Is the baryon asymmetry of the Universe related to galactic magnetic fields?

V. B. Semikoz,1,2∗ D.D. Sokoloff,3† and J. W. F. Valle1‡

1AHEP Group, Institut de Física Corpuscular – C.S.I.C./Universitat de València
Edificio Institutos de Paterna, Apt 22085, E-46071 Valencia, Spain
2IZMIRAN, Troitsk, Moscow region, 142190, Russia
3Department of Physics, Moscow State University, 119999, Moscow, Russia
(Dated: May 20, 2009)

A tiny hypermagnetic field generated before the electroweak phase transition (EWPT) associated to the generation of elementary particle masses can polarize the early Universe hot plasma at huge redshifts $z > \sim 10^{15}$. The anomalous violation of the right-handed electron current characteristic of the EWPT converts the lepton asymmetry into a baryon asymmetry. Under reasonable approximations, the magnetic field strength inferred by requiring such “leptogenic” origin for the observed baryon asymmetry of the Universe matches the large-scale cosmological magnetic field strengths estimated from current astronomical observations.

PACS numbers: 14.60.-z, 95.30.Qd, 98.80.Cq, 12.15.-y, 98.80.Es, 11.30.Er

I. INTRODUCTION

Magnetic fields seem crucial for our understanding the Universe [1,2], as they may fill intracluster and interstellar space, affect the evolution of galaxies and galaxy clusters, playing an important role for the onset of star formation and determining the distribution of cosmic rays in the interstellar medium. Cosmologists support the view that the early Universe indeed hosted strong primordial magnetic fields which could survive under some conditions after recombination and serve as seed fields for galactic dynamo. The ultimate origin of such fields could be traced to very early phase transitions predicted by particle physics.

In contrast, the astronomical community tends to believe that we do not need seed fields in order to explain the origin of large-scale galactic magnetic fields whose formation is associated to physical processes in galaxies, protogalaxies etc [3,4].

Fortunately, upcoming radio telescopes such as the Square Kilometre Array (SKA) [5] are expected shed light on this issue and help distinguish the two options.

Here we focus on the cosmology camp, further developing the suggestion made in Refs. [6,7] where further alternative realizations are also mentioned. The basic assumption is the existence of a primordial seed hypermagnetic field, and its interaction with neutrinos.

Thanks to their unique properties, neutrinos provide the only “messenger” capable of probing the early Universe at high redshifts, $z > z_{\text{recomb}} \sim 1100$. The fact that they are required to be massive in order to account for neutrino oscillations [8] opens ways for them to play a role in cosmology. For example, if neutrino masses arise via the seesaw mechanism [9] the baryon asymmetry of the Universe may be easily accounted for through the so-called leptogenesis mechanism [10]. Alternatively, the generation of neutrino masses may also shed light on the dark matter problem [11,12,13].

Here we consider a tiny primordial seed hypermagnetic field $B_0^Y$ generated at $T_0 \gg T_{\text{EWPT}}$. We show that its presence induces a nonzero lepton asymmetry in the early phases of the evolution of the Universe which, thanks to the anomaly [14,15], can be “leptogenic” without directly invoking nonzero neutrino masses. At present times such asymmetry can exist only in neutrinos and its possible detection remains a challenge [16]. However it may have cosmological implications for the cosmic microwave background [17] as well as for Big Bang nucleosynthesis. For example the latter constrain the electron neutrino asymmetry, $|\xi_{\nu_e}(T_{\text{BBN}})| \ll 0.07$, $T_{\text{BBN}} \sim 0.1$ MeV. Thanks to the large mixing angles indicated by neutrino oscillation data [8] a similar restriction also applies to the chemical potential of the other flavors at temperatures $T \sim 3$ MeV [18].

It is especially interesting to consider the possible effects of a neutrino asymmetry at much earlier times, say, at $T \gg 3$ MeV or $z \gg 10^9$. It is easy to generalize

---

*Electronic address: semikoz@ifpc.csic.es
†Electronic address: sokoloff@dds.srcc.msu.su
‡Electronic address: valle@ifpc.csic.es
Maxwell’s equations for the hypercharge field $Y_\mu$ present in the Standard Model by the addition of the parity violation pseudovector current term $J_5 \sim \alpha B_Y$, where

$$\alpha(T) = \frac{4g^2 \mu_\nu(T)}{1512\pi^2 \sigma_{\text{cond}}(T)},$$

$\mu_\nu(T) = \sum_{l=e,\mu,\tau} \mu_\nu^{(l)}(T)$ being the net neutrino chemical potential, and $\sigma_{\text{cond}}(T) \sim T$ denotes the hot plasma conductivity. A nonzero net neutrino asymmetry results, which may lead to a strong amplification of the primordial hypermagnetic field in the early Universe hot plasma [3], e.g.

$$B_Y(x) = B_0^Y \exp \left[ 32 \int_x^{x_0} \frac{dx'}{x'^2} \left( \frac{\xi_\nu(x')}{0.001} \right)^2 \right]$$

where we introduced the new variable $x = T/T_{EW}$ and $B_0^Y$ is the assumed amplitude of the seed hypermagnetic field. Note that this result relies only on the standard plasma physics and Standard particle physics, as it follows simply from the basic parity violating nature of the Standard Model.

Neglecting in MHD the displacement current $\partial E_Y/\partial t$ and using Maxwell’s equation $\partial_t B = -\nabla \times E_Y$ we easily derive Faraday’s equation in the rest frame of the Universe $\mathbf{V} = 0$, as

$$\partial_t B_Y = \nabla \times \alpha B_Y + \eta \nabla^2 B_Y,$$

where $\eta = (4\pi\sigma_{\text{cond}})^{-1}$ is the hypermagnetic diffusion coefficient.

We now turn to discuss the subsequent evolution of the asymmetry. As the theory passes from the unbroken gauge symmetry phase to the broken one, the anomalous violation of fermion number plays a key role [14,17]. In the presence of the anomaly for right-handed electrons one has

$$\partial_t \varphi_\nu^{e_R} = -\frac{g^2 g_\nu^2}{64\pi^2} Y_{\mu\nu} \bar{Y}^{\mu\nu}, \quad y_R = -2,$$

so that

$$\frac{dL_{eR}}{dt} \approx -\frac{g^2}{2\pi^2} E_Y \cdot B_Y, \quad 2\frac{dL_{eL}}{dt} = -\frac{dL_{eR}}{dt} + \frac{1}{8} \dot{B},$$

where $s = 2\pi^2 g^* T^3/45$ is the entropy density; $L_l = (n_l - n_{\bar{l}})/s, B = (n_B - n_{\bar{B}})/s$ are the lepton and baryon numbers correspondingly and we have neglected the collision integrals associated with decay (inverse decay) of Higgs bosons, e.g. $\phi^{(0)} \leftrightarrow e_\nu \bar{e}_R$.

Substituting the conservation law $L_{eR} = B/3 - 2L_{eL}$ that follows from the second equation in (3) into the first equation in (3), and taking into account the adiabatic approximation, $s \approx \text{const}$, so that chemical potentials change very slowly, $\partial_t \mu_{eL} = \partial_t \mu_{eR} \approx 0$, or $dL_{eL}/dt \approx 0$, one gets the change of the baryon asymmetry in the presence of hypercharge fields as

$$\frac{1}{3} \frac{\partial(n_B - n_{\bar{B}})/s}{\partial t} = -\frac{g^2}{4\pi^2 s} \mathbf{E}_Y \cdot \mathbf{B}_Y.$$  (6)

Substituting the hyperelectric field $E_Y$ from Maxwell’s equations we get the baryon asymmetry at $T_{EW}$ expressed as

$$\eta_B(t_{EW}) = \frac{3g^2}{4\pi^2 s} \int_{t_0}^{t_{EW}} \left[ \alpha B_Y^2 - \eta \left( \nabla \times \mathbf{B}_Y \right) \cdot \mathbf{B}_Y \right] dt,$$

where the baryon asymmetry $\eta_B = (n_B - n_{\bar{B}})/s$. This is our main result. It also follows by considering the change of the Chern-Simons number density released in the form of fermions due to the anomaly, $\eta_B(t_{EW}) = (3/2s) \Delta n_{CS}$, where $\Delta n_{CS}$ is given by

$$\Delta n_{CS} = -\frac{g^2}{2\pi^2} \int_{t_0}^{t_{EW}} \left( \mathbf{E}_Y \cdot \mathbf{B}_Y \right) dt.$$

One notes from Eq. (7) in order to account for the observed baryon asymmetry $\eta_B \sim 10^{-10} > 0$ one requires a positive sign for the net neutrino asymmetry, $\mu_\nu = \sum_l \mu_{\nu}^{(l)} > 0$. Note also that the second diffusion term in Eq. (4) must be less than the first one in $\alpha^2$ dynamo mechanism [21].

Let us give estimates of baryon asymmetry (7) for the topologically non-trivial hypermagnetic field configuration [19], $Y_0 = Y_2 = 0, Y_3 = Y(t) \sin k_0 z, Y_0 = Y(t) \cos k_0 z$, which leads to exponential amplification of the amplitude $Y(t)$ (compare with [4,7]),

$$Y(t) = Y_0 \exp \left[ \int_{t_0}^{t} \left[ k_0 \alpha(t') - k_0^2 \eta(t') \right] dt' \right].$$

Following Ref. [19] we find $B_Y = \nabla \times Y = B_Y(t)(\sin k_0 z, \cos k_0 z, 0)$, where $B_Y(t) = k_0 Y(t)$, or we should substitute in Eq. (7) $B_Y^2 = B_Y^2(t), (\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y = B_Y^2(t) k_0$.

Substituting the helicity parameter $\alpha(T)$ given by Eq. (1) and keeping all parameters including conductivity $\sigma_{\text{cond}}(T)$ and the hypermagnetic field strength $B(t_{EW})$ as constants at $T_{EW}$ due to the adiabatic regime with entropy $s \approx \text{const}$ or $T \sim T_{EW} \approx \text{const}$ we estimate the integral in Eq. (7) as the integrand $\times t_{EW}$ where $t_{EW} = (2H)^{-1} = M_0/2T_{EW}^2$, $M_0 = M_{Pl}/1.66\sqrt{g^*}$, or
we find from Eq. (7):
\[
\eta_B(t_{EW}) = \frac{135\alpha'(M_0/2T_{EW})}{8\pi^4(\sigma_{cond}/T_{EW})g^*} \left( \frac{B^2_\nu(t_{EW})}{T_{EW}^4} \right) \times \\
\left[ \frac{\mu_0}{T_{EW}} \frac{47\alpha}{94.5} - \frac{\kappa_0}{T_{EW}} \right].
\]
(9)
Substituting numbers $\eta_B(t_{EW}) \sim 10^{-10}$, $\sigma_{cond}/T_{EW} \sim 10^2$, $g^* \sim 10^2$, $\alpha' \sim 10^{-2}$, $M_0/T_{EW} = 7 \times 10^{15}$, $47\alpha/94.5 \sim 5 \times 10^{-3}$ and neglecting the second negative diffusion term one gets from Eq. (9)
\[
\left( \frac{\xi_\nu(T_{EW})}{0.001} \right) \frac{B^2_\nu(T_{EW})}{T_{EW}^4} \approx 3.3 \times 10^{-14}.
\]
(10)
which constrains the product of hypermagnetic field and asymmetry at the EWPT, and hence magnitude of the subsequent Maxwellian magnetic field, obtained from the boundary condition $A_j^{(em)}(0) = \cos \theta_W Y_j$ at EWPT.

In Ref. [7] a stringent upper bound on the net neutrino chemical potential
\[
\xi_\nu(T_{EW})/0.001 < 0.12
\]
was obtained by requiring field survival against Ohmic dissipation. This implies a lower bound on the strength of the hypermagnetic field at EWPT, $B_\nu(t_{EW}) \gtrsim 5.24 \times 10^{17}$ G $< T_{EW}^2 \sim 10^{24}$ G. We use this bound in order to fix the magnitude of the initial value of the Maxwellian magnetic field $B(t_{EW}) \sim 5 \times 10^{17}$ G.

The subsequent evolution of the Maxwellian magnetic field after EWPT as a result of cosmological expansion is illustrated by the solid (red) lines in Fig. 1 in terms of its dependence on the redshift $z$. The astronomical relevance of this field depends on its spatial scale. The plot presented can be directly used in order to estimate the magnitude of the seed field for the galactic dynamo, provided the field is homogeneous on scales larger than the horizon size at the epoch of the phase transition. Moreover one must assume that the field has been created by a noncausal mechanism.

An alternative option considered here is that the field becomes causal at the instant of the phase transition. This means that its spatial scale $l$ is very small at later epochs in comparison with galactic scales $L_{gal}$ and should be considered as a small-scale magnetic field in the context of galactic dynamos. Such magnetic field corresponds to smaller large-scale fields,
\[
B_{\text{mean}} = BN^{-1/2}
\]
inferred as statistical mean field, where $N = (L_{gal}/l)^3$. This magnitude is indicated by the dotted lines in Fig. 1.

Note that the mean field $B_{\text{mean}}$ is causal at the BBN time, but not at earlier times, in contrast to the initial Maxwellian field. This is the reason why the blue (dotted) line for the mean field does not extend to higher redshifts $z > z_{BBN}$.

Both estimates should be compared with the results predicted by the galactic dynamo theory [4]. The latter assumes that first seed fields for galactic dynamos were created at the epoch of protogalaxies, as indicated by the dashed and dot-dashed lines in Fig. 1. One concludes from the above considerations that cosmological magnetic fields can at least provide an important, if not the leading, contribution to the early stages of galactic magnetic field formation.

In summary, here we have considered the effect of a tiny hypermagnetic field generated by early Universe processes taking place before the electroweak phase transition. They can polarize the early Universe hot plasma so that, as the Universe undergoes the EWPT the anomalous violation of the right-handed electron current con-

![FIG. 1: Magnetic field evolution after EWPT. The solid (red) line represents the Maxwellian magnetic field evolved from the hypermagnetic one as frozen-in plasma, while the dotted (blue) line represents the large-scale (1 pc at the epoch of galaxy formation) component of magnetic field which becomes causal at a moment after the EWPT. The dashed line denotes the galactic magnetic field generated by mean-field dynamo, while the dash-dotted one represents the galactic magnetic field generated by small-scale and then mean-field dynamo, starting from $10^{-9}$ G. For comparison we also show in the boxes the magnetic field models in Refs. [22].](image)
vert the lepton asymmetry into the observed baryon asymmetry. Under simplifying model assumptions we have inferred the magnetic field strength at the EWPT by requiring that it reproduces the observed baryon asymmetry of the Universe. Within this picture one can also account for the large-scale cosmological magnetic field strengths estimated from current astronomical observations.

The topologically nontrivial solution Eq. (8) can be reconciled with homogeneity and isotropy of the Universe by considering a domain structure with topologically nontrivial $Y$-fields in each domain and random isotropic orientations of the $z$-axis. Such Universe is homogeneous and isotropic on scales $L \ll l_H$ where $L$ is the typical domain size, $l_H = (2H)^{-1}$ is the horizon size. Here we have focused our attention on the time evolution of the hypermagnetic field inside a given domain, the full picture will be given elsewhere.

Acknowledgments

The authors wish to thank Sergio Pastor and Timur Rashba for helpful discussions. This work was supported by the Spanish grant FPA2008-00319/FPA and the CSIC-RAS exchange agreement. DS acknowledges financial support from RFBR under grant 07-02-00127a.

[1] R. M. Kulsrud, Ann. Rev. Astron. Astrophys. 37, 37 (1999).
[2] R. Beck, A. Brandenburg, D. Moss, A. Shukurov and D. Sokoloff, Ann. Rev. Astron. Astrophys. 34, 155 (1996).
[3] R. Beck and R. Wielebinski, Cosmic magnetic fields (Springer, 2005).
[4] T. G. Arshakian, R. Beck, M. Krause and D. Sokoloff, Astronomy & Astrophys. 494 21 (2009), arXiv:0810.3114.
[5] S. Johnston et al., Science with askap - the australian square kilometre array pathfinder, 2008.
[6] V. B. Semikoz and D. D. Sokoloff, Phys. Rev. Lett. 92, 131301 (2004), astro-ph/0312567.
[7] V. B. Semikoz and J. W. F. Valle, JHEP 03, 067 (2008), [0704.3978].
[8] For a review see, e. g. M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 6, 122 (2004); updated neutrino oscillation results are given in T. Schwetz, M. Tortola and J. W. F. Valle, New J. Phys. 10, 113011 (2008), arXiv:0808.2016.
[9] H. Numokawa, S. J. Parke and J. W. F. Valle, Prog. Part. Nucl. Phys. 60, 338 (2008), arXiv:0710.0554 [hep-ph].
[10] M. Fukugita and T. Yanagida, Phys. Lett. B174, 45 (1986).
[11] V. Berezinsky and J. W. F. Valle, Phys. Lett. B318, 360 (1993), hep-ph/9309214.
[12] M. Lattanzi and J. W. F. Valle, Phys. Rev. Lett. 99, 121301 (2007), arXiv:0705.2406 [astro-ph].
[13] F. Bazzocchi, M. Lattanzi, S. Riemer-Sorensen and J. W. F. Valle, JCAP 0808, 013 (2008), [0805.2372].
[14] G. ’t Hooft, Phys. Rev. D 14, 3432 (1976).
[15] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B155, 36 (1985).
[16] G. B. Gelmini, Phys. Scripta T121, 131 (2005), hep-ph/0412305.
[17] J. Lesgourgues and S. Pastor, Phys. Rev. D60, 103521 (1999), hep-ph/9904411.
[18] A. D. Dolgov et al., Nucl. Phys. B632, 363 (2002), hep-ph/0201287.
[19] M. Giovannini and M. E. Shaposhnikov, Phys. Rev. D57, 2186 (1998), hep-ph/9710234.
[20] J. M. Cline, K. Kainulainen and K. A. Olive, Phys. Rev. D 49, 6394 (1994).
[21] I. B. Zeldovich, A. A. Ruzmaikin and D. D. Sokolov, Magnetic fields in astrophysics (New York, Gordon and Breach Science Publishers), The Fluid Mechanics of Astrophysics and Geophysics. Volume 3, 1983, 381 p. Translation.
[22] E. R. Harrison, Generation of magnetic fields in radiation era, MNRAS, 147, 279-286 (1970); I.N.Mishustin, A. A. Ruzmaikin, Occurrence of priming magnetic fields during the formation of protogalaxies, JETP, 61, 441-444 (1971), Sov. Phys. JETP, 34, 233-235 (1972).