Enhanced diamagnetism and Nernst signal from a chiral d-density wave state in the pseudogap regime of the cuprates

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Abstract. In this paper we propose an alternative explanation for the pseudogap regime of the cuprate superconductors. Specifically, a solution to this puzzle can be provided by associating at least a part of the pseudogap regime with a chiral $d_{xy} + id_{x^2-y^2}$ density wave. As we demonstrate for the first time, violation of parity and time reversal from this unconventional density wave gives rise to the Topological Meissner effect and the concomitant Anomalous Nernst effect. As a matter of fact, the enhanced diamagnetism and Nernst signal, recently observed in the cuprates, could be simultaneously explained in terms of a chiral d-density wave.

1. Introduction
Undoubtedly, understanding the pseudogap regime [1] of the cuprates is a key to ultimately resolve the mechanism of high $T_c$ superconductivity. The main question to be answered is the recent experimental data demonstrating enhanced diamagnetism and Nernst signal in this regime [2, 3, 4]. Such a behaviour is typical for the vortex state of superconductors. However, these properties are extremely peculiar for the pseudogap, as it is clearly a non superconducting state. There have been various attempts to explain this unconventional behaviour, among which there are two dominating trends. According to the first, the anomalous response can be associated with singular superconducting fluctuations that persist up to a high temperature [5]. In fact in this picture we have a realization of a Kosterlitz-Thouless type of topological phase transition [6]. On the other hand, there are theories which associate the pseudogap with a non superconducting ordered state, the d-density wave [7], which strongly competes with d-wave superconductivity. Although this state may provide a good explanation for several experimental results, it cannot interpret the recent findings concerning the enhanced diamagnetic and Nernst signals.

From our point of view, a solution can be naturally provided, if we associate the pseudogap with a chiral $d_{xy} + id_{x^2-y^2}$ density wave state. This unconventional density wave violates parity ($\mathcal{P}$) and time reversal ($\mathcal{T}$) giving rise to an anomalous thermo-magneto-electric coupling. As we demonstrate for the first time, a chiral d-density wave exhibits the Topological Meissner effect, which constitutes a source of enhanced diamagnetism without the necessary presence of any kind of superconductivity. Moreover, we demonstrate that the presence of non-zero chirality, also generates an Anomalous Nernst signal, with a temperature profile which is in perfect agreement...
with the experimental observations. As a matter of fact, the above features, render the chiral d-density wave scenario as a prominent alternative in understanding the pseudogap regime.

2. The Model

To demonstrate how the Topological Meissner effect and the concomitant Anomalous Nernst effect arise, we consider the following mean field Hamiltonian $H = \sum_k \{ \varepsilon(k)c^\dagger_k c_k + \varepsilon(k + Q)c^\dagger_{k+Q} c_{k+Q} + \Delta(k)c^\dagger_k c_{k+Q} + \Delta^*(k)c^\dagger_{k+Q} c^\dagger_k \}$, which describes a $d_{xy} + id_{x^2−y^2}$ density wave state characterized by the commensurate wave-vector $Q = (\pi, \pi)$. Since spin degrees of freedom do not get involved we have considered spinless electrons, so that all our results will refer to one spin component. We have introduced the single band energy dispersion $\varepsilon(k)$, which can be decomposed into the periodic $\delta(k) = 4t' \cos k_x \cos k_y - \mu$ and anti-periodic $\gamma(k) = -2t(\cos k_x + \cos k_y)$, parts. Moreover, $\Delta(k)$ is the chiral d-density wave order parameter defined as $\Delta(k) = \eta \Delta \sin k_x \sin k_y + i \Delta \cos k_x - \cos k_y \\text{.}$ According to our notation, $\Delta$ is the modulus of the $id_{x^2−y^2}$ order parameter and $\eta$ determines the relative magnitude of the two components and the direction of the chirality.

To reveal the topological structure of the Hamiltonian, it is eligible to introduce the spinor $\Psi_k = (c^\dagger_k, c^\dagger_{k-Q})$ and the $\tau$ isospin Pauli matrices. Moreover, we define the isospin vector $g(k) = (Re\Delta(k), -Im\Delta(k), \gamma(k))$. Under these conditions, the Hamiltonian obtains the form $H = \sum_k \Psi_k^\dagger \{ g(k) \cdot \sigma + \delta(k) \} \Psi_k$, which is reminiscent of the Weyl Hamiltonian describing chiral fermions [8]. As a matter of fact, in our case the $\mathcal{P} − \mathcal{T}$ violation induced by the $d_{xy} + id_{x^2−y^2}$ state, gives rise to an intrinsic angular momentum, as if the particles of our theory were chiral.

3. Topological Meissner Effect

The $\mathcal{P} − \mathcal{T}$ violation generated by the formation of the chiral d-density wave state, induces the so called Chern-Simons term in the effective action of the U(1) gauge field [8, 9]. These terms provide a topological mass to the electromagnetic field, in a gauge invariant manner [10], leading to perfect electromagnetic screening. As a consequence, a chiral d-density wave exhibits the Topological Meissner effect which in the particle-hole symmetric case ($\delta(k) = 0$), is identical to the usual superconducting diamagnetism [11]. The coefficient of the Chern-Simons action $S_{CS} = \int d^4 x \frac{\sigma_{\mu\nu}}{4} F^{\mu\nu} F_{\nu\lambda}$, is the Hall conductance $\sigma_{xy}$, where $A_\mu$ is the U(1) gauge field and $\sigma_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, with $\mu = 0, 1, 2$. In the case of particle-hole symmetry and zero temperature, the Hall conductance becomes a topological invariant [13, 14, 15] and is equal to $\sigma_{xy} = e^2/2\pi = e^2/\pi^2$, where we have introduced the unit vector $\mathbf{g} = \mathbf{g}/|\mathbf{g}|$ and the winding number $\tilde{N} = \frac{1}{4\pi} \int d^2 k \mathbf{g} \cdot \left( \frac{\partial}{\partial k_x} \times \frac{\partial}{\partial k_y} \right)$, which is related to the chirality of the state. In the
latter case, the equation which describes the Topological Meissner effect is $\frac{\partial^2 B_z}{\partial x^2} - \frac{1}{\lambda^2} B_z = 0$, with $\lambda = [\sqrt{\varepsilon / \mu}] / \sigma_{xy} d$, the penetration depth, $\varepsilon$ the electric permittivity, $\mu$ the magnetic permeability and $d$ the $z$-axis thickness [11]. This equation is identical to the one we find in a superconductor. However, we notice that only the $z$-component of the magnetic field is involved in the Meissner effect, implying that the latter occurs only for magnetic fields which are perpendicular to the plane. This anisotropy of the Meissner effect is totally consistent with the experimental observations in the cuprates [2, 3, 4].

4. Anomalous Nernst effect

Similarly to the Spontaneous Quantum Hall effect [13, 14, 15], where a Hall response can be generated with the sole application of an electric field, a chiral d-density wave can also exhibits an anomalous thermoelectric behaviour. Particularly, the application of an external temperature gradient can lead to a Hall response without the simultaneous presence of an external magnetic field. This necessity is fully settled by the intrinsic angular momentum of the state. This Anomalous Nernst effect arises because the Hall thermoelectric conductance $\alpha_{xy}$ (Peltier coefficient), is non-zero.

One may derive microscopically the expression for the thermoelectric Hall conductance by calculating a charge-heat current-current correlation function. Based on the equation of continuity the energy density $\mathcal{H}(q) = \sum_k \Psi_k \{ g(k) \cdot \tau + \delta(k) \} \Psi_k$, we can define an energy current by the relation $J_E(q) = \text{i} \lim_{q \to 0} \nabla_q \mathcal{H}(q) = \sum_k \Psi_k \{ g(k) \cdot \tau + \delta(k) \} \nabla_k \{ g(k) \cdot \tau + \delta(k) \} \Psi_k$. The corresponding heat current is determined as $J_Q(q) = J_E(q) - \tilde{\mu} J_C(q)$, where $\tilde{\mu}$ represents an external spatially varying chemical potential. Taking into account that our Hamiltonian already contains the term $\delta(k)$ which plays the role of a chemical potential, we can ignore external concentration gradients and chemical potentials, so we set $\tilde{\mu} = 0$ and consequently $J_Q = J_E$. The thermoelectric Hall conductance $\alpha_{xy}$ is determined by the Kubo formula $\alpha_{xy}(\omega) = \frac{1}{\omega^2} \Pi_{Q}^{xx} (q \to 0, i\omega \to \omega + i0^+)$. It is straightforward to obtain the expression for the d.c. thermoelectric Hall conductance

$$\alpha_{xy} = \frac{e}{T} \sum_k n_F [\delta(k) - E(k)] - n_F [\delta(k) + E(k)] g(k) \cdot \left[ \frac{\partial g(k)}{\partial k_x} \times \frac{\partial g(k)}{\partial k_y} \right] \delta(k). \quad (1)$$

In a Nernst setup, one applies simultaneously a temperature gradient in the $x$-direction and a magnetic field in the $z$-direction. The Nernst signal is the arising voltage in the $y$-direction in order to have zero electric currents along the $y$-axis. The charge and heat currents are given by the relations $J_C = \sigma \cdot E + \alpha \cdot (-\nabla T)$ and $J_Q = \tilde{\alpha} \cdot E + \kappa \cdot (-\nabla T)$, where $E$ is the electric field and $\sigma, \alpha \ (\tilde{\alpha})$ and $\kappa$ are the electric, thermoelectric and thermal conductivity tensors. The requirement of vanishing electric current yields the following formula for the Nernst signal $N \equiv E_y / \nabla T = (\alpha_{xx} \sigma_{xy} - \alpha_{xy} \sigma_{xx}) / (\sigma_{xx} \sigma_{yy} + \sigma_{xy}^2)$. Usually the Hall angle, $\tan \Theta_H = \sigma_{xy} / \sigma_{xx}$, is small and the Nernst signal is approximately $N \approx \rho \alpha_{xy}$, where we have introduced the longitudinal conductance $\rho = \sigma_{xx}^{-1} = \sigma_{yy}^{-1}$ and omitted the minus sign. For the case of the chiral d-density wave, this approximation stands when the chemical potential is large compared to the $d_{xy}$ which is generally assumed smaller than the $d_{x^2-y^2}$ component. The temperature dependence of the Anomalous Nernst signal in this limit has been studied by C. Zhang et al [16]. According to their results, the Nernst signal has a linear temperature dependence in this limit, which is incompatible with the experimental findings.

On the contrary, the tilted hill profile temperature dependence for the Anomalous Nernst signal can be obtained when the chemical potential lies between the two bands of the chiral d-density wave. In this case we cannot consider the Hall angle negligible. In fact the system is
Figure 2. Anomalous Nernst Effect. The tilted hill profile of the Anomalous Nernst signal not only matches the temperature dependence of the observed enhanced Nernst signal in the pseudogap regime, but also implies that this state could coexist with d-wave superconductivity. We notice that the increase of the $d_{xy}$ gap increases the temperature scale of the Anomalous Nernst signal’s peak. ($t = 250\text{meV}, d_{x^2-y^2} = 50\text{meV}$)

dominated by Hall transport. Calculating the Nernst signal for various $d_{xy}$ gaps, illustrates that the temperature profile of the Nernst signal is in perfect agreement with the experimental data. The chiral d-density wave is employed by us mainly in order to explain the enhanced Nernst signals only in the pseudogap regime, but the tilted hill profile suggests that this state could also be a hidden order deep into the d-wave superconducting state. Moreover, we observe that the tilted hill’s peak moves to larger temperatures with the increase of $d_{xy}$, a feature that is anticipated, as the increase of this gap improves the topological robustness of the system. The magnitude of the Nernst signal $\sim 100\mu\text{V/K}$, is about 100-1000 larger than the observed. However, we expect this result to decrease when departing from the nearly half-filled case. Finally, we should expect additional contributions by the usual quasi-particle Nernst response when an external magnetic field is applied.

Note Added
We should point out that the expression that we have obtained for $\alpha_{xy}$ is different from the one obtained in Ref. [16]. The reason is that in our treatment we have also kept intact the orbital magnetization currents. Nonetheless, the hill profile of the Anomalous Nernst signal persists also after subtracting this contribution.

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