Calculation of deformations of concrete with indirect reinforcement according to limit state design

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Abstract. Currently, the issue of increasing the efficiency of the use of traditional materials in building structures due to rational structural solutions and improving design methods and material models is relevant. The use of indirect reinforcement improves the strength and deformation characteristics of concrete. The article considers the influence of indirect reinforcement in the form of transverse welded meshes on the strength and ultimate compressibility of concrete. The main attention is paid to the determination of the strain values determining the stress-strain diagram for concrete with indirect reinforcement. It is known that with an increase in the strength of concrete, the value of its ultimate compressive strains decreases. Indirect reinforcement can significantly increase the ultimate compressibility of concrete and makes the difference in ultimate deformations even more significant for concrete of different strengths. In this context, when calculating structures in accordance with the limit state design and decreasing the strength values of concrete in order to obtain the necessary security, it is possible to overestimate the ultimate compressibility of concrete. This is unacceptable in calculations based on the deformation model, since the achievement of ultimate deformations of the material is the main criterion for structural failure. To confirm the hypothesis, the results of tests for central compression of concrete with indirect reinforcement in the form of transverse welded meshes were selected. The strain values at the peak of the diagram were calculated for the experimental samples. Calculations were also performed for the characteristic compressive strength values and the design values of strength. The results are compared with experimental data. A significant overestimation of the values of strain at the peak of the diagram was noted in the calculations based on the characteristic compressive strength values and the design values of strength.

1. Introduction

There is a large amount of experimental studies showing that, in the case of a triaxial stress state under conditions of lateral compression, the compressive strength and deformation characteristics of concrete are improved (Figure 1) [1-6]. In building structures, the triaxial stress state can be created by an indirect reinforcement that limits the transverse deformations of concrete. So far developed various design solutions for indirect reinforcement: pipe-concrete elements [7,8], the spiral reinforcement [9], welded mesh [11-14], etc. Each option has its own advantages and disadvantages depending on the specific application conditions. In particular, transverse welded mesh due to the relatively small size of the cell are included in the work on the sectional part of the element where compressive forces arise,
so that they can be effectively used both in compressed [10,11] and in bent elements [12,13]. There are proposals for the use of welded wire mesh indirect reinforcement to increase the survivability of building structures [15-17].

The use of reserves of bearing capacity and deformability of traditional building materials through the use of rational engineering solutions contributes to the optimal design and saving of material resources [18-20]. But if it is necessary to carry out practical calculations of structures with indirect reinforcement, a number of difficulties arise that do not have proper coverage in the Russian building code for concrete and reinforced concrete structures [21].

2. Materials and Methods

The Russian building code as the main tool for the calculation of reinforced concrete structures with indirect reinforcement offer a deformation model (Appendix I [21]), which is based on the choice of a stress-strain diagram [22-24]. To build a stress-strain relationship, it is necessary to determine the main points, the most important of which are the stress $R_{c3}$ and strain $\varepsilon_{c03}$ at the peak of the diagram. And, since structural failure in such calculations is determined by the achievement of ultimate strains by the material, the question of assigning ultimate compressibility $\varepsilon_{cu3}$ is especially relevant. The value of $\varepsilon_{cu3}$ is often taken as a function of increasing $\varepsilon_{c03}$:

$$\varepsilon_{cu3} = f\left(\frac{\varepsilon_{c03}}{\varepsilon_{c0}}\right),$$

where $\varepsilon_{c0}$ is a relative deformation corresponding to the peak of the unreinforced concrete compression diagram.

These values of $R_{c3}$ and $\varepsilon_{cu3}$ are usually calculated on the basis of empirical formulas obtained by processing a large sample of experimental data. This approach stems from the fact that the estimation of triaxial compression is a very difficult task and there is no generally accepted universal theory of strength.

The Russian building code [21] propose formulas:

$$\begin{align*}
R_{c3} &= R_c + \varphi \cdot \mu_{xy} \cdot R_{xy}; \\
\varepsilon_{c03} &= \varepsilon_{c0} + 0.02 \cdot \alpha_{red}; \\
\varphi &= \frac{1}{(0.23 + \alpha_{red})}; \\
\alpha_{red} &= \mu_{xy} \cdot R_{xy} / (R_c + 10)
\end{align*}$$

where $R_c$ is a prismatic concrete compressive strength; $R_{xy}$ is a tensile strength of indirect reinforcement rods; $\mu_{xy}$ is an indirect reinforcement ratio.

In [14] it was proposed to determine $R_{c3}$ and $\varepsilon_{cu3}$ by the formulas:
where $\psi_b = 0.375$ is a lateral compression unevenness coefficient; $\sigma_{c,xy}$ is a lateral compression stress. The expression for $\varepsilon_{c,3}$ was obtained on the basis of data from the study [1], in which a large amount of experimental data was processed for concrete samples with a strength of 20-110 MPa. The expression for $R_{c,3}$ is accepted based on [8].

According to the results of the analysis of experimental data [14], it was found that with increasing concrete compressive strength, the use of indirect reinforcement more effectively increases the strength characteristics, but less effectively increases the $\varepsilon_{c,3}$ strain. This fact is illustrated in Figure 2. Similar results were obtained when analyzing the calculation results both by formulas (2) and by formulas (3).

$$R_{c,3} = \left[ \frac{\rho_{c,xy}}{2} + \sqrt{\left(\frac{\rho_{c,xy}}{2}\right)^2 + 9\rho_{c,xy}} \right] R_c;$$

$$\rho_{c,xy} = \psi_b \mu_{c,xy} R_{c,xy} / R_c;$$

$$\varepsilon_{c,3} = \varepsilon^0 \varepsilon_{c,0};$$

$$n = (2.9224 - 0.00408 R_c) \left( 0.9 \sigma_{c,xy} / R_c \right) \left( 0.3124 + 0.0022 R_c \right);$$

$$(1 + 1.64 \gamma_c) = \frac{R_{ck}}{R_{cm}}$$

Figure 2. Stress-strain relationship for concrete with different compressive strength and equal reinforcement coefficient.

Figure 3. Concrete strength according to Gauss distribution curve.

In real structural engineering, while performing calculations in the framework of limit state design [25], it is necessary to assign values of the strength of materials with the required security. For these purposes carry out the transition from the average experimental value of the strength $R_{\text{fin}}$ to the characteristic compressive strength $R_{ck}$ and the design value of strength $R_{cd}$, which are less than the average value (Figure 3). The transition to the characteristic compressive strength is carried out through the coefficient of variation $\nu$, which is usually assumed to be equal to 13.5% in calculations (according to the results of statistical processing of quality control results at concrete plants):

$$R_{ck} = R_{cm} (1 - 1.64 \nu).$$

The design value of strength is obtained from the characteristic strength as a result of dividing by the partial factor for concrete:

$$R_{cd} = R_{ck} / \gamma_c.$$

In view of the foregoing, a contradiction arises due to the fact that initially the formulas for determining the compressibility limit were obtained for experimental strength values, but it is
proposed to use the characteristic strength and the design value of strength, which can lead to incorrect
determination of the $\varepsilon_{\text{crit}}$ strain.

It is known that the main factor that increases the compressibility limit of concrete is the indirect
reinforcement ratio, but the studies [10] also noted the influence of longitudinal high-strength
reinforcement. Taking this into account, for comparison with the results of calculations by formulas
(2) and (3), experimental data were selected for centrally compressed samples with indirect
reinforcement in the form of transverse welded meshes and longitudinal reinforcement of class A-III
(yield strength 400 MPa) and lower (or without longitudinal reinforcement), processed in [14].

Table 1. Parameters of experimental samples.

| No | Author | Longitudinal reinforcement | $R_{cm}$, MPa | $R_{ck}$, MPa | $R_{cd}$, MPa | Indirect reinforcement | $\mu_{cy}$ | $\varepsilon_{\text{exp}}^{\text{e}_c}$ |
|----|--------|-----------------------------|--------------|--------------|--------------|-----------------------|---------|----------------|
| 1. | Henzel | -                           | 13.1         | 10.2         | 7.8          | St.IVb                | 0.040   | 0.0134        |
| 2. |        | -                           | 23.8         | 18.5         | 14.3         | St.IVb                | 0.040   | 0.0089        |
| 3. |        | -                           | 30.9         | 24.1         | 18.5         | St.IVb                | 0.040   | 0.0068        |
| 4. |        | -                           | 13.1         | 10.2         | 7.8          | St.IVb                | 0.045   | 0.0130        |
| 5. |        | -                           | 23.8         | 18.5         | 14.3         | St.IVb                | 0.045   | 0.0087        |
| 6. |        | -                           | 30.9         | 24.1         | 18.5         | St.IVb                | 0.045   | 0.0058        |
| 7. | Filippov| A-III                       | 30.0         | 23.4         | 18.0         | A-III                | 0.016   | 0.0050        |
| 8. |        | A-III                       | 30.0         | 23.4         | 18.0         | A-III                | 0.016   | 0.0060        |
| 9. |        | A-III                       | 31.5         | 24.5         | 18.9         | A-III                | 0.049   | 0.0130        |
| 10.|        | A-III                       | 34.5         | 26.9         | 20.7         | A-III                | 0.049   | 0.0140        |
| 11.|        | A-III                       | 49.0         | 38.2         | 29.3         | A-III                | 0.016   | 0.0035        |
| 12.|        | A-III                       | 49.2         | 38.3         | 29.5         | A-III                | 0.016   | 0.0037        |
| 13.|        | A-III                       | 49.2         | 38.3         | 29.5         | A-III                | 0.049   | 0.0090        |
| 14.|        | A-III                       | 49.2         | 38.3         | 29.5         | A-III                | 0.049   | 0.0095        |
| 15.|        | A-III                       | 42.7         | 33.2         | 25.6         | A-III                | 0.021   | 0.0043        |
| 16.|        | A-III                       | 58.0         | 45.2         | 34.7         | A-III                | 0.020   | 0.0044        |
| 17.|        | A-III                       | 58.0         | 45.2         | 34.7         | A-III                | 0.052   | 0.0060        |
| 18.|        | A-III                       | 58.0         | 45.2         | 34.7         | A-III                | 0.102   | 0.0080        |
| 19.|        | A-III                       | 58.0         | 45.2         | 34.7         | A-III                | 0.075   | 0.0081        |
| 20.| Matkov| A-I                         | 22.3         | 17.4         | 13.4         | A-III                | 0.054   | 0.0080        |
| 21.| Bakirov| A-III                       | 42.0         | 32.7         | 25.2         | A-III                | 0.019   | 0.0043        |
| 22.|        | A-III                       | 42.0         | 32.7         | 25.2         | A-III                | 0.019   | 0.0045        |
| 23.|        | A-III                       | 41.2         | 32.1         | 24.7         | A-III                | 0.019   | 0.0043        |
| 24.|        | A-III                       | 42.2         | 32.9         | 25.3         | A-III                | 0.042   | 0.0055        |
| 25.| Karnet| A-I                         | 56.3         | 43.8         | 33.7         | A-I                  | 0.026   | 0.0042        |
| 26.|        | A-I                         | 53.4         | 41.6         | 32.0         | A-I                  | 0.026   | 0.0042        |
| 27.|        | A-I                         | 58.5         | 45.5         | 35.0         | A-I                  | 0.026   | 0.0037        |
| 28.|        | A-I                         | 46.0         | 35.8         | 27.6         | A-I                  | 0.052   | 0.0052        |
| 29.|        | A-I                         | 57.2         | 44.5         | 34.3         | A-I                  | 0.052   | 0.0053        |
| 30.|        | A-I                         | 58.1         | 45.2         | 34.8         | A-I                  | 0.052   | 0.0053        |

3. Results and discussion

The results of calculating the $\varepsilon_{\text{crit}}$ strain using formulas (2) and (3) and a comparison with the
experimental data are shown in Table 2. Strains were calculated for the average strength values
determined by the experimental results, for the characteristic compressive and for the design value of
strength.

The median values $\varepsilon_{\text{exp}}^{\text{e}_c} / \varepsilon_{\text{e}_c}$ for the processed sample of 30 elements:
and the average deviations of the calculation results from the experimental data:

$$\bar{\Delta} = \frac{1}{30} \sum_{i=1}^{30} |\varepsilon_{e,03}^{\exp} / \varepsilon_{e,03,i} - 1| \cdot 100\%,$$

were also calculated.

Table 2. Comparison of calculation results with experimental data.

| No  | According to (2) | According to (3) |
|-----|------------------|------------------|
|     | (\varepsilon_{e,03}^{\exp} / \varepsilon_{e,03}) | (\varepsilon_{e,03}^{\exp} / \varepsilon_{e,03}) |
|     | \varepsilon_{e,03} = f(R_{e,03}); | \varepsilon_{e,03} = f(R_{e,03}); |
|     | (\varepsilon_{e,03}^{\exp} / \varepsilon_{e,03}) | (\varepsilon_{e,03}^{\exp} / \varepsilon_{e,03}) |
| 1.  | 0.0220; (0.61) | 0.0297; (0.48) |
| 2.  | 0.0157; (0.57) | 0.0221; (0.42) |
| 3.  | 0.0133; (0.51) | 0.0182; (0.37) |
| 4.  | 0.0245; (0.53) | 0.0311; (0.42) |
| 5.  | 0.0174; (0.50) | 0.0234; (0.37) |
| 6.  | 0.0147; (0.39) | 0.0202; (0.29) |
| 7.  | 0.0058; (0.86) | 0.0074; (0.81) |
| 8.  | 0.0058; (1.03) | 0.0062; (0.96) |
| 9.  | 0.0121; (1.08) | 0.0165; (0.79) |
| 10. | 0.0114; (1.23) | 0.0156; (0.89) |
| 11. | 0.0046; (0.76) | 0.0052; (0.68) |
| 12. | 0.0046; (0.81) | 0.0059; (0.63) |
| 13. | 0.0091; (0.99) | 0.0126; (0.71) |
| 14. | 0.0091; (1.05) | 0.0126; (0.75) |
| 15. | 0.0055; (0.79) | 0.0072; (0.60) |
| 16. | 0.0046; (0.95) | 0.0070; (0.63) |
| 17. | 0.0090; (0.67) | 0.0126; (0.48) |
| 18. | 0.0152; (0.53) | 0.0221; (0.36) |
| 19. | 0.0122; (0.67) | 0.0174; (0.46) |
| 20. | 0.0154; (0.52) | 0.0205; (0.39) |
| 21. | 0.0049; (0.88) | 0.0062; (0.69) |
| 22. | 0.0049; (0.93) | 0.0062; (0.72) |
| 23. | 0.0049; (0.88) | 0.0063; (0.68) |
| 24. | 0.0084; (0.65) | 0.0115; (0.48) |
| 25. | 0.0044; (0.96) | 0.0056; (0.75) |
| 26. | 0.0045; (0.93) | 0.0057; (0.73) |
| 27. | 0.0043; (0.85) | 0.0055; (0.67) |
| 28. | 0.0076; (0.69) | 0.0088; (0.59) |
| 29. | 0.0066; (0.79) | 0.0090; (0.58) |
| 30. | 0.0066; (0.80) | 0.0090; (0.59) |

As can be seen from table 2, according to formulas (3), a better agreement is obtained between the calculated strain values and the experimental ones. The formulas from the Russian building code for most prototypes overestimate the ultimate compressibility and the average deviation was from 24.5%
for the average concrete strength values obtained from the test results to 41.2% for the design value of strength. According to formulas (3), the average deviation was from 16.9% to 27.5%, and the median value $\varepsilon_{c03}^{exp} / \varepsilon_{c03}$ in the calculations for average strength tends to 1. Perhaps the difference is due to the fact that when deriving formulas (2), the influence of longitudinal reinforcement, which increases the deformability, was not taken into account, while formulas (3) were originally obtained for unreinforced concrete under lateral compression.

4. Conclusions

Studies have shown that in practical calculations of reinforced concrete structures with indirect reinforcement in accordance with the limit state design, the use of characteristic and design values of concrete strength in the calculation formulas can lead to overestimated values of deformability $\varepsilon_{c03}$. This should be taken into account when setting stress-strain relationship for concrete with indirect reinforcement and to limit the ultimate compressibility.

The proposed formulas provide a better correspondence between the calculated values of the deformability of concrete with indirect reinforcement and experimental data in comparison with formulas from the Russian building code. When using the average experimental values of concrete strength in the calculations, the average deviation from the experimental results is within 17%, and the median value $\varepsilon_{c03}^{exp} / \varepsilon_{c03}$ for 30 processed samples tends to 1.

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