Construction of polar fronts of reactions in gas-air flows of mine workings near the centers of self-heating

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Abstract. Gas and thermodynamic phenomena often occur during the operation of coal plants, especially at deep horizons. These are sudden emissions of coal and gas, deflagration and detonation processes in gas and dust-gas-air mixtures that generate explosions and shock waves in the atmosphere of mining operations, which are complex movements of a continuous medium containing strong rips. The peculiarity of these movements is that for their theoretical study, it is necessary to take into account not only the fundamental laws of continuum mechanics, but also the basic provisions of thermodynamics and thermophysics. This article considers a mathematical model of a stationary oblique reaction front based on the laws of conservation of mass, momentum, and energy, assuming that the gas-air flow in which the reaction occurs is an ideal gas whose flow is stationary. Formulas are obtained and polars of reaction fronts are constructed, both in supersonic and subsonic gas-air flows in the atmosphere of mine workings. Based on the analysis of the constructed polars, a number of regularities of reactions in oblique fronts at different Mach and Damkeler numbers have been established.

1. Introduction

In recent years, the coal industry has regularly faced negative factors that have hindered the rate of underground coal production. One of these factors is sudden emissions of coal, rocks, and gas, rock impacts, and vented gas emissions [1-3].

Other factors are caused by the presence of coal dust formed during the operation of mining equipment, and self-heating centers in coal-rock accumulations. Coal dust, interacting with the atmosphere of mine workings, forms dust and gas-air mixtures that are capable of chemical reactions of coal oxidation in various forms [4, 5].

So, in [6] the conditions of appearance of a chemical reaction in the form of dust-Laden flue gas ignition microheterogeneous mixtures, and in the papers [7-10] discussed combustion processes fine and coarse dust-Laden flue gas mixtures. The authors of the study [11] experimentally investigated the processes of chemical reaction in the form of burning and detonation of dust-gas-air mixtures at different dust concentrations and different stoichiometric ratios. In the article [12], the propensity of brown coal dust to form an explosive mixture in the atmosphere of mining operations was revealed, and the authors of the work [13] investigated the detonation combustion of coal dust in a methane-air mixture with a small content of coal particles.

Foci of self-heating [14, 15], which are heat sources with an increased temperature, lead to a change in the temperature field of rocks surrounding mine workings [16, 17]. Rocks, having an in-
creased temperature, in turn, transfer heat to the atmosphere of the mine workings in its local area, called the heat transfer zone.

The zone of heat supply is a fairly narrow front, within which chemical reactions can occur, so that when passing through the front, the parameters of the mixture can change, and in an abrupt manner [18]. In [19], gas-dynamic and thermo physical conditions for the flow of deflagration and detonation modes of fine dust-gas-air mixtures in the atmosphere of mine workings that have zones of heat supply from nearby self-heating centers are considered. The article [20] considers the problem of forced detonation formed as a result of gas flow from an underground reservoir into a mine at supersonic speed.

In all these works, it was assumed that the heat supply zone and the chemical reaction front that occurs inside it are normal for the longitudinal axis of production. However, in general, the heat supply zones and reaction fronts may not be located perpendicular to the direction of the dust and gas flow. Because of this, when the reaction front is crossed, the flow changes its direction, which is a fundamental difference from the flow through the normal reaction front.

In this paper, we discuss the problem of an oblique reaction front in a gas-air flow modeled by an ideal gas whose flow is stationary. In our opinion, this task is relevant, since it will allow us to identify more complex gas-dynamic and thermo physical conditions for chemical reaction in mining operations, not only in straight sections, but also in the areas of turning workings and their interfaces.

2. Solution of the problem of reaction fronts in gas and air flows in mine workings near self heating centers

Let us assume that there is a zone of heat supply in a certain vicinity of the mine, which is a consequence of the self-heating hearth, and which is inclined to the longitudinal axis of the mine by an angle $\phi$ (Figure 1). Let us assume that when the mixture intersects the zone under consideration, a chemical reaction front is formed inside it, which will also be tilted by an angle $\phi$ to the longitudinal axis of the formation. The state of the gas-air mixture will be described by its velocity vector, density $\rho$, specific volume and pressure $p$, which in the first region will be supplied with the index 1, and in the second region – with the index 2. We need to find the parameters of the mixture in region 2, if we know its parameters in region 1.

![Figure 1. Flow of gas-air mixture in mining through the oblique reaction front.](image)

When constructing a solution to the problem, we will take into account the following assumptions:

1) the heat supply zone is a fairly narrow reaction front, through which the parameters of the mixture change abruptly;

2) we neglect the friction of the gas-air mixture on the walls of the workings and assume that the mixture is close to the ideal gas with a constant specific heat capacity;

3) the flow of the gas-air mixture in the mining operation is assumed to be one-dimensional.

To solve this problem, we will use the fundamental laws of conservation: mass, momentum, and energy [21, 22].

From the law of conservation of mass of the gas-air mixture crossing the reaction front, the equality follows
\[
\rho_1 v_{1n} = \rho_2 v_{2n}, \tag{1}
\]

representing the continuity equation for the oblique front of the reaction.

Two equalities follow from the law of conservation of momentum in the direction of the axis \( \tau \) coinciding with the reaction front

\[ p_1 = p_2, \quad (\rho_1 v_{1n}) v_{1\tau} = (\rho_2 v_{2n}) v_{2\tau}, \]

given the continuity equation (1) in the second of them, we get the equality

\[ v_{1\tau} = v_{2\tau} = v_{\tau}, \tag{2} \]

meaning that the velocity projections and on the axis are equal to each other. In formulas (1), (2), the values \( p_1, p_2 \) and \( \rho_1, \rho_2 \) are respectively the pressures and densities of the gas-air flow on opposite sides of the reaction front.

From figure 2 it is seen that for the normal and tangential components of the velocities \( \vec{v}_1 \) and \( \vec{v}_2 \) the relations take place

\[ v_{1n} = v_1 \sin \varphi, \quad v_{1\tau} = v_1 \cos \varphi, \quad v_{2n} = v_2 \sin(\varphi - \theta), \quad v_{2\tau} = v_2 \cos(\varphi - \theta), \tag{3} \]

where \( \varphi \) is the angle of inclination of the reaction front, and \( \theta \) is the angle of deviation of the velocity vector from the longitudinal axis of production.

\[ \begin{array}{c}
\text{Figure 2. Plan of velocities in the gas-air flow on the oblique front of the reaction.}
\end{array} \]

To analyze the state of the gas-air flow behind the “oblique” reaction front, add the momentum equation to the continuity equation (1)

\[ p_2 + \rho_2 v_{2n}^2 = p_1 + \rho_1 v_{1n}^2 \tag{4} \]

and the energy equation

\[ \frac{v_{2n}^2}{2} + i_2 = \frac{v_{1n}^2}{2} + i_1 + q, \tag{5} \]

where \( i_1, i_2 \) is the enthalpy of the gas-air mixture in regions 1 and 2, \( q \) is the amount of heat supplied per unit mass of the mixture from the heat supply zone, which will be considered positive.

\[ i_1 = k \frac{p_1}{k - 1 \rho_1}, \quad i_2 = k \frac{p_2}{k - 1 \rho_2}, \]
we can rewrite the energy equation (5) as follows

\[ \frac{v_{2a}^2}{2} + \frac{k}{k-1} \frac{p_2}{\rho_2} = \frac{v_{1a}^2}{2} + \frac{k}{k-1} \frac{p_1}{\rho_1} + q, \]

where do we find the ratio between the squares of the normal component speeds

\[ \frac{v_{2a}^2}{v_{1a}^2} = 1 + \frac{1}{v_{1a}^2} \left[ 2q - \frac{2k}{k-1} \left( \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right) \right], \]

(6)

where \( k = c_p/c_v \) is the Poisson’s adiabatic index, and \( c_p \) and \( c_v \) are the specific heat capacities of the gas-air mixture, respectively, at constant pressure and at constant volume. In the article, the Poisson’s adiabatic index is assumed to be \( k = 1.4 \).

Then we get the formula from the continuity equation (1) and the momentum equation (4)

\[ p_2 = p_1 + \rho_2 v_{1a}^2 \left( 1 - \frac{v_{2a}}{v_{1a}} \right), \]

(7)

by virtue of which, formula (6) is converted to the form

\[ \left( \frac{v_{2a}}{v_{1a}} \right)^2 = 1 + \frac{2}{v_{1a}^2} \left[ q - \frac{a_1^2}{k-1} \left( 1 - \frac{v_{2a}}{v_{1a}} \right) \left( M_{1a}^2 \frac{v_{2a}}{v_{1a}} - 1 \right) \right], \]

(8)

where \( a_1 = \sqrt{k \rho_1/\rho} \) is the speed of sound in the region 1, and the value \( m_{1n} \) included in the formula (8), defined as \( M_{1a} = v_{1a}/a_1 \), is converted taking into account the first formula (3) to the following form

\[ M_{1a} = M_1 \sin \varphi, \]

(9)

where \( M_1 = v_1/a_1 \) is the Mach number.

Note that the enthalpy of the mixture can be expressed by the formula [23]

\[ i = \frac{a^2}{k-1} - c_p T, \]

where \( T \) is the Kelvin temperature of the mixture, and so the formula (8) is reduced to the square equation

\[ \left( \frac{v_{2a}}{v_{1a}} \right)^2 - \frac{2 k M_1^2 \sin^2 \varphi + 1}{M_1^2 \sin^2 \varphi \cdot (k+1)} \frac{v_{2a}}{v_{1a}} + \frac{(k-1)M_1^2 \sin^2 \varphi + 2 \bar{q}}{M_1^2 \sin^2 \varphi \cdot (k+1)} = 0, \]

the roots of which are expressions

\[ \frac{v_{2a}}{v_{1a}} = 1 - \frac{1}{(k+1)M_1^2 \sin^2 \varphi} \left[ (M_1^2 \sin^2 \varphi - 1) \pm \sqrt{(M_1^2 \sin^2 \varphi - 1)^2 - 2 \bar{q}(k+1)M_1^2 \sin^2 \varphi} \right], \]

(10)

where \( \bar{q} = q/(c_p T) \) is the Damkeler number, which is the dimensionless value of the supplied heat.

Next, from the continuity equation (1), we find the relation
\[ \bar{V}_2 = 1 - \frac{1}{(k+1)M_i^2 \sin^2 \varphi} \left[ (M_i^2 \sin^2 \varphi - 1) \pm \sqrt{(M_i^2 \sin^2 \varphi - 1)^2 - 2q(k+1)M_i^2 \sin^2 \varphi} \right], \quad (11) \]

where \( \bar{V}_2 = V_2 / V_1 \), and the values \( V_1, V_2 \) – represent the specific volumes of the gas-air mixture, respectively, before and behind the reaction front. Given that the specific volumes are related to the densities of the mixture \( \rho_1 \) and \( \rho_2 \) by the formulas \( V_1 = 1/\rho_1, V_2 = 1/\rho_2 \), we transform formula (11) to the form

\[ \bar{\rho}_2 = \frac{(k+1)M_i^2 \sin^2 \varphi}{(k+1)M_i^2 \sin^2 \varphi - \left[ (M_i^2 \sin^2 \varphi - 1) \pm \sqrt{(M_i^2 \sin^2 \varphi - 1)^2 - 2q(k+1)M_i^2 \sin^2 \varphi} \right]}, \quad (12) \]

where \( \bar{\rho}_2 = \rho_2 / \rho_1 \).

Next, the formula (7) is first reduced to the form

\[ \bar{p}_2 = 1 + kM_i^2 \sin^2 \varphi \left( 1 - \frac{v_{2x}}{v_{1x}} \right) \]

and given the ratio (10), we find the relative pressure \( \bar{p}_2 = p_2 / p_1 \) behind the reaction front

\[ \bar{p}_2 = 1 + \frac{k}{k+1} \left[ (M_i^2 \sin^2 \varphi - 1) \pm \sqrt{(M_i^2 \sin^2 \varphi - 1)^2 - 2q(k+1)M_i^2 \sin^2 \varphi} \right], \quad (13) \]

Since, according to formula (2), the tangential components of the gas-air flow velocity at the transition through the reaction front are equal to each other, and the normal components have different values, the velocity vector \( \vec{v}_1 \) at the transition through the reaction front changes its direction by an angle \( \theta \) (see figure 1).

Polars are used not only for the analysis of oblique reaction fronts, but also for the study of oblique shock waves. However, unlike the latter, polars of reaction fronts can occur not only in supersonic, but also in subsonic flows.

### 3. Analysis of the results obtained

It follows from the above that to discuss the polar fronts of the reaction on the plane \( \bar{p}_2, \theta \) it is necessary to identify the relationship between the relative pressure \( \bar{p}_2 \) and the angle of rotation \( \theta \). For this purpose, we use the equalities (1) – (3), with which we find

\[ \tan \theta - \frac{1 - \bar{p}_2}{1 + \bar{p}_2 \tan^2 \varphi}, \quad (14) \]

and with the help of figure 2 define

\[ \tan \varphi = \frac{v_{1y} - v_{2y}}{v_{2y}}, \quad (15) \]

where the velocity components \( v_{2y}, v_{2x} \) are expressed as follows (see figure 2)

\[ v_{2y} = v_{1y} \cos \varphi, \quad v_{2x} = \frac{v_{2y} \cos \theta}{\sin(\varphi - \theta)}. \quad (16) \]
From the joint consideration of formulas (11) and (13), we establish a relationship between the relative specific volume $\bar{V}_2$ and relative pressure $\bar{p}_2$:

$$\bar{V}_2 = 1 - \frac{\bar{p}_2 - 1}{kM_1^2 \sin^2 \varphi},$$

by virtue of which, formula (14) is converted to the form

$$\tan \theta = \frac{\bar{p}_2 - 1}{kM_1^2 - (\bar{p}_2 - 1)} \frac{1}{\tan \varphi}. \quad (17)$$

And finally, substituting the expressions (15), (16) and (10) into formula (17), and using formula (10) again, after rather cumbersome transformations, we get the formula

$$\tan \theta = \pm \frac{\bar{p}_2 - 1}{kM_1^2 - (\bar{p}_2 - 1)} \sqrt{\frac{M_1^2}{\bar{p}_2 - 1} - \frac{1 - kq}{2k(\bar{p}_2 - 1) + 1}}, \quad (18)$$

which is the desired relation of angle $\theta$ and pressure $\bar{p}_2$, provided that the Mach numbers $M_1$ and Damkeler $q$ are known.

The analysis of formula (18) shows that it is valid for both supersonic flow ($M_1 > 1$) and subsonic flow ($M_1 < 1$). Graphs based on formula (18) on the plane $\bar{p}_2, \theta$, which are the polars of the reaction fronts, are very convenient for solving problems with specified conditions for pressure changes at the front boundaries.

In this article, we have constructed two types of polars – for supersonic gas-air flow ($M_1 = 2.5$) (figure 3a) and for subsonic ($M_1 = 0.55$) (figure 3b). In both cases, the number Damkaer taken the following $q = 0, 0.25, 0.5$.

Due to the presence of “plus” and “minus” signs in formula (18), the polars of the reaction fronts are symmetrical relative to the vertical axis $\bar{p}_2$. Therefore, any glade is a curve consisting of two symmetrical branches relative to the vertical axis (figure 3).

Thus, in the absence of heat supply ($q = 0$) to the flow area of the gas-air mixture, both in supersonic and subsonic flow, the polar represents a single “heart-shaped” curve and corresponds to the limiting case of an adiabatic jump of compaction.

If the Damkeler number is not zero, then the reaction front polar consists of two curves, one of which is closed, and the second is a convex-concave curve with two inflection points. The maximum value of the function $\bar{p}_2(\theta)$ on the second curve is $\bar{p}_2 = 1$ at the point $\theta = 0$, both in the supersonic flow and in the subsonic flow (see figure 3).

As the amount of heat supplied increases $\bar{q}$, the closed curves decrease and, at a certain maximum value $\bar{q} = \bar{q}_{\text{max}}$, degenerate into a point. If, then there is no angle of inclination of the front $\varphi$ at which the flow will be stationary.

The polars of subsonic reaction fronts (figure 3b), where the Mach number $M_1$ is less than one, differ from the polars of supersonic reaction fronts (figure 3A). In particular, note that if $M_1 < 1$, then its normal component $M_{1n} < 1$ and so the reaction proceeds in the form of deflagration combustion [19], which occurs only when the thickness of the combustion front is negligible compared with the dimensions of the flow region of the gas mixture.
Figure 2. Polars of the supersonic (a) and subsonic (b) reaction front at a constant Mach number of flow $M_1$ and variable heat supply in the axes $\theta$, $\bar{p}_2$ – plane.

4. Conclusions
The problem of the oblique reaction front in gas-air flows of mine workings is formulated. Formulas for determining the density, specific volume, and pressure behind the oblique front of the reaction are obtained. The relationship between the deflection angle of the gas-air flow crossing the reaction front and the pressure behind the front is established. On the basis of the obtained formulas and the constructed polar oblique fronts of the reaction it is established:

- with a fixed Mach number of the incoming gas-air flow and no heat supply to the flow area, the oblique front polar of the reaction is a single «heart-shaped» curve and corresponds to the limiting case of an adiabatic jump;
- if the Damkeler number is not zero, then the reaction front polar consists of two curves, the first of which is a closed curve, and the second is a convex-concave curve with two inflection points;
- as the Damkeler number increases, the size of closed curves decreases and at the maximum value $\bar{q} = \bar{q}_{\text{max}}$, the curves degenerate to a point;
- polars of subsonic fronts are characterized by the fact that the chemical reaction takes place in the form of deflagration combustion, which occurs only in the case when the thickness of the combustion front is negligible compared to the size of the flow region of the gas-air mixture.

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