Mechanism analysis and optimum control of negative airgap eccentricity effect for in-wheel switched reluctance motor driving system

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Received: 23 October 2021 / Accepted: 13 February 2023 / Published online: 25 February 2023 © The Author(s), under exclusive licence to Springer Nature B.V. 2023

Abstract In this paper, the generation mechanism of the negative airgap eccentricity effect for the in-wheel switched reluctance motor (SRM) driving system is analyzed. An independent current chopping control strategy is proposed to achieve optimum control between the response characteristic of the in-wheel motor driving system and the dynamic performance of electric vehicle (EV). Firstly, the electromagnetic characteristic of the studied SRM under airgap eccentricity is studied based on electromagnetic coupling model and circuit driving equation, and the radial electromagnetic force under different airgap eccentricity is verified by adopting the built experiment device. Then, combined with the excitation characteristics of the radial electromagnetic force, the negative dynamic effect of the in-wheel motor driving system is analyzed in the time–frequency domain. Finally, an independent current chopping control strategy for the in-wheel SRM driving system based on vehicle vibration feedback is proposed. The controller parameters including the turn-off angle and chopping current threshold are optimized by data interpolation. Results show that the proposed control strategy can achieve the optimum control between the response characteristics of the in-wheel motor driving system and the vehicle dynamic performance, especially to suppress the vehicle sprung mass acceleration and tire bounce while starting EV.

Keywords Electric vehicle · In-wheel motor driving system · Switched reluctance motor · Airgap eccentricity

1 Introduction

Vehicle electrification has become an inevitable trend in the development of the automobile industry due to the transformation of the global energy framework [1–3]. In particular, the in-wheel motor driving electric vehicle (EV) has a brilliant engineering prospect because of the virtue of the efficient mechanical transmission and rapid control response [4, 5]. The EV dynamic performance is determined by the in-wheel motor mounting directly on the wheel [6, 7]. Among different in-wheel motor types, the rotor of the switched reluctance motor (SRM) has neither windings nor permanent magnets. The motor torque is

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generated by the tendency of magnetic circuit choosing minimum reluctance structure [8], which embraces the application potential for the field of in-wheel motor driving EV [9, 10].

On the other hand, the special architecture of the SRM also leads to nonlinear electromagnetic characteristics, which causes serious problems with vibration and noise [11, 12]. It is noticed that the radial electromagnetic force between the stator and rotor is the primary factor causing vibration and noise [13, 14]. In recent years, the calculation about radial electromagnetic force based on the virtual displacement principle and Maxwell stress tensor has been investigated by many researchers, especially in the area of the radial electromagnetic force based on the virtual displacement principle and Maxwell stress tensor [15, 16]. These studies focused on the motor electromagnetic characteristic under ideal mechanical conditions, but do not fully consider the variety about the electromagnetic characteristic after airgap eccentricity occurs.

The airgap eccentricity of in-wheel motor caused by external disturbances such as road excitation and driving behavior is unavoidable, as shown in Fig. 1a. The resulting imbalance of radial electromagnetic force will produce residual unbalanced radial force [17, 18], which will further affect the EV dynamic performance [19, 20]. It is significant to clarify the influence mechanism of this negative airgap eccentricity effect of the in-wheel motor driving system with respect to the unbalanced radial force [21]. Most of the existing researches analyze its response features from the time domain or frequency domain alone. However, comprehensive analysis in the time–frequency domain combined with the radial electromagnetic force excitation characteristics is still not so much.

To suppress the negative airgap eccentricity effect for in-wheel motor driving system, the existing research mainly focused on the active suspension, multi-objective optimization and driving control [22–24]. Active suspension can improve the ride comfort and handling stability for in-wheel driving EV [25], but the increase in unsprung mass will destroy the structure stability of in-wheel driving system. Similarly, while the multi-objective optimization can build up the response characteristics of in-wheel motor driving system significantly, the improvement of the vehicle dynamic performance is limited [26]. Distinguishing from above two methods, this negative airgap eccentricity effect can be suppressed by driving control. For driving control, it is still a scientific problem that many scholars actively explore to seek an appropriate driving control strategy to suppress the negative airgap eccentricity effect.

Considering the shortages of researches mentioned above, this paper focuses on the mechanism analysis and optimum control of the negative airgap eccentricity effect for in-wheel motor driving system, so as to realize the optimum control between the response characteristic of the in-wheel motor driving system and the vehicle dynamic performance. Firstly, the electromagnetic characteristic of the SRM under

![Fig. 1](image-url) Schematic diagram of the switched reluctance motor. a external rotor motor structure; b motor airgap eccentricity
different airgap eccentricity are investigated based on electromagnetic coupling model and circuit driving equation. Then, the generation and influence mechanism of the negative airgap eccentricity effect for in-wheel motor is analyzed in time–frequency domain based on the radial electromagnetic excitation characteristics. Furthermore, an independent current chopping control strategy for the in-wheel SRM driving system based on vehicle vibration feedback is proposed, and the controller parameters are selected by data interpolation. Finally, some important conclusions are summarized in Sect. 5.

2 Modeling and calculation for switched reluctance motor

The dynamic response of in-wheel motor driving system is directly determined by in-wheel motor performance. In this section, the electromagnetic characteristic and response characteristics of the studied SRM under airgap eccentricity are analyzed by establishing the electromagnetic coupling model and circuit driving equation.

2.1 Electromagnetic coupling model

Fourier series fitting is a common method to obtain the electromagnetic characteristic of SRM, and the original data required for fitting can be required by experimental measurement or finite element analysis. In this paper, a four-phase 8/6 external rotor SRM as shown in Fig. 1b is employed as the research object [27]. The external rotor motor structure can be mounted on wheels to directly drive the vehicle. All structure parameters about the studied motor are listed in Appendix A.

The initial position is defined as the aligned position between stator salient pole and rotor groove, and the rotor angular displacement under this position is 0°. As the rotor rotated half pole pitch, the center lines of the stator and rotor salient poles will overlap, which is defined as the alignment position. At this time, the winding inductance \( L(\theta, i) \) can be expanded by Fourier series [28, 29] as follows:

\[
L(\theta, i) = L_0(i) + L_1(i) \cos(N_r \theta + \phi_1) + \sum_{n=2}^{\infty} L_n(i) \cos(nN_r \theta + \phi_n) \]

(1)

where \( \phi_n = n \pi \). Fourier series coefficients \( L_n \) can be derived from the winding inductance of the motor under a particular position. These coefficients are calculated based on aligned position inductance \( L_a \), unaligned position inductance \( L_u \) and semi-aligned position inductance \( L_m \) in this paper.

\[
\begin{align*}
L_0(i) &= \frac{1}{2} \left[ \frac{1}{2} (L_a + L_u) + L_m \right] \\
L_1(i) &= \frac{1}{2} (L_a - L_u) \\
L_2(i) &= \frac{1}{2} \left[ \frac{1}{2} (L_a + L_u) - L_m \right]
\end{align*}
\]

(2)

Since the motor airgap is relatively large at the unaligned position, the inductance \( L_a \) at this position can be regarded as a constant. For \( L_a \) and \( L_m \), it can be expressed as a polynomial with respect to winding current, and the phase winding inductance \( L(\theta, i) \) of SRM can be expressed as follows:

\[
L(\theta, i) = \sum_{n=0}^{\infty} L_n(i) \cos(nN_r \theta + \phi_a) = \frac{1}{2} \left[ \cos^2(N_r \theta) - \cos(N_r \theta) \right] \sum_{n=0}^{N} a_n i^n + \sin^2(N_r \theta) \sum_{n=0}^{N} b_n i^n + \frac{1}{2} L_m \left[ \cos^2(N_r \theta) + \cos(N_r \theta) \right]
\]

(3)

where \( a_n \) and \( b_n \) are polynomial fitting coefficients. By solving the integral of winding inductance with respect to winding current, the expression of winding flux linkage is obtained as follows:

\[
\psi(\theta, i) = \int_0^i L(\theta, i) \, di = \frac{1}{2} \left[ \cos^2(N_r \theta) - \cos(N_r \theta) \right] \sum_{n=1}^{N+1} c_n i^n + \sin^2(N_r \theta) \sum_{n=1}^{N+1} d_n i^n + \frac{1}{2} L_m \left[ \cos^2(N_r \theta) + \cos(N_r \theta) \right]
\]

(4)
where $c_n = a_n - 1/n$ and $d_n = b_n - 1/n$ are integral coefficients about $a_n$ and $b_n$, respectively.

Based on the above modeling and calculation, the winding inductance and flux linkage of the motor are shown in Fig. 2.

Both the inductance and flux linkage are changed nonlinearly with respect to angular position and winding current. The inductance reaches its maximum value at the aligned position, and the winding current is about 4A. Similarly, the flux linkage reaches its maximum value at the aligned position and the maximum winding current.

According to the virtual displacement principle and electromagnetic energy conversion rule, the magnetostatic torque and radial electromagnetic force of the studied motor are the partial derivatives of winding magnetic co-energy with respect to angular position and airgap length, respectively.

\[
\left\{ \begin{array}{l}
T_m = \left. \frac{\partial W_m}{\partial \theta} \right|_{i=\text{const}} = \int_0^1 \frac{\partial \psi(\theta, i)}{\partial \theta} \, di \\
F_r = \left. \frac{\partial W_m}{\partial l_g} \right|_{i=\text{const}} = \int_0^1 \frac{\partial \psi(\theta, i)}{\partial l_g} \, di 
\end{array} \right. \\
(5)
\]

The magnetostatic torque and radial electromagnetic force are shown in Fig. 3. It should be noticed that although the magnetostatic torque at the unaligned position and aligned position are both close to zero, the reasons behind this phenomenon are completely different.

At the unaligned position, the motor reluctance in the closed magnetic circuit is the highest among all rotating positions; therefore, the lowest winding magnetic co-energy and limited magnetostatic torque can be generated. However, at the aligned position, only radial mutual attraction exists between the two salient poles at this position, because the radial flux density between the stator and rotor is the largest, while the tangential flux density is zero.

2.2 Circuit driving equation

According to the analysis above, it is found that the current excitation is an important factor that affect the motor electromagnetic characteristic and response characteristics. Meanwhile, the current excitation is controlled by driving circuit [30]. The asymmetric half bridge power converter which is employed in this paper is shown in Fig. 4. According to Faraday’s law of electromagnetic induction, the winding voltage balance equation of the SRM can be expressed as follows:

\[
U_k = R_k i_k - e_k = R_k i_k + \frac{d\psi_k(\theta, i_k)}{dt} \\
(6)
\]

where $U_k$, $R_k$ and $i_k$ are the impressed voltage, resistance and current, respectively, and $e_k$ is induced electromotive force in phase $k$ winding.
Because the inductance and flux linkage are both multivariate functions with rotating position and current excitation, Eq. (6) can be rewritten as follows:

\[
U_k = R_k i_k + L_k(\theta, i_k) \frac{di_k}{dt} + i_k \frac{\partial L_k(\theta, i_k)}{\partial \theta} \frac{d\theta}{dt} + \frac{\partial i_k}{\partial \theta} \frac{d\theta}{dt} + \phi_k( \theta, i_k) \frac{d\theta}{dt}
\]

(7)

Equation (7) shows that the impressed voltage of the winding equals total voltage of the three parts in the driving circuit. The first term of Eq. (7) is the resistance voltage dropping in the winding circuit; the second term is the electromotive force caused by the current excitation, that is, transformer electromotive force; the third term is the electromotive force caused by the rotating position, namely moving electromotive force, which is directly related to the electromechanical energy conversion.

Therefore, the current excitation of phase \( k \) winding at different rotation speeds can be expressed from Eq. (7) as follows:

\[
i_k = \int \frac{U_k - R_k i_k - i_k \frac{\partial L_k(\theta, i_k)}{\partial \theta} \frac{d\theta}{dt} - \phi_k( \theta, i_k) \frac{d\theta}{dt}}{L_k(\theta, i_k)} \, dt
\]

(8)
where $\omega = d\theta/dt$ presents the angular velocity of motor. Furthermore, the real-time electromagnetic characteristic of the SRM under different driving conditions can be calculated using Eq. (8) and electromechanical coupling model shown in Sect. 2.1.

2.3 Characteristics analysis under airgap eccentricity

In order to accurately quantify the variation of airgap eccentricity between the stator and rotor, the airgap eccentricity ratio is defined as follows:

$$e = (\Delta g/L_g) \times 100\%$$  \hspace{1cm} (9)

where $\Delta g$ and $L_g$ are the airgap eccentricity displacement and initial airgap length of the motor, respectively. The inductance and flux linkage under fixed current excitation and different airgap eccentricity are obtained using the electromechanical magnetic coupling model and circuit driving equation as shown in Fig. 5.

Combining Eq. (4) and (5), the magnetostatic torque of the SRM under different airgap eccentricity is expressed as follows:

$$T_e = \int_0^i \frac{\partial \psi(\theta, i)}{\partial \theta} \, di$$

$$= \sin(N_r \theta) \sum_{n=1}^{N} \frac{1}{n} e_{n-1} f_{n}^n + \sin(2N_r \theta) \sum_{n=1}^{N} \frac{1}{n} f_{n} f_{n}^n$$  \hspace{1cm} (10)

where $e_n = N_r c_n/2$, $e_0 = 0$, $e_1 = N_r(c_1 - L_n)/2$; $f_n = N_r d_n - e_n$, $f_0 = 0$, $f_1 = N_r(2d_1 - c_1 - L_n)/2$.

Ignoring the end effect and winding mutual inductance, the radial electromagnetic force under different airgap eccentricity can be expressed as follows:

$$F_r = \int \frac{\partial \psi(\theta, i)}{\partial \theta} \, di = \frac{1}{2} L(\theta, i) - \Delta g^2 \quad \text{(11)}$$

Similarly, the magnetostatic torque and radial electromagnetic force under different airgap eccentricity are shown in Fig. 6. In addition, the comparison of peak results about electromagnetic characteristic and response characteristics under different airgap eccentricities are shown in Table 1.

Form Fig. 6 and Table 1, it can be observed that all motor characteristics results increase obviously with the aggravation of airgap eccentricity displacement. In particular, the radial electromagnetic force changes most significantly and its trend shows a linear increase with the slope of approximately one. Additionally, the magnetostatic torque is the partial derivative of flux linkage with respect to angular position according to Eq. (4), which leads to a similar trend between the above varieties. In summary, the imbalance of the radial electromagnetic force caused by airgap eccentricity on the SRM should be paid attention in engineering practice.

2.4 Measurement verification of radial electromagnetic force

Accurate calculation results of radial electromagnetic force are the prerequisite to suppress the negative airgap eccentricity effect for in-wheel motor driving system. Therefore, radial electromagnetic force measurements under different airgap eccentricity are completed by designing a measurement device as shown in Fig. 7.

The measurement device comprises a platform chassis, a support shaft, two rotor holders, two stator

![Fig. 5](image-url)  
**Fig. 5** Electromagnetic characteristic under airgap eccentricity. (a) inductance; (b) flux linkage

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holders, two sensor holders, two force sensors, the studied SRM and so on. The airgap eccentricity is adjusted by the fine thread bolts attached to the stator holder which converts the circumferential rotational motion of bolts into vertical linear motion.

During the measurement process, the airgap length is firstly adjusted to the target value by the fine thread bolts to get the airgap eccentricity to be measured. Meanwhile, by adjusting the height of the pressure supporter, the force sensor and the support shaft will not be affected by any external forces except the motor gravity. Then, the fine thread bolts are pushed back to a certain distance to ensure that its tail is out of contact with the support shaft. Finally, the radial electromagnetic force under airgap eccentricity is measured and recorded by adjusting current excitation. The radial electromagnetic force measurement device of SRM mentioned above is shown in Fig. 8.

### Table 1 Results comparison under different airgap eccentricities

| / | Inductance/mH | Flux linkage/Wb | Torque/Nm | Force/N |
|---|---|---|---|---|
| Initial | 103.21 | 0.8003 | 14.93 | 4000 |
| 10% | 105.26 (1.99%) | 0.8121 (1.47%) | 14.99 (0.40%) | 4230 (5.8%) |
| 20% | 107.32 (3.98%) | 0.8230 (2.84%) | 15.08 (1.00%) | 4440 (11.0%) |
| 30% | 109.45 (6.05%) | 0.8331 (4.10%) | 15.25 (2.14%) | 4627 (15.7%) |
| 40% | 111.70 (8.23%) | 0.8428 (5.31%) | 15.58 (4.35%) | 4806 (20.2%) |
| 50% | 114.09 (10.54%) | 0.8520 (6.46%) | 15.92 (6.63%) | 4970 (24.3%) |

The bold marks represent the variable change ratio under different eccentricities.

### Fig. 6 Response characteristics under different airgap eccentricity. a magnetostatic torque; b radial electromagnetic force

### Fig. 7 Schematic diagram of radial electromagnetic force measurement device
The measurement results of radial electromagnetic force and unbalanced radial force for the SRM under different airgap eccentricity are obtained using the measurement device as shown in Fig. 9. As shown in Fig. 9, the radial electromagnetic force and unbalanced radial force under different airgap eccentricity agree well with the finite element simulation results. It proved the accuracy of the radial electromagnetic force calculation results.

3 Vibration analysis for in-wheel motor driving system

To clarify the negative dynamic effect mechanism of the in-wheel SRM driving system caused by airgap eccentricity, this section analyzes the influence of unbalanced radial force on vehicle dynamic from time domain and frequency domain on the basis of the coupling relationship between road excitation, airgap eccentricity and unbalanced radial force.

3.1 Quarter vehicle dynamic modeling

The in-wheel motor airgap eccentricity caused by road excitation is unavoidable, which will generate residual unbalanced radial force and affect the dynamic response of the in-wheel motor driving system. A quarter vehicle dynamic model established in this paper is shown in Fig. 10. Different from the traditional quarter vehicle dynamic model, this model divides the vehicle unsprung mass into $m_s$ (total mass...
of rotor, rim and tire) and \( m_r \) (total mass of stator and housing). \( m_s \) and \( m_r \) are connected to each other by the motor and hub bearings with total stiffness \( k_r \). All vehicle parameters involved are listed in Appendix A.

According to Newton’s second law, the motion equation of the quarter vehicle dynamic model can be expressed as follows:

\[
\begin{align*}
mb \ddot{z}_b &= k_s(\dot{z}_s - \dot{z}_b) + c_s(\ddot{z}_s - \ddot{z}_b) \\
ms \ddot{z}_s &= k_r(\dot{z}_r - \dot{z}_s) - k_s(\dot{z}_s - \dot{z}_b) - c_s(\ddot{z}_s - \ddot{z}_b) - f_v \\
m_r \ddot{z}_r &= k_t(\dot{z}_g - \dot{z}_r) - k_r(\dot{z}_r - \dot{z}_s) + f_v
\end{align*}
\]

(12)

where \( z_b, z_s, z_r \) and \( z_g \) are displacement of the sprung mass, stator assembly, rotor assembly and road excitation, respectively. \( k_t, k_s \) and \( c_s \) are tire stiffness, suspension stiffness and damping coefficient, respectively. \( m_b \) is sprung mass. \( f_v \) is unbalanced radial force.

Vehicle vibration input and road roughness are described by road power spectral density \( G_q(n) \), and its fitting expression is as follows:

\[
G_q(n) = G_q(n_0) \left( \frac{n}{n_0} \right)^{-w}
\]

(13)

where \( n \) and \( n_0 \) are spatial frequency and reference spatial frequency, respectively. \( G_q(n_0) \) is the power spectral density value under the reference spatial frequency \( n_0 \), which is called the road roughness coefficient. \( w \) is frequency index.

Filtered white noise road excitation is set as input, and the vehicle travels on class A and class B road, respectively, with a speed of 60 km/h. The variation about the airgap eccentricity and unbalanced radial force of in-wheel motor driving system is shown in Fig. 11. It can be observed that both airgap eccentricity and unbalanced radial force will increase significantly with the deterioration of road excitation. Additionally, the airgap eccentricity will be further increased due to the mutual attraction of radial electromagnetic force between stator and rotor. It means that the road excitation, airgap eccentricity and unbalanced radial force present a positive proportional relationship.

3.2 Vibration mechanism analysis

It is of great theoretical significance to analyze the vibration mechanism of in-wheel motor driving system in suppressing its negative dynamic effect. The quarter vehicle dynamic model established in this paper can be expressed by the state-space equation as follows:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

(14)

First, the displacement and velocity of each component in the quarter vehicle model are selected as the state vector \( x \) as follows:

\[
x = [z_b \quad \dot{z}_b \quad z_s \quad \dot{z}_s \quad z_r \quad \dot{z}_r]^T
\]

(15)

Then, according to the dimension of the state vector and external excitation factors, the input vector \( u \) is defined as follows:

\[
u = [0_{1 \times 3} \quad f_v \quad z_g \quad \dot{z}_g]^T
\]

(16)

Finally, the output vector \( y \) of the state-space equation is composed of sprung mass acceleration, suspension dynamic deflection and tire bounce as follows:

![In-wheel motor driving system](image-url)
Based on the state vector, input vector and output vector, the system matrices of the state-space equation are listed in Appendix B. As shown in Table 2 and Fig. 12a, vibration energy distribution and vibration modes are obtained by energy decoupling for the quarter vehicle dynamic model. Furthermore, the transfer characteristics of the in-wheel motor driving system with respect to unbalanced radial force are shown in Fig. 13.

The vibration modes of the in-wheel motor driving system concentrates on the first mode with 1.24 Hz for sprung mass, the second mode with 8.19 Hz for rotor assembly and the third mode with 64.39 Hz for stator assembly. In the first mode, the vibration influence of the unbalanced radial force is similar to the low-order vibration response caused by road excitation in the traditional quarter vehicle model. Its vibration energy is mainly concentrated on the vehicle sprung mass. The vibration response caused by road excitation is related to vehicle mass and suspension parameters, and the vibration response caused by unbalanced radial force is concerned with airgap eccentricity and current excitation.

On the other hand, road excitation is generally random excitation, but unbalanced radial force is harmonic excitation with respect to vehicle driving speed and rotor salient pole number. The excitation frequency of unbalanced radial force can be expressed as follows:

\[ f_e = \frac{i_s n_v}{60} N_r = \frac{i_s \omega}{2\pi} N_r \approx i_s \frac{v}{2\pi R_e} N_r \]  

where \( i_s \) is vehicle transmission ratio, \( n_v \) is driving speed, and \( N_r \) is rotor salient pole number.

The unbalanced radial force excitation frequency under different salient pole number and vehicle driving speed is shown in Fig. 12b. Within the low driving speed, especially in the vehicle starting condition (0–5 km/h), the unbalanced radial force excitation frequency is concentrated on the low-frequency zone. It has significant influence on the vehicle riding comfort. In the second and third modes, the vertical vibration of the rotor assembly and stator assembly is the main cause. There is mutual strong coupling between the road excitation and the unbalanced radial force excitation frequency. Moreover, the vibration energy is mainly concentrated on the unsprung mass in these frequency bands. It indicated that in a higher frequency, the unbalanced radial force has greater effects on the unsprung mass.

In addition, because the in-wheel is mounted directly in the rim and connected to the tire, the second mode vibration energy distribution is large in the unsprung mass rotor assembly at the low driving speed.

\[
y = \begin{bmatrix} \dot{z}_b & z_b - z_s & 0 & z_r - z_g & 0 \end{bmatrix}^T
\]  

\( (17) \)

The bold marks represent the maximum proportion of vibration energy distribution under different modes.
Fig. 12  Frequency-domain characteristics of the in-wheel motor driving system. a vehicle vibration modes; b radial electromagnetic force excitation frequency.

Fig. 13  Transfer characteristics of the unbalanced radial force on vehicle vibration. a sprung mass acceleration; b suspension dynamic deflection; c tire bounce.
speed (5–30 km/h). The above frequency band is close to the tire resonance frequency. It deteriorates the road adhesion condition of tire and disturbs the wheel load distribution. In the common driving speed (30–80 km/h), the unbalanced radial force excitation frequency varies widely. The vibration energy is concentrated in the unsprung mass stator assembly. It threatens the in-wheel motor structure stability seriously.

4 Formulation and optimization of control strategy

An independent current chopping control strategy for the in-wheel motor driving system based on vehicle vibration feedback is proposed in this section. Furthermore, aiming at the time-varying of vehicle driving speed, the controller parameters are optimized by data interpolation to achieve a synergetic optimal performance between the in-wheel motor driving system and vehicle.

4.1 Control strategy formulation

For the in-wheel motor driving system, the driving torque and unbalanced radial force are positively correlated with the current excitation. It means that the increase in driving torque will inevitably lead to the aggravation of unbalanced radial force. It is an urgent engineering problem to achieve the maximum output of driving torque while suppressing unbalanced radial force. On the other hand, it can be found from Eq. (5) that the driving torque and unbalanced radial force can be expressed as partial derivatives of winding current. Therefore, the negative dynamic effect of in-wheel motor driving system caused by unbalanced radial force can be effectively improved by controlling current excitation.

Common control methods for SRM include angular position control (APC), current chopping control (CCC) and pulse width modulation control (PWM). These control methods regulate the force characteristics of SRM by controlling current excitation. The implementation of various control strategies can be attributed to the expansion and derivation of the above control methods.

For CCC modulation, its turn-on angle \( \theta_{on} \) and turn-off angle \( \theta_{off} \) will remain unchanged. The winding current will be limited between the given upper threshold \( i_H \) and lower threshold \( i_L \) by multiple executions of switching devices, and the electromagnetic torque will be changed accordingly. Figure 14a shows the typical winding current waveform under CCC. Similarly, when keeping the turn-on angle and turn-off angle constant, the PWM changes the average excitation voltage by modulating the effective time width of applied voltage \( u_s \) on the conduction phase. Typical PWM winding current waveform is shown in Fig. 14b.

In addition, by considering the vehicle driving conditions, it can be found that the airgap eccentricity mostly occurs at the vertical position of in-wheel motor affected by road excitation, while the airgap eccentricity displacement in the driving direction is neglected. In particular, for the external rotor SRM
where winding phase position is constant, the control of its driving torque and unbalanced radial force should be precise.

Based on the characteristics mentioned above, an independent current chopping control strategy for in-wheel motor is proposed in this section. The winding current is chopped and limited in the vertical phase to suppress the unbalanced radial force caused by the road excitation. Meanwhile, the current chopping threshold is not set in the remaining phases to improve the driving torque. Furthermore, a full chopping control strategy is compared to verify the effectiveness of the proposed control strategy. In the full chopping control strategy, the current chopping threshold is set on all winding phases without considering the position particularity of unbalanced radial force. The logic diagram of the control strategy is shown in Fig. 15.

The proposed control strategy is mainly composed and achieved by PWM signal generator, APC angle controller and CCC signal generator. In the proposed control strategy, the turn-on angle is set to $5^\circ$, turn-off angle to $25^\circ$, voltage excitation to $240$ V and chopping current threshold to $22$ A for in-wheel motor driving system.

For PWM signal generator, it is primarily related to the predetermined vehicle driving speed. Specially, the driving torque required by the vehicle is calculated according to the speed error between the actual vehicle driving speed and the given control strategy speed. The duty cycle data which is calculated by PID controller will be transmitted to PWM signal generator, and the voltage driving signal is generated to realize driving torque control.

As for APC signal generator, it plays a role of executive element for the whole control strategy and regulates the response characteristic with voltage driving signal and current chopping signal as inputs simultaneously. Furthermore, based on the dynamic transfer relationship of in-wheel motor driving electric vehicle, the vehicle driving model and vehicle vibration model are established, so as to realize the closed-loop design of the proposed control strategy.

Last but most important, CCC signal generator, through the flexible regulation for each phase circuit,
is an indispensable component of the built control system. In the independent chopping control strategy, phase A is set as the acting position of road excitation, and the winding current is chopped and limited in this phase to suppress unbalanced radial force caused by road excitation. On the contrary, the chopping current threshold is not set in the other phases (B, C, D) to improve the in-wheel motor driving torque.

4.2 Control strategy implementation

It is assumed that the vehicle drives at 20 km/h on class B road and the phase A is the active phase of unbalanced radial force for the in-wheel motor driving system. The dynamic response of in-wheel motor driving system under independent control strategy is shown in Figs. 16 and 17. It is conspicuous to find that the driving torque under the independent control strategy is obviously better than the full control strategy. In addition, the unbalanced radial force under both control strategies is basically at the same level, and it does not deteriorate due to the independent chopping for winding current.

Furthermore, the vehicle driving speed error and average voltage duty cycle under the control strategy are shown in Fig. 18. It is obvious that the average voltage duty cycle under the full current chopping is larger than that under the independent current chopping. However, the increase in the duty cycle cannot effectively avoid driving torque loss. On the contrary, the independent current chopping control can suppress the vehicle driving speed error by setting the current chopping selection reasonably. In summary, the independent current chopping control strategy for in-wheel motor driving system can improve the driving torque and ensure vehicle power performance while suppressing the unbalanced radial force.

4.3 Control strategy optimization

The vehicle often faces the demand of changing driving speed, especially in starting condition. It is difficult to achieve synergetic optimal performance of the in-wheel motor driving system under fixed controller parameters. To accomplish the optimal control of the in-wheel motor driving system and reduce the negative airgap eccentricity effect, an optimization strategy is discussed in this section. The controller parameters including the turn-off angle $\theta_i$ and chopping current threshold $i_t$ are selected by data interpolation under different vehicle driving speed intervals. The vehicle sprung mass acceleration is taken as feedback signal.

Furthermore, the vehicle driving acceleration is set to 1.25 m/s$^2$ in the optimization process. Since the driving speed and acceleration are low in this case, road excitation can be neglected. The unbalanced radial force $F_u$ minimization, vehicle sprung mass acceleration $v_s$ minimization, tire bounce $b_t$ minimization and the driving torque $T_d$ maximization are set together as the optimization objectives. According to the established optimization objective, the multi-objective optimization function is defined as follows:

![Fig. 16 Controlled response characteristics of in-wheel motor driving system. a average driving torque; b average unbalanced radial force](image)
\[
F(x_{opt}) = \min \left\{ w_1 \frac{F_u}{(F_u)_{int}} + w_2 \frac{v_s}{(v_s)_{int}} + w_3 \frac{b_t}{(b_t)_{int}} \right\} + \max \left\{ w_4 \frac{T_d}{(T_d)_{int}} \right\}
\]
\[\begin{align*}
x &= [\theta_t \in (20, 26)^\circ, \ i_t \in (18, 26)/A]
\end{align*}\]

where \(w_1-4\) are the weight factors and its values are all equal, and \(\theta_{int}\) is the amplitude of different optimization variable with initial controller parameters.

In the optimization process, the linear interpolation and spline interpolation methods are employed, respectively, to select the turn-off angle and chopping current threshold using NSGA-II (elitist Non-nominated Sorting Genetic Algorithm) optimization algorithm. The controller parameters obtained by the linear interpolation and spline interpolation are shown in Fig. 19. Compared with the fixed controller parameters, the interpolated optimal controller parameters can effectively improve the response characteristics of the in-wheel motor driving system and the vehicle dynamic performance. The simulation results after optimization are shown in Figs. 20 and 21 and Table 3.

In terms of the response characteristics of in-wheel motor driving system, the RMS value of the unbalanced radial force decreased by 18.67% and 20.26%, respectively, under linear and spline interpolation control. Due to the coupling conflict between the unbalanced radial force and driving torque, the increase in one term will inevitably lead to the decrease in the other. Therefore, the driving torque of in-wheel motor driving system is sacrificed by 3.32% and 5.50%, respectively, under different interpolation method. Combined with Fig. 20, it can be observed that both the unbalanced radial force and driving torque decrease during vehicle running, but the sacrificing of driving torque is far worth that the suppression of unbalanced radial force.

The vehicle dynamic performance after optimum control is also improved, and its sprung mass
Fig. 19  Optimized in-wheel motor controller parameters.  

(a) turn-off angle;  
(b) chopping current threshold

---

Fig. 20  Optimized response characteristics of in-wheel motor driving system.  

(a) unbalanced radial force;  
(b) driving torque

---

Fig. 21  Optimized vehicle dynamic performance.  

(a) sprung mass acceleration;  
(b) tire bounce
The acceleration RMS value decreased by 37.23% and 28.72%, respectively, and the sprung mass acceleration shock at the vehicle starts is well-restrained. Correspondingly, the tire bounce is also well-suppressed. Since the airgap length of in-wheel motor is very small, the reduction in tire bounce plays an important role in the structural impact for in-wheel motor driving system.

5 Conclusions

This paper focuses on the negative airgap eccentricity effect for in-wheel motor driving system caused by road excitation. Firstly, the electromagnetic characteristics and response characteristics of the SRM under airgap eccentricity are explored, and the radial electromagnetic force under different airgap eccentricity is measured by the laboratory-built device. Then, the vibration analysis of the in-wheel motor driving system is implemented according to the coupling relationship between the road excitation, airgap eccentricity and unbalanced radial force. Both time domain and frequency domain are studied. Finally, an independent current chopping control strategy for the in-wheel SRM driving system is proposed based on vehicle vibration feedback. The main contributions of this paper are shown as follows:

1. The electromagnetic characteristic and response characteristics of the SRM are highly nonlinear, and its magnitude varies nonlinearly with the change of angular position and current excitation. Airgap eccentricity leads to the increase in magnetostatic torque and radial electromagnetic force at the same time. Therefore, the output balance of them should be considered in engineering practice.

2. With the deterioration of road excitation, the airgap eccentricity and unbalanced radial force of the in-wheel motor driving system both increase significantly. The frequency-domain analysis results show that the unbalanced radial force affects the vehicle sprung mass at low frequency, while the vibration response at middle and high frequencies concentrate on the vehicle unsprung mass.

3. The proposed independent current chopping control strategy of the in-wheel motor driving system can effectively improve the driving torque while suppressing the vehicle driving speed fluctuation by controlling the current excitation. For control strategy optimization, the optimized controller parameters can achieve the optimum response between the driving torque and the unbalanced radial force. It effectively suppressed the vehicle sprung mass acceleration and tire bounce under vehicle starting condition.

Starting from the coupling mechanism about negative dynamic effect of the in-wheel SRM driving system, this paper seeks optimum control between the response characteristic of the in-wheel motor driving system and the vehicle dynamic performance, which lays an engineering foundation for the application and promotion of the EVs driven by in-wheel motor.

Acknowledgements This research is supported by the National Natural Science Foundation of China (Grant No. 52072054), the Science and Technology Research Program of Chongqing Municipal Education Commission (KJQN202100728), the Technology Innovation and Application Development of Chongqing Municipality (cstc2019jscx-zdztxxX0047).

Funding This work was supported by National Natural Science Foundation of China (52072054).

Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.
Appendix A: Structure parameters about the studied SRM and EV

| Parameter | Value | Unit | Meaning |
|-----------|-------|------|---------|
| $D_r$     | 382   | mm   | Outer diameter of rotor |
| $D_s$     | 266   | mm   | Outer diameter of stator |
| $\beta_r$ | 23    | deg  | Rotor pole arc angle |
| $\beta_s$ | 22    | deg  | Stator pole arc angle |
| $L_g$     | 0.5   | mm   | Airgap length |
| $L_s$     | 46    | mm   | Thickness of stator back iron |
| $L_r$     | 32    | mm   | Thickness of rotor back iron |
| $H$       | 74    | mm   | Motor axial length |
| $N$       | 136   | /    | Number of turns per phase |
| $m_b$     | 337.5 | kg   | Sprung mass of vehicle |
| $m_s$     | 37.5  | kg   | Total mass of stator and shell |
| $m_r$     | 65    | kg   | Total mass of rotor and tire |
| $c_s$     | 1450  | N/s  | Suspension damping |
| $k_s$     | 23500 | N/m  | Suspension stiffness |
| $k_r$     | 250000| N/m  | Tire stiffness |
| $k_e$     | 385000| N/m  | Total motor and hub bearing stiffness |

Appendix B: State-space equation system matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -k_m m_r^{-1} - c r & k_m r^{-1} - c m_r^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ k_m m_r^{-1} & c r m_r^{-1} - (k_s + k_r) m_r^{-1} - c m_r^{-1} & k_m r^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & k_m r^{-1} & 0 & - (k_s + k_r) m_r^{-1} - c m_r^{-1} \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ -m_r^{-1} & 0 & 0 \\ 0_{3 \times 3} & 0 & 0 \\ m_r^{-1} & k_m r^{-1} & 0 & 0 \end{bmatrix} ; C = \begin{bmatrix} -k_m m_r^{-1} - c m_r^{-1} & k_m r^{-1} & c m_r^{-1} & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0_{3 \times 6} \\ 0_{3 \times 6} \\ 0_{3 \times 6} \\ 0_{3 \times 6} \\ 0_{2 \times 6} \\ 0_{2 \times 6} \end{bmatrix}$$

References

1. Fernandes, J.C.M., Gonçalves, P.J.P., Silveira, M.: Interaction between asymmetrical damping and geometrical nonlinearity in vehicle suspension systems improves comfort. Nonlinear Dyn. 99(2), 1561–1576 (2020)

2. Rezig, A., Boudendouna, W., Djerdir, A., Diaye, N.: Investigation of optimal control for vibration and noise reduction in-wheel switched reluctance motor used in electric vehicle. Math. Comput. Simul. 167, 267–280 (2020)

3. Li, Y., Sun, W., Huang, J., Zheng, L., Wang, Y.: Effect of vertical and lateral coupling between tyre and road on vehicle rollover. Veh. Syst. Dyn. 51(8), 1216–1241 (2013)

4. Jamil, M.U., Kongprawechnon, W., Chayopitak, N.: Active fault diagnosis of a switched reluctance motor using sliding mode observer and average torque estimator for light electric vehicle applications. Int. Trans. Electr. Energy 30(11), 1-2602 (2020)

5. Pomponi, C., Scalzi, S., Pasquale, L., Verrelli, C.M., Marino, R.: Automatic motor speed reference generators for cruise and lateral control of electric vehicles with in-wheel motors. Control Eng. Pract. 79, 126–143 (2018)

6. Zhang, D., Qi, T., Wang, S., Ling, Z.: Effect of series/parallel circuits of eccentric switched reluctance motor on vehicle ride comfort. J. Braz. Soc. Mech. Sci. 43(4), 1–12 (2021)

7. Sun, X., Ban, W., Lei, G., Tian, X., Guo, Y., Zhu, J.: Multi-objective and multiphysics design optimization of a switched reluctance motor for electric vehicle applications. IEEE Trans. Energy Conver. 36, 3294–3304 (2021)

8. Chen, X., Chen, R., Deng, T.: An investigation on lateral and torsional coupled vibrations of high power density PMSM rotor caused by electromagnetic excitation. Nonlinear Dyn. 99(3), 1975–1988 (2020)

9. Chen, X., Deng, Z., Hu, J., Deng, T.: An analytical model of unbalanced magnetic pull for PMSM used in electric vehicle: numerical and experimental validation. Int. J. Appl. Electrom. 54(4), 583–596 (2017)

10. Zhu, Y., Wu, H., Zhen, C.: Regenerative braking control under sliding braking condition of electric vehicles with switched reluctance motor drive system. Energy 230, 120901 (2021)

11. Wang, Y., Li, Y., Sun, W., Zheng, L.: Effect of the unbalanced vertical force of a switched reluctance motor on the stability and the comfort of an in-wheel motor electric vehicle. Proc. Inst. Mech. Eng. Part D J. Autom. Eng. 229(12), 1569–1584 (2015)

12. Liu, F., Xiang, C., Liu, H., Han, L., Wu, Y., Wang, X.: Nonlinear vibration of permanent magnet synchronous motors in electric vehicles influenced by static angle eccentricity. Nonlinear Dyn. 90(3), 1851–1872 (2017)

13. Zuo, S., Liu, Z., Hu, S.: Influence of rotor eccentricity on radial electromagnetic force characteristics in switched reluctance motors and compensation. Electr. Power Compos. Syst. 48(4–5), 388–398 (2020)

14. Hu, S., Zuo, S., Liu, M., Wu, H., Liu, Z.: Modeling and analysis of radial electromagnetic force and vibroacoustic behaviour in switched reluctance motors. Mech. Syst. Signal Pr 142, 106778 (2020)
15. Wang, Y., Li, P., Ren, G.: Electric vehicles with in-wheel switched reluctance motors: coupling effects between road excitation and the unbalanced radial force. J. Sound Vib. 372, 69–81 (2016)

16. Torkaman, H., Afjei, E., Yadegari, P.: Static, Dynamic, and mixed eccentricity faults diagnosis in switched reluctance motors using transient finite element method and experiments. IEEE Trans. Magn. 48(8), 2254–2264 (2012)

17. Li, Z., Zheng, L., Ren, Y., Li, Y., Xiong, Z.: Multi-objective optimization of active suspension system in electric vehicle with in-wheel-motor against the negative electromechanical coupling effects. Mech. Syst. Signal PR 116, 545–565 (2019)

18. Ahmed, F., Kalita, K., Nemade, H.B.: Torque and controllable radial force production in a single winding bearingless switched reluctance motor with a speed controlled drive operation. Int. Trans. Electr. Energy 30(5), 12312 (2020)

19. Chen, X., Yuan, S., Peng, Z.: Nonlinear vibration for PMSM used in HEV considering mechanical and magnetic coupling effects. Nonlinear Dyn. 80(1–2), 541–552 (2015)

20. Tan, D., Lu, C.: The Influence of the magnetic force generated by the in-wheel motor on the vertical and lateral coupling dynamic of electric vehicles. IEEE Trans. Veh. Technol. 65(6), 4655–4668 (2016)

21. Sun, W., Li, Y., Huang, J., Zhang, N.: Vibration effect and control of in-wheel switched reluctance motor for electric vehicle. J. Sound Vib. 338, 105–120 (2015)

22. Shao, X., Naghdy, F., Du, H., Qin, Y.: Coupling effect between road excitation and an in-wheel switched reluctance motor on vehicle ride comfort and active suspension control. J. Sound Vib. 443, 683–702 (2019)

23. Li, Z., Zheng, L., Gao, W., Zhan, Z.: Electromechanical coupling mechanism and control strategy for in-wheel-motor-driven electric vehicles. IEEE Trans. Ind. Electron. 66(6), 4524–4533 (2019)

24. Deng, Z., Li, X., Liu, T., Zhao, S.: Modeling and suppression of unbalanced radial force for in-wheel motor driving system. J. Vib. Control 28, 3108–3119 (2021)

25. Qin, Y., He, C., Shao, X., Du, H., Xiang, C., Dong, M.: Vibration mitigation for in-wheel switched reluctance motor driven electric vehicle with dynamic vibration absorbing structures. J. Sound Vib. 419, 249–267 (2018)

26. Xiong, S.: Study on optimization technology of instantaneous torque control strategy for switched reluctance motor. Chem. Eng. Trans. 66, 1267–1272 (2018)

27. Xue, X.D., Cheng, K.W.E., Ng, T.W., Cheung, N.C.: Multi-objective optimization design of in-wheel switched reluctance motors in electric vehicles. IEEE Trans. Ind. Electron. 57(9), 2980–2987 (2010)

28. Khalil, A., Husain, I.: A Fourier series generalized geometry-based analytical model of switched reluctance machines. IEEE Trans. Ind. Appl. 43(3), 673–684 (2007)

29. Fahimi, B., Suresh, G., Mahdavi, J., Ehsami, M.: A new approach to model switched reluctance motor drive application to dynamic performance prediction, control and design. IEEE, pp. 2097–2102. (1998)

30. Ma, C., Qu, L.: Multiobjective optimization of switched reluctance motors based on design of experiments and particle swarm optimization. IEEE Trans. Energy Conver. 30(3), 1144–1153 (2015)

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