On the possibility of cosmological gravimetry in general relativity

V. I. Yudin\textsuperscript{1,2,3,*} and A. V. Taichenachev\textsuperscript{1,2}

\textsuperscript{1}Novosibirsk State University, ul. Pirogova 2, Novosibirsk, 630090, Russia
\textsuperscript{2}Institute of Laser Physics SB RAS, pr. Akademika Lavrent’eva 13/3, Novosibirsk, 630090, Russia
\textsuperscript{3}Novosibirsk State Technical University, pr. Karla Marks’a 20, Novosibirsk, 630073, Russia

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In the framework of the parametrized post-Newtonian (PPN) formalism, we propose and substantiate a hypothesis according to which we can measure the value of the cosmological gravitational potential \(\Phi_{\text{CP}}\) at the location of the Solar System. By the cosmological gravitational potential, we mean the potential formed by all the matter of the Universe (including dark matter and dark energy). At the same time, we believe that this global potential is spatially uniform on the scales of the solar system, and its value significantly exceeds the Newtonian gravitational potential of the Sun \(\varphi_S\) (where \(|\varphi_S|/c^2 \sim 10^{-8}\) in orbit of the Earth). Our method is based on the experimental measurement of the PPN parameters \(\beta\) and \(\gamma\), where information on \(\Phi_{\text{CP}}\) is contained in the differences \((\beta - 1)\) and \((\gamma - 1)\). Starting from the cosmological description of the Universe within the framework of the standard general relativity (i.e., \(\gamma_0 = \beta_0 = 1\) in the initial cosmological reference frame), we have shown that from the viewpoint of the local (laboratory) observer, the following relations hold for the post-Newtonian parameters \(\gamma\) and \(\beta\): \(|\gamma - 1| \sim \Phi_{\text{CP}}/c^2\) and \(|\beta - 1| \sim |\Phi_{\text{CP}}|/c^2\). This leads to the estimate: \(|\beta - 1| > 10^{-6}\) and \(|\gamma - 1| > 10^{-6}\) for gravitational experiments in the Solar system. The obtained results open up new unique opportunities for exploring the Universe, testing various theories of gravity and cosmological models. In particular, we determined an experimental cosmological test for Einstein’s general relativity. In addition, a unique post-Newtonian model of gravity has also been found, which is insensitive to cosmological potential and for which \(\gamma = \gamma_0 = \beta = \beta_0 = 1\) is always from the viewpoint of the cosmological and any laboratory observers. However, this model differs from Einstein’s theory.

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Introduction

More than a hundred years have passed since the advent of general relativity (GR), developed by A. Einstein at the beginning of the 20th century \cite{1,4}. However, all this time GR continues to be an area of active research, both theoretical and experimental (see the reviews \cite{2,9}). The verification of general relativity was carried out with increasing accuracy, and today it successfully explains almost all the available data from numerous experiments. The revolution in the experimental testing of GR has occurred over the past 50 years due to great progress in various fields of science and technology. First of all, this is due to the development of space research, the emergence of high-precision space navigation methods for spacecrafts, a significant improvement in the accuracy of astronomical observations, the development of laser range-finding of the Moon, etc.

Recent advances in applied physics have made available new instrumentation and technologies. For example, a new generation of quantum sensors (ultra-stable atomic clocks, accelerometers, gyroscopes, gravimeters, gravity gradiometers) significantly increases the potential of the modern measurement base, allowing us to achieve extremely high levels of accuracy in testing the fundamentals of modern physics, in general, and GR, in particular.

The latest achievements in this area include the results of Refs. \cite{2,8}, in which authors report on successful tests (at the level of \(10^{-5}\)) of the gravitational redshift and thus of local position invariance, as an integral part of the Einstein equivalence principle, which is the foundation of GR and all metric theories of gravitation.

Another main direction in the verification of general relativity is the experimental measurement of the so-called post-Newtonian parameters. This method is based on the phenomenological parametrization of the metric tensor of the gravitational field and is called the parametrized post-Newtonian (PPN) formalism, which is suitable for working with a wide class of metric theories, including GR as a special case. Within the PPN formalism, the specific metric theory of gravity is completely characterized by ten PPN parameters \(\beta, \gamma\) (\(\beta, \gamma\)). Formalism uniquely predicts the values of these parameters for each particular theory. Gravity experiments can be also analyzed in the framework of the PPN formalism. In this case, the experimental results make it possible to determine the values of the post-Newtonian parameters and, therefore, to conclude in favor of a particular theory.

Of particular importance are two PPN parameters \((\beta, \gamma)\), which have a clear physical meaning: \(\gamma\) is a measure of the curvature of space-time created by an unit rest mass; \(\beta\) is a measure of the nonlinearity of the law of superposition of gravitational fields. For example, the GR in the standard PPN calibration yields \(\beta = \gamma = 1\), and all other eight parameters disappear.

To date, much empirical data have already been ac-
cumulated on the measurement of post-Newtonian parameters ($\beta, \gamma$), obtained both using ground-based high-precision astrometric equipment and in experiments using spacecraft (see reviews [5, 6]). Such experiments include: measuring the anomalous perihelion shift of Mercury's orbit, measuring the deflection of light near the Sun, the time delay of radar signals sent across the Solar System past the Sun to a planet or satellite and returned to the Earth (due to the gravitation of the Sun), laser ranging of the Moon, and some others. Analyzing the obtained results, we can with high probability assert that within the Solar System there is: $|\beta - 1| < 10^{-4}$ and $|\gamma - 1| < 10^{-4}$. On the one hand, this indicates the high reliability of GR predictions. On the other hand, the achieved level of accuracy is still not so high as to guarantee, for example, the incorrectness of some alternative gravity theories for which $\beta \neq 1$ and $\gamma \neq 1$ (see reviews [5, 6]).

In this paper, we study the influence that the cosmological background has on the value of post-Newtonian parameters $(\beta, \gamma)$ in the interpretation of gravitational experiments within the Solar System. By cosmological background, we mean the gravitational contribution from all the celestial bodies of the Universe, located far beyond the Solar System (including dark matter and dark energy). Using the general PPN formalism, we show that for a local observer (i.e., an experimentalist in the Solar System) the values of post-Newtonian parameters $(\beta, \gamma)$ differ from their initial values $(\beta_0, \gamma_0)$, which appear in the theoretical description of gravitation from the viewpoint of the virtual Cosmological Observer. Moreover, information on the cosmological background is contained in the quantities $(\beta - \beta_0)$ and $(\gamma - \gamma_0)$. In particular, in the case of GR ($\beta_0 = \gamma_0 = 1$) we get a lower bound: $|\beta - 1| > 10^{-6}$ and $|\gamma - 1| > 10^{-6}$ in the Solar System. We also find a previously unknown post-Newtonian model of gravitation, which is insensitive to the cosmological background (in this model, we have $\gamma = \gamma_0 = \beta = \beta_0 = 1$ for any Laboratory Observer). The obtained results can be considered a theoretical basis for experimental cosmological gravimetry.

1. General relativity formalism

Let us consider some basic formulas of general relativity, which are necessary for our further considerations. In this case, the physical picture of gravitation is described by a metric in four-dimensional space-time, when the relationship between two infinitely close space-time events is given using the interval:

$$ds^2 = g_{jk}(\vec{x})dx^jdx^k \quad (j, k = 0, 1, 2, 3),$$

where $g_{jk}(\vec{x}) = g_{jk}(t, \vec{r})$ is the covariant metric tensor depending on the coordinates of the four-vector $\vec{x} = \{x^0 = ct, x^1, x^2, x^3\} = \{ct, \vec{r}\}$. This metric tensor defines the standard equations of the geodesic line along which the test body of small mass moves:

$$\frac{\partial^2 x^i}{\partial s^2} + \Gamma^i_{\alpha\beta} \frac{\partial x^\alpha}{\partial s} \frac{\partial x^\beta}{\partial s} = 0, \quad (i, \alpha, \beta = 0, 1, 2, 3),$$

where $\Gamma^i_{\alpha\beta}$ are the Christoffel symbols:

$$\Gamma^i_{\alpha\beta} = \frac{1}{2}g^{ik}(\frac{\partial g_{\alpha\lambda}}{\partial x^\beta} + \frac{\partial g_{\beta\lambda}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\lambda}) \quad (g^{\alpha\beta}g_{\lambda\kappa} = \delta^\alpha_\kappa).$$

The equations (2) related to the coordinate time $t$ can be rewritten in the following form (see, for example, Ref. [12]):

$$\ddot{x}^i + \Gamma^i_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta - c^{-1}g^0_\alpha \dot{x}^\alpha \dot{x}^0 = 0.$$  

Note that there are only three independent equations for $i = 1, 2, 3$, because the equation (3) for $i = 0$ reduces to the identity: $0 = 0$. In the absence of gravitation, the tensor $g_{jk}$ describes the metric of Minkowski space-time for the special relativity:

$$g^{(M)}_{jk} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

in which geodesic lines correspond to a uniform rectilinear motion: $\vec{r} = 0$.

For simplicity, we will consider the case of a static Universe, i.e. when the distribution of masses in the Universe does not change over time. Moreover, the metric tensor $g_{jk}$ will be described in the initial flat time space $\{t, \vec{r}\}_{\text{Cosm}}$, which we associate with the “Cosmological Observer”. In this case, to describe a weak gravitational field, the following expression for the interval is a good approximation:

$$ds^2 = g_{00}(\vec{r})(c\, dt)^2 - g_{11}(\vec{r})\, d\vec{r}^2,$$

when there is a diagonal spatially isotropic metric tensor:

$$g_{jk} = \begin{pmatrix} g_{00}(\vec{r}) & 0 & 0 & 0 \\ 0 & -g_{11}(\vec{r}) & 0 & 0 \\ 0 & 0 & -g_{11}(\vec{r}) & 0 \\ 0 & 0 & 0 & -g_{11}(\vec{r}) \end{pmatrix},$$

and the corresponding dual tensor $g^{jk}$:

$$g^{jk} = \begin{pmatrix} \frac{1}{g_{00}(\vec{r})} & 0 & 0 & 0 \\ 0 & \frac{1}{-g_{11}(\vec{r})} & 0 & 0 \\ 0 & 0 & \frac{1}{-g_{11}(\vec{r})} & 0 \\ 0 & 0 & 0 & \frac{1}{-g_{11}(\vec{r})} \end{pmatrix}.$$  

For such a metric tensor, the equation of the geodesic line (4) in the nonrelativistic limit on the velocity of the test body $|\vec{v}|/c \ll 1$ leads to the following basic approximation:

$$\ddot{x}^i \approx -\Gamma^i_{\alpha\beta} \frac{c^2}{2g_{11}(\vec{r})} \frac{\partial g_{00}(\vec{r})}{\partial x^\alpha}, \quad (i = 1, 2, 3),$$

where $\Gamma^i_{\alpha\beta}$ are the Christoffel symbols.
which can be written in vector form:

$$\frac{d^2 \mathbf{r}}{dt^2} = -\frac{c^2}{2\gamma_{11}(\mathbf{r})} \nabla_{\mathbf{r}} g_{00}(\mathbf{r}),$$  \hspace{1cm} (10)$$

where $$\nabla_{\mathbf{r}} = e_1(\partial/\partial x_1) + e_2(\partial/\partial x_2) + e_3(\partial/\partial x_3)$$ is the standard spatial gradient operator. Note that equation (10) will play a key role in our further considerations.

2. The concept of cosmological gravitational potential

In this paper, we assume that the source of gravitation in a static Universe is an arbitrary number of point masses $$M_1, M_2, ..., M_n, ...$$ which are stationary in space, i.e. we will neglect the movement of celestial bodies. In this case, the Newtonian gravitational potential is described in standard way:

$$U(\mathbf{r}) = -\sum_n \Gamma_0 M_n \frac{1}{|\mathbf{r} - \mathbf{R}_n|},$$  \hspace{1cm} (11)$$

where $$\Gamma_0$$ is the gravitational constant from the viewpoint of the Cosmological Observer, and $$\mathbf{R}_n$$ is the coordinate of a point source with mass $$M_n$$. Note that in our work we will follow the classical definition of the Newtonian attraction potential, which has a negative value, $$U(\mathbf{r}) < 0$$.

We will consider a small area of the Universe far from very massive bodies when the condition of small gravitational potential, $$|U(\mathbf{r})/c^2| < 1$$, is fulfilled. As a specific example, we will explore the spatial region within the Solar system. In this case, the Newtonian potential (11) can be represented as follows:

$$U(\mathbf{r}) = U_{\text{loc}}(\mathbf{r}) + \Phi_{\text{CP}}(\mathbf{r}),$$  \hspace{1cm} (12)$$

where the local potential

$$U_{\text{loc}}(\mathbf{r}) = -\sum_{n = 1}^{N_{\text{loc}}} \Gamma_0 M_n \frac{1}{|\mathbf{r} - \mathbf{R}_n|}$$  \hspace{1cm} (13)$$

includes summation over the indices $$\{1, 2, ..., N_{\text{loc}}\}$$, which number only the sources of gravitation inside the planetary system under consideration. In our particular case, $$U_{\text{loc}}(\mathbf{r})$$ includes the Newtonian potentials of the Earth $$U_E(\mathbf{r})$$, the Sun $$U_S(\mathbf{r})$$, the Moon $$U_M(\mathbf{r})$$, the Jupiter $$U_J(\mathbf{r})$$ and other bodies of the Solar system. The second contribution in Eq. (12) corresponds to the cosmological potential $$\Phi_{\text{CP}}(\mathbf{r})$$:

$$\Phi_{\text{CP}}(\mathbf{r}) = -\sum_{n > N_{\text{loc}}} \Gamma_0 M_n \frac{1}{|\mathbf{r} - \mathbf{R}_n|},$$  \hspace{1cm} (14)$$

including potentials from all other bodies in the Universe, which are at huge distances from the Solar System, as well as the effects of dark matter.

Let us now evaluate the order of magnitude of the various contributions. Near the Earth’s surface, we have an estimate of the Earth’s potential $$|U_E|/c^2 \approx 0.7 \times 10^{-9}$$, and the estimate $$|U_S|/c^2 \approx 10^{-8}$$ corresponds to the potential of the Sun in the Earth’s orbit. To estimate the lower bound of the cosmological potential $$\Phi_{\text{CP}}$$, we consider the orbital motion of the Solar system around the center of the Galaxy, which, as we know, occurs at a speed of $$v_S \approx 240$$ km/s. Therefore, the Newtonian galactic potential of attraction $$U_{\text{Galaxy}}$$ can be estimated as $$|U_{\text{Galaxy}}|/c^2 \approx v_S^2/c^2 \approx 0.7 \times 10^{-6}$$. As a result, we have a rough lower bound for the cosmological potential, $$|\Phi_{\text{CP}}|/c^2 > 10^{-6}$$. Moreover, since our Galaxy is only a small part of the Universe, we can expect a stronger condition, $$|\Phi_{\text{CP}}|/c^2 \gg 10^{-6}$$. Also, if we describe gravitational phenomena within the Solar system, then on such small scales we can absolutely neglect the spatial non-uniformity of the cosmological potential, i.e. $$\Phi_{\text{CP}}(\mathbf{r}) \approx \text{const}$$.

Thus, we formulate the main approximation that we will use further:

$$|\nabla_{\mathbf{r}} U_{\text{loc}}(\mathbf{r})| \gg |\nabla_{\mathbf{r}} \Phi_{\text{CP}}(\mathbf{r})|,$$  \hspace{1cm} (15)$$

where for the cosmological gravitational potential within the Solar System, the following lower bound holds:

$$|\Phi_{\text{CP}}|/c^2 > 10^{-6},$$  \hspace{1cm} (16)$$

which was determined from the orbital velocity of the Sun around the center of the Galaxy. Moreover, summarizing the above numerical estimates, we have

$$|U_{\text{loc}}(\mathbf{r})| \ll |\Phi_{\text{CP}}(\mathbf{r})|,$$  \hspace{1cm} (17)$$

at the distance of planetary orbits.
Obviously, a similar concept of cosmological potential [see expressions (12)- (17)] are universal and can be applied to most planetary systems with relatively small central stars.

3. Post-Newtonian parameters $\beta$ and $\gamma$

In this section, we show that, from the viewpoint of the Laboratory Observer, the main post-Newtonian parameters $\beta$ and $\gamma$ depend on the cosmological potential $\Phi_{\text{CP}}$. In the case of a small value, $|\Phi_{\text{CP}}/c^2|\ll 1$, these dependencies can be represented as:

$$
\beta = \beta_0 + \beta_1 \Phi_{\text{CP}}/c^2 + O\left(\Phi_{\text{CP}}^2/c^4\right),
\gamma = \gamma_0 + \gamma_1 \Phi_{\text{CP}}/c^2 + O\left(\Phi_{\text{CP}}^2/c^4\right),
$$

where $\beta_0$ and $\gamma_0$ are PPN parameters from the viewpoint of the Cosmological Observer.

Our approach is based on the following scheme. We start with a cosmological description, in which we initially include some local system of small gravitating bodies (e.g., the Solar System) as part of the global picture of the Universe. Further, using the general expression for the metric tensor written in the framework of the PPN formalism, we consider a relatively small area of space for the analysis of gravitation only inside of our chosen system. This allows us to distinguish the cosmological background using the concept of cosmological potential [see Eqs. (12) and (14)], which is then reduced going to the reference frame of the local Laboratory Observer. The result of such a reduction is the expression (18), which shows the difference between measured (within the Solar System) PPN parameters $(\beta, \gamma)$ from their initial (cosmological) values $(\beta_0, \gamma_0)$.

However, from a methodological point of view, it makes sense to first consider the simplest linear model of the metric tensor. This will avoid cumbersome formulas and make the presentation of our method as transparent as possible.

3.1. Simplified linear model of metric tensor

To demonstrate our approach, we consider a simplified model of gravitation in which the metric tensor (7) linearly depends on the weak gravitational potential $U(r)$. For this we use well-known formulas from GR:

$$
g_{00}(r) = 1 + \frac{2U(r)}{c^2}, \quad g_{11}(r) = 1 - \frac{2U(r)}{c^2},
$$

where $|U(r)/c^2|\ll 1$. Then, using the representation (12), we obtain for the spatial region of the Solar System:

$$
g_{00}(r) = 1 + \frac{2\Phi_{\text{CP}}}{c^2} + \frac{2U_{\text{loc}}(r)}{c^2},
\quad g_{11}(r) = 1 - \frac{2\Phi_{\text{CP}}}{c^2} - \frac{2U_{\text{loc}}(r)}{c^2},
$$

which defines the metric from the viewpoint of the Cosmological Observer in the initial flat cosmological spacetime $\{t, r\}_{\text{Cosm}}$.

Based on Einstein’s principles, we introduce a reference frame associated with a local “Laboratory Observer”, which, unlike the global and abstract “Cosmological Observer”, is closely tied to real physical measurements in a relatively small area of the space. In our case, to describe gravitational phenomena within the Solar System, spacetime for the Laboratory Observer is defined as follows. Neglecting the weak contribution of $U_{\text{loc}}(r)$ in Eqs. (20), the expression for the interval has the form:

$$
\frac{\text{d}s^2}{c^2} = \left(1 + \frac{2\Phi_{\text{CP}}}{c^2}\right)\left(c\, \text{d}t\right)^2 - \left(1 - \frac{2\Phi_{\text{CP}}}{c^2}\right)\left(\text{d}r\right)^2.
$$

According to Einstein’s basic postulate, a real Laboratory Observer is not able to detect the presence/absence of gravitation based on strictly local physical measurements (i.e., under constant gravitational potential). In particular, such local measurements include the measurement of the speed of light during its propagation between two arbitrarily close spatial points. Therefore, for the Laboratory Observer, the local speed of light in any region of the Universe should always be equal to $c$, regardless of the cosmological potential $\Phi_{\text{CP}}(r)$ for this region. Based on Einstein’s postulate and the formula (21), we find the following transformation law:

$$
\frac{\text{d}t'}{c} = \sqrt{1 + \frac{2\Phi_{\text{CP}}}{c^2}}, \quad \text{d}r' = \sqrt{1 - \frac{2\Phi_{\text{CP}}}{c^2}},
$$

which connects the space-time of the Laboratory Observer $\{t', r'\}_{\text{Lab}}$ with the space-time of the Cosmological Observer $\{t, r\}_{\text{Cosm}}$. In this case, the expression for the interval (21) in the reference frame $\{t', r'\}_{\text{Lab}}$ takes the following form:

$$
\frac{\text{d}s^2}{c^2} = (c\, \text{d}t')^2 - (\text{d}r')^2,
$$

i.e., laboratory space-time corresponds to the ideal Minkowski space-time, in which information about the cosmological potential of $\Phi_{\text{CP}}$ is quite deeply “hidden” from the observer. Setting $\text{d}s = 0$ in Eq. (23), we find that the speed of light for the Laboratory Observer is always equal to $c$ in any area of the Universe.

Thus, the Laboratory Observer is a observer studying gravitation in a relatively small region of space near relatively small celestial bodies. In our case, we associate the reference frame of the Laboratory Observer with the spatial region of the Solar System, where the continuum $\{t', r'\}_{\text{Lab}}$ is the flat space-time for a description of gravitational (and other physical) phenomena and experiments within the given system.

Accounting now of the gravitational contribution $U_{\text{loc}}(r)$ in Eqs. (20) leads to the following expression for the interval $\text{d}s^2$, rewritten in the variables of the laboratory reference frame $\{t', r'\}_{\text{Lab}}$:

$$
\frac{\text{d}s^2}{c^2} = \tilde{g}_{00}(r')(c\, \text{d}t')^2 - \tilde{g}_{11}(r')(\text{d}r')^2,
$$

where $\tilde{g}_{00}(r') = 1 + \frac{2\Phi_{\text{CP}}}{c^2} + \frac{2U_{\text{loc}}(r')}{c^2}$ and $\tilde{g}_{11}(r') = 1 - \frac{2\Phi_{\text{CP}}}{c^2} - \frac{2U_{\text{loc}}(r')}{c^2}$.
where
\[
\tilde{g}_{00}(r') = 1 + \frac{2U_{\text{loc}}(r')}{c^2(1 + 2\Phi_{\text{CP}}/c^2)}.
\]
\[
\tilde{g}_{11}(r') = 1 - \frac{2U_{\text{loc}}(r')}{c^2(1 - 2\Phi_{\text{CP}}/c^2)}.
\]
(25)
The spatial dependence \(U_{\text{loc}}(r')\), in accordance with the transformation law (22), is defined as:
\[
U_{\text{loc}}(r') = U_{\text{loc}}(r)|_{r \to r'}/\sqrt{1 - 2\Phi_{\text{CP}}/c^2}.
\]
(26)
However, in an experimental study of gravitation and testing of GR, the Laboratory Observer receives information about the gravitational potential not from purely theoretical calculations, but from experimental data on the motion of bodies under action of gravity. In particular, information on the Sun's gravitational potential is obtained from an analysis of the orbital motion of the planets and other small bodies. Thus, for the Laboratory Observer, the experimental determination of the gravitational potential \(\varphi(r')\) is connected with the equation of free motion in the framework of classical mechanics, which has the standard form:
\[
\frac{d^2 r'}{dt^2} = -\nabla_r \varphi(r'),
\]
(27)
where \(\varphi(r')\) differs from the theoretical function \(U_{\text{loc}}(r')\) defined in the framework of metric theory. Moreover, in addition to equation (27), the gravitational potential of \(\varphi(r')\) should also satisfy the additivity condition:
\[
\varphi(r') = \sum_a \varphi_a(r' - R'_a),
\]
(28)
where \(\varphi_a(r' - R'_a)\) is the gravitational contribution from the \(a\)-th point source with the coordinate \(R'_a\).
In order to determine the relationship between the functions \(\varphi(r')\) and \(U_{\text{loc}}(r')\), we write the equation for the geodesic line (10) in the cosmological reference frame \(\{t', r'\}_\text{Cosm}\) and in the linear approximation in \(U_{\text{loc}}(r')\):
\[
\frac{d^2 r'}{dt^2} = -\frac{1}{1 - 2\Phi_{\text{CP}}/c^2} \nabla_r U_{\text{loc}}(r'),
\]
(29)
where we used the components of the metric tensor (20) subject to the conditions (15). Rewriting equation (29) in the laboratory reference frame \(\{t', r'\}_\text{Lab}\) using the linear transformations (22), we obtain:
\[
\frac{d^2 r'}{dt^2} = -\frac{1}{1 + 2\Phi_{\text{CP}}/c^2} \nabla_r U_{\text{loc}}(r'),
\]
(30)
where \(\nabla_r = e_1(\partial/\partial x'_1) + e_2(\partial/\partial x'_2) + e_3(\partial/\partial x'_3)\) is the spatial gradient operator in the laboratory reference frame \(\{t', r'\}_\text{Lab}\). Comparing Eq. (29) with the equation for free motion (27) and the additivity condition (28), we find the relationship between \(\varphi(r')\) and \(U_{\text{loc}}(r')\):
\[
\varphi(r') = \frac{U_{\text{loc}}(r')}{1 + 2\Phi_{\text{CP}}/c^2}.
\]
(31)
The same result can be obtained immediately if, instead of equation (10) in the cosmological reference frame \(\{t, r\}_\text{Cosm}\), we use the following equation for the geodesic line:
\[
\frac{d^2 r'}{dt^2} = -\frac{c^2}{2\tilde{g}_{11}(r')} \nabla_r \tilde{g}_{00}(r'),
\]
(32)
written in the laboratory reference frame \(\{t', r'\}_\text{Lab}\).
The relationship (31) allows us to express the components of the metric tensor (20) in terms of the laboratory gravitational potential \(\varphi(r')\):
\[
\tilde{g}_{00}(r') = 1 + \frac{2\varphi(r')}{c^2}, \quad \tilde{g}_{11}(r') = 1 - \frac{2\gamma \varphi(r')}{c^2},
\]
\[
\gamma = \frac{1 + 2\Phi_{\text{CP}}/c^2}{1 - 2\Phi_{\text{CP}}/c^2}.
\]
(33)
This expression clearly demonstrates the appearance of the post-Newtonian parameter \(\gamma \neq 1\) for the Laboratory Observer even when we start with the cosmological description with the expression (10) obtained in the framework of the standard general relativity. Obviously, the difference (\(\gamma - 1\)) contains information about the cosmological potential of \(\Phi_{\text{CP}}\). However, for more accurate calculations of the post-Newtonian parameters \(\gamma\) and \(\beta\), it is necessary to consider a more general nonlinear post-Newtonian model of gravitation (see below).

3.2. General nonlinear post-post-Newtonian model of gravitation

In order to correctly calculate the post-Newtonian parameters \(\gamma\) and \(\beta\) for the Laboratory Observer in the linear approximation with respect to the small cosmological parameter \(\Phi_{\text{CP}}/c^2 \ll 1\) [see Eq. (15)], it is necessary to consider a nonlinear model of the metric tensor in the reference frame of the Cosmological Observer \(\{t, r\}_\text{Cosm}\). In this paper, we will use the PPN expression, which for the static distribution of masses has the form (see Refs. [6, 13]):
\[
\tilde{g}_{00}(r) = 1 + 2U(r)/c^2 + 2\beta_0 U^2(r)/c^4 + 2\zeta_0 Q(r)/c^6 + 2k_0 U^3(r)/c^8,
\]
\[
\tilde{g}_{11}(r) = 1 - 2\gamma U(r)/c^2 + 3\delta_0 U^2(r)/2c^4,
\]
(34)
where \(U(r)\) is determined in accordance with formula (11), and the function \(Q(r)\) has the following form:
\[
Q(r) = \sum_{n} \frac{\Gamma_0 M_n}{|r - R_n|} \sum_{\{\varphi_n\}} \frac{\Gamma_0 M_l}{|R_n - R_l|}.
\]
(35)
In expression (35), in addition to the standard post-Newtonian parameters \(\gamma_0\) and \(\beta_0\), there are two post-post-Newtonian parameters \(\delta_0\) (see, for example, [6]) and \(k_0\). For standard GR, there is \(\beta_0 = \gamma_0 = \delta_0 = 1\), while...
information about $\kappa_0$ is not available in the scientific literature, to our knowledge. It should be noted that the coefficient $\xi_0$ is not independent, but is expressed in terms of $\gamma_0$ and $\beta_0$. However, there are discrepancies in publications. So, for example, according to Ref. [3], we should set $\xi_0 = (2\beta_0 - 3\gamma_0 - 1)$, while Ref. [6] uses the other expression: $\xi_0 = (2\beta_0 - 1)$. Therefore, for simplicity, we will consider the coefficient $\xi_0$ as independent.

Using the representation for the potential in the form (37), we now rewrite the expressions (34) up to the quadratic term $U_{loc}^2(r)$ for the component $g_{00}(r)$ and up to the linear term $U_{loc}(r)$ for the component $g_{11}(r)$:

$$g_{00}(r) = G_{00} + A \frac{2U_{loc}(r)}{c^2} + B \frac{2U_{loc}^2(r)}{c^4} + \frac{2\xi_0 Q_{loc}(r)}{c^4},$$

$$g_{11}(r) = G_{11} - D \frac{2U_{loc}(r)}{c^2},$$

where we introduced a series of cosmological coefficients:

$$G_{00} = 1 + \frac{2\Phi_{CP}}{c^2} + \frac{2\xi_0 \Pi_{CP}}{c^4} + O \left( \frac{\Phi_{CP}}{c^4} \right),$$

$$G_{11} = 1 - \gamma_0 \frac{2\Phi_{CP}}{c^2} + O \left( \frac{\Phi_{CP}}{c^4} \right),$$

$$A = 1 + (2\beta_0 + \xi_0) \frac{\Phi_{CP}}{c^2} + O \left( \frac{\Phi_{CP}^2}{c^4} \right),$$

$$B = \beta_0 + 3\xi_0 \frac{\Phi_{CP}}{c^2} + O \left( \frac{\Phi_{CP}^2}{c^4} \right),$$

$$D = \gamma_0 - 3\xi_0 \frac{\Phi_{CP}}{c^2} + O \left( \frac{\Phi_{CP}^2}{c^4} \right).$$

Here we used that the expression (30) for $Q(r)$ can be represented with good accuracy in the following form (see Appendix A):

$$Q(r) \approx U_{loc}(r) \Phi_{CP} + \Pi_{CP} + Q_{loc}(r),$$

where $\Pi_{CP}$ is another cosmological contribution that can also be considered as a constant within the local planetary system ($\Pi_{CP} = const$), and its value is estimated in order of magnitude as

$$\Pi_{CP} \approx \frac{\Phi_{CP}}{c^4} \ll \frac{|\Phi_{CP}|}{c^2},$$

and it is a contribution to the $G_{00}$ [see Eqs. (30) and (37)]. The expression for $Q_{loc}(r)$ in Eq. (39) has the form:

$$Q_{loc}(r) = \sum_{n=1}^{N_{loc}} \Gamma_0 \frac{M_n}{|r - R_n|} \sum_{l \neq n} \frac{\Gamma_0 M_l}{|R_n - R_l|},$$

where, unlike Eq. (30), the summation is only over the indexes $\{1, 2, ..., N_{loc}\}$, which number the sources of gravitation only inside of the considered planetary system (the Solar System in our case).

By analogy with (22), we determine the following transformation law:

$$dt' = dt \sqrt{G_{00}}, \quad dr' = dr \sqrt{G_{11}},$$

which connects the space-time of the Laboratory Observer $\{t', r'\}_{Lab}$ with the space-time of the Cosmological Observer $\{t, r\}_{Cosm}$. As a result, we give the following expression for the interval $ds^2$, rewritten in the variables of the laboratory reference frame, $\{t', r'\}_{Lab}$:

$$ds^2 = \bar{g}_{00}(r')(c dt')^2 - \bar{g}_{11}(r')(dr')^2,$$

where, based on Eq. (30), we have:

$$\bar{g}_{00}(r') = 1 + 2A \frac{U_{loc}(r')}{G_{00} c^2} + 2B \frac{U_{loc}^2(r')}{G_{00} c^4} + \frac{2\xi_0 Q_{loc}(r')}{G_{00} c^4},$$

$$\bar{g}_{11}(r') = 1 - 2A \frac{U_{loc}(r')}{G_{11} c^2}.$$
The post-Newtonian parameters \( \beta, \gamma, \) and \( \xi \) in Eq. (48) from the viewpoint of the Laboratory Observer are defined as follows:

\[
\beta = \frac{G_{00}B}{A^2}, \quad \gamma = \frac{G_{00}D}{G_{11}A}, \quad \xi = \frac{G_{00}\xi_0}{A^2}.
\]

(51)

It is these quantities that can be measured in real experiments, rather than the initial \( \beta_0, \gamma_0, \) and \( \xi_0 \) [see in Eq. (34)] that appear in the theoretical description of gravitation from the viewpoint of the virtual Cosmological Observer.

Thus, we have shown the principal possibility of cosmological gravimetry, information about which is contained in the post-Newtonian parameters \( \beta, \gamma, \) and \( \xi. \) Moreover, as follows from Eq. (15), the cosmological background has absolutely no effect on the main Newtonian term [see \( 2v(r')/c^2 \) in the expression for \( \tilde{g}_{00}(r') \)].

Note that the above results were obtained in the model for a weak cosmological potential, \( |\Phi_{cp}/c^2| \ll 1. \) However, expressions (33) and (34)–51 can be considered in a more general context, as a universal phenomenological generalization independent of the initial equations for \( g_{jk}. \) In this case, the parameters \( G_{00} \) and \( G_{11} \) should be interpreted as components of the cosmological metric tensor \( G_{jk}^{(\text{Cosm})}: \)

\[
G_{jk}^{(\text{Cosm})} = \begin{pmatrix}
G_{00} & 0 & 0 & 0 \\
0 & -G_{11} & 0 & 0 \\
0 & 0 & -G_{11} & 0 \\
0 & 0 & 0 & -G_{11}
\end{pmatrix},
\]

(52)

which describes the Universe as a whole and is the subject of research of various cosmological models, including models with “dark” energy and matter, and even models that go beyond the standard Einstein equations (see, for example, the review [3]). Other cosmological parameters, \( A, B, \) and \( D, \) are due to the nonlinearity of the initial equations for \( g_{jk} \) in the cosmological reference frame. Moreover, from a cosmological viewpoint, all the specified parameters \{\( G_{00}(t, r), G_{11}(t, r), A(t, r), B(t, r), D(t, r) \}\} depend on coordinates (for example, due to the inhomogeneous distribution of matter in the Universe) and time (due to the general cosmic evolution, as well as changes in the mutual spatial arrangement of star systems and neighboring galaxies). However, within the local planetary system these parameters can be considered as constants, and their values are determined by the spatio-temporal location of this planetary system (for example, the Solar System in our case) within the framework of the general cosmological picture of the evolving Universe in general, and our Galaxy in particular.

4. Cosmological gravimetry in the framework of Einstein’s general relativity

If we now substitute the expressions for coefficients (37) into Eq. (51), then, up to a linear contribution of \( \Phi_{cp}/c^2, \) we write the expressions for \( \beta: \)

\[
\beta = \beta_0 + \beta_1 \frac{\Phi_{cp}}{c^2} + O \left( \frac{\Phi_{cp}^2}{c^4} \right); \\
\beta_1 = 2\beta_0(1 - 2\beta_0 - \xi_0) + 3\kappa_0,
\]

(53)

and for \( \gamma: \)

\[
\gamma = \gamma_0 + \gamma_1 \frac{\Phi_{cp}}{c^2} + O \left( \frac{\Phi_{cp}^2}{c^4} \right); \\
\gamma_1 = \gamma_0(2\beta_0 + 2 - 2\beta_0 - \xi_0) - 3\delta_0/2.
\]

(54)

In the case of Einstein’s general relativity, \( \beta_0 = \gamma_0 = \delta_0 = 1, \) we obtain:

\[
\beta - 1 = -(2 + 2\xi_0 - 3\kappa_0) \frac{\Phi_{cp}}{c^2} + O \left( \frac{\Phi_{cp}^2}{c^4} \right), \\
\gamma - 1 = -(\xi_0 - 0.5\kappa_0) \frac{\Phi_{cp}}{c^2} + O \left( \frac{\Phi_{cp}^2}{c^4} \right).
\]

(55)

(56)

In particular, if we assume \( \xi_0 = (2\beta_0 - 3\gamma_0 - 1) = -2 \) as in Ref. [3], then from Eq. (56) we find:

\[
\gamma - 1 \approx -0.5\frac{\Phi_{cp}}{c^2}.
\]

(57)

However, if we use \( \xi_0 = (2\beta_0 - 1) = 1 \) as in Ref. [6], then from Eq. (56) we have another expression:

\[
\gamma - 1 \approx -0.5\frac{\Phi_{cp}}{c^2}.
\]

(58)

Nevertheless, taking into account the estimate (10) for the cosmological potential \( \Phi_{cp}, \) we can give a general estimate for the experimentally measured post-Newtonian parameter \( \gamma: \)

\[
|\gamma - 1| > 10^{-6},
\]

(59)

which should be satisfied if Einstein’s general relativity is true.

Because the value \( \kappa_0 \) for GR is still unknown, it is impossible to draw a final conclusion about the post-Newtonian coefficient \( \beta \) in (55). However, we believe with high probability that for Einstein’s theory an estimate similar to (59) is fulfilled:

\[
|\beta - 1| > 10^{-6},
\]

(60)

Moreover, if the condition \( |\Phi_{cp}/c^2| \ll 1 \) is satisfied, then Eqs. (53) and (54) allow us to formulate a cosmological test for any post-Newtonian theory of gravitation:

\[
\frac{\beta - \beta_0}{\gamma - \gamma_0} \approx \frac{\beta_1}{\gamma_1},
\]

(61)

which, in the case of Einstein’s theory, has the form:

\[
\frac{\beta - 1}{\gamma - 1} \approx \frac{2 + 2\xi_0 - 3\kappa_0}{\xi_0 - 0.5}.
\]

(62)
Thus, the magnitude of this ratio can be the experimental cosmological test for standard GR.

Note that the above results were obtained in the framework of the mathematical model \((64)\), where we took into account only two post-post-Newtonian contributions: the contribution \(U^3(r)/c^8\) for the component \(g_{00}\), and the contribution \(U^2(r)/c^4\) for the component \(g_{11}\). In this case, the possible existence of other post-post-Newtonian contributions can lead to some correction of the expression (7), whose components have the following form:

\[
g_{00}(r) = e^{2U_{loc}(r)/c^2}, \quad g_{11}(r) = e^{-2U_{loc}(r)/c^2},
\]

where \(G_{00}\) and \(G_{11}\) should be considered as cosmological components. Using the transformations \((11)\), we go to the reference frame of the Laboratory Observer \(\{t', r'\}_\text{Lab}\) with the metric tensor \((12)\). In this reference frame, using Eq. \((64)\), we obtain:

\[
\tilde{g}_{00}(r') = e^{2U_{loc}(r')/c^2}, \quad \tilde{g}_{11}(r') = e^{-2U_{loc}(r')/c^2},
\]

where the spatial dependence \(U_{loc}(r')\) is determined in accordance with \((41)\).

Using expressions \((63)\), we write the equation for the geodesic line \((32)\) in the laboratory reference frame \(\{t', r'\}_\text{Lab}\):

\[
\frac{d^2r'}{dt'^2} = -\nabla_r U_{loc}(r'),
\]

where we restricted ourselves to a linear contribution of \(U_{loc}(r')\). Comparing this equation with the equation for free motion \((27)\), we find the relationship between \(U_{loc}(r')\) and the gravitational potential \(\varphi(r')\) from the viewpoint of the Laboratory Observer:

\[
\varphi(r') = U_{loc}(r').
\]

As a result, we obtain the final expressions:

\[
\tilde{g}_{00}(r') = e^{2\varphi(r')/c^2}, \quad \tilde{g}_{11}(r') = e^{-2\varphi(r')/c^2},
\]

in which there is no “trace” (information) of the cosmological potential \(\Phi_{CP}\). Moreover, in the case of \(|\varphi(r')/c^2| \ll 1\), the decomposition takes place:

\[
\tilde{g}_{00}(r') = 1 + \frac{2\varphi(r')}{c^2} + \frac{2\varphi^2(r')}{c^4} + \frac{4\varphi^3(r')}{3c^6} + ...,
\]

\[
\tilde{g}_{11}(r') = 1 - \frac{2\varphi(r')}{c^2} + \frac{2\varphi^2(r')}{c^4} + ...,
\]

from which Newton’s theory for weak gravitational fields follows [see the main (Newtonian) term, \(2\varphi(r')/c^2\), in the expression for \(\tilde{g}_{00}(r')\)].

Thus, we have proved that the metric model \((63)\) fully satisfies Postulates 1-3. It is also obvious that this model is invariant with respect to the transformation:

\[
U(r) \to U(r) + C_0,
\]

where \(C_0\) is an arbitrary constant. In essence, it is precisely the non-invariance with respect to the transformation \((70)\) that makes cosmological gravimetry possible for other models (including Einstein’s theory), which was demonstrated in our work.

Moreover, from expressions \((69)\), \((68)\) and \((66)\) we find the values of the post-Newtonian and post-post-Newtonian parameters [see in Eqs. \((33)\) and \((34)\)]:

\[
\beta = \beta_0 = \gamma = \gamma_0 = 1, \quad \xi = \xi_0 = 0, \quad \delta = \delta_0 = 4/3, \quad \kappa = \kappa_0 = 2/3,
\]

5. Cosmologically insensitive metric model of gravitation

In the previous sections, we have shown the possibility of cosmological gravimetry in the framework of metric theories of gravitation, including Einstein’s general relativity. The essence of this cosmological gravimetry is that the gravitational potential of the entire Universe \(\Phi_{CP}\), despite its highest degree of spatial homogeneity on small scales, can nevertheless have a certain effect on some gravitational measurements carried out within a relatively local system of small gravitating bodies (the Solar System, in our case). This allows us to measure (at least in principle) the value of \(\Phi_{CP}\) for a given region of space. However, as will be shown below, there is at least one distinguished metric model in which the possibility of such cosmological gravimetry is absent.

Abstracting from Einstein’s theory, we will build a nonlinear PPN model that satisfies the following three postulates:

1. Gravitation is described by the metric tensor \(g_{jk}\) [see Eq. \((1)\)].
2. In the case of a small gravitational potential, \(|U(r)/c^2| \ll 1\), the model should coincide with the classical Newtonian theory of gravitation.
3. The model should not allow us to measure the cosmological potential \(\Phi_{CP}\).

These postulates are satisfied by a model with a diagonal metric tensor \((7)\), whose components have the following form:

\[
g_{00}(r) = e^{2U(r)/c^2}, \quad g_{11}(r) = e^{-2U(r)/c^2}.
\]

Indeed, taking into account the cosmological potential \(\Phi_{CP}\) in the expression \((12)\), we rewrite the metric components \((63)\) as:

\[
g_{00}(r) = G_{00} e^{2U_{loc}(r)/c^2}, \quad g_{11}(r) = e^{-2\Phi_{CP}(r)/c^2},
\]

\[
g_{11}(r) = G_{11} e^{-2U_{loc}(r)/c^2}, \quad G_{11} = e^{-2\Phi_{CP}(r)/c^2},
\]

where \(G_{00}\) and \(G_{11}\) should be considered as cosmological components. Using the transformations \((11)\), we go to the reference frame of the Laboratory Observer \(\{t', r'\}_\text{Lab}\) with the metric tensor \((12)\). In this reference frame, using Eq. \((64)\), we obtain:

\[
\tilde{g}_{00}(r') = e^{2U_{loc}(r')/c^2}, \quad \tilde{g}_{11}(r') = e^{-2U_{loc}(r')/c^2},
\]

where the spatial dependence \(U_{loc}(r')\) is determined in accordance with \((41)\).

Using expressions \((63)\), we write the equation for the geodesic line \((32)\) in the laboratory reference frame \(\{t', r'\}_\text{Lab}\):

\[
\frac{d^2r'}{dt'^2} = -\nabla_r U_{loc}(r'),
\]

where we restricted ourselves to a linear contribution of \(U_{loc}(r')\). Comparing this equation with the equation for free motion \((27)\), we find the relationship between \(U_{loc}(r')\) and the gravitational potential \(\varphi(r')\) from the viewpoint of the Laboratory Observer:

\[
\varphi(r') = U_{loc}(r').
\]

As a result, we obtain the final expressions:

\[
\tilde{g}_{00}(r') = e^{2\varphi(r')/c^2}, \quad \tilde{g}_{11}(r') = e^{-2\varphi(r')/c^2},
\]

in which there is no “trace” (information) of the cosmological potential \(\Phi_{CP}\). Moreover, in the case of \(|\varphi(r')/c^2| \ll 1\), the decomposition takes place:

\[
\tilde{g}_{00}(r') = 1 + \frac{2\varphi(r')}{c^2} + \frac{2\varphi^2(r')}{c^4} + \frac{4\varphi^3(r')}{3c^6} + ...,
\]

\[
\tilde{g}_{11}(r') = 1 - \frac{2\varphi(r')}{c^2} + \frac{2\varphi^2(r')}{c^4} + ...,
\]

from which Newton’s theory for weak gravitational fields follows [see the main (Newtonian) term, \(2\varphi(r')/c^2\), in the expression for \(\tilde{g}_{00}(r')\)].

Thus, we have proved that the metric model \((63)\) fully satisfies Postulates 1-3. It is also obvious that this model is invariant with respect to the transformation:

\[
U(r) \to U(r) + C_0,
\]

where \(C_0\) is an arbitrary constant. In essence, it is precisely the non-invariance with respect to the transformation \((70)\) that makes cosmological gravimetry possible for other models (including Einstein’s theory), which was demonstrated in our work.

Moreover, from expressions \((68)\), \((67)\) and \((66)\) we find the values of the post-Newtonian and post-post-Newtonian parameters [see in Eqs. \((33)\) and \((34)\)]:

\[
\beta = \beta_0 = \gamma = \gamma_0 = 1, \quad \xi = \xi_0 = 0, \quad \delta = \delta_0 = 4/3, \quad \kappa = \kappa_0 = 2/3,
\]
which are the same for both Cosmological and Laboratory Observers. It follows that all the currently known successful experimental tests of GR within the Solar System (such as: measuring the anomalous perihelion shift of Mercury’s orbit, measuring the deflection of light near the Sun, the time delay of radio signals sent across the Solar System past the Sun to a planet or satellite and returned to the Earth (due to the gravitation of the Sun), laser ranging of the Moon, etc.) can also be considered, from a formal viewpoint, as confirmation of the cosmologically insensitive model of gravitation presented above, because all these experiments show: $\beta \approx 1$ and $\gamma \approx 1$. In this context, the experimental detection of deviations, $|\beta - 1| > 10^{-6}$ and $|\gamma - 1| > 10^{-6}$, will not only be a demonstration of cosmological gravimetry, but it will also be a critical test for Einstein’s general relativity. Indeed, if experiments will show $|\beta - 1| < 10^{-6}$ and $|\gamma - 1| < 10^{-6}$, then the cosmologically insensitive model of gravitation presented above will be more preferred.

We stress that the requirement of the invariance with respect to transformation (70) (or Postulate 3) is so very strong that it allows us to find a solution (63) without appeal to any equations for the metric tensor $g_{\beta k}$, which are unknown. Therefore, a special theoretical interest may be the problem of constructing a more complete and closed theory, in which expressions (63) are the solution of some equations (at least for the static distribution of masses). Some interesting and unusual features of the cosmologically insensitive model of gravimetry are presented in Appendix C.

6. Discussion and conclusion

As shown above, the key factor for the possibility of cosmological gravimetry is the non-invariance of the theory with respect to transformation (70). Indeed, if some theory is non-invariant with respect to Eq. (70), then in such a theory the gravitational potential is a completely definite (fixed) quantity. This, in turn, allows us to formulate the concept of the cosmological gravitational potential $\Phi_{\text{CP}}$ as a well-defined physical quantity and raises the question of its experimental measurement. This circumstance contrasts with the classical theory of gravitation, in which the basic concept is the gravitational force, defined as $F_{\text{grav}} = \nabla U(x)$, which leads to invariance with respect to transformation (70) and therefore the impossibility of cosmological gravimetry.

We note that the idea of the possibility of cosmological gravimetry in the case of non-invariance with respect to transformation (70) was first proposed in Ref. [14,15], where the gravitational redshift in atomic clocks was considered in the framework of the nonmetric description (due to the mass defect effect in quantum atomic physics, e.g., see Refs. [14,15]). However, in our paper, we substantiated the possibility of cosmological gravimetry for the metric post-Newtonian theory of gravitation, which includes Einstein’s general relativity. Such gravimetry is based on the experimental measurement of two main post-Newtonian parameters, the values of which are different for the Laboratory Observer $(\beta, \gamma)$ and for the Cosmological Observer $(\beta_0, \gamma_0)$. For instance, in the case of GR, information on cosmological gravitation is contained in the values $(\beta - 1)$ and $(\gamma - 1)$. Moreover, a lower estimate was obtained for the Solar System: $|\beta - 1| > 10^{-6}$ and $|\gamma - 1| > 10^{-6}$, based on the velocity of the Sun’s orbital motion around the Galaxy center. Also, a cosmological test for GR is formulated, in which the ratio $(\beta - 1)/(\gamma - 1)$ has a quite certain value, which can be analytically calculated if all post-Newtonian and post-post-Newtonian contributions in the metric tensor are known (this work still needs to be completed). The relationship between the gravitational constant $\Gamma$ for the laboratory reference frame and its initial value $\Gamma_0$ in the cosmological reference frame is also shown.

In addition, the motion of celestial bodies and the Sun’s orbital motion can, in principle, affect the presented results obtained in the framework of the static distribution of masses in the Universe and the motionless Solar System, if the cosmological potential $\Phi_{\text{CP}}$ does not far exceed the gravitational potential of our Galaxy, $U_{\text{Galaxy}}$. However, in any case, this does not negate the concept of cosmological gravimetry developed here.

Moreover, a unique PPN model was constructed, which is invariant with respect to transformation (70) and therefore has no possibility for cosmological gravimetry. The most unusual is that for this model we have, $\beta = \beta_0 = \gamma = \gamma_0 = 1$, which is very close to Einstein’s theory. Therefore, the experimental detection of deviations, $|\beta - 1| > 10^{-6}$ and $|\gamma - 1| > 10^{-6}$, becomes critical for GR. Indeed, if experiments will show $|\beta - 1| < 10^{-6}$ and $|\gamma - 1| < 10^{-6}$, then the cosmologically insensitive model of gravitation will be more preferred.

In the context of the above, it can be argued that all experiments on measuring post-Newtonian parameters $(\beta, \gamma)$ should be considered as experimental cosmological gravimetry. Such gravimetry, in the case of deviations of $|\beta - 1| > 10^{-6}$ and $|\gamma - 1| > 10^{-6}$, will open up new unique opportunities for investigation of the Universe and will be additional test of Einstein’s general relativity. At the same time, we note that the indicated level of accuracy ($10^{-6}$) is quite achievable using modern measuring equipment. Therefore, the obtained results can be used as additional motivation for new space-based experiments with the aim of more accurate measurement of the PPN parameters $(\beta, \gamma)$.

[1] A. Einstein, Jurbuch Radioaktivität Electron. 4, 411 (1907).
[2] A. Einstein, Annal. Physik (Liepzig) 35, 898 (1911).
Appendix A

We represent expression (35) in the form of the following three contributions:

\[ Q(r) = Q_1(r) + \Pi(r) + Q_{loc}(r), \]  

\[ Q_1(r) = \sum_{n=1}^{N_{loc}} \frac{\Gamma_0 M_n}{|r - R_n|} \sum_{l > N_{loc}} \frac{\Gamma_0 M_l}{|R_n - R_l|}, \]

\[ \Pi(r) = \sum_{n > N_{loc}} \frac{\Gamma_0 M_n}{|r - R_n|} \sum_{l \neq n} \frac{\Gamma_0 M_l}{|R_n - R_l|}, \]

\[ Q_{loc}(r) = \sum_{n=1}^{N_{loc}} \frac{\Gamma_0 M_n}{|r - R_n|} \sum_{l \neq n} \frac{\Gamma_0 M_l}{|R_n - R_l|}. \]

The first term \(Q_1(r)\) can be written as:

\[ Q_1(r) = \sum_{n=1}^{N_{loc}} \frac{-\Gamma_0 M_n}{|r - R_n|} \sum_{l > N_{loc}} \frac{-\Gamma_0 M_l}{|R_n - R_l|} = \sum_{n=1}^{N_{loc}} \frac{-\Gamma_0 M_n \Phi_{CP}(R_n)}{|r - R_n|}, \]  

where \(\Phi_{CP}(R_n)\) is the cosmological potential [see Eq. (14)] at the point where the gravitation source \(R_n\) is inside of our local system of celestial bodies (i.e., the Solar System). However, because the cosmological potential is almost uniform within the studied local system \(\Phi_{CP}(r) \approx const\), then, using (33), we can write:

\[ Q_1(r) \approx \Phi_{CP} \sum_{n=1}^{N_{loc}} \frac{-\Gamma_0 M_n}{|r - R_n|} = U_{loc}(r) \Phi_{CP} . \]

The second term in (A1) can be represented as:

\[ \Pi(r) = \sum_{n > N_{loc}} \frac{-\Gamma_0 M_n}{|r - R_n|} \sum_{l \neq n} \frac{-\Gamma_0 M_l}{|R_n - R_l|} = \sum_{n > N_{loc}} \frac{-\Gamma_0 M_n \Phi_{CP}(R_n)}{|r - R_n|} = \sum_{n > N_{loc}} \frac{\Gamma_0 M_n |\Phi_{CP}(R_n)|}{|r - R_n|}. \]

Because only deep space bodies \((n > N_{loc})\) are presented here, \(\Pi(r)\) should be considered as an additional cosmological contribution, which is spatially homogeneous within the considered local system, \(\Pi(r) \approx const.\) To estimate the value of \(\Pi(r)\), we divide expression (A1) by \(c^2\), that leads to the following inequality:

\[ \frac{\Pi(r)}{c^2} = \sum_{n > N_{loc}} \frac{\Gamma_0 M_n |\Phi_{CP}(R_n)|}{|r - R_n|} \ll \sum_{n > N_{loc}} \frac{\Gamma_0 M_n}{|r - R_n|} = |\Phi_{CP}(r)|, \]

where we used the condition \(|\Phi_{CP}(R_n)/c^2| \ll 1\). Going to dimensionless quantities, the inequality (A5) can be rewritten as:

\[ \frac{\Pi(r)}{c^2} \ll \frac{|\Phi_{CP}(r)|}{c^2}. \]

The expression for \(Q_{loc}(r)\) in Eq. (A1) is not changed. Thus, we finally write:

\[ Q(r) \approx U_{loc}(r) \Phi_{CP} + \Pi(r) + Q_{loc}(r), \]

where for the cosmological contribution \(\Pi(r)\) the estimate (A6) holds.

Appendix B

Let us consider the gravitational potential created by the point mass \(M\) with the coordinate \(R\) in the cosmological reference frame \(\{t, r\}_{\text{Cosm}}:\)

\[ U(r) = -\frac{\Gamma_0 M}{|r - R|}. \]
Then, taking into account definition (44), we find:

\[
U(r') = -\frac{\Gamma_0 M \sqrt{G_{11}}}{|r' - R|}. \tag{B2}
\]

At the same time, the Newtonian gravitational potential \(\varphi(r')\) for the Laboratory Observer has the standard form:

\[
\varphi(r') = -\frac{\Gamma M'}{|r' - R'|}. \tag{B3}
\]

where \(\Gamma\) and \(M'\) are the gravitational constant and mass, respectively, in the reference frame \((t', r')_\text{Lab}\). Substituting now expressions (B2) and (B3) in Eq. (46), we obtain:

\[
\frac{\Gamma M'}{|r' - R'|} = \frac{AV\sqrt{G_{11}}}{G_{00}} \Gamma_0 M. \tag{B4}
\]

that leads to the transformation law:

\[
\Gamma M' = \frac{AV\sqrt{G_{11}}}{G_{00}} \Gamma_0 M. \tag{B5}
\]

If we assume that the rest masses are equal for any reference systems \((M = M')\), then we finally write the following transformation law for the gravitational constant:

\[
\Gamma = \frac{AV\sqrt{G_{11}}}{G_{00}} \Gamma_0. \tag{B6}
\]

Using the expressions from Eq. (37), we find:

\[
\frac{\Gamma}{\Gamma_0} = 1 + (2\beta_0 - \gamma_0 + \xi_0 - 2) \frac{\Phi_{\text{CP}}}{c^2} + O(\Phi_{\text{CP}}^2/c^4). \tag{B7}
\]

In the case of GR \((\beta_0 = \gamma_0 = 1)\), there is:

\[
\frac{\Gamma}{\Gamma_0} = 1 + (\xi_0 - 1) \frac{\Phi_{\text{CP}}}{c^2} + O(\Phi_{\text{CP}}^2/c^4). \tag{B8}
\]

In particular, if we assume \(\xi_0 = (2\beta_0 - 3\gamma_0 - 1) = -2\) as in Ref. 8, then from Eq. (B8) we have:

\[
\frac{\Gamma}{\Gamma_0} = 1 - 3 \frac{\Phi_{\text{CP}}}{c^2} + O(\Phi_{\text{CP}}^2/c^4). \tag{B9}
\]

However, if we use \(\xi_0 = (2\beta_0 - 1) = 1\) as in Ref. 8, then from Eq. (B8) we obtain another estimate:

\[
\frac{\Gamma}{\Gamma_0} = 1 + O(\Phi_{\text{CP}}/c^4), \tag{B10}
\]

which means a significantly weaker dependence on \(\Phi_{\text{CP}}\) compared to Eq. (B9).

**Appendix C**

Below we consider some features of the cosmologically insensitive metric model of gravitation. First of all, we note that from a formal viewpoint there is a more general formulation than in Eq. (13), which also satisfies the Postulates 1-3:

\[
\left\{
\begin{array}{l}
g_{00}(r) = e^{2U(r)}/c^2, \\
g_{11}(r) = e^{-2wU(r)}/c^2,
\end{array}
\right. \tag{C1}
\]

where \(w\) is an arbitrary real coefficient. However, taking into account known empirical data, only a model with \(w = 1\) can be correct, i.e. Eq. (13).

### a. Chronometric gravimetry

According to general relativity [3], the passage of time in a gravitational field slows down. Consider the well-known gravitation-chronometric experiments (see, for example, [18, 19]) on measuring of the ratio

\[
\frac{\omega(r'_1) - \omega(r'_2)}{\omega(r'_1)} = \frac{\Delta \omega}{\omega} = 1 - \frac{\omega(r'_2)}{\omega(r'_1)}, \tag{C2}
\]

which describes the relative difference between the frequencies \(\omega(r'_1)\) and \(\omega(r'_2)\) for the same atomic transition at two different spacial points \(r'_1\) and \(r'_2\) with different gravitational potentials \(\varphi(r'_1)\) and \(\varphi(r'_2)\). In the framework of the metric theory of gravitation, the following relation holds:

\[
\frac{\omega(r'_2)}{\omega(r'_1)} = \sqrt{\frac{g_{00}(r'_2)}{g_{00}(r'_1)}}. \tag{C3}
\]

Then, using Eq. (65) for a cosmologically insensitive model, we obtain the expression:

\[
\frac{\Delta \omega}{\omega} = 1 - e^{-\Delta \varphi/c^2} = \frac{\Delta \varphi}{c^2} - \frac{(\Delta \varphi)^2}{2c^4} + \frac{(\Delta \varphi)^3}{6c^6} - \ldots, \tag{C4}
\]

\(\Delta \varphi = \varphi(r'_1) - \varphi(r'_2)\),

which depends only on \(\Delta \varphi\). For comparison, if we use for \(g_{00}(r')\) the expression (18) obtained in the framework of the PPN formalism, then up to quadratic contributions of \(\varphi(r')\) we have the following result:

\[
\frac{\Delta \omega}{\omega} \approx \frac{\Delta \varphi}{c^2} - \frac{(\Delta \varphi)^2}{2c^4} + (\beta - 1) \frac{\varphi^2(r'_1)}{c^4} - \varphi^2(r'_2)/c^4. \tag{C5}
\]

This formula shows the principal possibility of cosmological gravimetry in space-based chronometric experiments using ultra-precision atomic clocks for spacecraft at low and high orbits around the Sun.

### b. Free fall acceleration in nonlinear regime. Schwarzschild radius

Using expressions (68), we write the equation for the geodesic line (12) in the nonrelativistic low-velocity limit:

\[
\mathbf{a} = \frac{d^2 \mathbf{r}}{dt^2} = -e^{4\varphi(r')/c^2} \nabla_{r'} \varphi(r'), \tag{C6}
\]

which describes the free fall acceleration \(\mathbf{a}\) for the cosmologically insensitive model of gravitation. For the point mass \(M\) placed in the center of the spatial reference frame, the potential has the standard form: \(\varphi(r') = -\Gamma M/r\), where \(r = |r'|\). We introduce the Schwarzschild radius, \(r_S = MT/c^2\), which allows us to express the gravitational potential as \(\varphi(r') = -c^2r_S/r\). In this case, from (C6) we obtain:

\[
\mathbf{a} = -e^{-4r_S/r} \frac{c^2 r_S}{r^2} \mathbf{n}, \tag{C7}
\]
where $\mathbf{n} = r' / |r'|$ is the unit radial vector. We now define the parameter $a_S = c^2 / r_S$, which has the dimension of acceleration and which we will call the Schwarzschild acceleration. As a result, we obtain a universal expression for the value of $a = |a|$ in the form of a dependence on the dimensionless parameter $\rho = r / r_S$:

$$\frac{a}{a_S} = e^{-4/\rho^2},$$

which is presented in Fig. 2. This figure shows that there is a prominent maximum of $a \approx 0.034a_S$ for $r = 2r_S$ and at distances $r < 2r_S$ we see a sharp decrease in the gravitational effect. Thus, in spite of the fact that the cosmologically insensitive model of gravitation has no singularities at all (in contrast to GR for $r \leq r_S$), nevertheless, the concept of the Schwarzschild radius $r_S$ for this model is also relevant when the gravitational effect at distances $r > 2r_S$ is radically different from the behavior at small distances, $r < 2r_S$. 

FIG. 2: Dependence of the free fall acceleration for the cosmologically insensitive model of gravitation (solid thick line) [see Eq. (C8)]. The dashed line corresponds to the classical dependence for Newtonian gravity: $a = \Gamma M / r^2$. 

$\rho=r/r_S$

$a/a_S$

0 5 10 15 20 25

0.01 0.02 0.03 0.04 0.05