Type Ibn Supernova SN 2010al: Powerful Mass Loss Half Year Prior to the Explosion

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Abstract—Type Ibn supernova SN 2010al is explored to infer parameters of supernova and a circumstellar (CS) shell. The CS interaction model combined with the spectral model of 4600 Å blend suggests the explosion of a WR star with the energy of $(1-1.5) \times 10^{51} \text{erg}$ inside a dense confined CS shell with the mass of $\sim 0.1 M_\odot$ and kinetic energy of $\sim 10^{48} \text{erg}$. The confined CS shell has been formed during the last 0.4 yr prior to the core collapse.

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1. INTRODUCTION

Supernova SN 2010al (type Ibn) is the core-collapse supernova (CSSN) associated with the explosion of a Wolf-Rayet (WR) star with the signature of a dense circumstellar (CS) environment (Pastorello et al. 2015). The light curve of SNe Ibn around the light maximum is powered by the ejecta interaction with the dense CS gas (Moriya and Maeda 2016) likewise in the case of SN 2006jc, another SN Ibn (Chugai 2009).

The recent study of SNe Ibn light curve sample including SN 2010al (Maeda and Moria 2022) led authors to conclude that the fast luminosity decline after the light maximum reflects a steep CS density drop $\rho \propto r^{-\omega}$ with $\omega \sim 3$ for $r > 10^{15} \text{cm}$ and the low $^{56}$Ni mass in supernova ejecta. The steep CS density gradient in turn implies that the mass loss rate increases as preSN approaches the explosion (Maeda and Moria 2022). The presence of confined CS shell with the boundary radius of $\sim 10^{15} \text{cm}$ has been found earlier in type IIL SN 1998S (Chugai 2001) and SNe IIP, e.g., SN 2013fs (Yaron et al. 2017). These facts imply that some universal process in the core of massive stars gives rise to a heavy mass loss year-decade prior to the core collapse.

The lack of clarity in the understanding the origin of the vigorous mass loss shortly before the CCSN explosion, and the indication of the confined CS shell in SNe Ib motivate us to explore the well observed SN 2010al to probe parameters of the confined CS shell and the supernova envelope. Among the appropriate tools for this task is the CS interaction model (cf. Chugai 2001). Note that this requires the description of both the light curve and the expansion velocity; the latter is omitted in the recent model of SN 2010al. Moreover, the first spectrum of SN 2010al with the emission blend of He II 4686 Å, N III 4634, 4641 Å (Pastorello et al. 2015) could provide us with an additional observational constraint for model parameters. The point is that lines show narrow core and broad wings that is a signature of the line emission and Thomson scattering in an opaque CS shell (Chugai 2001). The line profile modeling could permit us to recover the Thomson optical depth of the CS shell and thus to validate the model of the CS shell.

I start with an overview of the CS interaction model including an extension on the case of the adiabatic forward shock in the CS shell with a steep density decline $\omega > 3$. I then describe the model aimed at the description of the 4600 Å blend. Thereafter modeling results are presented with the discussion of implications.

This study is based on SN 2010al spectrum (Pastorello et al. 2015) retrieved from the WISEREP data base (Yaron and Gal-Yam 2012).

2. CS INTERACTION MODEL

2.1. Thin Shell Approximation

The hydrodynamics of the CS interaction will be described based on the thin shell approximation that treats the swept up mass between the reverse and...
forward shock as a thin shell driven by the ejecta dynamical pressure (Giuliani 1981, Chevalier 1982, Chugai 2001). In the relevant conditions a bulk of the thin shell is cold (~10⁴ K) and thus can be dubbed the cold dense shell (CDS).

The kinetic luminosity of the forward shock \( L_{k,f} \) and reverse shock \( L_{k,r} \) is converted into X-rays, which are partially absorbed by the unshocked ejecta, CDS, and CSM thus giving rise to the observed optical luminosity. The X-ray luminosity at the age \( t \) for a certain shock, e.g., forward shock, is calculated as \( L_{X,f} = \eta_f L_{k,f} \) with the radiation efficiency \( \eta_f = (t)/(t + t_{c,f}) \), where \( t_{c,f} \) is the cooling time of the postshock gas in the forward shock. The cooling time is calculated assuming \( T_e = T_i \), the shock density of \( \times 4 \) (preshe shock density), and the cooling function for the hydrogen abundance \( X = 0.2 \) typical of WN stars (Hamann et al. 1991). The fraction of X-rays from the forward shock of the radius \( r_f \) that is intercepted by the unshocked ejecta and the CDS of the radius \( r_{CDS} \) is equal to the dilution factor \( W = 0.5[1 - (1 - (r_{CDS}/r_f)^2)^{1/2}] \). The absorbed fraction of X-rays is calculated assuming thermal bremsstrahlung spectrum for the shock temperature and absorption coefficient \( k_x = 100(E/1 \text{ keV})^{8/3} \text{ cm}^2 \text{ g}^{-1} \).

The model bolometric luminosity at the age \( t \) suggests the instant re-emission of the absorbed X-rays, provided the dilution factor \( W \). The fraction of X-rays from the forward shock of the radius \( r_f \) that is intercepted by the unshocked ejecta and the CDS of the radius \( r_{CDS} \) is equal to the dilution factor \( W = 0.5[1 - (1 - (r_{CDS}/r_f)^2)^{1/2}] \). The absorbed fraction of X-rays is calculated assuming thermal bremsstrahlung spectrum for the shock temperature and absorption coefficient \( k_x = 100(E/1 \text{ keV})^{8/3} \text{ cm}^2 \text{ g}^{-1} \).

The model bolometric luminosity at the age \( t \) suggests the instant re-emission of the absorbed X-rays, provided the diffusion time for the CSM \( t_{diff}(t) < t \); otherwise the luminosity is assumed to be \( 10^{40} \text{ erg s}^{-1} \) following the preSN luminosity of SN 2020tlf with the enhanced pre-explosion mass loss (Jacobson-Galan et al. 2022). The typical age \( t \), when \( t_{diff}(t) = t \), is \( \sim 2 \text{ d} \).

The CS density is set by the power law \( \rho(r) = Ar^{-\omega} \) with \( \omega < 3 \) for \( r < R_k \) and \( \omega > 3 \) for \( r > R_k \). A possible clumpiness of the CSM is ignored. The SN ejecta is set as a homologously expanding envelope \( (v = r/t) \) with the density distribution \( \rho(v) = \rho_0[1 + (v/v_0)^8] \). Parameters \( \rho_0 \) and \( v_0 \) are specified via the ejecta mass \( M \) and kinetic energy \( E \).

The radiation output of the ejecta/CSM interaction is determined by the kinetic energy of ejecta external layers, which for the power law SN density distribution is the same for the infinite properly adjusted combinations of \( E \) and \( M \). Particularly, for the ejecta density distribution \( \rho \propto 1/r^n \) the effect of the CS interaction will be the invariant provided \( M \) and \( E \) obey the relation \( E \propto M^{(n-5)/(n-3)} \), which reduces parameter degeneracy to the single parameter, e.g., \( M \). Adopting some value of the ejecta mass we are able to recover the ejecta kinetic energy and CS density distribution based on the light curve and the CDS velocity. With another choice of \( M \) we immediately find the corresponding \( E \) using the above relation. For the fiducial model we consider the case of \( 5 \, M_\odot \) ejecta, which corresponds to the helium core of \( 6.5 \, M_\odot \) for the main sequence star of \( 21 \, M_\odot \) (Woosley et al. 2002).

**Forward shock in the case of \( \omega > 3 \).** Simulations reveal that to describe the steep luminosity decline of SN 2010al after \( t > t_{cr} \sim 40 \text{ d} \) the thin shell model requires a steep CS density gradient \( (\omega > 5) \) in the outer zone \( r > 10^{15} \text{ cm} \). In the case of strongly radiative forward shock the thin shell model is able to cope with this situation. However, if the forward shock becomes adiabatic the thin shell model is not applicable anymore, since for \( \omega > 3 \) the adiabatic forward shock accelerates (Sedov 1959), whereas the CDS does not.

In order to treat the adiabatic forward shock in the case \( \omega > 3 \) we apply a hybrid model. Specifically, the reverse shock and the CDS expansion are treated using the thin shell model, whereas the forward shock is described by the self-similar Sedov solution for the blast wave in a non-uniform medium \( \rho = Ar^{-\omega} \). In this approach the shock radius is \( r = Bt^{3/5}/(3-\omega) \) (Sedov 1959), where \( B \) depends on the blast wave energy \( E \), CS density parameter \( A \), and adiabatic index. However, we fix \( B \) via matching the luminosity of the thin shell model with the luminosity of the detached adiabatic shock at \( t = t_{cr} \sim 40 \text{ d} \), when the forward shock enters the adiabatic regime.

The X-ray luminosity of the accelerating forward shock is estimated as follows. In the case \( \omega > 3 \) the swept up mass (total number of particles \( N \)) remains almost constant since the most of the CS mass already swept up soon after formation of the accelerated forward shock. This means that the average post-shock electron temperature \( T_e \propto E/N \sim \text{ const} \) and thus the cooling function \( \Lambda(T_e) \) remains constant as well. Therefore, the X-ray luminosity of the forward shock \( L_{X,f} \propto r^{-3}N^2A \propto r^{-3} \propto t^{-(6/5-\omega)} \), while the power absorbed by the CDS and unshocked ejecta is \( L_f \propto WL_{X,f} \). This is the maximal bolometric luminosity attributed to the forward shock.

For \( r_f/r_{CDS} \gg 1 \) one gets \( W = (1/4)(r_{CDS}/r_f)^2 \). With almost constant CDS velocity at the late stage \( W \propto t^{(6-2\omega)/5-\omega} \), so asymptotically \( L_f \propto t^{-(2\omega)/5-\omega} \). E.g., in the case \( \omega = 4.5 \) one gets \( L_f \propto t^{-18} \), a steep decline with the negligible contribution of the forward shock to the bolometric luminosity at the late time. This behavior can be described via the guillotine factor \( g = 1 \) at \( t < t_{cr} \) and zero otherwise. The bolometric luminosity related to the accelerating forward shock is then obtained by the multiplication of \( g \) and the luminosity related to forward shock of the thin shell model. It is reasonable to use smooth version of the factor \( g \)

\[
g(t) = 1/[1 + (t/t_{cr})^3],
\]
where we adopt $s \sim 15$ and $t_{cr}$ is the moment when cooling time $t_c$ meets the condition $t_c/t_{cr} = 0.5$.

2.2. Model for 4600 Å Emission Blend

The 4600 Å emission blend in the first spectrum at about 10 days after the explosion is composed by the He II 4686 Å, N III 4634, 4641 Å, and possibly C III 4647, 4650 Å (Pastorello et al. 2015). We model the blend as a linear superposition of lines with the same normalized profile. The spectrum of a single line is calculated using the Monte Carlo technique. The model suggests that photons are emitted and scattered on electrons in the shell with the inner radius $r_1$ coinciding with the CDS and the outer radius $r_2 = 2.5r_1$. The photosphere coincides with the CDS and is able to diffusively reflect photons with the albedo $\Omega = 0.5$. The density distribution corresponds to $\omega = 1$ in the inner zone of the CS interaction model, $r < R_k$. We adopt $n_e \propto \rho$, and emissivity $j \propto \rho^2$. The electron temperature in the shell is assumed to be constant $T_e = 25 000$ K.

The CSM velocity recovered from absorption minima of narrow lines on days 12, 16, and 26 d is 1000–1100, 1050–1150, and 1300–1400 km s$^{-1}$, respectively (Pastorello et al. 2015). The systematic velocity increase with time indicates that the velocity increases along the radius. We set the radial dependence of CSM velocity by the linear relation

$$u = (u_2 - u_1)(r - r_1)/(r_2 - r_1) + u_1,$$

where $u_1$ is the CS gas velocity at the radius $r_1$ and $u_2$ is the velocity at $r_2$.

The Thomson scattering takes into account Doppler shift between subsequent scatterings and the frequency redistribution in the comoving frame caused by the electron thermal motion. The latter is treated assuming angle-averaged frequency redistribution function for the Thomson scattering on thermal electrons (Hummer and Mihalas 1967).

3. PARAMETERS OF SUPERNOVA AND CS SHELL

The bolometric light curve and the expansion velocity are described by the optimal model (Fig. 1) with parameters presented in Table 1. Table 1 includes ejecta mass, ejecta energy, power law index of the CS density in inner ($r < R_k$) and outer zones, $R_k$ value, the CS shell mass in the range $r \leq R_k$, and the Thomson optical depth of the CS shell outside the CDS on day 10. At the stage $t \lesssim 40$ d the luminosity related to the reverse and forward shocks are comparable, whereas at the later stage the luminosity is determined entirely by the reverse shock. Remarkably, the model velocity of the CDS and boundary velocity of unshocked ejecta are consistent with the maximal expansion velocity estimated from He I 10 830 Å and calcium triplet Ca II 8600 Å in the spectra on day 60.

The light curve in combination with the CDS velocity permit us to find the explosion energy for the adopted ejecta mass of $5 M_\odot$. It is already emphasized that for the outer ejecta power law density $\rho \propto 1/r^n$, the energy should scale as $E \propto M^{(n-5)/(n-5)}$. Particularly, for $n = 8$ and twice as high ejecta mass the energy must be by 1.516 times larger, i.e., $10 M_\odot$ ejecta with the energy $E = 1.52 \times 10^{51}$ erg produces the same result as the model with $M = 5 M_\odot$, the fact we also confirmed numerically. The $10 M_\odot$ ejecta corresponds to $11.5 M_\odot$ preSN or $\approx 40 M_\odot$ main sequence progenitor (Woosley et al. 2002). A successful explosion of CCSNe with the formation of neutron star occurs only for stars with the initial mass <40 $M_\odot$, while the mass range $\lesssim 25 M_\odot$ gives rise to SNe IIP (Heger et al. 2003). We conclude therefore that SNe Ibn progenitors originate from the mass range $25 \lesssim M < 40 M_\odot$. This means that the explosion energy of SN 2010al is in the range of $(1-1.5) \times 10^{51}$ erg.

In the context of the progenitor mass of a high interest is the $^{56}$Ni mass in SN 2010al ejecta. Based on the CS interaction model we find that $M_{Ni} < 0.01 M_\odot$. The model independent estimate follows from the late observational bolometric luminosity, $M_{Ni} \leq 0.015 M_\odot$, which is consistent with the upper limit found earlier $M_{Ni} < 0.02 M_\odot$ (Maeda and Moriya 2022).

The CS interaction model is supported by the 4600 Å blend modeling (model A, Fig. 2, Table 2). Apart from He II 4686 Å, N III 4637 Å, the model includes N III 4515, 4544, and 4592 Å lines, C III emission, and Hβ. Table 2 includes the CS shell optical depth, $C/N$ that stands for the flux ratio C III 4648/N III 4637, the CS velocity at the radii $r_1$ and $r_2$. The CS density distribution ($\omega = 1$) and the optical depth of the CS shell outside the CDS ($\tau = 3.4$) are consistent with parameters of the CS interaction model.

The line profile is weakly sensitive to the electron temperature variation in the range 20 000–30 000 K; we adopt $T_e = 25 000$ K. By 3.6 days later the black body temperature is 21 000 K (De la Rosa et al. 2016), which is in line with the adopted electron temperature at the earlier age. Photospheric albedo ($\Omega$) also does not significantly affect the profile either; we adopt $\Omega = 0.5$. The model B (Fig. 2, Table 2) shows the pronounced effect of the low Thomson optical depth ($\tau = 1$). The model C without C III line suggests the lack of strong evidence for the presence of C III line,
although the fit at about 4650 Å is somewhat worse. The model D with the constant CS expansion velocity of 1000 km s$^{-1}$ fits the red wing of the He II line noticeably worse compared to the model A.

4. DISCUSSION AND CONCLUSIONS

The goal of this paper has been to explore the well observed type Ibn supernova SN 2010al in order to infer parameters of the supernova and the confined CS shell. The CS interaction model and the model of 4600 Å emission provide us with a picture of the WR progenitor explosion with the energy of $(1-1.5) \times 10^{51}$ erg inside a confined CS shell ($\sim 10^{15}$ cm) with the mass of 0.14 $M_\odot$. Remarkably, the explosion energy range is well within the neutrino-driven explosion $E \lesssim 2 \times 10^{51}$ (Janka 2017).

The confined CS shell with the mass of $M_{CS} = 0.14$ $M_\odot$ and the expansion velocity of $u \approx 1100$ km s$^{-1}$ within the radius $R_k = 1.4 \times 10^{15}$ cm is therefore produced during the last $t_{CS} = R_k/u \sim 0.4$ yr due to tremendous mass loss rate $M_{CS}/t_{CS} \sim 0.3$ $M_\odot$ yr$^{-1}$. The overall energy of this event is $E_{CS} = (1/2)M_{CS}u^2 \sim 1.7 \times 10^{48}$ erg and the average kinetic luminosity of the mass loss thus is $E_{CS}/t_{CS} \sim 10^{41}$ erg s$^{-1}$.

SN 2010al shows a maximal energy of the confined CS shell among core-collapse SNe with similar CS shell (here we do not include events similar to SN 1994W and SN 2006gy). Table 3 presents parameters of three well studied CCSNe of different types with confined CS shell. The Table includes the mass of the confined CS shell, velocity of the CS gas, kinetic energy of the CS shell, $^{56}$Ni mass in the supernova ejecta, and the duration of the heavy mass loss responsible for the CS shell formation. These supernovae compose the sequence along the energy of CS shell: SN IIP $\rightarrow$ SN IIL $\rightarrow$ SN Ibn with a large energy increment. It is sensible to suggest that the order reflects the growing progenitor mass along the sequence. In that case the central source responsible for the mass loss operates according to

![Graph](image-url)

**Fig. 1.** (a) Model bolometric light curve (thick solid line) overplotted on two different versions of the observational data: the pseudo-bolometric light curves inferred from the broad-band fluxes in the optical and near infrared domains (crosses) and the light curve obtained including the near ultraviolet contribution (circles). The thin line is the bolometric luminosity powered by the forward shock in the thin shell approximation, whereas the dashed line is the latter luminosity multiplied by the guillotine factor. The luminosity without CS interaction assuming preSN radius of 10 $R_\odot$ is shown by dotted line. Inset shows the CS density distribution. (b) The model CDS velocity (thick line) and the boundary velocity of the unshocked ejecta (thin line). The maximal velocity recovered from Ca II IR triplet and He I 10,830 Å line at about at the age of 60 days is shown by the circle. Inset shows the model CDS radius.

**Table 1.** Parameters of the CS interaction model

| $M$ ($M_\odot$) | $E$ ($10^{51}$ erg) | $\omega_{in}/\omega_{out}$ | $R_k$ ($10^{15}$ cm) | $M_{CS}$ ($M_\odot$) | $\tau$ |
|---|---|---|---|---|---|
| 5 | 1 | 1/4.9 | 1.4 | 0.14 | 3.4 |
the rule: the larger progenitor mass, the larger energy of hydrodynamic perturbations is transferred from the core to outer layers.

The theory of massive star evolution predicts that the oxygen burning in the core takes less time for progenitor with larger initial mass. For the 25 $M_\odot$ star the oxygen burning time is of 0.4 yr (Woosley et al. 2002), comparable to the time for the CS shell formation in SN 2010al. This indicates that the high energy of the CS shell of SN 2010al could be related to the main sequence star of $\sim 25 M_\odot$.

Processes involved in the generation of hydrodynamic perturbations responsible for the powerful mass loss of pre-collapse supernovae are far from clear. An interesting possibility is that the vigorous core convection might generate powerful flux of acoustic waves (Quataert and Shiode 2012). The WR presupernova of SN 2010al with the high energy of the expelled shell indicates that a slow mass loss is highly unlikely. A more appropriate mass loss regime is the shell ejection by a shock wave with the energy of the order of $10^{48}$ erg.

If the energy of perturbations responsible for the vigorous mass loss shortly before the collapse increases with the progenitor mass, then the small amount of $^{56}$Ni in SN 2010al ejecta could be related to the fallback most of $^{56}$Ni onto the neutron star, which is the case for massive progenitor (Woosley et al. 2002). Note that the fallback of $\lesssim 0.4 M_\odot$ can be consistent with the existence of massive neutron stars with masses up to 2.1 $M_\odot$ (Fonseca et al. 2001).

### Table 2. Model parameters for the 4600 Å blend

| Model | $\tau$ | C/N | $u_1$ (km/s) | $u_2$ (km/s) |
|-------|-------|-----|-------------|-------------|
| A     | 3.4   | 0.19| 400         | 1300        |
| B     | 1     | 0.19| 400         | 1300        |
| C     | 3.4   | 0   | 400         | 1300        |
| D     | 3.4   | 0.19| 1000        | 1000        |
Table 3. Confined CS shell in CCSNe

| SN type | SN     | $M_{CS} (M_{\odot})$ | $u_{CS} (\text{km/s})$ | $E_{CS} (\text{erg})$ | $^{56}\text{Ni} (M_{\odot})$ | $t_{CS} (\text{yr})$ |
|---------|--------|----------------------|------------------------|-----------------------|-----------------------------|---------------------|
| SN IIP  | 2013fs | 0.003\textsuperscript{a} | 50                     | $7 \times 10^{43}$   | 0.05\textsuperscript{d}     | $\sim 10$          |
| SN IIL  | 1998S  | 0.1\textsuperscript{b}  | 40                     | $2 \times 10^{45}$   | 0.15\textsuperscript{c}     | $\sim 10$          |
| SN Ibn  | 2010al | 0.14\textsuperscript{c}  | $10^3$                 | $10^{48}$            | <0.015\textsuperscript{c}   | 0.4                |

\textsuperscript{a} Yaron et al. (2017), \textsuperscript{b} Chugai (2001), \textsuperscript{c} this paper, \textsuperscript{d} Chugai (2020), \textsuperscript{e} Fassia et al. (2000).

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