The Effect of General Relativistic Precession on Tidal Disruption Events from Eccentric Nuclear Disks

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Abstract

An eccentric nuclear disk consists of stars moving on apsidally aligned orbits around a central black hole. The secular gravitational torques that dynamically stabilize these disks can also produce tidal disruption events (TDEs) at very high rates in Newtonian gravity. General relativity, however, is known to quench secular torques via rapid apsidal precession. Here we show that for a disk-to-black-hole mass ratio of $M_{\text{disk}}/M_\bullet \gtrsim 10^{-3}$, the system is in the full loss-cone regime. The magnitude of the torque per orbital period acting on a stellar orbit means that general relativistic precession does not have a major effect on the dynamics. Thus we find no evidence that TDE rates from eccentric nuclear disks in the full loss-cone regime are affected by general relativistic precession. Furthermore, we show that orbital elements between successive TDEs from eccentric nuclear disks are correlated, potentially resulting in unique observational signatures.

Key words: celestial mechanics – galaxies: kinematics and dynamics – galaxies: nuclei

1. Introduction

A tidal disruption event (TDE) occurs when a star is violently ripped apart by a black hole’s tidal forces (Hills 1975). When a star is tidally disrupted, roughly half of the stellar debris remains bound to the black hole while the other half of the debris escapes. The gravitationally bound debris forms an accretion disk that feeds the black hole, producing a flare (Rees 1988). The current detection rate of flares from TDEs is about two per year (van Velzen 2018) and this is expected to increase with new surveys such as the Large Synoptic Survey Telescope (van Velzen et al. 2011).

TDE flares can provide insight into the mysteries of many areas of astrophysics. They illuminate central black holes in otherwise quiescent galaxies (Maksym et al. 2013; MacLeod et al. 2014). We can use their observations to test theories of accretion physics and relativistic jets (Bloom et al. 2011; Zauderer et al. 2011; van Velzen et al. 2016; Alexander 2017). Tidal disruptions of white dwarfs should even produce gravitational waves detectable by the Light Interferometer Space Antenna (Zalamea et al. 2010; MacLeod et al. 2014). Additionally, we can test our understanding of gravitational stellar dynamics near supermassive black holes (SMBHs) by comparing theoretical TDE rates with observations. We begin by introducing loss-cone dynamics, which are important for understanding how general relativistic precession in eccentric nuclear disks could cause TDE rates to vary.

1.1. Loss-cone Dynamics

The rate of TDEs due to stellar two-body relaxation has been studied extensively (Frank & Rees 1976; Lightman & Shapiro 1977; Cohn & Kulsrud 1978; Shapiro & Marchant 1978; Stone & Metzger 2016). Two-body relaxation is the diffusive process by which stars exchange energy and angular momentum among themselves, sometimes scattering a star onto a tidally disrupting orbit. It is faster to reach such an orbit through diffusion in angular momentum than in energy (Frank & Rees 1976).

In order for a star in these systems to get close enough to the SMBH to tidally disrupt, it must enter the loss cone. The loss cone defines the region containing orbits with pericenters inside the tidal disruption radius of the black hole. The tidal disruption radius is

$$r_t = \left( \frac{M_\bullet}{M_*} \right)^{1/3} R_*, \hspace{1cm} (1)$$

where $M_\bullet$ is the mass of the black hole, $M_*$ is the mass of the star, and $R_*$ is the radius of the star (Rees 1988). Orbits within the loss cone have angular momenta less than the angular momentum of an orbit with a pericenter equal to the tidal radius,

$$J < J_{LC} \approx \sqrt{2GM_\bullet R_t}. \hspace{1cm} (2)$$

There are two loss-cone regimes, defined by the parameter $q$:

$$q = \left( \frac{\Delta J}{J_{LC}} \right)^2, \hspace{1cm} (3)$$

where $\Delta J_p$ is the change in angular momentum per orbital period (Lightman & Shapiro 1977). If $q \ll 1$, stars take multiple orbital periods to enter the loss cone. This is known as the empty loss-cone regime or the diffusion limit because the time for a star to enter the loss cone is greater than the time for the star to be destroyed. If $q \gg 1$, stars can jump into and out of the loss cone within one orbital period. This is known as the full loss-cone regime or the pinhole limit, because the loss cone is continuously populated by stars. The division between the two loss-cone regimes for a spherical nuclear star cluster lies close to the radius of influence of the black hole (Lightman & Shapiro 1977). The TDE rate, in this case, is also dominated by stars coming from this region.

1.2. Status of Observations of TDEs

In deriving theoretical TDE rates, we typically assume that stars come from an isotropic, spherical distribution around the
black hole and are driven to the black hole through two-body relaxation (Wang & Merritt 2004; Stone & Metzger 2016). Many TDE rates have been calculated theoretically for spherical nuclear star clusters. For example, Wang & Merritt (2004) calculated a TDE rate of $2.1 \times 10^{-4}$ yr$^{-1}$ gal$^{-1}$, and more recently Stone & Metzger (2016) calculated a rate of $2.0 \times 10^{-4}$ yr$^{-1}$ gal$^{-1}$. In observations, however, TDEs are preferentially found in post-merger or post-starburst galaxies (K+A/E+A galaxies) at much higher rates (Arcavi et al. 2014). K+A/E+A galaxies are a relatively rare subtype of elliptical galaxy that underwent a major starburst about 1–1.5 Gyr ago (Couch & Sharples 1987; Poggianti 2004). K +A/E+A galaxies make up 0.2% of the galaxies in the local universe, and yet, the observed TDE rates in these K+A/E+A galaxies are $(1-3) \times 10^{-3}$ yr$^{-1}$ gal$^{-1}$, which pushes the observed TDE rate of “normal” galaxies down to $(1-5) \times 10^{-6}$ yr$^{-1}$ gal$^{-1}$ (French et al. 2016). There is even (tentative) evidence that the TDE rate could be as high as $10^{-1}$ yr$^{-1}$ gal$^{-1}$ in ultraluminous infrared galaxies, which are typically in the process of merging (Dou et al. 2017; Tadhunter et al. 2017). We learn from these observations that merging galaxies and post-merger galaxies tend to have elevated TDE rates.

Several dozen TDE candidates have been identified in the last two decades, from UV/optical to X-ray. TDE candidates are generally identified as flaring events, inconsistent with supernovae, at the centers of galaxies. Candidates are typically excluded if the host galaxy shows signs of active galactic nucleus (AGN) activity. There have been a number of alternative ideas to explain these flaring events at galactic centers. Proposed TDE impostors include supernovae in AGN disks and black hole accretion disk instabilities (Saxton et al. 2016). One distinguishing feature that can be used to discriminate between real TDEs and impostors is the critical black hole mass beyond which a TDE will not be observable, known as the Hills mass (Hills 1975). The Hills mass results from the fact that the tidal radius and Schwarzschild radius of a black hole scale differently with the mass of the black hole. The Schwarzschild radius is given by

$$r_s = \frac{2GM}{c^2},$$

where $G$ is the gravitational constant and $c$ is the speed of light. Equating the tidal radius to the Schwarzschild radius yields a Hills mass of $\sim 10^8 M_\odot$ for a solar-type star. Above this limit, the star plunges into the black hole without emitting a flare. A rapidly spinning black hole can raise this limit to $\sim 10^9 M_\odot$ (Kesden 2012). Recently, van Velzen (2018) presented the black hole mass function of optical/UV-selected TDE candidates and showed a sharp decrease in the number of candidates above $M_\bullet = 10^5 M_\odot$. This is consistent with the direct capture of stars when the black hole is above the Hills mass and provides strong evidence that we are seeing TDEs rather than impostors.

1.3. Secular Dynamics and Eccentric Nuclear Disks

Two-body relaxation is not the only form of relaxation present in galactic nuclei. Resonant relaxation arises in near-Keplerian potentials (Rauch & Tremaine 1996). In resonant relaxation, a particle on a near-Keplerian orbit traces out the same path repeatedly. On a timescale less than the precession timescale, the orbits remain fixed, and exert mutual gravitational torques on each other. Thus, the angular momentum relaxation can be greatly enhanced, while the energy relaxation is unaffected (Rauch & Tremaine 1996). Resonant relaxation is most effective for stars orbiting close to the central SMBH (in the absence of general relativity). This means that in an isotropic, spherical stellar distribution, where TDEs come most often from near the radius of influence, resonant relaxation will not greatly increase the rate or number of TDEs (Rauch & Ingalls 1998). Not all galactic nuclei, however, are spherically like our galactic center. The nucleus of our nearest galactic neighbor, Andromeda (M31), has a very different configuration. The Andromeda galaxy (M31) has an elongated nucleus that resolves into two distinct brightness peaks. The double-nucleus can be explained by a thick, apsidally aligned eccentric nuclear disk of Keplerian orbits around a SMBH (Tremaine 1995). The two brightness peaks correspond to apopsis and periapsis of the eccentric nuclear disk.

While the central disk in M31 may seem like an unusual and unlikely arrangement, the fact that we see it in our closest major galaxy suggests that it is a common configuration. In fact, despite observational challenges, Lauer et al. (2005) found that about 20% of nearby, early-type galaxies have features consistent with eccentric nuclear disks seen from different angles on the sky.

1.4. TDEs from Eccentric Nuclear Disks

The stability of eccentric nuclear disks has long been a mystery. One would expect that the apsidal precession of individual orbits would spread out the disk into an axisymmetric structure on a timescale much shorter than the age of the stars (Tremaine 1995). In a recent paper (Madigan et al. 2018), we proposed that the same secular mechanism that stabilizes eccentric nuclear disks is responsible for producing high rates of TDEs.

In Madigan et al. (2018), we explain that the forces that cause precession in eccentricity vectors also result in a buildup of gravitational torques between orbits. These torques change the eccentricities of individual orbits as they are perturbed ahead of, or behind, the disk. Differential precession driven by these eccentricity changes holds the disk together. We also showed in Madigan et al. (2018) that the orbits in an eccentric nuclear disk undergo oscillations in eccentricity. During the high-eccentricity phase of an oscillation, a star can be tidally disrupted as it moves through pericenter. The gravitational torques due to secular dynamics are much more efficient at refilling the loss cone than two-body relaxation (Madigan et al. 2018), which has typically been used to determine TDE rates. We proposed that secular torques in eccentric nuclear disks can produce the observed high rate of TDEs in K+A/E+A galaxies (Madigan et al. 2018). Hopkins & Quataert (2010a, 2010b) show that eccentric nuclear disks can form via the merging of gas-rich galaxies, meaning that it would be likely to find eccentric nuclear disks in post-merger, K+A/E+A galaxies. TDE rates in eccentric nuclear disks could be as high as $\sim 1$ yr$^{-1}$ gal$^{-1}$ at early times in the life of the disk (Madigan et al. 2018).
Several other mechanisms have been theorized to explain the enhanced TDE rates in K+\(A/E+A\) galaxies. One of these theories is an enhanced rate due to SMBH binaries after the starburst. Chen et al. (2011) showed that the TDE rate should scale weakly with the SMBH mass ratio. This would indicate that TDEs would be seen primarily after minor mergers, which are more common. TDEs are preferentially observed, however, in mergers with a more equal SMBH mass ratio, indicating that the TDE rate is not driven by SMBH binaries (French et al. 2017). Another theory to explain the enhanced TDE rates in K+\(A/E+A\) galaxies involves more dense spherical star clusters resulting in enhanced two-body relaxation (Stone & van Velzen 2016; Stone et al. 2018).

1.5. This Work

In Madigan et al. (2018), we evolved eccentric nuclear disks with \(N\)-body simulations in Newtonian gravity. We showed that the secular torques in eccentric nuclear disks result in extremely high TDE rates. This previous work did not include rapid apsidal precession due to general relativity, however, which can quench secular dynamical mechanisms. A well-known example of this is the Kozai–Lidov effect, which can be quenched by general relativistic precession as low angular momentum orbits apsidally precess too fast for the gravitational torques to build up (Kozai 1962; Lidov 1962; Ford et al. 2000; Blaes et al. 2002; Naoz et al. 2013). Resonant relaxation in a spherical cluster also gets quenched at low semimajor axes by general relativistic precession as the orbits move too rapidly to allow torques to build up coherently (Rauh & Tremaine 1996; Madigan et al. 2011).

Similarly, one might expect general relativistic precession to disrupt the secular torques of the eccentric nuclear disk, greatly decreasing the TDE rate. As eccentricity increases due to secular torques, the general relativistic precession rate also increases as given by the orbit-averaged precession rate due to general relativity:

\[
\dot{\omega}_{GR} = \frac{6\pi GM}{ac^2(1 - e^2)}. \tag{5}
\]

Equation (5) is a first-order post-Newtonian approximation in general relativity yielding corrections to Newtonian accelerations of \(O(v^2/c^2)\) (Einstein 1916). One would therefore expect eccentric orbits to precess ahead of the disk, escaping completely until joining back up on the other side and recircularizing (Madigan et al. 2018). In this case, general relativistic precession would completely shut down the resonant relaxation in the eccentric nuclear disk and we would expect to see very few TDEs.

The goal of this work is to explore the effects of general relativity on TDEs occurring in eccentric nuclear disks, and to quantify the distribution of orbital elements of TDEs that originate in eccentric nuclear disks. We do this using \(N\)-body simulations with and without general relativity. We present the paper in the following manner. In Section 2 we describe the initial conditions and parameters for our simulations, and compare the number of TDEs that occur with and without general relativity. We track the orbital elements of a single tidally disrupted star in order to show how quickly the orbit is torqued to an extreme eccentricity. In Section 3 we explore the unique orbital elements of tidally disrupted stars from eccentric nuclear disks, including the penetration factor, inclination distribution, and change in eccentricity vector between TDEs. In Section 4 we summarize and discuss our results.

2. \(N\)-body Simulations of Eccentric Nuclear Disks with General Relativistic Precession

We run \(N\)-body simulations of eccentric nuclear disks with REBOUND (Rein & Liu 2012) and the IAS15 integrator (Rein & Spiegel 2015). We implement general relativity as a post-Newtonian approximation with REBOUNDX.\(^2\) In this paper, we show results from simulations with the following general disk parameters: \(N = 100\) stars,\(^3\) each with an initial eccentricity of 0.8, a range of semimajor axes (\(a = 1–2\)) with a surface density of \(\Sigma \propto a^{-2}\), Rayleigh distributed inclinations with mean 0°, and a disk mass of \(10^{-2}M_\odot\). We want to qualitatively understand the effects of general relativistic precession rather than obtain an exact number for the TDE rate.

In each of these simulations, we examine the effect that general relativity has on the number of TDEs. The orbit-averaged precession rate due to general relativity is given by Equation (5). We track the general relativistic precession rate in our simulations by calculating the change in the orientation of the eccentricity vector at each time step.

A star is considered tidally disrupted if at any point in the simulation its radius \(\leq r_t\). We treat stars as point masses and do not extract them from our simulation after they are disrupted, but they are counted only once as a TDE.

2.1. Effects of General Relativity

We find that the TDE rate with general relativistic precession is the same as in Newtonian gravity. About 12% of disk stars are tidally disrupted\(^4\) for a \(10^5M_\odot\) black hole during a time of 1000 orbital periods, where each orbital period is roughly 1000 yr. We have compiled results from \(\sim 45\) simulations with general relativistic precession and \(\sim 100\) simulations without general relativistic precession. The mean percent and standard deviation of tidally disrupted disk stars is shown in Figure 1. In an isotropic, homogeneous case, we would expect to see about 1% of disk stars tidally disrupted. We are focused, however, on comparing the number of TDEs in an eccentric nuclear disk with and without general relativistic precession. The number of TDEs is approximately equal for both general relativistic simulations and Newtonian simulations. This means that general relativistic precession does not shut down the secular torques in eccentric nuclear disks, as we initially expected.

In order to understand this, we track the orbital elements of a single star (with general relativistic precession), which suffers a TDE, in Figure 2. We see a star that develops an eccentricity such that its orbital angular momentum is less than the loss-cone angular momentum. The star also passes through pericenter while it is at a high eccentricity, meaning that the star is close enough to the black hole to be tidally disrupted. We also see that the star’s orbital inclination flips by \(\sim 180^\circ\) as it reaches extreme eccentricity (see discussion in Section 3.2). Panel 4 shows the general relativistic precession rate, which we

\(^2\) https://github.com/dhmayo/reboundx
\(^3\) In Madigan et al. (2018), we used a range of \(N = 100–1000\) stars. The Madigan et al. (2018) simulations in Newtonian gravity gave the same qualitative results for the different \(N\). Here we use \(N = 100\) stars in order to reduce computing time.
\(^4\) This percentage is smaller than that in Madigan et al. (2018) because we have a more rigorous TDE criterion and exclude partial disruptions from our analysis.
track by calculating the change of \(i_e\) in each time step. \(i_e\) tracks the orientation of the eccentricity vector in the plane of the disk and is given by

\[
i_e = \arctan\left(\frac{e_x}{e_y}\right).
\]

(Madigan & McCourt 2016). Here \(e_x\) and \(e_y\) are the x and y components of the eccentricity vector. We use \(i_e\) instead of \(e_x\) instead of the argument of periapsis, \(\omega\), or the longitude of periapsis, \(\varpi\), to avoid effects of changing inclination. As an orbit rolls over its major axis, the eccentricity vector remains close to the \(x-y\) plane, even though the inclination grows. \(\omega\) and \(\varpi\), however, will change with the flipping inclination. We see that the rate of change of \(i_e\) is very small until the star reaches pericenter at an extreme eccentricity where there is a large jump due to general relativity. This jump in precession rate is only present for a fraction of an orbital period. The final panel of Figure 2 shows the torque acting on the orbit in units of the circular angular momentum, which we explore in the next section.

### 2.2. Magnitude of Torque from Disk

Here we calculate the magnitude of the torque exerted on a typical orbit by the disk. The orbit is described by its specific angular momentum and energy

\[
j^2 = GM_a \left(1 - e^2\right),
\]

\[
E = \frac{GM_a}{2a}.
\]

For an eccentric orbit, the specific torque is given by

\[
\tau = j' = r \times f.
\]

\(r\) is the orbital radius and \(f\) is the specific gravitational force felt by an orbit due to the rest of the disk. This force is defined by

\[
|f| = \frac{GM_{\text{disk}}}{r^2},
\]

where \(M_{\text{disk}}\) is the mass of the eccentric nuclear disk. Approximating \(r\) by the semimajor axis \(a\) yields a torque

\[
|\tau| \approx \frac{GM_{\text{disk}}}{a}.
\]

Normalizing the torque by the circular angular momentum \((j_c = \sqrt{GM_a})\) yields

\[
\frac{|\tau|}{j_c} \approx \frac{M_{\text{disk}}}{M} \frac{2\pi}{P},
\]

where \(P = 2\pi\sqrt{a^3}/GM\) is the orbital period. Hence, in our \(N\)-body simulations, in which \(M_{\text{disk}}/M = 10^{-2}\), the normalized torque per orbital period should be on the order of \(6 \times 10^{-2}\). The final panel in Figure 2 shows that indeed our example star experiences a torque of \(O(\text{few} \times 10^{-2})\). This magnitude of torque can change an orbit’s eccentricity from \(e \approx 0.998\) to \(e \approx 1\) within one orbital period. That is, the change in angular momentum required to produce a TDE can occur within one orbital period, suggesting that our system is in the full loss-cone regime. By assuming a \(10^6 M_\odot\) black hole and a solar-type star, we find from Equation (3) that \(q \approx 40\) in our simulations, putting the system well within the full loss-cone or pinhole regime. This explains why general relativity is ineffective at shutting down the TDE production. In the full loss-cone regime, a stellar orbit can be propelled from outside the loss cone to inside in less than an orbital period. General relativistic precession only acts strongly when the star approaches pericenter, at which point it is too late to avoid disruption.

Not all eccentric nuclear disks will be in the full loss-cone regime. The transition from full loss cone to empty loss cone occurs when \(q = 1\) such that

\[
\frac{M_{\text{disk}}}{M} = \sqrt{\frac{r_i}{2\pi^2 a}}.
\]

For a SMBH of \(10^6 M_\odot\), solar-type stars, and a disk inner edge of \(a = 0.05\) pc, we find that disks with \(M_{\text{disk}}/M \geq 1.5 \times 10^{-3}\) are in the full loss-cone regime. The M31 disk has an observed disk mass ratio of \(\sim 10^{-1}\) (Tremaine 1995), putting it in the full loss-cone regime.

### 3. Unique Orbital Elements

Two-body relaxation predicts that the time between individual TDEs (~\(10^4\) yr) is much greater than the time it takes for a TDE disk to accrete onto the black hole. If stars come from eccentric nuclear disks, however, the typical timescale between individual TDEs can be much shorter (~1–10 yr; Madigan et al. 2018), and TDE disks could potentially overlap with one another. This could have interesting observational consequences, especially if the orbital parameters of TDEs are correlated.
3.1. Penetration Factor

The strength of a tidal disruption may be quantified by the dimensionless penetration factor,

$$ \beta = \frac{r_t}{r_p}, $$

(13)

where \( r_t \) is the tidal radius and \( r_p \) is the pericenter of the star’s orbit (Press & Teukolsky 1977). In Figure 3 we show the distribution of penetration factors in our simulations.

If the penetration factor is greater than or equal to one, the star will be tidally disrupted. If the penetration factor is less than but close to one, the star may have its outer layers stripped, with a stellar core remaining intact (Ivanov & Novikov 2001; Guillochon & Ramirez-Ruiz 2013; Bogdanović et al. 2014; Mainetti et al. 2017). If the penetration factor is too large, however, the star will fall straight into the black hole without emitting an electromagnetic flare. This occurs when the \( r_p < r_c \). For a non-spinning, \( 10^6M_\odot \) black hole, and solar-type stars, this occurs at \( \beta = 23.5 \).

We see in Figure 3 that the probability distribution function, \( P \propto \beta^{-2} \), is fully consistent with the full loss-cone or pinhole regime (Lightman & Shapiro 1977). This is significant because the critical radius (where \( q = 1 \)) is typically found near the radius of influence of the black hole. We find that eccentric nuclear disks bring the critical radius orders of magnitude closer to the black hole’sSchwarzschild radius. We see in this figure that our results are consistent with the pinhole regime in which orbits have large steps in angular momentum, allowing stars to jump into the loss cone within an orbital period. The probability distribution function of the impact parameter is well fit by the curve \( \propto \beta^{-2} \), as shown by the solid maroon line. The inset shows the same histogram with the curve \( \propto \beta^{-2} \) on a log–log scale.
within the radius of influence, to a radius smaller than the inner edge of the disk.

3.2. Inclination Distribution

In a spherical, isotropic stellar system dominated by two-body relaxation, there should be no correlation between the orbital angular momentum vectors of consecutive TDEs, so we would expect to see an isotropic distribution of TDE inclinations. This is quite different from the case in which stars are originating in an eccentric nuclear disk.

We find that whenever a star reaches a high eccentricity in our simulations, it undergoes an inclination flip of 180°. Figure 4 is an example of a double peak in eccentricity corresponding to a double 180° flip in inclination. As the orbit is negatively torqued by the disk to extreme eccentricity (in blue), its angular momentum vector decreases until it passes through zero. At this point, the inclination (in green) flips 180° and the orbit switches from a prograde orientation (with respect to the disk) to a retrograde orientation. The orbit now feels a positive torque causing it to circularize and precess quickly back toward the disk. On the other side of the disk, the angular momentum will again decrease, pass through zero, and change direction. The orbit is prograde again after the second flip. These double peaks of inclination were also seen in our Newtonian simulations (Madigan et al. 2018).

As the orbits flip from prograde to retrograde and back, the percentage of stars on retrograde orbits fluctuates throughout a given simulation. Figure 5 is an example of the percentage of retrograde orbits in a disk for a single simulation. Most of these retrograde orbits lie near the inner edge of the disk. There have been many kinematic studies of the M31 disk that give differing precession values for the disk (Sambhus & Sridhar 2000, 2002; Bacon et al. 2001; Jacobs & Sellwood 2001; Salow & Statler 2001, 2004; Lockhart et al. 2018). Current dynamical models of the eccentric nuclear disk in M31 (e.g., Peiris & Tremaine 2003) do not include retrograde moving stars. Our results indicate that the stars on retrograde orbits could be very important for both observations and models.

The flipping of orbits in inclination results in an anisotropic distribution of TDE inclinations (see Figure 6). Stars preferentially tidally disrupt at orbital inclinations of 0° and 180° with respect to the disk mid-plane. More disruptions occur at 0°. This is because the stars get the first opportunity to disrupt at an inclination of 0°, while their orbit is ahead of the disk. The probability for a star to disrupt in one orbital period is

\[ P_{\text{TDE}} = \frac{J_{\text{LC}}}{\Delta t_p} = \frac{1}{\sqrt{2 \pi}} \sqrt{\frac{r_H}{a}} \left( \frac{M_*}{M_{\text{disk}}} \right). \]  

We estimate \( a \approx 10^{-2} r_H \) for the inner edge of the disk, where \( r_H \) is the radius of influence of the black hole, based on the disks in M31 and the Galactic center (Madigan et al. 2018). We take \( r_H \) to be 5pc from observations of the Galactic center (Lu et al. 2009). We find that \( P_{\text{TDE}} = 0.15 \). Out of 100 stars vulnerable to disruption, \(~15\) will tidally disrupt at an inclination of 0°. Then, 85 stars will flip inclinations and have a 15% chance \((~12-13)\) of tidally disrupting at an inclination of 180°. Therefore, we find that the number of TDEs at 180° is 85% of the number at 0°, or in general,

\[ N_{\text{TDE}}(i=180°) = (1 - P_{\text{TDE}}) N_{\text{TDE}}(i=0°). \]  

This explains the height difference that we see in the inclination distribution in Figure 6.

For a spinning Kerr black hole, the tidal and capture cross sections shift toward negative angular momenta (Beloborodov...
et al. 1992). The asymmetric cross sections make it easier for
torques push the orbits of stars to extremely high eccentricities
or stream width, we

3.3. Eccentricity Vector

We plot the eccentricity vectors of tidally disrupted stars at
the time of TDE in the top panel of Figure 7. The eccentricity
vectors precess together in a prograde direction while
remaining in the plane. This means that when a stellar orbit
flips in inclination, it flips over its major axis. The bottom panel
of Figure 7 shows the angular momentum vectors of the same
tidally disrupted stars at the time of TDE. The spread in angular
momentum vectors confirms that the orbits roll over their
major axes.

With TDEs preferentially occurring in the plane with
inclinations of 0° or 180°, the debris streams from two
sequential TDEs could cross and produce unique observational
signatures. Bonnerot & Rossi (2019) derived the conditions
necessary for a tidal stream crossing to occur, which depend on
disk properties, tidal stream widths, and the time between
consecutive TDEs. If the pericenter shift between tidally
disrupted stars is positive, the time delay is small, and the
inclination offset is less than the width of the tidal streams,
crossing of the tidal streams could occur. While our simulations
do not allow for the calculation of accurate time delays (due to
the low N nature of our simulations) or stream width,
we calculate the pericenter shift between two TDEs with an angle,

\[ \Delta \theta = i_{e1} - i_{e2}. \] (16)

We show, in Figure 8, a distribution of the \( \Delta \theta \) between pairs
of TDEs in our simulations. We see that about 20% of
consecutive TDEs occur with a small (\(<20°\)), positive change in \( i_e \). This
is partially due to the prograde precession of the disk. The clustering of \( \Delta \theta \)
around 0 and slightly greater than 0 show that the first condition for tidal
streams crossing is met for many consecutive TDEs. Prograde precession is in
the positive (counterclockwise) direction.

\( \Delta \theta \). This \( \Delta \theta \) tracks the orientation of the orbit of the first TDE
with respect to the orientation of the orbit of the second TDE. We
track the orientation of the orbits in our simulation with \( i_e \),
defined in Equation (6). We therefore calculate \( \Delta \theta \) between
TDEs as

4. Discussion

This paper focuses on the dynamics of eccentric nuclear
disks with general relativistic precession. In Madigan et al.
(2018), we showed that the same secular mechanism that keeps
eccentric nuclear disks stable results in extremely high TDE
rates. This work did not include general relativity, however,
which is known to quench secular torques via rapid apsidal
precession. In this paper, we show that secular gravitational
torques push the orbits of stars to extremely high eccentricities
within one orbital period (full loss-cone regime). This does not
allow general relativistic precession enough time to suppress
the TDE rate. The geometry of eccentric nuclear disks is key:
the torques acting on an orbit from the rest of the disk stars are
coherent. Our results point to the following conclusions and
implications:

1. General relativistic precession does not significantly
affect the TDE rate from eccentric nuclear disks as stars
at the inner edge of the disk are in the full loss-cone
regime. TDEs occur in simulations with general relativity
approximately as often as they occur in Newtonian
simulations.
2. TDEs from eccentric nuclear disks do not follow an isotropic distribution of inclinations; they preferentially disrupt at inclinations of 0° and 180° with respect to the mid-plane of the disk. Overlapping TDE disks may have similar (or opposing) angular momenta that can build up (or cancel each other out).

3. The probability of disrupting stars on prograde orbits is higher than the probability of disrupting stars on retrograde orbits for a non-spinning, Schwarzschild black hole in an eccentric nuclear disk. Similarly, the number of prograde captured stars (within the Schwarzschild radius so that a flare will not be observed), will also be greater than the number of retrograde captured stars for a non-spinning, Schwarzschild black hole.

4. If an eccentric nuclear disk forms during a gas-rich major merger (Hopkins & Quataert 2010a, 2010b), it is likely that the central gaseous accretion event that produces the disk aligns the disk angular momentum with that of the central SMBH. Spinning, Kerr black holes have asymmetric tidal and capture cross sections (Beloborodov et al. 1992). For black holes with mass greater than the Hills mass, the only observable TDEs are those on prograde orbits aligned with the black hole spin. The preference for TDEs from eccentric nuclear disks to have ~0° orbital inclination puts them in the perfect orientation to be observably disrupted by such black holes. There may be evidence of TDEs by extremely massive black holes already. ASSASN-15lh is a TDE candidate found in a galaxy with a central SMBH much more massive than the Schwarzschild Hills mass (~10^{8.24}M_\odot) (Leloudas et al. 2016).

5. In steady state, eccentric nuclear disks have a non-negligible fraction of retrograde orbiting stars (~10%). Most of these should lie at the inner edge of the disk. This will lead to interesting observational signatures in the velocity moments of eccentric nuclear disks.

Finally, we look back to our nearest neighbor, Andromeda (M31). To date, no TDEs have been observed from this center. This may be due to the fact that the eccentric nuclear disk in M31 is very old, on the order of Gyr (Sil’chenko et al. 1998). Unless continuously replenished, an eccentric nuclear disk loses mass due to stars being destroyed by tidal forces, but it does not lose significant angular momentum. The disk, therefore, becomes less eccentric with time, causing the TDE rate to decrease (Madigan et al. 2018). Another possible explanation for the absence of TDEs in M31 is that more massive stars have a greater chance of disrupting than less massive stars (due to their location in the eccentric nuclear disk; H. Foote et al. 2019, in preparation) and the more massive stars will die off first, causing the TDE rate to decrease with time. At $M \approx 1.4 \times 10^5M_\odot$, the mass of the M31 black hole is also greater than the Hills mass (Bender et al. 2005). We should not expect to observe TDEs of solar-type stars, unless the black hole is spinning.

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Software: REBOUND (Rein & Liu 2012), REBOUNDX
https://github.com/dtamayo/reboundx.

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