I. THE MASS GAP AND SOLUTION OF THE QUARK CONFINEMENT PROBLEM IN QCD

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Using the previously derived confining gluon propagator, the corresponding system of equations determining the quark propagator is derived. The system of equations consists of the Schwinger-Dyson equation for the quark propagator itself, which includes the zero momentum transfer quark-gluon vertex. It is complemented by the Slavnov-Taylor identity for this vertex. The quark equation depends explicitly on the mass gap, determining the scale of the truly nonperturbative dynamics in the QCD ground state. The obtained system of equations is manifestly gauge-invariant, i.e., does not depend explicitly on the gauge-fixing parameter. It is also free from all the types of the perturbative contributions (“contaminations”), which may appear at the fundamental quark-gluon level.

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I. INTRODUCTION

To say today that Quantum Chromodynamics (QCD) is the nonperturbative (NP) theory of quark-gluon interactions is almost a tautology. The problem is how to define it exactly, since we know for sure that QCD has a nontrivial perturbative (PT) phase as well because of asymptotic freedom (AF) [1]. The corresponding asymptotic mass scale parameter \( \Lambda_{QCD} = \Lambda_{PT} \) is responsible for its PT dynamics (scale violation, AF, etc.). On the other hand, if QCD itself is a confining theory then a characteristic scale is very likely to exist. It should be directly responsible for the large-scale structure of the true QCD vacuum in the same way as \( \Lambda_{QCD} \) is responsible for its short-scale one. However, the Lagrangian of QCD does not contain explicitly any of the mass scale parameters which could have a physical meaning even after the corresponding renormalization program is performed.

The only place where the regularized version of the mass scale parameter (the mass gap in what follow, for simplicity) may appear is the dynamical system of quantum equations of motion of QCD. It is known as the Schwinger-Dyson (SD) equations. They should be complemented by the corresponding Slavnov-Taylor (ST) identities, which relate the different Green functions, entering the SD equations, to each other [1]. To solve this system means to solve QCD itself and vice-versa, since it contains the full dynamical information on QCD (and even more than that). Some solutions of these equations reflect the real structure of a QCD ground state, which is necessary to know in order to understand such an important physical phenomena as color confinement, spontaneous breakdown of chiral symmetry (SBCS) and many other NP effects. There is a close intrinsic link between these phenomena and the true structure of the QCD vacuum [2, 3, 4, 5].

Contrary to Quantum Electrodynamics (QED), in QCD the Green’s functions are essentially modified from their free counterparts due to the strong response of the highly complicated structure of the true QCD vacuum. Such a substantial modification can be neglected in the simplest cases only: in the weak coupling limit due to AF or for heavy quarks. In other words, it is not enough to know the Lagrangian of the theory. In QCD it is also necessary and important to know the true structure of its ground state. This knowledge comes just from the investigation of the above-mentioned system of the SD equations and ST identities. Although this system of dynamical equations can be reproduced by an expansion around the free field vacuum, the final equations make no reference to the vacuum of the PT. They are sufficiently general and should be treated beyond the PT, and thus serve as an adequate and effective tool for the NP approach to QCD [1].

Also, we need these solutions for the Green’s functions in order to calculate the physical observables in QCD from first principles. One of the main roles in the realization of this program belongs to the solution for the gluon Green’s function which describes their propagation in the QCD vacuum. In the presence of a mass gap responsible for the true NP QCD dynamics it has been exactly established in our previous work [2] (for a brief review see below).

The main purpose of this work is to derive the confining quark propagator on the basis of this solution by using the above-mentioned system of the corresponding SD equations and the quark-gluon ST identity.

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II. THE CONFINING GLUON PROPAGATOR

In our previous work \cite{2} in order to realize the above-mentioned mass gap responsible for the true NP QCD dynamics, we propose not to impose the transversality condition on the full gluon self-energy, while preserving the color gauge invariance condition for the full gluon propagator. Since due to color confinement the gluon is not a physical state, none of physical observables/processes in QCD will be directly affected by such a temporary violation of color gauge invariance/symmetry (TVCGI/S). In order to make the existence of a mass gap perfectly clear the corresponding subtraction procedure has been introduced. All this allowed us to establish the general structure of the full gluon propagator in the presence of a mass gap as follows (Euclidean signature here and everywhere below):

\[ D_{\mu\nu}(q) = i \left\{ T_{\mu\nu}(q)d(q^2) + \xi L_{\mu\nu}(q) \right\} \frac{1}{q^2}, \]

(2.1)

where \( \xi \) is the gauge-fixing parameter and \( T_{\mu\nu}(q) = \delta_{\mu\nu} - q_{\mu}q_{\nu}/q^2 = \delta_{\mu\nu} - L_{\mu\nu}(q) \). Evidently, it satisfies the color gauge invariance condition \( q_{\mu}q_{\nu}D_{\mu\nu}(q) = i\xi \) as mentioned above (the color group indices, which in this case is simply reduced to the trivial \( \delta \)-function, have been omitted). The full gluon form factor or equivalently the full effective charge \( d(q^2) = \alpha_s(q^2) \) is then

\[ d(q^2) = \frac{1}{1 + \Pi(q^2; D) + (\Delta^2(\lambda; D)/q^2)}. \]

(2.2)

Here \( \Pi(q^2; D) \) is the subtracted full gluon self-energy, while \( \Delta^2(\lambda; D) \) is the difference between the full gluon self-energy and its subtracted counterpart. Obviously, it is nothing but the sum of all possible quadratic divergences parameterized as the mass gap and regulated by \( \lambda \). Rewriting Eq. (2.2) as the corresponding transcendental equation for the effective charge, we were able to formulate and develop its nonlinear iteration solution \cite{2}. Finally it made it possible to exactly decompose the regularized full gluon propagator (2.1) as the sum of the two principally different terms

\[ D_{\mu\nu}(q; \Delta^2) = D^{\text{INP}}_{\mu\nu}(q; \Delta^2) + D^{\text{PT}}_{\mu\nu}(q), \]

(2.3)

where

\[ D^{\text{INP}}_{\mu\nu}(q; \Delta^2) = iT_{\mu\nu}(q) \frac{\Delta^2}{(q^2)^2} f(q^2), \]

(2.4)

and the superscript "INP" means intrinsically NP, while \( f(q^2) \) is determined by the corresponding Laurent expansion as follows:

\[ f(q^2) = \sum_{k=0}^{\infty} (\Delta^2/q^2)^k \Phi_k(\lambda, \alpha, \xi, g^2). \]

(2.5)

The mass gap \( \Delta^2 \equiv \Delta^2(\lambda, \alpha, \xi, g^2) \) depends on the same set of parameters as the residues \( \Phi_k(\lambda, \alpha, \xi, g^2) \) in the Laurent expansion (2.5), where in addition \( \alpha \) and \( g^2 \) are the dimensionless subtraction point and the coupling constant squared, respectively.

The PT gluon propagator

\[ D^{\text{PT}}_{\mu\nu}(q) = i \left[ T_{\mu\nu}(q)d^{\text{PT}}(q^2, \xi) + \xi L_{\mu\nu}(q) \right] \frac{1}{q^2}, \]

(2.6)

remains undetermined within our approach. This was the price we have had to pay to fix the functional dependence of the INP part of the full gluon propagator (up to the arbitrary, in general, residues). The only thing we know about the PT gluon form factor \( d^{\text{PT}}(q^2, \xi) \) is that it is a regular function at \( q^2 \to 0 \) and should satisfy AF at \( q^2 \to \infty \). Let us also note that it includes the free gluon propagator \( D^{\text{PT}}_{\mu\nu}(q) = i\left[T_{\mu\nu}(q) + \xi L_{\mu\nu}(q)\right](1/q^2) \) as well.

We distinguish between the two terms in the full gluon propagator (2.3) first by the explicit presence of the mass gap (when it formally goes to zero then the only PT term survives). Secondly, the INP part of the full gluon
propagator is characterized by the presence of severe power-type (or equivalently NP) infrared (IR) singularities \((q^2)^{-2-k}\), \(k = 0, 1, 2, 3, \ldots\). So these IR singularities are defined as more singular than the power-type IR singularity of the free gluon propagator \((q^2)^{-1}\), which thus can be defined as the PT IR singularity. Due to the character of the IR singularity the longitudinal component of the full gluon propagator should be included into its PT part, so its INP part becomes automatically transversal.

Both terms in Eq. \((2.3)\) are valid in the whole energy/momentum range, i.e., they are not asymptotics. At the same time, we have achieved the exact and unique separation between the two terms responsible for the NP (dominating in the IR at \(q^2 \to 0\)) and the nontrivial PT (dominating in the ultraviolet (UV) at \(q^2 \to \infty\)) dynamics in the true QCD vacuum. Thus it is really beset with severe IR singularities. Within the general nonlinear iteration solution they should be summarized (accumulated) into the full gluon propagator and effectively correctly described by its structure in the deep IR domain, exactly represented by its INP part. Concluding, let us emphasize that in performing the general nonlinear iteration procedure no truncations/approximations and no special gauge choice have been made in the corresponding regularized skeleton loop integrals, which represent the different terms contributing to the full gluon self-energy and hence to its subtracted counterpart.

### A. Subtraction(s)

As emphasized in our previous works [2, 6], many important quantities in QCD, such as the gluon and quark condensates, the topological susceptibility, the Bag constant, etc., are defined only beyond the PT. This means that they are determined by such S-matrix elements (correlation functions) from which all types of the PT contributions should be, by definition, subtracted. Anyway, to calculate correctly any truly NP quantity from first principles in low-energy QCD one has to begin with making subtractions at the fundamental quark-gluon level. Using the exact decomposition \((2.3)\), let us define the INP gluon propagator by the corresponding subtraction as follows:

\[
D_{\mu\nu}^{\text{INP}}(q; \Delta^2) = D_{\mu\nu}(q; \Delta^2) - D_{\mu\nu}(q; \Delta^2 = 0) = D_{\mu\nu}(q; \Delta^2) - D_{\mu\nu}^{\text{PT}}(q), \tag{2.7}
\]

so that the full gluon propagator becomes an exact sum of the two different terms in complete agreement with Eq. \((2.3)\). The principal difference between the full gluon propagator \(D_{\mu\nu}(q; \Delta^2)\) and the INP gluon propagator \(D_{\mu\nu}^{\text{INP}}(q; \Delta^2)\) is that the latter one is free of the PT contributions, while the former one, being also NP, is "contaminated" by them. Also, the INP gluon propagator is manifestly transversal, i.e., does not depend explicitly on the gauge-fixing parameter. Since the formal PT limit \(\Delta^2 = 0\) is uniquely defined in the framework of our method, the separation between the INP and PT gluon propagators is uniquely defined as well. Evidently, the subtraction \((2.7)\) is equivalent to the subtraction made at the level of the full gluon form factor in Eq. \((2.1)\) as follows: \(d(q^2) = d(q^2) - d^{\text{PT}}(q^2) + d^{\text{PT}}(q^2) = d^{\text{INP}}(q^2) + d^{\text{PT}}(q^2)\). It is worth emphasizing once more, that making the above-defined subtraction, we are achieving the two goals simultaneously: the transversality of the gluon propagator relevant for the truly NP QCD, and it automatically becomes free of the PT contributions ("PT contaminations") as well. So our prescription for the subtraction at the fundamental gluon level is simply reduced to the replacement of the general iteration solution by its INP part everywhere, i.e.,

\[
D_{\mu\nu}(q; \Delta^2) \to D_{\mu\nu}^{\text{INP}}(q; \Delta^2), \tag{2.8}
\]

and/or equivalently

\[
d(q^2; \Delta^2) \to d^{\text{INP}}(q^2; \Delta^2). \tag{2.9}
\]

Their explicit expressions are given below. The necessity of such kind of the subtraction and other types ones has been discussed and justified in our papers [2, 6] (see also references therein), where some concrete examples are present as well. Let us emphasize in advance that the replacements \((2.8)\) and \((2.9)\) for the full gluon propagator and the similar one for the full ghost self-energy (see below) mean omitting their corresponding PT parts in which their corresponding free PT counterparts are to be included.

Concluding, the replacements \((2.8)\) and/or \((2.9)\) are necessary to be made first at the fundamental gluon level in order to correctly calculate from first principles any truly NP physical quantities and processes in low-energy QCD.

### B. Multiplicative renormalizations

Thus the full gluon propagator, which is relevant for the description of the truly NP QCD dynamics, is as follows:
\[ D_{\mu\nu}(q, \Delta^2) = iT_{\mu\nu}(q) \frac{\Delta^2}{(q^2)^2} f(q^2), \]  
(2.10)

and

\[ f(q^2) = \sum_{k=0}^{\infty} (\Delta^2/q^2)^k \Phi_k(\lambda, \alpha, \xi, g^2). \]  
(2.11)

Evidently, after making the above described subtraction (2.7) or equivalently the replacement (2.8) the superscript "INP" has been omitted in order to simplify notations.

A new surprising feature of this solution is that its both asymptotics at zero \((q^2 \to 0)\) and at infinity \((q^2 \to \infty)\) are to be determined by its \((q^2)^{-2}\) structure only. This structure determines the behavior of the solution (2.10) at infinity, since all other terms in this expansion are suppressed in this limit. So the main problem with our solution (2.10) is its structure in the deep IR region \((q^2 \to 0)\). The function \(f(q^2)\) is defined by its Laurent expansion, and thus it has an isolated essentially singular point at \(q^2 = 0\). Its behavior in the neighborhood of this point is regulated by the Weierstrass-Sokhockey-Kazorati (WSK) theorem [7] which tells that

\[ \lim_{n \to \infty} f(q_n^2) = Z, \quad q_n^2 \to 0, \]  
(2.12)

where \(Z\) is an arbitrary number, and \(\{q_n^2\}\) is a sequence of points \(q_1^2, q_2^2, \ldots, q_n^2\) along which \(q^2\) goes to zero, and for which this limit always exists. Of course, \(Z\) remains arbitrary (it depends on the chosen sequence of points), but in general it depends on the same set of parameters as the residues, i.e., \(Z \equiv Z(\lambda, \alpha, \xi, g^2)\). This theorem thus allows one to replace the Laurent expansion \(f(q^2)\) by \(Z\) when \(q^2 \to 0\) independently from all other test functions in the corresponding integrands, i.e.,

\[ f(0; \lambda, \alpha, \xi, g^2) \to Z(\lambda, \alpha, \xi, g^2). \]  
(2.13)

There is no doubt that the only real severe (i.e., NP) IR singularity of the full gluon propagator (2.10) is the \((q^2)^{-2}\) NP IR singularity, while the Laurent expansion \(f(q^2)\) should be treated in accordance with the WSK theorem.

Our consideration at this stage is necessarily formal, since the mass gap remains unrenormalized yet as well as all other quantities. So far it has been only regularized, i.e., \(\Delta^2 \equiv \Delta^2(\lambda, \alpha, \xi, g^2)\). However, due to the above-formulated WSK theorem, the full gluon propagator (2.10) effectively becomes

\[ D_{\mu\nu}(q; \Delta^2) = iT_{\mu\nu}(q) \frac{1}{(q^2)^2} Z(\lambda, \alpha, \xi, g^2) \Delta^2(\lambda, \alpha, \xi, g^2), \]  
(2.14)

so just its \((q^2)^{-2}\)-structure is all that matters, indeed. Before going to the \(\lambda \to \infty\) limit in this expression, let us note that in general the coupling constant squared \(g^2\) may also depend on \(\lambda\), becoming thus the so-called ”running” effective charge \(g^2 \sim \alpha_s(\lambda)\). Let us now define the renormalized (R) mass gap in the strong coupling regime as follows:

\[ \Delta_R^2 = Z(\lambda, \alpha_s(\lambda)) \Delta^2(\lambda, \alpha_s(\lambda)), \quad \lambda \to \infty, \quad \alpha_s(\lambda) \to \infty, \]  
(2.15)

at any arbitrary \(\alpha\) and \(\xi\), the explicit dependence on which was omitted as unimportant. So that we consider \(Z(\lambda, \alpha_s(\lambda))\) as the multiplicative renormalization constant for the mass gap, and \(\Delta_R^2\) is the physical mass gap within our approach. Precisely this quantity should be identified with the Jaffe and Witten mass gap [8] (due to the WSK theorem, we can always choose such \(Z\) in order to make \(\Delta_R^2\) positive, finite, gauge-independent, etc.). The two other possible types of the effective charge’s behavior when \(\lambda \to \infty\) have been discussed in our previous work [2].

Thus the full gluon propagator relevant for the description of truly NP QCD dynamics and expressed in terms of the renormalized quantities finally becomes

\[ D_{\mu\nu}(q; \Delta_R^2) = iT_{\mu\nu}(q) \frac{\Delta_R^2}{(q^2)^2}. \]  
(2.16)
The renormalization of the mass gap is an example of the NP renormalization (let us remind that an infinite number of iterations (all iterations) invokes each severe IR singularity labelled by $k$ in Eq. (2.11)). The corresponding initial renormalization constant $Z(\lambda, \alpha, \xi, g^2)$ appears naturally, so the general renormalizability of QCD is not affected.

Since we were able to accumulate all the quadratic divergences (parameterized as the initial ("bare") mass gap) into its renormalization, the $(q^2)^{-2}$-type behavior of the relevant gluon propagator (2.16) at infinity is not dangerous any more, i.e., it cannot undermine the general renormalizability of QCD. It is worth reminding that in Ref. [2] it has been already explained why we call the potential (2.16) confining. In our next papers we will show explicitly that it leads to the confining quark propagator, indeed.

However, the real problem with our solution (2.16) is the behavior at the origin $(q^2 \to 0)$, since its IR singularity represents the so-called severe IR singularity, and the PT fails to deal with it. It should be treated by the distribution theory (DT) [9] into which the dimensional regularization method (DRM) [10] is to be correctly implemented (for a brief review of this program see our previous work [2] and references therein). In order to show that our expression (2.16) is an exact result, i.e., it is neither IR nor UV asymptotic, it is instructive to begin with the initial expressions (2.10) and (2.11), which are valid in the whole energy/momentum range. Because of the summation over $k$, nothing should depend on it. This is in agreement with what we already know from the WSK theorem. Thus the only NP IR singularity of Eq. (2.10) is its $(q^2)^{-2}$-structure. If $q$ is an independent skeleton loop variable, then the dimensional regularization of this NP IR singularity is given by the expansion [2,9]

$$(q^2)^{-2} = \frac{1}{\epsilon} \left[ \pi^2 \delta^4(q) + O(\epsilon) \right], \quad \epsilon \to 0^+.$$  

Here and below $\epsilon$ is the IR regularization parameter (which determines the deviation of the number of dimensions from four [2,9,10]). It should go to zero at the final stage only. Due to the $\delta^4(q)$ function in the residue of this expansion, all the test functions which appear under corresponding skeleton loop integrals should be finally replaced by their expression at $q = 0$. So Eq. (2.10) effectively becomes

$$D_{\mu\nu}(q; \Delta^2_R) = \frac{1}{\epsilon} i T_{\mu\nu}(q) \Delta^2 f(0) \delta^4(q) = \frac{1}{\epsilon} i T_{\mu\nu}(q) \Delta^2_R \delta^4(q),$$

where the the replacement (2.13) (i.e., the result of the WSK theorem) and the definition (2.15) have been used (the finite number $\pi^2$ as usual is included into the renormalized mass gap). For simplicity, the terms of the order $O(\epsilon)$ are not shown. Evidently, substituting the expansion (2.17) into Eq. (2.16), one obtains the same Eq. (2.18). This clearly shows that the previous Eq. (2.16) is exact, i.e., it is not IR asymptotic, and thus remain valid in the whole energy/momentum range.

The only problem remaining to solve is how to remove the pole $1/\epsilon$ which necessarily appears in the full gluon propagator. As emphasized in Ref. [2], in the presence of severe IR singularities, which are to be regularized in terms of the IR regularization parameter $\epsilon$ via the expansion (2.17), in general, all the Green’s functions and parameters depend on it. The only way to remove the pole in $\epsilon$ from the full gluon propagator (2.18) is to define the IR renormalized mass gap as follows:

$$\Delta^2_R = X(\epsilon) \Delta^2_R = \epsilon \Delta^2_R, \quad \epsilon \to 0^+,$$

where $X(\epsilon) = \epsilon$ is the IR multiplicative renormalization (IRMR) constant for the mass gap, and the IR renormalized mass gap $\Delta^2_R$ exists as $\epsilon \to 0^+$, by definition, contrary to $\Delta^2_R$. In both expressions for the mass gap the dependence on $\epsilon$ is assumed but not shown explicitly. Thus the IR and UV renormalized gluon propagator becomes

$$D_{\mu\nu}(q; \Delta^2_R) = i T_{\mu\nu}(q) \Delta^2_R \delta^4(q),$$

and it is instructive to compare it with the initial solution (2.10), which was neither UV nor IR renormalized. It has been only regularized. However, it survived both renormalization programs. In this paper we will show that the IR renormalization of the full gluon propagator or equivalently of the mass gap is completely sufficient to remove all severe IR singularities from all the skeleton loop integrals which may appear in the INP QCD. However, let us note in advance that beyond the one-loop skeleton integrals the analysis should be done in a more sophisticated way, otherwise the appearance of the product of at least two $\delta$ functions at the same point is possible. However, this product is not defined in the DT [9]. So in the multi-loop skeleton diagrams instead of the $\delta$ functions in the residues their derivatives may appear [2,8]. They should be treated in the sense of the DT.

Concluding, Eq. (2.16) is an exact result, i.e., it is neither UV nor IR asymptotic, manifestly transversal and even implicitly does not depend on the gauge-fixing parameter. If $q$ is an independent skeleton loop variable, then Eq. (2.20) is to be used from the very beginning.
C. The ZMME quantum structure of the true QCD ground state

The true QCD ground state is in principle a very complicated confining medium, containing many types of gluon field configurations, components, ingredients and objects of different nature [1, 2, 4, 11, 12]. Its dynamical and topological complexity means that its structure can be organized at both the quantum and classical levels. It is definitely “contaminated” by such gluon field excitations and fluctuations, which are of the PT origin, nature and magnitude. Moreover, it may contain such extra gluon field configurations, which cannot be described as possible solutions to the QCD dynamical equations of motion, either quantum or classical, for example, the vortex-type ones [13]. The only well known classical component of the QCD ground state is the topologically nontrivial instanton-antiinstanton type of fluctuations of gluon fields, which are solutions to the Euclidean Yang-Mills (YM) classical equations of motion in the weak coupling regime [14, 15]. However, they are by no means dominant but, nevertheless, play a special role in the QCD vacuum. In our opinion their main task is to prevent quarks and gluons to freely propagate in the QCD vacuum. It seems to us that this role does not contradict their standard interpretation as tunneling trajectories linking vacua with different topology ([1, 15] and references therein).

Our quantum-dynamical approach to the true QCD ground state is based on the existence and the importance of such kind of the NP excitations and fluctuations of virtual gluon fields which are mainly due to the NL interactions between massless gluon modes without explicitly involving some extra degrees of freedom. It analytically takes into account such gluon field configurations which can be described by the general nonlinear iteration solution (in the form of the corresponding skeleton loops expansion) to the QCD quantum equation of motion for the full gluon propagator in the presence of a mass gap. This solution inevitably becomes plagued by severe IR singularities, which thus play an important role in the large-distances behavior of QCD. They are to be summarized (accumulated) into the purely transversal part of the full gluon propagator, and are to be effectively correctly described by its severely singular structure in the deep IR domain, Eq. (2.10). We will call them the purely transversal singular gluon fields. In other words, they represent the purely transversal quantum virtual fields with the enhanced low-frequency components/large scale amplitudes due to the NL dynamics of the massless gluon modes.

At this stage it is difficult to identify actually which type of gauge field configurations can be finally formed by the purely transversal singular gluon fields in the QCD ground state, i.e., to identify relevant field configurations: chromomagnetic, self-dual, stochastic, etc. However, if these gauge field configurations can be absorbed into the gluon propagator (i.e., if they can be considered as solutions to the corresponding SD equation), then its severe IR singular behavior is a common feature for all of them. Being thus a general phenomenon, the existence and the importance of quantum excitations and fluctuations of severely singular IR degrees of freedom inevitably lead to the general zero momentum modes enhancement (ZMME) effect in the QCD ground state (or equivalently ZME which means simply zero momentum enhancement). Thus our approach to the true QCD ground state, based on the general ZMME phenomenon there, can be analytically formulated in terms of the full gluon propagator (2.10). Moreover, it has been clearly shown that our solution survives both renormalization programs, and is explicitly given in Eq. (2.19). At the same time, the above-mentioned possible complications due to the multi-loop skeleton diagrams should be always kept in mind.

Working always in the momentum space, we are speaking about the purely transversal singular gluon fields responsible for color confinement in our approach. Discussing the relevant field configurations, we always will mean the functional space. Speaking about relevant field configurations (chromomagnetic, self-dual, stochastic, etc), we mean all the low-frequency modes of these virtual transversal fields. Only large scale amplitudes of these fields (“large transversal gluon fields”) are to be taken into account by the INP part of the full gluon propagators. All other frequencies are to be taken into account by corresponding PT part of the gluon propagators. Apparently, it is not correct to speak about specific field configurations that are solely responsible for color confinement. The low-frequency components/large scale amplitudes of all the possible in the QCD vacuum the purely transversal virtual fields are important for the dynamical and topological formation of such gluon field configurations which are responsible for color confinement and other NP effects within our approach to low-energy QCD. For convenience, we will call them the purely transversal severely singular gluon field configurations as mentioned above.

The ZMME (or simply ZME) mechanism of quark confinement is nothing but the well forgotten IR slavery (IRS) one, which can be equivalently referred to as a strong coupling regime [1, 10]. Indeed, at the very beginning of QCD the general idea [10, 17, 18, 19, 20, 21, 22, 23] was expressed that because of the self-interaction of massless gluons in the QCD vacuum, the quantum excitations of the IR degrees of freedom enable us to understand confinement, dynamical (spontaneous) breakdown of chiral symmetry and other NP effects. In other words, the importance of the deep IR structure of the true QCD vacuum has been emphasized as well as its relevance to the above-mentioned NP effects and the other way around. This development was stopped by the wide-spread wrong opinion that severe IR singularities cannot be put under control. Here we have explicitly shown (see also our recent papers [2, 24, 25] and references therein) that the adequate mathematical theory of quantum YM physical theory is the DT (the theory of generalized functions) [9], complemented by the DRM [10]. Together with the theory of functions of complex variable
they provide a correct treatment of these severe IR singularities without any problems. Thus, we come back to the old idea but on a new basis that is why it becomes new ("new is well forgotten old"). In other words, we put the IRS mechanism of quark confinement on a firm mathematical ground.

Concluding, there is no doubt that the purely transversal severely singular virtual gluon field configurations play an important role in the dynamical and topological structure of the true QCD ground state, leading thus to the general ZMME effect there. The quark, ghost Green’s functions and the corresponding ST identities, etc. should be then reconstructed on the basis of this effect. This makes it possible to take into account the response of the NP QCD vacuum.

III. QUARK SECTOR

Together with the full gluon propagator, the full quark propagator also plays one of the most important roles in QCD. After establishing the confining gluon propagator in the previous section, the next step is to derive the confining quark propagator. It allows one to make further necessary steps in the realization of the program to calculate physical QCD. After establishing the confining gluon propagator in the previous section, the next step is to derive the confining vacuum.

Concluding, there is no doubt that the purely transversal severely singular virtual gluon field configurations play an important role in the dynamical and topological structure of the true QCD ground state, leading thus to the general ZMME effect there. The quark, ghost Green’s functions and the corresponding ST identities, etc. should be then reconstructed on the basis of this effect. This makes it possible to take into account the response of the NP QCD vacuum.

\[ S^{-1}(p) = S_0^{-1}(p) - C_F \int \frac{id^3q}{(2\pi)^3} \Gamma_\mu(p,q)S(p-q)\gamma_\mu D_{\mu\nu}(q), \]  

(3.1)

and \( C_F \) is the eigenvalue of the quadratic Casimir operator in the fundamental representation (for \( SU(N_c) \), in general, \( C_F = (N_c^2 - 1)/2N_c = 4/3 \) at \( N_c = 3 \)). \( \Gamma_\mu(p,q) \) is the quark-gluon proper vertex, while \( S(p) \) is the full quark propagator. Here and everywhere below the dependence on the coupling constant in the corresponding powers which comes from the corresponding point-like vertices has been included in the corresponding proper vertices. Let us remind that in the presence of the mass gap it plays no any role, anyway. The free quark propagator is

\[ S_0^{-1}(p) = i(\not{p} + m_0) \]  

(3.2)

with \( m_0 \) being the current ("bare") quark mass.

Since \( q \) is the independent skeleton loop variable and the number of skeleton loops coincides with the number of the full gluon propagators, we can directly substitute our solution for the confining gluon propagator (2.20), which yields

\[ S^{-1}(p) = S_0^{-1}(p) + \bar{\Delta}_\mu^2 \Gamma_\mu(p,0)S(p)\gamma_\mu, \]  

(3.3)

and, for convenience, all other finite numerical factors have been included into the mass gap with retaining the same notation. In deriving this equation, we have used the confining gluon propagator which was already UV and IR renormalized, i.e., free from all types of UV divergences and IR singularities, parameterized in terms of \( \epsilon \). In other words, the quark SD equation (3.3) is free from all these problems. So, we can consider all other Green’s functions entering this equation, namely the quark-gluon proper vertex and th full quark propagator, as the UV and IR renormalized from the very beginning, and omitting the corresponding subscripts and bars, for simplicity. In what follows we will always replace any Green’s functions by their IR renormalized counterparts when there will be no explicit dependence on \( \epsilon \) like it was in this case.

However, one important issue should be discussed in more detail in advance. In passing from Eq. (3.1) to Eq. (3.3) it was implicitly assumed that the vertex function \( \Gamma_\mu(p,0) \) can be simply obtained from \( \Gamma_\mu(p,q) \) in the \( q \to 0 \) limit. Evidently, this is only possible if the vertex is a regular function of the momentum transfer \( q \). In principle, we did not specify the analytical properties of all the vertex functions with respect to their gluon momenta transfer when the confining gluon propagator has been derived in Ref. 2. At the level of the gluon SD equation and within its nonlinear iteration solution the analytical properties of the vertex functions were not crucial. However, beyond the gluon sector they may be important. For example, if the proper vertex in Eq. (3.1) has additional singularities with respect to the gluon momentum \( q \), then they can be effectively incorporated into the gluon propagator itself. The initial singular structure \( (q^2)^{-2} \) of Eq. (2.10) becomes more complicated, so instead of the exponent \( -2 \) a more general exponent \( -2 - k, \ k = 0, 1, 2, 3... \) will appear (and there is no summation over \( k \), i.e., each \( k \) is to be investigated independently). At the same time, the different \( k \) mean different solutions, and different solutions mean different vacua (see discussion in Appendix B of Ref. 2). The Lagrangian of QCD formally remains the same, while the theory is
completely different from the normal QCD. By it we mean QCD in which the zero momenta transfer limit exists in all QCD vertex functions. Anyway, the normal QCD (which obviously corresponds to $k = 0$) should be investigated independently from QCD with additional singularities in the vertex functions. In what follows it is assumed that all severe IR singularities can be summarized by the full gluon propagator, and thus all the vertex functions are regular functions of the corresponding momenta transfer. Let us emphasize once more that this is obviously not a restriction, moreover important it may be the most realistic case. It is worth noting as well in advance that the smoothness properties of the corresponding test functions (which will be established in the subsequent paper) are in complete agreement with the above-mentioned regularity of all the QCD vertices.

IV. GHOST SECTOR

The information about the quark-gluon vertex function at zero momentum transfer, needed for the evaluation of the confining quark propagator (3.3), can be provided by the quark ST identity [1, 17, 26, 27] (and references therein), which contains unknown ghost contributions in the covariant gauge. For this reason let us consider in this section the SD equation for the ghost self-energy $b(k^2)$, which also obeys a simple SD equation with Euclidean signature [17, 27].

$$ik^2b(k^2) = -CA\int \frac{idq}{(2\pi)^4}G_\mu(k, q)G(k - q)(k - q)_\nu D_{\mu\nu}(q),$$

where $C_A$ is the eigenvalue of the quadratic Casimir operator in the adjoint representation (for $SU(N_c)$, in general $C_A = N_c = 3$). The full ghost propagator is

$$G(k) = \frac{i}{k^2[1 + b(k^2)]}$$

and

$$G_\mu(k, q) = k^\lambda G_{\lambda\mu}(k, q)$$

is the ghost-gluon proper vertex ( $G_{\lambda\mu} = g_{\lambda\mu}$ in the PT).

As for the quark SD equation, in the gluon self-energy the momentum transfer $q$ is the independent skeleton loop variable. This allows one to directly substitute again Eq. (2.20) which yields

$$ik^2b^{\mathrm{INP}}(k^2) = \bar{\Delta}_R^2G_\mu(k, 0)G(k)k_\mu,$$

where again all finite numerical factors have been included into the mass gap. We also retain the superscript "INP" for the gluon self-energy in the left-hand-side of this equation for future purpose (see next section). This is instructive to do in order to indicate that its right-hand-side has been obtained by replacing the full gluon propagator by its INP counterpart in accordance with our method.

It is convenient to rewrite Eq. (4.4) in the equivalent form as follows:

$$-\bar{\Delta}_R^2G_\mu(k, 0)G(k) = ik_\mu b^{\mathrm{INP}}(k^2).$$

Just this equation will be used in order to investigate the quark-gluon vertex function at zero momentum transfer. In the corresponding ST identity the momentum transfer goes through the ghost momentum (see next section). For that very reason, let us assume that the ghost self-energy $b(k^2)$ exists and is finite at $k^2 = 0$. Evidently, this means that both terms, namely $b^{\mathrm{INP}}(k^2)$ and $b^{\mathrm{PT}}(k^2)$, which appear in the formal decomposition $b(k^2) = b(k^2) - b^{\mathrm{PT}}(k^2) + b^{\mathrm{PT}}(k^2) = b^{\mathrm{INP}}(k^2) + b^{\mathrm{PT}}(k^2)$, also exist and are finite at zero point (in agreement with the above-mentioned regularity of the QCD vertex functions with respect to their momenta transfer). This can be directly shown, but we will not complicate the context of this section, since our final results will not depend explicitly on this auxiliary technical assumption. Concluding, let us only note that the above-mentioned decomposition follows from the exact and unique subtraction (2.7) after its substitution into the ghost self-energy SD equation (4.1).
V. QUARK- GHOST SECTOR

Though nothing should explicitly depend on ghost degrees of freedom in QCD, nevertheless, the ghost-quark sector contains a very important piece of information on quark degrees of freedom themselves through the corresponding quark ST identity. Precisely this information should be self-consistency taken into account. Otherwise any solutions to the dynamical equations will be plagued by unusual analytical properties (unphysical singularities), since in the absence of ghosts the unitarity of S-matrix is violated. The ST identity for the quark-gluon vertex function \( \Gamma_\mu(p,k) \) is ([1], [17], [26], [27], [28], [29], [30] and references therein)

\[
- i k_\mu \Gamma_\mu^a(p,k) \left[ 1 + b(k^2) \right] = \left[ T^a - B^a(p,k) \right] S^{-1}(p+k) - S^{-1}(p) \left[ T^a - B^a(p,k) \right],
\]

where \( b(k^2) \) is the ghost full self-energy and \( B^a(p,k) \) is the ghost-quark scattering amplitude. \( T^a \) are the color group generators. From the ST identity (5.1) one recovers the standard QED-type Ward-Takahashi (WT) identity in the formal \( b = B = 0 \) limit.

The ghost-quark scattering kernel \( B^a(p,k) \) is determined by its skeleton expansion

\[
B^a(p,k) = \sum_{n=1}^{\infty} B_n^a(p,k),
\]

which diagrammatically representation can be found, for example in Refs. [17], [27], [29], [30]. In the Landau gauge \( (\xi = 0) \) and at \( k = 0 \) Taylor [28] has shown that it is zero, i.e.,

\[
B^a(p,0) = 0,
\]

and this is valid for each skeleton term in the skeleton expansion (5.2), i.e., the relation (5.3) is valid because each \( B_n^a(p,0) = 0 \) in the Landau gauge.

Let us begin with the investigation of the first term \( B_1(p,k) \) of the \( B(p,k) \) skeleton expansion (5.2). After the evaluation of the color group factors its analytical expression becomes (Euclidean space)

\[
B_1(p,k) = \frac{1}{2} C_A \int \frac{i d^4 q}{(2\pi)^4} S(p-q) \Gamma_\nu(p-q,q) G_\mu(k,q) G(k+q) D_{\mu\nu}(q).
\]

Before proceeding further, let us show explicitly that it satisfies the Taylor’s general relation (5.3). In the Landau gauge \( D_{\mu\nu}(q) \sim T_{\mu\nu}(q) \) and at \( k = 0 \) the ghost-gluon vertex \( G_\mu(0,q) \sim q_\mu \), so \( q_\mu T_{\mu\nu}(q) = 0 \) leads to \( B_1(p,0) = 0 \), indeed, in the Landau gauge. These arguments are valid term by term in the skeleton expansion (5.2).

As in previous cases the gluon momentum \( q \) is independent skeleton loop variable, so again Eq. (2.20) can be directly substituted, which yields

\[
B_1(p,k) = \frac{1}{2} \Delta_k^2 S(p) \Gamma_\mu(p,0) G_\mu(k,0) G(k),
\]

and using further the ghost SD equation (4.5), one finally obtains

\[
B_1(p,k) = - \frac{1}{2} i S(p) \Gamma_\mu(p,0) b^{IJN\Phi}(k^2) k_\mu,
\]

which clearly shows that it is of order \( k (\sim O(k)) \) when \( k \) goes to zero since \( b^{IJN\Phi}(0) \) exists and finite (see previous section). Let us emphasize that this final expression does not depend on the mass gap as it should be. Moreover, in the expression (5.5) the mass gap (after the inclusion of all finite numerical factors) is the same as in Eq. (4.5), since the ghost line in the expression (5.4) is the same as in Eq. (4.1).

The analytical expression of the second skeleton diagram for the ghost-quark scattering kernel \( B(p,k) \) is

\[
B_2(p,k) = A \int \frac{i d^4 q}{(2\pi)^4} \int \frac{i d^4 l}{(2\pi)^4} S(p-q+l) \Gamma_\mu(p-q+l,l) S(p-q) \Gamma_\nu(p-q,q) G_\mu(k,-l) G(k-l,q) G(k-l+q) D_{\mu\nu}(l) D_{\alpha\beta}(q),
\]
where the constant $A$ is a result of the summation over color group indices (its explicit expression is not important here and below). Since both gluon momenta $q$ and $l$ are independent skeleton loop variables, we again can use Eq. (2.20) twice, which yields

$$B_2(p, k) = A_0 \hat{\Delta}_0 S(p) \Gamma_\beta(p, 0) S(p) \Gamma_\nu(p, 0) G_\beta(k, 0) G(k) G_\nu(k, 0) G(k),$$  \quad (5.8)$$

and again using the ghost SD equation (4.5) twice, one finally obtains

$$B_2(p, k) = A_0 S(p) \Gamma_\beta(p, 0) S(p) \Gamma_\nu(p, 0) [b^{INP}(k^2)]^2 k_\beta k_\nu,$$

which clearly shows that this term is of order $k^2$ as it goes to zero, since $b^{INP}(k^2)$ is finite at zero point.

In the same way it is possible to show that the third term $B_3(p, k)$ is of the order $k^3$ as $k$ goes to zero. These arguments are valid term by term in the skeleton expansion for the ghost-quark scattering kernel $B(p, k)$ (5.2). So, we have an exact estimate

$$B_n(p, k) = O(k^n), \quad k \to 0.$$  \quad (5.10)

It means that we maintain Taylor’s general result (5.3). It is worth emphasizing, however, that our confining gluon propagator is automatically transversal, i.e., we did not choose the Landau gauge by hand.

Differentiating now the quark ST identity (5.1) with respect to $k_\mu$ and passing to the limit $k = 0$, one obtains ($d_\mu = d/dp_\mu$, by definition)

$$-i \Gamma_\mu(p, 0) [1 + b(0)] = d_\mu S^{-1}(p) - \Psi_\mu(p) S^{-1}(p) + S^{-1}(p) \Psi_\mu(p),$$  \quad (5.11)

where $\Psi_\mu(p)$ is defined as

$$\Psi_\mu(p) = \left[ \frac{\partial}{\partial k_\mu} B(p, k) \right]_{k=0} = -\frac{1}{2} b^{INP}(0) S(p) \Gamma_\mu(p, 0),$$  \quad (5.12)

since due to an estimate (5.10) the first term (5.6) survives only in the $k = 0$ limit. Substituting it back into the ST identity (5.11), one obtains that it becomes

$$\left[ 1 + b^{PT}(0) + \frac{1}{2} b^{INP}(0) \right] \Gamma_\mu(p, 0) = id_\mu S^{-1}(p) - \frac{1}{2} b^{INP}(0) S(p) \Gamma_\mu(p, 0) S^{-1}(p),$$  \quad (5.13)

where the above-mentioned formal decomposition $b(0) = b^{PT}(0) + b^{INP}(0)$ has been also used (let us recall, however, that this decomposition is exact and unique, since it is due to the substitution of the subtraction (2.7) into the ghost self-energy SD equation (4.1)). In this form the quark ST identity first has been obtained by Pagels in his pioneering paper on NP QCD [17]. However, this form is not acceptable, since it depends explicitly on the PT part of the ghost self-energy, i.e., it is not completely free yet from the PT contributions ("contaminations").

Fortunately, we already know how to solve this problem. In accordance with our subtraction prescription (2.8) the full ghost self-energy at zero $b(0)$ should be replaced by its INP part $b^{INP}(0)$, which is equivalent to omit in the quark ST identity (5.13) the PT part of the ghost self-energy in which its free PT counterpart $b^{PT}_0 = 1$ is to be included. In other words, the sum $1 + b^{PT}(0) = b^{PT}_0 + b^{PT}(0) \to b^{PT}(0)$ should be omitted in the left-hand-side of the quark SD identity (5.13). So one gets

$$\frac{1}{2} b^{INP}(0) \Gamma_\mu(p, 0) = id_\mu S^{-1}(p) - \frac{1}{2} b^{INP}(0) S(p) \Gamma_\mu(p, 0) S^{-1}(p),$$  \quad (5.14)

and thus it becomes free of all types of the PT contributions, indeed. At the same, the necessary information on quark degrees of freedom important for the INP QCD dynamics has been completely extracted from the initial ST identity (the second term in Eq. (5.14), while the first term is the standard WT-type one). In a more sophisticated way this procedure is described in Appendix A.
VI. INTRINSICALLY NONPERTURBATIVE (INP) QCD

Let us now write down the system of equations obtained in the quark sector

$$ S^{-1}(p) = S_0^{-1}(p) + \Delta^2_{NP} \Gamma_\mu(p,0) S(p) \gamma_\mu, $$

$$ \frac{1}{2} b^{INP}(0) \Gamma_\mu(p,0) = id_\mu S^{-1}(p) - \frac{1}{2} b^{INP}(0) S(p) \Gamma_\mu(p,0) S^{-1}(p). \quad (6.1) $$

This system still suffers from the explicit presence of the unknown number, namely $b^{INP}(0)$. To resolve this difficulty, let us rescale the proper vertex as follows:

$$ \frac{1}{2} b^{INP}(0) \Gamma_\mu(p,0) \implies \Gamma_\mu(p,0), \quad (6.2) $$

which makes it possible to include this unknown number into the final mass gap, which we denote as $\Lambda^2_{NP}$. The initial system of equations (6.1) then becomes

$$ S^{-1}(p) = S_0^{-1}(p) + \Lambda^2_{NP} \Gamma_\mu(p,0) S(p) \gamma_\mu, $$

$$ \Gamma_\mu(p,0) = id_\mu S^{-1}(p) - S(p) \Gamma_\mu(p,0) S^{-1}(p). \quad (6.3) $$

Let us emphasize once more that the obtained system of equations (6.3) is exact, i.e., no approximations/truncations have been made so far. Formally it is valid in the whole energy/momentum range, but depends only on the mass gap responsible for the true NP QCD dynamics. It is free from all the types of the PT contributions (“PT contaminations”) at the fundamental quark-gluon level. Also, it is manifestly gauge-invariant, i.e., does not depend explicitly on the gauge-fixing parameter. In the part II of this paper it will be our primary goal to solve this system. For the first time the system of equations (6.3) has been published in our preliminary papers [29, 30].

We consider the INP QCD as a true theory of low-energy QCD, which makes it possible to calculate the physical observables/processes in QCD from first principles. Let us recall that we define INP QCD (see Refs. [2, 6] and section II in this work) by the subtractions of all the types and at all levels of the PT contributions from the corresponding QCD expressions, equations, relations, etc. Symbolically this can be shown as follows:

$$ QCD \implies INP \ QCD = QCD - GPT \ QCD, \quad (6.4) $$

where, evidently, GPT QCD symbolically stands for the general PT (GPT) QCD, and which includes all of the mentioned PT contributions. The first necessary subtraction has been done at the fundamental gluon level in Eq. (2.7). All other related subtractions have been also made in the quark, ghost and quark ST identity sectors in order to get to the final system of equations (6.3) at the fundamental quark-gluon level. It allows one to derive the full quark propagator in closed form and then to apply such a quark propagator for the calculation of any physical observable/process from first principle in terms of the mass gap $\Lambda^2_{NP}$ in low-energy QCD.

Before going to some conclusions, it is worth making a few remarks. Contrary to ghost and gluon degrees of freedom in which their free PT counterparts have been included into the their nontrivial PT parts, the free PT quark propagator has not been subtracted in Eq. (6.3). Evidently, it has to be retained in order to maintain the chiral limit physics in QCD, which is important to correctly understand the structure of QCD at low energies.

Concluding, using the confining gluon propagator the corresponding system of equations in the quark sector (6.3) has been derived in a self-consistent way. It is free from all types of the PT contributions, and thus is UV finite (i.e., free from the UV divergences). It does not depend explicitly on the gauge-fixing parameter. It has been derived for the Green’s functions which have been treated as the IR renormalized from the very beginning, since the confining gluon propagator used (2.20) was the UV and IR renormalized as well. However, the nontrivial IR renormalization program can be performed. In this way one obtains the system of the IR convergence conditions for the corresponding IRMR constants, which relate the regularized quantities to their renormalized counterparts. This makes it possible to remove all severe IR singularities parameterized in terms of the IR regularization parameter $\epsilon$ from all the equations, identities, etc. in a self-consistent way. Its solution will lead finally to the same system of equations (6.3), of course. That is why there is no need in these technical complications if it is not really necessary. This necessity may only appear in the multi-loop skeleton diagrams, containing the three- and four-gluon proper vertices.
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APPENDIX A: RESCALING PROCEDURE

Let us formulate the rescaling procedure in a more sophisticated and general ways. Its final goal is to get the system of equations, consisting of the quark SD equation (3.3) and quark ST identity (5.13), free of all types of the PT contributions ("PT contaminations"). For this purpose, it makes sense to rescale the vertex in Eq. (5.13) as follows:

\[ \left[1 + b^{PT}(0) + \frac{1}{2} b^{JNP}(0) \right] \Gamma_{\mu}(p, 0) \implies \Gamma_{\mu}(p, 0). \]  
(A1)

The ST identity (5.13) then becomes

\[ \Gamma_{\mu}(p, 0) = id_{\mu}S^{-1}(p) - (1 + \delta)^{-1}S(p)\Gamma_{\mu}(p, 0)S^{-1}(p), \]  
(A2)

where

\[ \delta = \frac{2[1 + b^{PT}(0)]}{b^{JNP}(0)}. \]  
(A3)

Let us note here that \( b^{PT}_{0} = 1 \), where \( b^{PT}_{0} \) is the form factor of the free ghost propagator (see Eq. (4.2) at \( b(k^2) = 0 \)). As in the case of the PT part of the full gluon propagator, let us include it into the PT part of the full gluon self-energy with retaining the same notation, i.e., the replacement \( 1 + b^{PT}(0) = b^{PT}_{0} + b^{PT}(0) \rightarrow b^{PT}(0) \) is understood in what follows.

Expanding formally in powers of \( \delta \), one gets

\[ (1 + \delta)^{-1} = 1 + \sum_{n=2}^{\infty} (-1)^{n-1}\delta^{n-1}, \quad \delta = 2 \frac{b^{PT}(0)}{b^{JNP}(0)}. \]  
(A4)

Substituting this back into the ST identity (A2), one obtains

\[ \Gamma_{\mu}(p, 0) = id_{\mu}S^{-1}(p) - S(p)\Gamma_{\mu}(p, 0)S^{-1}(p) + \Gamma'_{\mu}(p, 0), \]  
(A5)

where, obviously,

\[ \Gamma'_{\mu}(p, 0) = -S(p)\Gamma_{\mu}(p, 0)S^{-1}(p) \sum_{n=2}^{\infty} (-1)^{n-1} \left( 2 \frac{b^{PT}(0)}{b^{JNP}(0)} \right)^{n-1}. \]  
(A6)

Making the same rescaling trick (A1) in the quark SD equation (3.3), one obtains

\[ S^{-1}(p) = S^{-1}_{0}(p) + \Lambda_{NP}^{2} \Gamma_{\mu}(p, 0)S(p)\gamma_{\mu} + i\Sigma'(p), \]  
(A7)

where, evidently,

\[ i\Sigma'(p) = \Lambda_{NP}^{2} \Gamma_{\mu}(p, 0)S(p)\gamma_{\mu} \sum_{n=2}^{\infty} (-1)^{n-1} \left( 2 \frac{b^{PT}(0)}{b^{JNP}(0)} \right)^{n-1}. \]  
(A8)
We also include the finite numerical factor \((2/b^{\text{INP}}(0))\) into the mass gap \(\Delta_R^2\) and denote it as \(\Lambda_{NP}^2\). Just this quantity will be treated in what follows as the physical mass gap responsible for the truly NP QCD dynamics within our approach. Thus with the help of the formulated rescaling procedure we were able to exactly identify (decouple) the terms which are “contaminated” by the PT contributions due to ghosts in the quark ST identity and the quark SD equation in Eqs. (A5) and (A7), respectively. At the same, the necessary information on quark degrees of freedom important for the INP QCD dynamics has been completely extracted from the initial ST identity (the second term in Eq. (A5), while the first term is the standard WT-type one).

Let us now write down the system of equations obtained in the quark sector

\[
S^{-1}(p) = S_0^{-1}(p) + \Lambda_{NP}^2 \Gamma_{\mu}(p, 0) S(p) \gamma_{\mu} + i\Sigma(p),
\]

\[
\Gamma_{\mu}(p, 0) = id_{\mu} S^{-1}(p) - S(p) \Gamma_{\mu}(p, 0) S^{-1}(p) + \Gamma'_{\mu}(p, 0),
\]  

(A9)

and where the terms which are “contaminated” by the PT contributions due to ghosts \(i\Sigma(p)\) and \(\Gamma'_{\mu}(p, 0)\) are shown explicitly in Eqs. (A8) and (A6), respectively. The first of this system of equations, namely the quark SD one depends explicitly on the mass gap \(\Lambda_{NP}^2\), which determines the large-scale structure of the true QCD vacuum. In deriving this system no approximation/truncations have been made by hand. However, the two serious problems still remain to solve. The first one is its above-mentioned “contamination” with the PT contributions due to ghosts. The second one is in close relation with the first one, namely the PT part of the ghost self-energy \(b^{PT}(0)\) may still depend explicitly on the gauge-fixing parameter. Let us recall that we define INP QCD (see symbolic Eq. (6.4)) by the subtractions of all the types and at all levels of the PT contributions from the corresponding QCD expressions, equations, relations, etc. Equivalently, this can be achieved by simply dropping the terms “contaminated” by the PT contributions in all equations, relations, etc. Doing so in the system of equations (A9), one finally arrives to the same system of equations (6.3) as it should be.

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