Flavor changing $Z$ decay $Z \to b\bar{s}(\bar{b}s)$ in topcolor-assisted technicolor models

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Abstract

In the context of topcolor-assisted technicolor (TC2) models, we examine the flavor changing (FC) $Z$ decay $Z \to b\bar{s}(\bar{b}s)$ and calculate the contributions of the new particles predicted by TC2 models to the branching ratio $\text{Br}(Z \to b\bar{s} + \bar{b}s)$. We find that the contributions mainly come from the top-pions. In most of the parameter space, the $\text{Br}(Z \to b\bar{s} + \bar{b}s)$ can reach $10^{-5}$, which may be detected in near future experiments such as Giga-Z version of the TESLA. Thus, the FC $Z$ decay $Z \to b\bar{s}(\bar{b}s)$ can be used to test TC2 models.

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I. Introduction

It is widely believed that, in the standard model (SM), the flavor-changing neutral currents (FCNC’s) are absent at tree-level and at one-loop level they are GIM suppressed. The SM results of the rare decays, which are induced by FCNC, are very small and can not be detected in the current or future experiments. The rare top decays \[1\] and rare \[Z\] decays \[2\] are two classes of such examples. Thus, rare decays provide a very sensitive probe of new physics beyond the SM. Detection of rare decays at visible levels by any of the future colliders would be instant evidence of new physics. Searching for rare decays is one of the major goals of the next generation of high energy collider experiments.

Among rare decays the flavor-changing (FC) \[Z\] decay \[Z \rightarrow q_iq_j\], which \(q_i\) and \(q_j\) are fermions of different flavors, is one interest subject. Experimentally, high energy \(e^+e^-\) colliders can be used as \(Z\) factory, providing an opportunity to examine the decay properties of the neutral electroweak gauge boson \(Z\) in detail. The improved experimental measurement at present stimulates the studies of these decays. For example, with the Giga-\(Z\) option of the TESLA linear collider project, one may expect the production of about \(10^9\) \(Z\) bosons at resonance \[3\]. This huge rate allows one to study a number of problems with unprecedented precision. Among them is the search for the FC \(Z\) decays.

We know that the dominate mode of the FC \(Z\) decays is \(Z \rightarrow b\bar{s}(\bar{b}s)\). The branching ratio \(Br(Z \rightarrow b\bar{s} + \bar{b}s)\) in the SM has been calculated in Ref.[2] and a lot of theoretical studies involving the FC \(Z\) decay \(Z \rightarrow b\bar{s}(\bar{b}s)\) have been given within specific popular models beyond the SM. For instance, the branching ratio \(Br(Z \rightarrow b\bar{s} + \bar{b}s)\) has been calculated in the two Higgs doublets models (2HDM’s)[4], in super-symmetry (SUSY)[5], SUSY with \(R\)-parity violation [6], and in other beyond standard models [7]. Recently, Ref.[8] has studied the FC \(Z\) decay \(Z \rightarrow b\bar{s}(\bar{b}s)\) in the context of 2HDM’s and SUSY with flavor violation. They find that, within the SUSY scenarios for flavor violation, the branching ratio \(Br(Z \rightarrow b\bar{s} + \bar{b}s)\) can reach \(10^{-6}\) for large tan \(\beta\) values.

The FC \(Z\) decays may be useful in searching for new physics beyond the SM at the TESLA collider or any other future colliders, which are designed to run on the \(Z\)-pole with high luminosities. With advances in technology, i.e., improved b-tagging efficiencies,
the FC $Z$ decay $Z \rightarrow b\bar{s}(\bar{b}s)$, which is the easiest to detect among the FC hadronic $Z$ decays, may be accessible to a Giga-$Z$ option even for branching ratios as small as $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s) \sim 10^{-7} - 10^{-6}$[8]. Thus, it is very interesting to study this decay in various models beyond the SM. The aim of this paper is to point out that, in the context of topcolor-assisted technicolor (TC2) models [9], the branching ratio $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ also can be significantly enhanced, which can reach the detectability threshold for near future experiments such as Giga-$Z$ version of the TESLA.

To completely avoid the problems arising from the elementary Higgs field in the SM, various kinds of dynamical electroweak symmetry breaking (EWSB) models have been proposed, and among which the topcolor scenario is attractive because it explains the large top quark mass and provides possible dynamics of EWSB. TC2 models [9], flavor-universal TC2 models [10] and top see-saw models [11] are three of such examples. These kinds of models generally predict the existence of colored gauge bosons (top-gluons, colorons), color-singlet gauge boson $Z'$ and Pseudo-Goldstone bosons (technipions, top-pions). These new particles are most directly related to EWSB and generation of fermion masses. Thus, studying the effects of these new particles in various processes would provide crucial information for EWSB and fermion flavor physics as well. In this paper, we will calculate the contributions of these new particles to the branching ratio $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$. We find that the contributions of gauge bosons $B_{\mu}^A$ and $Z'$ to the $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ are small. The largest value is only $10^{-8}$. The main contributions to the FC $Z$ decay $Z \rightarrow b\bar{s}(\bar{b}s)$ come from the top-pions via the FC scalar couplings. In most of the parameter space, the branching ratio $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ varies in the range of $3.9 \times 10^{-5} \sim 3.8 \times 10^{-6}$.

The paper is organized as follows: In section 2 we discuss the contributions of Pseudo-Goldstone bosons (PGB’s) to $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$. The effects of topcolor gauge bosons on $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ are studied in section 3. Discussions and conclusions are given in section 4.

II The contributions of PGB’s

In TC2 models [9], the TC interactions play a main role in breaking the electroweak
The ETC interactions give rise to the masses of the ordinary fermions including a very small portion of the top quark mass, namely $\varepsilon m_t$ with a model dependent parameter $\varepsilon \ll 1$. The main part of the top quark mass is dynamically generated by topcolor interactions at a scale of order $1 TeV$, which also make small contributions to EWSB. Thus, for TC2 models, there is the following relation:

$$v^2_\pi + F^2_t = v^2_w, \quad (1)$$

where $v_\pi$ represents the contributions of TC interactions to EWSB, $v_w = v/\sqrt{2} = 174 GeV$, and $F_t\approx 50 GeV$ is the top-pion decay constant, which can be estimated from the Pagels-Stokar formula. This means that the associated top-pions $\pi^\pm_0$ are not the longitudinal bosons $W$ and $Z$, but are separately physically observable objects. The presence of physics top-pions in the low-energy spectrum is an inevitable feature of topcolor scenario that purports to avoid fine-tuning [12].

The flavor-diagonal couplings of top-pions to quarks can be written as [9,12,13]:

$$m_t(1 - \varepsilon) \frac{\sqrt{v^2_w - F^2_t}}{v_w} [i\gamma^5 t\pi^0_t + \sqrt{2}t_R b_L \pi^+_t + \sqrt{2}b_L t_R \pi^-_t]$$

$$+ \frac{m^*_b}{\sqrt{2}F_t} [i\gamma^5 b\pi^0_t + \sqrt{2}t_L b_R \pi^+_t + \sqrt{2}b_R t_L \pi^-_t], \quad (2)$$

where the factor $\frac{\sqrt{v^2_w - F^2_t}}{v_w}$ reflects the effect of the mixing between top-pions and the Goldstone bosons of EWSB. From Eq.(2) we can see that the couplings of top-pions $\pi^\pm_t$ to the right-handed b-quark ($b_R$) are very small, which are proportional to $\frac{m^*_b}{\sqrt{2}F_t}(m^*_b \leq m_b \ll v_w)$. So, in our following calculation, we will ignore the couplings of $\pi^\pm_t$ to $b_R$.

For TC2 models, the underlying interactions, topcolor interactions, are non-universal and therefore do not posses GIM mechanism. When one writes the non-universal interactions in the quark mass eigen-basis, it can induce the tree-level FC couplings. The FC couplings of top-pions to quarks can be written as [14,15]:

$$\frac{m_t}{\sqrt{2}F_t} \frac{\sqrt{v^2_w - F^2_t}}{v_w} [iK^{t_t^c} K^{L_L^t_c R_R^t_c} \pi^0_t + \sqrt{2}t_R b_L \pi^+_t + \sqrt{2}b_L t_R \pi^-_t]$$

$$+ \sqrt{2}K^{t_t^c} K^{L_L^t_c R_R^t_c} \pi^+_t + \sqrt{2}K^{L_L^t_c R_R^t_c} \pi^-_t, \quad (3)$$
where $K_{UL(R)}$ and $K_{DL(R)}$ are rotation matrices that diagonalize the up-quark and down-quark mass matrices $M_U$ and $M_D$, i.e., $K_{UL}^+ M_U K_{UR} = M_D^{\text{dia}}$ and $K_{DL}^+ M_D K_{DR} = M_D^{\text{dia}}$, for which the CKM matrix is defined as $V = K_{UL}^+ K_{DL}$. To yield a realistic form of the CKM matrix $V$, it has been shown [14] that the values of the coupling parameters can be taken as:

$$K_{UL}^{tt} \approx K_{DL}^{bb} \approx K_{DL}^{ss} \approx 1, \quad K_{UR}^{tc} \leq \sqrt{2\varepsilon - \varepsilon^2}. \quad (4)$$

In the following calculation, we will take $K_{UR}^{tc} = \sqrt{2\varepsilon - \varepsilon^2}$ and take $\varepsilon$ as a free parameter, which is assumed to be in the range of $0.03 - 0.1$ [9].

From Eq.(2) and Eq.(3), we can see that the FC $Z$ decay $Z \rightarrow b\bar{s} + bs$ can be induced through charged top-pion loops in TC2 models. The relevant Feynman diagrams for the contributions of the $\pi_t^+$ to $Z \rightarrow b\bar{s}$ are shown in Fig.1.

![Feynman diagrams](image)

Figure 1: Feynman diagrams that contribute to the flavor-changing (FC) $Z$ decay $Z \rightarrow b\bar{s}$, due to charged top-pions $\pi_t^+$ exchange.
After a straightforward calculation, we can give the effective coupling vertices $Zb\bar{s}$ in the on-shell renormalization scheme:

$$g_{L,\pi t}^{bs} = \frac{m_t^2(1 - \varepsilon)}{32\pi^2 F_t^2}(1 - \frac{F_t^2}{v_w^2}) e \frac{e}{S_W C_W} K_{UR} A, \quad g_{R,\pi t}^{bs} \approx 0,$$

with

$$A = \frac{2 S_W^2 - 1}{m_b^2 - m_s^2} [m_b^2 B_1(m_b, m_t, m_{\pi t}) + m_b^2 B_0(m_b, m_t, m_{\pi t})$$

$$+ m_s^2 B_0(m_s, m_t, m_{\pi t}) - m_s^2 B_0(m_s, m_t, m_{\pi t})$$

$$- m_s^2 B_1(m_s, m_t, m_{\pi t}) - m_s^2 B_0(m_s, m_t, m_{\pi t})]$$

$$- (1 - \frac{4 S_W^2}{3} m_t^2 C_0 - \frac{4 S_W^2}{3} B_0(m_Z, m_t, m_t) - 2 C_{24}$$

$$+ m_s^2 C_0) - 2(1 - 2 S_W^2) C_{24}^*]$$

where $S_W = \sin \theta_W$ and $C_W = \cos \theta_W$ which $\theta_W$ is the Weinberg angle. $B_i$, $C_0$, and $C_{ij}$ are the standard two-point and three-point Feynman integrals [16]. $C_0 = C_0(m_s, m_Z, m_{\pi t}, m_t, m_t)$, $C_{24} = C_{ij}(m_s, m_Z, m_{\pi t}, m_t, m_t)$ and $C_{24}^* = C_{ij}(m_s, m_Z, m_{\pi t}, m_{\pi t})$. The total decay width of the FC $Z$ decay $Z \to b\bar{s} + \bar{b}s$ can be written as:

$$\Gamma (Z \to b\bar{s} + \bar{b}s) = \frac{\alpha_e m_t^2 m_Z (1 - \varepsilon)^2}{96\pi^4 F_t^4} \frac{1}{(4 S_W C_W)^2} (1 - \frac{F_t^2}{v_w^2})^2 (K_{UR}^t)^2 A^2.$$

In above equation, we have taken the approximations such as $m_Z^2 - m_b^2 - m_s^2 \approx m_Z^2$.

In Fig.2 we plot $\text{Br}(Z \to b\bar{s} + \bar{b}s)$ as a function of $m_{\pi t}$ for three values of the parameter $\varepsilon$: $\varepsilon = 0.03, 0.05$ and 0.08. In our calculation, we have taken: $\Gamma_Z = 2.49 GeV$, $m_t = 175 GeV$, $m_b = 4.8 GeV$, $m_Z = 91.18 GeV$, $m_s = 0.15 GeV$, $\alpha_e = \frac{1}{128}$, and $S_W = 0.2322$ [17]. From Fig.2 we can see that $\text{Br}(Z \to b\bar{s} + \bar{b}s)$ increases with the parameter $\varepsilon$ increasing. On the other hand, the branching ratio is sensitive to the top-pion mass $m_{\pi t}$ and strongly suppressed by large $m_{\pi t}$. For $200 GeV \leq m_{\pi t} \leq 450 GeV$ and $0.03 \leq \varepsilon \leq 0.08$, $\text{Br}(Z \to b\bar{s} + \bar{b}s)$ varies in the range of $3.9 \times 10^{-5} \sim 3.8 \times 10^{-6}$.

To solve the phenomenological difficulties of the traditional TC models [13,18], TC2 models [9] were proposed by combing technicolor interactions with the topcolor interactions for the third generation quark at the scale about 1TeV. Thus, TC2 models predict
number of technipions in the technicolor sector. These new particles also have contributions to the FC $Z$ decay $Z \to b\bar{s}$ via the coupling $\pi^+ u_i d_j$. However, in TC2 models, the technipion-top-bottom coupling is proportional to $\frac{\varepsilon m_t}{F_{\pi}}$ and the technipion contributions to $\text{Br}(Z \to b\bar{s} + \bar{b}s)$ are proportional to $(\frac{\varepsilon m_t}{F_{\pi}})^2$. Furthermore, the coupling $\pi^+ t s$ is proportional to CKM matrix element $V_{ts}$, which is smaller than $K_{UR}^{tc}$. Thus, the contributions of technipions to $\text{Br}(Z \to b\bar{s} + \bar{b}s)$ are very small, which can be ignored.

![Diagram showing the branching ratio Br($Z\to b\bar{s} + \bar{b}s$) contributed by top-pions $\pi^\pm_t$ as a function of $m_{\pi_t}$ for the parameter $\varepsilon = 0.03$ (solid line), $\varepsilon = 0.05$ (dash line), and $\varepsilon = 0.08$ (dotted line).](image)

**Figure 2:** The branching ratio $\text{Br}(Z \to b\bar{s} + \bar{b}s)$ contributed by top-pions $\pi^\pm_t$ as a function of $m_{\pi_t}$ for the parameter $\varepsilon = 0.03$ (solid line), $\varepsilon = 0.05$ (dash line), and $\varepsilon = 0.08$ (dotted line).

### III. The contributions of topcolor gauge bosons

The key feature of TC2 models [9] is that a large part of the top quark mass is dynamically generated by topcolor interactions at a scale of order $1TeV$, which is flavor non-universal. To ensure that the top quark condenses and receives a large mass while
the bottom quark does not, a non-universal extended hyper-charge group $U(1)$ is often invoked, so that the topcolor gauge group is usually taken to be a strong coupled $SU(3) \times U(1)$. At the $\Lambda \sim 1 TeV$, the dynamics of a general TC2 model involves the following structure [13,15]:

$$SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \times SU(2)_L \rightarrow SU(3)_{QCD} \times U(1)_{EM},$$

(8)

where $SU(3)_1 \times U(1)_{Y_1}(SU(3)_2 \times U(1)_{Y_2})$ generally couples preferentially to the third (first and second) generations. The $U(1)_{Y_i}$ are just strongly rescaled versions of electroweak $U(1)_Y$. This breaking scenario gives rise to the topcolor gauge bosons including the color-octet colorons $B^A_\mu$ and color-singlet extra $U(1)$ gauge boson $Z'$. As a result, these new massive gauge bosons $B^A_\mu$ and $Z'$ couple predominantly to the third generation quarks and the third generation fermions, respectively. The flavor-diagonal couplings of these new gauge bosons to quarks, which are related to the FC $Z$ decay $Z \rightarrow \bar{b}s(\bar{b}s)$, can be written as:

$$\mathcal{L}_{B}^{FD} = \frac{1}{2} g_3 B^A_\mu [\cot \theta b\gamma^\mu \lambda^a b - \tan \theta \bar{s}\gamma^\mu \lambda^a s],$$

(9)

$$\mathcal{L}_{Z'}^{FD} = \frac{1}{6} g_1 \cot \theta' Z'_\mu (\bar{b}_L \gamma^\mu b_L - 2 \bar{b}_R \gamma^\mu b_R)
- \frac{1}{6} g_1 \tan \theta' Z'_\mu (\bar{s}_L \gamma^\mu s_L - 2 \bar{s}_R \gamma^\mu s_R),$$

(10)

with

$$k_3 = \frac{g_3^2 \cot^2 \theta}{4\pi}, \quad k_1 = \frac{g_1^2 \cot^2 \theta'}{4\pi}.$$  

(11)

Where $g_3(g_1)$ is the QCD ($U(1)_Y$) coupling constant at $\Lambda_{TC}$, $\theta$ and $\theta'$ are mixing angles. To select the top quark direction for condensation and not form a $\bar{b}b$ condensation, there must be $\cot \theta \gg 1$ and $\cot \theta' \gg 1$. To obtain proper vacuum tilting, the coupling constants $k_3$ and $k_1$ should satisfy certain constraint. There is a region of $k_3$ and $k_1$, i.e., $k_3 = 2$, $k_1 \leq 1$, satisfying requirement of vacuum tilting and the constraints from $Z$-pole physics and $U(1)$ triviality shown in Refs.[10,15]. We shall take $k_3 = 2$ and $k_1 = 1$ in the following calculation.

Similar to the top-pions, when one writes the non-universal interactions in the quark mass eigen-basis, these interactions can result in the FCNC vertices of the new gauge
bosons. The FC couplings of the gauge bosons $B^A_\mu$ and $Z'$ to quarks, which are related to our calculation, can be written as:

$$\mathcal{L}_B^{FC} = \frac{1}{2} g_3 K_B^{bs} B^A_\mu \bar{b} \gamma^\mu \lambda^a s,$$  \hspace{1cm} (12) \\

$$\mathcal{L}_{Z'}^{FC} = -\frac{1}{6} g_1 K_Z^{bs} Z'_\mu (\bar{b}_L \gamma^\mu s_L - 2 \bar{b}_R \gamma^\mu s_R),$$  \hspace{1cm} (13) \\

where $K_B^{bs}$ and $K_Z^{bs}$ are the flavor fixing factors. In the following estimation, we will assume $K_B^{bs} = K_Z^{bs} = K$.

The Feynman diagrams, which represent the new gauge boson exchange contributions to the process $Z \rightarrow b \bar{s}$, are depicted in Fig.3.

![Feynman diagrams for the contributions of the topcolor gauge bosons $B^A_\mu$ and $Z'$ to the FC $Z$ decay $Z \rightarrow b \bar{s}$.

Figure 3: Feynman diagrams for the contributions of the topcolor gauge bosons $B^A_\mu$ and $Z'$ to the FC $Z$ decay $Z \rightarrow b \bar{s}$.

Similar to Ref.[19], we can calculate these diagrams straightforwardly. In our calculation, we have taken $m_b \approx 0$, $m_s \approx 0$. It is easy to see that, relative to the contributions of Fig.3(a), the contributions of Fig.3(b) to $Z \rightarrow b \bar{s}$ are suppressed by the factors $\tan^2 \theta$ and $\tan^2 \theta'$, which correspond gauge bosons $B^A_\mu$ and $Z'$, respectively. Then the effective couplings of $Zb\bar{s}$, which arise from the gauge bosons $B^A_\mu$ and $Z'$ can be written as:

$$g_{L,B}^{bs} = g_{R,B}^{bs} = \frac{2 k_3 \tan \theta K}{9 \pi} g_b^{bs} \frac{m_b^2}{M_B^2} \ln \left( \frac{M_B^2}{m_Z^2} \right),$$  \hspace{1cm} (14) \\

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\[ g_{L,Z}^{bs} = \frac{k_1 \tan \theta' K}{54\pi} g_L^b \left[ \frac{m_Z^2}{m_Z^2} \ln \frac{M^2}{m_Z^2} \right], \quad g_{R,Z}^{bs} = \frac{2k_1 \tan \theta' K}{27\pi} g_R^b \left[ \frac{m_Z^2}{M^2} \ln \frac{M^2}{m_Z^2} \right], \] (15)

with

\[ g_L^b = \frac{e}{S_W C_W} \left( -\frac{1}{2} + \frac{1}{3} S_W^2 \right), \quad g_R^b = \frac{e}{S_W C_W} \left( \frac{1}{3} S_W^2 \right). \] (16)

The limits on the masses of the topcolor gauge bosons \( B^A_\mu \) and \( Z' \) can be obtained via studying their effects on various experimental observable [13]. For example, Ref.[20] has shown that \( B\bar{B} \) mixing provides stronger lower bounds on the masses of \( B^A_\mu \) and \( Z' \), one must have \( M_B > 3.1 \text{TeV} \) (4.8 TeV) and \( M_Z > 6.8 \text{TeV} \) (9.6 TeV) if ETC does (does not) contribute to the CP-violation parameter \( \epsilon \). Recently, Ref.[21] restudy the bound placed by the electroweak measurement data on the extra \( U(1) \) gauge boson \( Z' \). They find that \( Z' \) predicted by TC2 models must be heavier than about 1 TeV. As estimation the contributions of the topcolor gauge bosons to the FC \( Z \rightarrow b\bar{s} \) decay \( Z \rightarrow b\bar{s}(\bar{b}s) \), we take \( M_Z = M_B = M \) and take the mixing factor \( K_{Z}^{bs} = K_{B}^{bs} = K \) as free parameters in this paper.

Using Eq.(14)-(16), we can give the values of the branching ratio \( \text{Br}(Z \rightarrow b\bar{s} + \bar{b}s) \) arised from the topcolor gauge boson exchange. Our numerical results are shown in Fig.4 and Fig.5, in which we plot \( \text{Br}(Z \rightarrow b\bar{s} + \bar{b}s) \) as a function of the mass \( M \) for

![Graph](image_url)

Figure 4: \( \text{Br}(Z \rightarrow b\bar{s} + \bar{b}s) \) contributed by \( B^A_\mu \) and \( Z' \) as a function of \( M \) for \( K = \lambda = 0.22 \).
Figure 5: $\text{Br}(Z \to b\bar{s} + b\bar{s})$ contributed by $B^A_{\mu}$ and $Z'$ as a function of the flavor mixing factor $K$ for $M = 1.5\,\text{TeV}$ (solid line), $M = 2.5\,\text{TeV}$ (dash line), and $M = 3.5\,\text{TeV}$ (dotted line).

$K = \lambda(\lambda = 0.22$ is the Wolfenstein parameter [22]) and as a function of the mixing factor $K$ for $M = 1.5\,\text{TeV}$, $2.5\,\text{TeV}$ and $3.5\,\text{TeV}$, respectively. We can see from Fig.4 and Fig.5 that the branching ratio $\text{Br}(Z \to b\bar{s} + b\bar{s})$ decreases with $M$ increasing and $K$ decreasing. In all of the parameter space, the $\text{Br}(Z \to b\bar{s} + b\bar{s})$ is smaller than $10^{-7}$. For $M = 1.5\,\text{TeV}$, the value of $\text{Br}(Z \to b\bar{s} + b\bar{s})$ can reach $2.1 \times 10^{-9}(4.3 \times 10^{-8})$ for $K = 0.22(1)$.

The flavor-universal TC2 models [10] also predict the presence of color-octet gauge bosons (topgluons) and color-singlet gauge boson $Z'$. However, the topcolor interactions are flavor-universal and all quarks carry the same $SU(3)$ charge, the topgluons couple with equal strength to all quarks. As a result, topgluons can not cause tree-level FCNC’s and have no contributions to the FC $Z$ decay $Z \to b\bar{s}(b\bar{s})$. To ensure that top quark condenses and receives a large mass while the b-quark does not, a non-universal extended
hyper-charge group $U(1)$ is involved in the flavor-universal TC2 models. Thus, the $Z'$ predicted by these models treats the third generation fermions differently than those in the first and second generations and can cause the tree-level FCNC’s. So the gauge boson $Z'$ can give contributions to $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$. The numerical results are plotted in Fig.6 for the values of the $Z'$ mass: $M = 1.5\text{TeV}$, $2.5\text{TeV}$ and $3.5\text{TeV}$. We can see from Fig.6 that the contributions of $Z'$ to $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ are very small. The largest allowed value for the $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ is $\sim 10^{-11}$. For example, the value of the $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ is only $9.5 \times 10^{-13}$ for $M_Z = 1.5\text{TeV}$ and $K = \lambda = 0.22$.

![Figure 6](image.png)

Figure 6: $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ contributed by $Z'$ as a function of the flavor mixing factor $K$ for $M = 1.5\text{TeV}$ (solid line), $M = 2.5\text{TeV}$ (dash line), and $M = 3.5\text{TeV}$ (dotted line) in the flavor-universal TC2 models.

The extra $U(1)$ gauge bosons $Z'$ predicted by non-commuting ETC model [23] and un-unified standard model [24], which couples differently to fermions, also has contributions to the FC $Z$ decay $Z \rightarrow b\bar{s}(\bar{b}s)$. However, its contributions to $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ are also very small.
IV. Discussions and conclusions

The top quark, with a mass of the order of the electroweak scale, is singled out to play a key role in the dynamics of EWSB and flavor symmetry breaking. There may be a common origin for EWSB and top quark mass generation. The idea is first accomplished in the top-condensation model [25]. However, this model can not fully break the electroweak symmetry and also generate a large top quark mass. To address this problem, various kinds of strong top dynamical models have been proposed, including TC2 models [9], flavor-universal TC2 models [10], top see-saw models [11], and the top flavor seesaw models [26]. The common feature of such type of models is that the topcolor interactions generate the main part of top quark mass and also make small contributions to EWSB. EWSB is mainly generated by TC interactions or other interactions. Then, the presence of physical top-pions in the low-energy spectrum is an inevitable feature of these models. The effects of top-pions on low-energy observables are governed by its mass $m_{\pi_t}$, while the large couplings of top-pions to quarks and to gauge bosons are to large degree model-independent [12]. Thus, our calculations about the contributions of top-pions to the FC $Z$ decay $Z \rightarrow b\bar{s}$ also apply to other models. Certainly, the relevant flavor mixing factor may has different values for different models.

Another common feature of strong top dynamical models is that they extended one or more of the SM $SU(N)$ gauge groups to an $SU(N) \times SU(N)$ structure at energies well above the electroweak scale [27]. All of these models propose that the gauge groups should be flavor non-universal. For example, $SU(3)$ gauge group is flavor non-universal in TC2 models and $U(1)$ gauge group is flavor non-universal in the flavor-universal TC2 models and TC2 models. When the non-universal interactions are written in the mass eigen-states, the corresponding gauge bosons can induce the tree-level flavor changing couplings. Then these new gauge bosons may have significant contributions to some FCNC’s processes. Our numerical results show that the extra color-octet gauge boson $B_\mu^A$ predicted by TC2 models and the extra $U(1)$ gauge boson $Z'$ predicted by TC2 models or the flavor-universal TC2 models can indeed give contributions to the FC $Z$ decay $Z \rightarrow b\bar{s}$. With reasonable values of the parameters, the branching ratio $\text{Br}(Z \rightarrow b\bar{s}+\bar{s}b)$
can reach $4.3 \times 10^{-8}$ for TC2 dynamics. However, it is smaller than the reach of a Giga-$Z e^+e^-$ collider.

To summarize, we have examined the FC $Z$ decay process $Z \rightarrow \bar{b}s(\bar{b}s)$ in the framework of TC2 models and calculated the contributions of the new particles predicted by TC2 models to the branching ratio $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$. We find that the contributions of top-pions are larger than those of topcolor gauge bosons. In reasonable parameter space of TC2 models, the value of $\text{Br}(Z \rightarrow b\bar{s} + \bar{b}s)$ can be reach $3.9 \times 10^{-5}$, which may be detected by the Giga-$Z$ TESLA colliders. Thus, a signal of $Z \rightarrow b\bar{s}(\bar{b}s)$ in a Giga-$Z$ TESLA or any other colliders will be consistent with the underlying mechanisms for EWSB and top quark mass generation in topcolor scenario.

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