Double Andreev reflections and double electron transmissions in a normal-superconductor-normal junction based on type-II Weyl semimetal

Xue-Si Li¹, Shu-Feng Zhang¹,²,①, Xue-Rui Sun¹ and Wei-Jiang Gong¹,³
¹ College of Sciences, Northeastern University, Shenyang 110819, People’s Republic of China
² School of Physics and Technology, University of Jinan, Jinan, Shandong 250022, People’s Republic of China
³ Authors to whom any correspondence should be addressed.

E-mail: sps_zhang@ujn.edu.cn and gwj@mail.neu.edu.cn

Keywords: Andreev reflection, type-II Weyl semimetal, NSN junction

Abstract
We study the quantum transport behavior of a normal-superconductor-normal junction based on a type-II Weyl semimetal, which is arranged in the tilting direction of the Weyl semimetal. We find that both the crossed Andreev reflection and normal reflection are forbidden, while there will be double Andreev reflections and double electron transmissions for the incident electron from the semimetal side. Andreev reflections and transmissions occur both in the retro and specular directions simultaneously, symmetric about the normal of the interface but with different amplitudes, depending on the angle and energy of incident electrons. These transport processes make the junction here quite different from that based on the normal metal or graphene. In addition, the differential conductance is studied for experimental signatures. We find that the conductance is almost unaffected by the electrostatic potential in the normal region and it is enhanced with increasing junction length.

1. Introduction

The interface between a normal and a superconducting material has been a widely used platform to realize exotic transport behaviors, especially in recent years when plenty of new materials with massless linear excitations or nontrivial topological properties are predicted and used to fabricate these interfaces [1–9]. In a conventional normal metal-superconductor (NS) junction, there exists not only a normal reflection (NR) process but also an Andreev reflection (AR) process, in which an incident electron from the normal side is reflected back as a positive charged hole, and a Cooper pair forms in the superconductor (SC) [10]. It is also known as retro AR, since the hole retraces almost the path of the incident electron. Following the successful fabrication of some new low-dimensional materials, novel AR phenomena have been reported from various aspects. One typical result is the specular AR, which has been discovered with the hole reflected along a specular path of the incident electron in the NS junction based on the graphene-like materials [1,11]. And the perfect AR has been proposed and discovered in the NS junction of topological insulator [12,13]. Besides, in the SNS junction of topological SCs, the fractional Josephson effect is allowed to come into being [14,15].

Very recently, the type-II Weyl semimetal (WSM) has been predicted and observed in several materials, e.g., MoTe₂ [16,17], LaAlGe [18], WTe₂ [19,20] and MoₓW₁₋ₓTe₂ [21,22]. Like the type-I WSM, the low-energy excitations can be described by Weyl equation, but with a tilted anisotropic energy spectrum. For the type-I WSM, the conduction and valence bands intersect at several Weyl nodes and thus the Fermi surface is point-like and the spectra around the nodes are coniclike [2]. However, the spectra of the type-II WSM are tilted by rotating around the Weyl nodes and there will exist electron and hole pockets near the line-like Fermi surface with a large density of states [23–25]. Therefore, the type-II WSMs will show a lot of interesting properties, such as the anomalous Hall effect, chiral anomaly and other intriguing properties [26–31]. Further more, the tilted energy bands of type-II WSM have opportunities to bring new transport mechanisms. According to a preceding report
The phenomenon of double ARs has been predicted in the NS junction based on type-II WSM, where the retro and specular ARs coexist and the NR process is forbidden. In view of the AR result contributed by the type-II WSM, it is more likely that novel transport behavior will emerge in a normal-superconductor-normal (NSN) junction based on type-II WSM. Motivated by such a topic, in the present work we concentrate on the NSN junction arranged in the tilting direction of the type-II WSM and perform the discussion of the transport properties, with the help of the scattering matrix method. As is known, in a conventional NSN junction of normal metal, four transport processes coexist, i.e., NR, normal electron transmission (ET), AR, and crossed AR, as shown in figure 1(a) [33–38], where the crossed AR process refer to the incident electron enters into the SC accompanied by a hole in the other metal side. However, our study indicates that the NR and crossed AR are forbidden in the NSN junction based on the type-II WSM. Instead, the double ARs and double ETs happen simultaneously, including retro AR ($A_1$), specular AR ($A_2$), normal transmission ($T_1$), and specular transmission ($T_2$), just as shown in figure 1(b). It shows that two ARs (ETs) have the same reflection (refraction) angle but with different amplitudes in general. The prediction of ‘double ARs and double ETs’ phenomenon is the main result of our work. For more detail, the relationships between the amplitudes of these four scattering processes and incident angle, junction length and electrostatic potential in the normal region, are fully investigated in this paper. Just as important, the differential conductance is studied, and it has been found to have a larger magnitude considering the large momentum mismatch between the normal and SC region. Moreover, it is almost unaffected by the electrostatic potential in the normal region and enhanced with the increase of junction length, which can be viewed as the obvious signatures for experimental observation.

The rest of the paper is organized as follows. In section 2, we give the model Hamiltonian and introduce the phenomenon of double ARs and double ETs. In section 3, we calculate the amplitudes of these four transport processes with the scattering matrix method. In section 4, we study the differential conductance. In section 5, we give a brief conclusion.

2. The phenomenon of double Andreev reflections and double electron transmissions

2.1. Model
The schematic of the NSN junction based on type-II WSM is shown in figure 1(b). We model the type-II WSM with a most simple low-energy effective Hamiltonian which respects time reversal symmetry while breaks spatial inversion symmetry [39, 40]. Near Weyl point $k_0$, the Hamiltonian is written as [32]
via time reversal symmetry, and it takes the form
\[ \Delta_{\pm}(k) = \hbar v_{\pm} k, \]

in the basis \((\psi_{K_0 + k, s}, \psi_{K_0 - k, s})^T\). \(k\) is a wave vector small enough to ensure that the low-energy model is effective. \(\sigma = (\sigma_x, \sigma_y, \sigma_z)\) denotes the Pauli matrix, \(\sigma_0\) is the identity matrix. \(v_1\) describes the Fermi velocity and \(v_1\) determines the tilt along \(x\)-direction. It is a type-II WSM for \(|v_1| > v_2\) while a type-I WSM for \(|v_1| < v_2\). \(\nu\) is the electrostatic potential, which can be tuned by a gate voltage or doping. The Hamiltonian near the \(-K_0\) node is related with that near \(K_0\) via time reversal symmetry, and it takes the form
\[ H_{-}(k) = -\hbar v_{\pm} k, \]
in the basis \((\psi_{K_0 + k, s}, \psi_{K_0 - k, s})^T\). In the SC region, Cooper pairing forms between the electrons from the \(K_0\) and \(-K_0\) Weyl points. We consider the BCS pairing for simplicity. Then, the pairing Hamiltonian can be written as
\[ H(p) = \hbar \omega k, \sigma \cdot \nabla \phi, \]
where the wave vector has been replaced with \(k = -i\nabla_x\), since the translation invariance is broken along \(x\)-direction. \(\mu\) is the chemical potential of the whole system. In the absence of any bias voltage, the system will be in equilibrium, and we will be allowed to set \(\mu = 0\) for numerical calculation. Both the superconducting order parameter \(\Delta\) and electrostatic potential \(\nu\) is spatially dependent, which can be expressed as
\[ \Delta(r) = \begin{cases} 0 & \text{if } x < 0, \text{ or } x > L, \\ \Delta & \text{if } 0 < x < L. \end{cases} \]
\[ \nu(r) = \begin{cases} -V_g & \text{if } x < 0, \text{ or } x > L, \\ -U & \text{if } 0 < x < L. \end{cases} \]

The energy dispersion for quasiparticles in the SC region is written as
\[ E_{\pm}(k) = \pm \sqrt{\Delta^2 + (\hbar v_{\pm} k + \hbar \nu k - \tilde{U})^2}, \]
with \(\tilde{U} = U + \mu\). The dispersion for eigenstates with \(k_y = 3\), \(k_z = 0\) is plotted in figure 2(b) and it indicates that there are two right-moving and two left-moving modes for a fixed energy. In the WSM region, superconducting order \(\Delta\) vanishes and there are two electron modes and two hole modes with energy dispersions given as
\[ E_{e\pm}(k) = \hbar v_{\pm} k, \pm \hbar \nu k - \tilde{V}_g, \]
\[ E_{h\pm}(k) = -\hbar v_{\pm} k, \pm \hbar \nu k + \tilde{V}_g, \]
in which \(k = \sqrt{k_x^2 + k_y^2 + k_z^2}\) and \(\tilde{V}_g = V_g + \mu\). \(E_{e\pm}(E_{h\pm})\) is the conduction band for electrons (holes), while \(E_{e-}(E_{h+})\) describes the valence band. The group velocity is the gradient of dispersion \(v = \frac{1}{\hbar} \nabla_k E(k)\) and can be derived as

![Figure 2. Energy spectra with finite \(k_x\) and \(k_y\) for (a), (c) the type-II WSM and (b) SC region. In panel (a), the conduction/valence bands are colored with red/blue, and the solid/dashed lines denote electrons/holes. The arrows denote the direction of the incident, reflected and transmitted quasiparticles. Parameters: \(k_x = 3, k_z = 0, v_1 = 2, v_2 = 1, \Delta = 0, U = 10, \mu = 0\).](image-url)
\[ v_{x+}^e = v_1 + v_2 k_x / k, \quad v_{x-}^e = v_2 k_x / k, \]
\[ v_{x+}^h = -v_1 + v_2 k_x / k, \quad v_{x-}^h = v_2 k_x / k, \]
\[ v_{y+}^e = v_1 - v_2 k_y / k, \quad v_{y-}^e = -v_2 k_y / k, \]
\[ v_{y+}^h = -v_1 - v_2 k_y / k, \quad v_{y-}^h = v_2 k_y / k. \]

The \( z \)-component, which is missed here, shares a similar form as the \( y \)-component since the system is rotation invariant about \( x \)-axis. It is clear that both electron modes are right-moving while both hole modes are left-moving for type-II WSM with \(|v_1| > v_2\), as is shown in figures 2(a) and (c).

### 2.2. Double ARs and double ETs

Now we consider an incident electron from the left WSM region in the channel \( \psi_1 \) with energy \( E \) and wave vectors \( k_x \) and \( k_y \), as shown in figure 2(a). Because of the translation invariance, \( k_x \) and \( k_y \) are good quantum numbers and keep invariant in the transport process. Double ARs occur at the interface between the left WSM and SC regions. The incident electron will be retro reflected \((A_i)\) or specular reflected \((A_s)\) as a hole of modes \( \psi_{h+} \) or \( \psi_{h-} \), respectively. These two Andreev modes are symmetric about the normal of the interface in the incident plane with the same reflection angle \( \theta_{h} = \arctan \left( \frac{v_y}{\sqrt{v_x^2 - v_y^2}} \right) \) but different amplitudes depending on the incident angle and energy. Double ETs occur at the interface between the right WSM and SC regions. The incident electron transmits into the intraband and interband modes \( \psi_1 \) and \( \psi_2 \), respectively. The intraband (interband) ET corresponds to the normal ( specular) ET with amplitude \( T_1 \) ( \( T_3 \)). The intraband mode is identical to the incident mode while the specular ET is symmetric with the normal ET, with the same angle \( \theta_e = \arctan \left( \frac{v_x}{\sqrt{v_x^2 + v_y^2}} \right) \). On the other hand, we note that the NR and crossed AR are forbidden. These novel transport behaviors make this junction distinguished from the junction based on normal metal or graphene [33–38].

### 3. Amplitudes of double ARs and double electron transmissions

In this section, we study in detail the amplitudes of each transport process of the double ARs and double ETs via the scattering matrix method. The effect of electrostatic potential and junction length is also investigated. In the numerical calculations, we set \( v_1 = 2, v_2 = 1, \Delta = 1, U = 100 \) and \( V_g = 0.5 \) unless otherwise stated.

#### 3.1. Formalisms

The amplitude of each transport process and the differential conductance can be solved by the scattering matrix method. Since this system is rotation invariant about \( x \)-axis, we set \( k_z = 0 \) in the following discussion. Consider an electron incidents with energy \( E \) and wave vector \( k_x \). The wave-functions in the three regions can be expressed as

\[
\begin{align*}
\Psi_{\text{I}}(r) &= \Psi_{-}(r) + \eta \Psi_{h+}(r) + \zeta \Psi_{h-}(r), \\
\Psi_{\text{II}}(r) &= a \Psi_{0}(r) + b \Psi_{h+}(r) + c \Psi_{h-}(r), \\
\Psi_{\text{III}}(r) &= d \Psi_{-}(r) + e \Psi_{h+}(r),
\end{align*}
\]

where I, II, III denotes the left, central, and right region, respectively. \( \psi_{\pm} \) \( (\psi_{h\pm}) \) are the eigenvectors of two electron (hole)-modes and \( \Psi_{0,h,c,d} \) are corresponding modes in the SC region. The expressions of these eigenvectors are given in the appendix B. \( r_{1/2} \) is the retro/specular AR coefficient, and \( r_{1/2} \) is the normal/specular ET coefficients. \( a, b, c, d \) are coefficients of the quasiparticle modes in the SC region. These coefficients are determined by the boundary conditions at the two interfaces

\[
\begin{align*}
\Psi_{\text{I}}(r) \big|_{k=0^+} &= \Psi_{\text{II}}(r) \big|_{k=0^+}, \\
\Psi_{\text{II}}(r) \big|_{k=\infty} &= \psi_{\text{III}}(r) \big|_{k=\infty}.
\end{align*}
\]

The current density operator in the normal WSM region can be derived via \( J = \gamma \frac{1}{\hbar} [r, H_{\text{bdg}}] \). The \( x \)-component is

\[
J_x = \tau_2 (v_1 \sigma_0 + v_2 \sigma_3),
\]

where \( \tau_2 \) is the \( z \)-component of Pauli matrix which acts in the electron–hole space. Then the retro and specular AR coefficients \( A_1 \) and \( A_2 \) are evaluated by

\[
A_{1/2} = \left| \frac{\langle \Psi_{h\pm}|J_x|\Psi_{h\pm} \rangle}{\langle \Psi_{-}|J_x|\Psi_{-} \rangle} \right|^2.
\]
And the normal and specular ET coefficients $T_1$ and $T_2$ are evaluated by

$$T_{1/2} = \frac{|\langle \Psi_{\pm|L}\rangle \langle \Psi_{\pm|}\rangle|}{|\langle \Psi_{-|L}\rangle \langle \Psi_{-|}\rangle|} |\phi_{1/2}|^2.$$

(13)

Because of the conservation of current, we have $A_1 + A_2 + T_1 + T_2 = 1$.

### 3.2. Numerical results

In figure 3, we calculate the amplitudes of double ARs and double ETs, respectively, figure 3(a) shows the normal incidence case with junction length $L = \xi = \hbar (v_1 - v_2)/\Delta$. The electrostatic potential in the normal region is taken to be $V_g = 0.5$, which can be tuned by adjusting the gate voltage in experiment. One can find that the specular ET is forbidden, i.e., $T_2 = 0$, and the incident electron is Andreev reflected in the $A_1$ or $A_2$ mode for $E < V_g$ and $E > V_g$, respectively. However, the retro and specular modes coincide in the normal incident case to contribute identically to the quantum transport processes. Next, with the increase of incident energy, the ETs are enhanced while ARs are weakened monotonically in the energy regime $E < 2\Delta$. In the oblique incidence case shown in figures 3(b),(c), the four transport processes co-contribute to the transport through this junction. The retro AR coefficient $A_1$ decreases monotonically when the incident energy increases in the SC-gap regime $E < \Delta$, while the specular AR $A_2$ and normal ET coefficient $T_1$ exhibit increments in this regime, for junctions with length $L = \xi$ and $2\xi$ (see figures 3(b),(c)). However, the specular ET coefficient $T_2$ can decrease or increase in the cases of $L = \xi$ (figure 3(b)) and $L = 2\xi$ (figure 3(c)) in the low-energy regime $E < \Delta$. In the regime $E > \Delta$, all these scattering processes exhibit an oscillatory behaviors due to the coherent tunneling determined by the standing-wave condition. For the specular ET $T_2$, the valley (peak) of the oscillation emerges outside the gap regime leading to the increase (decrease) behavior in the gap regime for $L = \xi$ and $2\xi$, respectively. In figure 3(d), we present the total ET coefficient $T = T_1 + T_2$, the specular ET coefficient $T_2$ and the specular AR coefficient $A_2$ as functions of the incident energy for $V_g = 0.5$ and 5. It can be clearly found that the change of electrostatic potential in the normal region does not affect the ET coefficients $T$ and $T_1$, but only alters the relative weight of retro and specular ARs.

In what follows, we calculate the dependence of both double ARs and double ETs coefficients on incident angle $\theta$ and energy $E$, as shown in figure 4. The incident angle is related to the group velocity by the formula $\tan \theta = \frac{v_1}{v_2}$. By the velocity expressions in equation (8), we know that $\theta$ has a upper limit $\theta^* = \arctan \frac{v_1}{v_2} = 0.17\pi$, since the equi-energy surface is a hyperbola in the WSM region. Next, in figure 4(a) it shows that the retro AR $A_1$ decreases or increases monotonically with the increment of incident angle $\theta$ for electrons with incident energy $E < V_g$ and $E > V_g$, respectively. However, compared with $A_1$, the specular AR $A_2$ exhibits an opposite dependence on $\theta$, as displayed in figure 4(b). With respect to the double ETs, the results in figures 4(c),(d) show that $T_1$ and $T_2$ will decrease or increase monotonically if $\theta$ is increased in the whole energy regime $0 < E < 2\Delta$. Therefore, according to these results, we can find that the retro AR $A_1$ contributes mainly in the $E < V_g$ regime while specular AR $A_2$ mainly in the $E > V_g$ regime. This suggests that the AR can be
tuned via a gate voltage. Alternatively, the intraband ET $T_1$ contributes almost for all incident energy and angles, while the specular ET $T_2$ occurs mainly for large incident angles.

In figure 5, we study the dependence of AR and ET coefficients on the length of the SC region. Both the normal (figure 5(a)) and oblique incidences (figure 5(b)) are considered, by taking the incident energy to be $E = 0.6$. Firstly, in figure 5(a) one can find that in the normal incident case, only one AR mode and one ET mode appear, respectively, i.e., $A_2$ and $T_1$. This is exactly consistent with the result in figure 3(a) where both $A_1$ and $T_2$ modes vanish for $E > V_g$. In the short-junction limit $L \to 0$, only the intraband ET $T_1$ is allowed with its amplitude $T_1 = 1$, just as expected, since no scattering potential exists. With the increase of $L$, ET is weakened gradually while AR enhanced. However, in the long-junction limit, ET is suppressed and the complete AR takes place, in accordance with the result of type-II WSM-SC junction [32]. Next, in the oblique incidence case, both retro and specular AR coefficients, $A_1$ and $A_2$, increase monotonically until reach the saturation value, with the increase of junction length. In contrast, the ET coefficients will decrease accompanied by oscillation. As a consequence, the peak of the normal ET curve meets the specular ET valley, in the short-junction limit $L \to 0$.

The two ETs share an identical oscillation period, which can be approximated by $L = \frac{2\pi}{|k_{xx1} - k_{xx2}|}$ determined from the standing-wave condition. $k_{xx1}$ ($k_{xx2}$) is the real component of the wave vector of left (right) moving mode.
in the SC region. Most interestingly, there exists a phase shift of $\pi$ between normal and specular ETs, which vanishes the oscillation of the total transmission coefficient $T = T_1 + T_2$.

4. Conductance

Following the previous analysis of double ARs and double ETs, we investigate the properties of differential conductance in this section. We apply a bias voltage to the left WSM region, while the SC region and the right WSM region are grounded. Thus, the differential conductance can be calculated by the well-known BTK formula\cite{32,42}:

$$G(eV) = \frac{e^2S}{\pi\hbar} \sum_m \int dk_ydk_z [1 + A^{(m)}(k_y, k_z, eV)],$$

where $S$ is the cross-sectional area of the junction, $m = \pm$ is to distinguish the contribution of transport processes due to incident modes $\psi_{\pm}$, and $A^{(m)} = A_1^{(m)} + A_2^{(m)}$ is the total AR coefficient. The integration over the wave vectors is constrained near the Weyl node with a cut-off $q_m$, i.e., $\sqrt{(k_y^2 + k_z^2)} < q_m$. The contributions of ETs and ARs to the conductance, i.e., $G_T$ and $G_A$, can be written as

$$G_T(eV) = \frac{2e^2S}{\pi\hbar} \int dk_ydk_z T(k_y, k_z, eV),$$

$$G_A(eV) = \frac{2e^2S}{\pi\hbar} \int dk_ydk_z 2A(k_y, k_z, eV).$$

(15)

If we denote $G_N(eV) = \frac{2e^2S}{\pi\hbar} \cdot \pi q_m^2$, the conductance of a normal junction, there will be

$$G + G_T = 2G_N,$$

$$G_A = 2G - 2G_N.$$ 

(16)

This indicates that it is impossible to enhance both the total conductance and ET conductance at the same time.

The conductance $G$ and its ET component $G_T$ as functions of bias for several electrostatic potentials and SC region size $L$. Parameters: $v_1 = 2, v_2 = 1, U = 100, \mu = 0, \Delta = 1, q_m = 10$.

Figure 6. The conductance $G$ and its ET component $G_T$ as a function of the bias $eV$ for several electrostatic potentials and SC region size $L$. Parameters: $v_1 = 2, v_2 = 1, U = 100, \mu = 0, \Delta = 1, q_m = 10$.
occurrence of the perfect AR in the SC-gap regime $E < \Delta$. One can then find the result that $G/G_{TV} = 2$. If the bias is further increased, $G/G_T$ decreases (increases) gradually with an oscillation, due to the coherent tunneling determined by the standing-wave condition. However in the short-junction limit, both $G$ and $G_T$ are approximately equal to $G_{TV}$ in the SC-gap regime, because of the vanishing of AR. In addition, we notice that the conductance here has a large magnitude, considering that the large momentum mismatch between the normal and SC region contributes an effective scattering potential. In fact, it is exactly the absence of NR in our model leads to this phenomenon.

5. Summary

In summary, we have studied the transport property of the NSN junction based on type-II WSM, and predicted the novel phenomenon of double ARs and double ETs. It means that four scattering processes for incident electrons coexist in this system, i.e., the retro and specular ARs, the normal and specular ETs. However, the NR and crossed AR are forbidden. The retro and specular AR modes are symmetric about the normal of the interface but with different amplitudes, which is also the case for normal and specular ET processes. The dependences of the amplitudes on the incident angle and energy have been studied in detail with the effect of electrostatic potential and junction length considered. In addition, we have studied the differential conductance. It is found that the conductance is independent of electrostatic potential of the WSM region, suggesting its robustness. The normalized conductance will increase as enlarging junction length. The conductance has a large value considering the scattering potential due to the momentum mismatch between the normal and SC region.

Acknowledgments

This work was supported by the Liaoning BaiQianWan Talents program, the National Natural Science Foundation of China (Grant No. 11747122), the Natural Science Foundation of Shandong Province (Grant No. ZR2018PA007), and the Doctoral Foundation of University of Jinan (Grant No. 160100147).

Appendix A. BdG Hamiltonian

In this appendix, we derive the BdG Hamiltonian in equation (3).

Let us first define the operator $\hat{\psi}_{\nu,\mathbf{k}+\mathbf{k}'}$ to annihilate an electron with momentum $\pm \mathbf{k}_0 + \mathbf{k}$ and spin $\nu$. In the electron representation $\hat{\psi}_{\mathbf{k}+\mathbf{k}} = (\hat{\psi}_{\mathbf{k}+\mathbf{k},\uparrow}, \hat{\psi}_{\mathbf{k}+\mathbf{k},\downarrow})^T$, the second-quantized Hamiltonian near Weyl point $\mathbf{k}_0$ is

$$\hat{H}_+ = \sum_{\mathbf{k}} \hat{\psi}^\dagger_{\mathbf{k}+\mathbf{k}} (H_e(k) - \mu) \hat{\psi}_{\mathbf{k}+\mathbf{k}},$$

in which we have added the term of chemical potential. Similarly, the second-quantized Hamiltonian near the Weyl point $-\mathbf{k}_0$ can be expressed as

$$\hat{H}_- = \sum_{\mathbf{k}} \hat{\psi}^\dagger_{-\mathbf{k}+\mathbf{k}} (H_e(k) - \mu) \hat{\psi}_{-\mathbf{k}+\mathbf{k}},$$

in the electron representation $\hat{\psi}_{-\mathbf{k}+\mathbf{k}} = (\hat{\psi}_{-\mathbf{k}+\mathbf{k},\uparrow}, \hat{\psi}_{-\mathbf{k}+\mathbf{k},\downarrow})^T$.

$\hat{H}_+$ and $\hat{H}_-$ is related by time reversal symmetry. The time reversal operator $\hat{T}$ demands $\hat{T}\hat{\psi}_{\mathbf{k}+\mathbf{k}}\hat{T}^{-1} = T\hat{\psi}_{-\mathbf{k}+\mathbf{k}}$, where $T = -i\sigma_y K$ and $K$ is the complex conjugation operator. Then it is straightforward to prove that $\hat{H}_- = \hat{T}\hat{H}_+\hat{T}^{-1}$.

In the hole representation $(\hat{\psi}^\dagger_{-\mathbf{k}+\mathbf{k},\uparrow}, -\hat{\psi}^\dagger_{-\mathbf{k}+\mathbf{k},\downarrow})^T$, the Hamiltonian near $-\mathbf{k}_0$ becomes

$$\hat{H}_- = \sum_{\mathbf{k}} (\hat{\psi}^\dagger_{-\mathbf{k}+\mathbf{k},\uparrow}, -\hat{\psi}^\dagger_{-\mathbf{k}+\mathbf{k},\downarrow}) (-H_e(k) + \mu) \left( \begin{array}{c} \hat{\psi}_{-\mathbf{k}+\mathbf{k},\downarrow} \\ \hat{\psi}_{-\mathbf{k}+\mathbf{k},\uparrow} \end{array} \right) + \text{constant},$$

where the constant is a c-number and can be neglected.

Suppose that one electron near Weyl point $\mathbf{k}_0$ with quantum number $(\mathbf{k}_0 + \mathbf{k}, \nu)$ pairs with the other electron near Weyl point $-\mathbf{k}_0$ with quantum number $(-\mathbf{k}_0 - \mathbf{k}, \nu)$. The superconducting pairing term can be written as
The Hamiltonian of a bulk SC can be written as
\[ \hat{H} = \sum_k [\hat{\psi}^+_k \sigma^+_k \hat{\psi}_k + \text{h.c.}] \] (A4)

The Hamiltonian of a bulk SC can be written as \( \hat{H} = \hat{H}_\text{in} + \hat{H}_\text{Nambu} + \hat{H}_\text{out} \). In the Nambu representation \( \hat{\Psi}_k = (\hat{\psi}_{k_x+k_y}, \hat{\psi}_{k_x-k_y}, \hat{\psi}^+_{-k_x+k_y}, -\hat{\psi}^+_{-k_x-k_y}) \), it can be expressed as
\[ \hat{H} = \frac{1}{2} \sum_k \hat{\Psi}_k \mathcal{H}_\text{BdG} \hat{\Psi}_k + \text{constant}, \] (A5)

where the constant is a \( c \)-number. The BdG Hamiltonian can be derived straightforwardly with the equation above. The final result says that
\[ H_{\text{BdG}} = \begin{pmatrix} H_+(k) - \mu\sigma_0 & \Delta \sigma_0 \\ \Delta^\dagger \sigma_0 & -H_-(k) + \mu\sigma_0 \end{pmatrix}, \] (A6)

When it comes to the NSN junction, \( k \) should be replaced by \( i\nabla_x, v(r) \) and \( \Delta(r) \) will become space-dependent. Then we derive equation (3) in the text.

### Appendix B. Eigenvectors

In this appendix, we give the eigenvectors in the WSM and SC regions.

The BdG equation in WSM region is represented as
\[ \begin{pmatrix} H_+(k) - \mu & 0 \\ 0 & -H_-(k) + \mu \end{pmatrix} \Psi = E \Psi. \] (B1)

Two incident and two reflected modes with energy \( E \) and wave vector \( k_x \) and \( k_y \) can be given as [32]

\[ \begin{align*}
\Psi_{\text{inc.}} &= \begin{pmatrix} k_x + k_y \\ \nabla_x + ik_y \\ 0 \\ 0 \end{pmatrix} \exp(ik_{\text{inc.}}x + ik_{\text{inc.}}y + ik_{\text{inc.}}z), \\
\Psi_{\text{refl}} &= \begin{pmatrix} -k_x + k_y \\ k_x + ik_y \\ 0 \\ 0 \end{pmatrix} \exp(ik_{\text{refl.}}x + ik_{\text{refl.}}y + ik_{\text{refl.}}z), \\
\Psi_{\text{h.}} &\equiv \begin{pmatrix} k_x' + k_y' \\ k_x' + ik_y' \\ 0 \\ 0 \end{pmatrix} \exp(ik_{\text{h.}}'x + ik_{\text{h.}}'y + ik_{\text{h.}}'z), \\
\Psi_{\text{h.}} &\equiv \begin{pmatrix} k_x'' + k_y'' \\ k_x'' + ik_y'' \\ 0 \\ 0 \end{pmatrix} \exp(ik_{\text{h.}}''x + ik_{\text{h.}}''y + ik_{\text{h.}}''z).
\end{align*} \] (B2)

In equation (B2), the wave vectors and their \( x \)- and \( y \)-components of each energy band are, respectively, given as

\[ k_{\text{(x,y)}} = \frac{\nu_1(E + V_g) \mp \nu_2 \sqrt{(E + V_g)^2 + \hbar^2 (\nu_1^2 - \nu_2^2)(k_x^2 + k_y^2)}}{\hbar (\nu_1^2 - \nu_2^2)}, \]
\[ k_{\text{(x,y)}}' = \frac{-\nu_1(E - V_g) \pm \nu_2 \sqrt{(E - V_g)^2 + \hbar^2 (\nu_1^2 - \nu_2^2)(k_x^2 + k_y^2)}}{\hbar (\nu_1^2 - \nu_2^2)}, \]
\[ k_\pm = \sqrt{k_{\text{xx}}^2 + k_{\text{yy}}^2}, \quad k_\pm' = \sqrt{k_{\text{xx}}'^2 + k_{\text{yy}}'^2}, \]

where \( k_{\text{xx}} \) and \( k_{\text{xx}}' \) are corresponding \( x \)-component of wave vectors, shown in figure 2.

The BdG equation of the SC region is given in equation (3). In the large- \( U \) limit [1], the four excited modes with energy \( E > 0 \) and wave vector \( k_x \) and \( k_y \) can be approximated as [32]
\[ \Psi_{a}(r) = \begin{cases} e^{-i\beta} & \text{if } E < \Delta, \\ 1 & \text{if } E > \Delta, \end{cases} \]

\[ \Psi_{b}(r) = \begin{cases} e^{i\beta} & \text{if } E < \Delta, \\ 1 & \text{if } E > \Delta, \end{cases} \]

\[ \Psi_{c}(r) = \begin{cases} -e^{-i\beta} & \text{if } E < \Delta, \\ 1 & \text{if } E > \Delta, \end{cases} \]

\[ \Psi_{d}(r) = \begin{cases} -e^{i\beta} & \text{if } E < \Delta, \\ 1 & \text{if } E > \Delta, \end{cases} \]

\( \Psi_{b/d}(\Psi_{a/c}) \) are right (left) moving modes with positive slope as shown in figure 2. And the parameters are

\[ \beta = \begin{cases} \arccos(E/\Delta) & \text{if } E < \Delta, \\ -i \arccosh(E/\Delta) & \text{if } E > \Delta, \end{cases} \]

\[ k_{xL-} = k_{xL} - i\eta, \quad k_{xL+} = k_{xL} + i\eta, \]

\[ k_{xR-} = k_{xR} - i\eta, \quad k_{xR+} = k_{xR} + i\eta, \]

\[ k_{xL} \approx \frac{U}{\sqrt{v_1 + v_2}}, \quad k_{xR} \approx \frac{U}{\sqrt{v_1 - v_2}}, \]

\[ \eta = \frac{\Delta \sin \beta}{\hbar (v_1 + v_2)}, \quad \tau_2 = \frac{\Delta \sin \beta}{\hbar (v_1 - v_2)}, \]

where \( k_{xL-} \) and \( k_{xR+} \) are corresponding to the \( x \)-component of wave vectors, as shown in figure 2.

ORCID iDs

Shu-Feng Zhang @ https://orcid.org/0000-0001-5510-7591

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