Note on universal description of heavy and light mesons

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Abstract

The experimental spectrum of excited $S$-wave vector mesons with hidden quark flavor reveals a remarkable property: For all flavors, it is approximately linear in mass squared, $m_n^2 \approx a(n + b)$, $n$ is the radial quantum number. We draw attention to the fact that such a universal behavior for any quark mass cannot be obtained in a natural way within the usual semirelativistic potential and string-like models — if the Regge-like behavior is reproduced for the mesons made of the light quarks, the trajectories become essentially nonlinear for the heavy-quark sector. In reality, however, the linearity for the heavy mesons appears to be even better than for the light ones. In addition, the slope $a$ is quite different for different quark flavors. This difference is difficult to understand within the QCD string approach since the slope measures the interaction strength among quarks. We propose a simple way for reparametrization of the vector spectrum in terms of quark masses and universal slope and intercept. Our model-independent analysis suggests that the quarks of any mass should be regarded as static sources inside mesons while the interaction between quarks is substantially relativistic.

1 Introduction

Since the appearance of hadron spectroscopy it has been widely believed that the dynamics responsible for the formation of hadrons is more or less universal at all scales where the hadron resonances are observed. The discovery of QCD provided a powerful support for this belief. Unfortunately, QCD is still not amenable to analytical calculation of the hadron spectrum and for this purpose one resorts to simplified dynamical models simulating QCD. In practice, for description of the light and heavy hadrons, one exploits usually different models. The main problem in the hadron spectroscopy, however, is the building of a universal solvable model describing the whole meson or baryon spectrum as a function of quark masses. Despite a great deal of interesting attempts (see, e.g., [1, 2]), such a model that would satisfy the absolute majority of the spectroscopic community was not constructed.
The best test of any phenomenological model is the agreement with experimental data. A reliable test is possible only if the available data is rich enough. In the experimental hadron spectroscopy, the most studied sector is given by the unflavored vector mesons having the quantum numbers of the photon. These resonances are intensively produced in the $e^+e^-$-annihilation which represents a traditional laboratory for the discovery of new quark flavors and for precise measurements of the vector spectrum. Since the mechanism of creation of the vector mesons in the $e^+e^-$-annihilation is expected to be identical for any flavor and abundant experimental data is available, the case of the unflavored vector mesons is likely the most appropriate for analyzing the manifestations of universality of the strong interactions in the resonance formation. These manifestations must be reproduced by any viable dynamical model. It turns out that many models seem not to pass such a test.

In the present paper, we scrutinize thoroughly the spectroscopic universality among the vector mesons with hidden flavor — the $\phi$, $\psi$, $\Upsilon$ mesons and their analogues in the sector of $u,d$ quarks, the $\omega$-mesons (the spectrum of $\rho$-mesons is similar and does not bring anything new in our discussions). Based on the experimental data, the spectroscopic manifestations of the universality in question are summarized. We argue that these manifestations may be explained if the total meson mass is composed (in the first approximation) of two contributions — a nonrelativistic one due to the static quark masses and a relativistic one stemming from the gluon interactions. The mass spectrum can be described by a simple formula which is universal for all considered mesons. The found relation is checked and its parameters are estimated. We indicate then the reason which does not allow to obtain the proposed relation in the usual potential and string-like models and speculate on possible ways for modifying these models.

2 The vector spectrum

Let us plot the masses squared of known $\omega$, $\phi$, $\psi$, and $\Upsilon$-mesons [3] as a function of consecutive number $n = 0, 1, 2, \ldots$ called also the radial quantum number in the potential models. In all cases (except the $\phi$-meson where the data is scarcer) we omit the heaviest state as the least reliable one and try to exclude the $D$-wave resonances (they decouple from the $e^+e^-$-annihilation as it is a point-like process). The Figs. (1a)–(1d) demonstrate a universal linear behavior of the kind

$$M_n^2 = a(n + b),$$

(1)
with the ground state lying below the linear trajectory. The spectrum (1) is a typical prediction of the hadron string (flux-tube) models with massless quark and antiquark at the ends (see, e.g., [5] and references therein). The experimental observation of the behavior (1) in the light non-strange mesons [6] is often used as an argument in favor of the string-like models proposed by Nambu [7]. The hadron strings loaded by massive quarks predict that $M_n^2$ becomes essentially non-linear function of $n$ (see, e.g., an example below). In reality, however, we observe the linear spectrum (1) for heavy quarkonia with the same (or even better) accuracy [1]. In addition, the slope $a$ is proportional to the string tension (or energy per unit length in the potential models with linearly rising confinement potential) related to the pure gluodynamics.

The first observation of this phenomenon was made in Ref. [8]. The authors of [8] interpreted the radial spectrum of the $\psi$-mesons as an almost linear while that of the $\Upsilon$-mesons as having significant deviations from the linearity. We believe that, within the experimental uncertainty, the spectra of all unflavored vector mesons are nearly linear except the state $n = 0$ — the mass of a ground state lies noticeably below the corresponding linear trajectory, see Figs. (1a)–(1d).
Table 1: The masses of known $\omega$, $\varphi$, $\psi$ and $\Upsilon$ mesons (in MeV) [3]. The experimental error is not displayed if it is less than 1 MeV. The following least reliable states are omitted: $\omega(2330)$ (and another candidate $\omega(2290)$) [3], all $D$-wave $\psi$-mesons [3] and $\psi(4615)$ [4], and also $\Upsilon(11023)$ [3] (the last resonance has a small coupling to the $e^+e^-$-annihilation in comparison with $\Upsilon(10860)$ — this suggests a strong $D$-wave admixture in this resonance). The data indicates that we should ascribe $n = 3$ to the $\varphi(2175)$-state (the $\varphi$-meson corresponding to $n = 2$ with the mass of about $1920 \pm 20$ MeV is thus a prediction).

| $n$ | 0 | 1 | 2 | 3 | 4 |
|-----|---|---|---|---|---|
| $M_\omega$ | 783 | 1425 ± 25 | 1670 ± 30 | 1960 ± 25 | 2205 ± 30 |
| $M_\varphi$ | 1020 | 1680 ± 20 | — | 2175 ± 15 | — |
| $M_\psi$ | 3097 | 3686 | 4039 ± 1 | 4421 ± 4 | — |
| $M_\Upsilon$ | 9460 | 10023 | 10355 | 10579 ± 1 | 10865 ± 8 |

Thus, theoretically the slope $a$ is expected to be universal for all flavors. The fits in Table 2 show that this is not the case. We find reasonable to display two fits — the Fit I includes all states from Table 1 and in the Fit II the ground states are excluded. The reason is that the ground states lie systematically below the linear trajectory and the physics behind this effect is obscure, at least for us. The relation (1) holds in many relativistic models for light quarkonia where the boson masses enter quadratically. The data shows that the relativistic universality (1) takes place qualitatively but is strongly broken quantitatively for heavy flavors.

It is instructive to see how the relation (1) emerges in the string-like models. There exist plenty of models of this kind in the literature but some common basic steps can be extracted and exposed in a simplified manner. One assumes that a massless quark-antiquark pair (produced, e.g., in the $e^+e^-$-annihilation) moves back-to-back creating an elongated flux-tube of chromoelectric field which one treats as a string. The energy of the system (the meson mass) is

$$ M = 2p + \sigma r, \quad (2) $$

where $p$ denotes the quark momentum, $r$ is the distance between the quark and antiquark, and $\sigma$ represents a constant string tension. One assumes also that the quarks perform the oscillatory motion inside the flux-tube and that the semiclassical Bohr-Sommerfeld quantization condition can be applied to this motion,

$$ \int_0^l pdr = \pi(n + b), \quad n = 0, 1, 2, \ldots \quad (3) $$
Table 2: The radial Regge trajectories (1) (in GeV$^2$) for the data from Table 1 (see text).

| $M_2^2$  | Fit I     | Fit II    |
|----------|-----------|-----------|
| $M_\omega^2$ | 1.03$(n + 0.74)$ | 0.95$(n + 1.04)$ |
| $M_{\phi}^2$  | 1.19$(n + 1.07)$  | 0.95$(n + 1.96)$  |
| $M_{\psi}^2$  | 3.26$(n + 3.03)$  | 2.98$(n + 3.53)$  |
| $M_{\Upsilon}^2$ | 6.86$(n + 11.37)$ | 5.75$(n + 16.54)$ |

Here $l$ is the maximal quark-antiquark separation and the constant $b$ equals to $\frac{3}{4}$ for the $S$-wave states and to $\frac{1}{2}$ for the others. Substituting the momentum $p$ from (2) to (3) and using the definition $\sigma = \frac{M}{l}$ one arrives at the expression

$$M_n^2 = 4\pi \sigma (n + b),$$

which has the form of (1).

The massive quarks are introduced via the replacement

$$p \rightarrow \sqrt{p^2 + m^2}.$$  

If we make the substitution (5) in the simplistic model above (for two quarks of equal mass $m$) we obtain

$$M_n \sqrt{M_n^2 - 4m^2} + 4m^2 \ln \frac{M_n - \sqrt{M_n^2 - 4m^2}}{2m} = 4\pi \sigma (n + b).$$

In the relativistic limit, $M_n \gg m$, the relation (6) reduces to (1) while in the nonrelativistic one, $M_n - 2m \ll 2m$, the relation (6) yields the spectrum of linearly rising potential $2$, $M_n \sim n^{\frac{5}{2}}$. The spectrum of $\psi$ and $\Upsilon$-mesons cannot be well fitted by the relation (6) because of strong non-linearity at typical masses of heavy quarks and reasonable values of $\sigma$.

This example demonstrates a general problem of practically all string-like models (see, e.g., [5] and references therein), semirelativistic potential models (the Refs. [2, 10] show only a few of them), and of related relativistic approaches based on Bethe–Salpeter like equations [11] which we could find in the literature since 70-th. The problem consists in the use of the substitution (5) in order to include the quark masses into the models. This step results in a non-linear behavior of the spectrum and practically close

\[E_n \sim n^{\frac{2\alpha}{2\alpha + 1}}\] at large enough $n$ [9].
any possibility for reproducing the universality of the light and heavy vector spectra seen in Figs. (1a)–(1d). Superficially, the substitution (5) looks natural indeed, but upon a closer view it becomes somewhat questionable: The dispersion relation $E^2 = p^2 + m^2$ holds for the on-shell particles while the confinement makes quarks off-shell inside hadrons. For the constituent quarks, the applicability of this relation looks even less evident since the constituent quark mass is not a fundamental quantity and represents just a model parameter. It is interesting to assume that the absence of strong nonlinearity in Figs. (1c) and (1d) can be related with the change of the standard dispersion relation between the quark mass, energy and momentum in the excited $S$-wave mesons. The question arises which form of the dispersion relation leads to the correct excited spectrum?

3 Nonrelativistic vs. relativistic universality

The universality of light and heavy vector spectra seen in Figs. (1a)–(1d) is not the end of the story. Consider the mass differences $\Delta_i = M_i - M_0$, where $M_i$ is the mass of the $i$-th radial excitation and $M_0$ is that of the ground state. They are displayed in Table 3. The quantities $\Delta_i$ turn out to be approximately universal. We call this nonrelativistic universality because one deals with linear boson masses. Such a universality looks violated for the highly excited $\psi$-mesons. This violation seems to be triggered by a strong contamination of data by the presence of $D$-wave states which are mixed with the $S$-wave ones. The mixing is caused by relativistic effects and shifts the masses (see, e.g., [12]). The nonrelativistic universality implies that the mass of vector meson with hidden flavor is given by the relation

$$M_n = 2m + E_n,$$

where $E_n$ is a universal excitation energy and $m$ represents a constant depending on the quark flavor. Below we show that with a good accuracy this
constant can be identified with the quark mass. The existence of relativistic
universality suggests that $E_n$ should be given by a relativistic theory, namely
$E_n^2 \sim n$. Due to this feature the relation (7) is different from the predictions
of potential models, both semirelativistic and non-relativistic. Making use of
Figs. (1a)–(1d) as a hint, we put forward the following ansatz

$$(M_n - 2m)^2 = a(n + b), \quad (8)$$

where the slope $a$ and the intercept parameter $b$ are universal for all quark
flavors. The formula (8) generalizes the radial meson trajectory (1) to the
case of unflavored vector mesons made up of massive quarks. Taking the
square root of (8), we can write this relation in the nonrelativistic form (7),

$$M_n = 2m + \sqrt{a(n + b)}. \quad (9)$$

As we know, the confinement physics leads to the positive sign in the r.h.s.
of (9). The choice of the opposite sign would lead to a unphysical picture in
which the mesons look like the deuterium nucleus with unrealistically large
quark masses.

Let us estimate the parameters $a$, $b$, $m$ in the relation (8) (or in (9)). Two
different methods will be exploited. In the first one, we look for the best fit
taking the data from Table 1. For the sake of demonstration of sensitivity to
initial assumptions, we consider two cases — with the light quark mass set to
zero and with all quark masses unfixed. The results are given in Table 4. The
ensuing two variants for the spectrum (8) are depicted in Figs. (2a) and (2b).
The closer are the points the better works the universality.

The results in Table 4 demonstrate that the radial meson trajectories are
able to ”measure” the current quark masses with surprisingly good accuracy.
If we set $m_{u,d} = 0$, the current masses of other quarks turn out to be very close
to their phenomenological values [3]. If we keep all quark masses unfixed, they
acquire an additional contribution about 360 MeV. This contribution may
be interpreted as an averaged value of (momentum dependent) constituent
quark mass emerging due to the chiral symmetry breaking in QCD.

In the second method, we fix the parameter $b$ in the interval $0 \leq b \leq 1$
and for points in this interval we calculate the best value of $a$ and $m$ using the
experimental data. The results are displayed in Figs. (3a) and (3b) (Fit I)
and Figs. (4a) and (4b) (Fit II). The comparison of Figs. (3b) and (4b) shows
that a universal slope (with reservation above concerning the $\psi$-mesons) can
be achieved if the Fit I is used, i.e. if we use all data from Table 1 for our
fitting procedure. The Fig. (3a) tells us that in the interval $0.3 \lesssim b \lesssim 0.7$
the parameter $m$ can be indeed interpreted as the quark mass. The typical
phenomenological values of $b$ (Table 4) for fixed light quark masses lie in this
interval.
Table 4: The quark masses (in GeV), the slope \(a\) (in GeV\(^2\)) and the dimensionless intercept parameter \(b\) in the relation (8).

|            | \(m_{u,d}\) fixed | \(m_{u,d}\) unfixed |
|------------|-------------------|---------------------|
| \(m_{u,d}\) | 0                 | 0.36                |
| \(m_s\)    | 0.13              | 0.49                |
| \(m_c\)    | 1.17              | 1.55                |
| \(m_b\)    | 4.33              | 4.69                |
| \(a\)      | 1.10              | 0.49                |
| \(b\)      | 0.57              | 0.00                |

![Fig. (2a)](image1.png) The spectrum (8) for \(m_{u,d}\) fixed.

![Fig. (2b)](image2.png) The spectrum (8) for \(m_{u,d}\) unfixed.

4 Discussions and outlook

We have arrived at the conclusion that the spectrum of light and heavy unflavored vector mesons can be parametrized in a universal way by the relation (9). This simple relation is of course approximate\(^3\) and within its accuracy the parameter \(m\) represents the quark mass, \(a\) and \(b\) are universal parameters encoding the gluodynamics responsible for the formation of resonances. The slope \(a\) is presumably related to \(\Lambda_{\text{QCD}}\) — a renorminvariant scale parameter appearing due to the dimensional transmutation in QCD. \(\Lambda_{\text{QCD}}\) slightly decreases if a new quark flavor is added (this could explain the behavior of the mass difference \(\Delta_1\) in Table 3). The constant \(b\) seems to parametrize the mass gap in QCD.

The ansatz (9) correctly reproduces the spectroscopic universality in the unflavored vector mesons and shows qualitatively how masses of the vector meson resonances are formed by masses of quarks and the gluon interactions: There is a nonrelativistic contribution from two static quarks and a relativis-

\(^3\)The relation (9) cannot be an exact result in QCD because the quark mass is not renorminvariant while the hadron masses are. However, within the accuracy of the narrow-width approximation, the running of quark masses can be safely neglected.
Fig. (3a). The quark masses as a function of \( b \) in (8) for the Fit I. The horizontal lines show the experimental quark masses in GeV (at the scale 2 GeV for the heavy quarks and at the scale 1 GeV for the light ones). The range of \( m_b \) from the perturbative to the 1S value is shaded.

Fig. (3b). The slope \( a \) as a function of \( b \) in (8) for the Fit I.

Fig. (4a). The quark masses as a function of \( b \) in (8) for the Fit II.

Fig. (4b). The slope \( a \) as a function of \( b \) in (8) for the Fit II.

To the best of our knowledge, the relation (9) (or (8)) cannot be reproduced in the commonly used potential and string-like models. The basic problem is that such a separation of relativistic and nonrelativistic contributions is absent. In the potential models, one encounters the following dichotomy: The quark masses give an additive contribution to the meson mass in the nonrelativistic models, this is correct, but the gluodynamics is then nonrelativistic, this is not correct; in the relativistic potential models, the gluodynamics becomes relativistic, this is correct, but the quark masses do not yield an additive contribution because of the replacement (6), and this seems not to be correct. An ad hoc way out for the latter models could consist in imposing the linear dispersion relation

\[
E = |p| + m, \tag{10}
\]
for the quarks. The relation (10) should somehow arise due to the off-shell nature of quarks. The same ad hoc prescription can be used in the flux-tube models — if, instead of (5), one makes the replacement $p \rightarrow p + m$ in (2), the final relation for meson masses will have the form (8). Another possibility for the string-like models could be the requirement to base such models on the picture of static quarks, i.e. the quantization should be performed with fixed endpoints and the object of quantization should be only the field between the quark and antiquark. The simplest effective toy-model might look as follows: The quark and antiquark interact by the exchange of some (nearly) massless particle, say the pion or (perhaps massive) gluon, and one mimics this exchange by oscillatory motion of the particle to which the quantization condition (3) is applied. This exchange picture looks more natural from the point of view of the resonance production. Indeed, when the relativistic quark and antiquark are created and move back-to-back, it is very unlikely that a "turning point" could emerge. Rather the gluon field between quarks "resonates" at certain production energies of quark-antiquark pairs and something like a quasi-bound state appears.

The relation (10) can be interpreted as a reflection of the fundamental property that in any dynamical model one cannot calculate absolute energies but only energy differences. The constant $m$ embraces the contributions which are not described by the confinement mechanism (quark mass, spin-spin and spin-orbital contributions, etc.). Our fits show that in the unflavored vector mesons of all kinds, the dominant contribution to $m$ stems from the quark masses. Perhaps this is related to the fact that the quarks (being fermions) give negative contribution to the QCD vacuum energy. This effect may provide an heuristic understanding of our conclusion on the static nature of quarks: Injecting a quark-antiquark pair (with the quark mass $m$) into the QCD vacuum, one lowers its minimum roughly by the value of $2m$. And, in the first approximation, this seems to be the only quark effect that should be taken into account in the dynamical quark models. For this reason, any model constructed for the description of the light-meson spectroscopy should be equally (within the accuracy of the narrow-resonance approximation) good for the heavy mesons and vice versa, at least in the vector sector. For instance, the linear spectrum (1) is reproduced in the soft-wall holographic models for QCD [13]. Our analysis shows that these models can be applied without change of initial input parameters to the heavy vector mesons — one just shifts the obtained spectrum by $2m$, as in the relation (8).

In summary, we believe that the description of heavy and light hadrons basically should be very similar, if not identical. We have shown how to reveal the spectroscopic universality in the case of unflavored vector mesons where a sufficient amount of experimental data is available. It would be interesting
to study the manifestations of universality for other quantum numbers. This may yield, for instance, plenty of spectroscopic predictions.

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