Scalability of increase in spall threshold in the presence of cylindrical protrusions on metals surface

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Abstract. The article analyzes the extent to which the previously detected effect of increase in spall threshold due to nanorelief on the surface can be scaled to large dimensions of the target, the impactor and the relief elements. Earlier, it was shown on the basis of molecular dynamics simulation that the presence of a relief can significantly increase the spall threshold due to plastic deformation in the surface layer of metal and energy dissipation of the reflected shock wave. To study the scalability of this effect on the case of macroscopic targets, and not only on nanoscale systems, an analytical estimation is constructed for decrease in the shock amplitude wave due to plastic dissipation during flattening of cylindrical protrusions on the surface. On the basis of this estimation, it is shown that an increase in the splitting threshold of the rear surface of metals with relief is scalable.

1. Introduction

The use of subpicosecond lasers makes it possible to obtain shock pulses of picosecond duration [1–4]. Nanoscale areas of metals are investigated using molecular dynamic simulation under tension [5–9] and under condition of occurrence and propagation of shock waves [5, 10–12]. Resistance of metals to fracture under shock-wave action is of great interest.

In our recent papers [13, 14], the effect of the rear surface nanorelief on spall strength of metals at interaction with a shock wave was investigated with the help of molecular dynamics (MD). It was shown that a part of the energy of the compression pulse dissipates due to plastic deformation of the nanorelief. As a result, the amplitude of the rarefaction wave decreases, which leads to an increase in the spall threshold. The effect of increasing the threshold amplitude of the shock wave at which spallation begins is significant if the height of the protrusions is comparable to the thickness of the impactor, therefore, to the width of the compression pulse (the width of the compression pulse is approximately twice the thickness of the impactor). Simulations showed that in the case of cylindrical protrusions on the rear surface of copper, the amplitude of the incident wave leading to spallation increased at a maximum by 15 GPa, and the threshold impact velocity by 500–600 m/s. The effect of increase in spall threshold for aluminium is even more significant. Note that the effect was less for nanorelief elements of other forms, but it was also observed in MD simulations.

The study of the increase in the spall threshold in the presence of relief on the surface from which the shock wave is reflected is of great interest not only for nanoscale systems, but for other scales as well. To study the scalability of this effect on the case of macroscopic targets, an
analytical estimation of the decrease in the amplitude of the shock wave due to plastic dissipation during the flattening of cylindrical protrusions on the surface is constructed.

2. Estimation of increase in threshold amplitude of shock wave

An impactor of finite thickness $H$ generates compression pulse along the $x$ axis consisting of a shock wave followed by an unloading wave. A cylinder-shaped protrusion with a height $l_0$ and a diameter $d$ on the plain is considered as the initial state of the rear surface. According to the results of the previous MD simulation [13], the cylindrical protrusions are almost completely flattened when the shock wave interacts with the rear surface. Therefore, we assume that the protrusion transforms into a flat compacted layer with transverse dimensions $D$ and thickness $l$ as the final state of the system.

Consider the deformation of the protrusion along $x$-axis—the direction of propagation of the compression pulse:

$$\varepsilon_x = \ln \left(\frac{l}{l_0}\right),$$

(1)

where $l_0$ is the initial height of the protrusion, and $l$ is the height of the compacted flat layer.

In the initial state, the protrusion occupies the volume $V = (\pi d^2/4)l_0$. In the final state, we obtain a compacted layer with a volume $V = D^2l$ equal to the initial protrusion volume. Then the deformation of the protrusion can be written as

$$\varepsilon_x = \ln \left(\frac{\pi d^2}{4D^2}\right),$$

(2)

which has a negative value. Due to the symmetry of the problem, the deformations along the coordinate axes perpendicular to the direction of the compression pulse propagation can be estimated as $\varepsilon_y = \varepsilon_z = -\varepsilon_x/2$. Then the maximum shear deformation is

$$\varepsilon_\tau = \frac{\varepsilon_y - \varepsilon_x}{2} = -\frac{3}{4}\varepsilon_x = -\frac{3}{4}\ln \left(\frac{4D^2}{\pi d^2}\right).$$

(3)

Let us evaluate the work of plastic deformation of the protrusion. Suppose that the shear stress is constant in time during the plastic deformation of the protrusion ($\sigma_\tau = \text{const}$). The work of plastic deformation is defined as

$$A_{PL} = \sigma_\tau \varepsilon_\tau V = \sigma_\tau \varepsilon_\tau \frac{\pi d^2}{4}l_0.$$

(4)

We write the law of energy conservation in the form $E_{0SW} = E^{SW} + A_{PL}$, where $E^{SW}$ is the shock wave energy connected with its amplitude $\sigma$ as follows

$$E^{SW} = HD^2 \frac{\sigma^2}{\rho c^2},$$

(5)

where $\rho$ is the density of the material, $c$ is the speed of sound. Here we take into account that a half of the shock wave energy is in the elastic field, while the second half is in kinetic energy.

Spallation occurs when the amplitude of the reflected wave (minus the energy of plastic deformation of the protrusions) is equal to the spall strength $\sigma = \sigma_{sp}$. Then, to estimate the increase in the spallation threshold with a protrusion from the energy conservation law, we obtain

$$\sigma_0^2 = \sigma_{sp}^2 + \frac{3}{4}\rho c^2 \sigma_\tau \frac{\pi d^2 l_0}{4D^2H} \ln \left(\frac{4D^2}{\pi d^2}\right),$$

(6)

where $\sigma_{sp}$ is the spall strength of the material, $\sigma_\tau$ is the dynamic shear strength of the material, $\sigma_0$ is the threshold stress of spall fracture of the surface with protrusions. The spallation threshold
It is sensitive to the dependence of the spall and shear strength on the strain rate, which changes during the transition from nanoscale to micro- and macroscales. We use for these quantities the available experimental data.

In paper [15], on the basis of experimental data for aluminum, the following estimate of the dependence of the spall strength on the strain rate for polycrystalline Al was constructed:

\[ \sigma_{sp} = \sigma_1 + \sigma_2 \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_1} \right)^{0.2}, \]  

(7)

where \( \sigma_1 = 0.4 \) GPa, \( \sigma_2 = 3.75 \) GPa, and \( \dot{\varepsilon}_1 = 10 \) ns\(^{-1}\). In [16], the following expression was obtained also based on the analysis of experimental data for the dynamic shear strength of Al:

\[ \sigma_{\tau} = \sigma_3 \left( \dot{\varepsilon} \right)^{0.364}, \]

(8)

where \( \sigma_3 = 0.99 \) MPa. Because these approximations can give strength higher than theoretical limit at ultra-high strain rates, we use the spall strength limited from above by 10 GPa [15] and the shear strength limited by 2.8 GPa [17].

### 3. Results and discussion

Using (7), (8) and (6), one can calculate the dependence of the threshold stress on the thickness of the impactor and the geometric parameters of the protrusions. We consider the protrusions with an initial height equal to the thickness of the impactor \( l_0 = H \), and assume that the strain rate is \( \dot{\varepsilon} = 0.1c/H \), where 0.1 is a typical deformation in a shock wave; \( H/c \) is a typical time of the shock loading. Our consideration is valid if \( l_0 \) and \( d \) have the same order of magnitude.

Figure 1 shows the results for a nanoscale impactor, which coincides in thickness with that studied using MD simulation [13], for the case \( d = 2D/3 \). A comparison with [13] shows that there is an agreement between the results of analytical evaluation and MD simulations. The greatest effect of increasing the spall threshold according to both the estimate (see figure 1) and the MD simulations [13] is achieved when the cross-sectional area of the protrusion is about 0.4
Figure 2. Analytical estimation of the protrusion effect on spall threshold for various dimensions of the impactor (shown near curves). Threshold stress versus the ratio of the cross-section of the protrusions to the surface area.

Figure 3. The scalability of the effect of increasing the spall threshold in the presence of a protrusion on the back surface. Shows a relative increase in the spall threshold \((\sigma_0 - \sigma_{sp})/\sigma_{sp}\) is shown as a function of the thickness of the impactor.

of the surface area. The maximum increase in the spallation threshold is of the order of 12 GPa, which also corresponds to MD simulation [13]. It should be emphasized that here we present the estimate for aluminum, while in the MD simulation [13], the results for copper are mainly presented.

Coincidence of the results of MD simulations [13] with the analytical estimation allows us to generalize the results to other scales (figure 2) and to estimate the scalability of the effect of increasing the spall threshold in the presence of a protrusion on the back surface (figure 3).
4. Conclusion
The relief on the back surface can significantly increase the threshold of the spall fracture. The reason is that the reflection of a shock wave from a surface with protrusions is accompanied by unloading on the lateral surfaces of protrusions, which leads to severe plastic deformation and energy dissipation. As a result, part of the energy of the compression pulse is spent on plastic deformation, which limits the amplitude of the reflecting stretching pulse and suppresses the spallation. According to the constructed analytical estimation, the effect studied earlier using MD simulation for nanoscale systems is scalable on the micro- and macroscales.

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