Jordan duality and Freudenthal duality

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Abstract. We define a Jordan duality \( A \rightarrow A^\star \) on the quantised charges \( A \) of five dimensional black holes and strings and an analogous Freudenthal duality \( x \rightarrow \tilde{x} \) on the quantised charges \( x \) of four dimensional stringy black holes, both of which are distinct from U-duality but leave the lowest order entropy invariant. We discuss the action of these maps on discrete U-duality invariants. Based on joint work with L. Borsten, M. J. Duff, and W. Rubens.

1. Introduction
The quantised charges \( A \) of five dimensional black strings and quantised charges \( B \) of five dimensional black holes are assigned to elements of an integral Jordan algebra \( J \) whose cubic norm \( N \) determines the lowest order entropy
\[
S_5(\text{black string}) = 2\pi \sqrt{N(A)}, \quad S_5(\text{black holes}) = 2\pi \sqrt{N(B)},
\]
and whose reduced structure group \( \text{Str}_0(J) \) is the U-duality group. Integral cubic Jordan algebras are defined in section 2.1 and the reduced structure group is described in section 2.2. We proceed to define the Jordan duality operation in section 2.3.

Similarly the quantised charges \( x \) of black holes of the four dimensional supergravities arising from string and M-theory are assigned to elements of an integral Freudenthal triple system (FTS) \( \mathcal{M}(J) \) (where \( J \) is the integral cubic Jordan algebra underlying the corresponding 5D supergravity [1, 2, 3, 4, 5, 6, 7]) whose quartic form \( \Delta(x) \) determines the lowest order entropy
\[
S_4 = \pi \sqrt{\Delta(x)},
\]
and whose automorphism group \( \text{Aut}(\mathcal{M}(J)) \) is the U-duality group. FTSs are defined in section 3.1 and the automorphism group is described in section 3.2. We proceed to define the Freudenthal duality operation in section 3.3.

Examples of integral Jordan algebras and FTSs with the corresponding U-duality groups \( \text{Str}_0(J) \) and \( \text{Aut}(\mathcal{M}(J)) \) are given in Table 1. In particular this includes the cases \( \mathcal{N} = 2 \) STU, \( \mathcal{N} = 2 \) coupled to \( n \) vector multiplets, magic \( \mathcal{N} = 2 \), and \( \mathcal{N} = 8 \) [8, 9, 10, 11, 12, 13, 14, 15, 16]. The \( \mathcal{N} = 4 \) heterotic string with \( \text{SL}_2 \times \text{SO}_{6,22} \) U-duality may also be included using \( J = \mathbb{Z} \oplus Q_{5,21} \)[15, 17]. The notation \( J_3^A \) denotes sets of \( 3 \times 3 \) Hermitian matrices defined over the four division algebras \( A = \mathbb{R}, \mathbb{C}, \mathbb{H} \) or \( O \) (or their split signature cousins). The notation \( \mathbb{Z} \oplus Q_n \) denotes the infinite sequence of spin factors \( \mathbb{Z} \oplus Q_n \), where \( Q_n \) is an \( n \)-dimensional vector space over \( \mathbb{Z} \)[18, 19, 7, 20, 21].

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Second the trilinear map defines the following four maps
\[ \text{Tr}(X) := 3N(X, 1, 1), \]
\[ S(X) := 3N(X, X, 1), \]
\[ S(X, Y) := 6N(X, Y, 1), \]
\[ \text{Tr}(X, Y) := \text{Tr}(X) \text{Tr}(Y) - S(X, Y). \]
Third the trace bilinear form uniquely defines the quadratic adjoint map $\#: \mathfrak{J} \to \mathfrak{J}$ and its polarisation

$$\operatorname{Tr}(X\#, Y) := 3N(X, X, Y),$$  \hfill (7a)

$$X\#Y := (X + Y)^\# - X^\# - Y^\#. \hfill (7b)$$

Finally the Jordan product is defined as

$$X \circ Y := \frac{1}{2}(X\#Y + \operatorname{Tr}(X)Y + \operatorname{Tr}(Y)X - S(X, Y)\mathbb{1}). \hfill (8)$$

The result is a cubic Jordan algebra provided the cubic form is Jordan cubic, to wit

(i) The trace bilinear form (6d) is non-degenerate.

(ii) The quadratic adjoint map (7a) satisfies

$$(X^\#)^\# = N(X)X, \quad \forall X \in \mathfrak{J}. \hfill (9)$$

For example, in the $J_3^A$ case the Jordan product is $X \circ Y = \frac{1}{2}(XY + YX)$, where $XY$ is just the conventional matrix product. See [19] for a comprehensive account. In all cases, one defines the Jordan triple product as

$$\{X, Y, Z\} := (X \circ Y) \circ Z + X \circ (Y \circ Z) - (X \circ Z) \circ Y. \hfill (10)$$

In general an integral Jordan algebra is not closed under the Jordan product, but the cubic norm and trace bilinear form are integer valued, which are the crucial properties for our purposes. Furthermore $\mathfrak{J}$ is closed under the (un)polarised quadratic adjoint map as required.

2.2. Jordan ranks

The structure group, $\text{Str}(\mathfrak{J})$, is composed of all linear bijections on $\mathfrak{J}$ that leave the cubic norm $N$ invariant up to a fixed scalar factor,

$$N(g(X)) = \lambda N(X), \quad \forall g \in \text{Str}(\mathfrak{J}). \hfill (11)$$

The reduced structure group $\text{Str}_0(\mathfrak{J})$ leaves the cubic norm invariant and therefore consists of those elements in $\text{Str}(\mathfrak{J})$ for which $\lambda = 1$ [22, 19, 9]. The usual concept of matrix rank may be generalised to cubic Jordan algebras and is invariant under both $\text{Str}(\mathfrak{J})$ and $\text{Str}_0(\mathfrak{J})$ [18, 7]. The ranks are specified by the vanishing or not of three rank polynomials linear, quadratic, and cubic in $A$ (resp. $B$) as shown in Table 2. Large BPS black holes and strings correspond to rank 3 with $N(A), N(B) \neq 0$ and small BPS correspond to ranks 1 and 2 with $N(A), N(B) = 0$. In Table 2 we have listed the fraction of unbroken supersymmetry for the $N = 8$ case.

**Table 2.** Partition of the space $\mathfrak{J}$ into four orbits of $\text{Str}_0(\mathfrak{J})$ or ranks.

| Rank | Condition | $A$ | $A^\#$ | $N(A)$ | $N = 8$ BPS |
|------|-----------|-----|--------|--------|-------------|
| 0    | = 0       | = 0 | = 0    | = 0    | -           |
| 1    | $\neq 0$  | = 0 | = 0    | = 0    | 1/2         |
| 2    | $\neq 0$  | $\neq 0$ | = 0    | 1/4    |
| 3    | $\neq 0$  | $\neq 0$ | $\neq 0$ | 1/8    |
2.3. Jordan dual

Given a black string with charges $A$ or black hole with charges $B$, we define its Jordan dual by

$$A^* := A^# N(A)^{-1/3}, \quad B^* := B^# N(B)^{-1/3}. \quad (12)$$

J-duality is well defined for large rank 3 strings for which both $A^#$ and $N(A)$ are nonzero and large rank 3 holes for which both $B^#$ and $N(B)$ are nonzero. It can be shown [33] that the Jordan dual leaves the cubic form invariant $N(A) = N(A^*)$ and satisfies $A^{**} = A$. For a valid dual $A^*$, we require that $N(A)$ is a perfect cube. Despite the non-polynomial nature of the transformation, the J-dual scales linearly in the sense

$$A^*(nA) = nA^*(A), \quad B^*(nB) = nB^*(B), \quad n \in \mathbb{Z}. \quad (13)$$

The U-duality integral invariants $\text{Tr}(X,Y)$ and $N(X,Y,Z)$ are not generally invariant under Jordan duality while $\text{Tr}(A^*,A)$ and $N(A)$, and hence the lowest-order black hole entropy are. However, higher order corrections to the black hole entropy depend on some of the discrete U-duality invariants, to which we now turn.

2.4. Discrete U-invariants

J-duality commutes with U-duality in the sense that $A^*$ transforms contragredient to $A$. This follows from the property that a linear transformation $s$ belongs to the norm preserving group if and only if

$$s(A)^# s(B) = s'(A^# B) \quad (14)$$

where $s'$ is given by

$$\text{Tr}(s(A), s'(B)) = \text{Tr}(A, B) \quad (15)$$

and always belongs to the norm preserving group if $s$ itself does [34]. This implies

$$(s(A))^* = s'(A^*). \quad (16)$$

The gcd of a collection of not all zero integral Jordan algebra elements is defined to be the greatest integer that divides them. By definition gcd is positive. The gcd may be used to define the following set of discrete U-duality invariants [6]:

$$d_1(A) = \gcd(A), \quad d_2(A) = \gcd(A^#), \quad d_3(A) = |N(A)|, \quad (17)$$

which are the gcds of the rank polynomials. Clearly $d_3(A)$ is conserved as expected, but this is not necessarily the case for $d_1(A)$ and $d_2(A)$. Nevertheless the product $d_1(A)d_2(A)$ is preserved.

While we require that $N(A)$ is a perfect cube for a valid J-dual this is not a sufficient condition because we further require that

$$d_3(A) \equiv \left( \frac{d_2(A)}{d_1(A^*)} \right)^3 = \left( \frac{d_2(A^*)}{d_1(A)} \right)^3 = d_3(A^*). \quad (18)$$

For $J_3^A$ with $A \in \{C^*, H^*, O^*\}$ the orbit representatives of all black strings (holes) have been fully classified [6] and it can be shown that the three $d_i$ uniquely determine the representative.
3. 4D black holes and the Freudenthal dual

3.1. Freudenthal triple systems

Given an integral cubic Jordan algebra $\mathcal{J}$, one is able to construct an integral FTS by defining the vector space $\mathcal{M}(\mathcal{J})$,

$$\mathcal{M}(\mathcal{J}) = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathcal{J} \oplus \mathcal{J}.$$  \hspace{1cm} (19)

with elements $x \in \mathcal{M}(\mathcal{J})$ written

$$x = (\alpha, \beta, A, B), \quad \alpha, \beta \in \mathbb{Z}, \quad A, B \in \mathcal{J}.$$  \hspace{1cm} (20)

For convenience we identify the quadratic quantity

$$\kappa(x) := \frac{1}{2}(\alpha\beta - \text{Tr}(A, B)).$$  \hspace{1cm} (21)

The FTS comes equipped with a non-degenerate bilinear antisymmetric quadratic form, a quartic form and a trilinear triple product [8, 9, 35, 36, 7]:

(i) Quadratic form $\{x, y\}: \mathcal{M}(\mathcal{J}) \times \mathcal{M}(\mathcal{J}) \to \mathbb{Z}$

$$\{x, y\} = \alpha\delta - \beta\gamma + \text{Tr}(A,D) - \text{Tr}(B,C),$$

where $x = (\alpha, \beta, A, B), \quad y = (\gamma, \delta, C, D).$  \hspace{1cm} (22a)

(ii) Quartic form $\Delta: \mathcal{M}(\mathcal{J}) \to \mathbb{Z}$

$$\Delta(x) = -4[\kappa(x)^2 + (\alpha N(A) + \beta N(B) - \text{Tr}(A^\#, B^#))].$$  \hspace{1cm} (22b)

The quartic norm $\Delta(x)$ is either $4k$ or $4k + 1$ for some $k \in \mathbb{Z}$.

(iii) Triple product $T: \mathcal{M}(\mathcal{J}) \times \mathcal{M}(\mathcal{J}) \times \mathcal{M}(\mathcal{J}) \to \mathcal{M}(\mathcal{J})$ which is uniquely defined by

$$\{T(w, x, y), z\} = 2\Delta(w, x, y, z),$$  \hspace{1cm} (22c)

where $\Delta(w, x, y, z)$ is the fully polarised quartic form such that $\Delta(x, x, x, x) = \Delta(x)$.

Note that all the necessary definitions, such as the cubic and trace bilinear forms, are inherited from the underlying Jordan algebra $\mathcal{J}$.

3.2. FTS ranks

The automorphism group $\text{Aut}(\mathcal{M}(\mathcal{J}))$ is composed of all invertible $\mathbb{Z}$-linear transformations that leave both the antisymmetric bilinear form $\{x, y\}$ and the quartic form $\Delta(x)$ invariant [9].

The conventional concept of matrix rank may be generalised to Freudenthal triple systems in a natural and $\text{Aut}(\mathcal{M}(\mathcal{J}))$ invariant manner. The rank of an arbitrary element $x \in \mathcal{M}(\mathcal{J})$ is uniquely defined by the vanishing or not of four rank polynomials linear, quadratic (in essence), cubic, and quartic in $x$ as shown in Table 3 [36, 7]. Large BPS and large non-BPS black holes correspond to rank 4 with $\Delta(x) > 0$ and $\Delta(x) < 0$, respectively. Small BPS black holes correspond to ranks 1, 2 and 3 with $\Delta(x) = 0$. In Table 3 we have listed the fraction of unbroken supersymmetry for the $\mathcal{N} = 8$ case.

3.3. Freudenthal dual

Given a black hole with charges $x$, we define its Freudenthal dual by

$$\tilde{x} := T(x)|\Delta(x)|^{-1/2},$$  \hspace{1cm} (23)

where $T(x) \equiv T(x, x, x) \in \mathcal{M}(\mathcal{J})$. F-duality is well defined for large rank 4 black holes for which both $T(x)$ and $\Delta(x)$ are nonzero. It can be shown that the Freudenthal dual leaves the quartic...
form invariant $\Delta(x) = \Delta(\tilde{x})$ and satisfies $\tilde{x} = -x$. For a valid $\tilde{x}$ we require that $\Delta(x)$ is a perfect square. Despite the non-polynomial nature of the transformation, the F-dual scales linearly in the sense

$$\tilde{x}(nx) = n\tilde{x}(x), \quad n \in \mathbb{Z}. \quad (24)$$

The U-duality integral invariants $\{x, y\}$ and $\Delta(x, y, z, w)$ are not generally invariant under Freudenthal duality while $\{\tilde{x}, x\}$, $\Delta(x)$, and hence the lowest-order black hole entropy, are invariant. However, higher order corrections to the black hole entropy depend on some of the discrete U-duality invariants, to which we now turn.

### 3.4. Discrete U-invariants

We make the important observation that since

$$T(\sigma(x), \sigma(y), \sigma(z)) = \sigma(T(x, y, z)), \quad \forall \sigma \in \text{Aut}(\mathfrak{M}(\mathfrak{j})), \quad (25)$$

F-duality commutes with U-duality

$$\sigma(\tilde{x}) = \sigma(\tilde{x}). \quad (26)$$

The gcd of a collection of not all zero integral FTS elements is defined to be the greatest integer that divides them. By definition gcd is positive. The gcd may be used to define the following set of discrete U-duality invariants [7, 26]:

$$d_1(x) = \gcd(x), \quad d_3(x) = \gcd(T(x, x, x)),$$

$$d_2(x) = \gcd(3T(x, y) + \{x, y\} x) \quad \forall y, \quad d_4(x) = |\Delta(x)|,$$

$$d_2'(x) = \gcd(B^\# - \alpha A, A^\# - \beta B, 2\kappa(x)C + 2\{A, B, C\}) \quad \forall C, \quad d_4'(x) = \gcd(x \wedge T(x)), \quad (27)$$

where $\wedge$ denotes the antisymmetric tensor product. As in the 5D case, the unprimed invariants are gcds of the rank polynomials. Clearly $d_4(x)$ is conserved as expected and it can be shown [23] that $d_4'(x), d_2(x)$, and $d_2'(x)$ are also invariant, but this is not necessarily the case for $d_1(x)$ and $d_3(x)$. Nevertheless the product $d_1(x)d_3(x)$ is preserved.

Typically, the literature on exact 4D black hole degeneracies [37, 38, 39, 40, 41, 42, 43, 44, 23, 41, 44, 45, 46, 25, 26] deals only with primitive black holes $d_1(x) = 1$. We are not required to impose this condition and generically do not do so. More generally a quantity is termed primitive if it has unit gcd. A related simplifying concept is projectivity, wherein a charge vector for the cases $\mathfrak{j} = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$, $J_3^{\varphi}$, $J_3^{H}$, $J_3^{\rho}$ is projective if the components of it’s quadratic rank tensor are primitive.

1 In the $\mathcal{N} = 8$ case Sen [26] denotes $d_2(x)$ by $\psi$, and $d_4(x)$ by $\chi$. 

### Table 3. Partition of the space $\mathfrak{M}(\mathfrak{j})$ into five orbits of Aut$(\mathfrak{M}(\mathfrak{j}))$ or ranks.

| Rank | Condition | $T(x, x, x)$ | $\Delta(x)$ | $\mathcal{N} = 8$ BPS |
|------|-----------|--------------|-------------|----------------------|
| 0    | $x = 0$   | $= 0 \forall y$ | $= 0$       | $= 0$                |
| 1    | $\neq 0$ | $= 0 \forall y$ | $= 0$       | = 0                  |
| 2    | $\neq 0$ | $\neq 0$     | $= 0$       | $= 0$                |
| 3    | $\neq 0$ | $\neq 0$     | $\neq 0$   | $= 0$                |
| 4    | $\neq 0$ | $\neq 0$     | $\neq 0$   | $> 0$                |
| 5    | $\neq 0$ | $\neq 0$     | $\neq 0$   | $< 0$                |
While we require that $|\Delta(x)|$ is a perfect square for a valid F-dual this is not a sufficient condition because we further require that

$$d_4(x) = \left[\frac{d_3(x)}{d_1(x)}\right]^2 = \left[\frac{d_3(\tilde{x})}{d_1(\tilde{x})}\right]^2 = d_4(\tilde{x}). \quad (28)$$

Unlike in 5D the invariants $[27]$ are insufficient to uniquely determine the orbit representatives for the $J = J^A_4$ with $A \in \{C^s, H^s, O^s\}$ cases.

4. Conclusions

A subset of 5D black holes admit a Jordan dual $A^*$ preserving some, but not all, discrete U-invariants. Similarly, a subset of 4D black holes admit a Freudenthal dual $\tilde{x}$ preserving some, but not all, discrete U-invariants. In both the 4D and 5D cases, if the discrete invariant $d_1$ is preserved by the $J/F$-duality map, then all the listed discrete invariants are preserved. When $d_1$ isn’t conserved the $J/F$-dual is not U-related to the original charge vector. In the simpler 5D case the preservation of $d_1$ ensures that $A^*$ is U-related to $A$, but in 4D the analogous conclusion only holds in the projective case. For non-projective 4D black holes the situation is complicated by the absence of a complete orbit classification and uncertainty regarding what invariants are relevant to the higher-order corrections. This situation is summarised in Table 4. It would be interesting if in the non-projective case there were configurations with the same precision entropy that are F-related but not U-related.

From a physical standpoint F-duality and J-duality are defined on charge vectors rather than component fields of the lowest order action so their microscopic stringy interpretation remains unclear, but we remark that two black holes related by F-duality in 4D are related by J-duality when lifted to 5D. The 4D/5D lift [47] relates the entropy of non-rotating 4D black holes to the entropy of rotating 5D black holes and it can be shown [33] that the lift of the F-dual is related to the J-dual of the lift thus:

$$\begin{align*}
4D \text{ black hole } x & \xrightarrow{4D/5D \text{ lift}} 5D \text{ black string } A \sim \tilde{B}^* \\
\text{Freudenthal dual} \downarrow & \quad \downarrow \text{Jordan dual} \\
\text{dual 4D black hole } \tilde{x} & \xrightarrow{4D/5D \text{ lift}} \text{dual 5D black hole } \tilde{B} \sim A^*.
\end{align*} \quad (29)$$

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