Critical density of urban traffic

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A modified version of the Intelligent Driver Model was used to simulate traffic in the district of Afogados, in the city of Recife, Brazil, with the objective to verify whether the complexity of the underlying street grid, with multiple lane streets, crossings, and semaphores, is capable of exhibiting the effect of critical density: appearance of a maximum in the vehicle flux versus density curve. Numerical simulations demonstrate that this effect indeed is observed on individual avenues, while the phase offset among the avenues results in damping of this effect for the region as a whole.

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I. INTRODUCTION

Over the past decades various models have been proposed for simulating traffic flow (see e.g. [1–6] and references therein), which range on the phenomenon treatment detail level (microscopic, mesoscopic and macroscopic models), are based on individual events or global temporal behavior (discrete and/or stochastic), implementing real time or off-line simulations.

Among these, the so called “car-following” models play an important role, as they capture much of the realism of the phenomenon, with a high level of detail. In particular, the “Intelligent Driver Model” (IDM) proposed by Treiber and collaborators [7, 8] has attracted much attention, as it is capable of reproducing diverse realistic effects. This model is based on a system of coupled non-linear differential equations, where motion of each vehicle depends on the position and speed of the adjacent vehicles. The solution of this system of equations is only possible through numerical methods.

However, none of these models has up to date shown the ability to reproduce the empirically observed effect of a maximum in the vehicle flux versus density curve, which represents a fundamental phenomenon from the point of view of urban traffic planning.

In this work we implement a modified version of the IDM model, by assuming a truncated Gaussian distribution for the “desired” individual vehicle speed values, on top of the real street map of the suburb of Afogados in the city of Recife, Brazil, where we also implement real traffic rules. It is found that the composite effect of intermittent traffic lights, multiple merging lanes, and random “desired routes” of individual vehicles does produce a maximum (albeit weak) in the flux versus density curve when a single avenue is considered as part of a larger composite system, whereas this effect vanes off when the measurements are made on the district system as a whole.

To this end, we have developed a simulator with a “Graphical User Interface” (GUI), using animation to facilitate the correction of eventual problems, visualize the operation and the simultaneous interactions, and aid in a presentation which gives credibility to the model. Most importantly, this approach enables one to interactively adapt the model variables, which greatly aids the realization of numerical experiments.

In the following section we give a brief overview of the necessary aspects of the traffic theory and the IDM model, in the subsequent section we describe the numerical experiment and present the results, and finally, we draw the conclusions.

II. MODELING URBAN TRAFFIC

A. Basic concepts

There are many different aspects of vehicle properties, driver behavior, and the surrounding traffic infrastructure that may be taken into account when modeling urban traffic. For example, the vehicle weight has direct impact on the ability of changing the vehicle speed [9]. The movement of a vehicle α in a given lane of an avenue may be described by considering the vehicle size lα, its position in relation to the lane beginning xα, speed vα and the acceleration aα. In addition, individual driver characteristics influence the unique form in the vehicle conduction [8, 10], where the driver perception and the reaction time are the key elements for the efficient vehicle operation [9]. Furthermore, individual driver factors in the physiologic context [11] may be taken into account.

Generally speaking, urban traffic represents a highly complex non-linear phenomenon, directly related with the vehicle and driver properties, the interactions between the vehicles, the properties and configuration of the traffic infrastructure (streets, lanes, traffic lights etc.) and the quantity of vehicles that enter and exit the observed region per unit time [12].

The traffic flow q is defined [9, 12, 13] as the number of vehicles N which pass through a detector during a given
time interval $\Delta t$

$$q = \frac{N}{\Delta t}, \quad (1)$$

where in the case of multiple lanes, the composite vehicle flow on $L$ lanes is simply given by

$$q = \frac{1}{\Delta t} \sum_{i=1}^{L} N_i. \quad (2)$$

Depending on the vehicle density and the street capacity, traffic flow may be classified as belonging to one of the three basic regimes: free flow, synchronized traffic flow (when the street capacity is approached), and “wide moving jam” (when the street capacity is exhausted) [1, 14].

Vehicles in a given lane are ordered due to the impossibility of overtaking within the lane [1], the first in will be the first out, unless lane switching is permitted. The possibility of overtaking within the lane [1], the first in will end up extracting different functional forms from the same data sets. Also, there seems to be no general consensus up to date as to the relevance of individual components of various existing models for the description of diverse effects of interest for the understanding of the urban vehicular traffic phenomenon.

### B. Intelligent Driver Model

The “Intelligent Driver Model” (IDM) [7, 8] is governed by a system of non-linear differential equations for the individual vehicles indexed by $\alpha$, given by

$$\frac{\partial v_\alpha}{\partial t} = a \left[ 1 - \left( \frac{v_\alpha}{v_0} \right)^\delta - \left( \frac{S^*(v_\alpha, \Delta v_\alpha)}{S_\alpha} \right)^2 \right], \quad (6)$$

where $a$ is maximum (or intrinsic) acceleration, $\delta$ is the parameter controlling the rate at which the desired speed is approached, $v_0$ is the “desired” vehicle speed, $S_\alpha$ is the distance to the preceding vehicle $\alpha - 1$

$$S^*(v_\alpha, \Delta v_\alpha) = S_0 + \max \left( \frac{T v_\alpha + \frac{v_\alpha \Delta v_\alpha}{2 \sqrt{ab}}}{2} \right), \quad (7)$$

is the “safety distance” between the vehicles $\alpha$ and $\alpha - 1$, $S_0$ is the minimum security distance, $T$ is the breaking reaction time [10], $\Delta v_\alpha$ is the difference between the velocities of the two vehicles, and $b$ is the deceleration parameter governing the influence of the velocity difference on the safety distance.
Beside an abundance of parameters (perhaps, over-abundance for the current purpose), this model contains intuitively attractive components for the description of vehicular behavior, and has been shown to display rather realistic behavior in diverse situations (see [7, 8] and citing references). The solution of the model does not present physically unacceptable situations (such as negative velocities, vehicle overlapping etc.), and stable solutions are attainable already with the Euler’s method, while application of the Runge-Kutta method yields accurate results for describing effects such as e.g. traffic density waves propagation.

On the other hand, this model yields realistic effects only in the presence of obstacles or conflicting situations such as confluent lanes, whereas in the case of free lane traffic it demonstrates trivial behavior (all vehicles transit in parallel with the same speed), without limit for the street capacity. It turns out that using a truncated Gaussian distribution [17, 18] for maximum desired speeds of individual vehicles, is sufficient to create a traffic congestion in an avenue without obstacles [10], where street capacity becomes apparent through saturation of the flow curve. However, the maximum of the flow curve at the critical density was not observed with this generalization of the original IDM model [19].

In order to investigate whether this phenomenon may be brought about by the additional confusion stemming from the complexity of the environment (multiple lanes, vehicle types, traffic rules etc.), in what follows we implement a numerical simulation implementing such a situation.

### III. NUMERICAL EXPERIMENT

The area of study is located in the Recife Metropolitan Region (RMR), where a network of urban highways connects municipal districts and suburbs. According to Pernambuco State Traffic Department (DETRAN) data, in the year of 1990 the RMR was traversed by some 250000 vehicles, which has grown to over 700000 in 2007, where over 400000 of these are found in the city of Recife alone. This violent vehicular density increase has been causing heavy traffic jams in several parts of the city over the past years, augmenting the attention of the authorities and researchers. The study region itself represents one of the two principal transport corridors between the south metropolitan region and Recife downtown.

As already mentioned, the IDM model by itself does not yield results demonstrating critical behavior on the flux versus density scatter plot, and the generalization of IDM which considers a distribution of desired velocities leads only to saturation of the flux versus density curve, with no sign of diminishing flux when the density is further increased. In order to verify whether the complexity of an urban traffic system is able to reproduce such an effect, in the current numerical experiment we have implemented various components encountered in the real life situation. In particular, we consider the following basic elements:

- The avenues are considered in terms of length, number of lanes, crossings, and traffic lights;
- Vehicles of different size and maximum acceleration are considered (trucks, buses and cars), with individual maximum and minimum “desired” speed. Upon entry, each vehicle is (randomly) assigned a “desired” itinerary.
- Traffic rules are implemented in each avenue and lane, maneuvering is implemented in accord with the existing signalization, considering the possibility of changing lanes and overtaking slower vehicles, while avoiding collisions.

![FIG. 3: Street map of the Afogados suburb with indicated traffic flow directions.](image)
table taking into account all the possible combinations of the entry and exit avenues that comply with the traffic rules. While attempting to follow the desired route, the vehicle may also change lanes due to advantageous conditions (e.g., when directly in front of it there is a slower vehicle, and the adjacent lane is empty), while honoring traffic rules. This basic setup creates a rather realistic behavior of individual vehicles, and it seems to capture the collective behavior of the vehicle fleet, which is observed on the graphical interface, as shown in Fig. 4.

FIG. 4: A snapshot of the graphic interface of the simulator during the simulation.

In order to achieve a real time simulation on the current scale that permits realistic traffic behavior, we have opted for the Euler algorithm with a 0.05s time increment, and a full scene refresh at the rate of two frames per second. While a higher order Runge-Kutta method yields higher precision results, the computational demand on this simulation scale takes it out of the real time observation range on the current hardware, and we have verified that the overall behavior of the traffic system is not affected by the precision gain.

For the model parameters we adopt the values $a = 1.5m/s^2$, $b = 2.0m/s^2$, $T = 1.2s$, $S_0 = 2.0m$, and $\delta = 4$, while the values of the desired maximum speed for each vehicle are drawn from a normal distribution truncated at $V_{min}$ and $V_{max}$, depending on the vehicle type, and in accordance with the local traffic rules, with parameters given in Tab. I.

| Vehicle type | $V_{max}$ | $V_{min}$ | $\mu$ | $\sigma$ |
|--------------|-----------|-----------|------|-------|
| cars         | 80        | 40        | 60   | 20    |
| buses        | 50        | 30        | 40   | 5     |
| trucks       | 40        | 30        | 35   | 2.5   |

The simulation is performed by inserting vehicles into all the incoming avenues (and lanes) at the edge of the simulation zone. The insertion rate is gradually increased from 0.125 cars per second per lane (8s delay between incoming vehicles), to continuous influx (no delay between inserting the vehicles), with successive decrements of 0.1s for the delay between incoming vehicles. At each rate (altogether eighty values), ample equilibration time of 1800s (30 min) is used to attain equilibrium, after which the vehicle count and individual speeds are recorded in each avenue segment bounded by street crossings and confluence points. This process was repeated 100 times in order to establish the average values.

IV. RESULTS

Results of the simulation corresponding to the avenue labeled “00” in Fig. 4 are presented in Fig. 5 where it is observed that the flux versus occupation density scatter plot does display signs of a maximum (and thus existence of critical density), while the average speed versus occupation demonstrates large fluctuations in the low density regime, which diminish with density increase.

In order to alleviate the effect of large fluctuations and pinpoint the critical occupation density, in Fig. 6 we display the same data as in Fig. 5 together with the results of averaging the data in 2% bins, and polynomial regression which indicates a critical occupation of 41.63%, with a maximum flow of 0.91998 cars/s (or 3312 cars/h).

Results integrated over the entire simulation region are
FIG. 6: Polynomial regression of the third order for simulation data, avenue 00.

displayed in Fig. 6 which does not reveal a maximum in the flux versus occupation density scatter plot. Also, variation of the average velocity is pronounced over the whole observed range of density occupation values. This findings may be attributed to the fact (observed on the graphical interface during the simulation) that the entire region never becomes entirely congested, rather, different regions take turns in demonstrating free flow, and traffic jam states.

FIG. 7: Collected gross data for the entire region.

V. CONCLUSIONS

In this work we implement a modified version of the Intelligent Driver Model on top of a real street map of a suburb in Recife, Brazil, in order to verify whether the additional complexity brought about by the synchronized collective behavior of vehicles that attempt to follow individual routes, is capable of producing the critical occupation density effect with a maximum of the flux versus density curve. This behavior is empirically observed, and represents a fundamental effect from the point of view of urban traffic planning, but has not been reproduced up to date by existing mathematical and numerical models.

The modification of the IDM model implemented in this work consists of several ingredients. In particular, we consider three types of vehicles with different characteristics (length, maximum acceleration, and individual desired speed), multiple lanes with traffic rules (merging traffic, semaphores etc.), and predefined individual routes for every vehicle.

It turns out that different parts of the observed region go intermittently through phases of free flow and congested traffic behavior. If only a single avenue is observed, the composite effect of interaction with the neighboring regions is reflected in behavior that may be identified as critical density (albeit weak in the current simulation). However, the phase offset between the regions (while a given region is congested, a neighboring region displays free flow, and vice versa) leads to canceling out of individual congestion effects, and the maximum in the flux versus density curve is not observed. Also, the average velocity as a function of occupation displays heteroscedastic behavior, with large fluctuations at low densities that diminish as the density is increased, whereas large average velocity fluctuations are observed for the entire occupation density range for the suburb as a whole.

We may conclude that the critical density phenomenon should be regarded as a local effect, brought about by the interplay of a given region with the neighboring regions, as an exit point for one region represents an entry point for another. To the best of our knowledge, the current work represents the first report in the literature regarding a mathematical/numerical model capable of exhibiting this effect.

Further studies should be made as to the contribution of each of the implemented model components to the observed critical density effect, and whether it may be enhanced by inclusion of some others. It should also be investigated whether other microscopic models are capable of demonstrating this phenomenon in a similar setup.

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