Differential Game Model of Dispersed Material Drying

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Abstract. Continuous and discrete game-theoretic models of dispersed material drying process are formalized and studied in the paper. The existence of optimal drying strategies is shown through application of results from the theory of differential games and dynamic programming. These optimal strategies can be found numerically.

INTRODUCTION

Physical processes involving energy or material transfer, including mechanical movement, have long been described with the help of evolution equations, especially differential ones. This is the standard method of modelling electromagnetic systems, including electromechanical and computer components, power units, grids, antennas, etc. [1-6]. Such framework also allows to set and research the problems of control, stability and reliability [7-10].

Another less popular approach to modelling of physical processes is to use the game theory, which is mainly used in studies of socioeconomic systems and cooperation [11-23]. Examples of game theory application to the problems of physics can be found in [24-33].

In this paper, the dispersed material drying process is formalized as a two player game between the “nature” and a drying apparatus operator. The paper is organized as follows. In section “the continuous model of dispersed material drying process” we introduce the general framework of the model and study the optimal strategies of both players. In section “the discrete model of dispersed material drying process” the framework introduced in the previous section is extended to the case of \( n \)-step drying process. The conclusions are drawn in the last section.

DIFFERENTIAL GAME MODEL OF DISPERSED MATERIAL DRYING PROCESS

Measurable function \( r(\tau), \tau : [0, J^*] \rightarrow T \) is an admissible control of the operator. Here \([0, J^*]\) is the time interval on which the drying process takes place and \( T \subset [0, \infty) \) is a compact set of all possible temperatures the dispersed materials can be dried at in the apparatus. The measurable function \( \alpha(\tau), \alpha : [0, J^*] \rightarrow Q \) is an admissible control of the “nature”, where \( Q \) is a compact set of all possible parameters relevant to the drying process that can not be controlled by the operator. Denote by \( x \) the humidity of the dispersed material. Trajectory of the process,
corresponding to the pair of controls \((t, \alpha)\), is an absolutely continuous function \(x : [0, J^*] \rightarrow \mathbb{R}^n_+\) such that for almost all \(\tau : [0, J^*]\) the equation \(x(\tau) = f(x(\tau), t(\tau), \alpha(\tau))\) holds.

It is assumed that the drying process proceeds over the interval \([0, J^*]\) in such a way that the trajectory reaches the given set \(\tilde{X} \subset \mathbb{R}^n_+\), called the terminal set of the game before or at the time moment \(J^*\).

Let the energy cost of the drying process be characterized by a given function \(e(t, \alpha, x, \tau)\). If \(t(\tau), \alpha(\tau)\) are control functions of the operator and the “nature” over the interval \([0, J^*]\), \(x(\tau)\) is the trajectory of the drying process corresponding to them, then \(\int_0^J e(t(\tau), \alpha(\tau), \tau) d\tau = E(t, \alpha, x(t, \alpha))\) is energy cost for the drying process.

Within the framework of our model the following two problems are formalized:

1. to find the control function \(t(\tau)\) – thermal regime of the drying process – that brings the humidity of the dispersed material from the initial conditions to the final conditions \(x(\tau) \in \tilde{X}\) with minimal energy cost under all possible external parameters – controls of the “nature”;

2. to find the control function \(t(\tau)\) that brings the humidity of the dispersed material to the final conditions \(x(\tau) \in \tilde{X}\) in the minimal time under all possible external parameters.

For definiteness the first problem is considered. It is natural to consider the upper (majorant) game \(\Gamma\) (cf. [23, 29]) within the framework of the problem. Then the piecewise-program strategies are defined as follows. The strategy of the minimizing operator \(\vec{T}\) is a pair \(\vec{T} = (\sigma', \varphi_{\sigma'})\), where \(\sigma'\) is the finite partition of the interval \([0, J^*]\), \(\varphi_{\sigma'}\) is the lower strategy corresponding to the partition \(\sigma'\), that is the mapping which associates the informational status of the operator at the time moment \(\tau_i \in \sigma'\) with the measurable control \(t_i(\cdot)\) over the interval \([\tau_i, \tau_{i+1}]\).

The strategy of the “nature” \(\vec{\alpha}\) is the set \(\{\psi_{\sigma}\}_{\sigma \in \Sigma}\), where \(\Sigma\) is the set of all finite partitions of the interval \([0, J^*]\), \(\psi_{\sigma}\) is the upper strategy of the “nature” in the multi-step upper game \(\Gamma^{\sigma'}\) corresponding to the partition \(\sigma'\), that is the mapping which associates the informational status of the “nature” at the time moment \(\tau_i \in \sigma'\) with the measurable control \(\alpha_i(\cdot)\) on the interval \([\tau_i, \tau_{i+1}]\). Recall that in upper game \(\Gamma^{\sigma'}\) the operator knows the state of the process at any time moment \(t_i(\cdot)\). The “nature” knows the state of the process at any time moment \(t_i(\cdot)\) as well, along with the control of the operator at the interval \([t_i, t_{i+1}]\). Having found the pair of strategies \((\vec{T}, \vec{\alpha}\)) one can construct the single set of controls \(t(\cdot), \alpha(\cdot)\) and the corresponding trajectory of the process \(x(\tau) = \overrightarrow{h}(\vec{T}, \vec{\alpha})\) over the whole interval \([0, J^*]\). It is known [23], that there exists \(\varepsilon\)-saddle point for all \(\varepsilon > 0\) in the game \(\Gamma\), that is there exists such a pair of strategies \((\vec{T}, \vec{\alpha})\), that for all \(\vec{T} \in T, \vec{\alpha} \in Q\) the inequality

\[
E \left( \vec{T}, \vec{\alpha}, h \left( \vec{T}, \vec{\alpha} \right) \right) + \epsilon \leq E \left( \vec{T}, \vec{\alpha}, \varepsilon \left( \vec{T}, \vec{\alpha} \right) \right) \leq E \left( \vec{T}, \vec{\alpha}, -\varepsilon \left( \vec{T}, \vec{\alpha} \right) \right) - \epsilon
\]

holds. The value \(E \left( \vec{T}, \vec{\alpha}, \varepsilon \left( \vec{T}, \vec{\alpha} \right) \right) - \epsilon\) is the upper limit of the energy needed to dry the dispersed material regardless of the strategy the “nature” plays. Here \(E(\cdot)\) is payoff in the situation \((\vec{T}, \vec{\alpha})\).

**THE DISCRETE MODEL OF DISPERSED MATERIAL DRYING PROCESS**

Assume that the dispersed material is to be processed in a fixed sequence \(i_n\) in drying apparatuses \(i = 1, 2, \ldots, n\). At the \(i\)-th step \((i = 1, n)\) the drying process consumes the amount of energy \(e_i(t_i, \alpha_i, x_i)\) which depends on the thermal regime chosen by the operator and the vector of external parameters chosen by the “nature”. The temperature is assumed be constant at each \(i\)-th step. During the whole drying process the amount of energy
The dispersed material drying process is formalized as a continuous two-player game-theoretic model, the discrete model is considered as well. An algorithm for finding optimal strategies in the model is given. The results obtained in this paper can be used in further studies of the drying process including numerical computation of optimal strategies.

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