Constraining the coupling constant between dark energy and dark matter

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Abstract

We have investigated constraints on the coupling between dark matter and the interacting Chaplygin gas. Our results indicate that the coupling constant $c$ between these two entities can take arbitrary values, which can be either positive or negative, thus giving arbitrary freedom to the inter-conversion between Chaplygin gas and dark matter. Thus, our results indicate that the restriction $0 < c < 1$ on the coupling constant occurs as a very special case. Our analysis also supports the existence of phantom energy under certain conditions on the coupling constant.

Keywords: Chaplygin gas; Coupling constant; Dark matter; Dark energy.

1 Introduction

It is well known that the expansion of the universe is accelerated. It has been confirmed by numerous observations taken by various scientific groups across the globe using WMAP [1], distant supernova type 1a data [2, 3], large-scale structure and galaxy distribution [4] and the gravitational lensing phenomenon of high-redshift galaxies [5]. These observations clearly suggest that the universe is spatially flat and is dominated by some sort of vacuum energy having negative pressure commonly called ‘dark energy’. This energy has been interpreted in various forms like the cosmological constant [6, 7], quintessence models
based on the ideas of a spatially homogeneous and time-dependent scalar field \[8\], phantom energy \[9, 10, 11, 39, 13\], quintom models \[14, 15, 16\], k-essence \[17\], holographic dark energy \[18\] and the Chaplygin gas (CG) \[19, 20\] (see also \[21\] for a recent review on dark energy).

The CG is represented by an equation of state (EoS) of the form \( p = -\frac{A}{\rho} \), where \( A \) is a constant parameter \[22\]. The CG gives rise to a simple cosmological model that interpolates between the earlier matter-(or dust-) dominated to the later dark energy dominated phase of the universe. Due to its effectiveness in explaining the evolution of the universe, several generalizations of CG have been proposed in the literature \[23, 24, 25, 26\]. The observational evidence in support of cosmological models based on the CG-EoS is also very encouraging \[27, 28\]. The CG also possesses the property of giving accelerated expansion even if it gets coupled with other scalar fields like quintessence or dissipative matter fields \[29\]. It also yields traversable wormhole solutions to the Einstein field equations if the pressure and density of CG violates the null energy condition \[30\]. Besides its various useful implications in cosmology, CG has the drawback of producing oscillations or exponential blow up of dark matter power spectrum which is inconsistent with observations \[31\]. Similar results are obtained in later generalizations of CG \[32\]. But later it was proved that such oscillations can be avoided and structure formation can proceed (which is strongly supported by dark matter) if the phantom-like dark energy is excluded, thereby proceeding with only dark matter and dark energy \[33\]. It was further suggested that CG behaved like a passive background in the early evolution of the universe and that only dark matter leads to nonlinear growth of structures but later the evolution is dominated by CG \[34\]. The inhomogeneities generated by dark matter were stabilized by the CG which is compatible with the observations \[35\]. Later studies on supernovae data put constraints on CG leading to cosmological models based on CG to behave just like the cosmological constant \[36\].

Modern cosmology is plagued with numerous theoretical and observational problems: among them is the cosmic-coincidence problem which can be stated thus \[37\]: why are the energy densities of matter and dark energy almost of the same order at present? In the standard cosmological model, the ratio of the energy densities of matter and dark energy should fall rapidly as the universe expands, but observationally the corresponding ratio turns out to be almost constant or minutely fluctuating around unity, a phenomenon commonly called the ‘soft coincidence’. It leads to the possibility that energy might be exchanged to keep such a delicate balance in the densities. This interaction is generally studied in the models so-called ‘interacting dark energy’ \[38, 39, 40, 41\]. Unified models based on dark matter and CG have been widely investigated (see \[33\] and references therein) but the fundamental question dealing with the interaction between these two entities is not satisfactorily answered and requires further investigation. A cosmological
model based on the interacting CG had been proposed \cite{42} to investigate this interaction. This model yields the result that the universe is to cross the phantom divide i.e. the transition from the state $\omega > -1$ to $\omega < -1$ or more simply $\omega = -1$, which is not possible in the models based on pure CG. Furthermore, this leads to the scaling solutions of the cosmological dynamical system, which helps in explaining the coincidence problem effectively. Moreover, it is used in cosmological models to investigate dissipative effects of van der Waal’s fluid and dark energy \cite{43}. In fact it is later suggested that without interaction with other species, any hydrodynamical or k-essence like model in general relativity cannot cross $\omega = -1$ \cite{44}. The interacting CG also yields stable scaling solutions of Friedmann-Lemaitre-Robertson-Walker (FLRW) equations at late times of the universe. This model was later extended to the case of an interacting generalized Chaplygin gas (GCG) \cite{45}. There are some proposals that this interaction can be observed if cubic corrections are provided to the Hubble law, measured by distant supernovae of type 1a \cite{46}. It is worthwhile to understand the role of the coupling constant in the interacting models which we have investigated using a modified Chaplygin gas (MCG).

The outline of this paper is as follows: In the next section, we model our dynamical system on the pattern of \cite{45} and determine the critical points corresponding to that system. In the third section, we perform stability analysis corresponding to each critical point. In the fourth section, we perform analysis to determine constraints on the coupling constant. Finally we present conclusion of our paper.

2 Modeling of dynamical system

We start by assuming the background to be a spatially homogeneous and isotropic FLRW spacetime,
\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right), \]  
(1)
where $a(t)$ is the scale factor and the curvature parameter $k = -1, 0, +1$ describes spatially open, flat or closed spacetimes. It is assumed that the spacetime is filled with the a two-component fluid, namely dark matter and dark energy. The corresponding energy-momentum tensors are specified by
\[ T_{\mu\nu}^{(dm)} = \rho_{dm} u'_\mu u'^\nu, \quad T_{\mu\nu}^{(mcg)} = (\rho_{mcg} + p_{mcg}) u_\mu u_\nu + p_{mcg} u_\mu u_\nu. \]  
(2)
Here $u'_\mu$ and $u_\nu$ is the comoving four-velocity of dark matter and dark energy respectively. The notations $dm$ and $mcg$ corresponds to dark matter and dark energy respectively. In our model, the dark energy is specified by the modified Chaplygin gas EoS \cite{47}
\[ p_{mcg} = A\rho_{mcg} - \frac{B}{\rho_{mcg}^\alpha}, \]  
(3)
where $A$ and $B$ are constant parameters and $0 \leq \alpha \leq 1$. The MCG reduces to GCG if $A = 0$ and to the CG if furthermore $\alpha = 1$. It reduces to the standard linear EoS for a perfect fluid if $B = 0$. Recently it has been deduced, using latest supernova data, that models with $\alpha > 1$ are also possible [48]. For our analysis the positivity of $\alpha$ is sufficient. In our further discussion, the MCG and dark energy are used interchangeably.

The density evolution of MCG is given by

$$\rho_{mcg} = \left(\frac{B}{1 + A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right)^{\frac{1}{1+\alpha}},$$

(4)

where $C$ is the constant of integration. Note that Eq. (4) holds only when the interaction is absent. The equations of motion corresponding to FLRW spacetime filled with a two-component fluid are

$$\dot{H} = -\frac{\kappa^2}{6}(p_{mcg} + \rho_{mcg} + \rho_{dm}),$$

(5)

$$H^2 = \frac{\kappa^2}{3}(\rho_{mcg} + \rho_{dm}).$$

(6)

Here $\kappa^2 = 8\pi G$ is the Einstein gravitational constant and $H = H(t)$ is the Hubble parameter. We assume $k = 0$, representing a flat model of the universe. Furthermore, the energy conservation for the two-component perfect fluid is obtained from

$$\nabla\nu T_{\mu\nu} = \nabla\nu(T_{\mu\nu}^{(dm)} + T_{\mu\nu}^{(mcg)}) = 0.$$  

(7)

Here $\nabla\nu$ refers to the covariant derivative with respect to $x^\nu$ coordinate. Eq. (7) yields

$$\dot{\rho}_{mcg} + \dot{\rho}_{dm} + 3H(p_{mcg} + \rho_{mcg} + \rho_{dm}) = 0.$$  

(8)

Due to interaction, the energy will not independently be conserved for the interacting components, and therefore

$$\nabla\nu T_{\mu\nu}^{(mcg)} = -Q_{\mu}, \quad \nabla\nu T_{\mu\nu}^{(dm)} = Q_{\mu}.$$  

(9)

Here $Q_{\mu}$ is the interaction term that corresponds to energy exchange between dark energy and dark matter. Solving Eqs. (9) using (2), we obtain the so-called energy-balance equations corresponding to MCG and dark matter as:

$$\dot{\rho}_{mcg} + 3H(p_{mcg} + \rho_{mcg}) = -Q,$$  

(10)

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q.$$  

(11)

The function $Q \equiv Q_t$, $\mu = t$ has dependencies on the energy densities and the Hubble parameter, i.e. $Q(H\rho_{dm})$, $Q(H\rho_{mcg})$ or $Q(H\rho_{dm}, H\rho_{mcg})$ [49]. Because of the unknown nature of both dark energy and dark matter, it is not possible to derive $Q$ from first
principles. In order to deduce a reasonable $Q$, we may expand like $Q(H\rho_{dm}, H\rho_{mcg}) \approx \alpha_{dm}H\rho_{dm} + \alpha_{mcg} H\rho_{mcg}$. Since the coupling strength is also not known, we may adopt just one parameter for our convenience; hence we take $\alpha_{dm} = \alpha_{mcg} = c$ \[50\]. We here choose the following coupling function $Q$ given by \[39\]:

$$Q = 3Hc(\rho_{mcg} + \rho_{dm}).$$  \hspace{1cm} (12)

The choice of $Q$ is completely arbitrary but care must be taken that it must satisfy the energy conservation (see Ref. \[52\] for various other forms of $Q$). Here $c$ is the corresponding coupling constant (also called the ‘transfer strength’) for the interaction.

To study the dynamics of our system, we proceed by setting

$$x = \ln a = -\ln(1 + z),$$ \hspace{1cm} (13)

where $z$ is the redshift parameter. Moreover, the density and pressure of MCG can be expressed by dimensionless parameters $u$ and $v$ as follows:

$$u = \Omega_{mcg} = \frac{\rho_{mcg}}{\rho_{cr}} = \frac{\kappa^2 \rho_{mcg}}{3H^2}, \quad v = \frac{\kappa^2 p_{mcg}}{3H^2}. \hspace{1cm} (14)$$

The EoS parameter $\omega$ is conventionally defined as

$$\omega(x) \equiv \frac{p_{mcg}}{\rho_{mcg}}, \hspace{1cm} (15)$$

which becomes

$$\omega(x) = \frac{v}{u}. \hspace{1cm} (16)$$

The density parameters of MCG and dark matter are related as

$$\Omega_{dm} = \frac{\kappa^2 \rho_{dm}}{3H^2} = 1 - \Omega_{mcg} = 1 - u. \hspace{1cm} (17)$$

Since $\Omega_{dm} \sim 0.3$ \[2\], it constrains $u \in (0, 1)$ for a flat universe. The following system of differential equations governing the dynamics of our cosmological model is determined by using the above equations:

$$\frac{du}{dx} = -3c - 3v + 3uv, \hspace{1cm} (18)$$

$$\frac{dv}{dx} = -3 \left[ \frac{v}{u} + \left( A - \frac{v}{u} \right) (1 + \alpha) \right] (u + v + c) + 3v(1 + v). \hspace{1cm} (19)$$

Note that for $A = 0$, the above system reduces to the case for interacting generalized Chaplygin gas \[45\]. By equating Eqs. (18) and (19) to zero, we obtain the three critical
points \((u_{ic}, v_{ic})\), with \(i = 1, 2, 3\), given by

\[
\begin{align*}
u_{1c} &= 1 - c, \\
v_{1c} &= -1, \\
u_{2c} &= \frac{1}{2} \frac{\sqrt{A + 4c}}{2\sqrt{A}}, \\
v_{2c} &= \frac{1}{2}(A - \sqrt{A}\sqrt{A + 4c}), \\
u_{3c} &= \frac{1}{2} \left(1 + \frac{\sqrt{A + 4c}}{\sqrt{A}}\right), \\
v_{3c} &= \frac{1}{2}(A + \sqrt{A}\sqrt{A + 4c}).
\end{align*}
\]

For the critical points to be real valued, we require \(A + 4c \geq 0\). Notice that the first critical point is the same as discussed in \cite{45}. There it was proposed that the coupling constant \(c \in [0, 1]\). Since there is a transition from CG to dark matter (i.e. \(c \to 1\)) as the universe evolves, it implies that the future universe will contain only dark matter and might have no trace of CG. Our analysis in the next two sections suggests that \(c\) cannot necessarily be restricted in the range \(0 < c < 1\) and can take values outside this range.

\section{3 Stability analysis}

To perform a stability analysis of our dynamical system, we linearize the system of equations (18) and (19) about the critical points to get

\[
\begin{align*}
\frac{d\delta u}{dx} &= 3v_c\delta u + 3(-1 + u_c)\delta v, \\
\frac{d\delta v}{dx} &= \frac{3}{u_c^2}\[\alpha v_c(c + v_c) + Au_c^2(1 + \alpha)]\delta u \\
&\quad + \frac{3}{u_c}[(c + 2v_c)\alpha + u_c(1 + 2v_c + \alpha - A(1 + \alpha))]\delta v.,
\end{align*}
\]

The eigenvalues of the above dynamical system (26) and (27) corresponding to the three critical points Eqs. (20 - 25) are

\[
\begin{align*}
\lambda_1 &= \frac{3}{2(c - 1)}(2 + A - 2c - Ac + \alpha + A\alpha - A\alpha - A\alpha) \\
&\quad + \sqrt{4(-1 + A(c - 1))(c - 1)^2(1 + \alpha) + (2 - 2c + \alpha - A(c - 1)(1 + \alpha)^2)), \\
\mu_1 &= \frac{-3}{2(c - 1)}(-2 - A + 2c + Ac - \alpha - A\alpha + A\alpha) \\
&\quad + \sqrt{4(-1 + A(c - 1))(c - 1)^2(1 + \alpha) + (2 - 2c + \alpha - A(c - 1)(1 + \alpha)^2)),
\end{align*}
\]


The parameter $A$ for the interacting modified Chaplygin gas with the coupling constant fixed at $c = 0.5$ arises when

$$q = -\frac{\ddot{a}}{aH^2} < -1,$$

which holds for the first critical point $(u_{1c}, v_{1c})$ only, since $q_1 = -1$. While for $(u_{2c}, v_{2c})$ and $(u_{3c}, v_{3c})$, we require

$$q_{2,3} = \frac{1}{2} \left[ 1 + \frac{3}{2} (1 + 3 \sqrt{A + 4c}) \right] < -1.$$ 

As $A + 4c \geq 0$, the inequalities in Eqs. (35) do not hold and hence the accelerated-expansion solution is not obtained from the second and third critical points. Hence the valid attractor solution is obtained from the first critical point only. We shall, henceforth, deal with the first critical point only.

As shown in figure 1, the first critical point $(u_{1c}, v_{1c})$ is the stationary attractor solution for the interacting modified Chaplygin gas with the coupling constant fixed at $c = 0.5$. The parameter $A$ can assume values in the range $-0.35 \leq A \leq 0.025$; while we...
choose $A = 0.025$ for our numerical work. Also note that if the parameter $\alpha < 0$, then it yields a polytropic equation of state for dark energy but for the MCG, we take $\alpha = 0.004$. It is evident that all the solutions of the dynamical system with four different initial conditions converge to the same final state. As $q_1 = -1$, the first critical point gives rise to an accelerated-expansion solution of the universe which is consistent with the observations.

Moreover, the attractor solution corresponding to $(u_{1c}, v_{1c})$ is also possible if $c$ takes values outside the usual considered range of $0 \leq c \leq 1$. In figures 2 and 3, the parameter $c$ is given values 1.7 and $-1.5$, respectively, with the same initial conditions. Curiously, all the four solutions converge to the same single final state. It draws to the fact, that at least theoretically, the coupling constant $c$ can take values outside the interval $[0,1]$. This result is further deduced in the next section using a different formalism.

4 Constraints on coupling constant

We can determine the constraints on the coupling constant $c$ by using the first critical point of our dynamical system. For this purpose, we shall adopt the formalism of Guo and Zhang [55]. We define new parameters corresponding to MCG and dark matter by

$$\gamma_{mcg} \equiv 1 + \omega = \frac{\rho_{mcg} + p_{mcg}}{\rho_{mcg}}$$  \hspace{1cm} (36)

and

$$\gamma_{dm} \equiv \frac{\rho_{dm} + p_{dm}}{\rho_{dm}}.$$  \hspace{1cm} (37)

Note that $\gamma_{dm} = 1$ since $p_{dm} = 0$. Moreover, the parameter $\gamma_{mcg}$ will be determined corresponding to the first critical point. To find how the density ratio $R$ evolves with time, we differentiate it with respect to $t$ to get

$$\dot{R} = \frac{dR}{dt} = \frac{\rho_{dm}}{\rho_{mcg}} \left[ \frac{\dot{\rho}_{dm}}{\rho_{dm}} - \frac{\dot{\rho}_{mcg}}{\rho_{mcg}} \right].$$  \hspace{1cm} (38)

Using Eqs. (10) and (11), Eq. (38) becomes

$$\dot{R} = R \left[ \frac{Q}{\rho_{dm}} + \frac{Q}{\rho_{mcg}} + 3H(\gamma_{mcg} - 1) \right].$$  \hspace{1cm} (39)

Using Eq. (12) in (39), we get after simplification

$$\dot{R} = 3H[c(1 + R)^2 + R(\gamma_{mcg} - 1)].$$  \hspace{1cm} (40)
In order to get stationary solutions, we solve for $\dot{R} = 0$ to get

$$R^\pm = \frac{1 - \gamma_{mcg}}{2c} - 1 \pm \sqrt{\left(\frac{1 - \gamma_{mcg}}{2c} - 1\right)^2 - 1}. \quad (41)$$

Now, to get real valued solutions, we require $(\frac{1 - \gamma_{mcg}}{2c} - 1)^2 - 1 \geq 0$, which yields

$$\left(\frac{1 - \gamma_{mcg}}{2c} - 1\right)^2 - \left(\frac{1 - \gamma_{mcg}}{2c}\right) \geq 0. \quad (42)$$

The above inequality holds if the quantities in the brackets are either both positive or both negative. We shall take $c$ to be a free parameter which can take values other than zero.

Case (1)

Assume both quantities in the brackets in (42) to be positive, i.e.

$$\frac{1 - \gamma_{mcg}}{2c} - 2 \geq 0, \quad \frac{1 - \gamma_{mcg}}{2c} \geq 0. \quad (43)$$

Case (1a)

Now take $c > 0$; thus Eq. (43) gives

$$\gamma_{mcg} \leq 1 - 4c, \quad \gamma_{mcg} \leq 1, \quad (44)$$

which yields

$$\gamma_{mcg} \leq 1 - 4c. \quad (45)$$

Case (1b)

If $c < 0$, then Eq. (43) yields

$$\gamma_{mcg} \geq 1 - 4c, \quad \gamma_{mcg} \geq 1, \quad (46)$$

which implies

$$\gamma_{mcg} \geq 1 - 4c. \quad (47)$$

Case (2)

Now take both quantities in the brackets in Eq. (42) to be negative, i.e.

$$\frac{1 - \gamma_{mcg}}{2c} - 2 \leq 0, \quad \frac{1 - \gamma_{mcg}}{2c} \leq 0. \quad (48)$$

Case (2a)
Take $c > 0$; after solving Eq. (48), which gives

$$\gamma_{m_{cg}} \geq 1 - 4c, \quad \gamma_{m_{cg}} \geq 1,$$

which yield

$$\gamma_{m_{cg}} \geq 1.$$ (50)

Case (2b)

If $c < 0$, then Eq. (48) yields

$$\gamma_{m_{cg}} \leq 1 - 4c, \quad \gamma_{m_{cg}} \leq 1,$$

which results in

$$\gamma_{m_{cg}} \leq 1.$$ (52)

Now we shall use the definition $\gamma_{m_{cg}} = 1 + \omega_1 = 1 + v_{1c}/u_{1c}$ in each of the above four cases.

Case (1a)

Using $\omega_1 = \frac{v_1}{u_{1c}} = \frac{1}{1 - c}$ in Eq. (45), we have $(1 - 2c)^2 \geq 0$, which is satisfied for all values of $c$. Notice that from Eq. (44), we have an additional constraint $c < 1$; therefore, $0 < c < 1$, which is the range usually considered for $c$ in the literature.

Case (1b)

Using $\omega_1$ in Eq. (47) we get $(1 - 2c)^2 \leq 0$, which is satisfied only for $c = 1/2$. Since $c < 0$, we do not have an acceptable solution.

Case (2a)

Here for $\omega_1$, Eq. (50) implies $\frac{1}{1 - c} \geq 0$, which holds for all $c > 1$. Apparently it implies that the coupling constant between MCG and dark matter can take arbitrary value; thus, the mutual interaction can be more dynamic. It yields arbitrary freedom for the conversion of MCG into dark matter.

Case (2b)

For $\omega_1$, Eq. (52) implies $\frac{1}{1 - c} \leq 0$, which is viable if $c < 0$. Thus, the coupling constant can take arbitrary negative values. This case apparently supports the conversion of dark matter into MCG with arbitrary coupling. Note that $\omega_1$ represents the EoS of phantom energy ($\omega_1 < -1$).
5 Conclusion

We have investigated the possible interaction between dark matter and the Chaplygin gas and we deduced that the coupling constant involved can take values outside the range usually considered, $0 < c < 1$. This range arises as a special case in the Case (1a). Our analysis suggests that $c$ can take arbitrary positive or negative values. If $c > 1$, as in Case (2a), then it supports the conversion of MCG into dark matter. Conversely, if $c < 0$ as in Case (2b), it allows for the conversion of dark matter into MCG. It also supports the existence of phantom energy through the final case. Moreover, the present work may serve as the generalization of the earlier work by Zhang and Zhu [42] for the interacting Chaplygin gas and by Wu and Yu [45] for the interacting generalized Chaplygin gas.

In a recent investigation, Feng et al [49] have presented observational constraints on the coupling parameter and have deduced that small positive values for $c$ are most probable. This conclusion is drawn in order to alleviate the cosmic-coincidence problem. Also the negative values of $c$ are excluded to avoid the violation of the second law of thermodynamics [50]. We have deduced from our analysis that $c$ has no such theoretical constraints, and the usual choice $[0,1]$ is not a true range for the coupling parameter.

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Figure 1: The phase diagram of the interacting modified Chaplygin gas model with $c = 0.5$. The model parameters are fixed as $A = 0.025$ and $\alpha = 0.004$. The curved lines from left to right correspond to the initial conditions $u(-2) = 1.2, v(-2) = -0.2$ (green); $u(-2) = 1.3, v(-2) = -0.3$ (blue); $u(-2) = 1.4, v(-2) = -0.4$ (red); $u(-2) = 1.5, v(-2) = -0.5$ (black).
Figure 2: The phase diagram of the interacting modified Chaplygin gas model with $c = 1.7$. The model parameters are fixed as in Fig. 1. The curved lines from left to right correspond to the initial conditions as given in Fig. 1.
Figure 3: The phase diagram of the interacting modified Chaplygin gas model with $c = -1.5$. The model parameters are fixed as in Fig. 1. The curved lines from left to right correspond to the initial conditions as given in Fig. 1.