Phantom field dynamics in loop quantum cosmology

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We consider a dynamical system of phantom scalar field under exponential potential in background of loop quantum cosmology. In our analysis, there is neither stable node nor repeller unstable node but only two saddle points, hence no Big Rip singularity. Physical solutions always possess potential energy greater than magnitude of the negative kinetic energy. We found that the universe bounces after accelerating even in the domination of the phantom field. After bouncing, the universe finally enters oscillatory regime.

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I. INTRODUCTION

Recently, present accelerating expansion of the universe has been confirmed with observations via cosmic microwave background anisotropies \(^1\), \(^2\) large scale galaxy surveys \(^3\) and type Ia supernovae \(^4\), \(^5\). However, the problem is that the acceleration can not be understood in standard cosmology. This motivates many groups of cosmologists to find out the answers. Proposals to explain this acceleration made till today could be, in general, categorized into three ways of approach \(^6\).

In the first approach, in order to achieve acceleration, we need some form of scalar fluid so called dark energy with equation of state \(p = w\rho\) where \(w < -1/3\). Various types of model in this category have been proposed and classified (for a recent review see Ref. \(^7\), \(^8\)). The other two ways are that accelerating expansion is an effect of backreaction of cosmological perturbations \(^9\) or late acceleration is an effect of modification in action of general relativity. This modified gravity approach includes braneworld models (for review, see \(^10\)). Till today there has not yet been true satisfied explanation of the present acceleration expansion.

Considering dark energy models, a previous first year WMAP data analysis combined with 2dF galaxy survey and SN-Ia data and even a previous SN-Ia analysis alone favor \(w < -1\) than cosmological constant or quintessence \(^11\), \(^12\). Precise observational data analysis (combining CMB, Hubble Space Telescope, type Ia Supernovae and 2dF datasets) allows equation of state \(p = w\rho\) with constant \(w\) value between -1.38 and -0.82 at the 95 \% of confident level \(^13\). The recent WMAP three year results combined with Supernova Legacy Survey (SNLS) data when assuming flat universe yields \(-1.06 < w < -0.90\). However without assumption of flat universe but only combined WMAP, large scale structure and supernova data implies strong constraint, \(w = -1.06_{-0.08}^{+0.13}\) \(^14\). While assuming flat universe, the first result from ESSENCE Supernova Survey Ia combined with SuperNova Legacy Survey Ia gives a constraint of \(w = -1.07_{-0.09}^{+0.07}\) \(^15\). Interpretation of various data brings about a possibility that dark energy could be in a form of phantom field-a fluid with \(w < -1\) (which violates dominant energy condition, \(\rho \geq |p|\)) rather than quintessence field \(^16\), \(^17\), \(^18\). The phantom equation of state \(p < -\rho\) can be attained by negative kinetic energy term of the phantom field. However there are some types of braneworld model \(^19\) as well as Brans-Dicke scalar-tensor theory \(^20\) and gravitational theory with higher derivatives of scalar field \(^21\) that can also yield phantom energy. There has been investigation on dynamical properties of the phantom field in the standard FRW background with exponential and inverse-power law potentials by \(^22\), \(^23\), \(^24\), \(^25\) and with other forms of potential by \(^26\), \(^27\). These studies describe fates of the phantom dominated universe with different steepness of the potentials.

A problem for phantom field dark energy in standard FRW cosmology is that it leads to singularity. Fluid with \(w\) less than -1 can end up with future singularity so called the Big Rip \(^28\) which is of type I singularity according to classification by \(^29\), \(^30\). The Big Rip singularity corresponds to \(a \to \infty, \rho \to \infty\) and \(|p| \to \infty\) at finite time \(t \to t_s\) in future. Choosing particular class of potential for phantom field enables us to avoid future singularity. However, the avoidance does not cover general classes of potential \(^29\). In addition, alternative model, in which two scalar fields appear with inverse power-law and exponential potentials, can as well avoid the Big Rip singularity \(^31\). The higher-order string curvature correction terms can also show possibility that the Big-Rip singularity can be absent \(^32\).

Since phantom dominated FRW universe possesses singularity problem as stated above, in this work, instead of using standard FRW cosmology, the fundamental background theory in which we are interested is Loop Quantum Gravity-LQG. This theory is a non-perturbative type of quantization of gravity and is background-
It has been applied in cosmological context as seen in various literatures where it is known as Loop Quantum Cosmology-LQC (for review, see Ref. 33). Effective loop quantum modifies standard Friedmann equation by adding a correction term $-\rho^2/\rho_{\text{lc}}$ into the Friedmann equation 36, 37, 38, 39, 40. When this term becomes dominant, the universe begins to bounce and then expands backwards. LQG can resolve of singularity problem in various situations 34, 37, 41, 42. However, derivation of the modified term is under a condition that there is no matter potential otherwise, in presence of a potential, quantum correction would be more complicated 43. Nice feature of LQC is avoidance of the future singularity from the correction quadratic term $-\rho^2/\rho_{\text{lc}}$ in the modified LQG Friedmann equation 44 as well as the singularity avoidance at semi-classical regime 45. The early-universe inflation has also been studied in the context of LQC at semi-classical limit 40, 46, 47, 48, 49, 50. We aim to investigate dynamics of the phantom field and its late time behavior in the loop quantum cosmological context, and to check if the loop quantum effect could remove Big Rip singularity from the phantom dominated universe. The study could also reveal some other interesting features of the model. We organize this article as follows: in section II, we introduce LQC Friedmann equation, after that we briefly present relevant features of the phantom scalar field in section III. Section IV contains dynamical analysis of the phantom field in LQC background with exponential potential. The potential is a simplest case due to constancy of its steepness variable $\lambda$. Two real fixed points are found in this section. Stability analysis yields that both fixed points are saddle points. Numerical results and analysis of solutions can be seen in section V where we give conditions for physical solutions. Finally, conclusion is in section VI.

II. LOOP QUANTUM COSMOLOGY

LQC naturally gives rise to inflationary phase of the early universe with graceful exit, however the same mechanism leads to a prediction that present-day acceleration must be very small 46. At late time and at large scale, the semi-classical approximation in LQC formalisms can be validly used 51. The effective Friedmann equation can be obtained by using an effective Hamiltonian with loop quantum modifications 38, 44, 52:

$$C_{\text{eff}} = \frac{3M_p^2}{\gamma^2\mu^2} \sin^2(\mu\tau) + C_m.$$ (1)

The effective constraint (1) is valid for isotropic model and if there is scalar field, the field must be free, massless scalar field. The equation (1), when including field potential, must have some additional correction terms 43. In this scenario, the Hamilton’s equation is

$$\dot{p} = \{p, C_{\text{eff}}\} = -\frac{\gamma}{3M_p^2} \frac{\partial C_{\text{eff}}}{\partial \gamma},$$ (2)

where $\epsilon$ and $p$ are respectively conjugate connection and triad satisfying $\{\epsilon, p\} = \gamma/3M_p^2$. Dot symbol denotes time derivative. These are two variables in the simplified phase space structure under FRW symmetries 33. Here $M_p^2 = (8\pi G)^{-1}$ is square of reduced Planck mass, $G$ is Newton’s gravitational constant and $\gamma$ is Barbero-Immirzi dimensionless parameter. There are relations between the two variables to scale factor as $p = a^2$ and $\epsilon = \gamma a$. The parameter $\mu$ is inferred as kinematical length of the square loop since its order of magnitude is similar to that of length. The area of the loop is given by minimum eigenvalue of LQG area operator. $C_m$ is the corresponding matter Hamiltonian. Using the Eq. (2) with constraint from realization that loop quantum correction of effective Hamiltonian $C_{\text{eff}}$ is small at large scale, $C_{\text{eff}} \approx 0$, 35, 38, 39, 44, one can obtain (effective) modified Friedmann equation in flat universe:

$$H^2 = \frac{\rho}{3M_p^2} \left(1 - \frac{\rho}{\rho_{\text{lc}}}\right),$$ (3)

where $\rho_{\text{lc}} = \sqrt{3/(16\pi\gamma^2 G^2 \hbar)}$ is critical loop quantum density, $\hbar$ is Planck constant and $\rho_1$ is total density.

III. PHANTOM SCALAR FIELD

The energy density $\rho$ and the pressure $p$ of the phantom field contain negative kinetic term. They are given as 10

$$\rho = -\frac{1}{2} \dot{\phi}^2 + V(\phi),$$ (4)

$$p = -\frac{1}{2} \dot{\phi}^2 - V(\phi).$$ (5)

The conservation law is

$$\dot{\rho} + 3H(\rho + p) = 0.$$ (6)

Using the Eqs. (4), (5) and (6), we obtain Klein-Gordon equation:

$$\ddot{\phi} + 3H \dot{\phi} - V' = 0,$$ (7)

where $V' = dV/d\phi$ and the negative sign comes from the negative kinetic terms. The phantom equation of state is therefore given by

$$w = \frac{p}{\rho} = \frac{\dot{\phi}^2 + 2V}{\dot{\phi}^2 - 2V}.$$ (8)

From the Eq. (8), when the field is slowly rolling, as long as the approximation, $\dot{\phi}^2 \sim 0$ holds, the approximated value of $w$ is -1. When the bound, $\dot{\phi}^2 < 2V$ holds, $w$ is always less than -1. As mentioned before in sections I and III, there is not yet been a derivation of effective LQC Friedmann equation in consistency with a presence of potential. Even though, the Friedmann background is valid only in absence of field potential, however, investigation of a phantom field evolving under a potential is a challenged task.
Here we also neglect loop quantum correction effect in the classical expression of Eqs. (4) and (5) (see Refs. 43 and 53 for discussion).

IV. DYNAMICAL ANALYSIS

Differentiating the Eq. (3) and using the fluid Eq. (6), we obtain
\[ \dot{H} = -\frac{(\rho + p)}{2M_p^2} \left(1 - \frac{2\rho}{\rho_c}\right). \] (9)

The Eqs. (3), (6) and (9), in domination of the phantom field, become
\[ H^2 = \frac{1}{3M_p^2} \left(\frac{\dot{\phi}^2}{2} + V\right) \left(1 - \frac{\rho}{\rho_c}\right), \] (10)
\[ \dot{\rho} = -3H\rho \left(1 + \frac{\dot{\phi}^2}{\phi^2} + 2V\right), \] (11)
\[ \dot{H} = \frac{\phi^2}{2M_p^2} \left(1 - 2\frac{\rho}{\rho_c}\right). \] (12)

We define dimensionless variables following the style of 54
\[ X \equiv \frac{\dot{\phi}}{\sqrt{6M_pH}}, \quad Y \equiv \frac{\sqrt{V}}{\sqrt{3M_pH}}, \quad Z \equiv \frac{\rho}{\rho_c}, \] (13)
\[ \lambda \equiv -\frac{M_pV'}{V}, \quad \Gamma \equiv \frac{V''}{(V')^2}, \quad \frac{d}{dN} \equiv \frac{1}{H}\frac{dt}{dt}, \] (14)

where \( N \equiv \ln a \) is e-folding number. Using new variables in Eqs. (8) and (10), the equation of state is rewritten as
\[ w = \frac{X^2 + Y^2}{X^2 - Y^2}, \] (15)

in expression of the new variables, is therefore
\[ 3X^2(2Z - 1) < 1. \] (19)

Divided by the Eq. (10), the acceleration condition under the constraint is
\[ \frac{3}{1 - (Y^2/X^2)} \left(\frac{1 - 2Z}{1 - Z}\right) < 1, \] (20)

where the conditions \(|X| \neq |Y|\) and \(Z \neq 1\) must hold. As we consider \(Z = \rho/\rho_c\) with \(\rho = -\left(\dot{\phi}^2/2\right) + V\), we can write
\[ \frac{\rho_cZ}{3M_p^2H^2} = -X^2 + Y^2. \] (21)

With the condition \(|X| \neq |Y|\), clearly from Eq. (21), we have one additional condition, \(Z \neq 0\).

A. Autonomous system

Differential equations in autonomous system are
\[ \frac{dX}{dN} = -3X - \sqrt{\frac{3}{2}} \lambda Y^2 - 3X^3 (1 - 2Z), \] (22)
\[ \frac{dY}{dN} = -\sqrt{\frac{3}{2}} \lambda XY - 3X^2Y (1 - 2Z), \] (23)
\[ \frac{dZ}{dN} = -3Z \left(1 + \frac{X^2 + Y^2}{X^2 - Y^2}\right), \] (24)
\[ \frac{d\lambda}{dN} = -\sqrt{6}(\Gamma - 1)X^2. \] (25)

Here we will apply exponential potential,
\[ V(\phi) = V_0 \exp\left(-\frac{\lambda}{M_p}\phi\right), \] (26)
to this system. The potential is known to yield power-law inflation in standard cosmology with canonical scalar field. Its slow-roll parameters are related as \(\epsilon = \eta/2 = 1/P\) where \(\lambda = \sqrt{2/P}\) and \(P > 1\) 55 56. Although the potential has been applied to the quintessence scalar field with tracking behavior in standard cosmology 55, the quintessence field can not dominate the universe due to constancy of the ratio between densities of matter and quintessence field (see discussion in Ref. 5). In case of phantom field in standard cosmology under this potential, a stable node is a scalar-field dominated solution with the equation of state, \(w = -1 - \lambda^2/3\) 24 27 58. In our LQC phantom domination context, from Eq. (25), we can see that for the exponential potential, \(\Gamma = 1\). This yields trivial value of \(d\lambda/dN\) and therefore \(\lambda\) is a non-zero constant otherwise the potential is flat.

B. Fixed points

Let \(f \equiv dX/dN, g \equiv dY/dN\) and \(h \equiv dZ/dN\). We can find fixed points of the autonomous system under

1 The relation \(\Omega_\phi = \rho/3H^2M_p^2 = -X^2 + Y^2 = 1\) can not be applied here since it is valid only for standard cosmology with flat geometry.
FIG. 1: Three-dimensional phase space of $X$, $Y$ and $Z$. The saddle points (a) (-0.40825, 1.0801, 0) and (b) (-0.40825, -1.0801, 0) appear in the figure. $\lambda$ is set to 1. In region $Z < 0$, the solutions (red and blue lines) are non physical. In this region, $Z \rightarrow -\infty$ when $(X, Y) \rightarrow (0, 0)$. The green lines (class I) are in region $|X| > |Y|$ and $Z > 1$ but they are also non physical since they correspond to imaginary $H$ values. The only set of physical solutions (class II) is presented with black lines. They are in region $|Y| > |X|$ and range $0 < Z < 1$. This is the region above (a) and (b) of which $H$ takes real value. There are separatrices $|X| = |Y|, Z = 0$ and $Z = 1$ in the system (see section V B).

Special condition:

$$(f, g, h) |(x_c, y_c, z_c) = 0.$$  \hspace{1cm} (27)

The are two real fixed points of this system: \footnote{The other two imaginary fixed points $(i, 0, 0)$ and $(-i, 0, 0)$ also exist. However they are not interesting here since we do not consider model that includes complex scalar field.}

- Point (a): $$\left( -\frac{\lambda}{\sqrt{6}}, \sqrt{1 + \frac{\lambda^2}{6}}, 0 \right),$$  \hspace{1cm} (28)

- Point (b): $$\left( -\frac{\lambda}{\sqrt{6}}, -\sqrt{1 + \frac{\lambda^2}{6}}, 0 \right).$$  \hspace{1cm} (29)

C. Stability Analysis

Suppose that there is a small perturbation $\delta X$, $\delta Y$ and $\delta Z$ about the fixed point $(X_c, Y_c, Z_c)$, i.e.,

$$X = X_c + \delta X, \quad Y = Y_c + \delta Y, \quad Z = Z_c + \delta Z.$$  \hspace{1cm} (30)

From Eqs. (22), (23) and (24), neglecting higher order terms in the perturbations, we obtain first-order differential equations:

$$\frac{d}{dN} \begin{pmatrix} \delta X \\ \delta Y \\ \delta Z \end{pmatrix} = \mathcal{M} \begin{pmatrix} \delta X \\ \delta Y \\ \delta Z \end{pmatrix}. \hspace{1cm} (31)$$

The matrix $\mathcal{M}$ defined at a fixed point $(X_c, Y_c, Z_c)$ is given by

$$\mathcal{M} = \begin{pmatrix} \frac{\partial f}{\partial X} & \frac{\partial g}{\partial X} & \frac{\partial h}{\partial X} \\ \frac{\partial f}{\partial Y} & \frac{\partial g}{\partial Y} & \frac{\partial h}{\partial Y} \\ \frac{\partial f}{\partial Z} & \frac{\partial g}{\partial Z} & \frac{\partial h}{\partial Z} \end{pmatrix} (X = X_c, Y = Y_c, Z = Z_c). \hspace{1cm} (32)$$

We find eigenvalues of the matrix $\mathcal{M}$ for each fixed point:

- At point (a):

$$\mu_1 = \lambda^2, \quad \mu_2 = -\lambda^2, \quad \mu_3 = -3 - \frac{\lambda^2}{2}.$$  \hspace{1cm} (33)

- At point (b):

$$\mu_1 = \lambda^2, \quad \mu_2 = -\lambda^2, \quad \mu_3 = -3 - \frac{\lambda^2}{2}.$$  \hspace{1cm} (34)

From the above analysis, each point possesses eigenvalues with opposite signs, therefore both point (a) and (b) are saddle. Results from our analysis are concluded in TABLE I. Location of the points depends only on $\lambda$ and
FIG. 3: Phase space of the kinetic part $X$ and potential part $Y$ in standard general relativistic case. The location of points (a) and (b) in Fig. 2 are on the trajectory solutions here. This plot shows dynamics of phantom field in standard cosmological background without any other fluids. In presence of a barotropic fluid with any equation of state, the point (a) and (b) correspond to the Big Rip [23, 25].

The points exist for all values of $\lambda$. Both points correspond to the equation of state $-1 - \lambda^2/3$, that is to say, it has phantom equation of state for all values of $\lambda \neq 0$. Since there is no any attractor in the system, a phase trajectory is very sensitive to initial conditions given to the system. The stable node (the Big Rip) of the standard general relativistic case in presence of phantom field and a barotropic fluid, disappears here (see [23]).

V. NUMERICAL RESULTS

Numerical results from the autonomous set (22), (23) and (24) are presented in Figs. 1 and 2 where we set $\lambda = 1$. Locations of the two saddle points are: point (a) $(X_c = -0.40825, Y_c = 1.0801, Z_c = 0)$ and point (b) $(X_c = -0.40825, Y_c = 1.0801, Z_c = 0)$ which match our analytical results. In Fig. 3 we present a trajectory solution of a phantom field evolving in standard cosmological background for comparing with the trajectories in Fig. 2 when including loop quantum effects. The standard case has only simple two trajectories corresponding to a constraint $-X^2 + Y^2 = 1$. This is attained when taking classical limit, $Z = 0$. In loop quantum case, since there is no any stable node and the solutions are sensitive to initial conditions, we need to classify solutions according to each domain region separated by separatrices $|X| = |Y|, Z = 0$ and $Z = 1$ so that we can analyze them separately. Note that the condition, $Z > 0$ must hold for physical solutions since the density can not be negative or zero, i.e. $\rho > 0$. The blue lines and red lines in Figs. 1 and 2 are solutions in the region $Z < 0$ hence are not physical and will no longer be of our interest. From now on we consider only the region $Z > 0$. In regions with $|X| > |Y|$, the solutions therein are green lines (hereafter classified as class I). The other regions with $|Y| > |X|$ contain solutions seen as black line (classified as class II). Note that all solutions can not cross the separatrices due to conditions in Eqs. (10), (20) and (24).

A. Class I solutions

Consider the Friedmann equation (10), the Hubble parameter, $H$ takes real value only if

$$\frac{1}{3M_P^2}\left(\frac{\dot{\phi}^2}{2} + V\right)\left(1 - \frac{\rho}{\rho_{c0}}\right) \geq 0. \quad (35)$$

Divided by $H^2$ on both sides, the expression above becomes

$$(-X^2 + Y^2)(1 - Z) \geq 0. \quad (36)$$

It is clear from (30) that, in order to obtain real value of $H$, class I solutions (green line) must obey both conditions $|X| > |Y|$ and $Z > 1$ together. However, when imposing $|X| > |Y|$ to the Eq. (24), we obtain $Z < 0$ instead. This contradicts to the required range $Z > 1$. Therefore this class of solutions does not possess any real values of $H$ and hence not physical solutions.

B. Class II solutions

Proceeding the same analysis done for class I, we found that in order for $H$ to be real, class II solutions must obey both $|Y| > |X|$ and $0 < Z < 1$ together. Moreover when imposing $|Y| > |X|$ into Eq. (21), we obtain $Z > 0$. Therefore as we combine both results, it can be concluded

| Name | $X$ | $Y$ | $Z$ | Existence | Stability | $w$ | Acceleration |
|------|-----|-----|-----|-----------|----------|----|--------------|
| (a)  | $-\frac{1}{\sqrt{2}}\sqrt{1 + \frac{\lambda^2}{2}}$ | 0 | All $\lambda$ | Saddle point for all $\lambda$ | $-1 - \frac{\lambda^2}{2}$ | For all $\lambda$ (i.e. $\lambda^2 > -2$) | |
| (b)  | $-\frac{1}{\sqrt{2}}\sqrt{1 + \frac{\lambda^2}{2}}$ | 0 | All $\lambda$ | Saddle point for all $\lambda$ | $-1 - \frac{\lambda^2}{2}$ | For all $\lambda$ (i.e. $\lambda^2 > -2$) | |

TABLE I: Properties of fixed points of phantom field dynamics in LQC background under the exponential potential.
that class II solutions can possess real \( H \) value in the region \(|Y| > |X|\) and \(0 < Z < 1\), i.e., \(0 < \rho < \rho_{lc}\).

The bound is slightly different from the case of canonical scalar field in LQC (see Ref. [59]) of which the bound is \(0 \leq \rho \leq \rho_{lc}\). The class II is therefore the only class of physical solutions.

For class II solutions, we consider another set of autonomous equations from which the evolution of cosmological variables are conveniently obtained by using numerical approach. In the new autonomous set, instead of using \(N\), which could decrease after the bounce from LQC effect, time is taken as independent variable. We define new variable as

\[
\dot{\phi} = S.
\]

The Eqs. (37) and (12) are therefore rewritten as

\[
\dot{H} = \frac{S^2}{2M_P^2} \left[ 1 - \frac{2}{\rho_{lc}} \left( -\frac{S^2}{2} + V(\phi) \right) \right],
\]

\[
\dot{S} = -3HS + V'.
\]

The Eqs. (37), (38) and (39) form another closed autonomous system. Numerical integrations from the new system yield result plotted in Figs. 4 and 5 in which set values are \(\lambda = 1, \rho_{lc} = 1.5, V_0 = 1\) and \(M_P = 2\). From Eq. (3) the slope of \(H\) with respect to \(\rho\), \(dH/d\rho\), is flat when \(\rho = \rho_{lc}/2\) [59]. Another fact is

\[
\left( \frac{d^2H}{d\rho^2} \right)_{\rho = \rho_{lc}/2} = \frac{-2}{M_P \sqrt{3\rho_{lc}}} < 0,
\]

hence, as \(\rho = \rho_{lc}/2\), \(H\) takes maximum value, \(H_{\text{max}} = \sqrt{\rho_{lc}/12M_P^2}\). This result is valid in LQC scenario regardless of types of fluid. In Figs. 4 and 5 with set parameters given above, as \(\rho = \rho_{lc}/2 = 0.75\), \(H\) is maximum, \(H_{\text{max}} = 0.17678\). When \(H \approx 0\), i.e., \(\rho\) is approximately \(\rho_{lc} = 1.5\), the expansion halts and then bounces. At this bouncing point, the dynamics enters loop quantum regime which is a quantum gravity limit. Beyond the
boune, $H$ turns negative, i.e. contracting of scale factor. The universe undergoes accelerating contraction until reaching $H_{\text{min}}$. After that it contracts decelerately until bouncing at $H \approx 0$. The universe goes on faster bouncing forward and backward. The faster bounce in $H$ is an effect from the faster bounce in $\rho$ as illustrated in Fig. 5 where the red line represents potential energy density $V(\phi)$, the black line represents kinetic energy density $-\phi^2/2$ and the blue line is total energy density $\rho$. Oscillation in $\rho$ is from oscillation in the field speed $\phi$ and therefore oscillation in K.E. as shown in Fig. 6. This hence contributes to oscillation in $\rho$. The negative magnitude of kinetic energy density becomes larger and larger as the field rolling faster and faster up the potential. The exponential potential energy density therefore becomes larger and larger. This results in oscillation of $\rho$ and affects in oscillation of $H$ about the bounce $H = 0$. With a different approach, recently a similar result in $H$ oscillation is also obtained by Naskar and Ward [60].

VI. CONCLUSION

A dynamical system of phantom canonical scalar field evolving in background of loop quantum cosmology is considered and analyzed in this work. Exponential potential is used in this system. Dynamical analysis of autonomous system renders two real fixed points $(-\lambda/\sqrt{6}, \sqrt{1+\lambda^2/6}, 0)$ and $(-\lambda/\sqrt{6}, -\sqrt{1+\lambda^2/6}, 0)$, both of which are saddle points corresponding to equation of state, $w = -1 - \lambda^2/3$. Note that in case of standard cosmology, the fixed point $\left(X_c, Y_c\right) = (-\lambda/\sqrt{6}, \sqrt{1+\lambda^2/6})$ is the Big Rip attractor with the same equation of state, $w = -1 - \lambda^2/3$ [22]. Due to absence of stable node, the late time behavior depends on initial conditions given. Therefore we do numerical plots to investigate solutions of the system and then classify the solutions. Separatrix conditions $|X| \neq |Y|$, $Z \neq 1$ and $Z \neq 0$ arise from equation of state [15], Friedmann constraint [19] and definition of $Z$ in Eq. (21). At first, we consider solutions in region $Z > 0$, i.e. $\rho > 0$ for physical solutions. Secondly, within this $Z > 0$ region, we classify solutions into class I & II. Solutions in region $|X| > |Y|$ and $Z > 1$ are of class I. However, in order to obtain real value of $H$ in class I, $Z$ must be negative which contradicts to $Z > 1$. Therefore the class I solutions are non physical. Class II set is identified by $|Y| > |X|$ and $0 < Z < 1$. It is an only set of physical solutions since it yields real value of $H$. In class II set, the universe undergoes accelerating expansion from the beginning until $\rho = \rho_{\text{c}}/2$ where $H = H_{\text{max}} = \sqrt{\rho_{\text{c}}/12M_p^2}$. After that the universe expands decelerately until it bounces, i.e. stops expansion $H \approx 0$ at $\rho \approx \rho_{\text{c}}$. At the bounce the universe enters quantum gravity regime. Contraction with backward acceleration happens right after the bounce, however the contraction does not go on forever. When the universe reaches minimum value of negative $H$, the contraction turns decelerated, i.e. contracts slower and slower down. The universe, after undergoing contraction to minimum spatial size, bounces again and starts to expand acceleratingly. Our numerical results yield that oscillation in $H$ becomes faster as time passes.

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