Successive Cancellation Soft Output Detector For Uplink MU-MIMO Systems With One-bit ADCs

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Abstract—In this paper, we present a successive-cancellation-soft-output (SCSO) detector for an uplink multiuser multiplet-output-multiple-input (MU-MIMO) system with one-bit analog-to-digital converters (ADCs). The proposed detector produces soft outputs (e.g., log-likelihood ratios (LLRs)) from one-bit quantized observations in a successive way: each user’s message is sequentially decoded from a channel decoder in that order, and the previously decoded messages are exploited to improve the reliabilities of LLRs. Furthermore, we develop an efficient greedy algorithm to optimize a decoding order. Via simulation results, we demonstrate that the proposed ordered SCSO detector outperforms the other detectors for the coded MU-MIMO systems with one-bit ADCs.

Index Terms—One-bit ADC, MU-MIMO detection, Massive MIMO.

I. INTRODUCTION

A massive multiple-input-multiple-output (MIMO) is one of the promising techniques to cope with the predicted wireless data traffic explosion [1]-[4]. Whereas, the massive MIMO can considerably increase the hardware cost and the radio-frequency (RF) circuit consumption [5]. Among all the components in a RF chain, a high-resolution analog-to-digital converter (ADC) is particularly power-hungry as the power consumption of an ADC is scaled exponentially with the number of quantization bits and linearly with the baseband bandwidth [6] and [7]. To overcome this challenge, low-resolution ADCs (e.g., 1~3 bits) for massive MIMO systems have been considered as a low-power solution over the past years. The one-bit ADC is particularly attractive due to its lower hardware complexity.

Numerous MIMO detectors have been developed for uplink MU-MIMO systems with one-bit ADCs. The optimal maximum-likelihood (ML) detector was introduced in [8] and some low-complexity detectors were also proposed in [8]-[10]. Also, new MIMIO detection frameworks based on supervised learning and coding theory were recently presented in [11] and [12], respectively. In spite of their attractive uncoded performances, they yield a poor performance for the coded systems since their hard-decision outputs degrade the performances of the following channel codes (e.g., Turbo [13], low-density-parity-check (LDPC) [14] and polar codes [15]).

In our recent work, the above problem has been addressed in [16] by presenting a soft-output (SO) detector. In this method, the soft values are derived from the hard-decision observations by exploiting a novel distance measure, named weighted Hamming distance. Therefore, the SO detector can be naturally incorporated into a state-of-the-art soft channel decoder (e.g., belief-propagation decoder [14]) while the hard-output detectors in [8]-[12] should be combined with highly suboptimal hard channel decoder (e.g., bit-flipping decoder [17]). In [16], it was shown that the SO detector provides a substantial coded gain (about 10dB) over the optimal (hard-output) ML detector.

In this paper, we propose a successive-cancellation-soft-output (SCSO) detector which enhances the SO detector in [16] by exploiting a priori knowledge. In the SO detector, the LLRs are computed using the relative distances among the current observation and all possible noiseless channel outputs (say, codewords) where the set of such codewords are referred to as a spatial-domain code \( \mathcal{C} \). The key difference of the proposed SCSO detector is that the code \( \mathcal{C} \) is refined using a priori knowledge. To be specific, the SCSO detector produces the LLRs in a successive way: each user’s message is sequentially decoded from a channel decoder in that order and the soft inputs (LLRs) of the channel decoder are computed from the refined code. Here, the refined code is constructed by eliminating some codewords from the \( \mathcal{C} \) using the previously decoded messages. Since the codewords in the refined code can have larger distances than those in the \( \mathcal{C} \), a detection ambiguity can be reduced. We further improve the proposed detector by optimizing a decoding order. We notice that due to the non-linearity of the effective channel, it is not possible to employ the ordering idea in V-BLAST [18]. Instead, we determine a decoding order in a greedy fashion such that the resulting subcodes have a better structure, i.e., the distances of the remaining codes are as far as possible. Via
numerical results, it is demonstrated that the ordered SCSO detector can attain 1.5dB coded gain over the SO detector for the polar coded MU-MIMO systems with one-bit ADCs.

**Notation**: Lower and upper boldface letters denote column vectors and matrices, respectively. Let \([a] \triangleq \{1, \ldots, a\}\) for any positive integer \(a\). Also, for any vector \(x\), let \(x_a^b = [x_a, x_{a+1}, \ldots, x_{b-1}, x_b]\) denote the part of the vector \(x\) for some positive integers \(a\) and \(b\) with \(b > a\). For any \(\ell \in \{0,1,\ldots,L-1\}\), we let \(g(\ell) = [b_0, b_1, \ldots, b_L-1]^T\) denote the \(m\)-ary expansion of \(\ell\) where \(\ell = b_0n^{m^0} + \cdots + b_L-1n^{L-1}\). We also let \(g^{-1}(\cdot)\) denote its inverse function. For a vector, \(g(\cdot)\) is applied in an element-wise manner.

**II. Preliminaries**

In this section, we describe a system model and define its equivalent channel to be used in the sequel.

**A. System Model**

We consider a single-cell uplink MU-MIMO system in which there are \(K\) single-antenna users and one base station (BS) equipped with an array of \(N_r > K\) antennas. We use \(t\) to indicate a time-index label. Let \(w_k[t] \in \mathcal{W} = \{0, \ldots, m-1\}\) represent the user \(k\)'s message at time slot \(t\) for \(k \in [K]\), each of which contains \(\log m\) information bits. We also denote \(m\)-ary constellation set by \(\mathcal{S} = \{s_0, \ldots, s_{m-1}\}\). For the ease of expression, it is assumed that \(m = 2^p\) for some positive \(p\). However, the proposed detectors in this paper can be immediately extended to an arbitrary \(m\). Then the transmitted symbol of the user \(k\) at time slot \(t\), \(\tilde{x}_k(w_k[t])\) can be obtained by a modulation function \(f\) as

\[
\tilde{x}_k(w_k[t]) = f(w_k[t]) \in \mathcal{S}.
\]

When \(K\) users transmit the symbols \(\tilde{x}(w[t]) = [\tilde{x}_1(w_1[t]), \ldots, \tilde{x}_K(w_K[t])]\), the discrete-time complex-valued baseband received signal at the BS is

\[
\tilde{r}[t] = \tilde{H}\tilde{x}(w[t]) + \tilde{z}[t] \in \mathbb{C}^{N_r},
\]

where \(\tilde{H} \in \mathbb{C}^{N_r \times K}\) is the channel matrix between the BS and the \(K\) users, i.e., \(h_k^I \in \mathbb{C}^{1 \times K}\) is the channel vector between the \(i\)-th receiver antenna at the BS and the \(K\) users. In addition, \(z[t] = [\tilde{z}_0[t], \ldots, \tilde{z}_{N_r}[t]]^T \in \mathbb{C}^{N_r}\) is the noise vector whose elements are distributed as circularly symmetric complex Gaussian random variables with zero-mean and unit-variance, i.e., \(\tilde{z}[t] \sim \mathcal{CN}(0,1)\).

In the MU-MIMO system with one-bit ADCs, each receive antenna of the BS has RF chain which consists of two one-bit ADCs that separately applied to real and imaginary part (see Fig. 1). Let \(\text{sign}(\cdot) : \mathbb{R} \to \{0,1\}\) represent the one-bit ADC quantizer function with

\[
\text{sign}(\tilde{r}[t]) = \begin{cases} 
0 & \text{if } \tilde{r}[t] \geq 0 \\
1 & \text{if } \tilde{r}[t] < 0. 
\end{cases}
\]

For a vector, it is applied element-wise. After applying ADC quantizers, the BS observes the quantized received output vector as

\[
\tilde{r}_I[t] = \text{sign}(|\tilde{r}[t]|) \in \{0,1\}^{N_r},
\]

\[
\tilde{r}_I[t] = \text{sign}(|\tilde{r}_I[t]|) \in \{0,1\}^{N_r}.
\]

In this paper, we only consider a real-valued channel for the ease of representation, and we can remodel the complex-valued input-output relationship in (2) into the equivalent real-valued representation as

\[
r[t] = \text{sign}(\text{Re}(\tilde{H})) \in \{0,1\}^{N_r},
\]

where

\[
\begin{align*}
\tilde{H} &= \begin{bmatrix} \text{Re}(\tilde{H}) & -\text{Im}(\tilde{H}) \\ \text{Im}(\tilde{H}) & \text{Re}(\tilde{H}) \end{bmatrix} \\
r[t] &= [\tilde{r}_I[t], \tilde{r}_I[t]]^T \\
x(w[t]) &= [\text{Re}(\tilde{x}(w[t])), \text{Im}(\tilde{x}(w[t]))]^T \\
\tilde{z}[t] &= [\text{Re}(\tilde{z}[t]), \text{Im}(\tilde{z}[t])].
\end{align*}
\]

This real system representation will be used in the sequel.

A block fading channel is assumed where a channel matrix \(H\) remains constant during \(n\) time slots (i.e., coded block length) and changes independently across coded blocks. Also, it is assumed that the channel matrix \(H\) is perfectly known to a BS. It is remarkable that the proposed soft-output detector can also be performed with an estimated channel matrix \(\hat{H}\) by simply changing \(H\) into \(\hat{H}\) in the following sections.

**B. Equivalent \(N\) parallel B-DMCs**

In [12], it was shown that a real system representation in (4) can be transformed into an equivalent \(N = 2N_r\) parallel binary discrete memoryless channels (B-DMCs). In this section, we define the channel input/output and channel transition probabilities of the \(N\) parallel B-DMCs (see Fig. 2). Due to the equivalence, the channel output is clearly \(r[t]\).

**Channel input**: For a given \(H\), we define a spatial-domain code \(C = \{c_0, \ldots, c_{m-1}\}\) where each codeword \(c_\ell\) is determined as a function of \(H\) as

\[
c_\ell = [\text{sign}(h_1^I x(g(l)), \ldots, h_{N_r}^I x(g(l))]^T \in C,
\]

This code is solely based on \(H\). From Fig. 2, the output of auto-encoding function \(E\) is generated by

\[
q = E(w) = c_\ell,
\]
the LLR values from the one-bit quantized observation
Fig. 3 describes the SO detector for the proposed in [16]. Some useful definitions that will be used
of length y and corresponding codeword c_{i,t}, are defined as
\[ \mathbb{P}(r_i[t] | q_i = c_{i,t}) = \begin{cases} \epsilon_{i,t} & \text{if } r_i[t] \neq c_{i,t} \\ 1 - \epsilon_{i,t} & \text{if } r_i[t] = c_{i,t} \end{cases} \]
(8)
where the error probability of the i-th channel is computed as
\[ \epsilon_{i,t} = Q(|b_1^T x(g(\ell))| < 0) \]
(9)
and
\[ Q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left(-u^2/2\right) du. \]
(10)

III. THE SO DETECTOR

In this section, we review the soft-output (SO) detector proposed in [16]. Some useful definitions that will be used throughout the paper are provided as follows.

Definition 1 (distance measure): For any two vectors x and y of length N, we define a weighted Hamming distance \( d_{wh}(x,y; \alpha) \) with the weights \( \alpha_i \) as
\[ d_{wh}(x,y; \alpha) = \sum_{i=1}^{N} \alpha_i 1(x_i \neq y_i). \]
(11)

Definition 2 (subcode): For any fixed \( \{w_k[j]\} \), the subcode of \( C = \{c \in \mathbb{E}(w) : w \in \mathbb{W}^K\} \) is defined as
\[ C_{\{w_k[j]\}} = \{c \in \mathbb{E}(w) : w \in \mathbb{W}^K, w_k[j] = j\}. \]

We are now ready to explain how the SO detector computes the LLR values from the one-bit quantized observation \( r[t] \). Fig. 3 describes the SO detector for \( p = 2 \) (i.e., 4-QAM) where the red lines (i.e., feedbacks from channel decoders) are not used. Let \( \{\tau_k[1], \ldots, \tau_k[n]\} \) denote the coded output of the user \( k \)'s channel encoder. To make a notation simpler, we define:
\[ [b]_p \triangleq \sum_{i=1}^{p} b_i 2^{p-i}, \]
(12)
for any binary vector \( b = (b_1, \ldots, b_p) \). Then, the user \( k \)'s channel input message at time slot \( t \) is obtained as
\[ w_k[t] = [\tau_k[pt], \tau_k[pt-1], \ldots, \tau_k[pt-p+1]], \]
(13)
t for \( t \in [n/p] \), where \( n \) is assumed to be a multiple of \( p \). As illustrated in Fig. 3, each user \( k \) transmits the message \( \{w_k[t] : t \in [n/p]\} \) to the BS during the \( n/p \) time slots. At each time slot \( t \), the BS computes the LLRs from the current observation \( r[t] \) as
\[ L_{pt-1}^{k} = \log \frac{\mathbb{P}(r[t] = 0 | r[t])}{\mathbb{P}(r[t] = 1 | r[t])} \]
\[ = \log \frac{\sum_{b \in \{0,1\}} \mathbb{P}(w_k[t] = [b]_p | r[t])}{\sum_{b \in \{0,1\}} \mathbb{P}(w_k[t] = [b]_p | r[t])}, \]
(14)
for \( i \in [p] \). In [16], it was shown that the above LLRs can be efficiently computed using the weighted Hamming distance in Definition 1 as
\[ L_{pt-1}^{k} = \min_{c_{i,t} \in B^1_{i,t}(c)} d_{wh}(r[t], c_{i,t}; \log \epsilon_{i,t}^{-1}) \]
\[ - \min_{c_{i,t} \in B^0_{i,t}(c)} d_{wh}(r[t], c_{i,t}; \log \epsilon_{i,t}^{-1}), \]
(14)
where \( \epsilon_{i,t} \) is given in (9) and the associated subcodes are defined as
\[ B_{i,t}^{k} = \bigcup_{b \in \{0,1\}} C_{\{w_k[t] = [b]_p\}} \text{ for } j \in \{0,1\}. \]
(15)
During the \( n/p \) time slots, the BS collects the LLRs \( \{L_{pt-1}^{k}(r[t]) : i \in [p], t \in [n/p]\} \) for \( k = 1, \ldots, K \) and they are embedded into the corresponding channel decoder \( k \), for \( k = 1, \ldots, K \).
IV. THE PROPOSED SCSO DETECTOR

In this section, we present the SCSO detector which can enhance the SO detector in Section III by exploiting a priori knowledge conveyed by a channel decoder. The overall structure of the proposed detector is depicted in Fig. 3. Furthermore, we develop an efficient greedy algorithm to optimize a decoding order. The corresponding detector is referred to as an ordered SCSO detector.

A. The SCSO Detector

Recall that in the SO detector, LLRs are computed by searching all the codewords in the $C$ (see (14)). In contrast, the proposed SCSO detector produces the LLRs using a refined code, where the refined code (or subcode) contains some part of the codewords in the $C$ according to the previously decoded messages, as shown in Fig. 4. Since in the subcode, the distances among the codewords are larger than those in the $C$, an ambiguity of the detection can be reduced.

The detailed procedures of the SCSO detector are provided as follows. Focus on the LLR computations of the channel decoder $k$, with the knowledge of the $[k-1]$ users’ decoded messages $\tilde{w}_1^{k-1}[t] = \{\tilde{w}_1[t], \ldots, \tilde{w}_{k-1}[t]\}$. We first define a refined subcode of the $C$ as

$$C_{\{\tilde{w}_1^{k-1}[t] = \tilde{w}_k^{k-1}\}} \triangleq \{c = E(w) : w \in W, \tilde{w}_1^{k-1}[t] = \tilde{w}_k^{k-1}[t]\},$$

where its size is equal to $|C|/2^{k-1}$. Since the distances of the remaining codewords can be larger as $k$ increases, the SCSO detector can produce more reliable LLRs as the cancellation step proceeds (see Fig. 4). Then, using the refined subcode and (14), the enhanced LLRs are obtained as

$$\hat{L}_{pt-(i-1)}^k(r[t], \tilde{w}_1^{k-1}[t]) = \min_{c \in B^k_{(i,0)\tilde{w}_1^{k-1}[t]}} d_{wh}(r[t], c; \log \epsilon_{\ell,i}),$$

where the refined subcodes are constructed according to the known messages as

$$B^k_{(i,j)\tilde{w}_1^{k-1}[t]} = \bigcup_{b \in \{0,1\}^{p-b_i=j}} C_{\{w_h[t] = |b|, \tilde{w}_1^{k-1}[t] = \tilde{w}_k^{k-1}[t]\}}.$$  

By embedding the above LLRs $\{\hat{L}_{pt-(i-1)}^k(r[t], \tilde{w}_1^{k-1}[t]) : i \in [p], t \in [n/p]\}$ into the channel decoder $k$, the user $k$’s message is decoded as the bit-stream $B_k$. Also, the the $\{\tilde{w}_k[t] : t \in [n/p]\}$ are obtained from $B_k$, using the channel encoder and modulation function. Leveraging the decoded messages $\tilde{w}_1^{k-1}[t]$ and $\tilde{w}_k[t]$, then, the enhanced LLRs for the channel decoder $k + 1$ are computed. This process is repeatedly applied to (16)-(18) until all the $K$ users’ messages are decoded.

Remark 1: The complexity of LLR computation is proportional to the size of associated subcodes because the detector need to search them for the minimum operation in (14) and (17). Since the SO detector examines every codewords for each user, $K|C|$ number of comparison is needed. However, the SCSO detector reduces the size of subcodes by half each time. Therefore, total number of comparison is $|C| + |C|/2 + \cdots + |C|/2^{K-1} \approx 2|C|$. The ratio of the SCSO detector to the SO detector in the total number of comparison is $2/K$, so the complexity decreases as $K$ increases.

B. The Ordered SCSO Detector

We present an ordered SCSO detector which further improves the SCSO detector by carefully determining a decoding order. First of all, we notice that due to the non-linearity of the effective channel, it is not possible to use the SNR-based ordering in V-BLAST [18]. In the proposed method, we determine a decoding order in a greedy fashion: for each decoding step $i$, a user index $k_i$ is chosen from the remaining user indices such that the distance between two disjoint subcodes, where one is associated with $\tau_k[t] = 0$ and the other is associated with $\tau_k[t] = 1$, is maximized. This is motivated by the fact that LLRs with higher reliability tends to be obtained from far-off subcodes as if a lower-rate channel code performs better than a higher-rate channel code. In detail, a decoding order is determined as follows.

A user index $k_1 \in [K]$ to be decoded at the first step is chosen as

$$k_1 = \arg\max_{k \in [K]} \sum_{i \in [p]} \left\{ D_k \left( \bigcup_{|b| = 0} C_{|w_h[b]|} + \bigcup_{|b| = 1} C_{|w_h[b]|} \right) \right\}.$$  

where $D_k(C_1, C_2)$ represents the set distance between two codes $C_1$ and $C_2$. It is not obvious to find an optimal set distance $D_k(\cdot, \cdot)$, which is left for a future work. For the time being, we resort to using the mean distance (e.g., distance
between the centroids) as
\[
D_s(C_1, C_2) \triangleq \frac{1}{|C_1|^2} \sum_{c \in C_1} c - \frac{1}{|C_2|^2} \sum_{c \in C_2} c^2.
\] (20)

Next, a user index \(k_2 \in [K] \setminus \{k_1\}\) to be decoded in the second step is chosen using the previously decoded user \(k_1\)'s message \(\{\hat{w}_{k_1}^t : t = 1, \ldots, n/p\}\). Since \(\hat{w}_{k_1}^t\) can have a different value for various \(t\), the corresponding best user index can be different. Only one user index, however, should be selected for all time slots, in order to perform a sequential channel decoding. To meet this requirement, we select the user index to be chosen most frequently for the \(n/p\) time slots. Namely, we choose a \(k_2\) as
\[
k_2(\hat{w}_{k_1}) = \arg\max_{k \in [K] \setminus \{k_1\}} \sum_{i \in [p]} \left\{ D_s\left( \bigcup_{[b] \neq [0]} C_{[w_i]=[b],[w_{k_1}]=[\hat{w}_{k_1}]} \right) \right\}
\]
where \(\hat{w}_{k_1} = \text{Majority}\{\hat{w}_{k_1}^t : t = 1, \ldots, n/p\}\) represents the most frequent values in \(\{\hat{w}_{k_1}^t : t = 1, \ldots, n/p\}\).

Likewise, a user index \(k_\ell\) to be decoded at the \(\ell\)-th step is selected using the previously decoded messages \(\{\hat{w}_{k_1}^{\ell-1} : t = 1, \ldots, n/p\}\) as
\[
k_{\ell}(\hat{w}_{k_1}^{\ell-1}) = \arg\max_{k \in [K] \setminus \{k_1, \ldots, k_{\ell-1}\}} \sum_{i \in [p]} \left\{ D_s\left( \bigcup_{[b] \neq [0]} C_{[w_i]=[b],w_{k_1}^{\ell-1}=\hat{w}_{k_1}^{\ell-1}} \right) \right\}
\]
where \(\hat{w}_{k_1} = \text{Majority}\{\hat{w}_{k_1}^t : t = 1, \ldots, n/p\}\) for \(i = 1, \ldots, n/p\)

The above process is repeatedly applied to all the \(K\) users and the subsequent process is exactly identical with the proposed SCSSO detector (see Algorithm 1).

V. NUMERICAL RESULTS

In this section, we compare the proposed ordered SCSSO detector with the other detectors for the polar coded MU-MIMO systems with one-bit ADCs. A Rayleigh fading channel is assumed in which each element of a channel matrix \(H\) is drawn from an independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variable with zero mean and unit variance. We adopt a polar code of the blocklength 128 (e.g., \(n = 128\)) in [15]. The soft inputs (e.g., LLRs) of the polar decoder are computed from (14) and (17) for SO and ordered SCSSO detectors, respectively. For the simulation, a rate 1/2 polar code is used and SC polar decoder is used (see [15] for details). We would like to notice that similar trends in Figs. 5 and 6 are observed when using a more powerful SC list (SCL) polar decoder in [19]. We compare the coded MU-MIMO systems with various parameters \(K\) and \(N_r\). Each simulation results shows the coded frame-error-rate (FER) performance where the FER represents the average FER over \(K\) users. A perfect channel state information is assumed for all simulations.

A. Gain of successive cancellation

Fig. 5 shows the performance improvement of the proposed ordered SCSSO detector over the SO detector in [16] with respect to the number of receive antennas \(N_r\). From this simulation, we observe that the proposed detector can provide a suitable coded gain over the SC detector. For a target FER (e.g., \(10^{-1}\)), we can see that the performance gap between the proposed detector and the SO detector becomes larger as \(N_r\) decreases. When \(N_r\) is smaller for a fixed \(K\), the length of each codeword in the \(C\) is smaller and the number of \(|C|\) codewords are more densely located. Therefore, a refined subcode has more effect on the computation of LLRs.

B. Comparison with other detectors

Fig. 6 shows the coded FER performances for various soft-output MIMO detectors as the ZF-type detector [8], SO detector [16], and proposed ordered SCSSO detector. Note that ML detector [8], near-ML detector [8], and supervised-learning detector [11] are excluded in the comparison, since they cannot produce soft-outputs. Also, as already shown in [16], their performances are much worse than that of SO detector. We observe that the proposed detector significantly outperforms the ZF-type detector and the gap becomes larger as SNR increases. As observed in Figs. 5 and 6, it provides the additional coded gain over the SO detector. As noticed in Remark 2, the complexity of the proposed detector can be reduced as similar to that of ZF-type detector, by maintaining the performance. Due to its good performance and low-complexity, therefore, the proposed detector can be a good.
candidate as a MIMO detector for the uplink MU-MIMO system with one-bit ADCs.

Remark 2: The computational complexities of the both SO and SCSO detectors are problematic for a large $K$ since the size of the code $C$ (i.e., search-space) grows exponentially with $K$. Recently, the authors in [20] and [21] developed the efficient methods to significantly reduce the search-space, where the reduced search-space only takes the codewords from the $C$ that lie inside the sphere centered at the current observation $r(t)$ with a certain radius. Since these methods can be directly applied to the both SO and SCSO detectors, their complexities can be considerably reduced.

VI. CONCLUSION

We proposed the ordered successive-cancellation-soft-output (SCSO) detector for MU-MIMO systems with one-bit ADCs. The main idea of the proposed detector is that the previously decoded messages (fed by the channel decoders) are exploited to improve the reliabilities of the soft inputs of a subsequent channel decoder. Furthermore, we developed an efficient greedy algorithm to find a good decoding order. Via simulation results, it was shown that the ordered SCSO detector significantly outperforms the other detectors for the coded MU-MIMO systems with one-bit ADCs. One possible extension of this work is to optimize a distance measure between two subcodes defined over Hamming space.

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