Decoherence of quantum wavepackets due to interaction with
conformal spacetime fluctuations

W.L. Power, I.C. Percival

Department of Physics, Queen Mary and Westfield College, University of London, Mile End
Road, London E1 4NS, United Kingdom
Telephone: +44 171 775 3292, Fax: +44 181 981 9465
email: i.c.percival@qmw.ac.uk

(April 24, 2018)

Abstract

One of the biggest problems faced by those attempting to combine quantum theory and general relativity is the experimental inaccessibility of the unification scale. In this paper we show how incoherent conformal waves in the gravitational field, which may be produced by quantum mechanical zero-point fluctuations, interact with the wavepackets of massive particles. The result of this interaction is to produce decoherence within the wavepackets which could be accessible in experiments at the atomic scale.

Using a simple model for the coherence properties of the gravitational field we derive an equation for the evolution of the density matrix of such a wavepacket. Following the primary state diffusion programme, the most promising source of spacetime fluctuations for detection are the above zero-point energy fluctuations. According to our model, the absence of intrinsic irremoveable decoherence in matter interferometry experiments puts bounds on some of the parameters of quantum gravity theories. Current experiments give $\lambda > 18$, where $\lambda t_{\text{Planck}}$ is an effective cut-off for the validity of low-energy
I. INTRODUCTION

The relationship between gravity and quantum mechanics has been studied from many different perspectives. One way of approaching the problem is taken by those who are attempting to find a 'theory of everything' in which gravity is a necessary component of a theory which describes all of the fundamental forces in a unified way. This is the point of view taken by superstring theorists (see, for example, Green et al. 1987). A different approach to this problem has come from those who suggest that gravity may play a role in the quantum measurement process (Karolyhazy 1966; Diósi 1987; Ghirardi et al. 1990; Percival 1994; Penrose 1996), and who use this as their starting point for looking at the quantum properties of gravity.

In this paper we consider conformal gravitational waves and demonstrate that interactions with these waves can cause decoherence of quantum mechanical wavepackets. The mechanism by which this decoherence occurs is a consequence of the nonlinear nature of these waves. In a linear approximation the phase shifts produced by these waves cancel. Consequently there is no observable effect (although observable effects have been predicted in the linear approximation if the spacetime metric is non-commutative (Percival & Strunz 1997, Power 1998)). A related model for decoherence, with non-propagating conformal fluctuations has been studied by Sánchez-Gómez (1993).

It has been found in earlier work on primary state diffusion that modern matter interferometry experiments provide some of the best tests for decoherence due to spacetime fluctuations (Percival 1997). With this in mind we consider the possible sources of incoherent waves in the conformal gravitational field and examine whether these are likely to be sufficient to produce an experimentally observable effect. One possibility that we consider is that there may be cosmological sources of these waves, but an analysis, based on arguments originally proposed by Rosales and Sánchez-Gómez (1995), suggests that these are unlikely.
to be observable. Another possible source of conformal gravitational waves are the quantum zero-point energy fluctuations in the conformal field. By analysing these fluctuations we show that their decoherence effect on wavepackets may be detectable in atom interferometry experiments, although it should be noted that by using a classical model for quantum fluctuations we cannot be certain of including all of the essential physics. If such a decoherence effect is not observed this may then be used to set boundaries on some parameters of quantum gravity theories. Improvements in the sensitivity and time-of-flight in atom optics experiments could then be used to tighten the constraints on these parameters.

The structure of the paper is as follows. In section II we discuss the properties of conformal gravitational fluctuations and describe the effects of these fluctuations on a massive particle. In section III we describe the specific model which we use to describe the interaction of a wavepacket with the conformal fluctuations and define the coherence properties of the conformal field. In section IV the dynamics of the interaction are calculated, leading to an equation for the time evolution of the density matrix of the wavepacket which is dependent on the coherence properties of the conformal field. In section V we consider possible sources of fluctuations in the conformal field and the amplitudes and coherence properties of the resulting fluctuations. The equations derived in the paper are then applied to an atom interferometer, and it is shown that this type of experiment can be used to put limits on the properties of the conformal fluctuations which can then be used to put bounds on parameters appearing in some quantum theories of gravity. Finally in section VI we present our conclusions.

II. CONFORMAL SPACETIME FLUCTUATIONS

When we describe a fluctuation as conformal what we mean is that the deviation of the metric from the Minkowski metric is equal in all dimensions:

$$d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu$$

(1)

$$g_{\mu\nu} = f(x)\eta_{\mu\nu}.$$  

(2)
We choose to study conformal metric fluctuations because this simple structure makes them easy to deal with mathematically.

The equation for the action (in a given volume of D-dimensional spacetime) is

\[ I(D) = -K_D \int d^D x \sqrt{g(x)} R(x) \]  

(3)

in which \( K_D \) is a constant depending on the number of dimensions (for example, when \( D = 4 \) then \( K_4 = \frac{1}{16\pi G} \)), and \( R \) is the Ricci curvature scalar [1].

If we substitute the definition of the conformal fluctuations (Equation [1]) into the action integral (Equation [3]) we find after some algebra that

\[ I(D) = -\frac{4K_D(D-1)}{D-2} \eta^{\kappa\mu} \int d^4 x. \partial_\kappa A(x). \partial_\mu A(x), \]  

(4)

where

\[ A(x) = f(x)^{(D-2)/4} - 1 \]  

(5)

or equivalently

\[ f(x) = (1 + A(x))^{4/(D-2)}. \]  

(6)

\( A(x) \) is called the conformal field amplitude. From the variational principle we see that this field satisfies the Klein-Gordon equation

\[ \eta^{\kappa\mu} \partial_\kappa \partial_\mu A(x) = 0 \]  

(7)

for massless particles. When the conformal amplitude is zero \( g_{\mu\nu} \) is the Minkowski metric. Since the \( A \) field satisfies the massless Klein-Gordon equation it describes gravitational waves propagating at the speed of light and obeying the superposition principle.

At this point we should note that when \( A(x) = -1 \) the metric becomes singular because \( f(x) = 0 \). For \( A(x) < -1 \) the properties of \( f(x) \) depend on the value of \( D \). For \( D = 3, 4 \), \( f(x) \) defines a physical metric with a positive metric tensor. For \( D = 6 \), it changes the sign of the metric, and for \( D = 5 \) and \( D > 6 \) the metric is complex and non-physical. For the
important case of \( D = 4 \) a real plane sinusoidal wave of amplitude greater than unity has a set of singular planes with zero metric, but is otherwise well-behaved. In this paper we will deal principally with waves which have amplitudes much less than unity, and consequently these singularities will not play an important role.

A related point is that the absolute value of the field \( A(x) \) is important. This is unlike a potential where we can add a constant without changing the physics.

**A. Massive particle in a conformal field**

Now we wish to consider the effect of the conformal field on a particle of mass \( M \). First suppose the particle interacts with a general gravitational field described by a metric \( g_{\mu \nu} \). In the Newtonian approximation and with units such that \( c = 1 \), the Lagrangian of a particle of mass \( M \) is (Appendix A)

\[
L_M \approx \frac{1}{2} M \dot{x}^2 - \frac{1}{2} M(g_{00}(x) - 1). \tag{8}
\]

Next we consider only conformal spacetime fluctuations, in which case

\[
g_{\mu \nu}(x) = f(x) \eta_{\mu \nu}, \tag{9}
\]

so that in the Newtonian approximation,

\[
L_M \approx \frac{1}{2} M \dot{x}^2 - \frac{1}{2} M(f(x) - 1). \tag{10}
\]

In terms of the conformal amplitude (Equation 8) this is,

\[
L_M \approx \frac{1}{2} M \dot{x}^2 - \frac{1}{2} M((1 + A(x))^{4/(D-2)} - 1). \tag{11}
\]

the action on a particle moving between the spacetime points \( x_a \) and \( x_b \) is then

\[
I_M = \int_{x_a}^{x_b} ds \left[ \frac{1}{2} M \dot{x}^2 - \frac{1}{2} M((1 + A(x))^{4/(D-2)} - 1) \right]. \tag{12}
\]
III. MODEL OF THE CONFORMAL FLUCTUATIONS

We now restrict our analysis to the case of a four dimensional spacetime $D = 4$, and consider the effects of the conformal waves on wavepackets which are distributed along one spatial dimension. Our reason for doing this is that we ultimately intend to apply our model to the special case of an atomic wavepacket separated into two components in the arms of an interferometer and choose the spatial coordinates so that the $x$-axis is in the direction of the line joining the two components. However the theory as it is derived here is more general and applies to any one-dimensional distribution described by a density matrix $\rho$. In this approximation we consider that there are two types of conformal waves, those traveling from left to right, and those propagating from right to left. These are referred to as $A^+(x, t)$ and $A^-(x, t)$ respectively. We assume that the intensity of the waves is such that $|A^\pm(x, t)| < < 1$, and that there is a high-frequency cut-off at $f_{\text{cut-off}} < < 1/t_{\text{Planck}}$. This is consistent with the sources of conformal waves considered in section V, and allows us to make use of the Newtonian approximation in section II A. We will later find that it is important to define the correlation properties of the conformal field. We have done this by analogy to the treatment of correlations in quantum optics (alternative models for the correlation properties of the gravitational field have been presented by Diosi (1988), Sánchez-Gómez (1993) and Rosales and Sánchez-Gómez (1995)). Separating the fluctuations into an amplitude and a fluctuating part

\[
A^+(x, t) \equiv A_0^+\xi^+(x, t) = A_0^+\xi^+_0(t - x) \tag{13}
\]

\[
A^-(x, t) \equiv A_0^-\xi^-(x, t) = A_0^-\xi^-_0(t + x), \tag{14}
\]

the correlation properties of the fluctuating parts are

\[
\mathbf{M}\xi^\pm_0(t) = 0 \tag{15}
\]

\[
\mathbf{M}\xi^{s_1}_0(t)\xi^{s_2}_0(t') = \delta_{s_1, s_2}g^{(1)}(t - t') \tag{16}
\]

\[
= \delta_{s_1, s_2}\exp\left(-\frac{(t - t')^2}{\tau^2}\right) \tag{17}
\]
where \( s_i = + \) or \(-\). \( g^{(1)} \) is the first order correlation function, which is taken to be \( g^{(1)}(t-t') = \exp(-(t-t')^2/\tau^2) \). The numerical value of the correlation time \( \tau \) for physical sources of conformal fluctuations is discussed in section \[V\]. There is a further symmetry property which is very useful:

\[
M \left[ \prod_{i=1..n} \xi_{i}^{\pm}(t^{(i)}) \right] = 0 \text{ if } n \text{ is odd.} \tag{18}
\]

In addition the second order correlation function \( g^{(2)} \) is taken as

\[
M(\xi^{s_1(t)}(t)(\xi^{s_2}(t'))^2 = g^{(2)}(t-t') \text{ [if } s_1 = s_2]\tag{19}
\]

\[
= 1 \text{ [if } s_1 \neq s_2] \tag{20}
\]

\[
= 1 + \delta_{s_1,s_2}2\exp(-(t-t')^2/\tau^2). \tag{21}
\]

The choice of Gaussian correlation properties for the fluctuations simplifies the following analysis considerably. It is possible to work through the analysis that follows using a general form for the fluctuations, but we choose not to do this on the grounds of clarity.

### IV. INTERACTION OF THE FLUCTUATIONS WITH A WAVEPACKET

The derivation of the lowest significant order in the perturbation expansion for the interaction is relatively straightforward, but needs some care.

The potential energy of a particle of mass \( M \) in a conformal field \( A(x,t) \) is given by (equation \[12\])

\[
V(x,t) = \frac{M}{2}[(1 + A(x,t))^2 - 1] \tag{22}
\]

in four-dimensional space-time. Applying the model for the propagating fluctuations developed in the previous section gives us

\[
V(x,t) = \frac{M}{2}[(1 + A_0(\xi^+(x,t) + \xi^-(x,t)))^2 - 1] \tag{23}
\]

\[
= \frac{M}{2}A_0[2\xi^+(x,t) + 2\xi^-(x,t) + A_0(\xi^+(x,t)^2 + \xi^-(x,t)^2 + 2\xi^+(x,t)\xi^-(x,t))] \tag{24}
\]
Now we use perturbation theory to calculate the time evolution of a density matrix $\rho$ in such a potential in the time interval from $t = 0$ to $t = T$. $T$ is a long time compared to the time taken for light to travel across the wavepacket, and yet short compared to the time required to make a significant change in $\rho$.

We use a Dyson expansion to evaluate the time evolution

$$\rho(T) = M(\rho(0) + K^{(1)}(T,0)\rho(0) + \rho(0)K^{(1)\dagger} + K^{(2)}(T,0)\rho(0)$$

$$+ K^{(1)}(T,0)\rho(0)K^{(1)\dagger}(T,0) + \rho(0)K^{(2)\dagger}(T,0) \ldots)$$

(25)

and work with the terms up to second order. The operators $K^{(1)}$ and $K^{(2)}$ have the form

$$K^{(1)}(0,T) = -\frac{i}{\hbar} \int_0^T H(t) dt$$

(26)

$$K^{(2)}(0,T) = -\frac{1}{\hbar^2} \int_0^T H(t) dt \int_0^t H(t') dt'.$$

(27)

A. First-order terms

Using the form for the potential energy given by equation 24 and neglecting (for the moment) the kinetic energy, we can start to evaluate the terms in the Dyson expansion. We anticipate that the first-order terms will probably lead to no overall contribution as all parts of the wavepacket are subject to the same cumulative phase shifts for a single conformal wave (Percival & Strunz 1997).

The term $MK^{(1)}(T,0)\rho(0)$ is then given by

$$MK^{(1)}(T,0)\rho(0) = -\frac{i M A_0}{\hbar} M \int_0^T dt \int dx \left[ P(x) \left\{ 2\xi^+(x,t) + 2\xi^-(x,t) + A_0 \left( \xi^+(x,t)^2 + \xi^-(x,t)^2 \right) \right\} \rho(0) \tau \right].$$

(28)

Most terms are trivially eliminated using the property that $M\xi^\pm(m,n)^p = 0$ if $p$ is odd. Using $\xi^+(x,t) = \xi^+(0,t-x) \equiv \xi^+(t-x)$, this leaves

$$MK^{(1)}(T,0)\rho(0) = -\frac{i M A_0^2}{\hbar} M \int_0^T dt \int dx \left[ P(x)(\xi^+(t-x)^2 + \xi^-(t+x)^2)\rho(0) \tau \right]$$

(29)

$$= -2i M A_0^2 \int_0^T dt \rho(0).$$

(30)
The evaluation of the other first-order term follows in much the same way, and leads to the same answer except for a change of sign

$$M\rho(0)K^{(1)\dagger}(T, 0) = \frac{2iMA_0^2}{\hbar}T\rho(0).$$ \hfill (31)

These two terms cancel, leading to no overall change in the density matrix, as anticipated.

**B. Second-order terms**

We move on to consider the second-order terms $K^{(2)}(T, 0)\rho(0)$, $K^{(1)}(T, 0)\rho(0)K^{(1)\dagger}(T, 0)$ and $\rho(0)K^{(2)\dagger}(T, 0)$. The first of these is given by

$$MK^{(2)}(T, 0)\rho(0) = -\frac{M^2A_0^2}{4\hbar^2}M\left(\int_0^T dt \int dx \int_0^t dt' \int dx' [P(x)P(x')[2\xi^+(x, t) + 2\xi^-(x, t) + A_0(\xi^+(x, t)^2 + \xi^-(x, t)^2 + 2\xi^+(x, t)\xi^-(x, t))][2\xi^+(x', t') + 2\xi^-(x', t')] + A_0(\xi^+(x', t')^2 + \xi^-(x', t')^2 + 2\xi^+(x', t')\xi^-(x', t'))]\rho(0)\right).$$ \hfill (32)

Many of the terms in this equation are trivially eliminated using $M\xi^+(m, n)^p = 0$ if $p$ is odd, leaving

$$MK^{(2)}(T, 0)\rho(0) = -\frac{M^2A_0^2}{4\hbar}M\left(\int_0^T dt \int dx \int_0^t dt' \int dx' [P(x)P(x')4\xi^+(t - x)\xi^+(t' - x') + 4\xi^-(t + x)\xi^-(t' + x') + A_0^2(\xi^+(t - x)^2\xi^+(t' - x')^2 + \xi^+(t - x)^2\xi^-(t' + x')^2 + \xi^-(t + x)^2\xi^+(t' - x')^2 + \xi^-(t + x)^2\xi^-(t' + x')^2 + 4\xi^+(t - x)\xi^-(t + x)\xi^+(t' - x')\xi^-(t' + x')]\rho(0)\right).$$ \hfill (33)

The projection operators simplify to give:

$$MK^{(2)}(T, 0)\rho(0) = -\frac{M^2A_0^2}{4\hbar^2}M\left(\int_0^T dt \int dx \int_0^t dt' [P(x)4\xi^+(t - x)\xi^+(t' - x) + 4\xi^-(t + x)\xi^-(t' + x) + A_0^2(\xi^+(t - x)^2\xi^+(t' - x)^2 + \xi^+(t - x)^2\xi^-(t' + x)^2 + \xi^-(t + x)^2\xi^+(t' - x)^2 + \xi^-(t + x)^2\xi^-(t' + x)^2 + 4\xi^+(t - x)\xi^-(t + x)\xi^+(t' - x)\xi^-(t' + x))]\rho(0)\right).$$ \hfill (34)
Using the expressions for the correlation functions $M\xi_s(t)\xi_s(t') = \delta_{s,s}e^{-(t-t')^2/\tau^2}$ and $M\xi_s(t)\xi_s(t')^2 = 1 + 2\delta_{s,s}e^{-(t-t')^2/\tau^2}$ as discussed in section III gives

\[
MK_F^{(2)}(T, 0)\rho(0) = \frac{M^2A_0^2}{4\hbar^2}\left(\int_0^T dt \int_0^T dt' \int dxP(x)\left[8e^{-(t-t')^2/\tau^2} + A_0^2(4 + 4e^{-(t-t')^2/\tau^2} + 4e^{-2(t-t')^2/\tau^2})\right]\rho(0)\right) + A_0^2(4 + 4e^{-(t-t')^2/\tau^2} + 4e^{-2(t-t')^2/\tau^2})\rho(0). \tag{35}
\]

The term $\rho(0)K^{(2)\dagger}(T, 0)$ gives the same result

\[
Mrho(0)K^{(2)\dagger}(T, 0) = -\frac{M^2A_0^2}{4\hbar^2}\left(8T\sqrt{\pi} + A_0^2(2T^2 + 4T\sqrt{\pi} + T\sqrt{\pi}\sqrt{2})\rho(0). \tag{36}
\]

Finally we come to the term $MK^{(1)}(T, 0)\rho(0)K^{(1)\dagger}(T, 0)$. After elimination of the simpler terms, we are left with

\[
MK^{(1)}(T, 0)\rho(0)K^{(1)\dagger}(T, 0) = \frac{M^2A_0^2}{4\hbar^2} \left(M\left(\int_0^T dt \int dx \int_0^T dt' \int dx' \rho_{x,x'}[4\xi^+(t-x)\xi^+(t'-x') + 4\xi^-(t+x)\xi^-(t'+x')] + 4\xi^+(t-x)\xi^-(t'+x) + \xi^-(t+x)\xi^+(t'+x') + 4\xi^+(t-x)\xi^+(t'+x')\right]\right). \tag{38}
\]

Using the expressions for the correlations developed earlier this simplifies to

\[
MK^{(1)}(T, 0)\rho(0)K^{(1)\dagger}(T, 0) = \frac{M^2A_0^2}{4\hbar^2} \left(M\left(\int_0^T dt \int dx \int_0^T dt' \int dx' \rho_{x,x'}[4e^{-(t-x-t'+x')^2/\tau^2} + 4e^{-(t+x-t-x')^2/\tau^2} + A_0^2(4 + 2e^{-(t-x-t'+x')^2/\tau^2} + 2e^{-(t+x-t-x')^2/\tau^2} + 4e^{-(t-x-t'+x')^2/\tau^2}e^{-(t+x-t-x')^2/\tau^2})\right]\right) \tag{39}
\]

In evaluating the terms in this expression we need to remember that $T$ is large compared to the time taken for light to traverse the range $r \equiv x^{max} - x^{min}$ of the wavepacket. After evaluation of the integrals we then find

\[
MK^{(1)}(T, 0)\rho(0)K^{(1)\dagger}(T, 0) = \frac{M^2A_0^2}{4\hbar^2} \left(M\left(8\sqrt{\pi}T + A_0^2(4T^2 + 4T\sqrt{\pi})\right)\rho(0) + \int dx \int dx' \sqrt{\pi}T e^{-2(x-x')^2/\tau^2} \rho_{x,x'}\right). \tag{40}
\]
Summing up all of the second-order terms in the perturbation expansion leads to a significant amount of cancellation. The final result is that to second order in perturbation theory

$$\rho_{x,x'}(T) = \rho_{x,x'}(0) + \sqrt{\frac{\pi}{2}} \frac{M^2c^4A^4_0\tau}{\hbar^2} T (e^{-2(x-x')^2/\tau^2} - 1) \rho_{x,x'}(0), \quad (41)$$

where the dependence on $\tau$ is now made explicit.

Since the change in the density matrix is small over the time interval $T$ we may put this expression into a differential form

$$\dot{\rho}_{x,x'} = \sqrt{\frac{\pi}{2}} \frac{M^2c^4A^4_0\tau}{\hbar^2} (e^{-2(x-x')^2/\tau^2} - 1) \rho_{x,x'} - i\hbar \{H_0, \rho(t)\}_{x,x'} \quad (42)$$

where the first term represents the effect of spacetime fluctuations and the $H_0$ term the rest of the Hamiltonian.

Notice that this equation is in exactly the form as appears in the Ghirardi, Rimini and Weber (1986) theory of wavepacket reduction.

$$\dot{\rho}_{x,x'} = -\lambda(1 - e^{-\alpha^2/4}) \rho_{x,x'} - i\hbar [H_0, \rho(t)]_{x,x'} \quad (43)$$

This equation first appeared in the work of Barchielli et al. (1982), where it describes the evolution of a continuously observed quantum wavepacket. In the GRW model a quantum wavepacket undergoes occasional instantaneous quantum jumps which localise the position of the quantum system, in which case the ensemble average obeys this same density matrix equation. This equation also appears in the theory of continuous spontaneous localisation where the quantum wavepacket is continuously subject to interactions which localise its position (Diósi 1988, Gisin 1989, Pearle 1989).

We have identified a physical mechanism for equation (43) and supplied values for the undetermined constants

$$\lambda = \sqrt{\frac{\pi}{2}} \frac{M^2c^4A^4_0\tau}{\hbar^2} \quad (44)$$

and
\[ \alpha = \frac{8}{\tau^2}. \] 

(45)

This result is dependent on our previous choice of Gaussian correlation properties for the fluctuations; for the result in the general case of an unspecified correlation function see Appendix B. Pearle and Squires (1996) have made a different estimate of the parameters \( \alpha \) and \( \lambda \), starting with the theory of continuous spontaneous localisation.

V. INTERPRETATION

At this stage we must consider the possible sources of conformal fluctuations. There may be cosmological sources, the amplitude and frequency of which are hard to predict. Arguments given by Rosales and Sánchez-Gómez (1995) set a limit on the energy density that can be present in the form of conformal waves without adding sufficient extra mass to the universe that the universe would collapse in on itself. According to their argument the energy density of conformal waves must not exceed \( 10^{-29} \text{g/cm}^3 \), and, they argue, this limits the correlation length to being greater than \( 10^{-3} \text{cm} \) equivalent to having a correlation time of greater than \( 10^{-13} \text{s} \). Using dimensional arguments they relate the amplitude of the conformal fluctuations to the correlation length to set a maximum amplitude of \( 10^{-30} \). Substitution of these values into equation (41) suggests that the effects of these conformal waves would then probably never be observed in an interferometry experiment.

A second possible source of conformal waves is the quantum zero-point fluctuations of the gravitational field. We assume here that the classical conformal waves correspond to the zero-point fluctuations of quantum gravity, but also that the energy density normally associated with such classical waves is canceled by renormalization. Otherwise the fluctuations would be ruled out on the basis of the arguments of Rosales and Sánchez-Gómez.

According to many theories, such as the superstring theory, there is a length scale below which the universe in no longer effectively four-dimensional but has a higher number of dimensions. We may suppose that this length scale may effectively provide a cut-off wavelength for the conformal gravitational waves. Let us call this cut-off length \( l_{\text{cut-off}} \) and let
us relate this to the Planck length $l_{\text{Planck}}$ and define $\lambda$ such that

$$l_{\text{cut-off}} = \lambda l_{\text{Planck}}.$$  \hspace{1cm} (46)

This defines a cut-off frequency $\omega_M = 2\pi c \lambda_{\text{cut-off}}^{-1}$. The amplitude of the zero-point conformal fluctuations in the gravitational field is then approximately (see Appendix C)

$$A_0 \approx \omega_M^2 l_{\text{Planck}}^2 \approx \lambda^{-2}.$$  \hspace{1cm} (47)

Current theories place the value of $\lambda$ in the range of $10^2 - 10^6$. Within this range the Newtonian approximation that was made in section II A should be adequate for the order-of-magnitude estimates that we are interested in here.

Now we consider a matter interferometer which puts particles of mass $M$ into a superposition of states, separated by a distance which is very large compared to the correlation length of the conformal fluctuations, for a time $T$. Then from equation 41 we see that

$$\frac{\delta \rho}{\rho(0)} = \sqrt{\frac{\pi}{2}} M^2 c^4 A_0^4 \frac{\tau T}{\hbar^2}.$$  \hspace{1cm} (48)

By substitution of equation 47 and setting the correlation time $\tau = \lambda t_{\text{Planck}}$ (which follows from equation 46 when the fluctuations travel at the speed of light) we find that $\lambda$ is

$$\lambda = \left( \frac{\sqrt{\pi}}{2} M^2 c^4 t_{\text{Planck}} T}{\hbar^2 (\delta \rho/\rho(0))} \right)^{1/7}.$$  \hspace{1cm} (49)

In practice there will be other experimental causes for loss of contrast in the interference pattern, such as vibrational instabilities in the apparatus. Hence, by substituting the experimentally determined values for the loss of contrast into equation 49 we find a lower bound for $\lambda$. Notice that due to the $1/7$th power in equation 49 the bound on $\lambda$ only rises slowly with improvements in the experimental parameters.

In recent experiments by Peters et al. (1997) Caesium atoms (relative atom mass 132.9) were put into a superposition state for 0.32s before recombining to form an interference pattern. The observed loss of contrast over this period was approximately 3%. Substitution of these parameters into equation 49 leads to the setting of a lower bound on $\lambda$ of
\( \lambda > 18. \) \hfill (50)

Improvements in the time and resolution of atom interferometry experiments should enable the bound on \( \lambda \) to be raised. It is interesting to contrast this behaviour with that of particle accelerators which are effectively able to reduce the upper bound on \( \lambda \) by increasing particle energies, although current experiments still put an upper bound on \( \lambda \) well above the range of theoretical predictions.

VI. CONCLUSIONS

We have derived the leading terms in an expansion for the nonlinear interaction between the conformal gravitational field and wavepackets of massive particles. When the field contains incoherent conformal waves we have shown that this leads to the decoherence of the wavepackets. We have considered possible sources of conformal gravitational waves, and have shown how atom interferometry experiments can be used to try to detect their presence. If no decoherence is observed in these experiments then our model can be used to place limits on the properties of sources of these waves. If we have a theory for the sources of these waves, such as the model for quantum zero-point fluctuations presented here, then these limits can be used to place boundaries on the fundamental properties, such as the size of compactified dimensions, present in those theoretical models.

The authors would like to thank J. Charap, S. Thomas, W. Strunz and the QMW superstring group for very helpful discussions. We are also grateful to the EPSRC and the Leverhulme Foundation for providing the financial support that made this work possible.

APPENDIX A: DERIVATION OF LAGRANGIAN

Consider the motion of a mass \( M \) in a gravitational field specified by a metric \( g_{\mu\nu} \). The general geodesic equation for the velocity vector \( v^\mu = dz^\mu/ds \) is

\[
\frac{dv^\mu}{ds} + \Gamma^\mu_{\nu\sigma} v^\nu v^\sigma = 0. \quad (A1)
\]
If the field is slowly varying in time such that

$$\partial_0 g_{\mu\nu} \approx 0,$$  \hfill (A2)

and the field is assumed to be weak so that

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$  \hfill (A3)

where all the components of the tensor $h$ are much less than 1. Then for a particle moving slowly compared to the velocity of light

$$|v^m| \ll 1, \ (m = 1, 2, 3)$$  \hfill (A4)

we find by substitution into Equation A1 and neglecting second order quantities that

$$\frac{dv_m}{dx_0} = \partial_m (g_{00})^{1/2}. \hfill (A5)$$

From this it appears that the particle is moving in a potential $(g_{00})^{1/2}$; if we write

$$g_{00}(x) = 1 + 2 U_G(x)$$  \hfill (A6)

then for small $U_G$

$$(g_{00})^{1/2} \approx 1 + U_G(x)$$  \hfill (A7)

and we see that $U_G(x)$ is the classical gravitational potential. The gravitational potential energy of the particle of mass $M$ is therefore

$$M U_G \approx \frac{1}{2} M (g_{00}(x) - 1). \hfill (A8)$$

The Lagrangian of the particle, provided the above conditions hold (Equations A2 to A4), is then

$$L_M \approx \frac{1}{2} M \dot{x}^2 - \frac{1}{2} M (g_{00}(x) - 1). \hfill (A9)$$
APPENDIX B: EVOLUTION OF MATRIX ELEMENTS WITH GENERALIZED FLUCTUATIONS

If instead of assuming Gaussian correlation properties for the fluctuations, we work with a general expression for the fluctuations we find, after a considerable amount of algebra along the lines of section IV, that

\[ \rho_{x,x'}(T) \approx \rho_{x,x'}(0) + \frac{M^2 A_0^4}{\hbar^2} \left( \int_0^T dt \int_0^T dt' g^{(1)}(t - t' - x + x') g^{(1)}(t - t' + x - x') \rho_{x,x'}(0) \right. \]

\[ \left. -2 \int_0^T dt \int_0^t dt' \left( g^{(1)}(t - t') \right)^2 \rho_{x,x'}(0) \right) \]

(B1)

which when the correlation function is taken to be Gaussian

\[ M_{s_1 s_2}^{s_1 s_2} (t) = \delta_{s_1, s_2} g^{(1)}(t - t') \]

(B2)

\[ = \delta_{s_1, s_2} \exp \left( -\frac{(t - t')^2}{\tau^2} \right) \]  

(B3)

reduces to equation 41:

\[ \rho_{x,x'}(T) = \rho_{x,x'}(0) + \sqrt{\frac{\pi}{2}} \frac{M^2 A_0^4 \tau}{\hbar^2} T \left( e^{-2(x-x')^2/\tau^2} - 1 \right) \rho_{x,x'}(0). \]

(B4)

Note that the precise form of the second order correlation function is unimportant; all that we require is that \( g^{(2)}(t) \) should be an even function and that \( \int_{-\infty}^{\infty} (g^{(2)}(t) - 1) dt \) should be finite. Both of these conditions are features of physically realistic second-order correlation functions.

APPENDIX C: EVALUATION OF THE AMPLITUDE OF ZERO-POINT CONFORMAL GRAVITATIONAL WAVES

We first summarise the properties of conformal gravitational waves as derived in Section II

\[ d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu \]  

(C1)

\[ g_{\mu\nu} = f(x) \eta_{\mu\nu}. \]  

(C2)
and

\[ f(x) = (1 + A(x))^{4/(D-2)} \]  

(C3)

where \( A(x) \) is called the conformal field amplitude. This satisfies the Klein-Gordon equation

\[ \eta^{\kappa\mu} \partial_\kappa \partial_\mu A(x) = 0 \]  

(C4)

for massless particles. When the conformal amplitude is zero \( g_{\mu\nu} \) is the Minkowski metric. The Klein-Gordon equation is also known as the wave equation, and equation (C4) can be written as

\[ \frac{1}{c^2} \frac{\partial^2 A(x, t)}{\partial t^2} = \nabla^2 A(x, t). \]  

(C5)

This is essentially the same as the equation for the components of the electromagnetic potential in flat space, and the same methods can be used to obtain the energy density (see, for instance Loudon 1983).

By quantizing in a cube of volume \( V \), we find that for the scalar field \( A \) the density of modes (not taking into account supersymmetry) with frequency between \( \omega \) and \( \omega + d\omega \) is

\[ 4\pi \frac{\omega^2 d\omega}{(2\pi c)^3} V. \]  

(C6)

If each mode has a zero point energy of \( \frac{1}{2} \hbar \omega \) and we have a cut-off frequency of \( \omega_M \), then the total energy per unit volume is

\[ \int_0^{\omega_M} \frac{1}{2} \hbar \omega 4\pi \frac{\omega^2 d\omega}{(2\pi c)^3} d\omega = \frac{1}{16} \frac{\hbar \omega_M^4}{\pi^2 c^3} \]  

(C7)

Assuming that the squared amplitude of the gravitational field fluctuations is proportional to the energy density tells us that \( |A_0| \propto \omega_M^2 \). As we expect the (dimensionless) amplitude to be of the order of unity if the cut-off is set to be the Planck length this leads us to deduce that

\[ |A_0| \approx \omega_M^2 t_{\text{Planck}}^2. \]  

(C8)
REFERENCES

[1] Barchielli, A., Lanz, L. and Prosperi, G.M. 1982 A model for the macroscopic description and continual observations in quantum mechanics. Nuovo Cimento B 72, 79-121.

[2] Diósi, L. 1987 Universal master equation for the gravitational violation of quantum mechanics. Phys. Lett. A 120, 377-381.

[3] Diósi, L. 1988 Continuous quantum measurement and Itô formalism. Phys. Lett. A 129, 419-423.

[4] Diósi, L. 1989 Models for universal reduction of macroscopic quantum fluctuations. Phys. Rev. A, 40, 1165-1174.

[5] Ghirardi, G.C., Rimini, A. and Weber, T. 1986 Phys. Rev. D 34, 470-491.

[6] Ghirardi, R., Grassi, G.-C. and Rimini, A. 1990 A continuous spontaneous reduction model involving gravity. Phys. Rev. A 42, 1057-1064.

[7] Gisin, N. 1988 Stochastic quantum dynamics and relativity. Helvetica Physica Acta 62, 363-371.

[8] Green, M.B., Schwarz, J.H., & Witten, E. 1987 Superstring Theory. Vols I and II. Cambridge: Cambridge University Press.

[9] Károlyházy, F. 1966 Gravitation and Quantum Mechanics of Macroscopic Objects. Nuovo Cimento A 42, 390-402.

[10] Kasevich, M. & Chu, S. 1992 Measurement of the gravitational acceleration of an atom with a light-pulse atom interferometer. Appl. Phys. B 54, 321-332.

[11] Kenyon, I.R. General Relativity Oxford: Oxford University Press

[12] Loudon, R. The quantum theory of light Oxford: Oxford University Press.

[13] Pearle, P. 1989 Combining stochastic dynamical state-vector reduction with spontaneous
localisation. Phys. Rev. A 39 2277-2289.

[14] Pearle, P. and Squires, E. 1996 Gravity, energy conservation, and parameter values in collapse models. Foundations of Physics, 1996, 26, 291-305.

[15] Percival, I.C. 1994 Primary State Diffusion. Proc. R. Soc. Lond. A 447, 189-209.

[16] Penrose, R. 1996 On gravity’s role in state vector reduction. Gen. Rel. Grav. 28, 581-600.

[17] Percival, I.C. 1995 Quantum spacetime fluctuations and primary state diffusion. Proc. R. Soc. Lond. A 451, 503-513.

[18] Percival, I.C. & Strunz, W.T. 1997 Detection of spacetime fluctuations by a model matter interferometer. Proc. R. Soc. Lond. A 453, 431-446.

[19] Perival, I. 1997 Atom interferometry, spacetime and reality. Physics World 10, March issue, 43-48.

[20] Percival, I.C. 1998 Quantum State Diffusion. Cambridge: Cambridge University Press.

[21] Peter, A., Chung, K.Y., Young, B., Hensley, J. and Chu, S. 1997 Precision atom interferometry. Phil. Trans. R. Soc. Lond. A 355 2223-2233.

[22] Power, W.L. 1999 The interaction of propagating spacetime fluctuations with wavepackets. Accepted for publication in Proc. R. Soc. Lond. A 455.

[23] Rosales, J.L. & Sánchez-Gómez, J.L. 1995 A note on some physical and cosmological implications of conformal metric fluctuations. Physics Letters A 199, 320-322.

[24] Sánchez-Gómez, J.L. 1993 Decoherence through stochastic fluctuations of the gravitational field. In Stochastic evolution of quantum states in open systems and in measurement processes (ed. Diósi, L. & Lukács), 88-93. Singapore: World Scientific Publishing.