Signatures of $\alpha$-particle condensation in $N=Z$ nuclei

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Abstract. At the thresholds for multi-$\alpha$ particle decay a phase transition of second order appears in $N=Z$ nuclei, because the chemical potentials for nucleons in nuclear matter and in $\alpha$-particles become equal. The Bose gas created by the $\alpha$'s strongly interacting via $0^+$ states in $^8$Be and $^{12}$C, has coherent properties because of the fact that the de-Broglie wavelength is larger than the nuclear radius. Experimental observables are discussed, which show these coherent properties, like large radial extensions and the coherent emission of several $\alpha$'s.

1. Alpha-particles in nuclei

The $\alpha$-particle is the most strongly bound cluster in nuclei, it’s binding energy in nuclei follows the curve of binding energy per nucleon. The binding energy of nucleons in alphas is 7.03 MeV, a value which is somewhat smaller than in medium size nuclei (8.3 MeV/nucleon). Nuclei can have a certain probability of $\alpha$-clustering in their medium, as a mixed phase of nucleons and alphas [1, 2]. The nucleon and $\alpha$-binding energies depend on the excitation energy. At the thresholds for cluster decay (for multiple clusters of different size) a cluster-sub-structure [3] can appear, as predicted with the Ikeda diagram. The excitation energies, where the binding energy for only $\alpha$’s approaches zero, is at $E_x = 7.03$ MeV/nucleon, at these values an $\alpha$-particle condensates will be formed in $N=Z$ nuclei. Although for heavier ($N=Z$) nuclei these $\alpha$-condensed states are at rather high excitation energies in the continuum of nucleonic states, they may have collective properties [5, 6], which can give them a smaller observable width. The $\alpha$-particle condensed states have a larger radial extension due to the properties of the $\alpha$-$\alpha$ potential, a fact which can be observed in inelastic scattering to the excited $0^+$-states. For example in $^{16}$O we expect 5-6 such $0^+$ states, the state at 15.1 MeV having the most pronounced properties of a condensed state.

The basic equation for the formation of $\alpha$’s is the “reaction” of four “free” nucleons (two protons and two neutrons coupled to total values of spin and isospin of zero) :

$\left(N_1 + N_2 + N_3 + N_4\right) \leftrightarrow \alpha$-particle + 28.3 MeV. The free nucleons, $N_i$, should have a definite volume and pressure, in order to define thermodynamic quantities and where the density allows the occurrence of the mentioned reaction. We can assume that the particles interact in a well defined volume created by a self consistent mean field for the $\alpha$-clusters (the Gross-Piatevski approach for bosons). For the case that the difference in chemical potentials vanishes a second order phase transition will occur.

This process occurs also in models like the anti symmetrized molecular dynamics, AMD [7]. A certain number of nucleons are confined in a volume with a positive kinetic energy, as suggested in Fig. 1. In this model a cooling method is applied to find the states of the lowest energy. In the AMD approach during the cooling process a certain $\alpha$-cluster phase is observed.
before the formation of states with bound fermions, normal nuclei, with a binding energy per nucleon of 8.2 MeV or more, a value which is higher than in the $\alpha$-cluster (7.073 MeV). These two values define the difference in the chemical potentials in the two phases (Fig. 1). Starting from the ground states of normal nuclei the nucleons will form an $\alpha$-cluster phase with increasing temperature of the nucleus, e.g. with increasing excitation energy, see Fig. 1, (“heating”). We expect the decay of these excited states into many $\alpha$-particles, a decay not described by the Hauser-Fehsback formalism for statistical compound nucleus decay (see below).

![Figure 1](image)

**Figure 1.** Schematic illustration of the relative values of the energies of free nucleons, their binding energies in nuclei (8.2 MeV), and in $\alpha$-clusters (7.07 MeV). The difference $\Delta G$ between these two binding energies decreases in normal (N=Z) nuclei with increasing excitation energy, by (“heating”). At a critical value the two binding energies become equal, $\Delta G = 0$, a collective state of bosons (potentially mixed with fermions), the coherent $\alpha$-particle gas can be formed. In the AMD this state is approached by “cooling” from a fermion gas. From ref. [2]

## 2. Properties of $\alpha$-particle condensed states

In medium size nuclei ($Z<20$) the $\alpha$-condensates, calculated using the known alpha-alpha-potential with a self-consistent approach (based on the Gross-Platevski equation [4]), will have Coulomb barriers for the decay into multiple $\alpha$’s. With these barriers the states will have sufficiently small width for potential studies by inelastic scattering, they can be excited via monopole transitions. However, the heaviest nucleus for which this barrier can create a quasi-bound state [4], is estimated to be around $^{40}$Ca. In heavier nuclei these states will be embedded high in the continuum of the fermionic states, their decay is expected to be non statistical, as the most characteristic property to study. In heavier nuclei we further expect the formation of condensates, with the $\alpha$-gas outside strongly bound cores, like $^{16}$O or $^{40}$Ca.

The coherent $\alpha$-particle emission should occur into the same (identical) angle. This will lead to the situation that the observation of unbound resonances becomes very enhanced, such as $^8$Be($0^+, 2^+$) and the excited states of $^{12}$C, the $^{12}$C*(0, 0, 3)-clusters. This feature in fact has been observed in the recent data [2, 8, 9, 10], where the multi- $\alpha$-decay does not follow the independent decay steps predicted in compound decay by the Hauser-Fehsback statistical theory.

Most important for the properties of the $\alpha$-particle gas is, that they do not represent the “ideal” gas, they interact via an interaction which has similarities with a van der Waals interaction, with a strongly repulsive core due to the Pauli principle. Two $\alpha$-particles form,
as the lowest state, the ground state of $^8$Be, a resonance at $E^*_x = 92$ keV. We can calculate the de Broglie wave length, $\lambda = \frac{h}{\sqrt{2\mu E^*_x}}$, for this case we have $\lambda = 67$ fm (relative motion between the two $\alpha$-particles). If for higher excitation we incorporate the $2^+$ at 3.04 MeV the value of $\lambda$ is still 12.4 fm. Similarly for three $\alpha$-particles which form the Hoyle-state just above the three $\alpha$-particle threshold in $^{12}$C, the $0^+$ at 7.654 MeV (288 keV above the threshold), we again get a similarly large de Broglie wave length of relative motion. Overall we have values for $\lambda$ in the condensed states larger (by factors 2-5) then the radial extension of the nucleus. The condensed states at the binding energy threshold consisting of $\alpha$-particles will form coherent super-fluid states. The calculations of THSR-wave function based on a local $\alpha-\alpha$ potential reproduce the states of $^8$Be, and the threshold states in other light nuclei. The local potentials for the system of $^{16}$O+$\alpha$-particle, supports the formation of $\alpha$-condensates with a $^{16}$O-core, but also with a $^{40}$Ca-core (e.g. $^{52}$Fe = $^{40}$Ca+$6\alpha$) [10, 11]. The coherent decay of an $\alpha$-gas will be identified by the fact that the $\alpha$-particles are emitted preferentially into the same angle, and by the enhanced formation of $^8$Be and the $^{12}$C$(0^+_2)$ states.

![Figure 2](image-url)

Figure 2. Coincident $\gamma$-spectra gated with the $\alpha$-particles from $\Delta E-E$-telescopes with the emission of three random $\alpha$'s at different angles in different detectors upper part, in comparison with that obtained by the triple $\alpha$ emission with $^{12}$C$(0^+_2)$-gate (lower panel). The reaction is $^{28}$Si + $^{24}$Mg $\rightarrow ^{52}$Fe $\rightarrow ^{40}$Ca + 3$\alpha$ at 130 MeV. Note the additional lines for $^{36}$Ar in the lower panel, which indicates that a 4-rth $\alpha$ has been emitted (courtesy of Tz. Kokalova). Right side: The kinematics for the triple $\alpha$, the $^{12}$C$(0^+_2)$-observation.

3. Coherent multiple emission of $\alpha$-particles
The best way to study such decays is the combination of multi-detector arrays for particle detection with $\Delta E-E$ detectors and a “calorimeter” to observe the remaining compound nucleus residue via its $\gamma$-decay [2]. Such experiments have been performed with the large $\gamma$-detector array GASP at the Legnaro National Laboratory LNL at Padua (Italy), combined with the charged particle detector ball ISIS (details are given in ref. [9]) consisting of 42 $\Delta E-E$-telescopes. These experiments were performed in a study of $\gamma$-decays of compound nuclei selected with a
particular choice particle decays [8]. The large opening angle of the individual ISIS-\(\Delta E\)-E telescopes, which was 27°, allows to select the spontaneous decay of the weakly unbound states, namely of \(^{8}\)Be into two \(\alpha\)'s and the \(^{12}\)C\(^{+}(0^{+})\) decaying into three \(\alpha\)-particles. With the rather modest kinetic energy of these fragments and the small decay energies of a few 100 KeV, the opening angles between the \(\alpha\)'s are in the range of 10° - 25°, which fit into these solid angles of individual telescopes. Therefore these prompt multiple \(\alpha\)-decays are observed by the pile-up of the signals produced by individual three alpha-particles. In Fig. 2 the corresponding coincident (particle gated) \(\gamma\)-decays are compared with the spectra obtained from statistical emission (with the same \(\alpha\)-multiplicity), but into different angles of the \(\Delta E\)-E telescopes [9].

The observation of the emission of a 4-th \(\alpha\), contradicts strongly to the prediction of the statistical compound decay (3 \(\alpha\)'s, Hauser-Fehsbach prediction), the effect is due to a strongly lowered barrier for the \(\alpha\)-particle emission, which is due to the larger diffusivity and the larger radial extension of the \(\alpha\)-condensed state. This fact has been reproduced in calculations by Kokalova, Itagaki et al. in ref. [10], where the \(\alpha\)-folding model for a \(\alpha\)-condensed state with a \(^{40}\)Ca core has been performed. The result is strongly lowered emission barrier (by 10 MeV) for \(^{12}\)C\(^{+}(0^{+})\) emission.

![Figure 3. Break up of \(^{16}\)O into \(\alpha\)'s and \(^{8}\)Be at relativistic energies, courtesy of P.Zarubin [12](image)](image-url)

Another spectacular effect is observed in the collective decay into \(\alpha\)'s and \(^{8}\)Be after monopole excitation in high energy scattering of \(^{16}\)O. This measurement shown in Fig. 3 has been obtained at 4.5 GeV/N at Dubna [12]. using emulsions. With a clear visibility of tracks at small relative angles. The break-up occurs via Coulomb excitation at the smallest angles on a Ag-nucleus in the emulsion A sequence of three panels shows the total track.

4. References

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