Self gravity affects quantum states

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We study the self gravitation of a quantum system and how it affects the quantum coherence present in its state. The system is initially prepared in a spatial superposition of two different positions of a field excitation, and we employ semiclassical gravity to find its time evolution. Self gravitation affects the coherence initially present in the state of large and heavy systems, whereas light or massless particles are unaffected. The transition between the two scenarios is determined by the ratio of the characteristic size of the system and its Compton length. Furthermore, the phase of the coherence determines the increase or decrease of the probability of locating the system in each of the initial positions. All effects increase linearly with the mass of the particle and time, while they vanish for vanishing coherence in the initial state, for large distances in the spatial superposition, and for massless particles. We believe that our results can explain simultaneously two important aspects of physical systems: the possibility of coherently placing individual particles or photons in distant positions, and the difficulty of maintaining quantum coherence between massive objects.

I. INTRODUCTION

Do quantum systems gravitate as classical ones? Answering this question would have profound consequences on our understanding of physics at the overlap of relativity and quantum mechanics. More importantly, it would aid our efforts to obtain a unified theory of Nature.

To date, it is unknown how a quantum system gravitates. There has been extensive work on this topic, such as the first proposals of quantum state reduction and spontaneous collapse of the wave function due to gravity \cite{1–4}, the studies of gravitational bound states \cite{2, 3}, stochastic gravity \cite{7}, gravitational radiation due of coherent superpositions \cite{8}, and proposals of more advanced theories of (quantum) gravity \cite{9, 10}. Regardless of the advances achieved so far, it is still unclear how genuine quantum features, such as coherence and entanglement, affect the gravitational field of a physical system.

In this work we employ semiclassical gravity and previously developed tools \cite{11} to compute directly the time evolution of a quantum state in the presence of its own gravitational field. The system consists of a localized low-energy field excitation initially prepared in one of two possible positions that are separated by a given distance. Our approach is free from ad-hoc assumptions.

We find that the coherence initially present in the state is affected for macroscopic, heavy objects, while it is preserved for light or massless particles. This supports similar conclusions put forward in the literature \cite{4}. We are able to obtain the timescale and length scales that regulate the transition between the two different regimes. The timescale at which the effects vanish coincides with that obtained in previous work \cite{2, 3}. However, we find that the transition between the massive and massless regimes is characterized solely by the ratio of the characteristic size of the excitation and its Compton wavelength. Systems that occupy a volume defined by a length scale comparable or smaller than their Compton wavelength do not witness the effects of their gravitational field. On the contrary, systems whose characteristic linear size is much larger than their Compton wavelength are affected. Furthermore, we obtain that the increase of the system’s size, mass and initial mean separation degrades the initial coherence, as also shown in previous work \cite{1, 4}. We also find explicitly that the change in expectation value of the position is proportional to the amount of quantum coherence present. Surprisingly, the sign of the coherence determines the sign of the change of this expectation value, which we conjecture might be related to fundamental symmetries of spacetime. Finally, the initial mixedness of the subsystems (i.e., initial positions) changes linearly with time. However, it is unexpectedly unaffected when the diagonal elements of the density matrix are equal. This seems to indicate the presence of a fundamental relation between the mixedness of the system and its self gravity.

Our results confirm that mass, distance between the initial positions, and time decrease the quantum character (coherence) of extended heavy systems. On the contrary, quantum coherence present in the state of photons and small quantum systems is preserved. We believe that these conclusions have important implications for both quantum and relativistic theories. In particular, they validate the overwhelming evidence that macroscopic physical systems are found in classical states, and gravitate as classical objects.

Background\textsuperscript{4} We assume that particles are excitations of quantum fields propagating on a classical background spacetime \cite{12}. For simplicity, we consider a mas-

\textsuperscript{1} All computations are left to the appendices. Details omitted can be found in \cite{11}. We use the natural convention \( c = \hbar = 1 \), unless explicitly stated, as well as Einstein’s summation convention. The metric has signature \( (-, +, +, +) \). We work in the Schrödinger picture.
sive scalar quantum field $\hat{\phi}(x^\mu)$ with mass $m$ in (3+1)-dimensional spacetime with metric $g_{\mu\nu}$. We work in linearized gravity \[13\], and assume that the spacetime is a perturbation of flat Minkowski with metric
\[g_{\mu\nu} = \eta_{\mu\nu} + \xi h_{\mu\nu},\]
where $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$ is the flat metric and $\xi \ll 1$ is a small control parameter. Here, we will consider only effects that are proportional to $\xi$ and ignore $O(\xi^2)$ contributions. The parameter $\xi$ is uniquely determined by the relevant physical energy scales of the problem \[11\], and we will define it later on. Finally, the perturbation $h_{\mu\nu}$ depends on the spacetime coordinates $x^\mu$ and, crucially, is generated by the field excitations as discussed below.

Since we work in the Schrödinger picture, operators are time independent. Therefore, we do not find and solve the field equations for $\hat{\phi}(x^\mu)$ but consider the time evolution operator $\hat{U}(t)$ instead. It is well known that, in a general curved spacetime, there is no unique and well defined way to choose a Hamiltonian with which to evolve the system \[12\]. This is a consequence of the fact that, in general relativity, no notion of time is preferred \[12\]. In our case, however, we are looking at small perturbations of Minkowski spacetime. It is therefore possible to define the time evolution operator in meaningful way as $\hat{U}(t) = e^{-i\int_0^t dt' \hat{H}(t')}$, where the Hamiltonian $\hat{H}(t)$ is defined as $\hat{H}(t) := \int d^3x : \hat{T}_{\mu\nu} :$, and $\hat{T}_{\mu\nu}$ is the stress-energy tensor of the field defined as
\[T_{\mu\nu} := \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \frac{1}{2} g_{\mu\nu} [\partial^\rho \hat{\phi} \partial_\rho \hat{\phi} + m^2 \hat{\phi}^2].\]

We choose to decompose the field as $\hat{\phi} = \int d^3k \hat{a}_k u_k + \hat{a}_k^\dagger u_k^\dagger$, where $u_k(x^\mu) = (2\pi)^{-3/2} (2\omega_k)^{-1/2} \exp[i k_\mu x^\mu]$ are plane waves in flat spacetime, we have $k_\mu x^\mu = -\omega_k t + k \cdot x$ with the frequency $\omega_k := \sqrt{k^2 + m^2}$, and the annihilation and creation operators $\hat{a}_k, \hat{a}_k^\dagger$ satisfy the canonical commutation relations $[\hat{a}_k, \hat{a}_k^\dagger] = \delta^3(k - k')$, while all other vanish. The annihilation operators $\hat{a}_k$ define the vacuum state $|0\rangle$ through $\hat{a}_k |0\rangle = 0 \forall k$.

The perturbative expression for the time evolution operator $\hat{U}(t)$ reads $\hat{U}(t) \approx \hat{U}(0)(t) (1 - i\xi \int_0^t dt' \hat{U}(0)(t') \hat{H}(1)(t') \hat{U}(0)(t'))$, where we have defined $\hat{U}(0)(t) := \exp[-i \int d^3k \omega_k \hat{a}_k^\dagger \hat{a}_k] t$ and $\hat{H}(1)(t) := \int d^3x : \hat{T}^{(1)}_{\mu\nu} :$, and we have also used the perturbative expressions $\hat{H}(t) \approx \int d^3k \omega_k \hat{a}_k^\dagger \hat{a}_k + \xi \hat{H}^{(1)}(t)$ for the Hamiltonian and $\hat{T}^{(1)}_{\mu\nu} \approx T_{\mu\nu}^{(1)} + \xi \hat{T}^{(1)}_{\mu\nu}$ for the stress-energy tensor.

The evolution of the initial state is determined by the stress energy tensor, which in turn is determined by the metric induced by the systems itself. Such back reaction of the quantum field on the metric, and therefore the metric on the time evolution of the state, can be taken into account in a consistent way by employing the framework of semiclassical gravity, an approach that requires modifying Einstein equations. The utility of this approach as well as its limitations \[17\, 18\] have been already discussed in the literature for the specific case of interest here \[11\]. For the purposes of this work, we note that the regimes considered are well within the limits of validity of semiclassical gravity \[11\].

Back reaction of the quantum field on the gravitational field can be implemented through Einstein’s semiclassical equations
\[G_{\mu\nu} = 8\pi G_N \langle \hat{T}_{\mu\nu} \rangle ,\]
where $G_{\mu\nu}$ is Einstein’s tensor and $G_N$ is Newton’s constant. The average $\langle \cdot \rangle$ is done over the initial field state $\rho$. In this perturbative regime \[11\], it was shown that the semiclassical Einstein equations satisfy
\[G_{\mu\nu}^{(1)} = 8\pi \langle \hat{T}_{\mu\nu}^{(1)} \rangle ,\]
which, if solved explicitly, provide the expression for $h_{\mu\nu}$. Here, $G_N$ has been absorbed in the perturbative parameter $\xi := G_N E_0 / l_s \ll 1$, where $l_s := 1/\sigma$ is the length scale of the problem, i.e., the characteristic linear size of the particle as we will see below \[11\]. In addition, $E_0 := \text{Tr}[\rho \hat{H}^{(0)}]$ is the average energy of the particles.

Once we find an expression for $h_{\mu\nu}$ through \[2\] we can obtain $\tilde{H}^{(1)}(t)$ as $\tilde{H}^{(1)}(t) = \int d^3k d^3k' [A_{kk'} \hat{a}^\dagger_k \hat{a}^\dagger_{k'} + B_{kk'} \hat{a}_k \hat{a}_{k'} + \text{h.c.}]$, and the time-dependent coefficients $A_{kk'}(t)$ and $B_{kk'}(t)$ are given explicitly in \[A10\].

Particle states—The time evolution of the system requires us to specify the initial state of the system $S$ under consideration. We assume that it has the form
\[\rho_S(\alpha, \beta) = \alpha |01_R\rangle \langle 01_R| + (1 - \alpha)|1L_0\rangle \langle 1L_0| + |1R_0\rangle \langle 1R_0| + |1L_0\rangle \langle 1R_0| + |1R_0\rangle \langle 1R_0| ,\]
where we have $0 \leq \alpha \leq 1$ and $-1/2 \leq |\beta| \leq 1/2$, and $(\alpha - 1/2)^2 + |\beta|^2 \leq 1/4$ in order for $\rho(\alpha, \beta)$ to represent a physical state. On the one hand, the maximally coherent state has to be pure, requiring that $(\alpha - 1/2)^2 + |\beta|^2 = 1/4$, and we want it to be maximally nonclassical, requiring $\alpha = 1/2 = |\beta| = 1/2$ and $\beta = 0$. On the other, for $\alpha = 1/2$ and $\beta = 0$ the state is maximally mixed. Importantly, $\beta \in \mathbb{C}$.

We are interested in particles that have a finite size. Therefore, we introduce the normalized states $|01_R\rangle$ and $|1L_0\rangle$ of particles located on the ‘right’ and ‘left’ of the chosen origin of the coordinates respectively by
\[|01_R\rangle := \int d^3k F_{k_0}(k) e^{-i L \cdot k} a^\dagger_{k_0} |0\rangle \]
\[|1L_0\rangle := \int d^3k F_{k_0}(k) e^{i L \cdot k} a^\dagger_{k_0} |0\rangle ,\]
where the functions $F_{k_0}(k)$, peaked around $k_0$ with width $\sigma$, determine the shape of the wave packet of the particle in momentum space. The vector $\pm L$, on the other hand, defines the location of the peaks in position space. These are located at a distance of $2L := 2 \sqrt{L \cdot L}$. Normalisation implies that $\int d^3k |F_{k_0}(k)|^2 = 1$. We also choose the defining parameters $(k_0, \sigma, L)$ such that the particle states $|0\rangle$ are orthogonal, i.e., $\langle 1L_0|01_R\rangle \equiv 0$. Two examples are rectangle functions and sinc functions \[11\].
key observation here is that the vectors represent the same particle in two different positions.

We are not interested here in the exact nature of the peaked functions \( F_{\lambda}(k) \), but we note the following: we have changed from the plane wave mode basis \( u_{k} \), parametrized by \( k \), to a new basis that includes the modes defined by \( F_{\lambda}(k) e^{\pm i L \cdot k} \). This is common in quantum field theory, and analogous transformations appear, for example, in the transformation between Minkowski and Unruh modes \([13]\). In our case, we need to construct a full orthonormal basis of modes \( \{F_{\lambda}(k) e^{\pm i L \cdot k}\} \), where \( \lambda \) labels the remaining elements of the basis. We anticipate that our results are independent of the form and cardinality of such basis, and we just need to assume that \( \int d^{3}k F_{\lambda}^{*}(k) F_{\lambda}(k) e^{\mp i L \cdot k} = 0 \) and \( \int d^{3}k F_{\lambda}^{*}(k) F_{\mu}(k) = \delta_{\lambda \mu} \) for all \( \lambda, \mu \).

Gravity affects quantum states—We now proceed to outline our main results. We focus on the massive and static regime for the system \( S \), defined by \( \sigma/m = \lambda C / l_{\sigma} \ll 1 \) and \( k_{0} = 0 \). In this regime, a single excitation of size \( l_{\sigma} \) is much larger than its Compton length \( \lambda C := 1/m \), and has vanishing average momentum. This approximation crucially implies that \( U_{0}(t) |0_{Q} \rangle \approx \exp[\omega m t] |0_{Q} \rangle \) for \( Q = R, L \), which means that the particle states \( |n_{L} m_{R} \rangle \) approximate eigenstates to lowest order, and importantly \( U_{0}(t) |\rho_{S}(\alpha, \beta) \rangle U_{0}^{\dagger}(t) \approx |\rho_{S}(\alpha, \beta) \rangle \).

The total state of the system \( S \) and remaining degrees of freedom reads \( |\rho_{S}(\alpha, \beta) \rangle = |\rho_{S}(\alpha, \beta) \rangle \otimes |0_{\Delta} \rangle \), where all modes, except the ones of the system \( S \), are in the vacuum, i.e., \( |0_{\Delta} \rangle := \bigotimes_{\Delta} |0 \rangle \) and \( \rho_{S}(\alpha, \beta) \). We can compute the reduced state of the system \( S \) at time \( t \) by tracing out the unwanted degrees of freedom parametrized by \( \Delta \) from \( \rho(\alpha, \beta) \). We obtain

\[
\hat{\rho}_{S}(\alpha, \beta)(t) = U_{\text{eff}}(t) \hat{\rho}_{S}(\alpha, \beta) U_{\text{eff}}^{\dagger}(t),
\]

where we have defined the effective time-evolution operator \( U_{\text{eff}}(t) := U_{0}(t) U_{BS}(t) U_{C}^{(L)} \rho_{\text{SMs},+}(t) U_{C}^{(R)} \rho_{\text{SMs},-}(t) U_{\text{TMS}}(t) \) of the system \( S \), the effective free evolution \( U_{0}(t) \approx \exp[\omega m t] \), and the unitary operators

\[
\begin{align*}
U_{\text{BS}}(t) & \approx e^{-i \lambda (K^{(A,+)}(t) a_{R}^{\dagger} a_{L} + K^{(A,+)^{\dagger}}(t) a_{R}^{\dagger} a_{L})} \\
U_{C}^{(L)}(t) & \approx e^{-i \lambda (K^{(B,+)}(t) a_{R}^{\dagger} a_{L} + K^{(B,+)^{\dagger}}(t) a_{R}^{\dagger} a_{L})} \\
U_{C}^{(R)}(t) & \approx e^{-i \lambda (K^{(B,-)}(t) a_{R}^{\dagger} a_{L} + K^{(B,-)^{\dagger}}(t) a_{R}^{\dagger} a_{L})}.
\end{align*}
\]

Here, \( \omega := m \). The full expression of \( [5] \) to first order in \( \xi \), together with that of the coefficients that appear in \( [6] \), can be found in \([13]\) and \([14]\).

The reduced states of both position states can be easily computed using the main expression \([5]\), and we find that they satisfy the Lindblad-like equation

\[
\frac{d}{dt} \hat{\rho}_{Q}(\alpha, \beta) = -i \left[ \hat{H}_{Q}^{(1)}, \hat{\rho}_{Q}(\alpha, \beta) \right] \xi + \hat{L}_{Q} \hat{\rho}_{Q}(\alpha, \beta),
\]

where \( Q = R, L \). The operator \( \hat{H}_{Q}^{(1)}(t) \) has the expression

\[
\hat{H}_{Q}^{(1)}(t) := \omega \hat{a}_{Q}^{\dagger} \hat{a}_{Q} + \hat{K}_{Q}^{(B,+)}(t) \hat{a}_{Q}^{\dagger} \hat{K}_{Q}^{(B,+)}(t) \hat{a}_{Q} + \beta \mu_{Q} \hat{K}_{Q}^{(B,-)}(t) \hat{a}_{Q} \left( \mathbb{1} - \hat{a}_{Q}^{\dagger} \hat{a}_{Q} \right) + \text{h.c.},
\]

while the time-dependent Lindbladian \( \hat{L}_{Q} \hat{\rho}_{Q}(\alpha, \beta) \) has the form \( \hat{L}_{Q} \hat{\rho}_{Q}(\alpha, \beta) = \kappa_{L}^{(Q)} \hat{\rho}_{Q} \hat{a}_{L}^{\dagger} \hat{a}_{L} - \frac{1}{2} \hat{a}_{L}^{\dagger} \hat{a}_{L} \hat{\rho}_{Q} - \frac{1}{2} \hat{\rho}_{Q} \hat{a}_{L}^{\dagger} \hat{a}_{L} \),

\[
\text{with } \mu_{L} = \alpha, \gamma_{L} = \beta \kappa_{L}^{(Q)} \quad \xi = \hat{K}_{Q}^{(A,+)}(t) \hat{a}_{Q}^{\dagger} \hat{K}_{Q}^{(A,+)}(t),
\]

\[
\text{where } \mu_{L} = 1 - \mu_{L} + \frac{1}{2} \kappa_{L}^{(Q)} \text{ and } \hat{A}_{L} = \hat{a}_{L} \text{, while }
\]

\[
\mu_{L} = 1 - \mu_{L} + \frac{1}{2} \kappa_{L}^{(Q)} \text{ and } \hat{A}_{L} = \hat{a}_{L} \text{, while }
\]

\[
\text{This is expected, since the system’s global state } [5] \text{ contains coherence between the subsystems (i.e., the different degrees of freedom ‘L’ and ‘R’)}.
\]

With the states \([5] \) and \([7] \) we can now proceed to compute some quantities of particular interest. First of all we calculate the probability \( p_{R}(t) := \langle 1_{R} | \hat{\rho}_{R}(t) | 1_{R} \rangle \) and \( p_{L}(t) := \langle 1_{L} | \hat{\rho}_{L}(t) | 1_{L} \rangle \) of finding the excitations on the ‘right’ \( R \) or on the ‘left’ \( L \) respectively. We find

\[
p_{L}(t) = (1 - \alpha) / 2 - 2 \xi \left( \beta K^{(A,+)}(t) \right),
\]

and \( p_{R}(t) = 1 - p_{L}(t) \).

As expected for this ideal scenario (that is, where the “environment” is in its vacuum state), the probability of finding the system on the ‘right’ or ‘left’ is unity, i.e., \( p_{R}(t) + p_{L}(t) = 1 \). This property is key to ensure that the effect is not trivial since it cannot be described in another basis for the non interacting theory (it is not related to leakage or distortion to other modes).

The next step is to understand if the quantum coherence in the global state \([5] \) has changed. To do this, we note that the total entropy of the state does not change at first order in \( \xi \) since \([5] \) implies unitary evolution. Therefore, we can quantify the mixedness of a state \( \hat{\rho} \) through the purity \( \gamma(\hat{\rho}) := \text{Tr}(\hat{\rho}^{2}) \), where \( \gamma(\hat{\rho}) = 1 \) only for pure states and \( \gamma(\hat{\rho}) < 1 \) for mixed ones. We compute \( \gamma(\hat{\rho}) \) for both subsystems \( \hat{\rho}_{L}(\alpha, \beta) \) and \( \hat{\rho}_{R}(\alpha, \beta) \) and find that

\[
\Delta \gamma(\hat{\rho}_{Q}(\alpha, \beta)(t)) \approx -4(1 - 2 \alpha) \xi \beta K^{(A,+)}(t),
\]

where \( Q = L, R \) and \( \Delta \gamma(\hat{\rho}_{Q}(\alpha, \beta)(t)) := \gamma(\hat{\rho}_{Q}(\alpha, \beta)(t)) - \gamma(\hat{\rho}_{Q}(\alpha, \beta)(0)) \).

This means that the mixedness of the subsystems is affected by the evolution of the state, and therefore by gravity. Concretely, we can say that gravity is changing the quantum coherence initially present between the two subsystems.

Gravitational effects on quantum coherence—We have computed the time evolution of the state of a particle field excitation under the influence of its own gravitational field. We now proceed to analyze our results and understand which are the physical implications.

First of all, we see that all of the quantities of interest to us depend on \( \beta K^{(A,+)}(t) \). This prompts us to study how \( K^{A,\pm}(t) \) is determined as a function of \( \alpha, \beta \) and time \( t \). Their dependence on \( \alpha \) and \( \beta \) is to be expected,
since this quantity is determined by the metric $g_{\mu \nu}$, which in turn is determined by the initial state.

The function $K^{(A,+)}(t)$ can be computed and reads

$$K^{(A,+)}(t) = t \int \frac{d^3x}{(2 \pi)^3} h_{d(x)}^d \tilde{F}_0^r(L - x) \tilde{F}_0(L + x), \quad (11)$$

where $\tilde{F}_0(x) := \int \frac{d^3k}{(2 \pi)^3} F_0(k) e^{i k \cdot x}$ describes the spatial profile of the excitation.

From this expression, and the differential equations \cit{4} and \cit{9} that define the metric correction $h_{\mu \nu}$, we see that there are two possible scenarios. When $L/l_\sigma \gg 1$, the possible particle locations are very far apart compared to the characteristic size $l_\sigma$ of the particle, and $K^{(A,+)}(t) \to 0$ for $L/l_\sigma \to +\infty$. In physical terms, this means that when the separation $L$ between the positions `left' $L$ and `right' $R$ largely exceeds the size of the particle, all self gravitating effects vanish.

On the other hand, we have $L/l_\sigma \sim 1$ (either smaller or larger than unity, but not significantly). Combining all of the above we find

$$K^{(A,+)}(t) \approx t \Re(\beta \kappa(\sigma L)), \quad (12)$$

for an appropriate function $\kappa(\sigma L)$ that can be obtained explicitly once the shape $\tilde{F}_0(x)$ of the particle is known. Therefore, in this case we have direct proof that $K^{(A,+)}(t) = 0$ for $\beta = 0$, see \cit{11}.

This allows us to obtain a final expression for the main quantities that characterize the state of the system at time $t$. In particular, we have

$$\Delta \gamma(\bar{\rho}_Q(\alpha, \beta)(t)) \approx -4 (1 - 2 \alpha) \xi \Im(\beta) \Re(\beta \kappa(\sigma L)) \ t \ p_R(t) = 1 - p_L(t) = \alpha + 2 \xi \Im(\beta) \Re(\beta \kappa(\sigma L)) \ t, \quad (13)$$

where $Q = L, R$ indifferently and these expressions apply to massive and static particles. We can also show that for $\lambda_\sigma/l_\sigma \gg 1$ all effects vanish with time and therefore the state is unaffected. These are our main results.

**Physical significance and considerations**—We have shown that light or massless particles are not affected by their own gravitational field, while massive particles are.

The two quantities that regulate the transition are the Compton length $\lambda_C = 1/m$ of the particle and its characteristic size $l_\tau = 1/\sigma$. Objects that greatly exceed in size their own Compton length, i.e., for which $\lambda_C/l_\sigma \ll 1$, witness significant self gravitational effects on the quantum coherence present in the state. This is equivalent to say that larger and heavier particles are more affected, in accordance with experience. Furthermore, attempts to obtain massive objects in coherent superpositions of position degrees of freedom closer to each other are met with increasing back reaction from gravity that reduces the quantum coherence.

Importantly, notice the role of the diagonal terms governed by $\alpha$. When $\alpha = 1/2$, the coherence related effects \cit{13} vanish, irrespectively of the amount of coherence present (i.e., the value of $|\beta|$). The only surviving effect is the probability \cit{9} of finding the particle on the `Left' $L$ or the `Right' $R$. We do not have an explanation for this prediction, but we can say that the exchange symmetry between $R$ and $L$ — modulo a complex conjugation of $\beta$ — might forbid the gravitational field to act in a preferential way on the coherences. Furthermore, the phase of the parameter $\beta$ that quantifies the coherence (i.e., off-diagonal terms), crucially affects how the probabilities \cit{9} change, as already noted in previous work \cit{11}. For example, given a choice of $\beta$, then $p_R(t)$ increases and $p_L(t)$ decreases with time. However, by sending $\beta \to -\beta$, the opposite occurs. In this sense, we conjecture that the phase of the coherences might be related to symmetries of the spacetime, such as Lorentz invariance. We leave it to further work to investigate this intriguing aspect.

Another important aspect of our setup is that it can be used to represent two distinct physical scenarios: a single excitation of mass $m$ of the same particle in different positions that occupies an approximate volume $l_0^3$ or $N$ particles of mass $\mu = m/N$ in different positions that occupy an approximate volume $l_0^3$ each, which effectively we treat as one excitation of mass $m = N \mu$ in different positions occupying an approximate volume $l_0^3$.

Finally, all effects are proportional to the dimensionless parameter $\xi$ and change linearly with time. The parameter $\xi$ is fully determined by the relevant physical scales of the scenario \cit{11}, and most importantly by Newton's constant $G_N$. Restoring dimensions we have $\xi = \frac{G_N \sigma}{c^3}$, where we have noted that $E_0 \approx m c^2$ for massive static particles. This means that all effects increase with mass (energy), which confirms our initial hypothesis. In addition, if we assume that our work can be used to model composite systems with many constituent to good accuracy, we can write $m \sim \rho_0 l_\sigma^3$, where $\rho_0 = m/l_\sigma^3$ is the average mass density. Therefore, $\xi \sim \frac{G_N \sigma}{c^3}$ and we expect size to increase the effects as well.

There are two timescales that regulate the validity of the predictions of this work: $t_0 := \frac{\lambda_\sigma}{3} \left( \frac{\lambda_C}{c} \right)$ and $t_0' := \frac{\lambda_\sigma}{3} \frac{(h_{\lambda_\sigma})}{(G_N m^2)} \approx 2 (\lambda_\sigma/l_\sigma)/(r_S c)$, where $r_S := 2 G_N m c^2$ is the Schwarzschild radius associated to an object of mass $m$. Our techniques can be applied for $t/t_0 \ll 1$ and $t/t_0' \ll 1$. For times that approach, or are longer, than the smallest of these two timescales, the predictions presented here need to be improved.

The timescale $t_0'$ has already been obtained in the literature for a spherical object of mass $m$ and radius $l_\sigma$ that is prepared into a superposition of different locations, which is expected to lose coherence due to gravity \cit{8}. The fact that we also find such timescale further corroborates our results, but we stress here that we have an additional timescale. We note that $t_0$ is due to quantum mechanical effects alone, while $t_0'$ is due to the existence of relativistic and quantum mechanical effects. The natural question at this point is: which timescale applies first? This can be answered by looking at the ratio $t_0/t_0'$, which reads $t_0/t_0' = (l_\sigma r_S)/\lambda_\sigma^2$, and can take any positive value. Therefore, it allows us to understand which timescale is relevant in a given scenario. The interplay between the two timescales can be used for experimental
verification of the predictions of the theory. It is reasonable to believe that, for realistic systems composed of many interacting constituents, the timescale $t_0$ would not be present, since it arises a consequence of the inevitable spread of idealized wavepackets predicted by field theory.

To evaluate the magnitude of the effects, we consider two cases: a massive object with rest mass $m \sim 10^{-18}$ kg and one of rest mass $m \sim 10^{-16}$ kg. Their Compton lengths are $\lambda_C = \frac{h}{mc} \sim 2.2 \times 10^{-24}$ m and $\lambda_C \sim 2.2 \times 10^{-32}$ m, which are much smaller than the sizes $l_0 \sim 10^{-23}$ m and $l_0 \sim 10^{-9}$ m that we select. For these two cases we find $\xi \sim 7 \times 10^{-24}$ and $\xi \sim 7 \times 10^{-29}$, both very small. However, since $m \propto l_0^2$ for macroscopic objects, we have $\xi \propto G_N l_0^2 / c^2$ and therefore we can expect that the increase of size also will increase the effects. Turning to the timescales, we find $t_0 \sim 1.5 \times 10^{-48}$ s and $t_0 \sim 1.5 \times 10^{-8}$ s, and also $t_0 \sim 10^{-8}$ s and $t_0 \sim 1.5 \times 10^{-12}$ s.

The semiclassical gravity method employed here is valid in our regimes and is expected to give correct predictions. Relevant ways to judge the validity have been proposed in the past [17, 18] and have already been discussed in previous work [11]. Furthermore, we have restricted ourselves to states that do not have coherent superpositions or mixtures of single particle states with different mass (energy), since there is work that suggests that gravity should collapse states containing coherent superpositions of states with different energy (in line with [2]). In addition, our choice guarantees that the states $\rho(\alpha, \beta)$ are the most general one-particle states that we can consider. Therefore, more work must be carried out in order to claim that our methods can be extended to all states, quantum fields, energy and length scales.

Finally, we stress that our results occur in the basis of the localized modes [4] as we have shown here only when $\beta$ is nonzero. The vanishing of $\beta$ implies the vanishing of all effects, which corroborates our claims.

**Conclusion**—To summarize, we have studied the effects of self gravitation of a quantum system on the coherence present in its state in the form of quantum coherence between two positions.

Light and massless particles are unaffected, while very massive ones witness significant effects. In particular, we have found that increasing the distance at which any spatial superposition is created, compared to the size of the particle, decreases the effects. On the contrary, states that are created in superpositions of locations that are extremely close are greatly affected. The scale that governs this transition is the ratio between the Compton wavelength of the particle and its size. Systems of much greater size than their Compton wavelengths experience significant effects. In other words, we have shown directly that increasing mass and size of physical systems results in a greater back-reaction effect of their own gravitational field on the quantum coherence present in the state. When no quantum coherence is present initially, no effects occur. Furthermore, we noticed that the sign of the effects depends on the phase of the coherences. We conjectured that this phase can be intimately related to Lorentz invariance. More work is necessary to investigate this aspect. Finally, all effects increase linearly with the system’s mass and with time. All together this corroborates the claim that ‘more classical’ objects are expected to be found in ‘more classical’ states.

We believe that our conclusions can be used to resolve the fundamental conundrum of the apparent difficulty of producing macroscopic massive quantum systems.

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Appendix A: Preliminaries

In this work we use extensively the results obtained in [11], which we do not reprint in full here. Details of the derivations are left to the reader. Importantly for our study here, we work in the Schrödinger picture.

1. Self gravity-induced time evolution of macroscopically distinct systems: field quantization

The theory of field quantization necessary for this work is standard, and therefore we refer the reader to the literature for an in depth introduction to the quantum field theory in curved spacetime [12, 20].

Particles in this work are excitations of the uncharged, massive, scalar quantum field \( \phi(x^\mu) \) with mass \( m \), which is defined on a curved background spacetime with coordinates \( x^\mu \) and metric \( g_{\mu\nu} \) that has the expression

\[
g_{\mu\nu} = \eta_{\mu\nu} + \xi_h \delta_{\mu\nu},
\]

where \( \xi \ll 1 \) is the perturbative parameter that governs the regimes considered in this work, and \( \eta_{\mu\nu} = \text{diag}(-1,1,1,1) \) is the Minkowski metric. This parameter has been computed in the literature [11], and its expression in terms of the parameters of the problem is \( \xi = \frac{\ell_\pi m}{c} \). Here, \( \ell_\pi \) is the length scale of the problem. Importantly for our work, the metric correction \( \xi_h \) exists solely as a consequence of the self-gravitation of the low energy quantum system itself [11]. Obtaining an exact expression for \( \xi_h \) is, in general, very difficult and it depends on the wave-packet model for the of the particle excitations.

The corrections \( \xi_h \) are determined by the semiclassical Einstein equations

\[
G^{(1)}_{\mu\nu} = \langle : \hat{T}^{(0)}_{\mu\nu} : \rangle \rho,
\]

where \( \langle \hat{A} \rangle_\rho := \text{Tr}(\hat{A} \hat{\rho}) \) is the average computed over the state \( \hat{\rho} \). Note that we use the superscript \( (n) \) to denote the order \( n \) in the perturbative parameter \( \xi \) to which the contribution \( A^{(n)} \) appears in the perturbative expansion \( A = \sum_n A^{(n)} \xi^n \) of the full quantity \( A \). Here, \( \hat{T}_{\mu\nu} \) is the stress energy tensor, which in our case reads

\[
\hat{T}_{\mu\nu} := \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \frac{1}{2} g_{\mu\nu} \left[ \partial^\rho \hat{\phi} \partial_\rho \hat{\phi} + m^2 \hat{\phi}^2 \right].
\]

The time evolution of a state \( \hat{\rho} \) in the Schrödinger picture can be given an operationally meaningful definition when the spacetime is endowed with a global timelike Killing vector field, or at least a Killing vector field that is asymptotically timelike [12]. In our case, since we look at perturbations around the flat Minkowski metric, the natural choice is the timelike vector field \( \partial_t \). This means that, in this perturbative regime, the notion of time does not deviate from that of flat spacetime. Therefore, we can meaningfully define a Hamiltonian \( \hat{H}(t) \) through the stress energy tensor \( \hat{T}_{\mu\nu} \) as

\[
\hat{H}(t) := \int d^3 x \sqrt{|g_{3}|} \hat{T}_{00}. \quad \text{Here, } g_{3} \text{ is the determinant of the projection of the metric on the spacelike hypersurfaces orthogonal to } \partial_t, \text{ i.e., the hypersurfaces of constant time } t.
\]

The field quantization can be done in a standard fashion and it allows us to find the expression for the field in terms of the Fourier coefficients, later to be promoted to creation and annihilation operators [12].

Given our choice of time, it is also natural to choose a time-dependent decomposition of the field \( \hat{\phi} \) in the Schrödinger picture as

\[
\hat{\phi} = \int d^3 k \left[ \hat{a}_k u_k + \hat{a}_k^\dagger u_k^* \right],
\]

where \( u_k(x^\mu) = (2 \pi)^{-3/2} \omega_k^{-1/2} \exp[i k_\mu x^\mu] \) are plane waves in Minkowski spacetime, we have \( k_\mu x^\mu = -\omega_k t + \mathbf{k} \cdot \mathbf{x} \), the frequency \( \omega_k := \sqrt{|k|^2 + m^2} \), and the annihilation and creation operators \( \hat{a}_k, \hat{a}_k^\dagger \) satisfy the canonical commutation relations \( [\hat{a}_k, \hat{a}_k^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{k}') \), while all other vanish.

We use this field expansion within the definition of the stress energy tensor [A3], and the expansion \( \hat{T}_{\mu\nu} = \hat{T}^{(0)}_{\mu\nu} + \hat{T}^{(1)}_{\mu\nu} \xi \) of the stress energy, to then remove the time dependence and find that

\[
\hat{H} := \int d^3 x \sqrt{|g_{3}|} : \hat{T}_{00} := \hat{H}^{(0)}(t) + \hat{H}^{(1)}(t) \xi,
\]

where we have defined the expressions

\[
\hat{H}^{(0)}(t) := \int d^3 k \omega_k \hat{a}_k^\dagger \hat{a}_k
\]

\[
\hat{H}^{(1)}(t) := \int d^3 k d^3 k' A_{kk'}(t) \hat{a}_k^\dagger \hat{a}_{k'} + \int d^3 k d^3 k' B_{kk'}(t) \hat{a}_k \hat{a}_{k'} + \text{h.c.},
\]

with

\[
A_{kk'}(t) = \int d^3 x \sqrt{|g_{3}|} \left[ \partial_\mu \hat{\phi} \hat{\phi} \partial_\nu \hat{\phi} - \frac{1}{2} g_{\mu\nu} \left[ \partial^\rho \hat{\phi} \partial_\rho \hat{\phi} + m^2 \hat{\phi}^2 \right] \right]_{kk'}
\]

\[
B_{kk'}(t) = \int d^3 x \sqrt{|g_{3}|} \left[ \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \frac{1}{2} g_{\mu\nu} \left[ \partial^\rho \hat{\phi} \partial_\rho \hat{\phi} + m^2 \hat{\phi}^2 \right] \right]_{kk'}
\]
with the general form of the coefficients $A_{kk'}(t)$ and $B_{kk'}(t)$ that is computed below.

The stress energy tensor $\hat{T}_{\mu \nu}$ has the perturbative contributions

$$T_{\mu \nu}^{(0)}(t) := \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \eta_{\mu \nu} \left[ \partial \phi \partial_\nu \phi + m^2 \phi^2 \right]$$

$$T_{\mu \nu}^{(1)}(t) := - \frac{1}{2} \eta_{\mu \nu} \left[ \partial \phi \partial_\nu \phi + m^2 \phi^2 \right]. \tag{A7}$$

We then need the expression for $\sqrt{|g_{33}|} = |g_{33}^{(0)} + g_{33}^{(1)} \xi_{3}^{1/2}$. We note that $g_{33}^{(0)} \equiv \text{diag}(1, 1, 1)$, and that we have the following perturbative identity for the determinant of a matrix $M$ that has a perturbative expansion $M = \det \left( M^{(0)} + M^{(1)} \xi_{3} \right)$, namely $\det(M) = \det \left( M^{(0)} + M^{(1)} \xi_{3} \right) = \det(M^{(0)})^{-1} \left( 1 + \text{Tr}(M^{(0)^{-1}} M^{(1)}) \xi_{3} \right)$. In our case, since $g_{33j} = h_{jk}$, this means that

$$\sqrt{|g_{33}|} = 1 + \frac{1}{2} \text{Tr}(g_{33}) \xi_{3} = 1 + \frac{1}{2} h_{jk} \xi_{3}, \tag{A8}$$

where $j = 1, 2, 3$.

Therefore, the Hamiltonian contributions have the expression

$$\hat{H}^{(0)}(t) := \int d^3 k \omega_k \hat{a}_k^\dagger \hat{a}_k$$

$$\hat{H}^{(1)}(t) := \int d^3 x h_{jk}(t, x) : T_{00}^{(0)}(t) : - \frac{1}{2} \int d^3 x h_{00}(t, x) : \left[ \partial \phi \partial_\nu \phi + m^2 \phi^2 \right], \tag{A9}$$

which can be used to obtain $A_{kk'}(t)$ and $B_{kk'}(t)$ when the field expansion is used. One finally finds

$$A_{kk'}(t) = \frac{\omega_k \omega_{k'} + k \cdot k' + m^2}{2 (2 \pi)^3 \sqrt{\omega_k \omega_{k'}}} \int d^3 x h_{jk}(t, x) e^{i (k u_{-} s') x^\mu} - \frac{\omega_k \omega_{k'} - k \cdot k' - m^2}{2 (2 \pi)^3 \sqrt{\omega_k \omega_{k'}}} \int d^3 x h_{00}(t, x) e^{i (k u_{-} s') x^\mu}$$

$$B_{kk'}(t) = - \frac{\omega_k \omega_{k'} + k \cdot k' - m^2}{4 (2 \pi)^3 \sqrt{\omega_k \omega_{k'}}} \int d^3 x h_{jk}(t, x) e^{i (k u_{+} s') x^\mu} + \frac{\omega_k \omega_{k'} - k \cdot k' + m^2}{4 (2 \pi)^3 \sqrt{\omega_k \omega_{k'}}} \int d^3 x h_{00}(t, x) e^{i (k u_{+} s') x^\mu}. \tag{A10}$$

It is very important to note here that the dimensions of the quantities, once integrated over momenta as in (A10), correctly provide us with a quantity (i.e., $\hat{H}^{(1)}(t)$) that has dimension of mass.

### 2. Self gravity-induced time evolution of macroscopically distinct systems: initial state

Our main aim is to compute the time evolution of the family of initial state

$$\rho(\alpha, \beta) = \hat{\rho}_S(\alpha, \beta) \otimes \prod_{\Delta} \langle \Delta | 0_{\Delta} \rangle \langle 0_{\Delta} |, \tag{A11}$$

where we have defined

$$\hat{\rho}_S(\alpha, \beta) = \alpha |0_{1R} \rangle \langle 0_{1R}| + (1 - \alpha) |1_{L} \rangle \langle 1_{L}| + \beta |1_{L} \rangle \langle 1_{L}| + \beta^* |0_{1R} \rangle \langle 1_{L}|$$

$$\hat{a}_{R} := \int d^3 k F_{\alpha}(k) e^{- i L \cdot k} \hat{a}_k$$

$$\hat{a}_{L} := \int d^3 k F_{\alpha}(k) e^{i L \cdot k} \hat{a}_k$$

$$|0_{1R} \rangle := \int d^3 k F_{\alpha}(k) e^{- i L \cdot k} \hat{a}_k^\dagger |0 \rangle$$

$$|1_{L} \rangle := \int d^3 k F_{\alpha}(k) e^{i L \cdot k} \hat{a}_k^\dagger |0 \rangle$$

$$|0_{\Delta} \rangle := \int d^3 k F_{\alpha}(k) \hat{a}_k^\dagger |0 \rangle. \tag{A13}$$
Here, we have that \( \hat{a}^\dagger_R \) and \( \hat{a}^\dagger_L \) create a localized particle on the ‘right’ \( R \) and ‘left’ \( L \) of the origin of the coordinates respectively, and we assume that the particles occupy a finite volume defined by the functions \( F_{k_0}(k) \), which are normalized in the sense that \( \int d^3k |F_{k_0}(k)|^2 = 1 \). Also, \( k_0 \) is the peak momentum in the momentum distribution, \( L \) defines the location of the peak of the distribution in position space and we assume that \( F_{k_0}(k) \) have width \( \sigma \).

Notice that the Fourier transform \( \tilde{F}(x = L) \propto \int d^3k e^{-i\mathbf{k} \cdot \mathbf{x}} F_{k_0}(k) \) is a peaked function in position space of width \( l_\rho := 1/\sigma \) that we can interpret as the “shape” of the excitation in position space.

The functions \( F_{\lambda}(k) \) are parametrized by the set of degrees of freedom \( \lambda \) and they represent the modes that are orthogonal to the modes \( F_{k_0}(k) e^{\pm iL \cdot k} \). This means that \( \int d^3k F_{k_0}(k) e^{\pm iL \cdot k} F_{\lambda}(k) = 0 \) and we also assume that \( \int d^3k F_{\lambda}(k) F_{\lambda}^*(k) = \delta_{\lambda\lambda} \). For the moment we are uninterested in the exact shape of the functions \( F_{\lambda}(k) \), nor in the nature of the set of parameters \( \lambda \).

Therefore, a particle state in this notation reads \( |m_1, m_2, \{ n_\lambda \} \rangle \), which means that there are \( m_1 \) excitations on the ‘right’ \( R \), \( m_2 \) excitations on the ‘left’ \( L \), and \( n_\lambda \) excitations for each other mode labeled by \( \lambda \) which we consider as an ‘Environment’. This is the meaning of the notation of the product over \( \lambda \) in the set of parameters \( E \).

We also note that \( 0 \leq \alpha \leq 1 \), that \( -1/2 \leq |\beta| \leq 1/2 \) and \( (\alpha - 1/2)^2 + |\beta|^2 \leq 1/4 \) in order for \( \rho(\alpha, \beta) \) to represent a physical state. Notice that for \( \alpha = 1/2 \) and \( \beta = 0 \) one has a maximally mixed state while for \( \alpha = |\beta| = 1/2 \) one has a maximally coherent state.

Altogether, this implies that the state \( \rho_S(\alpha, \beta) \) is constructed starting from the reduced state \( \rho_S(\alpha, \beta) \) of the system \( S \) of the two excitations ‘left’ and ‘right’, tensor product the vacuum for all other modes of excitation labeled by \( \lambda \).

### 3. Self gravity-induced time evolution of macroscopically distinct systems: time evolution

The time evolution of quantum fields in curved spacetime is not uniquely defined for arbitrary spacetimes \[12\]. The main problem is to be able to define a unique notion of time with respect to which to define the Hamiltonian. Such problem does not arise here, since we are perturbing flat Minkowski spacetime parametrized by coordinates \((t, x, y, z)\). This allows us to find a well defined notion, that is, to employ the global timelike vector field \( \partial_t \) present in Minkowski spacetime and introduce the Hamiltonian \( \hat{H}(t) \) with respect to this time choice.

The time evolution operator \( \hat{U}(t) \) is defined by

\[
\hat{U}(t) = e^{\hat{T} \int_0^t dt' \hat{H}(t')}.
\]

The Hamiltonian and the time evolution have perturbative expansion

\[
\hat{H}(t) = \hat{H}^{(0)} + \xi \hat{H}^{(1)}(t)
\]

\[
\hat{U}(t) = \hat{U}^{(0)}(t) \left( 1 - i \xi \int_0^t dt' \hat{U}^{(0\dagger)}(t') \hat{H}^{(1)}(t') \hat{U}^{(0)}(t') \right)
\]

(A15)

to first order respectively. Here we have introduced \( \hat{U}^{(0)}(t) := \exp[-i \hat{H}^{(0)} t] \). In this work we define

\[
\hat{H}_1(t) := \int_0^t dt' \hat{U}^{(0\dagger)}(t') \hat{H}^{(1)}(t') \hat{U}^{(0)}(t')
\]

(A16)

for notational convenience.

We have already argued that the Hamiltonian can be computed given the stress energy tensor, see the previous subsection.

### 4. Massive static particles: approximation and timescales

We continue by introducing here the ultramassive and static regime for the states \[A12\]. We assume that the particle excitations are ultramassive, i.e., \( m/\sigma \gg 1 \), and we assume that they are also static, that is, \( k_0 \equiv 0 \). It is therefore not difficult to see that, in this regime, we have \( E_0 = m \) and therefore

\[
\hat{U}^{(0)}(t)|pn\{0_\lambda\}\rangle \approx e^{-i(p+n)\omega t}|pn\{0_\lambda\}\rangle
\]

(A17)

for a time \( t \) such that also \( t/t_0 \ll 1 \). Here we have introduced the important frequency \( \omega = m \) that, restoring units, reads \( \omega = m c^2/\hbar \), and \( t_0 \) is a timescale that will be defined below. The frequency \( \omega \) is nothing but the
Given the approximate frequency \( A18 \), it is easy to show that this requirement is fulfilled as long as \( |\text{current approximation}| \), this means that we can normalize \( \hat{g} \) to govern the effects, we start by recalling that we noted before that the evolution operator is dimensionless. To extract the dimensionless ratio of the meaningful physical parameters that at the correction \( \hat{t}/t \) upheld for \( \omega_k \), \( \omega_k \leq m (1 + 1/2 \sigma^2/m^2) \) in this regime. Here, we have that \( |k| \) is peaked around 0 with width \( \sigma \), and we expect that beyond \( \sim 3\sigma - 4\sigma \) the contributions to the frequency distribution become negligible due to the peaked nature of the function \( F_{k_{\text{in}}}(k) \).

Most importantly, in this regime we have seen that \( \hat{U} (0) (t) |1_L, 0 \rangle \approx \exp[-i m t] |1_L, 0 \rangle \) and \( \hat{U} (0) (t) |01_R \rangle \approx \exp[-i m t] |01_R \rangle \), which therefore implies

\[
\hat{U} (0) (t) \rho (\alpha, \beta) \hat{U} (0) ^\dagger (t) \approx \rho (\alpha, \beta).
\]

Finally, the timescale at which this approximation remains valid is that for which we have \( \hat{U} (0) (t) \hat{a}_K \hat{U} (0) (t) \approx \hat{a}_K \), where \( K = R, L, \Lambda \). Given the approximate frequency \( A18 \), it is easy to show that this requirement is fulfilled as long as \( |\omega_k - \omega| t \ll 1 \), which means

\[
\frac{\sigma^2}{m} t \ll 1.
\]

Therefore, we can define the timescale

\[
t_0 := \frac{\sigma^2}{\lambda_C} t,
\]

or \( t_0 = \frac{\sigma^2}{\lambda_C c} \) when units are restored, at which the effects start vanishing, and the validity of our results therefore is upheld for \( t/t_0 \ll 1 \).

The timescale \( t_0 \) is not the only timescale that occurs in our system in this approximation. We also need to look at the time correction \( \hat{H}_1 (t) \) that has the expression \( A16 \) that appears in the perturbative expression for the time evolution operator is dimensionless. To extract the dimensionless ratio of the meaningful physical parameters that govern the effects, we start by recalling that we noted before that \( \hat{H} (1) (t) \) has dimension of mass (energy). In the current approximation, this means that we can normalize \( \hat{H} (1) (t) \) by the mass \( m \) and we have that the correction \( \xi \hat{H}_1 (t) \) in \( A16 \) reads

\[
\xi \hat{H}_1 (t) = \xi \int_0^t dt' \hat{U} (0) (t') \hat{H} (1) (t') \hat{U} (0) (t')
\]

\[
= m \xi \int_0^t dt' \hat{U} (0) (t') \frac{1}{m} \hat{H} (1) (t') \hat{U} (0) (t')
\]

\[
= m \xi t \frac{1}{t} \int_0^t dt' \hat{U} (0) (t') \frac{1}{m} \hat{H} (1) (t') \hat{U} (0) (t')
\]

\[
= m \xi t \hat{H} (t),
\]

where \( \hat{H} (t) := \int_0^1 dx' \hat{U} (0) (t x') \frac{1}{m} \hat{H} (1) (t x') \hat{U} (0) (t x') \) is a dimensionless quantity.

Clearly, this means that, if the effects increase with time, our predictions remain valid as long as the correction \( A22 \) remains small. We anticipate that we can expect \( |\hat{H} (t)| \) not to grow in time but to oscillate instead. This
implies that, unless $|\mathcal{H}(t)| \to 0$ for $t \to \infty$, we must require that at least $m \xi t = t/t_0' \ll 1$ where we have introduced the second timescale of the system

$$t_0' := \frac{l_0}{G_N m^2}, \quad (A23)$$

or $t_0' := \frac{\hbar L}{G_N m}$ when units are restored. Clearly, when $t/t_0' \sim 1$ we see that the validity of the perturbative calculations breaks down. To obtain this we have used the explicit expression of $\xi$ in terms of the physical parameters. We will show later on that, indeed, $\mathcal{H}(t)$ does not vanish for large times, does not grow polynomially with time but is expected to oscillate between finite values.

It is reassuring to note that the timescale $t_0'$ is the same as the one computed in the literature for a spherical object of mass $m$ and radius $l_0$, that is forced into a superposition of different locations and that is expected to lose coherence due to gravity \cite{2,3}. This corroborates further the claims of our work. However, it is also worth noting that we have found an additional timescale.

**Appendix B: Time evolution due to self gravity**

In this section we present all the computations necessary to obtain our main result.

1. **Time evolution due to self gravity: preliminary equation**

   We now compute the time evolution of the state.
   We define the time evolved state $\rho(\alpha, \beta)(t)$ as $\rho(\alpha, \beta)(t) := \hat{U}(t) \rho(\alpha, \beta) \hat{U}^\dagger(t)$, which reads
   \[ \rho(\alpha, \beta)(t) = \hat{U}(t) \rho(\alpha, \beta) \hat{U}^\dagger(t) \]
   \[ = \hat{U}^{(0)}(t) \left( \mathbb{1} - i \xi \hat{H}_1(t) \right) \rho(\alpha, \beta) \left( \mathbb{1} + i \xi \hat{H}_1(t) \right) \hat{U}^{(0)}\dagger(t) \]
   \[ = \hat{U}^{(0)}(t) \rho(\alpha, \beta) \hat{U}^{(0)}\dagger(t) + i \xi \hat{U}^{(0)}(t) \left[ \rho(\alpha, \beta), \hat{H}_1(t) \right] \hat{U}^{(0)}\dagger(t). \quad (B1) \]

   The reduced state $\hat{\rho}_S(\alpha, \beta)(t)$ of the modes of the system $S$ can be computed as
   \[ \hat{\rho}_S(\alpha, \beta)(t) := \text{Tr}_E[\rho(\alpha, \beta)(t)] \]
   \[ = \prod_\lambda \sum_{n_\lambda} \langle \{n_\lambda\} | \rho(\alpha, \beta)(t) | \{n_\lambda\} \rangle, \quad (B2) \]
   where $|\{n_\lambda\}\rangle$ is a short notation for $|\{n_\lambda\}\rangle = |n_\lambda n_\lambda \ldots \rangle$.

   Therefore, inserting the definition (B1) of the evolved state into the definition of the reduced state (B2) we find
   \[ \hat{\rho}_S(\alpha, \beta)(t) = \prod_\lambda \sum_{n_\lambda} \langle \{n_\lambda\} | \hat{U}(t) \rho(\alpha, \beta) \hat{U}^\dagger(t) | \{n_\lambda\} \rangle \]
   \[ = \prod_\lambda \sum_{n_\lambda} \langle \{n_\lambda\} | \hat{U}^{(0)}(t) \rho(\alpha, \beta) \hat{U}^{(0)}\dagger(t) + i \xi \hat{U}^{(0)}(t) \left[ \rho(\alpha, \beta), \hat{H}_1(t) \right] \hat{U}^{(0)}\dagger(t) | \{n_\lambda\} \rangle \]
   \[ = \prod_\lambda \sum_{n_\lambda} \langle \{n_\lambda\} | \hat{U}^{(0)}(t) \rho(\alpha, \beta) \hat{U}^{(0)}\dagger(t) | \{n_\lambda\} \rangle + i \xi \prod_\lambda \sum_{n_\lambda} \langle \{n_\lambda\} | \hat{U}^{(0)}(t) \left[ \rho(\alpha, \beta), \hat{H}_1(t) \right] \hat{U}^{(0)}\dagger(t) | \{n_\lambda\} \rangle \]
   \[ = \prod_\lambda \sum_{n_\lambda} \langle \{n_\lambda\} | \hat{U}^{(0)}(t) \rho(\alpha, \beta) \hat{U}^{(0)}\dagger(t) | \{n_\lambda\} \rangle \]
   \[ + i \xi \prod_\lambda \sum_{n_\lambda} \langle \{n_\lambda\} | \left[ \hat{U}^{(0)}(t) \rho(\alpha, \beta) \hat{U}^{(0)}\dagger(t), \hat{U}^{(0)}(t) \hat{H}_1(t) \hat{U}^{(0)}\dagger(t) \right] | \{n_\lambda\} \rangle. \quad (B3) \]
At this point, we use the massive static approximation in the calculation, which allows us to continue to have

$$\hat{\rho}_S(\alpha, \beta)(t) \approx \prod_{\Delta} \sum_{n_\Delta} \langle \{n_\Delta\} | \hat{\rho}_S(\alpha, \beta) \otimes \prod_{\Delta} |0_{\Delta}\rangle \langle 0_{\Delta}| \{n_\Delta\} \rangle + i \xi \prod_{\Delta} \sum_{n_\Delta} \langle \{n_\Delta\} | \left[ \rho(\alpha, \beta), \hat{U}^{(0)}(t) \hat{H}_1(t) \hat{U}^{(0)\dagger}(t) \right] \{n_\Delta\} \rangle$$

$$\approx \hat{\rho}_S(\alpha, \beta) + i \xi \prod_{\Delta} \sum_{n_\Delta} \langle \{n_\Delta\} | \left[ \rho(\alpha, \beta), \hat{U}^{(0)}(t) \hat{H}_1(t) \hat{U}^{(0)\dagger}(t) \right] \{n_\Delta\} \rangle$$

$$\approx \hat{\rho}_S(\alpha, \beta) + i \xi \langle \{0_{\Delta}\} | \left[ \rho(\alpha, \beta), \hat{U}^{(0)}(t) \hat{H}_1(t) \hat{U}^{(0)\dagger}(t) \right] \{0_{\Delta}\} \rangle$$

where, to obtain the last line, we have used the fact that $\langle \{n_\Delta\} | \rho(\alpha, \beta) = \langle \{0_{\Delta}\} | \rho(\alpha, \beta)$ – see the definition of the total initial state.

Therefore, the main formula for the time evolution of the reduced state in the massive static regime is

$$\hat{\rho}_S(\alpha, \beta)(t) \approx \hat{\rho}_S(\alpha, \beta) + i \xi \langle \{0_{\Delta}\} | \left[ \rho(\alpha, \beta), \hat{U}^{(0)}(t) \hat{H}_1(t) \hat{U}^{(0)\dagger}(t) \right] \{0_{\Delta}\} \rangle.$$  

(B5)

A crucial question to ask is: why does it not occur that $\hat{H}_1(t) \hat{U}^{(0)\dagger}(t) \{0_{\Delta}\} = 0$? This could be expected, since we know that the sharp frequency operators $\hat{a}_k$ annihilate the Minkowski vacuum, that is, $\hat{a}_k |0\rangle = 0$. In addition, since we have constructed the modes $|m_1\{0\}_0\rangle$, $|m_2\{0\}_0\rangle$ and $|00\{\lambda\}_0\rangle$ from operators that are linear combinations of creation operators $\hat{a}_k^\dagger$ only, the new vacuum $|0'\rangle$ is the same as the original vacuum, i.e., $|0'\rangle = |0\rangle$. The point to make here is that we are left with the state $|\{0_{\Delta}\}\rangle$, and not the full vacuum state $|0'\rangle$. Therefore, the interaction Hamiltonian needs to be first decomposed as an operator acting on the whole Fock space, which can then be projected on the vector $|\{0_{\Delta}\}\rangle$. Computations of this sort are common in quantum filed theory in curved spacetime. For example, the occur in the procedure to obtain the Unruh effect [19].

It is important to note now that the resolution of the identity in this notation reads

$$1 := \prod_{\Delta} \sum_{m_1, m_2} \sum_{n_\Delta} |m_1 m_2 \{n_\Delta\}\rangle \langle m_1 m_2 \{n_\Delta\}|.$$  

(B6)

Then, we note that

$$\langle \{0_{\Delta}\} | \left[ \rho(\alpha, \beta), \hat{U}^{(0)}(t) \hat{H}_1(t) \hat{U}^{(0)\dagger}(t) \right] \{0_{\Delta}\} \rangle = 2 i \Im \Gamma(\alpha, \beta, t),$$  

(B7)

where we have introduced $\Gamma(\alpha, \beta, t) := \langle \{0_{\Delta}\} | \rho(\alpha, \beta) \hat{H}_1(t) \{0_{\Delta}\} \rangle$ and $\hat{H}_1(t) := \hat{U}^{(0)}(t) \hat{H}_1(t) \hat{U}^{(0)\dagger}(t)$ for notational convenience. Therefore, we can write

$$\Gamma(\alpha, \beta, t) := \langle \{0_{\Delta}\} | \rho(\alpha, \beta) \hat{H}_1(t) 1 \{0_{\Delta}\} \rangle$$

$$= \langle \{0_{\Delta}\} | \rho(\alpha, \beta) \hat{H}_1(t) \prod_{\Delta} \sum_{m_1, m_2} \sum_{n_\Delta} |m_1 m_2 \{n_\Delta\}\rangle \langle m_1 m_2 \{n_\Delta\}| \{0_{\Delta}\} \rangle$$

$$= \sum_{m_1, m_2} \left[ \alpha \langle 01 | \hat{H}_1(t) |m_1 m_2 \{0_{\Delta}\} \rangle |01\rangle \langle m_1 m_2| + (1 - \alpha) \langle 10 | \hat{H}_1(t) |m_1 m_2 \{0_{\Delta}\} \rangle |10\rangle \langle m_1 m_2| \right]$$

$$+ \beta \langle 01 | \hat{H}_1(t) |m_1 m_2 \{0_{\Delta}\} \rangle |10\rangle \langle m_1 m_2| + \beta^* \langle 10 | \hat{H}_1(t) |m_1 m_2 \{0_{\Delta}\} \rangle |01\rangle \langle m_1 m_2| \right].$$  

(B8)

We can employ the given expression [B8] with [B7] in the main expression [B5] for the reduced state $\hat{\rho}_S(\alpha, \beta)(t)$ to obtain, after some algebra

$$\hat{\rho}_S(\alpha, \beta)(t) \approx \hat{\rho}_S(\alpha, \beta) + i \xi \sum_{m_1, m_2} \left[ \alpha K_{01 m_1 m_2}(t) |01\rangle \langle m_1 m_2| + (1 - \alpha) K_{10 m_1 m_2}(t) |10\rangle \langle m_1 m_2| \right]$$

$$+ \beta K_{01 m_1 m_2}(t) |10\rangle \langle m_1 m_2| + \beta^* K_{10 m_1 m_2}(t) |01\rangle \langle m_1 m_2| \right] + \text{h.c.}.$$  

(B9)
where we have defined

\[
K_{mmn'}(t) := \langle mn\{0\_\}\vert \hat{H}_1(t)\vert m'n'\{0\_\}\rangle
\]

\[
= \langle mn\{0\_\}\vert \hat{U}^{(0)}(t) \hat{H}_1(t) \hat{U}^{(0)\dagger}(t)\vert m'n'\{0\_\}\rangle
\]

\[
= \langle mn\{0\_\}\vert \hat{U}^{(0)}(t) \left( \int_0^t dt' \hat{U}^{(0)\dagger}(t') \hat{H}_1(t') \hat{U}^{(0)}(t') \right) \hat{U}^{(0)\dagger}(t)\vert m'n'\{0\_\}\rangle
\]

\[
\approx \int_0^t dt' e^{i (m+n'-m-n) \omega(t-t')} \langle mn\{0\_\}\vert \hat{H}_1(t')\vert m'n'\{0\_\}\rangle.
\]  

(B10)

for convenience of presentation. This formula is the preliminary to our main result. In the following we proceed to simplify it using explicit expressions for \(K_{mmn'}(t)\).

2. Time evolution due to self gravity: explicit dependence on self gravity

We can now employ our main result in conjunction with the explicit expression of \(\hat{H}_1(t)\) in terms of the self gravitational field contributions \(h_{\mu\nu}\).

We start by recalling that \(\hat{H}_1(t)\) has the generic expression

\[
\hat{H}_1(t) = \int d^3k d^3k' A_{kk'}(t) \hat{a}_k \hat{a}_{k'} + \int d^3k d^3k' B_{kk'}(t) \hat{a}_k \hat{a}_{k'} + h.c.,
\]  

(B11)

which, in the massive static regime, implies that

\[
K_{mmn'}(t) \approx \int_0^t dt' e^{i (m+n'-m-n) \omega(t-t')} \langle mn\{0\_\}\vert \hat{H}_1(t')\vert m'n'\{0\_\}\rangle
\]

\[
= \sqrt{m + 1} \delta_{mm'} \delta_{nn'} \delta_{n-1} K^{(A,+)}(t) + m \delta_{m-1m'} \delta_{nn'} K^{(A,-)}(t)
\]

\[
+ n \delta_{mm'} \delta_{n-1} K^{(A,+)}(t) + \sqrt{n + 1} \sqrt{m + 1} \delta_{nn'} K^{(A,+)}(t)
\]

\[
+ \sqrt{m + 1} \sqrt{n + 1} \delta_{mm'} \delta_{nn'} K^{(A,+)}(t) + 2 \sqrt{n + 1} \sqrt{m + 1} \delta_{nn'} K^{(B,-)}(t)
\]

\[
+ \sqrt{m + 1} \sqrt{n + 1} \delta_{mm'} \delta_{nn'} K^{(B,+)}(t).
\]  

(B12)

Here, we have introduced

\[
K^{(A,\pm)}(t) := \int_0^t dt' e^{iL(k' \pm k)} F_0^*(k) F_0(k') A_{kk'}(t')
\]

\[
K^{(B,\pm)}(t) := \int_0^t dt' e^{\pm 2i \omega(t-t')} \int d^3k d^3k' F_0(k) F_0(k') B_{kk'}(t') e^{iL(k' \pm k)}
\]

(B13)

Importantly, notice that \(K^{(A,-)}(t) = K^{(A,+)}(t)\) since \(A_{kk'}(t) = A_{k'k}(t)\). Therefore, \(K^{(A,-)}(t) \in \mathbb{R}\) for all \(t\).
3. Time evolution due to self gravity: main equation

We now proceed by combining the expression (B12) for the coefficients \( K_{mnm' n'}(t) \) with the preliminary expression for the reduced state \([B9]\). We have

\[
\hat{\rho}_S(\alpha, \beta)(t) \approx \hat{\rho}_S(\alpha, \beta)
\]

\[
+ 2 \xi \Im(\beta K(A, +)(t)) |01\rangle\langle 01| - 2 \xi \Im(\beta K(A, +)(t)) |10\rangle\langle 10|
\]

\[
- (1 - 2 \alpha) i \xi K(A, +)(t) |01\rangle\langle 10| + (1 - 2 \alpha) i \xi K(A, +)(t) |10\rangle\langle 01|
\]

\[
+ \sqrt{2} i \xi \left[ \alpha K_+^{(B, +)}(t) + 2 \beta^* K_+^{(B, -)}(t) \right] |01\rangle\langle 21| + \text{h.c.}
\]

\[
+ \sqrt{2} i \xi \left[ 2 \alpha K_+^{(B, -)}(t) + \beta^* K_+^{(B, +)}(t) \right] |01\rangle\langle 12| + \text{h.c.}
\]

\[
+ \sqrt{2} i \xi \left[ (1 - \alpha) K_+^{(B, +)}(t) + 2 \beta K_+^{(B, -)}(t) \right] |10\rangle\langle 12| + \text{h.c.}
\]

\[
+ \sqrt{6} i \xi \alpha K_+^{(B, +)}(t) |01\rangle\langle 03| + \text{h.c.}
\]

\[
+ \sqrt{6} i \xi \beta^* K_+^{(B, +)}(t) |01\rangle\langle 30| + \text{h.c.}
\]

\[
+ \sqrt{6} i \xi \beta K_+^{(B, +)}(t) |10\rangle\langle 03| + \text{h.c.}
\]

\[
+ \sqrt{6} i \xi (1 - \alpha) K_+^{(B, +)}(t) |10\rangle\langle 30| + \text{h.c.,}
\]

which is our main result.

4. Time evolution due to self gravity: effective time evolution

We now take our main expression \([B14]\) and note that it can be re-written as

\[
\hat{\rho}_S(\alpha, \beta)(t) \approx \hat{U}_{Sq}(t) \hat{U}_{BS}(t) \hat{U}_0(t) \hat{\rho}_S(\alpha, \beta) \hat{U}_0^\dagger(t) \hat{U}_{BS}(t) \hat{U}_{Sq}(t),
\]

where we have defined

\[
\hat{U}_0(t) \approx \exp \left[ -i \omega \left( \hat{a}_R^\dagger \hat{a}_R + \hat{a}_L^\dagger \hat{a}_L \right) t \right]
\]

\[
\hat{U}_{BS}(t) \approx \exp \left[ -i \xi \left( K(A, +)(t) \hat{a}_R^\dagger \hat{a}_L + K(A, +)^*(t) \hat{a}_L^\dagger \hat{a}_R \right) \right]
\]

\[
\hat{U}_{Sq}(t) \approx \exp \left[ -i \xi \left( K(B, +)(t) \hat{a}_L^2 + K(B, +)^*(t) \hat{a}_R^2 \right) \right] \exp \left[ -i \xi \left( K(B, +)^*(t) \hat{a}_L^2 + K(B, +)(t) \hat{a}_R^2 \right) \right] \times \exp \left[ -i \xi \left( K(B, -)^*(t) \hat{a}_R \hat{a}_L + K(B, -)(t) \hat{a}_R \hat{a}_L \right) \right].
\]

Recall that the ‘Compton frequency’ \( \omega \) is just \( \omega \equiv m \) in natural units \( (\omega = mc^2/\hbar \text{ with restored physical units}) \).

5. Time evolution due to self gravity: reduced states

Next, we can look at the reduced state of the ‘right’ and ‘left’ systems R and L, which we call \( \hat{\rho}_L(\alpha, \beta)(t) \) and \( \hat{\rho}_R(\alpha, \beta)(t) \) respectively. These states are simply defined as

\[
\hat{\rho}_L(\alpha, \beta)(t) := \text{Tr}_R[\hat{\rho}_S(\alpha, \beta)(t)]
\]

\[
\hat{\rho}_R(\alpha, \beta)(t) := \text{Tr}_L[\hat{\rho}_S(\alpha, \beta)(t)].
\]
It is immediate to show that we have
\[ \hat{\rho}_L(\alpha, \beta)(t) \approx \left( \alpha + 2 \xi \Im(\beta K^{(A,+)}(t)) \right) |0\rangle\langle 0| + \left( (1 - \alpha) - 2 \xi \Im(\beta K^{(A,+)}(t)) \right) |1\rangle\langle 1| + \sqrt{2} i \xi \left[ \alpha K^{(B,+)}_+(t) + 2 \beta^* K^{(B,-)}(t) \right] |0\rangle\langle 2| + \sqrt{6} i \xi (1 - \alpha) K^{(B,+)}_+(t) |1\rangle\langle 3| + \text{h.c.} \]
\[ \hat{\rho}_R(\alpha, \beta)(t) \approx \left( (1 - \alpha) - 2 \xi \Im(\beta K^{(A,+)}(t)) \right) |0\rangle\langle 0| + \left( \alpha + 2 \xi \Im(\beta K^{(A,+)}(t)) \right) |1\rangle\langle 1| + \sqrt{2} i \xi \left[ (1 - \alpha) K^{(B,+)}_+(t) + 2 \beta K^{(B,-)}(t) \right] |0\rangle\langle 2| + \sqrt{6} i \xi \alpha K^{(B,+)}_+(t) |1\rangle\langle 3| + \text{h.c.} \]

(B18)

The evolution of these states can be re-cast in the form
\[ \frac{d}{dt} \hat{\rho}_L(\alpha, \beta)(t) \approx -i \left[ \hat{H}_L^{(1)}(t), \hat{\rho}_L(\alpha, \beta) \right] + \hat{\mathcal{L}}_L \hat{\rho}_L(\alpha, \beta) \]
\[ \frac{d}{dt} \hat{\rho}_R(\alpha, \beta)(t) \approx -i \left[ \hat{H}_R^{(1)}(t), \hat{\rho}_R(\alpha, \beta) \right] + \hat{\mathcal{L}}_R \hat{\rho}_R(\alpha, \beta), \]
where we have introduced the first order Hamiltonians \( \hat{H}_L^{(1)}(t) \) and \( \hat{H}_R^{(1)}(t) \) through
\[ \hat{H}_L^{(1)}(t) \xi := i \left( \frac{d}{dt} \hat{U}_L(t) \right) \hat{U}_L^\dagger(t) \]
\[ \hat{H}_R^{(1)}(t) \xi := i \left( \frac{d}{dt} \hat{U}_R(t) \right) \hat{U}_R^\dagger(t), \]
(B20)

and the unitary operators \( \hat{U}_L(t) := \hat{U}_{NL,L} \hat{U}_{Sq,L} \hat{U}_{0,L}(t) \) and \( \hat{U}_R(t) := \hat{U}_{NL,R} \hat{U}_{Sq,R} \hat{U}_{0,R}(t) \), which require the following expressions
\[ \hat{U}_{0,L}(t) \approx \exp \left[ -i \omega \hat{a}_L^\dagger \hat{a}_L t \right] \]
\[ \hat{U}_{0,R}(t) \approx \exp \left[ -i \omega \hat{a}_R^\dagger \hat{a}_R t \right] \]
\[ \hat{U}_{Sq,L}(t) \approx \exp \left[ -i \xi \left( K^{(B,+)*}_+(t) \hat{a}_L^2 + K^{(B,+)}_+(t) \hat{a}_L^2 \right) \right] \]
\[ \hat{U}_{Sq,R}(t) \approx \exp \left[ -i \xi \left( K^{(B,+)*}_-(t) \hat{a}_R^2 + K^{(B,+)}_-(t) \hat{a}_R^2 \right) \right] \]
\[ \hat{U}_{NL,L}(t) \approx \exp \left[ -2 i \xi \left( \frac{\beta^*}{\alpha} K^{(B,-)*}_-(t) \hat{a}_L^2 \left( \mathbb{1} - \hat{a}_L^\dagger \hat{a}_L \right) + \frac{\beta}{\alpha} K^{(B,-)}(t) \left( \mathbb{1} - \hat{a}_L^\dagger \hat{a}_L \right) \hat{a}_L^2 \right) \right] \]
\[ \hat{U}_{NL,R}(t) \approx \exp \left[ -2 i \xi \left( \frac{\beta^*}{1 - \alpha} K^{(B,-)*}_-(t) \hat{a}_R^2 \left( \mathbb{1} - \hat{a}_R^\dagger \hat{a}_R \right) + \frac{\beta}{1 - \alpha} K^{(B,-)}(t) \left( \mathbb{1} - \hat{a}_R^\dagger \hat{a}_R \right) \hat{a}_R^2 \right) \right] \]
\[ \hat{\mathcal{L}}_L \hat{\rho}_L(\alpha, \beta) := 2 \frac{\Im(\beta K^{(A,+)}(t))}{1 - \alpha} \xi \left[ \hat{a}_L \hat{\rho}_L(\alpha, \beta) \hat{a}_L^\dagger - \frac{1}{2} \hat{\rho}_L(\alpha, \beta) \hat{a}_L^\dagger \hat{a}_L - \frac{1}{2} \hat{a}_L^\dagger \hat{a}_L \hat{\rho}_L(\alpha, \beta) \right] \]
\[ \hat{\mathcal{L}}_R \hat{\rho}_R(\alpha, \beta) := 2 \frac{\Im(\beta K^{(A,+)}(t))}{\alpha} \xi \left[ \hat{a}_R \hat{\rho}_R(\alpha, \beta) \hat{a}_R^\dagger - \frac{1}{2} \hat{\rho}_R(\alpha, \beta) \hat{a}_R^\dagger \hat{a}_R - \frac{1}{2} \hat{a}_R^\dagger \hat{a}_R \hat{\rho}_R(\alpha, \beta) \right]. \]
(B21)

This also allows us to write the Hamiltonians \( \text{[B20]} \) explicitly to lowest order as
\[ \hat{H}_L^{(1)}(t) := \omega \hat{a}_L^\dagger \hat{a}_L + K^{(B,+)*}_+(t) \hat{a}_L^2 + K^{(B,+)}_+(t) \hat{a}_L^2 + \frac{\beta^*}{\alpha} K^{(B,-)*}_-(t) \hat{a}_L^2 \left( \mathbb{1} - \hat{a}_L^\dagger \hat{a}_L \right) + \frac{\beta}{\alpha} K^{(B,-)}(t) \left( \mathbb{1} - \hat{a}_L^\dagger \hat{a}_L \right) \hat{a}_L^2 \]
\[ \hat{H}_R^{(1)}(t) := \omega \hat{a}_R^\dagger \hat{a}_R + K^{(B,+)*}_-(t) \hat{a}_R^2 + K^{(B,+)}_-(t) \hat{a}_R^2 + \frac{\beta^*}{\alpha} K^{(B,-)*}_+(t) \hat{a}_R^2 \left( \mathbb{1} - \hat{a}_R^\dagger \hat{a}_R \right) + \frac{\beta}{\alpha} K^{(B,-)}(t) \left( \mathbb{1} - \hat{a}_R^\dagger \hat{a}_R \right) \hat{a}_R^2. \]
(B22)

The expressions \( \text{[B19]} \) are written in the standard Lindblad form which, as expected, states that the evolution of each subsystem is coherently dependent on the “environment”: in this case, the environment is just the other subsystem. This is a consequence of the fact that the remaining modes of the field, those labelled by \( \mathbf{\lambda} \) which we would consider as the environment proper, are all initially in the vacuum state.

Finally, notice the presence of the nonlinear unitary operations \( \hat{U}_{NL,L}(t) \) and \( \hat{U}_{NL,R}(t) \) which are required to construct the Lindblad equation to this perturbative order.
We can now ask a first question: what are the probabilities \( p_R(t) \) and \( p_L(t) \) of finding the excitation at time \( t \) on the ‘right’ \( R \) or on the ‘left’ \( L \) ? Such probabilities can be easily computed as \( p_R(t) = \text{Tr}(\hat{\rho}(\alpha, \beta)(t)|0_1\rangle\langle 0_1|) \) and \( p_L(t) = \text{Tr}(\hat{\rho}(\alpha, \beta)(t)|1_1\rangle\langle 1_1|) \), and we find
\[
\begin{align*}
p_L(t) &= (1 - \alpha) - 2 \xi \Im \left( \beta K^{(A,+)}(t) \right) \\
p_R(t) &= \alpha + 2 \xi \Im \left( \beta K^{(A,+)}(t) \right). \tag{B23}
\end{align*}
\]

Interestingly, the probability of detecting the excitation on the ‘right’ or ‘left’ is affected by the quantum coherence initially present and the strength of the self gravitational field, and it depends crucially on the parameter \( \beta \). More interesting still, the sum of the probabilities satisfies \( p_R(t) + p_L(t) = 1 \). This means that the mode structure of the field is preserved, and there is no deformation due to the time evolution. In turn, this implies that the Hilbert space partition is also preserved to lowest order.

7. Time evolution due to self gravity: measuring entropy

Given the effective time evolution expression \( [B15] \), and the expressions \( [B18] \) for the reduced states we can make some considerations about the entropy of the system \( S \).

First of all, the total entropy of the state does not change at first order in \( \xi \). We can compute the Von Neumann entropy \( S_N := - \sum_n \lambda_n \ln \lambda_n \) of each reduced state, where \( \lambda_n \) are the eigenvalues of the chosen state. We first use the fact that \( \hat{\rho}_L(\alpha, \beta)(0) = \hat{\rho}_L(\alpha, \beta) \) and \( \hat{\rho}_R(\alpha, \beta)(0) = \hat{\rho}_R(\alpha, \beta) \). Then we use first order perturbation theory to compute the changes in the eigenvalues of the state, therefore computing the change \( \Delta S_N := S_N(t) - S_N(0) \) of Von Neumann entropy for both reduced states. We find
\[
\begin{align*}
\Delta S_N(\hat{\rho}_L(\alpha, \beta)(t)) &= 2 \xi \Im \left( \beta K^{(A,+)}(t) \right) \ln \left( \frac{1 - \alpha}{\alpha} \right) \\
\Delta S_N(\hat{\rho}_R(\alpha, \beta)(t)) &= - 2 \xi \Im \left( \beta K^{(A,+)}(t) \right) \ln \left( \frac{1 - \alpha}{\alpha} \right). \tag{B24}
\end{align*}
\]

This means that the mixedness of the subsystems is affected by the evolution of the state, and therefore in turn by gravity. Furthermore, since the total state is pure, the entropies of both states are affected in such a way that the total entropy is conserved, i.e., \( S_N(\hat{\rho}_L(\alpha, \beta)(t)) + S_N(\hat{\rho}_R(\alpha, \beta)(t)) = S_N(\hat{\rho}_L(\alpha, \beta)(0)) + S_N(\hat{\rho}_R(\alpha, \beta)(0)) \).

8. Light and massless particles

We ask ourselves what occurs when the particle excitations are massless, or very light. In this case, we are looking at the regime where \( m/\sigma = l_\sigma/\lambda_C \ll 1 \).

In the calculations that we performed to compute the time evolution of the full state we would need to compute a quantity of the form
\[
\hat{U}^{(0)}(t) \rho(\alpha, \beta) \hat{U}^{(0)}(t)\{n_\Delta\} = \prod_{\Delta} \sum_{m_1, m_2} \sum_{n_{\Delta}} \hat{U}^{(0)}(t) \rho(\alpha, \beta) \hat{U}^{(0)}(t) |m_1 m_2 \{n_\Delta\}\rangle \langle m_1 m_2 \{n_\Delta\}|, \tag{B25}
\]
which contains expressions that, after some algebra, read
\[
\begin{align*}
\int \frac{d^3k}{(2 \pi)^3} |F_{k_0}(k)|^2 e^{i(k \cdot k')} L e^{i\omega_k t} \\
\int \frac{d^3k}{(2 \pi)^3} F_{k_0}^*(k) F_{\Delta}(k) e^{i(k \cdot k')} L e^{i\omega_k t}. \tag{B26}
\end{align*}
\]
Due to the Riemann-Lebesgue lemma, all terms of this form vanish as \( t \to \infty \). This implies that for any particle that is very light (or massless), the effects described in this work become negligible with time.
Appendix C: Effects on the quantum coherence in the state due to self gravity

In the previous section we have found how the state evolves due to the interaction with its self gravitational field. Our main result is the expression (B14), which is then complemented by the coefficients (A10) for the specific case of a massive and static object.

1. Gauge invariance of the effects

In order to obtain the desired expressions we start by noting that, in the massive static approximation, the expressions (A8) and (A10) together reduce to

$$: \tilde{T}^{(0)}_{00} : \approx \int \frac{d^3k \, d^3k'}{(2 \pi)^3} \, e^{-i \mathbf{k} \cdot \mathbf{x}} \hat{a}_k \hat{a}_{k'} + \mathcal{O}((\sigma/m)^2),$$

(C1)

which in turn, employing the perturbative semiclassical Einstein equation [11], gives us

$$G^{(1)}_{00} \approx \int \frac{d^3k \, d^3k'}{(2 \pi)^3} \, e^{-i \mathbf{k} \cdot \mathbf{x}} \langle \hat{a}_k \hat{a}_{k'} \rangle \hat{\rho}$$
$$G^{(1)}_{0d} \approx \mathcal{O}(\sigma/m)$$
$$G^{(1)}_{df} \approx \mathcal{O}((\sigma/m)^2),$$

(C2)

and $d, f = 1, 2, 3$.

Notice the crucial time independence on the right hand side of (C2), and the fact that we ignore terms of the order $\mathcal{O}(\sigma/m)^n$ for $n \geq 1$ since $\sigma/m \ll 1$. After some algebra we find

$$G^{(1)}_{00} \approx \alpha \bar{F}_0(L-x) \bar{F}_0(-(L-x)) + (1-\alpha) \bar{F}_0(-(L+x)) \bar{F}_0(L+x) + 2 \Re \left( \beta \bar{F}_0(L-x) \bar{F}_0(L+x) \right)$$
$$G^{(1)}_{0d} \approx \mathcal{O}(\sigma/m)$$
$$G^{(1)}_{df} \approx \mathcal{O}((\sigma/m)^2),$$

(C3)

where we have introduced the functions

$$\bar{F}_0(x) := \int \frac{d^3k}{(2 \pi)^{3/2}} F_0(k) \, e^{i \mathbf{k} \cdot \mathbf{x}}.$$  

(C4)

Here we see that, if the functions $F_0(k)$ represent the frequency distribution that characterizes an excitation in momentum space, then the functions $\bar{F}_0(x)$ are directly related to the “probability” of finding the particle at position $x$ in position space.

The linearized first-order Einstein tensor can be computed by computing $G^{(1)}_{00}$ and using the fact that $G^{(1)}_{xx} \approx 0$, and reads

$$G^{(1)}_{00} \approx -\frac{3}{4} \nabla^2 h_{00},$$

(C5)

where $\nabla^2 := \partial_x \partial^x$.

This expression is now used in conjunction with the following properties and observations: (i) the expression (C4) is time independent; (ii) we are interested only in first order effects in $\xi$. Therefore, we do not look at the second-order back-reaction of the affected state on the metric. This implies that the source of the self gravitational effects considered here is and remains the zero order stress energy tensor $: \tilde{T}^{(0)}_{00} :$ as discussed in the literature [11]. Finally, this means that we can expect the metric $h_{\mu \nu}$ itself to be time independent, since the mass-energy distribution $\tilde{T}^{(0)}_{00}$ is time independent and unaffected by the changes in the metric to this order. All together this implies that $\partial_0^2 h_{00} = 0$ and $\int_0^t dt' h_{\mu \nu}(x) = h_{\mu \nu}(x) \, t$.

This can be used to address how the effects that we are looking for change under gauge transformations. If the quantities that are involved are not gauge invariant, then it is unclear to what degree we can use them to make absolute statements. Let us first compute

$$g^{0 \mu} = 4 + \xi h^{0 \mu}$$
$$= 4 - \xi h_{00} + \xi \hat{\xi}.$$  

(C6)
It is known that the Einstein tensor $G_{\mu\nu}$ is gauge invariant [21, 22]. Therefore, since $G_{\mu\nu}$ is gauge invariant then $G_{00}$ is also gauge invariant. Since $G_{00}$ is gauge invariant, also $\nabla^2 h_{00}$ is through (C9). Since $\nabla^2 h_{00}$ is, also $h_{00}$ is. Since $h_{00}$ and $g^{0}_{\mu}$ are, then $h^{c}_{c}$ is, through (C6).

2. Contribution of self gravity

We proceed to look at the key quantity that governs our problem, namely $K^{(A,+)}(t)$. First we need the expressions (A10) for the massive static regime, which read

\[ A_{kk'}(t) \approx \int \frac{d^3x}{(2\pi)^3} h_d^d(x) e^{-i(k-k')\cdot x} + \mathcal{O}((\sigma/m)^2) \]

\[ B_{kk'}(t) \approx \int \frac{d^3x}{(2\pi)^3} h_{00}(x) e^{i(k+k')\cdot x} + \mathcal{O}((\sigma/m)^2). \]  

We then combine this with the definition (B13) of the function $K^{(A,+)}(t)$ to obtain

\[ K^{(A,+)}(t) = \int_{0}^{t} dt' \int \frac{d^3x}{(2\pi)^3} h_d^d(t, x) \tilde{F}_0^*(L - x) \tilde{F}_0(L + x) \]

\[ = t \int \frac{d^3x}{(2\pi)^3} h_d^d(x) \tilde{F}_0^*(L - x) \tilde{F}_0(L + x) \]  

(C8)

As argued before, the functions $F_0(k)$ are peaked around the value $k = 0$, with a width of $2\sigma$. The functions $\tilde{F}_0(L \pm x)$ are peaked around $x = \pm L$, and have a width of $2/\sigma$.

3. Full result

We are now ready to discuss our full result. We will assume that the ‘shape’ functions $\tilde{F}_0(z)$ are symmetric, that is, $\tilde{F}_0(z) = \tilde{F}_0(-z)$. This assumption can be relaxed for more realistic calculations.

There are two scenarios possible at this point.

i) Here $L/l_\sigma \gg 1$. In this case the two possible particle positions are separated much more than their size, which means that for $x = L$ we have $\tilde{F}_0(L + x) = \tilde{F}_0(2L)$ and $\tilde{F}_0(L - x) = \tilde{F}_0(0)$. Since $L/l_\sigma \gg 1$, and $\tilde{F}_0(z)$ is becomes negligible for $|z|/\sigma \gg 1$, this means that $\tilde{F}_0(2L) = 0$. Therefore, $\tilde{F}_0^*(L - x) \tilde{F}_0(L + x) = 0$ because the overlap of the two functions $\tilde{F}_0^*(L \pm x)$ is negligible, and therefore $K^{(A,+)}(t) = 0$. The argument runs exactly the same if $x = -L$.

ii) Here we have $L/l_\sigma \sim 1$, that is, $L/l_\sigma$ is not far from unity (either from below or above). Let us consider $L/l_\sigma \geq 1$ for simplicity. In this case the two possible particle positions have some overlap around $x = 0$. Without specifying the exact shape of the wave function, we assume that $\tilde{F}_0^*(L - x) \tilde{F}_0(L + x)$has a local maximum around $x = 0$. This is not impossible. In fact, considering the simple case of one dimension for illustration purposes, if we used Gaussian packets we could have $\tilde{F}_0(L \pm x) \propto \exp[-(L \pm x)^2 \sigma^2]$. Therefore the function $P(x) := \tilde{F}_0^*(L + x) \tilde{F}_0(L - x)$ would have a maximum at $x = 0$, and the maximum $P(0) = \exp[-2L^2 \sigma^2]$ would be $\exp[-2L^2 \sigma^2]$ times larger than the values $P(\pm L) = \exp[-4L^2 \sigma^2]$ of the function at $x = \pm L$. With different choices of peak functions, such as the rectangle functions considered in the literature [11], one might look at the condition $L/l_\sigma \leq 1$ instead.

The crucial point here is to note that there is a configuration $L/l_\sigma \sim 1$ where we can assume that $G^{(1)}_{00}$ in (C2) has the form

\[ G^{(1)}_{00} \approx 2 \Re \left( \beta \tilde{F}_0^*(L - x) \tilde{F}_0(L + x) \right) \]  

(C9)

since the terms governed by $\alpha$ and $(1 - \alpha)$ are smaller (exponentially smaller in the example of the Gaussian packets above). This approximation is also justified for another reason. We have seen that $G^{(1)}_{00}$ in (C2) has three contributions, which have three “peaks”: two peaks at $x = \pm L$ (i.e., the locations of the particle excitations), and one at $x = 0$. Therefore, even if we cannot compute the form of the metric explicitly, we expect the
three-metric $h_{ij}$ to also have three local maxima at the same locations. Since we are interested in computing $\mathcal{C}_8$, and this requires an integral of the form $\int d^3x \, h^j_j(t, x) \tilde{F}_0(L-x) \tilde{F}_0(L+x)$, in the present regime we have that the expression $\tilde{F}_0(L-x) \tilde{F}_0(L+x)$ in the integral will “select” only the contribution proportional to $\beta$, since the contributions to the metric that peak at $x = \pm L$ are much smaller.

Putting all together, we have that

$$K^{(A,+)}(t) \approx t \Re (\beta \kappa(L))$$
$$K^{(B,-)}(t) \approx 0,$$

where the function $\kappa(L)$ depends on the specific shape of the particle excitation, and the explicit form of the metric. An explicit form is not important here.

This concludes our analysis. Most importantly, we note that $K^{(A,+)}(t)$ grows linearly with time, and that $K^{(A,+)}(t) = 0$ for $\beta = 0$. Finally, $\mathcal{C}_{10}$ applies if $h^d_d(x) = h^d_d(-x)$, which is to be expected if $F_0(k) = F_0(-k)$. We leave it to further work to study the cases where this does not occur.