Role of the non-resonant background in the $\rho^0$-meson diffractive electro- and photoproduction.

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Abstract

The background due to the direct diffractive dissociation of the photon into the $\pi^+\pi^-$-pair to the "elastic" diffractive $\rho^0$-meson production in electron-proton collisions is calculated. The amplitude for the background process $\gamma p \rightarrow \pi^+\pi^-p$ is proportional to the $\pi$-meson - proton cross section. Therefore, describing the HERA data, we can estimate $\sigma(\pi p)$ at energy $s_{\pi p} \sim (2 - 3) \cdot 10^3$ GeV$^2$ that is considerably higher the existing data. At large $Q^2$ the interference between resonant and non-resonant $\pi^+\pi^-$ production leads also to the more slow increase of the $\sigma_L/\sigma_T$ ratio with the mass of the $2\pi$ (i.e. $\rho^0$-meson) state.

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1 Introduction

It was noted many years ago that the form of the $\rho$-meson peak is distorted by the interference between resonant and non-resonant $\pi^+\pi^-$ production. For the case of "elastic" $\rho^0$ photoproduction the effect was studied by P. Söding in [1] and S. Drell [2] (who considered the possibility to produce the pion beam via the $\gamma \rightarrow \pi^+\pi^-$ process). At high energies the main (and the only) source of background is the Drell-Hiida-Deck process [3] (see fig. 1). The incoming photon fluctuates into the pion pair and then $\pi p$-elastic scattering takes place. Thus the amplitude for the background may be written in terms of the pion-proton cross section. Recently the diffractive elastic production of $\rho^0$-mesons was measured at HERA [4, 5, 6, 7] both for the cases of photoproduction i.e. $Q^2 = 0$ and of $Q^2 \geq 4 \text{ GeV}^2$ (the so called deep inelastic scattering, DIS, regime). It was demonstrated [4, 6] that the interference with some non-resonant background is indeed needed to describe the distribution over the mass $M$ of $\pi^+\pi^-$ pair.

It was proposed by M. Arneodo that this effect can be used to estimate the value of $\sigma_{\pi p}$ from HERA data at high energies [9] ($\sqrt{s} = W \sim 40 - 55 \text{ GeV}$), in the range which is not otherwise acceptable.

In Sect. 2 the formulae for the $2\pi$ background which are valid for the DIS as well as for the photoproduction region are presented. The expression differs slightly from the Söding’s one as we take into account the pion form factor and the fact that one pion propagator is off-mass shell. We consider also the absorption correction coming from the diagram where both pions ($\pi^+$ and $\pi^-$) directly interact with the target proton. The role of the interference in photoproduction is discussed in Sect. 3, where we estimate the $\pi$-proton cross section $\sigma_{\pi p} \simeq 30 \text{ mb}$ at $\sqrt{s} \sim 50 \text{ GeV}$. Finally in Sect. 4 we compute the amplitude for a pion pair production in DIS. At large $Q^2 \sim 10 - 30 \text{ GeV}^2$ the background amplitude becomes relatively small, but still not negligible. It changes the ratio of the longitudinal to transverse $\rho$-meson production cross section and leads to the more slow increase of $R = \sigma^L/\sigma^T$ with $M^2$. 


2 Production amplitudes

The cross section of $\rho^0$ photo- and electroproduction may be written as:

$$\frac{d\sigma^D}{dM^2 dt} = \int d\Omega |A_\rho + A_{n.r.}|^2,$$

where $A_\rho$ and $A_{n.r.}$ are the resonant and non-resonant parts of the production amplitude, $D = L, T$ for longitudinal and transverse photons, $t = -q^2$ is the momentum transferred to the proton and $d\Omega = d\phi d\cos(\theta)$, where $\phi$ and $\theta$ are the azimuthal and polar angles between the $\pi^+$ and the proton direction in the $2\pi$ rest frame.

2.1 Amplitude for resonant production

The dynamics of vector meson photo- and electroproduction was discussed in the framework of QCD in many papers (see, e.g. [10-13]). However here we will use the simple phenomenological parametrization of the production amplitude because our main aim is the discussion of the interference between resonant and non-resonant contributions. So the amplitude for resonant process $\gamma p \rightarrow \rho^0 p; \rho^0 \rightarrow \pi^+ \pi^-$ reads:

$$A_\rho = \sqrt{\sigma_\rho} e^{-b_\rho q^2 t/2} \frac{\sqrt{M_0 \Gamma}}{M^2 - M_0^2 + iM_0 \Gamma} \frac{H^D(\theta, \phi)}{\sqrt{\pi}}.$$

To take into account the phase space available for the $\rho \rightarrow \pi^+ \pi^-$ decay we use the width $\Gamma = \Gamma_0 \left(\frac{M^2 - 4m^2}{M_0^2 - 4m^2}\right)^{3/2}$ (with $\Gamma_0 = 151$ MeV and $M_0 = 768$ MeV – its mass); $b_\rho$ is the $t$-slope of the ”elastic” $\rho$ production cross section $\sigma_\rho \equiv d\sigma(\gamma p \rightarrow \rho^0 p)/dt$ (at $t = 0$) and the functions $H^D(\theta, \phi)$, $D = T, L$ describe the angular distribution of the pions produced through the $\rho$-meson decay:

$$H^L = \sqrt{\frac{3}{4\pi}} \cos \theta,$$

$$H^T = \sqrt{\frac{3}{8\pi}} \sin \theta \cdot e^{\pm i\phi}.$$

Note that for transverse photons with polarization vector $\vec{e}$ one has to replace the last factor $e^{\pm i\phi}$ in eq. (4) by the scalar product $(\vec{e} \cdot \vec{n})$, where $\vec{n}$ is the unit vector in the pion transverse momentum direction.
2.2 Amplitude for non-resonant production

The amplitude for the non-resonant process $\gamma p \rightarrow \pi^+ \pi^- p$ is:

$$A_{n.r.} = \sigma_{\pi p} F_\pi(Q^2)e^{bt/2} \sqrt{\alpha} \sqrt{16\pi^3} B^D \sqrt{z(1-z)} \left| \frac{dz}{dM^2} \left( \frac{M^2}{4} - m^2_\pi \right) \right| \cos\theta,$$

where $b$ is the $t$-slope of the elastic $\pi p$ cross section, $F_\pi(Q^2)$ is the pion electromagnetic form factor ($Q^2 = |Q^2_\gamma| > 0$ is the virtuality of the incoming photon), $\alpha = 1/137$ is the electromagnetic coupling constant and $z$ – the photon momentum fraction carried by the $\pi^-$-meson; $\sigma_{\pi p}$ is the total pion-proton cross section.

The factor $B^D$ is equal to

$$B^D = \frac{(e^D_\mu \cdot k_{\mu-}) f(k_-^2)}{z(1-z)Q^2 + m^2_\pi + k^2_-} - \frac{(e^D_\mu \cdot k_{\mu+}) f(k_+^2)}{z(1-z)Q^2 + m^2_\pi + k^2_+}$$

For longitudinal photons the products $(e^L_\mu \cdot k_{\mu\pm})$ are: $(e^L_\mu \cdot k_{\mu-}) = z\sqrt{Q^2}$ and $(e^L_\mu \cdot k_{\mu+}) = (1-z)\sqrt{Q^2}$, while for the transverse photons we may put (after averaging) $e^T_\mu \cdot e^T_\nu = \frac{1}{2}\delta^T_{\mu\nu}$.

Expressions (5) and (6) are the result of straightforward calculation of the Feynman diagram fig. 1. The first term in (6) comes from the graph fig. 1 (in which the Pomeron couples to the $\pi^+$) and the second one reflects the contribution originated by the $\pi^-p$ interaction. The negative sign of $\pi^-$ electric charge leads to the minus sign of the second term. We omit here the phases of the amplitudes. In fact, the common phase is inessential for the cross section, and we assume that the relative phase between $A_{\rho}$ and $A_{n.r.}$ is small (equal to zero) as in both cases the phase is generated by the same 'Pomeron' exchange.

The form factor $f(k^2)$ is written to account for the virtuality ($k^2 \neq m^2_\pi$) of the t-channel (vertical in fig. 1) pion. As in fig. 1 we do not deal with pure elastic pion-proton scattering, the amplitude may be slightly suppressed by the fact that the incoming pion is off-mass shell. To estimate this suppression we include the form factor (chosen in the pole form)

$$f(k^2) = 1/(1 + k^2/m^2)$$

Better to say – 'vacuum singularity'.

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The same pole form was used for $F_2(Q^2) = 1/(1 + Q^2/m^2_\rho)$. In the last case the parameter $m_\rho = M_0$ is the mass of the $\rho$-meson – the first resonance on the $\rho$-meson (i.e. photon) Regge trajectory, but the value of $m'$ (in $f(k'^2)$) is expected to be larger. It should be of the order of mass of the next resonance from the Regge $\pi$-meson trajectory; i.e. it should be the mass of $\pi(1300)$ or $b_1(1235)$. Thus we put $m'^2 = 1.5 \text{ GeV}^2$.

Finally we have to define $k'^2_\pm$ and $k_t^\pm$.

$$k'^2_- = -(\vec{K}_t + z\vec{q}_t)$$  
$$k'^2_+ = \vec{K}_t + (1 - z)\vec{q}_t$$  

and

$$k'^2_+ = \frac{z(1 - z)Q^2 + m^2_\pi + k'^2_+}{1 - z}, \quad k'^2_- = \frac{z(1 - z)Q^2 + m^2_\pi + k'^2_-}{z(1 - z)}.$$  

In these notations

$$M^2 = \frac{K^2_t + m^2_\pi}{1 - z}, \quad dM^2/dz = (2z - 1)\frac{K^2_t + m^2_\pi}{z^2(1 - z)^2}$$

and $z = \frac{1}{2} \pm \sqrt{1/4 - (K^2_t + m^2_\pi)/M^2}$ with the pion transverse (with respect to the proton direction) momentum $\vec{K}_t$ (in the $2\pi$ rest frame) given by expression $K^2_t = (M^2/4 - m^2_\pi)\sin^2\theta$. Note that the positive values of $\cos\theta$ correspond to $z \geq 1/2$ while the negative ones $\cos\theta < 0$ correspond to $z \leq 1/2$.

### 2.3 Absorptive correction

To account for the screening correction we have to consider the diagram fig. 2, where both pions interact directly with the target. Note that all the rescatterings of one pion (say $\pi^+$ in fig. 1) are already included into the $\pi p$ elastic amplitude. The result may be written in form of eq. (5) with the new factor $\tilde{B}^D$ instead of the old one $B^D = B^D(\vec{K}_t, \vec{q})$:

$$\tilde{B}^D = B^D(\vec{K}_t, \vec{q}) - \int C\frac{\sigma_{\pi p e^{-bl^2}}}{16\pi^2}B^D(\vec{K}_t - z\vec{l}_t, \vec{q})d^2l_t$$

where the second term is the absorptive correction (fig. 2) and $l_\mu$ is the momentum transferred along the 'Pomeron' loop. The factor $C > 1$ reflects the contribution of the enhancement graphs with the diffractive exitation of the target proton in intermediate state. In accordance with the HERA data \[8\], where the cross section of "inelastic" (i.e. with the proton diffracted) $\rho$ photoproduction was estimated as $\sigma^{inel} \simeq 0.5\sigma^{el}$ we choose $C = 1.5 \pm 0.2$. 

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3 Photoproduction cross section

The results at $Q^2 = 0$ (photoproduction) are shown in fig.3. Both slope parameters, $b_\rho$ in eq.(2) and $b$ in eq.(5) were assumed to be equal to 10 GeV$^{-2}$. The solid curve corresponds to the resonant production contribution. It is normalized to the experimental ZEUS data \cite{4} near the $\rho$ peak.\footnote{The H1 Coll. cross section \cite{6} near the $\rho$ peak is about 1.5 times smaller than ZEUS Coll. cross section \cite{4}, so we can not describe both experiments simultaneously.} The contributions of non-resonant production and its interference with resonant one depend on the form factor $f(k'^2)$, the screening corrections and the value of $\sigma_{\pi p}$. To demonstrate their role we present in fig.3 six variants of non-resonant (dashed curves) and interference (dotted curves) contributions. The upper dashed curve at the right-hand-side of fig.3 corresponds to the simplest calculation for $f(k'^2) \equiv 1$ and without screening correction. Second (to the down) dashed curve shows the same calculation but with screening correction. Here the value of $\sigma_{\pi p} = 28$ mb was used. Third and fourth curves show the results of calculations with form factor $f(k'^2)$, eq.(7), and $\sigma_{\pi p} = 30$ mb, without and with screening corrections, respectively. Fifth (which is practically coincide with the fourth one) and sixth curves correspond to the calculations with form factor $f(k'^2)$, eq.(7), $\sigma_{\pi p} = 28$ mb, without and with screening corrections.

For interference contribution the upper curve at left-hand-side of fig.3 presents the case $f(k'^2) \equiv 1$, $\sigma_{\pi p} = 28$ mb and without screening correction. Next two curves correspond to the calculations with form factor eq.(7), without screening correction and for $\sigma_{\pi p} = 30$ mb and $\sigma_{\pi p} = 28$ mb, respectively. The fourth curve shows the variant with $f(k'^2) \equiv 1$, $\sigma_{\pi p} = 28$ mb and with screening correction. Fifth and sixth curves correspond to the calculations with form factor $f(k'^2)$, eq.(7), with screening correction and for $\sigma_{\pi p} = 30$ mb and $\sigma_{\pi p} = 28$ mb, respectively.

The sum of all three contributions for the same six variants for background and interference terms are presented in fig.4. Solid and dashed curves here show the calculations without and with screening correction, respectively. The difference between the curves is as a rule smaller than the experimental errors. One can see quite reasonable agreement with ZEUS experimental data \cite{4} and some difference between the considered variants that can allow one to choose the best variant if the accuracy of the data will increase. It
is necessary to note that the background and interference contribution have the same sign at $M_{\pi^+\pi^-} < M_\rho$ and different signs at $M_{\pi^+\pi^-} > M_\rho$ where these contributions cancel each other in part. So the last region is lesser sensitive to the value of $\sigma_{\pi p}$. The to-day HERA data [4] are collected from the range of $\gamma p$ energy $W = 60 - 80$ GeV. Mean energy of a pion, produced directly by the photon, is $E_\pi \sim E_\gamma/2$. Thus the cross section $\sigma_{\pi p}$ corresponds to $s_{\pi p} \sim W^2/2 \sim (1.8 - 3.2) \times 10^3$ GeV$^2$ and one can see that cross section $\sigma_{\pi p} = 28 \div 30$ mb (which is extrapolation by hand from lower energy region) is in more or less reasonably agreement with the data. Of course, more accurate analysis is needed for the extraction of $\sigma_{\pi p}$ with error bars.

4 Electroproduction cross section

At very large $Q^2$ the background amplitude (5) becomes negligible as, even without the additional form factor (i.e. at $f(k^2) \equiv 1$), the non-resonance cross section falls down as $1/Q^8$ [4] while experimentally [5, 7] the $Q^2$ dependence of the elastic $\rho$ cross sections has been found to be $1/Q^n$, with $n \sim 5 (< 8!)$. Nevertheless, numerically at $Q^2 \sim 10$ GeV$^2$ the background as well as interference contributions are still important. The results of our calculations of $(Q^2 + M_\rho^2)^2 d\sigma/dM$ with $\sigma_{\pi p} = 28$ mb, with form factor, eq.(7), and with screening correction are presented in fig.5. Dashed curve show the case of $\pi^+\pi^-$ pair photoproduction, four solid curves correspond to the electro production at $Q^2 = 1, 4, 10$ and $30$ GeV$^2$, the dotted curve to the value $Q^2 = 30$ GeV$^2$ and with $f(k^2) \equiv 1$. The dash-dotted curve corresponds to only resonant contribution at $Q^2 = 30$ GeV$^2$ and with form factor. This value of $(Q^2 + M_\rho^2)^2 d\sigma/dM$ should be independent on $Q^2$ in the case of pure $\rho$-dominant model. Really it has some slight $Q^2$ dependence.

The background as well as interference contributions lead to the nontrivial behaviour of the ratio $R = \sigma^L/\sigma^T$ with the two pion mass $M_{2\pi} = M$. In the theoretical formulae which we used the index $D = L, T$ denotes the polarization of the incoming photon. On the other hand experimentally one measures the $\rho$-meson polarization, fitting the angular distribution of decay

\footnote{In the amplitude $A_{n,r}$ one factor $1/Q^2$ comes from the electromagnetic form factor $F_n(Q^2)$ and another one – from the pion propagator (term $- z(1-z)Q^2$ in the denominator of $B^D$ (see eq.(6)).}
pions. To reproduce the procedure we take the flows of initial longitudinal and transverse photons to be equal to each other ($\epsilon = N^L/N^T = 1$, which is close to HERA case) and reanalyse the sum of cross sections ($\sigma = \sigma^L + \sigma^T$) in a usual way, selecting the constant and the $\cos^2\theta$ parts.

$$I_0 = \int_{-1}^{1} \sigma(\theta) d\cos\theta; \quad I_2 = \int_{-1}^{1} \sigma(\theta) \frac{15}{4} (3\cos^2\theta - 1) d\cos\theta$$

In these terms the density matrix element $r_{00} = (2I_2 + 3I_0)/9I_0$ and

$$R = \frac{\sigma^L}{\sigma^T} = \frac{r_{00}}{1 - r_{00}}$$

(11)

Namely these last ratios (12) are presented in fig.6 for different $Q^2$ values. One can see that they depend both on $Q^2$ and $M$. These ratios increase with $M$ for $M > M_\rho$ and this increase becomes more weak for large $Q^2$.

5 Conclusion

We presented simple formulae for the background to 'elastic' $\rho$-meson photoproduction which account for the absorptive correction and virtuality of the pion. The role of $\rho$-meson – background interference is not negligible even at $Q^2 \sim 10$ GeV$^2$, especially for the $\sigma^L/\sigma^T$ ratio. A reasonable description of the ZEUS photoproduction data is obtained with a value of the $\pi p$ total cross section $\sigma_{\pi p} = 28 \div 30$ mb at energy $s_{\pi p} \sim (2 - 3) \cdot 10^3$ GeV$^2$. We consider this as an indication to the growth of $\pi p$ cross section with energy in the energy region higher than the region of existing direct measurements.

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Figure captions

Fig. 1. Feynman diagram for the two pion photo-(electro)production.

Fig. 2. Diagram for the absorbtive correction due to both pions rescat-tering.

Fig. 3. Resonant (Breit-Wigner, solid curve), non-resonant background (dashed curves) and their interference (dotted curves) contributions to $\gamma p \rightarrow \pi^+\pi^- p$ reaction. The two last contributions are calculated with six different assumptions, see text.

Fig. 4. Distribution over the mass of two pions in photoproduction. Solid and dashed curves correspond to the calculations without and with screening correction, respectively. ZEUS Coll. data points [4] are presented also.

Fig. 5. $\pi^+\pi^-$ mass distribution in photo- (dashed curve 1) and electroproduction at $Q^2 = 1$ GeV$^2$ (solid curve 2), 4 GeV$^2$ (curve 3), 10 GeV$^2$ (curve 4) and 30 GeV$^2$ (curves 5) with (solid curve) and without (dotted curve) form factor. The only resonant contribution at $Q^2 = 30$ GeV$^2$ is shown by dash-dotted curve.

Fig. 6. The ratio $R = \sigma^L/\sigma^T$ in electroproduction process as a function of pion pair mass at $Q^2 = 1$ GeV$^2$ (curve 1), 10 GeV$^2$ (curve 2) and 30 GeV$^2$ (curves 3) with (solid curve) and without (dashed curve) form factor.
Fig. 1

Fig. 2
Fig. 3

$Q^2 = 0$
Fig. 4
Fig. 5
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