Article
Prediction of Vibration-Mode-Induced Noise of Structure–Acoustic Coupled Systems

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Abstract: The exposure of a structure to an acoustic domain induces a sound field owing to the interaction of the air-fluid and structure at the acoustic–structure boundaries. It is difficult to predict sound pressure level through vibration mode, due to the acoustic mode of the coupling effect between vibration and sound in addition to the acoustic mode induced by vibration mode generated by external force. In this study, the acoustic mode induced by structural vibration modes were predicted through a numerical analysis. A finite element model of a reverberation chamber with a shell at one side was constructed, and modal parameters of the vibration and acoustic modes were evaluated through an eigenvalue analysis. In addition, the sound pressure generated by impact loading of the shell were predicted by vibration mode through a time-domain structure–acoustic coupling analysis. The vibration and acoustic modal responses were identified from the measured responses, and the acoustic mode associated with a specific vibration mode was examined. The results showed that the acoustic mode from the coupling effect was verified, and sound pressure prediction from vibration mode was possible if considered as the coupling effect. The proposed approach can be applied to predict the heavy-weight floor impact sound from the vibration of slabs in apartments.

Keywords: acoustic mode; structural vibration mode; finite element model; time domain structure–acoustic coupling analysis; prediction of acoustic mode; heavy weight floor impact sound

1. Introduction

The floor impact sound generated in apartments considerably affects the comfort of residents, as indicated by research across Korea, Japan, and Europe, among other countries [1–3]. Particularly, most Korean people live in apartments built with concrete and their indoor lifestyle is that they live in bare feet. This lifestyle and structure showed that with the amplification of impact sounds, which had low frequencies, and the reduction of impact sound, it was made more difficult because of the propagation characteristics of low frequency [4–9]. Specifically, these sounds can adversely influence the physical and psychological health of the residents and lead to property-related issues. Although many researchers have examined floor impact sounds [4–9] and the corresponding influence of floor structures of public residences [10–13] using several evaluation indices [14–18], the problem of floor impact sounds remains to be solved. In this context, it is necessary to accurately predict floor impact sounds, especially in the Korean environment, as a relevant law named floor impact sound post-check system, and is expected to be enforced in July 2022.

Several researchers have attempted to predict the floor impact noise using the impedance methods and finite element method (FEM) [19], finite difference time domain (FDTD) [20–24], and statistical energy analysis method (SEA) [25]; however, the accuracy of these methods is limited. In general, the impact generated in a source room is transmitted through a slab in the form of vibrations and converted to sound on the ceiling surface. According to Kim et al. [26], the noise with high frequencies can be represented by the vibration
mode. Furthermore, the maximum sound level of heavy impact sound pertains to the acoustic mode, which is influenced by vibration transferred through a structure or wall, depending on the frequencies [27]. Therefore, to predict the sound level in a room, the relationship between the vibration and sound must be examined. In practice, structural vibrations generated a unique sound field in the acoustic domain according to the sound environment and structural characteristics. The sound field induced by the interaction between the vibration and fluid at the acoustic–structure boundary depends on the level of coupling; each coupling is expected to generate a sound field that is different from the corresponding individual vibration and sound fields. The heavy weight floor impact sound encountered in apartment buildings is representative of the structure-induced sound pressure often observed in structure–acoustic physical spaces. Since this sound pressure is generated by the vibration of a structure (slab), the two physical quantities are expected to be strongly correlated.

In an acoustic domain with structure and sound coupling, the prediction of structural-vibration-induced sound pressure or noise is expected to involve uncertainties owing to the variables in the sound environment (e.g., size and shape of the structure and acoustic boundary conditions). Nevertheless, the correlation between the two physical quantities is widely investigated owing to the ease of vibration measurement, which can facilitate comprehensive noise control by the vibration magnitude and realization of noise alarm functions. Moreover, as mentioned previously, the introduction of the floor impact sound post-verification system has necessitated the development of a method to predict the noise in a room caused by floor vibrations by estimating the floor impact sound in the design stage and to establish noise-reduction measures.

Certain researchers attempted to predict the noise by measuring the slab vibration to examine the correlation between the slab vibration and generated noise levels [28,29]. The light-weight floor impact sounds exhibited an extremely high correlation coefficient (approximately 0.95). In contrast, the correlation coefficient for heavy-weight floor impact sound varied between 0.46 and 0.9, which indicated that the use of the acceleration level to evaluate the noise between floors was associated with a limited accuracy. Other researchers used the relationship between the vibration and acoustic modes to increase the correlation between the vibration and heavy-weight floor impact sound [30]. The correlation between the vibration and sound pressure levels in a free field was extremely high (0.91), but that in a structure–acoustic combined space varied owing to the uncertainties in the modal response contribution levels and modal characteristics of the sound environment containing the structure. Several studies were conducted to examine the structural vibration and sound. For example, Bessac et al. [31] used the eigenvalue method to analyze a coupled system, and Desmet et al. [32] used the Trefftz approach to predict the wave propagation in a coupled system. FEM and other methods were also used to examine the influence of vibration–acoustic coupling on structures [33–36]. Notably, although these studies clarified the correlation between the structural vibration and acoustic domain noise, a quantitative causal relationship between the two physical quantities remains to be presented.

Considering these aspects, this study was aimed at examining the mechanism by which structural vibration was induced as noise from a structure–acoustic coupled system. Moreover, the influence of the vibration and acoustic modal parameters, which reflect the sound environment, on the formation of the acoustic mode in a receiving room was evaluated to predict the noise in the receiving room from the structural vibration mode. A time-domain numerical analysis was performed to assess the plausibility of the structural vibration–acoustic modal response predictions. In general, compared to a frequency-domain analysis, which could be performed to examine a stationary sound field, a time-domain analysis was more suitable for investigating the vibration induced by an impact load or the characteristics of the sound pressure induced by such vibrations (nonstationary signals, such as heavy-weight floor impact sound). Additionally, since raw data of the time history could be incorporated in the time-domain analysis, the maximum responses such as the maximum vibration and sound pressure against the impact load could be
determined. A secondary analysis, such as a 1/3 octave band spectrum analysis of the

time zone at the point of maximum response occurrence, can be performed to
determine the maximum acceleration level and sound pressure level. In this study,
the numerical analysis was performed on a reverberation chamber with a shell
structure on one side. A reverberation chamber with this configuration was
considered because it can provide ideal sound boundary conditions, enabling
clear observation of the relationship between the vibration and noise. The shell
structure was simulated to allow the induction of vibration by the impact load,
leading to the generation of heavy-weight impact sound in the reverberation
chamber. The time histories of the vibration and sound pressure at
the measurement locations on the shell and in the reverberation chamber were
extracted, and the vibration and acoustic modes were identified by applying
mode decomposition to validate the noise-induced acoustic mode formation
characteristics and proposed noise prediction framework.

2. Induction Characteristics of Acoustic Modes

2.1. Structure–Acoustic Coupling System

The acoustic domain coupled with a structure can be expressed as a structure–acoustic
coupled system by using an FEM model involving sound boundary conditions [37–39].

\[
\begin{align*}
\ddot{z} + c \dot{z} + k z + W^T p &= E \delta(t) \\
-\rho_a \ddot{\tilde{z}} + M_a \dot{\tilde{p}} + C_a \tilde{p} + K_a p &= 0
\end{align*}
\]  

\( (1) \)

where \( z \) and \( p \) represent the structural displacement and sound pressure, respectively; 
\( m, c, \) and \( k \) are the structural mass, damping, and stiffness matrices, respectively; 
\( W \) is the boundary matrix of the structure and acoustic domain; and \( E \) is the location matrix
pertaining to the impact load (\( \delta(t) \)). In addition, \( \rho_a, M_a, C_a, \) and \( K_a \)
represent the air density and mass, damping, and stiffness matrices of the acoustic domain, respectively.

Using a modal matrix and modal responses with the structure and acoustic domain in
an independent state, the structural displacement and sound pressure can be expressed
as follows:

\[
\begin{align*}
z &= \Phi \eta \\
\ddot{p} &= \Psi \dot{q}
\end{align*}
\]  

\( (2) \)

where \( \eta \) and \( \dot{q} \) are the structure and acoustic modes, respectively, and \( \Phi \) and \( \Psi \)
are the corresponding modal matrices. By substituting Equation (2) into Equation (1) and using
the component mode synthesis technique, the system combining the vibration and acoustic
modes can be expressed as follows:

\[
\begin{align*}
\ddot{\eta} + \sum_s \ddot{\xi}_s \eta + \Omega_s \eta + \Phi^T W \Psi \dot{q} &= \Phi^T E \delta(t) \\
-\rho_a \ddot{\tilde{q}} &+ \sum_a \ddot{\xi}_a \dot{q} + \Omega_a q = 0
\end{align*}
\]  

\( (3) \)

The mass matrix of the structure and acoustic domain is normalized as a unit matrix
\( (I) \) for each of the mode shapes.

\[
\begin{align*}
\Phi^T M \Phi &= I \\
\Psi^T M_a \Psi &= I
\end{align*}
\]  

\( (4) \)

The variables used in Equation (3) can be defined as follows:

\[
\begin{align*}
\sum_s &= \text{diag}[2 \ddot{\xi}_s \omega_s], \quad \Omega_s = \text{diag}[\omega_s^2] \\
\sum_a &= \text{diag}[2 \ddot{\xi}_a \omega_a], \quad \Omega_a = \text{diag}[\omega_a^2]
\end{align*}
\]
where \( \text{diag}[\cdot] \) represents a diagonal matrix; subscripts \( s \) and \( a \) represent the structural vibration and acoustic modes, respectively; and \( \omega_s, \omega_a, \xi_s, \xi_a \) denote the natural frequency and damping ratio, respectively.

Both sides of \( W \), which represents the boundary between the structure and acoustic domains, are multiplied by the mode shape to represent the coupling effect of the vibration and acoustic modes. The coupling matrix is defined as follows:

\[
\gamma = \Psi^T W \Phi
\]  (5)

The size of the coupling matrix \( \gamma \) depends on the disparity of the acoustic domain and structural model in the FEM analysis. Although the acoustic mode is influenced by all vibration modes, the influence is minimal when the natural frequencies of the two modes are significantly different. Therefore, only the nearby vibration mode is considered. The second-order coupling motion equation, formulated considering only the vibration and acoustic modes near those in Equation (3), is

\[
\ddot{\eta}_i + 2\xi_s \omega_s \dot{\eta}_i + \omega_s^2 \eta_i + \gamma_{ij} q_j = \Phi_i^T E \delta(t)
\]

\[-\rho a \gamma_{ji} \ddot{\eta}_j + \ddot{q}_j + 2\xi_a \omega_a \dot{q}_j + \omega_a^2 q_j = 0
\]  (6)

where \( \Phi_i^T \) is the mode shape of the \( i \)-th vibration mode \( (\eta_i) \), which is coupled with the \( j \)-th acoustic mode \( (q_j) \) by the elements of the coupling matrix \( (\gamma_{ij}) \). Only these two modes are assumed to be strongly coupled, as their natural frequencies are similar, and in the subsequent descriptions, the subscripts denoting specific modes are omitted to facilitate equation expansion.

The coupling matrix element \( (\gamma_{ij}) \) is defined as a new variable representing the coupling effect:

\[
g_s = \gamma_{ij}
\]

\[
g_a = \gamma_{ji}
\]  (7)

For the coupling matrix \( \gamma \) in Equation (5), \( g_s = g_a \) owing to the characteristics of the transposed matrix; however, a different subscript is used to distinguish the mode being applied. By substituting Equation (7) into Equation (6), the motion equation of the two coupled modes can be expressed as follows:

\[
\ddot{\eta} + 2\xi_s \omega_s \dot{\eta} + \omega_s^2 \eta + g_a q = \Phi_i^T E \delta(t)
\]

\[-\rho a g_a \ddot{\eta} + \ddot{q} + 2\xi_a \omega_a \dot{q} + \omega_a^2 q = 0
\]  (8)

By applying the Laplace transformation to Equation (8), the vibration (acceleration) and acoustic modes can be expressed as follows:

\[
\ddot{\eta}(s) = \frac{s^2 F_a(s)}{F_s(s) F_a(s) + \rho a g_a s^2} X(s)
\]

\[
q(s) = \frac{\rho a g_a s^2}{F_s(s) F_a(s) + \rho a g_a s^2} X(s)
\]  (9)

where

\[
F_s(s) = s^2 + 2\xi_s \omega_s + \omega_s^2
\]

\[
F_a(s) = s^2 + 2\xi_a \omega_a + \omega_a^2
\]

Note that \( s \), which is not a subscript, denotes the Laplace variable, and \( X(s) \) represents the Laplace transformation of the modal impact load \( (\Phi_i^T E \delta(t)) \). Equation (9) shows the transfer function from the impact load to the vibration (acceleration) and acoustic mode (sound pressure).

According to the equations, the vibration–acoustic coupling effect is influenced by the \( g_s \) and \( g_a \) values in the denominator. In the denominator in Equation (9), the two
modes are combined owing to the coupling effect, and these modes can be decomposed into two single degree-of-freedom modes. Specifically, the denominator in Equation (9) can be decomposed into two independent forms:

\[ F_s(s)F_a(s) + \rho_a g_s g_a s^2 = (s^2 + 2\xi_1 \omega_1 s + \omega_1^2)(s^2 + 2\xi_2 \omega_2 s + \omega_2^2) \]  

(10)

In this case, the original vibration and acoustic modes have two natural frequencies \((\omega_1, \omega_2)\) and damping ratios \((\xi_1, \xi_2)\), which are altered by the coupling effect. If \(g_s\) and \(g_a\) are large, the natural frequencies and damping ratios of the original vibration and acoustic modes will differ from the new frequencies and damping ratios owing to the coupling effect.

2.2. Prediction of Vibration-Mode-Induced Noise in the Acoustic Domain

For a given vibration mode, the nearby acoustic mode can be obtained through the Laplace transformation of Equation (8).

\[ q(s) = \frac{\rho_a g_a}{F_a(s)} \bar{\eta}(s) \]  

(11)

In other words, the response of the single degree-of-freedom system to the vibration mode (acceleration) is the acoustic mode.

The vibration mode or acoustic mode exhibit a wide spectrum at frequencies approaching the natural frequency. The sum of the power spectra near the corresponding frequency represents the vibration or acoustic magnitude. Therefore, the dispersion of the vibration mode and the acoustic mode induced from it can be used to estimate the acceleration level or sound pressure level, as follows:

\[ E[\bar{\eta}(s)\bar{\eta}(\tilde{s})] = \int |\bar{\eta}(\omega)|^2 d\omega \]

\[ E[q(s)\bar{\eta}(\tilde{s})] = \frac{\rho_a^2 g_a^2}{F_a(\omega)} \int |\bar{\eta}(\omega)|^2 d\omega \]  

(12)

Here, \( E[] \) is a symbol for equalization; specifically, \( s = i\omega \), where \( \omega \) denotes the angular velocity. Equations (12) can be substituted into Equation (9), respectively, to calculate the dispersion of the vibration and acoustic modes. If the external load in Equation (9) is assumed to be an impact load, the leading term is a quartic equation for \( s \), and thus, the dispersion can be calculated with a closed form [40]. When the dispersion of the vibration and acoustic modes in Equation (12) is expressed in a closed form, it consists of parameters including \( g_s \) and \( g_a \), representing the natural frequencies, damping ratios, and coupling effect. The dispersion of the vibration and acoustic modes is determined using these parameters, and then, the acceleration and sound pressure levels are calculated. Through this process, the correlation between the two modes can be identified in the frequency domain.

If the planar density of the structure in the acoustic domain is considerably greater than the density of the acoustic domain medium, the damping effect diminishes. Extremely large masses, such as slabs in apartment buildings, may exhibit a diminished coupling effect owing to the minimal influence by the sound pressure of the surrounding air. If the coupling between the structure and acoustic domain is negligible and only the structural vibration induces a sound field in the acoustic domain, the dispersion of the vibration and acoustic modes can be determined in a facile manner.
Specifically, if the effect of the sound field on the structure is ignored, $g_s = 0$ in Equation (8). Since the vibration affects the acoustic domain in a cascading manner, the vibration and acoustic modes can be expressed by maintaining $g_a$ in Equation (8) as follows:

$$\ddot{\eta}(s) = \frac{g^2}{F_s(s)} X(s)$$

$$q(s) = \frac{\rho_0 g_a s^2}{F_s(s) F_a(s)} X(s)$$

(13)

If the ideal impact load on the structure is a constant (band-limited white noise) at a specific frequency domain ($\omega_L < \omega < \omega_H$), the following expression holds:

$$S_{xx}(\omega) = |X(i\omega)|^2 = S_0 \text{ (constant)}$$

(14)

By substituting Equation (13) into Equation (12), respectively, and integrating them based on the assumption of a constant impact load, the dispersion of the vibration and acoustic modes can be determined in the closed form as follows (7):

$$E[\ddot{\eta}(s)\ddot{\tilde{s}}] = S_0 \int \left[ \frac{i\omega^2}{F_s(i\omega)} \right]^2 d\omega = \frac{\pi S_0}{2} \frac{\omega_s \left(1 + 4\xi_s^2\right)}{\xi_a} \approx \frac{\pi S_0}{2} \frac{\omega_s}{\xi_a}$$

$$E[q(s)q(s)] = S_0 \int \left[ \frac{\rho_0 g_a(i\omega)^2}{F_s(i\omega) F_a(i\omega)} \right]^2 d\omega = \pi S_0 \rho_a g_a^2 \frac{N}{D}$$

(15)

where $N = 2\omega_s \omega_a \left(\xi_s^2 \omega_s + \xi_a \omega_a\right) D = 4\xi_s^2 \xi_a \omega_s \omega_a \left[\left(\omega_s^2 - \omega_a^2\right) + 4\omega_s \omega_a \left(\omega_s \left(\xi_s^2 + \xi_a^2\right) + \xi_s \xi_a \left(\omega_s^2 + \omega_a^2\right)\right)\right]$

The dispersion ratio ($R$) of the acoustic mode against the vibration mode dispersion is

$$R(\xi_s, \xi_a, \omega_s, \omega_a, g_a) = \frac{\rho_a g_a^2}{\omega_s} \frac{\xi_a^2}{\xi_s^2} \frac{N}{D}$$

(16)

Figure 1 shows the normalized dispersion ratio, expressed as the ratio of the natural frequency of the acoustic mode to that of the vibration mode ($\omega_a / \omega_s$), as indicated in Equation (16). The ratio decreases as the difference in the frequencies of the acoustic and vibration modes increases, and the value is 40% of the maximum dispersion ratio when the frequency ratio varies by 5%.

If the vibration and acoustic modes are close, with $\omega_s = \omega_a$, the maximum dispersion ratio can be expressed as

$$R(\xi_s, \xi_a, \omega_s, g_a) = \frac{\rho_a g_a^2}{4\omega_s \xi_a \left(\xi_s^2 + \xi_a^2\right)}$$

(17)

In this manner, the acoustic mode dispersion can be predicted from the vibration mode dispersion as follows:

$$E[q(s)q(s)] = RE[\ddot{\eta}(s)\ddot{\tilde{s}}]$$

(18)

As mentioned previously, $R$ depends on the natural frequency and damping ratio of the vibration and acoustic modes and coupling coefficient $g_a$. The influence of the damping ratio of the acoustic mode on the ratio is significant. Therefore, in an acoustic domain with minor damping, such as a reverberation chamber, a greater sound pressure can be expected to be induced for the same vibration.
Figure 1. Covariance ratio: The graph was given by normalization from theory. If the difference between vibration mode and acoustic mode was larger, the frequency ratio became larger numerically.

3. Modeling

To simulate the structure–acoustic coupling system, a system consisting of a reverberation chamber with a shell on one side was configured. COMSOL was used to perform the time-domain coupling analysis. The modeled structure is shown in Figure 2. Figure 2a shows the shell (3.8 × 2.6 m) on one side of the reverberation chamber, locations of nine accelerometers installed at the intersections of the four quarters horizontally and vertically on the shell plane, and vibration point (red triangle) in the bottom left part of the shell. Figure 2b shows the plane of the reverberation chamber, along with the locations of five microphones (at a height of 1.2 m). Table 1 presents the properties of the material used for modeling. The shell thickness was varied to examine the vibration behaviors associated with the 10 Hz band, and the primary natural frequency was set as approximately 17 Hz. The Rayleigh damping parameters $\alpha$, $\beta$ were used to represent the shell’s damping, and the modal damping ratio in the band of 10–100 Hz was set as 2.5–4.5%. The reverberation chamber’s damping was implemented as an impedance (wall side) and set as 400 times the speed of sound. Table 2 lists the elements of the FEM model used in the analysis, mesh maximum size, and time gap for the time history analysis. The frequency range was up to 170 Hz, set considering the mesh size. The impact load was a pulse in the half wave form of a sine wave, the maximum load was 400 N, and the loading duration was 20 ms.

Table 1. Material property information of the modeled structure, which was necessary for FEM analysis.

| Domain | Item                  | Value                      |
|--------|-----------------------|----------------------------|
| Acoustic | Density                | 1.225 kg/m³                |
|         | Speed of Sound        | $c = 340$ m/s              |
|         | Impedance              | 400 c Pa s/m               |
| Shell  | Density                | 2400 kg/m³                |
|         | Poisson’s ratio        | 0.17                       |
|         | Young’s modulus        | $2 \times e^{10}$ Pa       |
|         | Size                   | $3.8 \times 2.6$ m         |
|         | Thickness              | 50 mm                      |
|         | Rayleigh Damping       | $\alpha = 4.5$, $\beta = 1.5 \times e^{-4}$ |
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4. Results and Discussion

4.1. Coupling Analysis

4.1.1. Mode Analysis

Table 3 summarizes the results of eigenvalue analysis of the structure–acoustic coupled FEM model. To show how the coupling effect influences the eigenvalues, the eigenvalues in 12 cases in which the shell and reverberation chamber were independent and coupled were compared. In the Table 3, Bold-underline values showed eigenvalues induced by coupling effect. They were induced by vibration, but their aspects were different from vibration and acoustic.

The damping ratios were determined by dividing the imaginary part with the real part. The vibration and acoustic mode damping ratios of the shell range from 2.5% to 4.5% and from 0.2% to 0.4%, respectively. The structure-induced eigenvalues (bold and underlined) exhibited decreased natural frequencies and damping ratios, although the differences were extremely small. For the coupled acoustic mode, the natural frequencies were similar, and the difference in the damping ratio was insignificant, except for the low-order acoustic modes. The minor changes in the natural frequency and damping ratio are attributable to changes in the solution of the quartic polynomial function owing to the coupling effect between the acoustic mode and nearby vibration mode. The damping ratio of the acoustic mode increased because of the effect of the damping ratio of the vibration mode, which was 10 times larger.
Table 3. Eigenvalues of vibration mode, acoustic mode from measured data, and mode generated by coupling effect. Bold-underline means eigenvalues made from coupled effect.

| Shell Only | Acoustic Only | Coupled |
|------------|---------------|---------|
| 16.714 + 0.48985i | 25.227 + 0.10865i | 16.686 + 0.48736i |
| 26.053 + 0.67818i | 32.718 + 0.11787i | 25.207 + 0.12469i |
| 39.543 + 1.09555i | 36.003 + 0.11248i | 26.038 + 0.66012i |
| 41.418 + 1.1671i | 42.469 + 0.14444i | 32.720 + 0.11852i |
| 47.871 + 1.4390i | 45.235 + 0.12918i | 36.001 + 0.11533i |
| 61.860 + 2.1636i | 48.826 + 0.14361i | 39.502 + 1.0900i |
| 62.149 + 2.1805i | 50.127 + 0.12359i | 41.379 + 1.1629i |
| 72.567 + 2.6434i | 56.491 + 0.16549i | 42.469 + 0.14531i |
| 80.413 + 3.4108i | 60.204 + 0.14039i | 45.239 + 0.13026i |
| 81.437 + 3.4891i | 61.792 + 0.14674i | 47.822 + 1.4333i |
| 87.588 + 3.9807i | 64.827 + 0.12166i | 48.829 + 0.14539i |
| 93.539 + 4.4907i | 67.742 + 0.12857i | 50.135 + 0.12421i |

The real part of the eigenvalue is the damped natural frequency ($\omega \sqrt{1 - \xi^2}$) and the imaginary part (marked as i) is $\xi \omega$.

4.1.2. Time History Analysis

The shell vibration caused by the impact load changes the sound pressure in the reverberation chamber. Figure 3 shows the propagation of sound pressure within the reverberation chamber over time, using COMSOL’s isosurface function. One millisecond after the impact, sound pressure was locally generated at the impact area at the bottom left of the shell plane. After 3 ms and 9 ms, the impact area exceeded the size of the shell, and the pressure propagated to the front part of the reverberation chamber. At $t = 21$ ms, the pressure propagated across the entire reverberation chamber. According to these analysis results, the sound pressure was generated by vibration of shell, and propagated the whole reverberant chamber gradually as the shape of sound pressure. It also showed that the propagation time was influenced by room characteristics, such as damping ratio. The sound pressure level was changed as time lapsed. In Figure 3, the sound pressure was denser near the impact points, and it propagated to the surrounding area to the 9 ms. After 9 ms, the sound pressure level was scattered whole area of reverberation chamber, but the level of that was weaker due to damping.

Figure 4 shows the time history of the acceleration and sound pressure at the accelerometer and microphone locations shown in Figure 2. Figure 4a shows the time history of acceleration at the bottom left corner of the shell plane, and Figure 4b shows the time history of the sound pressure recorded by the microphones installed in the center. The acceleration responses are in the form of free vibration and disappear within 2 s; however, the sound pressure responses can be observed for up to 5 s. This phenomenon likely occurred because the damping ratio of the acoustic domain was small owing to the large impedance of the reverberation chamber.

4.2. Acoustic Mode Prediction

Mode decomposition was applied to the time histories of nine acceleration and five sound pressure levels obtained through numerical analysis to extract the vibration and acoustic modes. Figure 5 shows the decomposed time history of the vibration and acoustic modes as a power spectrum of the 10–70 Hz band through the Fourier transform [41]. Figure 5a shows the spectrum of the vibration mode (acceleration), which represents a stable mode decomposition. The shape of the spectrum does not change significantly because the vibration mode is not considerably influenced by the sound pressure. This phenomenon can be explained considering Equation (9): When the coupling effect is small ($g_s = 0$), $F_a(s)$ is eliminated from the numerator and denominator, and only the single degree transfer function remains.
Figure 3. The aspect of sound pressure propagation: Time series of the propagating sound pressure level after the impact point (impact hammer) was shocked. The vibration was converted into sound pressure and the sound were scattered gradually as propagation whole room. The sound pressure level also lessens with the lapse of time.

Figure 4 shows the time history of the acceleration and sound pressure at the accelerometer and microphone locations shown in Figure 2. Figure 4a shows the time history of acceleration at the bottom left corner of the shell plane, and Figure 4b shows the time history of the sound pressure recorded by the microphones installed in the center. The acceleration responses are in the form of free vibration and disappear within 2 s; however, the sound pressure responses can be observed for up to 5 s. This phenomenon likely occurred because the damping ratio of the acoustic domain was small owing to the large impedance of the reverberation chamber.

Figure 5b shows the spectrum of the acoustic mode. The solid and dashed lines represent the acoustic- and vibration-mode-induced pressure, respectively. Since the damping ratio of the acoustic mode was smaller than that of the vibration mode, the spectrum appeared sharp. The sound pressure spectrum was not smooth at frequency bands exceeding 40 Hz because the vibration and acoustic modes were close to each other and cannot be completely decomposed using the mode decomposition method adopted in this study.

Moreover, in the sound pressure spectrum shown in Figure 5b, both the vibration and acoustic modes appeared because, as indicated in Equation (9), the acoustic mode was expressed as a two degrees-of-freedom system, which was then decomposed into two single degree-of-freedom modes (vibration and acoustic modes), contributing to the sound pressure.
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Mode decomposition was applied to the time histories of nine acceleration and five sound pressure levels obtained through numerical analysis to extract the vibration and acoustic modes. Figure 5 shows the decomposed time history of the vibration and acoustic modes as a power spectrum of the 10–70 Hz band through the Fourier transform \[41\].

Figure 5a shows the spectrum of the vibration mode (acceleration), which represents a stable mode decomposition. The shape of the spectrum does not change significantly because the vibration mode is not considerably influenced by the sound pressure. This phenomenon can be explained considering Equation (9): When the coupling effect is small \(g_0 = 0\), \(F_\omega(s)\) is eliminated from the numerator and denominator, and only the single degree transfer function remains.

Figure 5b shows the spectrum of the acoustic mode. The solid and dashed lines represent the acoustic- and vibration-mode induced pressure, respectively. Since the damping ratio of the acoustic mode was smaller than that of the vibration mode, the spectrum appeared sharp. The sound pressure spectrum was not smooth at frequency bands exceeding 40 Hz because the vibration and acoustic modes were close to each other and cannot be completely decomposed using the mode decomposition method adopted in this study.

Moreover, in the sound pressure spectrum shown in Figure 5b, both the vibration and acoustic modes appeared because, as indicated in Equation (9), the acoustic mode was expressed as a two degrees-of-freedom system, which was then decomposed into two single degree-of-freedom modes (vibration and acoustic modes), contributing to the sound pressure.

As mentioned previously, the vibration mode affected the nearby acoustic mode. In practice, the vibration mode affected all acoustic modes. However, if the natural frequencies of the two modes were highly deviated, the influence diminished drastically, as shown in Figure 1. Therefore, only the acoustic mode near the vibration mode was considered in this analysis.

An analysis was performed to verify if the acoustic mode could be predicted from the decomposed vibration mode. The 6th acoustic mode \((47.871 + 1.4390i)\) from the “Acoustic only” column in Table 3, which was near the 5th vibration mode \((47.871 + 1.4390i)\) from the “Shell only” column, was selected as the analysis object. The two modes generated the coupled eigenvalues \((47.822 + 1.4333i, 48.829 + 0.14539i)\) owing to the coupling effect.

Since the time history of the vibration mode was known, it was introduced in the differential equation (Equation (8)) to determine the time history of the acoustic mode. This value was compared with the decomposed acoustic mode to evaluate the prediction performance. The findings indicated that the proposed predictive formula can reliably predict the heavy-weight sound, which could not be accurately predicted (large deviations between 0.46 and 0.9) in the existing studies. A previous study reported a high performance in predicting the impact sound with the FDTD approach \[22, 23\]. The structures considered in this study were different from those in the previous study. The model performance can be enhanced if the damping coefficient condition is known. Moreover, the vibration data are less influenced by the boundary conditions. A previous study \[42\] reported that although
the impact sound could be accurately predicted using acoustic data, the performance was limited in different boundary conditions. In general, low frequencies corresponded to a high prediction complexity. A previous study based on the FEM-SEA method [25] indicated that the error rate was approximately 10% at a low frequency, although the predictions at other frequencies were reasonable. Although the proposed approach requires additional verification, it can effectively obtain predictions for low and high frequencies.

Figure 5. Power spectrum of modes: (a) Vibration mode from measured data; (b) Acoustic mode from vibration; The frequencies contained two modes. (1) Induced by vibration mode directly (blue dotted lines); (2) induced from vibro-acoustic coupling effect (red lines); (Dashed: vibration mode induced pressure. Solid: acoustic mode induced pressure).

Figure 6 shows the time history and spectrum of the acoustic mode. The comparison was conducted between decomposed data from measurement and analyzed data from FEM. Figure 6a presents the acoustic mode decomposed from the measured sound pressure and that obtained using the vibration mode. Figure 6b shows the magnified view of the maximum pressure. The results indicated that the most of two modes were similar except for some parts. Figure 6c shows the spectra of the two modes, which was shown that two spectra were almost similar. This demonstrated that the predictions for the acoustic mode are reliable.
reported that although the impact sound could be accurately predicted using acoustic data, the performance was limited in different boundary conditions. In general, low frequencies corresponded to a high prediction complexity. A previous study based on the FEM-SEA method [25] indicated that the error rate was approximately 10% at a low frequency, although the predictions at other frequencies were reasonable. Although the proposed approach requires additional verification, it can effectively obtain predictions for low and high frequencies.

Figure 6 shows the time history and spectrum of the acoustic mode. The comparison was conducted between decomposed data from measurement and analyzed data from FEM. Figure 6a presents the acoustic mode decomposed from the measured sound pressure and that obtained using the vibration mode. Figure 6b shows the magnified view of the maximum pressure. The results indicated that most of the two modes were similar except for some parts. Figure 6c shows the spectra of the two modes, which was shown that two spectra were almost similar. This demonstrated that the predictions for the acoustic mode are reliable.

4.3. Limitations and Scope for Future Work

The simulation results demonstrated that the acoustic mode can be predicted from the vibration mode of a structure. Notably, this research was conducted using a shell structure to verify the proposed predictive formula. Therefore, the proposed prediction method must be validated by applying it to actual structures. Moreover, to precisely predict the acoustic mode, the acoustic characteristics of the acoustic domain (natural frequency and damping ratio of the acoustic mode) must be considered. In this study, the acoustic characteristics were considered through an eigenvalue analysis of the acoustic domain (reverberation chamber). However, the determination of these values in practice may
be challenging. Accordingly, future work must be focused on estimating the acoustic characteristics in actual environments. The impedance and reverberation time pertaining to the acoustic boundary are key factors, and the damping ratio for the acoustic mode considerably influences the prediction of the acoustic mode from the vibration mode. The impedance considerably influences the damping [43,44]. Therefore, future work must identify a method to calculate the damping ratio from the impedance of the acoustic boundary and reverberation time, among other factors.

5. Conclusions

This study was aimed at identifying the correlation between the vibration and acoustic modes for predicting the sound pressure of the acoustic domain involving a structure, considering the structural vibration. The results demonstrated that the acoustic domain could be predicted from the vibration mode through a time domain structure–acoustic coupled simulation.

The vibration and acoustic modes were found by applying mode decomposition to the shell vibration and sound pressure measured by an FEM analysis of a reverberation chamber offering sound barrier conditions. Through the mode decomposition, the relationship vibration mode to acoustic mode and the fact that coupling system between vibration and sound affected the former were verified. Through the time-domain analysis, it was also proved that vibration diffused in shell structure after converting a shape of the sound. Therefore, coupling effect by predictive formula was verified and it was confirmed that the acoustic mode can be predicted from the vibration mode by the correlation between the two modes based on the former result. More precise prediction of the acoustic mode would require natural frequency and damping ratio of the given acoustic mode, and in particular, the damping ratio at the sound barrier domain.

To expand the acoustic mode prediction technique presented in this study from structure acceleration levels to heavy weight floor impact sound levels, a field experimental technique is required to identify the number of measurement sensors, the contribution level of vibration and sound pressure from each mode, and the modal characteristics of the acoustic domain. For this, it is judged that continuous research in this area is needed.

Author Contributions: Conceptualization, J.H.; writing—original and draft preparation, J.H. and S.K.; writing—review and editing, J.H., J.R., M.S. and S.K.; Supervision, J.H.; All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by a grant from Technology Advancement Research Program funded by Ministry of Land, Infrastructure and Transport of Korean government (grant 21CTAP-C164107-01). And this work was also supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (2020R1F1A1070349).

Conflicts of Interest: The authors declare no conflict of interest.

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