X(3915) and X(4350) as new members in P-wave charmonium family

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The analysis of the mass spectrum and the calculation of the strong decay of P-wave charmonium states strongly support to explain the newly observed X(3915) and X(4350) as new members in P-wave charmonium family. In this letter we adopt another point of view to explore whether the newly observed X(3915) and X(4350), under the P-wave charmonium assignment to X(3915) and X(4350), the J(PC) quantum numbers of X(3915) and X(4350) must be 0++ and 2++ respectively, which provide the important criterion to test P-wave charmonium explanation for X(3915) and X(4350) proposed by this letter. The decay behavior of the remaining P-wave charmonium states with the second radial excitation is predicted, and experimental search for them is suggested.

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Recently two new charmonium-like states X(3915) and X(4350) were released by the Belle Collaboration in the γγ fusion process \cite{1,2}. X(3915) is observed by the invariant mass spectrum of J/ψω in γγ → J/ψω channel. The mass and width of X(3915) are M = 3915 ± 3(stat) ± 2(sys) MeV and Γ = 17 ± 10(stat) ± 3(sys) MeV \cite{1}. X(4350), a new charmonium-like state found in the invariant mass spectrum of J/ψΦ, is of m = 4350 ± 5(stat) ± 0.7(sys) MeV and Γ = 13.3 ± 1.9(stat) ± 4.1(sys) MeV \cite{2}.

Until now, Belle experiment has reported three charmonium-like states via the γγ fusion. Besides X(3915) and X(4150), Z(3930) is a charmonium-like state observed in γγ → DD∗, which is of mass m = 3929 ± 5 ± 2(syst) MeV and Γ = 29 ± 10(stat) ± 2(syst) MeV \cite{3}. The angular distribution in the γγ center of mass frame shows J^PC = 2++, which indicates that Z(3930) is a good candidate of χc2, i.e., a charmonium with n^2_s+1J_L = 2^3P_2 \cite{3}.

The observations of X(3915) and X(4350) not only make the spectroscopy of charmonium-like state observed by the γγ fusion process become abundant, but also help us further reveal the underlying structure of charmonium-like states observed by the γγ fusion. Before illustrating the underlying structure of X(3915) and X(4350), one first gives a brief review of the established P-wave charmonium states or their possible candidates by Fig. 1, where h_c(3525) is a P-wave state with spin 0 is not listed. Three P-wave states without the radiative excitation are χ_{c0}(3415), χ_{c1}(3510), and χ_{c2}(3556) \cite{4}. For the radial excitation of P-wave charmonium, the candidate for 0++ state χ’_{c0} is still absent while X(3872) \cite{5,6} and Z(3930) \cite{5} can be recommended as 1++ state χ’_{c1} and 2++ state χ’_{c2}, respectively.

Usually the γγ fusion process provides a good environment to create charmonium by γγ fusion into a pair of c ¯ c. Although X(3915) seems to be explained as an exotic state indicated in Ref. \cite{3}, in this letter we adopt another point of view to explore whether the newly observed X(3915) can fill in the blank of the remaining P-wave charmonium with the first radial excitation shown in Fig. 1. The mass of X(3915) is consistent with the result of the potential model, which once predicted the mass of the first radial excitation X’_{c0} is around 3916 MeV according to Godfrey-Isgur relativized potential model \cite{3}.

Since there exists the vanishing coupling of X(3915) − DD∗ and the week interaction of Z(3930) − D D∗ while X(3872) with J^PC = 1++ interacting with D D∗ via S-wave is very strong, the coupled channel effect on bare χ’_{c0} and χ’_{c1,2} is weaker than that on bare χ’_{c1,2} \cite{5,6}, which explains why the mass difference between X(3915) and Z(3930) is smaller than that between X(3915) and X(3872).

![FIG. 1: (Color online.) The established P-wave state charmonium states without the radial excitation \cite{4} and the candidate for the first radial excitation of P-wave state. A comparison between newly observed X(4350) and the candidate of P-wave charmonium states is given.](image)

By the comparison of X(4350) with the existed P-wave states, one notices that the mass difference between X(4350) and Z(3930) is about 420 MeV, which is similar to that between Z(3930) and χ_{c2}(1P). The regularity of the mass gaps existing χ_{c2}(1P), Z(3930) and X(4350) is consistent with the estimate from Resonance Spectrum Expansion (RSE) model, which indicates the mass gap between the states with the radial quantum numbers n and n + 1 is 380 MeV \cite{1, 11}. Thus,
one further proposes that $X(4350)$ is as the second radial excitation of P-wave charmonium state. We also notice the prediction of the mass of $\chi''_{cJ}$ in Ref. \[9\] by Godfrey-Isgur relativized potential model, which is about 4337 MeV. This value is consistent with the mass of $X(4350)$.

| State | Modes | Decay channels |
|-------|-------|----------------|
| $\chi'_{c0}$ | $0^++0^-$ | $DD$ |
| | $0'+0^-$ | $D\bar{D}, D_1\bar{D}_1, D_2\bar{D}_2$ |
| | $1'^+1^-$ | $D_1\bar{D}_1, D_2\bar{D}_2$ |
| | $0'+1^+$ | $D_1\bar{D}_1(2430) + h.c., D_2\bar{D}_2(2420) + h.c.$ |
| $\chi''_{c0}$ | $0^++0^-$ | $DD, D_1\bar{D}_1, D_2\bar{D}_2$ |
| | $0'+0^-$ | $D_1\bar{D}_1(2430) + h.c., D_2\bar{D}_2(2420) + h.c.$ |
| $\chi'_{c1}$ | $0^++0^-$ | $DD, D_1\bar{D}_1, D_2\bar{D}_2$ |
| | $0'+0^-$ | $D_1\bar{D}_1(2430) + h.c., D_2\bar{D}_2(2420) + h.c.$ |
| $\chi''_{c1}$ | $0^++0^-$ | $DD, D_1\bar{D}_1, D_2\bar{D}_2$ |
| | $0'+0^-$ | $DD, D_1\bar{D}_1(2430) + h.c., D_2\bar{D}_2(2420) + h.c.$ |

TABLE I: The allowed open-charm strong decays of $\chi'_{c0}$ and $\chi''_{c0}$ ($J = 0, 1, 2$). Here, we take 4350 MeV as the upper limit of the mass of $\chi''_{c0}$. $D_1(2420)$ is the $1^+$ state in the $T = (1^+, 2^+)$ doublet while $D_1(2430)$ is the $1^+$ state in the $S = (0^+, 1^+)$ doublet as indicated in Ref. \[17\].

The decay modes of the created charmonium state from the $\gamma\gamma$ fusion include open-charm and hidden-charm decays, which are the observed decay channel of $Z(3930)$ and $X(3915)/X(4350)$, respectively. For testing the proposal for the structure of $X(3915)$ and $X(4350)$, in the following, we further study open-charm decay of the radial excited P-wave charmonium $\chi'_{c0}$ and $\chi''_{c0}$ ($J = 0, 1, 2$) by the Quark Pair Creation (QPC) model \[12,14\], which is a successful phenomenological model to calculate Okubo-Zweig-Iizuka (OZI) allowed strong decays of hadron.

The allowed decay modes of $\chi'_{c0}$ and $\chi''_{c0}$ are presented in Table I. The QPC model provides us an effective approach to study the two-body strong decays of the radial excited P-wave charmonium $\chi'_{c0}$ and $\chi''_{c0}$.

The transition matrix element of the process $A(c(1)\bar{c}(2)) \rightarrow B(c(1)\bar{q}(3)) + C(\bar{c}(2)q(4))$ in the center of mass frame of charmonium $A$ is written as $\langle BC|T|A\rangle = \delta^K(g_B + K_C)M^{M_B,M_C}_{M_A,M_B,M_C}(K)$, where the transition operator $T$ in the QPC model reads as

$$T = -3\gamma \sum_m \langle 1m|1-m|00 \rangle \int dk_3 \, dk_4 \, \delta^3(k_3 + k_4) \times Y_{lm}\left(\frac{k_3 - k_4}{2}\right) \chi_{1,m}^{34} \bar{\psi}_0^{34} d_i^{33}(k_3) b_j^{34}(k_4),$$

where $i$ and $j$ denote the SU(3) color indices of the created quark and anti-quark from the vacuum. $\varphi_0^{34} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\omega_0^{34} = \delta_{ij}\bar{s}/\sqrt{3}$. The strength of the quark pair creation from the vacuum is given by the fit in the data. In this letter, $\gamma = 6.3$ \[13\]. The strength of $s\bar{s}$ creation satisfies $\gamma_s = \gamma/\sqrt{3}$. The helicity amplitude $M^{M_B,M_C}_{M_A,M_B,M_C}(K)$ is extracted by the transition matrix element, which is related to the partial wave amplitude by \[16\].

$$M^{ij}(A \rightarrow BC) = \frac{\sqrt{2L + 1}}{2J_i + 1} \sum_{M_{B,C}} \langle 0LJM_i | J_i M_B \rangle \times \langle J_B M_B | C M_C | J_M \rangle M^{M_A,M_B,M_C}_{M_A,M_B,M_C}(K),$$

where $i$ and $j$ denote the SU(3) color indices of the created quark and anti-quark from the vacuum.
where \( J = J_F + J_S + J_L \) and \( J_\ell + J_F = J_B + J_C + L \). A detailed review of the QPC model was illustrated in Ref. \[17\]. The partial wave amplitude corresponding to the open-charm decays shown in Table \[11\] is presented in Table \[11\]. The concrete expressions of \( Q_{i,j}, P_{i,j} \) and \( P_{i,j,k} \) are extracted from the spatial integral \( \int_{\mathbb{R}^3}^{n \in T} T_{\mu_\ell \mu_\ell} \) (K), which describes the overlap of the initial meson (A) and the created pair with the two final mesons (B and C). Here, the harmonic oscillator (HO) wave function \( \Psi_{n,m}(k) = R_{\ell}(R,k) \) is involved in the calculation of the spatial integral. Parameter \( R \) in the HO wave function is obtained by reproducing the realistic root mean square (RMS) radius by solving the schrödinger equation with the linear potential \[13\]. The relevant mass values are taken in PDG \[4\]. We take \( R = 1.52, 1.41, 1.85, 1.69, 1.85, 1.75, 2.00, 2.00 \) GeV\(^{-1}\) corresponding to \( D, D_s, D^*, D^*_c, D_{s0}(2400), D_{s0}^*(2317), D_1(2430) \) and \( D_{1}(2420) \), respectively. Besides, other parameter inputs include \( m_c = 1.6 \) GeV, \( m_u = m_d = 0.22 \) GeV and \( m_s = 0.419 \) GeV.

In the right diagram of Figs. \[2\] one presents the total width of \( \chi_{c0}^\prime \) with the variation of \( R \). The node effect from the wave function of higher radial excited states results in the decay width calculated by the QPC model being dependent on the \( R \) value. When taking \( R = 1.80 \sim 1.99 \) GeV\(^{-1}\), the obtained total open-charm decay width of \( \chi_{c0}^\prime \) falls in the range of total width of \( X(3915) \) released by Belle \[1\] (the calculation result with the typical value \( R = 1.92 \) GeV\(^{-1}\) corresponds to the central value of the width of \( X(3915) \)). As a \( \chi_{c2}^\prime \) charmonium state \[3\], \( Z(3930) \) can be as a realistic test of the reasonability of the range of \( R \) for \( \chi_{c0}^\prime \). Our theoretical result of the open-charm decay of \( \chi_{c2}^\prime \) dependent on \( R \) and the comparison of the calculation result with Belle data of \( Z(3930) \) \[3\], which are shown in the left diagram of Figs. \[2\] indicate that the upper limit of \( R \) for \( \chi_{c2}^\prime \) is very close to the lower limit of \( R \) for \( \chi_{c0}^\prime \) as marked by the red arrows in Fig. \[2\] which further shows the reliability of investigating \( \chi_{c2}^\prime \) open-charm decay in the range of \( R = 1.80 \sim 1.99 \) GeV\(^{-1}\). Thus, explaining \( X(3915) \) as a \( \chi_{c0}^\prime \) charmonium is tested through the open-charm decay of \( \chi_{c0}^\prime \).

Considering an exotic \( D^*D^* \) molecule explanation to \( X(3915) \) suggested in Ref. \[8\], one proposes the experimental study of the open-charm decay \( DD \) to be as a best way to distinguish between the exotic and the conventional states for the controversial \( X(3915) \) since the \( DD \) decay of \( X(3915) \) under the \( D^*D^* \) molecule assignment occurs via hadronic loop effect as indicated in Ref. \[18\], which results in the decay width of \( X(3915) \to DD \) under the \( D^*D^* \) molecule assignment being far smaller than that under \( \chi_{c0}^\prime \) explanation to \( X(3915) \).

The results of the open-charm decays of \( \chi_{c0}^\prime \) (\( J = 0, 1, 2 \)) are presented in Fig. \[3\] via scanning the parameter space \( R = 1.8 \sim 3.0 \) GeV\(^{-1}\), which is due to the radius \( R \) of \( \chi_{c0}^\prime \) being fatter than that of \( \chi_{c2}^\prime \). We need to emphasize that the \( \gamma \gamma \) fusion process determines the most possible quantum number of \( X(4350) \) to be \( 0^{++} \) or \( 2^{++} \), which makes us choose \( X(4350) \) as the candidate of \( \chi_{c0}^\prime \) or \( \chi_{c2}^\prime \) and fully exclude \( \chi_{c3}^\prime \) assignment to \( X(4350) \) \[2\]. The open-charm decays of \( \chi_{c0}^\prime \) and \( \chi_{c2}^\prime \), two candidates of \( Y(4350) \), display different behaviors as illustrated in the left and right diagrams of Fig. \[3\]. The total open-charm decay of \( \chi_{c2}^\prime \) with \( R = 1.9 \sim 2.3 \) GeV\(^{-1}\) is well consistent with Belle data \[2\] as shown in the right diagram of Fig. \[3\] while the total open-charm decay of \( \chi_{c0}^\prime \) is far away from the Belle data \[2\], which shows that we can fully exclude \( \chi_{c0}^\prime \) explanation for \( X(4350) \) and finally establish \( X(4350) \) as a good candidate of \( \chi_{c2}^\prime \). Our numerical result demonstrates that \( 1^{+} \to 0^{+} \to 1^{+} \) (\( DD^* + h.c., D_1^*D_1^* + h.c., D_1^*D^* \)) are the dominant decay channels of \( X(4350) \).
Meanwhile, the predicted properties of the remaining two P-wave charmonium states with the second radial excitation can be as the guidance of experimental search for $\gamma^{\prime\prime}_{10}$ and $\gamma^{\prime\prime}_{11}$. The predicted total decay width of $\gamma^{\prime\prime}_{10}$ is around 82 ~ 110 MeV corresponding to $R = 2.0 ~ 3.0$ GeV$^{-1}$, which is not strongly dependent on the $R$ values. As the dominant decay mode of $\gamma^{\prime\prime}_{10}$, $0^{-+}$ channel including $D\bar{D}$ and $D^{*+}D^{-}$ is a golden channel to find $\gamma^{\prime\prime}_{10}$. For $\gamma^{\prime\prime}_{11}$, its total decay width is of large span from 47 MeV to 140 MeV corresponding to $R = 2.0 ~ 3.0$ GeV$^{-1}$. Among its partial decay channels, $0^{-1}$ channel is always the main decay channel of $\gamma^{\prime\prime}_{11}$ under taking different values of $R$, which indicates that $D\bar{D}^* + h.c.$ and $D_s^0D_s^* + h.c.$ can be as the suggested decay channel of searching for $\gamma^{\prime\prime}_{11}$. The detail of the open-charm decay behaviors of $\gamma^{\prime\prime}_{11}$ are listed in Fig. 3.

In summary, the newly observed $X(3915)$ and $X(4350)$ are firstly explained as $\gamma^{\prime\prime}_{00}$ and $\gamma^{\prime\prime}_{01}$ extremely well respectively by analyzing the mass spectrum of P-wave charmonium family as well as by calculating the open-charm strong decay of $X(3915)$ and $X(4350)$, which are consistent with the existed experimental findings. Just because of our explanations to $X(3915)$ and $X(4350)$, the spectroscopy of P-wave charmonium becomes abundant. Under the assignment of $X(3915)$ to $X(4350)$, the $J^{PC}$ quantum numbers of $X(3915)$ and $X(4350)$ will be definite, i.e. $J^{PC} = 0^{++}$ for $X(3915)$ and $J^{PC} = 2^{++}$ for $X(4350)$, which provides the powerful criterion to test the P-wave charmonium assignment for $X(3915)$ and $X(4350)$ since the experimental analysis of the angular distribution of $X(3915)$ and $X(4350)$ can give the concrete information of their quantum numbers with model independent. Additionally, in this work, we also predict the decay behaviors of the two remaining second radial excited P-wave charmonium states $\gamma^{\prime\prime}_{00}$ and $\gamma^{\prime\prime}_{11}$. These findings are expected to be revealed in future experiment.

This study can be extended to include the theoretical study of the hidden-charm decay, the radiative decay and the double-photon decay of $X(3915)$ and $X(4350)$, which will provide us valuable information of their underlying structure.

Note added: Recently, a theoretical work using QCD sum rule [19] shows that it is not possible to describe the $X(4350)$ structure as a $1^{-+}D_s^0D_s^{*+}$ molecular state, which supports our effort to explain $X(4350)$ under the conventional charmonium to some extent.

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