Symmetry Restoration at High Temperature in Little Higgs Models?

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Abstract

In this work, we show that the apparent symmetry non-restoration at high temperature for Little Higgs (LH), is not intrinsic feature for LH. We show that when including such dominant thermal corrections, the EW symmetry gets restored.

Keywords: Little Higgs, symmetry breaking, symmetry restoration at high temperature.

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1 Introduction

Recently the little Higgs (LH) mechanism has been proposed as a way to stabilize the weak scale from the radiative corrections of the Standard Model. In little Higgs models, the Standard Model Higgs doublet appears as pseudo-Goldstone in a global symmetry breaking; and it is kept light by approximate non-linear symmetries [1]. The little Higgs mechanism requires that two separate couplings communicate to the Higgs sufficient breaking of the non-linear symmetry to generate a Higgs mass. The weak scale is radiatively generated two loop factors below a cut-off estimated using naive dimensional analysis (NDA), $\Lambda \sim 4\pi f$, where $f \sim 1−10$ TeV, is the vev that breaks the global symmetry. The Higgs mass, in this setup, is protected from quadratic corrections at one-loop above the cut off. The scalar fields correspond to the broken generators of the global symmetry; and the scalar potential is just a combination of effective operators whose gauge and Yukawa origins. The electroweak symmetry breaking (EWSB) is triggered by large one-loop Yukawa contributions [1][2]. There are many variants of this scenario depending on the chosen global symmetry. The famous version is the so-called Littlest Higgs [1], which is based on a global $SU(5)$ symmetry spontaneously broken to $SO(5)$. These models are phenomenologically consistent [3].

However the EW symmetry restoration at temperature seems to be problematic as shown for the Littlest Higgs [4]. This can be understood due to the fact that thermal corrections ($\sim T^2/12$); and quadratic corrections ($\sim 3\Lambda^2/16\pi^2$) are generated at one-loop from the same interactions in any gauge theory. This means that the Higgs thermal corrections will cancel each other also; and the electroweak symmetry will not be restored at high temperature. Indeed, it was shown in Ref. [4], that above such critical temperature $T_c \sim f$, the thermal
corrections become negative and the absolute minimum \((h \neq 0)\), gets deeper at higher temperatures instead of being relaxed to zero.

In this work, taking the Littlest Higgs as an example, we will show that this unusual behavior of the effective scalar potential is realistic but due to incomplete computations, and the symmetry can be restored at high temperatures by taking some ignored corrections. In the next section, we briefly introduce the Littlest Higgs model. In the third section, we discuss the electroweak symmetry breaking and the non-restoration at high temperature. In section 4, we introduce some important higher order corrections to the effective potential and show that the symmetry is restored at high temperatures. Finally we give our conclusion.

## 2 Basics of the Littlest Higgs

The Littlest Higgs model \([1]\) is based on an \(SU(5)/SO(5)\) nonlinear sigma model, where the \(SU(5)\) symmetry is spontaneously broken down to \(SO(5)\) by the vacuum expectation value of a \(5 \times 5\) symmetric matrix scalar field

\[
\Sigma_0 = \begin{pmatrix}
0 & 0 & 1_{2 \times 2} \\
0 & 1 & 0 \\
1_{2 \times 2} & 0 & 0
\end{pmatrix}.
\]

The breaking of the global \(SU(5)\) symmetry results 14 Goldstone bosons: 4 are identified as the Standard Model (SM) Higgs doublet, 6 as complex triplet and 4 as the Goldstone bosons that give masses to the new heavy gauge bosons. These Goldstone bosons can be parameterized through the nonlinear

\[
\Sigma = e^{2\Pi/f} \Sigma_0 e^{2\Pi/f} = e^{2\Pi/f} \Sigma_0,
\]

\[
\Pi = \pi^a X^a.
\]

Here \(X^a\) are the \(SU(5)\) broken generators. The \(SU(5)\) subgroup \((SU(2) \times U(1))_1 \times (SU(2) \times U(1))_2\); is gauged, its diagonal subgroup being the SM electroweak group \(SU(2)_L \times U(1)_Y\). The gauge interactions can be seen from the kinetic term in the Lagrangian

\[
\mathcal{L}_\Sigma = \frac{f^2}{8} \text{Tr} \left\{ (D_\mu \Sigma) (D^\mu \Sigma)^\dagger \right\},
\]

\[
D_\mu \Sigma = \partial_\mu \Sigma - i \sum_i [g_i W^a_i (Q^a_i \Sigma + \Sigma Q^a_i \Sigma^T) + g'_i B_i (Y_i \Sigma + \Sigma Y_i \Sigma^T)],
\]

and

\[
Q^1_i = \begin{pmatrix} \sigma^a/2 \\ h^a_1 \end{pmatrix}, \quad Q^2_i = \begin{pmatrix} -\sigma^a/2 \\ -h^a_2 \end{pmatrix},
\]

\[Y_1 = \text{diag} (-3, -3, 2, 2, 2)/10, \quad Y_2 = \text{diag} (-2, -2, -2, 3, 3)/10; \]

are the gauge generators, \(\sigma^a\) are the Pauli matrices. The gauge couplings \(g_{1,2}\) and \(g'_{1,2}\) are related to the SM ones as \(1/g^2 = 1/g_{1}^2 + 1/g_{2}^2\) and \(1/g'^2 = 1/g_{1}'^2 + 1/g_{2}'^2\). The quark sector involves a new heavy singlet quark \(U\), where the Yukawa interaction is given by

\[
\mathcal{L}_Y = -\frac{\lambda_1 f}{\sqrt{2}} \bar{u}_R \text{Tr} \left\{ (\Sigma_{1\ell} \epsilon \Sigma^\dagger_{1\ell}) [\chi_{1\ell}^T] \right\} - \lambda_2 f \bar{U}_R U_L + h.c.,
\]

with \(\Sigma_1 = \{\Sigma_{im}\}\) and \(m,n = 4,5, i,j = 1,2,3; [\chi_{ij}] = \epsilon_{ijk} \chi_{kk}\) and \(\chi^T_L = (u_L, b_L, U_L)\), where \(u\) and \(U\) are mixtures of the top quark \(t\) and the new heavy quark \(T\). Here \(\lambda_{1,2}\) are related to the top Yukawa coupling as \(1/\lambda_T^2 = 1/\lambda_T^2 + 1/\lambda_T^2\).

In general, the neutral components of both doublet and triplet can develop a real vev. Then the Coleman-Weinberg potential \([2]\), \(V_{CW}(\Sigma)\), is a result of such effective operators coming from two sources: gauge and fermions interactions

\[
V_{CW}(\Sigma) = a_V f^4 \left( g_1^2 \text{Tr} \left\{ (Q^a_i \Sigma) (Q^a_i \Sigma)^\dagger \right\} + g_2^2 \text{Tr} \left\{ (Y_i \Sigma) (Y_i \Sigma)^\dagger \right\} \right)
\]

\[= \frac{a_F}{4} \chi_4^2 f^4 \text{Tr} \left\{ [\Sigma_{1\ell} \epsilon \Sigma^\dagger_{1\ell}] [\Sigma_{1\ell} \epsilon \Sigma^\dagger_{1\ell}] \right\}, \]

where \(a_V, a_F \sim O(1)\) are unknown parameters associated with the effective operators; their values depend on the UV completion of the theory. The Coleman-Weinberg potential is given in the \(h\) and \(\phi\) directions by

\[
V_{CW}(h) = \Theta f^4 \sin^4 \left( h/\sqrt{2}f \right), \quad V_{CW}(\phi) = \Theta f^4 \sin^2 \left( \sqrt{2}\phi/\sqrt{2}f \right),
\]

respectively, with \(\Theta = a_V [g_1^2 + g_2^2 + g_1'^2 + g_2'^2]/4 + a_F \chi_4^2/2\). It is clear that since \(\Theta > 0\), the theory ground state is stable in both of \(h\)- and \(\phi\)-directions; and the EW symmetry is not broken. But when including the one-loop corrections (especially the Yukawa’s), the EW symmetry gets broken and the SM fields develop masses \([1,2]\).
3 Symmetry Breaking and Restoration at High Temperature

In Ref. [4], it was shown in figures (1) and (2) how the EW symmetry gets broken in a slight way after including the one-corrections. While in figure (3), it was shown how thermal corrections help to relax the minimum to zero especially for low temperature values ($T \leq 0.44\, f$); but the maximum of the potential (at $h = \pi f/\sqrt{2}$) is getting down when increasing the temperature until becomes degenerate together with the absolute minimum at a critical temperature $T_c \sim 0.96\, f$. Above this temperature value ($T > f$), this new minimum gets deeper and deeper when increasing temperature. This unusual behavior had lead to the conclusion that symmetry can not be restored at high temperatures in these models.

The scalar potential behavior at high temperature that is mentioned in Ref. [4] can be understood by estimating the field-dependent masses, especially the Yukawa’s, within one period $h \in [0, \sqrt{2}\, \pi f]$. Naively, the gauge field masses vary below the squared of the gauge couplings in units of $f^2$, scalar masses vary below a combination of $a_V g_i^2$ and $a_F \lambda^2$ in units of $f^2$. These masses are significantly small when compared with the two Yukawa eigenmasses which vary between $\{0 \sim \lambda_i^2\}$ and $\{\lambda_i^2 \sim \lambda_1^2 + \lambda_2^2\}$. According to these values, the thermal integral [6], that involves $(m^2/T^2)$ as a variable, will not be suppressed for all fields; and the unsuppressed fermionic contributions, that have a large negative multiplicity ($n_T = n_T = -12$), will dominate the thermal scalar potential since they are $T^4$-proportional. In other gauge theories, SM as an e.g., most of the fields contributions are suppressed for large values of the scalar field $h$, but this not the case here because the scalar sector is periodic. Therefore one concludes that this behavior is a consequence of periodicity of the non-linear nature of the scalar representation; in addition to the largeness of negative fermionic contributions to the thermal effective potential, that can not be compensated by other bosonic contributions. But were all the significant contributions taken into account to obtain this behavior?

4 Thermal Corrections and Symmetry Restoration at High Temperature

The nonlinear nature of the scalar sector implies the existence of a new type of interactions like in Fig.1 when expanding the field matrix $\Sigma$ in powers of $1/f$ in the Lagrangian. [7]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The fields $\varphi_\alpha$ and $\varphi_\beta$ could be scalars, gauge fields or a fermion-antifermion pair.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Higher order loops corrections to the mass-squared matrix element $M_{\alpha\beta}^2$, that come from the interactions in Fig.1}
\end{figure}

These vertices which are proportional to $1/f^{n-2}$; could result higher order loop corrections as shown in Fig.2. The corrections in Fig.2 are not the only possible contractions of the vertices in Fig.1 but they are the dominant contributions since each scalar loop gives $T^2/12$. Therefore, they lead to a thermal correction to the field mass-squared matrix element $[\alpha, \beta]$, of the form

$$m^2(T) \sim m^2 + T^2 \sum_n c_n \left(\frac{T^2}{f^2}\right)^n,$$

where the zeroth order corresponds to the usual thermal corrections. The computation of the $c_n$ parameters for each mass-squared matrix element tends to determine the vertices of the interactions with scalar degrees of freedom [7]. This type of contributions (9) could be very important at temperatures $T \gtrsim f$. One can take these novel corrections into account by doing such a resummation where the field-dependant masses in the thermal
integral [6], could be replaced by thermally corrected masses [9]. In Fig. 3 we show how does the inclusion of these novel corrections affect the thermal effective potential behavior at \( T = 1.7f \) as an example.

\[ V_{\text{eff}}(h)/f^4 - 0.8 \]

\[ V_T(n=0) \]

\[ V_T(n=1) \]

\[ V_T(n=2) \]

\[ V_T(n=3) \]

As it is clear, it is enough to consider only the order \((n = 2)\) in [9] to see that the EW symmetry is restored at this temperature. Here is this example, we have chosen the cut-off scale to be \( \Lambda = 1.3f \), if it is taken to be the NDA value \( \Lambda = 4\pi f \), then we need more corrections \((n > 2)\) to show that the symmetry is restored, and for \( n = 2 \), the EW symmetry is restored at \( T \approx 2.854f \). This mean that the thermal effective potential behavior shown in Ref. [4], is not intrinsic but due the incomplete theory above such a validity scale.

\[ \text{Figure 3: The effective potential at } T=0 \text{ (solid line), and at } T=1.7f \text{ computed in the standard way, and using the resumed thermal masses } [9] \text{ taking into account 1-loop (n=0), 2-loop (n=1), and 3-loop (n=2) corrections.} \]

5 Conclusion

In this work, we have shown that the symmetry non-restoration at high temperature behavior for Little Higgs is not an intrinsic feature for LH, but just due incomplete computation setup. By taking some high order dominant corrections, we found that the symmetry is restored at high temperature for LH.

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