Correlation femtoscopy of multiparticle processes

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Abstract

Recent results on particle momentum and spin correlations are discussed in view of the role played by the effects of quantum statistics, including multiboson and coherence phenomena, and final state interaction. Particularly, it is demonstrated that the latter allows for (i) correlation femtoscopy with unlike particles; (ii) study of the relative space–time asymmetries in the production of different particle species (e.g., relative time delays or spatial shifts due to collective flows); (iii) study of the particle strong interaction hardly accessible by other means (e.g., in ΛΛ system).

1 Introduction

The momentum correlations of particles at small relative velocities are widely used to study space-time characteristics of the production processes, so serving as a correlation femtoscope. Particularly, for non-interacting identical particles, like photons or, to some extent, pions, these correlations result from the interference of the production amplitudes due to the symmetrization requirement of quantum statistics (QS) [1, 2]. There exists [3] a deep analogy of the momentum QS correlations of photons with the space–time correlations of the intensities of classical electromagnetic fields used in astronomy to measure the angular radii of stellar objects based on the superposition principle - so called HBT intensity interferometry [4].

The momentum QS correlations were first observed as an enhanced production of the pairs of identical pions with small opening angles (GGLP effect [1]). Later on, Kopylov and Podgoretsky [2] settled the basics of correlation femtoscopy; particularly, they suggested to study the interference effect in terms of the correlation function and clarified the role of the space–time characteristics of particle production in various physical situations.

The momentum correlations of particles emitted at nuclear distances are also influenced by the effect of final state interaction (FSI) [6, 7, 8]. Thus the effect of the Coulomb interaction dominates the correlations of charged particles at very small relative momenta (of the order of the inverse Bohr radius of the two-particle system), respectively suppressing or enhancing the production of particles with like or unlike charges. Though the FSI

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2 This analogy is sometimes misunderstood and the momentum correlations are mixed up with the space–time HBT correlations in spite of their orthogonal character and the failure of the superposition principle for correlations of identical fermions. In fact, in spite of the common QS origin of the momentum correlations of identical particles and the space–time HBT correlations (allowing for a generalization of the latter to any type of identical bosons or fermions), the corresponding correlation measurements differ in principle [3] (see also [5]). The former, being the momentum–energy measurement, yields the space–time picture of the source, while the latter does the opposite. In particular, the dependence of the number of coincident two–photon counts on the distance between detectors (a quantum analogy of the HBT measurement) provides the information on the characteristic relative three-momenta of emitted photons and so, when divided by the mean detected momentum, on the angular size of a star but, of course, - no information on the star radius or lifetime.
effect complicates the correlation analysis, it is an important source of information allowing for the coalescence femtoscopy (see, e.g., [9, 10, 11, 12]), the correlation femtoscopy with unlike particles [8, 13] including the access to the relative space–time asymmetries in particle production [14] and a study of particle interaction hardly accessible by other means.

We do not touch here the fluctuation measures which are closely related with particle correlations in momentum space and carry an important information on the dynamics and space-time evolution of the production process (see [15] for a recent review).

The rest of the report is organized as follows. In section 2, we briefly review the formalism of particle correlations at small relative velocities. The basic concepts of femtoscopy with identical and nonidentical particles, including the access to the relative space-time shifts in the emission of various particle species, and some recent results are reviewed in sections 3, 5 and 7. In section 4, we discuss the present theoretical and experimental status of the multiboson and coherence phenomena in multiparticle production. Recent results from correlation measurements of the strong interaction in various two-particle systems are reviewed in section 6. In section 8, we briefly discuss spin correlations as a new femtoscopy tool. We conclude in Section 9.

2 Formalism

The ideal two-particle correlation function $R(p_1, p_2)$ is defined as a ratio of the measured two-particle distribution to the reference one which would be observed in the absence of the effects of QS and FSI. In practice, the reference distribution is usually constructed by mixing the particles from different events of a given class, normalizing the correlation function to unity at sufficiently large relative velocities.

Usually, it is assumed that the correlation of two particles emitted with a small relative velocity is influenced by the effects of their mutual QS and FSI only and that the momentum dependence of the one-particle emission probabilities is inessential when varying the particle four-momenta $p_1$ and $p_2$ by the amount characteristic for the correlation due to QS and FSI (smoothness assumption). Clearly, the latter assumption, requiring the components of the mean space-time distance between particle emitters much larger than those of the space-time extent of the emitters, is well justified for heavy ion collisions.

The correlation function is then given by a square of the properly symmetrized Bethe-Salpeter amplitude in the continuous spectrum of the two-particle states, averaged over the four-coordinates $x_i = \{t_i, r_i\}$ of the emitters and over the total spin $S$ of the two-particle system [8]. After the separation of the unimportant phase factor due to the c.m.s. motion, this amplitude reduces to the one depending only on the relative four-coordinate $\Delta x \equiv x_1 - x_2 = \{t, r\}$ and the generalized relative momentum $\tilde{q} = q - P(qP)/P^2$, where $q = p_1 - p_2$, $P = p_1 + p_2$ and $qP = m_1^2 - m_2^2$; in the two-particle c.m.s., $P = 0$, $\tilde{q} = \{0, 2k^*\}$ and $\Delta x = \{t^*, r^*\}$. At equal emission times of the two particles in their c.m.s. $(t^* \equiv t_1^* - t_2^* = 0)$, the reduced non–symmetrized amplitude coincides with a stationary solution $\psi^{S(+)}(\mathbf{r}^*)$ of the scattering problem having at large distances $r^*$ the asymptotic form of a superposition of the plane and outgoing spherical waves (the minus

\[3\] Besides the events with a large phase-space density fluctuations, this assumption may not be justified also in low energy heavy ion reactions when the particles are produced in a strong Coulomb field of residual nuclei. To deal with this field a quantum adiabatic (factorisation) approach can be used [16].
The Bethe-Salpeter amplitude can be usually substituted by this solution (equal time approximation) for identical particles, the amplitude in Eq. (1) enters in a symmetrized form:

$$\rho_S = \frac{S}{2 S + 1}.$$ 

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$$\psi_S^{(+)}(r^*) \rightarrow [\psi_S^{(+)}(r^*) + (-1)^S \psi_S^{(+)}(r^*)]/\sqrt{2}. \quad (2)$$

The averaging in Eq. (1) is done over the four-coordinates of the emitters at a given total spin $S$ of the two-particles, $\rho_S$ is the corresponding population probability, $\sum_S \rho_S = 1$. For unpolarized particles with spins $s_1$ and $s_2$ the probability $\rho_S = (2S+1)/[(2s_1+1)(2s_2+1)]$. Generally, the correlation function is sensitive to particle polarization. For example, if two spin-1/2 particles are initially emitted with polarizations $\mathbf{P}_1$ and $\mathbf{P}_2$ then [8]

$$\bar{\rho}_0 = (1 - \mathbf{P}_1 \cdot \mathbf{P}_2)/4, \quad \bar{\rho}_1 = (3 + \mathbf{P}_1 \cdot \mathbf{P}_2)/4. \quad (3)$$

### 3 Femtoscopy with identical particles

For identical pions or kaons, the effect of the strong FSI is usually small and the effect of the Coulomb FSI can be in first approximation simply corrected for (see [17] and references therein). The corrected correlation function is determined by the QS symmetrization only (see Eq. (2) and substitute the non-symmetrized amplitude by the plane wave $e^{i\mathbf{q}z}$):

$$\mathcal{R}(p_1, p_2) = 1 + \langle \cos(q\Delta x) \rangle. \quad (4)$$

Its characteristic feature is the presence of the interference maximum at small components of the relative four-momentum $q$ with the width reflecting the inverse space-time extent of the effective production region. For example, assuming that for a fraction $\lambda$ of the pairs, the pions are emitted independently according to one-particle amplitudes of a Gaussian form characterized by the space-time dispersions $r_0^2$ and $\tau_0^2$ while, for the remaining fraction $(1 - \lambda)$ related to very long-lived sources ($\eta, \eta', K_0, \Lambda, \ldots$), the relative distances $r^*$ between the emitters in the pair c.m.s. are extremely large, one has

$$\mathcal{R}(p_1, p_2) = 1 + \lambda \exp \left( -r_0^2 q_T^2 - \tau_0^2 q_L^2 \right) = 1 + \lambda \exp \left( -r_0^2 q_T^2 - (r_0^2 + v^2 \tau_0^2) q_L^2 \right), \quad (5)$$

where $q_T$ and $q_L$ are the transverse and longitudinal components of the three-momentum difference $\mathbf{q}$ with respect to the direction of the pair velocity $\mathbf{v} = \mathbf{P}/P_0$. One may see that, due to the on-shell constraint [2] $q_0 = vq \equiv vq_L$ (following from the equality $qP = 0$), strongly correlating the energy difference $q_0$ with the longitudinal momentum difference $q_L$.

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Footnote: For non-interacting particles, the non-symmetrized Bethe-Salpeter amplitude reduces to the plane wave $e^{i\mathbf{q}z}$ which is independent of the relative time in the two-particle c.m.s. and so, coincides with the corresponding equal-time amplitude. For interacting particles, the equal time approximation is valid on condition [8] $|t^*| \ll m_2 t^2$ for sign$(t^*) = \pm 1$ respectively. This condition is usually satisfied for heavy particles like kaons or nucleons. But even for pions, the $t^* = 0$ approximation merely leads to a slight overestimation (typically < 5%) of the strong FSI effect and, it doesn’t influence the leading zero-distance ($r^* \ll |a|$) effect of the Coulomb FSI.
that with the increasing energy of heavy ion collisions from AGS and SPS up to the allows to disentangle all the freeze-out characteristics (see the review [20]). It appears simultaneous analysis of correlations and single particle spectra for various particle species out temperature and the transverse flow determine also the shapes of the decrease of the side radius and the spatial part of the out radius with \( \tau \) due to the finite emission duration \( \Delta \).

The correlation function can be used to determine both the duration of the emission and the form of the emission region [2], as well as - to reveal the details of the production dynamics (such as collective flows; see, e.g., [18, 19] and the reviews [20, 21]). For this, the correlation functions can be analyzed in terms of the out (x), side (y) and longitudinal (z) components of the relative momentum vector \( \mathbf{q} = \{q_x, q_y, q_z\} \) [22, 23]; the out and side denote the transverse, with respect to the reaction axis, components of the vector \( \mathbf{q} \), the out direction is parallel to the transverse component of the pair three–momentum. The corresponding correlation widths are usually parameterized in terms of the Gaussian correlation radii \( R_i \),

\[
\mathcal{R}(p_1, p_2) = 1 + \lambda \exp(-\frac{R_x^2 q_x^2 + R_y^2 q_y^2 + R_z^2 q_z^2 - R_{xy} q_x q_y}{2})
\]

and their dependence on pair rapidity and transverse momentum is studied. The form of Eq. (6) assumes azimuthal symmetry of the production process [20, 22]. Generally, e.g., in case of the correlation analysis with respect to the reaction plane, all three cross terms \( q_i q_j \) contribute [24].

It is well known that particle correlations at high energies usually measure only a small part of the space-time emission volume, being only slightly sensitive to its increase related to the fast longitudinal motion of particle sources. In fact, due to limited source decay momenta \( p^{(s)} \) of few hundred MeV/c, the correlated particles with nearby velocities are emitted by almost comoving sources and so - at nearby space–time points. In other words, the maximal contribution of the relative motion to the correlation radii in the two–particle c.m.s. is limited by the moderate source decay length \( \tau p^{(s)}/m \). The dynamical examples are sources-resonances, colour strings or hydrodynamic expansion. To substantially eliminate the effect of the longitudinal motion, the correlations can be analyzed in terms of the invariant variable \( q_{inv} \equiv Q = (-\vec{q}^2)^{1/2} = 2k^* \) and the components of the momentum difference in pair c.m.s. (\( \mathbf{q}' \equiv Q = 2k^* \)) or in the longitudinally comoving system (LCMS) [25]. In LCMS each pair is emitted transverse to the reaction axis so that the generalized relative momentum \( \vec{q} \) coincides with \( \mathbf{q}' \) except for the component \( q_z = \gamma q_z^* \), where \( \gamma \) is the LCMS Lorentz factor of the pair.

Particularly, in the case of one–dimensional boost invariant expansion, the longitudinal correlation radius in the LCMS reads [19] \( R_z \approx (T/m_t)^{1/2} \), where \( T \) is the freeze-out temperature, \( \tau \) is the proper freeze-out time and \( m_t \) is the transverse particle mass. In this model, the side radius measures the transverse radius of the system while, similar to Eq. (5), the square of the out radius gets an additional contribution \( (p_1/m_t)^2 \Delta \tau^2 \) due to the finite emission duration \( \Delta \). The additional transverse expansion leads to a slight modification of the \( p_t \)–dependence of the longitudinal radius and - to a noticeable decrease of the side radius and the spatial part of the out radius with \( p_t \). Since the freeze-out temperature and the transverse flow determine also the shapes of the \( m_t \)-spectra, the simultaneous analysis of correlations and single particle spectra for various particle species allows to disentangle all the freeze-out characteristics (see the review [20]). It appears that with the increasing energy of heavy ion collisions from AGS and SPS up to the
highest energies at RHIC, the data show rather weak energy dependence [26] and point to the kinetic freeze-out temperature somewhat below the pion mass, a strong transverse flow (with the mean transverse flow velocity at RHIC exceeding half the velocity of light [27]), a short evolution time of 8-10 fm/c and a very short emission duration of about 2 fm/c. The short evolution and emission duration at RHIC are also supported by the correlation analysis with respect to the reaction plane [28]. The small time scales at RHIC were not expected in transport and hydrodynamic models [29, 30] and may indicate an explosive character of particle production (see, e.g., [31, 32]). In fact, the RHIC data can be described in so called blast wave model [33, 34] assuming a strong three-dimensional expansion with a sharp boundary of the freeze-out density profile in transverse plane. The same model with $\sim 15\%$ lower mean transverse flow velocity is also consistent with the SPS data [35].

4 Multiboson and coherence effects

In present and future heavy ion experiments at SPS, RHIC and LHC many hundreds or thousands of pions can be produced per a unit rapidity interval. Since pions are bosons there can be multiboson effects enhancing the production of pions with low relative momenta thus increasing the pion multiplicities, softening their spectra and modifying the correlation functions (see [36, 37, 38] and references therein). In particular, it was shown [37] that the width of the low-$p_t$ enhancement due to BE condensation decreases with the system size as $r_0^{-1/2}$ and this narrowing makes easier the identification of this effect among others. For the events of approximately fixed multiplicity, the multiboson effects can be triggered by decreasing correlation strength and a dip in the two–pion correlation function at intermediate relative momenta [37, 38].

Though the present data does not point to any spectacular multiboson effects, one can hope to observe new interesting phenomena like boson condensation or speckles in some rare events or in eventually overpopulated kinematic regions with the pion density in the 6-dimensional phase space, $f = (2\pi)^3d^6n/d^3p d^3x$, of the order of unity. An example is a rapidly expanding system with the entropy much smaller than in the case of total equilibrium. Then a strong transverse flow can lead to rather dense gas of soft pions in the central part of the hydrodynamic tube at the final expansion stage (see, e.g., [39]). Another reason can be the expected formation of quark-gluon plasma or mixed phase. Due to large gradients of temperature or velocity the hydrodynamic layer near the boundary with vacuum can decay at a large phase space density and lead to pion speckles even at moderate transverse momenta [40].

In the low-density limit ($f \ll 1$), the mean phase space density at a given momentum $\mathbf{p}$ can be estimated as the mean number of pions interfering with a pion of momentum $\mathbf{p}$ (rapidity $y$ and transverse momentum $p_t$) and building the Bose-Einstein (BE) enhancement in the two-pion correlation function [41, 42]: $\langle f \rangle_p \sim \pi^{3/2} N(\mathbf{p})/V$, where $N(\mathbf{p}) = d^3n/d^3p$ and $V = r_x r_y r_z$ is the interference volume defined in terms of the outward ($r_x$), sideward ($r_y$) and longitudinal ($r_z$) interferometry radii. Typically $\langle f \rangle_p \sim 0.1$ for mid-rapidities and $p_t \sim \langle p_t \rangle$ [41]. The data are also consistent with the phase space density of pions near the local thermal equilibrium [43, 44].

At AGS and SPS energies the interference volume $V$ seems to scale with $dn/dy$ (see, e.g., [44, 45]) pointing to the freeze-out of pions at a constant phase space density. This
trend is however questioned by recent STAR data from RHIC, indicating an increase of the freeze-out phase space density with energy (a slight increase of $V$ is not sufficient to balance $\sim 50\%$ increase of $dn/dy$ as compared with SPS) and centrality [46]. Extrapolation of the RHIC phase space density measurements to low transverse momenta predicts $\langle f \rangle_p$ close to unity for central events, suggesting that significant multiboson effects can be present at low $p_t$ at RHIC.

According to lattice Monte Carlo calculations including dynamical fermions, deconfining phase transition leading to a quark-gluon plasma (QGP) phase of matter is accompanied by restoration of chiral symmetry. Subsequent phase transition into the hadronic phase can be revealed, particularly, through substantial delays in particle emission and/or, through the coherent component of the pion radiation. This component would be characterized by a narrow Poisson multiplicity distribution, contrary to wide multiplicity fluctuations in the usual BE condensate. The pions in the coherent state may appear from the decay of a quasi-classical pion field (the order parameter of the phase transition), the latter possibly related to the spontaneous chiral symmetry breaking via the formation of the disoriented chiral condensate (DCC) (see [47] and a review [48]).

The most plausible mechanism of DCC formation is a fast expansion of hot QGP resulting in a rapid supression of thermal fluctuations (quenching), which in turn triggers a dramatic amplification of soft pion modes. The detection and study of DCC is expected to provide valuable information about the chiral phase transition and vacuum structure of strong interactions. DCC formation is usually expected to be associated with large event-by-event fluctuations in the ratio of neutral to charged pions in a certain phase–space domain. The search for these fluctuations at CERN SPS has so far resulted in setting only an upper limit on the production of a single DCC domain [49]. The absence of experimental evidence for isospin fluctuations has been however recently claimed to be in agreement with presumably more realistic picture of an “unpolarized” DCC with the Fourier modes of the field randomly oriented in isospin space (instead of being aligned as in the original DCC scheme) [50]. The search for other DCC signatures like low momentum pion clusters is therefore important. Particularly, one can exploit the impact of the admixture of coherent radiation on the QS and Coulomb correlations of like and unlike pions [51]. Other possibilities of experimental investigations of BE condensate and DCC phenomena have been discussed, e.g., in [52, 53].

The presence of the coherent pions (or pions emitted in the same quantum state) manifests itself also as a suppression of the BE correlations of two or more identical pions [7, 54, 55, 56]. Unfortunately, there are also other reasons leading to the suppression of particle correlations. Besides the experimental effects like finite resolution and particle misidentification (that can be corrected for), presumably the most important one is the contribution of the particles emitted by long-lived sources [57], leading to the appearance of the parameter $\lambda < 1$ in Eqs. (5) and (6). Also the usual Gaussian parameterizations of the QS correlation functions may be inadequate and lead to $\lambda < 1$ in the presence of the sources with moderate but very different space-time characteristics [57, 58, 59].

In principle, the effect of long-lived sources can be eliminated in a combined analysis of two–pion and three–pion correlation functions. The measured quantity is the genuine three–pion correlation normalized with the help of the three two–pion contributions - its intercept measures the chaotic or coherent fraction [60]. First such measurements have been done only recently in heavy ion experiments at CERN SPS [61, 62] and RHIC [63] and, in $e^+e^-$ collisions at LEP [64]. The most accurate ones at RHIC and LEP indicate
a dominant chaotic fraction though the systematic errors allow for a substantial coherent component. Some sources of the systematic errors, e.g., the simplified treatment of the two-body Coulomb and strong FSI, can be overcome. However others, e.g., the approximate (factorization) treatment of the multiparticle FSI or the insufficiently differential analysis of the three-pion correlation function, can hardly be avoided at present computational and experimental possibilities.

5 Femtoscopy with unlike particles

The complicated dynamics of particle production, including resonance decays and particle rescatterings, leads to essentially non–Gaussian tail of the distribution of the relative distances $r^*$ of the particle emitters in the pair rest frame. Therefore, due to different $r^*$–sensitivity of the QS, strong and Coulomb FSI effects, one has to be careful when analyzing the correlation functions in terms of simple models. Thus, the QS and strong FSI effects are influenced by the $r^*$–tail mainly through the suppression parameter $\lambda$ already for distances of the order of inverse $q$–resolution (typically some tens fm) while, the Coulomb FSI is sensitive to the distances as large as the pair Bohr radius $|a|$; for $\pi\pi$, $\pi K$, $\pi p$, $KK$, $Kp$ and $pp$ pairs, $|a| = 387.5, 248.6, 222.5, 109.6, 83.6$ and $57.6$ fm, respectively. Clearly, the usual Gaussian parameterizations of the distributions of the components of the distance vector $r^*$ may lead to inconsistencies in the treatment of QS and FSI effects (the Coulomb FSI contribution requiring larger effective radii). These problems can be at least partially overcome with the help of transport code simulations accounting for the dynamical evolution of the emission process and providing the phase space information required to calculate the QS and FSI effects on the correlation function.

Thus, in a preliminary analysis of the NA49 correlation data from central $Pb+Pb$ 158 AGeV collisions [65, 66], the freeze–out phase space distribution has been simulated with the RQMD v.2.3 code [67]. The correlation functions have been calculated using the code of Ref. [8], weighting the simulated pairs by squares of the corresponding wave functions. The dependence of the correlation function on the invariant relative momentum $Q = 2k^*$ was than fitted according to the formula [65]

$$R(Q) = \text{norm} \cdot \text{purity} \cdot \text{RQMD}(r^* \to \text{scale} \cdot r^*) + (1 - \text{purity}); \quad (7)$$

to account for a possible mismatch in $\langle r^* \rangle$, the dependence on the $r^*$–scale parameter has been introduced using the quadratic interpolation of the points simulated at three scales chosen at 0.7, 0.8 and 1. The fitted values of the purity parameter are in reasonable agreement with the expected contamination of $\sim 15\%$ from strange particle decays and particle misidentification. The fitted values of the scale parameter indicate that RQMD overestimates the distances $r^*$ by 10-20%. Similar overestimation has been also observed when comparing RQMD predictions with the NA49 data on $pp$ and $\pi^+\pi^+$ correlations [68, 69, 70].

Recently, there appeared data on $pA$ correlation functions from $Au + Au$ experiment E985 at AGS [71] and $Pb + Pb$ experiment NA49 at SPS CERN [72]. As the Coulomb FSI is absent in $pA$ system, one avoids here the problem of its sensitivity to the $r^*$–tail. Also, the absence of the Coulomb suppression of small relative momenta makes this system more sensitive to the radius parameters as compared with $pp$ correlations [73]. In spite of rather large statistical errors, a significant enhancement is seen at low relative momentum,
consistent with the known singlet and triplet pA s-wave scattering lengths. In fact, the fits using the analytical expression for the correlation function (originally derived for pn system [8]) yield for the AGS data [66] the purity of $0.5 \pm 0.2$ and the Gaussian radius of $4.5 \pm 0.7$ fm. For the NA49 data the fitted parameters are [72] $0.17 \pm 0.11$ and $2.9 \pm 0.7$ fm. The fitted AGS purity is consistent with the estimated one, while the NA49 purity is about one standard deviation too low. Fixing the NA49 purity at the estimated value of 0.33, the Gaussian radius increases by about 1 fm and becomes $3.8 \pm 0.4$ fm [72]. The fitted AGS and NA49 radii are in agreement with the radii of 3-4 fm obtained from $pp$ correlations in heavy ion collisions at GSI, AGS and SPS energies.

6 Correlation measurement of strong interaction

In case of a poor knowledge of the two–particle strong interaction, which is the case for meson–meson, meson–hyperon or hyperon–hyperon systems, it can be improved with the help of correlation measurements. In heavy ion collisions, the effective radius $r_0$ of the emission region can be considered much larger than the range of the strong interaction potential. The FSI contribution is then independent of the actual potential form [75]. At small $Q = 2k^*$, it is determined by the s-wave scattering amplitudes $f^S(k^*)$ [8]. In case of $|f^S| > r_0$, this contribution is of the order of $|f^S/r_0|^2$ and dominates over the effect of QS. In the opposite case, the sensitivity of the correlation function to the scattering amplitude is determined by the linear term $f^S/r_0$.

The possibility of the correlation measurement of the scattering amplitudes has been demonstrated [66] in a recent analysis of the NA49 $\pi^+\pi^-$ correlation data within the RQMD model. For this, the strong interaction scale has been introduced (similar to the $r^*$-scale), redefining the original s-wave $\pi^+\pi^-$ scattering length $f_0 = 0.232$ fm: $f_0 \rightarrow \text{sisca} \cdot f_0$. The fitted parameter sisca $= 0.63 \pm 0.08$ appears to be significantly lower than unity. To a similar shift ($\sim 20\%$) point also the recent BNL data on $K_{l4}$ decays [76]. These results are in agreement with the two–loop calculation in the chiral perturbation theory with a standard value of the quark condensate [77].

Recently, also the singlet $\Lambda\Lambda$ s-wave scattering length $f_0$ has been estimated [66, 72] based on the fits of the NA49 $\Lambda\Lambda$ data. Using the analytical expression for the correlation function [78] (originally derived for $nn$ system [8]) and fixing the purity of direct $\Lambda$–pairs at the estimated value of 0.16 and varying the effective radius $r_0$ in the acceptable range of several fm, one gets [72] e.g., $f_0 = 2.4 \pm 2.1$ and $3.2 \pm 5.7$ fm for $r_0 = 2$ and 4 fm respectively (we use the same sign convention as for meson–meson and meson–baryon systems). Though the fit results are not very restrictive, they likely exclude the possibility of a large positive singlet scattering length comparable to that of $\sim 20$ fm for the two–nucleon system.

The important information comes also from $\Lambda\Lambda$ correlations at LEP [79]. Here the effective radius $r_0$ is substantially smaller than the range of the strong interaction potential, so the $\Lambda\Lambda$ correlation function is sensitive to the potential form and requires the account of the waves with orbital angular momentum up to $l \sim 20$ [80]. In Ref. [79], the strong interaction has been neglected and the observed decrease of the $\Lambda\Lambda$ correlation function

$^5$The $\Lambda\Lambda$ system is of particular interest in view of an experimental indication on the enhanced $\Lambda\Lambda$ production near threshold [74] and its possible connection with the 6-quark H dibaryon problem.
at small $Q$ has been attributed solely to the effect of the QS (Fermi-Dirac) suppression. The correlation function has been fitted by the expression\(^6\)

$$\mathcal{R} = 1 - \frac{1}{2} \lambda (1 + \mathcal{P}^2) \exp(-r_0^2 Q^2)$$

(8)

corresponding to the simple Gaussian distribution of the components of the relative distance vector $\mathbf{r}^*$ characterized by a dispersion $2r_0^2$. The fit results are however unsatisfactory for two reasons [80]: (i) the parameter $\lambda = 1.2 \pm 0.2$ (neglecting in Eq. (8) the $\mathcal{P}^2$ polarization term on a percent level) is significantly higher than the value of $\sim 0.5$ expected due to the feed-down from $\Sigma^0$ and weak decays; (ii) the parameter $r_0 = 0.11 \pm 0.02$ fm appears to be smaller than the string model lower limit of $\sim 0.2$ fm. Therefore, the observed anti-correlation at small $Q$ can be considered as a direct evidence for a repulsive core in the $\Lambda\Lambda$ interaction potential.\(^7\) In fact, reasonable fits can be achieved using the Nijmegen singlet potential NSC97e [81], rescaling the triplet one from Ref. [82] and, neglecting spin-orbit and tensor couplings. For example, at a fixed $\lambda = 0.6$, the fitted radius takes an acceptable value $r_0 = 0.29 \pm 0.03$ fm [80].

7 Accessing relative space–time asymmetries

The correlation function of two non–identical particles, compared with the identical ones, contains a principally new piece of information on the relative space-time asymmetries in particle emission such as mean relative time delays in the emission of various particle species [14]. It can be particularly useful in searches for the effects of the quark-gluon plasma phase transition like delays between the emission of strange and antistrange particles due to the process of strangeness distillation from the mixed phase. The important information is contained also in the spatial part of the asymmetry related, in particular, with the intensity of the collective flow [66].

Since the information on the relative space–time shifts enters in the two–particle wave function through the terms odd in $k^*\mathbf{r}^* = \mathbf{p}^*_i (\mathbf{r}^*_i - \mathbf{r}^*_2)$, it can be accessed studying the correlation functions $\mathcal{R}_{+i}$ and $\mathcal{R}_{-i}$ with respectively positive and negative projection $k^*_i$ of the momentum $k^* = \mathbf{p}^*_i = -\mathbf{p}^*_2$ on a given direction $i$ or, - the ratio $\mathcal{R}_{+i}/\mathcal{R}_{-i}$. For example, $i$ can be the direction of the pair velocity or, any of the out (x), side (y), longitudinal (z) directions. Note that in the LCMS system,

$$r^*_x \equiv \Delta x^* = \gamma_t (\Delta x - v_t \Delta t), \quad r^*_y \equiv \Delta y^* = \Delta y, \quad r^*_z \equiv \Delta z^* = \Delta z,$$

(9)

where $\gamma_t = (1 - v_t^2)^{1/2}$ and $v_t = P_t/P_0$ are the pair LCMS Lorentz factor and velocity. One may see that the asymmetry in the out (x) direction depends on both space and time asymmetries $\langle \Delta x \rangle$ and $\langle \Delta t \rangle$. In case of a dominant Coulomb FSI, the intercept of the correlation function ratio is directly related with the asymmetry $\langle r^*_i \rangle$ [83, 84] (see also [85]):

$$\mathcal{R}_{+i}/\mathcal{R}_{-i} \approx 1 + 2 \langle r^*_i \rangle/a,$$

(10)

\(^6\)The singlet and triplet contributions to the correlation function $\mathcal{R} = \mathcal{R}_s + \mathcal{R}_t$ are $\mathcal{R}_{s,t} = \tilde{\rho}_{s,t}[1 \pm \lambda \exp(-r_0^2 Q^2)]$, where $\tilde{\rho}_{s,t}$ depend on the $\Lambda$-polarization $\mathcal{P}$ according to Eq. (3) with $\mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}$.

\(^7\)The repulsive core arises due to the exchange of vector mesons and is present, e.g., in various Nijmegen potentials used for the analysis of the double $\Lambda$ hypernuclei. The core height and width are about 9 GeV and 0.4 fm respectively. The s–wave scattering length (effective radius) ranges from about 0.3 (15) fm to 11 (2) fm.
where \( a = (\mu z_1 z_2 e^2)^{-1} \) is the Bohr radius of the two-particle system taking into account the sign of the interaction (\( z_i e \) are the particle electric charges, \( \mu \) is their reduced mass).

At low energies, the particles in heavy ion collisions are emitted with the characteristic emission times of tens to hundreds fm/c so that the observable time shifts should be of the same order [14]. Such shifts have been indeed observed with the help of the \( R_+ / R_- \) correlation ratios for proton-deuteron systems in several heavy ion experiments at GANIL [86] indicating, in agreement with the coalescence model, that deuterons are on average emitted earlier than protons.

For ultra-relativistic heavy ion collisions, the sensitivity of the \( R_+ / R_- \) correlation ratio to the relative time shift \( \langle \Delta t \rangle \) (introduced \textit{ad hoc}) was studied for various two-particle systems simulated using the transport codes [85]. The scaling of the effect with the space-time asymmetry and with the inverse Bohr radius \( a \) was clearly illustrated. It was concluded that the \( R_+ / R_- \) ratio can be sensitive to the shifts in the particle emission times of the order of a few fm/c. Motivated by this result, the correlation asymmetry for the \( K^+ K^- \) system has been studied in a two-phase thermodynamic evolution model and the sensitivity has been demonstrated to the production of the transient strange quark matter state even if it decays on strong interaction time scales [87]. The method sensitivity to the space-time asymmetries arising also in the usual multiparticle production scenarios was demonstrated for AGS and SPS energies using the transport code RQMD [65, 83, 84].

At AGS energy, the \( Au + Au \) collisions have been simulated and the \( \pi p \) correlations have been studied in the projectile fragmentation region where proton directed flow is most pronounced and where the proton and pion sources are expected to be shifted relative to each other both in the longitudinal and in the transverse directions in the reaction plane. It was shown [84] that the corresponding \( R_+ / R_- \) ratios are sufficiently sensitive to reveal the shifts; they were confirmed in the directional analysis of the experimental AGS correlation data [88].

At SPS energy, the simulated central \( Pb + Pb \) collisions yield practically zero asymmetries for \( \pi^+ \pi^- \) system while, for \( \pi^+ p \) systems, the LCMS asymmetries are \( \langle \Delta x \rangle = -6.2 \) fm, \( \langle \Delta y \rangle = \langle \Delta z \rangle = 0 \), \( \langle \Delta t \rangle = -0.5 \) fm/c, \( \langle \Delta x^* \rangle = -7.9 \) fm in the symmetric midrapidity window\(^8\) [83] and, \( \langle \Delta x \rangle = -5.2 \) fm, \( \langle \Delta y \rangle = 0 \), \( \langle \Delta z \rangle = -6.5 \) fm, \( \langle \Delta t \rangle = 2.9 \) fm/c, \( \langle \Delta x^* \rangle = -8.5 \), for the NA49 acceptance (shifting the rapidities into the forward hemisphere) [65]. Besides, \( \langle x \rangle \) increases with particle \( p_t \) or \( u_t = p_t / m \), starting from zero due to kinematic reasons. The asymmetry arises because of a faster increase with \( u_t \) for heavier particle. The non-zero positive value of \( \langle x \rangle = \langle r_t \hat{x} \rangle \) (\( \hat{x} = p_t / p_t \) and \( r_t \) is the transverse radius vector of the emitter) and the hierarchy \( \langle x_\pi \rangle < \langle x_K \rangle < \langle x_p \rangle \) is a signal of a universal transversal collective flow [65, 66]. To see this, one should simply take into account that the thermal transverse velocity \( \beta_T \) is smaller for heavier particle and thus washes out the positive shift due to the transversal collective flow velocity \( \beta_F \) to a lesser extent. More explicitly, in the non-relativistic approximation, the transverse velocity \( \beta_t = \beta_F + \beta_T \); in the out-side decomposition, \( \beta_t = \beta_t \{ 1, 0 \} \), \( \beta_F = \beta_F \{ \cos \phi_r, \sin \phi_r \} \), \( \beta_T = \beta_T \{ \cos \phi_T, \sin \phi_T \} \). Due to the azimuthal symmetry, the vector of the transversal collective flow velocity \( \beta_F \) is parallel to the transverse radius vector \( r_t = r_t \{ \cos \phi_r, \sin \phi_r \} \) and, its magnitude depends only on \( r_t \): \( \beta_F = \beta_F (r_t) \). To calculate \( \langle x \rangle \), one has to average over four variables \( r_t, \phi_r, \beta_T \) and \( \phi_T \). At a fixed transverse velocity vector \( \beta_t \), only two of them (e.g., \( r_t, \phi_r \) or \( r_t, \beta_T \)) are independent. In particular, \( \beta^2_t = \beta^2_F + \beta^2_T - 2 \beta_F \beta_T \cos \phi_r \),

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\(^8\langle \Delta y \rangle = 0 \) due to the azimuthal symmetry and \( \langle \Delta z \rangle = 0 \) in a symmetric mid-rapidity window due to the symmetry of the initial system.
so the destructive effect of the thermal velocity $\beta_T$ on the out shift is clearly seen:

$$\langle x \rangle = \langle r_t \cos \phi_r \rangle = \left( r_t \frac{\beta_t^2 + \beta_F^2 - \beta_T^2}{2\beta_t \beta_F} \right).$$

(11)

The maximal out shift $\langle x \rangle_{\text{max}} = \langle r_t \rangle$ corresponds to zero thermal velocity. The shift vanishes when the width of the contributing interval $|\beta_t - \beta_F| \leq \beta_T \leq \beta_t + \beta_F$ becomes negligible compared with the characteristic width of the thermal distribution, e.g., at $\beta_t \to 0$ or $\beta_F \to 0$ or, for very light particles; the angle $\phi_r$ is then decorrelated from $\beta_T$ and so distributed uniformly in the full angular interval $(-\pi, \pi)$.\(^9\) As a result, in case of a locally equilibrated expansion process, one expects a negative asymmetry $\langle \Delta x \rangle \equiv \langle x_1 - x_2 \rangle$ provided $m_1 < m_2$. Moreover, this asymmetry vanishes in both limiting cases: $\beta_F \ll \beta_T$ and $\beta_F \gg \beta_T$.

These conclusions agree with the calculations in the longitudinal-boost invariant hydrodynamic model. Thus, assuming a linear non-relativistic transversal flow velocity profile $\beta_F = \beta_0 r_t / r_0$, the local thermal momentum distribution characterized by the kinetic freeze-out temperature $T$ and the Gaussian density profile $\exp(-r_t^2/(2r_0^2))$, one confirms a faster rise of $\langle x \rangle$ with $\beta_t$ for heavier particles (see the non-relativistic limit of Eq. (30) in Ref. [89]):

$$\langle x \rangle = r_0 \frac{\beta_t \beta_0}{\beta_t^2 + T/m_t}.$$  

(12)

The maximal magnitude of the asymmetry $\langle x_1 - x_2 \rangle$ at $\beta_{1t} = \beta_{2t} = \nu_t$ is achieved for an optimal value of the flow parameter $\beta_0 = T/(m_1 m_2)^{1/2} = T/(\gamma_t^2 m_1 m_2)^{1/2}$; e.g., for $\pi p$ pairs at $\nu_t = 0.6$ (close to a mean LCMS velocity of low-$Q$ $\pi p$ pairs in the NA49 experiment at SPS [65]) and $T = 120$ MeV, the optimal value $\beta_0 = 0.27$. The SPS data on particle spectra and interferometry radii in central $Pb + Pb$ collisions at 158 AGeV are consistent with the parameters $\beta_0 \approx 0.35$, $r_0 \approx 6$ fm and $T \approx 120$ MeV with the uncertainties of $10 - 20\%$ [20, 35, 90]. The corresponding out asymmetry for $\pi p$ pairs $\langle \Delta x \rangle \equiv \langle x_1 - x_2 \rangle \approx -4$ fm at $\nu_t = 0.6$. As for the longitudinal and time shifts, in the longitudinal-boost invariant hydrodynamic model $z = \tau \sinh \eta$ and $t = \tau \cosh \eta$, where $\tau$ is the proper freeze-out time and $\eta$ is the emitter rapidity. At a given $p_t$, the LCMS $\eta$-distribution of the contributing emitters is given by the thermal law $\exp(-m_t \cosh \eta/T)$. Being symmetric, it predicts vanishing longitudinal shift: $\langle z \rangle = \langle \tau \sinh \eta \rangle = 0$. To estimate the time shift, for $m_t > T$ one can write $\cosh \eta \approx 1 + \eta^2/2$ and get $\langle t \rangle \approx \tau(1 + \frac{1}{2}T/m_t)$.\(^10\) For the central $Pb + Pb$ collisions at SPS, $\tau \approx 8$ fm/c and the relative time shift $\langle \Delta t \rangle = \langle t_\pi - t_p \rangle \approx 3$ fm/c. This shift is about the same as predicted by RQMD for the asymmetric NA49 rapidity acceptance. The magnitude of the relative out shift in pair rest frame (determining the observable asymmetry), $\langle \Delta x^* \rangle \approx -7$ fm, is however lower than in RQMD due to $\sim 20\%$ lower magnitude of $\langle \Delta x \rangle$.

In fact, the NA49 data on $R_{+x}/R_{-x}$ ratio for $\pi^+ p$ and $\pi^- p$ systems show consistent mirror symmetric deviations from unity, their size of several percent and the $Q$-dependence being in agreement with RQMD calculations corrected for the resolution and purity [66, 70, 72]. Similar pattern of the correlation asymmetries has been reported.

\(^9\)Note that, irrespective of the thermal width, the side shift $\langle y \rangle = \langle r_t \sin \phi_r \rangle = 0$ since, due to azimuthal symmetry, the angles $\phi_r$ and $-\phi_r$ contribute with the same weights.

\(^10\)One also recovers the expression for the LCMS interferometry longitudinal radius squared [19]: $R^2 = \langle (z - \langle z \rangle)^2 \rangle \approx \tau^2 T/m_t$ up to a relative correction $O(T/m_t)$.\(^{10}\)
also for $\pi^\pm K^\mp$ and $\pi^\pm K^\mp$ systems in experiment STAR at RHIC. They seem to be in agreement with the hydrodynamic type calculations with a stronger transverse flow than at SPS and a box-like density profile (blast wave), and - somewhat lower than RQMD predictions [34, 46].

The finite widths of particle rapidity distributions require however a violation of the boost invariance. It can be parameterized by a Gaussian dispersion $\Delta \eta^2$ of the LCMS $\eta$-distribution centered at $-Y$, where $Y$ is the CMS pair rapidity; e.g., the data on central $Pb + Pb$ collisions at 158 AGeV are consistent with $\Delta \eta = 1.3$ [20]. As a result,

$$\langle z \rangle \approx -\tau Y (1 + \Delta \eta^2 m_t/T)^{-1}$$

(13)

and $\langle t \rangle$ acquires a $Y$-dependent contribution $\frac{1}{2} \tau Y^2 (1 + \Delta \eta^2 m_t/T)^{-2}$. For the asymmetric NA49 rapidity acceptance, the mean $\pi p$ pair rapidity $Y \sim 1.5$, $\langle z_\pi - z_p \rangle \approx -2.8$ fm and the $\pi p$ time shift at $Y = 0$ is increased by $\sim 0.7$ fm/c. This is in qualitative agreement with the RQMD predictions for the rapidity dependence of the longitudinal and time shifts. The magnitude of the $Y$-dependent shifts in the hydrodynamic model is however substantially smaller. Besides, the LCMS emission times in RQMD are by a factor of $2-3$ larger and show substantial dependence on the transverse velocity [65]. These differences may point to the oversimplified space-time evolution picture in the hydrodynamic model. Particularly, the neglect of $r_t$-dependence of the proper freeze-out time and of the longitudinal acceleration during the evolution may not be justified [20, 89].

8 Spin correlations

The information on the system size and the two–particle interaction can be achieved also with the help of spin correlation measurements using as a spin analyzer the asymmetric (weak) particle decay [65, 91, 92]. Since this technique requires no construction of the uncorrelated reference sample, it can serve as an important consistency check of the standard correlation measurements. Particularly, for two $\Lambda$–particles decaying into the $p\pi^-$ channel characterized by the asymmetry parameter $\alpha = 0.642$, the distribution of the cosine of the relative angle $\theta$ between the directions of the decay protons in the respective $\Lambda$ rest frames allows one to determine the triplet fraction $\rho_t = \mathcal{R}_t/\mathcal{R}$, where $\mathcal{R}_t$ is the triplet part of the correlation function (see the footnote in connection with Eq. (8)):

$$dN/d \cos \theta = \frac{1}{2} \left[ 1 + \alpha^2 \left( \frac{4}{3} \rho_t - 1 \right) \cos \theta \right].$$

(14)

Both the correlation and spin composition measurements were recently done for two–$\Lambda$ systems produced in multihadronic $Z^0$ decays at LEP [79, 93]. Except for a suppression at $Q < 2$ GeV/c, the triplet fraction $\rho_t$ was found to be consistent with the value 0.75, as expected from a statistical spin mixture. Such a suppression, as well as similar suppression of the usual correlation function, is expected due to the effects of QS and a repulsive potential core, and points to a small correlation radius $r_0 < 0.5$ fm [80].

The spin correlations allow also for a relatively simple test of the quantum–mechanical coherence, based on Bell–type inequalities derived from the assumption of the factorizability of the two–particle density matrix, i.e. its reduction to a sum of the direct products of one–particle density matrices with the nonnegative coefficients [92]. Clearly, such a form of the density matrix corresponds to a classical probabilistic description and cannot
account for the coherent quantum–mechanical effects, particularly, for the production of two Λ-particles in a singlet state. Thus the suppression of the triplet ΛΛ fraction observed in multihadronic $Z^0$ decays at LEP indicates a violation of one of the Bell-type inequalities $\rho_t \geq 1/2$.

9 Conclusions

Thanks to the effects of quantum statistics and final state interaction, the particle momentum and, recently, also spin correlations give unique information on the space–time production characteristics and the collective phenomena like multiboson and coherence effects and collective flows. Besides the flow signals from single-particle spectra and like-meson interferometry, rather direct evidence for a strong transverse flow in heavy ion collisions at SPS and RHIC comes from unlike particle correlation asymmetries. Being sensitive to relative time delays and collective flows, the correlation asymmetries can be especially useful to study the effects of the quark–gluon plasma phase transition. The correlations yield also a valuable information on the particle strong interaction hardly accessible by other means.

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