Problem-based learning for measuring representation ability

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Abstract. The research is descriptive qualitative which aims to describe mathematical representation ability of students in class 11 with problem-based learning models on geometric sequences and series. The instruments used in this study were students' worksheets, test questions, lesson plans, and teaching materials. In this research, it can be concluded that mathematical representation ability of students through the learning with Problem Based Learning (PBL) method comes up. The students do all the aspects of mathematical representation, which are the visual representation, numeric representation, symbolic representation, and algebraic representation. The dominant parts which appear are the visual representation, symbolic representation, and algebraic representation. The mathematical representation ability which rarely shows is the numeric representation. It also means that the form of the question also affects students in deciding what mathematical representation aspect to be used.

1. Introduction
In learning mathematics, one important aspect is mathematical thinking, especially in learning mathematics in Indonesia. Mathematical thinking is a way of thinking related to mathematical processes or ways of thinking in solving mathematical problems in simple or complex ways [1]. The same thing with [2] discusses that one of the activities carried out in the human mind is mathematical thought which he describes as a mathematical tool using abstraction, symbolic manipulation, and symbolic representation. Making conjectures, reasoning, proof, abstraction, generalization is an important aspect in mathematical thinking [3]. According to [4] mathematical thinking is very necessary for students, related to the needs of students to solve the problems they face every day.

Mathematical thinking is divided into 7 aspects which one aspect is representation [5]. Representation is pouring mathematical ideas from students that are used as a form of substitute problem situations to find solutions to the problems being faced by students as a result of the interpretation of their thoughts [6, 7, 8]. In line with [7] representation is the core of mathematics learning. This is reinforced by the opinion [9] which states that representation has an important role in increasing understanding of mathematical concepts. When students make, compare, and use their representations, students can develop their understanding of relationships and mathematical concepts. Representations can be in the form of graphs, pictures, symbols, etc. that can help students communicate their thoughts [9, 10]. Therefore, representation is one of the key mathematical communication skills.

From the findings [11] which states that students have difficulty in solving word problems since they do not understand what is known from the word problem or change the word problem into a mathematical equation. This causes students to be confused in determining the complete steps of the given word problem. It is supported by [12] that the students' common mistakes in doing mathematical
assignments are one of them due to lack of knowledge about symbols and errors in the calculation process.

Sequences and series are one of the materials contained in mathematics lessons. In everyday life, many problems can be solved using the rules of sequences and series. One of the competencies in this material expects students to be able to apply the concepts of sequences and series. In the matter of national exams, this material is classified as a material that often appears. Therefore, sequences and series are very important to be taught so that students can solve problems properly. According to [13] in the analysis of students’ error in solving sequence and series material, it was found that students were misunderstanding in answering geometric sequence and series.

However, the fact is learning mathematics in schools is still not able to develop students' mathematical representation abilities [14]. This can be seen from the delivery of mathematics learning by teachers who are still fixated on textbooks, and ways of teaching mathematics that is still accustomed to presenting material, giving examples of questions, and asking students to work on practice questions, so it is not yet possible to grow or develop representations optimally. Based on [15], students’ mathematical representation ability is low because they have difficulty to translate real-world problems into mathematical representation problems. In learning the material sequence and series, according to [16], it is need more emphasis on memorization of formula of the \( n^{th} \)-term (\( U_n \)) and sum of the first \( n \)-term of series (\( S_n \)). This is in line with research findings [17] which state that teachers are more likely to emphasize general formulas without understanding where they came from. Therefore, the teacher should plan an appropriate model to be applied to the material to be taught before carrying out learning. The learning model that is expected to be appropriate for developing Mathematical Thinking aspects of student representation is to use the Problem Based Learning (PBL) learning model.

According to [18, 19], PBL is a learning model that starts with giving a problem related to daily real life. Next students solve these problems to find new knowledge. In addition, [19] revealed that in solving real-world problems, students were still unable to represent real-world problems in the form of representative mathematical problems. Correspondingly, [21] revealed PBL learning there was a merging of the ideas of each student they expressed in the form of images, graphics, mathematical models, written text words, as well as drawing conclusions that are all produced by combining ideas in solving one problem. Based on the explanation above, this study aims to describe the mathematical representation ability of students with problem-based learning models on geometric sequences and series.

2. Method
This type of research used in this research is descriptive qualitative research. The purpose of this research is to use qualitative descriptive research to analyze mathematical thinking. Class 11 student representations use the Problem Based Learning (PBL) learning model. The indicators of the mathematical representation aspect are visual representation, numerical representation, symbolic representation, and algebra representation [5]. Visual representation is the making of images in expressing or interpreting an argument [22]. Numerical representations are representations of problems using tables or matrices through numbers and involve arithmetic calculations [23, 24]. Symbolic representations represent communication representations or represent opposition to mathematical language [25]. While algebraic representation is an amalgamation of arithmetic symbols that require variables in representing a possibility [23]. The procedure of this research was carried out in four steps, namely preparation, implementation, and data analysis, and the last was discussion. While the instruments used in this study are students' worksheets, test, lesson plans, and teaching materials. This study was conducted at one of the high schools in South Sumatra.

3. Result and Discussion
3.1. Research Preparation Phase
In the preparatory phase carried out in this study, namely taking care of research administration, observation to school, preparing research instruments and validating research instruments with 2
lecturers of mathematics education at Universitas Sriwijaya. Researchers improve the research instrument based on suggestions and input from the validator.

3.2. Implementation Phase Of the Study
This research was conducted during 3 meetings and 1 interview, which consisted of 2 meetings where the researcher delivered the learning material and geometric sequence using the Problem Based Learning model and 1 meeting where the researcher conducted a test to see the Mathematical Thinking aspects of student Representation. This research was conducted from 29 October 2019 to 14 November 2019.

3.2.1. Meeting 1 and 2
In the learning process, researchers use the Problem Based Learning model to Develop Mathematical Thinking aspects of student representation. There are five stages of learning the Problem Based Learning model, that are: 1) organizing students against problems, 2) organizing students to study, 3) guiding individual and group investigations, 4) develop and present the work, 5) analyze and evaluate the problem solving process. First stage, learning begins by asking students to read, observe, and understand the problems that exist in students’ worksheets by displaying the problem in Power Point starting from problem 1 to problem 3. After students read, observe and understand these problems, the teacher allows students to ask questions that have not been understood related to the problem that has been given. Second stage, students are assisted by the teacher to organize and define student learning tasks related to the problems that have been given by researchers. The teacher asks students to write down what information is in the problem and to determine the problem that needs to be solved. Third stage, in the guiding stage of individual and group investigators, the teacher asks students to determine what steps/plans of solution need to be taken to solve the problem and ask students to gather appropriate information and conduct experiments on information that students have been able to solve the problems given. Fourth stage, the teacher directs students to solve the problems that have been given by implementing the steps that students have designed. After all groups have finished solving the problem, the teacher asks one of the group representatives to present the results of the discussion. The last stage, in the activity of analyzing and evaluating, the teacher allows other groups by asking whether there are different answers from problem 1 to problem 3. Since there are no different answers, the teacher agrees that the results of the three problems are the same as the exposure group. After students explain the results of their discussion, and the teacher together with students agree on the results of the settlement of the problems that exist in the students' worksheets.

3.2.2. Test
At this meeting, researchers conducted written tests on students. The time given to do the test questions is for 30 minutes. The given problem is related to geometric sequences and series that can be seen in Figure 1.

![Figure 1](image)

3.2.3. Interview
After working on the written test, the results of student work are collected and analyzed based on indicators of mathematical representation. Six students became as research subjects, namely AY, RE, HN, WF, EL, and RP. Each research subject was interviewed by the researcher. This interview was...
conducted to find out more about how mathematical representation when working on geometrical sequences and series.

3.3. Data Analysis Stage

At this stage, the answers of six research subjects on the test were analyzed. The following are the emergence of mathematical representation aspects of the research subject.

| Subject | Visual Representation | Symbolic Representation | Numeric Representation | Algebraic Representation |
|---------|----------------------|------------------------|-----------------------|------------------------|
| AY      | ✓                    | ✓                      | -                     | ✓                      |
| RE      | ✓                    | ✓                      | ✓                     | ✓                      |
| HN      | ✓                    | ✓                      | ✓                     | ✓                      |
| WF      | ✓                    | ✓                      | ✓                     | ✓                      |
| EL      | ✓                    | ✓                      | ✓                     | ✓                      |
| RP      | ✓                    | ✓                      | ✓                     | ✓                      |

Based on the results of the test answers to the research subjects, there are several different solutions, namely the first by subject WF. Subject WF’s answers are shown in Figure 2.

The subject WF answered the question completely. In the results of the completion of subject WF, mathematical indicators of thinking aspects of representation that appear are visual representation, symbolic representation, and algebraic representation. To find out the mathematical thinking of subject WF when solving the problem, the researcher conducted an interview. On the results of the subject WF’s answers and mathematical thinking indicators interview results that appear in the settlement are visual representation, symbolic representation, and algebraic representation indicators. Visual representation indicator appears visible in the picture where the subject WF illustrates a square motif illustrated from the problem that has been given. The second indicator is the symbolic representation indicator that appears on the subject WF when the subject WF symbolizes the area of the first square.
into \( a \) and \( U_1 \). Moreover, the algebraic representation indicator seems visible when the subject WF looks for the value of the number of squares created and subject WF uses algebraic calculations.

The next solution is by subject EL. The EL subject’s answer is almost the same as subject WF, but subject EL experienced a mistake when writing the \( S_n \) formula. He works on problems with visual representation, symbolic representation, and algebraic representation. EL’s answer is shown in Figure 3.

![Figure 3. Subject EL’s answer.](image)

From the results of the EL subject’s answers are the same as subject WF where subject EL bring up three indicators of mathematical thinking aspects of representation, namely visual representation by describing the motives of the tablecloth, symbolic representation by symbolizing each side of the square into \( S_{b1} - 4 \), and algebraic representation by determining the number of squares to 2 cm\(^2\) using algebraic calculations. In contrast to the results of the completion of subject WF and subject EL where the subject HN did not finish the problem. The following is a snapshot of the results of subject HN completion.

![Figure 4. Subject HN’s answer.](image)

In this matter, the subject HN did not answer the question completely. However, there are 3 indicators of mathematical thinking aspects of representation that emerge from the process. These indicators are visual representation, symbolic representation, and numeric representation. To find out
why the work on subject HN was incomplete, the researchers conducted interviews as follows. From the interview results, the subject HN was confused about finding a ratio. After the subject HN re-understood the problem and the image that was made, the subject HN found the ratio of each square by finding the area of the second square using the Pythagorean formula to find its side. So that the researcher guides subject HN to complete the answer, subject HN finds the number of squares to 2 cm². During the interview, the algebraic representation indicator appears when subject HN determines the total area of the square.

In addition, three different answers from three subjects were analyzed based on table 1, it can be seen that subjects WF and EL answer the questions by bringing up indicators of visual representation, symbolic representation, and algebraic representation. The reason the subject brings up an indicator of visual representation in the problem is to find out the shape of the square motif contained in the problem which is useful to get the ratio of the square. Meanwhile, according to the analysis of researchers, symbolic representation and algebraic representation appear in all subjects because they think of how to get the sum of squares to squares with an area of 2 cm² correctly, so that the end is certain, not the result of guesses.

Moreover, the subject HN appears indicators of visual representation, numeric representation, and symbolic representation. The same reason with the subject WF and EL, visual representation indicators appear because to know the shape of the square motif contained in the problem. The symbolic representation indicator appears because the subject symbolizes the area of the first square with U1. At the subject HN's answer, it rises the numeric representation indicator. The reason subject HN gave rise to numeric representation was to look for the next terms. The algebraic representation indicator does not appear on subject HN because the subject has not completed it. However, the subject HN looks for the results of what is asked from the problem after the subject HN knows for certain that the ratio of the square is 12. In looking for these results an indicator of algebraic representation is seen. So the explanation above shows that the indicators of visual representation, symbolic representation, and algebraic representation are dominant representations that appear in students.

Based on the explanation above, we can detail that the most dominant aspects of mathematical thinking appearing by students are visual representation, symbolic representation, and algebraic representation. This representation was carried out by all research subjects. First, according to [22], visual representation is the making of images in stating or interpreting a problem. The reason the subject performs this visual representation is that the student sees the form of the problem and the order of the problem so the subject represents or illustrates it in a visual form to illustrate what the student is imagining. Furthermore, symbolic representation is represented in abstract symbols or mathematical symbols, which are agreement symbols used in mathematics, and verbal symbols [26]. According to [27] which states that numerical representations are substitutes for problem situations through numbers and involve arithmetic calculations. The reason for the subject to do symbolic representation is to make it easier for the subject to make equations and then the equation is solved by algebraic representation. The ability of mathematical thinking aspects of representation that is sometimes done by students is numeric representation.

From the learning that has been done with the Problem Based Learning model, it can be seen that the mathematical thinking aspects of student representation on the material sequence and geometry series develop. Problems given in Problem Based Learning are interpreted and represented in the form of mathematics, and the process of interpretation and representation becomes essential because people can obtain mathematical ideas by representing those ideas so that they understand mathematics [28]. Where in solving the problems that researchers provide, students pour their ideas in the form of visual representation, numeric representation, symbolic representation, and algebraic representation.

4. Conclusion
In this study, it can be concluded that students’ mathematical representation by learning using the Problem Based Learning (PBL) model on geometric sequences and series are three dominant aspects of representation that emerge, namely visual representation, symbolic representation, and algebraic representation. The dominant parts which appear are the visual representation, symbolic
representation, and algebraic representation. It is can be said that the student's representation aspect of mathematical thinking is developing. The mathematical representation which rarely shows is the numeric representation. It also means that the form of the question also affects students in deciding what mathematical representation aspect to be used.

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