Behavior of compressed reinforced concrete columns under thermodynamic influences taking into account increased concrete deformability

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Abstract. The behaviour of compressed reinforced concrete columns has been of great importance for the stability and safety of the entire structure. Therefore, the identification of additional resources that increase the load-carrying capacity of such structures under thermodynamic influences is the goal for solving this problem. The combined effect of dynamic loadings and high temperatures on the compressed element has its own specificity, expressed in the reduction of dynamic hardening coefficient, which is of no small importance for the actual evaluation of the resistance of buildings to progressive fracturing. In some cases, concrete is under volumetric compression, and therefore has increased strength and deformation properties, which can be expressed in greater fire resistance. A correct assessment of the effect of the combined factor of impacts on the compressed element, taking into account the revealed properties of limited concrete, is an actual scientific problem that solves issues related to ensuring the safety and reliability of building and structure’s operation. The article presents analytical calculations of reinforced concrete elements for fire impacts in conditions of volumetric compression of concrete, thanks to the presence of spiral, annular reinforcement, using experimental data. The article gives the calculation of a reinforced concrete column of a high-rise building taking into account the increased deformability of concrete in normal conditions and in conditions of fire impacts. Dynamic calculation of the column in fire conditions shows that taking into account the increased deformation properties of concrete leads to an extension of the time of formation of cracks; the operating time of the element increases in the elastically plastic and in the plastic stages. Calculations show that, under fire impacts, the increase in the concrete deformability affects the opening angle of the plastic hinge, which in turn leads to a change in the nature of the element fracture. To assess the fire resistance of reinforced concrete columns under dynamic influences, a loading varying in time with respect to a linearly increasing function was adopted. Calculations show that taking into account the properties of limited concrete when calculating structural elements for thermodynamic influences leads to an increase in fire resistance of structures by an average of 25-30%.

1. Introduction
Reinforced concrete buildings make up most of the civil buildings under construction and already built. In many skyscrapers, the frames of buildings are composed of two supporting systems: monolithic reinforced concrete and metal structures. And, as is common, in the structures the main supporting part is the reinforced concrete frame. Increased phenomena of the occurrence of man-made impacts on the structural elements of the frame give importance to the issue of studying the strength
and stability of structural elements and various thermodynamic effects. As many researches [1, 2, 3] show, the strength of reinforced concrete elements operating in high-temperature conditions grows down. At the same time, the dynamic strength at the same temperature decreases significantly more than the static one [4]. The reduction coefficient \( K_{d,t} = \gamma_t \) is a criterion, after which the dynamic strength of the element becomes less than static, is the starting point for the development of the hazardous area of the element's operation under dynamic loading in the fire impact conditions (Figure 1).

The diagram shows that the strength of the compressed reinforced concrete element decreases with rising temperature; a sharp decrease in dynamic strength occurs when compared with the static strength. In the calculated example at a temperature of 468.9 °C, the static strength of the element becomes equal to the dynamic strength, and in the subsequent increase in temperature, the dynamic strength shows lower values.

Thus, many researches reveal the need for research and development of methods for increasing the strength of building structural elements under thermodynamic influences.

Columns of multi-storey buildings are the most loaded elements in the structure of the building's frame, therefore they are vulnerable structures in the occurrence of man-caused phenomena. Structural vertical elements, in particular columns, can experience: as a vertical dynamic load - due to the instant removal of one structural element (progressive fracturing), and lateral dynamic load - due to impact of vehicle impact and etc. All these scenarios are in most cases accompanied by fire impacts. Registration of such special combinations of loads in the design phase will lead to an increase in the strength of the frame of the building. Studies devoted to increasing the strength and stability of compressed reinforced concrete elements under different thermodynamic influences without increasing the geometric characteristics are an urgent task for engineers.

Figure 1. Diagram of changing
1 - Dynamic; 2 - Static strength of concrete columns under standard temperature regime; 3 - fire temperature according to ISO 834
One of the techniques to increase the strength of compressed concrete elements is to use the property of limited concrete [5].

There are various techniques for creating a three-axis stress-strain state to increase the strength of reinforced concrete compressed elements: by gluing composite materials, compressing columns with metal sheets, compressing by mending plates, using prestressed spiral reinforcement. All of them are in various ways effective, for example, the use of spiral reinforcement in the columns can achieve a gain effect of up to 40%. The reinforcement with biaxial pre-compression of concrete increases the bearing capacity by 30.8%.

Under certain types of loads, the concrete may experience a constrained deformation, as experiments show, creates a kind of hooping and increases the strength up to 2.5 times in comparison with the prismatic strength, depending on the ratio of the maximum calculated area to the area of collapsing [6]. Thus, the development of ways to create a limited state of concrete in compressed reinforced concrete elements by means of lateral rods and to reveal the effect on dynamic strength is a task for further research.

To analyze the effect of creating a limited state of concrete in compressed reinforced concrete elements on the dynamic strength in normal conditions and in fire conditions, the column was calculated with different support fastenings: for anchorage and for hinged support (Figure 2a) and b). For a calculated example, consider the calculation of a centrally compressed column for the action of a lateral impact applied near the lower section (the case of a vehicle impact).

![Figure 2. Calculation schemes of reinforced concrete columns](image)

To do this, we divide the movement of the structure into three stages: 1) before the formation of cracks; 2) after the formation of cracks before reaching the fluidity in the longitudinal reinforcement; 3) after achieving fluidity before fracturing.

2. Development of calculation procedure
The deflection is determined by taking into account the shift on the deformation of the lower sections of the column [7]

\[ y = y_1 + y_2 \]  

(1)

where \( y_1 = \sum_{n} T_n X_n(x) \); \( y_2 = \sum_{n} \bar{T}_n X_n(x) \);

(2)

where \( X_n \), \( n \)-form of the natural vibration of the structures, meeting the given boundary conditions. \( T_n \) - corresponding to \( n \)-form of the required time function.

Taking the lateral displacement of the column in the form

\[ y(x,t) = \sum_{n} X_n(x) T_n(t). \]

(3)

The work of the longitudinal force in the calculation of the potential energy is determined by:

\[ W = \frac{N}{2} \int (\frac{\partial y}{\partial x})^2 dx. \]

(4)

The expressions for the bending moments and lateral forces in this case have the form:

\[ M = -B_1 \frac{\partial^2 y}{\partial x^2} + Ny; Q = -B_1 \frac{\partial^3 y}{\partial x^3} + N \frac{\partial y}{\partial x}. \]

(5)

The achievement of the maximum value \( F(t) \) is determined from the condition \( \alpha(t) = 0 \), where \( \alpha \) - is the solution of the column motion equation system in this stage, taking into account the work of the longitudinal force according to the formula 4. After the termination of the contact between the ram-tester and the column at the time \( t'' \), it is necessary to consider the free movement of the structure. The moment \( t'' \) can be determined from the condition \( p(t'') = 0 \).

The condition for the transition from the conventionally elastic stage to the elastic-plastic stage has the form: \( M(x, t'_1) = M_{u,d} \), where \( M(x, t'_1) \) - is determined by (5), \( M_{u,d} \) - the limiting moment in the normal section \( X \), corresponding to the achievement of fluidity in the stretched reinforcement; \( e \) - the eccentricity of the longitudinal static load; sign - is used if the moment created by the longitudinal force increases the bending moment from the dynamic load.

Elastically plastic stage. First consider the deformation of the column, which is supported with joint hinge on two ends. Suppose that at the beginning of the elastically plastic stage, the impactor's contact with the structure has not stopped stop. In this expression, the kinetic and potential energy takes the form:

\[ K = \frac{m}{2} \int (\dot{y}')^2 dx + \frac{m}{2} \int (\dot{y}^*)^2 dx + \frac{M}{2} \left[ \alpha + \phi x \right]^2; \]

\[ U = M_{u,d} \phi_1 \frac{1}{1-x} + \frac{k_e x^2}{2} \]

(6)

The work of the longitudinal force at the approach of the column ends caused by lateral deformation

\[ W = N[\phi(x)(1-x)] \]

(7)

Substituting the above expressions into Lagrange's equations and supposing that
\[
\sin \varphi_1 \cong \varphi_1; \sin(\varphi_1 \frac{x}{1-x}) = \varphi_1 \frac{x}{1-x}, \text{ we obtain a system of differential equations of motion:}
\]
\[
\begin{align*}
\phi_1 - \lambda^2 \phi_1 + \eta \alpha &= -\phi_3 \\
x \phi_1 + \alpha + \omega^2 \alpha &= 0
\end{align*}
\] (8)
\[
\lambda^2 = \frac{3Nl}{(1-x)x(ml + 3M_s)}; \eta = \frac{3M_s}{x(ml + 3M_s)}; \phi_3 = \frac{3M u_d d \cdot 1}{(ml + 3M_s)x (1-x)}; \\
t_0 = 0, \phi_1 = \phi_{10}; \phi = \phi_{10}; \alpha = \alpha_0; \alpha = \alpha_0.
\] (9)

Where \( \alpha \) and \( \alpha_0 \) are determined from the condition of continuity of implementation and speed of implementation, \( \phi_{10} = \frac{y(x,t)}{x} \); \( \varphi_{10} \) - is determined from the balance of kinetic energy at the end of the conditionally elastic and initially elastic-plastic stage.

The solution of system (8) has the form:
\[
\varphi_1 = (\omega_1^2 + r_1^2)(C_1 e^{r_1 t} + C_2 e^{-r_1 t}) + (\omega_2^2 - r_2^2)(C_3 \sin r_2 t + C_4 \cos r_2 t) + \frac{\phi_3}{\lambda^2}; \\
\alpha = -x r_1^2 (C_1 e^{r_1 t} + C_2 e^{-r_1 t}) + x r_2^2 (C_3 \sin r_2 t + C_4 \cos r_2 t); \\
\text{Here } C_1 = \frac{(\varphi_{10} - \theta_1) r_1 + \phi_{10} - \theta_1}{2 r_1 \theta_1}; C_2 = \frac{(\varphi_{10} - \theta_2) r_1 + \phi_{10} + \theta_2}{2 r_2 \theta_2}.
\] (11)

\[
\begin{align*}
C_3 &= \frac{\alpha_0}{x \cdot r_1^2} + \frac{r_1^3}{r_2^3} (C_1 - C_2); C_4 &= \frac{\alpha_0}{x \cdot r_2^2} + \frac{r_2^3}{r_1^3} (C_1 + C_2);
\theta_1 &= (\omega_1^2 + r_1^2) + \frac{r_1^2 (\omega_1^2 - r_2^2)}{r_2^2}; \theta_2 &= (\omega_2^2 + r_2^2) + \frac{r_2^2 (\omega_2^2 - r_2^2)}{r_2^2};
\end{align*}
\] (14)

where \( r_2 \geq r_1 \) - a discharge condition in the contact zone.

In the case \( t > t \) the system (8) will be valid in the condition of replacing \( \omega_1^2 = \frac{K_2}{M_s} \) and adding \( \phi_3 \), here all known are resulted as the conditions of continuity of movements and their velocities.

These equations will in turn be valid until the moment of termination of the contact \( \bar{t} \), determined from the condition \( F(t) = 0 \). After this, it is necessary to consider the free movement of the column. Defining the deflection and using the Lagrange procedure
\[
\phi - \lambda^2 \phi = -\phi_3;
\] (16)
\[
\lambda^2 = \frac{-3Nl}{(1-x)x ml}; \phi_3 = \frac{3M u_d d 1}{(1-x)x^2 ml}.
\] (17)

When \( N = 0 \), the equation of motion is transformed into the equation of free motion.
The solution is found in the form
\[ \varphi = E_1 e^{\lambda t} + E_2 e^{-2\lambda t} + \frac{\theta_0}{\lambda_3}. \]  

(18)

Initial conditions:
\[ t = 0; \quad \varphi(t) = \varphi_0; \quad \varphi'(t) = \varphi_1, \]
where \( \varphi_0 \) and \( \varphi_1 \) are resulted by the condition of continuity of the angles of rotation and angular velocity at the end of the previous one at the beginning of the stage under consideration.

Using the initial conditions gives
\[ E_1 = \frac{\varphi_0}{2} + \frac{\varphi_1}{2\lambda}, \quad E_2 = \frac{\varphi_0}{2} - \frac{\varphi_1}{2\lambda^2}. \]

(19)

The stopping time of the structure \( t_m \) can be found from the condition \( \varphi(t_m) = 0 \) or
\[ \varphi = E_1 \lambda e^{2\lambda t_m} - E_2 \lambda e^{-2\lambda t_m} = 0, \]
wherefrom:
\[ t_m = \frac{1}{2\lambda} \ln \frac{E_2}{E_1}. \]

(20)

The maximum rotation angle is obtained by substituting \( t_m = t. \)

The strength of the system will be ensured if the following condition is met:
\[ F_{st}(t) \leq F_{st,u}, \]

\( F_{st}(t) \) - the maximum dynamic axial force in a compressed inclined strip, determined from the dynamic calculation, \( F_{st,u} \) - is the limiting force corresponding to the cracking.

Stiffness of the column is determined by:
\[ C_d = \sum_{i=1}^n \frac{12E_s I_s}{l_3^3}. \]

(22)

Concrete stiffness of the compressed strip:
\[ C_{St} = \frac{E_s A_b \sin \theta}{h_b}. \]

(23)

Stiffness of clamps:
\[ C_W = \frac{E_s A_w (x-a)}{1an}. \]

(24)

\( l_n \) - a length of active section of deformation of lateral rods.

The total stiffness of the deformable element [8] will be:
\[ C0 = \frac{C_d C_m C_d'}{C_d C_m + C_d C_d' + C_m C_d'}, \]

(25)

where \( C_m = C_{St} + C_W. \)

We note that in conditions of frequent arrangement of the clamps and the effect of the engagement forces, one can neglect, that is, the creation of the effect of limited concrete leads to simplification. And in the formula for determining the potential energy, one can take \( C_{esk} = 0. \)

The calculation of the column in normal conditions shows that the movement at rigid fixing of the column from below and with the hinged support is different depending on the time.
In the conditions of fire impacts, as shown by the experimental analysis, the static strength and deformation characteristics of concrete and reinforcement decrease. Also, based on the experimental analysis, it was found that the coefficient of dynamic hardening of concrete, which is the ratio of dynamic strength to static strength, varies with the speed of application of the load when heated up to 900 °C (according to the diagram shown in Figure 3).

![Figure 3. Dependence $K_{b,d}$ on temperature](image)

The figure shows that the coefficient of dynamic hardening decreases to $K_{b,d} = 0.441$.

Dynamic calculation of a reinforced concrete column in temperature conditions begins from a thermo-technical calculation. As in normal conditions, the calculation is divided into 3 stages. Taking into account the change in dynamic strength and deformation characteristics, the transition time from one stage to another, the frequency of natural-vibration frequency at each stage of the element operation is determined with taking into account the change in the element stiffness.

The temperature at the setting out point (in the reinforcement) in the section of the reinforced concrete element is determined by the formula:

$$t_{n, x} = 1250 - (1250 - T_o) \left( \frac{K_b}{2F_E} \right)^2 + \frac{1}{2F_E} \left( \frac{2 - \zeta}{2F_E} \right)^2$$

(26)

Previously, the section of column is divided into 5 parts, and the temperatures in the middle of each part are defined. The temperature in the middle of the parts is determined by the formula:

$$\zeta(n) = 1 - \frac{(5 - n) \cdot b / 10 + b / 20}{0.5b + K \cdot \alpha_{rail}}$$

(27)

After determining the temperature in the column, the dynamic calculation is divided into 2 parts. In the case when the column receives a dynamic lateral impact and the temperature does not reach 100 °C in the reinforcement, the calculation is carried out according to the existing procedure.

Otherwise, when the temperature in the reinforcement shows large values, it is necessary to use coefficients that take into account the change in the strength and deformation properties of the materials.

3. Results of calculation

Thus, the calculation of the column for a lateral dynamic effect, with 2 types of fastenings, in fire conditions, taking into account the properties of limited concrete, shows the following values (see Figure 4).
Figure 4. Comparison of theoretical calculations of the deflections of a reinforced concrete column under lateral dynamic impact

a) The column is embedded from bottom (μ = 0.01)

b) Hinged support (μ = 0.01)

1 - a step of installation of 90mm clamps (at 20 °C); 1' - a step of installation of 45mm clamps (at 20 °C); at a temperature of 900 °C; 2 - a step of installation of 90mm clamps (at 900 °C); 2' - a step of installation of 45mm clamps 45mm (at 900 °C).

4. Conclusion

As shown by the dynamic calculation of the reinforced concrete column of 2 scenarios, the greatest effect from the constrained operation of concrete as by the element in fire impacts is resulted when the column is rigidly embedded. In this case, the deflection of the column at 900 °C is reduced by 24%. Consequently, the creation of the effect of constrained operation of concrete in a reinforced concrete column in the frame structure leads to a significant increase in the strength of a lateral impact in fire conditions.

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