On the amount of peculiar velocity field information in supernovae from LSST and beyond

Karolina Garcia,¹ ² Miguel Quartin¹ ³ and Beatriz B. Siffert⁴
¹Observatório do Valongo, Universidade Federal do Rio de Janeiro, 20080-090, Rio de Janeiro, RJ, Brazil
²Department of Astronomy, University of Florida, 32611, Gainesville, FL, USA
³Instituto de Física, Universidade Federal do Rio de Janeiro, 21941-972, Rio de Janeiro, RJ, Brazil
⁴Campus Duque de Caxias, Universidade Federal do Rio de Janeiro, 25265-970, Duque de Caxias, RJ, Brazil

ABSTRACT
Peculiar velocities introduce correlations between supernova magnitudes, which implies that the supernova Hubble diagram residual contains valuable information on both the present matter power spectrum and its growth rate. In this paper, by a combination of brute-force exact computations of likelihoods and Fisher matrix analysis, we parameterize how this estimation depends on different survey parameters such as its covered area, depth, and duration. This allows one to understand how survey strategies impact these measurements. For instance, we show that although this information is peaked in the range \( z \in [0, 0.15] \), there is also plenty of information in \( z \in [0.15, 0.4] \), and for very high supernova number densities there is even more information in the latter range. We show that LSST could measure \( \sigma_8 \) with a precision of \( 13\% \) (7.6%) with 5 (10) years of observations. This precision could increase further if low-redshift supernova completeness is improved. We also forecast results considering the extra parameter \( \gamma \), and show that this creates a non-linear degeneracy with \( \sigma_8 \) that makes the Fisher matrix analysis unsuitable. Finally, we discuss the possibility of achieving competitive results with the current Zwicky Transient Facility.

Key words: supernovae – large-scale structure – peculiar velocity – LSST

1 INTRODUCTION

In the late 90’s, Type Ia supernovae (SNe) confirmed the presence of dark energy, which opposes the attractive force of gravity and accelerates the Universe’s rate of expansion (Riess et al. 1998; Perlmutter et al. 1999). More than two decades later, SNe remain the only established high-redshift standard candles. Because of their high luminosity and low-scatter after light curve standardization (Hamuy et al. 1996), they help determine the properties of the dark energy component and constrain cosmological parameters.

Many supernova (SN) surveys – including the Dark Energy Survey (DES, Abbott et al. 2016), the Large Synoptic Survey Telescope (LSST, Abell et al. 2009), and the Zwicky Transient Facility (ZTF, Bellm 2014) – are being conducted or planned for the next decade, which will increase the number of observed explosions from the current \( \sim 10^3 \) (Betoule et al. 2014; Scolnic et al. 2018) to over \( \sim 10^6 \) (Abell et al. 2009), allowing for new, unprecedented tests on the \( \Lambda \)CDM model. However, systematic errors in cosmological parameters measurements with SNe are already of the same order of magnitude as the statistical ones (Davis et al. 2011). This means that in order to fully exploit the immense future dataset, we will have to make important improvements on our understanding of SNe. On the other hand, this huge increase in data numbers allows for brand new tests using new observables, which are subject to a different systematics. Thus, even if our understanding of the cosmological expansion becomes severely limited by systematics, we may still be capable to use SNe to learn about cosmological perturbation quantities.

One such new observable is SN lensing. This can be achieved by cross-correlating SNe and galaxy surveys, testing whether the SNe brightness fluctuates as expected with the matter density along the line-of-sight (Smith et al. 2014; Scovacricchi et al. 2017). Even though these cross-correlation studies will be very important in the next years as we keep covering the sky with different deep surveys, it is likewise interesting to have independent constraints from each cosmological observable. This allows one to check for the consistency of methods and look for hidden systematics using methods such as the External (March et al. 2011) and Internal Robustness (Amendola et al. 2013) tests or the Surprise concordance test (Seehars et al. 2014). With this in mind, the Method of the Moments (MeMo) was proposed in Quartin et al. (2014) and further discussed in Macaulay et al. (2017), which allows the measurement of quantities like \( \sigma_8 \) and the growth-rate index \( \gamma \) (see below for definitions)
by studying the higher moments (to wit: variance, skewness and kurtosis) of the residual Hubble diagram. The MeMo was applied to current data by Castro & Quartin (2014), yielding the measurement $\sigma_8 = 0.84^{+0.28}_{-0.65}$, using nothing except the SN magnitudes. It was also used by Castro et al. (2016) to put constraints in the Halo Mass Function. With future surveys, the precision should improve greatly due to increased statistics, as discussed by Quartin et al. (2014); Scovacricchi et al. (2017).

SN peculiar velocities (PVs) represent another new observable. They introduce measurable correlations in the SN magnitudes, an effect discussed in detail by Hui & Greene (2006) and Davis et al. (2011). Gordon et al. (2007) in particular discussed a method to extract this information and made preliminary forecasts. We summarize here the main idea. The SN PVs are traditionally just modeled as Gaussian random terms in SN studies (see e.g. Betoule et al. 2014). However, SN PVs are not actually random: they follow the large-scale gravitational potential wells. Any two SNs separated by up to $\sim 100$ Mpc (see Hoffman et al. 2015) should have significantly correlated magnitude fluctuations. In other words, if a given SN has below-average brightness because it is moving away from us, another SN close to it should have significantly correlated magnitude fluctuations. Because it will be in the same velocity flow (Hui & Greene 2006).

This effect can be expressed as a perturbation in the luminosity distance ($d_L$) given by

$$\frac{\delta d_L}{d_L} = \hat{x} \cdot \left( v - \frac{1 + z_i^2}{H(z)d_L} \left[ v - v_0 \right] \right),$$

where $\hat{x}$ is the position of the SN at redshift $z$, $H(z)$ is the Hubble function, and $v_0$ and $v$ are the PVs of the observer and SN respectively. The CMB dipole is usually taken as a direct and clean measurement of $v_0$.

This way, a SN survey can estimate the projected peculiar velocity (PV) field.

Using linear theory and considering that the velocity correlation function must be rotationally invariant, the velocity correlation function between objects located in positions $\mathbf{r}_i$ and $\mathbf{r}_j$ is expressed as (Castro et al. 2016):

$$\xi_{\parallel, \perp} = G'(z_i)G'(z_j) \int_0^\infty \frac{dk}{2\pi^2} \frac{P(k)}{3} K_{\|, \perp}(k|\mathbf{r}_i - \mathbf{r}_j),$$

where $G'$ is the derivative of the growth function with respect to $\ln a$, the symbols $\|, \perp$ denote the component parallel or perpendicular to $\mathbf{r}_i - \mathbf{r}_j$, $K_{\|, \perp}$ are combinations of the first two spherical Bessel functions, and $P(k)$ the matter power spectrum. The peculiar-motion covariance matrix is then given by

$$C_v(i, j) = \left[ 1 - \frac{(1 + z_i)^2}{H(z_i)d_L(z_i)} \right] \left[ 1 - \frac{(1 + z_j)^2}{H(z_j)d_L(z_j)} \right] \xi(x_i, x_j).$$

Since the amplitude of the correlations between SN PVs is directly related to the 2-point correlation function of matter, it is also proportional to the amplitude of the matter power spectrum, from which we can derive $\sigma_R$, the standard deviation of density perturbations on spheres of radius $R$:

$$\sigma_R \equiv \sqrt{\int dk \frac{k^2}{2\pi^2} \frac{P(k)}{3} [\sin(kR) - kR\cos(kR)]^2}.$$  \hspace{1em} (4)

It is common to use $R = 8$ Mpc/h. This defines the quantity $\sigma_z$, which will be the focus of our forecasts in this work.

If we extend the analysis for beyond the ACDM model, we can account for a different growth history through an extra parameter $\gamma$, the growth-rate index, which parametrizes the (linear) growth-rate $f$ as (Lahav et al. 1991):

$$f(z) = -\frac{d\ln G(z)}{d\ln(1+z)} \simeq \Omega_m(z)\gamma,$$

where the matter density parameter at redshift $z$ is

$$\Omega_m(z) = \Omega_m(1+z)^3 \frac{H_0^2}{H^2(z)}.$$  \hspace{1em} (5)

From $f(z)$, we can directly compute the growth function

$$G(z) = \exp \left[ -\int_0^z \frac{dz'}{1+z'} f(z') \right].$$  \hspace{1em} (7)

Since $\gamma$ does not depend strongly on the equation of state parameter of dark energy, it is often employed as a simple way of describing the growth rate in modified gravity models. Within General Relativity (GR) and for the ACDM model, $\gamma = \gamma_{\Lambda\text{CDM}} \approx 0.55$. Using this value, Planck CMB spectrum puts tight constraints in $\sigma_8$ (Aghanim et al. 2018). But when $\gamma$ is left free, the CMB constraints exhibit a large degenerescence between both parameters, as shown by Mantz et al. (2015).

Castro et al. (2016) showed that PV and gravitational lensing effects in SNe provide complementary constraints on $\sigma_8$ and $\gamma$. Thus, employing both methods to extract this extra information from SN data could help complement CMB constraints. This combination of both methods was also investigated by Macaulay et al. (2017). Both observables are nevertheless independent, and on this paper we focus exclusively on how much information future surveys can extract from the PV field using SN data alone.

The challenging aspect of SN PV studies is that PVs of $\sim 300$ km/s are typically much smaller than the Hubble expansion velocity; the two are similar in value only at the very lowest redshifts: $z \sim 0.001$. That is why PV studies so far have focused on low-redshift sources. However, the lower the redshift limit considered, the smaller is the volume sampled; finding out up to what redshift the PVs can be measured is one of the aims of this project. Moreover, it is not immediately clear whether for a given survey duration it is better to cover more area or to go deeper if one is interested in measuring these PV effects.

It is important to stress that the most common method to obtain information from clustering of galaxies, the Redshift Space Distortions (RSD, Kaiser 1987), suffers from the confounding factor due to the galaxy bias, i.e., the statistical relation between the distribution of galaxies and total matter. The degeneracy between the bias (specially if it turns out to be both redshift and scale-dependent) and galaxy power spectrum measurements is one of the main difficulties in probing the growth of structures. Direct PV measurements such as the ones in SN PVs, on the other hand, provide measurements on linear perturbation parameters that does not depend on the galaxy bias (Zheng et al. 2015). To wit,
following Burkey & Taylor (2004); Howlett et al. (2017), we can write the density-density, density-velocity and velocity-velocity power spectra as

\begin{align}
P_{\delta\delta}(k, \mu, z) &= (1 + \beta_\mu^2) \delta^2 D_\mu^2 G^2 P_{\mm}(k), \\
\Sigma_{\delta\nu}(k, \mu, z) &= \left( \frac{H_\mu}{k(1+z)} \right)^2 D_\mu^2 f^2 G^2 P_{\mm}(k), \\
\Sigma_{\nu\nu}(k, \mu, z) &= \left( \frac{H_\mu}{k(1+z)} \right)^2 D_\mu^2 f^2 G^2 P_{\mm}(k),
\end{align}

where \( v \) is the radial velocity \( v \cdot \hat{z} \), \( b \) is the galaxy bias, \( \beta \equiv f/b \), \( \mu \equiv k \cdot \hat{z} \), \( D_\nu \) and \( D_\mu \) are damping terms due to the non-linear RSD (which we will ignore throughout this work for simplicity), and \( P_{\mm} \) is the matter power spectrum at \( z = 0 \).

Clearly, measuring all three spectra above with the same tracer (SNe) allows us to measure independently both the cosmological and bias contributions. This was explored by Howlett et al. (2017), who simulated SNe from LSST to make predictions on its power to measure the growth of structure. They focused on measurements of \( f(z) \sigma_\delta(z) \) using a FM analysis and concluded that information could be gained up to a moderately high \( z \) of 0.5, ending up with very competitive results.

In this paper, we investigate in detail how the duration, depth, and area covered by SN surveys influence on the PV signals, focusing in particular on the estimation of \( \sigma_\delta \) and \( \gamma \). We first use a set of ideal SN catalogs (considering all exploded SNe in a given volume), and then make simulations based on the LSST survey to test our predictions and analyze how well they will perform on measuring these parameters. For our LSST survey forecasts, we strive to employ more realistic assumptions than Howlett et al. (2017). Moreover, we explore the applicability of Fisher matrices for this particular problem. To this end, we computed our likelihoods using a brute-force grid analysis and all our covariances in configuration space in the range \( z < 0.25 \), which considers all possible pairs of SNe. We then tested how good the FM approximation turns out to be, and used it to subsequently extend our forecasts to higher redshift values, where brute-force computation becomes impractical due to the high number of SNe.

Throughout this paper, we assume the following fiducial cosmological model: a CDM universe with \( \Omega_{\text{m}} h^2 = 0.3 \), \( h_0 = 100h \text{ km/s/Mpc} \) with \( h_{\text{fid}} = 0.7 \) and \( \sigma_8, f_{\text{fid}} = 0.83 \). Since CDM assumes GR, we also assume that the SN will have a total scatter in the Hubble diagram given by the quadrature sum of an intrinsic scatter \( \sigma_{\text{int}} = 0.13 \text{ mag} \) (which corresponds to a relative distance error of 6%) and a non-linear PV scatter corresponding to 150 km/s. All the other parameters were kept at values in line with current data (see e.g. Bennett et al. 2014; Aghanim et al. 2018; Iocco et al. 2009): \( \Omega_\Lambda = 0.046 \), \( n_s = 0.96 \), and \( \tau = 0.089 \). In any case, their effect on the PV observable is weak, as discussed by Castro et al. (2016). We also adopt the following broad uniform priors: \( 0 \leq \sigma_\delta \leq 2 \) and \(-1 \leq \gamma \leq 2.5 \).

This paper is organized as follows: in Section 2, we present the theory behind the estimation of \( \sigma_\delta \) and \( \gamma \) based on the Fisher matrix. In Section 3, we discuss how different observational parameters affect the observations of PVs, focusing on the effects of the maximum redshift, total area, and survey duration. In Section 4, we present forecasts for the precision with which we can estimate \( \sigma_\delta \) and \( \gamma \) from PV studies for LSST; we also briefly discuss the capabilities of ZTF. Finally in Section 5 we discuss our results.

Four appendices provide further details: Appendix A explains the construction of the ideal catalogs; Appendix B describes a technique applied to estimate standard deviations of the likelihood curves; Appendix C discusses details on the simulated LSST catalog; and Appendix D likewise for our DES catalog.

## 2 Fisher matrix applied to \( \sigma_\delta \) and \( \gamma \)

The Fisher matrix (FM) measures the amount of information that an observable carries about specific parameters under the assumption that the posterior is a Gaussian function of these parameters. Tegmark et al. (1997) gives an overview on the Fisher information matrix formalism applied to cosmological parameters, and Sellentin et al. (2014) discusses its interpretation in both frequentist and Bayesian frameworks. In our case, we are interested in studying how much information the velocity power spectrum carries about \( \sigma_\delta \) and \( \gamma \). Although our main results are not based on a FM analysis (but on a brute-force estimation of these parameters for different survey strategies), computing the FM is interesting to test how good an approximation it consists in practice. This is also important because it allows one to quickly test the amount of information at intermediate and high redshifts besides the ones we calculated by hand, and the dependence in other parameters that were not explored using brute force. We predict to observe a huge amount of SN in the next decade, and a brute force forecast with more than \( 10^8 \) objects is very computationally expensive. Finally, the FM allows us to comment on the results of Howlett et al. (2017) which were entirely based on this approximation.

The FM is defined as:

\[
F_{\ell m} \equiv - \frac{\partial^2 \ln P}{\partial \ell_1 \partial \ell_m},
\]

where the posterior \( P \) depends on a vector of \( p_i \) and \( p_m \), which represent hypothetical cosmological parameters to be estimated. The inverse of the FM \( (F^{-1}) \) is the covariance matrix of the model parameters, and the uncertainty \( \sigma_i \) in a parameter \( p_i \) marginalized over all others is simply given by \( (F^{-1})_{ii}^{1/2} \). For a 1-dimensional case (which is one of the cases considered here), \( (F^{-1})_{ii} \) is \( (F_{ii}^{-1}) \), and the uncertainty \( \sigma \) in that parameter is simply \( F_{ii}^{-1/2} \).

In the case of an experiment that measures the density power spectrum at a given redshift bin, the integral form of the FM was derived by Seo & Eisenstein (2003) based on the work of Tegmark (1997):

\[
F_{\ell m} = \frac{1}{8\pi^2} \int_{-1}^{+1} d\mu \int_{k_{\min}}^{k_{\max}} k^2 dk \left( \frac{\partial \ln P(k, \mu, z)}{\partial \mu} \frac{\partial \ln P(k, \mu, z)}{\partial \mu} \right) V_{\text{survey}},
\]

where \( P(k, \mu, z) \) is the density power spectrum \( P_{\delta\delta} \), \( n_{\text{SN}} \) is the number density of SNe in this region, and \( V_{\text{survey}} \) is the total volume observed by the survey in the given redshift bin. The full FM of a survey is given by summing the FMs.
of each redshift bin, which can be generalized to an integral of
(12) over \(z\). For the case of the velocity power spectrum the
equation is the same with \(P_{\delta \delta} \rightarrow P_{\nu \nu}\), and the shot-noise
term \(1/n_{\text{SN}} \rightarrow \sigma^2_{v,\text{eff}}/n_{\text{SN}}\) (Burk & Taylor 2004; Howlett
et al. 2017), where
\[
\sigma^2_{v,\text{eff}} \equiv \left[ \frac{10 \log 10}{5} H_0 d_C \sigma_{\text{int}} \right]^2 + \sigma^2_{v,\text{nonlin}}.
\]  

Here \(d_C\) is the comoving distance. As mentioned before, we
assume \(\sigma_{v,\text{nonlin}} = 150\) km/s.

The second line of Eq. (12) is often referred to as the ef-
ficent survey volume \(V_{\text{eff}}\), which is conveniently rewritten in
terms of the matter power spectrum using (10). Expanding
the functions \(f(z)\), \(G(z)\) and \(d_C(z)\) in units of Mpc/h (for
which \(H_0 = 1/3000\)), and assuming our fiducial cosmological
model, we found that we can approximate numerically
\(V_{\text{eff}}\) to within 1% in the range \(0 \leq z \leq 1\) by
\[
V_{\text{eff}} \simeq \left[ \frac{P_{\nu \nu}(k)}{P_{\nu \nu}(k) + \frac{k^2}{\pi^2} 10^{8.7} \left[ 7.17 \left( z^2 - 3.7 z^3 + 2 z^4 \right) \sigma_{\text{int}}^2 \right]^{\frac{2}{3}} \nu_{\text{SN}}(z)} \right]^2 V_{\text{survey}},
\]  

where above and henceforth \(P_{\nu \nu}(k)\) refers to the monopole
term \(P_{\delta \delta}(k, \mu = 0)\) of any \(x x\) power spectrum. Note that the
\(\sigma_{v,\text{nonlin}}\) term makes negligible contributions for \(z > 0.05\), so
it can be dropped at higher redshifts. For low-z and other
values of \(\sigma_{v,\text{nonlin}}\), the very last term can be just generalized
to \(20 \sigma^2_{v,\text{nonlin}}/(150\text{~km/s})^2\).

In order to understand how much information on \(P_{vv}\)
can be obtained in each redshift, it is more useful to write the
differential form of the FM for a bin of width \(dz\). In this case,
\(dV_{\text{survey}} = dz \Omega dC(z) z^2/H(z)\) (where \(\Omega\) is the solid angle
representing the sky area being observed). This can itself be
approximated also to within 1% in the range \(0 \leq z \leq 1\) by
\[
dV_{\text{survey}} \simeq dz \Omega (z^2 - 0.96 z^3 + 0.33 z^4) 27 10^9.
\]  

Finally, the integral over \(\mu\) can be done analytically yielding
\[
\int_{-1}^{1} d\mu dV_{\text{eff}} = dV_{\text{survey}} \left[ 3 - \frac{1}{1 + a} - 3 \sqrt{a} \arccot(\sqrt{a}) \right],
\]  

where
\[
a \equiv k^2 \frac{10^8 [7.17 z^2 - 3.7 z^3 + 2 z^4] \sigma_{\text{int}}^2 + 8.7}{n_{\text{SN}}(z) P_{\nu \nu}(k)}.
\]  

The extra \(k^2\) term in the denominator of \(V_{\text{eff}}\) for the \(P_{vv}\)FM (as compared to the \(P_{\delta \delta}\) FM) makes it clear that most of
the PV information is on large scales. This means that even for a very dense catalog (like the one from the 10-year
LSST survey), one can just set \(k_{\text{max}} = 0.1 h/\text{Mpc} with no
loss of information. We checked this numerically and only for
a very large \(n_{\text{SN}} \gtrsim 10^{-3} (h/\text{Mpc})^3\) (which corresponds to
over 10 years of an ideal survey and around 100 of LSST)
could one gain important extra information by going beyond
\(k_{\text{max}} = 0.1 h/\text{Mpc}.

For the parameter \(\sigma_8\) in particular, the derivative is
trivial: \(\partial \ln P/\partial \sigma_8 = 2/\sigma_8,\text{fid}\). The FM thus becomes:
\[
F_{\sigma_8 \sigma_8} = \frac{1}{2 \pi^2 \sigma_8,\text{fid}} \int_{k_{\text{min}}}^{k_{\text{max}}} \int_{k_{\text{min}}}^{k_{\text{max}}} k^2 dk dV_{\text{survey}} \left[ 3 - \frac{1}{1 + a} - 3 \sqrt{a} \arccot(\sqrt{a}) \right].
\]  

For \(\gamma\) instead the derivative of the power spectrum is more
complicated, and in particular it depends on \(z\):
\[
\frac{\partial \ln P}{\partial \gamma} = 2 \left[ \ln \Omega_m(z) - \int_0^z dz' \Omega_m(z')^{\text{int}} \ln \Omega_m(z') \right].
\]  

But this can be approximated by this series for our fiducial
model to within 2% in the range \(z \leq 0.8:\)
\[
\frac{\partial \ln P}{\partial \gamma} \simeq -2.39 + 5.27 z - 4.28 z^2 + 1.53 z^3.
\]

3 STRATEGIES TO OBSERVE PV CORRELATIONS

One of our goals here is to make a comparison among survey
parameters, so we started by considering SN simulations
based on idealized mock surveys. We thus assumed that all
SNe which explode in a certain volume are included in the
final catalog (i.e. a completeness of unity). We then selected
the area and depth of the survey and used a SN rate given by
\(2.6 \times 10^{-8} (1 + z)^{1.5}\) SN yr\(^{-1}\) Mpc\(^{-3}\) (Dilday et al. 2008;
Rodney et al. 2014; Cappellaro et al. 2015) to create a mock
Hubble diagram. For the LSST survey, we used instead the
full LSST collaboration SNANA .SIMLIB file, which contains
the observational strategy in all details, as described in
Section 4.

In order to add the PV effects and compute the full co-
variance among the SNe, we started by employing the pairV
code developed by Hui & Greene (2006). This code takes as
input a catalog of sources’ angular positions and redshifts
(which we generated for the mock surveys and for LSST)
and returns the full linear-order PV covariance matrix. We
added to this matrix a diagonal covariance matrix containing
the intrinsic dispersion of \(\sigma_{\text{int}} = 0.13\) mag, and a non-
linear velocity scatter \(\sigma_{v,\text{nonlin}}\) corresponding to 150 km/s,
which is in agreement with current SN data (Castro et al.
2016). From the resulting total covariance, we created mocks
by drawing random distance modulus realizations from the
Corresponding multi-normal distribution and adding them
to the fiducial SN distance moduli.

For the idealized surveys, we first simulated the mother
catalogs: 40 versions of 6-year catalogs, covering an area of
600 deg\(^2\), and reaching a maximum redshift of 0.25. This
resulted in 11,285 SNe. We later divided these catalogs into
children catalogs with different field areas, survey durations
and maximum redshifts in order to see how the uncertainty
on the measurement of \(\sigma_8\) scales with those observational
parameters. In Appendix A we provide details on the con-
struction of these catalogs. We constrained the value of \(\sigma_8\)
for each of these catalogs using the likelihood function (see
details Castro et al. 2016):
\[
L_{PV} \propto \frac{1}{\sqrt{|C_{PV}|}} \exp \left[ -\frac{1}{2} \delta_m^T (C_{PV})^{-1} \delta_m \right],
\]  

where \(\delta_m \equiv DM - DM_{\text{fid}},\text{ and } DM\) is the distance modulus.
The matter power spectrum was evaluated numerically using CAMB (Lewis et al. 2010) for our fiducial cosmology (discussed in Section 1). The likelihoods themselves were computed using a simple 1-dimensional parameter space sampled by a grid. Although we would ideally like to leave all parameters free, the large number of SNe here considered make this likelihood evaluation very slow (and memory consuming). Thus, employing full Markov chains Monte Carlo (MCMCs – as in Castro et al. (2016)) or multi-dimensional grids is completely unfeasible unless in a large computer cluster. Nevertheless, since our objective here is to make relative comparisons between survey strategies, there is no need to let all parameters free. So we fixed all our parameters in the fiducial values and varied only \( \sigma_8 \) (we will add also \( \gamma \) as a second variable in the next section).

Since a precise extraction of the PV signal requires a very large number of SNe, several of our likelihoods were broad enough that \( \sigma_8 = 0 \) was still allowed by the mock data. However, as \( \sigma_8 < 0 \) is non-physical, it is ruled out by our prior. This meant that the forecast error bars were sensitive to our prior, and not only to the data, which could bias the comparison between smaller samples (larger uncertainty and higher probability of having part of the curve below zero) and larger samples (smaller uncertainty and lower probability of having a truncation in zero). Here we are interested in the amount of information in the data only (and in any case this issue would be suppressed with more data), but since our brute-force configuration-space likelihood is computationally very expensive we chose not to use larger mock catalogs. Instead, we employed a simple Gaussian continuation technique (see Appendix B) which removes the prior sensitivity. After applying this technique, we computed \( \sigma_{\text{mean}}(\sigma_8) \) as the mean value of the uncertainty in \( \sigma_8 \) for the 40 versions of each children catalog.

The effect of survey area is the simplest one to understand, as \( F_{\text{conf}} \propto \Omega \). Because we are working with one-parameter likelihoods, this means that \( \sigma = 1/\sqrt{F_{\text{conf}}} \propto \Omega^{1/2} \). We tested numerically in our full (non-FM) likelihood that this expectation holds in our results: in average among the 40 versions the uncertainty indeed scaled as \( \Omega^{-1/2} \). The effect of maximum redshift is less straightforward, since there are two competing terms: the PV effect itself, which becomes relatively smaller at higher redshifts, and the volume, which increases rapidly. Hui & Greene (2006) and Gordon et al. (2007) considered that the correlations between SN PVs contribute significantly to the overall error budget only up to \( z \lesssim 0.1 \). Howlett et al. (2017) on the other hand considered redshifts up to 0.5.

We present our results of the redshift dependency in Figure 1, where we depict the uncertainty of \( \sigma_8 \) by taking the mean value on our 40 simulations. On the left panel, we show the behavior for each redshift bin centered around \( z_{\text{bin}} \) with width \( \Delta z = 0.05 \) for different survey durations. The right panel shows likewise the integrated \( \sigma_{\text{mean}}(\sigma_8) \) up to a maximum redshift \( z_{\text{max}} \). In both panels, the dashed lines represent the FM approximation of Eq. (18). One can see in those figures that the total information on \( \sigma_8 \) is peaked around \( z \sim 0.1 \) and diminishes slowly at higher redshifts. Thus, there is a good amount of information in the whole range \( 0 \leq z \leq 0.25 \). Similarly, we present in Figure 2 the dependency on survey duration for varying \( z_{\text{max}} \).

These figures indicate that the FM can be a reasonable approximation to the full likelihood, yielding forecasts which approximate within 25% of the exact ones. One can thus use the FM to extend these forecasts to higher redshifts, where
the very large number of SNe quickly makes the exact full likelihood calculation too computationally intensive.

In order to understand how the survey duration affects the final performance of the PV analysis, one should inspect Eq. (14), which gives the effective volume $V_{\text{eff}}$ in the FM formula. Similarly to what happens for measurements of $P_{\delta\delta}$, measurements of $P_{vv}$ have two asymptotic regimes, which are the limiting cases for $n_{\text{SN}}P_{vv}(k)$. When $n_{\text{SN}}P_{vv}(k) \gg \sigma_{v,v,\text{eff}}^2$, it means that the sampling is good enough to derive all the cosmological information that can be extracted from the survey; in other words, detecting more SNe will not bring any advantage. This is referred to as the cosmic variance limited regime. On the other hand, when $n_{\text{SN}}P_{v,v}(k,\mu) \ll \sigma_{v,v,\text{eff}}^2$, the effective volume is severely reduced, meaning that even a small amount of SN added can bring a lot of information. In particular, we see in this case that $F_{\text{SN}} \propto V_{\text{eff}} \propto n_{\text{SN}}^2$. And since $F_{\text{SN}} \propto 1/\sigma^2$, in this limit $\sigma \propto 1/n_{\text{SN}}$. This is dubbed the shot noise limited regime.

The same analysis extends directly to the survey duration as the number of SNe detected is directly proportional to the time spent revisiting a fixed observational area. This means in principle that if the survey duration is short in a given area, one gains much more information on the power spectra with SNe by extending the observation time in that area ($\sigma \propto t^{-1}$) than by observing more area ($\sigma \propto \Omega^{-1/2} \propto t^{-1/2}$). For $P_{vv}$ however this happens only for very short durations, as we now discuss.

Our differential FM approximation (16) is the key to explore further how the information scales with $n_{\text{SN}}$ and $z$ at higher redshifts. Figure 3 illustrates the FM predictions for different redshift bins with $\Delta z = 0.1$ as a function of $n_{\text{SN}}$ for a very large range of $n_{\text{SN}}$. We also depict the expected values of $\sigma_{v,v}$ for the ideal survey with 1 to 5 years of duration, as well as for the 5-year LSST survey (see Section 4 for more details on the LSST numbers). These predictions show that, for very high $n_{\text{SN}}$, the amount of information on the higher $z$ bins become relatively larger, but for lower densities most of the information is in the region $z \lesssim 0.3$. Figure 4 shows the same quantities for the case of $P_{\delta\delta}$. For $\gamma$, it is clear that the information is more concentrated on the lower redshift bins. However, as we will discuss in Section 4, $\sigma_{\delta\delta}$ and $\gamma$ are highly correlated in a non-linear fashion, and thus the FM forecasts with both variables free become less reliable. All the other cosmological parameters (including the nuisance ones) are either not considerably degenerate with $\sigma_{\delta\delta}$ and $\gamma$, or they are going to be very well estimated by standard SN distance measurements, as is the case of $\Omega_{m,0}$. Therefore, it is reasonable to fix those parameters at their best fit. One also has motivations for fixing $\gamma$ (it is fixed in GR) to analyze $\sigma_{\delta\delta}$, as we did in figures 1 and 2, but it is a bit unnatural to fix $\sigma_{\delta\delta}$ in order to study $\gamma$.

Figure 5 combines all the information on $F_{\sigma_v\delta}$ in the range $z \lesssim 0.25$ to illustrate the asymptotic regimes of $V_{\text{eff}}$. The inclined dashed lines represent power laws of the form $\sigma \propto (n_{\text{SN}})^{-1}$ (the shot-noise dominated regime) and $\sigma \propto (n_{\text{SN}})^{-1/2}$ (the transition between regimes), that serve as reference for the rate of gained information as a function of $n_{\text{SN}}$. The thin vertical lines represent the average number density of SNe in this redshift range for both LSST and ideal surveys with different durations. The conclusion from this figure is that (contrary to what happens with $P_{\delta\delta}$) for PVs the transition from the shot-noise dominated regime to the saturated ($\sigma \propto (n_{\text{SN}})^{1/2}$) regime is much more gradual. Therefore, a survey like LSST remains for the most part in the $\sigma \propto (n_{\text{SN}})^{-1/2}$ regime, for which increasing either the observational area or duration yield approximately the same gain in information. This also means that, if LSST could observe for a longer time, it would keep getting more PV information, and saturation would only start to kick in after around 100 years.

## 4 FORECAST FOR FUTURE SURVEYS

### 4.1 LSST

Currently, most of the available data on SNe come from the Sloan Digital Sky Survey (SDSS; Sako et al. (2018)), the Supernova Legacy Survey (SNLS; Conley et al. (2011)), and the Pan-STARRS1 Survey (Rest et al. 2014). Combined, those surveys make up to more than 80% of the Pantheon sample (Scolnic et al. 2018), which contains a total of 1,048 spectroscopically confirmed events. This scenario is about to change drastically in the next years with the upcoming results from current and future surveys, such as LSST and

| zbin | $n_{\text{SN}}$ (h/Mpc)$^3$ | relative $\sigma(\gamma)$ (arb. units) |
|------|-----------------|----------------------------------|
| 0.05 | $10^{-5}$ | 0.05 |
| 0.10 | $10^{-4}$ | 0.10 |
| 0.25 | $10^{-3}$ | 0.25 |
| 0.50 | 0.001 | 0.50 |
| 1.00 | 0.010 | 1.00 |
| 5.00 | 0.05 | 5.00 |

Figure 3. Uncertainty scaling (in arbitrary units) as a function of the number density $n_{\text{SN}}$ of observed SNe. Each curve represents a given redshift bin with $\Delta z = 0.1$. The black dots represent the corresponding number densities for an ideal survey of 1 or 5 years. The purple dots likewise for the LSST 5 year survey.

Figure 4. Same as Figure 3 for the variable $\gamma$. Note that for $\gamma$ the constraining power is more concentrated on the first redshift bins compared to the case of $\sigma_{\delta\delta}$, even at high $n_{\text{SN}}$.
ZTF. In this section, we present forecasts on $\sigma_8$ and $\gamma$ for LSST, and discuss how the current survey of the ZTF could perform. The DES observational area and redshift range makes it uncompetitive in measuring $P_{\nu\nu}$, but for completeness we also computed a forecast for it in Table 1 – see Appendix D for details on the survey itself.

LSST will look for transients on all its observation area of 18,000 deg$^2$, with redshifts up to 1.2 and exposure times of 30 seconds. We simulated SNe as observed by LSST in 5 years using the SuperNova ANAlysis (SNANA) package (Kessler et al. 2009). SNANA simulates light curves, coordinates and redshifts according to the characteristics of the survey, and assuming a redshift dependent SN explosion rate as described in (Dilday et al. 2008). SNANA contains specific files with the observing characteristics of LSST, and we used them to simulate SN light curves as observed by this survey during 5 and 10 years. For LSST, the quality cuts applied (Abell et al. 2009) are the following:

- at least 7 epochs of observation between −20 and +60 rest-frame days, counting from the B-band peak;
- at least one epoch before −5 rest-frame days;
- at least one epoch after +30 rest-frame days;
- largest gap between two subsequent observations of 15 rest-frame days, near the B-band peak (−5 to +30 rest-frame days);
- at least two observations in different filters with signal-to-noise ratio above 15.

All the above observations must satisfy $3000 \, \AA < \lambda_{\text{filter}}/(1+z) < 9000 \, \AA$. After applying these quality cuts, we ended up with $\sim 19,500$ (± 39,000) events, for 5 (10) years.

Figure 6 illustrates the completeness curves for LSST before and after the quality cuts were applied. We estimated the LSST maximum completeness (with no cuts) using a limiting magnitude of 24.5 mag for the r broad-band filter (Abell et al. 2009) and assuming an absolute magnitude of $−19.25 ± 0.50$ mag for the SNe. Note that less than ~15% of LSST SNe survive these cuts in the range $z \leq 0.5$, and even less beyond this range. These results were based on a 5-year survey. For a 10-year survey, the completeness is ~1.2 times higher. We discuss the LSST strategy in more detail in Appendix C: this difference between 5 and 10 years of survey is due to an increase in the general amount of SNe to be observed by LSST along the time, by basing our simulations on the LSST input files in SNANA.

As for the idealized case in Section 3, we computed the LSST uncertainty in $\sigma_8$ for different redshift bins (with $\gamma$ fixed) up to $z = 0.25$, as illustrated in Figure 7, where we also plotted the FM reaching up to $z = 0.55$. Note that in this case the FM forecasts are lower than the brute-force numbers by as much as 70% for $z = 0.225$. The FM curve shows that the uncertainty on $\sigma_8$ is roughly constant from
In the case of LSST results, despite the fact that the FM is derived from the FM. In the case of the ideal catalogs, besides \( \sigma_8 \) and \( \gamma \), we made a comparison with howwell et al. (2017). Moreover, the non-linearity of this degeneracy also makes the FM a very crude final approximation in this case, which we also illustrate in Figure 8.

The final numbers for LSST, as well as for a couple of ideal surveys, can be seen in Table 1. The uncertainties – \( \sigma(\sigma_8) \) and area(\( \sigma_8, \gamma \)) – for DES 5-yr, LSST 5-yr and Ideal 5-yr with \( z \leq 0.25 \) were calculated directly from our brute-force computations. All the other uncertainty values were derived from the FM. In the case of the ideal catalogs, because the FM formalism fits well the data (see figures 1 and 2), the derivation for higher redshifts was straightforward. In the case of LSST results, despite the fact that the FM for \( \sigma_8 \) (with \( \gamma \) fixed to its fiducial, GR value) is sometimes off by up to 70% and that it breaks down even worse in the \( \sigma_8 \times \gamma \) plane, it can still be used to infer relative differences. Moreover, the full likelihood method in configuration space becomes computationally prohibitive for \( z > 0.25 \), so unless some reliable approximations are found we cannot currently compute the full forecast for \( 0.25 < z < 0.5 \) (the FM tells us that the PV information is negligible for \( z > 0.5 \)). We therefore quote our forecasts for both \( 0.25 < z < 0.5 \) and 10 years LSST duration by using the FM forecasts scaled up by a constant so as to match the brute-force results for \( z_{\text{max}} = 0.25 \). For the case of variable \( \gamma \), we quote the total area of the 1σ ellipse of the FM in the \( \sigma_8 \times \gamma \) plane also scaled up in order to agree with the brute-force results at \( z_{\text{max}} = 0.25 \). We leave an extended brute-force analysis for higher redshifts for future work.

In the same table, we also illustrate how much better would an ideal survey (with unity completeness) be. We discuss two cases: one for 10000 deg\(^2\) and one which would cover the entire sky (galaxy plane included). The latter is faced with obvious practical difficulties, but it is interesting nevertheless as it puts an upper limit and allows one to see how close one is from it. Although these results were computed by fixing all other parameters besides \( \sigma_8 \) and \( \gamma \), we made a comparison with

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**Figure 8.** 1, 2 and 3σ confidence-level contours for \( \sigma_8 \times \gamma \) for the 5-year LSST survey using \( z_{\text{max}} = 0.25 \) for two different random realizations. The yellow dot denotes the fiducial parameter values. The green contours are the LSST forecasts with the full, configuration-space likelihood. The orange contours are for the corresponding (almost degenerate) Fisher matrix. The dashed white contours are for the FM with \( z_{\text{max}} = 0.5 \), after which all PV information has been extracted.

| Survey       | \( \Omega(\text{deg}^2) \) | \#SNe | \( \sigma(\sigma_8) \) | 1σ area(\( \sigma_8, \gamma \)) |
|--------------|-----------------------------|-------|-------------------------|----------------------------------|
| DES 5 yr     | 27                          | 173   | \( \sim 2 \)            | \( \gg 1 \)                       |
| LSST 5 yr    | 18000                       | 32k   | 0.55                    | 0.11                             |
| LSST 10 yr   | 18000                       | 75k   | 0.70                    | 0.063                            |
| Ideal 5 yr   | 10000                       | 175k  | 0.23                    | 0.060                            |
| Ideal 5 yr   | 41250                       | 720k  | 0.055                   | 0.021                            |

**Table 1.** Forecast on the final uncertainties in either \( \sigma_8 \) (with \( \gamma \) fixed) or the 1σ area for the pair \( \{ \sigma_8, \gamma \} \). We also show the observed area and the total number of SNe detected both in the range \( z \leq 0.25 \) (upper rows) and \( z \leq 0.5 \) (bottom rows). These numbers do not account for marginalization over nuisance parameters such as the ones in SALT2, but this should result in only a \( \sim 10\% \) increase in the error bars.
the results in Castro et al. (2016) where a full MCMC was run over many parameters. Marginalization over the other parameters change little the contours on \( \sigma_b \) and \( \gamma \): the increase in the uncertainties are only \( \sim 10\% \).

Finally, it is important to note that even though the PV signal exhibits this strong non-linear degeneracy between \( \sigma_b \) and \( \gamma \), Castro et al. (2016) demonstrated that the PV degeneracy is almost orthogonal to the degeneracy in CMB and cluster data, and almost at \( 45^\circ \) with the one from galaxy data.

### 4.2 ZTF

The Zwicky Transient Facility is a time-domain survey being held at Palomar Observatory since 2017. Due to its very large field of view of \( 47 \) deg\(^2\), ZTF is able to scan more than \( 3.750 \) deg\(^2\) in one hour, to a depth of \( 20.5 \) mag for the \( r \) broad-band filter, with 30 seconds exposure time (Bellm 2018). The SNANA package does not contain information on the ZTF survey, and in any case ZTF does not observe SNe with a full filter set, so the SNe detected will need follow-up from different surveys. Nevertheless, it is interesting to estimate the completeness achievable by ZTF. Using the \( 20.5 \) mag limiting magnitude, we derived its completeness as a function of redshift for SNe.

The results can be seen in Figure 6, which also shows the expected completeness for four ZTF coadded images, corresponding to an effective exposure time of 120 seconds. Using this curve, we estimated that ZTF will be able to detect \( 76,000 \) SNe with \( z \leq 0.25 \) in 5 years if it scans \( 10,000 \) deg\(^2\). Given the high scan rate of this survey, they could in principle cover an even greater area with high cadence.

### 5 DISCUSSION

We showed in this paper how different observational parameters affect the measurement of SN PVs. By studying the FM of the velocity power spectrum \( P_{vv} \) we found that, for most reasonable futuristic expectations of the observed number of SNe, the error bars scale roughly as \( n_{\text{SN}}^{1/2} \). This means that SN PVs will typically operate right in the transition from the shot-noise dominated regime and the cosmic variance dominated one (where information saturates).

We also discussed the limitations of the FM approach by computing the full, non-Gaussian likelihood based on brute-force in configuration space, i.e. by computing the PV correlation between all possible pairs of SNe. We found out that when the growth-rate index \( \gamma \) is fixed, the FM can be employed with caveats. Its estimated errors lay between 20 and 70\%. When considering both \( \sigma_b \) and \( \gamma \) simultaneously, the FM breaks down in much worse manner, as together these parameters exhibit a strong non-linear degeneracy.

Based on the official SNANA observational strategy and all the traditional quality cuts to the simulated light-curves, we forecast that LSST will be able to measure \( \sigma_b \) with a precision of 13\% (7.6\%) with 5 (10) years of observations. We also computed forecasts when considering also \( \gamma \), but their non-linear degeneracy makes it hard to summarize this in a single meaningful number. We chose to quote these results in terms of the \( 1\sigma \) confidence-level area in the \( \sigma_b \times \gamma \) plane, and found that LSST precision would be 0.55 (0.18) after 5 (10) years.

When studying LSST, it became clear that the traditional quality cuts imposed to the data severely constrain the SNe in \( z < 0.1 \). Since we showed that this range contains a considerable amount of PV information, if LSST completeness can be improved in this range, it will be able to perform considerably better. This should be possible, as in principle the closest SNe are the easier ones to follow-up and obtain spectra. This also means that the ZTF survey is capable of making important contributions to this measurement as it is capable of discovering most SNe within \( z \leq 0.3 \).

In this work we wanted to avoid assuming any model or parametrization for the galaxy bias, which meant that we did not use the information content on the spectra \( P_{bb} \) and \( P_{bb} \). Assuming a bias model however allows one to extract more information and better constrain \( \sigma_b \) and \( \gamma \) by combining in the final likelihood all three spectra.

One often finds in the literature that PV is only important for \( z \leq 0.1 \), and that for objects further out the effect of PVs can be disregarded. The truth is that, for high density surveys, the increase in numbers compensates the diminishing signal; the amount of information on the higher \( z \) bins even surpasses the one in the lower \( z \). For LSST, there is a good amount of information in the whole range (\( z \approx 0.4 \)).

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APPENDIX A: IDEAL CATALOG

We here illustrate the idealized survey children catalogs that were used to compare observational parameters and optimize the study of SN PVs. As explained in Section 3, we divided the mother catalog in different area sizes, survey durations, and reached depth.

The variations of time were taken by randomly picking SNe from the mother catalogs. For the 1-year catalogs, we took 1/6 of the SNe; for the 2-year ones, we took 2/6, etc. Given that, we constructed 40 versions of 6 children catalogs of 1, 2, 3, 4, 5 and 6 years, covering the whole 600 deg$^2$ survey area and reaching up to $z = 0.25$. Figure A1 depicts this. For the area variations, we sampled 2 subareas by taking 300 deg$^2$ and 450 deg$^2$ from the central region of the 600 deg$^2$ catalogs, and produced 2×40 children catalogs (40 for

Figure A1. Representation of the different duration children catalogs used in our brute-force likelihoods for an ideal survey. Area and redshift are fixed while survey duration is varying.

Figure A2. Similar to Figure A1, but for fixed duration and redshift but variable observed solid angle.

Figure A3. Similar to Figure A1, but for fixed duration and area but variable redshift depth.
300 deg$^2$ + 40 for 450 deg$^2$). Figure A2 illustrates the area variations. We also made variations in the maximum redshift considered, resulting in 40 full-area, full-time children catalogs for maximum $z = \{0.05, 0.1, 0.15, 0.2, 0.25\}$, which are represented in Figure A3.

We constrained the value of $\sigma_8$ for each of the 6×40 different-time catalogs, 3×40 different-area catalogs, and 5×40 different-maximum-z ones (three of these 14 combinations represent the same catalog with maximum values of area, survey duration and redshift) using the likelihood function given in Eq. (21).

APPENDIX B: GAUSSIAN CONTINUATION

In this appendix, we present the Gaussian continuation technique that was used to obtain all results from the $\sigma_8$ likelihood analysis presented in sections 2 and 4.

Negative values of $\sigma_8$ can appear in likelihoods curves obtained from samples with a low number of SNe (such as the children catalogs with low-$z$, small area and/or small duration), as an statistical fluctuation (see the blue curves in Figure B1). Although this cut at $\sigma_8 = 0$ is physically motivated as $\sigma_8$ should not be negative, the direct analysis of these truncated curves yield artificially low values for the uncertainty of $\sigma_8$ due to the prior. We are interested here, however, only on the information on the data.

In order to avoid this dependence on the prior, we chose to evaluate standard deviations from Gaussian curves fitted to the real likelihood curves (see yellow curves in Figure B1). Gaussianity can be assumed in those cases since the FM analysis adopted throughout this paper (see Section 2) also relies on this assumption.

The impact of the use of the Gaussian continuation on the uncertainty of $\sigma_8$ can be seen in Figure B2, where we show a comparison between the results obtained with this approach and the ones obtained from the real likelihood curves, as a function maximum redshift and survey duration, respectively. One can see that differences between the two approaches are greater for low-redshift, low-duration surveys, while for high-redshift long surveys (where the prior becomes irrelevant) the two approaches yield the same results.

APPENDIX C: THE NUMBER OF SNE ON LSST SIMULATIONS

The software we used to simulate SNe, SNANA, uses different input files for different surveys. The two main files are the .INPUT and the .SIMLIB ones, which come along with the package for the case of large known surveys, such as DES and LSST. While .INPUT contains general details on survey specifics, such as maximum redshift, covered area and quality cuts, .SIMLIB contains a list of pointings for each filter in different epochs, along with expected observation conditions.

By simulating light curves based on LSST strategy for different durations, we noticed that the number of SNe did not grow linearly with time, as naively expected. For example, the number of SNe observed in 10 years (≃ 800,000, after cuts) is not 2 times the number of SNe observed in 5 years (≃ 300,000, after cuts), and it is not 10 times the number of SNe observed in 1 year (≃ 40,000, after cuts). This is depicted in Figure C1, where we plot the number of observed SNe for the 10 years of survey.

This cannot be accounted for solely due to the quality cuts, which introduce a border effect in observational time. It is not either due to a change in survey depth along the years, as the maximum redshift remains constant. This is shown in Figure C2. We thus analyzed the list of observations in the .SIMLIB files, and realized that the number of pointings grows with time. Although we do not know the reason for such behavior, this explains the observed growth on the SN detection rate – and thus of the survey completeness.

APPENDIX D: DES SURVEY DETAILS

DES is expected to observe a few thousands of SNe within $z < 1.2$ during its full operation time. The DES SN survey will observe an area of 27 deg$^2$, divided into 8 “shallow” fields (exposures between 175 s and 400 s, depending on the filter), and 2 “deep” ones (exposures ranging from 600 s and 3630 s), each of which will be observed 20–30 times each survey year (Kessler et al. 2015). We also used SNANA to...
Figure C1. Histogram for the number of SNe to be observed by LSST along the years of survey. Note that the general number tends to increase with time, which explains why the completeness for 10 years of survey is $\sim 20\%$ higher than the one for 5 years, as stated in Figure 6.

Figure C2. Redshift histogram for the number of SNe to be observed by LSST, for 5 years (blue) and 10 years of survey. The similarity in both distributions rules out the possibility of a variation in survey depth to be the reason for the increasing general number of SNe along the years.

simulate the DES SN catalogs. The specifications of DES were already encoded into the SNANA package, through files that contain information on the seeing, zeropoint, CCD, filter, and cadence. The SNANA package include also the quality cuts adopted by DES (Bernstein et al. 2012), which select lightcurves with:

- at least one epoch before B-band peak;
- at least one epoch after +10 rest-frame days;
- at least one observation with signal-to-noise ratio above 10;
- at least two observations in additional filters with signal-to-noise ratio above 5.

All the above observations must satisfy $3,200 \text{ Å} < \lambda_{\text{filter}}/(1+z) < 9,500 \text{ Å}$, where $\lambda_{\text{filter}}$ is the filter’s mean wavelength. After applying these quality cuts, we ended up with 173 SNe with $z \leq 0.25$. 

$\frac{\lambda_{\text{filter}}}{\lambda_{\text{filter}}/(1+z)}$