OBSERVED SMOOTH ENERGY IS ANTHROPICALLY EVEN MORE LIKELY AS QUINTESSENCE THAN AS COSMOLOGICAL CONSTANT

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Abstract

For a universe presently dominated by static or dynamic vacuum energy, cosmological constant (LCDM) or quintessence (QCDM), we calculate the asymptotic collapsed mass fraction as function of the present ratio of vacuum energy to clustered mass, $\Omega_{Q0}/\Omega_{m0}$. Identifying these collapsed fractions as anthropic probabilities, we find the present ratio $\Omega_{Q0}/\Omega_{m0} \sim 2$ to be reasonably likely in LCDM, and very likely in QCDM.

1 A Cosmological Constant or Quintessence?

Absent a symmetry principle protecting its value, no theoretical reason for making the cosmological constant zero or small has been found. Inflation makes the universe flat, so that, at present, the vacuum or smooth energy density $\Omega_{Q0} = 1 - \Omega_{m0} < 1$, is $10^{120}$ times smaller than would be expected on current particle theories. To explain this small but non-vanishing present value, a dynamic vacuum energy, quintessence, has been invoked, which obeys the equation of state $w_Q \equiv P/\rho < 0$. (The limiting case, $w_Q = -1$, a static vacuum energy or Cosmological Constant, is homogeneous on all scales.)

Accepting this small but non-vanishing value for static or dynamic vacuum energy, the Cosmic Coincidence problem now becomes pressing: Why do we live when the clustered matter density $\Omega(a)$, which is diluting as $a^{-3}$ with cosmic scale $a$, is just now comparable to the static vacuum energy or present value of the smooth energy:

$$w_0^Q \equiv \Omega_{Q0}/\Omega_{m0} \sim 2.$$ 

The observational evidence is for a flat, low-density universe:

1. $\Omega_m + \Omega_Q = 1 \pm 0.2$
   - (Location of first Doppler peak in the CBR anisotropy at $l \sim 200$);
2. $\Omega_{m0} = 0.3 \pm 0.05$.
   - (Slow evolution of rich clusters, mass power spectrum, CBR anisotropy, cosmic flows);
3. $\Omega_{Q0} = 1 - \Omega_{m0} \sim 2/3$ (curvature in SNIa Hubble diagram, dynamic age, height of first Doppler peak, cluster evolution).

Of these, the SNIa evidence is most subject to systematic errors due to precursor intrinsic evolution and the possibility of grey dust extinction.

The combined data implies a flat, low-density universe with $\Omega_{m0} \sim 1/3$, with negative pressure $-1 \leq w_Q \leq -1/2$. In this paper, we use the evolution of large-scale structure to distinguish the

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two limiting cases:

**LCDM:** Cosmological constant: \( w_Q = -1, \quad n_Q = 3(1 + w_Q) = 0 \quad \Omega_\Lambda = 2/3 \)

**QCDM:** Quintessence: \( w_Q = -1/2, n_Q = 3/2, \quad \Omega_{Q0} = 1/3. \)

### 2 Evolution of a Low Density Flat Universe

The Friedmann equation in a flat universe with clustered matter and smooth energy density is

\[
H^2(x) \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left( \rho_m + \rho_Q \right),
\]

or, in units of \( \rho_{cr}(x) = 3H^2(x)/8\pi G, \) \( 1 = \Omega_m(x) + \Omega_Q(x), \) where the reciprocal scale factor \( x \equiv a_0/a \equiv 1 + z \rightarrow \infty \) in the far past, \( \rightarrow 0 \) in the far future.

With the EOS \( w \equiv P/\rho, \) different kinds of energy density dilute at different rates \( \rho \sim a^{-n}, \) \( n \equiv 3(1 + w), \) and contribute to the deceleration at different rates \((1 + 3w)/2\) shown in the table:

| Substance      | \( w \) | \( n \) | \((1 + 3w)/2\) |
|----------------|--------|--------|----------------|
| Radiation      | 1/3    | 4      | 1              |
| Normal Matter  | 0      | 3      | 1/2            |
| Quintessence   | -1/2   | 3/2    | -1/4           |
| Cosmological Const. | -1    | 0      | -1             |

The expansion rate in present Hubble units is

\[
E(x) \equiv H(x)/H_0 = (\Omega_{m0}x^3 + (1 - \Omega_{m0})x_Q^n)^{1/2}.
\]

The Friedmann equation has an unstable fixed point in the far past and a stable attractor in the far future. (Note the tacit application of the anthropic principle: Why does our universe expand, rather than contract?)

The second Friedmann equation is \(-\ddot{a}/a^2 = (1 + 3w_Q\Omega_Q)/2.\) The ratio of smooth energy to matter energy, \( \Omega_Q/\Omega_m \equiv \omega^2 = u_Q^2x^{3w_Q}, \) where \( \Omega_{Q0}/\Omega_{m0} \equiv u_Q^3 \approx 2 \) is the present ratio. As shown by the inflection points in the middle curves of the figure, for fixed \( \Omega_{Q0}/\Omega_{m0}, \) QCDM (upper middle curve) expands faster than LCDM (lower middle curve), but begins accelerating only at the present epoch. The top and bottom curves refer respectively to a De Sitter universe (\( \Omega_m = 0 \)), which is always accelerating, and an SCDM universe (\( \Omega_m = 1 \)), which is always decelerating.

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**Recent Past & Future Scale Evolution**

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2
As summarized in the table below, quintessence dominance begins 3.6 Gyr earlier and more gradually than cosmological constant dominance. (In this table, the deceleration \( q(x) \equiv -\ddot{a}/aH_0^2 \) is measured in present Hubble units.) The recent lookback time

\[ H_0 t_L(z) = z - (1 + q_0)z^2 + ..., \quad z < 1, \]

where \( q_0 = 0 \) for QCDM and \( = -1/2 \) for LCDM.

| Comparative Evolution of LCDM and QCDM |
|--------------------------------------|
| \( u \approx \Omega_m/\Omega_m = u_0^3x^3w - 2 \) | increases as the matter density decreases. The matter-smooth energy transition \( \Omega_m/\Omega_m = 1 \) took place only recently at \( x* = u_0 = 1.5874 \) for QCDM and, even later, at \( x* = 1 + z* = u_0 = 1.260 \) for LCDM. Because, for the same value of \( u_0 \), a matter-QCDM freeze-out would take place earlier and more slowly than a matter-LCDM freeze-out, it imposes a stronger constraint on structure evolution. To permit evolution to the same present structure, QCDM would require a smaller value of \( \Omega_m/\Omega_m \) than does LCDM.

### 3 Growth of Large Scale Structure

The background density for large-scale structure formation is overwhelmingly Cold Dark Matter (CDM), consisting of clustered matter \( \Omega_m \) and smooth energy or quintessence \( \Omega_Q \). Baryons, contributing only a fraction to \( \Omega_m \), collapse after the CDM and, particularly in small systems, produce the large overdensities that we see.

Structure formation begins and ends with matter dominance, and is characterized by two scales:

The horizon scale at the first cross-over, from radiation to matter dominance, determines the power spectrum \( P(k, a) \), which is presently characterized by a scale factor \( \Gamma = \Omega_m h = 0.25 \pm 0.05 \). The horizon scale at the second cross-over, from matter to smooth energy, determines a second scale factor, which for quintessence, is \( \Gamma_Q \) at \( \sim 130 \, Mpc \), the scale of voids, superclusters. A cosmological constant is smooth at all scales.

Quasars formed as far back as \( z \approx 5, \) galaxies at \( z \geq 6.7 \), ionizing sources at \( z = (10 - 30) \). The formation of any such structures, already sets an upper bound \( x* < 30 \) or \( \Omega_Q/\Omega_m < 1000 \), for any structure to have formed. A much stronger upper bound, \( w_0 < 5 \), is set by when typical galaxies form i.e. by using the observed LSS, not to fix \( \Omega_m/\Omega_Q \), but to estimate the probability of our observing this ratio \( \Omega_Q/\Omega_m \) at the present epoch.

For LCDM, Martel et al. [1] and Garriga et al. [2] calculate the asymptotic mass fraction that ultimately collapses into galaxies to be

\[ f_{c, \infty} = \text{erfc}(\beta^{1/2}), \]

remarkably a broad function of only \( \beta \equiv \delta_{c,c}^2/2(\sigma_i)^2 \), where \( \sigma_i^2 = (1.7 - 2.3)/(1 + z_i) \) is the variance of the mass power spectrum and \( \delta_{c,c} \) is the minimum density contrast which will make
an ultimately bound perturbation. This minimum density contrast grows with scale factor $a$, and is approximately unity at recombination. Thus, except for a numerical factor of order unity $\delta_{i,c} \sim x^* / (1 + z_i)$, the freeze-out projected back to recombination. Both numerator and denominator in $\beta$ refer to the time of recombination, but this initial time or red-shift cancels out in the quotient.

4 $\Omega_Q \sim \Omega_m$ is Quite Likely for Our Universe

For a cosmological constant, an anthropic argument has already been given [2, 3, 4, 5], assuming a universe of subuniverses with all possible values for the vacuum energy $\rho_V$ or $\Omega_\Lambda$. In each of these subuniverses, the probability for habitable galaxies to have emerged before the present epoch, is a function of $\Omega_\Lambda$ or the present ratio $\Omega_\Lambda/\Omega_{m0}$

$$P(\rho_V) \propto (\text{prior distribution in } \rho_V) \times (\text{asymptotic mass fraction } f_{c,\infty}).$$

MSW, assuming nothing about initial conditions, assume a prior flat in $\Omega_\Lambda$. GLV argue that the prior should be determined by a theory of initial conditions and is not flat for most theories.

Following MSW, we assume a flat prior, so that the differential probability $P$ for our being here to observe a value $\rho_V$ in our universe is simply proportional to the asymptotic collapsed mass fraction for this $\rho_V$. For LCDM,

$$\delta_{i,c} = 1.1337 u_0 / (1 + z_i), \quad 1.1337 = (27/2)^2/3/5.$$ 

As function of the ratio $\Omega_\Lambda/\Omega_{m0} = u_3^0$, the LCDM probability distribution has a broad peak about $u_3^0 \approx 12 - 30$. The value observed in our universe $u_3^0 \approx 2$ has reasonable probability $4 - 10%$.

This argument [4, 5] for LCDM ($w_Q = -1$) is easily extended to QCDM ($w_Q = -1/2$). The variance of the power spectrum, $\sigma^2$, is insensitive to $w_Q$ for $w_Q < -1/3$, but for $w_Q = -1/2$, the numerical factor in $\delta_{i,c}$ is the same as for $w_Q = -1$, but $x^* = u_0^2$ in place of $u_0$, so that $\delta_{i,c} = 1.1337 u_0^2 / (1 + z_i)$. Thus $\beta_{QCDM}(u_0) = \beta_{LCDM}(\sqrt{u_0})$, so that the QCDM probability distribution now peaks at $u_0^3 \approx 3.5 - 5.5$. With QCDM, the probability for observing $u_0^3 \approx 2$ is now increased to about 50%.

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