Discrimination and characterization of Parkinsonian rest tremors
by analyzing long-term correlations and multifractal signatures

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Abstract

In this paper, we analyze 48 signals of rest tremor velocity related to 12 distinct subjects affected by Parkinson’s disease. The subjects belong to two different groups, formed by four and eight subjects with, respectively, high- and low-amplitude rest tremors. Each subject is tested in four settings, given by combining the use of deep brain stimulation and L-DOPA medication. We develop two main feature-based representations of such signals, which are obtained by considering (i) the long-term correlations and multifractal properties, and (ii) the power spectra. The feature-based representations are initially utilized for the purpose of characterizing the subjects under different settings. Our results show that the effect of medication is clearly recognizable in the signals. In addition, when medication is used the related signals switch from anti-correlated to long-term, positively correlated. In agreement with previous studies, we show that deep brain stimulation does not significantly characterize neither of the two groups, regardless of the adopted representation. On the other hand, the medication effect yields statistically significant differences in both high- and low-amplitude tremor groups. We successively test several different instances of the two feature-based representations of the signals in the setting of supervised classification and (nonlinear) feature transformation. We consider three different classification problems, involving the recognition of (i) the presence of medication, (ii) the use of deep brain stimulation, and (iii) the membership to the high- and low-amplitude tremor groups. Classification results show that the use of medication can be discriminated with higher accuracy, considering many of the feature-based representations. Notably, we show that the best results are obtained with a parsimonious, two-dimensional representation encoding the long-term correlations and multifractal character of the signals.

Keywords—Long-term correlations; Multifractal spectra; Parkinsonian rest tremor; Classification; Feature transformation.

1 Introduction

Long-memory processes describing complex systems [1, 40] and the analysis of long-term correlations (LTC) in related signals [21] play an important role in many research contexts. For instance, it is possible to cite investigations in EEG signals [20], in the analysis subthalamic nucleus of patients with Parkinson’s disease [16], in DNA sequences [31], in postural sway of humans [8], and construction engineering [30]. LTC are usually a primary source of fractality in signals (in this paper, time series are signals used interchangeably) describing the observed variables of the system over time. Fluctuations in a fractal time series present a power-law scaling, denoting thus the absence of a characteristic time/space scale describing the underlying system. Multifractal time series are fractal time series that require more than one scaling exponent to be effectively described [15]. This happens when the underlying process is characterized by different scalings of the fluctuations, which in turn require a continuum of

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scaling exponents. Several different methods have been developed in the past years to detect fractal and multifractal signatures. Among the many we can cite multifractal detrended fluctuation analysis (MF-DFA) [15], adaptive fractal analysis [22], wavelet transform modulo maxima [29], and wavelet leaders [23]. Fractal and multifractal analysis of time series play a pivotal role in many scientific contexts, such as neuroscience and medicine in general [9, 10, 55]. Just to mention a few, it is possible to cite applications in human gait analysis [11], background neuronal noise-like activity in human and mouse hippocampus [32], analysis of cervical tissue samples [13], MRIs for tumor characterization [17], EEG signals [24], protein contact networks [26], and electromyograms for neuro-muscular disease diagnosis [25, 34].

Parkinson's disease (PD) is a neuro-degenerative disorder that targets the central nervous system. PD is characterized by the progressive loss of dopaminergic neurons in the substantia nigra of the midbrain. The most evident symptoms associated with PD are tremors, bradykinesia, rigidity, and postural instability, while at more advanced stages of the disease other factors might present, such as different types of cognitive impairments (e.g., dementia) and changes in behavior and/or emotional states [3, 14]. The causes of PD are however still largely unknown. This fact opened the way to multi-disciplinary researches, involving for instance the use of artificial neural networks for the purpose of prediction of related signals [12, 39] and mutual information based methods for detecting upper limb motor dysfunction [7]. Deep brain stimulation (DBS) [27, 28, 33] is a neurosurgical procedure that involves a surgical intervention to implant electrodes in brain areas suitable for receiving electrical impulses. DBS proved to be effective in the treatment of PD and other diseases, such as obsessive-compulsive disorders [4].

In this paper, we study 48 signals recorded from 12 subjects affected by PD [3]. Tremor signals are recorder by means of a velocity laser targeted to their index finger. For each subject, signal recording is performed in four different settings, given by the combination of the use of DBS and L-DOPA medication. The original experiment [3, 56] was performed on a larger (16) set of subjects. However, not all subjects were recorded in the four aforementioned conditions. Therefore, here we consider four subjects affected by high-amplitude tremors and eight affected by low-amplitude tremors. On the same data, Yulmetyev et al. [41] performed a wide analysis by using the statistical theory of non-Markov processes and flicker-noise spectroscopy. In addition, the attenuation effects of DBS on locomotion and tremor over different time scales were further investigated by Beuter and Modolo [2], developing a computational model of biological neural networks. Here we take a different route by using LTC and multifractal signatures as analysis and representation tools. To our knowledge, a characterization of such signals for the purpose of discrimination in terms of LTC and multifractal signatures has never been developed in the literature. LTC and multifractal properties are derived here by means of the MF-DFA technique. We develop two low-dimensional, feature-based representations (FBRs) of such signals, which are obtained by considering (i) the LTC and multifractal properties and (ii) the power spectra. The power spectra are principally used for comparison, since they were analyzed by Beuter et al. [3]. The FBRs are initially utilized for the purpose of characterizing the subjects under different test settings. All signals present a clear multifractal signature and different forms of LTC. Our results show that the effect of medication is clearly recognizable in the signals. In addition, the use of medication indicates a qualitative change of LTC from anti-persistent to persistent. In agreement with previous studies [3], we show that DBS does not characterize neither of the two groups, regardless of the adopted FBR. On the other hand, the medication effect yields statistically significant differences in both high- and low-amplitude tremor groups. We successively test several different instances of the two FBRs of the signals in the setting of supervised classification and (nonlinear) feature transformation. We consider three different classification problems, involving the recognition of (i) the presence of medication, (ii) the use of deep brain stimulation, and (iii) the high- and low-amplitude tremor groups. Classification results show that the use of medication can be discriminated with higher accuracy, considering many of the FBRs. Notably, we show that the best results are obtained with the herein developed parsimonious, two-dimensional representation encoding the LTC and multifractal character of the signals. Overall, our results highlight the usefulness of LTC and multifractal signatures in the analysis Parkinsonian rest tremors.

The remainder of this paper is structured as follows. In Section 2 we introduce the 48 signals, describing the developed FBRs. Details related to the experimental setting on which the original signals were obtained are omitted in this paper, referring the reader to the original study [3] for details. In Section 3 we present and discuss the results related to the developed FBRs of such signals. In Section 4 we draw the conclusions pointing at future directions. The paper includes Appendix A providing the essential details about MF-DFA.
2 Feature-based representation of the rest tremor signals

The original experiment [3] consisted in recording rest tremor velocity from 16 subjects affected by Parkinson’s disease. Rest tremor was recorded via a velocity laser under four different main conditions, given by combining the use of medication (L-DOPA) and high-frequency DBS. Participants received DBS of the internal globus pallidus, the subthalamic nucleus, or the ventrointermediate nucleus of the thalamus. Unfortunately, not all subjects were recorded in all four conditions; only for 12 subjects all four recording conditions (i.e., medication Off–On, DBS Off–On) are available. Among those 12 subjects, four belong to the high-amplitude rest tremor group (originally denoted as g2, s6, s7, and s8) while the remaining eight to the low-amplitude rest tremor group (originally denoted as g9, g10, g11, g12, g13, s14, s15, and s16). In the following, we study the related 48 signals of rest tremors.

The 48 signals are here represented according to two main FBRs, i.e., as numeric vectors. Notably, we represent such signals by using (i) LTC and MFS properties and (ii) the estimated power spectra. In the first case, we develop three, low-dimensional representations by means of the coefficients derived via MF-DFA – see Appendix A for technical details. MF-DFA is executed by considering 40 equally-spaced (temporal) scales in-between 16 and 512; we remove quadratic (local) trends; we consider 101 equally-spaced values in-between -5 and 5 for the fluctuation weighting factor q. The FBR shown in Fig. 1(a) uses only the Hurst (H) coefficient (4) and the multifractal spectrum width (MFSW) (10) elaborated from the available time series. In the following we denote such a representation as H-MFSW. Next, we consider the information provided by the entire multifractal spectrum (MFS). Notably, we initially represent each time series as a high-dimensional vector encoding the domain (9) and co-domain of the calculated MFS (5). Such high-dimensional vectors are then transformed by considering both principal component analysis (PCA) and its nonlinear extension [15], known as kernel PCA (kPCA); a Gaussian kernel is adopted. We noted that the first four principal components (PCs) usually explain more than 80% of the data variance, so in both cases they are retained for the embedding. The MFS coefficients transformed via the linear PCA are denoted as MFS-PCA, while MFS-kPCA is used to denote the four-dimensional vectors obtained by the corresponding nonlinear transformation – see respectively Fig. 1(b) and Fig. 1(c).

Power spectrum is estimated using the well-known Welch’s method [37]. We initially represented each time series as a high-dimensional vector containing the amplitude values at 1025 (normalized) frequencies. Such high-dimensional representations are then transformed by using PCA and kPCA; four dimensions are retained as in the previous case. Fig. 2 shows the first two PCs of such representations, denoted in following as POWER-PCA and POWER-kPCA, respectively.

Please note that the herein developed FBRs of the original signals are not “labeled” yet, in the sense that here we did not consider any specific discrimination problem. In Sec. 3 we will deal with several characterizations and discrimination problems aptly conceived over such FBRs.

3 Analysis of experimental results

Our principal goal here is to show that LTC and MFS can be adopted as effective features when representing the considered biomedical signals. We contrast the results with FBRs elaborated from the more conventional power spectrum. We show that the two FBRs provide comparable results, although the former allows also to infer important qualitative information regarding the signals. We proceed by analyzing the different FBRs developed in Sec. 2 by first taking account the independent contribution of the features, considering the contribution of each feature in the discrimination of the two groups of high- and low-amplitude rest tremor subjects (Sec. 3.1), and successively by evaluating the FBRs in terms of recognition performances over three suitably-defined classification problems (Sec. 3.2).

3.1 Characterization and statistical analysis

Let us first take into account the results delivered in Tab. 1. For the first FBR, we assessed the Hurst exponent (H) and MFSW, while for the second one we considered the first two PCs obtained from the PCA and kPCA, respectively. We categorize the data by considering the use of medication, the use of DBS, and subjects with high- and low-amplitude tremors. p-values are obtained by evaluating the t-test over each categorization; we consider the usual 5% as the threshold. The use of medication can
be suitably recognized when taking into account most of the considered variables. Notably, both H and MFSW result in a statistically significant discriminator for such a categorization. The use of DBS does not allow for any statistically significant discrimination – in terms of t-test. This suggests that DBS does not have a significant global impact on the subject tremors when the high/low rest tremor conditions of the subjects are not taken into account. Finally, only two of six considered variables (namely, MFSW and POWER-kPC1) produce statistically significant differences when considering the characterization in terms of high/low amplitude rest tremors.
Fig. 3 offers a visual representation of such statistics (for H and MFSW only) in the three different categorizations. According to the $p$-values in Tab. 1, LTC properties of the signals offer statistically significant differences only when considering medications. It is worth noting that the use of medication switches the LTC properties toward positively correlated signals, while in absence of medication the signals are clearly anti-correlated (upper panels). All signals appear as multifractal, with a relevant multifractal signature quantified by the MFSW (middle panels). It is worth noting that the multifractal signature is sufficiently preserved after shuffling the time series (lower panels), suggesting that LTC are not the only source for the observed multifractality. In fact, shuffling destroys LTC and any deterministic trend that might influence the degree of multifractality of the series. To conclude, we note that differences between the MFSW of the original and shuffled time series are statistically significant ($p < 0.0008$). Nonetheless, a more detailed verification shows that when medication is Off (On) differences are (not) statistically significant between the original and shuffled time series ($p < 0.9291$ ($p < 0.5251$); a similar scenario holds for the use of DBS ($p < 0.0022$ ($p < 0.1004$), and for high- (low-) amplitude tremors, $p < 0.0037$ ($p < 0.0816$).

Table 1: $p$-values – statistically significant results are in bold. The columns named “med-Off / med-On” and “DBS-Off / DBS-On” consider differences between all subjects in the respective settings, while the column “High-tremor / Low-tremor” considers the differences between the two groups taking into account all combinations of DBS and medication.

| Feature   | med-Off / med-On | DBS-Off / DBS-On | High-tremor / Low-tremor |
|-----------|------------------|------------------|--------------------------|
| H         | $p < 0.0001$     | $p < 0.9291$     | $p < 0.3860$             |
| MFSW      | $p < 0.0272$     | $p < 0.7727$     | $p < 0.0134$             |
| POWER-PC1 | $p < 0.2427$     | $p < 0.4869$     | $p < 0.1122$             |
| POWER-PC2 | $p < 0.0114$     | $p < 0.4879$     | $p < 0.0507$             |
| POWER-kPC1| $p < 0.0001$     | $p < 0.4721$     | $p < 0.0059$             |
| POWER-kPC2| $p < 0.4614$     | $p < 0.0675$     | $p < 0.0932$             |

Let us now take into account the results in Tab. 2 which show the contribution of, respectively, DBS and medication on the characterization of subjects having high- and low-amplitude tremors. As suggested by the original experiments conducted by Beuter et al. [3], DBS does not seem to have a statistically significant impact on such a characterization. In fact, in both groups, regardless of the considered feature, we obtain $p$-values far from denoting statistically significant results. On the other hand, when considering the impact of medication on the two groups, we obtain some statistically significant results. Notably, for the group having high-amplitude rest tremors (indicated as “High-tremor” in the table), features H, POWER-PC2, and POWER-kPC1 produce statistically significant results, while for the low-amplitude group we have H and POWER-kPC1.

Table 2: $p$-values are calculated to assess differences of each group (high and low tremor subjects) when considering the effects of DBS and medication. DBS does not produce statistically significant differences, while medication typically does yield statistically significant differences (reported in bold).

| Feature   | High-tremor | Low-tremor |
|-----------|-------------|------------|
| DBS-Off / DBS-On |             |             |
| H         | $p < 0.9587$ | $p < 0.8444$ |
| MFSW      | $p < 0.6068$ | $p < 0.4481$ |
| POWER-PC1 | $p < 0.4159$ | $p < 0.0692$ |
| POWER-PC2 | $p < 0.6833$ | $p < 0.4167$ |
| POWER-kPC1| $p < 0.3856$ | $p < 0.8968$ |
| POWER-kPC2| $p < 0.1485$ | $p < 0.3881$ |

| med-Off / med-On |             |             |
|------------------|-------------|------------|
| H         | $p < 0.0001$ | $p < 0.0001$ |
| MFSW      | $p < 0.0596$ | $p < 0.1753$ |
| POWER-PC1 | $p < 0.2402$ | $p < 0.8188$ |
| POWER-PC2 | $p < 0.0382$ | $p < 0.0823$ |
| POWER-kPC1| $p < 0.0001$ | $p < 0.0139$ |
| POWER-kPC2| $p < 0.8006$ | $p < 0.2780$ |
3.2 Classification of rest tremor signals

In order to evaluate the discrimination capability of each FBR, we considered three supervised classification problems. We face the problem of discriminating between the two groups (high- and low-amplitude tremors) and recognizing the use of medication and DBS, respectively. We have chosen the well-known support vector machine (SVM) \[35\] as supervised classification systems, configured with the Gaussian kernel. Notably, we used the version known as C-SVM, where C is a hyper-parameter controlling the complexity of the resulting model – in SVM the structural complexity of the model is measured by considering the number of support vectors (SVs) computed during the synthesis. Both hyper-parameters, i.e., C and the width of the Gaussian kernel, are determined by preliminary tests using a grid-search scheme.

Since our dataset, regardless of the adopted FBR, is limited to 48 patterns, we tested the recognition capability of C-SVM in the leave-one-out setting: each pattern is tested by synthesizing the C-SVM model on the remaining 47 patterns. Tab. 3 summarizes the results for all three classification problems. We report for each classification problem the results obtained with five FBRs. For POWER-PCA and POWER-kPCA we use, in both cases, the first 2, 3, and 4 PCs in order to evaluate the performances by varying the dimensionality of the representation. We show the number of errors (for each class), the area under the ROC curve (AUC), and the average number of SVs as an indicator of structural complexity of C-SVM. Results show that the effect of medication allows in general for a more accurate classification. It is worth underlying that the best result (AUC is 0.85) is obtained with the parsimonious, two-dimensional representation denoted as H-MFSW; the per-class errors are also more balanced with respect to the results obtained with the two-dimensional version of POWER-kPCA. In general, results for the last two problems, namely “DBS-Off / DBS-On” and “High-tremor / Low-tremor”, are not convincing – we obtain results compatible with a random classifier. This fact confirms that the effect of medication on the 48 subjects seems to be more characterizing, allowing for a good discrimination regardless of the use of DBS or the membership to the high- or low-amplitude rest tremor groups.
Table 3: Classification results with leave-one-out on the low-dimensional representations of the signals. Three different classification problems are faced by considering several feature-based, low-dimensional representations: recognition of (i) medication Off–On, (ii) DBS Off–On, and (iii) high–low amplitude tremor.

| Representation | Dimension | Errors | AUC | Avg. SVs |
|----------------|-----------|--------|-----|----------|
| **med-Off / med-On** |         |        |     |          |
| H-MFSW         | 2         | 7 (5/24, 2/24) | 0.85 | 35.2     |
| MFS-PCA        | 4         | 16 (8/24, 8/24) | 0.67 | 44.2     |
| MFS-kPCA       | 4         | 16 (10/24, 6/24) | 0.67 | 31.6     |
| POWER-PCA      | 2         | 22 (22/24, 0/24) | 0.54 | 46.0     |
| POWER-PCA      | 3         | 22 (22/24, 0/24) | 0.54 | 46.5     |
| POWER-PCA      | 4         | 23 (23/24, 0/24) | 0.52 | 46.5     |
| POWER-kPCA     | 2         | 13 (13/24, 0/24) | 0.73 | 31.5     |
| POWER-kPCA     | 3         | 9 (9/24, 0/24) | 0.81 | 31.1     |
| POWER-kPCA     | 4         | 8 (8/24, 0/24) | 0.83 | 31.1     |
| **DBS-Off / DBS-On** |         |        |     |          |
| H-MFSW         | 2         | 48 (24/24, 24/24) | 0.00 | 47.0     |
| MFS-PCA        | 4         | 29 (14/24, 15/24) | 0.40 | 47.0     |
| MFS-kPCA       | 4         | 26 (8/24, 18/24) | 0.46 | 44.6     |
| POWER-PCA      | 2         | 48 (24/24, 24/24) | 0.00 | 46.9     |
| POWER-PCA      | 3         | 48 (24/24, 24/24) | 0.00 | 46.9     |
| POWER-PCA      | 4         | 48 (24/24, 24/24) | 0.00 | 46.9     |
| POWER-kPCA     | 2         | 23 (3/24, 21/24) | 0.52 | 45.2     |
| POWER-kPCA     | 3         | 23 (3/24, 21/24) | 0.52 | 45.3     |
| POWER-kPCA     | 4         | 23 (3/24, 21/24) | 0.52 | 45.9     |
| **High-tremor / Low-tremor** |     |        |     |          |
| H-MFSW         | 2         | 16 (16/16, 0/32) | 0.50 | 32.7     |
| MFS-PCA        | 4         | 21 (14/16, 7/32) | 0.45 | 39.2     |
| MFS-kPCA       | 4         | 15 (12/16, 3/32) | 0.58 | 32.2     |
| POWER-PCA      | 2         | 16 (16/16, 0/32) | 0.50 | 31.3     |
| POWER-PCA      | 3         | 16 (16/16, 0/32) | 0.50 | 31.5     |
| POWER-PCA      | 4         | 16 (16/16, 0/32) | 0.50 | 32.2     |
| POWER-kPCA     | 2         | 13 (12/16, 1/32) | 0.61 | 29.2     |
| POWER-kPCA     | 3         | 14 (13/16, 1/32) | 0.58 | 30.2     |
| POWER-kPCA     | 4         | 14 (13/16, 1/32) | 0.58 | 28.9     |

4 Conclusion and future directions

We have studied 12 subjects divided in two groups affected by, respectively, high- and low-amplitude Parkinsonian rest tremors. Each subject has been tested in four settings, given by combining the use of deep brain stimulation and L-DOPA medication for relieving tremors and other symptoms. As a result, our initial dataset was formed by 48 signals related to the rest tremors measured via a velocity laser pointing at the index finger of the participants. We developed two main feature-based representations of such signals, which have been obtained by considering (i) the long-term correlations and multifractal properties and (ii) the power spectra. Initially, we have used such feature-based representations for the purpose of characterizing the subjects under different test settings. We have shown that the effect of medication is clearly recognizable in the represented signals. In agreement with previous studies, we have found that deep brain stimulation does not discriminate the two groups, regardless of the adopted representation. On the other hand, our results suggested that the effects of medication produce statistically significant differences in both groups having high and low-amplitude tremor. We successively tested several different instances of the two feature-based representations of the signals in the setting of supervised classification and (nonlinear) feature transformation. Three different classification problems have been considered, involving the recognition of (i) the presence of medication, (ii) the use of deep brain stimulation, and (iii) the membership to the high- and low-amplitude tremor groups. Classification results demonstrated that the use of medication can be discriminated with higher accuracy. Interestingly, the best results were obtained with a parsimonious, two-dimensional representation encoding the long-term correlations and multifractal character of the original signals. We believe that such results could be useful to neuroscientists, suggesting the potential of using LTC and multifractal signatures in the analysis Parkinsonian rest tremors.
Both long-term correlations and multifractal properties have been derived by using the “classical” multifractal detrended fluctuation analysis procedure. Future research works might exploit the direct estimation of the multifractal spectrum and related time-dependent Hurst exponent for the same aim of signal characterization and discrimination. Direct estimation of the multifractal spectrum might provide a compelling alternative for this purpose, especially when processing time series spanning a limited time frame.

A Multifractal detrended fluctuation analysis

Full details about the MF-DFA procedure are provided in Ref. [19]. Given a time series \( x_k \) of length \( N \) with compact support, the following steps are performed: The profile of the series, \( Y(i) \), is computed as:

\[
Y(i) \equiv \sum_{k=1}^{i} [x_k - \langle x \rangle], \quad i = 1, \ldots, N. \tag{1}
\]

The profile \( Y(i) \) is then divided in \( N_s \equiv \text{int}(N/s) \) non-overlapping segments of equal length \( s \). Since \( N \) might not be a multiple of \( s \), the operation is repeated by starting from the opposite end, obtaining thus a total of \( 2N_s \) segments.

Successively, a local detrending operation is executed by calculating a polynomial fit on each of the \( 2N \) segments. Then the local variance is determined as

\[
F^2(\nu, s) \equiv \frac{1}{s} \sum_{i=1}^{s} \left\{ Y[(\nu - 1)s + i] - y_\nu(i) \right\}^2, \tag{2}
\]

for each segment \( \nu = 1, \ldots, N_s \) and

\[
F^2(\nu, s) \equiv \sum_{i=1}^{s} \left\{ Y[N - (\nu - N_s)s + i] - y_\nu(i) \right\}^2 \tag{3}
\]

for \( \nu = N_s + 1, \ldots, 2N_s \), where \( y_\nu(i) \) is the fitted polynomial in segment \( \nu \). The order \( m \) of the fitting polynomial affects the capability of removing trends in the series; it has to be tuned according to the expected trending order of the time series. The \( q \)-th-order average of the variance over all segments is evaluated as

\[
F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [F^2(\nu, s)]^{q/2} \right\}^{1/q}, \tag{4}
\]

with \( q \in \mathbb{R} \). The \( q \)-dependence of the fluctuations function, \( F_q(s) \), allows to highlight the contributions of both high and low fluctuation magnitudes. Notably, for \( q > 0 \) only the larger fluctuations have higher impact in Eq. \( 4 \); conversely, for \( q < 0 \) the impact of the smaller fluctuations is enhanced. The case \( q = 0 \) cannot be computed with the averaging form in Eq. \( 4 \) and so a logarithmic form has to be used. The last steps are repeated for different scale sizes, \( s \).

The scaling behavior of the fluctuations can be determined by analyzing the slope of the doubly-logarithmic plot of \( F_q(s) \) versus \( s \), computed for each value of \( q \). If the series \( x_i \) is long-term correlated, then \( F_q(s) \) is approximated – for large values of \( s \) – by the power-law form:

\[
F_q(s) \sim s^{H(q)}. \tag{5}
\]

The \( H(q) \) exponent is the generalization of the Hurst exponent, \( H \), which is obtained for \( q = 2 \). When \( H > 1/2 \) the time series possesses long-term, positive correlations; for \( H < 1/2 \) the series is anti-correlated; \( H = 1/2 \) indicates that the series is compatible with uncorrelated noise – e.g., white Gaussian noise. An equivalent scaling over all fluctuation magnitudes, indicates that \( H(q) \) is independent from \( q \) and thus the series is mono fractal. On the contrary, when the small fluctuations scale differently from the large ones to a relevant degree, then the series can be considered as multifractal.

Starting from Eq. \( 4 \) and using Eq. \( 5 \) it is possible to obtain

\[
Z_q(s) = \sum_{\nu=1}^{N/s} [F(\nu, s)]^q \sim s^{qH(q) - 1}, \tag{6}
\]
where
\[ \tau(q) = qH(q) - 1 \] (7)
is the q-order mass exponent (also called Rényi scaling exponent) of the generalized partition function, \( Z_q(s) \). The multifractal spectrum, denoted as \( f(\cdot) \), provides a compact description of the multifractal character of the time series. Such a function can be obtained via the Legendre transform of \( \tau(q) \),
\[ f(\alpha) = q\alpha - \tau(q), \] (8)
where \( \alpha \) is equal to the derivative \( \tau'(q) \) – it corresponds to the Hölder exponent, also called singularity exponent. Using Eq. 7 it is possible to put in direct relation the generalized Hurst exponent, \( H(q) \), with \( \alpha \) and \( f(\alpha) \),
\[ \alpha = H(q) + qH'(q) \] and \[ f(\alpha) = q[\alpha - H(q)] + 1. \] (9)
The multifractal spectrum ([9] encodes important information regarding the degree of multifractality and the specific sensitivity of the time series to fluctuations of high/low magnitudes. Let \( q_- \) and \( q_+ \) be, respectively, the lower and upper values chosen for the \( q \) range. The width of the support of \( f(\cdot) \) is defined as
\[ \Delta \alpha = \alpha(q_-) - \alpha(q_+) \] (10)
and it offers an important quantitative indicator of the multifractal signature present in the series.

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