Lambda flow in heavy-ion collisions: the role of final-state interactions

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Abstract

Lambda flow in Ni+Ni collisions at SIS energies is studied in the relativistic transport model (RVUU 1.0). It is found that for primordial lambdas the flow is considerably weaker than proton flow. The inclusion of final-state interactions, especially the propagation of lambdas in mean-field potential, brings the lambda flow close to that of protons. An accurate determination of lambda flow in heavy-ion experiments is shown to be very useful for studying lambda properties in dense matter.
I. INTRODUCTION

Collective flows of hadrons and light fragments in heavy-ion collisions have been observed unambiguously at various incident energies [1–7]. Detailed comparisons of theoretical predictions with the experimental data have already provided valuable information about the nuclear equation of state [8,9], the in-medium nucleon-nucleon cross sections [10], and the medium modification of hadron properties [11]. It has been found that proton and fragment flows are largely caused by the compressional pressure generated at high densities and thus carry the information about the nuclear equation of state. Proton flow at intermediate energies has also been found to be sensitive to medium modifications of the nucleon-nucleon cross sections. Transport model calculations without including mean-filed potentials have shown that pion flow [12–14], as well as antikaon and antiproton flow [15,16], are mostly determined by their large absorption cross sections by nucleons, and thus usually show ‘antiflow’ with respect to nucleons. These results are, however, qualitatively changed when their mean-field potentials are included in the transport model. Both the antiflows of pions [17] and antikaons [18] are found to be significantly reduced by their attractive potentials in nuclear medium. A similar effect is expected for the antiflow of antiprotons when their attractive mean-field potential is included in the study. Also, kaon flow has been shown to be sensitive to the kaon potential in nuclear medium [11]. A comparison with recent experimental data from the FOPI collaboration at GSI has shown that a kaon feels both an attractive scalar and a repulsive vector potential [19], consistent with the prediction of effective chiral Lagrangians [20,21]. The resulting kaon potential is weakly repulsive and thus makes the kaon flow extremely weak compared to nucleon flow.

More recently, collective flow of lambdas has been measured through the reconstruction of $p\pi^-$ pairs by the FOPI collaboration in Ni+Ni collisions at 1.93 GeV/nucleon [5], and by the EOS collaboration in Ni+Cu collisions at 2.0 GeV/nucleon [4]. Although the magnitude of lambda flow differs in the two measurements, both collaborations have found that lambdas flow in the same direction as nucleons, and its strength is similar to that of proton flow. The
quantitative difference between the two measurements is due to different acceptance cuts
and centrality selections in the two experiments.

In heavy-ion collisions around 1-2 GeV/nucleon, lambdas are mainly produced in as-
association with kaons from baryon-baryon and meson-baryon collisions. In Refs. [11,19] we
have studied kaon flow from Ni+Ni collisions at similar energies, and found that the flow of
primordial kaons is considerably weaker than proton flow. This is also true for primordial
lambdas, as we shall show later. In the case of kaons, the inclusion of final-state interactions,
mainly their propagation in mean field, tends to repel them away from nucleons, and this
reduces kaon flow or even changes its sign if the kaon potential is very repulsive.

The final-state interactions of a lambda are different from that of a kaon. At low energies,
the lambda-nucleon cross section is relatively large compared to the kaon-nucleon cross
section. Also, instead of a weak repulsive potential as for a kaon, the lambda potential
is attractive in the density and momentum range relevant for heavy-ion collisions at beam
energies around 1-2 GeV/nucleon. These differences are expected to make the final lambda
flow quite different from the kaon flow, although they are very similar without final-state
interactions. In particular, we shall show that the attractive potential between a lambda
and the nuclear matter makes its flow similar to the nucleon flow.

The purpose of this paper is to study quantitatively the effects of final-state interactions
on lambda flow in heavy-ion collisions at SIS energies. We also study the sensitivity of
lambda flow to its properties in dense matter, which are important for understanding the
properties of ‘strange’ stars and the possible kaon condensation [22,24]. In Section II, we
briefly review the relativistic transport model (RVUU 1.0) that has been used extensively
in studying heavy-ion collisions at SIS energies. We also discuss how we treat lambda
production, scattering, and propagation in the transport model. The results from this study
are presented in Section III. The paper ends with a short summary in Section IV.
II. THE RELATIVISTIC TRANSPORT MODEL

The relativistic transport model (RVUU 1.0), first developed in Ref. [25] and further extended in Refs. [26,27], is based on the non-linear $\sigma$-$\omega$ model and includes explicitly the nucleon, delta resonance, and pion. It can also treat the production of eta, kaon, antikaon, hyperon, antiproton, and dilepton in heavy-ion collisions at SIS energies using the perturbative test particle method.

At SIS energies, lambdas are mainly produced from baryon-baryon ($BB \rightarrow B\Lambda K$, with $B$ denotes either a nucleon or a delta resonance) and pion-nucleon ($\pi N \rightarrow \Lambda K$) collisions. The process $\pi\Delta \rightarrow \Lambda K$ is neglected since its cross section is much smaller than that of $\pi N \rightarrow \Lambda K$, as shown in the resonance model of Ref. [28]. For the isospin-averaged cross sections $\sigma_{BB \rightarrow B\Lambda K}$, we use the parameterization of Ref. [29], e.g.,

$$\sigma_{NN \rightarrow N\Lambda K} = 0.072 \frac{p_{\text{max}}}{m_K} \text{ mb},$$

with

$$p_{\text{max}} = \frac{1}{2\sqrt{s}} \left[ (s - (m_N + m_\Lambda + m_K)^2)(s - (m_N + m_\Lambda - m_K)) \right]^{1/2}. \quad (2)$$

The cross section for $\pi N \rightarrow \Lambda K$ is taken from the parameterization in Ref. [30], i.e.,

$$\sigma_{\pi N \rightarrow \Lambda K} = 4.94(\sqrt{s} - \sqrt{s_0}) \text{ mb}, \quad \sqrt{s} - \sqrt{s_0} \leq 0.091 \text{ GeV}$$

$$= \frac{0.045}{0.01 + (\sqrt{s} - \sqrt{s_0})} \text{ mb}, \quad \sqrt{s} - \sqrt{s_0} > 0.091 \text{ GeV}, \quad (4)$$

with $\sqrt{s_0} = m_\Lambda + m_K$.

In the following calculations, we always include the medium modification of kaon properties by using its in-medium (pole) mass $m_k^*$ determined from the mean-field approximation to the chiral Lagrangian [26], i.e.,

$$m_k^* \approx m_K \left[ 1 - \frac{\Sigma_{KN}}{f^2m_K^2}\rho_S + \frac{3}{4f^2m_K}\rho_N \right]^{1/2}, \quad (5)$$

where $f = 93 \text{ MeV}$ is the pion decay constant, and $\Sigma_{KN} \approx 350 - 450 \text{ MeV}$ is the $KN$ sigma term, which depends on the strangeness content of a nucleon. The scalar and nuclear densities are denoted, respectively, by $\rho_S$ and $\rho_N$. 
The lambda mean-field potential in nuclear medium has been extracted from the properties of hypernuclei [31,32]. It has also been studied in the Dirac-Brueckner approach using a boson-exchange model for the $\Lambda N$ potential [33], or by extending the Walecka model from SU(2) to SU(3) [34,35]. In the naive constituent quark model, the lambda mean-field potential is about $2/3$ of that of a nucleon, and is in qualitative agreement with the empirical observation [31,32]. To see the sensitivity of lambda flow to the lambda potential in dense matter, we adopt the constituent quark relation but introduce a parameter $\alpha$ in the lambda vector potential, i.e.,

$$\Sigma^\Lambda_S \approx 2/3 \Sigma^N_S,$$

$$\Sigma^\Lambda_V \approx 2/3 \alpha \Sigma^N_V,$$

(6)

where $\Sigma^N_S$ and $\Sigma^N_V$ are the nucleon scalar and vector potentials in the non-linear $\sigma$-$\omega$ model [27]. The lambda optical-model potential can then be defined as

$$U_\Lambda(p, \rho) = \left( (m_\Lambda - \Sigma^\Lambda_S)^2 + p^2 \right)^{1/2} + \Sigma^\Lambda_V - \left( m_\Lambda^2 + p^2 \right)^{1/2}.$$  

(7)

We consider three values of $\alpha$, i.e., 0.85, 1.0, and 1.2 so that the lambda potential is varied within a reasonable range. The corresponding lambda potential is shown in Fig. 1. The nucleon potential used in this study is based on the parameter set in Ref. [27] that corresponds to a soft equation of state with a compression modulus of 200 MeV and a nucleon effective mass of $0.83 m_N$ at normal nuclear matter density. The solid circle with error bars is the currently determined lambda potential in nuclear matter from both the structure of hypernuclei [31,32] and the Dirac-Brueckner calculation [33].

Including the medium modification of the lambda, as well as the nucleon and kaon, $\sqrt{s}$ in Eq. (2) is replaced by $\sqrt{s^*} = (m_{N_1}^2 + p^2)^{1/2} + (m_{N_2}^2 + p^2)^{1/2} + (1 - 2/3 \alpha) \Sigma^N_V$, while $m_N$, $m_\Lambda$, and $m_K$ are replaced by $m_N^* = m_N - \Sigma^N_S$, $m_\Lambda^* = m_\Lambda - \Sigma^\Lambda_S$, and $m_K^*$, respectively. Similarly, $\sqrt{s}$ and $\sqrt{s_0}$ in Eq. (3) are replaced by $\sqrt{s^*} = (m_N^2 + p^2)^{1/2} + (m_\pi^2 + p^2)^{1/2} + (1 - 2/3 \alpha) \Sigma^N_V$ and $\sqrt{s_0^*} = m_\Lambda^* + m_K^*$, respectively. In calculating lambda production, we have thus included the change of the threshold due to the medium modification of hadron properties.

After production, a lambda interacts with surrounding baryons. This includes both $\Lambda N$ scattering and lambda propagation in the mean field generated by nuclear medium. In
principle, the $\Lambda N$ scattering is modified in medium, but we have not attempted to include this effect in a consistent fashion. As in most transport models, the free cross section will be used. However, we will show results from varying the $\Lambda N$ cross section. Furthermore, we will not address the question of consistently separating the interactions into a mean-field and a scattering part in the transport model simulation, which has recently been raised in Ref. [36].

The $\Lambda N$ cross section has been measured experimentally [37] and also studied in a boson-exchange model [33]. For beam energies considered in the present study, the $\Lambda N$ inelastic scattering, mainly $\Lambda N \to \Lambda \Delta$, is relatively unimportant and is neglected. We thus consider only $\Lambda N$ elastic scattering. The experimental data can be fitted by

$$\sigma_{\Lambda N} = 12.0 + 0.43 \frac{p_{\text{lab}}}{\text{GeV}} \text{ mb},$$

(8)

where $p_{\text{lab}}$, in units of GeV, is the momentum of the lambda in nucleon rest frame. This cross section is shown in Fig. 2, where circles are data from Ref. [37] and the solid curve is the parameterization. The angular distribution for the $\Lambda N$ scattering is assumed to be isotropic in its center-of-mass frame, which is consistent with the results from the boson-exchange model of Ref. [33]. To see how the lambda flow might depend on the angular distribution in $\Lambda N$ scattering, we have carried out a calculation by taking the $\Lambda N$ differential cross section as the proton-proton one [38], and we find that this has very little effect on the final lambda flow.

Including a mean-field potential, the lambda equations of motion between collisions are then given by

$$\frac{dx}{dt} = \frac{p^*}{E^*}, \quad \frac{dp}{dt} = -\nabla_x U_{\Lambda}(p, \rho),$$

(9)

where $E^* = \sqrt{p^*^2 + (m_\Lambda - \Sigma_{CS})^2}$. 


III. RESULTS AND DISCUSSIONS

Using the model described in the above, we have carried out a calculation of lambda production in Ni+Ni collisions at 1.93 GeV/nucleon and impact parameter $b \leq 4$ fm that corresponds approximately to the centrality selection in the FOPI experiment. A direct comparison of the predicted lambda flow with the preliminary FOPI and EOS data requires also knowledge on the experimental acceptance cuts. Since these corrections apply similarly to protons and lambdas, we have not attempted to include them in this study. What is significant in both FOPI and EOS data is the fact that the observed lambda flow is in the same direction as proton flow, and both have similar strengths. We thus compare instead the predicted lambda and proton flows from the transport model.

The results for lambda flow, i.e., the average transverse momentum $\langle p_x \rangle$ as a function of rapidity $y$ in the nucleus-nucleus center-of-mass frame, are shown in Fig. 3. The dashed curve is the flow of primordial lambdas. The dotted-dashed curve gives the flow of lambdas after including elastic $\Lambda N$ scattering with a cross section given by Eq. (8). The lambda flow including both scattering and lambda propagation in mean field (with $\alpha = 1.0$) is shown by the solid curve. For comparison, we also show by dotted curve the proton flow.

To be more quantitative, we introduce the flow parameter defined as the slope of the average transverse momentum at mid-rapidity, i.e.,

$$F = \frac{d\langle p_x \rangle}{dy} \bigg|_{y=0},$$

(10)

As in the case of kaons, the flow of primordial lambdas, which arises mainly from Lorentz boost in the direction of the baryon-baryon or meson-baryon pairs that produce the antikaon, is considerably weaker than the proton flow. Their flow parameter is about 60 MeV as compared to the proton flow parameter of about 140 MeV. Including elastic $\Lambda N$ scattering increases the flow parameter to about 80 MeV. This is due to the effect of thermalization that increases the magnitude of lambda momentum. The propagation of lambdas in mean-field potential further enhances their flow in the direction of nucleons, and the flow parameter is
now about 115 MeV, and is close to that of protons. Overall, final-state interactions enhance the lambda flow parameter by about a factor of two.

The importance of final-state interactions can also be seen from \( \langle p_x \rangle_{\text{max}} \), which occurs near the projectile and target rapidities. The magnitude of \( \langle p_x \rangle_{\text{max}} \) for primordial lambdas is about 45 MeV, which increases to about 60 MeV when \( \Lambda N \) scattering is included, and further increases to about 100 MeV after including also lambda propagation. Final-state interactions thus increase \( \langle p_x \rangle_{\text{max}} \) by about a factor of two as well.

To learn quantitatively the lambda potential from lambda flow requires not only more accurate experimental data but also better theoretical understandings on how the flow is affected by a possible change of \( \Lambda N \) cross section in nuclear medium. For this purpose, we have carried out two calculations, one with \( \sigma_{\Lambda N} \) reduced by 50% and the other with \( \sigma_{\Lambda N} \) increased by 50% (\( \alpha \) is always kept at 1.0). The results are shown in Fig. 4. It is seen that changing \( \Lambda N \) cross section by a factor of 3 does not affect significantly the lambda flow, with the flow parameter modified by only about 15 MeV. On the average, each lambda undergoes about 1.3, 2.5 and 3.6 collisions for 0.5\( \sigma_{\Lambda N} \), \( \sigma_{\Lambda N} \), and 1.5\( \sigma_{\Lambda N} \), respectively. This implies that with 1.3 \( \Lambda N \) scattering, lambdas are already thermalized by nucleons, so further increase of the cross section does not have significant effects on the lambda momentum distribution.

To see the sensitivity of lambda flow to the lambda potential we have done two additional calculations with \( \alpha = 1.2 \) and 0.85, while always using the \( \Lambda N \) cross section given by Eq. (8). The results for lambda flow are shown in Fig. 5 together with that using \( \alpha = 1.0 \). We see that with an increasingly attractive lambda potential, the lambda flow becomes stronger. The flow parameter \( F \) changes from about 95 MeV for \( \alpha = 1.2 \) to about 135 MeV for \( \alpha = 0.85 \). It is thus possible to differentiate the scenario with a shallow lambda potential (as given by \( \alpha = 1.2 \)) from that with a deep lambda potential (as given by \( \alpha = 0.85 \)) if the lambda flow can be measured with good accuracy.

Similarly, one can also look at the azimuthal distribution of lambdas to examine the effects of lambda potential. The results for the lambda azimuthal distribution near the target rapidity are shown in Fig. 6 for three different potentials, as well as for the case
without lambda potential. We have normalized these results around \( \phi = 0^0 \). In all four cases, the lambda azimuthal distribution exhibits a peak around \( \phi = 180^0 \), as in the nucleon distribution shown in the figure by the dotted curve. The lambda azimuthal anisotropy gets more pronounced as the lambda potential becomes more attractive. To show this more quantitatively, we introduce an anisotropy parameter \( R \) defined by

\[
R = \frac{dN/d\phi(\phi = 180^0 \pm 15^0)}{dN/d\phi(\phi = 0^0 \pm 15^0)}.
\] (11)

It is seen that \( R \) changes from about 1.9 to about 2.6 when \( \alpha \) changes from 1.2 to 0.85. Without lambda potential \( R \) is about 1.4. For comparison, we note that the anisotropy parameter for nucleons is \( R \approx 3.0 \). Thus, the lambda mean-field potential also enhances its azimuthal anisotropy in the direction of nucleons.

In Fig. 7, we summarize the dependence of the flow parameter \( F \) and the anisotropy parameter \( R \), as well as the lambda yield, on the strength parameter \( \alpha \) of the lambda vector potential. It is seen that the lambda yield also depends on the strength of lambda potential. A stronger repulsive lambda vector potential increases the threshold for its production and thus decreases its yield. As a result, we see a similar dependence of the lambda yield on the strength parameter \( \alpha \) as for the lambda flow parameter and anisotropy anisotropy.

**IV. SUMMARY**

In summary, we have studied lambda flow in heavy-ion collisions at SIS energies. We have found that the primordial lambdas show a relatively weak flow as compared with the nucleon flow. The inclusion of final-state interactions, especially the propagation in mean-field potential, enhances the lambda flow in the direction of nucleons, and brings the theoretical results in agreement with the preliminary data from both FOPI [5] and EOS [4] collaboration. Significant differences in both lambda in-plane and out-of-plane flows are found between the results with and without lambda potential. On the other hand, the final lambda flow is relatively insensitive to changes of the \( \Lambda N \) cross section.
within a reasonable range. Accurate measurements of both lambda yield and flow in heavy-ion collisions thus allow us to determine the lambda potential in the dense matter formed in heavy-ion collisions. This information cannot be obtained from studies of hypernuclei which provide only the lambda potential at and below normal nuclear matter density. For understanding the properties of neutron stars, lambda properties at higher densities than normal nuclear matter are needed. The information one derives from lambda flow in heavy-ion collisions is thus very useful for studying neutron star properties [22–24].

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**Figure Captions**

Fig. 1: The lambda optical potential. The solid circle is the empirically determined lambda potential at normal nuclear matter density.

Fig. 2: The elastic $\Lambda N$ cross section. Circles are experimental data from Ref. [37], and the curve is the parameterization given by Eq. (8).

Fig. 3: The average transverse momentum of lambdas as a function of rapidity. The dashed, dotted-dashed, and solid curves are for primordial lambdas, lambdas with scattering, and lambdas with both scattering and propagation, respectively. The dotted curve is for protons.

Fig. 4: Effects of $\Lambda N$ cross section on lambda average transverse momentum distribution in rapidity space. The dotted, solid, and dashed curves are obtained with $0.5\sigma_{\Lambda N}$, $\sigma_{\Lambda N}$, and $1.5\sigma_{\Lambda N}$, respectively.

Fig. 5: Effects of lambda potential on its average transverse momentum distribution in rapidity space. The dotted, solid, and dashed curves are obtained with $\alpha=0.85$, 1.0 and 1.2, respectively.

Fig. 6: The azimuthal distribution of lambdas near target rapidity. The dotted curve gives that of nucleons.

Fig. 7: The lambda yield, flow parameter, and azimuthal anisotropy parameter as functions of the strength parameter $\alpha$ for the lambda vector potential.