Attractor structure of a class of non-newtonian fluids based on generalized vector in riemannian manifold

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Abstract. In this paper, the canonical representation of point stress state and strain state is studied. Based on the introduction of the concept of the intrinsic elastic modulus of anisotropic elastomer, the concept of the opposite direction of the main body and the structure of the norm space is proposed, to study the orbital attractor. The structure opens up a new approach. The concepts of modal stress and modal strain are discussed. The classical Hooke's law and spectral analysis theory are used to discuss the topological structure of orbital attractors. A new theory of attractor structure-finite strength theory is established, and a class of equations is analysed. The attractor structure.

1. Introduction

The classical Newtonian fluid mechanics believe that in parallel flow, the shear force is proportional to the shear rate, and the proportional coefficient is called the viscosity coefficient. On this basis, the famous NS equation can be obtained. In recent years, people have a growing recognition of the important properties of Newtonian fluids. It has been found that there are a large number of fluids in daily life that do not obey Newton's law of constant viscosity, i.e. non-Newtonian fluids. Non-Newtonian fluids are ubiquitous in nature. This type of fluid, under stress, will continuously change its motion state, and its constitutive relation is significantly different from Newton's law of constant viscosity. However, there are few results in this respect, so the study of non-Newtonian Flow is very necessary.

In the fixed point and application of the ambiguity operator, we will discuss the condition that the nonlinear monotone operator with concavity and convexity has a unique positive fixed point and give the relationship of each concave operator, as well as each convex operator and order convex. On the basis of this, we prove that the necessary and sufficient conditions for the existence of the only positive fixed point of the concave addition operator are given. This paper discusses the existence of unique positive fixed points for monotone operators with ordered concave (convex) and mixed monotone operators with concavity and convexity.

In the study of fluid motion, the vacuum state is often involved and the problem becomes difficult and complicated. The existing results show that the Cauchy problem of the system of equations containing the constant coefficient of the vacuum state is ill-posed. This ill-posedness is reflected in the fact that the solution of this system has no continuous dependence on the initial value. When the initial
density has a tight support set, the system cannot have an overall regular solution. Based on physical considerations, Liu Taiping, Xin Zhouping, and Yang Ke studied the Cauchy problem of the viscosity-dependent equations and proved its local well-posedness. In fact, only when the temperature and density change within the appropriate range, the real fluid can be well described by the ideal fluid viscosity coefficient Zhao and human constant. In the case of large changes in temperature and density, the viscosity coefficient of the real fluid will change greatly with temperature and density. On the other hand, we know that the equation can be derived by the second-order expansion by the equation, and in the derivation process. We will find that the viscosity coefficient is temperature dependent. For example, for a hard ball collision model, the viscosity coefficient should be proportional to the square root of the temperature. If we consider the motion of the helium fluid, the second law of thermodynamics can be used to derive that the viscosity coefficient is density dependent. Therefore, when studying the problem of initial density containing vacuum, we need to consider the effect of density or temperature on the viscosity coefficient. In addition, in geophysics, many of the mathematical models used to study fluid motion are similar to those of viscosity-dependent density equations. For example, the study of shallow water wave systems studies the existence of local smooth solutions of the boundary problem of shallow water wave equations and the global smooth solution of initial values near equilibrium states. For the shallow water wave equation or the overall existence of the large initial value weak solution of the viscosity-dependent high-dimensional equations, there are still many problems that remain unresolved.

In recent years, fractional differential equations have been widely used in various fields of science: for example, mechanics (viscoelastic and viscoplastic theory), biochemistry (polymer and protein models), electrical engineering (propagation of ultrasound), medicine (in machinery) Loaded human tissue models) etc. [1-2]. Some classic works have emerged for example, Oldham and Spanier (1974) [3], and Samko, Kilba, and Marichev 01993) [4], Miller and Ross (1993) [5], Podlubny (1999) [6] written fractional calculus theory monograph. Mer "Application of Fractional Calculus in Physics" [7], Mainardi (2010) "Linear viscoelastic fractional micro Integral and Fluctuation: An Introduction to Mathematical Models [8], Tarasov (2010) on Fractional Dynamic Systems: Application of Fractional Calculus for Particles, Fields, and Media [9], and Klafter and Sokolov (2011) for Power Laws The discussion of continuous-time random walk models with waiting time distributions leads to several types of fractional differential equation models [10], Meerschaert and Sikorskii (2012) of the stochastic model of fractional calculus [11], Atanackovic et al. (2014) Fractional order micro product "Application in Mechanics". Includes "Fractal Derivative Modeling of Mechanics and Engineering Problems" prepared by Chinese scholar Chen Wen and others (2010) [7] and Academician Guo Boling et al. (2011) Differential equations and their numerical solutions [4] and other monographs.

Due to the non-local nature of fractional-order operators, most solutions of fractional-order differential or integral equations are difficult to represent with analytical expressions. With the extensive establishment and application of fractional-order calculus models in different fields, how to solve the score efficiently numerical algorithms for order differential or integral equations have become an urgent issue. Fortunately, there are some important numerical algorithms for solving fractional differential equations, such as finite difference methods [10] finite element method, spectral method... [11] and fast algorithm.

We apply the non-differentiable vector optimization problem and the generalized weak vector variational inequality method on Riemannian manifolds. We discuss a class of nonlinear equations in Banach space, combining nonlinear operator equations with nonlinearities and nonlinear integral fusion differential equations. A half-order method of the attractor structure will be given.

In order to clarify the structure of the attractor, we allow the appearance of vacuum. In order to overcome the strong coupling and singularity of the equation, we have regularized the viscous term with singularity and proved the attractor without vacuum under spectral analysis. The existence of the equation proves the topology of the equation attractor. The equation we discuss is as follows:

$$ \rho_t + (\rho u)_x + \text{div} u = \Delta u \quad (x,t) \in \Omega_T $$  \hspace{1cm} (1)
The unknown quantity \( \rho, u, \eta, P(\rho) \) represents the fluid density, velocity, density and pressure of the particles in the mixture, \( a > 0, \gamma > 1, 1/3 < p < 2 \), given the function \( f \) represents external potential energy, such as gravity and buoyancy. In particular, if \( f \) is the gravitational potential, then \( \alpha \) is the gravity constant, \( \gamma > 0 \) is the viscosity coefficient, \( \eta \) is a constant.

2. Main Result

In the analysis of the Riemannian manifold, a large part is the study of the harmonicity of the functions on the manifold, and the properties of these functions are obtained from them. For example, the derivation of the mean inequality of the hypoharmonic function, the existence of the harmonic function Proof, etc. In 2000, Peter Li proposed another function, the so-called \( \psi \)-harmonic function, which is much more extensive than the harmonic function, but at the same time has similar properties to the harmonic function. Some basic properties of the harmonic function and its \( \psi \)-harmonic function, some of which are simple extensions of the corresponding properties of the harmonic function. Finally, we will get the mean inequality of the \( \psi \)-harmonic function. First, we give the relevant definition.

Definition 1.1 M is an n-dimensional Riemannian manifold, \( \psi \in C^\infty(M), u \in C^\infty(M) \) is called \( \psi \)-harmonic if \( \text{div}(e^\psi \Delta u) = 0 \) is established on M. Similarly, if \( \text{div}(e^\psi \Delta u) \geq 0 \) is established on M, then \( u \) is called \( \psi \)-harmonic.

Definition 2 Let \( K \) be a non-empty subset of \( M \), \( \eta: M \times M \to TM \) is a vector-valued function, and \( \eta(x, y) \in T_yM \) is established for any \( x, y \in M \). If there is a unique geodesic \( \gamma: [0, 1] \to M \), such that \( \gamma(0) = y, \gamma'(0) = \eta(x, y), \gamma(t) \in K, t \in [0, 1] \) for any \( x, y \in K \), then the \( K \) is called Invariant convex set for \( \eta \).

Lemma 2 Real square \( A \) is positive definite if and only if \( H = (A + A^T)/2 \) is positive definite. Let \( H \in R^{n \times n} \) be a symmetric matrix, then \( H \) is positive definite if and only if all the eigenvalues of \( H \) are positive. Let \( A \in C^{n \times n} \), \( H = (A + A^T)/2 \), then the real part \( R(\lambda(A)) \) of all eigenvalues satisfy

\[
\lambda_{\min}(H) \leq \Re(\lambda(A)) \leq \lambda_{\max}(H)
\]  

(4)

Where, \( \lambda_{\min}(H) \) and \( \lambda_{\max}(H) \) are the minimum eigenvalue and the maximum eigenvalue of \( H \), respectively.

Theorem 1 Under conditions in the above description, there exists a unique global \( H^2 \)-solution \( (\rho(x, t), u(x, t)) \) to the problem (1)-(3), such that for any \( T > 0 \),

\[
\|\rho(t)\|_{H^1}^2 + \|u(t)\|_{H^1}^2 + \int_0^T (\|\rho(t)\|_{L^2}^2 + \|u(t)\|_{L^2}^2) dt \leq C(T)
\]  

(5)
3. Proof of Main Result

For the non-viscous system (1)-(3), the comparison principle can be studied under the condition that the boundary kernel \( K_{ij}(x, y) \) satisfies various conditions; and in (4)-(5), the following boundary is satisfied.

Conditional semilinear, non-viscous systems we can establish a comparison principle:

\[
A_i u_i = \int_{\Omega} K_{ij}(x, y) u_i(t, y) \, dt + \varphi_i(x) \quad (t > 0, x \in \partial \Omega)
\]  

For each of \( i = 1, 2, \ldots, n \), \( A_i \) is a function defined on \( x \). Here we note that if we do not care about the influence of function \( \varphi_i(x) \), the boundary conditions of the system (1)-(3) are wider than the boundary condition (4) and it is suitable. In fact, we can see from the discussion that follows. Out, our comparison results can be generalized to a limited boundary

\[
B_{ij}(x, y) \leq \int_{\Omega} \sum_{i=1}^{n} B_{ij}(x, t) \, dt < 1 \quad (x \in \partial \Omega, \ t \in [0, T])
\]

3.1. Proofs of Theorem

For the two systems of reaction-diffusion equations on unbounded regions, we can study its comparison principle under Dirichlet boundary conditions, and the established comparison principle is a more general result. In particular, our results are to some extent. The monotonic hypothesis about the response function was removed.

Let \( \Omega \) be an unbounded area of \( R^n \) with an unbounded inner diameter, that is

\[
d(\Omega) = \sup_{x \in \Omega} \text{dist}(x, \partial \Omega) = + \infty
\]

Including the whole space, the outer area of the bounded area, and other unbounded areas, such as the positive half space, by the quasi-monotonicity of \( f_i(u, v), \ i = 1, 2 \) and the quasi-monotonic non-existence of \( g_i(u, v), \ i = 1, 2, 3, 4 \), it is known that for each \( i = 1, 2, 3, 4 \), \( I_i(u_1, u_2, u_3, u_4) \) is intended to be monotonous. Less about \( u_i \). We can calculate directly further

Inspired by the above work, under some cycle assumptions about operators and response functions, we can use the comparison principle established above, combined with the monotonic method and the Bootstrap regularization technique of parabolic equations, we can study non-autonomous systems) The asymptotic behavior of the solution of the periodic system finds the maximum periodic solution and the minimum periodic solution of the system. The reaction function has quasi-monotonic and non-decreasing properties and the systems (1)-(3) have a pair of ordered upper and lower solutions. Then the system has a maximum periodic solution and a minimum periodic solution and is determined by the maximum periodic solution and the minimum period of the system. The fan shape is an attractor of the system.

First, we assume

\[
P_{ij}(x, y) \geq 0
\]

\[
\iint_{\Omega} \sum_{i=1}^{n} F_{ij}(x, t) \, dx \, dt \leq 1 \quad (x \in \Omega, t \in [0, T])
\]
Assume that \( f(x,t) \) is not monotonous with respect to \( u \), and use the principle of comparison, one can get

\[
\iint_{\Omega} P^2(x,y) \leq \frac{1}{\|\Omega\|} \tag{11}
\]

There exist constant \( C > 0 \), \( \alpha > 0 \), such that the solution of (1)-(3) satisfies:

\[
\| u(x,t) \| \leq C e^{-\alpha t} \tag{12}
\]

\[
B_i u_i(x,t) \leq \iint_{\Omega} \sum_{j=1}^{n} P_{ij} (x,t) u_i(x,t) dxdt \quad (x,t) \in \Omega \tag{13}
\]

Since the function sequence is given by a linear parabolic system, it is known from the basic theory of classical parabolic partial differential equations that each component can be represented by an integral equation, using the basic solution of the corresponding parabolic operator and the Green function. Let \( m \to \infty \) in the integral equation, and use (10), similar to the discussion above, one can get

\[
p_i(x,t) = u_{T_i}(x,t) - u_{T_i}(x,t+T), i = 1, 2, \cdots, n \tag{14}
\]

\[
\left. \frac{\partial p_i(x,t)}{\partial t} \right|_{t=0} - \Gamma_i(t) y_i(x,t) = \iint_{\Omega} \sum_{i=1}^{n} L_{ij}(x,y) \sigma_i(x,t) dxdt \quad (x,t) \in \Omega \tag{15}
\]

\[
\lim_{i \to \infty} \sup (u(x,t) - u_{T_i}(x,t)) \geq 0 \tag{16}
\]

Using the integral estimation of the parabolic partial differential equation and the convergence of the function columns, similar to the above discussion, the function column is converged in the middle. Schauder Estimation from Linear Parabolic Equations

\[
\| u_i^k - \omega_T \|_{C^1(T^{1+\alpha})} \to 0 \tag{17}
\]

Differentiating (1.4) with respect to \( x \), exploiting (1.8), we have

\[
u_{xt} = (\rho^{x} + \rho^{1+\rho} u_{xx}) \gamma (\gamma - 1) \rho^{x} \rho^{x-1} + (\theta + 1) \theta \rho^{\alpha-1}
\]

\[
\leq \rho^{2x} u_x + (\theta + 1) \rho^{\rho} \rho^{x} u_x + 2(\theta + 1) \rho^{\rho} \rho^{x} u_x + \rho^{\rho+1} u_{xx} x \tag{18}
\]

Integrating (14) with respect to \( t \) over \([0, t]\), using Lemmas 1 and the interpolation inequality, we derive

\[
\| \rho_{xx}(t) \|^2 + \iint_{\Omega} \| \rho_{xx}(s) \|^2 d\Omega
\]
The proof is complete.

We pass to the local attractor invariance. Namely, the above theorem holds.

4. Conclusion

This paper mainly studies the long-term behavior of solutions of nonlinear diffusion equations, which is an important part of the study of nonlinear partial differential equations. The diffusion equations are derived from the widespread diffusion phenomena in nature, in seepage theory, phase transition theory, image processing, in the fields of biochemistry and biological group dynamics, such equations are proposed. In view of the fact that nonlinear equations can more realistically describe actual phenomena than linear equations, in many cases, people need to consider nonlinear equations, so that the study of nonlinear problems appears. Especially important, it also attracts many mathematicians at home and abroad. In these equations, nonlinearity can come from many aspects, such as diffusion term, reaction term, convection term, etc. Due to the existence of nonlinearity, the research of the problem has been greatly difficulties; the problem processing methods are also very different for different nonlinear terms. In recent decades, there have been many advances in the study of nonlinear diffusion equations, and many results have been achieved, among which the seepage equation and the study of the non-percolation equation as a typical nonlinear diffusion equation have attracted the attention of many mathematicians. Classic work surface is still relatively rare, and research on issues such attractors of equations with the source of the problem has been one of the key workers in the study of mathematics.

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