Analytic approach to ALP emission in core-collapse supernovae

Ana Luisa Foguel\textsuperscript{1} \textsuperscript{*} and Eduardo S. Fraga\textsuperscript{2} \textsuperscript{†}

\textsuperscript{1}Instituto de Física, Universidade de São Paulo, 05508-090 São Paulo, SP, Brazil
\textsuperscript{2}Instituto de Física, Universidade Federal do Rio de Janeiro, CEP 21941-909 Rio de Janeiro, RJ, Brazil

We investigate the impact of a presumed core-collapse supernova explosion ALP emission on neutrino luminosities and mean energies employing a relatively simple analytic description. We compute the nuclear Bremsstrahlung and Primakoff axion luminosities as functions of the PNS parameters and discuss how the ALP luminosities compete with the neutrino emission, modifying the total PNS thermal energy rate. Our results are publicly available in the python package ARTiSANS, which can be used to compute the neutrino and axion observables for different choices of parameters.

I. INTRODUCTION

The Standard Model (SM) of particle physics is currently the best description of the fundamental interactions between elementary particles, with an overwhelming number of confirmed experimental predictions \cite{3}. Nevertheless, we know that it cannot be the final theory of Nature, and should be regarded as an effective theory \cite{2}. The reason being the existence of a plethora of different issues that the SM cannot address satisfactorily, e.g., the absence of a dark matter candidate and the evidence for nonzero neutrino masses. Such questions call for the on-going search for Beyond Standard Model (BSM) signatures.

Given the variety of possible scenarios for new physics, axion-like particles (ALPs) have shown to be among the best candidates so far (for a recent review see e.g. \cite{3}). Such particles arise as the pseudo Nambu-Goldstone bosons in any theory with a spontaneously broken global symmetry, which makes them a very general SM extension. Just to mention some of the best motivated ALP examples, we can cite the QCD axion \cite{4–7}, related to the breaking of the Peccei-Quinn symmetry and possible solution of the strong CP problem, the familons \cite{8–11}, related to family symmetry breaking, and the Majoron \cite{12, 13}, related to lepton number symmetry and that can provide a mechanism for neutrino mass generation. In addition, ALP models also present a rich phenomenology, with masses and couplings running over many orders of magnitude. Depending on the model, the ALP candidate can constitute part of the dark matter content in our Universe or even act as a dark portal to a given hidden sector \cite{14, 17}. Such interesting properties put them in the focus of several present and future experimental programs \cite{18, 21}.

Besides collider searches, another way to probe ALP models is via core-collapse supernovae (SNe) explosions \cite{22, 26}. These events are among the most extreme and powerful astrophysical phenomena in the Universe, making them perfect laboratories to test new physics. During the explosion, one would expect a significant emission of light ALPs as a result of the dominant axion-nucleon Bremsstrahlung interaction \cite{27} and the Primakoff process \cite{28}. Therefore, the ALP emission would compete with neutrinos in the dissipation of the SN binding energy, such that, by measuring the neutrino luminosities, one could impose constraints on the ALP model couplings. In this context, supernova SN1987A neutrino detection was of great importance for axion physics, and the SN neutrino burst measurement was able to put strong constraints on both the nucleon- and photon-axion couplings \cite{22, 29, 32}. Considering that, in the last few decades, we had several detector upgrades and new neutrino experiments (see Ref. \cite{33} for a nice review and Refs. \cite{34–37} for novel neutrino detection techniques), we expect that a future SN neutrino detection would be crucial not only to shed some light onto the explosion mechanism and SN neutrino physics \cite{38, 39} but also for the search of new physics.

In this paper we analyze the impact of a presumed supernova ALP emission on the neutrino luminosities and mean energies. In contrast with the usual methods, which rely on computationally expensive and complex SN numerical simulations \cite{23, 24, 40}, we provide a relatively simple analytic description. On the one hand, the analytic computation has the benefit of being more transparent to interpretation, besides depending on few free parameters. On the other hand, due to its simplicity, it can miss specific features that only a complete simulation can provide. Nevertheless, the analytic evaluation can still reproduce the main properties of axion and neutrino emissivities, as we shall show.

Regarding the axion BSM extension, we consider a generic ALP model where the axion-like particles couple with nucleons and photons via the following Lagrangian:

\[ \mathcal{L}_{\text{ALP}} = \frac{g_{\alpha NN}}{2m_N} (\bar{\psi}_N \gamma_\mu \gamma_5 \psi_N) \partial^\mu a - \frac{1}{4} a_{\alpha \gamma \gamma} F_{\mu \nu} \mathcal{F}^{\mu \nu} a, \]  

where \( g_{\alpha NN} \) is the axion-nucleon coupling, \( m_N \) is the nucleon mass, \( \psi_N \) represent the nucleon Dirac field, \( a \) represents the pseudo-scalar ALP field, \( a_{\alpha \gamma \gamma} \) is a (energy)\(^{-1}\) dimension coupling that parametrizes the axion-photon interaction and \( F_{\mu \nu} \) is the electromagnetic field strength tensor. The first term in this Lagrangian describes the
nuclear Bremsstrahlung axion interaction, which is the dominant emission process in the SN explosion. The second term gives rise to the Primakoff process, i.e., the axion-photon conversion that occurs in the presence of external electromagnetic fields.

For the case of the SN neutrino light curves, we follow the analytic calculations described in Ref. [41], where the authors considered a Lane-Emden solution with polytropic index $n = 1$ for the description of the protoneutron star (PNS) equation of state, and also applied the diffusion approximation for the neutrino transport equations. We compute the nuclear Bremsstrahlung and Primakoff axion luminosities as functions of the PNS parameters to evaluate the impact of the axion emission on several variables, such as the neutrino luminosities and mean energies. The ALP luminosities compete with the neutrino emission, modifying the total PNS thermal energy rate.

Finally, we also provide an user-friendly Python package for the analytic calculation of the neutrino and axion luminosities and mean energies in the presence of ALP nuclear Bremsstrahlung and/or Primakoff emission. The ARtiSANS (Analytic Re-evaluation of Supernova Axion and Neutrino Streaming) code is publicly available at https://github.com/anafoguel/ARtiSANS where one can also find a Jupyter Notebook tutorial for usage information.

This paper is organized as follows. In the next section, we explain in detail the analytic calculation of the nuclear Bremsstrahlung and Primakoff axion luminosities as a function of the PNS parameters. These luminosities will compete with the neutrino emission, modifying the total PNS thermal energy rate. In section III we show our results on the impact of the axion emission on several variables, such as the neutrino luminosities and mean energies, for different choices of ALP nuclear and photon couplings. We conclude in section IV with our final remarks and outlook.

II. ANALYTIC SETUP

Following Ref. [41], we can express the one-flavor neutrino luminosity as

$$L_\nu \approx 1.2 \times 10^{50} \left( \frac{M_{\text{PNS}}}{1.4M_\odot} \right)^{4/5} \left( \frac{R_{\text{PNS}}}{10\text{km}} \right)^{-6/5} \left( \frac{g_3}{3} \right)^{-4/5} \times \left( \frac{s}{1k_B\text{baryon}^{-1}} \right)^{4/5} \text{erg} \text{s}^{-1}, \quad (2)$$

where $M_{\text{PNS}}$ and $R_{\text{PNS}}$ are the protoneutron star (PNS) mass and radius, $g$ is a parameter that accounts for the deviation from the solution of the Lane-Emden equation of state with polytropic index $n = 1$, $\beta$ is a boosting factor necessary to include the effects of heavy nuclei that amplify the neutrino cross-section due to coherent scattering, $s$ is the entropy per nucleon, and $k_B$ is the Boltzmann constant. As usual, we normalize our stellar masses using the solar mass $M_\odot$, and use typical values for the mass and radius of the protoneutron star.

The total PNS thermal energy can be written as [41]

$$E_{\text{th}} = 2.5 \times 10^{52} \left( \frac{M_{\text{PNS}}}{1.4M_\odot} \right)^{5/3} \left( \frac{R_{\text{PNS}}}{10\text{km}} \right)^{-2} \times \left( \frac{s}{1k_B\text{baryon}^{-1}} \right)^2 \text{erg}. \quad (3)$$

Given that the neutrinos carry away the PNS thermal energy, we can relate these two quantities by

$$\frac{dE_{\text{th}}}{dt} = -6L_\nu, \quad (4)$$

where the factor of 6 comes from the number of neutrino and anti-neutrino flavors. Now, if we include the ALP emission, the above equation is modified to

$$\frac{dE_{\text{th}}}{dt} = -6L_\nu - L_a, \quad (5)$$

where $L_a$ is the total axion luminosity, which includes the axion emission due to nucleon-nucleon Bremsstrahlung, $L_{a\nu}^{\text{brem}}$, and to the Primakoff process, $L_a^{\text{prim}}$. In what follows, we derive the analytic expressions for these two contributions.

A. Nuclear Bremsstrahlung

The total nuclear Bremsstrahlung energy-loss rate per unit mass can be expressed as [42]

$$\epsilon_a^{\text{brem}} = \alpha_{aNN} 1.69 \times 10^{35} \left( \frac{\rho}{10^{15}\text{g cm}^{-3}} \right) \times \left( \frac{T}{\text{MeV}} \right)^{7/2} \text{erg} \text{g}^{-1} \text{s}^{-1}, \quad (6)$$

where $\alpha_{aNN} = g_{aNN}^2/4\pi$ is the axion-nucleon fine-structure constant, $T$ is the PNS temperature and $\rho$ is the density, which follows the Lane-Emden equation of state with $n = 1$ [41]

$$\rho = \frac{M_{\text{PNS}}}{4\pi^2} \left( \frac{\pi}{R_{\text{PNS}}} \right)^3 \frac{\sin \xi}{\xi} \left( \frac{r}{R_{\text{PNS}}} \right)^{-\pi}, \quad (7)$$

where we defined the dimensionless radius

$$\xi \equiv \frac{r}{R_{\text{PNS}}} \pi, \quad (8)$$

with radial coordinate $r$.

Using that we can also write the temperature as a function of the PNS mass, total radius, entropy and dimensionless radial parameter $\xi$ as [41]

$$T = 30 \left( \frac{M_{\text{PNS}}}{1.4M_\odot} \right)^{2/3} \left( \frac{R_{\text{PNS}}}{10\text{km}} \right)^{-2} \left( \frac{s}{1k_B\text{baryon}^{-1}} \right)^2 \left( \frac{\sin \xi}{\xi} \right)^{2/3} \text{MeV}, \quad (9)$$

II. ANALYTIC SETUP

Following Ref. [41], we can express the one-flavor neutrino luminosity as

$$L_\nu \approx 1.2 \times 10^{50} \left( \frac{M_{\text{PNS}}}{1.4M_\odot} \right)^{4/5} \left( \frac{R_{\text{PNS}}}{10\text{km}} \right)^{-6/5} \left( \frac{g_3}{3} \right)^{-4/5} \times \left( \frac{s}{1k_B\text{baryon}^{-1}} \right)^{4/5} \text{erg} \text{s}^{-1}, \quad (2)$$

where $M_{\text{PNS}}$ and $R_{\text{PNS}}$ are the protoneutron star (PNS) mass and radius, $g$ is a parameter that accounts for the deviation from the solution of the Lane-Emden equation of state with polytropic index $n = 1$, $\beta$ is a boosting factor necessary to include the effects of heavy nuclei that amplify the neutrino cross-section due to coherent scattering, $s$ is the entropy per nucleon, and $k_B$ is the Boltzmann constant. As usual, we normalize our stellar masses using the solar mass $M_\odot$, and use typical values for the mass and radius of the protoneutron star.

The total PNS thermal energy can be written as [41]

$$E_{\text{th}} = 2.5 \times 10^{52} \left( \frac{M_{\text{PNS}}}{1.4M_\odot} \right)^{5/3} \left( \frac{R_{\text{PNS}}}{10\text{km}} \right)^{-2} \times \left( \frac{s}{1k_B\text{baryon}^{-1}} \right)^2 \text{erg}. \quad (3)$$

Given that the neutrinos carry away the PNS thermal energy, we can relate these two quantities by

$$\frac{dE_{\text{th}}}{dt} = -6L_\nu, \quad (4)$$

where the factor of 6 comes from the number of neutrino and anti-neutrino flavors. Now, if we include the ALP emission, the above equation is modified to

$$\frac{dE_{\text{th}}}{dt} = -6L_\nu - L_a, \quad (5)$$

where $L_a$ is the total axion luminosity, which includes the axion emission due to nucleon-nucleon Bremsstrahlung, $L_{a\nu}^{\text{brem}}$, and to the Primakoff process, $L_a^{\text{prim}}$. In what follows, we derive the analytic expressions for these two contributions.

A. Nuclear Bremsstrahlung

The total nuclear Bremsstrahlung energy-loss rate per unit mass can be expressed as [42]

$$\epsilon_a^{\text{brem}} = \alpha_{aNN} 1.69 \times 10^{35} \left( \frac{\rho}{10^{15}\text{g cm}^{-3}} \right) \times \left( \frac{T}{\text{MeV}} \right)^{7/2} \text{erg} \text{g}^{-1} \text{s}^{-1}, \quad (6)$$

where $\alpha_{aNN} = g_{aNN}^2/4\pi$ is the axion-nucleon fine-structure constant, $T$ is the PNS temperature and $\rho$ is the density, which follows the Lane-Emden equation of state with $n = 1$ [41]

$$\rho = \frac{M_{\text{PNS}}}{4\pi^2} \left( \frac{\pi}{R_{\text{PNS}}} \right)^3 \frac{\sin \xi}{\xi} \left( \frac{r}{R_{\text{PNS}}} \right)^{-\pi}, \quad (7)$$

where we defined the dimensionless radius

$$\xi \equiv \frac{r}{R_{\text{PNS}}} \pi, \quad (8)$$

with radial coordinate $r$.

Using that we can also write the temperature as a function of the PNS mass, total radius, entropy and dimensionless radial parameter $\xi$ as [41]

$$T = 30 \left( \frac{M_{\text{PNS}}}{1.4M_\odot} \right)^{2/3} \left( \frac{R_{\text{PNS}}}{10\text{km}} \right)^{-2} \left( \frac{s}{1k_B\text{baryon}^{-1}} \right)^2 \left( \frac{\sin \xi}{\xi} \right)^{2/3} \text{MeV}, \quad (9)$$
we can obtain the axion luminosity $L^\text{brem}_{a}$ by integrating the energy-loss rate per unit volume $\epsilon^\text{brem}_a \times \rho$ through the PNS radial coordinate. Assuming spherical symmetry:

$$L^\text{brem}_a = 4\pi \int_0^{R_{\text{PNS}}} \epsilon^\text{brem}_a(r) \rho(r) r^2 dr,$$

and the integration gives

$$L^\text{brem}_a = \alpha_{aN} N \times 2.93 \times 10^{36} \left( \frac{M_{\text{PNS}}}{1.4M_\odot} \right)^{13/3} \left( \frac{R_{\text{PNS}}}{10^{10}\text{km}} \right)^{-10} \times \left( \frac{s}{1k_B\text{baryon}^{-1}} \right) \text{ erg s}^{-1}.$$ (10)

### B. Primakoff process

For the case of the Primakoff process, the energy loss rate per unit volume is given by [12]

$$Q_P = \frac{g_{a\gamma\gamma}^2 T^7}{4\pi} F(\kappa^2),$$

where $T$ is the temperature, $\kappa = k_S/2T$ with $k_S$ being the screening scale, and

$$F(\kappa^2) = \frac{k^2}{2\pi^2} \int_0^\infty dx \left[ (x^2 + \kappa^2) \ln \left( 1 + \frac{x^2}{\kappa^2} \right) - x^2 \right] \frac{x}{e^x - 1},$$

where $x$ is the ratio between the photon frequency and the temperature, i.e., $x \equiv \omega/T$.

For a non-degenerate medium, we can obtain the screening scale via the Debye-Hückle formula

$$k_S^2 = \frac{4\pi \Omega_{\text{EM}}}{T} n_N (Y_e + \sum_j Z_j^2 Y_j),$$

where $n_N = \rho/m_N$ is the nucleon number density, $Y_e$ is the electron fraction per baryon, and the sum goes over the different nuclear species $j$, with fraction $Y_j$, present in the medium. Since the electrons are highly degenerate, their phase-space is Pauli blocked, which implies that their contribution to the screening scale is negligible. Considering the non-degenerate regime, we can approximate the screening length to [31]

$$k_S^2 \approx \frac{4\pi \Omega_{\text{EM}}}{T} n_N Y_p = \frac{4\pi \Omega_{\text{EM}}}{T} \rho_p,$$ (15)

where $Y_p$ is the proton fraction per baryon. However, since protons are partially degenerate we expect that the effective number of targets is reduced, so that we need to consider $n_{p}^{\text{eff}}$. Although this number can vary depending on the PNS radius and time after bounce, following Ref. [22], here we assume that $n_{p}^{\text{eff}} = n_p/2$ for simplicity. Hence, we can write

$$k_S^2 \approx \frac{4\pi \Omega_{\text{EM}}}{T} \rho \frac{Y_p}{m} \frac{Y_p}{2}.$$ (16)

### III. RESULTS

According to equation [5], the inclusion of the nuclear Bremsstrahlung and Primakoff axion emission processes can impact the PNS energy dissipation. Hence we can numerically solve this differential equation to obtain the entropy as a function of time $s(t)$ provided that we fix an initial condition, the PNS parameters and dark axion couplings (for details on the computation, see appendix [A]). Since coherent scattering is suppressed for earlier times, we will consider an initial and a final phase in the PNS evolution, with different boosting factors $\beta_i$ and $\beta_f$ and total emitted neutrino energies $E_{\nu}\text{tot}$ and $E_{\nu}\text{f}$. For the results and plots presented in this section we always fix the PNS parameters to $M_{\text{PNS}} = 1.5 M_\odot$, $R_{\text{PNS}} = 12 \text{km}$, $g = 0.04$, $\beta_i = 3$, $\beta_f = 40$, $E_{\nu}\text{tot} = 4 \times 10^{52} \text{erg}$ and $E_{\nu}\text{f} = 10^{53} \text{erg}$. We also consider a proton fraction of $Y_p = 0.3$ [24], which corresponds to an intermediate scenario between very neutron-rich matter ($Y_p < 0.05$) and symmetric nuclear matter ($Y_p = 0.5$) [43]. Let us emphasize that all these parameters can be easily changed in the ARTISANS code.

Fig. [I] shows the time evolution of the one-flavor neutrino luminosity (solid) and mean energy (dashed) for two different benchmark values of axion-nucleon coupling $g_{aN} = 5 \times 10^{-11}$ (blue), $g_{aN} = 5 \times 10^{-10}$ (green), and

![Graph showing neutrino luminosity and mean energy evolution](image-url)
Concerning the luminosity released by the ALPs, we show in Fig. 4 the time evolution of $L_a$ for distinct benchmark values of axion-nucleon and axion-photon couplings. From the figure we can see that the nuclear Bremsstrahlung (Primakoff) axion emission is more relevant for earlier (later) times. Finally, the upper (lower) plot of Fig. 5 shows the PNS binding energy that was carried away by neutrinos (ALPs) as a function of time. We also display in Table I the total emitted energy by neutrinos $E_\nu$ and axions $E_a$, integrated up to 20s post-bounce, for the different choices of dark couplings, labeled by the model names in the first column.

\begin{table}
\centering
\begin{tabular}{|l|l|l|l|l|}
\hline
\textbf{Model} & \textbf{g_{aNN}} & \textbf{g_{a\gamma\gamma} [MeV^{-1}]} & \textbf{E_\nu [10^{52} erg]} & \textbf{E_a [10^{52} erg]} \\
\hline
SM & 0 & 0 & 10.8 & - \\
MP1 & 0 & 5 \times 10^{-13} & 10.7 & 0.27 \\
MN1 & 10^{-11} & 0 & 10.5 & 0.64 \\
MNP1 & 10^{-11} & 5 \times 10^{-13} & 10.3 & 0.9 \\
MN2 & 5 \times 10^{-11} & 0 & 6.47 & 6.48 \\
\hline
\end{tabular}
\caption{Comparison of the total energy emitted by neutrinos $E_\nu$ and axions $E_a$ for different axion parameters choices. The energies were computed by integrating the respective luminosities up to 20s post-bounce.}
\end{table}
FIG. 5. The upper (lower) plot shows the time evolution of
the SN total energy carried away by neutrinos (axions). The
colors indicate different choices of dark couplings as indicated
by the labels. The neutrino emission takes into account all
six flavors of neutrinos.

IV. OUTLOOK

In the present work we introduced an analytic method
to estimate the impact of the inclusion of axion-like-
particle emission in supernovae explosions on neutrino
observables, such as the luminosity $L_\nu$ and average en-
ergy $\langle E_\nu \rangle$. In particular, we considered the ALP emis-
sion via axion-nucleon Bremsstrahlung and axion-photon
Primakoff interactions, and derived semi-analytic expres-
sions for the respective axion luminosities $L_a$, given by
equations (11) and (17), following the same arguments
discussed in Ref. [41]. With such expressions we could
solve the first-order differential equation (5), which re-
lates the total SN binding energy loss rate with the
neutrino and axion luminosities, numerically to obtain the
PNS entropy as a function of time $s(t)$. Afterwards, we
used $s(t)$ to compute the final expressions of $L_\nu$, $L_a$, and
$\langle E_\nu \rangle$.

In order to solve the differential equation we need to
fix six PNS free parameters (mass $M_{\text{PNS}}$, radius $R_{\text{PNS}}$,
density correction factor $g$, opacity boosting factor $\beta$,
total energy emitted by neutrinos $E_{\nu \text{tot}}$ and proton frac-
tion $Y_\text{p}$) plus two axionic couplings (axion-nucleon cou-
ing $g_{a\text{NN}}$ and axion-photon coupling $g_{a\gamma\gamma}$). With such
choices we can analyze the impact on the neutrino lumini-
sity and mean energy and study the deviations from the
SM case. We found that, for the current allowed
ALP parameter space, the axion nuclear Bremsstrahlung
emission is dominant in comparison with the Primakoff
interaction, which, in turn, is more relevant for later
times in the PNS evolution. We showed that for axion-
nucleon(-photon) couplings larger than $g_{a\text{NN}} \sim 10^{-11}$
($g_{a\gamma\gamma} \sim 10^{-13}$ MeV$^{-1}$) the effect on the neutrino ob-
servables is larger than 5%. We also analyzed the neu-
trino and ALP luminosity and energy profiles for different
choices and combinations of axionic couplings.

We provide all our results and computations in
the user-friendly python package ARTiSANS, which is
publicly available on GitHub at https://github.com/
anafoguel/ARTiSANS together with a Jupyter tutorial
Notebook. The code allows the user to compute the neu-
trino luminosity and average energy as well as the ALP
nuclear Bremsstrahlung and/or Primakoff emission $L_a$
for any choice of PNS and ALP input parameters. It can
also compute the time evolution and the total binding
energy carried by neutrinos and ALPs.

Although the analytic calculations described in this
work possess computational limitations when compared
to complete SN simulations, they profit from other ben-
efits, such as the need of few free parameters, which
makes them simpler and faster. We have shown that
such computations can be effectively employed to exam-
ine the main features and properties of the ALP and neu-
trino emissivities and mean energies for different model
choices.

ACKNOWLEDGMENTS

This work was partially supported by INCT-FNA
(Process No. 464808/2014-5), CAPES (Finance Code
001), CNPq, and FAPERJ. A.L.F was supported by
FAPESP under contract 2022/04263-5.

Appendix A: Details of the analytic computation

In section 11 we obtained the expression for the axion
emission luminosity $L_a = L_a^{\text{bren}} + L_a^{\text{prim}}$ as a function
of the PNS variables, the entropy $s$ and the dark ax-
ionic couplings. For a given choice of PNS parameters
($M_{\text{PNS}}, R_{\text{PNS}}, g\beta, Y_\text{p}$) and dark couplings ($g_{a\text{NN}}, g_{a\gamma\gamma}$),
we can solve the differential equation (5) to obtain the
evolution of the entropy with time

$$\frac{ds}{dt} = \left( \frac{dE_{\text{th}}}{ds} \right)^{-1} (-6L_\nu - L_a). \quad (A1)$$

For instance, when considering only nuclear
Bremsstrahlung, which is dominant for $g_{a\gamma\gamma} \leq$
$10^{-13}$ MeV$^{-1}$, we end up with

$$
\frac{d\hat{s}}{dt} = -\left[ 14.4 \times 10^{-3} (\hat{M}_{\text{PNS}})^{\frac{12}{5}} (\hat{R}_{\text{PNS}})^{\frac{4}{5}} \left( \frac{g\beta}{3} \right)^{\frac{4}{5}} \hat{s} - \frac{4}{5} \hat{s} \right] - \frac{\alpha_{\text{NN}}}{5.9} \times 10^{20} (\hat{M}_{\text{PNS}})^{\frac{8}{5}} (\hat{R}_{\text{PNS}})^{-\frac{8}{5}} s^{-1},
$$

where the hat indicates dimensionless variables, normalized to their respective typical values as considered in the main text.

In order to solve the differential equation above, we need to provide an initial condition for the entropy. For this purpose we use, as an approximation, the analytic expression for the entropy that solves the differential equation without the axionic contributions, i.e. [11],

$$
\hat{s}(t) = 4 (\hat{M}_{\text{PNS}})^{\frac{12}{5}} (\hat{R}_{\text{PNS}})^{-\frac{2}{5}} \left( \frac{g\beta}{3} \right)^{\frac{2}{5}} \left( \frac{t + t_0}{100 s} \right)^{-\frac{2}{5}}, \quad (A2)
$$

where $t_0$ is the time origin, given by

$$
t_0 = 210 (\hat{M}_{\text{PNS}})^{\frac{2}{5}} (\hat{R}_{\text{PNS}})^{\frac{8}{5}} \left( \frac{g\beta}{3} \right)^{\frac{8}{5}} (\hat{E}_{\text{tot}})^{-\frac{1}{5}} s, \quad (A3)
$$

in terms of the total energy emitted by neutrinos $\hat{E}_{\text{tot}} \equiv E_{\text{tot}} / (10^{52} \text{erg})$.

Hence, once we fix the set of PNS parameters $(\hat{M}_{\text{PNS}}, \hat{R}_{\text{PNS}}, g\beta, \hat{E}_{\text{tot}})$, we can use $t_0$ to obtain the initial entropy $\hat{s}(t_0) \equiv \hat{s}_i$ at a given initial time $t_i$. With this information we can numerically solve the differential equation (A1) to obtain the time evolution of the entropy for the chosen set of PNS and dark parameters. Finally, we can use the calculated entropy evolution to compute the neutrino and ALP luminosities, given by equations (2), (11) and (17), and also the average energy of neutrinos [11]

$$
\langle E_{\nu} \rangle \simeq 7 (\hat{M}_{\text{PNS}})^{\frac{4}{5}} (\hat{R}_{\text{PNS}})^{-\frac{5}{4}} \left( \frac{g\beta}{3} \right)^{\frac{5}{4}} (\hat{s})^{\frac{5}{4}} \text{ MeV}. \quad (A4)
$$

One last point important to highlight is that the computation of the differential equation should be divided into two phases. The reason is due to the fact that for early times in the PNS evolution there are no heavy nuclei in the crust [31], which implies that the boosting factor $\beta$ acquires a time-dependence. Here, to simplify, we will follow [11] and consider an initial phase with small boosting ($\beta = 3$) and a later phase where the coherent scattering due to the heavier nuclei is relevant and, hence, the opacity boost increases ($\beta \gg 1$).

Since we divide the evolution in two components, the luminosities and neutrino mean energies will be expressed by

$$
L_{\nu,i} = L_{\nu,i}^{(i)} + L_{\nu,i}^{(f)}, \quad (A5)
$$

$$
\langle E_{\nu} \rangle = \frac{L_{\nu,i}^{(i)}}{L_{\nu,i}^{(i)} + L_{\nu,i}^{(f)}}, \quad \langle E_{\nu} \rangle = \frac{L_{\nu,f}^{(f)}}{L_{\nu,f}^{(i)} + L_{\nu,f}^{(f)}}, \quad (A6)
$$

where the superscripts $(i, f)$ indicate the computation of the luminosity and average energy in the initial and final phases, respectively.

---

[1] P. A. Zyla et al. (Particle Data Group), PTEP 2020, 083C01 (2020)
[2] L. Brivio and M. Trott, Phys. Rept. 793, 1 (2019), arXiv:1706.08945 [hep-ph]
[3] K. Choi, S. H. Im, and C. Sub Shin, Ann. Rev. Nucl. Part. Sci. 71, 225 (2021), arXiv:2012.05029 [hep-ph]
[4] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977)
[5] S. Weinberg, Phys. Rev. Lett. 40, 223 (1978)
[6] F. Wilczek, Phys. Rev. Lett. 40, 279 (1978)
[7] G. Grilli di Cortona, E. Hardy, J. Pardo Vega, and G. Villadoro, JHEP 01, 034 (2016), arXiv:1511.02867 [hep-ph]
[8] C. D. Foggart and H. B. Nielsen, Nucl. Phys. B 147, 277 (1979)
[9] A. Davidson and K. C. Wali, Phys. Rev. Lett. 48, 11 (1982)
[10] F. Wilczek, Phys. Rev. Lett. 49, 1549 (1982)
[11] J. Jaeckel, Phys. Lett. B 732, 1 (2014), arXiv:1311.0880 [hep-ph]
[12] Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. B 98, 265 (1981)
[13] G. B. Gelmini and M. Roncadelli, Phys. Lett. B 99, 411 (1981)
[14] J. Preskill, M. B. Wise, and F. Wilczek, Phys. Lett. B 120, 127 (1983)
[15] M. Dine and W. Fischler, Phys. Lett. B 120, 137 (1983)
[16] P. Arias, D. Cadamuro, M.Goodsell, J. Jaeckel, J. Redondo, and A. Ringwald, JCAP 06, 013 (2012), arXiv:1201.5902 [hep-ph]
[17] A. Ringwald, Phys. Dark Univ. 1, 116 (2012), arXiv:1210.5081 [hep-ph]
[18] P. W. Graham, I. G. Irastorza, S. K. Lamoreaux, A. Lindner, and K. A. van Bibber, Ann. Rev. Nucl. Part. Sci. 65, 485 (2015), arXiv:1602.00039 [hep-ex]
[19] I. G. Irastorza and J. Redondo, Prog. Part. Nucl. Phys. 102, 89 (2018), arXiv:1801.08127 [hep-ph]
[20] P. Sikivie, Rev. Mod. Phys. 93, 015004 (2021), arXiv:2003.02206 [hep-ph]
[21] E. Bertuzzo, A. L. Foguel, G. M. Salla, and R. Z. Funchal, (2022), arXiv:2202.12317 [hep-ph]
[22] J. S. Lee, (2018), arXiv:1808.10136 [hep-ph]
[23] P. Caretta, T. Fischer, M. Giannotti, G. Guo, G. Martínez-Pinedo, and A. Mirizzi, JCAP 10, 016 (2019), [Erratum: JCAP 05, E01 (2020)], arXiv:1906.11844 [hep-ph]
[24] T. Fischer, P. Caretta, B. Fore, M. Giannotti, A. Mirizzi, and S. Reddy, Phys. Rev. D 104, 103012 (2021), arXiv:2108.13726 [hep-ph]
[25] F. Calòre, P. Caretta, M. Giannotti, J. Jaeckel, G. Lucente, and A. Mirizzi, Phys. Rev. D 104, 043016 (2021), arXiv:2107.02186 [hep-ph]
[26] K. Mori, T. J. Moriya, T. Takiwaki, K. Kotake, S. Horiuchi, and S. I. Blinnikov, (2022), arXiv:2209.03517 [astro-ph.HE]
[27] G. G. Raffelt, Lect. Notes Phys. 741, 51 (2008), arXiv:hep-ph/0611350
[28] G. Raffelt and L. Stodolsky, Phys. Rev. D 37, 1237 (1988)
[29] A. Burrows, M. T. Ressell, and M. S. Turner, Phys. Rev. D 42, 3297 (1990)
[30] G. Raffelt and D. Seckel, Phys. Rev. Lett. 60, 1793 (1988)
[31] A. Payez, C. Evoli, T. Fischer, M. Giannotti, A. Mirizzi, and A. Ringwald, JCAP 02, 006 (2015), arXiv:1410.3747 [astro-ph.HE]
[32] G. Lucente, P. Carenza, T. Fischer, M. Giannotti, and A. Mirizzi, JCAP 12, 008 (2020), arXiv:2008.04918 [hep-ph]
[33] K. Scholberg, Ann. Rev. Nucl. Part. Sci. 62, 81 (2012), arXiv:1205.6003 [astro-ph.IM]
[34] A. L. Foguel, E. S. Fraga, and C. Bonifazi, Astropart. Phys. 127, 102534 (2021), arXiv:2005.13068 [hep-ph]
[35] A. Mirizzi, I. Tamborra, H.-T. Janka, N. Saviario, K. Scholberg, R. Bollig, L. Hudepohl, and S. Chakraborty, Riv. Nuovo Cim. 39, 1 (2016), arXiv:1508.00785 [astro-ph.HE]
[36] J. Olsen and Y.-Z. Qian, Phys. Rev. D 105, 083017 (2022), arXiv:2202.09975 [astro-ph.HE]
[37] B. Abi et al. (DUNE), Eur. Phys. J. C 81, 423 (2021), arXiv:2008.06647 [hep-ex]
[38] S. Horiuchi and J. P. Kneller, J. Phys. G 45, 043002 (2018), arXiv:1709.01515 [astro-ph.HE]
[39] Y. Koshio, G. D. Orebi Gann, E. O’Sullivan, and I. Tamborra, (2022), arXiv:2209.04298 [hep-ph]
[40] T. Fischer, S. Chakraborty, M. Giannotti, A. Mirizzi, A. Payez, and A. Ringwald, Phys. Rev. D 94, 085012 (2016), arXiv:1605.08780 [astro-ph.HE]
[41] Y. Suwa, A. Harada, K. Nakazato, and K. Sumiyoshi, PTEP 2021, 013E01 (2021), arXiv:2008.07070 [astro-ph.HE]
[42] G. G. Raffelt, Stars as laboratories for fundamental physics: The astrophysics of neutrinos, axions, and other weakly interacting particles (1996).
[43] D. P. Menezes, Universe 7, 267 (2021), arXiv:2106.09515 [astro-ph.HE]
[44] Y. Suwa, Publ. Astron. Soc. Jap. 66, L1 (2014), arXiv:1311.7249 [astro-ph.HE]