Brane world solutions of perfect fluid in the background of a bulk containing dust or cosmological constant

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The paper presents some solutions to the five dimensional Einstein equations due to a perfect fluid on the brane with pure dust filling the entire bulk in one case and a cosmological constant (or vacuum) in the bulk for the second case. In the first case, there is a linear relationship between isotropic pressure, energy density and the brane tension, while in the second case, the perfect fluid is assumed to be in the form of chaplygin gas. Cosmological solutions are found both for brane and bulk scenarios and some interesting features are obtained for the chaplygin gas on the brane which are distinctly different from the standard cosmology in four dimensions.

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I. INTRODUCTION

Recently there has been a proliferation of brane world models, where the standard model matter is confined in a four dimensional space time said to be a singular hypersurface or 3-brane embedded in a (4 + d)-dimensional space time. This (4 + d)-dimensional space time is said to be the bulk. Though the matter is all confined to a 3-brane, the gravitational field can propagate in the higher dimensional bulk. The d extra dimensions, however, need not be small or even compact in this theory. In fact, Randall and Sundrum [1] have shown that for d = 1 the fifth dimension may be infinite. The effective equations for gravity in four dimensions were obtained by Shiromizu et al [2] using Israel’s boundary conditions and Z²-symmetry for the bulk space time, in which the brane is embedded. It has been demonstrated by Binétruy et al [3,4] that generically the equations governing the cosmological evolution of the brane will be different from those corresponding to the analogous Friedmann equations of standard cosmology. Essentially the difference lies in the fact that the energy density of the brane appears in quadratic form in the brane equations, whereas it is linear in the standard cosmology.

In our paper, section II presents the derivation of brane equations starting from the Einstein equations in the bulk with a general form of matter there. Then in section III, assuming pure dust in the bulk, Einstein equations in the brane show that only a kind of barotropic equation of state \(p_b = -\frac{2}{3}\rho_b\) holds between the effective isotropic pressure \(p_b\) and effective energy density \(\rho_b\) (i.e., a linear relation among the actual pressure and energy density in the brane and the brane tension). Cosmological solutions are obtained for both bulk and brane scenario. Section IV deals with perfect fluid in the form of chaplygin gas in the brane model. When the bulk is vacuum, then the dynamics shows initial deceleration of the brane with subsequent acceleration going over to the ΛCDM model. On the other hand, when the bulk has negative 5D cosmological constant, i.e., an anti de-Sitter bulk, there occur two different cases. Depending on the parameters in the field equations, the brane may either transit from a decelerating stage to an accelerating stage.

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in course of time or it may simply exhibit a recollapsing behaviour. The paper ends with the concluding remarks.

II. BASIC EQUATIONS

We shall assume that the geometry of the five dimensional bulk is characterized by the space time metric of the form

\[ ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = g_{ij} dx^i dx^j + b^2 dy^2 \] (1)

with \( y \) as the fifth coordinate. The hypersurface \( y = 0 \) is identified as the world volume of the brane that forms our universe. As usual, for simplicity, the explicit form of the metric (1) is taken to be

\[ ds^2 = -n^2(t, y) dt^2 + a^2(t, y) \delta_{ij} dx^i dx^j + b^2(t, y) dy^2 \] (2)

where for simplicity we choose the usual spatial section of the brane to be flat.

Now for cosmological solutions, we shall have to specify the matter explicitly. In fact, we shall classify the matter as of two distinct parts namely

i) matter confined to our brane universe and
ii) matter distributed over the 5D bulk.

Hence, the energy momentum tensor can be decomposed into

\[ \bar{T}_{\mu\nu} = \tilde{T}_{\mu\nu}^{\text{bulk}} + T_{\mu\nu}^{\text{brane}} \] (3)

where the explicit form of the matter is [11, 3, 4]

\[ \tilde{T}_{\mu\nu}^{\text{bulk}} = \text{diag}(\rho_B, p_B, p_B, p_B, p_5) \] (4) and

\[ T_{\mu\nu}^{\text{brane}} = \frac{\delta(y)}{b} \text{diag}(-\rho_b, p_b, p_b, p_b, 0) \] (5)

In general, energy-momentum tensor in the bulk depends on \( t \) and \( y \) (i.e, \( \rho_B = \rho_B(t, y) \)) but to recover a homogeneous cosmology in the brane, matter field in the brane is assumed to be the function of time alone. Also the brane is assumed to be infinitely thin. Now the explicit form of the five dimensional Einstein equations

\[ \bar{G}_{\mu\nu} = \kappa^2 \bar{T}_{\mu\nu} \] (6) for the above metric (2) having matter distribution in the form (3) are [11, 3]

\[ \frac{3}{n^2} \left[ \frac{\ddot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left\{ \frac{a''}{a} + \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) \right\} \right] = \kappa^2 \left( \rho_B + \frac{\delta(y)}{b} \rho_b \right) \] (7)

\[ \frac{1}{b^2} \left[ \frac{\dot{a}'}{a} \left( \frac{\dot{a}}{a} + 2 \frac{n'}{a} \right) - \frac{\dot{b}'}{b} \left( \frac{n'}{n} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right] \]

\[ + \frac{1}{n^2} \left[ \frac{\ddot{a}}{a} \left( -\frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - 2 \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \left( -\frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\dot{b}}{b} \right] = \kappa^2 \left( p_B + \frac{\delta(y)}{b} p_b \right) \] (8)
\[
3 \left( \frac{n' \dot{a}}{n a} + \frac{a' \dot{b}}{a b} - \frac{\dot{a}'}{a} \right) = 0
\]

(9)

and

\[
\frac{3}{b^2} \left[ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{n^2} \left( \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right) \right] = \kappa^2 p_5
\]

(10)

where an overdot stands for differentiation with respect to time \( t \) and a prime indicates derivative with respect to the fifth coordinate \( y \).

For brane cosmology, we need solution of the Einstein’s equations (7)-(10) as \( y \to 0 \). Now, for a well defined geometry, the metric tensor should be continuous across the brane (i.e., \( y = 0 \)) while its derivative with respect to \( y \) may be discontinuous at the hypersurface \( y = 0 \). As a result, Dirac delta function will appear in the second derivatives of the metric coefficients with respect to \( y \) namely [3]

\[
a'' = (a'') + [a'] \delta(y)
\]

(11)

Here \((a'')\) is the non-distributional part of the second derivative i.e, the usual second order derivative and \([a']\) is the measure of discontinuity of first derivative across \( y = 0 \) (with \( \delta(y) \), the Dirac delta function) as

\[
[a'] = a'(0+) - a'(0-)
\]

(12)

If we now proceed to the limit as \( y \to 0 \) and match the Dirac delta function in both sides of the field equations (7) and (8) we obtain

\[
\frac{[a']}{a_0 b_0} = -\frac{\kappa^2}{3} \rho_b
\]

(13)

\[
\frac{[n']}{n_0 b_0} = \frac{\kappa^2}{3} (3 \rho_b + 2 \rho_5)
\]

(14)

where subscript '0' in the scale factors indicate their values on the brane.

One may note that the jump of the field equation (9) using equations (13) and (14) results in the energy conservation relation on the brane namely

\[
\dot{\rho}_b + 3 (\rho_b + p_b) \frac{\dot{a}_0}{a_0} = 0
\]

(15)

Further, if we take the average value of the field equation (10) for \( y \to 0^+ \) and \( y \to 0^- \) and impose the \( Z_2 \)-symmetry we get [3]

\[
\frac{\dot{a}_0}{a_0} + \frac{a_0^2}{a_0^5} = -\frac{\kappa^4}{36} \rho_b (\rho_b + 3 \rho_5) - \frac{\kappa^2 p_5}{3 \rho_b}
\]

(16)

Here we have chosen \( n_0 = 1 \), which is allowed by a suitable time transformation and hence \( t \) is now the usual cosmic time on the brane. This equation can be termed as generalized Friedmann type equation on the brane. The basic difference with the corresponding equation in standard cosmology is that, here the square of the Hubble parameter depends quadratically on the brane energy density, in contrast with the usual linear dependence for Friedmann universe.
Moreover, by introducing a function $\xi(t, y)$ defined by

$$\xi(t, y) = \frac{(a'a)^2}{b^2} - \frac{(\dot{a}a)^2}{n^2}$$

(17)

the five dimensional field equations (7) and (10) can be written in compact form as (for $y \neq 0$)

$$\xi' = -\frac{2}{3} a'a^3 \kappa^2 \rho_B$$

(18)

$$\dot{\xi} = \frac{2}{3} \dot{a}a^3 \kappa^2 \rho_B$$

(19)

Now equating the time derivative of equation (18) with the $y$-derivative of equation (19) we have

$$a'\rho_B' + \dot{\rho_B} + (\rho_B + p_5) \left( \ddot{a'} + 3 \frac{\dot{a}a'}{a} \right) = 0$$

(20)

As we have assumed $\bar{T}_{05} = 0$ indicating that there is no flow of energy along the extra dimension, the Bianchi identity $\nabla_\mu G^{\mu 0} = 0$ results

$$\rho_B' + \frac{3}{a} (\rho_B + p_B) + \frac{b}{b'} (\rho_B + p_5) = 0$$

(21)

Also taking average value of the equation (17) for $y \to 0^+$ and $y \to 0^-$, imposing the $Z_2$-symmetry and using the junction condition (13) we obtain the generalized Friedmann equation

$$\frac{a_0^2}{a_5^2} = \frac{\kappa^4}{36} \rho_B^2 - \frac{\xi_0(t)}{a_5^2}$$

(22)

where $\xi_0$ is the value of $\xi$ on the brane.

Therefore, the cosmological evolution on the bulk is completely characterized by the Einstein field equations (7)-(10) (also (18), (19)) and by the conservation equations (20) and (21). On the other hand, brane cosmology will be determined by equations (16) and (22), junction conditions (13) and (14) and by the matter conservation on the brane (15).

III. BRANE COSMOLOGY FOR DUST FILLED BULK

If the bulk matter is assumed to be in the form of pure dust i.e, $p_B = p_5 = 0$, then integrating the conservation equation (21) we get

$$\rho_B(t, y) = \rho_1(y)/a^3 b$$

(23)

where $\rho_1(y)$ is function of the coordinate $y$ alone.

Also comparing the conservation equations (20) and (21) and using the field equation (9) we have

$$n' = 0 \text{ (so that } n \text{ may be assumed to be unity without the loss of generality)} \text{ and } a' = \alpha(y) b \text{ (24)}$$

with $\alpha$, an arbitrary function of $y$. 

As $p_b = 0$, the equation (19) yields $\xi = \beta(y)$, a function of $y$ alone which again in view of (18) satisfies the relation

$$\beta'(y) = -\frac{k^2 \rho_b}{6} (a^4)'.$$

(25)

Now using (24) in (17), we obtain the differential equation for $a$ in the form

$$a^2 a'^2 = \alpha^2 a^2 - \beta$$

which on integration gives the solution for the scale factor on brane in the form

$$a^2(t, y) = \frac{1}{\alpha^2(y)} \left[ \beta(y) + \{\alpha^2(y)t + t_0(y)\}^2 \right]$$

(26)

with $t_0(y)$, an arbitrary integration function.

In order to obtain cosmological solutions on the brane, we now consider the generalized Friedmann equation (22) written earlier

$$\frac{\dot{a}_b^2}{a_5^2} = \frac{k^4}{36 \rho_b^2} - \frac{\beta_0}{a_5^2}$$

(27)

Further, combining (24) with the junction condition (14) we get for the effective equation of state for the brane matter

$$p_b = -\frac{2}{3} \rho_b$$

(28)

Equivalently, as

$$p_b = p - \lambda \quad \text{and} \quad \rho_b = \rho + \lambda$$

(29)

with $\lambda$ representing the brane tension, the above equation of state yields the following linear relation

$$p = \frac{2}{3} \rho + \frac{1}{3} \lambda$$

(30)

Thus for dust bulk, the fluid in the brane must satisfy the above linear relation between isotropic pressure, energy density and brane tension. Using now the above equation of state (28) into the brane conservation equation (15) we get

$$\rho_b = \rho_0 / a_0, \quad \rho_0 \text{ here is simply a constant.}$$

(31)

Now since $a' = \alpha(y)b$ as is evident from the equation (24), the brane must satisfy the following relation

$$[a'] = 2b_0 \alpha_0$$

and hence using equations (13) and (31), the arbitrary constants $\rho_0$ and $\alpha_0$ above are shown to be related as

$$\alpha_0 = -\frac{k^2 \rho_0}{6}$$

(32)

Further, for the dust model in the bulk and the equation of state (28) in the brane, the Friedmann type equation (16) simplifies to
\[ 2a_0 \dot{a}_0 + a_0^2 = \mu, \quad \text{where} \quad \mu = \frac{\kappa^4 \rho_0^2}{18} \] (33)

which has the first integral

\[ \dot{a}_0^2 = \frac{\mu}{2} + \frac{\mu_0}{a_0^2} \] (34)

where \( \mu_0 \) is an integration constant. This is fully consistent with the equation (27) provided \( \mu_0 = -\beta_0 \).

Now further integration of the equation (34) yields the explicit expression for the brane scale factor \( a_0 \) as

\[ a_0^2 = 2\mu (t_1 \pm t)^2 - \frac{2\mu_0}{\mu} \] (35)

Here since \( \ddot{a}_0 = -\frac{\mu_0}{a_0^4} \), we always get either an accelerating model or a decelerating model of the brane universe depending on the sign of the constant \( \mu_0 \), but there can not be any midway transition from the decelerating phase to the accelerating one (or the reverse). The choice of sign in front of \( t \) in equation (35) leads to an expanding (+ve sign) or a contracting (-ve sign) brane world.

**IV. CHAPLYGIN GAS BRANE MODEL**

We consider here a modified chaplygin gas [12], which satisfies an equation of state in the following form

\[ p_b = A\rho_b - \frac{B}{\rho_b^\alpha} \] (36)

where \( A \) and \( B(> 0) \) are constants and \( 0 < \alpha \leq 1 \).

So in this case from the energy conservation equation (15), the expression for matter density is given by [12]

\[ \rho_b = \left\{ \frac{1}{1 + A} \left\{ \frac{\rho_2}{a_0^{(1+A)/(1+\alpha)}} + B \right\} \right\}^{1/\alpha} \] (37)

with \( \rho_2 \), an arbitrary integration constant.

**Case-I: Vacuum Bulk**

As there is no matter in the bulk we have \( \tilde{T}_{\mu \nu}^{\text{bulk}} \equiv 0 \) and the field equations (18) and (19) show that \( \xi \) must be a constant (say, \( C \)). Hence we have the generalized Friedmann equation (see eq. (22))

\[ \frac{a_0^2}{a_0^2} = \frac{\kappa^2}{36} \rho_0^2 + C \] (38)

Also the generalized Friedmann type equation (16) now simplifies to

\[ \frac{\dot{a}_0^2}{a_0^2} = -\frac{\kappa^4}{36} \rho_b (\rho_b + 3p_b) \] (39)

Further, our calculation will be much simplified if we choose \( \alpha = 1 \) in the equation of state (36) and hence from (37) we get the differential equation for the scale factor \( a_0 \) in the form
\[ \ddot{a}_0 = -\frac{\kappa^4 (3A + 2)}{36 (A + 1)} \frac{\rho_0}{a_0^{6(A+5)}} + \frac{\kappa^4 Ba_0}{36(1 + A)} - \frac{C}{a_0^3} \]  

(40)

Note that as \( a_0 \to 0 \) (early epoch) \( \rho_b \to \infty \), \( \ddot{a}_0 \to -\infty \) while as \( a_0 \to \infty \), \( \rho_b \to -\left(\frac{B}{1 + A}\right)^{1/2} \) which in view of (36) leads to \( \rho_b \to -\left(\frac{B}{1 + A}\right)^{1/2} \) and \( \ddot{a}_0 \to \infty \). This shows that asymptotically the equation of state reduces to \( p_b = -\rho_b \), which points to a \( \Lambda \)CDM model in the corresponding FRW universe of standard cosmology. But for some special cases of the brane world models (shown in what follows) the situation is different, since here \( \ddot{a}_0 \) may remain negative throughout even if the scale factor \( a_0 \) increases indefinitely. Such behaviour is not consistent in the four dimensional Friedmann model containing chaplygin gas, which always asymptotically goes over to the \( \Lambda \)CDM model with the increase of the scale factor.

We now proceed to solve equation (40) for \( A = \frac{1}{3} \) (for which, solution in closed form is possible). The first integral of equation (40) can be written in an integral form as

\[ \frac{1}{4} \int \frac{dz}{\sqrt{bz^2 + Cz + d}} = \pm (t - t_0) \]  

(41)

with \( z = a_0^4 \), \( b = \frac{\kappa^4 B}{48} \), \( d = \frac{\kappa^4 \rho_0}{48} \), solutions of which are given by

\[ a_0^4 = \frac{\sqrt{4bd - C^2}}{2b} \sinh \left[ \pm 4\sqrt{b} (t - t_0) \right], \quad \text{(when } 4bd - C^2 > 0) \]  

(42)

\[ a_0^4 = \frac{\sqrt{C^2 - 4bd}}{2b} \cosh \left[ \pm 4\sqrt{b} (t - t_0) \right], \quad \text{(when } 4bd - C^2 < 0) \]  

(43)

and

\[ a_0^4 = \frac{1}{2b} e^{\pm 4\sqrt{b} (t - t_0)} - \frac{C}{2b}, \quad \text{(when } 4bd - C^2 = 0) \]  

(44)

Graphically, it can be seen that for equation (42), the scale factor expands exponentially whereas for equation (43), it decreases to a minimum before further expansion.

**Case-II: Bulk with Cosmological Constant**

In this case we choose in the bulk \( p_B = p_5 = -\rho_B \). It follows then from Bianchi identity, i.e, the conservation relation (21) demands \( \dot{\rho}_B = 0 \), which in turn in view of (20) yields \( \rho_B = 0 \). Hence \( \rho_B \) is purely a constant said to be the cosmological constant in five dimension (\( \equiv \Lambda_5 \)). Now the generalized Friedmann like equations (38) and (40) are modified in this case as

\[ \frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^2}{6} \Lambda_5 + \frac{\kappa^4}{36} \rho_b^2 + \frac{C}{a_0^3} \]  

(45)

and

\[ \ddot{a}_0 = -\frac{\kappa^4 (3A + 2)}{36 (A + 1)} \frac{\rho_0}{a_0^{6(A+5)}} + \frac{\kappa^4 Ba_0}{36(1 + A)} - \frac{\kappa^2}{6} \Lambda_5 a_0 - \frac{C}{a_0^3} \]  

(46)

We choose here \( A = 1/3 \), so that at the beginning, the brane world may be said to be filled up with (\( p = \rho/3 \)) radiation.
Fig. 1(a) shows the variation of the scale factor (given by eq. (42)) over the time for \(\kappa = 2\), \(B = \rho_0 = 3\), \(C = 1\) and \(t_0 = 1\) while Fig. 1(b) shows the same (for eq. (43)) for \(\kappa = 2\), \(B = \rho_0 = 3\), \(C = 3\) and \(t_0 = 1\). In both the cases, (+)ve sign is taken.

We now examine the following cases:

a) When \(\frac{\kappa^4 A}{48} - \frac{\kappa^2}{6} |\Lambda_5| < 0\)

For this restriction \(\ddot{a}_0 < 0\) for all time while \(\dot{a}_0\) vanishes at some finite \(a_0\). It is therefore, a decelerating model of the universe with only a maximum but no minimum. It is a recollapsing brane model (see fig. 2(a)) with \(a_0 \to 0\) and \(\ddot{a}_0 \to -\infty\) at both ends (radiation to radiation).

b) When \(\frac{\kappa^4 B}{48} - \frac{\kappa^2}{6} |\Lambda_5| > 0\)

Here \(H_0\) is non-zero throughout the evolution. As \(a_0 \to 0\), \(\dot{a}_0 \to -\infty\) and as \(a_0 \to \infty\), \(\dot{a}_0 \to \infty\). So there is a transition from deceleration to acceleration in course of evolution of the brane (Graphically it is shown in fig. 2(b)).

c) When \(\frac{\kappa^4 B}{48} - \frac{\kappa^2}{6} |\Lambda_5| = 0\) i.e., \(|\Lambda_5| = \frac{\kappa^2 B}{8}\)

In this case there is always \(\ddot{a}_0 < 0\) with \(H_0^2 \to 0\) as \(a_0 \to \infty\). So this has a typical Einstein-de Sitter like behaviour for the spatially flat standard model (presented in fig. 2(c)). The interesting feature of such a brane world model is that even though at large \(a\) the fluid has an equation of state \(p_b = -\rho_b\), the model does not show the character of a \(\Lambda\)CDM model as in standard cosmology. Rather it still remains a decelerating model. As we note that in one of the above three models of the brane universe, for example in the second model, there occurs a transition from a decelerating to an accelerating phase. It may be worthwhile to locate the said transition point where \(\ddot{a}_0 = 0\). It is, however, not difficult to find out from the equation (46) that the brane world model with \(A = 1/3\) remains accelerating (\(\ddot{a}_0 > 0\)) as long as

\[-\frac{\kappa^4 \rho_b}{6a_0} + \left(\frac{\kappa^4 B}{48} - \frac{\kappa^2}{6} |\Lambda_5|\right) a_0 - \frac{C}{a_0^3} > 0\]

that is,

\[a_0^8 > \frac{\kappa^4 \rho_b + Ca_0^4}{\frac{\kappa^4 B}{48} - \frac{\kappa^2}{6} |\Lambda_5|}\]

which finally reduces to the following condition by a straightforward calculation:

\[a_0^4 > \frac{C}{2\left(\frac{\kappa^4 B}{48} - \frac{\kappa^2}{6} |\Lambda_5|\right)} \left[1 + \sqrt{\frac{\kappa^4 \rho_b}{4}} \left(\frac{\kappa^4 B}{48} - \frac{\kappa^2}{6} |\Lambda_5|\right)\right]\]

In the standard cosmology with the chaplygin gas in FRW universe, the corresponding inequality is
Fig. 2(a) shows the recollapsing model of the universe for $\kappa = 1$, $B = 1$, $\rho_0 = 48$, $C = 1$, $|\Lambda_5| = 1$ and $t_0 = 24$.

Fig. 2(b) shows a transition from deceleration to acceleration for $\kappa = 1$, $B = 16$, $\rho_0 = 24$, $C = \frac{1}{2}$, $|\Lambda_5| = 1$ and $t_0 = 1$.

Lastly, Fig. 2(c) shows the Einstein-de Sitter universe for $\kappa = 1$, $B = 8$, $\rho_0 = 48$, $C = 1$, $|\Lambda_5| = 1$ and $t_0 = 1$.

\[ a_0^8 > \rho_0 / B. \]

Note that, this is only a formal comparison, as $\rho_0$, $B$ etc. in the brane and the corresponding quantities in FRW universe are not really identical.

V. CONCLUDING REMARKS

When the bulk is filled with only dust, there are certain restrictions imposed on the metric as well as on the matter content of the brane. As results of such restrictions the metric coefficient $g_{tt}$ becomes independent of the y-coordinate, while the perfect fluid in the brane satisfies an effective equation of state $p_B = -\frac{2}{3}\rho_B$, which apparently indicates that matter in the brane is in the form of dark energy.

On the other hand, if the matter in the bulk is in the form of cosmological constant or vacuum, there is neither any restriction on the metric nor on the matter contained by the brane. Binétruy et al and Leon [3, 4] have shown perfect fluid solution with barotropic equation of state for brane matter. In this paper, we have shown that for brane matter, it is possible to have chaplygin gas form of fluid and obtained some interesting conclusions. The recollapsing model of the brane universe shows that in course of evolution it
reaches a maximum size and finally collapses to a singularity. So for such a model, there is deceleration through the entire evolution period and it has singularities at both ends. This difference in behaviour of the model is caused by the modifications introduced in the field equations by the presence of an anti-de Sitter bulk ($\Lambda_5 < 0$). Another point of significance for a chaplygin gas in the brane world model is that, the density does not vanish as the scale factor increases indefinitely, rather it reaches a finite magnitude with an equation of state $p_b = -\rho_b$. This behaviour still remains valid in one of the brane world models, which expands indefinitely with deceleration throughout and resembles the standard Einstein-de Sitter model. Such behaviour of the brane is in clear contradiction to the standard 4D Friedmann universe.

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