Updated Estimate of the Muon Magnetic Moment
Using Revised Results from $e^+e^-$ Annihilation

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A new evaluation of the hadronic vacuum polarization contribution to the muon magnetic moment is presented. We take into account the reanalysis of the low-energy $e^+e^-$ annihilation cross section into hadrons by the CMD-2 Collaboration. The agreement between $e^+e^-$ and $\tau$ spectral functions in the $\pi\pi$ channel is found to be much improved. Nevertheless, significant discrepancies remain in the center-of-mass energy range between 0.85 and 1.0 GeV/$c^2$. The deviations from the measurement at BNL are found to be $(22.1 \pm 7.2 \pm 3.5 \pm 8.0) \times 10^{-10} (1.9 \sigma)$ and $(7.4 \pm 5.8 \pm 3.5 \pm 8.0) \times 10^{-10} (0.7 \sigma)$ for the $e^+e^-$ and $\tau$-based estimates, respectively, where the second error is from the nonhadronic contributions and the third one from the BNL measurement. Taking into account the $\rho^- - \rho^0$ mass splitting determined from the measured spectral functions increases the $\tau$-based estimate and leads to a worse discrepancy between the two estimates.

1. Introduction

Hadronic vacuum polarization in the photon propagator plays an important role in the precision tests of the Standard Model. This is the case for the muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$ where the hadronic vacuum polarization component, computed from experimentally determined spectral functions, is the leading contributor to the uncertainty of the theoretical prediction.

Spectral functions are directly obtained from the cross sections for $e^+e^-$ annihilation into hadrons. The accuracy of the calculations has therefore followed the progress in the quality of the corresponding data [1]. Because the latter was not always suitable, it was deemed necessary to resort to other sources of information. One such possibility was the use of the vector spectral functions [2] derived from the study of hadronic $\tau$ decays [3] for the energy range less than $m_\tau \sim 1.8$ GeV. Also, it was demonstrated that perturbative QCD could be applied to energy scales as low as 1-2 GeV [4], thus offering a way to replace poor $e^+e^-$ data in some energy regions by a reliable and precise theoretical prescription [5-7,9].

A complete analysis including all available experimental data was presented in Ref. [10], taking advantage of the new precise results in the $\pi\pi$ channel from the CMD-2 experiment [11] and from the ALEPH analysis of $\tau$ decays [12], and benefiting from a more complete treatment of isospin-breaking corrections [13-14]. In addition to these major updates, the contributions of the many exclusive channels up to 2 GeV center-of-mass energy were completely revisited. It was found that the $e^+e^-$ and the isospin-breaking corrected $\tau$ spectral functions were not consistent within their respective uncertainties, thus leading to inconsistent predictions for the lowest-order hadronic contribution to the muon magnetic anomaly. The leading contribution to the discrepancy originated in the $\pi\pi$ channel with a difference of $(-21.2 \pm 6.4_{\text{exp}} \pm 2.4_{\text{rad}} \pm 2.6_{\text{SU(2)}} (\pm 7.3_{\text{total}})) \times 10^{-10}$. The estimate based on $e^+e^-$ data agreed with another analysis using the same input data [15]. When compared to the world average of the muon magnetic anomaly
measurements, 
\[ a_\mu = (11.659 \pm 203 \pm 8) \times 10^{-10}, \]
which is dominated by the 2002 BNL result using positive muons [16], the respective \( e^+e^- \)-based and \( \tau \)-based predictions disagreed at the 3.0 and 0.9 \( \sigma \) level, respectively, when adding experimental and theoretical errors in quadrature.

Our analysis had to be updated [17] since the CMD-2 Collaboration at Novosibirsk discovered that part of the radiative treatment was incorrectly applied to the data and produced a complete reanalysis [18]. As the CMD-2 data dominate the \( e^+e^- \)-based prediction, the changes produce a significant effect in the final result. No significant change occurred for the \( \tau \)-based prediction. The only relevant fact is a new result [19] for the branching ratio of the \( \tau^- \rightarrow \nu_\tau h^-\pi^0 \) mode (\( h^- \) stands for a charged pion or kaon).

2. Muon Magnetic Anomaly

It is convenient to separate the Standard Model (SM) prediction for the anomalous magnetic moment of the muon into its different contributions,
\[ a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{weak}, \]
with
\[ a_\mu^{had} = a_\mu^{had,LO} + a_\mu^{had,HO} + a_\mu^{had,LBL}, \]
and where \( a_\mu^{QED} = (11.658 \pm 470.6 \pm 0.3) \times 10^{-10} \) is the pure electromagnetic contribution (see [20,21] and references therein [1]), \( a_\mu^{had,LO} \) is the lowest-order contribution from hadronic vacuum polarization, \( a_\mu^{had,HO} = (-10.0 \pm 0.6) \times 10^{-10} \) is the corresponding higher-order part [21,22], and \( a_\mu^{weak} = (15.4 \pm 0.1 \pm 0.2) \times 10^{-10} \), where the first error is the hadronic uncertainty and the second is due to the Higgs mass range, accounts for corrections due to exchange of the weakly interacting bosons up to two loops [25]. For the light-by-light (LBL) scattering part we add the values for the pion-pole contribution [26,27,28] and the other terms [27,28] to obtain \( a_\mu^{had,LBL} = (8.6 \pm 3.5) \times 10^{-10} \).

Owing to the analyticity of the vacuum polarization correlator, the contribution of the hadronic vacuum polarization to \( a_\mu \) can be calculated via the dispersion integral [29]
\[ a_\mu^{had,LO} = \frac{\alpha^2}{3\pi^2} \int ds \frac{K(s)}{s} R(s), \]
where \( K(s) \) is a well-known QED kernel. In Eq. (4), \( R(s) = R^{(0)}(s) \) denotes the ratio of the ‘bare’ cross section for \( e^+e^- \) annihilation into hadrons to the pointlike muon-pair cross section. The ‘bare’ cross section is defined as the measured cross section, corrected for initial-state radiation, electron-vertex loop contributions and vacuum polarization effects in the photon propagator (note that photon radiation in the final state (FSR) is included in the ‘bare’ cross section). The reason for using the ‘bare’ (i.e. lowest order) cross section is that a full treatment of higher orders is anyhow needed at the level of \( a_\mu \), so that the use of ‘dressed’ cross sections would entail the risk of double-counting some of the higher-order contributions.

The function \( K(s) \) decreases monotonically with increasing \( s \). It gives a strong weight to the low energy part of the integral [4]. About 91% of the total contribution to \( a_\mu^{had,LO} \) is accumulated at center-of-mass energies \( \sqrt{s} \) below 1.8 GeV and 73% of \( a_\mu^{had,LO} \) is covered by the two-pion final state which is dominated by the \( \rho(770) \) resonance.

3. Changes to the Input Data

The CMD-2 data, published in 2002 for the \( \pi \pi \) channel [11], have been completely reanalyzed [15] following the discovery of an incorrect implementation of radiative corrections in the analysis program. Overall, the pion-pair cross section increased by 2.1% to 3.8% in the measured energy range (cf. Fig. 1), well above the previously quoted total systematic uncertainty of 0.6%. Specifically, the leptonic vacuum polarization contribution in the \( t \)-channel had been inadvertently left out in the calculation of the Bhabha

\[ a_\mu^{weak} \]
cross section. This effect produced a bias in the luminosity determination, varying from 2.2% to 2.7% in the 0.60-0.95 GeV energy range. The problem consequently affected the measured cross sections for all hadronic channels. Another problem was found in the radiative corrections for the muon-pair process, ranging from 1% to 2% in the same region. A more refined treatment of hadronic vacuum polarization was performed, with changes not exceeding 0.2% for most data points. The effects in the Bhabha- and muon-pair channels also affected the event separation and the measured ratio of pion pairs to electron and muon pairs changed by typically 0.7%.

The correction of the bias in the luminosity determination increases all hadronic cross sections published by CMD-2. The changes are 2.4% and 2.7% on the ω and φ resonance cross sections, respectively.

New published data by SND on the ω resonance [31] and the 2π+2π− as well as π+π−2π0 modes [31] (unchanged cross sections for the latter two, but reduced systematics with respect to previous publications) have been included in this update.

A detailed discussion of radiative corrections, in particular the effect of final-state radiation by the charged hadrons was given in Ref. [11]. Also given therein is a compilation of all input data (with references) used to calculate the integral [4].

4. Comparison of $e^+e^-$ and $\tau$ Spectral Functions

The new $e^+e^-$ and the isospin-breaking corrected $\tau$ spectral functions can be directly compared for the $\pi\pi$ final state. The $\tau$ spectral function is obtained by averaging ALEPH [3], CLEO [32] and OPAL [33] results [10]. The $e^+e^-$ data are plotted as a point-by-point ratio to the $\tau$ spectral function in Fig. 2 and enlarged in Fig. 3 to better emphasize the region of the $\rho$ peak. The central bands in Figs. 2 and 3 give the quadratic sum of the statistical and systematic errors of the $\tau$ spectral function obtained by combining all $\tau$ data. The $e^+e^-$ data have moved closer to the $\tau$ results: they are now consistent below and around the peak, while, albeit reduced, the discrepancy persists for energies larger than 0.85 GeV.

A convenient way to assess the compatibility between $e^+e^-$ and $\tau$ spectral functions proceeds with the evaluation of $\tau$ decay fractions using the relevant $e^+e^-$ spectral functions as input. All the isospin-breaking corrections detailed in Ref. [10] are included. This procedure provides a quantitative comparison using a single number. The weighting of the spectral function is however different from the vacuum polarization kernels. Using the branching fraction $B(\tau^- \to \nu_\tau e^- \bar{\nu}_e) = (17.810 \pm 0.039)\%$, obtained assuming leptonic universality in the charged weak current [12], the result for the $\pi\pi$ channel is

$$B^{\pi\pi^0}_{\text{CVC}} = (24.52 \pm 0.26_{\text{exp}} \pm 0.11_{\text{rad}} \pm 0.12_{\text{SU(2)}})\%.$$  \hfill (5)

where the errors quoted are split into uncertainties from the experimental input (the $e^+e^-$ annihilation cross sections) and the numerical integration procedure, the missing radiative corrections applied to the relevant $e^+e^-$ data, and the isospin-breaking corrections when relating $\tau$ and $e^+e^-$ spectral functions. Even though the corrections to the CMD-2 results have reduced the discrepancy between [5] and the world average of the direct $B(\tau^- \to \nu_\tau \pi^-\pi^0)$ measurements from
Figure 2. Relative comparison of the $\pi^+\pi^-$ spectral functions from $e^+e^-$ and isospin-breaking corrected $\tau$ data, expressed as a ratio to the $\tau$ spectral function. The band shows the uncertainty on the latter. The $e^+e^-$ data are from CMD-2 [18], CMD, OLYA and DM1 (references quoted in [17]).

Figure 3. Relative comparison in the $\rho$ region of the $\pi^+\pi^-$ spectral functions from $e^+e^-$ and isospin-breaking corrected $\tau$ data, expressed as a ratio to the $\tau$ spectral function. The band shows the uncertainty on the latter. The references for $e^+e^-$ data are given in Fig. 2.

Figure 4. The measured branching ratios for $\tau^{-}\rightarrow \nu_{\tau}\pi^+\pi^-\pi^0$ compared to the prediction from the $e^+e^-\rightarrow \pi^+\pi^-$ spectral function applying the isospin-breaking correction factors discussed in Ref. [10]. The measured branching ratios are from ALEPH [12], CLEO [34] and OPAL [35]. The $L3$ and OPAL results are obtained from their $h\pi^0$ branching ratio, reduced by the small $K\pi^0$ contribution measured by ALEPH [36] and CLEO [37].

4.6 to 2.9 standard deviations (adding all errors in quadrature), the remaining difference of $-0.94 \pm 0.10 \pm 0.26 \pm 0.11_{\text{rad}} \pm 0.12_{\text{SU(2)}} \pm 0.32_{\text{total}}$% is still problematic. Since the disagreement between $e^+e^-$ and $\tau$ spectral functions is more pronounced at energies above 850 MeV, we expect a smaller discrepancy in the calculation of $a_{\mu}^{'\text{had,LO}}$ because of the steeply falling function $K(s)$. More information on the comparison is displayed in Fig. 4 where it is clear that ALEPH, CLEO, L3 and OPAL all separately, but with different significance, disagree with the $e^+e^-$-based CVC result.

5. Results

The integration procedure and the specific contributions – near $\pi\pi$ threshold, the $\omega$ and $\phi$ resonances, the narrow quarkonia and the high energy QCD prediction – are treated as in our previous analysis [10]. The contributions from the different processes in their indicated energy ranges are
Table 1

| Modes                  | Energy [GeV] | $e^+e^-$                      | $a_{\mu}^{\text{had,LO}} (10^{-10})$ | $\Delta(e^+e^- - \tau)$ |
|------------------------|--------------|--------------------------------|--------------------------------------|-------------------------|
| Low s exp. $\pi^+\pi^-$| $[2m_{e,\pi} - 0.500]$ | $58.04 \pm 1.70 \pm 1.17$ | $56.03 \pm 1.60 \pm 0.28$ | $+2.0 \pm 2.6$ |
| $\pi^+\pi^-$           | $[0.500 - 1.800]$ | $450.16 \pm 4.89 \pm 1.57$ | $646.03 \pm 2.95 \pm 2.34$ | $-13.9 \pm 6.4$ |
| $\pi^0\gamma$, $\eta\gamma$ (1) | $[0.500 - 1.800]$ | $0.93 \pm 0.15 \pm 0.01$ | - | - |
| $\omega$               | $[0.300 - 0.810]$ | $37.96 \pm 1.02 \pm 0.31$ | - | - |
| $\pi^+\pi^-\pi^0$ [below $\phi$] | $[0.810 - 1.000]$ | $4.20 \pm 0.40 \pm 0.05$ | - | - |
| $\phi$                 | $[1.000 - 1.055]$ | $35.71 \pm 0.84 \pm 0.20$ | - | - |
| $\pi^+\pi^-\pi^0$ [above $\phi$] | $[1.055 - 1.800]$ | $2.45 \pm 0.26 \pm 0.03$ | - | - |
| $\pi^+\pi^0$           | $[1.020 - 1.800]$ | $16.76 \pm 1.31 \pm 0.20$ | $21.45 \pm 1.33 \pm 0.60$ | $-4.7 \pm 1.8$ |
| $2\pi^+2\pi^-$         | $[0.800 - 1.800]$ | $14.21 \pm 0.87 \pm 0.23$ | $12.35 \pm 0.96 \pm 0.40$ | $+1.9 \pm 2.0$ |
| $2\pi^+2\pi^-\pi^0$   | $[1.019 - 1.800]$ | $2.09 \pm 0.43 \pm 0.04$ | - | - |
| $\pi^+\pi^-3\pi^0$ (2) | $[1.019 - 1.800]$ | $1.29 \pm 0.22 \pm 0.02$ | - | - |
| $3\pi^+3\pi^-$         | $[1.350 - 1.800]$ | $0.10 \pm 0.10 \pm 0.00$ | - | - |
| $2\pi^+2\pi^-2\pi^0$  | $[1.350 - 1.800]$ | $1.41 \pm 0.30 \pm 0.03$ | - | - |
| $\pi^+\pi^-4\pi^0$ (2) | $[1.350 - 1.800]$ | $0.06 \pm 0.06 \pm 0.00$ | - | - |
| $\eta(\to \pi^+\pi^-\gamma$, $2\gamma)\pi^+\pi^-$ | $[1.075 - 1.800]$ | $0.54 \pm 0.07 \pm 0.01$ | - | - |
| $\omega(\to \pi^0\gamma)\pi^0$ | $[0.975 - 1.800]$ | $0.63 \pm 0.10 \pm 0.01$ | - | - |
| $\omega(\to \pi^0\gamma)(\pi\pi)^0$ | $[1.340 - 1.800]$ | $0.08 \pm 0.01 \pm 0.00$ | - | - |
| $K^+K^-$                | $[1.055 - 1.800]$ | $4.63 \pm 0.40 \pm 0.06$ | - | - |
| $K_0^0K_0^*$            | $[1.097 - 1.800]$ | $0.94 \pm 0.10 \pm 0.01$ | - | - |
| $K_0^0K_0^\pm$         | $[1.340 - 1.800]$ | $1.84 \pm 0.24 \pm 0.02$ | - | - |
| $K^0K^0$ (2)           | $[1.440 - 1.800]$ | $0.60 \pm 0.20 \pm 0.01$ | - | - |
| $K^0\pi^0\pi^0$ (2)   | $[1.441 - 1.800]$ | $2.22 \pm 1.02 \pm 0.03$ | - | - |
| $R = \sum$ excl. modes | $[1.800 - 2.000]$ | $8.20 \pm 0.66 \pm 0.10$ | - | - |
| $R$ [Data]             | $[2.000 - 3.700]$ | $26.70 \pm 1.70 \pm 0.03$ | - | - |
| $\psi(2S)$             | $[3.088 - 3.106]$ | $5.94 \pm 0.35 \pm 0.00$ | - | - |
| $\omega(2S)$           | $[3.658 - 3.714]$ | $1.50 \pm 0.14 \pm 0.00$ | - | - |
| $R$ [Data]             | $[3.700 - 5.000]$ | $7.22 \pm 0.28 \pm 0.00$ | - | - |
| $R_{\text{had,LO}}$ [QCD] | $[5.000 - 9.300]$ | $6.87 \pm 0.10 \pm 0.00$ | - | - |
| $R_{\text{had,LO}}$ [QCD] | $[9.300 - 12.00]$ | $1.21 \pm 0.05 \pm 0.00$ | - | - |
| $R_{\text{had,LO}}$ [QCD] | $[12.0 - \infty]$ | $1.80 \pm 0.01 \pm 0.00$ | - | - |

\[ \sum (e^+e^- \to \text{hadrons}) [2m_{e,\pi} - \infty] = 696.3 \pm 6.2_{\exp} \pm 3.6_{\text{had}} \pm 0.8_{\text{had}} \pm 2.8_{SU(2)} -14.7 \pm 7.9_{\text{tot}} \]

1 Not including $\omega$ and $\phi$ resonances (see Ref. [10]).
2 Using isospin relations (see Ref. [11]).

$e^+e^-$ data are used above 1.6 GeV (see Ref. [10]).

Table 1. Summary of the $a_{\mu}^{\text{had,LO}}$ contributions from $e^+e^-$ annihilation and $\tau$ decays. The uncertainties on the vacuum polarization and FSR corrections are given as second errors in the individual $e^+e^-$ contributions, while those from isospin breaking are similarly given for the $\tau$ contributions. These 'theoretical' uncertainties are correlated among all channels, except in the case of isospin breaking which shows little correlation between the $2\pi$ and $4\tau$ channels. The errors given for the sums in the last line are from the experiment, the missing radiative corrections in $e^+e^-$ and, in addition for $\tau$, $SU(2)$ breaking.
listed in Table [1]. Wherever relevant, the two $e^+e^-$ and $\tau$-based evaluations are given. The discrepancies discussed above are now expressed directly in terms of $a_{\mu,\text{had,LO}}$, giving smaller estimates for the $e^+e^-$-based data set by $(-11.9 \pm 6.4_{\exp} \pm 2.4_{\text{rad}} \pm 2.6_{\text{SU}(2)} (\pm 7.3_{\text{total}})) \times 10^{-10}$ for the $\pi\pi$ channel and $(-2.8 \pm 2.6_{\exp} \pm 0.3_{\text{rad}} \pm 1.0_{\text{SU}(2)} (\pm 2.9_{\text{total}})) \times 10^{-10}$ for the sum of the $4\pi$ channels. The total discrepancy $(-14.7 \pm 6.9_{\exp} \pm 2.7_{\text{rad}} \pm 2.8_{\text{SU}(2)} (\pm 7.9_{\text{total}})) \times 10^{-10}$ amounts to 1.9 standard deviations. The difference could now be considered to be acceptable, however the systematic difference between the $e^+e^-$ and $\pi\pi$ spectral functions at high energies precludes one from performing a straightforward combination of the two evaluations.

5.1. Results for $a_{\mu}$

The results for the lowest order hadronic contribution are $(\times 10^{-10})$

$$a_{\mu,\text{had,LO}}^{\text{SM}} = 696.3 \pm 6.2_{\exp} \pm 3.6_{\text{rad}}$$

$$a_{\mu,\text{had,LO}}^{\text{rad}} = 711.0 \pm 5.0_{\exp} \pm 0.8_{\text{rad}} \pm 2.8_{\text{SU}(2)}$$

Adding to these the QED, higher-order hadronic, light-by-light scattering and weak contributions as given in Section 2, we obtain the SM predictions $(\times 10^{-10})$, with an additional common uncertainty of $\pm 3.5_{\text{LBL}} \pm 0.4_{\text{QED+EW}}$

$$a_{\mu,\text{had,LO}}^{\text{SM}} = 11659 \ 180.9 \pm 7.2_{\text{had,LO}}$$

$$a_{\mu,\text{had,LO}}^{\text{SM}} = 11659 \ 195.6 \pm 5.8_{\text{had,LO}}$$

These values can be compared to the present measurement [11]. Adding experimental and theoretical errors in quadrature, the differences between measured and computed values are found to be $(\times 10^{-10})$, with an additional common uncertainty of $\pm 3.5_{\text{other}} \pm 0.8_{\exp}$ from contributions other than hadronic vacuum polarization and the BNL g-2 experimental error$

$$a_{\mu,\text{had,LO}}^{\exp} - a_{\mu,\text{had,LO}}^{\text{SM}} = 22.1 \pm 7.2_{\text{had,LO}}$$

$$a_{\mu,\text{had,LO}}^{\exp} - a_{\mu,\text{had,LO}}^{\text{rad}} = 7.4 \pm 5.8_{\text{had,LO}}$$

where the error quoted is specific to each approach. The differences [5] correspond to 1.9 and 0.7 standard deviations, respectively. A graphical comparison of the results [5] with the experimental value is given in Fig. [5]. Also shown are

![Figure 5. Comparison of the results [5] with the BNL measurement [10]. Also given are our estimates [10-12] obtained before the CMD-2 data were available. For completeness, we show as triangles with dotted error bars the $e^+e^-$-based results [10, 11] derived with the previously published CMD-2 data [11].]

our estimates [10-12], obtained before the CMD-2 and the new $\tau$ data were available (see discussion below), and the $e^+e^-$-based evaluations of Refs. [10, 11], obtained with the previously published, uncorrected CMD-2 data [11].

6. Discussion

Although the new corrected CMD-2 $\pi^+\pi^-$ results are now consistent with $\tau$ data for the energy region below 850 MeV, the remaining discrepancy for larger energies is unexplained at present. Hence, one could question the validity of either $e^+e^-$ data with their large radiative corrections, $\tau$ data, or the isospin-breaking corrections applied to $\tau$ data. We shall briefly discuss these points below.

- The CMD-2 experiment is still the only one claiming systematic accuracies well below 1%. It is thus difficult to confront their data with results from other experiments. Whereas the measurements from OLYA are systematically lower than the new CMD-2
results in the peak region, there is a trend of agreement above, as seen in Fig. 4. This behaviour appears to be confirmed by preliminary data from the KLOE experiment at Frascati using the radiative return method from the $\phi$ resonance [38].

- The most precise results on the $\tau \pi\pi$ spectral function come from the ALEPH and CLEO experiments, operating in completely different physical environments. On the one hand, the main uncertainty in CLEO originates from the knowledge of the relatively low selection efficiency, a consequence of the large non-$\tau$ hadronic background, while the mass spectrum is measured with little distortion and good resolution. On the other hand, ALEPH has both large efficiency and small background, the main uncertainty coming from the $\pi^0$ reconstruction close to the charged pion, necessitating to unfold the measured spectrum from detector resolution and acceptance effects. A comparison of the $\tau$ spectral functions from ALEPH, CLEO and OPAL is given in Fig. 6 and in Fig. 7 for the $\rho$ peak region. Agreement is observed within quoted errors, in particular in the high mass region, although CLEO results are a bit closer to $e^+e^-$ data there. Overall, the $\tau$ data appear to be consistent.

- The last point concerns isospin corrections applied to the $\tau$ spectral functions. The basic components entering SU(2) breaking are well identified. The long-distance radiative corrections and the quantitative effect of loops have been addressed by the analysis of Ref. [14] showing that the effects are small. The overall effect of the isospin-breaking corrections (including FSR) applied to the $\tau \pi\pi$ data, expressed in relative terms, is $(−1.8 \pm 0.5)\%$. Its largest contribution $(−2.3\%)$ stems from the uncontroversial short-distance electroweak correction [39]. One could question the validity of the chiral model used. The authors of Ref. [14] argue that the corrections are insensitive to the details of their model and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Relative comparison of the $\pi^+\pi^−$ spectral functions extracted from $\tau$ data from different experiments, expressed as a ratio to the average $\tau$ spectral function.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Relative comparison in the $\rho$ region of the $\pi^+\pi^−$ spectral functions extracted from $\tau$ data, expressed as a ratio to the average $\tau$ spectral function.}
\end{figure}
essentially depend only on the shape of the pion form factor. As the latter is known from experiment to adequate accuracy, it seems difficult to find room for a $\sim 10\%$ effect as observed experimentally. Nevertheless, considering the situation regarding the first two experimental points, it would seem worthwhile to invest more theoretical work into the problem of isospin breaking.

The particular point of the $\rho^- - \rho^0$ mass splitting deserves some further discussion. This possibility was considered from the beginning when it was proposed to use $\tau$ spectral functions to compute vacuum polarization integrals \[2\]. However, at that time, the experimental investigation led to a splitting consistent with 0 within 1.1 MeV, supported by theoretical investigations \[40\] which indicated a value less than 0.7 MeV. Consequently, in further analyses, we assumed $m_{\rho^0} = m_{\rho^-}$ to hold within an uncertainty of 1 MeV. A re-analysis of this question with the current more precise data on $e^+e^-$ and $\tau$ spectral functions \[41\] leads to the conclusion that the mass splitting favoured by the data is $m_{\rho^-} - m_{\rho^0} = (2.3 \pm 0.8$ MeV. A similar conclusion was more recently reached \[42\]. Unfortunately, it is not yet clear whether such a result must be taken as definitive evidence for a mass splitting and if the corresponding correction must be applied to the $\tau$ spectral function. The significance of the result is still limited, but the most worrisome problem is the fact that the $e^+e^-$ and the mass-corrected $\tau$ spectral functions still disagree in magnitude by $3.3\%$ \[41\]. Therefore, while the consideration of the mass splitting improves the line shape comparison, it leaves us with a major normalization discrepancy. In other words, the $\rho^-$ mass correction spreads the large local difference observed in the 0.85-1. GeV range almost uniformly across the mass spectrum. We thus disagree with the conclusions reached in Ref. \[42\] that taking into account the $\rho$ mass difference solves the current discrepancy. If we take the point of view that the remaining difference is basically a normalization problem (either from the data or the isospin breaking corrections), then correcting for the apparent mass difference increases the $\tau$-based estimate of $a_{\mu}^{\text{had,LO}}$ by $5.4 \times 10^{-10}$, bringing it even closer to the BNL result, but further away from the $e^+e^-$-based estimate for a total of 2.5 $\sigma$. More experimental investigation with CMD-2, KLOE and BABAR is needed to consolidate the $e^+e^-$ picture.

7. Conclusions

An update of our analysis of the lowest-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment has been performed following a reevaluation by the CMD-2 Collaboration of their $e^+e^-$ annihilation cross sections. Part of the previous discrepancy between the $e^+e^-$ and $\tau \pi \pi$ spectral functions has now disappeared so that the corresponding evaluations of the lowest-order hadronic polarization contribution to the muon magnetic anomaly are closer. However, incompatible measurements remain between 0.8 and 1 GeV so that we do not proceed with an average of the two evaluations. The $e^+e^-$ and $\tau$-based predictions are respectively 1.9 and 0.7 standard deviations below the direct measurement from the g-2 Collaboration at BNL. Considering the $\pi \pi$ discrepancy from the point of view of the $\rho^- - \rho^0$ mass splitting turns out to increase the difference between the two estimates. The forthcoming results from radiative return with KLOE \[38\] and BABAR \[35\] will be important to sort out possibly remaining problems in the $\pi \pi$ and $4\pi$ spectral functions.

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