Targeted Synthesis for Programming with Data Invariants

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Abstract. Data structures in a program are frequently subject to data invariants—relations that must be maintained throughout program execution. Traditionally, invariants are implicit, and are enforced by manually-crafted code. Manual enforcement is error-prone, as the programmer must account for all locations that might break an invariant. Moreover, implicit invariants are brittle under code evolution: when the invariants and data structures change, the programmer must repeat the process of manually repairing all of the code locations where invariants are violated. In this work, we introduce programming with data invariants, a new programming model, where invariants are exposed to the programmer as a language feature, and statically checked by the compiler. Importantly, whenever programmer’s code breaks an invariant, the compiler synthesizes a patch to restore it. The two main challenges for implementing such a compiler are to make patch synthesis efficient and to avoid reverting changes made by the programmer. To tackle these challenges, we introduce Targeted Synthesis, an efficient patch synthesis algorithm, which exploits structural similarity between invariants and code to localize and simplify the synthesis problem. We evaluate our programming model and synthesis algorithm on a prototype language, Spyder, which is a core imperative language with collections, and supports a restricted but useful class of data invariants, which we term iterator-based invariants. We evaluate the succinctness and performance of Spyder on a variety of programs inspired by web applications, and show that Spyder allows concise specifications and implementation, and efficiently compiles and maintains data invariants.

1 Introduction

Programmers routinely face the task of enforcing data invariants. Prominent examples of data invariants include well-formedness of data structures, model-view relations in interactive GUI applications, and consistency between application data and the database. Failure to properly enforce invariants is a common source of serious bugs and security vulnerabilities [5]. Traditionally, programmers do not state invariants explicitly. Instead, they tacitly maintain invariants by sprinkling invariant-restoring snippets across their code. This ad-hoc practice is error-prone because the programmer must maintain a mental model of which invariants
(a)  

```javascript
for (i = 0; i != rows.length; ++i) {
    var r = rows[i];
    if (r > 0) {
        r.day = r.day * COLA;
    }
    // Targeted synthesis:
    // assigns to
    // r.week, r.total
}
// Naive synthesis: after loop
// regenerate rows and preserve days
```

(b)  

Fig. 1: GUI application for building a budget from recurring expenses and incomes.

An attractive alternative to this traditional model is to let programmers state the desired invariants explicitly, and have the programming language take responsibility for both checking the invariant satisfaction, as well as enforcing the invariants by updating the necessary data structures. Static checking of invariants is the subject of much prior work in program verification \cite{35,41,47,9}; these techniques, however, can only identify the code locations where an invariant might be violated, but they do not help the programmer restore the invariant. On the other hand, declarative constraint programming \cite{51,28} automatically adjusts program state to satisfy the invariant; the downside, however, is that doing so at run time is both unpredictable and inefficient. Wouldn’t it be great if instead we could compile declarative constraints into imperative code? Importantly, this would make the semantics of constraints more predictable, since any ambiguity would have to be resolved at compile time, when the compiler can ask the programmer for help. In this work, we propose using program synthesis technology to compile declarative data invariants into imperative, invariant-enforcing patches.

Program synthesis is an active area of research \cite{24,26,59,49,18,61} that tackles the problem of generating programs from declarative constraints. In particular, synthesis from logical specifications \cite{34,31,46} takes as input a logical predicate over a program’s inputs and outputs, and searches for a program that satisfies the predicate. We describe how program synthesis enables language support for data invariants through a motivating example.

1.1 Motivating Example: Budget Planner

Consider a budget builder application for recurring expenses and incomes, shown in Fig. 1. The amount for each item in the budget plan is stored in two different formats, Weekly and Daily, so that the end-user can provide input in the most relevant period. For example, a budget for meals can be given in Daily units, rent can be given in Weekly units, etc. To see whether the planned budget is balanced, the daily budget items are added up: a running total stored in Totals; the final entry of Totals contains the expected overall surplus or deficit per
day. Revenues are distinguished from Expenses by rendering Revenues black and Expenses red.

Each of these application properties is a data invariant that the programmer has to maintain: (1) weekly and daily are unit-conversions of each other, (2) totals is a running sum of the daily values, and (3) if an entry is negative, its font color is red.

Consider a function that adjusts the income in an existing budget according to a cost-of-living index. This function, shown in Fig. 1b, multiplies each positive daily item by the Cost-of-Living-Adjustment (COLA) constant. The loop in Fig. 1b breaks invariants (1) and (2): the weekly and total values are stale. Our goal is to synthesize an invariant patch, i.e., a code snippet that, when inserted into the function body, will provably restore the broken data invariants.

At a first glance, it seems natural to insert the patch at the end of the function, using the programmer-provided data invariants as the specification for synthesis. Unfortunately generating such a function-level patch is nontrivial even for this simple example. Since each row of the table is modified, the patch must involve a loop over the rows of the table. Synthesizing loops is challenging, because the synthesis algorithm must generate an inductive loop invariant. Note, that the original data invariant is not suitable because it does not hold on entry to the new loop – the programmer’s loop broke the data invariant in the first place. Moreover, even if the synthesis algorithm is clever enough to generate a loop, it must be careful to preserve the programmer’s original logic. The simplest solution is to update the daily field of each row using the weekly values. Such a patch would be disastrous – the data invariant is erroneously “maintained” by undoing the programmer’s changes!

More generally, this simple example highlights the two main research problems for synthesis-based language support of data invariants: (a) complex patches: even for simple data invariants, the synthesis algorithm must calculate both inductive invariants and complex control flow, and (b) the frame problem: without frame conditions, the synthesis algorithm can enforce the invariant by simply reverting the programmer’s changes.

1.2 Targeted Synthesis and the Spyder Language

The technical contribution of this paper is a solution to the above two research problems. Our solution consists of co-designing a programming language with a novel targeted synthesis algorithm, which generates patches locally – as close as possible to the invariant violation – as opposed to at the function boundaries.

Targeted synthesis addresses the problem of complex patches by generating multiple patches that are as local as possible. For example, in Fig. 1b, a local patch updates r.week and r.total inside the loop. Local patches are typically much smaller; moreover, pushing a patch inside a loop often results in preserving the original data invariant between loop iterations, creating an inductive loop invariant. In our example, not only is the desired patch a short, straight-line code snippet, but also it maintains data invariant (1) as an inductive loop invariant.
Targeted synthesis also addresses the frame problem: enforcing invariants at basic block boundaries enables a simple syntactic check that disallows patching variables modified by the programmer in that block and thereby ensures that all programmer’s changes are preserved.

This paper presents Spyder, a core language with iterators and data invariants, which is designed to be amenable to targeted synthesis. In particular, Spyder offers iterator-based loops and iterator-based data invariants, which allows the synthesis algorithm to exploit their structural similarity and push synthesis specifications inside loops, in order to generate local patches.

The remainder of the paper is structured as follows. We use the domain of web GUI applications to give a high-level overview of Spyder in Sec. 2. Sec. 3 formalizes the semantics of the Spyder language, and Sec. 4 presents our targeted synthesis algorithm for extending Spyder programs with invariant-preserving patches. As part of our formalisms, we contribute a soundness guarantee that the targeted synthesis algorithm preserves the original invariants; this is summarized in Sec. 4. Sec. 5 evaluates our Spyder compiler on a series of benchmark and case studies. Finally, we conclude by reviewing related work in Sec. 6.

1.3 Main Contributions

The main contributions of this paper are:

1) Programming with data invariants: a new programming model, where the developer explicitly states relational data invariants, and a synthesis engine automatically generates code patches to maintain these invariants.

2) Targeted Synthesis: a sound and efficient algorithm for synthesizing patches for the restricted but useful class of data invariants we call iterator-based invariants.

3) Spyder, a prototype implementation of Targeted Synthesis; our empirical evaluation shows that Spyder programs are concise and compositional, and that Targeted Synthesis generates patches more efficiently than traditional program synthesis techniques.

2 Overview

We begin with an overview of Targeted Synthesis on the budgeting application shown in Fig. 1 in which the programmer uses data invariants to author an interactive GUI application. The rendering and logic of the application are relatively easy to express using the imperative, as we will discuss in Sec. 6, but this approach does not offer language support for statically enforcing application data invariants. We will demonstrate how Spyder supports data invariants by iteratively building the interactive logic for this example.

2.1 Data Invariants

The programmer starts with the logic for the Weekly and Daily columns, shown in Fig. 2. To do this, the programmer declares a collection of ints termed weeks,
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1 data weeks: int[];
2 data days: int[];
3 foreach w in weeks, d in days:
4    w = 7 * d;

// procedure adjustForCOLA (cola: int):
1 foreach d in days:
2    if (d > 0):
3        d = d * cola;
4        w = 7 * d;

(a) Source code in the Spyder language for the days and weeks columns of the budgeting application.

(b) Generated Spyder code for adjustForCOLA in the budgeting application.

Fig. 2: Programming with an invariant between Days and Weeks in the budgeting application.

shown on line 1 of Fig. 2a, as well as a collection of int's termed days (line 2). These two declarations introduce new mutable global variables days and weeks.

One invariant of the system is the unit-conversion invariant (invariant (1) in Sec. 1.1): each of the elements of weeks is 7 times greater than the corresponding element of days. This invariant should always hold and in particular, needs to be enforced whenever either weeks or days is mutated. To specify the unit-conversion invariant, the programmer uses a foreach construct on line 4, binding the elements of weeks to the local iterator w and the elements of days to d. Using these local bindings, they express the unit-conversion invariant using the formula on line 5: 7 * d.val = w.val.

Because this unit-conversion invariant is defined over elements of collections, traditional techniques would model collections as arrays and require a quantified relation over the indices of the arrays. Such relations are notoriously tricky to build by hand (and indeed, to verify), but in Spyder, the programmer can use the foreach abstraction. This abstraction builds an element-wise product relation by introducing fresh iterator bindings over the abstracted collections.

2.2 Maintaining Data Invariants with Spyder

Next, the programmer writes imperative code implementing the desired functionality, without correcting for the violated unit-conversion invariant, as Spyder will patch to maintain it. In the application, recall that the budget-builder needs to adjust all of the revenues (and only the revenues) in the budget by the Cost-of-Living-Adjustment (COLA). To implement this modification, the programmer writes a procedure called adjustForCOLA on line 7. This function iterates over the elements of days using the for loop on line 8, which binds each element of days to a local iterator variable d.

Since the COLA should only be applied to revenues, the programmer checks the value of the element d using a conditional on line 9, and then scales the daily revenue by an iterator update on line 10. The iterator semantics of Spyder are standard for object-oriented iterators; in particular, notice that the value of the iterator (e.g. d.val) is implicitly given by the iterator variable itself (e.g. d in the expression d > 0).
In this code snippet, the programmer has directly assigned an updated value to `d`, and by extension the values of `days`. On its own, this update breaks the unit-conversion invariant — in particular, the Weekly value of this row of the application depends on the concrete value of `d`. Using traditional techniques, the programmer would have to manually maintain the invariant by setting the corresponding value of `weeks`, i.e. by adding an extra snippet for correctly updating `weeks`.

Fortunately for the programmer, invariants are statically maintained in SPYDER and the compiler synthesizes and inserts a invariant-restoring snippet automatically, as shown in Fig. 2b. In this case, the compiler *extends* the original loop over `days` with an extra binding over `weeks`; in SPYDER, this has the semantics of a simultaneous iteration (analogous to a functional zip) so that `d` and `w` refer to elements of `days` and `weeks` at the same index.

More generally, in contrast to traditional programming, SPYDER enables the programmer to write modifications that are *agnostic* to the existing invariants. In this case, the programmer simply writes a direct update to the elements of `days` and SPYDER ensures that the overall system’s state is correct.

### 2.3 Program Composition Through Data Invariants

In this subsection we demonstrate code evolution with SPYDER. At some later date, the programmer adds a feature to the budget application: a running totals column to help track the state of the budget. To do this, they add a collection for the total values, and the data invariant to populate it, seen in Fig. 3a, lines 8-9. In order to define the running-sum property, SPYDER provides an iterator method called `prev`, which allows access to the previous value of the iterator. This is useful for defining accumulator properties or enforcing sortedness. SPYDER will generate the implementation of populating the totals column in its entirety.

However, since time has passed since the last change made to the system, the programmer has forgotten about `adjustForCOLA`, which breaks our new totals invariant. In a traditional imperative programming paradigm, it would be the programmer’s responsibility to track down every function that breaks the invariant and fix it. However, with SPYDER, the compiler checks the new invariant against all existing functions and generates a new patch to `adjustForCOLA` to ensure it is maintained.

The different invariants are compositional from the user’s perspective—in practice, each function is checked against all invariants in the code. It is the responsibility of TARGETED SYNTHESIS to find in a failed set if invariants the actual invariants that have failed, and to reduce those to a local specification that can be used to synthesize a patch. This is shown in Sec. 4.2.

In evolving the codebase, the programmer later adds another feature, coloring negative values in red. This is done using two sets of invariants: one for totals (lines 12-14), and one for days (lines 16-19). Notice that `adjustForCOLA` does not invalidate the days invariant. SPYDER checks this in compile time, resulting in no changes being made to `adjustForCOLA`—as opposed to dynamic techniques which would generate code that tests this in runtime.
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Fig. 3: Programming with an accumulator invariant between days and totals, as well as a font color invariant. Colors indicate the relationship between the invariant (a) and generated code (b).

The code generated by SPYDER is seen in Fig. 3b. The fixes introduced by SPYDER are nontrivial in several ways: (1) the fixes are extensive, accounting for the majority of the code in Fig. 3b, (2) the fixes are nonlocal, meaning that each fix is spread out over (and interleaved with) the original code, (3) the fixes have to add new variables just to maintain the invariants, and (4) each invariant requires multiple fixes.

2.4 Generalization of Technique

From the programmer’s perspective, the process of invariant patching is invisible – SPYDER accepts the original, invariant-oblivious code. More generally, the cognitive load of imperative programming with invariants in SPYDER is significantly less than traditional techniques. Using TARGETED SYNTHESIS, when writing imperative code, the programmer needs to only reason about local code properties (e.g. the value of \( d \)) and does not need to reason about global code invariants (e.g. the relation between days and rowFontColors).

The structure of the remainder of this paper is as follows: We first describe the SPYDER source language in Sec. 3 and show how to translate from SPYDER terms to a well-studied imperative language. We also give a hoare-style axiomatic semantics to SPYDER programs, and provide a formal guarantee that SPYDER verification triples are equivalent to standard triples. In Sec. 4 we present synthesis rules for patching and extending SPYDER programs, and provide a formal guarantee that our synthesis rules are sound with respect to our SPYDER verification triples. Finally, we present several case studies and a benchmarking evaluation in Sec. 5 and conclude with a discussion of related work in Sec. 6.
3 The Spyder Language

We present the syntax and semantics of the Spyder source language. In this section, we do not describe how to maintain data invariants. Instead, we just provide the formal framework for both expressing collection-manipulation programs, as well as axiomatically verifying properties over programs. We will build on the results of this section in Sec. 4 to show how to maintain data invariants through Targeted Synthesis.

First, we introduce the core syntax of Spyder in Sec. 3.1. Then, we give a semantics to the syntax by translating Spyder terms to a well-studied standard imperative language in Sec. 3.2. Next, we demonstrate how to mechanically verify when invariants are maintained or violated by defining a Hoare-style axiomatic logic for Spyder in Sec. 3.4. Finally, we give a proof of soundness for this verification logic by reduction to the standard axiomatic semantics for imperative array programs (i.e. Hoare logic) in Theorem 1.

3.1 Surface Syntax for Spyder

At its core, Spyder is an imperative collection-manipulation language. The focus in Spyder is to support data invariants for mutable, finite collections. To this end, we formalize and define a core calculus for iterating over and mutating collections, which we present in Fig. 4a.

Values and Types Spyder has three datatypes: integers, collections, and iterators. Integers are standard and we denote a variable declaration of type int as `data x: int`.

Collections Collections hold elements and are analogous to ordered containers, e.g. lists or arrays. For variable declarations, we denote a collection of T by `data col: T[]`. Collections are homogeneous and for clarity of presentation, our core syntax and formalisms assume that all collections are 1-dimensional (i.e. collections of integers). In our implementation, however, collections can nest arbitrarily (and extending the formalisms to arbitrary nesting is straightforward). For example, the list `[1, 2, 3]` is a collection of integers and the list `[[1, 2], [3, 4]]` is a collection of integer collections. In contrast, the list `[1, [2, 3]]` has mixed element types and is not valid. Collections expose a single method, `size`, which returns the number of elements in the collection. For simplicity, we assume all collections have a statically known size which does not vary at runtime.

We also assume that collection sizes are homogeneous, for example, the list `[[1, 2], [3, 4, 5]]` would not be a valid Spyder collection. A key difference between collections and arrays is that collections do not support subscription (i.e. `col[idx]` is not a valid Spyder term). Instead, to access the elements of a collection, Spyder exposes the `for (x, y)` statement, which iterates over the values of the collection `y`. In addition to iteration, the `for` statement creates a new variable binding for an iterator variable.
Iterators Iterators allow access to the underlying elements of a collection. Iterator variables are not explicitly declared using `data` but are instead created in for loops. Spyder supports several standard iterator methods: `val`, which returns the value of the iterated collection; `idx`, which returns the current iteration index; `prev`, which returns the previous value; and the `x ← E` operator (termed “put”), which destructively updates the value of the iterator `x` with the expression `E`. For example, after the evaluation of the term `for x in xs: x ← x + 1`; each element of `xs` is incremented by exactly one.

Statements For control-flow, Spyder has mostly standard imperative statements. A key exception is the `for` term, which as discussed above, iterates over a collection. In addition, the `for` loop iterates over multiple collections simultaneously, similar to a zip in function programming. For example the term `for x in xs, y in ys: x ← x.val + 1; y ← y.val + 1;` replaces each element in `y` with the corresponding element in `x`. Furthermore, iteration is only well-defined when the iterated collections have the same size.

Specifications To express specifications for Spyder terms, Spyder exposes a rich specification language, the Spec terms. To ease the burden of synthesis and verification, we syntactically phrase specifications in a conjunctive normal form. At the top level are conjunctions of specifications using the `∧` operator. Each conjunct can be either a bare expression, or a quantification term. For quantification, Spyder supports two quantifiers: (1) An existential quantifier through the `exists` keyword. This quantifier is not present in the surface syntax of Spyder and is only used in the axiomatic semantics, which we present in Sec. 3.4. (2) A universal quantifier through the `foreach` keyword, which quantifies over the elements of a collection. For example, the specification `foreach x in xs, y in ys: x.val > y.val` states that each element of `xs` is greater than the corresponding element of `ys`. Similar to the `for` statement, the `foreach` term is only well-defined when the bound collections have the same size. We discuss the details of specifications more in Sec. 3.4.

3.2 Imperative Target Language

We formalize the semantics of Spyder by translating to an idealized imperative verification language, which we call Imp-Array. The syntax of this verification target language is shown in Fig. 13. This language is very similar to Boogie and indeed, in our implementation, we compile and synthesize to Boogie.

Although Spyder and Imp-Array have similar syntax, there are several major differences. Broadly speaking, Imp-Array does not have language support for either collections or iterators. Imp-Array instead offers mutable low-level arrays, which map (integer) indices to values. At the statement level Imp-Array supports mutable updates to both variables and arrays, as well as general while loops. For expressions, Imp-Array enables rich quantification through the `∀` quantifier, but in contrast to Spyder, does not support iterator methods.
\[
\text{Spec} ::= \text{foreach } (v, u) \text{ Spec} \\
\text{exists } v \text{ Spec} \\
\text{Spec} \land \text{ Spec} \\
\text{Expr} \\
\text{Block} ::= \text{skip} \mid \text{ Stmt } \mid \text{ Block} \\
\text{Stmt} ::= v := \text{ Expr} \\
\text{if } \text{ Expr} \text{ then Block else Block} \\
\text{for } (v, u) \text{ Block} \\
\text{Expr} ::= v \mid i \mid \text{true} \mid \text{false} \\
\text{Expr bop } \text{Expr} \\
\text{uop } \text{Expr} \\
\text{v.prev(Expr)} \\
\text{v.idx} \\
\text{v.size} \\
\text{bop} ::= + \mid \times \mid \% \mid \implies \mid \Leftrightarrow \mid \ldots \\
\text{uop} ::= \neg \mid ! 
\]

(a) Syntax for the Spyder language.

\[
\text{Stmt} ::= v := \text{ Expr} \\
\text{if } \text{Expr} \text{ then Stmt else Stmt} \\
\text{while } \text{Expr} \text{ Stmt} \\
\text{Stmt} \mid \text{ Stmt} \\
\text{Expr} ::= v \mid i \mid \text{true} \mid \text{false} \\
\text{Expr bop } \text{Expr} \\
\text{uop } \text{Expr} \\
\text{v.prev(Expr)} \\
\text{v.size} \\
\text{bop} ::= + \mid \times \mid \% \mid \implies \mid \Leftrightarrow \mid \ldots \\
\text{uop} ::= \neg \mid ! 
\]

(b) Syntax for the Imp-Array language.

Fig. 4: Syntax for Spyder and Imp-Array.

Fig. 5: Source code and translation maintaining a product invariant. In contrast to the examples in Sec. 2, the source code maintains the invariant, and the translation step must soundly produce ImpArray code which also maintains it.

To support collections and iterators, the translation from Spyder to Imp-Array must implement collection and iterator logic in terms of arrays and indices. We show an example of this in Fig. 5 in which a Spyder program for calculating a product is translated into Imp-Array. In this case, the integer collections values and product in Spyder map 1-to-1 to arrays in Imp-Array, and the...
for loop in SPYDER is desugared into a while loop with an explicit index in IMP-ARRAY. At a high-level, collections in SPYDER correspond 1-to-1 with arrays in IMP-ARRAY, and iterator variables in SPYDER correspond to indices in IMP-ARRAY. This example is similar to a subproblem of the TARGETED SYNTHESIS algorithm (discussed in detail in Sec. 4), which reasons about candidate programs like multiValues.

### 3.3 Overview of Translation Semantics

We formalize translation as a syntax-directed recursive function over SPYDER terms given in Fig. 6. Since for loops bind iterator variables, the translation must be stateful. We choose to explicitly pass the state using finite mathematical maps, which we term translation contexts and we generally denote as $\Gamma$. We denote the translation of a term $t$ using the context $\Gamma$ as the IMP-ARRAY term $\text{trans}(t, \Gamma)$; we refer to this as “the translation of $t$ in the context of $\Gamma$”.

| SPYDER Term | IMP-ARRAY Term |
|-------------|----------------|
| $\text{trans}(v, \Gamma)$ | $v$ |
| $\text{trans}(i, \Gamma)$ | $i$ |
| $\text{trans}(\text{true}, \Gamma)$ | true |
| $\text{trans}(\text{false}, \Gamma)$ | false |
| $\text{trans}(E\ bop\ E_1, \Gamma)$ | $\text{trans}(E, \Gamma)\ bop\ \text{trans}(E_1, \Gamma)$ |
| $\text{trans}(\text{uop}\ E, \Gamma)$ | $\text{uop}\ \text{trans}(E, \Gamma)$ |
| $\text{trans}(\text{v.val}, \Gamma)$ | $\Gamma(v)[v]$ |
| $\text{trans}(\text{v.prev}(E), \Gamma)$ | if $v > 0$ then $\Gamma(v)[v - 1]$ else $\text{trans}(E, \Gamma)$ |
| $\text{trans}(\text{v.size}, \Gamma)$ | $\text{size}(v)$ |

(a) Translation rules for Spyder Expressions to ImpArray Epressions.

| SPYDER Term | IMP-ARRAY Term |
|-------------|----------------|
| $\text{trans}(\text{exists}\ v\ i, \Gamma)$ | $\exists v.\ \text{trans}(i, \Gamma)$ |
| $\text{trans}(I_1 \land I_2, \Gamma)$ | $\text{trans}(I_1, \Gamma) \land \text{trans}(I_2, \Gamma)$ |

(b) Translation rules for Spyder Specifications to ImpArray Epressions.

| SPYDER Term | IMP-ARRAY Term |
|-------------|----------------|
| $\text{trans}(v::=E, \Gamma)$ | $v := \text{trans}(E, \Gamma)$ |
| $\text{trans}(\text{if}\ E\ \text{then}\ B_1\ \text{else}\ B_2, \Gamma)$ | if $\text{trans}(E, \Gamma)$ then $\text{trans}(B_1, \Gamma)$ else $\text{trans}(B_2, \Gamma)$ |
| $\text{trans}(\text{for}\ (x,y)\ B, \Gamma)$ | $x := 0$ ; while ($x < \text{size}(y)$) $\text{trans}(B, \Gamma) @ x \rightarrow y) ; x := x + 1$ |

(c) Translation rules for Spyder Statements to ImpArray Statements.

Fig. 6: Translation rules for Spyder to ImpArray

**Well-formedness of Translation Contexts** In general, the translation process is only well-defined if the translation context $\Gamma$ is well-formed. Intuitively, there must be no name-collisions; a collection must not be iterated over multiple times; an iterator variable must not be directly written to (i.e. using assignment := instead of the iterator $\leftarrow$ operator); etc. We formalize these well-formedness constraints in Fig. 7, which relates SPYDER terms $t$ to translation contexts that are well-formed for translating $t$. We denote a well-formed term using $\text{wf}(t, \Gamma)$ and we say $\Gamma$ is well-formed with respect to $t$. 

Fig. 7: Well-formedness rules for Spyder. For exposition, when rules bind a variable (e.g. for) we only formalize the well-formedness for a single binding. The extension to multiple bindings is straightforward.

Semantics for Spyder terms Since Imp-Array is well-studied and has a well-understood semantics (replicated in Sec. A.1), we define the semantics of Spyder by translating into Imp-Array. For details, see Sec. A.2.

A keen observer will notice that Spyder’s semantics are focused on alias-free iterator-based programs. Imp-Array has actually been studied in the context of verifying more exotic language features, such as object-oriented invariants [36], concurrency [11], general arrays [33], heap-manipulation [45], etc. Because other systems have verified exotic programs using the rich, low-level semantics of Imp-Array, in the future the semantics of Spyder can be extended to handle relational invariant maintenance for more complicated languages.

3.4 Verification in Spyder and ImpArray

We next define and present an axiomatic semantics for Spyder that Targeted Synthesis will use to mechanically verify when invariants are preserved or
violated by a statement. In addition, we prove that SPYDER axiomatic semantics are sound with respect to the standard axiomatic semantics for IMP-ARRAY (i.e. Hoare triples).

**Hoare Triples for ImpArray** We start by briefly reviewing axiomatic semantics in IMP-ARRAY which are well-known [26]. The standard approach, called Hoare triples, are deduction rules for relating three terms: a precondition $P$, a statement $S$, and a postcondition $Q$, denoted by $\{P\} S \{Q\}$. Intuitively, the rules derive a triple if and only if given the precondition $P$, the postcondition $Q$ holds after executing the statement $S$. We replicate these rules in Fig. 16.

Notice that in standard axiomatic semantics, the loop rule requires an inductive invariant $I$ to be maintained on every iteration. Furthermore, the axiomatic rules do not contain a notion of termination. As a result, the triple $\{P\} S \{Q\}$ should only be interpreted as valid if the statement $S$ terminates. In our case, termination is orthogonal. Our well-formedness constraints ensure that all loops over finite collections terminate, and so in practice, this is not an issue for our use of the axiomatic semantics of IMP-ARRAY.

**Hoare Triples for Spyder** We next provide a similar axiomatic semantics for SPYDER terms. In this case, we derive a triple $\langle P \rangle S \langle Q \rangle$, which has the same intuitive interpretation, that given $P$, $Q$ holds after executing $S$. As part of our contribution, we prove that the logic of Fig. 8 is relatively sound: given a well-formed translation context, the axiomatic rules are sound with respect to Hoare logic. Intuitively, if we prove a triple in the SPYDER semantics, then the corresponding translated triple holds in Hoare’s axiomatic semantics. More formally, let $P$ and $Q$ be SPYDER Expressions, let $S$ be a SPYDER Statement, and let $\Gamma$ be a translation context. If $\Gamma$ is well-formed with respect to $P$, $Q$, and $S$, and we derive the triple $\langle P \rangle S \langle Q \rangle$, then there exists a Hoare Triple for the corresponding translated terms in IMP-ARRAY:

**Theorem 1 (Relative Soundness).**

$$\forall P, S, Q, \Gamma. \text{wf}(P \land Q, \Gamma) \land \text{wf}(S, \Gamma) \implies \langle P \rangle S \langle Q \rangle \implies \{\text{trans}(P, \Gamma)\} \text{trans}(S, \Gamma) \{\text{trans}(Q, \Gamma)\}$$

We prove this property by induction over the derivation of the SPYDER Triple $\langle P \rangle S \langle Q \rangle$, given in Sec. A.3. The key parts of the proof are the soundness of the Put and For rules which we discuss in detail below.

*Strong Iterator Updates* Put is interesting because under the hood, the update $x \leftarrow E$ translates to an array write (namely $\Gamma(x)[x] := E$). This is potentially problematic because standard array semantics assume indices can alias and so all information about the collection $\Gamma(x)$ is lost after the update. However, SPYDER has no variable aliasing. Moreover, the well-formedness rules ensure that values of the collection $\Gamma(x)$ can only be referenced through exactly one iterator $x$ and
Consequence
\[ P \implies P' \quad Q' \implies Q \]
\[ (P') S (Q') \quad \text{Conditional} \quad (P) S (Q) \]
Assign
\[ P \quad (\text{fresh } v') \]
\[ (P) \quad (\exists v'. P[v \mapsto v'] \land v = E[v \mapsto v']) \]
Put
\[ (P) \quad (\text{fresh } v') \]
\[ v \leftarrow E \quad (\exists v'. P[\text{val}(v) \mapsto v'] \land \text{val}(v) = E[\text{val}(v) \mapsto v']) \]
For
\[ (\text{weaken prev}(I) \land 0 \leq \text{idx}(s) < \text{size}(y)) \quad B_i \quad (I) \]
\[ (\text{foreach } (x, y) I) \quad (x, y) B_i \quad (\text{foreach } (x, y) I) \]
Sequence
\[ (P) S (Q) \quad (P) B (R) \]
\[ (Q) S (R) \]
\[ \text{Skip} \quad (P) \quad \text{skip} (P) \]

Fig. 8: Hoare-style verification logic for SPYDER. For exposition, we only formalize the relation loops with a binding. Since loops are only well-defined when the iterated collections have the same statically known size, the extension to multiple bindings is straightforward.

Consequently, in the Put rule we reason about the value of \( x.\text{val} \) while soundly retaining information about the collection \( \Gamma(x) \).

Quantifier introduction and maintenance A key requirement of the axiomatic semantics is to soundly reason about when loops maintain (or violate) universally quantified invariants (i.e. foreach terms). To that end, we provide a For rule, which is similar to a standard while rule in that the inductive invariant is on both sides of the statement. Unlike the Hoare while rule, however, the For rule for a loop \( \text{for } x \text{ in } xs \) requires a top-level foreach \( x \text{ in } xs \) as well.

In order to show that a foreach invariant is maintained by a for loop, it suffices to reason about each iteration of the loop in isolation. Due to the well-formedness constraints, the only way to modify the elements of a collection is through the \( \leftarrow \) operator. As a consequence the execution of a loop iteration cannot invalidate the results of previous iterations. Since the loop is guaranteed to execute for each element of the collection, the rule introduces a foreach quantifier after the loop is complete.

Furthermore, it’s tempting to assume the specialized invariant as a precondition to verifying the loop body. If the invariant does not contain the prev method, this is completely valid. However, the prev method complicates matters because each iteration does not necessarily establish prev for the next iteration. To address this situation, we use the weaken_prev helper function to soundly weaken an expression with respect to prev. As a result, the For rule retains as much information as is soundly possible, and enables automated verification and synthesis by removing a layer of quantification.

1 In particular the well-formedness relation prohibits a foreach quantifier over a collection y from entering the body of a loop over y.

2 If a top-level term is not in this form but is equivalent under renaming and quantifier shuffling, the Consequence rule can be used to rewrite the term to make progress.
3.5 Maintaining Data Invariants

With an axiomatic semantics for SPYDER programs, we now consider several techniques for maintaining data invariants. We use a simple midpoint program in Fig. 9 in which two variables \( l \) and \( r \) sum to 10, to demonstrate these techniques.

(a) Imperative: program with no additional specifications.

```
data l: int;
data r: int;
// invariant: \( l + r = 10 \)
```

```
procedure incrL():
l = l + 1;
procedure incrR():
r = r + 1;
```

(b) FRP: functional specifications for \( l \) and \( r \).

```
data l: int;
data r: int;
// invariant: \( l + r = 10 \)
```

```
l + r = 10
```

```
procedure incrL():
l = l + 1;
procedure incrR():
r = r + 1;
```

(c) SPYDER: a single relational specification for \( l \) and \( r \).

```
data l: int;
data r: int;
// invariant: \( l + r = 10 \)
```

```
l + r = 10
```

```
procedure incrL():
l = l + 1;
procedure incrR():
r = r + 1;
```

Fig. 9: Three different specification techniques used to implement a midpoint program in which \( l \) and \( r \) sum to 10.

**Imperative Invariant Maintenance** The most common technique for invariant maintenance is to manually track invariants and provide a patch that maintains the invariant. This is tedious and error prone because the programmer must manually remember 1) what the invariant is, and 2) how to maintain the invariant when it breaks. For example, in the midpoint program (Fig. 9a), the programmer must remember that \( l \) must be decremented after \( r \) is incremented, and vice-versa.

From the programmer’s perspective, this is also the least compositional approach to invariant maintenance. If the invariant changes, it is up to the programmer to find all the patches and fix them. However it is also the most performant technique; the runtime system simply executes the code.

**Functional Invariant Maintenance** An alternative approach to manual maintenance is the Functional-Reactive programming (FRP) paradigm, in which the programmer provides a functional specification for solving the invariant, and the language runtime detects when the functional specification should be invoked. In this example (Fig. 9b) the programmer gives two functional specifications for \( l \) and \( r \), each in terms of the other. In return, the language runtime uses these specifications to perform invariant maintenance, saving the programmer the need to reason about maintenance within the implementation of \( \text{incrL}() \) or \( \text{incrR}() \).

The downside of this approach is that the runtime system must dynamically track data-dependencies, incurring a runtime overhead compared to the imperative approach. We discuss FRP in more detail in Sec. 6.

**Spyder Invariant Maintenance** Finally, TARGETED SYNTHESIS enables automatic relational invariant maintenance. In contrast to a functional specification, a
relational specification does not easily admit a clear resolution for the specification. From the programmer’s perspective, relational specifications are much more clear and concise. Consider in this example the specification in Fig. 9c; it clearly and unambiguously captures the data invariant that $l$ and $r$ sum to 10.

The power and expressiveness of relational specifications comes at a cost. One way to handle these rich relational invariants is to dynamically solve the relational specification, similar to FRP. This incurs a significant runtime overhead (see Sec. 6) and moreover, when the specification is erroneous, dealing with the error falls to the end-user of the code and not the programmer.

Instead, we take the approach of solving these invariants at compile time using program synthesis. In the next section, we detail exactly how TARGETED SYNTHESIS enables the programmer to use relational specifications automatically within the SPYDER language.

4 Targeted Synthesis for Spyder

In this section, we detail the automatic enforcing of data invariants. We motivate and formalize the problem in Sec. 4.1 then, in Sec. 4.2 we present its solution in the TARGETED SYNTHESIS algorithm. We prove the algorithm sound in Sec. 4.3.

Recall the budgeting example introduced in Sec. 1.1. Sec. 2 showed the specific case of the unit-conversion data invariant, which establishes the required relationship between daily and weekly values, seen in Fig. 2a. Throughout this section we will demonstrate our algorithm on this invariant.

4.1 Automatic Enforcement of Data Invariants

Let $\Pi$ be a Spec term, and $S$ be a SPYDER statement (i.e. a Stmt term). We say that $\Pi$ is a data invariant for $S$ if and only if $S$ maintains $\Pi$:

$$\langle \Pi \rangle S \langle \Pi \rangle.$$

For example, the specification

```python
foreach x in xs: x.val > 0
```

is a data invariant for a loop which increments each value of $xs$, for $x$ in $xs$: $x <- x.val + 1$, but it is not a data invariant for decrement loop

```python
for x in xs: x <- x.val - 1.
```

This definition extend straightforwardly to statement blocks $B$.

Let $B, B'$ be two SPYDER blocks. We say that a block $B'$ is an extension of $B$ ($B \prec B'$) if $B$ and $B'$ have identical semantics on variables modified by $B$.

An invariant enforcement problem is a pair $\langle B, \Pi \rangle$ of a block $B$ and a specification $\Pi$. A solution to the enforcement problem is a block $B'$ such that $B \prec B'$ and $\langle \Pi \rangle B' \langle \Pi \rangle$. In other words, the goal is to find an extension of $B$ such that $\Pi$ is a data invariant for the extended block.

In our example, we wish the unit-conversion invariant on lines 4 and 5 to be a data invariant. This means the invariant enforcement problem is to enforce this specification on the body of adjustForCOLA.

To find a solution, our algorithm analyses $B$ and insert local patches whenever the invariant needs to be restored. Since there are many candidate patches to
explore, the key challenge is to make the search efficient. To this end, our algorithm: (1) a-priori restricts the search to extensions of \( B \), by keeping track of the set of variables that a patch is allowed to modify; (2) targets the invariant \( \Pi \) to \( B \), producing a specification for each patch that is as local as possible.

In this example, because \texttt{adjustForCOLA} modifies the elements of \texttt{days}, our algorithm must find an extension that has an equivalent effect on \texttt{days}. Further, since \texttt{adjustForCOLA} iterates over \texttt{days}, our algorithm will target the data invariant on lines 4 and 5 to a local specification, specific to just the loop body on lines 9 and 10. We next explain the details of our algorithm.

### 4.2 Targeted Synthesis Algorithm

We formalize Targeted Synthesis as a completion judgment \( \text{md} \vdash \langle \Pi \rangle B \langle \Phi \rangle \rightarrow B' \). Intuitively, given a pre- and post-condition \( \Pi \) and \( \Phi \), and the set of variables \( \text{md} \) modified so far, an input block \( B \) should be completed into \( B' \). In this case, we say that \( B' \) is a completion for \( B \), and the intension is that \( B' \) satisfies the specification \( (\Pi) \ (B' \ (\Phi)) \) and does not modify any variables in \( \text{md} \) (i.e. \( \text{mod}(B) \cap \text{md} = \emptyset \)). We present the inference rules for this judgment in [Fig. 10](#).

**Patch Generation** The rule \texttt{Synth-Base} fires once we reach the end of the input block and performs the actual patch generation. It non-deterministically picks a patch satisfying the specification, and can only update “stale” variables, which are not modified but depend on modified variables via the specification \( \Pi \) (we formalize this dependency in [Fig. 11](#)). Our implementation realizes the non-deterministic choice via constraint-based synthesis in the space of all blocks that only contain assignments and put-statements. \texttt{Synth-Loop} is similar to \texttt{Synth-Base} but allows generating looping patches when the postcondition contains quantification.

**Accumulating Modifications** \texttt{Assign} and \texttt{Put} simply accumulate modifications made by the input block. In these rules, the variable modified by the current statement is added to \( \text{md} \), and the precondition of the subproblem is updated to reflect the result of the modification. Note that while the top-level completion problem is always symmetric (i.e. of the form \( \text{md} \vdash \langle \Pi \rangle B \langle \Pi \rangle \)), where \( \Pi \) is the data invariant we are trying to enforce, the pre- and the post-condition might become different as a result of applying \texttt{Assign} or \texttt{Put}. Sometimes these differences must be reconciled, because rules like \texttt{For-Specialize} only apply to symmetric goals. The rule \texttt{Inv} allow us to do just that: restore the invariant \( \Phi \) by inserting a patch in the middle of a block.

**Targeting** The central rule of our system is \texttt{For-Specialize}. If a data invariant and a loop have the same syntactic structure (i.e. iterate over the same collections), this rule targets the data invariant to the loop body: i.e. strips both loop and quantification from the subgoal. One complication here is the role of \texttt{prev} terms. As discussed in [Sec. 3](#) terms with \texttt{prev} cannot be used as an assumption for the body of a targeted loop. In this case, we first patch the current loop iteration into the term \( B_{\text{prev}} \), and then continue to the remainder of the loop body.
Fig. 10: Inference rules for the SPYDER algorithm, with explicit blocks.

**Alignment** Finally, a crucial necessity for the **For-Specialize** rule is that the data invariant and the loop are syntactically similar. To reach this state, the **Foreach-Extend** and **For-Extend** rules syntactically search for an alignment. Both of these rules are semantics-preserving and are performed so that the targeting rule can be applied.
Fig. 11: Inference rules for variable data-dependency relation. We relate two variables $x$ and $y$ by $\sim$ if a modification to $x$ might affect $y$.

Patching the Example  We next give a derivation for a patch for the running example, in which we extend the loop by iterating over `weeks` and introduce a maintenance `Put` to the new `weeks` iterator.

Recall that we wish the unit-conversion invariant on lines 4 and 5 to be a data invariant for the body of `adjustForCOLA`, lines 8 through 10.

In this case, the pre- and post-conditions are

```plaintext
foreach w in weeks, d in days: 7 * d.val = w.val,
```

and the block to be patched is

```plaintext
for d in days: if (d.val > 0): d <- d.val * cola;
```

First, to make the loop iterate over the same variables as the `foreach` term, we introduce a new iterator over `weeks` by applying `For-Extend`, producing the new loop `for w in weeks, d in days: ....`

Next, we target the specification to the loop by applying `For-Specialize`, which has the effect of stripping the `foreach` and `for` terms. As a consequence our new data invariant is $7 * d.val = w.val$, and our new block is `if (d.val > 0): d <- d.val * cola`.

We next apply `Conditional` to simplify the loop. The false-branch is empty and so satisfies the data invariant. We now only need to patch the true-branch.

Because the statement is a `Put`, we apply the `Put` rule, which logically embeds the effects of $d <- d.val * cola$ into the precondition, and adds $d$ to the set of modified variables $md$. At this point, we’re left with a logical specification, an empty block, and a set of modified variables with just one member, $md = \{d\}$.

Finally, we apply two rules. First, we find a maintenance patch for the data invariants by the `Synth-Base` rule. This produces a snippet $B'$ (in this case $w <- d.val * 7$;) such that if we add $B'$ at line 11, the resulting conditional (and loop) will maintain the invariant. We will discuss this further in a moment, but for now, we will produce an extension from $B'$ and the current block $d <- d.val * cola$; using the `Inv` rule.

Now we demonstrate how to find $B'$ using the `Synth-Base` rule. In this case, because $w$ and $d$ both appear in the precondition, and $d$ is in $md$ the candidate variables for a patch are $\{w, d\}$. However, since $B'$ is not allowed to modify any of the variables in $md$ (i.e. $d$), it’s forced to produce a patch that modifies $w$, which further satisfies the invariant $w.val = d.val * 7$. One such patch is $w <- d.val * 7$;, and so the `Synth-Base` rule calculates this patch for $B'$.
4.3 Soundness of Synthesis Rules

In all cases, if the SPYDER extension rules produce a new program, the program must satisfy the input data invariants. We formalize the synthesis soundness using the axiomatic semantics of Sec. 3:

\[ \forall \Pi, B, B', \emptyset \vdash (\Pi) B \langle \Pi \rangle \hookrightarrow B' \implies (\Pi) B' \langle \Pi \rangle \]

We prove this by generalizing to \( \text{md} \vdash (\Pi) B \langle \Phi \rangle \hookrightarrow B' \) and then by induction on the derivation. More detail is in Sec. A.5 and the proof is straightforward.

5 Evaluation

In this section, we detail the experiments run to evaluate SPYDER. We assessed SPYDER quantitatively via a set of benchmarks and using several case studies.

Research questions We test the following questions:

(RQ1) Is programming with SPYDER and data invariant is more succinct (and therefore easier) than maintaining data invariants manually?

(RQ2) Does SPYDER make code evolution easier? We test this by examining the necessary changes to implementation and invariants in order to implement new functionality.

(RQ3) Does TARGETED SYNTHESIS enable fast, scalable synthesis? To test this, we measure the performance of synthesizing with SPYDER.

Implementation We evaluate SPYDER and TARGETED SYNTHESIS using a prototype compiler that targets Boogie [32]. Our prototype implements the contents of Sec. 3 by compiling to Boogie, and we implement the contents of Sec. 4 by extending SPYDER terms using our own synthesis and CEGIS algorithms.

5.1 Case Studies

We first examine RQ1 and RQ2 using three detailed case studies.

The invariant language of SPYDER, targeted towards expressing relations over collections, is a perfect fit for many useful idioms in web programming. Using SPYDER, we implemented three applications inspired by real-life web programs.

Game of Life John Conway’s Game of Life [12] is a popular visualization of a cellular automaton with applications in Chemistry, Physics, Math, and Computer Science. In this game, a discrete world of cells obeys particular evolutionary behavior. At each time step of the application, the cells in the world change state according to the rules of the game. We looked at several interactive applications of the game of life online, such as [1]. In all of these applications, the programmer manually maintained an invariant between the visual cells of the board and the
internal data structure for the cells. To implement this in SPYDER, we encoded the internal state of the game and its visual state as two integer arrays. An element-wise invariant relates the internal state of the game to its visual state. We implemented procedures for 1. making transitions in the internal state according to the rules of the game, 2. interactive logic that allows the user to change the state of a cell by clicking on the board, and 3. a button for starting and stopping the game. SPYDER was able to synthesize a patch that re-synchronizes the model and the view for each of these procedures.

**Budgeting Application** Our second case study is a spreadsheet-style budgeting application, described in detail in [Sec. 1.1](#). For this benchmark, the programmer builds a financial application which takes in periodic revenues and deficits. This application takes amounts in three periodic intervals—weekly, monthly, and yearly—and converts between the amounts. In this way, the end-user can input data in the most convenient format.

A difficult feature of this benchmark was summing up the rows of the budget and presenting a total value. In traditional programming, this would require a procedure and would not be easy to compose. In contrast, in SPYDER, this invariant is easily expressible using the prev calculus and indeed composes very well with the other invariants of the system.

**Shared Expenses Application** Our final case study is an extension of the Budgeting Application. Anecdotally, one of the co-authors actually uses this type of application in real-life. The idea here is that two people who live in the same household want to split shared expenses equally at the end of the month. In this application, each row has 4 entries: in the first two cells store the expenses paid by person A and person B, respectively; in the third cell, stores the average cost for the expense (i.e. the final cost for each person), and in the fourth cell, the amount person A owes to person B (i.e. how much person B over/underpaid on the particular expense). Similar to the budgeting application, we can express each row of this application in SPYDER and further, we can conditionally render the amount owed between the participants.

### 5.2 Quantitative Evaluation

In addition to the qualitative evaluation, we empirically evaluate questions 1-3 on a series of benchmarks and compare them to two traditional techniques, manual invariant maintenance, and dynamic maintenance of functional specifications (i.e. Functional-Reactive Programming, FRP).

To compare SPYDER against these two techniques in a language-agnostic, apples-to-apples way, we implement each benchmark in all three paradigms using SPYDER’s syntax. For the imperative paradigm, we manually maintain invariants without using specifications. For the FRP paradigm, we write functional specifications for each variable in the program.
Table 1: Benchmarks comparing implementation in SPYDER to other techniques. 
Impl is the size of implementation (non-invariant) code and Spec is the size of invariants. All sizes are in AST nodes, and all implementations are in the SPYDER language. Each benchmark has invariants maintained manually (Impl), in the FRP paradigm, and with SPYDER. Synthesis times of SPYDER are given compared to Sketch (SK), on collections of size 3, 10, and 50 (SK-3, SK-10 and SK-50 resp.). We report a timeout (−) after ten minutes and we use N/A to denote a Sketch program that doesn’t use collections. Patches reports the size of patches synthesized by SPYDER and the number of patches (locs) per benchmark.

RQ1: Succinctness We measure the amount of code necessary to implement a set of benchmarks in three different techniques: manually (Imperative), FRP and SPYDER. We show the results in Tab. 1. As expected, the size of manually implemented code for both FRP and SPYDER is considerably smaller than Imperative. However, SPYDER specifications are as much as three times smaller than FRP specifications. Additionally, we see that patches are generated in a number of locations. This means manually maintaining the invariants would have required to keep track of all these locations. We do see that the size of the patches generated by SPYDER is much larger than the size of the manual implementation. There are two main reasons for this: 1) SPYDER patches are not meant for human consumption and so are unoptimized, and 2) patches are synthesized in the target language (i.e., Boogie), which is not as concise as SPYDER. The results show that SPYDER invariants provide a succinct way of specifying what would otherwise be a much larger piece of enforcement code.

RQ2: Ease of modification We measure the amount of modification required to evolve existing code, again comparing SPYDER to imperative and FRP invariant maintenance. Fig. 12a shows for each benchmark the size of the modification (in AST nodes) required to implement the new functionality portion of a new feature, without fixing broken data invariants, and Fig. 12b shows the size of the modification to invariant-preserving code.

As seen in our discussion of RQ1 we see in Fig. 12a that writing new functionality with SPYDER is more succinct than either writing it imperatively in the SPYDER language. Fig. 12b shows that the same is true for invariant maintenance: modifications to the invariants in SPYDER are considerably smaller than the manual changes in imp and the changes to code and invariants in FRP.

These benchmarks show that code evolution is also easier in SPYDER.
(a) Initial implementation effort: difference in size to implement a new feature (in AST nodes) while breaking invariants in all three techniques.

(b) Invariant maintenance effort: effort repairing broken invariants. For FRP and Spyder indicates updates to invariants and for Imperative this is manually written invariant maintenance code.

Fig. 12: Effort required to add new features to existing programs.

\textit{RQ3: Performance} We evaluated the scalability of the Spyder compiler (and by extension, the Targeted Synthesis algorithm) by compiling our benchmarks and comparing the performance against a standard synthesis technique, Sketch \cite{55}. For each of our benchmarks, we reimplemented the benchmark in Sketch and compared the performance. In contrast to Targeted Synthesis, Sketch performs bounded enumeration for verification, and as a consequence, quantified invariants scale in proportion to the size of the verified array. To measure the scalability of bounded verification, for Sketch programs with arrays we varied the number of elements in the concrete Sketch arrays from 3 elements to 50 elements.

Overall, we find that Sketch outperforms Spyder on the synthetic benchmarks but does not complete within the time limit in two of our three case studies. For the case studies, Sketch could solve these problems if the programmer wrote a synthesis template tailored to the specific study. In contrast, Spyder programmers do not have to develop an application-specific sketch.

\section{Related Work}

This paper builds upon two lines of prior work, which until now have developed independently: \textit{declarative constraint programming}, where the goal is to enforce global constraints at run time, and \textit{program synthesis and repair}, which enforces traditionally local, end-to-end functional specifications at compile time. We first discuss the trade-offs between static and dynamic constraint solving, and then we detail each of these areas.

\textit{Static and Dynamic Constraint Solving} Two of the longstanding research problems for constraint solving are performance \cite{22,17}, as well as debugging over- and under-constrained systems \cite{17,27,52,38}. In essence, the choice of static
vs dynamic constraint solving boils down to a tradeoff between issues at compile time vs issues at run time.

For performance, solving constraint statically results in (notoriously) long synthesis and compilation times, but produces fast code. Conversely, dynamic constraint solving does not require an expensive compilation pass but results in large runtime overheads, as high as 10x-100x (as reported in [17]). Consequently, the choice of static vs. dynamic for performance is a tradeoff between compilation time and runtime performance.

Debugging constraint systems is a similar story in that static systems can report a compile-time error when the system is over- or under-constrained. Conversely, dynamic systems generally attempt to resolve ill-posed systems anyway, using techniques such as constraint hierarchies [8], which results in unintuitive solutions – unintuitive because the solution does not satisfy the constraints. In either case, the ill-posed system must be debugged. In the static case, it is strictly the programmer who debugs the system, while in the dynamic case, the end user might be exposed to the ill-posed system. Consequently, the choice of static vs. dynamic for debugging is a tradeoff between programmer time and user time.

**Dynamic Invariant Enforcement** There are two closely related research arcs on dynamically enforcing invariants: the field of constraint imperative programming, and the work of functional reactive programming. Both of these areas provide mechanisms for dynamically solving invariants, and both are orthogonal to our efforts because we solve constraints statically through program synthesis.

**Constraint Imperative Programming.** The field of constraint solving is rich and storied [3,15], as constraint solvers excel at calculating global solutions. Despite their power, constraint solvers are traditionally relegated to libraries. The field of Constraint Imperative Programming aims to provide first-class language support for constraint solving [23,17,43], but again, fundamentally our work is orthogonal because we solve constraints statically.

**Functional Reactive Programming** The field of Functional Reactive Programming (FRP) provide a dataflow language for building graphical systems [60]. Although inspired by animations, FRP quickly became popular as a tool for taming web application logic [39,13]. The most popular recent work in this field are Elm [14] and its imperative cousin React [57], which provide a language and runtime for building client-side web applications. Although popular and powerful, FRP is a general, dynamic technique for abstracting over dataflow – in contrast, our work focuses on the problem of first-class data invariants, and solves for invariant patches statically.

**Program Synthesis and Repair** In recent years, *program synthesis* has emerged as a promising technique for automating tedious and error-prone aspects of programming [24,51,59]. The two main directions in this area are synthesis from informal descriptions (such as examples, natural language, or hints) [24,45,19,13,18,61,42,10] and synthesis from formal specifications, where the goal is to synthesize a program
that is provably correct relative to the specification. Both directions have been mainly focusing on synthesizing standalone programs from complete end-to-end functional specifications of their inputs and outputs. Instead, the present work focuses on synthesizing code snippets from incomplete, global specifications (data invariants) and integrating them with hand-written code.

The only prior techniques we are aware of for generating snippets from declarative specifications and inserting them into hand-written code is the in the context of information-flow security. Enforcement of data invariants brings a different set of challenge, since invariants are deep semantic properties.

Our work is related to sound program repair, where the problem is, given a formal specification and a program that violates it, modify the program so that it provably satisfies the specification. Program repair, however, is a very general problem, and so lacks a-priori restrictions on modifications the algorithm is allowed to make. As a result, if the given specification is incomplete, the problem is ill-defined. In this work we show that in the setting of enforcing data invariants, the space of possible modifications can be sufficiently restricted to make repair both predictable and efficient. Where efficiency is concerned, the deductive program repair technique of does not scale with the number of patches generated in one function, whereas Spyder leverages the restrictions to solve each synthesis task independently, hence avoiding a combinatorial explosion with the number of patches.

**Program Verification**

The programming and invariant language of Spyder is purposefully simple, allowing us to explore the idea of automatic invariant maintenance without getting distracted by challenges of program verification in the presence of aliasing, dynamic object structures, and arbitrary quantified invariants. There is a rich body of prior work in program verification that deals with these challenges, both in general and in the specific context of object invariants. Extending Targeted Synthesis to support one of these verification methodologies is an interesting direction for future work, but we consider it orthogonal to the initial exploration of programming with invariants.

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Appendix

A.1 ImpArray Syntax and Semantics

A.2 Spyder Semantics

Let $\sigma$ be a IMP-ARRAY state, $E$ a SPYDER Expression, and $\Gamma$ a well-formed translation context with respect to $E$. We define the denotational semantics of $E$ as the denotational semantics of the corresponding IMP-ARRAY expression:
\( v, u \in \text{Vars}, \quad i \in \mathbb{Z} \)

\[
\text{Stmt} ::= v := \text{Expr} \\
| \text{if} \text{Expr} \text{then} \text{Stmt} \text{else} \text{Stmt} \\
| \text{while} \text{Expr} \text{Stmt} \\
| \text{Stmt} ; \text{Stmt} \\
| \text{skip}
\]

\[
\text{Expr} ::= v \mid i \mid \text{true} \mid \text{false} \\
| \text{Expr} \text{bop} \text{Expr} \\
| \text{uop} \text{Expr} \\
| \text{Expr} [\text{Expr}] \\
| \text{size}(v) \\
| \forall v \cdot \text{Expr} \\
| \exists v \cdot \text{Expr}
\]

\[
bop ::= + \mid \times \mid \% \mid \Rightarrow \mid \Leftarrow \mid \ldots
\]

\[
uop ::= \neg \mid !
\]

Fig. 13: Syntax for the Imp-Array language.

\[
[v]_\sigma = \sigma[v] \\
[i]_\sigma = i \\
[\text{true}]_\sigma = \top \\
[\text{false}]_\sigma = \bot \\
[E_1 \text{bop } E_2]_\sigma = [E_1]_\sigma \text{bop } [E_2]_\sigma \\
[uop E]_\sigma = uop[E]_\sigma \\
[\forall v \cdot E]_\sigma = \forall x \in \sigma. [E[v \mapsto x]]_\sigma = \top
\]

Fig. 14: Denotational semantics for Imp-Array expressions

**Definition 1 (Spyder Expression Semantics).**

\[
[E]_\sigma ::= [\text{trans}(E, \Gamma)]_\sigma
\]

We similarly define the operational semantics of a Spyder statement \( S \) as the operational semantics of the corresponding Imp-Array statement \( \text{trans}(S, \Gamma) \):

**Definition 2 (Spyder Statement Semantics).**

\[
\Gamma \vdash S, \sigma \leadsto \sigma'
\]

A.3 Soundness of Spyder Triples

**Lemma 1 (Bindings).**

\[
\forall P, B, \Gamma . \text{wf}(\text{for}(x, y)B, \Gamma) \land \text{wf}(P, \Gamma) \implies x \notin \text{free}(P)
\]
\[ v := \text{Expr}, \sigma \mapsto [\text{Expr}]_{\sigma} \]

\[ v[\text{Expr}_1] := \text{Expr}_2, \sigma \mapsto [v[\text{Expr}_1]]_{\sigma} \mapsto [\text{Expr}_2]_{\sigma} \]

\[ [\text{Expr}]_{\sigma} = \top \]

\[ \text{Stmt}_1, \sigma \mapsto \sigma' \]

\[ \text{Stmt} \]

\[ \text{while} \text{Expr} \text{Stmt}, \sigma \mapsto \sigma' \]

\[ [\text{Expr}]_{\sigma} = \bot \]

\[ \text{Stmt} ; \text{while} \text{Expr} \text{Stmt}, \sigma \mapsto \sigma' \]

\[ \text{Stmt}_1, \sigma \mapsto \sigma' \text{Stmt}_2, \sigma' \mapsto \sigma'' \]

\[ \text{Stmt}_1 ; \text{Stmt}_2, \sigma \mapsto \sigma'' \]

\[ \text{skip}, \sigma \mapsto \sigma \]

Fig. 15: Operational semantics for IMP-ARRAY statements

Proof. Induction over the derivation of \( \text{wf}(P, \Gamma) \).

Lemma 2 (Assignment).

\[ \forall B, \Gamma. \text{wf}(\text{for}(x, y)B, \Gamma) \implies x \notin \text{assign}(B) \]

Proof. Induction over the derivation of \( \text{wf}(\text{for}(x, y)B, \Gamma) \).

Lemma 3 (Array substitution).

\[ \forall P, x, y, \Gamma. \text{wf}(P, \Gamma) \land \Gamma(x) = y \implies \forall \sigma. \sigma(\text{trans}(P, \Gamma)[y \mapsto y']) \implies \sigma(\text{trans}(P, \Gamma)[y[x] \mapsto x']) \]

Proof. Structural induction over \( P \).

Theorem 3 (Relative Soundness).

\[ \forall P, S, Q, \Gamma. \text{wf}(P \land Q, \Gamma) \land \text{wf}(S, \Gamma) \implies \langle P \rangle S \langle Q \rangle \implies \{\text{trans}(P, \Gamma)\} \text{trans}(S, \Gamma) \{\text{trans}(Q, \Gamma)\} \]

Proof. By induction over the derivation of \( \langle P \rangle S \langle Q \rangle \); for each case of \( S \), we build a corresponding derivation for \( \{\text{trans}(P, \Gamma)\} \text{trans}(S, \Gamma) \{\text{trans}(Q, \Gamma)\} \).

In all cases we start by assuming \( \text{wf}(P \land Q, \Gamma) \land \text{wf}(S, \Gamma) \).

Cases of \( S \):
\[ P \Rightarrow P' \quad Q' \Rightarrow Q \]

### Consequence

\[
\begin{array}{c}
\{P'\} S \{Q'\} \\
\{P\} S \{Q\}
\end{array}
\]

### Skip

\[
\begin{array}{c}
\{P\} \text{skip} \{P\}
\end{array}
\]

### Sequence

\[
\begin{array}{c}
\{P\} S_1 \{Q\} \\
\{Q\} S_2 \{R\}
\end{array}
\]

### Conditional

\[
\begin{array}{c}
\{P \land e\} S_t \{Q\} \\
\{P \land \neg e\} S_f \{Q\}
\end{array}
\]

### Assign-Var

\[
\begin{array}{c}
(fresh\ v') \\
\{P\} v := E \{\exists v'. P[v \mapsto v'] \land v = E[v \mapsto v']\}
\end{array}
\]

### Assign-Array

\[
\begin{array}{c}
(fresh\ v') \\
\{P\} v[E_i] := E_r \{\exists v'. P[v \mapsto v'] \land v = v'[E_i[v \mapsto v'] := E_r[v \mapsto v']]\}
\end{array}
\]

### While

\[
\begin{array}{c}
(I \land E) S \{I\} \\
(I) \text{while} E S \{I \land \neg E\}
\end{array}
\]

Fig. 16: Standard axiomatic semantics (Hoare logic) for Imp-Array.

1. **Base case**, in which the last step of the derivation is \(\langle P \rangle \text{skip} \{Q\}\). From the structure of \(\text{skip}\), it must be the case that \(P\) and \(Q\) are structurally identical, i.e. the derivation is \(\langle P \rangle \text{skip} \{P\}\). Since \(\text{trans}\) is a function, it maps \(\text{skip}\) to exactly one statement (namely \(\text{skip}\)), and \(P\) to exactly one expression \(\text{trans}(P, \Gamma)\). Finally, we apply the \(\text{Skip}\) Hoare rule to obtain \(\{\text{trans}(P, \Gamma)\} \text{skip} \{\text{trans}(P, \Gamma)\}\).

2. **Inductive case**, in which the last step of the derivation is \(\text{Consequence}: \langle P \rangle S \{Q\}\). We will use the corresponding \(\text{Consequence}\) rule of Hoare logic to build a derivation for \(\{\text{trans}(P, \Gamma)\} \text{trans}(S, \Gamma) \{\text{trans}(Q, \Gamma)\}\).

Since the case is \(\text{Consequence}\), there must be \(P'\) and \(Q'\) such that \(P \Rightarrow P', \quad Q' \Rightarrow Q,\) and \(\langle P'\rangle S \{Q'\}\). From \(\text{??}\), we know that \(\text{trans}(P, \Gamma) \Rightarrow \text{trans}(P', \Gamma)\) and \(\text{trans}(Q', \Gamma) \Rightarrow \text{trans}(Q, \Gamma)\). From the inductive hypothesis, we have the ImpArray triple

\[
\{\text{trans}(P', \Gamma)\} \text{trans}(S, \Gamma) \{\text{trans}(Q', \Gamma)\},
\]

and so we apply the \(\text{Consequence}\) ImpArray rule to obtain

\[
\{\text{trans}(P, \Gamma)\} \text{trans}(S, \Gamma) \{\text{trans}(Q, \Gamma)\}.
\]

3. **Inductive case**, in which the last step of the derivation is \(\text{Conditional}: \langle P \rangle \text{if } E \text{ then } S_t \text{ else } S_f \{Q\}\). This follows from the inductive hypothesis applied to \(E, S_t, \) and \(S_f\), as well as the \(\text{Conditional}\) ImpArray rule.
4. Inductive case, in which the last step of the derivation is **Sequence**: \( \langle P \rangle S_1 \ ; \ S_2 \ (R) \).
This follows from the inductive hypothesis applied to \( S_1 \), and \( S_2 \), as well as the **Sequence** ImpArray rule.

5. Inductive case, in which the last step of the derivation is **Assign**: \( \langle P \rangle \ v := E \ (Q) \).
In this case, the translation produces a Imp assignment to \( v \).
Since the last step is **Assign**, there must be a fresh variable \( v' \) such \( Q \) is the strongest postcondition of the assignment to \( v \):
\[
\exists v'. P[v \mapsto v'] \land v = E[v \mapsto v']
\]
From the inductive hypothesis, we know that translating the Spyder triple produces an equivalent Imp Hoare triple
\[
\{\text{trans}(P, \Gamma)\} v := \text{trans}(E, \Gamma) \{\exists v'. P[v \mapsto v'] \land v = E[v \mapsto v'], \Gamma\}.
\]
If you consider the translated term \( \text{trans}(\exists v'. P[v \mapsto v'] \land v = E[v \mapsto v'], \Gamma) \), using ?? and the definition of translation, you’ll find that it is exactly the ImpArray postcondition for assignment with \( \text{trans}(P, \Gamma) \) as a precondition:
\[
\exists v'. \text{trans}(P, \Gamma)[v \mapsto v'] \land v = \text{trans}(E, \Gamma)[v \mapsto v'].
\]
So, we apply **Assign** with \( P \) as a precondition to obtain
\[
\{\text{trans}(P, \Gamma)\} v := \text{trans}(E, \Gamma) \{\exists v'. \text{trans}(P, \Gamma)[v \mapsto v'] \land v = \text{trans}(E, \Gamma)[v \mapsto v']\},
\]
6. Inductive case, in which the last step of the derivation is **Put**: \( \langle P \rangle \ v \leftarrow E \ (Q) \).
For this, we will show that the translation of the put \( v \leftarrow E \) takes the precondition \( \text{trans}(P, \Gamma) \) to the translation of the Spyder post-condition \( \exists v'. \text{weaken}_\text{foreach}(P, v, \Gamma)[\text{val}(v) \mapsto v'] \land \text{val}(v) = E[\text{val}(v) \mapsto v'] \).
Consider the translation of \( \text{val}(v) \) in the context of \( \Gamma \). Since \( \Gamma \) is well-formed with respect to the Put to \( v \), it must be the case that \( v \in \Gamma \) and \( \Gamma(v) = y \) for some variable \( y \). Furthermore, the Spyder expressions \( \text{val}(v) \) and \( \text{iter}(v) \) are translated to \( y[v] \) and \( v \) respectively.
Next, consider the Hoare postcondition of the translated put statement. The **Put** statement is translated to \( y[v] := \text{trans}(E, \Gamma) \), and we can apply the **Assign-Array** rule to obtain the postcondition of \( \text{trans}(P, \Gamma) \):
\[
\{\text{trans}(P, \Gamma)\} y[v] := \text{trans}(E, \Gamma) \{\exists y'. \text{trans}(P, \Gamma)[y \mapsto y'] \land y = y'[v := \text{trans}(E, \Gamma)[y \mapsto y']\}],
\]
where \( y' \) is some fresh variable.
Because the case is **Put**, we have just derived the Spyder triple
\[
\langle P \rangle v \leftarrow E \ (\exists y'. P[\text{val}(v) \mapsto v'] \land \text{val}(v) = E[\text{val}(v) \mapsto v']),
\]
where \( v' \) is some free variable.
Let \( \sigma \) be an ImpArray state such that
\[
\llbracket \exists y'. \text{trans}(P, \Gamma)[y \mapsto y'] \land y = y'[v := \text{trans}(E, \Gamma)[y \mapsto y']] \rrbracket_\sigma = t
\].

Consider the Hoare term \( P' \), \( \exists v'. \text{trans}(P)[\text{val}(v) \mapsto v'] \land \text{val}(v) = E[\text{val}(v) \mapsto v'] \), or equivalently,
\[
\exists v'. \text{trans}(P, \Gamma)[y[v] \mapsto v'] \land y[v] = \text{trans}(E, \Gamma)[y[v] \mapsto v']
\].

We claim that \( J P' K \sigma = t \). Since \( P \) is well-formed with respect to \( \Gamma \), and \( \Gamma(x) = y \), it must be the case that the substitution of \( y \mapsto y' \) only affects translations of \( \text{val}(v) \). As a result, if \( y' \) is an (array) witness for \( J \exists y'. \text{trans}(P, \Gamma)[y \mapsto y'] \land y = y'[v := \text{trans}(E, \Gamma)[y \mapsto y']][v] = t \), we can use the value \( y[v] \) as a (variable) witness for \( P' \).

Since \( J P' K \sigma = t \), we can apply Consequence to obtain the triple
\[
\{ \text{trans}(P, \Gamma) \} \text{trans}(v \leftarrow E, \Gamma) \{ P' \}.
\]

7. Inductive case, in which the last step of the derivation is For: \( \langle P \rangle \) for \( (x, y)B_i \langle P \rangle \), where \( P \) is of the form \( \text{foreach}(x, y)P_i \).

At a high-level, this rule is introducing a quantification over the elements of \( y \). This is sound because the body \( B \) can only adjust the elements at the current iteration, because the loop cannot modify variables captured in \( I \), and because the translated loop is guaranteed to execute exactly once for every element of \( y \).

Let \( \Gamma' \) be \( \Gamma \) extended with the loop binding \( x \mapsto y \). Since \( \Gamma \) is well-formed with respect to the loop, it must be the case that \( \Gamma' \) is well-formed as well.

Recall that the translated loop is
\[
x := 0; \text{while}(x < \text{size}(y)) \text{trans}(B_i, \Gamma') ; x := x + 1.
\]

Consider the translated \( \text{foreach} \) predicate \( I \)
\[
\forall x'. 0 \leq x' < \text{size}(y) \implies \text{trans}(P_i, \Gamma')[x \mapsto x'].
\]

We will use the \( \text{While} \) rule with three helper predicates: intuitively, we keep three predicates around to 1) quantify \( I \) for previous iterations 2) safely weaken \( I \) for the current iteration and 3) quantify \( I \) for future iterations. Let \( I_{\text{pre}} \) restrict \( I \) up to the current iteration,
\[
\forall x'. 0 \leq x' < x \implies \text{trans}(P_i, \Gamma')[x \mapsto x'].
\]

Let \( I_{\text{post}} \) weaken \( I \) using \( \text{weaken}_{\text{prev}}(P_i) \) for future iterations:
\[
\forall x'. x < x' < \text{size}(y) \implies \text{trans}(\text{weaken}_{\text{prev}}(P_i), \Gamma')[x \mapsto x'].
\]
Finally, let $I_{\text{curr}}$ be the weakening of $I$ for the current iteration: $\text{trans}(\text{weaken}_{\text{prev}}(P_i), \Gamma')$. We will use the While rule with the combined predicate $I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}}$ as the loop invariant, and in particular, we will show the following Hoare triple holds:

$$\{I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}} \land 0 \leq x < \text{size}(y)\} \text{trans}(B_i, \Gamma') \; ; \; x := x + 1 \{I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}}\}.$$  

From the inductive hypothesis we have the triple

$$\{I_{\text{curr}} \land 0 \leq x < \text{size}(y)\} \text{trans}(B_i, \Gamma') \{\text{trans}(P_i, \Gamma')\}.$$  

Since $I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}} \implies I_{\text{curr}}$, we apply Consequence on the precondition to obtain

$$\{I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}} \land 0 \leq x < \text{size}(y)\} \text{trans}(B_i, \Gamma') \{\text{trans}(P_i, \Gamma')\}.$$  

From Lemma 2, since the loop with $B_i$ is well-formed with respect to $\Gamma'$, it must be the case that $x$ is not assigned within $B_i$. As well, since $B_i$ is restricted from writing to free variables of $B_i$, the only way for $I_{\text{pre}}$ and $I_{\text{post}}$ to be invalidated by $\text{trans}(B_i, \Gamma')$ is through a tt Put. Since Spyder does not have aliasing, each Put within $B_i$ with $x$ as a target only writes to the current iteration (i.e. each Put only invalidates $I_{\text{curr}}$). Furthermore, since $B_i$ does not have nested loops over $y$, $x$ is the only possible target to write to $y$, and so it must be the case that $I_{\text{pre}}$ and $I_{\text{post}}$ are not invalidated by $\text{trans}(B_i, \Gamma')$.

As a result, we can safely strengthen the postcondition of this triple with $I_{\text{pre}}$ and $I_{\text{post}}$:

$$\{I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}} \land 0 \leq x < \text{size}(y)\} \text{trans}(B_i, \Gamma') \{I_{\text{pre}} \land I_{\text{post}} \land \text{trans}(P_i, \Gamma')\}.$$  

Finally, consider the increment of $x$ after the loop. Given the precondition $I_{\text{pre}} \land I_{\text{post}} \land \text{trans}(P_i, \Gamma')$, we apply the Assign rule to obtain

$$\{I_{\text{pre}} \land I_{\text{post}} \land \text{trans}(P_i, \Gamma')\} x := x + 1 \{\exists \mathbf{v}. (I_{\text{pre}} \land I_{\text{post}} \land \text{trans}(P_i, \Gamma'))[x \mapsto \mathbf{v}] \land x = \mathbf{v} + 1\},$$  

where $\mathbf{v}$ is a fresh variable. This postcondition is logically equivalent to $I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}}$, and so we apply Consequence and Sequence to obtain

$$\{I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}} \land 0 \leq x < \text{size}(y)\} \text{trans}(B_i, \Gamma') ; x := x + 1 \{I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}}\}.$$  

Finally, we apply While with the condition $0 \leq x < \text{size}(y)$ to obtain the triple.
\{I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}}\} \text{ while } (0 \leq x < \text{size}(y)) \text{ trans}(B_i, \Gamma') \\
\{I_{\text{pre}} \land I_{\text{post}} \land I_{\text{curr}} \land x = \text{size}(y)\}.

From here, it remains to use Consequence, and Sequence to build a triple for the loop initialization.

A.4 Soundness of Targeted Synthesis

Lemma 4 (Block Append).

\[ \forall B, B', P, Q, R. (\langle P \rangle B \langle Q \rangle \land \langle Q \rangle B' \langle R \rangle) \implies (P) B \mp B' \langle R \rangle. \]

Proof. By structural induction over the arguments of \( \mp \).

Theorem 4.

\[ \forall \Pi, \Phi, B, B'. \text{cn}; \text{md} \vdash (\Pi) B \langle \Phi \rangle \hookrightarrow B' \implies (\Pi) B' \langle \Phi \rangle \]

Proof. Induction over the derivation of \( \text{cn}; \text{md} \vdash (\Pi) B \langle \Phi \rangle \hookrightarrow B' \). In all cases we show that \( (\Pi) B' \langle \Phi \rangle \).

1. Base case, in which the last step is Synth-Base: \( \text{cn} ; \text{md} \vdash (\Pi) \text{skip} \langle \Phi \rangle \hookrightarrow B \) Because a side-condition for Synth-Base is \( (\Pi) B \langle \Phi \rangle \), this is trivially true.

2. Base case, in which the last step is Synth-Loop: \( \text{cn} ; \text{md} \vdash (\Pi) \text{skip} (\text{foreach}(v_i, u_i) \phi \land \Phi) \hookrightarrow B \). This is true from the inductive hypothesis.

3. Recursive case, in which the last step is Consequence: \( \text{cn} ; \text{md} \vdash (\Pi) B \langle \Phi \rangle \hookrightarrow B' + B'' \). From Lemma 4 and the inductive hypothesis, it is the case that \( (\Pi) B' + B'' \langle \Phi \rangle \).

4. Recursive case, in which the last step is Assign: \( \text{cn} ; \text{md} \vdash (\Pi) v := E ; B \langle \Phi \rangle \hookrightarrow v := E ; B' \). We apply the Hoare rule for Assign to the inductive hypothesis.

5. Recursive case, in which the last step is Put: \( \text{cn} ; \text{md} \vdash (\Pi) v \leftarrow E ; B \langle \Phi \rangle \hookrightarrow v \leftarrow E ; B' \). This is analogous to Assign.

6. Recursive case, in which the last step is one of the Extension rules. These are all trivially sound from the inductive hypothesis.

7. Recursive case, in which the last step is Foreach-Specialize:

\[ \{\} \vdash (\text{foreach}(v_i, u_i) \phi \land \Phi) \text{ for } (x_i, y_i)B_i ; B (\text{foreach}(v_i, u_i) \phi \land \Phi) \leftrightarrow \\
\text{for}(x_i, y_i)(B_{\text{pre}} + B'_i) ; B'. \]

In this case, we use the inductive hypothesis to establish the triple for \( B'_i \). Next, we use the inductive hypothesis and Lemma 4 to establish the triple for \( B'_i \) and \( B_{\text{pre}} \):

\( (\text{weaken}_\text{prev}(\Phi)) (B_{\text{pre}} + B'_i) \langle \Phi \rangle \).
On this, we apply the For Hoare logic rule to introduce the foreach term, and we appeal to the inductive hypothesis for the remainder $B'$.

8. Recursive case, in which the last step is Conditional: $\{\} \vdash \langle \theta \rangle$ if $E$ then $B_i$ else $B_f$; $B \langle \theta \rangle \Rightarrow$ if $E$ then $B_i'$ else $B_f'$; $B'$. This follows from the inductive hypothesis and the Conditional Hoare rule.

A.5 Soundness of Targetted Synthesis

Theorem 5.

$$\forall \Pi, \Phi, B, B'. \text{cn}; \text{md} \vdash \langle \Pi \rangle B \langle \Phi \rangle \Rightarrow B' \implies \langle \Pi \rangle B' \langle \Phi \rangle$$

Proof. Induction over the derivation of $\text{cn}; \text{md} \vdash \langle \Pi \rangle B \langle \Phi \rangle \Rightarrow B'$. In all cases we show that $\langle \Pi \rangle B' \langle \Phi \rangle$.

1. Base case, in which the last step is Synth-Base: $\text{cn}; \text{md} \vdash \langle \Pi \rangle \text{skip} \langle \Phi \rangle \Rightarrow B$. Because a side-condition for Synth-Base is $\langle \Pi \rangle B \langle \Phi \rangle$, this is trivially true.

2. Base case, in which the last step is Synth-Loop: $\text{cn}; \text{md} \vdash \langle \Pi \rangle \text{skip} \langle \text{foreach}(v_i, u_i) \phi \land \Phi \rangle \Rightarrow B$. This is true from the inductive hypothesis.

3. Recursive case, in which the last step is Consequence: $\text{cn}; \text{md} \vdash \langle \Pi \rangle B \langle \Phi \rangle \Rightarrow B' + B''$. From Lemma 4 and the inductive hypothesis, it is the case that $\langle \Pi \rangle B' + B'' \langle \Phi \rangle$.

4. Recursive case, in which the last step is Assign: $\text{cn}; \text{md} \vdash \langle \Pi \rangle v := E ; B \langle \Phi \rangle \Rightarrow v := E ; B'$. We apply the Hoare rule for Assign to the inductive hypothesis.

5. Recursive case, in which the last step is Put: $\text{cn}; \text{md} \vdash \langle \Pi \rangle v \leftarrow E ; B \langle \Phi \rangle \Rightarrow v \leftarrow E ; B'$. This is analogous to Assign.

6. Recursive case, in which the last step is one of the Extension rules. These are all trivially sound from the inductive hypothesis.

7. Recursive case, in which the last step is Foreach-Specialize:

$$\{\} \vdash \langle \text{foreach}(v_i, u_i) \phi \land \Phi \rangle \text{ for } (x, y_i) B_i ; B \langle \text{foreach}(v_i, u_i) \phi \land \Phi \rangle \Leftarrow \text{for } (x_i, y_i) (B_{\text{pre}} + B_i') \odot B'. \text{pre}$$

In this case, we use the inductive hypothesis to establish the triple for $B_i'$. Next, we use the inductive hypothesis and Lemma 4 to establish the triple for $B_i'$ and $B_{\text{pre}}$:

$$\langle \text{weaken } \text{prev}(\Phi) \rangle (B_{\text{pre}} + B_i') \langle \Phi \rangle.$$