Multi-component fractional quantum Hall states in graphene: $SU(4)$ versus $SU(2)$

C Tőke$^{1,2}$ and J K Jain$^3$

$^1$ Institute of Physics, University of Pécs, H-7624 Pécs, Budapest, Hungary
$^2$ BME-MTA Exotic Quantum Phases ‘Lendület’ Research Group, Budapest University of Technology and Economics, Institute of Physics, H-1111 Budapest, Hungary
$^3$ Department of Physics, 104 Davey Laboratory, Pennsylvania State University, University Park, PA 16802, USA

E-mail: tcsaba@phy.bme.hu

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Abstract
Because of the spin and Dirac-valley degrees of freedom, graphene allows the observation of
one-, two- or four-component fractional quantum Hall effects in different parameter regions.
We address the stability of various states in the $SU(2)$ and $SU(4)$ limits. In the $SU(4)$ limit, we
predict that new low-energy Goldstone modes determine the stability of the fractional
quantum Hall states at $2/5$, $3/7$, etc; $SU(4)$ skyrmions are not found to be relevant for the
low-energy physics. These results are discussed in light of experiments.
(Some figures may appear in colour only in the online journal)

1. Introduction
Experiments have raised the issue of the relative stability of various fractional quantum Hall states in graphene. The earliest results in suspended graphene in the two-terminal geometry [1] gave indications of the fractional quantum Hall effect (FQHE) at $\nu = 1/3$. The more recent experiments by Dean et al and Ghahari et al [2] in the four-terminal Hall bar geometry for graphene on a boron nitride hexagonal substrate have revealed a much richer structure. Specifically, they observe FQHE at filling factors $|\nu| = 2/3, 4/3$ and in the central ($n = 0$) Landau level, and at $|\nu| = 2 + 1/3, 2 + 2/3, 2 + 4/3, 2 + 5/3, 2 + 7/3$ in the $|\nu| = 1$ Landau level. In addition, they also see evidence of FQHE at $|\nu| = 1/3$ in the $n = 0$ LL, but none at $|\nu| = 2 - 1/3$. Hansel et al have also reported $4/3$ FQHE in a four-terminal geometry [3]. More recently, Feldman et al [4] have observed incompressible states in suspended flakes in the sequence $\nu = p/(2p + 1)$ for all $p \leq 4$ in the $|\nu| \leq 1$ range, but only at $\nu = 2 - 2/3, 2 - 4/3, 2 - 4/7$ and $2 - 4/9$ in the $1 < |\nu| < 2$ range. The difference of the $0 < |\nu| < 1$ and $1 < |\nu| < 2$ interval, and the absence of FQHE at $2 - 1/3$ in particular, remains a puzzle.

Our treatment below builds upon our understanding of the FQHE in GaAs systems. In GaAs systems, the FQHE is associated with the formation of composite fermions (CFs), bound states of electrons and an even number of quantized vortices [5]. The electronic LL splits into Landau-like levels of composite fermions, called $\Lambda$ levels (ALs), and the integer quantum Hall effect of composite fermions produces the prominently observed FQHE at $\nu = m/(2p \pm 1)$, where $m$ is the number of completely filled $\Lambda$ levels and $2p$ is the number of vortices bound to electrons. The CF theory has been extended to include spin [6, 7]: with spin, FQHE occurs at the same fractions as before, namely $\nu = m/(2p \pm 1)$, but now, in general, FQHE states with different spin polarizations are available at each fraction, analogous to the situation at integer fillings where different spin polarizations would occur, depending on the strength of the Zeeman splitting relative to the cyclotron energy. Experiments have measured the transitions between differently polarized states, the actual spin polarizations of the states, CF $\Lambda$ level fan diagram and the phase boundaries [8], which are all found to be in good agreement with the CF theory. (For the integer quantum Hall effect, this physics is not relevant for GaAs,
because the Zeeman energy is very small compared to the cyclotron energy, thus producing the state with the smallest spin polarization. However, for FQHE, the Zeeman energy can be of the order of the effective cyclotron energy of composite fermions, and therefore many different spin polarizations occur.) We note that some of the states were written down in an early work of Halperin on multi-component wavefunctions for the FQHE [9].

The FQHE problem in graphene differs from that in GaAs in two respects. First, in graphene, each electron has four components, because of two spin projections and two valleys, producing an approximate SU(4) symmetry when the Zeeman energy and the valley splittings are negligible. Second, the linear dispersion leads to an interaction that is, in general, different from that in GaAs [10–14]. Our previous theoretical work on FQHE in graphene [11, 13, 15] has shown that composite fermions are formed in both graphene, each electron has four components, because of two spin projections and two valleys, producing an approximate SU(4) symmetry when the Zeeman energy and the valley splittings are negligible. Our previous theoretical work on FQHE in graphene [11, 13, 15] has shown that composite fermions are formed in both n = 0 and |n| = 1 LLs. The FQHE is thus still predicted to occur at v = m/(2pm ± 1), where v = v − (4n − 2) is the fractional part of the filling factor in LL n but, in general, many more kinds of incompressible states are now available at each fraction, depending on the system in the SU(2) limit or SU(4). The actual ground state is determined by a competition between the CF kinetic energy, which favors occupation of the lowest available Λ levels, the CF exchange energy, which favors occupation of Λ levels with the same SU(4) index, and the Zeeman and valley splittings [13].

Some of the observations on FQHE in graphene are consistent with theoretical expectations. (i) The observed fractions have the form m/(2pm ± 1) as measured relative to a filled Landau level. (ii) The strength of FQHE in the |n| = 1 LL is comparable to that in the |n| = 0 LL, to be contrasted with GaAs systems [11, 12], where FQHE in the second LL is much weaker than that in the lowest LL. (iii) As predicted in [14–16], no FQHE has been observed at even denominator fractions in the |n| = 1 LL. Some other observations are unexpected, however. Noteworthy is the absence of fractions such as 2 − 1/3 and 2 − 2/5, while 2 − 2/3 and 2 − 4/3 are strong. Also puzzling is the appearance of FQHE at 1/3; one would have expected 2 − 1/3 to be stronger given that its reference state at v = 2 does not require any spontaneous symmetry breaking. In contrast, in the |n| = 1 LL, 2 + 1/3 and 2 + 2/3 are both observed strongly. These fractions do not appear, at first sight, to be consistent with either SU(2) or SU(4) symmetry, thus posing a well-defined puzzle whose resolution will shed important light on the nature of the FQHE in graphene.

Our aim in this paper is to determine the stability of various states by evaluating the gaps using the CF theory. The gaps in the SU(2) limit were evaluated previously for states of the form v = m/(2pm + 1) for the lowest LL of GaAs (these results carry over to the n = 0 Landau level of graphene) but not for the n = 1 Landau level of graphene. In addition, gaps for the reverse-flux attached states at v = p/(2pm − 1) have not been calculated for technical reasons.

\[ \hat{H} = \frac{e^2}{\epsilon} \sum_{i,j} \frac{1}{|r_i - r_j|} - \sum_j \frac{\Delta_Z}{2} \hat{\sigma}_z^j + \sum_j \frac{\Delta_V}{2} \hat{\tilde{e}}_j. \] (1)

Here \( \epsilon \) is the dielectric constant of the background material, \( \hat{\sigma}_z^j \) and \( \hat{\tilde{e}}_j \) are Pauli matrices related to spin and Dirac-valley indices, and \( \Delta_Z \) and \( \Delta_V \) are Zeeman and valley splittings. For \( \Delta_Z = 0 \) and \( \Delta_V = 0 \), this Hamiltonian possesses an SU(4) symmetry, with \( \hat{\sigma}_z \otimes 1, 1 \otimes \hat{\tilde{e}}, \hat{\tilde{e}}_z \otimes \hat{\tilde{e}}_z \) (or any appropriate linear combinations thereof) defining the weights in SU(4) multiplets. Composite fermions acquire additional labels (quantum numbers) \( a^{(1)}, \ldots, a^{(4)} \), which come from a basis chosen in the fundamental representation of SU(4), and can be chosen conveniently to be \( \uparrow \uparrow, \uparrow \downarrow, \downarrow \uparrow, \downarrow \downarrow \), where the first arrow refers to valley and the second to spin.

Let \( (m_1, m_2, m_3, m_4) \) denote the incompressible ground state in which \( m_1, \ldots, m_4 \) Λ levels with label \( a^{(1)}, \ldots, a^{(4)} \) are fully occupied. Which of these states is the ground state depends [13] on the values of \( \Delta_Z \) and \( \Delta_V \). A coupling between the spin and valley degrees of freedom and LL mixing are neglected in what follows.

We neglect the anisotropy of the interaction due to the underlying sublattice structure of the basis states, because such effects are small, being proportional to the ratio of the lattice constant \( a \) to the magnetic length \( \ell = \sqrt{\hbar c/EB} \). The study of the residual lattice effects is beyond the scope of this paper.

Another important assumption of our model is the neglect of disorder. This is a standard assumption in most quantitative studies of FQHE, because it is not known how to include the effect of disorder in a reliable manner. It is possible that...
disorder might be affecting different FQHE states differently, thus affecting their relative stability. In the absence of a reliable quantitative treatment of disorder, while comparing our results with experiment we will make the simplest assumption that while the disorder causes a significant reduction of the excitation gap, it does not change the order of stability of various FQHE states. Strictly speaking, our results below ought to be considered as a prediction for future experiments in the limit of low disorder.

The kinetic term still determines the Coulomb matrix elements via the single-particle orbitals when the Landau level is fixed. For the elements via the single-particle orbitals when the Landau level experiments in the limit of low disorder.

The Monte Carlo evaluation of the energy of variational wavefunctions, on the other hand, requires a real-space interaction. To simplify calculation, we construct the wavefunctions in the lowest LL and use an effective real-space potential corresponding to the effective pseudopotentials $V_m^{(1)gr}$ in the lowest LL basis. Following a well-tested procedure [23, 7], we use the form

$$V_{\text{eff}}(r) = \frac{1}{r} + \sum_{i=0}^{M} c_i r^i e^{-r},$$  \hspace{1cm} (4)

and fit the coefficients $c_i$ to reproduce the first $M + 1$ pseudopotentials $V_m^{(1)gr}$ of equation (3). The coefficients $c_i$ obtained for $M = 6$ are given in [13].

3. SU(2) analysis

To weigh the relative importance of the relevant parameters, it is appropriate to consider the ratio of the Zeeman splitting to some relevant energy scale in the problem. We consider $\kappa = \Delta_2/(0.02e^2/\ell \epsilon)$, where $\Delta_2$ is the energy in the denominator is the Fermi energy of the single-component CF Fermi sea and $g$ is the Landé factor for the background semiconductor material [7, 8]. (The quantity $\ell = \sqrt{\hbar/eB}$ is the magnetic length. We could alternatively have chosen the gap of a prominent FQHE state.) The FQHE states in GaAs are typically fully spin-polarized for $B > 5$ T, i.e., for the ratio $\kappa > 0.5$ [7, 8]. For graphene, this ratio is 1.8 (0.6) at $B = 36$ T (4 T), suggesting that the spin degree of freedom may be frozen in the experiments of [2], which are performed at rather large $B$. The valley symmetry breaking in graphene is of the order of $a/\ell$, where $a$ is the lattice spacing [14]. At $B = 4$ and $36$ T, this ratio is $a/\ell \approx 0.02$ and 0.06, respectively. While this issue requires further investigation, it cannot be ruled out that the spin is fully polarized but the valley symmetry may not be broken.

We therefore first consider the situation where the spin degree of freedom is frozen but the valley symmetry is intact. (The analysis of the SU(4) limit is given in section 4.)

In the SU(2) limit, the orbital part of the wavefunctions at the maximal weight states [25] of an SU(2) multiplet at monopole strength $Q$ is given by equation (2), with

$$\Phi_m = \Phi_m^{(1)} \Phi_m^{(2)}$$  \hspace{1cm} (5)

with $m_1 + m_2 = m$ and the superscript denotes the electron species. In other words, the state consists of $m_1$ and $m_2$ Landau bands of up and down pseudospins occupied, respectively. (Here, ‘pseudospin’ refers to the valley SU(2) symmetry.) One can show that this wavefunction satisfies Fock’s cyclic condition [25], i.e., it is annihilated by any attempt at further antisymmetrization; this ensures that it is an eigenstate of the total pseudospin operator. The energies of various valley-polarized states can be evaluated using standard methods, from which the phase diagram as a function of the valley Zeeman energy can be constructed [7].

5 We note that this issue deserves further investigation. Recent experiments suggest that the valley symmetry breaking is stronger than the spin splitting for the integral quantum Hall plateaus. See, for example, [24].
The valley degree of freedom behaves very much like the spin degree of freedom. In fact, the predictions of the CF theory for two-component systems have been verified semiquantitatively [26] for another system with two valleys, namely ALAs quantum wells, where the valley splitting (analogous to the Zeeman energy) can be controlled by application of in-plane symmetry breaking strain.

With spin frozen, 1/3 is valley-polarized, whereas 2/3 and 2/5 are valley singlets. (These states are related by particle–hole symmetry to 5/3, 8/5 and 4/3.) That ̃ν = 2/5 and 2/3 are singlets in the n = 0 Landau level follows from the analogous result established in conventional two-dimensional electron systems such as GaAs [7, 6].

For the ̃ν = 2/5 in the |n| = 1 LL of graphene the energies of the 2/5 FQHE states in the thermodynamic limits can be obtained using the CF wavefunctions, and are given by –0.4450(2) ̵e^2/ ̵ea for the singlet and –0.4401(2) ̵e^2/ ̵ea for the polarized ground state energies, as shown in figure 1(a). The polarization energy, i.e. the energy difference between the polarized and the SU(2) singlet states, has been estimated to be 0.004 97(9) ̵e^2/ ̵ea in the thermodynamic limit as shown in figure 1(b). This confirms that, to the extent the wavefunctions are accurate, the energy ordering of states predicted by the CF theory remains the same in the |n| = 1 LL as in the n = 0 LL. We note that Shibata and Nomura [27] conclude, based on a density matrix renormalization group calculation, that the ground state here is polarized even in the absence of a symmetry breaking field; this conclusion, different from ours, is based on a study of smaller systems and without extrapolation to the thermodynamic limit.

We assume in what follows that the valley splitting is small but large enough to suppress the CF valley skyrmions (CF skyrmions are rapidly suppressed by the Zeeman/valley splitting [28]) and the relevant excitation is a particle–hole pair of composite fermions. If composite fermions were completely noninteracting, the 1/3 gap would be the smallest, as it does not involve CF cyclotron energy. The composite fermions, however, do feel a residual interaction, which necessitates a quantitative evaluation of the gaps, which can be accomplished using the standard methods of CF theory [22].

The energy gaps of the lowest charged excitations shown in table 1 have several unexpected features [28]. The 2/3 and 4/3 states in the n = 1 LL are the strongest, followed by 2/3 and 4/3 in the n = 0 LL, and 1/3 and 5/3 in the n = 1 LL. The next most stable are 1/3 and 5/3 in the n = 0 LL. These calculations are consistent with the observations that 2/3 and 4/3 are the strongest in the n = 0 LL (producing FQHE at 2 – 2/3 and 2 – 4/3), both 1/3 and 2/3 are strong in the n = 1 LL (resulting in FQHE at 2 + 1/3, 2 + 2/3, 4 ± 1/3 and 4 ± 2/3), and 2/5 and 8/5 are not yet observed.

While the observed order of stability is consistent with theory, the actual values of gaps are not. The largest observed gap [2] of ~16 K at ν = 4/3 and B = 35 T is much smaller than the theoretical estimate ~0.1 ̵e^2/ ̵ea ~110 K (for ̵e = 3). The significant discrepancy is no doubt in large part due to disorder and suggests that improvements in sample quality should reveal much further FQHE structure.

Table 1. The smallest gaps for the creation of a far-separated particle–hole pair of composite fermions. The 1/3 ground state is fully spin- and valley-polarized, whereas the 2/3 and 2/5 states are spin-polarized but valley singlets. Neglecting LL mixing, the gaps at ν and 2 – ν are equal. All energies are quoted in units of ̵e^2/ ̵ea. The 2/3 gaps come from a combination of exact diagonalization and CF theory; the rest comes from CF theory only.

| ̃ν | n = 0 | |n| = 1 |
|----|------|---|---|
| 1/3 | 0.073(1) | 0.095(2) |
| 5/3 | 0.11(4) | 0.21(14) |
| 2/3 | 0.0390(6) | 0.0475(9) |

The significant discrepancy is no doubt in large part due to disorder and suggests that improvements in sample quality should reveal much further FQHE structure.

6 The gaps shown are thermodynamic extrapolations from N = 10–30 for 1/3, N = 20–40 for 2/5, N = 24–45 for 3/7 and N = 8–12 for 2/3 (which requires reverse-flux attachment).

7 The gaps for 1/3 and 2/5 in the n = 0 LL were reported previously in [29]. The gap values here are somewhat larger, but agree within statistical error.
4. SU(4) analysis

We next turn to the SU(4) limit, achieved when the valley and the spin states are all approximately degenerate and may be applicable to experiments at very low B. In this limit, if composite fermions were completely noninteracting, only FQHE states of the form \( m/(2pm ± 1) \) with \( m = 4, 8, \ldots \) would be observable, but as seen above the residual interaction between composite fermions results in a spontaneous symmetry breaking, producing FQHE at all of these fractions [13]. We now show that new Goldstone modes become available, which are an SU(4) generalization of the spin waves of composite fermions considered previously in the context of GaAs FQHE [29]. (A similar mechanism has been considered for QHE at \( \nu \neq 4n - 2 \) integer fillings in graphene [10] and in bilayer systems [30].) As a result of the presence of these modes, fractions such as \( 2/N \) graphene [10] and in bilayer systems [30].) As a result of the presence of these modes, fractions such as \( 2/5 \) are expected to be much weaker than their SU weight states \([25]\) of an electron of the SU sector of the ground state, which in the context of GaAs FQHE \([29]\). (A similar mechanism has been considered for QHE at \( \nu \neq 4n - 2 \) integer fillings in graphene [10] and in bilayer systems [30].) As a result of the presence of these modes, fractions such as \( 2/5 \) are expected to be much weaker than their SU weight counterparts.

In the SU(4) generalization \([13]\) of CF theory \([19, 20]\), the orbital part of the ground state wavefunction at \( \nu = m/(2pm ± 1) \) for correlated electrons in the maximal weight states \([25]\) of an SU(4) multiplet at monopole strength \( Q \) is still given by equation (2), but with

\[
\Phi_{m} = \Phi_{m_{1}}^{(1)}\Phi_{m_{2}}^{(2)}\Phi_{m_{3}}^{(3)}\Phi_{m_{4}}^{(4)}
\]

(6)

where \( \Phi_{m_{i}}^{(j)} \) is the Slater determinant wavefunction of \( N_{i} \) electrons of the \( \sigma^{(i)} \) species at monopole strength \( q = Q - p(N - 1) \), with \( N = \sum N_{i} \) and \( m = \sum m_{i} \). With \( m_{1} \geq m_{2} \geq m_{3} \geq m_{4} \), it is easy to see that \( \Psi \) satisfies Fock’s cyclic condition \([25]\), i.e. it is annihilated by any attempt to antisymmetrize an electron \( l \) of type \( u \) (\( \min \leq l \leq \max_{u} \)) with respect to the electrons of type \( i < u \):

\[
1 - \sum_{k=\min}^{\max} \prod_{j=1}^{l} \Psi(z_{j}) = 0,
\]

(7)

where \( (k, l) \) permutes indices \( k \) and \( l \). For the CF particle–hole excitation, one must, in general, choose \( \Phi \) to be an appropriate linear combination of Slater determinant states to ensure the Fock condition. In each SU(4) multiplet, represented by a Young tableau (YT) \([M_{1}, M_{2}, M_{3}]\), where \( M_{1} \) is the length of the \( i \)th row (\( M_{i} \geq M_{i+1} \) and the last \( M_{3} \)s are not shown if zero), the maximal weight states can be constructed with the help of elementary group theory and the eigenstates of the total orbital angular momentum \( L \) are obtained by standard Clebsch–Gordan expansion. We show the appropriate combinations pictorially for all single-exciton excitations at 2/5, 3/7 and 4/9, along with their YT representations, in the subsequent figures. The interaction energies of the explicit wavefunctions are evaluated by Monte Carlo integration. For CF particle–hole excitation we identify their orbital angular momentum \( L \) with their wavevector \( k \) according to \( k = L/\ell \), which yields the dispersion for the excitations, with the \( k \to \infty \) limit giving the energy of a far-separated CF particle–hole pair. All states in a given multiplet are degenerate for \( \Delta_{Z} = 0 = \Delta_{V} \); for nonzero \( \Delta_{Z} \) and \( \Delta_{V} \) the energy splittings are straightforwardly determined.

The CF ground state at \( \tilde{\nu} = 1/3 \) has a single fully occupied \( n = 0 \) AL. As excitation to an unoccupied \( n = 0 \) AL is already possible in SU(2) system, we expect no new physics at this fraction.

The \( \tilde{\nu} = 2/5 \) state has two \( n = 0 \) ALs occupied in the ground state; assuming for definiteness that \( \Delta_{Z} > \Delta_{V} \), the occupied ALs are \( \uparrow \uparrow \) and \( \uparrow \downarrow \). It has two kinds of excitations depicted in figure 2. (i) The highly degenerate SU(4) spin wave, in which a CF is excited ‘sideways’ to an unoccupied AL as shown in figure 2(a). This mode, represented by the YT \([N/2, N/2 - 1, 1]\), is available only because of the SU(4) symmetry. (ii) Excitations within the SU(2) subspace of the ground state, in which a CF is raised ‘up’ to the second AL either flipping or preserving its valley index, represented, respectively, by YT’s \([N/2 + 1, N/2 - 1] \) and \([N/2, N/2] \). The four cases give rise to a valley triplet (figure 2(b)) and a valley singlet (figure 2(c)), which are both gapped at all wavelengths. The triplet is split by the \( \Delta_{V} \) and destabilizes the FQHE state when it reaches the ‘valley-roton’ gap \([29]\) of \( \Delta_{\text{roton}} \) \([29]\). The energy of the ‘singlet-roton’ gap in mode (c) is unaffected by valley or Zeeman splittings. Transitions that raise a CF to the first excited level of an otherwise empty band are ignored, as these involve a loss of exchange energy and a kinetic energy cost at the same time and are likely to represent higher excitations.

Figure 2 shows that the smallest gap to creating a far-separated pair of charged excitations, which is the one relevant for transport experiments, corresponds to a CF excitation in mode (a). Because this energy is approximately half the gap in GaAs for either a fully spin-polarized or spin singlet 2/5 state \([29]\), the new Goldstone mode thus results in a substantial weakening of the 2/5 state in the SU(4) limit. From the SU(4) spin wave dispersion at small \( k \) we estimate the energy of the SU(4) skyrmion, using the methods of \([31]\), to be 0.030(6) and 0.037(9) in the \( N = 0 \) and \( |n| = 1 \) Landau levels; this has a higher energy than the other excitations, so will not be relevant to either transport or spin polarization.

At \( \tilde{\nu} = 3/7 \) there exist ‘sideways’ excitations (figure 3(a)) as well as those that stay within the SU(3) sector of the ground state (figures 3(b) and (c)). The nine possible excitons within the SU(3) sector can be reduced to a [2, 1] octet of SU(3) (see the figure above figure 3(b)) and an SU(3) singlet. The latter SU(3) multiplets are embedded in \([N/3 + 1, 2, 1]\) and \([N/3] \) SU(4) multiplets, respectively, and are all gapped. The SU(4) spin wave mode of figure 3(a) is not gapped in the \( k \to 0 \) limit and has the smallest energy at all wavevectors. The rather small gap to creating a far-separated CF particle–hole pair in mode (a) makes the 3/7 state very delicate in the SU(4) limit. The stiffness of the SU(4) spin wave in (a) could not be determined with sufficient accuracy, precluding an estimate of the SU(4) skyrmion energy at 3/7.

Weak symmetry breaking fields \( \Delta_{Z} \) and \( \Delta_{V} \) split the multiplet and select a state of reduced symmetry out of the SU(2) \( \times \) SU(2) decomposition \([32]\) of the SU(4) multiplet for both the ground state and the exciton. To illustrate we consider \( \tilde{\nu} = 2/5 \) with \( \Delta_{Z} > \Delta_{V} \). The spin and pseudospin (valley)
Figure 2. Excitations and gaps (see footnote 4) of the $\tilde{\nu} = 2/5$ fractional quantum Hall effect in the $|n| = 0, 1$ graphene Landau levels. The $A$ level structure for the maximum weight state in each mode is indicated in the diagrams at the top. Mode (a), which has no analog in GaAs, determines the smallest charge gap. The modes (b) and (c) correspond to singlet and triplet excitations in an $SU(2)$ sector embedded in a $SU(4)$ multiplet. The transport and roton gaps shown near the largest momenta and the minimum, respectively, correspond to the thermodynamic limit. The rotons in mode (b) destabilize the state when $1/2Z$ or $1/V$ becomes greater than the roton energy.

Figure 3. Excitations and gaps (see footnote 4) of the $\tilde{\nu} = 3/7$ fractional quantum Hall effect in the $|n| = 0, 1$ graphene Landau levels. The spin-wave-like mode (a) produces the smallest charge gap. Modes (b) and (c) produce an $SU(4)$ roton; the roton in (b) causes an instability with increasing $1/Z$ or $1/V$. Roton and transport gaps are indicated. $2\times$ denotes weight multiplicity.

Quantum numbers are $(S, P) = (N/2, 0)$ for the ground state and $(N/2 - 1, 1)$ or $(N/2, 1)$ and $(N/2, 0)$, respectively, for modes (a)–(c). Thus the fourfold-degenerate mode (a) is raised by $\Delta_Z$ and split by $-\Delta_V$, $0, 0, \Delta_V$; mode (b) is split similarly but it is not shifted; mode (c) is neither split nor shifted. For each mode as well as the ground state there are other copies displaced to higher energies in multiples of $\Delta_Z$, which in turn must merge into the continuum of other excitations; the structure of these higher copies follows from the $SU(4)$ to $SU(2) \times SU(2)$ decomposition [32]. We have also studied $\tilde{\nu} = 4/9$, where there is no spin wave but the 16 possible excitations into the second $AL$
decouple into an $SU(4)$ singlet (b) and the 15-fold degenerate mode (a) labeled by the $YT[2,1,1]$, see figure 4. The large $k$ limits of lowest energy are 0.021(1) and 0.026(2) for the $n=0$ and 1 LLs. As a function of $\Delta_Z$ or $\Delta_V$ there are several possible transitions into CF states with other spin and valley quantum numbers [13]. The roton and transport gaps in mode (b) could not be obtained with sufficient accuracy in the thermodynamic limit, but in any finite systems they are not smaller than in mode (a). It is uncertain if mode (b) truly develops a roton gap. Rotons in mode (a) destabilize the state at sufficiently high Zeeman energy.

5. Conclusion

To conclude, we have argued that the spin degree of freedom may be frozen at high fields in graphene, and that many features of the FQHE in graphene at high magnetic field are consistent with $SU(2)$ CF theory. In particular, quantitative calculations are consistent with the observed order of stability of states in $n=0$ and $|n|=1$ LLs, although the observed gaps are much smaller than the theoretical values. We have further shown that, in the $SU(4)$ limit, the FQHE states at $\tilde{\nu}=2/5$ and $3/7$ have a greatly reduced charge gap due to the new $SU(4)$ spin wave mode, which implies more stringent conditions for their observation.

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