A "Freely Coasting" Universe

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Abstract

A strictly linear evolution of the cosmological scale factor is surprisingly an excellent fit to a host of cosmological observations. Any model that can support such a coasting presents itself as a falsifiable model as far as classical cosmological tests are concerned. This article discusses the concordance of such an evolution in relation to several standard observations. Such evolution is known to be comfortably concordant with the Hubble diagram as deduced from current supernovae Ia data, it passes constraints arising from the age and gravitational lensing statistics and just about clears basic constraints on nucleosynthesis. Such an evolution exhibits distinguishable and verifiable features for the recombination era. The overall viability of such models is discussed.

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1 1. INTRODUCTION

Large scale homogeneity and isotropy of matter and radiation observed in the universe suggests the following [Friedmann-Robertson-Walker (FRW)] form for the space-time metric:

$$ds^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

(1)

Here $K = \pm 1, 0$ is the curvature constant. In standard “big-bang” cosmology, the scale factor $a(t)$ is completely determined by the model for the equation of state of matter and Einstein’s equations. The scale factor, in turn, determines the response of a chosen model to cosmological observations. Four decades ago, the main “classical” cosmological tests were (1) The galaxy number count as a function of red-shift; (2) The angular diameter of “standard” objects (galaxies) as a function of red-shift; and finally (3) The apparent luminosity of a “standard candle” as a function of red-shift. Over the last two decades, other tests that have been perfected, or are fast approaching the state of perfection, are: the early universe nucleosynthesis constraints, estimates of age of the universe in comparison to ages of old objects, statistics of gravitational lensing and finally, the physics of recombination as deduced from cosmic microwave background anisotropy.

In this article we explore concordance of the above observations with a FRW cosmology in which the scale factor evolves linearly with time: $a(t) \propto t$, right from the creation event itself. The motivation for such an endeavor comes from several considerations. First of all, such a cosmology does not suffer from the horizon problem. Horizons occur in models with $a(t) \approx t^\alpha$ for $\alpha < 1$ [see eg. [1, 2]]. As a matter of fact, a linearly evolving model is the only power law model that has neither a particle horizon nor a cosmological event horizon. Secondly, linear evolution of the scale factor is supported in alternative gravity theories where it turns out to be independent of the matter equation of state [3, 4, 5]. The scale factor in such theories does not constrain the matter density parameter. This contrasts with the Standard FRW model where the Hubble parameter determines a critical value of density which turns out to be a dynamical repeller. This is the root cause of the “flatness” or fine tuning problem. Finally, such a linear coasting cosmology, independent of the equation of state of matter, is a generic feature in a class of models that attempt to dynamically solve the cosmological constant problem [3, 4, 5].
Such models have a scalar field non-minimally coupled to the large scale scalar curvature of the universe. With the evolution of time, the non-minimal coupling diverges, the scale factor quickly approaches linearity and the non-minimally coupled field acquires a stress energy that cancels the vacuum energy in the theory.

There have been other gravity models that also account for a linear evolution of the scale factor. Notable among such models is Allen’s [7] in which such a scaling results in an SU(2) cosmological instanton dominated universe. Yet another possibility arises from the Weyl gravity theory of Manheim and Kazanas [8]. Here again the FRW scale factor approaches a linear evolution at late times.

Although any of the above are good enough reasons for exploring the concordance of a linear coasting, we add to this list the following reason of our own. The averaging problem in General Relativity has never been properly addressed, let alone solved [9, 10]. This is in contrast with the corresponding problem in classical electromagnetic theory [11]. There one can (i) start with multi-singular solutions to the Laplace equation, (ii) smear each charge over a large enough sphere, and (iii) if the overall distribution satisfies Dirichlet / Neumann boundary conditions at infinity, the average potential can be defined and coincides with the solution to the Poisson equation. In General Relativity the corresponding construction has not been carried out. All precision tests of General Relativity strictly involve vacuum (source free region) solutions of Einstein equations. Strictly speaking, there are no tests of Einstein theory with matter. In the interior of all astrophysical sources, either the weak field (Newtonian) limit is put to test or, where the weak field limit is expected to break down, one assumes General Relativity to parametrize the equation of state (eg. for neutron / quark stars etc.).

On the other hand, the above problems could be circumvented by taking Einstein’s equations with the source terms as the defining equations for a gravity theory. The justification for such an approach could rely on its correct Newtonian limit. Such an attitude comes with its own problems.

First of all, one encounters a related averaging problem again when one applies the theory to cosmology. Is it justified to assume that the large scale behavior of the lumpy universe to be the same as that predicted by the smoothed out FRW models? The essential issue is that averaging the metric does not commute with determining the local connection followed by the determination of the local Ricci tensor and finally forming the field equations to determine the metric. There have been several attempts to
resolve this issue [9, 10], but with limited success. Moreover, reliance on an ansatz just because of its Newtonian limit may in fact be flawed. Newtonian gravity does not offer unique cosmological solutions in the continuum limit for an open cosmology [12].

All studies on the averaging problem and the continuum limit have not considered the retarded effects in their full generality. Newtonian cosmology, applied to an exploding *Milne ball* in a flat space-time [see eg. [13, 14]] gives a unique linear coasting cosmology viz. the FRW [Milne] metric with \(a(t) = t\).

Finally, we recall an approach to General Relativity starting from a spin two field interacting with a source in a flat space-time. Incorporating back reaction on the source in a gauge invariant manner and to all orders of perturbations yields Einstein’s theory [15, 16, 17, 18]. However, the entire analysis relies on canonical propagation of gravity and fails for a distribution of particles across horizons if one has a cosmological creation event. Equivalence Principle tells us that the natural way to describe a distribution of particles just after a creation event, in case one demands gravity not to have globally set in on account of event horizons, is a distribution in a flat space-time. This again takes one back to Milne’s cosmology.

Indeed, consider the universe just after its “creation event”, defined at \(t = 0\), at a small enough time \(t = \epsilon\) after its creation. In a classical description, let the matter be distributed as a swarm of particles in a Reimannian manifold. One may accept Einstein’s theory as a local theory and invoke Einstein’s equations at the location of each particle, viz.: \(G_{\mu \nu} = -8\pi T_{\mu \nu}\). In the inter-particle spaces, the equations read: \(G_{\mu \nu} = 0\). For \(\epsilon\) small enough, there is no reason to expect the global space-time dynamics to be governed by an average stress energy distribution: \(< G_{\mu \nu} >= -8\pi < T_{\mu \nu} >\). This is particularly unreasonable on account of horizons in the theory. There is absolutely no dynamical reason to expect an *average* gravity, described by Einstein’s equations on the average, to have globally “set in”. It is much more reasonable to expect gravity not to have set in globally on account of *retarded effects*. Global matter distribution on large scale, in the absence of global gravitation set in, is naturally described as a distribution in a flat space-time. Such a general homogeneous and isotropic distribution of matter in a flat space-time, described in Co moving coordinates, is just the Milne ball. This reduces to an open FRW universe with the scale factor \(a(t) = t\).

We may take any of the above as the basis for our linear coasting conjecture. In what follows, we assume that an homogeneous background FRW universe is born and evolves as a Milne Universe about which a matter dis-
tribution and standard General Relativity would determine the growth of perturbations. Thus we conjecture that Einstein equations give a correct \textit{microscopic} description of gravitation. This being so, the global dynamics of a FRW Universe, at a small time $\epsilon$ after a creation event, is not described by the averaged Einstein equations but as a freely coasting Milne Universe.

Interestingly, a universe born as a Milne model provides just the right initial condition required to sort out the cosmological constant problem. It is straightforward to formulate an action principle for gravity where the determinant is not a dynamical quantity. Trace of the stress tensor of any matter field does not contribute to the dynamics of gravitation \[6\]. Although this sorts out the naturalness problem of the cosmological constant, an effective cosmological constant appears as an integration constant in this formulation. What is needed is some physical reason that demands a flat space-time solution to describe cosmology at any instant of time and our conjecture does precisely that.

The following section reviews the concordance of linear evolution in relation to standard cosmological observations.

\section{2. A linearly coasting cosmology}

\subsection{2.1 Classical Cosmology tests}

To our knowledge, the first exploration of concordance of a linearly evolving scale factor with observations was conducted by Kolb \[19\]. Kolb obtained a linear evolution by a judicious choice of “K-matter” that makes the universe curvature dominated at low red-shifts. At sufficiently high red-shifts, normal matter becomes increasingly dominant. One could thus manage to have a linear coasting at low red-shifts without giving up several nice results of standard cosmology such as cosmological nucleosynthesis. Kolb demonstrated that data on Galaxy number counts as a function of red-shift as well as data on angular diameter distance as a function of red-shift do not rule out a linearly coasting cosmology. Unfortunately, these two tests are marred by effects such as galaxy mergers and galactic evolution. For these reasons these tests have fallen into disfavor as reliable indicators of a viable model.

The variation of apparent luminosity of a “standard candle” as a function
of red-shift is referred to as the Hubble test. The discovery of Supernovae type Ia [SNe Ia] as reliable standard candles, raised hopes of elevating the status of this test to that of a precision measurement that could determine the viability of a cosmological model. The main reason for regarding these objects as reliable standard candles are their large luminosity, small dispersion in their peak luminosity and a fairly accurate modeling of their evolutionary features. Recent measurements on 42 high red-shift SNe Ia's reported in the supernovae cosmology project [20] together with the observations of the 16 lower red-shift SNe Ia's of the Callan-Tollolo survey [21, 22] have been used to determine the cosmological parameters $\Omega_\Lambda$ and $\Omega_M$ for the standard model. The data eliminates the “minimal inflationary” prediction defined by $\Omega_\Lambda = 0$ and $\Omega_M = 1$. The data can however, be used to assess a “non-minimal inflationary cosmology” defined by $\Omega_\Lambda \neq 0$, $\Omega_\Lambda + \Omega_M = 1$. The maximum likelihood analysis following from such a study has yielded the values $\Omega_M = 0.28 \pm 0.1$ and $\Omega_\Lambda = 0.72 \pm 0.1$ [23, 24, 25, 26].

To explore the concordance of a linear coasting cosmology, it is convenient to consider a power law cosmology with the scale factor $a(t) = \bar{k}t^\alpha$, with $\bar{k}$, $\alpha$ arbitrary constants. It is straightforward to discover the following relation between the apparent magnitude $m(z)$, the absolute magnitude $M$ and the red-shift $z$ of an object for such a cosmology:

$$m(z) = M + 5\log H_0 + 5\log(\frac{\alpha}{H_0})^\alpha(1+z)\bar{k}\bar{S}[\frac{1}{(1-\alpha)\bar{k}}(\frac{\alpha}{H_0})^{1-\alpha}(1-(1+z)^{1-\frac{1}{\alpha}})]$$

(2)

Here $\bar{S}[X] = X, \text{Sin}(X)$ or $\text{Sinh}(X)$ for $K = 0, \pm 1$ respectively, and $M = M - 5\log(H_0) + 25$. The best fit turns out to be $\alpha = 1.001 \pm .0043$, $K = -1$. [27]. The minimum $\chi^2$ per degree of freedom turns out to be 1.18. This is comparable to the corresponding value 1.17 reported by Perlmutter et al for non-minimal inflationary cosmology parameter estimations. Linear coasting is as accommodating even for the largest red-shift supernova [1997ff] as the standard non-minimal inflationary model. The concordance of linear coasting with SNe1a data finds a passing mention in the analysis of Perlmutter [20] who noted that the curve for $\Omega_\Lambda = \Omega_M = 0$ (for which the scale factor would have a linear evolution) is “practically identical to bestfit plot for an unconstrained cosmology”.

The age estimate of the ($a(t) \propto t$) universe, deduced from a measurement of the Hubble parameter, is given by $t_o = (H_o)^{-1}$. The low red-shift SNe1a data [21, 22] gives the best value of 65 km sec$^{-1}$ Mpc$^{-1}$ for the Hubble pa-
rameter. The age of the universe turns out to be $15 \times 10^9$ years. This is $\approx 50\%$ greater than the age inferred from the same measurement in standard (cold) dark matter dominated cosmology (without the cosmological constant). Such an age estimate is comfortably concordant with age estimates of old clusters.

A study of consistency of linear coasting with gravitational lensing statistics has recently been reported [28]. The expected frequency of multiple image lensing events is a sensitive probe for the viability of a given cosmology. A sample of 867 high luminosity optical quasars projected in a power law FRW cosmology gives an expected number of five lensed quasars for a power $\alpha = 1.09 \pm 0.3$. This indeed matches observations. Thus a strictly linear evolution of the scale factor is comfortably concordant with gravitational lensing statistics.

2.2 "The precision" tests

a) The Nucleosynthesis Constraint What makes linear coasting particularly appealing is a recent demonstration of primordial nucleosynthesis not to be an impediment for a linear coasting cosmology [29, 30, 31]. A linear evolution of the scale factor may be expected to radically effect nucleosynthesis in the early universe. Surprisingly, the following scenario goes through.

Energy conservation, in a period where the baryon entropy ratio does not change, enables the distribution of photons to be described by an effective temperature $T$ that scales as $a(t)T = \text{constant}$. With the age of the universe estimated from the Hubble parameter being $\approx 1.5 \times 10^{10}$ years, and $T_0 \approx 2.7K$, one concludes that the age of the universe at $T \approx 10^{10} K$ would be some four years [rather than a few seconds as in standard cosmology]. The universe would take some $10^3$ years to cool to $10^7 K$. With such time periods being large in comparison to the free neutron life time, one would hardly expect any neutrons to survive. However, with such a low rate of expansion, weak interactions remain in equilibrium for temperatures as low as $10^8 K$. The neutron - pro-ton ratio keeps falling as $n/p \approx e^{[15/T_9]}$. Here $T_9$ is the temperature in units of $10^9 K$ and the factor of 15 comes from the n-p mass difference in these units. There would again hardly be any neutrons left if nucleosynthesis were to commence at (say) $T_9 \approx 1$. However, as weak interactions are still in equilibrium, once nucleosynthesis commences, inverse beta decay would replenish neutrons by converting protons into neutrons and pumping them into the nucleosynthesis channel. With beta decay in equilib-
rium, the baryon entropy ratio determines a low enough nucleosynthesis rate that can remove neutrons out of the equilibrium buffer at a rate smaller than the relaxation time of the buffer. This ensures that neutron value remains unchanged as heavier nuclei build up. It turns out that for baryon entropy ratio $\eta \approx 5 \times 10^{-9}$, there would just be enough neutrons produced, after nucleosynthesis commences, to give $\approx 23.9\%$ Helium and metallicity some $10^8$ times the metallicity produced in the early universe in the standard scenario. This metallicity is of the same order of magnitude as seen in lowest metallicity objects.

The only problem that one has to contend with is the significantly low yields of deuterium in such a cosmology. Though deuterium can be produced by spallation processes later in the history of the universe, it is difficult to produce the right amount without a simultaneous over production of Lithium. However, as pointed out in [29], the amount of Helium produced is quite sensitive to $\eta$ in such models. In an inhomogeneous universe, therefore, one can have the helium to hydrogen ratio to have a large variation. Deuterium can be produced by a spallation process much later in the history of the universe. If one considers spallation of a helium deficient cloud onto a helium rich cloud, it is easy to produce deuterium as demonstrated by Epstein [32] - without overproduction of Lithium.

Interestingly, the baryon entropy ratio required for the right amount of helium corresponds to $\Omega_b \approx 0.2$. Here $\Omega_b$ is the ratio of the baryon density to a “density parameter” determined by the Hubble constant: $\Omega_b \equiv \rho_b/\rho_c = 8\pi G \rho_b/3H_o^2$. $\Omega_b \approx 0.2$ closes dynamic mass estimates of large galaxies and clusters [see eg [33, 34]]. In standard cosmology this closure is sought to be achieved by taking recourse to non-baryonic cold dark matter. Thus in a linearly scaling cosmology, there would be no need of non-baryonic cold dark matter at all.

b) The recombination epoch

We describe this in some detail as the peculiarities of the recombination epoch in a linearly coasting cosmology are not covered in any standard (curvature dominated) cosmology description.

Salient features of a linear coasting cosmology at the recombination epoch can be deduced by making a simplifying assumption of thermodynamic equilibrium just before recombination. As in standard cosmology, a recombination process that directly produces a Hydrogen atom in the ground state releases a photon with energy $B = 13.6eV$ in each recombination. $n_\gamma(B)$,
the number density of photons in the background radiation with energy $B$, is given by [see eg. [35, 34]]:

$$\frac{n_\gamma(B)}{n} = \frac{16\pi}{n} T^3 \exp\left(\frac{-B}{T}\right) \approx \frac{3 \times 10^7}{\Omega_B h^2} \exp\left(\frac{-13.6}{\tau}\right)$$

(3)

Where $\tau$ is the temperature in units of eV. This ratio is unity at $\tau \approx .8$ for $\Omega_B h^2 \approx 1$ and decreases rapidly at lower temperatures. Any 13.6 eV photons released due to recombination have a high probability of ionizing neutral atoms formed a little earlier. [In the following, we shall quote all results by our favored values $\Omega_b \approx 0.2$ and the Hubble parameter 65 km/sec/Mpc] This process is therefore not very effective for producing a net number of neutral atoms. The dominant recombination process proceeds through an excited state: $(e + p \rightarrow H^* + \gamma; H^* \rightarrow H + \gamma_2)$. This produces two photons, each having lesser energy than the ionization potential of the hydrogen atom. The 2p and 2s levels provide the most rapid route for recombination. The 2p decay produces a single photon, while the decay from the 2s is by two photons. As the reverse process does occur at the same rate, this is a non-equilibrium recombination that proceeds at a much slower rate. The thermally averaged cross section for the process of recombination $(p + e \leftrightarrow H + \gamma)$ is given by [33, 34]:

$$\frac{<\sigma_v>}{c} \approx 4.7 \times 10^{-24} \left(\frac{T}{1 eV}\right)^{1/2} \text{cm}^2$$

(4)

This gives the reaction rate:

$$\Gamma = n_p <\sigma v> = 2.374 \times 10^{-10}\tau^{7/4} \exp(-6.8/\tau)(\Omega_b h^2)^{1/2} \text{ cm}^{-1}$$

(5)

This is to be compared to the Hubble expansion rate at that epoch, $H = H_0(T/T_0)$. Given the Hubble constant $(H_0 = 100h \text{ km/sec/Mpc})$ and CMB effective temperature $T_0 = 2.73K$ now, the Hubble parameter at any temperature turns out to be: $H = 4.7 \times 10^{-25}h\tau \text{ cm}^{-1}$. This equals $\Gamma$ at

$$\tau^{-3/4} \exp(6.8/\tau) = 1.96 \times 10^{15}(\Omega_b)^{1/2}$$

(6)

A straightforward iteration gives:

$$\tau^{-1} \approx 5.17 - 0.11ln(\tau^{-1}) + .074ln(\Omega_b) \approx (.2)^{-1}$$

(7)

corresponding to a redshift given by:

$$1 + z \approx 874.5[1 + .015ln(\Omega_b)]^{-1}$$

(8)
The residual fraction of electrons turns out to be [34]:

\[ x_e \approx \left(\frac{\pi}{4\xi(3)\sqrt{2}}\right)\frac{1}{2}\eta^{-1/2}\left(\frac{T}{m_e}\right)^{-1/4}\exp\left(-\frac{6.8\tau}{\tau}\right) \] (9)

From eqn.(6), we have

\[ x_e \approx 7.9 \times 10^{-9}\frac{\tau^{-3/2}}{\Omega_b h} \] (10)

For the red-shift range \(800 < z < 1200\), the approximate fractional ionization is:

\[ x_e = \frac{2.4 \times 10^{-3}}{\Omega_b h^2}(\frac{z}{1000})^{12.75} \] (11)

After decoupling at \(\tau = .2\), this gives a residual ionization:

\[ x_{e,\text{res}} \approx 9 \times 10^{-8}(\Omega_b h)^{-1} \] (12)

The only process that may still be effective at such low temperatures is the Thompson scattering with a cross section \(\sigma_T = 6.7 \times 10^{-25} \text{ cm}^2\). The optical depth for photons would be:

\[ \tau_{\gamma} = \int_0^t n_b(t)x_e(t)\sigma_T dt = -\int_0^z n_b(z)x_e(z)\sigma_T \left(\frac{dt}{dz}\right)dz \] (13)

With \(n_b(z) = \eta n_{\gamma}(z) = \eta \times 421.8(1 + z)^3 \text{ cm}^{-3}\), and

\[ \frac{dt}{dz} = -\frac{1}{H_0(1 + z)^2} \] (14)

one can find the red-shift at which the optical depth goes to unity.

If one considers the residual ionization \(x_{e,\text{res}}\), we get

\[ \tau_{\gamma} = 4.7 \times 10^{-2} \times (\frac{z}{1000})^2 \] (15)

From this optical depth, we can compute the probability that a photon was last scattered in the interval \((z, z + dz)\). This is given by:

\[ P(z) = e^{-\tau_{\gamma}}\frac{d\tau_{\gamma}}{dz} \approx .94 \times 10^{-5}(\frac{z}{1000})\exp[-0.047(\frac{z}{1000})^2] \] (16)

\(\tau_{\gamma}\) becomes unity at \(z \approx 4610\). This implies that the residual ionization has insufficient optical depth to scatter photons from the decoupling epoch.
From the expression for fractional ionization eqn(11), the optical depth of the last scattering surface can be deduced to be:

$$\tau_\gamma = 170 \times \left(\frac{z}{1000}\right)^{14.75} \quad (17)$$

This gives:

$$P(z) \approx 2.5 \left(\frac{z}{1000}\right)^{13.75} \exp[-170 \left(\frac{z}{1000}\right)^{14.75}] \quad (18)$$

$\tau_\gamma$ goes to unity at $z_R \approx 703$. This $P(z)$ can be approximated by a Gaussian centered at $z_R \approx 703$ with a width $\Delta z \approx 51.8$.

An important scale that determines the nature of CMB anisotropy is the curvature scale which is the same as the Hubble radius for the linear coasting. The angle subtended today, by the Hubble radius at $z_R = 703$, is determined by

$$\frac{1 + z_R \theta}{2} = \sinh\left[\frac{d(\theta)(1 + z_R)}{2a_0}\right] \quad (19)$$

Here $d(\theta) = d_H(t_R) = H(t_R)^{-1} = [H_0(1 + z_R)]^{-1}$. This gives:

$$\left(\frac{1 + z_R}{2}\right) \frac{\theta}{2} = \sinh\left(\frac{1}{2}\right) \quad (20)$$

or $\theta_H \approx 10$ minutes.

In standard cosmology, the sound horizon is of the same order as the Hubble length. The Hubble length determines the scale over which physical processes can occur coherently. In a linear coasting, the Hubble length is precisely the inverse of the curvature scale. However, the sound horizon ($s^*$) is much larger. Strictly speaking, the particle as well as the sound horizon are infinite for a linear coasting cosmology. For our purpose, it suffices to take the epoch of birth of pressure waves as the epoch of baryon production. We take this to be the QGP phase transition epoch $T_{QGP} \approx 10^{12}$K. The distance a sound wave travels from this epoch till recombination, would subtend an angle which can be refereed to as the sound horizon angle:

$$\theta^* \approx \frac{1}{\sqrt{3}} \ln\left(\frac{T_i}{T_f}\right) \times \frac{2}{1 + z^*} \quad (21)$$

This is $\approx 2^\circ$ for $T_i = T_{QGP}$ and $T_f \approx 10^3$K corresponding to $z^* \approx 705$. The angle subtended by the sound horizon scale is thus roughly 12 times that subtended by the curvature length scale of ten minutes. The photon
diffusion scale is determined by the thickness of the LSS. With $z^* \approx 705$ and $\Delta z \approx 51$, this gives an angular size which is roughly one fourteenth of the Hubble length at the LSS. This subtends an angle of 43" at the current epoch.

The above scales in principle determine the nature of CMB anisotropy. The CMB effectively ceases to scatter when the optical depth to the present drops to unity. After last scattering, the photons effectively free stream. On the LSS, the photon distribution may be locally isotropic while still possessing inhomogeneities i.e. hot and cold spots, which will be observed as anisotropies in the sky today [see eg. \[36, 37\]. As described in the Appendix, temperature fluctuations, determined by the potential and density perturbations, are expressible by an expansion in terms of eigenmodes of the generalized Laplace operator $\nabla^2$ with eigenvalues $-k^2$. The phase of oscillation is frozen in at last scattering. The critical wave number $k_A \equiv \pi/s^*$ corresponds to the sound horizon at that time. Longer wavelengths will not have evolved from the initial conditions and possess $\psi/3$ gravitational potential fluctuations after gravitational red-shift \[36, 37\]. This combination of the intrinsic temperature fluctuation and the gravitational red-shift is the “Sachs - Wolfe effect”. Shorter wavelengths can be frozen at different phases of the $\cos(ks^*)$ oscillation for adiabatic perturbative modes and as $\sin(ks^*)$ for isocurvature fluctuation modes. For adiabatic modes as a function of $k$ there will be a harmonic series of temperature fluctuation peaks with $k_m = m k_A = m \pi/s^*$ for the $m$th peak. Odd peaks represent compression phase (temperature crests), whereas even peaks represent the rarification phase (temperature troughs), inside potential wells. In the isocurvature case, just as in the adiabatic case, the self gravity of the photon baryon fluid essentially drives the oscillations. Unlike the adiabatic case, it is the sine rather than the cosine oscillations that are driven now. Peaks occur at $k = (m - 1/2)k_A$ with all even peaks being enhanced by the baryon drag. More exotic models might produce a phase shift leading to a fluctuation $\cos(ks^* + \phi)$. This would shift the location of the first peak while leaving the spacing between the peaks the same: $k_m - k_{m-1} = k_A$. Thus the sound horizon at last scattering should be measurable from the CMB.

Subtle complications that arise in our CMB anisotropy study can be tackled in the same manner that deals with them in the standard model. For example, in the total variance of temperature fluctuation, it can be seen that the photon density and potential fluctuations cancel the velocity (Doppler) fluctuations were the sound speed exactly $c_s = 1/\sqrt{3}$. However, for $c_s <
1/√3, the locations of the peaks for the temperature variance coincides with those of the photon density and potential fluctuations [see eg [37]]. The wave number $k = 1$, in units of the curvature scale, would correspond to a length on the LSS that subtends an angle of 10′. It is straightforward to determine the peak location for the adiabatic and isocurvature perturbations for the primary SW effect. For adiabatic modes, compression peaks occur for odd values of $m$ at angles $\theta_{m}^{\text{ad}} = 120/m\pi$ minutes. For isocurvature modes they occur at even $m$ at $\theta_{m}^{\text{iso}} = 120/(m - \frac{1}{2})\pi$ minutes. Fluctuations would have a decreasing amplitude for smaller angles due to photon diffusion that makes the coupling between the baryon - photon fluid bleed for small scales as it vanishes at 43″.

All modes corresponding to angles greater than 10 minutes correspond to eigenmodes $0 < k < 1$. These are supercurvature modes. The location of the largest (adiabatic) wavelength peak is $k = \pi/12 \approx 1/4$. As explained in the appendix [39, 38], the eigenfunctions of supercurvature modes are suppressed for open models. However, for $k$ as low as 1/4 the suppression of the eigenfunction is merely by a factor of the order unity. The relative amplitudes of the $k$ modes is determined by an initial power spectrum that is set by an ab initio ansatz. The suppression of the supercurvature mode with $k \approx 1/4$ can be countered by a corresponding change in the initial power spectrum.

The exact profile of the anisotropy would be determined by the choice of the nature of initial conditions (adiabatic or isocurvature), the chosen initial power spectrum, and the growth of perturbations after $z^*$ (decoupling). These determine the late or the integrated SW effect, aspects of reionization etc.

The main point we make in this article is that in spite of a significantly different evolution, the recombination history of a linearly coasting cosmology gives the location of peaks for the primary acoustic peaks in the same range of angles as that given in Standard Cosmology. Given that none of the alternative anisotropy formation scenarios provide a compelling ab initio model [41], it is perhaps best to keep an open mind to all possibilities. As the large scale structure and CMB anisotropy data continue to accumulate, one could explore the general principles for an open coasting cosmology to aid in the empirical reconstruction of a consistent model for structure formation.

Finally, we are tempted to mention that a linear coasting cosmology presents itself as a falsifiable model. It is encouraging to observe its concordance!! In standard cosmology, falsifiability has taken on a backstage -
one just constrains the values of cosmological parameters subjecting the data to Bayesian statistics.

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**Appendix:** Subsequent to decoupling, perturbations of the last scattering surface [LSS] and the intervening space, leave an imprint on the streaming microwave background photons observed at the present epoch. To describe the *gross* features of perturbations of the model we start by writing the background line element as

$$ds^2 = (0)_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j = a^2(\eta)(d\eta^2 - \gamma_{ij}dx^i dx^j) \quad (A.1)$$

where $\eta$ is the conformal time $d\eta \equiv a^{-1}dt$.\[477x408] \hspace{1cm} (A.1)

$$\gamma_{ij} = \delta_{ij}[1 + 1 \frac{4}{K}(x^2 + y^2 + z^2)]^{-2} \quad (A.2)$$

where $K = -1$ for the $\eta =$ constant hypersurface describing an open model’s space-like section.

Assuming the perturbations to be described by the perturbed Einstein Equations: $\delta G_{\mu\nu} = \delta T_{\mu\nu}$, the metric can be expanded as usual in terms of the scalar, vector and tensor modes [see eg. [42]]. The gauge invariant scalar perturbation equations are:

$$\nabla^2 \Phi - 3H\phi' - 3(H^2 - K)\Phi = 4\pi Ga^2 \delta \epsilon^{\sigma i} \quad (A.3a)$$

$$(a\Phi)'_i = 4\pi Ga^2 (\epsilon_o + p_o)\delta u_i^{\sigma i} \quad (A.3b)$$

$$\Phi'' + 3H\Phi' + (2H' + H^2 - K)\Phi = 4\pi Ga^2 \delta p^{\sigma i} \quad (A.3c)$$

Here, $\nabla^2 \Phi \equiv \gamma_{i,j} \Phi_{,i,j}$, is the wave operator for the open model. $H \equiv a'/a$, where $'$ is a derivative with respect to conformal time, and finally the $\delta \epsilon^{\sigma i}$, $\delta u_i^{\sigma i}$ and $\delta p^{\sigma i}$ are the gauge invariant density, velocity and pressure parameters respectively [12]. These equations are valid whenever linear perturbation theory is valid. This requires $|\Phi| << 1$ but not necessarily $|\delta \epsilon/\epsilon| << 1$. The above equations combine to give:

$$\Phi'' + 3H(1 + c_s^2)\Phi' - c_s^2 \nabla^2 \Phi + [2H' + (1 + 3c_s^2)(H^2 - K)]\Phi = 4\pi Ga^2 \tau \delta S \quad (A.4)$$
Here the parameters $c_s$, $\tau$ are determined in terms of the matter, radiation and entropy densities $\epsilon_m$, $\epsilon_\gamma$, $S$ and are given by:

$$c_s^2 = \frac{1}{3} \left( 1 + \frac{3}{4} \frac{\epsilon_m}{\epsilon_\gamma} \right)^{-1}, \quad \tau = \frac{c_s^2 \epsilon_m}{S}$$ (A.5)

Entropy perturbations, $\delta S$, also called isocurvature perturbations, can be generated if the different matter components are distributed non-uniformly in space but with uniform total energy density and hence uniform curvature at the beginning.

For a radiation dominated epoch, the evolution of adiabatic perturbations ($\delta S = 0$) is given by putting $c_s \approx 1/\sqrt{3}$ when eqn(A.4) reduces to:

$$\Phi'' + 4\Phi' + \frac{k^2}{3} \Phi + 4\Phi = 0$$ (A.6)

where we define $-k^2$ as the eigenvalue for $\nabla^2$. A straightforward solution to this equation is: $\Phi \rightarrow t^{-2} \exp(ik\eta/\sqrt{3})$. This form for $\Phi$, together with eqn(A.3a) determine the density perturbations in the radiation dominated epoch provided we have an ansatz for an initial power spectrum. It is also straightforward to solve the potential equations in the matter dominated epoch as well.

In general [see eg [36]] it is convenient to expand cosmological perturbations in a series of eigenfunctions of the Laplacian. Firstly, each mode (each term in the series) evolves independently with time. This makes it easy to evolve a given initial perturbation forward in time. Secondly, by assigning a Gaussian probability distribution to the amplitude of each mode, one can generate a homogeneous Gaussian random field. Such a field consists of an ensemble of possible perturbations. It is supposed that the perturbations seen in the observable universe is a typical member of the ensemble. The stochastic properties of a Gaussian random field are determined by its two point correlation function $\langle f(1)f(2) \rangle$, where $f$ is the perturbation and the brackets denote the ensemble average. For a homogeneous field, the correlation depends only on the distance between the two points.

For the expansion of perturbations in terms of the Laplacian with eigenvalues $-k/a^2$, modes with real $k^2 > 1$ provide a complete orthonormal basis for $L^2$ functions [40, 39]. They vary appreciably on scales less than the curvature scale $a$ and are called subcurvature modes. A related wave number and a related radial coordinate are defined as:

$$q^2 \equiv k^2 - 1, \quad \chi \equiv \sinh^{-1} r$$
A typical expansion of the wave mode is:

\[ f(\chi, \theta, \phi, t) = \int_0^\infty dq \sum_{lm} f_{klm}(t) Z_{klm}(\chi, \theta, \phi) \]  

Where \( Z_{klm} \equiv \Pi_{kl}(\chi)Y_{lm}(\theta, \phi) \), and the radial functions are:

\[ \Pi_{kl} = \frac{\Gamma(l + 1 + iq)}{\Gamma(iq)} \frac{1}{\sqrt{\sinh \chi}} P_{iq-1/2}^{-l-1/2}(\cosh \chi) \]  

normalized as:

\[ \int_0^\infty \Pi_{kl}(\chi)\Pi_{k'l'}(\chi)\sinh^2 \chi d\chi = \delta(q - q')\delta_{l'l'} \]

\[ \int Z_{klm}^* Z_{k'l'm'} dV = \delta(q - q')\delta_{l'l'}\delta_{mm'} \]  

The constant non-zero phase of \( \Pi_{kl} \) can be dropped by defining the real function:

\[ \Pi_{kl} \equiv N_{kl} \hat{\Pi}_{kl} \]

\[ \hat{\Pi}_{kl} \equiv q^{-2}(\sinh \chi)^l \left( \frac{-1}{\sinh \chi} \frac{d}{d\chi} \right)^{l+1} \cos(q\chi) \]

\[ N_{kl} \equiv \sqrt{\frac{2}{\rho^2}} q^2 [\Pi_{n=0}^l(n^2 + q^2)]^{-1/2} \]  

The problems with these modes is that they are inadequate to describe perturbations over scales larger than the curvature scale. For this purpose, while considering perturbations in an open universe, one should retain not only the subcurvature modes (defined as eigenfunctions of the Laplacian with eigenvalues less than -1 in units of curvature scale), but also the supercurvature modes whose eigenvalues lie between 0 and -1. All modes must be included to generate the most general homogeneous Gaussian random field even though they may not be linearly independent. The reason for this is the following:

With cosmological perturbations assumed to be Gaussian in the regime of linear evolution, a Gaussian perturbation is defined as one whose probability distribution functions are multivariate Gaussians and its stochastic properties are completely determined by its correlation function. The perturbation turns out to be homogeneous with the correlation function depending only on the distance between the points.
If one merely includes the subcurvature modes, it is easy to deduce the form for the correlation function \[39, 40\]:

\[
\xi_f = \int_1^\infty \frac{dk}{k} P_f(k) \frac{\sin(qr)}{qr} \sin hr \tag{A.11}
\]

Setting \(r = 0\) gives the mean square value:

\[
\xi_f(0) \equiv \langle f^2 \rangle = \int_1^\infty \frac{dk}{k} P_f(k) \tag{A.12}
\]

Therefore, by expanding a perturbation in terms of subcurvature modes, the correlation is bounded by:

\[
\frac{\xi_f(r)}{\xi_f(0)} < \frac{r}{\sin hr} \tag{A.13}
\]

\(q \rightarrow 0\) does not correspond to infinitely large scales, but to scales of the order of the curvature scale.

Thus including only the subcurvature modes generates a Gaussian perturbation whose correlation function necessarily falls off faster than \(r/\sin hr\). This reflects the fact that each supercurvature mode varies strongly on a scale no bigger than the curvature scale. A random superposition of such modes will hardly ever be nearly constant on a scale much bigger than the curvature scale. This is precisely what the lack of correlation on large scales tells us.

One could consider correlation on arbitrarily large scales by including the super curvature modes. For \(-1 < q^2 < 0\) the analytic continuation of the radial function \(\Pi_{kl}\) gives the supercurvature modes:

\[
\Pi_{kl} \equiv N_{kl} \hat{\Pi}_{kl}
\]

\[
\hat{\Pi}_{kl} \equiv |q|^{-2}(\sin hr)^l \left(\frac{-1}{\sin hr} \frac{dr}{dr}\right)^{l+1} \cosh(|q|r)
\]

\[
N_{k0} \equiv \sqrt{\frac{2}{\pi} |q|}
\]

\[
N_{kl} \equiv \sqrt{\frac{2}{\pi} |q| |\Pi_{n=1}^l (n^2 + q^2)|^{-1/2}} \quad (l > 0) \tag{A.14}
\]

These supercurvature modes go as \(\exp[-(1 - |q|)r]\) at large \(r\). With the volume element \(dV = \sinh^2 r \sin \theta \, dr \, d\theta \, d\phi\) the integral over all of space
of a product of any two of them diverges. The modes are therefore not orthogonal let alone orthonormal. In a finite region of space they are not linearly independent of the subcurvature eigenfunctions. None of this matters for the purpose of generating a Gaussian perturbation. The supercurvature modes add to the expansion (A.7), an additional:

\[ f^{SC}(r, \theta, \phi, t) = \int_0^1 d(q) \sum_{lm} f_{klm}(t) Z_{klm}(r, \theta, \phi) \]  

(A.15)

From this, the supercurvature contribution to the correlation function is seen to be [39]:

\[ \xi_f^{SC}(r) = \int_0^1 \frac{dk}{k} P_f(k) \frac{\sinh(|q|r)}{|q|\sin hr} \]  

(A.16)

Consider a supercurvature mode corresponding to a peak at \( k \approx 1/3 \) or \( q = 2\sqrt{2}i/3 \) in units of curvature scale. For such a mode, the correlation function is suppressed by a factor \( \sinh(|q|r)/(|q|\sinh(r)) \approx 2/3 \). This is a suppression by a factor of the order unity and can be compensated by an appropriate initial power spectrum.

The spectrum of initial fluctuations can be characterized by a power law \( |\delta_k|^2 = VAk^n \) where \( n \) is a spectral index and \( A \) is the amplitude at very early epochs. The values of these parameters should emerge from the physical model which describes the the production of the initial spectrum. In the absence of any reliable theoretical prediction for \( A \) and \( n \), it is best to treat them as free parameters which can be determined by comparison with observations.

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