Electrostriction enhancement in metamaterials

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We demonstrate a controllable enhancement in the electrostrictive properties of a medium using dilute composite artificial materials. Analytical expressions for the composite electrostriction are derived and used to show that enhancement, tunability and suppression can be achieved through a careful choice of constituent materials. Numerical examples with Ag, As$_2$S$_3$, Si and SiO$_2$ demonstrate that even in a non-resonant regime, artificial materials can bring more than a threefold enhancement in the electrostriction.

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I. INTRODUCTION

Optoacoustic interactions have gained considerable attention in recent years in the context of nanophotonics. One of the strongest and most important of these is Stimulated Brillouin Scattering (SBS), which is a coherent interaction between the electromagnetic and acoustic fields occurring in an optical waveguide. SBS has been demonstrated in a number of areas within nanophotonics, notably in the design of nanoscale devices for Brillouin lasers, signal processing and microwave generation. The strength of SBS is principally determined by the electrostriction, which is the induced strain arising from an electromagnetic field within the waveguiding material. The magnitude of the electrostrictive effect, as well as that of the related photo-elastic effect, has widely been considered a property of the material used, and as a consequence, the materials that have been used in SBS studies have been mostly limited to those with naturally large electrostriction constants.

At the same time, it is well-established in the metamaterials literature that large enhancements in the nonlinear properties of a medium can be achieved through the use of composites that have sub-wavelength structural features. Metamaterials have been used to enhance nonlinear scattering effects such as the Raman effect, to achieve nonlinear diffraction, and have been used in optomechanical systems at microwave frequencies. However, nonlinear metamaterials have yet to be designed for the enhancement and suppression of electrostriction and photoelasticity, particularly in the optical range.

In this paper we demonstrate that artificial materials can be designed for the tuneable enhancement or suppression of electrostriction. We investigate materials consisting of a dilute suspension of spheres embedded in a dielectric matrix, as presented in Fig. 1. We consider both dielectric and metallic inclusions, and derive a mixing formula that describes the effective electrostriction of the composite. The electrostriction for a selection of practically realisable examples is then evaluated, and used to show that enhancement or suppression of electrostriction can be achieved. To our knowledge, we are the first to explore modifications in the optoacoustic material properties of a medium. It has been shown previously that even very simple composite material designs can enhance the nonlinear susceptibility beyond that of either constituent materials, and therefore, we expect similar enhancements here with the electrostriction.

To determine the electrostrictive properties of a composite material, we must first obtain electrostriction values for all constituent media. Expressions for these constituents can differ depending on whether dispersion and loss are incorporated in their derivation, and less obviously, on other mechanical and thermodynamic assumptions that are imposed. These considerations play an important part in determining regimes over which estimates for the electrostriction are appropriate, and a discussion of these relevant approximations can be found.

FIG. 1. Schematic view of the metamaterial geometry investigated; a primitive cubic array of spheres in a host medium. Inlaid: fundamental unit cell for a cubic lattice of spheres.
The outline of this paper is as follows. In Section II we derive a general expression for the electrostriction, including the effects of dispersion, and apply this to uniform dielectrics and metals. In Section III we obtain the electrostriction for composite materials. In Section IV we consider a series of practical examples before concluding remarks in Section V.

II. ELECTROSTRICTION FOR CONSTITUENT MEDIA

In this section, we derive a general expression for the electrostriction of a homogeneous material. Typically, estimates for the electrostriction of materials are made under the assumption of zero loss and dispersion, zero shear stress, and that variations in the permittivity arise from changes in density alone (i.e. an isothermal process). In a generalisation of the standard procedure, we incorporate the effects of dispersion in our derivation. We begin by considering the electromagnetic energy density,

$$ u = \frac{1}{2} \varepsilon_0 \frac{\partial (\omega \varepsilon_r)}{\partial \omega} |E|^2, $$ (1)

where $\varepsilon_0$ denotes the free-space permittivity, $\varepsilon_r$ is the relative permittivity of the material, $\omega$ is the frequency and $E$ is the electric field. The change in energy corresponding to a small change in the density $\rho$ is therefore

$$ \Delta u = \frac{1}{2} \varepsilon_0 \frac{\partial (\omega \varepsilon_r)}{\partial \omega} \left[ \frac{\partial (\omega \varepsilon_r)}{\partial \omega} \right] |E|^2 \Delta \rho, $$ (2a)

where $\Delta$ denotes an infinitesimal quantity. This change in the internal energy can be equated to the work done $W$ per unit volume $V$ by the system

$$ \Delta W = P \frac{\Delta V}{V} = -P \frac{\Delta \rho}{\rho}, $$ (2b)

where $P$ is the induced pressure, to obtain

$$ P = -\frac{1}{2} \varepsilon_0 \gamma |E|^2, $$ (3)

and we define the electrostriction parameter

$$ \gamma = \rho \frac{\partial^2 (\varepsilon_r \omega)}{\partial \rho \partial \omega}, $$ (4)

as a nondimensional measure of the induced electrostrictive stress. From (4), we obtain expressions for the electrostriction of both dielectric and metallic media which are used in our composite model shown later.

A. Electrostriction for dielectric media

For a dielectric medium that is nondispersive and lossless, the electrostriction parameter simplifies to the familiar form

$$ \gamma = \rho \frac{\partial \varepsilon_r}{\partial \rho}. $$ (5a)

It is then usual to express this in terms of other well-known material response tensors for practical evaluation. For example, for isotropic and homogeneous materials, (5a) is given by

$$ \gamma = \frac{1}{3} \varepsilon_r^2 (p_{11} + 2p_{12}), $$ (5b)

where $p_{ij}$ denotes the elastooptic coefficients of the medium. These $p_{ij}$ coefficients are well-tabulated for a range of materials, and a selection of values for dielectric solids are presented in Table I for reference. However, to our knowledge no experimental data has been published on elastooptic coefficients for metallic media,

B. Electrostriction for metallic media

For metals, we return to (4), and use a simplified Drude model for the permittivity,

$$ \varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}, $$ (6)

to obtain a form for $\gamma$ which is useful for practical evaluation. Here, $\omega_p^2 = q^2 N/(\varepsilon_0 m_e)$ is the square of the plasma frequency, $q$ is the electric charge, $m_e$ is the effective mass of a constituent electron, $N = \rho/m$ is the number density, and $m$ is the mass density of the metal. Subsequently (4) and (6) give the estimate

$$ \gamma_{DM} = \frac{\omega_p^2}{\omega^2}. $$ (7)

We note that in the derivation of the metallic $\gamma$ above, we have neglected dissipation effects, which is consistent with the isothermal assumption made in the derivation of (4). The validity of this assumption is considered in the results section, with a discussion of attenuation.

Having derived evaluable expressions for dielectrics and metals, we now proceed to the electrostriction of composite materials.

| Table I. Material parameters for a selection of dielectric materials at specified wavelengths, $\gamma$ from Eq. (5b). |
|-----------|-----|-----|-----|-----|-----|-----|
| Material  | $\lambda$ (nm) | $\varepsilon_r$ | $p_{11}$ | $p_{12}$ | $\gamma$ | Ref |
| SiO$_2$   | 660 | 2.12 | 0.12 | 0.27 | 1.00 | 11 |
| As$_2$S$_3$ | 1150 | 6.06 | 0.31 | 0.30 | 11.1 | 15 |
| Si        | 3300 | 11.8 | -0.09 | 0.02 | -2.77 | 12 |
III. ELECTROstriction FOR COMPOSITE MATERIALS

In this section, we derive $\gamma$ for our composite material using the Maxwell-Garnett (MG) model. The effective permittivity given by this model is valid for a dilute array of spheres embedded in a host material, and has the form:

$$\varepsilon_r = \varepsilon_m + \frac{3\varepsilon_m(\varepsilon_i - \varepsilon_m)f}{(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f},$$  \hspace{1cm} (8)

Here $\varepsilon_{i,m}$ denotes the relative permittivities of the constituent materials, and we define the filling fraction

$$f = \frac{V_i}{V_i + V_m},$$  \hspace{1cm} (9)

where $V_{i,m}$ represent corresponding volumes. The subscript $i$ denotes the inclusion and $m$ denotes the matrix (constrained by the boundaries of the unit cell), as shown by the fundamental cell in Fig. 1.

To begin, we consider a fully nondispersive model for the composite electrostriction.

A. Nondispersive model

Under the assumption that all constituent materials are nondispersive, the $\gamma$ expression reduces to the form given in (12). Consequently, from the MG model we write

$$\gamma_{\text{ND}} = \rho \left[ \frac{\partial \varepsilon_r}{\partial \varepsilon_m} \frac{\partial \varepsilon_m}{\partial \rho} + \frac{\partial \varepsilon_r}{\partial \varepsilon_i} \frac{\partial \varepsilon_i}{\partial \rho} + \frac{\partial \varepsilon_r}{\partial f} \frac{\partial f}{\partial \rho} \right],$$  \hspace{1cm} (10)

where from (8) we have the partial derivatives

$$\frac{\partial \varepsilon_r}{\partial \varepsilon_m} = \frac{[(\varepsilon_i + 2\varepsilon_m)^2 + 2f(\varepsilon_i - \varepsilon_m)^2](1-f)}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2},$$  \hspace{1cm} (11a)

$$\frac{\partial \varepsilon_r}{\partial \varepsilon_i} = \frac{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)^2]f}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2},$$  \hspace{1cm} (11b)

$$\frac{\partial \varepsilon_r}{\partial f} = \frac{3\varepsilon_m(\varepsilon_i - \varepsilon_m)(\varepsilon_i + 2\varepsilon_m)}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2}.$$  \hspace{1cm} (11c)

However, to evaluate the remaining three derivatives in (10) a boundary condition is required.

In the presence of the electric field, we impose the condition that the pressure fields in both media remain continuous across the interface. Thus, the resulting perturbations satisfy

$$\Delta P_i|_{\partial \Omega} = \Delta P_m|_{\partial \Omega},$$  \hspace{1cm} (12)

where $P_{i,m}$ denotes the interior and exterior pressure fields and $\partial \Omega$ is the boundary of the inclusion. We can then evaluate Taylor series for the constituent volumes $V_{i,m}$ with respect to $P_{i,m}$ to obtain

$$\Delta V_i = \frac{\partial V_i}{\partial P_i} \Delta P_i,$$  \hspace{1cm} (13a)

$$\Delta V_m = \frac{\partial V_m}{\partial P_m} \Delta P_m,$$  \hspace{1cm} (13b)

and express (12) in the form

$$\frac{\Delta V_i}{V_i} = \frac{\Delta V_m}{V_m} \bigg|_{\partial \Omega},$$  \hspace{1cm} (14)

where we have introduced the compressibility constant

$$\beta = -\frac{1}{V} \frac{\partial V}{\partial P} = K^{-1},$$  \hspace{1cm} (15)

and $K$ denotes the bulk modulus. Integrating both sides of (14) we obtain the interface condition

$$V_m = A[V_i]^{\beta_m/\beta_i},$$  \hspace{1cm} (16)

for some constant $A$. With this condition, and using the definition for the composite density

$$\rho = \rho_i f + \rho_m (1 - f) = \frac{m_i + m_m}{V_i + V_m},$$  \hspace{1cm} (17)

we evaluate the remaining three derivatives to obtain

$$\frac{\partial \varepsilon_m}{\partial \rho} = \frac{\partial \varepsilon_m}{\partial \rho_i} \frac{\partial \rho_i}{\partial \rho} \frac{\partial V_i}{\partial \rho} = \gamma_m \frac{\beta_m}{\beta_c},$$  \hspace{1cm} (18a)

$$\frac{\partial \varepsilon_i}{\partial \rho} = \frac{\partial \varepsilon_i}{\partial \rho_i} \frac{\partial \rho_i}{\partial \rho} \frac{\partial V_i}{\partial \rho} = \gamma_i \frac{\beta_i}{\beta_c},$$  \hspace{1cm} (18b)

$$\frac{\partial f}{\partial \rho} = \frac{\partial f}{\partial V_i} \frac{\partial V_i}{\partial \rho} = f(1-f) \frac{\beta_i - \beta_m}{\beta_c},$$  \hspace{1cm} (18c)

where analogously to (14a) we introduce $\gamma_m = \rho_m \varepsilon_m/\partial \rho_m$ and $\gamma_i = \rho_i \varepsilon_i/\partial \rho_i$ as the electrostriction values of the constituent media, and $\beta_\ast = \beta_if + \beta_m(1-f)$ denotes the volume-averaged compressibility over the unit cell. Consequently, the nondispersive electrostriction for our composite is given by
\[ \gamma_{ND} = \frac{\beta_f}{\beta_c} \left[ \frac{(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)}{(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f} \right]^2 \gamma_i + \frac{\beta_m(1 - f)}{\beta_c} \left[ \frac{(\varepsilon_i + 2\varepsilon_m)^2 + 2f(\varepsilon_i - \varepsilon_m)^2}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2} \right] \gamma_m \]
\[ + \frac{(\beta_m - \beta)f(1 - f)}{\beta_c} \left[ \frac{3\varepsilon_m(\varepsilon_i - \varepsilon_m)(\varepsilon_i + 2\varepsilon_m)}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2} \right], \quad (19) \]

which is a weighted linear function of the constituent electrostriction values \( \gamma_i \) and \( \gamma_m \) plus a new artificial electrostriction term (highlighted). The latter term can be understood by considering the limit \( \gamma_i = \gamma_m = 0 \); if the two materials have different compressibility values, then compression leads to a change in the filling fraction \( f \), which, if \( \varepsilon_i \neq \varepsilon_m \), alters the effective dielectric constant \( \varepsilon \). Another interesting feature of (19) is the second-order pole present in all terms at
\[ f = (\varepsilon_i + 2\varepsilon_m)/(\varepsilon_i - \varepsilon_m), \quad (20) \]
giving a theoretically infinite value for the composite electrostriction. However, this resonance can only be obtained with a change in sign for either \( \varepsilon_i \) or \( \varepsilon_m \) for dilute, positive \( f \). A discussion of the asymptotic behaviour of (19) with respect to \( \beta_i, \beta_m \) and \( \varepsilon_i, \varepsilon_m \) is presented in Appendix A for completeness.

Next we consider the composite \( \gamma \) expression when dispersion is included.

### B. Dispersive corrections

In this section, we incorporate dispersion in the derivation of the composite \( \gamma \). We begin by returning to (14), which from the MG model \( \varepsilon \), has the form
\[ \gamma = \rho \frac{\partial \varepsilon_m}{\partial \omega} \left[ \frac{\varepsilon_i + \varepsilon_m}{\partial \varepsilon_i} \frac{\partial \varepsilon_m}{\partial \varepsilon_i} + \frac{\varepsilon_i + \varepsilon_m}{\partial \varepsilon_i} \frac{\partial \varepsilon_m}{\partial \varepsilon_i} \right] + \rho \omega \frac{\partial}{\partial \rho} \left[ \frac{\partial \varepsilon_m}{\partial \varepsilon_i} \frac{\partial \varepsilon_m}{\partial \omega} + \frac{\partial \varepsilon_i}{\partial \varepsilon_i} \frac{\partial \varepsilon_i}{\partial \omega} + \frac{\partial \varepsilon_i}{\partial \varepsilon_i} \frac{\partial \varepsilon_i}{\partial \omega} \right]. \quad (21) \]

This composite expression is then decomposed in the form \( \gamma = \gamma_{ND} + \gamma^D \), where \( \gamma_{ND} \) and \( \gamma^D \) represent the nondispersive and dispersive contributions, respectively.

The nondispersive contribution is given by the first three terms of (21) and has been evaluated in the previous section as (19), where we introduce the substitution \( \gamma_{i,m} = \gamma_{i,m}^{ND} \) therein.

Next, we evaluate the remaining terms in (21), and note that we have
\[ \frac{\partial f}{\partial \omega} = 0, \quad (22) \]
as all mechanical parameters, such as \( \beta \) and \( \rho \), are independent of the optical frequency. Accordingly, we decompose the dispersive term \( \gamma^D \) into a matrix and inclusion

### IV. NUMERICAL EXAMPLES

In this section, we investigate the composite electrostriction expressions (19) and (25) for our structure, using...
combinations of different materials. We accompany this investigation with an analysis of the losses for these designs, which is necessary for realistic applications. Accordingly, we return to the MG model and define the attenuation length

\[ \alpha_L = \left[ \frac{4\pi}{\lambda} \text{Im} (\sqrt{\varepsilon_r}) \right]^{-1}, \]  

which we emphasise, is completely independent from the electrostriction analysis. From (26), a threshold of \( \alpha_L \geq 0.1 \text{mm} \) is imposed as a tolerance for omitting dissipation effects, which is also a typical interaction length for SBS.

We begin by investigating the composite electrostriction for a cubic array of silver spheres embedded in a silica matrix, where we use (7) for \( \gamma \). In Fig. 2(a) we present a contour plot of \( \log_{10} |\text{Re}(\gamma)| \) over the wavelength range \( 350 \text{ nm} \leq \lambda \leq 4000 \text{ nm} \) for filling fraction \( 0 \leq f \leq 0.3 \).

A striking feature of this figure is the region corresponding to \( \log_{10} |\text{Re}(\gamma)| > 0.8 \), which contains the permittivity resonance (20). This region simply denotes \( \gamma \) values over a cut-off threshold, which is introduced to ensure that features of the contour plot are not dominated by the singularity in (25).

We note that the extremely strong enhancements in \( \gamma \) courtesy of (20) are associated with strong attenuation (26), and we highlight this by superposing a solid white curve over these contours, which represents an attenuation length threshold of \( \alpha_L = 0.1 \text{ mm} \) (where to the right of this curve we have longer \( \alpha_L \), and to the left, a region of shorter lengths).

Also shown is a dashed white curve, which represents our diluteness threshold of \( f = 15\% \). Accordingly, inside the region bound by these two curves (the region of validity (ROV)), we find a maximum composite electrostriction value of \( \gamma = 3.27 \) at \( (\lambda, f) = (1003 \text{ nm}, 0.15) \), which corresponds to the intersection of the \( \alpha_L \) and \( f \) curves. This point gives an enhancement factor of 3.36 relative to the electrostriction for the silica background at the same wavelength. It is also clear from these contours that the electrostriction is tuneable over a wide wavelength interval, but that these enhancements are ultimately constrained by the diluteness requirement of the MG model.

In Fig. 2(b) we present a contour plot of the effective permittivity over the same \( (\lambda, f) \) range, where the plasmon resonance is clearly visible. For our maximum electrostriction value at \( (\lambda, f) = (1003 \text{ nm}, 0.15) \), we have a composite permittivity of \( \varepsilon_r = 3.4 + 0.003i \), which is an enhancement factor of 1.6 relative to the background value of \( \varepsilon_m = 2.10 \) at the same wavelength. As one would expect, this contour plot features similar curvature to that of \( \gamma \), and a low degree of frequency dependence within the ROV \((2.08 < \text{Re}(\varepsilon_r) < 3.02)\).

We now consider silver spheres embedded in a chalcogenide matrix (amorphous As_{2}S_{3}). In Fig. 3(a) we present the composite \( \gamma \) for this configuration, and observe qualitatively similar behaviour to the previous example for a silica matrix in Fig. 3(a). The primary difference here is the much more restrictive \( \alpha_L \) threshold, which now extends to much longer wavelengths.

If one searches inside the ROV constrained by our \( \alpha_L \) and \( f \) bounds, we discover a maximum electrostriction value of \( \gamma = 27.4 \) at \( (\lambda, f) = (2064 \text{ nm}, 0.15) \), corresponding to the intersection of the \( \alpha_L \) and \( f \) curves as before, with an enhancement factor of 2.63 (c.f., \( \gamma_m = 10.44 \)). Fig. 3(b) reveals that this coordinate point has an effective permittivity value of \( \varepsilon_r = 9.31 + 0.01i \). This corresponds to a similar permittivity enhancement factor as the previous example (c.f., \( \varepsilon_m = 5.89 \)). A slightly higher level of frequency dependence is observed in the ROV also \((6.31 < \text{Re}(\varepsilon_r) < 8.91)\).

For these examples, we find that the composite electrostriction expression (25) gives a 10 – 20\% increase in the maximum electrostriction value compared to the nondispersive expression (19). This suggests that the omission of dispersion can give rise to a small but non-negligible correction to the composite electrostriction. Furthermore, a similar investigation with Au spheres embedded in these matrix materials reveals a comparable level of enhancement to Ag.
FIG. 3. Contour plots of (a) $\log_{10}|\text{Re}(\gamma)|$ from (25), and (b) $\log_{10}|\text{Re}(\varepsilon_r)|$ from (8), for an array of Ag spheres embedded in an $\text{As}_2\text{S}_3$ matrix, against filling fraction $f$ and wavelength $\bar{\lambda}$. A diluteness threshold of $f = 15\%$ (dashed white curve) and an attenuation length threshold of $\alpha_L = 0.1$ mm (solid white curve) are also shown.

In Fig. 4(a) we consider $\log_{10}|\text{Re}(\gamma)|$ from (19) for silica spheres embedded in a silicon matrix. This figure exhibits strong frequency dependence for $\bar{\lambda} < 1000\text{ nm}$ (courtesy of a material resonance for Si at $\bar{\lambda} \approx 370\text{ nm}$) and a near-horizontal arc of zero electrostriction which spans the entire ROV. That is, this metamaterial design can completely suppress electrostriction over an exceptionally wide frequency range. For this particular composite the attenuation length threshold is reached at approximately $\bar{\lambda} = 1000\text{ nm}$. In Fig. 4(b) we present a contour plot of $\log_{10}|\text{Re}(\varepsilon_r)|$ for completeness, which exhibits reassuringly minimal frequency dependence over the ROV.

In Fig. 5(a) we show a cross section of the composite $\text{Re}(\gamma)$ from Fig. 4(a) at $\bar{\lambda} = 1550\text{ nm}$. This gives confirmation that complete suppression of electrostriction is achieved at $f \approx 10\%$, and shows that we have sign-changing electrostriction from this metamaterial design. We note that the composite $\text{Re}(\gamma)$ (blue curve) exceeds that of the constituent electrostriction values (dashed curves) at a filling fraction of $f = 16.6\%$, which is reminiscent of earlier work which showed the nonlinear parameters of composite materials can exceed the values of the constituents but we note that care must be taken as the dilute lattice assumption breaks down in this region of enhancement here. An investigation using $\text{As}_2\text{S}_3$ spheres in a Si matrix demonstrated an identical result to that shown in Figures 4(a), 4(b) and 5(a), but at much lower filling fractions.

To emphasise the result presented in Fig. 5(a) we show the composite $\gamma$ curve for an array of chalcogenide ($\text{As}_2\text{S}_3$) spheres embedded in a silica matrix at $\bar{\lambda} = 1550\text{ nm}$ in Fig. 5(b). This shows a simple linear enhancement from the background electrostriction, to a maximum realisable value of $\gamma = 1.918$ at the threshold of $f = 15\%$ (i.e. an enhancement factor of approximately 2).

V. CONCLUDING REMARKS

We have presented an analytical representation for the electrostriction of a composite material by incorporating the simplest and analytically most transparent model from effective index theory, the Maxwell–Garnett model,
to the problem of electrostriction.

We show that expressions for the electrostriction of a composite material feature artificial electrostriction terms, which contribute to the enhancement or suppression of this material property, as observed for a selection of composites here. The presence of this term points towards the possibility that large enhancements in $\gamma$, beyond both material values, could be achieved for more sophisticated metamaterial designs. We also show that sign-switching electrostriction is achievable, and that resonant enhancements in the electrostriction of metal-dielectric composites are unrealistic, as they are associated with strong attenuation. Incorporating dispersive effects in the model is shown to give a small but non-negligible correction to estimates for the composite electrostriction.

It is important to emphasise that this work is a first step in the study of the electrostriction of composites, and so other considerations such as thermally-induced electrostriction and scattering losses, are not addressed here. These effects are more prominent for high-intensity wave problems, where more elaborate models are required to accurately evaluate the material response. Also, we note that our estimate for the electrostriction of metals is a low-order approximation, which requires experimental data for validation.

As a final comment, we emphasise that other homogenisation procedures\cite{[6]} can be used to determine the electrostriction for a periodic composite, which should remove several constraints of the present MG model, and open the way to investigations of exciting metamaterial designs.

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\section*{Appendix A: Asymptotic analysis of the nondispersive composite electrostriction}

In this appendix we examine several asymptotic limits for the composite electrostriction expression \cite{9}. First, assuming $\varepsilon_m \gg \varepsilon_i$, we obtain

$$\gamma \sim \frac{9\beta_i f - 2\gamma_m f - 6(\beta_m + \beta_i) f(1 - f)}{\beta_c(f + 2)^2 \beta_c(f + 2)^2 - \beta_c(f + 2)^2},$$

(A1)

where we have a persistent, but simplified, contribution from all terms in \cite{9}. By contrast, for $\varepsilon_i \gg \varepsilon_m$ we have

$$\gamma \sim \frac{\beta_m(1 + 2f)}{(1 - f)\beta_c} \gamma_m,$$

(A2)

revealing that for high permittivity inclusions, the composite electrostriction depends only on the $\gamma_m$ directly. The limit $\beta_m \gg \beta_i$ gives the asymptotic form

$$\gamma \sim \frac{[(\varepsilon_i + 2\varepsilon_m)^2 - 2f(\varepsilon_i - \varepsilon_m)^2]}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2} \frac{\gamma_m}{\gamma_i}$$

$$+ \frac{3f\varepsilon_m(\varepsilon_i - \varepsilon_m)(\varepsilon_i + 2\varepsilon_m)}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2} + \frac{3(1 - f)\varepsilon_m(\varepsilon_i - \varepsilon_m)(\varepsilon_i + 2\varepsilon_m)}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2}.$$

(A3)

and $\beta_i \gg \beta_m$ leads to

$$\gamma \sim \frac{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)]^2}{(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f} \gamma_i$$

$$- \frac{3(1 - f)\varepsilon_m(\varepsilon_i - \varepsilon_m)(\varepsilon_i + 2\varepsilon_m)}{[(\varepsilon_i + 2\varepsilon_m) - (\varepsilon_i - \varepsilon_m)f]^2}.$$

(A4)

These differences in sign in the artificial electrostriction contributions above suggest that the relative magnitudes of $\beta$ are relevant in establishing whether enhanced or suppressed electrostriction is observed.
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