ENHANCED ACCRETION RATES OF STARS ON SUPERMASSIVE BLACK HOLES BY STAR–DISK INTERACTIONS IN GALACTIC NUCLEI

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ABSTRACT

We investigate the dynamical interaction of a central star cluster surrounding a supermassive black hole (SMBH) and a central accretion disk (AD). The dissipative force acting on stars in the disk leads to an enhanced mass flow toward the SMBH and to an asymmetry in the phase space distribution due to the rotating AD. The AD is considered as a stationary Keplerian rotating disk, which is vertically extended in order to employ a fully self-consistent treatment of stellar dynamics including the dissipative force originating from star–gas ram pressure effects. The stellar system is treated with a direct high-accuracy N-body integration code. A star-by-star representation, desirable in N-body simulations, cannot be extended to real particle numbers yet. Hence, we carefully discuss the scaling behavior of our model with regard to particle number and tidal accretion radius. The main idea is to find a family of models for which the ratio of two-body relaxation time and dissipation time (for kinetic energy of stellar orbits) is constant, which then allows us to extrapolate our results to real parameters of galactic nuclei. Our model is derived from basic physical principles and as such it provides insight into the role of physical processes in galactic nuclei, but it should be regarded as a first step toward more realistic and more comprehensive simulations. Nevertheless, the following conclusions appear to be robust: the star accretion rate onto the AD and subsequently onto the SMBH is enhanced by a significant factor compared to purely stellar dynamical systems neglecting the disk. This process leads to enhanced fueling of central disks in active galactic nuclei (AGNs) and to an enhanced rate of tidal stellar disruptions. Such disruptions may produce electromagnetic counterparts in the form of observable X-ray flares. Our models improve predictions for their rates in quiescent galactic nuclei. We do not yet model direct stellar collisions in the gravitational potential well of the black hole, which could further enhance the growth rate of the black hole. Our models are relevant for quiescent galactic nuclei, because all our mass accretion rates would give rise to luminosities much smaller than the Eddington luminosity. To reach Eddington luminosities, outflows, and feedback as in the most active QSOs, other scenarios are needed, such as gas accretion after galaxy mergers. However, for AGNs close to the Eddington limit, this process may not serve as the dominant accretion process due to the long timescale.

Key words: accretion, accretion disks – celestial mechanics – galaxies: active – galaxies: nuclei – methods: numerical

Online-only material: color figures

1. INTRODUCTION

Kinematic and photometric data of galactic nuclei have revealed that supermassive central black holes (SMBHs) are ubiquitous in most galaxies (see, e.g., Kormendy & Richstone 1995 for a review). Detailed information on their photometric profiles and the shapes of spectral line profiles allow us, in certain limits, to deduce the true shape of the distribution function in phase space of such systems. The distribution function determines the rate at which stars will come close to the SMBH, be tidally disrupted or destroyed by direct collisions, and eventually accrete onto the black hole (Frank & Rees 1976; Bahcall & Wolf 1976). Quasars, however, which are the most luminous witnesses of accretion activity onto SMBHs, are already observed in the young universe at redshifts of $z > 5$. It is difficult to explain how black holes can grow so quickly to the observed high masses ($10^6–10^9 M_{\odot}$) by pure star accretion. It has therefore been argued that massive seed black holes already form by dissipative and viscous collapse, possibly accompanied by the formation of massive stars and their coalescence, at the time of galaxy formation (Colgate 1967; Spitzer & Saslaw 1966; Spitzer & Stone 1967; Sanders 1970; Rees 1984). In the hierarchical picture of galaxy formation, the most massive dark halos with small angular momentum can account for the early formation of the most massive black holes. The SMBH forms by a centrally focused collapse entering a phase of a dense supermassive gaseous object which is supported by star–gas interactions (Bisnovatyi-Kogan & Syunyaev 1972; Vilkoviskij 1975; Hara 1978; Langbein et al. 1990). The interaction of a compact stellar cluster (CSC) with a massive central object in the form of a superstructures or SMBH was considered by Vilkoviskij (1975), Hills (1975), and Hara (1978). The evolution of the dense non-rotating stellar cluster was studied by Spitzer & Saslaw (1966) and Bisnovatyi-Kogan (1978) among others, and the evolution of the gas sphere was considered by Langbein et al. (1990).

These studies have commonly neglected angular momentum, which should not be neglected during mergers nor in the intrinsic structure of galaxies. Spectra of active galactic nuclei (AGNs) suggest that there should be gas in the form of a massive central
accretion disk (AD) in which all interstellar matter settles before it may be accreted. The origin of the AD could be inflowing cool gas during mergers and/or debris by direct stellar collisions or stellar evolutionary processes, depending on the evolutionary state of the host galaxy. Artyomowicz et al. (1993) provide a theoretical model framework in which star–gas interactions and the build-up of massive stars generate a massive AD around the central SMBH.

Recent work shows that gaseous disks in galactic centers around SMBHs are important for the dynamics and the morphology of central regions of galaxies and for the evolution of single or multiple central black holes in many ways. For example, Cuadra et al. (2009) and Callegari et al. (2009, 2011) study the role of small-scale disks for the acceleration of binary black hole mergers, through dissipation of kinetic energy in the disk, after a galactic merger. Baruteau et al. (2011) discuss the hardening of stellar binaries in circumnuclear disks and their subsequent interactions with central black holes, which may lead to high velocity stellar escapers.

Shakura & Sunyaev (1973) developed a model for a stationary AD, which has been the basis of many investigations since then. Close to the inner boundary of the AD, at a few Schwarzschild radii $r_s$, general relativistic effects must be taken into account. Novikov & Thorne (1973) extended the standard disk model in that regime.

If the inner part of the AD reaches a critical surface density, then the interaction of orbits with the gas (or more correctly, energy and momentum transfer of stars due to ram pressure effects, henceforth denoted as dissipation) cannot be neglected anymore. Stellar interactions with ADs were also considered by Vilkoviskij & Bekbolosarov (1982), Vilkoviskij (1983), and Syer et al. (1991). More detailed investigations of the stellar orbits crossing ADs were presented in Vokrouhlicky & Karas (1998) and in Šubr & Karas (1999), Šubr & Karas (1999) and Šubr et al. (2004) assumed an infinitely thin disk interacting with stars; they found that the interaction between the disk and the stars (star–gas dissipation) will deplete counter-rotating stars, create a flattening of the large-scale structure of the system, and initiate anisotropies (i.e., changes of the eccentricity distributions). These studies did not take into account any feedback onto the disk or any finite-thickness effects of the disk. They tried to find a stationary state in which the transport of stars into the central galactic region is balanced by removal of stars from the central disk. Evolution timescales and initial conditions to reach this equilibrium were missing.

Around SMBHs, the AD may extend to the parsec scale, orders of magnitude larger than $r_s$. Rauch (1995, 1999) studied in detail the impact of the AD on the orbits and the distribution of stars with test particle simulations. Their work includes relativistic effects, even for the case of rotating Kerr black holes. They find that the orbital inclination with respect to the AD declines quickly as soon as the dissipative force becomes effective. Additionally, they find a steepening of the central density cusp, due to the combined effect of relativistic orbit migration and stellar collisions.

The semi-analytic model by Vilkoviskij & Czerny (2002) raises the point that two-body relaxation of the stars within and near the disk will tend to elevate trapped stars out of the disk again, and that the competition between relaxation and dissipation will define a stationary state of the system, with some well-defined stationary flux of stars going down to the black hole. Vilkoviskij & Czerny (2002) compared the star–star two-body interactions with the star–disk interactions and concluded that the latter is stronger in the inner parts of the AD, and vice versa in the outer parts based on nearly circular orbits of the stars. They derived analytic approximations for the effective inclination of stellar orbits, where the inflow to the SMBH takes place. They also derived a critical radius, inside which direct stellar collisions must be taken into account.

All of the mentioned papers so far have both strong and weak points. Rauch (1999) is the only paper to combine general relativity effects and stellar collisions, but considered only the central region where the (spherically symmetric) potential is dominated by the SMBH. They used an approximate model of stellar dynamics based on a Monte Carlo technique, which requires a spherically symmetric central star cluster. In Rauch (1995), they combined relativistic effects with star–disk interactions, but the disk is assumed to be infinitely thin. The infinitely thin disk approximation was also used in other work by Šubr & Karas (1999), Šubr et al. (2004), and Vilkoviskij & Czerny (2002).

Following the ideas of Vilkoviskij & Czerny (2002), we investigate by self-consistent direct N-body simulations the interaction of a central CSC with the AD in AGNs. We focus on the mutual interplay of two-body relaxation and the depletion of stars by the dissipative force in the AD as a secular long-term evolution of the stellar mass distribution and the velocity distribution function of the CSC. In this paper, we report results on a new model of star–disk interactions in galactic nuclei. Our focus lies on the correct and accurate representation of the stellar orbital motion crossing the disk, by implementing a disk with its density distribution in a full three-dimensional $N$-body simulation. We have added the force and force time derivative to the standard Hermite scheme (see below) as a function of local disk density and velocity in three dimensions. This is the most general approach, and later it will allow us to include evolving models of the AD and appropriately model the mass and energy transfer between the AD and CSC.

Our first results concern the enhanced accretion rate onto the central SMBH due to the interaction with the AD, in the regime where the relaxation timescale is comparable to the dissipation timescale. This is, to our knowledge, the first approach to study the competition between relaxation and star–gas dissipation in a direct simulation. Since the direct simulations are not yet able to reach realistic particle numbers and spatial resolution, we will perform a careful scaling analysis to show in which way our numerical simulations have to be interpreted for real astrophysical galactic nuclei.

This paper is organized as follows: in Section 2, we describe the accretion disk and the dissipative force in detail, Section 3 gives the numerical realizations of the system, in Section 4 the results are discussed, and in Section 5 a summary and conclusions are presented.

In follow-up papers, we will discuss the dependence of the dissipation on the orbital parameters and the phase space evolution of the cusp stars in detail and take into account the feedback of the star–gas interaction on the AD properties. Detailed studies of the migration of stars, binaries, and black holes inside the disk toward the center, and its observational consequences will also be included in future work.

2. PHYSICS OF THE ACCRETION DISK AND THE STAR–GAS INTERACTIONS

We consider an AGN model consisting of three subsystems: (1) a CSC with mass $M_*$ describing the inner part of the galactic center. It is spherically symmetric, non-rotating, and
in dynamical equilibrium. (2) An AD with mass \( M_d = \mu_d M_{\text{cl}} \). It is vertically extended, non-evolving, and has a Keplerian rotation curve. (3) A central SMBH with mass \( M_{\text{bh}} = \mu_{\text{bh}} M_{\text{cl}} \). The motion of each star \( m_i \) of the CSC is determined by the mutual gravitational interaction of the stars, the gravitational force of the SMBH, and by a dissipative force \( F_d \) from the AD. The equation of motion is given by

\[
\mathbf{r}_i = -\sum_{i \neq j} \frac{G m_i m_j}{r_{ij}^3} \mathbf{r}_{ij} - \frac{G M_{\text{bh}} m_i}{r_i^3} \mathbf{r}_i + F_d \quad ,
\]

where \( r_{ij} = r_i - r_j \), with \( r_i, r_j \) as the positions of stars \( i \) and \( j \), respectively. Since the AD has a small mass compared to the black hole and the enclosed stellar cluster, we neglect the gravitating effect of the disk on the system. Numerical details for computing the forces are given in Section 3.

### 2.1. The Accretion Disk

We are interested in the dynamical action of the AD on the stellar component. Therefore, we adopt a three-dimensional axisymmetric stationary disk model, which is differentially rotating with the local circular velocity. For the inner structure and evolution of such disks, see the review of Park & Ostriker (2001), and for the basic physics of ADs, see, e.g., Frank et al. (2002). A widely used model for the AD is the model of Shakura & Sunyaev (1973), with the radial scaling described in detail in Novikov & Thorne (1973, hereafter NT), which was also used by Rauch (1995) and Vil’kovskij & Czerny (2002).

For the radial profile of the AD, the surface density \( \Sigma \) is given by

\[
\Sigma(R) = \Sigma_d \left( \frac{R}{R_d} \right)^{-\alpha} \quad \text{with} \quad \alpha = 3/4 ,
\]

where \( R^2 = x^2 + y^2 \), \( R_d \) is the cutoff radius of the disk, and \( \Sigma_d \) is the surface density at the cutoff radius. The value \( \alpha = 3/4 \) corresponds to the outer disk range of NT. The mass \( M_d \) of the disk is

\[
M_d = 2\pi \int_0^{R_d} \Sigma(R) \, R \, dR = \frac{2\pi}{2 - \alpha} \Sigma_d R_d^2 .
\]

For the numerical integration using the fourth-order Hermite scheme, we need a smooth force in space. Therefore, we introduce a continuous but steep outer cutoff by

\[
\Sigma(R) = \Sigma_d \left( \frac{R}{R_d} \right)^{-\alpha} \exp \left[ -\beta_s \left( \frac{R}{R_d} \right)^{s} \right] .
\]

In order to retain exactly Equation (3) for the total disk mass (with the integral now up to \( \infty \)), we choose

\[
\beta_s = \left[ \Gamma \left( 1 + \frac{2 - \alpha}{s} \right) \right]^{-1/(2 - \alpha)}
\]

with the Gamma-function \( \Gamma(\alpha) \). For \( s \to \infty \), the cutoff is discontinuous at \( R_{\text{cut}} = R_d \) with \( \beta_s^{R_{\text{cut}}} \to 1 \). We will use \( s = 4 \), leading to \( \beta_s = 0.70 \) for \( \alpha = 3/4 \). In this case, the surface density at \( R_d \) is \( \Sigma(R_d) = 0.49 \Sigma_d \).

An inspection of the scaling relations in NT shows that in the case of SMBHs, self-gravity of the AD for the vertical structure becomes important for radii larger than \( \sim 100 \text{r}_* \). Since we cannot resolve the innermost part of the AD, we simplify the model of the vertical disk structure. We adopt a self-gravitating isothermal profile given by

\[
\rho_\text{g}(R, z) = \frac{\Sigma(R)}{2\pi \sqrt{\gamma} h_z} \exp \left( -\frac{z^2}{2h_z^2} \right) .
\]

In the NT model, the (half-)thickness \( h_z \) increases with distance according \( \propto R^{3/8} \). Since this is altered by self-gravitation and since we cannot resolve the vertical structure of such a thin disk close to the inner boundary, we decided to adopt a constant thickness \( h_z \), taking an appropriate value at some intermediate radius of the AD. To simplify the equations, we define the dimensionless value \( h = h_z/R_d \), which is also constant.

The effective sound speed \( c_s \) (which may be dominated by the turbulent motion in a clumpy gas) is given by (assuming a vertically isothermal model)

\[
c_s^2(R) = 8\pi G \rho_\text{g}(R, 0) h_z^2 = 4\sqrt{2\pi} G \Sigma h_z
\]

\[
= \left[ \frac{8}{\pi} (2 - \alpha) \frac{\mu_{\text{bh}}}{\mu_d} h \right] \left( \frac{R}{R_d} \right)^{1-\alpha} v_{\text{circ}}^2 (R) ,
\]

where \( v_{\text{circ}}(R) \) is approximated by the Kepler rotation of the SMBH only. In a thin disk, the radial pressure support can be neglected. Equation (7) shows that the rotation curve \( v_{\text{circ}} \) is highly supersonic \( (v_{\text{circ}}/c_s \approx 100) \) at the outer boundary \( R_d \) for the parameters used in our simulations: \( \mu_{\text{bh}} = 0.1 \), \( \mu_d = 0.01 \), and \( h = 10^{-3} \). The Mach number is increasing inward. The sound speed decreases with increasing radius and the stability of the AD is a function of radius. The Toomre \( Q \) stability parameter is given by

\[
Q^2 = \left( \frac{\kappa c_s}{\pi G \Sigma} \right)^2 = \frac{8}{2 - \alpha} \left( \frac{\mu_{\text{bh}}}{\mu_d} h \right) \left( \frac{R}{R_d} \right)^{a-3}
\]

with epicyclic frequency \( \kappa \), which shows that heavy and thin ADs are unstable near the outer boundary. For the case of \( \mu_d/\mu_{\text{bh}} = 0.1 \) and \( h = 10^{-3} \), the AD is formally unstable at \( R > 0.26 R_d \). In the framework of our simplified AD model with constant thickness, we may ignore this instability, since it could easily be avoided by adopting an increasing thickness with distance \( R \). With Equations (2) and (3), the density distribution of the AD with constant thickness (Equation (6)) is given by

\[
\rho_\text{g}(R, z) = \frac{2 - \alpha}{2\pi \sqrt{2\pi} h R_d^3} M_d \left( \frac{R}{R_d} \right)^{-\alpha} \exp \left[ -\beta_s \left( \frac{R}{R_d} \right)^{s} \right] \exp \left( -\frac{z^2}{2h^2 R_d^2} \right) .
\]

### 2.2. Dissipation of Stellar Kinetic Energy

A detailed theory of the dissipative force working on the stars crossing the AD depends on the details of differential rotation, density profile, turbulent motion in the disk, and possible resonance effects. Stars interact with the AD typically many times supersonically before they are finally trapped in it. Hence, we restrict our investigation to supersonic motion only, neglecting the details of the last few passages before trapping. In this case, we can use for the dissipative force the geometrical cross section \( F_{\text{geo}} = Q_{\text{geo}} \pi r_s^2 \rho_\text{g} v_{\text{geo}}^2 \) (determined by the effective area \( Q_{\text{geo}} \pi r_s^2 \) of the bow shock with stellar radius \( r_s \) and \( Q_{\text{geo}} ~ 5 \)) enhanced by dynamical friction, which is the gravitational pull by the overdensity in the Mach cone due
to gravitational focusing (Ostriker 1999). The dissipative force $F_d$ can be written in the form (Spurzem et al. 2004)

$$F_d = -\pi r_s^2 \rho_g |V_{rel}| V_{rel} \left[ Q_d + \left( \frac{v_{esc}}{V_{rel}} \right)^4 \ln \Lambda \right]$$

for $V_{rel} > c_s$, (10)

Here, $v_{esc}$ is the escape velocity at the surface of the star ($v_{esc} = 620 \text{ km s}^{-1}$ for the Sun) and the relative velocity $V_{rel} = V_s - V_d$ is the velocity of the star in the frame co-rotating with the disk. $\Lambda$ corresponds to the length of the Mach cone in units of the star radius $r_*$, leading to a Coulomb logarithm $\ln \Lambda \sim 10–20$.

Figure 1 shows the dissipative force $|F_d|$ for a range of values of $c_s/v_{esc}$ (full red lines). The dashed green line shows the approximation given in Equation (10) for $c_s/v_{esc} = 0.2$. The contribution to the dissipative force by $F_{geo}$ is shown by the dotted blue line. From the strong dominance of the dynamical friction for velocities $V_{rel}$ smaller than $v_{esc}$, we expect that stars are quickly decelerated to $V_{rel} < c_s$ and onto co-rotating circular orbits which then move slowly to the center with a radial decay speed comparable to $c_s$. The dissipative force is anti-parallel to the relative velocity and can also be accelerating with respect to the rest frame of the CSC.

For a measure of the efficiency of the dissipative force, we first introduce a dissipative timescale $t_{diss,\star}$, which describes the energy change $E_{\text{diss}}^{(\nu)}$ of a single star due to the dissipative force $F_d$ arising from interaction with the AD at the outer part of the disk. Our ansatz is

$$t_{diss,\star} = \frac{\xi_k m_\star \sigma^2}{P_d E^{\nu}_d}\left(\frac{r}{r_s}\right)^{4}\ln \Lambda,$$

with the stellar mass $m_\star$ and the three-dimensional stellar velocity dispersion $\sigma_\star$; the dissipative timescale as defined above depends strongly on radius. At the outer edge $R_d$ of the AD, we have for example $E_d^{\nu} = Q_d \pi r_s^2 \rho_g \sigma^4$, $\rho_g = \Sigma_d/R_d$, $t_{diss} = R_d/\sigma_\star$, and one gets

$$t_{diss}(R_d) = \frac{\xi_k}{Q_d P_d} \frac{\Sigma_d}{\Sigma_\star} \cdot t_{diss}.$$

In the second form of the above equation, we have defined the surface density of stars $\Sigma_\star = m_\star/(\pi r_s^2)$, which provides useful insights. Note that this dissipative timescale related to a single star is a strongly increasing function with radius, so the longest dissipation time can be found at the outer edge of the disk, in our case at $R_d$.

We now consider the quantities $\xi_k$ and $P_d$. The former is implicitly defined by $2E_* := \xi_k m_\star \sigma^2(R_d) = Gm_\star M_d/R_d$. The latter, $P_d$, accounts for the number of disk passages that a star will need before its full kinetic energy is dissipated, and it also includes an average over the orbital parameters of the stars, which gives

$$P_d = \left( g(e,i,R_d/p) \frac{t_{dyn}}{t_{rel}} \right),$$

The average in the equation above is taken over the disk crossing events of all stars with the proper efficiency function $g(e,i,R/p)$ which depends on the orbital eccentricity $e$, the inclination with respect to the AD $i$, and the focal parameter $p$ (the properties of $g(e,i,R/p)$ will be discussed in detail in a follow-up paper). For simplicity, we neglect here the contribution by dynamical friction. It can easily be included by replacing $Q_d$ with the velocity-dependent factor $(Q_d + (v_{esc}/V_{rel})^4 \ln \Lambda)$ in the definition of $P_d$.

In energy space, the dissipation process leads to a stationary flow of stars to highly negative values; at some point, stars will be absorbed by the central black hole, for example by tidal disruption at the tidal radius $r_t$. As discussed in Vilkovisky & Cerny (2002), the dissipation process will bring stars down into the plane of the disk, but two-body relaxation will reheat them in the vertical direction. Both effects drive the stars inward to the black hole, where they are finally tidally disrupted and accreted. Here, we use a simple parameterized model; stars are absorbed onto the central black hole at some accretion radius $r_{acc}$, for which we demand $R_d \gg r_{acc} \gg r_t$, but otherwise treat it as a free parameter, varied in our simulations later on.

For scaling purposes with particle number $N$ in the simulations, we use the standard half-mass relaxation time of Spitzer (1987):

$$t_{rx} = \frac{0.14 N}{\ln(0.4 N)} t_{dyn} \text{ with } \ t_{dyn} = \left( \frac{r_{\text{half}}}{G M_d} \right)^{1/2},$$

where $r_{\text{half}}$ is the half-mass radius of the CSC. The half-mass relaxation time $t_{rx}$ is given in Table 1 for a series of galactic nuclei covering the observed range of SMBH masses.

The local relaxation time $t_{rel}$ due to two-body encounters is given by (Binney & Tremaine 1987, Equation (8)–(71))

$$t_{rel} = \frac{0.34 \sigma^3}{G^2 m_\star \rho_\star \ln \Lambda} = \frac{18 \text{ Gyr}}{\ln \Lambda} \frac{\sigma}{(1 \text{ km s}^{-1})^3} \left( \frac{M_\odot}{m_\star} \right) \left( \frac{M_\odot \text{ pc}^{-3}}{\rho_\star} \right),$$

with one-dimensional velocity dispersion $\sigma$, mean particle mass $m_\star$, and density $\rho_\star$ of the CSC. This equation can be applied at each radius or to the central region of a system by using mean values inside some radius $r$ and is also called the core relaxation time. Inside the gravitational influence radius $r_0$ of the black hole (which is in our model of the same order as the disk outer radius $R_d$), we assume $\sigma^2 \propto r^{-1}$ and the standard Bahcall–Wolf (BW) density cusp solution for the stellar density profile in the CSC is $\rho \propto r^{-7/4}$ (Bahcall & Wolf 1976). Hence, we get for the local relaxation time (cf. Equation (16)) $t_{rel} \propto r^{1/4}$. For the galactic nuclei in Table 1, the core relaxation time at the
influence radius \(r_{\text{acc}}(r_{\text{bh}})\) is a factor of 1.5–2 shorter than \(t_{\text{rx}}\). This shows that two-body relaxation in the central part of galactic nuclei is well represented by our confined model with a CSC mass only 10 times larger than the SMBH mass.

For comparison with measurements in our simulations, as described later, it is more practical to define a global dissipation timescale \(t_{\text{diss}}\) and a global dimensionless dissipation timescale \(\eta\) in the following way:

\[
\eta = \frac{t_{\text{diss}}}{t_{\text{rx}}} \quad (16)
\]

\[
t_{\text{diss}} = \frac{E_k}{E_{\text{sd}}} = \frac{M_{\text{cl}} \sigma_{\text{init}}^2 (r_{\text{acc}})}{2E_{\text{sd}}} = \frac{G M_{\text{bh}} M_{\text{cl}}}{2r_{\text{acc}} E_{\text{sd}}}.
\]

Here, \(E_k\) is the kinetic energy and total energy dissipation rate of all stars at the accretion radius. An accretion time scale for the black hole growth is defined by \(t_{\text{acc}} = M_{\text{bh}} / M_{\text{bh}},\) or in dimensionless form \(\nu = t_{\text{acc}} / t_{\text{rx}}\). We remove stars at \(r_{\text{acc}}\), so in a stationary state the energy dissipation rate required at \(r_{\text{acc}}\) to sustain the black hole mass accretion rate is \(E_{\text{sd}} = M_{\text{bh}} G M_{\text{bh}} / (2r_{\text{acc}})\).

With Equation (16) we get

\[
\nu = \frac{r_{\text{acc}}}{t_{\text{rx}}} = \frac{E_{\text{sd}}}{E_{\text{diss}}} = \frac{M_{\text{bh}}}{M_{\text{cl}}} \frac{E_{\text{sd}}}{E_k} = \eta \mu_{\text{bh}}. \quad (17)
\]

Hence, the black hole mass accretion rate does not depend on the choice of \(r_{\text{acc}}\), as will be verified in our numerical simulations later.

### 3. The Numerical Model

For the high resolution direct \(N\)-body simulations of the CSC, we used the specially developed \(\phi\)GRAPE (= Parallel Hermite Integration with GRAPE) code. The program uses the fourth-order Hermite integration scheme for the particles with hierarchical individual block time steps, together with the parallel usage of GRAPE6 (or nowadays the GPU: Graphics Processing Unit) cards for the hardware calculation of the acceleration \(a\) and the first time derivative \(\dot{a}\) of the acceleration.

The code itself and also the special GRAPE hardware is described in more detail in Harfst et al. (2007). For all the new calculations, we use the different NVIDIA CUDA/GPU hardware which emulate the standard GRAPE library calls.\(^6\)

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\(^6\) ftp://ftp.mao.kiev.ua/pub/users/berczik/STARDISK/

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Table 1: Some Examples for the Physical Scaling of Galactic Nuclei

| Object | \(M_{\text{bh}}\) \((M_\odot)\) | \(\sigma_\text{cl}\) \((\text{km s}^{-1})\) | \(r_h\) (pc) | \(N\) | \(t_{\text{dyn}}\) \((\text{Myr})\) | \(t_{\text{rx}}\) \((\text{Gyr})\) | \(Q_{\text{tot}}\) | \(Q_{\text{tot}(8k)}\) | \(L_{\text{max}}\) \((L_\odot)\) |
|--------|-----------------|-----------------|-------|----|----------------|----------------|--------|----------------|----------------|
| M 87   | \(6.6 \times 10^6\) | 312              | 291   | 6.6 \times 10^{10} | 1.5             | 6 \times 10^5  | 2.1 \times 10^{-3} | 5.7 \times 10^{-3} | 1 \times 10^6 |
| NGC 3115 | \(9.6 \times 10^5\) | 230              | 78    | 9.6 \times 10^9    | 0.58            | 3.4 \times 10^4 | 4.2 \times 10^{-3} | 1.8 \times 10^{-3} | 2.8 \times 10^7 |
| NGC 4291 | \(3.2 \times 10^5\) | 224              | 24    | 3.2 \times 10^7    | 0.16            | 3400           | 1.5 \times 10^{-3} | 2.4 \times 10^{-3} | 9.4 \times 10^7 |
| M 31    | \(1.5 \times 10^6\) | 160              | 25    | 1.5 \times 10^8    | 0.26            | 2690           | 6.2 \times 10^{-3} | 4.7 \times 10^{-3} | 5 \times 10^8  |
| NGC 4486A | \(1.3 \times 10^6\) | 111              | 4.5   | 1.3 \times 10^8    | 0.067           | 68.8           | 1.7 \times 10^{-1} | 1.2 \times 10^{-1} | 1.9 \times 10^9 |
| MW     | \(4 \times 10^6\) | 110              | 1.4   | \(4 \times 10^7\)  | 0.021           | 7.2            | 5.2 \times 10^{-8} | 1.2 \times 10^{-8} | 5 \times 10^8  |
| M 32    | \(3 \times 10^6\) | 75               | 2.3   | \(3 \times 10^7\)  | 0.050           | 12.9           | 1.5 \times 10^{-5} | 2.8 \times 10^{-5} | 2 \times 10^8  |

**Notes.** For the CSC, we adopt \(\mu_{\text{bh}} = 0.1\), solar-type stars with \(m_\odot = 1 M_\odot, r_\odot = 2.3 \times 10^{-8} \text{pc}, v_\odot = 620 \text{ km s}^{-1}\), \(R_d = r_h\) and \(r_\text{def} = 3 r_h\), and \(Q_{\text{d}}\) for the bow shock size. Most observational data (Columns 2 and 3) are taken from Gültekin et al. (2009), improved values for M 87 are from Murphy et al. (2011), and \(M_{\text{bh}}\) for the Milky Way (MW) is from Gillessen et al. (2009). The influence radius (Column 4) is derived from \(r_{\text{acc}} = GM_{\text{bh}} / \sigma_{\text{cl}}^2\), with the core velocity dispersion \(\sigma_{\text{cl}}\) of the CSC; \(N\) is the number of stars in the CSC derived from \(M_{\text{bh}} / M_{\text{cl}}\); the dynamical and half-mass relaxation timescales (Columns 6 and 7) are derived by Equation (14); the physical and scaled cross sections \(Q_{\text{tot}}\) and \(Q_{\text{tot}(8k)}\) (Columns 8 and 9) by Equations (19) and (21); and the maximum luminosity by accretion (Column 10) assuming \(L_{\text{max}} = 0.1 M_{\text{bh}} c^2\).

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Here, we mention briefly the most important features added to the Hermite scheme of the \(N\)-body \(\phi\)GRAPE code in order to model the ram pressure dissipative force and the star accretion to the SMBH.

1. A dissipative force routine, where we calculate the acceleration caused by the interaction with the gaseous disk \(a_d\) (Equation (20)) and its first time derivative \(\dot{a}_d\).
2. We reduce the time steps of stars when they come close to the disk plane. Otherwise, stars with a big individual time step may miss the disk and would not feel the effect of the disk at all.
3. A simple accretion scheme of stars onto the SMBH, where the stellar mass is added to the central black hole if they get inside a certain accretion radius. The accretion radius is used as a free scaling parameter; results for different accretion radii can be used to scale to real parameters of galactic nuclei (see Section 4.2). This algorithm has been described and used in Fiestas et al. (2011) and Li et al. (2012).
4. In order to control integration error, we add a careful bookkeeping of energy changes caused by removing stars in the process of accretion and by the dissipative force of the stars–gas interaction. In all our runs, the total energy error does not exceed \(10^{-4}\).

In the simulations, we use standard \(N\)-body units given by

\[
G = M_\odot = 4 \pi |E_{\text{tot}}| = 1, \quad (18)
\]

where \(|E_{\text{tot}}|\) is the total energy of the initial Plummer sphere for the CSC without a perturbing SMBH (Heggie & Mathieu 1986). Note that in this scaling, an increase of the particle number takes place at constant total mass; hence, the stellar mass \(m_\star \propto 1/N\) decreases with particle number, and it also decreases with respect to, e.g., the disk mass and black hole mass.

Our model consists of three components: the CSC, the SMBH, and the AD. The CSC with initial mass \(M_{\text{cl}}\) is realized by \(N\) particles of mass \(m_\star\). The initial density distribution is a Plummer model, which is only in dynamical equilibrium if the gravity of the central SMBH is ignored. The system adjusts quickly in a few dynamical timescales. The mutual gravitational interactions and the dissipative force on the stars in the AD are calculated fully by the \(\phi\)GRAPE code.
The SMBH with initial mass $M_{\text{bh}} = \mu \nu M_{\odot}$ is modeled by an analytic Kepler potential fixed at the origin. We allow accretion of stars, which are effectively captured by the inner part of the AD or scattered by two-body relaxation into the loss cone, to grow the mass of the SMBH. Physically, the accretion radius $r_{\text{acc}}$ is given by the tidal radius $r_t$, where the stars are disrupted, or the Schwarzschild radius $r_S$. Both radii are orders of magnitude below our numerical resolution. Therefore, we need to analyze the scaling of the accretion with decreasing accretion radius. The orbits of the stars accreted by the interaction with the AD are circularized before being accreted. In contrast, stars accreted by two-body scattering into the loss cone are predominantly accreted on hyperbolic orbits. In order to simulate the effect of the different eccentricity distribution, we apply a second accretion criterion based on the velocity of the stars. We define an accretion criterion to merge stars with the SMBH using two criteria: (1) distance $r < r_{\text{acc}}$ and (2) $V^2 < k_{\text{acc}} v_{\text{circ}}^2$. Stars well inside the influence radius of the SMBH are moving on (local) Kepler orbits, where the velocity is given by $V^2 = v_{\text{circ}}^2 (2 - r/a)$ with semimajor axis $a$. In the limit of large $k_{\text{acc}}$, all stars reaching $r_{\text{acc}}$ would be accreted. For $k_{\text{acc}} = 1$, stars inside $r_{\text{acc}}$ are accreted if $a < r_{\text{acc}}$, i.e., all stars with energy below $-GM_{\odot}/r_{\text{acc}}$ (neglecting the potential energy of the CSC) are accreted in one orbital time. In all simulations, we use $k_{\text{acc}} = 1$. (Note for completeness that in some runs with AD we accreted all stars with $r < r_{\text{acc}}/10$ independent of their energy. We have tested that the accretion rates are not changed significantly by this additional accretion criterion.)

The properties of the AD are fixed by the normalized mass $\mu_\Delta$ with analytic density distribution according to Equation (6) with $\alpha = 3/4$, $s = 4$, and constant thickness $h = 10^{-2}$. We use a Kepler rotation in the AD in the potential of the SMBH, neglecting the gravity of the CSC and pressure gradients in the AD. We set the outer cutoff initially at $R_\Delta = r_h$. Using a Kepler rotation curve underestimates the real circular speed at the outer boundary by a factor of 1.4, but this has no significant influence on the dynamics of the stars. The reason is that the gas density and thus the friction force is very small in the outer regions of the AD. The part of the AD with significant dissipation of energy of crossing stars is deep inside the influence radius of the SMBH. Here, the approximation of Kepler rotation is very good. We have chosen the large cutoff radius only in order to avoid another free parameter.

It is helpful for calibration of our models with respect to real systems in galactic nuclei to define an effective dissipative parameter

$$Q_{\text{tot}} = Q_\Delta \frac{N \pi r_{\Delta}^2}{\pi R_\Delta^2}. \quad (19)$$

Note that $Q_{\text{tot}}$ enhances $Q_\Delta$ by a factor, which describes the dimensionless total dissipative cross section of $N$ stars, normalized to the disk area. With this definition, we can rewrite the original dissipative force in Equation (10) as acceleration

$$a_\Delta = F_\Delta/m_\star = -Q_{\text{tot}} \frac{\pi r_{\Delta}^2 \rho_\Delta}{M_\star} |V_{\text{rel}}| V_{\text{rel}}. \quad (20)$$

The local parameters like $r_\Delta$, $m_\star$, and $Q_\Delta$, whose scaling in terms of $N$-body units is not straightforward, are transformed in this way to the global quantities such as $R_\Delta$, $M_\star$, and $Q_{\text{tot}}$.

Due to numerical limitations, direct $N$-body simulations are performed here with particle numbers smaller than that in a real galactic nucleus. This leads to the relaxation time being a function of the particle number, $N$, whereas it is a fixed quantity for a given nucleus. To correct for this, the simulations are run for a multiple of the relaxation time and are then rescaled to the correct time units to compare to a physical galactic nucleus. For this reason, all physical processes, including the star–disk drag, need to be rescaled as well. In our models, all quantities except $Q_{\text{tot}}$ are invariant when changing the particle number in the simulation. Therefore, for calibration to real systems, we just have to compute the value of $Q_{\text{tot}}$ and adjust it correspondingly in our model system. But this step alone is not sufficient; our model system has a much shorter two-body relaxation time than in reality. We want to study numerical systems at smaller particle number, which have all the same ratio $\eta$ of dissipation to relaxation timescale. Therefore, we need to choose $Q_{\text{tot}}$ such that the same value for $\eta$ is achieved:

$$Q_{\text{tot}}(N_2) = \frac{t_{\text{rx}}(N_1)}{t_{\text{rx}}(N_2)} Q_{\text{tot}}(N_1). \quad (21)$$

Our model aims to discover numerically the secular evolution of the coupled star gas system in galactic nuclei. Therefore, we are interested in the interplay between dissipative processes (star–gas drag) and two-body relaxation processes—the semi-analytic model of Vilkoviskij & Czerny (2002) defines the setup in which we want to put our numerical models. Since we are not yet able to simulate star-by-star, two different effects have to be taken into account for this: (1) each simulation particle effectively represents many stars, not a single star. Therefore, we cannot study the individual star–gas drag effects here, only the collective one; this is shown by the definition of $Q_{\text{tot}}$ in Equation (19), which shows how for one simulation particle an effective cross section is realized, which corresponds to the effect of the much larger stellar system we have in mind. (2) In our system with lower particle number, the two-body relaxation time has a “wrong” ratio to other timescales. Therefore, we rescale $Q_{\text{tot}}$ in such a way that the relaxation time and the dissipation time have the correct ratio for the larger system, which is done in Equation (21). Equation (21) shows, for example, that for a system with $10^8$ particles, to be simulated by only $N = 10^4$, we need to increase $Q_{\text{tot}}$ artificially by a factor of $\sim 5000$. In Table 1, the real values of $Q_{\text{tot}}$ and simulated values for $N = 8000$ are both given for the selected galaxies. It should be stressed that we do NOT intend to model star–gas interactions in detail. Rather, we apply a standard model of it, scale it up in a collective way, and calibrate it with respect to the relaxation time. Our main goal is to check the increase of star accretion rates due to the presence of a central AD.

In our starting configuration, we added the central SMBH to the Plummer distribution of the CSC. In order to start the analysis of the system in dynamical equilibrium, we let the CSC evolve for $\sim 5t_{\text{dyn}}$ with the SMBH. After the initializing phase, the cusp around the SMBH shows a density slope $\gamma \approx 2.5$ which still differs from a BW cusp (top panel of Figure 2). The difference of the set-up Plummer profile and the cumulative mass profile at $t = 0$ demonstrates the effect of virialization in the potential of the SMBH. Then, we determine $R_\Delta$ such that at $t = 0$ the enclosed CSC mass at $R = R_\Delta$ equals the SMBH mass.

We performed a series of simulations combining different particle numbers $N = 4k$, $8k$, $16k$ and different accretion radii $r_{\text{acc}} = 0.04$, $0.02$, $0.01$, $0.005$ (see Table 2) until $t = 10t_{\text{rx}}$. The identifiers of the simulations in Table 2 are coding the particle number and accretion radius. For each parameter combination, a run without dissipative force is done for comparison. For the different $N$, we applied the scaling according to Equation (21).
For the simulations with dissipative force, our choice of \( Q_{\text{tot}} \) corresponds to the case of M 87 (compare Tables 1 and 2). From the last column in Table 1 it is obvious that for lower mass black holes the value of \( Q_{\text{tot}} \) in our simulations is unrealistically large.

For the calculation of the CSC dynamics, we used the parallel GRAPE systems built at the Astronomisches Rechen-Institut in Heidelberg,\(^7\) and at the Fesenkov Astrophysical Institute in Almaty. The code has recently been ported to large clusters with GPU hardware (in Beijing, China and Heidelberg, Germany, see acknowledgments) and results from these facilities have partly been used for this project.

4. RESULTS

In our starting configuration, the CSC is in dynamical equilibrium including the gravitational potential of the SMBH and has a steep cusp in the density profile. The cumulative mass profiles in the top panel of Figure 2 shows that after one relaxation time, the BW cusp is in place and remains stable over the full simulation. The lower panel of Figure 2 shows the Lagrange radii of the CSC for 1%, 5%, 25%, 50%, 90% enclosed mass with respect to the total mass \( M_d(t) \).

\(^7\) GRACE: http://www.uni-heidelberg.de/grace

### Table 2

| Model | \( N \) | \( \epsilon/R_d \) | \( r_{\text{acc}}/R_d \) | \( Q_{\text{tot}}(N) \) |
|-------|--------|----------------|----------------|----------------|
| 04K-005 | 4k | \( 4.55 \times 10^{-4} \) | \( 5 \times 10^{-3} \) | \( 10^{-2} \) |
| 04K-01 | 4k | \( 4.55 \times 10^{-4} \) | \( 10^{-2} \) | \( 10^{-2} \) |
| 04K-02 | 4k | \( 4.65 \times 10^{-4} \) | \( 2 \times 10^{-2} \) | \( 10^{-2} \) |
| 04K-04 | 4k | \( 4.55 \times 10^{-4} \) | \( 4 \times 10^{-2} \) | \( 10^{-2} \) |
| 08K-005 | 8k | \( 4.55 \times 10^{-4} \) | \( 5 \times 10^{-3} \) | \( 5.47 \times 10^{-3} \) |
| 08K-01 | 8k | \( 4.55 \times 10^{-4} \) | \( 10^{-2} \) | \( 5.47 \times 10^{-3} \) |
| 08K-02 | 8k | \( 4.55 \times 10^{-4} \) | \( 2 \times 10^{-2} \) | \( 5.47 \times 10^{-3} \) |
| 08K-04 | 8k | \( 4.55 \times 10^{-4} \) | \( 4 \times 10^{-2} \) | \( 5.47 \times 10^{-3} \) |
| 16K-005 | 16k | \( 4.55 \times 10^{-4} \) | \( 5 \times 10^{-3} \) | \( 2.97 \times 10^{-3} \) |
| 16K-01 | 16k | \( 4.55 \times 10^{-4} \) | \( 10^{-2} \) | \( 2.97 \times 10^{-3} \) |
| 16K-02 | 16k | \( 4.55 \times 10^{-4} \) | \( 2 \times 10^{-2} \) | \( 2.97 \times 10^{-3} \) |
| 16K-04 | 16k | \( 4.55 \times 10^{-4} \) | \( 4 \times 10^{-2} \) | \( 2.97 \times 10^{-3} \) |

**Notes.** Column 1 gives the identification label of the model, Columns 2–4 are number of particles, smoothing length, and accretion radius. Column 5 gives the total cross section, \( Q_{\text{tot}} \), scaled according to Equation (21). Common parameters for all models are \( \mu_{0\text{th}} = 0.1, \mu_{\text{d}} = 0.01, h = 10^{-3} \), and \( k_{\text{acc}} = 1 \).

For the model 16K-005 (see Table 2 for model parameters). Additionally, the position of the innermost particle (1p) shows that stars crossing \( r_{\text{acc}} \) on highly eccentric orbits are quickly circularized and accreted. The influence radius \( r_h \) of the SMBH is increasing during the simulation by an order of magnitude due to accretion, and the size of the BW cusp is growing accordingly.

#### 4.1. Scaling with Particle Number

In simulations without AD, stars reaching \( r_{\text{acc}} \) are immediately accreted onto the SMBH. The corresponding loss cone in phase space has a maximum angular momentum for each energy defining the opening angle of the loss cone in phase space. The regime of an empty loss cone depends on \( t_{\text{rx}}/t_{\text{dyn}} \) and the width of the loss cone (Amaro-Seoane & Spurzem 2001). In the regime with a high probability of a star to be scattered in or out of the loss cone, the loss cone is full. In the empty loss-cone regime, all stars scattered by two-body encounters into the loss cone are accreted and the accretion rate scales with \( t_{\text{rx}} \). In the full loss-cone regime, the accretion rate depends on \( t_{\text{dyn}} \). In a given system, the empty part of the loss cone increases with increasing particle number \( N \) leading to an \( N \) dependence of the accretion rate. We used an additional energy criterion accreting stars with semimajor axis \( a < r_{\text{acc}} \), which cuts the loss cone in energy space.

In Figure 3, the lower set of thin lines show the growing mass of the SMBH for different particle numbers for the cases without dissipation due to the AD (top panel: \( r_{\text{acc}} = 0.04 \) and bottom panel: \( r_{\text{acc}} = 0.005 \)). The accretion rate per relaxation time is seen to slowly increase with particle number.

The upper set of thick lines in Figure 3 show that accretion is significantly larger when the dissipative force of the AD is included. The accretion rate is found to be independent of particle number \( N \).

In Figure 4, the ratio of the accretion rates with and without dissipative force is quantified for the 8k simulations at different accretion radii. After a few relaxation times, an equilibrium with an enhancement factor of \( \sim 4 \) is established for all accretion radii.

#### 4.2. The Accretion Radius

The physical accretion radius \( r_{\text{acc}} \) is much smaller than the numerical resolution. Therefore, the scaling of the accretion rate with \( r_{\text{acc}} \) is very important. Without the AD, the standard...
loss cone becomes thinner with decreasing $r_{\text{acc}}$, leading to a decreasing accretion rate in the limit of an empty loss cone (e.g., Amaro-Seoane & Spurzem 2001). On the other hand, the BW cusp is characterized by a constant mass and energy flow to the SMBH and high binding energy, respectively. As a consequence, the accretion rate based on an energy criterion should be independent of $r_{\text{acc}}$. In Figure 5, the independence of the accretion rate in terms of the accretion timescale $\nu$ (Equation (17)) on $r_{\text{acc}}$ is shown for all $N = 16k$ runs with dissipative force. For $t > t_{\text{rx}}$, the accretion rate is also independent of the particle number $N$.

With the chosen parameters for the AD and the CSC the growth timescale of the SMBH by accretion of stars is of the order of the half-mass relaxation time $t_{\text{rx}}$ of the CSC, which is long compared to the Eddington accretion timescale of $\approx 50$ Myr (see Table 1). Therefore, our model applies to quiescent galactic nuclei. The accretion rate can be converted to a maximum luminosity by adopting $L_{\text{max}} = 0.1M_{\odot}c^2$. This upper limit is for all SMBH masses in the range of $2 \ldots 50 \times 10^7 L_{\odot}$ (last column of Table 1).

### 4.3. Energy Dissipation

Our measurement of the *global* normalized dissipative timescale $\eta$ in the simulations uses the definition as given in Equation (16).

In a stationary state, the timescale of transport of stars through the AD toward the black hole is dominated by the outer edge of the disk, where the dissipation timescale is longest. Therefore, a measurement of the total dissipated energy in our system should not depend on the choice of $r_{\text{acc}}$, where we actually remove the stars and add their mass to the central SMBH, as it has been shown by Equation (17). The normalized dissipation timescale $\eta$ is a measure of the dissipated energy. It is determined by the numerical evaluation of $E_{\text{kin}}(t)$ and $E_{\text{ad}}^{(\text{sd})}(t)$, the latter via smoothing of the cumulative function $E_{\text{ad}}^{(\text{sd})}(t)$ to derive the slope. After an initial adaption phase, $\eta$ is approximately constant (Figure 6) showing the constant energy flow in a stationary BW cusp. As seen in Figure 6, the normalized dissipation timescale is independent of $N$ and depends only weakly on $r_{\text{acc}}$. The long-term trend of increasing $\eta$ is due to the evolution of the SMBH and the CSC by the accretion of stars. The SMBH mass is increasing and the mass of the CSC is decreasing leading to an increasing $\mu_{\text{bh}}$ and virial factor $\xi_{\text{v}}$, and a decreasing total cross section $Q_{\text{tot}}$ and efficiency $P_{\text{eff}}$ with time. By comparing the dissipation timescale $\eta$ shown in Figure 6 and the accretion timescale $\nu$ shown in Figure 5, we observe that the relation derived in Equation (17) is approximately fulfilled (taking into account the particle number $N$).
account the growth of the black hole and the decreasing CSC mass for $\mu_{\text{th}}$.

5. DISCUSSION AND CONCLUSIONS

In galactic nuclei, SMBHs coexist with a dense stellar cluster; galaxy mergers and the quasar phenomenon indicate that at least for some time there should also be large amounts of interstellar gas present in the nuclear regions around the black holes. In this paper, we have examined the interaction and co-evolution of a dense star cluster surrounding a star-accreting SMBH with an assumed central gaseous disk. Interactions of such disks with the surrounding dense star clusters have been proposed as a source of gas supply to the central disk (Miralda-Escudé & Kollmeier 2005), as agents to enhance the tidal star accretion rate (Vilkoviskij & Czerny 2002), and to cause feedback on the orbital parameters of stars (Rauch 1995), including a modification of sources of gravitational waves (Rauch 1999).

Our model is the first self-consistent long-term simulation of a dense star cluster, surrounding a star-accreting SMBH and subjecting the stars to the dissipative forces from a resolved central gaseous disk. We resolve effects of two-body relaxation, dissipation of stellar kinetic energy in the disk, and star accretion onto the central black hole in a numerical study based on direct high-accuracy simulations. Since star-by-star modeling of a galactic nucleus down to the realistic tidal radius is not yet possible, despite the modern GPU hardware used for simulations, a careful scaling analysis is presented as a function of the particle number in the simulations and of an assumed star accretion radius, to allow conclusions for the real astrophysical situation in galactic nuclei. But our model still has a number of serious drawbacks. First, it assumes a stationary AD, so energetic feedback to the disk structure by star–disk interactions is neglected; second, the physics of star–gas interactions is modeled approximately. Finally, there is no distinction between properties of different stars (main sequence, giants, remnants), but rather a single stellar species with solar radius is assumed. In that sense, our study should be considered as a pathfinder and exploratory.

This investigation is a direct continuation of a semi-analytic study by Vilkoviskij & Czerny (2002), extending it by a more detailed and numerical study of the stellar dynamical evolution of the central stellar cluster. Our paper uses a numerical approach based on direct N-body simulations, including particle–particle forces as well as a dissipative force in the disk. Here, we resolve the dissipation of stellar kinetic energy along the stellar paths in the vertically extended AD. Particle numbers in our simulations and an adopted star accretion radius (onto the central SMBH) are used as free parameters in our model, while for other important parameters of the problem fiducial values are assumed. These are, e.g., the initial SMBH mass (10% of the initial central stellar cluster mass), the gaseous AD mass (10% of the initial black hole mass), and the outer radius of the AD (set equal to the black hole gravitational influence radius). Finally, all star particles are equal in the simulation, and their total effective cross section is used as a parameter to obtain the physically correct ratio of dissipation to relaxation time.

We show that the accretion rate of stars onto the SMBH is strongly enhanced by the dissipative force of the AD (a factor of four with our parameters). The accretion process is determined by an equilibrium of diffusion by two-body encounters and energy loss by the dissipative force. Consistently, there is also an energy deposition in the central AD; we find that most stars accreted or trapped in the disk are quickly accreted also to the black hole, because there is no stable co-rotation of the stars with the disk. Our results are robust with respect to variation of particle number and adopted accretion radius, therefore they should hold under realistic conditions in galactic nuclei. Our star accretion rate does not depend strongly on the adopted star accretion radius; it supports the idea of Vilkoviskij & Czerny (2002) that there is a stationary flow of stars within the disk toward the central black hole, which is determined by an equilibrium between dissipation and relaxation time. In spite of the enhanced number of stars accreted through the disk, we still find that there is a BW central density cusp present in the system, which is not significantly perturbed.

Central densities in star clusters near SMBHs can reach $10^8 M_\odot \text{pc}^{-3}$ or more. At such high stellar densities, direct, disruptive stellar collisions may produce gas in the gravitational potential well of the black hole. The gas production rate could be larger than the one obtained from tidal disruption of stars (Spitzer & Saslaw 1966; Spitzer & Stone 1967, see also Begelman & Rees 1978; Frank & Rees 1976; and numerical models in, e.g., Freitag & Benz 2002 and Freitag et al. 2006a, 2006b). But there is little doubt that a large fraction of this gas is finally accreted to the SMBH; some fraction of it though may escape. The same is true for the gas produced by tidal accretion. Our models do not yet resolve the very central regions of the star cluster, where stellar collisions may occur predominantly. We anyway treat the accretion radius as a free parameter used for scaling studies; here, it is usually large compared to the...
astrophysically defined tidal radius, where stars are destroyed by tidal forces. Our results should not depend strongly on whether the stars inside $r_{\text{acc}}$ are finally disrupted by tidal forces or destroyed by stellar collisions with subsequent accretion of the gas onto the SMBH. Less is known about the effect of induced stellar collisions due to low relative velocities of stars in the disk (due to dissipation). We will study the effects of direct stellar collisions in future work.

Direct stellar collisions produce another source of gas deep in the gravitational well of the central SMBH. As in the case of tidal disruptions of stars, we do not know exactly how much mass is accreted to the SMBH, and how much is ejected due to magnetic fields (jets) and radiation pressure near it (see, e.g., one improved model by Kasen & Ramirez-Ruiz 2010). Our assumption to add 100% of the material from tidal accretion to the black hole clearly is an upper limit, the real growth rate may be lower due to some mass loss in the process, also for stellar collisions. However, even in our case with possibly overestimated accretion rates, assuming a typical value of 10% mass to energy conversion, the luminosity obtained for stellar collisions due to low relative velocities of stars in the disk and star and gas accretion to the central black hole is quasi-periodic and highly non-stationary (as suggested by the ubiquitous time variability of radiation from AGNs).

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