Skeleton-based perpendicularly scanning: a new scanning strategy for additive manufacturing, modeling and optimization

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Abstract
Actually, additive manufacturing (AM) is considered as a major class of complex parts manufacturing technologies. Including a wide range of materials, a huge set of physico-chemical phenomena are involved and adapted to control and master the variety of materials processing. On the other side, AM still knows a low productivity rate, due to different reasons that are mainly related to the material-process interaction control. In this paper, the authors propose a novel 2D scanning strategy that could be adapted to AM processes such as Laser Beam Powder Bed Fusion of Metals (PBF-LB/M) and polymers (PBF-LB/P), Electron Beam Powder Bed Fusion for metals (PBF-EB/M), and Material Extrusion-based (MEx) (ISO/ASTM 52,900 standards). The novelty presented corresponds to a Skeleton Based Perpendicularly (SBP) scanning strategy that aims to reduce the scanning lengths, and thus the production time and processing energy. The competitiveness of the new technique is mainly discussed according to the hatch space distance and the dimensions of a rectangular shape that was selected for the proof of concept of this new scanning strategy. In other words, it is proposed to investigate the competitiveness of the new scanning technique compared to four classical scanning strategies that are widely used in AM in term of process productivity. A detailed benchmark analysis has been applied to the following strategies: chess, stripe, spiral, and contour scanning. An analytical mathematical modeling was developed leading to the evaluation of the performance of SBP scanning compared to the scanning benchmark strategies by means of two proposed geometrical indices: the “gain of length” and the “specific gain of length per surface unit”. These were exploited in two separate study cases. The simulation showed that the SBP scanning length exhibits an increasing quadratic dependence on rectangle dimensions and a decreasing hyperbolic behavior according to hatch space distance. The lengths of the benchmark scanning strategies also presented a hyperbolic decreasing behavior according to hatch space distance. After that, it was proved that the SBP strategy is absolutely competitive compared to chess and stripe scanning; the competitiveness fluctuates around 95%, but it is highly concentrated around 100%. Otherwise, for the contour and stripe strategies, it has been shown that the competitiveness is strongly affected by the hatch space distance and by the dimensions of the shape being scanned. A particular behavior of the feasibility percentage of decision variable combinations, was detected as power laws or polynomials of the hatch space distance in the case of SBP/spiral comparison. In this case, the competitiveness (feasibility) ranged from 20 to 98% while it ranged from 0 to 75% in the case of SBP/contour comparison. The results of this study could also constitute a major contribution to related scientific and technical fields concerned with optimal area or volume control.

Keywords Additive manufacturing · Process optimization · Productivity · Skeleton-based scanning strategy · Processing time · Processing energy

1 Introduction and context

Additive manufacturing (AM) is now considered as one of the leading classes of manufacturing processes for building complex volume and surface geometries. The size of the AM market has grown phenomenally over the past decade. According to Wohlers Associates (2020), the turnover has grown exponentially from 2007 to 2019, during which
time the market size grew 21.2% to $11.867 billions of US Dollars [1]. AM is applied in most industries and the corresponding processes are dedicated especially to parts with complex designs [2–5]. These are used both as structural features and as decorative utensils. Currently, the construction industries are also increasingly interested in 3D printing because it allows the inclusion of oriented mechanical properties and complex geometries [6–9]. In addition, the integration of 3D construction has evolved exponentially over the last decade [10].

Coupled with topology optimization (TO) and generative design (GE), AM processes are nowadays exploited to build very complex entities when several functional and technical constraints are taken into account [11]. From a performance point of view, an optimized choice of the process parameters combinations can achieve the desired performance in terms of final mechanical and material features [5, 12–19]. In general, semi-empirical models have been used to model and characterize AM processes based on existing physical phenomena and specific engineering areas (welding, sintering, diffusion, metallurgy, polymers, etc.). In addition, AM process designations are proposed by manufacturers based on the physical and technical specifications that are involved in the processing steps [20].

Since AM processes use layer-by-layer printing, the characteristics of the layer could be considered as common features for the majority of AM technologies. In general, these characteristics are:

- The layer thickness,
- Physics of scanning,
- Geometry parameters of the scanning strategy.

These latter are selected according to each technology and the precision required by the machine. For instance, the layer thickness in selective laser melting (SLM) varies from 20 to 50 µm [21–23], while in selective laser sintering (SLS), the thickness varies from 16 to 150 µm. In addition, other researchers have gone down to 1 mm for some aluminum alloys to qualify the effect of the powder layer thickness on the macroscale and mechanical properties of the produced parts [23].

Apart from the layer properties and scanning strategies, the input combinations varied depending on the physical phenomena to be controlled. For instance, in PBF-LB/M and PBF-LB/P processes, the main inputs generally correspond to laser properties and laser-matter interaction, while for the Electron Beam AM, the parameters to be mastered are related to the electron beam physics, EB-matter interaction, and wire characteristics [24]. For the material extrusion-based AM (MEx), the input vector mainly deals with the physics of extrusion, whether in the case of polymers or other extrudable materials, e.g., cement, concrete, etc.

The support and scanning strategies are also important parameters that have a great influence on the heat transfer from the built material to the printing plate, generating residual stresses or deflection of the layer or printed part. In the case of polymeric materials, the main role of the support structures is to avoid the collapse of the built parts [25]; while for metallic materials, in addition to the role of avoiding the layer’s collapse, the support structures strongly influence the final microstructure and the final mechanical properties through coupled metallurgical and thermomechanical transformations especially related to cooling rates of the printed materials [26–28].

The post-process analysis is generally concerned with the correlations between the process inputs and required process output:

- the mechanical properties:
  - Static resistance [29, 30]
  - Tolerance of the products in term of roughing and finishing needs [31–33]
  - Fatigue behavior [34–36]
  - Residual stresses and deformation rates [27, 28]
- Metallurgical properties and chemical resistance [37–40].

In practice, the effects of input parameters are coupled, and it would not be so easy to control each parameter separately. Thus, for AM processes, controlling the interactions between the input parameters is a key to controlling the overall “geometry-material-process” interactions. That is why, researchers have tried to understand the complexity of the inputs interactions using design of experiments and robust statistical methods. The Taguchi approach is widely used to reach this objective as a black box modeling process to summarize the manufacturing control [12, 14, 17–19].

A wide range of scanning strategies have been used and analyzed in the literature and are subjects to characterization from different perspectives, mechanical, physico-chemical, and metallurgical standpoints. The most strategies that are used are meander, spiral, chessboard, stripe, contour, checkerboard, etc. [33, 41–46]. Novel and more complex scanning strategies have also been developed recently like fractal scan strategies based on Hilbertand and Peano-Gosper curves. These were applied to SLM process to reduce the residual stresses and avoid cracks in the case of unweldable nickel superalloys [46].

In addition, scanning strategies should be considered as a vector of parameters that can be summarized in pattern style [13], orientation and placement strategies [16, 47], hatch distance, and hatch orientations [16, 47]. These latter are combined and must be meticulously selected to control the fabrication output that are:
– Density of the locally processed matter: it is directly affected by the processing energy for powder bed fusion or sintering, and thus it is inversely proportional to the hatch space distance as described by the energy density expressions [15, 22, 48, 49]; while for extrusion processes, the filling percentage is controlled by means of hatch space distance \( h \), higher is \( h \) lower is the density of the build material. This systematically reduces the mechanical properties and the mass of the products [50, 51].

– Residual stress generated during the printing: The literature categorizes the residual stresses into two categories: microscopic residual stresses and macroscopic residual stresses [15]. The microscopic residual stresses are measured using XRD analysis [13, 38, 52], while the macroscopic residual stresses are measured by means of parts deflections [15, 33, 53], or via microhardness measurement [40, 54]. Residual stresses interpretation is a complicated subject, since it involves several coupled physico-chemical transformations [55, 56]. For instance, for powder bed fusion processes, the complexity starts from local fusion/solidification cycles [57, 58] to reach intrinsic heat treatment caused by the laser or electron beams of the next shots. Same phenomena exist in multi-pass welding and the related physical models are actually expended to PBF-LB/M [53, 56, 59] and PBF-EB/M processes [59, 60]. In these cases, a particular attention must be given to the microstructural evolution that constitute the key of a deep interpretation and comprehension of residual stress generation phenomena [53, 61, 62];

– Heat effect of the neighbor areas during the processing: during the processing, the choice of the hatch space, the scanning pattern, and scanning speed highly affect the microstructures of the adjacent matter of the scan in the case of laser and electron beams. In situ repetitious heat treatments occur during the thermal cycles that lead to microstructural evolution of the processed matter [15, 53, 63]. A lot of issues appears in that case as, undercooling, crystal phases precipitations, carbides, nitrides, and oxides dissolutions [64], unstable phase change [56, 59, 61, 62], etc.;

– Mechanical anisotropy according to the scanning directions: for powder bed fusion, most references cited that the mechanical anisotropy comes from a solidification that is directed along the building directions [65], as it could be detected via EBSD analysis [29]. Same phenomenon occurs during casting and welding (multi-pass welding) depending on the solidification modes and locally cooling rates [55, 62]. The anisotropy majorly causes high or low gradients of properties depending on the built parts parameters and adopted post-heating treatment [30]. In addition to the metallurgical transformations, the mechanical anisotropy also deals with the hatch orientations and scan patterns where it was experimented that small patterns causes the reduction of the residual stress but increases the anisotropy of the build materials [46]. Hence, heat treatments are to be applied on the printer parts as post-process operations to relax the residual stresses and recrystallize the microstructure [63, 66]. For MEx AM processes like fused filament fabrication (FFF) or fused deposit modeling (FDM), the anisotropy is mainly caused by the differential directions of filament deposition [50]. Moreover, the shrinkage of the extruded filament can be seen as a kind of random phenomenon resulting in mechanical anisotropy [67].

From a technical perspective, the hatch distance must be greater than or equal to the resolution of the printer, but less than a critical value that should characterize some real constraints of the process, especially the final matter density, the re-melting and re-heating occurrence. This remark will be exploited in the modeling section of the paper.

In this paper, the authors present the mathematical design and formulation of a new scanning strategy. The proposed technique is based on the skeleton of 2D shapes, which should to be initially produced by the slicing procedure. This corresponds to a scanning perpendicularly to the skeleton of the shape edges. Therefore, the proposed scanning strategy will be referred as “Skeleton Based Perpendicular” (SBP) strategy.

Since the SBP scanning technique is proposed for the first time in the field of AM, the paper will analyze the case of a rectangle as a simple geometry to be analyzed. The main goal of the study is to enhance the productivity of a given AM process by minimizing the total displacement along the printed layer and thus minimizing the processing time and energy.

To present the proof of concept for this new technique, a benchmark analysis was conducted by comparing the SBP strategy to four existing AM patterns: chessboard, stripe, spiral and contour. Further works will focus on more complex geometries while others will be dedicated to mechanical properties of real printed parts by this new scanning pattern.

But, before starting the development, it is necessary to give a brief view on what the mathematical skeleton of a shape is.

### 1.1 Geometrical skeleton definition

The skeleton of a given shape construction represents a fieldwork of different researchers. It is based on several studies and applications, such as the reconstruction of forms and image processing. This geometrical feature finds its origin as a topological derivative of 2D and 3D mathematical object [68, 69]. The skeleton represents a physical or virtual object for the reconstitution of any form. It
is based on a body of research in biology and physics. The skeleton has many applications, especially in the field of recognition of shapes and imagery [70, 71]. In 2D (resp. 3D) analysis, the skeleton is composed of the set of centers of the maximal disks (resp. maximal balls) of a given shape frontier as it is shown in Fig. 1 [72]. For 2D shapes, the corresponding skeleton is considered as the continuous line of barycenters. Consequently, the skeleton is, by its topological definition, the set of the points closest to the boundary of a given shape [69]. It is worth mentioning that the first definition of the skeleton was proposed by Blum [73]; the skeleton was defined as the lines along which fire fronts meet when it started from the border of a given body [74]; the fire propagates along the vector normal to the shape border. Figure 2 shows the skeleton of a simple rectangle. Therefore, the skeletonization is defined as an application that relates a given shape to its skeleton.

For more details on the construction and use of the skeleton, the reader is referred to [69, 70, 75].

Given the above, this work was initiated by the idea that scanning of a sliced layer in AM processes, starting from and to the skeleton, should allow the reduction of the scanning trajectory, and thus increase the productivity of the printing process.

The rest of the paper will be organized as follows: the Sect. 2 will discuss the methodology of the work; the Sect. 3 will present the mathematical development in terms of the total scanning lengths of the SBP and benchmark strategies and the performance indices adopted; the Sect. 4 will present the construction of the optimization problem related to the pairwise comparison of the strategies in competition; the Sect. 5 will detail the simulation results, and the Sect. 6 will conclude the work and present the future developments.

2 Methodology

From a processing productivity perspective and in order to compare the SBP scanning strategy to the other strategies adopted, the following points was developed:

– Geometrical parametrization of the SBP and the other strategies patterns;
– The analytical formulation of the scanning lengths of the SBP and the benchmark strategies applied on a rectangular shape;
– The construction of two geometrical indicators that are:
  o The gain of length.
  o The specific gain of length (or specific length per surface unit).
– The development of a constrained optimization problem that allowed to express mathematically the focus of the proposed work;
– The discussion of the optimization problem in two separate cases of study that highlighted the SBP scanning advantages compared to the other techniques in term of:
  o Objective function behavior that corresponds to the total length of the SBP strategy;
  o Space decision feasibility of the optimization problem by means the indicators listed above.

All the programing, simulations, regressions fitting, and figures plotting were implemented in Matlab 2019.

Figure 3 presents the workflow proposed in the modeling and simulations of this study.

3 Modeling and comparison indicators

This section presents the summary of the analytical formulation of the total scanning length related to the SBP strategy and the other selected scanning strategies in the benchmark.
The details of the mathematical developments are presented in Appendix I and II. This section also introduces the comparison indicators that are used in the benchmark analysis.

### 3.1 Scanning strategies length modeling

#### 3.1.1 SBP strategy analysis

In the SBP scanning strategy, the skeleton is constructed for each sliced layer based on the layer contour. Subsequently, the trajectory scanning must be perpendicular to the skeleton of the layers. Figure 4 shows a typical SBP scanning pattern for a $L_1 \times L_2$ rectangle. The skeleton of the rectangle is represented by the set of blue discrete lines. The patterns for areas A and B are detailed in Figs. 5 and 7.

Since the angles of the rectangle are equal to 90°, the bisector slices the angle into 45° angles. Indeed, from Fig. 4a, an elementary geometrical identification allows us to write Eqs. (1.1) and (1.2) in order to quantify a proportional identification of the scanned regions A, B, and C of the rectangle.

The following Sects. 3.1.1.1–3.1.1.5 present the stepwise formulation of the scanning length $L_{\text{SBP}}$ of the SBP strategy, according to the parameterization detailed in the Figs. 4, 5, 6, and 7. The $L_{\text{SBP}}$ is progressively constructed by summing a productive length $L_p$ and jump length $L_j$ of each zone. A jump describes an active or inactive scan that corresponds to a displacement between two adjacent active scans that are perpendicular to the correspondent skeleton.
The detailed geometrical analysis and the algebraic developments are reported in Appendix I:

\[
\alpha_1 L_1 = L_2 / 2, \quad (1.1) \\
\alpha_2 L_1 = L_1 - L_2, \quad (1.2)
\]

### 3.1.1.1 Geometrical parametrization and calculus variables

Figure 6 shows the required characteristic points, vectors (\(\mathbf{v}\) and \(\mathbf{n}\)) and bisector (B) that are used in the analytical modeling of the scan distance series \(CH\). \(CH\) is considered as a characteristic distance of the problem since it describes the series of active scanning displacements in contrast to the jump displacements that are performed along the hatch space jumps. Once the distance \(CH\) expression is formulated, the calculation of the total productive scanning length \(L_p\) of the areas A and B of the rectangle is straightforward;
the total productive length corresponds to the sum of the series of $\overline{CH_i}$ as expressed by the Eq. (2):

$$L_{P_1,A \cup B} = \sum_{i \in \{\text{index set}\}} \overline{CH_i},$$  \hfill (2)

Figures 6 and 7 show the geometrical parameters that allow the calculation of the distances $\overline{CH_i}$ as follows:

- The parameters and notations utilized are:
  - $(x_i, y_i)$ and $(x_H, y_H)$: resp. are the coordinates of the points C and H;
  - $\vec{n}$: normal unit vector to the skeleton at any point H; it corresponds the unitary vector of the line (CH);
  - $\vec{v}$: carrier vector of the bisector (B) that is perpendicular to the vector $\vec{n}$;
  - The bisector at the upper-left vertex of the rectangle is denoted (B). Its equation is written as:

$$B : \ ax - y + b = 0,$$

According to the axis convention of Fig. 4, it is easy to argue that:

$$a = -1$$
$$b = \frac{L_2}{2}.$$  \hfill (4)

The modeling procedure of the distances $\overline{CH_i}$ begins with the decomposition of the first quadrant related to the left upper vertex as shown in the Fig. 7. Two horizontal axes are shown in this figure:

- the first (upper) horizontal axis describes the true abscissa $x$ (control abscissa);
- the second (lower) horizontal axis describes the index abscissa $k$.

Figure 7 shows the characteristic abscissa that will help in the formulation of the total scanning length of region A.

3.1.1.2 Productive and total jumps lengths of area (A) According to the results of Eq. (74) in the Appendix, the productive length of area (A) is expressed by the expression (5):

$$L_{P_1,A} = -a \sqrt{1 + a^2} \left[ \frac{\theta}{2}, k_{A_1}, (k_{A_c} + 1) + x_{H_A} \right].$$  \hfill (5)⇔(74)

where (Fig. 10): $k_{A_1}$ is the correction of the value of the limit of the index $k_A$ in the area (A); $x_{H_A}$ is the abscissa of the point H at the limit of the area (A);

$$k_{A_c} = E \left( \frac{\sqrt{2}aL_1}{(a^2 + 1) e} \right),$$  \hfill (6)

$$x_{H_A} = \frac{a_1L_1}{1 + a^2}.$$  \hfill (7)

Equations (5) and (6) correspond to Eqs. (72) and (73) in Appendix I.A, respectively.

According to the results of Eqs. (75) and (76) of the Appendix I.A, the total jumps length of the area (A) is expressed by the system of expressions (8.1) and (8.2).

- If $k_{A_c}$ is an impair:

Fig. 6 Positions and coordinates of the characteristic points {A, C, H}

Fig. 7 Decomposition of the first quadrant related to the left upper corner of the rectangle
at each corner of the rectangle; the area (C) is studied as a sole compartment.

Thus, the expression of the total productive length is given by:

\[ L_p = 8 \left( L_{P_1} + L_B \right) + L_{P_2}. \]

Since

\[ L_{P_2} = L_{P_1}. \]

Thus

\[ L_p = 16 L_{P_1} + L_{P_2}. \] (13)

Using the same approach, the total jump length is expressed by the Eq. (14):

\[ L_J = 16 L_{J_1} + L_{J_2}. \] (14)

Finally, the total scanning length of the SBP strategy \( L_{SBP} \) related to one layer of a rectangular layer is computed using the expression (15).

\[ L_{SBP} = L_p + L_J. \] (15)

The next paragraphs present the strategies involved in the benchmark study, their corresponding pattern, and total scanning length.

### 3.1.2 Chess strategy analysis

The analytical formulation of the chess strategy total length is detailed in the Appendix II.A. The related parameterization is depicted in Fig. 8.

The total scanning length related to the chess strategy \( L_{ch} \) is expressed by the Eq. (16):

\[ L_{ch} = N_{1c} L_{ch_1} + N_{2c} L_{ch_2}, \] (16)

where

\[ L_{ch_1} = a_x \left( E \left( \frac{n_{C_x}}{e} \right) + 1 \right) + a_y, \] (17.1)

\[ L_{ch_2} = a_y \left( E \left( \frac{n_{C_y}}{e} \right) + 1 \right) + a_x. \] (17.2)

\( L_{ch_1} \) and \( L_{ch_2} \) are, resp., the scanning length of patterns 1 and 2 of the chess strategy:
3.1.3 Stripe strategy analysis

The analytical formulation of the total length of the stripe strategy is detailed in the Appendix II.B. The parametrization of the stripe strategy is shown in Fig. 9.

The total scanning length of the stripe strategy $L_{\text{str}}$ is expressed by the Eq. (19):

$$L_{\text{str}} = N_1 L_{\text{str}1} + N_2 L_{\text{str}2},$$

where $L_{\text{str}1}$ and $L_{\text{str}2}$ are, resp., the scanning length of patterns 1 and 2 of the stripe strategy. Their expressions are detailed in the Appendix II.B; $N_1$ and $N_2$ are, resp., the number of repetitions of patterns 1 and 2. These latter are computed by the system (19).

3.1.4 Spiral strategy analysis

The analytical formulation of the spiral strategy total length is detailed in the Appendix II.C. The spiral strategy parametrization is presented in Fig. 10.

The total scanning length of the spiral strategy $L_{\text{spir}}$ is expressed by the Eq. (20):

$$L_{\text{spir}} = N_1 L_{\text{spir}1} + N_2 L_{\text{spir}2},$$

where $L_{\text{spir}1}$ and $L_{\text{spir}2}$ are, resp., the scanning length of patterns 1 and 2 of the spiral strategy. Their expressions are detailed in the Appendix II.C; $N_1$ and $N_2$ are, resp., the number of repetitions of patterns 1 and 2. These latter are computed by the system (19).
3.1.5 Contour strategy analysis

The analytical formulation of the contour strategy total length is detailed in Appendix II.D. The contour strategy parametrization is presented in Fig. 11.

The total scanning length of the contour strategy $L_{\text{cont}}$ is expressed by the Eq. (21):

$$L_{\text{cont}} = N_1L_{\text{cont}_1} + N_2L_{\text{cont}_2},$$

where $L_{\text{cont}_1}$ and $L_{\text{cont}_2}$ are, resp., the scanning length of patterns 1 and 2 of the contour strategy. Their expressions are detailed in the Appendix II.D; $N_1$ and $N_2$ are, resp., the number of repetitions of patterns 1 and 2. These latter are computed by the system (19).

3.2 Performance indicators

The pairwise comparisons between the SBP strategy and the benchmark strategies length were made by computing the “gain of length” (G) and the “specific gain of length per surface unit” (SG) as two major indicators of SBP competitiveness assessment. The indicator (G) was applied in the case of study n°1, while the indicator (SG) was applied in the case of study no 2.

3.2.1 Gain of length

The gains of length $G_i$ between the SBP and a given benchmark scanning strategy $i$ were computed using the Eq. (22):

$$G_i [\%] = \left( \frac{L_{\text{SBP}} - L_i}{L_i} - 1 \right) \times 100 .$$

where $L_i$ expresses the total length of a given strategy selected for the benchmark analysis.

Two different analysis of the gains $G_i$ are proposed:

- A pessimistic analysis of $G_i$ by the computing of the minimal gains $G_i^{\text{min}}$ that correspond to the maximization of the scanning length of the SBP strategy:
\[ G_i^{\text{max}}[\%] = \left( \frac{\min(n_{C_x}, n_{C_y})(L_{\text{SBP}})}{L_i} - 1 \right) \times 100. \quad (23) \]

An optimistic analysis of \( G_i \) by the computing of the maximal gains \( G_i^{\text{max}} \) that correspond to the minimization of the scanning length of the SBP strategy:

\[ G_i^{\text{min}}[\%] = \left( \frac{\max(n_{C_x}, n_{C_y})(L_{\text{SBP}})}{L_i} - 1 \right) \times 100. \quad (24) \]

For each hatch spaces distance \( e \), \( \max(n_{C_x}, n_{C_y})(L_{\text{SBP}}) \) and \( \min(n_{C_x}, n_{C_y})(L_{\text{SBP}}) \), resp. express the maximal and minimal lengths of \( L_{\text{SBP}} \) related to \( (n_{C_x}, n_{C_y}) \) parameters.

### 3.2.2 Specific gain per surface unit

The specific gains or the gains per surface unit \( SG_i \) were computed by considering a series of rectangles \((L_1 \times L_2)\) and a vector of hatch space distances \( e \).

The specific gains of lengths were computed using the following procedure:

- A set of \((L_1, L_2)_{L_i>0} \) rectangles with the corresponding areas \( A \) are generated as:

\[
\forall j, k \in \{1, \ldots, n_j\} \times \{1, \ldots, n_k\} \\
\begin{cases} 
L_1(j) = 100 + 10j \\
L_2(k) = 50 + 10k \quad (25.1) \\
\text{if } L_1(j) > L_2(k) : A(j, k) = L_1(j) \times L_2(k) \quad (25.2) \\
\text{else } A(j, k) = \text{NaN} \quad (25.3) \\
\end{cases}
\]

- \( n_j \) : is the number of decomposition of \( L_1 \) dimension

- \( n_k \) : is the number of decomposition of \( L_2 \) dimension

- The hatch space distance varies from 25 \( \mu \)m to 1 mm according to the bibliography reported in Table 6 of Appendix III.

- For each scanning strategy \( i \):

For each rectangle \((L_1, L_2)\), computing:

\[ SG_i[\% \text{mm}^{-1}] = \frac{G_i}{L_1 \times L_2}. \quad (26) \]

According to the definition (26), \( SG_i \) expresses the gain of length per surface unit.

Consequently, according to the expression (26), the SG index is a function of:

- The arguments of \( L_{\text{SBP}} \) function that are: \( n_{C_x}, n_{C_y}, e \);
- The arguments of \( L_i \) function that are: \( n_x, n_y, e \);
- The rectangle dimensions \( L_1 \) and \( L_2 \);

Hence, it is possible to express this dependency as proposed by the expression (27):

\[ SG = f(n_{C_x}, n_{C_y}, n_x, n_y, e, L_1, L_2). \quad (27) \]

From the Eqs. (22) to (26), one can argue that three possible scenarios could emerge from the simulation:

- \( SG_i < 0 \) : SBP strategy is more competitive than the compared strategy
- \( SG_i > 0 \) : the compared strategy is more competitive than the SBP
- \( SG_i = 0 \) : the two strategies are similar in total scanning length

Given the above (Eq. 27), a full visualization of the SG function is impossible to exhibit graphically since SG is a 7D function. Thus, the most important projections are presented in the results section, where we have adopted pessimistic and optimistic approaches to set the limits of the proposed SBP scanning strategy:

- The pessimistic approach considers minimal specific gains \( SG_i^{\text{min}} \) that are computed according to the maximal scanning lengths of SBP strategy (expression 28).

\[
SG_i^{\text{min}}[\% \text{mm}^{-1}] = \frac{1}{L_1 L_2} \left( \frac{L_{\text{SBP}}(n_{C_x}^{\text{max}}, n_{C_y}^{\text{max}}, n_x, n_y, e)(L_1, L_2)}{L_i} - 1 \right) \times 100. \quad (28)
\]

- The optimistic approach considers maximal specific gains that are computed according to the minimal scanning lengths of SBP strategy (expression 29):

\[
SG_i^{\text{max}}[\% \text{mm}^{-1}] = \frac{1}{L_1 L_2} \left( \frac{L_{\text{SBP}}(n_{C_x}^{\text{min}}, n_{C_y}^{\text{min}}, n_x, n_y, e)(L_1, L_2)}{L_i} - 1 \right) \times 100, \quad (29)
\]

where

\[
\left( n_{C_x}^{\text{min}}, n_{C_y}^{\text{min}} \right) = \arg \min (n_{C_x}, n_{C_y}) \left( L_{\text{SBP}}(n_x, n_y, e)(L_1, L_2) \right) \\
\left( n_{C_x}^{\text{max}}, n_{C_y}^{\text{max}} \right) = \arg \max (n_{C_x}, n_{C_y}) \left( L_{\text{SBP}}(n_x, n_y, e)(L_1, L_2) \right). \quad (30)
\]

The case of study 2 is devoted to the analysis of the specific gains \( SG_i \), by adopting the following setting:

- Variables to fix:

\((n_x, n_y)\): meaning that the decomposition policy of the benchmark strategies is fixed.
Variables to maintain (to vary):
\( (e, L_1, L_2) \).

Then, it is worth mentioning that the specific gains \( SG_i \) were discussed according to the 4D space \( (SG_i, L_1, L_2, e) \) or as a family of 3D spaces \( (SG_i, L_1, L_2) \) that are parametrized by the hatch space distance \( e \).

### 3.2.3 Gain of length as gain of time, energy, and transferred mass

For a pairwise comparison and considering the initial formula of the gain, the nominator and denominator were divided by the scanning speed \( V \):

\[
G_i \% = \left( \frac{L_{SBP} - L_i}{\frac{L_i}{V}} \right) \times 100 = \left( \frac{T_{SBP} - T_i}{T_i} \right) \times 100, 
\]

where
\[
T_{SBP} : \text{Production time related to SBP scanning strategy} \\
T_i : \text{Production time related to } i \text{ scanning strategy}
\]

Now, the nominator and denominator of Eq. (32) are multiplied by the power of scanning:

\[
G_i \% = \left( \frac{T_{SBP} - T_i}{T_i} \right) \times P \times 100 = \left( \frac{E_{SBP} - E_i}{E_i} \right) \times 100. 
\]

Thus,
\[
G_i[\%] = \left( \frac{E_{SBP}}{E_i} - 1 \right) \times 100, 
\]

where
\[
E_{SBP} : \text{energy consumption related to SBP scanning strategy} \\
E_i : \text{energy consumption related to } i \text{ scanning strategy}
\]

For the MEx AM processes, if the nominator and denominator of Eq. (32) are multiplied by the transferred mass flux \( M \), the gain will also be equivalent to the extruded gain of mass; Eqs. (35) and (36) expresses this equivalence:

\[
G_i \% = \left( \frac{T_{SBP} - T_i}{T_i} \times M}{T_i \times M} \right) \times 100 = \left( \frac{M_{SBP} - M_i}{M_i} \right) \times 100. 
\]

Thus,
\[
G_i[\%] = \left( \frac{M_{SBP}}{M_i} - 1 \right) \times 100, 
\]

where
\[
M_{SBP} : \text{Quantity of extruded mass related to SBP scanning strategy} \\
M_i : \text{Quantity of extruded mass related } i \text{ scanning strategy}
\]

Thus, it can be seen that the modified formula of the gain of length \( G_i \) is equivalent to the gain in production time, the gain in processing energy and mass to be transferred.

### 4 Optimization problem formulation

#### 4.1 Objective function

This section presents the mathematical formulation of the optimization problem (OP) related to the minimization of the total production length of the rectangular layer to be printed using the proposed SBP scanning strategy. In summary, the independent decision variables to be included in the analysis are:

- \( e \): expresses the hatch space distance;
- \( (n_x, n_y) \): expresses the number of divisions resp. according to \( x \) and \( y \) directions of the rectangle related to the selected benchmark strategies;
- \( (n_{Cx}, n_{Cy}) \): expresses resp. the divisions of the area \( C \) related to the SBP technique;

The function to minimize is to the total scanning length of the SBP strategy. It is then expressed as:

\[
f(e, n_{Cx}, n_{Cy}) = L_{SBP}(e, n_{Cx}, n_{Cy}).
\]

#### 4.2 Optimization problem definition

##### 4.2.1 Feasible space definition in term of scanning lengths

In order to ensure the competitiveness of the SBP scanning strategy, \( L_{SBP} \) must be smaller than the total scanning length of a given benchmark strategy “\( i \)” in competition. This condition is expressed by the inequality (38):

\[
L_{SBP} \leq L_i.
\]

where \( i \) is the index of a benchmark strategy in competition;

This inequality leads to the definition of the corresponding constraint function \( g_i \) as follows:

\[
g_i = L_{SBP} - L_i \leq 0,
\]

By adopting the modeling variables detailed in the previous paragraph, the inequality (39) can be written as:

\[
g_i(e, n_{Cx}, n_{Cy}) \leq 0.
\]
In consequence, the feasible space can be formulated by the expression (41):

\[
\left( e^*_s, n^*_{C_x}, n^*_{C_y} \right) = \arg\left( g_i(n_{x,i}, n_{y,i}) (e_s, n_{C_x}, n_{C_y}) \leq 0 \right) .
\] (41)

### 4.2.2 Correspondence between the feasible space, the gain of lengths \(G_i\) and the specific gain \(SG_i\)

Starting from the inequality (39), it can be argued that, for each benchmark strategy \(i\), the feasible space can be expressed in term of gain of length \(G_i\) or specific gain of length \(SG_i\), where:

\[
G_i = \frac{g_i}{L_i}, \quad SG_i = \frac{G_i}{L_1 L_2} = \frac{g_i}{L_1 L_2} .
\] (42)

Hence, the feasible space corresponds to the negative regions of \(g_i\), \(G_i\), or \(SG_i\) alike as described by Eqs. (43) and (44).

\[
\left( e^*_s, n^*_{C_x}, n^*_{C_y} \right) = \arg\left( g_i(n_{x,i}, n_{y,i}) (e_s, n_{C_x}, n_{C_y}) \leq 0 \right) \Leftrightarrow \left( e^*_s, n^*_{C_x}, n^*_{C_y} \right) = \arg\left( G_i(n_{x,i}, n_{y,i}) (e_s, n_{C_x}, n_{C_y}) \leq 0 \right) ,
\] (43)

\[
\Leftrightarrow \left( e^*_s, n^*_{C_x}, n^*_{C_y} \right) = \arg\left( SG_i(n_{x,i}, n_{y,i}) (e_s, n_{C_x}, n_{C_y}) \leq 0 \right) .
\] (44)

### 4.2.3 Hatch space distance constraining

In this study, the hatch space distance is considered as a geometrical feature but it is mandatory to recall the importance of this parameter from a physical perspective. For instance, in PBF-LB/M, two major effects could occur during and after the laser shot [53]:

- Remelted zones for lower values of hatch space distance. The remelted zone is characterized by its width \(D_{MZ}\);
- Variation in the width of the Heat Affected Zone (HAZ) according to the hatch space distance and thus, variation in the material’s metallurgy, microstructure, and density.

The modeling of the heat affected zone elaborated by Andreyavna et al. [76] led to the proposal of a preliminary constraining approach related to the maximal hatch distance \(e_c\):

- \(e_c > HAZ\): two parallel and adjacent laser spots will not have interdependent effects since the hatch distance \(e_c\) distance is important;
- \(D_{MZ} \leq e_c \leq HAZ\): for two parallel and adjacent laser spots, the HAZ and the melted zone that are realized by the first will be heat-treated by the thermal effect of the second laser beam which is called intrinsic heat treatment (IHT) [13];
- \(0 < e_c \leq D_{MZ}\): the melted zone will be remelted partially by the second laser scanning.

In general, the choice of the parameter \(e_c\) affects the homogeneity of the material, its density, its microstructure and consequently the mechanical and physical properties. In the literature, the choice of the hatch distance is usually considered as a process parameter that belongs to hatch style characteristics [47].

In the material extrusion-based AM, the hatch space distance mainly affects the infill density of the part and consequently the resulting macroscale characteristics of the produced parts.

From a technical standpoint, the hatch distance of both SBP and the benchmark strategies must be greater than the printer resolution \(e_{res}\) and less than the maximal limit value \(e_c\) as expressed by the inequality (45):

\[
e_{res} \leq e \leq e_c. \] (45)

The selection of \(e_c\) can be done according to the three remarks above.

### 4.3 Optimization problem formulation

Given the above, the optimization problem (OP) is proposed as follows:

- Considering a given rectangle \((L_1, L_2)\);
- Considering \((n_{x,i}, n_{y,i})\) divisions for the benchmark scanning strategy \(i\).
Table 1 Geometrical and scanning parameters used in the case of study 1

| Parameters                              | Values                                      |
|-----------------------------------------|---------------------------------------------|
| Geometry of the rectangle               | \( L_1 = 100 \text{ mm} \) \( L_2 = 50 \text{ mm} \) |
| Benchmark strategies decomposition      | \( n_x = 100 \) \( n_y = 50 \)             |
| Limit conditions for SBP strategy\( a \) | \( \min(n_{Cx}) = 1 \) \( \max(n_{Cx}) = 30 \) \( \min(n_{Cy}) = 1 \) \( \max(n_{Cy}) = 30 \) |
| Hatch limit conditions\( b \)          | \( \min(e) = e_{res} = 25 \mu \text{m} \) \( \max(e) = e_c = 1 \text{mm} \) |

\( a \)Setting \((n_{Cx}, n_{Cy})\) to (1.1) corresponds to the meander scanning strategy of the area \( (C) \)

\( b \)The values of \( e_{res} \) and \( e_c \) were selected according to the bibliography (see Table 6 of Appendix III)

5.1 Objective function evaluation

The objective function \( L_{SBP} \) corresponds to the expression (48.1) of the (OP). Figure 12 displays the objective function as a family of surfaces \( L_{SBP}(n_{Cx}, n_{Cy}) \) that are parametrized by the hatch space \( e \). The plotting space is the 3D space \((L_{SBP}, n_{Cx}, n_{Cy})\). Moreover, least square estimation of the first order variations of \( L_{SBP} \) were computed to approach its first order behavior.

The following notation are adopted:

\( \sim L_{SBP}(n_{Cx}, n_{Cy}) \): is the linear regressive approximation of \( L_{SBP} \) according to \((n_{Cx}, n_{Cy})\) at each value of hatch space \( e \);

\( b_0(e) = L_{SBP}(0,0) \) is the estimated intercept;

\( b_1(e) \) is the estimated slope of \( L_{SBP} \) according to \( n_{Cx} \);

\( b_2(e) \) is the estimated slope of \( L_{SBP} \) according to \( n_{Cy} \);

\( b_0, b_1, \) and \( b_2 \) were modeled by hyperbolic functions of \( e \) as detailed below. In consequence, Fig. 12a to e display the variations of the \( \sim L_{SBP} \), intercept \( b_0 \), and the first order slopes \((b_1, b_2)\) according to the hatch distance \( e \).

From Fig. 13a–e, one can observe that:

1. The parameters \((b_0, b_1, b_2)\) are positive, especially \( b_1 \) and \( b_2 \) which means that the scanning length \( L_{SBP} \) increases according to \( n_{Cx} \) and \( n_{Cy} \);

2. The evolution of the parameters \((b_0, b_1, b_2)\) strongly depends on the hatch spacing \( e \) and are hyperbolically decreasing according to it. Hence, \((b_0, b_1, b_2)\) were fitted according to the hatch space \( e \) using an hyperbola models (Fig. 12c–e); the fitting coefficient of determination \( R^2 \) is higher than 98\% , and the p-values are less than \( 10^{-6} \) (less than 5\%) which means that the fitting is highly significant;

5 Results and discussion

5.1 Case of study 1

The parameters that are adopted for the case of study 1 are described in Table 1.
Fig. 12 Total scanning length of the SBP strategy $L_{SBP}(n_{Cx}, n_{Cy})$

Fig. 13 Variation of the $L_{SBP}$ surfaces parameters. a Superposition of the surfaces family $L_{SBP}$. b 3D-plot of $(b_0, b_1, b_2)$. c Evolution of the intercepts $b_0$ according to hatch space $e$. d Evolution of first order variation $b_1$ according to hatch space $e$. e Evolution of first order variation $b_2$ according to hatch space $e$. 
Table 2  Approached variations signs of $\hat{L}_{SBP}$ surfaces

| Variations | Sign   | Behavior |
|------------|--------|----------|
| $\frac{\partial \hat{L}_{SBP}}{\partial e}$ | $-$     | Hyperbolic |
| $b_0 = \hat{L}_{SBP} (0,0)$      | $+$     | Hyperbolic |
| $b_1 (e) = \frac{\partial \hat{L}_{SBP}}{\partial n_{Cx}} (e)$ | $-$     |
| $b_2 (e) = \frac{\partial \hat{L}_{SBP}}{\partial n_{Cy}} (e)$ | $+$     |

3. The full linear behavior between the parameters $(b_0, b_1, b_2)$ of the (Fig. 12b) and the modeling computation produced the following statistics: $R^2_{adj} = 1$, $p_{value} = 0$.

It is therefore possible to summarize the evolution of the scanning length $L_{SBP}$ as follows:

- $L_{SBP}$ increases according to the decomposition parameters $(n_{Cx}, n_{Cy})$ of the C area. Different statistical fittings were tested to propose a regressive function $L_{SBP}$, but the fitting statistics, especially the R squared ($R^2$) statistics, were estimated as worst ($R^2 < 60\%$);
- Hyperbolic decrease of the $L_{SBP}$ according to the hatch distance $e$ (Fig. 13);
- $L_{SBP} (n_{Cx}, n_{Cy})$ constitute a family of surfaces that are parametrized by the hatch space distance;

Table 2 summarizes the effect of the decision variables $(n_{Cx}, n_{Cy}, e)$ on the scanning length $L_{SBP}$.

Hence, it is worth mentioning that the SBP total scanning length minimization requires the minimization of $(n_{Cx}, n_{Cy})$ and the maximization of the hatch space $e$. On the one hand, the minimization of $(n_{Cx}, n_{Cy})$ to the lower limit value of $(1,1)$ leads to a meander policy of the area C of the rectangle. Nevertheless, a meander scanning strategy could cause residual stress or excessive deflection of the printed layers due to the important course along the scanning axis. On the other hand, the maximization of the hatch space distance $e$ directly leads to the reduction of the matter density which affects the mechanical properties of the final products. Therefore, more generally, it will be necessary to couple the mathematical analysis with the physical characters in a deeper step of SBP process competitiveness assessment. This last remark will be discussed in detail in the conclusion and perspectives section.

5.1.2 Benchmark scanning strategies lengths

It is also interesting to simulate the total scanning lengths of the benchmark strategies before proceeding with the SBP pairwise comparisons. Figure 14 shows the scanning strategies lengths according to the hatch space distance $e$.

The hyperbolic decreasing behaviors are very salient. Furthermore, it is clear that the contour strategy presents the lowest scanning lengths, followed by chess and spiral strategies which are quasi-superposed. Finally, the stripe strategy presents the higher scanning lengths, which expresses its low productivity competitiveness compared to the other benchmark strategies.

Hyperbolic fittings of the Fig. 14 curves are presented in the Appendix IV.A.

Fig. 14  Total scanning lengths $L_i$ according to the hatch space distance $e$
Fig. 15  $g_i$ functions representation according to $(n_{Cx}, n_{Cy}, \epsilon)$: a SBP VS Chess, b SBP VS Stripe, c SBP VS Spiral, d SBP VS Contour
Fig. 15 (continued)
5.1.3 Feasible space evaluation and SBP strategy performance

5.1.3.1 Feasible space evaluation: \( g_1 \) constraint functions
The feasible space is computed according to the systems (47) and (48); it is computed for each benchmark strategy in competition with the SBP technique. Figure 13a–d presents the plot of the \( g_1 \) surfaces at different hatch space values \( e \) and area C decompositions \((n_{C_x}, n_{C_y})\).

As discussed above, the feasible space related to \( g_1 \) constraint function corresponds to the negative region of the \((n_{C_x}, n_{C_y}, g_1)\) spaces. The horizontal plans \( g_1 = 0 \) are depicted to easily distinguish the feasible and unfeasible regions of the \( g_1 \) constraints.

For all combinations of the triplet \((n_{C_x}, n_{C_y}, e)\), the computation of the constraint function \( g_1 \) led to the following conclusions:

- For the \( g_1 \) constraints functions related to chess (Fig. 15a), stripe (Fig. 15b), and spiral (Fig. 15c) scanning, the feasible domains are wide and scan the majority of space \((n_{C_x}, n_{C_y}, e)\):
  
  - For SBP/chess comparison: 91.80% of the decision space is feasible;
  - For SBP/stripe comparison: 87.08% of the decision space is feasible;
  - For SBP/spiral comparison: 89.96% of the decision space is feasible;

- For the \( g_1 \) constraints functions related to the contour scanning strategy (Fig. 15d), the feasible spaces are defined as the hatch space distances exceeds 0.325 mm: for hatch spaces lower than 0.325 mm, the contour strategies are more competitive than SBP. Conversely, the SBP strategy is more competitive and the feasibility reaches 70%.

### Table 3 Feasibility in terms of gains \( G_{\text{min}}^i G_{\text{max}}^i \)

|Gain Strategies involved| Feasible space \( e \) (mm) | SBP competitiveness (%) |
|--------------------------|-----------------------------|-------------------------|
|\( G_{\text{min}}^1 \) | SBP/Chess | All the domain | 100 |
|\( G_{\text{max}}^1 \) | | | 100 |
|\( G_{\text{min}}^2 \) | SBP/Stripe | \([0.025,1]\) \(\backslash\) \([0.675,0.785]\) | 86.73 |
|\( G_{\text{max}}^2 \) | All the domain | | 100 |
|\( G_{\text{min}}^3 \) | SBP/Spiral | \([0.025,1]\) \(\cup\) \([0.685,0.875]\) | 83.67 |
|\( G_{\text{max}}^3 \) | All the domain | | 100 |
|\( G_{\text{min}}^4 \) | SBP/Contour | \( e > 0.325 \) | 70.41 |
|\( G_{\text{max}}^4 \) | | \( e > 0.245 \) | 74.50 |

- For SBP/stripe comparison: 87.08% of the decision space is feasible;
- For SBP/spiral comparison: 89.96% of the decision space is feasible;

Fig. 16 \( G_{\text{min}}^i \) and \( G_{\text{max}}^i \) gains representations according to the hatch space distance \( e \): a SBP VS chess, b SBP VS stripe, c SBP VS spiral, d SBP VS stripe
5.1.3.2 SBP gain analysis  Figure 16 displays the minimal and maximal gains, resp. $G_i^{\text{min}}$ and $G_i^{\text{max}}$, as a function of hatch space $e$. The feasible or “competitive” space corresponds to the negative spaces $\{\{G_i^{\text{min}}(e) < 0\}\}$ and $\{\{G_i^{\text{max}}(e) < 0\}\}$ as denoted in Fig. 16.

According to Fig. 16a–d, Table 3 groups some conclusive remarks on the behavior of the gains $G_i^{\text{min}}$ and $G_i^{\text{max}}$ in term of the competitive and uncompetitive zones related to the hatch space distance $e$, resp. feasible and non-feasible spaces zones.

It can be deduced from Table 3 that the SBP scanning strategy is competitive with the benchmark scanning strategies that have been adopted, particularly for chess, stripe, and spiral strategies where more than 80% of the decision space is feasible. For SBP/Contour comparison, SBP strategy is more competitive for hatch space distances higher than 0.325 mm for $G_i^{\text{min}}$ and higher than 0.245 mm for $G_i^{\text{max}}$.

According to the detected feasible zones, it should be noted that the $G_i$ gains present different tendencies according to the hatch space values:

- The dispersion of the absolute values of $G_1$ increases according to hatch space;
- The dispersion of the absolute values of $G_2$ decreases according to hatch space;
- The dispersion of the absolute values of $G_3^{\text{min}}$ presents a kind of parabola that the maximum 9% is reached around 0.4 mm of hatch space. It is also remarkable that $G_3^{\text{min}}$ is stable at this level between 0.25 and 0.5 mm of hatch space. Hence, the following global behaviors of $G_3^{\text{min}}$ can be listed as:
  - $G_3^{\text{min}}$ gain increases for hatch space distances less than 0.4 mm
  - $G_3^{\text{min}}$ gain decreases for hatch space distances higher than 0.4 mm

- Regarding $G_3^{\text{max}}$ gain, it increases hyperbolically according to the hatch space to reach a maximum of 38% at $e = 1$ mm
- The dispersion of the absolute values of $G_4$ increases with hyperbolic behavior according to the hatch space distance;

In addition to the global tendencies described in the above list, local stepwise hyperbolic decreasing behaviors were observed. Indeed, for the chessboard, stripe, and spiral strategies, several distinct appearances emerge while for the contour strategy, only two or three distinct stepwise hyperbolic intervals can emerge from a preliminary observation. It is worth mentioning that this stepwise evolution is caused by the integer parts of several terms.

| Table 4 Absolute values of gain $G_i^e$ and the correspondent hatch space distance $e$ |
|---------------------------------|----------|-----------------|-------------|
| Max ($|G_i|$) $e_{\text{max}}$ [mm]$^a$ | Min ($|G_i|$) $e_{\text{min}}$ [mm]$^b$ |
| $G_1^{\text{min}}$ | 18.27% | 1 | 0.78% | 0.025 |
| $G_1^{\text{max}}$ | 45.33% | 1 | 2.12% | 0.025 |
| $G_2^{\text{min}}$ | 49.15% | 0.025 | 3.15% | 0.805 |
| $G_2^{\text{max}}$ | 49.40% | [0.025, 0.125] | 8.62% | 0.685 |
| $G_3^{\text{min}}$ | 8.096% | 0.385 | 0.55% | 0.025 |
| $G_3^{\text{max}}$ | 37.57% | 1 | 0.78% | 0.025 |
| $G_4^{\text{min}}$ | 27.09% | 0.885 | 1.53% | 0.335 |
| $G_4^{\text{max}}$ | 5.38% | 1 | 2.22% | 0.255 |

$^a$ $e_{\text{max}} = \arg \max |G_i^e|$ or $e_{\text{max}} = \arg \max G_i^e$

$^b$ $e_{\text{min}} = \arg \min |G_i^e|$ or $e_{\text{min}} = \arg \min G_i^e$

that are present in the expressions of all strategies length that have been modeled and studied in this work.

5.1.4 Conclusive remarks of case of study 1

According to the results of the case of study 1, it can be stated that the SBP scanning strategy is competitive regarding the benchmark scanning strategies, especially for chess, stripe, and spiral strategies. From the feasibility point of view, the competitiveness reaches more than 85% of the total decision space $(n_{C_x}, n_{C_y}, e)$. For the contour scanning strategy, more than 65% of the decision space $(n_{C_x}, n_{C_y}, e)$ is feasible.

Table 4 summarizes the details of the variation range of $G_i^{\text{min}}$ and $G_i^{\text{max}}$ for the stepwise comparisons that were performed.

From the expression (39), the system (42), and the objectives drawn by the (OP) at Sect. 4.3, it is clear that the of the objective function $L_{SBP}$ behave like the constraint function $g_i$ and the gains $G_i$. That is to say that the minimization of $g_i$, or $G_i$ leads to the minimization of $L_{SBP}$. Hence, the discussion developed in the paragraph 5.1.3.2 and the Table 4 can be extended to $L_{SBP}$. Taking into account the absolute values $G_i$, the maximal gains correspond to the minimal values of $L_{SBP}$. A particular attention will be given to this point in the conclusion section of the article since the minimization of $L_{SBP}$ leads to the maximization of the hatch space, especially in the case of chessboard, spiral and contour strategies.

5.2 Case of study 2

In the case of study 2, the specific gains $SG_i$ are computed according to Sect. 3.2.2. The pessimistic and optimistic approaches led to the comparison of the higher and the
lower values of lengths $L_{SBP}$ with the total lengths of the benchmark strategies, respectively. In the following sections, the feasibility (competitiveness) of the decision space $(L_1, L_2, e)$ will be discussed according to the inequality (44) of Sect. 4.2.2. Moreover, for each benchmark scanning strategy, the feasibility of the decision space was also computed and discussed according to the hatch space distance.

5.2.1 Objective function evaluation

It has been demonstrated in the modeling section that $L_{SBP}$ is a function of five variables $(L_1, L_2, n_C, n_C', e)$; i.e. neither 2D nor 3D visualization are able to give a reliable and correct evaluation to it. Thus, partial 3D visualizations are presented in this paragraph in order to appreciate some characters of the $L_{SBP}$ surfaces.

At first sight, some remarks can emerge from Fig. 17:

- high rate decrease of $L_{SBP}$ according to the hatch space distance $e$;
- linear increase of $L_{SBP}$ according to $(L_1, L_2)$;
- higher variation rate of $L_{SBP}$ according to $L_1$ than the variation rate according to $L_2$;

In order to figure out the effect of the decision variables $(L_1, L_2, n_C, n_C', e)$, Fig. 18 is presented to show the evolution of the estimated first order variations (slopes) of $L_{SBP}$ according to the hatch space distance.

First, it is shown that all the slopes are positive, which means that the $L_{SBP}$ objective increases according to all variables $(L_1, L_2, n_C, n_C')$. Second, hyperbolic decreases in all the slopes are clearly seen with very high variation coefficients $R^2$.

The adopted hyperbolic models are detailed in Appendix IV.B.

Hence, Table 5 summarizes the effect of the decision variables $(L_1, L_2, n_C, n_C')$ on the scanning length $L_{SBP}$ for the case of study 2.

Remark The overall behavior of $L_{SBP}$ was statistically modeled and found to be full quadratic according to the $(L_1, L_2, n_C, n_C')$ decision variables with $R^2$ statistics exceeding 75%. For more details, the reader may refer to Appendix IV.C.

5.2.2 Specific gain, competitiveness, and feasibility of SBP strategy

Figure 16a–d show the specific gains $SG_{min}$ families related to the chess, stripe, spiral, and contour scanning strategies.
It can be seen that SBP strategy is fully competitive with chess and stripe strategies (see Fig. 19a and b, respectively) since $SG_{\text{min}}$ is negative over the whole decision space. Nevertheless, for spiral (Fig. 19c) and contour (Fig. 19d) strategies, the competitiveness, and consequently the feasibility of the decision space, depends strongly on the values of the decision variables $s_1$. Indeed, the feasibility percentage was computed by dividing the number of feasible points by the total number of the decision space combinations. The maximal specific gains $SG_{\text{max}}$ surfaces are displayed in Fig. 25 of Appendix V.

More generally, Fig. 20 exhibits the distribution of the percentage of feasibility according to the specific gains according to the hatch space distance.

### 5.2.3 Main remarks on case of study 2

Figure 20 clearly shows that the SBP strategy presents high competitiveness compared to the chess and stripe strategies, since 100% of the points are feasible for $SG_{\text{max}}$, while in the case of $SG_{\text{min}}$, the percentage of decision space feasibilities is bounded between 88 and 100% for the chess strategy, and between 95 and 100% for the stripe strategy. Figure 21 shows a remarkable high density of $SG_{\text{min}}$ feasibility dispersion around 100%.

For spiral and contour strategies, the feasibility depends heavily on:

- Rectangle dimensions $L_1$ and $L_2$ (Fig. 19c and d);
- Hatch space distance $e$ (Fig. 20);
- the parameters of the area C decomposition of the SBP strategy $(n_{c_x}, n_{c_y})$ since the minimal and maximal specific gains $SG_{\text{min}}$ and $SG_{\text{max}}$ were computed at different value $(n_{c_x}, n_{c_y})$, resp. (20,20) and (1,1);

For the spiral-SBP (Fig. 20c), the feasibility behavior shows typical power laws according to the hatch space $e$ as
Fig. 19 Specific gains $\text{SG}_{\text{min}}$ according to $(L_1, L_2, \epsilon)$: a SBP VS Chess b SBP VS Stripe c SBP VS Spiral d SBP VS Contour. $\text{SG}_{\text{min}}$ are computed for the higher values of $L_{SBP}$ at $\left(\max(n_C^x), \max(n_C^y)\right)$. 

$\epsilon = 0.025\text{mm} ; \%\text{feasibility} = 88.2143\%$ 

$\epsilon = 0.125\text{mm} ; \%\text{feasibility} = 99.2857\%$ 

$\epsilon = 0.225\text{mm} ; \%\text{feasibility} = 100\%$ 

$\epsilon = 0.325\text{mm} ; \%\text{feasibility} = 100\%$ 

$\epsilon = 0.425\text{mm} ; \%\text{feasibility} = 100\%$ 

$\epsilon = 0.525\text{mm} ; \%\text{feasibility} = 99.2857\%$ 

$\epsilon = 0.625\text{mm} ; \%\text{feasibility} = 100\%$ 

$\epsilon = 0.725\text{mm} ; \%\text{feasibility} = 97.8571\%$ 

$\epsilon = 0.825\text{mm} ; \%\text{feasibility} = 96.5714\%$
Fig. 19 (continued)
Fig. 20 Percentage of \((L_1, L_2, e)\) decision space feasibility

Fig. 21 Histograms of the \(SG_{min}\) related to a chess–skeleton couple b stripe–skeleton couple
demonstrated by the Eqs. 49 and 50 with values of \( R^2 \) statistics higher than 95%:

\[
SG^{\text{spir}}_{\text{min}}(e) = ae^n
\]

\[
SG^{\text{spir}}_{\text{max}}(e) = ae^n
\]

\[
R^2 = 95.36\% \\
P_{\text{value}} = 7.91 \times 10^{-66}
\]

\[
SG^{\text{spir}}_{\text{max}}(e) = ae^n
\]

\[
R^2 = 98.70\% \\
P_{\text{value}} = 3.78 \times 10^{-93}
\]

While for the contour strategy, the feasibility (the competitiveness) can be described by a polynomial or a power law function according to the hatch space distance \( e \).

Finally, it is worth mentioning that the minimization of the objective function according to the proposed constraints depends on the combinations of the decision variables \( L_1, L_2, n_{C_1}, n_{C_2}, e \). In one hand, the feasibility of the decision space was discussed for the extreme values \( (n_{C_1}, n_{C_2}) \) based on the results of Figs. 19 and 25. In other hand, the global feasibility is analyzed related to the hatch space distance (Fig. 20). In addition, the objective function \( L_{\text{SBP}} \) showed a quadratic increase according to the hatch space distance, while it decreases hyperbolically according to the hatch space as it is highlighted in the previous section.

### 6 Conclusion and perspectives

In this paper, a new AM scanning strategy is proposed and it is compared with a set of conventional strategies, which are used in different AM processes, from the point of view of productivity. The new scanning design can be applied to PBF-LB/M, PBF-LB/P, PBF-EB/M, and MEx processes. The SBP strategy is based on scanning perpendicular to the skeleton of a given flat geometry. The modeling was formulated in the case of a rectangular shape \( L_1, L_2 \) to present a first proof of concept. Then, the productivity was evaluated by computing the total production length for the SBP strategy and for the other benchmark scanning strategies. The development of the production length formulation led to the design of a constrained optimization problem which is the minimization of the total SBP scanning length using an optimal choice of the decision space variables \( (L_1, L_2, n_{C_1}, n_{C_2}, e) \). The feasibility of the proposed (OP) was assessed by a set of constraint functions that were demonstrated to be equivalent to the gain of length \( G[\%] \) and to the specific length gain per surface unit \( SG[\%] \).

For the pairwise comparisons “SBP VS Benchmark strategy \( i^* \)”, it was shown that the lengths of all scanning strategies behave like a hyperbole according to the hatch space distance.

The analysis of the gains of length \( G_i[\%] \) for the case of study 1 showed that the SBP scanning is highly competitive compared to chess, stripe, and spiral scanning strategies, reaching more than 83% of competitiveness or feasibility of the decision space \( (n_{C_1}, n_{C_2}, e) \). Otherwise, in the case of the SBP/contour comparison, the SBP strategy is considered to be competitive for hatch space values exceeding 0.30 mm, reaching 70–74% of feasibility of the decision space. The effect of \( (n_{C_1}, n_{C_2}) \) has been discussed in detail in the paper, but the effect of the hatch space distance remained sharper.

In addition \( L_{\text{SBP}} \) increases according to the decomposition strategy \( (n_{C_1}, n_{C_2}) \) of the area \( (C) \) with variation rates \( \frac{\partial L_{\text{SBP}}}{\partial n_{C_1}} \) and \( \frac{\partial L_{\text{SBP}}}{\partial n_{C_2}} \) that are hyperbolically decreasing according to the hatch space distance.

The case of study 2 investigated the effects of the variation of the rectangular shape dimensions \( L_1, L_2 \) on the competitiveness of the proposed SBP strategy. Indeed, it was proved that the competitiveness of the new scanning strategy also depends on the rectangle dimensions; i.e. that the feasibility of the (OP) is affected mainly by the dimensions of the shape to be scanned. The \( L_{\text{SBP}} \) has been modeled using a quadratic model according to the decision variables which is detailed in the Appendix IV.C. It was also highlighted that the parameters of the models, specifically the first-order variations (slopes), are positive and are hyperbolic-decreasing according to the hatch space distance, which also means that the \( L_{\text{SBP}} \) length is hyperbolic-decreasing according to the hatch space distance. From the point of view of productivity competitiveness, the SBP strategy was shown to be valued 100% higher than chess and stripe scanning; and the comparison of SBP to spiral and contour patterns was discussed in terms of the decision space variable combinations. A global SBP competitiveness was assessed and the (OP) feasibility was modeled according to the hatch space distance. This latter varies from 20 to 98% for spiral scanning and from 0 to 75% for the contour scanning depending on all the decision variables combinations \( (L_1, L_2, n_{C_1}, n_{C_2}, e) \).

Finally, it is important to recall that this paper presented two major contributions to the analysis of AM scanning strategies:

- The proposal of a new pattern, SBP scanning strategy, which could improve the productivity of classes of AM...
processes, such as PBF-LB/M, PBF-LB/P, PBF-EB/M, and MEx based AM;

- A new mathematical framework or template for pair-wise comparisons of AM scanning strategies using new indices that are:

  - the gain of length: which also expresses the gain in AM production time, energy, or extruded matter;
  - the specific gain or the gain of length per surface unit: which also expresses the gain in AM production time, energy, or extruded matter per surface unit;

Further work will focus on the characterization of the mechanical and metallurgical properties of rectangular parts printed by SBP compared to other benchmark scanning strategies, while other works will be dedicated to the application of the SBP scanning to more complex geometries.

Appendix I: SBP scanning modeling

A. Study of the area (A)

a. Bisectors parametrization

According to Spain 1963 [77], the coordinates of the point \( H \) which is the projection of the point \( C \) onto the bisector \( (B) \) can be calculated from the point \( C \) coordinates and the carrier vector \( \vec{v} \) of the line \( (B) \), as shown in Fig. 7.

Since

\[
\begin{cases}
H \in (B) \\
(B) : f(x, y) = ax - y + b = 0.
\end{cases}
\]  

(51)

Then

\[ ax_H - y_H + b = 0. \]

(52)

b. Expression of the distance \( d_{CH/A} \)

This section is developed according to the results of Sect. 3.1.1.1.

For the calculation of the distance \( d_{CH/A} = ||CH|| \), the Cartesian distance is utilized according to the coordinates of the point \( C(x_C, y_C) \) and \( H(x_H, y_H) \), where:

\[ d_{CH/A} = ||CH|| = \sqrt{(x_H - x_C)^2 + (y_H - y_C)^2}. \]

Since

\[ \{C\} = (CH) \cap (D), \]

and

\[ (D) : y = L_2/2, \]

thus

\[ y_C = L_2/2. \]  

(53)

The vector \( \vec{n} \) which is normal to the bisector \( (B) \) is expressed by the following coordinates [54]:

\[
\vec{n} = \nabla f(x, y) = \begin{bmatrix} n_x = \frac{-a}{\sqrt{1 + a^2}} \\ n_y = \frac{-1}{\sqrt{1 + a^2}} \end{bmatrix}.
\]  

(54)

where \( f \) is the implicit function that defines the bisector \( (B) \) as described in Eq. (51).

To express the \( x_c \) coordinate, we need to determine the equation of the line \( (CH) \).

Since the line \( (CH) \) is carried by the vector \( \vec{n} \):

\[
(CH) : -\frac{1}{\sqrt{1 + a^2}}x - \frac{a}{\sqrt{1 + a^2}}y + Cst = 0.
\]  

(55)

The constant “Cst” could be calculated according to the point \( H \) which belongs to the line \( (CH) \).

Replacing the coordinates of point \( H \) in Eq. (51), we obtain:

\[
-\frac{1}{\sqrt{1 + a^2}}x_H - \frac{a}{\sqrt{1 + a^2}}y_H + Cst = 0.
\]

where

\[
\begin{align*}
(13) \iff y_H &= a x_H + b \\
&\Rightarrow -\frac{1}{\sqrt{1 + a^2}}x_H - \frac{a}{\sqrt{1 + a^2}}(a x_H + b) + Cst = 0 \\
&\Rightarrow Cst = \frac{x_H (1 + a^2) + ab}{\sqrt{1 + a^2}}.
\end{align*}
\]

So the equation of the line \( (CH) \) becomes:

\[
(CH) : \frac{-1}{\sqrt{1 + a^2}}x - \frac{a}{\sqrt{1 + a^2}}y + \frac{x_H (1 + a^2) + ab}{\sqrt{1 + a^2}} = 0.
\]

Since \( \forall a \in \mathbb{R} \)

\[ \sqrt{1 + a^2} \neq 0. \]

We find

\[
(CH) : x + ay - (x_H (1 + a^2) + ab) = 0.
\]

(56)

Since \( C \in (CH) \):

\[ x_C + ay_C - (x_H (1 + a^2) + ab) = 0. \]

According to the expression (53):
\[ x_c + \frac{aL_2}{2} - (x_H(1 + a^2) + ab) = 0. \]

Finally

\[ x_c - x_H = a(x_H + b - \frac{L_2}{2}), \quad (57) \]

\[ y_c - y_H = -\left(ax_H + b - \frac{L_2}{2}\right). \quad (58) \]

In addition to the Eqs. (57) and (58), we remark that:

\[ x_c - x_H = -a(y_c - y_H). \quad (59) \]

Or

\[ y_c - y_H = -\frac{1}{a}(x_c - x_H). \quad (60) \]

Thus:

\[ d_{CH/A} = ||\overrightarrow{CH}|| = \sqrt{a^2(y_H - y_c)^2 + (y_H - y_c)^2} \]

\[ \Rightarrow d_{CH/A} = |y_H - y_c|\sqrt{1 + a^2}. \]

Since

\[ y_H \leq y_c \Rightarrow |y_H - y_c| = y_c - y_H. \]

Hence

\[ d_{CH/A} = ||\overrightarrow{CH}|| = -ax_H\sqrt{1 + a^2}. \quad (61) \]

c. Expression of the command parameter \( x_H \)

Let \( x_H \) be the command scanning variable. To apply the jumps of the scanning, \( x_H \) must be considered as a discrete parameter. Thus \( x_H \) must be expressed according to the projection jump step on the \( \overrightarrow{y} \) axis.

The jump along the bisector (\( B \)) is denoted \( e_s \), its projection, perpendicularly to the skeleton, on the horizontal axis is denoted \( e'_s \) (Fig. 8).

So, the formulation of \( x_H \) is given by in the expression (62).

\[ x_H = ke'_s. \quad (62) \]

The geometrical link between \( e_s \) and \( e'_s \) is described by the Fig. 22.

Thus

\[ (\text{fig. 9}) \Rightarrow \cos(\theta) = e_s/e'_s \]

\[ (\text{fig. 6}) \Rightarrow \cos(\theta) = \frac{a_1L_1}{\Lambda} \]

\[ \Lambda = \sqrt{(a_1L_1)^2 + \left(\frac{L_2}{2}\right)^2}. \quad (63) \]

It implies that:

\[ e'_s = e_s\frac{\Lambda}{a_1L_1} = e_s\sqrt{(a_1L_1)^2 + \left(\frac{L_2}{2}\right)^2}. \]

Finally

\[ e'_s = e_s\sqrt{1 + \left(\frac{L_2}{2a_1L_1}\right)^2}. \quad (64) \]

Let us denote

\[ \beta = \sqrt{1 + \left(\frac{L_2}{2a_1L_1}\right)^2}. \quad (65) \]

And

\[ e'_s = \beta e_s. \quad (66) \]

In the case of this study, the angle \( \theta \) is equal to \( \frac{\pi}{4} \), so the link between the jumps \( e_s \) and \( e'_s \) could be directly expressed by the expression (67):

\[ \cos(\theta) = \frac{\sqrt{2}}{2} = e_s/e'_s. \quad (67) \]

For more genericity of the modeling approach, the author wanted to express the final expression of the total scanning length according to all geometrical parameters as dummy variables. Subsequently, in the simulation, the numerical values will appropriately replace the problem parameters.

Thus, according to (62), the expression of the command \( x_H \) becomes \( x_{H/k} \) such that:

\[ \begin{cases} x_{H/k}(k) = x_{H/A}(k) \\ x_{H/k}(k) = ke'_s = k\beta e_s, \quad 1 \leq k \leq k_A \end{cases} \quad (68) \]

where \( x_{H/A}(k) \) is the restriction of \( x_H \) on the area \( A \), function of the index \( k \).

The index \( k \) is the increment of the command, it begins by \( k_1 = 1 \) till a limit value \( k_A \) that limits the area \( (A) \) as depicted in Fig. 7. From the same figure, the \( k \) index takes \( k_A \) value.
when the point C reaches the right limits of the area (A). This condition is expressed by:

\[ x_C(k_A) = \alpha_L L_1, \]  

(69)

\[ x_H(k_A) = k_A \beta e_s. \]  

(70)

Replacing (69) and (70) in the Eq. (57):

\[ \alpha_L L_1 = (a^2 + 1) k_A \beta e_s + a \left( b - \frac{L_2}{2} \right). \]

Thus, \( k_A \) is computed using the expression (71).

\[ k_A = \frac{\alpha_L L_1}{(a^2 + 1) e_s \beta}. \]  

(71)

d. Correction of the index \( k_A \)

Since \( k_A \) is an integer number, the corrected \( k_{Ac} \) must is expressed by:

\[ k_{Ac} = E \left( \frac{\alpha_L L_1}{(a^2 + 1) e_s \beta} \right), \]  

(72)

and

\[ x_{Ha} = \frac{\alpha_L L_1}{1 + a^2}. \]  

(73)

e. Productive length of the area (A)

On the area (A), the productive length \( L_{P_A} \) is corresponds to the following summation:

\[ L_{P_A} = \sum_{k=1}^{k_{Ac}} \left( d_{CH/A} \right) + \left( d_{CH/A} \right)_f, \]

where \( \left( d_{CH/A} \right) \) is the distance \( CH \) at the limit of the area (A). According to the expression (61):

\[ \left( d_{CH/A} \right) = -ax_{Ha} \sqrt{1 + a^2}, \]

where \( x_{Ha} \) is expressed by the Eq. (73).

So

\[ L_{P_A} = \sum_{k=1}^{k_{Ac}} \left( d_{CH/A} \right) + \left( d_{CH/A} \right)_f = \sqrt{1 + a^2} \sum_{k=1}^{k_{Ac}} (-ax_{Ha}) + (-ax_{Ha}) \sqrt{1 + a^2}. \]

Since \( x_H = k e'_s \) (62)

\[ L_{P_A} = \sqrt{1 + a^2} \left[ -\frac{a}{2} k_{Ac} (k_{Ac} + 1) e'_s + (-ax_{Ha}) \right], \]

(74)

\[ L_{P_A} = \sqrt{1 + a^2} \left[ \frac{a}{2} e_s k_{Ac} (k_{Ac} + 1) + (-ax_{Ha}) \right], \]

f. Jump length of the area (A)

On the other hand, the non-productive or jump length \( L_{Ja} \) is calculated by multiplying the jumps lengths \( e_s \) and \( e'_s \) by their respective number of repetitions.

The author analyzes two cases:

- \( k_{Ac} \) is an impair number:

\[ L_{Ja} = \sum_{k=1}^{k_{Ac}} (e_s + e'_s) = \sum_{k=1}^{k_{Ac}} (1 + \beta), \]  

(75)

\[ L_{Ja} = E \left( \frac{k_{Ac}}{2} \right)(1 + \beta) e_s, \]

\[ k_{Ac} \] is pair number:

\[ L_{Ja} = \sum_{k=1}^{\frac{k_{Ac}}{2} - 1} e_s + \sum_{k=1}^{\frac{k_{Ac}}{2}} e'_s, \]  

(76)

\[ L_{Ja} = \left( \frac{k_{Ac}}{2} - 1 \right) e_s + \frac{k_{Ac}}{2} e'_s. \]

B. Study of the area (B)

Given the rectangular geometry analyzed in this paper, the region (A) and (B) are symmetrical according to the red line of Fig. 7. Then the productive and non-productive lengths formulated for the area (A) are adopted for the area (B).

c. Study of the area C

According to Fig. 4, the area (C) corresponds to a simple rectangle. The scanning schema to adopt can be composed by the classical chess policy or it is possible to create a second level of skeleton scanning on this area. In order to proceed as simple as possible for this first stage of skeleton scanning modeling, the authors decided to adopt the classical chess strategy on the area (C). This is consistent with the scanning principal adopted in this study, as the chess schema remains perpendicular and parallel to the skeleton. Thus, the constraint of the perpendicularity to the skeleton is well assured. It is possible to adopt a meander scanning perpendicular to the \( X \) axis which will correspond to a scanning perpendicularly to the skeleton of the region (C). In this case the scanning parameters of the (C) region will be \( (n_{C}, n_{C_x}) = (1, 1). \)
The productive length $L_{PC}$ and the non-productive or jump length $L_{JC}$ can be similarly formulated from Eqs. (81) and (82) that are developed for the chess scanning Appendix II.A. As expressed by these equations, the scanning schema can be fixed according to the number or the dimensions of the patterns of Figs. 5 and 6.

After manual development adaptation, the productive and the jump lengths of the area ($C$) are expressed by the Eqs. (77) and (78), respectively:

$$L_{PC} = \frac{N_1 a_1 L_1}{2} \left( \frac{1}{n_c} \left( \frac{L_2}{2 n_c e_s} + 1 \right) + \frac{1}{n_c} \left( \frac{a_2 L_1}{2 n_c e_s} + 1 \right) \right).$$

Since

$$\begin{cases} a_1 L_1 = \frac{L_2}{2} \\ a_2 L_1 = L_1 - L_2 \\ \end{cases},$$

the productive and non-productive lengths of $C$ area become:

$$L_{PC} = \frac{N_1 C L_2}{4} \left( \frac{1}{n_c} \left( \frac{L_2}{2 n_c n_c e_s} + 1 \right) + \frac{1}{n_c} \left( \frac{(L_1 - L_2)}{2 n_c e_s} + 1 \right) \right).$$

$$L_{JC} = \frac{L_2}{2} \left( \frac{N_1 C}{n_c} + \frac{N_2 C L_2}{2 n_c} \right).$$

$$\begin{cases} \text{if }\left( n_{c_x} \text{ is pair } \right) \text{ or } \left( n_{c_y} \text{ is pair } \right): N_1 C = N_2 C = \frac{n_c n_c}{} \\ \text{if chess begins with pattern 1: } N_1 C = E \left( \frac{n_c n_c}{2} \right) + 1 \\ \text{if chess begins with pattern 2: } N_2 C = E \left( \frac{n_c n_c}{2} \right) + 1 \\ N_1 + N_2 = n_c n_c \end{cases}.$$

where $N_1 C$ is the total number of chess pattern 1 (Fig. 8a); $N_2 C$ is the total number of pattern 2 (Fig. 8b); $n_C$ is the number of divisions of the dimension of the rectangle according to the pattern along the axis $\overrightarrow{x}$; $n_C$ is the number of divisions of the dimension of the rectangle according to the pattern along the axis $\overrightarrow{y}$.

### Appendix II: Benchmark strategies lengths modeling

#### A. Classical chess strategy: production time formulation

**a. Geometry parametrization**

Figure 23 presents a typical example of the classical chess-board scanning strategy. According to this figure, the system (80) presents the possible count of pattern 1 and pattern 2 as displayed in Fig. 8 of Sect. 3.1.2. The system (80) expresses also the divisions of the $(L_1 \times L_2)$ rectangle into regular $(n_1 \times n_2)$ portions.

$$\begin{cases} n_x = E \left( \frac{L_2}{a_x} \right) ; n_x = \frac{L_2}{a_x} \in \mathbb{N}^*; a_x = \frac{L_1}{n_x} \\ n_y = E \left( \frac{L_2}{a_y} \right) ; n_y = \frac{L_2}{a_y} \in \mathbb{N}^*; a_y = \frac{L_1}{n_y} \end{cases}$$

where $a_x$ is the length according to the $\overrightarrow{x}$ axis; $a_y$ is the length according to the $\overrightarrow{y}$ axis; $e$ is the hatch space of the chess strategy.

#### b. Total scanning length

**i. Productive time: active scanning** The productive time of the pattern 1 and pattern 2 could be determined directly from Fig. 8a and b, respectively.

The productive lengths of patterns 1 and 2, are respectively noted as $L_{P_1}$ and $L_{P_2}$, and are expressed by the Eqs. (81.1) and (81.1):

$$L_{P_1} = a_x (m_x + 1)$$

$$L_{P_2} = a_y (m_y + 1)$$

$$m_x = E \left( \frac{a_x}{e} \right)$$

$$m_y = E \left( \frac{a_y}{e} \right)$$

s.t. $m_x$ and $m_y$ are, respectively, the number of divisions, resp. of the patterns 1 and 2 along the axis $\overrightarrow{x}$ and $\overrightarrow{y}$.

**ii. Inactive/jump scanning lengths** The inactive or jump scanning could be defined as the scanning during which the laser or the electron beam is switched off in the case of PBF-LB/M, PBF-LB/P, PBF-EB/M processes. For Material Extrusion-
Based AM, this step is considered as productive. The jump scanning lengths, \(L_{j1}\) and \(L_{j2}\), related to pattern 1 and pattern 2 respectively are expressed by expressions (82.1) and (82.2):

\[
L_{j1} = a_x \quad (82.1) \\
L_{j2} = a_y \quad (82.2)
\]

**c. Total scanning length**

Hence, the total lengths \(L_{ch1}\) and \(L_{ch2}\) of resp. pattern 1 and pattern 2 of the chess strategy are expressed by the system (83):

\[
\begin{align*}
L_{ch1} &= a_x \left( E \left( \frac{a_x}{2} \right) + 1 \right) + a_y (83.1) \\
L_{ch2} &= a_y \left( E \left( \frac{a_y}{2} \right) + 1 \right) + a_x (83.2) .
\end{align*}
\]

Thus the total scanning time of the filling area related to the chess scanning strategy in the case of a rectangle \(L_1 \times L_2\) is given by the Eq. (84) and the system (85):

\[
L_{ch} = N_1 L_{ch1} + N_2 L_{ch2} ,
\]

\[
\text{if } \left( n_x \text{ is pair} \right) \text{ or } \left( n_y \text{ is pair} \right) : N_1 = N_2 = \frac{n_x n_y}{2} \quad (85.1) \\
\text{else} \left\{ \begin{array}{l}
\text{if the scanning begins} \\
\text{with pattern 1} \\
N_1 = E \left( \frac{n_x}{2} \right) + 1 (85.2) \\
\text{else} \\
\text{if the scanning begins} \\
\text{with pattern 2} \\
N_2 = E \left( \frac{n_y}{2} \right) + 1 (85.3) \\
N_1 + N_2 = 1
\end{array} \right.
\]

where \(N_1\) is the total number of pattern 1; \(N_2\) is the total number of pattern 2; \(n_x\) is the number of divisions of the dimension of the rectangle along the axis \(\bar{x}\); \(n_y\) is the number of divisions of the dimension of the rectangle along the axis \(\bar{y}\);

**B. Stripe strategy**

Figure 9 of Sect. 3.1.3 presents the geometry parametrization of a stripe strategy scanning.

The distance “\(d\)” can be negative in the case of scanning overlap, positive or zero. In the case of this study, “\(d\)” is considered zero, which means that no overlap is considered. Other stripes strategies could be analyzed by the same methodology.

**a. Stripe strategy parallel to \(x\) axis: pattern 1**

The geometry analysis allows the calculation of the length of scanning according to the procedure described by the Eq. (86) and the system (87). The geometry parametrization if detailed in Fig. 9, Sect. 3.1.3:

\[
L_{st1} = m_x \left( m_x e + a_x \right) + L' x ,
\]

\[
m_x = \begin{cases} 
& m_{x0} + 1 \text{ if } a_x - m_{x0} e < e \\
& m_{x0} \text{ if } a_x - m_{x0} e \geq e
\end{cases} \quad (87.1)
\]

\[
m_y = \begin{cases} 
& m_{y0} + 1 \text{ if } a_y - m_{y0} e < e \\
& m_{y0} \text{ if } a_y - m_{y0} e \geq e
\end{cases} \quad (87.2)
\]

\[
m_{x0} = E \left( \frac{a_x}{e} \right) \quad (87.3)
\]

\[
m_{y0} = E \left( \frac{a_y}{e} \right) \quad (87.4)
\]

\[
\begin{cases} 
L' x = a_x + e' x \text{ if } a_x - m_x e \geq e \\
& 0 \text{ else}
\end{cases}
\]

\[
e' x = \begin{cases} 
& a_x - m_x e \text{ if } a_x - m_x e \geq e \\
& 0 \text{ else}
\end{cases} \quad (87.5)
\]

\(L' x\) is the length of the last raw for which the height does not necessarily correspond to the hatch spacing \(e\) but to a residual hatch \(e_r\).

**b. Stripe strategy parallel to \(y\) axis: pattern 2**

The analytical expression for the length of pattern 2, that is denoted \(L_{st2}\), is systematically deduced from pattern 1 by permuting \(x\) and \(y\) indices (see Eq. (87)):

\[
L_{st2} = m_y \left( m_y e + a_y \right) + L' y ,
\]

where

\[
L' y = a_y + e' y \text{ if } a_y - m_y e \geq e \\
0 \text{ else}
\]

\[
e' y = \begin{cases} 
& a_y - m_y e \text{ if } a_y - m_y e \geq e \\
& 0 \text{ else}
\end{cases} \quad (89)
\]

**c. Total stripe strategy scanning length**

The numbers \(N_1\) and \(N_2\) of patterns 1 and 2 respectively are calculated by the same procedure like chess scanning strategy.

Thus, the total scanning length of \((L_1 \times L_2)\) rectangle is calculated by the Eq. (90):

\[
L_{st} = L_{st1} N_1 + L_{st2} N_2 .
\]

**C. Spiral scanning strategy**

Figure 10 of Sect. 3.1.4 presents the geometry parametrization of spiral strategy scanning.

The calculation procedure can be applied to both outer or internal spiral scanning strategy.
a. Spiral strategy starting by y axis: pattern 1

i. Sum of lengths along the x axis: $L_x$ Following the same modeling procedure and according to Fig. 10:

\[ a_{x_1} = a_x \]
\[ a_{x_2} = a_{x_1} - e \]
\[ a_{x_3} = a_{x_2} - e \]

\[ \vdots \]
\[ a_{x_i} = a_{x_{i-1}} - e. \]

**Summation:**

\[ a_{x_i} = a_x - \sum_{j=2}^{i} e \]

So:

\[ \forall i \geq 2 : a_{x_i} = a_x - e(i-1) \]
\[ a_{x_1} = a_x \]

\[ L_x = \sum_{i=1}^{m_x} a_{x_i} = a_x + \sum_{i=2}^{m_x} e(i-1) \]
\[ L_x = m_x a_x - e \sum_{i=2}^{m_x} (i-1) \]
\[ L_x = m_x a_x - e \left( \frac{m_x (m_x + 1)}{2} - 1 \right) \]
\[ L_x = m_x a_x - e \frac{m_x (m_x - 1)}{2}. \]

$m_x$ is computed according to the system (87).

ii. Sum of lengths along the y axis: $L_y$ Following the same modeling procedure and according to Fig. 10:

\[ a_{y_1} = a_y \]
\[ a_{y_2} = a_y \]
\[ a_{y_3} = a_{y_2} - e \]

\[ \vdots \]
\[ a_{y_i} = a_{y_{i-1}} - e. \]

**Summation:**

\[ a_{y_i} = a_y - \sum_{j=3}^{i} e \]

So:

\[ \forall i \geq 3 : a_{y_i} = a_y - e(i-2) \]
\[ a_{y_1} = a_y \]

\[ L_y = \sum_{i=1}^{m_y} a_{y_i} = a_y + \sum_{i=3}^{m_y} a_{y_i} = 2a_y + \sum_{i=3}^{m_y} (a_y - e(i-2)) \]
\[ L_y = 2a_y + (m_y - 2)a_y - e \sum_{i=3}^{m_y} (i-2) \]
\[ L_y = m_y a_y + 2e(m_y - 2) - e \left( -1 - 2 + \sum_{i=1}^{m_y} i \right) \]
\[ L_y = m_y a_y + 2em_y - 4e + 3e - e \frac{my}{2} (m_y + 1) \]
\[ L_y = m_y a_y + e \left( 2m_y - 1 - \frac{m_y}{2} (m_y + 1) \right) \]
\[ L_y = m_y a_y - e \frac{y}{2} (-4m_y + 2 + m_y (m_y + 1)) \]
\[ L_y = m_y a_y - e \frac{y}{2} (m_y^2 - 3m_y + 2). \]  
(92)

$m_y$ is computed according to the system (87).

iii. Total length of pattern 1 The total length of pattern 1 is expressed by Eq. (93):

\[ L_1 = L_x + L_y \]
\[ L_{sp1} = m_x a_x + m_y a_y - e \frac{2}{2} \left( m_x (m_x - 1) + \left( m_y^2 - 3m_y + 2 \right) \right). \]  
(93)

b. Spiral strategy starting by y axis: pattern 2

Pattern 2 of the spiral strategy corresponds to scanning starting parallel to the x axis. The total scanning length of spiral pattern 2 can be deduced from expression (93) by swapping the indices $x$ and $y$ resulting in the Eq. (94):

\[ L_{sp2} = m_x a_x + m_y a_y - e \frac{2}{2} \left( m_y (m_y - 1) + \left( m_x^2 - 3m_x + 2 \right) \right). \]  
(94)

c. Total spiral strategy scanning length

The numbers $N1$ and $N2$ of patterns 1 and 2, respectively, are calculated by the same procedure as the chess scanning strategy.

The total spiral strategy scanning length of the $L_1 \times L_2$ is calculated by the Eq. (95):

\[ L_{sp} = N_1 L_{sp1} + N_2 L_{sp2}. \]  
(95)
D. Contour scanning strategy

Figure 11 of Sect. 3.1.5 presents the geometry parametrization of contour strategy scanning.

a. Contour strategy starting by $y$ axis: pattern 1

Pattern 1 of the contour strategy corresponds to scanning starting parallel to the $y$ axis. The total scanning length of contour pattern 1 is modeled according to the following sections.

i. Sum of lengths along the $x$ axis: $L_x$ Following the same modeling procedure and according to Fig. 11:

- $a_{x_1} = a_x$
- $a_{x_2} = a_{x_1}$
- $a_{x_3} = a_{x_2} - 2e$
- $a_{x_4} = a_{x_3} - 2e$
- $a_{x_5} = a_{x_4} - 2e$

Summation:

\[
\forall i \geq 3 : a_i = a_{i-1} - 2e(i-2)
\]

\[
L_x = \sum_{i=1}^{m_x} a_i = a_{x_1} + a_{x_2} + \sum_{i=3}^{m_x} (a_x - 2e(i-2))
\]

ii. Sum of lengths along the $y$ axis: $L_y$ Following the same modeling procedure and according to Fig. 11:

\[
\begin{align*}
  a_{y_1} &= a_y \\
  a_{y_2} &= a_{y_1} - e \\
  a_{y_3} &= a_{y_2} - 2e \\
  a_{y_4} &= a_{y_1} + e \\
  a_{y_5} &= a_{y_4} - 2e \\
  a_{y_6} &= a_{y_5} - e \\
  a_{y_7} &= a_{y_6} - 2e \\
  a_{y_8} &= a_{y_7} + e \\
  \vdots
\end{align*}
\]

Summation:

\[
L_y = \sum_{i=1}^{m_y} a_i = a_{y_1} + a_{y_2} + \sum_{i=3}^{m_y} (a_y - 2e)\left(\frac{i-3}{2}\right) + 1)
\]

$m_y$ is computed according to the system (87).

iii. Total length of scanning along the diagonal The total length along the diagonal is the sum of the $e'$ distances (Fig. 11).

Flowing the same techniques of indexing, it can be stated that:

\[
e'_i = \begin{cases} 
\sqrt{2}e' & \text{if } i \text{ pair} \\
\text{NaN} & \text{else}
\end{cases}
\]

Thus, the total length along the diagonal is expressed by:

\[
L_{Diag} = \sum_{i=1}^{m} e'_i + \bar{e}'
\]

The parameter $m$ and $e'$ are calculated using the procedure (100) and (101). $e'$ is a residual distance to scan in order to avoid a lack of matter in the last processing step which can be caused by the integer count $m_x$ and $m_y$ of the scanning steps.
iv. Total length of contour pattern 1

The total length of contour pattern is computed according to Eq. (102):

\[ L_{\text{cont}}^1 = L_x + L_y + L_{\text{Diag}}. \]

b. Contour strategy starting by x axis: pattern 2

Pattern 2 of the contour strategy corresponds to scanning starting parallel to the x axis. The total scanning length of contour pattern 2 can be deduced from the previous paragraph by swapping the indices x and y of Eq. (102) and related equations.

c. Total contour strategy scanning length

The total contour strategy scanning length of the \( L_1 \times L_2 \) is calculated by Eq. (103).

The numbers \( N_1 \) and \( N_2 \) of patterns 1 and 2, respectively, are calculated by the same procedure like chess scanning strategy:

\[ L_{\text{cont}} = N_1 L_{\text{cont}}^1 + N_2 L_{\text{cont}}^2. \]

Appendix III: Scanning parameters according to the bibliography

See Table 6.

Appendix IV: Curves and surfaces modeling

A. Case of study 1: Modeling of benchmark strategies lengths \( L_i \) according to the hatch space distance \( e \) (Fig. 14)

According to Fig. 14, the authors propose to fit hyperbolic curves to the \((e, L_i)\) dispersions for the set of benchmark scanning strategies. Equation (104) present the proposed model:

\[ L_i \simeq a e^n, \]

where \( \{ a > 0, n > 0 \} \) is the approximation of the \( L_i \) distribution; \( i \) is a given strategy from the set: \{ chess \((i = 1)\), stripe \((i = 2)\), spiral \((i = 3)\), contour \((i = 4)\) \}.

A preliminary linearization of the distribution was applied by means of the log function, as follows:

\[ \log(L_i) = \log(a) - n \log(e). \]

Hence, linear regression was applied for all \( L_i \) curves. The models parameters estimation is presented in Table 7 with the correspondent \( R^2 \) statistics related to the linear fitting.

B. Case of study 2: modeling of the first order slopes of \( L_{\text{SBP}} \) according to the hatch space distance \( e \) (Fig. 18)

According to Fig. 18, the authors propose to fit hyperbolic curves to the first order variations or slopes of \( L_{\text{SBP}} \) that were noted in Fig. 18 as:

Table 6  AM filling parameters—case of PBF-LB/M processes

| Hatch space (mm) | Material       | References |
|------------------|----------------|------------|
| 0.150            | Al-Si-10 Mg    | [55]       |
| 0.090            | Invar          | [56]       |
| 0.300            | Invar          | [57]       |
| 0.08–0.320       | Invar          | [58]       |
| 0.150            | SS 316L        | [59]       |
| 0.070–0.140      | SS 316L        | [60]       |
| 0.050–0.070      | SS 316L        | [30]       |
| 0.080–0.120      | SS 316L        | [15]       |
| 0.030–0.060      | SS 316L        | [45]       |
| 0.080            | SS 316L        | [16]       |
| 0.100            | Ti-6Al-4 V     | [61]       |
| 0.085            | Ti-6Al-4 V     | [33]       |
| 0.040–1.000      | Ti-6Al-4 V     | [13]       |
| 0.074–0.100      | Ti-6Al-4 V     | [62]       |
| 0.105            | Pure Titanium CpTi | [63]     |
The hyperbolic models that are proposed are expressed by Eq. (106):

\[
\begin{align*}
 b_1 &= \frac{a}{n}, \\
 b_2 &= \frac{a L_2}{L_1}, \\
 b_3 &= \frac{a n_{C_x}}{L_1}, \\
 b_4 &= \frac{a n_{C_y}}{L_1}.
\end{align*}
\]

where \( i \in \{1, \ldots, 4\} \)

\( a > 0 \)

\( n > 0 \)

The hyperbolic models that are proposed are expressed by Eq. (106):

\[
b_i = \frac{a}{e^n}.
\]

Table 7 Hyperbolic fitting of scanning benchmark strategies lengths—case of study 1 (Fig. 14)

| Strategy | \( a \) | \( n \) | \( R^2 \) | \( p \_value \) |
|----------|--------|--------|--------|-------------|
| Chess    | 2.24 \( 10^3 \) | 0.8074 | 98.91\% | 7.8 \( 10^{-88} \) |
| Stripe   | 7.76 \( 10^3 \) | 1.0874 | 98.95\% | 4.88 \( 10^{-83} \) |
| Spiral   | 7.57 \( 10^3 \) | 0.8592 | 99.28\% | 7.54 \( 10^{-105} \) |
| Contour  | 9.58 \( 10^3 \) | 0.5931 | 98.08\% | 2.12 \( 10^{-84} \) |

The same linearization and fitting procedures were applied similarly to the modeling of the previous paragraph. The models parameters estimation is presented in Table 8 with the correspondent \( R^2 \) statistics related to the linear fitting.

Table 8 Hyperbolic fitting of \( L_{SBP} \) slopes—case of study 1 (Fig. 18)

| \( L_{SBP} \) Slope | \( a \) | \( N \) | \( R^2 \) (%) | \( p \_value \) |
|---------------------|--------|--------|-------------|-------------|
| \( b_1 \)           | 169.92 | 0.9800 | \( \sim 100\% \) | 2.18 \( 10^{-200} \) |
| \( b_2 \)           | 205.03 | 0.9871 | \( \sim 100\% \) | 1.05 \( 10^{-211} \) |
| \( b_3 \)           | 905.29 | 0.9302 | \( 99.99\% \) | 2.2 \( 10^{-148} \) |
| \( b_4 \)           | 874.10 | 0.9435 | \( 99.99\% \) | 3.43 \( 10^{-157} \) |

C. Case of study 2: modeling of \( L_{SBP} \) according to \((L_1, L_2, n_{C_x}, n_{C_y})\)

For each hatch space, linear, interactions, pure quadratic, and full quadratic models were tested in order to propose a regression modeling for \( L_{SBP} \) as simple as possible. Figure 24 shows the \( R^2 \) statistics related to each regressive model. It is interesting to see that full quadratic models produce high values of \( R^2 \), generally higher than 75%.

Hence, one can argue that the family of functions \( L_{SBP}(\hat{L}_1, L_2, n_{C_x}, n_{C_y}) \) could be described as a set of quadratics of \((\hat{L}_1, L_2, n_{C_x}, n_{C_y})\) according to the formulation (107).

For each hatch space distance \( e \):

\[
\tilde{L}_{SBP}(\hat{x}) = \frac{1}{2} \hat{x}^T A_e \hat{x} + \hat{b}_e^T \hat{x} + \gamma_e.
\]

where \( \hat{x} = (L_1, L_2, n_{C_x}, n_{C_y})^T \) is the model geometrical features; \( A_e \) is the family of symmetric matrices associated to

\[
\text{Fig. 24 $R^2$ statistics VS hatch space for}
\]
the quadratic terms of the models $\tilde{L}_{SBP}$; $\overline{d}_e$ is the family of the multiplications of the first order terms of the function $L_{SBP}$; $\gamma_e$ is the family of constant terms associated to the function $L_{SBP}$ (see Fig. 24).

Appendix V: Specific gains $SG_{\text{max}}$ plot

The maximal specific gains $SG_{\text{max}}$ were computed at minimal values of $L_{SBP}$ corresponding to $(\min(n_C), \min(n_C))$ (see Fig. 25).

Fig. 25 Specific gains $SG_{\text{min}}$ according to $(L_1, L_2, e)$ a SBP VS Chess b SBP VS Stripe c SBP VS Spiral d SBP VS Contour. $SG_{\text{min}}$ are computed for the higher values of $L_{SBP}$ at $(\max(n_C), \max(n_C))$.
Author contributions MEJ developed the approach, carried out the analytical modeling, the implementation of the code, the discussion and interpretation of the results. IA contributed to the discussion on the material science aspect, the choice of processing parameters to be adopted, and the constraining of the optimization problem formulation in term of hatch space distance. NS initiated the discussion on the proposal of a new scanning technique based on the notion of 2D shape skeleton generation.

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Declaration

Conflict of interests

The authors declare that they have no competing interests.

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