Structural Design and Inertial Impact Analysis of Vehicle ISD Suspension

Yanling Liu, Long Chen, Xiaofeng Yang*, Yi Yang

School of Automotive and Traffic Engineering, Jiangsu University, Zhenjiang 212013, China

*Corresponding author e-mail: yangxf18@ujs.edu.cn

Abstract. This paper concerns the impact of inerter element on the vehicle suspension system. On the basis of the analysis of three simple elements’ networks including one inerter, one spring and one damper, the improved networks are further proposed. A quarter-vehicle model is considered, and the three performance indexes, namely the body acceleration, suspension working space and dynamic tire load are taken into account. With the increase of the inerterance value, numerical simulations are carried out and the influences on the performance indexes, the partial frequencies and the dominant frequencies are discussed in detail. Results indicate that the value of the partial frequencies and the dominant frequencies will reduce effectively by increasing the inerterance value of the suspension system. At last, two type suspension layouts are analysed by comparing to a passive suspension, and their performance advantages are also validated.

1. Introduction

Inerter as a newly proposed vibration isolation component with the property of two terminals in 2001[1], it makes up for the lack of inertial element and promotes the structural development of vehicle suspension [2]. With the appearance of inerter, each mechanical network component can correspond to an electrical network component in the electromechanical similarity theory absolutely. In this theory, inerter corresponds to the capacitor[3], damper corresponds to the resistor and spring corresponds to the inductor. The applied force at the two terminals of inerter is proportional to the relative acceleration between them. [4,5,6].

There are many types of inerter, such as the rack-and-pinion inerter in [7], ball-screw inerter in [8], hydraulic inerter in [9]. According to [10,11,12,13], the suspension employing inerter element performs better than the traditional passive suspension and inerter has been proven that it is effective in increasing the performance of mechanical vibration isolation systems [14,15,16]. The application of inerter to the vehicle isolation system is positive and profound to the theoretical study of vehicle suspension.

In the vehicle suspension system, the spring is used as an energy storing element and determines the natural frequency. The damper is used as a dissipative element and it limits the vibration amplitude of vehicle body. When the inerter is used as a mass component in the passive suspension with the spring and damper, this passive suspension can be referred to ISD (inerter-spring-damper) suspension [17,18]. The performance of different structures of ISD suspensions is various. In recent years, many scholars focused on the study of the structure and performance of inerter. But the research of the influences of
inertance on the natural frequencies is few. In [19], Michael Chen introduced the fact that the values of the natural frequencies of vibration systems can be reduced by increasing the value of inertance $b$. This fact has been confirmed by algebraically deducting the natural frequencies of vibration systems.

In section 2 of this paper, eight $3E$-ISD networks (3-Element ISD networks) are proposed and improved into $3E'$-ISD networks. In section 3, a quarter-vehicle model is established to analyze the convergence of $3E'$-ISD suspensions, and the body acceleration, suspension working space and dynamic tire load are taken into account as performance indexes. In section 4, $3E'$-ISD suspension $1'$, $3'$, $6'$, $7'$ and $2'$ are classified into two types by mechanical impedance equations when the value of damper is 0. Reference to the conclusion in [19], the influence of inertance $b$ on the natural frequencies of two type suspensions is analyzed and one type suspension is singled out to discuss in detail. Then the performance of $3E'$-ISD suspension $6'$ and $7'$ is studied in section 5 with the traditional suspension as a reference standard. Last, main conclusions and findings are drawn in section 6.

2. $3E$-ISD network and $3E'$-ISD network

In the light of the circuit topology and electromechanical similarity theory [20,21], ISD networks can be classified into different types by the number of elements. Different combination of the springs, dampers and inerter can make numerous different structures of the ISD networks. 3-Element ISD network ($3E$-ISD network) including one inerter, one spring and one damper is the basic and representative type of the networks. Based on the permutation and combination, there are 8 effective implementations showed in Figure 1.

![Figure 1. Eight 3E-ISD networks](image)

Inerter and damper are the components without the characteristic of static loading. When ISD network works in a vehicle suspension, it should be supported by the spring to avoid overload and work in the effective travel. This means that in a vehicle suspension system, the inerter and damper should connect a spring or springs in parallel to guarantee the normal operation of each component [22].

In Figure 1, it is clear that $3E$-ISD network 2 and 4 can work normally in the vehicle suspension. In order to completely reflect the performance of each $3E$-ISD network, a spring is added to the $3E$-ISD networks in parallel to improve the practicability of ISD networks.

As is shown in Figure 2, 8 optimized $3E'$-ISD networks are presented.
On the basis of the characters of mechanical impedance[23,24], mechanical impedance equations of 8 $3E'$-ISD networks are presented as $L_i(s)$, $L_n(s)$, ..., $L_v(s)$ as following.

**Table 1. Mechanical impedance equations of 8 $3E'$-ISD networks**

| $3E'$-ISD network | 1' | 2' | 3' | 4' |
|------------------|----|----|----|----|
| Equation $L(s)$  | $\frac{k_1}{s} + \frac{1}{s + \frac{1}{k_2 + \frac{1}{c + bs}}}$ | $\frac{k_1}{s} + \frac{1}{s + \frac{1}{c + bs}}$ | $\frac{k_1}{s} + \frac{1}{c + \frac{k_2}{s + \frac{1}{bs}}}$ | $\frac{k_1}{s} + c + bs$ |

| $3E'$-ISD network | 5' | 6' | 7' | 8' |
|------------------|----|----|----|----|
| Equation $L(s)$  | $\frac{k_1}{s} + bs + \frac{1}{s + \frac{1}{k_2 + \frac{1}{c}}}$ | $\frac{k_1}{s} + c + \frac{1}{s + \frac{1}{k_2 + \frac{1}{bs}}}$ | $\frac{k_1}{s} + c + \frac{1}{s + \frac{1}{k_2 + \frac{1}{bs}}}$ | $\frac{k_1}{s} + \frac{1}{c + bs}$ |

In these equations, $k_1$ represents the stiffness of spring 1, $k_2$ represents the stiffness of spring 2, $b$ is inertance, $c$ is damping coefficient.

### 3. Convergence of 8 $3E'$-ISD suspensions

A quarter-vehicle model was established to analyze the convergence of $3E'$-ISD suspensions from the view of three performance indexes.

**Figure 3. A quarter-vehicle model**
The dynamical equations of the quarter-vehicle model can be expressed as following:

\[
\begin{align*}
& m_1 \cdot s^2 \cdot Z_1 + s \cdot L(s) \cdot (Z_1 - Z_2) = 0 \\
& m_2 \cdot s^2 \cdot Z_2 + k_t \cdot (Z_2 - Z_1) - s \cdot L(s) \cdot (Z_1 - Z_2) = 0
\end{align*}
\]  

(1)

Where \( m_1 \) stands for sprung mass, \( m_2 \) is on behalf of unsprung mass, \( z_3 \) is the input signal from the random road, \( z_2 \) is unsprung mass displacements and analogously \( z_1 \) represents sprung mass displacements. \( Z_3, Z_2 \) and \( Z_1 \) are the Laplace transformations of \( z_3, z_2 \) and \( z_1 \), \( k_t \) is tire stiffness, and \( L(s) \) is the vehicle suspension impedance in complex frequency.

Three performance indexes, namely the RMS (Root-Mean-Square) value of body acceleration, the RMS value of suspension working space and the RMS value of dynamic tyre load were used to evaluate the performance of the suspension[25]. The simulation was taken on the assumption that the vehicle passed the road with 20 m/s speed and road roughness coefficient \( G_0=5\times10^{-6} \text{ m}^3\cdot\text{cycle}^{-1} \). Random road served as input signal \( z_3 \). It can be expressed in the shape of time domain as the following equation:

\[
\dot{z}_3(t) = 2\pi \sqrt{G_0} \cdot v \cdot w(t).
\]  

(2)

In expression 2, \( w(t) \) is delegated to zero gauss white noise; \( v \) represents the vehicle speed.

In the process of the simulation, the parameters of the model remained the same except inertance. The value of inertance increased linearly from 0kg to 1000kg and the main parameters of the simulation are showed in Table 2. The results of the convergence of \( 3E' \) - ISD suspensions are presented from Figure 4 to Figure 6.

**Table 2. Main parameters of model**

| Parameters                  | Values |
|-----------------------------|--------|
| Sprung mass, \( m_1/\text{kg} \) | 320    |
| Unsprung mass, \( m_2/\text{kg} \) | 45     |
| Tire stiffness, \( k_t/(\text{kN} \cdot \text{m}^{-1}) \) | 190    |
| Spring stiffness, \( k_1/(\text{kN} \cdot \text{m}^{-1}) \) | 22     |
| Spring stiffness, \( k_2/(\text{kN} \cdot \text{m}^{-1}) \) | 10     |
| Damping coefficient, \( c/ (\text{Ns} \cdot \text{m}^{-1}) \) | 1500   |
| Inertance, \( b/\text{kg} \) | 0–1000 |

**Figure 4.** Influence of inertance \( b \) on body acceleration RMS values

(a) Convergency

(b) Divergence
From the results of simulation, it is clear that when inertance $b$ increased, the performance of $3E'$-ISD suspension 4', 5' or 8' becomes worse. However, $3E'$-ISD suspension 1', 3', 6', 7' or 2' has the trend stably.

### 4. Influence of inertance $b$ on the natural frequencies

Even though $3E'$-ISD suspension 1’, 3’, 6’, 7’ or 2’ performs better with the increment of inertance, the vibration in the sensitive resonance frequency range of human will destroy the organ. In order to keep human from the scathe of sympathetic vibration, influence of inertance $b$ on the natural frequencies is worth studying in a deep going way.

As showed in Table 3, $3E'$-ISD suspension 1’, 3’, 6’, 7’ and 2’ were classified into two types by mechanical impedance equations when the value of damper is 0.

| $3E'$-ISD suspensions | Undamped impedance | Suspension type |
|-----------------------|--------------------|----------------|
| 1’, 2’, 3’             | $\frac{k}{s}$     | L1             |
| 6’, 7’                 | $\frac{k_s}{s} + \frac{1}{bs}$ | L2             |
The expression for the undamped impedance of suspension L1 and L2 are substituted into equations (1) and \(Z_2\) and \(Z_1\) are set to zero respectively for computing the natural frequencies. Body partial frequency \(\omega_0\) and tire partial frequency \(\omega_t\) of suspension L1 and L2 can be obtained as following:

\[
L_1: \quad \omega_0 = \sqrt{\frac{k_L}{m_1}} \quad (3)
\]

\[
L_2: \quad \omega_0 = \sqrt{\frac{A^2 - 4m_1b_kk_2}{2m_1b}} , \quad A = m_2k_2 + bk_k + bk_2 \quad (4)
\]

\[
L_1: \quad \omega_t = \sqrt{(k_i + k_i)/m_2} \quad (5)
\]

\[
L_2: \quad \omega_t = \sqrt{\frac{B^2 - 4m_2b(k_kk_2 + k_2k_2)}{2m_2b}} , \quad B = m_2k_2 + bk_k + bk_2 + bk_2 \quad (6)
\]

From the equation (4) and (6), inertance \(b\) plays the same role as \(m_1\) or \(m_2\) for the body partial frequency \(\omega_0\) and tire partial frequency \(\omega_t\). But inertance \(b\) does not work in the partial frequencies of suspension L1.

For reaching the results and researching the influence of inertance \(b\) on the natural frequencies all-around, dominant frequencies are also studied. Assuming \(m_1\) and \(m_2\) are in a simple harmonic vibration with the same circular frequency \(\omega\) and the same identical phase angle \(\varphi\). The amplitudes of two simple harmonic vibrations are \(z_{10}\) and \(z_{20}\). The representations of \(z_{10}\) and \(z_{20}\) are:

\[
z_1 = z_{10} \cdot e^{(\alpha t + \varphi)} , \quad z_2 = z_{20} \cdot e^{(\alpha t + \varphi)} \quad (7)
\]

Taking the equation (7) into equation (1), dominant frequencies \(\omega\) of suspension L1 and L2 are got by the following equations. In this part

\[
L(s) = \frac{k}{s^2 + \omega^2 s + \frac{1}{b}} , \quad s = j \omega .
\]

\[
\begin{cases}
  m_1 \cdot s^2 \cdot z_{10} \cdot e^{(\alpha t + \varphi)} + s \cdot L(s) \cdot (z_{10} \cdot e^{(\alpha t + \varphi)} - z_{20} \cdot e^{(\alpha t + \varphi)}) = 0 \\
  m_2 \cdot s^2 \cdot z_{20} \cdot e^{(\alpha t + \varphi)} + k_1 \cdot z_{20} \cdot e^{(\alpha t + \varphi)} - s \cdot L(s) \cdot (z_{10} \cdot e^{(\alpha t + \varphi)} - z_{20} \cdot e^{(\alpha t + \varphi)}) = 0
\end{cases} \quad (8)
\]

A simulation was carried out in Matlab to emerge the influence of inertance \(b\) on the natural frequencies. The main parameters of simulation were the same as the parameters showed in Table 2 except \(c=0\). The results of simulation are showed in Figure 7.
As Figure 7 showing, the partial frequencies and dominant frequencies of suspension L1 are constant. They will not be affected by the value changes of inertance $b$. However, for suspension L2, inertance $b$ can reduce the partial frequencies and dominant frequencies effectively to go far away from the range of the personal most sensitive frequencies (4~12.5 Hz) in the vertical direction. When inertance $b$ becomes larger, partial frequencies and dominant frequencies become smaller.

The number of natural frequencies of suspension L2 is more than suspension L1. No matter tire partial frequencies, body partial frequencies or dominant frequencies of suspension L2 distribute on both sides of the natural frequencies of suspension L1 at all time. With the increment of inertance, the larger value of tire partial frequencies, body partial frequencies and dominant frequencies of suspension L2 has the trend to approach the value of tire partial frequencies, body partial frequencies and dominant frequencies of suspension L1.

5. Performance of 3E'-ISD suspension 6' and 7'

The simulation results of Figure 7 indicate that the natural frequencies of suspension L2 can be regulated by inertance $b$. So 3E'-ISD suspension 6' and 7' were further studied.

Multi-objective genetic algorithm was selected to acquire the optimal values of inertance $b$ of 3E'-ISD suspension 6' and 7'. When multi-objective genetic algorithm was used, objective function was $F$, maximal genetic algebra was 50, generation gap was 0.9 and weight coefficient was $w$.

$$w = [w_1, w_2, w_3]^T = [0.4, 0.3, 0.3]^T$$  \hspace{1cm} (9)
\[ F' = \sqrt{\frac{w \cdot J}{w_1 + w_2 + w_3}} \]  \hspace{1cm} (10)

\[ J = [BA, DTL, SWS] \]  \hspace{1cm} (11)

In equation (11), \( J \) represents evaluating indicator, \( BA \) is variance of body acceleration, \( DTL \) is variance of dynamic tire load and \( SWS \) is variance of suspension working space.

A quarter-vehicle model in Figure 3 was run in MATLAB to compare the performance between 3E'-ISD suspension 6' and 7'. The main parameters were uniform in values shown in Table 2 except the value of inertance \( b \) of 3E'-ISD suspension 6' was 482 kg and the value of inertance \( b \) of 3E'-ISD suspension 7' was 216 kg. As a reference, the structure of traditional suspension is as Figure 8 shown and the parameters \( k_1=22 \) kN/m, \( c=1500 \) Ns/m. For the sake of the fair and effective analysis result, the basic parameters of the traditional suspension are the same as the 3E'-ISD suspension 6' and 7'. The amplitude-frequency response characteristics of 3E'-ISD suspension 6' and 7' are showed in Figure 9 and Table 4.

![Figure 8. Structure of traditional suspension](image)

(a) Body acceleration gain  \hspace{1cm} (b) Suspension working space gain
Figure 9. Amplitude-frequency response characteristic of 3E'-'ISD suspension 6' and 7'

Table 4. RMS values of three suspensions

| Performance Index                      | Traditional Suspension | 3E'-'ISD Suspension 6 | Improvement | 3E'-'ISD Suspension 7 | Improvement |
|----------------------------------------|------------------------|-----------------------|-------------|-----------------------|-------------|
| Body Acceleration RMS / (m·s⁻²)        | 2.8143                 | 3.7704               | -33.97%     | 2.5963                | +7.75%      |
| Suspension Working Space RMS /m        | 0.0360                 | 0.0353               | +1.94%      | 0.0339                | +5.83%      |
| Dynamic Tire Load RMS /kN              | 1.5933                 | 1.7145               | +12.12%     | 1.5017                | +5.75%      |

As shown in Figure 9, when the value of frequency exceeds 5 Hz, the performance of 3E'-'ISD suspension 6’, 7’ and traditional suspension is similar. When the value of frequency from 1.5 Hz to 4 Hz, 3E’-'ISD suspension 7’ and traditional suspension reduce the value of body acceleration and suspension working space compared to 3E’-'ISD suspension 6’. For 3E’-'ISD suspension 6’, it is regrettable that the resonance frequency of body acceleration, suspension working space and dynamic tire load moves backward with the increment of inertance b.

On the whole, 3E’-'ISD suspension 6’ performs much better than the traditional suspension except the frequency from 1.5 Hz to 4 Hz. In this range, the performance of 3E’-'ISD suspension 6’ is bad.

For 3E’-'ISD suspension 7’, the performance of body acceleration, suspension working space and dynamic tire load are superior to the traditional suspension and 3E’-'ISD suspension 6’. In Figure 9 (a) and (c), 3E’-'ISD suspension 7’ almost does not have resonance peak in the low frequency section. In Figure 9 (b), the first formant appeared around 1 Hz and the second formant appeared around 2 Hz. Compared to traditional suspension, the second formant of 3E’-'ISD suspension 7’ is too gradual to ignore. But the value of the first formant is larger than the traditional suspension. It can be considered that the second formant was moved forward to the first formant of 3E’-'ISD suspension 7’ to keep resonance frequency away from the sensitive frequency of human to improve the vehicle ride comfort.

6. Conclusion

With the increment of inertance b, performance of 3E’-'ISD suspension 1’, 2’, 3’, 6’ and 7’ is convergence on three indexes, RMS of the body acceleration, RMS of the suspension working space and RMS of the dynamic tire load. Compared to 3E’-'ISD suspension 4’, 5’ and 8’, their performance is much steadier.

The partial frequencies and dominant frequencies of suspension L2 can be reduced by increasing the value of inertance b. Inertance b can be applied to change the natural frequencies of vehicle.
The performance improves obviously on restraining the vehicle vertical vibration in low frequency by 3E’-ISD suspension 7’. It perfects the performance of the ride comfort and road friendly. To sum up, the performance of 3E’-ISD suspension 7’ is much better than other 7 3E’-ISD suspensions.

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