Effect of micropolar fluids on the squeeze film elliptical plates

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Abstract. This paper elaborates on the theoretical analysis of squeeze film characteristics between elliptical plates lubricated with non-Newtonian micro-polar fluid on the basis of Eringen's micropolar fluid theory. The modified Reynold's equations governing flow of micro-polar fluid is mathematically derived and the outcome reveals distribution of film pressure which determines the dynamic performance characteristics in terms of load and squeezing time for various values of coupling number and micro structure size parameter. Based on the results reported, The influence of non-Newtonian micropolar fluids is examined in enhancing the time of approach and load carrying capacity to the case of classical Newtonian lubricant.

1. Introduction

The various characteristics of micropolar fluids have been discussed elaborately in Theory of micropolar fluids and Theory of continuum propounded by Eringen [1-2]. The theory of micropolar fluid according to Eringen is the rotation vector called micro rotation vector and gives the equations relating to rate of stress and strain. The analysis of the finite width journal bearings lubricated with micropolar fluids is discussed by Tsai–Wang Huang et.al [3]. It is noted that the prominent feature of micropolar fluid is that by increasing load capacity, and coefficient of friction reduces. Albert and Thamir [4] have discussed the effect of micropolar fluids on lubrication of infinitely long thrust bearings. They studied the laminar flow of micropolar fluid on thin film increased with the effect of viscosity. The Dynamic characteristics of finite-width journal bearings with micropolar fluids analysed by Tsai-Wang Huang and Cheng-I Weng [5], indicates that the lower limiting value of the Sommerfeld number is stable operation for increasing the load carrying capacity with increasing effective viscosity for micropolar fluid. Naduvinamani and Santosh [6] have presented the dynamic and static characteristics of squeeze film lubrication in finite porous journal bearings lubricated with micropolar fluid. They found that the micropolar fluid effect significantly increases the characteristics of bearings. The study of characteristics of one dimensional journal bearings on micropolar fluid lubrication is analysed by Zaheeruddin and Isa [7]. They showed that by the presence of base oil load capacity increases and decreases friction coefficient. The three dimensional micropolar fluid for Reynold’s equation is derived by Chandan Singh and Prawal Sinha [8]. The derived expressions helped to solve the equations of velocity distributions and micro rotation velocities. Allen and Kline [9] and Prakash and Sinha [10] have studied the lubrication theory for micropolar fluid and its application to journal
bearings. They have expressed the view that lubrication theory was developed for two dimensional bearings. Khonsari et.al [11] studied lubrication characteristics of micropolar fluids on the effect of viscous dissipation. They have analysed that the heat generated because of viscous dissipation in journal bearing.

This paper elaborates on the theoretical analysis of squeeze film characteristics between elliptical plates lubricated with non-Newtonian micro-polar fluid on the basis of Eringen's micropolar fluid theory.

2. Mathematical Formulation

The configuration of geometry between elliptical plates lubricated with pressure dependent viscosity with micropolar fluid is shown in figure 1. \(a\) and \(b\) are the semi major and semi minor axes of the elliptical plate, and assumed that lower plate is to be fixed while the upper plate moving with normal velocity \(v = \frac{dh}{dt}\) towards the lower plate as depicted in Figure 1.

![Figure1: The configuration of elliptical plates with Micro-polar fluids](image)

The basic equations governing the micro-polar fluid film of lubricant are derived in the form

\[
\begin{align*}
\left( \mu + \frac{\chi}{2} \right) \frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial v}{\partial y} &= \frac{\partial p}{\partial x} \\
\frac{\partial p}{\partial y} &= 0 \\
\left( \mu + \frac{\chi}{2} \right) \frac{\partial^2 w}{\partial y^2} - \chi \frac{\partial v}{\partial y} - \frac{\partial p}{\partial z} &= 0 \\
\gamma \frac{\partial^2 v_1}{\partial y^2} - \chi \frac{\partial u}{\partial y} - 2\chi v_3 &= 0 \\
\gamma \frac{\partial^2 v_1}{\partial y^2} - 2\chi v_1 + \chi \frac{\partial w}{\partial y} &= 0
\end{align*}
\]
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (6)
\]

\(u, v, \) and \(w\) represent the components of velocity in three directions of Cartesian co-ordinate system.

Where \(p\) is the film pressure, \(\mu\) is the lubricant viscosity, \(\eta\) is material constant responsible for couple stresses.

Applying the boundary conditions,

At the upper surface \(y = h\)

\[
u = v = w = 0, \quad v_1 = v_3 = 0 \quad (7a)
\]

At the porous surface \(y = 0\)

\[
u = v = w = 0, \quad v_1 = v_3 = 0 \quad (7b)
\]

Solving equations (1), (2), (3) and (4) subject to the given boundary conditions as stated in (7a), (7b), (7c) and (7d) we obtain

\[
u = \frac{1}{\mu} \frac{\partial p}{\partial x} \left[ \frac{y^2}{2} - \frac{N^2 h}{m} \frac{\text{Cosh}y - 1}{\text{Sinh}y} \right] + \frac{D_1}{(1 - N^2)} \left[ y - \frac{N^2}{m} \left\{ \frac{\text{Sinh}y - (\text{Cosh}y - 1) (\text{Cosh}y - 1)}{\text{Sinh}y} \right\} \right] \quad (8)
\]

\[
w = \frac{1}{\mu} \frac{\partial p}{\partial z} \left[ \frac{y^2}{2} - \frac{N^2 h}{m} \frac{\text{Cosh}y - 1}{\text{Sinh}y} \right] + \frac{D_1}{(1 - N^2)} \left[ y - \frac{N^2}{m} \left\{ \frac{\text{Sinh}y - (\text{Cosh}y - 1) (\text{Cosh}y - 1)}{\text{Sinh}y} \right\} \right] \quad (9)
\]

Where \(m = \frac{N}{l} \quad l = \left( \frac{\gamma}{4\mu} \right)^{\frac{1}{2}}\)

Substituting the value of \(u\) and \(w\) in equation (6) and integrating using boundary conditions (7a) to (7c), we get

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^3 p}{\partial x^2 \partial z} = \frac{12\mu dh}{dt} \frac{f(N, L, h)}{f(N, L, h)} \quad (10)
\]

Relevant pressure boundary conditions are

\(p(x_1, z_1) = 0 \quad (11)\)

Where \(\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1 \quad (12)\)
Solving equation (10) for elliptical plates with the boundary conditions as stated in equation (11), gives the expression for the elliptical plates for pressure distribution is

\[
p = -\frac{12\mu dh}{f(N, L, h)} \left( \frac{a^2 b^2}{2(a^2 + b^2)} \left( 1 - \frac{x_i^2}{a^2} - \frac{z_i^2}{b^2} \right) \right)
\]

(13)

The dimensionless pressure is

\[
p^* = -\frac{ph_0^3}{\mu(dh/dt)ab} \left( \frac{a^*}{a^* + 1} \right) \left( 1 - \frac{x_i^*}{a^*} - \frac{z_i^*}{b^*} \right) \frac{6}{F(N, L^*, h^*)}
\]

(14)

where

\[
h^* = \frac{h}{h_0}, \quad L^* = \frac{L}{h_0}, \quad a^* = \frac{a}{b}, \quad x_i^* = \frac{x_i}{a}, \quad z_i^* = \frac{z_i}{b}
\]

The expression for non-dimension load carrying capacity in form is

\[
W^* = -\frac{Wh_0^3}{\mu(dh/dt)a^2 b^2} \left( \frac{a^*}{a^* + 1} \right) \frac{3}{F(N, L^*, h^*)}
\]

(15)

The dimensionless squeeze film time is given by

\[
T^* = -\frac{1}{\mu a^2 b^2} \left( \frac{a^*}{a^* + 1} \right) \int_1^{h/\mu} \frac{3}{F(N, L^*, h^*)} \, dh
\]

(16)

3. Results and Discussions

The behaviour of squeeze film between elliptical plates lubricated with micro-polar fluid is analysed. The two dimensionless parameters are of great interest to study the lubricating effectiveness of the micro-polar fluid viz., the coupling number \(N = \left( \frac{\chi}{\chi + 2\mu} \right)^{1/2}\) which characterizes the linear coupling and rotational motion arising from the micro-motion of the lubricant additives. Therefore \(N\) stands the coupling between the rotational and Newtonian viscosities. As \(\chi \to 0\), we have \(N \to 0\) and the expressions for the characteristics of the bearing in this paper decreases to their analogues in classical Newtonian theory. The second dimensionless parameter \(L' = \left( \frac{1}{h_0} \right)^{1/2}\) with \(l = \left( \frac{y}{4\mu} \right)^{1/2}\) characterizes the performance between the configuration of the bearing and fluid.

**Pressure:** Figure 2 describes the effect of variation in dimensionless radius \(x^*\) on dimensionless squeeze film pressure \(P^*\) for various values of coupling number \(N\) with \(a=1, z=0, h^*=0.6\) and \(L^*=0.3\). It is observed that \(P^*\) is maximum at \(r^*=0\) and further \(P^*\) reduces for increasing value of \(x^*\) whereas \(P^*\) increases with increasing values coupling parameter \(N\). The influence of micro-polar effect is visibly apparent. Figure 3 represents the non-dimensional squeeze film pressure \(P^*\) as a function of dimensionless radius \(x^*\) for various values of \(L'\) with \(h^*=0.6\) and \(N=0.4\). It can be seen from the figure, maximum pressure \(P^*\) reduces as the value of \(x^*\) is increased. Further, pressure increases with increasing \(L'\). Contrary to the case of Newtonian lubricant.
Load-carrying capacity: Figure 4 depicts the variation of non-dimensional film thickness $h^*$ with non-dimensional load carrying capacity $W^*$ for different values of $N$, with $a=1$, $z=0$ and $L^*=0.3$. As per the observation the $W^*$ decreases for increasing values of $h^*$. It is also observed that $W^*$ increases with increasing values of $N$. Figure 5 indicates the variation of $W^*$ with non-dimensional film thickness $h^*$ for various values of $L^*$ and $N=0.3$. It is observed that the non-dimensional load carrying capacity $W^*$ decreases for increasing values of $h^*$. It can be realized that in the presence of micro-polar fluid enhances load carrying capacity $W^*$.

Squeeze Film Time: The variation of non-dimensional squeeze film time $T^*$ with the non-dimensional film thickness $h_1^*$ for different values $N$, with $a=1$, $z=0$ and $L^*=0.3$ is depicted in Figure 6. The micro polar effects signify an enhancement of squeezing time, especially for small values of film thickness. Further, it is observed that $T^*$ increases with the increasing values of $N$. Figure 7 indicates the variation of $T^*$ with $h_1^*$ for various values of $L^*$ and $N=0.4$. It is noted that $T^*$ decreases for increasing values of $h_1^*$. In order to illustrate the salient features of elliptical plates lubricated with micro-polar fluid, the numerical results are presented in Figures 2-7.
4. **Conclusion:** The behaviour of micropolar fluid film squeezed between two elliptical plates with lubrication of micro polar fluid is presented. On the basis of Eringen’s micro polar theory, the modified Reynolds equation is derived. Based on the theoretical results presented, the following conclusions are drawn.

- The squeeze film characteristics of elliptical plates are significantly affected in the presence of micro-polar fluid.
- The non-dimensional load carrying capacity $W^*$ and squeeze film time $T^*$ decrease with the increasing values of the fluid film thickness $h^*$.
- Non-dimensional pressure, load carrying capacity and squeeze film time increase with increasing coupling number $N$ and micro structure size parameter $L^*$. 

Figure 4: Variation of nondimensional load $W^*$ with $h^*$ for different values of $N$ with $a = 1, z = 0$ and $L = 0.3$

Figure 5: Variation of nondimensional load $W^*$ with $h^*$ for different values of $L^*$ with $a = 1, z = 0$ and $N = 0.4$

Figure 6: Variation of squeeze film time with $h_1^*$ for different values of $N, a=1, z=0$

Figure 7: Variation of squeeze film time $T^*$ with $h_1^*$ for different values of $L^*$ with $a = 1, z = 0$ and $N = 0.4$
Non-dimensional pressure is maximum at non-dimensional parameter \( x^* = 0 \) and decreases for increasing values of \( x^* \).

Variation of parameters \( N \) and \( L^* \) and the analogous calculations for elliptical plates illustrate that the above deductions are stable and may correspondingly have practical importance in the field of tribology.

**Nomenclature**

- \( 2a, 2b \) dimensions of the bearing
- \( h \) thickness in the film region
- \( h_0 \) initial film thickness
- \( h^* \) dimensionless film thickness \( ( = h/h_0 ) \)
- \( L \) micro structure size parameter \( (\eta/\mu)^{1/2} \)
- \( L^* \) non-dimensional couplestress parameter \( ( = L/h_0 ) \)
- \( N \) coupling number \( ( = \chi/\chi + 2\mu )^{1/2} \)
- \( p \) pressure in the film region
- \( P^* \) non-dimensional pressure \( ( = -ph_0^3/\mu(dh/dt)ab ) \)
- \( t \) squeeze film time
- \( T^* \) non-dimensional time of approach \( -\int_{1}^{t} \frac{Wh_0^3}{\mu a^2 b^2} dt \)
- \( W \) load carrying capacity \( -Wh_0^3/\mu(dh/dt)a^2 b^2 \)
- \( W^* \) non-dimensional load carrying capacity
- \( \mu \) viscosity of the fluid
- \( \eta \) material constant responsible for couplestress

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