Implementation of adaptive fault-tolerant tracking control for robot manipulators with integral sliding mode

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Abstract
This article presents an adaptive fault-tolerant tracking control for uncertain robot manipulators. By fully considering the effects of uncertainties and actuator effectiveness faults, an integral sliding manifold with a novel auxiliary function is first proposed for tracking control of robot manipulators. Then, a novel fault-tolerant tracking control based on the proposed integral sliding manifold is developed to acquire robustness in both reaching and sliding phase. In order to further reduce the effects of uncertainties on tracking performance, an adaptive law is performed to estimate unknown parameters of the upper bounded uncertainties, which further reduces the control torque input in case of the similar tracking performance. The stability of the proposed approaches is accomplished by Lyapunov stable theory and homogeneous technique. The key contributions of the proposed approach are as follows: (i) the reaching phase of sliding mode control is removed completely in the control design and then the tracking system enters the sliding mode from the initial states; (ii) the nominal control term is eliminated in the design of integral sliding surface and then the algebraic loop problem is also avoided in the proposed approach for robot manipulators; (iii) the simple control structure with an adaptive law is obtained better tracking performance by utilizing lower control torque input and then the effects of time-delay and computational burden are also restrained from the proposed approach. Simulation and experimental comparisons have been accomplished for verifying the effectiveness of the proposed approach.

KEYWORDS
actuator effectiveness faults, adaptive control, fault-tolerant control, homogeneous technique, integral sliding mode control, robot manipulators

1 INTRODUCTION

Inspiriting by high efficiency, high reliability, low cost, and high precision, robot manipulators have widely been utilized in various industrial application fields such as drilling, welding, and painting. In particular, the safety of robot manipulators is gradually promoted on an important place with the increased control precision requirements. The faults of robot manipulators derive mainly from two aspects including the sensor faults and the actuator faults, which not only reduce

Abbreviations: AFTC-ISM, Adaptive FTC with integral sliding mode control; FTC, fault-tolerant tracking control; ISMC, integral sliding mode control; UDEDs, uncertain dynamics and external disturbances.
the control precision, but also threaten human safety. Among them, the actuator faults have a great impact on the tracking performance of robot manipulators than sensor faults and thus attracted more attention in recent research results. In addition, uncertain dynamics and external disturbances (UEDEs) are another kind of factors affecting the control performance of robot manipulators. Accordingly, it is still a challenging issue to develop a robust control of robot manipulators subject to uncertainties and actuator effectiveness faults (UAEFs).11-13

With its simplicity and strong robustness, sliding mode control (SMC) is commonly applied for the elimination of matched uncertainties of nonlinear systems.4,9,14-19 Among them, the applications of the SMCs suffer from a large obstacle, that is, the chattering phenomenon. To this end, the integral sliding mode control (ISMNC) plays an important role for suppressing the chattering magnitude and enhancing the tracking precision.20-25 The ISMC is classified into two categories, that is, the ISMC with/without the reaching phase.16,26,27 Among them, the ISMCs without the reaching phase has an effectiveness of chattering suppression than that with the reaching phase. A nonlinear integral-type SMC without involving the reaching phase is proposed in the work20 for both matched and unmatched uncertain systems, which give rise to a common definition of ISMC along with its stability analysis. The ISMC without the reaching phase reveals an enormous advantages in both tracking precision and chattering suppression.28 On the other hand, actuator faults of robot manipulators are another major factor affecting the tracking performance and system stability. The actuator faults are offer eliminated by a fault-tolerant tracking control (FTC) with/without the faults observer.29,30 As a result, FTCs are always decomposed into the passive FTC (PFTC) and active FTC (AFTC). For AFTC, the actuator faults are obtained by an fault observer,31,32 but the calculated amount of controller will be increased vastly.14 On the contrary, the actuator effectiveness faults (AEF) for PFTC are determined as uncertainties of robot manipulators. Meanwhile, the PFTC does not claim the feedback of faults observer and indeed decreases the computational burden of controller. In conclusion, the PFTC guarantees rapid transient response than the active one, but employs larger control input than the active one.33 With the advantages of the SMCs and FTCs, it is still a challenging work to develop a strong robustness control to overcome the effects of the UAEFs for acquiring the high-precision tracking performance.

Based on these seminal works, several approaches combining SMCs and FTCs have been developed to overcome the effects of the UAEFs on the tracking performance of nonlinear systems. SMCs are always injected into the FTCs to overcome the effects of UAEFs on tracking performance.34,35 Upon the advantages of time-delay estimation and backstepping techniques, Mien et al.14,15 propose a novel SMC for the finite-time stable FTC systems. However, these approaches14,15,34,35 neglect the effects of the algebraic loop problem on tracking performance of robot manipulators, in which the joint acceleration or its upper bound is involved into the control design. In the applications of robot manipulators, the algebraic loop problem not only amplifies the control torque input but also destroys the stability of robot tracking system.36 Inspired by the chattering-suppression capability, several ISMCs20,28,37-40 have been proposed for FTC system to the practical applications of various nonlinear systems. In particular, the work37 provides an ISMC for FTC of a linear time-invariant system; while the detailed analysis of ISMC for a nonlinear FTC system is proposed in the work.38 In the approaches,39,40 the ISMCs are further developed in the control formulation of attitude FTC of spacecraft. By utilizing a fuzzy logic system, Mien Van et al.41 proposed an ISMC for FTC of robot manipulators, in which the disturbance observer is introduced to estimate the unknown nonlinear terms. These approaches are of great significance to the development of ISMCs uninvolved the reaching phase. However, in practical applications above ISMCs20,28,37-40 without the reaching phase ignore several facts as follows: (i) the integral sliding surface includes the nominal control input, which may cause the new algebraic loop problem;36 (ii) the stability analysis on sliding phase is not provided in these existing approaches.

It is well-known that the magnitude of controller chattering is proportional to the control gain which is commonly chosen as a larger value than the upper bound of uncertainties. Several approaches such as the boundary method,37 the neural network, the fuzzy logic system and adaptive SMC,42-47 have been developed to reduce the effects of chattering phenomenon of SMC/ISMNC. In these approaches, the chattering-restraining is mainly divided into two categories, that is, the adaptive estimation of the control gain related to the upper bound of uncertainties48-51 and the whole estimation of the lumped uncertainties. Among them, the neural network and the fuzzy logic techniques6-8,42,43 are always acted as the whole estimation of the lumped uncertainties for SMC/ISMNC of robot manipulators. Nevertheless, these approaches42-46 bring great tracking performance and meanwhile introduces several limitations such as high time-delay, large calculated amount, and complex control structure.

From above overview, there exists several factors which limits the practical application of ISMC for robot manipulators. These factors are as follows: (i) the nominal control term have been involved in the formulation of the integral sliding mode manifold without the reaching phase;20,41,52 (ii) the detailed stability analysis in sliding phase is not completed yet carefully; (iii) the integral sliding mode control is not introduced into the tracking control of robot manipulators in case of the effects of uncertain dynamics, external disturbance and actuator effectiveness faults. It is well-known that SMCs
maintain the invariant property to external disturbances and uncertain dynamics only in sliding phase rather than that of reaching phase. In other words, the strong robustness of conventional SMC is only acted on the sliding phase rather than in both reaching and sliding phases. Accordingly, our design is focused on a novel integral sliding mode control uninvolved the reaching phase which removes the nominal control term in the design of integral sliding manifold and is applying to the FTC system of robot manipulators by combining the adaptive technique. The main control design is included as follows: (i) a novel integral sliding manifold uninvolved the reaching phase and nominal control term is first constructed to the formulation of the proposed ISMC for FTC system of robot manipulators; (ii) an adaptive law is adopted to reduce the effects of UAEFs on tracking performance in which lower efforts of actuators are utilized to acquire better tracking performance than other integral sliding mode control. Finally, the simulation and experimental results have been accomplished on two-DOFs robot manipulators to verify the superior performance of the proposed approach in comparison with other state-of-the-art controllers for the FTC systems. In summary, the contributions of the proposed approach are as follows

(i) A novel integral sliding manifold with an auxiliary function is first proposed to the proposed ISMC uninvolved the reaching phase for FTC system of robot manipulators. For the proposed approach, the detailed stability analysis in sliding phase has been accomplished completely by utilizing the homogeneous technique. Then, an adaptive law has been adopted to the proposed approach in order to obtain better tracking performance than other SMCs in case of lower control torque input.

(ii) Distinguished from the approaches, the proposed integral sliding manifold removes the nominal control term in the proposed ISMC design. Then, the lower control efforts are adopted to obtain high steady-state tracking precision than other SMCs.

(iii) Compared with the works, the proposed approach removes the assumption that the inertia matrix $M(q)$ is exactly known in advance. In practical application, the exact knowledges of robot model cannot be acquired precisely. The proposed approach only requires the knowledges of the constant matrix $M_0$.

(iv) An adaptive technique is developed to assess unknown parameters of the upper bounded uncertainties. As a result, the simple control structure with an adaptive law is to acquire better tracking performance by utilizing lower control torque input and then the effects of time delay and computational burden are also restrained from the proposed approach.

The remainder of article are organized as follows: some fundamental facts and robot properties are illustrated in Section 2. The adaptive fault-tolerant approaches along with their stability analysis are developed in Section 3. The simulation and experimental results have been accomplished in Sections 4 and 5, respectively.

2 PRELIMINARIES

To benefit for the control formulation, the homogeneity definition and some lemmas are introduced for robot manipulators.

**Definition 1.** For a proper positive definite function $V : \mathbb{R}^n \to \mathbb{R}$, if $V (u^1 z_1, \ldots, u^n z_n) = u^k V (z)$, $\forall z \in \mathbb{R}^n$ where an arbitrary positive constant $u > 0$ and $r = (r_1, \ldots, r_n) \in \mathbb{R}_+^n$, $V$ is homogeneous of degree $k$. If the function $f_i$ is a homogeneous function of degree $r_i + k$ for all $1 \leq i \leq n$, then the vector $f = [f_1 \ldots f_n]^T$ is homogeneous of degree $k$.

**Lemma 1.** Consider a system

$$z(t) = h(t, z), \quad z(0) = z_0, \quad (1)$$

where $z \in \mathbb{R}^n$ and $h : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function. The solutions of (1) are understood in the sense of Filippov if $h(t, z)$ is discontinuous. Suppose the origin is an equilibrium point of (1). If there exists a continuous positive definite function $V(z) : \mathbb{R}^n \to \mathbb{R}$ such that $V(z) + k(V(z))^r \leq 0$, $z \in \mathbb{R}^n \setminus \{0\}$ with a real number $k > 0$ and $r \in (0, 1)$. Then the origin is a globally finite-time stable equilibrium of (1) and the settling time is

$$T(z_0) \leq \frac{1}{k(1 - r)} V^{1-r}(z_0). \quad (2)$$
2.1 | Robot manipulators and properties

Consider the following robot manipulators with \(n\)-DOFs as

\[
\overline{M}(q)\ddot{q} + \overline{C}(q, \dot{q})\dot{q} + \overline{G}(q) = \Gamma \tau + \overline{d},
\]

(3)

where \(q, \dot{q}, \ddot{q} \in \mathbb{R}^n\) define the joint position, velocity and acceleration vectors, respectively, \(\overline{M}(q), \overline{C}(q, \dot{q}) \in \mathbb{R}^{n \times n}\) represent the symmetric positive definite inertia matrix and the centrifugal-Coriolis matrix, respectively, \(\overline{G}(q) \in \mathbb{R}^n\) stands for the gravitational torque vector, \(\overline{d} \in \mathbb{R}^n\) is the bounded external disturbances satisfying the condition \(\|\overline{d}\| \leq \overline{d}_m\) with a unknown constant \(\overline{d}_m\), \(\overline{\tau} \in \mathbb{R}^n\) denotes the control input, \(\Gamma = \text{diag} \{\overline{\tau}_i(t)\} \cdot i = 1 \ldots n\) stand for the actuator health condition with \(\overline{\tau}_0 \leq \overline{\tau}_i(t) \leq 1\), and \(\overline{\tau}_0 \in (0, 1]\) represents a known positive constant.

**Assumption 1.** The robot manipulators model (3) is modified as

\[
\overline{M}(q) = \overline{M}_0(q) + \Delta \overline{M}(q), \quad \overline{C}(q, \dot{q}) = \overline{C}_0(q, \dot{q}) + \Delta \overline{C}(q, \dot{q}), \quad \overline{G}(q) = \overline{G}_0(q) + \Delta \overline{G}(q),
\]

(4)

where \(\overline{M}_0(q), \overline{C}_0(q, \dot{q}), \text{ and } \overline{G}_0(q)\) represent known dynamics, and \(\Delta \overline{M}(q), \Delta \overline{C}(q, \dot{q}), \text{ and } \Delta \overline{G}(q)\) illustrate uncertainties.

To benefit for subsequent control development and stability analysis, the following vectors have been illustrated.

\[
\text{Sgn}(\kappa) = \begin{bmatrix} \text{sign}(\kappa_1) \\ \vdots \\ \text{sign}(\kappa_n) \end{bmatrix} \in \mathbb{R}^n, \quad \text{Sig}^{\alpha}(\kappa) = \begin{bmatrix} \text{sign}^{\alpha}(\kappa_1) \\ \vdots \\ \text{sign}^{\alpha}(\kappa_n) \end{bmatrix} \in \mathbb{R}^n,
\]

(5)

where \(\alpha > 0\) denotes the designed positive constant, \(\kappa = [\kappa_1 \ldots \kappa_n]^T \in \mathbb{R}^n\), \(i = 1 \ldots n\) denote a vector, \(\kappa_i\) is the \(i\)th component of the vector \(\kappa\) and \(\text{sig}^{\alpha}(\kappa_i) = |\kappa_i|^{\alpha}\text{sign}(\kappa_i), \ i = 1 \ldots n\) with \(\text{sign}(\kappa_i)\) stands for the standard sign function.

The key designs of this article are composed of two stages as follows: (i) an novel integral manifold uninvolved the reaching phase with its robust controller is proposed to overcome the UAEFs; (ii) combining with an adaptive technique, the proposed approach is further extended as an adaptive FTC for robot manipulators. In this article, the errors in position and velocity tracking are represented as

\[
e = q - q_d, \quad \dot{e} = \dot{q} - \dot{q}_d,
\]

(6)

where \(q_d\) and \(\dot{q}_d\) denote the desired position and velocity of joints, respectively.

2.2 | Error system development

Upon the results of Assumption 1 and (6), robot system (3) is further degenerated as

\[
\overline{M}_0(q)\ddot{e} = \Gamma \tau + \overline{\rho} + \overline{\eta},
\]

(7)

where the nominal part \(\overline{\eta} \in \mathbb{R}^n\) and the lumped uncertainty \(\overline{\rho} \in \mathbb{R}^n\) are given as

\[
\overline{\rho} = -\Delta \overline{M}(q)\ddot{q} - \Delta \overline{C}(q, \dot{q})\dot{q} - \Delta \overline{G}(q),
\]

(8)

\[
\overline{\eta} = -\overline{C}_0(q, \dot{q})\dot{q} - \overline{G}_0(q) - \overline{M}_0(q)\ddot{q}_d.
\]

(9)

In light of Assumption 1 and system (3), it follows that

\[
\Delta \overline{M}(q)\ddot{q} = \Gamma \tau - \overline{C}(q, \dot{q}) - \overline{G}(q) + d - \overline{M}_0(q)\ddot{q}.
\]

(10)
By utilizing (10) and system (3), we have obtained
\[
\Delta \ddot{M}(q)\ddot{q} = \ddot{\Gamma} - \dddot{C}(q, \dot{q}) - \dddot{G}(q) + d - \dddot{M}_0(q)\dddot{M}^{-1}(q) \left( \dddot{\Gamma} - \dddot{C}(q, \dot{q}) - \dddot{G}(q) + d \right) \\
= \left( I_n - \dddot{M}_0(q)\dddot{M}^{-1}(q) \right) \left( \dddot{\Gamma} - \dddot{C}(q, \dot{q}) - \dddot{G}(q) + d \right) \\
= E(\dddot{\Gamma} - \dddot{C}(q, \dot{q})\dot{q} - \dddot{G}(q) + d),
\]
(11)
where \( I_n \) stands for the unit matrix and \( E \in \mathbb{R}^{n \times n} \) illustrated by Zhu \(^{36} \) is
\[
\dddot{E} = I_n - \dddot{M}_0(q)\dddot{M}^{-1}(q).
\]
(12)
Inspired by the work, \(^{36} \) the nominal matrix \( \dddot{M}_0(q) \) is represented as
\[
\dddot{M}_0 = \frac{2}{\bar{\gamma}_1 + \bar{\gamma}_2} I_n,
\]
(13)
where \( \bar{\gamma}_1 \) and \( \bar{\gamma}_2 \) are illustrated as follows \(^{36} \)
\[
\bar{\gamma}_1 \leq \left\| \dddot{M}^{-1}(q) \right\| \leq \bar{\gamma}_2.
\]
(14)
then
\[
\left\| \dddot{E} \right\| \leq \bar{\sigma}
\]
(15)
with \( \bar{\sigma} > 0 \) given as
\[
\bar{\sigma} = \frac{\bar{\gamma}_2 - \bar{\gamma}_1}{\bar{\gamma}_1 + \bar{\gamma}_2},
\]
(16)
where the proof of the fact related to (15) and (16) can be found in work. \(^{36} \)
In light of Assumption 1, (11) and (15), the upper bound of \( \bar{\rho} \) given by (8) is \(^{54} \)
\[
\left\| \bar{\rho} \right\| \leq k_0 + k_1 \left\| \dot{q} \right\| ^2 + \bar{\sigma} \left\| \dddot{\Gamma} \right\| ,
\]
(17)
where \( k_0 > 0 \) and \( k_1 > 0 \) denote two positive constants, and \( \bar{\sigma} \) is given by (16).
The proof of (17) can be found in the work. \(^{54} \)

Remark 1. Different from the work, \(^{2} \) the proposed approach removes the assumption that the lumped uncertainties \( \bar{\rho} \) given by (8) are upper bound by a known constant. Moreover, the proposed approach employs an appropriate control gain than the work \(^{2} \) owing to the combination of the adaptive technique and model transformation of robot manipulators given by (7)–(16).

3 METHODOLOGY

3.1 Integral sliding mode control

3.1.1 Control development

For system (7), in this part we have aimed to develop a robust FTC with integral sliding mode (RFTC-ISM).
First, the following nonlinear function is introduced as

$$f(e_i) = \begin{cases} K_1 e_i + K_2 \text{sig}^2(e_i), & |e_i| < \kappa \\ \text{sig}'(e_i), & |e_i| \geq \kappa, \end{cases}$$  

(18)

where $0 < r < 1$ denotes a positive constant, $0 < \kappa < 1$ stands for an arbitrary small positive constant, two constants are defined as $K_1 = (2 - r)\kappa^{r-1}$ and $K_2 = (r - 1)\kappa^{r-2}$, $\text{sig}'(e_i)$ is given by (5), $e_i$ represents the $i$th component of $e$ given by (6), and $\frac{df(e_i)}{de_i}$ is represented as

$$\tilde{g}(e_i) = \frac{df(e_i)}{de_i} = \begin{cases} K_1 + 2K_2 |e_i|, & |e_i| < \kappa \\ r|e_i|^{r-1}, & |e_i| \geq \kappa. \end{cases}$$  

(19)

To obtain the proposed RFTC-ISM approach, an auxiliary function is first defined as

$$\mu = e + \alpha e_s,$$  

(20)

where $\alpha > 0$ is a positive constant, and

$$\dot{e}_s = \bar{F}(e), \quad e_s(0) = -e(0)\alpha,$$  

(21)

$$\bar{F}(e) = \left[ f(e_1), \ldots, f(e_n) \right]^T$$  

(22)

with $f(e_i)$ is illustrated by (18).

In virtue of (20) and (21), a novel integral sliding manifold is first proposed as

$$S = \dot{\mu} + \beta \mu_t$$  

(23)

with

$$\dot{\mu}_t = \text{Sig}^\gamma_1(\mu) + \text{Sig}^\gamma_2(\dot{\mu}),$$  

(24)

$$\mu_t(0) = -\frac{\dot{\mu}(0)}{\beta} = -\frac{\dot{\mu}(0) + \alpha \bar{F}(e(0))}{\beta},$$  

(25)

where $\text{Sig}^\gamma(\cdot)$ is defined by (5), $\gamma_1$ and $\gamma_2$ are two positive constants satisfying the conditions $0 < \gamma_1 < 1$ and $\gamma_2 = 2\gamma_1/(\gamma_1 + 1)$, $\beta$ denotes a known positive constant, and $e_i(0), \mu(0), e(0)$, and $\dot{e}(0)$ are the initial states of state variables, respectively.

Remark 2. Observed by (20)–(25), $S(0) = 0$ and $\mu(0) = 0$ are guaranteed in state space at the very beginning of tracking control. As a result, the purpose of our work is transformed into design an ISMC approach such that the proposed integral sliding manifold (23) is stablized at the origin for all time. Then, the proposed approach removes the reaching phase of SMC systems and then wiped out the reaching time. Compared with the works, the proposed integral sliding manifold (23)–(25) does not involve the nominal control term $u_0$ and employs a simple structure. Above advantage is one of the main contribution of the proposed approach.

Then, the proposed RFTC-ISM is represented as

$$\bar{r} = \bar{r}_0 + \bar{r}_1 + \bar{r}_2$$  

(26)

with

$$\bar{r}_0 = -\bar{\eta} - aM_0\bar{H}(e)\dot{e} - \beta M_0 \dot{\mu}_t - \left( \frac{2}{\gamma_1 + \gamma_2} \right)^{1/2} \delta b(S),$$  

(27)
\[
\tau_1 = -b(S)\bar{w},
\]
\[
\tau_2 = -\frac{1 - \bar{\tau}_0}{\bar{\tau}_0}b(S)\|\bar{\tau}_0 + \bar{\tau}_1\|,
\]
where \(\bar{n}\) and \(\bar{\mu}_1\) are involved from (9) and (24), respectively, \(\beta\) is defined by (25), \(\theta\) denotes a positive constant, \(\bar{M}_0\) is defined by (13), \(\bar{\tau}_1\) and \(\bar{\tau}_2\) are given by (14) and
\[
\bar{H}(e) = \text{diag}\{\bar{g}(e_i)\}, \quad i = 1 \ldots n,
\]
\[
b(S) = \begin{cases} \frac{S}{\|S\|}, & \|S\| \neq 0 \\ 0, & \|S\| = 0, \end{cases}
\]
\[
\bar{w} = \frac{1}{1 - \bar{\sigma}}(k_0 + k_1\|\dot{q}\|^2 + \bar{\sigma}\|\bar{\tau}_0 + \bar{\tau}_2\|).
\]
and \(\bar{g}(e_i)\) is represented in (19), \(k_0\) and \(k_1\) are given by (17), respectively, and \(\bar{\sigma}\) is defined by (16).

From (26) to (32), system (7) can be transformed as
\[
\bar{M}_0\bar{\nu} = \bar{\tau}_0 + \bar{\tau}_1 + \bar{\Gamma}\bar{\tau}_2 - (I_n - \bar{\Gamma})(\bar{\tau}_0 + \bar{\tau}_1) + \bar{\rho} + \bar{n}.
\]

**Remark 3.** It is well-known that the conventional SMC maintains the invariant property and strong robustness to external disturbances and uncertain dynamics only in sliding phase rather than that of reaching phase. On the other hand, the chattering phenomenon of SMC comes mainly from the high-frequency switching between the reaching phase and sliding phase. \(^{28}\) For our design, the proposed approach (26)–(32) guarantees the states of robot tracking system on the proposed sliding manifold for all time in case of the elimination of the reaching phase. Thus, from the perspective of sliding mode theory, the chattering phenomenon can be reduced in a certain extent. In other words, the states of robot system controlled by the proposed approach is still stabled at the proposed integral sliding manifold to origin for all time.

**Remark 4.** Observed by (26)–(32), their upper bounds \(\|\bar{\tau}_1\|\) and \(\|\bar{\tau}_2\|\) cannot be included in the design of control parts \(\bar{\tau}_1\) and \(\bar{\tau}_2\) given by (28) and (29), respectively. As a result, the algebraic loop problem \(^{36}\) is removed in the sliding mode tracking system of uncertain robot manipulators. Given full consideration to uncertain robot manipulators and actuator faults, the proposed approach reserves a simple control structure and advanced convergence property of ISMC system than other ISMC\(^{20,28,37-40}\) to implement the trajectory tracking.

### 3.1.2 Stability analysis

In this section, the stability analysis and discussion have been accomplished.

**Theorem 1.** For system (33), the designed controller (26)–(32) and the proposed integral sliding manifold (23), the errors in position and velocity tracking converge globally to zero with the finite-time stability. First, the tracking trajectories are still restrained on the sliding manifold (23) excluding the reaching phase for all time, that is, \(S = 0\) if \(t > 0\) and \(S(0) = 0\). Second, an auxiliary function \(\mu, \nu, \text{ and } \nu\) along with the proposed sliding manifold (23) are guaranteed to zero with finite-time stability.

**Proof.** First, there exists a Lyapunov function as
\[
V = \frac{1}{2}S^T\bar{M}_0S,
\]
where \(\bar{M}_0\) is given by (13).

Then, the time differential of \(V\) is
\[
\dot{V} = S^T\bar{M}_0\dot{S} = S^T\bar{M}_0(\ddot{\nu} + \beta\dot{\nu}_1)
\]
where (20), (23), (26), and (33) are utilized.

Upon substituting (27) into (35) yields

\[
\dot{V} = S^T(\tau_0 + \tau_1 + \Gamma \tau_2 - (I_n - \Gamma)(\tau_0 + \tau_1) + \tilde{\rho} + \frac{2}{\tilde{r}_1 + \tilde{r}_2})^{1/2} \theta \| S \|
\leq S^T \tau_1 + S^T \Gamma \tau_2 + \| S \| \left\| I_n - \Gamma \right\| \| \tau_0 + \tau_1 \| + \| S \| \| \tilde{\rho} \| - \theta V^{1/2},
\]

(36)

where \( V = \frac{2}{\tilde{r}_1 + \tilde{r}_2} \| S \|^2 \) is involved from (34).

In light of (17) and (28), \( S^T \tau_1 + \| S \| \| \tilde{\rho} \| \) is rewritten as

\[
S^T \tau_1 + \| S \| \| \tilde{\rho} \| \leq -S^T b(S)\tilde{w} + \| S \| \left( k_0 + k_0 \| \dot{q} \|^2 + \| \tau \| \right).
\]

(37)

Substituting (31) into (37), it follows that

\[
S^T \tau_1 + \| S \| \| \tilde{\rho} \| \leq -\tilde{w} \| S \| + \left( k_0 + k_1 \| \dot{q} \|^2 + \| \tau \| \right) \| S \|
= -(1 - \tilde{\sigma})\tilde{w} \| S \| - \tilde{\sigma} \tilde{w} \| S \| + \left( k_0 + k_1 \| \dot{q} \|^2 + \| \tau \| \right) \| S \|,
\]

(38)

where \(- (1 - \tilde{\sigma})\tilde{w} \| S \| - \tilde{\sigma} \tilde{w} \| S \| = -\tilde{w} \) is involved.

Applied (32) to (38), we have obtained

\[
S^T \tau_1 + \| S \| \| \tilde{\rho} \| \leq - \left( k_0 + k_1 \| \dot{q} \|^2 + \| \tau_0 + \tau_2 \| \right) \| S \| - \tilde{\sigma} \tilde{w} \| S \| + \left( k_0 + k_1 \| \dot{q} \|^2 + \| \tau \| \right) \| S \|
\leq - \left( k_0 + k_1 \| \dot{q} \|^2 + \| \tau_0 + \tau_2 \| \right) \| S \| - \tilde{\sigma} \tilde{w} \| S \| + \left( k_0 + k_1 \| \dot{q} \|^2 + \| \tau_0 + \tau_2 \| \right) \| S \| + \tilde{\sigma} \| \tau_1 \| \| S \|
= -\tilde{\sigma} \tilde{w} \| S \| + \tilde{\sigma} \| \tau_1 \| \| S \|,
\]

(39)

where \( \| \tau \| = \| \tau_0 + \tau_1 + \tau_2 \| \leq \| \tau_0 + \tau_2 \| + \| \tau_1 \| \) is involved from (26).

In light of the facts \( \| \tau_1 \| = \tilde{w} \) with \( \tilde{w} > 0 \) from (28) and (32), (39) can be simplified as

\[
S^T \tau_1 + \| S \| \| \rho \| \leq -\tilde{\sigma} \tilde{w} \| S \| + \tilde{\sigma} \| \tau_1 \| \| S \| = 0.
\]

(40)

By utilizing (40), (36) can be degenerated as

\[
\dot{V} \leq S^T \Gamma \tau_2 + \| S \| \left\| I_n - \Gamma \right\| \| \tau_0 + \tau_1 \| - \theta V^{1/2},
\leq S^T \Gamma \tau_2 + (1 - \overline{\tau}_0) \| \tau_0 + \tau_1 \| \| S \| - \theta V^{1/2},
\]

(41)

where \( \| \Gamma \| \geq \overline{\tau}_0 \) and \( \left\| I_n - \Gamma \right\| \leq 1 - \overline{\tau}_0 \) are utilized from (3).

Substituting (29) into (41), we have obtained

\[
\dot{V} \leq -\frac{1 - \overline{\tau}_0}{\overline{\tau}_0} S^T b(S) \| \tau_0 + \tau_1 \| + (1 - \overline{\tau}_0) \| \tau_0 + \tau_1 \| \| S \| - \theta V^{1/2}.
\]

(42)

Applied (31) to (42), it follows that

\[
\dot{V} \leq -\frac{1 - \overline{\tau}_0}{\overline{\tau}_0} \| \Gamma \| \| \tau_0 + \tau_1 \| \| S \| + (1 - \overline{\tau}_0) \| \tau_0 + \tau_1 \| \| S \| - \theta V^{1/2}
\leq - (1 - \overline{\tau}_0) \| \tau_0 + \tau_1 \| \| S \| + (1 - \overline{\tau}_0) \| \tau_0 + \tau_1 \| \| S \| - \theta V^{1/2}
\leq -\theta V^{1/2},
\]

(43)

where \( \| \Gamma \| \geq \overline{\tau}_0 \) is used from (3).
From (20) and (23), it is observed that the proposed integral SMC (26)–(32) removes the reaching phase, that is, $S = 0$ if $t > 0$, $S(0) = 0$ and $\dot{\mu}(0) = 0$. Combined with the result of (43), we have obtained a fact that the tracking trajectories are still restrained on the proposed sliding manifold (23) for all time (i.e., $S = 0$ if $t > 0$). The results of above analysis further verify our conclusions in Remarks 2–4.

**Remark 5.** Different from the works,20,41 another key contribution is that our design provides a detailed stability analysis on sliding phase. Thus, the subsequent content focus on the stability analysis of ISMC on sliding phase (i.e., $S = 0$).

If $S = 0$ for all time, the integral sliding manifold (23) and (24) is further degenerated as

$$\ddot{\mu} = \text{Sign}(\mu) + \text{Sign}(\dot{\mu}).$$

For system (44), we chose the following Lyapunov candidate function as

$$V_1 = \frac{1}{1 + \gamma_1} \sum_{i=1}^{n} |\mu_i|^{\gamma_1+1} + \frac{1}{2} \dot{\mu}^T \dot{\mu},$$

where $\gamma_1$ is given by (24).

By selecting the proper $\gamma_1$ and $\gamma_2$ given by (24), similar to Step 3 of work,16 system (44) with $k : (\mu, \dot{\mu}) \rightarrow (\zeta^{2/(\gamma_1+1)} \mu, \zeta \dot{\mu})$ is homogeneous where $k = (\gamma_1 - 1)/(\gamma_1 + 1) < 0$. By combining Lemma 1 and the asymptotically stable system from (44) and (45), $\mu = 0$ and $\dot{\mu} = 0$ are guaranteed within a finite time.

In virtue of the results of $\mu = 0$ and $\dot{\mu} = 0$, (20) is divided into Cases 1 and 2.

**Case 1.** If $|e_i| \geq \kappa$, (18), (20), and (23) are

$$e_i + a e_{\text{sl}} = 0, \quad \dot{e}_{\text{sl}} = \text{sign}(e_i).$$

The integrator (46) is degenerated as

$$\dot{e}_{\text{sl}} = \text{sign}(-a e_{\text{sl}}) = -\text{sign}(a e_{\text{sl}}).$$

Then, the time of $e_{\text{sl}}$ taken to the origin is obtained

$$T_f = \frac{|e_{\text{sl}}(0)|^{1-r}}{a'^{1-r}} = \frac{|e(0)|^{1-r}}{a(1-r)}.$$

**Case 2.** When $|e_i| < \kappa$, similarly, (18) and (20) can be further changed as

$$e_i + a e_{\text{sl}} = 0, \quad \dot{e}_{\text{sl}} = K_1 e_i + \overline{K}_2 \text{sign}^2(e_i).$$

For system (49), it is further converted as

$$\dot{e}_{\text{sl}} = -a K_1 e_i - \overline{K}_2 \text{sign}^2(a e_{\text{sl}}).$$

Given a Lyapunov function candidate $V_2 = 1/2 e_{\text{sl}}^2$ for (50), then its time differential is

$$V_2 = e_{\text{sl}} \dot{e}_{\text{sl}} = -a K_1 e_i^2 - a \overline{K}_2 |e_{\text{sl}}|^3 < 0.$$

Based on Lyapunov stability and the result of (51), the states $e_{\text{sl}}$ and $e_i$ arrived at zero asymptotically.

Inspired by Cases 1 and 2, the following conclusions can be obtained: (i) first the tracking trajectories of the fault-tolerant tracking system are restrained on the proposed integral sliding manifold for all time owing to the proposed approach removes the reaching phase of SMC; (ii) the auxiliary variables $\mu$ and $\dot{\mu}$ arrive at zero with finite-time stability; and (iii) with a finite-time stable auxiliary function $\mu$, the position tracking
errors reach to an arbitrarily small range \( B_\kappa = \{ e_i | |e_i| \leq \kappa \} \) with finite-time convergence, and then asymptotic stability is guaranteed. This completes the proof.

3.2 Adaptive fault-tolerant tracking control with integral sliding mode

In order to further reduce the control torque input on the tracking performance, in this section an adaptive FTC with integral sliding mode (AFTC-ISM) scheme is further developed to estimate unknown parameters \( k_0 \) and \( k_1 \) from (17). The control law is the same as (26)–(32) except for \( \tau_1 \). In addition, \( \tau_1 \) and an adaptive law are proposed as

\[
\tau_1 = -b(S)\overline{w}_1
\]

with

\[
\overline{w}_1 = \frac{1}{1-\sigma}(\dot{k}_0 + \dot{k}_1 \| \dot{q} \|^2 + \sigma \| \tau_0 + \tau_1 \|).
\]

where \( \sigma \) is defined by (16), \( \dot{k}_0 \) and \( \dot{k}_1 \) are the estimations of unknown parameters \( k_0 \) and \( k_1 \) of (17), and the adaptive law is

\[
\dot{k}_0 = \theta_0 \| S \|, \quad \dot{k}_1 = \theta_1 \| \dot{q} \|^2 \| S \|,
\]

where \( \theta_0 \) and \( \theta_1 \) stand for two estimated parameters.

For the proposed AFTC-ISM, there exists the following theorem to guarantee the stability.

**Theorem 2.** For system (3) and the lumped uncertainty (8) with its upper bound (17), the parameters \( k_0 \) and \( k_1 \) are assumed to be completely unknown. For the proposed controller (26)–(32), (52), and (53) and the adaptive law (54), the tracking trajectories are still restrained on the proposed sliding manifold (23) for all time, and \( e_i \) arrived at an arbitrarily small range \( B_\kappa \) with finite-time convergence, thereafter converges to the origin asymptotically.

**Proof.** Given a proper Lyapunov function as

\[
V_3 = \frac{1}{2} S^T M_0 S + \sum_{i=0}^{1} \frac{1}{2\theta_i} (\hat{k}_i - k_i)^2.
\]

In virtue of system (33), \( \dot{V}_3 = \frac{dV_3}{dt} \) is

\[
\dot{V}_3 \leq S^T \left( \tau_1 + \Gamma \tau_2 - (I_n - \Gamma) (\tau_0 + \tau_1) + \varphi + \alpha M_0 \dot{e}_s + \beta M_0 \mu_1 \right) + \sum_{i=0}^{1} \frac{1}{\theta_i} \left( \hat{k}_1 - k_1 \right) \dot{k}_i.
\]

Substituting (27) and (54) into (56) yields

\[
\dot{V}_3 \leq S^T \tau_1 + S^T \Gamma \tau_2 + \| S \| \left( \| I_n - \Gamma \| \| \tau_0 + \tau_1 \| + \| \varphi \| \| S \| + (\hat{k}_0 - k_0) \| S \| + (\hat{k}_1 - k_1) \| \dot{q} \|^2 \| S \| - \theta \left( \frac{2}{\gamma_1 + \gamma_2} \right) \| S \| \right).
\]

Applying (29) to (57), it follows that

\[
\dot{V}_3 \leq S^T \tau_1 - \frac{1-\tau_0}{\tau_0} \| \Gamma \| \| S \| \| \tau_0 + \tau_1 \| + \| S \| \| I_n - \Gamma \| \| \tau_0 + \tau_1 \| + \| \varphi \| \| S \| + (\hat{k}_0 - k_0) \| S \| + (\hat{k}_1 - k_1) \| \dot{q} \|^2 \| S \| - \theta \left( \frac{2}{\gamma_1 + \gamma_2} \right) \| S \|.
\]
By utilizing the facts \(\|\Gamma\| \geq \tilde{\tau}_0\) and \(\|I_n - \Gamma\| \leq 1 - \tilde{\tau}_0\) from (3), (58) is modified as

\[
V_3 \leq S^T \tau_1 + \|\bar{p}\| \|S\| + (\hat{k}_0 - k_0) \|S\| + (\hat{k}_1 - k_1) \|\dot{q}\|^2 \|S\| - \theta \left( \frac{2}{\tilde{\tau}_1 + \tilde{\tau}_2} \right) \|S\|. \tag{59}
\]

In virtue of (17) and (52), \(S^T \tau_1 + \|\bar{p}\| \|S\|\) can be degenerated as

\[
S^T \tau_1 + \|\bar{p}\| \|S\| = -\bar{w}_1 \|S\| + (k_0 + k_1 \|\dot{q}\|^2 + \bar{\sigma} \|\tau\|) \|S\|. \tag{60}
\]

Substituting (52) into (60), we have obtained

\[
S^T \tau_1 + \|\bar{p}\| \|S\| = -\bar{w}_1 \|S\| + (k_0 + k_1 \|\dot{q}\|^2 + \bar{\sigma} \|\tau_0 + \tau_2\|) \|S\| + \bar{\sigma} \bar{w}_1 \|S\|. \tag{61}
\]

where \(\|\tau\| = \|\tau_0 + \tau_1 + \tau_2\| \leq \|\tau_0 + \tau_2\| \leq \|\tau_1\|\) and the fact \(\|\tau_1\| \leq \bar{w}_1\) with \(\bar{w}_1 > 0\) are involved from (26) and (52), respectively.

Accordingly, (61) can be modified as

\[
S^T \tau_1 + \|\bar{p}\| \|S\| \leq -(1 - \bar{\sigma}) \bar{w}_1 \|S\| - \bar{\sigma} \bar{w}_1 \|S\| + (k_0 + k_1 \|\dot{q}\|^2 + \bar{\sigma} \|\tau_0 + \tau_2\|) \|S\| + \bar{\sigma} \bar{w}_1 \|S\|
\]

\[=- (1 - \bar{\sigma}) \bar{w}_1 \|S\| + (k_0 + k_1 \|\dot{q}\|^2 + \bar{\sigma} \|\tau_0 + \tau_2\|) \|S\|. \tag{62}
\]

Applying (53) to (62), it follows that

\[
S^T \tau_1 + \|\bar{p}\| \|S\| \leq -(\hat{k}_0 - k_0) \|S\| - (\hat{k}_1 - k_1) \|\dot{q}\|^2 \|S\|
\]

\[= -(\hat{k}_0 - k_0) \|S\| - (\hat{k}_1 - k_1) \|\dot{q}\|^2 \|S\|. \tag{63}\]

Substituting (63) into (59) yields

\[
V_3 \leq -\theta \left( \frac{2}{\tilde{\tau}_1 + \tilde{\tau}_2} \right) \|^1/2 \|S\|. \tag{64}\]

From (64), we have obtained a fact that \(S = 0\) and the estimation errors \(\hat{k}_1 - k_1 = 0\) are guaranteed in a finite time. When \(S = 0\), the sliding phase proof of the proposed approach can be found in (44)–(51).

Upon above stability analysis of the proposed AFTC-ISM, the tracking trajectories are still restrained on the proposed integral sliding manifold (23) for all time; then \(e\) and \(\hat{e}\) converge globally to zero.

This completes the proof.

**Remark 6.** For the proposed adaptive control (52)–(54), the proper control gains \(k_0\) and \(k_1\) are used for trajectory tracking of robot manipulators instead of the overestimated control parameters. Meanwhile, the simple structure with the proposed control (26)–(32) and (52)–(54) is guaranteed to acquire superior tracking performance compared with other approaches.\(^{38,41}\) In addition, the proposed approach further eliminates the effects of time delay and computational burden.

**Remark 7.** Compared with the existing ISMCs,\(^ {20,28,37-40}\) the proposed approach removes the effects of the singularity and algebraic loop problem completely, and meanwhile the nominal control term is eliminated in the design of integral sliding manifold. Moreover, the strong robustness of ISTM is reserved in the proposed approach which is easy to implement with a simple control structure.

**Remark 8.** In order to eliminate the parameter drift problem for accomplishing the adaptive law (54), the dead-zone technique is adopted in the following simulation and experimental comparisons.

\[
\hat{k}_0 = \begin{cases} 0, & \|S\| \leq \epsilon_0 \\ \delta_0 \|S\|, & \|S\| > \epsilon_0 \end{cases}, \quad \hat{k}_1 = \begin{cases} 0, & \|S\| \leq \epsilon_1 \\ \delta_1 \|\dot{q}\|^2 \|S\|, & \|S\| > \epsilon_1 \end{cases}\tag{65}\]

where \(\epsilon_0\) and \(\epsilon_1\) denote the dead zone size of the parameters \(\hat{k}_0\) and \(\hat{k}_1\), respectively.
Remark 9. For eliminating the chattering of sliding mode control, the boundary technique has been adopted for $b(S)$ of (31) as

$$b(S) = \begin{cases} \frac{S}{|S|}, & |S| \geq \delta \\ \frac{S}{\delta}, & |S| < \delta, \end{cases}$$

(66)

where $\delta$ is the thickness of the boundary layer which is also used in (27), (28), and (52).

4 | SIMULATION RESULTS AND DISCUSSIONS

In this section, the plane robot manipulators are used to reveal the advanced tracking performance. For robot tracking systems, in this comparison the actuator health condition $\Gamma$ given by (3) has been considered as $\tilde{\Gamma} = [2 \sin(t) + 0.5 \sin(200 \pi t), \cos(2t) + 0.5 \sin(200 \pi t)]^T$. The sampling period is 1 ms. The initial states are $[q(0)^T, \dot{q}(0)^T] = [1.0, 1.0, 0, 0]^T$. The desired trajectories $q_d = [q_{d1}, q_{d2}]^T$ (rad) are as

$$q_{d1} = 1.25 - 7/5 \exp(-t) + 7/20 \exp(-4t), \quad q_{d2} = 1.25 + \exp(-t) - 1/4 \exp(-4t).$$

4.1 | Tracking performance with actuator faults and uncertainties

For robot tracking systems, in this comparison the actuator health condition $\Gamma$ given by (3) has been considered as

$$\tilde{\Gamma} = \begin{cases} I_{2 \times 2}, & t < 8 \text{s} \\ \text{diag} \{0.7 + 0.01 \sin(10t), 0.65\}, & t > 8 \text{s}. \end{cases}$$

(72)

For the simulation comparisons, the effectiveness of the AFTC-ISM is completed over an adaptive fuzzy integral sliding mode control with disturbances observer (AFISMC-DO), adaptive backstepping nonsingular fast terminal sliding mode controller (ABNFTSMC), and an adaptive fault-tolerant tracking control (AFTC). AFISMC-DO is represented as

$$\tilde{\tau} = u_0 + u_s,$$  

(73)

$$u_0 = \Lambda^{-1} (-f(x_1, x_2) - W^T \Psi(Z) - \Delta - \dot{\alpha}_1 - e_1 - K_2 e_2),$$  

(74)

$$u_s = -\Lambda^{-1} \left( \mu_1 \sigma^{1/2} \text{sign} (\sigma) + \xi \right)$$  

(75)
with

\[ x_1 = q, \quad x_2 = q, \quad e_1 = x_1 - x_d, \quad e = e_1, \]  
\[ e_2 = x_2 - a_1, \quad a_1 = -K_1 e_1 + \dot{x}_d \]  

with \( x_d = q_d \) denotes the desired trajectories and

\[ \dot{\bar{W}}_i = \Gamma_i \left[ e_2 \Psi_i(Z) - 2\Psi_1 \right], \]  
\[ \Lambda = M^{-1}(x_1), \quad \dot{\xi} = k_s(t) \text{sign}(\sigma) \]  

with

\[ \delta_s(t) = k_s(t) - \frac{1}{v_1} |w_{eq}(t)| - v_0, \quad v_0, v_1 > 0 \]  
\[ \dot{k}_s(t) = -(\delta_0 + \delta_s(t)) \text{sign}(\delta_s(t)), \quad \delta_0 > 0 \]  
\[ \dot{\delta}_s(t) = \beta_s |\delta_s(t)|, \quad \beta_s > 0, \]  

and

\[ \sigma(t) = S(t) - S(0) - \int_0^t \left( \Lambda u_0 + \Pi \right) dt \]  
\[ \Pi = f(x_1, x_2) + \bar{W}^T \Psi(Z) + \lambda \dot{e} - \ddot{x}_d + \dot{\Delta} \]  
\[ f(x_1, x_2) = M^{-1}(x_1)(\bar{C}(x_1, x_2)x_2 - g(x_1)), \]  
\[ S = \dot{e} + \lambda e, \]  
\[ \dot{\Delta}(t) = p(t) + K_0 s \]  
\[ p(t) = -K_0 \left( \bar{f}(x_1, x_2, \bar{\tau}, \bar{e}, \bar{x}_d) + \dot{\Delta}(t) \right), \]  

where \( \lambda, \mu_1, v_0, v_1, \delta_0, \) and \( \beta_s \) represent some positive constants, and \( K_0, K_0, \) and \( K_3 \) are the positive definite diagonal matrix. ABNFTSMC\textsuperscript{14} is represented as

\[ \bar{\tau} = \Xi^{-1}(\dot{e})H(x_1)(u_n(t) - u_{\omega}(t)) \]  

with

\[ u_n(t) = -\Xi(\dot{e})(f(x_1, x_2) - \ddot{x}_d) - \Psi(e, \dot{e}) + \alpha_2 - \int (\xi_2 \theta_3 + \theta_2) \]  
\[ u_{\omega}(t) = (\dot{\Lambda} + \zeta) \text{sign}(\theta_3) \]  

with

\[ \dot{\Lambda} = \begin{cases} 0, & \text{if } |\theta_3| \leq \epsilon \\ \frac{1}{\delta} |\theta_3|, & \text{if } |\theta_3| > \epsilon \end{cases} \]  
\[ \theta_1 = \sigma_1, \quad \theta_2 = \sigma_2 - a_1, \quad \theta_3 = \sigma_3 - a_2 \]  
\[ a_1 = -\xi_1 \theta_1, \quad a_2 = -\xi_2 \theta_2 - \theta_1 - \xi_3 \theta_2 \]  
\[ \sigma_1 = \int (e + k_1 \text{Sign}(e) + k_3 \text{Sign}(\dot{e})) \]  
\[ \sigma_2 = \sigma_1, \quad \sigma_3 = \sigma_1, \]  

where \( x_1 = q, e, \) and \( \dot{e} \) are defined by (7), \( k_1, k_2, \lambda, p, q, \xi_1, \xi_2, \xi_3, \delta, \) and \( \epsilon \) stand for some positive constants.
TABLE 1 Controller parameters selection.

| Controllers       | Parameters                                      |
|-------------------|------------------------------------------------|
| AFTC-ISM          | $\theta_0 = 0.01, \overline{r}_1 = 0.09, \overline{r}_2 = 0.2, \kappa = 0.1, \tau = 0.5, \alpha = 3$ |
|                   | $\beta = 7, \theta_1 = 0.02, \gamma_1 = 0.5, \gamma_2 = 0.65$ |
| ABNFTSMC$^{14}$   | $k_1 = 1, k_2 = 0.5, \lambda = 1.4, p = 9, q = 7, \xi_1 = 1, \xi_2 = 0.5$ |
|                   | $\xi_3 = 0.1, \delta = 0.5, \epsilon = 0.01, m_1 = m_2 = m_3 = 20$ |
| AFISMDO$^{41}$    | $\lambda = 5, \mu_1 = 10, v_0 = 3, v_1 = 2, \delta_0 = 3, \beta_1 = 5$ |
|                   | $K_s = 5I_2, K_1 = K_2 = K_m, \Gamma = 2, \gamma = 1$ |
| AFTC$^{33}$       | $\delta = 0.3, \alpha = 0.7, \beta = 1.1, K_1 = 3I_2, K_2 = 0.8I_2, C_2 = \text{diag}(10, 2)$ |
|                   | $L_a = 1, \gamma_0 = 0.2, \hat{a}_0 = [8.5; 4.5], C_1 = I_2, \Gamma_d = 100I_2, \mu = 2.5$ |

While AFTC$^{33}$ is represented as

$$\overline{r} = r_0 + r_1,$$  \hspace{1cm} (89)

$$r_0 = Y_d \hat{a}_d - d_M b(S_r) - L_a \| Y_d b(S_r) - \left[ C_1 \text{Sig}^\alpha(S_r) + C_2 \text{Sig}^\beta(S_r) \right] \|,$$  \hspace{1cm} (90)

$$r_1 = -\frac{1 - \gamma_0}{\gamma_0} b(S_r) \| r_0 \|,$$  \hspace{1cm} (91)

where

$$b(S_r) = \begin{cases} 
\frac{S_r}{\| S_r \|}, & \| S_r \| \neq 0 \\
0, & \| S_r \| = 0.
\end{cases}$$  \hspace{1cm} (92)

$$\hat{a}_d = -\Gamma_d Y_d^T S_r, \quad S_r = \dot{q} - \dot{q}_r, \quad \dot{q}_r = \dot{q}_d - K_1 F(e) - K_2 \text{Sig}^\delta(e),$$  \hspace{1cm} (93)

$$F(e) = [f(e_1) \ldots f(e_n)]^T$$

$$f(e_i) = \begin{cases} 
K_a e_i + K_b |e| \text{sign}(e_i), & \text{if } |e_i| < \delta \\
|e_i|^\beta \text{sign}(e_i), & \text{if } |e_i| \geq \delta.
\end{cases}$$  \hspace{1cm} (94)

where $\delta, \alpha, \beta, L_a, \gamma_0,$ and $\mu$ denote some positive constants, $K_1, K_2, C_1, C_2,$ and $\Gamma_d$ represent the positive definite diagonal matrix, and $Y_d$ is defined by (57) and (58) of the work.$^{33}$

Remark 10. From AFISMDO$^{41}$ and ABNFTSMC$^{14}$, the dynamics of robot manipulators (3) including $\overline{M}(q)$, $\overline{C}(q, \dot{q})$, and $\overline{G}(q)$ are assumed to be known exactly. In practical application, the dynamics of robot manipulators cannot acquired exactly. For a fair comparison, $\overline{M}(q)$, $\overline{C}(q, \dot{q})$, and $\overline{G}(q)$ given by (73)–(94) are assumed as the nominal parts $\overline{M}_0(q)$, $\overline{C}_0(q, \dot{q})$, and $\overline{G}_0(q)$.

Note that $\overline{M}_0(q)$, $\overline{C}_0(q, \dot{q})$, and $\overline{G}_0(q)$ have been obtained by replacing $\overline{m}_1$ and $\overline{m}_2$ of (67)–(70) to $\overline{m}_1 = 0.4$ kg and $\overline{m}_2 = 1.2$ kg. And, the constant matrix $\overline{M}_0$ of the proposed AFTC-ISM is defined by (13).

The parameters of these simulation comparisons are selected on a trial-and-error procedure including transient speed, steady-state tracking precision and control torque input. Moreover, the tracking performance of ABNFTSMC, AFISMDO, AFTC, and AFTC-ISM should be accomplished on the similar control efforts. Then, the parameters of these controller are given by Table 1.

Figure 1 gives the position tracking and adaptive gains $\hat{k}_0$ and $\hat{k}_1$. In subplot (a) of Figure 1, the real position tracks the desired trajectories in both joints 1 and 2 correctly, which reveals a better tracking performance. Figure 2 represents the sliding manifolds of AFISMDO, ABNFTSMC, AFTC and the proposed AFTC-ISM. From (20) to (25), the sliding manifold of the proposed AFTC-ISM started from zero (i.e., $S(0) = 0$) has been still stabled at the origin for all time; meanwhile, Figure 2 gives the same result that the proposed AFTC-ISM can removes the reaching phase of the conventional SMC and then enhances the transient and steady-state convergence performance. In Figure 2, the sliding manifold (23) has been stabled at the origin with the steady-state tracking errors $1 \times 10^{-3}$ and $2 \times 10^{-3}$ for two joints, respectively. In contrast, the AFISMDO, ABNFTSMC, and AFTC has a larger initial value for the sliding manifold. Figures 3 and 4 give the errors in position and velocity tracking of AFISMDO, ABNFTSMC, AFTC and the AFTC-ISM with their zoomed
plots. Obviously, the proposed AFTC-ISM employs a better steady-state tracking precision than AFISMDO, ABNFTSMC, and AFTC. Then, the position tracking errors of the AFTC-ISM are stabled at $0.5 \times 10^{-3}$ (rad) and $1 \times 10^{-3}$ (rad) for joints 1 and 2, respectively, which provides higher steady-state tracking precision than the AFISMDO, ABNFTSMC, and AFTC. While for the velocity tracking errors given by Figure 4, the proposed AFTC-ISM provides higher velocity tracking precision than AFISMDO, ABNFTSMC, and AFTC. From Figure 4, AFISMDO, ABNFTSMC, and AFTC are oscillated on zero continually; while the proposed AFTC-ISM reveals the improved velocity tracking precision than other controllers. The reason behind this is that the proposed approach removes the reaching phase of ISMC completely; while the state trajectories are still acted on the sliding manifold for all time. In Figure 5, we have acquired a fact that such superior results of the proposed approach given by Figures 1–4 does not use the excessive control torque input than the AFISMDO, ABNFTSMC and AFTC.

To further give the quantitative comparisons between the proposed approach and other controller, the following tracking precision and torque input have been accomplished after 2 s at very beginning of the simulation comparisons. These comparison indexes are as follows

$$E_p = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||e(n)||^2}, \quad E_v = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||\dot{e}(n)||^2}, \quad E_s = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||S(n)||^2}, \quad E_\tau = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||\tau(n)||^2}. \quad (95)$$

$$E_p = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||e(n)||^2}, \quad E_v = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||\dot{e}(n)||^2}, \quad E_s = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||S(n)||^2}, \quad E_\tau = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||\tau(n)||^2}. \quad (96)$$

**FIGURE 1** Position tracking and adaptive gains $\hat{k}_0$ and $\hat{k}_1$ with their upper bounds of the proposed AFTC-ISM. (A) Position tracking of the proposed AFTC-ISM. (B) Adaptive gains of $\hat{k}_0$ and $\hat{k}_1$.

**FIGURE 2** Sliding manifold of AFTC-ISM, ABNFTSMC, AFISMDO and AFTC. (A) Sliding manifold of first link. (B) Sliding manifold of second link.
FIGURE 3  Position tracking errors of AFTC-ISM, ABNFTSMC, AFISMC-DO, and AFTC. (A) Position tracking errors of first link. (B) Position tracking errors of second link.

FIGURE 4  Velocity tracking errors of AFTC-ISM, ABNFTSMC, AFISMC-DO, and AFTC. (A) Velocity tracking errors of first link. (B) Velocity tracking errors of second link. (C) Zoomed velocity tracking errors of first link. (D) Zoomed velocity tracking errors of second link.
TABLE 2  Comparison of control performance.

| Controller       | $\bar{E}_p$       | $\bar{E}_e$       | $\bar{E}_S$       | $\bar{E}_r$       |
|------------------|-------------------|-------------------|-------------------|-------------------|
| AFTC-ISM         | $1.7 \times 10^{-3}$ | $5.7 \times 10^{-3}$ | $9.16 \times 10^{-4}$ | 13.54             |
| ABNFTSMC         | $9.61 \times 10^{-2}$ | $7.82 \times 10^{-2}$ | $10.14 \times 10^{-2}$ | 15.61             |
| AFISMC-DO        | $1.59 \times 10^{-2}$ | $1.17 \times 10^{-2}$ | $8.04 \times 10^{-2}$ | 14.02             |
| AFTC             | $3 \times 10^{-3}$ | $9.3 \times 10^{-3}$ | $2.03 \times 10^{-3}$ | 14.28             |

FIGURE 5  Control torque input of AFTC-ISM, ABNFTSMC, AFISMC-DO, and AFTC. (A) Control torque input of first link. (B) Control torque input of second link.

where $N$ stands for the total sampling number, $\bar{\tau}(n)$, $e(n)$, $S(n)$, and $\dot{e}(n)$ denote the control input, the position tracking errors, the sliding manifold, and the velocity tracking errors at the $n$th sampling instant. These four performance indexes are illustrated in Table 2.

From Table 2, the quantitative comparisons further verify the advanced convergence performance of the proposed approach. Owing to the AFTC-ISM removes the reaching phase of SMC, the trajectories are still stablised at the sliding manifold for all time. Observed by $\bar{E}_S$ of Table 2, the proposed AFTC-ISM also shows higher steady-state precision of sliding manifold over ABNFTSMC, AFISMC-DO, and AFTC. As a result, the position/velocity tracking errors of the proposed AFTC-ISM without utilizing the excessive control input $\bar{E}_r$, are still stablised at a lower range than ABNFTSMC, AFISMC-DO, and AFTC. The quantitative comparisons of Table 2 further testify the improved tracking performance of the proposed AFTC-ISM. Moreover, the proposed approach has further verified the advantages of the proposed ISMC caused by the elimination of the reaching phase.

4.2  Discussion and analysis

In this section, the simulation comparison as shown in Figures 1–5 have been further discussed for revealing the improved tracking performance of the proposed approach in aspect of the tracking precision in steady-state. From Figure 2, it can be seen that the sliding manifolds of first/second link of the proposed approach are stablised at $1 \times 10^{-3}$ and $2 \times 10^{-3}$ from the beginning of initial states to the end of simulation comparisons, respectively. In contrast, other SMCs including ABNFTSMC, AFISMC-DO, and AFTC, also have a larger fluctuation around the origin (i.e., ±4 for ABNFTSMC, ±4 for AFISMC-DO, and ±4 for AFTC). It can be seen from Figure 2 that the sliding variable of the ABNFTSMC, AFISMC-DO, and AFTC at the beginning of simulation comparison is hundreds of times of the proposed approach. The reason behind this is that the states of robot system controlled by the proposed approach is always stablised at the proposed integral sliding manifold for all time. Figures 3 and 4 give the joint position and velocity tracking performance with their zoomed plots. From the subplot of Figures 3 and 4 named as “zoomed position errors of first/second link” and “zoomed velocity errors of first/second link,” it can be clearly seen that the proposed approach (blue one) gains higher steady-state tracking precision than other controllers after a transient period. In addition, the quantitative evaluation of Table 2 further verifies above conclusion about the contributions of the proposed approach. Observer by Figure 5, such superior tracking performance given by Figures 1–5 can be acquired by lower control torque input (i.e., AFTC-ISM for ±50 Nm, ABNFTSMC for ±400 Nm, AFISMC-DO for ±300 Nm, and ±200 Nm) than other controllers. Table 2 gives the
TABLE 3 Comparison with different control parameters.

| Controllers | Parameters |
|-------------|------------|
| P1          | $\theta_0 = 0.01$, $\gamma_1 = 0.09$, $\gamma_2 = 0.2$, $\kappa = 0.1$, $r = 0.5$, $\theta_1 = 0.02$, $\gamma_1 = 0.5$, $\gamma_2 = 0.65$, $\alpha = 3$, $\beta = 4$ |
| P2          | $\theta_0 = 0.01$, $\gamma_1 = 0.09$, $\gamma_2 = 0.2$, $\kappa = 0.1$, $r = 0.1$, $\theta_1 = 0.02$, $\gamma_1 = 0.5$, $\gamma_2 = 0.65$, $\alpha = 3$, $\beta = 4$ |
| P3          | $\theta_0 = 0.01$, $\gamma_1 = 0.09$, $\gamma_2 = 0.2$, $\kappa = 0.1$, $r = 0.5$, $\theta_1 = 0.02$, $\gamma_1 = 0.5$, $\gamma_2 = 0.65$, $\alpha = 6$, $\beta = 8$ |
| P4          | $\theta_0 = 0.01$, $\gamma_1 = 0.09$, $\gamma_2 = 0.2$, $\kappa = 0.1$, $r = 0.5$, $\theta_1 = 0.02$, $\gamma_1 = 0.5$, $\gamma_2 = 0.65$, $\alpha = 1$, $\beta = 2$ |
| P5          | $\theta_0 = 0.01$, $\gamma_1 = 0.09$, $\gamma_2 = 0.2$, $\kappa = 0.1$, $r = 0.5$, $\theta_1 = 0.02$, $\gamma_1 = 0.1$, $\gamma_2 = 0.18$, $\alpha = 3$, $\beta = 4$ |

**FIGURE 6** Position tracking errors of AFTC-ISM with different control parameters. (A) Position tracking errors of first link. (B) Position tracking errors of second link.

**FIGURE 7** Sliding manifold of the proposed AFTC-ISM with different control parameters.

quantitative comparison between the proposed AFTC-ISM and other controllers. One can obtain a conclusion that the proposed approach gains greatly improvement in the aspect of steady-state tracking precision than other SMCs (i.e., $E_p$, $E_v$, and $E_s$). More importantly, such superior can be acquired by utilizing lower control torque input than other controllers observed by $E_\tau$ of Table 2.

### 4.3 Tracking control with different control parameters

In this article, in order to show the effects of control parameters on tracking performance, the simulation comparison of the proposed AFTC-ISM have been accomplished with different control parameters as show Table 3.

In order to analyse the effects of the control parameters on the tracking performance, the control parameters of the proposed AFTC-ISM excluding $r$, $\alpha$, $\beta$, and $\kappa$ have the same values as Table 1. These parameters are summary in Table 3.

Figures 6–8 give several comparison results including the position tracking errors, sliding manifold and velocity tracking errors of the proposed AFTC-ISM with different control parameters, respectively. Whereas Figure 9 depicts
FIGURE 8 Velocity tracking errors of the proposed AFTC-ISM with different control parameters. (A) Velocity tracking errors of first link. (B) Velocity tracking errors of second link.

FIGURE 9 Control torque input of AFTC-ISM with different control parameters. (A) Control torque of first link. (B) Control torque of second link.

the requested control torque obtaining such comparison results as shown in Figures 6–8. As shown in Figures 6–8, the tracking performance of the proposed AFTC-ISM is increased with the larger $r$, $\alpha$, and $\beta$. However, the larger $r$, $\alpha$, and $\beta$ of the proposed AFTC-ISM can achieve a better tracking performance given by Figures 6–8, but such superior results may enlarge the control torque input as shown Figure 9. As a results, we select the parameters of the proposed AFTC-ISM on a trade-off between the tracking performance and control torque inputs.

4.4 Tracking performance with measurement noise

In order to show the effects of the measurement noise on tracking performance, in this section the simulation comparisons under the impact of measurement noise have been accomplished for the proposed AFTC-ISM, ABNFTSMC, AFISMC-DO, and AFTC. For these comparisons, all simulation conditions including initial states, control parameters and the control law are the same as section “4.1 Tracking Performance with Actuator Faults and Uncertainties.” In addition, a white noise (mean value: 0; variance: 0.001) is introduced into the states channel of joint position and velocity.

Figure 10 gives the estimation of the parameters $\tilde{k}_0$ and $\tilde{k}_1$ of the proposed AFTC-ISM. Figure 11 denotes the sliding manifold of ABNFTSMC, AFISMC-DO, AFTC, and AFTC-ISM. Figures 12 and 13 stand for the position and velocity tracking errors with their zoomed plots of ABNFTSMC, AFISMC-DO, AFTC, and AFTC-ISM, respectively. In view of
Figures 3 and 12, the position tracking performance of ABNFTSMC, AFISMC-DO, AFTC, and AFTC-ISM in Figure 12 have been affected by the injected measurement noise of joint position. Similarly, Figures 13 also reveals the performance reduction obviously than the results of Figure 4 without the measurement noise. As a result, even under the influence of measurement noise, the proposed approach can still maintain better steady-state tracking performance than ABNFTSMC, AFISMC-DO, AFTC, and AFTC-ISM. From the simulation comparisons, one can obtain the same conclusion as the Section 4.1. From the above discussion between Figures 3–4 and 12–13, one has obtained a conclusion that both other SMCs and the proposed AFTC-ISM also is sensitivity to mismatched uncertainties from state channel such as the measurement noise.

Remark 11. From the discussion between Sections 4.1 and 4.4, both other SMCs and the proposed approach is sensitive to the measurement noise. The reason behind this is that the typical SMCs is only insensitive to matched uncertainties from the control channel. However, the measurement noise is a typical mismatched uncertainties from states channel. Accordingly, conventional SMCs generally regards the measurement noise as uncertain dynamics.
5 | EXPERIMENTAL COMPARISON

5.1 Tracking control comparisons with actuator faults and uncertainties

In this section we have accomplished by the improved tracking performance on the SCARA robot platform shown in Figure 14. For this experimental platform, the maximum actuators of two joints are summarized as $[\pm 51.2, \pm 16]^T$ (Nm) with the harmonic reduction ratio as 1 : 80 and 1 : 50, respectively. The torques are calculated though the high performance computer of the simulink of Matlab 2012b. The parameters of ABNFTSMC, AFISMC-DO, AFTC and the proposed AFTC-ISM are described in Table 4.

In this experimental comparisons, the sample period is chosen as $t_s = 2$ ms. The initial states are $q = [0; 0]$ (rad) and the desired trajectories are

$$q_d = \left[-\frac{\pi}{10} \sin\left(\frac{t}{2} + \frac{\pi}{2}\right), \frac{\pi}{10} \sin\left(\frac{t}{2} + \frac{\pi}{2}\right)\right]^T,$$

(97)

**Remark 12.** For the proposed approach (26)–(32), the velocity of joint will be introduced into the application of robot tracking system. However, in practical application only position information can be acquired from encoder. Accordingly, in this article the joint velocity can be acquired by the following observer. That is

$$\dot{\xi}_0 = v_0$$
$$v_0 = -c_1|\xi_0 - x_1|^{1/2}\text{sign}(\xi_0 - x_1) + \xi_1$$
$$\dot{\xi}_1 = v_1$$
$$v_1 = -c_2\text{sign}(\xi_1 - v_0),$$

(98)
where $x_1 = q$ is the known joint position, and $\xi_0$ and $\xi_1$ stand for the estimation of joint position $q$ and joint velocity $\dot{q}$, respectively.

**Remark 13.** For the proposed approach, it focus mainly on the strong robustness to uncertain dynamics, external disturbances and actuator effectiveness faults, which removes the reaching phase and the proposed integral sliding manifold uninvolved the control term. From the simulation and experimental results, the proposed approach reveal strong robustness especially instead y-statetracking precision. Nevertheless, in practical applications the joint velocity cannot be acquired from robot plant. In our experimental comparison, the joint velocity of robot plant can be observer from (98). In addition, in common this small estimation error will be regard as the part of uncertain dynamics of robot manipulators which is restrained by the proposed AFTC-ISM. From another point of view, the tracking control without utilizing the joint velocity is another major research field.

For a fair comparison, the same health condition (72) has been added in the following experimental comparisons. Figure 15 gives an real tracking from real position to the desired trajectories and the adaptive gains. Figure 16 illustrate the errors in position tracking of AFTC-ISM, ABNFTSMC, AFISMC-DO, and AFTC with their zoomed plots; while Figure 17 gives the requested control of AFTC-ISM, ABNFTSMC, AFISMC-DO, and AFTC. As shown in Figure 15, the proposed approach reveals rapid transient response and higher steady-state tracking precision than ABNFTSMC, AFISMC-DO, and AFTC. From the zoomed plots of Figure 16, the steady-state tracking errors of two joints are stabled at $5 \times 10^{-3}$ (rad) and $1 \times 10^{-3}$ (rad), respectively. From Figure 17, the advanced tracking performance of the proposed AFTC-ISM in transient...
\textbf{FIGURE 14} The experimental robot setup.

\textbf{TABLE 4} Controller parameters selection.

| Controllers | Parameters |
|-------------|------------|
| AFTC-ISM    | $\kappa = 0.01$, $r = 0.5$, $\alpha = \text{diag}(30, 10)$, $\theta_0 = 0.7$, $\theta_1 = 0.7$ |
|             | $\beta = \text{diag}(70, 40)$, $\gamma_1 = 0.5$, $\gamma_2 = 0.2$, $\gamma_0 = 0.65$ |
| ABNFTSMC    | $k_1 = 10$, $k_2 = 5$, $\lambda = 1.4$, $p = 9$, $q = 7$, $\xi_1 = 120$, $\xi_2 = 110$ |
|             | $\xi_3 = 20$, $\delta = 0.5$, $\epsilon = 0.01$, $m_1 = 150$, $m_2 = 50$, $m_3 = 50$ |
| AFISMC-DO   | $\lambda = 5$, $\mu_1 = 10$, $\nu_0 = 3$, $v_1 = 2$, $d_0 = 3$, $\beta_1 = 5$ |
|             | $K_a = \text{diag}(80, 30)$, $K_1 = K_2 = \text{diag}(40, 25)$, $\Gamma = 2$, $\gamma = 1$ |
| AFTC        | $\delta = 0.3$, $\alpha = 0.7$, $\beta = 1.1$, $K_1 = \text{diag}(1.5, 1)$, $K_2 = 0.1I_2$ |
|             | $L_a = 0.01$, $\gamma_0 = 0.2$, $d_{\text{mo}} = [6.5; 4.0]$, $C_1 = \text{diag}(0.1, 0.1)$ |
|             | $C_2 = \text{diag}(0.1, 0.1)$, $\Gamma_d = \text{diag}(2, 2)$, $\mu = 2.1$, $\kappa = 1.1$ |

\textbf{FIGURE 15} Positions and adaptive gains with their upper bounds of the proposed AFTC-ISM. (A) Position tracking of the proposed AFTC-ISM. (B) Adaptive gains $\hat{k}_0$ and $\hat{k}_1$.

and steady-state does not utilize the excessive control torque input. Upon the above comparisons shown in Figures 16 and 17, the experimental results further show a fact that the proposed approach gains fast transient speed and high steady-state tracking precision than ABNFTSMC, AFISMC-DO, and AFTC, which is also completed by a simple control structure.

\section{5.2 Tracking control comparisons with different health conditions}

In order to further the effects of the actuator effectiveness faults on the tracking performance, in this section the tracking control comparisons with different health conditions have been accomplished for revealing the advancement of the proposed approach subject to UAEFs. The parameters of the proposed AFTC-ISM are summarized in Table 1. The health conditions $\Gamma$ are given by Table 5 for four modes.
**FIGURE 16** Position tracking errors of AFTC-ISM, ABNFTSMC, AFISMC-DO, and AFTC. (A) Position tracking errors of first link. (B) Position tracking errors of second link.

**FIGURE 17** Control torque input of AFTC-ISM, ABNFTSMC, AFISMC-DO, and AFTC. (A) Control input of first link. (B) Control torque of second link.

**TABLE 5** Tracking control comparison with different health conditions.

| Modes  | Health conditions |
|--------|-------------------|
| Mode-1 | $\Gamma = \begin{cases} I_{2 \times 2}, & t < 8 s \\ \text{diag} \{0.7 + 0.01 \sin(10t), 0.65\}, & t < 8 s \end{cases}$ |
| Mode-2 | $\Gamma = \begin{cases} I_{2 \times 2}, & t < 8 s \\ \text{diag} \{0.45, 0.5 + 0.01 \sin(10t)\}, & t < 8 s \end{cases}$ |
| Mode-3 | $\Gamma = \begin{cases} I_{2 \times 2}, & t < 8 s \\ \text{diag} \{0.55, 0.55\}, & t < 8 s \end{cases}$ |
| Mode-4 | $\Gamma = \begin{cases} I_{2 \times 2}, & t < 8 s \\ \text{diag} \{0.55, 0.5 + 0.2 \cos(20t)\}, & t < 8 s \end{cases}$ |
Figure 18 gives the tracking performance of the proposed AFTC-ISM with different health conditions with their zoomed plots. With the effects of four modes health conditions given by Table 5, obviously, the proposed AFTC-ISM still has a higher tracking precision in steady-state given by the subplot “Zoomed position tracking errors of first/second link” of Figure 18. From the experimental results given by (18), one can concluded that the proposed AFTC-ISM can still maintains the higher tracking precision and strong robustness no matter how larger the robot system will be affected by the actuator effectiveness faults.

6 | CONCLUSION

In this article, an adaptive FTC featuring with strong robustness, chattering-restraining, rapid transient response, high steady-state precision and simple control structure, has been proposed for uncertain robot manipulators. The simulation and experimental comparisons gain the fact that the proposed approach greatly improves the steady-state tracking precision than other controllers observed from Figures 1–10 and Tables 3 and 5. Meanwhile, such favor tracking performance does not uses the excessive control input torque. Moreover, the proposed AFTC-ISM removes the
singularity and algebraic loop problem of SMC for uncertain robot manipulators; the reaching phase is removed by the proposed approach for chattering-restraining SMC. Then, the developed control strategy offers an alternative approach for improving the design of uncertain robot manipulators for fault-tolerant tracking problem. Our future efforts are focused on the output feedback control for uncertain robot manipulators.

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CONFLICT OF INTEREST
The authors declare that they have no conflict of interest.

DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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