Microwave Photoconductivity in Two-Dimensional Electron Systems due to Photon-Assisted Interaction of Electrons with Leaky Interface Phonons

V. Ryzhii

Computer Solid State Physics Laboratory, University of Aizu, Aizu-Wakamatsu 965-8580, Japan

(Dated: March 22, 2022)

We calculate the contribution of the photon-assisted interaction of electrons with leaky interface phonons to the dissipative dc photoconductivity of a two-dimensional electron system in a magnetic field. The calculated photoconductivity as a function of the frequency of microwave radiation and the magnetic field exhibits pronounced oscillations. The obtained oscillation structure is different from that in the case of photon-assisted interaction with impurities. We demonstrate that at a sufficiently strong microwave radiation in the certain ranges of its frequency (or in certain ranges of the magnetic field) this mechanism can result in the absolute negative conductivity.

PACS numbers: PACS numbers: 73.40.-c, 78.67.-n, 73.43.-f

A substantial interest in the transport phenomena in a two-dimensional electron system (2DES) subjected to a magnetic field has been revived after experimental observations by Mani et al. 1, Zudov et al. 2 3 4 (see also Ref. 1) of vanishing electrical resistance caused by microwave radiation. The occurrence of this effect is primarily attributed to the realization of the absolute negative conductivity (ANC) when the dissipative dc conductivity \( \sigma < 0 \) in certain ranges of the microwave frequencies and the magnetic and electric fields \( 2 3 4 5 6 7 8 9 10 11 12 13 14 \). Mechanisms of ANC in a 2DES subjected to a magnetic field and irradiated with microwaves have been studied theoretically over many years \( 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 \). The mechanisms in question are associated with the photon-assisted electron-impurity and electron-phonon interactions. As shown \( 16 \), the interaction of electrons with leaky interface phonons can essentially govern the transport in a 2DES. The two dimensional character of the spectrum of such phonons substantially affects the scattering selection rules. As a result, the contributions to dissipative dc conductivity of the electron scattering processes with 2D and 3D acoustic phonons can be markedly different. The dissipative dc conductivity of a 2DES in the magnetic field in “dark” conditions (without irradiation) determined by the electron interaction with 2D acoustic phonons was calculated in Refs. \( 15 16 18 19 20 21 22 23 24 25 26 27 28 29 30 \). The effect of the electron interaction with 3D-acoustic phonons on the dc conductivity and the dc microwave photoconductivity was considered recently \( 14 \).

In this paper, we calculate the dissipative dc photoconductivity of a 2DES in the magnetic field irradiated with microwaves considering the photon-assisted interaction of electrons with leaky interface phonons. In particular, it is shown that this mechanism can lead to ANC.

The dissipative transport in the situation under consideration is associated with the shifts (hops) of the electron Larmor orbit centers in the direction of the net dc electric field \( \mathbf{E} = (E, 0, 0) \) and in the opposite direction caused by the electron scattering. The length of such a shift equals \( \delta \xi = L^2 q_y \), where \( L = (\hbar c/eH)^{1/2} \) is the quantum Larmor radius, \( q_y \) is the variation the electron momentum component perpendicular to the direction of the electric field, \( e = |e| \) is the electron charge, \( \hbar \) is the Planck constant, \( H = (0, 0, H) \) is the magnetic field (directed perpendicular to the 2DES plane), and \( c \) is the velocity of light. Taking this into account, we start from the following sufficiently general expression for the dissipative dc current in a 2DES in the presence of microwave radiation:

\[
J_{ph}(E) = \frac{2\pi e}{\hbar L} \sum_{N,N'} f_N(1 - f_{N'})
\]

\[
\times \int dq_x dq_y | \Omega(q_x, q_y)|^2 |Q_{N,N'}(L^2 q_y^2/2)|^2
\]

\[
\times \{N_{q_y} \delta[h\Omega + (N - N')h\Omega_c + hsq_y + eEL^2 q_y] + (N_{q_y} + 1)\delta[h\Omega + (N - N')h\Omega_c - hsq_y + eEL^2 q_y]\} \tag{1}
\]

Here \( \Omega_c = eH/mc \) is the cyclotron frequency, \( f_N \) and \( N_{q_y} \), are the electron and phonon distribution functions, respectively, \( N = 0, 1, 2, \ldots \) is the Landau level (LL) index, \( q_y = \sqrt{q_x^2 + q_y^2} \), \( s \) is the velocity (its real part) of leaky waves, \( \delta(q) \) is the form-factor of LL’s which at small their broadening \( \Gamma \) can be assumed to be the Dirac delta function, \( V(q_{LL}) \) is the matrix element of the electron-phonon interaction, \(|Q_{N,N'}(L^2 q_y^2/2)|^2 = |P_{N,N'}(L^2 q_y^2/2)|^2 \exp(-L^2 q_y^2/2) \) is determined by the overlap of the electron wave functions corresponding to the initial and final states, and \(|P_{N,N'}(L^2 q_y^2/2)|^2 \) is proportional to a Laguerre polynomial. The electron and phonon distribution functions are assumed to be the Fermi and Planck functions, respectively, with the temperature \( T \) (in energy units) and the Fermi energy \( \xi \) reckoned from the lowest LL.
The quantity $I_{0}(q_{x}, q_{y})$ is proportional to the incident microwave field on the in-plane electron motion. Disregarding the influence of the microwave radiation polarization, one can set $I_{0}(q_{x}, q_{y}) = J_{0}L^{2}q_{\perp}^{2}$, where $J_{0} = (E_{\Omega}/\varepsilon_{\Omega})^{2}$, $E_{\Omega}$ is the microwave electric field amplitude, which is assumed to be smaller than some characteristic microwave field $\varepsilon_{\Omega}$, and use the following formula [18]:

$$\varepsilon_{\Omega} = \frac{\sqrt{2}m\Omega^{2}L/e}{c\Omega\varepsilon_{\Omega} + \Omega^{2}}$$  \hspace{1cm} (2)

Here $\varepsilon_{\Omega} = \sqrt{2}m\Omega^{2}L/e$. Equation (1), which describes the effect of microwave field by the inclusion of factor $I_{0}(q_{x}, q_{y})$, corresponds to the single-photon absorption of microwave radiation with the electron transitions between different LL’s. The processes associated with the emission of microwave photons involving phonons in which electrons do not transfer between LL’s are also be possible and can provide some contribution to the photoconductivity at $\Omega \ll \Omega_{c}$. However, the range $\Omega \ll \Omega_{c}$ is not considered here. Equations (1) and (2) are valid even in the vicinity of the cyclotron resonance $\Omega = \Omega_{c}$, if $\varepsilon_{\Omega}/\varepsilon_{\Omega} < 1/\Omega_{c}$, when the quantity $\varepsilon_{\Omega}$ is limited by the LL broadening. In this limit one can estimate $\varepsilon_{\Omega} \simeq \sqrt{2}m\Omega^{2}/c\varepsilon_{\Omega}\Gamma/\Omega_{c}$. In the case when $\varepsilon_{\Omega}/\varepsilon_{\Omega} > 1/\Omega_{c}$, the dependence of the photon absorption on the microwave electric field becomes more complex, particularly at the cyclotron resonance, because multiple-photon processes (both real and virtual) become important. In such a case, the probability of the processes involving $n = 0, 1, 2, \ldots$ real photons are proportional to $I_{n}\Omega(q_{x}, q_{y}) = J^{2}_{0}(\sqrt{\Omega_{c}Lq_{\perp}})$, where $J_{n}(q)$ is the Bessel function.

Using the variables $q_{\perp}$ and $\Theta$ (instead of $q_{x}$ and $q_{y}$), so that $q_{y} = q_{\perp} \sin \Theta$, from Eq. (1) we arrive at

$$J_{ph}(E) = J_{0}(\frac{2\pi e}{h^{2}L}) \sum_{N\Lambda>0} f_{N}(1 - f_{N+\Lambda})$$

$$\times \int_{0}^{2\pi} d\Theta \sin \Theta \int_{0}^{\infty} d q_{\perp}^{2} \text{exp}(-L^{2}q_{\perp}^{2}/2)|V(q_{\perp})|^{2}$$

$$\times |P_{N}^{A}(L^{2}q_{\perp}^{2}/2)|^{2}\{N_{q_{\perp}} \delta \{q_{\perp} - q_{\perp}^{A} + (eEL^{2}/hs)q_{\perp} \sin \Theta \}$$

$$\pm [(N_{q_{\perp}} + 1)\delta \{q_{\perp} - q_{\perp}^{A} - (eEL^{2}/hs)q_{\perp} \sin \Theta \}],$$  \hspace{1cm} (3)

where $q_{\perp}^{A} = (\Omega - \Omega_{c})/s$, and $\Lambda > 0$. At low electric fields, one can expand the expression in the right-hand side of Eq. (3) in powers of $(eEL^{2}/hs)$. Upon integrating over $\Theta$ we present the dissipative dc photoconductivity $\sigma_{ph} = J_{ph}/E$ in the following form:

$$\sigma_{ph} = J_{0}(\frac{\pi e^{2}L^{3}}{h^{2}s^{2}}) \sum_{N\Lambda>0} f_{N}(1 - f_{N+\Lambda})$$

$$\times \int_{0}^{\infty} d q_{\perp}^{2} \text{exp}(-L^{2}q_{\perp}^{2}/2)|V(q_{\perp})|^{2}\left[\frac{K_{N}^{A}(q_{\perp}) \text{exp}(hsq_{\perp}/T)}{\text{exp}(hsq_{\perp}/T) - 1}\right]_{q_{\perp} = -q_{\perp}^{A}}$$  \hspace{1cm} (5)

$$\times \int_{0}^{\infty} d q_{\perp}^{2} \text{exp}(-L^{2}q_{\perp}^{2}/2)|V(q_{\perp})|^{2}\left[\frac{K_{N}^{A}(q_{\perp}) \text{exp}(hsq_{\perp}/T)}{\text{exp}(hsq_{\perp}/T) - 1}\right]_{q_{\perp} = q_{\perp}^{A}}$$  \hspace{1cm} (6)

at $\Omega - \Omega_{c} < 0$ (i.e., when $q_{\perp}^{A} < 0$), and

$$\sigma_{ph} \simeq J_{0}(\frac{\pi e^{2}L^{3}}{h^{2}s^{2}}) \sum_{N=N_{m}-\Lambda+1}^{N_{m}} f_{N}(1 - f_{N+\Lambda})$$

$$\times \int_{0}^{\infty} d q_{\perp}^{2} \text{exp}(-L^{2}q_{\perp}^{2}/2)|V(q_{\perp})|^{2}\left[\frac{K_{N}^{A}(q_{\perp}) \text{exp}(hsq_{\perp}/T)}{\text{exp}(hsq_{\perp}/T) - 1}\right]_{q_{\perp} = q_{\perp}^{A}}$$

at $\Omega - \Omega_{c} > 0$ (when $q_{\perp}^{A} > 0$). Here $K_{N}^{A}(q_{\perp}) = q_{\perp}^{A} \text{exp}(-L^{2}q_{\perp}^{2}/2)|V(q_{\perp})|^{2}\left[P_{N}^{A}(L^{2}q_{\perp}^{2}/2)\right]^{2}$ and $N_{m}$ is the number of filled LL’s, i.e., $N_{m}h\Omega_{c} < \zeta < (N_{m} + 1)h\Omega_{c}$. For a large $N$, expressing the Laguerre polynomials via the Bessel functions and assuming that $|V(q_{\perp})|^{2} \propto 1/q_{\perp}$, we have $K_{N}^{A}(q_{\perp}) \propto q_{\perp}^{A} \text{exp}(-L^{2}q_{\perp}^{2}/2)J_{0}^{2}(\sqrt{2NLq_{\perp}})$. Thus, $K_{N}^{A}(q_{\perp}) \propto q_{\perp}^{A+2}$ at $q_{\perp} < 1/\sqrt{2N}$, and $K_{N}^{A}(q_{\perp}) \propto q_{\perp}^{A} \text{exp}(-L^{2}q_{\perp}^{2}/2)\cos^{2}(\sqrt{2NLq_{\perp}} - (2\Lambda + 1))$ at $Lq_{\perp} \gg 1/\sqrt{2N}$. 

![FIG. 1: Photoconductivity vs microwave frequency for different $b = hs/EL$.](image-url)
In the case of resonance detuning $1/\sqrt{2N} < L|\Omega - \Lambda \Omega_c|/s \lesssim (LT/\hbar s) \ll L \Omega_c/s$ (in the immediate vicinity of the resonance $|\sigma_{ph}|$ is very small), from Eqs. (4) and (5) we arrive at

$$
\sigma_{ph} \propto J_\Omega \left( LT/\hbar s \right) L(\Omega - \Lambda \Omega_c)/s. \tag{7}
$$

Here we have averaged an oscillatory factor in $K_N^2$ bearing in mind the finite broadening of the LL’s. In the range $(LT/\hbar s) < L|\Omega - \Lambda \Omega_c|/s \ll L \Omega_c/s$, Eqs. (5) and (6) yield

$$
\begin{align*}
\sigma_{ph} &\propto J_\Omega \left( LT/\hbar s \right) \frac{L^2(\Omega - \Lambda \Omega_c)^2}{s^2} \\
&\times \left[ \frac{L^2(\Omega - \Lambda \Omega_c)^2}{s^2} - \frac{\left( \hbar s/L \Omega \right) L(\Omega - \Lambda \Omega_c)}{s} - 3 \right] \\
&\times \exp \left[ \frac{\left( \hbar s - \Lambda \Omega_c \right)}{2T} \right] \exp \left[ \frac{L^2(\Omega - \Lambda \Omega_c)^2}{2s^2} \right], \tag{8}
\end{align*}
$$

at $\Omega - \Lambda \Omega_c < 0$, and

$$
\begin{align*}
\sigma_{ph} &\propto -J_\Omega \left( LT/\hbar s \right) \frac{L^2(\Omega - \Lambda \Omega_c)^2}{s^2} \\
&\times \left[ \frac{L^2(\Omega - \Lambda \Omega_c)^2}{s^2} - 3 \right] \exp \left[ \frac{L^2(\Omega - \Lambda \Omega_c)^2}{2s^2} \right], \tag{9}
\end{align*}
$$

at $\Omega - \Lambda \Omega_c > 0$.

The obtained formulas describe an oscillatory dependence of the dissipative dc photoconductivity associated with the photon-assisted interaction of electrons with leaky interface phonons on the microwave radiation frequency and the magnetic field. Despite of a marked difference in the dissipative dark conductivities associated with the 2D and 3D electron scattering on phonons, respectively, (compare Refs. [10] and [14]), the spectral dependence of the dissipative dc microwave photoconductivity calculated here is qualitatively similar to that obtained for the case of the photon-assisted interaction of electrons with 3D-acoustic phonons [14]. Figure 1 shows the dissipative dc photoconductivity as a function of microwave frequency calculated using Eqs. (5) and (6) for $N_m \gg 1$ and different values of parameter $b = \hbar s/LT$, i.e., for different temperatures. The frequency dependence of $J_\Omega$ was taken according to Eq. (2) with a proper modification at the immediate vicinity of the cyclotron resonance: $J_\Omega \propto \frac{\Omega^2}{\hbar s/\Omega c} \left[ \frac{\Omega^2}{\hbar s/\Omega c} + \frac{\Omega_c}{\hbar} \right]^2$ with $\gamma = \Gamma/\Omega_c = 0.1$. One can see that the dissipative dc photoconductivity associated with the scattering processes under consideration exhibits a pronounced minimum with $\sigma_{ph} < 0$ in the microwave frequency range between the first (cyclotron) and the second resonances ($\Omega_c < \Omega < 2\Omega_c$). At sufficiently strong radiation with the frequency in this range, the value of the dissipative dc photoconductivity can exceed the dark conductivity leading to ANC. The dissipative dc photoconductivity is negative also in the ranges between higher resonances ($\Lambda \Omega_c < \Omega < (\Lambda + 1) \Omega_c$, with $\Lambda > 1$). However, the amplitude of the photoconductivity oscillations in these ranges is markedly smaller due to a significant decrease in $J_\Omega$ with increasing ratio $\Omega/\Omega_c$. It is instructive that the “phonon” mechanisms (considered above and in Ref. [14]) and the “impurity” mechanism [10, 11] result in quite different behavior of the microwave photoconductivity as a function of the resonance detuning, particularly in the vicinities of the resonances.

The author is grateful to V. A. Volkov and V. V. Vyurkov for numerous discussions and R. R. Du and M. A. Zudov for providing Refs. [4, 15].

[1] R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayananmurti, W. B. Johnson, and V. Umansky, Nature 420, 646 (2002).
[2] M. A. Zudov, R. R. Du, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 90, 046807-1 (2003).
[3] C. L. Yang, M. A. Zudov, T. A. Knuttila, R. R. Du, L. N. Pfeiffer, and K. W. West, arXiv:cond-mat/0303472 (2003).
[4] M. A. Zudov, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. B 64, 201311 (2001).
[5] P. W. Anderson and W. F. Brinkman, arXiv:cond-mat/0302129 (2003).
[6] A. V. Andreev, I. L. Aleiner, and A. J. Millis, arXiv:cond-mat/0302063 (2003).
[7] P. S. Bergeret, B. Huckestein and A. F. Volkov, arXiv:cond-mat/0303530 (2003).
[8] R. Fitzgerald, Physics Today 56, 24 (2003).
[9] V. I. Ryzhii, Sov. Phys.-Solid State 11, 2078 (1970).
[10] V. I. Ryzhii, R. A. Suris, and B. S. Shchamkhalova, Sov. Phys.-Semicond. 20, 1299 (1986).
[11] A. C. Durst, S. Sachdev, N. Read, and S. M. Girvin, arXiv:cond-mat/0301569 (2003).
[12] V. Shikin, JETP Lett. 77, 236 (2003).
[13] X. L. Lei and S. Y. Liu, arXiv:cond-mat/03004687 (2003).
[14] V. Ryzhii and V. Vyurkov, arXiv:cond-mat/0305190 (2003).
[15] M. A. Zudov, I. V. Ponomarev, A. L. Efros, R. R. Du J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. 86, 3614 (2001).
[16] M. Sh. Erukhimov, Sov. Phys.-Semicond. 3, 162 (1969).
[17] A. D. Malov and V. I. Ryzhii, Sov. Phys.- Solid State 14, 1766 (1973).
[18] V. V. V'yurkov, A. D. Gladun, A. D. Malov, and V. I. Ryzhii, Sov. Phys.- Solid State 19, 2113 (1977).