Initial state characteristics of proton-nucleus collisions from Glauber Monte Carlo

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Abstract. Fluctuations in physics observables in high energy ion collisions have been a topic of particular interest in recent years, as they may provide important signals regarding the formation of quark-gluon plasma and the existence of a critical point. We provide predictions for basic initial state characteristics of proton-nucleus collisions from Glauber Monte Carlo models. The following systems were simulated and analysed: \(p + ^{12}C\), \(p + ^{14}N\), \(p + ^{63}Cu\), and \(p + ^{208}Pb\) at wide energy range. We apply GLISSANDO\textsuperscript{D0} accordingly fitted to tasks defined in this paper.

1. Introduction

The aim of this talk is to provide predictions for basic initial state characteristics of proton-nucleus collisions from Glauber Monte Carlo (GMC) models. To do this we used GLISSANDO\textsuperscript{D0} [1,2], accordingly fitted to tasks defined in this work. The following systems were studied: \(p + ^{12}C\), \(p + ^{14}N\), \(p + ^{63}Cu\), and \(p + ^{208}Pb\) at wide energy range from low SPS up to LHC.

Attention was paid for distributions of wounded nucleons from target nucleus and its fluctuations which were studied in few variants of GMC. In addition, a production cross section \(\sigma_{p+N}^{\text{prod}}\), for \(p + ^{14}N\) interactions at LHC energy was estimated. Moreover, the dependence of \(\sigma_{p+N}^{\text{in}}\) on total inelastic proton–proton cross section was found and described by power-law functions for hard–sphere and Gaussian nucleon–nucleon wounding profiles. The range of production \(\sigma_{p+N}^{\text{prod}}\) as well as range of total inelastic proton-proton cross section, \(\sigma_{\text{inel}}\), at \(\sqrt{s} = 57\) TeV was estimated varying reasonable assumptions from GMC. This may be useful for cosmic ray physicists community, where the knowledge of production \(\sigma_{p+N}^{\text{prod}}\) cross section for proton-air interactions is crucial in proper simulations of the extensive air showers development in Earth’s atmosphere. The calculated cross-sections was compared with the Pierre Auger Observatory data [3].

A brief description of Glauber-like models is given in Sect. 2. In Sect. 3 a concise depiction of the Glauber Monte Carlo approach with particular attention to the model used in this work is provided. In Sect. 4 results of proton-nucleus simulation are presented and discussed. Section 5 closes the article.

2. A brief description of Glauber-like models

The Glauber model [4] was formulated in late 1950s in order to find description of high-energy collision of atomic nuclei treated as composite structures. Until then there was no systematic calculations regarding nuclear systems as projectile or target. Glauber model containing quantum theory of collisions of composite particles allows to describe experimental results on collisions of protons with deuterons and heavier nuclei. In the mid 1970s Bialas et al. [5,6] used the Glauber model to describe inelastic nuclear collisions in their wounded-nucleon model. Bialas et al. [5] formulation allows to treat a collision between nuclei as a superposition of incoherent collisions between their nucleons. An excellent review of Glauber modelling of high-energy collisions was given in [7].

2.1. Nuclear charge densities

In the Glauber-like calculations some parameters must be provided from experiment. Usually these are the nuclear charge densities and the energy dependence of the nucleon-nucleon cross section. In the Glauber model, the nucleons in nuclei are distributed randomly according to given nuclear density distribution. The radial nuclear density distribution, \(\rho(r)\), determined from electron-nucleus scattering experiments is, for sufficiently high nuclei, usually in the Woods-Saxon type, modified to incorporate the shape of nuclei deformation [8–10]:

\[
\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r - R(1 + \beta_2 Y_{20} + \beta_4 Y_{40})}{a}\right)},
\]

where \(\rho_0\) corresponds to the nucleon density in the center of nucleus, \(R\) corresponds to the nuclear radius, \(a\) to the skin depth, \(Y_{lm}\) are spherical harmonic functions of degree \(l\) and order \(m\) represented as a complex exponential and associated Legendre polynomials, and \(\beta_2\) and \(\beta_4\) are deformation parameters [11].

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Table 1. Woods-Saxon parameters of heavy nuclei used in this work. The $R$ values are taken from Eq. (3), while values of deformation parameters $\beta_2$ and $\beta_3$ from [11].

| Nucleus | $R$ [fm] | $\beta_2$ | $\beta_3$ |
|---------|----------|-----------|-----------|
| $^{63}$Cu | 4.212 | 0.162 | -0.006 |
| $^{208}$Pb | 6.407 | 0.0 | 0.0 |

For heavy nuclei, with atomic mass $A > 16$ the nuclear radii are well described by:

$$R = (1.12A^{1/3} - 0.86A^{-1/3}) \text{ fm},$$

while $a = 0.54$ fm.

Nucleons are extended objects, so the centers of the nucleons cannot be closer than particular expulsion distance $d$ – this is the usual way to introduce the nucleon-nucleon repulsion in GMC models. The magnitude of $d$ should be proportional to the hard-core repulsion range in the nuclear potential. Since the repulsion implemented via the condition $d > 0$ increases (at the level of 1%) the size $R$ of the nucleus then it should be compensated by appropriate change of the distribution parameters used for generation of positions of nucleons centers. It was found in [12] that the following parametrization:

$$R = (1.1A^{1/3} - 0.656A^{-1/3}) \text{ fm},$$

$a = 0.459$ fm

(3)

taken for $d = 0.9$ fm works well. Alvioli, Drescher and Strikman [13,14] prepared distributions of nucleons in nuclei which contain the central two-body correlations between nucleons. The procedure was based on the Metropolis algorithm used to search for configurations satisfying limitations imposed by the nucleon-nucleon correlations. The one-body Woods-Saxon distribution as well as central nucleon-nucleon correlations, taken in the Gaussian form was reproduced. The effects of these correlations on heavy-ion observables turn out to be indistinguishable from the hard-core repulsion with $d = 0.9$ fm [12,15]. In this paper the nucleon expulsion distance $d = 0.9$ fm is used as standard for all considered nuclei. Table 1 collects the values of Woods-Saxon parameters of heavy nuclei used in this work.

For lighter nuclei, with mass number $3 \leq A \leq 16$ a harmonic oscillator shell model density is used [8,9,16]:

$$\rho (r) = \frac{4}{\pi^{3/2}C^3} \left[ 1 + \frac{A - 4}{6} \left( \frac{r}{C} \right)^2 \right] \exp \left( -r^2 / C^2 \right),$$

$$C^2 = \left( \frac{5}{2} - \frac{4}{A} \right) - (r_{ch}^2)_{\Lambda} - (r_{ch}^2)_{p},$$

where $(r_{ch}^2)_{\Lambda}$ and $(r_{ch}^2)_{p} = 0.7714 \text{ fm}^2$ are the mean squared charge radii of the nucleus and proton, respectively [17]. The values of the harmonic oscillator shell model parameter $(r_{ch}^2)_{\Lambda}$ for light nuclei used in this work are shown in Table 2.

| Nucleus | $(r_{ch}^2)_{\Lambda}$ [fm$^2$] |
|---------|------------------|
| $^{12}$C | 5.10 |
| $^{14}$N | 5.66 |
| $^{15}$O | 5.86 |
| $^{16}$O | 6.40 |

$\sqrt{s}$ is the center of mass energy for nucleon pair $p_{lab}$ is the beam momentum in the laboratory frame.

Table 2. Harmonic oscillator shell model parameter $(r_{ch}^2)_{\Lambda}$ for several light nuclei [17]. The values include the case with no NN repulsion $(d = 0)$ and with the repulsion implemented via expulsion radius of $d = 0.9$ fm and $d = 1.5$ fm.

| Nucleus | $(r_{ch}^2)_{\Lambda}$ [fm$^2$] |
|---------|------------------|
| $^{12}$C | 4.31 |
| $^{14}$N | 4.02 |
| $^{16}$O | 4.28 |

| $\sqrt{s}$ [GeV] | $p_{lab}$ [GeV/c] | $\sigma^{inel}$ [mb] |
|-----------------|-----------------|-----------------|
| 5.12 | 13.0 | 29.1 |
| 16.83 | 150.0 | 31.72 |
| 7000.0 | 2.61 · 10$^3$ | 73.5 |

Table 3. Total inelastic, $\sigma^{inel}$ proton-proton cross sections for center of mass energies used in this work [20,21]. $\sqrt{s}$ is the center of mass energy for nucleon pair and $p_{lab}$ is the beam momentum in the laboratory frame.

Usually, the two types of processes are regarded, inelastic and production reactions. The production processes are those which lead to the production of new hadrons. Among the inelastic processes there are also interactions which result only in the disintegration of the projectile or target nucleus (quasi-elastic interactions). The inelastic cross section $\sigma^{inel}$ is the sum of the processes due to strong p+A and A+A interactions except coherent nuclear elastic scattering. Thus it contains interactions which result in the production of new hadrons (production processes) and quasi-elastic interactions resulting in the break up of the projectile or target nucleus. In high-energy experiments the production cross section $\sigma^{prod}$ is usually determined from the inelastic cross section by subtraction of the cross section of quasi-elastic p+A and A+A reactions [18].

The Glauber model was prepared for elastic collisions, where the nucleus does not change its properties over individual collisions. Using the Glauber model for inelastic collisions, it is assumed that after a single inelastic collision an excited nucleon-like object is created that interacts with the same nucleon-nucleus cross section with other nucleons [19].

In the present work for the simulation of production processes, the total inelastic proton-proton cross section, $\sigma^{inel}$, was applied. This is the only way to introduce the beam energy dependence of Glauber calculations.

In the case of simulations at $\sqrt{s} = 7000$ GeV the value of total inelastic proton-proton cross section was taken from [20], while for all other energies – from PDG library [21]. The total inelastic proton-proton cross section was calculated as a difference between total collision and total elastic proton-proton cross section. The values of total inelastic proton-proton cross sections for center of mass energies used in this work are shown in Table 3.

2.3. Nucleon-nucleon wounding profile

The nucleon-nucleon wounding profile (NNWP), $p(b)$, defined by the probability density $f(b)$ of inelastic nucleon-nucleon collision at the impact parameter $b$, plays a very important role in all Glauber like approaches and especially in the wounding-nucleon model [5]. NNWP is
normalized to the total inelastic nucleon-nucleon cross section
\[ 2\pi \int b p(b) \, db = \sigma_{inel}^\text{tot}. \] (5)

Usually in Glauber Monte Carlo codes it is assumed that \( p(b) \) is given by step function
\[ p(b) = \Theta(R - b) \] (6)
with \( R = \sqrt{\sigma_{inel}^\text{tot}/\pi} \) and probability density distribution \( f(b) = \frac{1}{\pi} \Theta(R - b) \). Later in the text, a profile function given by Eq. (6) we shall call a hard-sphere approximation.

The importance of the nucleon-nucleon wounding profile shape was discussed in [22,23]. Following [24], where the CERN ISR experimental data for proton-proton differential cross section were properly parametrized with a combination of Gaussians, the profile function was proposed in the form [22]
\[ p(b) = G \exp \left( -\frac{Gb^2}{R^2} \right) \] (7)
with parameter \( G = 0.92 \). We shall call it a Gaussian approximation. In [23] there is a suggestion that cross section fluctuations given by the gamma distribution with relative variance \( \omega = \text{Var}(\sigma)/\langle \sigma \rangle \), leads to the profile function
\[ p(b) = \Gamma \left( \frac{1}{\omega}, \frac{b^2}{R^2\omega} \right) / \Gamma \left( \frac{1}{\omega} \right), \] (8)
where \( \Gamma(z) \) is the Euler gamma function, \( \Gamma(\omega, z) \) is incomplete gamma function, and parameter \( \omega \in (0, 1) \).

NNWP given by Eq. (8) – a Gaussian approximation – smoothly ranges between the both hard-sphere for \( \omega \to 0 \) and Gaussian, \( \omega \to 1 \) approximations.

The Gamma approximation with parameter \( \omega = 0.4 \) corresponds to the shape of profile function which well reproduces the TOTEM data [25,26] on elastic differential cross section measured in proton-proton interactions at \( \sqrt{s} = 7000 \text{ GeV} \) [23].

3. GLISSANDO: The Glauber Monte Carlo approach

In the last few decades the popular Glauber Monte Carlo approach started to be an essential tool in the analysis of relativistic heavy-ion collisions [7]. One of the most important application of the GMC simulation is the estimate of the number of participants dependence on the centrality, especially in the collider experiments [1,2,7]. On the physics side, the presence of the event-by-event fluctuations in the initial Glauber phase is a crucial aspect of the approach. These geometric fluctuations are carried over to the final distributions of the experimentally measured hadrons. In many analyses the Glauber-model initial state is used as an starting point for event-by-event hydrodynamics [27]. Among other aspects which are also considered with this approach, one can include the forward-backward correlations [28], the two-dimensional correlations in relative rapidity and azimuth [29], or jet quenching [30].

\[ \text{GLISSANDO} [1,2] \] is a GMC generator for initial stages of relativistic heavy-ion collisions, written in c++ and interfaced to ROOT [31]. The program can be used for simulation of large variety of colliding systems: p+A, d+A and A+A at wide spectrum of energies. Several models are implemented: the wounded-nucleon model [5], the binary collisions model, the mixed model [32,33], and the model with hot-spots [34].

The program generates, among others, the variable-axes (participant) two- and three-dimensional profiles of the density of sources in the transverse plane and their Fourier components. These profiles can be used in further analyses of physics phenomena, such as the jet quenching, event-by-event hydrodynamics, or analysis of the elliptic flow and its fluctuations. Details can be found in [1,2].

4. Results for proton-nucleus interactions

In this section results for some initial state characteristics of proton-nucleus interactions are described. Simulation of the production reactions of protons with \( ^{12}\text{C}, \, ^{14}\text{N}, \, ^{63}\text{Cu} \) and \( ^{208}\text{Pb} \) nuclei at \( \sqrt{s} = 5.12, 16.83 \) and \( 7000 \text{ GeV} \) was performed with few variants of wounded-nucleon model. Since the production processes are considered, thus the values of \( \sigma_{inel}^\text{tot} \) are used. The results for \( p + ^{208}\text{Pb} \) system may be in particular interest of NA61/SHINE collaboration which recently collected data on \( p + ^{208}\text{Pb} \) interactions at CERN SPS energies.

4.1. Number of target participants

In Fig. 1 the mean number of target participants \( \langle N_{p}^{\text{agg}} \rangle \) as a function of target mass \( A \) is shown. There are results of GLISSANDO simulation within wounded-nucleon model with NNWP given by hard-sphere approximation. For reactions at all energies a smooth increase of the average

\[ \langle N_{p}^{\text{agg}} \rangle \] (10)

\[ A \]

Figure 1. The mean number of target participants \( \langle N_{p}^{\text{agg}} \rangle \) as a function of target mass \( A \). The results of GLISSANDO simulation for production reactions of protons with \( ^{12}\text{C}, \, ^{14}\text{N}, \, ^{63}\text{Cu} \) and \( ^{208}\text{Pb} \) ions at \( \sqrt{s} = 5.12, 16.83 \) and \( 7000 \text{ GeV} \) Wounded-nucleon model with NNWP given by hard-sphere approximation fitted by Eq. (9).

\[ \langle N_{p}^{\text{agg}} \rangle = 0.76 \pm 0.05 \cdot A^{0.11 \pm 0.01}, \langle N_{p}^{\text{agg}} \rangle = 0.56 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.89 \pm 0.04 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.75 \pm 0.05 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.52 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.60 \pm 0.04 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.79 \pm 0.05 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.53 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.61 \pm 0.04 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.76 \pm 0.05 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.55 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.63 \pm 0.04 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.78 \pm 0.05 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.51 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.60 \pm 0.04 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.77 \pm 0.05 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.50 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.59 \pm 0.04 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.77 \pm 0.05 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.50 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.58 \pm 0.04 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.77 \pm 0.05 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.50 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.58 \pm 0.04 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.77 \pm 0.05 \cdot A^{0.11 \pm 0.01}. \]

\[ \langle N_{p}^{\text{agg}} \rangle = 0.50 \pm 0.02 \cdot A^{0.11 \pm 0.01}. \]
number of target participants with increasing mass of target nuclei is observed. In particular, at $\sqrt{s} = 7000$ GeV it starts with $\langle N_{p}^{\text{part}} \rangle \simeq 2.45$ for $p + ^{12}$C and ends with $\langle N_{p}^{\text{part}} \rangle \simeq 7.61$ for $p + ^{208}$Pb interactions. For both lower energies the increase of $\langle N_{p}^{\text{part}} \rangle$ with mass number $A$ is slower and similar each other.

The target mass dependence of $\langle N_{p}^{\text{part}} \rangle$ may be described by simple power-law formula

$$\langle N_{p}^{\text{part}} \rangle = N_{0} \cdot A^{\alpha},$$

with parameters $N_{0}$ and $\alpha$ given in Table 4. The $\alpha$ exponent slightly increases its value from $\alpha \simeq 0.3$ at $\sqrt{s} = 5.12$ GeV up to $\alpha \simeq 0.4$ for LHC energy, $\sqrt{s} = 7000$ GeV.

The resultant fluctuations of the number of target participants as a function of target atomic mass $A$ are presented in Fig. 2. Again, wounded-nucleon model with NNWP given by hard-sphere approximation.

The comparison of $\alpha_{p}^{\text{part}}$ values obtained for $p + ^{208}$Pb production reactions simulated with different shapes of NNWP, namely hard-sphere, Gaussian and gamma approximations is shown in Fig. 3. For LHC energy, $\sqrt{s} = 7000$ GeV as well as hard-sphere and Gaussian – the NNWP given by gamma approximation with parameter $\omega = 0.4$ [23] was used. There is a small difference between hard-sphere and Gaussian approximations, of the order of few percents, however Gaussian approximation leads to higher fluctuations of target participants for all considered energies. The Gamma approximation for LHC energy gives fluctuations of the number of target participants just in the middle between the above mentioned hard-sphere and Gaussian.

### 4.2. Production cross section

In the modelling of propagation of cosmic rays through the atmosphere a knowledge of production proton-air cross section, $\sigma_{p+a}^{\text{prod}}$ plays a very important role. As air is mainly composed with $^{14}$N nuclei, we simulated $p + ^{14}$N production reactions at $\sqrt{s} = 7000$ GeV. In order to estimate the possible range of $\sigma_{p+N}^{\text{prod}}$, we performed simulations with all: hard-sphere, Gaussian and gamma with parameter $\omega = 0.4$ nucleon-nucleon winding profiles. In addition, we also varied the value of nucleon expulsion distance $d$. Table 5 shows the obtained values of $\sigma_{p+N}^{\text{prod}}$. As the proton-proton total inelastic cross section at LHC energy is known with the uncertainty $\sigma_{p+N}^{\text{inel}} = 73.5^{+1.8}_{-1.3}$ mb [20] we simulated $p + ^{14}$N interactions with three different values of $\sigma_{p+N}^{\text{inel}} = 73.5, 75.3$ and 72.2 mb for standard, upper and lower limit. The corresponding uncertainty estimates are shown in Table 5 as positive and negative numbers next to the evaluated values of $\sigma_{p+N}^{\text{prod}}$.

Recently NA61/SHINE collaboration measured production and inelastic cross section for $p + ^{12}$C interactions at beam momentum $p_{\text{lab}} = 31$ GeV/c [18]. The measured values are $\sigma_{p+C}^{\text{prod}} = 229.3 \pm 9$ mb and $\sigma_{p+C}^{\text{inel}} = 257.2 \pm 8.9$ mb, respectively. Our simulation for $p + ^{12}$C

### Table 4. Parameters used in Eq. (9).

| $\sqrt{s}$ (GeV) | $N_{0}$ | $\alpha$ |
|------------------|---------|----------|
| 5.12             | 0.75 ± 0.05 | 0.3 ± 0.01 |
| 16.83            | 0.76 ± 0.05 | 0.31 ± 0.01 |
| 7000.0           | 0.89 ± 0.04 | 0.4 ± 0.01 |

Figure 2. Scaled variance of the distribution of target participants number, $\alpha_{p}^{\text{part}}$ as a function of target atomic mass $A$. The results of GLISSANDO simulation for production reactions of protons with $^{12}$C, $^{14}$N, $^{60}$Cu and $^{208}$Pb ions at $\sqrt{s} = 5.12$, 16.83 and 7000 GeV. Wounded-nucleon model with NNWP given by hard-sphere approximation.

Figure 3. Scaled variance of the target participant number distribution as a function of center of mass energy. The comparison of results of GLISSANDO simulation for $p + ^{208}$Pb production reactions. Wounded-nucleon model with NNWP given by hard-sphere, Gaussian and gamma with parameter $\omega = 0.4$ approximations.
interactions with NNWP given by hard-sphere approximation and nucleon-nucleon expulsion distance $d = 0.9$ mb results with the production cross section $\sigma_{p+p}^{\text{prod}} = 222.9$ mb, and $\sigma_{p+p}^{\text{inel}} = 253.4$ mb, respectively, what is in good agreement with NA61/SHINE data. This fact, together with the one stated in Sect. 2 leads us to use NNWP given by hard-sphere approximation with nucleon-nucleon expulsion distance $d = 0.9$ as standard. Thus, $\sigma_{p+p}^{\text{prod}} = 394.3 +5.3$ mb should be used as a reference point for $p + ^{14}\text{N}$ production cross section at $\sqrt{s} = 7000$ GeV. However, as other parameterisations also look reasonable, then larger variation of $\sigma_{p+p}^{\text{prod}}$ in (390, 460) mb may be measured experimentally.

In Fig. 4, we present proton-nucleus production cross section $\sigma_{p+A}^{\text{prod}}$ as a function of target mass $A$. As in the case of average number of target participants dependence on target mass and following [35], we use the power-law function

$$\sigma_{p+A}^{\text{prod}} = \sigma_0 \cdot A^\gamma$$

(11)

with parameters $\sigma_0$ and $\gamma$ given in Table 6 to fit the simulated values of $\sigma_{p+A}^{\text{prod}}$. Here, the $\gamma$ exponent slightly decreases its value from $\gamma \geq 0.68$ at $\sqrt{s} = 5.12$ GeV to $\gamma \geq 0.59$ for $\sqrt{s} = 7000$ GeV.

It may be interesting to look how the production cross section $\sigma_{p+p}^{\text{prod}}$ simulated in high-energy $p + ^{14}\text{N}$ interactions depends on the total inelastic proton–proton

\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$d$ [fm] & $\sigma_{p+p}^{\text{prod}}$ [mb] (hard-sphere) & $\sigma_{p+p}^{\text{prod}}$ [mb] (gamma) & $\sigma_{p+p}^{\text{prod}}$ [mb] (Gaussian) \\
\hline
0 & 391.5$^{+5.1}_{-5.3}$ & 413.7$^{+5.6}_{-5.2}$ & 451.7$^{+7.5}_{-5.1}$ \\
0.9 & 394.3$^{+5.3}$ & 415.1$^{+6.0}_{-5.1}$ & 452.4$^{+7.2}_{-5.5}$ \\
1.5 & 396.7$^{+4.9}_{-3.4}$ & 416.3$^{+5.9}_{-3.6}$ & 452.3$^{+6.9}_{-5.3}$ \\
\hline
\end{tabular}
\caption{Production cross section, $\sigma_{p+p}^{\text{prod}}$ for $p + ^{14}\text{N}$ interactions at $\sqrt{s} = 7000$ GeV. Simulation performed with total inelastic proton-proton cross section $\sigma_{p+p}^{\text{inel}} = 73.5^{+11.1}_{-10.7}$ mb [20]. The upper (positive) uncertainty comes from simulations with $\sigma_{p+p}^{\text{inel}} = 75.3$ mb, while the lower one – from $\sigma_{p+p}^{\text{inel}} = 72.2$ mb.}
\end{table}

\begin{figure}[h!]
\centering
\includegraphics[width=0.8\textwidth]{fig4.png}
\caption{Proton-nucleus production cross section $\sigma_{p+A}^{\text{prod}}$ as a function of target mass $A$. The results of the GLISSANDO simulation for production reactions of protons with $^{12}\text{C}$, $^{14}\text{N}$, $^{65}\text{Cu}$ and $^{208}\text{Pb}$ ions at $\sqrt{s} = 5.12$, 16.83 and 7000 GeV. Wounded-nucleon model with NNWP given by hard-sphere approximation fitted by Eq. (11).}
\end{figure}

\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$\sqrt{s}$ [GeV] & $\sigma_0$ [mb] & $\gamma$ \\
\hline
5.12 & 41.5$^{+3.7}_{-3.6}$ & 0.68$\pm 0.01$ \\
16.83 & 44.7$^{+3.8}_{-3.6}$ & 0.67$\pm 0.01$ \\
7000.0 & 84.7$^{+4.5}_{-4.3}$ & 0.59$\pm 0.01$ \\
\hline
\end{tabular}
\caption{Parameters used in Eq. (11).}
\end{table}

\begin{figure}[h!]
\centering
\includegraphics[width=0.8\textwidth]{fig5.png}
\caption{p + $^{14}\text{N}$ production cross section, $\sigma_{p+p}^{\text{prod}}$ as a function of total inelastic proton-proton cross section, $\sigma_{p+p}^{\text{inel}}$. Simulation for wounded-nucleon model with hard-sphere and Gaussian wounded-nucleon wounding profiles, both with expulsion distance $d = 0.9$ fm. See text for details.}
\end{figure}

\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|}
\hline
$\sigma_0$ [mb] & $\gamma$ & $\sigma_{p+p}^{\text{prod}}$ [mb] \\
\hline
5.12 & 41.5$^{+3.7}_{-3.6}$ & (40.61$^{+0.12}_{-0.10}$) · ($\sigma_{p+p}^{\text{inel}}$)$^{0.53\pm0.007}$ mb, \\
16.83 & 44.7$^{+3.8}_{-3.6}$ & (26.56$^{+0.72}_{-0.69}$) · ($\sigma_{p+p}^{\text{inel}}$)$^{0.66\pm0.006}$ mb, \\
7000.0 & 84.7$^{+4.5}_{-4.3}$ & (26.56$^{+0.72}_{-0.69}$) · ($\sigma_{p+p}^{\text{inel}}$)$^{0.66\pm0.006}$ mb, \\
\hline
\end{tabular}
\caption{p + $^{14}\text{N}$ production cross section, $\sigma_{p+p}^{\text{prod}}$ as a function of total inelastic proton-proton cross section, $\sigma_{p+p}^{\text{inel}}$. According simulation was performed and the results are shown in Fig. 5. There is a difference between two extreme cases given by hard-sphere and Gaussian wounded-nucleon wounding profiles; namely Gaussian wounding profile leads to the faster growth of $\sigma_{p+p}^{\text{prod}}$ with $\sigma_{p+p}^{\text{inel}}$. The power-law fits to the simulated points result in:

$$\sigma_{p+p}^{\text{prod,hard-sphere}} = (40.61 \pm 0.12) \cdot \left(\sigma_{p+p}^{\text{inel}}\right)^{0.53\pm0.007} \text{ mb},$$

(12)

and

$$\sigma_{p+p}^{\text{prod,Gaussian}} = (26.56 \pm 0.72) \cdot \left(\sigma_{p+p}^{\text{inel}}\right)^{0.66\pm0.006} \text{ mb},$$

(13)

for hard-sphere and Gaussian wounding profiles, respectively.

In Ref. [3] the Pierre Auger Collaboration calculated proton-air production cross section, $\sigma_{p+\text{air}}^{\text{prod}}$ at $\sqrt{s} = 57$ TeV from cosmic ray data, to be equal

\footnote{In order to obtain the presented result, the five different values of the total inelastic proton-proton cross section was chosen corresponding to center of mass energies $\sqrt{s}$ equal to 5.12, 6.27, 7.62, 16.83 and 7000 GeV. We performed also simulation for $\sigma_{p+p}^{\text{inel}} = 150$ mb which corresponds to energies available in cosmic ray physics.}
\[ \sigma_{\text{prod}} = 505^{+28}_{-36} \text{ mb}. \] There was also found the corresponding inelastic proton-proton cross section, \( \sigma_{\text{inel}} = 92^{+9}_{-11} \text{ mb}. \) From our fits, Eqs. ((12)–(13)) to \( p + ^{14}\text{N} \) GLISSANDO simulation, we found \( \sigma_{\text{prod, hard-sphere}}(92 \text{ mb}) \approx 446 \text{ mb} \) and \( \sigma_{\text{prod, Gaussian}}(92 \text{ mb}) \approx 525 \text{ mb} \) what determine the limits for \( p + ^{14}\text{N} \) production cross section at \( \sqrt{s} = 57 \text{ TeV} \) and is in quite good agreement with [3].

From the other side, with use of Eqs. ((12)–(13)) it is possible to estimate the possible ranges of \( \sigma_{\text{inel}} \) at \( \sqrt{s} = 57 \text{ TeV} \). We recovered Pierre Auger Collaboration \( \sigma_{\text{prod}} = 505 \text{ mb} \) [3] at \( \sigma_{\text{inel}} \approx 87 \text{ mb} \) (Gaussian approximation) and \( \sigma_{\text{inel}} \approx 116 \text{ mb} \) (hard-sphere approximation) see Fig. 6.

### 5. Summary

The initial state characteristics of proton-nucleus collisions were simulated and analysed with use of Glauber Monte Carlo code GLISSANDO. The following systems were studied: \( p + ^{12}\text{C}, p + ^{14}\text{N}, p + ^{63}\text{Cu}, \) and \( p + ^{208}\text{Pb} \) at wide energy range from low SPS up to LHC. Our main results are as follows:

- The mean number of target participants as a function of atomic mass of target nucleus was plotted and discussed. A simple power-law fit with the exponent around 1/3 was found to properly describe above dependence for SPS as well as LHC energies.

The value of this exponent slightly increase with the increasing energy of collision.

- Fluctuations of target participants described by scaled variance of target participants distributions was plotted as a function of target nucleus mass and collision center of mass energy. An influence of winding profile on scaled variance was discussed.

- Production cross section as a function of target mass was estimated and fitted by power-law formula with the exponent around 2/3. The value of the exponent slightly decrease with the increasing energy of collision.

- For \( p + ^{14}\text{N} \) interactions, the production cross section, \( \sigma_{\text{prod}} = 394.3^{+5.1}_{-3.3} \text{ mb} \) was estimated to be equal \( \sigma_{\text{prod}} = 505^{+28}_{-36} \text{ mb} \) as well as range of total inelastic proton-proton cross section \( \sigma_{\text{inel}} \approx (87, 116) \text{ mb} \) at \( \sqrt{s} = 57 \text{ TeV} \) was estimated.

### References

[1] W. Broniowski, M. Rybczynski and P. Bozek, Comput. Phys. Commun. 180, 69 (2009) [arXiv: 0710.5731 [nucl-th]]

[2] M. Rybczynski, G. Stefanek, W. Broniowski and P. Bozek, Comput. Phys. Commun. 185, 1759 (2014) [arXiv:1310.5475 [nucl-th]]

[3] P. Abreu et al. [Pierre Auger Collaboration], Phys. Rev. Lett. 109, 062002 (2012) [arXiv: 1208.1520 [hep-ex]]

[4] R. J. Glauber in *Lectures in Theoretical Physics* W. E. Brittin and L. G. Dunham eds., (Interscience, New York, 1959) Vol. 1, p. 315

[5] A. Bialas, M. Bleszynski and W. Czyz, Nucl. Phys. B 111, 461 (1976)

[6] A. Bialas, M. Bleszynski and W. Czyz, Acta Phys. Polon. B 8, 389 (1977)

[7] M. L. Miller, K. Reygers, S. J. Sanders and P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57, 205 (2007) [nucl-ex/0701025]

[8] L. R. B. Elton, Nuclear Sizes, Oxford Univ. Press, London, 1961

[9] H. De Vries, C. W. De Jager and C. De Vries, Atom. Data Nucl. Data Tabl. 36, 495 (1987)

[10] K. Hagino, N. W. Lwin and M. Yamagami, Phys. Rev. C 74, 017310 (2006) [nucl-th/0604048]

[11] P. Moller, J. R. Nix, W. D. Myers and W. J. Swiatecki, Atom. Data Nucl. Data Tabl. 59, 185 (1995) [nucl-th/9308022]

[12] W. Broniowski and M. Rybczynski, Phys. Rev. C 81, 064909 (2010) [arXiv:1003.1088 [nucl-th]]

[13] M. Alvioli, H.-J. Drescher and M. Strikman, Phys. Lett. B 680, 225 (2009) [arXiv:0906.2670 [nucl-th]]

[14] see http://www.phys.psu.edu/~malvioli/event generator/
[15] M. Rybczynski and W. Broniowski, Phys. Part. Nucl. Lett. 8, 992 (2011) [arXiv:1012.5607 [nucl-th]]

[16] H. Pi, Comput. Phys. Commun. 71, 173 (1992)

[17] I. Angeli and K. P. Marinova, Atom. Data Nucl. Data Tabl. 99, 69 (2013)

[18] N. Abgrall et al. [NA61/SHINE Collaboration], Phys. Rev. C 84, 034604 (2011) [arXiv:1102.0983 [hep-ex]]

[19] W. Florkowski, Phenomenology of Ultra-relativistic Heavy-ion Collisions, World Scientific, Singapore, 2010

[20] G. Antchev, P. Aspell, I. Atanassov, V. Avati, J. Baechler, V. Berardi, M. Berretti and E. Bossini et al., Europhys. Lett. 96, 21002 (2011) [arXiv:1110.1395 [hep-ex]]

[21] J. Beringer et al., [Particle Data Group Collaboration], Phys. Rev. D 86, 010001 (2012)

[22] M. Rybczynski and W. Broniowski, Phys. Rev. C 84, 064913 (2011) [arXiv:1110.2609 [nucl-th]]

[23] M. Rybczynski and Z. Włodarczyk, arXiv:1307.0636 [nucl-th]

[24] A. Bialas and A. Bzdak, Acta Phys. Polon. B 38, 159 (2007) [hep-ph/0612038]

[25] G. Antchev et al. [TOTEM Collaboration], Europhys. Lett. 95, 41001 (2011) [arXiv:1110.1385 [hep-ex]]

[26] G. Antchev et al. [TOTEM Collaboration], Europhys. Lett. 101, 21002 (2013)

[27] R. Andrade, F. Grassi, Y. Hama, T. Kodama and O. Socolowski, Jr., Phys. Rev. Lett. 97, 202302 (2006) [nucl-th/0608067]

[28] A. Bialas and W. Czyz, Acta Phys. Polon. B 36, 905 (2005) [hep-ph/0410265]

[29] P. Bozek and W. Broniowski, Phys. Rev. Lett. 109, 062301 (2012) [arXiv:1204.3580 [nucl-th]]

[30] R. J. Fries and R. Rodriguez, Nucl. Phys. A 855, 424 (2011) [arXiv:1012.3950 [nucl-th]]

[31] R. Brun et al. Root Users Guide 5.16, CERN, 2007

[32] B. B. Back et al. [PHOBOS Collaboration], Phys. Rev. C 65, 031901 (2002) [nucl-ex/0105011]

[33] B. B. Back et al. [PHOBOS Collaboration], Phys. Rev. C 70, 021902 (2004) [nucl-ex/0405027]

[34] M. Gyulassy, D. H. Rischke and B. Zhang, Nucl. Phys. A 613, 397 (1997) [nucl-th/9609030]

[35] J. Carvalho, Nucl. Phys. A 725, 269 (2003)