Modeling of nonlinear dynamic characteristics and experimental study of piezoelectric energy harvesters with a panel type structure

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Abstract. Panel flutter vibration is a typical aeroelastic phenomenon taking place in flying aircrafts as they move at high speed. The vibration has been a hot academic research aspect. The present paper focuses on converting this aerodynamic mechanical vibration into useful electricity by using piezoelectric patches. A mathematical Euler-Bernoulli distributed model of the piezoelectric energy harvesting system is established to investigate nonlinear dynamical behaviour of a piezoelectric cantilever plate. The dynamical responding ability of energy harvesting is numerically calculated. Influences of the length and the thickness of piezoelectric layer, the external resistance and wind velocity on the output performance are emphatically explored. A series of prototypes of piezoelectric energy harvesters with different physical dimensions were designed and manufactured. Wind tunnel experiments were setup and carried out at different wind velocities. The experimental results show that the harvesting power gets enhanced as increasing the wind velocity. The maximum power could be obtained at condition of a reasonable piezoelectric length and width. Maximum power is 2.0 mW at wind speed of 11.0 m/s. This work provides a strong theoretical and experimental basis for the study of aerodynamic energy harvesting from panel aircrafts.

1. Introduction
With the fast growth of micro energy equipment such as micro electro mechanical systems (MEMS), sensors and wireless devices, energy harvesting from ambient vibrations or environmental clean sources has drawn much attention to replace conventional chemical batteries in recent years [1, 2]. There are different mechanisms to collect electric energy such as electrostatic [3], electromagnetic [4] and piezoelectric [5, 6] conversions. Utilizing the piezoelectric conversion has attracted much attention because of the characteristics of easy fabrication, high power density and no electromagnetic interface. Based on the direct piezoelectric effect, piezoelectric energy harvester can generate electricity from mechanical vibrations such as flutter [7-9], galloping [10] or vortex-induced vibration [11-13] induced by aeroelastic force on the surfaces of Unmanned Air Vehicles (UAVs) when they fly at high speed. The researches on the plate structure on aircraft are mainly focused on the aeroelastic elastic vibration and vibration suppression, but there is little research in relationship to piezoelectric energy harvesting from the plate structure vibrating.

In terms of harvesting energy via piezoelectric harvesters, researchers have proposed various configurations. Taylor et al. [14] proposed an eel using the trail of vortices behind a bluff body to harvest water energy. Bryant et al. [15] designed a wing-shaped structure to capture vibration energy from aeroelastic flutter. The results showed the wing-shaped structures were significantly more efficient than
cantilever beams without wing-shaped blocks. Dias et al. [16] proposed a hybrid piezoelectric and electromagnetic energy harvester, the cut-in speed and maximum output power were calculated. In the field of piezoelectric plate, Doaré et al. [17] studied the linear stability analysis of a single piezoelectric plate and found that the energy convention efficiency could be maximized by modes destabilization. Several researchers begin to harvest energy from fluid-solid interaction vibrations. Song et al. designed an upright energy harvester with cylinder extension using bending vortex-induced vibration [18] or bending-torsion coupling vibration [19]. Yang et al. [20] studied diverse cross section for galloping energy harvesting and found the square section is best for small wind energy harvesters. However, these energy harvesters have bluff bodies, which are unsuitable for energy harvesting from moving aircrafts.

In this paper, we aim to investigate the non-linear dynamical behaviour of a piezoelectric cantilevered plate. In section 2, the energy harvesting system is proposed, and the analytical model is established. Numerical analysis and experiments are given in section 3. Conclusions part is given in section 4.

2. Physical construction and mathematical modeling
Figure 1 presents the structure of the piezoelectric energy harvesting system. This system comprises of the piezoelectric layer adhere to the copper substrate layer. A load resistance is connected to two surfaces of piezoelectric layer by two electric nodes. The piezoelectric cantilever is stationary in downstream and is unconstrained in upstream. The cantilever could vibrate under the excitation of air flowing force. When the wind speed reaches the critical flutter velocity, the wall plate will start to flutter. The output power can be obtained by a data acquisition system.

The length of piezoelectric sheet is $L_p$ and length of substrate sheet is $L_s$. $h_p$ and $h_s$ are the thicknesses of piezoelectric sheet and substrate sheet, respectively. The width of piezoelectric sheet is $b_p$ and width of substrate sheet is $b_s$. The parameters $t_a$, $t_b$ and $t_c$ are the relative position respect to the neutral axis.

![Figure 1. Schematic of a PEH system.](image)

To model this energy harvesting system, the following assumptions are expected to be adopted: the piezoelectric layer and substrate are bonded perfectly; the cantilever is Euler-Benoulli beam; the wind flow is laminar flow. The lateral displacement $w(x,t)$ can be obtained by Euler-Benoulli assumptions and expressed as

$$\frac{\partial^2 M(x,t)}{\partial x^2} + m \frac{\partial^4 w(x,t)}{\partial t^4} + c \frac{\partial^2 w(x,t)}{\partial t^2} = b_p \Delta P$$

where $m$ is the length density and expressed as $m = \rho_c h_b + \rho_s h_p [H(x-H(x-L_p))]$, $H(x)$ is the Heaviside step function. $c$ is damping coefficient. The strains referring to the neutral axis are given by

$$\varepsilon_p = -z \frac{\partial^2 w(x,t)}{\partial x^2}$$

The stresses in the $x$ direction of cantilever beam are expressed as

$$\sigma_b = \frac{E_b}{1-v_s} \varepsilon_p \quad \sigma_p = \frac{E_p}{1-v_p} \varepsilon_p - e_{31} E_3$$

where $E_b$ and $E_p$ are Young’s modulus of substrate sheet and piezoelectric layer, respectively. $e_{31}$ is piezoelectric stress coefficient and $E_3$ is the electric field. The Poisson’s ratio of substrate sheet and piezoelectric sheet are $v_b$ and $v_p$, respectively. The moment $M(x,t)$ is expressed as
\[
M(x,t) = \begin{cases} 
\int_0^L b_x \sigma_x dz - \int_0^\frac{L}{2} b_x \sigma_x dz & 0 \leq x < L_p \\
\int_0^\frac{L}{2} b_x \sigma_x dz & L_p \leq x < L
\end{cases}
\] (4)

By substituting Eqs. (3) to (5) into Eq. (2), the equation of energy harvesting system can be transformed into

\[
D(x) \frac{\partial^4 w(x,t)}{\partial x^4} + \int_0^L \frac{\partial^2 w(x,t)}{\partial x^2} + \nu \frac{\partial w(x,t)}{\partial t} + \theta \int_0^L \left[ \frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] = b_x \partial P
\] (5)

where \( V(t) \) is the output voltage and is given by

\[
V(t) = R_i(t) = R \frac{dq(t)}{dt}
\] (6)

The parameter \( \theta \) is electromechanical coupling coefficient given by \( \theta = -be_{31}[(t_b)^2-(t_c)^2]/(2h_p) \). The quantity of electric charge is expressed as

\[
q(t) = \int D \cdot ndA = \int_0^L \left( \frac{b_x + b_y}{2} e_{11} \right) dx + e_{12} b_x \int_0^L V(t) dx
\] (7)

The strain-voltage relation governing equation is

\[
\frac{V(t)}{R} + C_p \frac{dV(t)}{dt} + e_{12} b_x \int_0^L \frac{\partial^3 \omega(x,t)}{\partial x^3} dx = 0
\] (8)

The aerodynamic force per unit length on the plate [21] is

\[
\Delta p(x,t) = \frac{\rho_a}{\pi} \left[ \kappa(x) \left( \frac{\partial^2 w(x,t)}{\partial x^2} + 2U \frac{\partial w(x,t)}{\partial t} + U^2 \frac{\partial^2 w(x,t)}{\partial x^2} \right) + R(x) \right]
\] (9)

where the simplified parameters are given by

\[
\kappa(x) = (1 - \frac{x}{L})[\ln(1 - \frac{x}{L}) - 1] + \frac{x}{L} \ln \frac{x}{L} - 1
\] (10)

\[
R(x) = -\ln \left( 1 - \frac{w(x,t)}{l} \right) \left( U \frac{\partial w(x,t)}{\partial t} + U^2 \frac{\partial^2 w(x,t)}{\partial x^2} \right)
\] (11)

where \( U \) is wind velocity and \( \rho_a \) is the density of air. To solve the equation (9), the displacement of cantilever can be separated using Galerkin procedure as

\[
\omega(x,t) = \sum_{i=1}^N \phi_i(x) q_i(t)
\] (12)

where \( \phi_i(x) \) and \( q_i(t) \) are the model shapes and model coordinates, respectively. Considering the boundary condition of cantilever, the eigenfunction is expressed by [18]

\[
\phi_i(x) = \chi_{\beta} x - \cos \beta x - (sh \beta L - \sin \beta L) / (ch \beta L + \cos \beta L)(sh \beta x - \sin \beta x)
\] (13)

The governing equation of harvesting system can be obtained as

\[
\mathbf{M} \mathbf{q}'(t) + \mathbf{C} \mathbf{q}'(t) + \mathbf{K} \mathbf{q}(t) + \mathbf{A} \mathbf{V}(t) = \mathbf{F}(t)
\] (14)

where these parameters are as following

\[
\mathbf{M} = \int_0^L m(x) \phi_i^2(x) dx - \frac{\rho_a}{\pi} \int_0^L \kappa(x) \phi_i^2(x) dx
\]

\[
\mathbf{C} = -\frac{U \rho_a}{\pi} \int_0^L \left[ 2 \kappa(x) \phi_i(x) \phi_j(x) - \ln \left( 1 - \frac{x}{L} \right) \phi_i(x) \phi_j(x) \right] dx + \int_0^L c_i \phi_j(x) \phi_i(x) dx
\]

\[
\mathbf{K} = \int_0^L \frac{U^2 \rho_a}{\pi} k(x) \phi_i(x) \phi_j(x) - \ln \left( 1 - \frac{x}{L} \right) \phi_i(x) \phi_j(x)] + D(x) \phi_j^2(x) \phi_i(x) dx
\]

\[
\mathbf{A} = \theta \phi_i^2(L_p)
\]

\[
\mathbf{F}(t) = \int_0^L q_i(t) \phi_i(x) dx
\]

Substituting Eq. (12) into Eq. (8), the relationship between output voltage and bending displacement of cantilever can be obtained by
The governing equations are rewritten as
\[
\dot{Y} = \begin{pmatrix} Y_1' \\ Y_2' \\ Y_3' \\ Y_4' \\ V' \end{pmatrix} = \begin{pmatrix} q_i' \\ q_2' \\ q_3' \\ q_4' \\ V' \end{pmatrix} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ V \end{pmatrix} = \begin{pmatrix} G_1 - D_1 Y_1 - D_2 Y_4 - L_1 Y_1 - L_2 Y_2 - B Y_1 \\ G_2 - D_3 Y_1 - D_4 Y_4 - L_3 Y_1 - L_4 Y_2 - B Y_2 \\ Y_3 - \frac{1}{C_p} Y_3 Y_3 + \frac{1}{C_p} Y_4 Y_4 \\ Y_4 - \frac{1}{C_p} Y_3 Y_3 + \frac{1}{C_p} Y_4 Y_4 \\ \end{pmatrix}
\]
(18)
where
\[
D = M_y^T C_y \quad L_y = M_y^T K_y \quad G = M_y^T F_y \quad B = M_y^T A_y
\]
(19)

The space-state Eq. (18) can be solved numerically solved by ode45 solver in MATLAB software. The output power \( P \) can be calculated by \( P = V_{RMS}^2 / R \) where the RMS means root mean square voltage.

### 3. Numerical analysis

The physical properties of the energy harvesting system are presented in table 1. Figure 2 illustrates the influence of wind velocity \( U \) on the output power \( P \) with 100 kΩ load resistance. When the wind speed is lower than the critical flutter velocity at 10 m/s, the output energy is almost zero because of low vibration amplitude. After the wind speed reaches the critical flutter speed, the output power increases gradually as increasing the wind speed. It is a linear relationship between wind speed and output power. The maximum output power can reach almost 3.5 mW at the wind velocity of 11.0 m/s.

| Parameters                        | Values  |
|-----------------------------------|---------|
| Density of the piezoelectric layer \( \rho_p \) (kg/m\(^3\)) | 7850    |
| Density of the copper layer \( \rho_c \) (kg/m\(^3\))    | 8500    |
| Young modulus of the piezoelectric layer \( E_p \) (GPa)  | 66      |
| Young modulus of the substrate layer \( E_s \) (GPa)      | 105     |
| Poisson ratio of the copper layer \( v_b \) (Dimensionless) | 0.35    |
| Poisson ratio of the piezoelectric layer \( v_p \) (Dimensionless) | 0.3    |
| Length of the substrate layer \( L_s \) (mm)              | 200     |
| Width of the copper beam \( b_b \) (mm)                    | 20      |
| Width of the piezoelectric layer \( b_p \) (mm)            | 20      |
| Thickness of the piezoelectric layer \( h_p \) (mm)        | 0.2     |
| Thickness of the substrate layer \( h_s \) (mm)            | 0.2     |
| Piezoelectric stress coefficient \( e_{31} \) (C/m\(^2\)) | -6.6    |
| Air density \( \rho_a \) (kg/m\(^3\))                     | 1.2     |

Table 1. Geometric and physical properties of energy harvester.

Figure 3 shows the analytical output powers versus external resistance at different wind speed. The output powers are calculated at 10.3 m/s, 10.5 m/s and 11.0 m/s. With the increasing of the load...
resistance, the output power rises firstly until it reaches the maximum, and then it decreases. There is an optimal resistance for the energy harvesting system. The value of resistance is referred to as optimum resistance when output power is largest. It can be obtained from Figure 3 that there is an optimal resistance around 200 kΩ for all different wind speeds.

![Figure 2](image1.png)

**Figure 2.** Analytical output power $P$ versus wind speed $U$.

![Figure 3](image2.png)

**Figure 3.** Analytical output power $P$ versus load resistance $R$ at various wind speeds.

The analytical results of the output power versus the length-width ratio at different wind speeds are presented in Figure 4. With the length-width ratio increasing, the overall trend of output power goes up firstly and then drops gradually. When the length-width ratio is 0.2, the output powers get maximum values at all three wind speeds. The output power is mainly depended on the average stress of piezoelectric layer. When the length of piezoelectric layer is small, the total stress of the cantilever is little either. As the ratio $r$ increases, a slight drop in vibration amplitude will appear. With greater length of piezoelectric layer, the total stress is larger. However, when the length ratio increases to a certain value, but vibration amplitude drops rapidly although the piezoelectric sheet is longer. There is a peak output power when $r$ is a specific value. This result contributes to find the optimal $r$ in the design progress of energy harvester.

Figure 5 illustrates the relationship between the thickness of the piezoelectric sheet and the output power at three different wind speeds. When the thickness of piezoelectric plate increases continuously, the output power of the system has the tendency to increase first and then decrease. The optimal
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Thickness of piezoelectric sheet is 0.15 mm at wind velocity of 10.3 m/s while the optimal thickness is 0.175 mm at the speed of 10.5 m/s or 11 m/s. As the flow speed increases, the thickness of the piezoelectric plate increases to ensure the maximum output power. But too higher thickness of piezoelectric layer can reduce energy output.

\[
\begin{align*}
\text{Table:} \\
\text{Thickness of piezoelectric layer } h_p & \quad 0.15 \text{ mm} \\
\text{Output power } P (\text{mW}) & \quad 0.05, 0.10, 0.15, 0.20, 0.25, 0.35, 0.40 \\
\text{Wind speed } U & \quad 10.3 \text{ m/s}, 10.5 \text{ m/s}, 11 \text{ m/s} \\
\end{align*}
\]

Figure 4. Analytical results of the output power \( P \) versus the \( L_p/L_b \) ratio at different wind speeds.

Figure 5. Analytical results of the output power \( P \) versus thickness of piezoelectric sheet.

4. Experimental setup

The experimental study of wind speed on the harvested energy was shown in Figure 6. An adjustable load resistance is connected to the electric rods of harvester to study its effect on output power. The wind speed is measured by an anemometer in real time. The computer is utilized to store the voltage signal data and calculate the output power.

Figure 7 illustrates the relationship between the measured output power \( P \) and wind velocity \( U \). It can be found that the output power rises as the increasing of wind velocity. The trend of experimental results is consistent with theoretical analysis. The maximum output power is 2.0 mW at the wind velocity of 11.0 m/s. The main reason for the gap between theoretical and experimental results is that the experiment cannot fully satisfy the assumptions in theoretical deduction. The vibration of the cantilever beam contains twisting, which does not comply fully the assumption of the Euler Bernoulli beam. The
piezoelectric sheet is bonded to copper beam with epoxy glue and the thickness of epoxy layer is not taken into considered.

![Energy harvesting system](image)

**Figure 6.** Energy harvesting system.

**Figure 7.** Relationship between the measured output power $P$ and wind velocity $U$.

Figure 8 illustrates the measured output power $P$ versus resistance $R$. Consistent with the theoretical analysis, energy harvesting efficiency increases first and then decreases with the increasing of resistance. The experimental optimal harvesting resistance is 100 kΩ while the optimal resistance from theory analysis is 200 kΩ. The reason for this deviation is that the deduced theory is only taken the effect of piezoelectric static capacitance into consideration. The dynamic capacitance is ignored in the theory analysis, which results in the difference between theoretical and experimental optimal load resistance.
Figure 8. Measured output power $P$ versus load resistance $R$ at the wind speed of 12.0 m/s.

5. Conclusions
This work investigates the output power ability of piezoelectric energy harvester for converting wind energy to electricity. It is fabricated with a piezoelectric sheet and a copper layer. A mathematical model for numerically studying the output power of the energy harvester is established. Based on the mathematical model, the state-space equations are presented. The energy harvesting ability versus the wind velocities and the resistance are investigated numerically. Theoretical and experimental results show that the output power rises gradually as the increasing of the wind velocity after it reaches the critical flutter speed. The effects of the $L_p/L_b$ ratio and thickness of piezoelectric layer are further studied on the output power. Both the two parameters have optimal values with respect to maximum output power.

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