Revealing QCD thermodynamics in ultra-relativistic nuclear collisions

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Based on 1908.09728, 1909.11609, in collaboration with Fernando Gardim, Giuliano Giacalone, Matt Luzum
A family of equations of state for ultra-relativistic baryonless matter is considered. These equations of state allow for a discontinuous or continuous transition from a low-entropy hadron gas to a high-entropy quark–gluon plasma. One analyzes the influence of the nature of the transition on the hydrodynamical flow expected in ultra-relativistic heavy ion collisions. Implications concerning the correlations between multiplicities and average transverse momenta are discussed.
~ 20 years later, somewhere near Trento
Another ~ 5 years later
Can we measure the equation of state of QCD in heavy ion collisions?

The equation of state is accurately calculated in lattice QCD.

Can we confirm some of these results with heavy-ion data?

Borsanyi et al, 1309.5258
Consider a gas at rest in thermal equilibrium, in a container of volume $V$ isolated in vacuum. At time $t=0$, one lets the gas expand freely into the vacuum.

Ideal hydrodynamic expansion conserves both the total energy $E$ and the total entropy $S$.

Measuring the number of particles $N$ and the energy per particle $E/N$ in the final state, one reconstructs the initial thermodynamic state if one knows the entropy per particle $S/N$ and the initial volume $V$, irrespective of the details of the hydro expansion.
The idea of Van Hove (1982)

The mean transverse momentum of charged hadrons, $<p_t>$, is a fraction of the energy per particle: proportional to temperature $T$.

The multiplicity $N_{ch}$ is proportional to the entropy density $s$ if the volume is fixed.

Vary the collision energy: $<p_t>$ vs. $N_{ch}$ gives $T$ versus $s$. 
In heavy-ion collisions, we only observe a slice of fluid near mid-rapidity, which is not an isolated system. Its energy decreases according to $dE=-PdV$ as the system expands.

Early on, the energy of the rapidity slice decreases typically like $t^{-1/3}$ (Bjorken expansion).

Later, transverse expansion sets in, the pressure decreases rapidly and the energy rapidly converges to a final value.
What we did back in 1987

The temperature one measures through $\langle p_T \rangle$ is not the initial temperature at $t_0$, but the temperature after the entropy density $s$ has decreased by a factor $\sim t_0/R$ through longitudinal cooling.

$$(s \, t_0/R)^{1/3}$$
An equivalent idea, but simpler and more precise:

Because of longitudinal cooling, the relevant energy is no longer the initial energy, but the final energy. 

<pt> should tell us the temperature corresponding to the final energy and entropy.
Effective temperature, effective volume

We define the effective temperature, $T_{\text{eff}}$, and the effective volume, $V_{\text{eff}}$, of the quark-gluon plasma, as those of a uniform fluid at rest which would have the same energy $E$ and entropy $S$ as the fluid at freeze-out.

\[ E = \int_{\text{f.o.}} T^0_{\mu} d\sigma_{\mu} = \epsilon(T_{\text{eff}}) V_{\text{eff}}, \]

\[ S = \int_{\text{f.o.}} s u^\mu d\sigma_{\mu} = s(T_{\text{eff}}) V_{\text{eff}}, \]

I will show that $T_{\text{eff}}$ and $s(T_{\text{eff}})$ can be obtained from experiment.
Effective temperature, effective volume

Put the total energy and entropy contained in the system (just before it transforms into hadrons) into a uniform cylinder.

Effective temperature and volume are those of this cylinder.
Value of $T_{\text{eff}}$ in hydro simulations of Pb+Pb @ 5.02 TeV

We use the MUSIC code, where the initial temperature is tuned to reproduce the charged multiplicity measured by ALICE for each centrality.

- Ideal hydrodynamics
- Viscous hydro with shear viscosity, $\eta/s=0.2$
- Viscous hydro with bulk viscosity, Duke parametrization
Value of $<p_t>$ in hydro simulations of Pb+Pb @ 5.02 TeV

(after resonance decays)

Ideal hydrodynamics
Viscous hydro with shear viscosity, $\eta/s=0.2$
Viscous hydro with bulk viscosity, Duke parametrization
Reviving Van Hove’s idea

\(<p_t> = 3.07 \ T_{\text{eff}}\) for all centralities, irrespective of bulk and shear viscosity!

Ideal hydrodynamics

Viscous hydro with shear viscosity, \(\eta/s=0.2\)

Viscous hydro with bulk viscosity, Duke parametrization
Varying the freeze-out temperature

$T_{\text{eff}}$ is remarkably independent of the freeze-out temperature, which confirms that the longitudinal cooling is no longer active when $T<160$ MeV.
Value of $T_{\text{eff}}$ from experiment

Extraction from data is straightforward. ALICE measures $\langle p_t \rangle = 681$ MeV in Pb+Pb @ 5.02 TeV in 0-5% centrality bin.

This implies $T_{\text{eff}} = \langle p_t \rangle / 3.07 = 222 \pm 9$ MeV,

where the error is estimated by varying the freeze-out temperature.
Next step: entropy density at $T_{\text{eff}}$

Entropy density = $S/V_{\text{eff}}$

$S$ = entropy at freeze-out, by definition of $V_{\text{eff}}$ and $T_{\text{eff}}$

$S/N_{\text{ch}} = 6.7 \pm 0.8$ after resonance decays,

$N_{\text{ch}}$ is measured, therefore, $S$ is known

Effective volume $V_{\text{eff}}$ cannot be extracted from data. Comes from a hydrodynamic calculation.
Estimating the effective volume

\[ V_{\text{eff}} \text{ is proportional to } R_0^3, \text{ where } R_0 = \text{ initial transverse size} \]

Viscous hydro with bulk viscosity, Duke parametrization

Ideal hydrodynamics

Viscous hydro with shear viscosity, \( \eta/s=0.2 \)
Entropy density at $T_{\text{eff}}$

We obtain $S/V_{\text{eff}} = s(T_{\text{eff}}) = 20 \pm 5 \text{ fm}^{-3}$.

error : 40% from initial size $R_0$, which depends on the model of initial conditions

60% from transport coefficients, which modify $V_{\text{eff}}/R_0^3$
Comparison with lattice QCD

$\frac{s}{T^3}$

$\frac{\varepsilon}{T^4}$

Entropy density
Comparison with lattice QCD

\[ T_{\text{eff}} = 222 \pm 9 \text{ MeV} \]

\[ s(T_{\text{eff}})/T_{\text{eff}}^3 = 14 \pm 3.5 \]

compatible with lattice.

Confirms large number of degrees of freedom, implying that color is liberated: deconfinement observed!
Varying the collision energy

As $\sqrt{s}$ increases, $T_{\text{eff}}$ increases, $V_{\text{eff}}$ remains constant. Increasing energy amounts to heating the system at constant volume.
Varying the collision energy

The variation of \( \langle p_t \rangle \) still closely follows that of \( T_{\text{eff}} \)
Varying the collision energy

Deviations from $<p_t>=3.07$ $T_{\text{eff}}$ are negligible at LHC energy and beyond
Parenthesis: Thermodynamic details

At RHIC energies, $\langle p_t \rangle$ is slightly steeper than $T$.

Back in 1987, we showed that around the transition region, $\langle p_t \rangle$ follows the energy over entropy ratio $\varepsilon/s$, rather than $T$.

In a baryonless plasma, $\frac{3}{4} T < \varepsilon/s < T$ so $\varepsilon/s$ and $T$ are almost proportional. Hadron to QGP transition: $T$ is almost constant, but $\varepsilon/s$ keeps increasing. This is probably what we see here.
The physics:
Increasing the collision energy amounts to putting more energy into a fixed volume. Gives direct access to the compressibility=speed of sound.

The math:
\[ T_{\text{eff}} \] proportional to \( \langle p_t \rangle \)
\[ s(T_{\text{eff}}) \] proportional to \( dN_{\text{ch}}/d\eta \)

\[ c_s^2(T_{\text{eff}}) \equiv \frac{dP}{d\varepsilon} = \frac{sdT}{T ds} \bigg|_{T_{\text{eff}}} = \frac{d \ln \langle p_t \rangle}{d \ln (dN_{\text{ch}}/d\eta)} \]
Speed of sound $c_s$ in the QGP

We obtain $c_s^2(T_{\text{eff}}) = 0.24 \pm 0.04$ (error from variation of $V_{\text{eff}}$)
Comparison with lattice QCD

compatible with lattice
Predictions for ultracentral collisions

Ultracentral collisions: beyond the knee, impact parameter is close to 0 but multiplicity keeps increasing

\[ P_{\mu,k} \times \left[ f N_{\text{part}} + (1-f)N_{\text{coll}} \right] \]

\[ f = 0.801, \mu = 29.3, k = 1.6 \]

VZERO amplitude = quantity used by ALICE to determine the centrality

ALICE arXiv:1301.4361
Predictions for ultracentral collisions

Zoom of the V0 distribution in ultracentral collisions

In a model of initial state (Trento model) tuned to reproduce the V0 distribution, the transverse radius saturates beyond the knee

The entropy density increases beyond the knee: hence $T_{\text{eff}}$ increases
We predict an increase of $\langle p_t \rangle$ in ultracentral collisions. No hydro, no free parameter.
We take $c_s^2$ from lattice EOS.

$$c_s^2(T_{\text{eff}}) \equiv \frac{dP}{d\varepsilon} = \frac{sdT}{Tds} \bigg|_{T_{\text{eff}}} = \frac{d\ln \langle p_t \rangle}{d\ln(dN_{\text{ch}}/d\eta)}$$
Summary

• Even though the hydro evolution spans a wide interval of temperatures, only a narrow range matters in practice for $<p_t>$ (probably also $v_n$), corresponding to the time where the transverse expansion develops.

• One can reconstruct the thermodynamics from $<p_t>$ and the multiplicity $dN/d\eta$ in this temperature range, 200-220 MeV at LHC.

• Results agree with lattice QCD and clearly confirm that a quark-gluon plasma, where color degrees of freedom are liberated, is produced during the collision.

• While the observed $<p_t>$ is constant up to 1% centrality, we predict that it increases by 18 MeV between 1% and 0.001% centrality.
Supplementary material
Event-to-event fluctuations

$T_{\text{eff}}$ varies event to event, but the ratio $\langle p_t \rangle / T_{\text{eff}}$ is essentially constant.

Hence, event-to-event fluctuations do not change the determination of $T_{\text{eff}}$ from data.
Definition of initial radius $R_0$

$$(R_0)^2 \equiv \frac{2 \int |\mathbf{r}|^2 s(\tau_0, \mathbf{r})}{\int s(\tau_0, \mathbf{r})}.$$ 

where $s(\tau_0, \mathbf{r})$ is the entropy density profile at the beginning of the hydro evolution, and integration is over the transverse plane. The factor 2 ensures that one recovers the correct result for a uniform uniform density in a circle of radius $R_0$. 
System-size (in)dependence

In hydro, $<p_t>/T_{\text{eff}}$ is identical in Pb+Pb and Xe+Xe collisions.

In experiment, $<p_t>$ is essentially the same in both systems, therefore $T_{\text{eff}}$ is also the same.

$V_{\text{eff}}$ and the multiplicity are both proportional to $A$ at a given centrality percentile.