New RG-invariants of Soft Supersymmetry Breaking Parameters

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Abstract

We study new renormalization-group invariant quantities of soft supersymmetry breaking parameters other than the ratio of gaugino mass to gauge coupling squared by using the spurion method. The obtained invariants are useful to probe supersymmetry breaking and μ-term generation mechanisms at high-energy scale. We also discuss the convergence behavior of fixed points of supersymmetry breaking parameters.

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Several types of supersymmetry (SUSY) breaking and mediation mechanisms have been studied. Each mechanism leads to a proper structure of SUSY breaking terms. The renormalization group (RG) effects are not negligible to obtain SUSY breaking terms at low energy. It is known that the ratio $M/g^2$, where $M$ is the gaugino mass and $g$ is the gauge coupling, is RG-invariant at one-loop level. For example, if $M/g^2$ for different gauge groups are measured as similar values in future experiments, that implies the gaugino masses are universal at the unification scale of gauge couplings. On the other hand, when gauge interactions are relevant to the mediation mechanism of SUSY breaking [1], $M/g^2$ is given by the group-theoretical factors for each gauge group and generally shows disconnected structure with each other. Thus the RG-invariant quantity is important to probe SUSY breaking mechanisms.

Recently, the renormalization group equations of soft SUSY breaking parameters have been studied within the spurion formalism [2]-[9]. Renormalization of softly broken SUSY theories can be described by introducing the spurion fields into rigid SUSY theories. That implies the beta-functions of soft SUSY breaking parameters can be written by those of the gauge couplings and anomalous dimensions of the rigid SUSY theories. This aspect has been used to derive several important features and led to interesting applications, e.g. calculations of higher-order beta functions of soft SUSY breaking parameters, finiteness conditions, RG-invariant trajectories and analytical solutions. As another example of applications, in ref. [9] the power-law behavior of running soft SUSY breaking parameters has been derived within the framework of extra dimensions.

In this letter, we consider a SUSY gauge-Yukawa system and derive new RG-invariant quantities among soft SUSY breaking parameters through RG-invariants among gauge and Yukawa couplings by use of the spurion technique. The RG invariance holds for any arbitrary value of couplings, not only on a specific RG trajectory. Such invariants of soft SUSY breaking parameters are useful to probe the SUSY breaking mechanisms at high energy. Furthermore we discuss the convergence of infrared fixed points from the obtained RG invariants.

Let us consider the $N = 1$ SUSY gauge theory with the gauge coupling $g$ and the following superpotential,

$$W = y \Phi_1 \Phi_2 \Phi_3.$$  \hspace{1cm} (1)
The one-loop beta functions of the gauge and the Yukawa couplings are obtained

$$\frac{d\alpha}{dt} = b\alpha^2,$$

(2)

$$\frac{d\alpha_y}{dt} = \alpha_y(\alpha\alpha_y - c\alpha),$$

(3)

where $\alpha \equiv g^2/(16\pi^2)$ and $\alpha_y \equiv y^2/(16\pi^2)$. The coefficients, $a$, $b$ and $c$ are written in terms of group-theoretical factors and $a$ and $c$ are always positive. The asymptotically free (non-free) theory corresponds to $b < 0$ ($b > 0$).

The soft SUSY breaking terms are written

$$- L_{\text{soft}} = \left[ \frac{M}{2} \lambda \lambda + Ay\phi_1\phi_2\phi_3 + \text{h.c.} \right] + \sum_i m_i^2 |\phi_i|^2,$$

(4)

where $\lambda$ is the gaugino and $\phi_i$ is the scalar field in the chiral multiplet $\Phi_i$. The total Lagrangian with the soft SUSY breaking terms (4) can be written in terms of $N = 1$ superfields by introducing the spurion fields $\eta = \theta^2$ and $\bar{\eta} = \bar{\theta}^2$ [10]. Furthermore, the beta functions of $M$, $A$ and $\Sigma$, where

$$\Sigma \equiv m_1^2 + m_2^2 + m_3^2,$$

(5)

are obtained through those of $\alpha$ and $\alpha_y$. That is, in eqs. (2) and (3) we replace $\alpha \rightarrow \tilde{\alpha}$ and $\alpha_y \rightarrow \tilde{\alpha}_y$ as follows [5],

$$\tilde{\alpha} = \alpha(1 + M\eta + \bar{M}\bar{\eta} + 2M\bar{M}\eta\bar{\eta}),$$

(6)

$$\tilde{\alpha}_y = \alpha_y(1 - A\eta - \bar{A}\bar{\eta} + (A\bar{A} + \Sigma) \eta\bar{\eta}),$$

(7)

and then we obtain the one-loop beta functions of SUSY breaking parameters,

$$\frac{dM}{dt} = b\alpha M,$$

(8)

$$\frac{dA}{dt} = a\alpha_y A + c\alpha M,$$

(9)

$$\frac{d\Sigma}{dt} = a\alpha_y(\Sigma + A\bar{A}) - 2c\alpha M\bar{M}.$$  

(10)

The two-loop and higher-loop beta functions are obtained similarly, but those are scheme dependent [11]. Here we use the same technique to derive RG-invariants among the SUSY breaking parameters $M$, $A$ and $\Sigma$. 

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Comparing the beta functions (2) and (8), we can obtain the well-known RG-invariant, 
\[ \frac{M}{\alpha} = \text{RG–invariant}. \] (11)

As an alternative derivation, we show this fact can be understood from the spurion method.

From eq. (4), we have the RG equation of gauge coupling, \( d\alpha^{-1}/dt = -b \) and then in the softly broken case, the corresponding equation is

\[ \frac{d\tilde{\alpha}^{-1}}{dt} = -b. \] (12)

Note that the right-hand side of the equation does not include any couplings. The coefficient of \( \eta \) in \( \tilde{\alpha}^{-1} \) is \(-M/\alpha\). Thus it is found that \( M/\alpha \) is RG invariant. This derivation implies that if we have a relation among supersymmetric couplings, then we can extend it to the RG-invariants among soft SUSY breaking parameters. As noted before, the RG-invariant \( M/\alpha \) is important for the purpose to relate values of \( M \) between low and high energies and to probe the SUSY breaking mechanism at high-energy scale. Similarly, RG-invariants for other soft SUSY breaking parameters, if they exist, are also helpful to probe SUSY breaking mechanisms.

Now let us derive RG-invariants for \( A \) and \( \Sigma \) by use of the spurion technique. The above illustration tells us that if we have a relation of couplings like \( f(\alpha, \alpha_y) = \text{coupling-independent} \), we can derive additional RG-invariants by expanding over the Grassmannian variable \( \eta \). First the comparison between the beta functions of \( \alpha \) and \( \alpha_y \) (2, 3) leads to the RG-invariant \( \Gamma \) of the rigid SUSY theory,

\[ \Gamma \equiv \left[ 1 - \frac{(b + c)}{a} \frac{\alpha}{\alpha_y} \right] \alpha^{-(1+c/b)} = \text{RG–invariant}. \] (13)

Alternatively, we define a ratio between \( \alpha \) and \( \alpha_y \),

\[ R \equiv \frac{a}{b + c} \frac{\alpha_y}{\alpha}, \] (14)

then the RG-invariant \( \Gamma \) is written as \( \Gamma = (R - 1)/R \alpha^{1+c/b} \). The infrared fixed point of the ratio corresponds to the point \( R = 1 \). Now let us replace \( \alpha \) and \( \alpha_y \) in \( \Gamma \) by \( \tilde{\alpha} \) and \( \tilde{\alpha}_y \) in order to extend \( \Gamma \) to the quantity \( \tilde{\Gamma} \) including soft SUSY breaking parameters,

\[ \tilde{\Gamma} = \Gamma - \frac{X}{\alpha^{1+c/b}} \eta - \frac{\tilde{X}}{\alpha^{1+c/b}} \tilde{\eta} + \frac{Z}{\alpha^{1+c/b}} \eta \tilde{\eta} = \text{RG–invariant}, \] (15)
where

\[
X = \frac{A + M}{R} + \left(1 + \frac{c}{b}\right) \frac{R - 1}{R} M, \\
Z = \frac{\Sigma - M \bar{M} - (A + M)(\bar{A} + \bar{M})}{R} + \left(1 + \frac{c}{b}\right) \frac{A + M}{R} \bar{M} \\
+ \left(1 + \frac{c}{b}\right) \frac{\bar{A} + \bar{M}}{R} M + \left(1 + \frac{c}{b}\right) \frac{c}{b} \frac{R - 1}{R} M \bar{M}.
\]

Thus we obtain the two more RG-invariants, \(X/\alpha^{1+c/b}\) and \(Z/\alpha^{1+c/b}\), among the soft SUSY breaking parameters. Note that \(X\) is a complex-valued and the RG-invariant \(X/\alpha^{1+c/b}\) includes a \(CP\) phase.

These new RG-invariants are important as well as \(M/\alpha\) to probe the SUSY breaking mechanism at high energy from experimental values if soft SUSY breaking parameters would be measured in future. For example, a certain type of SUSY breaking mechanism such as the no-scale supergravity models \([12]\) and the gauge-mediated SUSY breaking models leads to suppressed values of \(A\) at definite energy scales (no-scale models also predict suppressions of \(\Sigma\)), while string-inspired supergravity theories \([13]\) lead to \(A = -M\) and \(\Sigma = M^2\) at high energy. The difference between these initial conditions provides us with the meaningful difference for the RG-invariants at low-energy scale, \(\Delta(X/\alpha^{1+c/b}) = M_0/(R_0 \alpha_0^{1+c/b})\) and \(\Delta(Z/\alpha^{1+c/b}) \simeq |M_0|^2/(R_0 \alpha_0^{1+c/b})\). Note that since \(X/\alpha^{1+c/b}\) and \(Z/\alpha^{1+c/b}\) are RG invariant, the low-energy differences can be written in terms of \(\alpha_0\) and \(M_0\) given at the scale at which the boundary condition is imposed in each case. For example, it is found from these that the difference is larger for \(b > 0\) case by about 1 order of magnitude. Therefore one can find the gauge-mediation scenarios could reveal rather different signatures than others.

Furthermore the RG-invariants take certain values under a definite condition. For example, on the RG-invariant trajectory we always have \(\Gamma = X = Z = 0\), i.e. \(R = 1\). The quasi fixed-point solution (the large Yukawa coupling case) corresponds to \(R_0 \gg 1\) and that leads to \(X = (1 + c/b)M_0\) and \(Z = (1 + c/b)c/b|M_0|^2\).

For another application, let us discuss the convergence behavior of fixed points. As said above, the infrared fixed point of \(R\) is obtained as \(R = 1\). Similarly in the softly broken case, the fixed points of \(X\) and \(Z\) correspond to the points \(X = 0\) and \(Z = 0\). That implies

\[
A = -M, \quad \Sigma = M\bar{M},
\]
at the point $R = 1$. From the beta functions of the gauge and Yukawa couplings, we have

$$\frac{R - 1}{R} = \xi \frac{R_0 - 1}{R_0},$$

(19)

with $\xi \equiv (\alpha_0/\alpha)^{1+c/b}$. The couplings with subscript 0 denote some initial values for them. Hence it is found that the convergence into $R = 1$ is described by the quantity $\xi$ \[14\]. In the asymptotically non-free gauge theories, we have a rapid convergence to the fixed point $R = 1$, i.e. a tiny value of $\xi$. Furthermore such analysis on the convergence of the Yukawa fixed points has been extended to the theories with extra spatial dimensions \[15\], where we can have a good convergence even for asymptotically free theories with a certain condition.

As for the soft SUSY breaking parameters, we obtain by replacing $R$ by $\hat{R}$ (or from the RG invariance discussed above),

$$X = \xi X_0, \quad Z = \xi Z_0.$$  

(20)

Remarkably, the convergence behaviors of $X = 0$ and $Z = 0$ are described by the same quantity $\xi$ as $R = 1$. Therefore in the models where the Yukawa couplings go to the fixed points rapidly, the soft breaking terms also have good convergence to their fixed points (at the same rate).

So far we have considered the case that a single gauge coupling is dominant in the anomalous dimensions of $\Phi$. In the minimal supersymmetric standard model for example, the three gauge couplings and the three gaugino masses contribute to RG equations, in particular around the grand unification scale. For that purpose it is useful to extend our analyses to the case with the gauge group $G = \prod G_i$ whose gauge couplings are $g_i$. The beta functions of the supersymmetric couplings $\alpha_i = g_i^2/(16\pi^2)$ and $\alpha_y$ are obtained

$$\frac{d\alpha_i}{dt} = b_i \alpha_i^2, \quad \frac{d\alpha_y}{dt} = \alpha_y \left( a\alpha_y - \sum_i c_i \alpha_i \right).$$  

(21)

(22)

In this case, the RG invariant quantity in the rigid theory is defined by

$$\Gamma \equiv \frac{E}{\alpha_y} + aF = \text{RG–invariant},$$  

(23)

where $E \equiv \prod \alpha_i^{-c_i/b_i}$ and $F(t) = \int^t E(t')dt'$. Note that in the case of one gauge coupling ($i = 1$), this expression just reproduces the previous one \[13\] (multiplied by a constant).

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We extend it to the softly broken RG-invariants by replacing $\alpha_i$ and $\alpha_y$ with the redefined couplings $\tilde{\alpha}_i$ and $\tilde{\alpha}_y$ where

$$\tilde{\alpha}_i = \alpha_i(1 + M_i \eta + \bar{M}_i \bar{\eta} + 2M_i \bar{M}_i \eta \bar{\eta}),$$

and $\tilde{\alpha}_y$ is given in eq. (7). From these, we can find two RG-invariants $\Gamma_\eta$ and $\Gamma_{\eta \bar{\eta}}$, which are the coefficients of $\eta$ and $\eta \bar{\eta}$ in $\tilde{\Gamma}$,

$$\Gamma_\eta = \frac{E}{\alpha_y} \left( A - \sum_i \frac{c_i}{b_i} M_i \right) - a \sum_i \int t E(t') \frac{c_i}{b_i} M_i(t') dt',$$

$$\Gamma_{\eta \bar{\eta}} = \frac{E}{\alpha_y} \left( A\bar{A} - \Sigma + E_{\eta \bar{\eta}} - A \sum_i \frac{c_i}{b_i} \bar{M}_i - \bar{A} \sum_i \frac{c_i}{b_i} M_i \right) + a \int t E(t') E_{\eta \bar{\eta}}(t') dt',$$

where $E_{\eta \bar{\eta}}$ is the coefficient of $\eta \bar{\eta}$ in $E$ and explicitly given by

$$E_{\eta \bar{\eta}} = - \sum_i \frac{c_i}{b_i} \left( 1 - \frac{c_i}{b_i} \right) |M_i|^2 + \sum_{i>j} \frac{c_i c_j}{b_i b_j} (M_i M_j + M_j M_i).$$

Finally, we consider the RG-invariants of the $\mu$ term and the corresponding $B$ term for the application to the supersymmetric standard models. The beta function of the $\mu$ term is given by

$$\frac{d\mu}{dt} = \mu \left( a_{\mu} \alpha_y - \sum_i c_{\mu i} \alpha_i \right),$$

The coefficients $a_{\mu}$ and $c_{\mu i}$ are written in terms of the group-theoretical factors and are always positive. The beta function of the soft breaking parameter $B$ can be obtained by replacing the supersymmetric couplings $\alpha_i$, $\alpha_y$ and $\mu$ by $\tilde{\alpha}_i$, $\tilde{\alpha}_y$ and $\tilde{\mu}$, where the $\eta$ coefficient of $\tilde{\mu}$ is given by $-B \mu$. In this case we have the RG-invariant among the supersymmetric coupling,

$$\mu E_\mu \alpha_y^{-a_{\mu}/a} = \text{RG–invariant},$$

where

$$E_\mu = \prod_i \alpha_i^{-c_i' / b_i},$$

with $c_i' = (a_{\mu} / a) c_i - c_{\mu i}$. By use of the spurion method, this equation provides us with the RG-invariant concerned with $B$ as

$$B + \sum_i \frac{c_i'}{b_i} M_i - \frac{a_{\mu}}{a} A = \text{RG–invariant}.$$
the $\mu$ term is generated at high-energy scale \cite{16}, we can use the RG-invariant \cite{11} for selecting the $\mu$-term generation mechanisms. Note that the invariant also includes a CP phase as $X$.

To summarize, we have presented several RG-invariants of soft SUSY breaking parameters, e.g. $X/\alpha^{1+c/b}$ and $Z/\alpha^{1+c/b}$ for the case with a single gauge coupling. The RG invariance holds not only on a specific RG-trajectory but also for arbitrary set of couplings. These are important to probe the SUSY breaking mechanisms and the origin of $\mu$-term at high energy when the parameters would be measured in future experiments. Our analyses have shown that if one has a RG-invariant among SUSY couplings, one can generally derive corresponding RG-invariants among soft SUSY breaking parameters. We have also discussed the convergence behavior of the fixed points of SUSY breaking parameters. The obtained results in this letter may be extended to the cases with some Yukawa couplings \cite{17}.

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