Direct Air-to-Underwater Optical Wireless Communication: Statistical Characterization and Outage Performance

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Abstract—In general, a buoy relay is used to connect the underwater communication to the terrestrial network over a radio or optical wireless communication (OWC) link. The use of relay deployment may pose security and deployment issues. This paper investigates the feasibility of direct air-to-underwater (A2UW) communication from an over-the-sea OWC system to an underwater submarine without deploying a relaying node. We analyze the statistical performance of the direct transmission over the combined channel fading effect of atmospheric turbulence, random fog, air-to-water interface, oceanic turbulence, and pointing errors. We develop novel analytical expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the resultant signal-to-noise ratio (SNR) in terms of bivariate Meijer-G and Fox-H functions. We use the derived statistical results to analyze the system performance by providing exact and asymptotic results of the outage probability in terms of system parameters. We use computer simulations to demonstrate the performance of direct A2UW transmissions compared to the relay-assisted system.

Index Terms—Atmospheric turbulence, BS distribution, fog, outage probability, underwater communications, UWOC.

I. INTRODUCTION

Optical wireless communication (OWC) is a potential technology for underwater applications such as oceanographic data collection, tactical surveillance, and offshore explorations [1]. The OWC system performs exceedingly well for underwater communication achieving ultra-high data rate secured transmission at a low-power consumption compared to legacy technologies such as acoustic waves and radio-frequency (RF). Still, the performance of underwater wireless optical communication (UWOC) is limited by the oceanic turbulence caused by air bubbles levels and variation in the temperature and pressure of the seawater in addition to other impairments such as scattering and absorption [2], [3], [4].

There has been an increased research interest in studying the heterogeneous underwater-terrestrial network to offload the underwater data using RF or OWC technologies. Cooperative relaying protocols such as amplify-and-forward (AF) and decode-and-forward (DF) are generally employed to interface the UWOC with terrestrial link using a buoy relay node [5], [6], [7], [8], [9]. In [7], the authors analyzed the performance of an unmanned-aerial-vehicle (UAV) assisted RF-UWOC system using both fixed-gain AF and DF relaying protocols. The authors in [9] used the OWC transmission over Gamma-Gamma turbulence to study the fixed-gain AF relaying for the mixed UWOC-OWC communication system.

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In the aforementioned and related research, the deployment of relaying devices, either floating buoys or mounted on a ship, may be challenging, and to the least, can impose security issues, especially if the underwater communication is carried out for tactical surveillance. Recently, there have been some preliminary studies for direct transmissions, specifically towards the statistical modeling of wavy water surface at the air-to-water interface [10], [11], [12]. The Birnbaum-Saunders (BS) distribution function was found to be a better fit to predict the statistical behavior of the fading channel in the presence of random aquatic waves at the air-to-water interface [11]. Further, Agheli et al. [13] presented a study for OWC transmission directly to an underwater submarine under the impact of air-to-water interface with log-normal turbulence model for underwater communication. The considered log-normal model for underwater is limited to weak oceanic turbulence. Further, Agheli et al. [13] used deterministic path loss and did not consider atmospheric turbulence for the OWC link. This resulted in cascading of three statistical effects of channel: air-to-water interface (BS distribution), oceanic turbulence (log-normal model), and pointing errors (zero-boresight model), thus excluding the statistical effect of atmospheric turbulence and randomness in the signal attenuation on the system performance. It is customary to include atmospheric turbulence in the air-to-water OWC link. Further, foggy weather conditions over the sea may randomly change the optical signal absorption, precluding deterministic path loss for the OWC link [14], [15]. In general, the direct transmission requires cascading of five independent channel components (atmospheric turbulence, random path loss, air-to-water interface, oceanic turbulence, and pointing errors) each distributed according to the corresponding channel characteristics. It requires novel approaches to analyze the system performance for direct transmission since the density and distribution functions of these channel components are mathematically complicated, precluding the direct application of statistical analysis for the product of random variables. As such, the foggy channel contains a logarithmic function, the generalized model for atmospheric turbulence has a Bessel function, the BS density function is a sum of two exponential functions, oceanic turbulence is a weighted sum of exponential and generalized gamma, and pointing errors is modeled using an algebraic function. To the best of the author’s knowledge, an exact analysis for direct transmission over the cascaded channel components with generalized fading models together with a comparison to the relayed system has not yet been studied.

In this paper, we investigate the feasibility of direct air-to-underwater (A2UW) communication from an over-the-sea OWC system to an underwater submarine under generalized channel conditions. The major contributions of the proposed work are summarized as follows:

1) We analyze the statistical performance of the A2UW communication by considering the combined channel effect of atmospheric turbulence, random fog, air-to-water interface, oceanic turbulence, and pointing errors.

2) We consider the generalized Malága distribution for the atmospheric turbulence, Gamma distributed attenuation coefficient for random fog, the BS distribution to model the aquatic waves at the air-water interface, the mixture exponential–generalized gamma (EGG) distribution for the oceanic turbulence, and zero-boresight for random misalignment errors between transmitter and detector.

3) We develop novel analytical expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the resultant signal-to-noise ratio (SNR) for the A2UW using
the bivariate Mellin-Barnes integrals. It should be mentioned that direct use of the theory for the product of random variables is not readily applicable.

4) We use the derived statistical results to analyze the system performance by providing exact outage probability in terms of system parameters. We also analyze the outage performance asymptotically to derive diversity order providing insights on the system behavior in the high SNR regime.

5) We validate our derived analysis and use computer simulations to demonstrate the performance of the direct A2UW scheme with a comparison to the DF-based relay-assisted system. We conclude that the direct A2UW can provide acceptable performance for various scenarios of interest when the double fading effect above-the-sea and underwater channels is not severe.

II. SYSTEM AND CHANNEL MODELS

We consider an A2UW transmission scheme where an over-the-sea UAV directly communicates with an underwater submarine (i.e., shown in Fig. 1). The UAV transmits a source signal using the intensity modulation/direct-detection (IM/DD) technique for the destination located in underwater. The transmitted signal encounters fading due to atmospheric turbulence, random fog, oceanic turbulence, and pointing errors. In addition to these channel fading impairments, the signal is affected by erratic random and non-random aquatic waves at the air-water interface. We consider link without breaking waves (i.e., no bubbles) at the air-water interface. Thus, the electrical signal received (denoted by $y$) at the underwater detector can be expressed as

$$y = h_{at}h_f h_{ws} H_a h_{ot} h_p s + n$$

where $s$ is the transmitted signal, $h_{at}$ is the channel coefficient for the atmospheric turbulence, $h_f$ models the randomness in the path gain due to the foggy condition, $h_{ws}$ models the effect of scattering and reflection of the signal by water waves at the air-to-water interface, $H_a$ denotes the oceanic path gain, $h_{ot}$ is the oceanic turbulence, $h_p$ is the channel coefficient due to the pointing errors, and $n$ denotes the additive noise at the detector with variance $\sigma_n^2$. The parameterized values of the channel coefficients in (1) are given in Table I.

We consider the generalized Malaga distribution to model the atmospheric turbulence as

$$f_{h_{at}}(x) = A_{mg} \sum_{m=1}^{\beta_M} a_m x^{K_M - m} \left(2^{\frac{\alpha M \beta_M x}{g \beta M + \Omega}}\right)^{\frac{-1}{\beta M - 1}}$$

where $A_{mg} = \frac{2^{\alpha M \beta_M \Omega}}{\Gamma(1/m)} \left(\frac{\beta M + \Omega}{\beta M + \Omega + 1/m}\right)^{1/m - 1} \left(\frac{\alpha M \beta_M x}{g \beta M + \Omega}\right)^{m/2}, a_m \triangleq \left(\frac{\beta M - 1}{m - 1}\right) \left(\frac{\alpha M \beta_M x}{g \beta M + \Omega}\right)^{m/2}, \Gamma(\cdot)$ denotes the Gamma function [16, Sec. (8.310)], $K_v(\cdot)$ is the $v^{th}$-order modified Bessel function of the second kind [16, Sec. (8.433)], and $(\alpha M, \beta M, \Omega)$ are fading parameters, as defined in [17]. As such, $\alpha M$ is the effective number of large-scale cells of the scattering process, $\beta_M$ denotes the amount of fading parameter in natural number, $g$ is the average power of the scattering component, which is received by off-axis eddies, and $\Omega'$ is defined as the average power from the coherent contributions. The probability density function (PDF) of the foggy channel is given as [14]:

$$f_{h_f}(x) = \frac{z^k}{\Gamma(k)} \left(\frac{1}{x^k}\right)^{k-1} x^{k-1}, 0 < x \leq 1$$

where $z = 4.343/\beta d_{air}$, $k > 0$ is the shape parameter, $\beta > 0$ is the scale parameter and $d_{air}$ (in km) is link distance from the water surface to the UAV. For the deterministic path gain, we can use the visibility model $h_f = e^{-\phi_{air} d_{air}}$, where $\phi_{air}$ is the attenuation coefficient [18].

We use the BS distribution to characterize statistically the air-to-water interface [19]. Originally, the BS distribution was proposed as a lifetime model for materials subject to cyclic stress and strain patterns, where the ultimate failure of the material comes from the growth of a prominent crack beyond a critical length. The BS distribution has also been used in different frameworks such as environmental, health metric, and neural spiking. We can adapt the lifetime model for the channel fading due to aquatic waves at the air-water surface. The $i$-th aquatic wave leads to an additional signal attenuation by $L_{i,ws}$, assumed to be a random variable with mean $\mu_0$ and variance $\sigma_0^2$. Thus, accumulated attenuation by $n$ aquatic waves is given by $\sum_{i=1}^{n} L_{i,ws}$, which is normally distributed with mean $n\mu_0$ and variance $n\sigma_0^2$. Assume that the outage (similar to the lifetime in the fatigue model) occurs when the signal attenuation exceeds a threshold attenuation $L_{th}$. Denoting by $h_{ws}$ the channel fading due to aquatic waves, the PDF of $h_{ws}$ is given as [19]

$$f_{h_{ws}}(x) = \frac{1}{2\sqrt{2\pi \alpha \beta}} \left[\left(\frac{\beta}{x}\right)^{1/2} + \left(\frac{x}{\beta}\right)^{3/2}\right]$$

$$\times \exp\left[-\frac{1}{2\alpha^2} \left(\frac{x}{\beta} + \frac{\beta}{x} - 2\right)\right]$$

where $\alpha = \sqrt{\frac{\mu_0}{L_{th}}} > 0$ and $\beta = \frac{L_{th}}{\mu_0} > 0$ are the shape and scale parameters to parameterize the BS distribution. Physically, the shape parameter $\alpha$ changes the shape of the PDF by modifying its slope and the scale parameter $\beta$ compresses or stretches the PDF. Note that the
PDF $f_{h_{uw}}(x) \to 0$ if $x \to 0$ as well as when $x \to \infty$ and the PDF is unimodal (with a median of $\beta$) for all values of $\alpha$ and $\beta$.

The path gain for the oceanic turbulence can be approximated using Beer-Lambert law $H_e = e^{-\phi_{water} d_{water}}$ (in m) [13], where $\phi_{water}$ is the extinction attenuation coefficient. Further, we use the recently proposed mixture EGG distribution for the oceanic turbulence (caused by air bubbles and temperature gradient) [4]:

$$f_{h_{uw}}(x) = \frac{\omega}{\lambda} \exp\left(-\frac{x}{\lambda}\right) + (1 - \omega) \frac{c x^{ac-1} \exp(-\frac{(x \gamma)\rho}{1})}{\Gamma(a)}$$ (5)

where $\omega$ is the mixture coefficient of the distributions satisfying $0 < \omega < 1$, $\lambda$ is the exponential distribution parameter, $a$, $b$, and $c$ are the generalized Gamma distribution parameters. Here, the weight factor $\omega$ quantifies the relative strength of irradiance fluctuations due to air bubbles and temperature-induced fading. Finally, we model the misalignment between the transmit aperture of UAV and detector at the submarine using the PDF $f_{h_{bp}}(x) = \frac{c x^{\rho-1}}{A_0}$, where $A_0$ and $\rho$ are pointing error parameters [20].

III. STATISTICAL CHARACTERIZATION

In this section, we statistically characterize the direct transmission by deriving PDF and CDF of the SNR for the combined channel effect consisting of over-the-sea and underwater fading channels.

We define $\gamma = |h|^2$ as the SNR for the system model of (1), where the combined channel $h = h_{uw} h_{bp}$, with $\gamma_0 = 2 \rho^2 \sigma_0^2 H_\omega \sigma_i^2$. $P_a$ is the average optical transmitted power, and $R$ is the responsivity in the following. We develop PDF of the SNR for the direct transmission scheme as given in (1), considering fading models of atmospheric turbulence, random fog, air-to-water interface, oceanic turbulence, and pointing errors. It should be mentioned that direct application of the product of random variables is not readily applicable to derive the PDF of SNR for the A2UW. We define $b_m = a_{\alpha m} [\beta M/(\gamma M + \Omega)]^{-(\alpha M + m)/2}$.

Theorem 1: If $\{\alpha M, \beta M, \Omega, \gamma\}$ models the atmospheric turbulence, $\{z = 4.343/|\beta|, \gamma, k\}$ models the random fog, $\{\alpha, \beta\}$ models the air-to-water interface, $\{\omega, \lambda, \alpha, b_a\}$ models the oceanic turbulence, and $\{\rho, A_0\}$ models the pointing errors, then the PDF of SNR $\gamma$ for the direct A2UW scheme is given in (6), shown at the bottom of the page.

Proof: The proof is presented in Appendix A. 

Remark 1: The derived PDF in (6) represented using bivariate Mellin–Barnes integral is due to the manifestation of cascaded behavior of five different channel components with generalized fading models. However, standard computational routines are available in MATLAB and MATHEMATICA to compute bivariate Meijer-G and Fox-H functions [21]. Further, the bivariate representation of the PDF of the SNR allows an exact statistical analysis and an excellent asymptotic representation using simpler Gamma functions to demonstrate the impact of different parameters on the system performance, which is not straightforwardly present through numerical integration. It should be mentioned that the Monte-Carlo simulation requires higher computation time to assess the performance of the considered system.

IV. OUTAGE PROBABILITY

We use the derived statistical results to study the outage probability of the system as a performance metric to compare with the relay-assisted transmissions. Note that other statistical performance metrics such as average BER and ergodic capacity can be similarly derived.

We can use (6) to develop an exact outage probability of the considered system for a given threshold SNR $\gamma_0$, as $P_{out} = P[\gamma < \gamma_0]$. Thus, we substitute (6) in $P_{out} = \int_{\gamma_0}^{\infty} f_\gamma(\gamma) d\gamma$ and apply standard Mathematical procedure as used in deriving the PDF to get the outage probability in (7), shown at the bottom of the next page.

Although (7) can provide outage probability over a wide range of $\gamma_0$, it is desirable to analyze the outage probability asymptotically at a high SNR $\gamma_0 \to \infty$. Thus, we use the asymptotic results of Meijer-G [22, eq. 07.34.06.0018.01] and Fox-H functions [23, Th. 1.7] to express the outage probability in the high SNR regime as presented in (8), shown at the bottom of the next page.

Remark 2: Compiling the dominant terms of $\gamma_0$ in (8) and using the definitions $A_i, P_i$, and $Q_i$, the diversity order for the A2UW can be expressed as $C_{out} = \min\{z, \frac{\gamma_0}{2}, \frac{\gamma_0}{4}, \frac{\gamma_0}{6}, \frac{\gamma_0}{8}\}$. It can be seen that the diversity order in [4], [9] becomes a special case of our generalized model, which includes random fog and air-to-water interface in addition to the atmospheric and oceanic turbulence with pointing errors.

Remark 3: The diversity order reveals several interesting behaviors of the outage probability for the direct A2UW transmission, specifically: (i) it is independent of fading parameters of the BS distribution modeled for the air-to-water interface; and (ii) the diversity order is $\min\{\frac{z}{2}, \frac{\gamma_0}{2}\}$ if $z < 1$ depending on the channel model.
on the density fog $\beta_f$ and $d_{\text{d}}$ since measurement data reveals that typically $ac > 1$, $\alpha_M > 1$, and $\beta_M > 1$. Further, the diversity order for the direct A2UW transmission becomes independent of pointing errors if $\rho > 1$ can be maintained with higher beam-width.

V. SIMULATION RESULTS AND DISCUSSIONS

We use MATLAB software to validate our analysis and demonstrate the performance of the direct A2UW transmission considering various system and channel configurations. We also compare the performance with the DF-based relaying to better assess use cases for the proposed scheme. We use Monte-Carlo simulations to evaluate the performance of relay-assisted system and validate the derived analytical expressions of A2UW transmission using Monte-Carlo simulations. We fix the air-to-water surface link distance $d_{\text{air}}$ from 20 m (for example, a UAV hovering over the sea), and consider underwater link range $d_{\text{water}}$ from 20m to 80m. We use three atmospheric turbulence (weak, medium, and strong) [24] and consider low $\rho = 5$ and high $\rho = 1$ pointing errors with $A_0 = 0.0032$. Other simulation parameters are listed in Table I. First, we demonstrate the effect of fading due to the aquatic waves at the air-to-water interface by comparing PDF of the combined BS and EGG distributions with the EGG, as shown in Fig. 2. The figure shows a higher probability of getting the fading coefficient $1 < h_{\text{sat}} < 1.7$ due to the oceanic turbulence of the relay-assisted system. However, the probability diminishes to zero when $h_{\text{sat}} > 1.7$. The effect of BS distribution on the resultant channel for the direct A2UW transmission can also be observed in Fig. 2. The probability of achieving higher channel gain $h_{\text{sat}} > 1$ diminishes compared with the oceanic turbulence for a specified range between 1 to 1.7, i.e., $P(1 < h_{\text{sat}} < 1.7) < P(1 < h_{\text{sat}} < 1.7)$, thus deteriorating the quality of transmission. On the other hand, the A2UW has a higher probability of achieving better channel coefficient $h_{\text{sat}} > 1.7$ compared with the relay-assisted system. Thus, the PDF plots in Fig. 2 depict that the direct A2UW communication may perform close to the relay-assisted under specific scenarios of practical interest.

Next, we compare the outage probability of the direct A2UW transmission with the DF relay system at three different underwater
parameters $(\alpha, \beta)$ on the outage probability is marginal, and the slope remains constant, which confirms our analysis for the diversity order.

Finally, in Fig. 4, we demonstrate the interplay of oceanic turbulence, pointing errors, atmospheric turbulence, and fog density on the outage performance. The figure shows that the A2UW performs close to the relayed system for both strong and weaker oceanic turbulence scenarios without atmospheric turbulence. However, the direct transmission incurs a penalty of 4 dBm transmit power to achieve the exact outage probability of the relayed system in the presence of atmospheric turbulence. The degradation in the A2UW performance occurs due to the double fading effect of turbulence and path gain of two mediums for the direct A2UW transmissions compared with the dominant single link performance using the DF protocol. The figure also confirms the relative effect of pointing errors ($\rho = 1$ and $\rho = 5$) and intensity of oceanic turbulence (EGG-1 and EGG-2) on wireless transmissions.

We can conclude that the direct A2UW achieves acceptable performance for various scenarios of interest and performs close to the relay-assisted transmissions when the double fading effect caused by the cascading of above-the-sea and underwater channels is not severe. Nevertheless, relay-assisted transmission is near-optimal in the mixed OWC-UWOC transmission with an additional relay at the water surface. Experimental demonstrations are further required to support the theoretical analysis developed in this paper. We envision that the proposed direct A2UW transmission may further substantiate underwater communications using optical carriers.

**APPENDIX A**

**PDF of SNR**

To derive the PDF of the product of 5 random variables, we express $h = h_1 h_f h_w h_{ut} h_p$, where $h_1 = h_{at} h_p$. The PDF of $h_1$ is given in [17]

$$f_{h_1}(x) = \frac{\rho^2 A_{\text{avg}}}{2x} \sum_{m=1}^{\beta_M} b_m G_{m,1}^{3,0}(\alpha_M, \beta_M, \frac{x}{g_{\beta_M} + \Omega A_0}, \rho^2, \alpha_M, m)$$

(9)

where the Meijer’s-G function is defined as $G_{p,q}^{m,n}(a_1, \ldots, a_p ; b_1, \ldots, b_q) = \frac{1}{\pi} \int_0^{\infty} \prod_{j=1}^{m} \Gamma(b_j - s) \prod_{i=1}^{q} \Gamma(1 - a_i + s) \prod_{k=1}^{n} \Gamma(1 - a_i - s) x^s ds [16, \text{eq. (9.301)}]$.  

Next, we express $h = h_1 h_{ut} h_w$, where $h_2 = h_1 h_f$. Substituting (3) and (9) in $f_{h_2}(x) = \int_0^\infty f_{h_1}(h_2 | h_1) f_{h_1}(h_1) dh_1$, we get

$$f_{h_2}(x) = \frac{\rho^2 A_{\text{avg}}}{2h_1} \int_0^\infty h_1^{-z-1} \left[ \ln\left( \frac{h_1}{x} \right) \right]^{\beta_M - 1} h_1 \sum_{m=1}^{\beta_M} b_m G_{m,1}^{3,0}(\alpha_M, \beta_M, h_1, \frac{x}{g_{\beta_M} + \Omega A_0}, \rho^2, \alpha_M, m) dh_1$$

(10)

Substituting $\ln(h_1/x) = t$, we solve the integral

$$\int_0^\infty e^{-t} t^{\beta_M - 1} \exp(-z s) dt = \Gamma(k) \Gamma(k (z + s))$$

in terms of the Gamma function, and apply the definition of the Meijer-G function to get the PDF of $h_2$:

$$f_{h_2}(x) = \frac{\rho^2 A_{\text{avg}}}{2x} \sum_{m=1}^{\beta_M} b_m G_{m,1}^{3,0}(\alpha_M, \beta_M, h_1, \frac{x}{g_{\beta_M} + \Omega A_0}, \rho^2, \alpha_M, m)$$

(11)

Note that the PDF in (11) can be verified using the unified expression in [25].
Next, we express $h = h_3 h_{aw}$, where $h_3 = h_3 h_{aw}$. The product distribution of $h_3$ can be expressed as

$$f_{h_3}(x) = \int_0^\infty \frac{1}{h_{aw}} f_{h_3}(h_3/h_{aw}) f_{h_{aw}}(h_{aw}) dh_{aw}$$

(12)

Since the direct use of (4) in (12) becomes intractable, we convert the PDF of BS distribution in (4) as

$$f_{h_{aw}}(h_{aw}) = \frac{1}{2\sqrt{\pi} \alpha \beta} \exp \left( \frac{1}{\alpha^2} \right) \left( \frac{\beta}{h_{aw}} \right)^{1/2}$$

$$G_{1,1}^{0,0} \left( \frac{-h_{aw}}{2\alpha \beta} \right) G_{0,1}^{0,0} \left( \frac{-\beta}{2\alpha h_{aw}} \right) + \frac{1}{2\sqrt{\pi} \alpha \beta} \exp \left( \frac{1}{\alpha^2} \right) \left( \frac{\beta}{h_{aw}} \right)^{3/2}$$

$$G_{1,1}^{0,0} \left( \frac{-h_{aw}}{2\alpha \beta} \right) G_{0,1}^{0,0} \left( \frac{-\beta}{2\alpha h_{aw}} \right)$$

(13)

Substituting (11) and (13) in (12) with the identity

$$C_{p,q}^{m,n} \left( \frac{a}{b} \right) = C_{p,q}^{m,n} \left( \frac{1-b}{1-a} \right)$$

and applying the identity [22, eq. 07.34.21.0081.01], we get

$$f_{h_3}(x) = \frac{z^k \beta A_{\text{avg}}}{4\sqrt{\pi} x} \exp \left( \frac{1}{\alpha^2} \right) \sum_{m=1}^{b_m} b_m G_{1,1}^{0,0} \left( \frac{1,0;1,3+k}{1,0;1,3+k} \right)$$

$$\left( \frac{1}{\alpha^2} \right) \left( \frac{\beta \rho}{\alpha \beta \rho \beta} \right)$$

$$\left( \frac{1}{\alpha^2} \right) \left( \frac{\beta \rho}{\alpha \beta \rho \beta} \right)$$

(14)

where $\zeta = \{1, - \rho, 1 - \alpha \rho, 1 - m \rho, \{1 - z \rho\}^k\}$, $\chi = -\rho^2$, $\{z \rho\}$.

Finally, we use (5) and (14) in $f_{h_3}(x) = \int_0^\infty \frac{1}{h_{aw}} f_{h_3}(h/h_{aw}) f_{h_{aw}}(h_{aw}) dh_{aw}$, and apply the line integral definition of Meijer-G and fox-H functions to get the PDF of $h = h_3 h_{aw}$ by solving four inner integrals as $I_1 = I_2 = \int_0^\infty h_3^k dh_3 \int_0^\infty h_{aw}^{1+k} dh_{aw} = \lambda^{1+k+2(1+1)}$, and $I_3 = I_4 = \int_0^\infty h_{aw}^{1+k} dh_{aw} = \frac{(1+1)^{1+k}}{2\pi \alpha^2}$, and apply the definitions of bivariate Meijer-G and Fox-H functions [26] with the transformation $h = \sqrt{\frac{h}{\alpha}}$ to get the PDF of SNR $\gamma$ in (6).

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