Universal Procedure for Correction of Plasticity Effect in Hole-Drilling Uniform Residual Stress Measurement

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Abstract
Background The hole-drilling method is a well-known and widely used technique for the determination of residual stresses, but is limited to materials with linear elastic behaviour. This can be a problem when high residual stresses are measured, since local yielding can occur due to stress concentration around the drilled hole.

Objective If the residual stress exceeds about 80% of the material yield stress, the error caused by the plasticity effect becomes significant. In order to correctly evaluate high uniform residual stresses, a universal procedure for a correction of the plasticity effect is introduced.

Methods The procedure uses a neural network and is capable of correcting any combination of uniform residual stresses with magnitudes up to the material yield stress. It also covers a wide range of material parameters, hole diameters, and strain gauge rosettes and it is independent of the orientation of the strain gauge rosette.

Results The correction procedure was tested by more than a million randomly generated stress states that covered the entire range of input parameters and performs remarkably well, since the error of the corrected residual stresses is negligible even for the states with residual stress magnitudes equal to the material yield stress.

Conclusions The proposed correction extends the application range of the hole-drilling method to high uniform residual stresses and therefore can be very useful for practical measurements.

Keywords Hole-drilling method · Residual stress · Plasticity effect · Correction procedure · Neural network · Finite element simulations

Introduction
The hole-drilling method is widely used technique for measuring near-surface residual stresses. The method is based on the measurement of relaxed deformations caused by drilling a small blind hole into the component surface in a series of steps (Fig. 1) [1]. Afterwards, residual stresses are evaluated from measured strains and the corresponding calibration coefficients [2].

Since the drilled hole represents a stress concentrator, local yielding can occur around the drilled hole when high residual stresses are present in a component material. Due to the fact that the hole-drilling method assumes linear elastic behaviour of the measured material, the evaluated residual stresses are overestimated for states with plastic deformation. Plastic deformations begin to form when the residual stresses are about 25–50% of the material yield stress, but the error of the evaluated stresses is relatively small. However, for residual stresses greater than 80% of the material yield stress, the error caused by the plasticity effect can be significant and can devalue the measurement results [4–6].

Several authors have tried to solve this problem and suggested correction procedures. However, these procedures require specific residual stresses to be present in the material

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1 Based on Kirsh equations [9], the plasticity onset happens for shear residual stress at 25% of the material yield stress, for uniaxial residual stress at 33% of the material yield stress, and for equibiaxial residual stress at 50% of the material yield stress. Since the equations are valid only for an infinite plate with a through hole, the plasticity onset limits for a thick body with a blind hole are slightly different, but they cannot be determined analytically.
(e.g. equibiaxial stresses) [7] or the knowledge of the principal stresses’ directions, since the strain gauge rosette must be aligned with the principal stresses [8]. These requirements are sometimes difficult to achieve, therefore, a new procedure for the correction of the plasticity effect in the evaluation of uniform residual stresses is introduced.

Evaluation of Residual Stresses

The strain gauge rosette used to measure relaxed deformations typically consists of three strain gauges oriented at angles: 0°, 45° (or 225°), and 90° (Fig. 2). Based on the convention, if the strain gauges are labelled as in Fig. 2, the rosette is called a counter-clockwise (CCW). If strain gauges \( a \) and \( c \) are interchanged, a clockwise (CW) rosette is used. For both types of rosettes, the evaluation process remains the same, but for the CW rosettes the angle \( \beta \) defining the direction of the principal stress \( \sigma_1 \) is reversed and is measured clockwise from the gauge labelled as \( a \).

In order to evaluate the residual stresses from the measured strains, calibration coefficients \( \bar{a} \) and \( \bar{b} \) depending on a hole diameter, a hole depth, and the strain gauge rosette geometry are required. These calibration coefficients for 6 commonly used strain gauge rosettes and for the range of hole depths and diameters considered in the correction procedure were calculated by finite element model described in the following section. Calibration coefficients \( \bar{a} \) and \( \bar{b} \) were obtained by applying a uniform equibiaxial load (equal to 1 MPa) and uniform shear load (equal to 1 MPa and -1 MPa) to the model, respectively. Thereafter, a polynomial function was fitted to the calculated data using a least squares method (root mean square error of fitting was less than 0.0004). Terms of polynomial function with only a hole diameter variable (and also a constant term) were omitted since the calibration coefficients are zero for zero drilled depth. Also, some higher order terms were omitted for the weak dependency of polynomial function on them. The variables of proposed polynomial function were used in dimensionless forms expressed as the ratio of the hole diameter and the hole depth to a diameter of gauge circle.

The values of calibration coefficients \( \bar{a} \) and \( \bar{b} \) for the evaluation of uniform residual stresses (stresses which do not vary significantly through the hole depth) can be calculated using the following equation:

\[
\bar{a}_j, \bar{b}_j = C_1 \left( \frac{h_j}{D} \right) + C_2 \left( \frac{h_j}{D} \right)^2 + C_3 \left( \frac{h_j}{D} \right) \left( \frac{D_0}{D} \right) + C_4 \left( \frac{h_j}{D} \right)^3
+ C_5 \left( \frac{h_j}{D} \right)^2 \left( \frac{D_0}{D} \right) + C_6 \left( \frac{h_j}{D} \right) \left( \frac{D_0}{D} \right)^2 + C_7 \left( \frac{h_j}{D} \right)^4
+ C_8 \left( \frac{h_j}{D} \right)^3 \left( \frac{D_0}{D} \right)
\]

(1)

where \( \bar{a}_j \) and \( \bar{b}_j \) are calibration coefficients for the \( j \)-th depth increment, \( D_0 \) is the hole diameter, \( D \) is the diameter of gauge circle, \( h_j \) is a hole depth at the \( j \)-th depth increment, and \( C_1, \ldots, C_8 \) are polynomial coefficients listed in Table 1. The equation (1) is applicable to hole diameters in the range \( 0.35 \leq D_0/D \leq 0.43 \) and hole depths in the range \( 0 \leq h_j/D \leq 0.2 \).

Following the procedure for the calculation of uniform residual stresses described in the ASTM E837-20 standard [10], the combinations of measured strains \( p_j, q_j, \) and \( t_j \) for the \( j \)-th depth increment are calculated as:

\[
p_j = \frac{\varepsilon_c + \varepsilon_a}{2}
\]

(2)
where $\varepsilon_a$, $\varepsilon_b$, and $\varepsilon_c$ are measured strains from the strain gauges labelled as $a$, $b$, and $c$. The combinations of stresses $P$, $Q$, and $T$ are calculated from the combinations of strains and corresponding calibration constants as follows:

$$P = -\frac{E}{1 + \nu} \sum \frac{(\bar{a}_j \cdot P_j)}{\bar{a}_j^2}$$  \hspace{1cm} (5)

$$Q = -E \sum \frac{(\bar{b}_j \cdot q_j)}{\bar{b}_j^2}$$  \hspace{1cm} (6)

$$T = -E \sum \frac{(\bar{b}_j \cdot t_j)}{\bar{b}_j^2}$$  \hspace{1cm} (7)

where $E$ and $\nu$ are Young’s modulus and Poisson’s ratio, respectively. The following equations are used to calculate in-plane stresses aligned with strain gauges:

$$\sigma_a = P - Q$$  \hspace{1cm} (8)

$$\sigma_c = P + Q$$  \hspace{1cm} (9)

$$\tau_{ac} = T$$  \hspace{1cm} (10)

where $\sigma_a$ and $\sigma_c$ are normal residual stresses in the directions of the strain gauges $a$ and $c$, respectively, and $\tau_{ac}$ is a shear residual stress. The principal stresses $\sigma_I$ and $\sigma_{II}$ are calculated using equation:

$$\sigma_I, \sigma_{II} = P \pm \sqrt{Q^2 + T^2}$$  \hspace{1cm} (11)

and the angle $\beta$ defining the direction of the principal stress $\sigma_I$ is calculated as:

$$\beta = \frac{1}{2} \arctan \left( -\frac{T}{Q} \right)$$  \hspace{1cm} (12)

For the calculation of the angle $\beta$ the two-argument arctan function, where the signs of the numerator and denominator are considered, should be used. Otherwise, the angle may need to be adjusted by adding or subtracting 90° to be placed in the range defined in Table 2.

### Simulation of Hole-Drilling Process

A finite element model of a hole-drilling process was created in ANSYS software. As the ASTM E837-20 standard recommends, 10 uniform depth increments were considered and the drilling process was simulated by sequential removal of elements in the area representing the hole. Due to the symmetry, only one quarter of the entire volume was simulated and symmetry boundary conditions were applied on the faces located in the section planes (Fig. 3). To ensure that the relieved strains are not affected by the boundaries, the length and width of the simulated body were equal to $60D_0$ and the thickness was equal to $12D_0$, while the diameter of the drilled hole $D_0$ depended on the simulated variant. The material behaviour was defined by the bilinear stress–strain curve and the isotropic strain hardening plasticity model, so

| Strain gauge rosette | Supplier | $D$(mm) | $\bar{a}$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7$ | $C_8$ |
|----------------------|----------|---------|----------|--------|--------|--------|--------|--------|--------|--------|--------|
| RY61-1.5/120S        | HBM      | 5.1     | -0.2881  | 5.0090 | -0.7101| -37.620 | 14.692 | 8.0349 | 148.71  | -101.51 |
| RY61-3.2/120S        | HBM      | 10.14   | -0.3293  | 5.4014 | -0.4937| -37.921 | 12.130 | 7.4817 | 142.12  | -90.761 |
| Type A               | Vishay   | 2.57    | -0.2431  | 4.4176 | -0.7296| -34.196 | 14.190 | 7.5260 | 137.61  | -95.723 |
|                      |          | 5.13    | -0.6729  | 4.8399 | 1.0652 | -40.582 | 31.098 | 9.0136 | 181.91  | -164.58 |
| Type B               | Vishay   | 5.13    | -0.3049  | 5.2190 | -0.6176| -38.350 | 13.721 | 7.9005 | 148.98  | -98.598 |
|                      |          |         | -0.8805  | 6.0345 | 1.8605 | -47.878 | 33.229 | 9.0472 | 205.83  | -179.17 |

Table 2 Placement of the angle $\beta$ [10]

| $Q > 0$ | $Q = 0$ | $Q < 0$ |
|---------|---------|---------|
| $T < 0$ | $45^\circ < \beta < 90^\circ$ | $45^\circ$ | $0^\circ < \beta < 45^\circ$ |
| $T = 0$ | $90^\circ$ | undefined | $0^\circ$ |
| $T > 0$ | $-90^\circ < \beta < -45^\circ$ | $-45^\circ$ | $-45^\circ < \beta < 0^\circ$ |
four material parameters were required for the simulations: Young’s modulus, Poisson’s ratio, the yield stress, and the tangent modulus. Because of plastic deformations, the load cannot be applied to the drilled hole. Therefore, the required stress state defined by the stresses $\sigma_a$ and $\sigma_c$, was generated by applying a pressure on the outside boundary faces of the model in $x$ and $y$ directions before removing the first depth increment. A fine mapped mesh with element size 0.04 mm was created around the hole and the size of elements gradually increased from the hole towards the boundaries (Fig. 4). The total number of nodes and quadratic elements was equal to 77,000 and 24,000, respectively. In order to efficiently obtain the relieved strains for different strain gauge rosettes and various orientations of these rosettes, the entire strain field around the drilled hole was saved after each drilled increment and a script for postprocessing of these strain fields was prepared in MATLAB software. The orientation of the strain gauge rosette was defined by the angle $\beta^s$, which is the angle between the strain gauge $a$ and the maximum principal stress. Prior to the creation of the final finite element model, initial studies of the outer boundary size, the element size, and the size of the area with fine mesh around the hole were performed to ensure that these parameters did not affect relieved strains in the evaluated area.

### Errors Caused by Plasticity Effect

In order to study the influence of plastic deformations on the residual stress evaluation, several finite element simulations were carried out. Thereafter, residual stresses $\sigma_a$, $\sigma_c$, and $\tau_{ac}$ were calculated from the relieved strains obtained from the simulations by equations (1) – (10). The influence of various parameters (a residual stress magnitude, an orientation of strain gauge rosette, material parameters, a hole diameter, etc.) on the residual stress evaluation error was studied by the comparison of evaluated $(\sigma_a, \sigma_c, \tau_{ac})$ and simulated $(\sigma_a^s, \sigma_c^s, \tau_{ac}^s)$ stresses and is discussed in more detail in the following section. For the quantification of residual stress level, the ratio of the simulated von Mises equivalent stress $\sigma_{eq}^s$ to the material yield stress $\sigma_Y$ was used. The equivalent stress $\sigma_{eq}^s$ was calculated from the simulated stresses as follows:

$$
\sigma_{eq}^s = \sqrt{\left(\frac{\sigma_a^s}{\sigma_Y}\right)^2 - \frac{\sigma_a^s}{\sigma_Y} + \left(\frac{\sigma_c^s}{\sigma_Y}\right)^2 + 3\left(\tau_{ac}^s\right)^2}
$$

where $\sigma_a^s$ and $\sigma_c^s$ are simulated normal residual stresses in the directions of the strain gauges $a$ and $c$, respectively, and $\tau_{ac}^s$ is a simulated shear residual stress.

When the stress states with the ratio $\sigma_{eq}^s/\sigma_Y < 0.5$ were examined, the evaluated stresses corresponded to the residual stresses present in the simulated body. However, as residual stresses increased and plastic deformations formed around the drilled hole, the difference between the evaluated and simulated stresses was clearly noticeable. This difference is demonstrated in Fig. 5, which depicts the evaluated and simulated stresses for the stress state with the ratio $\sigma_{eq}^s/\sigma_Y$ equal to 0.996.

Figure 5 shows only one specific stress state for one orientation of the strain gauge rosette. If all stress states with the ratio $\sigma_{eq}^s/\sigma_Y$ equal to 0.996 and one orientation of strain gauge rosette are plotted in the graph, the points from simulated stresses create an ellipse (Fig. 6). Since strain gauges $a$ and $c$ were aligned with the principal stresses ($\beta^s = 0$), the ellipse is placed in plane with zero shear stress. The points from evaluated stresses create a general 3D curve whose deviation from the simulated values depends on the ratio $\sigma_a/\sigma_c$ (Fig. 6). After including various orientations of the strain gauge rosette ($-45^\circ \leq \beta^s \leq 45^\circ$), points from the simulated stresses create an ellipsoid and the points from evaluated stresses create a general surface (Fig. 7).

Analogous surfaces can be created for stress states with various $\sigma_{eq}^s/\sigma_Y$ ratios (Fig. 8). It can be seen that the surfaces
created by the simulated and evaluated stresses are very similar for the states with ratios \( \sigma_{eq}^s/\sigma_Y \) close to 0.5. However, the higher the ratio \( \sigma_{eq}^s/\sigma_Y \), the bigger the difference between simulated and evaluated stresses. An important fact that can be observed from Fig. 8 is that the difference between evaluated and simulated stresses depends not only on residual stress level \( \sigma_{eq}^s/\sigma_Y \), but also on orientation of strain gauge rosette (\( \beta^s \)) and the ratio \( \sigma_Y/\sigma_x \).

During the initial investigation, it was discovered that the surfaces with the ratio \( \sigma_{eq}^s/\sigma_Y \) close to 1 intersect the surfaces with lower ratios (Fig. 9). That leads to a non-unique representation of the results, since two (or more) different simulated stress states have the same values of the evaluated residual stresses (points lying on the intersection of the surfaces shown in Fig. 9). The highest value of the ratio \( \sigma_{eq}^s/\sigma_Y \), which provides unique results, was found to be 0.996. Therefore, the data set described in the following section contained only stress states with the ratio \( \sigma_{eq}^s/\sigma_Y \leq 0.996 \).

All previous figures show stress states only for one particular material, one hole diameter, and one strain gauge rosette. Since the drilled hole represents the stress concentrator and the plastic deformations depend on the size of the concentrator, the hole diameter affects the value of evaluated stresses. The hole-drilling machine is a precise device, but small variations in the diameter of the drilled holes may still occur. The difference among the same stress states simulated with various hole diameters can be seen in Fig. 10.

Another factor that affects plastic deformations around the drilled hole is the measured material itself. If the material is defined by a bilinear stress–strain curve, the plastic deformation increases with decreasing tangent modulus \( E_t \) or increasing Young’s modulus \( E \). It was found that the important parameter for the interpretation of the plasticity effect is a ratio of the Young’s modulus to the tangent modulus, not the values of these modules themselves. The surfaces created by evaluated stresses for the same stress states but simulated with various ratios of \( E_t/E \) can be seen in Fig. 11. It was also
Fig. 8 Surfaces created by simulated (left) and evaluated (right) stresses for states with the various ratios $\sigma_{eq}/\sigma_Y$ and $\sigma_n/\sigma_Y$ and the angle $\psi$ in the range from -45° to 45°.
found that after dividing the evaluated and simulated stresses by the yield stress \( \sigma_y \), these stresses become independent of the yield stress of the measured material, so it is not necessary to examine materials with different yield stresses and the same \( E_t/E \) ratio separately.

The last investigated factor influencing the stress evaluation was the geometry of the strain gauge rosette. Since there are several suppliers that produce strain gauge rosettes with various dimensions, the differences were also observed among the same stress states evaluated by various strain gauge rosettes (Fig. 12).

**Correction of Plasticity Effect**

The correction of the plasticity effect can be defined as the difference between the positions of the evaluated stresses and the simulated stresses in the graph shown in Fig. 5. Because the stress state is defined by three stresses, the correction vector has three components: \( (\sigma_a - \sigma'_a)/\sigma_y \), \( (\sigma_c - \sigma'_c)/\sigma_y \), and \( (\tau_{ac} - \tau'_{ac})/\sigma_y \). The values of these components for various stress states and the orientations of the strain gauge rosette are illustrated in Figs. 13, 14, and 15. As can be seen from these figures, each evaluated stress state, represented by the specific point in the graph, has its own value of the correction vector.
During a detailed examination of the correction vectors, it was discovered that the entire volume of points shown in Figs. 13, 14, and 15 can be obtained from the stress states where $\sigma_a \geq \sigma_c$ and at the same time $\sigma_a \geq -\sigma_c$ by applying specific symmetric conditions. These symmetric conditions were based on the facts that the isotropic homogeneous material has the same behaviour in the tension and compression and that the material response is independent of the loading direction. Consequently, only one quarter of the simulations was needed to cover all possible stress conditions.

Because the effect of plasticity depends on many different factors, a large data set containing all parameters mentioned in previous section was needed to create a universal correction procedure. In order to cover all possible stress conditions in the measured materials, 861 different stress states were simulated. These simulated states, illustrated in Haigh-Westergaard stress space in Fig. 16, were defined by the ratio $\sigma_{eq}/\sigma_Y$ with values from 0.5 (negligible plasticity) to 0.996 (high plasticity) and by the ratio $\sigma_y/\sigma_x$ with values from -1 (shear stress) to 1 (equibiaxial stress). Due to the symmetric conditions and zero third principal stress, Haigh-Westergaard stress space is reduced only to the region shown in Fig. 16.

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**Fig. 13** Evaluated stress states with the correction component $(\sigma_a - \sigma_c)/(\sigma Y)$

**Fig. 14** Evaluated stress states with the correction component $(\sigma_y - \sigma_c)/(\sigma Y)$

**Fig. 15** Evaluated stress states with the correction component $(\tau_{ac} - \tau_{ac}')/(\sigma Y)$

**Fig. 16** Simulated stress states
Each stress state was simulated for 5 different values of the material parameter $E_t/E$ and 5 different ratios $D_0/D$ giving a total of 21,525 simulations. To make the correction procedure independent of the material yield stress, all stresses were divided by $\sigma_Y$. The strain measurement was assumed by 6 different strain gauge rosettes, including 91 various orientations of each rosette. For further processing, a numerical value expressed by the parameter $SGR$ was assigned to each considered strain gauge rosette. The main dimensions of the strain gauge rosettes and values of the parameter $SGR$ are listed in Table 3. A total of 7,835,100 different states were prepared by postprocessing of strain fields from the simulations. The summary of all considered parameters with their values is given in Table 4. Thereafter, two sets of parameters were extracted from each simulated state:

- the input parameters defining the particular state by the variables: $\sigma_a/\sigma$, $\sigma_c/\sigma$, $\tau_{ac}/\sigma$, $E_t/E$, $D_0/D$, $SGR$.
- the output parameters defining the components of correction vector for the specific stress state: $(\sigma_a - \sigma_a^i)/\sigma_Y$, $(\sigma_c - \sigma_c^i)/\sigma_Y$, $(\tau_{ac} - \tau_{ac}^i)/\sigma_Y$.

| Strain gauge rosette | Supplier | Diameter of gauge circle $D$ (mm) | Gauge length (mm) | Gauge width (mm) | Nominal hole diameter (mm) | Nominal hole depth (mm) | $SGR$ (-) |
|----------------------|----------|----------------------------------|-----------------|----------------|--------------------------|------------------------|----------|
| RY61-1.5/120S        | HBM      | 5.1                              | 1.5             | 0.77           | 2                        | 1                      | 0.1      |
| RY61-3.2/120S        | HBM      | 10.14                            | 3.2             | 3.2            | 4                        | 2                      | 0.4      |
| Type A (1/32 in.)    | Vishay   | 2.57                             | 0.79            | 0.79           | 1                        | 0.5                    | 0.2      |
| Type A (1/16 in.)    | Vishay   | 5.13                             | 1.59            | 1.59           | 2                        | 1                      | 0.2      |
| Type A (1/8 in.)     | Vishay   | 10.26                            | 3.18            | 3.18           | 4                        | 2                      | 0.2      |
| Type B               | Vishay   | 5.13                             | 1.59            | 1.14           | 2                        | 1                      | 0.3      |

| Parameter | Value | Quantity |
|-----------|-------|----------|
| $\sigma_a/\sigma_c$ | (-) | 41 |
| $\sigma_{eq}/\sigma_Y$ | (-) | 21 |
| $\beta$ | (-) | 91 |
| $E_t/E$ | (-) | 5 |
| $D_0/D$ | (-) | 5 |
| Strain gauge rosette | RY61-1.5/120S, RY61-3.2/120S, Type A (1/32 in., 1/16 in., 1/8 in.), Type B |

### Neural Network in Correction Procedure

Initially, a polynomial fitting technique (the method used to obtain equation (1)) was tried to approximate the general relationship between the input and the output parameters. However, high-order polynomial functions were required for satisfactory approximation due to the complex relationship among the parameters. Considering 6 variables and 3 functions (one for each correction component), higher order polynomial functions may contain thousands of coefficients (e.g. 9009 coefficients are required for 8th degree polynomials with all terms). Therefore, several other methods were tested and finally a neural network fitting technique, which is more suitable for multivariable problems, was chosen for the final approximation. This method was found to be more efficient than polynomial approximation, since it generated a much smaller fitting error with a significantly smaller number of required coefficients.

The neural network is a computational tool based on the interconnection of nodes called neurons in a layered structure that mimics a human brain. It can be trained by an appropriate training algorithm and suitable sample data to generalize any nonlinear relationship [12]. Because the appropriate number of neurons and layers depends on the complexity of the solved problem, the various designs and the setting of feedforward neural networks were tested. Before training, the data set was randomly divided into two subsets: the training subset, which contained 85% of the states and was used to compute the gradient and update the weights and biases of the network, and the validation subset, which contained 15% of the states and was used to monitor data overfitting. The training process, which involves adjusting network weights and biases to optimize network performance, was performed in MATLAB software by the Levenberg–Marquardt backpropagation training algorithm with the mean squared error performance function [13]. Since each network training session starts with different initial weights and biases and different divisions of the data set into the training and validation subsets, different solutions

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2 A desktop computer with 3.2 GHz Intel Core processor and 16 GB of RAM was used for the calculations. One simulation took approximately 12 min, which gives a total computation time about 4300 h. Multiple computers were used simultaneously to reduce the time required for the calculation of all simulated states.

3 One training process took approximately 200 h.
The reason why the input vector \( \mathbf{X} \) contains the parameters \( X_1, X_2, \) and \( X_3 \) instead of \( \sigma_a, \sigma_c, \) and \( \tau_m \) is discussed in the following section. The tangent sigmoid transfer function, which is equivalent to a hyperbolic tangent, is used to calculate the neuron output \( y_k^{(i)} \) for neurons in hidden layers:

\[
TF^{(1)}, TF^{(2)} : y_k^{(i)} = \tanh \left( \frac{a_k^{(i)}}{3} \right) = \frac{e^{2a_k^{(i)}} - 1}{e^{2a_k^{(i)}} + 1}
\]

The final output layer uses the linear transfer function, \( TF^{(3)} : y_k^{(3)} = a_k^{(3)} \), so the final output of the network \( \mathbf{Y} = [Y_1, Y_2, Y_3]^T \) can be written as:

\[
\mathbf{Y} = \mathbf{y}^{(3)} = \mathbf{a}^{(3)}
\]

Since the application of trained neural network requires only basic mathematical operations (matrix multiplication and addition, and the use of trigonometric function \( \tanh \)), the correction vector \( \mathbf{Y} \) can be calculated from the input vector \( \mathbf{X} \) by equations (14) – (18) in any calculation software with the weights \( w_{k,m}^{(i)} \) and biases \( b_k^{(i)} \) listed in Appendix Tables 9, 10, 11, and 12.

**Application of Correction Procedure**

Since only the stress states where \( \sigma_a \geq \sigma_c \) and at the same time \( \sigma_a \geq -\sigma_c \) were used to create the plasticity correction, the application of specific symmetric conditions is required to cover all possible stress combinations. That is provided by adjusting the input vector \( \mathbf{X} = \left[ \frac{X_1}{\sigma_y}, \frac{X_2}{\sigma_y}, \frac{X_3}{\sigma_y}, \frac{E_t}{E}, \frac{D_0}{D}, SGR \right]^T \), where the parameters \( X_1, X_2, \) and \( X_3 \) are determined from Table 5 based on the comparison of stresses \( \sigma_a, \sigma_c, \) and \( -\sigma_a \). This table also defines how the components of correction vector \( (Y_1, Y_2, Y_3) \) are used to calculate the corrected residual stresses \( \sigma_a^{cor}, \sigma_c^{cor}, \) and \( \tau_m^{cor} \). The table is applicable to clockwise, as well as counterclockwise strain gauge rosettes.

When the correction vector \( \mathbf{Y} \) is calculated (by equations (14) – (18)), corrected residual stresses can be determined by
the equations listed in Table 5. Afterwards, the combinations of corrected stresses \( P^{\text{cor}}, Q^{\text{cor}}, \) and \( T^{\text{cor}} \) are calculated as:

\[
P^{\text{cor}} = \left( \frac{\sigma_{\text{cor}}^a + \sigma_{\text{cor}}^c}{2} \right)
\]

(19)

\[
Q^{\text{cor}} = \left( \frac{\sigma_{\text{cor}}^c - \sigma_{\text{cor}}^a}{2} \right)
\]

(20)

\[
T^{\text{cor}} = \tau_{ac}^{\text{cor}}
\]

(21)

and the corrected principal stresses \( \sigma_{I}^{\text{cor}}, \sigma_{II}^{\text{cor}} \) and the corrected angle \( \beta^{\text{cor}} \) are calculated using:

\[
\sigma_{I}^{\text{cor}}, \sigma_{II}^{\text{cor}} = \sqrt{\left( Q^{\text{cor}} \right)^2 + \left( T^{\text{cor}} \right)^2}
\]

(22)

\[
\beta^{\text{cor}} = \frac{1}{2} \arctan \left( \frac{-T^{\text{cor}}}{Q^{\text{cor}}} \right)
\]

(23)

The same rule that applies to the calculation of \( \beta \) applies to \( \beta^{\text{cor}} \), i.e. if the two-argument arctan function is not used, the angle may need to be adjusted by adding or subtracting 90° to be placed in the range defined in Table 2.

The procedure for the correction of uniform residual stresses is summarized in the following steps:

1. One of the six strain gauge rosettes listed in Table 3 is applied to the surface of the measured body and three sets of strains \( \varepsilon_a, \varepsilon_b, \) and \( \varepsilon_c \) are measured during drilling a hole with the diameter in the range from 0.35D to 0.43D. Ten uniform depth increments are recommended for the final hole depth of 0.5, 1, or 2 mm, depending on the used strain gauge rosette.

2. The calibration coefficients \( a \) and \( b \) are calculated using equation (1) depending on the diameter of the gauge circle \( D \), the hole diameter \( D_0 \), and the individual depth increments \( h_i \).

3. The combinations of measured stresses \( P, Q, \) and \( T \) are calculated using equations (2) – (7).

4. The in-plane residual stresses \( \sigma_a, \sigma_c, \) and \( \tau_{ac} \) are calculated using equations (8) – (10).

5. The equivalent residual stress is calculated as

\[
\sigma_{eq} = \sqrt{\sigma_a^2 - \sigma_a \cdot \sigma_c + \sigma_c^2 + 3\tau_{ac}^2}
\]

If \( \sigma_{eq} \) is lower than half the material yield stress \( \sigma_Y \), the correction of the plasticity effect is not required and the principal stresses \( \sigma_I, \sigma_{II} \) and the angle \( \beta \) defining the direction of the maximum principal stress can be calculated by equations (11) and (12). If \( \sigma_{eq} \geq 0.5 \sigma_Y \), the correction of evaluated residual stresses using the following steps is recommended.

6. The parameters \( X_1, X_2, \) and \( X_3 \) are determined from Table 5 based on the comparison of \( \sigma_{eq}, \sigma_a, \) and \( -\sigma_c \).

The input vector \( \mathbf{X} = \begin{bmatrix} X_1 & X_2 & X_3 & E & D_0 & SGR \end{bmatrix}^T \) is created from the parameters \( X_1, X_2, \) and \( X_3 \), the material yield stress \( \sigma_Y \), the Young’s modulus \( E \), the tangent modulus \( E_t \), the diameter of the gauge circle \( D \), the hole diameter \( D_0 \), and the parameter \( SGR \) determined from Table 3 based on the used strain gauge rosette.

7. The correction vector \( \mathbf{Y} = \begin{bmatrix} Y_1 & Y_2 & Y_3 \end{bmatrix}^T \) is calculated from the input vector \( \mathbf{X} \) by equations (14) – (18).

8. The equations for the correction of the evaluated stresses are determined from Table 5 and the corrected residual stresses \( \sigma_a^{\text{cor}}, \sigma_c^{\text{cor}}, \) and \( \tau_{ac}^{\text{cor}} \) are calculated by these equations.

9. The combinations of corrected stresses \( P^{\text{cor}}, Q^{\text{cor}}, \) and \( T^{\text{cor}} \) are calculated using equations (19) – (21).

10. The corrected principal stresses \( \sigma_{I}^{\text{cor}}, \sigma_{II}^{\text{cor}} \) and the corrected angle \( \beta^{\text{cor}} \) defining the direction of the principal stress \( \sigma_{I}^{\text{cor}} \) are calculated using equations (22) and (23).

### Testing of Correction Procedure

The performance of the neural network was monitored during the training process and the fitting error was very small. However, to ensure that the network was not overtrained on the training data set and is able to generalize to unknown situations, new data sets were prepared to test the correction procedure.

First of all, a testing of proposed specific symmetric conditions was performed. Since any stress combination may occur in practical measurements (not only the combinations where \( \sigma_a \geq \sigma_c \) and at the same time \( \sigma_a \geq \sigma_c \)), a data set with 500 randomly generated stress states (Fig. 19) was
created by finite element simulations. The stress states in this data set had the ratio $\sigma_{eq}/\sigma_Y$ from 0.5 to 1 with emphasis on the states with $\sigma_{eq}/\sigma_Y > 0.8$, since these states have larger evaluation errors due to larger plastic deformations. Each simulated stress state was evaluated for 100 random orientations of the strain gauge rosette resulting in a total of 50,000 test states. Because this test was aimed at verifying only the specific symmetric conditions, all simulated states had $\sigma_Y = 500$ MPa, $E_t/E = 0.1$, $D_0/D = 0.39$, and the strain gauge rosette RY61-1.5/120S was assumed to measure the relieved strains.

The correction procedure described in the previous section was applied to the evaluated stresses and the errors of the correction were calculated as the absolute difference between the corrected and simulated principal residual stresses and as the absolute difference between the corrected and simulated angles defining the direction of the maximum principal stresses. The errors arranged according to the quadrants defined by the comparison of $\sigma_a$, $\sigma_c$, and $-\sigma_c$ are shown in Figs. 20 and 21. The figures also contain the errors of uncorrected principal residual stresses and uncorrected angle $\beta$. Based on the fact that the magnitudes of errors are similar in all quadrants, it can be stated that the application of specific symmetric conditions is a correct approach to cover stress states outside the region where $\sigma_a \geq \sigma_c$ and $\sigma_a \geq -\sigma_c$.

Since the previous test approved the application of specific symmetric conditions, more detailed testing was performed only in the region where $\sigma_a \geq \sigma_c$ and $\sigma_a \geq -\sigma_c$. In

| Parameter          | Min. value | Max. value |
|--------------------|------------|------------|
| $\sigma_a/\sigma_y$| -1         | 1          |
| $\sigma_{eq}/\sigma_Y$| 0.5       | 1          |
| $\sigma_Y$ (MPa)   | 300        | 800        |
| $E_t/E$            | 0.01       | 1          |
| $D_0/D$            | 0.35       | 0.43       |
| $\beta^\prime$    | -45        | 45         |
| Strain gauge rosette | RY61-1.5/120S, RY61-3.2/120S, Type A, Type B |
order to test the proposed correction procedure in the entire trained range of input parameters, 2,500 states were simulated by the finite element model. Each simulated input parameter was randomly selected from the range given in Table 6. As can be seen from Figs. 22 and 23 showing variations of selected input parameters, the emphasis was on the states with larger plastic deformations, i.e. states with higher ratio $\sigma_{eq}/\sigma_Y$ and lower ratio $E_t/E$. Each simulated state was evaluated for 100 random orientations of strain gauge rosettes: RY61-1.5/120S, RY61-3.2/120S, Type A, and Type B, giving a total of 1,000,000 test states.

The test states were evaluated using the method described in the ASTM E837-20 standard and the correction procedure was applied to each evaluated state. Thereafter, the errors of the corrected principal stresses $\sigma_{I}, \sigma_{II}$ and the angle $\beta$ as well as the uncorrected principal stresses $\sigma_{I}, \sigma_{II}$ and the angle $\beta$ were calculated as the absolute difference between the simulated value and the value obtained after and before the correction. For better illustration, the errors were arranged from largest to smallest and are shown in Fig. 24. It can be seen that without correction the error of evaluated stresses can reach hundreds of MPa, while the maximal error of corrected stresses is 15 MPa.

Unlike the principal stresses, the error of the evaluated angle $\beta$ is not a critical factor, because even without the correction it reaches only small values (Fig. 25). However, even for this angle, an improvement in the evaluation can be observed after applying the correction procedure. It is worth noting that the states with the ratio $\sigma_{II}/\sigma_I \geq 0.98$ and the ratio $\sigma_{II}^{cor}/\sigma_I^{cor} \geq 0.98$ were considered as equibiaxial stress states and the error of angles $\beta$ and $\beta^{cor}$ was set to zero. Even though, the largest errors were produced by the stress states with higher ratios of $\sigma_{II}/\sigma_I$ and $\sigma_{II}^{cor}/\sigma_I^{cor}$, so if a lower limit is chosen to consider stresses as equibiaxial, the maximal error of $\beta$ and $\beta^{cor}$ will be even smaller.

When the error of the evaluated residual stresses is assessed, the absolute value may be misleading because it does not give information about the magnitude of the evaluated stresses. Therefore, the relative errors of the uncorrected $\delta$ and corrected $\delta^{cor}$ equivalent residual stresses were calculated as:
where $\sigma_{eq}^{cor}$ is a corrected equivalent stress calculated according to the same formula as $\sigma_{eq}$ (equation (13)), but using corrected residual stresses $\sigma_a^{cor}$, $\sigma_c^{cor}$, and $\varepsilon_{ac}^{cor}$. The relative error was not calculated from the principal stresses because it could reach infinite values for states where $\sigma_{II}$ approaches zero. As can be seen from Fig. 26, which shows the relative errors of the corrected and uncorrected equivalent residual stresses, the correction procedure performs remarkably well as 99.95% of all corrected test states have a relative error of less than 1% and the maximal error is about 3%. On the contrary, the relative error of uncorrected equivalent residual stresses may reach up to 65%, which can devalue the measurement results.

The effect of plasticity depends on several different factors, but the residual stress magnitude has the greatest influence. Therefore, the relative error of the equivalent residual stresses was plotted as a function of $\sigma_{eq}/\sigma_Y$ in Fig. 27. Figure contains the mean values of the corrected and uncorrected relative errors, the minimum and maximum values, and also the percentiles indicated the ranges in which 90% or 99% of the states are placed. It is worth noting that due to the overestimation of the evaluated stresses, the ratio $\sigma_{eq}/\sigma_Y$ can reach values far beyond the point where the evaluated equivalent stress is equal to the material yield stress. The results confirm the limits stated in the ASTM E837-20 standard [10]: "satisfactory measurement results can be achieved providing the residual stresses do not exceed about $\sigma_Y$.

\[
\delta = \frac{|\sigma_{eq} - \sigma_{eq}^{cor}|}{\sigma_{eq}} \times 100\% \tag{24}
\]

\[
\delta_{cor} = \frac{|\sigma_{eq}^{cor} - \sigma_{eq}^{2}|}{\sigma_{eq}} \times 100\% \tag{25}
\]

Relative errors were determined for the ratio $\sigma_{eq}/\sigma_Y$ instead of $\sigma_{eq}^{cor}/\sigma_Y$, since $\sigma_{eq}^{cor}$ is not known in the measurement. The equivalent stress $\sigma_{eq}$ was calculated according to the same formula as $\sigma_{eq}$ (equation (13)), but using the stresses $\sigma_a$, $\sigma_c$, and $\varepsilon_{ac}$.
80% of material yield stress for blind-hole drilling", as the average relative error of uncorrected equivalent stress is about 4% at \(\sigma_{eq}/\sigma_Y = 0.8\), which is still an acceptable value. However, results at this limit should be considered with caution, since the error may be as high as 12%. Although the correction procedure is applicable from the ratio \(\sigma_{eq}/\sigma_Y = 0.5\), its greatest benefit is for the states with higher \(\sigma_{eq}/\sigma_Y\) ratios. For these states, the uncorrected stresses reach a relative error of tens of percent, which can be critical for the measurement accuracy. On the contrary, the corrected equivalent stresses have a negligible error over the whole range of \(\sigma_{eq}/\sigma_Y\), even for the highest \(\sigma_{eq}/\sigma_Y\) ratios.

Since all measurements are subject to error and uncertainty, the value of parameters used for residual stress evaluation may slightly differ from their true values. For that reason, the sensitivity analysis of the correction procedure focused on several sources of uncertainty was carried out. The changes of relative error of \(\sigma_{eq}^{cor}\) caused by the variation of the hole diameter, Young's modulus, Poisson's ratio, the yield stress, and the tangent modulus are listed in Table 7. The data were calculated for three different stress states (uniaxial, shear, equibiaxial) with high residual stresses \((\sigma_{eq}/\sigma_Y = 0.97)\). The mean value of studied parameters was \(D_0 = 2\text{ mm}, E = 210\text{ GPa}, E_t = 2100\text{ MPa}, \sigma_Y = 500\text{ MPa},\) and \(\nu = 0.3\). The relative uncertainty of hole diameter was based on the accuracy of MTS 3000-Restan, the device used for the measurement of residual stresses, which is capable of measuring drilled hole diameter with uncertainty \(\pm 0.01\text{ mm} [14]\) (that gives the relative uncertainty \(\pm 0.5\%\) considering the hole diameter of 2 mm). The relative uncertainty of material parameters was considered in the standard range of \(\pm 2\% - 5\%\). In addition, the calculations with the tangent modulus were performed for a value of \(\pm 20\%\), as the correction procedure shows a weak sensitivity to this parameter.\(^5\) The results show that the parameters \(D_0\) and \(E\) have greater influence on the accuracy of the correction procedure than parameters \(\sigma_Y\) or \(E_t\), since they are used not only in the correction procedure but also in the calculation of calibration coefficients and the calculation of stresses \(\sigma_a, \sigma_c\) and \(r_{ac}\). None of the studied parameters significantly affected the angle \(\beta^{cor}\), as the error was less than 1° for all investigated states.

The standard uncertainty of final hole depth for MTS 3000-Restan is reported to be \(\pm 0.01\text{ mm} [14]\), which is a value that does not affect the correction procedure. However, since the amount of plastic deformation increases with the drilled depth, the cases where the hole is not drilled up to the recommended depth \(^6\) were investigated. Table 8 shows the change of relative error of \(\sigma_{eq}^{cor}\) for the cases with a hole drilled only up to 90%, 80%, and 50% of recommended hole depth. Since the final hole depth affects the accuracy of the correction procedure, the recommended depth should be fulfilled.

The proposed correction procedure was designed for a uniform distribution of residual stresses in the depth direction. Since the perfectly uniform distribution is very rare in industrial practice, the effect of slightly non-uniform distribution was investigated. Three above-mentioned residual stress states (uniaxial, shear, equibiaxial) were simulated for two linear distributions of stresses in the depth direction. The differences between \(\sigma_{eq}^{cor}\) and the mean values of equivalent stresses calculated from simulated non-uniform stress distributions were less than 0.4% and 2.8% for stress distributions with gradients 10 MPa/mm and 50 MPa/mm, respectively. Relatively small error can be explained by

\(^5\) Various mean values of the tangent modulus were investigated to verify the weak sensitivity of the correction procedure to this parameter.

\(^6\) The recommended hole depth is given in Table 3.
equations (5) – (7), which average the relieved strains for all hole depths and thus reduce the effect of slightly non-uniform stress distribution.

**Conclusion**

The hole-drilling method is based on drilling a small hole into the component surface, which may lead to local yielding in the area around the drilled hole. Since the method assumes linear elastic behaviour of the measured material, the error occurs when the states with plastic deformations are evaluated. The error is still acceptable if the residual stresses are below 80% of the material yield stress, but beyond this limit the measurement may be quite inaccurate. The aim of this study was to propose a correction procedure that would remove these limitations and allow accurate measurement of uniform residual stresses with magnitudes up to the material yield stress.

First of all, the influence of various stress levels, stress combinations, strain gauge rosette orientations, hole diameters, material properties, and strain gauge rosette geometries on the evaluation error was investigated. It was found that after the application of specific symmetric conditions, only one quarter of all stress states was required to cover all possible stress combinations. Since all the above-mentioned parameters significantly affect the evaluated residual stresses, the large data set containing almost eight million states with plastic deformations was prepared to create the correction procedure. The data were obtained by the finite element simulations of the hole-drilling process and by the postprocessing of the results from these simulations. Each simulated state was characterized by the two sets of parameters defining the particular state and the correction vector for this state. For the generalization of the relationship among the parameters the neural network fitting technique was used. Since the application of the trained neural network requires only basic mathematical operations (matrix multiplication and addition, and the use of trigonometric function tanh), any calculation software can be used to calculate the correction vector from listed weights and biases. The proposed correction procedure is capable of correcting any combination of uniform residual stresses (the stresses which do not vary significantly through the depth of the drilled hole) with magnitudes up to the material yield stress. It also covers a wide range of material parameters, hole diameters, and strain gauge rosettes and it is independent of the orientation of the strain gauge rosette.

In order to test the response of the proposed correction procedure to new unknown data, more than one million random test states covering the entire range of input parameters were prepared. The testing confirmed that the correction procedure performs remarkably well, as 99.95% of the test states had the relative error of equivalent stresses below 1% and the maximal relative error was 3%. On the contrary, the relative error of uncorrected stresses reached up to 65% for the states with high residual stresses.

Although the testing of the correction procedure shows very good results, there are several uncertainties that may affect the accuracy of the method in practical measurements, e.g. uncertainty of drilled hole diameter, the hole eccentricity, uncertainty of material parameters or inaccuracy of strain gauge rosette dimensions. Also, the exact material yield stress may not always be known, especially in situation where the material has undergone some mechanical processes producing a large local plastic strain, in which case the local yield stress may be higher than the global material yield stress. In addition, the bilinear stress–strain curve assumed for the correction procedure may not be sufficiently characteristic for some materials. In order to ensure satisfactory results of the proposed correction procedure, the sensitivity analysis focused on several sources of uncertainty was carried out. Even though the error of the correction increased due to the uncertainty of parameters used in the calculation, it still remained within an acceptable range. It is worth noting that some of these sources of uncertainty affect not only the correction procedure, but also the evaluation process of the hole-drilling method itself.

In conclusion, the proposed correction procedure extends the application range of the hole-drilling method as it provides a more accurate evaluation of high uniform residual stresses and can therefore be very useful in practical measurements.
### Table 9 The biases of the first, second, and output layer of the neural network

| k   | $b_k^{(1)}$ | $b_k^{(2)}$ | $b_k^{(3)}$ |
|-----|-------------|-------------|-------------|
| 1   | -8.42919    | -2.70966    | -1.85226    |
| 2   | 4.20910     | -0.26996    | 1.18795     |
| 3   | 1.39061     | 2.57024     |             |
| 4   | -18.96214   | -4.77604    | -0.99788    |
| 5   | 22.26914    | 0.82816     |             |
| 6   | 0.16341     | 0.35284     |             |
| 7   | 3.78624     | -0.83109    |             |
| 8   | -5.30275    | -1.12741    |             |
| 9   | -17.13206   | -3.27310    |             |
| 10  | 4.15097     | 0.14138     |             |
| 11  | 0.24657     | -0.36166    |             |
| 12  | -2.73740    | 3.46989     |             |
| 13  | -1.78667    | -0.36930    |             |
| 14  | -6.35008    | 1.06765     |             |
| 15  | 2.52819     | 1.78342     |             |
| 16  | -0.75424    | 1.95923     |             |
| 17  | 4.95390     | -0.76692    |             |
| 18  | 0.79343     | 0.63498     |             |
| 19  | -2.76177    | 0.59359     |             |
| 20  | 6.66665     | -0.09140    |             |
| 21  | -0.04635    | 3.14663     |             |
| 22  | 0.73638     | 2.24901     |             |
| 23  | -1.72987    | 0.14175     |             |
| 24  | 10.51818    | -0.99313    |             |
| 25  | -0.75676    | 1.48166     |             |
| 26  | 1.01142     | 1.76186     |             |
| 27  | -12.18886   | -0.00740    |             |
| 28  | -5.23773    | 2.57131     |             |
| 29  | 0.72806     | -2.94732    |             |
| 30  | -0.12003    | -2.16723    |             |
Table 10  The weights of the connections between the input layer and the first layer of neurons: $w^{(1)}_{k,m}$

| $\mathbf{k}$ | 1     | 2     | 3     | 4     | 5     | 6     |
|------------|-------|-------|-------|-------|-------|-------|
| 1          | 0.15462 | 0.42567 | 0.23263 | 1.38670 | 23.87401 | -0.34062 |
| 2          | -0.52258 | 0.02896 | 0.18900 | -3.35518 | -0.72759 | -0.10885 |
| 3          | 0.49744 | -0.55005 | 3.60008 | 0.57841 | 0.02629 | 0.08490 |
| 4          | 8.81555 | 4.67539 | -0.92375 | 1.70655 | 10.24057 | -0.17766 |
| 5          | -4.26957 | 1.33593 | -0.40329 | -1.83380 | -42.17702 | 0.16339 |
| 6          | 1.38755 | -1.75759 | 0.14108 | -0.08115 | 1.21516 | -0.03710 |
| 7          | -1.76716 | -0.28854 | 1.86730 | 0.81621 | -1.61737 | 0.11845 |
| 8          | 4.92056 | -3.79039 | 0.69898 | 0.63667 | -2.10478 | 0.16990 |
| 9          | 2.37893 | -0.49444 | 0.16726 | 1.84797 | 38.20895 | -0.50975 |
| 10         | 0.41311 | -0.78845 | -0.49364 | -0.84399 | -14.02064 | 0.28658 |
| 11         | 0.25323 | -0.48135 | -0.31583 | -0.42940 | -0.52732 | -0.20812 |
| 12         | -0.83277 | 0.06568 | 0.77234 | 0.09113 | 0.03190 | 17.19741 |
| 13         | 0.41007 | -1.76027 | -0.88090 | 0.05351 | -0.07681 | 0.00595 |
| 14         | 0.60097 | -1.26104 | -0.34733 | 0.19825 | -0.93479 | 27.12769 |
| 15         | -0.76756 | 0.19985 | 1.96348 | -0.66620 | -1.09347 | -0.11976 |
| 16         | 0.76643 | 0.15858 | 1.93983 | -0.07382 | 0.04469 | 0.01846 |
| 17         | 0.73963 | -0.15554 | -0.75477 | -0.04011 | -0.27723 | -13.31410 |
| 18         | -0.44530 | -0.42925 | -0.21730 | 0.81224 | 0.69761 | 0.11076 |
| 19         | 0.14666 | -0.85405 | -6.23892 | -0.24392 | -0.64731 | -0.14861 |
| 20         | -5.52074 | 2.23579 | -0.31538 | 1.66201 | -9.32677 | 23.22802 |
| 21         | 0.96878 | 1.11940 | -3.07021 | -0.67161 | -0.19760 | -0.01656 |
| 22         | -2.34793 | 1.45457 | 0.04142 | 1.35242 | -3.09331 | 25.57065 |
| 23         | -0.24716 | 1.55889 | -0.19230 | -0.37001 | 1.13508 | 0.00031 |
| 24         | -2.07687 | 2.47688 | 0.98748 | -0.01114 | 1.04688 | -21.67945 |
| 25         | 0.99995 | 0.24979 | -0.91830 | -0.10381 | 0.15810 | -0.01790 |
| 26         | 2.15446 | -0.06714 | 0.98523 | 1.28506 | 8.03182 | -0.43294 |
| 27         | 3.32514 | 0.17778 | -0.35580 | -1.33314 | 33.43016 | -5.00804 |
| 28         | 1.36679 | -0.74647 | 4.31208 | 1.03245 | 4.46319 | 0.17590 |
| 29         | -0.03743 | 0.17035 | 0.05857 | -0.05103 | 0.12861 | -3.77991 |
| 30         | -1.15530 | -0.14249 | 0.58786 | 0.48976 | -0.02590 | 0.06603 |
### Table 11

The weights of the connections between the first and second layer of neurons: $w_{km}^{(2)}$

| $k$ | $m$ | $w_{km}^{(2)}$ |
|-----|-----|----------------|
| 1   | 1   | -0.15833      |
| 2   | 1   | -0.54661      |
| 3   | 1   | 0.26182       |
| 4   | 1   | -0.03497      |
| 5   | 1   | 0.28920       |
| 6   | 1   | -0.03003      |
| 7   | 1   | -0.02372      |
| 8   | 1   | 0.04591       |
| 9   | 1   | 0.01278       |
| 10  | 1   | -0.19889      |
| 11  | 1   | -0.05501      |
| 12  | 1   | -0.01184      |
| 13  | 1   | 0.17215       |
| 14  | 1   | -0.19824      |
| 15  | 1   | 0.05609       |
| 16  | 1   | 0.52681       |
| 17  | 1   | 0.18884       |
| 18  | 1   | -0.04682      |
| 19  | 1   | -0.14657      |
| 20  | 1   | -0.09700      |
| 21  | 1   | 0.19972       |
| 22  | 1   | 0.19428       |
| 23  | 1   | 0.00807       |
| 24  | 1   | -0.06825      |
| 25  | 1   | 0.11795       |
| 26  | 1   | -0.20008      |
| 27  | 1   | 0.03175       |
| 28  | 1   | 0.53878       |
| 29  | 1   | -0.03066      |
| 30  | 1   | 0.20965       |

| $k$ | $m$ | $w_{km}^{(2)}$ |
|-----|-----|----------------|
| 1   | 2   | -0.04776      |
| 2   | 2   | 0.40902       |
| 3   | 2   | -1.18027      |
| 4   | 2   | 2.44894       |
| 5   | 2   | -0.66122      |
| 6   | 2   | 3.34473       |
| 7   | 2   | -0.38797      |
| 8   | 2   | 0.35883       |
| 9   | 2   | 0.66877       |
| 10  | 2   | 0.37423       |
| 11  | 2   | -0.34339      |
| 12  | 2   | -0.89456      |
| 13  | 2   | -0.32361      |
| 14  | 2   | 3.28389       |
| 15  | 2   | -2.87200      |
| 16  | 2   | -0.24632      |
| 17  | 2   | -3.25153      |

The weights of the connections between the first and second layer of neurons: $w_{km}^{(2)}$.
| m   | 11          | 12          | 13          | 14          | 15          | 16          | 17          | 18          | 19          | 20          |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 18  | -0.22714    | 0.07876     | -0.20365    | 0.01077     | -0.02331    | -0.10783    | -0.07649    | -0.29887    | -0.00309    | 0.01934     |
| 19  | 0.25672     | -0.02980    | 0.24149     | -0.02935    | 0.56780     | -0.30227    | 0.01263     | 2.40900     | -0.06121    | -0.09074    |
| 20  | -0.59703    | 0.03430     | -0.18529    | 0.00611     | 0.07282     | -0.32141    | -0.04259    | -0.13313    | -0.11609    | -0.00414    |
| 21  | -2.88620    | -0.06295    | -0.10026    | 0.12257     | 1.37039     | -1.87358    | 0.06177     | -1.78929    | -0.29136    | 0.06574     |
| 22  | -2.33988    | 0.05853     | -0.78284    | -0.01089    | 2.25139     | -3.02218    | -0.04231    | -1.18942    | -0.50244    | 0.03582     |
| 23  | -0.11561    | 0.11735     | -0.10018    | 0.00667     | 0.12059     | -0.10481    | -0.10625    | 0.19333     | -0.04176    | 0.02855     |
| 24  | 0.53824     | -0.11191    | 1.85341     | -0.01141    | -1.10730    | 2.06545     | 0.10518     | -0.36681    | -0.06035    | 0.04093     |
| 25  | -2.94344    | -0.24710    | -0.43695    | 0.05854     | 1.81567     | -2.10983    | 0.25402     | -1.48435    | -0.16384    | 0.05050     |
| 26  | 4.78029     | 0.68461     | 1.78483     | 0.28813     | -4.58105    | 4.29874     | -0.72949    | 2.59591     | 0.15917     | -0.17407    |
| 27  | -0.10145    | -0.05296    | -0.05428    | 0.00168     | -0.04944    | -0.03781    | 0.02100     | -0.22813    | 0.02819     | -0.00422    |
| 28  | -5.95776    | -1.26169    | -1.20202    | -1.44309    | 4.75290     | -2.74440    | 1.38418     | -2.62256    | -0.36893    | 0.15147     |
| 29  | 3.73007     | 0.21298     | 0.17092     | -0.12795    | -1.90843    | 2.40649     | -0.22404    | 2.45401     | 0.31936     | -0.11965    |
| 30  | -4.67816    | -0.74461    | -1.77683    | -0.33318    | 4.45963     | -4.23327    | 0.79268     | -2.64956    | -0.16280    | 0.19691     |

Table 11 (continued)
Table 12 The weights of the connections between the second and the output layer of neurons: $w_{k,m}^{(3)}$

| $k,m$ | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | -3.10230 | 0.15750 | 1.95746 | 0.05922 | -1.28329 | -0.14503 | 1.88092 | 0.12433 | -2.71653 | 1.57339 |
| 2     | -1.83704 | 0.17497 | 2.20238 | 0.04793 | -1.63304 | 0.41442 | 0.18438 | 0.15429 | -4.11337 | 2.28669 |
| 3     | -6.79094 | -0.05385 | -3.58056 | 0.37138 | 2.50932 | -0.02312 | -1.18549 | 0.05251 | -1.49322 | 1.16264 |
| 4     | 0.52722 | -1.46001 | 2.80927 | 0.76079 | 0.03196 | -0.05693 | 0.93299 | -3.93913 | 1.72287 | -0.64298 |
| 5     | -6.79094 | -0.05385 | -3.58056 | 0.37138 | 2.50932 | -0.02312 | -1.18549 | 0.05251 | -1.49322 | 1.16264 |
| 6     | 0.61644 | -4.18242 | 5.42477 | 1.67486 | 0.42760 | 0.00937 | 2.26598 | 0.45360 | 3.33653 | -1.04187 |
| 7     | -8.88684 | -1.07640 | 0.67489 | 0.07791 | 4.36808 | 3.35665 | 1.17474 | 0.85451 | -1.87193 | 2.59810 |
| 8     | -1.83704 | 0.17497 | 2.20238 | 0.04793 | -1.63304 | 0.41442 | 0.18438 | 0.15429 | -4.11337 | 2.28669 |
| 9     | 2.50572 | -1.77518 | -0.87775 | 0.08761 | -0.35552 | 0.28100 | 3.38886 | -0.21290 | 2.02860 | 0.27396 |
| 10    | -8.88684 | -1.07640 | 0.67489 | 0.07791 | 4.36808 | 3.35665 | 1.17474 | 0.85451 | -1.87193 | 2.59810 |
| 11    | -0.01555 | -2.71114 | 5.21706 | 0.11676 | 0.51677 | 0.37292 | 9.16296 | -0.00614 | -0.45807 | 0.27594 |

Declarations

Conflict of Interest The authors declare that they have no conflict of interest.

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