Improving a family of Darboux methods for rational second order ordinary differential equations

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A B S T R A C T
We have been working in many aspects of the problem of analyzing, understanding and solving ordinary differential equations (first and second order). As we have extensively mentioned, while working in the Darboux type methods, the most costly step of our methods and algorithms of solution is the determination of Darboux polynomials for the associated differential operators. Here, we are going to apply a procedure to greatly reduce the time expenditure in determining these needed Darboux polynomials for a class of second order differential equations.

New version program summary
Program Title: FiOrDi
Catalogue identifier: AEQL_v2_0
Program Summary URL: http://cpc.cs.qub.ac.uk/summaries/AEQL_v2_0.html
Program obtainable from: CPC Program Library, Queen’s University, Belfast, N. Ireland
Licensing provisions: Standard CPC licence, http://cpc.cs.qub.ac.uk/licence/licence.html
No. of lines in distributed program, including test data, etc.: 1784
No. of bytes in distributed program, including test data, etc.: 35174
Distribution format: tar.gz
Programming language: Maple (release 17).
Computer: PC.
Operating system: Windows 7. Windows Vista
RAM: 128 Mb
Keywords: First integrals, Second order ordinary differential equations, Darboux type approach, Computer algebra, Darboux polynomials.
PACS: 02.30.Hq.
Classification: 4.3, 5.
Catalogue identifier of previous version: AEQL_v1_0
Journal reference of previous version: Compt. Phys. Commun. 185(2014)307
Does the new version supersede the previous version?: Yes
Nature of problem: Determination of first order differential invariants for rational second order ordinary differential equations.
Solution method: The method of solution is based on a Darboux type approach.
Reasons for the new version: We have been working on analyzing and solving systems of first and second order differential equations (1ODEs and 2ODEs, respectively) from a numerical point of view, using Lie
methods and Darboux type approaches. For this latter class of methods, we have been developing (semi) algorithms to deal with classes of ODEs. In these algorithms, one fact has been always present: the most (computationally) costly step is the determination of the associated Darboux polynomials.

Based on this realization, here we will be focused on speeding the process of finding Darboux polynomials for a class of ODEs of our interest. In particular, in this paper, we will talk about a class of rational 2ODEs.

Summary of revisions: We have realized that one can extract information regarding the Darboux polynomials, correspondent to the D-operator related to the 2ODE (please see [1]) being studied in a very straightforward way. This, although a simple procedure, will prove essential to solve (or at least reduce) some 2ODEs.

As mentioned in [1], the fact that our method uses the differential operator defines as in Eq. (1) is very advantageous.

So, let us first define the D-operator we have been using:

\[ D ≡ N \partial_x + z N \partial_y + M \partial_z, \]  

where \( z = \frac{\partial y}{\partial x}, \ y = y(x), \) and \( M \) and \( N \) are polynomials in \( (x, y, z). \)

Considering this \( D \)-operator, one can see by inspection on (1), that the cases that will be of interest to us are the ones listed below:

1. Darboux polynomials as factors of the numerator \((in \ M)\) that are polynomials on \((z)\) only.
2. Darboux polynomials as factors of the denominator \((in \ N)\) that are polynomials without \(z.\)

So, the main revision we introduce here is to implement routines, in our new Maple code, to look for the needed Darboux polynomials for 2ODEs belonging to the classes mentioned above, just by inspecting the general expression for the 2ODE. With this, we make it available for the researcher, using our package, a more powerful weapon. These implementation is basically done via a modification to the command \texttt{dsolve}\texttt{theneededDarbouxpolynomials,thearchitectwedevelopedin[1]findstheresults(3)and(5).Forabove),withtermsuptothepowerof10,maketheregularprocessofdeterminingitvery''expensive''intimeexpenditureandmemory.}\texttt{Runningtime:}This dependsonthely, which very quickly determined the needed Darboux polynomials, the algorithm we introduced in [1] finds the results (3) and (5).

For this particular instance, the in-built Maple (very powerful) \texttt{dsolve} command fails to reduce this ode. Our procedure takes some minutes but reduces it.

\textbf{Running time:} This depends strongly on the ODE, but usually under 4 seconds.
References:

[1] L.G.S. Duarte and L.A.C.P. da Mota, Finding Elementary First Integrals for Rational Second Order Ordinary Differential Equations, Journal of Mathematical Physics, Volume 50, Issue 1, pp. 013514-013514-17 (2009).

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