Anomalous U(1) Symmetry and Sparticles

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Abstract

Using anomalous $U(1)$ symmetry the quark mass texture is determined uniquely. We analyze squark mass spectrum based on the above mass matrices and discuss the possibility to solve the problems of FCNC and CP caused by complex phases of soft terms, including the viewpoint of M theory.

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Introduction

The minimal supersymmetric standard model (MSSM) does not contradict any experiment till now. However, it does not explain the origin of mass hierarchy and the problem of flavor-changing neutral current, the CP violation problem in soft terms, the origin of baryogenesis and so on. In string theory, which often has anomalous $U(1)$ gauge symmetry, there have been many attempts to explain the above problems by using this symmetry [1]. We investigate these subjects including a viewpoint of M-theory which has developed very rapidly. In the Hořava-Witten construction [2], 11-dimensional supergravity (low-energy limit of M-theory) is compactified on $M_4 \times S^1/\mathbb{Z}_2 \times X$, where $X$ is a 6-dimensional Calabi-Yau manifold, $M_4$ is a 4-dimensional Minkowski space, and the fifth dimension is compactified on a line segment $(S^1/\mathbb{Z}_2)$ whose length, $\pi \rho$, is larger than the “radius” of the Calabi-Yau volume. In this picture, the observable and hidden gauge degrees of freedom (the former is the surviving subgroup of $E_8$, and the latter is $E_8'$ or its subgroup) live on two distinct 4-dimensional “walls”, a distance $\pi \rho$ apart. Standard model gauge fields and charged matter are confined to one wall (or it may be on a “non-perturbative” five-brane). Gravity lives on the 5-dimensional “bulk”. Distinctive features to take account of M-theory are that the string coupling is strong and because there is a separating bulk between the observable and hidden sectors, there live vector- and hyper- multiplets.

Previously we examined mass spectrum of quarks and charged leptons [3] based on $\mathbb{Z}_3$ orbifold model using flipped $SO(10)$ symmetry because in this model flipped $SO(10)$ exists uniquely as a group of GUT. The mass spectrum could be reproduced, but simpler is to use anomalous $U(1)$ symmetry that acts as a horizontal symmetry [4]. Although various anomalous $U(1)_X$ charges of quarks have been assigned to fit the low-energy mass spectrum using the renormalization group method, they were rather devoid of definiteness. However, as pointed out in [3], $U(1)_X$ charges can be determined almost uniquely from the CKM angles. In [4] this $U(1)_X$ symmetry is embedded into a larger gauge group (GUT). We treat the anomalous $U(1)_X$ symmetry separately here and apply the result to the sector of superpartners. Because the $U(1)_X$ symmetry is not observed at low energies, it must be broken. The Higgs mechanism is used to break the $U(1)_X$ gauge symmetry. We denote this kind of Higgs field $\theta$. The electroweak singlet fields $\theta$ may appear in vectorlike pairs or as chiral individuals. If they appear in vectorlike pairs, they would obtain in general very large mass of order the Planck mass. We take $\theta$ to be a chiral superfield in this paper. Then the mixed anomalies of the $U(1)_X$ symmetry are necessarily nonzero and must be cancelled using the Green-Schwarz mechanism [6]. This incidentally fixes the weak mixing angle without recourse to GUT [7]. Moreover, the value of the parameter $\varepsilon \equiv \langle \theta \rangle/M_P$, where $M_P$ is the four-dimensional reduced Planck mass, is determined definitely from the $D$-term. The structure (texture) of mass matrices will change little if two or more chiral fields $\theta$ are included. So we consider a single chiral field in this paper. We denote the $U(1)_X$ chiral charge of $\theta$, $X(\theta) = -q_\theta < 0$. $U(1)_X$ charges of matter fields are fractional in general.

*When we embed the anomalous $U(1)_X$ group in a GUT group, the problem of the existence of an adjoint Higgs boson as well as complexity to include various representations of Higgs bosons and how to break the symmetry is difficult to solve. Here we consider only $U(1)_X$ group “simple-mindedly.”*
We assume chiral flavor charges of matter superfields $\phi_i$ are greater than or equal to zero, $X(\phi_i) \geq 0$. In most of the previous papers, in which the anomalous $U(1)$ symmetry was used, the mass term of $\theta$ was inserted explicitly and the mass scale was set at the electroweak scale not necessarily definite, and furthermore, in many cases supersymmetry is broken by the $D$-term using that mass term. However, taking supergravity into consideration at the same time, the situation improves. We consider here that supersymmetry is broken locally by the gaugino condensation in the hidden sector. Then $\theta$, squarks and gauginos in the observable sector get mass of the order of the gravitino mass. The $D$-term becomes nonzero concurrently with it, and first two generations of squarks will get much larger mass than the gravitino mass because the value of the $D$-term is greater than the gravitino mass.

In what follows we first define the form of the $U(1)_X$ charge. Next we determine the mass matrices of up- and down-quarks from the mass of the quarks and the CKM matrix. However, there remains a few versions at this stage. By considering the Higgs sector we can fix the charges definitely. These charges affect the mass spectrum of sparticles. We derive the values of masses of sparticles and discuss that this scheme will avoid the problems of FCNC and CP breaking by soft terms and suggest a model to solve the problem of baryogenesis.

$U(1)_X$ charge

The superpotential for the up quarks is given by

$$W \sim Q_i u_j^c H_2 \left( \frac{\theta}{M_P} \right)^{n_{ij}^{(u)}},$$

where $i$ represents the generation. We neglect numerical factors because in this approach we are interested in the order of magnitude. From charge conservation one obtains

$$q_0n_{ij}^{(u)} = X(Q_i) + X(u_j^c) + X(H_2).$$

As for top mass, there is an infrared stable quasi-fixed point as long as the Yukawa coupling of the top is of order one. The predicted value fits experimental data very well. So we assume $n_{33}^{(u)} = 0$, and $n_{ij}^{(u)} \geq 0$ for $(i, j) \neq (3, 3)$. Effective Yukawa couplings are given by

$$Y_{ij}^{(u)} \sim \left( \frac{<\theta>}{M_P} \right)^{n_{ij}^{(u)}}.$$  

The $3 \times 3$ mass matrix of the up quarks are written as

$$\frac{M_u}{m_t} \sim \left( \varepsilon n_{ij}^{(u)} \right).$$

Of course, the important thing here is the form of the superpotential and not the $U(1)_X$ charge. However, when considering the mass of sparticles and not to break the symmetries of the standard model at high energy, this assumption is necessary.
Denoting the $U(1)_X$ charges of quarks and electroweak Higgs fields as
\[ X(Q_i) = \alpha_i, \ X(u^c_i) = \beta_i, \ X(d^c_i) = \gamma_i, \]
\[ X(H_1) = h_1, \ X(H_2) = h_2, \]  
Eq.(2) is written as
\[ n^{(u)}_{ij} = \frac{1}{q_\theta} (\alpha_i + \beta_j + h_2). \]
If we factor out
\[ m_t = \lambda_t < H_2 > \varepsilon^{(u)}_{33} = \lambda_t < H_2 > \]
from the mass matrix, then
\[ (n^{(u)}_{ij}) = \frac{1}{q_\theta} \begin{pmatrix} (\alpha_1 - \alpha_3) + (\beta_1 - \beta_3) & (\alpha_1 - \alpha_3) + (\beta_2 - \beta_3) & \alpha_1 - \alpha_3 \\ (\alpha_2 - \alpha_3) + (\beta_1 - \beta_3) & (\alpha_2 - \alpha_3) + (\beta_2 - \beta_3) & \alpha_2 - \alpha_3 \\ \beta_1 - \beta_3 & \beta_2 - \beta_3 & 0 \end{pmatrix} \]
For the down quarks we obtain the same expression with $\gamma_i$ instead of $\beta_i$ except for the prefactors.

\[
\text{Hereafter, for simplicity, we assume that } -q_\theta = -1, \text{ and that the chiral charges of matter fields are integer.}
\]

**quark sector**

We neglect the complex (CP) phase in the quark sector, for simplicity. To determine the form of quark matrices, the form of the CKM matrix must be fixed first. The Wolfenstein parametrization of the CKM matrix is given by
\[ \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^5 & -\lambda^2 & 1 \end{pmatrix} \]
in terms of the Cabibbo angle $\lambda$ ($\lambda \simeq 0.22$), with all prefactors of order one. Although the (3, 1) component can be chosen $\lambda^4$ instead of $\lambda^3$ according to experimental data, $\lambda^4$ is excluded by the argument below of charge conservation. If we assume the Grand Desert Scenario, namely no significant matter between the TeV scale and string scale around $10^{16}$GeV, the fermion masses satisfy
\[ \frac{m_u}{m_t} \sim \lambda^{7-8}, \frac{m_c}{m_t} \sim \lambda^4, \frac{m_d}{m_b} \sim \lambda^4, \frac{m_s}{m_b} \sim \lambda^2. \]
As mixing angles are small, at the scale $M_{\text{string}}$
\[ M^{\text{diag}}_u \sim \begin{pmatrix} \lambda^8(\lambda^7) & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_t, \]
\[ M^\text{diag}_d \sim \begin{pmatrix} \lambda^4 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_b. \]  

The relation \( \varepsilon \sim \lambda \) should hold. We will remark on the appropriateness of it later. The CKM matrix is expressed as

\[ V_{CKM} = V_u^{(L)\dagger} V_d^{(L)}, \]

(13)

where unitary matrices \( V_u^{(L)} \) and \( V_d^{(L)} \) are defined as

\[ M_D M_D^\dagger = V_d^{(L)} (M^\text{diag}_d)^2 V_d^{(L)\dagger}, \]

(14)

\[ M_U M_U^\dagger = V_u^{(L)} (M^\text{diag}_u)^2 V_u^{(L)\dagger}. \]

(15)

We want to determine first the form of \( M_U \) and \( M_D \) from \( M^\text{diag} \) and \( V_{CKM} \). If the following relations

\[ [M_D M_D^\dagger, V_u^{(L)}] = 0, \quad [M_U M_U^\dagger, V_d^{(L)}] = 0 \]

(16)

are satisfied, \( M_D M_D^\dagger \) and \( M_U M_U^\dagger \) are uniquely determined as follows,

\[ M_U M_U^\dagger = V_{CKM}(M_u^\text{diag})^2 V_{CKM}^\dagger = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} m_t^2, \]

(17)

\[ M_D M_D^\dagger = V_{CKM}(M_d^\text{diag})^2 V_{CKM}^\dagger = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} m_b^2. \]

(18)

Or according to Elwood, Irges and Ramond [5], if we assume that \( V_u^{(L)} \) and \( V_d^{(L)} \) have the same form as that of \( V_{CKM} \) with different numerical factors, we get the same form as above, since

\[ M_U M_U^\dagger = V_u^{(L)} (M_u^\text{diag})^2 V_u^{(L)\dagger} \simeq V_{CKM}(M_u^\text{diag})^2 V_{CKM}^\dagger, \]

(19)

\[ M_D M_D^\dagger = V_d^{(L)} (M_d^\text{diag})^2 V_d^{(L)\dagger} \simeq V_{CKM}(M_d^\text{diag})^2 V_{CKM}^\dagger, \]

(20)

We note \( M_D M_D^\dagger \) and \( M_U M_U^\dagger \) have exactly the same form in the lowest order in \( \lambda \).

Next we will determine the form of \( M_U \) and \( M_D \). We define

\[ M_D = \begin{pmatrix} \lambda^4 & \lambda^x & \lambda^y \\ \lambda^o & \lambda^2 & \lambda^2 \\ \lambda^b & \lambda^c & 1 \end{pmatrix} m_b \]

(21)

and plug it into \( M_D M_D^\dagger \). Comparing it with Eq. (20), we obtain

\[ x \geq 3, \quad y = 3, \quad z = 2, \]

(22)
The superpotential of the down quarks is given by

\[ W \sim \lambda_{ij}^{(d)} Q_i d_j^{(c)} H_1 \left( \frac{\theta}{M_P} \right)^{n_{ij}^{(d)}}. \]  

(25)

From the conservation of chiral \( U(1)_X \) charge we get [5]

\[ n_{ij}^{(d)} + n_{ji}^{(d)} = n_{ii}^{(d)} + n_{jj}^{(d)} = \alpha_i + \alpha_j + \gamma_i + \gamma_j - 2(\alpha_3 + \gamma_3) \]  

(26)

and using this we can get the lower components \( n_{ij}^{(d)} (i > j) \).

Similar relations hold for the up quarks. The lower triangular components of the mass matrix are determined using these relations, and we obtain

\[ M_U = \begin{pmatrix} \lambda^8 & \lambda^x & \lambda^y \\ \lambda^a & \lambda^4 & \lambda^z \\ \lambda^b & \lambda^c & 1 \end{pmatrix} m_t, \]  

(23)

\[ M_U = \left( \begin{array}{ccc} \lambda^8 & \lambda^z & \lambda^y \\ \lambda^a & \lambda^4 & \lambda^z \\ \lambda^b & \lambda^c & 1 \end{array} \right) m_t. \]  

(27)

\[ M_D = \lambda^x \left( \begin{array}{ccc} \lambda^4 & \lambda^x & \lambda^z \\ \lambda^6 & \lambda^2 & \lambda^3 \\ \lambda & 1 & 1 \end{array} \right) m_t. \]  

(28)

where

\[ \lambda^x = \frac{m_b}{m_t} \]  

(29)

The (2,1) components and the prefactor of the matrix \( M_D \) are not determined yet. In order to satisfy Eq.(2), since the charge of the matter fields are non-negative, \( X(H_2) \equiv h_2 \leq 0 \). Without loss of generality we can assume \( h_2 = 0 \). Then we get

\[ X(Q_3) = X(u_3^c) = 0. \]  

(30)
Moreover we obtain $x_u = 5$ due to conservation of $U(1)_X$ charge. Furthermore, the (1,1) component of $M_U$ must be $\lambda^8$, because in the case of $\lambda^7$, $U(1)_X$ charge is not conserved. $M_U$ is then determined uniquely as

$$M_U = \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} m_t, \quad (31)$$

and we can determine the $U(1)_X$ charges of up quarks, as follows,

$$X(Q_2) = 2, X(Q_1) = 3, X(u_2^c) = 2, X(u_1^c) = 5. \quad (32)$$

Once $U(1)_X$ charges of up quarks are fixed, $x_d$ is determined as $x_d = 3$. Then we get

$$M_D = \lambda^x \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} m_t, \quad (33)$$

and $U(1)_X$ charges of down quarks are given by

$$X(d_{3L}) = 0, X(d_{2L}) = 2,$$

$$X(d_{1L}) = 3, X(d_3^c) = x - h_1 \equiv \gamma_3,$$

$$X(d_2^c) = x - h_1 = \gamma_3, \quad X(d_1^c) = x - h_1 + 1 = \gamma_3 + 1. \quad (34)$$

In this way quark mass matrices are determined uniquely. From the mass matrices determined this way, the $U(1)_X$ charges of quarks are determined excepting $x$ and $h_1$ as in Eqs. (30), (32) and (34).

### Higgs Sector

There remains undetermined the $U(1)_X$ charge of $H_1$, i.e., $h_1$. In the low energy effective superpotential, the term

$$W = \mu H_1 H_2 \quad (35)$$

is needed to avoid the (electroweak scale) axion and to get Higgsino mass. Phenomenologically, $\mu \sim m_Z$, where $m_Z$ is the mass of the neutral weak gauge boson. According to string theory, at least three fields are multiplied in the superpotential. Then $\mu$ must be a kind of vacuum expectation value of some field(s) which might be a (standard model) gauge singlet or a condensate or a product of several fields. We denote it as “N”. Then $\langle N \rangle = \mu$ and

$$W = \langle N \rangle H_1 H_2 \quad (36)$$

In the case of $N \sim \theta^n$, i.e.,

$$W = \lambda_H H_1 H_2 \frac{\theta^n}{M_P^3 1}, \quad (37)$$
the natural scale of $\mu$ would be the string scale. So we have to forbid this term. Since $h_2 = 0$, we get $h_1 \leq 0$. Then from Eq. (37), $X(N) \geq 0$ in general. When $h_1 < 0$, $m^2_{H_1}$ might be negative at the string scale, and then the vacuum becomes unstable. Even if it is not negative, because $m^2_{H_1} \ll m^2_{H_2}$ at the string scale, and $\tan\beta \equiv \langle H_0^0 \rangle / \langle H_0^1 \rangle$ might become much smaller than one, it is phenomenologically unacceptable. Furthermore, because of naturalness of the electroweak scale, mass of Higgses should be $\lesssim 1$ TeV [10]. If $h_1 = 0$, it can be satisfied. Hence we will assign $h_1 = 0$. Namely two (electroweak) Higgs supermultiplets have vanishing $U(1)_X$ charge. Then $X(N) = 0$. When “N” is in the observable sector (wall), the vacuum expectation value of which is generally string scale, $\mu$ is too large. When “N” is in the hidden sector (another wall), there is no direct coupling to Higgses. It has to be mediated by fields in the bulk. Then the $\mu$-term may become of electroweak scale, but the scenario is much model-dependent, and there is a shade of fine-tuning. We therefore assume as a more natural scheme that the $\mu$-term is generated through Kähler potential by the Giudice-Masiero mechanism [12]. There is a model [13], for example, in which

$$K(T, U) + K_{Higgs} \sim -\ln[(T + \bar{T})(U + \bar{U}) - (H_1 + \bar{H}_2)(\bar{H}_1 + H_2)].$$

(38)

where

$$K = K(S) + K(\Phi) + K_{Higgs} + K_{matter}(Q, Q^\dagger) + K(\theta, \theta^\dagger)$$

(39)

in which $S$ is dilaton and $\Phi$ denotes moduli $T_i$ or $U_m$. The effective superpotential is then

$$W \sim m^2_\mu H_1 H_2.$$

(40)

Thus we have determined the $U(1)_X$ charges of Higgses and also obtained the $\mu$-term of the electroweak scale consistently.

**The sparticle spectrum**

We assume (scalar) particles which have nonzero $U(1)_X$ charges are in flat directions, otherwise they get mass of the Planck scale. To be concrete, they are those particles of the standard model and $\theta$-field. Hence they are massless at high energy. The $D$-term contribution to the scalar potential is

$$V_D = \frac{1}{2} g^2_X D_X^2$$

(41)

where

$$D_X = \sum_i K_{ij} \phi_i^* \phi_j - \theta^* \theta + \xi^2 + \cdots$$

(42)

and $K_{ij}$ denotes $\partial^2 K/\partial \phi_i \partial \phi_j$. A Fayet-Iliopoulos term is generated at the one string loop level from the anomaly cancellation by the Green-Schwarz mechanism and is given by [14]

$$\xi^2 = \frac{1}{192\pi^2} M_P^2.$$

(43)
Since $\text{Tr}X \neq 0$ as a result of the $U(1)$ anomaly, $\langle \theta \rangle$ is nonzero and the value of which is near the string scale in order to be $D_X = 0$. The $U(1)_X$ symmetry is broken but SUSY is not broken yet.

We assume SUSY breaking occurs through gaugino condensation in the hidden sector, and it is communicated to the observable sector gravitationally. Gravitino mass is then given by

$$m_2^2 = e^{\frac{\Delta}{2} < K > |W|}. \quad (44)$$

When the cosmological constant vanishes, it is given by

$$m_2^2 = \frac{1}{3} \langle K_{ij} F^i F^j \rangle. \quad (45)$$

where $F^i$ is an auxiliary field of a chiral field $\phi_i$. Especially if SUSY breaking is caused by dilaton and moduli because either $F^S$ or $F^T$ gets nonzero value owing to gaugino condensation, $\langle \lambda \lambda \rangle \neq 0$, and if

$$K(S) + K(\Phi) = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}), \quad (46)$$

at the tree level as in string theory and M-theory in Eq.(39), then

$$m_2^2 = \frac{|F^S|^2}{3(S + \bar{S})^2} + \frac{|F^T|^2}{(T + \bar{T})^2} \sim \frac{\langle \lambda \lambda \rangle^2}{M_P^4}. \quad (47)$$

However, the magnitude or scale of $F^S$ or $F^T$ is not definite. They depend on various factors: the group structure of the hidden sector, the existence of hidden matter, gaugino condensation scale, the gauge coupling function, the structure of moduli space, the existence of five-branes and so on. In M-theory it is said that $F^S \sim F^T$ [15]. The value of $\langle S \rangle$ or $\langle T \rangle$ is not known yet. We assume here $m_2$ is around 1TeV. In other words, $F^S$ and/or $F^T$ should be of order $10^{11} - 10^{13}$ GeV. Because fifth dimension $\rho$ is near $\rho_{\text{crit}}$ [16], the gauge coupling constant in the hidden sector is rather large in the case of $E_8$ and the condensation scale may become too large. The non-standard gauge-embedding would be needed and the gauge group in the hidden sector would have to be broken to a smaller one in order to have the condensation scale of near $10^{11} - 10^{13}$ GeV.

Soft scalar masses are written as

$$m_{ij}^2 = m_{ij}^2 |F| + m_{ij}^2 |D|, \quad (48)$$

where

$$m_{ij}^2 |F| = Z_{ij} m_2^2 - F^i \bar{F}^j [\partial_i \bar{\partial}_j Z_{1i} - Z_{1i} \partial_i \bar{\partial}_j Z_{Lj}]. \quad (49)$$

and

$$K_{\text{matter}}(Q, \bar{Q}) = Z_{ij}(\Phi, \bar{\Phi}) \bar{Q}^I Q^J + \cdots. \quad (50)$$

\[1\] In M-theory the scale of $M_{\text{string}}$, $M_{\text{GUT}}$, or $M_{\text{pl}}$ should be sometimes replaced with five-dimensional Planck scale, five-dimensional radius, etc., but the change of scale is rather small.
Subscript $I$ denotes matter and $i$ denotes dilaton and moduli. Soft terms derived from Eq.(19) is model-dependent much. They depend on the modular weight of each quark, and on whether $F^S$ or $F^T$ is larger than the other, and on the form of the Kähler potential. Similarly scalar masses in Eq.(19) at low energy are affected strongly by the renormalization group and very much model-dependent. So even if two kinds of squarks have an identical mass, they are different at low energy in general. The mass difference of squarks as a result of Eq.(49) would cause the problem of the flavor-changing neutral current and many parameters would cause CP breaking. To avoid these problems it would be necessary to have very precise universality of squark masses or alignment with quark matrices. These seem to be rather fine-tuning if there is not a certain symmetry which requires them.

However, in our scheme, concurrent with the gaugino condensation the value of the $D$–term of the anomalous $U(1)$ symmetry becomes nonzero due to the existence of hidden matter, and the $D$-term contribution is much greater than the $F$-term contribution. Consequently the complicated situations or various parameters concerning $F$-terms do not matter.

As discussed before, the gauge group in the hidden sector would be broken to the subgroup smaller than $E_8$, say $SU(N)$, and hidden matter would also exist. Then the discussion of Binetruy and Dudas in [8] for the case of $SU(N) \times U(1)_X$ would be applicable also to this case here. The $D$-term becomes nonzero concurrently with gaugino condensation, and the position of the minimum of the scalar potential is shifted from $\langle \theta \rangle \simeq \xi$. Whereupon the squarks get soft terms of the following form,

$$m^2_i|_D = X(\phi_i)g_X^2 \langle D_X \rangle,$$

where the $\langle D_X \rangle$ is given by

$$\langle D_X \rangle \sim \left[ \frac{N_f \langle \lambda \lambda \rangle}{\xi^2} \right]^2$$

and $N_f$ denotes the number of flavors of hidden matter “quarks”. Then

$$\frac{m^2_{\tilde{t}}}{m_i|_D} \simeq \frac{\langle \lambda \lambda \rangle/M_P^2}{\langle \lambda \lambda \rangle/\xi^2} = \left( \frac{\xi}{M_P} \right)^2 \simeq \varepsilon^2.$$

Namely $m^2_{ij}|_D$ due to the anomalous $U(1)_X$ contributions are much larger than the supergravity-induced soft terms, $m^2_{ij}|_F$. Using the Eqs.(30) and (32), which were obtained from the discussion of the quark mass,

$$X(t) = X(t^c) = 0, X(c) = X(c^c) = 2, X(u) = 3, X(u^c) = 5,$$

the squark masses are obtained as the following:

$$m^2_{\tilde{t}, \tilde{t}^c} \simeq m^2_{\tilde{t}}, m^2_{\tilde{c}, \tilde{c}^c} \simeq m^2_{\tilde{c}}(1 + \frac{2}{\varepsilon^4}), m^2_{\tilde{u}} \simeq m^2_{\tilde{u}}(1 + \frac{3}{\varepsilon^4}), m^2_{\tilde{u}^c} \simeq m^2_{\tilde{u}^c}(1 + \frac{5}{\varepsilon^4}).$$

\[\text{§} \text{see also ref.[18]. It also uses the result of Binetruy and Dudas, considering both the } F \text{-term and } D \text{-term breaking.} \]

\[\text{¶} \text{The characteristic scale of the anomalous } U(1) \text{ symmetry is } \xi. \text{ Then also from dimensional analysis } m_i|_D \text{ would be of the order } \langle \lambda \lambda \rangle/\xi^2. \]

\[\text{∥} \text{In the case of SUSY breaking by the } D \text{-term, } m_{\tilde{u}}/m_i|_D \sim \varepsilon. \]
Similarly, since $h_1 = 0$, we could determine the $U(1)_X$ charges of down quarks as follows:

$$X(b) = 0, X(b^c) = x, X(s) = 2,$$

$$X(s^c) = x, X(d) = 3, X(d^c) = x + 1.$$  \hspace{1cm} (56)

where $x$ is defined in Eq.(29). We obtain

$$m_{\tilde{b}}^2 \simeq m_{\tilde{b}}^2, m_{\tilde{b}^c}^2 \simeq m_{\tilde{b}}^2 (1 + \frac{x}{\varepsilon^4}), m_{\tilde{s}}^2 \simeq m_{\tilde{b}}^2 (1 + \frac{2}{\varepsilon^4}),$$

$$m_{\tilde{s}^c}^2 \simeq m_{\tilde{s}}^2 (1 + \frac{x}{\varepsilon^4}), m_{\tilde{d}}^2 \simeq m_{\tilde{b}}^2 (1 + \frac{3}{\varepsilon^4}), m_{\tilde{d}^c}^2 \simeq m_{\tilde{b}}^2 (1 + \frac{x + 1}{\varepsilon^4}).$$  \hspace{1cm} (57)

To be consistent with observed quark masses, $1 \leq x \leq 3$, probably $x = 2$ or $3$ is better than the choice $x = 1$, because in our scheme $\tan \beta = v_2/v_1 \sim 1$.

From the above result, the mass of top and left-handed bottom squarks would be around the gravitino mass, i.e., around 1TeV, meanwhile other squarks would be roughly 10 – 100 TeV for $\varepsilon \sim 0.1 – 0.2$. Although the right-handed bottom squark is heavy, because of the smallness of the factor $\lambda^2$ in the Yukawa coupling ($\lambda \simeq 0.22$), it would not upset the naturalness of the electroweak scale [10]. Therefore we are able to avoid too large contribution of the flavor-changing neutral current (FCNC) as well [19]. Namely, because first and second generation squarks are heavy enough, their contributions to FCNC processes are very small [10].

As we have only a single chiral $\theta$ field, and not a vectorlike pair, the gaugino mass is not caused by $<\theta> \neq 0$ at the tree level. Accordingly the gaugino mass is generated by the F terms gravitationally and by loop effects. Gravitationally, the gaugino mass is given by [17], [20]

$$\tilde{m}_a = \frac{F^\phi}{2Re(f_a)} \frac{\partial f_a}{\partial \phi}$$  \hspace{1cm} (58)

where $f_a$ is a gauge kinetic function in the observable sector. When $f_a$ is expressed as

$$f_a = S + \alpha T,$$  \hspace{1cm} (59)

then

$$\tilde{m}_a = \frac{F^S + \alpha F^T}{2Re(S + \alpha T)}.$$  \hspace{1cm} (60)

It is expected to be of the order of the gravitino mass from M-theory in contrast to other scenarios where $m_\frac{1}{2}$ is estimated to be a much smaller value which is phenomenologically awkward.

In [9] it is stated that if the first two families of sparticles are heavier than $\sim 10m_{\tilde{g}}$ (where $m_{\tilde{g}}$ is the gluino mass and $m_{\tilde{g}} \sim m_\frac{1}{2}$) while third family squarks are heavier

**There may also occur problems in the above decoupling scenario taking higher-loop (two-loop) corrections into consideration. But the contribution of higher-loop effects is not clear yet.
than $\sim 550\text{GeV}$, then none of the complex phases of the soft terms lead to unacceptable EDMs for the electron or the neutron, even if CP violation is maximal. In the MSSM there are more than 40 physical CP violating phases. To solve the CP problem by the anomalous $U(1)$ symmetry seems much simpler than to assume “ad hoc” symmetries in order to make very many complex phases real. Furthermore, in the latter scenario there may be subtlety in running down from the string scale to the electroweak scale by the renormalization group method. The explanation which was done in this paper conforms to the conditions mentioned above.

\[ B \text{-term is given by [17]} \]

\[ B = F^i\{\partial_i\mu + \frac{1}{2}\mu K_i - Z_N^J\partial_iZ_{j(\mu,j)n} - m_2^2\mu \] 

\[ \cong \frac{1}{2}\mu(K_S F^S + K_T F^T_i) - m_2^2\mu - \mu F^T_i Z_{H_1} \bar{H}_2(\partial_{T_i}Z_{H_2H_1} ) \]  

In generic models typically $B \gg \mu^2$ where $B \sim \mu^2$ is needed phenomenologically. Here it is estimated to be of the order $m_2^2$ because $\mu \sim m_2$ was obtained as in Eq.(40).

As for the $A$ terms the largest one is $A^u_{33}$, since $\lambda^u_{33} \sim 1$. To evaluate $A^u_{33}$, the value of $\bar{F}\bar{\theta}$ is

\[ \bar{F}\bar{\theta} = e^{\frac{1}{2}K}K^{\bar{\theta}} \theta W + W \partial_b K \cong e^{\frac{1}{2}K}\theta^W \]  

because $\partial_b W = 0$ by assumption. Then

\[ A^u_{33} = F^\theta \{\partial_b(e^{\frac{1}{2}K} \lambda^u_{33} + \frac{1}{2}K_b e^{\frac{1}{2}K} \lambda^u_{33} ) \sim \varepsilon^2 m_2^2 e^{\frac{1}{2}K} , \]  

so it satisfies the condition that $A \lesssim 3$. The values satisfy the constraint on $A$ and $B$. It also affects the CP violation by soft terms to a better direction.

**Discussion**

From the definition of $\varepsilon$, the value of $\varepsilon$ is given by

\[ \varepsilon = \frac{\langle \theta \rangle}{M_P} \sim \frac{\xi}{M_P} \sim \sqrt{\frac{1}{192\pi^2}} \sum_i X(\phi_i) \sim 0.1 \]  

if we suppose the sum of the chiral charge of leptons are the same order as that of quarks. Thus we have obtained $\varepsilon$ to be the same order as the Wolfenstein parameter, $\lambda \sim 0.2$.

We did not mention to the mass of fermionic $\theta$. But its interaction with ordinary matter is very weak because the $U(1)_X$ gauge boson is very heavy.

The problem of the flavor- changing neutral current and CP phases may be solved simply using anomalous $U(1)$ symmetry besides having explained quark mass hierarchy almost uniquely. Because the $D$-term is nonzero, the cosmological constant seems to be nonzero. It is a very attractive idea that inflation is triggered by the $D$-term of the
anomalous $U(1)$ symmetry $[21]$. This is expected to occur also around $10^{13}$ GeV or so, namely near the scale of the gaugino condensation.

If there is an effective nonrenormlizable coupling between the gaugino in the hidden sector and “gauge-singlet” right-handed neutrinos such as $\lambda \nu^c \nu^c$, the right-handed neutrinos get mass of the order $10^{13}$ GeV. Then mass of the lepton sector fits well, too.

The most promising idea of baryogenesis is that it is generated by the Affleck-Dine (AD) mechanism $[22]$. If the texture of mass matrices of quarks and leptons are similar, $X(\nu^c_3)$ would be zero. In the early universe anti-sneutrino of the third generation might have a vacuum expectation value, $\langle \tilde{\nu}^c_3 \rangle \neq 0$, and break lepton number, and then by spherarion may be converted to baryon asymmetry $[23]$. Casas and Gelmini $[24]$ state that if the AD field is not charged under the inflationary $U(1)$, the AD mechanism works good. To regard the third generation sneutrino $\tilde{\nu}^c_3$ as the AD field may fit well.

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