Manifestly covariant action for symmetric Chern-Simons field theory

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Abstract

We study a three-dimensional symmetric Chern-Simons field theory with a general covariance and it turns out that the original Chern-Simons theory is just a gauge fixed action of the symmetric Chern-Simons theory whose constraint algebra belongs to fully first class constraint system. The Abelian Chern-Simons theory with matter coupling is studied for the construction of anyon operators without any ordering ambiguity with the help of this symmetric Chern-Simons action. Finally we shall discuss some connections between the present symmetric formulation of Chern-Simons theory and the St"ukelberg mechanism.

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1. INTRODUCTION

A Chern-Simons (CS) theory has been enormously studied in the (2+1)-dimensional quantum field theory [1] and applied to an anyon system [2,3] and the quantum gravity [4] apart from in its own right. One of the most intriguing feature in the CS theory is due to the symplectic structure of gauge fields which is related to a second class constraint system. The second class constraint structure in the CS theory does not give a closed constraint algebra in Poisson brackets even though a local gauge symmetry exists.

To quantize a second class constraint system, the Dirac method [5,6] may be used in the Hamiltonian quantization. However, Dirac brackets are generically field-dependent, nonlocal, and have a serious ordering problem between field operators. These are problematic and unfavorable in finding canonically conjugate pairs. Once the first class constraint system is realized, the usual Poisson bracket corresponding to the quantum commutator can be used. Quantization in this direction has been well appreciated in a gauge invariant manner using Becci-Rouet-Stora-Tyutin (BRST) symmetry [6,7]. So one might wonder how to convert the second class constraint system in the CS theory into a first class one, and what kind of symmetry is involved in the symplectic structure.

On the other hand, it would be interesting to interpret the original CS theory as a gauge fixed version of a symmetric CS theory similarly to the anomalous gauge theory in Ref. [8]. Then the symplectic structure of gauge fields naturally appears after unitary gauge fixing. We expect that this gauge fixing which gives the symplectic structure in CS theory is independent of the local gauge symmetry. In anomalous gauge theories, in fact, it is well known that the algebra of first class gauge constraints becomes second class after quantization. In the CS theory, however, the origin of second class constraint is more or less different in that it is not due to the anomalous breaking of symmetry but rather the symplectic structure of the CS term. Henceforth the Gauss’ law as a gauge constraint remains as a first class constraint in the CS theory. In this respect, there may exist some differences to convert the second class constraint system into the first class one. This problem has been extensively studied in the
context of Batalin-Fradkin-Tyutin (BFT) Hamiltonian embedding \[9\] in Refs. \[10–14\]. The BFT method also has been applied to the other interesting physical problems in Ref. \[15\]. However the unitary gauge exists only in the Hamiltonian defined in the phase space, and the Wess-Zumino (WZ) type action which makes the first class constraint system depends on the details of the matter action coupled to CS term. Since the origin of second class algebra in CS matter theory is of no relevance to the form of matter coupling, we expect the WZ type action should be independent of the matter fields in contrast to the previous result. So first of all the pure CS theory should be considered, and matter coupling will be a simple extension. Very recently, the second class constraint algebra of a (2+1)-dimensional Abelian CS term was converted into first class one, the BFT method has been applied and it turns out that the symmetry of relevance to the symplectic structure is a local translational symmetry \[16\]. Unfortunately, the final action corresponding to the first class Hamiltonian system is not generally covariant and how to couple to matter fields is uncertain.

In this paper, we study generally covariant (2+1)-dimensional symmetric CS theories. The proposed action is manifestly covariant and the unitary gauge fixing exists in the final action. In Sec. 2, we exhibit the constraint structure of the non-Abelian CS theory to reveal the second class algebra. In Sec. 3, we briefly review some of the recent study on the symmetric Abelian CS theory for the self-contained manner and find out a clue how to convert the second class constraint system into first class constraint one for the complicated system in the generally covariant fashion. We shall use a very simple method in order to convert constraint algebra through some observations. In Sec. 4, the symmetric non-Abelian CS theory whose constraint algebra is fully first class is presented and the usual Poisson brackets are well defined without recourse to Dirac brackets. The proposed action is generally covariant and has an additional local translational symmetry which is of relevance to the symplectic structure of the CS term. In Sec. 5, as an field-theoretic application, anyon operators are constructed by using the symmetric CS term coupled to complex scalar fields. After all, we find the anyon operator without resorting to the Dirac brackets and naturally circumvent the ordering ambiguity between gauge fields. Finally we discuss the present
method and its connection to the St"ukelberg mechanism which is apparently unrelated to our analysis in Sec. 6.

2. CONSTRAINT STRUCTURE OF 2+1 DIMENSIONAL CS THEORY

We now exhibit some of the salient features of the 2+1 dimensional nonAbelian CS theory whose action is given by

\[ S_{CS} = \kappa \int d^3x e^{\mu \nu \rho} \text{tr}(A_\mu \partial_\nu A_\rho - \frac{2}{3} i A_\mu A_\nu A_\rho), \]  

(2.1)

where the diagonal metric \( g_{\mu \nu} = \text{diag}(+, -, -) \) and \( \epsilon^{012} = +1 \). The Lie algebra-valued gauge field is defined by \( A_\mu = A_\mu^a T^a \) satisfying \([T^a, T^b] = i f^{abc} T^c \) and \( \text{tr}(T^a T^b) = \frac{1}{2} \delta^{ab} \) where \( T^a \) is a Hermitian generator. Note that the large gauge invariance of the nonAbelian Chern-Simons action requires the quantization of the dimensionless constant \( \kappa \), \( \kappa = \frac{n}{4\pi} (n \in Z) \).

The canonical momenta of the action (2.1) are given by

\[ \Pi^{0a} \approx 0, \]  

(2.2)

\[ \Pi^{ia} = \frac{\kappa}{2} \epsilon^{ij} A^a_j, \]  

(2.3)

which are all primary constraints. Performing the Legendre transformation, the canonical Hamiltonian is written as

\[ H_c = - \int d^2x A^{0a}(\partial_i \Pi^{ia} + \frac{\kappa}{2} \epsilon^{ij} \partial_i A^a_j + \frac{\kappa}{2} f^{abc} \epsilon^{ij} A^b_i A^c_j). \]  

(2.4)

At this stage, we define nonvanishing Poisson brackets as

\[ \{ A^a_\mu(x), \Pi^{ab}(y) \} = g^{\mu \nu} \delta^{ab} \delta^2(x - y). \]  

(2.5)

The time evolution of the primary constraint Eq. (2.2) yields Gauss’ law constraint as a secondary constraint, which is simply written by

\[ \partial_i \Pi^{ia} + \frac{\kappa}{2} \epsilon^{ij} \partial_i A^a_j + \frac{\kappa}{2} f^{abc} \epsilon^{ij} A^b_i A^c_j \approx 0. \]  

(2.6)
Further time evolution of the Gauss’ constraint (2.6) gives no more additional constraint. We therefore obtain the following set of constraints

\[ \Omega^0a = \Pi^0a \approx 0, \quad (2.7) \]
\[ \Omega^ia = \Pi^ia - \frac{\kappa}{2} \epsilon^{ij} A_j^a \approx 0, \quad (2.8) \]
\[ \Omega^a = \partial_i \Pi^ia + f^{abc} A^b_i \Pi^c + \frac{\kappa}{2} \epsilon^{ij} \partial_i A_j^a \approx 0. \quad (2.9) \]

By using the Poisson bracket, the first class constraint algebra is given by

\[ \{ \Omega^0a(x), \Omega^0b(y) \} = \{ \Omega^0a(x), \Omega^b(y) \} \approx 0, \]
\[ \{ \Omega^a(x), \Omega^b(y) \} = f^{abc} \Omega^c(x) \delta^2(x - y), \quad (2.10) \]

while the nonvanishing second class algebra is written as

\[ \{ \Omega^ia(x), \Omega^jb(y) \} = -\kappa \epsilon^{ij} \delta^{ab} \delta^2(x - y). \quad (2.11) \]

Note that the second class constraint algebra (2.11) is reminiscent of an anomalous commutator of the anomalous gauge theory which reflects a local gauge symmetry breaking, while the first class constraints (2.7) and (2.9) guarantee the local gauge symmetry. So one might wonder what kind of additional local symmetry is broken in the second class constraint algebra (2.11). Therefore, it is necessary to study how to convert the second class constraint algebra into the first class one to answer this question.

There exist largely two ways to recover the symmetry in an extended configuration space. One is the Stükelberg mechanism [17] which is done by performing a gauge transformation, \( A_\mu \rightarrow U^{-1} A_\mu U - i U^{-1} \partial_\mu U \) and identifying a new field \( U \) as a Stükelberg scalar field. The other one is the BFT method which converts a second class constraint algebra into a first class constraint algebra in the Hamiltonian formalism by introducing new conjugate pairs. Unfortunately, in the nonAbelian CS theory these methods may not be successful so far since we do not know what is the relevant symmetry to this kind of constraint algebra (2.11) for the case of the Stükelberg mechanism, and there are some arbitrariness to introduce conjugate fields for the BFT formalism. So the general covariance of the action is lost in the...
course of calculation in the latter formalism [16]. Therefore we suggest a method to convert
the second class constraint system into a first class one in a generally covariant fashion by
simply substituting the original gauge field by a new field. This method as a matter of fact
amounts to the St"ukelberg mechanism which will be discussed in later. In Sec. 4 and 5, we
shall consider this method, and apply to the (2+1)-dimensional nonAbelian CS theories and
CS matter coupling.

3. 2+1 DIMENSIONAL SYMMETRIC ABELIAN CS THEORY

In an Abelian CS theory, some new results are obtained in recent work [16] on the
symmetry of relevance to the symplectic structure of the CS theory. At this juncture we
recapitulate some of the results and will assume a symmetry in order to apply nonAbelian
cases in a generally covariant fashion.

One can now apply a BFT Hamiltonian embedding of the CS term by introducing aux-
iliary fields [9]. The starting (2+1)-dimensional Abelian CS Lagrangian is given by

\[ \mathcal{L}_{(0)} = \frac{\kappa}{2} \epsilon^{\mu\nu\rho} A^{(0)}_\mu \partial_\nu A^{(0)}_\rho, \]  

(3.1)

where \( A^{(0)}_\mu \) is an original CS gauge field and for simplicity the CS coefficient is set to \( \kappa = 1 \).

We introduce an auxiliary field \( A^{(1)}_i \) satisfying

\[ \{ A^{(1)}_i(x), A^{(1)}_j(y) \} = \delta_{ij}(x, y) \]  

(3.2)

which makes the second class constraint \( \omega^i = \Pi^i - \frac{\kappa}{2} \epsilon^{ij} A_j \approx 0 \) into a first class one where
\( \delta_{ij}(x, y) \) will be explicitly determined in later. Making use of the auxiliary field \( A^{(1)}_i \), we could
write the effective first class constraints as \( \tilde{\omega}^i(\pi^{(0)}_\mu, A^{(0)}_\mu; A^{(1)}_i) = \omega^i + \sum_{n=1}^\infty C^{(i)}_n \) satisfying the
boundary condition \( \tilde{\omega}^i(\pi^{(0)}_\mu, A^{(0)}_\mu; 0) = \omega^i \) as well as requiring the strong involution, \( i.e., \)
\( \{ \tilde{\omega}^i, \tilde{\omega}^j \} = 0 \). Here \( C^{(i)}_n \) is assumed to be proportional to \( (A^{(1)}_i)^n \). In particular, the first
order correction in these infinite series is given by

\[ C^{(1)}_i(x) = \int d^2 y X^{ij}(x, y) A^{(1)}_j(y), \]  

(3.3)
and the requirement of the strong involution gives the condition
\[-\kappa \epsilon_{ij} \delta^2(\mathbf{x} - \mathbf{y}) + \int d^2u d^2v X^{ik}(x, u) \partial_{k\ell}(u, v) X^{j\ell}(v, y) = 0. \tag{3.4}\]

We take the simple solution of \(\vartheta_{ij}\) and \(X^{ij}\) as
\[
\vartheta_{ij}(x, y) = \epsilon_{ij} \delta^2(\mathbf{x} - \mathbf{y}), \tag{3.5}
\]
\[
X^{ij}(x, y) = -\epsilon_{ij} \delta^2(\mathbf{x} - \mathbf{y}). \tag{3.6}
\]

By using Eqs. (3.5) and (3.6), we obtain the strongly involutive first class constraints which are proportional only to the first order of the auxiliary field as
\[
\tilde{\omega}_{(1)}^i = \pi_{(0)}^i - \frac{1}{2} \epsilon^{ij} A_{j(0)}^0 - \epsilon^{ij} A_{j(1)}^0 = 0, \tag{3.7}
\]
and the canonical Hamiltonian density
\[
\tilde{H}_c = -A_{0(0)}^0 \epsilon^{ij} \partial_i \left( A_{j(0)}^0 + A_{j(1)}^1 \right), \tag{3.8}
\]
satisfying \(\{\tilde{\omega}^i, \tilde{H}_c\} = 0\). The corresponding Lagrangian to Eq. (3.8) with the auxiliary field \(A_i^{(1)}\) is obtained through the usual Legendre transformation \([10-14]\) as
\[
\mathcal{L}_{(1)} = -\frac{1}{2} \epsilon^{ij} A_{j(0)}^i \dot{A}_{j(0)}^0 + A_{0(0)}^0 \epsilon^{ij} \partial_i A_{j(0)}^0
\]
\[
-\frac{1}{2} \epsilon^{ij} A_{j(1)}^i \dot{A}_{j(1)}^0 + A_{0(0)}^0 \epsilon^{ij} \partial_i A_{j(1)}^1 - \epsilon^{ij} A_{j(1)}^1 \dot{A}_{j(0)}^0. \tag{3.9}
\]

However, the first iteration of the BFT formalism is not satisfactory since the action (3.9) is not the genuine first class constraint system in the Poisson algebra, which is seen from the following reconsideration of Hamiltonian analysis. The canonical momenta from (3.9) are
\[
\pi_{(0)}^0 = 0, \quad \pi_{(0)}^i = \frac{1}{2} \epsilon^{ij} A_{j(0)}^0 + \epsilon^{ij} A_{j(1)}^0, \quad \text{and} \quad \pi_{(1)}^i = \frac{1}{2} \epsilon^{ij} A_{j(1)}^1. \]

From the time stability conditions of these primary constraints, we can get one more secondary constraint and after redefining the constraints we can easily obtain the maximally irreducible first class constraints as
\[
\tilde{\omega}^0 = \pi_{(0)}^0 \approx 0, \quad \tilde{\omega}^3 = \partial_i \pi_{(0)}^i + \frac{1}{2} \epsilon^{ij} \partial_i A_{j(0)}^0 \approx 0, \quad \text{and}
\]
\[
\tilde{\omega}_{(1)}^i = \pi_{(0)}^i - \frac{1}{2} \epsilon^{ij} A_{j(0)}^0 - (\pi_{(1)}^i + \frac{1}{2} \epsilon^{ij} A_{j(1)}^0) \approx 0, \tag{3.10}
\]
as well as the problematic constraint
\[
\omega_i^{(1)} = \pi_i^{(1)} - \frac{1}{2} \epsilon^{ij} A_j^{(1)} \approx 0, \tag{3.11}
\]
which is unfortunately second class. Therefore there remains still a second class constraint even after the first order of correction. The consistent bracket is defined by the Dirac bracket as
\[
\{A_i^{(1)}(x), A_j^{(1)}(y)\}_D = \epsilon_{ij} \delta^2(x - y), \tag{3.12}
\]
which is compatible with Eqs. (3.2) and (3.5) to make the second class constraint \(\omega_i\) into the first class one. Therefore the bracket defined in Eq. (3.2) is not the Poisson bracket but the Dirac one. This is reason why we do not obtain the first class constraint system in the Poisson algebra. This enforces the BFT Hamiltonian embedding of the CS theory not stopping any finite number of steps. Therefore the same step should be infinitely repeated until fully first class constraint algebra appears by introducing infinite auxiliary fields denoted by \(A_i^{(n)}\). The similar circumstances are already encountered in self-dual theory as a chiral boson theory, [18,19]. In this respect, all the previous results [10–14] of the BFT formalism applied to the CS matter coupling cases are also confronted with this kind of problem. After repeating the BFT formalism infinitely, the final action can be written in the form
\[
\mathcal{L}_{SCS} = -\frac{1}{2} \epsilon^{ij} A_i^{(0)} \dot{A}_j^{(0)} + A_0^{(0)} \epsilon^{ij} \partial_i A_j^{(0)} - \frac{1}{2} \epsilon^{ij} \sum_{n=1}^{\infty} A_i^{(n)} \dot{A}_j^{(n)} + A_0^{(0)} \epsilon^{ij} \sum_{n=1}^{\infty} \partial_i A_j^{(n)}

- \epsilon^{ij} \sum_{n=1}^{\infty} A_i^{(n)} \dot{A}_j^{(0)} - \epsilon^{ij} \sum_{n=1}^{\infty} \sum_{m=n+1}^{\infty} A_i^{(m)} \dot{A}_j^{(n)}. \tag{3.13}
\]
Then the symmetric CS theory (3.13) with the infinite number of auxiliary fields now completely gives the first class constraint system and strongly vanishing Poisson brackets between constraints in contrast to the finite iteration of BFT method. Remarkably the other convenient action of Eq. (3.13) is written in the compact form as
\[
\mathcal{L}_{SCS} = -\frac{1}{2} \epsilon^{ij} \left( A_i^{(0)} + \sum_{n=1}^{\infty} A_i^{(n)} \right) \left( \dot{A}_j^{(0)} + \sum_{n=1}^{\infty} \dot{A}_j^{(n)} \right) + A_0^{(0)} \epsilon^{ij} \partial_i \left( A_j^{(0)} + \sum_{n=1}^{\infty} A_j^{(n)} \right) \tag{3.14}
\]
after some resummations of auxiliary fields. This action is invariant under the following local gauge transformation
\[
\begin{align*}
\delta A_0^{(0)} &= \partial_0 \Lambda, \\
\delta A_i^{(0)} &= \partial_i \Lambda + \epsilon_i^{(1)}, \\
\delta A_i^{(n)} &= -\epsilon_i^{(n)} + \epsilon_i^{(n+1)}, \quad (n = 1, 2, \ldots).
\end{align*}
\tag{3.15}
\]

The transformation rule (3.15) is implemented by the usual U(1) gauge transformation with the gauge parameter \( \Lambda \) and a new type of local symmetry with \( \epsilon_i^{(n)} \). Note that the procedure to arrive the final result is cumbersome if one want to apply it to other case as an nonAbelian extension of the CS term. After all, we have obtained the first class constraint system from the second class original CS theory. Further the general covariance is lost in the course of our calculation in BFT Hamiltonian embedding. In fact, the general covariance of CS theory is an essential feature of the CS theory, which is a metric independent property. Therefore we overcome these problems with the help of some observations in the next section.

4. 2+1 DIMENSIONAL SYMMETRIC NONABELIAN CS THEORY

In this section, we shall generalize the previous Abelian result to the nonAbelian CS theory with maintaining general covariance. We observe that the time component of gauge fields is missing in the transformation rule (3.15), which becomes in fact a fundamental reason why we did not obtain the generally covariant first class constraint system. The resolution of the covariance problem does not appear in a natural way in the BFT Hamiltonian embedding of our model. Furthermore nonAbelian extension of the previous Abelian result within the BFT formalism may be intractable and seems to be cumbersome because of some complexities. Therefore, without further resort to the BFT formalism, at this stage we simply assume that the new local translational symmetry combined with the local Abelian gauge symmetry is promoted to the following form

\[
\begin{align*}
\delta A_\mu^{(0)} &= \tilde{D}_\mu \epsilon^{(0)} + \epsilon^{(1)}_\mu, \\
\delta A_\mu^{(n)} &= -\epsilon^{(n)}_\mu + \epsilon^{(n+1)}_\mu, \quad (n = 1, 2, \ldots) 
\end{align*}
\tag{4.1}
\tag{4.2}
\]
where the covariant derivative is defined by 
\[ \tilde{D}_\mu \epsilon^{(0)} = \partial_\mu \epsilon^{(0)} - i [\tilde{A}^{(n)}_\mu , \epsilon^{(0)}] \] and \( \tilde{A}_\mu = A^{(0)}_\mu + \sum_{n=1}^{\infty} A^{(n)}_\mu \). The matrix valued gauge and translational parameters are denoted by \( \epsilon^{(0)} \) and \( \epsilon^{(n)}_\mu \) respectively. Note that the covariant derivative is expressed in terms of not only an original CS gauge field but also auxiliary fields. Then the symmetric action under the transformation \((4.1)\) and \((4.2)\) is given by

\[ L_{SCS} = \kappa \epsilon^{\mu \nu \rho} \text{tr} \left( \tilde{A}_\mu \partial_\nu \tilde{A}_\rho - \frac{2}{3} i \tilde{A}_\mu \tilde{A}_\nu \tilde{A}_\rho \right). \] (4.3)

It is interesting to note that the symmetric CS action is apparently the same form as the original CS action except for a new gauge field which is composed of the infinite number of vector fields. Under the given local transformation \((4.1)\) and \((4.2)\), the Lagrangian \((4.3)\) is invariant up to a total divergence term as

\[ \delta L_{SCS} = \partial_\mu \left[ \kappa \epsilon^{\mu \nu \rho} \text{tr} \left( \tilde{A}_\nu \tilde{D}_\rho \epsilon^{(0)} - 2 i \tilde{A}_\nu \tilde{A}_\rho \epsilon^{(0)} \right) \right]. \] (4.4)

Note that the above total divergence does not depend on the translational parameter and the sufficient convergence condition of gauge group parameter \( \epsilon^{(0)} \) guarantees the invariance of the action. The two symmetries are controlled by the independent parameters and the symmetry transformation rules can be arbitrarily separated by a modified gauge transformation

\[ \delta A^{(0)}_\mu = \tilde{D}_\mu \epsilon^{(0)}, \quad \delta A^{(n)}_\mu = 0 \] (4.5)

and a translational symmetry

\[ \delta A^{(0)}_\mu = \epsilon^{(1)}_\mu, \quad \delta A^{(n)}_\mu = -\epsilon^{(n)}_\mu + \epsilon^{(n+1)}_\mu \] (4.6)

respectively. We should recall that the infinite number of vector fields are involved in the covariant derivative which is unusual.

On the other hand, the collective expression of the transformations from Eqs. \((4.3)\) and \((4.6)\) is written as

\[ \delta \tilde{A}_\mu = \tilde{D}_\mu \epsilon^{(0)} \] (4.7)
and the symmetric action (4.3) is automatically invariant under the transformation rule. This situation is very similar to the usual gauge invariance of the original CS term. Note that the concise expression (4.7) may not be decomposed into the transformations (4.1) and (4.2) since the decomposition is not unique. Hence the collective expression (4.7) is just only for convenience. As for the finite transformation of $\tilde{A}_\mu$, it is naturally written as $\tilde{A}_\mu \rightarrow U^{-1}\tilde{A}_\mu U - iU^{-1}\partial_\mu U$ where the finite transformation matrix is $U = e^{iT^a\epsilon^{(0)a}}$ and the quantization condition of CS coefficient is still valid. In our consideration, the general covariance has been maintained.

To check whether or not the symmetric CS action (4.3) gives a desired first class constraint system, the canonical momenta are derived from Eq. (4.3)

$$\Pi^0_{(n)} \approx 0,$$

$$\Pi^i_{(n)} = \frac{\kappa}{2} \epsilon^{ij} \tilde{A}_j,$$  \hspace{1cm} (4.8)

where the spatial momentum is a collection of all vector fields. So the primary Hamiltonian becomes

$$H_p = \int d^2x \left[ -\tilde{A}_0^a \left( \kappa \epsilon^{ij} \partial_i \tilde{A}_j^a + \frac{1}{2} f^{abc} \tilde{A}_i^b \tilde{A}_j^c \right) + \sum_{n=0}^{\infty} \lambda^{(n)a}_\mu \Omega^{(n)}_{\mu a} \right],$$  \hspace{1cm} (4.10)

where the two constraints are rewritten as for convenience

$$\Omega^{0a}_{(n)} = \Pi^{0a}_{(n)} \approx 0$$  \hspace{1cm} (4.11)

$$\Omega^{ia}_{(n)} = \Pi^{ia}_{(n)} - \Pi^{ia}_{(n+1)} \approx 0,$$  \hspace{1cm} (4.12)

where hereafter we assume $n = 0, 1, 2, \cdots$. The Gauss' law is given by

$$\Omega^{3a} = (\tilde{D}_i \Pi^i_{(0)})^a + \frac{\kappa}{2} \epsilon^{ij} \partial_i \tilde{A}_j^a \approx 0.$$  \hspace{1cm} (4.13)

At first sight, the primary constraint (5.4) and the Gauss constraint (4.13) seem to be a second class, however it is actually first class one as easily seen from the redefined form of Eq. (5.11).

To make this explicit in another way, we now rewrite the action (4.3) after some resumptions of terms in the action, which is given by
Then the canonical momenta for \( A^{(n)a}_\mu \) are

\[
\Pi^{(n)a}_\mu \approx 0, \quad (4.15)
\]

\[
\Pi^{ia}_\mu = \frac{\kappa}{2} \varepsilon^{ij} A^{(n)a}_j + \kappa \varepsilon^{ij} \sum_{m=n+1} A^{(m)a}_j, \quad (4.16)
\]

and the primary Hamiltonian is

\[
H_p = \int d^2 x \left[ -\sum_{n=0} A^{(n)a}_\mu (\partial_i \Pi^{ia}_\mu) + f^{abc} \sum_{m=0} A^{(m)b}_i \Pi^{ic}_\mu + \frac{\kappa}{2} \varepsilon^{ij} \partial_i A^{(0)a}_j \right] + \sum_{n=0} \Lambda^{(n)a}_\mu \Omega^{ca}_\mu. \quad (4.17)
\]

The constraints are written as

\[
\Omega^{0a}_\mu = \Pi^{0a}_\mu \approx 0, \quad (4.18)
\]

\[
\Omega^{ia}_\mu = \Pi^{ia}_\mu - \frac{\kappa}{2} \varepsilon^{ij} A^{(n)a}_j - \left( \Pi^{ia}_{(n+1)} + \frac{\kappa}{2} \varepsilon^{ij} A^{(n+1)a}_j \right) \approx 0, \quad (4.19)
\]

\[
\Omega^{3a} = \partial_i \Pi^{ia}_\mu + f^{abc} \sum_{n=0} A^{(n)b}_i \Pi^{ic}_\mu + \frac{\kappa}{2} \varepsilon^{ij} \partial_i A^{(0)a}_j \approx 0. \quad (4.20)
\]

Note that the constraint \((4.19)\) is obtained from a recombination and the last term in Eq. \((4.16)\) is written by the next order of Eq. \((4.16)\). The Poisson brackets between the constraints yield the desired first class constraint algebra after some calculations,

\[
\{ \Omega^{0a}_\mu(x), \Omega^{0b}_\mu(y) \} = \{ \Omega^{0a}_\mu(x), \Omega^{ib}_\mu(y) \} = \{ \Omega^{0a}_\mu(x), \Omega^{3b}(y) \} \approx 0, \quad (4.21)
\]

\[
\{ \Omega^{ia}_\mu(x), \Omega^{ib}_\mu(y) \} \approx 0, \quad (4.22)
\]

\[
\{ \Omega^{ia}_\mu(x), \Omega^{3b}(y) \} = f^{abc} \Omega^{ia}_\mu(x) \delta^2(x-y), \quad (4.23)
\]

\[
\{ \Omega^{3a}_\mu(x), \Omega^{3b}(y) \} = f^{abc} \Omega^{3c}(x) \delta^2(x-y), \quad (4.24)
\]

where we used the Jacobi identity to show the last algebra Eq.\((4.24)\).

As a result, the symmetric nonAbelian CS theory is obtained by simply redefining the original gauge field as the new tilda field. The constraint algebra is fully first class. If one
chooses an unitary gauge for the translational symmetry \( A_{\mu}^{(1)} = A_{\mu}^{(2)} = \cdots = 0 \), then the original CS theory recovers and the consistent bracket will be the Dirac bracket as (2.11). Further gauge fixing corresponding to the usual Coulomb type gauge fixing, the constraint algebra becomes fully second class.

5. CONSTRUCTION OF ANYON OPERATORS

The Abelian Chern-Simons theory coupled to the complex matter fields is reconsidered in our formalism, which is essentially first class constraint constraint system. By analyzing this model without any gauge fixing condition, we naturally obtain gauge-independent anyon operators which is also free from ordering problems between field operators.

The matter coupling to the CS term is given by the action

\[
L_0 = L_{\text{CS}}(A_\mu^{(0)}) + (\partial_\mu \phi + iA_\mu^{(0)}\phi)^\dagger(\partial_\mu \phi + iA_\mu^{(0)}\phi),
\]

(5.1)

where it is a second class constraint system already studied in Ref. [10,11] in terms of the BFT Hamiltonian embedding. In these works, the first class system was impossible when we assume the usual Poisson brackets. If one wants to quantize the system by using the Poisson bracket (commutator) instead of Dirac ones, then the symmetric action will be adopted by substituting \( A_\mu^{(0)} \) by \( \tilde{A}_\mu \) in the given action, which is simply written as

\[
\mathcal{L} = \mathcal{L}_{\text{SCS}}(\tilde{A}_\mu) + (\tilde{D}_\mu \phi)^*(\tilde{D}_\mu \phi),
\]

where the covariant derivative is defined by \( \tilde{D}_\mu \phi = \partial_\mu \phi + ie \sum_{n=0}^{\infty} A_\mu^{(n)}\phi \). This original action (5.1) can be recovered at the Lagrangian stage by choosing unitary gauge \( A_\mu^{(1)} = A_\mu^{(2)} = \cdots = 0 \). If we turns off the matter fields, then the pure symmetric CS theory appears. So our formulation on CS field theory is independent of matter contents, which is in contrast with the previous result.

The Lagrangian with the Klein-Gordon fields \( \phi \) and \( \phi^* \) is given by

\[
\mathcal{L} = \frac{\kappa}{2} \epsilon^{\mu \nu \rho} \sum_{n=0}^{\infty} \left( \sum_{m=0}^{n} A_\mu^{(n)} \partial_\nu A_\rho^{(m)} + \sum_{m=n+1}^{\infty} A_\mu^{(m)} \partial_\nu A_\rho^{(n)} \right) + (\tilde{D}_\mu \phi)^*(\tilde{D}_\mu \phi),
\]

(5.2)
through the resummations as Eq. (4.14). From this Lagrangian (5.2), the canonical momenta are obtained as

$$\Pi^0(n) \approx 0,$$

$$\Pi^i(n) = \frac{\kappa}{2} \epsilon^{ij} A_j^{(n)} + \kappa \epsilon^{ij} \sum_{m=n+1}^{\infty} A_j^{(m)},$$

$$\Pi = (\bar{D}_0 \phi)^*,$$

$$\Pi^* = (\bar{D}_0 \phi),$$

where $\Pi^0(n)$, $\Pi^i(n)$, $\Pi$, and $\Pi^*$ are the conjugate momenta of $A_j^{(n)}$, $A_i^{(n)}$, $\phi$, and $\phi^*$, respectively.

From the Legendre transformation, we obtain the canonical Hamiltonian

$$\mathcal{H}_c = |\Pi|^2 + |D\phi|^2 - \sum_{n=0}^{\infty} A_0^{(n)} G,$$

and Gauss’ law is written as

$$G = \kappa \epsilon^{ij} \sum_{n=0}^{\infty} \partial_i A_j^{(n)} + J_0,$$

where the source current

$$J_\mu = ie \left[ (\bar{D}_\mu \phi)^* \phi - (\bar{D}_\mu \phi) \phi^* \right]$$

is conserved. The primary constraints are

$$\Omega^0(n) = \Pi^0(n) \approx 0$$

$$\Omega^i(n) = \Pi^i(n) - \frac{\kappa}{2} \epsilon^{ij} A_j^{(n)} - \left( \Pi^{i(n+1)} + \frac{\kappa}{2} \epsilon^{ij} A_j^{(n+1)} \right) \approx 0,$$

where the constraints (5.11) are obtained by the recombination of the momenta (5.4). From the Gauss’ law (5.8) as a secondary constraint is rewritten as

$$\Omega^3 = \partial_i \Pi^i(0) + \frac{\kappa}{2} \epsilon^{ij} \partial_i A_j^{(0)} + ie (\Pi \phi - \Pi^* \phi^*) \approx 0.$$

The fundamental Poisson brackets are defined by

$$\{ A_{\mu}^{(n)}(x), \Pi^\nu_{(m)}(y) \} = \delta_{nm} \delta_\mu^\nu \delta^2(x - y),$$

$$\{ \phi(x), \Pi(y) \} = \{ \phi^*(x), \Pi^*(y) \} = \delta^2(x - y).$$
Note that all the constraints $\Omega^0$, $\Omega^i$, and $\Omega^3$ are first class and the problematic algebra (3.2) does not appear in our calculation, therefore the usual Poisson brackets can be used in the construction of anyon operators.

Following the procedure suggested in Ref. [2], a gauge-invariant anyon operator denoted by $\hat{\phi}$ is constructed as

$$
\hat{\phi}(x) = \exp \left( \int dy \, 2i \sin(\theta) \omega(x - y) J_0(y) + i \int_{x_0}^x dy_i \tilde{A}_i(y) \right) \phi(x),
$$

satisfying

$$
[J_0(x), \hat{\phi}(y)] = \hat{\phi}(y) \delta^2(x - y),
$$

where $\Omega$ is given by

$$
\omega(x - y) = \arctan \left( \frac{x^2 - y^2}{x^1 - y^1} \right)
$$

which is multivalued and statistics is characterized by $s(\theta) = 1/\theta$ for the Klein-Gordon field. Therefore, we can show that

$$
\hat{\phi}(x) \hat{\phi}(y) = e^{\pm 2i \sin(\theta)} \hat{\phi}(y) \hat{\phi}(x),
$$

using $\omega(x - y) - \omega(y - x) = \pm \pi$. Note that we have not assumed any path-ordering of gauge field in Eq. (5.14) since the gauge fields themselves are commuting in contrast to the conventional derivation of anyon operators. We have shown that the anyon operators are constructed in terms of the Poisson brackets in the enlarged field space. On the other hand, the Hamiltonian formalism and anyon operators are studied in many literatures so far [3] in the reduced field space, and it seems to be regarded the anyon system as an effective theory when the phase space is reduced. In our formulation, the construction of anyon operator is possible in the gauge-independent way and without any ordering problems.

6. DISCUSSION

Now it seems to be appropriate to comment on our symmetric action in the context of the St"ukelberg mechanism. Our derivation of the symmetric CS theory relies on a con-
jecture from the Abelian BFT method in some sense. One might wonder how to derive the symmetric CS action in terms of St"ukelberg mechanism. Here we briefly discuss how to obtain the symmetric action in this method. The internal gauge parameter of no relevance to our discussion is definitely independent of the translational symmetry parameter and we can simply write the transformation as $A^{(0)}_\mu \rightarrow A^{(0)}_\mu + \epsilon^{(1)}_\mu$ with simply setting $\epsilon^{(0)} = 0$. The above transformation rule is derived as usual directly from the spatial integration of constraint Eqs. (2.7) and (2.8) which is given by a symmetry generator expressed as

$$Q^{(0)} = -2 \int d^2 \mathbf{x} \text{tr}[\epsilon^{(1)}_0 \Pi^{(0)}_0 + \epsilon^{(1)}_i (\Pi^{(0)}_i - \frac{1}{2} \epsilon^{ij} A^{(0)}_j)].$$

The Poisson bracket of gauge fields with this generator $Q^{(0)}$ yields the above transformation rule, however the original CS action is not invariant under this transformation. To make the CS action invariant under this transformation rule according to the St"ukelberg mechanism, we substitute the original field $A^{(0)}_\mu$ with $A^{(0)}_\mu + \epsilon^{(1)}_\mu$ and identify $\epsilon^{(1)}_\mu$ as a new vector field $A^{(1)}_\mu$. Unfortunately, the transformed CS action which partially corresponds to the first iterated action (3.9) of BFT Hamiltonian embedding does not give the first class constraint system, which is easily checked with the help of constraint analysis. Therefore the second step of St"ukelberg mechanism similar to the above one is needed. In this second step, from the transformed action one can obtain the constraints as $\Pi^{(0)}_a = \Pi^{(1)}_a \approx 0$, $\Pi^{(0)}_a - \frac{k}{2} \epsilon^{ij} A^{(0)}_j A^{(1)}_a \approx 0$, and $\Pi^{(0)}_a - \frac{k}{2} \epsilon^{ij} A^{(1)}_a A^{(1)}_a \approx 0$ which gives the following transformation rule as $A^{(0)}_\mu \rightarrow A^{(0)}_\mu + \epsilon^{(1)}_\mu$ and $A^{(1)}_\mu \rightarrow A^{(1)}_\mu - \epsilon^{(1)}_\mu + \epsilon^{(2)}_\mu$ in terms of the generator $Q^{(1)} = -2 \int d^2 \mathbf{x} \text{tr}[\epsilon^{(1)}_0 (\Pi^{(0)}_0 - \Pi^{(1)}_0) + \epsilon^{(2)}_0 \Pi^{(1)}_0 + \epsilon^{(1)}_i (\Pi^{(0)}_i - \frac{1}{2} \epsilon^{ij} A^{(0)}_j - \Pi^{(1)}_i - \frac{1}{2} \epsilon^{ij} A^{(1)}_j) + \epsilon^{(2)}_i (\Pi^{(1)}_i - \frac{1}{2} \epsilon^{ij} A^{(1)}_j)]$ where $\epsilon^{(1)}_\mu$ and $\epsilon^{(2)}_\mu$ are local parameters. Performing the St"ukelberg method and identifying $\epsilon^{(2)}_\mu$ with $A^{(2)}_\mu$ again, we obtain the action corresponding to the second procedure of the BFT formalism. Note that we need not identify $\epsilon^{(1)}_\mu$ with another vector field since it cancels out under the transformation in this second step. After all, the infinite number of St"ukelberg substitution is expected to yield the desired action.
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