Introduction

A repeated measurement design (RMD) is one in which multiple, or repeated measurements are made on each experimental unit. The experimental unit could be a person or an animal. In repeated measurement design each subject is measured before and one or several times after an intervention. Areas where RMDs are widely used include medicine, pharmacology, animal sciences and psychology. A characteristic feature of RMD is that the effect which a treatment has during its period of application (its direct effect) may persist into the following period(s). If the effect persists only into the immediately following period, the effect is called the first-order residual effect or residual effect. The choice of RMD must be made in a way that the treatments can be efficiently compared after allowing for the residual effects. In this paper circular balanced and strongly balanced repeated measurement designs are constructed to extend the work of Iqbal and Tahir [1] for $v \leq 100$ in 3 periods.

Williams [2,3] first initiated repeated measurements designs. Magda [4] introduced the idea of a circular balance repeated measurement design when proper balance for different effects is considered, also proved its universal optimality over the subclass of designs with the same set of parameters. Roy [5] constructed strongly balanced uniform repeated measurements designs for $v=0,1$ or 3 modulo 4, by using the methods of differences and Hamiltonian decomposition of the lexicographic product of two graphs. Jones & Kenward [6] gave a thorough review of the design and analysis of cross-over trials. They also discussed its importance. Afarinejed [7] presented some construction methods for repeated measurement design. Cheng & Wu [8] explained two different types of repeated measurements designs (RMD), the balanced uniform RMD and the strongly balanced uniform RMD. Iqbal & Jones [9] constructed

i. Efficient RMDs with equal and unequal period sizes using method of cyclic shifts for $3 \leq v \leq 10, 3 \leq p \leq 10$,

ii. Strongly balanced RMDs for $3 \leq v \leq 10, 3 \leq p \leq 10$, 

iii. RMDs that are balanced for first and second order residual effect for $6 \leq v \leq 9, 4 \leq p \leq 6$ and 

iv. combinatorial balanced designs for two un equal number of periods for $5 \leq v \leq 10, 3 \leq p_i \leq 6$ and $3 \leq p_j \leq 10$.

Sharma et al. [10] introduced a general strategy of construction of balanced RMDs for odd number of treatments and their analysis. Iqbal& Tahir [1] constructed CSBMD (circular strongly balanced repeated measurements designs) for some classes. Iqbal et al. [11] constructed some first- and second-order CBRMD (circular balanced repeated measurements designs). They also constructed some CSBMDs. In this article, BRMDs and SBRMDs are constructed for linear periods in two different period sizes.

Definition 1.1: The set of all RMD with $p$ periods, $n$ experimental units and $v$ treatments is denoted by $RM(v,n,p)$. A repeated measurements design is balanced with respect to the first-order residual effects if each treatment is immediately preceded $\lambda'$ times by each other treatment (excluding itself).
Definition 1.2: A repeated measurements design is strongly balanced with respect to the first-order residual effects if each treatment is immediately preceded $\lambda^*$ times by each other treatment (including itself).

Definition 1.3: For given $v$ and $p$, a balanced or strongly balanced repeated measurements design is minimal if $\lambda^* = 1$.

In this paper, method of cyclic shifts is explained briefly in Section 2. BRMDs and SBRMDs are constructed in Section 3 and 4 respectively.

Method of Cyclic Shifts

Method of cyclic shifts is explained here briefly. For detail, see Iqbal and Tahir [1] and Iqbal et al. [11].

BRM Designs in Linear Blocks of two unequal Sizes

In this Section BRMDs are constructed for linear periods two different sizes (Table 1, 2).

| $V$ | $P_1$ | $P_2$ | Sets of Shifts |
|-----|-------|-------|----------------|
| 7   | 5     | 3     | [1,3,2,4]+[5,6] |
| 9   | 5     | 3     | [1,2,3,5]+[4,6]+[7,8] |
| 11  | 5     | 3     | [1,2,3,4]+[5,7,6,8]+[9,10] |
| 13  | 5     | 3     | [1,2,3,4]+[5,6,7,9]+[7,10]+[11,12] |
| 15  | 5     | 3     | [1,2,3,4]+[5,6,7,9]+[8,10,11,12]+[13,14] |
| 17  | 5     | 3     | [1,2,3,4]+[5,6,7,8]+[9,10,11,12]+[13,14]+[15,16] |
| 19  | 5     | 3     | [1,2,3,4]+[5,6,7,8]+[9,11,10,12]+[13,14,15,16]+[17,18] |
| 21  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,13,11]+[12,14,15,16]+[17,18]+[19,20] |
| 23  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,13]+[12,14,15,16]+[17,18,19,20]+[21]+[22] |
| 25  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,15,16]+[17,18,19,20]+[21,22]+[23,24] |
| 27  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,15,14,16]+[17,18,20,19]+[21,22,23,24]+[25,26] |
| 29  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26]+[27,28] |
| 31  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+[29,30] |
| 33  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+[29,30]+[31,32] |
| 35  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[33,34] |
| 37  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[33,34]+[35,36] |
| 39  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,28,27]+[29,30,31,32]+[33,34,35,36]+[37]+[38] |
| 41  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,19,20,21]+[22,23,24,25]+[26,27,28]+[29,30,31,32]+[33,34,35,36]+[37,38]+[39,40] |
| 43  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,19,20,21]+[22,23,24,25]+[26,27,28]+[29,30,31,32]+[33,34,35,36]+[37,38,39,40]+[41,42] |
| 45  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,19,20,21]+[22,23,24,25]+[24,26,27,28]+[29,30,31,32]+[33,34,35,36]+[37,38,39,40]+[41,42]+[43,44] |
| 47  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,20,21]+[22,23,25,24]+[26,27,28]+[29,30,31,32]+[33,34,35,36]+[37,38,39,40]+[41,42,43,44]+[45,46] |
| 49  | 5     | 3     | [1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,20,21]+[22,23,25,24]+[26,27,28]+[29,30,31,32]+[33,34,35,36]+[37,38,39,40]+[41,42,43,44]+[45,46]+[47,48] |

Table 2: BRMDs for $p_1 = 5$ and $p_2 = 4$ (Linear Periods).

| $V$ | $P_1$ | $P_2$ | Sets of Shifts |
|-----|-------|-------|----------------|
| 8   | 5     | 4     | [1,2,3,4]+[5,6,7] |
| 11  | 5     | 4     | [1,2,3,4]+[5,7,6]+[8,9,10] |
SBRM Designs in Linear Blocks of two unequal Sizes

In this Section SBRMDs are constructed for linear periods two different sizes (Table 3,4).

**Table 3:** BRMDs for \( p_1 = 5 \) and \( p_2 = 3 \) (Linear Periods).

| \( V \) | \( p_1 \) | \( p_2 \) | Sets of Shifts |
|-------|-------|-------|----------------|
| 6     | 5     | 3     | \([0,1,3,2] + [4,5]\) |
| 8     | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 10    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 12    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 14    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 16    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 18    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 20    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 22    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 24    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 26    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |
| 28    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13]\) |

**Table 4:** BRMDs for \( p_1 = 5 \) and \( p_2 = 4 \) (Linear Periods).

| \( V \) | \( p_1 \) | \( p_2 \) | Sets of Shifts |
|-------|-------|-------|----------------|
| 30    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29]\) |
| 32    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29]\) |
| 34    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33]\) |
| 36    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33] + [34,35]\) |
| 38    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33,34,35] + [36,37]\) |
| 40    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33,34,35] + [36,37,38,39]\) |
| 42    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33,34,35] + [36,37,38,39] + [40,41]\) |
| 44    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33,34,35] + [36,37,38,39] + [40,41,42,43]\) |
| 46    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33,34,35] + [36,37,38,39] + [40,41,42,43] + [44,45]\) |
| 48    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33,34,35] + [36,37,38,39] + [40,41,42,43] + [44,45] + [46,47]\) |
| 50    | 5     | 3     | \([0,1,2,3] + [4,5,6,7] + [8,9,10,11] + [12,13,14,15] + [16,17,18,19] + [20,21,22,23] + [24,25,26,27] + [28,29,30,31] + [32,33,34,35] + [36,37,38,40] + [39,41,42,43] + [44,45,46,47] + [48,49]\) |

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| v  | p₁ | p₂ | Sets of Shifts |
|----|----|----|----------------|
| 23 | 5  | 4  | [0.1, 2.3] + [6, 5.7, 8] + [9, 10, 11, 13] + [12, 14, 15, 16] + [4, 17, 18, 19] + [20, 21, 22] |
| 24 | 5  | 4  | [0.1, 2.3] + [6, 5.7, 8] + [9, 10, 11, 12] + [13, 14, 15] + [4, 16, 17] + [18, 19, 20] + [21, 22, 23] |
| 25 | 5  | 4  | [0.1, 2.3] + [6, 5.7, 8] + [9, 10, 11, 12] + [13, 14, 15, 4] + [16, 17, 18] + [19, 20, 21] + [22, 23, 24] |

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