NONLOCAL AND LOCAL GHOST FIELDS IN QUANTUM CORRELATIONS

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(February 5, 2022)

Abstract

Einstein Podolsky Rosen quantum correlations are discussed from the perspective of a ghost field introduced by Einstein. The concepts of ghost field, hidden variables, local reality and the Bell inequality are reviewed. In the framework of the correlated singlet state of two spin-$\frac{1}{2}$ particles, it is shown that quantum mechanics can be cast in a way that has the form of either a nonpositive and local ghost field or a positive and nonlocal ghost field.

PACS numbers: 03.65.Bz
I. INTRODUCTION

Despite the spectacular success of quantum mechanics as a fundamental theory and as a superb calculational tool for all modern physics, the scientific literature continuously carries a substantial number of papers devoted to the foundations and the interpretation of the theory. The introduction of probability in the sense used in quantum mechanics has been the key issue of debates. Since its early days the probabilistic interpretation of quantum phenomena has had its opponents, and Einstein in a letter to Born expressed the view [1]:

The theory produces a good deal but hardly brings us closer to the secret of the OLD ONE. I am at all events convinced that He does not play dice.

This statement is the best known example of the open criticism of the probabilistic interpretation of quantum mechanics.

In his fundamental paper devoted to the quantum theory of spontaneous and stimulated processes by radiation, Einstein expressed his well-known discomfort about the probabilistic description of spontaneous emission in the following words [2]:

The weakness of the theory lies on the one hand in the fact that it does not get us any closer to making the connection with wave theory; on the other hand, that it leaves the duration and direction of the elementary process to 'chance'.

In making the connection between waves and particles, Einstein noted that the emission of radiation (outgoing radiation) is directional. In the same paper he wrote:

Outgoing radiation in the form of spherical waves does not exist.

In the early 1920's, Einstein, in his unpublished speculations, proposed the idea of a "Gespensterfeld" or a ghost field which determines the probability for a light-quantum to take a definite path. In these speculations, the ghost field gives the relation between a wave field and a light-quantum by triggering the elementary process of spontaneous emission. The directionality of the elementary process is fully described by the will (dynamical properties) of the ghost field (see Fig. [1]).

The probabilistic interpretation of quantum mechanics as developed by Born has been considerably influenced by the idea of the ghost field [3]. Born realized that unlike the electromagnetic field, the Schrödinger wave function $\Psi$ has no direct physical reality. Only if one interprets $|\Psi|^2$ or the ghost field as a real probability, do all the weaknesses of the interpretation disappear because:

$$GHOST\ FIELD\ \rightarrow\ |\Psi|^2$$  \hspace{1cm} (1)

In a paper presented at Oxford, Born expressed the opinion that [4]:

But, of course, anybody dissatisfied with these ideas may feel free to assume that there are additional parameters not yet introduced into the theory which determine the individual event.

These additional parameters called, accordingly, hidden variables and denoted by $\lambda$, have been introduced in order to remove from quantum mechanics the indeterminacy introduced
by the Born interpretation of the wave function. According to hidden variables theories, all physical observables such as, for example the position and the momentum of a particle, the spin or the electric field are entirely deterministic, i.e., are objective realities given by functions \( O(\lambda) \). Due to the assumed unobservable nature of the hidden parameters \( \lambda \), only the following average
\[
\langle O \rangle_{HV} = \int d\lambda \, P(\lambda) \, O(\lambda)
\] (2)
can be measured in realistic experimental situations. The distribution of the as yet undiscovered hidden variables is positive (a well-behaved probability distribution) and normalized
\[
P(\lambda) \geq 0 \quad \text{and} \quad \int d\lambda \, P(\lambda) = 1.
\] (3)
In this interpretation the hidden parameters would be equivalent to a ghost field of some yet undisclosed nature, i.e.,
\[
\text{GHOST FIELD} \rightarrow P(\lambda)
\] (4)
In 1935, Einstein, Podolsky, and Rosen (EPR) published their famous paper \[5\] in which they argued that:

The wave function \( \Psi \) does not provide a complete description of physical reality.

The need for a revision of the interpretation of the wave function was strongly expressed in this paper. In fact, EPR questioned the completeness of quantum mechanics, analyzing a correlated system of two particles. The EPR arguments can be best summarized in Bohm’s version \[6,7\] which uses two correlated spin-\( \frac{1}{2} \) particles as the EPR physical system.

Let us consider an initial physical system consisting of two distinguishable spin-\( \frac{1}{2} \) particles \( a \) and \( b \) in a spin-0 quantum state (a spin singlet state). This system dissociates to a pair of spatially separated spin-\( \frac{1}{2} \) particles. Let assume that the dissociation process conserves the total spin of the system and that the separated particles are now moving to opposite Stern-Gerlach detectors (Fig. 2). In the framework of quantum mechanics the Bohm singlet wave function corresponding to the EPR dissociating system of two spin-\( \frac{1}{2} \) particles \( a \) and \( b \) is:
\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (|+\rangle_a \otimes |-\rangle_b - |-\rangle_a \otimes |+\rangle_b)
\] (5)
where \(|+\rangle\) and \(|-\rangle\) are the two eigenstates of the Pauli matrix \( \hat{\sigma}_z \) (quantum operators are denoted with carets) corresponding to the two eigenvalues +1 and −1. After the dissociation the particles \( a \) and \( b \) move to opposite directions without ever being disturbed in any way and various spin components of each of these particles may be measured independently.

The particle \( a \) is detected by a Stern-Gerlach apparatus which can be oriented along directions \( \vec{a}, \vec{b}, \) and \( \vec{c} \) respectively. The particle \( b \) is detected by a Stern-Gerlach apparatus which can be oriented along the same three directions \( \vec{a}, \vec{b}, \) and \( \vec{c} \) respectively (Fig. 2).

The EPR argument is based on the following observation involving such measurements. The Stern-Gerlach apparatus may record that the spin \( a \) is parallel or antiparallel to \( \vec{a} \), which one can take to be the z-axis. Let us assume that it has been detected as parallel
to \( \vec{a} \) meaning that we know that the value of the spin \( a \) is +1. Knowing that the spin of particle \( a \) is +1, common sense based on an objective reality associated with the spin (a singlet state consists of two antiparallel spins) predicts that the second particle \( b \) has its spin −1. This means that that the result of a measurement carried on the second particle by a Stern-Gerlach apparatus with the same orientation \( \vec{a} \) will predict an antiparallel spin.

The same argument based on objective realities for spin variables can be repeated for the orientations \( \vec{b} \) and \( \vec{c} \) of the Stern-Gerlach apparatus leading to the conclusion that the values of the spin are known in the three directions which is in violation of quantum mechanical properties of Pauli spin operators projected along \( \vec{a} \), \( \vec{b} \), and \( \vec{c} \) respectively.

Accordingly EPR concluded that one should search for a deeper-lying theory in which the following two very important criteria are satisfied:

- **The reality criterion**: *If without in any way disturbing the system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*

- **The locality criterion**: *If at the time of measurement... two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.*

The EPR analysis of quantum correlations based on the reality and the locality criteria have led to apparently contradictory conclusions with quantum mechanics which do not allow for commuting objective realities associated with the spin observables. EPR summarized their views in the following statement:

*This simultaneous [predictability] makes the reality of [the spins in Bohm’s version] dependent upon the process of measurement carried out on the first system which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this.*

If the spin singlet wave function (5) of the two particles is measured by two Stern-Gerlach analyzers, in the directions given by unit vectors \( \vec{a} \) and \( \vec{b} \), the single detection of the spins is denoted by \( E(\vec{a}) \) and \( E(\vec{b}) \), and the joint simultaneous detection of the particles \( a \) and \( b \) is denoted by \( E(\vec{a};\vec{b}) \). These expectation values for this wave function (5) are given by the following formulas:

\[
E(\vec{a}) = \langle \Psi | \hat{\sigma}(\vec{a}) | \Psi \rangle = 0, \\
E(\vec{b}) = \langle \Psi | \hat{\sigma}(\vec{b}) | \Psi \rangle = 0, \\
E(\vec{a};\vec{b}) = \langle \Psi | \hat{\sigma}(\vec{a}) \otimes \hat{\sigma}(\vec{b}) | \Psi \rangle = -\vec{a} \cdot \vec{b}, \tag{6}
\]

where \( \hat{\sigma}(\vec{a}) = \hat{\sigma}_a \cdot \vec{a} \) and \( \hat{\sigma}(\vec{b}) = \hat{\sigma}_b \cdot \vec{b} \) are the projections of the three spin-\( \frac{1}{2} \) Pauli matrices of the particle \( a \) and \( b \) on the Stern-Gerlach apparatus orientations characterized by the unit vectors \( \vec{a} \) and \( \vec{b} \).

Quantum mechanics predicts that for parallel detectors we have a perfect correlation:

\[
(\hat{\sigma}_{az} \otimes \hat{\sigma}_{bz}) | \Psi \rangle = (\pm 1)(\mp 1) | \Psi \rangle = - | \Psi \rangle \tag{7}
\]
i.e., the eigenvalue of $\hat{\sigma}_{az}$ is ±1 and as a result we have with probability one that the eigenvalue of $\hat{\sigma}_{bz}$ is ±1. This perfect correlation between these two measurements permits one to predict the outcome of the second measurement with probability one without ever disturbing the second system.

On other hand in the framework of hidden variables or an unknown ghost field these expectation values are given by the following expression [8]:

\[
E(\vec{a}) = \int d\lambda_a \int d\lambda_b P(\lambda_a; \lambda_b) \sigma(\vec{a}, \lambda_a), \\
E(\vec{b}) = \int d\lambda_a \int d\lambda_b P(\lambda_a; \lambda_b) \sigma(\vec{b}, \lambda_b), \\
E(\vec{a}; \vec{b}) = \int d\lambda_a \int d\lambda_b P(\lambda_a; \lambda_b) \sigma(\vec{a}, \lambda_a) \sigma(\vec{b}, \lambda_b). 
\]  
(8)

In this formula $P(\lambda_a; \lambda_b)$ describes a joint probability distribution of the ghost field characterized by the hidden variables $\lambda_a$ and $\lambda_b$. The objective realities of the spin variables are given by the deterministic functions $\sigma(\vec{a}, \lambda_a)$ and $\sigma(\vec{b}, \lambda_b)$.

The locality assumption leads to the property that the distribution function of the ghost field is independent of the surrounding experimental setup and does not depend on what other observables are measured simultaneously on the same system.

It is the purpose of this article to provide a quantum mechanical description of the ghost field. Using different descriptions and formulations of quantum mechanics, we shall discuss the same EPR correlation in different, though, equivalent forms of quantum theory. In short, we shall allow quantum mechanics to speak for itself on such issues as locality and the ghost field. We believe that letting quantum mechanics speak out via its different formulations can provide the most down-to-earth description of these fundamental and puzzling issues. As a result, our approach will be almost completely divorced from all of its philosophical or speculative aspects. Instead of this, we shall focus our effort on the question: How can we make quantum mechanics look like a ghost field and vice versa? These considerations demonstrate a much closer correspondence between the probabilistic interpretation of quantum mechanics and hidden variables or ghost fields. In our approach we shall show that the wave function $\Psi$ can always be replaced by a suitable quantum mechanical ghost field $P_{qm}(\lambda_a; \lambda_b)$ with properties that defy the EPR concepts of objective realities or locality.

\[
\text{GHOST FIELD} \rightarrow |\Psi|^2 \rightarrow P_{qm}(\lambda_a; \lambda_b). 
\]  
(9)

Expressing the formalism of quantum mechanics in a given and suitable form can provide a better understanding of the nature of the quantum mechanical ghost field. Instead of being vague in most of such descriptions, the formalism of quantum mechanics can provide a reasonable (if not ideal) description of quantum locality, reality, nonlocality and their interrelations.

II. LOCAL HIDDEN VARIABLES AND THE BELL INEQUALITY

The concept of local realism as introduced by EPR is based on the fundamental assumption that physical systems can be described by local objective properties that are independent of observation. Starting with the EPR article, questions devoted to local realism, to
measurements and the nonlocal character of quantum correlations, and to hidden variables
have been raised, debated and analyzed both theoretically and experimentally. This debate
has produced various alternative theories to quantum mechanics. These theories have been
considered over the years, hidden variables theories being prominent among these.

From the theoretical point of view, a real breakthrough came after Bell discovered [9,10]
in 1965 that certain classes of hidden variables theories limit the strength of spin or photon
correlations and that quantum mechanical correlations are not constrained by these limits.
These fundamental limitations or constraints inhibited by local realism lead to the famous
Bell inequality.

For the purpose of proving the Bell inequality we shall assume that Stern-Gerlach appa-
ratuses for the two particles are oriented along the three directions \( \vec{a}, \vec{b}, \vec{c} \) with angles
\( \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = \cos 120^\circ \) (see Fig. 2).

In order to prove the Bell inequality we introduce (following Cover [11]) the following
quantity built from objective spin realities:

\[
\Sigma(\lambda_a, \lambda_b) = -\left( \sigma(\vec{a}, \lambda_a) + \sigma(\vec{b}, \lambda_a) + \sigma(\vec{c}, \lambda_a) \right) \left( \sigma(\vec{a}, \lambda_b) + \sigma(\vec{b}, \lambda_b) + \sigma(\vec{c}, \lambda_b) \right)
\]  

(10)

We shall restrict the range of these objective realities to \( |\sigma(\vec{i}, \lambda_a)| \leq 1 \) and \( |\sigma(\vec{i}, \lambda_b)| \leq 1 \) for all possible orientations \( \vec{i} = \vec{a}, \vec{b}, \vec{c} \) of the Stern-Gerlach apparatus. Because the spins are
antiparallel we have with certainty that:

\[
\sigma(\vec{a}, \lambda_a) \cdot \sigma(\vec{a}, \lambda_b) = \sigma(\vec{b}, \lambda_a) \cdot \sigma(\vec{b}, \lambda_b) = \sigma(\vec{c}, \lambda_a) \cdot \sigma(\vec{c}, \lambda_b) = -1
\]  

(11)

for all the three orientations and for all the possible hidden variables \( \lambda_a \) and \( \lambda_b \) forming the
ghost field that governs the statistical properties of the local objective realities representing
the spin. This follows from the fact that if the first Stern-Gerlach apparatus oriented at \( \vec{a}, \vec{b}, \vec{c} \) detects the spin direction to be \( \pm 1 \), the second Stern-Gerlach apparatus oriented as
in Fig. 2 has to detect the other spin of the singlet state (5) as \( \mp 1 \). From the fact that the
objective realities are bounded by \( \pm 1 \) we note that (10) reaches a minimum value:

\[
\Sigma(\lambda_a, \lambda_b) \geq \Sigma_{min}(\lambda_a, \lambda_b) = -(1 - 1 - 1)(-1 + 1 + 1) = 1
\]  

(12)

From the fact that the spins are antiparallel i.e., (11) holds, we have the following inequality
involving the objective realities:

\[
3 - \sigma(\vec{a}, \lambda_a)\sigma(\vec{b}, \lambda_b) - \sigma(\vec{a}, \lambda_a)\sigma(\vec{c}, \lambda_b) - \sigma(\vec{b}, \lambda_a)\sigma(\vec{c}, \lambda_b)
- \sigma(\vec{b}, \lambda_a)\sigma(\vec{a}, \lambda_b) - \sigma(\vec{c}, \lambda_a)\sigma(\vec{a}, \lambda_b) - \sigma(\vec{c}, \lambda_a)\sigma(\vec{b}, \lambda_b) \geq 1
\]  

(13)

multiplying this inequality by \( P(\lambda_a; \lambda_b) \), and integrating over the ghost field variables we
obtain the following Bell inequality:

\[
E(\vec{a}; \vec{b}) + E(\vec{a}; \vec{c}) + E(\vec{b}; \vec{c}) \leq 1,
\]  

(14)

where we have used the fact that correlations are symmetric i.e., for example \( E(\vec{a}; \vec{b}) = E(\vec{b}; \vec{a}) \).

In order to disprove a theory based on local objective realities a series of three of different
correlation experiments is required (in our case correlations \( \vec{a} \vec{b}, \vec{a} \vec{c} \) and \( \vec{b} \vec{c} \)).
What we have seen in the process of deriving the Bell inequality (14) is that one could provide a much more accurate description of locality, local reality and correlations in terms of (8). What is even more important in this inequality is the fact that all these ideas could be put to experimental verifications.

For the three orientations from Fig. the inequality (14) reduces to
\[ 2E(120^\circ) + E(240^\circ) \leq 1. \]
and for the quantum correlation (6) we have:
\[ -2 \cos 120^\circ - \cos 240^\circ = \frac{3}{2} \leq 1. \]
which is false and clearly violates the Bell inequality (14) for the spin-$\frac{1}{2}$ singlet state given by (5).

These inequalities have helped to put a large class of hidden variables theories to experimental tests [12]. Theories based on local realism have all failed these tests. So far, all the experiments involving correlations constrained by the Bell’s inequality have been positive for quantum mechanics and negative for various versions of local-realism [13].

If only one correlation experiment is performed the odds against objective realities are not very good. Even if disproved in a series of three experiments, objective realities leave a puzzling question. If they come so close to the quantum predictions, perhaps there are some elements of truth in their descriptions given by (8). If so, which particular assumptions of a local hidden variable theory are in agreement or disagreement with quantum mechanics. In order to answer such a question, we shall describe in the following sections, genuine quantum correlations in a form which bears a lot of similarities to the objective reality description. Once quantum mechanics has been formulated in the form expressed by Eq. (8) a genuine effort can be made to address such issues as hidden variable objective realities and locality. Once this relation is established, the question why quantum mechanics violates the Bell inequality can be posed and investigated.

III. LOCAL QUANTUM GHOST FIELD

The classical Malus’ Law predicts an attenuation of an incident polarized light beam with intensity $I_0$, passing through a linear polarizer to obey $I = I_0 \cos^2 \alpha$. This attenuation depends on the relative angle $\alpha$ between the polarization direction $\vec{n}$ of the incoming wave and the orientation $\vec{a}$ of the polarizer, i.e, $\cos \alpha = \vec{n} \cdot \vec{a}$. If the incoming light beam consists of a statistical mixture of polarized light, the incoming intensity is:
\[ \langle I \rangle = \int d\vec{n} \, P_{cl}(\vec{n}) \cos^2 \alpha. \]
In this formula, the integration is over all possible angles of the random polarization direction $\vec{n}$ and the classical distribution function $P_{cl}(\vec{n})$ characterizes the statistical properties of the incident light beam polarization.

In quantum mechanics a similar Malus’ Law holds for spin-$\frac{1}{2}$ particles detected by a Stern-Gerlach apparatus. Let assume that the spin of the particle $a$ is oriented along an
arbitrary unit direction $\vec{n}_a$. The quantum expectation value of such a spin, if detected by the Stern-Gerlach apparatus oriented in the direction $\vec{a}$ is:

$$\langle \vec{n}_a | \hat{\sigma}(\vec{a}) | \vec{n}_a \rangle = \vec{n}_a \cdot \vec{a} = \cos \alpha(\vec{a}, \vec{n}_a)$$  \hspace{1cm} (18)

In this expression $\alpha(\vec{a}, \vec{n}_a)$ is the relative angle between the orientation of the detected spin state $|\vec{n}_a\rangle$ and the Stern-Gerlach apparatus $\vec{a}$. The corresponding expectation value of the spin-$\frac{1}{2}$ projection operator $\hat{P}(\vec{a}) = \frac{1}{2}(1 + \hat{\sigma}(\vec{a}))$ is:

$$\langle \vec{n}_a | \hat{P}(\vec{a}) | \vec{n}_a \rangle = \frac{1}{2}(1 + \vec{n}_a \cdot \vec{a}) = \cos^2 \frac{\alpha}{2}$$  \hspace{1cm} (19)

is just the quantum version of the Malus' Law for spin. Light (17) and spin-$\frac{1}{2}$ (19) differ in this formulation by a factor $\frac{1}{2}$ in the relative angle involved in the Malus law.

Following the classical Malus' Law for an unpolarized light beam (17), one can write the following quantum spin correlation function for an arbitrary quantum state of the two spin-$\frac{1}{2}$ particles detected by the Stern-Gerlach apparatus:

$$E(\vec{a}; \vec{b}) = \int d\vec{n}_a \int d\vec{n}_b \ P(\vec{n}_a; \vec{n}_b) \ \cos \alpha(\vec{a}, \vec{n}_a) \ \cos \alpha(\vec{b}, \vec{n}_b).$$  \hspace{1cm} (20)

where the distribution function $P(\vec{n}_a; \vec{n}_b)$ describes the quantum mechanical ghost field for an arbitrary state $|\Psi\rangle$ of the two spin-$\frac{1}{2}$ particles. This quantum mechanical expectation value (20) has a remarkable similarity to the hidden variable equation (8) if the following correspondence is made [14,15]:

$$\vec{n}_a \leftrightarrow \lambda_a \quad \text{and} \quad \vec{n}_b \leftrightarrow \lambda_b$$

$$\sigma(\vec{a}, \lambda_a) \leftrightarrow \cos \alpha(\vec{a}, \vec{n}_a) \quad \text{and} \quad \sigma(\vec{b}, \lambda_b) \leftrightarrow \cos \alpha(\vec{b}, \vec{n}_b).$$  \hspace{1cm} (21)

i.e., the $\cos \alpha(\vec{a}, \vec{n}_a)$ and $\cos \alpha(\vec{b}, \vec{n}_b)$ are now the directional cosine functions between the directions of the polarizers $\vec{a}$ and $\vec{b}$ and the “hidden-variable” directions $\lambda_a = \vec{n}_a$ and $\lambda_b = \vec{n}_b$ averaged with respect to a local, i.e., Stern-Gerlach apparatus independent ghost field $P(\lambda_a; \lambda_b) = P(\vec{n}_a; \vec{n}_b)$.

It is then clear that the quantum mechanical formula (20) has the form of a hidden variable theory with the local spin realities given by $\cos \alpha(\vec{a}, \vec{n}_a)$ and $\cos \alpha(\vec{b}, \vec{n}_b)$, and with a local ghost field represented by the distribution $P(\vec{n}_a; \vec{n}_b)$.

For the EPR correlated system of the two spin-$\frac{1}{2}$ systems, it is natural just to take the probability distribution in the form:

$$P(\vec{n}_a; \vec{n}_b) = \frac{1}{4\pi} \delta^{(2)}(\vec{n}_a + \vec{n}_b) \int d\vec{n}_a \int d\vec{n}_b \ P(\vec{n}_a; \vec{n}_b) = 1.$$  \hspace{1cm} (22)

where the two-dimensional Dirac delta function indicates that the direction $\vec{n}_a$ is antiparallel to the direction $\vec{n}_b$, i.e., $\vec{n}_a = -\vec{n}_b$.

The marginal properties of this distribution represent just an isotropic uniform distribution of the spin directions on the sphere:
\[\int d\vec{n}_b \ P(\vec{n}_a; \vec{n}_b) = \frac{1}{4\pi} \quad \text{and} \quad \int d\vec{n}_a \ P(\vec{n}_a; \vec{n}_b) = \frac{1}{4\pi}. \tag{23}\]

From these relations we conclude that individual ghost fields of the spins correspond to a uniform distribution of orientations. This leads to single spin averages equal to zero, i.e.:

\[E(\vec{a}) = \frac{1}{4\pi} \int d\vec{n}_a \ \vec{n}_a \cdot \vec{a} = 0,\]
\[E(\vec{b}) = \frac{1}{4\pi} \int d\vec{n}_b \ \vec{n}_b \cdot \vec{b} = 0. \tag{24}\]

On the other hand a direct integration of the Malus correlation (20) with the antiparallel distribution function given by Eq. (22) leads to:

\[E(\vec{a}, \vec{b}) = \frac{1}{4\pi} \int d\vec{n}_a \int d\vec{n}_b \ \delta^{(2)}(\vec{n}_a + \vec{n}_b) \ (\vec{n}_a \cdot \vec{a}) \ (\vec{n}_b \cdot \vec{b}) = -\frac{1}{3} \vec{a} \cdot \vec{b}. \tag{25}\]

This result does not violate the Bell inequality (14) and is off from the quantum mechanical result (14) by a factor of 3. This can be simply fixed by a change of the \(\frac{1}{4\pi}\)-prefactor in (22) into \(\frac{3}{4\pi}\). But this change has tremendous consequences for the normalization of the distribution function (22). In order to preserve the normalization we write:

\[P_{\text{qm}}(\vec{n}_a; \vec{n}_b) = \frac{3}{4\pi} \delta^{(2)}(\vec{n}_a + \vec{n}_b) - \frac{2}{(4\pi)^2}. \tag{26}\]

We check that indeed with this change:

\[\int d\vec{n}_a \int d\vec{n}_b \left( \frac{3}{4\pi} \delta^{(2)}(\vec{n}_a + \vec{n}_b) - \frac{2}{(4\pi)^2} \right) = \frac{3}{4\pi} - \frac{2}{(4\pi)^2} (4\pi)^2 = 1. \tag{27}\]

We calculate the marginals:

\[\int d\vec{n}_a \left( \frac{3}{4\pi} \delta^{(2)}(\vec{n}_a + \vec{n}_b) - \frac{2}{(4\pi)^2} \right) = \frac{3}{4\pi} - \frac{2}{(4\pi)^2} 4\pi = \frac{1}{4\pi},\]
\[\int d\vec{n}_b \left( \frac{3}{4\pi} \delta^{(2)}(\vec{n}_a + \vec{n}_b) - \frac{2}{(4\pi)^2} \right) = \frac{3}{4\pi} - \frac{2}{(4\pi)^2} 4\pi = \frac{1}{4\pi}. \tag{28}\]

Clearly the function (26) has the correct marginals (28), reproduces the right quantum mechanical prefactor and yet is not positive! The appearance of such a nonpositive probability distribution function (called for this reason a quasi-distribution function) is a typical quantum property, whenever a probabilistic phase-space is constructed for noncommuting quantum observables. The first nonpositive quasi-probability distribution was introduced into quantum mechanics by Wigner in 1932 (16) for the position and the momentum phase-space. The quasi-distribution function (26) corresponds to a Wigner-type distribution function for the spin-\(\frac{1}{2}\) phase-space variables. In fact it is possible to study and derive rigorously (17,18) all kinds of quantum mechanical spin-\(\frac{1}{2}\) quasi-distribution functions with (26) being just an example corresponding to the quantum ghost field of the EPR singlet state (5).
The quantum Malus Law [20], if applied to the joint correlations involving two Stern-Gerlach detectors, has the formal structure of a hidden variable theory, with the hidden parameters represented by "hidden directions" \( \vec{n}_a \) and \( \vec{n}_b \). There the analogy ends because the quantum distribution corresponding to the local ghost field is a non-positive function that leads to the failure of the Bell’s inequalities for such a correlated state (see Fig. 3).

Using the Malus form of spin correlations we have:

\[
\text{LOCAL GHOST FIELD} \implies \frac{3}{4\pi} \delta^{(2)}(\vec{n}_a + \vec{n}_b) - \frac{2}{(4\pi)^2}.
\]

(29)

IV. NONLOCAL QUANTUM GHOST FIELD

In this section we are going to show that EPR correlations can be described by a quantum nonlocal ghost field similar to the one used in a hidden-variables theory.

We can rewrite the spin correlation function (3) in terms of a ghost field if we use the following two integral identities for the Pauli matrices:

\[
\hat{\sigma}^{(\vec{a})} = \int d\lambda a \lambda a \delta(\lambda a - \hat{\sigma}^{(\vec{a})}) , \quad \hat{\sigma}^{(\vec{b})} = \int d\lambda b \lambda b \delta(\lambda b - \hat{\sigma}^{(\vec{b})}).
\]

(30)

Introducing these identities into (3) we obtain:

\[
E(\vec{a}; \vec{b}) = \langle \Psi | \hat{\sigma}^{(\vec{a})} \otimes \hat{\sigma}^{(\vec{b})} | \Psi \rangle = \int d\lambda a \int d\lambda b P(\vec{a}, \lambda a; \vec{b}, \lambda b) \lambda a \lambda b.
\]

(31)

where the distribution function is given by the following quantum mechanical average:

\[
P(\vec{a}, \lambda a; \vec{b}, \lambda b) = \langle \Psi | \delta(\lambda a - \hat{\sigma}^{(\vec{a})}) \otimes \delta(\lambda b - \hat{\sigma}^{(\vec{b})}) | \Psi \rangle.
\]

(32)

With the help of this distribution function we have rewritten the quantum mechanical correlation function in a form which has remarkable similarities to the correlation function given by Eq. (8). Because the spin operators \( \hat{\sigma}^{(\vec{a})} \) and \( \hat{\sigma}^{(\vec{b})} \) have eigenvalues equal to +1 or −1, i.e., can represent only “parallel” (“p”) or “antiparallel” (“a”) outcomes, \( \lambda a \) and \( \lambda b \) can take only values equal to +1 and −1. The bivalued distribution given by Eq. (32) is positive everywhere and normalized:

\[
\int d\lambda a \int d\lambda b P(\vec{a}, \lambda a; \vec{b}, \lambda b) = \langle \Psi | \Psi \rangle = 1.
\]

(33)

This function depends on the polarization directions \( \vec{a} \) and \( \vec{b} \). The distribution function which depends on the orientation \( \vec{a} \) of the first Stern-Gerlach apparatus and on the orientation \( \vec{b} \) of the second (possibly even remote) Stern-Gerlach apparatus is nonlocal. Because of this property we shall call an analyzer-dependent distribution function a nonlocal distribution function. The nonlocality of this distribution function makes the Bell’s inequality void, because in order to obtain this inequality the existence of a universal, local (polarization independent) distribution in the parameters \( \lambda a \) and \( \lambda b \) (hidden parameters in this case) like that appearing in (8) was essential [19]. Quantum mechanics tells us that if we insist on a
distribution of the form given by Eq. (32), we can do it but only under the condition that
the statistical distribution of the ghost field with the parameters $\lambda_a$ and $\lambda_b$ is nonlocal \[20\].

This joint distribution function (32) can be written in the following form (Bayes theorem):

$$P(\lambda_a; \lambda_b) = P(\lambda_a|\lambda_b)P(\lambda_b)$$

(34)

where the distribution $P(\lambda_a|\lambda_b)$ is the conditional of the event $\lambda_a$ to occur under the condition
that $\lambda_b$ has occurred. The distribution $P(\lambda_b)$ is a one-fold marginal of (32) and is:

$$P(\vec{a}, \lambda_b) = \int d\lambda_a P(\vec{a}, \lambda_a; \vec{b}, \lambda_b) = \langle \Psi | \delta(\lambda_b - \hat{\sigma}(\vec{b})) | \Psi \rangle = \frac{1}{2}.$$ 

(35)

This means the the one-fold marginal is local (apparatus independent) and that the outcomes are equally probable ie., $P(\lambda_b = +1) = \frac{1}{2}$ and $P(\lambda_b = -1) = \frac{1}{2}$.

In order to elucidate the nonlocal distribution further we shall rewrite the nonlocal conditional distribution function in the following matrix form:

$$P(\lambda_a|\lambda_b) = \begin{pmatrix} P(+1|+1) & P(+1|-1) \\ P(-1|+1) & P(-1|-1) \end{pmatrix}$$

(36)

From Eq.(32) and the form of the EPR singlet state we obtain that the conditional probabilities for the particular transitions are

$$P(\lambda_a|\lambda_b) = \begin{pmatrix} \sin^2 \alpha & \cos^2 \frac{\alpha}{2} \\ \cos^2 \frac{\alpha}{2} & \sin^2 \frac{\alpha}{2} \end{pmatrix}$$

(37)

This result shows that one can regard the EPR correlations as just correlations of two sequences of random numbers jumping between values +1 and −1 (“p” and “a” answers) for polarization measurements performed with linear analyzers. The distribution $P(\lambda_a|\lambda_b)$ is the conditional of the event $\lambda_a$ (“p” or “a”) to occur under the condition that $\lambda_b$ (“a” or “p”) has occurred.

This positive and nonlocal distribution leads to a simple statistical interpretation of the spin transitions and of the violation of Bell’s inequality in terms of random numbers ±1 for the variables $\lambda_a$ and $\lambda_b$. The quantum mechanical average in this case is represented by an ensemble average of two sequences of random numbers “a” and “p”. The random character of these variables can be applied to the description of the EPR correlations measured by two polarizers. To each polarizer there corresponds a sequence of random variables denoted by $\lambda_a$ and $\lambda_b$. These are the only possible outcome of the experiment. On each single polarizer the outcomes are completely random and the “p” and “a” answers occur with equal probability ($\frac{1}{2}$ in this case). The nonlocality of the EPR correlations shows up in the fact that these two perfectly random sequences (on the first and the second analyzer) are correlated and the correlations are given by Eq. (3). These formulas predict that the EPR wave function can be understood as a nonlocal correlation between two random sequences $\lambda_a = (p, a, a, \ldots)$ and $\lambda_b = (a, a, p, \ldots)$. The nonlocality of these correlations follows from the fact that whenever $\lambda_b = p$ or $\lambda_b = a$ on the polarizer $\vec{b}$, we must have $\lambda_a = p$ or $\lambda_a = a$ on the analyzer $\vec{a}$ with the probability $\sin^2 \frac{\alpha}{2}$, i.e., the outcomes on $\vec{a}$ (possibly even a remote analyzer) are determined by the outcomes on the analyzer $\vec{b}$. 

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For antiparallel correlations $\alpha = 0$ and the conditional matrix has the following form:

$$P(\lambda_a | \lambda_b) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{38}$$

This ghost field correspond to the following two outcomes at the Stern-Gerlach analyzers:

$$\lambda_b = \begin{pmatrix} p \\ a \\ a \\ \vdots \end{pmatrix} \implies \lambda_a = \begin{pmatrix} a \\ p \\ p \\ \vdots \end{pmatrix}. \tag{39}$$

We see that the outcomes correspond to two random sequences with perfect correlation corresponding to $a \leftrightarrow p$ with conditional probability one and $a \leftrightarrow a$ and $p \leftrightarrow p$ with conditional probability zero.

Let us illustrate, using these random sequences, the violation of Bell’s inequality. Let us assume that in the first series of experiments we set $\alpha = 120^\circ$ and $\alpha = 240^\circ$. According to the formulas (37) we have for these angles:

$$P(\lambda_a | \lambda_b) = \begin{pmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{pmatrix}. \tag{40}$$

This means that with a 75% confidence we shall have that the outcomes on $\vec{a}$ will be the same as the outcomes on $\vec{b}$ and a 25% confidence that the outcomes are different. This leads to the spin correlation:

$$E(120^\circ) = E(240^\circ) = \sum_{\lambda_a = \pm 1, \lambda_b = \pm 1} \lambda_a \lambda_b P(\lambda_a | \lambda_b) \frac{1}{2}$$

$$+ 1 \cdot 0.75 - 1 \cdot 0.25 = -\cos(120^\circ) = \frac{1}{2} \tag{41}$$

We already have shown (15) that these correlation do violate the Bell inequality (14). The nonlocal ghost field contradicts the idea of objective realities because the probability of reproducing a sequence on the polarizer $\vec{a}$ if the sequence on the polarizer $\vec{b}$ is known is nonlocal if the ghost field distribution depends on the relative orientation $\alpha$ of the polarizers. This is how the EPR quantum nonlocal probabilities violate local realism (Fig. 3):

$$\textit{NONLOCAL GHOST FIELD} \Rightarrow P(\lambda_a | \lambda_b). \tag{42}$$

In fact it is easy to give an example of a ghost field that will not violate the Bell inequality in this case. If we select:

$$P(\lambda_a | \lambda_b) = \begin{pmatrix} 5/12 & 7/12 \\ \frac{7}{12} & \frac{5}{12} \end{pmatrix} \tag{43}$$

we obtain that:

$$E(120^\circ) = +1 \cdot \frac{5}{12} - 1 \cdot \frac{7}{12} = -\frac{1}{3} \cos 120^\circ \tag{44}$$

which does not violate the Bell inequality. The quantum nonlocal ghost field predicts in this case that the outcomes on the two Stern-Gerlach detectors is the same with a 75% confidence while hidden variables will lead to $\frac{5}{12} \approx 41.6$% confidence. The quantum correlations induced by the nonlocal quantum ghost (37) are ”stronger” compared to the one obtained in the frame work of local objective realities (8).
V. CONCLUSIONS

The two different representations of quantum mechanics given by a local and nonpositive ghost field and by a nonlocal and positive ghost field are equivalent descriptions of spin-\(\frac{1}{2}\) correlations and both have a hidden-variable look-a-like form. These descriptions are different in form, but are really equivalent to the standard formulation of quantum mechanics. Instead of spin observables, projection operators and quantum states, the quantum ghost field description of the EPR correlations involves local (or nonlocal), positive (or nonpositive) distributions for random spin orientations satisfying (or not satisfying) the Malus law. The relations (26) and (37) show that the violation of Bell’s inequality can be due either to a nonpositive and local or to a positive and nonlocal distribution function.

We have exhibited in this paper the two extreme cases which show that quantum mechanics is equivalent to a hidden-variable theory with nonpositive probabilities or to a hidden-variable theory with nonlocal distribution functions. Which view we adopt is quite irrelevant because the two pictures are equivalent and represent simply different aspects of the same quantum mechanical reality.

ACKNOWLEDGMENTS

The author is indebted to Prof. P. L. Knight for the encouragement to write in this form my thoughts about the ghost field, which I presented for the first time at Imperial College in 1986. Over these years I have benefited enormously discussing with I. Bialynicki-Birula, C. Caves, T. Cover, J. H. Eberly, G. Herling, P. Meystre, G. Milburn, P. Milonni, K. Rzążewski, M. O. Scully, J. Töke and A. Zeilinger.

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FIGURES

FIG. 1. According to the Born interpretation of the wave function $\Psi$, the time and direction of the spontaneously emitted light-quantum (from an excited state to the ground state) is left to ‘chance’. In order to overcome the ‘chance’ and the probabilistic nature of such a spontaneous emission act, Einstein speculated in the early 1920’s that a ghost field of a yet unspecified nature determines the probability for the spontaneously emitted light-quantum to take a definite path.

FIG. 2. A system of two distinguishable spin-$\frac{1}{2}$ particles $a$ and $b$ with total spin $= 0$, dissociates into a pair of spatially separated particles. Spin components of each of these particles are then measured independently by Stern-Gerlach detectors. The EPR joint correlations are detected by two Stern-Gerlach apparatuses oriented along the three directions $\vec{a}$, $\vec{b}$ and $\vec{c}$.

FIG. 3. Spin correlations in the EPR system of two spin-$\frac{1}{2}$ particles can be described to a local objective ghost field that determines the probability of the individual spin orientations. In quantum mechanics this ghost field is local and not positive and is given as $P_{qmn}(\vec{n}_a; \vec{n}_b) = \frac{3}{4\pi} \delta^{(2)}(\vec{n}_a + \vec{n}_b) - \frac{2}{(4\pi)^2}$. The negative regions of the ghost field, denoted by $-$, make the Bell inequality void in this case.

FIG. 4. Spin correlations in the EPR system of two spin-$\frac{1}{2}$ particles can be described by a nonlocal objective ghost field that determines the probability of the individual spin orientations. In quantum mechanics this positive and nonlocal, apparatus dependent, ghost field is given as $P(\lambda_a|\lambda_b)$. The nonlocality of the ghost field makes the Bell inequality void in this case.
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