Nonsingular bouncing cosmologies in light of BICEP2

Yi-Fu Cai, a Jerome Quintin, a Emmanuel N. Saridakis b,c and Edward Wilson-Ewing d

a Department of Physics, McGill University, 
3600 rue University, Montréal, QC, H3A 2T8 Canada
b Physics Division, National Technical University of Athens, 
15780 Zografou Campus, Athens, Greece
c Instituto de Física, Pontificia Universidad Católica de Valparaíso, 
Casilla 4950, Valparaíso, Chile
d Department of Physics and Astronomy, Louisiana State University, 
Tower Drive, Baton Rouge, 70803 U.S.A.
E-mail: yifucai@physics.mcgill.ca, jquintin@physics.mcgill.ca, Emmanuel.Saridakis@baylor.edu, wilson-ewing@phys.lsu.edu

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Abstract. We confront various nonsingular bouncing cosmologies with the recently released BICEP2 data and investigate the observational constraints on their parameter space. In particular, within the context of the effective field approach, we analyze the constraints on the matter bounce curvaton scenario with a light scalar field, and the new matter bounce cosmology model in which the universe successively experiences a period of matter contraction and an ekpyrotic phase. Additionally, we consider three nonsingular bouncing cosmologies obtained in the framework of modified gravity theories, namely the Hořava-Lifshitz bounce model, the $f(T)$ bounce model, and loop quantum cosmology.

Keywords: alternatives to inflation, gravitational waves and CMBR polarization

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1 Introduction

Very recently, the BICEP2 collaboration announced the detection of primordial B-mode polarization in the cosmic microwave background (CMB), claiming an indirect observation of gravitational waves. This result, if confirmed by other collaborations and future observations, will be of major significance for cosmology and theoretical physics in general. In particular, the BICEP2 team found a tensor-to-scalar ratio \( r = 0.20^{+0.07}_{-0.05} \),\(^{(1.1)}\) at the \(1\sigma\) confidence level for the \(\Lambda\)CDM scenario. Although there remains the possibility that the observed B-mode polarization could be partially caused by other sources [2–4], it is indeed highly probable that the observed B-mode polarization in the CMB is due at least in part to gravitational waves, remnants of the primordial universe.

The relic gravitational waves generated in the very early universe is a generic prediction in modern cosmology [5, 6]. Inflation is one of several cosmological paradigms that predicts a roughly scale-invariant spectrum of primordial gravitational waves [6–8]. The same prediction was also made by string gas cosmology [9–12] and the matter bounce scenario [13–15]. (Note that the specific predictions of \( r \) and the tilt of the tensor power spectrum can be used in order to differentiate between these cosmologies.) So far, a lot of the theoretical analyses of the observational data have been in the context of inflation (see, for instance, [16–34]).

In this present work, we are interested in exploring the consequences of the BICEP2 results in the framework of bouncing cosmological models. In particular, we desire to study the production of primordial gravitational waves in various bouncing scenarios, in both the settings of effective field theory and modified gravity. First, we show that the tensor-to-scalar ratio parameter obtained in a large class of nonsingular bouncing models is predicted to be quite large compared with the observation. Second, in some explicit models this value can be suppressed due to the nontrivial physics of the bouncing phase, namely, the matter bounce curvaton [35] and the new matter bounce cosmology [36–38]. Additionally, for bounce models where the fluid that dominates the contracting phase has a small sound velocity, primordial gravitational waves can be generated with very low amplitudes [39]. We show that the current
Planck and BICEP2 data constrain the energy scale at which the bounce occurs as well as the slope of the Hubble rate during the bouncing phase in these specific models.

The paper is organized as follows. In section 2, we focus on matter bounce cosmologies from the effective field theory perspective. In particular, we explore the matter bounce curvaton scenario [35] and the new matter bounce cosmology [37]. In section 3, we explore another avenue for obtaining nonsingular bouncing cosmologies, that is modifying gravity. We comment on the status of the matter bounce scenario in Hořava-Lifshitz gravity [40], in \( f(T) \) gravity [41], and in loop quantum cosmology [42]. We conclude with a discussion in section 4.

2 Matter bounce cosmology

As an alternative to inflation, the matter bounce cosmology can also give rise to scale-invariant power spectra for primordial density fluctuations and tensor perturbations [13, 14]. In the context of the original matter bounce cosmology, both the scalar and tensor modes of primordial perturbations grow at the same rate in the contracting phase before the bounce. As a result, this naturally leads to a large amplitude of primordial tensor fluctuations [15], greater than the observational upper bound. However, an important issue that has to be additionally incorporated in these calculations is how the perturbations pass through the bouncing phase, which can drastically decrease the tensor-to-scalar ratio. One example is the matter bounce in loop quantum cosmology, where the tensor-to-scalar ratio is suppressed by quantum gravity effects during the bounce [43]. Also, for some parameter choices in the new matter bounce model, \( r \) can be suppressed by a small sound speed of the matter fluid [37].

In this section, we focus on two particularly interesting matter bounce cosmological models in the effective field theory setting. First, we consider the matter bounce curvaton model, in which the primordial curvature perturbation can be generated from the conversion of entropy fluctuations seeded by a second scalar field [35]. Secondly, we investigate the new matter bounce cosmology, where the primordial curvature perturbations can achieve a gravitational amplification during the bounce [36].

2.1 The matter bounce curvaton model

The matter bounce curvaton model was originally studied to examine whether the bouncing solution of the universe is stable against possible entropy fluctuations [35] and particle creation [44]. In a toy model studied in [35], a massless entropy field \( \chi \) is introduced such that it couples to the bounce field \( \phi \) via the interaction term \( g^2 \phi^2 \chi^2 \). The entropy field evolves as a tracking solution in the matter contracting phase and its field fluctuations are nearly scale-invariant provided the coupling parameter \( g^2 \) is sufficiently small. The amplitude of this mode is comparable to the tensor modes and scales as the absolute value of the Hubble parameter \( H \) before the bounce. Afterwards, the universe enters the bouncing phase and the kinetic term of the entropy field varies rapidly in the vicinity of the bounce. As in the perturbation equation of motion this term effectively contributes a tachyonic-like mass, a controlled amplification of the entropy modes can be achieved in the bouncing phase. Since this term does not appear in the equation of motion for tensor perturbations, the amplitude of primordial gravitational wave is conserved through this phase. After the bounce, the entropy modes will be transferred into curvature perturbations, and this increases the amplitude of
the power-spectrum of the primordial density fluctuations. An important consequence of this mechanism is its suppression of the tensor-to-scalar ratio.

In this subsection, we briefly review the analysis of [35], in light of the BICEP2 results. In the simplest version of the matter bounce curvaton mechanism, there are only three significant model parameters, namely the coupling parameter $g^2$, the slope parameter of the bouncing phase $\Upsilon$ (which is defined by $H \equiv \Upsilon t$ around the bounce), and the maximal value of the Hubble parameter $H_B$. The value of $H_B$ is associated with the mass of the bounce field $m$ through the following relation

$$H_B \simeq \frac{4m}{3\pi}. \quad (2.1)$$

The propagation of primordial gravitational waves depends only on the evolution of the scale factor, and it is possible to calculate the power spectrum for primordial gravitational waves in this scenario

$$P_T = \frac{2H_m^2}{9\pi^2 M_p^2}, \quad (2.2)$$

from which we see that the amplitude is determined solely by the maximal Hubble scale $H_m$. However, the amplitude of the entropy fluctuations is increased during the bounce. Since tensor perturbations do not couple to scalar perturbations, the entropy perturbations do not affect the power spectrum of gravitational waves, whereas the entropy modes are amplified and act as a source for curvature perturbations. This asymmetry leads to a smaller tensor-to-scalar ratio of

$$r \simeq \frac{35}{F^2}, \quad (2.3)$$

where the amplification factor is given by

$$F \simeq e^{\sqrt{y(2+y)} + \frac{1}{\sqrt{2}} \sinh^{-1} \left( \frac{\sqrt{2}}{y} \right)}, \quad \text{with} \quad y \equiv \frac{m^2}{\Upsilon}, \quad (2.4)$$

and $\Upsilon$ is the slope parameter of the bouncing phase as defined before eq. (2.1). Since the exponent in the above equation is approximately linear in $y$ in the regime of interest, we see that the tensor-to-scalar ratio can be greatly suppressed for large values of $y$, that is for large $m$ or small $\Upsilon$. Also, we see that $r$ will reach a maximal value in the massless limit or in the limit where the bounce is instantaneous (i.e., $\Upsilon \rightarrow \infty$), in which case entropy perturbations are not enhanced.

We recall that, according to the latest observation of the CMB (Planck + WP), the amplitude of the power spectrum of primordial curvature perturbations is constrained to be [45]

$$\ln \left(10^{10} A_s \right) = 3.089^{+0.024}_{-0.027} \ (1 \sigma \ CL), \quad (2.5)$$

at the pivot scale $k = 0.002 \text{Mpc}^{-1}$. Moreover, the recently released BICEP2 data indicate that [1]

$$r = 0.20^{+0.07}_{-0.05} \ (1 \sigma \ CL). \quad (2.6)$$

$M_p \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass.
Figure 1. Constraints on the mass parameter $m$ and the slope parameter $\Upsilon$ of the bounce phase from the measurements of Planck and BICEP2 in the matter bounce curvaton scenario. The blue bands show the $1\sigma$ and $2\sigma$ confidence intervals of the tensor-to-scalar ratio and the red bands show the confidence intervals of the amplitude of the power spectrum of curvature perturbations.

By making use of the above data, we performed a numerical estimate and derived the constraint on the model parameters $\Upsilon$ and $m$ shown in figure 1. From the result, we find that the mass scale $m$ and the slope parameter $\Upsilon$ appearing in the matter bounce curvaton model have to be in the following ranges

\[ 2.5 \times 10^{-4} \lesssim m/M_p \lesssim 4.5 \times 10^{-4}, \]
\[ 7.0 \times 10^{-8} \lesssim \Upsilon/M_p^2 \lesssim 3.5 \times 10^{-7}, \]

respectively. The resulting constraints suggest that if the universe has experienced a nonsingular matter bounce curvaton, then the energy scale of the bounce should be of the order of the GUT scale with a smooth and slow bouncing process.

2.2 New matter bounce cosmology

In the new matter bounce scenario, as first developed in [36], the universe starts with a matter-dominated period of contraction and evolves into an ekpyrotic phase before the bounce. This scenario combines the advantages of the matter bounce cosmology, which gives rise to scale-invariant primordial power spectra, and the ekpyrotic universe, which strongly dilutes primordial anisotropies [46]. The model can be implemented by introducing two scalar fields, as analyzed in the context of effective field theory [37].

In the effective model of the two field matter bounce [37], one scalar field is introduced to drive the matter-dominated contracting phase and the other is responsible for the ekpyrotic phase of contraction and the nonsingular bounce. Therefore, similar to the matter bounce curvaton scenario, there also exist curvature perturbations and entropy fluctuations during the matter-dominated contracting phase. However, the main difference between these two models is that, in the present case, the entropy modes have already been converted into curvature perturbation when the universe enters the ekpyrotic phase before the bounce, while in the matter bounce curvaton mechanism, this process occurs after the bounce.
In this model, when the universe evolves into the bouncing phase, the kinetic term of the scalar field that triggers the bounce could vary rapidly which is similar to the analysis of the matter bounce curvaton mechanism. This process can also effectively lead to a tachyonic-like mass for curvature perturbations, and therefore, the corresponding amplitude can be amplified. For the same reason as the matter bounce curvaton mechanism, this effect only works on the scalar sector. Correspondingly, the tensor-to-scalar ratio is suppressed when primordial perturbations pass through the bouncing phase in the new matter bounce cosmology. We would like to point out that this effect is model dependent, namely, it could be secondary if the kinetic term of the background scalar evolves very smoothly compared to the bounce phase [47, 48].

Following [37], one can write the expression of the power spectrum for primordial tensor fluctuations as

$$P_T \simeq \frac{F_\psi^2 H^2_E}{16\pi^2 (2q - 3)^2 M_p^2},$$

(2.9)

with

$$\gamma_\psi \simeq \frac{1}{2(1 - 3q)},$$

$$F_\psi \simeq \exp \left[ 2\sqrt{\Upsilon} t_{B+} + \frac{2}{3} \Upsilon^{3/2} t_{B+}^3 \right],$$

(2.10)

up to leading order. In the above expression, $H_E$ is the value of the Hubble rate at the beginning of the ekpyrotic phase and $q$ is an ekpyrotic parameter which is much less than unity. Note that we have assumed that the bouncing phase is nearly symmetric around the bounce point with the values of the scale factor before and after the bounce being comparable. We denote the time at the end of the bounce phase by $t_{B+}$.

At leading order, the power spectrum for curvature fluctuations is given by

$$P_\zeta \simeq \frac{F_\zeta^2 H^2_E a^2_E}{8\pi^2 M_p^4} \gamma^2 \zeta^2 m^2 |U_\zeta|^2,$$

(2.11)

with $\gamma_\zeta \simeq \gamma_\psi$ and

$$U_\zeta = -(25 + 49q)\frac{H_E}{24m} - \frac{27q}{24},$$

$$F_\zeta \simeq e^{2\sqrt{2+\Upsilon^2} \left(t_{B+}^{i_{B+}} + \frac{2(2+\Upsilon^2+\Upsilon^4)}{3\sqrt{2+\Upsilon^2}} \left(t_{B+}^{i_{B+}}\right)\right)}.$$  

(2.12)

Similarly to $H_E$, we introduced $a_E$, which is the value of the scale factor at the beginning of the ekpyrotic phase (in the pre-bounce branch of the universe). Also, we introduced the mass $m$ of the scalar field responsible for the phase of matter contraction. We also note the presence of the variable $T$, which comes into play in the evolution of the bounce field (see [37] for more details). From eqs. (2.9) and (2.11), the tensor-to-scalar ratio in this model then takes the form of

$$r \equiv \frac{P_T}{P_\zeta} \simeq \frac{F_\psi^2 M_p^2}{2(2q - 3)^2 F_\zeta^2 a_E^2 m^2 |U_\zeta|^2}.$$  

(2.13)
Similarly to the previous subsection, we perform a numerical estimate to investigate the consequences of the observational constraints on the parameter space of the new matter bounce scenario. Note that, although the model under consideration involves a series of parameters, there are three main parameters that are most sensitive to observational constraints, i.e., the slope parameter $\Upsilon$, the Hubble rate at the beginning of the ekpyrotic phase $H_E$, and the dimensionless duration parameter $t_{B+}/T$ of the bouncing phase.

We first look at the correlation between $\Upsilon$ and $t_{B+}/T$, with the numerical result shown in figure 2. One can read that the slope parameter $\Upsilon$ and the dimensionless duration parameter $t_{B+}/T$ are slightly negatively correlated. This implies that one expects either a slow bounce with a long duration or a fast bounce with a short duration. However, it is easy to find that the constraint on the dimensionless duration parameter $t_{B+}/T$ is very tight with a value slightly less than 2. Therefore, it is important to examine whether the model predictions accommodate with observations by fixing the parameter $t_{B+}/T$.

Then, we analyze the correlation between $\Upsilon$ and $H_E$ after setting $t_{B+}/T = 1.86$. The allowed parameter space is depicted by the intersection of the blue and red bands shown in figure 3. From the result, we find that the Hubble scale $H_E$ and the slope parameter $\Upsilon$ introduced in the new matter bounce cosmology are constrained to be in the following ranges

$$1.9 \times 10^{-8} \lesssim H_E/M_p \lesssim 1.9 \times 10^{-6},$$

$$4.9 \times 10^{-7} \lesssim \Upsilon/M_p^2 \lesssim 8.5 \times 10^{-7},$$

respectively. One can easily see that the constraints on the slope parameter $\Upsilon$ in the new matter bounce cosmology and in the matter bounce curvaton scenario are in the same ballpark, i.e., $\Upsilon \sim \mathcal{O}(10^{-7})$. For the new matter bounce cosmology, if we assume that the bounce occurs at the GUT scale, then the duration of the bouncing phase is roughly $\mathcal{O}(10^8)$ Planck times. We also note that the amplitude of the Hubble scale $H_E$ in the new matter
bounce cosmology is much lower than the GUT scale. This allows for a long enough ekpyrotic contracting phase that can suppress the unwanted primordial anisotropies as addressed in [46].

In summary, from the analysis of the matter bounce curvaton and the new matter bounce cosmology scenarios, we can conclude that, in general, a nonsingular bouncing cosmology has to experience the bouncing phase smoothly for it to agree with latest observational data. In other words, the Hubble parameter cannot grow too fast during the bounce phase since the constraints that we find favor a small value of Υ. Depending on the detailed bounce mechanism, the observed amplitude of the spectra of the CMB fluctuations may be determined by the bounce scale or the value of the Hubble parameter at the moment when primordial perturbations were frozen at super-Hubble scales. For the matter bounce curvaton, the mass scale of the bounce is of the order of $O\left(10^{-4}\right) M_p$ which is close to or slightly lower than the GUT scale ($O\left(10^{16}\right)$ GeV). On the other hand, for the new matter bounce cosmology, due to the introduction of the ekpyrotic phase, it is the Hubble parameter $H_E$ at the onset of the ekpyrotic phase that determines the amplitude of the primordial spectrum of the perturbations, and it must be much lower than the GUT scale in order to agree with observations. These interesting results encourage further study of bouncing cosmologies following the effective field approach.

3 Implications for modified-gravity bouncing cosmology

In the previous section, we performed numerical computations to constrain two representative bounce cosmologies that are described by the effective field approach. It is interesting to extend the analysis to bouncing cosmology models where the bounce occurs due to modified-gravity theories. In the following, we shall focus on three specific models. The first one is to obtain the matter bounce solution in the framework of a non-relativistic modification to
Einstein gravity, namely the Hořava-Lifshitz bounce model \cite{40, 49}. The second is to realize the nonsingular bounce by virtue of torsion gravity, i.e., the $f(T)$ bounce model \cite{41}. And the third is a study of the new matter bounce cosmology in the setting of loop quantum cosmology \cite{42}.

### 3.1 Matter bounce in Hořava-Lifshitz gravity

Hořava-Lifshitz gravity is argued to be a potentially UV complete theory for quantizing the graviton, and it has important implications in the physics of the very early universe. In particular, a nonsingular bouncing solution can be achieved in this theory when a non-vanishing spatial curvature term is taken into account \cite{50, 51}. In this case, the higher order spatial derivative terms of the gravity Lagrangian can effectively contribute a stiff fluid with negative energy, which can trigger the nonsingular bounce as well as suppressing unwanted primordial anisotropies. Thus, the bouncing solutions obtained in this picture are marginally stable against the BKL instability.

As was shown in \cite{40}, if the contracting phase is dominated by a pressure-less matter fluid, Hořava-Lifshitz gravity can provide a realization of the matter bounce scenario. Moreover, for primordial perturbations in the infrared limit, the corresponding power spectrum for both the scalar and tensor modes are almost scale-invariant \cite{52}. However, the paradigm derived in this framework belongs to the traditional matter bounce cosmology, and so the amplitude of the tensor power spectrum is of the same order as the scalar power spectrum. In this regard, the corresponding tensor-to-scalar ratio is too large to explain the latest cosmological observations. To address this issue, one needs to enhance the amplitude of the curvature perturbations generated in the contracting phase in the infrared limit, for example by applying the matter bounce curvaton mechanism.

### 3.2 The $f(T)$ matter bounce cosmology

We briefly describe the realization of the matter bounce in the $f(T)$ gravitational modifications of general relativity. The $f(T)$ gravity theory is a generalization of the formalism of the teleparallel equivalent of general relativity \cite{53–55}, in which one uses the curvature-less Weitzenböck connection instead of the torsion-less Levi-Civita one, and thus all of the gravitational information is included in the torsion tensor.

We use the vierbeins $e_A(x^\mu)$ (Greek indices run over the coordinate space-time and capital Latin indices run over the tangent space-time) as the dynamical field, related to the metric through $g_{\mu\nu}(x) = \eta_{AB} e^A_\mu(x) e^B_\nu(x)$, with $\eta_{AB} = \text{diag}(1, -1, -1, -1)$. Thus, the torsion tensor is $T^\lambda_{\mu\nu} = e^A_\lambda \left( \partial_\mu e^A_\nu - \partial_\nu e^A_\mu \right)$, and the torsion scalar is given by

$$T = \frac{1}{4} T^\mu_{\lambda\nu\lambda} T_{\mu\nu\lambda} + \frac{1}{2} T^\mu_{\nu\lambda\mu} T_{\lambda\nu\mu} - T^\nu_{\nu\mu} T^\lambda_{\lambda\mu}. \quad (3.1)$$

Thus, inspired by the $f(R)$ modifications of Einstein-Hilbert action, one can construct the $f(T)$ modified gravity by taking the gravity action to be an arbitrary function of the torsion scalar through $S = \int d^4x \sqrt{-g} f(T + f(T))$. One interesting feature of the $f(T)$ gravity is that the null energy condition can be effectively violated, and thus nonsingular bouncing solutions are possible. In particular, it has been shown that the matter bounce cosmology can be achieved by reconstructing the form of $f(T)$ under specific parameterizations of the scale factor \cite{41}.
In this model the power spectrum of primordial curvature perturbations is also scale-invariant if the contracting branch is matter-dominated. Its form is given by

$$P_\zeta = \frac{H_m^2}{48\pi^2 M_{Pl}^2},$$  \hspace{1cm} (3.2)

where $H_m$ is the maximal value of the Hubble parameter throughout the whole evolution \cite{41} (and thus its definition is the same as that introduced in the matter bounce curvaton scenario).

Now we investigate the evolution of primordial tensor fluctuations in the $f(T)$ matter bounce. The perturbation equation for the tensor modes can be expressed as \cite{54}

$$\left( \ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2} h_{ij} \right) - \frac{12H\dot{H}f_{TT}}{1 + f_T} h_{ij} = 0,$$

where the tensor modes are transverse and traceless. It is interesting to note that the last term appearing in eq. (3.3) plays a role of an effective “mass” for the tensor modes which may affect their amplitudes along the cosmic evolution. However, as it was pointed out in \cite{41}, the effect brought by $f_{TT}$ is negligible since $f(T)$ is approximately a linear function of $T$ in the matter contracting phase. Hence, for primordial tensor fluctuations at large length scales, although the power spectrum is also scale-invariant, the amplitude takes of the form of

$$P_T = \frac{H_m^2}{2\pi^2 M_p^2}.$$  \hspace{1cm} (3.4)

Thus, this result is already ruled out by the present observations unless one introduces some mechanism to magnify the amplitude of scalar-type metric perturbations. This issue can be resolved by the matter bounce curvaton mechanism. Doing so, the tensor-to-scalar ratio in our model can be suppressed by the kinetic amplification factor in the bouncing phase as described in the previous section.

### 3.3 Loop quantum cosmology

The realization of bouncing cosmologies becomes very natural in the frame of loop quantum cosmology (LQC) since the classical big-bang singularity is generically replaced by a quantum bounce when the space-time curvature of the universe is of the order of the Planck scale \cite{56, 57}. Several cosmological models have been studied in the context of LQC, including inflation \cite{58–60}, the matter bounce \cite{43}, and the ekpyrotic scenario \cite{61}. Note however that anisotropies are generically expected to become important near the bounce point — with the exception of the ekpyrotic scenario — and, while the bounce is robust in the presence of anisotropies \cite{62, 63}, the analysis of the cosmological perturbations becomes considerably more complex when anisotropies are important.

There are two realizations of the matter bounce scenario that have been studied so far in LQC: the “pure” matter bounce model, where the dynamics are matter-dominated at all times, including the bounce, and the new matter bounce model (also called the matter-ekpyrotic bounce) where the space-time is matter-dominated at the beginning of the contracting phase, while an ekpyrotic scalar field dominates the dynamics during the end of the contracting era and also the bounce.
In LQC, the dynamics of cosmological perturbations are given by the effective equation of motion for the Mukhanov-Sasaki variables that include quantum gravity effects,

\[ v''_k + \left( c_s^2 k^2 - \frac{z_i''}{z_i} \right) v'_k = 0, \tag{3.5} \]

where \( k \) labels the Fourier modes and the index \( i = \{ S, T \} \) denotes the scalar and tensor modes respectively. The detailed forms of the sound speed parameter \( c_s \) and the coefficient \( z_i \) for holonomy-corrected LQC are \( c_s^2 = 1 - 2\rho/\rho_c \), \( z_T^2 = a^2/c_s^2 \), and \( z_S^2 = a^2(\rho + P)/H^2 \). Here \( \rho_c \sim M_p^4 \) is the critical energy density of LQC where the bounce occurs. These are the equations of motion that were used to determine the observational predictions of the pure matter bounce and the matter-ekpyrotic bounce models in LQC.

In the pure matter bounce model in LQC, tensor perturbations are strongly suppressed during the bounce due to the quantum gravity modification of \( z_T \) and this gives a predicted tensor-to-scalar ratio of \( r \sim O \left( 10^{-3} \right) \), well below the signal detected by BICEP2 \cite{43}. In addition, the amplitude of the spectrum of scalar perturbations is of the order of \( \rho_c/M_p^4 \), and therefore in order to match observations, it is necessary for \( \rho_c \) to be several orders of magnitude below the Planck energy density. This is problematic as heuristic arguments relating LQC and the full theory of loop quantum gravity indicate that \( \rho_c \) is expected to be at most one or two orders of magnitude below \( \rho_P \).

This last problem is avoided in the new matter bounce model for the following reason: when the universe evolves into the ekpyrotic phase, all the perturbation modes at super-Hubble scales freeze and thus the amplitude of the perturbations are entirely determined by the value of the Hubble parameter at the beginning of the ekpyrotic phase, \( H_E \) \cite{42}. Because of this, the observed amplitude of the scalar perturbations determines \( H_E \), not \( \rho_c \). In addition, the ekpyrotic phase also dilutes the anisotropies before the bounce occurs and hence the BKL instability is avoided in this model. In order to determine how the recent results of the BICEP2 collaboration constrain the matter-ekpyrotic bounce in LQC, it is necessary to determine the amplitude of the primordial tensor fluctuations in this scenario.

The dynamics of scalar perturbations in the LQC matter-ekpyrotic bounce model have been studied in detail in \cite{42}, and this analysis is easy to extend to tensor perturbations as their evolution is given by a very similar differential equation as seen in eq. (3.5). Due to the fact that their equations of motion are very similar at times well before the bounce, the amplitude of the spectra of the scalar and tensor modes are of the same order, and it is easy to check that if the ekpyrotic scalar field dominates the dynamics during the bounce, both the scalar and tensor modes evolve trivially through the bounce (note that this is very different to what happens if the matter field dominates the dynamics during the bounce). The result is that, as in other matter-ekpyrotic bounce models without entropy perturbations, the resulting amplitude of the tensor perturbations is significantly larger than for the scalar perturbations and therefore this particular model is ruled out by observations.

However, if there is more than one matter field then entropy perturbations may become important, and they have been neglected in the above analysis. As explained in section 2, entropy perturbations can significantly increase the amplitude of scalar perturbations, while not affecting the dynamics of tensor perturbations in any way, thus decreasing the tensor-to-scalar ratio. Therefore, for the matter-ekpyrotic bounce model to be viable in LQC, it will be necessary to include entropy perturbations in some manner, perhaps as is done in the new matter bounce model presented in section 2.2.

Finally, it is possible (at least for the flat FLRW space-time) to express LQC as a teleparallel theory, which leads to slightly different equations of motion for cosmological
perturbations [64]. In this setting, as there exist solutions with a large range of tensor-to-scalar ratios, \( r \in [0.1243, 1.34375] \) [65], it is possible to obtain a value of \( r \) that is compatible with the results of the BICEP2 collaboration.

4 Conclusion

In this work, we confronted various bouncing cosmologies with the recently released BICEP2 data. In particular, we analyzed two scenarios in the effective field theory framework, namely the matter bounce curvaton scenario and new matter bounce cosmology, and three modified gravity theories, namely Hořava-Lifshitz gravity, the \( f(T) \) theories, and loop quantum cosmology. In all of these models, we showed their capability of generating primordial gravitational waves.

Since matter bounce models typically produce a large amount of primordial tensor fluctuations, specific mechanisms for their suppression are needed. In the matter bounce curvaton scenario, introducing an extra scalar coupled to the bouncing field induces a controllable amplification of the entropy modes during the bouncing phase, and since these modes will be transferred into curvature perturbations the resulting tensor-to-scalar ratio is suppressed to a value in agreement with the observations of the BICEP2 collaboration.

Another possibility, called the new matter bounce cosmology, is to have two scalar fields, one driving the matter contracting phase and the other driving the ekpyrotic contraction and the nonsingular bounce. Thus, the entropy modes are converted into curvature perturbations when the universe enters the ekpyrotic phase before the bounce, and the resulting tensor-to-scalar ratio is again suppressed to observed values.

Furthermore, in both of these models we used the BICEP2 and the Planck results in order to constrain the free parameters in these models, namely the energy scale of the bounce, the slope of the Hubble rate during the bouncing phase, or the Hubble rate at the beginning of the ekpyrotic-dominated phase for the new matter bounce cosmology.

Finally, we considered bouncing cosmologies in the framework of modified gravity. In particular, in both the Hořava-Lifshitz bounce model and the \( f(T) \) gravity bounce, we have argued that the presence of a curvaton field may suppress the tensor-to-scalar ratio to its observed values. We leave the detailed analysis of this topic for a follow-up study.

In loop quantum cosmology, two realizations of the matter bounce have been studied. In the simplest matter bounce model where there is only one matter field, the amplitude of the tensor perturbations is significantly diminished during the bounce due to quantum gravity effects; this process predicts a very small value of \( r \sim O(10^{-3}) \), well below the value observed by BICEP2. The other model that has been studied is the new matter bounce scenario, which in the absence of entropy perturbations predicts a large amplitude for the tensor perturbations (in this case quantum gravity effects do not modify the spectrum during the bounce). Therefore, for the new matter bounce scenario in LQC to be viable, it is also necessary to include entropy perturbations in order to lower the value of \( r \) to a value in agreement with the results of BICEP2. Also, as can be seen here, the dominant field during the bounce significantly affects how the value of \( r \) changes during the bounce and therefore it seems likely that by carefully choosing this field, it may be possible to obtain a tensor-to-scalar ratio in agreement with observations. We leave this possibility for future work.

In summary, the predictions of the matter bounce cosmologies where entropy perturbations significantly increase the amplitude of scalar perturbations remain consistent with observations, and thus these models are good alternatives to inflation.
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References

[1] BICEP2 collaboration, P.A.R. Ade et al., Detection of B-Mode Polarization at Degree Angular Scales by BICEP2, Phys. Rev. Lett. 112 (2014) 241101 [arXiv:1403.3985] [SPIRE].

[2] J. Lizarraga, J. Urrestilla, D. Daverio, M. Hindmarsh, M. Kunz and A.R. Liddle, Can topological defects mimic the BICEP2 B-mode signal?, Phys. Rev. Lett. 112 (2014) 171301 [arXiv:1403.4924] [SPIRE].

[3] A. Moss and L. Pogosian, Did BICEP2 see vector modes? First B-mode constraints on cosmic defects, Phys. Rev. Lett. 112 (2014) 171302 [arXiv:1403.6105] [SPIRE].

[4] C. Bonvin, R. Durrer and R. Maartens, Can primordial magnetic fields be the origin of the BICEP2 data?, Phys. Rev. Lett. 112 (2014) 191303 [arXiv:1403.6768] [SPIRE].

[5] L.P. Grishchuk, Amplification of gravitational waves in an isotropic universe, Sov. Phys. JETP 40 (1975) 409 [Zh. Eksp. Teor. Fiz. 67 (1974) 825] [SPIRE].

[6] A.A. Starobinsky, Spectrum of relict gravitational radiation and the early state of the universe, 30 Years of the Landau Institute — Selected Papers. Volume 11, World Scientific Publishing Co. (1996) [JETP Lett. 30 (1979) 682] [Pisma Zh. Eksp. Teor. Fiz. 30 (1979) 719] [SPIRE].

[7] V.A. Rubakov, M.V. Sazhin and A.V. Veryaskin, Graviton Creation in the Inflationary Universe and the Grand Unification Scale, Phys. Lett. B 115 (1982) 189 [SPIRE].

[8] A.A. Starobinsky, Cosmic Background Anisotropy Induced by Isotropic Flat-Spectrum Gravitational-Wave Perturbations, Sov. Astron. Lett. 11 (1985) 133 [SPIRE].

[9] R.H. Brandenberger and C. Vafa, Superstrings in the Early Universe, Nucl. Phys. B 316 (1989) 391 [SPIRE].

[10] R.H. Brandenberger, A. Nayeri, S.P. Patil and C. Vafa, Tensor Modes from a Primordial Hagedorn Phase of String Cosmology, Phys. Rev. Lett. 98 (2007) 231302 [hep-th/0604126] [SPIRE].

[11] R.H. Brandenberger, A. Nayeri and S.P. Patil, Closed String Thermodynamics and a Blue Tensor Spectrum, arXiv:1403.4927 [SPIRE].

[12] T. Biswas, T. Koivisto and A. Mazumdar, Atick-Witten Hagedorn Conjecture, near scale-invariant matter and blue-tilted gravity power spectrum, arXiv:1403.7163 [SPIRE].

[13] D. Wands, Duality invariance of cosmological perturbation spectra, Phys. Rev. D 60 (1999) 023507 [gr-qc/9809062] [SPIRE].

[14] F. Finelli and R.H. Brandenberger, On the generation of a scale invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase, Phys. Rev. D 65 (2002) 103522 [hep-th/0112249] [SPIRE].
[15] Y.-F. Cai, T.-t. Qiu, R.H. Brandenberger and X.-m. Zhang, A Nonsingular Cosmology with a Scale-Invariant Spectrum of Cosmological Perturbations from Lee-Wick Theory, Phys. Rev. D 80 (2009) 023511 [arXiv:0810.4677] [nSPIRE].

[16] A. Kehagias and A. Riotto, Remarks about the Tensor Mode Detection by the BICEP2 Collaboration and the Super-Planckian Excursions of the Inflaton Field, Phys. Rev. D 89 (2014) 101301(R) [arXiv:1403.4811] [nSPIRE].

[17] Y.-Z. Ma and Y. Wang, Reconstructing the Local Potential of Inflation with BICEP2 data, arXiv:1403.4585 [nSPIRE].

[18] K. Harigaya and T.T. Yanagida, Discovery of Large Scale Tensor Mode and Chaotic Inflation in Supergravity, Phys. Lett. B 734 (2014) 13 [arXiv:1403.4729] [nSPIRE].

[19] J.-O. Gong, Blue running of the primordial tensor spectrum, JCAP 07 (2014) 022 [arXiv:1404.5163] [nSPIRE].

[20] V. Miranda, W. Hu and P. Adshead, Steps to Reconcile Inflationary Tensor and Scalar Spectra, Phys. Rev. D 89 (2014) 101302 [arXiv:1403.5231] [nSPIRE].

[21] M.P. Hertzberg, Inflation, Symmetry and B-Modes, arXiv:1403.5253 [nSPIRE].

[22] D.H. Lyth, BICEP2, the curvature perturbation and supersymmetry, arXiv:1403.7323 [nSPIRE].

[23] J.-Q. Xia, Y.-F. Cai, H. Li and X.-m. Zhang, Evidence for bouncing evolution before inflation after BICEP2, Phys. Rev. Lett. 112 (2014) 251301 [arXiv:1403.7623] [nSPIRE].

[24] D.K. Hazra, A. Shafieloo, G.F. Smoot and A.A. Starobinsky, Whipped inflation, arXiv:1404.0360 [nSPIRE].

[25] M.W. Hossain, R. Myrzakulov, M. Sami and E.N. Saridakis, A class of quintessential inflation models with parameter space consistent with BICEP2, Phys. Rev. D 89 (2014) 123513 [arXiv:1405.7491] [nSPIRE].

[26] B. Hu, J.-W. Hu, Z.-K. Guo and R.-G. Cai, Reconstruction of the primordial power spectra with Planck and BICEP2, arXiv:1404.3690 [nSPIRE].

[27] W. Zhao, C. Cheng and Q.-G. Huang, Hint of relic gravitational waves in the Planck and WMAP data, arXiv:1403.3919 [nSPIRE].

[28] Y.-F. Cai, Testing quantum gravity effects with latest CMB observations, Phys. Lett. B 735 (2014) 108 [arXiv:1404.6672] [nSPIRE].

[29] Y.-F. Cai and Y. Wang, Sterile neutrinos help reconcile the observational results of primordial gravitational waves from Planck and BICEP2, arXiv:1403.7028 [nSPIRE].

[30] M.W. Hossain, R. Myrzakulov, M. Sami and E.N. Saridakis, Eviding Lyth bound in models of quintessential inflation, arXiv:1405.7491 [nSPIRE].

[31] Y.-F. Cai, R.H. Brandenberger and X.-m. Zhang, The Matter Bounce Curvaton Scenario, JCAP 03 (2011) 003 [arXiv:1101.0822] [nSPIRE].
[36] Y.-F. Cai, D.A. Easson and R.H. Brandenberger, Towards a Nonsingular Bouncing Cosmology, JCAP 08 (2012) 020 [arXiv:1206.2382] [nSPIRE].

[37] Y.-F. Cai, E. McDonough, F. Duplessis and R.H. Brandenberger, Two Field Matter Bounce Cosmology, JCAP 10 (2013) 024 [arXiv:1305.5259] [nSPIRE].

[38] Y.-F. Cai, Exploring Bouncing Cosmologies with Cosmological Surveys, Sci. China Phys. Mech. Astron. 57 (2014) 1414 [arXiv:1405.1369] [nSPIRE].

[39] D. Bessada, N. Pinto-Neto, B.B. Siffert and O.D. Miranda, Stochastic background of relic gravitons in a bouncing quantum cosmological model, JCAP 11 (2012) 054 [arXiv:1207.5863] [nSPIRE].

[40] R.H. Brandenberger, Matter Bounce in Hořava-Lifshitz Cosmology, Phys. Rev. D 80 (2009) 043516 [arXiv:0904.2838] [nSPIRE].

[41] M. Osipov and V. Rubakov, Galileon bounce after ekpyrotic contraction, JCAP 11 (2013) 031 [arXiv:1303.1221] [nSPIRE].

[42] M. Koehn, J.-L. Lehners and B.A. Ovrut, A Cosmological Super-Bounce, Phys. Rev. D 90 (2014) 025005 [arXiv:1310.7577] [nSPIRE].

[43] E. Wilson-Ewing, The Matter Bounce Scenario in Loop Quantum Cosmology, JCAP 03 (2013) 026 [arXiv:1211.6269] [nSPIRE].

[44] Y.-F. Cai, R.H. Brandenberger and P. Peter, Anisotropy in a Nonsingular Bounce, Class. Quant. Grav. 30 (2013) 075019 [arXiv:1301.4703] [nSPIRE].

[45] E. Kiritsis and G. Kofinas, Hořava-Lifshitz Cosmology, Nucl. Phys. B 821 (2009) 467 [arXiv:0904.1334] [nSPIRE].

[46] Y.-F. Cai and X.-m. Zhang, Primordial perturbation with a modified dispersion relation, Phys. Rev. D 80 (2009) 043520 [arXiv:0906.1789] [nSPIRE].

[47] J.B. Dent, S. Dutta and E.N. Saridakis, Are loop quantum cosmos never singular?, Class. Quant. Grav. 26 (2009) 125005 [arXiv:0901.2750] [nSPIRE].
[58] A. Ashtekar and D. Sloan, *Loop quantum cosmology and slow roll inflation*, Phys. Lett. B 694 (2010) 108 [arXiv:0912.4093] [nSPIRE].

[59] L. Linsefors, T. Cailleteau, A. Barrau and J. Grain, *Primordial tensor power spectrum in holonomy corrected Ω loop quantum cosmology*, Phys. Rev. D 87 (2013) 107503 [arXiv:1212.2852] [nSPIRE].

[60] I. Agulló, A. Ashtekar and W. Nelson, *The pre-inflationary dynamics of loop quantum cosmology: Confronting quantum gravity with observations*, Class. Quant. Grav. 30 (2013) 085014 [arXiv:1302.0254] [nSPIRE].

[61] E. Wilson-Ewing, *Ekpyrotic loop quantum cosmology*, JCAP 08 (2013) 015 [arXiv:1306.6582] [nSPIRE].

[62] A. Ashtekar and E. Wilson-Ewing, *Loop quantum cosmology of Bianchi type I models*, Phys. Rev. D 79 (2009) 083535 [arXiv:0903.3397] [nSPIRE].

[63] B. Gupt and P. Singh, *Quantum gravitational Kasner transitions in Bianchi-I spacetime*, Phys. Rev. D 86 (2012) 024034 [arXiv:1205.6763] [nSPIRE].

[64] J. Haro, *Cosmological perturbations in teleparallel Loop Quantum Cosmology*, JCAP 11 (2013) 068 [Erratum ibid. 1405 (2014) E01] [arXiv:1309.0352] [nSPIRE].

[65] J. de Haro and J. Amorós, *Viability of the matter bounce scenario in Loop Quantum Cosmology from BICEP2 last data*, arXiv:1403.6396 [nSPIRE].