Nuclear Weak Processes and Astrophysical Applications

Toshio Suzuki\textsuperscript{1,2} and Michio Honma\textsuperscript{3}
\textsuperscript{1}Department of Physics, College of Humanities and Sciences, Nihon University
Sakurajosui 3-25-40, Setagaya-ku, Tokyo 156-8550, Japan
\textsuperscript{2}Center for Nuclear Study, University of Tokyo, Hirosawa, Wako-shi, Saitama 351-0198, Japan
\textsuperscript{3}Center for Mathematical Sciences, University of Aizu
Aizu-Wakamatsu, Fukushima 965-8580, Japan
E-mail: suzuki@chs.nihon-u.ac.jp

Abstract. Nuclear weak processes are investigated based on new shell model Hamiltonians, which give successful description of spin responses in nuclei, and applied to astrophysical problems. Neutrino-induced reactions on $^{12}$C and synthesis of light elements by supernova neutrinos, and effects of contamination of $^{13}$C, whose natural isotopic abundance is 1.1\%, on inclusive $\nu$-$^{12}$C reactions are discussed. Spin-dipole transitions and $\nu$-induced reactions on $^{16}$O are studied by using a new Hamiltonian with proper tensor components, and compared with conventional calculations and previous CRPA results. Gamow-Teller transition strength in $^{40}$Ar and $\nu$-induced reactions on $^{40}$Ar by solar neutrinos are studied based on monopole-based-universal interaction (VMU). We finally discuss electron capture reactions on Ni isotopes in stellar environments.

1. Introduction
Neutrino-nucleus reactions and electron capture reactions, which are dominantly induced by excitations of spin modes in nuclei, are investigated with the use of new shell model Hamiltonians, SFO[1] and SFO-tls [2] in $p$-shell and $p$-$sd$ shell, GXPF1 [3] in $fp$-shell and monopole-based-universal interaction (VMU[4]) in $sd.fp$ shell. The new Hamiltonians properly take into account important roles of the tensor interactions.

Neutrino nucleus reactions on $^{12}$C, $^{13}$C and $^{16}$O are studied in Sect. 2. Neutrino-induced reactions on $^{40}$Ar are studied in Sect. 3. Electron capture reactions on Ni isotopes in stellar environments are discussed in Sect. 4.

2. Neutrino-Nucleus Reactions in Carbon and Oxygen
2.1. Neutrino-induced Reaction on $^{12}$C and Light Element Synthesis
Neutrino-induced reactions in $p$-shell nuclei are investigated based on new shell model Hamiltonians[1], which take into account important roles of tensor interaction properly[9] and give good account of spin-dependent modes in $p$-shell nuclei. Magnetic moments of $p$-shell nuclei are systematically well described, and Gamow-Teller (GT) transition strengths in $^{12}$C and $^{14}$C are also well explained[1].

The GT strength in $^{12}$C is enhanced for the new Hamiltonian, SFO[1], compared to conventional Hamiltonians, and the cross sections of $\nu$-induced exclusive charge-exchange
reaction, $^{12}\text{C} (\nu, e^-)^{12}\text{N}_{\text{g.s.}}$, are reproduced by SFO. The enhancement of the $\nu$-$^{12}\text{C}$ reaction cross sections is found to lead to the enhancement of the production yields of light elements, $^{11}\text{B}$ and $^7\text{Li}$, in supernova explosions[5, 6]. The element $^{11}\text{B}$ is produced by both $^{12}\text{C} (\nu, \nu'p)^{11}\text{B}$ and $^7\text{Li} (\alpha, \gamma)^{11}\text{B}$ reactions while $^7\text{Li}$ is produced through $^4\text{He} (\nu, \nu'p)^{3}\text{H} (\alpha, \gamma)^{7}\text{Li}$. Therefore, in addition to the enhancement of the reaction cross sections in $^{12}\text{C}$, the enhancement of the $\nu$-$^4\text{He}$ reaction cross sections with the use of recent Hamiltonians such as WBP[7] is also important to get the enhancement of the enhancement of the production yields of $^7\text{Li}$ and $^{11}\text{B}$. The enhancement factor for the production yield is found to be about 20% and 30% for $^{11}\text{B}$ and $^7\text{Li}$, respectively, for WBP+SFO Hamiltonians in a supernova explosion model[5]. Here, neutral current reactions induced by heavy-flavor neutrinos with higher energies play dominant roles, while in case of neutrino oscillations charge-exchange reactions induced by $\nu_e$ also become important[6].

2.2. Neutrino-induced Reactions on $^{13}\text{C}$

Neutrino-induced reactions on $^{13}\text{C}$ are studied by shell model calculations with the use of SFO. The carbon target is one of a very few examples, on which neutrino induced reactions are measured. The target is usually assumed to be pure $^{12}\text{C}$ isotope while it has contamination of $^{13}\text{C}$ with 1.1% of the natural isotopic abundance. As the Q-values for the $\nu$-$^{13}\text{C}$ reactions are low compared to $^{12}\text{C}$ case, the cross sections for $^{13}\text{C}$ are larger than those of $^{12}\text{C}$ and the small admixture of $^{13}\text{C}$ in the target can have non-negligible contributions to the inclusive $\nu$-carbon cross sections at low neutrino energies.

Figure 1. (a) Inclusive reaction cross sections for $^{12}\text{C} (\nu, e^-)^{12}\text{C}$ and $^{13}\text{C} (\nu, e^-)^{13}\text{N}$ obtained by shell model calculations with the use of SFO. The sum of $^{12}\text{C}$ and 1.1% of $^{13}\text{C}$ cross sections are also shown. (b) Ratio of the cross sections; $[^{12}\text{C}+^{13}\text{C}(1.1\%)]/^{12}\text{C}$.

Calculated inclusive cross sections for carbon isotopes are shown in Fig. 1. The contributions from GT and spin-dipole transitions are included; multipoles up to $J^\pi = 4^+$ are taken into account. The effects of the mixing of $^{13}\text{C}$ on the $\nu$-$^{12}\text{C}$ cross sections are about 20% $\sim$ 4% at $E_\nu$ =30 $\sim$ 80 MeV, and get larger at lower $E_\nu$ (see Figs.1(a) and 1(b)). The inclusive cross sections for supernova temperatures $T_\nu$ are affected by about 30% $\sim$ 5% at $T_\nu$ =4$\sim$ 10 MeV. While the
effects are minor at high temperatures, they can be as large as $(50\%, 30\%, 15\%)$ for $T_\nu=(3.5, 4, 6)$ MeV, which correspond to a typical set of $T_\nu$ for ($\nu_e, \bar{\nu}_e, \nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$).

2.3. Neutrino-induced Reactions on $^{16}$O

The $^{16}$O nucleus plays an important role in producing $^{15}$N by $^{16}$O ($\nu, \nu'p$) $^{15}$N reaction in supernova explosions. It is also contained in water, which is used as target for neutrino detections. Here, $\nu$-$^{16}$O reactions are studied by using the modified SFO interaction, SFO-tls[2], where the tensor components of the $\pi + \rho$ meson exchange potential are used for the $p$-$sd$ cross shell two-body matrix elements. As the $\nu$-$^{16}$O reactions are mainly induced by spin-dipole transitions, it is important to take full account of the tensor force properly in the $p$-$sd$ cross shell part. Energies of the spin-dipole states are rather well reproduced. Calculated spin-dipole strength is more fragmented and shifted toward lower energy region for SFO-tls compared to SFO.

![Figure 2](image)

**Figure 2.** (a) Calculated reaction cross sections for $^{16}$O ($\nu, e^-$) $^{16}$F obtained by shell model calculations with the use of SFO-tls and SFO, as well as CRP A calculations. (b) Ratios of the cross sections; SFO-tls vs CRPA, and SFO vs CRPA.

Calculated cross sections for charged-current and neutral-current reactions are shown in Figs. 2 and 3, respectively. The cross sections obtained by SFO-tls are compared with those of SFO and CRPA[8]. The charged-current (neutral-current) cross sections for SFO-tls are enhanced compared with those of SFO and CRPA by about $30\% \sim 5\%$ ($20\% \sim 10\%$) and $15\%$, respectively, at $E_\nu =30\sim 60$ MeV. It would be interesting to study effects of the enhancement of the cross sections for SFO-tls on nucleosynthesis and neutrino detections.

3. Neutrino-induced Reactions on $^{40}$Ar

Liquid Ar is an important target to measure solar neutrinos. Neutrino-induced reactions on $^{40}$Ar are studied by using the monopole-based-universal interaction (VMU) which has proper tensor components[4, 9]. The sdpf-m[10] and Gxpf1j[11] interactions are used for $sd$-shell and $fp$-shell part, respectively, while the VMU is adopted for the $sd$-$fp$ cross shell part. This interaction will be referred as SDPF-VMU. Configurations are restricted within $2\hbar \omega$.
The same as in Fig. 2 for neutral current reaction; $^{16}$O ($\nu$, $\nu'$) $^{16}$O.

Excitations, $(sd)^{-2}(fp)^2$, and the GT transition strengths are obtained for SDPF-VMU and WBT[7] interactions with the quenching factor of $f = 0.775[12]$. For SDPF-VMU, the energy of the first $1^+$ state of $^{40}$K is well reproduced and the strength distribution is also rather well described. Calculated $B(GT_1)$ and the sum of $B(GT_1)$ up to the excitation energy of $^{40}$K ($E_x$) are shown in Fig. 4 as well as experimental values obtained by recent ($p$, $n$) reaction[13]. In case of WBT, the GT strength distribution is shifted toward lower energy by 1.81 MeV so that the $E_x$ for the first $1^+$ state coincides with the experimental value; this case is referred as WBT-$\Delta E$. The experimental GT strength is rather well described by SDPF-VMU as shown in Fig. 4(b). The sum of the GT strength in Ref. [12] is smaller than the observed strength[13].

Calculated cross section for $^{40}$Ar ($\nu$, $e^-$) $^{40}$K is shown in Fig. 5 for SDPF-VMU. The GT transitions and the transition to the isobaric analog state (IAS) are included. The contributions from the transitions where the final electron energy is larger than 5 MeV, which is consistent with the experimental condition for ICARUS[14], are taken into account. The calculated cross section is consistent with that obtained from experimental GT strength from the ($p$, $n$) reaction[13].

Cross sections folded over $^8$B $\nu$ spectrum[15] are shown in Table I. Here, the contributions from $E_1^5$ (axial electric dipole), $M1$ (magnetic dipole), $C_1^5$ (axial Coulomb) and $L_1^1$ (axial longitudinal) for GT and $C0$ (Coulomb) and $L0$ (longitudinal) for IAS are taken into account, except for the case of Ref. [12] where only the $E_1^5$ and $C0$ contributions are included. Note that $C0 + L0 = (2^2 - \omega^2)C0$ with $q$ ($\omega$) the momentum (energy) transfer. The GT contributions are found to be enhanced for SDPF-VMU and WBT-$\Delta E$ about by 40% compared with those of Ref. [12].

**4. Electron Capture Reactions in Ni Isotopes**

Electron capture reactions in Ni isotopes in stellar environments are studied by using the new shell model Hamiltonian, GXPF1J[11]. Spin properties of $fp$-shell nuclei are well described by GXPF1J with the universal quenching factors: $g_A^{E1}/g_A = 0.74$ and $g_A^{M1}/g_A = 0.75$. Experimental GT$_+$ strengths in the $\beta^+$ channel in $^{58}$Ni [16] and $^{60}$Ni [17] are well reproduced by GXPF1J [18]. Thus, the electron capture rates in $^{58}$Ni and $^{60}$Ni obtained from the experimental GT$_+$ strengths
Figure 4. (a) GT strengths for $^{40}\text{Ar} \rightarrow ^{40}\text{K}$ transition obtained by shell model calculations with the use of SDPF-VMU and WBT-$\Delta E$ interactions. (b) The sum of $B(GT)$ values up to excitation energy $E_x$ of $^{40}\text{K}$ for SDPF-VMU and WBT-$\Delta E$ as well as the experimental data[13]. The values in Ref. [12] are also shown.

Table 1. Calculated cross sections for $^{40}\text{Ar}$ ($\nu$, $e^-$) $^{40}\text{K}$ reaction folded over the neutrino spectrum of $^8\text{B}$[15]. Contributions from GT, IAS and GT+IAS transitions are shown in units of $10^{-43}\text{cm}^2$ for SDPF-VMU, WBT-$\Delta E$ interactions and Ref. [12].

|       | GT   | IAS  | GT+IAS |
|-------|------|------|--------|
| SDPF-VMU | 10.99 | 2.10 | 13.1   |
| WBT-$\Delta E$ | 10.78 | 2.10 | 12.9   |
| Ref. [12] | 7.7  | 3.8  | 11.5   |

are well reproduced by GXPF1J at high temperatures, $T = T_9 \times 10^9$ K with $T_9 = 1$~10, and at high densities, $\rho Y_e = 10^7 \sim 10^{10}$ g/cm$^3$ with $Y_e$ the lepton-to-baryon ratio in stars[18]. The GXPF1J is also promising in neutron-rich Ni isotopes. The capture rates in $^{56}\text{Ni}$ for GXPF1J are found to be smaller compared to those for KB3G [19] due to larger fragmentation of the GT strength for GXPF1J. The extension of the present work to other isotopes such as Co and Mn is now under way[20].

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Figure 5. Calculated reaction cross sections for $^{40}$Ar ($\nu$, $e^-$) $^{40}$K reaction obtained by shell model calculations with the use of SDPF-VMU. The GT and IAS transitions are taken into account. Results are shown up to $E_x = (a)$ 40 MeV (log scale) and (b) 100 MeV.

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