Impact of finite temperature on a quasi one-dimensional BEC interferometer

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Abstract. We consider a time-dependent atom interferometer based on splitting and merging an elongated Bose-Einstein condensate (BEC) along the weakly bound axis. The interference phase is imprinted at maximum splitting, and both arms are then recombined. We discuss a framework to include thermally populated low-energy excitations. Previous studies performed at zero temperature in the mean field description are extended using a truncated Wigner method. First results are discussed for the dipole mode oscillations in the recombined trapping potential.

PACS number(s): 03.67.Lx, 32.80.Pj, 34.90.+q

Dated: 13 July 2005

Recent advances in atom cooling and manipulation have promoted matter wave interferometers to serious competitors of optical ones for precision measurements of inertial forces or gravity. This potential is expected to increase further when using Bose-Einstein condensates (BECs) because they provide intense and coherent sources [1]. Several groups are now pursuing technological applications of BECs in atom interferometers. It has been shown that when the matter wave circuit is integrated onto a solid-state substrate, one can achieve compact and rapidly operating devices like atomic clocks or sensitive magnetic field sensors [2, 3].

In a BEC, the interatomic interaction energy plays a significant rôle due to the high density. The main impact of these interactions on the dynamics of a BEC has been successfully understood, to a large extent, using a nonlinear Schrödinger equation, the Gross-Pitaevskii equation (GPE), that allows to compute density profiles, solitons, vortices, collective excitations etc. [4]. In condensate interferometry, the interaction energy translates into ‘nonlinear’ (density-dependent) phase shifts that are often difficult to distinguish from the sought-for ‘linear’ phase shifts. We have shown recently that the nonlinearity can also be exploited usefully to amplify a phase shift, in particular when a coherently split condensate is slowly recombined [5]. We discuss what happens to this process when low-energy excitations are thermally populated in the BEC. We use the truncated Wigner framework for the calculation [6, 7], outline it and present first numerical results.

1. The model

1.1. Zero temperature

We consider a quasi one-dimensional condensate, where the transverse (radial) trap frequency $\omega_\perp$ is much larger than the longitudinal (axial) one $\Omega$. If the radial degrees of freedom are
Figure 1. Left: Excitation spectrum of the lowest odd eigenstate of the nonlinear Schrödinger equation in the potential (2), for fixed separations $d$ between the trap minima. Blue circles (red squares): real (imaginary) part of the eigenvalues. Harmonic oscillator units relative to $d = 0$ are used. At large splitting, the vibration frequency in the minima is $\sqrt{2}\Omega$. Interaction strength $gN = 10$.

Right: Distribution of the number $N_0$ of condensate atoms for a given total atom number $N$. Temperature $k_B T = 30\hbar\Omega$.

Frozen to the ground state, the relevant dynamics is one-dimensional. We have first considered the problem at zero temperature (pure Gross-Pitaevskii regime), namely

$$i\hbar \frac{\partial \Psi(x, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) + Ng|\Psi(x, t)|^2 \right] \Psi(x, t)$$  \hspace{1cm} (1)

where the interaction constant is related to the three-dimensional scattering length $a_{3D}$: $g \approx 2\hbar\omega_\perp a_{3D} > 0$ (repulsive interactions). We use as trapping potential

$$V(x, t) = \frac{m\Omega^2}{2} \frac{(x^2 - d(t)^2)^2}{x^2 + d(t)^2}$$  \hspace{1cm} (2)

with $d(t) \equiv 2a\sin^2(\pi t/\tau)$ (half) the separation between the trap minima. The interferometer corresponds to the following scenario: at time $t = 0$ we have a condensate in the ground state of a harmonic trap (oscillation frequency $\Omega$). During $0 < t < \tau/2$ we adiabatically split the condensate by creating a barrier in the middle of the trap. At maximum splitting, $t = \tau/2$, a relative phase $\Theta$ is imprinted between the two parts of the BEC, e.g., by flashing a laser pulse onto one part [8]. During $\tau/2 < t < \tau$, we lower the barrier and restore the initial harmonic trap. For $t > \tau$ the condensate is left to evolve in the same harmonic trap as the initial one.

As we have seen in Ref.[5], a dark soliton is created when the two condensate parts merge. For an ideal gas, the merging produces a superposition of ground and first excited trap states that produces a dipole mode oscillation at the trap frequency. To read out the BEC interference, two ways are thus open: measure the amplitude of the condensate’s dipole mode or the amplitude of the oscillating gray soliton. In both cases, there is a steep slope in amplitude near a phase shift $\Theta = \pi$. This sensitivity can be understood from the Bogoliubov spectrum around the first excited state of the GPE at a fixed splitting $d(t)$. As can be seen in Fig.1, the spectrum contains a mode whose frequency is purely imaginary for some values of $d(t)$ (a slightly different spectrum was presented in Ref.[9]). The odd condensate state created by the phase imprint $\Theta = \pi$ is thus dynamically unstable while the traps are merging, and the even (ground) state rapidly grows.
1.2. Finite temperature

In order to describe the effects of the temperature on the performance of the interferometer, we use a classical field method based on the truncated Wigner approach [6, 7]. The standard GPE describes a pure condensate, neglecting the interactions between condensate particles and noncondensate ones. In the truncated Wigner approach, both condensate and noncondensate particles are described by a single classical field \( \Psi(x, t) \) that evolves according to the GPE. The noncondensate part is taken into account by the thermal occupation, at \( t = 0 \) of the corresponding condensate excitations.

We work in the \( U(1) \) symmetry-preserving Bogoliubov approach [10], where the total number of particles is conserved. The non-condensate part of the field operator at \( t = 0 \) is expanded as

\[
\hat{\Psi}_\perp(x) = \sum_k \left[ \hat{b}_k u_k(x) + \hat{b}_k^\dagger v_k(x) \right]
\]  

where the mode functions \( u_k(x), v_k(x) \) for the excitations are the eigenfunctions (with eigenvalue \( \epsilon_k \)) of the Bogoliubov operator

\[
\mathcal{L} \begin{pmatrix} u_k(x) \\ v_k(x) \end{pmatrix} = \begin{pmatrix} H_{\text{GP}} - \mu + QNg|\Phi|^2Q & QNg\Phi^2Q^* \\ -Q^*Ng(\Phi^2)^2Q & -[H_{\text{GP}} - \mu + Q^*Ng|\Phi|^2Q^*] \end{pmatrix} \begin{pmatrix} u_k(x) \\ v_k(x) \end{pmatrix}
\]

\[
H_{\text{GP}} = -\frac{k^2}{2m} \partial_x^2 + V(x, t = 0) + Ng_{1D}|\Phi(x)|^2
\]

\[
Q = 1 - |\Phi\rangle \langle \Phi|.
\]

Here, \( N \) is the total number of atoms, \( \mu \) the chemical potential and \( \Phi(x) \) the condensate wave function. The projector \( Q \) ensures that the excitations are orthogonal to the condensate wave function.

At thermal equilibrium in the canonical ensemble, the noncondensate atoms are described by the density operator

\[
\hat{\sigma}_{\text{ex}} = \frac{1}{Z} \exp \left[ -\beta \sum_k \epsilon_k \hat{b}_k^\dagger \hat{b}_k \right], \quad \beta = \frac{1}{k_B T}.
\]

where \( Z \) is the partition function and \( k_B \) is the Boltzmann constant. As already studied in [6], the Wigner representation gives the smallest errors when representing the density matrix of a Bose field in terms of quasi-distributions (compared to the Glauber \( P \) and the positive \( P \) distribution). The Wigner function of the operator \( \hat{\sigma}_{\text{ex}} \) is the Gaussian

\[
W(\{b_k\}, \{b_k^\dagger\}) = \prod_k \frac{2}{\pi} \tanh \left( \frac{\beta \epsilon_k}{2} \right) \exp \left[ -2|b_k|^2 \tanh \left( \frac{\beta \epsilon_k}{2} \right) \right].
\]

The Wigner function allows us to retrieve expectation values of symmetrically ordered operators \( \hat{b}_k, \hat{b}_k^\dagger \) in terms of an average over c-numbers \( b_k, b_k^\dagger \) whose probability distribution is the Wigner function. We can thus construct the random complex function \( \Psi_\perp(x) \) to represent the excited field operator. The non-condensed part of the density operator, e.g., is given by

\[
\frac{1}{2} \text{Tr} \left[ \hat{\sigma}_{\text{ex}} \left( \Psi_\perp^\dagger(x_1)\Psi_\perp(x_2) + \Psi_\perp(x_2)\Psi_\perp^\dagger(x_1) \right) \right] = \Psi_\perp(x_1)\Psi_\perp^\dagger(x_2)_{\text{W}}
\]

where the overbar means the statistical average. The dynamics of the finite-temperature sample is now obtained by generating an initial field with amplitudes \( b_k \) distributed according to Eq.(8). One can show within the Wigner framework that the time evolution is given approximately by the GPE (1) [6, 7], which we solve numerically using a split-step/fast Fourier method. The results are averaged over a sufficient number of initial data.
1.3. Practical remarks

The first step is to obtain the ground state of the condensate \( \Phi \) by solving the time-independent GPE with the total number of particles \( N \). Secondly we diagonalize the Bogoliubov operator (4), truncating the space of eigenfunctions to \( N_{\text{mod}} \) modes such that the largest eigenvalue does not exceed a few \( k_B T \). As explained in [12], this cutoff must also satisfy \( k_B T \gg \hbar \Omega \), where \( \Omega \) is the frequency of the initial harmonic trap. We randomly generate the excited field \( \Psi^\perp(x) \), and get as initial value for the classical matter field

\[
\Psi(x, t = 0) = \sqrt{N_0} \left[ \Phi(x) + \frac{\phi^{(2)}(x)}{N} \right] + \Psi^\perp(x),
\]

where \( N_0 \) is the number of condensate atoms, and the function \( \phi^{(2)}(x) \) is a correction that describes the depletion of the condensate. The number \( N_0 \), for a given excited field configuration \( \Psi^\perp(x) \), is computed from [12]

\[
N_0 = N - 1 + \frac{1}{4} \sum_{k=0}^{N_{\text{mod}}} \left[ 2 - \tanh(\beta \epsilon_k/2) \right] - \int dx \left[ |\Psi^\perp(x)|^2 - |\varphi(x)|^2 \right]
\]

\[
\varphi(x) = \sum_k \tanh(\epsilon_k \beta/2) \left( b_k u_k(x) - b_k^* v_k(x) \right),
\]

Averaging over \( \Psi^\perp(x) \), the full distribution of \( N_0 \) comes out correctly, not only its mean value. An example is shown in Fig.1. The depletion correction to the condensate wave function is computed from Eqs. (D2,D3) of Ref. [12], rotated into imaginary time. This calculation involves quantum averages that are computed with the help of Eq. (9) and a sample of initial conditions for \( \Psi^\perp(x) \).

2. Results

The following plots illustrate the sensitivity of the time-dependent interferometer to the initial temperature of the sample. The atom density is shown in Fig.2 for a single realization of the excitation field: one sees a significant amount of sound waves propagating through the condensate. These excitations are sufficiently strong to dissipate the dark soliton that would have been created at lower temperature (see Fig.1 of Ref. [5]). A more systematic investigation is plotted in Fig.2 where one of the interferometer readout signals, the amplitude of the condensate dipole oscillation, is shown for different phase shifts. Note the strong contrast to Fig.2 of Ref. [5] (zero temperature): the steep features around \( \Theta = \pi \) have disappeared, although the oscillation amplitude is slightly larger. There is still a phase dependence at finite temperature, though. We attribute this to the instability of the system during the merging stage. A similar effect has been found when the interferometer is subject to a fluctuating potential, as would be the case with thermal magnetic fields on a integrated atom chip device [5].

3. Conclusion

In this article we have discussed a quasi 1D matter wave interferometer using Bose Einstein condensates when finite temperature effects are relevant. In case of a genuine condensate the output signal can be measured either from the amplitude of the condensate’s dipole mode or from the oscillation of the created dark soliton. We have seen that the strong phase sensitivity of the interferometer is connected to a dynamical instability of the system when the BEC parts are recombined for a phase shift near \( \pi \).

The temperature also plays an important rôle for the output signal of the system. Though the soliton still exists, its oscillations are damped, because it mixes with elementary excitations...
Figure 2. Left: Atomic density as the interferometer arms are recombined. The gray lines crossing the condensate background correspond to thermally excited sound waves. Operation time $\Omega \tau = 70$, interaction strength $gN = 50$, temperature $k_B T = 30 \hbar \Omega$, maximum splitting $\max d = 4.4$, phase imprint $\Theta = 0.99 \pi$, simulation with $N = 10000$ atoms. Harmonic oscillator units are used.

Right: Amplitude of the condensate’s dipole oscillation mode after recombining the interferometer arms, vs. the phase shift $\Theta$ imprinted at maximum splitting. Same parameters as for the left figure. Average over 2000 realizations, with the error bars representing the statistical errors.

(sound waves). However, the dipole mode oscillations survive and their amplitude is even larger than for a pure condensate. But the sensitivity to a phase shift is significantly reduced.

In the future we shall investigate the effects of asymmetry in the potential, the free expansion when the axial confinement is shut down at maximum separation. A better understanding of how the output signal depends on the number of particles would also be desirable to estimate the interferometer’s ultimate sensitivity.

Acknowledgements.
We acknowledge financial support from the European Union under the contracts HPRN-CT-2002-00304 (FASTNet) and IST-2001-38863 (ACQP). A. N. is grateful to Alice Sinatra, Iacopo Carusotto and Yvan Castin for very helpful discussions on the truncated Wigner method. C. H. thanks Chris Westbrook and Klaus Mølmer for helpful comments.

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