Shelving according to the nRules

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Abstract

When a weak photon decay competes with a strong photon decay in a 3-level atom, the weak occasionally prevails over the strong photon. This is the shelving phenomenon. It is assumed in this paper to occur objectively and autonomously, independent of external observation – or lack of observation.

We here consider an auxiliary rule-set called the nRules that triggers a non-unitary mechanism that applies to all macroscopic measurements, as well as to many microscopic processes including shelving. These rules are shown below to describe the shelving process in complete detail without resorting to the notion of null measurement, or to an external measurement of any kind. They describe the V and the Λ shelving configurations, as well as the two cascade configurations.

A Retraction

It is assumed in Eq. 1 of this paper that a laser acting on a ground state atom will create a ready component that includes the excited atom. I here assume that Rabi oscillations begin only after the atom has reached the excited state – contrary to the conclusions of a previous paper. However, the previous paper was correct. Upon laser stimulation, the ground state atom will enter immediately into Rabi oscillation with the excite state. So the conclusions of this paper following Eq. 1 are wrong. A correct treatment of shelving is given in another paper called “Optical Shelving: Suppressed Fluorescence” at quant-ph/0701098.

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Introduction

A single three-level atom has a ground state 0 and excited states 1 and 2. The “strong” decay (1-0) has a lifetime of about $10^{-8}$ s, and the “weak” decay (2-0) has a lifetime of about 2 s. The atom is exposed to two laser beams where the first is tuned to the transition 0-1 and the second is tuned to 0-2.

The atom will oscillate rapidly between the ground state and excited state 1 for a period of time, giving off a visible light. It will then stop radiating and become dark for a period of time, after which it will resume the visible radiation. This light/dark intermittent fluorescence is sometimes thought to be due to the atom being trapped in the weak state 2 during the dark time [1, 2]. The fluorescent radiation is said by Dehmelt to be suspended for a time because the atom is temporarily “shelved” in state 2.

The question is: What is the quantum mechanical mechanism that causes this shelving? It is shown below that this mechanism is implicit in the nRules. These four rules are listed in the next section. Their origin and numerous applications are found in other papers [3, 4].

The nRules

We define ready components to be the basis components of state reduction. These are the components that are chosen to survive the collapse of the wave function during a measurement. They are underlined throughout the paper. Components that are not ready are called realized components and appear without an underline.

The first nRule describes how ready components are introduced into solutions of Schrödinger’s equation.

**nRule (1):** If an irreversible interaction produces a complete component that is discontinuous with its predecessor in some variable, then it is a ready component. Otherwise a component is realized.

*Note:* A complete component is one that includes all the symmetrized objects in the universe. Each included object is itself complete in that it is not a partial expansion in some representation.

The second rule establishes the existence of a stochastic trigger. The flow per unit time of square modulus is given by the square modular current $J$, and the total square modulus of the system is given by $s$. 
**nRule (2):** A systemic stochastic trigger strikes a ready component with a probability per unit time equal to the positive probability current J/s flowing into it. A realized component is not stochastically chosen.

*note:* The division of J by s automatically normalizes the system at each moment of time. Currents rather than functions are normalized in these rules.

The collapse of a wave is given by nRule (3)

**nRule (3):** When a ready component is stochastically chosen it will become realized component, and all other (non-chosen) components will go immediately to zero.

The fourth nRule has a less obvious meaning.

**nRule (4):** A ready component cannot transmit probability current to other components or advance its own evolution.

*note:* The fourth nRule is enforced by withholding a ready component’s Hamiltonian as explained in the next section.

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**The Quantum Algorithm**

To understand nRule (4) we go back to the Hamiltonian formalism, adopting a modification that I call the “Quantum Algorithm”. Instructions for the application of the classical Hamiltonian are: Beginning with the initial boundary conditions of a closed system, the Hamiltonian drives all of the system’s particles and all of its interactions into the indefinite future. This defines a classical unitary evolution that goes on forever.

The quantum algorithm modifies these instructions to read:

**QUANTUM ALGORITHM:** Beginning with the initial boundary conditions of a closed system, the Hamiltonian drives all of the system’s particles and all of its interactions up to but not beyond the next ready component(s). This introduces a non-unitary process at each ready component that locates sites (or possible sites) of quantum measurement.

The fourth nRule is implicit in this algorithm. In fact, the quantum algorithm can be used in place of nRule (4).

It works like this.

The horizontal line labeled $S_0$ in Fig. 1 represents the initial state of a system that evolves continuously in time. It is a single (complete) component driven by the Hamiltonian $H_0 + H_{01}$ until it encounters and includes the discontinuous irreversible gap $G_{01}$. The interaction Hamiltonian $H_{01}$ will drive probability
current across this gap to the first ‘ready’ component in the figure (i.e., the first shaded area), but nRule (4) will not allow current to pass beyond that point. As a result, the shaded component in $G_{01}$ will accumulate square modulus. This blocking of probability current is accomplished by truncating the initial Hamiltonian so it does not contain the term $H_1$.

A stochastic hit on the ready component in $G_{01}$ will change it to a ‘realized’ component according to nRules (3), thereby launching a new solution $S_1$ of Schrödinger’s equation. It will also make the initial system state $S_0$ go to zero. The shaded ready component is called a launch component, for it contains all of the initial conditions of the launched solution $S_1$. These initial conditions are continuously updated by current flow into the launch component prior to the stochastic hit. We therefore say that the collapse of the wave across $G_{01}$ is characterized by a new solution together with its new Hamiltonian, where the initial conditions of this solution at the moment of collapse are contained in the launch component at that moment. All of this is accomplished if, in place of nRule (4), the quantum algorithm is directly applied: for in that case, the second Hamiltonian $H_1 + H_{12} + H_{13}$ will substitute for $H_0 + H_{01}$ the moment the ready component in $G_{01}$ is stochastically chosen. The Hamiltonian $H_1$ is always distinguishable from $H_0$ in that the variables of each are indexed to the variables of $S_1$ and $S_0$ respectively.

Once $S_1$ is launched, the new Hamiltonian will carry the system up to and across the next discontinuous and irreversible gap. Let this consists of the two parallel gaps $G_{12}$ and $G_{13}$ as shown in Fig. 1, so the launch components in $G_{12}$ and $G_{13}$ present the stochastic chooser with two possibilities. The Hamiltonian driving $S_1$ will be $H_1 + H_{12} + H_{13}$, where $H_{12}$ and $H_{13}$ are discontinuous interaction terms, so positive current will flow into both of the parallel ready components in Fig. 1 (i.e., the two parallel shaded areas). However, only one
will be chosen. Suppose it is $S_3$ together with Hamiltonian $H_3$ with everything else going to zero. In Fig. 1 we conclude with this solution and its Hamiltonian.

**The Two Level Atom**

Applying this scheme to the case of a two level atom, the atom in its ground state interacts with a laser field $\gamma_N(t)$ containing $N$ photons of the excitation frequency. This is a field of frequency 0-1, where 0 refers to the ground state $a_0(t)$, and 1 refers to the excited state $a_1(t)$. The nRules then give

$$\Phi(t \geq t_0) = \gamma_N a_0 + \gamma_{N-1} a_1$$ (1)

where the second component is zero at time $t_0$ and increases in time. Only one state in this ready component needs to be underlined. It is understood that every state represented here is a function of time. The atom in this equation absorbs a photon and advances to the excited state. With a stochastic hit on $a_1$ following nRule (2) at time $t_{sc}$, and after the collapse required by nRule (3) we get

$$\Phi(t \geq t_{sc} > t_0) = \gamma_{N-1} a_1 \leftrightarrow \gamma_N a_0 + \gamma_{N-1} a_0 \otimes \gamma$$ (2)

where the only the first component in the first row is non-zero at $t_{sc}$. The double arrow ($\leftrightarrow$) represents the reversible Rabi oscillation of stimulated emission that begins the moment the atom reaches the excited state. The third component in Eq. 2 shows the spontaneous emission of the photon $\gamma$. When that ready component (containing $a_0$) is stochastically chosen following nRule (2), the atom will go back to $a_0$, completing a cycle in which the laser field has given up one of its $N$ particles to become an emitted photon of higher entropy. The process can repeat itself until all $N$ photons are converted to a higher entropy state.

The collapse at time $t_{sc}$ occurs with or without the benefit of an external detector or observer of any kind. The nRules provide for objective state reductions that significantly modify atomic ontology.

**A Three Level Atom**

In addition to the $N$ photons of frequency 0-1, the atom is now given a second excited state 2 that is exposed to another laser field containing $M$ photons $\gamma'$ of the frequency 0-2. Then beginning in the ground state $a_0$, Eq. 1 becomes

$$\Phi(t \geq t_0) = \gamma_N \gamma' M a_0 + \gamma_{N-1} \gamma' M a_1 + \gamma_N \gamma' M-1 a_2$$ (3)
where the ready components are zero at \( t_0 \) and increase in time. If \( a_1 \) is stochastically chosen at time \( t_{sc1} \), then

\[
\Phi(t \geq t_{sc1} > t_0) = \gamma_N^{-1}\gamma_M^'a_1 + \gamma_N^{-1}\gamma_M^'a_0 \otimes \gamma
\]  

(4)

where the second component is zero at \( t_0 \) and increases in time. Stimulated emission from an excited state is not a factor in the following, so it has been dropped from this equation. If \( a_0 \) in Eq. 4 is stochastically chosen at time \( t_{sc11} \), then

\[
\Phi(t \geq t_{sc11} > t_{sc1} > t_0) = \{ \gamma_N^{-1}\gamma_M^'a_0 + \gamma_N^{-1}\gamma_M^'a_1 + \gamma_N^{-1}\gamma_M^'a_2 \} \otimes \gamma
\]

which is the same as Eq. 3 except that the strong laser beam has given up another 0-1 photon, and a spontaneous 0-1 photon \( \gamma \) has been emitted by the atom. The cycle is then repeated, resulting in a rapid-fire release of many strong photons giving us the fluorescent period. Since probability current in Eq. 3 flows into \( a_1 \) much faster than it does into \( a_2 \), a stochastic hit on the \( a_2 \) component in Eq. 3 rarely interrupts this fluorescence.

However, the ready component containing \( a_2 \) in Eq. 3 is occasionally stochastically chosen. When that happens at time \( t_{sc2} \) we get

\[
\Phi(t \geq t_{sc2} > t_0) = \gamma_N\gamma_M^'-a_0 + \gamma_N\gamma_M^'-a_2 \otimes \gamma'
\]

\[
\Rightarrow \gamma_N\gamma_M^'a_0 \Rightarrow \gamma_N^{-1}\gamma_M^'a_1 \Rightarrow \gamma_N^{-1}\gamma_M^'a_2 \otimes \gamma
\]

(5)

where the first row is a spontaneous decay to ground plus the emission of a weak photon, and the second row ends in a spontaneous decay to ground plus the emission of a strong photon. The second row begins with a stimulated decay to \( a_0 \), followed by a three-level resonance that carries it to \( a_1 \), and that is followed by the final spontaneous decay to ground. The three-level resonance begins when the atom is in the prolonged excited state \( a_2 \). During this time the two unoccupied states \( a_0 \) and \( a_1 \) are stimulated by the 0-1 laser, and this produces a coherent superposition between the two states. When the atom does finally go coherently into \( a_0 \) (second row of Eq. 5) it is immediately swept up in this resonance that carries it on to \( a_1 \). Hence, the arrows \( \Rightarrow \) in Eq. 5. The three level atomic resonance is given by

\[
\gamma_N\gamma_M^'-a_2 \Rightarrow \gamma_N\gamma_M^'a_0 \Rightarrow \gamma_N^{-1}\gamma_M^'a_1 \Rightarrow \gamma_N^{-1}\gamma_M^'a_2
\]

(6)

which is radiationless. Evidently the state \( a_2 \) in Eq. 5 goes over to \( a_1 \) by means of coherent population transfer \([5]\). However, when the atom reaches the level \( a_1 \) in this cycle it is subject to a fast spontaneous decay back to ground, terminating the second row of Eq. 5.
The two decays in Eq. 5 (given by the top and bottom rows) represent dark periods, or shelved times, that compete with one another to be stochastically chosen. The time from $\gamma_{N-1}\gamma'_M a_1$ to $\gamma_{N-1}\gamma'_M a_0 \otimes \gamma$ the second row of Eq. 5 is negligible compared to this dark-time. As soon as $a_1$ acquires any amplitude at all, it tends to go spontaneously to ground at the rate of a strong photon decay.

**Null Measurement**

When the third component in Eq. 3 is stochastically chosen, fluorescent radiation is suppressed until the dark period has run its course. The idea of a null measurement is commonly offered to account for this suppression. Porrati and Puttermann say “even the observation of no photons produces a reduction of the wave function of the atomic system” [6]. Accordingly, the lack of a strong photon arriving at the detector is claimed to initiate the dark period. But one should not take this idea too literally, for a “non-photon” cannot do anything. In this section we will discuss the appearance of the dark period from the point of view of standard theory, and compare it with the above nRule account.

Certainly the Schrödinger equation cannot (by itself) suppress fluorescence prior to a dark period. Schrödinger gives a superposition of all the possible scenarios, but it cannot choose between them. These competing scenarios have been calculated, where the options are found to involve long and short time intervals between weak and/or strong photons [7]. In Eqs. 5.9a,b of this paper the authors find an expression for the time distribution of the next strong/weak photon to be

\[
\begin{align*}
\text{next strong photon} & \propto \exp\left[-\beta_1 t/2\right] + \exp\left[-\lambda_2 t\right] \\
\text{next weak photon} & \propto \exp\left[-\lambda_2 t\right]
\end{align*}
\]

after a reset that occurs when a photon of either kind is received by the detector. The constants $\beta_1$ and $\lambda_2$ are short and long decay constants, where the first gives the probability of a fluorescent photon following reset and the second gives the probability of a weak photon (after a $\lambda_2$ dark period) following reset. Of the many emission scenarios that Eq. 7 shows are possible in an individual trial, it is the detector rather the Schrödinger equation that chooses between them – according to standard collapse theory.

When stated in this way, it sounds as though the dark period (i.e., the selection of one of the $\lambda_2$ exponentials) results because the detector has made a null measurement. However, the detector does not measure the lack of a photon. It simply chooses one of the many possible scenarios. It selects from
among photons that have the probability of appearing at different times after the reset event. Considered in this way, the behavior of the detector seems very ordinary. There is some truth to the null measurement idea, but it is not a literal truth.

Compare Eq. 7 with the nRule account given in Eqs. 3 and 5. First, the nRules do not require the presence of a detector to choose from among the many possibilities. Those choices are made by the stochastic trigger of nRule (2), giving an objective account of the collapses within the dressed atom that are independent of any external detector or observer. Second, the many possible scenarios implicit in Eq. 7 are also found in Eqs. 3 and 5. Equation 7 compares the competing scenarios quantitatively, whereas Eqs. 3 and 5 give a qualitative description of what is happening within the atom. They explain why Schrödinger’s equation yields the otherwise non-intuitive distributions in Eq. 7. Equation 3 explains why the fluorescence is objectively suppressed long enough for the dark period to run its course. Dehmelt’s expression “shelved” is a good description of the atom when \( g_2 \) is stochastically chosen in Eq. 3.

**Other Configurations**

The energy level scheme of the previous section is known as the \( V \) configuration because the 0 level is a ground state that is energetically below the other two. There are a total of four configurations shown in Fig. 2: (a) the \( V \) configuration, (b) the \( \Lambda \) configuration, (c) the cascade configuration with \( E_1 > E_0 \), and (d) the cascade configuration with \( E_1 < E_0 \). The Rabi oscillations of stimulated emission are not shown. In all cases, the transitions between 1 and 0 are strongly driven and those between 2 and 0 are weakly driven (Ref. 2).

(Fig. 2a) \( V \) Configuration: The figure shows the absorption from 0 to 1 (left arrow up) and the subsequent spontaneous emission (left dashed arrow down). Similar lines appear between the 0 to 2 levels. In this configuration, a photon is released only after the dark ‘shelving’ time in level 2 is concluded.

(Fig. 2b) \( \Lambda \) Configuration: The figure shows the strong absorption between level 0 and 1 (left arrow up), the strong spontaneous emission from level 0 to 1 (left dashed arrow down). Similar lines appear between the 0 to 2 levels. The dark time of the atom occurs while it is in level 2. In this case, a photon is released before the dark ‘shelving’ time in level 2 is begun. Comparing this diagram with the first one, it is easy to see how nRule (4) works in each of these transitions to get the claimed result.
Figure 2: Energy level schemes

(Figs. 2c and 2d): The \( E_1 > E_0 \) and \( E_1 < E_0 \) Cascade Configurations: There are two ways that the 0 level can appear between levels 1 and 2. These are the cascade configurations in Figs. 2c and 2d. The weak photon is released \textit{before} the dark time in the cascade of Fig. 2c, and it is released \textit{after} the dark time in the cascade of Fig. 2d. In both of these results the nRules correctly predict the experimental outcome.

Conclusion

Shelving is here understood to result from a non-unitary pair of atomic state reductions that occur objectively and autonomously, independent of any external observation. The advantage of the nRules is that they give a precisely defined algorithm for the introduction of \textit{every} non-unitary process. Where macroscopic instruments are involved, the nRules define a non-unitary wave collapse for \textit{every} operation that is normally called a “measurement”. When the nRules are projected into the microscopic domain they predict the existence of non-unitary events on that level too, but I would not call these measurements. I would say only that they are microscopic state reductions. Shelving results from two of these, one associated with Eq. 3 and one associated with Eq. 5.
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