An OPERA inspired classical model reproducing superluminal velocities

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Abstract

In the context of the sensational results concerning superluminal velocities, announced recently by the OPERA Collaboration, we have proposed a classical model yielding a statistically calculated measured velocity of a beam, higher than the velocity of the particles constituting the beam. The two key elements of our model, necessary and sufficient to obtain this curious result, are a time-dependent “transmission” function and statistical method of the maximum-likelihood estimation.

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PACS: 06.30.Gv Velocity, acceleration, and rotation, 06.20.Dk Measurement and error theory, 07.05.Kf Data analysis: algorithms and implementation; data management, 13.15.+g Neutrino interactions, 14.60.Lm Ordinary neutrinos, 03.30.+p Special relativity

Inspired by the amazing results of the OPERA Collaboration [1], claiming that the velocity of light \(c\) has been beaten by a beam of neutrinos, we propose a simple classical model, which yields the curious effect of a seeming increase of “effective” velocity (see [2], for an earlier independent approach, and also [3], for a wave version). There are dozens of papers, which have recently appeared, adopting various attitudes towards the results presented by the OPERA Collaboration, for example [4, 5, 6, 7, 8] (see also an example of an older work on superluminal velocities [9]). The attitude of our paper is...
definitely skeptical. In view of the fact that we have found a (“mathematical”) model providing an “artificial” increase of real velocity, we are forced to put the results announced by the OPERA Collaboration in doubt. Moreover, we have also proposed a simple working example, such that appropriately fitting its parameters one can simulate the controversial results of the OPERA Collaboration. Strictly speaking, our paper does not indisputably invalidates the conclusions drawn by the OPERA Collaboration, but it seriously weakens their argumentation, indicating a logical gap in their reasoning.

Our model is purely classical and dynamics free. No new physics, nor quantum mechanics, nor even (classical) wave mechanics is involved. Only standard classical kinematics notions as well as the statistical method of the maximum-likelihood estimation (MLE) are used in our approach.

The assumptions of our model are the following. A spatially homogeneous, lasting the period $T$, beam of classical particles (“extraction”) moving with a constant speed $u \leq c$ travels from a source to a detector. The distance between the source and the detector is $d$. The probability density function (PDF) of the time of emission of the particles within the duration $T$ of production of the beam is given by the function $w(t)$. In an ideal situation (none of the particles is lost) we would obtain, according to classical kinematics, the measured data waveform $y(t) = w(t - t_0)$, where $t_0 = d/u$. Now, let as suppose that the fraction of the particles emitted, measured by the detector, due to some physical mechanism, is given by the (non-negative) “transmission” function

$$f(t) \equiv \frac{N_d(t)}{N_e(t)},$$

where $N_e(t)$ is the number of the particles emitted at the time instant $t$, and $N_d(t)$ is the number of the particles detected, which have been sent at the same time instant $t$. In other words, in general, not all particles emitted are detected ($f(t) < 1$) — obvious, and moreover $f(t) \neq \text{const}$ — conceivable. For simplicity, we will assume that the numbers of the particles, $N_d(t)$ and $N_e(t)$, are so large that we are allowed to use a continuous approximation. Then, the transmission function $f(t)$ is a (continuous) function satisfying the condition $0 \leq f(t) \leq 1$.

To be able to draw conclusions from experimental data, we should implement some statistical methodology. To be so precise as possible, in our approach, we adopt the method of the MLE, as the OPERA Collaboration has done [1]. In the framework of the MLE, we introduce the likelihood function $L$. 

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1. [1]
The logarithm \( l \) of \( L \) is given by the formula

\[
l(\delta t) \equiv \log [L(\delta t)] \equiv \sum_j \log [w(t_j + \delta t)],
\]

where \( t_j \) are the time instants corresponding to the measurement events at the detector, and the time deviation \( \delta t \) we are interested in (see [1], for details) provides the maximum of \( l \) (and also of \( L \)). As the numbers of the measured events \( t_j \) are large, in our continuous approach, instead of summation in (2), we should use integration with an appropriate integration measure \( z(t)dt \), where \( z(t) \) represents the time distribution of the experimental events detected by the OPERA. In fact, \( z(t) \) is determined by product of two factors. The first factor, \( y \), is proportional to the number of the particles sent, i.e. \( y(t + t_0) = w(t) \), and the second one is proportional to the transmission function \( f(t) \). Then,

\[
z(t) = f(t)y(t + t_0) = f(t)w(t).
\]

Finally,

\[
l(\delta t) = \int \log [w(t + \delta t)] f(t)w(t)dt.
\]

To demonstrate that our idea actually works, we propose a specific example. The parameters of the example are so fitted that it yields the time deviation \( \delta t \approx +75.5 \) ns (for comparison, the OPERA result is 60.7 ns). For calculational simplicity, we assume the Gaussian form of the PDF (see the solid line in Fig.1),

\[
w(t) = \exp \left( -\frac{t^2}{2} \right),
\]

as well as the Gaussian form of of the transmission function,

\[
f(t) = 1 - \frac{1}{10} \exp \left[ - (t - 1)^2 \right],
\]

where the one tenth in front of the exponent is reminiscent of “10% variation” in [2]. The time distribution of the “detected experimental data” \( z(t) \), corresponding to \( w(t) \) of the form (5) and to the transmission function \( f(t) \) of the form (6) is presented in Fig.1 by the dashed curve. In this (doubly) Gaussian case it is even possible to solve the problem analytically (see Appendix), but
The presented specific example is intended to “mathematically” qualitatively simulate the OPERA experiment, yielding the time deviation $\delta t \approx +75.5 \text{ ns}$. The solid curve represents the probability density function (PDF) $w(t)$, whereas the dashed one corresponds to the time distribution of the “detected experimental data” $z(t)$.

for our purposes a numerical value will do. It appears, that the maximum of (4) is attained for the time deviation

$$\delta t \approx +0.0288 \text{ s.d.},$$

(7)

where s.d. means the standard deviation. One can easily translate the dimensionless (7) into a dimensionfull entity. In the OPERA experiment $T \approx 10.5 \mu s$, whereas in our example we can reasonably assume, for definiteness, that $T$ equals (in dimensionless units, or in the units of the standard deviation) twice the two standard deviations, i.e. $T \approx 2 \cdot 2 = 4$ (see two vertical intervals in Fig.1). Then, the dimensionfull time deviation corresponding to (7) is

$$\delta t \approx \frac{0.0288 \cdot 10.5}{4} \mu s \approx 75.5 \text{ ns} \approx 60.7 \text{ ns (OPERA)}.$$

(8)

In particular, our analysis confirms the findings of [2] that the time deviation $\delta t$ is independent of the distance $d$, but depends on the shape of the
beam. Obviously, the numerical coincidence (8) has only an illustrative purpose. The non-zero time deviation $\delta t$ directly implies a modified velocity $v$ according to the elementary formula

$$\frac{v - u}{u} \approx \frac{\delta t}{t_0}.$$  

(9)

In our model the sign of $\delta t$ (positive in our example) depends on details of the form of the transmission function $f(t)$. E.g., reversing the sign in front of the exponent in (6) reverses the sign of $\delta t$.

We would like to stress that our specific example is not supposed to mimic a real situation in the OPERA experiment, but only to demonstrate that it is, in principle, possible to easily come to its conclusions without any reference to superluminal particles and/or some exotic phenomena. Any considerations concerning a possible physical mechanism governing the time-dependence of the “transmission” function $f(t)$ are outside the scope of our paper. We can only speculate that $f(t) \neq \text{const}$ could be a property of the source, or of the detector or it could follow from interactions in the Earth’s crust. Moreover, measurement errors, always encountered in real experiments, do not enter our considerations as they have nothing to do with the discussed effect.

It is also possible to intuitively explain the non-zero value of $\delta t$. Namely, deforming the shape of the PDF $w(t)$ with an appropriate (time asymmetric) transmission function $f(t)$ shifts a portion (its “upper” part) of the “waveform” $w(t)$ forward or backward, yielding positive or negative $\delta t$, respectively, which is next erroneously interpreted by the method of the MLE as a “real” time deviation. In a sense, our approach is a “corpuscular” analog of the well-known situation of “superluminal” velocity of light in material media [10], where to some extent, the role of $f(t)$ plays dispersion.

In conclusion, we would like to emphasize that there are two elements of our model, responsible for the curious effect of the superluminal velocity: the time-dependent transmission function $f(t)$, which should “favor” leading edges and “disfavor” trailing edges of the beam, and statistical methodology, erroneously assuming the method of the MLE. This is why it is conceivable that the paradoxical results of the OPERA Collaboration could be possibly avoided upon another approach, e.g. an approach giving preference to the front velocity or to another statistical method. Thus, no superluminal particles are necessary to explain the “superluminal” velocities derived by the OPERA Collaboration. Moreover, independently of the further fate of the
conclusions drawn by the OPERA Collaboration (acceptance or refutation), we have demonstrated that standard statistical methods, e.g. MLE, should be used with great care, as they can provide erroneous output. Therefore, the results of our paper do not rely on the final solution of the OPERA paradox.

Appendix A. Analytical form

For the Reader’s convenience, we present here analytical form of the expressions used in the main part of the paper. For $w(t)$ given by (5) and $f(t)$ given by (6), we obtain the integral (4)

$$
l(\delta t) = \int_{-\infty}^{+\infty} \left[ -\frac{(t+\delta t)^2}{2} \right] \left[ 1 - e^{-\frac{(t-1)^2}{10}} \right] \exp \left( -\frac{t^2}{2} \right) dt
$$

$$= \frac{\sqrt{\pi} \left[ \sqrt{3} (7 + 12 \delta t + 9 \delta t^2) - 270 \sqrt{e} (1 + \delta t^2) \right]}{270 \sqrt{2} \sqrt{e}}. \quad (A.1)$$

The maximum of $l(\delta t)$ is determined by its derivative

$$l' (\delta t) = \frac{\sqrt{\pi} \left[ \sqrt{3} (12 + 18 \delta t) - 540 \sqrt{e} \delta t \right]}{270 \sqrt{2} \sqrt{e}}, \quad (A.2)$$

and is attained for

$$\delta t = \frac{2}{30 \sqrt{3} \sqrt{e} - 3}. \quad (A.3)$$

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