Optomagnonically induced RoF chaotic synchronization

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Abstract
Optomagnonics is a good platform for the interplay of radio frequency and optical signals, which are the primary communication carriers in the present day. On the basis of optomagnonics, we provide a multi-scale method for analyzing its behavior, including frequency comb and RoF chaotic synchronization at the microwave scale. Adjusting the pump light intensity permits the transition of RoF signals between harmonics, frequency combs, and chaotic movements. The dual optomagnonical device enables the synchronization of RoF signals between different cavities. Our study will contribute to the use of multiscale electromagnetic wave coupling in both conventional and quantum information applications.

1. Introduction
Optomagnonics enable strong interactions between optical photons and microwave magnons [1–11]. Millimeter-scale yttrium–iron–garnet (YIG, Y₃Fe₅O₁₂) resonators [1, 5, 9, 12] that can support the optical whispering gallery mode (WGM) [13–16] and magnon mode are optomagnonic materials with excellent optical and magnetic parameters. The YIG resonator can be a various structures such as planar [7], cylindrical [12], and ellipsoidal [17, 18]. Various phenomena and applications such as frequency combs [19, 20], chaos [21–24], photon blockade [25], spectral gaps and beam deflections [40], and the optical excitation of spin waves [41]. This optomagnonical material is also highly effective as a nonlinear microwave device. Over the past few years, microwave frequency combs and chaotic magnonic devices have also been gradually realized [42–44]. Microwaves and optics are the major method of information transmission in the present society. The interaction of these two bands has numerous applications in actual application scenarios. Strong coupling between microwave and optical signals provides a platform for optical and microwave information conversion in YIG optomagnonics [7, 31]. Therefore, it is crucial to explore the hybrid microwave-optical characteristics of optomagnonics.

The significant order-of-magnitude difference between optical and magnon frequencies in optomagnonics systems makes even numerical solutions difficult. The usual way to deal with it is the rotational frame transformation [19, 21, 23, 45]. The rotation frame transformation requires accurate knowledge of the optical frequency before solving. When a system has a periodic structure, the time-Floquet method can be used to increase the system’s solvability [46]. However, the system is not always periodic, and when the system is nonlinear, the periodicity is typically broken. Thus, we need a new way to explore system characteristics more deeply.

For a more in-depth explanation of the nature of optomagnonics, we provide a multi-time-scale method that enables us to explore the characteristics of optomagnonics at any timescale of interest. We get the dynamic equations of radio over fiber (RoF) and microwave magnon oscillation using this method.

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Figure 1. Scheme of a YIG optomagnonics. All of the system works in a magnetostatic environment. The TE and TM photons are coupled through the scattering of magnons. \( a \) indicates the optical TE mode, \( b \) indicates the optical TM mode, and \( m \) corresponds to the magnon mode in this figure. (a) Shows the equipment schematic diagram. (b) Shows the schematic of the physical mechanism.

RoF signal may produce frequency comb and chaos in a dynamic magnonic condition. In addition, we discover that the RoF chaotic signal may be synced by using the TE mode. The findings will further research on RoF frequency combs, chaos, and chaotic synchronization.

2. Multi-timescale coupling of the optomagnonics

We consider the optomagnonics illustrated in figure 1. The TE and TM modes have distinct resonance frequencies for permittivity anisotropy in the YIG sphere. In optomagnonics, when a TE photon with frequency \( \omega_a \) is dispersed by magnons, a TM photon with frequency \( \omega_b \) and a magnon with frequency \( \omega_m \) are produced; hence, \( \omega_m = \omega_b - \omega_a \), as required by the law of conservation of energy. Classically, this process can be described as the permittivity tensor \( \epsilon(M) \) of the material is related to the magnetic field \([1, 47]\) with interaction energy \( \frac{1}{4} \int E^*(r)\epsilon(M)E(r)dr \), where \( E \) shows the electric field. YIG spheres typically range in size from hundreds of micrometers to several millimeters. To improve the interaction between the microwave field and YIG, the sphere of YIG is put in a microwave cavity with a scale of a few centimeters to tens of centimeters. The fiber excites the WGMs mode of the optomagnonics, whereas the microwave excites the magnon mode.

The magnon works in the surface magnetostatic modes in this article \([9]\). Nonetheless, it also works for the uniform magnetostatic mode, i.e., Kittel mode \([48]\). And the chosen TE and TM modes satisfy the angular-momentum selection rules \([5]\). The Hamiltonian of the system is:

\[
H_S = \frac{\hbar}{2} \left( \omega_a a^\dagger a + \omega_m m^\dagger m + \omega_b b^\dagger b + g(ab^\dagger m + a^\dagger bm) \right)
\]  

(1)

where \( a \) (\( a^\dagger \)), \( b \) (\( b^\dagger \)) and \( m \) (\( m^\dagger \)) are, in order, the creation and annihilation operators for TE photons, TM photons, and magnons. \( g \) refers to the optomagnonic coupling rate. From the Hamiltonian, the dynamic equations of the system can be derived as follows:

\[
\frac{da}{dt} = i\omega_a a + igb^\dagger m + \mathcal{P}_a e^{-i\omega_L t}
\]

(2a)

\[
\frac{db}{dt} = i\omega_b b + igm + \mathcal{P}_b e^{-i\omega_L t}
\]

(2b)

\[
\frac{dm}{dt} = i\omega_m m + ig a^\dagger b + \mathcal{P}_m e^{-i\omega_m t}.
\]

(2c)

Where \( \mathcal{P}_i, i = a, b, m \) is the pump power. \( \omega_L \) and \( \omega_{\text{mw}} \) represent the optical drive frequency and the microwave drive frequency, respectively. The optical frequency is \( 10^5 \rightarrow 10^8 \) times the microwave or radio frequency. Therefore, mathematically modelling their interaction directly is difficult. The multi-timescale method, on the other hand, provides a way to analyze the optical envelope (RoF) coupled with microwave magnons without fully solving the preceding equations.

The slow timescale of the optical object may be controlled by the frequency difference between the TE and TM modes, which can be of the same order of magnitude as the magnonic frequency. When referring
respectively. Multi-timescale supports the term $e^{i(\omega_a - \omega_m)t}$ to equations (11) and (8) in appendix, the dynamic equations of the whole system is:

\[
\frac{\partial A(\tau)}{\partial \tau} = i\gamma A(\tau) + i\gamma_m A(\tau)M(\tau) + P_o
\]

\[
\frac{\partial B(\tau)}{\partial \tau} = -\gamma B(\tau) + i\gamma B(\tau)A(\tau) + P_o
\]

\[
\frac{\partial M(\tau)}{\partial \tau} = i(\omega_m + i\gamma_m)M(\tau) + i\gamma^2 A(\tau)B(\tau) + P_m \exp(-i\Omega t).
\]

$A$, $B$ and $M$ represent the slow component of the optical TE, TM, and magnonic mode amplitude, respectively. Multi-timescale supports the term $e^{i(\omega_a - \omega_m)t}$ more than the rotating frame technique. Note that we neglected the damping term and pumping in this derivation. Here we alter the $a^{(1)}$ and $b^{(1)}$ to $A^{(1)}B^{(1)}e^{i(\omega_a - \omega_m)t}$. The $A(\tau)$ and $B(\tau)$ is the slow part of the system. As we discussed before, the $a^{(1)}$ and $b^{(1)}$ are also the slow part of the system, then we can directly use $A(\tau)$ and $B(\tau)$ and a relative phase term $e^{i(\omega_a - \omega_m)t}$ to replace $a^{(1)}$ and $b^{(1)}$.

### 3. Chaos in the magnonic oscillation region

When the magnonic frequency is near to the RoF, the system operates in a dynamic state. The magnons’ dynamics should also be considered. The equations for the dynamic state with damping and pumping are:

\[
\frac{\partial A(\tau)}{\partial \tau} = -\gamma A(\tau) + i\gamma M(\tau)B(\tau) + P_o
\]

\[
\frac{\partial B(\tau)}{\partial \tau} = -\gamma B(\tau) + i\gamma M(\tau)A(\tau) + P_o
\]

\[
\frac{\partial M(\tau)}{\partial \tau} = i(\omega_m + i\gamma_m)M(\tau) + i\gamma^2 A(\tau)B(\tau) + P_m \exp(-i\Omega t).
\]

$\gamma$ and $\gamma_m$ gamma and gammam represent the optical damping and the magnetic damping, respectively. $P_o$ (Pm) signifies the optical (magnonical) pump amplitude; this is a typical method for added mode drive and dissipation term while studying cavity dynamics. $s$ is the scale factor. We set 1 GHz as one unit in this section. We use the experimentally achievable parameters as follows [1, 22, 48, 49]: the optomagnon coupling strength is $1 \times 10^{-4}$ (0.1 MHz), the magnon frequency is 1.1 units (1.1 GHz) with 0.3 unit of dissipation (0.3 GHz), the optical dissipation is 10 MHz. Under these parameters, figure 2 depicts the development of the system with varying microwave pump strengths. The microwave pump strength is $5 \times 10^{3}$ (10 μW) in figures 2(a)–(c) and $1 \times 10^{4}$ (40 μW) in figures 2(d)–(f). The figures illustrate, from left to right, the TE mode, the magnonic mode, and the TM mode. The $\gamma$-axis represents the real part of mode. Under various microwave pump strengths, the system exhibits radically different behavior: the system operates in the harmonic area shown in figures 2(a)–(c), and the TM and magnonic modes oscillate in a Rabi-like form when the TE mode enters its stable zone. The stable TE mode may be viewed as a TM-magnon coupling factor, allowing the system to convert a microwave magnon signal into a TM
Figure 3. Frequency spectrum of the system under different microwave pump strengths. The microwave pump powers are $8 \times 10^3$ (25.6 $\mu$W), $10 \times 10^3$ (40 $\mu$W), $20 \times 10^3$ (160 $\mu$W), and $50 \times 10^3$ (1 mW) from (a) to (d). Other parameters are the same as in previous figures.

envelope signal in this area; when the pump power is raised to $1 \times 10^4$ (40 $\mu$W), the system enters a chaotic state, as seen in figures 2(d)–(f). All modes’ oscillations are chaotic.

To investigate the chaotic condition of the system, the spectrum of the TM mode is shown in figure 3. The microwave pump power is $5 \times 10^3$ (10 $\mu$W), $10 \times 10^3$ (40 $\mu$W), $20 \times 10^3$ (160 $\mu$W), and $50 \times 10^3$ (1 mW) from (a) to (d). The system features frequency comb teeth solely at the pump frequency for a
pump power of $5 \times 10^3$ (10 $\mu$W) in figure 3(a). When the pump power is increased to $10 \times 10^3$ (40 $\mu$W), the spectrum becomes chaotic in figure 3(b). However, there are three obvious comb teeth in the chaotic system: the pump peak in the center, a minor peak to the left of the main peak corresponding to the slow oscillation in figure 3, and a wide-range peak to the right, which is the signal’s primary comb teeth multiplier region. The spectrum under $20 \times 10^3$ (160 $\mu$W) pumping is shown in figure 2(c) which is the frequency comb region. Note, that this is a RoF comb and not an optical frequency comb. In figure 3(d), when the pump power is raised to $50 \times 10^3$ (1 mW), the pump is excessively powerful and totally dictates the nature of the system; the system again displays a single tooth.

A important measure for the study of chaotic behavior is the trajectory separation rate of trajectory evolution in the starting state with minor deviation. In order to determine trajectory separation, the starting intensities of the system’s TE modes are set to 0 and $10^{-6}$. The evolution trajectories under these two initial conditions are given by $b(t)$ and $b'(t)$, respectively. Then we define the trajectory separation as $\log(|b(t) - b'(t)|)$ and show it in figure 4. In figure (a), the pump strength is set to $5 \times 10^3$ (1 $\mu$W), and in
Figure 5. Poincaré section of the RoF TM signal under different magnonic dampings. The pump power is $10 \times 10^3$ (40 $\mu$W).

- (a) The magnonic damping is 0.5 units,
- (b) the magnonic damping is 0.1 units (100 MHz),
- (c) and (d) the magnonic damping is 0.06 units (60 MHz), where (d) is a zoom-in of (c),
- (e) and (f) The magnonic damping is 0.05 units (50 MHz), where (f) is a zoom-in of (e),
- (g) and (h) The magnonic damping is 0.01 units (10 MHz), where (h) is a zoom-in of (g).

In the harmonic region, the divergence between trajectories reduces gradually with time, while it initially grows exponentially in the chaotic region. To quantify this distinction, the Lyapunov exponent is investigated. Figure 4(c) shows the Lyapunov exponent for various magnon pump powers. This figure shows that when the pump power is between $9.8 \times 10^3$–$18 \times 10^3$, the system operates in the chaotic area.

Poincaré cross-sections are essential for analyzing the characteristics of multivariable systems. In figure 5, we illustrate the Poincaré cross-sections for various dampings with a pump power of $10 \times 10^3$ (40 $\mu$W); other parameters are the same as figure 2. To create the Poincaré cross-section, we use $pi$ as the period and $\{\text{Re}(b), \text{Im}(b)\}$ as the Poincaré points. We set the dissipation in figure 5(a) to 0.5 units, and there is a conventional chaotic section. When the dissipation is reduced to 0.1 units, a chaotic ring appears in figure 5(b). We can only establish the system’s probable amplitude range, but we cannot determine the system’s state at a precise time under this condition. In figure 5(c), when the dissipation is reduced to 0.06 units, the system is still chaotic, but there is a dot in the center of the chaotic ring. When we zoom in on this center, a torus structure with 20 islands can be observed in figure 5(d). There are twenty islands because the system is pumped at 1/10 the system’s frequency and Poincaré points are measured every half cycle. When the damping is reduced to 0.05, the chaos points become weaker in figure 5(e). The island at the center of the section is maintained in figure 5(f). When we continue to reduce the damping to 0.01 in figures 5(g) and (h), only the torus region exists. In addition, the shape of the islands changes from needle-like to round. This indicates that the system has changed total regularity. The above results show that even if the system is in a chaotic state, periodic components may exist, and it is best to avoid this state when using chaotic encryption for communication.

4. Optically mediated chaotic synchronization

Chaotic secure communication is the most essential use of chaos, and chaotic synchronization is a crucial occurrence for this application. Typically, chaotic synchronization includes dual systems. As demonstrated in figure 6, we create two identical optomagnonic cavities pair. The TE mode of the first cavity is connected to the second cavity in a unidirectional manner. The second cavity has a rapid decay channel for the TE mode. Then the RoF dynamical equations are as follows:

\[
\begin{align*}
\frac{\partial A(\tau)}{\partial \tau} &= -\gamma A + \text{i}gM^\dagger(\tau)B(\tau)e^{\text{i}(\omega_b - \omega_a)t} + P_o \tag{5a} \\
\frac{\partial B(\tau)}{\partial \tau} &= -\gamma B + \text{i}gM(\tau)A(\tau)e^{\text{i}(\omega_a - \omega_b)t} + P_o \tag{5b} \\
\frac{\partial M(\tau)}{\partial \tau} &= \text{i}(\omega_m + \text{i}\gamma_m)M(\tau) + \text{i}g^2A^\dagger(\tau)B(\tau)e^{\text{i}(\omega_b - \omega_a)t} + P_m \exp(-\text{i}\Omega t) \tag{5c}
\end{align*}
\]
Figure 6. A scheme to achieve twin cavity system, the TE mode of the first cavity is unidirectional coupling to the second cavity. And the second cavity have the same TE mode as the first cavity.

Figure 7. Synchronization of the RoF TM and microwave signals under a pump power of $10 \times 10^4$ (40 μW).
(a) Synchronization of the TM mode in the twin systems. (a) Synchronization of the microwave mode in the twin systems.

\[
\frac{\partial B'(\tau)}{\partial \tau} = -\gamma B' + igM'(\tau)A(\tau)e^{i(\omega_a - \omega_b)t} + P_o \quad (5d)
\]

\[
\frac{\partial M'(\tau)}{\partial \tau} = i(\omega_m + i\gamma_m)M'(\tau) + ig^2A^\dagger(\tau)B'(\tau)e^{i(\omega_b - \omega_a)t} + P_m \exp(-\Omega t). \quad (5e)
\]

$M'$ and $B'$ are the corresponding RoF signal strengths of the second cavity, respectively. In figure 7, we show the separation rate of the two different cavities under an initial separation $10^{-6}$. Figures 7(a) and (b) exhibit the separation rates of the TM mode $|B'(\tau) - B(\tau)|$ and magnonic mode $|M'(\tau) - M(\tau)|$, respectively. In 300 time units, the rate of separation between the TM and magnonic modes decreases exponentially to $10^{-20}$. The signal differential between the two cavities is small, and both systems are perfectly synced. Therefore, we achieved chaotic synchronization, which may be used to synchronize the RoF signal in various resonators.

The synchronization speed is an important index for signal synchronization. In figure 8, we utilize the slope of the trajectory separation to show the synchronization speed. As the pump energy rises, the slope of the system linearly increases. The energy transfer between the systems increases as the pump energy grows.
the mutual influence between the systems increases. So, synchronization is rapid with the higher pump power system. This result of chaotic synchronization of the optomagnonics is of great application significance. Firstly, RoF signals flow via fiber, therefore we can accomplish synchronization at the fiber communication distance. Secondly, our system can create chaotic synchronization of optical signals and microwave signals at the same time, and the system is scalable in microwave-fiber communication interface situations.

5. Conclusion

In summary, we studied the envelope and microwave features of an optomagnonics with a multi-timescale method. We find the optically carried microwave signal, RoF frequency comb and chaos can be tuned by changing the pump power and damping under magnon excitation. We have studied the Lyapunov exponent and Poincaré cross section of the system. The Poincaré cross section shows that the chaos state may contain periodic signals. Furthermore, we discover that the TM mode and the magnon mode may be synchronized in the twin optomagnonics by sharing the TE mode. Our findings have the potential to open the way for optical signal processing, chaotic information encryption, and chaos-assisted secure communication.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix

To implement the multi-timescale method, we set $\omega_a \approx \omega_b \gg \omega_m$ and $x = x^{(0)} + sx^{(1)}$, $s$ is the scale factor, $x$ is $a$ or $b$. We separate differential symbols for different timescales as $d/dt = \partial/\partial t + s\partial/\partial \tau (x = a, b)$, where $\tau = st$, $s \ll 1$. We directly substitute $a, b, m$, and the differential symbol $d/dt$ in the equation. Then, we obtain:

\begin{align}
(\partial/\partial t + s\partial/\partial \tau)(a^{(0)} + sa^{(1)}) &= i\omega_a(a^{(0)} + sa^{(1)}) + ig(b^{(0)} + sb^{(1)})m^\dagger + Pa e^{-i\omega_a t} \tag{6a} \\
(\partial/\partial t + s\partial/\partial \tau)(b^{(0)} + sb^{(1)}) &= i\omega_b(b^{(0)} + sb^{(1)}) + ig(a^{(0)} + sa^{(1)})m + Pb e^{-i\omega_b t} \tag{6b} \\
\partial m/\partial \tau &= i\omega_m m + ig(a^{(0)} + sa^{(1)})^\dagger(b^{(0)} + sb^{(1)}) + P_m e^{-i\omega_m t}. \tag{6c}
\end{align}
The magnonic and photonic parts are discussed separately below. First, we will look at the magnonic component. We divide equation (6c) by $T$ after integrating it on the time scale $t$ from 0 to $T$. Then we may obtain:

$$\frac{\partial m}{\partial \tau} = i\omega_mm + \left(\int_0^T ig(sa^{(0)}b^{(1)} + sa^{(1)}b^{(0)} + a^{(0)}b^{(0)})dt\right)/T + ig^2a^{(1)}b^{(1)} + \mathcal{P}_m e^{-i\omega_mt}. \tag{7}$$

Since both $a^{(0)}(t)$ and $b^{(0)}(t)$ are quasi-harmonic, its average value equals 0, i.e. $\int_0^T a^{(0)}dt/T = \int_0^T b^{(0)}dt/T \approx 0$ with $T$ is the long time period on the $t$ time scale. We can explain that the effective influence on the system driven with the high frequency is 0. Then we can eliminate the $a^{(0)}, b^{(0)}$ and $a^{(0)}b^{(0)}$ term in equation (7) and get:

$$\frac{\partial m}{\partial \tau} = i\omega_mm + ig^2a^{(1)}b^{(1)} + \mathcal{P}_m e^{-i\omega_mt}. \tag{8a}$$

This equation demonstrates that only the slow component of the photonic evaluation will have an effect on the magnon.

Then we focus on the photonic component. The microwave frequency is much lower than the optical frequency, and the magnonic component is constant over fast timescales. We combine terms of the same order according to $s$ for equations (6a) and (6b), because $s$ is small and the above equations hold for different values of $s$; hence, equations of the same order of $s$ are equivalent. The optical portion of the system consists of:

$$(\partial/\partial t + s\partial/\partial \tau)(a^{(0)} + sa^{(1)}) = i\omega_a(a^{(0)} + sa^{(1)}) + ig(b^{(0)} + sb^{(1)})m^\dagger + \mathcal{P}_a e^{-i\omega_at} \tag{9a}$$

$$(\partial/\partial t + s\partial/\partial \tau)(b^{(0)} + sb^{(1)}) = i\omega_b(b^{(0)} + sb^{(1)}) + ig(a^{(0)} + sa^{(1)})m + \mathcal{P}_b e^{-i\omega_bt}. \tag{9b}$$

The zero-order of $s$ corresponding to the fast part, while the first order of $s$ corresponding to the slow part. The fast part of the optical field is harmonic. As the upper bound of optomagnonic coupling is approximately 1 MHz, which is far weaker than the optical frequency, the fast part of the system is:

$$\frac{\partial a^{(0)}}{\partial t} = i\omega_a a^{(0)} + \mathcal{P}_a e^{-i\omega_at}, \tag{10a}$$

$$\frac{\partial b^{(0)}}{\partial t} = i\omega_b b^{(0)} + \mathcal{P}_b e^{-i\omega bt}. \tag{10b}$$

This equation demonstrates that the system’s oscillation is independent of the magnetic frequency. Notice that the $m$ term includes a $s$ term by definition. The first-order term may thus be expressed as:

$$\frac{\partial a^{(0)}}{\partial \tau} + \frac{\partial a^{(1)}}{\partial t} = i\omega_a a^{(1)} + igb^{(0)}m^\dagger \tag{11a}$$

$$\frac{\partial b^{(0)}}{\partial \tau} + \frac{\partial b^{(1)}}{\partial t} = i\omega_b b^{(1)} + iga^{(0)}m. \tag{11b}$$

Take the partial differential of the first two equations with respect to time $t$, apply the operator $\partial/\partial t$ to both sides of the first two equations, we obtain:

$$\partial^2 a^{(0)}/\partial \tau^2 + \partial a^{(1)}/\partial t^2 = i\omega_a(a^{(0)} + sa^{(1)}) + igm\partial^2 a^{(0)}/\partial t^2 \tag{12a}$$

$$\partial^2 b^{(0)}/\partial \tau^2 + \partial b^{(1)}/\partial t^2 = i\omega_b(b^{(0)} + sb^{(1)}) + igm\partial^2 b^{(0)}/\partial t^2. \tag{12b}$$

To simplify the calculation of system properties, we set $x^{(0)}(t, \tau) = sX(\tau)e^{i\omega_\tau t} (x = a, b)$. For the sake of computational simplicity, we take $m$ as a constant and just analyze equations (11a) and (11b), from which we can derive:

$$\partial^2 a^{(1)}/\partial \tau^2 + \omega_a^2a^{(1)} = -2i\omega_sxe^{i\omega_\tau t}\partial A(\tau)/\partial \tau - g\partial m\omega_a + \omega_b\partial B(\tau)/\partial \tau - gm\partial A(\tau)/\partial \tau \tag{13a}$$

$$\partial^2 b^{(1)}/\partial \tau^2 + \omega_b^2b^{(1)} = -2i\omega_sxe^{i\omega_\tau t}\partial B(\tau)/\partial \tau - g\partial m\omega_a + \omega_b\partial A(\tau)/\partial \tau \tag{13b}.$$
Assuming that the optical frequency relationship is \( \omega_a \cong \omega_b \), we can get \( \omega_a + \omega_b = 2\omega_{a0} \) and the optical slow scale equation is:

\[
-2i\omega_a e^{i\omega t} \frac{\partial A(\tau)}{\partial \tau} - 2g m_1 \omega_a B(\tau) e^{i\omega t} = 0 \quad (15a)
\]

\[
-2i\omega_a e^{i\omega t} \frac{\partial B(\tau)}{\partial \tau} - 2g m_2 \omega_b A(\tau)e^{i\omega t} = 0. \quad (15b)
\]

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