QCD-BAG MASS SPECTRUM AND PHASE TRANSITIONS

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We obtain the hadronic mass spectrum in the ‘bag of bags’ statistical bootstrap model (BBSBM), implementing the colorless state condition, aside of baryon and strangeness conservation, using group projection method. We study the partition function, investigate the properties of dense hadronic matter, and determine the conditions under which the system undergoes a phase transition to a deconfined quark-gluon plasma. We show that a phase transition cannot occur in the $N = 1$ (Abelian) limit of our model, and is first order for QCD-like case $N = 3$.

Dedicated to the memory of Peter A. Carruthers

1 Introduction

Considerable theoretical and experimental effort is being devoted to the study of the phase transition between a hadron gas and a quark-gluon plasma (QGP). This phase transition is believed to have occurred in the early Universe, and is searched for in high energy nuclear collisions in laboratory. The fundamental theoretical treatment of the hadronic phase transition relies on lattice QCD approximation. This approach exhibits a phase transition and gives the equation of states of both phases. The case of nonzero baryon density (finite chemical potential) comprises not yet solved technical difficulties. In a phenomenological study of this case one starts from the low temperature phase, i.e., the hadron phase, and implement the high temperature phase in a more or less ad hoc way. This was the case of the statistical bootstrap model (SBM) when a quark component is added to it beyond the critical curve limiting the existence domain of the hadron phase. Here we show that it is possible to extend SBM in a natural way to encapsulate the high temperature deconfined phase. We next recall the SBM and in particular illustrate derivation of the hadronic mass spectrum. We introduce the modifications.

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Supported in part by the U.S. Department of Energy, grant DE-FG03-95ER40937.
leading to a SBM-type model which describes both hadron phase and QGP phase in section 3, and we study the phases and properties of the system at finite baryochemical potential in section 4.

2 Statistical bootstrap

2.1 The hadronic mass spectrum

The SBM approach can be characterized in three statements: 1) hadrons are made of hadrons; 2) compound hadrons and ‘elementary’ hadrons have to be treated on the same footing: this is the bootstrap; 3) the main effect of strong interactions is hadron production. Hadron-hadron interactions in thermally and chemically equilibrated system can be accounted for (nearly) completely by considering the abundances of particles present, i.e., a strongly interacting hadron gas can be described as a mixture of infinitely many ideal gases, weighted by the hadronic mass spectrum. The dynamical problem in SBM is the theoretical determination of the hadronic mass spectrum, i.e., the number of hadron states per unit interval of rest mass.

For pointlike particles in a box of volume $V$ the bootstrap equation for this density reads

$$
\rho_{\text{out}}(m) = \sum_{n=2}^{\infty} \left( \frac{V}{(2\pi)^3} \right)^{n-1} \frac{1}{n!} \prod_{i=1}^{n} \int \rho_{\text{in}}(m_i) \, dm_i \
\times \int \delta \left( \sum_{i=1}^{n} E_i - m \right) \delta^3 \left( \sum_{i=1}^{n} \vec{p}_i \right) \, d^3 p_i .
$$

(1)

In this equation, $\rho_{\text{out}}$ represents the total density of states in the box, while $\rho_{\text{in}}(m_i)$ counts the number of mass states of particle $i$. Note that the density of levels is written with center of mass at rest because this is the density to be identified with the number of hadron states per unit interval of rest mass.

One has to solve Eq. (1) with the bootstrap condition:

$$
\rho_{\text{out}}(m) = \rho_{\text{in}}(m) \equiv \rho(m).
$$

(2)

The solution is of the form

$$
\rho(m) \sim cm^a e^{m/T_0}, \quad \text{with} \quad a = -3.
$$

(3)
2.2 The partition function

The partition function in the Boltzmann approximation can be written

\[ \ln Z(V, T) = \frac{VT}{2\pi^2} \int_0^\infty \rho(m) m^2 K_2 \left( \frac{m}{T} \right) \, dm, \]

where \( K_2 \) is the modified Hankel function. With a mass spectrum of the form (3), one finds for \( T \to T_0 \),

\[ \ln Z(V, T \to T_0) \approx cV \left( \frac{T_0}{T} \right)^2 \left( 1 - \frac{T_0 - T}{T_0 T} \right)^{(a + 5/2)} \Gamma \left( a + \frac{5}{2}, \ldots \right), \]

and the energy density,

\[ \varepsilon(T) = \frac{T^2}{V} \frac{\partial \ln Z(V, T)}{\partial T}, \]

behaves for \( T \to T_0 \) as

\[ \varepsilon \approx \begin{cases} \left( \frac{T_0 - T}{T_0 T} \right)^{(a + 7/2)} & \text{for } a > -7/2, \\ \ln \left( \frac{T_0 - T}{T_0 T} \right) & \text{for } a = -7/2, \\ \text{Const.} & \text{for } a < -7/2. \end{cases} \]

One sees that for \( a \geq -7/2, T_0 \) is attained with infinite energy density and appears as a maximal limiting temperature. All energy delivered is converted into particle multiplicity rather than thermal motion. But for \( a < -7/2 \), \( T_0 \) is reached at finite energy density and the system may undergo a phase transition. In the original SBM \( a > -7/2 \) and thus \( T_0 \) is a limiting temperature.

Interestingly, the hadronic partition function in SBM is singular (a square-root singularity) even for finite volume. This last feature can be traced back to the assumption of pointlike hadrons. However when the individual hadron volumes are taken into account through the so called ‘excluded volume’ correction, \( T_0 \) is reached at finite energy density but not in the usual thermodynamic limit. When baryon number is considered, there is not a single transition point \( T_0 \), but a transition curve \( T(\mu) \) in the plane \( T - \mu \), where \( \mu \) is the baryonic chemical potential, and \( T_0 = T(\mu = 0) \). Beyond this critical curve the SBM partition function becomes complex and extraneous assumptions are needed to implement the new QGP phase. This results in a two component model, one component describing the hadron side and another the quark-gluon side. However, a single closed analytic model unifying both phases can be built, provided that we introduce from the start the quark structure of hadrons as extended objects.
3 Bag of bags SBM

3.1 The bag grand canonical partition function

Hadrons are considered now as bags of quarks and gluons occupying finite volumes. They have to be colorless objects, i.e., $SU_c(3)$ (or $SU_c(N)$) singlets. This is an expression of color confinement. To be realistic, we also implement baryon number and strangeness conservation effects. With these constraints, we will determine the hadronic mass spectrum in such a bag-of-bags-SBM (BB-SBM). Consider a colorless bag of volume $v$ filled with quarks, anti-quarks and gluons at sufficiently high temperature to neglect their interactions (asymptotic freedom). The density of states of this bag can be determined by taking the inverse Laplace transform of its grand canonical partition function (GCPF).

But quarks and gluons transform respectively under the fundamental and the adjoint representations of $SU_c(3)$. In the GCPF one has to retain only the singlet states. Furthermore, as we have noted in Sec. 2.1, the density of levels is defined with the center of mass at rest.

With the preceding constraints the GCPF, can be derived, using projection technique, from the function

$$\tilde{Z}_Q(\beta, v, \mu_q, \mu_s) = \int \frac{d\mu(g)}{SU_c(3)} \int \frac{d^3 R}{v} \left\{ Tr_G U_G(g) e^{-\beta H_G + i \vec{P}_G \cdot \vec{R}} \right\}$$

$$\times \left\{ Tr_q U_q(g) e^{-\beta H_q + i \vec{P}_q \cdot \vec{R} - i \mu_q \hat{N}_q} \right\} \left\{ Tr_{\bar{q}} U_{\bar{q}}(g) e^{-\beta H_{\bar{q}} + i \vec{P}_{\bar{q}} \cdot \vec{R} + i \mu_q \hat{N}_{\bar{q}}} \right\}$$

$$\times \left\{ Tr_s U_s(g) e^{-\beta H_s + i \vec{P}_s \cdot \vec{R} - i \mu_s \hat{N}_s} \right\} \left\{ Tr_{\bar{s}} U_{\bar{s}}(g) e^{-\beta H_{\bar{s}} + i \vec{P}_{\bar{s}} \cdot \vec{R} + i \mu_s \hat{N}_{\bar{s}}} \right\},$$

where $G$ stands for gluons, $q$ for the quarks $u$ and $d$ and $s$ for the strange quark. $\hat{N}_q$ and $\hat{N}_s$ are the quark number operators and $\mu_q$ and $\mu_s$ the corresponding chemical potentials. $\vec{P}_G$, $\vec{P}_q$ and $\vec{R}$ are the momenta of gluons and quarks and their space position inside the bag. $U(g)$ is a unitary representation of $SU_c(3)$ in the Hilbert space of states, $g$ being an element of $SU_c(3)$. The physical grand partition function $Z_Q(\beta, v, \mu_q, \mu_s)$ is obtained from $\tilde{Z}_Q(\beta, v, \mu_q, \mu_s)$ by the so-called ‘Wick rotation’ $\mu \rightarrow -i \beta \mu$. In spite of its apparent complexity Eq. (8) corresponds simply to the product of five partitions functions (five traces) for non interacting five particle species ($G$, $q$, $\bar{q}$, $s$ and $\bar{s}$). The integration over the $SU_c(3)$ group with the Haar measure $d\mu(g)$ selects the $SU_c(3)$ singlet states and the integration over $\vec{R}$ in the volume $v$ selects the bag states of zero momentum.

$Z_Q(\beta, v, \mu_q, \mu_s)$ can be written in form

$$\tilde{Z}_Q(\beta, v, \mu_q, \mu_s) = \int \frac{d\mu(g)}{SU_c(3)} \int \frac{d^3 r}{v/\beta^3} e^{\Theta},$$

(9)
where \( \vec{r} = \vec{R}/\beta \) and

\[
\Theta = \frac{v}{\pi^2 \beta^3 (1 + r^2)^2} \left[ d_G \mathcal{G}(g) + d_q \mathcal{Q}(g, \mu_q) + d_s \mathcal{Q}(g, \mu_s) \right],
\]

with

\[
\mathcal{G}(g) = \sum_{k=0}^{\infty} \frac{1}{k^4} \chi_{1,1}(g^k),
\]

for gluon contribution and

\[
\mathcal{Q}(g, \mu) = 2\Re \sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{k^4} e^{ik\mu} \chi_{1,0}(g^k),
\]

for quark contribution. \( \chi_{1,0}(g) \) and \( \chi_{1,1}(g) = |\chi_{1,0}(g)|^2 - 1 \) are respectively the characters of the fundamental and the adjoint representation of \( SU_c(3) \). As a class function the characters depend on the eigenvalues of \( e^{i\theta_i} \) of \( g \), \( i=1, 2, 3 \): \( \chi_{1,0}(g) = \sum_{i=1}^{i=3} e^{i\theta_i} \) with \( \sum_{i=1}^{i=3} \theta_i = 0 \). In Eq. (11), \( d_G = 2, d_q = 4 \) and \( d_s = 2 \) are the number of degrees of freedom of gluons and quarks, not counting the color multiplicity, already taken into account in the characters since \( \chi_{1,1}(0,0) = 8 \) and \( \chi_{1,0}(0,0) = 3 \).

For high temperature, \( (\beta \to 0) \), integral (9) can be performed by the saddle point method. The maximum of \( \Theta \) is reached for \( r = 0 \) and \( \theta_i = 0 \). Expanding \( \Theta \) near this point to second order in \( r \) and \( \theta_i \) one finds

\[
\Theta = \frac{1}{3 \beta^3} u(\mu_q, \mu_s)(1 - 2r^2) - \frac{v}{2 \beta^3} C(\mu_q, \mu_s) \sum_{i=0}^{i=3} \theta_i^2.
\]

Finally, integration over \( r \) and \( \theta_i \) gives in the saddle point approximation the physical canonical partition function (after Wick rotation)

\[
Z_Q(\beta, v, \mu_q, \mu_s) = \frac{3}{4} \sqrt{2\pi C^{-\frac{4}{3}}(\mu_q, \mu_s)} u^{-\frac{1}{2}} \left( \frac{\beta^3}{v} \right)^{\frac{12}{3}} \exp \left[ \frac{1}{3} \frac{uv}{\beta^3} \right],
\]

where \( u(\mu_q, \mu_s) \) are known functions of quark chemical potentials and gluon and quark degrees of freedom (see Eqs. (31) and (32) below for general case of \( N \) colors).

3.2 The bag density of states and BBSBM mass spectrum

The density of states \( \sigma(W, v, \mu_q, \mu_s) \) of a bag of volume \( v \), at energy \( W \), is derived from its partition function \( Z_Q(\beta, v, \mu_q, \mu_s) \), Eq. (14), through inverse
Laplace transform:

\[
\sigma(W, v, \mu_q, \mu_s) = \frac{1}{2\pi i} \int_{c+i\infty}^{c+i\infty} d\beta \exp(\beta W) Z_Q(\beta, \mu_q, \mu_s). \tag{15}
\]

For large \( W \) this integral can be evaluated by the steepest descent method to obtain the asymptotic bag level density:

\[
\sigma(W, v, \mu_q, \mu_s) = \frac{3}{8} C^{-4} u^{7/2} v^{-3/2} W^{-11/2} \exp\left[\frac{4}{3} u^{1/4} v^{1/4} W^{3/4}\right]. \tag{16}
\]

The GCPF of a gas of hadrons (bags) with mass \( m_i \) and proper volume \( v_i \) can written in the Boltzmann approximation:

\[
Z_H(T, V, \lambda_q, \lambda_s) = \sum_{N=0}^{\infty} \frac{1}{N!} \int \prod_{i=1}^{N} \left[ \frac{d^3 p_i}{(2\pi)^3} dm_i dv_i \left( V - \sum_{j=1}^{N} v_j \right) \right] e^{-\beta \sqrt{p_i^2 + m_i^2}} \tau(m_i, v_i, \lambda_q, \lambda_s) \theta\left( V - \sum_{j=1}^{N} v_j \right), \tag{17}
\]

where \( \tau(m, v, \lambda_q, \lambda_s) dm dv \) is the number of bag states in the mass interval \((m, m+dm)\) and volume interval \((v, v+dv)\). It has been shown that the \( v \) dependence of \( \tau(m, v, \lambda_q, \lambda_s) \) is obtained through the derivative of \( \sigma(W, v, \lambda_q, \lambda_s) \), as given by Eq. (16) with respect to the volume \( v \),

\[
\tau(m, v, \lambda, \lambda_s) = \left. \frac{\partial}{\partial v} \sigma(W, v, \lambda, \lambda_s) \right|_{W=m-Bv}, \tag{18}
\]

where the bag model relation \( W = m - Bv \) between the energy \( W \) and the mass \( m \) is used, \( B \) being the bag constant. This yields the BBSBM asymptotic hadronic mass spectrum:

\[
\tau(m, v, \lambda, \lambda_s) \approx \frac{1}{8} C^{-4} u^{15/4} v^{-9/4} (m-Bv)^{-19/4} \exp\left[\frac{4}{3} u^{1/4} v^{1/4} (m-Bv)^{3/4}\right]. \tag{19}
\]

This is the asymptotic expression of \( \tau(m, v, \lambda_q, \lambda_s) \) derived from the asymptotic expression of \( \sigma(W, v, \lambda, \lambda_s) \). For small masses a phenomenological function \( \tau_0 \) is introduced

\[
\tau_0(m, v, \lambda_q, \lambda_s) = \sum_i g_i(\lambda_q, \lambda_s) \delta(m - m_i) \delta(v - v_i), \tag{20}
\]

where \( g_i(\lambda_q, \lambda_s) \) is the multiplicity modified by fugacities of a hadron of mass \( m_i \) and volume \( v_i \).
4 Description of the HG \rightarrow QGP phase transition

4.1 Pressure partition function

We study here the analytic properties of the GCPF to show that the system undergoes a phase transition from hadron to QGP phase, and we implement the generalization to N-colors. The analytic properties of the GCPF $Z_H(T, V, \lambda_q, \lambda_s)$, Eq. (17), in the thermodynamic limit, ($V \rightarrow \infty$), are most easily investigated by considering the corresponding pressure-GCPF which is its Laplace transform with respect to the volume:

$$\Pi(T, s, \lambda_q, \lambda_s) \equiv \int_0^\infty dV e^{-sV} Z_H(T, V, \lambda_q, \lambda_s).$$ (21)

Substituting Eq. (17) in Eq. (21), one obtains

$$\Pi(T, s, \lambda, \lambda_s) = \frac{1}{s - f(\beta, s, \lambda, \lambda_s)},$$ (22)

where

$$f(\beta, s, \lambda_q, \lambda_s) = f_0 + \frac{1}{2\pi^2 \beta} \int_{V_0}^{\infty} dv \int_{M_0 + Bv}^{\infty} d\tau \int_0^\infty dm^2 K_2(m\beta) \tau(m, v, \cdot \cdot \cdot) e^{-sv}.$$ (23)

and where $f_0 \equiv f_0(\beta, s, \lambda_q, \lambda_s)$ corresponds to the contribution coming from $\tau_0$. The lower limits $M_0$ and $V_0$ have values above which the asymptotic expression (19) begins to hold.

Because of the properties of the inverse Laplace transform the behavior of $Z_H(T, V, \lambda_q, \lambda_s)$ in the thermodynamic limit is governed by the rightmost singularity of $\Pi(T, s, \lambda_q, \lambda_s)$. From Eq. (22) it can be shown that $\Pi$ has two singularities on the real axis of the $s$-plane: a pole $s_0$ given by

$$s_0(\beta, \lambda_q, \lambda_s) = f(\beta, s_0(\beta, \lambda_q, \lambda_s)),$$ (24)

and a singularity $s_c(\beta, \lambda_q, \lambda_s)$ induced by $f(\beta, s, \lambda_q, \lambda_s)$ which remains finite for $s \rightarrow s_c$. Furthermore $s_c$ is found to be given by

$$s_c(\beta, \lambda_q, \lambda_s) = \frac{1}{3} u(\lambda_q, \lambda_s)(\beta^{-3} - B\beta).$$ (25)

For given values of $\lambda_q$ and $\lambda_s$ the transition temperature $T_C(\lambda_q, \lambda_s) = \beta_c^{-1}$ is determined by matching of these two singularities, namely when

$$s_c(\beta_c(\lambda), \lambda) = s_0(\beta_c(\lambda), \lambda) = f(\beta_c(\lambda), s_c(\beta_c(\lambda), \lambda), \lambda).$$ (26)

where we have written $\lambda$ instead of $(\lambda_q, \lambda_s)$ to simplify the notation.
4.2 Properties of hadronic phases

For $T < T_C$, in the hadron phase, the rightmost singularity is $s_0(\beta, \lambda_q, \lambda_s)$ and the partition function is governed by this singularity in the thermodynamic limit. Then the pressure and the energy density are given by

$$P_H(T, \lambda) \equiv \lim_{V \to \infty} \frac{T}{V} \ln Z_H(T, V, \lambda) = T s_0(\beta, \lambda),$$

$$\varepsilon_H(T, \lambda) \equiv -\lim_{V \to \infty} \frac{\partial}{\partial \beta} \ln Z_H(T, V, \lambda) = -\frac{\partial}{\partial \beta} s_0(\beta, \lambda).$$

(27)

For $T > T_C(\lambda_q, \lambda_s)$, the leading singularity is $s_c$ and we have in the thermodynamic limit

$$P_Q(T, \lambda) \equiv T s_c(\beta, \lambda) = \frac{1}{3} u(\lambda) T^4 - B,$$

$$\varepsilon_Q(T, \lambda) \equiv -\frac{\partial}{\partial \beta} s_c(\beta, \lambda) = u(\lambda) T^4 + B.$$  

(28)

Eqs. (28) are the equations of a non-interacting QGP. Of course other quantities like baryon number density, quark number densities can be derived form the expression of the partition function in the thermodynamic limit. Fig. 1 displays the energy density and the pressure for both phases for a value of $B^{1/4} = 225$ MeV, corresponding to $T_C = 150$ MeV. Fig. 2 is the phase diagram obtained by solving Eq. (26) for different values of the strange quark chemical potential $\mu_s$.

4.3 Extension to $N$-colors

The previous results can naturally be extended to $SU_c(N)$. By the same method as in Sec. 3.1 and Sec. 3.2, for $N = 3$, we obtain for the grand canonical partition function

$$Z_Q(\beta, v, \mu_q, \mu_s) = A C^{(N^2 - 4)/2} u^{-3/2} \left( \frac{v}{\beta^3} \right)^{N^2/2} \exp \left[ \frac{1}{3} \frac{uv}{\beta^3} \right],$$

(29)

where

$$A = \frac{3 \sqrt{3} \prod_{i=1}^{N-1} i \sqrt{i}}{8 (2\pi)^{(N-4)/2} \sqrt{N}},$$

(30)
Figure 1: Energy density and pressure versus temperature: solid curve gives the EoS in the case \( \mu_q = \mu_s = 0 \) and for \( B^{1/4} = 225 \) MeV corresponding to \( T = 150 \) MeV. Dashed line is the pressure in the same conditions.

\[
u(\mu_q, \mu_s) = \frac{\pi^2}{30} \left[ (N^2 - 1) d_g + \frac{7}{4} N (d_q + d_s) + \frac{15}{2} d_g \left( \frac{\beta \mu_q}{\pi} \right)^2 \left( 1 + \frac{\beta^2 \mu_q^2}{2\pi^2} \right) + \frac{15}{2} N d_s \left( \frac{\beta \mu_s}{\pi} \right)^2 \left( 1 + \frac{\beta^2 \mu_s^2}{2\pi^2} \right) \right], \tag{31}
\]

and

\[
C(\mu_q, \mu_s) = \left[ \frac{N}{3} d_g + \frac{d_q + d_s}{6} + \frac{d_q}{2} \left( \frac{\beta \mu_q}{\pi} \right)^2 + \frac{d_s}{2} \left( \frac{\beta \mu_s}{\pi} \right)^2 \right]. \tag{32}
\]

By inverse Laplace transform of \( Z_Q(\beta, v, \mu_q, \mu_s) \), we obtain the bag density of levels \( \sigma(W, v, \mu_q, \mu_s) \) for large \( W = m - Bv \), from which the hadronic mass spectrum \( \tau(m, v, \mu_q, \mu_s) \) for large \( m \) is derived, according to Eq. (18)

\[
\tau(m, v, \mu_q, \mu_s) \sim C^{(1-N^2)/2} u^{3(1+N^2)/8} v^7 W^8 \exp \left[ 4/3 u^{1/4} v^{1/4} W^{-3/4} \right], \tag{33}
\]
Figure 2: Critical curves in the plane $T-\mu_q$ for $\mu_s = 0, 100, 200, 300, 400, 500, 536, 560$ MeV, right to left. The lower curve corresponds to $\bar{K}$ condensation.

where $\gamma = -(N^2 + 9)/8$ and $\delta = -(11 + 3N^2)/8$. Furthermore it can be shown that the conditions for a phase transition to occur are $\gamma + \delta = -(5 + N^2)/2 < -3$ and $\delta = -(11 + 3N^2)/8 < -7/4$. Both conditions give $N^2 > 1$. This can also be seen on the mass dependence of the spectrum. From the bag relations, $m = 4Bv = W + Bv$, we have $v = m/4B$ and $W = 3m/4$. Substituting for $W$ and $v$ in Eq. (33) we find

$$\tau(m, \mu_q, \mu_s) \sim m^{-(N^2+5)/2} C^{(1-N^2)/2} u^{3(N^2+3)/8} B^{(9+N^2)/8} \exp \left[ \left( \frac{u}{3B} \right)^{1/4} m \right],$$

where $u$ and $C$ are given by Eq. (31) and Eq. (32). Comparison with a spectrum of the form in Eq. (3) shows that $a = -(N^2 + 5)/2$ which is, for $N > 1$, smaller than $-7/2$ as required for a phase transition to occur (see Sec. 2.2). Thus for $N = 1$ there is no phase transition. For $N = 2$ our approach is not equivalent to lattice-QCD where one find a second order phase transition, a result requiring further study.
5 Conclusions

Needless to repeat here, lattice QCD and its continuum limit is the fundamental theory for describing in a unified scheme the deconfinement phase transition, also for finite baryon density, and we are looking forward to future developments in this direction. As we await these advances, we have offered here a relatively simple extension of the statistical bootstrap model, motivated by some well understood properties of strong interactions. We have shown that in theoretically consistent manner we can study the properties of deconfinement phase transition at finite baryon density. There are several simplifications in our approach which need attention: for example we assumed vanishing quark mass even for the s-quark, and we ignored QCD interactions in the bag. Even so, we find several interesting results: the properties of the latent heat of the phase transition, and the general features of the hadronic mass spectrum are in as one could wish for, and there is no phase transition when $N = 1$.

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