Study of the kaon electromagnetic form factors in $e^+e^-$ annihilation and $\tau$ decays

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Abstract. The recent precise measurements of the $e^+e^-\to K^+K^-$ and $e^+e^-\to K_SK_L$ cross sections and the hadronic spectral function of the $\tau^-\to K^-K_S\nu_\tau$ decay are used to extract the isoscalar and isovector electromagnetic kaon form factors and their relative phase in a model independent way. The experimental results are compared with a fit based on the vector-meson-dominance model.

1. Introduction
The presented analysis is based on $e^+e^-\to KK$ data from BABAR [1, 2], SND [3], CMD-3 [4, 5] and $\tau^-\to K^-K^0_\nu_\tau$ decay data from BABAR [6]. The $e^+e^-\to K^+K^-$ and $e^+e^-\to K_SK_L$ production cross sections are parametrized in terms of the charged $F_{K^+}$ and neutral $F_{K^0}$ kaon form factors as follows

$$\sigma_{K^+K^-}(s) = \frac{\pi\alpha^2\beta^3}{3s} |F_{K^+}|^2, \quad \sigma_{K_SK_L}(s) = \frac{\pi\alpha^2\beta^3}{3s} |F_{K^0}|^2,$$

where $s$ is the center of mass energy, $\beta = \sqrt{1 - 4m^2/s}$, and $m$ is the charged or neutral kaon mass. The charged and neutral kaon form factors, obtained from (1), are shown in figure 1. Assuming the conserved vector current (CVC) hypothesis and isospin invariance [7, 8], one can obtain the relations between the isovector ($I=1$) and isoscalar ($I=0$) form factors:

$$F_{K^+} = F^{I=1}_{K^+} + F^{I=0}_{K^+}, \quad F_{K^0} = F^{I=1}_{K^0} + F^{I=0}_{K^0}, \quad F^{I=0}_{K^+} = F^{I=0}_{K^0}, \quad F^{I=1}_{K^0} = -F^{I=1}_{K^+}.$$ (2)

The additional information about the isovector form factor is taken from the $K^-K^0_\nu_\tau$ mass spectrum (figure 2) in the $\tau^-\to K^-K^0_\nu_\tau$ decay [6]:

$$\frac{dB}{BdM} = C(M)|F_{K^-K^0}(M)|^2$$ (3)

where $B$ is the $\tau^-\to K^-K^0_\nu_\tau$ branching ratio, $M$ is the $K^-K^0$ mass, $C(M)$ is a theoretically known function [8], and $F_{K^-K^0} = -2F^{I=1}_{K^+}$. 

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1. Extraction of the isovector and isoscalar form factors

Using Eqs. (1), (2), (3) and $e^+e^-$ and $\tau$ data (figures 1, 2), we can obtain the isovector $|F_{K^+}^{I=1}|$ and isoscalar $|F_{K^+}^{I=0}|$ form factors and relative phase between them $\Delta \phi_{K^+} = \phi_{K^+}^{I=1} - \phi_{K^+}^{I=0}$:

$$|F_{K^+}^{I=1}|^2 = \frac{|F_{K-K^0}|^2}{4}, \quad |F_{K^+}^{I=0}|^2 = \frac{|F_{K^+}|-|F_{K^0}|^2}{2} - |F_{K^+}^{I=1}|^2, \quad \cos(\Delta \phi_{K^+}) = \frac{|F_{K^+}|^2 - |F_{K^0}|^2}{2|F_{K^+}^{I=1}|^2|F_{K^+}^{I=0}|^2},$$

whose distributions are shown in figures 3, 4, 5. Both isoscalar and isovector form factors decrease monotonically in the range below 1.5 GeV. An unexpected feature of the form factors is the almost constant, close-to-zero the relative phase $\Delta \phi_{K^+}$ in the energy range from 1.06 to 1.5 GeV, where there are significant contributions of the $\rho(1450)$ and $\omega(1420)$ resonances.

2. Fit to the form factor data

The second part of this article is devoted to the simultaneous fitting $e^+e^-$ and $\tau$ two-kaon data. In the vector meson-dominance model [8], the amplitude of the single-photon transition...
$A_{\gamma^*\rightarrow p\bar{p}}$ is described as a sum of amplitudes of vector-meson resonances of the $\rho$, $\omega$, and $\phi$ families. The isovector resonances $\rho(770)$, $\rho(1450)$, $\rho(1700)$, and $\rho(2150)$ and isoscalar resonances $\omega(782)$, $\omega(1420)$, $\omega(1680)$, $\omega(2150)$, $\phi(1020)$, $\phi(1680)$, and $\phi(2170)$ are included into the fit. The resonance line shapes are described by the Breit-Wigner function

$$BW(s) = \frac{M_V^2}{M_V^2 - s - iM_V\Gamma_V(s)},$$

where $M_V$ and $\Gamma_V(s)$ are the resonance mass and energy dependent width. The relative contribution of different resonances are free fit parameters.

The results of the fit are shown by the solid curves in figures 2 - 6. It is seen that $K^-K^0$ mass spectrum, both isovector and isoscalar form factors, the relative phase $\Delta\phi_{K^+}$ and measured $e^+e^-\rightarrow K_SK_L$ and $e^+e^-\rightarrow K^+K^-$ cross sections are described by this model rather well. The $\chi^2/\nu = 183/142$ of the fit is not quite good, but reasonable, taking into account that the systematic uncertainties of the measurements are not included into the fit. More details about fit results can be found in [8]. A large deviation from the quark model prediction is observed for the contributions of the $\rho(1450)$ and $\omega(1420)$ resonances. This deviation leads to the almost constant value of the relative phase $\Delta\phi_{K^+}$ in the energy range 1.06–1.5 GeV (figure 5).

![Figure 5. The cosine of the relative phase $\Delta\phi_{K^+}$ between the isoscalar and isovector form factors. The solid curve represents the fit described in the text.](image1)

![Figure 6. The $e^+e^-\rightarrow K^+K^-$ (top) and $e^+e^-\rightarrow K_SK_L$ (bottom) cross sections. The curves represent the fits described in the text.](image2)
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