Constraints on Early Dark Energy from the Axion Weak Gravity Conjecture

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A popular proposal for resolving the Hubble tension involves an early phase of dark energy, driven by an axion field with a periodic potential. In this paper, we argue that these models are tightly constrained by the axion weak gravity conjecture: for typical parameter values, the axion decay constant must satisfy \( f < 0.008M_{Pl} \), which is smaller than the axion decay constants appearing in the vast majority of early dark energy models to date. We discuss possible ways to evade or loosen this constraint, arguing that its loopholes are small and difficult to thread. This suggests that it may prove challenging to realize early dark energy models in a UV complete theory of quantum gravity.

I. INTRODUCTION

Early dark energy (EDE) was introduced in [1], and in [2] it was shown to resolve the Hubble tension [3–6]. The model features an axion \( \chi \) sitting in a potential of the form

\[
V(\chi) = \Lambda^4 (1 - \cos \chi/f)^n .
\]

(1)

The axion begins during radiation domination near its maximum at \( \chi \approx \pi f \), and it remains there for a while due to Hubble friction. During this phase, \( \chi \) has an equation of state of \( w_\chi \approx -1 \), and contributes to the vacuum energy (hence the name, early dark energy).

When \( H \) decreases sufficiently, the field begins to oscillate around its minimum at \( \chi = 0 \). For \( n = 1 \), this is the classic axion dark matter scenario: coherent oscillations of the axion act as a source of dark matter, with \( w = 0 \). For more general \( n \), one instead has [2]:

\[
w_n = \frac{n-1}{n+1} .
\]

(2)

For \( n = 2 \), for instance, \( w_n = 1/3 \), and coherent oscillations decay like radiation. Resolving the Hubble tension requires \( n \geq 2 \). Some typical values for the parameters are [2]:

\[
f \approx 0.1M_{Pl}, \quad \Lambda^4 \approx 10^{10}M_{Pl}^2H_0^2,
\]

(3)

with \( \chi_0 \lesssim \pi f \), and \( n = 2 \) or 3.

In this paper, we will address the question of whether the above model can be embedded into a UV complete theory of quantum gravity. Said differently, we want to know if EDE resides in the landscape or the swampland.

Several aspects of EDE models point in favor of a possible UV completion. Axions are ubiquitous in string compactifications [8–10], and their connection to the absence of global symmetries in quantum gravity suggests that this is not merely a lamppost effect [11]. Axions have periodic potentials, \( V(\chi) = V(\chi + 2\pi f) \) which naturally allows them to be much lighter than the Planck mass and protects their potentials from Planck-suppressed higher-dimension operators, such as \( \chi^n/M_{Pl}^{n-4} \).

On the other hand, some aspects of EDE models point against the possibility of a UV completion. The most glaring issue is the \( n \geq 2 \) constraint: the potential must be approximately quartic near its minimum (i.e., the axion must be approximately massless). This is hard to arrange for the following reason: axion potentials in UV complete frameworks admit an expansion in higher harmonics of the form

\[
V(\chi) = V_0 + \Lambda_{UV}^4 \sum_{k=1}^\infty e^{-S_k} \left( 1 - \cos \left( \frac{\chi}{kf} + \delta_k \right) \right) .
\]

(4)

Here, \( \Lambda_{UV} \) is some UV scale, such as the Planck scale or the string scale, and \( \delta_k \) is a phase. In a controlled scenario, \( S_1 \) is large, and \( S_k \) typically grows roughly linearly with \( k \), so the dominant contribution to the axion potential comes from the lowest harmonic \( k = 1 \), and all higher harmonics can be ignored. This situation readily occurs, for instance, if the potential \( V(\chi) \) is generated by Euclidean D-branes or worldsheet instantons wrapping cycles of a Calabi-Yau manifold in a string compactification, or if the potential is generated by Yang-Mills instantons. This controlled scenario will not give the necessary potential for EDE, however: it yields a potential of the form in [1], but with \( n = 1 \).

A potential satisfying the \( n \geq 2 \) constraint therefore requires a delicate cancelation between distinct harmonics of the potential, such that the \( \chi^4 \) term of the potential vanishes to very high precision, while the \( \chi^2 \) or \( \chi^0 \) terms remain relatively large. Meanwhile, the instanton actions \( S_k \) must remain large, or else the UV scale \( \Lambda_{UV} \) must be

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1 This periodicity condition is modified in axion monodromy models, as the potential develops multiple branches \( V_i(\chi) \) satisfying \( V_i(\chi + 2\pi f) = V_{i+1}(\chi) \). For further explanation, see e.g. [12–15].

2 Barring significant fine-tuning, these effects will spoil EDE models with scalar fields that are not axions, so even power-law EDE potentials \( V(\chi) \propto \chi^{2n} [2] [16] \) must involve axion fields or else violate naturalness.

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tuned significantly for agreement with \[3\]. Within string theory, it is unclear how to do all of this while maintaining perturbative control of the instanton expansion.

One way around this issue is to add a second scalar field \(\phi\), sometimes referred to as a saxion, which couples to the axion \(\chi\) via a potential of the form \[17\]

\[V(\phi, \chi) = \Lambda^4_{UV} e^{-(S_0 + \beta \phi)} (1 - \cos(\chi/f)) + V_0 e^{\chi/\Lambda_{UV}^4}. \]  

(5)

Note that the potential here has \(n = 1\): it is quadratic near its minimum. Potentials of this form do show up in string compactifications, with an additional caveat that the axion decay constant \(f\) is \(\phi\)-dependent, \(f = f(\phi)\). \[17\] showed that a potential of this form could produce a cosmologically viable model of early dark energy: an important step towards a UV completion of EDE.

We will see below, however, that even under the most favorable circumstances, there is an additional obstacle to UV completion: the axion weak gravity conjecture. In the following section, we will introduce this conjecture. We will then show that this conjecture leads to a tight, \(n\)-independent constraint on EDE models, which for typical parameter values amounts to a bound \(f < 0.008M_{Pl}\) on the axion decay constant. This value is smaller than those typically considered in previous studies of EDE, and the resulting bound is tighter than those previously noted in discussions of swampland constraints on EDE \[18\] \[20\]. We will elaborate on possible ways to evade this constraint by taking advantage of various “loopholes” and uncertainties in the parameter values, and we will conclude with a discussion of directions for future research.

II. THE AXION WEAK GRAVITY CONJECTURE

The axion weak gravity conjecture is part of a larger swampland program, which seeks to delineate the boundary between the landscape of effective field theories that are compatible with quantum gravity and the swampland of effective field theories that are not. This task is nontrivial due to the enormity of the landscape and the sophisticated nature of quantum gravity, and as a result, few conclusions about quantum gravity can be drawn with certainty. Instead, there has been a proliferation of conjectures suggesting that certain features of known quantum gravity theories should be universally present in the landscape.

Not all of these conjectures are on equal footing. Some, such as the absence of global symmetries in quantum gravity \[21\] \[23\], are supported by very general arguments in string theory (the best candidate for a UV complete theory of quantum gravity) as well as semiclassical arguments (which remain agnostic about the UV completion of quantum gravity). Others, such as the de Sitter conjecture \[24\], are evidently violated in the standard model \[25\] \[26\] and are very difficult to test outside certain controlled regimes of string theory, so their regime of validity may be limited to a portion of the landscape.

The axion weak gravity conjecture is somewhere in between. It is a close relative of the weak gravity conjecture \[27\], which was originally motivated by black hole physics and is now supported by very general arguments in perturbative string theory \[28\] \[29\], many examples in nonperturbative string theory \[30\] \[32\], qualitative arguments in effective field theory \[33\] \[34\], and several (not entirely convincing) semiclassical arguments \[35\] \[42\]. The axion weak gravity conjecture, in contrast, does not have a direct link to black hole physics, and the primary evidence for it comes from many examples in string theory \[43\] \[47\], plus the lack of a counterexample, and from its close connection to the ordinary weak gravity conjecture \[27\].

The precise statement of the axion weak gravity conjecture is also debated \[48\] \[50\]. In four dimensions, the conjecture holds that in a consistent theory of quantum gravity with an axion field with decay constant \(f\), there must exist an instanton with action \(S\) carrying integer charge \(k\) under the axion field such that

\[\frac{fS}{k} \leq cM_{Pl}. \]  

(6)

Here, the coefficient \(c\) is some \(O(1)\) number, but its precise value is debated. In theories without exactly massless scalar fields, however, the only finite value of \(c\) which has been proposed in the literature is

\[c = \frac{\sqrt{5\pi}}{4}, \]  

(7)

which was introduced in \[49\] \[50\] by analogy with black hole physics, and subsequently studied in \[51\]. For more details, see section 3.7 of \[52\].

In what follows, we will therefore define the axion weak gravity conjecture to be equation \[4\] with \(c\) given by \[7\], though the reader should keep in mind that the precise value of \(c\) is a matter of ongoing discussion and research.

III. CONSTRAINTS ON EARLY DARK ENERGY

Consider an EDE model with action

\[S = \int d^4x \left[ -\frac{1}{2} (\partial \mu \chi)^2 - \Lambda^4_{UV} e^{-S} \left(1 - \cos \left(\frac{k \chi}{f}\right)\right)^n + \ldots \right]. \]  

(8)

Here, the instanton action \(S = S(\phi)\) may be a function of one or more scalar fields in the theory, as in \[3\]. \(\Lambda_{UV}\) is some UV scale, such as the Planck scale or the string scale, \(k\) is the integer charge of the instanton (a.k.a. instanton number), and \(f\) is the axion decay constant.

We define the effective decay constant as \(f_{\text{eff}} = f/k\) and the axion mass parameter as

\[m^2_x = \frac{\Lambda^4_{UV} e^{-S}}{f_{\text{eff}}^2}. \]  

(9)
For $n = 1$, $m_\chi$ is precisely the axion mass, whereas for $n > 1$, it is a conventionally-defined parameter of the theory with dimension of mass.

The axion weak gravity conjecture [6] then translates into a bound on the axion mass parameter

$$f_{\text{eff}} \log \left( \frac{\Lambda_{\text{UV}}}{f_{\text{eff}}^2 \pi^2} \right) \leq \frac{\sqrt{6}}{4} M_{\text{Pl}}, \quad (10)$$

where we have used the value $c = \sqrt{6}/4$ from [7]. Note that this bound is independent of $n$. Plugging in a typical value of $m_\chi \approx 10^{-27} \text{ eV}$ and setting $\Lambda_{\text{UV}} = M_{\text{GUT}} \approx 10^{16} \text{ GeV}$, we find a bound on the decay constant $f_{\text{eff}}$ of

$$f_{\text{eff}} \lesssim 0.008 M_{\text{Pl}}, \quad (11)$$

which is smaller than the decay constants appearing in EDE models to date (see e.g. [2] [20] [53]).

It is worth comparing these bounds to bounds on natural inflation from the axion weak gravity, as discussed in e.g. [27] [44] [45] [58]. In natural inflation, the axion mass will be relatively large, of order $m_\chi \sim 10^{-5} M_{\text{Pl}}$. As a result, the instanton action $S$ can be rather small. Setting $S \gtrsim 1$ for perturbative control of the instanton expansion, the axion weak gravity conjecture implies the familiar constraint $f_{\text{eff}} \lesssim M_{\text{Pl}}$, which is indeed satisfied by axions in string theory [43]. In the case of slow-roll, however, the axion must be very light, which implies $S \gg 1$ in the absence of significant fine-tuning of the UV scale $\Lambda_{\text{UV}}$. This in turn leads to a tighter bound on the axion decay constant, $f_{\text{eff}} \ll M_{\text{Pl}}$.

\section{IV. CONSTRAINTS ON MODEL PARAMETERS}

So far, we have seen that the axion weak gravity conjecture places constraints on the axion decay constant $f$ in terms of the axion mass parameter $m_\chi$ and the energy scale $\Lambda_{\text{UV}}$. In cosmological studies of early dark energy, however, the relevant parameters are instead the critical redshift $z_c \equiv a_c^{-1} - 1$, before which the axion is frozen near the maximum of its potential and effectively acts as a cosmological constant, and the fraction $f_{\text{EDE}}$ of the total energy density stored in the axion condensate at the time $z_c$, $f_{\text{EDE}} \equiv \Omega_\chi(a_c)/\Omega_{\text{tot}}(a_c)$.

In this section, we translate the bound $f > 0.008 M_{\text{Pl}}$ derived in the previous section into the space of parameters $a_c$, $f_{\text{EDE}}$ and compare it to the observational constraints on these parameters found in [2]. We do this by taking the standard $\Lambda$CDM model of cosmology, with cosmological parameters

$$\Omega_{0,M} = 0.31, \quad \Omega_{0,K} = 9 \times 10^{-5}, \quad \Omega_\Lambda = 0.69, \quad (12)$$

and adding an axion $\chi$ with potential (11), which begins at rest near the top of the cosine hill with an initial position of $\chi_0 = (\pi - 0.01) f$. Following [2], we then approximate the axion energy density with an EDE energy density:

$$\Omega_\chi(a) = \frac{2\Omega_{\text{EDE}}(a_c)}{(a/a_c)^{3(1+w_n)} + 1}, \quad (13)$$

where $w_n = (n-1)/(n+1)$. Using a best-fit analysis, we determine the parameters $a_c$, $f_{\text{EDE}} = \Omega_\chi(a_c)/\Omega_{\text{tot}}(a_c)$ of the EDE model that most closely approximates the energy density of the axion field condensate. The results of this fitting procedure for a model with $n = 3$ are shown in figure 1.

In general, the EDE parameter $f_{\text{EDE}}$ is approximately half the axion decay constant $f$ (in Planck units) and depends weakly on the axion mass parameter $m_\chi$, whereas the critical redshift $z_c$ is more sensitive to changes in $m_\chi$. The bound $f \lesssim 0.008 M_{\text{Pl}}$ thus roughly translates into a bound $f_{\text{EDE}} \lesssim 0.004$ for typical values of $m_\chi$ for $n = 2, 3$, and this bound gets tighter as $n$ increases. More precise bounds are shown in figure 2 (right), which can be compared with the experimental bounds on $f_{\text{EDE}}$ and $a_c$ from [2]. A large portion of the parameter space is inconsistent with the axion weak gravity conjecture.

Aside from the question of a UV completion, another prominent issue facing EDE is its incompatibility with LSS data; namely, EDE seems to exacerbate the $S_8$ tension [19] [59]. In [53], it was argued that this tension could be alleviated by taking $f_{\text{EDE}}$ sufficiently large, $f_{\text{EDE}} \gtrsim 0.09$. Here, however, we see that such large values of $f_{\text{EDE}}$ are incompatible with the axion weak gravity conjecture, so consistency between EDE and LSS data likely requires a further late-time alteration of $\Lambda$CDM [60].

\section{V. LOOPHOLES}

Above, we found a tight constraint on axion decay constants in EDE models. We now ask: can this constraint be loosened?
To begin, it is worth noting that this bound depends only logarithmically on the mass $m_\chi$ and the UV scale $\Lambda_{\text{UV}}$, so it is quite robust to changes of these parameters. Decreasing $\Lambda_{\text{UV}}$ by a factor of $10^{-10}$, for instance, only loosens the bound $\langle f S_{\text{eff}} \rangle \approx 0.13 M_{\text{Pl}}$.

Rather than changing the parameters of the model, another way to loosen the constraint would be to change the parameter $c$ appearing in the axion weak gravity conjecture. The maximum value of the axion decay constant scales linearly with $c$, so if $c$ were increased by a factor of 3, the maximal value of $f_{\text{eff}}$ would also grow by a factor of 3, accommodating a wider range of parameter space. However, at present, there is no competing proposal for $c$ in theories without massless scalars other than the one introduced in [7], so there is no compelling justification for loosening the constraint in this way. Clearly, the relevance to EDE models makes the task of determining the correct value of $c$ even more urgent.

Constraints from the axion weak gravity conjecture on natural inflation models feature a pair of famous loopholes, which ostensibly could be used as loopholes for EDE models as well. Most prominent is the extra instanton loophole [54, 55, 67, 64], in which one instanton satisfies the axion weak gravity conjecture, while another gives the dominant contribution to the axion potential. Since the latter instanton controls the phenomenology of the model, the constraints from the axion weak gravity conjecture are rendered vacuous.

To be more explicit, let us suppose that the dominant contribution to the axion potential comes from an instanton of charge 1, with action $S_1$. Let us suppose that the axion weak gravity conjecture is satisfied by an instanton of charge $k$, with action $S_k$. Thus, we have

$$\frac{f S_k}{k} \leq \frac{\sqrt{6} \pi}{4} M_{\text{Pl}}.$$  \hspace{1cm} (14)

In order to suppress the contributions from the charge $k$ instanton relative to those from the dominant charge 1 instanton, we need $S_k > S_1$, which therefore implies

$$\frac{f S_1}{k} \leq \frac{\sqrt{6} \pi}{4} M_{\text{Pl}}.$$ \hspace{1cm} (15)

This weakens the bound [11] by a factor of $k$, so by taking $k$ sufficiently large, the axion weak gravity conjecture bounds on $S_1$ become negligible. However, in practice, every string compactification studied thus far satisfies the weak gravity conjecture with a state of charge $k \leq 3$ [28, 51], so parametric enhancement of $k$ seems difficult to achieve in a UV complete theory. Furthermore, this loophole introduces additional parameters into the theory (namely, the instanton actions $S_2, \ldots, S_k$) which must be tuned appropriately to ensure consistency with the axion weak gravity conjecture without spoiling the phenomenology of the model. Such tuning lowers the prior probability of the model in a Bayesian framework.

The other prominent loophole in the natural inflation literature is known as the small action loophole. In some cases, such as extranatural inflation [62], it may be possible to suppress higher harmonics to the axion potential.
even for $S < 1$, which in principle could lead to a para-
metrically large decay constant \[54\]. In the case of EDE, how-
ever, $S > 1$ is required not only for perturbative con-
trol of the instanton expansion, but also for the lightness of
the axion field, $m_\chi \ll eV$. The small action loophole is
therefore unavailable in the EDE context.

Finally, let us remark on the possibility evading the
constraint \[10\] using multi-axion models of EDE. His-
torically, multi-axion models of natural inflation \[53-62\]
have been a popular approach to evading axion decay
constant constraints from string theory \[45\] and the ax-
ion weak gravity conjecture \[27\]. However, more recent
studies have shown that the axion weak gravity conjec-
ture constraints extend straightforwardly to multi-axion
models \[53-68\], so these models offer no additional path
to EDE aside from the loopholes discussed above.

We conclude that constraints on EDE from the axion
weak gravity conjecture are quite robust. Loopholes do
exist, but it may well prove more difficult to thread such
a loophole than it is to build an EDE model satisfying
the axion weak gravity conjecture constraint \[10\].

VI. DISCUSSION

We have seen that the requirement of a consistent UV
completion places significant constraints on models of
early dark energy. To fully understand how significant
these constraints are, however, several questions need to
be answered.

First of all, is it possible to generate an EDE model
with $n \geq 2$ through a delicate cancelation of terms in
an instanton expansion? How delicate does this cancela-
tion have to be to produce a phenomenologically viable
model of EDE? In other words, how fine-tuned would the
mass of an EDE axion have to be to resolve the Hubble
tension?

Is $c = \sqrt{6\pi}/4$ the correct value to use in the axion weak
gravity conjecture? If not, what is the correct value of $c$?
If the answer is much larger (smaller) than this value, the
constraint on EDE models will be much weaker (stronger)
than the one proposed in this paper.

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From a model-building perspective, is there a way to
thread the extra instanton loophole detailed above, in
which one instanton satisfies the axion weak gravity con-
jecture while another produces a model of early dark en-
ergy? Is there a way to tune the scale $\Lambda_{UV}$ by many
orders of magnitude, thereby loosening the bounds on
the axion decay constant $f$? Can phenomenologically vi-
able models satisfy the bound $f < 0.008M_{Pl}$? Is there
a phenomenologically viable axio-dilaton model of EDE
with $n = 1$ and $f < 0.008M_{Pl}$?

Besides the axion weak gravity conjecture, do other
swampland conjectures give significant constraints on
EDE models? In the same paper that first introduced
the swampland distance conjecture \[63\], Ooguri and Vafa
also argued that scalar moduli space should have non-
trivial 1-cycles with minimum length in a given homotopy
class. This means that any periodic axion field $\chi$ must be
accompanied by a dilaton $\phi$, such that the decay constant
of $\chi$ depends on $\phi$, $f = f(\phi)$, with $f \to 0$ as $\phi \to \infty$.
The swampland distance conjecture constrains the limit
$\phi \to \infty$, implying a tower of light states with masses be-
ginning at the scale $e^{-\alpha\phi}$, where $\alpha = O(M_{Pl})$. It would be
worthwhile to study how introducing dilaton-dependence
via the axion decay constant, $f = f(\phi)$, affects the analy-
sis of \[17\]. It may also prove interesting to study the cos-
mological consequences of an $e^{-\alpha\phi}\bar{\psi}\psi$ dilatonic coupling
to dark matter in such an axio-dilaton EDE model\[4\].

Of course, these questions are particularly important
insofar as the Hubble tension remains unresolved. The
most important question—will the Hubble tension stand
up to increased experimental scrutiny?—remains to be
seen.

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3 One recent paper \[20\] considered the cosmological consequences
of a coupling $e^{-\alpha\chi}\bar{\psi}\psi$ between a fermionic dark matter field $\psi$
and the EDE axion $\chi$. Such couplings have been observed in
string compactifications \[67\] and are a prediction of the refined
swampland distance conjecture \[68\], but they are not necessarily
required by the original swampland distance conjecture \[69\];
because the periodicity of the axion under $\chi \to \chi + 2\pi f$
means that $\chi \to \infty$ is not an infinite distance limit in moduli space.

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