The Effect of a Magnetic Field on the Profile of Sessile Magnetic Nanofluid Droplets

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ABSTRACT
Surface wettability plays an important role in droplet formation, removal, and resistance to fouling. The sessile droplet profile is one of the most important parameters characterizing surface wettability. In this study, an investigation was carried out into the effects of a magnetic field on the profile of sessile droplets of magnetic nanofluid. We started with a revision of the classic Young–Laplace equation according to normal stress balance principles by considering a magnetic nanofluid droplet in an applied magnetic field gradient. The secant method was then used to solve the non-linear differential equation and predict the sessile droplet profile. The results showed that a downward magnetic force caused an expansion of the profile and an upwards magnetic force caused contraction. In other words, droplets on an hydrophilic or hydrophobic surface could be tuned by magnetic field gradients of different strength and direction. This magnetowetting effect could be further adjusted by selection of different fluid, material, and particle concentration.

KEYWORDS
Wettability; magnetic nanofluid; sessile droplets; surface tension; contact angle

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1. Introduction
The demand for tuneable droplets in fluidic devices for both research and practical application is increasing. Numerous mechanisms have been proposed for the improvement of surface wettability by the application of external potential, such as thermocapillary, optowetting, electrowetting, and electrowetting-on-dielectric [1–6]. Electrowetting-on-dielectric has become an enormously useful research tool because of advantages such as fast-acting response, reversible function, and low power requirement [7]. Reactive wetting, controlled by an external magnetic field (so-called magnetowetting) offers another option for the improvement of surface wettability and can be of particular advantage for magnetic control of the wettability of droplets in fluidic devices.

The steady trend toward the development of micro/nano manufacturing technology, requires micro/nanoparticle materials and these can be made by physical grinding or chemical synthesis. The static and dynamic behavior of
magnetic micro/nanoparticles dispersed in a carrier liquid can be tuned by a magnetic field or more than one field. In previous reports, Egatz-Gómez showed the motion of water droplets with paramagnetic microparticles by a permanent magnet [8]. Recently, Nguyen et al. further investigated the magnetowetting and sliding motion of magnetic nanofluid droplets driven by a permanent magnet [9]. In droplet fluidics, a proper understanding of wettability is very important for accurate static and dynamic control of droplet-based fluids [10]. Surface tension and droplet geometry, including the surface contact angle and profile of a droplet, are very important properties of a fluid. Aim et al. [11] used the perpendicular field instability to calculate surface tension in magnetic nanofluids. They found that measuring the incipient fluid instability peaks of Taylor wavelength can give analogous tensiometer values. Huminic et al. [12] investigated the surface tension of a water-based FeC magnetic nanofluid, prepared by laser pyrolysis. Their results showed that surface tension increases with nanoparticle mass concentration. Manukyan and Schneider [13] studied the wetting behavior of an oil-based magnetic nanofluid on a hydrophobic surface. The experimental results showed that when the external magnetic field strength was increased, the contact angle of the oil-based magnetic nanofluid droplet decreased to start with and then increased until it reached a turning point. Furthermore, when the external magnetic field strength was increased, the oil-based magnetic nanofluid droplets took on a pointy shape. Rigoni et al. [14] studied the static magnetowetting behavior of water-based γ − Fe₃O₄ magnetic nanofluid droplets deposited on a flat substrate. Their results showed that the flattened droplets, extended normally to the substrate, resulted from the magnetic field. In measurements of droplet profile, the pendant and sessile droplet techniques are the two basic methods used [15]. However, both methods require a balance between surface and body forces to achieve mechanical equilibrium of the droplets. Recently, Weng et al. [16] revised the Young–Laplace equation, according to the normal stress balance principle, to predict the pendant droplet profile of magnetic nanofluids. Their results revealed that the presence of an applied magnetic field gradient could result in the contraction of the neck of the pendant droplet. Furthermore, increasing the magnetic saturation of the fluid could increase the magnetic field effect. Although there have been many interesting studies of related phenomena, no theoretical or predictive investigation of the profile of sessile magnetic nanofluid droplets, with respect to magnetic field effect, has so far been published.

In this study, numeric investigation of the sessile droplet profile of magnetic nanofluids on solid surfaces was carried out under an applied magnetic field gradient. The purpose being to report the magnetowetting behavior of magnetic nanofluid sessile droplets of different composition. The classical Young–Laplace equation was revised by the normal stress balance principle and the Secant method was then used to numerically solve the non-linear differential equation. Sessile droplet profiles under different magnetic field gradient strengths and directions are presented and discussed. The results from this study can help devise procedures for the magnetic control of wettability in droplet-based fluidic applications, such as coating, sputtering, dusting, spray painting, and spray cooling.

2. Methodology

2.1. Governing Equation

Revisions of the governing equations of stationary magnetic nanofluids were done less often than those for fluids in motion [17]. This study of the effect of a magnetic field on the profile of sessile magnetic nanofluid droplets was conducted as shown in Figure 1. The droplet is subject to an applied magnetic force as well as that of gravity. Let \( x \) and \( z \) represent the horizontal and vertical coordinates, \( R_1 \) and \( R_2 \) the two principal curvature radii, \( \theta \) the tangential angle, and \( g \) and \( \nabla h \) gravity and the magnetic field gradient. The profile of a sessile droplet can be described by the Young–Laplace equation, a non-linear partial differential equation, that describes the pressure difference \( \Delta p \) at the surface between two fluids:

\[
\Delta p = 2\gamma H
\]  
(1)

where \( \gamma \) is the surface tension between the fluids and \( H \) is the mean curvature and could be written as

\[
H = \frac{1}{2} \left( \frac{-d^2x/dz^2}{(1 + (dx/dz)^2)^{3/2} + \frac{1}{x((dx/dz)^2 + 1)^{1/2}}} \right)
\]  
(2)

The curvature geometry, as shown in Equation (2), is related to three kinds of field acting on the surface element

![Figure 1. Profile geometry of a sessile droplet.](image-url)
of a droplet. Surface tension is the main force responsible for the surface element:

\[ f_s = 2\pi R_0 \gamma \sin^2 \theta \]  

(3)

where \( R_0 \) is the curvature radius of the profile at the droplet apex. This presents a force in opposition to the downward force of gravity:

\[
f_G = \Delta \rho V g
\]  

(4)

where \( \Delta \rho \) is the density difference between the magnetic nanofluid and air, \( V \) is the volume of the surface element, and \( g \) is gravitational acceleration. The density difference can be denoted as

\[
\Delta \rho = \rho_{\text{mnf}} - \rho_{\text{air}} = \rho_c \varphi + \rho_{\text{cl}} (1 - \varphi) - \rho_{\text{air}}
\]  

(5)

where \( \rho_{\text{mnf}} \) is the magnetic nanofluid density, \( \rho_{\text{air}} \) is the air density, \( \rho_c \) is the solid phase density, \( \rho_{\text{cl}} \) is the carrier liquid density, and \( \varphi \) is the particle volume fraction. In addition, the surface element also bears a force due to the applied magnetic field gradient:

\[
f_M = \mu_0 \varphi m_l V \frac{dL(\xi)}{dz}
\]  

(6)

where \( \mu_0 \) is the magnetic permeability in a vacuum (\( \mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2} \)), \( m_l \) is the saturation magnetization of magnetic nanoparticles, \( L(\xi) \) is the Langevin function, and \( \xi \) is the Langevin argument of the local magnetic field strength \( h \), and \( dh/\text{dz} \) is the magnetic field gradient. It should be noted that the direction of the magnetic force is downward when the value of \( dh/\text{dz} \) is positive, and the direction is upward when it is negative.

Since the pressure difference shown in Equation (1) is determined by the surface force \( f_s \), gravity \( f_G \), and magnetic force \( f_M (\Delta \rho = f_s / \pi (R_0 \sin \theta)^2 - f_G/V + f_M/V) \), the classical Young–Laplace equation could be revised as follows:

\[
\left( \frac{-d^2 x}{dz^2} \right)^{3/2} + \frac{1}{x((dx/dz)^2 + 1)^{1/2}} = \frac{2}{R_0} - \left( \frac{\Delta \rho g}{\gamma} + \frac{\mu_0 \varphi m_l L(\xi)}{\gamma} \frac{dh}{dz} \right)z
\]  

(7)

\[
\left( \frac{-d^2 x}{dz^2} \right)^{3/2} + \frac{1}{x((dx/dz)^2 + 1)^{1/2}} = \frac{2}{R_0} - \left( \frac{\Delta \rho g}{\gamma} + \frac{\mu_0 \varphi m_l L(\xi)}{\gamma} \frac{dh}{dz} \right)z
\]  

(8)

\[
\left( \frac{-d^2 x}{dz^2} \right)^{3/2} + \frac{1}{x((dx/dz)^2 + 1)^{1/2}} = \frac{2}{R_0} - \left( \frac{\Delta \rho g}{\gamma} + \frac{\mu_0 \varphi m_l L(\xi)}{\gamma} \frac{dh}{dz} \right)z
\]  

where \( i = 2, 3, 4, \ldots, N + 1 \). The parameter values of \( R_0, \gamma, \rho_{\text{mnf}}, \rho_c, \rho_{\text{cl}}, \varphi, \xi, \) and \( dh/\text{dz} \) were selected, and the initial condition \( x(z = 0) = 0 \) was input at \( i = 1 \).

To solve the above problem for a particular sessile droplet, we first set \( i = 2 \) and solved Equation (8) through the secant method to determine the value of \( x_\text{i} \). Then \( i = i + 1 \) was set and the procedure was repeated until \( i = N + 1 \). Finally, the \( x \) and \( z \) data were used to draw the sessile droplet profile. Note: at the beginning of the numerical procedure (while \( i = 2 \)), the value of \( x_{i-2} \) was unknown, and so the first principle curvature radius \( r_1 \) was replaced by the apex curvature radius \( R_0 \). Furthermore, because changes in the value of \( x \) are more significant when the value of \( z \) approaches 0, the total droplet height was divided into five sections to allow more accurate description of the profile. The grid sizes used for each section are, respectively, 0.000001, 0.0000011, 0.0000013, 0.0000016, and 0.0000025 m. The grid number in the previous four sections was set to 24, and the grid number in the last section depended on the total height of the sessile droplet. These grid settings were validated by comparison with experimental observations obtained using a contact angle meter with the sessile drop method for determining the profile of a deionized water droplet (volume: \( 1.31 \times 10^{-8} \text{ m}^3 \)) on an alumina surface in air at 25 °C.

### 3. Results and Discussion

A numeric study of the influence of a magnetic field gradient on the sessile droplet profiles of magnetic nanofluids, vertically magnetized in the downward and upward directions was carried out. To do this, a sessile droplet of a magnetic nanofluid with a volume of \( 1.31 \times 10^{-8} \text{ m}^3 \) and an apex curvature radius of 0.0020137 m on a flat surface in air at 25 °C was employed. Material parameter settings were \( \rho_{\text{air}} = 1.185695 \text{ kg m}^{-3}, \rho_c = 5240 \text{ kg m}^{-3}, \rho_{\text{cl}} = 996.95 \text{ kg m}^{-3}, \) and \( m_l = 4.77 \times 10^9 \text{ A}^{-1} \text{ m}^{-1} \). For
of a downward magnetic force could be used to expand
the droplet profile and decrease the angle of contact. For
example, from $81^\circ$ to $66^\circ$ for $\mu = 0.075 \, \text{N m}^{-1}$ or from
$84^\circ$ to $70^\circ$ for $\mu = 0.085 \, \text{N m}^{-1}$. Furthermore, a compar-
ison between the two profiles at different surface tension
indicates that this downward magnetic effect could be
increased if a nanofluid with lower surface tension was
used. This is because surface tension is the result of inter-
molecular attractive forces at the surface of the fluid, and
lower surface tension will offer less resistance to geometric
changes caused by an external force such as gravity or a
magnetic field. An applied upward magnetic force can
be used to cause the profile to contract and this changes
the contact angle. In Figure 3 it can be seen that the angle
changes from $81^\circ$ to $110^\circ$ for $\mu = 0.075 \, \text{N m}^{-1}$ and from
$84^\circ$ to $105^\circ$ for $\mu = 0.085 \, \text{N m}^{-1}$. A comparison between
the two different droplet surface tension profiles also

simplicity, the saturated magnetization state $I(\xi) = 1$
was assumed to have been reached. The ranges
covered for the downward magnetic force were:
$0.075 \, \text{N m}^{-1} \leq \gamma \leq 0.085 \, \text{N m}^{-1}$, $0.01 \leq \varphi \leq 0.02$, and
$0 \leq dh/dz \leq 3.0 \times 10^6 \, \text{A m}^{-2}$ and for the upward mag-
netic force the range was $-3.0 \times 10^6 \, \text{A m}^{-2} \leq dh/dz \leq 0$.  

3.1. Magnetic Field Effect with Different Surface
Tension

The sessile droplet profiles of magnetic nanofluids of
particle volume fraction $\phi = 0.01$ for different values of
magnetic field gradient $dh/dz$ at low and high values of
the surface tension $\gamma$ are shown in Figures 2 and 3. A
positive value of $dh/dz$ indicates a downward magnetic
force, while a negative value indicates an upward mag-
netic force. From Figure 2, it can be seen that application
of a downward magnetic force could be used to expand
the droplet profile and decrease the angle of contact. For
example, from $81^\circ$ to $66^\circ$ for $\gamma = 0.075 \, \text{N m}^{-1}$ or from
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Figure 2. Sessile droplet profiles for different downward magnetic forces at lower and higher surface tensions.

Figure 3. Sessile droplet profiles for different upward magnetic forces at lower and higher surface tensions.
and 5 which show the downward and upward magnetic forces. Figure 4 shows that the contact angle changes from 90° to 70° for $\varphi = 0.01$ and from 86° to 70° for $\varphi = 0.02$, as the profile is expanded by a downward magnetic force. In addition, a more significant magnetic effect (for a higher particle volume fraction nanofluid) can be seen by comparing two profiles of different particle volume fractions. This is because a nanofluid with higher particle concentration responds more strongly to an external magnetic field. Figure 5 shows that an upward magnetic force contracts the droplet profile and increases the contact angle from 84° to 113° for $\varphi = 0.01$ and from 83° to 157° for $\varphi = 0.02$. It is clear, from a comparison of two different particle volume fraction profiles, that nanofluid with a high concentration of particles will show a more significant magnetic effect than one with fewer particles. The results of these experiments show that magnetowetting can be adjusted by the selection of particle concentration.

4. Conclusions

Surface wettability is an important characteristic of droplets on a solid surface. The wettability properties of a fluid are affected by surface tension and droplet geometry such as the profile and contact angle. A study of the theoretical prediction of the profile of sessile magnetic nanofluid droplets in an applied magnetic field gradient has been carried out. The classical Young–Laplace equation, revised on the basis of the normal stress balance principle, and the secant method were used to solve the non-linear differential equation. This allowed a prediction of sessile drop profile and the numerical results of the influence of magnetic force on droplet profile and contact angle to be made and presented. A downward or upward magnetic force can be used to expand or contract the droplet profile which reduces or increases the contact angle. This magnetowetting effect can be further increased if a nanofluid with low surface tension, or high particle concentration, is used. This study may cast more light on the matter and be useful for investigators who have a need to control wettability properties in droplet fluidic applications, such as in coating, sputtering, dusting, spray painting, and spray cooling.

Disclosure Statement

No potential conflict of interest was reported by the authors.

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