On the Nature of Core-Collapse Supernova Explosions

Adam Burrows and John Hayes
Departments of Physics and Astronomy, University of Arizona, Tucson, AZ 85721

and

Bruce A. Fryxell
Goddard Space Flight Center, NASA, Greenbelt, MD 20771

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ABSTRACT

We investigate in this paper the core-collapse supernova explosion mechanism in both one and two dimensions. With a radiation/hydrodynamic code based upon the PPM algorithm, we verify the usefulness of neutrino-driven overturn ("convection") between the shock and the neutrinosphere in igniting the supernova explosion. The 2-D simulation of the core of a 15M⊙ star that we present here indicates that the breaking of spherical symmetry may be central to the explosion itself and that a multitude of bent and broken fingers is a common feature of the ejecta. As in one-dimension, the explosion seems to be a mathematically **critical** phenomenon, evolving from a steady-state to explosion after a critical mass accretion rate through the stalled shock has been reached. In the 2-D simulation we show here, the pre-explosion convective phase lasted \( \sim 30 \) overturns (\( \sim 100 \) milliseconds) before exploding. The pre-explosion steady-state in 2-D is similar to that achieved in 1-D, but, in 2-D, due to the higher dwell time of matter in the overturning region, the average entropy achieved behind the stalled shock is larger. In addition, the entropy gradient in the convecting region is flatter. These effects, together with the dynamical pressure of the buoyant plumes, serve to increase the steady-state shock radius \( R_s \) over its value in 1-D by 30\%-100\%. A large \( R_s \) enlarges the volume of the gain region, puts shocked matter lower in the gravitational potential well, and lowers the accretion ram pressure at the shock for a given \( \dot{M} \). The critical condition for explosion is thereby relaxed. Since the "escape" temperature \( T_{\text{esc}} \) decreases with radius faster than the actual matter temperature \( T \) behind the shock, a larger \( R_s \) puts a larger fraction of the shocked material above its local escape temperature. \( T > T_{\text{esc}} \) is the condition for a thermally-driven corona to lift off of a star. In one, two, or three dimensions, since supernovae are driven by
neutrino heating, they are coronal phenomena, akin to winds, though initially bounded by an accretion tamp. Neutrino radiation pressure is unimportant.

We find that large and small eddies coexist, both before and after explosion. In the unstable region before explosion, columnar downflows are quasi-periodically formed and break up. These plumes excite non-linear internal g-modes that feed back onto the convection and cause the plumes to meander over the neutrinosphere. The radial neutrino flux fluctuates with angle and time in response to the anisotropic mass flux onto the neutrinospheres by as much as a factor of three. The boiling motion of the unstable region interior to the shock is epitomized by neutrino-heated bubbles that rise and collide episodically with the shock, whose radius oscillates in angle and time by as much as 30%. The angle-averaged neutrino luminosities vary by as much as 60% and decrease by a factor of two right after the explosion in a characteristic way.

The region interior to the neutrinosphere has weakly unstable lepton and entropy gradients that drive persistent convective motions after core bounce. However, the effects of this convection on the driving neutrino luminosities seem dwarfed by the effects of convective dredge up and by the wildly varying accretion component.

We see no evidence of a “building” or accumulation of energy before explosion, save in the kinetic energy, in response to the decaying accretion ram. In fact, the total energy in the overturning region decreases steadily before explosion. In addition, we have noted a non-trivial dependence on the neutrino transport algorithm. We would eschew terms such as “robust” when referring to the effect of convection on the outcome of collapse.

Neutrino energy is pumped into the supernova during the shock’s propagation through the inner many thousands of kilometers and not “instantaneously.”
Curiously, just after the explosion is triggered, the matter that will eventually be ejected is still **bound**. In addition, for a given asymptotic explosion energy, the amount of mass that reaches explosive nucleosynthesis temperatures is less than heretofore assumed. This may help to solve the $^{56}$Ni overproduction problem encountered in previous models of explosive nucleosynthesis.

The high-speed fingers that emerge from our model core seem a natural explanation for the nickel bullets seen in SN1987A and the shrapnel inferred in some supernova remnants. In addition, the vigorous convective motions interior to the shock can impart to the residue recoil velocities and spins. The magnitudes of the former might be within reach of the observed pulsar proper motions, but extensive new calculations remain to be done to verify this. Within 100 milliseconds of the explosion, a strong, neutrino-driven wind is blowing outward from the protoneutron star that clears the interior of mass and, while operative, does not allow fallback. At the base of the rising explosion plumes (in the early wind), a few high entropy ($\sim 60$) clumps are ejected, whose subsequent evolution may prove to be of relevance to the r-process.
1. Introduction

The supernova problem has been solved many times in the last thirty years, but never yet for long. Its persistence as a puzzle has many causes: 1) galactic supernovae are rare (\(\sim 1/30-100\) years), 2) expertise in a broad range of scientific and numerical subdisciplines seems to be required, 3) the launching of the explosion is obscured by a massive envelope impenetrable by photons, and 4) extragalactic supernovae have not until recently been observed in sufficient detail to provide theorists with hard constraints. Furthermore, the community has been divided into those emphasizing as primarily important either nuclear physics, progenitor models, neutrino interactions, multi-dimensional effects, numerical rigor, or exotic processes. This fragmentation may merely reflect the richness of a subject that involves most of twentieth-century physics. However, its complexity and highly nonlinear nature have left supernova theory vulnerable in the past to simplistic solutions that are only slowly refuted. Nevertheless, there has been much progress in the last few years in distinguishing the essential from the non-essential ingredients in a detailed supernova simulation, in discovering what does not work, and in establishing what will be required to satisfy the growing number of observational constraints. In particular, an understanding of hydrodynamic instabilities and overturn before, during, and after core collapse now seems to be central to the resolution of some or all of the supernova problem. This paper is the first in a new series of papers on the mechanism of supernova explosions. We present new one- and two-dimensional radiation/hydrodynamic simulations of the collapse, bounce, and explosion of the cores of massive stars, with special emphasis on multi-dimensional effects. In \(\S\) II, we describe the numerical scheme we have developed to perform supernova simulations in one and greater dimensions. In \(\S\) III, we discuss, in the context of new 1-D hydrodynamic simulations of stellar collapse, bounce, shock formation, and accretion, the demise of the prompt mechanism and quantities relevant in the study of the supernova
mechanism. The roles and characteristics of the progenitor models are discussed in §IV. This is followed in §V by a discussion of the neutrino-driven mechanism in one-dimension. The general physics of convective instability is described in §VI, in which we also survey the status of multi-dimensional supernova work. In §VII, our new two-dimensional supernova explosion calculations are depicted, described, and analyzed. This is followed in §VIII with a précis of our conclusions concerning multi-dimensional supernovae.

2. The Numerical Scheme

The multi-dimensional computer code we have used to simulate supernova explosions is a robust tool that can follow in a self-consistent fashion evolution from stellar collapse, to bounce, to explosion, through to the propagation of the blast out into the progenitor. To achieve flexibility and some speed, we have made many approximations in the neutrino transport module. In particular, we solve the neutrino diffusion equations, not the Boltzmann or transport equations, we have simplified the neutrino cross sections and sources, and we have dropped all relativistic terms. In addition, the gravity is Newtonian and only the monopole term in the potential is retained. The basic hydrodynamics code that we have embellished is PROMETHEUS (Fryxell, Müller, and Arnett 1989, 1991), itself a realization of the Piecewise-Parabolic-Method (PPM) of Colella and Woodward (1984). This hydro portion of the code is automatically conservative, explicit, Eulerian, and second-order accurate in space and time (error of $O(\Delta x)^2$ and $O(\Delta t)^2$), with a Riemann solver that enables us to resolve the shock position to within two zones. The diffusive transport is done implicitly to avoid severe time step limitations in the core, where neutrino processes achieve thermal and chemical equilibrium very quickly during post-bounce phases.

An Eulerian collapse code has intrinsic virtues and vices. Since the grid is fixed in
space and not in mass, winds and explosions can be more realistically handled than in Lagrangian codes. This is particularly important for supernova simulations. However, since the zones do not ride on the collapsing mass, one must pre-zone the core quite accurately to resolve the violence of bounce and shock formation. Typically, we have 100–400 radial zones interior to 100 kilometers during these dynamical phases, out of a total of 600–700 zones that span the entire inner 4500 kilometers (for models w*n or w*t). We have installed a rezoning feature that, while not “dynamical,” allows us to redistribute zones at various phases during a calculation, should we so desire. This 1-D→1-D mapper is augmented by a 1-D→2-D mapper that allows us to follow spherical (1-D) phases in 1-D and map to a 2-D calculation just before multi-dimensional effects are expected. In this way, we do not waste CPU time. We can also cut out an inner sphere in the middle of a calculation and replace it with a new inner boundary at the sphere’s outer surface that automatically uses the excised zones’ densities, pressures, energies, and fluxes for the new inner boundary. More powerfully, we can do the inner core’s hydrodynamics in one dimension, while continuing to follow the rest of the star in 2-D. As long as the inner core is spherical, this “restricted-2-D” allows us to avoid severe CFL restrictions, due to the convergence of the angular zones at the stellar center, while still following the core hydro and transport. A full 2-D run typically requires 50 trillion floating point operations. On the C90 at Pittsburgh, we achieve speeds of about 300 MFlops. The timestep during most of a 2-D calculation is 1–3 microseconds.

The nuclear equation of state (EOS) we have employed for the calculations discussed in this paper is similar to that used by Burrows and Lattimer (1985). The only difference is that in the current version the symmetry energies we employ are those of Lattimer and Swesty (1991). The EOS assumes NSE and a single representative heavy nucleus, includes alphas, neutrons, protons, photons, pairs, and neutrinos (only in the opaque regions), and is fully vectorized for optimal operation on CRAY architectures. Recently, we have tabulated for hydrodynamic use the full Lattimer and Swesty (1991) nuclear equation of state, which
after sufficient speed and accuracy are achieved, we plan to use in future calculations. The differences between the two EOS’s are slight, except near the phase transition at \( \sim 0.5\rho_{\text{nuc}} \) and at low densities \(( \leq 10^8 \text{ gm/cm}^3)\) and temperatures \(( < 0.7 \text{ MeV})\), where our alpha fractions are a bit too high.

The basic transport scheme is as described in Burrows and Lattimer (1986) and Burrows and Fryxell (1993). Radial transport along different angular rays is followed independently, but angular transport is suppressed, thereby suppressing various neutrino viscosity effects that are ignored in the basic hydro structure of PPM anyway. The transport is coupled to the hydro in operator-split fashion. Three neutrino fluids, \( \nu_e \)'s, \( \bar{\nu}_e \)'s, and “\( \nu_{\mu} \)”’s \((\nu_{\mu}, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau \)'s\) are followed in the diffusive approximation. In the semi-transparent regions \((\tau_\nu < 0.5)\), the rates of neutrino absorption and emission are assumed to be of the following simple forms.

For \( \nu_e (n, p) e^- \) and \( \bar{\nu}_e (p, n) e^+ \),

\[
\dot{\epsilon}_+ = 2.0 \times 10^{18} T_{\nu_e}^6 \left( \frac{F_5(\eta_{\nu_e})}{F_5(0)} Y_n + \left( \frac{T_{\bar{\nu}_e}}{T_{\nu_e}} \right)^6 \frac{F_5(\eta_{\bar{\nu}_e})}{F_5(0)} Y_p \right) \frac{\text{ergs}}{\text{gm} \cdot \text{s}} \tag{1}
\]

and

\[
\dot{\epsilon}_- = 2.0 \times 10^{18} T_e^6 \left[ \frac{F_5(\eta_e)}{F_5(0)} Y_p + \frac{F_5(-\eta_e)}{F_5(0)} Y_n \right] \frac{\text{ergs}}{\text{gm} \cdot \text{s}}, \tag{2}
\]

where \( T_{\nu_e} \) and \( T_{\bar{\nu}_e} \) are the electron neutrino and antineutrino temperatures at their respective neutrinospheres, \( \eta_{\nu_e} \) and \( \eta_{\bar{\nu}_e} \) are the assumed spectral pinch factors (Myra and Burrows 1990), \( \eta_e \) is \( \frac{4\pi}{3} T_e \), and \( T_e \) is the matter temperature. Guided by the work of Myra and Burrows (1990), we assume in these calculations that the emergent \( \nu_e \) and \( \bar{\nu}_e \) spectra are Fermi-Dirac in shape, with a default \( \eta \) of 2 and a temperature equal to the matter temperature at decoupling. The \( F_n(x) \)'s are the standard relativistic Fermi integrals. \( f \) is the spherical dilution factor that has the correct values at the neutrinospheres \((R_{\nu})\) and for \( r >> R_{\nu} \) and which we set equal to \( \frac{1}{2}(1 + (\frac{R_{\nu}}{r})^2)(1 - \sqrt{1 - (\frac{R_{\nu}}{r})^2}) \). The pair loss rate is
taken to be

\[ \dot{\epsilon}_p = 1.5 \times 10^{25} \frac{T_9^3}{\rho} \left[ \frac{F_4(\eta_e)F_3(-\eta_e) + F_4(-\eta_e)F_3(\eta_e)}{2F_4(0)F_3(0)} \right] \text{ergs} \left[ \text{gm} \cdot \text{s} \right]. \]  

(3)

As in Schinder and Shapiro (1982), neutrino-electron scattering is handled for each \( \nu_i - e^\pm \) combination as a Compton scattering problem:

\[ \dot{\epsilon}_{\nu_i e} = \frac{1.64 \times 10^{24}}{\rho} \Lambda_i \left( 1 + \frac{\eta_e}{4} \right) \frac{F_2(\eta_e)F_4(\eta_{\nu_i})}{F_2^2(0)} (T_{\nu_i} - T_e) T_{\nu_i} T_e^4 \frac{\text{ergs}}{\text{gm} \cdot \text{s}}, \]  

(4)

where \( \Lambda_i \) is a function only of the Weinberg angle. The electron number sources and sinks are derived using the same approach that gave eqs. (2) and (3), with the appropriate changes of sign, Fermi indices, and powers of \( T \). Note that \( \dot{\epsilon}_p \) is greater than \( \dot{\epsilon}_+ \) only for entropies above \( \sim 70 \).

The neutrino-matter cross sections used in the diffusive regime are those due to the standard neutral- and charged-current neutrino-nucleon interactions, with no corrections for ion-ion correlation or screening (Lattimer and Burrows 1991). Neutrino-electron scattering is ignored in the opacity and the matter and neutrino temperatures are assumed equal when \( \tau_\nu > 2/3 \). Levermore-Pomraning flux limiters are used. In addition, our Eulerian scheme does not capture the rapid break-out phenomenon very well and our neutrino luminosities between bounce and 10 milliseconds are not to be trusted. However, after break-out, our energy luminosities are competitive. Though this approach is highly simplified and a multi-group treatment would be preferable, such a more detailed scheme would multiply our CPU requirements by more than a factor of twenty. Nevertheless, our general results in 1-D are similar to those obtained by others who employ more precise neutrino transport schemes.
3. The Fall of the Prompt Mechanism

This section will not be a review of the details of stellar collapse, but a discussion and listing of some of its essentials. For a broader overview, the reader is referred to Burrows (1990), Bruenn (1985, 1989a,b), Woosley and Weaver (1986), and Bethe (1988).

At collapse, depending on the specific calculation and the progenitor mass \( M_{\text{ZAMS}} > 8 \, M_\odot \), the central density \( \rho_c \), temperature \( T_c \) and entropy are \( 4 \times 10^9 - 10^{10} \) gm/cm\(^3\), \( \sim 0.5 \) MeV, and \( 0.5 - 1.2 \) per baryon per Boltzmann’s constant, respectively. Due to electron capture during and after core carbon and oxygen burning, the central electron fractions \( Y_e \) hover around 0.43 and the electron chemical potential \( \mu_e \) is \( 8 - 10 \) MeV (Aufderheide et al. 1994; Nomoto, Thielemann, and Yokoi 1985). The “Chandrasekhar” mass of the iron \( M_{\text{ZAMS}} > 10 \, M_\odot \) or ONeMg \( 8 \, M_\odot < M_{\text{ZAMS}} \leq 10 \, M_\odot \) cores ranges between 1.2 \( M_\odot \) and \( \sim 2.0 \, M_\odot \) and the density structures show a correlated variation that determines the outcome of collapse (see §IV). The more compact configurations collapse to nuclear densities more quickly than those that are more extended. To illustrate stellar collapse, we have started with the 15 \( M_\odot \) (s15s7b) and 20 \( M_\odot \) (s20s7b) core models of Weaver and Woosley (1995). Figures 1a and 1b depict the evolution in 1-D of the mass density profiles versus interior mass from when a velocity near minus \( 10^8 \) cm/s is first approached (“a”) to 50–60 milliseconds after core bounce (“d”). Seven hundred radial zones were used. To ensure absolute consistency when comparing one-dimensional and two-dimensional calculations, we always used exactly the same code (see §II). Model w15t takes 209 milliseconds to bounce at \( \rho_c \sim 3.8 \times 10^{14} \) gm/cm\(^3\) and model w20t takes \( \sim 547 \) milliseconds, but in both \( \rho_c \) has climbed 4.5–5 orders of magnitude. The evolution of the central density for various 1-D models mentioned in this study is summarized in Table 1. The central density takes 145–167 milliseconds to go from \( 10^{10} \) to \( 10^{11} \) gm/cm\(^3\), then 20–25 milliseconds to achieve \( 10^{12} \) gm/cm\(^3\), then \( \sim 5 \) milliseconds to reach \( 10^{13} \) gm/cm\(^3\), and
finally only $\sim 1.5$ milliseconds to attain $10^{14}$ gm/cm$^3$. Within one millisecond of bounce, a strong sound wave that steepens into a shock wave near 0.6–0.8 $M_\odot$ is generated and penetrates the electron neutrinosphere near $10^{11}$ gm/cm$^3$ and 0.9–1.0 $M_\odot$. However, within ten milliseconds of bounce, the shock stalls near 90–110 kilometers and 1.2 $M_\odot$ due to neutrino losses at shock break-out and the dissociation of nuclei, the former being the more important. The bounce-shock fails to explode. Figures 2a and 2b depict the evolution of velocity versus interior mass and nicely illustrate the growth of the shock and its stall into accretion. Similarly, the evolution of the entropy and $Y_e$ profiles to $\sim 60$ milliseconds after bounce is depicted in Figures 3a, 3b, 4a, and 4b. A few generic features deserve mention. The entropy behind the shock first peaks near 0.9–0.95 $M_\odot$ (and near shock break-out of the electron neutrinospheres) at a value of 8–10. As the shock stalls, it leaves behind a negative entropy gradient that is unstable to overturn, but is smoothed by neutrino heat transport within $\sim 15$ milliseconds. However, though the entropy at $\sim 0.9$ $M_\odot$ decreases within tens of milliseconds to $\sim 5$, a region of negative entropy gradient between $\sim 0.9$ $M_\odot$ and 1.2–1.4 $M_\odot$ develops in the same region that the neutronization trough (Burrows and Mazurek 1983) appears. In this mass range, $Y_e$ and entropy gradients that would be Rayleigh-Taylor unstable in two or three dimensions are formed, in equilibrium with the neutrino diffusion of heat and leptons. Just behind the shock, electron capture on newly-liberated protons is rapid and can leave the matter with $Y_e$’s less than 0.1. In these calculations, the central trapped lepton fraction ($Y_e$) is 0.35–0.38, a bit higher than the preferred values ($\sim 0.33–0.35$, Bruenn 1985, 1992). If our electron capture and neutrino transport algorithm is modified to allow more capture on infall and delayed trapping, the smaller consequent $Y_\ell$ ($\sim 0.34$) results in a weaker bounce and shifts the first entropy peak to a value of $\sim 7$ near 0.8 $M_\odot$. In this case, the diffusion of heat does not smooth the negative entropy gradient imposed on the matter as the shock stalls. Thus, we see that the trapped lepton fraction has a quantitative effect on where and when matter is unstable to overturning motions, but that unstable
lepton and entropy gradients are always obtained early and maintained. This is important in the subsequent development of hydrodynamic instabilities. The sharp entropy spike seen just behind the shock in Figures 3 is a consequence of neutrino heating and has been seen by others (e.g. Bruenn 1992). In our calculations, the position of the shock is resolved to better than four kilometers. However, there are a number of vices in our approach that should be mentioned (see also §II). Capture on heavies is not included, electrons are assumed to be relativistic, general relativity is ignored, the phase transition near nuclear density is not done consistently, and neutrino transport is done in the diffusive approximation. Nevertheless, the generic character of the more detailed calculations of Bruenn (1992), Wilson (1985), Wilson and Mayle (1993), and Swesty et al. (1994) is reproduced. The luminosities, shock radii, velocities, and timescales, etc. are not precisely the same as obtained in those other papers. In particular, the advection terms in our transport module are not robust enough to accurately handle the rapid and violent break-out phenomenon. As a result, our break-out luminosities, in particular our anti-electron neutrino luminosities, are not correct during the first ten milliseconds after shock formation. Within twenty milliseconds of break-out, the scheme is back on track and any differences between our luminosities and those of others at this time are predominantly due to the different opacities, neutrino processes, and progenitors employed by the various groups. In particular, the opacities that we use result in slightly larger (but not extraordinarily so) core luminosities during the delay phase of the evolution. It is important to note that the differences after break-out between our results and those of other theorists are smaller than the range of results still possible due to the ambiguities in the nuclear symmetry energies, the nuclear incompressibility, and the neutrino opacities (Swesty et al. 1994; Bruenn and Mezzacappa 1994).

The positions of the shock and the various neutrinospheres (defined by $\tau = 2/3$) are depicted in Figures 5a and 5b for models w15t and w20t, respectively. For comparison, the angle-averaged shock radius versus time for the 2-D star calculation is depicted in Figure
5c and compared to the corresponding curve for its 1-D analog. This important plot will be discussed in §VII. In the 1-D calculations, the shock settles to near 80 kilometers and sinks at a rate of 0.3–0.4 kilometers per millisecond. The neutrinospheres are 20–30 kilometers interior to the shock, between 35 and 60 kilometers from the center, and sink at an average initial rate of ~0.4 kilometers per millisecond. The differences between the characteristics of the 15 M⊙ and 20 M⊙ models are not as important as their similarities. Figures 6a and 6b show the evolution of the neutrino luminosities 60 milliseconds before and after bounce for 1-D models w15t and w20t.

As stated above, the radiation module of the code does not handle the shock break-out phenomenon particularly well since this phase is fundamentally dynamical and rapid. As a consequence, the ringing seen in Figures 6 (with a period of ~5 milliseconds) is real, but its amplitudes may be smaller and the ratio of the $L_{\nu_e}$ to $L_{\nu_{\mu}}$ within ~10 milliseconds of break-out may be corrupted. In addition, our breakout $L_{\nu_{\mu}}$’s are probably too large. Nevertheless, the rapid rise time of $L_{\nu_{\mu}}$ and $L_{\nu_e}$ and the subsequent suppression of $L_{\nu_e}$ relative to $L_{\nu_{\mu}}$ due to the preponderance of electron neutrinos near the neutrinospheres are generic. The ringing is damped out after 3–4 cycles. The electron neutrino luminosity has a pre-bounce rise time of 10–16 milliseconds that reflects the nuclear symmetry energy employed (Swesty et al. 1994). Due to higher accretion rates, the luminosities in the 20 M⊙ model are larger than in the 15 M⊙ model, and they decay more slowly. A slow convergence of $L_{\nu_{\mu}}$ and $L_{\nu_e}$ (not seen in Figures 6) is a consequence of the high post-shock capture rates for models with small equilibrium shock radii ($R_s \sim 80$ km, as opposed to 150 km).

Figures 1–6 demonstrate, as has been concluded many times by others (e.g. Mazurek 1982; Burrows and Lattimer 1985; Bruenn 1985; Bruenn 1989a; Bruenn 1992; Wilson 1985; Baron and Cooperstein 1990) that the direct mechanism of supernova explosions (Colgate and Johnson 1960) does not work. The pressure deficits due both to electron capture and
the radiation of neutrinos of all species and to the photo-dissociation of infalling nuclei can not be overcome. The shock must stall within $\sim 10$ milliseconds of its birth. This conclusion does not depend on the incompressibility of nuclear matter, the trapped lepton fraction, or general relativity (Swesty et al. 1994; Burrows and Lattimer 1985; Bruenn 1985). Another simple way to see why the shock stalls is to compare the hydrodynamic power ($L_H = 4\pi r^2 P v$ or $4\pi r^2 (\frac{1}{2}) \rho v^3$) to the sum of the neutrino luminosities ($L_\nu$) and $L_D = \dot{M}_s \epsilon_d$, where $\epsilon_d$ is the energy required to dissociate a unit mass of “iron” ($\approx 8.8$ MeV/b) and $\dot{M}_s$ is the rate at which the shock encompasses mass (see below). While $L_H$ near break-out is a few times $10^{53}$ ergs/s, $L_T$ hovers near $10^{54}$ ergs/s and $L_D$ is also a few times $10^{53}$ ergs/s. Crudely, the shock stalls because $L_H \lesssim L_T + L_D$. Various codes and progenitors will result in different entropy and lepton profiles and stalled shock positions, but the fundamental outcome is the same. There has been considerable effort to prove otherwise to no avail. Even if the bounce shock were to succeed in a situation with “all the physics” ostensibly included, such an explosion would leave behind too small a neutron star ($\sim 1.1 \, M_\odot$ gravitational) and eject too much neutron-rich material ($\sim 0.2 - 0.3 \, M_\odot$) (Thielemann, Hashimoto, and Nomoto 1990).

There is nothing particularly esoteric or controversial about this conclusion. The explosion must occur after the shock stalls and is a function of the behavior of the quasi-hydrostatic protoneutron star, bounded by an accretion shock. The heating of the shocked envelope by the neutrinos from the core has been suggested as the driver of the explosion (Bethe and Wilson 1985 and §V), but it is yet to be shown that explosions with the required energy can occur after a delay without some sort of convective instability (§VI and §VII).

Before we turn to a short discussion of the progenitor models, some numbers derived from simulations w15t and w20t will prove useful. Though both the “15 $M_\odot$” and “20
"progenitor cores behave similarly, there are interesting differences that stem almost exclusively from their core structures. The 15 \(M_\odot\) model has an iron core mass of 1.28 \(M_\odot\) and a steep density gradient exterior to it that initially follows a \(1/r^{(3.5-3.8)}\) power law. The iron core of the 20 \(M_\odot\) star is 1.74 \(M_\odot\) and has a shallower density gradient that follows a \(1/r^3\) power law. The rate at which mass accumulates interior to the shock is very different for the two progenitors, each representative of the two basic classes of structures that modelers have published. Within 50 milliseconds of bounce, while the shock in model w15t bounds \(\sim 1.35 M_\odot\), that in model w20t already bounds \(\sim 1.5 M_\odot\). For model w20t, the mass accretion rate through the shock (\(\dot{M}_s\)) at this time is twice that for model w15t. Figure 7 depicts this mass accretion rate through the stalled shock and the mass interior to the shock (\(M_s\)) for models w15t and w20t during the first 60 milliseconds after bounce. Near break-out, \(\dot{M}_s\) is near 100 \(M_\odot/s\), but within 10 milliseconds it is below 10 \(M_\odot/s\) for both models. At \(\sim 50\) milliseconds, \(\dot{M}_s\) is \(\sim 5 M_\odot/s\) in model w20t. Importantly, \(\dot{M}_s\) and \(M_s\) after the shock stalls are functions, not of the physics of bounce or neutrinos, but of the initial progenitor structures (and the electron capture algorithm employed on infall). This is a consequence of the fact that the unshocked mantle is in supersonic infall and is not causally (sonically) connected to the core. All else being equal, \(\dot{M}_s\) and \(M_s\) after the shock settles into accretion are vital in determining the outcome of collapse (Burrows and Goshy 1993), the delay to explosion necessary to prevent the contamination of the ISM with exotic nuclear species, and whether a neutron star of the requisite gravitational mass is left. For instance, if the gravitational mass of the residual neutron star is 1.35 \(M_\odot\), its baryon mass must be \(\sim 1.5 M_\odot\). Model w15t would take \(\sim 500\) milliseconds to achieve such an \(M_s\), while model w20t would require only 50 milliseconds. This does not include the effect of fallback either early (within 20 seconds) or late, due to the creation of a reverse shock at the hydrogen/helium interface after many minutes to hours. However, such considerations set the timescales for action in these models. In addition, the times after bounce to accrete the
iron core edges if an explosion has not occurred are 25 and \( \sim 600 \) milliseconds for models w15t and w20t, respectively. Table 2 shows the edge radii for various 1-D models at various epochs. The locations of the edges of the iron cores at bounce are 540 and 1430 kilometers, respectively, and of the silicon outer edges are 2460 and 3270 kilometers, respectively. During infall, the iron core edges moved 600 kilometers and 800 kilometers for w15t and w20t, respectively. These numbers, and numbers like them for other supernova codes and progenitor models, provide the context in which to diagnose the various explosion scenarios. Another number of relevance is the mass \( (\Delta M) \) between the electron neutrinosphere and the shock versus time, both before and after explosion. In our models w15t and w20t, \( \Delta M \) at 50 milliseconds after bounce is only \( \sim 0.01 \, M_\odot \) and is decreasing with a mean life of 25–50 milliseconds. This is rather small and implies (with the considerations of §V) that in the 1-D calculations what has been accreted through the shock before explosion will remain, to within no more than 0.01\( M_\odot \).

The code that we have constructed can also be made to generate a variety of explosions by changing a variety of parameters. Such artificial explosions can be followed to very large radii and can be used to understand blast propagation through the rest of the progenitor in a natural and consistent way that partitions the internal and kinetic energies realistically (Aufderheide et al. 1994; Burrows and Hayes, in preparation). One such calculation is model w15n in which we have turned off all neutrino processes and frozen \( Y_e \). The velocity versus radius profiles for this model are depicted in Figure 8. Here, the shock succeeds and has reached 4100 kilometers within 180 milliseconds of bounce, despite the dissociation “losses.” Clearly, neutrino losses change the outcome in a qualitative way (cf. model w15t). The total kinetic energy of the blast is \( 0.85 \times 10^{51} \) ergs at 190 milliseconds, but troughs near \( 0.3 \times 10^{51} \) ergs earlier in the explosion. It grows due to the reassociation of the heavies and alphas previously broken up by the shock. The evolution of the kinetic energy with time is depicted in Figure 9. It plateaus off the graph near \( \sim 1 \times 10^{51} \) ergs. However, the
same sort of calculation with the 20 M⊙ model (w20n) did not explode. In that simulation, the shock reached ∼800 kilometers before stalling. In the 20 M⊙ model, there was too much matter at small radii to dissociate and too high a gravitational binding energy to overcome.

### 4. Progenitor Structures

Despite significant progress in recent years, our understanding of massive star progenitor structures is still incomplete. This is illustrated in Figure 10, which depicts the iron core masses (M_{Fe}) recently calculated by Weaver and Woosley (1993) and Weaver and Woosley (1995) under various assumptions and by Nomoto and Hashimoto (1988, NH). The former now prefer the “sb” series, but there is as yet no sign of convergence between the groups for a given ZAMS mass. Near 20 M⊙, M_{Fe} has ranged 0.6 M⊙ during the 1980’s and 1990’s, depending on the value of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate, the degree of overshoot, the handling of semi-convection, the degree of electron capture during burning, etc. (Arnett 1991; Woosley and Weaver 1986; NH). Bazan and Arnett (1994) have recently shown that after the onset of core oxygen burning convection is dynamical, the burning timescales are comparable to the convective overturn times, and the mixing-length prescription currently employed by the stellar modelers is suspect. As Weaver and Woosley (1993) have shown, the mapping of ZAMS mass to M_{Fe} and core structure is very sensitive to inputs, in what is perhaps a chaotic way. In addition, mass loss from the most massive stars (not included in the calculations of Weaver and Woosley 1993,1995) will change the core mass systematics in as yet unknown ways (Woosley, Langer, & Weaver 1994). In short, we can not yet say with any certainty what density profiles at collapse and what M_{Fe}’s correspond to what ZAMS mass progenitors. However, there are certain systematics that bear mentioning. As Figure 10 shows, there is a trend in M_{Fe} with M_{ZAMS} that reflects the higher core entropies expected at a given burning temperature for higher masses (NH). With increasing mass, the
relative carbon yields shrink and the duration and degree of carbon and neon shell burning decreases. This results in higher core entropies, fatter core envelopes, and higher effective Chandrasekhar masses. Overshoot, semi-convection, and a high $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate generally do the same (Woosley 1986). Though $M_{\text{Fe}}$ need not be monotonic in $M_{\text{ZAMS}}$ (Barkat 1975), there do seem to be two families of core structures at collapse: 1) low entropy ($< 1$), compact, low mass iron or ONeMg cores with outer mantle density profiles that are steep and 2) high entropy ($\geq 1$), fatter, high mass cores with shallow density profiles in their outer mantles. The latter generally have significantly more intermediate mass elements (Si, Ca, Ar, S, etc.) at collapse and higher oxygen yields. The oxygen yields increase systematically with ZAMS mass in a less ambiguous way. Mass density profiles from Weaver and Woosley (1995) representative of these two classes of cores are depicted in Figure 11. The two families are clearly distinguished, though the specific ZAMS mass indexing each profile should be given a lower significance. As we discussed in §III, the smaller cores accumulate mass more slowly, but explode more easily (at least in 1-D). The relative ease with which the smaller cores might explode can be explained in part by Figure 12, which depicts the behavior of the binding energy versus interior mass. The total binding energies of the models are the intercept values at $M = 0$. However, the mass cut is further out (a neutron star or black hole is left). For the fatter cores, the mass cut must be much further out for a "given" amount of energy of order $1 - 2 \times 10^{51}$ ergs, transferred either hydrodynamically or by neutrinos, to yield an explosion. The larger $\dot{M}_{\nu}$'s associated with the larger cores may delay the explosion sufficiently so that only lower neutrino luminosities are available to drive the explosion after it commences (see §V), resulting in an explosion energy that can not be much higher than that for the smaller cores. The delay to explosion for the more massive cores (if they explode) is followed by a slower expansion as the larger envelope binding energies are overcome, during which time the shocked matter can achieve higher entropies via neutrino heating than can matter in an exploding core of lower mass (Woosley).
et al. 1994a). These higher entropies might be relevant to the r-process (Meyer et al. 1992) and may not be achievable in 1-D for the “slimmer” cores (cf. 15 M⊙) (Burrows and Hayes in preparation). The fat cores can explode only after sufficient binding energy is accreted. If such cores reach the maximum mass of a neutron star before exploding, a black hole is the likely result. At any rate, binding energy arguments alone make it next to impossible for fat cores to yield the low-mass neutron stars that are observed in binary pulsar systems (Thorsett et al. 1993; Arzoumanian et al. 1994). Furthermore, the fatter cores generally explosively overproduce 56Ni by factors of 3–5, if the SN1987A, SN1993J, and SN1994I data are representative (0.05–0.09 M⊙; Bouchet et al. 1991; Nomoto, K., et al. 1993; Woosley et al. 1994b). The overproduction of 56Ni is correlated with the overproduction in such models of intermediate mass elements (Timmes, Woosley, and Weaver 1994), particularly of silicon. In addition, to avoid the overproduction of neutron-rich species, the mass cut must be at or exterior to the edge of the iron core, which for the fat models used here is at values of 1.74 and 1.78 M⊙, again too large to yield the measured pulsar masses (but, for a possible way out, see Fuller and Meyer 1994).

It might be thought that the mass cut of the explosion is out in the freshly synthesized 56Ni region and that much of the overproduction is buried in the core (Thielemann, Hashimoto, and Nomoto 1990). However, such behavior might require fine-tuning: the mass cut in “realistic” 1-D explosions is often determined either at explosion or as a result of reverse-shock reimplosion. To make matters worse, the best fit oxygen yield of SN1987A is ∼ 1.3 M⊙ (Chugai 1994; Spyromilio and Pinto 1991), just what some of the models which overproduce intermediate mass elements and 56Ni say the 6 M⊙ helium core of Sanduleak –69° 202 should produce (Woosley, Pinto, and Ensmann 1988; Thielemann, Hashimoto, and Nomoto 1990). One might be led to speculate that the fat cores either can not explode or if they explode must leave behind black holes or very fat neutron stars (Bethe and Brown 1994). One may also speculate that fat cores arise in stars with ZAMS masses higher than
the ∼ 18–20 Mₜ cut of models wwsb (Figure 10). The smaller cores of 18 – 20 Mₜ stars of Nomoto and Hashimoto may be favored for SN1987A, but the pure Schwarzschild condition that they employ in their calculations has recently been called into question (Stothers and Chin 1994). In a recent paper by Maeder (1992), it is argued on the basis of the observed $\frac{dV}{dz}$ of the galaxy that there must be a maximum mass above which a star can not be allowed to litter the ISM with all of its helium and metals. This may be germane to the fate of the fat cores identified above. Intriguingly, Maeder (1992) suggests a cutoff ZAMS mass of ∼ 20 – 25Mₜ (however, see Peimbert et al. 1994). (This conclusion depends sensitively on the IMF employed.) In short, there is as yet no consistent 1-D theory for massive star evolution and explosion that fits all the data. The problems with the progenitors are at least as severe as the problems with the 1-D explosion mechanism itself. In particular, the radii and masses of the various $Y_e$, entropy, and density ledges, that appear to be so sensitive to the convective algorithm, deserve further scrutiny. Though the $M_{ZAMS}$ to $M_{Fe}$ mapping is ambiguous, there do seem to be two classes of core structures with which a supernova modeler can profitably contend and which may yield different outcomes and products.

5. The Neutrino-Driven Mechanism (One-Dimension)

Fundamentally, the mechanism of a supernova explosion involves the transfer of energy from the core that remains to the mantle that is ejected. In the prompt mechanism, the piston action of the inner core was to do sufficient work on the outer core to unbind it. The residual protoneutron star would have been left more bound due to the work it performed. With the demise of the direct mechanism, the agency of energy transfer is now thought to be neutrinos (Wilson 1983; Bethe and Wilson 1985). Core electron- and antielectron-type neutrinos radiated from the neutrinospheres (see Figures 5a and 5b) deposit a small fraction of their energy in the outer shocked mantle as they emerge. The major processes are
neutrino-electron scattering can also heat the matter. We describe in this section some of the basics of this process that has yet to be definitively understood. In particular, without a convective boost in the driving neutrino luminosities or other convective effects, detailed 1-D calculations have failed to date to yield supernovae (Burrows 1987; Mayle and Wilson 1988; Bruenn 1987; Bruenn 1992; Burrows and Fryxell 1993; Herant, Benz, and Colgate 1992). The reader can refer to Janka and Müller (1993), Janka (1993), Burrows and Goshy (1993), and Bethe and Wilson (1985) for a more rounded perspective of 1-D neutrino-driven explosions.

The duration of the delay between the stall of the shock and its revitalization is unknown and could range between 50 milliseconds and a second. Delay has virtues: it allows low $Y_e$ material with $\eta$’s ($= 1 - 2Y_e$) greater than 0.01 to be buried, it fattens the residue so that observed neutron star gravitational masses ($M_G \sim 1.35 M_\odot$) can obviously be achieved, and it buries material with a high binding energy that would otherwise have to be overcome. However, if the delay is too long, once the supernova is launched the driving luminosities may have decayed too much to yield supernova energies in the required range ($1 - 2 \times 10^{51}$ ergs). It is yet to be determined whether a long delay is good (Woosley et al. 1994a) or bad (Takahashi, Witti, and Janka 1994) for the r-process, which seems to require high entropies (Meyer et al. 1992). As stated in §IV, in 1-D low-mass massive stars with thin envelopes may not be able to achieve the requisite entropies.

As Burrows and Goshy (1993) and Burrows (1987) have shown, the supernova is a coronal phenomenon: when temperatures reach “escape” temperatures a wind is driven off of the protoneutron star. The neutrino Eddington luminosity is $\sim 10^{55}$ ergs/s, too large by one to two orders of magnitude to be relevant. The protoneutron star bounded by an accretion shock can (before explosion) be treated quasi-statically like any star; in particular,
it can be subjected to a stability analysis. Burrows and Goshy (1993) showed that in 1-D there is a critical curve in the $L_\nu$ versus $\dot{M}$ plane, above which the protoneutron star is unstable to a bifurcation to an explosive solution. ($\dot{M}$ is the accretion rate at a radius of 200 kilometers, always exterior to the shock before explosion and not conceptually equal to $\dot{M}_s$. However, it is numerically very close to $\dot{M}_s$ during the quasi-static phase.) In this analysis, the shock radius was treated as the eigenvalue of the quasi-static problem. Above the critical curve, no solution to the steady-state can be found. The critical $\dot{M}$ is roughly proportional to $(L_{\nu e}/M_s)^{2.3}$ and $R_s$ in the steady-state is roughly proportional to $L_{\nu e}^2/\dot{M}$.

The gain radius (Mayle 1985) exterior to which heating ($H$) predominates over cooling ($C$) is where $H = C$. This is almost exactly at the entropy peak, since in the steady state, $u_1 T \frac{ds}{dr} \cong H - C$, where $u_1$ and $T$ are the post-shock settling velocity and the temperature, respectively, and $s$ is the specific entropy. Before explosion, the integral (exterior to the electron neutrinosphere) of the cooling function is always greater than that of the heating function. Before explosion, the energy deposited in the gain region by neutrinos bears no obvious relationship to the explosion energy. In spherical accretion, it is advected into the core and reradiated. At the onset of instability (explosion), the “optical” depth to $\nu_e$‘s at the gain radius is 0.05 to 0.15, not 0.01, and the peak entropy is less than 25 (Burrows and Goshy 1993). As the explosion develops, the optical depth decreases. Before explosion, oscillations of the shock (à la AM Her stars (Langer et al. 1981; Chevalier and Imamura 1982)) are overcritically damped both by neutrino losses and shock damping (with a characteristic time that scales with $M_s/\dot{M}$) and the matter is never radiation-dominated.

Though the eigenvalue analysis of Burrows and Goshy (1993) assumed nothing about the actual neutrino luminosities, they were able to show that accretion power alone could not ignite a supernova in spherical symmetry. The critical $L_\nu$ vs $\dot{M}$ curve intersected the $L_{\text{accretion}}$ vs $\dot{M}$ curve at values of $\dot{M}$ that were higher by as much as an order of magnitude than those obtained after the stalled shock settles (see Figure 7). To produce a
neutrino-driven explosion in “spherical” symmetry, some other source of luminosity must be available, be it due to rapid diffusion out of the core (modified neutrino opacities) or “convective” enhancement.

The rate at which heat is deposited via the charged-current absorptions on free nucleons at a radius $r$ is approximately given by,

$$
\dot{\epsilon}_\nu = 200 \frac{\text{MeV}}{\text{baryon} \cdot \text{s}} \left( \frac{T_{\nu_e}}{4.5 \text{MeV}} \right)^2 \left( \frac{L_{\nu_e}}{10^{52} \text{ergs/s}} \right) \left( \frac{10^2 \text{km}}{r} \right)^2, \tag{5}
$$

where it was assumed that $L_{\nu_e} = L_{\nu_e}$, $Y_n + Y_p = 1$ and $T_{\nu_e} = T_{\nu_e}$, the temperature of the electron neutrinosphere ([Bethe and Wilson 1985]). At $L_{\nu_e} = 4 \times 10^{52} \text{ergs/s}$, $T_{\nu_e} = 4.5 \text{ MeV}$, and $r = 10^2 \text{ kilometers}$, the time to raise the temperature of the matter by 1 MeV ($\tau \sim \epsilon_T/\dot{\epsilon}_\nu$) (including the electrons in the specific heat) is roughly 10 milliseconds. This is a measure of the time to achieve a steady-state or the timescale of explosive expansion in $R_s$, once the mantle of the protoneutron star is unstable. In the steady-state it is also “equal” to $\frac{R_s}{u_1}$, where, again, $u_1$ is the post-shock infall speed. Since $L_{\nu_e} \propto T_{\nu_e}^4 R_\nu^2$, where $R_\nu$ is the neutrinosphere radius, $\dot{\epsilon}_\nu$ is proportional to $T_{\nu_e}^6 (\frac{R_\nu}{r})^2$, a very steep power of $T_{\nu_e}$.

If we assumed that the cooling function is proportional to $T^6$, the equilibrium matter temperature would be a weak power of $r$ ($\propto 1/r^3$). Actually, the advective term in the heat equation forces the gradient in $T$ to be steeper in the steady state ([Bruenn 1992], but $T_{\nu_e}$ always provides an upper bound to the shocked mantle temperatures and the gradient of the matter temperature exterior to $R_\nu$ is always “shallow.” Since $T_{\nu_e}$ is $4 - 5 \text{ MeV}$ from quite general neutrino opacity arguments, matter temperatures behind the shock of $1 - 4 \text{ MeV}$ are easily understood. These are not the temperatures of 100 MeV one would get by setting $\epsilon_g (= \frac{GM}{R})$ equal to $\epsilon_T$ (thermal) at a typical cold neutron star radius of 10 kilometers. The “escape” temperature, $T_{esc}$, that one gets by setting $\epsilon_g = \epsilon_T$ is much larger than the real temperature near the neutrinosphere. Since it decreases as $1/r$ and the matter temperature actually decreases more slowly, the two curves intersect at some radius exterior
to $R_\nu$, typically 100–200 kilometers, at a value near 2–3 MeV. This temperature, not the core temperatures or $T_{\nu_e}$, sets the specific energy scale of the explosion, and hence, along with the binding energy of the progenitor envelope (Figure 12), sets the magnitude of the total supernova explosion energy. As with any corona, when the local temperature reaches the “escape” temperature, a powerful wind is driven off the star. Semi-quantitatively, the shock radius must be exterior to this radius (at which $\epsilon_T \sim \epsilon_g$). These arguments can be shown to be conceptually equivalent to those in the eigenvalue study of Burrows and Goshy (1993) when the entire star, with bounding accretion shock, is analyzed.

When thinking about supernova explosion energies, a few basic facts are useful. About a third of a solar mass of ideal gas nucleons at a temperature of 1 MeV has an internal energy of $10^{51}$ ergs. A spherical volume with a radius of 1000 kilometers filled with radiation (with pairs) at a temperature of 1 MeV has an internal energy of $1.6 \times 10^{51}$ ergs. The thermonuclear energy derived from the complete burning of 1 M$_\odot$ of oxygen to $^{56}$Ni is $10^{51}$ ergs and of 1 M$_\odot$ of silicon to $^{56}$Ni is approximately $0.4 \times 10^{51}$ ergs. The burning of 0.07 M$_\odot$ (87Al) of oxygen completely to $^{56}$Ni yields only $0.7 \times 10^{50}$ ergs.

Both Janka (1993) and Burrows and Goshy (1993) have shown that a large driving neutrino luminosity radiated over a short time (a flash) is more efficient at igniting a supernova and ejecting the envelope than the same total neutrino energy radiated over a longer time. It has been suggested (Burrows 1987) that such a flash might be the consequence of convective motions near the neutrinosphere, but this has not been convincingly demonstrated (Burrows and Fryxell 1993). To simulate an early explosion with such a character, we have altered our model with artificially hardening the emergent electron neutrino spectra to have a Fermi-Dirac $\eta$ of 3.0. Note that, guided by the work of Myra and Burrows (1990), we assume in these calculations that the emergent $\nu_e$ and $\bar{\nu}_e$ spectra are Fermi-Dirac in shape, with a default $\eta$ of 2 and a temperature equal to the
matter temperature at decoupling (see §II). The velocity vs. radius profiles of this model (w15e3) are depicted in Figure 13. This figure shows the progress of the supernova shock to a radius of \( \sim 4000 \) kilometers and the collision of the protoneutron star wind with the inner ejecta. The kinetic energy versus time for model w15e3 is depicted in Figure 9. At the end of the calculation it has reached \( 1.2 \times 10^{51} \) ergs. There are interesting differences between models w15e3 and w15n, both of which exploded. In model w15e3, the peak entropies achieved in most of the matter reach \( \sim 34 \), while in model w15n without neutrinos the number was 10–15 (Figure 14). In model w15n, when the shock had reached \( \sim 4000 \) kilometers, most of the supernova energy was in kinetic energy, while at the same radius in model w15e3 it was more evenly divided between internal and kinetic energy. In neither blast was the internal energy overwhelmingly in photons and pairs when 4000 kilometers was reached. The post-shock temperatures in the two cases were interestingly different, with consequences for explosive nucleosynthesis. In particular, the ansatz often employed in explosive nucleosynthetic studies that all the explosion energy is in radiation when the silicon and oxygen zones are encountered will have to be reexamined (Aufderheide et al. 1991; Thielemann, Hashimoto, and Nomoto 1990). A major conclusion of the comparison of models w15e3 and w15n is that the mechanism of explosion affects the subsequent nucleosynthesis, etc., even when the explosion energies for the different explosion scenarios are comparable. Note also that only a neutrino-driven explosion can yield entropies above 30. If such entropies (and higher) are required by nucleosynthetic arguments (Meyer et al. 1992; Takahashi, Witti, and Janka 1994), then such arguments are telling us something about the supernova mechanism itself. Note that the entropy profile of model w15e3 at late times is grossly Rayleigh-Taylor unstable. This means that the calculations should have been done from an earlier time in more than one dimension. This may be directly relevant to the high speed \(^{56}\)Ni bullets observed in SN1987A (see §VII). In addition, since (as stated in §III) the mass between the shock and the neutrinosphere becomes small early after
bounce (< 0.01M⊙), explosion calculations in 1-D, such as w15e3, suggest that the mass cut might be determined at the \textit{onset} of explosion, not later. The outgoing protoneutron star wind would assure this (modulo fallback due to reverse shocks on much longer timescales).

It was not our purpose in this section to explore the many interesting consequences of 1-D models such as w15e3. These will be postponed to a later paper. Rather, in this section we hoped to communicate a few of the salient and important features of generic neutrino-driven explosions to establish the context both of such a mechanism and of our new multi-dimensional calculations discussed in §VII.

6. Instabilities and Convection in Supernova Explosions: Preliminaries

Most hydrodynamic problems in astrophysics involve instabilities that are either central or decorative, but are often ignored. Rayleigh-Taylor and Kelvin-Helmholtz instabilities during supernova explosions are both of the former and can no longer be the latter. The wildly varying composition, entropy, density, and pressure gradients behind the supernova shock before and after it explodes drive violent overturning in which the Mach numbers can approach unity (Burrows 1987; Burrows and Lattimer 1988; Burrows and Fryxell 1992; Janka and Müller 1993a; Herant, Benz, and Colgate (1992, HBC); Benz, Colgate, and Herant 1994; Fryxell 1994; Müller, Fryxell, and Arnett 1991). The question that is now being addressed is whether such instabilities are crucial to the supernova mechanism itself and attendant issues and on this there are contending opinions.

The basic physics of convection in supernovae was first seriously discussed by Epstein (1979), who employed a mixing-length prescription and emphasized lepton-driven convection. No conclusions concerning the role of such motions were reached, but this work stimulated a flurry of activity in the early 1980’s (Colgate and Petschek 1980; Livio
Buchler, and Colgate 1980; Bruenn, Buchler, and Livio 1979; Smarr et al. 1981). In these papers, lepton-driven, as opposed to entropy-driven, instabilities were the focus, since it was known that the protoneutron star would neutronize from the outside in (however, see Smarr et al. 1981). Negative lepton gradients were supposed to drive either violent overturn of the entire core or to accelerate the loss of lepton number and energy. The paper by Bruenn, Buchler, and Livio (1979) is especially noteworthy in its suggestion that lepton-driven convection might enhance the neutrino luminosity that could thereby ignite the supernova (see their model 5). However, though multi-dimensional hydro codes were used in some of these papers, the initial models, transport, hydro algorithms, boundary conditions, and physical assumptions often left much to be desired. With the more rigorous work of Lattimer and Mazurek (1981), it was realized that violent core overturn was suppressed by both the shock-imposed positive entropy gradients (Figures 3) and the shallowness of the lepton gradients in the inner core (Figures 4), and interest in the role of hydrodynamic instabilities in supernova explosions waned. More recently, Wilson (1985) and Mayle and Wilson (1988) advanced a neutrino heating mechanism (§V) that in their calculations required “neutron finger” instabilities to sufficiently boost the driving neutrino luminosity. This “neutron finger” instability is not the lepton-driven instability, but is akin to the salt-finger instability seen in the oceans (in particular at the mouth of the Mediterranean) and relies for its existence on a large ratio of heat diffusivity to composition diffusivity. In the terrestrial example, the layering of hot, salty water over cold, fresh water, which is stable by the normal Ledoux or Brunt-Väisälä analysis, is unstable because a dimple at the slab interfaces allows enhanced heat transfer without a correspondingly rapid transfer of salt. The result is “cold” salty water over cold fresh water (a density inversion) and overturn, but its rate is slaved to the relative rate of heat and salt diffusion. This is not the classic convection, and laboratory experiments and ocean observations frequently show that a layered structure, not churning mixing, results (Turner 1974). In the protoneutron
star case, it was thought that since heat was transported by six neutrino species, whereas lepton number (“salt”) was transported only by $\nu_e(+) \text{ and } \nu_e(-)$, the hot neutron star was similarly unstable. The low opacities of the $\nu_\mu$-matter interaction were to enhance the effect, which in the calculations of Mayle and Wilson (1988) was simulated with a mixing-length algorithm. That the matter was importantly unstable to neutron finger instabilities was immediately challenged by Burrows (1987) and Burrows and Lattimer (1988) and has recently been challenged in a more comprehensive work by Bruenn, Mezzacappa, and Dineva (1994). In the latter, it is pointed out that once the “$\nu_\mu$’s” start to move quickly, they uselessly decouple from the matter.

Burrows (1987) suggested that the negative entropy gradient originally created by the stalling shock would, because of its proximity to the neutrinospheres, drive convective overturn that because of this proximity would churn up heat, boost the neutrino luminosities, and by the neutrino heating mechanism ignite the supernova. This effect was shown to exist ([Burrows 1987], [Burrows and Lattimer 1988], [Burrows and Fryxell 1993]), but it now seems that it exists for only about 10–20 milliseconds, since the transport of heat by neutrinos smooths the gradients out (see Figures 3, [Burrows and Fryxell 1993], and [Bruenn and Mezzacappa 1994]). However, as was shown by Burrows and Fryxell (1993), right under and around the neutrinospheres the lepton gradients in particular, but also the entropy gradients, are unstable to standard convection, which should persist. Using a mixing-length approach, Bruenn and Mezzacappa (1994) have suggested that this effect on the neutrino luminosities is marginal, but Burrows and Fryxell (1993) find using a 2-D hydrodynamics code that the effect can be important. When the Mach numbers and scales of the flow are so large, a mixing-length calculation will underestimate such effects. In addition, the work done by the hydrodynamic overturn can be large and can readjust the flow (e.g. move the shock outward ([Burrows and Lattimer 1988], [Burrows and Fryxell 1992])). These effects cannot be adequately treated by mixing-length.
Bethe (1990) argued that the region just behind the shock should be a theorist’s focus and that neutrino heating “from below” of this material would naturally create negative and unstable entropy gradients, whatever the initial gradients. The material exterior to the gain radius would overturn and the energy deposited by the core neutrinos, instead of being reradiated and lost as the matter plummets into the pit, would be available to do work on the shock. The stalled shock might thereby be revitalized and driven during the explosion. HBC and Herant, Benz, Hix, Fryer, and Colgate (1994, HBHFC) have recently lent support to this view in an extensive and important series of detailed 2-D simulations. In a careful 2-D parameter study with spherical light-bulb heating, Janka and Müller (1993a) partially support Bethe’s general idea, but, along with Burrows and Fryxell (1993) challenge some of HBC’s notions of the flow structure. This outer entropy-driven instability near the shock joins the inner entropy- and lepton-driven instabilities and the neutron-finger instability as the major instabilities suggested to be important in reviving and energizing a stalled shock. The effects of rotation have been addressed by Mönchmeyer and Müller (1989), Shimizu et al. (1993), and Shimizu et al. (1994). The latter emphasize the rotation-induced asymmetry of the neutrinosphere, which the authors contend will produce jets and large pulsar kick velocities. Müller (1993) has compared 2-D flow structures with the 3-D flow structures and finds that overshooting is weaker in 3-D. It is important to understand the differences between 2-D and 3-D flow behavior, but, currently, full 3-D rad/hydro simulations are beyond the reach and patience of supernova modelers.

Before we turn in §VII to the results of our new two-dimensional supernova simulations, a few simple analytic arguments can help guide the discussion. The Brunt-Väisälä angular frequency $N$ is given by

$$N^2 = -g \left( \frac{\partial \ln \rho}{\partial r} - \frac{1}{\gamma} \frac{\partial \ln P}{\partial r} \right),$$

where $g$ is the effective gravity defined to be positive in the negative $r$ direction and $\gamma = \left. \frac{\partial \ln P}{\partial \ln \rho} \right|_s$. This is exactly the frequency derived in the linear, compressible Rayleigh-Taylor
analysis. The two are equivalent and the standard incompressible Rayleigh-Taylor result is seen in the $\gamma \to \infty$ limit. Eq. (6) can be rewritten using a few thermodynamic identities (Lattimer and Mazurek 1981) to yield,

$$N^2 = \frac{\gamma}{\rho,\gamma_e} \left[ \frac{\partial \ln P}{\partial s} \rho,\gamma_e \frac{ds}{dr} + \frac{\partial \ln P}{\partial Y_e} \rho,s \frac{dY_e}{dr} \right],$$

where, for specificity, we have ignored the effects of neutrinos. The neutrinos can easily be included, but care must be taken when moving from the neutrino-opaque, beta-equilibrium region to the neutrino-transparent region. Eq. (7) shows that when (as is common) the pressure derivatives are positive, negative entropy and $Y_e$ gradients are unstable ($N^2 < 0$). $\frac{\partial \ln P}{\partial Y_e} \rho,s$ can be negative, but only at sufficiently high $s$’s and low $Y_e$’s to be rare (Lattimer and Mazurek 1981). Note that $\frac{\partial \ln P}{\partial s} \rho,Y_e = \frac{1}{C_v}$, where $C_v$ is the specific heat at constant volume. The overturn timescales from this linear analysis in the protoneutron star context are expected to be 3–10 milliseconds. Eq. (7) encapsulates the Ledoux criterion for convection.

Hydrostatic equilibrium structures are minimum total energy structures, subject to the constraint of mass conservation. In a more general thermodynamic sense, they are minimum free energy structures. The constraint of sphericity may inhibit the object’s attempt to achieve this minimum free energy. When $N^2$ in eq. (7) is negative, the structure can achieve a lower free energy via convective overturn. The free energy difference between the relaxed 1-D (spherical)-configurations and “3-D”-configurations is available to do useful work. This work can help push the shock outward, even if a concomitant neutrino luminosity boost is ignored. Thus, all convective motions are reservoirs, temporary or otherwise, of useful energy. This can be seen for the region of negative entropy created by the stalling shock in the calculations of Burrows and Fryxell (1992). There it is shown that, due only to overturn, the shock can receive a significant boost.

If an unstable region is modeled as two constant-density slabs of equal thickness,
Δx, then it can easily be shown that the gravitational energy available due to the simple interchange of the slabs is,

\[ W = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} g \Delta M \Delta x, \]

(8)

where ΔM is the total mass of the unstable region of thickness 2Δx(= Δh) and g is the local gravitational acceleration. If we assume that \( \rho_1 - \rho_2 = \frac{d\rho}{dr} \frac{\Delta h}{2} \) and that \( \rho_1 \) and \( \rho_2 \) are not very different, we obtain

\[ W \sim \frac{1}{8} \frac{d \ln \rho}{dr} g \Delta M (\Delta h)^2. \]

(9)

Using the correspondence between eqs. (6) and (7), we can substitute \( \frac{1}{\gamma C_v \frac{ds}{dr}} \) for \( \frac{d \ln \rho}{dr} \) in eq. (9) to find,

\[ W \sim \frac{1}{8} \frac{g \Delta s}{\gamma C_v \frac{ds}{dr}} \Delta M (\Delta h)^2, \]

(10)

where, for simplicity, we have ignored the \( \frac{d\nu}{dr} \) term. From eq. (10), we can derive two interesting relationships. The first gives simply and very approximately the energy available in the overturn of a region of mass ΔM and thickness Δh with a given unstable Δs at a distance r from the center of a protoneutron star of mass M:

\[ W \sim 10^{51} \text{ ergs} \left( \frac{\Delta M}{0.1 M_\odot} \right) \left( \frac{\Delta h}{10^2 \text{ km}} \right) \left( \frac{\Delta s}{10} \right) \left( \frac{135 \text{ km}}{r} \right)^2 \left( \frac{M}{1.3 M_\odot} \right). \]

(11)

We see that if ΔM \sim 0.3M_\odot, and Δs \sim 5, W is of order 10^{51} ergs. These numbers are realistic for the region behind the stalled shock with the initially unstable entropy gradient.

Note again that this is free energy available, ignoring neutrino heating. If on the other hand, the unstable Δs is created by neutrino heating (as in Bethe convection), eq. (10) can be transformed, assuming that \( \langle T \rangle \Delta s = \frac{\Delta E_\nu}{\Delta M_2} \) (where \( \langle T \rangle \) is an average temperature in a heated region of mass ΔM_2 and ΔE_ν is the neutrino heat deposited), into

\[
W = \frac{g \Delta s}{8\gamma C_v} \Delta M \Delta h = \frac{g \Delta E_\nu}{8\gamma \langle T \rangle \Delta M_2} \frac{\Delta M \Delta h}{C_v} \\
= \frac{\Delta E_\nu}{\langle T \rangle} \frac{\Delta M}{\Delta M_2} \left[ \frac{g \Delta h}{8\gamma C_v} \right].
\]
If we use the Bernoulli integral \( \left( \frac{1}{2} v_i^2 + \Phi(r_i) + \frac{\gamma P_i}{(\gamma - 1) \rho_i} \right) = \text{const.} \) for matter flowing from the shock to the heated region (where it presumably turns around) and assume that \( v_1 = v_2 \), that \( g \Delta h = \Delta \Phi \), that \( P = \rho RT \), that \( C_v = \frac{3}{2} R \), that \( \Delta M_2 = \frac{1}{2} \Delta M \), and that \( \langle T \rangle = \frac{T_h}{2} \), we obtain

\[
W = \frac{1}{2} \frac{\Delta T}{T_h} \Delta E_{\nu},
\]

(11)

where \( T_h \) is the temperature at the gain radius and \( \Delta T \) is the temperature difference between the shock “slab” and the gain radius “slab.” Eqs. (8) and (11) were derived by assuming the interchange of slabs of thickness \( \Delta h/2 \). If instead, we say that the cycle is from the top of the top slab (at the shock) to the bottom of the bottom slab (at the gain radius), we obtain the Carnot relation,

\[
W = \frac{\Delta T}{T_h} \Delta E_{\nu}.
\]

(12)

Notice that the \( \gamma \) has dropped out. A relation such as eq. (12) might suggest a more general validity. Very crudely, eq. (12) would give the optimum efficiency with which the deposited neutrino energy can be converted into useful work. Such a relation was implicitly posited in HBHFC. However, this relation assumes that heating occurs on the “power stroke” and not at all during early compression and that matter turns around at the gain radius. We find in our 2-D simulations that neither of these assumptions is particularly good. Furthermore, the analysis above ignores electron capture and composition changes which turn out to be important. The “convective engine” is very leaky. In addition, the implication of the engine analogy that the work done in a cycle has something to do with the explosion condition and the explosion energy is misleading. Since the boiling phase before explosion is a succession of quasi-steady states, neither the total neutrino energy deposited nor the work done in putative cycles during this waiting phase are remembered by the matter or are relevant to the subsequent explosive evolution.
7. New Multi-Dimensional Supernova Explosion Results

To focus the discussion and to avoid juggling an avalanche of plots and numbers, we highlight in this paper only one full 2-D simulation of a model of a massive star progenitor, s15s7b of Weaver and Woosley (1995). This 15 $M_\odot$ model was discussed previously in the context of models w15t and w15n and is typical of the compact class with smaller iron cores ($M_{Fe} \sim 1.28 M_\odot$) lighter envelopes (Figure 11), and lower binding energies (Figure 12). With our default neutrino transport scheme, this core does not explode in 1-D (see Figure 2a). However, as we will demonstrate, in 2-D the character of the flow and the final outcome are qualitatively different.

We refer to the two-dimensional simulation of the full radiation/hydrodynamic evolution of the core of the s15s7b model as our star run. The inner 4500 kilometers was included on the grid, which was 500 (radial) times 100 (angular) zones. Spherical coordinates with azimuthal symmetry were used, and the angular zones were distributed evenly in the $45^\circ < \theta < 135^\circ$ interval (see, however, Janka and Müller 1993a). The inner 15 kilometers was followed in one-dimension using our restricted-2-D algorithm. The angular boundary conditions were periodic. To resolve the pre-explosion convective stage, we put 300 radial zones interior to 150 kilometers and distributed the remaining 200 radial zones in a quadratically expanding fashion so that the outer zone was $\sim 50$ kilometers thick ($\delta r / r \sim 0.01$).

The star run can be compared to the 1-D w15t run and its outcome. In model w15t, the core bounced at $\sim 209$ milliseconds (Table 2) and the shock quickly settled to a radius of $\sim 80$ kilometers (Figure 5a). (Note that, unless otherwise stated, the quoted times are the times since the start of the calculation, not the times since bounce. This is the common, if unfortunate, convention.) Electron capture in the shocked zone was copious and low $Y_e$’s were achieved in a trough between 0.8 and 1.3 $M_\odot$ (Figure 4a). Within 60 milliseconds of
bounce, the shock radius was at $\sim 60$ kilometers. The small shock radius almost guarantees failure since the gain region is either small or non-existent, electron capture is faster at the consequently higher post-shock densities, and the accretion ram pressure is larger, for a given $\dot{M}$. In model w15t, the shock radius continues to sink and there is no explosion. A black hole would eventually form.

Since a finite-difference code such as PPM is almost noiseless, we have to add seed perturbations to start any hydrodynamic instabilities. For the 2-D star calculation, this perturbation was in density and was sinusoidal with an amplitude of 2% and a period of $\pi/5$. An additional random perturbation of 1% was superposed. Memory of the specific perturbation was completely lost after $\sim 10$ milliseconds. In sprinkling the seed perturbations, we were guided by the convective oxygen burning study of Bazan and Arnett (1994). In that paper, density fluctuations as large as a few percent and Mach numbers near 0.25 were obtained. We assumed that similar excursions from quiescent mixing-length parameters obtained during the silicon burning phase that immediately precedes core collapse. Since the inner edge of the convective silicon burning shell is always between 0.8 and 1.1 $M_\odot$ (interior) in the progenitor models and these zones are always encountered within a millisecond of bounce, we get an immediate start to the Rayleigh-Taylor instabilities that is physically motivated.

It is difficult to communicate all the characteristics and behaviors of a 2-D hydrodynamic simulation in the standard paper format. We have derived velocity, $Y_e$, temperature, entropy, pressure, Mach-number, composition, and density (etc.) maps and profiles and their evolutions, but we can not hope to convey all the nuances of the flow in only words or tables. Therefore, we present in this paper representative color snapshots from the star run of contour plots of various quantities at interesting and illustrative phases of the simulation, both before and after explosion.
Table 3 lists the 2-D plots that we have included to illustrate our discussion.

a. Multi-Dimensional Hydrodynamics Before Explosion

Figure 15 depicts the inner 150 kilometers of the distribution at \( t = 238 \) milliseconds (about 30 milliseconds after bounce). Velocity vectors that trace the flow are superposed. The mildly aspherical shock is at the juncture of the blue (\( Y_e \sim 0.25 - 0.35 \)) and red (\( Y_e \sim 0.48 \)) regions at radii between 84 and 94 kilometers. Due to electron capture on the shock-liberated protons, \( Y_e \) plummets to near 0.1 at \( \sim 30 \) kilometers in a fashion similar to that depicted in Figure 4a for model w15t. The immediate post-shock temperatures and densities are \( \sim 2 \text{ MeV} \) and \( \sim 10^{10} \) gm/cm\(^3\), respectively. The pre-shock temperature is near 0.7 MeV. As Figure 15 shows, the material between the shock and \( \sim 45 \) kilometers is unstable, as is the region between 15 and 30 kilometers. Convection in the latter region is driven by negative lepton and entropy gradients as discussed in §VI, but is weak in the star series. The spherically distributed accretion flow exterior to the shock is channelled into predominantly downward-moving plumes that hit the inner core near the electron neutrinosphere in a very aspherical fashion. The mass accretion flux into the inner core can vary with angle by factors of three and the accretion neutrino luminosity varies accordingly. There are hot spots of neutrino emission that dance over the inner core. Before the explosion, the mass accretion rate through the shock is almost equal to that on the inner core. As a consequence, the local mass flux per unit area at the neutrinospheres and the local neutrino fluxes can be much larger than in the 1-D calculations. Figure 16a depicts the variation of the neutrino luminosities (fluxes times area at large \( r \)) with angle at various times. Figure 16b portrays the variation of the angle-averaged neutrino luminosities versus time for both the star calculation and its 1-D analog. The flickering depicted in Figures 16 is significant and dwarfs in magnitude the convective boost championed by Burrows and Fryxell (1993).

In the star calculation, the pre-explosion, post-bounce phase lasts almost 100
milliseconds (~ 30 turnover times). During this time, the character of the unstable flow changes. Figures 17 through 21 depict this evolution in $Y_e$ maps at $t = 270, 299, 304, 307,$ and 311 milliseconds. The initial instability is a result of the negative entropy gradient behind the shock, imposed as it stalls (Burrows 1987). As the neutrinospheres recede, the unstable region grows until the region between the shock and ~ 35 kilometers is engulfed. This thickening of the unstable region can be seen by comparing Figures 15 and 18 at 238 and 299 milliseconds, respectively. As suggested by Bethe (1990) and HBC, after about 10 milliseconds into the instability, it is neutrino heating from below that drives the subsequent “convective” motions. The flow takes on the character of vigorous “boiling,” but before explosion there is always a net mass flux through the region. There is no secular tendency for the shock to move monotonically outward during this pre-explosion phase. The convective region encompasses 4–6 pressure scale-heights.

Early after bounce, the flows interior to the shock are predominantly inward, at speeds of $1 - 2 \times 10^4$ km/s. There are some partially stagnated accretion streams at lower speeds and a few upward plumes and swirls at a few times $10^3$ km/s. As time advances towards explosion, more vigorous convective eddies appear and along with the downward plumes, upward bubbles rise with speeds that may exceed $3 \times 10^4$ km/s. Their Mach numbers are near one. However, despite the bubbles, at all times before explosion the average energy and mass fluxes are inward. As the external accretion rate decays, the unstable motions become more vigorous. Early in the convective phase, the bubbles created by neutrino heating are at lower entropies of 10–15, but those formed later on can reach entropies of 25–35. These local entropies are larger than can be achieved before explosion in 1-D (Burrows and Goshy 1993). In fact, small hot blobs with entropies near 60 are formed when the rising bubbles encounter the shock and are turned around. Dwelling in the gain region longer than is possible in 1-D allows small clouds of gas to achieve entropies unachievable in 1-D before explosion. However, the swirling motions return these hot blobs to the cooling regions
where their entropies drop below 35.

A major feature of the unstable flow is the formation of many bubbles that rise and collide with the shock. These collisions warp and bow out the shock surface episodically on timescales of 1–5 milliseconds. The shock radius can vary with angle by as much as 30% (see Figure 18). The bubbles give the shock a botryoidal appearance, much like an aggregate of hematite or an oscillating drop of charged liquid mercury. In addition, the average radius of the shock varies between 85 and 120 kilometers in response to the rising bubbles and the gradual decay of the mass accretion rate. This is in contrast to a 1-D shock radius below 60 kilometers and is an index of the important difference between one- and multi-dimensional simulations. This difference is most clearly seen in Figure 5c in which the 1-D and 2-D shock radii versus time are depicted. In 2-D, a gain region of respectable size is maintained so that when the accretion rate has subsided sufficiently, the core can explode into a supernova, as the star series eventually does near 310 milliseconds (Figures 20 and 21).

When a particularly vigorous bubble encounters the shock, if the explosion is not imminent, the bubble material is forced along the shock and then downward and can encounter similar streams of matter to form a coherent columnar downflow. (During this collision, the shock is pushed to slightly larger radii.) Such plumes are seen in Figures 19–21 and similar plumes have been identified by HBC. The entropy of matter in these downward plumes is generally ten to fifteen units lower than in the bubbles. Figure 22 depicts the entropy structure at \( t = 304 \) milliseconds when such a plume is in evidence. The plume is as much as four times denser than adjacent matter at the same radius. However, plumes seen earlier in the calculation broke up within 5–10 milliseconds and the flow resumed its more chaotic appearance. We do not see a final merger of small eddies into larger ones, or the appearance with time of a dominant “\( \ell = 1 \)” mode. The flow is always fluctuating
and complicated, with equivalent $\ell$ mode numbers of 2–10 generally in evidence. We never see families of simple convective cells. The origin of the differences between our results and those in the HBC and HBHFC series is not clear to us. (The two groups do almost everything differently, with different base hydro codes). However, the fact that we allow the neutrino fluxes to vary with angle is certainly a factor, as is the fact that our shock stalls at smaller radius.

Difficult to see in Figures 15 through 21, but obvious in our analysis, are quasi-periodic gravity waves (g-modes) at the base of the unstable region near the neutrinospheres. These internal modes propagate perpendicularly in the theta direction with speeds of five to fifteen kilometers per millisecond and with a variety of periods around 3 milliseconds. They have wavelengths between 5 and 20 kilometers. Their displacement amplitudes can be greater than 5 kilometers and as the gravity waves propagate over the neutrinospheres they ripple them perceptibly (Figure 23). Penetration of the neutrinosphere by as much as 5–10 kilometers is a result. The existence and persistence of numerous strong gravity modes is another major hydrodynamic feature of this pre-explosive phase and is correlated with the angular variations in the neutrino fluxes and the undulating motions of the accretion shock. In fact, there seems to be a strong coupling between the meandering columnar plumes and the g-modes that they excite. Such feedback has been seen in other simulations of compressible convection (Hurlbert, Toomre, and Massaguer 1986). The boost in the average neutrino luminosities due to the overshoot and dredge-up is difficult to distinguish from the wildly varying fluxes due to aspherical accretion, but from a comparison with the 1-D calculation seems to grow to be as much as 20% just before explosion (Figure 16b).

Before explosion, we see no evidence of an “accumulation” or “building” of total energy. In fact, in the star calculation before explosion, but after bounce, the total energy (kinetic plus internal plus gravitational) in the overturning region actually decreases (though the
star model does eventually explode!). Since $\dot{M}$ changes only slowly, the “boiling” shocked material is in quasi-hydrostatic equilibrium. Furthermore, the average shock radius does not increase monotonically with time (see Figure 5c). If $\dot{M}$ is held constant, the average shock radius and the kinetic and thermal energies are roughly constant and there is no tendency to explode. We checked this by putting a constant density, velocity, pressure, and energy outer boundary condition at 200 kilometers about 50 milliseconds after bounce. The mass accretion rate stabilized and the increase in the kinetic energy stopped. There was no explosion. The increase in “convective” vigor with time that we see in the simulations is in some sense in response to the decay of $\dot{M}$ and the accretion ram.

It is important to note that the accuracy of the neutrino transport algorithm employed can make a qualitative difference in the outcome. Our “st2d” series used different approximations for the neutrino-matter coupling above the neutrinospheres and the resulting accretion shock radii were as much as ten kilometers smaller. This “st2d” series did not explode. Another series that we performed (the “bang” series) employed a neutrino transfer algorithm that enhanced the neutrino-matter coupling in the semi-transparent region. Its pre-explosion accretion shock radii varied between 100 and 150 kilometers, it exploded within 50 milliseconds, and it involved less vigorous convective motions, that did not plunge all the way to the neutrinospheres. In the “bang” simulation, two uncoupled convective regions emerged, an inner one driven by shallowly negative entropy and lepton gradients between 20 and 50 kilometers and an outer one driven à la Bethe by neutrino heating between 100 and 150 kilometers. The inner convection boosted the core neutrino luminosity by as much as 10% (Burrows and Fryxell 1993), but does not seem to have been the crucial factor in igniting the explosion. (This model almost exploded in 1-D!) The lesson we draw from these ancillary studies is that, though “Bethe” convection helps a core to explode, it does not guarantee it. Bethe convection results in larger accretion shock radii and gain regions (see §VI), but neutrino energy and lepton losses and the accretion ram
can still be too much to overcome. Explosion is a quantitative question that hinges for the theorist on the physical assumptions employed. In particular, a model with the best neutrino transport, but a dense envelope and large consequent ˙M's need not explode, even when multi-dimensional effects are included. We find that Bethe convection exists and is very useful, but that it does not guarantee explosion.

b. The Explosion

As we have stated, we believe that whether a “theoretical” core explodes is a quantitative question. Our default 2-D calculation of s15s7b, the star simulation, exploded magnificently 100 milliseconds after bounce. We are compelled to ask why. Figure 20 shows the flow field at $t = 307$ milliseconds, a few milliseconds before explosion. The shock is between 100 and 120 kilometers in radius, and there are two strong downward plumes near 5° and 45°. The “boiling” is more vigorous than 50–100 milliseconds earlier and the excursions in $\Delta R_s$ are a bit larger. However, in many ways, nothing extraordinary is occurring. Yet, by 311 milliseconds (Figure 21), the core is obviously exploding. Just before the explosion, the shock moves inward in places by about 10 kilometers and the accretion luminosity rises fractionally, but it does so undramatically. We are led to conclude that just as in the 1-D case described in §V the shocked envelope reaches a critical state and becomes unstable. Note, however, that the 1-D calculation w15t did not explode, despite the steady decay of the mass accretion rate. “Convection” is obviously an important ingredient in this supernova explosion. Before the explosion, the positive influences of neutrino heating and convection are “balanced” by the negative effects of electron capture, neutrino energy losses, and the accretion ram. Eventually, when the critical condition is reached this balance is tipped in favor of dynamical expansion. Note that, at the onset of explosion, the matter that will eventually be ejected may still be bound. Neutrino heating during the explosion is crucial to the eventual achievement of supernova energies. The aggregate energy deposited
before explosion is almost completely irrelevant to the final supernova energy.

Since the precise nature of the trigger is so important, we digress a bit here to summarize the role of 2-D in triggering the explosion. In 1-D, the heated parcel would perforce fall directly into the cooling region interior to the gain radius (where cooling = heating) and lose its just recently gained energy. (Curiously, the cooling region lies closer to the neutrinospheres where the temperatures are higher.) However, the rising balloons, upon encountering the shock, are immediately advected inward by the powerful mass accretion flux raining down. They do not dwell near the shock. In fact, the net mass flux through the shock is approximately equal to the mass flux onto the core and mass does not accumulate in the convective zone. The mass between the shock and the neutrinospheres decreased by about a factor of three in the star calculation during the pre-explosion boiling phase that lasted $\sim 100$ milliseconds ($\sim 30$ convective turnover times). All the matter that participates in the “convection” before explosion eventually leaves the convection zone and settles onto the core. A given parcel of matter may “cycle” one or two times before settling inward (and a large fraction never rises), but more than three times is rare. The boiling zone is resupplied with mass by mass accretion through the shock and a secularly evolving steady-state is reached. This steady-state is similar to that achieved in 1-D, but due to the higher dwell time the average entropy in the envelope is larger and its entropy gradient is flatter. These effects, together with the dynamical pressure of the buoyant plumes, serve to increase the steady-state shock radius over its value in 1-D by $30\%-100\%$ (Figure 5c). It is this effect of boiling that is central to its role in triggering the explosion, for it thereby lowers the critical luminosity threshold (Burrows and Goshy 1993). The lowering of the effective critical curve allows the actual model trajectory in $L_{\nu_e}$ vs. $\dot{M}$ space to intersect it. Even if in 1-D it can be shown that the two curves can intersect, they would intersect earlier and more assuredly with the multi-dimensional effects included. The physical reasons for the lowered threshold are straightforward: a large $R_s$ enlarges the volume of the gain region,
puts shocked matter lower in the gravitational potential well, and lowers the accretion ram pressure at the shock for a given $\dot{M}$. Since the “escape” temperature ($T_{\text{esc}} \propto \frac{GM\mu}{kR}$) decreases with radius faster than the actual matter temperature ($T$) behind the shock, a larger $R_s$ puts a larger fraction of the shocked material above its local escape temperature. $T > T_{\text{esc}}$ is the condition for a thermally-driven corona to lift off of a star. In one, two, or three dimensions, since supernovae are driven by neutrino heating, they are coronal phenomena, akin to winds, though initially bounded by an accretion tamp. Neutrino radiation pressure is unimportant.

We conclude that the instability that leads to explosion in $\geq$2-D is of the same character as that which leads to explosion in 1-D. Since the explosion succeeds a quasi-steady-state phase, neither the total neutrino energy deposited during the boiling phase nor any putative coeval thermodynamic cycle is of relevance to the energy of the explosion or the trigger criterion. Energy does not accumulate in the overturning region before explosion (it in fact decreases) and the increasing vigor (speed) of convection is in response to the decay of $\dot{M}$. If $\dot{M}$ were held constant, the overturning would not grow more vigorous with every “cycle” and a simple, stable convective zone would be established. In fact, before explosion the average total energy fluxes ($\langle \epsilon + P/\rho + \frac{1}{2}v^2 - \frac{GM}{r} \rangle \dot{M}$) due to the overturning motions are inward, not outward, since the net direction of the matter is onto the core.

The average electron-neutrino luminosity of the star model at explosion was lower than that required by the 1-D theory of Burrows and Goshy (1993). However, a major consequence of the breaking of spherical symmetry and the resulting increase in the steady-state shock radius is the lowering of the critical explosion threshold. In addition, as we have shown, the fluctuations in the neutrino fluxes are quite large (Figures 16). Since in 1-D it has been demonstrated that a “flash” of neutrinos is more efficient at igniting an explosion than the same energy radiated over a longer time (Janka 1993), it could be that
the observed variation in $L_{\nu}$ with angle plays a role in triggering the explosion. In addition, as Figure 16b implies, the effects of dredge-up on the average luminosities are not small just before the explosion. Once the explosion commences, it runs away, since neutrino cooling is a stiff function of temperature and the temperature decreases slightly as the envelope expands. Though the analytic development that led to equations 8–12 is of qualitative interest, the flow complexity demonstrated in Figures 17–21, the quasi-steady-state nature of the pre-explosion object, the influence of the g-modes, copious electron capture, the simultaneous existence of bubbles and downward accretion plumes, and the heterogeneous entropy structure in the pre-explosion envelope make its serious application of dubious use. The “convective engine” analogy (HBHFC) does not capture the essential physics of the phenomena we see.

Figures 24 through 30 depict the development of the explosion from 318 to 408 milliseconds. Figures 24 through 27 show the entropy distribution at 318, 348, 378, and 408 milliseconds, while Figures 28, 29, and 30 show the $Y_e$, radial velocity, and $\log \rho$ distributions, respectively, at $t = 408$ milliseconds. Also displayed on Figure 28 are velocity vectors that trace the flow. This set of figures encapsulates a rich variety of results and conclusions.

When the explosion commences, the high entropy bubbles drive it and they are not distributed isotropically. Plumes and fingers (some at times almost jet-like) emerge from the core and push the shock outward. The working surfaces of the plumes near the shock spread out and back in the classical fashion of jets and experience Kelvin-Helmholtz instabilities not well resolved in these calculations. The anisotropy of the material behind the shock is much larger than that of the shock itself, which may be only $10$–$20\%$ ($= \frac{R_s}{R}$). The entropies of the fingers vary from 10 to 40, even at the same radius. The vigorous finger at $\sim 60^\circ$ has entropies that range from 20 to 40, Mach numbers near 3, outward
velocities as high as 50,000 km/s (Figure 29), and densities that can be a factor of four smaller than those of adjacent matter. The shock wave itself moves outward at speeds from 20,000 to 25,000 km/s. The density distribution is very heterogeneous, with low and high density regions at the same radii and regions in which the density at small radii is smaller than the density at larger radii, along the same angular ray. In our $90^\circ$ wedge, at least three fingers are in evidence. Such fingers may be part of the explanation for the high velocities and asymmetrical gamma-ray and infrared line profiles of the iron-peak elements seen in SN1987A (McCray 1993; Tueller et al. 1990).

Early during the explosion, while most of the shocked matter is moving outward, some of it is still falling inward. There is simultaneously explosion and accretion. The cooler material at $90^\circ$ in Figure 27 is plunging inward even 40 milliseconds after the explosion is ignited, though by this time very little additional matter is sinking and the general flow is explosive. The continuous accretion early during the explosion is not necessary to power the blast wave, and subsides after $\sim 70$ milliseconds without any effect on its viability. Specifically, there are no accretion fingers that dive into the core after $\sim 70$ milliseconds and the wind is clearing out the interior. However, this “broken symmetry” not seen in the 1-D simulations may be the means by which nature fattens the core to observed neutron star masses ($M_g \sim 1.35 \, M_\odot$, $M_B \sim 1.5 \, M_\odot$), while exploding quickly within 100 milliseconds of bounce. An “early” 1-D explosion of a 15 $M_\odot$ star leaves behind too little mass (Figure 7). However, at $t = 408$ milliseconds into the 2-D star simulation, the mass left behind is still only 1.32 $M_\odot$.

It is into the channels created by the rising plumes that the protoneutron star wind material flows. This anisotropic wind continues to drive the inner explosion. Two hundred milliseconds after bounce, a clump in the wind has an entropy near 60 (Figure 27), the highest in the flow and higher than achieved in 1-D (Figure 14). Such high entropies are
achieved because wind material emerging from the core is slowed down temporarily by the more slowly moving material it encounters further out. This allows such matter to be heated longer by the core neutrinos and thereby to achieve higher entropies. How these hot spots evolve has yet to be determined and, as stated previously, may be of relevance to the r-process (Takahashi, Witti, and Janka 1994; Woosley et al. 1994a). The hot spot in Figure 27 has a mass near $10^{-5} \pm 1 M_\odot$, depending on how its third dimension is treated.

At $t = 408$ milliseconds, the region interior to 120 kilometers that was vigorously unstable before explosion is now host to a stable, spherical wind, though there are still interacting non-radial streams in the region between 150 and 600 kilometers. By 348 milliseconds, as Figure 25 depicts, the shock has reached $\sim 700$ kilometers and the inner edge of the oxygen zone that has fallen in to meet it. The post-shock temperatures are $\sim 0.8$ to 0.9 MeV and the post-shock densities are $\sim 10^8$ gm/cm$^3$. The immediate post-shock entropies are between 8 and 11 and $Y_e$ equals $\sim 0.5$. In material with these thermodynamic characteristics, alpha particles are $\sim 90\%$ by mass, and the rest is neutrons. However, in the high-entropy, “low” density plumes depicted in Figures 25–27, the alphas are dissociated into neutrons and protons. It is on the free neutrons and protons that the neutrinos from the core can be absorbed. Therefore, despite the fact that the matter is at progressively larger radii, the high entropy fingers continue to be heated, albeit at a decreasing rate. Nevertheless, from 318 to 408 milliseconds, the entropy of the leading edges of the $60^\circ$ plume increases by no more than a few units, while that of the material at the base of the $60^\circ$ plume (at smaller radii) is increased by as much as ten units (and starts from higher values).

A major concern of our star results is the electron fraction of some of its ejecta. Most of the exploding material depicted in Figure 28 has an unoffending $Y_e$ near 0.5. However, the most vigorous and obvious plume at $60^\circ$ from the vertical in Figure 29 has a $Y_e$ in
most of its mass of 0.43 to 0.46. What is more, \( Y_e \) approaches 0.4 near its base in the inner wind-fed region. The mass of the neutron-rich plume may total \( 10^{-3}M_\odot \), which may be too much to be consistent with the observed abundances of neutron-rich isotopes near the iron-peak. Since the ejected \( Y_e \) is modified by the electron- and anti-electron-neutrino fluxes, the solution to this dilemma may simply be further evolution or better neutrino physics (Fuller and Meyer 1994). It may also be solved by a longer delay to explosion, since the accreting matter is gradually thinning out with time and the post-shock electron capture rates are decreasing. This has the effect of ejecting less mass, and with higher \( Y_e \)’s.

At the end of the calculation at \( t = 408 \) milliseconds, the leading edge of the shock is at 2200 kilometers. The matter temperatures between a radius of 1000 kilometers and the shock are between 0.45 and 0.6 MeV at all angles. Though the final explosion energy has not yet saturated, the kinetic energy is \( 0.42 \times 10^{51} \) ergs, the internal energy interior to the shock is \( 0.06 \times 10^{51} \) ergs, and the dissociation energy is \( 0.38 \times 10^{51} \) ergs, two hundred milliseconds after bounce. The total energy is rising at an instantaneous rate of \( 4 \times 10^{51} \) ergs/s.

c. Pulsar Kicks

The convective fluctuations in mass accretion flux and neutrino luminosity before and during explosion suggest a natural mechanism for imparting kicks to neutron stars. The observed average kick speed of radio pulsars is \( \sim 450 \) km/s (Lyne and Lorimer 1994). The inner 15 kilometers of the star model was pinned and done in 1-D and a calculation with an angular range of \( 360^\circ \) (or \( 180^\circ \)) is called for. However, with the fluctuating momentum in the \( y \) direction (at \( \theta = 0^\circ \)), we can estimate the recoil velocities that would have been imparted to the core. The recoil speed in the \( y \) direction fluctuated on a short convective timescale of 3 milliseconds, but grew on average to peak at 295 milliseconds (\( \sim 15 \) milliseconds before explosion) at a value of \( \sim 180 \) km/s. Convection deep in the potential well causes the core
to shake significantly. The recoil due to the integrated anisotropic neutrino emissions before explosion is smaller than that due to the mass motions before explosion by a factor of roughly $\frac{v}{c}$, as can be demonstrated simply by noting that the accretion $L_\nu$ is proportional to $\frac{v^2}{2}$, while the recoil due to mass accretion scales as $v$, where $v$ is the post-shock accretion speed. If we multiply this peak one-dimensional speed of 180 km/s by $\sqrt{3}$ to account for three dimensions, we obtain a peak recoil speed of 300 km/s due merely to the oscillations of the core near explosion. This estimate does not include the effects of mass motions at the polar caps omitted in the star calculation. Note that the symmetry of our star calculation does not allow us to estimate the precise magnitude of this effect with any confidence. However, we think that we have in the vigorous convective motions of the material between the stalled shock and the neutrinospheres a natural mechanism for imparting to many young pulsars high proper motions. This is accomplished without the use of magnetic fields. However, velocities above 500 km/s may still be difficult to explain, though we surmise that the peak vigor of the convection is larger for the fatter cores associated with more massive progenitors. This would suggest that the RMS kick speed of a neutron star may increase with progenitor mass, all else being equal. Very mild support for this hypothesis comes from the smaller proper motion observed for the Crab pulsar ($v_t \sim 150$ km/s; [Harrison, Lyne, and Anderson 1993]), which Nomoto et al. (1982) conclude originated from a smaller 8–10 $M_\odot$ progenitor. However, for a perfectly spherical progenitor, a long delay to explosion would cancel the asymptotic recoil speed convection could impart (Janka and Müller 1994), since the initial total momentum would be zero and the mass in the shocked envelope is gradually decreasing with time. For a given progenitor, there is a critical explosion time for peak pulsar recoil, after which the convective effect we have outlined diminishes. What the actual explosion and recoil systematics are has yet to be determined.

A related and intriguing effect of neutrino-driven convection is its tendency to impart to the residue not only linear momentum, but angular momentum as well. The g-mode
modulated accretion plumes strike at a variety of angles and torque the core stochastically. The numbers of our star simulation suggest that the core is easily left with rotation periods of less than one second. Thus, even if the Chandrasekhar core of a massive star is not rotating at collapse, the neutron star residue should be left rotating with respectable pulsar periods. If the kicks and periods are convection-induced and the white-dwarf cores are rotating very slowly at collapse, we would expect a roughly linear correlation between the rotation frequencies of pulsars at birth and their proper motions \( \omega \sim \frac{\Delta v}{R} \), where \( R \sim 30 \) kilometers. We would also expect that the kick direction and spin axis would be very approximately at right angles. However, any significant initial angular momentum just before collapse might easily confuse and suppress this correlation, as might a significantly aspherical core (cf. Bazan and Arnett (1994)). In addition, any systematic trend of initial core rotation period with progenitor mass would have to be understood (as would the effect of rotation itself on neutrino-driven convection) to extract this convective correlation unambiguously. Nevertheless, it is clear that a core need not be rotating before collapse for a pulsar to be born with an interesting period.

8. Conclusions

Our 2-D calculations imply that the breaking of spherical symmetry may be central to the supernova phenomenon. The explosion does not erupt spherically, but in crooked fingers and is fundamentally heterogeneous. This conclusion is consistent with the nickel bullets and the ragged and skewed infrared iron, nickel, and cobalt line profiles observed in SN1987A, the “shrapnel” observed in CAS A, N132D, and the Vela supernova remnant (Aschenbach et al. 1994), and the large observed pulsar proper motions. In this paper, we have attempted to understand and extend the current theory of supernova explosions. To do so, we have performed new one- and two-dimensional radiation/hydrodynamic simulations
of all phenomena from collapse through explosion. The 1-D calculations allowed us to analyze the context in which the 1-D neutrino-driven mechanism should be understood and to highlight its limitations. The role of progenitor structure on the 1-D development of collapse was explored. It was shown that the two classes of core structure, compact and extended, can evolve in quantitatively, and perhaps qualitatively, different ways.

With our 2-D star simulation of the core of a $15 \, M_\odot$ progenitor, we have verified the potential importance of neutrino-driven convection in igniting a supernova explosion ([Bethe 1990] and HBC). After a delay of $\sim 100$ milliseconds, the core exploded aspherically driven by rising neutrino-heated bubbles that developed after another 100 milliseconds into tangled fingers. These fingers seem to be generic features of supernova explosions. We do not see a cascade to a dominant $\ell = 1$ mode, either before or after explosion, and both small- and large-scale structures are always visible. Before explosion, vigorous columnar downflows develop and dance over the neutrinosphere, but they break up within at most 20 milliseconds. The flow fluctuates between ordered and “chaotic” motion and modes of $\ell$ equal 2 to 10 are generally in evidence. Internal gravity waves (g-modes) are excited by the downflows and easily achieve non-linear amplitudes of 5 to 10 kilometers. These g-modes feed back onto the downflows and modulate both the convective motions between the shock and the neutrinospheres and the positions of the downflowing plumes. The convective motions get progressively more vigorous with time. The rising bubble speeds reach values of 30,000 km/s within 50 milliseconds. These bubbles hit the shock and result in oscillations with time and variations with angle of its radius that can be 30%. The convective zone can overshoot by as much as ten kilometers and the consequent enhancement in neutrino luminosity can be 10–20%. Lepton-driven convection beneath the neutrinospheres starts early and is maintained even after explosion. Its boosting effect on the neutrino luminosities that drive the convection is persistent, but seems swamped by the variations with angle and time of the fluctuating accretion luminosity. The latter varies on a convective timescale
of \(\sim 3\text{-}5\) milliseconds and by as much as a factor of three in angle. The angle-averaged luminosities vary by as much as 60\% (Figure 16b).

As in 1-D, the explosion appears to be a critical phenomenon with a critical condition. In 2-D, the rising balloons, upon encountering the shock, are immediately advected inward by the powerful mass accretion flux raining down. They do not dwell near the shock. In fact, the net mass flux through the shock is approximately equal to the mass flux onto the core and mass does not accumulate in the convective zone. The mass between the shock and the neutrinospheres decreased by about a factor of three in the star calculation during the pre-explosion boiling phase that lasted \(\sim 100\) milliseconds (\(\sim 30\) convective turnover times). All the matter that participates in the “convection” before explosion eventually leaves the convection zone and settles onto the core. A given parcel of matter may “cycle” one or two times before settling inward (and a large fraction never rises), but more than three times is rare. The boiling zone is resupplied with mass by mass accretion through the shock and a secularly evolving steady-state is reached. This steady-state is similar to that achieved in 1-D, but, due to the higher dwell time in the gain region, the average entropy in the 2-D envelope is larger and its entropy gradient is flatter. These effects, together with the dynamical pressure of the buoyant plumes, serve to increase the steady-state shock radius over its value in 1-D by 30\%–100\%. It is this effect of boiling that is central to its role in triggering the explosion, for it thereby lowers the critical luminosity threshold. The lowering of the effective critical curve (Burrows and Goshy 1993) allows the actual model trajectory in \(L_{\nu e}\) vs. \(\dot{M}\) space to intersect the new critical curve. The physical reasons for the lowered threshold are straightforward: a large \(R_s\) enlarges the volume of the gain region, puts shocked matter lower in the gravitational potential well, and lowers the accretion ram pressure at the shock for a given \(\dot{M}\). Since the “escape” temperature \(T_{\text{esc}}\) decreases with radius faster than the actual matter temperature \(T\) behind the shock, a larger \(R_s\) puts a larger fraction of the shocked material above its local escape temperature. \(T > T_{\text{esc}}\) is
the condition for a thermally-driven corona to lift off of a star. Since supernovae are driven by neutrino heating, they are coronal phenomena, akin to thermally-driven winds, though initially bounded by an accretion tamp. Neutrino radiation pressure is unimportant.

We see no evidence that convection in and of itself leads to a “building” or accumulation of energy before explosion. In fact, in the star calculation, the total energy in the overturning region actually decreased monotonically before explosion. The kinetic energy alone increased. If almost anytime before explosion the accretion rate was not allowed to decay further, but was frozen, the convection would boil in a steady state and the star would not explode. Furthermore, we find that the accuracy and specifics of the neutrino scheme employed can make a qualitative difference in the outcome. We conclude that even with Bethe convection, the explosion is not assured and the outcome is still a quantitative question requiring further exploration.

After explosion, the ejecta emerge in many contorted fingers in which the density can vary with angle by more than a factor of four. Early during the 2-D explosion, unlike in the 1-D calculations, some matter is still accreting in long, thin, bent, lower-entropy plumes. Later, all of the matter is exploding, though the entropy in the outer ejecta varies with angle from 10 to 40. However, as late as 70 milliseconds after explosion, some matter is still crashing down to within 200 kilometers of the core. By 100 milliseconds, a stable, neutrino-driven wind is clearly being blown from the protoneutron star. This wind is partially channelled by the low-density fingers and helps to drive the explosion. It is roughly spherical, but roils the matter it encounters 200 to 600 kilometers away (see also Figure 13) and does not allow matter to penetrate onto the core. The entropy in hot spots at the base of the plumes and in the wind has reached 60, 100 milliseconds into the explosion. What its entropy is asymptotically (the r-process?) remains to be seen.

The kinetic energy of the explosion at 408 milliseconds is $0.42 \times 10^{51}$ ergs, the internal
energy interior to the shock is $0.06 \times 10^{51}$ ergs, and the stored dissociation energy is $0.38 \times 10^{51}$ ergs. At the end of the calculation, the kinetic energy is growing at a rate of $2.2 \times 10^{51}$ ergs/s and the total explosion energy is growing at twice that rate. The supernova energy will be the sum of the above with the neutrino energy yet to be deposited, the gravitational energy of the ejecta, the binding energy of the unshocked mantle, and the thermonuclear component. The scale of a supernova’s explosion energy may ultimately be set by the initial binding energy of the progenitor envelope (Figure 12). It is very important to note that in the star calculation, and perhaps generically in supernovae, energy deposition is not instantaneous. Even after the inner edge of the oxygen zone is encountered by the shock, neutrinos continue to heat the ejecta. The total supernova energy is determined only after the shock has propagated many thousands of kilometers. This means among other things that the post-shock temperatures are lower during explosive nucleosynthesis than heretofore assumed and as a consequence that the explosive yield of $^{56}\text{Ni}$ is lower. This may help solve the $^{56}\text{Ni}$ overproduction problem alluded to in §IV.

We see evidence that pulsar kick velocities of hundreds of kilometers per second can be imparted hydrodynamically to the core by the chaotic convective motions before and during explosion. However, the precise magnitude and systematics with progenitor of these recoils remain to be determined. In addition, due solely to neutrino-driven convection, a core can be left with a rotational period less than one second (even if it is not rotating at collapse). These intriguing possibilities of direct relevance to pulsar properties will be explored with future calculations, which will ultimately have to be done in 3-D.

Two major concerns of the star model are the low baryon mass of its residue and the large amount of neutron-rich material ejected (perhaps $10^{-3} M_\odot$). The latter may have embarrassing nucleosynthetic consequences, but may be solved if neutrino capture on nuclei is included in the calculation (Fuller and Meyer 1994). However, both problems
might be solved if the delay to explosion is longer. The time between bounce and explosion is a quantitative question that rests in no small part on the accuracy of the neutrino algorithm and the progenitor structure. A major index of a supernova calculation is the steady-state shock radius it achieves. Variations with progenitor and among modelers of the hydrodynamics and outcomes might be traced to its value, which ranges in the theories between 60 and 150 kilometers.

Figure 16b depicts the average neutrino luminosities versus time for the star calculation and its 1-D analog. Apart from demonstrating the large fluctuations alluded to previously and the growing effect of overshoot before explosion, it shows that the average luminosities drop by about a factor of two over a period of $\sim 10^{-15}$ milliseconds right at the explosion. The detection of this explosion signature by underground neutrino telescopes may be feasible and would shed needed light on supernova dynamics (Burrows, Klein, and Gandhi 1992).

The calculations summarized in this paper are but the beginning of a long series of numerical explorations by us and by others to understand the multi-dimensional aspects of supernova explosions. The systematics with progenitors, the possibility of dynamo action (Thompson and Duncan 1993), the recoil speeds, the gravitational radiation signatures, the effects of neutrino viscosity (Burrows and Lattimer 1988), the formation of black holes, and the nucleosynthetic yields have yet to be adequately derived or investigated. Furthermore, the influence of resolution, better neutrino transport, and the third dimension have not been clarified. Our experience with this extensive set of numerical simulations convinces us that most future issues of supernova theory should be pursued with multi-dimensional tools.
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Figure Captions

Figure 1a: Logarithm of the density vs. interior mass for selected times during the evolution of model sequence w15t. The curves labeled “a” through “d” correspond to times of 0.0200, 0.2075, 0.2106, and 0.2691 seconds, respectively, where time is measured from the beginning of the calculation. Core bounce occurred near $t = 0.209$ seconds.

Figure 1b: Logarithmic density profiles as in Fig. 1a; but for model sequence w20t. The curves labeled “a” through “d” are at times $t = 0.0200$, 0.5465, 0.5594, and 0.6020 seconds, respectively. Core bounce occurred near $t = 0.546$ seconds.

Figure 2a: Profiles of velocity vs. mass for model sequence w15t. The labeled curves (a-d) are at times of 0.0200, 0.2091, 0.2106, and 0.2691 seconds, respectively.

Figure 2b: Profiles of velocity vs. mass for model sequence w20t. The labeled curves (a-d) are at time of 0.0200, 0.5472, 0.5500, and 0.6020 seconds, respectively.

Figure 3a: Profiles of entropy (per Boltzmann’s constant per baryon) vs. mass for model sequence w15t. The labeled curves (a-d) are at times of 0.0200, 0.2106, 0.2187, and 0.2691 seconds, respectively.

Figure 3b: Profiles of entropy vs. mass for model sequence w20t. The labeled curves (a-d) are at times of 0.0200, 0.5486, 0.5700, and 0.6020 seconds, respectively.

Figure 4a: Profiles of electron fraction vs. mass for model sequence w15t. The labeled curves (a-d) are at times of 0.0200, 0.2087, 0.2187, and 0.2691 seconds, respectively.

Figure 4b: Profiles of electron fraction vs. mass for model sequence w20t. The labeled curves (a-d) are at times of 0.0200, 0.5465, 0.5551, and 0.6020 seconds, respectively.

Figure 5a: Plots of the shock radius and neutrinosphere radii vs. time for model sequence w15t. The zero point of time is taken to be the time of core bounce, $t = 209$ ms.
Figure 5b: Plots of the shock radius and neutrinosphere radii vs. time for model sequence w20t. The zero point of time is taken to be the time of core bounce, $t = 547$ ms.

Figure 5c: Plots of the shock radius vs. time for the 2-D star model and its 1-D analog. The zero point of time is taken to be the beginning of the calculation. This plot illustrates one of the major multi-dimensional effects.

Figure 6a: Plots of the neutrino luminosities vs. time for model sequence w15t (One “foe” is $10^{51}$ ergs.). The zero point of time is that of core bounce, as in Fig. 5a.

Figure 6b: Plots of the neutrino luminosities vs. time for model sequence w20t (One “foe” is $10^{51}$ ergs.). The zero point of time is that of core bounce, as in Fig. 5b.

Figure 7: Mass accretion rate through the shock and the total mass interior to shock vs. time for model sequences w15t and w20t. The zero point of time is that of core bounce in all cases.

Figure 8: Profiles of velocity vs. radius for model sequence w15n. The total evolution time is approximately 429 ms.

Figure 9: The total kinetic energy vs. time from core bounce for model sequences w15n and w15e3.

Figure 10: Plots of iron core mass vs. ZAMS mass for models from Weaver and Woosley (1995, ww) and Nomoto and Hashimoto (1988, NH) (see text). Note the large spread in iron core mass (several tenths of a solar mass) for a given total mass. wwsb is the preferred model of ww, while wwna with severely reduced semi-convection is disfavored.

Figure 11: Profiles of density vs. mass for the inner 2 M$_\odot$ of five different initial models from the Weaver and Woosley (1995) “sb” series. Locations of the iron core edges are indicated by the open circles.
Figure 12: Profiles of the binding energy vs. mass for four of the Weaver and Woosley (1995) progenitor models. Locations of the iron core edges are indicated by open circles.

Figure 13: Profiles of velocity vs. radius for the model sequence w15e3. Note the prominent second shock developing at approximately 800 km, driven by neutrino heating from below and the steady neutron star wind.

Figure 14: Entropy vs. radius for the final timestep in the w15e3 model sequence. The peak at 800 km is due to the shock at the interface between the neutron star wind and the more slowly moving material beyond it. The peak at 1800 km is that which was created at core bounce, and the peak near 2400 km was created when the bounce shock encountered the inner edge of the silicon shell.

Figure 15: The $Y_e$ distribution at $t = 238$ milliseconds ($\sim 30$ milliseconds after bounce). The figure is 150 km×150 km. The dominant flows are still inward and the degree of overshoot is significant. Velocity vectors are superposed.

Figure 16a: Neutrino luminosities vs. angle at three different times in the star model sequence. The luminosity is defined to be the flux along the indicated angular coordinate times the total area at infinity. The times are 0.2502, 0.2703, and 0.2977, seconds; the first time is shortly after the onset of the instability, and the final time is immediately prior to the explosion. The curves for the final time are labeled with “e.”

Figure 16b: The three neutrino luminosities vs. time for the star model sequence and its 1-D analog. The luminosities plotted for the 2-D star model are the angle-averaged quantities.

Figure 17: Same as Figure 15, but for 270 milliseconds. Multiple plume and bubble structures can be seen.
Figure 18: The $Y_e$ distribution at $t = 299$ milliseconds, $\sim 10$ milliseconds before explosion. The plot is 150 km on a side and velocity vectors trace the flow. Note the vigorous bubble at $\sim 20^\circ$ that has pushed the shock out to $\sim 135$ kilometers and the corrugations at the base of the convective zone.

Figure 19: The $Y_e$ distribution with velocity vectors at $t = 304$ milliseconds, $\sim 5$ milliseconds before explosion. The scale is as in Figure 15. The size of the convective zone has increased significantly since bounce (or since in Figure 15). A strong downward plume can be seen at $\sim 60^\circ$ penetrating deep below the neutrinospheres.

Figure 20: The $Y_e$ distribution for the star calculation at $t = 307$ milliseconds, just before explosion. The scale is 150 km×150 km. The material near $\theta = 0^\circ$ is moving inward and the chaotic motions are quite vigorous and varied (see the velocity vectors that follow the flow).

Figure 21: The $Y_e$ distribution at $t = 311$ milliseconds calculation, within milliseconds of exploding. Some material is still falling in. The scale is as in Figure 20 and velocity vectors indicate the direction and magnitude of the motions.

Figure 22: Same as Figure 19, but the entropy distribution (at $t = 304$ milliseconds) without velocity vectors. The hot spots in blue have reached entropies of $\sim 32$ units.

Figure 23: Neutrinosphere radii at the times indicated in Figure 16a. For each neutrino species, the uppermost curve corresponds to the first time indicated (0.2502 sec), and the trend with time is uniformly downward. Thus the lowest curves on this plot correspond to the curves labeled “e” in Figure 16a.

Figure 24: The entropy distribution at $t = 318$ milliseconds, ($\sim 10$ milliseconds into the explosion). The scale is 1500 km×1500 km. The hot blue bubbles driving the explosion are clearly seen.
Figure 25: Same as Figure 24, but at $t = 348$ milliseconds. The shock is encountering the inner edge of the oxygen zone near 700 kilometers.

Figure 26: Same as Figure 24, but at $t = 378$ milliseconds. The highest entropies (at $\sim 38$ units) are found in clumps.

Figure 27: The entropy distribution at $t = 408$ milliseconds ($\sim 100$ milliseconds into the explosion) for the star model. The horizontal and vertical scales are 2500 kilometers. Note the hot spots near $\theta = 45^\circ$, where the entropy is $\sim 60$ units. Note also the warm fingers (with mushroom caps) and the thin cooler tendrils.

Figure 28: Same as Figure 27 (at $t = 408$ milliseconds), but for $Y_e$, with velocity vectors superposed. The tongue of material with $Y_e'$s of 0.43–0.46 is in evidence, as is a slower plume near $\theta = 20^\circ$.

Figure 29: Same as Figure 27, but for the radial velocity. The “jets” in places are moving faster than $5 \times 10^9$ cm/s ($\equiv 50,000$ km/s).

Figure 30: The same as Figure 27 ($t = 408$ milliseconds), but for log$_{10}$ $\rho$. The blue dots in the most vigorous finger have densities below $10^6$ gm/cm$^3$, and entropies near 35. Their densities are below those immediately in front of the shock.
Table 1

Central Density Rise Times for 1-D Models

| Model Sequence | w15t | w15e3 | w15n | w20t | w20n |
|----------------|------|-------|------|------|------|
| *Δt 10-11 (ms) | 167  | 167   | 186  | 145  | 192  |
| Δt 11-12 (ms)  | 25.1 | 25.1  | 35.9 | 20   | 32.5 |
| Δt 12-13 (ms)  | 5.25 | 5.25  | 8.5  | 4.75 | 8.48 |
| Δt 13-14 (ms)  | 1.53 | 1.53  | 2.4  | 1.30 | 2.51 |

*Time for the central density to go from $10^{10}$ to $10^{11}$ gm/cm$^3$, other rows correspondingly.
Table 2

Evolution of Fe and Si Edges for 1-D Models

| Model Sequence | w15t      | w15e3     | w15n      | w20t      | w20n      |
|----------------|-----------|-----------|-----------|-----------|-----------|
| R(Fe) at t=0 (cm) | $1.15 \times 10^8$ | $1.15 \times 10^8$ | $1.15 \times 10^8$ | $2.21 \times 10^8$ | $2.21 \times 10^8$ |
| R(Fe) at Bounce (cm) | $5.38 \times 10^7$ | $5.38 \times 10^7$ | $1.89 \times 10^7$ | $1.43 \times 10^8$ | $8.74 \times 10^7$ |
| R(Si) at t=0 (cm) | $2.46 \times 10^8$ | $2.46 \times 10^8$ | $2.46 \times 10^8$ | $3.27 \times 10^8$ | $3.27 \times 10^8$ |
| R(Si) at Bounce (cm) | $1.71 \times 10^8$ | $1.71 \times 10^8$ | $1.71 \times 10^8$ | $2.54 \times 10^8$ | $2.13 \times 10^8$ |
| R(Si) at Finish (cm) | $1.61 \times 10^8$ | $3.45 \times 10^8$ | $3.63 \times 10^8$ | $2.41 \times 10^8$ | $9.51 \times 10^7$ |
| Time at Bounce (ms) | 208       | 209       | 244       | 547       | 744       |
| $t_{sh}$(Fe) (ms) | 233 (24.7) | 232 (23.3) | 245 (1.93) | ...       | 769 (25.3) |
| $t_{sh}$(Si) (ms) | ...       | 302 (93.0) | 298 (54.3) | ...       | ...       |
| Time at Finish (ms) | 270 (61.7) | 401 (192) | 432 (188) | 602 (55.3) | 905 (161) |

* $t_{sh}$ is the time the shock encounters the composition boundary.

(numbers in parentheses are times after bounce in milliseconds.)
## Table 3

### Index of Color Figures

| Figure # | File Name | Epoch (ms) | Scale (km×km) | Variable |
|----------|-----------|------------|---------------|----------|
| **Before Explosion** | | | | |
| 15 | starrcx | 238 | 150×150 | $Y_e$ |
| 17 | starrdm | 270 | 150×150 | $Y_e$ |
| 18 | starree | 299 | 150×150 | $Y_e$ |
| 19 | starrei | 304 | 150×150 | $Y_e$ |
| 20 | starrek | 307 | 150×150 | $Y_e$ |
| 21 | starren | 311 | 150×150 | $Y_e$ |
| 22 | starrei | 304 | 150×150 | Entropy |
| **After Explosion** | | | | |
| 24 | starres | 318 | 1500×1500 | Entropy |
| 25 | starrfm | 348 | 1500×1500 | Entropy |
| 26 | starrgf | 378 | 1500×1500 | Entropy |
| 27 | starrgy | 408 | 2500×2500 | Entropy |
| 28 | starrgy | 408 | 2500×2500 | $Y_e$ |
| 29 | starrgy | 408 | 2500×2500 | $v_r$ |
| 30 | starrgy | 408 | 2500×2500 | log $\rho$ |