APPLICATION OF THE ROBUST APPROACH TO INCREASE THE ACCURACY OF DETERMINING THE COORDINATES OF THE ELEMENTS OF WIRELESS SENSOR NETWORKS
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Background. Modern methods for determining the coordinates of elements of wireless sensor networks allow solving the problems of determining the mutual distances between the elements of a wireless network under the assumption that the errors in measuring the mutual distances between network elements are distributed according to the normal law. With increasing requirements for the accuracy of determining coordinates, these methods do not allow solving the problem.

Objective. The purpose of the paper is to improve the accuracy of determining the coordinates of elements of wireless sensor networks by using robust estimation methods.

Methods. Determination of coordinates of elements of wireless sensor networks is implemented on the basis of two robust methods.

The first is the use of a median estimate based on multiple measurements of the mutual distances between elements to determine their coordinates.

The second is based on multiple measurements of the mutual distances between elements, determining their coordinates based on the Huber influence function and comparing two robust methods.

Results. The use of a robust method based on the Huber influence function makes it possible to increase the accuracy of determining the coordinates of elements of wireless sensor networks by 5-10% compared to classical estimation methods.

Conclusions. The proposed robust approach to determining the coordinates of elements of wireless sensor networks can be implemented in modern ground-based sensor networks for various purposes.

Keywords: wireless sensor network; determination of coordinates of elements of wireless sensor networks; robust estimation methods.

Introduction

A “wireless sensor network” (WSN) is a distributed, self-organizing network of miniature electronic devices resistant to failures of individual elements. In this case, the exchange of information between the elements (nodes) of the network occurs wirelessly (Fig. 1). All nodes of the sensor network are autonomous. BSS belong to networks of the WPAN class (Wireless Personal Access Network - wireless personal area networks).

Fig. 1

In recent years, WSN are increasingly used [1,2], and with their improvement, much attention is still paid to the issues of collecting information, energy efficiency, and routing, as well as improving the accuracy of determining the coordinates of the nodes of the BSS. The latter task is still relevant today, especially when specifying the location of the mobile sensor nodes after their next movement.

A brief overview of methods for determining the location of the WSN nodes is given in [3], from which it follows that the known classical methods based on the assumption of the normality of the distribution laws of measurement errors do not solve at this stage the problem of increasing the accuracy of determining the coordinates.

Formulation of the problem

There is a WSN consisting of \( n \) fixed elements after moving. The coordinates of the BSS elements are determined by measuring the mutual distances. It is known that measurement errors are distributed according to a law that is close to normal. It is necessary to improve the accuracy of determining the coordinates by taking into account the difference in the distribution of measurement errors from the normal one.

If we consider the law of distribution of errors in coordinate measurements to be normal and the accuracy of determining the coordinates suits us, then the problem of increasing the accuracy of measuring coordinates is solved due to the redundancy of measurements by averaging the measured values of the coordinates of each sensor.

In this case, the vector of estimating the coordinates of the \( k \) sensor based on the results of \( n \) measurements can be written

\[
\mathbf{\bar{x}}_k = \frac{\sum_{i=1}^{n} X_i}{n},
\]

(1)

here \( X_i \) is the vector of the \( k \)-sensor coordinate estimate, obtained, for example, as a result of measuring the mutual distances to two neighbouring sensors at the \( i \)-th measurement step.

Then the root-mean-square value of the error (RMS) of the estimate of the sensor coordinate \( k \) with equal measurement errors will have the form

\[
\sigma_k = \sqrt{\frac{\sum_{i=1}^{n} \sigma_i^2}{n}},
\]

(2)
The larger the number of measurements, the higher the estimation accuracy. Note that the arithmetic mean of the \( X_k \) coordinate is an unbiased estimate for any symmetric distribution law, besides being consistent, efficient and sufficient (a characteristic of the completeness of using all information contained in the sample).

However, estimate (1.2) with a normal distribution of measurement errors is weakly protected from the influence of anomalous measurements. It is weakened only by a factor of \( t \), where \( t \) is the number of dimensions, while its possible sample size \( n \) is unlimited. The arithmetic mean \( \bar{X}_t \) (over a limited number of measurements) can be determined

\[
\bar{X}_t = \frac{1}{n-2t} \sum_{i=t+1}^{n} X_i, \quad (3)
\]

When solving the problem of estimating the parameters of the location of fixed elements, the arithmetic mean of a 90\% sample is often used (denoted by the symbol \( \bar{X}_{0.9} \)). We represent this sample in the form

\[
\bar{X}_r = \frac{1}{n-2r} \sum_{i=r+1}^{n-r} X_i, \quad (4)
\]

where \( r \) - is the number of measurements not taken into account.

Then the arithmetic mean \( \bar{X}_{0.9} \), has the form

\[
\bar{X}_{0.9} = \frac{1}{n-2r} \sum_{i=r+1}^{n-r} X_{m_i} m_{i}, \quad (5)
\]

where \( m_i \) is the frequency of the \( i \) value hitting the \( m \)-th interval (at interval representation of the variation series).

Estimate (5) is less sensitive to results with gross errors than the sample arithmetic mean \( X_t \), since during processing 90\% of the sample size is discarded from the ends of the variation series \( X_1 \leq X_2 \leq X_3 \leq \ldots X_n \) for 5\% of the most distant results, in which may contain gross errors. Therefore, relations (4.5) can be applied only after preliminary analysis of the sample and determination of anomalous measurements in the analysed sample by accumulating measurement results and rejecting measurements whose errors exceed the permissible value.

The disadvantages of this approach are the inability to work with a small sample and obtain estimates of the location of the FSS element in real time; moreover, when the measurements are rejected according to (1.2), the estimation accuracy also decreases.

Suppose the coordinates of two sensors are known, it is necessary to determine the rest. To simplify, the problem of finding the coordinates of the third and other sensors will be carried out using the rangefinder method [3], according to which, to determine the coordinates of the next BSN sensor, the values of the mutual distances to two neighbouring sensors are required. The coordinates of the next sensor are determined using the Newton-Raphson method [4].

\[
X_{k+1} = X_k + W_{r_k}^{-1} f_k, \quad (6)
\]

where \( f_k \) is the vector of measured ranges \( r_{1k}, r_{2k} \) to the \( k \)-th sensor;

\( W_{r_k}^{-1} \) - matrix of partial derivatives \( r_{1k}, r_{2k} \) in coordinates \( x, y \);

\( X_{k+1} \) - desired coordinates of the current sensor.

The sensor coordinates are determined with the required accuracy for 3-5 iterations. It is assumed that the errors in measuring the mutual distances are subject to the normal distribution law with zero mean and known variance.

Taking into account the complexity of calculating the coordinates of the WSN sensor (6), a complex nonlinear transformation is required to determine the coordinate vector, which leads to a deviation of the distribution law of errors in determining the coordinates of the next sensor and all subsequent ones from the normal one. In addition, the presence of abnormal measurements is not excluded when determining the mutual distances between sensors.

If under these conditions the coordinates of the sensor are determined according to (1), then the estimate of the coordinates will not be optimal, consistent and effective. In the case when the accuracy of determining the coordinates of the sensors suits, then for calculations you can use (1). Let us further consider the possibilities of using robust methods [4,5] for estimating the coordinates of sensors. If the law of distribution of errors in measurements of mutual distances is not known, one can use Huber's minimax approach [4].

Its main idea is that the law with the worst distribution, for example, with an asymmetric distribution, is taken as the distribution law of measurement errors. As an estimate of the coordinates, one is taken that will have the minimum error of the maximum possible. Note that if the distribution law is symmetric, Huber's robust estimate becomes unstable.

Then the most robust (stable to the law of distribution of errors) is the median estimate, and when solving the problem of increasing the accuracy of determining the coordinates of the location of the WSN, the use of influence functions [4, 5], which show what influence the anomalous measurement has on the estimate of the coordinates, is taken. The main idea is that the law with the worst distribution, for example, with an asymmetric distribution, is taken as the distribution law of measurement errors. As an estimate of the coordinates, one is taken that will have the minimum error of the maximum possible. Note that if the distribution law is symmetric, Huber's robust estimate becomes unstable.

For example, Huber's influence function is written as

\[
\Psi(x) = \begin{cases} -\alpha, & x < -\alpha \\ x, & |x| \leq \alpha \\ \alpha, & x > \alpha \\ \end{cases}
\]

where \( \alpha \) is a constant, (for \( \alpha = 1.345 \), 95\% of the M-estimate efficiency is achieved, with a known variance). The essence of the approach based on the use of the influence function lies in the assumption that in a real sample the measurement errors are distributed according
to two distribution laws. The first basic \( N(x, D_1) \) is a normal distribution law with zero mean and variance of measurement errors \( D_1 \), the second - \( R(x, D_2) \), a normal distribution law with zero mean and variance of measurement errors \( D_2 \).

The resulting distribution law with "pollution" has the form

\[
P(x) = (1 - \varepsilon)N(x, D_1) + \varepsilon R(x, D_2),
\]

(7)

where \( \varepsilon \) is the pollution parameter, which in practice is taken equal to 0.05-0.1.

As applied to the distribution law (7) as \( \varepsilon \to 0 \), the arithmetic mean (1) is taken as an estimate; in the case \( \varepsilon \to 1 \), the median of the sample.

**Research results**

Let's consider the results of simulation. The initial data for modelling are the reference values of the coordinates of two sensors: \((X_{e1}, Y_{e1})\), \((X_{e2}, Y_{e2})\) on the basis of which the estimate of the coordinates of the next sensor is determined (Fig. 1). The estimation was carried out using relation (6) at \( n = 100 \).

Fig. 2 shows histograms (blue) and a graph of the normal distribution law of coordinate estimates (red) with the reference values \( X_{e3} = 199.98 \) (Fig. 2a), \( Y_{e3} = 691.61 \) (Fig. 2b) distributed according to the normal law with zero mean and RMS \( S_{D1} = 4m \).

Fig. 2 shows that the maximum of the histograms is clearly manifested and coincides with the average value. Coordinate estimates are determined as the arithmetic mean over \( n \) measurements in accordance with (1). In this case, the median estimates \( X_{med}, Y_{med} \) coincide with the mean.

Further, for the same reference values of estimates \( X_{e3} = 199.98, Y_{e3} = 691.61 \) in Fig. 3 shows the simulation results using a robust estimation method at \( \varepsilon = 5\% \) with a slight difference in the variance of the basic distribution law \( N(x, D_1) \) c CKO \( S_{D1}=3m \) from the “polluted” \( R(x, D_2) \) with \( S_{D2} = 4m \). In the first case, the median is taken as an estimate, and in the second, the estimate using the Huber influence function.
Fig. 3

Analysis of the histograms in Fig. 2 and 3 shows that the histogram using the distribution law (7) (Fig. 3.a, c) does not have a clearly defined maximum, which, moreover, does not coincide with the average value. Fig. 3.c, d shows histograms using the Huber influence function, which has two manifested maxima and is due to the high frequency of anomalous measurements and the limitation of the sample from n measurements. Further, the average value for this sample is taken as an estimate. The estimates obtained using the Huber influence function $X_{hub}$, $Y_{hub}$ are closest to the reference values of the X, Y sensor coordinates estimates compared to the median estimate.

Fig. 4-6 show the histograms of the distribution of estimation errors with a given parameter $\varepsilon$ ($\varepsilon = 10\%$, (Fig. 4), $\varepsilon = 20\%$, (Fig. 5) and $\varepsilon = 40\%$ (Fig. 6)) and after rejection of measurements using Huber’s influence functions.
a) Coordinate X

\[ \epsilon = 0.2 \]

\[ \text{Xmed} = 199.86 \]

\[ \text{Xet} = 199.98 \]

\[ \text{Xhub} = 200.01 \]

b) Function Huber coordinate X

\[ \text{Yet} = 691.61 \]

\[ \text{Yhub} = 691.67 \]

c) Coordinate Y

\[ \epsilon = 0.2 \]

\[ \text{Ymed} = 691.25 \]

\[ \text{Yet} = 691.61 \]

\[ \text{Yhub} = 691.67 \]

d) Coordinate X

\[ \epsilon = 0.4 \]

\[ \text{Xmed} = 200.37 \]

\[ \text{Xet} = 199.98 \]

\[ \text{Xhub} = 200.32 \]

d) Function Huber coordinate Y

\[ \epsilon = 0.2 \]

\[ \text{Ymed} = 691.25 \]

\[ \text{Yet} = 691.61 \]

\[ \text{Yhub} = 691.67 \]
E. Yakornov, O. Tsukanov. Application of the Robust Approach to Increase the Accuracy of Determining the Coordinates of the Elements of Wireless Sensor Networks

The histogram of the distribution of coordinate errors at $\varepsilon = 10\%$, shown in Fig. 4 is also asymmetrical and weighted tails are present. Fig. 5 shows the distribution of errors at $\varepsilon = 20\%$ and, in comparison with the previous diagram, the maximum is “blurred” and shifted from the centre of the probability density function. The histogram of the distribution of coordinate errors Fig. 6 at $\varepsilon = 40\%$ also does not have a clearly defined maximum, and the “weighted tails” are even larger in comparison with Fig. 4.5. And under these conditions, the estimate based on the influence function also provides higher accuracy than the median estimate. This is explained by the rejection of anomalous measurements and, accordingly, by a decrease in their influence on the estimation accuracy.

Conclusions
1. The results of simulation have confirmed that when the error distribution law differs from the normal one, the estimate based on the arithmetic mean is unstable and cannot be used to solve the problem of high-precision determination of the coordinates of the BSN sensors.
2. To obtain estimates if the error distribution law differs from the normal one, robust methods should be used.
3. Comparison of robust estimation methods based on the median estimate and the estimate based on the influence function shows that the latter has higher estimation accuracy.
4. The presented results of estimation modelling assume estimation on a sample basis. Therefore, to improve the accuracy of determining the coordinates when moving the sensor sensors relative to the base ones, it is necessary to store information about the location of the latter.
5. The use of methods of robust estimation methods makes it possible to increase the accuracy of determining the coordinates of stationary sensors of the BSS by an average of 5-10%.

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Застосування робастного підходу для підвищення точності визначення координат елементів бездротових сенсорних мереж

Проблематика. Сучасні методи визначення координат елементів бездротових сенсорних мереж дозволяють вирішувати завдання визначення взаємних відстаней між елементами бездротової мережі в припущеннях, що помилки вимірювань взаємних відстаней між елементами мережі розподілені по нормальному закону. При підвищенні вимог до точності визначення координат ці методи не дозволяють вирішити поставлену задачу.

Мета досліджень. Підвищення точності визначення координат елементів бездротових сенсорних мереж шляхом використання робастних методів оцінювання.

Методика реалізації. Визначення координат елементів бездротових сенсорних мереж реалізується на основі двох робастних методів.

Перший - використання медианної оцінки на основі багаторазових вимірювань взаємних відстаней між елементами для визначення їх координат.

Другий - на основі багаторазових вимірювань взаємних відстаней між елементами визначення їх координат на основі функції впливу Хьюбера і порівняння двох робастних методів.

Результати досліджень. Застосування робастного методу на основі функції впливу Хьюбера, дозволяє підвищити точність визначення координат елементів бездротових сенсорних мереж в порівнянні з класичними методами оцінювання на 5-10%.

Висновки. Запропонований робастний підхід визначення координат елементів бездротових сенсорних мереж може бути реалізований в сучасних наземних сенсорних мережах різного призначення.

Ключові слова: бездротова сенсорна сеть; визначення координат елементів бездротових сенсорних мереж; робастні методи оцінювання.

Якорнов Е.А., Цуканов О.Ф.
Применение робастного подхода для повышения точности определения координат элементов беспроводных сенсорных сетей

Проблематика. Современные методы определения координат элементов беспроводных сенсорных сетей позволяют решать задачи определения взаимных расстояний между элементами беспроводной сети в предположении, что ошибки измерений взаимных расстояний между элементами сети распределены по нормальному закону. При повышении требований к точности определения координат эти методы не позволяют решить поставленную задачу.

Цель исследований. Повышение точности определения координат элементов беспроводных сенсорных сетей путем использования робастных методов оценивания.

Методика реализации. Определение координат элементов беспроводных сенсорных сетей реализуется на основе двух робастных методов.

Первый - использование медианной оценки на основе многократных измерений взаимных расстояний между элементами для определения их координат.

Второй - на основе многократных измерений взаимных расстояний между элементами определение их координат на основе функции влияния Хьюбера и сравнение двух робастных методов.

Результаты исследований. Применение робастного метода на основе функции влияния Хьюбера, позволяет повысить точность определения координат элементов беспроводных сенсорных сетей по сравнению с классическими методами оценивания на 5-10%.

Выводы. Предложенный робастный подход определения координат элементов беспроводных сенсорных сетей может быть реализован в современных наземных сенсорных сетях различного назначения.

Ключевые слова: беспроводная сенсорная сеть; определение координат элементов беспроводных сенсорных сетей; робастные методы оценивания.