What affects the performance of POD for the simulation of heat transfer through building component?

Tianfeng Hou¹,²,³,⁴, Staf Roels¹, Hans Janssen¹

¹KU Leuven, Department of Civil Engineering, Building Physics Section, 3001 Leuven, Belgium
²Data Assimilation Research Team, RIKEN Center for Computational Science, Kobe, Japan
³RIKEN Prediction Science Laboratory, Kobe, Japan
⁴RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program, Kobe, Japan

Abstract The capability of the proper orthogonal decomposition for the simulation of heat transfer in building components is investigated via three applications: linear heat transfer (not coupled to mass transfer), mildly non-linear heat transfer (coupled to air and moisture transfer (hygroscopic)) and highly non-linear heat transfer (coupled to moisture transfer (capillary)). It is shown that increasing non-linearity leads to an increasing number of required construction modes. To further investigate the reason for this degrading performance of POD, the singular values’ decay progress from the different training snapshots is addressed in this paper. The results confirm that a fast decay of the singular values implies a high interrelation of the snapshots and a better performance of the POD method.

1. Introduction

Numerical simulations are widely used for building hygrothermal performance assessment, because they allow a large number of scenarios and factors to be assessed easily when judging heat and moisture transport processes in built structures. The simulation tools are often based on the conventional methods for spatial and temporal discretization though, and can hence be very time consuming, because of the strongly nonlinear equations, the multi-dimensional and large sets of built structures, and/or the long simulation periods needed.

For this paper a well-known model order reduction method (proper orthogonal decomposition, POD) has been studied, to lower the computational costs of these building thermal performance simulations, from the assessment of linear heat transfer through a building materials (not coupled to mass transfer, case 1) [1], over the study of mildly non-linear heat transfer through a light-weight wall (coupled to air and hygroscopic moisture transfer, case 2) [2], to the judgment of highly non-linear heat transfer through a composite wall (coupled to capillary moisture transfer, case 3) [2]. It is shown that POD performs very well for case 1 (linear), as it results in a very good accuracy with a relatively low number of construction modes. However, relative to that linear case 1, (much) more construction modes are required in order to obtain an accurate result for the mildly and highly nonlinear cases 2 and 3. To illustrate what may affect the performance of POD for the simulation of hygrothermal performance, the results of the above three case studies are firstly presented and the singular values’ decay progress from the different training snapshots is analyzed.
2. Case studies

For investigating the performances of POD for building thermal analysis with different degrees of non-linearity and complexity, three applications – case 1, case 2 and case 3 – are simulated below. The first one is a simple case of thermal conduction through a massive wall [1], the latter two are HAMSTAD Benchmarks 3 and 4 [2]. All of them are 1-D building component, and have different transfer mechanisms (with different degrees of coupling to mass transfer) with different combinations of boundary conditions and material properties. More details of these three case studies can be found in [1-3].

For assessing the performance of POD against the number of POD modes, for all the three cases the results derived by POD models with different number of construction modes are compared to a reference solution calculated by the conventional Finite Element Method (FEM), the results of which are presented in Figure 1. Figure 1 shows the average temperature deviations between the POD and FEM solution, as an indicator of the accuracy of the POD simulations.

![Figure 1: Average temperature differences between the reference solution and results of the POD models constructed by different number of construction modes.](image)

It is shown in Figure 1, for all three cases, the accuracy of the POD models improves as the number of their modes increases, as well as that the increasing non-linearity results in an increasing number of required construction modes. For case 1, the POD method shows to perform very well: with POD a rather accurate result can be obtained with a relatively small number of construction modes (the average temperature differences for POD models are below 0.1 °C with 3 modes or more). Relative to case 1, more construction modes are required to obtain an accurate result for case 2: to get absolute errors below 0.1 °C, over 30 modes are necessary. For case 3, POD requires respectively 100 modes to reach the same accuracy level for the prediction of the temperature profiles. In such cases, due to the relatively large number of construction modes, the computational complexity of POD could be much larger than the conventional methods. As a result, the POD method loses its superiority.

3. Decay progress of the singular values

The main propose of POD is approximating a high-dimensional system to its ‘most important component’ only, and the information of the physical field can be represented by all the snapshots’ singular values. As already mentioned in many publications [4-5], the interrelation of the snapshots is key, and the singular values’ decay speed suggests the level of the interrelation among the snapshot, which can thus be treated as a performance indicator of the POD models. Theoretically, a fast decrease of the singular values indicates a strong snapshots interrelation, and therefore a relatively small size of POD model is required for delivering an accurate result.
Figure 2: The singular values’ decay progress of the snapshots of all three case studies.

To further illustrate the interrelation of the snapshots, the snapshots’ singular values’ decay process for all three case studies are shown in Figure 2. The figure shows that, for all three case studies, the snapshots’ singular values decline as their numbers raise. Different from the singular values of case 1, for cases 2 and 3 the decline speeds of their singular value curves are (much) lower. That implies that the snapshots of cases 2 and 3 have relatively smaller interrelations than the snapshots of case 1. Therefore, relative to case 1, more POD modes are needed for a POD model to accurately calculate the case 2 and 3.

4. Conclusions

We conclude that the accuracy and performance of POD appears to strongly depend on the interrelation of the snapshots. Relative to the case 1, the interrelations of the snapshots of the case 2 and 3 were shown to be a lot weaker. Hence, for the POD method a (much) larger number of construction modes is needed to predict the temperature profile. In such cases, due to the larger number of needed construction modes, the computational complexity of POD could be (far) larger than the conventional methods. As a result, the POD method may lose its superiority. Complementary to the results of the thermal behaviour, we have also analysed the hygric behaviour, and similar conclusions come about, with the remark that their singular values are even larger, and that the impact of non-linearity there is even larger.

References
[1] Hou, T., Roels, S., & Janssen, H. (2020). Model order reduction for efficient deterministic and probabilistic assessment of building envelope thermal performance. Energy and Buildings, 110366.

[2] Hagentoft, C.-E. (2002). Hamstad wp2-benchmark package.

[3] Hou, T., Roels, S., & Janssen, H. (2019). The use of proper orthogonal decomposition for the simulation of highly nonlinear hygrothermal performance. CESBP'2019, 282.

[4] Fic, A., Bialecki, R. A., & Kassab, A. J. (2005). Solving transient nonlinear heat conduction problems by proper orthogonal decomposition and the finite-element method. Numerical Heat Transfer, Part B: Fundamentals, 48(2), 103–124.

[5] Hou, T., Meerbergen, K., Roels, S., & Janssen, H. (2020). POD–DEIM model order reduction for nonlinear heat and moisture transfer in building materials. Journal of Building Performance Simulation, 13(6), 645-661.