Distribution of the time of the maximum for stationary processes

FRANCESCO MORI1(a), SATYA N. MAJUMDAR1 and GRÉGORY SCHEHR2

1 LPTMS, CNRS, Univ. Paris-Sud, Université Paris-Saclay - 91405 Orsay, France
2 Sorbonne Université, Laboratoire de Physique Théorique et Hautes Energies, CNRS, UMR 7589
4 Place Jussieu, 75252 Paris Cedex 05, France

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Abstract – We consider a one-dimensional stationary stochastic process \(x(\tau)\) of duration \(T\). We study the probability density function (PDF) \(P(t_m|T)\) of the time \(t_m\) at which \(x(\tau)\) reaches its global maximum. By using a path integral method, we compute \(P(t_m|T)\) for a number of equilibrium and nonequilibrium stationary processes, including the Ornstein-Uhlenbeck process, Brownian motion with stochastic resetting and a single confined run-and-tumble particle. For a large class of equilibrium stationary processes that correspond to diffusion in a confining potential, we show that the scaled distribution \(P(t_m|T)\), for large \(T\), has a universal form (independent of the details of the potential). This universal distribution is uniform in the “bulk”, i.e., for \(0 \ll t_m \ll T\) and has a nontrivial edge scaling behavior for \(t_m \to 0\) (and when \(t_m \to T\)), that we compute exactly. Moreover, we show that for any equilibrium process the PDF \(P(t_m|T)\) is symmetric around \(t_m = T/2\), i.e., \(P(t_m|T) = P(T - t_m|T)\). This symmetry provides a simple method to decide whether a given stationary time series \(x(\tau)\) is at equilibrium or not.

The properties of extremes of stochastic processes are of fundamental importance in a wide range of practical situations, including finance, computer science, and climate science [1]. For instance, in the context of climate change, it is paramount to estimate the probability of extreme climate events, such as heat waves, hurricanes, and tsunamis. Even if in many cases one is interested in the magnitude of such anomalous events, it is often also relevant to study the time \(t_m\) at which they occur within some fixed time period \([0, T]\) (see fig. 1). This observable \(t_m\), the time of the maximum, is a central quantity in several applications [1–7]. For instance, in finance, the distribution of the time at which the price of a stock attains its maximum value within a fixed time period is a quantity of clear practical interest.

Within the framework of extreme value theory, the statistical properties of \(t_m\) have been investigated for a wide range of stochastic processes [8–33]. For instance, in the case of an overdamped Brownian particle in one dimension, the full probability density function (PDF) \(P(t_m|T)\) of \(t_m\) was computed analytically by Lévy and is given by [8–10]

\[
P(t_m|T) = \frac{1}{\pi \sqrt{t_m(T-t_m)}},
\]

with \(0 \leq t_m \leq T\). More recently, the PDF of \(t_m\) was also computed for constrained Brownian motions (BM) [11,12,14–18,29,31], Bessel process [17], and run-and-tumble particles (RTPs) [28,30,33], amongst others. However, to the best of our knowledge, the time of the maximum has never been studied for stationary processes, i.e., stochastic processes whose statistical properties are invariant under a time shift.

Stationary phenomena are ubiquitous in nature and appear in a wide range of systems, including Brownian engines [34], active matter [35] and climate systems [36]. They are divided into two main categories: equilibrium and out-of-equilibrium ones. Equilibrium processes satisfy the detailed balance condition, implying that all currents vanish and that the dynamics is time reversible. Standard techniques from statistical physics can be applied to study equilibrium systems and thus their behavior is generally well understood. In contrast, nonequilibrium phenomena are characterized by the presence of currents in the steady state and very few general results exist in this case [37–42].

The distribution of the maximal value \(M\) has been studied for several stationary processes of fixed duration \(T\), including fluctuating interfaces [43,44], the Ornstein-Uhlenbeck process [1], and BM with stochastic resetting [1,45,46]. BM with stochastic resetting has become a rather popular subject of late both theoretically and
exponentially — for a recent review see [47]. Notably, in the case where the autocorrelation function of the process decays sufficiently fast the distribution of $M$, properly centered and scaled, approaches a universal Gumbel form at late times, i.e., for $T \gg T_{\text{micro}}$, where $T_{\text{micro}}$ is a microscopic correlation time [1,44,48]. However, it is not clear if this universality also extends to the distribution of the time $t_m$ at which the maximum $M$ is reached. Moreover, since the statistical properties of stationary processes by definition do not evolve in time, one could naively expect the distribution of $t_m$ to be uniform, i.e., $P(t_m|T) = 1/T$. Quite surprisingly, we show that this is not true in general, due to the presence of nonzero temporal correlations of the process.

In this letter, we consider a one-dimensional stationary stochastic process $x(\tau)$, evolving in the time interval $[0,T]$ (see fig. 1). At the initial time, we assume that the process has already reached its stationary state $P_0(x)$. This is equivalent to preparing the system in some initial condition at time $\tau = -\infty$ and starting to observe it at $\tau = 0$. Using a path integral approach, we compute exactly the distribution $P(t_m|T)$ of the time $t_m$ at which $x(\tau)$ attains its maximal value for several stationary models, both equilibrium and nonequilibrium. Notably, in the case of the equilibrium motion of a Brownian particle in a confining potential, we show that $P(t_m|T)$ becomes universal at late times. Moreover, we demonstrate that for any equilibrium process the PDF $P(t_m|T)$ is symmetric around the midpoint $t_m = T/2$. For two nonequilibrium processes, namely the resetting BM and a single confined RTP, we verify by computing $P(t_m|T)$ exactly that this symmetry is not present (see fig. 2). Thus, the measurement of the distribution of $t_m$ provides a simple recipe to detect nonequilibrium dynamics in a stationary time series.

We start by investigating $P(t_m|T)$ in the case of equilibrium systems. The process that we consider is an overdamped 1d Brownian particle (with diffusion coefficient $D$ and friction coefficient $\Gamma = 1$) moving in a symmetric confining potential that grows for large $|x|$ as $V(x) \approx \alpha |x|^p$, where $\alpha > 0$ and $p > 0$. In this system, the case of the Ornstein-Uhlenbeck process, obtained from numerical simulations with $\alpha = D = T = 1$. The PDF $P(t_m|T)$ is symmetric around the middle point $t_m = T/2$ (dashed black line), since the process is at equilibrium. (b) Probability distribution $P(t_m|T)$ vs. $t_m$ in the case of Brownian motion with resetting, obtained from numerical simulations with $D = T = 1$ and $r = 10$. The PDF $P(t_m|T)$ is asymmetric around the middle point $t_m = T/2$ (for further proof, see [49]), signaling the nonequilibrium nature of the process.

The PDF $P(t_m|T)$ can be also derived exactly for $p = 1$ (eq. (3) of [49]). In addition to proving the symmetry $P(t_m|T) = P(T - t_m|T)$, the exact result in eq. (2) for $p = 2$, and the analogous one for $p = 1$ (see eq. (8) in [49]), can be used to extract the asymptotic behaviour of $P(t_m|T)$ in the large $T$ limit. In this limit, we find that there is a “bulk” regime where $P(t_m|T) \approx 1/T$ is essentially flat. However, near the two “edges” $t_m = 0$ and $t_m = T$ (symmetrically), the distribution $P(t_m|T)$ has a nontrivial shape (see fig. 2(a)). Moreover, near the edges, once appropriately scaled, the scaling form of $P(t_m|T)$
turns out to be identical for both $p = 1$ and $p = 2$! This “universality” is rather unexpected and naturally leads us to wonder whether the edge behavior of $P(t_m|T)$ for general $p > 0$ is also universal. We show that indeed this universality holds for any $p \geq 1$, i.e., for sufficiently confining potentials. However, for $0 < p < 1$, i.e., for “shallow” potentials, there is no universal edge behavior.

In the absence of an exact result for general $p \geq 1$, we develop a real-space “blocking argument” (à la Kadanoff), which demonstrates clearly this universality of the edge behavior for $p \geq 1$. More precisely, we find that, for $p \geq 1$,

$$P(t_m|T) \simeq \begin{cases} \frac{1}{T} G \left[ \frac{t_m}{\lambda(T)} \right], & \text{for } t_m \ll \lambda(T), \\ \frac{1}{T} G \left[ \frac{T - t_m}{\lambda(T)} \right], & \text{for } \lambda(T) \ll t_m \ll T - \lambda(T), \\ \frac{1}{T} G \left[ \frac{T - t_m}{\lambda(T)} \right], & \text{for } t_m \gg T - \lambda(T), \end{cases}$$

with $\lambda(T) = \frac{4D}{\alpha p^{2}} \left[ \frac{2}{p} \log(T) \right]^{-2(p-1)/p}$ denoting the width of the edge region and the universal scaling function is given by

$$G(z) = \frac{1}{2} \left[ 1 + \frac{e^{-z}}{\sqrt{\pi z}} + \text{erf}(\sqrt{z}) \right],$$

where $\text{erf}(x) = (2/\sqrt{\pi}) \int_{0}^{x} e^{-u^{2}} du$. The late-time distribution $P(t_m|T)$ in eq. (3) is manifestly symmetric around $t_m = T/2$ for all $p \geq 1$ and the dependence on the parameters $p$ and $\alpha$ appears only through the width $\lambda(T)$. When $z \rightarrow 0$, $G(z)$ diverges as $1/(2\sqrt{\pi z})$. On the other hand, for large $z$, $G(z)$ goes to the limit value 1, smoothly connecting with the central part where $P(t_m|T) \simeq 1/T$. The scaled distribution $TP(t_m|T)$ is shown as a function of $t_m/\lambda(T)$ in fig. 3. The numerical curves obtained for different values of $p$ collapse onto the same theoretical curve, given in eq. (3). We have checked that the deviations from the theoretical curve are a consequence of finite-size effects [49]. Note that for $p = 1$ the width $\lambda(T)$ is a constant independent of $T$, while for $p > 1$ it shrinks as $\log(T)^{-2(p-1)/p}$ for large $T$.

We next focus on nonequilibrium stationary processes. One of the simplest nonequilibrium models is BM with stochastic resetting [45,47]. Here, we consider a one-dimensional BM, whose position is reset to the origin randomly in time with constant rate $r$. The resetting dynamics induces a nonzero net probability current towards the origin, driving the system to a nonequilibrium stationary state where the position distribution, in 1d, is known to be $P_{\text{st}}(x) = \sqrt{r/4D} e^{-\sqrt{rD|x|}}$, where $D$ is the diffusion constant [45]. The distribution $P(t_m|T)$ for this process has been recently studied where the starting position is fixed [51]. Here, instead, we assume that the initial position of the particle is drawn from the stationary state $P_{\text{st}}(x)$. We show that $P(t_m|T) = rF_{R}(r_{m}, r(T - t_m))$, where the scaling function $F_{R}(t_1, t_2)$ is given in eq. (8) of [49]. In this case, we find that $F_{R}(t_1, t_2) \neq F_{R}(t_2, t_1)$.

![Fig. 3: The scaled distribution $TP(t_m|T)$ as a function of $t_m/\lambda(T)$ in the region $t_m \in [0, 10 \lambda(T)]$. The symbols depict the results of numerical simulations with the potential $V(x) = |x|^p$, with $p = 1, 2$ and 3, and large $T$ ($T = 6400$ for $p = 1$ and $T = 800$ for $p = 2, 3$). The continuous line corresponds to the analytical result in eqs. (3) and (4).](image)

implying that $P(t_m|T)$ is not symmetric around $t_m = T/2$. This asymmetry is confirmed by numerical simulations (see fig. 2(b)) and analytically (see eqs. (9) and (10) as well as fig. 2 in [49]).

The second nonequilibrium process that we consider is a single RTP with fixed velocity $v_{0}$, moving in a one-dimensional potential $V(x) = \alpha|x|$, with $\alpha > v_{0}$ (for the details of the model, see [49]). In the context of active matter, the RTP model has been widely studied [35,52–56]. We compute exactly $P(t_m|T)$ for this model, showing that it is not symmetric around $t_m = T/2$ [49].

Interestingly, the fact that for all equilibrium processes the distribution $P(t_m|T)$ is symmetric around $t_m = T/2$ provides a simple criterion to detect nonequilibrium dynamics in stationary time series. More precisely, imagine that one has access only to a long stationary time series $x(\tau)$ as a function of time $\tau$, e.g., from experimental measurements (see fig. 4), but with no other additional information. This setup is motivated by the increasing interest in single-particle tracking, which provides individual-particle trajectories with high space-time resolution [57–63]. For instance, this time series $x(\tau)$ could represent the location of a confined active particle or the position of a BM in an optical trap. Then, a natural question arises: is there a simple way to determine whether or not $x(\tau)$ is at equilibrium, without any a priori knowledge of its underlying dynamics? In recent years, several attempts to answer this question have been made [64]. One possibility is the verification of the so-called fluctuation-dissipation theorem, which is only valid at equilibrium [64–68]. As an example, this method has been employed to show the nonequilibrium nature of red blood cells [68]. Several other methods, based, e.g., on the detection of probability currents in the phase space or the breakdown of time-reversal symmetry, have also been developed [36,42,64,69–85].
Here, we propose the following simple recipe which consists of two steps. a) Divide the long time series $x(\tau)$ into $N$ blocks each of duration $T$ (see fig. 4) and measure the time $t_m$ at which the maximum of $x(\tau)$ within the $i$-th block is reached (see inset). From the histogram of these $N$ variables $t_{m1}, \ldots, t_{mN}$, one can estimate the distribution $P(t_m|T)$, defined in the text. If $P(t_m|T)$ is not symmetric around $t_m = T/2$, then the process is out of equilibrium.

Let us also mention that there exist nonequilibrium processes for which our criterion is inconclusive. For instance, let us consider another model of active matter, namely a single one-dimensional active Ornstein-Uhlenbeck particle (AOUUP) in a harmonic potential [86,87]. In this case, the joint process $(x,v)$, where $x(\tau)$ is the position of the particle and $v(\tau)$ is the force acting on it (for the details of the model see [49]), violates the detailed balance condition in the $(x,v)$ plane. This is easy to see from the fact one can go from $(x,v)$ to $(x = x + dvt, v)$ in a small time dt but not from $(x = x + dvt, v)$ to $(x, v)$ without changing $v$. This indicates a nonzero current in the $(x,v)$ plane, indicating its nonequilibrium nature. However, for the component process $x(\tau)$ one finds that the distribution $P(t_m|T)$ of the time of the maximum is symmetric around $t_m = T/2$ [87]. This is just a consequence of the fact that the AOUUP in a harmonic potential is a Gaussian stationary process. Indeed, it is possible to show that for any Gaussian stationary process the distribution of $t_m$ is symmetric around $T/2$ [49]. This then provides an example where the process is nonequilibrium, yet the distribution $P(t_m|T)$ is symmetric around $t_m = T/2$. In other words, if $P(t_m|T)$ is symmetric, our test is inconclusive since the underlying dynamics of the process can be either equilibrium or nonequilibrium.

We start by sketching the blocking argument that leads to the universal result in eqs. (3) and (4) for all $p \geq 1$. We consider the position $x(\tau)$ of a single overdamped Brownian particle in a confining potential growing as $V(x) \simeq \alpha |x|^p$ for large $|x|$, with $\alpha > 0$ and $p \geq 1$. The Langevin equation that describes the evolution of $x(\tau)$ is

$$\frac{dx(\tau)}{d\tau} = -V'(x) + \eta(\tau),$$

(5)

where $\eta(\tau)$ is Gaussian white noise with zero mean and correlator $\langle \eta(\tau)\eta(\tau') \rangle = 2D\delta(\tau - \tau')$ and $V'(x) = dV(x)/dx$. For $p \geq 1$, one can show that the autocorrelation function $\langle x(\tau)x(\tau') \rangle - \langle x(\tau) \rangle \langle x(\tau') \rangle$ decays exponentially in $|\tau - \tau'|$ over a typical time $T_B \sim O(1)$ [88]. For $T \gg T_B$, we can divide the time interval $[0,T]$ into $N_B$ blocks of identical size $T_B$, which are essentially uncorrelated. Let $m_i$ be the maximal position reached in the $i$-th block. Clearly the variables $m_i$’s are independent of each other (since they belong to different blocks), but they are identically distributed due to the stationarity of the process. This implies that the probability that the maximum is reached in the $i$-th box is the same for each box and thus it is simply $1/N_B = T_B/T$. This argument suggests that the probability distribution of $t_m$ is approximately given by the uniform measure $P(t_m|T) \approx 1/T$. However, this argument is only valid in the bulk of the distribution $P(t_m|T)$, i.e., when $T_B \ll t_m \ll T - T_B$. In the regions $0 < t_m < T_B$ and $T - T_B < t_m < T$, a detailed analysis, taking into account edge effects, is required.

To show this, we consider the interval $[0,T_B]$ and condition on the event that the maximum is reached in this first block. Since the position $x(\tau)$ in this block will be very close to the maximal position $M$, we can linearize the potential $V(x)$ around $x = M$. To leading order, the Langevin equation (5) becomes

$$\frac{dx(\tau)}{d\tau} = -V'(M) + \eta(\tau).$$

(6)

In first approximation, the particle is subject to a constant negative drift $\mu = -V'(M) < 0$. For large $T$, the maximum $M$ typically grows as $(D/\alpha \log(T))^{1/p}$ [49]. Consequently, the constant drift $\mu$ is given by

$$\mu \simeq -\alpha p \left( \frac{D}{\alpha \log(T)} \right)^{(p-1)/p}.$$  

(7)
The PDF of the time \( t_m \) of the maximum in a time interval \([0, T_B]\) of a BM with constant drift \( \mu \) has been computed in ref. [15] and is given by

\[
P(t_m|T_B) = \frac{h_\mu(t_m) h_\mu(T_B - t_m)}{\pi \sqrt{t_m(T_B - t_m)}},
\]

where

\[
h_\mu(\tau) = e^{-\mu^2 \tau / 4D} + \mu \sqrt{\frac{\tau}{4D}} \left[ 1 + \text{erf} \left( \mu \sqrt{\frac{\tau}{4D}} \right) \right].
\]

Thus, for \( 0 \leq t_m \ll 1 \) and \( T \gg 1 \), the distribution of \( t_m \) can be written as

\[
P(t_m|T) \simeq \frac{T_B h_\mu(t_m) h_\mu(T_B - t_m)}{T \pi \sqrt{t_m(T_B - t_m)}},
\]

where the drift \( \mu \) is given in eq. (7). We recall that the term \( T_B/T \) is the probability that the maximum falls in the first block. Note that, since we do not know the precise value of \( T_B \), the result in eq. (10) gives us the edge behavior of \( P(t_m|T) \) up to a multiplicative constant. In particular, in the region where \( t_m \ll T_B \), we obtain

\[
P(t_m|T) \propto \frac{1}{T} \frac{h_\mu(t_m)}{\sqrt{t_m}}.
\]

Finally, the multiplicative factor can be obtained by imposing that the edge expression in eq. (11) matches for large \( t_m \) with the bulk result \( P(t_m|T) \simeq 1/T \) and, using the expression of \( h_\mu(\tau) \) in eq. (9), we obtain the result in eq. (3). An analogous derivation can be carried out for the right edge of \( P(t_m|T) \). In the special cases \( p = 1 \) and \( p = 2 \), where we could compute \( P(t_m|T) \) exactly [89], the asymptotic analysis for large \( T \) is fully consistent with the approximate block argument developed above for arbitrary \( p \geq 1 \). Note that in the case \( 0 < p < 1 \) the result in eq. (3) is not valid since the autocorrelation function of \( x(t) \) does not decay exponentially in time [88].

We next present the derivation of the fact that \( P(t_m|T) \), for any equilibrium stationary process on the interval \([0, T]\), is symmetric around \( t_m = T/2 \). For simplicity, we consider a discrete-time process \( x_k \), with \( 1 \leq k \leq T \). It is easy to generalize the following derivation to continuous time. Note that here \( t_m \) and \( T \) are integer numbers. Denoting by \( P\{\{x_k\}\} \) the probability of observing the trajectory \( \{x_k\} = \{x_1, \ldots, x_T\} \), the distribution of the time \( t_m \) of the maximum can be written as

\[
P(t_m|T) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \text{d}x_T \Theta_{t_m}(\{x_k\}) P\{\{x_k\}\},
\]

where \( \Theta_{t_m}(\{x_i\}) = \prod_{x_k \neq x_i} \theta(x_k - x_i) \) and \( \theta(z) \) is the Heaviside step function, i.e., \( \theta(z) = 1 \) for \( z > 0 \) and \( \theta(z) = 0 \) otherwise. In other words, \( \Theta_{t_m}(\{x_i\}) \) is one if the maximum of the trajectory \( \{x_i\} \) is reached at step \( k \) and zero otherwise. Thus, in eq. (12), we integrate over all possible trajectories for which the time of the maximum is \( t_m \). Let us denote by \( \{\tilde{x}_k\} = \{x_{T-k}\} \) the time-reversed trajectory associated to \( \{x_k\} \). For an equilibrium process, it is possible to show that, as a consequence of the detailed balance condition, \( P\{x_k\} = P\{\tilde{x}_k\} \) (this is not true in general for nonequilibrium processes). Using this result in eq. (12) and performing the change of variables \( x_i \rightarrow \tilde{x}_i = x_{T-i} \), we obtain

\[
P(t_m|T) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \text{d}x_T \Theta_{t_m}(\{\tilde{x}_k\}) P\{\{\tilde{x}_k\}\}.
\]

It is easy to show that \( \Theta_{t_m}(\{\tilde{x}_k\}) = \Theta_{T-t_m}(\{\tilde{x}_k\}) \) and thus we find

\[
P(t_m|T) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \text{d}x_T \Theta_{T-t_m}(\{\tilde{x}_k\}) P\{\{\tilde{x}_k\}\}.
\]

Recalling the expression for \( P(t_m|T) \), given in eq. (12), we obtain our desired result \( P(t_m|T) = P(T-t_m|T) \), which is thus a necessary, but not a sufficient, condition for a stationary process to be at equilibrium.

To conclude, we have investigated the distribution \( P(t_m|T) \) of the time \( t_m \) at which a stationary process of duration \( T \) reaches its global maximum. Using path integral techniques, we have computed exactly \( P(t_m|T) \) for several stationary processes. In particular, for a diffusive particle in a trapping potential, we have further shown that \( P(t_m|T) \), suitably scaled, is universal at late times, i.e., independent of the details of the potential. Moreover, we have presented a simple sufficiency test to detect whether a stationary time series has nonequilibrium dynamics. Our method is based on estimating the PDF \( P(t_m|T) \). If it is asymmetric the dynamics is necessarily nonequilibrium. The test proposed in this letter is very general and can be applied to any stationary process.

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