Democratic Neutrino Mixing Reexamined

Harald Fritzsch
Sektion Physik, Universität München, Theresienstrasse 37A, 80333 Munich, Germany

Zhi-zhong Xing
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100039, China
(Electronic address: xingzz@mail.ihep.ac.cn)

Abstract

We reexamine the democratic neutrino mixing ansatz, in which the mass matrices of charged leptons and Majorana neutrinos arise respectively from the explicit breaking of $S(3)_L \times S(3)_R$ and $S(3)$ flavor symmetries. It is shown that a democracy term in the neutrino sector can naturally allow the ansatz to fit the solar neutrino mixing angle $\theta_{\text{sun}} \approx 33^\circ$. We predict $\sin^2 2\theta_{\text{atm}} \approx 0.95$ for atmospheric neutrino mixing and $J \approx 1.2\%$ for leptonic CP violation in neutrino oscillations without any fine-tuning. Direct relations between the model parameters and experimental observables are also discussed.

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The recent solar [1], atmospheric [2], KamLAND [3] and K2K [4] neutrino oscillation experiments provide us with very compelling evidence that neutrinos are massive and lepton flavors are mixed. To account for the observed neutrino mass-squared differences ($\Delta m_{\text{sun}}^2 \sim 6.9 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{\text{atm}}^2 \sim 2.3 \times 10^{-3} \text{ eV}^2$) and mixing factors ($\sin^2 2\theta_{\text{sun}} \sim 0.84$ and $\sin^2 2\theta_{\text{atm}} \sim 1.0$ [5]), many phenomenological models of lepton mass matrices have been proposed in the literature [6]. Some of them take advantage of the idea of flavor democracy, from which the largeness of two lepton mixing angles, the smallness of three quark mixing angles, and the wide mass gaps between ($m_\tau, m_\mu, m_b$) and their lighter counterparts can simultaneously be understood.

The original ansatz of democratic neutrino mixing [7] is based on the phenomenological conjecture that charged lepton and Majorana neutrino mass matrices may arise from the breaking of $S(3)_L \times S(3)_R$ and $S(3)$ flavor symmetries, respectively:

$$
M_l = \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \Delta M_l ,
$$

$$
M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \Delta M_\nu ,
$$

where $c_l$ and $c_\nu$ measure the corresponding mass scales of charged leptons and neutrinos. The explicit symmetry-breaking term $\Delta M_l$ is responsible for the generation of muon and electron masses, and $\Delta M_\nu$ is responsible for the breaking of neutrino mass degeneracy. A very simple form of $\Delta M_l$ and $\Delta M_\nu$ reads [7]

$$
\Delta M_l = \frac{c_l}{3} \begin{pmatrix} -i\delta_l & 0 & 0 \\ 0 & i\delta_l & 0 \\ 0 & 0 & \varepsilon_l \end{pmatrix} ,
$$

$$
\Delta M_\nu = c_\nu \begin{pmatrix} -\delta_\nu & 0 & 0 \\ 0 & \delta_\nu & 0 \\ 0 & 0 & \varepsilon_\nu \end{pmatrix} ,
$$

where ($\delta_l, \varepsilon_l$) and ($\delta_\nu, \varepsilon_\nu$) are real dimensionless perturbation parameters of small magnitude, and the imaginary phase of $\Delta M_l$ is a natural source of leptonic CP violation in neutrino oscillations. Because $M_\nu$ is already diagonal, we only need to diagonalize $M_l$ by means of the orthogonal transformation $V M_l V^T = \text{Diag}\{m_e, m_\mu, m_\tau\}$, in order to express the leptonic charged-current interactions in terms of the mass eigenstates of charged leptons and neutrinos. The lepton flavor mixing matrix is just given by the unitary matrix $V$; i.e.,

$$
V \approx \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} + \sqrt{\frac{m_e}{m_\mu}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & 0 \end{pmatrix} + \frac{m_\mu}{m_\tau} \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} \\ 0 & 0 & 0 \end{pmatrix} .
$$

Given $m_e/m_\mu \approx 0.00484$ and $m_\mu/m_\tau \approx 0.0594$ [8], the mixing factors of solar and atmospheric neutrino oscillations turn out to be
\[ \sin^2 2\theta_{\text{sun}} \approx 1 - \frac{4}{3} \frac{m_e}{m_\mu} \approx 0.99 , \]
\[ \sin^2 2\theta_{\text{atm}} \approx \frac{8}{9} \left( 1 + \frac{m_\mu}{m_\tau} \right) \approx 0.94 . \]  
(4)

The result of \( \sin^2 2\theta_{\text{sun}} \) is obviously disfavored by current solar neutrino data, and that of \( \sin^2 2\theta_{\text{atm}} \) apparently deviates from the maximal atmospheric neutrino mixing.

A simple way to suppress the afore-obtained value of \( \sin^2 2\theta_{\text{sun}} \) and enhance that of \( \sin^2 2\theta_{\text{atm}} \) is to add another \( S(3) \)-symmetry term, which was not included in Eq. (1), into the neutrino mass matrix \( M_\nu \) [9]. In this case, we have

\[
M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \Delta M_\nu ,
\]
(5)

where \( r_\nu \) is in principle an arbitrary parameter. To get large lepton mixing angles, however, \(|r_\nu| \ll 1\) must be satisfied. It is shown in Ref. [9] that the \( r_\nu \)-induced corrections to \( \sin^2 2\theta_{\text{sun}} \) and \( \sin^2 2\theta_{\text{atm}} \) can both be constructive, and \( \Delta m^2_{\text{sun}} \sim (1 - 2) \times 10^{-4} \text{ eV}^2 \) is predicted by taking the appropriate parameter space of \((c_\nu, r_\nu, \delta_\nu, \varepsilon_\nu)\) \(^1\). Note that \( \Delta m^2_{\text{sun}} \sim \mathcal{O}(10^{-4}) \text{ eV}^2 \) is no more favored by today’s experimental data on solar neutrino oscillations. It is therefore necessary to reexamine whether a favorable bi-large neutrino mixing pattern can naturally be derived from the explicit breaking of \( S(3)_L \times S(3)_R \) symmetry of charged leptons and \( S(3) \) symmetry of Majorana neutrinos. If the answer remains affirmative, then direct and testable relations between the model parameters and experimental observables should be established.

The main purpose of this short paper is to demonstrate that the \( r_\nu \)-modified version of our phenomenological ansatz is actually compatible with current neutrino oscillation data. We find that the experimentally-favored value of \( \sin^2 2\theta_{\text{sun}} \) can naturally be achieved. We derive a simple relation between \( \sin^2 2\theta_{\text{atm}} \) and \( \cos 2\theta_{\text{sun}} \), and then arrive at the prediction \( \sin^2 2\theta_{\text{atm}} \approx 0.95 \) without any fine-tuning. We also show how to relate the model parameters to the relevant observables. Our analytical results will be very useful to test the democratic neutrino mixing scenario, when more accurate experimental data are available in the near future.

For simplicity, we take \( c_\nu, r_\nu, \delta_\nu \) and \( \varepsilon_\nu \) in Eq. (5) to be real and positive. Then \( M_\nu \) can be diagonalized by means of a real orthogonal transformation \( U^T M_\nu U = \text{Diag}\{m_1, m_2, m_3\} \). It is obvious that three neutrino masses must be nearly degenerate. Taking the convention \( m_1 < m_2 < m_3 \), we obtain

\[
m_1 \approx c_\nu \left( 1 + r_\nu - \sqrt{r_\nu^2 + \delta_\nu^2} \right) ,
\]
\[
m_2 \approx c_\nu \left( 1 + r_\nu + \sqrt{r_\nu^2 + \delta_\nu^2} \right) ,
\]
\[
m_3 \approx c_\nu \left( 1 + r_\nu + \varepsilon_\nu \right) .
\]
(6)

\(^1\)Note that the diagonal perturbation term of \( M_\nu \) in Ref. [9] is not exactly the same as our \( \Delta M_\nu \) given in Eq. (2).
The near degeneracy of three neutrino masses implies that the effective mass-squared term of the tritium beta decay, defined as \( \langle m^2 \rangle_i = \sum (m_i^2 |V_{ei}|^2) \) for \( i = 1, 2 \) and 3, approximately amounts to \( c_{\nu}^2 \). In other words, \( c_{\nu} \approx \langle m \rangle_e \) holds. Then we obtain \( c_{\nu} < 2.2 \text{ eV} \) from the direct-mass-search experiments [8] and \( c_{\nu} < 0.23 \text{ eV} \) from the recent WMAP observational data [10]. In view of \( \Delta m^2_{\text{sun}} \gg \Delta m^2_{\text{atm}} \), we require \( \varepsilon_{\nu} \gg r_{\nu} \) and \( \varepsilon_{\nu} \gg \delta_{\nu} \). Therefore,

\[
\begin{align*}
\Delta m^2_{\text{sun}} &= \Delta m^2_{21} \approx 4c_{\nu}^2 \sqrt{r_{\nu}^2 + \delta_{\nu}^2}, \\
\Delta m^2_{\text{atm}} &= \Delta m^2_{32} \approx 2c_{\nu}^2 \varepsilon_{\nu}.
\end{align*}
\]

(7)

As for the orthogonal matrix \( U \), its nine elements \( U_{ii}, U_{2i} \) and \( U_{3i} \) (for \( i = 1, 2, 3 \)) have the following relations:

\[
\begin{align*}
U_{2i} &= c_{\nu} \frac{(1 - \delta_{\nu}) - m_i}{c_{\nu} (1 + \delta_{\nu}) - m_i} U_{1i}, \\
U_{3i} &= -r_{\nu} \frac{U_{1i} + U_{2i}}{c_{\nu} (1 + r_{\nu} + \varepsilon_{\nu}) - m_i}.
\end{align*}
\]

(8)

To the accuracy of \( \mathcal{O}(r_{\nu}/\varepsilon_{\nu}) \), the expression of \( U \) is found to be

\[
U \approx \begin{pmatrix}
\cos \theta & \sin \theta & \frac{r_{\nu}}{\varepsilon_{\nu}} \\
-\sin \theta & \cos \theta & \frac{\varepsilon_{\nu}}{r_{\nu}} \\
\frac{r_{\nu}}{\varepsilon_{\nu}} (\sin \theta - \cos \theta) & -\frac{\varepsilon_{\nu}}{r_{\nu}} (\sin \theta + \cos \theta) & 1
\end{pmatrix},
\]

(9)

where \( \tan 2\theta \equiv r_{\nu}/\delta_{\nu} \). Without loss of generality, \( \theta \) is required to lie in the first quadrant. It is clear that \( U \) becomes the unity matrix in the limit \( r_{\nu} = 0 \). In the case of \( r_{\nu} \neq 0 \), the lepton flavor mixing matrix takes the form \( \hat{V} = UV \), where \( V \) has been given in Eq. (3). We explicitly obtain \( \hat{V} \approx \hat{V}_0 + \hat{V}_1 \) as a good approximation, in which

\[
\hat{V}_0 = \begin{pmatrix}
\frac{1}{\sqrt{2}} (\cos \theta + \sin \theta) & \frac{1}{\sqrt{2}} (\cos \theta - \sin \theta) & 0 \\
\frac{1}{\sqrt{6}} (\cos \theta - \sin \theta) & \frac{1}{\sqrt{6}} (\cos \theta + \sin \theta) & \frac{2}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} (\cos \theta - \sin \theta) & \frac{1}{\sqrt{3}} (\cos \theta + \sin \theta) & \frac{1}{\sqrt{3}}
\end{pmatrix},
\]

(10)

and

\[
\hat{V}_1 = i \sqrt{\frac{m_e}{m_\mu}} \begin{pmatrix}
\frac{1}{\sqrt{6}} (\cos \theta - \sin \theta) & \frac{1}{\sqrt{6}} (\cos \theta + \sin \theta) & \frac{2}{\sqrt{6}} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} + \frac{m_\mu}{m_\tau} \begin{pmatrix}
\frac{1}{\sqrt{12}} (\cos \theta - \sin \theta) & \frac{1}{\sqrt{12}} (\cos \theta + \sin \theta) & \frac{1}{\sqrt{12}} \\
0 & 0 & 0 \\
\frac{1}{\sqrt{12}} (\cos \theta - \sin \theta) & \frac{1}{\sqrt{12}} (\cos \theta + \sin \theta) & \frac{1}{\sqrt{12}}
\end{pmatrix}.
\]
Comparing $\hat{V}$ with $V$, we see that $\hat{V}_{e3} \approx V_{e3}$ holds. This result implies that the mixing angle $\theta_{13}$ in the standard parametrization of $\hat{V}$ [8] is rather small:

$$\sin \theta_{13} = |\hat{V}_{e3}| \approx \frac{2}{\sqrt{3}} \sqrt{m_e / m_\mu} \approx 0.057,$$

or $\theta_{13} \approx 3.2^\circ$. On the other hand, eight other elements of $\hat{V}$ may get appreciable $r_\nu$-induced corrections.

With the help of Eqs. (10) and (11), the solar neutrino mixing factor is obtained as

$$\sin^2 2\theta_{\text{sun}} = 4|\hat{V}_{e1}|^2|\hat{V}_{e2}|^2 \approx \cos^2 2\theta .$$

It follows that $\theta \approx (45^\circ - \theta_{\text{sun}})$ holds. In other words, $\theta$ measures the deviation of $\theta_{\text{sun}}$ from $45^\circ$. As observed in Ref. [11], the sum $\theta_{\text{sun}} + \theta_C \approx 45^\circ$ with $\theta_C$ being the Cabibbo angle of quark mixing is favored by current experimental data. In this case, we are then left with $\theta \approx \theta_C \approx 12^\circ$. The ratio $r_\nu / \delta_\nu$ can in turn be determined in terms of the mixing angle $\theta_{\text{sun}}$: $r_\nu / \delta_\nu \approx \cot 2\theta_{\text{sun}}$. Typically taking the best-fit value $\theta_{\text{sun}} \approx 33^\circ$, we arrive at $r_\nu / \delta_\nu \approx 0.44$. One may also estimate the magnitude of $r_\nu / \varepsilon_\nu$ with the help of Eq. (7). The result is

$$\frac{r_\nu}{\varepsilon_\nu} \approx \frac{\Delta m^2_{\text{sun}}}{\Delta m^2_{\text{atm}}} \cdot \frac{\cos 2\theta_{\text{sun}}}{2} \approx 6.1 \times 10^{-3} ,$$

where $\Delta m^2_{\text{sun}} / \Delta m^2_{\text{atm}} \approx 3 \times 10^{-2}$ and $\theta_{\text{sun}} \approx 33^\circ$ have been used. It is then clear that $\varepsilon_\nu \gg \delta_\nu \sim r_\nu$ holds.

Now let us calculate the atmospheric neutrino mixing factor $\sin^2 2\theta_{\text{atm}}$ by using Eqs. (10) and (11). We obtain

$$\sin^2 2\theta_{\text{atm}} = 4|\hat{V}_{\mu 3}|^2 (1 - |\hat{V}_{\mu 3}|^2) \approx \frac{8}{9} \left( 1 + \frac{m_\mu}{m_\tau} + \frac{\Delta m^2_{\text{sun}}}{\Delta m^2_{\text{atm}}} \cos 2\theta_{\text{sun}} \right) \approx 0.95 .$$

Comparing between Eqs. (4) and (15), we find that the $r_\nu$-induced correction to $\sin^2 2\theta_{\text{atm}}$ is constructive but suppressed by $\Delta m^2_{\text{sun}} / \Delta m^2_{\text{atm}} \sim \mathcal{O}(10^{-2})$. We conclude that the maximal atmospheric neutrino mixing cannot be achieved in a simple and natural way, unless the ratio $\Delta m^2_{\text{sun}} / \Delta m^2_{\text{atm}}$ is as large as of $\mathcal{O}(10^{-1})$. A more precise determination of $\Delta m^2_{\text{sun}}$, $\Delta m^2_{\text{atm}}$, $\theta_{\text{sun}}$ and $\theta_{\text{atm}}$ will test the validity of Eq. (15).

The consequences of this phenomenological ansatz on the neutrinoless double beta decay and CP violation in neutrino oscillations are interesting. A straightforward calculation yields $\langle m \rangle_{ee} = |\sum (m_i \hat{V}_{ei}^2)| \approx c_\nu$ for the effective mass of the neutrinoless double beta decay. It becomes obvious that $\langle m \rangle_{ee} \approx \langle m \rangle_e \approx c_\nu$ holds. The absolute neutrino mass scale in our ansatz can be fixed either from a measurement of the tritium beta decay or from a positive
signal of the neutrinoless double beta decay. The Jarlskog invariant of CP violation [12] is found to be

\[ J \approx \frac{1}{3\sqrt{3}} \sqrt{\frac{m_e}{m_{\mu}}} \sin 2\theta_{\text{sun}} \approx 0.012. \] (16)

Such a strength of leptonic CP violation is likely to be observed in a long-baseline neutrino oscillation experiment.

One can see that the democratic neutrino mixing ansatz under discussion is compatible with all of current neutrino data. Its prediction for \( \sin^2 2\theta_{\text{atm}} \) can easily be tested in the near future. Of course, part of our results depend on the explicit symmetry breaking patterns (i.e., \( \Delta M_L \) and \( \Delta M_{\nu} \)). Let us comment on the effects of S(3) flavor symmetry breaking terms in some detail:

(a) The lepton flavor mixing matrix \( \hat{V} \) is insensitive to the form of \( \Delta M_L \), as already observed in Ref. [7]. The point is simply that the strong mass hierarchy of three charged leptons makes the contribution of \( \Delta M_L \) to \( \hat{V} \) insignificant, no matter whether \( \Delta M_L \) is diagonal or off-diagonal.

(b) If a contrived and fine-tuned pattern of \( \Delta M_{\nu} \) is taken, it should be possible to obtain a “proper” (2,3)-rotation angle from \( M_{\nu} \) in order to arrive at \( \theta_{\text{atm}} \sim 45^\circ \). However, it is more natural to consider the simple forms of \( \Delta M_{\nu} \) such as the diagonal perturbation given in Eq. (2), at least from the point of view of model building [13].

(c) A remarkable advantage of the diagonal perturbation \( \Delta M_{\nu} \) is that it guarantees \( M_{\nu} \) to be stable against radiative corrections [9,14], although three mass eigenvalues of \( M_{\nu} \) are almost degenerate. This feature makes sense for model building too, because the S(3)\(_L\) \( \times \) S(3)\(_R\) symmetry of \( M_L \) and the S(3) symmetry of \( M_{\nu} \) are most likely to manifest themselves at a high energy scale (e.g., the seesaw scale [15], where three heavy right-handed neutrinos might also have an approximate flavor democracy [16]).

Finally, it is worth emphasizing that four free parameters of \( M_{\nu} \) may all be determined in terms of the relevant observable quantities. We obtain \( c_{\nu} \approx \langle m \rangle_e \approx \langle m \rangle_{ee} \). Then \( \varepsilon_{\nu} \approx \Delta m^2_{\text{atm}}/(2\langle m \rangle^2_e) \) can straightforwardly be derived from Eq. (7). With the help of Eq. (14), we further arrive at

\[
\begin{align*}
  r_{\nu} & \approx \frac{\Delta m^2_{\text{sun}}}{\langle m \rangle^2_e} \cdot \frac{\cos 2\theta_{\text{sun}}}{4}, \\
  \delta_{\nu} & \approx \frac{\Delta m^2_{\text{sun}}}{\langle m \rangle^2_e} \cdot \frac{\sin 2\theta_{\text{sun}}}{4}. 
\end{align*}
\] (17)

Note that the magnitudes of \( \varepsilon_{\nu} \) and \( \delta_{\nu} \) should be of or below \( O(0.1) \), because they are perturbative parameters of \( \Delta M_{\nu} \). Taking \( \varepsilon_{\nu} \sim 0.1 \), for instance, we may get \( \langle m \rangle_e \approx \langle m \rangle_{ee} \sim 0.1 \text{ eV} \). To measure such a small \( \langle m \rangle_e \) in the tritium beta decay is practically difficult (but not impossible) in the near future [17]. In comparison, \( \langle m \rangle_{ee} \sim 0.1 \text{ eV} \) is definitely accessible in a number of planned experiments for the neutrinoless double beta decay [18]. This numerical example indicates that \( \varepsilon_{\nu} \sim O(0.1) \) is most plausible. A much smaller \( \varepsilon_{\nu} \) would make \( m_i \approx c_{\nu} \) (for \( i = 1, 2, 3 \)) too large to be compatible with the WMAP upper limit on \( m_i \), while a much bigger \( \varepsilon_{\nu} \) would lose its physical meaning as a perturbative parameter.
In summary, we have reexamined the democratic neutrino mixing ansatz by taking into account an extra $S(3)$-symmetry term in the Majorana neutrino mass matrix. After explicit symmetry breaking induced by the diagonal perturbations, we obtain the mass spectrum of charged leptons with a strong hierarchy and that of neutrinos with a near degeneracy. The suppressed democracy term in the neutrino sector can naturally permit the model to fit current solar neutrino oscillation data with $\theta_{\text{sun}} \approx 33^\circ$. We have derived a simple relation between $\sin^2 2\theta_{\text{atm}}$ and $\cos 2\theta_{\text{sun}}$, and achieved the prediction $\sin^2 2\theta_{\text{atm}} \approx 0.95$ without any fine-tuning. Whether this atmospheric neutrino mixing factor is really maximal or not will provide a sensitive test of our phenomenological ansatz. We have also established the direct relations between the model parameters and relevant experimental observables. We remark that the democratic neutrino mixing scenario is simple, viable and suggestive. It could be useful for model building, in particular at a high energy scale at which the $S(3)_L \times S(3)_R$ symmetry of charged leptons and the $S(3)$ symmetry of Majorana neutrinos are expected to become relevant.

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