CCT determination using a method with automated iterative calculation

I Ázara 1 T Menegotto 2 J Gomes 1

1 Divisão de Metrologia Óptica, Inmetro, Duque de Caxias, RJ

2 Superintendência Regional do Rio Grande do Sul, Inmetro, Porto Alegre, RS
tmenegotto@inmetro.gov.br

Abstract. Correlated Colour Temperature (CCT) of light sources has been usually determined using the Robertson’s method, that is based on a table with values of chromatic coordinates corresponding to various values of the blackbody locus in the CIE u,v diagram. Robertson’s method requires interpolation, normally linear, that led to an acceptable accuracy but that could be improved. The proposed method is based directly on the CCT as defined by the CIE. It allows a better accuracy, in the determination of CCT and Duv, that is the distance of the chromaticity to the blackbody locus in the CIE u,v diagram. The Excel® Solver resource is used, and the method permits easily updating the universal constants to the recently defined values by the BIPM. 

Keywords: colorimetry, CCT, chromatic coordinates

1. Introduction
In 1931 the CIE (Commission Internationale de l’Éclairage) adopted a system for characterization of colours using three numerical values, called chromaticity coordinates. These are represented by the symbols x, y and z, such that \(x + y + z = 1\). This relation allows the representation of colours in a bi-dimensional diagram, called x,y diagram, or CIE 1931 diagram (figure 1). The values shown in the edges of the diagram represent the wavelengths, in nm, of the visible monochromatic radiations.

The x,y diagram is not uniform. In figure 1, the ellipses correspond to equal visual sensation differences, but their areas are different.

The u,v diagram, or CIE 1960 diagram, is a uniform colour diagram obtained by mathematical relations between the u,v and x,y coordinates, demonstrated by the equations (1) and (2). When one says chromaticity of a light source, this means the x,y or u,v coordinates in the respective diagrams.

\[
\begin{align*}
    u &= \frac{4x}{12y - 2x + 3} \\
    v &= \frac{6y}{12y - 2x + 3}
\end{align*}
\]

In the diagram of figure 2, the chromaticities of the colours emitted by a blackbody are shown in the line called blackbody locus (BBL). The chromaticity of any other light colour can be in any local of the colour space.
A blackbody (also called planckian radiator) is an ideal radiator, what means that, in thermal equilibrium, absorbs all energy incident on it (i.e. it doesn’t reflect or transmit radiation), and emits the same amount of incident energy.
Planck established the relation between the blackbody temperature and the spectral distribution $S(\lambda)$ of its emission, that depends only on its temperature (equation (3) and figure 3).

$$S(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$  \hspace{1cm} (3)

where

- $h$: Planck’s constant ($6,626 \ 068 \cdot 10^{-34} \text{m}^2 \text{kg} \text{s}^{-1}$)
- $c$: Light speed ($2,997 \ 924 \ 58 \cdot 10^8 \text{m/s}$)
- $k$: Boltzman’s constant ($1,380 \ 650 \ 3 \cdot 10^{-23} \text{J/K}$)
- $T$: thermodynamic temperature (K)
- $\lambda$: wavelength (m)

The above set of values for $h$ and $k$ are the new ones defined by the BIPM (Bureau International des Poids et Mesures) in 2018 [1].

![Figure 3 – Blackbody spectral emission from 4 000 K to 8 000 K, at 1 000 K intervals](image)

The characterization of colours emitted by light sources is made mainly by two quantities identified as CCT (Correlated color temperature) and Duv. This is the smaller distance of the point of coordinates $u, v$ to the BBL in the CIE 1960 diagram, and CCT is the temperature associated to the point in the BBL that is the nearest to chromaticity of the light source.

To determine the $x, y$ coordinates of a particular spectral distribution, one must use spectral functions defined by the CIE [2]. Once determined the $x, y$ coordinates and transformed to the $u, v$ coordinates, using equations (1) and (2), the Duv and the CCT can be calculated. A method largely used is the one proposed by Robertson [3]. This paper presents another method that allows better accuracy in determining the Duv and the CCT parameters.
2. Definition of CCT and Duv

The CIE definition of CCT is, presented in a simplified manner, the temperature of the planckian radiator with the chromaticity nearest to the chromaticity of the spectral distribution being analyzed.\(^1\)

So, it is necessary to find the smaller distance, in the CIE 1960 diagram, between the chromaticity associated with the test source spectral distribution and the BBL. Calling \(u_t, v_t\), the coordinates of the test source, and \(u_p, v_p\), the coordinates of the nearest point pertaining to BBL, the distance between them, called Duv, is calculated by equation (4).

\[
Duv = \left[ (u_t - u_p)^2 + (v_t - v_p)^2 \right]^{1/2}
\] (4)

Equation (4) leads to an iterative procedure, assigning an initial arbitrary value to the temperature of blackbody, calculating its coordinates \(u, v\) and the distance to \(u_t, v_t\), followed by the assignment of other values until the minimal distance is found. This is a laborious task, however. Robertson calculated the \(u, v\) coordinates for 586 isothermals along the BBL, from 135 K to 100 000 K, and with this he aided to improve the CCT and Duv determination. In the more usual range of CCT in colorimetry (2 000 K to 15 000 K), the intervals in the Robertson’s table vary from 40 K to 3 000 K. Inside each interval, interpolations must be made. Nowadays, due to the new values of the constants \(h\) and \(k\) it would be necessary to recalculate the Robertson’s table, but this would not lead to significative changes in the resulting CCT values.

Using accessible computational processes, the use of optimization algorithms is possible, allowing to perform in a effective way the iterative procedure described in the previous paragraph. This leads to more accurate results, which is important to CCT and Duv determination, because the definition of CCT only applies if Duv is smaller than 0,05.\(^4\) The straight segments normal to the BBL (figure 2) represents the range of Duv for which the CCT can be determined, from 2 000 K to 10 000 K, intervals of 2 000 K.

An inaccurate calculation of Duv can cause the erroneous determination of the CCT of a light source. The Duv, preceded by + or – signal, indicates the colour difference from the test source to the corresponding colour in the BBL. The + signal means that \(u, v\) is above the BBL. The classification of a light source in a nominal CCT class, that corresponds to a range of CCT values, can be affected by calculation inaccuracy.

3. The iterative method using Solver

First of all, the chromaticity \(u_t, v_t\) of the test light source must be determined, as explained in Section 1. The Excel® program has a resource called Solver, which purpose is the execution of iterative calculation to the optimization of a value in function of one or more variable parameters. To use it in the calculation of CCT and Duv, a calculation worksheet was prepared based on equation (3), in which one can enter with a value for the blackbody temperature, and the corresponding spectral distribution is calculated. Using this spectral data, the corresponding chromaticity \((u, v)\) is calculated, which determines its position on the BBL, in the CIE 1960 diagram. Another worksheet, based on equation (4) calculates the distance between this point and the chromaticity of the test light source. This distance is the control parameter. The Solver is used for searching the smallest distance value, by the variation of the value attributed to the blackbody temperature, starting with an arbitrary value (e.g 1 000 K). The smaller distance is the Duv. If \(|Duv| \leq 0,05\), the corresponding value of the blackbody temperature is deemed to be the CCT of the source \(^4\). This calculation can be made for the spectral distribution of any light source, but the wavelength intervals must be 5 nm or less, since the greater the wavelength intervals, the lower the results accuracy. CIE does not have a recommended method to use intervals greater than 5 nm \(^2\).

\(^1\) The complete definition can be found in the International Lighting Vocabulary \(^4\)
4. Application examples

The calculations performed with the Solver and presented in sections 4.1 to 4.2 were made using the old values of the constants \( h \) and \( k \). This was performed to easily compare the results of the method with those obtained using currently available Robertson’s tables, that were not yet updated to the new constants \( h \) and \( k \).

4.1. Illuminant A

To calculate CCT and Duv for the illuminant A, the \( x,y \) chromaticity coordinates values from CIE 015:2018 were used [2]. These values are \( x_A = 0.44758 \); \( y_A = 0.40745 \), corresponding to \( u_A = 0.25597 \); \( v_A = 0.34953 \), following equations (1) and (2).

The CCT and the Duv results obtained by Solver and Robertson methods, for the illuminant A, are presented in Table 1

| Method    | CCT [K]  | Duv       |
|-----------|----------|-----------|
| Solver    | 2 855,456| \( 5.2 \times 10^{-7} \) |
| Robertson | 2 855,525| \( 6.7 \times 10^{-5} \) |

The CCT attributed by the CIE to the illuminant A is 2 855.5 K. We can see in Table 1 that the method Solver allowed better optimization of the parameter, i.e. smaller absolute value of Duv. In practical terms, the illuminant A Duv is zero, because the CIE distribution of the illuminant A and that of the blackbody at 2 855.5 K are identical.

These results show that Excel® Solver method is effective in the determination of CCT and Duv. This can be evidenced by the convergence of the results, and by the fact that Duv value calculated by the Solver, that is approximately one hundredth of the value obtained by Robertson’s method.

4.2. Illuminants CIE D50 and D65

The illuminants CIE D50 and CIE D65 correspond to conditions of daylight that present CCT near 5 000 K and 6 500 K, respectively. Their spectra are tabulated.

The CCT and Duv results obtained using Solver and Robertson methods, for the CIE illuminants D50 and D65, are presented in Table 2.

| Illuminant D50 | CCT [K]  | Duv       |
|----------------|----------|-----------|
| Solver         | 5 001,622| \( 3.195 \times 10^{-3} \) |
| Robertson      | 4 999,576| \( 3.200 \times 10^{-3} \) |

4.3. Led artifacts

The CCT and Duv results, obtained using Solver and Robertson methods, for two LED artifacts whose spectral distributions were previously measured in the laboratory, identified as LED 5k and LED 8k, are presented in Table 3.

| LED artifact | CCT [K]  | Duv       |
|--------------|----------|-----------|
| LED 5k       | 6 501,684| \( 3.197 \times 10^{-3} \) |
| LED 8k       | 6 502,681| \( 3.220 \times 10^{-3} \) |
Table 3 – Comparison of CCT and Duv for the artifacts LED 5k e LED 8k

|         | LED 5k          | LED 8k          |
|---------|-----------------|-----------------|
|         | CCT [K] | Duv        | CCT [K] | Duv        |
| Solver  |        |            |        |            |
|         | 4 703,292 | 1,154 × 10⁻² | 8 011,291 | 4,165 × 10⁻³ |
| Robertson | 4 702,733 | 1,156 × 10⁻² | 8 007,396 | 4,575 × 10⁻³ |

The correct estimation of Duv gain importance as CCT grows due to the fact that the lines of greater CCT are closer to each other, to equal intervals of CCT, as the value of CCT increases. This can be observed in the figure 2. For example, by calculating the angular difference between the slopes of the isothermals for the lines 2 000 K and 4 000 K, the angular difference is 29,5° whereas for the lines 6 000 K and 8 000 K, the difference is 8,4°. So, the sensitivity of CCT to Duv errors increases as the CCT grows.

4.4. Illuminant A with the new values of h and k

As explained previously, all the results presented above were obtained using the old values of h and k, for a fair comparison between the two methods. Using the new values adopted after the redefinition of the International System of Units [1], the CCT of the illuminant A was calculated by the Solver method to be 2 855,496 K, leading to a difference of +0,04 K relative to the value calculated with the old constants. In practice, the CCT for illuminant A is rounded to 2 855,5 K.

5. Conclusion

This paper presents fundamental concepts associated to the colorimetric quantities CCT and Duv for light sources and introduces an iterative method for their determination, based directly on the CIE definition. This method uses computing resources available in virtually any laboratory and is easy to deploy even without advanced programming skills. It is based on the resource Solver of the Excel® program and allows better accuracy than the Robertson method with immediate utilization of the new values of universal constants defined in 2018 by the BIPM.

Acknowledgement

To Inmetro, for supporting the research of the author I Ázara via the program Pronametro.

References

[1] BIPM 2018 Proc. of the 26th meeting of the CGPM, (Sèvres: BIPM)
[2] CIE 2018 CIE 015:2018 Colorimetry, 4th Edition (Vienna: CIE)
[3] Robertson A R 1968 Computation of correlated color temperature and distribution temperature J. Opt. Soc. Am. 58 1528
[4] CIE 2016 CIE DIS 017/E:2016 ILV: International Lighting Vocabulary 2nd Edition (Vienna: CIE)