Detection of Single vs Multiple Antenna Transmission Systems Using Pilot Data

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Abstract—In this paper, we consider the problem of classifying the transmission system when it is not known a priori whether the transmission is via a single antenna or multiple antennas. The receiver is assumed to be employed with a known number of antennas. In a data frame transmitted by most multiple input multiple output (MIMO) systems, some pilot or training data is inserted for symbol timing synchronization and estimation of the channel. Our goal is to perform MIMO transmit antenna classification using this pilot data. More specifically, the problem of determining the transmission system is cast as a multiple hypothesis testing problem where the number of hypotheses is equal to the maximum number of transmit antennas. Under the assumption of receiver having the exact knowledge of pilot data used for timing synchronization and channel estimation, we consider maximum likelihood (ML) and correlation test statistics to classify the MIMO transmit system. When only probabilistic knowledge of pilot data is available at the receiver, a hybrid-maximum likelihood (HML) based test statistic is constructed using the expectation-maximization (EM) algorithm. The performance of the proposed algorithms is illustrated via simulations and comparative merits of different techniques in terms of the computational complexity and performance are discussed.

Index Terms—Blind signal recognition, MIMO, SIMO, ML estimation, GLRT detector, Correlation detector

I. INTRODUCTION

The rapidly growing interest in software defined and cognitive radios, and pervasive ubiquitous sensing has been fuelled by the development of intelligent communication systems [1]–[3]. In spite of the tremendous advances made, practical and efficient realization of these systems for envisaged applications in different domains requires that several obstacles be overcome. Signal recognition with minimal amount of prior information is one of the key requirements in most intelligent communication systems [4]–[8]. With the increasingly diverse radio frequency signal population ranging from simple narrow-band analog and digital modulations to wideband digital modulation schemes utilizing multiple transmit antennas, the problem of blind signal recognition continues to be extremely challenging.

Multiple-input-multiple-output (MIMO) systems is a physical layer technology that can provide many benefits through multiple antennas and advanced signal processing. With the availability of perfect timing and frequency information, and when the parameters related to the observation model, such as, channel state information (CSI), the number of transmit antennas and encoding schemes are available at the receiver, the problem of recovering the transmitted signal has been studied widely [9]. However, when the receiver and the transmitter are noncooperative or have limited cooperation, MIMO signal recognition is challenging [10]. More specifically, when the transmitters are employed with sophisticated communication technologies, there is a need for new blind signal recognition algorithms that are able to operate in such environments. There are recent efforts that consider how to determine the modulation schemes used by MIMO transmit antennas when the transmitter and receiver are equipped with multiple antennas [11]–[18]. However, in most of these works, it was assumed that the number of transmit antennas is known at the receiver.

In a typical MIMO system, multiple antennas at the transmitter require multiple RF chains which consist of amplifiers, analog to digital converters, mixers, etc., that are typically very expensive. An approach for reducing the cost while maintaining a high potential data rate of a MIMO system is to employ a reduced number of RF chains at the transmitter and attempt to optimally allocate each chain to one of the larger number of transmit antennas which are usually cheaper [19]. Thus, when the transmitter changes the number of transmit antennas depending on the application or the requirement, the receiver finds it is challenging to determine the transmission scheme in order to perform a given inference task.

The problem of determining the number of sources/signals in array signal processing has been addressed in early works [20]–[23]. Determining the number of transmit antennas used in a MIMO system has been addressed by several authors in [24]–[50]. In [20], the authors consider the problem of determining the number of transmit antennas utilizing objective information theoretic criteria such as Akaike’s information criterion (AIC) and minimum descriptor length (MDL). Another approach to estimate the number of transmit antennas in a MIMO orthogonal frequency division multiplexing (OFDM) system based on pilot patterns is presented in [24], [25]. In these works, the problem of determining the number of transmit antennas, $n$, is formulated as that of detecting $n$ pilot patterns in an OFDM system. In [27], [28], the authors consider the determination of the number of transmit antennas in a MIMO-OFDM system when each burst is preceded by a preamble. Their algorithm is based on the estimation of channels with different hypothetical numbers of transmit antennas. In [29], blind detection of the number of transmit antennas is studied by exploiting the time-diversity of fading channels. Classification of multiple antenna systems in the presence of possible transmission impairments exploiting cyclostationarity

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property of space-time block codes (STBCs) was considered in [30]. In all these works, it is assumed that the symbol timing synchronization at the receiver has been achieved. In [19], [31], the problem of antenna selection at the transmitter is considered in which the goal is to decide which set of antennas are to be used for transmission based on different performance criteria at the receiver.

Our goal in this paper is to study the problem of determining the number of transmit antennas prior to performing symbol timing synchronization and channel estimation when the transmitter and the receiver have limited cooperation. In a practical MIMO system, frequency and symbol timing synchronization, and channel estimation need to be performed at the receiver before recovering data. While there are several techniques proposed for symbol timing and channel estimation [32], the use of pilot symbols in MIMO systems has been studied extensively for symbol timing synchronization [33], [34] and channel estimation [33], [35]–[38]. The knowledge of the pilot sequences assigned to each transmit antenna may not be available exactly at the receiver depending on the level of cooperation between the transmitter and the receiver. In this paper, we study the problem of MIMO transmit system classification based on pilot data. In particular, we propose several test statistics, develop algorithms and discuss relative merits of different algorithms considering asynchronous (in the absence of symbol timing synchronization) and synchronous cases (in the presence of symbol timing synchronization). In the asynchronous case, maximum likelihood (ML) and correlation classifiers are considered assuming that an exact knowledge of the pilot sequence assignment for each antenna is available at the receiver. The results are then specified for the synchronous case. When the exact knowledge of the pilot data assignment of each transmit antenna is not known at the receiver, we develop hybrid maximum likelihood (HML) based classification schemes using the expectation-maximization (EM) algorithm for both asynchronous and synchronous cases. We further specify the results when the transmission system is binary; i.e., transmission can be performed either with a single antenna or multiple antennas with a known number of antennas. Then, the MIMO transmit system classification problem reduces to the problem of MIMO vs. single-input multiple-output (SIMO) detection. The performance of each classifier/detector is illustrated via simulations and comparative merits of different techniques are discussed.

The rest of the paper is organized as follows. In Section II the background and the problem formulation are presented. In Section III the problem of asynchronous MIMO transmit antenna classification is discussed when the receiver has the exact knowledge of the pilot sequences. The results are provided for the synchronous case as well for the case when the transmit system is binary. In Section IV the analyses are extended to the case where the exact knowledge of the pilot sequences is not known at the receiver. Simulation results are given in Section V and concluding remarks are given in Section VI.

Notation and Terminology

We use 'Tx' and 'Rx' to denote 'transmit' and 'receive', respectively. Lower case letters, e.g., \( x \), are used to denote scalars and functions while boldface lower case letters, e.g., \( X \), are used to denote vectors. Boldface upper case letters, e.g., \( X \), are used to denote matrices. The \((j,k)\)-th element of a matrix \( X \) is denoted by \((X)_{jk}\). Matrix transpose and Hermitian transpose operators are denoted by \((\cdot)^T\) and \((\cdot)^H\), respectively. The notation \(|\cdot||F\) is used to denote the Frobenius norm. The trace operator is denoted by \(tr(\cdot)\).

II. BACKGROUND AND PROBLEM FORMULATION

Consider a MIMO communication system with \( m \) receiver (Rx) antennas and \( n \) transmit (Tx) antennas (often called \( m \times n \) MIMO) as depicted in Fig. 1. The symbols transmitted by each Tx antenna are assumed to undergo independent fading.

Let \( x_i[l] \) be the \( l \)-th symbol transmitted by the \( i \)-th antenna. Note that \( x_i[l] \) can be either a pilot/training symbol or (encoded) data symbol. After pulse shaping, the transmitted signal by the \( i \)-th Tx antenna can be expressed as,

\[
r_i(t) = \beta l \sum x_i[l] g(t - lT_s)
\]

where \( T_s \) is the symbol period, and \( g(\cdot) \) denotes the transmitted pulse. We denote the symbol rate as \( R_s = \frac{1}{T_s} \). Further, in general, we assume different Tx powers for data symbols and pilot symbols. Thus, \( \beta = \beta_d \) if \( x_i[l] \) represents a data symbol and \( \beta = \beta_p \) if \( x_i[l] \) represents a pilot symbol. We assume that the transmit power is equally distributed across the transmitted antennas, thus, the amplitudes are identical across them. The transmitted pulse is assumed to be a square root raised-cosine (\( \sqrt{RC} \)) pulse [39] which is given by,

\[
g(t) = \frac{4e^{-\epsilon^2/2}}{\pi \sqrt{T_s}} \cos((1 + \epsilon)\pi t/T_s) + \frac{\sin((1-\epsilon)\pi t/T_s) + (4\epsilon t/T_s)}{1 - (4\epsilon t/T_s)^2}
\]

where \( \epsilon \) is the roll-off factor.

With a given delay \( \tau_q \), the received signal at the \( q \)-th Rx antenna is given by [39],

\[
y_q(t) = \sum_{i=1}^{n} h_{qi} \sum x_i[l] g(t - lT_s - \tau_q) + w_q(t)
\]
for $0 < t < T_0$ where $h_{qi}$ is the complex fading channel coefficient from the $i$-th Tx antenna to the $q$-th Rx antenna (assumed to be fixed for given $T_0$), and $w_q(t)$ is the additive noise. For a given number of Tx antennas, we further assume that $\tau_q = \tau$ for all $q$ which is a reasonable assumption when the Rx antennas are located not very far from each other. If perfect timing and frequency information is available at the receiver, the $l$-th received symbol in (1) after matched filtering can be written as

$$y_q[l] = \sum_{i=1}^{n} h_{qi} \beta x_i[l] + w_q[l]$$

for $l = 1, 2, \ldots$ and $q = 1, \ldots, m$.

However, when the time delay is unknown, it has to be estimated before performing matched filtering. Further, even if the symbol timing is available or symbols are synchronized, it is required to estimate the channel state information at the receiver before recovering the data symbols. In order to perform all these operations, the number of Tx antennas used by the transmitter should be known at the receiver.

In this paper, we classify the Tx system in terms of the number of transmit antennas before recovering or extracting required information about the transmitted signal. When the maximum number of Tx antennas is known, this problem can be formulated as a multiple hypothesis testing (or classification) problem. Classification is performed based on training/pilot data used for symbol timing synchronization and channel estimation. In many communication systems, timing synchronization and channel estimation are achieved with the aid of pilot data [33]-[38]. For example, in Fig. 2 one general format of the data transmitted by each Tx antenna is illustrated in a time division multiple access (TDMA) framework [40]. With this timing and framing structure, each Tx antenna transmits a burst of length $L_b$ which contains pilot data for symbol timing, pilot data for channel learning and encoded (informative) data. This particular format (with/without slight modifications) with the same pilot sequence repeating periodically at a given antenna is used in commercial systems such as narrowband TDMA/STCM-based modems [41].

We consider several scenarios. In the asynchronous case, we use the pilot data used for timing synchronization to classify the Tx system. On the other hand, when perfect timing synchronization is achieved at the receiver (without the use of pilot symbols) or when time delay is negligible, we use pilot sequences used for channel estimation to classify the Tx system. Depending on which pilot data is used and the knowledge available at the receiver about the pilot data, the decision statistics used for Tx system classification are different.

III. TX SYSTEM CLASSIFICATION WITH EXACT KNOWLEDGE OF PILOT DATA AT THE RECEIVER

Each Tx antenna is assumed to use an equal number of training/pilot symbols (say $L_s$) for timing synchronization in each transmitted frame of length $L_b$. Channel estimation is performed periodically within a burst of length $L_b$ with a different set of pilot data of length $L_p$. We assume that the pilot sequences are selected from a given pool of orthogonal sequences. The use of orthogonal sequences for symbol timing and channel learning has been used in many MIMO systems [41]. Although it is reasonable to assume to have different number of pilot symbols per antenna [42], here we consider that the pilot symbols are of the same length.

In this section, we assume that the pilot sequences $x_i[l]$’s for $l = 1, \ldots, L_s$ (or $L_p$) assigned to the $i$-th antenna for $i = 1, \ldots, n$, are known at the receiver. This is a valid assumption when the Tx and Rx have limited cooperation.

Let the vector representation of $y_q(t)$, $w_q(t)$ and $r_i(t)$ be

$$y_q[1], \ldots, y_q[L_s] \equiv y_q, \quad w_q[1], \ldots, w_q[L_s] \equiv w_q$$

$$r_i[1], \ldots, r_i[L_s] \equiv r_i$$

respectively for $q = 1, \ldots, m$ and $i = 1, \ldots, n$ and $L_s = \frac{T_0}{\tau}$, where $T_0$ is the observation time period for pilot symbols. Then the received signals at the $m$ receivers can be written in matrix form as,

$$Y = HR_r + W$$

where $Y = [y_1, \ldots, y_m]^T$, $R_r$ represents the equivalent discrete time representation of $r_i(t - \tau)$ for $i = 1, \ldots, n$ similar to $R = [r_1, \ldots, r_n]^T$, and $W = [w_1, \ldots, w_m]^T$. The noise process is assumed to be uncorrelated and Gaussian (and the elements of $W$ are assumed to be iid Gaussian with mean zero and variance $\sigma_w^2$). The matrix $H$ is the channel matrix where $(H)_{qi} = h_{qi}$ for $q = 1, \ldots, m$ and $i = 1, \ldots, n$.

After matched filtering with a given delay $\tau$, (3) is equivalently represented by,

$$Y_r = HR + W$$

where the $(q, k)$-th element of $Y_r$ is given by,

$$(Y_r)_{qk} = y_q[k] = \int_{T_0}^{T_0} y_q(t)g(t - kT_s - \tau)dt.$$

Let there be a maximum of $n_{\text{max}}$ antennas possible for the MIMO system. The transmission system can choose any number of antennas, $n$, from $[1, n_{\text{max}}]$ with equal probability. Then, the MIMO Tx system classification problem can be treated as a multiple hypothesis testing problem with $n_{\text{max}}$ number of hypotheses. Each hypothesis $H_j$, for $j = 1, \ldots, n_{\text{max}}$, represents a MIMO system with a given number of antennas (including SIMO system when $n = 1$). The multiple hypothesis testing problem is given by,

$$H_j : Y_{r_j} = H_j R_j + W$$

for $j = 1, \ldots, n_{\text{max}}$. It is noted that, [43] has to be solved in the presence of unknown parameters $H_j$ and $\tau$ under $H_j$ for $j = 1, \ldots, n_{\text{max}}$.

In the case where timing information is available at the receiver (i.e. the synchronous case), the received signal matrix can be represented by,

$$Y = HR + W$$

where now $(Y)_{qk} = y_q[k] = \sum_{i=1}^{n} \beta_p h_{qi} x_i[k] + w_q[k]$ and $(R)_{ik} = r_i[k] = \beta_p x_i[k]$ for $q = 1, \ldots, m$ and $k = 1, \ldots, L_p$. Then, similar to [43], the synchronous MIMO classification problem can be cast as,

$$H_j : Y = H_j R_j + W.$$

(6)
Thus, in the synchronous case, MIMO classification is required to be performed based on \( (\hat{y}_t|H_j, \hat{\tau}_j, H_j) \) in the presence of unknown \( H_j \) under \( H_j^j \) for \( j = 1, \ldots, n_{\text{max}} \).

### A. ML Classifier

In the maximum likelihood (ML) framework, the unknown parameters are found so that the likelihood function under each hypothesis is maximized. For the asynchronous case, let \( \hat{\tau}_j (\tau \text{ under } H_j \text{ is denoted by } \tau_j \text{ for clarity}) \) and \( H_j \) denote the ML estimates of \( \tau \) and \( H_j \), respectively. Under \( H_j \), given \( H_j \) and \( \tau_j \), \( Y \) has the following matrix variate normal distribution:

\[
p(Y_{\tau_j}|H_j, \tau_j, H_j) = \frac{1}{(2\pi\sigma_w^2)^{nM/2}} e^{-\frac{1}{2\sigma_w^2}(Y - H_jR_j)^H(Y - H_jR_j)}. \tag{7}\n\]

The ML estimates (MLEs) of \( H_j \) and \( \tau_j \) are computed so that the probability density function (pdf) in (7) is maximized with respect to \( H_j \) and \( \tau_j \). They can be obtained as the solutions to the following two equations:

\[
H_j = Y_{\tau_j}R_j^H(R_jR_j^H)^{-1} \tag{8}
\]

and

\[
\sum_{i,k} (H_jR_j)_{ik} \frac{\partial y_{\tau_j}^i[k]}{\partial \tau_j} = 0 \tag{9}
\]

where \( y_{\tau_j}^i[k] = \int_0^{T_0} y_i(t)g(t - kT_s - \tau_j)dt \). It is noted that (8) and (9) are obtained by letting the partial derivatives of \( p(Y_{\tau_j}|H_j, \tau_j, H_j) \) in (7) with respect to \( H_j \) and \( \tau_j \) to be zero. From (8) and (9), \( \hat{\tau}_j \) can be computed as the solution for \( \tau_j \) to the following equation:

\[
\sum_{i,k} (Y_{\tau_j}P_j)_{ik} \frac{\partial y_{\tau_j}^i[k]}{\partial \tau_j} = 0 \tag{10}
\]

where

\[
P_j = R_j^H(R_j^H)^{-1}R_j. \tag{11}
\]

Once \( \hat{\tau}_j \) is obtained, \( \hat{H}_j \) is given by

\[
\hat{H}_j = Y_{\hat{\tau}_j}R_j^H(R_jR_j^H)^{-1}
\]

which is the least squared (LS) channel estimator for the estimated time delay \( \hat{\tau}_j \).

Then, the ML classifier selects the hypothesis which gives the maximum likelihood function:

\[
\hat{j} = \arg\max_j \frac{1}{\sigma_w^2} \left\{ \text{tr}(R_j^H\hat{H}_jY_{\tau_j}) - \frac{1}{2} \text{tr}(R_j^H\hat{H}_j^H\hat{H}_jR_j) \right\}. \tag{12}
\]

For the synchronous case, the ML classifier reduces to

\[
\hat{j} = \arg\min_j \text{tr}(P_j^HYH) \tag{12}
\]

where \( P_j \) is as defined in (11) and \( (Y)_{qk} = \sum_{l=1}^n \beta_p h_q x_l[k] + w_q[k] \).

1) **MIMO vs. SIMO detection**: In this section, we specify the results when the Tx system is binary; i.e., transmission can be via either a single antenna or multiple antennas (i.e., SIMO vs. MIMO). This becomes a binary detection problem with:

\[
H_1(\text{MIMO}) : Y_\tau = H_M R_M + W
\]

\[
H_2(\text{SIMO}) : Y_\tau = H_S R_S + W \tag{13}
\]

where the subscripts \( M \) and \( S \) are used to denote the multiple antenna and single antenna cases, respectively. It is noted that \( H_S \) is a \( m \times 1 \) vector and \( R_S \) is a row vector. In this case, the ML test statistic, which is known as the generalized likelihood ratio test (GLRT), reduces to

\[
\Lambda_{\text{GLRT,async.}} = \frac{1}{\sigma_w^2} \left\{ \text{tr}(R_M^H\hat{H}_M Y_{\hat{\tau}_M} - R_S^H\hat{H}_S Y_{\hat{\tau}_S}) - \frac{1}{2} \text{tr}(R_M^H\hat{H}_M^H\hat{H}_M R_M - R_S^H\hat{H}_S^H\hat{H}_S R_S) \right\}. \tag{14}
\]

In the synchronous case, the GLRT test statistic can be expressed as

\[
\Lambda_{\text{GLRT,sync.}} = \text{tr}(P_S^H - P_M^H)YH \tag{14}
\]

where \( P_S^H = I - P_S \) and \( P_M^H = I - P_M \) with \( P_S = R_S^H(R_S R_S^H)^{-1}R_S \) and \( P_M = R_M^H(R_M R_M^H)^{-1}R_M \), as defined before. It is noted that the GLRT test statistic in the synchronous case can be computed fairly easily.

2) **Design of threshold for GLRT in the synchronous case**: In the following, we design the threshold of the synchronous GLRT detector so that the probability of false alarms is kept under a desired value. The probabilities of false alarm and detection of the GLRT are given by,

\[
P_f = Pr(\Lambda_{\text{GLRT,sync.}} \geq \tau_0|H_2)
\]

and

\[
P_d = Pr(\Lambda_{\text{GLRT,sync.}} \geq \tau_0|H_1)
\]
respectively, where \( \tau_g \) is the threshold. With Gaussian approximation, we can show that the threshold \( \tau_g \) that maintains the probability of false alarm at \( \alpha \) can be approximated as given in the following proposition.

**Proposition 1.** The threshold of the synchronous GLRT detector which ensures \( P_f \leq \alpha \) can be approximated by

\[
\tau_g \approx (\hat{\sigma}Q^{-1}(\alpha) + \hat{\mu}),
\]

where

\[
\hat{\mu} \approx (\bar{R}_S^H(P_M - P_S)\bar{R}_S) \sum_{l=1}^{m} |\bar{H}_S(l)|^2 + m \sigma_w^2 \text{tr}(P_M - P_S)
\]

and

\[
\hat{\sigma}^2 \approx 2\sigma_w^2 (\bar{R}_S^H(P_M + P_S - 2P_MP_S)\bar{R}_S) \sum_{l=1}^{m} |\bar{H}_S(l)|^2 + m\sigma_w^2 \text{tr}(P_M + P_S - 2P_MP_S)
\]

where \( \sigma_w^2 = \mathbb{E}[(1/W)_{ij}] \) and \( \bar{R}_S = R_S^T \).

**Proof:** See Appendix A.

The computation of the test statistic for the ML classifier (and the GLRT detector) in a closed-form is difficult in the asynchronous case since it is difficult to obtain a closed-form solution to (10). However, the ML classifier has a simple closed-form expression in the synchronous case. In both cases, (asynchronous/synchronous) the performance of the classifier depends on the ML estimates of unknown parameters which depend on the length of the pilot sequences. In the following, we consider statistics for a suboptimal detector, known to be correlation detector, to perform MIMO system classification where estimation of all the unknown parameters is not required.

**B. Correlation Classifier**

For a single-input single output (SISO) link with AWGN channels, ML estimation of the delay involves the maximization of the correlation between the received signal and the transmitted signal (however, with an unknown channel matrix, this optimality may no longer holds). Motivated by this, we expect that for MIMO system as considered in this paper, the correlation between the received signal matrix and the transmitted pilot symbols will provide a reasonable decision statistic for the classification problem in (13) in the presence of unknown delay. In the following, we consider correlation classifier for the multiple hypothesis testing problem which does not require the estimation of \( H_j \). For the asynchronous case, the correlation classifier is given by,

\[
\hat{j} = \arg \max_j \text{tr}(Y_{\tau_j}(R_{\tau,j}^H R_j)Y_{\tau_j}^H)
\]

where

\[
\hat{\tau}_j = \arg \max_{\tau_j} ||C_{\tau_j}||^2_F.
\]

and \( C_{\tau_j} = Y_{\tau_j} R_{\tau,j}^H \). It is noted that, this decision statistic in (18) is fairly easy to compute. For the synchronous case, (18) reduces to,

\[
\hat{j} = \arg \max_j \text{tr}(Y(R_{\tau,j}^H R_j)Y^H).
\]

**IV. TX System Classification When the Exact Knowledge of Pilot Data is Not Available**

For both ML and the correlation classifiers as considered in Section III (in both asynchronous and synchronous cases), the exact knowledge of the pilot data assigned to each Tx antenna is assumed to be available at the receiver. However, when the cooperation between the transmitter and the receiver is too limited, this assumption may be too restrictive. Next, we consider the case where the receiver has only probabilistic information regarding the pilot sequences assigned to each Tx antenna. First, we consider the asynchronous case.

Let each Tx antenna use a pilot sequence \( x_i = [x_i[1], \ldots , x_i[L_s]]^T \) for symbol timing synchronization drawn as a column of a given orthogonal matrix \( Q \) where \( Q \) is known to the receiver. However, the receiver is not aware of the exact column assigned to a given antenna from \( Q \). Thus, the receiver assumes that any column of \( Q \) is assigned to \( x_i \) randomly with the same probability for given \( i \).

Let

\[
X = \begin{bmatrix} x_1^T \\ \vdots \\ x_n^T \end{bmatrix}
\]

be the \( n \times L_s \) matrix containing all the different pilot sequences used by \( n \) antennas. Then, there can be \( L_s^n \) possibilities for \( X \). The \( k \)-th realization of \( X \) is denoted by \( X_k \). The joint pdf of \( Y_r \) marginalized over \( X_k \) under \( \mathcal{H}_j \) is given by,

\[
p(Y_{\tau_j}|H_j, \tau_j, \mathcal{H}_j) = \sum_{X_k} p(Y_{\tau_j}|H_j, \tau_j, X_k; \mathcal{H}_j)p(X_k)
\]

where \( p(X_k) \) is the probability that \( X_k \) being selected. When \( X_k \) is selected uniformly, we have \( p(X_k) = \frac{1}{L_s^n} \). In order to perform likelihood ratio based classification, we consider the hybrid maximum likelihood (HML) based approach, where the unknown parameters are estimated so that the marginalized pdf under each hypothesis is maximized. Since finding these estimators is computationally intractable due to marginalization, we provide a numerical technique based on the expectation-maximization (EM) algorithm.

**A. HML Based MIMO Antenna Classification via EM**

The EM algorithm is an iterative numerical method which can be used to compute ML estimates. It is well suited when ML estimation is intractable due to the presence of hidden (unobserved) data. The outline of the EM algorithm is given in (13), and the use of the EM algorithm in HML based classification is discussed in our previous work in (14) in a different application scenario. For the problem addressed in this paper, the actual pilot sequences \( X \) can be treated as hidden data. Then, complete data can be expressed as \([Y, X] \).
Starting from initial estimates \( \hat{H}_j^{(0)}, \hat{\tau}_j^{(0)} \), the two operations as in (22) are performed at the \( r \)-th iteration under \( H_j \).

Let \( \alpha_j^{(r)} = p(X = X_i|Y_{\hat{\tau}_j}^{(r)}, \hat{H}_j^{(r)}) \) which can be expressed as,

\[
\alpha_j^{(r)} = \frac{p(Y_{\hat{\tau}_j}^{(r)}|\hat{H}_j^{(r)}, X = X_i)}{\sum_{k=1}^{L_p^2} p(Y_{\hat{\tau}_j}^{(r)}|\hat{H}_j^{(r)}, X = X_k)}
\]

where \( \left(Y_{\hat{\tau}_j}^{(r)} \right)_{jk} = \int_{T_0} y_j(t)g(t-kT_s-\hat{\tau}_j^{(r)})dt \). Then, the estimates \( \hat{H}_j^{r+1} \) and \( \hat{\tau}_j^{r+1} \) at the \((r+1)\)-th iteration can be found as the solution for \( H_j \) and \( \tau_j \) in the following two equations:

\[
H_j = Y_{\hat{\tau}_j} \left( \sum_{l=1}^{L_p^2} \alpha_l^{(r)} \beta_p X_i^H \right)^{-1} \left( \sum_{l=1}^{L_p^2} \alpha_l^{(r)} \beta_p^2 X_i X_i^H \right)
\]

and

\[
\sum_{l=1}^{L_p^2} \sum_{i,k} \alpha_l^{(r)} (\beta_p H_j X_i)_{i,k} \frac{\partial y_i^{\tau_j}}{\partial \tau_j} = 0
\]

where \( y_i^{\tau_j}[k] = \int_{T_0} y_j(t)g(t-kT_s-\tau_j)dt \). Once the unknown parameters under both hypotheses are found via the EM algorithm, the HML based asynchronous MIMO Tx system classifier is given by,

\[
\hat{\gamma} = \arg \max_{1 \leq j \leq n_{\text{max}}} p(Y_{\hat{\tau}_j}|\hat{H}_j^{(r)}, \hat{\tau}_j; H_j)
\]

V. Simulation Results

In this section, we provide numerical results to illustrate the performance of MIMO system classification schemes developed in this paper. We assume that there is a maximum of \( n = 4 \) Tx antennas. The pilot sequences are chosen to be the columns of \( L_s \times L_s \) \((L_p \times L_p)\) Hadamard matrix for symbol timing (channel) estimation. In all the simulations, the following values for specific parameters are used unless otherwise specified. The transmitted pulse \( g(t) \) is assumed to be symmetrically truncated with a roll-off factor \( \epsilon = 0.3 \) and duration \( 6T_s \). We let \( \beta_p = 1 \), and the pilot sequences with multiple antenna Tx are normalized so that both multiple antenna Tx and single antenna Tx have the same transmit power. The symbol duration \( T_s = 1 \).

A. Performance of Asynchronous MIMO vs. SIMO Detection

First, we show the classification performance when the Tx system is binary; i.e., the problem is to detect SIMO vs. MIMO. In Fig. 3, we plot receiver operating characteristic (ROC) curves to depict the performance of asynchronous GLRT and asynchronous correlation based MIMO vs. SIMO detection using pilot data used for symbol timing synchronization. For the ROC curves, we plot the probability of detection \( P_d \) vs. probability of false alarms \( P_f \) which are defined below:

\[
P_f = Pr(\hat{H} = \mathcal{H}_1|\mathcal{H}_2), \quad P_d = Pr(\hat{H} = \mathcal{H}_1|\mathcal{H}_1)
\]

where \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) are as defined in (13) and \( \hat{H} \) denotes the detection decision. For a given signal-to-noise ratio (SNR), the ROC curves are obtained by varying the threshold and 500 Monte Carlo runs were used to generate each curve. We assume in Fig. 3 that the pilot sequence assigned to each Tx antenna is known at the receiver. For an \( m \times n \) MIMO system, the first \( n \) columns of the Hadamard matrix are selected. In the correlation detector, the delay is estimated as in (19) under each hypothesis. We further plot the performance of the correlation detector when the timing delay is ignored; i.e., with the assumption that \( \hat{\tau}_j = 0 \) for \( j = 1,2 \). Different subplots in Fig. 3 are for different SNR values and antenna systems. The SNR is defined as \( 10 \log_{10} \left( \frac{E[|H|^2]}{E[|W|^2]} \right) \). As expected, it can be seen that the GLRT detector performs better than the correlation detector, especially when the SNR is low and the number of antennas used for MIMO is small. Further, \( 4 \times 4 \) MIMO vs. SIMO has a better classification performance compared to \( 4 \times 2 \) MIMO vs. SIMO with both types of detectors. The correlation detector, which has a simpler implementation compared to GLRT in the asynchronous case, is more promising in classifying \( 4 \times 4 \) MIMO vs. SIMO compared to classifying \( 4 \times 2 \) MIMO vs. SIMO. Further, with \( 4 \times 2 \) MIMO, the performance of the correlation detector degrades quite significantly when the time delay is ignored,
Fig. 3. ROC curves for asynchronous MIMO vs. SIMO classification via GLRT and correlation detectors using pilot data for symbol timing; \( \tau_j \) for \( j = 1, 2 \) are selected uniformly randomly from \([0, \frac{1}{2} T_s]\) where \( T_s = 1, \ L_s = 8 \) compared to that with \( 4 \times 4 \) MIMO. When the SNR is changed (e.g from 0 dB to –10 dB), a significant performance degradation is not observed.

In Fig. 4 we show the performance of the correlation detector with ignored time delay (which has a simpler implementation than that with estimated time delay) as the number of Tx antennas for MIMO varies for a given SNR. We further show the performance with the estimated time delay. It can be observed that, as the number of Tx antennas in MIMO increases, the performance of the correlation detector with the ignored time delay becomes very close to that with the estimated time delay.

In Fig. 5 we illustrate the performance of the HML based classification scheme when the exact knowledge of the pilot sequences is not available at the receiver. The EM algorithm as discussed in Subsection IV-A is implemented to get the ML estimates of \( H_j \), and \( \tau_j \) for \( j = 1, 2 \). It is well known that the performance of the EM algorithm depends on the initial values of the unknown parameters. In this work, we consider that the initial values are selected randomly. For \( \tau_j \), the initial values are obtained uniformly from \([0, \frac{1}{2} T_s]\) while for \( H_j \), the initial values are drawn from a complex normal

Fig. 4. ROC curves for asynchronous MIMO vs. SIMO classification via correlation detector with estimated and ignored time delay; \( \tau_j \) for \( j = 1, 2 \) are selected uniformly randomly from \([0, \frac{1}{2} T_s]\) where \( T_s = 1, \ L_s = 8 \), SNR = –10 dB
in the asynchronous case, it is seen that GLRT performs better and SNR values for MIMO vs. SIMO detection. As observed than the correlation detector and this performance gain is more case, the correlation detector is pretty simple since it does not require the estimation of any unknown parameters while GLRT estimates the channel matrix. It can be seen that, the GLRT detector for both synchronous and asynchronous cases provides similar performance while the synchronous correlation detector performs better than the asynchronous correlation detector with a considerable performance gain. It is noted that, the implementation of the asynchronous correlation detector with the estimated time delay is more computationally complex than the synchronous correlation detector. With the asynchronous correlation detector with ignored time delay (which has a similar implementation complexity as that of the synchronous correlation detector), a significant performance loss can be observed compared to the synchronous correlation detector. Thus, neglecting the unknown time delay leads to poor MIMO/SIMO classification performance especially when $n$ is small. However, as seen in Fig. 4 the correlation detector with ignored time delay is promising when the number of Tx antennas is not too small.

In Fig. 5 we plot the symbol error rate (SER) when the data is recovered via ML estimation after the detection decision is made via GLRT in the synchronous case. The channel matrix for ML estimation of data is taken as the one estimated in the GLRT algorithm. Fig. 8 depicts the recovery performance depending on the operating point of GLRT; i.e. SER vs. $P_f$ and $P_d$ is plotted. We further plot the SER when it is exactly known what transmission scheme used (single or multiple). As a reference, we also plot SER with known channel matrix. It can be seen that, when SNR is high (Figs. 8 (b) and (d)), SER obtained after making the detection decision based on GLRT with a proper selection of threshold almost coincides with that when the transmission system is exactly known. However, as the probability of false alarms increases, the SER also increases. Thus, it is important to design the threshold of the GLRT so that a desired probability of false alarm is achieved. The design of the threshold for synchronous GLRT in closed-form was discussed in Section III-A.2.

In Fig. 7 we compare the performance of asynchronous and synchronous classification schemes when the exact pilot data is available at the receiver. We consider $4 \times 2$ MIMO vs. SIMO classification and for both synchronous and asynchronous cases, we assume that the pilot sequences are of the same length (i.e. $L_s = L_p = 8$). Further, we let $SNR = -10 dB$. It can be seen that, the GLRT detector for both synchronous and asynchronous MIMO/SIMO classification via HML using pilot data for symbol timing; $\tau_j$’s for $j = 1, 2$ are selected uniformly randomly from $[0, \frac{1}{2} T_s]$ where $T_s = 1$, random initialization is used for the EM algorithm.
Fig. 6. ROC curves for synchronous MIMO/SIMO with pilot data used for channel estimation; $T_s = 1$

Fig. 8. SER when data is recovered via ML after detection decision is made via GLRT with pilot data used for channel estimation
Fig. 7. Asynchronous and synchronous MIMO/SIMO classification using GLRT and correlation detectors; \( L_p = 8 \), \( \sigma^2_w = 10 \), SNR = \(-10\) dB

Fig. 9. Probability of detection (SIMO/MIMO) vs. SNR via GLRT in the synchronous case; \( L_p = 8 \), varying \( \alpha \)

VI. DISCUSSION AND SUMMARY

In a typical MIMO system, symbol timing synchronization and channel estimation are required to be performed before signal demodulation. One common approach to achieve these two tasks is to utilize some pilot data assigned to each Tx antenna. In this work, we have explored how pilot data used for symbol timing synchronization and channel estimation can be exploited to solve the Tx system classification problem. When perfect timing information is available at the receiver, Tx system classification can be performed with the pilot data used for channel estimation. Then, designing ML based classifiers that estimate unknown parameters so that the likelihood function is maximized is not computationally challenging.

In the case where Tx system is binary (i.e., SIMO vs MIMO with a known number of Tx antennas), it has been shown that, the synchronous GLRT detector with a suitable threshold classifies MIMO vs. SIMO perfectly when the SNR exceeds a certain level. Correlation detector is another simple suboptimal approach which shows comparable performance to GLRT in the high SNR region and when the number of antennas used for MIMO is not very small. However, when the number of Tx antennas in the MIMO system is small, the correlation detector has a significant performance gap compared to the GLRT detector. When the perfect timing information is not available at the receiver, it also has to be estimated. In a system

C. Performance of ML Classifier

In Fig. 10 we show the performance of the ML classifier when the exact number of antennas used for MIMO is not known. We consider the synchronous case, and assume that the exact knowledge of the pilot sequences is available at the receiver. In simulations, the true value for the number of antennas is selected uniformly among \( \{1, \cdots, n_{\text{max}}\} \) while two values were considered for \( n_{\text{max}} \). Classification is performed based on the pilot data used for channel estimation with \( L_p = 12 \) and \( L_p = 16 \). As expected, it can be observed that, the correct classification rate with 3 Tx antennas is higher than that with when there are 4 Tx antennas. As \( n_{\text{max}} \) increases, the number of hypotheses also increases leading to weaker distinguishably among hypotheses. However, when the SNR exceeds a certain threshold \( \approx 0\) dB, the ML classification method provides perfect classification on the Tx system for both values of \( n_{\text{max}} \).
where time delay is also estimated via pilot data, we compute decision statistics for asynchronous MIMO/SIMO detection based on that pilot data. We developed asynchronous GLRT and correlation detectors where the estimation of the time delay for both detector requires one-dimensional numerical optimization step. It was observed that the asynchronous correlation detector becomes promising compared to GLRT when as the number of antennas used in the MIMO model increases, even when the time delay is ignored.

In the general multiple hypothesis testing case where the transmission system can take more than two possibilities, it was shown that the performance of the ML classifier degrades as the number of hypotheses increases. However, as the SNR exceeds a certain threshold, perfect classification can be achieved.

In summary, we discussed several algorithms to classify Tx system based on pilot data used for timing synchronization and channel estimation. Test statistics for classification were constructed from this pilot data without additional overhead. This joint processing is promising in applications where the receiver and the transmitter have limited cooperation and when multiple antennas are used in an adaptive manner to maintain the required quality of service. An interesting future direction is to explore novel techniques to determine the transmission scheme when the transmitter and the receiver are not cooperative at all.

**APPENDIX A**

**Proof of Proposition 1**

Let $\mathbf{P}_S = \mathbf{U}_S \mathbf{R}_S^H$ and $\mathbf{P}_M = \mathbf{U}_M \mathbf{R}_M^H$, where $\mathbf{U}_S = \mathbf{R}_S^H (\mathbf{R}_S \mathbf{R}_S^H)^{-1/2}$ and $\mathbf{U}_M = \mathbf{R}_M^H (\mathbf{R}_M \mathbf{R}_M^H)^{-1/2}$ so that $\mathbf{U}_S \mathbf{U}_S = \mathbf{I}$ and $\mathbf{U}_M \mathbf{U}_M = \mathbf{I}$. Then we can write,

$$\text{tr}(\mathbf{P}_S^{-1} \mathbf{P}_M) y^H y = m \sum_{l=1}^{m} y_l^H \mathbf{U}_M y_l - \sum_{l=1}^{m} y_l^H \mathbf{U}_S y_l$$

$$= \sum_{l=1}^{m} z_l^H z_l - \sum_{l=1}^{m} v_l^H v_l = \tilde{Z}_1 - \tilde{Z}_2$$

where $\tilde{Z}_1 = \sum_{l=1}^{m} z_l^H z_l$ and $\tilde{Z}_2 = \sum_{l=1}^{m} v_l^H v_l$, $y_l$ is the $l$-th row vector of $\mathbf{Y}$, $z_l = \mathbf{U}_M^H y_l$ and $v_l = \mathbf{U}_S^H y_l$. We can show that the two random variables $\frac{1}{\sigma_w} \tilde{Z}_1$ and $\frac{1}{\sigma_w} \tilde{Z}_2$ have Chi-squared distributions under both hypotheses.

It is noted that $\tilde{Z}_1$ and $\tilde{Z}_2$ are two correlated chi squared random variables and the computation of the pdf of $\tilde{Z} = \tilde{Z}_1 - \tilde{Z}_2$ is difficult. Thus, in the following we approximate $\tilde{Z}$ to be a Gaussian random variable since using central limit theorem, we can approximate $\tilde{Z}_1$ and $\tilde{Z}_2$ to be Gaussian when $m \sigma_P$ is sufficiently large. Let $\tilde{Z} | \mathcal{H}_2 \sim \mathcal{N}(\tilde{\mu}, \tilde{\sigma}^2)$. It is noted that under $\mathcal{H}_2$, the $l$-th row vector of $\mathbf{Y}$ is Gaussian with $y_l \sim \mathcal{CN}(\mathbf{H}_S(l) \mathbf{R}_S, \sigma_w^2 \mathbf{I})$ where $\mathbf{H}_S(l)$ is the $l$-the element of $\mathbf{H}_S$ for $l = 1, \ldots, m$ and $\mathbf{R}_S = \mathbf{R}_S^T$ is a $L_P \times 1$ vector containing $L_P$ training symbols. Then $\tilde{y}_l = \frac{y_l}{\sigma_w} \sim \mathcal{CN}(\frac{\mathbf{H}_S(l) \mathbf{R}_S}{\sigma_w}, \mathbf{I})$. We can compute $\tilde{\mu}$ as,

$$\tilde{\mu} = (\mathbf{R}_S^H (\mathbf{P}_M - \mathbf{P}_S) \mathbf{R}_S) \sum_{l=1}^{m} |\mathbf{H}_S(l)|^2 + m \sigma_w^2 \text{tr}(\mathbf{P}_M - \mathbf{P}_S).$$

The variance of $\tilde{Z}$, $\tilde{\sigma}^2$, is given by,

$$\tilde{\sigma}^2 = \text{var}(\tilde{Z}_1) + \text{var}(\tilde{Z}_2) - 2\text{cov}(\tilde{Z}_1, \tilde{Z}_2).$$

We have, $\text{var}(\tilde{Z}_1) = \sum_{l=1}^{m} \text{var}(y_l^H \mathbf{P}_M y_l)$ and $\text{var}(y_l^H \mathbf{P}_M y_l) = E((y_l^H \mathbf{P}_M y_l)^2) - (E(y_l^H \mathbf{P}_M y_l))^2$. Using results in (45), we can show that,

$$\text{var}(\tilde{Z}_1) = 2 \sigma_w^2 \mathbf{R}_S^H \mathbf{P}_M \mathbf{R}_S \sum_{l=1}^{m} |\mathbf{H}_S(l)|^2 + m \sigma_w^4 \text{tr}(\mathbf{P}_M)$$

and similarly,

$$\text{var}(\tilde{Z}_2) = 2 \sigma_w^2 \mathbf{R}_S^H \mathbf{P}_S \mathbf{R}_S \sum_{l=1}^{m} |\mathbf{H}_S(l)|^2 + m \sigma_w^4 \text{tr}(\mathbf{P}_S).$$

Next we compute the covariance of $\tilde{Z}_1$ and $\tilde{Z}_2$. We have

$$\text{cov}(\tilde{Z}_1, \tilde{Z}_2) = E\{\tilde{Z}_1 \tilde{Z}_2\} - E\{\tilde{Z}_1\}E\{\tilde{Z}_2\}.$$  \hspace{1cm} (24)

We can compute $E\{\tilde{Z}_1 \tilde{Z}_2\}$ as,

$$E\{\tilde{Z}_1 \tilde{Z}_2\} = \sum_{l=1}^{m} y_l^H \mathbf{P}_M y_l \sum_{l=1}^{m} y_l^H \mathbf{P}_S y_l$$

and

$$E\{\tilde{Z}_1 \tilde{Z}_2\} = E\{\tilde{Z}_1\}E\{\tilde{Z}_2\}.$$  \hspace{1cm} (25)

We have,

$$\text{cov}(\tilde{Z}_1, \tilde{Z}_2) = \sigma_w^4 \sum_{l=1}^{m} E\{y_l^H \mathbf{P}_M y_l \tilde{y}_l^H \mathbf{P}_S \tilde{y}_l\}$$

$$+ \sigma_w^4 \sum_{l \neq j} E\{\tilde{y}_l^H \mathbf{P}_M \tilde{y}_l \tilde{y}_j^H \mathbf{P}_S \tilde{y}_j\}.$$  \hspace{1cm} (25)

It can be shown that,

$$\text{cov}(\tilde{Z}_1, \tilde{Z}_2) = 2 \sigma_w^2 \mathbf{R}_S^H \mathbf{P}_M \mathbf{P}_S \mathbf{R}_S \sum_{l=1}^{m} |\mathbf{H}_S(l)|^2 + m \sigma_w^4 \text{tr}(\mathbf{P}_M \mathbf{P}_S).$$

Then the variance is given by,

$$\tilde{\sigma}^2 = 2 \sigma_w^2 \mathbf{R}_S^H (\mathbf{P}_M + \mathbf{P}_S - 2 \mathbf{P}_M \mathbf{P}_S) \mathbf{R}_S \sum_{l=1}^{m} |\mathbf{H}_S(l)|^2 + m \sigma_w^4 \text{tr}(\mathbf{P}_M + \mathbf{P}_S - 2 \mathbf{P}_M \mathbf{P}_S).$$  \hspace{1cm} (26)

Since $\mathbf{H}_S$ is to be estimated, the threshold is found based on the estimated values for $\mathbf{H}_S$ and approximate values for $\tilde{\mu}$ and $\tilde{\sigma}$ are obtained accordingly. Thus, we have,

$$\text{Pr}(\Lambda_{GLRT, \text{sync}} \geq \tau_g | \mathcal{H}_2) \approx \text{Pr}(\tilde{Z} \geq \tau_g) \approx Q \left( \frac{\tau_g - \tilde{\mu}}{\tilde{\sigma}} \right)$$

completing the proof.
