The contributions of $qqqq\bar{q}$ components to the axial charges of proton and its resonances

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Abstract. In this paper we calculate the axial charges of the proton and its resonances in the framework of the constituent quark model, which is extended to include the $qqqq\bar{q}$ components. If $20\%$ admixtures of the $qqqq\bar{q}$ components in the proton are assumed, the theoretical value for the axial charge in our model is in good agreement with the empirical value, which can not be well reproduced in the traditional constituent quark model even though the $SU(6)\otimes O(3)$ symmetry breaking or relativistic effect is taken into account. We also predict an unity axial charge for $N^*(1440)$ with $30\%$ $qqqq\bar{q}$ components constrained by the strong and electromagnetic decays.

PACS. 12.39.Jh Nonrelativistic quark model – 14.20.Gk Baryon resonances with S=0

1 Introduction

Axial charges of the nucleon and nucleon resonances are fundamental quantities in QCD. They quantify spontaneous chiral symmetry breaking in the low energy QCD. The proton axial charge is well known from neutron beta decay experiment, $g_A=1.2670 \pm 0.0030$ (in units of vector charge $g_V$) \cite{1}. The conventional quark model in the $SU(6)\otimes O(3)$ symmetry scheme predicts that axial charge of the proton is exactly $5/3$, which is about $24\%$ larger than experimental value as given above. The diagonal axial charges of nucleon resonances can not be measured experimentally, but they can be calculated on the lattice QCD. Recently the lattice results show that $N^*(1535)$ has a very small axial charge and the axial charge of $N^*(1650)$ is about $0.55$ \cite{2}, which agree with constituent quark model (CQM) values under the $SU(6)\otimes O(3)$, 1/9 and 5/9, respectively. These values imply that the mixing between the two negative states is small, which conflicts with the popular view that the mixings are required by the data under very general assumptions about the $SU(6)$ symmetries of the decay amplitudes, differential cross section and other observables \cite{3}.

Recently Zou and Riska et.al. suggested that many puzzles surrounding baryon resonances in the CQM may be solved by extending 3-quark wave function to include multiquark configuration $qqqq\bar{q}$ \cite{4,5,6,7,8}. After introducing the five-quark components, an interesting question is how the amplitudes of the $qqq$ and $qqqq\bar{q}$ configurations are determined. Some authors have discussed $\Delta(1232)$, $N^*(1440)$ and $N^*(1535)$ electromagnetic and strong decay processes which provide constraints to the amplitudes of the $qqq$ and $qqqq\bar{q}$ configurations, but only give a wide range of probability of five-quark components, one reason of which is that these observables involve several adjustable parameters: constituent quark mass, harmonic oscillator strengths $\omega_3$ and $\omega_5$ of three quark and five quark components contained in the spatial wave function.

However, the nonrelativistic axial charge operator is independent of spatial part and given by $\hat{g}_A = \sum_i q_i^a \tau_i^a$. The axial charge can be approximately expressed as a sum of the diagonal matrix elements of all possible configurations, $g_A=C P_3+\sum_i C_i P_i$ \cite{9}, which is a sum of axial charge $C_i$ of each single configuration multiply by corresponding possibility $P_i$. Obviously axial charge is independent of adjustable parameters mentioned above appearing in the spatial coordinate of transition operator. In addition, another parameter, i.e., the relative phase factor $\delta$ between the $qqq$ and $qqqq\bar{q}$ components is not involed also. So it is easy and reliable to obtain the relation between the probability of the two components.

The present paper is organized as follows. In section 2, the wave function of nucleon and nucleon resonances will be...
be presented. In section 3, the axial charges of the proton and nucleon resonances will be calculated in the extended five-quark model. Conclusions and discussions are given in the Sec.4.

2 The five-quark wave functions of nucleon and its resonances

For the proton and nucleon resonances, the extended wave functions can be written as [8],

\[ |p, N^+(1440, 1710), s_z \rangle = A(p)_{3q}|qqq\rangle + A(p)_{5q} \times \sum_i A_i|qqqqq_i\rangle, \]
\[ |N^+(1535, 1650), s_z \rangle = A(N^+)_{3q}|qqq\rangle + A(N^+)_{5q} \times \sum_i A_i|qqqqq_i\rangle, \]

(1)

where \( A(p, N^+)_{3q} \) and \( A(p, N^+)_{5q} \) are the amplitudes factors for the 3-quark and 5-quark components, respectively. The sum over \( i \) runs over all the possible \( qqqq_i \) components, i.e., \( qqq\bar{u}, qqq\bar{d} \) and \( qqq\bar{s} \).

Here flavor-spin hyperfine interaction between quarks is assumed as [11],

\[ C_{FS} = - \sum_{ij} \lambda_i^F \cdot \lambda_j^F \sigma_i \cdot \sigma_j, \]

(2)

General wave function expressions in this spin-flavor scheme for \( qqq\bar{q} \) with positive parities are taken as

\[ |p, N^+(1440, 1710), s_z \rangle^{(i)} = \sum_{a,b,c,d,e,M,m,s,s_z} C_{JM,bs} C_{M,ss} C_{11}^{(i)} C_{11}^{(i)} C_{PA} \times \sum_{[211]} \langle O(a)[31]_{x,m}(b)[F]d(S)|S_z \rangle \bar{\xi}_{y,t} \xi_{ss} \psi(\kappa_i)(3) \]

where we propose that orbital state of four-quark subsystem is in the first excited state \( L = 1 \), whose Weyl tableaux is \([31]_{x,i}\). \( i \) is the number of the \( qqq\bar{q} \) configuration, as given in Table 1. \( \bar{\xi}_{y,t} \) and \( \xi_{ss} \) represent the wave function of flavor and the spin space of the antiquark respectively, and \( \psi(\kappa_i) \) represents the orbital symmetric function of the momentums of the five-quark component.

The first summation involves the \( S_2 \) Clebsch-Gordan coefficients for the indicated color \([211]\), orbital space \([31]\) and flavor \([F] \) and spin \([S] \) of the \( qqq\bar{q} \) system, and the second one runs over the spin indices in the standard \( SU(2) \) Clebsch-Gordan coefficient.

Similarly five-quark wave function of negative parity resonance with total angular momentum \( \frac{1}{2} \),

\[ |N^+(1535, 1650), t, s_z \rangle^{(i)} = \sum_{a,b,c,d,e,M,m,s,s_z} \sum_{Y,y,T,T_z} \sum_{S_z,s_z} \sum_{s,\bar{s}z} C_{[11]}^{[11]} \times \langle Y,T,t_z,y,t_{1/2},1/2,t(S,S_z,1/2,s,\bar{s}z,1/2,\bar{s}) \rangle \bar{\xi}_{y,t} \xi_{ss} \times \phi[5], \]

(4)

where we assume that orbital state of four-quark subsystem is in the ground state \( L=0 \), whose Weyl tableaux is \([4]_{x} \). So orbital wave function \( \phi[5] \) is completely symmetric.

These wave functions are given in explicit form in Ref. [8]. Now, we can order these configurations in the terms of increasing matrix element \( \langle F \rangle \) of chiral hyperfine interaction \((2) [11] \). The \( qqq\bar{q} \) configurations having appropriate quantum numbers in the proton, \( N^+(1440) \) and \( N^+(1710) \), and the corresponding axial charge coefficients \( C_{i} \) are listed in Table 1 and for the \( N(1650) \) and \( N(1535) \), the results can be found in Ref.[10].

3 The axial charges of the proton and its resonances with the extended five-quark wave function

It is straightforward to calculate the diagonal charges in the \( SU(6) \otimes O(3) \) symmetry scheme [9]. Then we consider the mixings between nonstrange baryons due to symmetry breaking and find that the prediction is not well improved for proton. The mixing coefficients for proton and \( N^+(1440) \) are taken from Ref.[3]. Without configuration mixings axial charges is given in the first row and with mixings axial charges given in the third row in Table 2. In the last row, the values are obtained in the extended quark model.

Table 1. The \( qqq\bar{q} \) configurations having appropriate quantum numbers in the proton, \( N^+(1440) \) and \( N^+(1710) \), and the corresponding coefficients \( C_i \) in the axial charge expression.

| configuration | flavor-spin | \( \langle C_{FS} \rangle \) | \( C_i \) |
|--------------|-------------|-----------------|---------|
| 1            | [4]_{FS}[22]_{FS}[22]_{FS} | -28             | -2/9    |
| 2            | [4]_{FS}[31]_{FS}[31]_{FS} | -64/3           | -4/15 \( J_{q^+} = 0 \) |
| 3            | [4]_{FS}[31]_{FS}[31]_{FS} | -64/3           | 28/45 \( J_{q^+} = 1 \) |
| 4            | [31]_{FS}[211]_{FS}[22]_{FS} | -16             | 0       |
| 5            | [31]_{FS}[211]_{FS}[31]_{FS} | -40/3           | 0 \( J_{q^+} = 0 \) |
| 6            | [31]_{FS}[211]_{FS}[31]_{FS} | -40/3           | 4/9 \( J_{q^+} = 1 \) |
| 7            | [31]_{FS}[22]_{FS}[31]_{FS} | -28/3           | +17/18  |

Table 2. Axial charges of the proton, \( N^+(1440) \) and \( N^+(1710) \) in the both ummixing and mixing cases.

|                | \( g_A(\text{proton}) \) | \( g_A(N(1440)) \) | \( g_A(N(1710)) \) |
|----------------|------------------------|-------------------|--------------------|
| Unmixing       | 5/3                    | 5/3               | 1/3                |
| Mixing         | 1.60                   | 1.67              | -                  |
| ECQM           | 1.25 \sim 1.46         | \sim 1            | -                  |

In the extended constituent quark model (ECQM), with the values in Table 1, the diagonal axial charge reads

\[ g_A(N(938, 1440)) = \frac{5}{3} p_3 - \frac{2}{9} p_5^{(1)} - \frac{4}{15} p_5^{(2)} J_{q^+} = 0 + \frac{28}{45} + \frac{4}{9} p_5^{(3)} J_{q^+} = 1, \]

(5)
where $P_3$ represents the probability for the $qqq$ configuration, while $P_5^{(i)}$ is the probability for $i$-th $qqqq$ configuration in Table 1.

The energy of the configuration with the spin-flavor symmetry $[4]_F S [22]_F [22]_S$ is about 140-200 MeV lower than others within the hyperfine interaction model of Eq. (2) and simultaneously can give the experimentally observed positive strangeness magnetic moment and strange electric radii [12], and negative strange electric form factor [13]. If only the first two terms in Eq. (5) are taken into consideration, $g_A(N(938))$ would be equal to the experimental value 1.26 with $P_5^{(1)} \sim 20\%$. Electromagnetic and strong decays of $\Delta(1232)$, $N^*(1440)$ [15,16] and $N^*(1535)$ [8] also put strong restrictions on five quark proportion in the proton and leads to the probability of $qqqq$ components in the proton about $10\% \sim 20\%$. This proportion is inserted into the expression (5) and the obtained numerical value for $g_A(N(938))$ falls in the range 1.25 to 1.45 which brackets 1.26. In the same time, the change of helicity amplitudes for $\Delta(1232)$ and $N^*(1440)$ is by only a few percent, as given in Table 3 if the probability of five-quark components in the proton is assumed to fall in the range $10\% \sim 20\%$.

For the $N^*(1440)$, by explicit introduction of about $30\%$ five quark component it becomes possible to improve the helicity amplitude and strong decay width of $N^*(1440)$. If only the configuration $[4]_F S [22]_F [22]_S$ is taken into account, inserting $P_5^{(1)} \sim 30\%$ into Eq. (5) will give axial charge of $N^*(1440)$ about unity. However, chiral restoration scheme predicts that the chiral partner of $N^*(1535)$, Roper state, also has the small axial charge, i.e., $\sim 0$.

The next-to-lowest-energy $qqqq$ configuration is $[4]_F S [31]_F [31]_S$. The $J^{P}=0^+$ and $J^{P}=3$ $qqqq$ states of $N^*(1440)$ are assumed to have the equal proportion, i.e.,

$$P_5^{(2)}=P_5^{(3)}=1/2,$$

where $P_5^{(2)}$ is the probability of spin-flavor symmetry $[4]_F S [31]_F [31]_S$ of $qqqq$ subsystem in $N^*(1440)$. With the same total proportion ($\sim 30\%$) for five-quark component in $N^*(1440)$ as before, but the probability of $[4]_F S [22]_F [22]_S$ and $[4]_F S [31]_F [31]_S$ taken as $80\%$ and $20\%$, respectively, the final $g_A(N(1440))$ is obtained as 1.04. In comparison to axial charge with only the configuration $[4]_F S [22]_F [22]_S$ considered, the new configuration does not lead to any obvious change.

As for $N^*(1535)$, lattice QCD results show that axial charge takes quite small value. It is possible for $N^*(1535)$ to have the very small or possibly vanishing axial charge after considering the sea quarks configurations [10]. In the CQM, after taking into account mixing angle between $N^*(1535)$ and $N^*(1650)$, the predictions on both axial charges have large deviations from lattice results. In the calculation, standard mixing angle $\theta_S$ from OGE in the lowest mass negative parity nucleons is $-32^\circ$. We take into consideration the experimental error of $10^\circ$ and calculate the axial charges, as given in the table 3, of $g_A(N(1535))$ and $g_A(N(1650))$ at mixing angles, $-22^\circ$ and $-42^\circ$, respectively. Without configuration mixings axial charges is given in the first column and with mixings axial charges given in the other three columns. Lattice results of axial charge is given in the sixth column [2]. In the last column, the values are axial charge obtained in the extended quark model.

### Table 3. Helicity amplitudes in units of GeV$^{-1/2}$. $10\% \sim 20\%$ five-quark components in the proton is assumed.

| $N^*(1440)$ | $A_{1/2}$ | $A_{1/2}$ | $A_{1/2}$ | $A_{1/2}$ |
|-------------|-----------|-----------|-----------|-----------|
| $N^*(1440)$ | $-0.065 \pm 0.004$ | $-0.045 \sim 0.056$ | $-$ | $-$ |
| $\Delta(1232)$ | $-0.135 \pm 0.006$ | $-0.093 \sim 0.091$ | $-0.255 \pm 0.008$ | $-0.167 \sim 0.171$ |

Physical resonances can be expressed as

$$|N^*(1535)| = \cos \theta_S |P_M| - \sin \theta_S |P_M|,$$

$$|N^*(1650)| = \sin \theta_S |P_M| + \cos \theta_S |P_M|.$$  

Here the $\theta_S$ is mixing angle, which is defined as Isgur and Karl [19]. The axial charges $N^*(1535)$ and $N^*(1650)$ are defined by

$$g_A(N(1535)) = \cos^2 \theta_S |P_M| \sum_i \sigma_i |\tau_i|^2 |P_M| + \sin^2 \theta_S \times \langle 4P_M | \sum_i \sigma_i |\tau_i|^2 |P_M| - 2 \cos \theta_S,$$

$$\times \sin \theta_S |P_M| \sum_i \sigma_i |\tau_i|^4 |P_M|,$$

$$g_A(N(1650)) = \sin^2 \theta_S |P_M| \sum_i \sigma_i |\tau_i|^2 |P_M| + \cos^2 \theta_S \times \langle 4P_M | \sum_i \sigma_i |\tau_i|^4 |P_M| + 2 \cos \theta_S,$$

$$\times \sin \theta_S |P_M| \sum_i \sigma_i |\tau_i|^4 |P_M|.$$  

The sum of both axial charges is a variable independent on the mixing angle, i.e., $g_A(N(1535)) + g_A(N(1650)) = \frac{\pi}{2}$ exactly is a constant. The lattice result is about 0.55. We see that it is impossible to improve the prediction of axial charges in the CQM by adding the effects of the $SU(6) \otimes O(3)$ symmetry breaking.

The strong couplings of $N^*(1535)\eta \phi$, $N^*(1535)\eta \phi$, and $N^*(1535)K\Lambda$, as predicted in Refs. [16,17], show that there may be large strangeness components in $N^*(1535)$ resonance. In addition, it is proved to appear possible to reach the value 0 for axial charge in the extended quark model [10]. And about 45$\%$ $qqqqq$ components taken into account, the helicity amplitudes for the electromagnetic transition $\gamma^* N \rightarrow N^*(1535)$ in the extended quark model have satisfactory description [8].
In the case of $N^*(1650)$, theoretical predictions for $\pi$ and $\eta$ decay widths of the $N^*(1650)$ in the traditional quark model are unsatisfactory [20]. A more realistic resonance wave function beyond pure three-quark bound state, including five-quark component, may improve the theoretical expectations. Like $N^*(1535)$ [10], we have the following expression of axial charge,

$$g_A(N(1650)) = \frac{5}{9} P_3 + \frac{5}{6} P_5^{(2)} - \frac{1}{9} P_5^{(3)} - \frac{4}{15} P_5^{(4)} - \frac{17}{18} P_5^{(5)}.$$  \hfill (9)

If large range $10\% \sim 30\%$ of proportion of five-quark component in $N^*(1650)$ is assumed, the numerical value for its axial charge falls in the range 0.39 to 0.54, which includes the lattice value $\sim 0.5$.

### 4 Conclusions

We conclude that axial charge of proton in the traditional quark model does not have the substantial improvement even though the breakdown of the $SU(6) \otimes O(3)$ symmetry is taken into account. After taking relativistic effects into account, for massless relativistic quarks in the MIT bag model, one obtains much smaller value $g_A(N(938))=1.09$, which is also in disagreement with experiment value. But the empirical data for the axial charge of the proton can be well described qualitatively in the extended constituent quark model including the five-quark component, and the probabilities of the five-quark component ($\sim 20\%$) we obtain in the proton are consistent with the previous results [7]. In addition, we predict a unity axial charge for $N^*(1440)$, if the contributions of $\sim 30\%$ five-quark components are considered. In the future more wealth lattice data of axial charge will put constrain on proportion of sea-quark components in Roper state and other nucleon resonances.

For $N^*(1650)$ it is unknown whether or not it has sizable five quark components. Its properties might be explored in the photoproduction of the kaon meson through analyzing the wealthy polarized and unpolarized observables data. And in the strong interaction process $pp \rightarrow pK^+\Lambda$, new measurements at excess energy 171 $MeV$ also clearly show the contribution of $S_{11}(1500)$ baryon excitation [21]. In order to obtain $S_{11}$ properties we could analyze the more rich data at CSR or COSY in the future.

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| Table 4. Axial charges of the two lowest-energy negative parity resonances. |
|----------------|----------------|----------------|----------------|----------------|----------------|
| $g_A(N(1535))$ | Unmixing | $\theta_S = -22^\circ$ | $\theta_S = -32^\circ$ | $\theta_S = -42^\circ$ | Lattice | ECQM |
| $g_A(N(1650))$ | 5/9 | 0.29 | 0.47 | 0.63 | $\sim 0$ | $-0.05 \sim +0.06$ |

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