Wilson lines in the operator definition of TMDs: spin degrees of freedom and renormalization

I.O. Cherednikov*,†, A.I. Karanikas** and N.G. Stefanis‡

*Departement Fysica, Universiteit Antwerpen, B-2020 Antwerpen, Belgium
†Joint Institute for Nuclear Research, RU-141980 Dubna, Russia
**Department of Physics, University of Athens, Nuclear and Particle Physics Section, Panepistimiopolis, GR-15771 Athens, Greece
‡Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

Abstract. A generalized idea of gauge invariance, that embodies into the Wilson lines the spin-dependent Pauli term \( \sim F_{\mu\nu}\gamma_{\mu\nu} \), is applied to set up a new framework for the operator definition of transverse-momentum-dependent parton densities (TMDs). We show that such a treatment of gauge invariance is justified, since it does not change the leading-twist behavior of the TMDs, albeit it contributes to their twist-three properties, in particular, to their anomalous dimensions. We discuss other consequences of this generalization and its possible applications to lattice simulations of the TMDs.

Keywords: Transverse-momentum-dependent PDFs; Wilson lines; renormalization
PACS: 11.10.Jj, 12.38.Bx, 13.60.Hb, 13.87.Fh

In order to render operator products and their (hadronic) matrix elements gauge-invariant, one usually uses a path- \((\mathcal{C})\) dependent gauge link (Wilson line) with an exponent containing only the gauge field \(A\):

\[
[y; x|\mathcal{C}] \equiv \mathcal{P} \exp \left[ -ig \int_{x[\mathcal{C}]}^{y} dz^\mu A_\mu^a(z) t^a \right].
\]

(1)

This is, however, the minimal option which simply reflects the fact that color vectors cannot be compared at a distance. The gauge potential \(A^a_\mu\) as such is spin-blind; hence one loses any information about the transfer of the spin degrees of freedom along the contour \(\mathcal{C}\). Therefore, to include a direct spin interaction, one has to include into the gauge link an additional term proportional to the gluon strength tensor \(F_{\mu\nu}^a\) (the so-called Pauli term) that explicitly accommodates the spin-dependent interactions. These non-minimal, i.e., enhanced gauge links, generalized with the inclusion of the Pauli contribution \(\sim F_{\mu\nu}^a J_{\mu\nu}\) (with \(J_{\mu\nu} = (1/4)[\gamma_{\mu}\gamma_{\nu}]\)) are normally ignored. This simplification appears natural in the case of, e.g., integrated (collinear) parton distribution functions, where the integration path is trivial and goes along a straight lightlike line.

On the other hand, operator definitions of the unintegrated transverse-momentum dependent parton distributions (TMDs) [1] contain a compound of longitudinal and

---

1 Talk given at the XIX International Workshop on Deep Inelastic Scattering (DIS 2011), 11 - 15 Apr 2011, Thomas Jefferson National Accelerator Facility, Newport News (VA)
transverse (at light-cone infinity) gauge links, the color structure and the space-time setup of which may be rather complicated [2]. In the latter case, the non-triviality of the integration contours makes it crucial to take into account contributions of the non-minimal spin-dependent terms. In lattice realizations of TMDs, the spin-dependent terms in the Wilson lines can also produce a significant effect for various choices of the integration path on the lattice [3, 4].

To justify the introduction of the enhanced Wilson lines, let us imagine two orthogonal “spaces”, with a cross-talking between a pair of quantum fields. The first “space” is the color space, where such a binary relation is accomplished in terms of the minimal Wilson lines in the fundamental or adjoint representation of $SU(3)_c$. The spin correlations (generated by the Pauli terms) are, in contrast, defined in the second “space”. From straightforward power-counting we conclude that the spin-dependent terms are of the nonleading-twist with respect to the spin-blind ones. However, our analysis demonstrates that the inclusion of the Pauli terms, though invisible in the completely unpolarized TMDs, can affect significantly a number of polarized distributions, e.g., those responsible for time-reversal-odd phenomena, such as single-spin asymmetries [4]. Adopting this encompassing idea of gauge invariance, we have to clarify whether the definition of TMDs, we proposed before in Refs. [5], has to be modified and to which extent the incorporation of the Pauli term has phenomenological consequences, for instance, for the UV evolution of TMDs.

In this talk, we report on the first results of our recent study of the renormalization-group properties (anomalous dimensions) of the TMD distribution functions with enhanced gauge links [6]. Because the UV properties of this matrix element are independent of specific hadronic states, we consider the “quark-in-a-quark” TMD. According to our generalized concept of gauge invariance, the unsubtracted distribution function (i.e., that without a soft-term supplement) of a quark with momentum $k$ and flavor $i$ in a quark with momentum $p$ reads

\[ \mathcal{F}_{\Gamma}^{i/q}(x, k_\perp) = \frac{1}{2} \text{Tr} \int dk^- \int \frac{d^4 \xi}{(2\pi)^4} e^{-ik^+ \xi} \langle p | \bar{\psi}_i(\xi) \left[ [\xi^-, \xi_\perp; \infty^-, \xi_\perp] \right] \Gamma \left[ [\infty^-, \xi_\perp; \infty^-, \infty_\perp] \right] \bar{\psi}(0) | p \rangle \]

where $\Gamma$ denotes one or more $\gamma$-matrices and corresponds to the particular distribution under consideration. The state $|p\rangle$ stands for the quark target state.

An important comment about definition (2) is in order. We started from the “fully unintegrated” correlation function, which depends on all four components of the parton’s momentum [7]. Thus, the TMD PDF is obtained after performing the $k^+$ integration that formally renders the coordinate $\xi^+$ equal to zero $\int dk^- e^{-ik^- \xi^+} = 2\pi \delta(\xi^+)$. However, this operation may produce additional divergences because, carrying it out, all quantum fields involved (quarks and gluons) are defined on the light ray $\xi^+=0$. This means that the plus light-cone coordinates of the product of two quantum fields always coincide. The appropriate treatment of this delicate issue is discussed in detail in [6].

We define the enhanced longitudinal gauge link along the $x^-$ direction:

\[ [[\infty^-, 0_\perp; 0^-, 0_\perp]] = \mathcal{P} \exp \left[ -i g \int_0^\infty d\sigma \ u_\mu A^\mu_{a}(u\sigma)t^a - i g \int_0^\infty d\sigma \ J_{\mu\nu} F^\mu\nu_{a}(u\sigma)t^a \right] \] (3)
and the enhanced *transverse* gauge link:

\[
[\mathcal{I}^-; \mathcal{I}^\perp; \mathcal{I}^-; 0] = \mathcal{P} \exp \left[ -ig \int_0^\infty d\tau \mathbf{l}_\perp \cdot \mathbf{A}^a\parallel (l\tau) t^a - ig \int_0^\infty d\tau J_{\mu\nu} F^{\mu\nu}_a (l\tau) t^a \right],
\]

where the two-dimensional vector \( l \equiv l_\perp \) is arbitrary. We make use of the following reparameterization of the (initially dimensionless) vectors defining the path of the integration as \( n^+_{\mu} \rightarrow u^+_{\mu} = \frac{1}{p^+} n^+_{\mu} , \quad n^-_{\mu} \rightarrow u^-_{\mu} = p^+ n^-_{\mu} \), which implements boosts in the longitudinal directions. The plus-component of the momentum \( p \) is large in the given kinematics and is the only momentum scale in our reparameterization\(^2\). The enhanced Wilson lines, introduced above, viz., Eqs. (3) and (4) set up a corner stone of the concept of generalized gauge invariance in the operator formalism of the TMDs.

Let us now briefly describe some of the quantitative results obtained within this framework (see \([6]\) and our previous works in \([5]\) for technical details). To analyze the UV divergences in the leading \( \alpha_s \)-order, one has to evaluate the diagrams displayed in Fig. 1. The corresponding Feynman rules are summarized in Fig. 2.

**FIGURE 1.** One-loop Feynman graphs contributing to the TMD (2). Double lines denote minimal gauge links, while those with a ring represent enhanced gauge links with Pauli contributions. Fermions and gluons are shown as solid and curly lines, respectively. Graphs (a), (b), (c), and (d) describe virtual gluon corrections; graphs (e), (f), and (g) represent real-gluon exchanges.

As a result, the non-trivial UV-singular (in the dimensional regularization) contribution arises from the cross-talking of the gauge fields belonging to the transverse *minimal* and longitudinal *enhanced* Wilson lines, e.g., from diagram Fig. 1(d). In the case of the twist-two Dirac structures \( \Gamma_{\text{tw} - 2} = \{ \gamma^+, \gamma^+ \gamma^5, i\sigma^i \gamma^5 \} \), the corresponding singular terms cancel by their “mirror” (Hermitean conjugated) counterparts. In contrast, the twist-three TMDs (say, \( \Gamma_{\text{tw} - 3} = \gamma^i \)) get from these terms non-trivial UV divergent

\(^2\) An analogous definition holds for the \( x^+ \) direction, provided one makes the replacement \( u \rightarrow u^* \).
contributions of the type

$$\Gamma_{\text{tw}-3} \langle A_{\perp}^+ F^- \rangle + \langle A_{\perp}^+ F^- \rangle \Gamma_{\text{tw}-3} = -C_F \frac{1}{4\pi} \left[ \gamma^+, \gamma^+ \right] \Gamma(\epsilon) \left( 4\pi \frac{\mu^2}{\lambda^2} \right)^{\epsilon},$$

where $$\langle A_{\perp}^+ F^- \rangle$$ stands for the result of the calculation of the diagram in Fig. 1(d).

To conclude, we presented a new framework [6] for completely gauge-invariant TMDs which takes into account explicitly the spin degrees of freedom of quantum particles by means of the Pauli term in the Wilson lines. Let us stress that because the spin-dependent terms contribute to the UV anomalous dimensions of the twist-three TMDs, their evolution appears to be non-trivial, compared to the RG-properties of the TMDs with minimal Wilson lines. This calls for the modification of the renormalization procedure to preserve the parton-number interpretation of TMDs that deserves a dedicated investigation in the future.

REFERENCES

1. D. E. Soper, Phys. Rev. Lett. 43, 1847 (1979); J. C. Collins and D. E. Soper, Nucl. Phys. B 193, 381 (1981); Erratum ibid. B 213, 545 (1983); Nucl. Phys. B 194, 445 (1982); J. C. Collins, Acta Phys. Pol. B 34, 3103 (2003); arXiv:1107.4123 [hep-ph].

2. C. J. Bomhof and P. J. Mulders, Nucl. Phys. B 795, 409 (2008); C. J. Bomhof, P. J. Mulders and F. Pijlman, Eur. Phys. J. C 47, 147 (2006).

3. B. U. Musch, Ph. Hägler, J. W. Negele and A. Schäfer, AIP Conf. Proc. 1350, 321 (2011); Phys. Rev. D 83, 094507 (2011).

4. D. Boer et al., arXiv:1108.1713 [nucl-th].
5. I. O. Cherednikov and N. G. Stefanis, Phys. Rev. D 77, 094001 (2008); Nucl. Phys. B 802, 146 (2008); Phys. Rev. D 80, 054008 (2009); N. G. Stefanis and I. O. Cherednikov, Mod. Phys. Lett. A 24, 2913 (2009).

6. I. O. Cherednikov, A. I. Karanikas and N. G. Stefanis, Nucl. Phys. B 840, 379 (2010); N. G. Stefanis, I. O. Cherednikov and A. I. Karanikas, PoS LC2010, 053 (2010).

7. R. D. Tangerman and P. J. Mulders, Phys. Rev. D 51, 3357 (1995); J. C. Collins, T. C. Rogers and A. M. Stasto, Phys. Rev. D 77, 085009 (2008).