Evolution of cooperation driven by individual disguise in the public goods game with pool punishment

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Abstract. The phenomenon of individual disguise is pervasive in the real world. But in theory it is unclear what roles it plays in the evolution of cooperation. In this work we introduce individual disguise into a public goods game with pool punishment, and we assume that defectors choose to probabilistically disguise to escape punishment in interaction groups. By using the replicator equations, we show that the introduction of individual disguise hinders public cooperation in the conditions of low fine and low disguise cost no matter whether second-order punishment is considered or not. Besides, we find that the positive role of punishment is completely undermined in the absence of second-order punishment, whereas the situation is improved in the presence of second-order punishment.

1. Introduction

Evolution of cooperation is an important studied subject across biological system ranging from animal world to human society [1-2]. And cooperation can be destroyed or even cooperation collapses because of the existence of free-riders [3-4]. Evolutionary game theory has been able to provide a powerful and effective tool for studying the evolution of cooperation [5-7]. Besides, the study on mechanisms of the evolution of cooperation, such as punishment [8-9], reward [10-11], exclusion [12-13], reputation [14-15] and so on, has attracted wide publicity.

In particular, aiming at pool punishment mechanism in theory, ample research efforts have already been shown on its positive role in promoting the evolution of cooperation [16-17]. For example, it is worth noting that Sigmund [18] et al compared the prevailing model of peer-punishment with pool-punishment, which reveals that pool punishment can prevail over peer punishment only if second-order punishment is considered. In addition, Czako et al compared the efficiencies of pool punishment and peer punishment in a spatial public goods game, which shows that pool punishment can be only maintained in the limit of weak peer punishment by numerical simulations [19]. Subsequently, Szolnoki et al also studied the effects of pool-punishment and compared the reported previously results for peer-punishment in the spatial public goods game, which concludes that pool punishment can be maintained even without the necessity of punishing second-order free-riders [20]. Furthermore, Traulsen et al showed that humans prefer pool punishment for maintaining the commons by an economic experiment, despite being costly even in the absence of defectors [21].

However, sometimes the effectiveness of pool punishment may be reduced for the evolution of cooperation due to the existence of second-order free-riders. For example, Ref.[18] also showed that peer punishment do better than pool punishment in the absence of second-order punishment. In addition, Liu et al studied the direct competition between pool punishment and pool exclusion, and found that
pool exclusion always outcompetes pool punishment, no matter whether second-order punishment is considered or not [22]. Furthermore, disguise behavior of individuals can be often found in the real world, and it may also weaken the enforcement of pool punishment. For example, drunk driving is often encountered in human society, but some people can evade the punishment by changing seats. But in theory it is unclear what roles it plays in the evolution of cooperation.

Inspired by the above consideration, we introduce individual disguise into the public goods game (PGG) with an additional strategy of pool punishment, and aim to study how disguise cost and fine influence the evolutionary dynamics of cooperators, defectors, and punishers in a large well-mixed population. In particular, we focus on whether pool punishment can effectively promote the evolution of cooperation in a disguising environment. Besides, we assume that defectors choose to probabilistically disguise to escape punishment in interaction groups. Considering this frame of population composition, by using replicator equations, we show that the introduction of disguise hinders public cooperation in the conditions of low fine and low disguise cost, no matter whether second-order punishment is considered or not. Besides, we find that the positive role of punishment is completely undermined in the absence of second-order punishment, whereas the situation is improved in the presence of second-order punishment.

2. Model and method
We introduce individual disguise into the PGG with pool punishment in an infinitely large, well-mixed population. In each round of the game, \( N \) individuals of population are randomly chosen and formed a group. In the game, each cooperator (\( C \)) contributes a fixed amount \( c \) to the common pool, whereas defectors (\( D \)) contribute nothing. Simultaneously, each punisher (\( P \)) also contributes a fixed amount \( c \) to the common pool, and then provides a fixed amount \( \beta \) to the punishment pool and imposes a fine \( \alpha \) on each defector. The accumulated contribution is multiplied by the enhancement factor \( r \) (\( 0 < r < N \)) measuring the intensity of a dilemma [23-24], and the total contributions are equally allocated by all the \( N \) group members. In addition, defectors expect to escape punishment in interaction groups, and thus try to disguise themselves with probability \( q \) (\( 0 < q < 1 \)) by paying a cost \( \gamma \), then defectors can successfully avoid punishment with probability \( p \) (\( 0 < p < 1 \)).

2.1. Without second-order punishment
Here we assume that pool punishment does not punish individuals (second-order free-riders) who contribute to the public goods but do not play the role of punishment, on the basis of the above description, accordingly the payoffs \( \pi \) of three strategies (\( i = C, D, \) or \( P \)) in the group can be given as, respectively

\[
\pi_C = \frac{rc(N_e + N_r + 1)}{N} - c, \\
\pi_D = \frac{rc(N_e + N_r)}{N} - q\gamma - (1 - pq)\alpha N_r, \\
\pi_P = \frac{rc(N_e + N_r + 1)}{N} - c - \beta,
\]

where \( N_i (i = C, D, \) or \( P \)) represents the number of players with strategy \( i \) among the \( N-1 \) individuals in the group.

2.2. With second-order punishment
Different from above model, here pool punishment also resorts to punish cooperators (second-order free-riders) who only cooperate in producing public goods and imposes a fine \( \alpha \) on each cooperator. In addition, we assume that there exists a weight \( k \) (\( 0 < k < 1 \)) for cooperators’ fine. Accordingly the payoffs \( \pi_i \) of three strategies (\( i = C, D, \) or \( P \)) in the group can be calculated as, respectively
\( \pi_c = \frac{rc(N_c + N_o + 1)}{N} - c - k\alpha N_p, \)

\( \pi_d = \frac{rd(N_d + N_o)}{N} - q\gamma - (1 - pq)\alpha N_p, \)

\( \pi_p = \frac{rc(N_p + N_o + 1)}{N} - c - \beta, \)

where \( N_i (i = C, D, \text{ or } P) \) represents the number of players with strategy \( i \) among the \( N - 1 \) individuals in the group.

2.3. Replicator equations

In infinitely large populations, the frequencies of cooperator, defector, and punisher in the population can be denoted by \( x, y, \) and \( z, \) respectively. Consequently, the evolutionary dynamics of strategies can be studied by replicator equations \((25-26)\),

\[ \dot{x} = x(P_c - \bar{P}), \]
\[ \dot{y} = y(P_o - \bar{P}), \]
\[ \dot{z} = z(P_p - \bar{P}), \]

where \( P_i (i = C, D, \text{ or } P) \) represents the average payoff of the strategy \( i \), and it can be calculated by

\[ P_i = \sum_{N_c=0}^{(N-1)!} \frac{(N-1)!}{N_c!N_o!N_p!} x^{N_c} y^{N_o} z^{N_p} \pi_i, \]

in which \( \pi_i \) denotes the probability of finding \( N - 1 \) co-players which includes \( N_i \) players.

In addition, \( \bar{P} = xP_c + yP_o + zP_p \) denotes the average payoff of the population.

3. Evolutionary dynamics without second-order punishment

3.1. Stability analysis of equilibrium points

Substituting the payoff equations \((1)\) of three strategies into Eq.\((3)\), correspondingly we have the following equation system

\[ \dot{x} = x[(1-x)\beta - y(\sigma + \beta - q\gamma - (1-pq)\alpha(N-1)(1-x-y))], \]
\[ \dot{y} = y[(1-y)(\sigma + \beta - q\gamma - (1-pq)\alpha(N-1)(1-x-y)) - x\beta], \]

where \( \sigma = c - \frac{rc}{N} \). Therefore, we let \( \dot{x} = 0 \) and \( \dot{y} = 0, \) then system \((5)\) has at most four boundary fixed points in the condition of \( \sigma = q\gamma \) which are \( (0,0) \), \( (0,1) \), \( (1,0) \), and \( (0,1 - \frac{\sigma + \beta - q\gamma}{(1-pq)\alpha(N-1)}) \), respectively. In addition, we analyze stability of four equilibria by means of the sign of eigenvalues of Jacobian matrix \([27]\), then the Jacobian matrix of system \((5)\) can be obtained by calculating the first order partial derivatives as

\[ J = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{bmatrix} \]

We accordingly have the following results. And we show that the stability of equilibrium points in Table 1.

**Proposition 3.1.1.** In the conditions of \( \sigma + \beta - q\gamma < 0 \) or \( (1-pq)\alpha(N-1) \leq \sigma + \beta - q\gamma \), the system \((5)\) has three fixed points as \((0,0), (0,1), \) and \((1,0), \) respectively.

For \( \sigma < q\gamma \), the largest eigenvalues of \( J(0,0) \) and \( J(0,1) \) are positive, whereas the largest eigenvalue of \( J(1,0) \) is negative. Consequently, the fixed points \((0,0)\) and \((0,1)\) are unstable, while the fixed point \((1,0)\) is stable.
For $\sigma > q\gamma$, the largest eigenvalues of $J(0,0)$ and $J(1,0)$ are positive, whereas the largest eigenvalue of $J(0,1)$ is negative. Consequently, the fixed points $(0,0)$ and $(1,0)$ are unstable, while the fixed point $(0,1)$ is stable.

Table 1. Stability of equilibrium points.

| Condition                        | $(0,0)$ | $(1,0)$ |
|----------------------------------|---------|---------|
| $\sigma + \beta - q\gamma \leq 0$ or $0 < \sigma + \beta - q\gamma < (1 - pq)\alpha(N - 1)$ | Unstable | Stable  |
| $\sigma < q\gamma$               | $(0,1)$ | $(0,0)$ |
| Stable                           | $(0,1)$ | $(0,0)$ |
| $\sigma > q\gamma$               | $(0,1)$ | $(0,0)$ |
| Stable                           | $(0,1)$ | $(0,0)$ |

Proposition 3.1.2. In the conditions of $0 < \sigma + \beta - q\gamma < (1 - pq)\alpha(N - 1)$, the system (5) has four fixed points as $(0,0)$, $(0,1)$, $(1,0)$, and $(0,1, \frac{\sigma + \beta - q\gamma}{(1 - pq)\alpha N - 1})$, respectively.

For $\sigma < q\gamma$, the largest eigenvalues of $J(0,0)$, $J(0,1)$, and $J(0,1, \frac{\sigma + \beta - q\gamma}{(1 - pq)\alpha N - 1})$ are positive, whereas the largest eigenvalue of $J(1,0)$ is negative. Consequently, the fixed points $(0,0)$, $(0,1)$, and $(0,1, \frac{\sigma + \beta - q\gamma}{(1 - pq)\alpha N - 1})$ are unstable, while the fixed point $(1,0)$ is stable.

For $\sigma > q\gamma$, the largest eigenvalues of $J(0,0)$, and $J(0,1, \frac{\sigma + \beta - q\gamma}{(1 - pq)\alpha N - 1})$ are positive, whereas the largest eigenvalue of $J(0,1)$ is negative. Consequently, the fixed points $(0,0)$, $(1,0)$, and $(0,1, \frac{\sigma + \beta - q\gamma}{(1 - pq)\alpha N - 1})$ are unstable, while the fixed point $(0,1)$ is stable.

3.2. Numerical example

A representative evolutionary dynamics of strategies in the absence of second-order punishment for $0 < \sigma + \beta - q\gamma < (1 - pq)\alpha(N - 1)$ is plotted in Figure 1. For $\sigma > q\gamma$, we find that there is only one global stable fixed point where $x = 0$, $y = 1$, and $z = 0$ corresponding to the D node in Figure 1(a). In other words, it shows that the introduction of individual disguise causes public cooperation to collapse for low disguise cost. In addition, while pool punishment is considered in a disguising environment, but its positive role in the evolution of cooperation is undermined. Furthermore, for $\sigma < q\gamma$, the system converges to the state of all cooperators, as is shown in Figure 1(b), which indicates that cooperation emerges in the condition of high disguise cost. These results demonstrate that pool punishment becomes useless for the evolution of cooperation in the absence of second-order punishment.

Figure 1. Evolutionary dynamics of strategies in the absence of second-order punishment. Panel (a): $\sigma > q\gamma$. Panel (b): $\sigma < q\gamma$. Nodes C, D, and P of the simplex denote homogeneous populations of cooperators, defectors, and punishers, respectively. Correspondingly, filled cycles denote stable equilibria and open cycles denote unstable equilibria. Parameters are $N = 5$, $r = 3$, $c = 1$, $\alpha = 0.5$, $\beta = 0.2$, $p = 1$, $q = 0.5$, $\gamma = 0.2$ for (a), and $\gamma = 0.9$ for (b).
4. Evolutionary dynamics with second-order punishment

4.1. Stability analysis of equilibrium points
Substituting the payoff equations (2) of three strategies into Eq.(3), we have the following equation system

\[
\begin{cases}
\dot{x} = x[(1-x)[\beta - k\alpha(N-1)[1-x-y]] - y(\sigma + \beta - q_r - (1-pq)\alpha(N-1)(1-x-y))] , \\
\dot{y} = y[(1-y)[\sigma + \beta - q_r - (1-pq)\alpha(N-1)(1-x-y)] - x(\beta - k\alpha(N-1)(1-x-y))] , \\
\end{cases}
\]

(6)

where \( \sigma = c - \frac{R}{N} \). Therefore, we let \( \dot{x} = 0 \) and \( \dot{y} = 0 \), then system (6) has at most five boundary fixed points in the condition of \( \sigma \neq q_r \), which are \((0,0)\) , \((0,1)\) , \((1,0)\) , \((0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)})\), and \((1 - \frac{\beta}{k\alpha(N-1)},0)\), respectively.

Similarly, we accordingly have the following results. And we show the stability of equilibrium points in Table 2.

**Proposition 4.1.1.** In the conditions of \( 0 < \sigma + \beta - q_r < (1-pq)\alpha(N-1) \) and \( \beta < k\alpha(N-1) \), the system (6) has five fixed points as \((0,0)\) , \((0,1)\) , \((1,0)\) , \((0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)})\), and \((1 - \frac{\beta}{k\alpha(N-1)},0)\), respectively.

For \( \sigma < q_r \), the largest eigenvalues of \( J(0,1) \), \( J(0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)}) \), and \( J(1 - \frac{\beta}{k\alpha(N-1)},0) \) are all positive, whereas the largest eigenvalues of \( J(0,0) \) and \( J(1,0) \) are both negative. Consequently, the fixed points \((0,1)\) , \((0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)})\), and \((1 - \frac{\beta}{k\alpha(N-1)},0)\) are unstable, while the fixed points \((0,0)\) and \((1,0)\) are stable.

For \( \sigma > q_r \), the largest eigenvalues of \( J(0,1) \), \( J(0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)}) \), and \( J(1 - \frac{\beta}{k\alpha(N-1)},0) \) are all positive, whereas the largest eigenvalues of \( J(0,0) \) and \( J(1,0) \) are both negative. Consequently, the fixed points \((1,0)\) , \((0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)})\), and \((1 - \frac{\beta}{k\alpha(N-1)},0)\) are unstable, while the fixed points \((0,0)\) and \((0,1)\) are stable.

**Table 2. Stability of equilibrium points.**

| Condition                  | Fixed Points                          | Stability |
|----------------------------|---------------------------------------|-----------|
| \( 0 < \sigma + \beta - q_r < (1-pq)\alpha(N-1) \) and \( \beta < k\alpha(N-1) \) | \((0,0)\) , \((0,1)\) , \((1,0)\) | Stable: \((0,0)\) , \((0,1)\) , \((1,0)\) |
| \( \sigma > q_r \)        | \((0,0)\) , \((0,1)\) , \((1,0)\) | Stable: \((1,0)\) |
| \( \sigma < q_r \)        | \((0,0)\) , \((0,1)\) , \((1,0)\) | Stable: \((0,0)\) , \((0,1)\) , \((1,0)\) |

**Proposition 4.1.2.** In the conditions of \( 0 < \sigma + \beta - q_r < (1-pq)\alpha(N-1) \) and \( \beta = k\alpha(N-1) \), the system (6) has four fixed points as \((0,0)\) , \((0,1)\) , \((1,0)\) , and \((0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)})\), respectively.

For \( \sigma < q_r \) and \( \beta = k\alpha(N-1) \), the largest eigenvalues of \( J(0,1) \) and \( J(0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)}) \) are positive, whereas the largest eigenvalue of \( J(1,0) \) is negative. Consequently, the fixed points \((0,1)\) and \((0,1 - \frac{\sigma + \beta - q_r}{(1-pq)\alpha(N-1)})\) are unstable, while the fixed point \((1,0)\) is stable. Besides, one eigenvalue of the Jacobian matrix at the fixed point \((0,0)\) is zero and the other one is negative. Accordingly, we study its stability by means of the
center manifold theorem [27]. We have \( y = h(x) \) is a center manifold, and start to try \( h(x) = \alpha y \), which yields the reduced system \( \dot{x} = (x^2 - x')\beta + O[y'] \).

Since \( \beta \neq 0 \), and thus the fixed point \( x = 0 \) of the reduced system is unstable. Consequently, the fixed point \((0,0)\) is unstable.

For \( \sigma < q \) and \( \beta = 0 \), the largest eigenvalues of \( J(0,0), J(0,1), \) and \( J(0,1) - \frac{\alpha + \beta - q}{(1 - pq) \alpha(N - 1)} \) are positive, whereas the largest eigenvalue of \( J(1,0) \) is negative. Consequently, the fixed points \((0,0), (0,1), \) and \((0,1) - \frac{\alpha + \beta - q}{(1 - pq) \alpha(N - 1)} \) are unstable, while the fixed point \((1,0)\) is stable.

For \( \sigma > q \) and \( \beta = 0 \), the largest eigenvalues of \( J(0,0), J(0,1), \) and \( J(0,1) - \frac{\alpha + \beta - q}{(1 - pq) \alpha(N - 1)} \) are positive, whereas the largest eigenvalue of \( J(1,0) \) is negative. Consequently, the fixed points \((0,0), (0,1), \) and \((0,1) - \frac{\alpha + \beta - q}{(1 - pq)(N - 1)} \) are unstable, while the fixed point \((1,0)\) is stable. (Proof is similar as that for (i) in the Proposition 4.1.2).

For \( \sigma > q \) and \( \beta = 0 \), the largest eigenvalues of \( J(0,0), J(0,1), \) and \( J(0,1) - \frac{\alpha + \beta - q}{(1 - pq) \alpha(N - 1)} \) are all positive, whereas the largest eigenvalue of \( J(1,0) \) is negative. Consequently, the fixed points \((0,0), (0,1), \) and \((0,1) - \frac{\alpha + \beta - q}{(1 - pq)(N - 1)} \) are unstable, while the fixed point \((1,0)\) is stable. (Proof is similar as that for (ii) in the Proposition 4.1.3).
For \( \sigma > q_f \) and \( \sigma + \beta - q_f r > 0 \), the largest eigenvalues of \( J(0,0), J(1,0) \) and \( J(1,0) - \frac{\beta}{k\sigma(N-1)} \) are positive, whereas the largest eigenvalue of \( J(0,1) \) is negative. Consequently, the fixed points \((0,0), (1,0), (1-kN_0)\) are unstable, while the fixed point \((0,1)\) is stable.

**Proposition 4.1.4.** In the conditions of \( \sigma + \beta - q_f r \leq 0 \) or \( \sigma + \beta - q_f r \geq (1-pq)\alpha(N-1) \) and \( \beta \geq k\alpha(N-1) \), the system (6) has three fixed points as \((0,0), (0,1), \) and \((1,0)\), respectively.

Similarly, the following conclusions can be easily proved according to above methods.

For \( \sigma < q_f \), equilibria \((0,0)\) and \((0,1)\) are unstable, while the fixed point \((1,0)\) is stable.

For \( \sigma > q_f \), equilibria \((0,0)\) and \((1,0)\) are unstable, while the fixed point \((0,1)\) is stable.

### 4.2. Numerical example

A representative evolutionary dynamics of strategies in the presence of second-order punishment for \( 0 < \sigma + \beta - q_f r < (1-pq)\alpha(N-1) \) and \( \beta \geq k\alpha(N-1) \) is plotted in Figure 2. For \( \sigma > q_f \), the evolutionary dynamics exhibit bistability of the two homogeneous states of all P-players (P node) and all D-players (D node), whereas evolutionary dynamics exhibit the bistability of the two homogeneous states of all P-players (P node) and all C-players (C node) for \( \sigma < q_f \), as is presented in Figure 2(a) and (b), respectively. These results indicate that the positive role of pool punishment is improved in the presence of second-order punishment no matter whether disguise cost is high or low.

![Figure 2](image)

**Figure 2.** Evolutionary dynamics of strategies in the presence of second-order punishment. Panel (a): \( \sigma > q_f \). Panel (b): \( \sigma < q_f \). Nodes C, D, and P of the simplex denote homogeneous populations of cooperators, defectors, and punishers, respectively. Correspondingly, filled cycles denote stable equilibria and open cycles denote unstable equilibria. Parameters are \( N = 5, r = 3, c = 1, \alpha = 0.5, \beta = 0.2, p = 1, q = 0.5, k = 0.2, \gamma = 0.2 \) for (a), and \( \gamma = 0.9 \) for (b).

### 5. Conclusions

So far many previous theoretical works have also revealed that pool punishment can maintain high level of cooperation [18-19]. However, in order to protect own or collective interests, some individuals choose to disguise, then it may weaken the enforcement of punishment and affect cooperative behavior in the population. And few studies have thus far considered this factor despite it is common in real society. In this paper, we have introduced individual disguise into the public goods games with pool punishment. By means of replicator equations, in the absence of second-order punishment, we find that the positive role of punishment is eliminated in promoting the evolution of cooperation for low disguise cost. However, the positive role of punishment is maintained in the presence of second-order punishment. In addition, for a large disguise cost, cooperation always emerges, no matter whether second-order punishment is considered or not. We finally demonstrate that considering second-order punishment is more effective in the evolution of cooperation when individual disguise is introduced in the public goods game.

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