Multigroup-Decodable STBCs from Clifford Algebras

Sanjay Karmakar and B. Sundar Rajan
Department of ECE, Indian Institute Of Science
Bangalore, India-560012
Email: {sanjay,bsrajan}@ece.iisc.ernet.in

Abstract—A Space-Time Block Code (STBC) in $K$ symbols (variables) is called $g$-group decodable STBC if its maximum-likelihood decoding metric can be written as a sum of $g$ terms such that each term is a function of a subset of the $K$ variables and each variable appears in only one term. In this paper we provide a general structure of the weight matrices of multi-group decodable codes using Clifford algebras. Without assuming that the number of variables in each group to be the same, a method of explicitly constructing the weight matrices of full-diversity, delay-optimal $g$-group decodable codes is presented for arbitrary number of antennas. For the special case of $N_1 = 2^n$ we construct two subclass of codes: (i) A class of $2n$-group decodable codes with rate $\frac{n}{2^{n-1}}$, which is, equivalently, a class of Single-Symbol Decodable codes, (ii) A class of $(2n-2)$-group decodable with rate $\frac{n}{2^{n-2}}$, i.e., a class of Double-Symbol Decodable codes. Simulation results show that the DSD codes of this paper perform better than previously known Quasi-Orthogonal Designs.

I. PRELIMINARIES AND INTRODUCTION

An $N_1 \times N_2$ linear dispersion STBC [5] with $K$ real variables, $x_1, x_2, \cdots, x_K$ can be written as

$$S(X) = \sum_{i=1}^{K} x_i A_i$$

where $A_i \in \mathbb{C}^{N_1 \times N_2}$ and $X = [x_1, x_2, \cdots, x_K] \in \mathbb{R}^{1 \times K}$ ($\mathbb{C}$ and $\mathbb{R}$ denote respectively the complex and the real field). Now if $X \in \mathcal{A}$, a finite subset of $\mathbb{R}^K$, then assuming that perfect channel state information is available at the receiver the maximum likelihood (ML) decision rule minimizes the metric,

$$M(S) \triangleq \min_{S} \text{tr}((Y - SH)(Y - SH))^H = \| Y - SH \|^2.$$ 

(2)

It is clear that, in general, ML decoding requires $|\mathcal{A}|$ number of computations, one for each codeword. Suppose we partition the set of weight matrices of the above code into $g$ groups, the $k$-th group containing $n_k$ matrices, and also the information symbol vector as, $X = [X_1, X_2, \cdots, X_g]$, where $X_k = [x_{j_1+1}, x_{j_2+2} \cdots x_{j_k+n_k}]$, $j_1 = 0$ and $j_k = \sum_{i=1}^{k-1} n_i$, $k = 2, \cdots, g$. Now, $S(X)$ can be written as,

$$S(X) = \sum_{k=1}^{g} S_k(X_k), \quad S_k(X_k) = \sum_{i=1}^{n_k} x_{j_k+i}A_{j_k+i}.$$ 

If the weight matrices of (1) are such that,

$$S^H(X)S(X) = \sum_{k=1}^{g} S_k^H(X_k)S_k(X_k)$$ 

(3)

and $X_k \in \mathcal{A}_k \subset \mathbb{R}^{n_k}$, $\forall k$ take values independently then using (3) in (2) we get,

$$M(S) = \sum_{k=1}^{g} \| Y - S_k(X_k)H \|^2 - (g - 1) \| Y \|^2$$

(4)

from which it follows that minimizing the metric in (4) is equivalent to minimizing,

$$M(S)_k = \| Y - S_k(X_k)H \|^2$$

for each $1 \leq k \leq g$ individually.

Definition 1: A linear dispersion STBC as in (1) is called $g$-group decodable if its decoding metric in (2) can be simplified as in (4) and the information symbols in each group takes values independent of information symbols in other groups. It is easily that the ML decoding of a $g$-group decodable code requires only $\sum_{k=1}^{g} |\mathcal{A}_k|$ computations which in general is much smaller than $|\mathcal{A}| = \prod_{k=1}^{g} |\mathcal{A}_k|$. Single-Symbol Decodable (SSD) and Double-Symbol Decodable (DSD) codes have been studied extensively [2],[4],[8]. Note that a SSD code in $K$ variables is nothing but a $K$-group decodable code and a DSD code is nothing but a $\frac{K}{2}$-group decodable code. For example, a $4 \times 4$ SSD code [3],[4],[6] or CIOD [6] is $4$-group decodable code, each group containing two real information symbols to be decoded together.

In this paper we study general $g$-group decodable codes using Clifford algebras. The main contributions of this paper can be summarized as follows:

- All known results for $g$-group decodable codes so far, including the most recent one [1] study $g$-group decodable codes in which each group contain the same number of information symbols. In this paper we give a general algebraic structure of the weight matrices of $g$-group decodable codes, where different groups can have different number of information symbols to be decoded together.

- Recently $g$-group decodable codes, for $g = 4, 6, 8$ have been reported in [1]. Due to the recursiveness of the reported construction procedure delay optimal codes for number of transmit antennas, which is not a power of 2 are not obtainable from the techniques of [1]. Whereas due to our general construction procedure being non-recursive, we can construct delay-optimal $g$-group decodable codes even for $N_1 \neq 2^n$. Example (1) of Section (II) is one such code.
• An analytic expression for the diversity product of our code is given (Section IV), using which the full diversity property of the codes is established.

The remaining part of the paper is organized as follows: In Section II we present construction of weight matrices of a class of linear dispersion codes which will facilitate the design of multi-group decodable codes in the following section. In Section III we present our explicit construction of g-group decodable codes for all values of g. A closed form expression for the diversity product of our codes is obtained in Section IV and in Section V we present SSD and DSD codes obtainable from our construction of Section III along with simulation results for one such code.

The proofs of all the theorems and lemmas have been omitted due to lack of space.

II. GENERAL STRUCTURE OF MULTIGROUP DECODABLE CODES

In this section, we describe a construction of weight matrices of a linear dispersion code which will greatly facilitate the design of multi-group decodable codes subsequently. Let there be \((g + 1)\) number of collection of matrices \(G_0, G_1, \cdots G_g\), where

\[
G_0 = \{ A_{0,k} \in \mathbb{C}^{n \times m} \mid k = 1, 2, \cdots g \}
\]

\[
G_1 = \{ A_{1,i} \in \mathbb{C}^{n \times n} \mid i_1 = 1, 2, \cdots n_1 \}
\]

\[
\vdots
\]

\[
G_g = \{ A_{g,i_g} \in \mathbb{C}^{n \times n} \mid i_g = 1, 2, \cdots n_g \}
\]

Now from the above set of matrices we form a new set of following matrices, which can be used as weight matrices of LD codes subsequently:

\[
W = \{ A_{0,k} \otimes A_{i,k} \mid 1 \leq k \leq g; 1 \leq i_k \leq n_k \}
\]

Let \(K = |W| = \sum_{k=1}^{g} n_k\) be the cardinality of \(W\) and the information bits to be transmitted is mapped to a real vector \(X = [x_1, \cdots x_K] \in \mathbb{R}^K\), where \(\mathbb{R}^K\) is finite. Then we construct the corresponding STBC as follows,

\[
S(X) = \sum_{k=1}^{g} \sum_{i=1}^{n_k} x_{j_k+i} A_{0,k} \otimes A_{i,k} = \sum_{k=1}^{g} S_k(X_k);
\]

\[
S_k(X_k) = \sum_{i=1}^{n_k} x_{j_k+i} A_{0,k} \otimes A_{i,k}
\]

where \(X = [X_1, X_2, \cdots X_g]\) and \(X_k = [x_{j_k+1}, \cdots x_{j_k+n_k}] \in \mathbb{R}^{n_k}, 1 \leq k \leq g\).

**Theorem 1:** The linear dispersion code given in (7) is a g-group decodable code, the kth group involving \(n_k\) information symbols of \(X_k\), if the following conditions are satisfied,

\(X_1, X_2, \cdots X_g\) are mutually independent

\(A_{0,j}^H A_{0,j} + A_{0,j} A_{0,j}^H = 0, \forall 1 \leq i \neq j \leq g\)

\(A^H B = B^H A, \forall A \in G_i, B \in G_j, 1 \leq i \neq j \leq g\).

**Theorem 2:** Suppose the \((g + 1)\) set of matrices of (5) satisfy Theorem 1 and moreover, the weight matrices corresponding to \(G_k, 1 \leq k \leq g\) can be subdivided into \(g_k\) subgroups, i.e., \(G_k = G_{k,1} \cup G_{k,2} \cup \cdots \cup G_{k,g_k}\), such that

\[
A_{k}^H B_k + B_k^H A_k = 0, \forall 1 \leq k \leq g,
\]

where \(A_k \in G_{k,i}, B_k \in G_{k,j}, 1 \leq i \neq j \leq g_k\) and the corresponding information vectors, \(X_{k,1} \cdots X_{k,g_k}\), are independent, where \(X_k = [X_{k,1}, X_{k,2} \cdots X_{k,g_k}]\) and \(X_{k,i} \in \mathbb{R}^{G_{k,j}}\). Then the ML decoding of \(X_k\) can further be separated into \(g_k\) subgroups, for each \(1 \leq k \leq g\).

If the collection of matrices in (5) of the STBC given in (7) satisfies Theorem 1 and Theorem 2 simultaneously, then the code is \((\sum_{k=1}^{g} g_k)\)-group decodable.

III. EXPLICIT CONSTRUCTION OF MULTIGROUP DECODABLE CODES

In this section we construct g-group decodable codes for any value of g.

**Theorem 3:** Let \(\tilde{G}\) be a set of \(n \times n\) mutually commuting Hermitian complex matrices and \(G_0\) is a set of weight matrices such that, for any \(A, B \in G_0, A^H B + B^H A = 0\). Now if we choose \(G_1 = \tilde{G}_2 = \cdots G_g = \tilde{G}\), where \(g = |G_0|\) and construct the weight matrices as in (6) and further construct a STBC as in (7), then the resulting code will be a g-group decodable STBC with rate,

\[
R_r = \frac{|\tilde{G}| |G_0|}{mn}
\]

real information symbols per channel use.

Now if we want to construct a g-group decodable code, according to Theorem 3 above we need to select the collection of matrices \(G_0\) with cardinality at least \(g\). But from (9) the rate is dependent on choice of \(G_0\) through \(|G_0|\) and \(m\). So for larger rate it is better to choose \(m\) as small as possible as \(g\) is fixed. This is a very hard problem in general to solve. So we will assume \(G_0\) to be a collection of unitary matrices since the answer to the above question is available in [8] for these cases. The answer is, for \(g\) matrices the minimum value of \(m\) is given by \(m = 2\left \lceil \frac{g-1}{2} \right \rceil\). Note that with this result, we have for every \(g, a g\)-group decodable code for \(N_i = 2\left \lceil \frac{g-1}{2} \right \rceil\) transmit antennas in [8]. Here \(\tilde{G} = \{1\}\) is the trivial set. Now suppose we want a g-group decodable code for \(N_i\) transmit antennas, where \(N_i = m = 2\left \lceil \frac{g-1}{2} \right \rceil\), \(n \geq 2\). Then \(\tilde{G}\) must contain \(n \times n\) Hermitian, mutually commuting complex matrices, according to Theorem 3. But again from (9) the rate of the code (that we are going to construct) depends on the choice of \(\tilde{G}\) through \(|\tilde{G}|\) and \(m\). As \(n\) is fixed \((N_i = mn\) is given and we have found \(m\) during the choice of \(G_0\), we need to make the cardinality of \(\tilde{G}\) as large as possible. Again at this stage we will assume unitarity of the matrices in \(\tilde{G}\). With this assumption we obtain the following lemma on the cardinality of \(\tilde{G}\).

**Lemma 1:** The cardinality of the set \(\tilde{G}\) of Theorem 3 is \(n\) under the unitarity assumption, and the assumption that the resulting code is uniquely decodable.

With this result we see that the code constructed following Theorem 3 will be of rate \(R_r = \frac{|\tilde{G}| |G_0|}{mn} = \frac{2}{m}\) real information symbols per channel use. Note that the construction suggested
in the description above is far from general and the weight matrices of the codes constructed by this method will be unitary.

To explain the construction of the set \( G_0 \) we need irreducible matrix representation of Clifford Algebra.

**Definition 2:** The Clifford algebra, denoted by \( CA_L \), is the algebra over the real field \( \mathbb{R} \) generated by \( L \) objects \( \gamma_k \), \( k = 1, 2, \ldots, L \) which are anti-commuting, \( (\gamma_k \gamma_j = -\gamma_j \gamma_k, \forall k \neq j) \) and squaring to \(-1, (\gamma_k^2 = -1 \ \forall k = 1, 2, \ldots, L) \).

Let
\[
\sigma_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \quad \text{and} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},
\]
\[
\sigma_4 = -j \sigma_2 \quad \text{and} \quad A \otimes^m = A \otimes A \otimes A \ldots \otimes A.
\]

From [8] we know that the representation \( R(\gamma_i), j = 1, 2, \ldots, L \) of the generators of \( CA_{g-1} \) are obtainable in terms of \( \sigma_i \), \( i = 1, 2, 3, 4 \), and explicitly shown in [8].

**A. Construction of \( G_0 \)**

If \( g \) is even, say \( (g - 1) = (2a + 1) \), find the irreducible representation of \( CA_{g-1} \) as described in [8]. Then our required set \( G_0 \) is,
\[
G_0 = \{ R(\gamma_i) = I_{m \times m}, R(\gamma_i), R(\gamma_2), \ldots, R(\gamma(g - 1)) \}
\]

Here \( R(\gamma_i) \in \mathbb{C}^{m \times m} \) and \( m = 2^g - 1 \). Similarly for \( g = 2a + 1 \) odd we find the irreducible representation of \( CA_{g} \) and add to this set the identity matrix. Thus we will get \( g + 1 \) matrices. We can use any \( g \) of them (or we can use all \( g + 1 \) of them and consider any two groups as a single one, this way we can increase the rate).

**B. construction of \( \tilde{G} \)**

Lemma 1 above suggest a construction method of the set \( \tilde{G} \). Following that method, for a given \( n \) we will first find \( n \) linearly independent vectors \( \beta_i \in \{+1, -1\}^n, 1 \leq i \leq n \). Then we will choose an \( n \times n \) unitary matrix. The choice of this matrix is important as explained in Note 1 below. Now we can construct the set as follows,
\[
\tilde{G} = \{ U \text{Diag}(\beta_i) U^H | i = 1, 2, \ldots, n \}.
\]

**NOTE 1:** Note that in the above construction \( U = I_{n \times n} \) may be a choice. But then the resulting matrices will be diagonal and will contain a large number of zero entries. This will lead to a large PAPR of the code. So \( U \) need to be chosen in such a way that the matrices in \( G \) have as small number of zero entries as possible.

As an example we will construct below a 4-group decodable code for \( N_t = 6 \) transmit antennas which is delay optimal. As mentioned earlier this code can’t be obtained following the approach of [1]. This code also has rate 1.

**Example 1:** According to the construction procedure of \( G_0 \) described above we choose,
\[
G_0 = \{ I_2, \sigma_1, \sigma_2, j \sigma_3 \}.
\]

For this example we don’t take the trouble to find an appropriate \( U \) as explained in Note 1. Instead we choose \( U = I_{3 \times 3} \). Thus our set \( \tilde{G} \) is,
\[
G_i = \tilde{G} = \{ \text{Diag}((1, 1, 1)), \text{Diag}((1, 1, 1)), \text{Diag}([-1, 1, 1]), i = 1, 2, \ldots, 4 \}.
\]

With this set of matrices and an information vector \( X = [x_1, \ldots, x_{12}] \), we construct the STBC according to (7) as given in [10] at the top of the next page, where \( z_1 = x_1 + jx_{10}, z_2 = x_2 + jx_{11}, z_3 = x_3 + jx_{12}, z_4 = x_4 + jx_7, z_5 = x_5 + jx_{8}, z_6 = x_6 + jx_9 \).

In the next section we prove that this code is of full diversity by showing that every code constructed according to the Theorem 3 achieve full diversity.

**IV. DIVERSITY PRODUCT OF MULTI-GROUP DECODABLE CODES**

Let \( S(X) \) be a \( g \)-group decodable code, constructed according to Theorem 3 where \( X = [X_1, \ldots, X_g], X_k \in \mathcal{A}_k \subset \mathbb{R}^n, \forall 1 \leq k \leq g \). Let’s also denote \( \mathcal{A}_1 \times \cdots \times \mathcal{A}_g = \mathcal{A} \). Now suppose, \( X \neq \mathcal{X} \) and \( \Delta X = X - \mathcal{X} \). Then,
\[
S(X) - S(\mathcal{X}) = S(\Delta X) = \sum_{k=1}^{g} \sum_{i=1}^{n} \Delta x_{(k-1)n+i} A_{0,k} \otimes A_i, A_{0,k} \in G_0, A_i \in \tilde{G}
\]
and
\[
S^H(\Delta X) S(\Delta X)
\]
\[
= \sum_{i=1}^{n} \left\{ \sum_{i=1}^{n} \Delta x_{(k-1)n+i} A_{m \times m} \otimes A_i \right\}^H \sum_{i=1}^{n} \Delta x_{(k-1)n+i} A_{m \times m} \otimes A_i \right\}.
\]

Now according to construction,
\[
\tilde{G} = \{ A_i = U \text{Diag}(b_i) U^H, i = 1, 2, \ldots, n \}
\]
using which in (11) we get,
\[
S^H(\Delta X) S(\Delta X)
\]
\[
= I_{m \times m} \otimes U \sum_{k=1}^{g} \left\{ \sum_{i=1}^{n} \Delta x_{(k-1)n+i} A_{m \times m} \right\}^2 \otimes \text{Diag}(b_i) \right\} I_{m \times m} \otimes U^H.
\]

Now let,
\[
Y_k = \left[ y_{(k-1)n+1}, y_{(k-1)n+2}, \ldots, y_{(k-1)n+n} \right]
\]
\[
= \begin{bmatrix} b_1^T, b_2^T, \ldots, b_n^T \end{bmatrix}^T [x_{(k-1)n+1}, x_{(k-1)n+2}, \ldots, x_{(k-1)n+n}]^T.
\]

Then, if \( X_k \in \mathcal{A}_k \) then \( Y_k \in \mathcal{B}_k \subset \mathbb{R}^n \). Here \( c \) is chosen so that the average energy of both the constellations \( \mathcal{A}_k \) and \( \mathcal{B}_k \) is same. And as the transform \( T \) is non singular, there is a one-to-one correspondence between the points in \( \mathcal{A}_k \) and
Let's now define, real dimensional constellation, i.e., \( \Delta \), that all the above expression is minimized when \( \Delta \) takes values from the same \( \mathcal{A}_y \). Hence, for only two real symbols. This imply that all \( X_k \) take their values. Now our strategy will be to select a \( \mathcal{A}_y \) with its CPD being maximal [9]. Then we apply a linear transform \( T^{-1} \) to get \( \mathcal{A}_y \). Now allow \( X_k \in \mathcal{A}_y, \forall k \). Thus the resulting code will achieve the maximal (non-zero) diversity product.

**V. CONSTRUCTION OF SSD AND DSD CODES**

In this section we construct SSD codes [6] and DSD codes using a modified version of the construction described in Section III. Towards this end, we first give an alternative construction of the set \( \tilde{G} \) for \( N_t = 2^a, a \in \mathbb{N} \).

**A. Alternative Construction of \( \tilde{G} \)**

For \( N_t = 2^a, a \in \mathbb{N} \) we take the matrices \( \{R(\gamma_1), R(\gamma_2), \ldots R(\gamma_{2a+1})\} \) as given in [8]. From this set we construct \( \{\tilde{A}_1 = jR(\gamma_1)R(\gamma_{a+1}), \tilde{A}_2 = jR(\gamma_2)R(\gamma_{a+2}), \ldots \tilde{A}_a = jR(\gamma_a)R(\gamma_{2a})\} \). It can be easily verified that these matrices are commuting and Hermitian. Now from this we construct the set we require containing \( 2^a \) matrices as follows,

\[
\tilde{G} = \{I_{n \times n}\} \cup \{\pm \tilde{A}_k | k = 1, \ldots a\} \cup \{\pm \prod_{j=2}^{a} \tilde{A}_k | 1 \leq k < k(i+1) \leq a\}
\]

(14)

Note that the matrices in (14) are all distinct, unitary, Hermitian and mutually commuting \( n \times n \) complex matrices.

**B. SSD codes**

Suppose we want to construct SSD code for \( N_t = 2^a, a \geq 2 \) transmit antennas. In other words the codes to be constructed are \( g \)-group decodable for some \( g \), where each group contain only two real symbols, This imply that \( n = 2 \). From the above construction we find, \( \tilde{G} = \{I_{2 \times 2}, \sigma_i \} \). Now from \( N_t = mn = 2m \) we get the value of \( m \). Next we need to find the set \( G_0 \). Following the discussion in Subsection III.A we can construct the set \( G_0 \), for this value of \( m \) which is illustrated in the following example.

Example 2: We take, \( N_t = 4 \). Then \( \tilde{G} = G_1, i = 1, 2, 3, 4 \) is as described above. For \( m = 2 \) we get, \( G_0 = \{I_2, \sigma_1, \sigma_2, j\sigma_3\} \). Next we construct the STBC according to
For any given above in Subsection V-A we get, we get a \( G \) the construction procedure in Subsection III-A we find the set \( \{ I_{2 \times 2} \otimes I_{2 \times 2}, \sigma_3 \otimes j \sigma_1, \sigma_1 \otimes \sigma_2, \sigma_4 \otimes \sigma_3 \} \). (16)

For any given \( N_t = 2^a \), we find \( m = N_t \). Then following the construction procedure in Subsection III-A we find the set \( G_0 \), and then construct the STBC according to \( 7 \).

Example 3: Let us take \( N_t = 8 \). Then \( G = G_i, i = 1, 2, 3, 4 \) is given by \( 16 \). For \( m = 2 \) we get, \( G_0 = \{ I_{2 \times 2}, \sigma_1, \sigma_2, j \sigma_3 \} \). Next we construct the LDSTBC according to \( 7 \) and is given in \( 15 \) at the top of this page. According to the construction this is a 4-group decodable code.

In general for any given \( N_t = 2^a \) number of transmit antennas, we get a \((2a - 2)\)-group decodable code, with rate \( \frac{(a - 1)}{2^a - 2} \) complex symbols per channel use.

D. Simulation Results of DSD codes

In Figure 1 we have compared the performance of QOSTBC [7] and DSD code for 8-transmit antennas. For QOSTBC we used two 7-ary constellation optimally rotated as in [7]. For DSD as \( \alpha_g \), we used a 16-point 4-real dimensional CPD-optimized constellation. And then obtained \( \alpha_g \) by transforming \( \alpha_g \) by \( T^{-1} \). Then we allowed \( X_k \in \alpha_x \), \( \forall k \).

ACKNOWLEDGMENT

This work was partly supported by the DRDO-IISc Program on Advanced Research in Mathematical Engineering, partly by the Council of Scientific & Industrial Research (CSIR), India, through Research Grant (22(0365)/04/EMR-II) and also by Beceem Communications Pvt. Ltd., Bangalore to B.S. Rajan.

REFERENCES

[1] C. Yuan, Y.L.Guan and T.T.Jhong, “A class of four-group Quasi-Orthogonal STBC achieving full rate and full diversity for any number of antennas,” Proceedings of PIMRC 2005, 11-14, Sept., 2005, Vol.1, pp.92-96.
[2] Sanjay Karmakar and B.Sundar Rajan, “Minimum-Decoding-Complexity, Maximum-rate Space-Time Block Codes from Clifford Algebras,” Proceedings of IEEE International Symposium on Information Theory (ISIT 2006), Seattle, U.S.A., July 9-15, 2006, pp.788-792.
[3] H.Wang, D.Wang and X.G.Xia, “On Optimal Quasi-orthogonal space-time block codes with minimum decoding complexity,” Proc. ISIT 2006, Adelaide, Nov. 2005, pp.1168-1172.
[4] C.Yuan, Y.L.Guan and T.T.Jhong, “Construction of quasi-orthogonal STBC with minimum decoding complexity,” Proc. ISIT 2004, Chicago, June/July 2004, p.308.
[5] B. Hassibi and B. Hochwald, “High-rate codes that are linear in space and time,” IEEE Trans. Inform. Theory, vol.48, no.7, pp.1804-1824, July 2002.
[6] Md. Zafar Ali Khan and B. Sundar Rajan, “Single-Symbol Maximum-Likelihood Decodable Linear STBCs,” IEEE Transactions on Information Theory, Vol.52, No.5, May 2006, pp.2062-2091.
[7] Weifeng Su and X.G.Xia, “Signal constellations for QOSTBC with full diversity”, IEEE trans. on Information Theory, Vol-50, Oct, 2004, pp.2331- 2347.
[8] O. Tirkkonen and A. Hottinen, “Square-matrix embeddable space-time block codes for complex signal constellations,” IEEE Trans. Inform. Theory, Vol.48, No.2, Feb. 2002, pp.384-395.
[9] Full Diversity Rotations, [http://www1.ilec.polito.it/~viterbo/rotations/rotations.html]