Modelling of a stirling cryocooler regenerator under steady and steady – periodic flow conditions using a correlation based method

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Abstract. The performance of a Stirling cryocooler depends on the thermal and hydrodynamic properties of the regenerator in the system. CFD modelling is the best technique to design and predict the performance of a Stirling cooler. The accuracy of the simulation results depend on the hydrodynamic and thermal transport parameters used as the closure relations for the volume averaged governing equations. A methodology has been developed to quantify the viscous and inertial resistance terms required for modelling the regenerator as a porous medium in Fluent. Using these terms, the steady and steady – periodic flow of helium through regenerator was modelled and simulated. Comparison of the predicted and experimental pressure drop reveals the good predictive power of the correlation based method. For oscillatory flow, the simulation could predict the exit pressure amplitude and the phase difference accurately. Therefore the method was extended to obtain the Darcy permeability and Forchheimer’s inertial coefficient of other wire mesh matrices applicable to Stirling coolers. Simulation of regenerator using these parameters will help to better understand the thermal and hydrodynamic interactions between working fluid and the regenerator material, and pave the way to contrive high performance, ultra-compact free displacers used in miniature Stirling cryocoolers in the future.

1. Introduction

The growing demand for short to long wavelength IR imaging instruments for space observational applications, imaging cameras, high temperature superconducting filters (HTS) and military detectors has led to the ongoing research on miniaturized free piston free displacer (FPFD) Stirling cryocoolers. Fast cool down, small size, light weight, lower power consumption and high reliability are some of the requirements of these cryocoolers \cite{1}. Additionally, they demand ultra-compactness, long life and a temperature of 60–80 K with cooling power varying from mW to a few Watts. The use of hermetically sealed linear drives with dry clearance seals provided a new concept for eliminating the side forces on the piston and the displacer which results in no wear and tear and leads to increased life and performance of the cooler.

Performance of a Stirling cooler mainly depends on the effectiveness of the regenerator used in the system. Theoretically, to produce refrigeration power, regenerator effectiveness of a cryocooler
operating at 80 K should be greater than \(-91\%\) but in actual case, it demands much more. The purpose of the regenerator is to deliver PV power from the compressor to the cold end of the regenerator with minimum losses. Thus the regenerator alternatively undergoes hot and cold blows and serves as a thermal flywheel. Made up of fine metallic wire meshes tightly stacked in a tube, an ideal regenerator demands minimum pressure drop, infinite heat capacity ratio between solid material and gas, large surface area, zero void volume and zero conduction loss from the warm end to the cold end. These conflicting requirements and their impact on the performance of the cryocooler have led to extensive research and great effort is being devoted to the design and material selection of regenerators.

Y. Zhao and H. Dang [2] established a two dimensional axis-symmetric CFD model of a miniature coaxial Stirling type pulse tube cooler operating at 128 Hz. Both thermal equilibrium and non-equilibrium mechanisms for the porous matrix are considered and various regenerator losses are calculated. S. C. Costa et al. [3] proposed a finite volume method (FVM) based non – thermal equilibrium porous media modeling approach for the derivation of the flow resistance coefficients and thermal non – equilibrium heat transfer coefficient for porous media of Stirling regenerators under unidirectional flow conditions. M. G. Pathak et al. [4] conducted steady and oscillatory flow experimental study and recorded the mass flow and pressure drop data of helium flow through Er\(_{50}\)Pr\(_{50}\) rare - earth regenerator material under ambient temperature conditions. They applied a CFD based method for the quantification of Darcy permeability and Forchheimer’s coefficient of porous structures under steady and periodic flow conditions. Tao et al. [5] investigated the hydrodynamic and heat transfer performances of various regenerator matrices and derived correlations for permeability K and inertia coefficient F in oscillating flow which include the effect of geometric parameters, oscillating flow, operating frequency and the effect of cryogenic temperature. Cha et al. [6] conducted experimental investigations and established an experiment based CFD assisted method for the quantification of directional permeability and Forchheimer’s coefficient of porous regenerator structures under steady – periodic flow conditions. Some previous studies including that of Cha et al. [6] reported that the steady flow friction factor values are identical to the oscillatory flow friction factor, at small Reynolds numbers. At higher Reynolds numbers, the values deviate significantly.

One major stumbling block in the CFD analysis of a Stirling cooler is the requirement of the hydrodynamic resistance parameters for modeling the momentum transfer in porous regenerator matrix. The reliability of the simulation results depends on the accuracy of viscous resistance \(D\) and inertial resistance \(C\) used for modeling the regenerator as a porous medium. Survey of the literature reveals that little attention has been paid to quantify the hydrodynamic and thermal transport parameters of commonly used regenerator meshes mainly because of the complexity of the experimental measurements. In this study, a methodology has been developed for the prediction of Darcy permeability and Forchheimer's coefficient of wire mesh screens relevant to Stirling cooler.

2. Method
In the present study, the regenerator is modelled as a porous medium. Based on an experimental study, Clearman et al. [7] developed a friction factor correlation for the steady flow of helium through porous media in terms of the viscous and inertial resistance as given in equation (1).

\[ f_K = \frac{2e}{Re_K} + 2Ce^2 \]  

\[ Re_K = \frac{pReR}{\mu} \]  

\[ Cf = \frac{CvR}{2e^3} \]  

\[ K = \frac{e^2}{D} \]  


In the above equations, $Re_K$ denotes the Reynolds number based on $\sqrt{K}$ as length scale and $\varepsilon$ represents porosity of the regenerator. The symbols $C$, $D$, $C_f$, $K$, $\rho_f$, $u$ and $\mu$ represent inertial resistance ($m^{-1}$), viscous resistance ($m^{-2}$), Forchheimer’s inertial coefficient, permeability of the matrix ($m^2$), density ($kg/m^3$), velocity ($m/s$) and viscosity ($N\cdot s/m^2$) of the working fluid, respectively.

Thomas et al. [8] summarized different friction factor correlations for regenerator of Stirling cycle machines. The friction factor correlations proposed by Gedeon et al., Tong et al., Blass, Miyabe and Tanaka are expressed in a general form,

$$f = M + \frac{N}{Re} \quad (5)$$

The values of correlation constants $M$ and $N$ are tabulated in table 1. The Reynolds number $Re$ in terms of mesh distance is defined as,

$$Re = \frac{\dot{m}l}{\beta Ap} \quad (6)$$

Here $\dot{m}$, $l$, $\beta$ and $A$ represent the mass flow rate of helium ($kg/s$), mesh distance ($m$) opening area ratio and area of regenerator ($m^2$), respectively.

**Table 1. Values of correlation constants.**

| Correlation       | $N$    | $M$    |
|-------------------|--------|--------|
| Gedeon/Wood       | 68.556 | 0.5274 |
| Tong/London       | 44.710 | 0.3243 |
| Blass             | 47.245 | 0.4892 |
| Miyabe            | 33.603 | 0.3370 |
| Tanaka            | 40.741 | 0.5315 |

Using the above correlations, the friction factor was calculated at different mass flow rates. Clearman and Landrum obtained the Darcy permeability, $K$ and Forchheimer’s inertial coefficient, $C_f$ for 325, 400 and 635 mesh matrices based on their experimental study. Using these empirical constants, friction factor $f_K$ was calculated for the above meshes. The friction factor $f$ obtained from different correlations is compared with $f_K$. The correlation which calculates friction factor values in close agreement with that obtained from equation (1) was identified. Subsequently, the friction factor data obtained from the matching correlation is substituted in equation (1) and the inertial resistance, $C$ and viscous resistance, $D$ were iteratively adjusted to get unique values of these parameters for a porous media. These values of $C$ and $D$ are used as input parameters in Fluent for modelling the regenerator as a porous medium. Pressure drop at different mass flow rates is obtained and compared with the reported experimental data.

2.1. Computational domain and boundary conditions

The steady flow regenerator test section [7] is modelled as a 2D axisymmetric system in Fluent. The model of 325 and 400 mesh regenerator test section is shown in figure 1 and their dimensions are tabulated in table 2. The porous zone of the model is filled with 325 and 400 mesh SS wire screens. A similar test section developed by Landrum et al. [9] was used to model the steady flow of helium through 635 mesh regenerator. The properties of the wire screens are tabulated in table 3.

For the steady flow analysis, a First Order Upwind Discretisation (FOUD) scheme with a simple pressure – velocity coupling was employed. The steady state solver uses the two equation $k-\varepsilon$ model for turbulence modelling. A reference pressure of 1 atm (101.325 kPa) was applied to the system. The
convergence criterion is $10^{-7}$ for continuity and $10^{-8}$ for energy. Mass flow inlet and pressure outlet boundary conditions were applied for steady flow simulation.

**Figure 1.** Steady axial flow model of 325 and 400 mesh regenerator test section.

**Table 2.** Different components of 325 and 400 mesh test sections and their dimensions.

| Component | Radius (mm) | Length (mm) |
|-----------|-------------|-------------|
| A         | 3.8100      | 20.32       |
| B         | 3.9688      | 38.10       |
| C         | 3.1750      | 30.48       |
| D         | 2.2860      | 123.19      |

**Table 3.** Properties of the wire mesh screens.

| Matrix Type          | Wire diameter (µm) | Porosity (%) |
|----------------------|--------------------|--------------|
| 325 mesh SS screen   | 42.00              | 69.69        |
| 400 mesh SS screen   | 36.96              | 69.69        |
| 635 mesh SS screen   | 20.30              | 63.12        |

**Figure 2.** Oscillatory flow model of 635 mesh regenerator test section.

The steady periodic flow test sections for 325, 400 and 635 mesh regenerators [6, 9] were modelled as 2D axisymmetric system in Fluent. In oscillatory flow model shown in figure 2, the porous zone is placed between sloped transition cones located between regenerator housing and sensor tap locations. It helps to avoid large step change in pipe diameter and associated flow disturbances. A UDF (User Defined Function) was developed and coupled to the main CFD code to apply a transient pressure boundary condition at the inlet. The steady periodic boundary condition was represented as a Fourier cosine series,

$$ P_{osc}(\theta) = \sum_{n=1}^{3} A_n \cos(n\omega t + \phi_n) $$  \hspace{1cm} (7)
where \( f \) is the frequency, \( A_n \) and \( \Phi_n \) are the \( n^{th} \) harmonic pressure magnitude and phase of the Fourier cosine series, respectively. Isothermal (300 K) boundary condition is applied for all other boundaries. Grid independence study was conducted for steady and steady - periodic flow models to obtain the optimum cell count.

Table 4. Different components of 635 mesh oscillatory flow test section and their dimensions.

| Component | Radius (mm) | Length (mm) |
|-----------|-------------|-------------|
| E         | 4.2164      | 5.08        |
| F         | -           | 22.86       |
| G         | 1.0030      | 11.43       |
| H         | 2.0066      | 12.7        |
| I         | 1.0033      | 30.48       |
| J         | -           | 22.86       |
| K         | 4.2164      | 30.48       |
| L         | 2.3495      | 33.274      |
| M         | 2.7940      | 32.893      |

3. Results and discussions
The steady flow friction factor calculated from different standard correlations was compared with the reported experimental data [7, 9]. For 325 and 400 mesh regenerators, the friction factor values obtained from Blass correlation are quite close to the reported experimental values. For 635 mesh regenerator, the friction factor calculated from Tong/London correlation is in good agreement with that of experimental data. For 325 and 400 mesh regenerators, the friction factor values calculated from Blass correlation are substituted in equation (1) and the values of \( C \) and \( D \) are adjusted iteratively to get unique values of these parameters for a wire screen matrix. The procedure was repeated for 635 mesh regenerator using the values obtained from Tong/London correlation. Table 5 demonstrates the hydrodynamic resistance parameters calculated for 325, 400 and 635 mesh regenerator matrices.

Figure 3. Variation of friction factor with Reynolds number (325 mesh regenerator).

Figure 4. Variation of friction factor with Reynolds number (635 mesh regenerator).

The aforementioned values of \( C \) and \( D \) are applied as the closure relations for the volume averaged governing equations to model the regenerator as a porous medium using Fluent. The steady flow of helium was simulated at different mass flow rates. Figure 5 compares the pressure drop obtained from the simulation and the experimental study [7, 9]. The average deviation of the simulated pressure drop...
from the experimental data is 9.91 %, 6.12 % and 3.6 % for 325 mesh, 400 mesh and 635 mesh regenerator respectively. From figure 5, it can be concluded that the CFD simulation using the hydrodynamic resistance terms obtained from the present method could predict the pressure drop with reasonable accuracy.

**Table 5.** Hydrodynamic resistance parameters of 325, 400 and 635 mesh regenerators.

| Matrix Type            | Viscous resistance, D (m⁻²) | Inertial resistance, C (m⁻¹) | Permeability, K (m²) | Forchheimer’s coefficient, C_f |
|------------------------|----------------------------|----------------------------|---------------------|-------------------------------|
| 325 mesh SS screen     | 1.955E+10                  | 68400                      | 2.48E-11            | 0.5036                        |
| 400 mesh SS screen     | 2.418E+10                  | 76050                      | 2.01E-11            | 0.5036                        |
| 635 mesh SS screen     | 7.575E+10                  | 89280                      | 5.26E-12            | 0.4071                        |

In actual working conditions of a cryocooler, the flow through the regenerator is not steady, but steady - periodic in nature. For the range of mass flow rate considered for investigation, the steady flow friction factor was assumed to be equal to the oscillatory flow friction factor. Therefore, the hydrodynamic resistance parameters calculated from the correlation based method was used for the oscillatory flow simulation. The steady - periodic flow of helium through 325, 400 and 635 mesh wire matrices were simulated using the viscous and inertial resistance given in table 5. The flow through 325 and 400 mesh regenerators were simulated with frequencies of 10, 20, 30, 40, 50 and 60 Hz. The flow through 635 mesh regenerator was simulated at 50, 100, 150 and 200 Hz, respectively.

**Figure 5.** Comparison of simulated and experimental pressure drop under steady flow conditions.

**Figure 6.** Pressure at P1 and P2 locations. (325 mesh, 50 Hz frequency).

Figures 6, 7 and 8 demonstrate the pressure at P1 and P2 locations of the 325, 400 and 635 mesh regenerator test sections respectively at 50 Hz. These figures show reasonable agreement between the predicted and experimental pressure amplitudes at P2. The phase difference between the pressure at P1 and P2 was also predicted accurately. The predicted and simulated pressures at the P2 location were also compared at other frequencies. Table 6, 7, and 8 show the deviation of the simulated pressure amplitude at location P2 from the experimental data. The average difference between the predicted pressure amplitude and the corresponding experimental values are 19.32 %, 15.83 % and 4.7 % for 325, 400 and 635 mesh regenerator respectively. The difference between the predicted and experimental pressure amplitude increases as the frequency increases. Table 6, 7 and 8 clearly show that the deviation of the predicted phase difference from the phase difference obtained from experiment (Δθ) is negligible.
Figure 7. Pressure at P1 and P2 locations (400 mesh, 50 Hz frequency).

Figure 8. Pressure at P1 and P2 locations (635 mesh, 50 Hz frequency).

Table 6. Comparison of experimental and predicted pressure amplitudes at location P2 (325 mesh).

| Frequency (Hz) | Pressure amplitude, $P_{2\text{Exp}}$ [kPa] | Pressure amplitude, $P_{2\text{Sim}}$ [kPa] | Difference between $P_{2\text{Exp}}$ and $P_{2\text{Sim}}$ (%) | $\Delta \theta$ (deg.) |
|---------------|---------------------------------|---------------------------------|-------------------------------------------------|------------------|
| 10            | 50.04                           | 45.36                           | 9.35                                           | 5.998            |
| 20            | 31.92                           | 26.36                           | 17.41                                          | 5.404            |
| 30            | 39.85                           | 32.07                           | 19.52                                          | 5.403            |
| 40            | 20.69                           | 16.15                           | 21.95                                          | 0                |
| 50            | 13.09                           | 10.03                           | 23.33                                          | 0                |
| 60            | 10.06                           | 7.61                            | 24.40                                          | 2.692            |

Table 7. Comparison of experimental and predicted pressure amplitudes at location P2 (400 mesh).

| Frequency (Hz) | Pressure amplitude, $P_{2\text{Exp}}$ [kPa] | Pressure amplitude, $P_{2\text{Sim}}$ [kPa] | Difference between $P_{2\text{Exp}}$ and $P_{2\text{Sim}}$ (%) | $\Delta \theta$ (deg.) |
|---------------|---------------------------------|---------------------------------|-------------------------------------------------|------------------|
| 10            | 7.22                            | 6.42                            | 11.08                                           | 8.995            |
| 20            | 4.17                            | 3.41                            | 18.32                                           | 5.403            |
| 30            | 5.37                            | 4.94                            | 8.00                                            | 0                |
| 40            | 2.94                            | 2.48                            | 15.65                                           | 0                |
| 50            | 1.88                            | 1.50                            | 20.23                                           | 5.403            |
| 60            | 1.43                            | 1.12                            | 21.72                                           | 0                |

The 325 and 400 mesh regenerators have a porosity of 0.6969 and 635 mesh regenerator has a porosity of 0.6312. Results from the present study indicate that for high porosity matrices, friction factor calculated from Blass correlation is in agreement with that of Clearman et al. As the mesh number increases and the mesh become less porous, the friction factor in Tong/London correlation is more similar with that of Clearman et al. The results further confirm that the methodology developed in the present study can be extended to other wire mesh screens relevant to miniature Stirling cryocooler. Therefore the above method was applied to determine the Darcy permeability and Forchheimer’s inertial coefficient of 200, 250, 300, 450 and 510 mesh regenerators. Blass correlation was employed for 200, 250 and 300 mesh regenerators and for 450 and 510 mesh regenerators,
Tong/London correlation was used. The geometrical properties and the hydrodynamic resistance parameters calculated from the correlation based method are presented in table 9.

**Table 8.** Comparison of experimental and predicted pressure amplitudes at location P2 (635 mesh).

| Frequency (Hz) | Pressure amplitude, $P_{2\text{Exp}}$ [9] (kPa) | Pressure amplitude, $P_{2\text{Sim}}$ (kPa) | Difference between $P_{2\text{Exp}}$ and $P_{2\text{Sim}}$ (%) | $\Delta \theta$ (deg.) |
|----------------|-----------------------------------------------|---------------------------------------------|-------------------------------------------------|-----------------------|
| 50             | 122.82                                        | 116.49                                      | 5.16                                            | 0                     |
| 100            | 35.23                                         | 33.01                                       | 6.32                                            | 0                     |
| 150            | 12.35                                         | 12.59                                       | -1.96                                           | 16.18                 |
| 200            | 8.59                                          | 9.05                                        | -5.37                                           | 0                     |

**Table 9.** Geometric Properties and hydrodynamic resistance parameters of selected wire screen matrices.

| Matrix Type | Wire diameter (µm) | Porosity (%) | Viscous resistance, $D$ (m$^2$) | Inertial resistance, $C$ (m$^1$) | Permeability, $K$ (m$^2$) | Forchheimer’s coefficient, $C_f$ |
|-------------|--------------------|--------------|---------------------------------|----------------------------------|--------------------------|----------------------------------|
| 200 mesh    | 58                 | 66.7         | 1.0215E+10                      | 49400                            | 4.36E-11                 | 0.5493                           |
| 250 mesh    | 41                 | 66.6         | 1.9235E+10                      | 67850                            | 2.31E-11                 | 0.5515                           |
| 300 mesh    | 30                 | 68.9         | 3.2450E+10                      | 88130                            | 1.46E-11                 | 0.5153                           |
| 450 mesh    | 25                 | 65.1         | 4.8690E+10                      | 71,500                           | 8.70E-12                 | 0.3823                           |
| 510 mesh    | 25                 | 60.1         | 4.9973E+10                      | 72490                            | 7.23E-12                 | 0.4489                           |

**4. Conclusions**

A methodology has been developed to quantify the Darcy permeability and Forchheimer’s coefficient of wire mesh structures applicable to miniature Stirling coolers. The results of the steady and oscillatory flow simulations using this methodology show good agreement with the experimental pressure drop data. The hydrodynamic resistance parameters of any wire mesh screen can be obtained from this method. The simulation of regenerator using these parameters will pave the way to contrive high performance, ultra-compact free displacers used in the miniature Stirling coolers in the future.

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