HIGHER DIMENSIONAL DUST COSMOLOGICAL IMPLICATIONS OF A DECAY LAW FOR $\Lambda$ TERM: EXPRESSIONS FOR SOME OBSERVABLE QUANTITIES

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In this paper we have considered the multidimensional cosmological implications of a decay law for $\Lambda$ term that is proportional to $\frac{\beta \dot{a}}{a}$, where $\beta$ is a constant and $a$ is the scale factor of RW-space time. We discuss the cosmological consequences of a model for the vanishing pressure for the case $k = 0$. It has been observed that such models are compatible with the result of recent observations and cosmological term $\Lambda$ gradually reduces as the universe expands. In this model $\Lambda$ varies as the inverse square of time, which matches its natural units. The proper distance, the luminosity distance-redshift, the angular diameter distance-redshift, and look back time-redshift for the model are presented in the frame work of higher dimensional space time. The model of the Freese et al. (Nucl. Phys. B 287, 797 (1987)) for $n = 2$ is retrieved for the particular choice of $A_0$ and also Einstein-de Sitter model is obtained for $A_0 = \frac{1}{3}$. This work has thus generalized to higher dimensions the well-know result in four dimensional space time. It is found that there may be significant difference in principle at least, from the analogous situation in four dimensional space time.

Keywords: Cosmology; higher dimensional space time; decaying cosmological constant; cosmological tests

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1. Introduction

It is widely believed that a consistent unification of all fundamental forces in nature would be possible within the space-time with an extra dimensions beyond those four observed so far. The absences of any signature of extra dimensions in current experiments is usually explained the compactness of extra dimensions. The idea of dimensional reduction or self compactification fits in particularly well in comology because if we belive in the big bang,
our universe was much smaller at the early stage and the present four dimensional stage could have been preceded by a higher dimensional one (Chodos and Detweiler\textsuperscript{1}). In this work we consider multidimensional Robertson Walker (RW) model as a test case. In RW type of homogenous cosmological model, the dimensionality has a marked effect on the time temperature relation of the universe and our universe appears to cool more slowly in higher dimensional space time (Chatterjee\textsuperscript{2}).

In recent year, models with a relic cosmological constant $\Lambda$ have received considerable attention among researchers for various reasons (see Refs.\textsuperscript{3} – \textsuperscript{5} and references therein). We should realize that the existence of a nonzero cosmological constant in Einstein’s equations is a feature of deep and profound consequence. The recent observations indicate that $\Lambda \sim 10^{-55} \text{cm}^{-2}$ while particle physics prediction for $\Lambda$ is greater than this value by a factor of order $10^{120}$. This discrepancy is known as cosmological constant problem. Some of the recent discussions on the cosmological constant “problem” an consequence on cosmology with a time-varying cosmological constant are investigated by Ratra and Peebles,\textsuperscript{6} Dolgov,\textsuperscript{7} - \textsuperscript{9} Sahni and Starobinsky,\textsuperscript{10} Padmanabhan\textsuperscript{11} and Peebles.\textsuperscript{12} For earlier reviews on this topic, the reader is referred to Zeldovich,\textsuperscript{13} Weinberg\textsuperscript{14} and Carroll, Press and Turner.\textsuperscript{15}

Recent observations of type Ia supernovae (SNe Ia) at redshift $z < 1$ provide startling and puzzling evidence that the expansion of the universe at the present time appears to be accelerating, behaviour attributed to “dark energy” with negative pressure. These observations (Perlmutter \textit{et al.},\textsuperscript{16} Riess \textit{et al.},\textsuperscript{17} Garnavich \textit{et al.},\textsuperscript{18} Schmidt \textit{et al.}\textsuperscript{19}) strongly favour a significant and positive value of $\Lambda$. The main conclusion of these observations is that the expansion of the universe is accelerating.

A number of authors have argued in favour of the dependence $\Lambda \sim t^{-2}$ first expressed by Bertolami\textsuperscript{20} and later on by several authors\textsuperscript{21} – \textsuperscript{26} in different context. Recently, motivated by dimensional grounds in keeping with quantum cosmology, Chen and Wu,\textsuperscript{27} Abdel-Rahaman\textsuperscript{28} considered a $\Lambda$ varying as $a^{-2}$. Carvalho \textit{et al.},\textsuperscript{29} Waga,\textsuperscript{30} Silveira and Waga,\textsuperscript{31} Vishwakarma\textsuperscript{32} have also considered/modified the same kind of variation. Such a dependence alleviates some problems in reconciling observational data with the inflationary universe scenario. Al-Rawaf and Taha and Al-Rawaf\textsuperscript{33} and Overdin and Cooperstock\textsuperscript{34} proposed a cosmological model with a cosmological constant of the form $\Lambda = \beta \frac{\ddot{a}}{a}$, where $\beta$ is a constant. Following the same decay law recently Arbab\textsuperscript{35} have investigated cosmic acceleration with positive cosmological constant and also analyze the implication of a model built-in cosmological constant for four-dimensional space time. The cosmological consequences of this decay law are very attractive. This law provides reasonable solutions to the cosmological puzzles presently known. One of the motivations for introducing $\Lambda$ term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

In this paper by considering cosmological implication of decay law for $\Lambda$ that proportional to $\frac{\ddot{a}}{a}$, we have calculated the deceleration parameter, age of the universe. We have also analyzed the cosmological tests pertaining proper distance, luminosity distance, angular diameter distance, and look back time in the framework of higher dimensional space time for dust model $p = 0$ and shown that Freese \textit{et al.}\textsuperscript{36} model is retrieved from our model for a particular choice of $A_0$ and $n = 2$. The Einstein-de Sitter (ES) results are also
obtained from our results for the case $A_0 = \frac{2}{3}$ and $n = 2$.

2. The Metric and Field Equations

Consider the $(n + 2)$-dimensional homogeneous and isotropic model of the universe represented by the space time

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 dX_n^2 \right],$$

(1)

where $a(t)$ is the scale factor, $k = 0$, $\pm 1$ is the curvature parameter and

$$dX_n^2 = d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + ... + \sin^2 \theta_1 \sin^2 \theta_2 ... \sin^2 \theta_{n-1} d\theta_n^2$$

. The usual energy-momentum tensor is modified by addition of a term

$$T_{ij}^{\text{vac}} = -\Lambda(t)g_{ij},$$

(2)

where $\Lambda(t)$ is the cosmological term and $g_{ij}$ is the metric tensor.

Einstein’s field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} - \Lambda(t)g_{ij}.$$  

(3)

The energy-momentum tensor $T_{ij}$ in the presence of a perfect fluid has the form

$$T_{ij} = (p + \rho)u_i u_j - pg_{ij},$$

(4)

where $p$ and $\rho$ are, respectively, the energy and pressure of the cosmic fluid, and $u_i$ is the fluid four-velocity such that $u^i u_i = 1$.

The Einstein field Eqs. (3) and (4) for the metric (1) take the form

$$\frac{n(n + 1)}{2} \left[ \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right] = 8\pi \rho + \Lambda(t),$$

(5)

$$\frac{n\ddot{a}}{a} + \frac{n(n - 1)}{2} \left[ \frac{\ddot{a}}{a^2} + \frac{k}{a^2} \right] = -8\pi p + \Lambda(t).$$

(6)

An over dot indicates a derivative with respect to time $t$. The energy conservation equation $T_{j;1}^i = 0$ leads to

$$\dot{\rho} + (n + 1)(\rho + p)H = -\frac{\dot{\Lambda}}{8\pi},$$

(7)

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter.

For complete determinacy of the system, we consider a perfect-gas equation of state

$$p = \gamma \rho, \quad 0 \leq \gamma \leq 1.$$  

(8)
It is worth noting here that our approach suffers from a lack of Lagrangian approach. There is no known way to present a consistent Lagrangian model satisfying the necessary conditions discussed in the paper.

3. Solution of the Field Equations

In case of the vanishing pressure i.e. $\gamma = 0$ in Eq. (8), Equations (6) with $k = 0$ reduces to

$$\frac{\ddot{a}}{a} + \frac{(n - 1)}{2} \left( \frac{\dot{a}}{a} \right)^2 = \frac{\Lambda(t)}{n}.$$  \hspace{1cm} (9)

We propose a phenomenological decay law for $\Lambda$ of the form$^{29,30}$

$$\Lambda = \beta \left( \frac{\dot{a}}{a} \right),$$  \hspace{1cm} (10)

where $\beta$ is constant. Overdin and Cooperstock$^{34}$ have pointed out that the model with $\Lambda \propto H^2$ is equivalent to above form.

Using Eq. (10) in Eq. (9) and by integrating we obtain

$$a(t) = \left[ \frac{K}{A_0} t \right]^{A_0},$$  \hspace{1cm} (11)

where $K$ is an integrating constant and the constant $A_0$ has the value

$$A_0 = \frac{2(\beta - n)}{2\beta - n(n+1)}. $$  \hspace{1cm} (12)

By using Eq. (11) in the field equations Eqs. (5) and (6) we obtain

$$\Lambda(t) = \frac{2n(n-1)\beta(\beta - n)}{[2\beta - n(n+1)]^2} \frac{1}{t^2}, \quad \beta \neq \frac{n(n+1)}{2}. $$  \hspace{1cm} (13)

$$\rho(t) = \frac{n(\beta - n)}{4\pi[2\beta - n(n+1)]} \frac{1}{t^2}, \quad \beta \neq \frac{n(n+1)}{2}. $$  \hspace{1cm} (14)

The deceleration parameter $q$ is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{(1-A_0)}{A_0} = \frac{n(n-1)}{2(n-\beta)}, \quad \beta \neq n$$  \hspace{1cm} (15)

The density parameter of the universe $\Omega_m$ is given by

$$\Omega_m = \frac{16\pi\rho}{n(n+1)H^2} = \frac{2\beta - n(n+1)}{(n+1)(\beta - n)}, \quad \beta \neq \frac{n}{2}, \quad n \geq 2.$$  \hspace{1cm} (16)

The density parameter due to vacuum contribution is defined as $\Omega_{\Lambda} = \frac{2\Lambda}{n(n+1)H^2}$. Employing Eq. (13), this gives

$$\Omega_{\Lambda} = \frac{(n-1)\beta}{(n+1)(\beta - n)}, \quad \beta \neq \frac{n}{2}, \quad n \geq 2.$$  \hspace{1cm} (17)
From Eqs. (16) and (17), we obtain

$$\Omega_m + \Omega_\Lambda = 1.$$  

According to high redshift supernovae and CMB, the preliminary results from the advancing field of cosmology suggest that the universe may be accelerating universe with a dominant contribution to its energy density coming in the form of cosmological $\Lambda$-term. The results, when combined with CMB anisotropy observations on intermediate angular scales, strongly support a flat universe

$$\Omega_m + \Omega_\Lambda = 1.$$  \hspace{1cm} (18)

The age of the universe is calculated as

$$t_0 = H_0^{-1} A_0.$$  \hspace{1cm} (19)

A high value of Hubble constant $H_0 \sim 80$ km/sec/Mpc predicts a short age of the universe which is incompatible with the ages of oldest stars (12-16 Gyr) unless the universe is open ($\Omega_m < 0.1$) or flat and $\Lambda$ is dominated by $\Omega_m + \Omega_\Lambda = 1$ (Sahni and Starobinsky\textsuperscript{10}).

4. Neoclassical Tests (Proper Distance $d(z)$)

A photon emitted by a source with coordinate $r = r_1$ and $t = t_1$ and received at a time $t_0$ by an observer located at $r = 0$. The emitted radiation will follow null geodesics on which $(\theta_1, \theta_2, ..., \theta_n)$ are constant.

The proper distance between the source and observer is given by

$$d(z) = a_0 \int_a^{a_0} \frac{da}{a\dot{a}},$$  \hspace{1cm} (20)

$$r_1 = \int_{t_1}^{t_0} \frac{dt}{a} = \frac{a_0^{-1} H_0^{-1} A_0}{(1 - A_0)} \left[ 1 - (1 + z) \frac{\Delta_{z_{\infty}}}{A_0} \right].$$

Hence

$$d(z) = r_1 a_0 = H_0^{-1} \left( \frac{A_0}{1 - A_0} \right) \left[ 1 - (1 + z) \frac{\Delta_{z_{\infty}}}{A_0} \right],$$  \hspace{1cm} (21)

where $(1 + z) = \frac{\Delta_{z_{\infty}}}{A_0} = \text{redshift}$ and $a_0$ is the present scale factor of the universe.

For small $z$ Eq. (21) reduces to

$$H_0 d(z) = z - \frac{1}{2} A_0 z^2 + ...$$  \hspace{1cm} (22)

By using Eq. (15)

$$H_0 d(z) = z - \frac{1}{2} (1 + q) z^2 + ...$$  \hspace{1cm} (23)

From Eq. (21), it is observed that the distance $d$ is maximum at $z = \infty$. Hence

$$d(z = \infty) = H_0^{-1} \left( \frac{A_0}{1 - A_0} \right) = \frac{2H_0^{-1} (n - \beta)}{n(n - 1)}$$  \hspace{1cm} (24)
Eq. (21) gives the Freese et al. results for the proper distance if we choose \( n = 2 \) and

\[
\frac{A_0}{(1 - A_0)} = \frac{1}{q} = \frac{2}{(3\Omega_m - 2)}
\]

and \( d(z) \) is maximum for \( \Omega_m \rightarrow 0 \) (de-Sitter universe) and minimum for \( \Omega_m \rightarrow 1 \) ES.

5. Luminosity Distance

Luminosity distance is another important concept of theoretical cosmology of a light source. The luminosity distance is a way of expanding the amount of light received from a distant object. It is the distance that the object appears to have, assuming the inverse square law for the reduction of light intensity with distance holds. The luminosity distance is not the actual distance to the object, because in the real universe the inverse square law does not hold. It is broken both because the geometry of the universe need not be flat, and because the universe is expanding. In other words, it is defined in such a way as generalizes the inverse-square law of the brightness in the static Euclidean space to an expanding curved space (Waga\cite{30}).

If \( d_L \) is the luminosity distance to the object, then

\[
d_L = \left( \frac{L}{4\pi l} \right)^{\frac{1}{2}}, \tag{25}
\]

where \( L \) is the total energy emitted by the source per unit time, \( l \) is the apparent luminosity of the object. Therefore one can write

\[
d_L = r_1 a_0 = d(1 + z). \tag{26}
\]

Using Eq. (21) equation (26) reduces to

\[
H_0 d_L = (1 + z) \left( \frac{A_0}{1 - A_0} \right) \left[ 1 - (1 + z) \frac{\Omega_m^{-1}}{\Omega_m} \right]. \tag{27}
\]

For small \( z \), Eq. (27) gives

\[
H_0 d_L = z + \frac{1}{2}(1 - q)z^2 + ... \tag{28}
\]

or by using Eq. (16)

\[
H_0 d_L = z + \left[ 1 - \left( \frac{n + 1}{2} \right) \Omega_m \right] z^2 + ... \tag{29}
\]

The luminosity distance depends on the cosmological model we have under discussion, and hence can be used to tell us which cosmological model describe our universe. Unfortunately, however, the observable quantity is the radiation flux density received from an object, and this can only be translated into a luminosity distance if the absolute luminosity of the object is known. There is no distant astronomical objects for which this is the cases.
This problem can however be circumvented if there are a population of objects at different distances which are believed to have the same luminosity; even if that luminosity is not known, it will appear merely as an overall scaling factor.

Such a population object is Type Ia supernovae. These are believed to be caused by the core collapse of white dwarf stars when they accrete material to take them over the Chandrasekhar limit. Accordingly, the progenitor of such supernovae are expected to very similar, leading to supernovae of a characteristic brightness. This already gives good standard candle, but it can be further improved as there is an observed correlation between the maximum absolute brightness of a supernova and the rate at which its brightens and faded. And because a supernova at maximum brightness has a luminosity comparable to an entire galaxy, they can be seen at great distance. Exactly such an effect has been observed for several dozen Type Ia high z supernovae \( z_{\text{max}} \leq 0.83 \) by two teams: the supernova cosmology project\(^{16}\) and the high-z supernova search team.\(^{17}\) The results delivered a major surprise to cosmologists. None of the usual cosmological models without a cosmological constant were able to explain the observed luminosity distance curve. The observations of Perlmutter et al.\(^ {16}\) indicate that the joint probability distribution of \((\Omega_m, \Omega_\Lambda)\) is well fitted by

\[
0.8\Omega_m - 0.6\Omega_\Lambda \simeq -0.2 \pm 0.1.
\]

The best-fit region strongly favours a positive energy density for the cosmological constant \(\Omega_\Lambda > 0\).

6. Angular Diameter Distance

The angular diameter distance is a measure of how large objects appear to be. As with the luminosity distance, it is defined as the distance that an object of known physical extent appears to be at, under the assumption of Euclidean geometry.

The angular diameter distance \(d_A\) of a light source of proper distance \(d\) is given by

\[
d_A = d(z)(1 + z)^{-1} = d_L(1 + z)^{-2}.
\]  (30)

Applying Eq. (27) we obtain

\[
H_0d_L = \frac{A_0}{1 - A_0} \left[ \frac{1 - (1 + z)\frac{A_0 - 1}{A_0}}{(1 + z)} \right].
\]  (31)

Usually \(d_A\) has a minimum (or maximum) for some \(Z = Z_m\). In Freese et al.\(^ {36}\) model, for example this occurs for \(n = 2\) and

\[
Z_m = \left( \frac{3}{2} \Omega_m \right)^{\frac{2}{2\Omega_m - 2}} - 1.
\]

The maximum \(d_A\) for our model, one can easily find by setting the value of \(\Omega_m\).

The angular diameter and luminosity distances have similar forms, but have a different dependence on redshift. As with the luminosity distance, for nearly objects the angular diameter distance closely matches the physical distance, so that objects appear smaller as
they are put further away. However the angular diameter distance has a much more striking
behaviour for distant objects. The luminosity distance effect dims the radiation and the
angular diameter distance effect means the light is spread over a large angular area. This is
so-called surface brightness dimming is therefore a particularly strong function of redshift.

7. Look Back Time

The time in the past at which the light we now receive from a distant object was emitted
is called the look back time. How long ago the light was emitted (the look back time) depends on the dynamics of the universe.

The radiation travel time (or look back time) \( (t - t_0) \) for photon emitted by a source at
instant \( t \) and received at \( t_0 \) is given by

\[
t - t_0 = \int_a^{a_0} \frac{da}{a},
\]

Equation (11) can be rewritten as

\[
a = B_0 t^{A_0}, \quad B_0 = constant.
\]

This follows that

\[
\frac{a_0}{a} = 1 + z = \left( \frac{t_0}{t} \right)^{A_0},
\]

The above equation gives

\[
t = t_0 (1 + z)^{-\frac{1}{A_0}}.
\]

From Eqs. (21) and (35), we obtain

\[
t_0 - t = A_0 H_0^{-1} \left[ 1 - (1 + z)^{-\frac{1}{A_0}} \right],
\]

which is

\[
H_0 (t_0 - t) = A_0 \left[ 1 - (1 + z)^{-\frac{1}{A_0}} \right].
\]

For small \( z \) one obtain

\[
H_0 (t_0 - t) = z - \left( 1 + \frac{q}{2} \right) z^2 + ....
\]

From Eqs. (35) and (37), we observe that at \( z \to \infty, H_0 t_0 = A_0 \) (constant). For \( n = 2 \) and
\( A_0 = \frac{2}{3} \) gives the well-known ES result

\[
H_0 (t_0 - t) = \frac{2}{3} \left[ 1 - (1 + z)^{-\frac{4}{3}} \right].
\]

8. Discussion

In the above we have presented the cosmological consequences of Eq. (10) for the
vanishing pressure in the frame work of higher dimensional space time. The zero pressure
is not strictly appropriate because it occurs at the latter stage of evolution when "extra"
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Dimension lose much of their significance and the dimensionality has a marked effect on the time temperature relation of the universe, and our universe appears to cool more slowly in higher dimensional space time suggested by Chaterjee. To solve the age parameter and density parameter one require the cosmological constant to be positive or equivalently the deceleration parameter to be negative. The nature of the cosmological constant $\Lambda$ and the energy density $\rho$ have been examined. We have found that the cosmological parameter $\Lambda$ varies inversely with the square of time, which matches its natural units. This supports the views in favour of the dependence $\Lambda \sim t^{-2}$ first expressed by Bertolami and later on observed by several authors.

We also attempted to investigate the well known astrophysical phenomena, namely the neoclassical test, the luminosity distance-redshift, the angular diameter distance-redshift, and look back time-redshift for the model in the frame work of multidimensional space time. The importance of the concept of luminosity distance to the study of theoretical astrophysics needs no further elaboration as the intrinsic luminosity of a source may be calculated with help of source’s redshift and apparent luminosity are known. However, as it is related to the scale factor, space curvature, etc. The results for the cosmological tests are compatible with the present observations. The model of the Freese et al. is retrieved from our model for $n = 2$ and the particular choice of $A_0$. Also the ES results are obtained for the case $A_0 = \frac{2}{3}$. These tests are found to depend on $\beta$. It is a general belief among cosmologists that more precise observational data should be achieved in order to make more definite statements about the validity of cosmological models (Charlton and Turner). We hope that in the near future, with the new generation of the telescope, the present situation could be reversed.

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