THE SOLAR CORE: NEW LOW-\(l\) \(p\)-MODE FINE-SPACING RESULTS FROM BiSON

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ABSTRACT

The fine-structure spacing \(d_l(n) = \nu_{l,n} - \nu_{l+2,n-1}\) for low-degree solar \(p\) modes of angular degree \(l\) and radial order \(n\) is sensitive to conditions in the deep radiative interior of the Sun. Here we present fine-structure spacings derived from the analysis of nearly 5 years of helioseismological data collected between 1991 July and 1996 February by the Birmingham Solar Oscillations Network (BiSON). These data cover \(9 \leq n \leq 28\) for \(d_l(n)\), and \(11 \leq n \leq 27\) for \(d_l(n)\). The measured spacings are much more precise and cover a greater range than earlier measurements from BiSON data (Elsworth et al. 1990a). The predicted fine-structure spacings for a “standard” solar model are clearly excluded by the BiSON data (at \(\approx 10\ \sigma\)); models that include helium and heavy-element settling provide a much better match to the observed spacings (see also Elsworth et al. 1995). Since the inclusion of core settling in solar models will tend to increase slightly the predicted neutrino flux, the BiSON fine-structure data appear to reinforce previous conclusions, i.e., an astrophysical solution to the solar neutrino problem seems unlikely.

Subject heading: stars: evolution — Sun: interior — Sun: oscillations

1. INTRODUCTION

The solar \(p\)-mode oscillations are manifestations of trapped standing acoustic waves in the solar interior. The low-degree (low-\(l\)) modes, which are globally coherent, possess radial wave functions that penetrate into the core of the Sun. Fine-structure spacings \(d_l(n) = \nu_{l,n} - \nu_{l+2,n-1}\) (where \(n\) is the modal radial order), constructed from the low-degree modes, are extremely sensitive to conditions in the deep solar interior. The use of these spacings to constrain models and theory is particularly appealing, since one avoids relying upon inadequate modeling of the outer solar layers.

Here we present fine-structure spacings \(d_l(n)\) and \(d_l(n)\) derived from the analysis of several years of solar Doppler velocity data collected between 1991 and 1996 by the Birmingham Solar Oscillations Network (BiSON). Elsworth et al. (1990a), with pre-1990 vintage BiSON data, demonstrated that the “standard” solar model provided a good match to the observed spacings. Furthermore, a variety of weakly interacting, massive particle (WIMP) models (e.g., Cox, Guzik, & Raby 1990) were shown to be in significant conflict with the observations. Elsworth et al. (1990a) therefore concluded that the helioseismological data provided evidence for the solar neutrino problem requiring a solution in particle—and not astro—physics, e.g., via mixing between different species of neutrinos or spin precession. The substantially superior quality of the BiSON data presented in this paper has allowed us to make even more precise—and demanding—comparisons with solar models. We demonstrate that models including helium and heavy-element settling are in much better agreement with the new BiSON data (see also Elsworth et al. 1995).

2. DATA AND ANALYSIS

We have used seven 8 month–long and one 32 month–long time series, all constructed from BiSON data collected between 1991 July and 1996 February, for the analyses discussed in this paper. The modes have been fitted in the frequency domain as rotationally split, Lorentzian multiplets with an associated flat background offset. The relative amplitudes of different \(m\) components within individual \(l\) multiplets were fixed with the whole-disc sensitivities of Christensen-Dalsgaard (1989). We have constrained the heights of peak components with the same \(|n|\) to be equal within each fitted multiplet. A Levenberg-Marquardt technique was used to perform the fitting, minimizing a maximum likelihood function that reflects the \(\chi^2\) 2 degrees of freedom statistics of the frequency spectrum (Chaplin et al. 1996). Dziembowski & Goode (1996) indicate that \(m\)-dependent asymmetries in the real mode-multilet structures might introduce systematic errors. Our mode-fitting procedure is most heavily weighted toward the sectoral components, which appear as the strongest peaks in our data. The calculations of these authors suggest that—considering, for example, \(d_l(n)\)—the frequency differences between the \(l = 0\) and \(l = 2\) components should not be strongly affected over the solar cycle.

Fine-structure spacings \(d_l(n)\) and \(d_l(n)\) were determined for each spectrum from the fitted frequencies. Formal errors on the spacings were derived from the formal uncertainties on the mode frequencies returned by the fitting program. Mean sets of spacings were constructed from the seven 8 month spectra by computing weighted averages according to

\[
\tilde{d}_l(n) = \frac{\sum_{i=1}^{N} d_l(n)_i / \sigma_i^2}{\sum_{i=1}^{N} 1 / \sigma_i^2},
\]

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where $N$ is the number of independent measures at each $n$, i.e., $N \leq 7$, and $\sigma_i$ are the errors associated with each spacing $d_i(n)$. Estimates of the external error on each weighted mean (e.g., see Topping 1962) were calculated according to

$$
\delta_{ext}[\bar{d}(n)] = t(N - 1) \sqrt{\frac{\sum_{i=1}^{N} [d_i(n) - \bar{d}(n)]^2 / \sigma_i^2}{(N - 1) \sum_{i=1}^{N} 1 / \sigma_i^2}},
$$

(2)

where, because of the small number of samples at each $\langle ln \rangle$, a suitable correction factor, $t(N - 1)$, appropriate to the desired 1 $\sigma$ significance interval, has been drawn from the $t$ distribution in order to assess correctly the confidence limits bracketing the mean (e.g., see Book 1978, p. 85). Constructing the errors in the manner shown above reflects the scatter in the observables, i.e., an analysis indicates that the external errors are a factor of $\approx 1.5$ larger than the internal errors, while preserving the additional information provided by the formal fitting uncertainties. A small number of data were omitted from a few of the mean spacing calculations on the basis of several “outlier” rejection criteria. We employed variants of the standard discordancy test (internally Studentized extreme deviation from the sample mean; e.g., see Barnett & Lewis 1984, p. 167), omitting each measure in turn from the sample in the same spirit as the “jackknife” technique and also taking into account the internal weights in one of the tests. In addition, we also used the Dixon $Q$ family of tests (Dixon 1951) in order to check for the presence of single (Dixon’s $r_0$) and double (Dixon’s $r_{m0}$ and $r_{21}$) outliers.

Final sets of fine-structure spacings were then constructed by combining the 32 month and mean 8 month data: for $d_0(n)$, 32 month spacings were used for $9 \leq n \leq 13$, and mean 8 month spacings were used for $14 \leq n \leq 28$; for $d_1(n)$, 32 month spacings were used for $11 \leq n \leq 13$, and mean 8 month spacings were used for $14 \leq n \leq 27$. The errors used for $n \leq 13$ are the formal uncertainties from the fitted 32 month spectrum, while those above are the external errors from the 8 month averages ($eq. [2]$). The averaged data sets can be found at http://bison.ph.bham.ac.uk.

3. RESULTS AND DISCUSSION

We have followed the convention of Elsworth et al. (1990a) and parameterized each set of data in terms of a straight line according to

$$
\bar{d}_i(n) = c_0 + c_1(n - 21),
$$

(3)

where $c_0$ and $c_1$ are the coefficients of the fit, suitably normalized to the radial order datum $n = 21$ on the abscissa. We have performed the fit over the range $15 \leq n \leq 27$ in order to facilitate a proper comparison with Elsworth et al. (1990a). The best-fit coefficients for the Elsworth et al. (1990a) and new BiSON data are given in Table 1. The new data are represented graphically in Figure 1: here the residuals, generated by subtracting the appropriate best straight-line fit from the mean fine-structure spacings $\bar{d}_i(n)$ and $d_i$, have been plotted. (See below for explanation and discussion of the solid, dot-dashed, and dotted line model-generated predictions.) The uncertainties on the new best-fit values are, as expected, substantially superior to those in Elsworth et al. (1990a), owing to the higher quality and quantity of the more recent data. The fitted $c_0$ coefficients for $d_i(n)$ differ by some $3 \sigma$ (combined error); this is because the $l = 3$ frequencies—the mode peaks of which are substantially weaker than those at $l = 0$, 1, and 2—were less well determined in the old BiSON spectra.

Fitting statistics indicate that a straight line is an inadequate representation of both sets of data, i.e., with reference to Figure 1, there is clearly rather more structure present in each plot. The use of higher order terms, or other more compli-
cated functions, in any parameterized description of $d_0(n)$ and $d_1(n)$ is therefore now required, given the accuracy of the measured spacings.

We have searched for solar cycle effects in both $d_0(n)$ and $d_1(n)$. No significant variations were found. For $d_0(n)$, we find that a variation in the mean spacing, as weighted by the global view, over the range $14 \leq n \leq 26$, is excluded at $\sim 0.08 \mu$Hz (3 $\sigma$); and for $d_1(n)$, over the same range in $n$, it is excluded at $\sim 0.09 \mu$Hz (3 $\sigma$). These values should be compared with the corresponding $\sim 0.45 \mu$Hz change observed in the eigenfrequencies themselves (Elsworth et al. 1990b, 1994; Libbrecht & Woodard 1991; Regulo et al. 1994; see also Evans & Roberts 1992). From an asymptotic description of the acoustic modes (e.g., Tassoul 1980), it can be shown that (for $n \gg l$) the fine spacings are proportional to the gradient of the sound speed. Since $n/l$ is quite small for some of the observed low-degree modes that make up the presented fine-structure spacings, care must be taken in interpreting these data through the use of asymptotics. This assumed, the solar cycle constraints on $d_0(n)$ and $d_1(n)$ imposed by the BiSON data—in each case an upper limit to any change of less than 1%—would appear to place similar tight constraints on any implied variation of the sound-speed gradient through the deep solar interior over the solar cycle.

We have compared the observed BiSON fine-structure spacings with those derived from three, slightly different solar models. These are referred to as models A, B, and C in Table 1. Model A is a “standard” solar model (model 1 from Christensen-Dalsgaard, Proffitt, & Thompson 1993; official designation is 14b.14), which neglects settling effects, and it was constructed with the CEFF equation of state (Christensen-Dalsgaard & Däppen 1992) and the Livermore (OPAL) opacity tables (Iglesias, Rogers, & Wilson 1992). Model B (model 4 from Christensen-Dalsgaard et al. 1993; official designation is 14b.d.20) again employs the CEFF equation of state and the OPAL opacities, in addition to helium and heavy-element settling, in addition to the OPAL equation of state (Rogers, Swensen, & Iglesias 1996).

The fine-structure data for each model were fitted, as per equation (3), over the range $15 \leq n \leq 27$. An inspection of the fitted $c_0$ coefficients listed in Table 1 clearly indicates that model A is excluded at a high level of significance. While models B and C provide a better match to the observed data, the $d_0(n)$ comparison favors model B, while that for $d_1(n)$ favors model C. The fitted slopes for the models are all fairly consistent: they lie roughly 3 and 1.5 $\sigma$ from the new BiSON values for $d_0(n)$ and $d_1(n)$, respectively.

The use of the fitted BiSON and model coefficients in assessing the relative merits of the models is, to some extent, misleading, i.e., the comparison is ultimately dependent upon the suitability of the parameterized description—here a simple straight line—used to describe the trend in the spacings. As previously noted, the plots in Figure 1 clearly show that a straight-line representation of the measured spacings is inadequate. We have therefore also employed a direct one-to-one comparison between the BiSON and model spacings, as indicated in Table 1. For each model under consideration, fine-spacing residuals were computed at each $n$, in the sense of BiSON minus model. Mean, weighted residuals were then computed for a variety of ranges in $n$; those for the range $9 \leq n \leq 28$ for $d_0(n)$ and $11 \leq n \leq 27$ for $d_1(n)$ are shown in Table 1. The uncertainties on each fine-structure datum were used to weight the computation of the mean, and its error determined from the weighted scatter of the residuals.

With reference to Table 1, the difference residuals imply
that the spacings for the “standard” model (A) are significantly larger than the BiSON values. The mean residuals for \( d_0(n) \) and \( d_1(n) \) exclude model A at the 13 \( \sigma \) and approximately 9 \( \sigma \) levels. The models incorporating settling provide a much better match to the observed spacings; however, marginally significant differences do remain. The comparative \( d_0(n) \) statistics imply that model B is in slightly better agreement with the BiSON spacings than model C. Figure 1 appears to show that model B (dot-dashed line) provides a better match to the BiSON \( d_0(n) \) data for \( n > 17 \), while at lower \( n \) models B and C are comparable. (The weighted difference residuals for \( 9 \leq l \leq 17 \) are significant at the \( \sim 3 \) and \( \sim 4 \) \( \sigma \) levels for models B and C, respectively, while for \( 18 \leq l \leq 28 \) they are both significant at the \( \sim 0.1 \) and \( \sim 5 \) \( \sigma \) levels, respectively.) For \( d_1(n) \), the difference residuals for models B and C are similar; model C is perhaps more in accordance with the observed values.

As an aside, we note that the inclusion of turbulence in model B has little effect on the spacings. The introduction of turbulence in the model changes the sound-speed profile, between approximately 0.5 \( \leq r/R_\odot \leq 0.7 \), relative to a model that neglects its effects. However, the sound-speed profile—and the hydrogen and helium abundances—are very similar in the deep radiative interior. Christensen-Dalsgaard et al. (1993) also constructed a settling model that neglected turbulent effects: this model produces \( c_0, c_s \), and mean residual values in good agreement with model B.

The plots in Figure 1 and the comparative residual statistics clearly show that model A is excluded—at a very high level of significance—by the BiSON data. While neither of the “settling” models considered provides a satisfactory match over the whole range in \( n \) for which the observed spacings are available, they do nevertheless match the BiSON data far better. The inclusion in solar models of nonstandard settling effects in the core will tend to raise the opacity of the deep radiative interior and increase the solar neutrino flux, albeit rather modestly (Christensen-Dalsgaard 1996). The results in this paper would therefore appear to reinforce the conclusions of Elsworth et al. (1990a), in arguing against an astrophysical solution to the solar neutrino problem.

Christensen-Dalsgaard (1996) does, however, point out that certain models can be constructed (e.g., Antia & Chitre 1995) that match observed solar neutrino capture rates and the helioseismological data, but at the cost of employing what appear to be unrealistic assumptions regarding the physics of the solar interior, for example, models that arbitrarily “juggle” the comparative contributions of a reduced core opacity (which will tend to reduce the fine-structure spacings) and mixing in the core (which will tend to increase them). At present there appears to be little justification for constructing such models. We add an additional note of caution by indicating that the characteristics of the \( p \) modes in themselves—which depend upon the mechanical properties of the solar interior—cannot uniquely define the internal solar temperature and, by implication, the expected neutrino flux. Nevertheless, helioseismological data, such as those presented in this paper, reinforce our previous conclusions, i.e., an astrophysical solution to the solar neutrino problem seems to be excluded. If the various nuclear cross sections are correct, then electron neutrinos would therefore seem to disappear between the solar core and terrestrial detectors. Possible causes are oscillations between various species or the precession of a magnetic moment, both of which require neutrinos to have mass (e.g., see Bahcall 1988).

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