Accelerated Expansion of the Universe based on Emergence of Space and Thermodynamics of the Horizon

Fei-Quan Tu,† Yi-Xin Chen,‡ Bin Sun,† and You-Chang Yang†

†School of Physics and Electronic Science, Zanji Normal University, Zanji 563006, China
‡Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou 310027, China

Researches in the several decades have shown that the dynamics of gravity is closely related to thermodynamics of the horizon. In this paper, we derive the Friedmann acceleration equation based on the idea of “emergence of space” and thermodynamics of the Hubble horizon whose temperature is obtained from the unified first law of thermodynamics. Then we derive another evolution equation of the universe based on the energy balance relation \( \rho V_H = TS \). Combining the two evolution equations and the equation of state of the cosmic matter, we obtain the evolution solutions of the FRW universe. We find that the solutions obtained by us include the solutions obtained in the standard general relativity (GR) theory. Therefore, it is more general to describe the evolution of the universe in the thermodynamic way.

1. INTRODUCTION

Numerous astronomical observations tell us that the universe is in accelerated expansion \([1–6] \). In order to explain the accelerated expansion, a cosmological constant is usually added to the Einstein field equation. In particular, Λ cold dark matter (ΛCDM) model with 4% usual matter, 23% cold dark matter and 73% dark energy describes the accelerated expansion of the universe well. In addition, the accelerated expansion can also be described by modifying the geometrical part of the field equation (For example, \( f(R) \) gravity \([7, 8] \) and Lanczos-Lovelock gravity).

Recently, Padmanabhan \([9, 10] \) have suggested that the difference between the number of the surface degrees of freedom \( N_{\text{sur}} \) and the number of the bulk degrees of freedom \( N_{\text{bulk}} \) in a region of space drives the accelerated expansion of the universe through a simple equation \( \frac{dV}{dN} = L_p^2 (N_{\text{sur}} - N_{\text{bulk}}) \), where \( V \) is the Hubble volume and \( L_p \) is the Planck length. The standard Friedmann equation of the FRW universe can be derived in this emergence of space scenario. Subsequently, emergent perspective of gravity was further investigated by many researchers \([11–13] \).

On the other hand, it is interesting and meaningful to study cosmology from the point of view of thermodynamics. In 1995, Jacobson \([14] \) argued that the Einstein equation can be derived from the proportionality of entropy and horizon area together with the Clausius relation \( \delta Q = TdS \) with \( \delta Q \) and \( T \) interpreted as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. He also pointed out that the Einstein equation is an equation of state. His paper revealed that thermodynamics of spacetime is closely related to dynamics of gravity. Since then, the relationship between thermodynamics of spacetime and dynamics of gravity have been investigated widely. It has been shown that the field equations are equivalent to the thermodynamic identity \( TdS = dE + PdV \) on the horizons in a very wide class of gravitational theories, such as on the static spherically symmetric horizons \([15] \), the stationary axisymmetric horizons and evolving spherically symmetric horizons \([16] \) in Einstein gravity, static spherically symmetric horizons \([17] \) and dynamical apparent horizons \([18] \) in Lanczos-Lovelock gravity, three dimensional BTZ black hole horizons \([19, 20] \) etc.

Einstein’s equations are equivalent to the unified first law of thermodynamics for the dynamical black hole when the notion of trapping horizon is introduced in the GR theory \([21–24] \). Inspired by this conclusion, our universe may be considered as a non-stationary gravitational system \([25] \). Thus we can obtain the temperature of Hubble horizon of the universe based on the unified first law. The advantage of obtaining the horizon temperature in this way is that the temperature has a definite physical origin and a clear mathematical expression.

In many references (e.g. Refs. \([11–13, 26] \) which describe the evolution of the FRW universe, the temperature of Hubble horizon is usually assumed to be \( T = H/2\pi \). However, the form of temperature cannot be obtained from an elegant physical principle. In this paper, we consider the accelerated expansion of the universe in the late time based on emergence of space and thermodynamics of the Hubble horizon. We employ the temperature obtained from the unified first law of thermodynamics as the temperature of the horizon. Furthermore, we obtain the number of modified bulk degrees of freedom and get the corresponding dynamical equations in the FRW universe based on emergence of space. Then we obtain another evolution equation of the universe based on the energy balance relation \( \rho V_H = TS \), where \( S \) is the entropy associated with the area of the Hubble sphere and \( TS \) is the heat energy of the boundary surface. Combining the two evolution equations and the equation of state of the cosmic matter, we determine the evolution laws of the universe. By analyzing the solutions of the evolution...
laws, we find the solutions obtained by us include the solutions obtained in the standard general relativity (GR) theory. Therefore, it is more general to describe the evolution of the universe in the thermodynamic way. Next we analyze the solutions in order for the completeness of the discussion. Finally, we make some comments on the alternative perspective for the evolution of the universe.

The present paper is organized as follows. In section 2, we give a simple review about Padmanabhan’s work and obtain the temperature of Hubble horizon based on a good motivation. Furthermore, we derive the modified Friedmann equations of the FRW universe based on emergence of space and thermodynamics of the Hubble horizon. In section 3, we obtain the solutions of the Friedmann equations of the FRW universe and analyze their physical meanings. In section 4, some comments on the alternative perspective for the evolution of the universe are made. Finally, we present our conclusions.

In this alternative perspective for the evolution of the universe, we give a simple review about Padmanabhan’s work and obtain the properties of the evolution of the FRW universe in the thermodynamic way. Next we analyze the solutions in order for the completeness of the evolution of the universe in the thermodynamic way. Fortunately, we can obtain the expression of the temperature on the horizon of the de Sitter space which gives
\[
\frac{\dot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p).
\]

This result is the same as that obtained by solving the Einstein field equation.

If the equation of state in the de Sitter space is \( p = -\rho \), then Eq. (4) can be reduced to \( H^2 = 8\pi L_p^2 \), which is the standard result of the evolution of the de Sitter universe in the GR theory. This result also shows that the evolution of the de Sitter space obeys the principle of holographic equipartition.

Furthermore, our universe is asymptotically de Sitter, so he thought that the difference between \( N_{\text{sur}} \) and \( N_{\text{bulk}} \) drives the universe towards holographic equipartition and the evolution of the universe is dominated by
\[
\frac{dV}{dt} = L_p^2 (N_{\text{sur}} - N_{\text{bulk}}).
\]

Thus substituting Eq. (2) and Eq. (3) into Eq. (5) and using the relations \( V = 4\pi/3H^3 \), \( T = H/2\pi \), we obtain the following relation
\[
\frac{\dot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p).
\]

This result is the same as that obtained by solving the Einstein field equation.

In Refs. [27, 28], the authors derived the temperature of the horizon of the de Sitter space \( T = H/2\pi \) by using the field theory where \( H^{-1} \) is the radius of the de Sitter space. However, our universe is asymptotically de Sitter rather than purely de Sitter, so the temperature of the horizon of the universe may not necessarily be expressed as \( H/2\pi \). Thus the following two questions arise: What is the expression of the temperature on the horizon of the FRW universe? How can we obtain this expression? Fortunately, we can obtain the expression of the temperature on the horizon of the FRW universe from an elegant physical principle.

The above FRW metric (Eq. (1)) can be written in the double-null form [29, 30] as
\[
ds^2 = -2d\xi^+ d\xi^- + R^2 d\Omega^2,
\]
where \( \partial_\pm = \frac{\partial}{\partial \xi^\pm} = -\sqrt{2} \left( \frac{\partial}{\partial t} \pm \frac{1}{r} \frac{\partial}{\partial r} \right) \) are future pointing null vectors.

The trapping horizon is defined as \( \partial_+ R|_{R=R_T} = 0 \), which gives
\[
R_T = \frac{1}{\sqrt{H^2 + \frac{8\pi}{3}}}/\sqrt{H^2} = R_A,
\]
where \( R_A \) is the apparent horizon. The surface gravity is defined as
\[
\kappa = \frac{1}{2\sqrt{-h}} \partial_a \left( \sqrt{-h} h^{ab} \partial_b R \right),
\]
so we can get
\[
\kappa = -\left( 1 + \frac{\dot{H}}{2H^2} \right) H.
\]
for the apparent horizon of the flat universe. Here we would like to make some explanations on why we choose the flat universe \((k = 0)\): (1) Our universe is flat according to astronomical observations; (2) We have chosen \(1/H\) as the radius of the horizon when we obtain the dynamical equation of the cosmic evolution, so we also choose Hubble horizon here in order for the consistency.

According to the relation between the surface gravity and the temperature, we obtain the temperature of the Hubble horizon here in order for the consistency. 

\[
T = \frac{|\kappa|}{2\pi} = \frac{H}{2\pi} \left(1 + \frac{\dot{H}}{2H^2}\right). \tag{11}
\]

It is emphasized that the definition of the surface gravity Eq.\((9)\) and the temperature on the horizon Eq.\((11)\) are from the unified first law. The unified first law is \(dE = A\Psi + WdV\) where \(E\) is Misner-Sharp energy, \(\Psi_a\) is energy-supply, \(W\) is work, \(A\) and \(V\) are the area and volume of a sphere. One can obtain the projection relation \(<dE, \xi> = \frac{2\pi}{\sqrt{\kappa}} <dA, \xi> + <WdV, \xi>\) where \(\xi\) is a projection vector and \(\kappa\) is defined as Eq.\((9)\) when projecting the unified first law along the inner trapping horizon of the FRW universe in the GR theory. On the inner trapping horizon, the projection relation can be expressed as \(<A\Psi, \xi> = \frac{2\pi}{\sqrt{\kappa}} <dA, \xi>\). Note that \(A\Psi\) when projected on the trapping horizon gives the heat flow \(\delta Q\) so the projection relation implies the Clausius relation \(\delta Q = TdS\) if we employ that \(T = |\kappa|/2\pi\) and \(S = A/4G\). Only when the temperature (or the surface gravity) is defined in this way can the unified first law imply the Clausius relation. Therefore, we can say that the expression \((11)\) of the temperature on the horizon of the FRW universe obtained by us originates from the fundamental physical relation, i.e. the unified first law.

Substituting Eq.\((2)\), Eq.\((3)\) and Eq.\((11)\) into Eq.\((5)\), we get

\[
\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p) \left(1 + \frac{\dot{H}}{2H^2}\right)^{-1}. \tag{12}
\]

This is the evolution equation of the FRW universe when we employ the idea of emergence of space and reconsider the temperature of the Hubble horizon. Comparing Eq.\((12)\) with the standard Friedmann acceleration equation \(\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p)\) in the GR theory, we find the term about the density of the energy and pressure has a correction in our model. Furthermore, Eq \((12)\) can be rewritten as

\[
\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p) - \frac{\dot{H}}{2} - \frac{\dot{H}^2}{2H^2}. \tag{13}
\]

In the GR theory, the Friedmann acceleration equation with cosmological constant \(\Lambda\) is \(\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p) + \frac{\dot{H}}{2} - \frac{\dot{H}^2}{2H^2}\), where the subscript \(m\) represents matter and the positive constant \(\Lambda\) is explained as the vacuum energy. Introducing cold dark matter again, this equation can describe the accelerated expansion of the universe well. In the model of entropic cosmology\([32,33]\), the Friedmann acceleration equation is given as \(\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p) + C_HH^2 + C_H\dot{H}\) where the coefficients are bounded by \(\frac{1}{2\pi} \leq C_H \leq 1\) and \(0 \leq \dot{C_H} \leq \frac{1}{H}\). The reason that the term \(C_HH^2 + C_H\dot{H}\) is added to the Friedmann acceleration equation is that the surface term in the gravitational action can not be ignored. The accelerated expansion of the late universe\([32]\) and the inflation of the early universe\([33]\) can be explained by adding this term. So the general formula of the Friedmann acceleration equation can be summarized as (See, for example, in Refs.\([32,33]\))

\[
\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p) + f(H, \dot{H}), \tag{14}
\]

where \(f(H, \dot{H})\) is a function of \(H\) and \(\dot{H}\). The specific forms of Eq.\((14)\) have been used to discuss cosmology widely, which result in some good results (For example in Refs.\([34,35]\)). However, \(\rho\) and \(p\) are the effective energy density and pressure which include the equivalent energy and pressure of the vacuum energy in our model.

If we compare Eq.\((13)\) with the Friedmann acceleration equation \(\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3}(\rho + 3p)\) in the GR theory, we find that the vacuum energy \(\Lambda\) has a shift \(-\frac{3\dot{H}}{2H^2}(H^2 + \dot{H}) = -\frac{3\dot{H}}{2H^2}\) which changes with time therefore it is not a constant in our model. Analyzing the shift \(-\frac{3\dot{H}}{2H^2}\), we find that the total vacuum energy is always positive as long as the universe is in accelerated expansion and \(H < 0\). On the other hand, the results that the vacuum energy is positive and the universe is in accelerated expansion are consistent with the present astronomical observations. Thus the acceleration equation \((13)\) has a good physical explanation.

Now we would like to make some comments about the Friedmann acceleration equation Eq.\((13)\). Firstly, the unambiguous derivation of Eq.\((13)\) is just based on emergence of space and the reasonable temperature of the Hubble horizon of the FRW universe. In contrast, the model in Ref.\([32]\) is a phenomenological model in which the surface term is introduced without rigorous derivation. Secondly, the temperature of the horizon of the de Sitter space have been derived\([27,28]\, but the temperature of the Hubble horizon of the FRW universe, which is not purely de Sitter space, is just assumed to be \(T = H/2\pi\) in many references as we have pointed out. However, we employ the temperature of the Hubble horizon which can be obtained from the unified first law\([29,30]\) in this paper. Thirdly, the specific forms of \(f(H, \dot{H})\) in Eq.\((14)\) have been studied widely and some good results have been obtained. Moreover, our equation is consistent with astronomical observations. Finally, we would like to stress that the derivation of the acceleration equation \((13)\) is based on an elegant principle (the unified first law) rather than an assumption and this is
an advantage of our model.

On the other hand, the Friedmann equation \( \rho = 3H^2/(8\pi G) \) can be rewritten as an energy balance relation
\[
\rho V_H = TS
\] (15)
in thermodynamic language, where \( S = A_H/(4L_p^2) = \pi H^{-2}/L_p^2 \) is the entropy associated with the area of the Hubble sphere \( V_H = 4\pi H^3 \) and \( TS \) is the heat energy of the boundary surface.

From now on, we take the energy balance relation (15) as the fundamental equation describing the change of the energy density of cosmic matter. Substituting Eq.(11) and \( S = \pi H^{-2}/L_p^2 \) into the energy balance relation (15), we obtain
\[
H^2 = \frac{8\pi L_p^2}{3} \left( 1 + \frac{\dot{H}}{2H^2} \right)^{-1} \rho.
\] (16)

This equation can be rewritten as
\[
H^2 = \frac{8\pi L_p^2}{3} \rho - \frac{\dot{H}}{2}.
\] (17)

Comparing Eq.(17) with the standard Friedmann equation \( H^2 = \frac{8\pi L_p^2}{3} \rho \) in the GR theory, we find that Eq.(17) has an extra term \(-\frac{\dot{H}}{2}\) in the right side. Combining Eq.(13) with Eq.(17), we obtain the following equation
\[
\dot{\rho} + 3H(\rho + p) = \frac{3}{8\pi L_p^2} \left( \frac{\dot{H}}{2} - \frac{H^2}{H} \right).
\] (18)

The R.H.S. of the above equation is nonzero, which indicates that the evolution of the universe is a nonadiabatic process from the thermodynamic point of view. This is understandable because there exist temperature and entropy on the Hubble horizon of the universe. The general form of the modified continuity equation \( \dot{\rho} + 3H(\rho + p) = g(H, \dot{H}) \) where \( g(H, \dot{H}) \) is the function of \( H \) and \( \dot{H} \) has been introduced in following three models. The first model is the bulk viscous cosmology \[38, 39\] where a bulk viscosity of cosmological fluid is assumed. The function \( g(H, \dot{H}) = 9H^2\dot{\zeta} \) in which \( \dot{\zeta} \) is the bulk viscosity is employed to investigate the accelerated expansion of the universe (See, for example, in Refs.\[40–42\]. The second model is the energy exchange cosmology in which the exchange of energy between two fluids is assumed. For example, the continuity equation for matter “m” is given as \( \rho_m + 3H(\rho_m + p_m) = -\Lambda(t) \) when the dynamical cosmological term \( \Lambda(t) \) decays linearly with the Hubble rate \( H \)[43–46]. The third model is the nonadiabatic entropic cosmology where the Hubble horizon is assumed to have an entropy and a temperature, and the function \( g(H, \dot{H}) \) is \[35\] \( \frac{3}{8\pi L_p^2} \dot{H} H \). However, in this study, the function \( g(H, \dot{H}) \) is \( \frac{3}{8\pi L_p^2} \left( \frac{\dot{H}}{2} - \frac{H^2}{H} \right) \) and the modified continuity equation (18) is the result of combination of Eq.(13) and Eq.(17). That is to say, the derivation of Eq.(18) is based on emergence of space and thermodynamics of the Hubble horizon.

At the end of this section, we would like to stress that Eq.(13) and Eq.(17) are the laws of evolution of the universe obtained by us. The derivation of Eq.(13) is based on the principle of asymptotically holographic equipartition and the unified first law, while the derivation of Eq.(17) is based on the energy balance relation on the Hubble horizon. Therefore, we describe the evolution of the universe in the language of thermodynamics in this section.

3. FORMULATIONS AND EXPLANATION OF ACCELERATED EXPANSION OF THE UNIVERSE

As usual, the form of the equation of state of the matter is
\[
p = \omega \rho,
\] (19)
where \( \omega \) is a parameter which may change over time. Combining Eq.(13), Eq.(17) with Eq.(19), we obtain the following equation
\[
\ddot{H}^2 + \frac{7 + 3\omega}{2}H^2\dot{H} + 3(1 + \omega)H^4 = 0.
\] (20)

This equation can be reduced to
\[
\left( H^2 + \frac{1}{2}H \right) \left[ 3(1 + \omega)H^2 + 2\dot{H} \right] = 0.
\] (21)

The equation can be divided into two equations \( H^2 + \frac{1}{2}H = 0 \) and \( 3(1 + \omega)H^2 + 2\dot{H} = 0 \). Now we discuss these equations and analyze their physical meanings.

First of all, we discuss the solution \( H^2 = -\frac{1}{2}\dot{H} \). we get the scale factor
\[
a(t) \sim t^{1/2}
\] (22)
under this solution. The deceleration parameter in cosmology is defined by
\[
q \equiv -\left( \frac{\ddot{a}}{aH^2} \right)_{t=t_0}.
\] (23)

Substituting Eq.(22) into Eq.(23), we obtain the deceleration parameter \( q = 1 \) which implies that our universe has a tendency of decelerated expansion. As we know, there exists attraction between any kinds of ordinary matter and gravity has a tendency to decelerate the expansion of the universe decelerate. Therefore, the universe has to have a tendency to decelerate as long as there exists ordinary matter, and this solution is a reflection of this property. This solution is consistent with that of the epoch dominated by the radiation in the GR theory.
Next, we shall analyze the equation $3(1 + \omega)H^2 + 2 \dot{H} = 0$ and find that this equation is the same as the solution of evolution of the universe in the GR theory. In order for the completeness of the discussion, we analyze the solution $H^2 = \begin{cases} \frac{2}{3(1 + \omega)} \dot{H} & \omega \neq -1 \\ \text{constant} & \omega = -1 \end{cases}$ and present the nature of evolution of the universe in the following paragraphs.

The “Hubble constant” $H$ is a true constant when $\omega = -1$. We can get the scale factor $a(t) \sim e^{\alpha t}$, which is known as the de Sitter model \cite{47, 48}. In addition, the solution is also related to inflation at the early stage of the universe. When $H^2 = -\alpha \dot{H}$ where $\alpha = \frac{2}{3(1 + \omega)}$, we get the scale factor

$$a(t) \sim t^\alpha. \quad (24)$$

Substituting Eq.\!(24)\! into Eq.\!(23), we obtain the deceleration parameter $q = \frac{1}{ \alpha } - 1$. Our universe is in accelerated expansion according to the astronomical observation, so the deceleration parameter $q$ has to be less than 0 which implies $\alpha > 1$ and $-1 < \omega < -\frac{1}{3}$. In order to see the nature of evolution of the universe clearly, we choose several specific values of $\omega$. We choose the following values:

| $\omega$ | $\alpha$ |
|----------|----------|
| -0.52    | 1.4      |
| -0.67    | 2.0      |
| -0.78    | 3.0      |

The simple relation between the age of the universe and the Hubble constant gives the age of the universe

$$t_0 = \frac{\alpha}{H_0}, \quad (25)$$

where $H_0$ is the current Hubble constant. Substituting the current Hubble constant $H_0 = 70 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ into Eq.\!(25), we obtain that the age of the universe is 14 billion year.

The luminosity distance $d_L$ has been widely used to investigate the accelerated expansion of the universe and it is given as \cite{35, 49}

$$d_L = \frac{c(1 + z)}{H_0} \int_1^{1+z} \frac{dy}{H_y/H_0} \quad (26)$$

where the variable $y$ is defined as $y = a_0/a$ and $z$ is the redshift defined by $z + 1 \equiv y = a_0/a$. Substituting $a(t) = t^\alpha$ into Eq.\!(26), we have

$$\frac{H_0}{c} d_L = \frac{1 + z}{1 - 1/\alpha} [(1 + z)^{1-1/\alpha} - 1] \quad (27)$$

Substituting $\alpha = 3.0$, $\alpha = 2.0$ and $\alpha = 1.4$ into Eq.\!(27) and plotting the dependent relation curves between the luminosity distance and the redshift factor, we can obtain the above three curves of FIG.1 respectively.

![FIG. 1. (color online). The above three curves correspond to $\alpha = 3.0$, $\alpha = 2.0$ and $\alpha = 1.4$ from top to bottom respectively, and the curve at the bottom is a fitting curve of the data obtained by astronomical observations.](image)

For the $\Lambda$CDM model, the luminosity distance of the flat universe is given by Refs.\cite{33, 50}, its form is

$$\left(\frac{H_0}{c}\right) d_L = (1 + z) \int_0^z dz' [(1 + z')^2 (1 + \Omega_m z') - z' (2 + z') \Omega_\Lambda]^{-1/2}, \quad (28)$$

where $\Omega_m$ and $\Omega_\Lambda$ are the ratios of the non-relativistic matter and the vacuum energy to the critical energy density respectively. It has been found that the relation curve between the luminosity distance and the redshift factor for $\Omega_m = 0.23$ and $\Omega_\Lambda = 0.73$ is consistent with the fitting curve of the data obtained by WMAP \cite{51, 52}, and the curve represented by Eq.\!(28) is the curve at the bottom of FIG.1. Analyzing FIG.1, we find that these curves are well in line with the result of astronomical observations. In particular, the curve represented by $\alpha = 1.4$ agrees very well with the curve fitted by the observed supernova data (see FIG.2) and the age of the universe will be 19.6 billion year if the universe has been evolving in this way.

### 4. COMMENTS ON THE ALTERNATIVE PERSPECTIVE FOR THE EVOLUTION OF THE UNIVERSE

The conventional way of describing the evolution of the universe is employing the GR theory and cosmological principle. In the GR theory, the evolution equations of the flat universe are $\ddot{a} = H^2 + \dot{H} = \frac{4\pi L^2}{3} (\rho + 3p)$ and $H^2 = \frac{8\pi L^2}{3} \rho$. Combining the two equations with the equation of state of the cosmic matter, we can obtain $3(1 + \omega)H^2 + 2 \dot{H} = 0$ which is only a solution of Eq.\!(21). Our results are more general because they can cover the results obtained in the GR theory. There has
always to be the solution \( a(t) \sim t^{1/2} \) throughout the evolutionary history of the universe if the evolution of the universe obeys the equations Eq.(13) and Eq.(17). We think that this solution is the evolution law of a possible universe in which the mass of matter equals exactly the mass of anti-matter. If we only consider the solution \( 3(1 + \omega)H^2 + 2H = 0 \), then the thermodynamical description in our model is equivalent to the dynamical description in the GR theory from the perspective of evolution of the universe. However, the physical picture is still different, the vacuum energy has a shift which changes with time in our model while the vacuum energy is constant in the GR theory. In our models, the entire evolutionary history of the universe can be described by the radiation dominated period, non-pressure dominated period and vacuum energy dominated period. The age of the universe is less than \( t_0 = 13.4^{+1.3}_{-1.0} \times 10^9 \) yr indicated by the data of the supernova\(^2\)\(^\text{[2]}\) if the universe evolves according to the laws of the radiation dominated period or non-pressure dominated period. The age of the universe is greater than \( t_0 = 13.4^{+1.3}_{-1.0} \times 10^9 \) yr when the universe evolves according to the laws of the vacuum energy dominated period. So the law of evolution of the universe has to change from a decelerated expansion to an accelerated expansion at a certain point in time. The result is compatible with astronomical observations\(^2\).

Now we discuss the physical meaning of the temperature of the Hubble horizon. Jacobson pointed out that the equilibrium thermodynamic relation \( \delta Q = TdS \) is equivalent to the Einstein equation. Moreover, the first law of thermodynamics \( TdS = dE + PdV \) is also equivalent to the field equations in a very wide class of gravitational theories as we have pointed out in the introduction. All these show that our universe can be viewed as an equilibrium system (that is to say, the evolution of the universe can be viewed as a series of quasi-static processes) when the temperature is taken to be the temperature of the horizon from the thermodynamic point of view. Hence, it is reasonable to employ the temperature of the Hubble horizon as the temperature of the universe. This is reasonable because numerous researches have shown that our universe can be viewed as an equilibrium system from the thermodynamic point of view.

Combining the two evolution equations and the equation of state of the cosmic matter, we determine the evolution equation of the FRW universe. Then we analyze the solutions and discuss their physical meaning. For the solution \( H^2 = -\frac{1}{2}H \), we think that this is the evolution law of a possible universe in which the mass of matter equals exactly the mass of anti-matter. For the solution \( 3(1 + \omega)H^2 + 2H = 0 \), we find that this solution is the same as the solution of the evolution equation of the universe in the GR theory. In order for the completeness of the discussion, we plot some relation curves with the parameters \( \alpha = 1.4, 2.0, 3.0 \) of the luminosity distance and the redshift factor, and find these curves are well in line with the fitting curve of data of astronomical observations. In particular, the curve represented by \( \alpha = 1.4 \) agrees very well with the curve fitted by the observed supernova data. The solutions of the evolution equation of the universe in our model include the solutions obtained in the GR theory, so it is more general to describe the evolution of the universe in the thermodynamic way.

5. CONCLUSIONS

In this paper, we study the accelerated expansion of the universe based on emergence of space and thermodynamics of the Hubble horizon. For this purpose, we derive two evolution equations of the universe from a thermodynamical point of view. One is the Friedmann acceleration equation which is derived from emergence of space and thermodynamics of the Hubble horizon whose temperature is obtained from the unified first law of thermodynamics. The other is obtained from the energy balance relation \( \rho V_H = TS \). In the process of derivation, we employ the temperature of the Hubble horizon as the temperature of the universe. This is reasonable because numerous researches have shown that our universe can be viewed as an equilibrium system from the thermodynamic point of view.

6. ACKNOWLEDGMENTS

This work is supported by the NNSF of China(Grant No.11375150) and Doctoral Foundation of Zunyi Normal University(Grant No.BS[2016]03). B.S. acknowledges the Science and Technology Foundation of Guizhou Province (Grant No.J[2015]2149). Y.-C.Y. acknowledges the NNSF of China (Grant No.11265017), the China Postdoctoral Science Foundation (Grant No.2015M571727), and the Guizhou province outstanding youth science and
technology talent cultivation object special funds (Grant No.QKHRZ(2013)28).

[1] S. Perlmutter et al., “Discovery of a supernova explosion at half the age of the Universe”, Nature 391, 51(1998)
[2] S. Perlmutter et al., “Measurements of Ω and A from 42 High-Redshift Supernovae”, Astrophys. J. 517, 565(1999)
[3] A. G. Riess et al., “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant”, Astron. J. 116, 1009(1998)
[4] A. G. Riess et al., “Type Ia Supernovae Discoveries at z > 1 from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution”, Astronomy. J. 607, 665(2004)
[5] A. G. Riess et al., “New Hubble Space Telescope Discoveries of Type Ia Supernovae at z ≥ 1: Narrowing Constraints on the Early Behavior of Dark Energy”, Astronomy. J. 659, 98(2007)
[6] M. Hicken et al. “Improved Dark Energy Constraints From 100 New CfA Supernova Type Ia Light Curves”, Astronomy. J. 700, 1097(2009)
[7] S. Nojiri, S. D. Odintsov, “Modified f(R) gravity consistent with realistic cosmology: from matter dominated epoch to dark energy universe”, Phys. Rev. D 74 086005(2006)
[8] S. Kapozziello, S. Nojiri, S. D. Odintsov, A. Troisi, “Cosmological viability of f(R)-gravity as an ideal fluid and its compatibility with a matter dominated phase”, Phys. Lett. B 639, 135(2006)
[9] T. Padmanabhan, “Emergence and Expansion of Cosmic Space as due to the Quest for Holographic Equipartition”, arXiv:1206.4916
[10] T. Padmanabhan, “Emergent perspective of Gravity and Dark Energy”, Res. Astro. Astrophys. 12, 891(2012)
[11] R. G. Cai, “Emergence of Space and Spacetime Dynamics of Friedmann-Robertson-Walker Universe”, JHEP 11, 016(2012)
[12] F. Q. Tu, Y. X. Chen, “Emergence of spaces and the dynamic equations of FRW universes in the f(R) theory and deformed Horava-Lifshitz theory”, JCAP 05, 024(2013)
[13] K Yang, Y. X Liu, Y. Q Wang, “Emergence of Cosmic Space and the Generalized Holographic Equipartition”, Physical Review D 86, 10(2012)
[14] T. Jacobson, “Thermodynamics of Space: The Einstein Equation of State”, Phys. Rev. Lett. 75, 1260(1995)
[15] T. Padmanabhan, “Classical and quantum thermodynamics of horizons in spherically symmetric space-times”, Class. Quant. Grav. 19, 5387(2002)
[16] D. Kothawala, S. Sarkar, T. Padmanabhan, “Einstein’s equations as a thermodynamic identity: The cases of stationary axisymmetric horizons and evolving spherically symmetric horizon”, Phys. Lett. B 652, 338(2006)
[17] A. Paranjape, S. Sarkar, and T. Padmanabhan, “Thermodynamic route to Field equations in Lanczos-Lovelock Gravity”, Phys. Rev. D 74, 104015(2006)
[18] R. G. Cai et al., “Generalized Vaidya spacetime in Lovelock gravity and thermodynamics on the apparent horizon”, Phys. Rev. D 78, 124012(2008)
[19] M. Akbar, A. A. Siddiqui, “Charged rotating BTZ black hole and thermodynamic behavior of field equations at its horizon”, Phys. Lett. B 656, 217(2007)
[20] M. Akbar, “Thermodynamic Interpretation of Field Equations at Horizon of BTZ Black Hole”, Chin. Phys. Lett. 24, 1158(2007)
[21] S. A. Hayward, “Gravitational energy in spherical symmetry”, Phys. Rev. D 53, 1938(1996)
[22] S. A. Hayward, “Unified first law of black-hole dynamics and relativistic thermodynamics”, Class. Quantum Grav. 15, 3147(1998)
[23] S. A. Hayward, S. Mukhoyama, M. C. Ashworth, “Dynamic black-hole entropy”, Phys. Lett. A 256, 347(1999)
[24] S. A. Hayward, “Energy Conservation for Dynamical Black Holes”, Phys. Rev. Lett. 93, 251101(2004)
[25] S. Mitra, S. Saha, S. Chakraborty, “Universal thermodynamics in different gravity theories: Modified entropy on the horizons”, Phys. Lett. B 734, 173(2014)
[26] R. G. Cai, S. P. Kim, “First Law of Thermodynamics and Friedmann Equations of Friedmann-Robertson-Walker Universe”, JHEP 02, 050(2005)
[27] G. W. Gibbons, S. W. Hawking, “Cosmological event horizons, thermodynamics, and particle creation”, Phys. Rev. D 15, 2738(1977)
[28] D. Lohiya, J. Phys. A, “Trace anomalies in a two-dimensional de Sitter metric and black-body radiation”, Jour. Phys. A 11, 1335(1978)
[29] R. G. Cai, L. M. Cao, “Unified first law and the thermodynamics of the apparent horizon in the FRW universe”, Phys. Rev. D 75, 064008(2007); M. Akbar, R. G. Cai, “Thermodynamic behavior of the Friedmann equation at the apparent horizon of the FRW universe”, Phys. Rev. D 75, 084003(2007)
[30] S. Mitra, S. Saha, S. Chakraborty, “Universal thermodynamics in different gravity theories: Modified entropy on the horizons”, Phys. Lett. B 734, 173(2014)
[31] M. Akbar, R. G. Cai, “Thermodynamic Behavior of Friedmann Equation at Apparent Horizon of FRW Universe”, Phys. Rev. D 75, 084003(2007)
[32] D. A. Easson, P. H. Frampton, G. F. Smoot, “Entropic accelerating universe”, Phys. Lett. B 696, 273(2011)
[33] D. A. Easson, P. H. Frampton, G. F. Smoot, “Entropic Inflation”, Int. J. Mod. Phys. A 27, 1250066(2012)
[34] T. S. Koivisto, D. F. Mota, M. Zumalacarregui, “Constraining entropic cosmology”, J. Cosmol. Astropart. Phys. 02, 027(2011)
[35] N. Komatsu, S. Kimura, “Non-adiabatic-like accelerated expansion of the late universe in entropic cosmology”, Phys. Rev. D 87, 043531(2013)
[36] F. Q. Tu, Y. X. Chen, “Emergence of space and cosmic evolution based on entropic force”, Gen. Relativ. Gravit. 47, 87(2015)
[37] T. Padmanabhan, “Do We really Understand the Cosmos?”, Comptes Rendus Physique, 18, 275(2017)
[38] W. Zimdahl, “Bulk viscous cosmology”, Phys. Rev. D 53, 5483(1996)
[39] R. Colistete, Jr., J. C. Fabris, J. Tossa, W. Zimdahl, “Bulk viscous cosmology”, Phys. Rev. D 76, 103516(2007)
[40] I. Brevik, “Viscous Cosmology, Entropy, and the Cardy-Verlinde Formula”, arXiv:gr-qc/0404095
[41] I. Brevik, O. Gorbunova, “Dark energy and viscous cosmology”, Gen. Relativ. Gravit. 37, 2039(2005)
[42] A. Avelino, U. Nucamendi, “Can a matter-dominated model with constant bulk viscosity drive the accelerated expansion of the universe?”, JCAP 04, 006(2009)
[43] S. Carneiro, C. Pigozzo, H. A. Borges, “Supernova constraints on decaying vacuum cosmology”, Phys. Rev. D 74, 023532(2006)
[44] J. S. Alcaniz, H. A. Borges, S. Carneiro, J. C. Fabris, C. Pigozzo, W. Zimdahl, “A cosmological concordance model with dynamical vacuum term”, Phys. Lett. B 716, 165(2012)
[45] C. Pigozzo, M. A. Dantas, S. Carneiro, J. S. Alcaniz, “Observational tests for Λ(t) CDM cosmology”, JCAP 08, 022(2011)
[46] H. Fritzsch, J. Sola, “Matter non-conservation in the universe and dynamical dark energy”, Class. Quantum Grav. 29, 215002(2012)
[47] W. de Sitter, “On the relativity of inertia: Remarks concerning Einstein’s latest hypothesis”, Proc. Kon. Ned. Acad. Wet. 19, 1217(1917)
[48] W. de Sitter, “On the curvature of space”, Proc. Kon. Ned. Acad. Wet. 20, 229(1917)
[49] K. Sato et al., “Cosmology I”, Modern Astronomy Series Vol.2, Nippon Hyoron Sha Co.(2008)
[50] S. M. Carroll, W. H. Press, “The Cosmological Constant”, Annu. Rev. Astron. Astrophys 30, 499(1992)
[51] N. Jarosik et al., “Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Sky Maps, Systematic Errors, and Basic Results”, Astrophys. J. Suppl. Ser. 192, 14(2011)
[52] E. Komatsu et al., “Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation”, Astrophys. J. Suppl. Ser. 192, 18(2011)
[53] S. Weinberg, “Cosmology”, Oxford university press(2008)