Mass for the graviton

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Abstract
Can we give the graviton a mass? Does it even make sense to speak of a massive graviton? In this essay I shall answer these questions in the affirmative. I shall outline an alternative to Einstein Gravity that satisfies the Equivalence Principle and automatically passes all classical weak-field tests \((GM/r \approx 10^{-6})\). It also passes medium-field tests \((GM/r \approx 1/5)\), but exhibits radically different strong-field behaviour \((GM/r \approx 1)\). Black holes in the usual sense do not exist in this theory, and large-scale cosmology is divorced from the distribution of matter. To do all this we have to sacrifice something: the theory exhibits prior geometry, and depends on a non-dynamical background metric.

Keywords: graviton, mass.

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1 Introduction

Can we give the graviton a mass? What would giving the graviton a mass mean? Does it really make any sense to even speak of a massive graviton? These are subtle issues which in the past have led to considerable confusion. In particular, it is far from clear how to extrapolate a graviton mass defined for weak fields back into the strong-field regime. In this essay I shall show that doing so entails some surprises.

Recall that there is a general uniqueness result for Einstein gravity [1, pages 417, 429, 431]. Any theory of gravity which:

1. is a metric theory, (roughly speaking: satisfies the Equivalence Principle),
2. has field equations linear in second derivatives of the metric,
3. does not have higher-order derivatives in the field equations,
4. satisfies the Newtonian limit for weak fields,
5. and, does not depend on any prior geometry, (has no background metric),

must be exactly Einstein gravity itself, thereby implying an exactly massless graviton. Thus introducing a graviton mass will clearly require some rather drastic mutilation of the usual foundations underlying Einstein gravity. To accommodate a massive graviton without sacrificing experimental results such as the Eötvös experiment and the Newtonian limit, and do so without the theoretical complications of a higher-derivative theory, I shall explore the option of adding prior geometry by introducing a background metric.

In the weak-field limit ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}; h \ll 1$) the field equations for a massless graviton (in the Hilbert–Lorentz gauge) are

\[ \Delta \left[ h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \right] + O(h^2) = 8\pi G T_{\mu\nu}. \]  

We can get this from the action

\[ S = \int dx \sqrt{-\eta} \left\{ \frac{1}{2} \left[ h^{\mu\nu} \Delta h_{\mu\nu} - \frac{1}{2} h \Delta h \right] + O(h^3) - 8\pi G h^{\mu\nu} T_{\mu\nu} \right\}. \]

In this same limit, it is natural to define the field equations for a massive graviton to be

\[ \Delta \left[ h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \right] + \frac{m^2 c^2}{\hbar^2} \left[ h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \right] + O(h^2) = 8\pi G T_{\mu\nu}. \]  

\[ \text{As we shall soon see, “natural” is a loaded word in this context.} \]
The relevant action is now
\[ S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} h_{\mu\nu} \Delta h_{\mu\nu} - \frac{1}{2} h \Delta h + \frac{1}{2} \frac{m_g^2 c^2}{\hbar^2} \left[ h_{\mu\nu} h_{\mu\nu} - \frac{1}{2} h^2 \right] + O(h^3) - 8\pi G h_{\mu\nu} T_{\mu\nu} \right\}. \] (4)

The first term is easily extrapolated back to strong fields: it is simply the quadratic term in the linearization of the usual Einstein–Hilbert Lagrangian \( \int dx \sqrt{-g} R(g) \). It is the second term—the mass term for the graviton—that does not have a clear extrapolation back to strong fields. The key is to introduce a background metric \( g_0 \), which will not be subject to a dynamical equation (at least not classically), and write
\[ S_{\text{mass}}(g, g_0) = + \frac{1}{2} \frac{m_g^2 c^2}{\hbar^2} \int \sqrt{-g_0} \left\{ (g_0^{-1})_{\mu\nu} (g - g_0)_{\mu\sigma} (g_0^{-1})_{\sigma\rho} (g - g_0)_{\rho\nu} - \frac{1}{2} [(g_0^{-1})_{\mu\nu} (g - g_0)_{\mu\nu}]^2 \right\}. \] (5)

This mass term depends on two metrics: the dynamical spacetime-metric, \( g \), and the non-dynamical background metric, \( g_0 \), and makes perfectly good sense for arbitrarily strong gravitational fields. The weak-field limit consists of taking \( g = g_0 + h \) with \( h \) small.

2 The model

The full action for the variant theory of gravity I will consider in this essay is
\[ S = \int d^4x \left[ \sqrt{-g} \frac{R(g)}{16\pi G} + \sqrt{-g_0} L_{\text{mass}}(g, g_0) + \sqrt{-g} L_{\text{matter}}(g) \right]. \] (6)

Note that the background metric shows up in only one place: in the mass term for the graviton. The equations of motion for arbitrarily strong gravitational fields

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2Note that the linearized mass term is not the Pauli–Fierz term that is the main center of interest in the Van-Dam–Veltman, Ford–Van-Dam, and Boulware–Deser analyses. This fact is essential to having a well-behaved classical limit as the graviton mass goes to zero, and will be the topic of a more extensive forthcoming publication. I wish to thank Larry Ford for emphasizing the importance of the consistency problems wrapped up in this issue.

3There is great deal of arbitrariness in writing down the mass term. Any algebraic function of the metric and background metric that has the correct linearized behaviour up to second order in \( h \) would do. (See also [5].)

4The original version of this essay discussed a variant of the current model that was seriously flawed by internal inconsistency — that version was actually a variant of massive Brans–Dicke theory in disguise. I wish to thank David Garfinkle for pointing out the serious problems in that model.
fields are

\[ G^{\mu\nu} = 8\pi G \left( T^{\mu\nu} - \frac{m_g^2 c^2}{h^2} \left\{ (g_0^{-1})^\mu_\sigma \left[ (g - g_0)_\sigma_\rho - \frac{1}{2} (g_0)_\sigma_\rho (g_0^{-1})^\alpha_\beta (g - g_0)_\alpha_\beta \right] (g_0^{-1})^\rho_\nu \right\} \right). \] 

(7)

As the mass of the graviton goes to zero we smoothly recover the ordinary Einstein field equations — the Lagrangian and field equations are in this limit both identical to the usual ones. The only effect, at the level of the field equations, is to introduce what is effectively an extra contribution to the stress-energy tensor:

\[ T_{\text{mass}}^{\mu\nu} = - \frac{m_g^2 c^2}{8\pi G h^2} \left\{ (g_0^{-1})^\mu_\sigma \left[ (g - g_0)_\sigma_\rho - \frac{1}{2} (g_0)_\sigma_\rho (g_0^{-1})^\alpha_\beta (g - g_0)_\alpha_\beta \right] (g_0^{-1})^\rho_\nu \right\}. \] 

(8)

The field equations can now be rearranged to look more like the usual Einstein equations:

\[ G^{\mu\nu} = 8\pi G \left[ T^{\mu\nu}_{\text{mass}} + T^{\mu\nu} \right]. \] 

(9)

3 Experimental tests: Weak field

To precisely specify the weak-field limit we will have to pick a particular background geometry for our non-dynamical metric. The most sensible choice for almost all astrophysical applications is to take \( g_0 \) to correspond to a flat space-time (Minkowski space), in which case we absorb all of the coordinate invariance in the theory by going to Cartesian coordinates to make the components of \( g_0 \) take on the canonical Minkowski-space values. Once we have done this there is no further coordinate invariance left. In particular, it is meaningless to attempt to impose the Hilbert–Lorentz gauge condition, which is at first a little puzzling since we needed the Hilbert–Lorentz condition to set up the linearized weak field theory in the first place. The resolution to this apparent paradox is that the conservation of stress-energy implies, among other things, that

\[ \nabla_\mu T_{\text{mass}}^{\mu\nu} = 0. \] 

(10)

Here \( \nabla \) denotes the covariant derivative calculated using the dynamical metric \( g \). If we now linearize this equation around the non-dynamical metric \( g_0 \) we find that the Hilbert–Lorentz condition emerges naturally as a consequence of the equations of motion, not as a gauge condition. (Exactly the same phenomenon

\footnote{There is of course also considerable ambiguity in this effective stress-energy term, and in the strong-field equations of motion. Any strong-field equation that exhibits the appropriate linearized behaviour around flat spacetime is a reasonable candidate for “massive gravity”. From the point of view espoused in this essay, anything that linearizes to equation \( \ref{eq:linearized_field_equation} \) is acceptable.}
occurs when we give the photon a small mass via the Proca Lagrangian. The Lorentz condition, $\partial_\mu A^\mu = 0$, then emerges as consequence of electric current conservation, instead of being an electromagnetic gauge condition.\footnote{See also the similar comments in \cite{5}.} The analysis of the weak field limit proceeds in exactly the same way as for ordinary Einstein gravity. The gravitational field surrounding a point particle of mass $M$ and four-velocity $V^\mu$ is approximated at large distances by\footnote{In obtaining this particular form of the weak-field metric it is absolutely essential that the mass term I have introduced is not the Pauli–Fierz term. Again, further details will be deferred to a forthcoming paper \cite{6}.}

$$g_{\mu\nu} \approx \eta_{\mu\nu} + \frac{2GM}{r} \exp\left(-\frac{m_g r}{\hbar}\right) \left[2V_\mu V_\nu + \eta_{\mu\nu}\right].$$ \hspace{1cm} (11)

The only intrinsically new feature here is the exponential Yukawa fall-off of the field at large distances.

From astrophysical observations the \textbf{Particle Data Group} is currently quoting an experimental limit of\footnote{See also the recent paper by Will for new solar system limits on the graviton mass \cite{10}.}

$$m_g < 2 \times 10^{-29} \text{ electron–Volts} \approx 2 \times 10^{-38} m_{\text{nucleon}},$$ \hspace{1cm} (12)
corresponding to a Compton wavelength of

$$\lambda_g = \frac{\hbar}{m_g c} > 6 \times 10^{22} \text{ metres} \approx 2 \text{ Mega-parsecs}. \hspace{1cm} (13)$$

However, insofar as these estimates are based on galactic dynamics\footnote{Though Will has recently argued that there might be measurable effects in the gravity wave chirps due to black hole coalescence \cite{10}.}, the continuing controversies surrounding the dark-matter/missing-mass problem (relevant already at distance scales of order kilo-parsecs) should inspire a certain caution concerning the possibly over-enthusiastic nature of this limit. Still, even with an uncertainty of a factor of a thousand or so in this bound it is clear that the Compton wavelength of the graviton should be much larger than the dimensions of the solar system. The relevant exponentials are all well approximated by 1 for solar system physics, and so this variant theory of gravity automatically passes all solar system tests of gravity.\footnote{The only intrinsically new feature here is the exponential Yukawa fall-off of the field at large distances.}

There will be small (too small to be observable) effects on the propagation of gravitational waves. The speed of propagation will be slightly less than that of light, and will depend on frequency, with

$$v(\omega) = c \sqrt{1 - \frac{m_g^2 c^4}{\hbar^2 \omega^2}} = c \sqrt{1 - \frac{\lambda^2}{\lambda_g^2}}.$$ \hspace{1cm} (14)

For astrophysically relevant frequencies, and given the limit on the graviton mass, effects due to this phenomenon are too small to be observable.\footnote{The only intrinsically new feature here is the exponential Yukawa fall-off of the field at large distances.}
4 Experimental tests: Medium field

Although often presented as strong-field tests of Einstein gravity, the binary pulsar tests\[^1\][^2\] are really medium-field tests ($GM/r \approx 1/5$). The present theory also automatically passes all these medium-field tests. This can be seen by working perturbatively around the Schwarzschild geometry and noting that the effective contribution to the stress-energy arising from the graviton mass can be approximated as

$$T_{\text{mass}}^{\hat{\mu}\hat{\nu}} \approx -\frac{\hbar}{\ell_{\text{Planck}}^2 \lambda_g^2 \ell} \times \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + O[(GM/r)^2]. \quad (15)$$

**Tricky points:** Here I have written the Schwarzschild geometry in harmonic coordinates (and started by working in the coordinate basis). This is needed to be compatible with the Einstein–Lorentz condition. In harmonic coordinates the horizon is at $r = GM$. I then transform to the orthonormal frame attached to the harmonic coordinates to obtain the physical components of the graviton mass contribution to the stress-tensor. This is a double perturbation expansion — first in the mass of the graviton and secondly in the field strength. It should only be trusted for $r \ll \lambda_g$ and $r$ greater than and not too close to $M$. To extend this to the regime $r \approx \lambda_g$ and greater simply make the substitution $M \rightarrow M \exp(-r/\lambda_g)$ to obtain

$$T_{\text{mass}}^{\hat{\mu}\hat{\nu}} \approx -\frac{\hbar}{\ell_{\text{Planck}}^2 \lambda_g^2 \ell} \times \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + O[(GM/r)^2]. \quad (16)$$

This can also be obtained directly from the weak-field solution, equation\[^{10}\].\[^{11}\]

Even if we are in medium-strength fields, $GM/r \approx 1/5$, the extreme smallness of the graviton mass, (or equivalently the extreme largeness of its Compton wavelength), is enough to render this effective contribution to the stress tensor completely negligible. In the medium-field regime the spacetime geometry of the spherically symmetric vacuum solution ($T^{\mu\nu} = 0$; $T^{\mu\nu}_{\text{mass}} \neq 0$) will not deviate appreciably from the Schwarzschild geometry. For the same reason, the production of gravity waves and consequent orbital decay will not be significantly affected\[^{11}\].

\[^{10}\]Note that this means that the effective contribution to the stress energy violates the null energy condition (NEC) and in fact all of the classical energy conditions. This should not come as a surprise since asymptotically we demand that the effective gravitational mass of any isolated system to be an exponentially decreasing function of distance: $m(r) \approx M \exp(-r/\lambda_g)$. The only way that this can happen is by having a negative effective stress energy in the asymptotic regime. This argument does not necessarily imply that the NEC violations persist in the strong-field regime.

\[^{11}\]Subtle non-leading order effects have recently been discussed by Will\[^{14}\].
5 Experimental tests: Strong field

It is in very strong fields $GM/r \approx 1$, that the first evidence of dramatic departure from Einstein gravity arises. I have identified two areas where the physics is radically altered: black holes and cosmology. In both these cases the variant of Einstein gravity that I am describing in this essay is compatible with the current experimental situation.

5.1 Black holes?

Suppose we look at a spherically symmetric-static spacetime and write the metric as

$$ds^2 = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + R(r)^2[d\theta^2 + \sin^2(\theta)d\varphi^2].$$

(17)

Because we have used up all the coordinate freedom in reducing $g_0$ to its Minkowski space form we no longer have the freedom to go to Schwarzschild coordinates by setting $R(r) = r$. It is now easy to see that black holes (of the usual type) do not exist in this theory. Simply take the trace of the equations of motion to calculate the Ricci scalar

$$\text{Ricci} = -8\pi G T + \frac{m^2 c^2}{\hbar^2} \left\{ g_{tt}(g_{tt} - 1) + g_{rr}(g_{rr} - 1) + 2 \frac{R(r)^2 - r^2}{r^2} \right\}. \quad (18)$$

A normal Schwarzschild-type event horizon, should one exist, is characterized by the gravitational potential $g_{tt}$ going to zero, while $g_{rr}$ tends to infinity. But by the field equations, if we assume finiteness of the stress-energy tensor, this implies that the Ricci curvature is going to infinity. Thus singularities cannot be surrounded by event horizons of the usual type—any singularity that is present in this theory must be either (1) naked and violate cosmic censorship, or (2) the horizon must be abnormal in the sense that both $g_{tt}$ and $g_{rr}$ must tend to zero and change sign at the horizon.

We can deduce roughly where all the interesting physics happens by working perturbatively around the Schwarzschild geometry and looking at the effective stress-energy attributable to the presence of a graviton mass, which becomes Planck scale once

$$r < GM[1 + 2(\ell_{\text{Planck}}/\lambda_g)].$$

(19)

This will occur in a thin layer, of proper thickness

$$\delta\ell = \ell_{\text{Planck}} \sqrt{GM/\lambda_g},$$

(20)

located just above where the event horizon would have been if the graviton mass were exactly zero. Thus there will be a thin layer near $r = GM$, typically much narrower than a Planck length, where the geometry is radically distorted away from the Schwarzschild metric. (Remember that we are in harmonic coordinates.) Even though the metric is extremely close to Schwarzschild for
$r \gg GM$, the global topology of the maximally-extended spacetime is nowhere near that of the Kruskal–Szekeres manifold. Because of the assumed existence of the flat background metric $g_0$, the maximally-extended spacetime is in this case topologically $R^4$.

This is compatible with all current observational evidence regarding the existence of black holes. The current observational data really only shows the existence of highly compact heavy objects and does not directly probe the behaviour or existence of the event horizon itself.

Those aspects of standard black hole physics that do not depend critically on the precise geometry at or inside the event horizon will survive in this theory. For instance, most of the Membrane Paradigm of black hole physics (and the observational consequences thereof) survives [13]. As long as the “stretched horizon” is more than a few $\delta \ell$ above $r = GM$, the near-field geometry will be indistinguishable from Schwarzschild.

On the other hand, the process of Hawking radiation (semi-classical black hole evaporation) depends critically on the precise features of the event horizon. This is one area where we can expect radical changes from the conventional picture.

The fundamental reason why horizons are so different in this theory is that with two metrics in the theory, there are now simple scalar invariants, such as $g_0^{\mu \nu} g_{\mu \nu} = \text{tr}(g_0^{-1} g)$, which blow up at the event horizon. Because the non-dynamical background metric “knows” about asymptotic spatial infinity it carries information down to the horizon to let the theory know in a local way that the horizon is a very special place. In Einstein gravity, absent the non-dynamical metric, there is no local way for the theory to “know” that the horizon is special.

Another interesting side-effect of the existence of prior geometry is that the object which in standard Einstein gravity is called the stress-energy pseudotensor of the gravitational field can now be elevated to the status of a true tensor object. This permits us to now assign a well-defined notion of stress-energy to the gravitational field itself.

### 5.2 Cosmology?

A second situation in which a small mass for the graviton can have big effects is in cosmology: The fundamental physics is that with the Yukawa fall-off providing a long distance cutoff on the inverse-square law the motion of galaxies separated by more than a few Compton wavelengths becomes uncorrelated and the large-scale expansion of the universe is no longer dependent on the cosmological distribution of matter.

In a cosmological setting it is no longer obvious that we should use the flat-space Minkowski metric as background. I will keep the discussion general by using the usual assumed symmetry properties to deduce that the dynamical metric and non-dynamical metric should both be Friedmann–Robertson–Walker.
we put the physical metric into the canonical proper-time gauge

\[ ds^2 = -dt^2 + a^2(t) \, g_{ij} \, dx^i \, dx^j, \]  

(21)

then we no longer have full freedom to do so with the non-dynamical background metric and must be satisfied by taking

\[ ds_0^2 = -b_0^2(t) dt^2 + a_0^2(t) \, g_{ij} \, dx^i \, dx^j. \]  

(22)

Here \( b_0(t) \) and \( a_0(t) \) are (for the time being) arbitrary functions of cosmological time \( t \). The graviton mass term in the effective stress energy tensor is

\[
T^\mu_\nu_{\text{mass}} \approx \frac{\hbar}{\ell_{\text{Planck}}^2} \left\{ \eta^{\mu\nu} + \frac{1}{2a_0^2 b_0^2} \begin{pmatrix}
3a^2b_0^2 - a_0^2 & 0 & 0 & 0 \\
0 & -a^2b_0^2 - a_0^2 & 0 & 0 \\
0 & 0 & -a^2b_0^2 - a_0^2 & 0 \\
0 & 0 & 0 & -a^2b_0^2 - a_0^2
\end{pmatrix} \right\}.
\]

(23)

If we treat \( a_0 \) as a completely arbitrary function of \( t \) then \( b_0 \) is determined (as a function of \( a_0 \) and \( a \) via stress-energy conservation. However this leaves us with a completely arbitrary contribution to the cosmological stress-energy. That is: an arbitrary background geometry, \( g_0 \), can be used to drive an arbitrary expansion for the physical metric, \( g \). Consequently the expansion of the universe is completely divorced from the cosmological distribution of matter unless we place some constraints on the choice of background geometry.

One particularly attractive choice of cosmological background is the Milne universe [14, pages 198–199]. This consists of a spatially open universe with \( b_0 \) constant and \( a_0(t) = b_0 ct \). Remarkably, this is just flat Minkowski space in disguise, and in this sense even cosmology can be performed with a flat background.

A second attractive choice of cosmological background is the de Sitter universe [14, pages 77–78, 307–310]. This consists of a spatially flat universe with \( b_0 \) constant and \( a_0(t) = b_0 \exp(\kappa t) \).

There are many options for the theoreticians to explore, in that the combination of choosing a background geometry and graviton mass can potentially influence many standard cosmological tests (primordial nuclear abundances, cosmic microwave fluctuations, etc.)

Observational cosmologists might like to view this as an opportunity to feel justified in measuring \( a(t) \) directly from the observational data without interference from theoretical prejudices of how \( a(t) \) should behave in normal Einstein gravity. Once \( a(t) \) has been measured, it can be inserted into the Einstein equations to determine \( T^\mu_\nu_{\text{mass}} \). With some independent estimate for \( m_g \) we could then deduce the geometry of the background spacetime \( g_0 \) by inference from the observational data.

In particular, this is one way of fixing the age-of-the-oldest-stars problem currently afflicting observational cosmology. (This is by no means the most
attractive solution, attributing the current crisis to observational error or to a non-zero cosmological constant are less radical and more attractive solutions.)

6 Discussion

The variant theory of gravity I have sketched in this essay—a specific proposal for giving the graviton a mass—passes all present tests of classical gravity. In fact, since we have more free variables to play with, it is in better agreement with empirical reality than the current theory. This should be balanced against the fact that with enough free parameters we can fit almost anything. The most interesting part of the theory is that it radically changes ideas concerning black holes and cosmology—but does so in a way that is compatible with what we currently know.

The most disturbing part of the theory is the role of the non-dynamical background metric. For asymptotically flat spacetimes it seems clear that the appropriate background metric to take is flat Minkowski space. For cosmological situations the issue is less clear-cut but the choice of the Milne universe or de Sitter universe for the background geometry seems particularly appealing.

Clearly, the theory presented in this essay is far from being completely and definitively understood: there are a lot of issues (such as quantization [2, 3, 4, 5, 8, 9]) ripe for further development. What is particularly intriguing here is the fact that asking such a simple and basic question can lead to such unexpected surprises—classical gravity still exhibits a great potential for confounding the unwary.

Finally, I would be remiss in not mentioning related work of the Russian school, such as that of Logunov and co-workers [15, 16, 17, 18, 19, 20], and that of Loskutov [21, 22]. Additionally, there have also been attempts at deriving and calculating a graviton mass from first principles using fundamental string theory [23].

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12 From the point of view of Van Dam and Veltman [2, 3], Ford and Van Dam [4], and Boulware and Deser [5] the mass term I have discussed in this note is viewed as pathological due to an unboundedness of the energy. However this unboundedness is not something particular to the mass term itself but is merely a manifestation of the well-known instability that formally afflicts even the kinetic terms. Thus I would argue that the instability in the mass term is no worse than the known (formal) instability in the kinetic terms, and amenable to similar treatment. I plan to develop these issues more fully in [6].
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