Nuclear matter parameters for finite nuclei using relativistic mean field formalism within coherent density fluctuation model

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We obtained an analytical expression by fitting the nuclear matter (NM) binding energy of the effective field theory motivated relativistic mean filed (E-RMF) model for different neutron-proton symmetry. The local density approximation is adopted to generate the expression in the coordinate space. This expression has an edge over the Brückner energy density functional [Phys. Rev. 171, 1188 (1968)] since it resolves the Coester-Band problem. The NM parameters like incompressibility, neutron pressure, symmetry energy and its derivatives are derived using the acquired expression of energy per nucleon. Further, the weight functions calculated by E-RMF densities are folded with newly constructed NM parameters within coherent density fluctuation model to find the effective surface properties of doubly closed shell densities are folded with newly constructed NM parameters within coherent density fluctuation model to find the corresponding surface quantities for finite nuclei. The results present the neutron pressure $P$, symmetry energy $S$ and its derivative $L_{sym}$ called as slope parameter which lie within a narrow domain whereas there is large variation in isoscalar incompressibility $K^A$ and surface incompressibility $K_{sym}^A$ while moving form light to heavy nuclei. The sizable variation in $K^A$ and $K_{sym}^A$ for light and heavy nuclei depicts their structural dependence due to peculiar density distribution of each individual nucleus.

I. INTRODUCTION

The correlations among the nuclear matter (NM) and finite nuclei in terms of symmetry energy and its coefficients play a crucial role not only in nuclear physics but also in astrophysics. The isospin dependence of symmetry energy imparts the information about the isovector component of the nuclear interaction, which is directly connected with the skin thickness of the nuclei. Eventually, different studies such as island of stability of exotic nuclei, the dynamics of heavy-ion collisions, dipole polarizability, properties of neutron stars, core collapse of compact massive stars and the nucleosynthesis process through neutrino convection at high density hinge upon the symmetry energy and its coefficients [1–10]. Therefore, it is indispensable to determine the symmetry energy and its coefficients for finite nuclei. Recently, many efforts have been made on theoretical as well as experimental fronts to probe the isospin dependence of symmetry energy and its coefficients, which is an ultimate bridge between finite nuclei and infinite nuclear matter [3, 6–10]. Moreover, in some recent works it is established that the kink in the symmetry energy of finite nuclei over the isotopic chain infers the appearance of shell/sub-shell closures [8–10].

It is worthwhile to mention that in earlier works, the Brückner energy density functional [11, 12] has been used within coherent density fluctuation model (CDFM) to calculate the surface properties of nuclei [9, 10]. This functional had been fitted to the kinetic and potential energy parts to get the analytical expression of binding energy per particle $E/A$ in the local density approximation (LDA) of Thomas-Fermi approach. It is relevant to point out that Brückner energy functional does not respect the "Coester-Band", i.e. NM saturates at $\rho \sim 0.2$ fm$^{-3}$ instead of $\sim 0.15$ fm$^{-3}$ [13, 14]. In order to have some meaningful correlations while extrapolating to higher densities, the nuclear equation of state (EOS) must satisfy the nuclear saturation properties, especially the "Coester Band". To address this crucial problem, we have fitted the NM saturation plots for different values of asymmetry parameter, using effective field theory motivated relativistic mean field (E-RMF) model [15, 16] with G3 and standard NL3 parameter sets for the first time. The different NM parameters such as incompressibility, symmetry energy and its derivatives are obtained using the fitted expression of $E/A$ using E-RMF density functional. Subsequently, theses NM parameters are used along with E-RMF densities within the CDFM to find the corresponding surface quantities for doubly magic $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{56}$Ni, $^{90}$Zr, $^{116}$Sn, and $^{208}$Pb nuclei.

The paper is organized as follows: The E-RMF approach and the fitting procedure to get the coefficients of the analytical expressions for binding energy per particle is discussed in Sec. II. The CDFM is also discussed in this section. Sec. III is assigned to the discussion of the results obtained from the calculations. A brief summary and conclusions are presented in Sec. IV.

II. THEORETICAL FRAMEWORK

A. E-RMF Formalism

In this sub-section, we briefly describe the formalism of recently developed E-RMF model. The E-RMF Lagrangian density is constructed by taking the interactions of isoscalar
(scalar $\sigma$, vector $\omega$) and isovector (scalar $\delta$, vector $\rho$) mesons with nucleons and among themselves. The E-RMF Lagrangian is discussed in Refs. [15–18]. The E-RMF is considered to be one of the most successful model to reproduce the ground state properties of not only $\beta$—stable nuclei but it also predicts quite reasonably the properties of drip-lines and superheavy nuclei [15, 16]. During last few decades, the application of this formalism to nuclear astrophysics is at forefront. It predicts the structure of neutron star and explains the tidal deformability satisfactorily [19]. The energy density functional for a nucleon-meson interacting system is given as [15]:

$$
\mathcal{E}(r) = \sum_\alpha \varphi_\alpha^\dagger(r) \left\{ -i \alpha \cdot \nabla + \beta [M - \Phi(r) - \tau_3 D(r)] + W(r) + \frac{1}{2} \tau_3 R(r) + \frac{1 + \tau_3}{2} A(r) \\
- \frac{i \beta \alpha}{2M} \left( f_\omega \nabla W(r) + \frac{1}{2} f_\rho \nabla R(r) \right) \right\} \varphi_\alpha(r) + \left( \frac{1}{2} + \frac{\kappa_3}{3!} \Phi(r) + \frac{\kappa_4}{4!} \Phi^2(r) \right) \frac{m^2}{g_\sigma^2} \Phi^2(r) \\
- \frac{\zeta_0}{4! g_\omega^2} W^4(r) + \frac{1}{2g_\omega^2} \left( 1 + \alpha_1 \frac{\Phi(r)}{M} \right) (\nabla \Phi(r))^2 - \frac{1}{2g_\omega^2} \left( 1 + \alpha_2 \frac{\Phi(r)}{M} \right) (\nabla W(r))^2 \\
- \frac{1}{2} \left( 1 + \eta_1 \frac{\Phi(r)}{M} + \frac{\eta_2 \Phi^2(r)}{2 M^2} \right) \frac{m^2}{g_\omega^2} W^2(r) - \frac{1}{2e^2} (\nabla A(r))^2 - \frac{1}{2} \left( \nabla R(r) \right)^2 \\
- \frac{1}{2} \left( 1 + \eta_\rho \frac{\Phi(r)}{M} \right) - \Lambda_v \left( R^2(r) \times W^2(r) \right) + \frac{1}{2g_\rho^2} (\nabla D(r))^2 + \frac{1}{2g_\delta^2} \left( D^2(r) \right),
$$

(1)

Here, $\Phi$, $W$, $R$ and $D$ are the re-defined fields for $\sigma$, $\omega$, $\rho$ and $\delta$ mesons given as $\Phi = g_\sigma \sigma$, $W = g_\omega \omega^\mu$, $R = g_\rho \rho^\mu$ and $D = g_\delta \delta$, respectively. $M$, $m_\sigma$, $m_\omega$, $m_\rho$ and $m_\delta$ are the masses of nucleon, $\sigma$, $\omega$, $\rho$ and $\delta$ mesons, respectively. From Eq. (1), we obtain the energy density $\mathcal{E}_{\text{nucl.}}$ [15, 16] by considering that the exchange of mesons create an uniform field, where the nucleon oscillates in a simple harmonic motion. From the E-RMF energy density, the equation of motions for the mesons and the nucleons are derived using the Euler-Lagrange equation. A set of coupled differential equations are obtained and solved self-consistently. The calculations are done within the mean field approximation and the $\mathcal{E}_{\text{nucl.}}$ is obtained as a function of baryon density given as:

$$
\mathcal{E}_{\text{nucl.}} = \frac{2}{(2\pi)^3} \sum_i \int d^3k E_i^2(k) + \frac{m^2 \Phi^2}{g_\sigma^2} \left( \frac{1}{2} + \frac{\kappa_3}{3!} \Phi + \frac{\kappa_4}{4!} \Phi^2 \right) + \rho_\omega W - \frac{\zeta_0 W^4}{4! g_\omega^2} \\
- \frac{1}{2} \frac{m^2}{g_\omega^2} W^2 \left( 1 + \frac{\eta_1 \Phi}{M} + \frac{\eta_2 \Phi^2}{2 M^2} \right) + \frac{1}{2} \rho_\omega R - \frac{1}{2} \left( 1 + \frac{\eta_\rho \Phi}{M} \right) \frac{m^2}{g_\rho^2} R^2 \\
- \Lambda_v (R^2 \times W^2) + \frac{1}{2} \frac{m^2}{g_\delta^2} (D^2),
$$

(2)

The scalar and vector densities,

$$
\rho_s(r) = \sum_i^A \psi_i(r) \overline{\psi_i}(r),
$$

(3)

$$
\rho_v(r) = \sum_i^A \psi_i(r) \psi_i(r),
$$

(4)

are evaluated from the converged solutions within spherical harmonics. The vector density $\rho_v(r)$ is further used within CDFM to find out the weight function $|F(x)|^2$, which is an important quantity to calculate the incompressibility ($K^A$), symmetry energy ($S^A$), neutron pressure ($P^A$) and surface symmetry coefficient ($K_{\text{sym}}^A$) for the doubly magic spherical nuclei.
B. Fitting procedure

The important part of the present calculation is to convert the nuclear matter quantities to coordinate space which were initially in the momentum space i.e. the reconstruction of NM quantities at local density. The results of our calculations are shown in Fig. 1 for NL3 and G3 parameter sets. The NL3 set gives a stiff equation of state (EOS) as compared to G3 force. This is because, the NM incompressibility $K$ at the saturation for NL3 is 271.76 MeV and that of G3 is 243.96 MeV. We consider that the NM is composed of tiny spherical pieces described by a local density function $\rho_0(x) = 3A/4\pi x^3$. Using this consideration, the fitted binding energy function (Fig. 1) of E-RMF is embedded in the following equation

$$\mathcal{E}(x) = C_k \rho_0^{2/3}(x) + \sum_{i=3}^{14} (b_i + a_i \alpha^2) \rho_0^{i/3}(x). \quad (5)$$

Then the NM parameters $K^{NM}, S^{NM}, L^{NM}_{sym}$ and $K^{NM}_{sym}$ are obtained from the following standard relations [15, 20]

$$K^{NM} = 9\rho_0^2 \frac{\partial^2}{\partial \rho^2} (\frac{\mathcal{E}(\rho)}{\rho}) \bigg|_{\rho=\rho_0}, \quad (6)$$
$$S^{NM} = \frac{1}{2} \frac{\partial^2 (\mathcal{E}(\rho)/\rho)}{\partial \alpha^2} \bigg|_{\alpha=0}, \quad (7)$$
$$L^{NM}_{sym} = 3\rho \frac{\partial S(\rho)}{\partial \rho} \bigg|_{\rho=\rho_0} = \frac{3P}{\rho_0}, \quad (8)$$
$$K^{NM}_{sym} = 9\rho_0^2 \frac{\partial^2 S(\rho)}{\partial \rho^2} \bigg|_{\rho=\rho_0}. \quad (9)$$

are given by

$$K^{NM} = -150.12 \rho_0^{2/3}(x) + \sum_{i=4}^{14} i (i-3) b_i \rho_0^{i/3}(x) \quad (10)$$
$$S^{NM} = 41.7 \rho_0^{2/3}(x) + \sum_{i=3}^{14} a_i \rho_0^{i/3}(x), \quad (11)$$
$$L^{NM}_{sym} = 83.4 \rho_0^{2/3}(x) + \sum_{i=3}^{14} i a_i \rho_0^{i/3}(x), \quad (12)$$
$$K^{NM}_{sym} = -83.4 \rho_0^{2/3}(x) + \sum_{i=3}^{14} i (i-3) a_i \rho_0^{i/3}(x), \quad (13)$$

where $C_k$ is the kinetic energy coefficient given as $C_k = 37.53[(1 + \alpha)^{5/3} + (1 - \alpha)^{5/3}]$ within the Thomas-Fermi approach. The densities of double closed shell spherical nuclei $^{16}$O, $^{40}$Ca, $^{56}$Ni, $^{90}$Zr, $^{116}$Sn and $^{208}$Pb are calculated using E-RMF formalism. These densities are used as input in the CDFM (described in the following sub-section) to calculate the weight function, which is a key quantity acting as a bridge between NM parameters in $x-$space and finite nuclei in $r-$space (using LDA). To match with the $r-$ and $x-$space together, we construct the total density of the nucleus with superposition of an infinite number of $F_{l u c t o n}$, following the approach of CDFM discussed below.

C. Coherent Density Fluctuation model

The CDFM of Antonov and collaborators [21–24] has been used to calculate the NM parameters of finite nuclei. Within CDFM, the one-body density matrix (OBDM) $\rho(\mathbf{r}, \mathbf{r}')$ of a finite nucleus is written as the coherently superposed of OBDM $\rho_x(\mathbf{r}, \mathbf{r}')$ for spherical pieces of NM termed as $Fluctons$ [9, 25],

$$\rho_x(\mathbf{r}) = \rho_0(x) \Theta(x - |\mathbf{r}|), \quad (14)$$

with $\rho_0(x) = \frac{3A}{4\pi x^3}$16. The generator coordinate $x$ is the radius of a sphere consisting of Fermi gas having all A nucleons distributed uniformly within it. It is suitable to apply for such a system the OBDM expressed as below [9, 22, 25, 26],

$$\rho(\mathbf{r}, \mathbf{r}') = \int_0^\infty dx |F(x)|^2 \rho_x(\mathbf{r}, \mathbf{r}') \quad (15)$$

where, $|F(x)|^2$ is the weight function (WF). The coherently superposition of OBDM $\rho_x(\mathbf{r}, \mathbf{r}')$ is given as:

$$\rho_x(\mathbf{r}, \mathbf{r}') = 3\rho_0(x) \frac{J_1(\gamma f(x)|x - \mathbf{r}'|)}{J_1(\gamma f(x)|x - \mathbf{r}'|)} \times \Theta \left( x - \frac{|\mathbf{r} + \mathbf{r}'|}{2} \right) \quad (16)$$

where $J_1$ is the first order spherical Bessel function and $k_f$ is the Fermi momentum of nucleons inside the Flucton having radius $x$ and $k_f(x) = (3\pi^2/2\rho_0(x))^{1/3} = \gamma/x$, ($\gamma \approx$...
is obtained [21]. The \( |F(x)|^2 \) for a given density \( \rho (r) \) is expressed as

\[
|F(x)|^2 = -\left( \frac{1}{\rho_0(x)} \frac{d\rho(r)}{dr} \right)_{r=x},
\]

with \( \int_0^\infty dx |F(x)|^2 = 1 \). We refer \([9, 21, 22, 25, 26]\) for a detailed analytical derivation. The CDFM allows us to make a transition from the properties of \( \text{NM} \) to those of finite nuclei. The finite nuclear incompressibility \( K^A \), symmetry energy \( S^A \), neutron pressure \( P^A \) and surface incompressibility \( K^A_{\text{sym}} \) for a finite nucleus are calculated by weighting the corresponding quantities for infinite nuclear matter within the CDFM, as given below \([24–28]\):

\[
K^A = \int_0^\infty dx |F(x)|^2 K^{\text{NM}}(\rho(x)).
\]

\[
P^A = \int_0^\infty dx |F(x)|^2 P^{\text{NM}}(\rho(x)),
\]

\[
S^A = \int_0^\infty dx |F(x)|^2 S^{\text{NM}}(\rho(x)),
\]

\[
L^A_{\text{sym}} = \int_0^\infty dx |F(x)|^2 L^{\text{NM}}_{\text{sym}}(\rho(x)),
\]

\[
K^A_{\text{sym}} = \int_0^\infty dx |F(x)|^2 K^{\text{NM}}_{\text{sym}}(\rho(x)),
\]

The \( K^A, P^A, S^A, L^A_{\text{sym}}, \) and \( K^A_{\text{sym}} \) in Eqs. \((20–24)\) are the surface weighted average of the corresponding \( \text{NM} \) quantities in the LDA limit for finite nuclei.

### III. RESULTS AND DISCUSSION

The results of our calculations are discussed in this section. It is important to note that earlier studies are carried out using Brückner density functional. But it has limitation since the minima in \( \text{NM} \) corresponds to \( \rho \sim 0.2 \text{ fm}^{-3} \) whereas the relativistic approaches produce this optimum value at \( \sim 0.15 \text{ fm}^{-3} \), which is in good agreement with the empirical \( \text{NM} \) assumption. In other words, this density corresponds to a binding energy per particle \( \sim 16 \text{ MeV} \) satisfying the Coester-Band problem \([13]\). The Coester-Band problem was rectified partially by adding the three-body force in the nucleon-nucleon potential \([29, 30]\).

Also, it is understood that the non-linear terms in the RMF Lagrangian mimics the effects of three-body force in the nuclear potential and acts as a remedy for the Coester-Band issue. Therefore, it is quite important to refit the same in the wake of solution of Coester-Band problem by RMF model, which is the main motivation of the present calculations. The Fig. 1 shows the \( \text{NM} \) binding energy per particle \( E/A \) for \( \text{NL3} \) and \( \text{G3} \) parameter sets along with fitted \( E/A \) curves at different

### TABLE I: The coefficients of the analytical expression for NM binding energy per particle as a function of density (\( \rho(x) \)) and the asymmetric factor \( \alpha = \frac{\rho_{\text{sym}} - \rho_0}{\rho_{\text{sym}} + \rho_0} \). The values are given for NL3 and G3 parameter sets.

|   | NL3     | G3     |
|---|---------|--------|
| b3 | -3449.92| -490.15|
| b4 | 93386.65| -465.80|
| b5 | -123357.10| 7107.17|
| b6 | 9041665.48| -53960.91|
| b7 | -41166214.95| 284155.27|
| b8 | 123164197.67| -938303.73|
| b9 | -248225071.34| 2066363.99|
| b10 | 33808737.81| -3133853.65|
| b11 | -305682367.52| 3246326.72|
| b12 | 174988663.34| -218895.63|
| b13 | -57095582.73| 861872.30|
| b14 | 8030800.13| -149719.19|
| a3 | -1098.99| 391.87|
| a4 | 43110.63| -5565.05|
| a5 | -636205.52| 80413.45|
| a6 | 5283025.49| -639847.05|
| a7 | -27638744.86| 3139872.96|
| a8 | 96251325.61| -10022304.90|
| a9 | -228719960.80| 21277231.89|
| a10 | 372188746.60| -30256280.52|
| a11 | -407653299.90| 28503817.38|
| a12 | 286972591.80| -17096240.48|
| a13 | -117150348.00| 5921783.83|
| a14 | 21061682.62| -903228.53|

1.52\,A^{1/3} \). The Wigner distribution function of the OBDM of Eq. \((16)\) is given by,

\[
W(r, k) = \int_0^\infty dx \, |F(x)|^2 \, W_x(r, k).
\]

Here, \( W_x(r, k) = \frac{1}{4\pi} \Theta(x-|r|) \Theta(k_F(x)-|k|) \). The density \( \rho(r) \) in terms of the WF within the CDFM approach is:

\[
\rho(r) = \int dk \, W(r, k) = \int_0^\infty dx \, |F(x)|^2 \, \frac{3A}{4\pi x^3} \Theta(x-|r|),
\]

which is normalized to \( A \), i.e., \( \int \rho(r) dr = A \). In the \( \delta \)-function limit, the Hill-Wheeler integral equation, that is the detailed analytical derivation. The CDFM allows us to make a transition from the properties of \( \text{NM} \) to those of finite nuclei. The finite nuclear incompressibility \( K^A \), symmetry energy \( S^A \), neutron pressure \( P^A \) and surface incompressibility \( K^A_{\text{sym}} \) for a finite nucleus are calculated by weighting the corresponding quantities for infinite nuclear matter within the CDFM, as given below \([24–28]\):
TABLE II: The surface properties - nuclear incompressibility $K^A$, symmetric energy $S^A$, neutron pressure $P^A = \rho_0 L^A_{sym}/3$, slope $L^A_{sym}$ and curvature $K^A_{sym}$ of the symmetry energy of nuclei for NL3 and G3 parameter sets. All values are in MeV.

|       | $^{16}$O | $^{40}$Ca | $^{48}$Ca | $^{56}$Ni | $^{90}$Zr | $^{116}$Sn | $^{208}$Pb |
|-------|----------|----------|----------|-----------|-----------|-----------|-----------|
| $K^A$ | 618.05   | 584.11   | 564.86   | 627.33    | 450.87    | 476.46    | 411.03    |
| $P^A$ | 9.70     | 8.30     | 7.84     | 8.68      | 6.90      | 7.24      | 6.68      |
| $S^A$ | 40.30    | 39.40    | 38.83    | 41.66     | 36.97     | 38.38     | 37.28     |
| $L^A_{sym}$ | 118.89   | 119.79   | 120.54   | 130.30    | 117.01    | 121.20    | 117.95    |
| $K^A_{sym}$ | 32.42    | -10.92   | -3.18    | -4.37     | 33.30     | 31.34     | 47.99     |

|       | $^{16}$O | $^{40}$Ca | $^{48}$Ca | $^{56}$Ni | $^{90}$Zr | $^{116}$Sn | $^{208}$Pb |
|-------|----------|----------|----------|-----------|-----------|-----------|-----------|
| $K^A$ | 258.87   | 262.09   | 270.26   | 279.33    | 253.87    | 249.74    | 238.85    |
| $P^A$ | 3.39     | 3.07     | 3.08     | 3.12      | 2.75      | 2.68      | 2.52      |
| $S^A$ | 30.12    | 30.43    | 31.15    | 31.98     | 30.88     | 30.87     | 30.53     |
| $L^A_{sym}$ | 51.98    | 51.92    | 52.71    | 53.57     | 51.40     | 51.18     | 50.28     |
| $K^A_{sym}$ | -103.23  | -89.43   | -89.52   | -90.00    | -91.55    | -93.07    | -96.39    |

FIG. 2: The density (dotted lines) and weight function (solid lines) of $^{16}$O and $^{208}$Pb for NL3 (upper panel) and G3 (lower panel) parameter sets.

values of $\alpha$, where $\alpha$ varies from 0 to 1. For symmetric NM, $\alpha = 0$ and for pure neutron matter $\alpha = 1$. The value of $\alpha$ determines the neutron-proton asymmetry in the NM system. Further, the density and weight function for $^{16}$O and $^{208}$Pb are shown in Fig. 2 as two representative cases. It is clear from Fig. 2 that maxima of weight function corresponds to the surface region of the density where the density is significantly reduced compared to the central region. Due to this reason the symmetry energy, its slope and curvature, neutron pressure etc. are labelled as surface properties.

The refitted E/A obtained by NL3 and G3 parameter sets (Eq. (5)) is obtained as a function of $x$ with the Flucton density as the expansion variable. It is important to note that Equation (5) is the equivalent energy expression as of Eq. (2). The Eq. (2) obeys the Coester-Band problem so also Eq. (5) and the subsequent expressions Eqs. (20–24) also follow the relativistic characteristics. The quantities, derived from these equations are shown in Table II for some double closed shell spherical nuclei. We get a wide range of finite nuclear incompressibility $K^A = 618.05$ to 411.03 MeV and $K^A = 258.87$ to 238.85 MeV for $^{16}$O - $^{208}$Pb with NL3 and G3 set, respectively. From these results, we cannot conclude about the mass dependence of the finite nuclear incompressibility, for example, $K^A = 618, 627$ and 411 MeV for $^{16}$O, $^{56}$Ni and $^{208}$Pb respectively. Similar uncertainty in $K^A$ for different nuclei is clearly visible in G3 set also.

In the case of $P^A, S^A$ and $L^A_{sym}$, the variation is in a narrow range from $^{16}$O to $^{208}$Pb, i.e., minimum $P^A$ is 2.52 MeV for $^{208}$Pb and maximum is 3.39 MeV for $^{16}$O with G3. The variations in $S^A$ and $L^A_{sym}$ is also small (see Table II). On the other hand, the $K^A_{sym}$ varies a lot depending on both force parameter as well as the mass of the nucleus. The very different values of $K^A$ and $K^A_{sym}$ for light and heavy nuclei indicate the structural dependence of the finite nuclei since the density distribution varies from nucleus to nucleus.

IV. SUMMARY AND CONCLUSION

In brief, we have fitted the nuclear matter saturation curves for different values of asymmetry parameter, employing the E-RMF density functional with two well-known G3 and NL3 parameter sets, in the wake of solution of Coester-Band problem by relativistic approach. The newly fitted expression of E/A is used to find nuclear matter parameters such as incompressibility, neutron pressure, symmetry energy. These NM parameters together with E-RMF densities are used in the calculations of surface properties of doubly magic nuclei ranging from light to heavy mass region, within the coherent density fluctuation model. The values of symmetry energy, neutron pressure, L-coefficient show small variation for different mass nuclei. On the other hand, the incompressibility and surface incompressibility show the large variation while moving from light ($^{16}$O) to heavy ($^{208}$Pb) nuclei which presents their dependence upon the density distribution of a particular nucleus.

In other words, we discuss here that the direct use of Brückner energy functional to evaluate the effective nuclear surface properties is not adequate in context of Coester-Band issue. The present method with fitted nuclear matter saturation curves using E-RMF density functional to find the effective nuclear surface properties opens up a new window for future calculations of other nuclei in the nuclear landscape including the dip-line regions.

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