Fault tolerant control of fixed wing uav based on adaptive method

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Abstract: with the rapid development of fixed-wing uav technology, the requirements for precision and fault tolerance are getting higher and higher. However, the control methods commonly used at present have shown the shortcomings in accuracy and fault tolerance. Especially when there is unknown interference in the controlled object, the commonly used control method such as PID control is far from the desired effect, and the adaptive control just can make up for this shortcoming. Adaptive control can automatically adjust the controller parameters or control laws in the control system according to the output of the system. The main content of this chapter is to design an adaptive law to directly estimate controller parameters in the case of partial actuator failure and certain disturbance, so as to construct an adaptive closed-loop system controller, which can compensate actuator failure failure and offset system disturbance, and ensure the asymptotic stability of the closed-loop system.

1. Introduction
The research background of this paper is fault-tolerant control of fixed wing UAV based on adaptive method. On the basis of establishing the lateral model of UAV and considering the failure of actuator (rudder surface), an adaptive parameter adjustment method is proposed and designed to stabilize the flight control system. The adaptive fault-tolerant method compensates the system bias by modifying the time-varying parameters of the controller online and assuming that the failure bias is bounded. Finally, the simulation model and actual output were successfully established using MATLAB/Simulink software, system tracking effect, parameter changes involved are listed and analyzed in detail, verifying the effectiveness of the adaptive fault-tolerant control law.

2. Problem description
Consider the following linear time-invariant system with actuator bias and external disturbance

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B_1 u(t) + B_2 \omega(t) \\
\dot{y}(t) &= Cx(t) + B_1 H u(t) + B_2 \omega(t) \\
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n\) represents the UAV system state variable, \(u(t) \in \mathbb{R}^m\) represents system control input, \(B_1 \in \mathbb{R}^{nxm}, B_2 \in \mathbb{R}^{nxm}\) respectively represents the coefficient matrices of state vector, control vector and external disturbance vector, simultaneously \((A,B)\) is steerable and \((A,C)\) is observable, \(\omega(t)\) represents outside disturbance.

A \(\in \mathbb{R}^{nxn}\), In order to ensure sufficient fault tolerance in the state feedback design, the following assumptions are made for the fault tolerant control system:
**Hypothesis 1** The state of all systems is measurable.

**Hypothesis 2** The external disturbance is piecewise continuous bounded equation, that is, there is an unknown normal number $\tilde{\omega}$, resulting in $\|\omega(t)\| \leq \tilde{\omega}$.

**Hypothesis 3** For linear time-invariant fault-tolerant control system Equation (1), there exists a matrix equation $F$ with appropriate dimensions, resulting in $B_2 = B_1 F$.

Observability means that the output can fully reflect the characteristics of the system state, so all the states of the system are guaranteed to be measurable by assumption 1.1. In the control system, the external disturbance is generally required to be bounded to ensure the effectiveness of the control system. Hypothesis 1.3 is a condition for disturbance matching. From the perspective of physical sense, the control and disturbance signals are in the same channel, which is the necessary hypothesis for the system to fully compensate the disturbance.

3. **Fault-tolerant controller design**

The control system in this paper only considers actuator (rudder surface) failure and external disturbance, and there is no parameter perturbation. Therefore, the design of fault-tolerant controller for linear time-invariant system is as follows

$$u(t) = \hat{K}_1(t)x(t) + K_2(t)$$

(2)

$$K_1(t) \in R^{m\times n}$$

can be obtained by the following adaptive law.

$$\frac{d\hat{K}_1(i)}{dt} = -J_i \beta \|x^TPB\|$$

(3)

$J_i$ is a normal number, $B$ is the $i$th column of $C$, and $P$ is a positive definite matrix. The main purpose of the direct adaptive fault-tolerant controller $K_1(i)$ is to ensure the stability of the control system, while the controller gain $K_2(t) \in R^{m\times n}$ is mainly used to compensate the partial failure of the system actuator and offset the external disturbance. The controller gain $K_2(t)$ is designed to make the control system asymptotically stable under actuator failure and disturbance.

The gain of the adaptive controller $K_2(t)$ is designed as follows

$$K_2(t) = -\frac{(x^TPB)^{\beta}}{\|x^TPB\|^\alpha}K_3$$

(4)

$\alpha, \beta > 0$ and the need to meet $\|x^TPB\|^\alpha \leq \|x^TPB\|^\beta$, $K_3$ is a constant, $\hat{K}_3(t)$ is an estimate of $K_3$. $\hat{K}_3(t)$ is adjusted by the following adaptive law.

$$\frac{d\hat{K}_3(t)}{dt} = r\|x^TPB\|$$

(5)

$r$ is a constant. If $\hat{K}_3(t_0) \geq 0$, it is $\hat{K}_3(t) \geq 0$.

According to Equations (1), (2) and the hypothesis, the closed-loop fault-tolerant control system can be rewritten into the following model

$$\dot{x}(t) = (A + B_1HK_1)x(t) + B_1K_2(t) + B_1F\omega(t)$$

(6)

Int

$$\hat{K}_1(t) = \hat{K}_1(t) - K_1$$

$$\hat{K}_3(t) = \hat{K}_3(t) - K_1$$

(7)
As both $K_1$ and $K_3$ are constants mentioned above, the error model of the closed-loop fault-tolerant control system is shown as follows

$$\frac{d\hat{K}_1(i)}{dt} = -J_i b_i P x x^T$$

$$\frac{d\hat{K}_3(t)}{dt} = r ||x^T P B||$$

**Theorem 1:** For the white adaptive closed-loop system Equations (6) and error system Equations (8), which satisfy the assumptions 1.1 ~ 1.3, if there is a positive definite symmetric matrix, and the control gain equation $K_i(t)$ mentioned in Equation (4) and the adaptive law $\hat{K}_i(t)$ and $\hat{K}_3(t)$ of Equations (3) and (5) are selected, then the obtained fault tolerant control system is asymptotically stable.

**Proof:** For the adaptive fault-tolerant control system (6), the Lyapunov function is designed as follows

$$V(x, \hat{K}_1, \hat{K}_3, t) = x^T P x + tr(\hat{K}_1^T J \hat{K}_1) + \frac{1}{r} \hat{K}_3^2$$

According to Equation (8), the derivative of trajectory time along Equation (9) of the closed-loop system is

$$\frac{dV(x, \hat{K}_1, \hat{K}_3, t)}{dt} = x^T \left[(A + B_1 \hat{K}_1)^T P + P(A + B_1 \hat{K}_1)\right] x + 2x^T P B_i (K_2 + \Delta + F \omega)$$

$$+ tr\left(\hat{K}_1^T \Gamma^{-1} \hat{K}_1 + \hat{K}_3^T \Gamma^{-1} \hat{K}_3\right) + 2r^{-1} \hat{K}_3 \hat{K}_3$$

$$= x^T \left[(A + B_1 \hat{K}_1)^T P + P(A + B_1 \hat{K}_1)\right] x + 2\left\|x^T P B\right\| \left\|\beta \right\| x^T P B \left\|\hat{K}_3\right\|$$

$$+ 2x^T P B_i (\Delta + F \omega) + tr\left(\hat{K}_1^T \Gamma^{-1} \hat{K}_1 + \hat{K}_3^T \Gamma^{-1} \hat{K}_3\right) + 2r^{-1} \hat{K}_3 \hat{K}_3$$

The above equation can be further rewritten as

$$\frac{dV(x, \hat{K}_1, \hat{K}_3, t)}{dt} \leq x^T \left[(A + B_1 \hat{K}_1)^T P + P(A + B_1 \hat{K}_1)\right] x + 2\left\|x^T P B\right\| \left\|\hat{K}_3\right\| + \left\|\Delta\right\| + \left\|F\right\| \left\|\omega\right\|$$

$$+ tr\left(\hat{K}_1^T \Gamma^{-1} \hat{K}_1 + \hat{K}_3^T \Gamma^{-1} \hat{K}_3\right) + 2r^{-1} \hat{K}_3 \hat{K}_3$$

$$= x^T \left[(A + B_1 \hat{K}_1)^T P + P(A + B_1 \hat{K}_1)\right] x + 2\left\|x^T P B\right\| \left\|K_3\right\| + \left\|\hat{K}_3\right\|$$

$$+ 2\left\|x^T P B\right\| \left\|\sigma + \left\|F\right\| \left\|\omega\right\|\right\| + tr\left(\hat{K}_1^T \Gamma^{-1} \hat{K}_1 + \hat{K}_3^T \Gamma^{-1} \hat{K}_3\right) + 2r^{-1} \hat{K}_3 \hat{K}_3$$

Since $\sigma$ and $\bar{\omega}$ are constants, there must be a constant $K_3$ that satisfies the inequality.

$$\left\|x^T P B\right\| K_3 \geq \left\|x^T P B\right\| (\sigma + \left\|F\right\| \left\|\omega\right\|)$$

It can be derived...
\[
\frac{dV(x, \tilde{K}_1, \tilde{K}_3, t)}{dt} \\
\leq x^T \left[ (A + B_1 \hat{K}_1)^T P + P \left( A + B_2 \hat{K}_1 \right) \right] x + 2 \| x^T P B_1 \| \tilde{K}_3 \\
\quad + \text{tr} \left( \tilde{K}_1^T \Gamma^{-1} \tilde{K}_1 + \tilde{K}_3^T \Gamma^{-1} \tilde{K}_3 \right) + 2r^{-1} \tilde{K}_3 \tilde{K}_3
\]

(13)

Since \((A, B)\) is controllable, there must be matrices \(K_i \in R^{n\times n}\) and \(P \in R^{n\times n}\) that satisfy the following inequality

\[
(A + B_1 K_1)^T P + P (A + B_2 K_1) < 0
\]

(14)

Int

\[-Q = (A + B_1 K_1)^T P + P (A + B_2 K_1)\]

(15)

Equation (15) can be expressed as

\[
\frac{dV(x, \tilde{K}_1, \tilde{K}_3, t)}{dt} \\
\leq -x^T Q x - 2 \| x^T P B_1 \| \tilde{K}_3 + 2x^T P B_1 \tilde{K}_1 x \\
\quad + \text{tr} \left( \tilde{K}_1^T \Gamma^{-1} \tilde{K}_1 + \tilde{K}_3^T \Gamma^{-1} \tilde{K}_3 \right) + 2r^{-1} \tilde{K}_3 \tilde{K}_3
\]

(16)

We know from the properties of the trace of the matrix, \(\text{tr}A = \text{tr}A^T\), \(x^T A x = \text{tr} \left( xx^T A \right)\), so we can get the following formula

\[
\text{tr} \left( \tilde{K}_1^T \Gamma^{-1} \tilde{K}_1 \right) = \text{tr} \left( \tilde{K}_3^T \Gamma^{-1} \tilde{K}_3 \right)
\]

(17)

Equations (16) and (17) are used to simplify Equation (4.18) into the following form

\[
\frac{dV(x, \tilde{K}_1, \tilde{K}_3, t)}{dt} \\
\leq -x^T Q x - 2 \| x^T P B_1 \| \tilde{K}_3 + 2x^T P B_1 \tilde{K}_1 x \\
\quad + \text{tr} \left( \tilde{K}_1^T \Gamma^{-1} \tilde{K}_1 + \tilde{K}_3^T \Gamma^{-1} \tilde{K}_3 \right) + 2r^{-1} \tilde{K}_3 \tilde{K}_3
\]

(18)

It is obvious from the above formula, when it is \(x \neq 0\), resulting in \(\frac{dV(x, \tilde{K}_1, \tilde{K}_3, t)}{dt} < 0\).

According to the Lyapunov stability principle, the solutions of the closed-loop fault-tolerant control system are uniformly bounded and \(x(t)\) asymptotically stable to 0.

To point out here that the above method can fully compensate the actuator failure or the influence of external disturbance, but there is such a situation, the control signal in the system state structure is inevitably appear chattering phenomenon, due to the situation of the chattering in the actual control system are not allowed to exist, because the phenomenon of the rudder surface can reduce operation efficiency, reduce the service life. Therefore, in practical application, to compensate for the influence of faults, the bounded stability and asymptotic stability of the system must be mutually adjusted. In order to avoid chattering, some asymptotic stability is often sacrificed and the bounded stability of the system is obtained. Even if the bounded stability is pursued, in the actual system, it has already met
the basic operating requirements. In the control equation, the control equation (2) will be improved below to obtain a bounded stability controller without chattering.

**Corollary 1** Considering the adaptive closed-loop system formula and the error system formula satisfy Hypotheses 1-3. Suppose there is a positive definite symmetric matrix P, and the following control gain equation is selected

\[
K_2(t) = -\left( x^T PB \right)^T \beta \left\| x^T PB \right\| K_3 \left( \alpha + \delta \right)
\]  

(19)

In the above equation, \( \delta \) is a small positive number, while \( \alpha \) and \( \beta \) satisfy the following inequalities:

\[
\left\| x^T PB \right\| \alpha + \delta \leq \left\| x^T PB \right\| \beta
\]

(20)

The adaptive law as shown in Equation (4.8) is selected to make the state of the closed-loop fault-tolerant control system tend to a sufficiently small region \( \varepsilon \) and satisfy the following inequalities:

\[
\lim_{t \to \infty} x(t) \geq \frac{1}{\left\| PB \right\| \sqrt{\frac{\delta}{\beta - \alpha}}
\]

(21)

Therefore, as \( x(t) \) approaches zero, the inequality cannot hold, and \( K_2(t) \) cannot make the system state asymptotically stable but tends to a sufficiently small region \( \varepsilon \) that satisfies Equation (21).

4. **Fault-tolerant control simulation and analysis**

In order to verify the effectiveness of the adaptive control algorithm proposed in this chapter, the control object adopts the lateral and lateral model of fixed-wing UAV and analyzes whether the attitude velocity curve of fixed-wing UAV under two conditions \( K_2(t) \) is contained in the controller.

The state space expression of the decoupled linearized lateral-lateral model of the down-order UAV is as follows

\[
x(t) = \left( A + B_1HK_3 \right)x(t) + B_1K_3(t) + B_1F \alpha(t)
\]  

(22)

Thereinto, \( x(t) = [\beta(t), p(t), r(t)]^T \), \( u(t) = [\delta_s, \delta_d]^T \).

\( \beta(t) \) represents the sideslip Angle, \( p(t) \) represents the roll angular velocity of the fixed wing, and \( r(t) \) is the yaw angular velocity of the fixed wing. When the UAV attack Angle is 29.73°, the velocity reaches 0.2M, and the altitude is 10,000 feet, the coefficient matrix of the linearized state space system of the fixed-wing UAV is:

\[
A = \begin{bmatrix}
-0.059 & 0.496 & -0.868 \\
-4.513 & -1.439 & 1.665 \\
0.098 & 0.026 & -0.104
\end{bmatrix}
\]

\[
B_1 = \begin{bmatrix}
0.006 & 0.006 \\
1.879 & 0.029 \\
-0.109 & -0.096
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
0.3 & -0.5 \\
5.5 & 3
\end{bmatrix}
\]

The design and initial conditions of system controller parameters during simulation are shown below
\[ \Gamma = \text{diag}[10, 10], \quad r = 10, \quad \alpha = 1, \]
\[ \beta = 100, \quad x(0) = [0.5, 1, -0.5], \]
\[ H = \text{diag}[1, 0.6], \]
\[ \dot{K}_{1i}(0) = [0, 0, 0]^T, \quad i = 1, 2, \]
\[ \dot{K}_3(0) = 0 \]

Disturbing \( \omega(t) = [\sin(0.2t), 1]^T, \quad t \geq 0 \)

The following is the comparison of the variation curves of the simulation parameters of fixed-wing UAV in two cases of whether the controller contains \( K_2(t) \).

Fig. 1 The first row of fault tolerant control \( K_1 \)

Fig. 2 The first row of non-fault tolerant control \( K_1 \)

Fig. 3 The second row of fault tolerant control \( K_1 \)
Fig. 4 The second row of non-fault tolerant control $K_1$

Fig. 5 Estimated value $K_3$ of fault tolerant control

Fig. 6 Estimated value $K_3$ of non-fault tolerant control

Figure 7 Controller output with fault tolerant control
According to the above analysis in Fig. 1-8, when there is partial failure of actuator (rudder surface) and external disturbance in the system, the simple controller gain $K_1$ can only guarantee a certain stability of the control system, while the sideslip Angle, roll angular velocity and yaw angular velocity of the system change greatly and show irregular changes. This means that the roll Angle and yaw Angle of the UAV will change with the change of time, and the actual course of the UAV cannot advance in accordance with the predetermined flight direction. With the addition of control quantity $K_2$, the system output tends to zero, although there is still a small fluctuation, but it is controlled in a small area.

After the fault occurs, the controller output of non-fault tolerant controller $K_2$ presents irregular and large changes, leading to large changes in the system output, while the controller output of fault-tolerant controller $K_2$ presents small fluctuations in the stable near zero $\delta_v$ and regular changes in $\delta_r$, ensuring the system state response tends to be stable.

5. Conclusion

This chapter system based on adaptive control theory, considering the rudder surface appears deviation outside the presence of disturbance and system failure, successfully fixed-wing UAV adaptive fault-tolerant controller was designed, and the control system is proved with Lyapunov stability theory of stability, so as to guarantee the closed-loop fault-tolerant control system can meet the requirements of system stability, fault tolerance eventually reached the expected control effect.

Compared with the fault-tolerant control (FTC) method based on fault diagnosis, the direct adaptive fault-tolerant controller designed in this chapter does not need to design redundant fault monitoring and diagnosis links, which greatly reduces the dependence of the actual system on fault diagnosis and the possible risk of misdiagnosis. At the same time, the adaptive fault-tolerant controller designed in this chapter effectively solves the problems related to the parameter uncertainty and is not only limited to the constant changes of some parameters but can even be extended to the time-varying and all parameter uncertainty problems.

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