Reconstruction of new holographic scalar field models of dark energy in Brans-Dicke universe

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Motivated by the work [K. Karami, J. Fehri, Phys. Lett. B \textbf{684}, 61 (2010)] and [A. Sheykhi, Phys. Lett. B \textbf{681}, 205 (2009)], we generalize their work to the new holographic dark energy model with $\rho_D = \frac{M^2}{16\pi}(\mu H^2 + \nu \dot{H})$ in the framework of Brans-Dicke cosmology. Concretely, we study the correspondence between the quintessence, tachyon, K-essence, dilaton scalar field and Chaplygin gas model with the new holographic dark energy model in the non-flat Brans-Dicke universe. Furthermore, we reconstruct the potentials and dynamics for these models. By analysis we can show that for new holographic quintessence and Chaplygin gas models, if the related parameters to the potentials satisfy some constraints, the accelerated expansion can be achieved in Brans-Dicke cosmology. Especially the counterparts of fields and potentials in general relativity can describe accelerated expansion of the universe. It is worth stressing that not only can we give some new results in the framework of Brans-Dicke cosmology, but also the previous results of the new holographic dark energy in Einstein gravity can be included as special cases given by us.

Keywords: holographic dark energy ; Brans-Dicke theory ; scalar field

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According to the recent observations of type Ia supernovae\textsuperscript{1}, we learn that the universe is undergoing an accelerated expansion, also along with the observations of CMBR anisotropy spectrum\textsuperscript{2} and large scale structure (LSS)\textsuperscript{3}. The fact implies that there must be some unknown component in universe which can drive the accelerated expansion of the universe. This component is named as dark energy (DE), and holds a large negative pressure. After this, lots of candidates to dark energy have been suggested: (i) the cosmological constant\textsuperscript{4}; (ii) scalar fields such as quintessence\textsuperscript{5}, phantom\textsuperscript{6}, K-essence\textsuperscript{7}, tachyon field\textsuperscript{8}, dilatonic ghost condensate\textsuperscript{9}, and so forth; (iii) The interacting DE models including Chaplygin gas\textsuperscript{10}, generalized Chaplygin gas\textsuperscript{11–13}, braneworld models\textsuperscript{14} and agegraphic DE models\textsuperscript{15,16}, etc.. Besides, holographic dark energy (HDE) model\textsuperscript{17} is proposed on the base of the holographic ideas. According to the holographic principle, the number of degrees of freedom...
of a physical system scales with the area of its boundary. In this context, Cohen et al.\(^\text{18}\) suggested that one uses the event horizon as the cosmological horizon, and the total energy in a region of size \(L\) should not exceed the mass of a black hole of the same size. The holographic DE not only gives the observational value of DE in the universe, but also can drive the universe to an accelerated expansion phase. In that case, however, an obvious drawback concerning causality appears in this propose. This motivated Granda and Oliveros\(^\text{19}\) to propose a new infrared cut-off for HDE, which is about the square of the Hubble parameter \(H^2\) and the time derivative of the Hubble parameter \(\dot{H}\), this model is called as new HDE (NHDE) model in this paper. The NHDE model can avoid the problem of causality which appears using the event horizon area as the cut-off. By use of the new infrared cut-off for HDE, Granda and Oliveros\(^\text{20}\) study the correspondence between the quintessence, tachyon, K-essence and dilaton energy density with NHDE model in the flat FRW universe. Subsequently, Karami and Fehri\(^\text{21}\) generalize their work to the non-flat case.

On the other hand, it is quite possible that gravity is not given by the Einstein action, at least at sufficiently high energies. In string theory, gravity becomes scalar-tensor in nature. The low energy limit of string theory leads to the Einstein gravity, coupled non-minimally to a scalar field\(^\text{22}\). Although the pioneering study on scalar-tensor theories was done by Brans and Dicke several decades ago who applied Mach’s principle into gravity\(^\text{23}\), it has got a new impetus as it arises naturally as the low energy limit of many theories of quantum gravity (for example, superstring theory, et al.), and scalar-tensor theories of gravity have been widely applied in cosmology\(^\text{24–26}\). For HDE as a dynamical model, we need a dynamical frame to accommodate it instead of general relativity. Recently, the studies on the HDE model in the framework of Brans-Dicke theory have been carried out\(^\text{27–31}\), the purpose of which is to construct a cosmological model of late acceleration based on the Brans-Dicke theory of gravity.

In this paper, we will try to generalize the previous work in Refs.[21] and [27] to the NHDE model with \(\rho_D = \frac{3\phi^2}{4\omega} (\mu H^2 + \nu \dot{H})\) in the framework of Brans-Dicke cosmology. Hence, the evolution of equation of state (EoS) for the NHDE model will be discussed in non-flat Brans-Dicke universe. Moreover, we will study the correspondence between the quintessence, tachyon, K-essence, dilaton and Chaplygin gas model with NHDE model in the non-flat Brans-Dicke universe. Furthermore, we will reconstruct the potentials and dynamics for these models. By analysis we can show that for new holographic quintessence and Chaplygin gas models, if the related parameters to the potentials satisfy some constraints, the accelerated expansion of universe can be achieved in Brans-Dicke cosmology.

We start from the action of Brans-Dicke theory, in the canonical form it can be written\(^\text{32}\)

\[
S = \int d^4x \sqrt{|g|} \left(-\frac{1}{8\omega}\phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M\right),
\]

(1)

where \(R\) is the scalar curvature and \(\phi\) is the Brans-Dicke scalar field. The non-minimal coupling term \(\phi^2 R\) replaces with the Einstein-Hilbert term \(R/G\) in such a way that \(G^{-1} = 2\pi\phi^2/\omega\), where \(G\) is the gravitational constant. The signs of the non-minimal coupling term and the kinetic energy term are properly adopted to \((+ - - -)\) metric signature. Here, we consider the non-flat Friedmann-Robertson-Walker (FRW) universe which is described by the line element

\[
ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right),
\]

(2)
where $a(t)$ is the scale factor, and $k$ is the curvature parameter with $k = -1, 0, 1$ corresponding to open, flat, and closed universes, respectively. Varying action (1) with respect to metric (2), one can obtain the following field equations

\[
\frac{3}{4\omega} \phi^2 (H^2 + \frac{k}{a^2}) - \frac{1}{2} \phi^2 + \frac{3}{2\omega} H \dot{\phi} \phi = \rho_D, \tag{3}
\]

\[
-\frac{1}{4\omega} \phi^2 (2\ddot{a} + H^2 + \frac{k}{a^2}) - \frac{1}{\omega} H \dot{\phi} \phi - \frac{1}{2} \phi \ddot{\phi} - \frac{1}{2} (1 + \frac{1}{\omega}) \dot{\phi}^2 = p_D, \tag{4}
\]

\[
\ddot{\phi} + 3H \dot{\phi} - \frac{3\alpha}{2\omega} \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) \phi = 0, \tag{5}
\]

where \(\dot{}\) is the derivative with respect to time and $H = \dot{a}/a$ is the Hubble parameter. Here $\rho_D$ and $p_D$ are, respectively, the density and pressure of DE. And we neglect the contributions from matter and radiation in the universe, that is, $\rho_{\text{total}} = \rho_D$. In addition, we shall assume that Brans-Dicke field can be described as a power law of the scale factor, $\phi \propto a^\alpha$. Taking the derivative with respect to time, one can get

\[
\dot{\phi} = \alpha H \phi, \tag{6}
\]

\[
\ddot{\phi} = \alpha^2 H^2 \phi + \alpha \dot{\phi} H. \tag{7}
\]

According to Ref.[16], the HDE density with the new infrared cut-off is given by

\[
\rho_D = \frac{3}{8\pi G} (\mu H^2 + \nu \dot{H}), \tag{8}
\]

this is just the NHDE model, where $H = \dot{a}/a$ is the Hubble parameter, $\mu$ and $\nu$ are constants which must satisfy the restrictions imposed by the current observational data.

In the framework of Brans-Dicke theory, using $\phi^2 = \omega/2\pi G_{\text{eff}}$, one can obtain

\[
\rho_D = \frac{3\phi^2}{4\omega} (\mu H^2 + \nu \dot{H}). \tag{9}
\]

Thus, Eq.(3) becomes

\[
\frac{3}{4\omega} \phi^2 (H^2 + \frac{k}{a^2}) - \frac{1}{2} \phi^2 + \frac{3}{2\omega} H \dot{\phi} \phi = \frac{3\phi^2}{4\omega} (\mu H^2 + \nu \dot{H}). \tag{10}
\]

Furthermore, we obtain

\[
\frac{dH^2}{dx} + \frac{2s}{3\nu} H^2 = \frac{2k}{\nu} e^{-2x}, \tag{11}
\]

where $x = \ln a$ and $s = 2\omega \alpha^2 - 6\alpha + 3\mu - 3$. Integrating the above equation with respect to $x$ yields

\[
H^2 = \frac{3k}{s - 3\nu} e^{-2x} + Ce^{-2sx/3\nu}, \tag{12}
\]

where $C$ is an integration constant and $s \neq 3\nu$. For the special case of $s = 3\nu$, from Eq.(11), we have

\[
H^2 = \frac{2k}{\nu} e^{-2x} + Ce^{-2x}. \tag{13}
\]

But, here we only make the discussion of $s \neq 3\nu$. For the flat case $k = 0$ and using $\dot{x} = H$, from Eq.(11), one can reduce to

\[
H^2 = \frac{3\nu}{s} \dot{H}, \tag{14}
\]

where the dot denotes the time derivative with respect to the cosmic time $t$. Furthermore, one has

\[
H = \frac{3\nu}{s} \frac{1}{t}. \tag{15}
\]
It is easy to see from Eq.(14) that when $\alpha = 0$, Hubble parameter $H = \frac{\mu}{\nu - 1}$, which exactly reduces to the case in Ref.[20].

In addition, according to the conservation equation

$$\dot{\rho}_D + 3H(1 + w_D)\rho_D = 0,$$

and $w_D = p_D/\rho_D$, EoS can be expressed as

$$w_D = -1 - \frac{2\alpha\mu H^3 + 2(\alpha \nu + \mu)H\dot{H} + \nu \ddot{H}}{3H(\mu H^2 + \nu \dot{H})},$$

which is just the evolution of EoS for the NHDE model in non-flat Brans-Dicke universe. When $\alpha = 0$, it is easy to see that the Brans-Dicke scalar field becomes trivial, and Eq.(16) can reduce to the case of NHDE model in general relativity$^{20,21}$, i.e.,

$$w_D = -1 - \frac{2\mu H\dot{H} + \nu \ddot{H}}{3H(\mu H^2 + \nu \dot{H})}.$$  

(17)

It follows that the results in Refs.[20,21] are included as the special cases of $\alpha = 0$ and $k = 0$ given by us in this paper.

Furthermore, from Eqs.(12) and (16) we obtain

$$w_D = \frac{27e^{2s/3\mu k\nu}(1 + 2\alpha)(\mu - \nu) + X[2s - 9\nu - 6\alpha \nu]}{9\nu [X + 9e^{2s/3\mu k(\nu - \mu)}]},$$

(18)

where $X = C e^{2s}(s - 3\mu)(s - 3\nu)$ (Here, $s \neq 3\nu$ and $\mu \neq \nu$). Eq.(18) shows that EoS parameter is time-dependent, and can cross the phantom divide $w_D = -1$.$^{33-35}$ Of course, the current observations from the seven-year WMAP data and the analysis of SN1a and CMB data$^{36}$ favor the present values $w_0$ of EoS bigger than $-1$.

The evolutionary trajectories of EoS in Eq.(18) are plotted in Fig.1 (Here we take $k = 1$, $\omega = -1000$ and $\alpha = -1/1000$). From Fig.1, it is easy to see that the EoS of NHDE model can cross the phantom divide $w = -1$. Furthermore, we also give the present values of EoS $w_0$ in Tab.1. The values of $w_0$ are basically in the ranges of $w_0 = -1.10 \pm 0.14$, which are supported by the seven-year WMAP data and the analysis of SN1a and CMB data$^{36}$.

![Figure 1](image-url)  

**Figure 1.** The evolutionary trajectories of EoS in NHDE model. (a) for fixed constants $C$ and $\mu$ but different coupling constants $\nu$; (b) for fixed constants $C$ and $\nu$ but different constants $\mu$; (c) for fixed constants $\mu$ and $\nu$ but different constants $C$.
We know that Brans-Dicke cosmology becomes standard cosmology when $\omega \to \infty$, in this case $\alpha \to 0$, according to this result, we can obtain

\[
 w_D = -1 - \frac{2\alpha}{3} + \frac{2s}{9\nu}.
\]  

(20)

In addition, by use of $w_D < -1/3$ and the present values $H_0$, $t_0$, $a_0$, we can obtain the constraints of the parameters (such as $\mu$, $\nu$, $\omega$ and $\alpha$ etc.). For the non-flat case ($k \neq 0$), in Eq.(12), when taking $H = H_0$, $a = a_0 = 1$ (i.e., $x_0 = \ln a_0 = 0$), the integration constant $C$ can be expressed as $C = H_0^2 - 3k/(s - 3\nu)$. Thus, by means of EoS parameter $w_D < -1/3$ in Eq.(18), we get the following constraint

\[
\frac{3k(-6\nu - 12\alpha - 6\alpha^2 + 4\alpha^2) - H_0^2(2\alpha^2 - 6\nu - 12\alpha - 6\alpha^2)}{27k + 9H_0^2(3 + 6\alpha - 2\alpha^2)} < -1/3.
\]

Specially, for the flat case of $k = 0$, from Eqs.(14) and (20), we can obtain $H_0t_0 = 3\nu/s$, and $w_D = -1 - \frac{2\alpha}{3} + \frac{2s}{9\nu} < -1/3$, which leads to $\alpha > \frac{1}{3\alpha} - 1$. Furthermore, according to $H_0t_0 = \frac{1}{3\alpha} \ln \left[ \frac{1 + \sqrt{3\nu}}{1 - \sqrt{3\nu}} \right]^{37.38}$, we find when $\Omega_D \to 1$, $H_0t_0 \to \infty$. Hence, we can conclude that for the flat case of $k = 0$, the constraint on $\alpha$ is $\alpha > -1$.

Contrary to the non-flat case, the EoS for the flat case is constant. Furthermore, when $\alpha = 0$, Eq.(20) reduces to the case of NHDE model in general relativity

\[
 w_D = -1 + \frac{2(\mu - 1)}{3\nu}.
\]  

(21)

In order to obtain accelerated expansion, the constants $\mu$ and $\nu$ must satisfy the restrictions: for the quintessence-like phase with $-1 < w_D < -1/3$, we obtain $\mu > 1$ and $\nu > \mu - 1$, or, $\mu < 1$ and $\nu < \mu - 1$. For $\mu < 1$ and $\nu > 0$, or, $\mu > 1$ and $\nu < 0$, it describes a phantom-like phase with $w_D < -1$.

Below, we will study the correspondence between NHDE model with quintessence, tachyon, K-essence and dilaton scalar field models as well as Chaplygin gas model in the non-flat Brans-Dicke universe. Also, we will reconstruct the potentials and dynamics for these scalar field models. Specially, when taking $k = 0$, we can give the related results of scalar fields and potentials for the NHDE model in the flat Brans-Dicke universe. Furthermore, we can obtain the fields and potentials in general relativity, which describe accelerated expansion of the universe. To establish this correspondence, we compare energy density of the NHDE model given by Eq.(9) with the corresponding energy density of scalar field model, and also equate the EoS for these scalar

| $w_0(C = -3, \mu = 0.85)$ | $w_0(C = -3, \nu = 0.45)$ | $w_0(\mu = 0.85, \nu = 0.45)$ |
|-------------------------|-------------------------|-------------------------|
| -1.04030($\nu = 0.25$) | -1.14700($\mu = 0.80$) | -1.99726($C = -2$) |
| -1.04433($\nu = 0.35$) | -1.05785($\mu = 0.85$) | -1.05785($C = -3$) |
| -1.05785($\nu = 0.45$) | -0.97081($\mu = 0.90$) | -1.09249($C = -4$) |
| -1.07338($\nu = 0.55$) | -0.88619($\mu = 0.95$) | -1.11491($C = -5$) |

Table 1. The present values $w_0$ of EoS in Fig. 1.
models with the EoS given by Eq.(18). Here, we take scalar field as φ in order to differ from the parameter φ in Brans-Dicke theory.

1. New holographic quintessence model

The energy density and pressure of the quintessence scalar field φ are as follows:

\[ \rho_Q = \frac{1}{2} \dot{\phi}^2 + V(\phi), \]  

\[ p_Q = \frac{1}{2} \dot{\phi}^2 - V(\phi). \]  

The EoS for the quintessence scalar field is given by

\[ w_Q = \frac{p_Q}{\rho_Q} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \]  

Therefore, we have

\[ w_D = \frac{3\dot{\phi}^2}{4\omega}(\alpha H^2 + \beta \dot{H}) = \frac{1}{2} \dot{\phi}^2 + V(\phi). \]  

Furthermore, the kinetic energy term and the quintessence potential energy are

\[ \dot{\phi}^2 = \frac{e^{2(\alpha - s/3\nu - 1)x}[27e^{2sx/3\nu}k(1 - \alpha)(\mu - \nu) + X(3\alpha \nu - s)]}{18\omega \nu (s - 3\nu)}, \]  

\[ V(\phi) = \frac{e^{2(\alpha - s/3\nu - 1)x}[27e^{2sx/3\nu}k(2 + \alpha)(\mu - \nu) + X(s - 9\nu - 3\alpha)]}{36\omega \nu (s - 3\nu)}. \]  

Using \( \dot{\phi} = H \phi' \), where prime denotes the derivative with respect to x, we obtain

\[ \phi' = \frac{1}{H} \sqrt{\frac{e^{2(\alpha - s/3\nu - 1)x}[27e^{2sx/3\nu}k(1 - \alpha)(\mu - \nu) + X(3\alpha \nu - s)]}{18\omega \nu (s - 3\nu)}}, \]  

where \( H \) is given by Eq. (12). Consequently, after integration with respect to x we can obtain the evolutionary form of the quintessence scalar field as

\[ \phi(a) - \phi(0) = \int_0^a \frac{1}{H} \sqrt{\frac{e^{2(\alpha - s/3\nu - 1)x}[27e^{2sx/3\nu}k(1 - \alpha)(\mu - \nu) + X(3\alpha \nu - s)]}{18\omega \nu (s - 3\nu)}} dx, \]  

where we take \( a_0 = 1 \) for the present time.

For the flat case of \( k = 0 \), assuming \( \phi(0) = 0 \) for the present \( t_0 = 0 \), then Eqs.(28) and (30) reduce to

\[ \phi(t) = \sqrt{\frac{(s - 3\mu)(3\alpha \nu - s)}{18\omega \nu}} \frac{3\alpha \nu/s}{\alpha} \quad (\alpha \neq 0), \]  

\[ V(\phi) = \frac{\nu(s - 3\mu)(s - 9\nu - 3\alpha \nu)}{4\omega s^2} \left[ \sqrt{\frac{18\omega \nu}{(s - 3\mu)(3\alpha \nu - s)\alpha^2}} \right]^{2(1-s/3\alpha \nu)}. \]  

It is easy to see that when taking \( 3\alpha \nu/s = -1 \), Eq.(32) can reduce to \( V = V_1 \phi^4 \) (\( V_1 \) is a constant) which describes the accelerated expansion of universe. By using the decelerated parameter \( q \equiv -\ddot{a}/\dot{a}^2 < 0 \) and \( a = t^{-2/\alpha} \), we have \(-2 < \alpha < 0 \) or \( \alpha > 0 \). Furthermore, \( w_D = -1 - 2\alpha/3 + 2s/9\nu < -1/3 \) can give \( \alpha > -1/2, \)
so we obtain \(-1/2 < \alpha < 0\) or \(\alpha > 0\). In addition, considering that \(\frac{18\omega \nu}{(s-3\nu)(3\nu-s)} > 0\) in Eqs.(31) and (32), thus we have \(\omega < -3/2\) for \(-1/2 < \alpha < 0\) and exclude the case of \(\alpha > 0\). Therefore we believe that the potential (32) for the new holographic quintessence model, which can reduce to \(V = V_1 \varphi^4\), can describe the accelerated expansion of universe if the parameters satisfy the constraints \(3\alpha \nu/s = -1\), \(-1/2 < \alpha < 0\) and \(\omega < -3/2\).

Specially, when taking \(k = 0\) and \(\alpha = 0\) in Eqs.(28) and (29), the scalar field and potential can reduce to the case of NHDE model in general relativity

\[
\varphi(t) = \sqrt{\frac{2\nu}{\mu - 1}} M_p \ln t, \tag{33}
\]

\[
V(\varphi) = \frac{3\nu - \mu + 1}{(\mu - 1)^2} M_p^2 \exp \left(-\sqrt{\frac{2(\mu - 1)}{\nu} \varphi} \right). \tag{34}
\]

According to Ref.[40], this potential can describe an accelerated expansion provided that \(\nu/(\mu - 1) > 1\), and also has cosmological scaling solutions.

2. New holographic tachyon model

The energy density and pressure for the tachyon field are as follows

\[
\rho_T = \frac{V(\varphi)}{\sqrt{1 - \varphi^2}}, \tag{35}
\]

\[
p_T = -\frac{V(\varphi)}{\sqrt{1 - \varphi^2}}, \tag{36}
\]

where \(V(\varphi)\) is tachyon potential. The EoS for the tachyon scalar field is obtained as

\[
w_T = \frac{p_T}{\rho_T} = \varphi^2 - 1. \tag{37}
\]

By means of \(w_D = w_T\) and \(\rho_D = \rho_T\), we can obtain the kinetic energy term and the tachyon potential energy in the new holographic tachyon model as follows

\[
\dot{\varphi}^2 = \frac{4\nu^2 e^{2s \nu/k} \mu \nu (\alpha - 1)(\mu - \nu) + 2X(s - 3\alpha \nu)}{9\nu [X + 9e^{2s \nu/k} \nu (\nu - \mu)]}, \tag{38}
\]

\[
V(\varphi) = \frac{e^{2(s - 3\alpha \nu)} [9e^{2s \nu/k} \nu (\mu - \nu) - X] Y}{12\omega(s - 3\nu)}, \tag{39}
\]

where \(Y = \frac{27e^{2s \nu/k} [12\omega(1 + 2\nu)(\nu - \mu) - X(2s - 6\nu - 6\alpha \nu)]}{\nu [X + 9e^{2s \nu/k} \nu (\nu - \mu)]}\).

Furthermore, the evolutionary form of the tachyon scalar field can be obtained

\[
\varphi(a) - \varphi(0) = \int_0^{\ln a} \frac{1}{H} \frac{54\nu e^{2s \nu/k} \mu \nu (\alpha - 1)(\mu - \nu) + 2X(s - 3\alpha \nu)}{9\nu [9e^{2s \nu/k} \nu (\nu - \mu) + X]} dx. \tag{40}
\]

For the flat case \(k = 0\), using the initial condition \(\varphi(0) = 0\), we have

\[
\varphi(t) = \sqrt{\frac{2(s - 3\alpha \nu)}{\nu}} t, \tag{41}
\]

\[
V(\varphi) = \frac{3(3\mu - s) \mu^2}{4s^2} \sqrt{\frac{9
u + 6\alpha \nu - 2s}{\nu}} \left[\frac{9\nu}{2(s - 3\alpha \nu)} \varphi \right]^{2(3\alpha \nu/s - 1)}. \tag{42}
\]

Furthermore, when taking \(\alpha = 0\), Eqs.(41) and (42) can reduce to the counterparts in general relativity

\[
\varphi(t) = \sqrt{\frac{2(\mu - 1)}{3\nu}} t, \tag{43}
\]
According to Refs.[20] and [40], the inverse square potential in Eq.(44) can describe the accelerated expansion of universe when the parameters satisfy the condition $-1/9(1+\sqrt{10})<(\mu-1)/\nu<1/9(-1+\sqrt{10})$, which gives the only viable late-time attractor solution.

3. New holographic K-essence model

The energy density and pressure for the K-essence DE model are given by\footnote{The EoS is obtained as}

$$\rho(\varphi, \chi) = f(\varphi)(-\chi + 3\chi^2),$$ \hspace{1cm} (45)

$$p(\varphi, \chi) = f(\varphi)(-\chi + \chi^2).$$ \hspace{1cm} (46)

The EoS is obtained as

$$w_{KE} = \frac{p(\varphi, \chi)}{\rho(\varphi, \chi)} = \frac{\chi - 1}{3\chi - 1}.\hspace{1cm} (47)$$

Equating Eq.(47) with the EoS of NHDE, $w_{KE} = w_D$, one has

$$\chi = \frac{27\kappa^{2/3\nu}k\alpha(\mu + 2)(\mu - \nu) + X(s - 9\nu - 3\alpha\nu)}{3[27\kappa^{2/3\nu}k\alpha(\mu + 1)(\mu - \nu) + X(s - 6\nu - 3\alpha\nu)]}.\hspace{1cm} (48)$$

By use of Eq.(48), $\dot{\varphi}^2 = 2\chi$, and $\varphi = \varphi' H$, we obtain the evolutionary form of the K-essence scalar field as

$$\varphi(t) - \varphi(0) = \int_0^t \frac{1}{H} \sqrt{\frac{2[27\kappa^{2/3\nu}k(\alpha + 2)(\mu - \nu) + X(s - 9\nu - 3\alpha\nu)]}{3[27\kappa^{2/3\nu}k(\alpha + 1)(\mu - \nu) + X(s - 6\nu - 3\alpha\nu)]}} dx.\hspace{1cm} (49)$$

Furthermore, using $\rho_D = \rho(\varphi, \chi)$, Eqs. (3) and (12), the expression of $f(\varphi)$ can be given as follows

$$f(\varphi) = \frac{\nu^{2(\alpha-s/3\nu-1)}(9\kappa^{2/3\nu}k(\mu - \nu) - X)}{4\omega(s - 3\nu)\chi(3\chi - 1)}.\hspace{1cm} (50)$$

For the flat case of $k = 0$, assuming $\varphi(0) = 0$, then Eqs. (48), (49) and (50) become into

$$\chi = \frac{s - 9\nu - 3\alpha\nu}{3(s - 6\nu - 3\alpha\nu)},\hspace{1cm} (51)$$

$$\varphi(t) = \sqrt{\frac{2(s - 9\nu - 3\alpha\nu)}{3(s - 6\nu - 3\alpha\nu)}}t,\hspace{1cm} (52)$$

$$f(\varphi) = \frac{9\nu(s - 3\mu)(s - 6\nu - 3\alpha\nu)^2}{4\omega s^2(s - 9\nu - 3\alpha\nu)} \left[ \sqrt{\frac{3(s - 6\nu - 3\alpha\nu)}{2(s - 9\nu - 3\alpha\nu)}} \varphi^\nu \right]^{2(3\alpha\nu/s-1)}\hspace{1cm} (53)$$

Thus, when taking $\alpha = 0$, Eqs.(52) and (53) can reduce to $\varphi(t) = \sqrt{\frac{t}{3}} \left( \frac{2s - 2\mu + 1}{2s - 2\nu + 1} \right)^t$ and $f(\varphi) = 6\rho_D^2 \nu \left[ \frac{2s - 2\mu + 1}{(\mu - 1)^2} \right]^2 \varphi^2$. Hence, the potential of new holographic K-essence corresponds to the power-law expansion like new holographic tachyon model.

4. New holographic dilaton model

The energy density and pressure of the dilaton DE model are given by\footnote{The EoS is obtained as}

$$\rho_{DI} = \chi - 3c^2 e^{\lambda\varphi} \chi^2,\hspace{1cm} (54)$$

$$p_{DI} = \chi - c^2 e^{\lambda\varphi} \chi^2,\hspace{1cm} (55)$$
where \( c' \) and \( \lambda \) are positive constants and \( \chi = \dot{\phi}^2/2 \). The EoS for the dilaton scalar field is given by
\[
w_{D1} = \frac{p_{D1}}{\rho_{D1}} = \frac{\chi - c' e^{\lambda \phi} \chi^2}{\chi - 3c' e^{\lambda \phi} \chi^2}. \tag{56}
\]

By means of \( \omega_{D1} = \omega_D \), we find the following solution
\[
c' e^{\lambda \phi} \chi = \frac{27 \epsilon^{2s/3} k \nu (\alpha + 2)(\mu - \nu) + X(s - 9\nu - 3\alpha \nu)}{3[27 \epsilon^{2s/3} k \nu (\alpha + 1)(\mu - \nu) + X(s - 6\nu - 3\alpha \nu)]}. \tag{57}
\]

Moreover, using \( \chi = \dot{\phi}^2/2 \) and \( \dot{\phi} = \phi' H \), we have
\[
e^{\lambda \phi}(s) = e^{\lambda \phi(0)} + \frac{\lambda}{\sqrt{6c'}} \int_0^{\ln a} \frac{1}{H} \sqrt{\frac{27 \epsilon^{2s/3} k \nu (\alpha + 2)(\mu - \nu) + X(s - 9\nu - 3\alpha \nu)}{27 \epsilon^{2s/3} k \nu (\alpha + 1)(\mu - \nu) + X(s - 6\nu - 3\alpha \nu)}} \, dx, \tag{58}
\]
where \( H \) is given by Eq. (12). Therefore, the evolutionary form of the dilaton scalar filed can be written as
\[
\phi(t) = \frac{2}{\lambda} \ln \left[ \sqrt{\frac{s - 9\nu - 3\alpha \nu}{6c'(s - 6\nu - 3\alpha \nu)}} \right] \cdot \chi', \tag{59}
\]

For the flat case \( k = 0 \), using \( \dot{x} = H \), assuming \( \phi(0) \rightarrow -\infty \) for the present time \( t_0 = 0 \), then one can give
\[
\phi(t) = \frac{2}{\lambda} \ln \left[ \sqrt{\frac{s - 9\nu - 3\alpha \nu}{6c'(s - 6\nu - 3\alpha \nu)}} \right]. \tag{60}
\]

By using \( \alpha = 0 \), Eq.(60) becomes into \( \phi(t) = \frac{2}{\lambda} \ln \left[ \sqrt{\frac{\mu + \nu}{2\nu - \mu + 1}} \right] \cdot \chi'. \) According to Ref.[40], we know that for the dynamics system of this model, the condition of accelerated expansion of universe is \(-2 < (\mu - 1)/\nu < 1.\)

5. New holographic Chaplygin gas model

Kamenshchik et al.\(^{10}\) proposed a model of DE involving a fluid known as a Chaplygin gas(CG). This fluid also leads to the acceleration of the universe at late times, and its simplest form has the following specific equation of state
\[
\rho_{CG} = -\frac{A}{\rho_{CG}}, \tag{61}
\]
where \( A \) is a positive constant. With the continuity equation, Eq. (61) can be integrated to give
\[
\rho_{CG} = \sqrt{A + Be^{-6x}}, \tag{62}
\]
where \( B \) is an integration constant and \( x = \ln a \). We will establish the correspondence between NHDE model and CG model, that is, the new holographic CG model. To do this, we treat CG as an ordinary scalar field \( \varphi \). So we have
\[
\rho_{\varphi} = \dot{\varphi}^2/2 + V(\varphi) = \sqrt{A + Be^{-6x}}, \tag{63}
\]
\[
p_{\varphi} = \dot{\varphi}^2/2 - V(\varphi) = \frac{-A}{\sqrt{A + Be^{-6x}}}. \tag{64}
\]

Hence, it is easy to obtain the kinetic energy terms and the scalar potential for the CG as
\[
\dot{\varphi}^2 = \frac{Be^{-6x}}{a^6\sqrt{A + Be^{-6x}}}, \tag{65}
\]
\[
V(\varphi) = \frac{2A + Be^{-6x}}{a^6\sqrt{A + Be^{-6x}}}. \tag{66}
\]
By use of $\rho_{CG} = \rho_D$, one can obtain

$$B = \frac{e^{-4x/3\nu}[M - 16Ae^{(6+4x/3\nu)x}x^2(s - 3\nu)]}{16\omega^2(s - 3\nu)^2}, \quad (67)$$

where $M = C^2e^{(6+4x)(s-3\mu)}(s-3\nu)^2 + 81e^{(2+4x+4s/3\nu)x}k^2(\mu - \nu)^2 + 18Ce^{2x(6+4s/3\nu)}k(s-3\mu)(s-3\nu)(\nu - \mu)$.

And using $p_{CG} = \rho_{CG}w_D$, we have

$$B = Ae^{6x}\left[ -1 + \frac{9\nu X + 81\nu e^{2x/3\nu}k(\nu - \mu)}{27e^{2x/3\nu}k\nu(1 + 2\alpha)(\mu - \nu) + X(2s - 9\nu - 6\alpha\nu)} \right]. \quad (68)$$

So it is easy to give the expression of $A$ and $B$

$$A = -\frac{e^{-4x/3\nu}M[27e^{2x/3\nu}k\nu(1 + 2\alpha)(\mu - \nu) + X(2s - 9\nu - 6\alpha\nu)]}{144\omega^2\nu(s - 3\nu)^2[X + 9e^{2x/3\nu}k(\nu - \mu)],} \quad (69)$$

$$B = \frac{e^{-4x/3\nu}M[27e^{2x/3\nu}k\nu(\alpha - 1)(\mu - \nu) + X(s - 3\alpha\nu)]}{72e^2\nu(s - 3\nu)^2[X + 9e^{2x/3\nu}k(\nu - \mu)].} \quad (70)$$

Thus we can obtain the kinetic energy terms and the scalar potential for the Chaplygin gas as

$$\varphi^2 = \frac{[27e^{2x/3\nu}k\nu(\alpha - 1)(\mu - \nu) + X(s - 3\alpha\nu)]N}{18\nu[X + 9e^{2x/3\nu}k(\nu - \mu)]}, \quad (71)$$

$$V(\varphi) = \frac{[27e^{2x/3\nu}k\nu(\alpha + 2)(\mu - \nu) - X(s - 9\nu - 3\alpha\nu)]N}{36\nu[X + 9e^{2x/3\nu}k(\nu - \mu)]}, \quad (72)$$

where $N = \sqrt{\frac{2(6+4x/3\nu)x}{\omega^2(s - 3\nu)^2}}$. Using $\dot{\varphi} = H\varphi'$ and integrating Eq.(72), one can obtain the evolutionary form as

$$\varphi(a) - \varphi(0) = \int_0^{lna} \frac{1}{H} \sqrt{\frac{[27e^{2x/3\nu}k\nu(\alpha - 1)(\mu - \nu) + X(s - 3\alpha\nu)]N}{18\nu[X + 9e^{2x/3\nu}k(\nu - \mu)]}} dx. \quad (73)$$

For the flat case $k = 0$, assuming the initial condition $\varphi(0) = 0$, one can obtain

$$\varphi(t) = \sqrt{(s - 3\mu)(3\alpha\nu - s)} \frac{18\omega\nu}{18\omega\nu} \left( \alpha \neq 0 \right), \quad (74)$$

$$V(\varphi) = \frac{\nu(s - 3\mu)(s - 9\nu - 3\alpha\nu)}{4\omega^2s^2} \left[ \sqrt{\frac{18\omega\nu}{(s - 3\mu)(3\alpha\nu - s)}} \alpha \varphi \right]^{2(1-s/3\alpha\nu)}, \quad (75)$$

The discussion about the physical statement of the potential is the same as one in the new holographic quintessence model because the both has the same potential.

In summary, by generalizing the previous work\textsuperscript{21,27} to the NHDE model with $\rho_D = \frac{3d^2}{d^2}(\mu H^2 + \nu \dot{H})$ in the framework of Brans-Dicke cosmology, we have obtained the evolution of EoS and given the present values of EoS $w_0$ (see Tab.1), which are basically consistent with the present observation data $w_0 = -1.10 \pm 0.14$. Furthermore, we have established the correspondence between NHDE model with the quintessence, tachyon, K-essence, dilaton, CG model in the non-flat Brans-Dicke universe. Also, we have constructed the potentials and the dynamics of these models, and found that the dynamics and potential in the new holographic quintessence model are the same as ones in the new holographic CG model, but they are completely different from each other in the non-flat universe. For the new holographic quintessence and CG models, if the parameters of the potentials satisfy the constraints $3\alpha\nu/s = -1, -1/2 < \alpha < 0$ and $\omega < -3/2$, the accelerated expansion can be achieved in Brans-Dicke cosmology. It is worth stressing that not only have we given some new results of the NHDE model in the framework of Brans-Dicke theory, but also the previous results of the new holographic dark energy in Einstein gravity\textsuperscript{20} can be included as special cases of $\alpha = 0$ or $k = 0$ given by us, which can describe the accelerated expansion.
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