Effective chiral restoration in the $\rho'$-meson in lattice QCD

L. Ya. Glozman, C. B. Lang, and Markus Limmer
Institut für Physik, FB Theoretische Physik, Universität Graz, A-8010 Graz, Austria

In simulations with dynamical quarks it has been established that the ground state $\rho$ in the infrared is a strong mixture of the two chiral representations $(0,1)+(1,0)$ and $(1/2,1/2)$. Its angular momentum content is approximately the $^3S_1$ partial wave which is consistent with the quark model. Effective chiral restoration in an excited $\rho$-meson would require that in the infrared these mesons couple predominantly to one of the two representations. The variational method allows one to study the mixing of interpolators with different chiral transformation properties in the non-perturbatively determined excited state at different resolution scales. We present results for the first excited state of the $\rho$-meson using simulations with $n_f=2$ dynamical quarks. We point out, that in the infrared a leading contribution to $\rho' = \rho(1450)$ comes from $(1/2,1/2)_b$, in contrast to the $\rho$. Its approximate chiral partner would be a $h_1(1380)$ state. The $\rho'$ wave function contains a significant contribution of the $^3D_1$ wave which is not consistent with the quark model prediction.

PACS numbers: 11.15.Ha, 12.38.Gc, 11.30.Rd

1. Introduction. A central question for QCD is how both, confinement and chiral symmetry breaking, are interrelated and influence the mass generation of hadrons. Chiral symmetry is dynamically broken in the QCD vacuum and this phenomenon is the most important for the mass origin of the ground state hadrons. It is believed, that a coupling of valence quarks with the quark condensate of the vacuum is responsible for the large constituent mass of quarks at low momenta. This large mass makes the problem effectively non-relativistic and the ground state $\rho$ is a $^3S_1$ state according to the quark model language [1-4]. Traditionally the excitation spectrum is also described with the quark model and, e.g., the first excited state of the $\rho$-meson, $\rho(1450)$, is believed to be the first radial excitation of the ground state, i.e., the $^3S_1$ state [3, 4].

The empirical spectrum of the highly excited hadrons in the light quark sector exhibits patterns of parity doublets. It has been suggested that these patterns signal effective restoration of both $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries in excited hadrons [3, 4] (for a review see [5]). This conjecture would imply that the mass generation mechanism in highly excited hadrons is very different and the quark condensate of the vacuum is of little importance. It would also imply that the constituent quark model language is not adequate for highly excited states.

Such an interpretation of the spectroscopic patterns is not unique and it is possible to imagine some alternative explanations of existing symmetry patterns [5-11]. Effective chiral and $U(1)_A$ restorations have a strong predictive power. Some of the chiral partners for existing highly excited states are missing and one obvious way to confirm or reject effective chiral and $U(1)_A$ restorations is to find experimentally missing chiral partners or establish their absence [12]. This is a difficult experimental task, though. Another empirical approach to answer this question is to find alternative experimental signatures that would correlate with the spectroscopic patterns. Indeed, the effective chiral restoration would require that the states with approximate chiral symmetry must almost decouple from pions and their diagonal axial coupling constants must be small [13, 14]. It is difficult, if not impossible, to measure experimentally diagonal axial coupling constants for highly excited hadrons. The effective chiral restoration also requires that the states which are assumed to be in approximate chiral multiplets, have a small strong decay coupling constants into the ground states and a pion. The analysis of empirical decays of excited nucleons shows that indeed all those excited nucleons that are approximate parity doublets have a very small decay coupling constant [13]. There are no other experimental tools that would help us to clarify this important question beyond reasonable doubts. To resolve the issue one needs direct information about the hadron structure, which can be supplied in ab initio lattice simulations.

A first attempt to address the problem on the lattice was Ref. [16]. The low-lying states are dominated by the near zero modes of the Dirac operator, which are directly related to the quark condensate of the vacuum. The role of these modes at small Euclidean times of the two-point correlation function is insignificant. This part of the correlator is dominated by the highly excited hadrons. This observation is consistent with effective chiral restoration but not sufficiently clear. A conclusive evidence would require to extract the high-lying chiral partners and see what the role of the near zero modes for the splitting of these states is. This is a difficult task, however. The other way is to measure the axial coupling constants of excited states [17]. For that purpose one would need reliable plateaus in two- and three-point functions for excited states near the chiral limit, which is not easy.

Here we suggest an alternative approach to the problem. Using the variational method [18] and a set of interpolators that scan a complete set of chiral representations for a given meson, we can study couplings of
different interpolators to a given meson at different resolution scales \[19\]. This method has successfully been applied to the ground state \(\rho\)-meson and the analysis has revealed that the ground state in the infrared is a strong mixture of the two possible representations \((0,1) + (1,0)\) and \((1/2,1/2)_b\) of \(SU(2)_L \times SU(2)_R\). Chiral symmetry is strongly broken in the state and this state is approxi-
mately a \(3S_1\) partial wave which is in agreement with the quark model language. Here we use this method for the first excited state of the \(\rho\)-meson, \(\rho(1450)\). We are able to present for the first time a clear evidence that in the infrared the structure of this state is different from the quark model prediction \((3S_1)\): it belongs predomin-
antly to the \((1/2,1/2)_b\) representation, which indicates a smooth onset of effective chiral restoration.

2. Elements of the formalism. There exist two different local operators with the \(\rho\)-meson quantum numbers \(I, J^{PC} = 1, 1^{-+}\), the vector current, \(\bar{q}\gamma^\mu\tau q\), and the pseudotensor “current”, \(\bar{q}\sigma^{\mu\nu}\tau q\). It is well established, both in quenched and dynamical (see, e.g., \[19\]−\[24\]) lattice simu-
lations, that both the ground state \(\rho\)-meson as well as its first excited state can be created from the vacuum by either of these operators. These two operators have distinct chiral transformation properties with respect to \(SU(2)_L \times SU(2)_R\) \[19\]−\[24\]. The vector current belongs to the \((0,1) + (1,0)\) representation and its chiral partner is the axial-vector current \(\bar{q}\gamma^\mu\gamma^5\tau q\), which creates from the vacuum the axial-vector meson \(a_1\) \((I, J^{PC} = 1, 1^{++})\). The pseudotensor interpolator transforms as \((1/2,1/2)_b\) and its chiral partner is the operator \(e^{ijk}\bar{q}\sigma^{jk}q\) that creates from the vacuum the \(h_1\)-meson \((I, J^{PC} = 0, 1^{-})\).

Assuming that chiral symmetry is not broken both explic-
itly and spontaneously, there would be two different groups of \(\rho\)-mesons. The first group would belong to the \((0,1) + (1,0)\) representation, would couple exclusively to the vector current (or to the non-local operator with the same chiral transformation properties) and each member would be mass degenerate with its corresponding axial-vector partner \(a_1\). The \(\rho\)-mesons of the second group would transform as \((1/2,1/2)_b\), could be created from the vacuum only by the pseudotensor operator (or by the non-local operator with the same chiral transformation properties) and would be systematically degenerate with their corresponding partners \(h_1\). A total amount of \(\rho\)-mesons in the spectrum would coincide with the combined amount of \(a_1\) and \(h_1\)-mesons.

Chiral symmetry breaking in the vacuum implies that the \(\rho\)-mesons are mixtures of both the \((0,1) + (1,0)\) and \((1/2,1/2)_b\) representations and can be created by both the vector and the pseudotensor interpolators. Effective chiral restoration in highly excited \(\rho\)-mesons would require that some of them predominantly couple to the vector interpolator, and the other ones would couple predominantly to the pseudotensor operator. Asymptoti-
cally each of these mesons would belong entirely to one of the two representations, \((0,1) + (1,0)\) or \((1/2,1/2)_b\), and could not be created by the operator that transforms according to the other representation.

In lattice simulations, using the variational method, it is possible not only to reliably separate a given excited state and measure its mass \[18\] but also to define and measure a ratio of couplings of different lattice operators to the given excited state \[19\]. The two chiral representations \((0,1) + (1,0)\) and \((1/2,1/2)_b\) form a complete and orthogonal basis (with respect to the chiral group) for the \(\rho\)-meson. Consequently using the variational method we are able to study a mixing of two representations in the excited \(\rho\)-meson and see whether or not a given excited \(\rho\)-meson at low resolution (infrared) scale couples predomi-
nantly to one of the representations and decouples from the other.

Assuming that the set of interpolating operators \(O_i(t)\) is projected to vanishing spatial momentum, the energies of the states \(E^{(n)}\) and the coefficients giving the overlap of operators with the physical state, \(a_i^{(n)} = \langle 0|O_i|n\rangle\), can be extracted from the cross-correlation matrix

\[
C(t)_{ij} = \langle O_i(t)O_j(0) \rangle = \sum_n a_i^{(n)} a_j^{(n)*} e^{-E^{(n)} t}. \tag{1}
\]

It can be shown \[18\] (for a recent discussion see \[26\]−\[27\]) that the generalized eigenvalue problem

\[
\tilde{C}(t)_{ij}u_j^{(n)} = \lambda^{(n)}(t,t_0)\tilde{C}(t_0)_{ij}u_j^{(n)} \tag{2}
\]

allows to recover the correct eigenvalues and eigenvectors within some approximation. At \(t = t_0\) all eigenval-
es are 1 and the eigenvectors are arbitrary. Energies of the subsequent states can be extracted from the leading exponential decays of each eigenvalue.

Ratios of couplings of the different operators to the physical states can be obtained as (cf., \[19\])

\[
\frac{C(t)_{ij}u_j^{(n)}}{C(t)_{kj}u_j^{(n)}} = \frac{a_i^{(n)}}{a_k^{(n)}}. \tag{3}
\]

A decomposition of a hadron depends on the resolution scale, i.e., what we see in our microscope, depends on its resolution. If one uses point-like lattice interpolators then the resolution scale \(1/R\) is determined by the lattice spacing \(a\). We are interested to study a decomposition of a hadron at a very low resolution scale, determined by the hadron size. For that we cannot use a large \(a\) because a proper matching with the ultraviolet (continuum) behavior of QCD will be lost. Given a fixed, reasonably small, value for \(a\), a small resolution scale \(1/R\) can be achieved by the gauge-invariant smearing of the pointlike interpolators \[19\]. We use the interpolator smeared over the size \(R\) in physical units such that \(R/a \gg 1\), so even in the continuum limit \(a \to 0\) we probe the hadron structure at the scale fixed by \(R\). Changing the smearing size \(R\), we can study the hadron content at different resolution scales of the continuum theory at \(a \to 0\). For
this purpose we use Jacobi smearing [28], which provides a Gaussian shape of interpolators in spatial directions. For this purpose we use Jacobi smearing [28], which provides a Gaussian shape of interpolators in spatial directions. For the fermions the Chirally Improved (CI) Dirac operator [24, 31] the Lüscher-Weisz gauge action is used [32]. For configurations [22, 29, 30] and recently for dynamical fermions [27] the Wilson Dirac operator [33].

In this study of the chiral decomposition of the excited \( \rho \)-meson we use three sets of dynamical configurations [27] with two mass-degenerate light sea quarks on lattices of \( 32 \times 32 \) (see Tab. I).

| Set \( \beta_{LW} \) | \( a m_0 \) | \#conf | \( a \) [fm] | \( m_\pi \) [MeV] | \( m_\rho \) [MeV] |
|-----------------|--------|-------|---------|----------------|----------------|
| A               | 4.70   | -0.050 | 200   | 0.1507(17)   | 526(7)         |
| B               | 4.65   | -0.060 | 300   | 0.1500(12)   | 469(5)         |
| C               | 4.58   | -0.077 | 300   | 0.1440(12)   | 323(5)         |

TABLE I: Specification of the data used here; for the gauge coupling only the leading value \( \beta_{LW} \) is given, \( m_0 \) denotes the bare mass parameter of the CI action. Further details on the action, the simulation and the determination of the lattice spacing and the \( \pi \)- and \( \rho \)-masses are found in [24, 31].

In Fig. 2 we show the \( R \)-dependence of the ratio \( a_V/a_T \) both for the ground state \( \rho \)-meson and its first excited state. This ratio of the vector to pseudotensor coupling gives us the information on the chiral decomposition of both states in terms of the representations \((0, 1) + (1, 0)\) and \((1/2, 1/2)_b\). We observe that this ratio for the ground state at the smallest possible resolution scale (largest smearing radius) \( R = 0.67 \) fm approaches a value close to 1.2. This implies a strong mixture of both representations in the \( \rho \)-meson wave function. Using a unitary transformation from the chiral basis to the \( ^2S+1L_J \) basis [34],

\[
| ^3S_1 \rangle = \sqrt{\frac{7}{3}} |(0, 1) + (1, 0); 1^-\rangle + \\
\sqrt{\frac{1}{3}} |(1/2, 1/2)_b; 1^-\rangle ,
\]

\[
| ^3D_1 \rangle = \sqrt{\frac{1}{3}} |(0, 1) + (1, 0); 1^-\rangle - \\
\sqrt{\frac{2}{3}} |(1/2, 1/2)_b; 1^-\rangle ,
\]

one obtains that the ground state \( \rho \) is a \( ^3S_1 \) state with a tiny admixture of the \( ^3D_1 \) wave, \( 0.997 | ^3S_1 \rangle - 0.073 | ^3D_1 \rangle \). This implies that at a resolution fixed by the \( \rho \)-size the \( \rho (770) \) is approximately a \( ^3S_1 \) state in agreement with the quark model.

Our main result in the present report is that the chiral decomposition of the first excited state, \( \rho' = \rho (1450) \), and its scale dependence is very different. In this case at large \( R \) a leading contribution comes from the \((1/2, 1/2)_b \)
representation. Given only two different $R$ values we cannot reliably extrapolate to the scale of 1 fm, as suggested by the size of excited rho-meson. It is clearly seen, however, that such a ratio at $R \sim 1$ fm is very small; it can be both positive and negative in sign.

This behavior has a clear interpretation. In the deep ultraviolet one expects from the conformal symmetry of QCD that the pseudotensor interpolator decouples from the $\rho$-mesons. Consequently towards small $R$ the ratio $a_T/a_V$ must increase. At large $R$ the ratio determines a degree of the chiral symmetry breaking in the infrared region, where mass is generated. One observes that such a breaking for the $\rho(1450)$ is insignificant, in contrast to the $\rho(770)$. Since the chiral decomposition of the $\rho'$ state is dominated by one of the chiral representations, it indicates a smooth onset of effective chiral restoration. One naturally arrives at the following identification $\rho(1450)$ is insignificant, in contrast to the $\rho(770)$. Since the chiral decomposition of the $\rho'$ state is dominated by one of the chiral representations, it indicates a smooth onset of effective chiral restoration. Given that this leading representation of $\rho'$ is $(1/2,1/2)_h$ one predicts that in the same energy region there must exist a $h_1$ (and not a $a_1$) meson. Inspecting the Particle Data Group[4] one finds that there is indeed the state $h_1(1380)$ and no $a_1$ in the same energy region.

While we do not know the precise value of the ratio $a_T/a_V$ for the $\rho(1450)$ at $R \sim 1$ fm, it is indicative that this ratio is very small. Then we can qualitatively estimate the angular momentum content of $\rho(1450)$ in the infrared. Assuming a vanishing ratio, one obtains the following partial wave content, $\sqrt{1/3}|^3S_1\rangle - \sqrt{2/3}|^3D_1\rangle$. A significant contribution of the $|^3D_1\rangle$ wave is obvious. A possible variation of the ratio at large $R$ changes slightly numbers for the partial wave decomposition, but does not change the qualitative result. This result is not consistent with $\rho'$ to be a radial excitation of the ground state $\rho$-meson, i.e., a $|^3S_1\rangle$ state, as predicted by the quark model.

We thank G. Engel and C. Gattringer for discussions. L.Ya.G. and M.L. acknowledge support of the Fonds zur Förderung der Wissenschaftlichen Forschung (P21970-N16) and (DK W1203-N08), respectively. The calculations have been performed on the SGI Altix 4700 of the Leibniz-Rechenzentrum Munich and on local clusters at ZID at the University of Graz.