Time-resolved spectral analysis, though a very promising method to understand the emission mechanism of gamma-ray bursts (GRBs), is difficult to implement in practice because of poor statistics. We present a new method for pulse-wise time-resolved spectral study of the individual pulses of GRBs, using the fact that many spectral parameters are either constants or smooth functions of time. We use this method for the two pulses of GRB 081221, the brightest GRB with separable pulses. We choose, from the literature, a set of possible models that includes the Band model, blackbody with a power law (BBPL), and a collection of blackbodies with a smoothly varying temperature profile, along with a power law (mBBPL), and two blackbodies with a power law (2BBPL). First, we perform a time-resolved study to confirm the spectral parameter variations, and then we construct the new model to perform a joint spectral fit. We find that any photospheric emission in terms of blackbodies is required mainly in the rising parts of the pulses and the falling part can be adequately explained in terms of the Band model, with the low-energy photon index within the regime of synchrotron model. Interestingly, we find that 2BBPL is comparable or sometimes even better, though marginally, than the Band model, in all episodes. Consistent results are also obtained for the brightest GRB of Fermi era—GRB 090618. We point out that the method is generic enough to test any spectral model with well-defined parameter variations.

Key words: gamma-ray burst: general – methods: data analysis – methods: observational

1. INTRODUCTION

The spectrum of the prompt emission of a gamma-ray burst (GRB) is generally fitted with the celebrated Band spectral model (Band et al. 1993). This model represents a non-thermal spectrum, and it can be described in terms of two smoothly joined power laws (PLs). Generally speaking, the Band model adequately fits most of the time-integrated prompt emission spectra of GRBs, though additional spectral components show up for some GRBs (e.g., Preece et al. 1996; González et al. 2003; Shirasaki et al. 2008; Abdo et al. 2009; Ackermann et al. 2010). But, to emphasize, the exceptions are very few in number in comparison with the large set of GRBs, which can be fitted with a simple Band only function (e.g., Kaneko et al. 2006; Nava et al. 2011). Zhang et al. (2011) have found that 15 out of 17 LAT detected GRBs could be fitted with the Band-only model (category I, in their notation). Hence, the Band function, to date, is the simplest, standalone model for GRB spectra, whether time-integrated or time-resolved.

Although the Band model is statistically the most appropriate model of GRB data, its physical origin is yet to be identified. Over the years, many authors have investigated the underlying mechanism of the prompt emission. In the fireball model of GRB, the observed radiation during the prompt emission is attributed to a highly relativistic optically thick outflow, which thermalizes photons due to random collisions. This thermal energy is expected to be seen from the photosphere (Goodman 1986; Paczynski 1986; Mészáros & Rees 2000; Pe’er 2008), where the fireball becomes optically thin and radiation decouples from the matter. In this scenario, the light curve (LC) is expected to be a single, smooth pulse and the spectrum should be a Lorentz boosted blackbody (BB), while the temperature of the BB adiabatically cools. Hence, the time-integrated spectrum should be a superposition of many BBs. Though some GRBs do have a single pulse, most GRB LCs are either superposition of many pulses or they are highly variable. Moreover, a BB in the Rayleigh–Jeans region has a photon index 1.0, which is much harder than that actually observed for GRBs ($\alpha \sim -1$). Many models have been proposed to overcome these difficulties. For example, the internal shock model (Rees & Meszaros 1994; Woods & Loeb 1995; Sari & Piran 1997; Kobayashi et al. 1997) assumes that the major radiation is not due to photospheric emission, but optically thin synchrotron radiation (SR) from internal shocks. One major problem with the SR is that the low-energy photon index is limited to $\alpha < \sim -2/3$ (Preece et al. 1998, Crider et al. 1999), using time-resolved spectra of 99 GRBs, have shown that the instantaneous spectra and their evolution cannot be explained by SR: $\alpha$ often crosses the line of death, set by the synchrotron model. Another possibility is that the radiation is due to inverse Compton (IC) of the thermal photons near the photosphere (Thompson 1994; Pe’er et al. 2005, 2006; Beloborodov 2010; Lazzati & Begelman 2009). This process can indeed produce a Band-like spectrum, but with a rather hard value of $\alpha \sim 0.4$ at best, assuming slow heating.

A unique prompt emission model of a GRB has yet to be determined. From the phenomenological point of view, the correct model can be identified by segregating the details from the average properties. For example, GRBs are superpositions of pulses (Nemiroff 2000; Norris et al. 2005). Hence, one should use the individual pulses for spectral study, instead of the full GRB. The next step is to study the spectral evolution within the individual pulses. Hence, one should do time-resolved spectroscopy in order to extract greater information than merely an average spectral property of a pulse, e.g., average peak energy, isotropic energy, etc. But, performing such a detailed study is difficult, as one loses photon counts. For example, Ghirlanda et al. (2010) have studied time-resolved spectra of nine selected GRBs, detected by Gamma-ray Burst Monitor (GBM) on board the Fermi satellite. As the photon count is low in each time bin, they could model the spectra only with a cutoff power law (CPL). Parameters of a more complicated model, such as Band, cannot be well constrained from the time-resolved data. The solution to this problem lies in the realization that spectral evolution is not totally unpredictable, and one can suitably parameterize this...
evolution in order to reduce the number of free parameters of the description. For example, the spectral evolution of a GRB pulse is generally described as a hard-to-soft evolution (e.g., Liang & Kargatis 1996; Kocevski & Liang 2003; Nemiroff 2012). Recently, Basak & Rao (2012a, 2012b) have assumed this hard-to-soft evolution of the individual pulses of the set of nine GRBs of Ghirlanda et al. (2010) to generate simultaneous spectral and timing model of the pulses, with essentially two parameters, namely, the peak energy at the start of the pulse ($E_{\text{peak},0}$) and the characteristic evolution parameter ($\phi_0$). The basic assumptions in this approach, however, are not well established. For example, it was assumed that the spectral softening happens throughout the pulse, though there is evidence that some GRB pulses show a different behavior like the intensity-tracking spectral evolution (see, e.g., Lu et al. 2012). Further, it was also assumed that the applicable model is the Band function throughout. Hence, it is essential to critically examine all the applicable spectral models and their evolution to arrive at a correct pulse-wise description of a GRB.

In this paper, we discuss a new method for pulse-wise spectral analysis where we parameterize the spectral evolution in order to arrive at the correct spectral description with a minimum set of free parameters. We apply this method to study GRB 081221, the brightest GRB with clean, separable pulses. We compare the results obtained for this GRB with those for GRB 090618—the brightest GRB in the Fermi era. The organization of this paper is as follows: in Section 2, we describe the analysis techniques and the basic assumptions of our model. Results are given in Section 3, and major conclusions are drawn in Section 4.

2. ANALYSIS METHOD

2.1. Data Selection and Analysis

The basic necessity for a good spectral analysis of GRB pulses is wide band coverage to identify additional spectral components. The GBM on board the Fermi satellite, with its wide band width and excellent sensitivity, provides a good data base for such studies. It has two scintillation detectors: the sodium iodide (NaI) detector is sensitive in the $\gtrsim 8$ keV to $\sim 900$ keV range while the bismuth germanate (BGO) energy range is $\sim 200$ keV to $\sim 40$ MeV (Meegan et al. 2009). We examined the Nava catalog (Nava et al. 2011) of Fermi/GBM GRBs and found that there are 112 bright (fluence $\gtrsim 10^{-6}$ erg), long ($\delta t \gtrsim 15$ s) GRBs and 11 of these GRBs have single/separable pulses. GRB 081221 is the brightest among them. In Figure 1, we have plotted the LC of this GRB with the Norris model (Norris et al. 2005) fitted for the two pulses. We have also made a systematic analysis of the other 10 GRBs and the results of the time-integrated spectral analysis for all of them are given later.

We use the CSPEC data for time-integrated study and the time tag event (TTE) data for the time-resolved spectral analysis. We choose two or more NaI detectors having high count rate and one/both BGO detector(s). For source selection and background subtraction, we use the rmfit v3.3p7 tool, developed by User Contributions of Fermi Science Support Center. The background exposure time is chosen before and after the burst. This background is modeled by a polynomial of different degrees, according to the need. The pulse-height analyzer (PHA) files are binned in energy channels so as to get a minimum count in each spectral bin. Typically, the NaI detectors are binned by minimum count $\geq 40$, while the BGO detectors are binned by requiring a minimum count of $\sim 50$–60. First, we perform time-integrated analysis for all the 11 bright, long GRBs having single/separable pulses. We implement both C-stat and $\chi^2$ minimization methods in rmfit. We fit either Band or CPL model.

In Table 1, we report the best-fit parameter values along with the corresponding 3$\sigma$ errors. The reduced C-stat and $\chi^2_{\text{red}}$ with degrees of freedom (dofs) are also reported. For comparison, we quote the results of Nava et al. (2011) for these GRBs. It is clearly seen that all these values are matching quite well. The sources of very minor deviations between the values of this work, done by C-stat minimization, and Nava et al. (2011), are (1) mismatch between actual start and stop time, (2) exact background selection and modeling, and (3) the exact number of detectors and the channels used. Comparing the deviation of the parameter values, it is clear that the deviation resulting from using different statistics other than C-stat, i.e., $\chi^2$ minimization, is much less than that resulting from these other reasons. Hence, we conclude that the statistics plays a minimal role in actual parameter estimation. In fact, the GRBs taken in our analysis are all bright GRBs (fluence $\gtrsim 10^{-6}$ erg). Hence, by default, the $\chi^2$ minimization is a correct technique for parameter estimation of GRBs with high count rates.

2.2. Spectral Models for Time-resolved Study

We select four models for the spectral study: (1) Band model, (2) blackbody with a power law (BBPL), (3) a modified blackbody with a power law (mBBPL), and (4) two blackbodies with a power law (2BBPL). The Band function can be written in terms of the spectral indices ($\alpha$ and $\beta$) and the peak energy ($E_{\text{peak}}$) as

$$I(E) = \begin{cases} A_b \left(\frac{E}{\Gamma}\right)^\alpha \exp \left[\frac{-(2+\alpha)E}{E_{\text{peak}}}\right] & \text{if } E \leq (\alpha-\beta)E_{\text{peak}}/(2\alpha) \\ A_b \left(\frac{E}{\Gamma}\right)^\beta \exp [\beta - \alpha] \left[\frac{(\alpha-\beta)E_{\text{peak}}}{100(2+\alpha)}\right]^{(\alpha-\beta)} & \text{otherwise.} \end{cases}$$

(1)

Here, $A_b$ is the normalization constant. A model consisting of thermal (BB with temperature $kT$) and non-thermal (PL with index $\Gamma$) components has been used earlier (see, e.g., Ryde 2004; Ryde et al. 2006; Ryde & Pe’er 2009; Pe’er & Ryde 2011).
name this function as BBPL. This can be written as

\[ I(E) = \frac{K_1 \times 8.0525E^2}{(kT)^4[\exp(E/kT) - 1]} + K_2 E^{-\gamma}, \]

(2)

where \( K_1 \) and \( K_2 \) are normalization constants. There are suggestions (e.g., Ryde et al. 2010) in the literature of a modified blackbody (mBB), which may exist due to angular dependence of the optical depth and the observed temperature (Pe’er 2008). Hence, we also investigate this model with a power law (mBBPL). The mBB model is a multi-color BB disk model; the local disk temperature \( kT(r) \) is proportional to \( r^{-\gamma} \). In several GRBs there are distinct additional thermal components (see, e.g., Shirasaki et al. 2008) and further, if the GRB spectrum is due to thermal IC of seed photons, then there may be, in principle, multiple

### Table 1

Results of Time-integrated Spectral Analysis of the GRBs

| GRB    | \( t_1, t_2 \)   | This Work                        | Nava et al. (2011) |
|--------|------------------|---------------------------------|--------------------|
|        | \( E_p \)        | \( \alpha \)                    | \( \beta \)        |
|        | \( \chi^2 \)     | \( \chi^2_{\text{red}} \)      | \( \chi^2_{\text{red}} \) |
| 080904 | -4.096, 21.504   | \( \alpha = 1.22^{+0.21}_{-0.20} \) | \( \alpha = 1.21^{+0.20}_{-0.19} \) |
| (CPL)  | \( E_p = 40.1^{+3.90}_{-3.56} \) | \( \chi^2 = 1.08 (597) \) | \( \chi^2_{\text{red}} = 1.23 (597) \) |
| 080925 | -3.840, 32.0     | \( \alpha = 1.06^{+0.11}_{-0.10} \) | \( \alpha = 1.06^{+0.11}_{-0.10} \) |
| (Band) | \( \beta = 2.34^{+0.30}_{-1.13} \) | \( \beta = 2.24^{+0.24}_{-0.74} \) | \( \beta = 2.29^{+0.22}_{-0.08} \) |
| 081118 | 0.003, 19.968    | \( \alpha = 0.42^{+0.70}_{-0.49} \) | \( \alpha = 0.37^{+0.70}_{-0.49} \) |
| (Band) | \( \beta = 2.19^{+0.36}_{-0.15} \) | \( \beta = 2.14^{+0.19}_{-0.19} \) | \( \beta = 2.29^{+0.05}_{-0.05} \) |
| 081207 | 0.003, 103.426   | \( \alpha = 0.58^{+0.10}_{-0.09} \) | \( \alpha = 0.58^{+0.12}_{-0.11} \) |
| (Band) | \( \beta = 2.15^{+0.17}_{-0.33} \) | \( \beta = 2.13^{+0.20}_{-0.04} \) | \( \beta = 2.22^{+0.07}_{-0.07} \) |
| 081217 | -28.672, 29.696  | \( \alpha = 1.09^{+0.12}_{-0.14} \) | \( \alpha = 1.10^{+0.16}_{-0.14} \) |
| (CPL)  | \( E_p = 193.0^{+65.9}_{-37.3} \) | \( E_p = 200.5^{+77.2}_{-41.7} \) | \( E_p = 189.7^{+11.2}_{-9.7} \) |
| 081221 | 0.003, 39.425    | \( \alpha = 0.84^{+0.06}_{-0.05} \) | \( \alpha = 0.84^{+0.06}_{-0.06} \) |
| (Band) | \( \beta = -4.24^{+0.03}_{-0.12} \) | \( \beta = -3.89^{+0.09}_{-0.1} \) | \( \beta = -3.73^{+0.20}_{-0.20} \) |
| 081222 | -0.768, 20.736   | \( \alpha = 0.89^{+0.14}_{-0.15} \) | \( \alpha = 0.89^{+0.14}_{-0.12} \) |
| (Band) | \( \beta = 2.46^{+0.37}_{-0.37} \) | \( \beta = -2.32^{+0.31}_{-0.98} \) | \( \beta = -2.33^{+0.10}_{-0.10} \) |
| 090129 | -0.256, 16.128   | \( \alpha = 1.43^{+0.10}_{-0.16} \) | \( \alpha = 1.46^{+0.18}_{-0.16} \) |
| (CPL)  | \( E_p = 170.4^{+130.0}_{-48.5} \) | \( E_p = 195.5^{+212}_{-63.5} \) | \( E_p = 166.0^{+15.1}_{-30.3} \) |
| 090709 | 0.003, 18.432    | \( \alpha = 1.04^{+0.38}_{-0.32} \) | \( \alpha = 1.08^{+0.37}_{-0.31} \) |
| (CPL)  | \( E_p = 116.7^{+76.9}_{-30.6} \) | \( E_p = 124.1^{+34.7}_{-101} \) | \( E_p = 137.5^{+12.5}_{-29.5} \) |
| 091020 | -3.584, 25.088   | \( \alpha = 1.31^{+0.29}_{-0.18} \) | \( \alpha = 1.35^{+0.22}_{-0.19} \) |
| (CPL)  | \( E_p = 255.7^{+322.0}_{-92.0} \) | \( E_p = 276.4^{+455.0}_{-107.0} \) | \( E_p = 186.8^{+24.8}_{-35.7} \) |
| 091221 | -2.048, 37.889   | \( \alpha = 1.52^{+0.27}_{-0.21} \) | \( \alpha = 1.62^{+0.34}_{-0.23} \) |
| (Band) | \( \beta = -2.40^{+0.50}_{-1.15} \) | \( \beta = -2.26^{+0.48}_{-2.80} \) | \( \beta = -2.22^{+0.10}_{-0.10} \) |
| 091221 | -2.048, 37.889   | \( \alpha = 1.24^{+0.44}_{-0.66} \) | \( \alpha = 1.44^{+0.44}_{-0.66} \) |

Notes.

a The errors quoted from Nava et al. (2011) are symmetric errors. Errors for this work are \( \sigma \) errors.
b C is the reduced C-stat value, the numbers in the parentheses are dof.s.
c The Band spectrum showed unbound \( \sigma \) errors; we found better fit with CPL for this GRB.
shown that the parameter of BBPL norms, or, we can parameterize one of the norms and norm of BBPL model is more complicated, as, unlike Band, we as the photon indices of the Band. The parameterization of the in each episode. This index can be determined in the similar way corrected for the actual start time, the actual values of chosen this start time to be zero for all the cases, except for the rising part of GRB 090618, where to account for the start time, but we have chosen it to be zero for this difficulty, let us assume (which will be justified later) that the BBPL more constrained, in the sense that the overall norm is can be made a free parameter by parameterizing the ratio of the BB norm and PL norm as $K_1/K_2 \sim \tau^{n/n_2} \sim \tau^\alpha$.

Thus, both the Band and BBPL models have the equal number of free parameters: $m + n + 8$. For the Band model, $m + n$ are norms of the Band function in $m + n$ time bins. The other eight parameters are $\alpha, \beta, \mu$, and the $E_{\text{peak}}$ at the starting bin, $E_{\text{peak}}(t_0)$, in the two episodes. Peak energy at any time $t$ is determined by $E_{\text{peak}}(t_0) \times (t/t_0)^\mu$. For the BBPL model, $m + n$ are overall norms $(K)$ and the other four parameters are PL index $(\Gamma)$, $\mu$, $\nu$, and $K_T$ at the initial bin, $kT(t_0)$, in the two episodes. For the mBBPL model, we have an added parameter, namely, $p$, in each episode. Hence, the number of parameters is $m + n + 10$. For the 2BBPL model, the photons are boosted by the same material, hence, the BB parameters cannot be arbitrary. We assume that the ratio of temperatures and norms of these BBs are fixed, which should be determined by tying the ratios in all bins. Hence, the number of parameters in this model is $m + n + 12$. As an example, if there are five time bins preceding to the peak of a pulse and 10 bins afterward, then the parameterization for Band and BBPL reduces the number of free parameters from 60 to 23; for a total of 25 bins this factor is 100 to 33 and so on.

To compare the significance of a model with respect to another, we have performed the F-test. The F value is defined in the most general case as $F = ((x_1^2/\text{dof}_1)/(x_2^2/\text{dof}_2))$, where index 1 is used for the primary model, while 2 is used for the alternative model. We compute the probably $(p)$ of a given $F$ value and thereby find the $\sigma$ significance (and the % confidence level—CL) of the alternative model preferred over the primary model. If the primary model is a subset of the alternative model, then the $F$ value is defined as, $F = ((x_1^2-x_2^2)/\text{dof}_1)/(x_2^2/\text{dof}_2))$ and the difference between the dofs (M) is deemed as the dof of the primary model.

3. RESULTS

3.1. Time-resolved Spectra of GRB 081221

We start with a time-resolved spectral analysis so that some of the assumptions sketched above can be examined and validated. The major challenge in time-resolved spectroscopy is to define the time bin size, which crucially depends on two factors: (1) timescale of spectral evolution and (2) the minimum bin size allowed by the data, which in turn depends on the subjective decision of signal-to-noise ratio (S/N). One cannot violate the latter condition for a given S/N. The peak count rate of this GRB is ~4000, while we have demanded that each PHA bin should have at least 40 counts (S/N ~ 6.3, i.e., total ~5000 counts, for 128 channels). Hence, we cannot choose smaller than ~1 s time bin. First, we choose uniform time bins of 3 s to extract time-resolved spectra. Later, we reduce the time bin to 1 s to check any improvement due to finer time bins.

3.1.1. Case I: Bin Size of 3.0 s

In Table 2, we report the results of the Band and BBPL fits to the time-resolved data of 3 s bin size. The time bin starts from $-1 s$, with 14 bins (numbered 0–13). Approximately, the first four bins belong to pulse 1, the last eight bins belong to pulse 2, and the two intermediate bins belong to the overlapping region. For the BBPL fit, we first fit the spectra with the PL index $(\Gamma)$ free. We note that $\Gamma$ is more or less constant for the major portion of the burst (bins 0–2 for pulse 1 and 6–11 for pulse 2). We take the average of $\Gamma$ over these bins, separately for the two pulses, and found in both cases, $\Gamma = 1.83$ with standard deviation $(\sigma) 0.14$ and 0.10 for pulses 1 and 2, respectively. $\Gamma$ has large error bars in the last bin of pulse 1 and last two bins of the second pulse. The values in the overlapping region (bins
Table 2

| Bin          | BBPL (Γ Free) | Band               |
|--------------|---------------|--------------------|
|              | $kT$ | $K_1$ | $Γ$ | $K_2$ | $χ^2_{red}$( dof) | $kT$ | $K_1$ | $K_2$ | $χ^2_{red}$( dof) | $α$ | $β$ | $E_{peak}$ | $χ^2_{red}$( dof) |
|--------------| ---- | ---- | --- | ---- | ---------------- | ---- | ---- | ---- | ---------------- | --- | --- | ---------- | ---------------- |
| 0            | 38.03 | 3.84 | 3.84 | 3.84 | 3.84 | 3.84 | 3.84 | 3.84 | 3.84 | 3.84 | 3.84 | 3.84 | 3.84 |
| 1            | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 | 16.26 |
| 2            | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 | 10.14 |
| 3            | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 | 10.98 |
| 4            | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  | 6.82  |
| 5            | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 | 11.36 |
| 6            | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 | 22.61 |
| 7            | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 | 23.69 |
| 8            | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 | 19.77 |
| 9            | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 | 14.41 |
| 10           | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 | 12.64 |
| 11           | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 | 11.33 |
| 12           | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 | 10.03 |
| 13           | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  | 7.61  |

Notes. For BBPL values are quoted for both power-law index (Γ) free and frozen to the mean value at the high count rate region: 1.83. $K_2$ is BB normalization, while $K_1$ is that of the PL. The norms should be used as the relative normalization, as there is another constant multiplication due to detector effective area. Bins are 0–13 with 0 denoting 13–7. The Astrophysical Journal time-resolved spectroscopy of GRB 081221. The symbols are explained in the paper. Than those of the BBPL fit in terms of the $χ^2_{red}$ of other models show superior fits compared with the BBPL model is either of these three.

3.2. Parameter Evolution

The spectral evolution in the pulses is not arbitrary. For example, the temperature of the BB evolves with time in a smooth way. This can be seen from Figure 3, where we have plotted the $kT$ of BBPL (both Γ free and frozen) by circles. The $χ^2_{red}$ of Band, BBPL, mBBPL, and 2BBPL in the time-resolved spectra of GRB 081221. The symbols are explained in the inset. For convenience, we draw lines to join the BBPL (Γ free) and dot-dashed lines to join 2BBPL points.

Figure 2. Comparison of $χ^2_{red}$ of Band, BBPL, mBBPL, and 2BBPL in the time-resolved spectroscopy of GRB 081221. The symbols are explained in the inset. For convenience, we draw lines to join the BBPL (Γ free) and dot-dashed lines to join 2BBPL points.

In Figure 2, we have plotted the $χ^2_{red}$ of BBPL with Γ thawed, BBPL with Γ frozen, and Band fit with filled circles, open circles, and stars, respectively. The BBPL model is clearly inferior to the Band model for a major portion of the second pulse, especially in the regions where photon counts are high. In the first pulse (−1 to 11 s), the BBPL model except for the one bin is comparable to the Band model. We also fit mBBPL and 2BBPL, which are shown by filled boxes and pluses. The mBBPL and 2BBPL are as good as the Band model, in terms of $χ^2_{red}$. Hence, the correct model for the major portion of the burst is either of these three.
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Table 3

| Method         | 3 s Time Bins | 1 s Time Bins |
|----------------|---------------|---------------|
|                | $\chi^2_{red}$ of Full GRB | $\chi^2_{red}$ (Second Pulse) | $\chi^2_{red}$ of Full GRB | $\chi^2_{red}$ (Second Pulse) |
| BBPL (Γ free)  | 1.31 ± 0.35   | 1.52 ± 0.32   | 1.11 ± 0.26   | 1.21 ± 0.28   |
| BBPL (Γ frozen)| 1.30 ± 0.35   | 1.50 ± 0.33   | 1.16 ± 0.27   | 1.22 ± 0.29   |
| Band           | 1.09 ± 0.14   | 1.17 ± 0.11   | 1.00 ± 0.16   | 1.04 ± 0.18   |
| mBBPL          | 1.15 ± 0.14   | 1.23 ± 0.13   | 1.07 ± 0.17   | 1.06 ± 0.19   |
| 2BBPL          | 1.09 ± 0.15   | 1.17 ± 0.13   | 1.02 ± 0.16   | 1.05 ± 0.17   |

In the falling part, the variation is clearly LK96 type, but in the rising part, the variation is rather “soft-to-hard.” This effect may be the result of overlap between two pulses. In fact, the first two bins of the second pulse belong to the overlapping region. Hence, they might be contaminated with the preceding pulse. However, the third bin, where there should not be any effect of the first pulse, also deviates from the LK96 law. This might indicate that the second pulse is genuinely intensity tracking. Kocevski & Liang (2003) argued that the “intensity-tracking” pulses for which this evolution does not appear very prominent are rather made of more than one short hard-to-soft pulse. Ghirlanda et al. (2011), on the other hand, have analyzed time-resolved spectra of 11 long and 12 short Fermi GRBs and found that the long GRBs appear to follow a “soft-hard-firm” trend, tracking the flux of the GRB, rather than a strict “hard-to-soft” evolution. Lu et al. (2012) have categorized GRB 081221 as one having a strict “hard-to-soft” pulse followed by “intensity-tracking” pulse. They have simulated overlapping pulses to show that in the overlapping region the spectral evolution may appear to be “intensity tracking.” However, they also found some single pulses to have “intensity-tracking” spectral evolution. Hence, the second pulse may be genuinely “intensity tracking.”

In Figure 4 (bottom panels), the temperature evolution is plotted against BB fluence. Here the same behavior is noticed. Hence, $kT$ evolution of the first pulse and the falling part of the second pulse can as well be described by a similar exponential decay:

$$kT(t) = kT_0 \exp\left(-\frac{\phi_{BB}(t)}{\phi_{BB,0}}\right),$$

where $\phi_{BB}(t) = \int_0^t f(t')dt'$ is the running fluence of the BB component at time $t$, $f(t')$ being the flux at $t'$. $\phi_{BB,0}$ and $kT_0$ are the constants of the evolution law. LK96 law is empirical and a simpler version, in principle, can be used instead, such as a simple PL of time. As fluence is a monotonically increasing function of time, either of them can be used for evolution study.

The BBPL model has two components. Hence, in order to parameterize the norms of this model one has to see the flux evolution of the individual components. In Figure 5, we have plotted both photon and energy flux of the individual components calculated for 8–900 keV energy range. The flux evolutions look similar for both Γ free and frozen cases. Interestingly, the PL flux is as smooth as the BB flux, in each pulse. Hence, as argued in Section 2.3, we can safely assume that the ratio of their evolutions is a smooth function of time. In Figure 6, we have shown the evolution of $\alpha, \beta$ of Band, and Γ of BBPL. The parameter $\beta$, in many cases, has either large error bars or only an upper limit could be derived. In some cases, they peg to the value $-10$. It is clear from Figure 6 that the parameters remain reasonably constant at all episodes (rising and falling part) of a pulse. Hence, we tie them over all the time bins in a given episode to determine their values with greater certainty.

Figure 3. Time evolution of $kT$ and $E_{peak}$ of GRB 081221. The filled circles are the BBPL with Γ free, while the open circles are BBPL with Γ frozen. The plus and error bars or only an upper limit could be derived. In some cases, they peg to the value $-10$. It is clear from Figure 6 that the parameters remain reasonably constant at all episodes (rising and falling part) of a pulse. Hence, we tie them over all the time bins in a given episode to determine their values with greater certainty.
Figure 4. Verification of Liang & Kargatis (1996, LK96) law in the pulses of GRB 081221. The first pulse (left panels) shows a strict hard-to-soft evolution, while the second pulse is intensity tracking. The x-axis represents the “running fluence,” defined by LK96. For the BBPL model, fluence here means that of the BB component.

Figure 5. Flux evolution of GRB 081221. Crosses represent BB flux, while triangles represent PL flux. The total flux is marked by open boxes. Energy flux, in the units of $10^{-7}$ erg cm$^{-2}$ s$^{-1}$, is plotted in the upper panels; photon flux, in the units of photon cm$^{-2}$ s$^{-1}$, is plotted in the lower panel, for both $\Gamma$ free and frozen cases.

accuracy. This reduces the number of free parameters of the description of spectral evolution to a great extent, as described in Section 2.3.

3.3. Results of the Parameterized Spectral Fitting of GRB 081221

The fact that the parameters are well-behaved functions of time makes the time-resolved spectroscopy more tractable, as we can reduce the number of free parameters in the pulse-wise description (see Section 2.3). Following the parameterization and tying scheme of Section 2.3, we do the spectral analysis for the individual pulses of the GRB. In the following analysis, we use the TTE data of NaI ($n_0$, $n_1$, $n_2$) and BGO ($b_0$) for our analysis. The constant, which takes care of the relative normalization of the detectors, should not vary throughout the burst. Hence, we freeze them to the values obtained in the time-integrated analysis, i.e., 2.25, 2.32, 2.34, and 3.24, respectively.
Additionally, we make the following changes compared with the time-resolved spectral analysis discussed earlier. We divide the data into spectra of equal total counts rather than equal time bins so that equal importance is given to all individual spectra. Further, the spectral data in each bin are regrouped into spectral channels to provide a uniform S/N. We also note that the 30–40 keV region of the spectrum of this GRB has the known calibration issues due to the K-edge of NaI (see, e.g., Guiriec et al. 2011). This does not matter much for parameter estimations, but, if one wants to compare different models in terms of $\chi^2$ then it is wise to neglect these bins. In the following, we have done the spectroscopy by neglecting the 30–40 keV band.

3.3.1. Analysis of Pulse 2

This pulse constitutes the major portion of the burst. The count rate is $\gtrsim 3$ times higher than pulse 1. Hence, we can analyze this pulse with greater accuracy and later use our experience to analyze the other one. We perform the analysis for two cases as follows.

**Case I: Analysis for count per time bin $\gtrsim 3000$.** This analysis is done by dividing the second pulse from 17.0 s onward, requiring $\gtrsim 3000$ counts per time bin. We divide the pulse into two parts. 17–21.45 s is the rising part and the rest up to 40.45 s is the falling part. In the rising part, we get three time bins and the falling part has nine of them. The spectral bins in the energies $> 100$ keV sometimes show less than $2\sigma$ count, while $<15$ keV show less than $3\sigma$ count. Hence, we merge the 8 keV to 15 keV bins to form one bin; 100–900 keV bins are merged into seven bins, with progressively higher binning at higher energies. Similarly, spectral bins of the BGO (200 keV to 30 MeV) are merged into five large bins. All the spectral fit parameters are listed in Table 4.

(a) **Rising part.** In the rising part (first four rows of Table 4), the BBPL, compared with the other models is inferior with regard to $\chi^2$ (dof): 455.92 (354). If we parameterize only the BB norm, and treat the PL norm as a free parameter, the corresponding $\chi^2$ (dof) is 487.68 (354). A BBPL fitting with no constraint gives $\chi^2$ (dof) = 451.09 (348). While we have gained dof, the $\chi^2_{\text{red}}$ remains of the same order, which confirms that the parameterization works. In comparison with BBPL, mBBPL is a better fit with $\chi^2$ (dof) = 355.68 (353), and significance of 2.55$\sigma$ (98.93% confidence). The Band model is better than the BBPL model, with $\chi^2$ (dof) = 364.41 (354), and significance of 2.37$\sigma$ (98.23% confidence). This suggests that the radiation mechanism in the rising part of the second pulse may be a photospheric emission, but the thermal part is an mBB rather

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**Figure 6.** Evolution of $\alpha$ (triangles), $\beta$ (stars) of Band, and $\Gamma$ (pluses) of BBPL throughout the GRB. The error bars of parameter $\beta$, being large and undetermined in many cases, are not shown. In some cases, the $\beta$ pegs at $-10$. Note that the parameters are more or less constant throughout the burst. Hence, they can be tied (see the text).

**Table 4.** Study of Spectral Evolution in Pulse 2 (17.0–40.55 s) of GRB 081221 (Neglecting 30–40 keV)

| Model  | $\chi^2$ (dof) | $\mu$  | $v$  | $\alpha$ | $\beta$  | $E_{\text{peak}}$ | $\rho$ | $\Gamma$ | $kT_i/kT_{\alpha}$ | $kT_i^+$ |
|--------|----------------|--------|------|----------|---------|------------------|-------|---------|-----------------|----------|
| Band   | 5.364.41 (354) | 1.0 ± 0.3 | ... | -0.44$^{+0.06}_{-0.03}$ | -7.61$^{+2.14}_{-3.02}$ | 98.59$^{+2.75}_{-3.89}$ | ... | ... | ... | ... |
| BBPL   | 595.92 (354)  | 0.5 ± 0.3 | 3.2 ± 1.0 | ... | ... | ... | ... | 1.89$^{+0.05}_{-0.04}$ | 24.16$^{+0.64}_{-0.63}$ | ... |
| mBBPL  | 355.68 (353)  | 0.9 ± 0.2 | 0.8 ± 1.0 | ... | ... | ... | ... | 0.81$^{+0.08}_{-0.04}$ | 1.81$^{+0.08}_{-0.11}$ | 40.02$^{+2.37}_{-2.04}$ |
| 2BBPL  | 351.78 (352)  | 0.6 ± 0.1 | 4.7 ± 1.0 | ... | ... | ... | ... | 1.94$^{+0.09}_{-0.11}$ | 28.73$^{+1.63}_{-1.44}$ | 9.75$^{+1.14}_{-1.03}$ |

**Note.** The values quoted are for the first time bin.

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**Table 4.** Study of Spectral Evolution in Pulse 2 (17.0–40.55 s) of GRB 081221 (Neglecting 30–40 keV)

| Model  | $\chi^2$ (dof) | $\mu$  | $v$  | $\alpha$ | $\beta$  | $E_{\text{peak}}$ | $\rho$ | $\Gamma$ | $kT_i/kT_{\alpha}$ | $kT_i^+$ |
|--------|----------------|--------|------|----------|---------|------------------|-------|---------|-----------------|----------|
| Band   | 1188.17 (993)  | -2.1 ± 0.1 | ... | -0.68 ± 0.05 | -3.55$^{+0.26}_{-0.44}$ | 115.5$^{+3.0}_{-3.2}$ | ... | ... | ... | ... |
| BBPL   | 1557.25 (993)  | -1.9 ± 0.1 | -3.2 ± 0.4 | ... | ... | ... | ... | 2.02$^{+0.03}_{-0.02}$ | 26.81$^{+0.58}_{-0.67}$ | ... |
| mBBPL  | 1223.39 (992)  | -2.0 ± 0.2 | 3.5 ± 0.3 | ... | ... | ... | ... | 0.74$^{+0.02}_{-0.03}$ | 2.03$^{+0.09}_{-0.06}$ | 49.93$^{+2.82}_{-4.44}$ |
| 2BBPL  | 1147.53 (991)  | -1.9 ± 0.1 | -3.1 ± 0.4 | ... | ... | ... | ... | 2.15$^{+0.06}_{-0.08}$ | 38.13$^{+1.63}_{-1.52}$ | 13.33$^{+0.71}_{-0.68}$ |

**Case I: Count per time bin $\gtrsim 3000$—Rising part (17.0–21.45 s, 3 bins)

**Case I: Count per time bin $\gtrsim 3000$—Rising part (21.55–40.55 s, 9 bins)

**Case I: Count per time bin $\gtrsim 1000$—Rising part (17.0–21.45 s, 10 bins)

**Case I: Count per time bin $\gtrsim 1000$—Falling part (21.55–40.55 s, 29 bins)

**Case II: Count per time bin $\gtrsim 1000$—Falling part (21.55–40.55 s, 29 bins)
than a simple BB. However, if we compare these values with a 2BBPL model, then we immediately see that this model is the best with $\chi^2 \text{(dof)} = 351.78 \ (352)$. As the set of parameters of BBPL model is a subset of 2BBPL, the significance of 2BBPL compared with BBPL is much higher: $9.29 \sigma$ (100% confidence). Compared with the Band model, 2BBPL model has a significance of $0.86 \sigma$ (60.94% confidence), which shows that 2BBPL is only marginally better than the Band model.

(b) Falling part. In the falling part (see Table 4), the Band model is better compared to the BBPL as well as the mBBPL model. Compared with the mBBPL model, the Band model has $35.22 \ less \ \chi^2$ with one more dof. A comparison with the 2BBPL model, on the other hand, shows that 2BBPL is better than the Band model at $1.03 \sigma$ significance (69.71% confidence). Compared with the Band model, 2BBPL has $40.64 \ less \ \chi^2$ with two less dofs. The Band model does not show much difference in terms of residuals of individual spectral fit. But, it is only when we perform a parameterized joint fit that we realize that the 2BBPL model is marginally better than the Band model (Table 4). Hence, in this region either Band or 2BBPL is the best model, with 2BBPL marginally better. In Figure 7, we have shown the significance of 2BBPL fitting over the BBPL fitting as a case study. The residual of the BBPL model shows excess at various channels. No such structure is visible in the residual of the 2BBPL fit. Note that the NaI K-edge is present in both the residuals between 30 and 40 keV. We have done the fitting both by including and excluding this band. When all the channels are used, the $\chi^2 \text{(dof)}$ of BBPL and 2BBPL are $340.85 \ (217)$ and $239.82 \ (215)$, respectively. 2BBPL is preferred over BBPL at a significance of $8.42\sigma$ (100% confidence, $p = 3.86 \times 10^{-17}$). If we exclude the 30–40 keV bins, the corresponding $\chi^2 \text{(dof)}$ are $300.26 \ (197)$ and $193.17 \ (195)$, while 2BBPL is preferred at a significance of $9.01\sigma$ (100% confidence, $p = 2.10 \times 10^{-19}$).

Case II: Analysis for count per time bin $\gtrsim 1000$. To check whether lowering the size of the time bins improves the BBPL fitting, we perform the same analysis for count per time bin $\gtrsim 1000$. As the count rate is lower, we merge the 100–900 keV of NaI detectors into five channels rather than seven. The rest of the binning remains the same. We obtain 10 bins in the rising and 29 bins in the falling part.

(a) Rising part. The values of $\chi^2 \text{(dof)}$ for BBPL, Band, mBBPL, and 2BBPL are $1247.91 \ (1187)$, $1328.50 \ (1187)$, $1239.20 \ (1186)$, and $1222.76 \ (1185)$, respectively. Compared with BBPL model, the Band model is preferred at $1.36 \sigma$ (82.76% confidence), mBBPL is preferred at $1.56 \sigma$ (88.17% confidence), while 2BBPL is preferred at $9.66 \sigma$ (100% confidence). Hence, the conclusions of Case I remains unaltered.

(b) Falling part. Similarly, in the falling part, the finer bin makes equal impact on all the models and hence, the conclusion remains the same. Note that compared with mBBPL, the 2BBPL model has $113.84 \ less \ \chi^2$, with the cost of one more dof.
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Table 5
Study of Spectral Evolution in Pulse 1 (−1.0 to 12.05 s) of GRB 081221

| Model   | $\chi^2$ (dof) | $\mu$ | $v$ | $\alpha$ | $\beta$ | $E_{\text{peak}}$ | $p$ | $\Gamma$ | $k\tau_b/k\tau_{\text{in}}/k\tau^*$ | $k\tau^*$ |
|---------|----------------|-------|-----|----------|---------|------------------|-----|---------|--------------------------------------|---------|
| Rising part (−1 to 2.15 s, 1 bin) |
| Band    | 115.78 (116)   | ⋯     | ⋯   | −0.55±0.26 | −10.0   | ⋯                | ⋯  | ⋯      | ⋯                                   | ⋯       |
| BBPL    | 109.67 (116)   | ⋯     | ⋯   | −0.52±0.20 | −10.0   | 170.3±30.7       | ⋯  | ⋯      | ⋯                                   | ⋯       |
| mBBPL   | 110.27 (115)   | ⋯     | ⋯   | −0.49±0.20 | −10.0   | 1.93±0.35        | ⋯  | 38.27±4.08 | ⋯                                   | ⋯       |
| 2BBPL   | 103.05 (114)   | ⋯     | ⋯   | −0.49±0.20 | −10.0   | 0.98±0.28        | ⋯  | 2.15±0.46 | 62.78±18.14                         | ⋯       |

| Falling part (2.25 to 12.05 s, 4 bins) |
| Band    | 544.65 (473)   | −0.7±0.1 | ⋯   | −0.86±0.22 | −3.61±0.69 | 82.02±7.53 | ⋯  | 174.3±3.04 | 38.47±4.36                          | 5.7±1.16 |
| BBPL    | 571.47 (473)   | −0.7±0.1 | ⋯   | −0.2±0.4   | −3.61±0.69 | 82.02±7.53 | ⋯  | 2.09±0.11 | 19.62±1.88                          | ⋯       |
| mBBPL   | 548.22 (472)   | −0.7±0.2 | ⋯   | 2.5±0.8    | −3.61±0.69 | 82.02±7.53 | ⋯  | 0.63±0.10 | 41.63±6.49                          | ⋯       |
| 2BBPL   | 544.79 (471)   | −0.7±0.2 | ⋯   | −0.1±0.5   | −3.61±0.69 | 82.02±7.53 | ⋯  | 2.04±0.14 | 28.59±5.99                          | 9.95±3.20 |

Notes. The bins are obtained by requiring $\gtrsim$1000 counts bin$^{-1}$ (30–40 keV bins are neglected).

(a) The values quoted are for the first time bin.

Hence, the 2BBPL model is preferred over mBBPL in the falling part (1.31σ with 68.99% confidence). Compared with the BBPL model, Band, mBBPL, and 2BBPL are preferred at 3.12σ (99.82% confidence), 2.67σ (99.24% confidence), and 19.61σ (100% confidence), respectively. 2BBPL is marginally better than Band at 0.95σ (65.64%).

### 3.3.2. Analysis of Pulse 1

The time-resolved spectra of this pulse are extracted by requiring $\gtrsim$1000 counts bin$^{-1}$, as the photon count is $\sim$1/3 of pulse 2. As before the spectral bins of NaI are binned in 8–15 keV and 100–900 keV, while BGO spectral channels are merged to form five broad channels. The 30–40 keV band is neglected. The results of different fits are reported in Table 5.

(a) Rising part. The rising part has only one bin from −1.0 to 2.15 s. Interestingly, the BBPL model is marginally better than the Band model in the rising part (see Table 5) at 0.87σ (61.46% confidence). The mBBPL model has comparable $\chi^2$ as BBPL, but with one more parameter. 2BBPL model has a significance of 1.04σ (70.17% confidence) compared with the Band model, while the same model has a significance of 2.19σ (97.12% confidence) compared with the BBPL model.

(b) Falling part. In the falling part of this pulse, the BBPL model is no longer the best model. Band is the best model with $\chi^2$ (dof) = 544.65 (473). The mBBPL and 2BBPL models are comparable to Band with $\chi^2$ (dof) = 544.22 (472) and 544.79 (471). Hence, the spectrum may be still thermal, though the thermal part is no longer a simple BB, but either a multi-color BB (mBB) or has multiple spectral component (one more BB) or simply synchrotron dominated (Band). Note also that the low-energy photon index ($\alpha = −0.86^{+0.22}_{-0.19}$) in the falling part is within the regime of synchrotron model, which is clearly in contrast with the rising part ($\alpha = −0.55^{+0.24}_{-0.23}$). This phenomenon of softening of photon index at the falling part of a pulse can be seen for all the pulses (see Tables 4 and 5). In Table 6, we have listed all the significance levels (in terms of $p$-value, sigma level, and CL) of a model over another. Model$_1$ is the primary model, while Model$_2$ is the alternative model. It is clear from the table that the 2BBPL model is preferred over the Band model, though marginally, in some cases. The $p$ values denote the probably that the alternative hypothesis is incorrect. Hence, the lower the value of $p$ is, the better the alternative model will be over the primary model. Only in one case, namely the falling part of pulse 1, Band is preferred over 2BBPL. But the $p$-value of this case is 0.48, which signifies that they are only comparable. Interestingly, if we use finer bin size, the significance of Band and mBBPL over BBPL decreases in the second pulse. The significance of 2BBPL, however, increases.

### 3.3.3. Connection between the Rising and Falling Parts

The smooth variations of the parameters demand that the temperature, $kT$ (of BBPL or mBBPL or 2BBPL), or peak

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### Table 6
Comparison of Different Model Fits at Different Episodes of GRB 081221

| Region       | Model$_2$/Model$_1$ | $p$   | $\sigma$ | CL  |
|--------------|---------------------|-------|----------|-----|
| Pulse 1, Rising part (−1 to 2.15 s) |
| Band/Band   | BBPL/Band           | 0.385 | 0.87     | 61.46% |
| mBBPL/Band  | BBPL/Band           | 0.415 | 0.81     | 58.50% |
| 2BBPL/Band  | BBPL/Band           | 0.298 | 1.04     | 70.17% |
| Pulse 1, Falling part (2.25 to 12.05 s) |
| Band/Band   | BBPL/Band           | 0.301 | 1.03     | 69.93% |
| mBBPL/Band  | BBPL/Band           | 0.334 | 0.96     | 66.57% |
| 2BBPL/Band  | BBPL/Band           | 0.480 | 0.705    | 51.95% |
| Pulse 2, Rising part (3000 counts bin$^{-1}$) |
| Band/Band   | BBPL/Band           | 0.018 | 2.37     | 98.23% |
| mBBPL/Band  | BBPL/Band           | 0.011 | 2.55     | 98.93% |
| 2BBPL/Band  | BBPL/Band           | 1.5×10$^{-20}$ | 9.29 | 100% |
| Pulse 2, Falling part (3000 counts bin$^{-1}$) |
| Band/Band   | BBPL/Band           | 0.390 | 0.86     | 60.94% |
| mBBPL/Band  | BBPL/Band           | 7.86×10$^{-5}$ | 3.95 | 99.99% |
| 2BBPL/Band  | BBPL/Band           | 1.99×10$^{-6}$ | 17.22 | 100% |
| Pulse 2, Rising part (1000 counts bin$^{-1}$) |
| Band/Band   | BBPL/Band           | 0.172 | 1.36     | 82.76% |
| mBBPL/Band  | BBPL/Band           | 0.118 | 1.56     | 88.17% |
| 2BBPL/Band  | BBPL/Band           | 4.55×10$^{-22}$ | 9.66 | 100% |
| Pulse 2, Falling part (1000 counts bin$^{-1}$) |
| Band/Band   | BBPL/Band           | 0.0018 | 3.12    | 99.82% |
| mBBPL/Band  | BBPL/Band           | 0.0075 | 2.67    | 99.24% |
| 2BBPL/Band  | BBPL/Band           | 1.23×10$^{-85}$ | 19.61 | 100% |
| 2BBPL/Band  | BBPL/Band           | 0.343 | 0.95     | 65.64% |

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energy, $E_{\text{peak}}$ (of Band model), should be a continuous function of time, even during the pulse peak time. Hence, these parameters should match at the peak within errors to the one predicted by the empirical law. We follow the evolution law of the rising part to predict these parameters in the first bin of the falling part. We compare these values with the corresponding observed values. In Figure 8, we have plotted the observed values with respect to the predicted values. Note that the error bars of the observed values are much less compared with those of the predicted values. The sources of errors in the predicted values are errors in the evolution parameter, $\mu$ and the errors in the actual parameter at the starting bin of the rising part. Generally, the parameter $\mu$ has large errors, which affects the errors of the predicted values considerably. The data points are essentially the same for both the wider bin (open circles) and the finer bin (filled circles). The dot-dashed line, which shows the equality of the observation and prediction, goes through all the points.

3.4. Comparison with GRB 090618

GRB 090618 is interesting in many aspects (for details see Ghirlanda et al. 2010; Rao et al. 2011; Basak & Rao 2012a), one being its very high fluence. The redshift ($z$) of this object is 0.54 and the total fluence is $3398.1 \pm 62.0 \times 10^{-7}$ erg cm$^{-2}$, when integrated over its duration (182.27 s). In terms of fluence, this is the brightest among all GRBs detected by Fermi. This is a very long GRB with multiple peaks. It has four broad pulses as follows. Pulse 1 is rather a clean precursor from $-1$ s to 40 s. The second pulse is well separated from this precursor, and occurs from 50 s to 75 s, with two structures in 50–61 s. The third pulse, occurring from 75 s to 100 s, is contaminated with the falling part of the second pulse, and the rising part of the fourth pulse. The fourth pulse occurs from 100 s to 124 s. Though the secondary pulses (i.e., other than the precursor) are sometimes overlapping, we can still examine the spectral variation in the first pulse and in the major portions of the other pulses. GRB 081221 has only one secondary pulse, which makes it more convenient. However, in contrast with GRB 090618, the precursor of GRB 081221 has overlaps with the secondary pulse. In order to compare the results of GRB 081221, we shall take the precursor and the second pulse of GRB 090618.

3.4.1. Precursor Pulse

The time-resolved spectra of this pulse are obtained by requiring minimum of 1000 counts bin$^{-1}$. We obtain 10 spectra in the rising part ($-1.0$ to 14.15 s) and 11 spectra in the falling part (14.15 to 40.85 s). For the rising part, the parameters, $\mu$ and $\nu$, are obtained by assuming the start time at $-10.0$ s (see Section 2.3). The results of spectral fitting by different models are reported in Table 7. The advantage of this pulse over GRB 081221 is it is longer and brighter, enabling us to parameterize the rising part. Also, this pulse is fully separated from the secondary events. It is clear from Table 7 that the BBPL model is inferior to the mBBPL model for this pulse. In the case of GRB 081221, we found that the BBPL and mBBPL models are comparable to each other, and marginally better than the Band model. In this case, we definitely need an mBBPL rather than a BBPL for a fit comparable to that of Band. Note that 2BBPL is only comparable, but not better than mBBPL. Hence, the spectrum in the rising part may be mBB dominated. In the falling part, mBBPL is again comparable to Band. The 2BBPL model is superior to all the models in the falling part. The same conclusion was drawn for GRB 081221. Hence, there is hardly any difference of spectral evolution in the precursor pulse between these two GRBs.

Table 7

| Model | $\chi^2$ (dof) | $\mu$ | $\nu$ | $\alpha$ | $\beta$ | $E_{\text{peak}}^a$ | $p$ | $\Gamma$ | $kT_0/kT_{\text{in}}/kT$ | $kt^b$ |
|-------|---------------|-------|-------|---------|--------|---------------------|----|---------|-------------------------|-------|
| Rising (−1 to 14.15 s, 10 bins) | | | | | | | | | | |
| Band | 623.94 (547) | −0.8 ± 0.2 | | | | | | | | |
| BBPL | 661.77 (547) | −0.6 ± 0.2 | | | | | | | | |
| mBBPL | 621.93 (546) | −0.7 ± 0.2 | | | | | | | | |
| 2BBPL | 624.55 (545) | −0.7 ± 0.2 | | | | | | | | |
| Falling (14.15 to 40.85 s, 11 bins) | | | | | | | | | | |
| Band | 602.53 (571) | −1.0 ± 0.3 | | | | | | | | |
| BBPL | 642.37 (571) | −0.9 ± 0.3 | | | | | | | | |
| mBBPL | 602.30 (570) | −1.1 ± 0.2 | | | | | | | | |
| 2BBPL | 593.74 (569) | −0.3 ± 0.3 | | | | | | | | |

Notes. The bins are obtained by requiring $\gtrsim 1000$ counts bin$^{-1}$. The values quoted are for the first time bin.

Figure 8. Predicted peak energy ($E_{\text{peak}}$) of Band, $kT$ of BBPL, $kT_{\text{in}}$ of mBBPL, and $kt^a$ of 2BBPL models are compared with the observed values. The open circles are the values obtained for wider bin size ($\gtrsim 3000$ counts bin$^{-1}$), while the filled circles represent values obtained for finer bin size ($\gtrsim 1000$ counts bin$^{-1}$). The dot-dashed line is the line of equality.
The values quoted are for the first time bin. Band 2012.93 (1707) 8
Rising part (61–64.35 s, 1 bin)
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We obtain the following
the 2BBPL model, which shows comparable or better fit than
part. In comparison, the Band model is much better. However,
various models are reported in Table 8. It is clear from this table
following the analysis of GRB 081221, we restrict ourselves to
principle, can be lowered as we have higher count rate, but,
Hence, the Band and 2BBPL models are comparable, though
spectra should be equally good at the peak position, except for
that while Band accounts for the peak position, with exponential
model and the models with thermal and non-thermal parts is
two parts in a GRB spectrum—the peak of the spectrum
This pulse is more difficult to analyze because it has two small
structures in the rising part and two pulses at the peak. Hence,
we ignore the 50–61 s of data and analyze only the 61–75 s of
data. The falling part covers 64.35–74.95 s, where we obtain
24 time-resolved spectra requiring 2000 counts bin$^{-1}$. This, in
principle, can be lowered as we have higher count rate, but,
following the analysis of GRB 081221, we restrict ourselves to
a moderate count per bin. The results of joint spectral fit with
various models are reported in Table 8. It is clear from this table
that BBPL and mBBPL are not the correct models in the falling
part. In comparison, the Band model is much better. However,
the 2BBPL model, which shows comparable or better fit than
Band in all cases, is only comparable to mBBPL in this particular
case. This may arise due to the fact that this pulse is actually
a combination of two highly overlapping pulses (see Rao et al.
2011). Hence, we redo the analysis on 11 spectra from 69.25 s
to 74.95 s, which covers only the falling part of the second pulse.
We obtain the following $\chi^2$ (dof): 1137.67 (991), 1179.13 (990),
and 1156.25 (989) for Band, mBBPL, and 2BBPL, respectively.
Hence, the Band and 2BBPL models are comparable, though
we cannot rule out the possibility of contamination even in this
falling part.

In the rising part (see Table 8), though mBBPL is better than
BBPL, it is inferior to 2BBPL and Band. Hence, the rising part of this pulse is probably synchrotron dominated. 2BBPL is comparable to Band. In summary of this pulse, the whole episode can be described by Band model. This is in contrast with the second pulse of GRB 081221, where mBBPL clearly dominates the rising part, and then it is taken over by Band.

### 4. DISCUSSION AND CONCLUSIONS

From a purely data analysis point of view, there are essentially
two parts in a GRB spectrum—the peak of the spectrum
and the wings, which extend to very low and very high
energies. The fundamental difference between a Band-only
model and the models with thermal and non-thermal parts is
that while Band accounts for the peak position, with exponential
fall in the wings, the other models have a thermal component
to account for the peak position, and a PL, falling slower than an
exponent, holding the spectra at the wings. In principle, all these
spectra should be equally good at the peak position, except for
the fact that Band and mBB have a broader peak than the simple
BB, while 2BBPL has a double hump. Hence, the difference
between these models arises mainly in the wings. The BB is
inferior to the others if the peak is not narrow. As photon count
at the peak is larger, the residual should show up immediately.
But, this is not easy to see if the difference occurs in the wings.
Consequently, the three very different models, namely, Band,
mBBPL, and 2BBPL show comparable $\chi^2$, while fitting
time-resolved data. Hence, re-binning at these wings plays a very
important role to pin down the correct model. However, we
cannot expect an order-of-magnitude improvement in the $\chi^2$,
because, binning in these wings gives 6–8 broad bins (see
Section 3.3.1) with large errors, while the major statistics comes
from the peak position.

We have found, in our analysis, that the spectrum changes
from one pulse to the other, and even within a pulse. The fact that
one of these four models is superior to the others, in a particular
episode, points to the fundamental radiation mechanism. We see
that this change of superiority is not random. For example, the
first pulse of both the GRBs has shown that an mBBPL model
is better, though marginally, than Band in the rising part. For GRB
081221, this could be described even by the BBPL model in the
rising part, which is really pointing toward the thermal origin
of radiation in the first pulses. Similar observations are reported in
the literature; e.g., Ryde & Pe’er (2009) showed, in the 1–3 s
time bin of BATSE detected GRB 981021, that a BBPL is better
fit than a Band model. Note that the low-energy photon index
($\alpha$) of the Band model crosses the synchrotron limit in the rising
part, where the thermal models are adequate. However, in the
falling part of all the pulses, where Band is better than mBBPL,
$\alpha$ is consistent with the synchrotron limit. Hence, we can safely
conclude that the radiation mechanism starts with a thermal
origin, but is rapidly overtaken by synchrotron mechanism. The
first pulse may be dominated by the photospheric emission in all
episodes, but the second pulse is mostly synchrotron dominated.
The second pulse may or may not have a thermal origin.
For example, the second pulse of GRB 081221 shows an mBBPL
model in the rising part, which then becomes synchrotron
-dominated in the falling part. On the other hand, the second
pulse of 090618 is always synchrotron dominated. Hence, the
transition between these different radiation paradigms is smooth
and repeatable.

In comparison to the mBBPL model, we note that the 2BBPL
is particularly better in all episodes. This model sometimes
shows superiority to the Band model, even at the falling part
Figure 9. Comparison of spectral fitting between the 2BBPL (upper panels) and the Band (lower panels) models for $-1.0$ to $2.15$ s time bin of GRB 081221. $\nu F_{\nu}$ has the units keV$^2$ (photon cm$^{-2}$ s$^{-1}$). The right panels show the fit with the 30–40 keV channels neglected. Note the structure in the residual of the Band model—positive excess near 15 keV and 150 keV, and negative excess near 40–60 keV. Compared to this, the 2BBPL model does not show any structures in the residual. The 2BBPL model is preferred over the Band model at 1.04 $\sigma$ with 70% confidence level and $p$-value = 0.298 for 30–40 keV neglected case and at 0.95 $\sigma$ with 65.5% confidence level and $p$-value = 0.341 for 30–40 keV included case, based on $F$-tests.

of a pulse, except for the second pulse of GRB 090618, though we cannot rule out the possibility of two highly overlapping pulses in this particular case. A softer component than Band was reported for a few BATSE GRBs by Preece et al. (1996). Shirasaki et al. (2008), using the time-resolved spectral data of GRB 041006, detected by HETE-2 (2 keV to 400 keV), found multiple spectral components, each having characteristic evolution. After the launch of Fermi satellite, these earlier claims were reconfirmed in some cases. For example, Guiriec et al. (2011), fitting the time-integrated spectrum of GRB 100724B, have shown the presence of an additional BB component along with the traditional Band spectrum (also see Burgess et al. 2011). In our analysis, we have used two BBs to account for the softer components. The origin of these two components is speculative. They might be different locations of the boosted front of the fireball having the same temperature, but different boosting factors. Alternatively, they can be different seed photon baths, upscattered by the bulk material. Irrespective of its origin, this model shows superiority to all other models in all episodes. Note that, although 2BBPL model has double hump in the peak, one of the peaks may occur in the lower wing (i.e., $<15$ keV). Hence, it is easy to identify this model, only if the difference occurs at the peak. Figure 7 clearly shows the double hump in the residuals of the BBPL fit. Hence, it is easy to visualize the 2BBPL model from this figure. The Band model, however, has similar residuals as the 2BBPL model. Hence, the data are not sufficient to distinguish between these two models, except when we perform a parameterized joint fit. In Figure 9, we have shown the marginal superiority of the 2BBPL fit over the Band model as a case study of the rising part of the first pulse ($-1.0$ to $2.15$ s) of GRB 081221. In the right panels, we have plotted the fitted data neglecting the 30–40 keV channels. The upper panels are 2BBPL fits, while the lower panels are Band model fits. Residuals of the Band model show structures with excesses in 15 keV, 50–60 keV, and 150 keV regions. These are not present in the residuals of the 2BBPL model. Of course, the difference is not as prominent as the case of Figure 7. The 2BBPL model is preferred over the Band model at 1.04 $\sigma$ ($p = 0.298$, 70.17% confidence). Hence, 2BBPL is only marginally better than the Band model.

To visualize the evolution of the lower BB component, we have plotted in Figure 10 the residuals of 2BBPL fit, with the lower BB omitted. This technique is well known for finding iron line profile in the inner accretion disks of black holes (see, e.g., Miller 2007). We fit the spectrum with the 2BBPL model and then omit the lower BB. The residual (expressed
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as normalized counts keV$^{-1}$ s$^{-1}$ of the fit clearly shows this BB component. Residuals of different detectors are shown by different markers. We have overplotted the lower BB model (in terms of normalized counts keV$^{-1}$ s$^{-1}$) on the residual to guide the eye. We have plotted these residuals for second, sixth, and ninth time bins from top to bottom panels to show that the BB peaks at lower energies at later times.

In summary, we have rigorously used the evolution of parameters in the pulses of a GRB to construct various spectral models with a minimum number of parameters. We have constructed the Band model with parameterized peak evolution and tied photon indices, BBPL with parameterized norm ratio of the BB and PL, parameterized temperature, and tied PL indices. Apart from these, we have used mBBPL and 2BBPL, which, other than the same parameterizations as BBPL, have tied $p$ indices, and tied ratios of temperatures and norms, respectively. This new method is quite general in the sense that any such model can be incorporated with suitable parameterization. The fact that the parameterization works demands a close look into the theoretical predictions of various radiation models. These models, irrespective of their complexities, should produce such smooth variations of parameters within a pulse of a GRB. Also, if there is really a transition from one radiation mechanism to another, one should correctly model the mechanism of such a transition. The fact that the synchrotron model is applicable at the falling part of the pulses, without invoking any other component, is intriguing and demands a close look at the predictions of the internal shock model.

One of the surprising result obtained in this work is that the 2BBPL model is statistically superior to the other models in most of the episodes in these two GRBs. Basak & Rao (2012c) have used this model for GRB 090902B. The residual of BBPL fit clearly shows double humps (see Figure 3 of Basak & Rao 2012c), which are taken care of by the two peaks of the 2BBPL model. We selected this model purely in a phenomenological and data analysis perspective: distinct BB components (apart from the main peak in the spectrum) are seen in a few GRBs and while looking at the residuals, two humps are clearly discernible in a few time bins. Since these two are the brightest GRBs for such analysis (GRB 081221 is the brightest GRB in the category of GRBs with single/separable pulses and GRB 090618 is the brightest GRB in the Fermi era), it is unlikely that we can reinforce this result by analyzing data from other GRBs. One method could be to get the pulse-wise spectral parameters of a sample of GRBs and relate them to other properties of GRBs like redshift, afterglow properties, etc. This will not only help us to identify the most appropriate spectral description but also to identify the emission mechanism operating during the prompt emission.

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Figure 10. Residuals in various time bins of the falling part of GRB 081221. The data used are those with ≥3000 counts bin$^{-1}$. The time bins used are second, sixth, and ninth bins (from top to bottom). The residuals are obtained by omitting the lower BB from the 2BBPL fit. The lower BB models are overplotted with the residuals to show the significance of this BB component. Different lines and symbols show different detectors. Note that the lower BB temperature shifts to the lower energy with time.
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