\propto t^{-\alpha}$) does not show strong dependence on $\gamma$ and the driving schemes. Throughout the whole simulations, the kinetic energy density decays with $0.8<\alpha<1.2$. The range is nearly same as what [Mac Low et al. (1998)] found ($0.85<\alpha<1.2$).

For polytropic $\gamma>1$ cases, $\alpha$ ranges from 1.0 to 1.2 in solenoidal turbulence and from 0.8 to 1.0 in compressively driven turbulence, with the largest value of $\alpha$ being obtained in the case of $M_s\sim1$ in both driving schemes. This result confirms the assumption of [Davidovits & Fisch (2017)] that for $\gamma=5/3$, $\alpha$ falls into the range $1.0\sim1.5$ with slight dependence on initial Mach number.

Even if no relationship between polytropic $\gamma$ and scaling relation of the decay law ($<v^2>\propto t^{-\alpha}$) is found through our study, there are several noticeable characteristics in the case of compressively driven turbulence. First, the slight increase of $<v^2>$ and the associated decrease of $\sigma_{\rho/\rho_0}$ are found in Figures 8 and 3 respectively. As we described earlier, those effects can be interpreted as the additional energy released from compressed regions via expansion. Second, in the case of $M_s\sim1$, the effect is most significant. This is possibly due to relatively strong pressure compared to that of supersonic cases, which results in the stronger expansion. Third, when $M_s$ is same, the bump and dip like features are more prominent in the case of $\gamma=0.7$. This is because when compressive driving is applied, turbulent gas with $\gamma=0.7$ is more easily compressed. Thus, they are also easily expanding as turbulence decays, which leads to the clearer feature in the case of $\gamma=0.7$. Lastly, $\sigma_{\rho/\rho_0}$ decays more quickly for $\gamma=0.7$ than for $\gamma>0.7$ in both transonic and supersonic turbulence driven by compressive driving. The reason is that when $\gamma$ is less than one, expansion increases internal temperature, which dissipates density structures quickly. Therefore, density decays faster for smaller $\gamma$.

More interestingly, compressive energy density decays faster than solenoidal energy density in the case of turbulence initially driven by compressive driving as shown in Section 3.2. We can interpret this as follows. When turbulence initially driven by compressive driving decays, the energy of compressive component is dissipated through both turbulent cascade and the dissipation at shocks, and a fraction of the energy would convert into solenoidal energy. On the contrary, it would be only turbulent cascade that allows energy of solenoidal component to be dissipated. Therefore, because of less channels for energy dissipation and the contribution from compressive component, solenoidal energy density in turbulence initially driven by compressive driving decays slower. However, more detailed analysis, such as what fraction compressive kinetic energy changes into solenoidal kinetic energy, is limited and beyond the scope of this paper; further studies will be required to understand this issue in the quantitative manner.
Let us discuss astrophysical implication of our result. As described earlier, the polytropic exponent $\gamma$ is useful to describe a variety of components in the ISM. For example, the polytropic EOS with $\gamma \approx 0.8$ can represent the density range of $10^{6}cm^{-3} \leq n \leq 10^{4}cm^{-3}$, where $n$ is hydrogen number density (Glover & Mac Low 2007b). Giant molecular clouds can fall into this density range (Ferri` ere 2001). Also, the EOS with $\gamma \sim 1.4$ could represent the center of protostellar cores, which corresponds to the density range of $10^{12}cm^{-3} \leq n \leq 10^{17}cm^{-3}$ (Masunaga & Inutsuka 2000). Therefore, our result suggests that when driving of turbulence ceases to act, turbulence quickly decays in a corresponding dynamical timescale of a certain system irrespective of its spatial scale. Moreover, even if turbulence is initially driven by compressive driving, such as by supernova explosions, solenoidal motions will dominate as the turbulence decays due to much faster decay of compressive motions.

In summary, we have studied the influence of polytropic EOS and driving schemes on decaying turbulence and its statistics and found the following results.

1. We have demonstrated that there is no significant correlation between scaling relation of the decay law ($E \propto t^{-\alpha}$) and polytropic $\gamma$ in the case of solenoidally driven turbulence.

2. We have found that driving schemes have non-negligible effect on the decay rate of turbulence: the power-law index $\alpha$ for turbulence initially driven by compressive driving is smaller than that for turbulence initially driven by solenoidal driving.

3. We have proven no significant effect of polytropic $\gamma$ on decay rate of velocity in compressively driven turbulence.

4. The polytropic $\gamma$ has small effect on the density fluctuations in compressively driven turbulence: the smaller polytropic $\gamma$ is, the faster standard deviation of density fluctuation of the turbulence decays.

5. When we consider decay of solenoidal and compressive velocity components in compressively driven turbulence separately, energy of compressive velocity component decays much faster.

6. Regarding statistics of compressively driven turbulence, we have shown deviation of the density PDF from a log-normal distribution, especially for $\gamma > 1$. In addition, we have found that skewness of the density PDF of the turbulence becomes zero as it decays.

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