Large scale circulation in turbulent Rayleigh-Benard convection of liquid sodium in cylindrical cell

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Abstract. In agreement with a recent experimental study by Xi et. al. [1], we also observed the sloshing mode of the large scale circulation (LSC) in our experimental investigation of turbulent Rayleigh-Benard convection of liquid sodium in a cylindrical cell of aspect ratio one. The Rayleigh and Prandtl numbers vary within the range of \((0.5 – 2.6) \times 10^7\) and \((8.7 – 9.9) \times 10^{-3}\) respectively. The characteristic times and corresponding Reynolds numbers of the general and sloshing modes of global circulation were estimated by analyzing the cross correlations of temperature oscillations and by determining the locations of peaks on the spectra of temperature fluctuations.

1. Introduction
Turbulent Rayleigh-Benard convection (RBC) depends strongly on the material properties of the studied fluid that are characterised by the Prandtl number, the ratio of kinematic viscosity of the fluid to thermal diffusivity, \(Pr = \nu/\chi\). Compared with the vast number of investigations at \(Pr \geq 1\) [2], the very-low-Pr regime appears almost as a less studied despite many applications. Turbulent convection in the Sun is present at Prandtl numbers \(Pr < 10^{-3}\) [3]. The Prandtl number in the liquid metal core of the Earth is \(Pr \sim 10^{-2}\) [4]. Convection in material processing [5], nuclear engineering [6], or liquid metal batteries [7] characterized by Prandtl numbers between \(3 \times 10^{-2}\) and \(10^{-3}\). At high Rayleigh number (say \(Ra_o = g\alpha\Delta T L^3/(\nu\chi) > 10^6\), here \(\alpha\) denotes the isobaric thermal expansion coefficient, \(g\) is the gravity acceleration, \(L\) is the layer height, and \(\Delta T\) is the temperature difference between the warm and cold isothermal boundaries), the RBC becomes fully turbulent and demonstrates a various flow dynamics [8].

One of the questions is of primary interest in studies of turbulent convection: what is the space-time structure of the large-scale circulation (LSC), which develops against the background of small-scale turbulence? Besides the heating intensity (Rayleigh number) it strongly depends on the cell geometry (the ratio of the diameter to height \(D/L\) if we confine ourselves to cylindrical cells) and properties of the fluid (Prandtl number). The LSC in case of RBC demonstrates various behaviours. Azimuthal motions, reorientations, cessations, and reversals are possible and frequency of the latter also depends on the cell aspect ratio [9]. All of these phenomena are now understood on the basis of a simple stochastic model derived from the Navier-Stokes equations [10, 11].

In addition to the general mode of flow, the torsional mode [12] and sloshing mode [1] are also possible. An extended study of RBC for a low Prandtl number fluid (mercury, \(Pr = 0.025\)) in a unit aspect ratio cell was performed by Cioni et al. [13]. Mercury has a relatively poor (for metals) thermal conductivity, which allowed the authors to achieve a wide range of Rayleigh numbers
(5·10^6 < Ra < 5·10^9) and obtain a dependence of the Reynolds number on the Rayleigh number. Dipolar LSC was observed persisting over the whole explored range. The global circulation never disappears, maintaining its strength as it changes in azimuth.

In this paper, we aim to study experimentally the convection of liquid sodium in a cylindrical cell of the unit aspect ratio. We consider the dependence of Reynolds number on Grashof number, and discuss the structure of LSC and its time evolution.

2. Experimental setup

The experimental setup consists of a closed cylindrical convective cell (1), bounded by two heat exchangers, heater (2), and cooler (3) (see figure 1a). The cell and heat exchangers are made of stainless steel pipe (inner diameter $D = 212$ mm with a 3.5 mm wall) and filled with liquid sodium. The length of the convective cell $L = 216$ mm.

The peculiarity of these heat exchangers is that instead of traditional thick copper plates, thin plates intensively washed with liquid sodium, are used [14]. The required flow of sodium in the chambers of the heat exchangers is provided by a travelling magnetic field, as in electromagnetic stirrers [15, 16]. Each heat exchanger is equipped with six induction coils (5), shifted to the outer end face of the corresponding heat exchanger. Experimental and numerical study showed that the electromagnetic influence of inductors on the metal in the convective cell is negligible [14].

![Figure 1](image.png)

Figure 1. (a) – Experimental setup: 1 – convective cell, 2 – hot heat exchanger, 3 – cold heat exchanger, 4 – expansion vessel, 5 – inductors. (b) – Convective cell: the bottom view (left); the cross-section M-M with five thermocouples for the A to E lines (centre); the cross-sections N-N with three thermocouples for the B, C, D, and F, G, H lines (right).

The electromagnetic force and, consequently, the stirring velocity intensity are determined by the electric current feeding the coils. A set of nine thermocouples is installed in each heat exchanger, allowing us to control the azimuthal and axial temperature distributions. The cell and heat exchanger chambers are separated by 1 mm thick copper discs, which are the end faces of the cell. A heater with a maximum power of 15 kVA is installed in the chamber of the hot heat exchanger (2). The cold heat exchanger has a copper needle-plate radiator in its casing and air flow is forced through it. The chamber (3) of the cold heat exchanger is connected to the expansion vessel (4). The entire setup is on a frame providing strongly vertical position of the cell.

The channel has 28 thermocouples with an isolated junction of 1 mm. The sample rate for each thermocouple is 75 Hz. The thermocouples are located on 8 lines of 3–5 pieces with an equal azimuthal step of 45° (figure 1b). These thermocouples are in five cross-sections of the cell, marked by numbers (1 to 5 from the hot face end to the cold one). All the thermocouples are installed at the same distance of 17 mm from the inner wall. The azimuthal position is marked clockwise by capital letters from A to H (looking from the cold end face). The line A has the upper position. The cross-sections 1, 3, and 5 contain eight thermocouples, and cross-sections 2 and 4 hold only two thermocouples (A and E).
3. Results
The experiments were carried out for strictly vertical position of the cell. The measurements were performed for 1 hour (270 000 data points per thermocouple). We use 28 thermocouples to analyze the structure of the temperature field in the convective cell. Temperature signals of all thermocouples in three cross-sections for $Ra = 1.49 \cdot 10^7$ are shown in figure 2 (averaging time is 20 s). Colour indicates the relative temperature: ascending (descending) flows of sodium are overheated (over cooled) and marked in shades of red (blue).

The azimuthal angle of the LSC plane characterized by the position of the maximum and minimum varies with time in a stochastic manner, but near some equilibrium state. Sometimes positions of the maximum and minimum of temperature swap places, which corresponds to a change in the LSC direction of rotation to the opposite (see figure 2 at 10 min). The so-called reversal of large scale circulation occurs in this case. In contrast to the reversal in common sense [9], it took place slowly for about 10 minutes and was not accompanied by a LSC stop (cessation). In our hourly experiments this event happened only once for $Ra = 1.49 \cdot 10^7$.

Another peculiarity can be observed in figure 2. The overheated and overcooled areas seem quite wide at any time. If we now reduce the time averaging step to 1 second, it becomes clear that the position of the local maximum and minimum changes with time, as shown in figures 3 and 4. These changes are periodic with frequency of about 0.1 Hz for $Ra = 1.96 \cdot 10^7$. This is so-called sloshing mode of the LSC, which was also observed in Ref. 1, for example, for water. This mode consists of a time-periodic lateral displacement of the entire LSC circulation plane away from the center line that is in phase along the entire cylinder axis.
Thermocouples were also used to measure the average axial velocity component in the area between adjacent thermocouples. This velocity is estimated from the position of the maximum of the cross-correlation function calculated for each pair of signals from adjacent sensors (small plots in figure 5). The fluctuations of temperature were sufficiently high in all experiments. The maximal correlation in all measurements of axial velocity was in the range of 0.6–0.8 and the peaks were sharp, allowing us to determine the mean velocity with uncertainty below 5%.

To characterize the mean intensity of the large-scale flow, we calculated the mean velocity $U_{cc}$, averaged over absolute values of velocities measured at eight triplets of thermocouples A1–A3–A5, B1–B2–B3, C1–C2–C3, D1–D2–D3, E1–E3–E5, F1–F2–F3, G1–G2–G3, and H1–H2–H3. For each of the triplet on small plots in figure 5 two cross-correlation functions are demonstrated. In the large polar plots the relative temperature profile is shown. Positive correlation times of the main peaks correspond to positive values of temperature and vice versa. Additional peaks of positive and negative correlation time correspond to sloshing mode oscillations with a period of about 10 seconds (see zooming in figure 6).

Another way to estimate the mean velocity of global circulation (or another flow mode, e.g. sloshing mode) is provided by power spectral density of temperature fluctuations. There are pronounced peaks on spectra of temperature signals of all thermocouples inside the cell. Takeshita et al. (1996) [17] have detected the same oscillation in mercury. They have checked that its frequency is proportional to the global circulation velocity, measured from the correlations between two temperature probes. Cioni et al. [13] have observed the same oscillation from temperature measurements in the plates.

Figure 7 demonstrates power spectral density of temperature fluctuations for thermocouple A3 for various Rayleigh numbers. Main peaks correspond to the oscillation frequency $f_p$ of the large-scale flow inside the cell, $f_p = U/X$ ($U$ is a time averaged velocity of flow motion and $X$ – is a characteristic size). The frequency $\omega_p = 2\pi f_p$, normalized by $\chi/D^2$, is plotted in figure 9 versus Rayleigh number. The best fit gives $\omega_p D^2/\chi = Ra^{0.43}$. Our result is close to the fit $\omega_p D^2/\chi = 0.47 Ra^{0.424}$, obtained by Cioni et al. for mercury [13].
Therefore we can calculate the Reynolds number in two ways: based on cross correlation of temperature fluctuations $Re_{cc} = U_{cc} D / \nu$ and based on Fourier spectrum peaks $Re_f = X D f_p / \nu$. Assuming first $X \cong 4D$ we obtain $Re_{cc} = 4D^2 f_p / \nu$. Figure 8 demonstrates Reynolds numbers $Re_f$ and $Re_{cc}$ versus Grashof number $Gr = Ra / Pr$ (we use $Gr$ instead $Ra$ because the mean temperature in the experiments was different). The best fit gives us $Re_{cc} \sim Gr^{0.46 \pm 0.01}$ and $Re_f \sim Gr^{0.42 \pm 0.01}$. These dependences have a similar slope and are shifted relative to each other by the same value. Assuming now $Re_{cc} = Re_f$ we can obtain true value of $X$.

Figure 9. The characteristic frequency $\omega_p$, normalized by $\chi / D^2$ vs. Rayleigh number (log-log scale).

Figure 10. The normalized value $X$ vs. the Grashof number.

Figure 8 also shows corrected Reynolds number $Re_{corr} = \bar{X} D f_p / \nu$ where $\bar{X}$ is the value averaged over all experiments. The difference in slopes of dependencies $Re_{cc}(Gr)$ and $Re_f(Gr)$ is ensured by the fact that the value of $X$ grows with increasing Grashof number (figure 10).

4. Conclusions

We have described the results of the experimental study of the large scale circulation structure and characteristics in case of turbulent Rayleigh-Bénard convection of liquid sodium in a cylindrical cell. The global circulation never disappears in all explored range ($Ra = (0.5 – 2.6) \cdot 10^7$), maintaining its strength as it changes in azimuth. The behaviour of LSC is characterized by the presence of a sloshing mode in the whole explored range of Rayleigh numbers. The characteristic times of the large-scale vortex circulation and the sloshing mode seem to be practically the same and, respectively, the dependences of the Reynolds numbers calculated at these times on the Rayleigh number are similar.
The Reynolds number was evaluated in two ways: by estimating velocities during the analysis of the cross correlations of temperature oscillations and by determining the locations of peaks on the spectra of temperature fluctuations. We believe that the first way corresponds to the global circulation and the second to the sloshing mode. This experimental work confirms for the first time the existence of a sloshing mode in low Prandtl number fluids (liquid sodium: \( Pr = (8.7 \pm 9.9) \times 10^{-3} \)).

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