Chiral Perturbation Theory for Hadrons Containing a Heavy Quark: The Sequel

Peter Cho
Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

Charm and bottom mesons and baryons are incorporated into a low energy chiral Lagrangian. Interactions of the heavy hadrons with light octet Goldstone bosons are studied in a framework which represents a synthesis of chiral perturbation theory and the heavy quark effective theory. The differential decay rate for the semileptonic process \( \Lambda_b^0 \to \Sigma_c^{++} + e^- + \nu_e + \pi^- \) is calculated at the zero recoil point using this hybrid formalism.
Chiral perturbation theory and the heavy quark effective theory represent two descriptions of hadronic physics that become exact in opposing limits of QCD \cite{1}. The first is based upon a global $SU(3)_L \times SU(3)_R$ symmetry which is spontaneously broken by the strong interactions to the diagonal subgroup $SU(3)_{L+R}$. The original chiral and residual flavor symmetries are only approximate, for they are explicitly violated by quark masses. However, since the masses of the three lightest quarks are small compared to the strong interaction scale $\Lambda_{QCD}$, these symmetries are reasonably accurate in the real world and are fully restored in the zero quark mass limit. The second is derived from an approximate $SU(6)$ spin-flavor symmetry which results from the masses of the three heavy quarks in the standard model being large relative to $\Lambda_{QCD}$. This spin-flavor $SU(6)$ becomes exact in the infinite quark mass limit.

Both of these effective theories are well-established and have been widely studied in separate contexts. Recently however, a synthesis of the two has been proposed \cite{2,3}. Interactions of heavy mesons with light Goldstone bosons have been discussed in a chiral Lagrangian framework. Applications of this new hybrid formalism to semileptonic $B$ and $D$ decays with slow pion emission have been considered. In addition, $SU(3)$ breaking contributions to heavy meson decay constant ratios as well as $B-\bar{B}$ mixing matrix elements have been analyzed \cite{4}. In this letter, we incorporate baryons containing a single heavy quark into this picture and investigate semileptonic transitions among these hadrons.  

To begin, we briefly review the standard procedure for constructing low energy chiral Lagrangians \cite{5,6,7}. The Goldstone bosons in the pion octet

$$\pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta \\ \pi^- \\ -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta \\ K^0 \\ K^- \\ K^0 \\ -\sqrt{\frac{2}{3}} \eta \end{pmatrix}$$

are first arranged into the exponentiated matrix functions $\Sigma = e^{2i\pi/f}$ and $\xi = \sqrt{\Sigma} = e^{i\pi/f}$. The parameter $f \approx 93$ MeV that enters into these definitions corresponds at lowest order to the pion decay constant. The exponentiated fields transform under the chiral symmetry group as

$$\Sigma \rightarrow L \Sigma R^\dagger$$

$$\xi \rightarrow L \xi U^\dagger = U \xi R^\dagger$$

\footnote{Similar work has recently been reported in ref. \cite{3}.}
where $L$ and $R$ represent global $SU(3)_L$ and $SU(3)_R$ transformations. The matrix $U$ is a complicated nonlinear function of $L,R$ and $\pi$ which acts like a local transformation under the diagonal flavor subgroup. Chiral invariant terms can then be built up from the fields in (1) and their derivatives. To leading order in a derivative expansion, the phenomenological Lagrangian that describes the self interactions of the Goldstone bosons is simply

$$\mathcal{L}^{(0)} = \frac{f^2}{4} \text{Tr}(\partial^\mu \Sigma \partial_\mu \Sigma) + \frac{f^2}{2} \text{Tr}(\Sigma^\dagger \mu M + \mu M^\dagger \Sigma).$$  \hspace{1cm} (2)$$

Explicit chiral and flavor symmetry breaking effects are represented in this Lagrangian by the constituent mass parameter $\mu$ and the current quark mass matrix

$$M = \begin{pmatrix} m_u & m_d & m_s \end{pmatrix}.$$  

Meson and baryon matter fields can generally be included into the effective Lagrangian. Their interactions with the pion octet are governed solely by their light flavor symmetry properties. The Goldstone bosons couple derivatively to matter fields through the vector and axial vector combinations

$$V^\mu = \frac{1}{2} (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger) = \frac{1}{2f^2} [\pi, \partial^\mu \pi] - \frac{1}{24f^4} [\pi, [\pi, [\pi, \partial^\mu \pi]]] + O(\pi^6)$$  \hspace{1cm} (3a)$$

$$A^\mu = \frac{i}{2} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger) = -\frac{1}{f} \partial^\mu \pi - \frac{1}{6f^4} [\pi, [\pi, \partial^\mu \pi]] + O(\pi^5).$$  \hspace{1cm} (3b)$$

The vector acts like an $SU(3)_{L+R}$ gauge field

$$V^\mu \to U V^\mu U^\dagger + U \partial^\mu U^\dagger$$

while the axial vector simply transforms as an $SU(3)_{L+R}$ octet:

$$A^\mu \to U A^\mu U^\dagger.$$  

We would specifically like to incorporate hadrons that contain a single heavy quark $Q$. Following the approach developed for the heavy quark effective theory [9], we work with velocity dependent fields whose interactions are constrained by an $SU(2)_v$ spin symmetry group. We start with the operators $P$ and $P^*$ that annihilate $J^P = 0^-$ and $1^-$ mesons with quark content $Q\bar{Q}$. If the heavy quark constituent is charm, the individual components of these fields are

$$P_1 = D^0 \quad P_2 = D^+ \quad P_3 = D^+_s \quad P^*_1 = D^{*0} \quad P^*_2 = D^{*+} \quad P^*_3 = D^{s*+}.$$  

2
The pseudoscalar and vector meson operators can be combined into the $4 \times 4$ matrices

$$H_i(v) = \frac{1 + \not{v}}{2} \left[ -P_i \gamma^5 + P^s_{i\mu} \gamma^\mu \right]$$

$$\bar{H}^i(v) = \left[ P^i \gamma^5 + P^s_{i\mu} \gamma^\mu \right] \frac{1 + \not{v}}{2}.$$  \hfill (4)

$H$ transforms as an antitriplet matter field under $SU(3)_{L+R}$

$$H_i \rightarrow H_j (U^\dagger)^j_i$$

and as a doublet under $SU(2)_v$:

$$H \rightarrow e^{i \vec{\epsilon} \cdot \vec{S}_v} H.$$  \hfill (5)

The spin symmetry rotates the $P$ and $P^s_{i\mu}$ operators in (4) into one another.

We also include baryons with quark content $Qqq$ into the chiral Lagrangian. The light degrees of freedom inside these hadrons carry either one or zero units of angular momentum. In the former case, the resulting $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ baryons are degenerate in the infinite quark mass limit and can be assembled into the matrices [12]

$$S^{ij}_\mu(v) = \sqrt{\frac{1}{3}} (\gamma_\mu + v_\mu) \gamma^5 \frac{1 + \not{v}}{2} B^{ij} + \frac{1 + \not{v}}{2} B^s_{i\mu}$$

$$\bar{S}^{ij}_\mu(v) = -\sqrt{\frac{1}{3}} B^{ij} \frac{1 + \not{v}}{2} \gamma_\mu (\gamma_\mu + v_\mu) + B^s_{ij} \frac{1 + \not{v}}{2}.$$  \hfill (5)

The $S$ field obeys the constraints $v_\mu S^{ij}_\mu = 0$ and $\not{v} S^{ij}_\mu = S^{'ij}_\mu$. It transforms as an $SU(3)_{L+R}$ sextet

$$S^{ij}_\mu \rightarrow U^i_i U^j_j S^{'ij}_\mu$$

and is an axial vector under parity. When the heavy quark is taken to be charm, the components of the Dirac spinor operators $B^{ij}$ in eqn. (6) are the $J^P = \frac{1}{2}^+$ baryons

$$B^{11} = \Sigma_c^{++} \quad B^{12} = \sqrt{\frac{1}{2}} \Sigma_c^+ \quad B^{22} = \Sigma_c^0$$

$$B^{13} = \frac{1}{2} \Xi^+_c \quad B^{23} = \sqrt{\frac{1}{2}} \Xi^0_c \quad B^{33} = \Omega^0_c.$$  \hfill (6)

\footnote{Chiral perturbation theory for baryons containing no heavy quarks has been thoroughly studied in ref. [11]. Many of the static fermion techniques described in that body of work are similar to those used here.}
Their spin-$\frac{3}{2}$ counterparts appear in the Rarita-Schwinger field $B^{\star}_{\mu i j}$ which satisfies $\gamma^\mu B^*_{\mu} = 0$. These different spin components are transformed into one another under the action of $SU(2)_v$:

$$S_\mu \rightarrow e^{i \vec{\epsilon} \cdot \vec{S}_v} S_\mu.$$

The remaining heavy baryons whose light spectator degrees of freedom have zero angular momentum are assigned to the matrix

$$T_i(v) = \frac{1 + \delta}{2} B_i$$

which is an $SU(3)_{L+R}$ antitriplet:

$$T_i \rightarrow T_j (U^\dagger)_i^j.$$

Its conjugate field is simply

$$\bar{T}^i(v) = \bar{B}_i \frac{1 + \delta}{2}.$$

The components of the singly charmed $B_i$ operators are the $J^P = \frac{1}{2}^+$ baryons

$$B_1 = \Xi^0_c \quad B_2 = -\Xi^+_c \quad B_3 = \Lambda^+_c.$$

The $SU(2)_v$ symmetry rotates the spins of these baryons which come entirely from their heavy quark constituents:

$$T \rightarrow e^{i \vec{\epsilon} \cdot \vec{S}_v} T.$$

Before displaying the heavy hadron contributions to the low energy Lagrangian, we should mention the power counting rules that can be used to estimate the sizes of their coefficients \[7,14\]. Each term begins proportional to $f^2 \Lambda^2$ and has the factors

\begin{itemize}
  \item $1/f$ for each strongly coupled light boson,
  \item $\sqrt{M}/f \sqrt{\Lambda}$ for each strongly coupled boson containing a heavy quark of mass $M$,
  \item $1/f \sqrt{\Lambda}$ for each strongly coupled light fermion,
  \item $1/\Lambda$ for each derivative or dimension one symmetry breaking term.
\end{itemize}

\[3\] In the absence of a universally accepted nomenclature convention for distinguishing between the isospin-$\frac{1}{2}$ $\Xi_Q$ states in the sextet and antitriplet multiplets, we have followed ref. \[13\] and denoted the heavier sextet states with a prime.
Here $\Lambda \approx 4\pi f \approx 1$ GeV represents the chiral symmetry breaking scale. The mass of a heavy meson can be substituted in place of the mass of its heavy quark at lowest order. All dependence upon heavy meson masses can subsequently be removed from the zeroth order Lagrangian via the redefinition $H' = \sqrt{M_H} H$. The meson field $H'$ then has mass dimension $\frac{3}{2}$ like the $S$ and $T$ baryon fields.

We can now write down all the leading order terms in the chiral Lagrangian which are hermitian, Lorentz invariant, parity even, symmetric under heavy quark spin $SU(2)_v$ and light flavor $SU(3)_{L+R}$, and baryon number conserving:

$$L_v^{(0)} = \sum_{\text{Heavy Flavors}} \left\{ -i \text{Tr}(H' v \cdot D H'_i) - i S_{ij}^\mu v \cdot D S_{ij}^\mu + i T^i v \cdot D T_i 
+ g_1 \text{Tr}(H'_i (A)^{i j}_j \gamma^5 T^j) 
+ i g_2 \varepsilon_{\mu\nu\sigma\lambda} S_{ik}^\mu v_\nu (A^\sigma)^i_j (S^\lambda)^{jk} 
+ g_3 [\varepsilon_{ijk} T^i (A^\mu)^j_l S^{kl}_\mu + \varepsilon^{ijk} S_{kl}^\mu (A_\mu)^i_j T_l] \right\}. \tag{8}$$

A few points should be noted. Firstly, the matter field covariant derivatives are constructed from the Goldstone boson vector current in (3a):

$$D^\mu H'_i = \partial^\mu H'_i - H'_j (V^\mu)^j_i$$
$$D^\mu S_{ij}^\nu = \partial^\mu S_{ij}^\nu + (V^\mu)^i_k S_{kj}^{ik} + (V^\mu)^j_k S_{ik}^{jk}$$
$$D^\mu T_i = \partial^\mu T_i - T_j (V^\mu)^j_i.$$ 

Partial derivatives acting on the velocity dependent fields which are only slightly off-shell yield small residual momenta. Secondly, the signs in front of the kinetic terms have been chosen so that the meson and baryon components of $H', S$ and $T$ are conventionally normalized. Notice that the spin-$\frac{3}{2}$ Rarita-Schwinger fields inside $S_{ij}^\mu$ enter into the kinetic part of (8) with opposite sign to their spin-$\frac{1}{2}$ counterparts. Thirdly, we have neglected the mass difference between the sextet and antitriplet multiplets in this zeroth order Lagrangian. The mass difference is phenomenologically comparable to the mass splittings within the multiplets. We consequently regard it as a small correction that should be included with $SU(3)$ breaking effects at next-to-leading order.

Finally, observe that there is no axial vector term for the antitriplet baryons like those for the mesons and sextet baryons in Lagrangian (8). Candidate terms such as $\overline{T} A \gamma^5 T$ or $\overline{T} v A \gamma^5 T$ either break the spin symmetry or vanish. One can understand why such an
axial vector interaction cannot exist by considering a representative process which it would mediate:

$$\Lambda_Q = Q(qq) \rightarrow \Lambda_Q = Q(qq) + \eta$$

| $S_{\text{heavy}}$ | $S_{\text{light}}$ | $P$ |
|------------------|------------------|-----|
| $\frac{1}{2}$    | 0                | +   |
| $\frac{1}{2}$    | 0                | +   |
| 0                | 0                | −   |

As a reminder, we have indicated the spins of the heavy quark and the residual light degrees of freedom as well as the intrinsic parities of the hadrons involved in this transition. In order to conserve angular momentum, the outgoing hadrons must emerge in an S-wave. But then the parity of the final state does not equal the parity of the initial state. So this hadronic process cannot take place in the infinite quark mass limit of QCD.

Feynman rules can be simply derived from the effective Lagrangian. The Dirac and Rarita-Schwinger spinor sums [15]

$$\Lambda_{+} = \sum_{s=1}^{2} u(v, s) \pi(v, s) = \frac{1 + \not{v}}{2}$$

$$\Lambda_{+}^{\mu \nu} = \sum_{s=1}^{4} U^\mu(v, s) \bar{U}^{\nu}(v, s) = \left[ -g_{\mu \nu} + v_{\mu} v_{\nu} + \frac{1}{3} (\gamma_{\mu} + v_{\mu})(\gamma_{\nu} - v_{\nu}) \right] \frac{1 + \not{v}}{2}$$

along with the polarization sum

$$\Lambda_{+}^{\mu \nu} = \sum_{s=1}^{3} e_{\mu}(v, s) e_{\nu}(v, s)^* = -g_{\mu \nu} + v_{\mu} v_{\nu}$$

appear in the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryon and vector meson propagators $i\Lambda_{+}/(vk)$, $i\Lambda_{+}^{\mu \nu}/(vk)$ and $i\Lambda^{\mu \nu}/(2vk)$ where $k$ denotes the heavy particles’ residual momenta. Interaction vertices are established by expanding the velocity dependent fields and Goldstone boson currents in (8). With the Feynman rules in hand, one may readily compute rates for strong interaction decays of heavy hadrons with single pion emission. Some representative examples are listed below:

$$\Gamma(D^{+*} \rightarrow D^{0} \pi^+) = \frac{g_1^2}{12\pi} \frac{|p_{\pi}|^3}{f^2}$$

$$\Gamma(\Sigma^{++*} \rightarrow \Sigma^{+} \pi^+) = \frac{g_2^2}{72\pi} \frac{|p_{\pi}|^3}{f^2}$$

$$\Gamma(\Sigma^{++*} \rightarrow \Lambda^{+} \pi^+) = \Gamma(\Sigma^{++} \rightarrow \Lambda^{+} \pi^+) = \frac{g_2^3}{12\pi} \frac{|p_{\pi}|^3}{f^2}.$$
In principle, these rates fix the three independent couplings $g_1$, $g_2$ and $g_3$. However, the parameters’ values cannot yet be determined given current experimental data. They are expected to be of order one.

Weak semileptonic $b \rightarrow c$ transitions can also be investigated in this chiral Lagrangian framework. Such processes are governed by the underlying four-fermion interaction

$$\mathcal{L}_{\text{weak}} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_{\ell=e,\mu,\tau} (\bar{\ell} \gamma^\mu P_- \nu_{\ell}) (\bar{c} \gamma_{\mu} P_- b)$$

where $P_- = \frac{1}{2}(1 - \gamma^5)$ denotes a left-handed projection operator. The hadronic current that enters into this weak vertex matches at zeroth order onto an effective current in the low energy theory which is specified in terms of four Isgur-Wise functions \cite{2,12}:

$$\bar{c} \gamma_{\mu} P_- b \rightarrow C_{cb} \left\{ -\xi(vv') \text{Tr}(\mathcal{H}_c(v')\gamma_{\mu} P_- H'_b(v)) ight.$$  
$$- \left[ g_{\alpha\beta} \eta_1(vv') - v_\alpha v'_\beta \eta_2(vv') \right] \mathcal{S}_c^\alpha(v') \gamma_{\mu} P_- S^\beta_b(v)$$  
$$+ \eta(vv') \mathcal{T}_c(v') \gamma_{\mu} P_- T_b(v) \right\}.$$  

Perturbative QCD scaling corrections are absorbed into the prefactor $C_{cb}$. When $v = v'$, the functions $\xi$, $\eta_1$ and $\eta$ equal unity while all dependence upon the remaining $\eta_2$ function disappears. We will confine our attention to the kinematic neighborhood around the zero recoil point in order to take advantage of this tremendous simplification.

As an illustration of the utility of chiral perturbation theory for hadrons containing a heavy quark, we consider $\Lambda_0^b$ semileptonic decays. Such processes are of phenomenological interest since they are among the more readily identifiable bottom baryon transitions that will be measured in the future. We are interested in studying generalizations of the pure semileptonic decay

$$\Lambda_0^b(P; v) \rightarrow \Lambda_+^c(p_1; v) + e^-(p_2) + \nu_e(p_3)$$

that have low momentum Goldstone bosons in the final state. The simplest possibility

$$\Lambda_0^b(P; v) \rightarrow \Lambda_+^c(p_1; v) + e^-(p_2) + \nu_e(p_3) + \eta(p_4)$$

does not occur at lowest order due to the absence of an axial vector coupling to the antitriplet baryons. This process is mediated by $O(1/M_Q)$ operators which break the heavy quark spin symmetry. But predictive power is diminished at next-to-leading order since those operators’ coefficients are unknown.
We consider instead the alternative

\[ \Lambda_b^0(P; v) \rightarrow \Sigma_c^{++}(p_1; v) + e^-(p_2) + \bar{\nu}_e(p_3) + \pi^-(p_4). \]  

(10)

The corresponding transition with no final state pion violates both isospin and strong parity of the light degrees of freedom within the heavy hadrons \[13, 16\]. Therefore, decay (10) most likely represents the dominant \( \Lambda_b^0 \rightarrow \Sigma_c^{++} \) semileptonic mode. It proceeds via the three pole diagrams illustrated in fig. 1. Adding these graphs together, squaring the resulting amplitude, and averaging and summing over fermion spins, we obtain the total squared amplitude

\[
\frac{1}{2} \sum_{\text{spins}} |A|^2 = -\frac{16}{27} G_F^2 \left( \frac{g_3}{f} \right)^2 |V_{cb}|^2 C_{cb}^2 \times \left\{ 5[p_2p_4 p_3p_4 - p_2p_4 v p_3 v p_4 - p_3p_4 v p_2 v p_4 + v p_2 v p_3 (v p_4)^2] \\
+ [2p_2p_3 + 9v p_2 v p_3] [p_3^2 - (v p_4)^2] \right\}/(v p_4)^2.
\]

(11)

The differential rate for (10) is then given by [17]

\[
d\Gamma = \frac{1}{8}(2\pi)^{-8} \frac{M_{\Sigma_c}}{M_{\Lambda_b}} \left( \frac{1}{2} \sum_{\text{spins}} |A|^2 \right) |\vec{p}_1| d\Omega_1 |\vec{p}'_1| d\Omega'_1 d\Omega_{234} |\vec{p}'_{234}| d\Omega''_{234} d\Omega_{234}.
\]

(12)

In this expression, \( m_{23} = \sqrt{p_{23}^2} = \sqrt{(p_2 + p_3)^2} \) is the invariant mass of the lepton pair, and \( (|\vec{p}'_{234}|, d\Omega''_{234}) \) stands for the electron’s three-momentum in the rest frame of its virtual \( W^* \) progenitor. Similarly, \( m_{234} = \sqrt{(p_2 + p_3 + p_4)^2} \), and \( (|\vec{p}'_1|, d\Omega_1) \) represents the emitted pion’s momentum in the \( W^* \) and \( \pi^- \) center of mass frame. Finally, \( (|\vec{p}_1|, d\Omega_1) \) denotes the momentum of the recoiling \( \Sigma_c^{++} \) in the \( \Lambda_b^0 \) rest frame which vanishes of course at the zero recoil point.

After boosting to the primed and doubleprimed frames to evaluate the dot products in (11) and performing the angular integrations in (12), we find for the differential width

\[
\frac{1}{|\vec{p}_1|} \frac{d\Gamma(\Lambda_b \rightarrow \Sigma_c e\bar{\nu}_e \pi^-)}{dm_{234} dm_{23}} \bigg|_{m_{234} = M_{\Lambda_b} - M_{\Sigma_c}} = \frac{2}{81}(2\pi)^{-5} G_F^2 |V_{cb}|^2 C_{cb}^2 \left( \frac{g_3}{f} \right)^2 \frac{M_{\Sigma_c}}{M_{\Lambda_b}} \frac{m_{23}}{M_{\Lambda_b} - M_{\Sigma_c}} \times \left[ (M_{\Lambda_b} - M_{\Sigma_c} + m_\pi)^2 - m_{23}^2 \right] \left[ (M_{\Lambda_b} - M_{\Sigma_c} - m_\pi)^2 - m_{23}^2 \right] \times \sqrt{[(M_{\Lambda_b} - M_{\Sigma_c})^2 - m_\pi^2]^2 - 2[(M_{\Lambda_b} - M_{\Sigma_c})^2 + m_\pi^2] m_{23}^2 + m_{23}^4} \times \left\{ [(M_{\Lambda_b} - M_{\Sigma_c})^2 - m_\pi^2]^2 + [20(M_{\Lambda_b} - M_{\Sigma_c})^2 - 2m_\pi^2] m_{23}^2 + m_{23}^4 \right\}.
\]
For comparison purposes, we normalize this result to the corresponding zero recoil rate for the pure semileptonic process in (9):

\[
\frac{1}{d\vec{p}_1} \left| \frac{d\Gamma(\Lambda_b \to \Lambda_c e\bar{\nu}_e)}{dm_{23}} \right|_{m_{23}=M_{\Lambda_b} - M_{\Lambda_c}} = 2(2\pi)^{-3}G_F^2 |V_{cb}|^2 C_{cb}^2 \frac{M_{\Lambda_c}}{M_{\Lambda_b}} (M_{\Lambda_b} - M_{\Lambda_c})^3.
\]

The dimensionless ratio of these two differential decay rates

\[
R = \frac{d\Gamma(\Lambda_b \to \Sigma_c e\bar{\nu}_e \pi)/dm_{234}dm_{23}}{d\Gamma(\Lambda_b \to \Lambda_c e\bar{\nu}_e)/dm_{23}} \bigg|_{m_{234}=M_{\Lambda_b} - M_{\Sigma_c}} \tag{13}
\]

is plotted in fig. 2 as a function of the invariant lepton pair mass over its range \(0 \leq m_{23} \leq m_{234} - m_4 = M_{\Lambda_b} - M_{\Sigma_c} - m_\pi\). Since the derivative expansion breaks down as the outgoing pion’s momentum approaches the chiral symmetry breaking scale, the plot can only be trusted near the high end of the \(m_{23}\) range where the leptons carry away most of the released energy. However, one can see from the figure that the rates for the \(\Lambda_0^b\) semileptonic decays with and without final state pion emission are comparable.

Other interesting interactions between very heavy and very light hadrons can be studied using the hybrid chiral Lagrangian formalism. Some questions which cannot be answered by either chiral perturbation theory or the heavy quark effective theory alone may be addressed by their union. The synthesis of the two effective theories therefore broadens the scope of QCD phenomena that can be sensibly investigated.

Acknowledgements

Helpful discussions with Eric Carlson, Howard Georgi, Liz Simmons, Mark Wise and Tung-Mow Yan are gratefully acknowledged. I am especially indebted to Mark Wise for communicating his results prior to publication and for bringing ref. [5] to my attention. (As indicated by the title, this letter is intended to be a close follow-on to Wise’s original work [2].) I would also like to thank Tung-Mow Yan for kindly communicating ref. [3]. Finally, I am grateful to Charles Wohl for providing access to heavy baryon Particle Group data.

This work was supported in part by the National Science Foundation under contract PHY-87-14654 and by the Texas National Research Commission under Grant # RGFY9106.

\[\text{We use the heavy hadron mass values } M_{\Lambda_c} = 2285 \text{ MeV, } M_{\Sigma_c} = 2453 \text{ MeV and } M_{\Lambda_b} = 5640 \text{ MeV [18]}.\]
References

[1] M. Wise, Caltech Preprint CALT-68-1721 (1991), Lectures presented at the Lake Louise Winter Institute.

[2] M. Wise, Caltech Preprint CALT-68-1765 (1992).

[3] G. Burdman and J. F. Donoghue, UMHEP-365 (1992).

[4] B. Grinstein, E. Jenkins, A. Manohar, M. Savage and M. Wise, UCSD/PTH 92-05 (1992).

[5] T.M. Yan, H.Y. Cheng, C.Y. Cheung, G.L. Lin, Y.C. Lin and H.L. Yu, CLNS-92/1138 (1992).

[6] S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239; C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247.

[7] A. Manohar and H. Georgi, Nucl. Phys. B234 (1984) 198; H. Georgi, Weak Interactions and Modern Particle Theory, (Benjamin/Cummings, Menlo Park, 1984).

[8] J. Gasser and H. Leutwyler, Ann. Phys. 158 (1984) 142; J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.

[9] E. Eichten and B. Hill, Phys. Lett. B234 (1990) 511; H. Georgi, Phys. Lett. B240 (1990) 447.

[10] H. Georgi, Heavy Quark Effective Field Theory, HUTP-91-A039 (1991), Lectures delivered at TASI.

[11] E. Jenkins and A. Manohar, Phys. Lett. B255 (1991) 558; E. Jenkins and A. Manohar, Phys. Lett. B259 (1991) 353; E. Jenkins, UCSD/PTH 91-12 (1991); E. Jenkins and A. Manohar, UCSD/PTH 91-30 (1991).

[12] H. Georgi, Nucl. Phys. B348 (1991) 293.

[13] N. Isgur and M. Wise, Nucl. Phys. B348 (1991) 276.

[14] H. Georgi and L. Randall, Nucl. Phys. B276 (1986) 241; H. Georgi, Nucl. Phys. B331 (1990) 311.

[15] H. Umezawa, Quantum Field Theory, (North-Holland, Amsterdam, 1956); T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B355 (1991) 38; A. Falk, SLAC-PUB-5689 (1991).

[16] H. D. Politzer, Phys. Lett. B250 (1990) 128.

[17] Particle Data Review, Phys. Lett. B239 (1990) III.45.

[18] Particle Data Review, to appear in Phys. Rev. D suppl. (1992).
Figure Captions

Fig. 1. Leading order pole diagrams that contribute to the semileptonic process $\Lambda_0^0 \rightarrow \Sigma_c^{++} + e^- + \bar{\nu}_e + \pi^-$. Strong and weak interaction vertices are denoted by solid circles and squares respectively.

Fig. 2. The dimensionless decay rate ratio $g_3^{-2}R$ defined in eqn. (13) plotted as a function of the invariant lepton pair mass $m_{23}$. 