THE QCD TRICRITICAL POINT: BEYOND MONOTONY IN HEAVY ION PHYSICS

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I first sketch recent developments concerning the phase diagram of strongly interacting matter as a function of temperature and baryon density, obtained using a model for two-flavor QCD in which the interaction between quarks is modelled on that induced by instantons. The phase diagram has a chiral symmetry breaking vacuum, a color superconductor phase at high density, and a mixed phase in between which can describe nuclear matter if the high density regions in the mixed phase form droplets. This is amusingly similar to the phase diagram of the copper oxide superconductors, although their critical temperatures are smaller by a factor of about $10^{-9}$ and their mixed phase seems to form stripes, rather than droplets. This and other approaches to QCD with two massless quarks suggests the existence of a tricritical point on the boundary of the phase with spontaneously broken chiral symmetry. In QCD with massive quarks there is then a critical point at the end of a first order transition line. We discuss possible experimental signatures of this point, which provide information about its location and properties. We propose a combination of event-by-event observables, including suppressed fluctuations in $T$ and $\mu$ and, simultaneously, enhanced fluctuations in the multiplicity of soft pions. As a control parameter (like the collision energy) is varied, the fluctuations of appropriate event-by-event observables should therefore have minima or maxima. Experimental detection of these non-monotonic signatures would confirm that the matter was initially above the chiral phase transition, increasing our confidence in the interpretation of monotonic changes in other observables, and would teach us much about the QCD phase diagram.

1 Introduction

In QCD with two massless quarks, a spontaneously broken chiral symmetry is restored at finite temperature. It can be argued\cite{5,6,7} that this phase transition is likely second order and belongs to the universality class of $O(4)$ spin models in 3 dimensions. If this transition is indeed second order, QCD with two quarks of nonzero mass has only a smooth crossover as a function of $T$. Although not yet firmly established, this picture is consistent with present lattice simulations and many models.

At zero $T$ several models suggest\cite{8,9,10,1,11,3,12} that the chiral symmetry restoration transition at finite $\mu$ is first order. Assuming that this is the case in QCD, one can easily argue that there is a tricritical point in the $T\mu$ phase

\footnote{\textsuperscript{a}Much of this work was done in collaboration with M. Alford and F. Wilczek\cite{1} with J. Berges\cite{2} and with M. Stephanov and E. Shuryak\cite{3}. Preprint MIT-CTP-2774.}
In QCD with massless quarks, there is a sharp boundary in the $T \mu$ plane separating regions with chiral symmetry manifest from those in which it is broken. The point on this boundary at which the transition changes from first to second order is by definition tricritical. The nature of this point can be understood by considering the Landau-Ginzburg effective potential for the order parameter of chiral symmetry breaking, $\phi = (\sigma, \pi) \sim \langle \bar{\psi} \psi \rangle$:

$$\Omega(\phi) = a\phi^2 + b(\phi^2)^2 + c(\phi^2)^3.$$  \hfill (1)

The coefficients $a$, $b$, and $c > 0$ are functions of $\mu$ and $T$. The second order phase transition line described by $a = 0$ at $b > 0$ becomes first order when $b$ changes sign. The critical properties of this point can be inferred from universality and are characterized by the exponents of the mean field theory, and calculable logarithmic corrections to scaling because the critical dimension above which $\phi^d$ theory has mean field exponents is $d = 3$. If two-flavor QCD has a second order transition at high temperatures and a first order transition at high densities, then it will have a tricritical point in this universality class.

Away from the chiral limit, the second order chiral transition turns into a smooth crossover, while the first order transition remains first order. Of particular interest is the fact that the tricritical point becomes an Ising second order transition. This raises the possibility that even though the pion is massive in nature, long correlation lengths and divergent susceptibilities may arise in heavy ion collisions which traverse the chiral transition with a suitable chemical potential. We describe the resulting non-monotonic signatures in Section 4. In Section 2, we first sketch one of the models within which the tricritical point arises explicitly. The color superconducting phase of QCD can also be studied within this model, and we note several analogies between the phase diagram which emerges and that of the cuprate superconductors. In Section 3 we discuss the effect of the strange quark on the position of the tricritical point.

2 A Model and an Amusing Analogy

Berges and I have done a qualitative exploration of the phase diagram of two-flavor QCD as a function of temperature and chemical potential in a context which allows us to describe likely patterns of symmetry breaking and to make rough qualitative estimates. As a tractable model, we consider a class of fermionic models for QCD where the quarks interact via a four-fermion interaction modelled on that induced by instantons which respects all the symmetries of QCD. There are two competing ordering possibilities in QCD at nonzero
Figure 1: Phase diagram as a function of $T$ and $\mu/3$, and as a function of $T$ and $n^{1/3}$ (all in GeV) for the model for QCD with two massless quarks described in Ref.1. This section can be viewed as one long caption for these figures. The solid curves are second order phase transitions; the dashed curves describe the first order phase transition.

At low temperatures and chemical potentials, we expect chiral symmetry breaking via a nonzero chiral condensate $\phi \sim \langle \bar{\psi}\psi \rangle$. At large chemical potentials and low temperatures, we expect the quark Fermi surfaces to be unstable to the formation of a condensate of diquark Cooper pairs, leading to a superconductor in which gauge symmetries are broken and there is a gap $\Delta \sim \langle \bar{\psi}\psi \rangle$ in the spectrum of fermionic excitations. In Ref. 1, the model is presented in detail and choices of parameters are discussed. We evaluate the thermodynamic potential as a function of $\phi$ and $\Delta$ for different $T$ and $\mu$, and obtain the phase diagram shown in Fig. 1.

At zero temperature, we find a first order phase transition between a phase
with density zero in which chiral symmetry is broken, namely the vacuum, and a phase with density \( n_0 \), in which chiral symmetry is restored. This has an interesting interpretation. For reasonable choices of parameters, \( n_0 \) is greater than nuclear matter density and comparable to the baryon density in a nucleon. Ordinary nuclear matter is then in the mixed phase of this transition and consists of nucleon-sized droplets within which \( n = n_0 \) and \( \phi = 0 \), surrounded by regions of vacuum within which \( n = 0 \) and \( \phi \neq 0 \). The model as presently analyzed does not give a complete description of nuclear matter. First, color must be gauged if the model is to yield droplets which are color singlets. Second, a short range repulsion and long range attraction between droplets must be included so that small droplets are favored over bigger ones, and so that the droplets attract each other to form nuclei. Were one to add these interactions, one could describe the liquid-gas transition (between the nuclear matter liquid and a low density gas of nucleons) observed in low energy nuclear collisions as discussed recently.

Once nuclear matter is squeezed enough that the transition to the phase with density \( n_0 \) and above is completed, one obtains a spatially homogeneous color superconductor. The Cooper pairs which condense are color 3, flavor singlet, Lorentz scalars. We find critical temperatures of order 25-40 MeV for the parameters in Fig. 1, and up to \( \sim 100 \) MeV for other choices of parameters. The gap \( \Delta \) is about 1.7 times \( T_c \). Only two of the three colors participate in the condensate, meaning that only 2/3 of the quark excitations at the fermi surface have a gap. The phase transition at \( T_c \) at which superconductivity is lost is second order in our mean field analysis, but may become either first order or a crossover when fluctuations are taken into account.

In QCD with three massless quarks, a phase diagram similar to that in Fig. 1 is plausible, with two qualitative changes: all the transition lines are likely first order as we discuss below, and the symmetry properties of the superconducting phase are strikingly different than in the two flavor theory. The quark pair condensate which forms at densities high enough that chiral symmetry is restored must break flavor symmetries, since two quarks cannot form a flavor singlet. In fact, the condensate which seems to be favored “locks” color and flavor rotations, breaking the color and flavor \( SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \) symmetries down to the diagonal subgroup \( SU(3)_{\text{color}+L+R} \). All quarks have a gap, and therefore if this condensate occurs within neutron stars, direct neutrino emission is blocked. All gluons get a mass. There is an octet of pseudoscalar approximate Nambu-Goldstone boson excitations of the diquark condensate reflecting the fact that chiral symmetry is broken by color-flavor locking, even in the absence of a \( \langle \bar{\psi}\psi \rangle \) condensate. There is a modified but still massless photon; with respect to this photon, one finds only integrally
charged excitations. The thermal properties are dominated by a neutral superfluid and by the Nambu-Goldstone excitations. The gaps characterizing this phase are of order 10-100 MeV. Including a nonzero strange quark mass \( m_s \) results in a mismatch between the Fermi momenta for the light and strange quarks given by \( \mu - \sqrt{\mu^2 - m_s^2} \), of order 10 MeV for \( \mu \sim 500 \) MeV. As long as the gap is larger than this mismatch, the strange quark mass should have only a quantitative effect. Were the gap smaller, one would return to the two-flavor condensate described above, with the light and strange quarks behaving independently.

The lower panel of Fig. 1 looks similar to the phase diagram which describes the cuprate superconductors. There, one can vary both the temperature and, by doping, the number density of holes in the copper oxide planes. At low densities, and at temperatures up to many hundred Kelvin, these materials are antiferromagnets. Antiferromagnetic order and chiral symmetry breaking are similar. Indeed, the development of antiferromagnetic order can be mapped onto chiral symmetry breaking in a 2+1 dimensional theory of Dirac fermions and gauge fields. Like QCD, and unlike a ferromagnet, an antiferromagnet has Goldstone bosons with a dispersion relation \( \omega \sim k \). At higher densities and lower temperatures, these materials are superconductors. In between the low density antiferromagnetic and high density superconducting phases, there is a range of densities in which one finds a spin glass, which has no obvious analogue in the QCD phase diagram. However, if these materials were fully 3-dimensional, rather than behaving 2-dimensionally in many ways, the antiferromagnetic phase would likely extend to higher densities.

There may be a further analogy between QCD and the cuprate superconductors. There are indications that on the low density side of the superconducting region in the cuprate phase diagram, there may be a phase characterized by alternating antiferromagnetic and superconducting stripes. In the mixed phase in QCD, the high density regions organize themselves into small nucleon-sized droplets, while in the copper oxide planes, it seems that the hole-rich regions may organize themselves into long stripes a single lattice spacing wide. As in QCD, this can be seen as a competition between phase separation which reduces the disruption of the magnetic (chiral) order by segregating the fermions, and repulsion between the fermions which tries to scatter them. For whatever reason, this competition results in narrow stripes in one case and

\[ T_c \text{ in these materials can be over } 100 \text{ K; hence they are called high temperature superconductors. However, as in Fig. 1, } T_c \text{ is still significantly lower than the temperature at which the antiferromagnetic order is lost at zero density. } T_c \text{ for the QCD superconductor is many orders of magnitude higher than that for any cuprate superconductor, but this is an unfair comparison! It is more appropriate to note that in QCD we find } T_c \text{'s of order } 10\% \text{ of the Fermi energy, reasonable for a BCS superconductor at reasonably strong coupling.} \]
small droplets in the other. Whereas the droplets in the QCD mixed phase are so small (quark number 3) that they cannot be superconductors in any meaningful sense, the stripes in the cuprates can be superconducting.

QCD and the cuprate superconductors are both strongly interacting systems with chiral symmetry breaking/antiferromagnetism at low densities and superconductivity at higher densities and relatively low temperatures. Furthermore, it may be that in both systems these phases are separated by complicated regions of the phase diagram describable as mixed phases: nuclear matter in one case, stripes in the other. The different microscopic Hamiltonians and, likely most important, the different dimensionalities make it unlikely that these analogies can be made more than amusing. Perhaps, though, qualitative ideas from one system could prove useful in the study of the other.

3 The (Tri)critical Point....

![Figure 2: The schematic phase diagram of QCD. The dashed lines represent the boundary of the phase with spontaneously broken chiral symmetry in QCD with 2 massless quarks. The point P is tricritical. The solid line with critical end-point E is the line of first order transitions in QCD with 2 quarks of small mass. The point M is the end-point of the nuclear liquid-gas transition probed in multifragmentation experiments. The superconducting phase of QCD, marked SC, is not relevant to the rest of our discussion.](image)

We now set aside models and analogies, and return to our general discussion of the tricritical point in the QCD phase diagram. This section and the next describe work done with M. Stephanov and E. Shuryak. In real QCD with nonzero quark masses, the second order phase transition becomes a smooth crossover and the tricritical point becomes a critical (second order) end-point of a first order phase transition line. Universality arguments also predict that the end-point E in QCD with small quark masses is shifted with respect to the tricritical point P towards larger $\mu$ as shown in Fig. 3. It can also be argued that the point E is in the universality class of the Ising model in 3 dimensions, because the $\sigma$ is the only field which becomes massless at this point. (The pions remain massive because of the explicit chiral symmetry...
breaking by quark masses.) In this paper we discuss experimental signatures of this critical end-point, which is in the same universality class as that in the standard liquid-gas phase diagram.

The position of the points P and E in two-flavor QCD was estimated using two different models (that of Ref. 3 and a random matrix model) as $T_p \sim 100$ MeV and $\mu_p \sim 600 - 700$ MeV. These are only crude estimates, since they are based on modeling the dynamics of chiral symmetry breaking only.

The third (strange) quark has an important effect on the position of the point P and, therefore, of the point E. At $\mu = 0$, if the strange quark mass $m_s$ is less than some critical value $m_{s3}$, the second order finite $T$ transition becomes first order. This leads to a tricritical point in the $T m_s$ plane. Theoretically, the origin of this point is similar to the one we are discussing. In terms of eq. (1), the effect of decreasing $m_s$ is similar to the effect of increasing $\mu$: the coefficient $b$ becomes negative. What is important is that, unlike $m_s$, $\mu$ is a parameter which can be experimentally varied.

Clearly, the physics of the $T \mu$ plane is as in Fig. 2 only for $m_s > m_{s3}$. For $m_s < m_{s3}$, the transition is first order already at $\mu = 0$, and, presumably, remains first order at all nonzero $\mu$. As $m_s$ is reduced from infinity, the tricritical point P of Fig. 2 moves to lower $\mu$ until, at $m_s = m_{s3}$, it reaches the $T$-axis and can be identified with the tricritical point in the $T m_s$ plane. The two tricritical points are continuously connected. We assume that $m_s > m_{s3}$ which is consistent with the lattice studies of Ref. 24. What is important for us is that the qualitative effect of the strange quark is to reduce the value of $\mu_p$, and thus of $\mu_E$, compared to that in two-flavor QCD, since $\mu_p = 0$ at $m_s = m_{s3}$. This shift may be significant, since lattice studies show that the physical value of $m_s$ is of the order of $m_{s3}$.

Analysis of particle abundance ratios observed in central heavy ion collisions indicates that chemical freeze-out happens near the phase boundary, at a chemical potential $\mu \sim 500 - 600$ MeV at the AGS (11 GeV·A), while at the SPS (160 - 200 GeV·A) it occurs at a significantly lower $\mu \lesssim 200$ MeV. In view of the effect of the strange quark just discussed, the estimated position of P and E should be shifted from $\mu_E \sim 600 - 700$ MeV to lower $\mu$. Thus, it may well be between the SPS and the AGS values of $\mu$, and therefore the point E may be accessible at lower energy or non-central collisions at the SPS.

4 .... and its Signatures

The strategy for finding the point E which we propose is based on the fact that this point is a genuine critical point. Such a point is characterized by enhanced long wavelength fluctuations of the order parameter which lead to singularities
in all thermodynamic observables. In the liquid-gas phase transition in water, the observation of critical opalescence is perhaps the easiest way to detect these unique properties characterizing physics near the critical point. The signatures we propose can play an analogous role in QCD.

It is important to have control parameters which can be adjusted so that the system explores different points on the phase diagram. For example, by increasing the energy of the collision one increases the initial $T$ and decreases the initial $\mu$. A qualitatively similar effect may be achieved by increasing centrality. We will generically call the control parameter which is varied “$x$”, and take increasing $x$ to mean increasing collision energy or increasing centrality. Scanning in centrality will almost certainly be the easiest, since in any given run events with all impact parameters are present. However, comparison of AGS and SPS results demonstrates that scanning in energy will yield a large variation in the $\mu$ at which the transition is crossed, whereas this has not been demonstrated for scanning in centrality, which may only provide fine tuning.

![Figure 3: Schematic examples of three possible trajectories for three values of $x$ on the phase diagram of QCD (see, Fig. 2). The points I, S, H and F on different trajectories are marked with different symbols. The dashed lines show the locations of the initial, I, and final, F, points as $x$ is increased in the direction shown by the arrows.](image)

In this work we do not discuss initial equilibration and we choose to define the initial point, I($x$), as the point at which compression has ended, most of the entropy is already produced, and approximately adiabatic expansion begins. The system will then follow some trajectory in the $T\mu$ plane characterized by the ratio of the baryon charge density to the entropy density, $n/s$, which is (approximately) conserved. Three trajectories are shown schematically in Fig. 3. (For realistic hydrodynamical calculations and discussion see, e.g., Refs. 26, 27).

Recall that the first order line in the $T\mu$ plane is actually a whole region of mixed phase, with the additional hidden parameter being the volume fraction.
of the two coexisting phases. The zig-zag shape occurs because, as the trajectories enter the mixed phase region, $n/s$ remains constant during the adiabatic expansion, latent heat is released and $T$ increases. In Fig. 3, we use the following notation: $S(x)$ for the “softest” point, $H(x)$ for the “hottest” point and $F(x)$ for the final thermal freeze-out after which no scattering occurs. (Note that at small values of $x$, at which the transition is first order, the trajectories are likely to begin within the mixed phase region. The special case when $I(x)$ coincides with $S(x)$ leads to a local maximum of the QGP lifetime.) Increasing $x$ will yield trajectories shifted to the left in Fig. 3, traversing the transition region at lower $\mu$ and higher $T$. The existence of the end-point singularity, $E$, leads to the phenomenon which we refer to as the “focusing” of trajectories towards $E$. The initial point $I(x)$ and the beginning of the zig-zag $S(x)$ depend on the control parameter $x$ more strongly than the zig-zag end-point $H(x)$. The reason for this is that the point $H(x)$ is always closer to $E$ than $S(x)$ (see Fig. 3). This focusing effect implies that exploring physics in the vicinity of the end-point singularity may not require a fine-tuned $x$. This situation resembles that in low energy nuclear collisions, in which the first order liquid-gas phase transition also has a critical end-point at a temperature of order 10 MeV (point M on Figs. 2 and 3). In such experiments, one varies control parameters to maximize the probability of multi-fragmentation. It was noticed long ago how surprisingly easy it is to hit the critical region. We believe that part of the reason is a focusing phenomenon analogous to the one we are describing.

Another aspect of the “focusing” arises via the divergence of susceptibilities, such as the specific heat capacity $c_V = T\delta s/\delta T$, at the endpoint $E$. As a result, the trajectories which pass near the critical point will linger there longer. This makes it likely that final freeze-out occurs at a temperature quite close to $T_F$, rather than below it. So, while scanning in some control parameter $x$ and measuring the positions of the points $F(x)$, we may expect to find a bump in the vicinity of the point $E$. (See the lower dashed curve on Fig. 3.) At this point it is instructive to consider the dependence on another control parameter, the atomic weight $A$ of the colliding nuclei. If $A$ were infinite, the point $F$ would be close to $T_F = 0, \mu_F = m_N$. Thus, for $A$ large enough, the dotted curve in Fig. 3 moves down and the bump and indeed all the signatures described below fade away. Experimentally, the $A$ dependence of the point $F$ has been established recently by the analysis of flow, Coulomb effects and pion interferometry. For example, in central S+S collisions at SPS $T_F \approx 140 - 150$ MeV, while for Pb+Pb it is only $T_F \approx 120$ MeV.

We shall now discuss the signatures which directly reflect thermodynamic properties of the system near its critical point. With the advent of wide-solid-
angle detectors like NA49 at CERN, it is now possible to make event-by-event measurements of observables which are proxies for the freeze-out $T$ and $\mu$.

We argue that the event-by-event fluctuations in both quantities should be anomalously small for values of $x$ such that the system freezes out near the critical point. As has been suggested recently, event-by-event fluctuations of $T$ can be related by basic thermodynamics to the heat capacity at freeze-out

$$\frac{(\Delta T)^2}{T^2} = \frac{1}{C_V}. \quad (2)$$

The quantity $C_V$ is extensive, so $\Delta T \sim 1/\sqrt{N}$ as expected, where $N$ is the number of particles in the system. If the specific heat $c_V$ diverges, the coefficient of $1/\sqrt{N}$ vanishes and fluctuations of $T$ are suppressed. For freezeout in the crossover region, or in the hadronic phase just below the first order transition, $c_V$ is finite. (If freeze-out occurs from the mixed phase, some linear combination of the two susceptibilities is relevant.) As the freeze-out point approaches the critical point from either the left or the right, $c_V$ diverges and $\Delta T \sqrt{N}$ is minimized. Other susceptibilities, in particular, $-\partial^2 \Omega/\partial \mu^2$, are also divergent. This implies that fluctuations of $\mu$ are also suppressed at the critical point.

Experimentally, $\Delta T$ can be found via event-by-event analysis of $p_T$ spectra. Fluctuations in $\mu$ correspond to event-by-event fluctuations in the baryon-number-to-pion ratio. The event-by-event fluctuations in experimental observables will receive contributions in addition to the thermodynamic ones we describe. For example, the fluctuations in the slope of the $p_T$ spectrum will receive a contribution from $\Delta T$ and a (likely small) contribution from fluctuations in the flow velocity. We therefore expect that as the collision energy is increased so that the freeze-out point moves from right to left past the critical point, we will find minima (but not zeroes) in the widths of the distributions of event-by-event observables which are well-correlated with $T$ and $\mu$.

Using universality, we can predict the exponents for the divergent susceptibilities at the point $E$. Very naively, one might think that the exponent describing the divergence of $C_V$ is $\alpha$, which is small for the 3-dimensional Ising model universality class: $\alpha \approx 0.12$. In fact the exponent for $C_V$ is significantly larger. This and the exponent for the $\mu$-susceptibility are determined by finding two directions, temperature-like and magnetic-field-like, in the $T\mu$ plane near point $E$, following the standard procedure for mapping a liquid-gas transition onto the Ising model. The two linear combinations of $T - T_E$ and $\mu - \mu_E$ corresponding to these directions should then be identified (in the sense of the universality) with the temperature, or $t = T - T_c$, and the magnetic field, $h$, in the Ising model. The $t$-like direction should be tangential to the first-order line at the point $E$. Then $C_V$ and $-\partial^2 \Omega/\partial \mu^2$ are different linear combinations.
of the $t$-like and $h$-like susceptibilities. In both linear combinations, the divergence of the $h$-like susceptibility will dominate because $\gamma \approx 1.2 \gg \alpha \approx 0.12$. The exponent for the divergence of the $h$-like susceptibility as a function of the distance, $\ell$, from the point $E$ will depend on the direction along which one approaches this point. For almost all directions it will be given by $\gamma/\beta \delta \approx 0.8$ (except for exactly the $t$-like direction, where it is $\gamma$). As a result, for points on the $T\mu$ plane along a generic line through $E$ one finds

$$ (\Delta T)^2 \sim (\Delta \mu)^2 \sim \ell^{0.8} \quad (3) $$

sufficiently close to $E$. Therefore, the fluctuations of $T$ and $\mu$ are considerably suppressed when the freeze-out occurs near the critical point.

We turn now to direct signatures of the long-wavelength fluctuations of the massless $\sigma$ field. For the choices of control parameters $x$ such that freeze-out occurs at (or near) the point $E$, the $\sigma$-meson is the most numerous species at freeze-out, because it is (nearly) massless and so the equilibrium occupation number of the long-wavelength modes ($T/\omega$) is large. Because the pions are massive at the critical point $E$, the $\sigma$’s cannot immediately decay into $\pi\pi$. Instead, they persist as the density of the system further decreases. It is important to realize that after freeze-out, one can (by definition) approximately neglect collisions between particles. Collective effects related to forward scattering amplitudes cannot be neglected. That is, although the particles no longer scatter, their dispersion relations will not be given by those in vacuum until the density is further reduced by continued expansion.

During the expansion, the in-medium sigma mass rises towards its vacuum value and eventually exceeds the $\pi\pi$ threshold. As the $\sigma\pi\pi$ coupling is large, the decay proceeds rapidly. This yields a population of pions with small transverse momentum, $p_T < m_\pi$. Because this process occurs after freeze-out, the pions generated by it do not get a chance to thermalize. Thus, the resulting pion spectrum should have a non-thermal enhancement at low $p_T$ which is largest for freeze-out near $E$ where the $\sigma$’s are most numerous.

These pions result from the (formerly) long wavelength modes of the $\sigma$ field, which (unlike $T$ and $\mu$) are expected to fluctuate at the critical point. For freeze-out close enough to $E$ that the sigma mass at freeze-out is less than $T$, the thermal fluctuations of the number, $N_\sigma$, of $\sigma$ particles are determined by the classical statistics of the field $\sigma$, rather than by Poisson statistics of particles. Therefore, $\langle N_\sigma^2 \rangle - \langle N_\sigma \rangle^2 \sim \langle N_\sigma \rangle^2$, rather than $\langle N_\sigma \rangle$. Thus, we expect large event-by-event fluctuations in the multiplicity and distributions of the soft pions: $N_\pi \approx 2N_\sigma$. Due to critical slowing down, non-equilibrium effects may further enhance these fluctuations. Thus, these pions could be detected either directly as an excess in the $p_T$-spectra at low $p_T$, or via increased event-by-
event fluctuations at low \( p_T \), or by an increase in HBT correlations due to the larger number of pions per phase space cell at low \( p_T \).

To conclude, we propose that by varying control parameters such as the collision energy and centrality, one may find a window of parameters for which the \( T\mu \) trajectories pass close to the critical point \( E \). Enhanced critical fluctuations of the \( \sigma \) field and the associated thermodynamic singularities lead directly to the signatures we propose. When the freeze-out occurs near the point \( E \), we predict large non-thermal multiplicity and enhanced event-by-event fluctuations of the soft pions. In contrast, the event-by-event fluctuations in both \( T \) and \( \mu \), as determined using pions with \( p_T \gtrsim m_\pi \), will be anomalously suppressed. Both effects should disappear if the atomic weight \( A \) is very large. No one of these signatures is distinctive in isolation and without varying control parameters. Several of them seen together and seen to turn on and then turn off again as a control parameter is varied monotonically would constitute a decisive detection of the critical point.

What would we learn about QCD if such a point is found? First, we would learn that there is a genuine critical point in the \( T\mu \) plane in nature. Second, we would learn that \( m_s > m_{\pi^0} \) in nature, and the \( \mu = 0 \) thermal transition is a crossover for physical quark masses, rather than a first-order phase transition. Third, the experimental discovery of the critical end-point \( E \) would mean that if the light quark masses were set to zero, there would be a tricritical point \( P \) in the phase diagram of QCD.

5 Beyond Monotony

Most of the signatures which have been proposed as means of searching for new phases of QCD are expected to change monotonically as the collision energy or centrality is increased. Once \( J/\Psi \)'s or \( \rho \)'s are gone, for example, they are not expected to come back. This makes these signatures susceptible to mimicry. The suppression of \( J/\Psi \) production or the broadening of the \( \rho \) peak can occur at least to some extent in a medium which is hot and/or dense, but is still on the hadronic side of the transition region. The effects of any new phase then simply augment a previously established monotonic trend, and may be hard to discern. This makes non-monotonic signatures valuable. One previous example arises from the softening of the equation of state near the phase transition, whether first or second order or a crossover, which can yield non-monotonic behavior of observables related to the lifetime and collective flow of the plasma. Here, we have shown how physics unique to the region of the critical point in the phase diagram can yield non-monotonic behavior of the event-by-event fluctuations of suitable observables. Successful detection
would demonstrate directly that freeze-out occurred near the critical point, and therefore indirectly that the initial conditions were well above the transition region. Having seen non-monotonic behavior characterizing the critical point, monotonic observables which can probe the initial conditions directly could then be interpreted with greater confidence.

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