A Closed-loop Supply Chain Inventory Model Considering Limited Number of Remanufacturing Generation and Environmental Investigation

W A Jauhari\textsuperscript{1,}\textsuperscript{*}, R D Septian\textsuperscript{1}, P W Laksono\textsuperscript{1}, A R Dwicahyani\textsuperscript{2}

\textsuperscript{1} Department of Industrial Engineering, Universitas Sebelas Maret, Surakarta, Indonesia
\textsuperscript{2} Department of Industrial Engineering, Adhi Tama Institute of Technology, Surabaya, Indonesia

E-mail: wakhidjauhari@gmail.com

Abstract. This study presents a mathematical inventory model in the Closed-loop Supply Chain (CLSC) system. We investigate a CLSC system that consists of a retailer, a manufacturer and a 3rd party collection dealer. The model aims to minimize the annual joint total inventory cost for the three parties and determine several decision variables, including number of shipments, manufacturer shipment lot size and number of remanufacturing generations. The number of remanufacturing generations is one of our concerns since we consider a type of product that could not be remanufactured in an unlimited number of time. Thus, how many times the product should be remanufactured becomes important. In this study, we also investigate the environmental impacts in term of carbon emissions, energy usage, as well as annual disposed items. The model can guide companies in the related industry to plan a system of CLSC inventory management with environmental considerations.

1. Introduction

A supply chain provides a network for business players and suppliers working together to deliver goods, services and information to consumers to be more effectively and efficiently [1]. Within a good planning and design of supply chain management, the company could achieve what we call by an “optimal supply chain system”. There are two streams in supply chain management, namely open-loop supply chain and closed-loop supply chain. Open-loop supply chain is noticed by no product returns from consumer to initial producer [2]. Whereas closed loop supply chain is characterized by the existence of product returns managed by the manufacturer or supplier by performing the recovery process on the returned product prior to resale. The recovery process can be categorized into repair, refurbishing, remanufacturing, cannibalization and recycling [3]. The categorization of the recovery process depends on the type of product and the treatment needed to recover the product.

In a supply chain system, inventory need to be controlled effectively and efficiently. Inventories play a role in eliminating the risk of delays and material deficiencies. Excessive inventory will be resulting in waste due to high storage and maintenance costs. While the lack of inventory leads to shortages that may disrupt the activities of production and distribution. There are many prior studies that focus on the development of an optimal inventory modelling in reverse logistics, including [4], [5], [6], [7], [8], [9], [10].

On the other hand, environmental issues that continue to be the world's attention have forced manufacturing companies to carry out sustainable production and distribution activities. As discussed formerly, a closed-loop supply chain is a sustainable operation that enables manufacturer to recover returned products from the market for future resale. The recovery of used items can reduce waste of end-of-life products in the market [11]. In addition, it can also reduce the use of raw materials from nature. Therefore, the application of closed loop supply chain in the world of
manufacturing industry is closely related to sustainable supply chain and environmental considerations.

El Saadany et al. [7] developed a single-echelon inventory model considering a number of limited remanufacturing generations. The number of remanufacturing generations indicates the number of times a product can be remanufactured. This relaxes the general assumption regarding products that can be continuously remanufactured at an unlimited number of times. The number of remanufacturing generations depends on the amount of the R&D investments made on the product. The higher number of remanufacturing generations, the lesser the waste would be. Bazan et al. [8] improved the model of [7] to consider carbon emissions, energy, and transportation costs. Dwicahyani et al. [10] then developed the model of [8] by including more parties among the supply chain, which are a distributor that acts as both retailer and collector of used items.

Based on the above discussion, many studies have accommodated the recovery process of used items. However, the company who collects used items from the market is always the manufacturer or the retailer itself. Moreover, there has been no research to include inspection of used items that returned from the market before they went through the recovery process. In this study, we developed a three-echelon closed-loop supply chain inventory model consisting of a manufacturer, a retailer and a 3rd party collection dealer as a collector of used items from the market. We consider a limited number of remanufacturing generations [7] and also includes freight costs as well as carbon emissions costs generated from the regular production process, remanufacturing process and transportation activities.

2. Model development

Here, we examine a CLSC system that consists of a retailer, a manufacturer and a used item collector. We seek to get the optimal solution regarding several inventory decisions, such as the shipment lot size, number of shipments per year, and also the number of remanufacturing generations. The material flow among those parties are depicted in Figure 1.

![Figure 1. The CLSC system investigated in the model.](image)

In this model, we consider the concept of remanufacturing generations firstly proposed by [7]. By this concept, items that classified as recoverable will go through several number of remanufacturing generations before they are completely disposed as waste. This study intends to determine the number of remanufacturing generations along with the determination of shipment lot size and number of shipment between investigated parties so the total profit can be maximized.

We consider two components of cost related to environmental implications, which are cost of carbon emission and energy usage. We adopt the functions of both costs from [8]. The following component of costs are considered in the development of model’s objective function:

a. Holding costs at retailer, manufacturer (finished product
The objective of this model is to minimize all of the above costs and determine the following decision variables:

- $k$: number of shipments from collector to manufacturer, w.r.t $k \geq 1$ and an integer
- $n$: number of shipments from manufacturer to retailer, w.r.t $n \geq 1$ and an integer
- $Q$: shipment lot size from manufacturer to retailer (unit)
- $\zeta$: number of remanufacturing generations, w.r.t $\zeta \geq 0$ and an integer

The parameters used in this model following with their notations and constraints are listed as follows:

- $\beta$: nominal proportion of used items returned for remanufacturing, when the number of remanufacturing generations is unlimited ($\zeta=\infty$), with $0 \leq \beta < 1$;
- $\beta_{i}$: actual proportion of used items returned for remanufacturing, when the number of remanufacturing generations is limited ($\zeta\geq0$), with $\beta_{i} = 1 - \frac{(1 - \beta_{p})}{(1 - \beta_{r})}$;
- $H_{1}$: holding cost per unit per unit of time at manufacturer finished product inventory ($$/unit/year$);
- $H_{2}$: holding cost per unit per unit of time at retailer inventory ($$/unit/year$);
- $H_{3}$: holding cost per unit per unit of time at collector inventory ($$/unit/year$);
- $H_{4}$: holding cost at manufacturer recoverable item inventory ($$/unit/year$);
- $H_{5}$: holding cost at manufacturer raw material inventory ($$/unit/year$);
- $A_{1}$: set-up cost of remanufacturing ($$/set-up$$);
- $A_{2}$: set-up cost of manufacturing ($$/set-up$$);
- $S_{1}$: ordering cost for manufacturer recoverable inventory ($$/order$$);
- $S_{2}$: ordering cost for retailer inventory ($$/order$$);
- $P_{m}$: retailer purchasing cost ($$/unit$$);
- $P_{b}$: collector used item collection cost ($$/unit$$);
- $C_{r}$: remanufacturing cost per unit ($$/unit$$);
- $C_{mn}$: manufacturing cost per unit ($$/unit$$);
- $a_{i}$: emissions function parameter for process $i$ (ton year$^{2}$/unit$^{i}$), with $i\in\{p, r\}$ where $i=p$ for manufacturing activity and $i=r$ for remanufacturing activity;
- $b_{i}$: emissions function parameter for for process $i$ (ton/year$^{2}$), with $i\in\{p, r\}$ where $i=p$ for manufacturing activity and $i=r$ for remanufacturing activity;
- $c_{i}$: emissions function parameter for process $i$ (ton/unit$^{2}$), with $i\in\{p, r\}$ where $i=p$ for manufacturing and $i=r$ for remanufacturing;
- $C_{0}$: the required energy at the machine to manufacture one unit (KWh/unit);
- $C_{1}$: the required energy per year when manufacturing is idle (KWh/year);
- $C_{0}'$: the required energy at the machine to remanufacture one unit (KWh/unit);
- $C_{1}'$: the required energy per year when remanufacturing is idle (KWh/year);
- $d$: annual demand rate (units/year);
- $v$: annual remanufacturing rate (units/year), with $v > d$;
- $\gamma$: annual manufacturing rate (units/year) with $\gamma > d$;
- $C_{GHG}$: carbon emission cost ($$/ton CO_{2}$$);
- $c_{en}$: cost of energy ($$/KWh$$);
- $c_{w}$: disposal cost per unit ($$/unit$$);
- $c_{is}$: inspection cost ($$/unit$$);
- $\rho$: collector defective item proportion, with $0 \leq \rho \leq 1$;
Asset Investment Factor that Governs the Ratio of Investment for Each Remanufactured Generation, with $0 \leq \theta < 1$

Annual Investment in the R&D Process to, theoretically, Be Able to Remanufacture It for an Indefinite Number of Time ($/Year$)

Here are Several Assumptions That Used to Develop the Model:

1. Manufacturing and Remanufacturing Processes Are Perfect
2. Remanufacturing Rate Is Less Than the Demand Rate ($v < d$)
3. Collector Will Inspect All of Its Items at the Beginning of Each Cycle
4. Demand ($d$) Is Deterministic
5. Equal Shipment Lot Size for Each Party
6. Lead Time for Transportation and Production Processes Are Not Considered (Equals to Zero)
7. Remanufactured Items Have the Same Quality as the New Items (As-Good-As-New-One)
8. Used Items Return from the Market Have Uniform Quality.

The Formulation of Cost Functions Are Referred to the Model of [7], [8], [9], and [10]. The Objective Function Is to Minimize the Total Inventory Related Costs Among the Supply Chain.

A. Retailer Inventory Cost Function

Inventory Cost Related to the Retailer ($TIC_{re}$) Consists of Holding Cost ($HC_{re}$), Ordering Cost ($SC_{re}$), Transportation Cost ($TC_{re}$) and Carbon Emission Cost from Transportation Activity ($CGHG_{re}$). The On-Hand Stock Level at Retailer Inventory Is Depicted in Figure 2.

![Figure 2. On-hand stock level at retailer inventory.](image)

The Function of $TIC_{re}$ Is Given By:

$$TIC_{re} = H_2 \frac{Q}{2} + S_2 \frac{d}{Q} + \frac{dF_t}{tc} + g_t c_GHG \left( \frac{d}{tc} \right)$$

B. Manufacturer Inventory Cost Function

Figure 3 Shows the On-Hand Stock Level at Finished Products, Recoverable Items and Raw Materials Inventory. As Mentioned Previously, the Holding Costs Belongs to Manufacturer ($HC_{m}$) Include the Cost of Storing the Finished Products, Recoverable Items and Raw Materials. The Annual Inventory Cost for Manufacturer ($TIC_{m}$) Is Consists of Holding Costs ($HC_{m}$), Order Cost ($OC_{m}$), Set-Up Cost ($SC_{m}$), Production Costs ($MC_{m}$), Transportation Cost ($TC_{m}$), R&D Investment Cost ($C_{inv}$), Carbon Emission Cost ($CGHG_{m}$), and Energy Usage Cost ($EC_{m}$).
**Figure 3.** On-hand stock level at manufacturer finished product inventory (a) and raw material-recoverable item inventory (b).

\( TIC_m \) is formulated as:

\[
TIC_m = HC_m + OC_m + SC_m + MC_m + TC_m + C_{in} + CGHG_m + EC_m
\]

\[
= \left( H_1 \frac{n}{2} + H_4 \frac{n}{2} \left( 1 - \frac{1-\beta}{1-\beta^2} \right)^2 + H_5 \frac{n}{2} \left( \frac{1-\beta}{1-\beta^2} \right)^2 \right) + \left( S_1 \frac{k}{nQ} \right) + \left( A_1 \frac{d}{nQ} \left( 1 - \frac{1-\beta}{1-\beta^2} \right) + A_2 \frac{d}{nQ} \left( \frac{1-\beta}{1-\beta^2} \right) \right)
\]

\[
+ \left( d \left( C_{ri} \left( 1 - \frac{1-\beta}{1-\beta^2} \right) + C_{mn} \left( \frac{1-\beta}{1-\beta^2} \right) \right) \right) + \frac{dF_i}{l_c} \left( 1 - \frac{1-\beta}{1-\beta^2} \right) + c_{inv} (1 - \frac{e^{-\beta t}}{1-\beta^2}) + c_{GHG} d \left( \left( c_r b_v y + a_p y^2 \right) \left( 1 - \frac{1-\beta}{1-\beta^2} \right) + \left( c_p b_p y + a_p y^2 \right) \left( \frac{1-\beta}{1-\beta^2} \right) + \left( \frac{r_c}{l_c} \right) \left( 1 - \frac{1-\beta}{1-\beta^2} \right) \right)
\]

\[
+ c_{en} d \left( \left( C_0 v + \frac{C_1 v}{y} \right) \left( 1 - \frac{1-\beta}{1-\beta^2} \right) + \left( C_0 y + \frac{C_1 y}{y} \right) \left( \frac{1-\beta}{1-\beta^2} \right) \right)
\]

\[
(2)
\]

**C. Collector Inventory Cost Function**

Inventory cost related to the collector (\( TIC_c \)) consists of holding cost (\( HC_c \)), collection cost (\( PC_c \)), inspection cost (\( C_{ins} \)) and waste disposal cost (\( WC_c \)). Figure 4 shows the on-hand stock level at collector used item inventory.

**Figure 4.** On-hand stock level at collector used item inventory.

The function of \( TIC_c \) is given by:

\[
TIC_c = H_3 \frac{nQ}{2} \left( 1 - \frac{1-\beta}{1-\beta^2} \right) + d \left( P_b + c_{isc} + c_w \right) \left( 1 - \frac{1-\beta}{1-\beta^2} \right) + p
\]

\[
(3)
\]

Finally, we get the function of annual joint total cost for the three parties as given by

\[
AJTC(n, Q, k, \zeta) = TIC_r + TIC_m + TIC_c
\]

**3. Solution Procedure**

The optimization problem of the proposed model is to determine the values of \( n, Q, k \) and \( \zeta \) that minimize the annual inventory related costs of all parties, \( AJTC(n, Q, k, \zeta) \). We get the optimal solutions of \( Q \) by taking the first partial derivative of Equation (4) with respect to \( Q \), and then setting the result to zero. The optimal value of \( Q \) denoted by \( Q^* \) is derived as follows.

\[
Q^*(n, k, \zeta) = \sqrt{\frac{2d k (\beta^{1+\zeta})^2 (A_2 (\beta-1) - A_1 \beta + A_1 \beta^{1+\zeta} - n S_h + n S_1 \beta^{1+\zeta} + k S_2 (\beta^{1+\zeta}))}{n H_2 (\beta^{1+\zeta})^2 + n (\beta (\beta^{1+\zeta}) (H_1 (\beta^{1+\zeta}) + H_k (\beta^{1+\zeta}) + H_k (\beta^{1+\zeta})))}}
\]

(5)
With the same procedure, we obtain \(n^*\) and \(k^*\) as given by Equation (6) and (7) respectively.

\[
n^*(Q, k, \xi) = \frac{2d^k(\beta^\xi-1)}{H_1k(\beta^\xi-1)^2 + dQ(\beta^\xi-1)(H_2(\beta^\xi-1)+H_4k(\beta^\xi-1)^2)}
\]

(6)

\[
k^*(n, Q, \xi) = \frac{H_1n^2\beta^\xi}{\sqrt{2dS_1(\beta^\xi-1)}}
\]

(7)

The optimal values of \(Q^*, n^*, k^*\) and \(\xi^*\) that minimize \(AJTC(Q, n, k, \xi)\) with \(1 \leq \xi \leq 10\) are derived by the following solution procedure.

**Step 1** Set \(\xi = 1\)

**Step 2** Substitute \(k^*(n, Q)\) in Equation (7) into Equation (6), by solving the equation we then obtain the function of \(n^*(Q)\)

**Step 3** Substitute \(n^*(Q)\) into Equation (5) and solve the equation. We then obtain the function of \(Q^*\)

**Step 4** Compute \(n^*(Q)\) with \(Q = Q^*\)

**Step 5** Compute \(k^*(n, Q)\) with \(Q = Q^*\) and \(n = n^*\)

**Step 6** Repeat steps 1 to 5 with \(\xi = \xi + 1\)

**Step 7** Find the value of \(\xi^*\) that gives the minimum value of \(AJTC(Q^*, n^*, k^*, \xi^*)\), and set as \(\xi^*\).

By solving the optimization problem analytically, we guarantee a global optimal solution for the problem.

**4. Numerical Example, Discussion, and Analysis**

To illustrate the model, we provide a numerical example using a set of data collected from the previous studies ([4], [8], and [10]). The parameters used to illustrate the model are:

\[
\begin{align*}
\beta &= 0.67 & C_{ren} &= $20/unit & C_{GHG} &= $2/ton CO_2 \\
d &= 100 \text{ units/year} & C_{re} &= $10/unit & C_0 &= 57.96 \text{ Kwh/unit} \\
\gamma &= 150 \text{ units/year} & P_t &= $10/unit & C_1 &= 1,855,744 \text{ Kwh/year} \\
v &= 100 \text{ units/year} & P_c &= $8/unit & C_2 &= 18.9 \text{ Kwh/unit} \\
H_1 &= $50/\text{unit/year} & P_b &= $5/unit & C_{1,0} &= 605,110 \text{ Kwh/year} \\
H_2 &= $60/\text{unit/year} & c_w &= $5/unit & c_m &= $0.000928/\text{Kwh} \\
H_3 &= $25/\text{unit/year} & \rho &= 0.1 & c_{ic} &= $2/unit \\
H_4 &= $40/\text{unit/year} & \alpha &= 8.33 \times 10^{-8} \text{ ton/year}^2/\text{unit}^3 & c_{m,0} &= $500/\text{year} \\
H_5 &= $40/\text{unit/year} & b_r &= 0.0002 \text{ ton/year}^2/\text{unit}^2 & \theta &= 0.2 \\
A_1 &= $100/\text{batch} & c_r &= 1.4 \text{ ton/unit} & F_t &= $100/\text{truck} \\
A_2 &= $200/\text{batch} & a_p &= 3 \times 10^{-7} \text{ ton/year}^2/\text{unit}^3 & t_r &= 20 \text{ unit/truck} \\
S_1 &= $30/order & b_p &= 0.0012 \text{ ton/year}^2/\text{unit}^2 & g_t &= 375 \text{ gallons/truck} \\
S_2 &= $50/order & c_p &= 1.4 \text{ ton/unit} & e_t &= 0.1008414 \text{ ton/gallon}
\end{align*}
\]

We solve the problem using the procedure given in Section 3. The solutions for the given problem are: 3 shipments/cycle with the size of 13 units/shipment from manufacturer to retailer; 2 shipments/cycle from collector to manufacturer; 3 production cycles/year; 2 remanufacturing generations. By setting \(Q, n, k,\) and \(\xi\) at the suggested values, the supply chain system minimizes its annual cost at $7,580.16.

More generation of remanufacturing will eventually end up in a higher amount of \(CO_2\) emission from the remanufacturing process and yet lower amount of \(CO_2\) emission from the manufacturing process (see Figure 5a). The more generations of remanufacturing will increase the amount of product returned to the system and the amount of product produced through remanufacturing process. Emissions generated from the manufacturing process remains decreased since the amount of products produced by the manufacturing activities is smaller. Lowest \(CO_2\) emission is obtained at \(\xi = 1\), and highest \(CO_2\) emission is obtained at \(\xi = 10\). The value of \(\xi = 1\) indicates that the product is only manufactured 1 time in its lifetime. Therefore, few emissions are generated from the
remanufacturing process. In addition, there is only one transportation activity to deliver the used items from the market, resulting in less amount of CO₂ emission generated from related activity.

![Figure 5. The amount of ton CO₂ emissions per year (a) and amount of energy (KWh) per year at different values of ζ.](image1)

Manufacturing energy usage gradually decreases as the number of remanufacturing generation increases (see Figure 5b). Whereas, the increased number of remanufacturing generations actually increases the amount of energy used to perform remanufacturing process. The lowest usage of energy came when the number of remanufacturing generations is at its highest. When ζ=10, the system reduce a significant amount of energy usage from manufacturing process because the energy required to produce one unit of product in the manufacturing process is higher than remanufacturing.

We understand that less number of remanufacturing generations will give less CO₂ emissions, but highest energy usage. Therefore, companies should reconsider what criteria to prioritize in order to investigate environmental effects. If the company is focusing more in reducing the CO₂ emissions, then the decision will be not to remanufacture more. Yet, if the company is more concerned to reduce the amount of energy usage, the decision should be to remanufacture more. However, this should also be considered in conjunction with the costs and other parameters associated and experienced by the company. Hence in the end, the decision can actually provide a best solution that gives lowest cost to the system.

![Figure 6. The effects of c_re on AJTC for ζ = {1, 2, 3, 8}.](image2)

Figure 6 shows the effects of different c_re on AJTC for ζ={1, 2, 3, 8}. The interesting finding is that, the optimal value of ζ will be increased from 2 to 3, when the cost to remanufacture is decreased to $2.5/unit. In addition, when the cost to remanufacture is increased to $12.5/unit, the optimal value of of ζ will be decreased from 2 to 1. It suggests that remanufacturing variable cost actually affects the optimal number of remanufacturing generations. Higher c_re will be resulting in lower ζ, while lower c_re will be resulting in higher ζ. Here, we can take a note that when the cost to remanufacture one unit of item is expensive, the system should consider designing its product to be less...
remanufacturable. It will be not profitable if there are more number of items that will go through the remanufacturing process.

5. Conclusion

This study develops a CLSC inventory model consisting a manufacturer, a retailer, and a used item collection dealer. Previous works on this problem have not yet included the third party collection dealer as one separate party among the reversed supply chain. Here, we investigate limited number of remanufacturing generations and consider carbon emission and energy effects as component of inventory costs. The results explain that in order to obtain minimum integrated supply chain inventory cost, the system should design a product in such a way so that it could be remanufactured at a desired number before it undergoes disposal. However, designing a product to be highly remanufacturable does not necessarily become a good strategy, since it actually gives higher cost to the system.

Future works for the related problems should include more recovery activities, such as disassembly and remanufacturing of individual components as well as assemblies. Another challenge is to consider that not all collected items have the same quality. Since it comes from the different consumers, the remaining quality of each item may not be uniformly distributed. As stated by [12] uncertainties in product acquisition may affect both manufacturing and remanufacturing strategies. It is explained that the return rate of used item will actually be possible to be controlled if the company performs a specific acquisition effort. As of it will affect the quality, quantity, cost, and lead time of the return item management activity.

References

[1] Akdogan S M and Coskun A 2012 Drivers of reverse logistics activities: an empirical investigation Int. Stra. Manag. Conf. Procedia-Soc. and Behav. Scie. 58 1640.
[2] Pujawan I N 2008 Supply Chain Management (2nd ed), Surabaya: Guna Widya
[3] Thierry M, Salomon M, Nunen J V and Wassenhove L V 1995 Strategic issues in product recovery management California Manag. Rev. 37 114.
[4] Mitra S 2009 Analysis of two echelon inventory system with returns The Int. J. Manag. Sci. 37 106.
[5] Chung S, Wee H M and Yang P 2008 Optimal policy for a closed-loop supply chain inventory system with remanufacturing J. Math. and Comp. Modelling 48 867.
[6] Yuan K F and Gao Y 2010 Inventory decision-making models for a closed-loop supply chain system Int. J. Prod. Res. 6155.
[7] El Saadany A M A, Jaber M Y and Bonney M 2013 How many times to remanufacture? Int. J. Prod. Econ. 143 598.
[8] Bazan E, Jaber M Y and El Saadany A M A 2015 Carbon emissions and energy effects on manufacturing-remanufacturing inventory model Comp.and Indast. Eng. 88 307.
[9] Dwicahyani A R, Jauhari W A and Jonrinaldi 2017 A regular production-remanufacturing inventory model for a two-echelon system with price-dependent return rate and environmental effects investigation J. Physics: Conf. Series 855.
[10] Dwicahyani A R, Jauhari W A, Rosyidi C N and Laksono P W 2017 Inventory decisions in a two-echelon system with remanufacturing, carbon emission, and energy effects Cogent Eng. 4 1.
[11] Fleischmann M, Bloemhof-Ruwaard J M, Dekker R, Van der Laan E, Van Nunen J A A E E and Van Wassenhove L N 1997 Quantitative models for reverse logistics: A review European J. Oper. Res.103 1.
[12] Lechner G and Reimann M 2014 Impact of product acquisition on manufacturing and remanufacturing strategies Prod. and Manufac. Res. 2 831.