Error Analysis of Integrated Six-Light-Screen Array Measurement Model

Rui Chen*1, Bowen Ji1, Chenxi Duan1

1 School of Optoelectronic Engineering, Xi’an Technological University, Xi’an 710032, China
*Corresponding author’s e-mail: chenrui_xatu@163.com

Abstract. The light-screen array measurement method is very suitable for measuring the coordinates of rapid-fire weapons, and the measurement error is determined by the measurement model. In this paper, the separated light-screen array is improved to an integrated light-screen array, which reduces the parameters and optimizes the measurement model. Three kinds of factors affecting the coordinate measurement error of the projectile under the integrated measurement model are analysed, and the influence of the factors on the distribution of coordinate measurement errors is simulated and analysed in the selected 1m×1m target area. Then the error distribution of the separated measurement model and the integrated measurement model is simulated and analysed under the same conditions based on the design values and current technology level. The result shows that compared with the separated measurement model under the same simulation conditions, the comprehensive coordinate measurement error is optimized by about 2.1mm within 1m×1m target area. The research can provide reference for the design and optimization of light-screen array and other similar photoelectric measurement systems, and provide new ideas for improving the coordinate measurement precision of rapid-fire weapons.

1. Introduction

With the development of barrel weapons, rapid-fire weapons have been widely used[1-3]. In order to evaluate the performance of such weapons, it is necessary to realize the coordinate measurement of continuous projectiles with high firing rate. Among existing non-contacting measurement equipment[4-7], light-screen array is very suitable for the measurement of rapid-fire weapons which has been used for the measurement of close-quarters cannons of shipborne weapon systems with high firing rate and large target surface.

By measuring the time that the projectile reaching each light-screen and combining with the known spatial structure of the light-screen array, the coordinates of the projectiles are measured[8-9]. The measurement results are influenced by combined effect by the calibration errors of spatial structure the light-screen array[10], the time extraction errors of the projectile reaching the light-screens[11] and so on. To study the influence of various factors on the measurement error for six light-screen array, which is of great significance to design optimization and engineering application of this kind of equipment, so as to effectively improve the measuring accuracy of rapid fire weapons.

The existing research of light-screen array is mainly taken the separated measurement model as research object. However, the structure parameters are difficult to achieve in engineering practice[12-13]. Combined with the actual situation, the integrated light-screen array measurement model with fewer structure parameters is not only easier to use, but also simplifies the measurement model and
theoretically. This paper analyzes the coordinate measurement error of the integrated measurement model. The error propagation formula of projectile coordinates is deduced firstly, and the influence of each factor are synthesized and analyzed. Then the coordinate measurement error distribution is compared under the separated measurement model and the integrated measurement model within the selected 1 m×1 m area.

2. Principle of measurement

Light-screen vertical target is a typical instrument which used to form the light-screen, and two sets of sky-screen vertical target are placed in parallel, respectively called front target and rear target. The double N shaped light-screen array is shown in Fig. 1.

In Fig. 1, \( s \) and \( h \) are the target distance and the differences in height between front and rear target respectively, collectively called to target parameters. The X-axis direction is the preset trajectory direction. The six-light-screen plane which formed by vertical target can be regarded as the ideal planes [14]. It shows the section view of double N shaped six-light-screen array at two planes of \( z=0 \) and \( y=b \ (b>0) \) in Fig.2.

In Fig. 2, \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) and \( \beta_1, \beta_2 \) represent the angles between the light-screens, which are collectively called the structural parameters of the light-screen array. The projectile reaches six light-screens in sequence, and the time is \( t_1, t_2, t_3, t_4, t_5 \) and \( t_6 \). Assuming the projectile makes a uniform linear motion within the light-screen array, the coordinates of the projectile in each light-screen plane and the flight velocity of the projectile can be calculated according to the plane equations in the coordinate system [15]. The measuring principle is shown in formula (1).

\[
X = M^{-1} \cdot N
\]
In the formula, $M$ is the related to items of coefficient on light-screen plane equations and determined by the angles between the light-screens. $N$ is related items of target parameters. $X$ is flight parameters of the projectile. And $x_1, y_1, z_1$ is the coordinate of the projectile in the light-screen $G_1$ and $(v_x, v_y, v_z)$ is the flight velocity of the projectile along the corresponding coordinate axes.

In order to reduce the parameters of the measurement model, the front target and the rear target are all fixed on the same pedestal in the integrated six-screen array measurement model, as shown in Fig. 3.

In Fig. 3, the front target and the rear target are fixed on the pedestal after adjusting by the laser, which is easy to be placed in the field. More importantly, after proper adjustment, the parameter $h=0$ in Formula (1) makes the measurement model more simplified. According to the measurement principle, the expression of projectile coordinate is

$$
\begin{align*}
\begin{bmatrix}
    x_1 \\
y_1 \\
z_1 \\
v_x \\
v_y \\
v_z
\end{bmatrix}
= & \begin{bmatrix}
    -s \cos \alpha_1 + h \sin \alpha_1 \\
    -s \cos \beta_1 \\
    -s \cos \alpha_2 + h \sin \alpha_2 \\
    0 \\
    0 \\
    0
\end{bmatrix}

\end{align*}
$$

And $(t_1, t_2, t_3, t_4, t_5, t_6)$ is the time when the projectile arrives at each light screen, which is collectively referred to as time series.

3. Measurement error analysis

It is generally considered that the spatial structure of the light-screen array is consistent with the design, that is, the angles between light-screens meet the condition of $\alpha_1=\alpha_2=\alpha_3=\alpha_4$ and $\beta_1=\beta_2$. The plane of $G_1, G_3, G_4$ and $G_6$ are perpendicular to the $XOY$ plane and $G_2$ and $G_5$ are perpendicular to the $XOZ$ plane. According to the measurement principle, the measurement error of integrated six-screen array measurement model mainly include: a) the calibration error of angles between light-screens; b) the incident angle of the projectile; c) the time extraction error of the projectile reaching the light-screen planes.

Combined with the design value and the actual condition, the parameters are taken $s=3m$, $h=0m$, $\alpha_1=\alpha_2=\alpha_3=\alpha_4=25^\circ$, $\beta_1=\beta_2=24^\circ$. The effects of the above factors on the projectile coordinate measurement error is analysed in a $1m \times 1m$ target plane with $z=[-500, 500]mm$, $y=[800,1800]mm$. As the measurement error of the horizontal and vertical coordinate of the projectile are different, we take the formula (3) as the coordinate comprehensive error.

$$
\delta = \sqrt{dz^2 + dy^2}
$$
And $dz, dy$ represents measurement error of horizontal and vertical coordinate respectively. In the target plane.

### 3.1 Calibration error of angles between light-screens

Considering the projectile incident along the direction of the preset trajectory, the projectile with velocity $v_x=720\text{m/s}$, $v_y=v_z=0\text{m/s}$ incident on different positions in the selected area. The coordinate error expression of Calibration error of angles between light-screens is

$$
\begin{align*}
\delta z^2 &= \left( \frac{\partial z}{\partial \alpha_1} \right)^2 + \left( \frac{\partial z}{\partial \alpha_2} \right)^2 + \left( \frac{\partial z}{\partial \alpha_3} \right)^2 + \left( \frac{\partial z}{\partial \alpha_4} \right)^2 + \left( \frac{\partial z}{\partial \beta_1} \right)^2 + \left( \frac{\partial z}{\partial \beta_2} \right)^2 \\
\delta y^2 &= \left( \frac{\partial y}{\partial \alpha_1} \right)^2 + \left( \frac{\partial y}{\partial \alpha_2} \right)^2 + \left( \frac{\partial y}{\partial \alpha_3} \right)^2 + \left( \frac{\partial y}{\partial \alpha_4} \right)^2 + \left( \frac{\partial y}{\partial \beta_1} \right)^2 + \left( \frac{\partial y}{\partial \beta_2} \right)^2
\end{align*}
$$

And then the coordinate measurement error can be obtained. It is assumed that the calibration error of angles between the six light-screens are the same and both are $0.1^\circ$, and the measurement error in the selected area are analysed respectively.

![Fig. 4 Coordinate Measurement Error Distribution of Calibration Error of Angles Between Light-Screens in Selected Area](image)

It can be seen that the influence of the calibration error of angles between the light screens can be divided into two types. When there are calibration errors of angles $\alpha_1, \alpha_2, \alpha_3$ and $\alpha_4$ in the vertical direction, the coordinate measurement error increases with the increase of ordinate axis. When there are the calibration errors of the angles $\beta_1, \beta_2$ in the horizontal direction, the coordinate measurement error is symmetrically distributed with the abscissa axis.

### 3.2 The time extraction error

When the projectile reaches the light-screens in the six light-screen model, the photoelectric sensor of each light-screens generate an output signal, and the time between these output signals is the time series in formula (2). The coordinate error expression of time extraction error is

$$
\begin{align*}
\delta z^2 &= \left( \frac{\partial z}{\partial t_1} \right)^2 + \left( \frac{\partial z}{\partial t_2} \right)^2 + \left( \frac{\partial z}{\partial t_3} \right)^2 + \left( \frac{\partial z}{\partial t_4} \right)^2 + \left( \frac{\partial z}{\partial t_5} \right)^2 + \left( \frac{\partial z}{\partial t_6} \right)^2 \\
\delta y^2 &= \left( \frac{\partial y}{\partial t_1} \right)^2 + \left( \frac{\partial y}{\partial t_2} \right)^2 + \left( \frac{\partial y}{\partial t_3} \right)^2 + \left( \frac{\partial y}{\partial t_4} \right)^2 + \left( \frac{\partial y}{\partial t_5} \right)^2 + \left( \frac{\partial y}{\partial t_6} \right)^2
\end{align*}
$$
When the experimental conditions are consistent, the time extraction errors in each light screen are the same. It is mainly determined by the time extraction algorithm and the chronoscope. According to the current technology level, the time time extraction errors are \( d_i = 2 \mu s, i = 1, 2, 3, \ldots, 6 \) which signal collection frequency is 20MHz. The measurement error in the selected area are analysed.

The results in Fig. 5 show that the abscissa measurement error basically increases with the increase of coordinates, which is related to the tilt direction of light-screen \( G_2 \) and \( G_5 \). The ordinate direction error increases with the increase of ordinates, and the increasing trend gradually increases.

3.3 Projectile incident angle

The error analysis of the coordinate measurement above is assumed that the flight direction angle \( \theta \) and pitch angle \( \gamma \) are both 0°, which means \( v_y = v_z = 0 \text{m/s} \). In this section, we consider the projectile incident with an angle and analyze the effect of the deviation of the coordinate measurement error. When the projectile is taken from a fixed position on the target plane, the projectile incidence angle \( \theta \) and pitch angle \( \gamma \) are increased from 0° to 5° respectively. Considering the 6 calibration errors of angle between light-screens are all 0.05°, and the time time extraction errors are \( d_i = 2 \mu s, i = 1, 2, 3, \ldots, 6 \), the measurement error as shown in Fig.6.

It can be seen that the error of coordinate measurement tends to enlarge when the incident angle of the projectile increases.

4. Error Comparison

We pick up the maximum of all the factors above based on design values and current technology level, when the target distance is \( s = 3 \text{m} \) and the height difference is \( h = 10 \text{mm} \); The maximum of projectile incident angle is \( \gamma = \theta = 5^\circ \); The time time extraction errors are \( d_i = 2 \mu s, i = 1, 2, 3, \ldots, 6 \), using four-channel data collector which signal collection frequency is 20MHz; The calibration error of angles between light-screens is not more than 0.1°. The horizontal placement error and parallel placement error are 0.03°; The target distance and height difference are 3mm in the separated six light-screen array. The coordinate measurement error is comprehensively analysed in the selected 1m × 1m target plane under
the separated six light-screen array model and the integrated six light-screen array model respectively, as shown in Fig. 7.

![Separated Six Light-screen Array Model](image1.png) ![Integrated Six Light-screen Array Model](image2.png)

Fig. 7 Coordinate Measurement Error Distribution Under Different Measurement Model

The error distribution trend of the two measurement models is similar. However, due to the fact that the integrated measurement model has fewer model parameters and introduces fewer error components, the comprehensive coordinate measurement error is optimized by about 2.1mm within 1m×1m target area comparing with the separated measurement model.

5. Conclusions

In this paper, the coordinate measurement error of the integrated six-screen array measurement model is analysed. Firstly, the measurement model is simplified by reducing parameters, and the error influencing factors under the integrated measurement model are analysed as follows: a) the calibration error of angles between light-screens; b) the incident angle of the projectile; c) the time extraction error of the projectile reaching the light-screen planes. Then the influence of the factors on the distribution of coordinate measurement errors is simulated and analysed in the selected 1m×1m target area. Based on the design values and current technology level, the error distribution of the separated measurement model and the integrated measurement model is simulated and analysed under the same conditions. The result shows that compared with the separated measurement model under the same simulation conditions, the comprehensive coordinate measurement error is optimized by about 2.1mm within 1m×1m target area because of the integrated measurement model has fewer model parameters and introduces fewer error components. The results of this paper can provide a reference for the design and optimization of light-screen arrays and provide new ideas for improving the coordinate measurement accuracy of rapid-fire weapons.

Acknowledgements

This work was supported by the Scientific Research Plan Projects of Shaanxi Education Department [grant number 20JK0692].

References

[1] China Association for Science and Technology, 2009, China Ordnance Society. Report on Advances in Science and Technology China Science and Technology Press, Beijing.

[2] N. A. Kazarinov. 2020 Experimental and numerical analysis of PMMA impact fracture, *INT. J. IMPACT. ENG.* 143:103597-1-6.

[3] Jason Angel. 2009 Methology for dynamic characterization of fragmenting warheads. *Weapons and Materials Research Directorate*, Army Research Laboratory,

[4] Sedunov A, Sutin A and Salloum H. 2013 Passive acoustic localization of small aircraft, *Journal of the Acoustical Society of America* 134 L5

[5] Y. Li. 2019 A new motion parameter estimation and relocation scheme for airborne three channel CSSAR-GMTI systems, *IEEE Trans. Geosci. Rem. Sens.* 57 4107-4120
[6] Saide Victor G.P., Viegas Gabriel M., Canuto André V.S., Barra Cristina M., Shimamoto Gustavo G., Tubino Matthieu and Rocha Junior José G. 2021 Rifle bullets comparison by wavelength dispersive X-ray fluorescence spectroscopy and chemometric analysis. Forensic Science International 325.

[7] He K.P., Xu D. and Li H. 2019 Measuring method for barrage weapons dispersion by line laser parallel detector array. Infrared and Laser Engineering 45 L10 1017004.

[8] Z.C. Wu and X.L. Zhang 2019 On-sate calibration method of target distance of the sky screen target velocity measuring system Optik 178 483-487.

[9] Hanshan Li. 2016 Research on Space Target Dection Ability Calculation Method and Spectral Filtering Technology in Sky-screen's Photoelectric system. Microwave and Optical Technology Letters 58 L5 1035-1041.

[10] R. Chen, D. Chen, B.W. Ji and J.P. Ni 2020 Inversion method of the key structure parameters of light screen array testing system using genetic algorithm Optik 206 164064.

[11] D. Chen and J.P. Ni. 2018 Pulse compression-based improvement on the estimation accuracy of time interval between two trigger signals in light screen array Optik 158 675–683.

[12] Z.C. Wu, J.P. Ni and X.L. Zhang. 2014 Study on verification device of screen spatial location parameters of sky screen target, Optik 125 3770-3773.

[13] R. Chen, J.P. Ni, J and L. Liu 2019 Uncertainty analysis of coordinate measurement of six-light-screen array sky screen vertical target based on engineering model, ACTA ARMAMENTARII 40 612-620.

[14] R. Chen and J.P. Ni. 2018 Calibration method of light-screen plane equation of sky screen vertical target. Optik 155 276-284.

[15] J.P. Ni 2014 Technology and Application of Measurement of the Light Screen Array, National Defense Industry Press.