CFAR Detection of High Grazing Angle Sea-Clutter Based on KR Distribution

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Abstract. Constant false alarm rate (CFAR) is a traditional method for radar target detection. A lot of relevant researches assume that the sea clutter obeys the K distribution. However, with the improvement of the radar resolution and the increase of the grazing angle, the distribution of sea clutter gradually deviates from the K distribution. Although good fitting results can be obtained through the more complicated sea clutter models such as KA distribution, K+Rayleigh distribution and so on, the computational efficiency of the detection algorithm is reduced obviously. For computational complexities are mainly caused by non-closed expression, A hybrid sea-clutter distribution (KK distribution) with closed-form expression for high-resolution and high grazing angle situation is proposed, which fits the measured data. Inspired by this, a new hybrid sea-clutter distribution (KR distribution) with 4 parameters is proposed. A new method of parameter estimation for KR distribution is given, that is, passing-parameter estimation method based on hybrid distribution. The results show that the fitting effect of KR distribution is better than that of both K distribution and KK distribution in case of high resolution and large grazing angle. And the CA-CFAR based on KR distribution of passing-parameter estimation performs relatively better in the aspect of target detection.

1. Introduction
The CFAR detector is performed based on a certain background clutter distribution. In the study of the statistical characteristics of sea clutter amplitude, Rayleigh distribution, Weibull distribution, logarithmic normal distribution and K distribution are often used in low-resolution and low grazing angle sea clutter. With the increase of resolution, the statistical distribution of sea clutter shows a long tail and significantly deviates from the Rayleigh distribution. At this time, the sea clutter is strongly correlated. The fitting models including compound K distribution, KK distribution [1], KA distribution [2], neural network method, and fractal model are commonly used. With the increase of the grazing angle, the equivalent backscattering in cross-sectional area of the sea surface rapidly increases and most of the sea clutter energy is projected onto a few distance units, resulting in an uneven spatial distribution of energy. At this time, the sea clutter distribution deviates from the original distribution. Luke Rosenberg and Simon Watts used the Ingara data to study the sea clutter from 15 gazing angle to 45 grazing angle. Then, K+Rayleigh distribution, K+noise distribution and Pareto+noise distribution [3-4] as well as the methods of parameters estimation are proposed, which solve the problem of amplitude fitting in high resolution and high grazing angle situation. However,
the three distributions without closed expression have brought great difficulties to application because of the calculation in CFAR algorithm.

2. Distributions of Sea Clutter

2.1. KK Distribution

In many detection scenarios, K distribution can not only fit the observed intensity measurement well. Because of the sea spikes, the sea clutter distribution may deviate from the K distribution. The higher the resolution is, the more obvious the deviation is. To solve the problem, KK distribution is proposed, which improves the fitting results at the clutter-tail portion where sea spikes exist. However, there is no closed expression for this distribution, and its probability density function and probability distribution function can only be numerically approximated, which brings great inconvenience to the application of KK distribution. A hybrid distribution model, the KK distribution model, is proposed and experiments show that this distribution can better fit the larger sea clutter tail caused by sea spikes [5-7].

The KK distribution is a mixed distribution model that assumes both the Bragg/Whitecap scattering center and the sea peak scattering center follow the K distribution, its expression is shown as follows:

\[
f(x,v,b) = \frac{b^{v+1}}{\Gamma(v)2^{v+1/2}} x^{v-1} \left( \frac{bx}{\sqrt{2}} \right) K_{v-1}\left( \frac{bx}{\sqrt{2}} \right)
\]

(1)

Where \( v \) is the shape parameter, \( b \) is the scale parameter. And \( K_{v-1}(z) \) is the Bessel function of the \( v \)-order second modified class. The weighted sum of the two parts of the clutter model gives the probability density function for the KK distribution:

\[
f_{KK}(x) = (1-k_0) f_1(x; v_1, b_1) + k_0 f_2(x; v_2, b_2)
\]

(2)

It can be seen that when the value of \( k_0 \) is 0, KK distribution is K distribution.

2.2. KR Distribution

More complicated models like KA distribution and GK distribution are proposed. The researchers applied these models to the amplitude distribution of high-resolution sea clutter with large grazing angle and found that the KK model has the best fitting effect. However, the estimation of five parameters as well as the calculation of the third-order moments lead to great difficulties. There is literature noticing a non-negligible Rayleigh part of radar echoes which meet the K distribution in the case of high resolution and large grazing angle based on the analysis of the measured data. Then, Luke Rosenberg and Simon Watts proposed a series of models such as K+Rayleigh distribution, and pointed out that the K distribution was a special form of K+Rayleigh distribution. However, it is regrettable that a closed expression of the new model is not given. The probability density function must be numerically calculated, which greatly increases the computational complexity, reduces the radar performance and the efficiency of CFAR algorithm. Therefore, the application of this distribution is very limited. It is of great significance to find a distribution which has less calculation and fits the measured data better than K distribution.

It is generally considered that the amplitude of low-resolution and low grazing angle sea clutter obeys Rayleigh distribution. Rayleigh distribution is one of Gaussian models, that is, assuming that the real part and imaginary part of random complex variables are subject to Gaussian distribution, so that the echo amplitude from each resolving unit obeys Rayleigh distribution[5]. Its probability density function (pdf) is as follows:

\[
f(x) = \frac{x}{\sigma^2} \exp\left( -\frac{x^2}{2\sigma^2} \right)
\]

(3)

The distribution has been widely used in the classical CFAR detection algorithm of point target as well as extended target.
Sea clutter shows a heavier tail as the resolution of radar increases. In order to fit the sea clutter better, some models for high-resolution and low grazing angle situation are proposed, such as the K distribution. K distribution is a multiplicative model, which is the most widely used sea clutter model. The model interprets sea clutter as a fast changing component which is modulated by a slowly varying component. The slow changing component is a texture component expressed in a square root gamma distribution. The fast changing component is speckle component expressed in Rayleigh distribution. Expression of the K distribution [8-10] is as follows:

\[
f(x) = \frac{2}{b \Gamma(v)} \left( \frac{x}{2b} \right)^{v-1} K_{v-1} \left( \frac{x}{b} \right)
\]

In equation (4), \(v\) is the shape parameter, which indicates the tail of the sea clutter, \(b\) is the scale parameter which is related to the level of sea clutter power and \(K_{v-1}(\cdot)\) is the modified Bessel function of \(v-1\) order. Its parameters can be obtained by maximum likelihood estimation or moment estimation. The K distribution proves to be a good fit in many scenarios.

The analysis of measured data shows that the K distribution has a good fitting effect on the tail of the pdf of high-resolution and large grazing angle sea clutter, while the Rayleigh distribution has a better fitting effect on the rest part of the pdf. Inspired by the hybrid distribution, KK distribution, a mixed distribution with 4 parameters is proposed based on the characteristics of the measured data, which is called the KR distribution.

The KR model is a combination of K distribution and R distribution, and its expression is as follows:

\[
f(x) = kf(x) = \frac{2}{b \Gamma(v)} \left( \frac{x}{2b} \right)^{v-1} K_{v-1} \left( \frac{x}{b} \right) + (1-k) \frac{x}{\sigma^2} \exp \left( -\frac{x^2}{2\sigma^2} \right)
\]

In equation (5), \(v\) is the shape parameter, which indicates the tail of the sea clutter, \(b\) is the scale parameter which is related to the level of sea clutter power and \(K_{v-1}(\cdot)\) is the modified Bessel function of \(v-1\) order. And \(\sigma\) is just the same as that in Rayleigh distribution.

### 3. Parameters Estimation

The parameters of KR distribution can be estimated by particle swarm optimization, but it usually plunges into local optima. Three methods are proposed as follows, which are more simple and efficient.

#### 3.1. Parameter Estimation Based on Numerical calculation

Please The KR model needs to estimate 4 parameters: weight parameter \(k\), shape parameter \(v\), scale parameter \(b\) and sigma. The sigma can be estimated by the first moment and the last three parameters \(k, v, b\) constitute the parameter space which is shown just as in (5). The parameter space contains most of the points in the cuboid and every point in the space corresponds to only one combination of the three parameters as its coordinate. The optimal parameter combination is a certain point in the parameter space, and the problem of parameter estimation becomes the problem of finding the best point in the entire parameter space. However, it is worth noting that not all points in the cuboid really mean something, such as the point F, and all the points on the line segments which contain point F, and they are not in the parameter space. Finally, the parameters will be found by calculating all the valid points in the cuboid. Every point that is calculated will be judged by its fitness which can be calculated by equation (6). \(x\) is the measured data, \(f\) is the fitting distribution. \(err\) is the “fitness” as well as the relative error. This process is called traversal search method.

\[
\text{err}(i) = \left( \frac{(x(i) - f(i))^2}{x(i)} \right)
\]
Or the point can be found by downsizing the parameter space. As is shown in Figure 2, there are 8 paths in the parameter cuboid, the fitness of the path would be smaller if the best points are near it and the path would be called the best path. A sub-cuboid can be found which includes the best path, the easiest one is a smaller cuboid. The process will be performed until the sub-cuboid is little enough (This experiment repeat the process 50 times). And the final parameter is the average of each coordinate value. All the process is called geometric segmentation estimation method.

\[
e = \min (err)
\]  

Figure 1. parameters space  

Figure 2. geometric segmentation estimation method

3.2. Passing-parameter Estimation Method Based on Hybrid Distribution

The appearance of mixed distribution brings great convenience to the sea clutter amplitude fitting and CFAR detection. Many of the derived sea clutter distribution models have no closure expressions, and their probability density function and probability distribution function can only be calculated by numerical calculation. This results in a huge efficiency loss in the CFAR detection algorithm. In order to reduce computation, a hybrid distribution generated by weighted combinations of two known distribution with closed expressions is applied to the CFAR detection and can obtain better clutter fitting results and CFAR detection performance. For example, the KK distribution is the best known mixed distribution for sea clutter. Mixed distribution has the following unified expression:

\[
F = kA + (1 + k)B
\]  

In equation(8), \( k \) is a weight parameter, \( A \) and \( B \) are distributions with closure expressions, which generally contain parameters that need to be estimated. The parameters within \( A \) or \( B \) are related with each other, in another word, the parameters between \( A \) and \( B \) are not related. This paper proposes a new method (Passing-parameter Estimation Method) for estimating the parameters based on the characteristics of hybrid distribution. The flow chart is as follows:

Figure 3. Passing-parameter Estimation Method.

In Figure 3, modules 1, 2, and 3 estimate the parameters, modules 1, 2 are the first stage estimation, and module 3 is the second stage estimation. Module 1 estimates all parameters of function \( A \) and module 2 estimates all parameters of function \( B \), module 3 estimates weight parameter \( k \). The data is
inputted into the module 1 and the module 2 at the same time, the estimation results of the two modules are passed to the module 3 for estimating the weight parameter. If there is only one parameter to be estimated in one module, it will be estimated by moment estimation method. If there are only two related parameters, they will be estimated by traversal search estimation method, but the parameters constitute a flat not a cuboid. In order to improve the calculation efficiency of each module, this method is not suitable for the use in a mixed parameter model with more than two inseparable related parameters, that is, each parameter estimation module only contains related parameters that cannot be further separated. For example, the function of the parameter to be estimated in the module 2 is a mixed one such as KK distribution, then module 2 should be separated into 2 modules, the schematic diagram after separation is shown in Figure 4.

![Figure 4. Passing-parameter Estimation Method after Separation.](image)

In Figure 4 module 2, module 3, and module 4 are all juxtaposed with the module 1, and they form the first layer of the estimation. The relationship between module 2, 3, and 4 is the same as the relationship between module 1, module 2, and module 3 in Figure 2. The rules are the same as in Figure 2. For example, if the function of module 2 in Figure 2 is KK distribution, the model can be expressed as in Figure 3, module 2 and module 3 in Figure 3 is K distribution, parameter to be estimated in module 4 is $k = 1$.

This paper fits the measured data by KR distribution, which is a weighted combination of K distribution and Rayleigh distribution. The KR distribution has the same weight parameter $k$, shape parameter $v$, and scale parameter $b$, where $v$ and $b$ are relevant parameters, they will be estimated by traversal search method. Sigma is an individual related parameter and estimated by moment estimation method.

### 4. Results of CFAR Detection Simulation of Measured Data

The most well-known CFAR methods are the mean level detectors. The top 3 mean level detectors that are used most widely are cell averaging (CA-) CFAR, the greatest of (GO-) CFAR and the smallest of (SO-) CFAR. CA-CFAR has the optimal performance in a homogeneous environment for various signal-to-noise ratios (SNR) but it suffers severe performance degradation in nonhomogeneous environment. GO-CFAR has a better performance in controlling probability of false alarm ($P_{fa}$) in the case of clutter edges, but it results in a great decrease in probability of detection ($P_{d}$) in multiple targets. SO-CFAR performs better in multiple targets but results in more $P_{fa}$ than CA-CFAR in clutter edges [11-13].

#### 4.1. Fitting Results of Measured Data

The data used in this experiment is measured from the calm sea, including 1000 pulses, every pulse is sampled 2048 times and target is a boat at the 1500th sampling point. The average correlative unit in distance is 1. The grazing angle is about 69 degree. The fitting result is shown in Figure 5 and Figure 6. All the comparison results are shown in table 1.
Figure 5. Fitting results of measured data at 69 grazing angle.

Figure 5 shows the fitting results of measured data at 69 grazing angle with K distribution, R distribution, KK distribution and KR distribution. And the parameters of KR distribution are estimated by passing-parameter estimation method (KRPP).

Figure 6. Fitting results of measured data at 69 grazing angle.

Figure 6 shows the fitting results of measured data at 69 grazing angle with KR distribution whose parameters are estimated by 2 methods. Compared with the results of Figure 5 and Figure 6, we can draw a conclusion just like Table 1 that fitting performance of KR distribution whose parameters are estimated by passing-parameter estimation method (KRPP) is 54.6% worse than that of traversal search method (KRT) and 33.4% worse than that of geometric segmentation estimation method (KRP). In another word, the fitting performance of KR distribution based on traversal search method estimation (KRT) is 31.8% better than KR distribution based on geometric segmentation estimation method (KRP), 54.6% better than KR distribution based on passing-parameter estimation method (KRPP), 56.6% better than KK distribution, 59.6% better than K distribution and 90.55% better than R distribution.
Table 1. Detection results of different distribution

| Fitting results | KRT   | KRP   | KRPP  | KK    | K     | R     |
|-----------------|-------|-------|-------|-------|-------|-------|
| Relative error  | 0.0799| 0.1172| 0.1759| 0.1842| 0.1981| 0.8456|
| Improvemen1     | ———  | 31.8% | 54.6% | 56.6% | 59.6% | 90.55%|
| Improvemen2     | -54.6%| -33.4%| ———  | 4.54% | 11.2% | 79.2% |

In a word, the fitting performance order of the measured data from the best to the worst is as follows: the KR distribution based on traversal search method estimation (KRT), the KR distribution based on geometric segmentation estimation method (KRP), the KR distribution based on passing-parameter estimation method (KRPP).

4.2. Fitting Results of Measured Data

Since the target is a boat, the length of it is about 9 meters, the target window is set to 10, and the protection window is set to 20. The threshold is calculate by Monte Carlo method. The measured data are fitted by K distribution and KR distribution separately. And the results of detection probability and false alarm rate of CA-CFAR, GO-CFAR, SO-CFAR, VI-CFAR and ACCA-CFAR are shown in Table 2 and Table 3.

Table 2. Detection results of K distribution

| Detection results | Numbers of false targets | Numbers of missed targets | Detection times | Pfa  | Pd   |
|-------------------|--------------------------|---------------------------|----------------|------|------|
| CA-CFAR           | 287                      | 249                       | 1000           | 28.7%| 75.1%|
| GO-CFAR           | 231                      | 257                       | 1000           | 23.1%| 74.3%|
| SO-CFAR           | 374                      | 211                       | 1000           | 37.4%| 78.9%|
| VI-CFAR           | 208                      | 207                       | 1000           | 20.8%| 79.3%|
| ACCA-CFAR         | 347                      | 101                       | 1000           | 34.7%| 89.9%|

Table 2 shows the detection results of K distribution at 69 grazing angle, the detection probability goes from high to low are as follows: ACCA-CFAR, VI-CFAR, SO-CFAR, CA-CFAR, GO-CFAR. And the highest detection probability comes from ACCA-CFAR which is 89.9%. The false alarm rate goes from high to low are as follows: VI-CFAR, GO-CFAR, PSVM-CFAR, CA-CFAR, ACCA-CFAR, SO-CFAR. And the lowest one comes from SO-CFAR, which is 20.8%.

Table 3. Detection results of KR distribution

| Detection results | Numbers of false targets | Numbers of missed targets | Detection times | Pfa  | Pd   |
|-------------------|--------------------------|---------------------------|----------------|------|------|
| CA-CFAR           | 281                      | 243                       | 1000           | 28.1%| 75.7%|
| GO-CFAR           | 228                      | 251                       | 1000           | 22.8%| 74.9%|
| SO-CFAR           | 369                      | 217                       | 1000           | 36.9%| 79.3%|
| VI-CFAR           | 202                      | 199                       | 1000           | 20.2%| 80.1%|
| ACCA-CFAR         | 342                      | 98                        | 1000           | 34.2%| 90.2%|

Table 3 shows the detection results of KR distribution at 69 grazing angle, the detection probability goes from high to low are as follows: ACCA-CFAR, VI-CFAR, SO-CFAR, CA-CFAR, GO-CFAR. And the highest detection probability comes from ACCA-CFAR which is 90.2%. The false alarm rate
goes from high to low are as follows: VI-CFAR, GO-CFAR, PSVM-CFAR, CA-CFAR, ACCA-CFAR, SO-CFAR. And the lowest one comes from SO-CFAR, which is 20.2%.

5. Conclusion
The fitting performance of the measured data rank from the best to the worst is as follows: the KR distribution based on traversal search method estimation (KRT), the KR distribution based on geometric segmentation estimation method (KRP), the KR distribution based on passing-parameter estimation method (KRPP). The highest detection probability of traditional CFAR detection studied in the paper comes from ACCA-CFAR and the lowest false alarm rate comes from VI-CFA. The detection probability of KR-distribution-fitted data is commonly higher than that of K-distribution-fitted data and the false alarm rate is on the contrary.

Acknowledgments
Received funds for covering the costs to publish in open access: National Science Foundation of China. (Number: 61471381, 61471380)

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