A study on lot-size dependence of the energy consumption per unit of production throughput concerning variable lot-size

Hironori HIBINO*, Takamasa HORIKAWA** and Makoto YAMAGUCHI***

* Department of Industrial Administration, Faculty of Science and Technology, Tokyo University of Science
2641 Yamazaki, Noda-shi, Chiba 278-8510, Japan
E-mail: hibino@rs.tus.ac.jp

** Department of Industrial Administration, Graduate School of Science and Technology, Tokyo University of Science
2641 Yamazaki, Noda-shi, Chiba 278-8510, Japan

*** Mechanical Engineering Course, Department of Systems Design Engineering, Faculty of Engineering Science, Akita University
1-1 Tegata-gakuenmachii, Akita-shi, Akita, 010-8502, Japan

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Abstract
Recently, in order to reduce energy consumption while maintaining productivity in lines, industries require pre-evaluation methods and production management methods that consider these aspects simultaneously. A pre-evaluation method that considers both productivity and energy consumption and simultaneously calculates productivity and energy consumption during the planning of a manufacturing system has already been proposed. In addition, with production management methods that consider productivity and energy consumption, the lot-size dependence of energy consumption per unit of production throughput has been formulated and verified. On the other hand, because the needs of consumers have become more diverse, mass customized or high-mix low-volume manufacturing systems, in which lot sizes vary, have been developed. We defined the condition in which lot size is not varied as “constant lot-size”, whereas the condition in which lot size is varied as “variable lot-size”. We define the variable lot-size that lot sizes are not constant during a certain continuous production time. However, production management methods that consider variable lot size have not developed. Therefore, we propose a formulation for the lot size dependence of the energy consumption per unit of production throughput for variable lot sizes. The key parameter of this formulation is the average lot size, as the throughput and energy consumption per unit of production throughput are independent of the lot size distribution. We also carry out case studies using a discrete event simulation to verify the proposed formulation.

Keywords: Manufacturing system simulation, Energy consumption, Productivity, Variable lot size, Formulation

1. Introduction

A revision of the Act on the Rational Use of Energy that was implemented in April 2014 institutionalised an annual reduction in energy consumption per unit of production throughput by an average of at least 1% (Agency for Natural Resources and Energy). Additionally, the Paris agreement at COP21 in December 2015 made all countries, including developing countries, responsible for framing measures to achieve targets as well as submission of the reduction targets with respect to greenhouse gases (United Nations Framework Convention on Climate Change). As a result, domestic measures involve the realization of a revolution in energy management technology for efficient energy usage by analyzing the status of energy use in manufacturing processes to realize energy savings in industries (Ministry of Economy). Thus, pre-evaluation and reduction of energy consumption in manufacturing activities correspond to important global topics.

Pre-evaluation methods using manufacturing system simulations with respect to productivity have already been developed (Kim et al., 2003) (Kim et al., 2005) (McLean et al., 2005) (Mitsuyuki et al., 2004) (Williams et al., 1998) (Hibino, 2014) (Hibino et al., 2015) (Kim et al., 2015). However, industries must reduce energy consumption while maintaining productivity by evaluating the energy consumption and productivity in the stages of design, implementation, and operation (Göschel et al., 2012). With respect to production management methods that consider productivity and...
energy consumption, previous studies have presented approaches for reducing the energy consumption per unit of production throughput that focus on either the machine level (Heilala et al., 2008) (Ghandimi et al., 2014) (Weinnert et al., 2014) (Frigerio et al., 2014) (Frigerio et al., 2015) or the line level (Murayama et al., 2005) (Sakuma et al., 2013) (Hibino et al., 2012) (Schultz et al., 2015) (Li et al., 2012) (Beier et al., 2017) (Herrmann et al., 2011) (Yamaguchi et al., 2016) (Kobayashi et al., 2016). Line-level approaches can focus on line designs, such as the number of facilities, buffer capacity, and processing time, while evaluating the energy consumption. Alternatively, they can focus on evaluation of the energy consumption per unit of production throughput considering the production management methods for the lot sizes.

However, these studies have focused only on the condition in which lot size is constant. Furthermore, because the needs of consumers have become more diverse, mass customized or high-mix low-volume manufacturing systems, in which lot sizes vary, have become increasingly important. In this study, we defined the condition in which lot size is not varied as “constant lot-size”, whereas the condition in which lot size is varied as “variable lot-size”. We define the variable lot-size that lot sizes are not constant during a certain continuous production time. For example, the following three cases are considered.

(1) with high-mixed production, we need to consider that the lot size is different for each type.
(2) with volume flexible production, we need to consider that the lot size of the same product is different.
(3) with mixing the high-mixed production and the volume flexible production, we need to consider both above cases.

Additionally, the lot size frequently varies for each lot within a factory. Thus, it is essential to establish production management methods that consider variable lot size. When the lot size varies, it is not clear how to evaluate the energy consumption per unit of production throughput because the preparation interval is not constant. Therefore, it is necessary to understand the behavior, such as the preparation of machines in lines, in order to evaluate the energy consumption per unit of production throughput.

Therefore, we propose a formulation for the lot size dependence of the energy consumption per unit of production throughput for variable lot sizes. The key parameter of this formulation is the average lot size, as the throughput and energy consumption per unit of production throughput are independent of the lot size distribution. We also carry out case studies to verify the proposed formulation. In order to carry out the case studies, we use virtual real data obtained from a developed simulation. The simulation can create virtual real data in response to the variable lot size. Through the case studies, we validate the proposed formulation.

2. Previous studies

2.1 Existing formulation of lot size dependence of energy consumption per unit of production throughput for constant lot size

Yamaguchi et al. focused on a constant lot size that corresponds to an important item that should be determined in production controls and conducted studies related to the lot size dependence of energy consumption per unit of production throughput and formulated the following relationships.

2.1.1 Energy consumption per unit of production throughput

The energy consumption per unit of production throughput is a representative index for evaluating the total production towards the total energy consumption. Energy consumption per unit of production throughput, \( U \), is expressed using the total production \( P \) [product] and the total energy consumption \( E \) [J] as follows:

\[
U = \frac{E}{P}
\]  

(Definition of variables)

\( U \): energy consumption per unit of production throughput
\( P \): total production of the line
\( E \): total energy consumption

2.1.2 Pre-requisite for formulation

The following conditions are pre-requisites for the existing formulation.

(1) Conditions for the production machines

- The production machines are in one of the following states: running state, setting-up state, or idle state.
(2) Conditions for manufacturing system
- Lot size is always fixed.
- Machines are in series.
- There is no parts assembly process.
- The total operating time is sufficiently long.
- The first machine does not occur idle states.
- A buffer after each machine has enough buffer capacity.

2.1.3 Works-in-process coefficient $q^k$

There exists a buffer between each machine. When there is a difference in the production capabilities of each machine, there are works-in-process in the buffer at the end of operation. Here, the works-in-process coefficient for machine $k$ is defined by dividing the total production from machine, $P^k$, by the total production, $P$ as follows:

$$q^k = \frac{P^k}{P} \quad (2)$$

(Definition of variables)
- $k$: $k$'th machine in a line
- $q^k$: works-in-process coefficient in machine $k$
- $P^k$: total production by machine $k$

The total production by machine $k$ is expressed by using works-in-process coefficient $q^k$ as the total production in the final machine, $P$.

2.1.4 Time occupied by machine $k$ in each state

2.1.4.1 Running time $T_r^k$

The total production by machine $k$, $P^k$ is the product of machine $k$’s throughput $p_r^k$ and running time $T_r^k$. Therefore, the running time $T_r^k$ is represented as shown below by using the total production $P^k$ and throughput $p_r^k$.

$$T_r^k = \frac{P^k}{p_r^k} \quad (3)$$

(Definition of variables)
- $T_r^k$: total running time of machine $k$
- $p_r^k$: production per unit time of machine $k$

2.1.4.2 Setting-up time $T_s^k$

The total setting-up time for machine $k$ corresponds to the product of the number of preparation events and the time required for setting-up $\lambda^k$. The number of setting-up corresponds to the total production in machine $k$, $P^k$, divided by the lot size $LS$ as follows:

$$T_s^k = \frac{P^k}{LS} \times \lambda^k \quad (4)$$

(Definition of variables)
- $T_s^k$: total setting-up time of machine $k$
- $LS$: lot size
- $\lambda^k$: time required for a setting-up

2.1.4.3 Idle time $T_i^k$

The total idle time of machine $k$ is subtracted the total running time $T_r^k$ and the total setting-up time $T_s^k$ from the total operating time $T$, and thus it is represented as follows:

$$T_i^k = T - T_r^k - T_s^k \quad (5)$$

(Definition of variables)
- $T_i^k$: total idle time of machine $k$
- $T$: total operating time
2.1.5 Energy consumption by machine \( k \)

If the energy consumption by each state of machine \( k \) (energy consumption per unit time) corresponds to \( e^k_r \), \( e^k_s \), and \( e^k_i \); then the total energy consumption is as follows:

\[
E = \sum_{k=1}^{n} E^k = \sum_{k=1}^{n} (e^k_r T^k_r + e^k_s T^k_s + e^k_i T^k_i)
\]

(6)

(Definition of variables)

\( E^k \): energy consumption by machine \( k \)
\( e^k_r \): energy consumption per time during running by machine \( k \)
\( e^k_s \): energy consumption per time during setting-up by machine \( k \)
\( e^k_i \): energy consumption per time during idle by machine \( k \)

\( n \): number of machines in a line

2.1.6 Machine \( k \)’s energy consumption per unit of production throughput

The total production \( P \) is divided by the total operating time \( T \) to yield the amount of production per unit time, i.e., the throughput \( p \). From Eq.(3–6), machine \( k \)’s energy consumption per unit of production throughput \( U^k \) is as follows if the throughput (for the entire line) \( p \) is used:

\[
U^k = \frac{E^k}{p} = \frac{1}{p} \left( e^k_r T^k_r + e^k_s T^k_s + e^k_i T^k_i \right)
\]

\[
= \frac{1}{p} \left( e^k_r \frac{p^k}{p^k} + e^k_s \frac{p^k \lambda^k}{LS} + e^k_i \left( T - \frac{p^k}{p^k} \frac{p^k \lambda^k}{LS} \right) \right)
\]

\[
= \frac{1}{p} \left( e^k_r \frac{q^k P}{p^k} + e^k_s \frac{q^k P \lambda^k}{LS} + e^k_i \left( T - \frac{q^k P}{p^k} \frac{q^k P \lambda^k}{LS} \right) \right)
\]

\[
= \frac{1}{p} \left( \frac{q^k e^k_r}{p^k} + \frac{q^k e^k_s \lambda^k}{LS} \right) + e^k_i \left( \frac{1}{p} - q^k \left( \frac{1}{p^k} + \frac{\lambda^k}{LS} \right) \right)
\]

(7)

(Definition of variables)

\( U^k \): energy consumption per unit of production throughput for machine \( k \)
\( p \): throughput of the line

Hence, the total production \( P \) is removed from the formulation. Therefore, if the works-in-process coefficient \( q^k \) and the throughput \( p \) in the formulation are calculated using only the machine variables, then Eq.(7) can be used to obtain machine \( k \)’s energy consumption per unit of production throughput \( U^k \).

2.1.7 Calculation method for works-in-process coefficient \( q^k \) and throughput \( p \) with respect to infinite buffer capacity

The works-in-process coefficient \( q^k \) is redefined from Eq.(2) as shown below if machine \( k \)’s throughput \( p^k \) and (of the whole line) throughput \( p \) are used as follows:

\[
q^k = \frac{p^k}{p} = \frac{p^k / T}{T / p} = \frac{p^k}{p}
\]

(8)

(Definition of variables)

\( p^k \): throughput of machine \( k \)

Equation (8) shows the relationship between works-in-process coefficient \( q^k \) and throughput \( p \). As shown in
Eq.(8), it is evident that even in cases when only one of the variables $q^k$ or $p$ is sought, the other variable can be obtained if $p^k$ is known.

Furthermore, the term appearing in Eq.(7) is defined as $p_0^k$, throughput when machine $k$ operates solo as follows:

$$p_0^k = \left( \frac{1}{p_i^k} + \frac{\lambda^k}{Ls} \right)^{-1}$$

(Definition of variables)

$p_0^k$: throughput when machine $k$ operates solo

Specifically, $p_0^k$ represents the throughput when machine $k$ operates independently.

When the buffer capacity is infinite, there is no state in which the machine cannot conduct processing owing to existing buffers after the machine is full, that is blocking. First, if it is assumed that work is supplied without a break in machine 1 ($k=1$), then machine 1’s throughput $p^1$ is as follows:

$$p^1 = p_0^1$$

(10)

Additionally, machine $k$’s throughput $p^k$ is such that if the capability of machine $k'(k' < k)$, which exists prior to machine $k$ is worse than that of machine $k$ (the throughput when machine $k$ operates solo $p_0^k$ is small, i.e., $p_0^k < p_0^k$), a situation arises in which there is no work to be supplied to machine $k$; thus processing stops (starving) and throughput $p^k$ then corresponds to $p_0^k$ and not to $p_0^k$. Moreover, when machine $k'(k' < k)$ is more capable than machine $k$ ($p_0^k > p_0^k$), then throughput $p^k$ corresponds to $p_0^k$. Therefore, the following relationship is exist between the throughput of machine $k$ and that of its predecessor, $k-1$.

$$p_0^k > p_0^{k-1} \rightarrow p^k = p_0^{k-1}$$

(11)

$$p_0^k < p_0^{k-1} \rightarrow p^k = p_0^k$$

(12)

From Eq.(10-12), it is possible to determine $p^k$ for all $k$. Thus, $p^k$ is rate-limited by the machine with the worst capability before machine $k$ and is therefore expressed as follows:

$$p^k = \min(p_0^1, p_0^2, \ldots, p_0^k)$$

(13)

Similarly, $p$ is rate-limited by the machine with the worst productivity in the entire line. Therefore, if there are $n$ machines in the line, then it is expressed as follows:

$$p = \min(p_0^1, p_0^2, \ldots, p_0^n)$$

(14)

From Eq.(14), the throughput $p$ when buffer capacity is infinite, is obtained using basic information. Similarly, the works-in-process coefficient $q^k$ is also obtained from Eq.(8) as follows:

$$q^k = \frac{p^k}{p} = \frac{\min(p_0^1, p_0^2, \ldots, p_0^k)}{\min(p_0^1, p_0^2, \ldots, p_0^n)}$$

(15)

As shown above, when the buffer capacity is set as infinite, machine $k$’s throughput $p^k$ and (the whole line) throughput $p$ are obtained from the machine variables to determine the works-in-process coefficient $q^k$ from machine variables only. Therefore, the lot size dependence of energy consumption per unit of production throughput $U^k$ can be theoretically expressed using Eq.(7).

3. Proposed Formulation of Lot Size Dependence and Energy consumption per unit of production throughput for Variable Lot Size

Equation (7) is a formulation of the lot-size dependence of energy consumption per unit of production throughput in a line in the case of constant lot size [27,28]. However, this methodology cannot be applied to variable lot size. Therefore, we propose a formulation of the lot-size dependence of energy consumption per unit of production throughput $U^k$ can be theoretically expressed using Eq.(7).

3.1 Pre-requisites for formulation

The following conditions are pre-requisites for the formulation involved in this study.

(1) Conditions for the production machines
• The production machines are in one of the following states: running state, setting-up state, or idle state.

(2) Conditions for manufacturing system
• The lot size varies.
• Machines are in series.
• There is no parts assembly process.
• The total operating time is sufficiently long.
• The setting-up always occurs whenever a lot is changed.
• The first machine does not occur idle states.
• A buffer after each machine has enough buffer capacity.

3.2 Lot size ratio
In this study, the lot size ratio is defined as the proportion held by a lot size relative to all lot sizes when the lot size varies in a manufacturing process. The lot size ratio of lot size $LS_i$, is denoted as $X(LS_i)$. Additionally, when there are $m$ lot sizes, the total sum of the lot size ratios satisfies the following condition.

$$\sum_{i=1}^{m} X(LS_i) = 1$$

(16)

(Definition of variables)
$X(LS_i)$: lot size ratio of lot size $LS_i$
$m$: total number of types of lot sizes

3.3 Average lot size
When the lot size varies, the setting-up time cannot be given by Eq.(4), because the processing time for each lot size is different. In order to calculate the energy consumption per unit of production throughput even when the lot size varies, an average lot size is proposed by modifying Eq.(4). When the total operating time is sufficiently long and the total production is high, the average lot size is obtained as the sum of the products of lot size and lot size ratio. The average lot size, $\bar{LS}$, with $m$ lot sizes is expressed as follows:

$$\bar{LS} = \sum_{i=1}^{m} LS(i) \times X(LS_i)$$

(17)

(Definition of variables)
$\bar{LS}$: average lot size

3.4 Setting-up time $T_s^k$
When the lot size varies, the number of setting-up events is approximated using the average lot size, $\bar{LS}$, to obtain the setting-up time, $T_s^k$, involved. Therefore, it is represented as follows:

$$T_s^k = \frac{p^k}{\bar{LS}} \times \lambda^k$$

(18)

3.5 Energy consumption per unit of production throughput of machine $k$ for variable lot size
When Eq.(7) is revised with Eq.(13–15) and Eq.(18) is substituted for (4), machine $k$’s energy consumption per unit of production throughput for variable lot size is

$$U^k = \min\left(\frac{p_1^1, p_2^1, \ldots, p_n^1}{p_1^0, p_2^0, \ldots, p_n^0}, \frac{e_1^k + e_2^k \lambda^k}{\bar{LS}}, \frac{1}{\min[p_1^1, p_2^1, \ldots, p_n^1]} - \frac{\min[p_1^1, p_2^1, \ldots, p_n^1]}{\min[p_1^0, p_2^0, \ldots, p_n^0]} \left(\frac{1}{p_r^0} + \frac{\lambda^k}{\bar{LS}}\right)\right)$$

(19)

From Eq.(19), it was possible to conduct a theoretical formulation of the lot-size dependence and energy consumption per unit of production throughput in a machine for a variable lot size.
4. Verification of the proposed formulation

4.1 Conditions for Simulation

This section examines the validity of the proposed formulation by using simulations of a semiconductor line. We use virtual real data obtained from a developed simulation. The simulation can create virtual real data in response to the simulation.

Electricity consumption is evaluated as energy consumption. A model of the semiconductor line is shown in Fig. 1. This model involves a line of three machines in series that correspond to a part of a semiconductor factory, namely, a solder printing machine \((k = 1)\), a mounter machine \((k = 2)\), and a reflow soldering machine \((k = 3)\). Furthermore, an infinite-capacity buffer exists before each machine. This buffer capacity was used to remove the dependency on buffer capacity, restricting the evaluation to only dependency on variable lot size. The machine variables for each machine are listed in Table 1. The total operating time corresponded to 144000 s \((8 \times 5 \times d)\), and the simulation was conducted with a sufficiently long time set. There were four lot sizes \((30, 90, 180, \text{and} 360)\), and work was to be produced with any of the lot sizes. The lot size ratio was set at 0.1 intervals from 0.0 to 1.0 and used as an input value in the simulation. Furthermore, setting-up occurred whenever a lot was changed.

We carried out two case studies for the semiconductor line. We examined sets of two and three lot sizes and compared the data calculated by using the proposed formulation to the virtual real data obtained from simulation. Additionally, we investigate how the fluctuation in lot size affects energy consumption per unit of production throughput.

![Fig.1 Simulation model of the semiconductor line.](image)

| Input parameter                  | Machine variable of each production machine. |
|----------------------------------|-----------------------------------------------|
| Running energy \(e^a\) \[kW\]   | Solder Printing \(k=1\) | Mounter \(k=2\) | Solder Rerflow \(k=3\) |
| Setting-up energy \(e^b\) \[kW\] | 1.25 | 3.75 | 3.0 |
| Idle energy \(e^c\) \[kW\]      | 3.0 | 1.5  | -   |
| Running time per unit of product \(1/p^d\) \[s\] | 0.2 | 0.2  | 3.0 |
| Setting-up time \(2^e\) \[s\]    | 120±10 | 120±10 | -   |

The simulation method simultaneously obtains the pre-evaluation of productivity and energy consumption by using the organization of required items and functionality (Sakuma et al., 2013) (Hibino et al., 2012). More specifically, as shown in Fig. 2, the status of a production machine and its transition model are generalized and modeled using UML (Unified Modeling Language). Additionally, energy consumption rate was calculated using a method that simultaneously calculates the energy consumption and the total production by applying the production machine status and its transition model to WITNESS (ITOCHU Techno-Solutions Corporation), which corresponds to a discrete event simulation. Moreover, the energy consumption per unit of production throughput was visualized per unit evaluation time to check the efficacy.
4.2 Verification through semiconductor line

4.2.1 Set of two lot sizes

Each combination of two lot sizes, A and B, from the four lot sizes, namely 30, 90, 180, and 360, is evaluated (6 combinations). The combinations of A and B are shown in Table 2. Additionally, the lot size ratio is varied from 0 to 1 with an interval of 0.1 (11 distributions). The combinations of lot size ratio and average lot size for cases with two variable lot sizes are listed in Table 3.

Table 2 Six combinations of two lot sizes, A and B.

| Case | Lot Size A | Lot Size B |
|------|------------|------------|
| Case1| 30         | 90         |
| Case2| 30         | 180        |
| Case3| 30         | 360        |
| Case4| 90         | 180        |
| Case5| 90         | 360        |
| Case6| 180        | 360        |

Table 3 Eleven distributions of two lot sizes.

| Lot Size A | Lot Size B | Lot Size Ratio | Average Lot Size |
|------------|------------|----------------|------------------|
| LS(1)      | LS(2)      | X(LS(1))       | X(LS(2))         | \(LS\)           |
| 0.0        | 1.0        | 0.0            | 1.0              | B                |
| 0.1        | 0.9        | 0.1            | 0.9              | A                |
| 0.2        | 0.8        | 0.2            | 0.8              | A                |
| 0.3        | 0.7        | 0.3            | 0.7              | B                |
| 0.4        | 0.6        | 0.4            | 0.6              | B                |
| 0.5        | 0.5        | 0.5            | 0.5              | B                |
| 0.6        | 0.4        | 0.6            | 0.4              | B                |
| 0.7        | 0.3        | 0.7            | 0.3              | B                |
| 0.8        | 0.2        | 0.8            | 0.2              | A                |
| 0.9        | 0.1        | 0.9            | 0.1              | A                |
| 1.0        | 0.0        | 1.0            | 0.0              | A                |

Therefore, we examine 66 patterns. There are two lot sizes, so \(m = 2\), and the average lot size is as follows:

\[
\bar{LS} = \sum_{i=1}^{2} LS(i) \times X(LS(i))
\]

Subsequently, we calculate the throughput when each machine operates solo, \(p^B_k\), in order to obtain machine \(k\)’s throughput, \(p^k\), and the throughput from the entire line, \(p\). The throughputs when each machine operates solo \(p^B_k\) are as follows:
Therefore, machine $k$’s throughput, $p^k$, and the entire line’s throughput, $p$, are expressed as follows:

$$p^1 = \min(p_0^1) = \left(10 + \frac{120}{L_S}\right)^{-1} \tag{24}$$

$$p^2 = \min(p_0^1, p_0^2) = \left(10 + \frac{120}{L_S}\right)^{-1} \tag{25}$$

$$p^3 = \min(p_0^1, p_0^2, p_0^3) = \left(10 + \frac{120}{L_S}\right)^{-1} \tag{26}$$

$$p = \min(p_0^1, p_0^2, p_0^3) = \left(10 + \frac{120}{L_S}\right)^{-1} \tag{27}$$

Therefore, the energy consumption per unit of production throughput of each machine, $U^k$, is as follows:

$$U^1 = 12.5 + \frac{360}{L_S} \tag{28}$$

$$U^2 = 37.5 + \frac{180}{L_S} \tag{29}$$

$$U^3 = 30 + \frac{360}{L_S} \tag{30}$$

These equations are compared to the virtual real data obtained from simulation.

First, we verify the throughput predicted by Eq.(27). Throughput corresponds to the amount of production per unit time. Hence, multiplying this by the period of simulation (144000 s) yields the total production $P$. Figure 3 compares the data calculated by using the proposed formulation and the virtual real data obtained from simulation. The vertical axis denotes the total production, and the horizontal axis denotes the inverse of average lot size.

![Figure 3](image-url)  
Fig.3 Relationship between total production and inverse of average lot size for two lot sizes.
As shown in Fig. 3, the total production decreases with a decreasing of the average lot size. This potentially occurs because a reduction in the average lot size increases the number of setting-up events, reducing the running time, thereby reducing total production. Furthermore, Fig. 3 confirms that the total production obtained from Eq. (27) matched the virtual real data obtained from simulation.

Next, we verify the formulation for machine $k$’s energy consumption per unit of production throughput. For an example, Table 4 shows the results of the relationship between the average lot size and the energy consumption per unit of production throughput in case 1. The results of each case shown in Table 2 are shown in Fig. 4–9. In these figures, the vertical axis represents the energy consumption per unit of production throughput, and the horizontal axis represents the inverse of average lot size. The solid lines in the figures represent the proposed formulations (Eq. (28–30)), and the symbols represent the virtual real data obtained from simulation ($\times$: soldering printing machine, $\Box$: mounter machine, $\triangle$: reflow soldering machine). As shown in Fig. 4–9, the data calculated by using the proposed formulation were agreed well with the virtual real data obtained by simulation.

Table 4  Average lot size and energy consumption per unit of production throughput in Case 1.

| Lot Size | Lot Size Ratio | Average Lot Size | Energy Consumption per Unit of Production Throughput | Case Sub Number |
|----------|----------------|------------------|-----------------------------------------------------|-----------------|
|          | $L_S(2)$ | $X(L_S(2))$ | $L_S^*$ | $U^*_1$ (formula) | $U^*_1$ (virtual real data) | $U^*_2$ (formula) | $U^*_2$ (virtual real data) | $U^*_3$ (formula) | $U^*_3$ (virtual real data) | $U^*_4$ (formula) | $U^*_4$ (virtual real data) |
| 0.0      | 1.0   | 90       | 16.493 | 16.500 | 39.507 | 39.500 | 34.042 | 34.000 | [1] |
| 0.1      | 0.9   | 84       | 16.796 | 16.786 | 39.661 | 39.643 | 34.362 | 34.362 | [2] |
| 0.2      | 0.8   | 78       | 17.124 | 17.115 | 39.826 | 39.808 | 34.693 | 34.693 | [3] |
| 0.3      | 0.7   | 72       | 17.533 | 17.500 | 40.025 | 40.000 | 35.092 | 35.092 | [4] |
| 0.4      | 0.6   | 66       | 17.951 | 17.955 | 40.240 | 40.227 | 35.525 | 35.525 | [5] |
| 0.5      | 0.5   | 60       | 18.459 | 18.500 | 40.498 | 40.500 | 36.044 | 36.044 | [6] |
| 0.6      | 0.4   | 54       | 19.095 | 19.167 | 40.817 | 40.833 | 36.680 | 36.680 | [7] |
| 0.7      | 0.3   | 48       | 19.898 | 20.000 | 41.215 | 41.250 | 37.475 | 37.475 | [8] |
| 0.8      | 0.2   | 42       | 20.957 | 21.071 | 41.747 | 41.786 | 38.540 | 38.540 | [9] |
| 0.9      | 0.1   | 36       | 22.444 | 22.500 | 42.475 | 42.500 | 40.000 | 40.000 | [10] |
| 1.0      | 0.0   | 30       | 24.499 | 24.500 | 43.491 | 43.500 | 42.072 | 42.000 | [11] |

Fig. 4 Energy consumption per unit of production throughput for each machine

Fig. 5 Energy consumption per unit of production throughput for each machine
4.2.2 Set of three lot sizes

Each combination of three lot sizes, A, B, and C, from the four lot sizes types, namely 30, 90, 180, and 360, is evaluated (4 combinations). The combinations of A, B, and C are shown in Table 5. Additionally, the lot size ratio is varied from 0.0 to 1.0 with an interval of 0.1 (66 distributions). The combinations of lot size ratio and average lot size for cases with three lot sizes are listed in Table 6.

Therefore, we examine 264 patterns. There are three lot sizes, so $m = 3$, and the average lot size is as follows:

$$\overline{LS} = \sum_{i=1}^{3} LS(i) \times X(\overline{LS}(i))$$

(31)
Furthermore, the throughput, $p$, and each machine’s energy consumption per unit of production throughput, $U^k$, for cases with three lot sizes are as follows:

$$p = \left(10 + \frac{120}{LS}\right)^{-1}$$  \hspace{1cm} (32)

$$U^1 = 12.5 + \frac{360}{LS}$$  \hspace{1cm} (33)

$$U^2 = 37.5 + \frac{180}{LS}$$  \hspace{1cm} (34)

$$U^3 = 30 + \frac{360}{LS}$$  \hspace{1cm} (35)

It is worth noting that Eq.(32–35) are identical to Eq.(27–30), which suggests that the methodology is applicable to any permutation of number of lot sizes and lot size distributions; the critical parameter is the average lot size. These equations are compared to the virtual real data obtained from simulation.

First, we verify the throughput predicted by Eq.(32). Multiplying the throughput by the period of simulation (144000 s) yields the total production $P$. Figure 10 compares the data calculated by using the proposed formulation and the virtual real data obtained from simulation. The vertical axis denotes the total production, and the horizontal axis denotes the inverse of average lot size. Figure 10 demonstrates that the virtual real data obtained from simulation matched the data calculated by using the proposed formulation for three lot sizes.

Next, we verify the formulation for machine $k$’s energy consumption per unit of production throughput with respect to three lot sizes. For an example, Table 7 shows the results of the relationship between the average lot size and the energy consumption per unit of production throughput in case 7. The results of each case shown in Table 5 are shown in Fig. 11–14. The vertical axis represents the energy consumption per unit of production throughput, and the horizontal axis represents the inverse of average lot size. The solid lines in the figures represent the proposed formulations (Eq.(33–35)), and the symbols represent the virtual real data obtained from simulation ($\times$: soldering printing machine, $\square$: mounter machine, $\triangle$: reflow soldering machine).
As shown in Fig. 4–14, the data calculated by using the proposed formulation agreed well with the virtual real data obtained by simulation. Comparing the cases with two and three lot sizes, we find that the formulations are only functions of the average lot size, regardless of the lot size distribution. For example, case sub numbers {25} and {26} in Table 7

Table 7  Average lot size and energy consumption per unit of production throughput in Case 7.

| Case Sub Number | Lot Size Ratio | Average Lot Size | Energy Consumption per Unit of Pprdct Throughput |
|-----------------|----------------|-----------------|-----------------------------------------------|
|                 | LS(1) | LS(2) | LS(3)   | X(LS(1)) | X(LS(2)) | X(LS(3)) | \(U^1\) virtual real data | \(U^1\) (formula) | \(U^2\) virtual real data | \(U^2\) (formula) | \(U^3\) virtual real data | \(U^3\) (formula) |
| 1               | 0.0  | 0.0  | 1.0     | 180     | 14.484  | 14.500  | 38.491  | 38.500  | 32.016  | 32.000  | 32.000  |
| 2               | 0.0  | 0.1  | 0.9     | 171     | 14.578  | 14.605  | 38.553  | 38.553  | 32.142  | 32.105  | 32.105  |
| 3               | 0.1  | 0.0  | 0.9     | 165     | 14.667  | 14.682  | 38.595  | 38.591  | 32.225  | 32.182  | 32.182  |
| 25              | 0.2  | 0.5  | 0.3     | 105     | 15.870  | 15.929  | 39.199  | 39.214  | 33.437  | 33.429  | 33.429  |
| 26              | 0.5  | 0.0  | 0.5     | 105     | 15.958  | 15.929  | 39.242  | 39.214  | 33.524  | 33.524  | 33.524  |
| 27              | 0.1  | 0.7  | 0.2     | 102     | 16.066  | 16.029  | 39.297  | 39.265  | 33.633  | 33.529  | 33.529  |
| 28              | 0.2  | 0.4  | 0.3     | 99      | 16.147  | 16.136  | 39.338  | 39.318  | 33.713  | 33.636  | 33.636  |
| 29              | 0.9  | 0.0  | 0.1     | 45      | 20.501  | 20.500  | 41.522  | 41.500  | 38.091  | 38.000  | 38.000  |
| 64              | 0.9  | 0.1  | 0.0     | 36      | 22.444  | 22.500  | 42.475  | 42.500  | 40.000  | 40.000  | 40.000  |
| 65              | 1.0  | 0.0  | 0.0     | 30      | 24.499  | 24.500  | 43.491  | 43.500  | 42.072  | 42.000  | 42.000  |
have different lot size distributions, but the average lot size and predicted energy consumption per product throughput are identical. Thus, the energy consumption per unit of production throughput is determined by the average lot size without depending on the lot size distribution.

As shown in Fig. 4–9 and 11–14, when the average lot size is small, the energy consumption per unit of production throughput is high because of the increasing number of setting-up events. In contrast, when the average lot size is large, the energy consumption per unit of production throughput is small. Through these two case studies, we have clarified the relation between energy consumption per unit of production throughput and lot size in lines with variable lot size.

5. Conclusion

This study examined the lot size dependence of energy consumption per unit of production throughput for variable lot size in a line. A formulation was developed to define this relationship. The formulation was verified using simulation results. The formulation was proposed such that it could deal with the case of variable lot size by defining the average lot size. With respect to the case study, it was possible to consider cases with two and three lot sizes and confirm that there was a match between the virtual real data obtained from simulation and the data calculated by using the proposed formulation. Future work will include applying this formulation to lot-size optimization.

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