Masonry elastic characteristics assessment by thermographic images

Federico Cluni · Vittorio Gusella · Gianluca Vinti

Abstract In the present paper, the elastic mechanical characteristics of masonry samples, whose texture is not visible due to plaster, are estimated by means of homogenization technique applied through thermographic images. In particular, three masonry samples with different textures have been purposely built. The chosen textures were periodic, quasi-periodic and random. The images, taken with a thermocamera, are analyzed in order to identify the texture. An homogenization technique, based on the application of appropriate boundary conditions, is used. The mechanical characteristics, obtained using the textures identified using photographic images and thermographic images, are compared. The influence of some parameters (such as the dimensions of the structural element used in morphological operator) are analyzed. The obtained results permit to point out the reliability of the masonry elastic characteristics assessment by the proposed procedure.

Keywords Homogenization · Masonry texture · Digital image processing · Thermography

1 Introduction

The assessment of the mechanical characteristics of the masonry assumes an important role with particular attention to structural analysis and restoration of historical constructions and cultural heritages.

Considering the masonry as a bi-phase composite, built by mortar and bricks (or stones), the mechanical behaviour is depending on the actual texture, both in elastic field [1, 2] and in plastic one [3, 4].

An adequate recognition of the texture is necessary to evaluate the Periodic Unit Cell (PUC) in periodic case [5]. This is also true to asses the Representative Volume Element (RVE) for quasi periodic or random arrangements [6, 7]. Moreover, the knowledge of the actual texture takes a fundamental role using the approach that involves the periodization of random media by statically equivalent periodic unit cell [8–12]. Eventually the texture knowledge is necessary for the homogenization residuals analysis [13–15].

Given the previous observations, it is obvious the importance of developing procedures for identifying the texture of the masonry in order to estimate the mechanical characteristics by non destructive testing.
In recent years, the possibility of estimating the masonry texture by means of digital (photographic) images has been explored by several authors [11, 16]; moreover digital images can be acquired by laser scanner [17].

These procedures are not applicable when the masonry walls are covered with plaster or, very frequently for historical building, with frescoes.

To overcome these situations, the thermographic tool has recently been proposed in [18, 19] together with some methods to enhance the images and subsequently to estimate the masonry characteristics.

Nevertheless a fundamental aspect to be investigated is the reliability assessment of the thermographic procedure for masonry texture recognition. To analyze this topic an ongoing research has been launched with the building of masonry wall samples, characterised by different textures, and the assessment of mechanical characteristics by homogenization theory, applied both on photographic and thermographic images.

This paper reports the first obtained results and it is organized in the following sections. After some considerations about the thermographic theory and the peculiarities of the apparatus used in the experimental tests (Sect. 2), the characteristics of masonry samples and how they were built are described in Sect. 3. The homogenization approach, used to obtain the elastic masonry characteristics, is briefly remembered in Sect. 4, and the procedure to identify the texture is recalled in Sect. 5. Finally, the comparison between photographic results and thermographic results is shown in Sect. 6, with comments on the reliability of the proposed procedure. The influence of some parameters involved in the proposed approach (such as the dimensions of the structural element used in morphological operators) are analyzed.

2 Thermographic images

The principle by which the thermography works is that every body emits electromagnetic radiation, some part of it being in infrared range depending on its temperature. If the sample is not in thermal equilibrium, there is an heat flux which pass trough it: since the thermal conductivity of mortar and bricks are different, the two phases have different temperatures which can be detected by an infrared sensor. Therefore, by means of thermography, it is possible to measure the infrared radiation emitted from every body without direct contact, so that it can be considered a non-destructive test technique.

A thermographic camera differs from a photographic camera on the wave lengths of the radiation which it can detect: the wave lengths of the radiation which can be measured by a thermographic camera are in the infrared range, between about 700–1 mm, which correspond to 430 THz–300 GHz in frequency, while the wave lengths of the radiation which can be measured by a photographic camera are in the visible range, between about 400–700 nm, which correspond to 190–430 GHz, see Fig. 1.

The image resolution is typically much smaller than that allowed by a photographic camera, and therefore reconstruction technique could be used to enhance the quality of the image [18, 19]. In the present paper, two thermocamera models have been used: the thermocamera model B360 from FLIR has been used for the quasi-periodic masonry sample and model 885-2 from Testo has been used for periodic and random masonry samples. Each of the model have a sensor of dimension 320×240 pixels. All the samples have been exposed to direct sunlight in order to improve the heat flux trough the body.

3 Building of samples

Three samples of masonry walls have been built; for all the samples, UNI bricks of dimensions 250×120×55 mm have been employed, either used as a whole or split in two or in four, as shown in Fig. 2. Mortar joint were made 1 mm in thickness.

In what follows, we denote width of the brick its horizontal length, height of the brick its vertical length. The bricks were assembled using three different textures:

![Radiation wave length range](image)

**Fig. 1** Radiation wave length range
The textures typologies of the masonry wall are shown in Fig. 3. For periodic and random textures, samples with 780×780 mm dimensions were built.

(i) a periodic texture, made with brick having the same widths and heights. In particular, a running bond texture has been used, where head joints of adjacent rows are half brick width apart;

(ii) a quasi-periodic texture, made with bricks having different widths but equal heights, arranged in horizontal rows in such a way to avoid the correspondence between vertical joints;

(iii) a random texture, made using bricks having different widths and heights (achieved by rotating some bricks).

The sample of quasi-periodic masonry has been built larger than the others, with 835×780 mm dimensions. In Fig. 3 the actual portion of samples is shown inside the dashed line.

The masonry samples, as built before the application of the plaster, are shown in Fig. 4.

As a final step, the samples have been covered with plaster, with 10 mm thickness, as can be seen in Fig. 5a for the quasi-periodic texture sample. An example of the image of the sample with quasi-periodic masonry obtained by the thermocamera is shown in Fig. 5b.

4 Homogenization of masonry

The mechanical characteristics of the masonry have been estimated as in [6]. The procedure is here recalled briefly. We are in a case of plain stress, and therefore

\[ \sigma_{xz} = \sigma_{xy} = \sigma_{yz} = 0 \]  

With the preceding, the relation between stress and strain can be expressed, using Voigt notation, with the following
The material is heterogeneous, the relations (2) and (5) are in terms of mean values averaged over the volume \( \Omega \) as follows

\[
\langle \sigma \rangle = C^H \langle \epsilon \rangle, \quad \langle \epsilon \rangle = S^H \langle \sigma \rangle
\]

where

\[
\langle \sigma \rangle = \frac{1}{|\Omega|} \int_{\Omega} \sigma \, d\Omega, \quad \langle \epsilon \rangle = \frac{1}{|\Omega|} \int_{\Omega} \epsilon \, d\Omega
\]

In (7), \( C^H \) is the effective stiffness matrix and \( S^H \) is the effective flexibility matrix. In general, if the domain is not the size of a representative volume element, the estimates of \( C^H \) and \( S^H \) depends on the applied boundary conditions [20].

Two type of boundary conditions are mainly used:

(i) essential boundary conditions, expressed in terms of displacements as

\[
\begin{align*}
\begin{cases}
ux &= \epsilon_{xx}^0 x + \epsilon_{xy}^0 y \\
u_y &= \epsilon_{xy}^0 x + \epsilon_{yy}^0 y
\end{cases}
\end{align*}
\]

where \((x, y)\) are the coordinates of the point on the boundary. By imposing \( \{\epsilon_{xx}^0, \epsilon_{yy}^0, 2\epsilon_{xy}^0\}^T = \{1, 0, 0\}^T \) we have \( \langle \epsilon \rangle = \{1, 0, 0\}^T \) \[21\] and therefore the first column of the estimate of \( C^H \) is given by \( \langle \sigma \rangle \). Proceeding in analogous way the other column of the estimate of \( C^H \) can be found. This estimate of \( C^H \) is denoted by \( C^E \).

(ii) natural boundary conditions, expressed in terms of forces as

\[
\begin{align*}
\begin{cases}
t_x &= \sigma_{xx}^0 n_x + \sigma_{xy}^0 n_y \\
t_y &= \sigma_{xy}^0 n_x + \sigma_{yy}^0 n_y
\end{cases}
\end{align*}
\]

where \((n_x, n_y)\) denotes the outward normal vector at the point on the boundary. By imposing \( \{\sigma_{xx}^0, \sigma_{yy}^0, \sigma_{xy}^0\}^T = \{1, 0, 0\}^T \) we have \( \langle \sigma \rangle = \{1, 0, 0\}^T \) \[21\] and therefore the first column of the estimate of \( C^H \) is given by \( \langle \epsilon \rangle \). Proceeding in analogous way the other column of the estimate of \( S^H \), and therefore the estimate of \( C^H \) as \( C^H = (S^H)^{-1} \) can be found. This estimate of \( C^H \) is denoted by \( C^N \)

It is possible to prove that \[22\]

\[
C^N \leq C^H \leq C^E
\]

In case that \( C^E \) and \( C^N \) are sufficiently close, \( C^H \) can be estimated through the following arithmetic mean:

\[
C^H = \frac{1}{2} (C^E + C^N)
\]

5 Identification of texture

The estimation of the texture from the photographic and the thermographic images follows the same steps.
In particular, the images are converted from color images to black and white images [19], in which black pixels identify mortar elements and white pixels identify brick elements. It is worth noting that, if necessary, the image was corrected in order to compensate the perspective.

An image can be seen as one or more discrete functions, $f_i(x, y)$, with $x = 1, 2, \ldots, N$ and $y = 1, 2, \ldots, N$, where $N$ is the image width in pixels (it is assumed that the image has equal width and height). In case of color image, there are three functions $f_i$, one for each channel (red, green and blue). In case of gray-scale image, there is only one function, corresponding to the gray level (usually expressed in the range 0–1).

At first the image is converted from color to gray-scale by eliminating hue and saturation information while retaining the luminance [23]. In particular, the following function is used

$$g(x, y) = 0.299f_R(x, y) + 0.587f_G(x, y) + 0.114f_B(x, y)$$  \hspace{1cm} (13)

where $g(x, y)$ is the gray level of the pixel at position $(x, y)$ and $\{f_R, f_G, f_B\}$ are the red, green and blue levels of the same pixel in the color image.

Thereafter a median filter to enhance the quality of image is applied. The median filter is defined by the following

$$\tilde{g}(x, y) = \text{median}\{g(s, t) \text{ for } (s, t) \text{ in } N^l_{(x,y)}\}$$  \hspace{1cm} (14)

where $N^l_{(x,y)}$ is a structural element, a square of size $l$ pixels centered in pixel at $(x, y)$. In the present case, $l = 3$.

The gray-scale image is converted to a black and white image by the following

$$b(x, y) = \begin{cases} 
0 & \text{if } \tilde{g}(x, y) \leq k \\
1 & \text{if } \tilde{g}(x, y) > k 
\end{cases}$$  \hspace{1cm} (15)

where $k$ is the gray level used as a threshold. In the current application, the value of $k$ is chosen using an adaptive approach [24] in order to compensate the effect of gradient of illumination present in the images. The value 1 (black) is associated with mortar pixels, the value 0 (white) with brick pixels.

As final steps, morphological operators are used. At first, mortar (black) region of pixels which are surrounded by brick (white) pixels are removed. Finally erosion and dilation operators are applied (in this succession) in order to smooth the contours of the inclusions.

In particular, erosion is defined by

$$b_e(x, y) = \text{maximum}\{b(s, t) \text{ for } (s, t) \text{ in } N^l_{(x,y)}\}$$  \hspace{1cm} (16)

and dilation by

$$b_d(x, y) = \text{minimum}\{b_e(s, t) \text{ for } (s, t) \text{ in } N^l_{(x,y)}\}$$  \hspace{1cm} (17)

The resulting black and white image has a consistent separation of phases, i.e. each stone is surrounded by mortar joints and unrealistic conjunction of inclusions is reduce as much as possible.

A portion with eight rows of stones and with a width equivalent to two entire bricks has been chosen, together with the corresponding head and bed joints, therefore the dimensions of the portion are 520 mm in width and 520 mm in height. The corresponding image size in pixel is 200×200.

In Figs. 6, 7 and 8, the obtained results for both the thermographic image and the photographic image of
the same portion of sample is shown. In particular, in the case of thermographic images two different black and white images image have been obtained, differing in the value of \( l \) used for dilation operator: \( l = 3 \) is the same used for erosion, \( l = 5 \) is slightly larger and used in order to obtain a concentration ratio, defined as the percentage of brick phase, closer to the one of photographic image.

Fig. 7 Identified texture of quasi-periodic masonry sample: photographic image image (a) and its identified texture (b), thermographic image (c) and its identified texture with \( l = 3 \) (d) and \( l = 5 \) (e).

Fig. 8 Identified texture of random masonry sample: photographic image image (a) and its identified texture (b), thermographic image (c) and its identified texture with \( l = 3 \) (d) and \( l = 5 \) (e).

Fig. 9 Finite element model of a portion with dimension 50×50 pixels: mesh, brick in red and mortar in green (a), applied forces at the boundary to estimate the first column of \( C^N \) (b) and corresponding stresses \( \sigma_{xx} \) (in MPa) (c). (Color figure online)

6 Numerical results

Two problems have to be considered in practical applications: first if the thermographic image can be assumed as RVE, second to assess the mechanical characteristics of masonry.

With regards to the first aspect, the image has been analyzed also considering its portions of increasing
dimensions; in particular since the image has dimensions $200 \times 200$, 16 portion of dimensions $50 \times 50$ and 4 portions of dimensions $100 \times 100$ have been taken into account. For each size, the portions do not overlap.

The mechanical characteristics used for the constituent phases are reported in Table 1.

In order to obtain the estimates of $C^E$ and $C^N$, the boundary value problems are solved through finite element method using 4-node elements with plane stress formulation, assuming that each pixel of the binary image is a finite element. This choice has been made to avoid a further application of erosion or dilation operators; if the finite element contains more pixels of different phases, then material of the element should be the prevalent one in order to have a mesh with elements of the same dimension.

A numerical example is shown in Fig. 9 for one of the portions of smaller size: the mesh is shown in (a), the forces applied at the boundary to estimate the first column of $C^N$ and the corresponding stresses $\sigma_{xx}$ (x being horizontal) are shown in (b) and (c) respectively.

In Fig. 10 are shown the mean and standard deviation of the norm of the stiffness matrices obtained in essential and natural boundary conditions and the estimate of the effective one for different dimensions of the portion as explained above. It can be observed that $C^H$ is quite independent from dimensions while the mean of the $C^E$ and $C^N$ is approaching the effective one quite quickly.

The results in terms of components of stiffness matrix relative to the whole sample are shown in Tables 2, 3 and 4.

In all of the cases the difference between the estimates in natural and essential boundary conditions are below the 3%, and therefore the portion of the sample is close to the representative volume element. As expected, the results show a dependency not only on the texture but also on the concentration ratio, $c_1$. The difference is about 25% for $l = 3$ and 12% for $l = 5$ in the case of periodic texture, 11% for $l = 3$ and 5% for $l = 5$ in the case of quasi-periodic texture, while is about 17% for $l = 3$ and 4% for $l = 5$ in the case of random texture.

The stiffness matrix or its inverse, namely the flexibility matrix, can be used to estimate the value of the engineering constants involved in orthotropic materials, $E_x$, $E_y$, $v_{xy}$ and $G_{xy}$ and their mutual relation.

In particular, in plane stress the flexibility matrix can be written as

$$
S = \begin{bmatrix}
\frac{1}{E_x} & -\frac{v_{xy}}{E_x} & 0 \\
-\frac{v_{xy}}{E_x} & \frac{1}{E_y} & 0 \\
0 & 0 & \frac{1}{G_{xy}}
\end{bmatrix}
$$

and the stiffness matrix can be written as

$$
C = \begin{bmatrix}
\frac{E_x^2}{E_x - E_y v_{xy}^2} & \frac{E_x E_y v_{xy}}{E_x - E_y v_{xy}^2} & 0 \\
\frac{E_x E_y v_{xy}}{E_x - E_y v_{xy}^2} & \frac{E_y}{E_x - E_y v_{xy}^2} & 0 \\
0 & 0 & G_{xy}
\end{bmatrix}
$$

The values of the engineering constants $E_x$, $E_y$ and $v_{xy}$ can be found by the following

$$
E_x = \frac{1}{S_{11}}, \quad E_y = \frac{1}{S_{22}}, \quad v_{xy} = -\frac{S_{12}}{S_{11}}
$$

The results are shown in Table 5. It can be seen that the differences between the values, obtained by photograhic and termographic images, are the largest in the case of periodic texture, while are the smallest in the case of random texture. This can be explained observing that, for periodic texture, an error in the phases identification easily corrupt the periodicity itself, while, in the case of the random texture, this is somehow mitigated.

Moreover, the following relation between the engineering constants can be established

$$
\frac{E_x}{E_y} = \frac{C_{11}}{C_{22}}, \quad v_{xy} = \frac{C_{12}}{C_{22}}, \quad G_{xy} = \frac{C_{33}}{C_{11}}
$$

and the results are show in Table 6.

As can be seen, the results in terms of ratio of engineering constants are in good agreement, since the effect of phase concentration is greatly reduced. Therefore, in terms of characterization of the
### Table 2  Concentration ratio (non-dimensional) and components of estimated stiffness matrix for periodic texture (in MPa)

|                | $c_1$ | $C_{11}$ | $C_{12}$ | $C_{22}$ | $C_{33}$ |
|----------------|------|---------|---------|---------|---------|
| Natural b.c.   |      |         |         |         |         |
| Photographic   | 0.684 | 13708   | 2225    | 11333   | 4417    |
| Therm. $l = 3$ | 0.482 | 10482   | 1770    | 8651    | 3433    |
| Therm. $l = 5$ | 0.592 | 12071   | 2022    | 9989    | 3948    |
| Essential b.c. |      |         |         |         |         |
| Photographic   | 0.684 | 14024   | 2243    | 11496   | 4539    |
| Therm. $l = 3$ | 0.482 | 10831   | 1794    | 8848    | 3533    |
| Therm. $l = 5$ | 0.592 | 12490   | 2051    | 10234   | 4080    |
| Effective      |      |         |         |         |         |
| Photographic   | 0.684 | 13866   | 2234    | 11415   | 4478    |
| Therm. $l = 3$ | 0.482 | 10657   | 1782    | 8749    | 3483    |
| Therm. $l = 5$ | 0.592 | 12280   | 2036    | 10111   | 4014    |

### Table 3  Concentration ratio (non-dimensional) and components of estimated stiffness matrix for quasi-periodic texture (in MPa)

|                | $c_1$ | $C_{11}$ | $C_{12}$ | $C_{22}$ | $C_{33}$ |
|----------------|------|---------|---------|---------|---------|
| Natural b.c.   |      |         |         |         |         |
| Photographic   | 0.620 | 12285   | 2050    | 10425   | 4044    |
| Therm. $l = 3$ | 0.538 | 10803   | 1928    | 9446    | 3709    |
| Therm. $l = 5$ | 0.646 | 12468   | 2209    | 11053   | 4304    |
| Essential b.c. |      |         |         |         |         |
| Photographic   | 0.620 | 12606   | 2064    | 10597   | 4163    |
| Therm. $l = 3$ | 0.538 | 11173   | 1944    | 9692    | 3846    |
| Therm. $l = 5$ | 0.646 | 12837   | 2231    | 11308   | 4455    |
| Effective      |      |         |         |         |         |
| Photographic   | 0.620 | 12446   | 2057    | 10511   | 4103    |
| Therm. $l = 3$ | 0.538 | 10988   | 1936    | 9569    | 3778    |
| Therm. $l = 5$ | 0.646 | 12653   | 2220    | 11181   | 4379    |

### Table 4  Concentration ratio (non-dimensional) and components of estimated stiffness matrix for random texture (in MPa)

|                | $c_1$ | $C_{11}$ | $C_{12}$ | $C_{22}$ | $C_{33}$ |
|----------------|------|---------|---------|---------|---------|
| Natural b.c.   |      |         |         |         |         |
| Photographic   | 0.642 | 12128   | 2076    | 11215   | 4154    |
| Therm. $l = 3$ | 0.496 | 9878    | 1788    | 9289    | 3485    |
| Therm. $l = 5$ | 0.610 | 11481   | 2042    | 10831   | 4037    |
| Essential b.c. |      |         |         |         |         |
| Photographic   | 0.642 | 12355   | 2085    | 11404   | 4262    |
| Therm. $l = 3$ | 0.496 | 10137   | 1796    | 9503    | 3575    |
| Therm. $l = 5$ | 0.610 | 11778   | 2057    | 11085   | 4151    |
| Effective      |      |         |         |         |         |
| Photographic   | 0.642 | 12241   | 2080    | 11310   | 4208    |
| Therm. $l = 3$ | 0.496 | 10008   | 1792    | 9396    | 3530    |
| Therm. $l = 5$ | 0.610 | 11629   | 2049    | 10958   | 4094    |
orthotropic behavior the estimates, obtained by thermographic images, can be considered quite reliable.

7 Conclusions

In the present paper we present the first results of an ongoing research relative to estimation, using nondestructive techniques, of the mechanical characteristics of masonry walls covered with plaster. The method is based on thermographic images of the masonry, to which digital image techniques are applied in order to obtain a black and white image with a consistent separation of the phases (brick and mortar in the present case study). Then, the mechanical characteristics are estimated by means of an homogenization procedure based on natural and essential boundary conditions. These mechanical characteristics have been compared with those obtained using photographic images of the same samples with a good agreement; then the obtained results highlight the reliability of thermographic procedure. As a further development, it is planned that the research will continue with laboratory tests on the masonry walls samples described in the present paper to experimentally estimate their elastic characteristics, in order to provide an additional validation of the proposed procedure. Furthermore, several tests will be performed in order to evaluate the most adequate heating conditions and the influence of the plaster thickness on

| Table 5 | Concentration ratio (non-dimensional) and estimated value of engineering constants ($E_x$, $E_y$ and $G_{xy}$ in MPa) |
|---------|-------------------------------------------------|
|         | $c_1$    | $E_x$  | $E_y$  | $v_{xy}$ | $G_{xy}$ |
| **Periodic** |
| Photographic | 0.684 | 13428 | 11055 | 0.196 | 4478 |
| Therm. $l = 3$ | 0.482 | 10293 | 8452 | 0.204 | 3483 |
| Therm. $l = 5$ | 0.592 | 11869 | 9774 | 0.201 | 4014 |
| **Quasi-periodic** |
| Photographic | 0.620 | 12043 | 10171 | 0.196 | 4103 |
| Therm. $l = 3$ | 0.538 | 10596 | 9228 | 0.202 | 3778 |
| Therm. $l = 5$ | 0.646 | 12212 | 10791 | 0.199 | 4379 |
| **Random** |
| Photographic | 0.642 | 11859 | 10956 | 0.184 | 4208 |
| Therm. $l = 3$ | 0.496 | 9666 | 9075 | 0.191 | 3530 |
| Therm. $l = 5$ | 0.610 | 11246 | 10597 | 0.187 | 4094 |

| Table 6 | Relation between engineering constants |
|---------|----------------------------------------|
|         | $c_1$  | $E_x/E_y$ | $v_{xy}$ | $G_{xy}/E_x$ |
| **Periodic** |
| Photographic | 0.684 | 1.215 | 0.196 | 0.323 |
| Therm. $l = 3$ | 0.482 | 1.218 | 0.204 | 0.327 |
| Therm. $l = 5$ | 0.592 | 1.214 | 0.201 | 0.327 |
| **Quasi-periodic** |
| Photographic | 0.620 | 1.184 | 0.196 | 0.330 |
| Therm. $l = 3$ | 0.538 | 1.148 | 0.202 | 0.344 |
| Therm. $l = 5$ | 0.646 | 1.132 | 0.199 | 0.346 |
| **Random** |
| Photographic | 0.642 | 1.082 | 0.184 | 0.344 |
| Therm. $l = 3$ | 0.496 | 1.065 | 0.191 | 0.353 |
| Therm. $l = 5$ | 0.610 | 1.061 | 0.187 | 0.352 |
masonry texture identification, in view of practical applications. The proposed procedure appears very valuable for single leaf masonry wall, nevertheless the possibility to extend its applicability to multi leaves masonries (for instance using GPR measurements together thermography) may represent a new interesting research frontier.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig10}
\caption{Estimates of mean and standard deviation of $C^N$, $C^E$ and $C^H$ at increasing dimensions of portion analyzed (random texture)}
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