Exploring the Capability and Limits of the Feedback Mechanism

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Abstract

Feedback is a most important concept in control systems, its main purpose is to deal with internal and/or external uncertainties in dynamical systems, by using the on-line observed information. Thus, a fundamental problem in control theory is to understand the maximum capability and potential limits of the feedback mechanism. This paper gives a survey of some basic ideas and results developed recently in this direction, for several typical classes of uncertain dynamical systems including parametric and nonparametric nonlinear systems, sampled-data systems and time-varying stochastic systems.

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1. Introduction

Feedback is ubiquitous, and exists in almost all goal-directed behaviors [1]. It is indispensable to the human intelligence, and is important in learning, adaptation, organization and evolution, etc. Feedback is also the most important concept in control, which is a fundamental systems principal when dealing with uncertainties in complex dynamical systems. The uncertainties of a system are usually classified into two types: internal (structure) and external (disturbance) uncertainties, depending on the specific dynamical systems to be controlled. Feedback needs information, and there are also two types of information in a control system: a priori information and posteriori information. The former is the available information before controlling a system, while the later is the information exhibited by the system dynamic behaviors. It is the posteriori information that makes it possible for the feedback to reduce the influences of the uncertainties on control systems. Two of the fundamental questions in control theory are: How much uncertainty can be

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dealt with by feedback? What are the limits of feedback? These are conundrums, despite of the considerable progress in control theory over the past several decades.

The existing feedback theory in control systems can be roughly classified into three groups: traditional feedback, robust feedback and adaptive feedback. In the ideal case where the mathematical model can exactly describe the true system, the feedback law that are designed based on the full knowledge of the model may be referred to as traditional feedback. Unfortunately, as is well known, almost all mathematical models are approximations of practical systems, and in many cases there are inevitable large uncertainties in our mathematical descriptions. The primary motivation of robust and adaptive control is to deal with uncertainties by designing feedback laws, and much progress has been made in these two areas. Robust feedback design allows that the true system model is not exactly known but lies in a “ball” centered at a known nominal model with reliable model error bounds (cf. e.g. [2], [3]).

By adaptive feedback we mean the (nonlinear) feedback which captures the uncertain (structure or parameter) information of the underlying system by properly utilizing the measured on-line data. The well-known certainty-equivalence principle in adaptive control is an example of such philosophy. Since an on-line learning mechanism is usually embedded in the structure of adaptive feedback, it is conceivable that adaptive feedback can deal with larger uncertainties than other forms of feedback can do. Over the past several decades, much progress has been made in the area of adaptive control (cf. e.g. [4]–[9]). For linear finite dimensional systems with uncertain parameters, a well-developed theory of adaptive control exists today, both for stochastic systems (cf. [5], [8], [9]) and for deterministic systems with small unmodelled dynamics (cf. [6]). This theory can be generalized to nonlinear systems with linear unknown parameters and with linearly growing nonlinearities (cf. e.g. [12]). However, fundamental difficulties may emerge in the design of stabilizing adaptive controls when these structural conditions are removed. This has motivated a series of studies on the maximum capability (and limits) of the feedback mechanism starting from [10].

To explore the maximum capability and potential limits of the feedback mechanism, we have to place ourselves in a framework that is somewhat different from the traditional robust control and adaptive control. First, the system structure uncertainty may be nonlinear and/or nonparametric, and a known or reliable ball containing the true system, which is centered at a known nominal model, may not be available a priori. Second, we need to consider the maximum capability of the whole feedback mechanism (not only a fixed feedback law or a special class of feedback laws). Moreover, we need to answer not only what the feedback can do, but also the more difficult and important question, what the feedback can not do. We shall also work with discrete-time (or sampled-data) feedback laws, as they can reflect the basic causal law as well as the limitations of actuator and sensor in a certain sense, when implemented with digital computers.

In this talk, we will give a survey of some basic ideas and results obtained in the recent few years ([10]–[17]), towards understanding the capability and limits of the feedback mechanism in dealing with uncertainties. For several basic classes of
uncertain nonlinear dynamical control systems, we will give some critical values and “Impossibility Theorems” concerning the capability of feedback. The reminder of the paper is organized as follows: the problem formulation will be given in Section 2, and some basic classes of discrete-time parametric and nonparametric nonlinear control systems will be studied in Sections 3 and 4, respectively. Other classes of uncertain systems, including sampled-data systems and time-varying linear systems with hidden Markovian jumps, will be considered in Section 5, and some open problems will be stated in the concluding remarks in Section 6.

2. Problem formulation

Let \( \{u_t, t \geq 0\} \) be an \( \mathbb{R}^m \)-valued input process of a discrete-time or sampled-data uncertain dynamical control system (whose structure is unknown or not fully known, and is subject to some noise disturbances), and let \( \{y_t, t \geq 0\} \) be the corresponding on-line observed \( \mathbb{R}^n \)-valued output process (see the following figure).

\[
\begin{align*}
  &u_t \quad \text{noise} \\
  &\text{uncertain system} \\
  &y_t
\end{align*}
\]

At any moment \( t \geq 0 \), the input signal \( u_t \) is said to be a feedback signal, if there is a Lebesgue measurable mapping \( h_t(\cdot) : \mathbb{R}^{(t+1)n} \rightarrow \mathbb{R}^m \) such that

\[ u_t = h_t(y_0, y_1, \ldots, y_t). \]

In other words, \( u_t \) is a function of the posteriori information observed up to time \( t \).

A feedback law \( u \) is a sequence of feedback signals, i.e., \( u = \{u_t, t \geq 0\} \). Furthermore, the feedback mechanism \( U \) is defined as the set of all possible feedback laws,

\[ U = \{u| \ u \text{ \ is any feedback law}\}. \]

For a complex system whose structure contains uncertainties, it is not a simple (and in fact difficult) problem to find a feedback law \( u \), such that the corresponding output process can achieve a desired goal. This involves questions like: what kind of properties of an uncertain dynamical system can be changed by feedback? how to construct a satisfactory feedback based on the available information? More fundamental questions are: how much uncertainty can be deal with by feedback? What are the maximum capability and limits of the feedback mechanism \( U \)? In the next three sections, we will present some preliminary results towards answering these questions. For further discussion, we need the following definition.

**Definition 2.1.** A dynamical control system is said to be globally stabilizable if there exists a feedback law \( u \in U \), such that the output process of the system is bounded in the mean square sense, i.e.,

\[ \sup_{t \geq 0} E\|y_t\|^2 < \infty, \quad \text{for any initial value } y_0. \]
3. Parametric nonlinear systems

Consider the following basic discrete-time parametric nonlinear system:

\[ y_{t+1} = \theta f(y_t) + u_t + w_{t+1}, \]  

(3.1)

where \( y_t, u_t \) and \( w_t \) are the scalar system output, input and noise processes, respectively. For simplicity, we assume that

A1) \( \{w_t\} \) is a Gaussian noise process;

A2) \( \theta \) is an unknown non-degenerate Gaussian random parameter;

A3) The function \( f(\cdot) \) is known and has the following growth rate:

\[ f(x) \sim M x^b, \quad \text{as} \quad x \to \infty, \]

where \( b \geq 0, M > 0 \) are constants. Obviously, if \( b \leq 1 \), then the nonlinear function \( f(\cdot) \) has a growth rate which is bounded by linear growth. This case can be easily dealt with by the existing theory in adaptive control (see e.g. [12]). Our prime concern here is to know whether or not the system can be globally stabilized by feedback for any \( b > 1 \)?

The following theorem gives a critical value of \( b \), which characterizes the maximum capability of the feedback mechanism.

**Theorem 3.1.** Consider the system (3.1) with Assumptions A1)–A3) holding. Then \( b = 4 \) is a critical case for feedback stabilizability. In other words,

(i). If \( b \geq 4 \), then for any feedback law \( u \in U \), there always exists a set \( D \) (in the basic probability space) with positive probability such that

\[ |y_t| \to \infty, \text{ on } D \]

at a rate faster than exponential.

(ii). If \( b < 4 \), then the least-squares-based adaptive minimum variance feedback control \( u_t = -\theta_t f(y_t) \) where \( \theta_t \) is the least-squares estimate for \( \theta \) at time \( t \), can render the system to be globally stable and optimal, with the best rate of convergence:

\[ \sum_{t=1}^{T} (y_t - w_t)^2 = O(\log T), \quad \text{a.s., as } T \to \infty. \]

**Remark 3.1.** This result is somewhat surprising since the assumptions in our problem formulation have no explicit relationships with the value \( b = 4 \). We remark that the related results were first found and established in a somewhat general framework in [10]. In particular, the first part (i) was contained in Remark 2.2 in [10], and was later extended to general unknown parameter case in [12] by using a conditional Cramer-Rao inequality. The second part (ii) is a special case of Theorem 2.2 in [10].

**Remark 3.2.** There are many implications of Theorem 3.1. For example, the limitation of feedback given in Theorem 3.1 (i) is readily applicable to general class of uncertain systems of the form

\[ y_{t+1} = f_t(y_t, ..., y_{t-p}, u_t, ..., u_{t-q}) + w_{t+1}, \]
as long as it contains the basic class \( (3.1) \) as a subclass. Theorem 2.1 can also be used to show the fundamental differences between continuous-time and discrete-time nonlinear adaptive control (see also, [12]). Note that the noise free case can be trivially controlled, regardless of the growth rate of the nonlinear function \( f(\cdot) \) (see [11]). This means that the noise effect in (3.1) plays an essential role in the non-stabilizability result of Theorem 3.1 (i): the noise effect gives estimation errors to even the “best” parameter estimates, which are then amplified step by step by the nonlinearity of the system, leading to the final instability of the closed-loop systems, despite of the strong consistency of the parameter estimates (10) .

Theorem 3.1 concerns with the case where the unknown parameter \( \theta \) is a scalar. To see what happens when the number of the unknown parameters increases, let us consider the following polynomial nonlinear regression:

\[
y_{t+1} = \theta_1 y_t^{b_1} + \theta_2 y_t^{b_2} + \cdots + \theta_p y_t^{b_p} + u_t + w_{t+1}.
\]

Again, for simplicity, we assume that

A1)' \( b_1 > b_2 > \cdots > b_p > 0 \);
A2)' \( \{w_t\} \) is a sequence of independent random variable with a common distribution \( N(0,1) \);
A3)' \( \theta \overset{\Delta}{=} [\theta_1 \cdots \theta_p]^T \) is a random parameter with distribution \( N(\overline{\theta}, I_p) \).

Now, introduce a characteristic polynomial

\[
P(z) = z^{p+1} - b_1 z^p + (b_1 - b_2) z^{p-1} + \cdots + (b_{p-1} - b_p) z + b_p,
\]

which plays a crucial role in characterizing the limits of the feedback mechanism as shown by the following “impossibility theorem”.

**Theorem 3.2.** If there exists a real number \( z \in (1, b_1) \) such that \( P(z) < 0 \), then the above system (3.2) is not stabilizable by feedback. In fact, for any feedback law \( u \in U \) and any initial condition \( y_0 \in \mathbb{R}^1 \), it is always true that

\[
E|y_t|^2 \to \infty, \quad \text{as} \quad t \to \infty
\]

at a rate faster than exponential.

**Remark 3.3.** The proof of Theorem 3.2 can be found in [11], and extensions to non-Gaussian parameter case can be found in [12]. An important consequence of this theorem is that the system (3.2) is not stabilizable by feedback in general, whenever \( b_1 > 1 \) and the number of unknown parameters \( p \) is large (see [11]). This fact implies that the uncertain nonlinear system

\[
y_{t+1} = f(y_t) + u_t + w_t
\]

with \( f(\cdot) \) being unknown but satisfying

\[
\|f(x)\| \leq c_1 + c_2 \|x\|^b, \quad b > 1,
\]

may not be stabilizable by feedback in general. This gives us another fundamental limits on feedback in the presence of parametric uncertainties in nonlinear systems, and motivates the study of nonparametric control systems with linear growth conditions in the next section.
4. Nonparametric nonlinear systems

Consider the following first-order nonparametric control system:

\[ y_{t+1} = f(y_t) + u_t + w_{t+1}, \quad t \geq 0, \quad y_0 \in \mathbb{R}^1, \]

where \( \{y_t\} \) and \( \{u_t\} \) are the scalar output and input, and \( \{w_t\} \) is an “unknown but bounded” noise sequence, i.e. \( |w_t| \leq w, \forall t \), for some constant \( w > 0 \). The nonlinear function \( f(\cdot) : \mathbb{R}^1 \to \mathbb{R}^1 \) is assumed to be completely unknown. We are interested in understanding how much uncertainty in \( f(\cdot) \) can be dealt with by feedback. In order to do so, we need to introduce a proper measure of uncertainty first.

Now, define \( \mathcal{F} \triangleq \{ f : \mathbb{R}^1 \to \mathbb{R}^1 \} \) and introduce a quasi-norm on \( \mathcal{F} \) as follows:

\[ \| f \| = \lim_{\alpha \to \infty} \sup_{(x,y) \in \mathbb{R}^2} \frac{|f(x) - f(y)|}{|x - y| + \alpha}, \quad \forall f \in \mathcal{F}. \]

Having introduced the norm \( \| \cdot \| \), we can then define a ball in the space \( (\mathcal{F}, \| \cdot \|) \) centered at its “zero” \( \theta_F \) with radius \( L \):

\[ \mathcal{F}(L) \triangleq \{ f \in \mathcal{F} : \| f \| \leq L \} \]

where \( \theta_F \triangleq \{ f \in \mathcal{F} : \| f \| = 0 \} \). It is obvious that the size of \( \mathcal{F}(L) \) depends on the radius \( L \), which will be regarded as the measure of the size of uncertainty in our study to follow.

The following theorem establishes a quantitative relationship between the capability of feedback and the size of uncertainty.

**Theorem 4.1.** Consider the nonparametric control system (4.1). Then the maximum uncertainty that can be dealt with by feedback is a ball with radius \( L = \frac{3}{2} + \sqrt{2} \) in the normed function space \( (\mathcal{F}, \| \cdot \|) \), centered at the zero \( \theta_F \). To be precise,

(i) If \( L < \frac{3}{2} + \sqrt{2} \), then there exists a feedback law \( u \in U \) such that for any \( f \in \mathcal{F}(L) \), the corresponding closed-loop control system (4.1) is globally stable in the sense that

\[ \sup_{t \geq 0} \{|y_t| + |u_t|\} < \infty, \quad \forall y_0 \in \mathbb{R}^1; \]

(ii) If \( L \geq \frac{3}{2} + \sqrt{2} \), then for any feedback law \( u \in U \) and any initial value \( y_0 \in \mathbb{R}^1 \), there always exists some \( f \in \mathcal{F}(L) \) such that the corresponding closed-loop system (4.1) is unstable, i.e.,

\[ \sup_{t \geq 0} |y_t| = \infty. \]

The proof of the above theorem is given in [14], where it is also shown that once the stability of the closed-loop system is established, it is a relatively easy task to evaluate the control performance.

**Remark 4.1.** The stabilizing feedback law in Theorem 4.1 (i) can be constructed as follows (see [14]):

\[ u_t = \begin{cases} \ u_t', & \text{if } |y_t - y_{i_t}| > \epsilon \\ \ u_t'', & \text{if } |y_t - y_{i_t}| \leq \epsilon \end{cases} \]
where \( \epsilon > 0 \) is any given threshold. In other words, \( u_t \) is a switching feedback based on a stabilizing feedback \( u'_t \) and a tracking feedback \( u''_t \), which are defined as follows:

\[
u'_t = -\hat{f}_t(y_t) + \frac{1}{2}(b_t + \tau_t)
\]

where \( \hat{f} \) is the nearest neighbor (NN) estimate of \( f \) defined by

\[
\hat{f}_t(y) \triangleq y_{i_t+1} - u_{i_t},
\]

with

\[
i_t = \text{argmin}_{0 \leq i \leq t-1} |y_t - y_i|
\]

and where

\[
b_t = \min_{0 \leq i \leq t} y_i, \quad \tau_t = \max_{0 \leq i \leq t} y_i
\]

The tracking feedback \( u''_t \) is defined

\[
u''_t = -\hat{f}_t(y_t) + y_{t+1}^*,
\]

where \( \{y^*_t\} \) is a bounded reference sequence. It is obvious that \( u_t \) depends on the observations \( \{y_0, y_1, \ldots, y_t\} \) only.

One may try to generalize Theorem 4.1 to the following high-order nonlinear systems (\( p \geq 1 \)):

\[
y_{t+1} = f(y_t, y_{t-1}, \ldots, y_{t-p+1}) + u_{t+1} + w_{t+1}
\]

where \( f(\cdot) : \mathbb{R}^p \to \mathbb{R}^1 \) is assumed to be completely unknown, but belongs to the following class of Lipschitz functions:

\[
\mathcal{F}(L) = \{ f(\cdot) : |f(x) - f(y)| \leq L\|x - y\|, \forall x, y \in \mathbb{R}^p \}
\]

where \( L > 0, \|x\| = \sum_{i=1}^{p} |x_i|, x = (a_1, \ldots, x_p)^T \in \mathbb{R}^p \). Again, \( \{w_t\} \) is a sequence of “unknown but bounded” noises. The following “impossibility theorem” is established in [17].

**Theorem 4.2.** If \( L \) and \( p \) satisfy

\[
L + \frac{1}{2} \geq (1 + \frac{1}{p})(pL)^1/p
\]

then there does not exist any globally stabilizing feedback law for the class of uncertain systems \( \{4.2\} \) with \( f \in \mathcal{F}(L) \).

It is easy to see that if \( p = 1 \) then the above inequality \( 4.3 \) reduces to \( L \geq \frac{3}{2} + \sqrt{2} \), which we know to be a critical value for the capability of feedback by Theorem 4.1. However, when \( p > 1 \) and \( 4.3 \) does not hold, whether or not there exists a stabilizing feedback for the uncertain system \( \{4.2\} \) with \( f \in \mathcal{F}(L) \) still remains as an open question.
5. Other uncertain systems

In this section, we briefly mention some related results on other basic classes of uncertain systems.

Let us first consider the following simple but basic continuous-time system:

$$\dot{x}_t = f(x_t) + u_t, \quad t \geq 0, x_0 \in \mathbb{R}^1. \tag{5.1}$$

The system signals are assumed to be sampled at a constant rate $h > 0$, and the input is assumed to be implemented via the familiar zero-order hold device (i.e., piecewise constant functions):

$$u_t = u_{kh}, \quad kh \leq t < (k+1)h, \tag{5.2}$$

where $u_{kh}$ depends on $\{x_0, x_h, \ldots, x_{kh}\}$.

The nonlinear function $f$ in (5.1) is assumed to be unknown but belongs to the following class of local Lipschitz (LL) functions:

$$G^L_c = \{ f \mid f \text{ is LL and satisfies } |f(x)| \leq L|x| + c, \forall x \in \mathbb{R}^1 \}, \tag{5.3}$$

where $c > 0$ and $L > 0$ are constants. The “slope” $L$ of the unknown nonlinear functions in $G^L_c$ may be regarded as a measure of the size of the uncertainty. Similar to the discrete-time case in Theorem 4.1, $L$ plays a crucial role in the determination of the capability and limits of the sample-data feedback [13].

**Theorem 5.1.** Consider the sampled-data control system (5.1)–(5.2). If $Lh > 7.53$, then for any $c > 0$ and any sampled-data control $\{u_{kh}, k \geq 0\}$ there always exists a function $f^* \in G^L_c$, such that the state signal of (5.1)–(5.2) corresponding to $f^*$ with initial point $x_0 = 0$ satisfies $(k \geq 1)$

$$|x_{kh}| \geq (\frac{Lh}{2})^{k-1}ch \quad k \to \infty.$$

**Remark 5.1.** This “impossibility theorem” shows that if $Lh$ is larger than a certain value, then there will exist no stabilizing sampled-data feedback. On the other hand, it is easy to show that if $Lh < \log 4$, then a globally stabilizing sampled-data feedback can be constructed (see [13]). An obvious open question here is how to bridge the gap between $\log 4$ and 7.53. Needless to say, Theorem 5.1 gives us some useful quantitative guidelines in choosing properly the sampling rate in practical applications.

Next, we consider the following linear time-varying stochastic model:

$$x_{t+1} = A(\theta_t)x_t + B(\theta_t)u_t + w_{t+1}, \quad t \geq 1; \tag{5.4}$$

where $x_t \in \mathbb{R}^n, u_t \in \mathbb{R}^m$ and $w_{t+1} \in \mathbb{R}^n$ are the state, input and noise vectors respectively. We assume that

H1) $\{\theta_t\}$ is an unobservable Markov chain which is homogeneous, irreducible and aperiodic, and which takes values in a finite set $\{1, 2, \cdots, N\}$ with transition matrix denoted by $P = (p_{ij})_{NN}$, where by definition $p_{ij} = P\{\theta_t = j | \theta_{t-1} = i\}$. 
H2) There exists some $m \times n$ matrix $K$ such that $\det[(A_i - A_j) - (B_i - B_j)K] \neq 0$, $\forall i \neq j, 1 \leq i, j \leq N$, where $A_i \triangleq A(i) \in \mathbb{R}^{nn}$, $B_i \triangleq B(i) \in \mathbb{R}^{nm}$ are the system matrices.

H3) $\{w_t\}$ is a martingale difference sequence which is independent of $\{\theta_t\}$, and satisfies $\sigma I \leq Ew_tw_t' \leq \sigma w$, $\forall t$, where $\sigma$ and $\sigma_w$ are two positive constants, and the prime superscript represents matrix transpose.

For simplicity of presentation, we denote $S \triangleq \{1, 2, \ldots, N\}$. The following theorem gives a fairly complete characterization of feedback stabilizability for the hidden Markovian model (5.4) [16].

**Theorem 5.2.** Let the above Assumptions H1)–H3) hold for the dynamical system (5.4) with hidden Markovian switching. Then the system is stabilizable by feedback if and only if the following coupled algebraic Riccati-like equations have a solution consisting of $N$ positive definite matrices $\{M_i > 0, i \in S\}$:

$$\sum_j A'_j p_{ij} M_j A_j - \left( \sum_j A'_j p_{ij} M_j B_j \right) \left( \sum_j B'_j p_{ij} M_j B_j \right)^+ \left( \sum_j B'_j p_{ij} M_j A_j \right) - M_i = -I,$$

where $i \in S$ and $(\cdot)^+$ denotes the Moore-Penrose generalized-inverse of the corresponding matrix.

**Remark 5.2.** Theorem 5.2 shows that the capability of feedback depends on both the structure complexity measured by $\{A_j, B_j, 1 \leq j \leq N\}$ and the information uncertainty measured by $\{p_{ij}, 1 \leq i, j \leq N\}$. To make it more clear in understanding how the capability of feedback depends on both the complexity and uncertainty of the system, we consider the simple scalar variable case with $B(\theta_t) = 1$, where the Markov chain $\{\theta_t\}$ has two states $\{1, 2\}$ only and $p_{12} = p_{21}$. It can be shown by Theorem 5.2 that the system is stabilizable if and only if $CP < 1$, where $C \triangleq (A_2 - A_1)^2$ and $P \triangleq (1 - p_{12})p_{12}$ can be interpreted as measures of the structure complexity (degree of dispersion) and the information uncertainty respectively (see [15] for details).

6. **Concluding remarks**

For several basic classes of uncertain dynamical control systems, we have given some critical values or equations to characterize the capability and limits of the feedback mechanism, and have shown that “impossibility theorems” hold even for some seemingly simple uncertain dynamical systems. Of course, many important problems still remain open. Examples are as follows:

(i) For general high-dimensional or high-order uncertain nonlinear control systems, to find critical conditions characterizing the capability of feedback, at least to find general sufficient conditions under which feedback stabilization is possible in the discrete-time case.

(ii) To characterize the maximum capability of feedback that is designed based on switched linear control models, in dealing with uncertain nonlinear dynamical systems.
(iii) To find a suitable mathematical framework within which the issue of establishing a quantitative relationship among *a priori* information, feedback performance and computational complexity can be addressed adequately.

**References**

[1] Wiener, N., *Cybernetics, or Control and Communication in the Animal and the Machine*, MIT Press, 1948.

[2] Zames, G., ‘Feedback and optimal sensitivity: Model reference transformations, weighted seminorms and approximate inverses’, *IEEE Trans. Automat. Contr.*, 23(1981), 301–320.

[3] Zhou, K., J. C. Doyle and K. Glover, *Robust and Optimal Control*, Prentice-Hall, 1996.

[4] Åström, K. J. and B. Wittenmark, *Adaptive Control*, Addison-Wesley, Reading, MA, 2nd ed., 1995.

[5] Chen, H., and L. Guo, *Identification and Stochastic Adaptive Control*. Boston, MA: Birkhäuser, 1991.

[6] Ioannou, P. A., and J. Sun, *Robust Adaptive Control*, Englewood Cliffs, NJ: Prentice-Hall, 1996.

[7] Krstić, M., I. Kanellakopoulos and P. V. Kokotović, *Nonlinear and Adaptive Control Design*. New York: John Wiley & Sons, 1995.

[8] Guo, L., and H. F. Chen, ‘The Åström-Wittenmark self-tuning regulator revisited and ELS-based adaptive trackers’, *IEEE Trans. Automat. Contr.*, 36(7)(1991), 802–812.

[9] Guo, L., ‘Self-convergence of weighted least-squares with applications to stochastic adaptive control’, *IEEE Trans. Automat. Contr.*, 41(1)(1996), 79–89.

[10] Guo, L., ‘On critical stability of discrete-time adaptive nonlinear control’, *IEEE Trans. Automat. Contr.*, 42(11)(1997), 1488–1499.

[11] Xie, L. L., and L. Guo, ‘Fundamental limitations of discrete-time adaptive nonlinear control’, *IEEE Trans. Automat. Contr.*, 44(9)(1999), 1777–1782.

[12] Xie, L. L., and L. Guo, ‘Adaptive control of discrete-time nonlinear systems with structural uncertainties’, *AMS/IP Studies in Mathematics*, 17(2000), 49–89.

[13] Xue, F. and L. Guo, ‘Stabilizability, uncertainty and the choice of sampling rate’, Proc. *IEEE-CDC*, December 2000, Sydney.

[14] Xie, L. L., and L. Guo, ‘How much uncertainty can be dealt by feedback?’ *,IEEE Trans. on Automatic Control*, 45(12)(2000), 2203–2217.

[15] Xue, F., L. Guo and M. Y. Huang, ‘Towards understanding the capability of adaptation for time-varying systems’, *Automatica*, 37(2001), 1551–1560.

[16] Xue, F. and L. Guo, ‘ Necessary and sufficient conditions for adaptive stabilizability of jump linear systems’, *Communications in Information and Systems*, 2(1),(2001), 205–224.

[17] Zhang, Y. X., and L. Guo, ‘A limit to the capability of feedback’, *IEEE Trans. Automatic Control*, Vol.47, No.4 (2002), 687-692.