Dark Energy, Dark Matter and Gravity

Orfeu Bertolami

Instituto Superior Técnico, Departamento de Física
Av. Rovisco Pais 1, Lisbon, 1049-001, Portugal

We discuss the motivation for high accuracy relativistic gravitational experiments in the Solar System and complementary cosmological tests. We focus our attention on the issue of distinguishing a generic scalar-theory of gravity as the underlying physical theory from the usual general relativistic picture, where one expects the presence of fundamental scalar fields associated, for instance, to inflation, dark matter and dark energy.

Keywords: Dark matter; dark energy; scalar fields; gravity.

1. Introduction

Present day experimental evidence indicates that gravitational physics is in agreement with Einstein’s theory of General Relativity to considerable accuracy; however, there are a number of reasons, theoretical and experimental, to question the theory as the ultimate description of gravity.

On the theoretical side, difficulties arise from various corners, most stemming from the strong gravitational field regime, associated with the existence of spacetime singularities and the difficulty to describe the physics of very strong gravitational fields. Quantization of gravity is a possible way to overcome these obstacles, however, despite the success of modern gauge field theories in describing the electromagnetic, weak, and strong interactions, the path to describe gravity at the quantum level is still to be found. Indeed, our two foundational theories, Quantum Mechanics and General Relativity, are not compatible with each other. Furthermore, in fundamental theories that attempt to include gravity, new long-range forces often arise in addition to the Newtonian inverse-square law. Even at the classical level, and assuming the validity of the Equivalence Principle, Einstein’s theory does not provide the most general way to establish the spacetime metric. There are also important reasons to consider additional fields, especially scalar fields. Although the latter appear in unification theories, their inclusion predicts a non-Einsteinian behavior of gravitating systems. These deviations from General Relativity include violations of

∗Talk presented at the International Workshop “From Quantum to Cosmos: Fundamental Physics Research in Space”, 22-24 May 2006, Warrenton, Virginia, USA.
†E-mail: orfeu@cosmos.ist.utl.pt
the Equivalence Principle, modification of large-scale gravitational phenomena, and variation of the fundamental “constants”. These predictions motivate new searches for very small deviations of relativistic gravity from General Relativity and drive the need for further gravitational experiments in space. These include laser astrometric measurements\(^1,^2,^3,^4\), high-resolution lunar laser ranging (LLR)\(^5\) and long range tracking of spacecraft using the formation flight concept, as proposed\(^6\) to test the Pioneer anomaly\(^7\). A broader discussion on the motivations to perform fundamental physics experiments in space can be found elsewhere\(^8\).

On the experimental front, recent cosmological observations does lead one to conclude that our understanding of the origin and evolution of the Universe based on General Relativity requires that most of the energy content of the Universe resides in the presently unknown dark matter and dark energy components that may permeate much, if not all spacetime. Indeed, recent Cosmic Microwave Background Radiation (CMBR) WMAP three year data\(^9\) indicates that our Universe is well described, within the framework of General Relativity, by a flat Robertson-Walker metric, meaning that the energy density of the Universe is fairly close to the critical one, \(\rho_c \equiv 3H_0^2/8\pi G \simeq 10^{-29} \text{g/cm}^3\), where \(H_0 \simeq 73 \text{ km s}^{-1}\text{Mpc}^{-1}\) is the Hubble expansion parameter at present. Moreover, CMBR, Supernova and large scale structure data are consistent with each other if, in the cosmic budget of energy, dark energy corresponds to about 73% of the critical density, while dark matter to about 23% and baryonic matter, the matter that we are made of, to only about 4%. Furthermore, it is generally believed that the ultimate theory that will reconcile Quantum Mechanics and General Relativity will also allow for addressing the cosmological questions related with the origin and destiny of the Universe.

It is our opinion that the crystallization of these fundamental questions is well timed with recent progress in high-precision measurement technologies for physics experiments in space. This puts us in position to realistically address crucial questions, such as the nature of dark energy and dark matter, the existence of intermediate range forces and the ultimate nature of gravity. Furthermore, given the ever increasing practical significance of General Relativity, for spacecraft navigation, time transfer, clock synchronization, weight and length standards, it is just natural to expect that the theory will be regularly tested with ever increasing accuracy. Thus, it seems legitimate to speculate that the present state of physics represents a unique confluence of important challenges in high energy physics and cosmology together with technological advances and access to space, a conjunction that is likely to yield major discoveries.

In what follows we shall address the key issue of distinguishing a generic scalar-theory of gravity, as the underlying fundamental physical theory, from the usual general relativistic picture, where one expects the presence of fundamental scalar fields associated to inflation, dark matter and dark energy. In order to concretely discuss the matter we will consider a fairly general scalar-tensor theory of gravity as an example, and indicate how its main features can be extracted from high-resolution measurements of the parametrized post-Newtonian (PPN) parameters.
$\beta$ and $\gamma$. As is well known, scalar-tensor theories of gravity mimic a plethora of unification models. For instance, the graviton-dilaton system in string/M-theory can be viewed as a specific scalar-tensor theory of gravity.

Of course, one should bear in mind that current experimental data shows an impressive agreement with General Relativity\textsuperscript{10,11}. Indeed, most stringent bounds arise from the Cassini’s 2003 radiometric experiment\textsuperscript{12}:

$$\gamma - 1 = (2.1 \pm 2.5) \times 10^{-5} ,$$

and

$$\beta - 1 = (1.2 \pm 1.1) \times 10^{-4}$$

that arises from limits on the Strong Equivalence violation parameter, $\eta \equiv 4\beta - \gamma - 3$, that are found to be $\eta = (4.4 \pm 4.5) \times 10^{-4}$, as inferred from LLR measurements\textsuperscript{13}.

As already mentioned, in cosmology, General Relativity allows for detailed predictions of the nucleosynthesis yields and of the properties of the CMBR, provided one admits the presence of fundamental scalar fields, the inflaton, the quintessence scalar field\textsuperscript{14} or the generalized Chaplygin gas model underlying scalar field, complex\textsuperscript{15} or real\textsuperscript{16}, to account for the late accelerated expansion of the Universe, and in the case of some candidates for dark matter, self-interacting\textsuperscript{17} or not\textsuperscript{18}.

It is worth remarking the generalized Chaplygin gas model corresponds to a unified model of dark energy and dark matter, based on the equation of state $p = -A/\rho^\alpha$, where, $p$ is the isotropic pressure, $\rho$ is the energy density, and $A$ and $\alpha$ are positive phenomenological constants. Its agreement with observational data has been extensively studied: CMBR\textsuperscript{15}, supernova\textsuperscript{16,20,21}, gravitational lensing\textsuperscript{21}, gamma-ray bursts\textsuperscript{22} and cosmic topology\textsuperscript{23}. A fully consistent picture for structure formation in the context of the generalized Chaplygin gas model remains still an open question\textsuperscript{24}.

Another interesting cosmological issue concerns the resemblance of inflation and the late accelerated expansion of the Universe, which has lead to proposals where the inflaton and the quintessence scalar field are related\textsuperscript{25}.

A scalar field with a suitable potential can be also the way to explain the Pioneer anomaly\textsuperscript{26}. It is interesting to point out that scalar fields can affect stellar dynamics and hence, specific measurements of, for instance, the central temperature of stars and their luminosity can allow for setting bounds on scalar field models\textsuperscript{27}.

2. Scalar-Tensor Theories of Gravity

In many alternative theories of gravity, the gravitational coupling strength exhibits a dependence on a field of some sort; in scalar-tensor theories, this is a scalar field $\varphi$. The most general action for a scalar-tensor theory of gravity up to first order in the curvature can be written as

$$S = \frac{c^3}{4\pi G} \int d^4x \sqrt{-g} \left[ \frac{1}{4} f(\varphi) R - \frac{1}{2} g(\varphi) \partial_{\mu} \varphi \partial^{\mu} \varphi + V(\varphi) + \sum_i q_i(\varphi) \mathcal{L}_i \right] ,$$
where \( f(\phi), g(\phi), V(\phi) \) are generic functions, \( q_i(\phi) \) are coupling functions and \( \mathcal{L}_i \) is the Lagrangian density of the matter fields.

For simplicity, we shall consider only the theories for which \( g(\phi) = q_i(\phi) = 1 \). Hence, for a theory for which the \( V(\phi) \) can be locally neglected, given that its mass is fairly small so that it acts cosmologically, the resulting effective model can be written as

\[
S = \frac{c^3}{4\pi G} \int d^4x \sqrt{-g} \left[ \frac{1}{4} \hat{R} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \sum_i \mathcal{L}_i(\hat{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}) \right],
\]

where \( A^2(\phi) \) is the coupling function to matter and the factor that allows one to write the theory in the Einstein frame.

It is shown that in the PPN limit, that if one writes

\[
\ln A(\phi) \equiv \alpha_0(\phi - \phi_0) + \frac{1}{2} \beta_0(\phi - \phi_0)^2 + O((\phi - \phi_0)^3),
\]

then

\[
\gamma - 1 = -\frac{2\alpha_0^2}{1 + \alpha_0^2},
\]

and

\[
\beta - 1 = \frac{1}{2} \frac{\alpha_0^2 \beta_0}{(1 + \alpha_0^2)^2}.
\]

Most recent bounds arising from binary pulsar PSR \( B1913 + 16 \) data indicate that

\[
\beta_0 > -4.5 \quad , \quad \alpha_0 < 0.060
\]

and

\[
\frac{\beta - 1}{\gamma - 1} < 1.1.
\]

These results are consistent with Solar System constraints and one expects that improvement of data may allow within a decade to achieve \( |\gamma - 1| \sim 10^{-6} \), an order of magnitude better than Cassini’s constraint. Notice that the PPN formalism for more general cases is available. For sure, gravitational experiments in space will allow to further constrain these models.

It is relevant to point out that scalar-tensor models have also been proposed to explain the accelerated expansion of the Universe, even though not quite successfully.

3. Gravitational Experiments in Space

Let us now give some examples of gravitational experiments that critically rely on space technology and that may crucially contribute to clarify some of the discussed issues.
3.1. Lunar Laser-Ranging: APOLLO Facility

The Apache Point Observatory Lunar Laser-ranging Operation (APOLLO) is a new LLR effort designed to achieve millimeter range precision and order-of-magnitude gains in the measurement of physical parameters\(^5\).

The major advantage of APOLLO over current LLR operations is a 3.5 m astronomical high quality telescope at a good site, the Sacramento Mountains of southern New Mexico (2780 m), with very good atmospheric quality. The APOLLO project will allow pushing LLR into the regime of millimeter’s range precision. For the Earth and Moon orbiting the Sun, the scale of relativistic effects is set by the ratio \((GM/rc^2) \sim v^2/c^2 \sim 10^{-8}\). Relativistic effects are small compared to Newtonian effects. The Apache Point 1 mm range accuracy corresponds to \(3 \times 10^{-12}\) of the Earth-Moon distance. The impact on gravitational physics is expected to yield an improvement of an order of magnitude: the Equivalence Principle would give uncertainties approaching \(10^{-14}\), tests of General Relativity effects would be smaller than 0.1%, and estimates of the relative change in the gravitational constant would be about 0.1% of the inverse age of the Universe.

Therefore, the gain in the ability to conduct even more precise tests of fundamental physics is enormous, thus this new instrument stimulates development of better and more accurate models for the LLR data analysis at a mm-level\(^{31}\).

3.2. The LATOR Mission

The proposed Laser Astrometric Test Of Relativity (LATOR)\(^{1,2,3,4}\) experiment is designed to test the metric nature of gravitation, a fundamental postulate of General Relativity. By using a combination of independent time-series of highly accurate gravitational deflection of light in the immediate vicinity of the Sun, along with measurements of the Shapiro time delay on interplanetary scales (to a precision respectively better than \(10^{-13}\) radians and 1 cm), LATOR will considerably improve the knowledge about relativistic gravity. Its main objectives can be summarized as follows: i) Measure the key post-Newtonian Eddington parameter \(\gamma\) with accuracy of a part in \(10^9\), a factor 30,000 beyond the present best result, Cassini’s radiometric experiment\(^{12}\); ii) Perform the first measurement of gravity’s non-linear effects on light to about 0.01% accuracy; including both the traditional Eddington \(\beta\) parameter via gravity effect on light to about 0.01% accuracy and also the never measured spatial metric’s second order potential contribution, \(\delta\); iii) Perform a direct measurement of the solar quadrupole moment, \(J_2\), to accuracy of a part in 200 of its expected size; iv) Measure the “frame-dragging” effect on light due to the Sun’s rotational gravitomagnetic field, to 0.1% accuracy. LATOR’s measurements will be able to push to unprecedented accuracy the search for relevant scalar-tensor theories of gravity by looking for a remnant scalar field. The key element of LATOR is the geometric redundancy provided by the laser ranging and long-baseline optical interferometry.

LATOR mission is the 21st century version of Michelson-Morley-type experi-
Orfeu Bertolami

ment particularly suitable for the search of effects of a scalar field in the Solar System. In spite of the previous space missions exploiting radio waves for spacecraft tracking, this mission will correspond to a breakthrough in the relativistic gravity experiments, as it allows to take full advantage of the optical techniques that have recently became available. LATOR has a number of advantages over techniques that use radio waves to measure gravitational light deflection. Indeed, optical technologies allow low bandwidth telecommunications with the LATOR spacecraft and the use of the monochromatic light enables the observation of the spacecraft at the limb of the Sun. The use of narrow band filters, coronagraph optics and heterodyne detection allows for suppression of background light to a level where the solar background is no longer the dominant source of noise. The short wavelength allows much more efficient links with smaller apertures, thereby eliminating the need for a deployable antenna. Finally, the use of the International Space Station enables the experiment to be above the Earth’s atmosphere, the major source of astrometric noise for any ground based interferometer. We think that these features fully justify LATOR as a fundamental mission in the search for gravitational phenomena beyond General Relativity.

3.3. A Mission to test the Pioneer Anomaly

Pioneer 10 and 11 were launched in 1972 and 1973 to study the outer planets of the Solar System. Both probes have followed hyperbolic trajectories close to the ecliptic to opposite outward directions in the Solar System. Due to their robust design, it was possible to determine their position with great accuracy. During the first years of its life, the acceleration caused by solar radiation pressure on the Pioneer 10 was the main effect. At about 20 AU (by early 1980s) solar radiation pressure became sub-dominant and it was possible to identify an unaccounted anomaly. This anomaly can be interpreted as a constant acceleration with a magnitude of \(a = (8.74 \pm 1.33) \times 10^{-10} \text{ms}^{-2}\) and is directed toward the Sun. This effect became known as the Pioneer anomaly. For the Pioneer spacecraft, it has been observed, at least, until 70 AU. The same effect was also observed in the Pioneer 11 spacecraft.

This puzzling deceleration has divided the space community in the last few years. If on one hand, skeptics have been arguing that the most likely solution for the riddle is some unforeseen on-board generated effect such as fuel leaking from the thrusters or non-symmetrical heat dissipation from the nuclear powered energy sources, the most optimistic point out to the fact that this effect may signal a new force or fundamental field of nature and hence an important window for new physics. The approach that has been advocated by some groups that answered to the recent European Space Agency (ESA) call Cosmic Vision 2015 - 2025 with proposals of missions to test the Pioneer’s anomalous acceleration is that whatever the cause

\(^a\)The demonstration that the gravitational field due to the Kuiper Belt is not the cause of the anomaly has been recently reanalyzed. The literature is particularly rich in proposals.

\(^b\)The demonstration that the gravitational field due to the Kuiper Belt is not the cause of the anomaly has been recently reanalyzed. The literature is particularly rich in proposals.
of the slowing down of the spacecraft, meeting the requirements of such a mission would give rise to developments that will be invaluable for building and designing noise-free spacecraft for future deep space missions. Actually, the theoretical concept of a mission to verify the anomalous acceleration has been suggested earlier in a study\textsuperscript{34} commissioned by ESA in 2002.

A dedicated mission would rely on a simple concept, which consists in launching into deep space a geometrically symmetric\textsuperscript{34} and spin-stabilized\textsuperscript{35} probe whose behavior (mechanical, thermal, electromagnetic, etc.) is carefully monitored. Accurate tracking of its orbit would allow for precise evaluation of the anomaly, as any deviation from the predicted trajectory would be used to examine the unmodeled anomalous acceleration. The exciting possibility of using laser ranging techniques and the flying formation concept to characterize the nature of the anomaly, and solar sailing propulsion has been more recently discussed\textsuperscript{6}. Particularly pleasing is the announcement that ESA is seriously considering such an ambitious and challenging undertaking in the period 2015 - 2025\textsuperscript{36}. Naturally, a mission of this nature can be particularly useful for testing the existence of any Solar System range new interaction as well as to explore, for instance, the structure of the Kuiper Belt.

4. Discussion and Conclusions

Let us now review the main points of our discussion. It seems evident that resolving the dichotomy dark energy - dark matter versus gravity will require a concerted effort and a whole new program of dedicated experiments in space.

It is an exciting prospect that dark matter can be directly detected in underground experiments or in the forthcoming generation of colliders. Even though it does not seem feasible to directly test the properties of dark energy, it is not impossible that indirect evidence can be found in laboratory. The most bold proposal suggests the existence of a cutoff frequency of the noise spectrum in Josephson junctions\textsuperscript{37}, while a more conventional approach is to investigate the effect that dark energy may have, for instance, on the variation of the electromagnetic coupling\textsuperscript{38}. It follows that the characterization of dark energy and dark matter will most likely be achieved via cosmological observations, most of them to be carried out by space-borne experiments. These encompass a large array of phenomena such as supernovae, gamma-ray bursts, gravitational lensing, cosmic shear, etc. The result of these observations will also provide increasingly detailed information on the adequacy of General Relativity at cosmological scales. It is quite exciting that existing supernova data\textsuperscript{39} together with latest CMBR data\textsuperscript{9} and the recently discovered baryon acoustic oscillations\textsuperscript{40} are sufficiently constraining to virtually rule out\textsuperscript{41}, for instance, most of the braneworld inspired gravity models put forward to account for the accelerated expansion of the Universe. The prospect of testing some of these models through the study of the orbital motion of planets in the Solar System has also been recently discussed\textsuperscript{42}.

We have seen how stands the situation in what concerns scalar-tensor gravity
models. Relevant results are expect within a decade from the observation of binary pulsar systems. To further test General Relativity and examine the implications of its contending theories or extensions (scalar-tensor theories, braneworld models, string inspired models, etc.) a new program of gravity experiments in space is clearly needed. We have discussed how LLR can be used to improve the knowledge of relativistic gravity and pointed out how the LATOR mission and a mission to test the Pioneer anomaly can play a key role in the search for evidence of a remnant scalar field in the Solar System, to identify new forces with ranges of a few decades of AU and, of course, to resolve the Pioneer anomaly puzzle. It is relevant to point out that the latter type of mission, besides its technological appeal, can also to used to gather information about the vicinity of the Solar System as well as to set relevant upper bounds on environmental parameters such as the density of interplanetary dust and dark matter.

Acknowledgments

It is a pleasure to thank the members of the Pioneer Science Team for the countless discussions on the questions related with this contribution. I am particularly in debt to Jorge Páramos, Slava Turyshev, Serge Reynaud, Clovis de Matos, Pierre Toubul, Ulrich Johann and Claus Lämmerzahl for their insights and suggestions.

References

1. S. G. Turyshev, M. Shao and K. L. Nordtvedt, Jr., *Int. J. Mod. Phys.* D 13 (2004) 2035, gr-qc/0410044.
2. S. G. Turyshev, M. Shao and K. L. Nordtvedt, Jr., *Class. Quantum Gravity* 21 (2004) 2773, gr-qc/0311020.
3. S. G. Turyshev, M. Shao and K. L. Nordtvedt, Jr., In Proc. “The XXII Texas Symposium on Relativistic Astrophysics”, Stanford University, December, 2004, eds. P. Chen, E. Bloom, G. Madejski and V. Petrosian. SLAC-R-752, Stanford e-Conf #C041213, paper #0306, eprint: [http://www.slac.stanford.edu/econf/C041213/](http://www.slac.stanford.edu/econf/C041213/), gr-qc/0502113.
4. LATOR Collaboration (S.G. Turyshev et al.), “Fundamental Physics with the Laser Astrometric Test of Relativity”, ESA Spec. Publ. 588 (2005) 11; gr-qc/0506104
5. T. W. Murphy, Jr., J. D. Strasburg,., C. W. Stubbs, E. G. Adelberger, J. Angle, K. Nordtvedt, J. G. Williams, J. O. Dickey and B. Gillespie, “The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO)”, Proceedings of 12th International Workshop on Laser, Ranging, Matera, Italy, November 2000, in press, 2002, [http://www.astro.washington.edu/murphy/apollo/matera.pdf](http://www.astro.washington.edu/murphy/apollo/matera.pdf)
6. PIONEER Collaboration (H. Dittus et al.), “A Mission to Explore the Pioneer Anomaly”, ESA Spec. Publ. 588 (2005) 3; gr-qc/0506139
7. J. D. Anderson, P. A. Laing, E. L. Lau, A. S. Liu, M. M. Nieto and S. G. Turyshev, *Phys. Rev.* D65 (2002) 082004.
8. O. Bertolami, C. J. de Matos, J. C. Grenouilleau, O. Minster and S. Volonte, *Acta Astronautica* 59 (2006) 490.
9. D. N. Spergel et al., “Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology”, astro-ph/0603449.
10. C. M. Will, Proceedings “100 Years of Relativity: Spacetime Structure - Einstein and Beyond”, ed. Abhay Ashtekar (World Scientific, Singapore), “Was Einstein Right? Testing Relativity at the Centenary,” gr-qc/0504086.

11. O. Bertolami, J. Páramos and S. G. Turyshev, Proceedings 359th WE-Heraeus Seminar: “Lasers, Clock, and Drag-Free: Technologies for Future Exploration in Space and Gravity Tests”, University of Bremen, ZARM, Bremen, Germany, 30 May - 1 June 2005, “General Theory of Relativity: Will it survive the next decade?”, gr-qc/0602016.

12. B. Bertotti, L. Iess and P. Tortora, Nature 425 (2003) 374.

13. J. G. Williams, S. G. Turyshev and D. H. Boggs, Phys. Rev. Lett. 93 (2004) 261101.

14. E. J. Copeland, M. Sami and S. Tsujikawa, “Dynamics of dark energy”, hep-th/0603057.

15. M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 66 (2002) 043507.

16. O. Bertolami, A. A. Sen, S. Sen and P. T. Silva, Mon. Not. R. Ast. Soc. 353 (2004) 329.

17. M. C. Bento, O. Bertolami, R. Rosenfeld and L. Teodoro, Phys. Rev. D 62 (2000) 043502; M. C. Bento, O. Bertolami and R. Rosenfeld, Phys. Lett. B 518 (2001) 276.

18. O. Bertolami and F. Nunes, Phys. Lett. B 452 (1999) 108.

19. M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 67 (2003) 063511; Phys. Lett. B 575 (2003) 172; Gen. Relat. Gravit. 35 (2003) 2063.

20. M. C. Bento, O. Bertolami, N. M. C. Santos and A. A. Sen, Phys. Rev. D 71 (2005) 063501.

21. P. T. Silva and O. Bertolami, Astroph. J. 599 (2003) 829.

22. O. Bertolami and P. T. Silva, Mon. Not. R. Ast. Soc. 365 (2006) 1149.

23. M. C. Bento, O. Bertolami, M. J. Rebouças and P. T. Silva, Phys. Rev. D 73 (2006) 043504.

24. M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 70 (2004) 083519.

25. P. J. E. Peebles and A. Vilenkin, Phys. Rev. D 59 (1999) 063505; K. Dimopoulos and J. W. F. Valle, Astropart. Phys. 18 (2002) 287; R. Rosenfeld and J. A. Frie mann, JCAP 0509 (2005) 003; O. Bertolami and V. Duvvuri, “Chaplygin Inspired Inflation”, astro-ph/0603366, to appear in Phys. Lett. B.

26. O. Bertolami and J. Páramos, Class. Quantum Gravity 21 (2004) 3309.

27. O. Bertolami and J. Páramos, Phys. Rev. D 71 (2005) 023521.

28. T. Damour and G. Esposito-Farèse, Phys. Rev. Lett. 70 (1993) 2220; Phys. Rev. D 54 (1996) 1474; D 58 (1998) 042001.

29. G. Esposito-Farèse, “Tests of scalar-tensor gravity”, gr-qc/0409081.

30. O. Bertolami and P. J. Martins, Phys. Rev. D 61 (2000) 064007.

31. J. G. Williams, S. G. Turyshev, and T. W. Murphy, Jr., Int. J. Mod. Phys. D 13 (2004) 567.

32. J. I. Katz, Phys. Rev. Lett. 83 (1999) 1892; L.K. Scheffer, Phys. Rev. D 67 (2003) 084201.

33. O. Bertolami and P. Vieira, Class. Quantum Gravity 23 (2006) 4625.

34. O. Bertolami and M. Tajmar, Report ESA CR(P) 4365 (2002).

35. J. D. Anderson, M. M. Nieto and S. G. Turyshev, Int. J. Mod. Phys. D 11 (2002) 1545.

36. European Space Agency Cosmic Vision: Space Science for Europe 2015 - 2025, BR-247, October 2005.

37. C. Beck and M. C. Mackey, Phys. Lett. B 605 (2005) 295.

38. K. A. Olive and M. Pospelov, Phys. Rev. D65 (2002) 085044; E. J. Copeland, N. J. Nunes, and M. Pospelov, Phys. Rev. D69 (2004) 023501; M. C. Bento, O. Bertolami
and N. M. C. Santos, *Phys. Rev.* **D70** (2004) 107304; O. Bertolami, R. Lehnert, R. Potting and A. Ribeiro, *Phys. Rev.* **D69** (2004) 083513.

39. Supernova Search Team Collaboration (A. G. Riess et al.), *Astroph. J.* **607** (2004) 665.

40. D. Eisenstein et al., *Astroph. J.* **633** (2005) 560.

41. R. Maartens and E. Majerotto, “Observational constraints on self-accelerating cosmology”, astro-ph/0603353; M. C. Bento, O. Bertolami, M. J. Rebouças and N. M. C. Santos, *Phys. Rev. D* **73** (2006) 103521.

42. L. Iorio, *JCAP* **0601** (2006) 008.