QCD Equation of State and Hadron Resonance Gas

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arXiv:0912:2541
Hadron Resonance Gas model

- EoS of interacting hadron gas well approximated by non-interacting gas of hadrons and resonances

\[ P(T) = \sum_i \int d^3p \frac{p^2}{3E} f(p, T) \]

- valid when
  - interactions mediated by resonances

- Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts
- Gerber & Leutwyler, NPB321, 387 (1989): chiral perturbation theory
  \Rightarrow \text{HRG good approximation at low temperatures}
  \rightarrow \text{lattice should reproduce HRG at } T \leq 120 - 140 \text{ MeV}

- practical problem: how to convert \text{fluid} to \text{particles}?
- energy conservation iff EoS is the same before and after freeze-out
EoS by hotQCD collaboration

Bazavov et al. arXiv:0903.4379 [hep-lat]

- evaluate interaction measure $(\epsilon - 3P)/T^4$
- obtain pressure via

$$ \frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\epsilon - 3P}{T'^5} $$

- What is $P(T_0)$?
- What is $(\epsilon(T_0) - 3P(T_0))/T_0^4$?
- How good is lattice below $T_c$?
Trace anomaly below $T_c$

Bazavov et al arXiv:0903.4379

\[(\epsilon-3p)/T^4\]

- asqtad: $N_\tau=8$
- $p4: N_\tau=8$

• Lattice EoS ≠ Hadron Resonance Gas EoS
Hadrons on lattice

• Hadron masses depend on lattice cutoff
  ⇒ i.e. on temperature:
  E.g. for pseudoscalar mesons

\[ m_{ps_i}^2 = m_{ps_0}^2 + \frac{1}{r_1^2} \frac{a_i}{b_{ps} x^2} \left( 1 + c_{ps} x \right)^{\beta_i} \]

\[ x = \left( \frac{a}{r_1} \right)^2 \]

\[ a = \frac{1}{N \tau T} \]

• 16 pseudoscalar mesons on lattice

• HRG with lattice mass spectrum?
Hadronic fluctuations

i.e. baryon number, strangeness and charge susceptibilities

\[
\chi_2^x = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_x/T)^2} = \frac{1}{T^2} \frac{\partial^2 P}{\partial \mu_x^2},
\]

where \( \mu_x = \mu_B, \mu_S \) or \( \mu_Q \)

- **Lattice masses** → fluctuations in resonance gas and lattice *similar*
• very little room for modifications in hadron gas
• **BUT**, what is physical mass spectrum?
• **conservative estimate**: free particle masses
Phenomenological EoS

- $T < T_{sw}$: HRG interaction measure (black)
- $T > T_{sw}$: Lattice interaction measure (red)
- lattice $N_{\tau} = 6$ data, Cheng et al. Phys. Rev. D 77, 014511 (2008)

- $\epsilon$ and $P$ overshoot Stefan-Boltzmann limit!
- Interaction measure too large, but where?
Interaction measure

**Cheng et al ('08)**

\[
\frac{(\varepsilon - 3p)}{T^4} \quad \text{Tr}_0
\]

\[
\text{p4: } N_T = 4 \quad \text{p4: } N_T = 6 \quad \text{p4: } N_T = 8
\]

\[
T \ [\text{MeV}]
\]

**Bazavov et al ('09)**

\[
\frac{(\varepsilon - 3p)}{T^4} \quad \text{Tr}_0
\]

\[
\text{asqtad: } N_T = 8 \quad \text{p4: } N_T = 6 \quad \text{p4: } N_T = 8
\]

\[
T \ [\text{MeV}]
\]

- peak region sensitive to \( N_T \)
Procedure for EoS

- **HRG below** $T \approx 180 - 190$ MeV
- **Parametrize** lattice using:

$$\frac{\epsilon - 3P}{T^4} = \frac{d_2}{T^2} + \frac{d_4}{T^4} + \frac{c_1}{T^{n_1}} + \frac{c_2}{T^{n_2}}$$

- **Require** that:

$$\left. \left| \frac{\epsilon - 3P}{T^4} \right| \right|_{T_0}, \quad \left. \frac{d}{dT} \left| \frac{\epsilon - 3P}{T^4} \right| \right|_{T_0}, \quad \left. \frac{d^2}{dT^2} \left| \frac{\epsilon - 3P}{T^4} \right| \right|_{T_0} \quad \text{are continuous}$$

$$\left. \left| \frac{P}{T^4} \right| \right|_{T=800\text{MeV}} = 0.95 \frac{S_{SB}}{T^4}$$

$\implies T_0, d_4, c_1, c_2$ fixed

- $\chi^2$ **fit to lattice above** $T = 250$ MeV + **one point** at $T = 206$ MeV

- We get $T_0 = 183.8$ MeV, $d_2 = 0.2660$, $d_4 = 2.403 \cdot 10^{-3}$, $c_1 = -2.809 \cdot 10^{-7}$, $c_2 = 6.073 \cdot 10^{-23}$, $n_1 = 10$, $n_2 = 30$
• obtain pressure via

\[ \frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\epsilon - 3P}{T'^5} \]
• no softening below the HRG!
Bazavov and Petreczky, arXiv:1005.1131

\[
\frac{(\varepsilon - 3p)}{T^4}
\]

HISQ, \(N_f=8\)
HISQ, \(N_f=6\)
asqtad
p4
Laine
s95p-v1
Bazavov and Petreczky, arXiv:1005.1131

\[ \frac{(\varepsilon-3p)}{T^4} \]

HISQ, \( N_t=8 \)

HISQ, \( N_t=6 \)

asqtad

p4

Laine

s95p-v1

Borsányi et al, arXiv:1005.3508

\[ \frac{(\varepsilon-3p)}{T^4} \]

HRG physical

HRG distorted stout \( N_t=8 \)

HRG distorted asqtad \( N_t=8 \)

stout \( N_t=10 \)

stout \( N_t=8 \)

asqtad \( N_t=8 \)

p4 \( N_t=8 \)
Ideal hydrodynamics

matter in local equilibrium: \( T^{\mu\nu} = (e + p)u^{\mu}u^{\nu} - pg^{\mu\nu} \), \( N^{\mu} = nu^{\mu} \)

local, macroscopic variables: energy density \( e(x) \)
pressure \( p(x) \)
flow velocity \( u^{\mu}(x) \) \( (u^{\mu}u_{\mu} = 1) \)
baryon density \( n(x) \)

energy-momentum and charge conservation:

\[
\begin{align*}
\partial_{\mu}T^{\mu\nu}(x) &= 0 \\
\partial_{\mu}N^{\mu}(x) &= 0
\end{align*}
\]

Unknowns: initial state, final state

matter characterized by: equation of state \( p(e, n) \)
Elliptic flow $v_2$

Spatial anisotropy $\rightarrow$ final azimuthal momentum anisotropy

$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$

$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$

Sensitive to speed of sound $c_s^2 = \frac{\partial p}{\partial e}$ and shear viscosity $\eta$
Effect on flow I

- ideal fluid, \( b = 7 \text{ fm} \)
- keep everything fixed:
  - \( \tau_0 = 0.6 \text{ fm/c}, T_{dec} = 125 \text{ MeV} \)

\[
\Rightarrow \text{harder EoS, flatter spectra}
\]
Effect on flow II

- ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV
- chemical equilibrium

- s95p: $T_{dec} = 140$ MeV
- EoS Q: first order phase transition at $T_c = 170$ MeV, $T_{dec} = 125$ MeV
Chemical non-equilibrium

- ideal fluid, $b = 7$ fm
- keep everything fixed:
  - $\tau_0 = 0.2$ fm/$c$, $T_{chem} = 150$ MeV, $T_{dec} = 120$ MeV

$\Rightarrow$ harder EoS, flatter spectra
Effect on flow III

- ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV
- $T_{\text{chem}} = 150$ MeV

- EoS Q: $T_{\text{dec}} = 120$ MeV, $s_{\text{ini}} \propto N_{\text{bin}}$, $\tau_0 = 0.2$ fm/c
- s95p, $\tau_0 = 0.8$: $T_{\text{dec}} = 120$ MeV, $s_{\text{ini}} \propto N_{\text{bin}}$, $\tau_0 = 0.8$ fm/c
- s95p, $\tau_0 = 0.2$: $T_{\text{dec}} = 120$ MeV, $s_{\text{ini}} \propto N_{\text{bin}} + N_{\text{part}}$, $\tau_0 = 0.2$ fm/c
Conclusions

- below $T_c$ lattice and HRG differ because of hadron mass spectrum

⇒ HRG **good description** below $T_c$

- some uncertainty in the parametrization of the EoS

⇒ but it **doesn’t matter**

- proton $v_2(p_T)$ may or may not be sensitive to EoS — **details matter!**

- EoS tables available at
  - https://wiki.bnl.gov/hhic/index.php/Lattice_calculations_of_Equation_of_State
  - and
  - https://wiki.bnl.gov/TECHQM/index.php/QCD_Equation_of_State