Inferences and Modal Vocabulary
Logical Understanding II

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Abstract

Deduction is one of the major forms of inferences and commonly used in formal logic. This kind of inference has the feature of monotonicity, which can be problematic. There are different types of inferences that are not monotonic, e.g., abductive inferences. The debate between advocates and critics of abduction as a useful instrument can be reconstructed along the issue, how an abductive inference warrants to pick out one hypothesis as the best one. But how can the goodness of an inference be assessed? Material inferences express good inferences based on the principle of material incompatibility. Material inferences are based on modal vocabulary, which enriches the logical expressivity of the inferential relations. This leads also to certain limits in the application of labeling in machine learning. I propose a modal interpretation of implications to express conceptual relations.

1 Introduction

1.1 Context – Commitments

To contextualize this paper, I will start with two commitments. (1.) I believe that philosophy can contribute to the debates in the field of artificial intelligence. The philosophy of language and the philosophy of logic provide us with many tools and insights of language use and how we use concepts.

(2.) I believe that inferential relations govern our concept use. So, if we label something, what we actually do, is drawing an inference. There are different kinds of inferential classifications: to label something or to describe something are actions that are governed by inferential relations. It could also be described as a difference between classificatory and conceptual inferences. [14]

[1] This paper is based on a talk that I gave at the International Conference on Information and Computer Technologies (ICICT 2020) in San Jose.

[2] Charles Sanders Peirce thinks that a perceptual judgment is really an abductive inference, that can be made explicit. (CP 5.3) In epistemological debates in philosophy it is an extensively discussed topic, whether to perceive something is to draw an inference or not.
1.2 Problem

There are different kinds of inferences, like e.g. induction, deduction, and abduction. – Deduction is the one of the major forms of inferences and commonly used in formal logic. This kind of inference has the feature of monotonicity, which can be problematic as the famous example of the “flying penguin” shows.\cite{6} Take the premises: (1.) Tweety is a penguin. (2.) Penguins are birds. (3.) Birds can fly. From these premises can be concluded, that Tweety can fly. If we add the statement that penguins cannot fly, we can draw the consequence that Tweety can’t fly, but the former statement that Tweety can fly is still valid. So, we have an inconsistent knowledge base.\cite{6} – And nothing is more awful for a logician, if there is an inconsistency. It is possible to construct a consistent metalanguage like a calculus for propositional logic with the introduction of deductive reasoning (modus ponens) and the feature of monotonicity, but the example above shows the problematic point of using deductive reasoning for natural language use and it is mainly connected with the feature of monotonicity, which therefore has to be omitted.

2 Nonmonotonic Inferences

There are different types of inferences that are not monotonic, e.g. abductive inferences do not require an exhaustive list of conditions or premises that might be in the end inconsistent, because it is more about proposing premises as hypotheses. Charles Sanders Peirce writes: “Deduction proves that something must be; Induction shows that something actually is operative; Abduction merely suggests that something may be.”\cite{10} (CP 5.171) The debate between advocates and critics of abduction as a useful instrument can be reconstructed along the issue, how an abductive inference warrants to pick out one hypothesis as the best one (given that there is an already established set of hypotheses to choose from).\cite{5} There are approaches that claim that the best hypothesis is picked out by applying principles like coherence or simplicity to the explanations.\cite{15} It needs to be shown, how these principles can be at least spelled out more concretely.

2.1 Abduction - Peirce

Peirce worked extensively on this kind of inference, although he admitted in several occasions that he confused induction and abduction. (He called abduction hypothetic inference, but understood later that it is actually an induction from qualities and that abduction is something else.\cite{3} Peirce describes the form of the inference in the following way:

\cite{3}\cite{11}\cite{12}
Long before I first classed abduction as an inference it was recognized by logicians that the operation of adopting an explanatory hypothesis – which is just what abduction is – was subject to certain conditions. Namely, the hypothesis cannot be admitted, even as a hypothesis, unless it be supposed that it would account for the facts or some of them. The form of inference, therefore, is this:

The surprising fact, C, is observed;
But if A were true, C would be a matter of course,
Hence, there is reason to suspect that A is true.

Thus, A cannot be abductively inferred, or if you prefer the expression, cannot be abductively conjectured until its entire content is already present in the premise, 'If A were true, C would be a matter of course.'

To admit or propose a hypothesis as a hypothesis is only possible with respect to the fact. The inference presupposes the relation of them in the premise. It is in fact not an act of cognition – of grasping the propositional content, that the premise expresses –, but an act of recognition of the relation.

In the end, what is important is to know which conditions should be fulfilled that the abductive inference is a good inference. According to him the “question of the goodness of anything is whether that thing fulfills its end.” And the end of the abductive inference is “to lead to the avoidance of all surprise and to the establishment of a habit of positive expectations that shall not be disappointed”. One needs to be capable to experimentally verify the expectations of the hypothesis. Peirce sees of course that this leads to the question: “What are we to understand by experimental verification?” – But experimental verification is governed by other principles, rules, and inferences and then we are talking about the “logic of induction”.

2.2 Abduction - Best Explanation

There are other suggestions for the goodness of an abductive inference (without diving in the vast discourse in the philosophy of science). Paul Thagard claims that a theory explains the facts better, if it does not make additional statements that might not seem plausible or hard to combine with other explanations, facts,

\[4\] Admitting, then, that the question of Pragmatism is the question of Abduction, let us consider it under that form. What is good abduction? What should an explanatory hypothesis be to be worthy to rank as a hypothesis? Of course, it must explain the facts. But what other conditions ought it to fulfill to be good? The question of the goodness of anything is whether that thing fulfills its end. What, then, is the end of an explanatory hypothesis? Its end is, through subjection to the test of experiment, to lead to the avoidance of all surprise and to the establishment of a habit of positive expectation that shall not be disappointed. Any hypothesis, therefore, may be admissible, in the absence of any special reasons to the contrary, provided it be capable of experimental verification, and only insofar as it is capable of such verification. This is approximately the doctrine of pragmatism. But just here a broad question opens out before us. What are we to understand by experimental verification? The answer to that involves the whole logic of induction."
or theories. He mentions the advantages of Charles Darwin’s theory of natural
selection to explain certain facts, like e.g. the geographic distribution of species.
Another example that the mentions is the “phlogiston theory” that “burning ob-
jects give off the substance phlogiston, whereas, according to Lavoisier, burning
objects combine with oxygen. The main point of Lavoisier’s argument is that
his theory can explain the fact that bodies undergoing combustion increase in
weight rather than decrease [...]. To explain the same fact, proponents of the
phlogiston theory had to make such odd assumptions as that the phlogiston
that was supposedly given off had ‘negative weight.” [15] (77/78) Thagard sug-
gests therefore that the “above arguments exemplify three important criteria for
determining the best explanation.” These criteria are: “consilience, simplicity,
and analogy.” Important is here to mention that by “criteria” he does “not mean
necessary or sufficient conditions.” [15] (77/78)

It is also argued that the hypothesis needs to yield an explanation that is
sufficiently good enough[9], but it is not clear what “sufficiently good enough”
might mean and involve. Another example of nonmontonic inferences is clearer
in this respect: material inferences. They express good inferences based on the
principle of material incompatibility. The claim, if p then q, is incompatible with
the claim that it is possible that p and not-q.[2] Instead of formal inconsistencies
(like in the case of deduction) a kind of material incoherence can be expressed
and is taken as a principle to distinguish good inferences from bad inferences,
but the same problem arises, because if it is not possible to deal with inconsistent
predicates or propositions like in the example of the flying penguin, it would be
also difficult to deal with incoherent or incompatible ones.

2.3 Material Inferences

Branden develops an incompatibility semantics in his book Between Saying and
Doing. Branden uses the deontic vocabulary “commitment” and “entitlement”.
Being committed to a claim does not only mean to state that the claim is true,
but also to commit yourself to be able to justify the claim. It is your responsi-
bility to give a reason for your commitment and if you have a reason you might
be also entitled to the claim. If you are e.g. committed to a plane figure being
rectangular, you also have to be committed to its being polygonal. “And the old
nautical meteorological homily, ‘Red sky at night, sailor’s delight; red sky in
morning, sailor take warning,’ tells us that anyone who sees a colorful sunrise is
entitled to the claim that a storm that day is probable.” Branden writes that
this reasoning is only probative, not dispositive, because the “colorful sunrise
provides some reason to predict a storm, but does not yet settle the matter.
Other considerations, such as a rising barometer, may license one not to draw
the conclusion one would otherwise be entitled to by the original evidence.” [2]
(119/120)

A deductive inferential relation means that if you are committed to one claim
you also have to be committed to the consequence of the claim. A rectangular
figure is also polygonal. Both predicates are not incompatible with each other.
An inductive inferential relation means that one who is entitled to one claim

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is also entitled to the consequence of the claim. Important is here that the “reasoning is only probative”. Brandom talks here about predictions. In machine learning one of the interests lies in making predictions from experience or data. If you commit yourself to the claim that “Pedro is a donkey”, then you have to be committed to the claim that “Pedro is a mammal.” All that is incompatible with the second claim is also incompatible with the first one.

3 Incompatibility

The inferential relations of deduction and induction can be made explicit by the deontic vocabulary of “commitment” and “entitlement”. For Brandom it seems possible to make “incompatibility-entailments” explicit by a specific “kind of inferences”: “modally qualified conditionals”. He writes that if “two properties (such as being a mammal and being an invertebrate) are incompatible then it is impossible for any object simultaneously to exhibit both.” This means therefore “it is impossible for anything to be a donkey and not be a mammal. That is why the incompatibility-entailment in question supports counterfactuals such as ‘If my first pet (in fact, let us suppose, a fish) had been a donkey, it would have been a mammal.’ We could say: ‘Necessarily, anything that is a donkey is a mammal.’” [2] (121/122)

The modal vocabulary is a specific tool of expressing inferential relations. Relations between species and genus can be made explicit by this “modally qualified conditionals”. If a genus has necessarily a certain characteristic, also the species has to have it. There are also certain characteristics that do not necessarily belong to the genus, because not all the species have it. Every mammal has necessarily a heart, but not every mammal has necessarily legs (e.g. whales or dolphins). This is connected with the semantic dimension of the concepts. The essential idea of Brandom is that incompatibility does not have to be mistaken with inconsistency on the formal, logical side. It can be that one claim is not incompatible with other claims individually, but it can be that it is incompatible with a set of these individual claims, e.g. “the claim that the piece of fruit in my hand is a blackberry is incompatible with the two claims that it is red and that it is ripe, though not with either individually”. [2] (123)

It is an important question, how incompatibility can be formalized. Maybe one can formalize it by introducing an operator for incompatibility, but then it has to be shown, how the this operator is primitive or how it can be formulated by definition with other primitive operators. Brandom writes that “p is incompatible with the set consisting of p → q and ~ q, but not with either individually.” [2] (123) I believe that after the arrow should be the set and that the implication does not belong to the set. The set can be expressed as a conjunctions of claims:

(1) x is a blackberry is incompatible with x is red ∧ x is ripe

Thus “Pedro is a donkey,” incompatibility-entails ‘Pedro is a mammal,’ for everything incompatible with Pedro’s being a mammal (for instance, Pedro’s being an invertebrate, an electronic apparatus, a prime number) is incompatible with Pedro’s being a donkey.” [2] (121)
y is a cherry is $\neg$ incompatible with y is red $\land$ y is ripe

Intuitively it could also be expressed in the following way:

$p \land q \rightarrow \neg r$ If x is a red and a blackberry, then it is not ripe.

Problematic is that it would also be true in the case that the antecedent is false.

There are other sets of sentences that are incompatible and the problem is here, that at the level of formal logic and universal statements, it leads to inconsistencies, that can not be solved on the formal level. The general statement that all birds fly has its problems with species that belong to the genus bird, but do not fly, like e.g. penguins.

$x$ is a dove is $\neg$ incompatible with $x$ is a bird $\land$ x flies

$y$ is a penguin is incompatible with $y$ is a bird $\land$ y flies

The statement in (5) on the left side is not incompatible with all sentences, but just with one. If the sets are interpreted in a probabilistic way then a weight is assigned to them according to how many sentences are compatible or incompatible. Of course, a lot of logical fine-tuning has to be done in order to understand the nature of concepts. The expressive logical tools to do so need to be examined and the analytic framework that contains them. The introduction of modal vocabulary can help here.

4 Modal Vocabulary

Possible worlds are sometimes interpreted as existing parallel worlds (David Lewis) or as worlds that are accessible from the actual world and they represent different states of the actual world, which can be expressed by counterfactuals (Saul Kripke). Possible worlds are logically consistent within themselves and to a certain degree the likelihood of the weights (predicates) can be adjusted. But to start just with logical consistency, will never get you to what is actually the case. All possible worlds seem to be equally likely. That is why I suggest that the actual world should be represented by a proposition that serves as the antecedent of a conditional. The antecedent identifies or picks out the object that is classified in the consequent.

$p \rightarrow q \land \neg r$ If x is a blackberry, then x is red and x is not ripe.

There is a widely used example that shows how a counterfactual statement expresses the unlikelihood of a possible world and this makes it at least seem difficult to take counterfactuals as adequate expressive tools. There is a difference between: “If Shakespeare did not write Hamlet, someone else did.” and “If Shakespeare had not written Hamlet, someone else would have.” The counterfactual expresses in way an idea that at least one person might have written the same piece of art word by word. The example is from Jonathan Bennett. For an analysis of conditionals and the difference between indicative and subjunctive conditionals see von Fintel (2012). The discussion of possible worlds revolves around the similarity of the worlds and the possible worlds are placed in the antecedent. This is different to the approach that is proposed here.
In the conditional the antecedent expresses a sufficient condition for the consequent, while the consequent expresses a necessary condition. Another example is: If Nellie is an elephant, she has a trunk. Being an elephant is a sufficient condition of her to have a trunk and having a trunk is a necessary condition that she is an elephant. But the consequent expresses possible or dispositional properties.

(7) \( p \rightarrow \Diamond(q \land \neg r) \) If x is a blackberry, then it is possible that x is red and x is not ripe.

The antecedent expresses here the starting point or the condition that forms the possible world (consequent). It is a set of propositions or properties which are possible, but also necessarily possible. It gives room for updating the properties, i.e. to change the proposition, but also to regard them as necessary properties of the object. The consequent expresses as a conjunction conceptual relations between sets of propositions. I would call it the intensional part and the possible world. Possible worlds express conceptual relations and not ontological worlds or other realities. They are models of conceptual relations. Even if the antecedent is not realized they are true, but without the antecedent the relation has no context. It makes no sense to say that something is possibly red and not ripe – but what exactly? – We are talking about blackberries and not about cherries.

5 Limits of Labeling

The underlying conceptual problem is that concepts, at least non-analytic concepts, have only necessary characteristics and not necessary and sufficient characteristics. For analytic concepts it might be possible to have a list of necessary features of the concept and together they are sufficient to grasp or describe the concept. We do not have an exhaustive list of features that constitute a concept. To every characteristic another can be added: e.g. that elephants have trunks, have four legs, are grey. Every feature is necessary, but not sufficient to describe the species elephant. What can be selected are only necessary features, when we label or cluster. Therefore, conceptually there is already a limitation involved. – I believe that this limitation lies in the underlying logical structure of concept use in classifying or labeling objects. (In the end, the properties are only possible properties, but necessarily possible.)

This ideas can be linked to labeling in machine learning. A learning algorithm can be supervised or unsupervised. Supervised learning means that the data engineer knows what is “correct” or “incorrect”, it is labeled like this. In

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7 Thomas Zoglauer writes that they express natural laws, but that is not correct. Dispositional properties are not natural laws. They have a different logical form. 17 (41)

8 One axiom of the modal system S5 is that something that is possible is necessarily possible \( (\Diamond p \rightarrow \Box \Diamond p) \). The necessary conditionality is expressed by the role the antecedent and the consequent play in the truth values of the conditional. Maybe the axiom of S5 stems actually from the logical structure of the implication.
unsupervised learning clusters are build by examples. John Guttag, in his book
*Introduction to Computation and Programming Using Python*, uses as an ex-
example how to classify animals as reptiles. Of course, we already know which
animal is a reptile and which is not. We use some examples to extract features
that, we suppose, are necessary conditions, like: is cold-blooded, has scales, lays
eggs, is poisonous, has no legs. These features are extracted from the examples:
cobra and rattlesnake. If we add boa constrictor (to the training data), then we
see that laying eggs and poisonous are not necessary features. Unfortunately,
a salmon would now also be classified as a reptile and an alligator would not
count as a reptile. So, the feature has no legs has to be discarded as irrelevant,
too. (Or, that a reptile has either no legs or four legs.)[7] (375-377) It depends
here not only on the fact that we know which animal is a reptile, but also on
what is a necessary condition for being a reptile.

“Does this mean that we should give up because all of the available
features are mere noise? No. In this case, the features *scales* and
cold-blooded are necessary conditions for being a reptile, but not
sufficient conditions. The rule that an animal is a reptile if it has
scales and is cold-blooded will not yield any false negatives, i.e., any
animal classified as non-reptile will indeed not be a reptile. However,
it will yield some false positives, i.e., some of the animals classified
as reptiles will not be reptiles.”[7] (377)

I think that he is correct that they are only necessary conditions, but the neces-
sity is based on a stipulation by us. – Observable and perceivable properties, like
to have scales, are only conditional generalities that are embedded in a context
or situation or bound to a singular thing. They are empirically verified[9] The
stipulations are nor arbitrary, because the are undergirded empirically. One
can now count on them and calculate with them. The classification encodes
knowledge that can be used to “calculate” with it.

6 Concept Use

The consequent of the conditional can then be used as the antecedent or premise
for another conditional.

(8) $\diamond (q \land \neg r) \rightarrow \diamond s$

If it is red and not ripe, then it is not eatable. – The formalization of a disposi-
tional predicate is much more difficult. Some use conditionals and others use
counterfactuals, but this would lead us to another topic. Brandom writes that in

“the most general sense, one classifies something simply by respond-
ing to it differentially. Stimuli are grouped into kinds by the response-
kinds they tend to elicit. In this sense, a chunk of iron classifies its

9This is the idea of abduction from Peirce. We stipulate a hypothesis and see if it applies
to a case, but it has to be undergirded empirically.
environments into kinds by rusting in some of them and not others, increasing or decreasing its temperature, shattering or remaining intact. As is evident from this example, if classifying is just exercising a reliable differential responsive disposition, it is a ubiquitous feature of the inanimate world.” [3] (200/201)

An algorithm would be a “good” algorithm, if it classifies the objects reliably. Problematic is for Brandom that “classification as the exercise of reliable differential responsive dispositions (however acquired) is not by itself yet a good candidate for conceptual classification, in the basic sense in which applying a concept to something is describing it.” [3] (202) He builds his epistemological approach on Sellars, who states that in order to describe objects we need to “locate these objects in a space of implications” and therefore we do not “merely label” them, or merely classify them. It means not just to know in which situation to respond reliably, but also to know the consequences of the application and to understand, when propositions or sets of propositions are used as antecedents or consequents in conditionals or implications. Canonized knowledge, which is based on empirical knowledge, can serve as a premise of a conditional. It is a necessary and general knowledge (a posteriori) – a possibility in the “space of implications” that can be used to “calculate” with it or to count on it. This means that it is reliable knowledge.

Space of implications: The classification is: \( p \rightarrow \Diamond (q \land \neg r) \) (If x is a blackberry, then it is possible that x is red and x is not ripe.) and the description is: \( \Diamond (q \land \neg r) \rightarrow \Diamond s \) (If it is red and not ripe, then it is not eatable.) One has to understand which role the (sets of) propositions play (being premises or consequences) in order to “move” competently in the web of inferences. That means also that there are different kinds of concept use that only make sense and have meaning within the “space of implications”.

The space of implications is a space of conceptual possibilities and has to be elaborated. Which kind of reasons are good reasons in the sense that they are employed in the right language game. The adequacy of concept use and its location in the right language game are crucial to distinguish between different kinds of reasons. This means that statements are embedded in a logical space of conditions. There are different kinds of conditions for statements: they can represent reasons, conceptual relations, causes, or also motivational states that explain actions. Inferring from the statements to its conditions is part of abductive reasoning.

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10. It is only because the expressions in terms of which we describe objects, even such basic expressions as words for the perceptible characteristics of molar objects, locate these objects in a space of implications, that they describe at all, rather than merely label.” [13] (section 108)

11. To learn what they mean is to learn, for instance, that the owner put a red label on boxes to be discarded, green on those to be retained, and yellow on those that needed further sorting and decision. Once I know what follows from affixing one rather than another label, I can understand them not as mere labels, but as descriptions of the boxes to which they are applied. Description is classification with consequences, either immediately practical (‘to be discarded/examined/kept’) or for further classifications.” [3] (203)
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