Optical lattice platform for the SYK model

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The tractability of the Sachdev-Ye-Kitaev (SYK) model at large $N$ limit makes it ideal to theoretically study its chaotic non-Fermi liquid behavior and holographic duality properties. We show that complex SYK Hamiltonian emerges from a system of spinless itinerant fermions in optical Kagome lattice with a strong disorder. We discuss the regimes supporting flat band spectra in Kagome lattice, where the system can be non-dispersive. Random interaction between non-dispersive fermions is induced due to randomly distributed immobile impurities in the optical lattice, that exclude the presence of itinerant fermions at their locations. We show that the proposed setup is a reliable experimental platform to realize the SYK model and study its exotic behavior. We show that the velocity distribution of the released fermions is a sensitive probe of the many-body Wigner-Dyson spectral density of states while the averaged many-body Loschmidt echo scheme can measure two-point out-of-time-ordered correlation functions of the SYK system.

The Sachdev-Ye-Kitaev (SYK) model, studied by Sachdev and Ye in Ref. 1 and reexamined recently by Kitaev in Refs. 2 and 3, has attracted much interest as a strongly interacting system, which exhibits many prominent properties of modern theoretical physics including non-Fermi liquid behavior, AdS/CFT duality, fully chaotic behavior, and aspects of integrability. The model is an excellent building block for systems where all these properties reveal themselves and lead to fascinating physics that can be studied effectively. 4–15 The SYK model is solvable, whose two-point correlation function shows non-Fermi liquid behavior, and the out-of-time-ordered correlation (OTOC) function displays a maximal Lyapunov exponent. 16 At low energy, the SYK model has an emergent conformal symmetry and is dual to an extremal black hole in near AdS2 space. 13

Experimental realization of the SYK model is an important task, which would allow testing the basic understanding of the physics behind it. There is already an activity in this direction. Several possible realizations of SYK model in experiments were proposed, such as Majorana SYK model at the interface of topological insulator and superconductor. 18 Majorana SYK model with quantum dot coupled to an array of topological superconducting wires 19, real SYK model in optical lattice loaded with atoms and molecules 20 and complex SYK model in graphene flake in magnetic field. 21 A vital ingredient of the SYK model is that it is zero space dimensional. Therefore, in order to fabricate the SYK model experimentally in real $d$-dimensional space, it is necessary to eliminate the momentum dependence of the spectrum and make a flat band. In Ref. 21 this problem is solved by introducing a strong magnetic field, which forces energy levels of the system to become flat Landau levels. Besides the flatness of the band, one also needs to generate random couplings for interactions of fermions. In Ref. 21 this problem is solved by considering an ensemble of samples with random boundary conditions. In this paper, we present another scheme of flat band formation and randomization of the interaction coupling. We propose a concrete realistic realization of the SYK model with cold atoms in an optical lattice of Kagome type, which gives straightforward ways of detecting its nontrivial properties experimentally. Among those is the measurement of the distribution of the particle velocities after deconfinement of the optical lattice. It will manifest the many-body Wigner-Dyson spectral density of states. The second possibility is the measurement of the averaged two-point OTOC function.

The Kagome optical lattice realized in Ref. 22 shows a flat band 23, which is an ideal playground for studying...
enhanced interaction effects of particles. In this paper, we show that the low energy effective theory of spinless fermions in optical Kagome lattice with strong disorder realizes the complex SYK model. Unlike previous proposals, this method does not need superconductors, strong magnetic fields, while disorder can be tuned.

We now concentrate on the details of the Hamiltonian describing the proposed scheme. It consists of three terms: $H = H_0 + H_{\text{imp}} + H_{\text{int}}$, with the tight-binding Hamiltonian $H_0$ of itinerant ultracold fermions supporting a flat band, the impurity Hamiltonian $H_{\text{imp}}$ describing a set of randomly distributed onsite $\delta$-function potentials, and a short-range interaction Hamiltonian $H_{\text{int}}$. The tight-binding Hamiltonian on Kagome lattice (see fig. 1a) is given by:

$$H_0 = -\mu \sum_m a_{r_m}^\dagger a_{r_m} - t e^{i\varphi} \sum_{<m,n>} a_{r_m}^\dagger a_{r_n}. \quad (1)$$

Here $a_{r_m}^\dagger$ and $a_{r_m}$ are creation and annihilation operators of spinless fermions loaded into optical lattice. $\mu$ is their chemical potential, $t$ is the hopping parameter and $\varphi$ is the phase of the hopping that can be tuned with the help of artificial gauge fields.\textsuperscript{27–30} The momentum space representation of the hopping Hamiltonian, $H_0$, reads:

$$H_0(k) = -\mu (a_k^{(a)\dagger} a_k^{(a)} + a_k^{(b)\dagger} a_k^{(b)} + a_k^{(c)\dagger} a_k^{(c)}) - 2 t e^{-i\varphi} \cos(k \cdot b) a_k^{(a)\dagger} a_k^{(b)} - 2 t e^{i\varphi} \cos(k \cdot c) a_k^{(a)} a_k^{(c)} - 2 t e^{-i\varphi} \cos(k \cdot c - k \cdot b) a_k^{(b)\dagger} a_k^{(c)} + h.c. \quad (2)$$

Here $a_k^{(a)}$ and $a_k^{(c)}$ ($a = a, b, c$) are creation and annihilation operators of fermion on sublattice $\alpha$ with momentum $k$, $b = (\frac{1}{3}, \frac{\sqrt{3}}{3})$, $c = (\frac{1}{3}, 0)$ and $k = (k_x, k_y)$. The spectrum of the hopping Hamiltonian $H_0$, which describes the lattice subject to the staggered flux depicted in fig. 1a, is found from the characteristic equation $\det(EI - H_0(k)) = 0$. The latter acquires the form

$$x^3 - \frac{3}{2} + A(k) x - \frac{1}{2} (\cos(3\varphi) + A(k) \cos(3\varphi)) = 0, \quad (3)$$

where $x = -\frac{E + \mu}{2t}$ and $A(k) = k_x + 2 \cos \frac{k_y}{\sqrt{3}} \cos \frac{\sqrt{3}k_x}{2}$. If one of three energy bands of the Hamiltonian is non-dispersive (flat), the corresponding $x$ should be independent of $k$. This implies $x + \cos(3\varphi) = 0$, $x^3 - \frac{3}{2} x - \frac{1}{2} \cos(3\varphi) = 0$. Noting that $\varphi$ and $\varphi + 2\pi/3$ are equivalent since $2\pi/3$ can be gauged out, the possible values for $\varphi$ supporting a flat band are $\varphi = 0, \frac{\pi}{3}, \pi$, and the corresponding band structures are shown in figs. 1b to 1d where flat bands located at top, middle and bottom respectively.

The real space expression of the impurity Hamiltonian corresponding to randomly distributed onsite impurities reads

$$H_{\text{imp}} = u \sum_{r_m \in R} a_{r_m}^\dagger a_{r_m}, \quad (4)$$

where $u$ is on-site potential, $R$ is a random set of $M$ sites.

The flat band of the Kagome lattice originates from the structural destructive interference containing degenerate localized states circulating the hexagons of the Kagome lattice.\textsuperscript{31–32} The impurity Hamiltonian, $H_{\text{imp}}$, in turn, connects wave-functions between hexagons that are next to each other while keeping the localization property intact in general. Hence $H_{\text{imp}}$ will remove a number of states from the flat band, but most of them will still remain there (fig. 2). With rotational symmetry, the spectrum of $H_0$ is six-folded. The impurity Hamiltonian $H_{\text{imp}}$ breaks the rotational symmetry making the spectrum smeared.
Finally, the interaction Hamiltonian is given by
\[ H_{\text{int}} = \frac{1}{2} \sum_{m} \rho_{r_m} V(r_m - r_n) \rho_{r_n}, \] (5)
where \( \rho_{r_m} = a_{r_m}^{\dagger} a_{r_m} \) is the particle number operator on site \( m \), \( V(r) \) is a short-range two-body interaction.

Below, we will concentrate on \( V(r) = V_0 e^{-r/\sigma} \).

Now we will show that the free Hamiltonian, \( H_0 + H_{\text{imp}} \), together with the perturbation \( H_{\text{int}} \), is capable of generating the SYK model. Consider \( N \) particles with wave-functions \( \phi_i(r_m), i = 1, \ldots, N \), that are accommodated within the flat band due to the fine-tuned chemical potential (e.g. keeping the lattice filling fraction \( \nu \leq 1/3 \) for \( \varphi = \pi; 1/3 < \nu \leq 2/3 \) for \( \varphi = \pi/6 \); and \( 2/3 < \nu \leq 1 \) for \( \varphi = 0 \)). The second quantized wave function of the fermion at cite \( r_m \) can be expanded over the basis of flat band wave functions as \( a_{r_m} = \sum_i \phi_i(r_m)c_i \), where the \( c_i \) is an annihilation operator of that state. In terms of these operators we can get the low-temperature effective Hamiltonian for the degenerate ground state\(^{21}\)
\[ H_{\text{eff}} = (2t - \mu) \sum_i c_i^{\dagger} c_i + \sum_{ijkl} \tilde{J}_{ijkl} c_i^{\dagger} c_j c_k c_l, \] (6)
with
\[ \tilde{J}_{ijkl} = \frac{1}{2} \sum_{r_1 r_2} [\phi_i(r_1) \phi_j(r_2)]^* V(r_1 - r_2) [\phi_k(r_1) \phi_l(r_2)], \] (7)
where \( \tilde{t} = t \) for \( \varphi = 0 \), \( \tilde{t} = 0 \) for \( \varphi = \pi/6 \) and \( \tilde{t} = -t \) for \( \varphi = \pi \). \( \phi_i(r) \) is the wave function of the \( i \)-th degenerate state, \( r_{i,j} \) is the lattice sites and \( c_i^{\dagger} \) and \( c_i \) are creation and annihilation operators of fermionic modes residing in the flat band. Using the anti-commutation relations of creation and annihilation operators, it is convenient to equivalently rewrite the effective Hamiltonian \( H_{\text{eff}} \) as
\[ H_{\text{eff}} = (2\tilde{t} - \mu) \sum_i c_i^{\dagger} c_i + \sum_{i>j,k>l} J_{ijkl} c_i^{\dagger} c_j c_k c_l, \] (8)
where
\[ J_{ijkl} = \tilde{J}_{ijkl} + \tilde{J}_{ikjl} - \tilde{J}_{ijlk} - \tilde{J}_{jilk}. \] (9)

Here we introduced ordering of indices in the Hamiltonian, and now will show that the resultant couplings, \( J_{ijkl} \), are fully random. Suppose \( J_{ijkl} \) and \( J_{ijkl} \) are calculated in the basis \( \{\phi_i\} \). Under basis transformation \( \phi'_i = \sum U_{vi} \phi_v \), where \( U_{vi} \) is a unitary matrix, \( U^{T} U = 1 \), the couplings transform as
\[ J'_{ij'j'k'k'} = \sum_{ijkl} U_{ij'} U_{j'j} U_{k'k} U_{l'k} J_{ijkl}, \]
\[ J'_{ij'j'k'k'} = \sum_{ijkl} U_{ij'} U_{j'j} U_{k'k} U_{l'k} J_{ijkl}. \]
We see that in the absence of impurities, the specific values of \( J_{ijkl} \) are basis dependent. Fortunately, if couplings \( J_{ijkl} \) are independent Gaussian random variables, \( J_{ijkl}'s \) are also independent Gaussian random variables with the same variance. So the distribution will be independent of basis.

Interestingly, from eq. (9), we have
\[ \sum_{i>j,k>l} J_{ijkl} = \frac{1}{2} \sum_{r_1 r_2} V(r_1 - r_2) \sum_{i>j,k>l} [\phi_i(r_1) \phi_j(r_2) - \phi_j(r_1) \phi_i(r_2)]^* [\phi_k(r_1) \phi_l(r_2) - \phi_l(r_1) \phi_k(r_2)] \]
\[ = \frac{1}{2} \sum_{r_1 r_2} V(r_1 - r_2) \sum_{i>j} (\phi_i(r_1) \phi_j(r_2) - \phi_j(r_1) \phi_i(r_2))^2 \]
\[ \geq 0. \] (10)

This property implies that we have a constraint on the \( J_{ijkl} \) couplings, which removes one degree of freedom. Fortunately, if the number of random \( J_{ijkl} \) is large (\( N \gg 1 \)), the degree of freedom is large, and hence the constraint will not affect the statistical properties.

Consider now \( n \) particles in the flat-band Kagome lattice described by the Hamiltonian \( H_0 + H_{\text{imp}} \) with chemical potential \( \mu \) that insures the Fermi surface to lie in the flat band. With given \( n \), the conserved charge \( Q = \sum_i (c_i^{\dagger} c_i - \frac{1}{2}) \) has the eigenvalue \( q = n - N/2 \) (\( N \) is the number of states in the flat band). For even \( N, q \) would be an integer, and for odd \( N, q \) would be a half-integer. Since \( Q \) commutes with the Hamiltonian eq. (8), we can diagonalize the Hamiltonian within a specific \( q \) subspace.

As the next step, we exactly diagonalize the Hamiltonian \( H_0 + H_{\text{imp}} \) and calculate the couplings \( J_{ijkl} \) using eq. (9) and then calculate the thermodynamic entropy of the effective Hamiltonian eq. (8). The results are plotted in figs. 3a and 3e with phase \( \varphi = 0 \) and figs. 3b and 3f with \( \varphi = \pi \). In all cases, the distribution of \( J_{ijkl} \) is nearly Gaussian, which is the defining property of the disordered couplings in the SYK model. Also, the entropy agrees with that of the SYK model. At the high-temperature limit \( T/J \gg 1 \), the entropy approaches its maximum value \( S_n = k_B \ln \binom{N}{n} \). As the temperature goes to zero, for infinite \( N \), the entropy of the SYK model tends to a finite number. While for finite \( N \), the entropy goes to zero as expected by the third law of thermodynamics. It can be shown that the averaged difference of entropies at finite \( N \) and infinite \( N \) is proportional to \( 1/N \).\(^{32}\)

The SYK Hamiltonian eq. (8) without a chiral symmetry will experience no extra constraint when \( q = 1 \).\(^{33}\) All the couplings \( J_{ijkl} \) with chosen parameters can be real in a certain basis. So the probability distribution of energy spacings (defined as distribution of \( r_n = (E_{n+1} - E_n)/(E_n - E_{n-1}) \) where \( E_n \) is the energy of the \( n^{th} \) level) will exhibit behavior inherent to the Gaussian orthogonal ensemble (GOE)\(^{34}\) as shown in the figs. 4a and 4b. For \( N = 15 \) case, \( q \) is always non-zero. The level statistics is the same as for \( q \neq 0 \) situation at \( N = 14 \).

For \( \varphi = \pi/6 \), the same quantities, namely distribution of couplings \( J_{ijkl} \), the entropy(figs. 5c and 5d), and level statistics(fig. 4c) are calculated with \( u = t \). Now one has complex couplings \( J_{ijkl} \), hence the distribution of \( |J_{ijkl}| \) becomes Chi distribution with degree of freedom.
two. Also the level statistics follows the distribution of the Gaussian unitary ensemble (GUE) \([21]\).

While the distribution of \(r_n\) shows the correlations between adjacent energy levels, the spectral form factor defined as

\[
Z(J/T+iJ\tau)Z(J/T-iJ\tau) = \sum_{n,m} e^{-(J/T+iJ\tau)E_n} e^{-(J/T-iJ\tau)E_m},
\]

where \(Z\) is the partition function of the model, characterizes correlations between all energy levels at all scales \([22]\). On the gravity side, this quantity describes the properties of the black hole in the dual AdS space \([23,24]\).

The averaged spectral form factor \(\langle |Z(J/T+iJ\tau)|^2 \rangle_f\) is shown in fig. 5. One can see that at short times, the slope regime is dominated by the decoupled SYK saddle points \([35]\), for which it decays with a power law. For relatively high temperature, the late time ramp and plateau originate from the statistics of the random matrix ensemble. Similar behavior exists in the Jackiw-Teitelboim gravity \([25]\). As temperature decreases, the height of the ramp tends to zero. The behavior can be captured by the Brownian SYK model, in which the random couplings are independent in time. One can notice that the flat-band model which breaks the time-reversal symmetry \((\varphi = \pi/6)\) is more robust against the temperature, which make it a better platform for experimental realization.

Importantly, the averaged two-point OTOC function, \(\int dA \langle A(0)A^\dagger(t) \rangle_f\), where \(A\) is a local unitary operator one can access in experiment and \(dA\) is the Haar measurement with respect to \([34]\) is proportional to the spectral form factor \(\langle |Z(2T,\tau)|^2 \rangle\). OTGC can be measured by many-body Loschmidt echo scheme \([35,36]\).

Upon releasing the trap and the optical lattice in time
of flight experiments, the fermion states are projected onto plain waves, and the many-body energy spectrum distribution of the flat-band system (given by the Wigner semicircle) is projected to the energy distribution of atoms. Hence, the velocity distribution of atoms, as free particles, can be determined from many-body Wigner-Dyson statistics, as seen in fig. 6. The latter can be observed in the time of flight experiments. We notice that the distribution of energy and the velocity predicted for the present scheme has a somewhat long tail, which is attributed to the small variation of $J$ when averaging over different disordered realizations.

In the following, we discuss aspects of the experimental realization of the proposed scheme: i) The randomly distributed on-site potential can be realized by heavy atoms randomly loaded in the optical lattice whose strength can be tuned by Feshbach resonances. ii) Positive or complex hopping can be realized with the help of artificial gauge fields created by applying a zero averaged homogeneous inertial force or Raman-assisted tunneling in asymmetric Kagome lattice. iii) The strength of $J$ can be estimated assuming the coarse grained single-particle wavefunctions are independent random variables. $J \approx \frac{2k_{\text{B}}}{\xi} (\frac{N}{L})^{3/2} \sqrt{\pi \Gamma(0, \frac{2a}{\xi})}$, where $a$ is the lattice constant, $\xi$ is the coarse-grained length in the unit of $a$, $N$ is the number of states in the flat band, $L$ is number of sites in optical lattice, $\Gamma(x, y)$ is the incomplete Gamma function. For Kagome lattice, $N/L \approx 1/3$ and $\xi$ is of the order 1. So one can tune the value of $J$ by tuning the interaction strength.

The realization of the SYK model within the described technique is not specific to the Kagome lattice. We believe these phenomena are quite universal, and the described technique can be applied to other lattices supporting a flat band. This, for example, can be seen upon investigating SYK physics from the interplay of disorder and interactions in the experimentally realized Lieb lattice using trapped fermions.

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**FIG. 5:** The spectral form factor is plotted at $\varphi = 0$ (left column) and $\varphi = \pi/6$ (right column) for $J/T = 2, 4, 6$ respectively from top to bottom for the system with the number of states in flat band $N = 14$.

**FIG. 6:** Energy spectral density and the velocity distribution after releasing the atoms (dotted curves correspond to the numerical results of the effective Hamiltonian eq. (5), dash-dotted curves correspond to the exact diagonalization results of the SYK model, and solid curves correspond to the Wigner semicircle law of the random matrix theory) at $N = 14$. Phases are $\varphi = 0$ for panels (a) and (c), and $\varphi = \pi/6$ for panels (b) and (d). The velocity is measured in the units of $v_0 = \sqrt{\frac{2k_{\text{B}}}{m}}$, where $m$ is the mass of the itinerant atoms.
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