Techniques for improved heavy particle searches with jet substructure

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We present a generic method for improving the effectiveness of heavy particle searches in hadronic channels at the Large Hadron Collider. By selectively removing, or pruning, protojets from the substructure provided by a $k_T$-type jet algorithm, we improve the mass resolution for heavy decays and decrease the QCD background. We show that the protojets removed are typical of soft radiation and underlying event contributions, and atypical of accurately reconstructed heavy particles.

PACS numbers: 13.87.-a, 29.85.Fj

The Large Hadron Collider (LHC) presents at once great opportunity and great challenge. Many scenarios for new physics involve heavy particles that decay, possibly through a cascade, to Standard Model (SM) light quarks and gluons. The resulting final states consist partly, or even entirely, of jets. If the new particles are not too heavy, they may often be produced with sufficient boost to appear in a single jet. Thus, in the search for new physics at the LHC, identifying those jets that contain the decay of a heavy particle may be an important tool. The key difficulty will be separating this signal from the SM background, namely QCD jets. Recently, several groups have suggested novel and effective techniques for separating hadronic decays of heavy particles from QCD making use of the expected differences in the internal structure of the jets \cite{1,2,3,4,5,6,7}. The procedures proposed tend to be “top-down” in the sense that they are tuned to specific properties of, say, the two-pronged decay of a Higgs boson, or the three-pronged decay of a top quark. Here we present a related approach, based, of course, on the same underlying differences between real decays and QCD, but of a simpler nature and intended for use in general searches for new (a priori unknown) heavy particles.

While historically the masses of jets have played little role in the analysis of collider data, this is likely to change at the LHC \cite{8}. The simplest way to search for heavy particle decays into single jets is to look for features (“bumps”) in the jet mass distribution for an observed jet sample. Since QCD lacks any intrinsic scale beyond $\Lambda_{QCD}$, the background will be featureless aside from statistical fluctuations. Further, if the heavy particle decay includes a chain of new heavy particles, it is natural to ask whether we can look for evidence of these other mass scales in the substructure of the jet. Consider, for example, searching for a top quark in a single jet (as in \cite{1,2,3}). (We will use the top quark as a surrogate for new particle searches in the studies outlined below.) We would not only expect to see an enhancement for jet masses near the top quark mass, but we would expect correlated evidence of the $W$ boson mass in the substructure of the jet. If we are using a recombination algorithm such as the $k_T$ algorithm, the natural choice is to identify the $W$ with one of the protojets involved in the final merging.

Our aim in this paper is to present a procedure that improves the effectiveness of this type of search. Our technique suppresses systematic effects of the jet algorithm, as well as generic features of hadron collider events, such as the underlying event. Both effects tend to obscure the mass scales present in a heavy particle decay as observed in a single jet. Our technique narrows the structure in the jet and protojet mass distributions for jets from heavy particle decays, and reduces the smooth background QCD jet mass distribution. The result is a substantially increased likelihood of identifying a new physics (heavy particle) signal in the measured jet and protojet mass distributions.

Jet algorithms are designed to interpret long-distance degrees of freedom observed in the detector in terms of short-distance degrees of freedom. The algorithms take a set of initial protojets, such as calorimeter towers, and group them into jets. Recombination algorithms are a special class of jet algorithms that specify a prescription to pairwise combine protojets in an iterative procedure, eventually yielding jets. This prescription is based on the dominant soft and collinear physics in the QCD shower, so that the algorithm can trace back to objects coming from the hard scattering. The pairwise merging scheme of recombination algorithms naturally gives substructure to a jet, which provides kinematic handles to determine whether the jet was produced by QCD alone or a heavy particle decay plus QCD.

A general recombination algorithm uses a distance measure $\rho_{ij}$ between protojets to control how they are merged. A beam distance $\rho_b$ determines when a protojet should be promoted to a jet. The algorithm iteratively finds the smallest of the $\rho_{ij}$ and the $\rho_b$. If the smallest is a $\rho_{ij}$, protojets $i$ and $j$ are merged into a new protojet. Otherwise, the protojet corresponding to the smallest $\rho_i$ is promoted to a jet. The algorithm terminates when no protojets remain.

For the $k_T$ \cite{9} and Cambridge-Aachen (CA) \cite{10} algo-
rithms, the metrics are
\[ k_T : \rho_{ij} \equiv \min(p_T^i, p_T^j) \frac{\Delta R_{ij}}{D}, \quad \rho_i \equiv p_T^i; \]
\[ CA : \rho_{ij} \equiv \frac{\Delta R_{ij}}{D}, \quad \rho_i \equiv 1; \]

where \( p_T^i \) is the transverse momentum of protojet \( i \) and \( \Delta R_{ij} \equiv \sqrt{(\phi_i - \phi_j)^2 + (y_i - y_j)^2} \) is a measure of the angle between two protojets, where \( \phi \) is the azimuthal angle around the beam direction and \( y \) is the rapidity along the beam direction. The angular parameter \( D \) governs when protojets should be promoted to jets: it determines when a protojet’s beam distance is less than the distance to other objects. The substructure arising from this pairwise merging procedure is straightforward.

In considering the kinematics of the substructure, two variables, \( z \) and \( \theta \), are particularly useful. For a recombination \( 1, 2 \to p \), we define
\[ z \equiv \min(p_T^1, p_T^2)/p_T^p, \quad \theta \equiv \Delta R_{12}. \]

To identify heavy particle decays reconstructed in a single jet, we are concerned with recombinations that occur at large \( \theta \), typically the final recombination. In general, small-\( \theta \) recombinations are likely to represent the QCD showering of the decay products. Similarly, small-\( z \), or soft, recombinations are typical for a QCD shower. Even the large-angle, but small-\( z \), recombinations that can appear in jets from a heavy particle decay will be unlikely to yield an accurate representation of the decay: if a heavy particle decays such that one decay product has a much lower \( p_T \) relative to the others, the parent particle is unlikely to be accurately reconstructed. So, while the variable \( z \) can be an effective discriminator between QCD and decays in principle, the substructure found by the jet algorithm often does not faithfully represent the differing dynamics. Soft radiation, as well as soft contributions from the underlying event and pileup, will be present in all jets. These contributions to the jet lead to broadened mass distributions, especially for \( k_T \) jets. In addition, due to the systematic effects of the jet algorithm, these soft contributions can often appear in the final recombination. This is particularly true for \( CA \) jets, because \( CA \) orders strictly by \( \theta \). The large number of soft protojets ensures that frequently one will appear at a large angle in the final recombination.

We now define a procedure that systematically removes these undesirable soft, large angle recombinations. The procedure operates by rerunning the algorithm and vetoing on these recombinations, i.e., removing, or pruning, them from the substructure of the jet. It is algorithmically similar to others [3, 5], which also modify the jet substructure to improve heavy particle identification. The key distinction is that pruning is applied to an entire jet from the bottom up, with no goal of finding a particular number of “subjets”. The pruning procedure is:

1. Rerun the jet algorithm on the set of initial protojets from the original jet, checking for the following condition in each recombination \( 1, 2 \to p \):
\[ z < z_{cut} \quad \text{and} \quad \Delta R_{12} > D_{cut}. \]

2. If this condition is met, do not merge the two protojets 1 and 2 into \( p \). Instead, discard the softer protojet and proceed with the algorithm. The resulting jet is the pruned jet.

The pruning procedure involves two parameters, \( z_{cut} \) and \( D_{cut} \), which determine how small \( z \) must be and the minimum angle \( \Delta R \) of the recombination for it to be pruned. In this study we use \( D_{cut} = m_J/p_T^J \) for both \( k_T \) and \( CA \), where \( m_J \) is the mass of the originally identified jet and \( p_T^J \) is its transverse momentum. This choice is both adaptive to the properties of the individual jet and IR safe. Pruning with a smaller \( D_{cut} \) degrades the mass resolution by significantly pruning the QCD shower of daughter partons of the heavy particle decay, and pruning with a larger \( D_{cut} \) does not take full advantage of the procedure. For the \( CA \) algorithm, we use \( z_{cut} = 0.10 \). Because the \( k_T \) algorithm orders recombinations partly in \( z \), very small-\( z \) recombinations are not expected at the end of the algorithm. This implies a more aggressive pruning procedure is needed for the \( k_T \) algorithm, so in this study we use \( z_{cut} = 0.15 \) for the \( k_T \) algorithm. We find that these values of the pruning parameters yield roughly optimal results, largely insensitive to small changes in their values [11].

We examine the effects of the pruning procedure in a study of top quark reconstruction and separation from the QCD background. The top quark serves as a surrogate for a heavy particle decay at the LHC, and lets us learn about the effects of pruning in identifying heavy particles.

We generate events using MadGraph/MadEvent v4.4.21 [12] interfaced with Pythia v6.4 [13] for showering and hadronization. For the QCD background, we produce a matched sample of 2, 3, and 4 hard partons (gluons and the four lightest quarks) using MLM-style matching implemented in MadGraph (see, e.g., [14]). We use the DWT tune [15] in Pythia to give a “noisy” underlying event. No detector simulation is performed so we can isolate the “best case” effects of our method.

The signal sample is \( t\bar{t} \) production with fully hadronic decays. We generate signal and background samples with a parton-level \( h_T \) cut for generation efficiency, where \( h_T \) is the scalar sum of all \( p_T \) in the event. Because we focus on single jet methods to identify heavy particles, we make samples defined by criteria on jets instead of events. For each sample, we select central jets (with pseudorapidity \(|\eta| < 2.5 \)) and divide them into four \( p_T \) bins: [200, 500], [500, 700], [700, 900], and [900, 1100] (all in GeV/c). These bins confine the top quark boost to a narrow range
within each bin and allow us to study the performance of pruning as the top quark $p_T$ varies. For each $p_T$ bin $[pT_{\text{min}}, pT_{\text{max}}]$, the parton-level $h_T$ cut is $pT_{\text{min}} - 25 \text{ GeV/c} \leq h_T / 2 \leq pT_{\text{max}} + 100 \text{ GeV/c}$. We take the matching scales ($Q_{\text{cut}}, Q_{\text{match}}$) to be $(20, 30) \text{ GeV}$ for the lowest $p_T$ bin and $(50, 70) \text{ GeV}$ in the other three bins.

From the hadron-level output of Pythia, we group final-state particles into “cells” based on the segmentation of the ATLAS hadronic calorimeter ($\Delta \eta = 0.1$, $\Delta \phi = 0.1$ in the central region). We sum the four-momenta of all particles in each cell and rescale the resulting three-momentum to make the cell massless. After a threshold cut on the cell energy of 1 GeV, cells become the inputs to the jet algorithm. Our implementation of recombination algorithms uses FastJet [16].

To quantify the effects of pruning in top identification and background separation, we define criteria for a jet to be labeled as reconstructing a top quark decay. For either the pruned or unpruned jet, a top jet is one whose mass is within the top mass window and whose heavier daughter protojet mass is within the $W$ mass window. Both windows come from fits to the mass distributions in the signal sample, and do not need to be known a priori. These are fit using a skewed Breit-Wigner distribution for the peak and a power-law continuum for the background. These functions are

$$f(m) = \frac{[a + b(m - M)]}{(m^2 - M^2)^2 + M^2 \Gamma^2}$$

continuum: $g(m) = \frac{c}{m} + \frac{d}{m^2}$. (4)

The fitted mass $M$, which is within a few GeV/$c^2$ of $m_{\text{top}}$, and the fitted width $\Gamma$ are the relevant parameters; the mass window is $M \pm \Gamma$. These mass windows are in general different for the pruned and unpruned samples. In Fig. 1 we plot the top and $W$ window widths for the $k_T$ and CA algorithms for both pruned and unpruned jets. We refer to the pruned version of algorithm A as $pA$.

The top and $W$ mass windows are significantly narrower for the pruned samples. Moreover, the widths for the pruned $k_T$ and CA algorithms are very similar, unlike the unpruned case. The narrower widths mean fewer jets from the QCD samples will be misidentified as tops.

We now discuss a more quantitative measure of the performance of pruning. From the found mass windows we count the number of top jets in the signal and background samples, $N_s(A)$ and $N_b(A)$, for algorithm A. Using these counters, we define a statistical measure, $S$, to quantify how pruning improves top identification and separation from QCD backgrounds. $S$ is defined as

$$S = \frac{N_s(pA)/\sqrt{N_b(pA)}}{N_s(A)/\sqrt{N_b(A)}}.$$ (5)

which is the improvement from pruning in the ratio of the signal size to the statistical fluctuations in the background, and is a measure of the expected improvement in significance of the signal. Values greater than one indicate an improvement in pruning versus not pruning. Note that while the significance of the signal depends on the relevant cross sections and the integrated luminosity, the improvement measure $S$ does not. Using a constant value of $D = 1.0$ for all $p_T$ bins, we plot $S$ in Fig. 2 for both the $k_T$ and CA algorithms.

![FIG. 1: Pruned and unpruned top (a) and $W$ (b) mass window widths (in GeV/$c^2$) versus $p_T$ window center (in GeV/c) for both $k_T$ and CA algorithms.](image)

![FIG. 2: $S$ vs. $p_T$ for the CA and $k_T$ algorithms, using $D = 1.0$. Statistical errors, due to limited QCD sample sizes after cuts, are shown.](image)

For both algorithms, the measure $S$ is in the range 1.2 to 1.4 in the lowest $p_T$ bin, and increases with increasing $p_T$, with a dramatically increased significance in the range of 3 to 6 in the highest bin. These large values of $S$ arise partially from using a fixed value of $D$ with varying $p_T$. The opening angle of the typical top quark decay varies as $\Delta R \approx 2m_{\text{top}}/p_T$, which is less than $D = 1.0$ in the larger $p_T$ bins. The large $D$ allows for extra radiation to be merged in the jet, which may sufficiently alter the order of the substructure reconstruction to render an actual top decay no longer identifiable as a top jet. Additionally, a larger $D$ at fixed $p_T$ leads to larger mass QCD jets and enhances the probability to fake top quarks. In both scenarios the extra radiation included...
within the larger $D$ jet is often soft and uncorrelated. Hence pruning tends to dramatically improve top finding at large $p_T$ in fixed $D$ jets.

In a real search, the mass of the heavy state is not known. Once an enhancement in the mass distribution has been observed, knowledge of the purported mass can be used to tune the analysis parameters, such as $D$. (Another approach, using “variable-R” jets, is discussed in [17].) Even if $D$ is tuned for each $p_T$ bin to maximize the performance of the unpruned algorithm, we would still expect pruning to show an improvement over the unpruned case. This can be seen in the lowest bin of Fig. 2 where the value of $D = 1.0$ is already roughly optimal and $S$ is still larger than 1.

Given that pruning always provides an improvement, the relevant question for designing a search procedure using single jets is whether pruned, tuned-$D$ jets provide much better results than pruned, fixed-$D$ jets. To answer this question, we compare signal-to-noise for pruned jets with fixed $D = 1.0$ to the case where $D$ is picked for each $p_T$ bin to match the typical opening angle of the top quark decay. In particular, we set $D$ to be approximately $2m_{top}/p_T^\text{min}$, where $p_T^\text{min}$ is the lower $p_T$ limit for the given bin, up to a maximum of 1.0. Thus we choose the $D$ values of $\{1.0, 0.7, 0.5, 0.4\}$ for our $p_T$ bins. This exercise leads to Fig. 3 where we see a ratio analogous to $S$ that we call $S_D$. For each $p_T$ bin, $S_D$ is the ratio of signal-to-noise for pruned jets with the value of $D$ from the above list to signal-to-noise for pruned jets with fixed $D = 1.0$. We see that the values of $S_D$ are close to one for all $p_T$ bins. This implies the important result that, as long as we prune the jets, using a tuned $D$ value for each $p_T$ bin provides little advantage over the simpler fixed $D$ analysis. Note also that in Fig. 3 the statistical uncertainties in $S_D$ are on the order of the improvements. This procedure, pruning, removes recombinations unlikely to represent an accurately reconstructed heavy particle, narrows mass distributions of reconstructed states and reduces the QCD background in a given mass bin. As we have demonstrated, heavy particle searches can benefit from all of these effects. While unpruned jets are sensitive to the specific choice of jet algorithm and the value of the parameter $D$, pruning removes much of this sensitivity. It is just as effective to use a large $D$ over a broad range in $m/p_T$ of the heavy state. When searching for a particle of unknown mass, pruning allows the use of a large fixed $D$ without losing statistical power.

The effects of pruning, and in general the application of jet substructure to find heavy particles, requires further study \[11\]. Pruning must be verified as an effective component of heavy particle searches at the LHC, including understanding the impact of using a realistic detector. An important test bed for pruning and other jet substructure tools will be early validation studies of the Standard Model at the LHC, where we expect to be able to observe top quarks, $W$’s and $Z$’s in the single jet data. Initial studies such as that described here give promising indications that these tools will prove useful in the search for new physics.

We gratefully acknowledge helpful discussions with Matt Strassler and Gavin Salam, and also with Karl Jacobs, Peter Loch, Michael Peskin, Tilman Plehn, and others in the context of the Joint Theoretical-Experimental Terascale Workshops at the Universities of Washington and Oregon, supported by the U.S. Department of Energy under Task TeV of Grant No. DE-FG02-96ER40956. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-96ER40956. J.W. was also supported in part by an LHC Theory Initiative Graduate Fellowship.

![Graph: $S_D$ vs. $p_T$ for the CA and $k_T$ algorithms. The line at $S_D = 1$ separates the regions where a tuned $D$ helps (above the line) and does not (below). The lowest $p_T$ bin is not shown because the $D$ value does not change ($S_D = 1$). Statistical errors are shown.](image)

In this work, we have introduced a generic procedure that modifies jet substructure to improve heavy particle identification and separation from QCD backgrounds.

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