1. INTRODUCTION

In wind engineering and in other fields, damping is one of the most important parameters. Damping though comes with high uncertainty. This consequently results in lower reliability for wind-resistant design. In the past, damping has been assigned a value that is constant with amplitude, based on Jacobsen’s equivalent viscous damping concept. But in real structures, it is known that there are no viscous elements. Meanwhile, it has been established that damping has a certain amplitude dependency and it is said to be due to stick-slip friction. It has also been discussed that the use of the hysteretic damping concept can be applied to non-viscous damping, providing an equivalent viscous damping value for a certain amplitude even in systems without viscous elements. For a structure without supplementary damping devices, damping can be viewed as a sum of structural and aerodynamic damping contributions. Aerodynamic damping is obviously also amplitude-dependent, and it has already been studied by Krishna et al., Marukawa et al., Holmes, and many others. It is already well-known that along-wind oscillation can result in positive aerodynamic damping, while across-wind oscillation can result in either positive or negative aerodynamic damping. Therefore, the focus in this paper is on structural damping, its estimation, and whether stick-slip mechanism can indeed describe it or not.

This study of structural damping is particularly for wind-resistant design purposes, where in current practice, the main structure is desired to remain within linear-elastic limits.

SUMMARY

Damping measurements on a 200m-high steel office building show an increase and then decrease of damping with amplitude. But current damping predictors for such a building does not account for the decreasing part. There is thus a big discrepancy between the predictor model and actual conditions that implicate unreliability in wind-resistant design. Meanwhile, damping is said to be based on stick-slip phenomenon. This study shows results of theoretical derivations and nonlinear time-history analysis of a one-degree-of-freedom system model with varying stick-slip components. Comparisons of damping estimates are then made between current damping predictor models and the stick-slip damping model.

key words: stick-slip, structural damping, amplitude dependency, wind-resistant design, steel building
the main structure goes beyond such limits and into the plastic region, the natural frequency of the main system more rapidly decreases, resulting in a significant increase in wind loads. Dynamic wind loads also generally have a mean component and their period of application is long (e.g. one hour). These are reasons why the main structure should remain in the linear-elastic region under the wind-resistant design process. The assumption of linear main structure discussed later in this study is therefore deemed reasonable.

As mentioned, stick-slip phenomenon has already been qualitatively discussed as a model for structural damping in buildings. But there is no previous research that has truly considered stick-slip components in their modeling and analysis. Thus, at first, this paper quantitatively discusses the validity of stick-slip phenomenon as primary mechanism for damping estimation. This is carried out by modeling multiple stick-slip components (SSC), theoretically examining their contributions to damping, and validating derived theoretical expressions against numerical results. In this paper, secondary elements are likewise assumed to be free from damage, effectively assuming that the sole source of damping is stick-slip mechanism only. Finally, structural damping estimation for the wind-resistant design of a 200m-high steel office building using current damping predictors and the stick-slip damping model are carried out and the results are compared against actual measurement data.

2. BUILDING DATA

The building investigated in this paper is the ORC200 Symbol Tower in Osaka, Japan. In damping measurements on the building, an active-control device was used to excite the building to a certain amplitude. The excitation was then stopped, and the free vibration response was recorded. The damping ratio and natural frequency for each amplitude level were then evaluated from the measurement data, step-by-step. For the fundamental mode of vibration, the results are shown in Fig. 1, which shows damping with respect to amplitude expressed as horizontal velocity at the tip of the building. Note how damping first increases with amplitude in the lower amplitude range. It then reaches a peak value between 1.2% to 1.5%, at around 7 mm/s velocity. Finally, it decreases again with amplitude. Meanwhile, the natural frequency continuously decreases with amplitude, although the decrease is small, at only around 4% from maximum to minimum value.

The maximum damping corresponds to what has been called a “critical tip drift ratio.” Tip drift ratio is the ratio between the horizontal displacement amplitude $X$ at the tip of the building to its height $H$. The critical tip drift ratio is defined as the point beyond which there is no further increase of damping with amplitude. These are observed to be around $10^{-4}$ to $10^{-5}$, based on measurements of 4 steel buildings. In Fig. 1, the critical tip drift ratio was labeled as $X_m/H$, and it is around $2 \times 10^{-5}$. This value has been approximated using the corresponding “critical” velocity $V_m/H$ of 7 mm/s, the height $H$ of 200m, the measured natural frequency $f$ of around 0.3 Hz, and Eq. 1:

$$\frac{X_m}{H} = \frac{V_m}{(2\pi f)H}$$

Note that this damping measurement on the ORC200 Symbol Tower was carried out up to almost 8 times the critical amplitude and this is perhaps why the decrease of damping (after the critical amplitude was reached) has been observed.
3. CURRENT DAMPING ESTIMATION PROCEDURE

A damping predictor for steel buildings has been derived based on a large database of measurements in Japan \(^1\), \(^2\) and following the general damping predictor model first proposed by Jeary \(^5\). This damping predictor model is said to be based on stick-slip friction. Note though that the current damping predictor is intended for wind-resistant design purposes where the main structure is desired to remain within linear-elastic limits. The model is illustrated in Fig. 2 and described by Eq. 2:

\[
\zeta_s(X) = \zeta_b + \zeta_s(X) \leq \zeta_{s,max}
\]

In Eq. 2, \(\zeta_s(X)\) is the amplitude-dependent structural damping ratio, \(\zeta_b\) is a constant “baseline” damping ratio, \(\zeta_s(X)\) is the amplitude-dependent component due to stick-slip phenomenon, \(X\) is displacement amplitude at the top of the building, and \(\zeta_{s,max}\) is an upper limit for the structural damping ratio. The current predictor model \(^1\), \(^2\) is effectively Eq. 2 used with Eqs. 3 to 5:

\[
\zeta_b = \begin{cases} 
0.013f + 0.0029 & \text{(Based on measurements)} \\
0.65/H + 0.0029 & \text{(Habitability design standard)} \\
0.54/H + 0.0029 & \text{(Safety design standard)} 
\end{cases}
\]

\[
\zeta_s(X) = \frac{400X}{H}
\]

\[
\zeta_{s,max} = \begin{cases} 
0.013f + 0.0109 & \text{(Based on measurements)} \\
0.65/H + 0.0109 & \text{(Habitability design standard)} \\
0.54/H + 0.0109 & \text{(Safety design standard)} 
\end{cases}
\]

In the above equations, \(H\) is building height and \(f\) is measured natural frequency. Based on the initial analysis of the database of measurements, the predictor formula had the natural frequency as a parameter, and the amplitude expressed as tip drift ratio. But it was later recommended that it should be based on height instead of natural frequency. \(^1\), \(^2\) The reasoning was that the term is indicative of soil-structure interaction, which has lesser effect on taller buildings.

Thus, for Habitability and Safety design, the original expressions have been modified. Note that the term \(0.65/H\) for Habitability design is directly based on the measurements where \(f = 50/H\). But \(0.54/H\) for Safety design is only an estimated value that is approximately 0.83 of \(0.65/H\). For the case of the example building in this paper, the ratio of the natural frequencies at the highest to the lowest amplitudes is 0.96, which is greater than 0.83.

A method for estimating \(X/H\) for Habitability design depending on height is also available. \(^2\) Here, Habitability design corresponds to a 1-year return period wind speed. \(^4\) For Safety design, \(X/H = 2 \times 10^{-5}\) is recommended when using the damping predictor formula. \(\zeta_{s,max}\) is thus obtained by using Eq. 2 to 4 with \(X/H = 2 \times 10^{-5}\). This was due to records in the Japanese Damping Database being mostly at amplitudes below \(X/H = 2 \times 10^{-5}\), even though at Safety level, \(X/H \gg 2 \times 10^{-5}\).

For the 200m steel building discussed here, \(X/H\) is calculated to be \(2 \times 10^{-5}\) for Habitability. This results in estimates of around 1.4% for both Habitability and Safety design. Based on the same formulas, it has been suggested that 1% and 1.5% are used as “standard” values for Habitability and Safety design, respectively. \(^2\) These are values rounded off to the nearest 0.5%. However, because of the dispersion in the data, 30% lower values have been recommended for design, or:

\[
\zeta_{s,des}(X) = 0.7\zeta_s(X)
\]

\[
\zeta_{s,max,des}(X) = 0.7\zeta_{s,max}(X)
\]

In Eqs. 6 and 7, \(\zeta_{des}\) is the structural damping ratio recommended for design, and \(\zeta_{s,max,des}\) is the maximum structural damping ratio recommended for design. If the recommended values are calculated using Eqs. 2 to 7, a value of 1% would be recommended for both Habitability and Safety.
design of 200m-high steel buildings. This number corresponds to the maximum value because the amplitude assumed for both was $X/H = 2 \times 10^{-5}$. But in the literature $^{12}$, 0.7% and 1% for Habitability and Safety design, respectively, has been recommended because of rounding off of values.

As mentioned, Jeary $^{5}$ only described damping qualitatively using stick-slip phenomenon as primary mechanism. The model suggests that damping has a minimum baseline value ($\zeta_b$) at the low amplitude range. From a certain point ($X_i$), it increases with amplitude. And finally, it reaches a maximum value ($\zeta_{\text{max}}$) that is assumed to be applicable up to the amplitude levels corresponding to wind-resistant design. Unfortunately, this model shows a very different situation from the one shown in Fig. 1 of an actual condition. Eq. 2 and Jeary’s model could be valid for the Fig. 1 results, but its validity would only be up to around $X/H = 2 \times 10^{-5}$, the upper limit of the measurements. Fig. 1 suggests that the assumption of constant damping until the Safety Level amplitudes, even with a 30% reduction in value, might not be sufficient. Thus, the validity of stick-slip phenomenon assumed as primary damping mechanism is quantitatively studied first.

4. THEORETICAL DERIVATION

A 1DOF system with one SSC (1DOF+1SSC) is first considered. The aim is to illustrate the stick-slip phenomenon and to understand damping contribution from one SSC. A 1DOF system is assumed to represent a single mode of vibration. As an example, a simple one-storey shear building with two columns and rigid mass (Fig. 3a) is considered. This behaves as a 1DOF system with mass $m_b$, stiffness $k_b$, and inherent damping $c_b$. Note that $c_b$ can be assumed as zero, but it is included here for generality. A non-structural wall as a secondary member is then introduced. The wall is assumed to have a stiffness $k_0$. The wall is in contact with the 1DOF system, and a friction force $Q$ is generated at the contact surface when the system moves. $Q$ is generally less than or equal to the friction capacity $Q_c$. $Q_c$ is defined as the product of coefficient of friction $\mu$ and normal force $N'$ at the contact surface. $\mu$ can be approximated for materials used in construction. But $N'$ is difficult to measure in actual structures.

This simple 1DOF+1SSC system can also be represented by the simple mass-spring-dashpot model in Fig. 3b. In Fig. 3b, the element $Q$ representing the friction at the contact surface can be defined by the force-displacement relationship as in Fig. 4a. Fig. 4a is the typical characteristic of a friction damper. This $Q$ element is actually a stick-slip surface. If the movement of the main structure is very small, $Q$ is less than $Q_c$, the wall is “stuck,” and its full stiffness $k_0$ contributes to the whole system. If the movement is very large, $Q$ stays constant at $Q_c$, and slipping at the contact surface occurs.

If combined with the wall, the $Q$ friction element and $k_0$ stiffness element actually result in a force-displacement relationship such as in Fig. 4b. This combination element is what is called here as a SSC. Note that Fig. 4b is similar to the force-displacement relationship of an elastoplastic material.

\[ Q = \beta N' \]

where $\beta$ is the friction coefficient and $N'$ is the normal force.

\[ \zeta_{\text{max}} = \frac{Q_c}{\sqrt{m_b k_b}} \]

It is clear here that the SSC cannot provide viscous damping. Thus, the hysteretic damping concept can be applied.\(^{40}\) Again, the aim is to arrive at equivalent viscous damping ratios for corresponding amplitudes. This well-known concept can be described by Fig. 5. The derived formulas will approximate the amplitude-dependent component of the structural damping ratio, $\zeta_b$. To get the total structural damping ratio, $\zeta_b$ will be added to the baseline damping ratio $\zeta_0$ as in Eq. 2. $\zeta_b$ is due to the assumed inherent damping $(c_b)$.

When applied to the force-displacement relationships in Fig. 4a and 4b, the derived formulas are Eq. 8 and 9, respectively:
In Eq. 9 and Fig. 4b, the quantity \( d_c \) has been introduced. \( d_c \) is called the slip trigger displacement. It corresponds to the amplitude when slipping starts to occur at the contact surface. Quantitatively, it is simply equal to \( Q_c/k_0 \).

Visually, Eq. 8 and 9 suggest example damping amplitude dependency curves as shown in Fig. 6. Fig. 6 uses damping normalized against the maximum value such that the maximum normalized damping is unity, and displacement amplitude normalized against the \( d_c \) value used in Eq. 9. Fig. 6a suggests that if only the stick-slip friction surface is considered, damping would start from a very high value and then continuously decrease with amplitude. Fig. 6b suggests that if the nonlinear SSC only is considered, damping would continuously increase with amplitude. Therefore, these two results (Eqs. 8 and 9) are incorrect. These are not reflective of actual conditions, like the one shown in Fig. 1.

Now, the force-displacement relationship of the whole system (linear main system plus nonlinear SSC) in Fig. 7a is considered next. The hysteretic damping analysis results in Eq. 10, and it can be visualized by the example plot in Fig. 7b.

\[
\zeta_c(X) = \frac{2}{\pi} \left( \frac{1}{1 + \frac{d_c}{X}} \right) \left( 1 - \frac{d_c}{X} \right) \left( 1 - \frac{d_c}{X} \right)
\]

(10)

Fig. 7b and Eq. 10 appear more realistic now in that it illustrates an increase and decrease of damping with amplitude. In terms of shape, it is also similar to plots shown earlier by Wyatt. Fig. 7b is also more similar to the plot in Fig. 1a of an actual measurement data, compared to Fig. 6a or 6b. In Eq. 10, there are three things to note:

- Eq. 10 suggests that damping due to stick-slip components can be viewed as a function of only two parameters: a stiffness ratio \( k_0/k_b \) and an amplitude ratio \( (d_c/X) \). \( k_0/k_b \) is the ratio of stiffness between the secondary member and the main structure.

- Damping is not directly proportional to the amplitude \( X \) (i.e. \( \zeta \propto X \) is not true). It is also not inversely proportional to \( X \) (i.e. \( \zeta \propto 1/X \) is not true). It is more proportional to \( 1/X^2 \). Eq. 10 is effectively like an inverse quadratic equation that provides the increasing and decreasing nature of damping with amplitude.
• The damping due to SSC is not a function of basic system mass $m_b$ nor system damping $c_b$ or $\zeta_b$. It is also not a function of basic natural frequency $f_b$ per se, except that $f_b = \sqrt{\frac{k_b}{m_b}}/2\pi$.

The problem now is that these are results for a 1DOF+1SSC system only. In real structures, there could be a very large number of SSC. A 1DOF+NSSC system should be considered, where $N$ is the number of SSC. For now, it is assumed that the damping contributions are simply additive. This is illustrated by Eq. 11, which is a more general form of Eq. 10.

$$\zeta_c(X) = \sum_{i=1}^{N} \frac{2}{\pi} \left( \frac{1}{1 + \frac{c_i}{\zeta_c}} \right) \left( 1 - \frac{d_{ci}}{X} \right) \left( 1 - \frac{d_{ci}}{X} \right)$$  \hspace{1cm} (11)

However, before Eq. 10 and 11 can be used, these need to be validated first.

5. VALIDATION OF THEORETICAL EQUATIONS

The results of nonlinear time-history analysis (NTHA) of a numerical model of Fig. 3a or 3b can be considered as theoretically accurate, if the errors can be minimized. Such numerical results can then be used to validate the Eq. 10 and 11, which are only based on the hysteretic damping concept. The 1DOF system for validation has a natural frequency of 9.8 Hz, and inherent damping ratio $\zeta_b = 0.3\%$. Eq. 10 is validated first, and thus one SSC is introduced to the 1DOF system. The system is then a 1DOF+1SSC system, as referred to in this paper. The SSC has assigned values of $k_0 = 6 \times 10^4$ kN/m and $d_{ci} = 6 \times 10^4$ m. The simple model is not meant to be a model of any actual building. Likewise, it is not known if the assigned values can represent any actual SSC. But these values are only necessary to proceed with the NTHA.

For the NTHA itself, there were a number of analysis techniques that can be used. For this study, the Fast Nonlinear Analysis or FNA method was chosen. This was because the model met the two criteria for using FNA. First, there was no material nonlinearity (i.e. the main system is linear). Second, there were discrete nonlinear elements (i.e. the SSC). The FNA also requires less computational resources and has better accuracy than other NTHA techniques. For added accuracy, a very small analysis time step of 1x10−2 sec was used. This was equivalent to a sampling rate of 10,000 Hz, or around 1,000 times the natural frequency. The SSC was modeled as a nonlinear spring element with force-displacement relationship similar to Fig. 4b. To arrive at damping ratios for various amplitudes, a very large impulse was applied. The analysis then ran for 104.8576 sec, equivalent to 220 analysis steps. The total computational duration though was about one hour, using an ordinary desktop computer. The impulse created free vibration response in the system. The logarithmic decrements were then obtained, step-by-step, to calculate the damping ratios.

The numerical and theoretical results are compared in Fig. 8. The figure shows a good match, and thus Eq. 10 is validated. The match is not perfect though. The imperfect match could be attributed to small errors in the NTHA that manifest only near the end of the analysis run, at the lower amplitudes.

To validate Eq. 11, a simple 1DOF+6SSC system and a 1DOF+441SSC system were analyzed. Note that in real structures, the number of SSCs can be much more than 6 or 441. But the 6SSC system can be viewed as representing a system with 6 groups of SSCs, with each group having distinct properties, but each group having little to no variability. The use of 441 SSCs is only due to the limitation in the computational power of the desktop machine used for numerical analysis. The main structures in these systems were the same as that used in the 1DOF+1SSC system discussed earlier. For the 6SSC system, the mean of the $k_0$ and $d_{ci}$ values were set to not have any variability (coefficient of variation or COV = 0%). The $d_{ci}$ values had a COV of around 70% and followed a uniform distribution. For the 441SSC system, $k_0$ and $d_{ci}$ had COV values of around 20% and 300%, respectively. But these did not necessarily follow any specific probability distribution.

The theoretical and numerical results for the 6SSC and 441SSC systems are shown in Fig. 9. In the numerical results, the plots were widely diverging at the lower amplitudes again due to the same reasons as for the 1DOF+1SSC case stated earlier, hence these were not included in Fig. 9. But the overall shapes show a sufficient match, thus validating Eq. 11.
6. PARAMETRIC NONLINEAR ANALYSIS WITH MULTIPLE STICK-SLIP COMPONENTS

As mentioned earlier, it is generally difficult to determine stick-slip properties in actual structures. But this is probably more true particularly for $d_i$ values. $k_0$ can be easier to estimate, since these are simply the stiffness values of secondary elements. The uncertainty in $d_i$ values is due to the fact that it is related to $Q_c$, or that $d_i = Q_c/k_0$. Meanwhile, $Q_c$ is related to the normal force $N$ at contact surfaces. As mentioned, $N$ is difficult to measure in real structures. This all becomes a larger problem with a very large number of SSC. For example, consider a system with 10,000 SSC. A probabilistic approach is therefore necessary. It is necessary to do a number of different cases using different probability distributions and COV. Fortunately, there is no need to perform NTHA because now, the validated Eq. 11 can be used.

A total of 672 analysis cases were performed following different possible combinations of parameters listed in Table 1. Positive numbers following the five different probability distributions, with a mean of 1 and standard deviation corresponding to the COVs listed in Table 1, were generated for each case and then multiplied to mean values of $k_0$ and $d_i$. The mean of $k_0$ was made constant for all cases. The lowest of all $d_i$ values, or $d_i*$, was likewise constant for all cases.

The univariate probability distributions were chosen such that they have different shapes (Fig. 10). The normal, log-normal, and Weibull distributions have double curvature, resembling a bell shape. The uniform distribution does not have any curvature. The Gamma distribution shows a single curvature that is distinctly different from a bell-shaped curve. The uniform and normal distributions have zero skew. The Weibull distribution has negative skew. The log-normal and Gamma distributions have positive skew. The multivariate normal distribution has the same shape as its univariate equivalent in each of the two variables. While all five distribution types have the mean and standard deviation (or the COV) as parameter, the Weibull and Gamma distributions have two additional shape parameters each, namely $\kappa$ and $\lambda$, and $\tau$ and $\theta$ respectively, which define the shape of the distributions. For their shapes to be distinct from the normal and log-normal distribution types, for the purposes of this analysis, these “shape parameters” have been set to specific values, namely: $\kappa = 5$, $\lambda = 1$, $\tau = 1$, and $\theta = 2$.

Before discussing the results, the quantities $d_{i*}$, $X_i$, and $X_m$, and the ratio $X_m/X_i$ are first described. The quantities can be visualized in the example result in Figure 11. $d_{i*}$ describes the lowest amplitude where damping starts to increase with amplitude. $X_i$ describes the amplitude where damping starts to significantly increase with amplitude. $X_m$ describes where the maximum damping ($\zeta_{max}$) occurs, before damping starts to decrease with amplitude. $X_m$ then corresponds to the critical tip drift ratio.

| Parameter | Values          |
|-----------|-----------------|
| $N$       | 10, 100, 1,000, 10,000 |
| Univariate probability distribution | Uniform, Normal, Log-Normal, Weibull, Gamma |
| COV for univariate cases (where possible) | $0\%$, $5\%$, $10\%$, $25\%$, $50\%$, $75\%$, $100\%$, $200\%$, $500\%$ |
| Univariate sub-cases | constant $d_{i*}$, constant $k_0$ constant $k_0\times d_{i*}$ but varying $k_0$ constant $k_0\times d_{i*}$ but varying $d_{i*}$ varying $k_0$ and $d_{i*}$ similarly (correlation coefficient $= 1$), varying $k_0$ and $d_{i*}$ similarly but inversely (correlation coefficient approaches $-1$) |
| Multivariate probability distribution | Multivariate Normal |
| Multivariate correlation coefficients | -95\%, -80\%, -50\%, -20\%, -10\%, -5\%, -1\%, 0\%, 1\%, 5\%, 10\%, 20\%, 50\%, 80\%, 95\% |

$^1$Except for the 0% correlation case, all other target correlation coefficients are applied two times, because each generation results in a different set of random numbers.

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Fig. 9. Validation of Eq. 11 against numerical results (1DOF+NSSC systems)
Fig. 10. Visualization of different probability distributions used: (a) uniform, (b) normal, (c) log-normal, (d) Weibull, and (e) Gamma. “ρ values” refers to random numbers generated.

Fig. 11. Visualization of $d_i$, $X_l$, $X_m$, and $\zeta_{max}$ overlain on one example case with $N = 10,000$, COV = 500% and log-normal distribution for $k_0$, $k_{id}$ kept constant.

$X_m/X_l$ describes the ratio between the amplitude at maximum damping and the amplitude where damping starts to noticeably increase with amplitude. $X_l$, $X_m$, and $X_{max}$ are particularly important because these are quantities that can be observed from damping measurements, as in Fig. 1.

The results for all 672 analysis cases can be arranged into 5 groups, for which 9 examples are shown in Figs. 12 to 15. The figures show amplitude normalized against $X_l$ and damping normalized against the maximum damping from all cases. The grouping is as follows:

- One clear peak with $X_m/X_l \approx 2$, and closely resembling results for a 1DOF+1SSC system (e.g., Fig. 12a).
- One clear peak with $2 < X_m/X_l < 900$ (for example, Figs. 12b to 12e).
- Two peaks (for example, Figs. 13a and 13b).
- Three peaks (Fig. 14).
- No clear peak (Fig. 15).

The first group (Fig. 12a) actually comprises 92 cases where the COV of $d_i$ values is zero. This suggests that a 1DOF+1SSC model could be valid if there is very small variability (COV) in $d_i$ values.
Damping. For the examples shown in Fig. 12, different shape from the first group, but likewise have one peak
around 3 to around 100. As mentioned, $X_m/X_l$ is between 2 and 900 for all cases that belong to the second group.

The third and fourth groups shown in Figs. 13a, 13b, and 14 show examples with $X_m/X_l$ of around 2, 60, and 300, respectively. But as mentioned, there are smaller secondary peaks. The results suggest that if we have observed one clear peak in damping measurements, it is possible that we might have not observed another peak at higher or lower amplitudes. This suggests that if damping was measured only over a narrow amplitude range, there is a huge uncertainty in the amplitude ranges that we do not have measurements for. For example, unless we have measurements of damping at amplitudes corresponding to wind-resistant design levels, the appropriate design damping value remains highly uncertain. It is in this regard that others have proposed higher wind load factors where dynamic wind response is significant.

The last grouping (Fig. 15) shows no clear peak. Instead, it shows that damping could have a maximum value that is almost constant over a wide range of amplitudes. This suggests that the current damping predictor (Fig. 2) is not invalid. But here, it is qualified that its validity is only for a narrow amplitude range, and possibly not for all building types. In this case, if damping at the amplitude level corresponding to wind-resistant design has not been measured, it is unsafe to assume that the maximum damping extends up to the design-level amplitudes. This again emphasizes the importance of measurements at design-level amplitudes, though it is recognized that it is highly impractical. Instead, perhaps it is better to use the stick-slip damping model discussed in this paper, so that the increase and decrease of damping is accounted for.

Note that, in this paper, amplitude is discussed in terms of displacements. But the measurements shown in Fig. 1 are in terms of velocity. Many measurements are in terms of acceleration. However, it is pointed out that the damping amplitude dependency is not much different whether amplitude is expressed in terms of displacement, velocity, or acceleration, as shown in the example in Fig. 16. Therefore, $X_m$ or $X_l$ could also refer to velocity or acceleration. It is only important that consistency is considered. For example, if amplitude is expressed as velocity, all amplitude-related quantities are expressed in velocity. In any case, systems such as tall buildings have low stiffness ratios $(k_m/k_n)$ between secondary members or SSCs and main frames, and small damping ratios. For those systems, Fig. 16a shows that damping amplitude dependency

Fig. 12. Five sample results out of 672 analysis cases of different 1DOF+NSSC systems showing one peak: (a) $N = 10,000$, COV = 500% and log-normal distribution for $k_{0s}$, constant $d_{c,s}$; (b) $N = 10,000$, COV = 25% and normal distribution for both $k_{0s}$ and $d_{c,s}$ varied similarly such that they are fully correlated; (c) $N = 10$, COV = 50% and Weibull distribution for both $k_{0s}$ and $d_{c,s}$; (d) $N = 100$, COV = 500% and log-normal distribution for $d_{c,s}$, $k_{0s}$, $k_{0d}$, kept constant; and (e) $N = 1,000$, COV = 200% and log-normal distribution for $k_{0s}$, $k_{0d}$, $d_{c,s}$ kept constant.

Fig. 13. Two sample results out of 672 analysis cases of different 1DOF+NSSC systems showing two peaks: (a) $N = 100$, COV = 500% and log-normal distribution for $k_{0s}$, constant $d_{c,s}$ and (b) $N = 10$, COV = 100% and log-normal distribution for both $k_{0s}$ and $d_{c,s}$ varied inversely such that their correlation coefficient approaches negative one.

Fig. 14. One result out of 672 analysis cases of different 1DOF+NSSC systems showing three peaks. ($N = 100$, COV = 500% and log-normal distribution for both $k_{0s}$ and $d_{c,s}$ varied similarly such that they are fully correlated.)

Fig. 15. One result out of 672 analysis cases of different 1DOF+NSSC systems showing no clear peak. ($N = 100$, COV = 200% and log-normal distribution for $k_{0s}$, $k_{0d}$, $d_{c,s}$ kept constant.)

The second group (Figs. 12b to 12e) can be of similar or of different shape from the first group, but likewise have one peak damping. For the examples shown in Fig. 12, $X_m/X_l$ ranges from~
expressed in terms of displacement, (pseudo-)velocity, or (pseudo-)acceleration is practically the same. For shorter buildings with higher stiffness ratios and higher damping because of SSCs, there is some difference between the damping amplitude dependencies if the amplitude quantity is changed. In Fig. 16, the terms “pseudo-velocity” and “pseudo-acceleration” are used. The prefix “pseudo-” simply implies that the velocity and acceleration were only approximated using the natural frequency as parameter in formulas such as Eq. 1.

7. COMPARISON OF DAMPING ESTIMATES

How the stick-slip damping model better estimates damping particularly for wind-resistant design purposes is now illustrated. In this paper, the 200m-high steel building is used as example.

First, it is considered that the building will be evaluated for habitability under 1-year and possibly even 20-year return period winds, and for safety under 50- and 500-year return period winds. The tip drift ratio corresponding to 1-year winds is first estimated [2], using the procedure mentioned earlier, as around $2 \times 10^5$. However, the AIJ Guidelines for Evaluation of Habitability to Building Vibration (AIJ-GBV-2004) [21] shows calculated wind-induce responses for different buildings including the ORC Symbol Tower. AIJ-GBV-2004 suggests that the response under 1-year wind speeds might be somewhere between 6 to 10 cm/s². Assuming a natural frequency of 0.26 Hz, this then translates to roughly around 37 to 61 mm/s in terms of velocity, or around $1.1 \times 10^4$ to $1.9 \times 10^4$ in terms of tip drift ratio. Note that this is practically an order of magnitude higher and are deemed more correct than the results from the earlier-mentioned estimation procedure. For the purposes of this comparison, the 1-year return period wind response is set to 8 cm/s² in terms of acceleration, 50 mm/s in terms of velocity, or $1.5 \times 10^4$ in terms of tip drift ratio. Certain ratios of this amplitude level for 20-, 50-, and 500-year return period winds are now assumed, using available procedures [4].

The results are shown in Table 2. In Table 2, the numbers for $U_r$ are approximations for Osaka where the example building is located. The calculations though are only approximate. The calculations assume that only the changes in design wind speed, natural frequency, and consequently the gust effect factor affect the wind loads and correspondingly the amplitudes. Changes in damping ratio and other parameters also affect the wind loads and response amplitudes. But these are not yet taken into account. Lastly, note that here it is effectively assumed that the main structure remains linear-elastic even under 1000-year return period wind loads.

![Normalized damping vs amplitude](attachment:image1.png)

**Fig. 16.** Comparison of results for an example 1DOF+1SSC case with amplitude as displacement, velocity, and acceleration: (a) $k_0/k_0 = 0.01$; $\zeta_{max} = 0.2\%$; and (b) $k_0/k_0 = 1$; $\zeta_{max} = 11\%$.

| $r$ | $k_{rw}$ | $U_r$ | $U_r^2/G_{ai}$ | $f$ | $G_{ai}/G_{ai}$ | $XH$ | $V$ |
|-----|----------|-------|----------------|----|----------------|------|----|
| $<< 1$ | - | - | 0.27 | - | $6 \times 10^6$ | 2 |
| $~1$ | - | - | 0.266 | - | $2 \times 10^5$ | 7 |
| 1 | 0.57 | 17 | 0.26 | 1 | $1.5 \times 10^4$ | 50 |
| 20 | 0.85 | 26 | 2.4 | 0.26 | 1.04 | $3.7 \times 10^4$ | 125 |
| 50 | 0.94 | 30 | 3.2 | 0.26 | 1.07 | $5.1 \times 10^4$ | 171 |
| 500 | 1.15 | 39 | 5.3 | 0.26 | 1.13 | $9 \times 10^4$ | 300 |

- $r$ = return period in years, $k_{rw}$ = return period conversion factor, $U_r$ = wind speed in m/s estimated for Osaka corresponding to $r$, $U_r$ = wind speed in m/s estimated for Osaka corresponding to $r = 1$ year, $f$ = measured natural frequency in Hz, $G_{ai}/G_{ai}$ = ratio of gust effect factor for $r$ relative to gust effect factor for $r = 1$, $V$ = approximate velocity amplitude in mm/s.
- Also in this list are very low amplitude ($V = 2$ mm/s) corresponding to baseline damping, and amplitude ($V = 7$ mm/s) corresponding to maximum damping and critical tip drift ratio. These return periods are not used for wind-resistant design. They are only included here to estimate damping at their corresponding amplitude levels.
- Only the $U_r$ values for $r = 1$ and $r = 100$ were taken from the wind speed maps in AIJ-RLB-2004.
- The $U_r$ values for $r = 20$ and $r = 50$ were approximated.
- The $U_r$ value for $r = 500$ was calculated using $k_{rw}$.
Table 2 shows the ratios $U_r^2/U_1^2$ and $G_r/G_e$ which approximate the ratio of response amplitude at the corresponding return period to the 1-year return period amplitude. Note that the velocity amplitudes under all design cases, except the one under 1-year return period winds, show velocity responses that are greater than 60 mm/s. These are all greater than the maximum recorded velocity during the damping measurements shown in Fig. 1. Note also that $X/H > 2 \times 10^{-5}$ for all design cases $(r \geq 1)$; the amplitudes for design are larger than most of the measurements used in the damping database. Based on Fig. 1, the first mode of the building appears to have $X_l = 2 \text{ mm/s, } X_m = 7 \text{ mm/s, } \zeta_b = 0.4\%$, and $\zeta_{s,\text{max}} = 1.3\%$. Thus, $X_m/X_l$ is between 3 and 4. It is possible that $\zeta_b < 0.4\%$, and $\zeta_{s,\text{max}} > 1.3\%$ at higher or lower amplitudes. But since there is no information at higher or lower amplitudes, those possibilities are not considered. Given only what is known, the curve of Fig. 12b can be used to model the damping amplitude dependency in Fig. 1. Fig. 12b is used because it shows $X_m/X_l$ of around 3, which is similar to the characteristics of Fig. 1. Fig. 12b is the result for a 1DOF+NSSC system with $N = 10,000$, and COV = 25% for normally distributed and fully correlated $k_0$ and $d_c$. In the actual building, the $N$ value or how many stick-slip components exist is not known. However, because of the large size of the building, it can only be deduced that it has a large $N$. Thus the assumption here of $N = 10,000$ maybe an acceptable approximation. A simpler 1DOF+1SSC model is also considered. But this will require adjustment since $X_m/X_l$ is around 2 for a 1DOF+1SSC system. In this case, $X_l$ is then set to around 3.5 mm/s.

The stick-slip models are now visually fitted to the measurement data of Fig. 1. As shown in Fig. 17, the stick-slip models can account for the increase and decrease of damping that the current damping model (Fig. 2) cannot. But at the higher amplitudes, the stick-slip curves for $\zeta_b = 0.4\%$ (Figs. 17a, 17b, 17c, 17g, 17h, and 17i) appear to greatly underestimate damping. Better fitting curves are thus used, using the same 1DOF+NSSC and 1DOF+1SSC models. These better fitting curves assume $\zeta_b = 0.6\%$ instead of 0.4%. Also, the data shows $\zeta_{s,\text{max}}$ ranging from around 1.2% to around 1.5% though. Thus, upper and lower bound curves are also included corresponding to $\zeta_{s,\text{max}}$ of 1.5% and 1.2%, respectively. After selecting these possible curve fits, damping ratios at different amplitudes can now be estimated. The results are compared with other methods listed in Table 3.
Table 3 Different methods used to estimate damping and corresponding Method IDs in this paper

| Method | Description |
|--------|-------------|
| 1      | As measured (as shown in Fig. 1) |
| 1a     | Upper bound of measured values as reported in literature (10, 11) |
| 1b     | Lower bound of measured values as reported in literature (10, 11) |
| 2      | Using 1DOF+NSSC model with $\zeta_b = 0.4\%$ (Fig. 17a) |
| 2a     | Using upper bound of Method 2 (Fig. 17b) |
| 2b     | Using lower bound of Method 2 (Fig. 17c) |
| 3      | Using 1DOF+NSSC model with $\zeta_b = 0.6\%$ (Fig. 17d) |
| 3a     | Using upper bound of Method 3 (Fig. 17e) |
| 3b     | Using lower bound of Method 3 (Fig. 17f) |
| 4      | Using 1DOF+1SSC model with $\zeta_b = 0.4\%$ (Fig. 17g) |
| 4a     | Using upper bound of Method 4 (Fig. 17h) |
| 4b     | Using lower bound of Method 4 (Fig. 17i) |
| 5      | Using 1DOF+NSSC model with $\zeta_b = 0.6\%$ (Fig. 17j) |
| 5a     | Using upper bound of Method 5 (Fig. 17k) |
| 5b     | Using lower bound of Method 5 (Fig. 17l) |
| 6      | Current damping predictor model applied to measurement data (Fig. 2 and Eq. 2 based on information from Fig. 1) |
| 6a     | 30% lower values compared to Method 6 |
| 6b     | Current damping predictor model using formulas based on measurements (i.e. calculation using Eqs. 2, 3, 4, and 5) |
| 7      | 30% lower values of Method 7 |
| 7a     | Tabulated “standard” values (2) |
| 8      | Tabulated “recommended” values (2), or approximately 30% lower values of Method 8 |

Table 4 Estimated total structural damping ratios

| $V$ | $r$ | $a$ | $b$ | 1 | 20 | 50 | 125 | 171 | 300 |
|-----|-----|-----|-----|---|----|----|-----|-----|-----|
| 2   | 0.4%| 1.3%| 0.8%| -- | -- | -- | --   | --   | --   |
| 1a  | 1.2%|     |     |    |    |    |      |      |      |
| 1b  | 0.8%|     |     |    |    |    |      |      |      |
| 2a  | 0.4%| 1.5%| 0.7%| 0.6%| 0.5%| 0.5%| 0.5% | 0.5% | 0.5% |
| 2b  | 0.4%| 1.2%| 0.6%| 0.5%| 0.5%| 0.5%| 0.5% | 0.5% | 0.5% |
| 3a  | 0.6%| 1.5%| 0.8%| 0.8%| 0.7%| 0.7%| 0.7% | 0.7% | 0.7% |
| 3b  | 0.6%| 1.2%| 0.8%| 0.7%| 0.7%| 0.7%| 0.7% | 0.7% | 0.7% |
| 4a  | 0.4%| 1.3%| 0.6%| 0.5%| 0.5%| 0.5%| 0.5% | 0.5% | 0.5% |
| 4b  | 0.4%| 1.2%| 0.6%| 0.5%| 0.5%| 0.5%| 0.5% | 0.5% | 0.5% |
| 5a  | 0.6%| 1.3%| 0.8%| 0.7%| 0.7%| 0.7%| 0.7% | 0.7% | 0.7% |
| 5b  | 0.6%| 1.2%| 0.8%| 0.7%| 0.7%| 0.7%| 0.7% | 0.7% | 0.7% |
| 6a  | 0.4%| 1.3%| 1.3%| 1.3%| 1.3%| 1.3%| 1.3% | 1.3% | 1.3% |
| 6b  | 0.4%| 1.2%| 1.3%| 1.3%| 1.3%| 1.3%| 1.3% | 1.3% | 1.3% |
| 7a  | 0.6%| 1.4%| 1.4%| 1.4%| 1.4%| 1.4%| 1.4% | 1.4% | 1.4% |
| 7b  | 0.6%| 1.0%| 1.0%| 1.0%| 1.0%| 1.0%| 1.0% | 1.0% | 1.0% |
| 8a  | 0.7%| 0.7%| 0.7%| 0.7%| 0.7%| 0.7%| 0.7% | 0.7% | 0.7% |
| 8b  | 0.7%| 0.7%| 0.7%| 0.7%| 0.7%| 0.7%| 0.7% | 0.7% | 0.7% |

* Corresponding to $\zeta_b$ at around $V = 2$ mm/s.  
* Corresponding to critical tip drift ratio ($V = 7$ mm/s).  
* No measurements at these amplitudes.  
* Maximum reported damping ratios range from 1% to 1.2% corresponding to $V = 8$–36 mm/s. Damping ratio of 1.2% corresponds to $V \approx 36$ mm/s.  
* Minimum reported value of 0.8% corresponds to $V \approx 0.2$ mm/s.

Errors and correlation coefficients are also calculated, but there are no measurements at the very high amplitudes ($V > 55$ mm/s). However, the 1DOF+NSSC model with $\zeta_b = 0.6\%$ shown in Fig. 17d (Method 3) appears to be the best fitting curve particularly going towards the high amplitude range.
Errors and correlation coefficients are then evaluated against the results for Method 3. The average errors and the correlation coefficients are listed in Table 5.

![Fig. 18. Ranges of damping estimates using different methods: (a) actual measurements (Method 1); (b) range of reported values in literature (Methods 1a and 1b); (c) range of predictions using stick-slip model (Methods 2 to 5, including upper and lower bound variants); (d) range of predictions using current damping model in Fig. 2 (Methods 6 and 6a); (e) range of predictions using current damping model using Eq. 2 (Methods 7 and 7a); and (f) range of predictions using current damping model as recommended in literature (Methods 8 and 8a). Refer to Table 3 for detailed descriptions of methods.](image)

| Method | Average Error | Correlation Coefficient |
|--------|---------------|-------------------------|
| 1      | 2%            | 0.98                    |
| 1a     | 2%            | 0                        |
| 1b     | 8%            | 0                       |
| 2      | -22%          | 1                        |
| 2a     | -19%          | 1                        |
| 2b     | -24%          | 1                        |
| 3      | --            | --                       |
| 3a     | 3%            | 1                        |
| 3b     | -3%           | 1                        |
| 4      | -27%          | 1                        |
| 4a     | -24%          | 1                        |
| 4b     | -28%          | 1                        |
| 5      | -4%           | 1                        |
| 5a     | -1%           | 1                        |
| 5b     | -5%           | 1                        |
| 6      | 75%           | 0.29                     |
| 6a     | 22%           | 0.29                     |
| 7      | 92%           | 0.3                      |
| 7a     | 34%           | 0.3                      |
| 8      | 84%           | -0.78                    |
| 8a     | 24%           | -0.78                    |

Note that there are no estimates and calculated errors under Methods 8 and 8a for certain return periods since the methods are based on tabulated values only. Those tabulated values indicated estimates only for Habitability (1-year return period) and Safety design (in this case, assumed as having return periods of 100 years or longer).

The results overall illustrate the following:

- The scatter in the estimates is larger in the higher amplitude range than in the lower amplitude range. This is probably because the measurements were made only at the lower amplitude range, and the current predictor models were based only on such measurements.
- The use of upper values of damping ratios measured at lower amplitudes (e.g. Method 1a) could be unsafe for wind-resistant design, which are at higher amplitudes. It is safer to use the lowest measured damping ratios (e.g. Method 1b), but in this case, it still over-estimates damping at the design amplitude levels.
- Stick-slip modeling (Methods 2 to 5, including upper and lower bound variants) results in an amplitude dependency that is most similar with the actual data. At least for the range of measurements, the correlation coefficient between data and stick-slip model is very high (near 1). Compare this to the zero, very low, or even negative correlation coefficients for Methods 6 to 8 (including variants) which are based on the current damping predictor model of Fig. 2 or Eq. 2. This is a validation of the use of the stick-slip damping model described here.
- A 1DOF+1SSC model can also be used to describe the amplitude-dependency. This is a much simpler model that can be used by engineers, considering also that the number and probability distributions of actual stick-slip parameters are not known.
- The median curve fits (Methods 2 to 5) appear sufficient. The upper or lower bound curves (Methods 2a, 2b, 3a, 3b, 4a, 4b, 5a, and 5b) are not necessary.
- The 1DOF+NSSC model in Methods 2 and 4, including their variants, have higher errors than their 1DOF+1SSC counterparts in Methods 3 and 5, and their variants. Models 2 and 4 also generally under-estimate damping at the higher amplitudes, which would be safe for design, but possibly too conservative. Thus, for the subject building, the baseline damping ratio $\zeta_b$ appears to be 0.6% and not 0.4%. Meanwhile, based on the database of measurements, Method 7, which is based on Eqs. 2, 3, 4, and 5, suggest that $\zeta_b \approx 0.6\sim0.9\%$, which in turn...
corresponds to the assumption of 0.6%. The lone data point in the actual data of Fig. 1 at \( V = 2 \text{ mm/s} \) and \( \zeta_s = 0.4\% \) might have been in error. Method 1b also suggests a damping ratio of 0.8% at very low amplitudes.

- The stick-slip models of Methods 2 to 5, including their variants, show good estimates of the damping ratio at the critical tip drift ratio. However, this is mostly because these were specifically fitted to the measurement data.
- Methods 6 to 8, including their variants, appear to overestimate damping at all design amplitudes, but this is assuming that no other damping mechanisms, such as damage to secondary elements, could increase damping again at around the design amplitudes. Since there are no measurements at such amplitude levels, this would be a safe conclusion.
- The use of 30% lower values in Methods 6a, 7a, and 8a provides better estimates of damping for design purposes than their counterparts (Methods 6, 7, and 8). But they still appear to overestimate damping by 25%–60% at amplitudes corresponding to wind-resistant design. But again, these estimates are still possibly accurate, for example if there is some secondary member damage. But again, there is no data on whether the likelihood of such can occur or not.
- The 30% lower values of Methods 6a, 7a, and 8a, as well as Method 8, would result in an underestimate of damping at the critical tip drift ratio.
- Methods 8 and 8a have very high negative correlation with the actual data. This suggests that the nature of these methods is qualitatively different from actual conditions.

7. SUMMARY AND RECOMMENDATIONS

This paper presented a theoretical expression for structural damping for use in wind-resistant design. The theoretical expression was based on stick-slip mechanism at multiple contact surfaces in buildings. For the example 200m-high building, damping was estimated using different methods, and the results were compared. The results showed that:

- A damping model based on stick-slip phenomenon can have much less errors and very high positive correlation with actual data such as in Fig. 1 for the example 200m-high steel office building. The stick-slip damping model can account for the increase and decrease of damping with amplitude. There is no need to assume lower values for design except to account for uncertainty.
- The current damping predictor model appears inadequate for estimating damping at amplitudes corresponding to wind-resistant design levels. The 30% lower values recommended for design can provide better estimates of damping than “standard” values, and for Habitability design, this may be sufficient. But these could still significantly overestimate damping at the high amplitude range corresponding to Safety design. The 30% lower values also tend to significantly underestimate damping at the critical tip drift ratio.

An assumption in this study is that the building would remain linear, which is the desired structural behavior in wind-resistant design. Hence, the conclusions here might be applicable only to steel buildings. However, it is believed that the stick-slip damping model is also applicable to concrete buildings. Likewise, it has been assumed that damages to secondary members would not occur that could increase damping again in the high amplitude range, considering that there is no data on whether such phenomenon occurs. Therefore, it is believed that ignoring such possibility is valid. Lastly, the possibility of aerodynamic damping should be checked, and considered if necessary for wind-resistant design.

The paper now recommends the following:

- Consider the stick-slip damping model described here for structural damping estimation of other steel buildings under various wind load levels.
- Consider not directly using damping values estimated via current damping predictor models. The currently recommended 30% lower values could still over-estimate actual damping at wind-resistant design amplitudes.
- As an interim measure, the lowest measured damping values at very low amplitudes can be used. For 200-meter high buildings, this ranges from 0.6% to 0.9%. For the building studied in this paper, it is found to be 0.6%. It is still possible though that these over-estimate damping at design levels.
- Perform similar analyses on concrete main systems where some material nonlinearity already starts to manifest even under small amplitudes.
- Consider the stick-slip damping model in all future damping measurements, or in re-evaluation of damping from previous measurements.
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REFERENCES

1) Tamura, Y., “Amplitude Dependency of Damping in Buildings and Critical Tip Drift Ratio”, Int. J. High-Rise Build., Vol.1, No.1, pp.1-13, (2012)
2) Jacobsen, L.S., “Steady forced vibration as influenced by damping”, Trans. ASME-APM-52-15, pp.169-181, (1930)
3) Wyatt, T.A., “Mechanisms of damping”, Symposium on dynamic behavior of bridges, Transport and Road Research Laboratory, pp.10-21, (1977)
4) Eyre, R., and Tilly, G.P., “Damping measurements on steel and composite bridges”, Proc. Symposium on dynamic behavior of bridges, Transport and Road Research Laboratory, pp.22-39, (1977)
5) Jeary, A.P., “Damping in tall buildings – a mechanism and a predictor”, J. Earthq. Eng. Struct. Dyn., Vol.14, No.5, pp.733-750, (1986)
6) Davenport, A.G, and Hill-Carroll, P., “Damping in tall buildings: its variability and treatment in design”, ASCE Spring Convention: Building Motion in Wind, pp.42-57, (1986)
7) Krishna, P., Ahmad, B., and Pande, P.K., “Role of damping in wind induced excitation of towers”, J. Wind Eng. Ind. Aerodyn., Vol.14, pp.319-330, (1983)
8) Marukawa, H., Kato, N., Fujii, K., and Tamura, Y., “Experimental evaluation of aerodynamic damping of tall buildings”, J. Wind Eng. Ind. Aerodyn., Vol.59, pp.177-190, (1996)
9) Holmes, J. D., “Along-wind response of lattice towers – II. Aerodynamic damping and deflections”, Engrg. Struct., Vol.18, No.7, pp.483-488, (1996)
10) Nakamura, Y., Okada, K., Yokota, H., Kitada, Y., Tsuji, E., Ukit, T., and Yamaura, N., “Dynamic Vibration Tests of ORC200 Symbol Tower: Part 2 Variation in Dynamic Characteristics”, Summaries of Technical Papers of Annual Meeting of Architectural Institute of Japan, Structures 1, pp.877-878, (1993)
11) Architectural Institute of Japan, “Database of dynamic properties of buildings and structures in Japan”, Working Group on Database (T. Arakawa, K. Morita, J. Ono, A. Sasaki, N. Satake, and K. Suda), Research Committee on Damping Evaluation (Chair: Y. Tamura), (2000)
12) Tamura, Y., Suda, K., and Sasaki, A., “Damping in buildings for wind resistant design”, International Symposium on Wind and Structures for the 21st Century, pp.115-130, (2000)
13) Tamura, Y., “Amplitude dependency of damping in buildings and estimation techniques”, 12th AWES Wind Engineering Workshop, (2006)
14) Architectural Institute of Japan, Recommendations for Loads on Buildings, AIJ-RLB-2004, (2004)
15) Aquino, R.E.R., and Tamura, Y., “Damping based on EPP spring models of stick-slip surfaces”, Proc. International Conference on Wind Engineering, on CD-ROM, (2011)
16) Aquino, R.E.R., and Tamura, Y., “On stick-slip phenomenon as primary mechanism behind structural damping in wind-resistant design applications”, J. Wind Eng. Ind. Aerodyn., doi:10.1016/j.jweia.2012.12.017, (2013)
17) Computers and Structures, Inc.: SAP2000 User’s Manual, Berkeley, Calif., (2000)
18) Ibrahimbegovic, A. and Wilson, E. L.: “Simple numerical algorithms for the mode superposition analysis of linear structural systems with non-proportional damping”, Computers and Structures, vol.33, no.2, pp.523-531, (1989)
19) Wilson, E. L.: “An Efficient Computational Method for the Base Isolation and Energy Dissipation Analysis of Structural Systems”, Proc. Seminar on Seismic Isolation, Passive Energy Dissipation, and Active Control, ATC17-1, Applied Technology Council, Redwood City, Calif., (1993)
20) Bashor, R., and Kareem, A., “Load factors for dynamically sensitive structures”, 11th Americas Conference on Wind Engineering, (2007)
21) Architectural Institute of Japan, Guidelines for the Evaluation of Habitability to Building Vibration, AIJ-GBV-2004, Maruzen, (2004)