NEW PERTURBATION THEORY IN QED
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The perturbation theory in QED used the exact solution with taking into account the special form of interaction is constructed. The mean electromagnetic field of charged particle is calculated. The possibility of elimination the problem with ultraviolet as well as infrared divergences is shown. The electromagnetic energy of the particle turns out to be regular and small.

1. Introduction

In the conventional Dyson-Feinman approach in quantum electrodynamics all quantities are calculated within the perturbation theory with using the zeroth order photon and electron propagators (see, e.g., the famous monographs ([1],[2],[3],[4],[5]). Fundamental difficulty of the approach is a divergence of integrals representing radiative corrections to the electron mass, electron charge or power of interaction between electrons and photons. A renormalization procedure allows us to eliminate the divergences with the help of any form of subtracting the infinitely large terms in the perturbation theory. Some famous physicists (including Feinman [6], Landau and Lifshitz, Gell-Mann [5]) were estimating this procedure as incorrect. There is a number of attempts to solve the problem with divergences in the framework of conventional theory. In the last few years some works was published where the authors find the possibility to avoid the divergence problem as in QED as well as in the other field theories with the help of ”clothing” procedure firstly proposed in [7] (see, e.g., [8], [9], [10], [11], [12] and references therein). In this connection one should note, that the ”clothing” is fulfilled within the conventional perturbation theory. On the other hand, it is interesting to
know, whether this difficulty is the property only of the perturbation theory or of the quantum electrodynamics itself? For answer this question one should construct any other form of the perturbation theory, quantitative different from the Dyson-Feinman approach. We would like to show that the appropriate form of the theory indeed can be constructed where as infrared as well as ultraviolet divergences turn out to be absent. The general scheme of the theory was described in [13]. The relativistic units (\( \hbar = 1, c = 1 \)) are used throughout the paper. The normalization volume is set equal to unity. We use the common-famous notations of four-dimensional relativistic theory of fields. The Greek indices run over 0,1,2,3; the Latin one - 1,2,3; \( g_{\alpha\beta} \) is the metric of the pseudo-Euclidean space-time with the signature (1,-1,-1,-1).

2. Basic Definitions

Consider the interaction of a free electron with the field of its own radiation. Consider electron as quantum of the spinor field with the wave operator in usual form

\[
\hat{\psi} = \sum_{\lambda, \vec{p}} [u_{\lambda\vec{p}} e^{-ipx} \hat{a}_{\lambda\vec{p}} + v_{\lambda\vec{p}} e^{ipx} \hat{d}_{\lambda\vec{p}}],
\]

where \( \vec{p} \) - is the momentum of the electron, \( \lambda = \pm 1/2 \) denotes the projection of the electron spin on the the specific axe of quantization (down we set this one is z-axe). Bispinors taken part in (1) are defined, e.g., in [5],

\[
u_{\lambda\vec{p}} = u_{-\lambda, -\vec{p}} = \frac{1}{\sqrt{2\varepsilon_{\vec{p}}}} \left( \frac{\sqrt{\varepsilon_{\vec{p}}} + m}{\sqrt{\varepsilon_{\vec{p}}} - m (\vec{n}_{\vec{p}} \vec{\sigma})} w_{\lambda} \right),
\]

\[
\left( \sqrt{\varepsilon_{\vec{p}} + m} w_{\lambda} \right).
\]

Here \( \vec{\sigma} \) - is the vector of Pauli matrices, \( p^0 = \varepsilon_{\vec{p}} = (p^2 + m^2)^{1/2} \) - is an energy of the electron obeying the momentum \( \vec{p} \). The unit spinors \( w_{\lambda}, w'_{\lambda} \) describe the states with the spin projections equal to \( \lambda \). In the equation (1) \( \hat{a}_{\lambda\vec{p}} \) - is the electron destruction operator in a state with the defined polarization, momentum and the energy \( \varepsilon_{\vec{p}}, \hat{d}^\dagger_{\lambda\vec{p}} \) - is the positron creation operator for the same state. All the operators \( \hat{a}_{\lambda\vec{p}}, \hat{d}^\dagger_{\lambda\vec{p}}, \hat{d}_{\lambda\vec{p}}, \hat{d}^\dagger_{\lambda\vec{p}} \) obey the standard Fermi commutation relations.

In the current density operator

\[
\hat{j}^\mu = e : \hat{\psi} \gamma^\mu \hat{\psi} :,
\]
where \( \ldots \) means the normal ordering, \( e \) - is the electron charge, \( \hat{\psi} = \psi^\dagger \gamma^0 \) is the usual Dirac conjugate operator. In the following we need the Fourier representation of the current density operator which in our case has the form

\[
\hat{j}_\mu^\mu (t) = e \sum_{\lambda, \sigma, \vec{p}} [\bar{u}_{\lambda \vec{p}} \gamma^\mu \sigma_{\vec{p}+\vec{q}} \hat{a}_{\lambda \vec{p}} \hat{a}_{\sigma_{\vec{p}+\vec{q}}} e^{i(\varepsilon_{\vec{p}-\vec{p}+\vec{q}})t} +
\bar{v}_{\lambda \vec{p}} \gamma^\mu \sigma_{-\vec{p}-\vec{q}} \hat{d}_{\lambda \vec{p}} \hat{d}_{\sigma_{-\vec{p}-\vec{q}}} e^{-i(\varepsilon_{\vec{p}+\vec{p}+\vec{q}})t} -
\bar{v}_{\lambda \vec{p}} \gamma^\mu \sigma_{-\vec{p}-\vec{q}} \hat{d}_{\lambda \vec{p}} \hat{a}_{\sigma_{-\vec{p}-\vec{q}}} e^{-i(\varepsilon_{\vec{p}-\vec{p}+\vec{q}})t}].
\]

(2)

Note, as a rule, the current density operator don’t conserves the spin of the particle. The conservation of the spin is possible only at \( \vec{q} = 0 \).

Now we need to construct the "approximate" current operators which are to be commute if related to different times. For the one electron problem one of the appropriate expression is given by the formula

\[
\hat{j}_\mu^\mu (0) (t) = f^\mu (\vec{q}, t) \hat{\rho}_{\vec{q}}.
\]

(3)

The 4-vector \( f^\mu (\vec{q}, t) \) is defined below and

\[
\hat{\rho}_{\vec{q}} = \sum_{\lambda, \vec{k}} \hat{a}_{\lambda \vec{k}}^\dagger \hat{a}_{\lambda \vec{k} + \vec{q}}
\]

is the Fourier component of the time-independent electron density operator.

Note, the operators (3) include only diagonal terms in polarization indices and if even refer to different times, obey the commutation relations

\[
[\hat{j}_\mu^\mu (0) (t), \hat{j}_\nu^\nu (0) (t')]_+ = 0.
\]

(4)

Down we consider (3) as the zeroth order approximation to (2). To define the vector \( f^\mu (\vec{q}, t) \) we impose the condition: the current mean value of (3) is to coincide with the mean value of (2), namely,

\[
(t | \hat{j}_\mu^\mu (\vec{q}, t) | t) = f^\mu (\vec{q}, t) (t | \hat{\rho}_{\vec{q}} | t).
\]

(5)

This condition ensures the been chosen \( f^\mu (\vec{q}, t) \) gives the sufficiently rapid convergence of series in the modified perturbation theory. Obviously, the equation (5) may be solved only approximately because the exact vector of state is unknown.

3. Solution to the Ground State Problem
Denote the deviation of the current from its "zeroth" value as
\[ \Delta \hat{j}^\mu_q(t) = \hat{j}^\mu_q(t) - \hat{j}^\mu_q(0)(t) \]
and represent the electromagnetic interaction as a sum of two parts, \( \hat{H}_{\text{int}}(t) = \hat{H}_{\text{int}}^{(0)}(t) + \hat{H}_{\text{int}}^{(1)}(t) \), where
\begin{align*}
\hat{H}_{\text{int}}^{(0)}(t) &= e \int \hat{j}^\mu_q(\vec{r},t) \hat{A}_\mu(\vec{r},t) dV; \\
\hat{H}_{\text{int}}^{(1)}(t) &= e \int \Delta \hat{j}^\mu_q(\vec{r},t) \hat{A}_\mu(\vec{r},t) dV.
\end{align*}

The 4-vector-potential operator of electromagnetic field is defined by the ordinary way as
\[ \hat{A}_\mu(\vec{r},t) = \sum_{\alpha,\vec{q}} g_q \{ \hat{b}_{\alpha\vec{q}} e^{\mu\alpha} e^{i\vec{q}\vec{r}} + \hat{b}_{\alpha\vec{q}}^\dagger e^{\mu\alpha} e^{-i\vec{q}\vec{r}} \}. \]

Here \( \hat{b}_{\alpha\vec{q}}^\dagger \) and \( \hat{b}_{\alpha\vec{q}} \) - are the operators of creation and destruction of photons in states possessed the polarization \( \alpha(\alpha = 0, 1, 2, 3) \), momentum \( \vec{q} \) and energy \( \omega = q \). The coupling function \( g_q = \sqrt{2\pi/\omega} \) is defined, e.g., in [5]. On definition the different unit vectors of polarization \( e^{\mu\alpha} \) are orthogonal to each other and obey the normalization conditions (see, e.g., [2])
\[ g_{\mu\nu} e^{\mu\alpha} e^{\nu\alpha'} = g_{\alpha\alpha'}; \quad \sum_{\alpha} e^{\mu\alpha} e^{\nu\alpha} g_{\alpha\alpha} = g_{\mu\nu}. \]

The conditions of the relativistic invariance (see, e.g., ([1],[2],[3])) are satisfied when the photon operators obey the commutation relations
\begin{align*}
[\hat{b}_{\alpha\vec{q}}, \hat{b}_{\beta\vec{q}}^\dagger]_- &= -g_{\alpha\beta} \cdot \Delta(\vec{q} - \vec{q}'); \\
[\hat{b}_{\alpha\vec{q}}, \hat{b}_{\beta\vec{q}}]_- &= [\hat{b}_{\alpha\vec{q}}^\dagger, \hat{b}_{\beta\vec{q}}^\dagger]_- = 0.
\end{align*}
Here \( \Delta(\vec{q} - \vec{q}') \) equal to unity only if \( \vec{q} = \vec{q}' \) and equal to zero in opposite case. It is well known that the non-usual commutation relations for the scalar photons lead to the indefinite metric in the photon state space. There are also some peculiarities in the definition of physical quantities calculated below.

By virtue of (4), the equation
\[ i \frac{d}{dt} |t\rangle = \hat{H}_{\text{int}}^{(0)}(t) |t\rangle \]
have the exact solution expressed as the direct product of extended coherent states [13], namely,
\[ |t\rangle_0 = \prod_{\alpha,\vec{q}} \exp\{ -i \hat{\chi}_{\alpha\vec{q}}(t) - \hat{b}_{\alpha\vec{q}} \hat{Q}_{\alpha\vec{q}}^\dagger(t) + \hat{b}_{\alpha\vec{q}}^\dagger \hat{Q}_{\alpha\vec{q}}(t) \} |0\rangle. \]
Here
\[ \hat{Q}_{\alpha \vec{q}}(t) = \hat{\rho}_q Q_{\alpha \vec{q}}(t), \quad \hat{\chi}_{\alpha \vec{q}}(t) = \hat{\rho}^\dagger_q \hat{\rho}_q \chi_{\alpha \vec{q}}(t) \] (11)
and
\[ Q_{\alpha \vec{q}}(t) = -ig_q \int_0^t dt' e^{\mu*} f_{\alpha \vec{q}}(\vec{q}, t') e^{i\omega t'}, \] (12)
\[ \chi_{\alpha \vec{q}}(t) = -\frac{i}{2} \int_0^t \{ \hat{Q}_{\alpha \vec{q}}^*(t') Q_{\alpha \vec{q}}(t') - Q_{\alpha \vec{q}}^*(t') \hat{Q}_{\alpha \vec{q}}(t') \} dt'. \] (13)

Within our approach an initial vector of state \(|0\rangle\) is a direct product of the electromagnetic field vacuum state \(|\text{vac}\rangle\) and the vector \(|\psi_0\rangle\) describing the initial state of a particle, \(|0\rangle = |\psi_0\rangle \otimes |\text{vac}\rangle\); below, for simplifications, we denote this state also as \(|\psi_0, \text{vac}\rangle\). Assume the vector \(|\psi_0\rangle\) is a free wave packet which extends its width in course of time. We must take into account that at \(t > 0\) in our interacted system each part of it isn’t independent and can not be described exactly. E. g., there isn’t the wave function for the electron but the mostly simplex way to get an explicit information about it consists in calculation its density matrix.

4. The New Perturbation Theory

If we neglect the corrections producing by the interaction \(\hat{H}_{\text{int}}^{(1)}\), then the equation (10) completely solves the problems with calculating the physical quantities of interest. The quantity \(|Q_{\alpha \vec{q}}(t)|^2\) has the direct physical meaning of the mean number of photons possessing the polarization \(\alpha\) and the momentum \(\vec{q}\) created by the particle to the given instant. Therefore,
\[ \Delta \vec{k}(t) = \sum_{\alpha, \vec{q}} \vec{q}|Q_{\alpha \vec{q}}(t)|^2 \] (14)
is the mean momentum loss and
\[ \Delta E(t) = \sum_{\alpha, \vec{q}} \omega|Q_{\alpha \vec{q}}(t)|^2 \] (15)
is the mean energy loss of the particle. Sometimes (e.g., in case of the resting particle) the energy loss as well as the momentum loss are zeros.

In the approximation considered we can calculate also many other physical quantities (e.g., the Green function) for the particle with taking into account the back influence of electromagnetic quanta on particle’s state. The corrections should be evaluated with the help of a modified perturbation theory. To
construct the new perturbation theory with respect to the interaction $\hat{H}_{int}^{(1)}$ we introduce the "zeroth" order evolution operator

$$\hat{U}_0(t) = \exp\{\sum_{\alpha, \vec{q}} \hat{Q}_{\alpha \vec{q}}(t) \hat{b}^\dagger_{\alpha \vec{q}} - \hat{Q}^{\dagger}_{\alpha \vec{q}}(t) \hat{b}_{\alpha \vec{q}} - i\chi_{\alpha \vec{q}}(t)\}. \quad (16)$$

The vector (10) now can be rewritten as $|t\rangle_0 = \hat{U}_0(t) |0\rangle$. Introduce the new representation of operators as follows

$$\tilde{A}(t) = \hat{U}_0^\dagger(t) \hat{A}(t) \hat{U}_0(t). \quad (17)$$

The vector of state $|t\rangle = \hat{U}_0^\dagger |t\rangle$ in this representation obeys the equation

$$i \frac{d}{dt} |t\rangle = \tilde{H}_{int}^{(1)}(t) |t\rangle. \quad (18)$$

The solution to this equation may be found in a rigorous way via T-exponent

$$|t\rangle = T \exp \left\{-i \int_0^t \tilde{H}_{int}^{(1)}(t') dt' \right\} |0\rangle. \quad (19)$$

5. Ultraviolet behavior of spectral functions

The approximate solution to the equation (5) can be found with the help of a self-consistent procedure, when we use as $|t\rangle$ an approximate expression which functionally depends on $f^\mu(\vec{q}, t)$. For the first approximation we suppose $|t\rangle \approx |t\rangle_0$. Generally speaking, the quantity $f^\mu(\vec{q}, t)$ must be chosen in a strong connection to the particular problem. We consider now only the evolution of an one-particle initial state on the background of photon vacuum. We begin with evaluation of the left hand of the equation (5).

Assume $\varphi_0$ is a Gauss wave packet

$$|\varphi_0\rangle = (8\pi\delta^2)^{3/4} \sum_{\vec{k}} e^{-\delta^2(\vec{k} - \vec{k}_0)^2} a^\dagger_{\lambda', \vec{k}} |vac\rangle, \quad (20)$$

where $\lambda'$ is a some initial polarization of the electron. Let us represent the equation (5) in a more convenient form. Using the well-known Baker-Hausdorff formula

$$\exp(\hat{A} + \hat{B}) = \exp(\hat{A}) \exp(\hat{B}) \exp(-1/2[\hat{A}, \hat{B}]),$$

applicable for the case $[\hat{A}, \hat{B}], \hat{A}] = [\hat{A}, \hat{B}], \hat{B} = 0$, we get

$$|t\rangle_0 = \exp \left[\sum_{\alpha, \vec{q}} \left(-i\chi_{\alpha \vec{q}}(t) + Q_{\alpha \vec{q}}(t) \hat{\rho}_{\alpha \vec{q}} \hat{b}^\dagger_{\alpha \vec{q}} - \frac{1}{2} |Q_{\alpha \vec{q}}(t)|^2 \right)\right] |0\rangle. \quad (21)$$
Note, operator $\hat{U}_0$ is diagonal in spin indices and therefore in our approximation only diagonal part of $\hat{j}_q^\mu(t)$ contribute to the right hand of (5). The straightforward calculation gives:

$$(t|\hat{j}_q^\mu(t)|t) \approx (0|\hat{U}_0(t)\hat{j}_q^\mu(t)\hat{U}_0(t)|0) \approx e \cdot \exp \left[ -\sum_{\alpha,\vec{q}'} |Q_{\alpha\vec{q}}(t)|^2 \right] \times$$

$$\sum_{\lambda,\vec{k}} \bar{u}_{\lambda,\vec{k}} \hat{\gamma}^\mu u_{\lambda,\vec{k}+\vec{q}/2} e^{i(\vec{\varepsilon}_{\vec{k}+\vec{q}/2} - \vec{\varepsilon}_{\vec{k}})t} (0|\exp \left\{ -\sum_{\beta,\vec{q}_1} Q^*_{\beta\vec{q}_1}(t) \hat{\rho}^\dagger_{\beta\vec{q}_1} \hat{b}_{\beta\vec{q}_1} \right\} |0), \quad (22)$$

or, equivalently,

$$(t|\hat{j}_q^\mu(t)|t) \approx (0|\hat{U}_0(t)\hat{j}_q^\mu(t)\hat{U}_0(t)|0) = e \cdot \exp \left[ -\sum_{\alpha,\vec{q}'} |Q_{\alpha\vec{q}}(t)|^2 \right] \times$$

$$\int \frac{d^3s}{(2\pi)^3} \sum_{\lambda,\vec{p}} \bar{u}_{\lambda,\vec{p}} \hat{\gamma}^\mu u_{\lambda,\vec{p}+\vec{q}/2} \exp \left[ i(\vec{\varepsilon}_{\vec{p}+\vec{q}/2} - \vec{\varepsilon}_{\vec{p}})t \right] \times$$

$$\sum_{\lambda,\vec{k}} e^{i(\vec{k}-\vec{p})\vec{s}} (0|\exp \left\{ -\sum_{\beta,\vec{q}_1} Q^*_{\beta\vec{q}_1}(t) \hat{\rho}^\dagger_{\beta\vec{q}_1} \hat{b}_{\beta\vec{q}_1} \right\} \times$$

$$\hat{a}^\dagger_{\lambda,\vec{k}-\vec{q}/2} \hat{a}_{\lambda,\vec{k}+\vec{q}/2} \exp \left\{ -\sum_{\beta,\vec{q}_2} Q_{\beta\vec{q}_2}(t) \hat{\rho}_{\beta\vec{q}_2} \hat{b}^\dagger_{\beta\vec{q}_2} \right\} |0), \quad (23)$$

Now, using the relation (see [13])

$$\sum_{\lambda,\vec{k}} e^{i\vec{k} \cdot \vec{x}} \hat{a}_{\lambda\vec{k}} \exp \left\{ -\sum_{\beta,\vec{q}} Q_{\beta\vec{q}}(t) \hat{\rho}_{\beta\vec{q}} \hat{b}^\dagger_{\beta\vec{q}} \right\} |\vec{k}_0, \text{vac}\rangle = e^{i\vec{k}_0 \cdot \vec{x}} |\text{vac}_p\rangle \otimes$$

$$\exp \left\{ -\sum_{\beta,\vec{q}} Q_{\beta\vec{q}}(t) e^{-i\vec{q} \cdot \vec{x}} \hat{b}^\dagger_{\beta\vec{q}} \right\} |\text{vac}\rangle, \quad (24)$$

where $|\text{vac}_p\rangle$ is a particle vacuum state, one can obtain

$$f^\mu(\vec{q}, t) = e \cdot \sum_{\vec{p}} \bar{u}_{\lambda,\vec{p}} \hat{\gamma}^\mu u_{\lambda,\vec{p}+\vec{q}/2} \exp \left[ i(\vec{\varepsilon}_{\vec{p}+\vec{q}/2} - \vec{\varepsilon}_{\vec{p}})t \right] \times$$

$$\int \frac{d^3s}{(2\pi)^3} \exp \left\{ i\vec{s}(\vec{k}_0 - \vec{p}) - \frac{s^2}{8\delta^2} - \sum_{\alpha',\vec{q}'} |Q_{\alpha'\vec{q}'}|^2 \left(1 - e^{-i\vec{q}' \cdot \vec{s}}\right) \right\}. \quad (25)$$
Because $Q_{\alpha q}$ depends on $f^\mu$, (25) is actually the integral equation for $f^\mu(q,t)$.

Let us assume the initial width of the particle’s wave packet is large compared to the Compton wave length, i.e., $\delta \gg 1/mc$. In this condition one can see in the right hand of (25) the main contribution to the sum occurs from the small neighborhood of the point $\vec{p} = \vec{k}_0$. Assuming in this neighborhood the quantity $|\vec{p} - \vec{k}_0|$ is small and comparable with $|\Delta \vec{k}|$, we can fulfill the series development in the power of exponent in the subintegral expression in the right hand of (25) with taking into account only terms of the first and the second order in $(\vec{p} - \vec{k}_0)$. In this case the equation (25) is reduced to

$$f^\mu(\vec{k}, t) = e^{(2\pi\delta^2)^{3/2}} \sum_{\vec{p}} \bar{u}_{\lambda',\vec{p} - \vec{q}/2} \gamma^\mu u_{\lambda',\vec{p} + \vec{q}/2} \times$$

$$\exp \left[ i(\varepsilon_{\vec{p} - \vec{q}/2} - \varepsilon_{\vec{p} + \vec{q}/2})t - 2\delta^2 \left( \vec{p} - \vec{k}_0 + \Delta \vec{k}(t) \right)^2 \right],$$

(26)

Consider the non-relativistic case, when $k_0 \ll m$, and $\varepsilon_{\vec{q}/2 + \vec{p}} - \varepsilon_{\vec{q}/2 - \vec{p}} \approx q\vec{p}/\varepsilon_{\vec{q}/2}$. The stationary point of exponent in (26) is found to be

$$\vec{p} = \vec{k}_0 - \Delta \vec{k}(t) + i \frac{\vec{q} t}{\delta^2 \varepsilon_{\vec{q}/2}}; \quad k_0 \ll m.$$  

(27)

At sufficiently large width $\delta$ and at $q < 2m$ the last imaginary term in the first approximation can be neglected. Moreover, because the width has the tendency to increase in course of time (in the non-relativistic case this increasing asymptotically develops proportional to time), the last term in the right hand of (27) don’t increase in course of time. We see the behavior of the subintegral function in (26) is defined mostly by the real part of the power in exponent. Accepting this assumption, in the first approximation we can calculate the sum involved in (26) in the stationary phase approximation and get

$$f^\mu(\vec{q}, t) \approx e \cdot \vec{u}_{\lambda',\vec{p}_m - \vec{q}/2} \gamma^\mu u_{\lambda',\vec{p}_m + \vec{q}/2} e^{-i(t(\varepsilon_+ - \varepsilon_-))},$$

(28)

where $\vec{p}_m = \vec{k}_0 - \Delta \vec{k}(\vec{k}_0, t)$, $\varepsilon_{\pm} = (\varepsilon^2 + (\vec{p}_m \pm \vec{q}/2)^2)^{1/2}$, $\Delta \vec{k}(\vec{k}_0, t)$ - is the mean momentum loss of the particle. In this paper we take the equation (28) as the basis for constructing the first simple variant of the modified perturbation theory.

6. Mean Electromagnetic Field

Consider the problem of corrections to the Coulomb field. The mean vector-potential of the field is given as $A^\mu = \langle t | A^\mu | t \rangle$. Calculate this vector in
the zeroth approximation setting \(|t\rangle \approx |0\rangle\). With the help of this assumption we obtain

\[
A^\mu = 2e \text{Re} \left\{ i \sum_{\alpha, \vec{q}} g_\alpha^2 g_{\alpha q} e^{i\bar{q}r - i\omega t} e^{\alpha q} e_{\alpha q}^\nu e^{i\omega t} f_\nu(\vec{q}, t') \rho_q(t') \, dt' \right\},
\]

(29)

Note, in this expression we have taken into account the negative eigenvalue sign for the destruction operator of the scalar photon (see Appendix). Substituting (28) and taking into account the summing rules for polarization vectors, we get

\[
A^\mu = 2e \text{Re} \left\{ i \sum_{\alpha, \vec{q}} g_\alpha^2 e^{i\bar{q}r - i\omega t} \int_{-\infty}^{t} \bar{u}_{\lambda'}\bar{p}_{\lambda' - \vec{q}/2} / 2 \gamma^\mu u_{\lambda' \vec{p} + \vec{q}/2} \times \hat{\rho}(t') \, dt' \right\},
\]

(30)

where the function \(\hat{\rho}(t)\) is the Fourier component of particle’s probability distribution in the rest frame.

For the uniformly moving particle we have \(\vec{p}' = \vec{k}_0\) and arrive at

\[
A^\mu = 2e \text{Re} \left\{ i \sum_{\vec{q}} g_\alpha^2 e^{i\bar{q}r - i\omega t} \int_{-\infty}^{t} \bar{u}_{\lambda'}\bar{p}_{\lambda' - \vec{q}/2} / 2 \gamma^\mu u_{\lambda' \vec{p} + \vec{q}/2} \times \hat{\rho}(t') \, dt' \right\},
\]

(31)

The potential \(A^\mu\) is represented by the convolution of one for the point particle and the probability distribution for the particular state. It is interesting to analyze the universal case, corresponding to the point charge, for which \(\rho_q = 1\). In this case the last formula gives

\[
A^\mu = 2e \text{Re} \left\{ \sum_{\vec{q}} g_\alpha^2 \bar{u}_{\lambda', \vec{k}_0 - \vec{q}/2} / 2 \gamma^\mu u_{\lambda' \vec{k}_0 + \vec{q}/2} \frac{\exp[i\bar{q}r - i(\epsilon_+ - \epsilon_-)t]}{\omega_q - \epsilon_+ - \epsilon_-} \right\},
\]

(32)

or, in 3-representation,

\[
A^0 = 2e \text{Re} \left\{ \sum_{\vec{q}} \frac{g_\alpha^2}{\sqrt{4\varepsilon_+ + \varepsilon_-}} w^\dagger_{\lambda'} \left( \sqrt{(\epsilon_+ + m)(\epsilon_- + m) + \sqrt{(\epsilon_+ - m)(\epsilon_- - m)} / |\vec{k}_0 - \vec{q}/2||\vec{k}_0 + \vec{q}/2|} \right) (k_0^2 - 4 + i\vec{r} \cdot [\vec{k}_0 \times \vec{q}]]) e^{i\bar{q}r - i(\epsilon_+ - \epsilon_-)t} / \omega_q - \epsilon_- - \epsilon_+ \right\};
\]

(33)
\[ \vec{A} = 2e \text{Re}\left\{ \sum_{\vec{q}} \frac{g_{\vec{q}}^2}{\sqrt{4\varepsilon_+\varepsilon_-}} w^\dagger_{\vec{q}} \left( \vec{\sigma}(\vec{p}_+ \cdot \vec{\sigma}) + (\vec{p}_- \cdot \vec{\sigma})\vec{\sigma} \right) w_{\vec{q}} \right\} \]

\[ \exp\left[ i\vec{q}\vec{r} - i(\varepsilon_+ - \varepsilon_-)t \right] / \omega_{\vec{q}} + \varepsilon_- - \varepsilon_+ \}, \]

(34)

where \( \vec{p}_{\pm} = (\vec{k}_0 \pm \vec{q}/2)\sqrt{(\varepsilon_+ + m)/(\varepsilon_\pm + m)} \).

Now first we evaluate the scalar potential for the particle found in the rest. We substitute \( \vec{k}_0 = 0 \) and get

\[ A^0 = 2e \text{Re}\left\{ \sum_{\vec{q}} g_{\vec{q}}^2 u^\dagger_{\vec{q},-\vec{q}/2} u_{\vec{q},\vec{q}/2} \frac{\exp i\vec{q}\vec{r}}{\omega_{\vec{q}}} \right\}. \]

(35)

Because of \( u^\dagger_{\vec{q},-\vec{q}/2} u_{\vec{q},\vec{q}/2} = m/\varepsilon_{\vec{q}/2} \) after some algebra the formula (35) gives

\[ A_0 = e \frac{2}{\pi r} \int_0^{2mr} K_0(\xi) d\xi, \]

(36)

where \( K_0(\xi) \) - is the MacDonald’s function of zero order. It is important to note that at \( r \to 0 \) the potential (36) diverges, but only logarithmical as \( (Zm/\pi) \ln(e/mr) \), \( r \ll 1/m \). At \( r \gg 1/m \) the potential coincides with the Coulomb one. The interaction with the electromagnetic field changes particle’s form-factor on distances which don’t exceed some Compton wave lengths. The same behavior manifests the potential which takes into account the radiative corrections in the Feynman perturbation theory.

If we attempt to represent the potential (36) as the usual retarded potential corresponded to any charge distribution, then will see, that it is possible only if this distribution sufficiently depends on the mass of the particle. The dependency of the field on the mass can be connected to the effect of the reaction force exited at photon radiation. This circumstance means not the charge distribution but the sample action of electromagnetic quanta wrapped around the charge creates the field which we observe as the source of the Coulomb force.

Consider the selfenergy for the resting particle. The electromagnetic contribution to it can be obtained in a usual manner as the energy of an electric field

\[ E = \frac{1}{8\pi} \int (-\nabla A_0)^2 dV = -\frac{1}{8\pi} \int A_0 \Delta A_0 dV. \]

The integral can be evaluated exactly. First we introduce \( \Phi = rA_0 \). Then, using the spherical symmetry and \( \Delta A_0 = (1/r)d^2\Phi/dr^2 \), bring the energy to
the form

\[
E = \frac{8}{\pi^2} (em)^2 \int_0^\infty K_0^2(2mr) \, dr = e^2 m.
\]

In the last expression the value of the integral was used

\[
\int_0^\infty K_0^2(\xi) \, d\xi = \frac{\pi^2}{4}.
\]

As we see, the electromagnetic contribution to the selfenergy of elementary particle is comparatively small. In the case of electron as well as proton the electromagnetic correction is equal to the selfenergy multiplied by the fine structure constant. We can confirm the famous result the main part of the selfenergy for elementary particles has the non-electromagnetic nature.

Now we should make once more important notation. We can calculate the general energy of all the photons wrapping around the particle and found that the total its energy exactly consists with the electromagnetic contribution to the selfenergy. This result allows us to consider one as a new method to calculate the energy of electromagnetic field, which don’t use the explicit expressions for the electric and magnetic fields.

Consider the case of non-relativistic particle. We can set \( \vec{k}_0 \approx m \vec{r}_0 \), where \( \vec{r}_0(t) \) - is the current mean coordinate of the particle,

\[
\varepsilon_+ - \varepsilon_- \approx \frac{m \vec{q} \vec{r}_0(t)}{\varepsilon_{q/2}}; \]

\[
\varepsilon_+ \varepsilon_- \approx m^2 \left( 1 + \frac{q^2}{4m^2} \right).
\]

In this case we arrive at

\[
A^0 = 2e \operatorname{Re}\left\{ \sum_{\vec{q}} \frac{g_q^2}{\varepsilon_{q/2}} \frac{\exp[i \vec{q} \vec{r} - i (\varepsilon_+ - \varepsilon_-)t]}{\omega_{\vec{q}} + \varepsilon_- - \varepsilon_+} \right\}; \quad (37)
\]

and

\[
\vec{A} = \vec{A}_s + 2e \operatorname{Re}\left\{ i \sum_{\vec{q}} \frac{g_q^2}{\varepsilon_{q/2}} \left( \vec{p} - \frac{\vec{q} (\vec{q} \vec{p})}{4\varepsilon_{q/2}(m + \varepsilon_{q/2})} \right) \times \frac{\exp[i \vec{q} \vec{r} - i (\varepsilon_+ - \varepsilon_-)t]}{\omega_{\vec{q}} + \varepsilon_- - \varepsilon_+} \right\}; \quad (38)
\]

Note, within the linear in \( \vec{k}_0 \) approximation the 4-potential \((37, 38)\) obeys the Lorentz condition

\[
\frac{\partial A^\mu}{\partial x^\mu} = 0.
\]
With the help of simple transformations one can reduce the potential (37) to the 3-dimensional integral

$$A^0 = \frac{2}{\pi^2} em \int_{-\infty}^{t} dt' \int_{0}^{\infty} \frac{dk}{|\vec{r} - \vec{m}\vec{x}_0(t')|} \int_{0}^{\infty} d\tau \cos(k\tau) \times$$

$$\left\{ K_0 \left( 2m|t - t' - \tau - |\vec{r} - \frac{m\vec{x}_0(t')}{\varepsilon_{k/2}}| \right) - K_0 \left( 2m|t - t' - \tau + |\vec{r} - \frac{m\vec{x}_0(t')}{\varepsilon_{k/2}}| \right) \right\}$$  \hspace{1cm} (39)

By setting $\vec{p}_m = 0$ the potential (39) reduces to the potential for resting particle (36).

Within the region $r \geq 1/2m$ the main contribution to the potential (39) occurs from the small $k \leq 2m$. In this region one can neglect all the nonlinear in $\gamma = k/2m$ terms in the subintegral function and get

$$A_\mu = \frac{2}{\pi} Zmc^2 \int_{0}^{\infty} d\tau \frac{\hat{r}_{(0)\mu}(t - \tau)}{|\vec{r} - \vec{r}_0(t - \tau)|} \left[ K_0 (2mc|c\tau - |\vec{r} - \vec{r}_0(t - \tau)||) - K_0 (2mc(c\tau + |\vec{r} - \vec{r}_0(t - \tau)|)) \right].$$  \hspace{1cm} (40)

Comparing this result with (39) we see that the previous expression contains the additional retardation time $\tau > 0$. The physical explanation of this effect can be found on the basis of action of the reaction force appearing at the virtual fast $k > 2m$ photon radiation. At this radiation the particle get the great (virtual) deflection from the center of the probability distribution. This deflections display themselves in (39) in form of the retardation time.

At the distances $r \gg 1/2m$ the MacDonald’s functions changes very fast compared to the characteristic distance of the potential inhomogeneity. If one replaces in (40) the MacDonald’s functions by the corresponded delta-functions, the Lienard-Wichert potentials are occur,

$$A^\mu = \frac{1}{\pi} \frac{2}{mc} \int_{-\infty}^{t} dt' \frac{\delta_\mu}{|\vec{r} - \vec{v}t'|} \{ \delta[t' - t + |\vec{r} - \vec{v}t'|] - \delta[t' - t - |\vec{r} - \vec{v}t'|] \}.$$  \hspace{1cm} (41)

### 7. Electron Magnetic Momentum Field

The previous consideration of the mean electromagnetic field in the sec.6 has shown, that the Coulomb field isn’t the all electromagnetic field of the
electron found in the rest. The additional contribution appears from the
mean value of the vector potential, which general expression is given by the
formula
\[
\vec{A}(\vec{r}) = 2e \Re \left\{ \sum q \tilde{\vec{u}}_{\lambda, \vec{q} \times \vec{q}} \tilde{u}_{\lambda, \vec{q}} \frac{\exp\{iq\vec{r}\}}{\omega_{\vec{q}}} \right\}.
\] (41)

After simple algebra one can reduce this expression to
\[
\vec{A}(\vec{r}) = \nabla \Phi \times \hat{\mu}_{el},
\] (42)
where
\[
\Phi(r) = \frac{4m}{\pi} \int_0^\infty \frac{\sin(x) \, dx}{x \sqrt{x^2 + 2mr^2}} = \frac{2}{\pi r} \int_0^{2mr} K_0(x) \, dx,
\] (43)
\[
\hat{\mu}_{el} = \frac{e}{m} \hat{s} - \text{is the electron magnetic momentum. Apparently, the expression}
\] (42) \text{represents himself the vector potential of the electron magnetic momentum magnetic field. At} \ r \to \infty \ \Phi(r) \to \frac{1}{r}, \text{what is right for the point magnetic momentum. But at} \ r < m^{-1} \ \text{the function} \ \Phi(r) \ \text{has only the logarithmic behavior, simulating the behavior of the main part of potential.}

In the sec.2 it has been noted, that the interaction with the photon field can produce the change in the electron spin direction. Indeed, the explicit evaluation of matrix elements \( (\tilde{u}_{\lambda', \vec{q}' / 2} \gamma^\mu \tilde{u}_{\lambda, \vec{q}} / 2) \) shows that if in the mostly probably processes the condition \( q \gg \varepsilon \vec{p} \) is fulfilled, then independently of any initial polarization in course of evolution the probabilities of all polarizations become approximately equal. For more explicit proof of this conclusion one have to calculate the above matrix elements at \( \vec{k}_0 = 0 \) :
\[
u_{\lambda', \vec{q}' / 2} \tilde{\vec{u}}_{\lambda, \vec{q}} \tilde{u}_{\lambda, \vec{q}} / 2 = \frac{m}{\varepsilon \vec{q}' / 2} \delta_{\lambda' \lambda} ;
\]
\[
u_{\lambda', \vec{q}' / 2} \tilde{\vec{u}}_{\lambda, \vec{q}} \tilde{u}_{\lambda, \vec{q}} / 2 \cdot \tilde{e}_{\beta \vec{q}} = i \frac{q}{2 \varepsilon \vec{q}' / 2} \tilde{w}_{\lambda', \vec{q}'} \tilde{\sigma} [\tilde{e}_{\beta \vec{q}} \times \tilde{n}_{\vec{q}}] \tilde{w}_{\lambda},
\] (44)
where \( \tilde{n}_{\vec{q}} \) - is the unit vector directed along \( \vec{q} \), and \( \tilde{e}_{\beta \vec{q}} \ (\beta = 1, 2) \) - are the polarization vectors of transversal photons. Right hand of the second equation in (44) occurs in consequence of interaction of the electron magnetic momentum with magnetic field of radiating photons. In our approximation, when only diagonal part of Hamiltonian included in the ground state definition, we don’t take into account the spin change during the radiation. In the more strict consideration one need to take into account the above speculations and to get the regular form of the magnetic momentum field.
8. Infrared asymptotics of the photon number

Consider the asymptotic behavior of the mean number of quanta at $\omega = q \to 0$. As it is known, in classical electrodynamics as well as in the semiclassical approach there is the characteristic behavior $n_{\alpha q} \sim 1/\omega^3$ at $\omega \to 0$. In consequence of this the total number of radiated photons is proved to be infinitely large. This result arises due to the rapid increasing the photon number at $\omega \to 0$. Now we note that taking into account the reactive radiation forces remove the singular behavior of the total photon number. As it was shown in [14], the number of radiating photons in collision between the moving particle and a resting scatterer is given by the expression

$$n_{\alpha q}(\infty) = e^2 g_q^2 \left| \frac{\vec{e}_{\alpha q} \vec{v}_2}{\omega - \vec{q} \vec{v}_2 + \Delta} - \frac{\vec{e}_{\alpha q} \vec{v}_1}{\omega - \vec{q} \vec{v}_1 + \Delta} \right|^2,$$  \hspace{1cm} (45)

where $\vec{v}_1$ is assuming the velocity before and $\vec{v}_2$ - after collision, $|\vec{v}_1 - \vec{v}_2| \ll v_1$,

$$\Delta = 2e^2 \sum_{q} g_q^2 \frac{\vec{p}_0^2 - (q' \vec{p}_0)^2/q^2}{\epsilon_- \epsilon_+(\omega' - \epsilon_+ + \epsilon_- + \Delta)}.$$

Here $\vec{p}_0 \approx m\vec{v}_1$. In the non-relativistic limit $v_0 \ll c$ from (46) we get $\Delta \approx (4/3)e^2 m v_1^2$. The formula (45) doesn’t contain the infrared singularity. The deviation from the law $n_{\alpha q} \sim 1/\omega^3$ at $\omega \to 0$ begins at $\omega \sim \Delta$. This energy is the less the less the energy of relative motion of the electron and the scatterer.

8. Conclusion

We see, that problems with divergences in the QED are more the problems of the perturbation theory than the QED itself. The divergences characteristic for the Feinman-Dyson perturbation theory are eliminated due to the physical effects: i) displacement of the particle as the reaction on the own radiation, ii) spin overturn processes due to radiation. In the Feinman approach the commonly used zeroth order propagators don’t take into account the electromagnetic interaction at all. Apparently, if we use the more correct propagators within the new perturbation theory (which is equivalent to the prior ”clothing” of particles), the infrared as well as ultraviolet divergences at the calculation of physical quantities of interest will be eliminated.
Appendix

**Coherent States for the Scalar Photon Field**

Because of the non-usual commutation relations for scalar photons, there are some peculiarities in formulating the coherent states in this case. Below we omit index $\vec{q}$ in notations of operators $\hat{a}_{0,\vec{q}}$ for convenience. To construct the coherent states for the scalar photon field, applicable within the Lorentz gauge and Gupta-Bleuer formalism, we have first to define the Hamiltonian for the free scalar photons. In opposite to the transversal and longitudinal photons it contains the negative sign in front of the usual expression (see, e.g., [3]).

$$\hat{H}_0 = -\omega \hat{b}_0^\dagger \hat{b}_0.$$  

Consider the Schroedinger equation for the forced oscillator

$$i \frac{d}{dt}|t\rangle = [ -\omega \hat{b}_0^\dagger \hat{b}_0 + \alpha(t) \hat{b}_0^\dagger + \alpha^*(t) \hat{b}_0 ] |t\rangle. \quad (47)$$

The vector of state in the interaction representation obeys the equation

$$i \frac{d}{dt}|t\rangle = [ \alpha(t) \tilde{b}_0^\dagger(t) + \alpha^*(t) \tilde{b}_0(t) ] |t\rangle. \quad (48)$$

Here the interaction representation of operators don't differ from usual, for example: $\tilde{b}_0(t) = b_0 e^{-i\omega t}$.

Consider the usual definition of coherent states via D-operator (see, e.g., [15])

$$|Q_0\rangle = \hat{D}(Q_0)|vac\rangle = \exp (Q_0 \hat{b}_0^\dagger - Q_0^* \hat{b}_0 ) |vac\rangle. \quad (49)$$

Taking into account $\hat{b}_0|vac\rangle = 0$, we get

$$\hat{D}(Q_0)|vac\rangle = e^{iQ_0^2/2} \cdot \sum_{n=0}^{\infty} \frac{Q_0^n}{n!} (\hat{b}_0^\dagger)^n |vac\rangle. \quad (50)$$

As we see, the sign in the power of the first exponent in the right hand of this equation is opposite compared to the ordinary coherent state. The coherent state (50) is the eigenvector of the annihilation operator but its eigenvalue has the different sign:

$$\hat{b}_0|Q_0\rangle = -Q_0|Q_0\rangle.$$ 

The solution to the equation (48) can be simply obtained with the help of famous Feinman-Schwinger approach (see, e.g., in [15]) in the form $|t\rangle =$
\[ e^{-i\omega_0 t} |Q_0\rangle \] Apply the simple algebra, we get

\[
|t\rangle = \exp\left\{ \frac{i}{2} \int_{t_0}^{t} \Im[\dot{Q}_0(t')Q_0(t')]dt' \right\} \cdot \hat{D}(Q_0(t)) |\text{vac}\rangle,
\]

where \( t_0 \) - is an instant, when we know the initial vacuum state of the photon field,

\[
Q_0(t) = -i \int_{t_0}^{t} \alpha(t')e^{i\omega t'} dt'.
\]

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References

[1] J.D. Bjorken, S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Company, 1965)

[2] C. Itzikson, J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill Book Company, 1978)

[3] L.H. Reider, *Quantum Field Theory* (University Press, Cambridge, 1995)

[4] S. Wienberg, *The Quantum Theory of Fields*, Vol.1 (University Press, Cambridge, 1995)

[5] V.B. Berestezkiy, E.M. Lifshitz, L.P. Pitajevskiy, *Quantum Electrodynamics* (Nauka, Moscow, 1980)

[6] R.P. Feinman, *QED the Strange Theory of Light and Matter* (Princeton Univ. Press, Princeton, 1985)

[7] Greenberg O.V., and Schweber S.S. *Clothed particle operators in simple models of quantum fields theory*, Nuovo Cimento 8 (1958), 378

[8] D.J. Hearn, M. McMillan, and A. Raskin, *Dressing the cloudy bag model: Second-order nucleon-nucleon potential*, Phys. Rev. C 28 (1983), 2489

[9] M. Kobayashi, T. Sato, and H. Ohtsubo, *Effective interaction for mesons and baryons in nuclei*, Progr. Theor. Phys. 98 (1997), 927

[10] A.V. Shebeco, M.I. Shirokov, *Unitary transformations in quantum field theory*, nucl-th/0102037

[11] V.Yu. Korda, and A.V. Shebeco, *Clothed particle representation in quantum field theory: Mass renormalization*, Phys.Rev. D70, (2004), 085011

[12] E.V. Stefanovich, *Renormalization and dressing in quantum field theory*, hep-th/0503076

[13] G.M. Filippov, *Extended coherent states and modified perturbation theory*, J.Phys.A: Math.Gen. 33 (2002), L293 [quant-ph/0003011]

[14] Filippov G.M., *Interaction of radiation and a relativistic electron in motion in a constant magnetic field*, Sov. Phys. JETP 86 (1998), 459 [quant-ph/9907064]
[15] Perelomov A.M. *Generalized Coherent States and Their Applications*, (Springer-Verlag, 1986)