Controllable spin-current blockade in a Hubbard chain

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We investigate the spin/charge transport in a one-dimensional strongly correlated system by using the adaptive time-dependent density-matrix renormalization group method. The model we consider is a non-half-filled Hubbard chain with a bond of controllable spin-dependent electron hoppings, which is found to cause a blockade of spin current with little influence on charge current. We have considered (1) the spread of a wave packet of both spin and charge in the Hubbard chain and (2) the spin and charge currents induced by a spin-dependent voltage bias that is applied to the ideal leads attached at the ends of this Hubbard chain. It is found that the spin-charge separation plays a crucial role in the spin-current blockade, and one may utilize this phenomenon to observe the spin-charge separation directly.

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In one-dimensional (1d) strongly correlated systems, an essential phenomenon is the spin-charge separation (SCS), which is believed to play a central role in 1d transport. To study transport problems in 1d systems, the characteristics of SCS must be taken into account. However, due to the limitations of existing methods, the discussion in the past has been limited to a few simple cases; an example is the significant work by Kane and Fisher on scaling properties of tunneling through a spin-symmetric point impurity in a fermion system. Hence more powerful methods are needed to compute transport properties beyond scaling and to treat a general interacting Hamiltonian. Furthermore, spin-dependent transport problems have attracted increasing interest in the last two decades: many proposed spintronics devices require the manipulation of spin currents, which are naturally decoupled from charge currents in 1d systems. On the other hand, a number of experimental works have sought to observe the phenomenon. Rapid progress in ultracold atomic gas experiments makes it possible to see SCS in a new context.

Recently, the adaptive time-dependent density-matrix renormalization group (t-DMRG) with quantum information concepts. The key idea of this method is to break up the evolution operator with Trotter decomposition, then apply it to the states within a DMRG configuration. At the same time, other real-time evolution methods within DMRG were also proposed. By use of these numerical methods, there have been investigations on transport properties in 1d strongly-correlated or impurity systems, including spin-1/2 chains, Bose-Hubbard model, and conductance analysis. The dynamical problems with impurities were also studied widely using static DMRG method embedding with persistent current and functional renormalization group. An interesting result that partly motivates our study was the study of SCS by Kollath, Schollwoeck, and Zwerger. In conventional treatment with the bosonization method, only low energy excitations were considered. The above study goes beyond the low energy excitation spectrum by considering the evolution of a "big" (multipartile) wave packet that shows the SCS phenomenon.

In this Letter, we propose to consider a non-half-filled Hubbard chain in which one special bond has controllable spin-dependent electronic hoppings, motivated by the development of optical lattices of ultracold atoms in which all hoppings can be controlled. By using the adaptive t-DMRG method, we simulate the spread of a wave packet as well as the spin and charge currents under a spin-dependent voltage bias. We find that the spin-current blockade can be realized by adjusting the spin-dependent hopping on that special bond while the charge current has not been affected. A possible application of this phenomenon is discussed.

The system we are considering is described by the following Hamiltonian,

\[ H_S = -\sum_{i,\sigma} t^\sigma_{i,i+1} (c^\dagger_{i,\sigma} c_{i+1,\sigma} + \text{h.c.}) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}, \]  

where \( c^\dagger_{i,\sigma}(c_{i,\sigma}) \) creates (annihilates) an electron with spin \( \sigma = \uparrow, \downarrow \) on the \( i \)-th site, \( n_{i,\sigma} = c^\dagger_{i,\sigma} c_{i,\sigma} \) is the corresponding electron number operator, the electron hopping constants \( t^\sigma_{i,i+1} \equiv t_0 \) on all bonds but a special bond \( (i = l_0) \), where \( t^\sigma_{i,i+1} = t_\sigma \) is an adjustable spin-dependent quantity, which introduces a local magnetic moment, like a spin-dependent Anderson impurity. Without the special bond, the Hamiltonian is nothing but the usual Hubbard model with \( t \) being the electron hopping constant between nearest neighbor sites and \( U(\geq 0) \) the on-site repulsive Coulomb interaction. Without loss of generality, we will always keep \( t_\uparrow = t_0 \) while \( t_\downarrow \) is adjusted from \( t_0 \) to 0. We present a proposal in Fig. 1 on
the realization of a Hubbard chain with a special bond of controllable spin-dependent electronic hopping in an optical lattice. The rotation of the left spin-down atoms around the spin-up atom axis reduces the spin-down atom hopping at the special bond, i.e., the $t_{ij}$ in Eq. (1) decreases as the angle $\theta$ increases.

First, we consider the spread of a wave packet as done by Kollath et al. [23], but in a Hubbard chain with a special bond which acts only on the spin up electrons, so that it carries both of spin and charge. Then the potential will be switched off and the wave packet will be spread according to the time-dependent many-body Schrödinger equation. Clearly $P$ determines the potential strength, $l_p$ the center of the induced wave packet and $l_q$ its width. A small value of $l_q$ (e.g., $\leq 1$) corresponds to a local potential at a single site as suggested by Anderson [2] the case we will consider here.

In the following, we apply the t-DMRG method with second order Trotter decomposition to simulate the dynamical evolution of this wave packet. The number of sites ($L$) is taken to be 176 while the number of electrons ($N_e$) is 116; the corresponding filling factor ($N_e/(2L)$) is about 1/3. The time step is taken as 0.04 (in unit of $\hbar/\tau_0$) and the number of kept DMRG states ($M$) is chosen to be large enough to ensure the error being less than $O(10^{-3})$.

The spin and charge currents, the quantities we focus on in this calculation, are defined as $J_{\sigma}(j) = J^\uparrow_{\sigma}(j) - J^\downarrow_{\sigma}(j)$ and $J_c(j) = J^\uparrow_{c}(j) + J^\downarrow_{c}(j)$, separately, where

$$J_{\sigma}(j) \equiv it^\sigma_{ij} t_{ij+1}^{\sigma'} (c_{ij+1,\sigma}^\dagger c_{ij,\sigma} + c_{ij,\sigma}^\dagger c_{ij+1,\sigma}) .$$

In Eq. (2), we show the spin and charge current at various times for $t_1 = 0$. The initial potential locally on the spin up generates simultaneously spin and charge wave packet, and then the wave packets will split into two parts and propagate to opposite directions respectively. [23]

If the potential strength $P$ we took is not large, the spin and charge density of this wave packet is very small, but the currents defined in Eq.(3) show the propagation of this wave packet very clear. In Fig. 2(a), we show the current distribution at a time before the spin and charge reach the special bond. The split of the two peaks indicates the different speeds of spin and charge excitations, so that the spin-charge separation is observed clearly. Next we’ll see clearly from the figure, the charge current goes through the special bond almost freely, while the spin current is blocked by the special bond. A spin current reflecting at the special bond is shown in the figure by its value being changed from positive to negative. The change of charge currents from negative to positive indicates the charge reflection at the left end since we use an open boundary condition for the chain.

We argue that, in the following, the spin-current blockade we observe here only happens in a strongly correlated system. It is different from the “spin blockade” effect [24] which is merely spin-related Coulomb blockade. To understand the phenomenon we observed above, we consider the large $U$ limit of the Hubbard-like model in Eq. (1), which leads to the so-called $t - J$ model $H_{i+J} = H_t + H_J$, where $H_t$ is the hopping term in Eq. (1) and $H_J = -\frac{1}{U} \sum_{ijkss'} t^s_{ij} t'^s_{jk} c_{i\sigma}^\dagger c_{j\sigma} n_{j\sigma} n_{k\sigma'} c_{k\sigma'} c_{ss'}$ (4) with $t^s_{ij} \equiv t^s_{i,j+1} \delta_{i,j-1} + t^s_{i-1,j} \delta_{i,j+1}$. [23] This model works on the space that has projected out all configurations.
FIG. 3: A spin-exchange process that is necessary for the spin current in the strong interaction limit.

with at least one doubly occupied site for a less-than-half filling system and is responsible for the low-energy excitations of the model in Eq. (1). It is well known that the hopping term $H_t$ is responsible for the charge excitations while $H_J$ controls the spin excitations, and that for the usual Hubbard model, $H_J$ corresponds to a Heisenberg spin chain. Writing

$$\vec{S}_i \cdot \vec{S}_j = S_i^z S_j^z + \frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+), \quad \text{Eq. (5)}$$

the second term is responsible for the spin-exchange process that is necessary for an $S_z$ spin current. Based on $H_J$ in Eq. (4) from the large-$U$ expansion, we show the spin-exchange process in Fig. 3, in which there is a virtual intermediate state, so electronic hoppings for both spins are necessary for the process via a virtual state. Then it is clear that spin current is blocked at the special bond when $t_1 = 0$ since only spin-up electrons can hop across the special bond.

Now, we come to the calculation of spin and charge currents under a spin-dependent voltage bias, for which we consider the system of a Hubbard chain with a special bond is attached with two ideal leads at its two end, that is described by the following Hamiltonian,

$$H = H_S + H_{\text{lead}} + H_{\text{int}}, \quad \text{Eq. (6)}$$

where $H_S$ has been given in Eq. (1) and the summation over site index runs from 1 to $L$ with $L$ being the size of the Hubbard chain, $H_{\text{lead}} \equiv H_L + H_R$ and

$$H_{\alpha} = -t_0 \sum_{i, \sigma \in \alpha} (c_{i, \sigma}^\dagger c_{i+\sigma, \sigma} + \text{h.c.}) - \sum_{i, \sigma \in \alpha} V_{\sigma}^\alpha n_{i, \sigma}, \quad \text{Eq. (7)}$$

with $\alpha = L$ or $R$ and the sign $\pm$ taking $-(+)$ for $\alpha = L$ ($R$), and

$$H_{\text{int}} = -t_0 \sum_{\sigma} (c_{0, \sigma}^\dagger c_{1, \sigma} + c_{L, \sigma}^\dagger c_{L+1, \sigma} + \text{h.c.}). \quad \text{Eq. (8)}$$

The spin-dependent voltage bias $V_\sigma \equiv V_\sigma^L - V_\sigma^R$ applied at the two leads turns on at $t = 0$, so that currents appear gradually at the same time and reach constant values finally. In the calculation, we take $V_1 = 0$ and $V_1 = 0.1t_0$,

without loss of generality, to induce both spin and charge currents through the Hubbard chain.

Now we show in Figure 4 the spin and charge currents at various times for different Hubbard chain lengths ($L$) or the numbers of kept DMRG states ($M$) in the calculations induced by the spin-dependent voltage bias. First the Hubbard interaction is switched off for the test of the calculation precision for a chain of $L = 40$ without the special bond. The lengths of ideal leads attached the chain are taken to be long enough so that the results are sufficiently insensitive to it. The incoming and outgoing currents are defined as the corresponding currents at the left and right interfaces according to Eq. (3), respectively. At $U = 0$, the spin and charge currents are the same since no interaction between electrons of different spins exists. Furthermore, the currents can be calculated exactly at the case, which is shown as solid lines in (a). A time lag between the incoming and outgoing currents is observed due to the spin/charge transportation from the left chain end to the right one. Finally, steady transport is reached at $t = 40(t_0/h)$. In Fig. 4(a), we also show the results obtained from the t-DMRG method by keeping various numbers of DMRG states. It can be seen that the curves obtained when the number of kept DMRG states ($M$) is

FIG. 4: The spin and charge currents at various times for different numbers of kept DMRG states ($M$) or Hubbard chain lengths ($L$). (a) Both incoming and outgoing spin/charge currents are shown for $U = 0$ and $L = 40$. (b) Only the outgoing currents are shown for $U = 8t_0$ and $L = 40$. (c) The outgoing currents for different chain lengths $L$, and $U = 8t_0$ and $M = 160$. 

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activation does not destroy the Mott-insulating behavior. But this is not required in our case since the Mott gap is zero. Furthermore, an important advantage is that the spin-current blockade could be realized by adjusting only one parameter \((t_1)\), which simplifies experimental observation.

In summary, we have investigated the spin and charge transport in a Hubbard chain with a bond of controllable electronic hopping. We find the spin current can be blocked by this special bond while charge current passes through the bond freely. It is found that a large Hubbard \(U\) is required for the observed blockade since the spin-charge separation plays a crucial role in it.

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FIG. 5: The dependence of transmission rates (see text for definition) on (a) \(t_1\) (\(U = 8t_0\)) and (b) \(U\) \((t_1 = 0\)).
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