Preparation of Pure Gaussian States via Cascaded Quantum Systems*

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Abstract—This paper provides an alternative approach to the problem of preparing pure Gaussian states in a linear quantum system. It is shown that any pure Gaussian state can be generated by a cascade of one-dimensional open quantum harmonic oscillators, without any direct interaction Hamiltonians between these oscillators. This is physically advantageous from an experimental point of view. An example on the preparation of two-mode squeezed states is given to illustrate the theory.

I. INTRODUCTION

Gaussian states are of great practical importance in quantum information processing and quantum computation [1]–[4]. They possess several distinctive properties that make them stand out from other quantum states. They are commonly encountered in quantum optics laboratories [5]. Vacuum states, thermal states, coherent states and squeezed states are all Gaussian states. Moreover, their Gaussian character is preserved under several common experimental operations. Examples include displacement, squeezing and phase rotation. If we trace out a mode from a multipartite Gaussian state, the reduced state is also Gaussian [6].

Recently, the problem of preparing pure Gaussian states has been studied in the literature [7]. The approach is based on engineering the dissipation of the system as a means of quantum state control. For this reason, the approach is often referred to as reservoir engineering [8]–[12]. Suppose the open quantum system is described by the following Markovian master equation [13]:

$$\frac{d}{dt} \rho = -i[\hat{H}, \rho] + \sum_{j=1}^{m} \left( \hat{b}_{j} \hat{\rho} \hat{b}_{j}^{\dagger} - \frac{1}{2} \hat{b}_{j}^{\dagger} \hat{b}_{j} \rho - \frac{1}{2} \rho \hat{b}_{j}^{\dagger} \hat{b}_{j} \right),$$

(1)

where $\hat{\rho}$ is the density operator, $\hat{H}$ is the (self-adjoint) Hamiltonian of the system and $\hat{b}_{j}$ is the coupling operator which describes the interaction between the system and the $j$th environment. Then it is shown in [7] that any pure Gaussian state can be uniquely generated by selecting suitable operators $\hat{H}$ and $\hat{b}_{j}$. This is also known in the field of quantum information, where a quantum state can be characterized via a set of operators known as nullifiers [14].

For general reservoir engineering problems including the above-mentioned Gaussian case, a common question is how to practically construct the system operators $(\hat{H}, \hat{b}_{j})$ satisfying the requirements to realize the desired dissipation for a large number of modes. One such construction is the so-called quasi-local system-reservoir interaction, which requires only a few system components interacting with the reservoir [15]–[18]. On the other hand, realization with the cascade connection has been investigated in [19], [20]; in this case all the system components interact with the reservoir fields in a one-way fashion; see also [21]. A clear advantage of this construction is that the subsystems can be placed at far away sites, which as a result yield a quantum communication channel.

This paper considers the problem of preparing pure Gaussian states in a quantum network synthesis setting [19], [20], using a cascade realization. Our result is that any pure Gaussian state can be generated by engineering a cascade of one-dimensional open quantum harmonic oscillators, without any direct interaction Hamiltonians between these oscillators. In addition to the above-mentioned advantage, this pure cascade realization may be easier to implement in practice than other realizations, and hence may serve as an alternative for the experimental preparation of pure Gaussian states.

Notation: Let $A = [A_{jk}]$ be a matrix (or a vector) whose entries $A_{jk}$ are complex numbers or operators. We define $A^{\dagger} = [A_{kj}^{\dagger}]$, $A^{*} = [A_{jk}^{*}]$, where the superscript * denotes either the complex conjugate of a number or the adjoint of an operator. Clearly, $A^{\dagger} = (A^{*})^{\dagger}$. $P_{n}$ is a $2n \times 2n$ permutation matrix defined by $P_{n}[a_{1} a_{2} \cdots a_{2n}]^{\dagger} = [a_{1} a_{3} \cdots a_{2n-1} a_{2} a_{4} \cdots a_{2n}]^{\dagger}$ for any column vector $a = [a_{1} a_{2} \cdots a_{2n}]^{\dagger}$.

II. PRELIMINARIES

In this section, we first review some relevant properties of pure Gaussian states [1], [22]. Then, we provide an introduction to the network synthesis of linear quantum systems [19], [23], [24].

A. Pure Gaussian States

Consider a bosonic system of $n$ modes. Each mode is characterized by a pair of quadrature field operators $\{\hat{q}_{j}, \hat{p}_{j}\}, j = 1, 2, \cdots, n$. If we collect them into a vector $\vec{\xi} = (\hat{q}_{1}, \cdots, \hat{q}_{n}, \hat{p}_{1}, \cdots, \hat{p}_{n})^{\dagger}$, the canonical commutation relations are written as

$$[\xi_{j}, \xi_{k}] = i \Sigma_{jk}, \quad j, k = 1, 2, \cdots, 2n,$$

(2)

where $\Sigma_{jk}$ is the $(j, k)$ element of the $2n \times 2n$ matrix $\Sigma = \begin{bmatrix} 0 & I_{n} \\ -I_{n} & 0 \end{bmatrix}$. 

*This work was supported by the Australian Research Council.

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A state is said to be a Gaussian state if its Wigner function (a function defined in the phase space; see e.g. [1]) is Gaussian, i.e.,

\[
W(\xi) = \frac{\exp[-1/2(\xi - \langle \xi \rangle)^\top V^{-1}(\xi - \langle \xi \rangle)]}{(2\pi)^n \sqrt{\det(V)}},
\]

where \(\langle \xi \rangle = \text{tr}(\hat{\xi} \hat{\rho})\) is the mean value vector, and \(V\) is the covariance matrix \(V = 1/2(\Delta \xi \Delta \xi^\top + (\Delta \xi \Delta \xi^\top)^\top)\). A Gaussian state is completely characterized by the mean vector \(\langle \xi \rangle\) and the covariance matrix \(V\). As the mean vector \(\langle \xi \rangle\) contains no information about entanglement and can be made to vanish via local unitary operations, we will restrict our attention to zero-mean Gaussian states in the sequel [22], [25]. The purity of a Gaussian state is defined by \(\mathbb{P} = \text{tr}(\hat{\rho}^2) = 1/\sqrt{2^n \det(V)}\). We see that a Gaussian state is pure if and only if its covariance matrix \(V\) satisfies \(2^n \det(V) = 1\). A more explicit and useful parametrization for the covariance matrix of a pure Gaussian state is as follows.

**Lemma 1** ([22], [26]): A matrix \(V_p\) is the covariance matrix of an \(n\)-mode pure Gaussian state if and only if there exist real symmetric \(n \times n\) matrices \(X\) and \(Y\) with \(Y > 0\), such that

\[
V_p = \frac{1}{2} \left[ Y^{-1} - Y^{-1}X Y^{-1}X + Y \right].
\]  

This lemma states that a (zero-mean) pure Gaussian state corresponds to a pair of real symmetric matrices \([X,Y]\) with \(Y > 0\). Given a pure Gaussian state, the matrices \(X\) and \(Y\) can be uniquely determined, and vice versa. This result will be repeatedly used in the following discussions.

**B. Network Synthesis of Linear Quantum Systems**

In the SLH framework developed in [23], [24], an open quantum system independently coupled to \(m\) environmental fields is characterized by a triple

\[
G = (\hat{S}, \hat{L}, \hat{H}),
\]

where \(\hat{S}\) is an \(m \times m\) unitary scattering matrix, \(\hat{L}\) is an \(m \times 1\) coupling operator vector and \(\hat{H}\) is the Hamiltonian operator. In the assumption, we assert that no scattering is involved between the system and the quantum fields, i.e., \(\hat{S} = I_m\). If we feed the output of an open quantum system \(G_1 = (\hat{S}_1, \hat{L}_1, \hat{H}_1)\) into the input of another open quantum system \(G_2 = (\hat{S}_2, \hat{L}_2, \hat{H}_2)\), the SLH model of this cascade system is

\[
G_2 \circ G_1 = \left( \hat{S}_2 \hat{S}_1, \hat{L}_2 + \hat{S}_2 \hat{L}_1, \hat{H}_2 + \hat{H}_1 + \text{Im}(\hat{L}_2^\top \hat{S}_1 \hat{L}_1) \right).
\]  

Here the notation \(G_2 \circ G_1\) denotes the cascade connection of \(G_1\) and \(G_2\).

Within this theory, we now consider a cascade of \(n\) one-dimensional open quantum harmonic oscillators \(G_j = (\hat{S}_j, \hat{L}_j, \hat{H}_j), j = 1, 2, \ldots, n\), as shown in Fig. 1. For each oscillator \(G_j\), the scattering matrix is \(\hat{S}_j = I_m\); the coupling operator has the linear form \(\hat{L}_j = K_j \hat{\xi}_j, \hat{x}_j = [\hat{\xi}_j, \hat{\rho}_j]^\top, K_j \in \mathbb{C}^{m \times 2}\) and the Hamiltonian \(\hat{H}_j\) has the quadratic form \(\hat{H}_j = \frac{1}{2} \hat{\xi}_j^\top R_j \hat{\xi}_j\), where \(R_j\) is a real symmetric \(2 \times 2\) matrix.

![Fig. 1. A cascade connection of \(n\) one-dimensional open quantum harmonic oscillators.](image)

In Fig. 1, \(\hat{A}(t) = [\hat{A}_1(t), \hat{A}_2(t), \ldots, \hat{A}_m(t)]^\top\) is the input of the cascade system \(G\). The entries \(\hat{A}_j(t), j = 1, 2, \ldots, m\), represent \(m\) environmental channels and satisfy the following Itô rules:

\[
\begin{align*}
\frac{d\hat{A}_j(t)}{dt} & = \delta_{jk} dt, \\
\frac{d\hat{A}_j(t)}{dt} & = \delta_{j1} dt \hat{A}_1(t) = \hat{d}_j(t) dt = 0.
\end{align*}
\]  

The output field \(\hat{Y}(t) = [\hat{Y}_1(t), \hat{Y}_2(t), \ldots, \hat{Y}_m(t)]^\top\) results from the interaction between the cascade system \(G\) and the input field \(\hat{A}(t)\). The entries \(\hat{Y}_j(t), j = 1, 2, \ldots, m\), also satisfy Itô rules similar to (5) [23], [24].

By using the formula (4), an explicit expression can be derived for the SLH model of the cascade system \(G\).

**Lemma 2** ([20]): The system \(G = (\hat{S}, \hat{L}, \hat{H})\) obtained by a cascade of \(n\) one-dimensional open quantum harmonic oscillators \(G_j = (\hat{S}_j, \hat{L}_j, \hat{H}_j)\), \(\hat{S}_j = I_m, j = 1, 2, \ldots, n\), has the following SLH model:

\[
\begin{align*}
\frac{\hat{S}}{dt} & = I_m, \\
\frac{\hat{L}}{dt} & = K \hat{\xi}, \\
\frac{\hat{H}}{dt} & = \frac{1}{2} \hat{\xi}^\top R \hat{\xi},
\end{align*}
\]

where \(M = [M_{jk}], j, k = 1, 2, \ldots, n\) with \(M_{jj} = R_j, M_{jk} = \text{Im}(K_j^\top K_k)\) whenever \(j > k\) and \(M_{jk} = M_{kj}^\top\) whenever \(j < k\).

Note that \(M_{jk}\) are all real \(2 \times 2\) matrices. Hence \(M\) is a real symmetric \(2n \times 2n\) matrix. Alternatively, we can use a quantum stochastic differential equation (QSDE) to describe the evolution of the entire cascade system \(G\). The QSDE model is as follows [19], [20], [27]:

\[
\begin{align*}
\frac{d\hat{\xi}(t)}{dt} & = \mathscr{A} \hat{\xi}(t) dt + \mathscr{B} \left[ \frac{d\hat{A}(t)}{dt} \right], \\
\frac{d\hat{Y}(t)}{dt} & = \mathscr{C} \hat{\xi}(t) dt + \mathscr{D} d\hat{A}(t),
\end{align*}
\]

where \(\mathscr{A} = \Sigma(R + \text{Im}(K^\top K)), \mathscr{B} = i\Sigma[-K^\top K^\top], \mathscr{C} = K, \mathscr{D} = I_m\).

In fact, a bijective correspondence exists between the SLH model and the QSDE model of an open quantum system in the case of \(\hat{S} = I\); see [28] for details.

From the QSDE (7), it follows that the time evolution of the mean value \(\langle \hat{\xi}(t) \rangle\) and the covariance matrix \(V(t)\) is as follows:

\[
\begin{align*}
\frac{d\langle \hat{\xi}(t) \rangle}{dt} & = \mathscr{A} \langle \hat{\xi}(t) \rangle, \\
\frac{dV(t)}{dt} & = \mathscr{A} V(t) + V(t) \mathscr{A}^\top + \frac{1}{2} \mathscr{B} \mathscr{B}^\top.
\end{align*}
\]

If the initial state of the system is given by a Gaussian state, then at any later time \(t \geq 0\) the system is in a Gaussian state.
every eigenvalue of $A$ over these constructions. Without any explicit optimization to implement than others in practice. We provide a feasible pure Gaussian state, and some realizations may be easier that there may exist different cascade realizations for a given $n$ state uniquely.

Given by

$$V = \sum_{j} X_j \otimes \hat{R}_j \xi_j,$$

Using Lemma 2,

$$K = [K_1 \ K_2 \ \cdots \ K_n] P_n^\top$$

$$= Y^{-1/2} \left[ -(X + iY), I_n \right] P_n \begin{bmatrix} I_2 & 0_{2 \times 2} & \cdots & 0_{2 \times 2} \\ 0_{2 \times 2} & I_2 & \cdots & 0_{2 \times 2} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{2 \times 2} & 0_{2 \times 2} & \cdots & I_2 \end{bmatrix} P_n^\top$$

$$= Y^{-1/2} \left[ -(X + iY), I_n \right].$$

On the other hand, when $j > k$, we have

$$\text{Im}(K_j^\dagger K_k)$$

$$= \text{Im} \left( [0_{2 \times (2j-2)} \ I_2 \ 0_{2 \times (2n-2j)}] P_n^\top \begin{bmatrix} -(X - iY) \\ I_n \end{bmatrix} Y^{-1/2} \right.$$

$$Y^{-1/2} \left[ -(X + iY), I_n \right] P_n \begin{bmatrix} 0_{(2k-2) \times 2} \\ I_2 \\ 0_{(2n-2k) \times 2} \end{bmatrix}$$

$$= [0_{2 \times (2j-2)} \ I_2 \ 0_{2 \times (2n-2j)}] P_n^\top \Sigma P_n \begin{bmatrix} 0_{(2k-2) \times 2} \\ I_2 \\ 0_{(2n-2k) \times 2} \end{bmatrix}$$

$$= [0_{2 \times (2j-2)} \ I_2 \ 0_{2 \times (2n-2j)}] \Theta \begin{bmatrix} 0_{(2k-2) \times 2} \\ I_2 \\ 0_{(2n-2k) \times 2} \end{bmatrix}$$

$$= 0,$$

where $\Theta = \text{diag}_{n}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Then we have $R = 0_{2n \times 2n}$. Therefore, the SLH model of the cascade system $G$ is

$$\hat{S} = I_n,$$

$$\hat{L} = K_j^\xi_j,$$

$$\hat{H} = \frac{1}{2} \xi_j^\dagger R_j \xi_j, \quad R_j = 0_{2 \times 2}.$$

Next, we show that the steady state of this cascade system $G$ is the desired pure Gaussian state with the covariance matrix $V$. We now calculate the SLH model of the cascade system $G$.

$$\begin{cases} \hat{S} = I_n, \\ \hat{L} = K_j^\xi_j, \\ \hat{H} = \frac{1}{2} \xi_j^\dagger R_j \xi_j. \end{cases}$$

Using Lemma 2,

$$K = [K_1 \ K_2 \ \cdots \ K_n] P_n^\top$$

The Hurwitz property of $A$ and the Lyapunov equation (11) guarantee that this pure Gaussian state is the unique steady state of the cascade system $G$.

Remark 1: In our construction, there is no need for a Hamiltonian contribution; the steady state is only determined by the coupling operators (dissipation). This is analogous to the manner in which a pure Gaussian state can be specified via its nullifier operators [14]. Note also that the constructed system $G$ is coupled to $n$ quantum noises (environmental
channels). That is, $\hat{L}$ is an $n \times 1$ coupling operator vector. This is sometimes unnecessary. In some cases, an $n$-mode pure Gaussian state may be generated by a cascade system with less than $n$ quantum noises. An example to illustrate this is given in Section IV.

IV. ILLUSTRATIVE EXAMPLE

This section studies the preparation problem of the so-called two-mode squeezed state [1]; this is a highly symmetric entangled state which is useful for several quantum information protocols such as teleportation. It is known that the covariance matrix $V$ of a two-mode squeezed state is

$$V = \frac{1}{2} \begin{bmatrix} \cosh(2\alpha) & \sinh(2\alpha) & 0 & 0 \\ \sinh(2\alpha) & \cosh(2\alpha) & 0 & 0 \\ 0 & 0 & \cosh(2\alpha) & -\sinh(2\alpha) \\ 0 & 0 & -\sinh(2\alpha) & \cosh(2\alpha) \end{bmatrix},$$

where $\alpha$ is the squeezing parameter. Then it can be calculated from (1) that

$$X = 0_{2 \times 2}, \quad Y = \begin{bmatrix} \cosh(2\alpha) & -\sinh(2\alpha) \\ -\sinh(2\alpha) & \cosh(2\alpha) \end{bmatrix}.$$

Our objective here is to engineer a cascade of two open quantum harmonic oscillators. The output of $G_1$ is fed into the input of $G_2$.

Here we provide two different realizations.

**Realization 1:** In this cascade realization, the SLH models for the subsystems $G_1$ and $G_2$ are

$$\begin{align*}
\hat{\Sigma}_1 &= 1, \\
\hat{L}_1 &= [i\hat{Q}_1, 1]\hat{x}_1, \\
\hat{H}_1 &= \frac{1}{2} \hat{x}_1^\dagger \begin{bmatrix} 2 & \hat{Q}_2 \\ \hat{Q}_2 & 2 \end{bmatrix} \hat{x}_1, \\
Q_1 &= \sinh(2\alpha) - \cosh(2\alpha), \\
Q_2 &= \frac{\sinh^2(2\alpha) - \sinh(2\alpha) \cosh(2\alpha)}{\cosh(2\alpha)},
\end{align*}$$

and

$$\begin{align*}
\hat{\Sigma}_2 &= 1, \\
\hat{L}_2 &= [i\hat{Q}_1, 1]\hat{x}_2, \\
\hat{H}_2 &= -\frac{1}{2} \hat{x}_2^\dagger \begin{bmatrix} 2 & \hat{Q}_2 \\ \hat{Q}_2 & 2 \end{bmatrix} \hat{x}_2,
\end{align*}$$

respectively.

Using Lemma [2], the Hamiltonian matrix $R$ and coupling matrix $K$ of the system $G$ are calculated as follows.

$$R = \begin{bmatrix} 2 & 0 & Q_2 & Q_1 \\ 0 & -2 & -Q_1 & -Q_2 \\ Q_2 & Q_1 & 2 & 0 \\ Q_1 & Q_2 & 0 & -2 \end{bmatrix},$$

$$K = \begin{bmatrix} i\hat{Q}_1 & i\hat{Q}_1 & 1 & 1 \end{bmatrix}.$$

Then we have

$$\mathcal{A} = \Sigma (R + \text{Im}(K^\dagger K)^\dagger) = \Sigma \begin{bmatrix} 2 & 0 & Q_2 - Q_1 & 0 \\ 0 & -2 & -2Q_1 & -(Q_2 + Q_1) \\ Q_2 + Q_1 & 0 & 2 & 0 \\ 2Q_1 & Q_1 - Q_2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} Q_2 + Q_1 & 0 & 2 & 0 \\ 2Q_1 & Q_1 - Q_2 & 0 & -2 \\ -2 & 0 & Q_1 - Q_2 & 0 \\ 0 & 2 & 2Q_1 & Q_2 + Q_1 \end{bmatrix}.$$

The characteristic polynomial of $\mathcal{A}$ is

$$\det(\lambda I - \mathcal{A}) = (\lambda^2 - 2Q_1\lambda + Q_2^2 - Q_1^2 + 4)^2.$$

Because $Q_1 < 0$ and $Q_2^2 - Q_1^2 + 4 > 0$, it follows from Vieta’s formulas that $\mathcal{A}$ is Hurwitz. Combining this fact with the Lyapunov equation [8] yields that the two-mode squeezed state is the steady state of the cascade system $G = G_2 \triangleleft G_1$. Based on the results in [19], a quantum optical realization of such a cascade system $G$ is given in Fig. 3. As shown in Fig. 3, the Hamiltonian of the system is realized by a pumped nonlinear crystal with specified pump intensity parameter $\varepsilon$ and cavity detuning parameter $\Delta$, and the coupling of the system is realized by implementing an auxiliary cavity. This auxiliary cavity interacts with the system via a cascade of a pumped nonlinear crystal and a beam splitter. The auxiliary cavity has a fast mode that can be adiabatically eliminated.

**Realization 2:** In this cascade realization, we use the result in Theorem [2]. By direct computations, the SLH models for the subsystems $G_1$ and $G_2$ are given by

$$\begin{align*}
\hat{\Sigma}_1 &= I_2, \\
\hat{L}_1 &= K_1 \hat{x}_1, \quad K_1 = \begin{bmatrix} -i\cosh(\alpha) & \cosh(\alpha) \\ i\sinh(\alpha) & \sinh(\alpha) \end{bmatrix}, \\
\hat{H}_1 &= \frac{1}{2} \hat{x}_1^\dagger R_1 \hat{x}_1, \quad R_1 = 0_{2 \times 2},
\end{align*}$$

and

$$\begin{align*}
\hat{\Sigma}_2 &= 1, \\
\hat{L}_2 &= [i\hat{Q}_1, 1]\hat{x}_2, \\
\hat{H}_2 &= -\frac{1}{2} \hat{x}_2^\dagger \begin{bmatrix} 2 & \hat{Q}_2 \\ \hat{Q}_2 & 2 \end{bmatrix} \hat{x}_2,
\end{align*}$$

respectively.
and

\[
\begin{align*}
\hat{S}_2 &= I_2, \\
\hat{L}_2 &= K_2 \hat{x}_2, \quad K_2 = \begin{bmatrix} i \sinh(\alpha) & \sin(\alpha) \\ -i \cosh(\alpha) & \cosh(\alpha) \end{bmatrix}, \\
\hat{H}_2 &= \frac{1}{2} K_2 R_2 \hat{x}_2, \quad R_2 = 0_{2 \times 2},
\end{align*}
\]

respectively.

A corresponding quantum optical realization of such a cascade system \( G = G_2 \otimes G_1 \) is given in Fig. 4. As \( R_1 = R_2 = 0_{2 \times 2} \), no optical crystals have to be implemented for the Hamiltonians of the system. On the other hand, as one component of the coupling operator vector \( \hat{L}_1 \) of the subsystem \( G_1 \) is

\[
\hat{L}_{11} = [-i \cosh(\alpha) \quad \cosh(\alpha)] \begin{bmatrix} \hat{q}_1 \\ \hat{p}_1 \end{bmatrix} = -i \sqrt{2} \cosh(\alpha) \hat{a}_1,
\]

where \( \hat{a}_1 \) denotes the annihilation operator, it can be simply implemented with a partially transmitting mirror without implementing an auxiliary cavity. This situation also occurs in the subsystem \( G_2 \).

As shown in Fig. 3 and Fig. 4, the engineered system in Realization 1 is coupled to only one quantum noise, while in Realization 2 the engineered system is coupled to two quantum noises.

**Remark 2:** It was shown in [7] that a pure entangled Gaussian state is generated in a simpler cascade system composed of standard optical parametric oscillators. However, the state generated is asymmetric, unlike the two-mode squeezed state. The point obtained above is that a symmetric (thus highly) entangled state is generated in a one-way (thus asymmetric) cascade system.

**V. CONCLUSIONS**

This paper has considered the problem of preparing pure Gaussian states in a linear quantum system. We have shown that any pure Gaussian state can be generated by a pure cascade of several one-dimensional open quantum harmonic oscillators. No interaction Hamiltonians have to be implemented between these oscillators. This pure cascade feature indicates that the proposed approach may be useful in quantum information processing. For instance, as mentioned in Section I, it yields a direct realization of a quantum communication channel where each subsystem corresponds to a quantum repeater [29]. More precisely, we can now dissipatively generate a long-distance entangled state by assigning a target pure Gaussian state to \( \hat{\rho}_{12} \otimes \hat{\rho}_{34} \otimes \cdots \otimes \hat{\rho}_{(n-1)n} \) with \( \hat{\rho}_{ij} \) a two-mode squeezed state generated among the \( r \)th and \( j \)th subsystems (repeaters) and then by performing the entanglement swapping via Bell-measurement on each site.

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