A MANIFESTLY GAUGE INVARIANT EXACT
RENNORMALIZATION GROUP

Stefano Arnone¹, Antonio Gatti², Tim R. Morris³

Department of Physics and Astronomy, University of Southampton
Highfield, Southampton SO17 1BJ, United Kingdom.

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A manifestly gauge invariant exact renormalization group for pure SU(N) Yang-Mills theory is proposed, allowing gauge invariant calculations, without any gauge fixing or ghosts. The necessary gauge invariant regularisation which implements the effective cutoff, is naturally incorporated by embedding the theory into a spontaneously broken SU(N|N) super-gauge theory. This guarantees finiteness to all orders in perturbation theory.

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1 ERG and gauge invariance

The basic idea of the exact renormalization group (ERG) is summarised in the diagram below. For a detailed review, and current developments, see for example [1, 2]. In the partition function for the theory, defined in the continuum and in Euclidean space, rather than integrate over all momentum modes in one go, one first integrates out modes between an overall cutoff \( \Lambda_0 \) and the effective Wilsonian cutoff \( \Lambda \ll \Lambda_0 \). The remaining integral can again be expressed as a partition function, but the bare action, \( S_{\Lambda_0} \), is replaced by an effective action, \( S \). The new Boltzmann factor, \( \exp(-S) \), is more or less the original partition function, modified by an infrared cutoff \( \Lambda \) [3]. When finally \( \Lambda \) is sent to zero, the full partition function is recovered and all the physics that goes with it (e.g. Green functions). In practice however, one does not work with this integral form but rather a differential equation for \( S \), the ERG equation, that expresses how \( S \) changes as one lowers \( \Lambda \).

The application of this technique to quantum field theory brings with it many advantages, because renormalization properties, which are normally subtle and complicated, are here —using Wilson’s insight [4]— straightforward to build in from the beginning. Thus solutions for the effective action may be found directly in terms of renormalized quantities (in fact without specifying a bare action at \( \Lambda_0 \), which is anyway, by universality, largely arbitrary), and within this framework almost any approximation can be considered (for example truncations [5], derivative

¹E-mail address: S.Arnone@soton.ac.uk
²E-mail address: A.Gatti@soton.ac.uk
³E-mail address: T.R.Morris@soton.ac.uk
expansion [6] etc.) without disturbing this property [1]. The result is that these ideas form a powerful framework for considering non-perturbative analytic approximations in quantum field theory [2].

In particle physics, all the interesting non-perturbative questions also involve gauge theory. However, in order to construct a gauge invariant ERG, we must overcome an obvious conflict: the division of momenta into large and small, according to the effective scale $\Lambda$, is not preserved by gauge transformations. (Explicitly, consider a matter field $\phi(x)$. Under a gauge transformation $\phi(x) \rightarrow \Omega(x) \phi(x)$ momentum modes $\phi(p)$ are mapped to a convolution with the modes from $\Omega$.) We only have two choices. Either we break the gauge invariance and try to recover it once the cutoff is removed, by imposing suitable boundary conditions on the ERG equation [7], or we generalise things so that we can write down a gauge invariant ERG equation.

We will go with the second choice [9, 10, 11]. Furthermore, we will find that we can continue to keep the gauge invariance manifest at all stages even when we start to compute the effective action. No gauge fixing or ghosts are required. The full power and beauty of gauge invariance thus shines through in the calculations, as will all become clear in the next talk [8].

2 Regularisation via $SU(N|N)$

2.1 General idea

As a necessary first step, we need a gauge invariant implementation of the non-perturbative continuum effective cutoff $\Lambda$. The standard ERG cutoff is implemented by inserting $c^{-1}(p^2/\Lambda^2)$ into the kinetic term of the action. $c$ is a smooth ultraviolet cutoff profile with $c(0) = 1$, decaying sufficiently rapidly as $p/\Lambda \rightarrow \infty$ that all quantum corrections are regularised. To restore the gauge invariance we covariantise so that the regularised bare action takes the form:

$$\frac{1}{2g^2} \text{tr} \int d^Dx \ F_{\mu\nu} \ c^{-1} \left( -D^2/\Lambda^2 \right) \cdot F^{\mu\nu}. \quad (1)$$
Here $F_{\mu\nu} = i[D_\mu, D_\nu]$ is the standard field strength, built from the covariant derivative $D_\mu = \partial_\mu - iA_\mu$. We scale out the coupling $g$ for good reason: since gauge invariance will be exactly preserved, the form of the covariant derivative is protected [12], which in this parametrisation simply means that $A$ suffers no wavefunction renormalization. Eq. (1) is nothing but covariant higher derivative regularisation and is known to fail at one-loop [13]. Slavnov solved this problem by introducing gauge invariant Pauli-Villars fields [14]. These appear bilinearly so that their one-loop determinants cancel the remaining divergences. We cannot use these ideas directly since the bilinearity property cannot be preserved by the ERG flow [9, 11]. Instead, we discovered a novel and elegant solution: we embed (1) in a spontaneously broken $SU(N|N)$ super-gauge theory [15]. We will see that the result has similar characteristics to Slavnov’s scheme but sits much more naturally in the effective action framework. Indeed the regularising properties will follow from the supersymmetry in the fibres of the high energy unbroken supergroup. We will then design an ERG in which the spontaneous breaking scale and higher derivative scale are identified and flow together as we lower $\Lambda$.

### 2.2 The $SU(N|N)$ super group

The graded Lie algebra of $SU(N|M)$ in the $(N+M)$-dimensional representation is given by

$$\mathcal{H} = \begin{pmatrix} H_N & \theta \\ \theta^\dagger & H_M \end{pmatrix}. \quad (2)$$

$H_N$ ($H_M$) is an $N \times N$ ($M \times M$) Hermitian matrix with complex bosonic elements and $\theta$ is an $M \times N$ matrix composed of complex Grassmann numbers. $\mathcal{H}$ is required to be supertraceless, i.e.

$$\text{str}(\mathcal{H}) = \text{tr}(\sigma_3 \mathcal{H}) = \text{tr}(H_N) - \text{tr}(H_M) = 0 \quad (3)$$

(where $\sigma_3 = \text{diag}(1_N, -1_M)$ is the obvious generalisation of the Pauli matrix to this context). The traceless parts of $H_N$ and $H_M$ correspond to $SU(N)$ and $SU(M)$ respectively and the traceful part gives rise to a $U(1)$, so we see that the bosonic sector of the $SU(N|M)$ algebra forms a $SU(N) \times SU(M) \times U(1)$ sub-algebra.

Specialising to $M = N$, we see that the $U(1)$ generator becomes just $1_{2N}$ and thus commutes with all the other generators. We cannot simply drop it however because it is generated by other elements of the algebra (e.g. $\{\sigma_1, \sigma_1\} = 21_{2N}$). Bars suggested removing it by redefining the Lie bracket to project out traceful parts [16]: $[,]_\pm \mapsto [\, ,]_\pm - \frac{1}{2N}\text{tr}[\, ,]_\pm$. We can use this idea but only on the gauge fields: the matter fields require the full commutator because invariance of the Lagrangian in this sector requires the bracket to be Leibnitz [15]. A simpler and equivalent solution [15] is to keep the $1_{2N}$ and note that the corresponding gauge field, $A_0^0$, which we have seen is needed to absorb gauge transformations produced in the $1_{2N}$ direction, does not however appear in the Lagrangian at all! (Its absence is then protected by a no-$A_0^0$ shift-symmetry: $\delta A_\mu^0 = \Lambda_\mu$.)

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**Gauge invariant ERG**

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2.3 Higher derivative $SU(N|N)$ super-gauge theory

We promote the gauge field to a connection for $SU(N|N)$:

$$A_\mu = A^0_\mu \mathbb{I} + \begin{pmatrix} A^1_\mu & B_\mu \\ \bar{B}_\mu & A^2_\mu \end{pmatrix},$$

where the $A^i_\mu$ are two the bosonic gauge fields for $SU(N) \times SU(N)$, and $B_\mu$ is a fermionic gauge field. The field strength $F_{\mu\nu}$ is now a commutator of the super-covariant derivative $\nabla_\mu = \partial_\mu - i A_\mu$. The super-gauge field part of the Lagrangian is then

$$L_A = \frac{1}{2g^2} F_{\mu\nu} \{c^{-1}\} F^{\mu\nu}.$$  \hspace{1cm} (5)

Here we take the opportunity to be more sophisticated about the covariantization of the cutoff. For any momentum space kernel $W(p^2/\Lambda^2)$, there are infinitely many covariantizations. The form used in (1) is just one of them. Another way would be to use Wilson lines [12, 9]. In general, the covariantization results in a new set vertices (infinite in number if $W$ is not a polynomial):

$$u(W)v = \sum_{n,m=0} \int x,y \int x_{i_1},y_{j_1} \ldots x_{i_n},y_{j_m} W_{\mu_1 \ldots \mu_n,\nu_1 \ldots \nu_m}(x_{i_1};y_{j_1};x,y)$$

$$\text{str}[u(x)A_{\mu_1}(x_1) \cdots A_{\mu_n}(x_n)v(y)A_{\nu_1}(y_1) \cdots A_{\nu_m}(y_m)].$$  \hspace{1cm} (6)

($u(x)$ and $v(y)$ are any two supermatrix representations.) These can be graphically represented as in Fig. 2.

Fig. 2. Expansion of the covariantization in terms of super-gauge fields.

2.4 Spontaneous breaking in fermionic directions

Now we add a super-scalar field, $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_C$,

$$\mathcal{C} = \begin{pmatrix} C^1 \\ D \\ C^2 \end{pmatrix} \in U(N|N),$$

with a Lagrangian that encourages spontaneous symmetry breaking:

$$\mathcal{L}_C = \frac{1}{2} \nabla_\mu \cdot C \{\tilde{c}^{-1}\} \nabla^\mu \cdot C + \frac{\lambda}{4} \text{str}\left[ (C^2 - \Lambda^2)^2 \right].$$  \hspace{1cm} (8)

Choosing the classical vacuum expectation value $\Lambda \sigma_3$ breaks all and only the fermionic directions, and expanding about this, by $C \mapsto C + \Lambda \sigma_3$, gives

$$\mathcal{L}_C = \frac{1}{2} \nabla_\mu \cdot C \{\tilde{c}^{-1}\} \nabla^\mu \cdot C - i \Lambda [A_\mu, \sigma_3] \{\tilde{c}^{-1}\} \nabla^\mu \cdot C$$

$$- \frac{1}{2} \Lambda^2 [A_\mu, \sigma_3] \{\tilde{c}^{-1}\} [A^\mu, \sigma_3] + \frac{\lambda}{4} \text{str}\left[ (\Lambda \sigma_3 + C^2)^2 \right].$$  \hspace{1cm} (9)
Since (fermionic) bosonic parts (anti)commute with $\sigma_3$, we see in the second line that $B$ gains a mass $\sqrt{2}\Lambda$ ($B$ eats $D$), and $C^1$ and $C^2$ gain masses $\sqrt{2}\lambda\Lambda$. These heavy fields play the rôle of Slavnov’s gauge invariant Pauli-Villars fields.

### 2.5 Proof of regularisation

A proof that this all adds up to a regularisation of four dimensional $SU(N)$ Yang-Mills theory has been given in [15]. We only have room to summarise the conclusions.

If $c^{-1}$ and $\tilde{c}^{-1}$ are chosen to be polynomials of rank $r$, $\tilde{r}$, we require $r > \tilde{r} - 1$ and $\tilde{r} > -1$ simply to ensure that at high momentum the propagators go over to those of the unbroken $SU(N|N)$ theory. The stronger constraints $r > 1$ and $r - \tilde{r} > 1$ then ensure finiteness in all perturbative diagrams except pure $A$ one-loop graphs with up to 4 external legs. This maximises the regularising power of the covariant higher derivatives and is ensured simply by power counting. The remaining diagrams can be shown to be finite within spontaneously broken $SU(N|N)$ gauge theory as follows. One-loop diagrams with 2 or 3 external $A$ legs are finite because of supersymmetric cancellations in group theory factors: $\text{str}A_\mu = \text{str} \mathbb{1} = 0$. Transverse parts of such diagrams with four external legs are finite by power counting, whilst the longitudinal parts are finite once gauge invariance properties are taken into account [15].

Finally, we need to show that at energies much lower than the cutoff, the theory we are supposed to be regularising is recovered, namely $SU(N)$ Yang-Mills. (In the ERG context the cutoff in question is $\Lambda_0 \to \infty$ where explicitly or implicitly, the partition function is defined.) There is a case to answer because the massless sector that remains, contains the second gauge field, $A^2$. In fact this gauge field is unphysical because the supertrace in (3) gives it a wrong sign action, as can be seen from eq. (5), leading to negative norms in its Fock space [15]. Fortunately, the Appelquist-Carazzone theorem saves the day: since the $A^1$ and $A^2$ live in disjoint groups, the lowest dimension interaction between $A^1$ and $A^2$ is proportional to $\text{tr}(F_{1\mu}^2) \text{tr}(F_{2\mu}^2)$. Since this is irrelevant, the $A^2$ sector decouples in the limit that $\Lambda_0 \to \infty$.

This completes the proof of finiteness to all orders of perturbation theory, in four (or less) dimensions. In the limit $N = \infty$, the scheme can be shown to regularise in any dimension [15].

### 3 Manifestly gauge invariant flow equation

#### 3.1 Polchinski’s equation

We are ready to write a gauge invariant flow equation. To motivate it consider Polchinski’s version of Wilson’s ERG [17]. We can cast it in the form

$$\Lambda \partial_\Lambda S = -\frac{1}{\Lambda^2} \frac{\delta S}{\delta \varphi} \cdot c' \cdot \frac{\delta \Sigma}{\delta \varphi} + \frac{1}{\Lambda^2} \frac{\delta}{\delta \varphi} \cdot c' \cdot \frac{\delta \Sigma}{\delta \varphi}.$$  

(10)

Here $\varphi$ is for example a single scalar field. $\Sigma$ is the combination $S - 2\tilde{S}$, where $\tilde{S}$ is the regularised kinetic term $\tilde{S} = \frac{1}{2} \partial_\mu \varphi \cdot c^{-1} \cdot \partial_\mu \varphi$. In this form it is clear that the ERG leaves the partition function invariant because the Boltzmann measure factor flows into a total functional derivative:

$$\Lambda \partial_\Lambda \exp -S = -\frac{1}{\Lambda^2} \frac{\delta}{\delta \varphi} \cdot c' \cdot \left( \frac{\delta \Sigma}{\delta \varphi} \exp -S \right).$$ 

(11)

The supertrace is a necessity since it is this, not the trace, that leads to invariants when supergroups are used [16, 15].
At this stage we recognize that there is nothing particularly special about the Polchinski / Wilson version. There are infinitely many other ERG flow equations with this property [18], the continuum analogue of the infinitely many possible blockings on the lattice. All we have to do is to choose a gauge invariant one by making a gauge covariant replacement for \( \Psi = c' \cdot \frac{\delta \hat{S}}{\delta \phi} \).

### 3.2 \( SU(N) \) gauge invariant ERG

Writing \( \varphi \rightarrow A_\mu \), this can be done simply by replacing \( c' \cdot \) with \( \{ c' \} \) and replacing \( \hat{S} \) with a gauge invariant generalisation. Thus:

\[
\Lambda \partial_\Lambda S = -\frac{1}{\Lambda^2} \frac{\delta S}{\delta A_\mu} \{ c' \} \frac{\delta \Sigma_g}{\delta A_\mu} + \frac{1}{\Lambda^2} \frac{\delta}{\delta A_\mu} \{ c' \} \frac{\delta \Sigma_g}{\delta A_\mu} + \cdots,
\]

where \( \hat{S} = \frac{1}{4} F_{\mu \nu} \{ c^{-1} \} F_{\mu \nu} + \cdots \). Recall that the coupling \( g \) was scaled out, cf. eq. (1). It must reappear somewhere in the flow equation and some thought shows that the appropriate place is in the combination \( \Sigma_g = g^2 S - 2 S \). We have added the ellipsis in the recognition that further regularisation will be needed over and above the gauge invariant higher derivatives.

### 3.3 \( SU(N|N) \) gauge invariant ERG

We get the remaining regularisation by promoting \( A \) to \( A \), adding the super-scalar sector, and then shifting \( C \) to the fermionic symmetry breaking vacuum expectation value. We want to ensure that under the ERG flow, this vacuum expectation value flows with the effective cutoff, i.e. as \( \langle C \rangle = \Lambda \sigma_3 \). One can show that this follows at the classical level if we work instead with a dimensionless superscalar, \( \hat{C} \rightarrow \Lambda^2 \hat{C} \), so that the shift becomes \( \hat{C} \rightarrow \hat{C} + \sigma_3 \). It is technically very convenient if the ERG equation allows for the classical two-point vertices to be equal to those coming from \( \hat{S} \) [11], as is true (10) and (12) above. To keep this property in the spontaneously broken phase we need different kernels for \( B \) and \( D \) which we can make by adding \( \hat{C} \) commutator terms. Constructing the appropriate \( \Psi \), we thus obtain in the symmetric phase, a fully manifestly \( SU(N|N) \) gauge invariant flow equation: \( \Lambda \partial_\Lambda S = -a_0[S, \Sigma_g] + a_1[\Sigma_g] \), where

\[
a_0[S, \Sigma_g] = \frac{1}{2 \Lambda^2} \left( \frac{\delta S}{\delta A_\mu} \{ c' \} \frac{\delta \Sigma_g}{\delta A_\mu} - \frac{1}{4} \{ C, \frac{\delta S}{\delta A_\mu} \} \{ M \} \{ C, \frac{\delta \Sigma_g}{\delta A_\mu} \} \right)
+ \frac{1}{2 \Lambda^2} \left( \frac{\delta S}{\delta C} \{ H \} \frac{\delta \Sigma_g}{\delta C} - \frac{1}{4} \{ C, \frac{\delta S}{\delta C} \} \{ L \} \{ C, \frac{\delta \Sigma_g}{\delta C} \} \right),
\]

\[
a_1[\Sigma_g] = \frac{1}{2 \Lambda^2} \left( \frac{\delta}{\delta A_\mu} \{ c' \} \frac{\delta \Sigma_g}{\delta A_\mu} - \frac{1}{4} \{ C, \frac{\delta}{\delta A_\mu} \} \{ M \} \{ C, \frac{\delta \Sigma_g}{\delta A_\mu} \} \right)
+ \frac{1}{2 \Lambda^2} \left( \frac{\delta}{\delta C} \{ H \} \frac{\delta \Sigma_g}{\delta C} - \frac{1}{4} \{ C, \frac{\delta}{\delta C} \} \{ L \} \{ C, \frac{\delta \Sigma_g}{\delta C} \} \right).
\]

In here, we can take \( \hat{S} = \int d^4 x (L_A + L_c) \), although there is considerable flexibility over the exact choice as there is with the covariantization, and recognising this, we were able to turn this to our advantage [8, 20]. The kernels \( M, H \) and \( L \) are then determined in terms of \( c, \tilde{c} \) and other parameters in \( \hat{S} \) (here \( \lambda \)) by the requirement that the classical solution \( \hat{S} \) can have the same two-point vertices as \( \hat{S} \). Although \( g \) appears explicitly as a parameter in these flow equations, it is
not yet defined as the running Yang-Mills coupling. As usual, this is done via a renormalisation condition: for the pure $A^1$ part we require

$$S = \frac{1}{2g^2(\Lambda)} \text{tr} \int d^4x (F^1_{\mu\nu})^2 + O(D^3).$$  \hspace{1cm} (14)$$

At first sight, it appears that we have specialized the kernels for the gauge fields so that no longitudinal terms appear. In fact, any longitudinal term $\sim \nabla_\mu \cdot \delta S / \delta A^\mu$ may be converted to $C$ commutator terms, $C \cdot \delta S / \delta C$, i.e. $L$ type terms, via $SU(N|N)$ gauge invariance.

The supermatrix functional derivatives are most easily computed by noting that they have a very simple effect on supertraces. Either we have ‘supersowing’, $\text{str} A \frac{\delta}{\delta X} \text{str} XB = \text{str} AB$, or ‘supersplitting’, $\text{str} \frac{\delta}{\delta X} AXB = \text{str} A \text{str} B$. Drawing single supertraces as closed curves, and using Fig. 2, we get a useful diagrammatic interpretation which counts supertraces, analogous to the ’t Hooft double-line notation [19]. These relations follow from the completeness relations for the supergenerators. Just as in the analogous formulae for $SU(N)$, generically there are $1/N$ corrections, but they involve ordinary traces (or equivalently $\sigma_3$) which would violate $SU(N|N)$. In the case of this $SU(N|N)$ gauge theory, they must all cancel out and they do [15, 20], so the double line notation is exact – even at finite $N$ [20].

Finally, shifting $C$ to $C + \sigma_3$, allows us to perform computations in which not only unbroken $SU(N) \times SU(N)$ gauge invariance, but also broken fermionic gauge invariance, is manifestly preserved at every step. As well as providing yet another beautiful balance in the formalism, one sees very clearly how a massive vector field ($B$) as created by spontaneous symmetry breaking, and its associated Goldstone mode ($D$), actually form a single unit, tied together by the underlying gauge invariance [20].

4 Summary and conclusions

A manifestly gauge invariant ERG, together with the necessary non-perturbative gauge invariant regularisation scheme, has been proposed. No gauge fixing is required to define it, and as we will see in the next talk no gauge fixing is needed to compute the solutions either [9, 10, 11], thus avoiding the Gribov problem [21]. Although there has been no room to explain it, especially the gauge sector (12), may be reinterpreted in terms of Wilson loops, the natural order parameter for gauge theory. The ERG then has an interpretation in the large $N$ limit as quantum mechanics of a single Wilson loop, with close links to the Migdal-Makeenko equation [10, 22].

As explained in the next lecture, we are in the midst of developing and testing perturbatively, powerful methods of computation in gauge theory based on the above ERG. For the future, we intend to include matter in the fundamental representation, and turn our attention to non-perturbative approximations and QCD. It also seems a simple matter to incorporate space-time supersymmetry, opening up intriguing possibilities for deeper investigations of Seiberg-Witten methods and the AdS/CFT correspondence [23, 24, 25].

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