Investigations on the Stability of a Two-Wheeled Vehicle with a Tendency to Topple Sideways

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Abstract. Two-wheeled vehicles are the most commonly used means of transportation. Every day many unspoken tragedies are happening on the roads that kill the hope of many families. The primary cause of this is the inherent instability of the two-wheeled vehicle. In this paper, the precession effect of the gyroscope is used to prevent the sideways toppling of the bicycle. When the bicycle begins to tilt, the torque created by the gyroscope’s precession effect is applied to the gimbal, and the ensuing reaction moment keeps the bicycle upright. The movements of a bicycle with a gimbal placed on the bottom are measured, and a three-dimensional model with a sliding mode controller is created and simulated.

1. Introduction
The rising number of deaths and injuries caused by traffic accidents is causing widespread concern around the world. Two-wheeler riders are involved in most traffic collisions, and the majority of them die on the spot[1]. Two-wheeled vehicles are becoming popular modes of transportation due to economy, time efficiency and practicality. Studies show that the accidents are only increasing over the years, and the primary cause of this is the inherent instability of the two-wheeled vehicle [2].

In India, people travel in two-wheeled vehicles with front and rear wheels. The number of accidents can be decreased if the bike turns over sideways is avoided. As a result, focused our research on self-stabilising a bicycle, and the results will be helpful in future research. It has been discovered through studies that a control moment gyroscope can be used to balance a bicycle[3]. The bicycle prototype with CMG demonstrated that the bicycle stabilises as the flywheel velocity increased[8]. A revolving rotor and two gimbals make up the CMG, which provides a large amount of constant angular momentum. The precession effect of the gyroscope can be used to prevent the bicycle from toppling sideways[6]. As the bicycle begins to tilt, the torque created by the gyroscope’s precession effect is applied to the gimbal, and the consequent reaction moment holds the bicycle upright.[4]

The strength of this paper is that it uses Solidworks to construct a three-dimensional model of the bicycle, analyses the dynamics of the bicycle at various flywheel speeds, and uses MATLAB and Simulink to create a sliding mode controller to control the bicycle’s posture so that it does not fall.
2. Methodology

The project began with the development of a two-dimensional sketch of the prototype in Solidworks. Then, using MATLAB and Simulink, a first-order sliding mode controller is created using the mathematical equation of the bicycle formulated[4] using the Lagrangian equation. It’s then used to keep the bicycle upright by controlling the roll angle. The design is then dynamically analysed using Solidworks motion analysis. The component’s structural rigidity was then tested for manufacturability using static analysis.

![Methodology Diagram](image1)

Figure 1. Methodology

3. System Description

3.1. Principle of Working

When an external force is applied to a rotating object such as a gyroscope, a torque-induced precession (gyroscopic precession) occurs, describing a cone in space. As the bicycle tilts away from vertical, a precision inducing torque is applied to the gyroscope. The gyroscopic reaction moment that results would bring the bicycle to a standstill. The essential idea is that the motion of the gyroscope relative to the bicycle is actively managed to provide a stabilising moment.

![Bicycle Diagram](image2)

Figure 2. Bicycle(Side View)
3.2. Mathematical Model

A Lagrangian function is a quantity used to describe the state of a physical system. The Lagrangian is the difference between kinetic and potential energy in mechanics[5]. It is often used to describe the state of a nonlinear system. If \( L \) represents Lagragian and \( \phi \) represents bicycle tilt angle, then the bicycle dynamics are calculated using the Euler-Lagrangian equation.

\[
\frac{\partial L}{\partial \phi} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}} \right) 
\]

If mass of the bicycle is \( m_b \) and height of the centre of mass of the bicycle is \( h_b \), then the potential energy of the bicycle

\[
= m_b g h_b \cos \phi 
\]

If mass of the gyroscope is \( m_g \) and height of centre of mass of the gyroscope is \( h_g \), then the potential energy of the gyroscope

\[
= m_g g h_g \cos \phi 
\]

Let \( V \) represent the bicycle’s velocity, \( \psi \) represent the yaw angle, and \( I_{b_x}, I_{b_y}, I_{b_z} \) represent the bicycle’s inertia about each axis. Then the kinetic energy of the bicycle

\[
= \frac{1}{2} \left[ V^2 + h_b^2 \psi^2 \sin^2 \phi + 2 V h_b \psi \sin \phi + h_b^2 \phi^2 \right] + \frac{1}{2} \left[ I_{b_x} \phi^2 + I_{b_y} \psi^2 \sin^2 \phi + I_{b_z} \psi^2 \cos^2 \phi \right] 
\]

Let \( \Omega \) represents the flywheel velocity, \( \alpha \) represents the gimbal angle, \( \sigma \) represents the curvature of path, and \( I_{g_x}, I_{g_y}, I_{g_z} \) represent the gyroscope’s inertia about each axis. Then the kinetic energy of the gyroscope

\[
= \frac{1}{2} m_g \left[ (V + \psi \sin \phi)^2 + \phi^2 h_g^2 \right] + \frac{1}{2} I_{g_x} (\phi \cos \alpha - \psi \cos \phi \sin \alpha)^2 \\
+ \frac{1}{2} I_{g_y} (\psi \sin \phi + \alpha)^2 + \frac{1}{2} I_{g_z} (\phi \sin \alpha + \psi \cos \phi \cos \alpha + \Omega)^2 
\]

The Langrangian is the difference between the overall kinetic energy of the bicycle and the overall potential energy of the gyroscope, which is then substituted in Equ(1) and the bicycle dynamics equation is formulated[4].

\[
\ddot{\phi} = f_1(\phi) + f_2(\phi, V) + f_3(\phi, V, \alpha) - f_4(\phi, V, \alpha, \alpha) 
\]

where, \( f_4(\phi, V, \alpha, \alpha) \) is the precession effect of the gyroscope. As a result, by manipulating the gyroscope’s precession effect, the bicycle can be made to stand upright.

4. Design and analysis

4.1. Three-dimensional model of the bicycle

A three-dimensional model of a bicycle is designed and developed from the 2D sketch of the prototype using Solidworks. The design was dynamically analysed in Solidworks utilising motion analysis. The bicycle model is made up of a flywheel installed on the bicycle’s bottom. The flywheel is a solid piece of mild steel in the shape of a disc with a geometric centre bore. This is an important component of the CMG because its rotation aids in the gyroscopic effect needed to hold the two-wheeler balanced. Gimbals made of mild steel are used to provide the gyroscope’s second axis with the requisite free rotation to aid in precession.
4.2. Static Analysis of the Structure
A finite element method study of the bicycle frame is carried out to measure the materials’ strength under static load. It helped recognise the design’s limitations. These models (Figures 4 to 6) depict the total stress distribution, displacement, and strain acting on the frame under an anticipated load of 2000N.

| Material Properties |
|---------------------|
| Name                |
| Alloy Steel         |
| Model Type          |
| Linear Elastic Isotope |
| Default Failure     |
| Max Von Mises Stress |
| Yield strength:     |
| 6.20422e+08 N/m²    |
| Tensile strength:   |
| 7.23826e+08 N/m²    |
| Elastic modulus     |
| 2.1e+11 N/m²        |
| Poisson’s ratio     |
| 0.28                |
| Mass density        |
| 7,700 kg/m³         |
| Shear modulus       |
| 7.9e+10 N/m²        |

Figure 3. Bicycle 3D Model

Figure 4. Stress Analysis of the bicycle model

Figure 5. Displacement Analysis of the bicycle model
4.3. Motion Analysis

The motion analysis of the bicycle model is carried out at various rpm values, and the model’s simulation (Figures 7 and 8) provided a rough idea of how the model will function in real life.

4.3.1. When bicycle is stationary. Case1: Flywheel at 0 RPM. This case is about establishing and analysing two-wheelers without a gyroscope. The time it takes to topple sideways completely is around 1.4 seconds, as seen in this Tilt vs Time graph.

Case2: Flywheel at 5000 RPM. The gyroscopic action slows the tilt of the body while the flywheel is rotating at a constant speed of roughly 5000 rpm. The precession force operates on the gimbal’s horizontal axis.
Case3: Flywheel at 20000 RPM. A higher flywheel rpm of 20000 is provided to study how the precession effect of gyroscope is affecting the bicycle model. Here, it is observed that as the flywheel velocity increases, the tilt action of the body slows down further, causing it to fall after 2 seconds.

4.3.2. When bicycle is moving with a constant velocity. The bicycle is moving with constant velocity’s motion along the x-y plane. The z-axis represents the toppling of the bicycle with varying flywheel velocities. From the graph, it is observed that at no flywheel velocity, the bicycle falls in 4 secs after many wobblings. As the flywheel velocity increases, its stability increases and wobbling reduces. But if the flywheel velocity is increased further, it will become more unstable and falls even faster.
5. Control Design

There will still be a difference between the actual plant and its statistical model due to plant parameters and other unknown external disturbances. It is a difficult challenge to design control laws that have the optimal output in the face of these disturbances/uncertainties. A sliding mode controller is a commonly used robust controller for naturally unstable and significant nonlinear systems[7]. The roll angle of the bicycle is regulated here by the gimbal angle rate. A first-order sliding mode controller is used to control the roll angle by choosing the sliding variable $s_\phi$ as [4]

$$s_\phi = c_\phi \dot{\phi} + c_{\phi^2} \phi^2 + \alpha$$  \hspace{1cm} (7)

A sliding mode controller is designed and simulated using MATLAB and Simulink. Then the variation of roll angle is analysed by providing a disturbance to the system with control and without control.

5.1. Without Sliding Mode Control

A disturbance is applied to a bicycle moving with a constant velocity. Then cycle first wobbles and then gradually falls down.
5.2. With Sliding Mode Controller
A disturbance is applied to a bicycle moving with a constant velocity. Then cycle first tilts, but the controller will asymptotically try to bring back the cycle in an upright position.

![Figure 14. Roll angle variation with control](image)

6. Conclusion
The concept of using the theory of gyroscopic precession to stabilise the bicycle is investigated in this paper. The bicycle frame is statically analysed to ensure that the material selection is sufficient and can withstand the anticipated loads. In addition, the motion of the bicycle is studied at various flywheel speeds. The result shows that as the flywheel RPM increases, the time it takes for the cycle to fully topple increases when the bicycle is stationary and an optimum flywheel speed is required when the bicycle is moving. Higher RPM motors are needed to make the bicycle stand upright due to the limited flywheel size of the built model. Without using high RPM motors, the time it takes for the bicycle to topple can be improved by increasing the flywheel size. A sliding mode controller is designed to control the bicycle’s roll angle to make it stand upright. The simulation results proved that the proposed control design was successful. Even in the face of uncertainties and disruptions, the controller’s output is found to be satisfactory.

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