Natural Heat Transfer Phenomenon in MHD Fractional Second Grade Fluid

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Abstract This paper aims at the heat transfer phenomenon and the effect of magnetic field on the second-grade fluid in a vertical oscillating cylinder. By applying a perpendicular magnetic field, the fluid gets magnetized. Fractional MHD flow was modeled with Caputo-Fabrizio non-integer derivative approach. Exact solution of the governing equations was obtained by Laplace and finite Hankel transforms. Mathematical computations and graphical plots were used to investigate the quantitative effects of emerging dimensionless physical parameters on the second-grade fluid flow, such as magnetic field and Prandtl number.

Keywords Magnetized Solution, Buoyancy Forces, Non-integer Derivative, Conventional Fluid, Fractional Fluid

1. Introduction

Nowadays BFD (biomagnetic fluid dynamics) and MHD (magneto hydrodynamics) are gaining significant attention in fractional-order electromagnetism, bio-engineering and neurons modeling in biology. Heat transfer has a major impact on the non-Newtonian flow problems in industry and engineering.

In the analytical study of Nehad and Ilyas [1], fractional parameter enhances the fluid velocity in the vertical oscillating plate. Das et al [2] applied Runge-Kutta sixth order method to the stretching heat model. The results conclude that thermal radiation significantly increases the boundary layer velocity and temperature. Rehman et al. [3] used a homotopy analysis method to find the Erying Powell fluid stagnation point inside the vertical cylinder. Keller box scheme was employed by Prasad et al. [4] to numerically simulate the incompressible second-grade fluid. Numerical results show that the heat transfer rate and velocity gradient decelerates with streamwise coordinate. Aalo et al. [5] solved the viscous dissipation model by spectral relaxation method. It was shown that thermal radiation rise resulted in the cooling plate. Fourth-grade thin-film flow was analytically studied by using Adomian decomposition method and Homotopy asymptotic method by Gul et al. [6]. Graphical results were compared and found in good agreement with both of the methods. Visco-elastic fluid flow inside the circular cylinder was investigated by Choudhury and Deka [7]. Meksyn application model of steepest descent method was applied, where it was found that Nusselt number and visco-elastic absolute value reduced the shearing stress. Hayat et al. [8] discussed the heat absorption and heterogeneous reactions due to a rotating disk for the second-grade fluid. Appropriate initial guesses were made to assure the solution convergence. Computed results depicted that visco-elastic parameter and Schmidt number were the increasing functions of concentration profile. Shear stress and velocity profile were evaluated by using the fractional derivative approach in Raza et al. [9] model. The hybrid technique involves semi-analytical fractional-order solutions condensed to the ordinary form. Non-Fourier heat flux and thermal conductivity for temperature-dependent fluid were numerically investigated by Hayat et al. [10]. HAM solution showed that the velocity profile accelerated with visco-elastic and curvature parameter. Moreover, temperature decayed with increasing thermal stratification and Prandtl number. Blood and fluid flow problems without singularity in the fractional domain were analytically studied by Uddin et al. [11], [12], [13]& [14]. Temperature distribution for solid oxide fuel cell studied
by Guk et al. [15] was subjected to various flow parameters impact. Open circuit voltage conditions and direct hydrogen oxidation was the most significant contributor for the average increment in the cell’s temperature. Zhang et al. [16] experimentally investigated lateral smoke extraction. Thermal behaviour across longitudinal ventilation was also studied. Experimental results displayed that the smoke temperature reduced exponentially along with the tunnel ceiling. Curved sidewall effect on flame characteristic and ceiling was experimentally studied by Liang et al. [17]. Inside the tunnel ceiling temperature distribution was found asymmetrical near the fire source region. Shojaei et al. [18] analytically examined the Soret and Dufour effects along stretching cylinder. Both were in negative correlation with heat and mass transfer rate. Xu et al. [19] used a fuel cell device for the direct conversion of chemical energy into electricity. Two-dimensional control orient model of the differential equation was established for the fuel cell device. Thermoelectric characteristics were reflected by the simulated results and especially the temperature distribution across the device.

In this paper heat transfer effect due to natural convection in a vertically oscillating cylinder is focused upon. Previously, analytic solutions were expressed in terms of generalized functions, which were inadequate for numerical simulations. Computer software Mathematica was used for simulations. Discussion and graphical illustrations have been made in the end.

2. Mathematical Model

Present taking the unusual Boussinesq approximation, the unsteady MHD second grade thermal fluid flow is governed by the following set of partial differential equations

\[
\frac{\partial u(r, t)}{\partial t} = \left( \nu + \frac{\alpha_3}{\rho} \frac{\partial}{\partial t} \right) \left( \frac{\partial^2 u(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r, t)}{\partial r} \right) - \frac{\sigma B_0^2 u(r, t)}{\rho} + g \beta T(T(r, t) - T_e); \quad t > 0, \quad r \in [0, R_0].
\]

\[
\frac{\partial T(r, t)}{\partial t} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, t)}{\partial r} \right); \quad t > 0, \quad r \in [0, R_0].
\]

The initial and boundary conditions are

\[
u(r, 0) = 0, \quad T(r, 0) = T_e; \quad r \in [0, R_0],
\]

\[
u(R_0, t) = V_0 U(t) \exp(i \omega t), \quad T(R_0, t) = T_e; \quad t > 0.
\]

Where in Eq.(1) & (2) \( u(r, t) \) is the fluid velocity, \( T(r, t) \) is fluid temperature, \( \nu \) is the fluid kinematic viscosity, \( \alpha_3 \) is the second grade fluid parameter, \( \rho \) is the fluid density, \( \beta_T \) is the fluid volumetric coefficient of thermal expansion, \( g \) is the gravitational acceleration, \( C_p \) is the fluid heat capacity at constant pressure and \( k \) is the fluid thermal conductivity.

2.1. Dimensionless Time Fractional Model

A Governing momentum and energy equations are non-dimensionalised by introducing dimensionless variables. After dropping * notations we have

\[
\frac{\partial}{\partial t} u(r, t) = \left[ 1 + \alpha_4 \frac{\partial}{\partial t} \right] \left( \frac{\partial^2 u(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r, t)}{\partial r} \right) - H a \nu^2 u(r, t) + Gr T(r, t); \quad t > 0,
\]

\[
\frac{\partial}{\partial t} T(r, t) = \frac{1}{Pr} \left( \frac{\partial^2 T(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, t)}{\partial r} \right); \quad t > 0, \quad r \in [0,1],
\]

\[
u(r, 0) = 0, \quad T(r, 0) = 0; \quad r \in [0,1],
\]

\[
u(1,t) = U(t) \exp(i \omega t), \quad T(1,t) = 1; \quad t > 0.
\]

Replacing the classical time partial derivative with the Caputo-Fabrizio time fractional derivative of non-integer order \( \alpha \in [0,1] \) in eqs (5) and (6) respectively, one obtains:

\[
D_t^{(\alpha)} u(r, t) = \left[ 1 + \alpha_4 D_t^{(\alpha)} \right] \left( \frac{\partial^2 u(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r, t)}{\partial r} \right) - H a \nu^2 u(r, t) + Gr T(r, t); \quad t > 0,
\]

\[
D_t^{(\alpha)} T(r, t) = \frac{1}{Pr} \left( \frac{\partial^2 T(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, t)}{\partial r} \right); \quad t > 0, \quad r \in [0,1],
\]

2.1.1. Temperature Field

Temperature of second grade fluid can be obtained by taking Laplace and finite Hankel transforms of the energy equation in Eq. (10) along with initial and boundary conditions in Eq. (7) & (8), and we obtain the final analytical form of the temperature profile in the local as well as fractional model \( 0 < \alpha \leq 1 \),

\[
T(r, t) = 1 + 2 \sum_{n=1}^{\infty} \frac{J_0(r_n) \beta}{J_1(r_n) \beta} \left[ 1 - \frac{\alpha}{b_2} \right] e^{-b_2 t}, \quad \text{for} \ 0 < \alpha \leq 1.
\]

Where \( b_1 = Pr + \frac{r_n^2}{\nu^2} \), \( b_2 = r_n^2 \alpha / b_1 \), \( J_0 \) and \( J_1 \) are the bessel functions of zero and first order with first kind and \( r_n \), \( n = 1, 2, 3, \ldots \) are the positive roots of \( J_0(\alpha) = 0 \).

2.1.2. Velocity Field

The Velocity profile of second grade fluid under the action of external applied magnetic field and heat transfer in the fractional model can be obtained by Eq. (9) along with initial and boundary conditions stated in Eq. (7) & (8). After taking Laplace and finite Hankel transforms on these equations we have,
\[ u(r, t) = e^{i\omega t} + 2 \sum_{n=1}^{\infty} \frac{r_n J_n(r r_n)}{f_2 J_1(r_n)} \left[ f_4 f_1 e^{i\omega t} - f_4 f_1 e^{i\omega t} + f_4 f_1 e^{i\omega t} \right] + a_4 e^{i\omega t} - a_4 f_4 e^{-f_1^t} + f_4 f_1 e^{i\omega t} + f_4 f_1 e^{i\omega t} + f_4 f_1 e^{i\omega t} + f_4 f_1 e^{i\omega t} + f_4 f_1 e^{i\omega t} + f_4 f_1 e^{i\omega t} + f_4 f_1 e^{i\omega t} \]

\[ f_{11} = \frac{-1}{f_3 + i\omega} \quad (23) \]

2.2. Numerical Results and Discussion

By using Eq. (11) & (12), influence of flow parameters like Prandtl number and external magnetic field on temperature and velocity profile is investigated. Fig (1) shows the flow geometry. For computer-based simulations other fixed flow parameters are \( a_4 = 0.5 \) and 1 (second grade fluid parameter), \( \text{Pr} = 1, 2, 4 \) and 6 (Prandtl number), \( Gr = 1 \) (Grashof number), \( \omega = \frac{Pr}{4} \) and \( Ha = 2, 3, 4 \) and 5.

\[ f_1 = r_n^2 + Ha^2, \quad (13) \]
\[ f_2 = 1 + f_1 - f_1 a + a_4 r_n^2, \quad (14) \]
\[ f_3 = \frac{f_1 a}{f_2}, \quad (15) \]
\[ f_4 = a - f_3 (1 - a), \quad (16) \]
\[ f_5 = \frac{1}{b_2 - f_3}, \quad (17) \]
\[ f_6 = \frac{1}{f_3 - b_2}, \quad (18) \]
\[ f_7 = \frac{1}{f_3 b_2}, \quad (19) \]
\[ f_8 = \frac{-1}{b_2 (f_3 - b_2)}, \quad (20) \]
\[ f_9 = \frac{-1}{f_3 (b_2 - f_3)}, \quad (21) \]
\[ f_{10} = \frac{1}{f_3 + i\omega}, \quad (22) \]

Figure 1. Fluid flow geometry inside oscillating walls

Figure 2. Temperature profile \( T(r, t) \) against \( r \)
In Fig. (2), the effect of Prandtl number on temperature profile for fractional second grade non-Newtonian fluid is shown. It is observed that heat transfer from oscillating cylinder towards the fluid is significant and fluid gets warm for small Pr and begins to cool by increasing values due to thickening of the thermal boundary layer.

Fluid flow was exposed to the external magnetic field at different strengths in Fig. (3), $Ha = 2, 3, 4, 5$ against $r$. Fluid velocity decreased with the magnetic field.

3. Conclusions

The article concluded by mentioning the following main points that:

1. Thickening of the thermal boundary layer decreases the fluid temperature.
2. Fluid flow can be controlled by applying sufficiently strong magnetic field.

It is expected that the present study can inspire other researchers as well. The study can be extended to the stretching flow models in the food process, blood flow, paper production and polymeric solutions.

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