A Model Study of the Strength Distribution for a Collective State Coupled with Chaotic Background System

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ABSTRACT

We consider a model in which a collective state couples to a large number of background states. The background states can be chosen to have properties which are classically characterized as regular or chaotic. We found that the dynamical nature of the background system considerably affects some fluctuation properties of the strength function.
1. Introduction

Over the last twenty years a number of new giant resonances have been found in a broad range of the nuclear table. They are coherent particle-hole excitations carrying a large fraction of the energy weighted sum rules. In most cases they are embedded in a continuum and are damped due to particle escape. At not much high excitation energies, however, the damping of giant resonances is dominated by spreading, i.e., by coupling to a huge number of background states. It is also known that there is in some cases a mechanism which reduces the width of resonances [1].

In many theoretical calculations the spreading of a collective particle-hole state is accounted for by coupling to more complicated states, e.g., two-particle two-hole states. As the number of these states in heavy nuclei becomes enormously large it is necessary to truncate the configuration space, or to introduce some kind of statistical treatment. For instance, in ref.2, the spreading was discussed by means of the random matrix theory. Aside from the reduction mechanism mentioned above, the coupling matrix element to each of the background states is expected to behave randomly, reflecting a random nature of the latter. In fact, the single-particle width distribution of neutron resonances obeys the Porter-Thomas distribution consistent with the random matrix theory [3]. We may note that level statistics of excited states have been investigated in realistic shell model calculations [4,5]. Not much is known, however, if or what properties of a strength distribution of collective state are related to the dynamical properties of the background states.

In the present note we consider a model which imitates a coupling of a collective state to background states, the latter being chosen to have chaotic or regular dynamical properties. In this manner we investigate whether there is a characteristic signature in the strength function which reflects a nature of the background system.

2. Model

In the study of giant resonances, the Hamiltonian is normally diagonalized within the truncated space of one-particle one-hole states plus background states,
mainly two-particle two-hole states. In the present model we replace the background states with eigenstates of a system whose classical counterpart is well studied, while a collective mode is represented by a single boson excitation independent of the background system. The structure of the collective strength function depends both on the coupling Hamiltonian of the collective mode to the background and also on the dynamics of the background system itself. We adopt a simple ansatz for the coupling in order to focus on the latter effect. We start with the Hamiltonian

\[ H = H_{\text{coll}} + H_{\text{bg}} + V_{\text{coupl}}, \]  

\[ H_{\text{coll}} = \epsilon B^\dagger B, \quad H_{\text{bg}} = \frac{1}{2}(p_x^2 + p_y^2 + x^4 + y^4) - k x^2 y^2. \]  

Here \( H_{\text{coll}} \) represents a Hamiltonian for the collective excitation, where \( B^\dagger \) and \( B \) represent boson creation and annihilation operators. We write the boson vacuum as \( |0\rangle \). As the Hamiltonian \( H_{\text{bg}} \) of the background system we choose that of a two-dimensional anharmonic oscillator characterized by a single parameter \( k \). The classical counterpart of this Hamiltonian describes a transition from an integrable to a chaotic dynamical system[6]. The quantum spectra and the wave function characteristics follow the same trend[7,8]: At \( k = 0 \) the system is separable while at \( k = 0.6 \), for instance, the nearest neighbor spacing distribution of the quantum levels shows the Wigner distribution which is typical for chaotic systems. We denote the eigenstate of \( H_{\text{bg}} \) as \( |n\rangle \) (\( |n = 0\rangle \) for the ground state). In the diagonalization of \( H_{\text{bg}} \), we take as the basis states of the background system the eigenstates of an uncoupled harmonic oscillator whose frequency \( \Omega \) is determined so as to optimize the diagonalization of \( H_{\text{bg}} \) [7,8]. They are denoted by \( |N\rangle \) (\( |N = 0\rangle \) for the ground state) where \( N \) stands for a pair of integers, i.e., numbers of oscillator quanta in the \( x-\) and the \( y-\)directions.

The interaction \( V_{\text{coupl}} \) represents the coupling between the collective state and
the background states. We take a simple ansatz for the coupling,

$$V_{\text{coupl}} = \chi \sum_{N \neq 0} (B^\dagger |0\rangle \langle N| + \text{h.c.})$$

characterized by the strength parameter $\chi$. Note that the state $|N\rangle$ is different for each value of $k$ because of the difference in the $\Omega$ value. Due to the simple form of eq.(3), regular or random behavior of the coupling matrix elements depends entirely on the dynamics of the background system which can be controlled by the parameter $k$. The assumption (3) may not be realistic, as the coupling strength of the actual nuclear system would decrease for complicated states (i.e., large $N$). We expect, though, eq.(3) is sufficient for the present purpose of studying a qualitative difference coming from the dynamical structure of the background system.

We divide the whole space into four parts:

1. The ground state of $H_{\text{coll}} + H_{\text{bg}}$: $|0; 0\rangle \equiv |0\rangle|0\rangle$.
2. The one collective boson state: $|1; 0\rangle \equiv B^\dagger|0\rangle|0\rangle$.
3. The background states: $|0; n\rangle \equiv |0\rangle|n(\neq 0)\rangle$.
4. Other states.

In diagonalizing the Hamiltonian (1), we neglect the coupling between space 1 and space 2, and also omit the space 4. These approximations produce negligible effect.

The eigenstates of $H_{\text{bg}}$ are classified into several symmetry classes with no coupling among them [7]. In the present calculation we consider only those states which belong to the class symmetric in the $x-$, $y-$ and the diagonal($x = y$)-directions. First $H_{\text{bg}}$ is diagonalized within a large space (the number $N_{\text{max}}$ of basis states is 5776) and then the lowest 800 states are included in the diagonalization of the total Hamiltonian. As for the values of $k$, we consider three typical cases, i.e., $k = 0.0$ for an integrable background system, 0.2 for a partially irregular system, and 0.6 for an almost chaotic system. The value of $\epsilon$ has been fixed to 220, so that the collective state is located in the middle of the background 800 states.
and thus a large number of background states can be found in the neighborhood. For a fair comparison of the role of integrable versus chaotic background system one might better adjust $\epsilon$ value for each $k$ to give a similar local level density. Although the asymmetry in the level density of the background states around $\epsilon$ does affect the third moment of the strength function as discussed later, its effect on the fluctuation properties of the strength distribution is expected to be small. The coupling strength is mostly fixed to $\chi = 1.0$ in the calculation below.

### 3. Distribution of Strength

Collective strengths are measured with respect to the operator $\hat{O} = B^\dagger + B$, i.e.,

$$S(E) = \sum_\alpha \delta(E - (E_\alpha - E_{g.s.}))|_{\text{tot}} \langle \alpha | \hat{O} | g.s. \rangle_{\text{tot}} |^2,$$

where $|\alpha\rangle_{\text{tot}}$ denotes an eigenstate of $H$, and $E_\alpha$ the corresponding eigenvalue.

Before discussing the strength function $S(E)$ in detail, we first examine the distribution of the coupling matrix elements $v_n \equiv \langle 1; 0 | V_{\text{coupl}} | 0; n \rangle$. According to our choice of the interaction it is expected that the coupling matrix elements $v_n$ would behave regularly for $k$ small and randomly for $k$ in the chaotic regime. This is indeed so as seen in Fig.1 where the distribution of the coupling matrix elements is shown.

It is seen that the matrix element values for $k = 0.0$ are concentrated at $\pm 1$, while for $k = 0.6$ they show almost the gaussian distribution centered at zero and with the width of around 1.1. In fact, for a variable $v_n$ composed of a large number of
random elements the distribution would be a gaussian,

\[ P(v_n) = \frac{1}{\sqrt{2\pi a_n}} \exp\left(-\frac{v_n^2}{2a_n^2}\right), \quad (5) \]

with \( a_n = 1 \) from the normalization of \( v_n \).

Figure 2 shows a strength function \( S(E) \). In spite of the difference in the coupling matrix elements as seen above, the shape of \( S(E) \) looks rather similar for \( k=0.0 \) and 0.6. For the sake of quantitative discussion of the gross structure, let us consider the cumulant \( \langle E^n \rangle_c \) of the strength function. Several low order cumulants are given by

\[ \langle E \rangle_c = \langle E \rangle, \quad \langle E^2 \rangle_c = \langle E^2 \rangle - \langle E \rangle^2, \quad \langle E^3 \rangle_c = \langle E^3 \rangle - 3 \langle E^2 \rangle \langle E \rangle + 2 \langle E \rangle^3, \quad (6) \]

where \( \langle E^n \rangle \) is the \( n \)-th energy-moment of the strength function defined as,

\[ \langle E^n \rangle \equiv \int E^n S(E) dE. \quad (7) \]

We show in Table 1 the calculated cumulants up to the third order at three \( k \) values and for \( \chi = 1.0 \). Let us consider the second cumulant which can be written as,

\[ \langle E^2 \rangle_c = \sum_{n(\neq 0)} v_n^2, \]

\[ v_n^2 = \chi^2 |\langle 0 | 0 \rangle|^2 \sum_{N,N'(\neq 0)} \langle N | n \rangle \langle n | N' \rangle. \quad (8) \]
Eq.(8) shows that, if we do not truncate the number of \( |n\rangle \), we would obtain almost the same values of \( \langle E^2 \rangle_c \) determined by \( N_{\text{max}} \) for any \( k \) [9]. Indeed, the actual values of \( \langle E^2 \rangle_c \), as listed in Table 1, are all close to 800 for the three values of \( k \). The third cumulant can be written as,

\[
\langle E^3 \rangle_c = \sum_{n(\neq 0)} (\omega_n - \epsilon)v_n^2,
\]

where \( \omega_n \) denotes the eigenvalue of \( H_{\text{bg}} \). From Table 1, we find that the value of \( \langle E^3 \rangle_c \) decreases as \( k \) increases. This trend is mostly due to the difference in the level densities around \( \epsilon \) as mentioned before: If we artificially use the same sequence of \( \omega_n \) for the three \( k \) values we obtain almost the same values for the third cumulant. The apparent similarity of the gross structure of the strength function may be understood in this manner. We mention that the shape of the strength function is not much different for different values of \( \chi \), except that the width of the distribution is scaled accordingly.

![Figure 3](image-url)

The similarity of the strength function is only superficial, however. This is seen in the strength distribution \( P(S) \) as given in the upper part of Fig.3. The smooth curve shows a Porter-Thomas distribution which is expected to hold in the chaotic system as given in the random matrix theory [3,4]. Difference due to the dynamics of the background system can be more clearly seen in the lower part of Fig.3, where the distribution of the amplitude \( \sqrt{S} \) corrected for the energy dependent factor \( ((E-\epsilon)^2 + (\Gamma/2)^2)^{1/2} \) is shown. The latter has been introduced to remove approximately the energy denominator contribution: If we assume constant coupling matrix elements \( v_n = v_c \) and an equal level distance \( D_c \), the strength function will be given by \( S(E) \simeq (\Gamma/2\pi)/\{(E-\epsilon)^2 + (\Gamma/2)^2\} \), where \( \Gamma = 2\pi v_c^2/D_c \) [9]. This may be contrasted to the present calculation where the mean square value
of $v_n$ gives $\bar{v}_n^2 \approx \chi^2$ (see Table 1) and the mean level distance is $\bar{D} \approx 0.5$. Thus we choose $\Gamma = 4\pi\chi$ in Fig.3. The result shows that at $k = 0.6$ the distribution follows a gaussian, while at $k = 0.0$ it is peaked at unity. Notice that the former distribution is generic, while the latter, corresponding to the regular background system, depends on the specific structure of the coupling.

The strength distribution is one of the signatures which characterize the underlying dynamics of the system. It reflects only a part of the structure in the strength function of Fig.2. For instance, the structure of the eigenstates of $H_{bg}$ as a function of energy changes regularly at small $k$ values, while at $k = 0.6$ it strongly fluctuates from state to state[8]. A similar behavior may be expected to occur in the present coupled system. For this purpose we perform a moment analysis similar to the one applied for a multifractal system[10]. This analysis takes into account some features of the energy-strength correlation, and therefore can be another characteristic measure of the strength function independent of the distribution $P(S)$. Here the whole energy interval $\Delta E$ is divided into $L$ segments each having a width $\delta E = \Delta E/L$. (We take $\Delta E = E_{800th} - E_{g.s.}$ in the present case.) The strength in each segment is summed up to give the strength $P_j (j = 1, \ldots, L)$ for the $j$–th segment, with the normalization condition $\sum_j P_j = 1$. The $m$–th moment at the scale $\delta E$ is defined by

$$M_m(\delta E) \equiv \sum_{j=1}^{L} P_j^m. \quad (10)$$

We then study its behavior as we refine the scale, e.g., as $L = 2 \to 2^2 \to 2^3$ etc.

Figure 4 shows the dependence of the moments $M_m$ for $m = 2$ to 5 on the scale $\delta E$ at $k = 0.0$ and 0.6. We also show, for the sake of comparison, the one for the ideal case of equidistant eigenvalues with equal strength. Since we have
a discrete spectrum, the moment $M_m$ for small $\delta E$, i.e., for a large number of segments eventually reaches a fixed value. It is seen that the slope becomes almost constant around $\delta E \simeq 2$. We now consider the fractal dimension $D_m$ defined by

$$D_m \equiv \lim_{\delta E \to 0} \frac{B_m(\delta E)}{m - 1}, \quad B_m(\delta E) = \frac{\log M_m}{\log \delta E}. \tag{11}$$

In practice, the expression for $B_m(\delta E)$ is replaced with the ratio of the difference of $\log M_m(\delta E)$ to that of $\log \delta E$ in the appropriate interval of $\delta E$. The quantity $D_m$ reflects a state-to-state fluctuation of the strength function. In Table 2 we show the calculated values of $D_m$ for $m = 2$ to 5.

| Table 2 |

The adopted interval of $\delta E$ is indicated in Fig.4. The result shows that the strength function at $k = 0.6$ has a smaller $D_m$ value than that for $k = 0.0$ and for the ideal case. We also see from Fig.4 that the dependence of $M_m$ on $\delta E$ is smoother for $k = 0.6$ than the other cases. These results may indicate that the scaled moments (10) provide another characteristic signature which reflects the underlying dynamics of the system.

4. Summary

In summary, we constructed a model which simulates a collective state coupled to a background. We took a two-dimensional anharmonic oscillator as the background system which exhibits a regular or a chaotic spectrum depending on the parameter $k$. We found that the difference in the dynamics of the background system causes a characteristic difference in the strength distribution. We also suggested that the scaled moment analysis might be useful in the study of an energy-strength correlation, although it requires a further study in order to make clear whether the conclusion survives in a more generic context. In the actual
study of the strength function of a physical system, e.g., of the nuclear giant resonances, the finite experimental resolution as well as the effect of a continuum put a restriction on the applicability of the method presented here. Other possible methods, such as the autocorrelation function and the Fourier transform, are now under investigation[11].

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TABLES

Table 1. Cumulants of the strength function from the first to the third order for $\chi = 1.0$.

| $k$ | $\langle E \rangle_c$ | $\langle E^2 \rangle_c$ | $\langle E^3 \rangle_c$ |
|-----|---------------------|---------------------|---------------------|
| 0.0 | 220.0               | 842.5               | 28516.5             |
| 0.2 | 220.0               | 779.5               | 15784.9             |
| 0.6 | 220.0               | 965.9               | -1942.3             |

Table 2. Fractal dimension $D_m$ ($m = 2$ to $5$) at $k = 0.0$ and 0.6 for $\chi = 1.0$. Also those for the ideal case of equidistant energy eigenvalues with equal strength are listed for comparison.

| $k$    | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|--------|-------|-------|-------|-------|
| 0.0    | 0.73  | 0.71  | 0.70  | 0.69  |
| 0.6    | 0.64  | 0.56  | 0.51  | 0.47  |
| Ideal case | 0.98  | 0.98  | 0.97  | 0.97  |
FIGURE CAPTIONS

Fig. 1 Distribution of the coupling matrix elements $v_n$ for $\chi = 1.0$ at three values of $k$. The smooth curves show a normalized gaussian distribution having the same width as that of $S(E)$ at each $k$ value.

Fig. 2 Strength function $S(E)$ for $\chi = 1.0$ at $k = 0.0$ and 0.6.

Fig. 3 Upper part: Strength distribution $P(S)$ for $\chi = 1.0$ at three values of $k$. The smooth curves show the Porter-Thomas distribution. Lower part: Distribution $P(\sqrt{\tilde{S}})$ where $\tilde{S} = S((E - \epsilon)^2 + (\Gamma/2)^2)$. $\Gamma$ is fixed to $4\pi$. The smooth curves show the gaussian distribution.

Fig. 4 $M_m(\delta E)$ versus $\delta E$ for $\chi = 1.0$ at $k = 0.0$ and 0.6 as well as for the ideal case of equidistant energies with equal strengths. The lines correspond to $m = 2$ to 5 from the upper to the lower ones. The arrows indicate the interval of $\delta E$ where the fractal dimensions are evaluated.
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