Scientific paper

The Shear Behavior of RC Tapered Short Beams with Stirrups

Teng Shuo1*, Kazumasa Okubo2 and Junichiro Niwa3

Abstract

This study is aimed at investigating the effects of stirrups on the shear behavior of RC tapered short beams. Static four-point bending tests were conducted to six RC tapered beams with various stirrup ratios and arrangements. Besides, nonlinear FEM analysis was also conducted to simulate the shear behavior and to confirm the properties of the inclined compressive strut. The results showed that the shear capacity and contribution of stirrups did not definitely increase with the increase in the stirrup ratio. Moreover, the condition of shear resistance mechanism varied due to the different arrangement of stirrups. Finally, a more rational evaluation method was proposed on the shear capacity of RC tapered short beams with stirrups based on experimental and analytical results.

1. Introduction

In the current construction field, reinforced concrete members (hereinafter referred to as RC tapered beams), whose effective depth of the cross section varies uniformly along their axes, are frequently used in structural portal frames, cantilevers and bridge structures. Compared to constant depth beams, these members are convenient in the way that each cross section along the member axis can be designed according to the required resistance against external loads. Many studies on the shear behavior of RC tapered beams without stirrups have been conducted. Ishibashi et al. (1983) claimed that for RC tapered deep beams (a/d from 0.33 to 1.0) without stirrups, there was barely any difference in the shear capacity with RC constant depth beams. On the other hand, according to the experimental results of Kakuta et al. (1997) and MacLeod et al. (1994), the shear capacity of RC tapered slender beams (a/d of 3.0 to 4.0) without stirrups increased as the slope increased compared to RC constant depth beams, even if the amount of concrete is reduced. Regarding tapered beams, since the cross sectional height varies along the longitudinal axis, it is usually considered necessary to define one critical section for evaluating the shear capacity with the effective depth and other shear behaviors. This section is defined as “critical section”. As for the shear resistance mechanism, Iwanaga (2011) and Hou et al. (2015) clarified the influence of various parameters on compressive strut and critical section, based on which equations for evaluating the shear capacity of RC tapered beams without stirrups were proposed. Hou et al. (2016) further developed those equations to be adaptable to RC tapered beams with stirrups with a/d within 2.71 to 3.75 by adopting a term of stirrup ratio, based on the relationship between the change of critical section’s location and the stirrup ratio.

However, since stirrups are essential in actual design to avoid shear failure, shear behavior of RC tapered beams with stirrups should also be clarified in order to determine the optimal stirrup usage. The current situation is that the rational and economical design method for such beams has not been established specifically in JSCE Standard Specifications for Concrete Structures (JSCE 2017). Only recommendations on evaluation section to be substituted into the existing equations for RC constant depth beams are provided, or vague instruction as “considering the vertical component of the inclined flexural stress” is mentioned in certain specifications. Moreover, experimental studies and data on RC tapered beams with stirrups are rather insufficient in the first place.

Since previous studies discovered the difference in the effects of taper slope on RC deep beams and slender beams, respectively, and how the stirrups further influence those effects regarding slender beams, this research focuses on the effects of stirrups on the shear behavior of RC tapered short beams with a/d as 2.5. The objectives of this study are to investigate the behavior and contribution of stirrups, to investigate the shear mechanism of RC tapered beams with stirrups, and to compare the experimental results with calculated results based on conventional evaluation methods and propose a comparatively more appropriate evaluation method based on both experimental and analytical results.

2. Experimental and analytical outline

2.1 Test specimens and materials

Six RC tapered beams with the same dimension were tested in total under four-point bending. The shear span was 625 mm and the effective depth at the support and
the mid span were 166 mm and 250 mm, making the shear span to effective depth ratio 2.5 and the angle of the taper 9.1°, which is close to the specimens’ angles in the previous studies that have been mentioned above in Section 1. The details of the specimens are shown in Table 1 and in Figs. 1 and 2.

For each specimen, the water-cement ratio of concrete was 0.57. The design compressive strength of concrete was 30 N/mm² and the mix proportion is shown in Table 2.

Two D22 deformed steel bars ($A_s=380.1$ mm²) with the yield strength of 1170 N/mm² were used as longitudinal tensile bars. The yield strength of D10 deformed steel bars used as compressive bars and stirrups was 375 N/mm² and that of D6 deformed steel bars for stirrups was 344 N/mm².

Formworks were removed after 48 hours from casting. Curing of beams and strength tests cylinders was done by covering moistened cloths. The loading tests were conducted after about 1 week of curing.

2.2 Loading method

Four-point bending tests under simply-supported condition were carried out for all six specimens. Steel support plates with 75 mm width were placed on the pin-hinge supports. Teflon sheets with silicon grease inserted were put on the support plates in order to reduce the horizontal friction.

2.3 Measurements

During the loading tests, the applied load, mid-span deflection and strains of tensile, compressive reinforcements and stirrups were measured (Fig. 3) and recorded by a data logger. Moreover, three digital cameras were set to take pictures during the loading tests for observation of crack propagation.

2.4 Nonlinear FEM analysis

The two-dimensional nonlinear FEM (finite element method) analysis using DIANA system (version 10.2) was also conducted to simulate the shear behavior of RC tapered beams in this study. In order to compare the analytical results in this study with previous studies (Iwanaga 2011; Hou et al. 2015), the settings were the same as those in previous studies as follows. The total strain fixed smeared crack model was applied as the crack model for concrete elements. The constitutive models used for concrete in tension and compression were that of Hordijk (1991) and the parabolic curve model modified by Feenstra (1993), respectively. The embedded reinforcement elements that have perfect bond

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**Table 1 Experimental parameters.**

| Specimens   | $r_\omega$ (%) | $s$ (mm) | $n$ | $D$ (mm) |
|-------------|----------------|----------|-----|----------|
| RW0         | 0              | 0        | 0   | 0        |
| RW18        | 0.18           | 176      | 3   | 6.35     |
| RW25        | 0.25           | 125      | 4   | 6.35     |
| RW33        | 0.33           | 96       | 6   | 6.35     |
| RW50-D6     | 0.50           | 62.5     | 9   | 6.35     |
| RW50-D10    | 0.51           | 143      | 9   | 9.53     |

$r_\omega$: Stirrup ratio ($A_s/(b\cdot s)$), $s$: Stirrup spacing, $n$: Number of stirrups in each test shear span, $D$: Nominal diameter of stirrups used in test shear span

**Table 2 Mix proportion of concrete.**

| Gmax (mm) | W/C | Unit weight (kg/m³) |
|-----------|-----|---------------------|
| 20        | 0.57| W: 172, C: 301, S: 785, G: 1031, AE: 3.01 |

Gmax: Maximum size of coarse aggregate, W: Water, C: High-early-strength Portland cement, with density of 3.14 g/cm³, S: Fine aggregate, with density of 2.59 g/cm³, G: Coarse aggregate, with density of 2.65 g/cm³, AE: Air-entraining water-reducing agent

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Fig. 1 Geometry of specimens.

Fig. 2 Sections of specimens.

Fig. 3 Arrangement of steel strain gauges (example of RW18).
with concrete were selected for steel reinforcements. The mesh size was approximately 20 mm for the squared mesh. However, due to the specimen’s geometry, meshes have different sizes at different locations. In order to eliminate the mesh size effect, concrete constitutive models that consider the crack band width \( h \) were selected, and calculated equal to \( \sqrt{A} \), where \( A \) is the area of the element.

In this analysis, a constant value of shear retention factor \( \beta \) as 0.1 was used. Displacement control with Quasi-Newton method (also called “Secant Method”) was adopted to solve equilibrium equations. In each step with 0.02 mm displacement increment, when the variation of internal energy had become less than 0.01% of the internal energy of the first iteration in the step or the iteration number of the step was more than the maximum setting of 400, the iteration process was terminated to move to the next step.

3. Results and discussion

3.1 Load-deflection relationships

Figure 4 shows the load-deflection curves of all six specimens. The solid parts indicate the phase of loading and the dashed parts indicate the phase of unloading.

It can be observed from Fig. 4 that most of the cases continued to carry load even after the peak. In the case of RW0, which is the specimen without stirrups, the obvious critical diagonal crack did not occur until the second peak load. As for the other specimens, the diagonal cracks happened at around 100 kN, judging from the earliest increase of strain in stirrups in each specimen (Fig. 5). Only RW50-D6 failed due to the crush of concrete near the loading points as shown in Fig. 6(e). As for the shear capacity, it did not necessarily increase as the stirrup ratio increased. The shear capacity of RW25 exceeded that of RW33, and RW50-D6 larger than RW50-D10.

![Fig. 4 Load-deflection relationships.](image)

![Fig. 5 Earliest strain in stirrups.](image)
3.2 Crack patterns
The crack patterns in the failed shear span at just after the peak of all six specimens are shown in Fig. 6, where the red cracks represent the critical diagonal cracks. It can be seen that the cracks become more distributed as the shear capacity increased. It was difficult to identify the critical diagonal cracks in RW50-D6 and RW50-D10 since there were multiple ones just before the peak. Therefore, the one with the earliest occurrence was considered as the critical one in each specimen. On the other hand, regarding these two specimens, the cracks in the former one were more distributed than in the latter one, which resulted in the larger shear capacity in the former one.

In order to compare the critical diagonal cracks in all specimens, they were turned to the left shear span and overlapped together as shown in Fig. 7, where the colors correspond to the color of the specimen names below. It can be seen that all the critical diagonal cracks end at almost the same location. As the stirrup ratio increased, the inclination of the critical diagonal crack between the compressive and the tensile reinforcement tended to be steeper.

3.3 Strain in compressive reinforcement
As mentioned earlier, the strain distribution of compressive reinforcement was measured in each specimen as shown in Fig. 3, with one in the center and four along each shear span in equal interval. The results of the strain gauges of the center and in failed shear span are displayed in Fig. 8, with the x-axis as strain in $10^{-6}$ and y-axis as applied load in kN. It can be noticed that the compressive strain of C2 increased noticeably after the peak load compared to other locations in each specimen. Moreover, in RW0, a slight decrease in C2 happened just before the first peak load. On the other hand, an obvious reduction of compressive strain in C2 of RW18 occurred at a similar load, and the same phenomenon happened in RW25 also but was less obvious. It can be considered that a transition of the dominant shear resistance mechanism from the beam action to the arch action took place at around the first peak load of the specimen without stirrups. According to Kani (1964), due to the formation of the arch action, the strain of the concrete compression fiber would shift to decreasing, and similar phenomenon was also observed in the experiments of Stefanou (1983) and those of Iwanaga (2011), regarding the tapered beam with $a/d = 1.4$ without stirrups. However, when the stirrup ratio rose up to more than 0.33%, a decrease in the inclination could be observed in C2 of RW33, RW50-D6, and C1 of RW50-D10. These phenomenon may indicate a more gradual transition of shear resistance mechanism compared to the specimens with less stirrup ratio. Possibilities might be that, the transition did not happen or that the transition could not be captured from the locations of the attached strain gauges.

3.4 Shear resistance mechanism
Quantitative analysis of beam action and arch action will
be discussed in this section. By focusing on the basis of the equilibrium in the shear span of the beam at the ultimate state, the shear mechanism can be divided into arch action and beam action based on Eq. (1) (Fenwick and Paulay 1968):

\[ V = \frac{dM}{dx} + jd \frac{dT}{dx} + T \frac{d(jd)}{dx} \]  

(1)

where \( V \) is the total shear resistance, \( M \) is the bending moment, \( x \) is the distance between the support and a certain cross section, \( jd \) is the distance between the resultants of the internal compressive and tensile forces on a cross section (namely, lever arm length), and \( T \) is the tensile force.

In Eq. (1), the first term represents the beam action. With a complete bonding, the lever arm length, \( jd \), is kept constant and only the internal tensile force changes along the beam to equilibrate with externally applied moment, ideal beam action will occur.

On the other hand, if the bonding force cannot work, there would be no change in the tensile force \( T \), and then the lever arm length has to change to equilibrate with the externally applied moment and resist the external shear force. In this case, the shear is transferred by the arch action.

There have been various understandings and processing methods based on this theory, while in this study, used the method by Nakamura and Watanabe (2008) as a reference, which assumes the tensile force distribution of the tensile reinforcement as linear to the distance from the support, and the arm length as quadratic to the distance from the support. After differentiation, average value within the range of the occurrence of the critical diagonal crack was taken. The processing was conducted at several loads discretely according to the condition of crack propagation. In this study, polynomial approximation was conducted to the actual experimental results to obtain the distribution of the tensile force and the internal lever arm. Differentiation was done based on those functions and the results of beam action and arch action in each beam are displayed in Fig. 9, where the orange and red solid polylines present the arch action (\( V_{arch} \)) and beam action (\( V_{beam} \)), respectively. The blue solid curves reveal the experimental results (\( V_{exp} \)) and dashed polylines reveal calculation results (\( V_{mech} \)) as the sum of \( V_{arch} \) and \( V_{beam} \).

The behavior of tensile reinforcement was unstable in the initial phase of loading, especially before the occurrence of initial crack at around 40 kN. Therefore, the results of the shear resistance mechanism tend to be scattering from the experimental result at first. It can be
seen that, except for the initial phase in RW0, the calculation value of shear capacity matched the experimental results quite well in all cases, thus making the discussion on the shear resistance mechanism plausible. The arch action governed in all cases and kept increasing until failure. According to Russo and Puleri (1997), when arch action governs, it is reasonable to assume that the stirrup inclusion does not affect the concrete shear stress, stirrups being almost not involved in the arch mechanism. Therefore, the arch action can be considered to be resulted from the short shear span as 2.5, causing the tied arch mechanism.

On the other hand, it is also known that stirrup inclusion is beneficial to beam action by increasing: 1) the dowel action, because of the support offered by stirrups to longitudinal bars; 2) the strength of the concrete tooth, due to an inclined compression field associated with the truss mechanism; 3) the interlock strength, by means of a crack opening control (Park and Paulay 1975). In this study, it could be verified that the beam action was certainly enhanced in RW18 and RW25, by comparing with RW0. Moreover, corresponding to the results of strain of compressive reinforcements in Section 3.3 above, the contribution of arch action did exceed that of beam action at around 60 kN of shear force (around 120 kN of applied load). However, when the stirrup ratio increased to 0.33%, the value of beam action was smaller than that in RW25, which might be the reason of the smaller shear capacity. Combining with what discussed previously regarding Fig. 8 that the transition of domination from beam action to arch action in RW33 was subtle, there is a possibility that the increase of arch action was subliminal compared to RW25, as shown in Fig. 9. In this study, this inexplicable phenomenon occurred only in one specimen, and since the crack width and the strain of concrete were not measured, it is difficult to affirm the reason of the decrease in beam action of RW33.

When the stirrup ratio rose up to 0.50%, the results differed with the arrangement of the stirrups. Regarding RW50-D6, both the value and the proportion of beam action were fairly enhanced, while the beam action in RW50-D10 was almost zero.

### 3.5 Behavior and contribution of stirrups

Besides dividing into beam action and arch action, the shear force can also be considered as resisted by both stirrups and concrete. Several perspectives to evaluate the contribution of stirrups will be discussed and the results are tabulated in Table 3. $V_{\text{exp}}$ shows the difference of shear capacity between each specimen with RW0, which is the evaluation of effect of stirrups directly on the shear capacity [Eq. (2)]. $V_{\text{exp}}$ is the experimental result of the shear force carried by stirrups calculated from the strain. As for the selection of stirrups and strain gauges for calculation, the closest (among the upper, middle and lower) strain gauges in the stirrups intersected by the critical diagonal cracks were used for calculation [Eq. (3)].

![Fig. 9 Shear resisting force carried by arch action and beam action.](image-url)
Table 3 Evaluation of contribution of stirrups.

| Specimens | $V_{\text{exp}}$ (kN) | $V_{\text{sys}}$ (kN) | $V_{\text{sys,JSCE}}$ (kN) | $V_{\text{sys}}/V_{\text{sys,JSCE}}$ | $V_{\text{exp}}/V_{\text{sys,JSCE}}$ | $V_{\text{exp}}/V_{\text{sys}}$ |
|-----------|------------------------|-----------------------|-----------------------------|---------------------------------|---------------------------------|-------------------------------|
| RW18      | 49.7                   | 40.8                  | 27.1                        | 1.08                            | 1.72                            | 0.88                          |
| RW25      | 94.2                   | 60.9                  | 38.1                        | 1.45                            | 1.54                            | 0.94                          |
| RW33      | 81.9                   | 70.7                  | 49.7                        | 0.97                            | 1.16                            | 0.84                          |
| RW50-D6   | 145.3                  | 112.9                 | 76.3                        | 1.12                            | 1.29                            | 0.87                          |
| RW50-D10  | 119.2                  | 160.5                 | 81.8                        | 0.93                            | 0.74                            | 1.27                          |

$V_{\text{exp}}$: Difference in shear capacity with RW0  
$V_{\text{sys}}$: Shear resistance carried by stirrups calculated from strain  
$V_{\text{sys,JSCE}}$: Shear resistance carried by stirrups calculated from Hou’s method (Hou et al. 2016)  
$V_{\text{sys}}$: Shear resistance carried by stirrups calculated from JSCE Specifications (JSCE 2017)

Table 4 Evaluation methods of shear capacity.

| Specimens | $f'_c$ (N/mm²) | $V_{\text{sys}}$ (kN) | $d_{\text{crack}}$ (mm) | $V_{\text{sys,JSCE}}$ (kN) | $V_{\text{sys}}/V_{\text{sys,JSCE}}$ | $V_{\text{sys}}/V_{\text{cal}}$ |
|-----------|----------------|-----------------------|---------------------------|-----------------------------|---------------------------------|-------------------------------|
| RW0       | 33.3           | 60.8                  | 220                       | 0.88                        | 0.71                            | 0.97                          |
| RW18      | 33.2           | 110.4                 | 226                       | 1.16                        | 0.71                            | 0.96                          |
| RW25      | 34.0           | 154.9                 | 215                       | 1.45                        | 0.84                            | 1.15                          |
| RW33      | 35.7           | 142.6                 | 212                       | 1.19                        | 0.68                            | 0.91                          |
| RW50-D6   | 39.9           | 206.1                 | 205                       | 1.38                        | 0.69                            | 0.98                          |
| RW50-D10  | 37.5           | 180.0                 | 213                       | 1.18                        | 0.62                            | 0.82                          |

$V_{\text{sys}}$: Shear resistance calculated from Hou’s method (Hou et al. 2016)  
$V_{\text{sys,JSCE}}$: Shear resistance calculated from JSCE Specifications (JSCE 2017)

$V_{\text{sys}}$ is the shear capacity, $V_{\text{sys,JSCE}}$ is the calculated shear capacity by method proposed in this study

$V_{\text{sys}} = V_{\text{u}} - V_{\text{u,RW0}}$  

$V_{\text{sys}} = \sum A_s f_s \varepsilon$, where $E_s \varepsilon \leq f_{\text{sys}}$  

where $V_{\text{sys}}$ is the shear capacity, $V_{\text{sys,JSCE}}$ is the shear capacity of RW0, $A_s$ is the total sectional area of one pair of stirrups, $E_s$ is the elastic modulus of stirrups, $\varepsilon$ is the strain in stirrups, $f_{\text{sys}}$ is the yield strength of stirrups.

$V_{\text{sys,JSCE}}$ and $V_{\text{sys,cal}}$ are the calculation results based on the method proposed by Hou et al. (2016) and JSCE Standard Specifications (JSCE 2017), respectively. These two evaluation methods are both based on the concept as Eqs. (4) and (5):

$V_s = A_s f_s \varepsilon \cot \theta_{\text{strut}} / s$  

$z = 0.875d$  

where $\theta_{\text{strut}}$ is the angle of compressive strut, $s$ is the spacing of stirrups, and $d$ is the effective depth of the largest cross section. $V_{\text{sys,cal}}$ was obtained by substituting $d$ and $\theta_{\text{strut}}$ as follows:

$d = d_{\text{without}} + \frac{6 \tan \alpha + 3.27}{3.27 - \tan \alpha} d_s$  

$\theta_{\text{strut}} = \arctan^{-1}(0.75 \tan \alpha + 0.409)$

where $d_{\text{without}}$ is the effective depth of the critical section when without stirrups, $d_s$ is the effective depth of the section at support, and $\alpha$ is the angle of taper slope. On the other hand, $V_{\text{sys,JSCE}}$ assumes $\theta_{\text{strut}}$ as 45 degree and the critical section as the largest section $d$.

Comparing these four results, it can be seen that the JSCE Standard Specifications (JSCE 2017) gave quite conservative evaluation, which is reasonable and understandable for the safety design. Except for RW50-D10, $V_{\text{exp}}$ were larger than $V_{\text{sys}}$ which was larger than $V_{\text{sys,JSCE}}$. On the other hand, the results given by Hou et al. (2016) were rather close to $V_{\text{exp}}$ compared to any other two results, except for RW25, which seemed to be stronger than expected due to the reason discussed in the previously in Section 3.4. This means that Hou’s method could evaluate the functional reinforcing effects of stirrups fairly well since it adopts the experimental value of compressive strut angle and effective depth of the critical section. Therefore, there is a strong possibility that an accurate evaluation method could be developed if the evaluation of the contribution of concrete could also be correctly done. However, it is slightly unconvincing to use the critical section of the case without stirrups even though there is a method defining the critical section with stirrups in the paper (Hou et al. 2016).

3.6 Discussion on evaluation methods and a proposal of new method

As mentioned in previous section, the method proposed by Hou et al. (2016) was rather satisfying when evaluating the contribution of stirrups. Therefore, in this section, proposal on the evaluation of shear capacity will be done and results are summarized in Table 4, where $V_{\text{sys,exp}}$ shows the experimental results, $V_{\text{sys,cal}}$ is the calculated
results of the shear capacity as the sum of the contribution of concrete and stirrups, following the methods proposed by Niwa et al. (1987) as expressed by Eq. (8), and the JSCE Standard Specifications (JSCE 2017) as expressed by Eq. (4), in which $\theta_{\text{strut}}$ is 45 degrees:

$$V_{\text{c-cal}} = a f'_{c}(1000/d)^{4/3} hbd$$

$$a = 0.2(0.75 + \frac{1.4}{a/d})$$

(8)

$$V_{\text{c-cal}} + V_{\text{c,JSCSE}}$$

(9)

where $f'_{c}$ is the compressive strength of concrete, $a$ is the length of the shear span, $p_{w}$ is the tensile reinforcement ratio percentage ($= 100 A_{s}/(bd)$), $b$ is the width of the beam, and $A_{s}$ is the cross section area of tensile reinforcement bars.

$V_{\text{c,JSCSE}}$ is the calculated result of the shear capacity based on Hou’s method (Hou et al. 2016), obtained by substituting $d_{\text{with}}$ for the effective depth of the largest cross section $d$ in Eq. (8) as follows:

$$d = d_{\text{with}} = d_{\text{without}}(0.667 r_{w} + 1)$$

(10)

where $d_{\text{with}}$ is the effective depth of the critical section when with stirrups, and $d_{\text{without}}$ is the stirrup ratio.

As indicated in Table 4, the JSCE Standard Specifications (JSCE 2017) still gave conservative evaluations on specimens with stirrups, while Hou’s method tended to overestimate the shear capacity in all cases. Since in Hou’s study, the shear span ratios of the RC tapered beams were larger than 2.7, Hou’s method may no longer be adaptable to RC tapered short beams. Therefore, it is necessary to reconsider a new evaluation method regarding this condition, especially the concrete contribution evaluation part.

In this study, a unique phenomenon happened that, cracks also occurred on the top of all specimens just before failure as shown in Fig. 10 with RW18 as example, while the locations of the cracks varied among them. In terms of the timing of the occurrence, the cracks happened just before the peak load, which made the possibility of being crucial for shear capacity greater. Moreover, judging from the results in Section 3.3 that the transition of shear resistance mechanism happened at C2 which was close to the top crack, and also referring to the ideas in previous research (Hou et al. 2016; Iwanaga 2011; Kani 1964; Kostovos 1988), the location of the cracks may indicate where the compressive strut met the upper fiber of the beam and finally penetrated and caused what seemed like buckling, thus resulted in the shear failure.

Consequently, it seems worth a try to assume the section with the occurrence of the top cracks to be the critical section and substitute the effective depth (hereinafter $d_{\text{crack}}$) for the ones in currently proposed evaluation methods. In this study, regarding the shear carried by concrete, $d_{\text{crack}}$ is used in the equation proposed by Niwa et al. (1987) in Eq. (8). As for the shear carried by stirrups, as mentioned before, the evaluation method proposed by Hou et al. (2016) was accurate while unconvincing in terms of the critical section. Therefore, the $d_{\text{crack}}$ is also used as the effective depth for the calculation of the contribution of stirrups as in Eq. (3) and (4). The value of $d_{\text{crack}}$ in each specimen is listed in Table 4. The calculated results are referred to as $V_{\text{c-crack}}$ compared with the experimental results $V_{\text{c-exp}}$ as in Table 4. It is clear that this proposed evaluation method showed rather good agreement with the experimental results.

It should be noticed that, in Hou’s study where the specimens’ shear span ratio was larger than 2.5, the location of the critical section tended to move toward the loading points with the increase in stirrup ratio, while in this study was the opposite as shown in Fig. 11, where $d_{\text{with}}$ and $d_{\text{without}}$ means the effective depth of the critical section measured from the experiment when with and without stirrups, respectively. The difference may ex-
plain the inapplicability of Hou’s method to RC tapered short beams in this study.

Since the position of the top cracks cannot be determined in practical design, prediction by using FEM was attempted, where similar trend could also be proved. As in the method used in the study by Iwanaga (2011), the tendency of inclination of the compressive strut flow initiated from the support was investigated using ideas by Lertsamattiayakul (2005). Verification of the model used in the FEM analysis is conducted by the comparison of the load-deflection relationships and crack patterns with the experimental results, as shown in Figs. 14 and 15. The load-deflection relationships obtained from FEM showed good agreement with experimental results. Similar results were gained that RW25 showed larger shear capacity than RW33 and RW50-D6 than RW50-D10. The distribution of principal tensile strain at just before the peak load was comparable with crack patterns observed from the experiment.

The location where the value of shear stress at the Gauss’s point became the maximum in the vertical layer’s elements have been chosen and it was found that the points having the maximum shear stress (red dots in Fig. 12) tended to be positioned in a linear line and the angle can be considered as the angle of compressive strut. Therefore, linear approximation was conducted to find the angle of the shear stress flow as presented in Fig. 12 with crack patterns as reference, and it can be observed that the angle of stress flow and the inclination of critical diagonal crack were nearly parallel. On the other hand, the inclination, angle of compressive strut and the effective depth of proposed critical section were summarized in Table 5. The accuracy of the FEM model will be verified in Section 3.7. It needs to be noted that the clear stress flow could not be obtained in RW0, thus it has been excluded from the discussion.

It could be seen that $d_0$, which is the effective depth calculated using Eqs. (11) and (12) from the compressive strut angle in Table 5 as shown in Fig. 13, could be seen very close to experimental result $d_{\text{crack}}$.

$$\frac{0.875d_0}{(d_0 - d_c)/\tan\theta} = \tan\theta$$  \hspace{1cm} (11)

$$d_0 = d_0 \tan\theta / (\tan\theta - 0.875 \tan\alpha_c)$$ \hspace{1cm} (12)

Comparison of the calculation results using the compressive strut angle obtained from FEM analysis and $d_0$ is also shown in Table 5. It can be seen that, except for RW25, which had been remarkably reinforced by stirrups, $d_0$ based on the FEM analysis could accurately evaluate the shear capacity due to the accurate simulation of compressive struts. Therefore, these results can point out the possibility of FEM analysis predicting the location of

![Fig. 13 Calculation of $d_0$.](image)

![Fig. 12 Inclination of shear stress flow of specimens with stirrups.](image)
critical section and angle of compressive strut, and thus the shear capacity of RC tapered short beams. The reproducibility of the shear performance and applicability of the proposed method were verified in the range as \(a/d = 2.5\), angle of taper = 9.1 degrees and stirrup ratio = 0 to 0.50%, by comparison between the experimental and analytical results. As for the other range, the accuracy of proposed method via FEM analysis should be verified in future studies.

4. Conclusions

Based on the experimental and analytical results of the shear behavior of RC tapered beams with stirrups with the shear span ratio as 2.5, the following conclusions were drawn.

1. The shear capacity did not necessarily increase as the stirrup ratio increased, due to the difference in shear resistance mechanism.
2. With all the critical diagonal cracks ended at almost the same location near the loading point in all cases, it has been observed that the critical diagonal cracks tended to be closer to the loading point and thus the inclination increased as the stirrup ratio increased.
3. When stirrup ratio was not larger than 0.33%, a transition of shear resistance mechanism from beam action to arch action was observed in the strain in compressive reinforcement at the failure load of the specimen without stirrups.
4. Beam action was enhanced due to the inclusion of stirrups in the cases of stirrup ratio as 0.18% and 0.25%, while when increased to 0.33%, the effect of

| Specimens | \(|k|\) | \(\Theta\) (°) | \(d_{\text{crack}}\) (mm) | \(d_{\text{o}}\) (mm) | \(d_{\text{crack}}/d_{\text{o}}\) | \(V_{\text{exp}}/V_{\theta}\) |
|-----------|-------|-------------|-----------------|-----------------|-------------------|------------------|
| RW18      | 0.48  | 25.6        | 226             | 234             | 0.97              | 0.91             |
| RW25      | 0.68  | 34.2        | 215             | 209             | 1.03              | 1.34             |
| RW33      | 0.63  | 32.2        | 212             | 214             | 0.99              | 1.02             |
| RW50-D6   | 0.50  | 26.6        | 213             | 231             | 0.92              | 0.93             |
| RW50-D10  | 0.65  | 33.0        | 205             | 212             | 0.97              | 0.96             |

\(|k|\): Absolute value of the inclination of shear stress flow  
\(\Theta\): Angle of compressive strut \((\Theta = \tan^{-1}(|k|))\)  
\(d_{\text{crack}}\): Effective depth of the critical section measured in the experiment  
\(d_{\text{o}}\): Effective depth of the critical section calculated from \(\Theta\)  
\(V_{\text{exp}}\): Experimental results of shear capacity  
\(V_{\theta}\): Calculated results of shear capacity using \(\Theta\) and \(d_{\theta}\)
beam action decreased. The situation also differed in cases of 0.50% in that beam action was greatly enhanced in denser arrangement while it was almost zero in the other.

(5) Regarding most cases, the shear carried by the functional reinforcing effects on the shear capacity from stirrups was larger than the shear force in stirrups along the critical diagonal cracks calculated from strain, which was larger than the stirrups’ contribution evaluated by JSCE Standard Specifications.

(6) FEM analysis can accurately simulate the compressive strut and the location of the critical section. Therefore, a new method to evaluate the shear capacity of RC tapered short beams with stirrups based on FEM analysis was proposed in this study and showed fairly good agreements with experimental results.

It is recommended to enlarge the cases of FEM analysis by further adopting more parameters (such as taper slope and $a/d$) to determine a more comprehensive and general evaluation function for compressive strut angle and effective depth of cross section. Consequently, the evaluation equation for the shear capacity of RC tapered short beams considering those two values is expected to be more accurate and rational for practical use for engineers.

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