An interpretation of saturation phenomena as Glauber-Gribov multiple parton scatterings

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Abstract. We compare two formalisms that describe minijet production in \( pA \) and \( AA \) collisions: \( pQCD \) supplemented by Glauber-Gribov multiple semi-hard parton scatterings (\( pQCD+Glauber \)), and the Colour Glass Condensate (CGC). We argue that in a suitable limit they are equivalent to each other, the \( pQCD+Glauber \) model being more accurate from a numerical point of view. Finally, we analyze RHIC data on Au-Au integrated charged multiplicities in the \( pQCD+Glauber \) framework, and conclude that at least at central rapidity there is no sign of gluon saturation.

Keywords: \( pQCD \), multiparton scatterings, gluon saturation, CGC

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1. \( pQCD \), Glauber-Gribov parton rescatterings and the colour dipole

The model of Ref. [1], hereafter labeled “\( pQCD+Glauber \)”, assumes minijet production in \( pA \) collisions to be dominated by semihard parton multiple scatterings. By “semihard scattering” we mean a process with a minimum transverse momentum exchange \( p_0 \approx 1 - 2 \text{ GeV} \), described by leading order (LO) \( pQCD \) parton-parton cross section. The model assumes the S-matrix for a collision of one parton on \( n \) partons from the target to be factorizable in terms of S-matrices for parton-parton elastic-scattering, and assumes generalized \( pQCD \) factorization. Considering only gluons for simplicity, the minijet transverse spectrum is then given by

\[
\frac{d\sigma_{pA}^{\text{mj}}}{d\eta d^2p_T} = G(x, p_T^2) \frac{d\sigma_{\text{hard}}}{d^2p_T} + A G(x', p_T'^2) \frac{d\sigma_{\text{hard}}}{d^2p_T},
\]

where \( G(x, Q^2) \) is the proton distribution function of a gluon with fractional momentum \( x \), and \( x(x') \approx (p_T/\sqrt{s}) \exp(\pm \eta) \). In the second term of Eq. (1), the A-nucleus
partons are assumed to undergo a single scattering on the proton with cross section

\[ \frac{d\sigma}{d^2p_T} = \int_{x'_\text{min}}^1 dx' \, G(x', p_T^2) \frac{d\hat{\sigma}}{d^2p_T} . \]  

(2)

The LO pQCD cross section for gluon-gluon scattering is

\[ \frac{d\hat{\sigma}}{d^2p_T} \approx \frac{9}{2} \alpha_s^2 \frac{1}{(p_T^2 + p_0^2)^2} \]  

and the limits of integration on \( x' \) are given by parton-parton kinematic constraints. The first term of Eq. (1) accounts for multiple semihard scatterings \( s \) of the proton partons on the nucleus. The proton is considered pointlike at an impact parameter \( b \). Nuclear effects are assumed to be due only to multiple semihard scatterings, and are included in \( d\sigma_\text{hard}^A/d^2p_T \), the transverse momentum distribution of a proton parton who suffered at least one semihard scattering. This is written as \([1–3]\):

\[ \frac{d\sigma_\text{hard}^A}{d^2p_T} = \sum_{n=1}^\infty \frac{1}{n!} \int d^2b \frac{d\sigma_\text{hard}^p T_A(b)}{d^2k_1} \times \cdots \times \frac{d\sigma_\text{hard}^p T_A(b)}{d^2k_n} e^{-\sigma_\text{hard}^p(p_0)T_A(b)} \]

\[ \times \delta^{(2)}(\sum k_i - p_t) , \]  

(3)

where \( \sigma_\text{hard}^p(p_0) = \int d^2k \frac{d\sigma_\text{hard}^p}{d^2k} \) is the integrated gluon-nucleon cross section, which depends explicitly on the infrared regulator \( p_0 \). \( T_A(b) \) is the target nucleus thickness function. The exponential factor in Eq. (3) represents the probability that the parton suffered no semihard scatterings after the \( n \)-th one. In such a way, unitarity is explicitly implemented at the nuclear level, as discussed in Ref. [1, 3]. The sum over \( n \) may be explicitly performed in Fourier space. The result reads:

\[ \frac{d\sigma_\text{hard}^A}{d^2p_T} = \int \frac{d^2r_T}{4\pi^2} e^{-i\vec{k}\cdot\vec{r}_T} S_{\text{hard}}^A(r_T; p_0) , \]  

(4)

where

\[ S_{\text{hard}}^A(r_T; p_0) = \int d^2b \left[ e^{-\tilde{\sigma}_\text{hard}(r_T; p_0) T_A(b)} - e^{-\sigma_\text{hard}^p(p_0) T_A(b)} \right] \]  

(5)

and

\[ \tilde{\sigma}_\text{hard}^p(r_T; p_0) = \int d^2k \left[ 1 - e^{-i\vec{k}\cdot\vec{r}_T} \right] \frac{d\sigma_\text{hard}^p}{d^2k} . \]  

(6)

Note that \( \sigma_\text{hard}^p(r_T) \propto r_T^2 \) as \( r_T \to 0 \) and \( \sigma_\text{hard}^p(r_T) \to \sigma_\text{hard}^p \) as \( r_T \to \infty \). This suggests the interpretation of \( \sigma_\text{hard}^p(r) \) as a dipole-nucleon “hard” cross section. This dipole is of mathematical origin, and comes from the square of the scattering amplitude written in the Fourier variable \( r_T \), which represents the transverse size of the dipole. Then, we can interpret \( S_{\text{hard}}^A \) as the dipole-nucleus “hard” cross section. Eq. (5) clearly incorporates Glauber-Gribov multiple scatterings of the colour dipole. Note that no nuclear effects on PDF’s are included, but shadowing is partly taken into account by the dipole multiple scatterings.
2. CGC and the colour dipole

The Colour Glass Condensate is an effective theory for the nucleus gluon field at small-$x$, which describes the high-density regime where gluon saturation comes into play to modify the free-nucleon PDF’s [7].

In this framework, minijet production in $pA$ collisions may be related to the cross section for the scattering of a colour dipole on a nucleus [8]. The basic assumption is that the density of projectile partons is low enough for parton correlations inside the proton to be due only to DGLAP evolution. In this case one is allowed to treat the proton in the collinear factorization limit, exactly as in the pQCD+Glauber model of the previous section, see Eqs. (1) and (4) compared to Eqs. (36) and (32) of Ref. [8].

However, the dipole-nucleus interaction is computed entirely in the CGC model for the target nucleus. By using the so-called “Gaussian approximation” [9], in which parton correlation are assumed only to be Gaussian, one may write the dipole-nucleon cross section appearing in Eq. (4) as follows:

$$\tilde{\sigma}_{pCGC}(r_T) = 4\pi\alpha_s N_c \int \frac{d^2k_T}{(2\pi)^2} \frac{\mu_T(k_T)}{k_T^4} [1 - e^{-ik_T \cdot r_T}] ,$$

where $N_c$ is the number of colours, and $\tau = \ln(1/x)$ is called “rapidity”. Next, $\mu_T$ is interpreted as the **unintegrated gluon distribution** and has two limits [9]:

$$\mu_T = \begin{cases} 
\frac{4\pi^2}{N_c^2 - 1} \left( \frac{dG(x, k_T^2)}{d\ln(k_T^2)} \right) \frac{k_T^2}{k_T^2 + Q_s^2} \\
\frac{\delta}{\alpha_s} \frac{k_T^2}{Q_s^2} \ln \left( \frac{Q_s}{k_T} \right) \frac{k_T^2}{(k_T^2 + Q_s^2)^2} 
\end{cases} ,$$

These high- and low-$k_T$ limits are defined relatively to the “saturation momentum” $Q_s^2(\tau) = Q_0^2 e^{4.84 \delta (\tau - \tau_0)}$, where $\tau_0$ is the minimum rapidity at which gluon saturation occurs and $Q_0$ is the corresponding saturation scale. Unfortunately, the constants $\delta$, $\tau_0$ and $Q_0$ cannot be computed in this approximation.

3. CGC is pQCD+Glauber

The similarity of the CGC model at high-$k_T$ with the pQCD+Glauber model becomes evident by comparing Eqs. (7) and (8) with Eqs. (5), (6) and (2). To make this relationship more precise we proceed in four steps.

**Step 1.** In the CGC, we separate “soft” and “hard” interactions:

$$\mu_T = \mu_T^S + \mu_T^H$$

with

$$\begin{align*}
\mu_T^H &= \frac{4\pi^2}{N_c^2 - 1} \left( \frac{dG(x, k_T^2)}{d\ln(k_T^2)} \right) \frac{k_T^4}{(k_T^2 + Q_s^2)^2} \\
\mu_T^S &= \mu_T - \mu_T^H
\end{align*}$$

so that $\mu_T^H$ vanishes quickly for $k_T < Q_s$. Accordingly, we have a soft and hard dipole-nucleon cross section: $\tilde{\sigma}_{pCGC}^S = \tilde{\sigma}_{eff}^S + \tilde{\sigma}_{eff}^H$. 
Step 2. In the dipole-nucleus cross section, we isolate contributions from processes with purely soft rescatterings, and processes with at least one hard scattering. These define, respectively, the soft and hard dipole-nucleus cross section:

\[
S_{CGC}^A = S_{S}^{\text{eff}} + S_{H}^{\text{eff}} = \int d^2b e^{-\tilde{\sigma}_{\text{eff}}(r_T) T_{A}(b)} e^{-\sigma_{\text{eff}}(r_T) T_{A}(b)} + \int d^2b e^{-\tilde{\sigma}_{\text{eff}}(r_T) T_{A}(b)} \left[ e^{-\sigma_{\text{eff}}(r_T) T_{A}(b)} - e^{-\sigma_{\text{eff}}(r_T) T_{A}(b)} \right].
\]

The first term includes any number of soft scatterings but no hard ones, and the second term processes with at least one hard scattering.

Step 3. Study observables for which it is possible to neglect the soft part, e.g., hadron spectra at large \( p_T \) (Cronin effect), or integrated charged multiplicities at large \( \sqrt{s} \), where contribution from hard processes should become dominant.

Step 4. Following Ref. [10], we use the DGLAP equation to approximate the unintegrated gluon distribution in Eq. (9) with the integrated one:

\[
\frac{d x G(x, k_T^2)}{d \ln k_T^2} \approx \frac{\alpha_s N_c}{\pi} \int_x^1 dx' G(x', k_T^2) (9/2) \frac{\alpha_s^2}{k_T^2 + Q_s^2} [1 - e^{-i k_T \cdot r_T}].
\]

Eqs. (10) and (11) are equal to the pQCD+Glauber Eqs. (5) and (6) with \( p_0 = Q_s \) and \( x = x_{\text{min}}' \).

4. What do RHIC data have to say?

Under the assumptions stated in the previous section, it is possible to quantitatively compute some observables of interest. While a detailed application of Eq. (4) to hadron \( p_T \)-spectra and the Cronin effect is under investigation [11], we may use it to study RHIC Au-Au data on the centrality dependence of charged particle pseudorapidity densities [12]. The model was derived in the case of \( pA \) collisions, but is generalizable to AB collisions under the assumption that in the regime of interest both nuclei are composed of a diluted enough system of partons.

As a first step, we need the average minijet multiplicity at fixed impact parameter and pseudorapidity. This is obtained by integrating Eq. (3) over \( p_T \), and introducing the thickness function \( T_B \) of projectile nucleus:

\[
\frac{dN^{mj}_{\text{ij}}}{d\eta}(b; p_0) = \int d^2 \beta G(x, p_0^2) T_B(b - \beta) \left[ 1 - e^{-K \sigma_{\text{hard}}(p_0) T_{A}(\beta)} \right] + A \leftrightarrow B,
\]

(12)
Saturation as Glauber-Gribov multiscatterings

Fig. 1 Charged particle multiplicity per participant pair at RHIC, Eq. (13), as a function of the number of participants at different center of mass energies. Solid lines are the model results for the hard plus soft component. In the numerical computations both the gluons and the quarks have been included. Dashed lines are the result of the KLN saturation model [14]. Experimental data are from the PHOBOS collaboration [15].

where the K-factor that simulates higher-order corrections to the pQCD cross section is explicitly shown.

The integrand in Eq. (12) may be interpreted as the average density of projectile partons (at a given $x$) times the probability of having at least one semi-hard scattering against the target. At low values of $p_0$ the semihard cross section is large and the target becomes more and more black to the projectile partons: the probability of scattering at least once becomes so high that nearly every projectile parton scatters and is extracted from the incoming nuclear wave-function. In this regime even if we use a lower cutoff $p_0$ no more partons are there to be extracted. For this reason the minijet multiplicity tends to saturate [4]. We call saturation cutoff $p_{sat}$ the largest value of $p_0$ at which this happens, and evaluate minijet multiplicities from Eq. (12) with $p_0 = p_{sat}$. We refer to [5, 12] for more details.

To apply this minijet-level computation to integrated charged multiplicities, we write the charged particle multiplicity per unit rapidity as the sum of a soft and a semi-hard part [12]. For the soft part we use the wounded-nucleon model, which postulates a scaling of $N^{ch}$ with the number of participants. The semi-hard part is assumed to be completely computable from the saturation criterion for minijet production described above. To convert the minijet multiplicity to charged particle multiplicity, we further assume isentropic expansion of the initially produced minijet plasma, and parton-hadron duality. Our final formula reads

$$\frac{1}{N_{part}(b)/2} \frac{dN^{ch}}{d\eta}(b) = n_{soft} + \frac{1}{N_{part}(b)/2} \frac{5}{3} \frac{dN^{mj}}{d\eta}(b; p_{sat}) \ .$$  \hspace{1cm} (13)

As the soft component $n_{soft} = n_{soft}(\sqrt{s})$ in Eq. (13) does not depend on the centrality of the collision, we may fit it to data for central collision only (we used data at $\sqrt{s} = 56$-200 GeV [13] and extrapolated the fit down to 19 GeV, see Ref. [5]). The behaviour of the observable in non-central collisions is then completely determined by the model. In Fig. 1 we show the results of Eq. (13) compared to PHOBOS data.
Note that in our computation we used standard DGLAP evolved PDF’s from the GRV group [6], with no nuclear modification of any sort.

5. Conclusions

The pQCD+Glauber formulation gives a very well defined framework in which detailed numerical calculations may be performed, and which is well-tested in the case of nucleon-nucleon collisions. It is equivalent to the CGC model in a kinematic region where parton densities are low enough for gluon saturation effects to be negligible, but high enough for multiple scatterings and unitarity effects to become important. RHIC data on charged multiplicities at central rapidity are very satisfactorily described by the pQCD+Glauber model with free-nucleon PDF’s. Therefore, at least at central rapidity, RHIC data do not show any sign of gluon saturation.

However, integrated multiplicities have very limited sensitivity to the details of the production processes, and alone do not allow any definitive conclusion. To assess the presence or absence of nuclear effects beyond multiscatterings and unitarity, it is necessary to study more differential observables in a cleaner environment than the hot and dense medium produced in Au-Au collisions. Novel effects may then be observed as a deviation of experimental data from the baseline given by pQCD+Glauber. The Cronin effect on hadron production in $dA$ collisions at RHIC is an ideal candidate.

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