SEIRDV MODEL FOR QATAR COVID-19 OUTBREAK: A CASE STUDY

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ABSTRACT

The Covid-19 outbreak of 2020 has required many governments to develop mathematical-statistical models of the outbreak for policy and planning purposes. This work provides a tutorial on building a compartmental model using Susceptibles, Exposed, Infected, Recovered, Deaths and Vaccinated (SEIRDV) status through time. A Bayesian Framework is utilized to perform both parameter estimation and predictions. This model uses interventions to quantify the impact of various government attempts to slow the spread of the virus. Predictions are also made to determine when the peak Active Infections will occur.

1 Introduction

Coronavirus Disease 2019 (COVID-19) (Wu et al. (2020); Rezabakhsh, Ala, and Khodaei (2020)) is a severe pandemic affecting the whole world with a fast spreading regime, requiring to perform strict precautions to keep it under control. Despite that people are getting vaccinated almost everyday, the spread of Covid-19 is still concerning. As there is no cure and target treatment yet, establishing those precautions become inevitable. These limitations are social distancing, mask wearing and hand-washing (Giuliani, et al. (2020)).

Corona Virus is a new human Betacoronavirus that uses densely glycosylated spike protein to penetrate host cells. The COVID-19 belongs to the same family classification with Nidovirales, viruses that use a nested set of mRNAs to replicate and it further falls under the subfamily of alpha, beta, gamma and delta Co-Vis. The virus that causes COVID-19 belongs to the Betacoronavirus 2B lineage and has a close relationship with SARS species. It is a novel virus since the monoclonal antibodies do not exhibit a high degree of binding to SARS-CoV-2. Replication of the viral RNA occurs when RNA polymerase binds and reattaches to multiple locations (McIntosh (2020); Fisher and Heyman (2020)).

Cases of COVID-19 started in December 2019 when a strange condition was reported in Wuhan, China. This virus has a global mortality rate of 3.4%, which makes it more severe in relation to flu. The elderly who have other pre-existing illnesses are succumbing more to the COVID-19. People with only mild symptoms recover within 3 to 7 days, while those with conditions such as pneumonia or severe diseases take weeks to recover. The recovery percentage of patients, for example, in China stands at 51%. The recovery percentage rate of COVID-19 is expected to hit 90% (WHO (2020)).

The virus has spread from China to 196 other countries and territories across the globe. From Wuhan, Hubei province, the virus spread to Mainland China, Thailand, Japan, South Korea, Vietnam, Singapore, Italy, Iran, and other countries. The State of Qatar was one of the countries that were affected by the COVID-19 spreading and the first infected case was reported on 29th of February 2020 and it could be considered the 2nd highest in the Arab World with the number of confirmed cases 362, 007 as of April 5, 2022.

Despite the various intervention measures taken by the Qatari governemrnt, we still have quite a large number of infected cases. The question is could it be that some interventions were more helpful than the others, or some weren’t even helpful at all? For example, on the 10th of March 2020, Qatari government announced closure of all schools and universities due to Covid-19 outbreak, and placed a travel ban on 15 countries. Furthermore, on 21st March 2020, the ministry of Municipality and Environment closed all parks and public beaches to try curb the spread of Covid-19. On March 23rd, the ministry of Commerce and Industry decided to temporarily closed all resturants, cafes, food outlets in sport area, Doha Corniche, Al Khor Corniche

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and Aspire park. On June 4th 2020, the cabinet decided to allow four people in a vehicle, but families were exempted. Also, the ministry of commerce and industry (MoCL) announced the permitted working hours for private sector from 7am until 8pm. However, between June 15th and September 2020, restrictions placed by the ministry of commerce and industry were lifted. Therefore, to understand the impact of intervention on the spread of Covid-19, and the effect of vaccination on the number of secondary infections, it is essential to incorporate vaccination in our previously developed SEIRD model. This will enable us to monitor the intervention measures taken by the government of Qatar. Thus, we develop an SEIRDV model and naturally incorporate interventions in the model, focusing on infected, deaths, recovered and vaccinated as those are the only data available.

This document is organized in the following manner. In Section 2, the Susceptible, Exposed, Infected, Recovered, Death and Vaccinated (SEIRDV) model that is employed is defined. Next, Section 3 provides the proof of existence and uniqueness of the model. Section 4 introduces the data available and gives description. Section 5 shows how interventions are incorporated into the model. The Bayesian inference model specification is given in Section 6. Summaries of the parameter estimation is shown in Section 7. Time varying reproduction number details is provided in Section 8. The model validation is demonstrated in Section 9. And finally, a discussion is found in Section 10.

2 The SEIRDV Model

Let $S(t)$ be the number of people Susceptible at time $t$, $E(t)$ be the number of people Exposed at time $t$, $I(t)$ be the total number of Infected at time $t$, $R(t)$ be the cumulative number of recovered at time $t$, $D(t)$ be the cumulative number of Deaths at time $t$ and $V$ be the number of those who are vaccinated at time $t$. This can be modeled with the following system of ordinary differential equations:

$$\begin{align*}
\frac{dS}{dt} &= -\alpha S(t)E(t) - \rho S(t) - \beta S(t) - \rho I(t)S(t) \\
\frac{dE}{dt} &= \alpha S(t)E(t) - \beta E(t) + \alpha(1 - k)V(t)E(t) - \gamma E(t) \\
\frac{dI}{dt} &= \beta E(t) - \gamma I(t) - \eta I(t) + \beta S(t) \\
\frac{dR}{dt} &= \gamma I(t) + \rho E(t) \\
\frac{dD}{dt} &= \eta I(t) \\
\frac{dV}{dt} &= \rho S(t) + \rho I(t)S(t) - \alpha(1 - k)V(t)E(t) + \rho R(t),
\end{align*}$$

with $S(0) = 2782000$, $E(0) = 3$, $I(0) = 1$, $R(0) = 0$, $D(0) = 0$, and $V(0) = 0$.

Where $\alpha$ is the transmission rate (per day\times individual) from Susceptible to Exposed, $\beta$ is the rate (per day) at which Exposed become Infected, $\gamma$ is the rate (per day) at which Infected become recovered, $\eta$ is the mortality rate (per day) for those Infected, $\rho$ is the rate (per day) at which the Susceptible become vaccinated, and $k$ is the inefficacy of the vaccine, hence, $(1-k)$ is the efficacy of the vaccine. Notice that, this model formulation makes several key assumptions:

1. Immigration, emigration, natural mortality and births are negligible over the time frame and hence are not in the model.
2. Once a person is in the Infected group, they are quarantined and hence they do not mix with the Susceptible population.
3. The Recovered and Deaths compartments are for those who first are infected.
4. Those who are exposed and never become infected(sick) recover at the same rate $\gamma$ as those who become sick and recover.
5. The vaccine efficacy is not 100% and this is included in the model.

Traditional analysis would include a steady state analysis, however, in this case the dynamics of the short term is of interest. Hence, this work does not address any steady state or equilibrium concerns. This work is concerned with fitting the model given in (1) to the Covid-19 data concerning the State of Qatar during the 2020 outbreak and using the model for forecasting several possible scenarios.

2.1 The Disease-free Equilibrium and the Reproduction Number

Disease free means that there is no disease in the system, hence no one is infected and invariably, no one is exposed. The disease-free equilibrium of the systems (1), denoted by $X^0$ is obtained by setting all the derivatives to zero with $I = 0$ and $E = 0$, as a consequence of the above explanation. Thus, the system (1) yields:

$$X^0 = \left( \begin{array}{c}
\frac{\lambda}{\rho + \beta}, 0, 0, 0, 0, \frac{\lambda}{\rho + \beta}
\end{array} \right)$$
From the above, we can deduce that if everyone is vaccinated, then the number of susceptible is equal to the number of vaccinated. Thus, the model suggests that if everyone is vaccinated, then we can maintain a disease-free state.

The reproduction number, \( R_0 \) is computed using the next-generation matrix approach (Van den Driessche, Pauline, and James Watmough (2002)). Let \( X = (E, I)^T \), thus, the systems (1) can be written as

\[
\frac{dX}{dt} = F(X) - W(X),
\]

where \( F(X) = (\alpha SE + \alpha(1 - \kappa)VE, 0)^T \) and \( W(X) = ((\xi + \gamma)E - \beta E, (\eta + \gamma)I)^T \). The corresponding Jacobian matrices of \( F(X) \) and \( W(X) \) are:

\[
J(F(X)) = F = \begin{bmatrix}
\alpha S + \alpha(1 - \kappa)V & 0 \\
0 & 0
\end{bmatrix}.
\]

\[
J(W(X)) = W = \begin{bmatrix}
(\xi + \gamma) & 0 \\
-\beta & (\eta + \gamma)
\end{bmatrix}.
\]

According to (Van den Driessche, Pauline, and James Watmough (2002)), \( R_0 \) is defined as the spectral radius of the next-generation matrix \((FW)^{-1}\), we need to compute \((FW)^{-1}\), and then its spectra radius.

\[
W^{-1} = \begin{bmatrix}
\frac{1}{(\xi + \gamma)} & 0 \\
\frac{1}{(\eta + \gamma)} & -1
\end{bmatrix}.
\]

Therefore,

\[
(FW)^{-1} = \begin{bmatrix}
\frac{\alpha S + \alpha(1 - \kappa)V}{(\xi + \gamma)} & 0 \\
0 & \frac{1}{(\eta + \gamma)}
\end{bmatrix}.
\]

Recall: \( R_0 = \rho(FW^{-1}) \), where \( \rho \) is the spectral radius (largest absolute value of its eigenvalues). Thus, the basic reproduction number for our model is given as:

\[
R_0 = \frac{\alpha(S + (1 - \kappa)V)}{(\xi + \gamma)},
\]

which is used to measure the transmission potential of a disease. And it is defined as the number of secondary cases or new infections produced by an infectious individual in a completely susceptible population. Thus, we expect an endemic state when \( R_0 > 1 \), and a disease-free state when \( R_0 < 1 \) (Van den Driessche, Pauline, and James Watmough (2002)).

### 3 Existence and Uniqueness of the Endemic Equilibrium

In this section, we want to investigate the existence and uniqueness of the endemic equilibrium when \( R_0 > 1 \). Notice that for generality, birth and death have been included in the model (1) to obtain model (3-8). Therefore, the modified systems of ordinary equations is:

\[
\frac{dS}{dt} = \lambda - \alpha S(t)E(t) - \beta S(t) - \rho I(t)S(t) - \rho S(t)
\]

\[
\frac{dE}{dt} = \alpha S(t)E(t) - \beta E(t) + \alpha(1 - \kappa)V(t)E(t) - \gamma E(t) - \rho E(t)
\]

\[
\frac{dI}{dt} = \beta E(t) - \gamma I(t) - \eta I(t) + \tilde{\beta} S(t) - \rho I(t)
\]

\[
\frac{dR}{dt} = \gamma I(t) - \gamma E(t) - \rho R(t)
\]

\[
\frac{dD}{dt} = \eta I(t) - \rho D(t)
\]

\[
\frac{dV}{dt} = \rho S(t) + \rho I(t)S(t) - \alpha(1 - \kappa)V(t)E(t) - \rho V(t).
\]

Let \( X^* = (S^*, E^*, I^*, D^*, V^*) \) be the endemic equilibrium of the model (3-8). To obtain this, we set the derivatives in model (3-8) to zero:
Solving the fifth equation of (9):
\[ \eta I(t) - \rho D(t) = 0 \]
\[ D^* = \frac{\eta I^*}{\rho} > 0 \]

Adding the second and sixth equation of (9):
\[ aS(t)E(t) - \beta E(t) - \gamma E(t) - \rho E(t) + \rho S(t) + \rho I(t)S(t) - \alpha(1 - \kappa)V(t)E(t) - \rho V(t) = 0 \]
substituting \( S^* \) and \( E^* \) into the above expression, we get
\[ a_0 V^{*2} - a_1 V^* + a_2 = 0, \]
where
\[ a_0 = \frac{(\alpha(1 - \kappa))^2 u_1}{\rho}, \]
\[ a_1 = \left( \frac{au_1(1 - \kappa)}{\rho} + (1 - \kappa) \right) I^* + \left( \frac{au_1(1 - \kappa) + 1}{\alpha \rho} \right) + (1 - \kappa) + \frac{u_2^2 \alpha(1 - \kappa)}{\rho}, \]
\[ a_2 = u_1 - \frac{u_1^2}{\alpha \rho}. \]  
(10)

Clearly, \( a_0 \) and \( a_1 \) are positive when \( \kappa < 1 \) and \( a_2 \) is positive when \( u_1 > \frac{u_1^2}{\alpha \rho} \). Therefore, there’s a unique positive value for \( V^* \), and consequently, a unique endemic equilibrium \( X^* \). Notice that our system doesn’t include birth and death, however the proof for existence and uniqueness is analogous.

4 Data

The Johns Hopkins University (JHU) Covid-19 Github site (https://github.com/CSSEGISandData/COVID-19) includes for every country for each day the cumulative number of confirmed infections, cumulative number of recovered and the cumulative number of deaths for each day starting 22 January 2020. The data for Qatar was obtained. Notice that in model (1) the Recovered and Death states are cumulative as once one enters the compartment their is no exit. However, the Infected compartment has transitions from Exposed and to Recovered and Deaths. Hence the data provided for confirmed infections is cumulative and included both Recovered and Deaths and will need to be removed from this compartment’s data. Let \( CI(t) \) be the Confirmed Infections as reported by JHU at time \( t \) and let Infected \( I(t) \) be defined as:

\[ I(t) = CI(t) - \rho(t) - D(t) \]  
(11)

For clarity the term “Active Infections” will be used to denote the derived variable, \( I(t) \), versus the Cumulative Infected, \( CI(t) \), provided in the data, and note that our model will contains the vaccine which became available on the 29th of April.
Figure 1 shows the plots of the Active Infections, Recovered and Deaths data for Qatar for the days since 29 February 2020, and vaccine was given on 29 April 2021. Notice that the Active Infections are very low until around day 35 when there is a large jump due to increased testing. The Active Infections then seems to plateau for until day 300, after which there is extreme growth in Active Infections. There seems to be a similar pattern for the Recovered with a delay showing the time of infection before recovery. The plot for Deaths shows no deaths until day 95 and then a steady increase in Deaths for the remaining days. And the plot for vaccinated shows that vaccination began on day 426 and there is a large jump due to increased number of vaccinated individuals.

![Figure 1: Plots of Number of Active Infections (a), Cumulative Number of Recovered (b) and Cumulative Number of Deaths (c) for the State of Qatar for the days since 29 February 2020 until 13 October 2021. And the plot of vaccinated (d) for the State of Qatar for the days since 29 April 2021 until 13 October 2021.](image)

The State of Qatar, prepared an excellent flexible plan for risk management, grounded on national risk assessment, taking account of the global risk assessment done by WHO, focuses on reinforce capacities to reduce or eliminate the health risks from COVID-19. Embed complete emergency risk management strategy in the health sector. Furthermore, Enabling and promoting multi-sectoral linkage and integration across the whole-of-government and the whole-of-society (DOH-UK, 2020), (DOH-Australia, 2020) and (WHO, 2020).

On March 9, 2020 (day 48), Qatar announced a closure of all universities and schools. It placed a travel ban on 15 countries: Bangladesh, China, Egypt, India, Iran, Iraq, Italy, Lebanon, Nepal, Pakistan, the Philippines, South Korea, Sri Lanka, Syria, and Thailand. On March 14, 2020 (day 53), Qatar expanded its travel ban to include three new countries: Germany, Spain and France (Hamad Medical Corporation 2020 and MPH-Qatar 2019). The Ministry of Municipal and Environment on March 19, 2020 (day 58), closed all parks and public beaches and put public sector employees at 80% of work from home to curb the spread of coronavirus. The Council of Ministers on April 1, 2020 (day 71) further required that private sector employees must conduct 80% of their work from home. On April 8, 2020 (day 78), the Ministry of Commerce and Industry decided to temporarily close all restaurants, cafes, food outlets, and food trucks at the main public era. On April 23, 2020 (day 93) the holy month of Ramadan began where people were not allowed the typical social mingling the month typically provides. In fact the government highly encouraged extremely limited social mingling. A detailed list of government interventions can be found at (Hamad Medical Corporation 2020 and MPH-Qatar 2019).

These interventions taken by the government change the dynamics of the system and hence need to be incorporated into the model. The next section details how we introduce interventions both from the government and interventions guided by the model. While the model interventions correspond to government policy changes, one cannot assume that any impact is solely due to those policies as other lurking variables may contribute to the impact.

## 5 Interventions

In Figure 1, one can see the jump at day 12 and a plateau until day 136. The model needs to be able to handle interventions made by the Government of the State of Qatar. The main parameter that policy can influence is $\alpha$, the rate of transmission from Susceptible to Exposed. One way to implement this the use of indicator functions $W_k(t)$ defined as:

$$W_k(t) = \begin{cases} 1 & \text{if } t > t_k \\ 0 & \text{otherwise} \end{cases}$$

where $t_k$ is the time where the $k^{th}$ intervention is taken and index $k = 1, 2, \ldots, K$. For each intervention there needs to be a change to the value of $\alpha$, denoted $\alpha_k$, that captures the impact of the intervention. Let the vector $W(t) = (W_1(t), W_2(t), \ldots, W_K(t))^T$ be the vector of the values of each $W_k(t)$ at time $t$. Let $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_K)^T$, where each of the $\alpha$’s are independent of each other,
which means that we do not need \( \alpha_i \) to obtain \( \alpha_{i+1} \). Thus, the transitions rates between \( S(t) \) and \( E(t) \) is formulated as follows:

\[
\alpha(t) = \begin{cases} 
\alpha_0 & \text{if } 0 < t_1 < t \\
\alpha_1 & \text{if } 0 < t_2 < t \\
\alpha_2 & \text{if } 0 < t_3 < t \\
... & .. \\
\alpha_K & \text{if } 0 < t_K < t 
\end{cases}
\]

which will require the following constraints due to the fact that \( \alpha(t) > 0 \) for all \( t \):

\[
\alpha_0 > 0 \\
\alpha_1 > 0 \\
\alpha_2 > 0 \\
... > .. \\
\alpha_K > 0
\]

In addition to changes in infection rates \( \alpha \), impulse functions can be used to model dramatic one time shifts in transitions between states. A Dirac delta function defined by

\[
\delta(x) = 0, \text{ if } x \neq 0
\]

which satisfies \( \int_{-\infty}^{\infty} \delta(x) dx = 1 \) (Dirac, 1958). This can be integrated in the model to capture spikes in the number of cases. In our case the State of Qatar data shows exhibits this type of behavior at day 35 where one can clearly see a large jump in the number of infections. This is incorporated into the model presented by a Dirac delta function, \( \delta(t - \tau) \), in transition rate between Exposed and Infected, which is coupled with a coefficient to \( \beta_A \) to capture the impact of the jump.

Since we aim to capture the interventions made by the government of Qatar, and the government intervenes daily, therefore, the assumption of steady state no longer hold in our model.

### 6 Bayesian Analysis

Due to the complexity of the model the Bayesian inferential framework is chosen. Recall, Bayes formula is given by (Bayes and Price, 1763):

\[
\pi(\theta|D) = \frac{\pi(\theta)L(D|\theta)}{\int_{\theta} \pi(\theta)L(D|\theta)d\theta}
\]

where \( \pi(\theta|D) \) is the posterior probability distribution for the parameters \( \theta \) given the data \( D \), \( \pi(\theta) \) is the prior distribution of \( \theta \) and \( L(D|\theta) \) is the likelihood of the data given \( \theta \).

In order to specify the likelihood of the model in equation (1) the model modified to model the mean abundance in each compartment and is given by:

\[
\begin{align*}
\frac{d\lambda_S(t)}{dt} &= -W(t)^T a\lambda_S(t)\lambda_E(t) - \rho\lambda_S(t) - \beta\lambda_S(t) - \rho\lambda_I(t)\lambda_S(t) \\
\frac{d\lambda_E(t)}{dt} &= W(t)^T a\lambda_S(t)\lambda_E(t) - \beta\lambda_E(t) - \gamma\lambda_E(t) + \alpha(1 - \kappa)\lambda_V(t)\lambda_E(t) - \beta_A\lambda_E(t)\delta(t - \tau) \\
\frac{d\lambda_I(t)}{dt} &= \beta\lambda_E(t) + \beta_A\lambda_I(t)\delta(t - \tau) - \gamma\lambda_I(t) - \eta\lambda_I(t) + \beta\lambda_S(t) \\
\frac{d\lambda_R(t)}{dt} &= \gamma\lambda_I(t) \\
\frac{d\lambda_D(t)}{dt} &= \gamma\lambda_E(t) \\
\frac{d\lambda_V(t)}{dt} &= -\rho\lambda_S(t) + \rho\lambda_I(t)\lambda_S(t) - \alpha(1 - \kappa)\lambda_V(t)\lambda_E(t). 
\end{align*}
\]

where \( \lambda_S(t), \lambda_E(t), \lambda_I(t), \lambda_R(t), \lambda_D(t) \) and \( \lambda_V(t) \) are the means of \( S(t), E(t), I(t), R(t), D(t) \) and \( V(t) \) respectively and the parameters have the same definition as provided in the system given in equation (1). Since there is no data for \( S(t) \) and \( E(t) \), these compartments will be latent variables and will not directly factor into the likelihood. The likelihood for \( I(t), R(t), D(t) \) and \( V(t) \) are given by:

\[
\begin{align*}
I(t) &\sim \text{Poisson}(\lambda_I(t)) \\
R(t) &\sim \text{Poisson}(\lambda_R(t)) \\
D(t) &\sim \text{Poisson}(\lambda_D(t)) \\
\end{align*}
\]
\( V(t) \sim \text{Poisson}(\lambda _V(t)) \)  

(17)

To specify the prior distributions for \( \alpha, \beta_A, \beta, \gamma, \eta, \rho \) and \( \kappa \) one must incorporate the following constraints \( \alpha > 0, \beta > 0, \gamma > 0, \eta > 0, \rho > 0 \) and \( \kappa > 0 \). Hence the following prior distributions are set:

\[
\begin{align*}
\alpha & \sim \text{Exp}(i) \\
\beta_A & \sim \text{Exp}(1) \\
\beta & \sim \text{Exp}(i) \\
\gamma & \sim \text{Exp}(i) \\
\eta & \sim \text{Exp}(1) \\
\rho & \sim \text{Exp}(1) \\
\kappa & \sim \text{Beta}(0, i)
\end{align*}
\]

(18)

The likelihood and prior distributions specifications lead to the following posterior distribution:

\[
\pi(\alpha, \beta_A, \beta, \gamma, \eta, \rho, \kappa | \mathbf{D}) \propto \pi(\alpha)\pi(\beta_A)\pi(\beta)\pi(\gamma)\pi(\eta)\pi(\rho)\pi(\kappa)|L(\mathbf{D} | \alpha, \beta_A, \beta, \gamma, \eta, \rho, \kappa)
\]

\[
\times \prod_{t=1}^{T} \lambda _V(t) I(t) \gamma _V(t) R(t) \beta _V(t) \gamma _D(t) R(t) \gamma _V(t) V(t) e^{-\lambda _V(t) - \lambda _D(t) - \lambda _V(t) - \lambda _V(t)}
\]

(19)

The posterior distribution does not lend to any analytic solution, hence Markov chain Monte Carlo (MCMC) techniques will be used to sample from the posterior distribution (Gelman et al., 2013). Specifically Metropolis-Hastings sampler is used to obtain samples from the posterior distribution (Gilks, Richardson, and Spiegelhalter, 1996) and (Albert, 2009). To tune the sampler, a series of short chains were generated and analyzed for convergence and adequate acceptance rates. These initial short chains were discarded as “burn-in” samples. The tuned sampler was used to generate 30,000 samples from \( \pi(\alpha, \beta_A, \beta, \gamma, \eta, \rho, \kappa | \mathbf{D}) \) and trace plots were visually examined for convergence and deemed to be acceptable. All inferences will be made from these 30,000 samples. The model and sampling algorithm is custom programmed in the R statistical programming language version 3.6.3. The computation takes approximately 290 seconds using a AMD A10-9700 3.50GHz processor with 16GB of RAM to obtain 30,000 samples from the posterior distribution. For more on statistical inference see Wackerly, Mendenhall, and Scheaffer (2008), Casella and Berger (2002), and Berger (1985).

7 Results

To apply the model the following initial conditions are specified: \( S(0) = 2,782,000, E(0) = 3, I(0) = 1, R(0) = 0, D(0) = 0 \) and \( V(0) = 0 \). Here \( S(0) \) is the current population of the State of Qatar, \( I(0) \), \( R(0) \) and \( D(0) \) are obtained directly from the data. The choice of \( E(0) \) was used as it a minimal value that would allow the disease to spread but not so large as to make the spread rapid. Several values of \( E(0) \) were explored and the value of 3 was found to have the best fit. Furthermore, model interventions were placed at days \( t_1 = 12, t_2 = 35, t_3 = 40, t_4 = 60, t_5 = 70, t_6 = 80, t_7 = 87, t_8 = 95, t_9 = 105, t_{10} = 124, t_{11} = 136, t_{12} = 350, t_{13} = 355 and t_{14} = 420 \) with an Dirac delta impulse at time \( t = 35 \).

Table 1 shows the means, standard deviations and the 0.025%, 0.5% and 0.975% quantiles for the model parameters based on the 30,000 samples from the posterior distribution. Notice that \( \alpha_2 = 1.28 \times 10^{-7} \) and \( \alpha_1 = 7.74 \times 10^{-8} \) are quite close, indicating that the first intervention resulted in a low transmission rate. Similarly we can see that the second and third interventions \( \alpha_2 = 1.22 \times 10^{-7} \) and \( \alpha_3 = 6.11 \times 10^{-8} \) have some overlap resulting in a very low transmission rate. Furthermore, \( \alpha_4 = 3.85 \times 10^{-8} \) is a low decrease, and a very small increase in \( \alpha_5 = 4.01 \times 10^{-8} \) which still resulted in a low transmission rate. However, \( \alpha_6 = 2.19 \times 10^{-8} \) is a moderate decrease with another moderate increase in \( \alpha_7 = 4.00 \times 10^{-8} \). Also, \( \alpha_8 = 5.39 \times 10^{-8} \) is a moderate increase, \( \alpha_9 = 2.74 \times 10^{-8} \) is a moderate decrease, \( \alpha_{10} = 4.38 \times 10^{-8} \) is a moderate increase, \( \alpha_{11} = 5.49 \times 10^{-8} \) is a moderate increase, but \( \alpha_{12} = 2.76 \times 10^{-9} \) is a very low decrease, while \( \alpha_{13} = 3.61 \times 10^{-8} \) and \( \alpha_{14} = 5.31 \times 10^{-8} \) both have a moderate increase which still leaves a final transmission rate of \( \sum_{k=0}^{\infty} \alpha_k \approx 8.00 \times 10^{-7} \). Of particular note is the mean mortality rate \( \eta = 0.00013 \approx 1/7692 \) which means that about 1 in 7,692 people die from the disease each day, which is quite low. Also note that the mean infection (confirmed) rate is \( \beta = 0.07640 \approx 1/13.08 \) which corresponds to about 1 in 13.08 exposed people become confirmed each day. The quantile intervals provide a 95% credible interval for the parameters and can be used to obtain a range of reasonable parameter values. For example for the parameter \( \beta \) the interval is \((0.07183, 0.07849)\) meaning that the probability that \( \beta \) is between \((0.07183, 0.07849)\) is 0.95. This can be used to create an interval for the risk interpretations as between \((1/0.07183 \approx 13.92)\) and \((1/0.07849 \approx 12.74)\) Exposed people are confirmed as infected each day. This also gives insight into how many people may be in the population who are Exposed and may be infectious but do not yet exhibit symptoms. Recall, \( \gamma \) corresponds to the rate at which people recover from the disease which is 0.04132. This corresponds to \((1/0.04132 \approx 1/24.20)\) which corresponds to 1 in 24.20 people recover from the disease each day. This value may be artificially low due to delays in reporting. However, this value seem reasonable, considering the total population in Qatar.
Table 1: Mean, Median, Standard Deviation and \((Q_{0.025}, Q_{0.5}, Q_{0.975})\) for \(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, \beta_D, \beta, \gamma, \eta, \kappa, \beta_D, \gamma, \eta, \kappa\). Based on 30,000 samples from the posterior distribution

| Parameter | Mean | Median | Std Dev. | \((Q_{0.025}, Q_{0.5}, Q_{0.975})\) |
|-----------|------|--------|----------|---------------------------------|
| \(a_0\) | 1.28 x 10^-7 | 1.29 x 10^-7 | 8.23 x 10^-7 | (1.04 x 10^-6, 1.29 x 10^-7, 1.35 x 10^-7) |
| \(a_1\) (day 12) | 7.74 x 10^-8 | 7.76 x 10^-8 | 1.82 x 10^-7 | (7.18 x 10^-8, 7.76 x 10^-8, 9.50 x 10^-8) |
| \(a_2\) (day 35) | 1.22 x 10^-7 | 1.23 x 10^-7 | 6.33 x 10^-9 | (1.07 x 10^-7, 1.23 x 10^-7, 1.25 x 10^-7) |
| \(a_3\) (day 40) | 6.11 x 10^-8 | 6.10 x 10^-8 | 2.19 x 10^-9 | (5.81 x 10^-8, 6.10 x 10^-8, 6.88 x 10^-8) |
| \(a_4\) (day 60) | 3.85 x 10^-8 | 3.71 x 10^-8 | 4.90 x 10^-9 | (3.56 x 10^-8, 3.71 x 10^-8, 5.72 x 10^-8) |
| \(a_5\) (day 70) | 4.01 x 10^-8 | 3.96 x 10^-8 | 2.61 x 10^-9 | (3.65 x 10^-8, 3.96 x 10^-8, 4.28 x 10^-8) |
| \(a_6\) (day 80) | 2.19 x 10^-8 | 2.27 x 10^-8 | 4.37 x 10^-9 | (1.89 x 10^-8, 2.27 x 10^-8, 2.44 x 10^-8) |
| \(a_7\) (day 87) | 4.00 x 10^-8 | 4.11 x 10^-8 | 3.01 x 10^-9 | (3.38 x 10^-8, 4.11 x 10^-8, 4.28 x 10^-8) |
| \(a_8\) (day 95) | 5.39 x 10^-8 | 5.31 x 10^-8 | 3.21 x 10^-9 | (4.77 x 10^-8, 5.31 x 10^-8, 5.99 x 10^-8) |
| \(a_9\) (day 105) | 2.74 x 10^-8 | 2.68 x 10^-8 | 4.95 x 10^-9 | (2.17 x 10^-8, 2.68 x 10^-8, 4.48 x 10^-8) |
| \(a_{10}\) (day 124) | 4.38 x 10^-8 | 4.25 x 10^-8 | 3.76 x 10^-9 | (3.79 x 10^-8, 4.35 x 10^-8, 5.06 x 10^-8) |
| \(a_{11}\) (day 136) | 5.49 x 10^-8 | 5.48 x 10^-8 | 1.20 x 10^-9 | (5.31 x 10^-8, 5.48 x 10^-8, 5.80 x 10^-8) |
| \(a_{12}\) (day 350) | 2.76 x 10^-9 | 8.43 x 10^-11 | 4.92 x 10^-9 | (1.62 x 10^-10, 8.43 x 10^-11, 1.14 x 10^-9) |
| \(a_{13}\) (day 355) | 3.61 x 10^-8 | 3.64 x 10^-8 | 1.40 x 10^-9 | (3.20 x 10^-8, 3.64 x 10^-8, 3.77 x 10^-8) |
| \(a_{14}\) (day 420) | 5.31 x 10^-8 | 5.29 x 10^-8 | 2.24 x 10^-9 | (5.23 x 10^-8, 5.29 x 10^-8, 6.24 x 10^-8) |

While many of the parameters do not lend well to the traditional \(H_0: \beta = 0\) hypothesis testing as they must be positive. We can conduct simple hypothesis tests on the \(\alpha\) parameters to look for significant changes due to interventions using contrasts. Specifically the sequential contrasts of \(a_1 - a_0, a_2 - a_1, a_3 - a_2, a_4 - a_3, a_5 - a_4, a_6 - a_5, a_7 - a_6, a_8 - a_7, a_9 - a_8, a_{10} - a_9, a_{11} - a_{10}, \ldots, a_{14} - a_{13}\). These contrasts quantify the changes that in transmission rate from Susceptible to Exposed due to the interventions and are what policy makers want to see. Furthermore, they want a statistical test on whether or not the intervention performed in a statistically significant manner. This can be done by simply subtracting the MCMC samples to generate the contrast of interest. Using these subtracted samples one can look at the mean, median, standard deviation, quantiles and the proportion of samples above 0, \(P(> 0)\). Table 2 shows these quantities for the contrasts listed above. Notice that the intervention at day 12 reduced the transmission rate by approximately \(5.05 \times 10^{-8}\) which is considerable and the proportion of samples above 0 was 0.000 indicating a statistically significant change due to the intervention. The intervention taken at day 35, \(a_2 - a_1\), actually increased the transmission rate, while the intervention taken at day 40, \(a_3 - a_2\), then reduced the transmission rate. The interventions at day 60, \(a_4 - a_3\) reduced the transmission rate, but the intervention at day 70, \(a_5 - a_4\) actually increased the transmission rate and the intervention at day 80, \(a_6 - a_5\) then reduced the transmission rate. Although the interventions at day 87, \(a_7 - a_6\) and at day 95, \(a_8 - a_7\) both increased the transmission rate, the intervention at day 105, \(a_9 - a_8\) was very helpful in reducing the transmission rate. Similarly the two interventions at days 124 and 136 increased the transmission rate where the intervention at day 350 then decreased the transmission rates. Lastly, the other two interventions increased the transmission rate, respectively. Furthermore, all interventions deemed statistically significant since \(P(> 0)\) is either 0.000 or 1.000 indicating significance.

In Table 2, we notice that \(a_1 - a_0, a_2 - a_1, a_3 - a_2, a_4 - a_3, a_5 - a_4, a_6 - a_5, a_7 - a_6, a_8 - a_7, a_9 - a_8\) and \(a_{12} - a_{11}\) increased the infection rates while the others decreased the infection rates. Connecting these contrast to the interventions deployed by the Qatari government, we notice that the intervention measure taken by closing schools and universities on the 10th of March falls between day 40 and day 60 (\(a_4 - a_3\)). This shows that this intervention was helpful in decreasing the transmission rate. Furthermore, the intervention deployed on 21st March 2020 by the ministry of Municipality and Environment to close all parks and public beaches, as well as the intervention deployed on 23rd March 2020 by the ministry of commerce and industry to temporarily close down all restaurants and cafes, food trucks and food outlets fall between day 60 and 62 (\(a_4 - a_3\)), hence, these interventions were helpful in the reduction of the transmission rate. The intervention deployed on June 4th, when the cabinet decided to allow four people
inside a vehicle, exempting only families, and the intervention by the ministry of commerce and industry (MoCL) to permit working hours for private sectors from 7am until 8pm falls on day 136 (α2 − α0). However, these two interventions were not effective as the transmission rate increased. Thus, we see that some of the interventions by the Qatari government was effective in reducing the transmission rate of Covid-19, and if more interventions are taken, then we would see a tremendous improvement in the reduction of transmission rate.

Report shows that some restrictions were lifted on June 15th (phase 1) through September 2020 (phase 4), and there hasn’t been any major interventions since June 4th, which could be why there are more cases, especially between July 2020 and March 2021. Furthermore, result in Table 2 shows that if an intervention took place on 5th January 2021, this would have reduced the transmission rate. Therefore, we deduce that to reduce the transmission rates, there’s urgent need for the government to place more interventions with the hope that these interventions would be effective.

**Table 2:** Mean, Median, Standard Deviation, (Q,0.025, Q,0.5, Q,0.975) and proportion of samples larger than zero P(>0) for sequential contrasts across α. Based on 30,000 samples from the posterior distribution

| Contrast | Mean     | Median   | Std Dev. | (Q,0.025, Q,0.5, Q,0.975) | P(>0)  |
|----------|----------|----------|----------|--------------------------|--------|
| α1 - α0  | -5.05 × 10⁻⁸ | -5.12 × 10⁻⁸ | 6.47 × 10⁻⁹ | (-5.56 × 10⁻⁸, -5.12 × 10⁻⁸, -3.25 × 10⁻⁸) | 0.000  |
| α2 - α1  | 4.48 × 10⁻⁸  | 4.63 × 10⁻⁸  | 5.10 × 10⁻⁹ | (3.56 × 10⁻⁸, 4.63 × 10⁻⁸, 4.76 × 10⁻⁸) | 1.000  |
| α3 - α2  | -6.11 × 10⁻⁸ | -6.32 × 10⁻⁸ | 7.69 × 10⁻⁹ | (-6.45 × 10⁻⁸, -6.32 × 10⁻⁸, -3.85 × 10⁻⁸) | 0.000  |
| α4 - α3  | -2.26 × 10⁻⁸ | -2.32 × 10⁻⁸ | 3.17 × 10⁻⁹ | (-2.43 × 10⁻⁸, -2.32 × 10⁻⁸, -1.17 × 10⁻⁸) | 0.000  |
| α5 - α4  | 1.57 × 10⁻⁹  | 2.44 × 10⁻⁹  | 5.22 × 10⁻⁹ | (-1.92 × 10⁻⁸, 2.44 × 10⁻⁹, 6.61 × 10⁻⁹) | 1.000  |
| α6 - α5  | -1.81 × 10⁻⁹ | -1.77 × 10⁻⁹ | 2.33 × 10⁻⁹ | (-2.76 × 10⁻⁹, -1.77 × 10⁻⁹, -1.63 × 10⁻⁹) | 0.000  |
| α7 - α6  | 1.81 × 10⁻⁸  | 1.81 × 10⁻⁸  | 2.32 × 10⁻⁹ | (1.63 × 10⁻⁸, 1.81 × 10⁻⁸, 2.60 × 10⁻⁸) | 1.000  |
| α8 - α7  | 1.38 × 10⁻⁸  | 1.37 × 10⁻⁸  | 2.55 × 10⁻⁹ | (1.14 × 10⁻⁸, 1.37 × 10⁻⁸, 1.82 × 10⁻⁸) | 1.000  |
| α9 - α8  | -2.65 × 10⁻⁸ | -2.63 × 10⁻⁸ | 7.79 × 10⁻⁹ | (-3.82 × 10⁻⁸, -2.63 × 10⁻⁸, -3.13 × 10⁻⁸) | 0.000  |
| α10 - α9 | 1.64 × 10⁻⁸  | 1.64 × 10⁻⁸  | 6.90 × 10⁻⁹ | (3.89 × 10⁻⁹, 1.64 × 10⁻⁸, 2.81 × 10⁻⁸) | 1.000  |
| α11 - α10| 1.10 × 10⁻⁸  | 1.13 × 10⁻⁸  | 4.52 × 10⁻⁹ | (3.24 × 10⁻⁹, 1.13 × 10⁻⁸, 1.84 × 10⁻⁸) | 1.000  |
| α12 - α11| -5.21 × 10⁻⁸ | -5.47 × 10⁻⁸ | 5.52 × 10⁻⁹ | (-5.80 × 10⁻⁸, -5.47 × 10⁻⁸, -4.17 × 10⁻⁸) | 0.000  |
| α13 - α12| 3.34 × 10⁻⁸  | 3.48 × 10⁻⁸  | 4.05 × 10⁻⁹ | (2.62 × 10⁻⁸, 3.48 × 10⁻⁸, 3.65 × 10⁻⁸) | 1.000  |
| α14 - α13| 1.70 × 10⁻⁸  | 1.66 × 10⁻⁸  | 3.20 × 10⁻⁹ | (1.53 × 10⁻⁸, 1.66 × 10⁻⁸, 3.04 × 10⁻⁸) | 1.000  |

We also conducted simple hypothesis tests on the γ and κ parameters to look for significant changes due to intervention using contrasts. For γ, we specifically used the sequential contrasts of γ1 − γ0, γ2 − γ1, γ3 − γ2, γ4 − γ3, γ5 − γ4, γ6 − γ5. This contrast quantify changes in the recovery rate from Infected to Recovery due to interventions. This would allow us to see if the interventions performed are statistically significant or not. The contrast is obtained similar to the transmission rate, where we subtracted the MCMC samples to generate the contrast of interest. Similar idea is deployed to understand the inefficacy of the vaccine, κ. Table 3 shows the result of the contrasts for both γ and κ.

**Table 3:** Mean, Median, Standard Deviation, (Q,0.025, Q,0.5, Q,0.975) and proportion of samples larger than zero P(>0) for sequential contrasts across γ and κ. Based on 30,000 samples from the posterior distribution

| Contrast | Mean     | Median   | Std Dev. | (Q,0.025, Q,0.5, Q,0.975) | P(>0)  |
|----------|----------|----------|----------|--------------------------|--------|
| γ1 − γ0  | 3.79 × 10⁻⁸ | 3.69 × 10⁻⁸ | 4.65 × 10⁻⁹ | (3.58 × 10⁻⁸, 3.69 × 10⁻⁸, 5.69 × 10⁻⁹) | 1.000  |
| γ2 − γ1  | 5.97 × 10⁻⁸ | 5.87 × 10⁻⁸ | 4.76 × 10⁻⁹ | (5.78 × 10⁻⁸, 5.87 × 10⁻⁸, 7.46 × 10⁻⁸) | 1.000  |
| γ3 − γ2  | 5.92 × 10⁻⁹ | 5.95 × 10⁻⁹ | 4.79 × 10⁻⁹ | (-1.38 × 10⁻⁹, 5.95 × 10⁻⁹, 1.35 × 10⁻⁹) | 1.000  |
| γ4 − γ3  | -2.60 × 10⁻⁸ | -2.51 × 10⁻⁸ | 3.86 × 10⁻⁹ | (-4.18 × 10⁻⁸, -2.51 × 10⁻⁸, -2.37 × 10⁻⁸) | 0.000  |
| γ5 − γ4  | -3.74 × 10⁻⁸ | -3.78 × 10⁻⁸ | 4.28 × 10⁻⁹ | (-4.40 × 10⁻⁸, -3.78 × 10⁻⁸, -3.10 × 10⁻⁸) | 0.000  |
| γ6 − γ5  | 7.01 × 10⁻⁸  | 7.11 × 10⁻⁸  | 4.58 × 10⁻⁹ | (6.21 × 10⁻⁸, 7.11 × 10⁻⁸, 7.16 × 10⁻⁸) | 1.000  |
| κ1 − κ0  | -8.66 × 10⁻⁷ | -9.03 × 10⁻⁷ | 1.64 × 10⁻⁶ | (-9.15 × 10⁻⁷, -9.03 × 10⁻⁷, -1.49 × 10⁻⁷) | 0.000  |

To assess the fit of the model the posterior predictive distribution was used and is given by:

\[
\pi(I_{\text{new}}(t), R_{\text{new}}(t), D_{\text{new}}(t), V_{\text{new}}(t)|\mathbf{D}) = \int L(I_{\text{new}}(t), R_{\text{new}}(t), D_{\text{new}}(t), V_{\text{new}}(t)|\alpha, \beta, \gamma, \eta, \kappa) \times \pi(\alpha, \beta, \gamma, \eta, \kappa|\mathbf{D}) \, d\alpha d\beta d\gamma d\eta d\kappa. \tag{20}
\]
Using the samples 30,000 samples from the posterior distribution, 30,000 samples were generated from the posterior predictive distribution. At each time \( t \) the median, 0.025 and 0.975 quantiles were obtained to form a posterior predictive interval.

Figure 2 shows the model fits for Active Infections, Recovered, Deaths and Vaccinated with posterior predictive bands. Notice that, the model does quite well at fitting the dynamics of the Active Infections including the jumps at days 95 and 420, and captures the plateau and the exponential growth after the plateau as well. The Recovered and the vaccinated models fit well as does the Deaths data. To assess the explained variance a pseudo-\( R^2 \) was formed using the median from the posterior predictive distribution at each time as the point estimates. This, resulted in a pseudo-\( R^2 \) of 0.999 which indicates the fitted model explains approximately 99.9% of the variance in the data. Based on this the model is deemed to fit well. It should be noted that standard data splitting procedures for model validation are difficult in this scenario as removing values from the system may cause unstable behavior.

![Figure 2: Plots of Total Active Infections (a), Cumulative Recovered (b), Cumulative Deaths (c) for the State of Qatar for the days since 29 February 2020 until until 13 October 2021 and Vaccinated (d) for the days since 29 April 2021 until 13 October 2021 with posterior predictive bands. Posterior predictive bands are based on the 0.025 and 0.975 Quantiles from 30,000 samples from the posterior predictive distribution.](image)

### 8 Time Varying Reproduction Number

The basic reproduction number, \( R_0 \) is defined as the average number of people an infectious person will infect, assuming that the rest of the population is susceptible. In this study, we are considering in the time varying \( R_0 \), which means that we are interested in knowing the number of secondary cases an infectious person can produce throughout the period of the infection.

Since the the number of susceptible, vaccinated and the parameters in (2) are time varying, we can write our time varying reproduction number as follows:

\[
R_0(t) = \frac{\alpha(t)(S(t) + (1 - \kappa(t))V(t))}{(\beta(t) + \gamma(t))}.
\]

Here, \( S(t), V(t) \) and \( \beta(t) \) remain as defined in (2), but the other parameters are now in vector form and are defined as follows:

- \( \alpha_i(t), i = 1, 2, \cdots, 15 \)
- \( \gamma_j(t), j = 1, 2, \cdots, 7 \)
- \( \kappa_l(t), l = 1, 2 \).

Since the \( R_0(t) \) is time varying, we might want to worry about the validity of the existence and the uniqueness theorem (3). Fortunately, our systems equation (3) is continuous and its derivatives are continuous and bounded, because \( S(t), E(t), I(t), R(t), D(t), V(t) \) are finite, therefore, the existence and uniqueness theorem still holds under the time varying reproduction number scenario. Thus, since \( S(t) \) and \( V(t) \) are both numbers and are finite and the parameters \( \alpha_i(t), \beta(t), \gamma_j(t) \) & \( \kappa_l(t) \) are all numbers, then the existence and uniqueness theorem is still valid.

Figure 3 shows that at the beginning, the intervals around the reproduction number, \( R_0(t) \) which is around the 95% confidence interval, the entire interval is above 1, which means that we are definitely in a pandemic state. When we come down, we see that 1 is in the interval, which means that it is hard to tell whether we are in the pandemic state or not. But then at the end it goes further down, which clearly shows if we are in the endemic state or the declining state. Furthermore, we can see the effect of the interventions by the Qatari government on the reproduction number. In section 7, we extensively discussed the effects of the interventions by the government on the transmission rate, but here, we want to observe the effect of these interventions on \( R_0(t) \). Between days 146 and 420, we couldn’t decide whether we were in the endemic or declining state, which could be as a result of the restrictions lifted by the government on June 15 (day 146) without any major intervention measures until April 29th (day 420), when vaccine was introduced in Qatar.
Comparing the reproduction number (Figure 3) with our result in Table 2, Figure 3 shows that at the beginning of the pandemic, the reproduction was about 2.4 which indicates a severe endemic state as we said earlier. That is, the number of secondary infections is pretty high and more people could get infected since the population is well mixed. However, when there was an intervention on day 60, we see a decline in the value of $R_0$ from about 2.4 to 2.3. This shows that the intervention on day 60 was helpful in reducing the spread of number of secondary infections. Furthermore, on day 80, we see a huge decline in the $R_0$ from about 2.3 to 1. Obviously, the intervention implemented on this day was very effective in controlling the spread of Covid-19, and this is consistent with the result in Table 2. Before vaccination, we see that the value of $R_0$ bounced between 0.9 and 1.2, but when vaccine was deployed on day 420 the reproduction number reduced drastically from about 1.1 to about 0.4! This decline in the spread of secondary infections is very impressive, with this result, if 95% of the population in Qatar could get vaccinated, then the value of $R_0$ might keep decreasing and the spread of Covid-19 might be well controlled.

9 Model Validation

We have developed an SEIRDV model that incorporate intervention, and we have seen in Section 7 that some of these interventions were significant, hence were helpful in the reduction of the transmission rate, while some weren’t helpful since the transmission rate increased after those interventions were deployed. We also learned that the transmission rate would have reduced if intervention measures were taken on 5th January 2021. However, we are unsure if this model only works for Qatar dataset, since we developed the model based on the situation of the country. Therefore, to be convinced that our model is valid and robust, that is, can be used on any Covid-19 dataset, we performed a validation test using another country’s dataset, Nigeria. Interestingly, our model fits this data quite well with a pseudo-$R^2 \approx 0.7$.

10 Discussion

This work has demonstrated how to build a SEIRDV model for the Covid-19 outbreak in the State of Qatar, include interventions, estimate model parameters and the time varying reproduction number using a Bayesian framework. Furthermore, the model is able to treat the Susceptible and Exposed compartment as latent variables, as no data is observed about them other than approximate initial values. The model fits the data quite well with a pseudo-$R^2 \approx 0.999$. One can also note that in the model definition, immigration, emigration, natural births and natural mortality were not included and based on the high pseudo-$R^2$ would have a negligible effect on fit.

The modeling paradigm is quite flexible for modeling the Covid-19 data as it easily incorporates interventions into the system and can quantify the impact of the intervention. Apparently, our model could detect the interventions which are effective in reducing and increasing the transmission rates. Our model shows the strict/severe interventions such as closure of schools, parks, restaurants and bars, and travel bans were effective in reducing the transmission rate, while non-strict/liberal interventions in-
increased the transmission rate. In addition, from this model we can infer that to successfully curb the fast spread of Covid-19, there is need for strict intervention, but the questions is, would the government be willing to do this again? After lifting travel bans and other severe restrictions in September 2020, there was no record of interventions until today. And our model shows that if the government had passed a strict intervention on January 5th 2021, that intervention would have reduced the fast spread of Covid-19 (reduced the transmission rate). Furthermore, it is interesting to see how the introduction of vaccine impacts the transmission rate positively. In Figure 3, it was hard to tell whether we are in the pandemic state or not before the introduction of Covid-19 (reduced the transmission rate). Furthermore, it is interesting to see how the introduction of vaccine impacts the transmission rate positively. In Figure 3, it was hard to tell whether we are in the pandemic state or not before the introduction of Covid-19 (reduced the transmission rate). Furthermore, it is interesting to see how the introduction of vaccine impacts the transmission rate positively. In Figure 3, it was hard to tell whether we are in the pandemic state or not before the introduction of Covid-19 (reduced the transmission rate).

We addressed possible sensitivity issues to the initial Exposed value $E(0) = 3$ by looking at the data values of $E(t)$. To study this the model was run again with values $E(0) = 5, 10, 15$ and $25$ and the model pseudo- $R^2$ was calculated for each. All runs were under the exact same prior distribution specifications and using the same MCMC procedures. For $E(0) = 5$ the model produces a pseudo- $R^2 = 0.999$, with $E(0) = 10$ produces a pseudo- $R^2 = 0.990$, using $E(0) = 15$ a pseudo- $R^2 = 0.967$ is found and finally, with $E(0) = 25$ a pseudo- $R^2 = 0.865$ is given. This suggests that the model is not sensitive to low values of $E(0)$ but values that are too large produce a poorer and poorer performance especially at the beginning time frame. Lastly, we performed a cross-validation test using a different dataset (Nigeria dataset) to check for the robustness of our model, and we found that our model fits the external dataset (Nigeria data) quite well as we have $R^2 = 0.7$.

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