INTRODUCTION

The desired route of a given ship is usually expressed using waypoints (Fossen, 2011). This description method is extremely attractive, since the route can be easily stored in the onboard computer’s memory. Waypoints can be designated and programmed before or during the cruise, taking into account such factors as weather conditions, avoiding obstacles, and mission planning (Smierzchalski & Lebkowski, 2002; Lazarowska, 2016; Lisowski, 2016). Each waypoint, defined in Cartesian coordinates \((x_i, y_i)\), is used for creating the desired route as a set of straight line segments connecting pairs of successive waypoints. When the ship reaches the designated acceptance circle surrounding the waypoint \(i\), the path is switched to the next line segment connecting waypoints \(i\) and \(i+1\). In a more advanced solution, arcs of circles connecting line segments of the route are defined around each waypoint, and are then used to determine the desired turning at this point (Kula & Tomera, 2017). Once the waypoints are established, it is usually desirable for the sea unit to track the waypoints as closely as possible, even in the presence of unknown environmental disturbances.

Analyzing the operation of ship control systems for surface ships moving along the desired route began in the 1980s. In principle, it is easy to design a system to control the ship course along the set trajectory passed from a conventional autopilot by using the information obtained from the positioning system (Amerongen & Nauta Lemke, 1986). However, better quality is obtained when considering the system as a whole, including the ship, the ship-acting environment, and the regulator, as in this case all relevant state variables can be used in control synthesis. The entire system can be analyzed through the use of techniques known as analytical control strategies, such as self-tuning control (Kallstrom, 1982), LQG (Holzhutter, 1990; Bertin, 1998; Morawski & Pomirski, 1998), adaptive control (Chociernowicz &
A common feature of all the above analytical control strategies is their dependence on the reliability of the mathematical model describing the maneuvering dynamics of the ship. In addition, it is often necessary to linearize the ship’s model before applying the above analytical control strategies.

In order to avoid the above difficulties associated with the accuracy of the applied mathematical model of ship dynamics, other control strategies, making use of the fuzzy set theory (Vukic et al., 1998; Velagic et al., 2003; Gierusz et al., 2007; Ahmed & Hasegawa, 2016; Yu & Xiang, 2017), artificial neural networks (Zhang et al., 1996; Kula, 2015; Zhuo & Guo) and nonlinear control (Pettersen & Lefeber, 2001; Do & Pan, 2003; Fredriksen & Pettersen, 2006; Baker et al., 2013; Witkowska & Śmierzchalski, 2018) have also been developed.

Conventional ships are usually equipped with one or two main propellers for controlling the surge velocity and fins for controlling the course. Even if additional transverse thrusters are installed, they do not provide a significant extra force at transit speeds. This means that independent control is possible with only two degrees of freedom (DOF): surge and yaw. In this article, the problem of control is defined in such a way that the ship follows straight line segments between the route waypoints at constant speed, and the ship motion control is performed using the rudder blade as a commanded parameter.

2 PROBLEM FORMULATION

The movement of a ship sailing on the water surface is described in three degrees of freedom. Two coordinate systems are used for its description (Figure 1). The first of them is the Earth-fixed coordinate system \((X_N, Y_N)\), related to the water map, in which the \(X_N\)-axis points north and the \(Y_N\)-axis points east. The other coordinate system \((X_N, Y_N)\) is associated with a moving ship, and its origin is on the waterline, at the point consistent with the position of the center of gravity of the ship. The state variables \(x\) describing the ship movement are collected in two vectors \(\eta = [x, y, \psi]^T\) and \(\nu = [u, v, r]^T\) (Fossen, 2011).

The components of the vector \(\eta\) are defined in the Earth-fixed coordinate system \((X_N, Y_N)\), while those of the vector \(\nu\) in the body-fixed coordinate system \((X_N, Y_N)\). The resultant vector of the ship’s movement status has the form

\[
x = [\eta^T \ \dot{\nu}^T]^T = [x, y, \psi, u, v, r]^T
\]  

(1)

The velocity vector \(\dot{\eta}\) defined in the Earth-fixed coordinate system is associated with the velocity vector \(\dot{\nu}\) determined in the body-fixed coordinate system using the following kinematic relationship

\[
\dot{\eta} = R(\psi)\dot{\nu}
\]  

(2)

![Figure 1. Quantities describing ship’s movement in the horizontal plane, \((X_0, Y_0)\) – Earth-fixed reference system, \((X_N, Y_N)\) – body-fixed reference system, \((x, y, \psi)\) – position coordinates, \(\psi\) – ship’s course, \(u\) – surge speed, \(v\) – sway speed, \(r\) – yaw rate, \(U\) – total speed, \(\delta\) – rudder angle, \(\beta\) – sideslip angle.](image-url)

where \(R(\psi)\) is the rotation matrix determined from the formula

\[
R(\psi) = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(3)

The ideal desired route of the ship consists of a number of linear segments \(N\) (Figure 2). The ship is assumed to move along straight line segments between the waypoints. The desired speed \(u_{des}\) is assumed constant over each individual route segment. The course \(\psi\) resulting from the current route segment is the right-handed angle, relative to the \(X_N\)-axis

\[
\psi = \operatorname{atan2}(y_{k+1} - y_k, x_{k+1} - x_k)
\]  

(4)

During the turning maneuver at a waypoint, the ship moves along a circular arc connecting the adjoining linear segments at this point. In order to be able to perform such a maneuver, it is necessary to start the turning maneuver at a distance \(L_k\) ahead of the waypoint, the length of which depends on the course difference between two consecutive linear route segments

\[
L_k = f(\Delta \psi_k) = f(\psi_{k+1} - \psi_k)
\]  

(5)

and can be determined experimentally.

To facilitate determining the deviation of ship’s position in relation to the implemented segment of
the desired route, the third coordinate system \((X_b, Y_b)\) is introduced. The origin of this system is at the starting point of the executed line segment of the desired route \((x_i, y_i)\), while the \(X_b\)-axis points towards the waypoint \((x_{i+1}, y_{i+1})\).

The control task consists in finding such an algorithm that will allow the ship to follow the ideal route of the passage (Figure 2). The control signal is assumed to be a two-element vector having the form

\[
\mathbf{s}(t) = [\delta(t) \ n_{\theta}(t)]^T
\]

where \(\delta(t)\) it is the commanded rudder blade deflection angle, whereas \(n_{\theta}(t)\) is the commanded rotational speed of the propeller.

3 MATHEMATICAL MODEL OF SHIP’S DYNAMICS AND ACTUATORS

The control plant is a 1:24 scale physical model of the tanker \textit{Blue Lady}. The most important parameters of this model are summarized in Table 1.

| Parameter                  | Value                  |
|----------------------------|------------------------|
| Total length               | \(L_{G4} = 13.78\) (m) |
| Breadth                    | \(B = 2.38\) (m)       |
| Draft (full load)          | \(T_d = 0.86\) (m)    |
| Displacement (full load)   | \(A = 22.83\) (m³)     |
| Position of center of gravity | \(x_G = 0.00\) (m) |

A complex mathematical model for this tanker was developed by Gierszus (2001). The model includes all actuators installed on the ship and allows to analyze its movement in the entire speed range.

In a general form, the mathematical model of ship’s dynamics is given as

\[
M \ddot{\mathbf{v}} + C(\mathbf{v}) \mathbf{v} + D(\mathbf{v}) \mathbf{v} = \tau
\]

The matrix \(M\) contains the parameters of inertia of the rigid body, its dimensions, weight, mass distribution, and volume, as well as the added weight coefficients

\[
M = \begin{bmatrix}
    m - X_u & 0 & 0 \\
    0 & m - Y_v & mx_G - Y_f \\
    0 & mx_G - N_v & I_z - N_f
\end{bmatrix}
\]

The centripetal and Coriolis force matrix \(C\) contains hydrodynamic coefficients associated with the liquid in which the ship moves

\[
C(\mathbf{v}) = \begin{bmatrix}
    0 & 0 & -m(x_G r + v) \\
    m(x_G r + v) & -(m - X_u)u & 0
\end{bmatrix}
\]

The damping matrix \(D\) is associated with hydrodynamic damping forces and makes it possible to determine these forces for high velocities.

\[
D(\mathbf{v}) = \begin{bmatrix}
    -d_{11}(\mathbf{v}) & 0 & 0 \\
    0 & -d_{22}(\mathbf{v}) & -d_{23}(\mathbf{v}) \\
    0 & -d_{32}(\mathbf{v}) & -d_{33}(\mathbf{v})
\end{bmatrix}
\]

where

\[
\begin{align*}
    d_{11}(\mathbf{v}) &= X_{\delta_{rudder}} \left| u \right|, \\
    d_{22}(\mathbf{v}) &= Y_{\delta_{rudder}} \left| u \right| + Y_{\delta_{rudder}} \left| v \right| + Y_{\delta_{rudder}} \left| r \right|, \\
    d_{23}(\mathbf{v}) &= Y_{\delta_{rudder}} \left| u \right| + Y_{\delta_{rudder}} \left| v \right| + Y_{\delta_{rudder}} \left| r \right|, \\
    d_{32}(\mathbf{v}) &= N_{\delta_{rudder}} \left| u \right| + N_{\delta_{rudder}} \left| v \right| + N_{\delta_{rudder}} \left| r \right|, \\
    d_{33}(\mathbf{v}) &= N_{\delta_{rudder}} \left| u \right| + N_{\delta_{rudder}} \left| v \right| + N_{\delta_{rudder}} \left| r \right|.
\end{align*}
\]

Table 2 presents all parameters related to the mathematical model of \textit{Blue Lady} dynamics given by Eq. 7. The vector of forces acting on the ship’s hull is composed of forces generated by the propeller and rudder blade and those generated by interacting external environmental disturbances.

\[
\mathbf{\tau} = [\tau_X, \tau_Y, \tau_N]^T = \mathbf{\tau}_{\text{in}} + \mathbf{\tau}_{\text{w}}
\]

3.1 Mathematical model of propeller and rudder blade

For a propeller with a fixed blade angle, the generated thrust force is more or less proportional to the square of the shaft speed \(n_p\). The propeller/rudder model can
be divided into two parts. The first part describes the nominal pressure (at rudder angle $\delta = 0$).

$$T = \begin{cases} k_T n_g^2 & n_g \geq 0 \\ k_T n_g & n_g < 0 \end{cases}$$

(12)

The second part concerns additional forces: drag and lift, produced by the rudder blade associated with the propeller.

$$D = \begin{cases} 0.5 F_R \sin \delta, & n_g \geq 0 \\ 0 & n_g < 0 \end{cases}$$

(13)

$$L = \begin{cases} F_R \cos \delta, & n_g \geq 0 \\ 0 & n_g < 0 \end{cases}$$

(14)

where $F_R$ is the operating force of the rudder blade, expressed as:

$$F_R = \begin{cases} k_{fp} \cdot u_\delta \sin(\delta + \beta_R) & u \geq 0 \\ k_{fn} \cdot u_\delta \sin(\delta + \beta_R) & u < 0 \end{cases}$$

(15)

The local rudder blade drift angle $\beta_c$ is determined as:

$$\beta_c = -\tan(2(v_\delta, u_\delta))$$

(16)

where $u_\delta$ is the effective inflow of the water jet to the rudder blade in the longitudinal direction

$$u_\delta = \begin{cases} \sqrt{k_1(k_3 u_k + \sqrt{k_3 u_k^2 + k_4 T})^2 + k_5 u_k^2} & T > 0 \\ u & T \leq 0 \end{cases}$$

(17)

and $v_\delta$ is the effective inflow of the water jet to the rudder blade in the transverse direction, determined from:

$$v_\delta = v - \frac{r L_n}{2}$$

(18)

For a system that includes a propeller and the associated rudder blade, the following force and torque vector is applied to the ship hull

$$\begin{bmatrix} \tau_X \\ \tau_Y \\ \tau_N \end{bmatrix} = \begin{bmatrix} T - D \\ k_{y} T + k_{y} L \\ (k_{s} T + k_{s} L) L_{sR} \end{bmatrix}$$

(19)

4 STRUCTURE OF CONTROL SYSTEM

The above defined control was implemented in the system shown in Figure 3. The input signal to this system is the desired route given by the path planning system as the safe path of ship movement. The desired route has the form of a broken line, defined by the coordinates of subsequent waypoints $(x_k, y_k)$. The ship motion on the water surface is described by the vector $x$ consisting of six state variables (1), where $(x, y)$ are the ship position coordinates measured by the DGPS system, $\psi$ is the ship's heading measured by the gyrocompass, $(u, v)$ are the linear body-fixed velocity components (surge, sway), and $r$ is the yaw rate. Usually, the velocity components are not measured. The measured coordinates of the ship motion state are collected in the vector $\eta = [x, y, \psi]^T$.

![Block diagram of ship's motion control system along desired route](image)

Figure 3. Block diagram of ship's motion control system along desired route

Based on the information received from the path planning system in the form of the desired route and the measured ship position coordinates and course collected in vector $\eta = [x, y, \psi]^T$, the waypoint path controller determines the commanded rudder angles $\delta_c$. The second commanded value, which is the main propeller revolutions $n_{gc}$, is constant and not regulated. The commanded rudder angle $\delta_c$ is determined using a set of two component controllers, as shown in Figure 4.

Two modes of waypoint path controller operation are considered. The first mode, called track-keeping, consists in controlling the ship's movement along a straight line segment of the route, while the second mode is used during the maneuver of changing to the next straight line route segment and is called the turning maneuver. Conditions for switching between these two operation modes are shown in Figure 5. In Mode 1 ($\sigma = 1$), both component controllers are involved in determining the commanded value of rudder blade deflection $\delta$. The PD component
controller minimizes the course error $\varepsilon_\psi$, while the PI controller minimizes the ship cross-track error $\varepsilon_y$. The path controller switches to the turning maneuver when the ship arrives at a distance $L_s$ from a waypoint, which is smaller than the distance $L_k$ for this waypoint ($L_s < L_k$).

The ahead distance $L_s$ at which the turning maneuver should be started depends on the course difference between two consecutive segments of the desired path $L_s = f(\Delta \psi)$). This distance was determined experimentally in the here reported tests. After switching to the turning maneuver, the integral in the PI controller is reset to zero using the signal $\text{Reset}$ and the switching signal $\sigma$ stops passing the side deviation $\varepsilon_y$ to the PI controller input ($\sigma = 2$). During the turning maneuver, the specified deflection of the rudder blade $\delta$ is determined with the assistance of the PD component controller. The turning maneuver is terminated when both the course error $\varepsilon_\psi$ and its derivative $\dot{\varepsilon}_\psi$ are smaller than their limits. In the present case, these limits were assumed as: $\varepsilon_\psi < 5$ and $\dot{\varepsilon}_\psi < 0.5$.

![Figure 4. Internal structure of the waypoint path controller](image)

For the purpose of path controller synthesis, the dynamics of the tested ship, given by Eq. (7), has been simplified, assuming the constant surge velocity of the ship, $u = u_0 = \text{constant}$, and low values of velocities $v$ and $r$. This allowed linearizing the nonlinear matrix $D$ given by Eq. (9) to the following form

$$D \approx D_L = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_r & -Y_r \\ 0 & -N_y & -N_r \end{bmatrix}$$

(20)

After the linearization of Eq. (7), the longitudinal ship dynamics was decomposed assuming its longitudinal symmetry. The longitudinal force, which depends on the rotational speed of the main propeller screw $n_o$, was linearized to the form $\tau = X_0 n_o$. The forces acting on the ship's hull are usually linearly dependent on the rudder deflection $\delta$, according to the relations $\tau = -Y \delta$ and $\tau = -N \delta$. As a result, the finally obtained maneuvering model consists of the excluded longitudinal ship dynamics

$$(m - X_a) \ddot{u} - X_a u - mv r - m x G r^2 = X_a n_o$$

(21)

and the angular-positive dynamics, which is the Davidson and Schiff model (1946) obtained from linearization of Eq. (7)

$$M_1 \dot{v}_1 + N(u_0) v_1 = B \delta$$

(22)

where $\mathbf{v} = [v, r]^T$ is the state vector, and $\delta$ is the rudder deflection. The matrices $M_1, N(u_0)$ and $B$ in Eq. (13) are defined as follows (Davidson & Schiff, 1946)

$$M_1 = \begin{bmatrix} m - Y_v & m x G - Y_r \\ m x G - N_y & I_z - N_r \end{bmatrix}$$

(23)

$$N(u_0) = \begin{bmatrix} -Y_v \\ -N_y + X_0 u_0 \end{bmatrix}$$

(24)

$$B = \begin{bmatrix} -Y_\delta \\ -N_\delta \end{bmatrix}$$

(25)

5 SYNTHESIS OF CONTROL ALGORITHM

The tested path controller is composed of two components connected in parallel. The first component is the PD course controller, used to minimize the course error, while the second is the PI controller, used to minimize the cross-track error from the desired path segment.

![Figure 5. Directed graph illustrating conditions for switching between path controller operation modes](image)

For the purpose of path controller synthesis, the dynamics of the tested ship, given by Eq. (7), has been simplified, assuming the constant surge velocity of the ship, $u = u_0 = \text{constant}$, and low values of velocities $v$ and $r$. This allowed linearizing the nonlinear matrix $D$ given by Eq. (9) to the following form

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and the angular-positive dynamics, which is the Davidson and Schiff model (1946) obtained from linearization of Eq. (7)

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where $\mathbf{v} = [v, r]^T$ is the state vector, and $\delta$ is the rudder deflection. The matrices $M_1, N(u_0)$ and $B$ in Eq. (13) are defined as follows (Davidson & Schiff, 1946)

$$M_1 = \begin{bmatrix} m - Y_v & m x G - Y_r \\ m x G - N_y & I_z - N_r \end{bmatrix}$$

(23)

$$N(u_0) = \begin{bmatrix} -Y_v \\ -N_y + X_0 u_0 \end{bmatrix}$$

(24)

$$B = \begin{bmatrix} -Y_\delta \\ -N_\delta \end{bmatrix}$$

(25)

The obtained linearized Davidson and Schiff model (1946) of ship dynamics, given by Eq. (22), has a sway velocity $v$ which in the proposed control algorithm (10) was not planned to be stabilized. Therefore, the next step was to eliminate this velocity, after which the Nomoto model was designated (Nomoto et al., 1957)
\[
\frac{r(s)}{\delta(s)} = \frac{K(sT_1 + 1)}{(sT_1 + 1)(sT_2 + 1)}
\]  (26)

where \( K \) is the static gain of angular speed, \( T_1, T_2 \) and \( T_3 \) are time constants, and \( r \) is the angular ship velocity \( r = \dot{\psi} \). The transmittance parameters (26) refer to hydrodynamic coefficients, according to the following relations

\[
T_1T_2 = \frac{|M_1|}{|N|}
\]  (27)

\[
T_1 + T_2 = \frac{n_{11}m_{22} + n_{22}m_{11} - n_{12}m_{21} - n_{21}m_{12}}{|N|}
\]  (28)

\[
K_R = \frac{n_{12}b_1 - n_{11}b_2}{|N|}
\]  (29)

\[
K_RT_3 = \frac{m_{21}b_1 - m_{11}b_2}{|N|}
\]  (30)

\[
K = -K_R
\]  (31)

where coefficients \( m_{ij}, n_{ij} \) and \( b_i (i=1,2; j=1,2) \) are the coefficients of matrices \( M_i, N \) and \( B \) (23)-(25), while \( |M_1| \) and \( |N| \) are the determinants of matrices \( M_1 \) and \( N \), respectively.

The identification of the Nomoto model parameters based on the sea maneuvering tests has shown that the parameter values \( T_2 \) and \( T_3 \) in Eq. (26) do not differ much from each other (Fossen, 2011). This allowed further simplification of the transfer function (26), after which the first order Nomoto model was obtained

\[
\frac{r(s)}{\delta(s)} = \frac{K}{sT + 1}
\]  (32)

where \( T = T_1 + T_2 - T_3 \) is the effective time constant of the angular velocity. The above model can be stored in the time domain as follows

\[
\begin{align*}
\dot{r} &= -ar + b\dot{\delta} \\
\dot{x} &= uc\cos\psi - v\sin\psi \\
\dot{y} &= u\sin\psi + v\cos\psi \\
\dot{\psi} &= r
\end{align*}
\]  (33-36)

The above equations are nonlinear and depend on the values of states \( u, v \) and \( \psi \). Nevertheless, linear approximations of these equations can be made, provided that the stationary coordinate system is rotated in such a way that the given course \( \psi_0 \) becomes equal to zero (\( \psi_0 = 0 \)). This way, the ship's control along the desired route will be carried out in the coordinate system \( (X_d, Y_d) \) related to the currently executed path segment. Hence, the ship's course \( \psi' \) will have a small value during the control along the desired route, and we can assume that

\[
\sin\psi' \approx \psi' \quad \cos\psi' = 1
\]  (37)

Then, assuming that \( u \approx U \), the kinematic equations of ship motion can be reduced to a set of linear equations

\[
\begin{align*}
\dot{x} &= U + d_x \\
\dot{y} &= U\psi' + v + d_y \\
\dot{\psi} &= r
\end{align*}
\]  (38-40)

Two additional elements \( (d_x, d_y) \) are introduced in the above equations. They describe the errors related to linearization and the slideslip angle caused by environmental disturbances. In Eq. (36), \( \psi' \) is the ship cross-track error from the desired route, determined from the formula

\[
y' = e\psi(t) = [x(t) - x_k]\sin\psi - [y(t) - y_k]\cos\psi
\]  (41)

This error depends very strongly on changes in ship's surge speed \( U \).

The task of the path regulator is to control the ship's movement along the current route segment with end coordinates \( (x_{k+1}, y_{k+1}) \) and \( (x_k, y_k) \), and minimizing the course \( \psi' \) and the cross-track error of ship's position from this segment, \( e_\psi = e\psi' \). The preset course resulting from a given route segment is determined using Eq. (4), and is changed after reaching a new waypoint. In the path controller, the integral of lateral ship deviation \( y' \) from the path is introduced, through coupling, to its input. Hence, a new state appears in the plant

\[
\dot{y}' = y'
\]  (42)

The designed waypoint path regulator will not control the surge velocity of the ship, therefore Eq. (38) can be omitted in further analysis. On the basis of Eqs. (40), (33), (39) and (42), we can write the dynamics equations of a simplified mathematical model of the process for the waypoint path controller design

\[
\begin{align*}
\dot{x} &= u\cos\psi - v\sin\psi \\
\dot{y} &= u\sin\psi + v\cos\psi \\
\dot{\psi} &= r
\end{align*}
\]  (34-35)

\[
\dot{y}' = y'
\]  (42)
The controlled variables \( \psi^r \) and \( y^r \) are determined as follows

\[
\begin{bmatrix}
\dot{\psi}^r \\
\dot{y}^r \\
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & a & 0 & 0 \\
U & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\psi^r \\
y^r \\
\end{bmatrix}
+ \begin{bmatrix}
b & \delta + \frac{1}{\nu_d} \end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\] (43)

The algorithm designed for the trajectory controller takes the form

\[
\delta_z = u_{PD} + u_{PI} = k_1 e^x + k_2 y^x + k_3 y^r + k_4 y^f
\] (45)

where

\[
e^x(t) = \psi^x(t) - \psi(t)
\] (46)

\[
r^x(t) = d e^x(t)/dt
\] (47)

The parameters of the trajectory controller (45), were determined using the pole placement method, based on the linearized process described by Eq. (43) for constant surge velocity \( u_0 = 1.1 \) (m/s). For further calculations, the following eigenvalues of the designed control system were adopted

\[
p_1 = -0.1834, \ p_2 = -0.0692 \pm j0.155, \ p_3 = -0.0047
\] (48)

The desired values of the trajectory controller gains (Table 5) were determined using the function `polyfit` included in the Matlab program function set (Mathworks, 2019).

The first carried out study aimed at experimental determination of the ahead distance \( L_k \) for starting the turning maneuver. For this purpose, several maneuvers were made for different course angle changes between two successive straight line segments of the desired route. The obtained test results are shown in Figure 6. The results of the experimental tests, marked as asterisks (*), were approximated using the following formula

\[
L_k = a_6 \Delta \psi^k_6 + a_5 \Delta \psi^k_5 + a_4 \Delta \psi^k_4 + a_3 \Delta \psi^k_3 + a_2 \Delta \psi^k_2 + a_1 \psi^k_1
\] (49)

The values of parameters \( a_1 \ldots a_6 \) in Eq. (49) were determined using the function `polyfit` included in the Matlab program function set (Mathworks, 2019). These values are collated in Table 6.

Next, the operation of the designed control algorithm along the desired route was tested experimentally. The results of these experiments are given in Figs. 7 and 8. Figure 7 shows the map of the water basin, with the desired route marked by 5 waypoints \((x_k, y_k)\) connected with straight lines (dotted lines in the figure).

This figure also shows the real path (solid line) of the ship sailing along the desired route.

Table 6. Values of parameters describing the approximation of the ahead distance for starting the turning maneuver (49)

| No | Variable | Value         |
|----|----------|---------------|
| 1  | \(a_6\)  | 5.987527e-09 |
| 2  | \(a_5\)  | -1.561371e-06|
| 3  | \(a_4\)  | 1.430259e-04 |
| 4  | \(a_3\)  | -0.00493527  |
| 5  | \(a_2\)  | 2.10745127   |
| 6  | \(a_1\)  | -0.02348713  |

Figure 6. Experimentally determined function \( L_k = f(\Delta \psi) \) for training ship Blue Lady

Table 5. Parameters calculated for the PDPI controller.

|     | \(k_1\) | \(k_2\) | \(k_3\) | \(k_4\) |
|-----|---------|---------|---------|---------|
| PDPI controller | 1.6 | 19.92 | 2.125 | 92.1 |

6 RESULTS

To check the correctness of the designed control system, both simulation and experimental tests were carried out. The experimental tests were carried out on the training ship Blue Lady at the Ship Handling, Research and Training Centre on the Słom lake in Iława/Kamionka. The ship was in full load. During the tests, the wind speed did not exceed 4 m/s. In the experimental tests, the rotational speed of the propeller was constant and equal to \( n_3 = 440 \) rpm.
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