Holographic Dark Energy Scenario and Variable Modified Chaplygin Gas

Surajit Chattopadhyay\textsuperscript{1*} and Ujjal Debnath\textsuperscript{2†}

\textsuperscript{1}Department of Computer Application, Pailan College of Management and Technology, Calcutta-104, India.
\textsuperscript{2}Department of Mathematics, Bengal engineering and Science University, Howrah-103, India.

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In this letter, we have considered that the universe is filled with normal matter and variable modified Chaplygin gas. Also we have considered the interaction between normal matter and variable modified Chaplygin gas in FRW universe. Then we have considered a correspondence between the holographic dark energy density and interacting variable modified Chaplygin gas energy density. Then we have reconstructed the potential of the scalar field which describes the variable modified Chaplygin cosmology.

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The holographic principle emerged in the context of black-holes, where it was noted that a local quantum field theory can not fully describe the black holes [1]. Some long standing debates regarding the time evolution of a system, where a black hole forms and then evaporates, played the key role in the development of the holographic principle [2,3,4]. Cosmological versions of holographic principle have been discussed in various literatures [e.g., 5,6,7]. Easther and (1999)[7] proposed that the holographic principle be replaced by the generalized second law of thermodynamics when applied to time-dependent backgrounds and found that the proposition agreed with the cosmological holographic principle proposed by Fischler and Susskind (Ref [5]) for an isotropic open and flat universe with a fixed equation of state. Verlinde [8] studied the holographic principle in the context of an \((n+1)\) dimensional radiation dominated closed FRW universe. Numerous cosmological observations have established the accelerated expansion of the universe [9,10]. Since it has been proven that the expansion of the universe is accelerated, the physicists and astronomers started considering the dark energy cosmological observations indicated that at about \(2/3\) of the total energy of the universe is attributed by dark energy and \(1/3\) is due to dark matter [11]. In recent times, considerable interest has been stimulated in explaining the observed dark energy by the holographic dark energy model [1,11,12]. An approach to the problem of dark energy arises from the holographic principle stated in the first paragraph. For an effective field theory in a box size \(L\) with UV cutoff \(\Lambda_c\), the entropy \(L^3\Lambda_c^2\). The non-extensive scaling postulated by Bekenstein suggested that quantum theory breaks down in large volume [11]. To reconcile this breakdown, Chohe et al [13] pointed out that in quantum field theory a short distance (UV) cut-off is related to a long distance (IR) cut-off due to the limit set by forming a black hole. Taking the whole universe into account the largest IR cut off \(L\) is chosen by saturating the inequality so that we get the holographic dark energy density as [11] \(\rho_H = 3c^2 M_p^2 L^{-2}\) where \(c\) is a numerical constant and \(M_p \equiv 1/\sqrt{8\pi G}\) is the reduced Plank mass. On the basis of the holographic principle proposed by [5] several others have studied holographic model for dark energy [10]. Employment of Friedman equation [14] \(\rho = 3M_p^2 H^2\) where \(\rho\) is the total energy density and taking \(L = H^{-1}\) one can find \(\rho_m = 3(1 - c^2)M_p^2 H^2\). Thus either \(\rho_m\) or \(\rho_\Lambda\) behaves like \(H^2\). Thus, dark energy results as pressureless. But, neither dark energy, nor dark matter has laboratory evidence for its existence directly. Thus, Cardone et al [15] and Bento et al [16] proposed unified dark matter / energy scenario in which two dark components are different manifestations of a single cosmic fluid. Some interesting examples of such an unification are the generalized Chaplygin gas, the tachyonic field, and the condensate cosmology [15].

The Chaplygin gas is characterized by an exotic equation of state \(p = -B/\rho\) [17], where \(B\) is a positive constant. Role of Chaplygin gas in the accelerated universe has been studied by several authors. The above mentioned equation of state has been modified to \(p = -B/\phi^\alpha\) with \(0 \leq \alpha \leq 1\). This is called generalized Chaplygin gas [18]. This equation has been further modified to \(p =Ap - B/\phi^\alpha\) with \(0 \leq \alpha \leq 1\). This is called modified Chaplygin gas [19]. This equation of state shows radiation era at one extreme and \(\Lambda CD M\) model at the other extreme. Correspondence between the holographic dark energy scenario and the Chaplygin gas is studied in [14,20]. Debnath [21] introduced a variable modified Chaplygin gas with \(B\) as a function of the scale factor \(a\).

\begin{flushright}
* surajit_2008@yahoo.co.in
† ujjaldebnath@yaghoo.com
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Thus, the equation of state is \( p = A \rho - \frac{B(a)}{\rho^\alpha} \). Present paper endeavors to establish a correspondence between the holographic dark energy scenario and the variable modified Chaplygin gas model.

The metric of a homogeneous and isotropic universe in an FRW model is

\[
ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]

(1)

where, \( a(t) \) is the scale factor, and \( k \) denotes the curvature of the space.

The first Friedman equation is given by

\[
H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} [\rho_{\Lambda} + \rho_m]
\]

(2)

Let us define \( \Omega_m = \frac{\rho_m}{3M_p^2H^2} \), \( \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{3M_p^2H^2} \), \( \Omega_k = \frac{k}{a^2H^2} \).

Assuming \( B(a) = B_0a^{-n} \) in the equation of state of the variable modified Chaplygin gas with \( B_0 > 0 \) and \( n(> 0) \) as constant we get the solution \( \rho \) as

\[
\rho_{\Lambda} = \left[ \frac{3(1 + \alpha)B_0}{3(1 + \alpha)(1 + A) - n a^n} + \frac{C_0}{a^{3(1 + A)(1 + \alpha)}} \right]^{\frac{1}{1 + \alpha}}
\]

(3)

The continuity equations for variable modified Chaplygin gas and cold dark matter are

\[
\dot{\rho}_{\Lambda} + 3H(1 + \omega_{\Lambda})\rho_{\Lambda} = -\delta \rho_{\Lambda}
\]

(4)

\[
\dot{\rho}_m + 3H(1 + \omega_m)\rho_m = \delta \rho_{\Lambda}
\]

(5)

where \( \delta \) is the interaction term. This corresponds to the decay of generalized Chaplygin gas component into CDM. Taking \( x = \frac{\rho_m}{\rho_{\Lambda}} \) it is obtained from equation (5) that

\[
\dot{x} = 3Hx \left[ \omega_{\Lambda} - \omega_m + \frac{1 + x}{x} \frac{\delta}{3H} \right]
\]

(6)

We define

\[
w_{\Lambda}^{eff} = w_{\Lambda} + \frac{\delta}{3H}
\]

(7)

\[
w_m^{eff} = -w_m - \frac{1}{x} \frac{\delta}{3H}
\]

(8)

Taking derivative of both sides of equation (3) with respect to cosmic time we obtain

\[
\dot{\rho}_{\Lambda} = 3H \left( C_0 a^{-(1 + A)(1 + \alpha)} + \frac{3(1 + \alpha)a^{-n}B_0}{3(1 + A)(1 + \alpha) - n} \right)^{\frac{1 - \alpha}{1 + \alpha}} \left( -(1 + A)C_0 a^{-3(1 + A)(1 + \alpha)} - \frac{na^{-n}B_0}{3(1 + A)(1 + \alpha) - n} \right)
\]

(9)

In non-flat universe, our choice for holographic dark energy density is

\[
\rho_{\Lambda} = 3c^2 M_p^2 L^{-2}
\]

(10)
Using holographic dark energy density we obtain

\[
\dot{\rho}_\Lambda = -3^{1-\alpha}H \left( (1 + A) C_0 a^{-3(1+\alpha)(1+A)} + \frac{n B_0 a^{-n}}{3(1 + A)(1 + \alpha) - n} \right) \left( c^2 L^2 M_p^2 \right)^{-\alpha} \quad (11)
\]

and

\[
w_\Lambda = A - \frac{B_0 a^{-n}}{(3c^2 M_p^2 L^{-2})^{1+\alpha}} \quad (12)
\]

Using the definition \( \Omega_\Lambda = \frac{\rho_\Lambda}{3 M_p^2 H^2} \) it can be obtained that

\[
HL = \frac{c}{\sqrt{\Omega_\Lambda}} \quad (13)
\]

Here, \( c \) is a positive constant in holographic model of dark energy \( c \geq 1 \). \( L \) is defined as

\[
L = a R(t) \quad (14)
\]

where \( a \) is the scale factor and \( R(t) \) is relevant to the future event horizon of the universe \([14]\) and it can be derived that

\[
L = a(t) \sinh \sqrt{|k|} \frac{R_h(t) / a(t)}{\sqrt{|k|}} \quad (15)
\]

where, \( R_h \) is the event horizon.

Now, equation (11) becomes

\[
\dot{\rho}_\Lambda = -3^{1-\alpha} \frac{c}{L \sqrt{\Omega_\Lambda}} \left( (1 + A) C_0 a^{-3(1+\alpha)(1+A)} + \frac{n B_0 a^{-n}}{3(1 + A)(1 + \alpha) - n} \right) \left( c^2 L^2 M_p^2 \right)^{-\alpha} \quad (16)
\]

Substituting this relation into the interaction equation (4) and taking \( \delta = 3b^2 H \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right) \) we get

\[
w_\Lambda = -1 - \frac{b^2(1 + \Omega_k)}{\Omega_\Lambda} + \left( C_0 (1 + A) - \frac{n a^{-n-3(1+\alpha)(1+A) B_0}{n - 3(1 + \alpha)(1 + A)} \right) \left( 3\Omega_\Lambda M_p^2 H^2 a^{3(1+A)} \right)^{-1-\alpha} \quad (17)
\]

Where, \( B_0 \) comes out to be

\[
B_0 = \frac{3(1 + \alpha)(1 + A) - n}{3(1 + \alpha)} \left( (3c^2 M_p^2 L^{-2})^{1+\alpha} - \frac{C_0}{a^{3(1+\alpha)(1+A)}} \right) a^n \quad (18)
\]

where,

\[
C_0 = \frac{(3H^2 M_p^2 \Omega_\Lambda a^{3(1+A)})^{1+\alpha}}{c(3(1 + A)(1 + \alpha) - n)} \left( 3b^2 (1 + \alpha) \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right) + c(n - 2(1 + \alpha)) + 2(1 + \alpha) \sqrt{\Omega_\Lambda - c^2 \Omega_k} \right) \quad (19)
\]

Using \( C_0 \) in \( B_0 \) we finally obtain

\[
B_0 = a^n (3M_p^2 H^2 \Omega_\Lambda)^{1+\alpha} \left( \frac{1 + 3A}{3} + \frac{b^2(1 + \Omega_k)}{\Omega_\Lambda} + \frac{2\sqrt{\Omega_\Lambda - c^2 \Omega_k}}{3c} \right) \quad (20)
\]

Now we can rewrite the scalar potential and kinetic energy terms as follows
\[ V(\phi) = 2H^2M_p^2\Omega_\Lambda \left[ 1 - 3A - \frac{3b^2(1 + \omega_k)}{\omega_\Lambda} + \frac{2\sqrt{\omega_\Lambda - c^2 \omega_k}}{c} \right] \]  

and

\[ \dot{\phi} = \frac{cM_p}{L} \sqrt{2 + \frac{3b^2(1 + \omega_k)}{2\omega_\Lambda} + \frac{\sqrt{\omega_\Lambda - c^2 \omega_k}}{c}} \]  

Since the models trying to provide a description of the cosmic acceleration are proliferating, there exists the problem of discriminating between the various contenders.

In this letter, we have considered that the universe is filled with normal matter and variable modified Chaplygin gas. Also we have considered the interaction between normal matter and variable modified Chaplygin gas in FRW universe. Then we have considered a correspondence between the holographic dark energy density and interacting variable modified Chaplygin gas energy density. From this, we have found the expressions of the arbitrary positive constants \( B_0 \) and \( C_0 \) of variable modified Chaplygin gas. We have seen that if we put \( n = 0 \) and \( A = 0 \), variable modified Chaplygin gas reduces to generalized Chaplygin gas and in this case equations (19) and (20) reduce to equations (31) and (32) of reference [22]. Then we have reconstructed the potential of the scalar field which describes the variable modified Chaplygin cosmology.

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