Energy dependence of average transverse momentum in hadron production due to collective effects

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Abstract

Motivated by the first measurements of the experiment CMS at the LHC at $\sqrt{s} = 0.9$ and 2.36 TeV, we discuss energy dependence of average transverse momentum of the secondary particles in hadron production in pp collisions. We suggest a possible explanation of this dependence as a result of collective rotation of the transient state and associate its further possible decrease with flattening off at higher energies with transition to the genuine QGP state of matter.
Introduction

Nowadays the LHC enters the initial phase of collecting data and providing first experimental results. Along with realization of its discovery potential, start of the LHC experimental program will definitely lead to the renewed interest to the well known problems and deepened insights into those issues. Multiparticle production and global observables such as average transverse momentum of produced particles provide an information on the mechanisms of deconfinement, hadronization as well as on the size and the temperature of an interacting system. The first experimental results obtained at the initial LHC energies $\sqrt{s} = 0.9$ and 2.36 TeV appear to be consistent with the previous experimental measurements, which demonstrate rising energy dependence of the average transverse momentum of secondaries. They might imply description in the thermodynamics terms [2]. Origin of the energy dependent behavior can be related to the observation of the possible new states of hadronic matter and therefore studies of this dependence can provide clues on the phase transition between them as it was noticed by Van Hove in the context of the starting the S$p$p$S$ experiments at CERN [3]. It is interesting to note that the experimentally observed one-particle transverse momentum spectrum with power-law tail is consistent with canonical Tsallis distribution associated with non-extensive thermodynamics (cf. e.g. [4]). Thus, the existing models relate energy dependence of transverse momentum with different mechanisms, but it is evident that such dependence has a non-trivial origin and reflects some gross features of the interaction dynamics.

In this note we suggest a new explanation of the energy dependence of the average transverse momentum emphasizing that the collective effects in in the initial state of the proton collisions might be a ground for this energy dependence. Proceeding this way, we adhere to the similarities between hadron and nucleus collisions and use the geometrical notions for description of multiparticle production in hadronic reactions as it was done long ago by Chou and Yang [5]. Of course, there should be obvious differences in hadron and nuclei cases related to the size of colliding systems, but those differences are mostly quantitative ones.

In the following section we discuss what kind of degrees of freedom are responsible for the transient state of matter in hadron interaction and after that we use these notions to make conclusion on the nature the energy dependence of the average transverse momentum.

1 Transient state of matter in hadron interactions

It might happen therefore that the transient states of matter in hadron and nuclei collisions have the same nature and originates from the nonperturbative sector of
QCD, which has degrees of freedom associated with the mechanism of spontaneous chiral symmetry breaking ($\chi$SB) in QCD [6]. Due to this mechanism transition of current into constituent quarks occur, which are the quasiparticles whose masses are comparable with a typical hadron mass scale. These constituent quarks interact via exchange of the Goldstone bosons. Goldstone bosons are collective excitations of the condensate and are represented by pions (cf. e.g. [7]). The general form of the effective Lagrangian ($L_{QCD} \rightarrow L_{\text{eff}}$) relevant for description of the non-perturbative phase of QCD includes the three terms [8]

$$L_{\text{eff}} = L_{\chi} + L_{I} + L_{C}.$$  

Here $L_{\chi}$ is responsible for the spontaneous chiral symmetry breaking and turns on first. To account for the constituent quark interaction and confinement the terms $L_{I}$ and $L_{C}$ are introduced. $L_{I}$ and $L_{C}$ do not affect the internal structure of the constituent quarks.

The picture of a hadron consisting of constituent quarks embedded into quark condensate implies that overlapping and interaction of peripheral clouds occurs at the first stage of hadron interaction. Nonlinear field couplings transform then some part of the kinetic energy to internal energy [9, 10]. As a result the massive virtual quarks appear in the overlapping region and the effective field is generated that way. This field is generated by $\bar{Q}Q$ pairs and pions strongly interacting with quarks. Pions themselves are bound states of constituent quarks. During this stage the part of the effective Lagrangian $L_{C}$ is turned off (it turns on again in the final stage of the reaction) and interaction is described by $L_{I}$, its possible form of $L_{I}$ was discussed in [11]. The effective field (transient phase) generation time $\Delta t_{\text{eff}}$

$$\Delta t_{\text{eff}} \ll \Delta t_{\text{int}},$$

where $\Delta t_{\text{int}}$ is the total interaction time. This assumption on the almost instantaneous generation of the effective field has obtained support in the very short thermalization time revealed in heavy-ion collisions at RHIC [12].

This picture assumes deconfinement at the initial stage of the hadron collisions and generation of common for both hadrons mean field during the first stage. Such ideas were used in the model [13] which has been applied to description of elastic scattering. Massive virtual quarks play a role of scatterers for the valence quarks in elastic scattering and their hadronization leads to production of secondary particles in the central region. The mechanism of multiparticle production in the central and fragmentation region was described in [14]. To estimate number of such scatterers one could assume that certain part of hadron energy carried by the outer condensate clouds is being released in the overlap region to generate massive quarks. Then this number can be estimated by:

$$\hat{N}(s, b) \propto \frac{(1 - \langle k_{Q} \rangle) \sqrt{s}}{m_{Q}} D_{c}^{b_{1}} \otimes D_{c}^{b_{2}} \equiv N_{0}(s)D_{C}(b), \quad (1)$$
where $m_Q$ – constituent quark mass, $\langle k_Q \rangle$ – average fraction of hadron energy carried by the constituent valence quarks. Function $D_h^c$ describes condensate distribution inside the hadron $h$, and $b$ is an impact parameter of the colliding hadrons.

In the following we will concern particle production in the central region. Since the quarks are constituent, it is natural to expect direct proportionality between an average number of secondary particles and number of virtual massive quarks appeared in collision of the hadrons with a given impact parameter value [14]:

$$\langle n \rangle(s, b) = \alpha N_0(s) D_C(b),$$

(2)

where $\alpha$ is a constant factor. The geometrical picture of hadron collision discussed above implies that at high energies and non-zero impact parameters the constituent quarks produced in overlap region carry large orbital angular momentum. It can be estimated as follows

$$L(s, b) \propto b \sqrt{s} D_C(b).$$

(3)

Due to supposed strong interaction between quarks this orbital angular momentum will lead to the coherent rotation of the quark system located in the overlap region as a whole. This rotation is similar to rotation of the liquid where strong correlations between particles momenta exist [15]. In what follows we argue that this collective coherent rotation would lead to the energy dependence of the average transverse momentum and can explain experimentally observed rising behavior of this quantity.

## 2 Average transverse momentum

To calculate average transverse momenta we need to know single-particle inclusive cross-section. We use unitarized expression for this quantity based on the rational representation for the scattering amplitude, recent discussion on this form of unitarization can be found in [16]. The rational form of unitarization in quantum field theory is based on the relativistic generalization [17] of the Heitler equation of the relativistic damping theory [18].

Unitarity condition for the elastic scattering amplitude $F(s, t)$ can be written in the form

$$\text{Im}F(s, t) = H_{el}(s, t) + H_{inel}(s, t),$$

(4)

where $H_{el, inel}(s, t)$ are the corresponding elastic and inelastic overlap function introduced by Van Hove [19]. The functions $H_{el, inel}(s, t)$ are related to the functions $h_{el, inel}(s, b)$ and via the Fourier-Bessel transforms, i.e.

$$H_{el, inel}(s, t) = \frac{s}{\pi^2} \int_0^\infty bdb h_{el, inel}(s, b) J_0(b \sqrt{-t}).$$

(5)
The elastic and inelastic cross-sections can be obtained as follows:

\[ \sigma_{el, inel}(s) \sim \frac{1}{s} H_{el, inel}(s, t = 0). \]  

(6)

In the \((U\text{-matrix})\) approach the elastic scattering matrix in the impact parameter representation has the form:

\[ S(s, b) = \frac{1 + iU(s, b)}{1 - iU(s, b)}, \]  

(7)

where \(S(s, b) = 1 + 2if(s, b)\) and \(U(s, b)\) is the generalized reaction matrix, which is considered to be an input dynamical quantity similar to the eikonal function. Unitarity equation rewritten at high energies for the elastic amplitude \(f(s, b)\) has the form

\[ \text{Im}\ f(s, b) = h_{el}(s, b) + h_{inel}(s, b) \]  

(8)

where the inelastic overlap function

\[ h_{inel}(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{inel}}{db^2} \]

is the sum of all inelastic channel contributions. Inelastic overlap function is related to \(U(s, b)\) according to Eqs. (7) and (8) as follows

\[ h_{inel}(s, b) = \frac{\text{Im}U(s, b)}{|1 - iU(s, b)|^2}, \]  

(9)

i.e.

\[ \sigma_{inel}(s) = 8\pi \int_0^{\infty} db \frac{b \text{Im}U(s, b)}{|1 - iU(s, b)|^2}. \]  

(10)

It should be noted that

\[ \text{Im}U(s, b) = \sum_{n \geq 3} \tilde{U}_n(s, b), \]  

(11)

where \(\tilde{U}_n(s, b)\) is a Fourier–Bessel transform of the function

\[ \tilde{U}_n(s, t) = \frac{1}{n!} \int \prod_{i=1}^{n} \frac{d^3q_i}{q_{i0}} \delta^{(4)}(\sum_{i=1}^{n} q_i - p_a - p_b) U_n^\ast(q_1, ..., q_n; p'_a, p'_b) \cdot \]  

(12)

\[ \tilde{U}_n(q_1, ..., q_n; p_a, p_b). \]

Here the functions \(U_n(q_1, ..., q_n; p_a, p_b)\) and \(U_n(q_1, ..., q_n; p'_a, p'_b)\) correspond to the ununitarized (input or “Born”) amplitudes of the process

\[ a + b \to 1 + .... + n, \]
and the process with the same final state and the initial state with different mo-
ments \( p'_a \) and \( p'_b \)
\[
a' + b' \rightarrow 1 + \ldots + n,
\]
respectively. They are the analogs of the elastic \( U \)-matrix for the processes \( 2 \rightarrow n \). The sum in the right hand side of the Eq. (12) runs over all inelastic final states \( |n\rangle \).

Then denoting via \( \zeta \) a set of kinematical variables which characterizes the kinematics of the final state particle \( c \), the expression for the inclusive cross-section of the process \( ab \rightarrow cX \) has the following form [20]:
\[
\frac{d\sigma}{d\zeta} = 8\pi \int_0^\infty bdb \frac{I(s,b,\zeta)}{|1 - iU(s,b)|^2},
\]
(13)
where \( I(s,b,\zeta) \) is the Fourier-Bessel transform of the functions which are defined similar to Eq. (4) but with the fixed kinematical variables \( \zeta \) related to the particle \( c \) in the final state. It should be noted that the impact parameter \( b \) is the variable conjugated to the variable \( \sqrt{-t} \), where \( t = (p_a - p'_a)^2 \) and that the following sum rule is valid for the \( I(s,b,\omega) \)
\[
\int d\zeta I(s,b,\zeta) = \langle n \rangle(s,b) \Im U(s,b).
\]
(14)
The initial impact parameter \( b \) is related to the impact parameters of the secondary particles by relation [21]
\[
b = \sum_{i=1}^n x_i b_i,
\]
(15)
where \( x_i \) stands for the Feynman variable \( x \) of the \( i \)-th particle. Combining Eqs. (9,13,14), one can easily obtain the following relation of the average transverse momentum \( \langle p_T(s) \rangle \) with \( \langle p_T \rangle(s,b) \):
\[
\langle p_T(s) \rangle = \frac{\int_0^\infty bdb \langle p_T \rangle(s,b)\langle n \rangle(s,b)h_{\text{inel}}(s,b)}{\int_0^\infty bdb \langle n \rangle(s,b)h_{\text{inel}}(s,b)}
\]
(16)
with inelastic overlap function \( h_{\text{inel}}(s,b) \) given by Eq. (9).

3 Coherent rotation of transient matter and energy
dependence of average transverse momentum

We are going now to evaluate energy dependence of the average transverse momentum and propose a possible mechanism leading to this dependence. First of
all it should be noted that the function $U(s,b)$ is represented in the model as a product of the averaged quark amplitudes $\langle f_Q \rangle$,

$$U(s,b) = \prod_{i=1}^{N} \langle f_{Q_i} \rangle(s,b).$$  \hspace{1cm} (17)$$

This factorization originates from an assumption of a quasi-independent nature of the valence quark scattering, $N$ is the total number of valence quarks in the colliding hadrons. The essential point here is the rise with energy of the number of the scatterers like $\sqrt{s}$. The $b$-dependence of the function $\langle f_Q \rangle$ has a simple form $\langle f_Q \rangle(s,b) \propto \exp(-m_Q b/\xi)$ which can be related to the formfactor of a constituent quark. The resulting generalized reaction matrix $U$ gets the following form

$$U(s,b) = g \left( 1 + \alpha \frac{\sqrt{s}}{m_Q} \right)^N \exp\left(-\frac{M b}{\xi}\right),$$  \hspace{1cm} (18)$$

where $M = \sum_{Q=1}^{N} m_Q$. Here $m_Q$ is the mass of constituent quark, which is taken to be 0.35 GeV. Other parameters have values obtained from the experimental data fitting. To make explicit calculations we model for simplicity the condensate distribution $D_C(b)$ by the exponential form. Then the mean multiplicity in the impact parameter representation has the form

$$\bar{n}(s,b) = \alpha N_0(s) \exp(-b/R_C)$$  \hspace{1cm} (19)$$

which resulting in the two-parametric power-like energy dependence of the experimentally measurable average multiplicity multiplicity

$$\langle n \rangle(s) = a s^\delta$$

This power-like dependence of mean multiplicity is in good agreement with the experimental data [14]. Note that power index has the following relation with parameters of the model

$$\delta = \frac{1}{2} \left( 1 - \frac{\xi}{m_Q R_C} \right).$$

It should be noted that discovery of the deconfined state of matter has been announced by four major experiments at RHIC [22]. Despite the highest values of energy and density have been reached, a genuine quark-gluon plasma QGP (gas of the free current quarks and gluons) was not found. The deconfined state reveals the properties of the perfect liquid, being a strongly interacting collective state and therefore it was labeled as sQGP. Using similarity between hadronic and nuclear interactions we assumed that transient state in hadron interactions is also a liquid-like strongly interacting matter. It means as it was already mentioned in
the beginning that the presence of large angular momentum in the overlap region will lead to coherent rotation of quark-pion liquid. Of course, there should be experimentally observed effects of this collective effect, one of them is the directed flow in hadron reactions, with fixed impact parameter discussed in [15]. It is not impossible task to measure impact parameter of collision in hadron reactions with the help of the event multiplicity studies. But effects averaged over impact parameter can be measured more easily using standard experimental technics. So, it is natural to assume that the rotation of transient matter will affect average transverse momentum of secondary hadrons in proton-proton collisions. Let for beginning do not take into account the other sources of the transverse momentum and temporarily suppose that all average transverse momentum is a result of a coherent rotation of transient liquid-like state. Then the following relation can be invoked

\[ \langle p_T \rangle(s, b) = \kappa L(s, b), \]  

where \( L(s, b) \) is given by Eq. (3) and \( \kappa \) is a constant which has dimension of inverse length. It is natural to relate it with inverse hadron radius, \( \kappa \sim 1/R_h \).

Now calculating the integrals in Eq. (16), we obtain the power-like dependence of the average transverse momentum \( \langle p_T \rangle(s) \) at high energies

\[ \langle p_T \rangle(s) = cs^{\delta_C}, \]

where

\[ \delta_C = \frac{1}{2} \left( 1 - \frac{\xi}{m_Q R_C} \right). \]

The value of \( \xi \) is fixed from the data on angular distributions [13] and for the mass of constituent quark the standard value \( m_Q = 0.35 \text{ GeV} \) is taken. Of course, besides collective effects average transverse momentum would get contributions from other sources such as thermal distribution proposed long time ago by Hagedorn [2]. This part has no energy dependence and we take it into account by simple addition of the constant term to the power-dependent one, i.e.:

\[ \langle p_T \rangle(s) = a + cs^{\delta_C} \]

Existing experimental data can be described well (cf. Fig.1) using Eq. (22) with parameters \( a = 0.337 \text{ GeV/c}, c = 6.52 \cdot 10^{-3} \text{ GeV/c} \) and \( \delta_C = 0.207 \). The numerical value of \( R_C \) is determined by pion mass with better than 10% precision, \( R_C \approx 1/m_\pi \). In the model the indices in the energy dependencies of average multiplicity and transverse momentum \( \delta \) and \( \delta_C \) are determined by the same expression and experimental data fitting with free parameters \( \delta \) and \( \delta_C \) confirms this coincidence with better than 10% precision also; note that the value \( \delta = 0.201 \) follows from the experimental data analysis for the average multiplicity.
Figure 1: Energy dependence of the average transverse momentum in $pp$-collisions, experimental data from [1, 23].

4 Genuine QGP formation

It should be noted that the dynamical mechanism of the average transverse momentum growth originates in collective effect of transient matter rotation, while dynamics of average multiplicity growth is related to the mechanism where a nonlinear field couplings transform the kinetic energy to internal energy. According to this difference one could expect divergent energy dependencies of these two observables at higher energies. Indeed, formation of a genuine quark-gluon plasma in transient state in the form of the noninteracting gas of free quarks and gluons would result in disappearance of the collective effect of rotation of the transient state. Orbital angular momentum presented in the initial state would lead then to the global polarization of produced particles as it was discussed in [24]. In [15] we noted that disappearance of collective effect of rotation would lead to the vanishing directed flow $v_1$. The simultaneous vanishing of the energy dependent contribution to the average transverse momentum should also be expected, i.e. average transverse momentum would reach maximum at some energy and after that will decrease till eventual flat behavior (cf. Fig.2). Of course, this picture is a rather qualitative one since the nature of phase transition from strongly interacting matter (associated with quark-pion liquid in the model) to the genuine quark-gluon plasma is not known. Currently, we do not even have any experimental indications that such transition will indeed take place at super high energies. It is interesting to note that inverse phase transition from parton gas to liquid could explain a saturation phenomena in deep inelastic processes [25].
**Conclusion**

We proposed here a new explanation of the energy dependent behavior of the average transverse momentum in pp collisions as a result of the presence of a collective effect in the transient state of interaction. Due to strong interaction nature of this transient state, the orbital angular momentum in the initial state would lead to collective rotation which in its turn contributes to average transverse momentum. Orbital angular momentum increases with energy and this increasing behavior leads to increasing energy dependence of the average transverse momentum. The main issue here is the assumption of the existence of the liquid intermediate state in hadron interactions, which is based on the analogy with nuclear collisions. Available experimental data are in good agreement with resulting model dependence. Transition to the genuine QGP formation would destroy then any coherence (rotation) in the transient state and result in decreasing and further flattening of energy dependence of the average transverse momentum.

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