A Heavy Glueball in a Bag Model at Finite Temperature

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March 26, 2022

Abstract

We obtain a heavy glueball (much heavier than the ones studied by others which usually are in the range of 1-2 GeV) in a bag model calculation with exact discrete single particle states of gluons at finite temperature. This heavy glueball, within the cosmological context, is what Abbas has recently predicted (hep-ph/9504430).
Since the establishment of the quark model [1] and Quantum Chromodynamics QCD [2], the nonobservability of quarks and gluons has necessitated the concept of confinement. QCD is non-perturbative at large distances. The confinement itself can not be treated perturbatively and is yet to be confirmed analytically. However lattice calculations [3] do indicate confinement. Based on confinement and due to the non-abelian nature of QCD, bound states of gluons, the so called glueballs, should exist. Much work has been done both theoretically [4,5] and experimentally [6] on the glueballs which suggests that the lowest state glueball must be light (1 − 2 GeV). Recently Abbas [7] has suggested that if we take the two distinct QCD phase transitions [8] in the early universe, wherein the gluon transition temperature is $T_g \sim 400$ MeV and the quark transition temperature is $T_q \sim 250$ MeV, then as the universe cool the gluon condensates to a weakly interacting massive glueball ($\geq 45$ GeV). These then decouple from the rest of the universe. Hence they become a natural candidate for the Dark-Matter (DM). Here we wish to understand this prediction of the existence of a heavy glueball in another framework discussed below.

A study of the glueballs was performed [9-14] in the framework of the MIT Bag model. Both the lattice Monte Carlo [15] and the Bag model [9-14] approaches predict a large number of glueballs between 1 − 2 GeV. In these bag studies, the bag constant $B^{\frac{3}{2}}$ was taken maximum upto 230 MeV. The finite size corrections to the bag containing only gluons were calculated by Jennings and Bhaduri and for quarks by Bhaduri et al. [16]. They calculated the effective smooth single particle density of states for gluons and quarks to include the finite size effects. They used an approximation. The discrete sum over single particle states in a finite bag was approximated by an integral with single particle density of states. These calculations are valid within the domain $RT \geq 1$, $R$ is the radius of the bag and $T$ is the temperature of the system. Later Dey et al. [17]
calculated the free energy ($F$) and the other thermodynamic quantities for the quark-gluon system by computing the discrete allowed states and taking appropriate weighted sums. In this discrete sum method, they have discussed the first order phase transition from hadrons to QGP. Later Ansari et al.[18] studied the quark-gluon system within a spherical bag and as well as in a deformed bag of spheroidal shape. They constructed a grand canonical partition function in terms of quark-gluon single particle states inside a bag. They found [18] a temperature range $T_s < T < T_c$ in the $\mu-T$ plane in which a superheated (supercooled) metastable state exists. Beyond $T_c$ there is no bag solution, indicating a first order phase transition from hadrons to QGP.

In this paper we plan to study the thermodynamics of a bag containing only gluons using the discrete allowed states in such a bag. These discrete single particle states have been obtained by solving the equation of motion with linearised boundary conditions in a MIT bag[19]. The solutions for the gluonic bag $R(T)$ is obtained from the extrema of the total free energy ($F_T$). The two extrema in $F_T$ correspond to a light glueball and a heavy glueball. For a particular bag pressure constant $B$, there is a transition temperature (say $T_s$) below which only the low mass excitation exists. For $T_s \leq T < T_c$ the two extrema case arises. There is a critical temperature $T_c$ above which no bag solution $R(T)$ exists indicating the deconfinement of glueballs.

In this model we consider that a system of gluons is placed in a heat bath with which it can exchange the energy and the particle number through which it achieves thermal equilibrium. We study the thermodynamic properties of a thermalized system.

The partition function for the gluonic system is given by,

$$lnZ = -8 \sum_i g_i \ln(1 - e^{-\frac{\epsilon_i}{T}})$$  

where $g_i$ is the spin degeneracy factor for the $i^{th}$ single particle state with energy $\epsilon_i$. The degeneracy factor 8 arises due to the $SU(3)$ color-group in which gluons correspond to
adjoint representation.

The energy and the free energy of the gluonic system are

\[ E = T^2 \left( \frac{\partial \ln Z}{\partial T} \right) \]
\[ = 8 \sum_i g_i \epsilon_i \left( e^{\frac{T}{\epsilon_i}} - 1 \right) \]  

(2)

\[ F = -T \ln Z \]
\[ = 8T \sum_i g_i \ln \left( 1 - e^{-\frac{T}{\epsilon_i}} \right) \]  

(3)

If we include the zero point energy of the bag \[19\] \( BV + \frac{C}{R} \) to Eq.(2) and (3), then the total energy and the free energy of the bag becomes

\[ E_T = E + BV + \frac{C}{R} \]  

(4)

\[ F_T = -T \ln Z + BV + \frac{C}{R} \]  

(5)

where the values of \( C \sim 0.36 \) is taken from ref.[20] and \( B \) is the conventional bag pressure constant and \( V \) is the volume of the system.

The pressure generated by gluons inside the bag

\[ P_{\text{gluons}} = -\left( \frac{\partial F_T}{\partial V} \right)_T \]
\[ = \frac{1}{3V} \left( E + \frac{C}{R} \right) \]  

(6)

Where \( V = \frac{4}{3} \pi R^3 \)
As we know, for a stable configuration\cite{19}, the total pressure on the surface of the bag vanishes.

\[ P = P_{\text{gluon}} - B = 0 \]

Hence,

\[ P_{\text{gluons}} = B \] (7)

So at the extremum of \( F_T \) where the total pressure vanishes, the energy is given by,

\[ E_T = 4BV \] (8)

The number of gluons, \( N_G \) of the glueball at a given \( T \) can be obtained as,

\[ N_G = 8 \sum_i g_i f_i \] (9)

where \( f_i = \frac{1}{\left( e^{\frac{T}{T_w}} - 1 \right)} \), the B - E distribution function.

Now treating the gluonic bag(glueball) at high \( T \) like a many-body system, its stability features are studied by plotting \( F_T \) as a function of bag radius \( R \), since the physical behaviour of the system at a given \( T \) is governed by the properties of its free energy \( F_T \). In Fig.1. a plot of total free energy(\( F_T \)) vs. \( R \) is displayed for \( B^+ = 250 \text{ MeV} \).

We see that there is a transition temperature \( T_s \) below which \( F_T \) has only a single minimum with a finite \( R \) value. But at the temperature \( T = T_s = 218.5 \text{ MeV} \), \( F_T \) has two extrema, one with smaller \( R \) and the other one is with larger \( R \). Now as the temperature increases, both the solutions approach each other and at the critical temperature \( T = T_c = 256.9 \text{ MeV} > T_s \), both solutions meet at one point and beyond \( T_c \) there is no extrema in \( F_T \) implying no bag solution and glueball does not exist. In Fig.2. and Fig.3. we plot the radius of bag (\( R \)) and mass (\( M \)) corresponding to the extrema of \( F_T \) as a function of the temperature \( T \). We see from Fig.2. & 3. that the
radius corresponding to minima in $F_T$ remains almost same (0.335 fm) upto $T_s$ (218.5 MeV) giving a stable state with very low mass (0.32 GeV).

Since physically a meaningful maximum appears in $F_T$ at $T_s$ with $R = 4.08$ fm (Fig.2), indicating a heavy metastable glueball with very high mass ($\sim 578$ GeV) (Fig.3). As the stable state has very low mass compared to the heavy metastable state at $T = T_s$, there is a sharp increase in the Fig.3. Now in the region $T_s < T < T_c$ the solution with lower $R$, corresponding to minimum in $F_T$ (Fig.1.), expands very slowly (in Fig.2.) with slow increase of mass (Fig.3.) whereas the solution with larger $R$, corresponding to maximum in $F_T$ (Fig.1.), contracts very fast (Fig.2) with the rapid decrease of mass (Fig.3). Since at $T = T_c$, both solutions converge at one point with $R = 0.422$ fm and mass $\sim 0.64$ GeV.

Note that for $B^\perp = 400$ MeV, $T_s$ becomes $\sim 350$ MeV with a heavy metastable glueball having mass $\sim 670$ GeV whereas that of the light glueball is $\sim 0.5$ GeV. The value of $T_c$ turns out to be $\sim 410$ MeV with $R \sim 0.25$ fm and mass $\sim 1$ GeV.

The heavier mass state tends to be highly collective, e.g. for $T = 220$ MeV with $B^\perp = 250$ MeV, the ratio of the gluon number (gluon number is calculated using $N_G$) in the heavy mass glueball state to that of the low mass glueball state is 10605. Though the collectivity does go down with the increase in temperature, e.g. at $T = 230$ MeV, the ratio comes down to 385 and at $T = 240$ MeV, the ratio is 62. One may speculate that this large collectivity of the heavy glueball may lead to a greater stability of the same.

The most interesting result here is the existence of the very massive glueball. This is exactly what was predicted by Abbas[7] recently in the context of his cosmological arguments. We feel that our calculations here should have much relevance to the early universe scenario as well as in QGP. This calls for further study.
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FIGURE CAPTIONS

Figure 1: The variation of the total free energy ($$F_T$$) as a function of the radius ($$R$$) of the gluonic system for different temperatures with $$B = 250 \, MeV$$ is shown. Here $$T_s = 218.5 \, MeV$$ and $$T_c = 256.9 \, meV$$.

Figure 2: The variation of the radius ($$R$$) of the system with temperature ($$T$$) is shown. Here $$B = 250 \, MeV$$, $$T_s = 218.5 \, MeV$$ and $$T_c = 256.9 \, MeV$$.

Figure 3: The mass ($$M$$) variation of the gluonic system with temperature ($$T$$) is displayed.
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