Quantum ballistic experiment on antihydrogen fall

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Abstract
We propose an approach to measuring gravitational mass of antihydrogen (H\textbar\textbar) based on interferometry of time distribution of free-fall events of antiatoms. Our method consists of preparing a coherent superposition of quantum states of H\textbar\textbar localized near a material surface in the gravitational field of the Earth, and then observing the time distribution of annihilation events after the free-fall of the initially prepared superposition from a given height to a detector plate. We show that the time distribution of interest is mapped to a precisely predictable velocity distribution of the initial wave packet. This approach is combined with production of a coherent superposition of gravitational states by inducing a resonant transition using an oscillating gradient magnetic field. We show that the relative accuracy of measuring the H\textbar\textbar atom gravitational mass can be achieved with this approach is 10^{-4}, with 10^3 antiatoms settled in lowest gravitational states.

Keywords: Antihydrogen, gravitational mass, quantum states

(Some figures may appear in colour only in the online journal)

1. Introduction

Antihydrogen atom (H\textbar\textbar), being the antimatter counterpart to hydrogen atom, is of particular interest due to the simplicity of its internal structure, stability (in vacuum), and neutrality. These properties open possibilities for precision tests of charge-parity-time-symmetry (CT) and the Equivalence Principle (EP) with cold antiatoms in ultimate quantum regime. Some existing and many forthcoming experiments \cite{1–7} aim to put limits on weak equivalence principle (WEP) violations in antimatter-gravity coupling, in particular by measuring gravitational mass (or free-fall acceleration) of H\textbar\textbar atoms. Such experiments are extremely challenging. On the one hand, this is due to the weakness of gravitational force compared to electromagnetic interactions, which means that careful elimination of any false effects related to electromagnetic forces is required. On the other hand, this is due to the very limited number of cold H\textbar\textbar atoms available experimental set-ups. Here we study a typical case of a ballistic experiment in which an initially prepared H\textbar\textbar state is released at a moment t_0 and falls down to a detector plate installed at a height downstream. The time distribution of annihilation events in the detector plate is related to the initial velocity and position distribution of antiatoms and is peaked around a mean value, corresponding to the classical–time of fall. We show that shaping the vertical velocities in such an initial state in order to reduce corresponding uncertainty improves the accuracy of free-fall time measurement in spite of statistics reduction. Such a decrease of uncertainty in velocity and spatial distributions meets quantum limitations.

In this paper we discuss the possibility of exploiting quantum properties of H\textbar\textbar atoms in the gravitational field in order to measure the gravitational mass of H\textbar\textbar. Such quantum properties manifest themselves in the existence of so-called gravitational states of H\textbar\textbar \cite{8}, which are are long-living quantum states of H\textbar\textbar in the gravitational field of the Earth above a material surface. In spite the naive expectation that any contact with material surface would result in prompt annihilation, such states have lifetimes of the order of a fraction of a second due to the phenomenon of quantum
(over-barrier) reflection of ultrashort antiatoms from steep antiatom-surface potential.

Shaping the vertical velocities can be performed by transmitting antiatoms through a slit between a bottom mirror and an upper absorber. We show that in the ultimate case of a slit size of a few dozen micrometers only a few quantum gravitational states can pass through. The time distribution of free-fall events in this case maps the momentum distribution in those gravitational states. The interference between different gravitational states, defined by the gravitational energy level difference, can be visualized via the time distribution of free-fall events, which enable us to evaluate the gravitational mass of \( \mathcal{H} \) with high accuracy using potentially very precise interferometric methods.

### 2. Galileo-type classical ballistic experiment

Classical ballistic experiments are designed to measure the gravitational free-fall acceleration \( g \) of \( \mathcal{H} \). They consist of preparing an initial state (in the case of a GBAR experiment [1] this state of \( \mathcal{H}^+ \) is settled in the ion trap), releasing \( \mathcal{H} \) atom from the trap at a moment \( t_0 \) in GBAR experiment this moment corresponds to a laser photo-detachment pulse \( \mathcal{H}^+ + h\omega \rightarrow \mathcal{H} + e^- \), and detecting the annihilation events in the detector plate installed at a height \( H \) below the trap. The measured quantity is the time distribution of annihilation events on the detector plate, which is related to the initial phase-space distribution of \( \mathcal{H} \).

To find this relation we assume the classical probability density of finding \( \mathcal{H} \) in a vicinity of a phase-space point \((z, v)\) (here \( v \) denotes the vertical velocity and \( z \) denotes the vertical position) is given by function \( P(z, v) \), while the relation between the classical–time to reach the plane \( z = 0 \) for initial values of vertical position \( z \) and vertical velocity \( v \) is given by the well-known expression:

\[
t = \sqrt{2z/g} + \sqrt{v^2/g^2} + \sqrt{g},
\]

where \( g \) is a free-fall acceleration.

We assume the probability density \( P(z, v) \) is peaked around \( z_0 = H \) and \( v_0 = 0 \), and the corresponding standard deviations \( \sigma_z \ll H \) and \( \sigma_v \ll \sqrt{2gH} \), so that in the first order:

\[
t - t_f \approx (z - H)/v_f + v/v_f.
\]

Here \( t_f = \sqrt{2H/g} \) is a classical–time of free-fall from the height \( H \) with zero initial velocity and \( v_f = \sqrt{2gH} \) is a classical velocity, gained by a body during free-fall from the height \( H \).

Using the above expressions we arrive at the free-fall event distribution as a function of \( \tau = t - t_f \):

\[
N(\tau) = g \int P(z, \tau g - gz/v_f)dz.
\]

The most typical case corresponds to the independent spatial and velocity Gaussian type distributions:

\[
P(z, v) = \frac{1}{\pi\sigma_z\sigma_v} \exp \left( -\frac{(z-H)^2}{\sigma_z^2} \right) \exp \left( -\frac{v^2}{\sigma_v^2} \right).
\]

In this case the free-fall event distribution has a form:

\[
N(\tau) = \frac{1}{\sqrt{\pi \sigma}} \exp \left( -\tau^2/\sigma^2 \right),
\]

where \( \sigma = \sqrt{\sigma_z^2/(2gH) + \sigma_v^2/g^2} \).

The ratio, which defines the relative accuracy, is given by:

\[
\delta = \frac{\sigma}{t_f} = \frac{\sigma_v}{\sqrt{2gH}} + \frac{\sigma_z}{\sqrt{2gH}}.
\]

As we can see from the above expression, a trivial way of improving accuracy is to increase the time of free-fall by increasing \( H \). However, this is limited by a dramatic increase in the cost of experimental installation with and an increase of its size, as well as due to the difficulty of eliminating false systematic effects during longer falls.

The main contribution to the time of free-fall uncertainty \( \sigma \) usually comes from the spread in initial velocity distribution \( \sigma_v \). In particular, for the design of the GBAR experiment, this value could be estimated as \( \sigma_v \approx 0.5 \text{ m/s} \) (at least at the early stages of the experiment). The corresponding relative accuracy in the evaluation of \( t_f \) is:

\[
\epsilon_t = \frac{\delta}{\sqrt{N_{tot}}} \approx \frac{\sigma_v}{\sqrt{2gH N_{tot}}},
\]

where \( N_{tot} \) is the total number of free-fall events.

The corresponding accuracy in the evaluation of the free-fall acceleration \( g = 2H/t_f^2 \) is:

\[
\epsilon_g = 2\epsilon_t = \frac{2\sigma_v}{\sqrt{2gH N_{tot}}},
\]

Selecting antiatoms with small enough vertical velocities from a certain range may provide a method to decrease the uncertainty \( \sigma_v \) and to improve the desired accuracy of evaluation of the value \( g \).

Such selection could be realized by passing \( \mathcal{H} \) through a slit of size \( h \) between two plates, where the lower acts as a mirror, while the upper plays the role of absorber [25]. This approach is equivalent to that used in experiments with ultra-cold neutrons (UCNs) in order to select certain states of vertical motion of neutrons in the gravitational field [14, 1, 17, 24]. We will discuss the physical principle of operating such a shaping device in a case of antiatoms in the next section. Here we note only that within a classical description, antiatoms released from a trap at the height of the bottom mirror with a vertical velocity \( |v| > 2gh \) hit the absorber and are lost. Thus only antiatoms with a typical spread in the vertical velocity \( \sigma'_v \approx \sqrt{2gh} \) would pass through the shaping device. We assume that \( \sigma'_v \ll \sigma_v \).
A sketch of the shaping device is shown in figure 1. The total number of antiatoms that would be detected in the detector plate after passing the shaping device is reduced to $N \approx N_{\text{tot}} \sigma / \sigma_v$. Then we get the following expression for improved accuracy of measuring the value of $g$:

$$\epsilon_g = \frac{\sqrt{2gH \sigma_v}}{\sqrt{2gHN_{\text{tot}}}}$$

(9)

The ratio $\epsilon'_g / \epsilon_g$ is thus:

$$\frac{\epsilon'_g}{\epsilon_g} = \frac{\sqrt{2gH}}{\sqrt{\sigma_v}}$$

(10)

We can see that the shaping of vertical velocities results in improvement of precision in spite of the reduction of statistics.

We note that the above semi-quantitative arguments are based on a classical description of $H$ dynamics. Improvement of the velocity and position uncertainty of the initial state by decreasing the slit size $h$ meets the quantum restrictions for sufficiently small values of $h$. When the slit size $h$ becomes smaller than $h_0 < 50 \mu$m account of quantum properties of antiatom motion in the gravitational field is essential. We discuss in the following sections the method needed to utilize the quantum properties for precision measurements of the gravitational mass of $H$.

3. $H$ states near material surface and vertical velocity shaping

In this section we discuss in more details the physics behind the $H$ interaction with the shaping device.

The bottom plate, being a well-polished metallic surface, plays the role of mirror for $H$ atoms with vertical velocities of a few cm/s. As shown in [8, 20, 22, 23] the interaction of slow antiatoms with a material surface results in reflection with a probability close to unity. This is explained by the effect of quantum (over-barrier) reflection of $H$ from the van der Waals/Casimir-Polder (further vdW/CP) potential between an (anti)atom and a material surface.

The $H$ reflection coefficient is given in figure 2 as a function of incident energy.

![Figure 1. Sketch of experimental device for shaping velocity distribution of antiatoms. Antiatom trap is shown as a red circle in the center of the shaping disks. The bottom disk is a mirror and, upper disk is an absorber. The disk radius is $R$, the trap radius $r \ll R$, the size of the slit between disks is $h$, and the height of a bottom mirror above detector plate is $H$. Classical trajectories of antiatoms are shown by the curved lines with arrows.](image)

![Figure 2. Coefficient of reflection of the ground state $H$ atom from different surfaces, including silica slabs of different width, as a function of the incident energy in units of the height of free-fall.](image)

In the limit of small incident velocities, the reflection coefficient is a linear function of $v$:

$$R = 1 - b_r v$$

(11)

The factor $b_r$ is related to the scattering length $a_{CP}$, which characterizes the antiatom-surface interaction by

$$b_r = 4m \left| \text{Im} a_{CP} \right| / \hbar.$$  

(12)

For an ideal reflecting surface it is equal to:

$$b_r = 0.911 \text{ s/m}.$$  

The above law is valid when $1 - R \ll 1$. We can verify that it is well justified for velocities below 0.1 m/s, which corresponds to the velocity gained during the free-fall from height $H = 0.5$ mm. For such velocities the reflection coefficient is around 90%.

Thus a polished surface can play the role of a mirror for (anti)atoms with small enough vertical velocities. This means, in particular, that the motion of $H$ in the gravitational field above such a reflecting surface is quantized (just like the case of UCNs [13–15]). Antiatoms are localized near a material surface in long-living gravitational states. A detailed study of such states can be found in [8].
For convenience we summarize here the main results concerning the gravitational states of H above a material surface. In the following we distinguish between the gravitational mass, which we refer to as M and the inertial mass, hereafter denoted by m. Confinement of H atoms above a material surface in the gravitational field of the Earth is achieved due to the quantum reflection from the vdW/CP atom-surface potential. Such a potential has significant atom-wall distance asymptotic behavior $-Cz^4$, where z is the atom-wall distance.

The characteristic length and energy scales of quantum states in the gravitational potential are given in the following expressions:

$$l_g = \sqrt{\frac{\hbar^2}{2mG}} = 5.871 \mu\text{m},$$

$$\varepsilon_g = \sqrt{\frac{\hbar^2 M^2 g^2}{2m}} = 2.211 \cdot 10^{-14} \text{ a.u.}.$$

The corresponding characteristic gravitational time scale is:

$$\tau_g = \frac{\hbar}{\varepsilon_g} = 0.001 \text{ s}.$$ (15)

The typical spatial scale of the vdW/CP potential is given by:

$$l_{CP} = \sqrt{2mC_4}/\hbar = 0.003 \mu\text{m}.$$ (16)

The hierarchy of the vdW/CP interaction scale and the gravitational scale ensures the energy levels and corresponding wave functions of gravitational states can be found using the ratio of the scattering length on vdW/CP potential and the gravitational length scale $a_{CP}/l_g \approx -i0.005$ as a perturbation parameter.

The energies and eigen functions of the gravitational states are given by the following equation:

$$E_n = \varepsilon_g (\lambda_n^0 + a_{CP}/l_g),$$

$$\Phi_n(z) = CAi(z/l_g - \lambda_n^0 - a_{CP}/l_g),$$

$$Ai(-\lambda_n^0) = 0.$$ (19)
In the following we study the quantum regime of a ballistic experiment, when a shaping device selects one or a few of the lowest gravitational states.

A scheme of such an experiment is explained below. Time zero is the moment of release of $\bar{H}$ atoms from the trap. In the GBAR experiment this moment is defined by a short photo-detachment laser pulse. $\bar{H}$ passes through a slit of a shaping device, formed by a mirror and an absorber, and then falls down from the edge of the mirror to the detection plate, positioned at a height $H_p = 30 \text{ cm}$ below the mirror, as shown in figure 4. The detection plate is designed to serve as position-sensitive detector. This is important for simultaneously measuring the horizontal velocity of $\bar{H}$ ($V_o = L_d/T$, where $L_d$ is the annihilation event radial distance, and $T$ is the time elapsed from the moment of release up to annihilation). With a corresponding correction for the time spent by the atom inside the shaping device $t = L/V = LT/L_d$, we can find the time distribution of annihilation events.

As we will show the distribution of interest is mapped onto the momentum distribution $F_0(p)$ of a wave packet taken at the moment $t = LT/L_d$. We assume the wave packet is initially centered around the origin $z = 0$ and calculate the flux through a detector plate in $z = -H_p$:

$$\Psi(z, t) = \frac{1}{\sqrt{2 \pi \hbar}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(p, p', t) F_0(p') \exp(ipz/\hbar) dp dp', \quad (24)$$

$$G(p, p', t) = \exp\left(-\frac{it}{2m\hbar} \left(p^2 + Mgpt + M^2 g^2 t^2 / 3\right)\right) \times \delta(p + Mg - p'). \quad (25)$$

The combination of both expressions gives the following:

$$\Psi(z, t) = \frac{1}{\sqrt{2 \pi \hbar}} \exp\left(-\frac{it}{2m\hbar} \left(p^2 + Mgpt + M^2 g^2 t^2 / 3\right)\right) \times F_0(p + Mg) \exp(ipz/\hbar) dp, \quad (26)$$

With the use of the stationary phase method, which is justified for large free-fall times $t \gg \tau_g$, expression (24) can be estimated as follows:

$$\Psi(z, t) \approx \frac{1}{\sqrt{2 \pi \hbar}} \exp\left(-\frac{ig^2 M^2 t^3}{24m\hbar} \right) \times F_0(p_0 + Mg) dp, \quad (27)$$

where $p_0 = mz/t - Mg/2$ is a stationary phase point.

For $t$ sufficiently large the integral in the right-hand side of the above expressions converges for $p - p_0 \ll \sqrt{2m/t}$. If this convergence momentum turns out to be much smaller than the characteristic width $\delta k$ of the momentum distribution $F_0(k)$, so that $\sqrt{2m/t} \ll \delta k$, the function $F_0(k)$ can be taken out of the integral. In this case we get:

$$\Psi(z, t) \approx \frac{\bar{m}}{\sqrt{it}} \exp\left(-\frac{ig^2 M^2 t^3}{24m\hbar} \right) \times F_0(p_0 + Mg), \quad (28)$$

The above expression shows that the momentum distribution $F_0(k)$ determines both the time and vertical position-distribution of the annihilation events, which are taken for sufficiently large free-fall times. This distribution is given by flux through a detector plane:

$$P = \frac{\hbar}{2} \left| \frac{d\Psi^*}{dz} - \Psi^* d\Psi}{dz} \right|_{z=-H_p} \quad (29)$$

$$P = \left(\frac{Mg}{2} + \frac{mH_p}{t^2}\right) \times \left| F_0 \frac{Mg}{2} - \frac{mH_p}{t} \right|^2 \approx Mg |F_0(Mg(t - t_f))^2|, \quad (30)$$

4. Free-fall of a wave-packet
Here $H_p$ is the height of the free fall and, $t_f$ is the classical—time of free-fall from height $H_p$.

The characteristic velocity dispersion in a ground grav-itational state is $\sigma_v = \sqrt{2\sigma_g/m} \approx 1.6 \text{ cm s}^{-1}$. The corresponding distribution is shown in figure 5, and the annihilation events time distribution is shown in figure 6. The free-fall time $t_f = \sqrt{2mH_p/(Mg)}$ from a height of 30 cm, extracted from a central maximum position in the annihilation time distribution, is $t_f = 247.4$ ms. The dispersion is $\delta = 1.9$ ms, which gives an accuracy for the ratio $M/m$ of the order of $10^{-4}$, provided that statistics is 1000 events.

The velocity dispersion of a ground gravitational state is more than the 30 times less than initial velocity dispersion of $\text{H}$ atoms released from GBAR trap. The corresponding increase of the accuracy of the $g$ measurement, with account of loss of statistics, is one order of magnitude compared to experiments without velocity shaping.

However, the use of quantum gravitational states creates wider possibilities by carefully studying the time of arrival distribution in the case of superposition of a few gravitational states used as the initial wave packet:

$$\Psi(z, t_0) = A g_1(z) + B g_6(z).$$  \hspace{1cm} (31)

Here $t_0$ is the moment the free-fall starts, $A, B$ are the amplitudes of gravitational states, and $g_1(z)$ are the gravitational state wave functions.

The distribution of arrival time events for a superposition of two gravitational states (ground state $g_1$ and sixth state $g_6$) is illustrated in figures 7 and 8 with different sets of amplitudes of gravitational states in both cases. As we can see the rich interference of the initial velocity distribution is mapped by the arrival time distribution, and it strongly depends on the relative amplitudes of the gravitational states.

In order to get useful information from this interference we should control the relative amplitudes of the gravitational states in their initial superposition, which can be done by inducing a transition from an initially prepared pure ground state into superposition of states by a resonant gradient magnetic field. The details of inducing resonant transitions with inhomogeneous oscillating magnetic field are discussed in [26, 27]. Here we are interested in using an analogous method to produce superposition of gravitational states with a controlled relative phase.
An external magnetic field applied to the shaping device has the following form:

\[
\vec{B}(z, x, t) = B_0 \vec{e}_z + \beta \cos(\omega t)(\vec{e}_x - x\vec{e}_z). \tag{32}
\]

Here \(B_0\) is a constant vertical guiding field, \(\beta\) is the gradient of magnetic field, and \(z\) and \(x\) are the vertical and horizontal positions.

An inhomogeneous magnetic field couples the spin states and the spatial states of motion.

The interaction between magnetic field and the magnetic moments of the antiproton and positron is described by:

\[
\vec{H}_m = -2\vec{B}(z, x, t) \left( \mu_p \vec{s}_p \times \hat{I}_p + \mu_\rho \vec{s} \times \hat{I}_p \right). \tag{33}
\]

Here \(\mu_p\) is the positron magnetic moment, \(\mu_\rho\) is the antiproton magnetic moment, \(\vec{s}_p\), \(\vec{s}\) is a corresponding particle spin operator, and \(\hat{I}_p\), \(\hat{I}_p\) is an identity operator, acting in the space of corresponding particle coordinates and spin variables.

The magnetic field is chosen to be weak enough such that the following inequalities hold: \(\mu_B B_0 \ll \alpha_{HF}\), where \(\alpha_{HF} = 2.157 \cdot 10^{-7}\) a.u. The following hierarchy

\[
\frac{me^4}{2\hbar^2} \gg \alpha_{HF} \gg \mu_B B_0 \gg \varepsilon_\nu \tag{34}
\]

justifies the adiabatic approximation, which results in an equation system for the amplitude \(C_\nu(t)\) of a gravitational state \(g_\nu(z)\):

\[
i\hbar \frac{dC_\nu(t)}{dt} = \sum_k C_k(t) V(t)_{\nu,k} \exp(-i\omega_{\nu,k} t). \tag{35}
\]

The transition frequency \(\omega_{\nu,k} = (\varepsilon_k - \varepsilon_\nu)/\hbar\) is defined by the gravitational energy level spacing.

The coupling potential \(V(t)_{\nu,k}\) is given by the energy of an atom in a fixed hyperfine state. Such an energy \(E(z, t)\) is a function of slowly varying parameter \(z\) and \(t\):

\[
V(t)_{\nu,k} = \int_0^\infty g_\nu(z) g_k(z) E(z, t) dz. \tag{36}
\]

In the above expression \(g_\nu(z)\) is the gravitational state wave function [9–12] and, \(E(z, t)\) is the eigenvalue of internal and magnetic interactions. This eigenvalue depends on slowly-varying c.m. coordinate \(z\) and time \(t\):

\[
E_{a,c}(z, t) = E_{1s} - \frac{\alpha_{HF}}{4} - \frac{1}{2} \left(\frac{\alpha_{HF}}{4} + \left| (\mu_B - \mu_\rho) B(z, t) \right|^2 \right), \tag{37}
\]

\[
E_{d,d}(z, t) = E_{1s} + \frac{\alpha_{HF}}{4} + \frac{1}{2} \left(\frac{\alpha_{HF}}{4} + \left| (\mu_B + \mu_\rho) B(z, t) \right|^2 \right). \tag{38}
\]

Subscripts \(a, b, c, d\) denote the hyperfine states. The dominant transitions happen between \(b\) and \(d\) type states, because of their linear dependence on the weak magnetic field.

A well-known approximative solution of (35) in the case of only two resonantly coupled states is given by the Rabi formula:

\[
\Psi(z, t) \sim \left[ 1 + e^{i\Delta \omega t/2} \left( \cos(\Omega t/2) - i\frac{\Delta \omega}{\Omega} \sin(\Omega t/2) \right) \right] \left[ g_{b}(z) - \frac{\hbar \omega}{2} e^{-i\Delta \omega t/2} \sin(\Omega t/2) g_{d}(z) \right] e^{-\Gamma t/2}, \tag{39}
\]

\[
P_{bd} = \frac{1}{2} \left( \frac{V_{bd}}{2} + h^2 (\Delta \omega)^2 \sin^2 \left( \frac{\sqrt{(V_{bd})^2 + h^2 (\Delta \omega)^2}}{2\hbar} \right) \right) \exp(-\Gamma t). \tag{40}
\]

Here \(\Delta \omega\) is detuning from the resonance frequency, \(\Omega = \sqrt{V_{bd}^2/h^2 + \Delta \omega^2}\), and \(P_{bd}\) is the probability of transition between the initial and final state as a function of time \(t\).

Next, we consider the production of a superposition of ground and sixth gravitational states. The corresponding resonant transition frequency is equal to \(\omega = 972.46\) Hz. The value of the field gradient \(\beta\), required to obtain the maximum probability of the \(1 \rightarrow 6\) transition during the time of flight \(t_\beta = \tau = 0.1\) s, is \(\beta = 2.72 \cdot 10^{-3}\) T. The required guiding field value is found to be no less than \(B_0 = 3 \cdot 10^{-3}\) T.

Thus a magnetic field induced transition creates the possibility of preparing the state superposition with controllable amplitudes of gravitational states.

Our experiment consists of releasing \(\vec{H}\) atoms at time \(t_0\), and selecting the ground gravitational state while passing through the slit between the mirror and absorber, positioned at height \(14 < h < 24\) \(\mu\)m above the mirror, preparing a superposition of ground and excited states by applying the oscillating magnetic field to \(\vec{H}\) atoms moving above the surface of the mirror in the gravitational field, and finally detecting the time of arrival events on the detector plate, installed at height \(h_p = 30\) cm below the mirror. The detector plate also allows measuring the position of annihilation events and thus makes it possible to account for the time spent by an antiatom in the shaping device. A corresponding sketch is shown in figure 4.

The interference method of measuring the gravitational mass of \(\vec{H}\) is based on the fact that narrow peaks of velocity distribution in initial wave packets are reproduced in the time-of-flight distribution of annihilation events. The initial wave packet is comprised of coherent superposition of gravitational states, thus the form of velocity distribution can be precisely predicted. The two narrowest peaks, as shown in figure 8, correspond to times \(t_1 = 246.9\) ms and \(t_2 = 247.8\) ms, and the width of the peaks is equal to \(0.5\) ms. Knowledge of the peaks in the time distribution of the free-fall events \(t_m\) may be related to the corresponding maxima in the momentum distribution \(p_m\), or with dimensionless variable \(k_m = p_m l_\xi/\hbar\) via
equation (29):
\[
\frac{\hbar k_m}{I_g} = Mg(t_m - t_0).
\]

(41)

The gravitational mass of antihydrogen could be then extracted using the following expression:
\[
M = \sqrt[3]{\frac{2m \hbar^2}{g^2(t_m - t_0)^3}}.
\]

(42)

The relative accuracy of this method is of the order of \(10^{-4}\) for the value of \(M\), provided that the number of annihilation events is \(10^3\).

The method is especially powerful, since it provides detailed information. We can control the frequency of the applied magnetic field, monitor the momentum distribution of a superposition of states mapped in the time of arrival distribution, and get these results for atoms with different horizontal velocities, and thus with different times spent in the shaping device.

5. Conclusion

We proposed a novel approach to study the properties of quantum motion of \(\bar{H}\) in a gravitational field of the Earth based on the interferometry of an initially prepared superposition of gravitational quantum states. The interference pattern was mapped in the time-of-arrival annihilation signal, obtained when such a superposition was dropped from the edge of the mirror down to the detector plate. The study of such a time distribution allows measuring the mean time of a fall, as well as the details of the momentum distribution of initial states superposition. We showed that forming an initial coherent state is possible by passing the flux of \(\bar{H}\) atoms through a shaping device, which consists of a material horizontal mirror and an absorber installed above it at a height of \(14 < h < 24 \mu m\). In spite of the unavoidable loss of statistics this procedure improves the accuracy of measurements due to a significant decrease of the width of the velocity distribution of initial state. The coherent superposition of two gravitational states is formed by inducing a resonant transition between gravitational states by oscillating the gradient magnetic field. We showed that measuring the positions of narrow interference maxima (minima) in the time distribution of the annihilation signal makes it possible to measure the gravitational mass \(M\) of the \(\bar{H}\) atom with relative accuracy of the order of \(10^{-4}\) with 1000 annihilation events.

In the above treatment we avoided consideration of false effects, related to roughness of a mirror, as well as the presence of impurities on the mirror surface (including charged particles). Such effects would result in quenching and mixing of gravitational states and will be examined in our forthcoming publications.

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