Observable Lepton Flavor Symmetry at LHC

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Abstract

I discuss a model of lepton flavor symmetry based on the non-Abelian finite group $T_7$ and the gauging of $B - L$, which has a residual $Z_3$ symmetry in the charged-lepton Yukawa sector, allowing it to be observable at the Large Hadron Collider (LHC) from the decay of the new $Z'$ gauge boson of this model to a pair of scalar bosons which have the unusual highly distinguishable final states $\tau^-\tau^-\mu^+ e^+$. 

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1 A Short History of $A_4$

In 1978, soon after the putative discovery of the third family of leptons and quarks, it was conjectured by Cabibbo\cite{1} and Wolfenstein\cite{2} independently that the $3 \times 3$ lepton mixing matrix may be given by

$$U_{\nu}^{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

(1)

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. This implies $\theta_{12} = \theta_{23} = \pi/4$, $\tan^2 \theta_{13} = 1/2$, and $\delta_{CP} = \pm \pi/2$. Thirty years later, we know that they were not completely correct, but their bold conjecture illustrated the important point that not everyone expected small mixing angles in the lepton sector as in the quark sector. The fact that neutrino mixing turns out to involve large angles should not have been such a big surprise.

In 2001, Ma and Rajsekaran\cite{3} showed that the non-Abelian discrete symmetry $A_4$ allows $m_{e,\mu,\tau}$ to be arbitrary, and yet $\sin^2 2\theta_{atm} = 1, \theta_{e3} = 0$ can be obtained. In 2002, Babu, Ma, and Valle\cite{4} showed how $\theta_{13} \neq 0$ can be radiatively generated in $A_4$ with the prediction that $\delta_{CP} = \pm \pi/2$, i.e. maximum $CP$ violation.

In 2002, Harrison, Perkins, and Scott\cite{5}, after abandoning their bimaximal and trimaximal hypotheses, proposed the tribimaximal mixing matrix, i.e.

$$U_{\nu}^{HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim (\eta_8, \eta_1, \pi^0),$$

(2)

which is easy to remember in terms of the meson nonet. This means that $\sin^2 2\theta_{atm} = 1$, $\tan^2 \theta_{sol} = 1/2$, $\theta_{e3} = 0$.

In 2004, I showed\cite{6} that tribimaximal mixing may be obtained in $A_4$, with

$$U_{\nu}^{\dagger} M_\nu U_{\nu} = \begin{pmatrix} a + 2b & 0 & 0 \\ 0 & a - b & d \\ 0 & d & a - b \end{pmatrix},$$

(3)
in the basis that $M_l$ is diagonal. At that time, the Sudbury Neutrino Observatory (SNO) data gave $\tan^2 \theta_{sol} = 0.40 \pm 0.05$, but it was changed in early 2005 to $0.45 \pm 0.05$. Thus tribimaximal mixing and $A_4$ became part of the lexicon of the neutrino theorist.

After the 2005 SNO revision, two $A_4$ models quickly appeared. (I) Altarelli and Feruglio\cite{7} proposed

$$U_{CW}^\dagger M_\nu U_{CW} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix},$$

i.e. $b = 0$, and (II) Babu and He\cite{8} proposed

$$U_{CW}^\dagger M_\nu U_{CW} = \begin{pmatrix} a' - d^2/a' & 0 & 0 \\ 0 & a' & d \\ 0 & d & a' \end{pmatrix},$$

i.e. $d^2 = 3b(b - a)$.

The challenge is to prove experimentally that $A_4$ or some other discrete symmetry is behind neutrino tribimaximal mixing. If $A_4$ is realized by a renormalizable theory at the electroweak scale, then the extra Higgs doublets required will bear this information. Specifically, $A_4$ breaks to the residual symmetry $Z_3$ in the charged-lepton sector, and all Higgs Yukawa interactions are determined in terms of lepton masses. This notion of lepton flavor triality\cite{9} may be the key to such a proof, but these exotic Higgs doublets are very hard to see at the LHC.

## 2 Frobenius Group $T_7$

The tetrahedral group $A_4$ (12 elements) is the smallest group with a real $3$ representation. The Frobenius group $T_7$ (21 elements) is the smallest group with a pair of complex $3$ and $3^*$ representations. It is generated by

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

\[6\]
where \( \rho = \exp(2\pi i/7) \), so that \( a^7 = 1 \), \( b^3 = 1 \), and \( ab = ba^4 \). It has been considered by Luhn, Nasri, and Ramond\[10\], Hagedorn, Schmidt, and Smirnov\[11\], as well as King and Luhn\[12\].

The character table of \( T_7 \) (with \( \xi = -1/2 + i\sqrt{7}/2 \)) is given by

| class | \( n \) | \( h \) | \( \chi_1 \) | \( \chi_1' \) | \( \chi_1'' \) | \( \chi_3 \) | \( \chi_3* \) |
|-------|-------|-------|---------|---------|---------|-------|-------|
| \( C_1 \) | 1 | 1 | 1 | 1 | 1 | 3 | 3 |
| \( C_2 \) | 7 | 3 | 1 | \( \omega \) | \( \omega^2 \) | 0 | 0 |
| \( C_3 \) | 7 | 3 | 1 | \( \omega^2 \) | \( \omega \) | 0 | 0 |
| \( C_4 \) | 3 | 7 | 1 | 1 | 1 | \( \xi \) | \( \xi* \) |
| \( C_5 \) | 3 | 7 | 1 | 1 | 1 | \( \xi* \) | \( \xi \) |

Table 1: Character table of \( T_7 \).

The group multiplication rules of \( T_7 \) include

\[
\begin{align*}
\mathbf{3} \times \mathbf{3} &= \mathbf{3}^*(23, 31, 12) + \mathbf{3}^*(32, 13, 21) + \mathbf{3}(33, 11, 22), \\
\mathbf{3} \times \mathbf{3}^* &= \mathbf{3}(21^*, 32^*, 13^*) + \mathbf{3}^*(12^*, 23^*, 31^*) + \mathbf{1}(11^* + 22^* + 33^*) \\
&\quad + \mathbf{1}^*(11^* + \omega 22^* + \omega^2 33^*) + \mathbf{1}^''(11^* + \omega^2 22^* + \omega 33^*). (8)
\end{align*}
\]

Note that \( \mathbf{3} \times \mathbf{3} \times \mathbf{3} \) has two invariants and \( \mathbf{3} \times \mathbf{3} \times \mathbf{3}^* \) has one invariant. These serve to distinguish \( T_7 \) from \( A_4 \) and \( \Delta(27) \).

3 \hspace{1cm} \textbf{\( U(1)_{B-L} \) Gauge Extension with \( T_7 \)}

Recently, the following model has been proposed by Cao, Khalil, Ma, and Okada\[13\]: Under \( T_7 \), let \( L_i = (\nu, l)_i \sim \mathbf{3} \), \( l_i^c \sim \mathbf{1}^*, \mathbf{1}^{'}, \mathbf{1}^{''} \), \( \Phi_i = (\phi^+, \phi^0)_i \sim \mathbf{3} \), which means that \( \bar{\Phi} = (\bar{\phi}^0, -\phi^-)_i \sim \mathbf{3}^* \). The Yukawa couplings \( L_i l_j^c \bar{\Phi}_k \) generate the charged-lepton mass matrix

\[
M_l = \begin{pmatrix}
    f_1 v_1 & f_2 v_1 & f_3 v_1 \\
    f_1 v_2 & \omega^2 f_2 v_2 & \omega f_3 v_2 \\
    f_1 v_3 & \omega f_2 v_3 & \omega^2 f_3 v_3
\end{pmatrix} = U_{CW}^\dagger \begin{pmatrix}
    f_1 & 0 & 0 \\
    0 & f_2 & 0 \\
    0 & 0 & f_3
\end{pmatrix} \sqrt{3} v, \quad (9)
\]

if \( v_1 = v_2 = v_3 = v \) as in the original \( A_4 \) proposal.
Let $\nu^c_i \sim \mathbf{3}^*$, then the Yukawa couplings $L_i \nu^c_i \Phi_k$ are allowed, with

$$M_D = f_D v \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$  \hfill (10)

Note that $\Phi$ and $\tilde{\Phi}$ have $B - L = 0$.

Now add the neutral Higgs singlets $\chi_i \sim \mathbf{3}^*$ and $\eta_i \sim \mathbf{3}^*$, both with $B - L = -2$. Then there are two Yukawa invariants: $\nu^c_i \nu^c_j \chi_k$ and $\nu^c_i \nu^c_j \eta_k$. Note that $\chi^*_i \sim \mathbf{\bar{3}}^*$ is not the same as $\eta_i \sim \mathbf{3}^*$ because they have different $B - L$. This means that both $B - L$ and the complexity of $T_7$ are required for this scenario. The heavy Majorana mass matrix for $\nu^c$ is then

$$M = h \begin{pmatrix} u_2 & 0 & 0 \\ 0 & u_3 & 0 \\ 0 & 0 & u_1 \end{pmatrix} + h' \begin{pmatrix} 0 & u'_3 & u'_2 \\ u'_3 & 0 & u'_1 \\ u'_2 & u'_1 & 0 \end{pmatrix} = \begin{pmatrix} A & 0 & B \\ 0 & A & 0 \\ B & 0 & A \end{pmatrix},$$  \hfill (11)

where $A = hu_1 = hu_2 = hu_3$ and $B = h'u'_2$ with $u'_1 = u'_3 = 0$ have been assumed, i.e. $\chi_i$ breaks in the (1,1,1) direction, whereas $\eta_i$ breaks in the (0,1,0) direction. This is the $Z_3 - Z_2$ misalignment also used in $A_4$ models. The seesaw neutrino mass matrix is now

$$M_\nu = -M_D M^{-1} M_D^T = -f_D^2 v^2 \begin{pmatrix} A^2 - B^2 & 0 & 0 \\ 0 & A^2 & -AB \\ 0 & -AB & A^2 \end{pmatrix},$$  \hfill (12)

i.e. the two-parameter tribimaximal form proposed by Babu and He, but without the auxiliary $Z_4 \times Z_3$ symmetry assumed there. Two limiting cases are (I) normal hierarchy ($d = -a$): $m_1 = m_2 = 0$, $m_3 = 2a$, and (II) inverted hierarchy ($d = 2a$): $m_1 = 3a$, $m_2 = -3a$, $m_3 = -a$, with the effective mass in neutrinoless double beta decay given by $m_{ee} = a = \sqrt{\Delta m^2_{\text{atm}}/8} = 0.02$ eV.

4 **Higgs Structure**

In the charged-lepton Yukawa sector, i.e. $L_i l^c_j \tilde{\Phi}_k$, a residual $Z_3$ symmetry exists so that linear combinations of $\Phi_k$ become $\phi_0, \phi_1, \phi_2 \sim 1, \omega, \omega^2$ together with $e, \mu, \tau \sim 1, \omega^2, \omega$. Their
interactions are given by

\[ \mathcal{L}_Y = (\sqrt{3}v)^{-1}[m_\tau \bar{L}_\tau \tau_R + m_\mu \bar{L}_\mu \mu_R + m_e \bar{L}_e e_R]\phi_0 + (\sqrt{3}v)^{-1}[m_\tau \bar{L}_\mu \tau_R + m_\mu \bar{L}_e \mu_R + m_e \bar{L}_e e_R]\phi_1 + (\sqrt{3}v)^{-1}[m_\tau \bar{L}_e \tau_R + m_\mu \bar{L}_\tau \mu_R + m_e \bar{L}_e e_R]\phi_2 + H.c. \]

(13)

As a result, the rare decays \( \tau^+ \rightarrow \mu^+ \mu^+ e^- \) and \( \tau^+ \rightarrow e^+ e^- \mu^- \) are allowed, but no others. For example, \( \mu \rightarrow e\gamma \) is forbidden. Here \( \phi_1^0, \bar{\phi}_2^0 \sim \omega \), mixing to form mass eigenstates \( \psi_{1,2}^0 = (\phi_1^0 \pm \bar{\phi}_2^0)/\sqrt{2} \). Using

\[ \frac{B(\tau^+ \rightarrow \mu^+ \mu^+ e^-)}{B(\tau \rightarrow \mu \nu \nu)} = \frac{m_\tau^2 m_\mu^2 (m_1^2 + m_2^2)^2}{m_1^2 m_2^2} < \frac{2.3 \times 10^{-8}}{0.174}, \]

(15)

the bound \( m_1 m_2/\sqrt{m_1^2 + m_2^2} > 22 \text{ GeV} \) \((174 \text{ GeV}/\sqrt{3}v)\) is obtained. Hence the production of \( \psi_{1,2}^0 \bar{\psi}_{2,1}^0 \) at the LHC with final states \( \tau^- e^+ \tau^- \mu^+ \) and \( \tau^+ \mu^- \tau^+ e^- \) would be indicative of this \( Z_3 \) flavor symmetry.

5 LHC Observations

The \( \phi_{1,2} \) scalar doublets have \( B - L = 0 \), so they do not couple directly to the \( Z'_{B-L} \) gauge boson, but they can mix, after \( U(1)_{B-L} \) breaking, with the \( \chi \) and \( \eta \) singlets \( (B - L = -2) \) which do. Thus this model can be tested at the LHC by discovering \( Z' \) from \( q\bar{q} \rightarrow Z' \rightarrow \mu^- \mu^+ \) and looking for \( Z' \rightarrow \psi_{1,2}^0 \bar{\psi}_{2,1}^0 \) with the subsequent decays \( \psi \rightarrow \tau^- e^+ \) and \( \bar{\psi} \rightarrow \tau^- \mu^+ \). We assume for simplicity that \( m_1 = m_2 \), and take the LHC energy as 14 TeV, which is expected to be reached in a year or two.

Let \( \Gamma_0 = g_{B-L}^2 m_{Z'}/12\pi \), then the partial decay widths of \( Z' \) are

\[ \Gamma_q = (6)(3)(1/3)^2 \Gamma_0, \]

(16)
\( \Gamma_l = (3)(-1)^2 \Gamma_0, \)  
\( \Gamma_\nu = (3)(-1)^2 (1/2) \Gamma_0, \)  
\( \Gamma_\psi \approx (2)(-2)^2 \sin^2 \theta (1/4) \Gamma_0, \)

where \( \sin \theta \) is an effective parameter accounting for the mixing of \( \psi \) to \( \chi \) and \( \eta \). The signature events are chosen to be \( \tau^- \tau^- \mu^+ e^+ \) with \( \tau^- \) decaying into \( l^- (e^- \) or \( \mu^- \) plus missing energy. The background events yielding this signature come from

\[ WWZ : \quad pp \rightarrow W^+ W^- Z, \quad W^\pm \rightarrow l^\pm \nu, \quad Z \rightarrow l^+ l^-, \]

\[ ZZ : \quad pp \rightarrow ZZ, \quad Z \rightarrow l^+ l^-, \quad Z \rightarrow \tau^+ \tau^-, \quad \tau^\pm \rightarrow l^\pm \nu \nu, \]
We require no jet tagging and consider only events with both $e^+$ and $\mu^+$ in the final states. Our benchmark points for $m_{Z'}, m_\psi$ (in GeV) are (A), (1000,100), (B) (1500,100), (C) (1000,300), (D) (1500,300), with $g_{B-L} = g_2 = e/\sin \theta_W$, and $\sin^2 \theta = 0.2$. We impose the following basic acceptance cuts:

$$p_{T,i}^{(1,2)} > 50 \text{ GeV}, \quad p_{T,j}^{(3,4)} > 20 \text{ GeV}, \quad |\eta_l| < 2.5, \quad \Delta R_{ij} \equiv \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2} > 0.4, \quad \text{missing } E_T > 30 \text{ GeV},$$

We impose

$$t\bar{t} : \quad pp \rightarrow t\bar{t} \rightarrow b(\rightarrow l^-)\bar{b}(\rightarrow l^+)W^+W^-, \quad w^\pm \rightarrow l^\pm \nu, \quad (22)$$

$$Zb\bar{b} : \quad pp \rightarrow Zb(\rightarrow l^-)\bar{b}(\rightarrow l^+), \quad Z \rightarrow l^+l^- \quad (23)$$

Figure 2: Distribution of the invariant mass of the $e^+$ and reconstructed $\tau^-$ pair.
where $\Delta R_{ij}$ is the separation in the azimuthal angle ($\phi$) – pseudorapidity ($\eta$) plane between $i$ and $j$. We also model detector resolution effects by smearing the final-state energy. To further suppress the backgrounds, we require

$$H_T \equiv \sum_i p_{T,i} + \text{missing } E_T > 300 \text{ GeV}, \quad (26)$$

where $i$ denotes the visible particles.

To reconstruct the scalar $\psi$, we adopt the collinear approximation that the $l$ and $\nu$’s from $\tau$ decays are parallel due to the $\tau$’s large boost, coming from the heavy $\psi$. Denoting by $x_{\tau_i}$ the fraction of the parent $\tau$ energy which each observable decay particle carries, the
transverse momentum vectors are related by

$$\text{missing } E_T = (1/x_{\tau_1} - 1)\vec{p}_1 + (1/x_{\tau_2} - 1)\vec{p}_2.$$  \hspace{1cm} (27)$$

When the decay products are not back-to-back, this gives two conditions for $x_{\tau_i}$, with the $\tau$ momenta as $\vec{p}_1/x_{\tau_1}$ and $\vec{p}_2/x_{\tau_2}$, respectively. We further require $x_{\tau_i} > 0$ to remove the unphysical solutions, and minimize $\Delta R_{e+\ell^-}$ to choose the correct $e^+\ell^-$ to reconstruct $\psi$ and then $Z'$. In Table 2 we show the signal and background cross sections (in fb) for the benchmark cases (A) and (C).

|                | (A) | (C) | $t\bar{t}$ | $WWZ$ | $ZZ$ | $Z\bar{b}$ |
|----------------|-----|-----|------------|-------|------|------------|
| no cut         | 5.14| 2.57| 1.22       | 0.21  | 27.11| 2.99       |
| basic cut      | 1.46| 1.05| 0.16       | 0.02  | 0.0052| 0.024      |
| $H_T$ cut      | 1.41| 1.04| 0.08       | 0.006 | 0.0  | 0.0        |
| $x_{\tau_i} > 0$ | 0.69| 0.52| 0.015      | 0.002 | 0.0  | 0.0        |

Table 2: Signal and background cross sections (in fb) for $m_{Z'} = 1$ TeV and $m_\psi = 100$ GeV (A) and 300 GeV (C).

We show in Fig. 1 the $H_T$ distribution in case (A) to demonstrate the separation of signal from background. We then show in Fig. 2 how the mass of $\psi$ may be obtained from $e^+\tau^-$, and in Fig. 3 how the $Z'$ mass may be reconstructed. The $5\sigma$ discovery contours for (A) to (D) are shown in Fig. 4 in the $m_{Z'} - \sin^2 \theta$ plane.

6 Conclusion

Using the non-Abelian discrete symmetry $T_7$ together with the gauging of $B - L$, a simple renormalizable two-parameter model of the neutrino mass matrix is obtained with tribimaximal mixing. The charged-lepton Higgs Yukawa interactions are predicted completely and exhibit a residual $Z_3$ symmetry which is verifiable at the LHC. The signature is $pp \rightarrow Z' \rightarrow \psi\bar{\psi}$
with the subsequent decays $\psi \rightarrow \tau^{-} e^{+}$ and $\bar{\psi} \rightarrow \tau^{-} \mu^{+}$. With 10 fb$^{-1}$ at $E_{cm} = 14$ TeV and $m_{Z'} \sim 1$ TeV, a 5$\sigma$ discovery is expected.

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References

[1] N. Cabibbo, *Phys. Lett.* B72, 333 (1978).

[2] L. Wolfenstein, *Phys. Rev.* D18, 958 (1978).

[3] E. Ma and G. Rajasekaran, *Phys. Rev.* D64, 113012 (2001).

[4] K. S. Babu, E. Ma, and J. W. F. Valle, *Phys. Lett.* B552, 207 (2003).

[5] P. F. Harrison, D. H. Perkins, and W. G. Scott, *Phys. Lett.* B530, 167 (2002).

[6] E. Ma, *Phys. Rev.* D70, 031901 (2004).

[7] G. Altarelli and F. Feruglio, *Nucl. Phys.* B720, 64 (2005).

[8] K. S. Babu and X.-G. He, arXiv:0507217 [hep-ph].

[9] E. Ma, *Phys. Rev.* D82, 037301 (2010).

[10] C. Luhn, S. Nasri, and P. Ramond, *Phys. Lett.* B652, 27 (2007).

[11] C. Hagedorn, M. A. Schmidt, and A. Yu. Smirnov, *Phys. Rev.* D79, 036002 (2009).

[12] S. F. King and C. Luhn, *JHEP* 0910, 093 (2009).

[13] Q.-H. Cao, S. Khalil, E. Ma, and H. Okada, arXiv:1009.5415 [hep-ph].