An Enhanced Differential Evolution Algorithm Using a Novel Clustering-based Mutation Operator

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Abstract—Differential evolution (DE) is an effective population-based metaheuristic algorithm for solving complex optimisation problems. However, the performance of DE is sensitive to the mutation operator. In this paper, we propose a novel DE algorithm, Clu-DE, that improves the efficacy of DE using a novel clustering-based mutation operator. First, we find, using a clustering algorithm, a winner cluster in search space and select the best candidate solution in this cluster as the base vector in the mutation operator. Then, an updating scheme is introduced to include new candidate solutions in the current population. Experimental results on CEC-2017 benchmark functions with dimensionalities of 30, 50 and 100 confirm that Clu-DE yields improved performance compared to DE.

I. INTRODUCTION

Optimisation problems exist in a variety of scientific fields, ranging from medicine to agriculture. While conventional optimisation algorithms are popular, they suffer from drawbacks such as getting stuck in local optima and being sensitive in the initial state [1]. To tackle these problems, population-based metaheuristic algorithms such as particle swarm optimisation [2] offer a powerful alternative thanks to their well-recognised characteristics, such as self-adaptation and being derivative free [3].

Differential evolution (DE) [4] is a simple yet effective population-based algorithm which has shown good performance in solving optimisation problems in areas including image processing [5], [6], pattern recognition [7]–[9], and economics [10], [11]. DE is based on three primary operators: mutation, which generates new candidate solutions based on scaling differences among candidate solutions, crossover, which combines a mutant vector with the parent one, and selection, which selects a better candidate solution from a new one and its parent.

The performance of DE is directly related to these operators [12]. Among them, the mutation operator plays a crucial role to generate new promising candidate solutions and significant recent work has focussed on developing effective mutation operators. [13] proposes a multi-population DE which combines three different mutation strategies including current-to-pbest/1, current-to-rand/1, and rand/1. [14] employs three trial vector generation strategies and three control parameter settings and randomly selects between them to create new vectors. In [15], \( k \)-tournament selection is used to introduce selection pressure for selecting the base vector. [16] proposes a neighbourhood-based mutation that is performed within each Euclidean neighbourhood. In [17], a competition scheme for generating new candidate solutions is introduced so that candidate solutions are divided into two groups, losers and winners. Winners create new candidate solutions based on standard mutation and crossover operators, while losers try to learn from winners.

In this paper, we propose a novel DE algorithm, Clu-DE, which employs a novel clustering-based mutation operator. Inspired by the clustering operator in the human mental search (HMS) optimisation algorithm [18], Clu-DE clusters the current population into groups and selects a promising region as the cluster with the best mean objective function value. The best candidate solution in the promising region is selected as the base vector in the mutation operator. An updating strategy is then employed to include the new candidate solutions into the current population. Experimental results on CEC-2017 benchmark functions with dimensionalities of 30, 50 and 100 confirm that Clu-DE yields improved performance compared to DE.

The remainder of the paper is organised as follows. Section II-A describes the standard DE algorithm and some preliminaries about clustering. Section III introduces our Clu-DE algorithm, while Section IV provides experimental results. Section V concludes the paper.

II. BACKGROUND

A. Differential Evolution

Differential evolution (DE) [4] is a simple but effective population-based optimisation algorithm based on three main operators: mutation, crossover, and selection. The mutation operator generates a mutant vector 
\[
\vec{v}_{i}^{1} = (v_{i,1}, v_{i,2}, ..., v_{i,T})
\]
for each candidate solution as
\[
\vec{v}_{i}^{1} = \vec{x}_{r_{1}}^{1} + F(\vec{x}_{r_{2}} - \vec{x}_{r_{3}}),
\]
where \( \vec{x}_{r_{1}}^{1}, \vec{x}_{r_{2}}^{1}, \text{ and } \vec{x}_{r_{3}}^{1} \) are three distinct candidate solutions randomly selected from the current population and \( F \) is a scaling factor.

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Crossover shuffles the mutant vector with the parent vector. For this, binomial crossover, defined as

\[
u_{i,j} = \begin{cases} v_{i,j} & \text{if } \text{rand}(0,1) \leq CR \text{ or } j == j_{\text{rand}} \\ x_{i,j} & \text{otherwise} \end{cases}
\]

is employed, where \( \overrightarrow{u} \) is called a trial vector, \( CR \) is the crossover rate, and \( j_{\text{rand}} \) is a random integer number between 1 and the number of dimensions.

Finally, the selection operator selects the better candidate solution from the new candidate solution and its parent to be passed to the new population.

B. Clustering

Clustering is an unsupervised pattern recognition technique to partition samples into different groups so that the members of a cluster share more resemblance compared to members of different clusters. \( k \)-means [19] is the most popular clustering algorithm based on a similarity measure (typically Euclidean distance). It requires to define \( k \), the number of clusters, in advance and proceeds as outlined in Algorithm 1.

**Algorithm 1: Pseudo-code of \( k \)-means clustering algorithm.**

**Input:** \( X \): samples

\( k \): number of clusters

**Output:** \( C = c_1, c_2, ..., c_k \): the set of clusters defined by their centres \( \mu_j \)

Initialise cluster centres \( \mu_1, \mu_2, ..., \mu_k \) randomly;

while stopping condition is not met do

Assign each sample \( x_i \) to the closest cluster centre, i.e. based on \( \text{argmin}_j ||x_i - \mu_j|| \);

For all clusters, calculate new cluster centre as \( \mu_j = \frac{\sum_{x_i \in j} x_i}{n_j} \), where \( n_j \) is the number of samples in the \( j \)-th cluster;

end

**III. PROPOSED CLU-DE ALGORITHM**

In this paper, we improve DE using a novel clustering-based mutation and updating scheme. Our proposed algorithm, Clu-DE, is given, in the form of pseudo-code in Algorithm 2 while in the following we describe its main contributions.

A. Clustering-based Mutation

For our improved mutation operator, Clu-DE first identifies a promising region in search space. This is performed, similar as in the HMS algorithm [18], using a clustering algorithm. We employ the well known \( k \)-means clustering algorithm to group the current population into \( k \) clusters so that each cluster represents a region in search space. The number of clusters is selected randomly between 2 and \( \sqrt{N_p} \) [8], [21].

After clustering, the mean objective function value for each cluster is calculated, and the cluster with the best objective function value is then used to identify a promising region in search space. Fig. 1 illustrates this for a toy problem with 17 candidate solutions divided into three clusters.

Finally, our novel clustering-based mutation is conducted as

\[
\overrightarrow{v_{i}^{\text{clu}}} = \overrightarrow{\text{winner}} + F(\overrightarrow{x_{r_1}^1} - \overrightarrow{x_{r_2}^1}),
\]

region in search space. Fig. 1 illustrates this for a toy problem with 17 candidate solutions divided into three clusters.
where \( x_{r1} \) and \( x_{r2} \) are two different randomly-selected candidate solutions, and \( \text{winner} \) is the best candidate solution in the promising region. It is worth noting that the best candidate solution in the winner cluster might not be the best candidate solution in the current population. Clustering-based mutation is performed \( M \) times following standard crossover and mutation.

### Table I: Summary of CEC2017 benchmark functions [20].

\( N \) indicates the number of basic functions to form hybrid and composite functions. The search range is \([-100, -100]^{D2}\) in all cases.

| Unimodal functions                  |                      |
|-------------------------------------|----------------------|
| F1 Shifted and Rotated Bent Cigar Function |                      |
| F2 Shifted and Rotated Sum of Different Power Function |                      |
| F3 Shifted and Rotated Zakharov Function |                      |

| Multimodal functions               |                      |
|------------------------------------|----------------------|
| F4 Shifted and Rotated Rosenbrock’s Function |                      |
| F5 Shifted and Rotated Rastrigin’s Function |                      |
| F6 Shifted and Rotated Expanded Schaffer’s Function |                      |
| F7 Shifted and Rotated Lunacek BiRastrigin Function |                      |
| F8 Shifted and Rotated Non-Continuous Rastrigin’s Function |                      |
| F9 Shifted and Rotated Levy Function |                      |
| F10 Shifted and Rotated Schwefel’s Function |                      |

| Hybrid multimodal functions        |                      |
|------------------------------------|----------------------|
| F11 Hybrid Function 1 (\( N = 3 \)) |                      |
| F12 Hybrid Function 2 (\( N = 3 \)) |                      |
| F13 Hybrid Function 3 (\( N = 3 \)) |                      |
| F14 Hybrid Function 4 (\( N = 4 \)) |                      |
| F15 Hybrid Function 5 (\( N = 4 \)) |                      |
| F16 Hybrid Function 6 (\( N = 4 \)) |                      |
| F17 Hybrid Function 7 (\( N = 5 \)) |                      |
| F18 Hybrid Function 8 (\( N = 5 \)) |                      |
| F19 Hybrid Function 9 (\( N = 5 \)) |                      |
| F20 Hybrid Function 10 (\( N = 6 \)) |                      |

| Composite functions                |                      |
|------------------------------------|----------------------|
| F21 Composition Function 1 (\( N = 3 \)) |                      |
| F22 Composition Function 2 (\( N = 3 \)) |                      |
| F23 Composition Function 3 (\( N = 4 \)) |                      |
| F24 Composition Function 4 (\( N = 4 \)) |                      |
| F25 Composition Function 5 (\( N = 5 \)) |                      |
| F26 Composition Function 6 (\( N = 5 \)) |                      |
| F27 Composition Function 7 (\( N = 6 \)) |                      |
| F28 Composition Function 8 (\( N = 6 \)) |                      |
| F29 Composition Function 9 (\( N = 3 \)) |                      |
| F30 Composition Function 10 (\( N = 3 \)) |                      |

### B. Population Update

After generating \( M \) new offsprings using clustering-based mutation, the population is updated for which we employ a scheme based on the generic population-based algorithm (GPBA) [22]. In particular, the population is updated in the following manner:

1. **Selection**: \( k \) candidate solutions are selected randomly. This corresponds to the initial seeds for \( k \)-means clustering.
2. **Generation**: \( M \) new candidate solutions are created as \( v^{clu} \). This is conducted by the clustering-based mutation.
3. **Replacement**: \( M \) candidate solutions are selected randomly from the current population as set \( B \).
4. **Update**: From \( v^{clu} \cup B \), the best \( M \) individuals are selected as \( B \). The new population is then obtained as \((P - B) \cup B\).

### IV. Experimental Results

To verify the efficacy of Clu-DE, we perform experiments on the CEC2017 benchmark functions [20], 30 functions with different characteristics including unimodal functions, multi-modal functions, hybrid multi-modal functions, and composite functions, summarised in Table I.

In all experiments, the maximum number of function evaluations is set to \( 3000 \times D \), where \( D \) is the dimensionality of the search space.

The population size, crossover rate, and scaling factor are set to 50, 0.9, and 0.5, respectively. For Clu-DE, \( M \) is set to 10. Each algorithm is run 25 times independently, and we report mean and standard deviation over 25 runs.

To evaluate if there is a statistically significant difference between two algorithms, a Wilcoxon signed-rank test [23] is performed with a confidence interval of 95% on each function.

Table III gives the results of Clu-DE compared to standard DE for \( D = 30 \). From the table, we can see that Clu-DE statistically outperforms DE for 16 of the 30 functions, while obtaining equivalent performance for 12 functions. Only for two of the multi-modal functions Clu-DE yields inferior results.

When increasing the number of dimensions to 50, for which the results are listed in Table III, Clu-DE retains its efficacy. As can be seen, it statistically outperforms standard DE for 12 of the 30 functions, while giving similar results for 16 functions.

For \( D = 100 \), the results are given in Table IV. As we can see from there, Clu-DE obtains better or similar results for 24 of the 30 functions, thus clearly outperforming DE also for high-dimensional problems.

Last but not least, Figure 2 shows convergence curves of our proposed algorithm compared to DE for, as representative examples, F10 and F15 and all dimensionalities. As we can observe, Clu-DE converges faster than standard DE.
TABLE II: Results for $D = 30$. The last column (WSRT) gives the results of the Wilcoxon signed-rank test. + indicates that Clu-DE outperforms DE, – the opposite, and = that there is no significant difference between the two algorithms.

| function | DE (avg.) | DE (std.dev.) | Clu-DE (avg.) | Clu-DE (std.dev.) | WSRT |
|----------|-----------|---------------|---------------|------------------|------|
| F1       | 8.68E+03  | 1.47E+04      | 5.35E+03      | 5.74E+03         | =    |
| F2       | 3.86E+19  | 1.93E+20      | 1.97E+14      | 9.35E+14         | +    |
| F3       | 9.43E+03  | 8.94E+03      | 3.01E+02      | 2.67E+00         | =    |
| F4       | 4.35E+02  | 2.08E+01      | 4.49E+02      | 3.14E+01         | =    |
| F5       | 6.85E+02  | 9.19E+00      | 5.61E+02      | 2.62E+01         | +    |
| F6       | 6.00E+02  | 1.60E-04      | 6.00E+02      | 2.25E-01         | =    |
| F7       | 9.12E+02  | 1.71E+01      | 7.94E+02      | 2.88E+01         | +    |
| F8       | 9.89E+02  | 1.21E+01      | 8.03E+02      | 3.03E+01         | +    |
| F9       | 9.00E+02  | 4.94E-01      | 9.26E+02      | 4.33E+01         | =    |
| F10      | 8.76E+03  | 3.83E+02      | 5.89E+03      | 9.66E+02         | =    |
| F11      | 1.13E+03  | 2.07E+01      | 1.13E+03      | 1.39E+01         | =    |
| F12      | 6.00E+02  | 3.62E+05      | 7.16E+04      | 4.20E+04         | =    |
| F13      | 1.82E+04  | 2.15E+04      | 2.16E+04      | 4.20E+04         | =    |
| F14      | 9.17E+03  | 6.76E+00      | 1.44E+03      | 1.12E+01         | =    |
| F15      | 1.61E+03  | 8.88E+01      | 1.38E+03      | 1.22E+02         | =    |
| F16      | 3.09E+03  | 2.67E+02      | 2.89E+03      | 3.33E+02         | =    |
| F17      | 2.17E+03  | 2.01E+02      | 2.22E+03      | 2.87E+02         | =    |
| F18      | 1.09E+04  | 8.97E+03      | 8.24E+03      | 7.25E+03         | =    |
| F19      | 1.92E+03  | 5.77E+00      | 1.92E+03      | 6.39E+00         | =    |
| F20      | 2.32E+03  | 2.28E+02      | 2.46E+03      | 2.20E+02         | =    |
| F21      | 2.48E+03  | 7.34E+00      | 2.35E+03      | 2.26E+01         | =    |
| F22      | 9.99E+03  | 3.09E+01      | 6.32E+03      | 2.36E+03         | =    |
| F23      | 2.83E+03  | 1.09E+01      | 2.72E+03      | 2.90E+01         | =    |
| F24      | 3.01E+03  | 9.53E+00      | 2.91E+03      | 4.32E+01         | =    |
| F25      | 2.88E+03  | 1.16E+00      | 2.88E+03      | 1.39E+00         | =    |
| F26      | 5.26E+03  | 2.03E+02      | 4.30E+03      | 2.35E+02         | =    |
| F27      | 3.20E+03  | 1.32E-04      | 3.20E+03      | 2.49E-04         | =    |
| F28      | 3.30E+03  | 1.75E-04      | 3.30E+03      | 3.47E-04         | =    |
| F29      | 3.83E+03  | 2.48E+02      | 3.32E+03      | 2.43E+02         | =    |
| F30      | 3.22E+03  | 8.07E+00      | 3.22E+03      | 1.90E+01         | =    |

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V. CONCLUSIONS

In this paper, we have proposed a novel differential evolution algorithm, Clu-DE, based on a novel clustering-based mutation operator. A promising region in search space
TABLE IV: Results for \( D = 100 \), laid out in same fashion as Table II.

| Function | DE      | Clu-DE   | WSRT |
|----------|---------|----------|------|
| F1       | avg. 9.77E+03, std.dev. 1.21E+04 | 4.09E+03, std.dev. 6.94E+03 |
| F2       | avg. 7.36E+02, std.dev. 3.68E+03 | 2.95E+05, std.dev. 3.35E+04 |
| F3       | avg. 1.38E+06, std.dev. 2.95E+06 | 0.15E+05, std.dev. 0.35E+04 |
| F4       | avg. 3.94E+02, std.dev. 5.55E+01 | 6.57E+02, std.dev. 4.92E+01 |
| F5       | avg. 1.21E+03, std.dev. 4.17E+02 | 8.95E+02, std.dev. 7.70E+01 |
| F6       | avg. 6.01E+02, std.dev. 4.44E-01 | 6.13E+02, std.dev. 3.06E+00 |
| F7       | avg. 4.17E+01, std.dev. 4.44E-01 | 1.92E+02, std.dev. 1.56E+03 |
| F8       | avg. 1.51E+03, std.dev. 3.02E+02 | 1.18E+03, std.dev. 1.20E+02 |
| F9       | avg. 1.91E+03, std.dev. 3.02E+02 | 1.00E+04, std.dev. 8.00E+03 |
| F10      | avg. 3.28E+04, std.dev. 5.66E+02 | 2.29E+04, std.dev. 1.56E+03 |
| F11      | avg. 3.50E+03, std.dev. 4.17E+02 | 1.61E+03, std.dev. 1.29E+02 |
| F12      | avg. 6.72E+06, std.dev. 4.00E+06 | 1.20E+07, std.dev. 6.59E+06 |
| F13      | avg. 7.94E+03, std.dev. 9.64E+03 | 9.00E+03, std.dev. 1.31E+04 |
| F14      | avg. 4.07E+05, std.dev. 9.64E+03 | 4.35E+05, std.dev. 1.31E+04 |
| F15      | avg. 6.13E+03, std.dev. 3.08E+02 | 8.32E+03, std.dev. 7.70E+02 |
| F16      | avg. 1.01E+04, std.dev. 3.08E+02 | 7.18E+03, std.dev. 5.66E+02 |
| F17      | avg. 7.43E+03, std.dev. 5.66E+02 | 6.15E+03, std.dev. 3.35E+04 |
| F18      | avg. 7.94E+03, std.dev. 5.66E+02 | 6.57E+03, std.dev. 3.35E+04 |
| F19      | avg. 3.88E+03, std.dev. 3.08E+02 | 5.25E+03, std.dev. 4.40E+03 |
| F20      | avg. 7.07E+03, std.dev. 7.07E+03 | 6.61E+03, std.dev. 7.70E+02 |
| F21      | avg. 3.08E+03, std.dev. 3.08E+02 | 2.72E+03, std.dev. 2.41E+03 |
| F22      | avg. 3.46E+04, std.dev. 5.29E+02 | 2.57E+04, std.dev. 2.41E+03 |
| F23      | avg. 3.05E+03, std.dev. 3.71E+02 | 3.32E+03, std.dev. 6.19E+01 |
| F24      | avg. 4.07E+03, std.dev. 3.71E+02 | 3.97E+03, std.dev. 6.19E+01 |
| F25      | avg. 3.26E+03, std.dev. 3.71E+02 | 3.29E+03, std.dev. 6.19E+01 |
| F26      | avg. 1.13E+04, std.dev. 3.71E+02 | 1.37E+04, std.dev. 6.19E+01 |
| F27      | avg. 3.20E+03, std.dev. 4.12E-01 | 3.20E+03, std.dev. 1.11E+01 |
| F28      | avg. 3.30E+03, std.dev. 4.12E-01 | 3.30E+03, std.dev. 1.11E+01 |
| F29      | avg. 8.30E+03, std.dev. 8.30E+03 | 6.81E+03, std.dev. 6.59E+06 |
| F30      | avg. 1.02E+04, std.dev. 9.45E+03 | 1.50E+04, std.dev. 9.45E+03 |

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is found using \( k \)-means clustering and some new candidate solutions are generated using the proposed cluster-based mutation. A population update scheme is introduced to include the new candidate solutions into the current population. Extensive experiments on the CEC-2017 benchmark functions for dimensionalities of 30, 50 and 100 verify that Clu-DE is a competitive variant of DE. In future work, we intend to extend Clu-DE for multi-objective optimisation problems.

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