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On Larkin-Imry-Ma State of $^3$He-A in Aerogel

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Abstract  Superfluid $^3$He-A shares the properties of spin nematic and chiral orbital ferromagnet. Its order parameter is characterized by two vectors $\hat{d}$ and $\hat{l}$. This doubly anisotropic superfluid, when it is confined in aerogel, represents the most interesting example of a system with continuous symmetry in the presence of random anisotropy disorder. We discuss the Larkin-Imry-Ma state, which is characterized by the short-range orientational order of the vector $\hat{l}$, while the long-range orientational order is destroyed by the collective action of the randomly oriented aerogel strings. On the other hand, sufficiently large regular anisotropy produced either by the deformation of the aerogel or by applied superflow destroys the Larkin-Imry-Ma effect leading to the uniform orientation of $\hat{l}$. The interplay of regular and random anisotropy allows us to study many different effects.

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1 Introduction

NMR experiments on liquid $^3$He confined in aerogel demonstrate a rich but still unexplained life in two regions of the phase diagram in Fig. 1: (i) in the region between the lines $T_{cb}$ and $T_{ca}$ of superfluid phase transition in bulk liquid and in aerogel correspondingly; and (ii) in the region of the so-called A-like phase which is metastable in the absence of external magnetic field. Here we concentrate on the A-like phase, which will be discussed in terms of the Larkin-Imry-Ma effect occurring in the anisotropic $^3$He-A superfluid when it is confined in aerogel.

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Fig. 1 Two main problems related to superfluid $^3$He in aerogel: (1) What is the state of $^3$He between the two lines of the superfluid phase transition? What is the state of $^3$He in the so-called A-like phase?

1.1 Doubly anisotropic superfluid $^3$He-A

Superfluid $^3$He-A shares the properties of spin nematic and chiral orbital ferromagnet. Its order parameter is characterized by two vectors $\hat{d}$ and $\hat{l}$:

$$A_{\mu i} = \Delta e^{ib} \hat{d}_\mu (\hat{m}_i + i\hat{n}_i) \quad , \quad \hat{l} = \hat{m} \times \hat{n} . \quad (1)$$

Unit vector $\hat{d}$ marks direction of anisotropy axis in spin space, and is responsible for anisotropy of spin susceptibility; unit vector $\hat{l}$ marks the direction of the orbital angular momentum of Cooper pairs and simultaneously the direction of axis of the orbital anisotropy and it is responsible for anisotropy of superfluid density:

$$F_s + F_o + F_{so} = \frac{(S \cdot \hat{d})^2}{2\chi_\parallel} + \frac{(S \times \hat{d})^2}{2\chi_\perp} + \frac{\rho_s}{2}(v_s \cdot \hat{l})^2 + \frac{\rho_{s\perp}}{2}(v_s \times \hat{l})^2 - g_D(\hat{d} \cdot \hat{l})^2 . \quad (2)$$

Here $S$ is spin density in the applied magnetic field, $S_{\alpha} = \chi_{\alpha\beta}H_\beta$; $v_s$ is the velocity of superfluid mass current $j^s_i = \rho_s^{ij}v^s_j$. Because of the chiral nature of the orbital order parameter $\hat{m} + i\hat{n}$, the superfluid velocity and its vorticity

$$v_s = \frac{h}{2m}(\nabla \phi + \hat{m}_i \nabla \hat{n}_i) \quad , \quad \nabla \times v_s = \frac{h}{4m}e^{ijk}\hat{l}_i \nabla \hat{l}_j \times \nabla \hat{l}_k . \quad (3)$$
can be generated by the texture of the $\mathbf{l}$-vector. The last term in Eq. (2) describes a tiny spin-orbit coupling $F_{so}$ between $\mathbf{d}$ and $\mathbf{l}$, which is however very important for the NMR measurements.

This doubly anisotropic superfluid, when it is confined in aerogel, represents the most interesting example of a system with continuous symmetry in the presence of random anisotropy. For such systems the surprising conjecture was made by Imry and Ma\textsuperscript{5} that even a weak disorder may destroy the long-range orientational order. This should occur in the same manner as impurities destroy the long-range translational order in a lattice of Abrikosov vortices in superconductors as was first predicted by Larkin\textsuperscript{6}. Recent NMR experiments allow us to discuss the state of $^3$He-A in aerogel in terms of the Larkin-Imry-Ma (LIM) state. The LIM state in $^3$He-A is characterized by short-range orientational order of the orbital vector $\mathbf{l}$, while the long-range orientational order is destroyed by the collective action of the randomly oriented aerogel strings. The order of magnitude theoretical estimations based on the model of aerogel in Sec. 2.1 and the NMR data suggest that in the aerogel samples under investigation, the LIM length $L_{LIM}$ at which the orientational order is destroyed is of order of a micron. This is much bigger than the microscopic scales of aerogel strands, but is smaller than dipole length $\xi_D \sim 10 \mu\text{m}$ characterizing spin-orbit coupling between $\mathbf{l}$ and $\mathbf{d}$ in Eq. (2). The latter leads to anomalously small values of the observed transverse NMR frequency shift and longitudinal NMR frequency as we shall discuss in Sec. 3.

A regular anisotropy may suppress the LIM effect, and this does take place when one applies a uni-axial deformation to the aerogel sample (Sec. 2.2). A linear deformation of about 1% leads to the restoration of the uniform orientation of the $\mathbf{l}$-vector in the sample\textsuperscript{3}. As a result the value of the transverse NMR frequency shift is enhanced by an order of magnitude compared to that in the LIM state. If the deformation leads to orientation of $\mathbf{l}$ along the magnetic field $\mathbf{H}$, the transverse NMR frequency shift becomes negative. These are the observations which support the interpretation of the A-phase like state in aerogel as the disordered LIM state.

Open problems related to the disordered LIM state including the superfluid properties of the LIM state are discussed in Sec. 4. Superfluidity in LIM state could be rather unusual because, according to the Mermin-Ho relation in Eq. (3) between $\mathbf{l}$ and $\psi$, the LIM texture generates superfluid vorticity. As a result, the disordered LIM state can be represented as a system of randomly distributed skyrmions – vortices with continuous cores. The characteristic distance between the vortex-skyrmions and the size of their cores are determined by the LIM length $L_{LIM}$. In principle, the superfluid density of $^3$He-A in aerogel may depend on the ability of the aerogel to pin the randomly distributed vortex-skyrmions.

2 Theory of Larkin-Imry-Ma effect

2.1 Larkin-Imry-Ma state in the model of random cylinders

We shall use a simple model of aerogel as a system of randomly oriented cylinders (see Ref.\textsuperscript{6} and Fig. 1) with diameter of aerogel strand $\delta \sim 3$ nm and the length $\xi_a \sim 20$ nm which is the distance between the strands. Since $\delta$ is much smaller than the superfluid coherence length $\xi_0$, the theory of Rainer and Vuorio for the microscopic body immersed in $^3$He-A\textsuperscript{6} can be applied. According to this
theory, the $\mathbf{l}$-vector remains homogeneous outside the cylindrical object (see Fig. 2 left), and the orientational energy acting from the $\mathbf{l}$-vector on a cylinder $i$ with the direction of axis along $\mathbf{n}_i$ is

$$E_i = E_a (\mathbf{l} \cdot \mathbf{n}_i)^2, \quad E_a \sim \frac{\Delta^2}{T_c k_F^2} \xi_a \delta > 0.$$  \hspace{1cm} (4)

Here $k_F$ is Fermi momentum; the parameter $E_a$ is positive as follows from the Rainer-Vuorio calculations.

For the infinite system of cylinders, the average orientational effect of the random cylinders on the $\mathbf{l}$-vector is absent, if the $\mathbf{l}$-vector is kept uniform. However, there is a collective effect of many cylinders, which makes the $\mathbf{l}$-vector inhomogeneous on large scales. Let us consider the box of size $L \times L \times L$, which contains a large but finite number $N \sim L^3/\xi_a^3 \gg 1$ of randomly oriented cylinders and still has a uniform orientation of the $\mathbf{l}$-vector. Due to fluctuations of orientational energies of randomly oriented cylinders, the $\mathbf{l}$-vector will find the preferred orientation in the box, with the energy gain proportional to $N^{1/2}$. The corresponding negative energy density is determined by the variance of the energy of random anisotropy:

$$E_{\text{ran}} \sim -\left( \sum_{i=1}^{N} (E_i - \langle E \rangle)^2 \right)^{1/2} L^{-3} \sim -N^{1/2} E_a L^{-3} \sim -E_a \xi_a^{-3/2} L^{-3/2}.$$  \hspace{1cm} (5)

The neighboring boxes prefer different orientations, as a result the effect of orientation by the aerogel strands will be opposed by the gradient energy $E_{\text{grad}} \sim K (\mathbf{Vl})^2 \sim K/L^2$. The optimal size of a box with a uniform orientation of $\mathbf{l}$ within the box – the LIM length $L_{\text{LIM}}$ – is found from competition between the energy density of random anisotropy and the gradient energy:

$$E_{\text{ran}} + E_{\text{grad}} \sim -E_a \xi_a^{-3/2} L^{-3/2} + KL^{-2}.$$  \hspace{1cm} (6)

Minimization of Eq. (6) gives

$$L_{\text{LIM}} \sim \frac{K^2 \xi_a^3}{E_a^2} \sim \frac{\xi_a \xi_a^2}{\delta^2}. \hspace{1cm} (7)$$
Here $\xi_0$ is the superfluid coherence length at $T=0$; we used Eq. (4) for $E_a$; and estimated the rigidity of the $\hat{l}$-vector as $K \sim (k_F/m)(\Delta^2/T_c^2)$. With $\xi_a \sim \xi_0 \sim 20$ nm and $\delta \sim 3$ nm, the length $L_{\text{LIM}} \sim 1$ $\mu$m. This means that the $\hat{l}$-vector is highly uniform at the scale of the coherence length, i.e. there is a well defined short-range orientational order in aerogel, while the long-range order is lost at much larger length $L_{\text{LIM}} \gg \xi_0$ (Fig. 2 right).

2.2 LIM effect killed by regular anisotropy

The uniaxial deformation of the cylindrical sample of aerogel along its axis $z$ induces the regular anisotropy for the $\hat{l}$-vector with the energy density:

$$E_{\text{reg}} = \left\langle \sum_i (E_i - \langle E \rangle) \right\rangle \sim \frac{\Delta l}{l} E_a \xi_a^{-3} \left( \hat{l} \cdot \hat{z} \right)^2 \frac{1}{3}.$$  \hspace{2cm} (8)

The squeezing ($\Delta l/l < 0$ (Fig. 3) makes $z$ the easy axis for $\hat{l}$, while stretching ($\Delta l/l > 0$) produces the easy plane. Comparing this energy of regular anisotropy with the LIM energy (in Eq. 6 at $L = L_{\text{LIM}}$), one finds that the ordered state with $\hat{l}$ oriented along the easy axis $\hat{z}$ becomes more energetically favorable than the LIM state when the squeezing is sufficiently large:

$$\frac{|\Delta l|}{l} > \left( \frac{\xi_a}{L_{\text{LIM}}} \right)^{3/2}.$$ \hspace{2cm} (9)

The LIM effect is so subtle, that the critical value of the deformation at which the first order phase transition from the LIM state to the uniform state occurs is rather small; $|\Delta l/l| \sim 10^{-3} - 10^{-2}$. Experimentally, deformation of about 1% is enough to obtain the uniform $\hat{l}$. In case of uniaxial stretching the polar phase is predicted to appear in vicinity of the superfluid transition.
3 Experiment

3.1 Estimation of Larkin-Imry-Ma length from NMR

Fig. 4 demonstrates results of the NMR experiments on $^3$He-A in conventional aerogel sample (left), and in a squeezed aerogel (right). In the latter, the transverse NMR line has negative frequency shift corresponding to the uniform $\hat{l}$-vector oriented along the magnetic field $H \parallel \hat{z}$.

In Ref. 2, two NMR lines have been observed in the transverse NMR spectrum in Fig. 4 (left). The line with a larger frequency shift, denoted as $f$-line, can be removed after application of the 180° pulse while cooling through $T_{ca}$ (the superfluid transition temperature in aerogel). The salient feature of the $c$-line which survives after the 180° pulse is the anomalously small frequency shift. It is about 30 times smaller than the magnitude of the negative frequency shift observed in Ref. 3 in the aerogel sample with the uniform $\hat{l}$-vector (we take into account that the frequency shift is $\propto 1/H$ with $H = 224$ G in Ref. 2 and $H = 290$ G in Ref. 3). This large difference can be easily understood if one identifies the $c$-state (the state with a single $c$-line in the spectrum) as the LIM state.

Properties of NMR on the LIM state depend on the ratio between $L_{\text{LIM}}$ and the dipole length $\xi_D \sim 10 \mu$m characterizing spin-orbit coupling between $\hat{l}$ and $\hat{d}$. The orbital vector $\hat{l}$ will be locked with the spin-space vector $\hat{d}$ if $L_{\text{LIM}} > \xi_D$ and unlocked from $\hat{d}$ if $L_{\text{LIM}} < \xi_D$. The NMR results are in favor of the almost completely unlocked case, since in the limit $L_{\text{LIM}}/\xi_D \ll 1$, the $\hat{l}$-texture is completely disordered, $\langle \hat{l}_x^2 \rangle = 1/3$, and the frequency shift is zero. The nonzero but relatively small magnitude of the frequency shift compared to the negative frequency shift, $\Delta \omega_{c-line} \sim |\Delta \omega_{\text{deformed}}|/30$, can be ascribed to the $(L_{\text{LIM}}/\xi_D)^2$ correction. This allows us to estimate the LIM length $L_{\text{LIM}} \sim 1 \mu$m, which is in a reasonable agreement with Eq. (7).
3.2 Tipping angle dependence of NMR

Lines $c$ and $f$ in the spectrum of transverse NMR in $^3$He-A in aerogel demonstrate different dependence of frequency shift on the tipping angle $\beta$ in Fig. [5]. This can be compared with the theoretical frequency shift in the limit of the almost completely disordered state, when $L_{\text{lim}}/\xi_D \ll 1$:

$$\Delta \omega(\beta)/\Delta \omega_{\text{max}} = b \cos \beta + a(1 + \cos \beta).$$  \hspace{1cm} (10)

Here $\Delta \omega_{\text{max}}$ is the maximum possible frequency shift occurring in the uniform $\hat{I}$-field, it coincides with the magnitude of the negative frequency shift when $\hat{I} \parallel H$; the parameter $a \ll 1$ and $b \ll 1$ are nonzero due to the deviations from the full disorder caused by spin-orbit interaction:

$$b = \frac{1}{2} \left( 1 - 3 \left\langle \ell_2^2 \right\rangle \right), \quad a = \frac{1}{6} \left( 1 - 2 \left\langle \sin^2 \Phi \right\rangle \right),$$  \hspace{1cm} (11)

and $\Phi$ is the angle between the components $\hat{I}_\perp$ and $\hat{d}_\perp$, which are transverse to magnetic field $H \parallel \hat{z}$. Parameters $a$ and $b$ are zero in the limit of strong disorder compared to the spin-orbit interaction, i.e. in the limit when $L_{\text{lim}}/\xi_D \rightarrow 0$, and are proportional to $(L_{\text{lim}}/\xi_D)^2$ with pre-factors depending on the type of disorder.

The disorder can be described in the following phenomenological way. In the $c$-state the transverse component $\hat{I}_\perp$ of the vector $\hat{I}$ is random, while the spin-space vector $\hat{d}$ is perpendicular to $H$ and is regular. This gives $a = 0$. On the other hand the magnetic field $H \parallel \hat{z}$, via the vector $\hat{d}$, produces a weak easy-plane anisotropy for the $\hat{I}$-vector. As a result, one has $b > 0$. The $f$-line represents regions with the $f$-state where the $\hat{I}$-vector is fully random, and thus $b = 0$, while $\hat{d}$ and $\hat{I}_\perp$ are not fully independent, producing $a > 0$.

However, it is not clear what is the microscopic background for such behavior of the parameters, and the detailed numerical simulations of the structure of different possible disordered states of $^3$He-A in aerogel are required. Another interpretation of the $\beta$-dependence of the two NMR lines in terms of the modified robust phase has been proposed by Fomin.\textsuperscript{11}
4 Discussion: superfluid properties of LIM state

In chiral anisotropic superfluids the LIM effect should influence the superfluid properties of the system. According to the Mermin-Ho relation in Eq. (3), circulation of superfluid velocity along the closed path $L$ is expressed in terms of the surface integral over the $\hat{l}$-field

$$N(L) = \frac{1}{\kappa} \oint_L d\mathbf{r} \cdot \mathbf{v}_s = \frac{1}{4\pi} \epsilon^{ijk} \int_S dS_k \hat{\mathbf{l}} \cdot \left( \frac{\partial \hat{\mathbf{l}}}{\partial x^j} \times \frac{\partial \hat{\mathbf{l}}}{\partial x^i} \right).$$

(12)

Here we normalized the circulation to the circulation quantum in $^3$He, $\kappa = \pi \hbar / m$, and the circulation number $N$ is not necessarily integer.

If we ignore for a moment that the $\hat{l}$-field is the part of the order parameter triad $\hat{\mathbf{m}}$, $\hat{\mathbf{n}}$ and $\hat{\mathbf{l}}$ in Eq. (1) and consider $\hat{l}$ as completely independent field, then the variance of $N$ for sufficiently large contours is

$$\langle N^2(L) \rangle \sim \frac{L^2}{L_{\text{LIM}}^2}, \quad L \gg L_{\text{LIM}}. \quad (13)$$

Such a LIM state can be represented as a system of randomly distributed skyrmions – vortices with continuous cores. The characteristic distance between the vortex-skyrmions and the size of their soft cores are determined by the LIM length $L_{\text{LIM}}$. In principle, these vortices can screen the applied superfluid velocity. If this happens, then while the local superfluid density $\rho_s$ on the scales below $L_{\text{LIM}}$ is of the same order as in $^3$He-B, it can be reduced and even become zero at large scales.

Experiments on $^3$He-A in aerogel\cite{12,13} demonstrate that $\rho_{s,\parallel} \neq 0$ though it is somewhat smaller as compared to $\rho_{s,\perp}$ in B-phase. This shows that in the consideration of the LIM effect, we cannot ignore the energy of superflow – the energy related to the gradient $\hat{\mathbf{m}} \cdot \nabla \hat{\mathbf{n}}$, which is the superfluid velocity according to Eq. (3). It was stated that the energy of superflow is not relevant for the LIM effect\cite{14}. However, it should suppress the abundance of vortex-skyrmions. The pinning of skyrmions would also restore superfluidity.

There are many open questions related to the LIM state. In particular, how rigid is the $\hat{l}$-texture at large length scales and how strongly are the vortex-skyrmions pinned by aerogel? The problem of rigidity in the LIM state is thirty years old, see the paper by Efetov and Larkin\cite{15} and recent publications\cite{16}.

The idea of quasi-long-range order (QLRO) with a power-law decay of correlators discussed for a vortex lattice in superconductors\cite{19} is applicable to $^3$He-A in aerogel. Probably the QLRO does not occur in isotropic aerogel because of the non-Abelian $SO(3)$ group of triad $\hat{\mathbf{m}}$, $\hat{\mathbf{n}}$, $\hat{\mathbf{l}}$, but it should take place in a stretched aerogel where the induced easy plane anisotropy for $\hat{l}$ reduces $SO(3)$ space to the Abelian $U(1) \times U(1)$ subspace. According to Ref. 18, the power law should depend on stiffness parameters $K$ in the gradient energy. The phase transition from LIM state with QLRO to disordered or to non-superfluid LIM state can be generated by increasing the density of skyrmions. The states $c$ and $f$ in NMR experiments\cite{2} and states in hydrodynamic experiments\cite{13} (see below) may differ by density of skyrmions and thus may have different superfluid properties.

The external superflow influences the LIM state due to anisotropy of the superfluid density in Eq. (2): since $\rho_{s,\parallel} < \rho_{s,\perp}$, the applied $\mathbf{v}_s$ produces the regular...
Fig. 6 Anisotropic superfluid density of $^3$He-A in aerogel according to Ref. 13. Superfluid density is history dependent and depends on the applied superfluid velocity.

easy-axis anisotropy for $\hat{l}$. This suggests that the sufficiently large $v_s$ destroys the LIM state, which is consistent with hydrodynamic experiment 13 demonstrating glassy behavior of $\hat{l}$-texture at low $v_s$ and more regular behavior at large $v_s$. If the initial state $A$ in Fig. 6 is fully disordered but rigid, its superfluid density must be $\rho_A^s = (1/3)\rho_\parallel + (2/3)\rho_\perp$. With increasing $v_s$ the state becomes more and more uniform. In the state $H$, which is reached at large $v_s$, the vector $\hat{l}$ is oriented along $v_s$, and one has $\rho_H^s = \rho_\parallel < \rho_A^s$. Because of the hysteresis, the uniform texture persists when velocity is reduced, but now with $\hat{l} \perp v_s$ as dictated by geometry of the sample; as a result in the subsequent state $I$ one has $\rho_I^s = \rho_\perp > \rho_A^s$.

Another open problem is the role of extended objects. Topological defects with hard cores (single-quantum and half-quantum vortices) may be pinned by aerogel, while $c$ and $f$ states may have different abundance of the defects. The network of connected aerogel strands may have the long range correlations, and the interplay of the long range and short range quenched disorder may also lead to two LIM states with different LIM scales 19.

The LIM effect is rather subtle in $^3$He-A, it is destroyed by relatively weak regular anisotropy produced by flow and/or by deformation of aerogel. This allows us to study novel phenomena which cannot be observed in bulk $^3$He-A. For example, orientation of $\hat{l}$ along the magnetic field $\mathbf{H}$ (not possible in a pure bulk $^3$He-A) stabilizes: (i) Alice strings (half-quantum vortices) in rotating samples 20; and (ii) the phase-coherent precession of magnetization 21. The latter was observed in $^3$He-A in deformed aerogel 22. The coherent precession in $^3$He-A is another realization of Bose-Einstein condensation (BEC) of magnons. The first example of magnon BEC found in 1984 in $^3$He-B is known as the homogeneously precessing domain (HPD, see review 23), and historically this was the first Bose condensate which was experimentally stabilized. Magnon condensates in $^3$He-A and condensates in $^3$He-B (HPD and $Q$-balls 24) all have different spin-superfluid properties.

In conclusion, $^3$He-A in aerogel is one of the most interesting objects for investigation of the Larkin-Imry-Ma and related effects.
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