Flux-flow Oscillator with Anticorrelated Noise on the Bias Current and the Magnetic Field

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Abstract. A Flux-flow oscillator based on a long Josephson junction is a local oscillator in fully superconducting integrated receivers (SIRs). The low frequency voltage fluctuations define the free-running FFO radiation linewidth, an important parameter for employing the SIR as a submm spectrometer. The DC voltage of the oscillator is independently controlled by both the dissipative DC bias current $I_B$ and the loss-free external magnetic field at the two FFO ends. It is mainly the natural wideband current noise of the Josephson barrier, which is converted into the fluctuations of voltage by the differential resistance $R_d$ on the current $I_B$. We have developed and experimentally demonstrated the method for suppression of the voltage fluctuations by convey of the bias current noise into the anticorrelated noise of the magnetic field. The experiments were made with Nb/AlN/Nb FFOs, having a specially designed shunting structure at the end, where flux quanta enter the junction.

1. Introduction
During the last decade the flux-flow oscillator (FFO) based on the long Josephson junction has been extensively studied for using it as an on-chip integrated local oscillator (LO) in superconducting integrated submm receivers (SIRs) for airborne missions, related to environmental monitoring of the atmosphere [1]. The magnetic flux quanta nucleated at the FFO entrance end are accelerated by the bias current, $I_B$. They travel along the junction, reach the opposite FFO end and radiate the RF-power into the network used to couple the LO signal to the SIS-mixer. The FFO voltage and thus its frequency is controlled independently by the overlap bias current, $I_o$, and the so called “control line” inline current, $I_{cl}$, which creates the magnetic field at the ends of the FFO.

For spectroscopic measurements of submm lines of gases the FFO is phase-locked to a certain harmonic of the reference oscillator and the fraction of the phase-locked power, the so-called spectral ratio (SR), is mainly determined by the PLL bandwidth and the autonomous FFO radiation linewidth $\Delta f$[2]. The residual part of the FFO power contributes to the phase noise and increases the uncertainty in SIR spectral measurements [2]. For this reason the FFO linewidth and the way to decrease it were one of the key issues in development of SIRs.

Experimental studies of the overlap-type FFOs have shown [3], that $\Delta f$ can be described by formula (1), containing the differential resistances both on the bias current $R_d = \partial V/\partial I_B$ and on the control line current, $R_{d_{cl}} = \partial V/\partial I_{cl}$:

$$\Delta f = 2\pi e \left(\frac{1}{\Phi_0}\right)^2 (R_d + K \cdot R_{d_{cl}}^4) \left[ I_{op} \coth \frac{eV}{2k_BT} + 2I_s \coth \frac{eV}{k_BT} \right]$$  \hspace{1cm} \hspace{1cm} (1)
which includes a nonlinear superposition of thermal and shot noise down-converted by the Josephson junction to low frequencies. $I_{QP}$ and $I_S$ are the quasi-particle and the superconducting components of $I_B$ respectively. $K$ - is a fitting parameter, depending mainly on the geometry of FFO electrodes. At $K = 0$ formula (1) takes the form of the expression for the lumped Josephson junction. In the presence of the so-called “self-field” effect the current $I_B$ also contributes to the magnetic field at one of the FFO ends and modifies the steepness of the I-V characteristic and therefore $R_d [4,5]$. In [4] the FFO is considered as an element, which voltage fluctuations originate only in internal low frequency current noise of the tunneling barrier. Under this assumption $I_B$ creates only the DC magnetic field and does not make any magnetic field noise, i.e. power spectrum of voltage fluctuations and therefore $\Delta f$ are not influenced by the self-field effect. In this case $K$-parameter relates $R_d$, we measure, and $R_d$ of the “bare” FFO, which relates the intensities of the bias current and the voltage fluctuations. The aim of our study was to couple the internal fundamental noise of the FFO bias current to an external network and employ it to reduce the FFO linewidth.

2. Concept of FFO voltage noise suppression

The DC voltage of the Flux-flow oscillator is a function of $I_B$ and the magnetic fields $H_{in} = \alpha_1 I_{CL1}$ and $H_{out} = \alpha_2 I_{CL2}$ at the FFO entrance end and the one radiating RF-power:

$$V = V(I_B, H_{in}, H_{out})$$  \hspace{1cm} (2)

Here the parameters $\alpha_1$ and $\alpha_2$ are defined by the FFO layout. They have the physical meaning of mutual inductances and can be computed by the solvers for inductance extraction. Normally, the noise-free inline currents $I_{CL1}$ and $I_{CL2}$ run in the base electrode and for the FFO of pure overlap bias type $I_{CL1} = I_{CL2} = I_CL [5]$.

![Figure 1. The noise equivalent diagram of the FFO with RL-shunt](image)

According to the Langevin noise diagram (figure 1), the noise current source is connected in parallel to the lossy FFO. Here we consider the case of spatially correlated low-frequency current noise, when all noise sources from the all elementary FFO parts separated by the junction’s geometric inductance can be represented by a single noise source. A part of the noise current can be split into an
external shunting structure. When such a shunt is placed at the entrance FFO end it creates noise in the magnetic field anticorrelated with the bias current noise and decreases the amplitude $V_{ij}(\omega)$ of the voltage fluctuations. The FFO noise current of the amplitude $I_{ij}$ is split into the current $I_{ij1}$ and $I_{ij2}$ running through the tunneling barrier and the shunt correspondingly. The shunt has the resistor $R_{sh}$ and the inductance $L_{sh}$. The shunt resistor has the second noise source, generating the noise current $\Delta I_{sh}$ independent to $I_{ij}$.

The FFO differential resistances $\partial V/\partial H_{in}$, $\partial V/\partial H_{out}$ are functions of $I_{B}$, $H_{in}$ and $H_{out}$. However, we normally measure $R_{dCL}$, which can be expressed as follows:

$$R_{dCL} = \frac{\partial V}{\partial I_{C1}} + \frac{\partial V}{\partial I_{C2}} = \frac{\partial V \cdot \alpha_1}{\partial H_{in}} + \frac{\partial V \cdot \alpha_2}{\partial H_{out}}$$  \hspace{1cm} (2)

It was demonstrated in [5], that the FFO entrance end is about three times more sensitive to variations of the magnetic field than the radiating end

$$A = \frac{\partial V}{\partial H_{in}} \left(\frac{\partial V}{\partial H_{out}}\right)^{-1} = 3.12$$  \hspace{1cm} (3)

Substituting (3) into (2) we express the FFO differential resistance on $H_{in}$ in terms of $R_{dCL}$

$$R_{dCL} = \left(\alpha_1 + \frac{\alpha_2}{A}\right) \frac{\partial V}{\partial H_{in}}$$  \hspace{1cm} (4)

The increase of the FFO bias voltage due to the increase of the bias current $I_{B}$ should be compensated by the shunt, which current $I_{sh}$ reduces $H_{in}$ (see figure 1):

$$H_{in} = \alpha_1 I_{C1} - \alpha_2 I_{sh}$$  \hspace{1cm} (5)

and therefore

$$\frac{\partial V}{\partial H_{in}} = \frac{\partial V}{-\alpha_2 I_{sh}}$$  \hspace{1cm} (6)

Substituting here the ratio $\partial V/\partial H_{in}$ from (4) we find

$$\frac{\partial V}{\partial I_{sh}} = \frac{A \alpha_2 R_{dCL}}{A \alpha_1 + \alpha_2}$$  \hspace{1cm} (7)

From the equivalent diagram shown in figure 1 a part of the current produced by the FFO noise source is split into the FFO itself and has the amplitude

$$\bar{I}_{ij1} = X_1 \ast \bar{I}_{ij}$$  \hspace{1cm} (8)

while another part running through the shunt is defined as
\[ T_{JJ2} = X_2 \cdot T_{JJ} \quad (9) \]

Here \( X_1 = \frac{(i\omega L_{sh} + R_{sh})(i\omega C_{JJ}R_d + 1)}{R_d + (i\omega L_{sh} + R_{sh})(i\omega C_{JJ}R_d + 1)} \) and \( X_2 = \frac{R_d}{R_d + (i\omega L_{sh} + R_{sh})(i\omega C_{JJ}R_d + 1)} \) are the FFO capacitance. The shunt resistor \( R_{sh} \) also contributes to the FFO noise current

\[ I_{sh2} = \frac{T_{sh} \cdot R_{sh} \cdot (i\omega C_{JJ}R_d + 1)}{R_d + (R_{sh} + i\omega L_{sh}) \cdot (i\omega C_{JJ}R_d + 1)} \quad (10) \]

In the presence of the resistive shunt having spectral density of voltage fluctuations \( S_{V_{sh}}(\omega) \) the FFO voltage fluctuations of the spectral density \( S_{V_{FFO\_shunted}}(\omega) \) consist of the two parts:

\[ S_{V_{\_FFO\_shunted}}(\omega) = \left| \bar{V}_{JJ}(\omega) \right|^2 + S_{V_{-sh}}(\omega) \quad (11) \]

The second term here accounts for the FFO noise current produced by \( R_{sh} \). Since \( \partial V = R_d \partial I_B \) and taking into account (6),(8) one can write the formula for the first contribution to the FFO voltage fluctuations:

\[ \bar{V}_{JJ}(\omega) = \left( R_d \cdot X_1 - \frac{A\alpha_{sh} R_d^{\text{Cl}}}{A\alpha_1 + \alpha_2} X_2 \right) \left( \frac{eI_B}{2\pi} \cdot \coth \left( \frac{eV}{2kT} \right) \right)^{1/2} \quad (12) \]

From (10) we find

\[ S_{V_{-sh}}(\omega) = \frac{4\pi \cdot kT}{R_{sh}} \left( \frac{R_d \cdot R_{sh} \cdot (i\omega C_{JJ}R_d + 1)}{R_d + (R_{sh} + i\omega L_{sh}) \cdot (i\omega C_{JJ}R_d + 1)} \right)^2 \quad (13) \]

Without a shunt the spectral density of FFO voltage fluctuations \( S_{V_{FFO}}(\omega) \) is given by

\[ S_{V_{FFO}} = R_d^2 \frac{eI_B}{2\pi} \coth \left( \frac{eV}{2kT} \right) \quad (14) \]

Finally, the parameter we need to minimize by implementing the RL-shunt is the FFO voltage noise gain \( G_N = \frac{S_{V_{FFO\_shunted}}}{S_{V_{FFO}}}. \)

3. Experimental check

In this section we present the results of linewidth measurements made for both the shunted and the reference Nb/AlN/Nb FFOs in the frequency range 500-700 GHz. The shunted junction (see figure 2) had the same layout as the reference one, but with a specific RL-structure at the entrance end.
A part of $I_B$ running via the shunt was reducing the value of $H_n$ and making the I-V characteristics steeper. The typical differential resistances of the reference unshunted FFO, measured at the fixed value of $I_B$ are shown in figure 3. These dependencies were used to calculate $G_N$ from the equations (11)-(14) at $R_{sh} = 0.02$ Ohm, $\alpha_1=200$, $\alpha_2=150$, $\alpha_{sh}=160$ a.u. (figure 3, green curve).

Normally, the shape of $\Delta f(V)$ dependency repeats the shape of $Rd(V)$, but $\Delta f$ of the FFO with the RL-shunt at the entrance end was found to be almost independent on frequency (figure 4a). The narrow radiation lines were observed at the voltages around 1mV close to the boundary of the Josephson self-coupling regime [6], where FFOs have large $Rd$, $Rd_{CL}$ and $\Delta f$ values.
There is some discrepancy between the calculated and the measure noise gain, shown in figure 4b. We just took the ratio between $\Delta f$ values for the shunted and the reference FFO obtained at the same DC voltage to define $G_N$ from the experiment. This discrepancy can be associated with the imbalance between $H_{in}$ and $H_{out}$, caused by the DC bias current running via the shunt. The difference between the magnetic field at the two FFO ends increases with $I_B$ and modifies the differential resistances $R_d$ and $R_dCL$.

4. Conclusion
We have developed and tested an experimental technique, which makes possible to couple internal fundamental FFO noise to an external shunt and create anticorrelated noise on the magnetic field. It has been experimentally proven that such a shunt does reduce the amplitude of the voltage fluctuations and therefore the autonomous linewidth. The method is promising for Josephson local oscillators with NbN electrodes, which would allow the SIR to operate above 1 THz.
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