Optical field modulation on the group delay of chiral tunneling in graphene

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Abstract. The influences of optical fields on the group delay of chiral tunneling in graphene are investigated in real time using the finite-difference time-domain method. The group delay of tunneling electrons irradiated by an optical field is significantly different from that observed in traditional quantum tunneling. We found that when the barrier width increases, the group delay becomes constant for the reflected wave packet, but increases linearly for the transmitted wave packet. This peculiar tunneling effect can be attributed to current leakage in a time-dependent barrier generated via the optical Stark effect.

Quantum tunneling time has received much attention since MacColl pointed out that it takes no approximate time ([1]; [2, 3] and references therein). The question of how long it takes an electron to tunnel through a potential barrier is still replete with controversy. The debates center around the definition of tunneling time and its exact physical meaning. Hartman [4] calculated the group delay or phase time and found that the group delay $\tau_g$ becomes constant, while the barrier length increases. Thus, with a wider barrier, superluminal group velocities can be observed. Recently, Winful [3, 5, 6] proposed that the group delay in tunneling represents a lifetime of stored energy escaping through both sides of the barrier and does not represent a transit time. However, a unlimited group velocity is not a meaningful concept in tunneling and does not imply superluminality.

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In the meantime, many optical and acoustic experiments have been carried out to determine group delay. Steinberg et al [7] measured the time delay for a photon to tunnel across a one-dimensional (1D) photonic band-gap material. They reported that the group velocity for single-photon tunneling is about $1.7c_0$, where $c_0$ is the speed of light in vacuum. The authors attributed superluminality to the fact that the wavepacket may be reshaped in the tunneling process. Even though the exact physical meaning of these experimental results is controversial, there is no doubt that there is a finite duration for the photon or phonon tunneling process [3–9]. However, few direct experiments have shown that there is finite group delay in the quantum particle tunneling process.

There are some difficulties in the measurement of the group delay of electrons. For instance, the coherence time of electrons must be long enough to ensure that the tunneling process is coherent transport. It seems that graphene is an ideal candidate material for this. The reported electron mobility in graphene at room temperature is in excess of $15,000 \text{cm}^2\text{V}^{-1}\text{s}^{-1}$, and the limit of electron mobility at room temperature is about $200,000 \text{cm}^2\text{V}^{-1}\text{s}^{-1}$ in theory [10–12]. In single-layer graphene, perfect transmission through a potential barrier in the normal direction is expected [13–17], and there is no Hartman effect [18, 19].

However, if the barrier is irradiated by an intense optical field, the conduction band and the valence band are mixed, the chiral symmetry of the Dirac electrons is broken and a dynamical gap opens [20–22]. For instance, at the resonant point, the magnitude of the light-induced gap is given by $2\Omega_k$, where $\Omega_k$ is the Rabi frequency [20–23], and the wave functions of these dressed states can be written as the superposition of the conduction wave function $\Psi_+$ and the valence wave function $\Psi_-$, i.e. $\Psi_{dr} = \Psi_+ + \Psi_- = (1, 0)^\top$ [20, 23], perfect tunneling is strongly suppressed. Thus, the Hartman effect may take place in this case and therefore the tunneling dynamics requires further exploration. Furthermore, because of the existence of zero-point fields, the vacuum state is not truly empty, but instead contains electromagnetic fields. The actual tunneling processes are combined with strong electromagnetic waves. Thus, the influence of electromagnetic fields on the group delay must be carefully treated. In addition, the external electromagnetic field leads to a time-dependent barrier potential, which is quite different from the steady one. For instance, for an opaque rectangular barrier with a small time-dependent modulation, the traversal time of Büttiker and Landauer [24] is proportional to the barrier width $D$.

In this paper, we investigate the influence of electromagnetic fields on the group delay of Dirac electrons in graphene. We consider a rectangular potential barrier with height $V_0$ and width $D$ (see figure 1(a)). The Fermi level lies in the valence band inside the barrier and in the conduction band outside the barrier. A $Y$-direction polarized laser beam is propagated perpendicular to the layer surface with a detuning $\Delta_0 = E_h - \hbar \omega$, where $E_h = 2(V_0 - E_k)$ (see figure 1(a)) with $E_k$ being the Fermi level. We choose $\Delta_0 > 0$ to ensure that there is no interband absorption inside the barrier and $\hbar \omega \ll 2E_k$ to guarantee that the influence of the optical field outside the barrier can be neglected. Thus, neglecting the scattering between different valleys [21, 25], the scattering process of Dirac electrons in the $K$ point is described by the time-dependent Dirac equation [20]

$$i\hbar \frac{\partial}{\partial t} \Psi(r, t) = [H_0 + V(r) + H_{\text{int}}] \Psi(r, t),$$

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where $\Psi(\mathbf{r}, t) = [C_A(\mathbf{r}, t), C_B(\mathbf{r}, t)]$ is the wave function, $\mathbf{H}_0 = -i\hbar\mathbf{v}_F\sigma \cdot \nabla$ is the unperturbed Dirac Hamiltonian, $\sigma = (\sigma_x, \sigma_y)$ are the Pauli matrices, $v_F \approx 10^6$ m s$^{-1}$ is the Fermi velocity, in the barrier $V(\mathbf{r}) = V_0$ and $V(\mathbf{r}) = 0$ for outside the barrier, $\mathbf{I}$ is the unit matrix and $\mathbf{H}_{\text{int}}$ is the interaction Hamiltonian. $\mathbf{H}_{\text{int}}$ can be written as [26]

$$\mathbf{H}_{\text{int}} = -\hbar e\mathbf{v}_F (A_x\sigma_x + A_y\sigma_y) = \hbar \begin{pmatrix} 0 & V_{12} \\ V_{21} & 0 \end{pmatrix},$$

(2)

where $e$ is the electron charge and $(A_x, A_y)$ are the vector potentials of the electromagnetic field. We should also point out that the interaction Hamiltonian $\mathbf{H}_{\text{int}}$ does not commute with the helicity operator $\hat{h} = \frac{i}{2} \sigma \cdot \mathbf{p}/|\mathbf{p}|$. Thus if the tunneling electrons are irradiated by an optical field, the helicity values are not good quantum numbers. Perfect chiral tunneling is suppressed, and the Hartman effect may take place.

In order to study such a time-dependent scattering process, we employ the finite-difference time-domain (FDTD) method to solve the time-dependent Dirac equation numerically [20]. Different from the Crank–Nicolson method and the Trotter–Suzuki formulae, no large matrix equations need to be stored and solved for the FDTD method. And as a time-domain technique, the FDTD method can demonstrate the propagation of a wave packet through a model in real time [27]. In the FDTD method, the time-dependent Dirac equation at the $K$ point is replaced by a finite set of finite differential equations [20]

$$C^{k+1/2}_A(i) \left[ \frac{1}{\Delta t} - \frac{V(i)}{2i} \right] = \left[ \frac{1}{\Delta t} + \frac{V(i)}{2i} \right] C^{k-1/2}_A(i) - \left[ \frac{v_F}{\Delta x} - \frac{V_{12}(i + 1/2)}{2i} \right] C^k_B(i + 1/2)$$

$$+ \left[ \frac{v_F}{\Delta x} + \frac{V_{12}(i - 1/2)}{2i} \right] C^k_B(i - 1/2),$$

(3a)
The group delay is about $10^{12}$, the barrier width $D$ is equal to the period of the optical field. This is because the amplitude of the reflected wave packet is modulated by the optical field very slightly with frequency equal to $\omega$. We can detect the distortion, a relatively long pulse is used: the peak position $t_0 = 1.2$ ps, and the pulse width $\tau = 0.8$ ps. From figure 1(c), we found that when the sample is irradiated by an intense nonresonant laser beam, a reflected wave packet appears and perfect transmission is suppressed.

Under an intense optical field, the light-induced band shift can also create a dynamic gap at small detuning. As is seen in traditional quantum tunneling (e.g., the Hartman effect), the group delay of Dirac electrons should be constant while the barrier width increases. To verify this, we studied the group delay (i.e. the delays of the peaks of the reflected and transmitted pulses) of Dirac electrons through a potential barrier generated by an intense light beam. In order to reduce the distortion, a relatively long pulse is used: the peak position $t_0 = 5.0$ ps and the pulse width $\tau = 3.3$ ps.

Figure 2(a) shows the group delay for the reflected wave packet as a function of barrier width $D$ for different pump intensities. As seen in traditional quantum tunneling, the group delay is saturated by increasing the barrier length. For $I_\omega = 20$ MW cm$^{-2}$, $I_\omega = 30$ MW cm$^{-2}$ and $I_\omega = 50$ MW cm$^{-2}$, the corresponding saturated delays are about 0.112, 0.0893 and 0.0662 ps, respectively. The delay can also be explained by the tunneling depth or dwell time. As shown in figure 2(c), the spatial probability density distributions in the barrier are well fitted by using the exponential function $F = A_0 e^{-x/x_0}$. For $I_\omega = 20$ MW cm$^{-2}$, $I_\omega = 30$ MW cm$^{-2}$ and $I_\omega = 50$ MW cm$^{-2}$, the corresponding tunneling depths $x_0$ are 55, 44 and 33 nm, respectively. In this way, we extract that the corresponding tunneling delays $t_0 = 2x_0/v_F$ are about 0.11, 0.088 and 0.066 ps, respectively. These results are consistent with the saturated delay obtained in figure 2(a). Another interesting phenomenon is that the group delay for the reflected wave packet takes on the quantized values $\tau = nT_1$ ($n = 1, 2, 3, \ldots$). We found that $T_1 \approx 0.011$ ps is equal to the period of the optical field. This is because the amplitude of the reflected wave packet is modulated by the optical field very slightly with frequency equal to $\omega$. We can detect the peak of the reflected pulse only when the Gaussian wave packet and the modulation induced by the optical field achieve the maximum simultaneously.

The case is quite different for the transmitted wave packet. As shown in figure 2(b), when the barrier width $D > 900$ nm, the group delay increases linearly with increasing barrier width. The group velocity is about $10^6$ m s$^{-1}$ and is the same as the Fermi velocity of the Dirac electrons in graphene. This result can be explained by the time-dependent potential barrier. In a time-dependent potential barrier, the energy storage depends on time. Variations in energy storage
Figure 2. (a) Group delay for the reflected wave packet and (b) group delay for the transmitted wave packet as a function of barrier width $D$ at different pump intensities for $\Delta_0 = 5$ meV. (c) Snapshots of the spatial probability density distributions on the left side of the barrier for different pump intensities. The solid lines are the best fit using exponential functions. (d) Snapshots of the spatial probability density distribution on the right side of the barrier for $I_\omega = 50$ MW cm$^{-2}$.

will lead to an extra leakage current. The dynamic gap caused by the optical field is therefore not a complete gap, and the probability density distribution on the right side of the barrier is no longer exponential decay (see figure 2(d)). Since the amplitude of the extra leakage current is quite small, it has little effect on the reflected wave packet but still determines the group delay for the transmitted wave packet at large barrier widths. For small barrier widths, since the tunneling current is much larger than the extra leakage current, the time delays of the reflected and transmitted wave packets are equal. From figure 2(b), we see that the amplitude of the extra leakage current strongly depends on the pump intensity. Specifically, for $I_\omega = 50$ MW cm$^{-2}$, the group delay increases linearly with increasing barrier width when $D > 700$ nm, but for $I_\omega = 20$ MW cm$^{-2}$ the linearly increasing delay appears when $D > 900$ nm. However, the group velocity of the extra leakage current is independent of pump intensity. Thus, even with quite a small time-dependent modulation, no ‘Hartman effect’ occurs.

The existence of current leakage in a time-dependent barrier can also be confirmed by the shape of tunneled pulses. In a thin barrier, the amplitude of the tunneling current is much larger than the amplitude of the extra leakage current induced by the time-dependent modulation. The group delay of the transmitted and reflected wave packets is equal (see figure 3(a)), and the distortion is quite small (see the inset of figure 3(a)). As the thickness of the barrier increases, the tunneling rates decrease rapidly. In a thick barrier, the amplitude of the extra leakage current is comparable with that of the tunneling current. Since the group delays of the extra leakage current and the tunneling current are different, a serious distortion can be found in the tunneled pulse (see figure 3(b)). However, if the width of the barrier is large enough (e.g. $D = 900$ nm), the tunneling rate is very small, and the extra leakage current is the main contributor to the
Figure 3. Incident (black lines), tunneled (green lines) and reflected (red lines) pulses with pump intensity $I_\omega = 30 \text{ MW cm}^{-2}$ for different barrier widths (a) $D = 100 \text{ nm}$, (b) $D = 500 \text{ nm}$ and (c) $D = 900 \text{ nm}$. The inset shows the normalized tunneled pulse overlaid with the incident pulse.

tunneled pulse. Thus, the group delays of the transmitted and reflected wave packets are different (see figure 3(c)), and there is no distortion on this scale (see the inset of figure 3(c)).

The tunneling delay for a time-dependent potential barrier might help us to understand quantum tunneling. For example, traditional definitions of group delays are unsuitable for the system we studied. In the traditional definitions, the group delay of the transmitted and reflected wave packets are equal for a symmetrical barrier. More importantly, if the quantum fluctuation or the zero-point field is included, all potential barriers are time dependent.

Finally, we turn to the discussion on the experimental realization for our theory prediction. To measure the group delay, the time-resolved optical spectroscopy techniques can be used, e.g. the injected electrons can be generated or controlled via a two-color optical coherence pump [26, 28–30], the barrier can be manipulated by the ultrafast laser beam as we have shown in this paper, and the tunneling current can also be detected directly in real time by using the two-color optical coherence absorption spectra [31]. The very long phase coherence length ($\sim 3–5 \mu\text{m}$) in graphene makes the detection of group delay feasible [32].

In conclusion, we have calculated the influence of optical fields of group delay of electrons in graphene using the FDTD method. We found that the group delay of the reflected packet is also saturated as the barrier width increases. However, the delay increases linearly with barrier length for the transmitted wave packet. This peculiar tunneling effect is attributed to current leakage in a time-dependent barrier generated via the optical field. Thus, the zero-point field may have a significant influence on the group delay of electrons, and should be carefully treated. These unique transport properties of dressed Dirac electrons in graphene might be important for understanding quantum tunneling.
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