Deformation of rock blocks with a cavity

AA Krasnovsky
Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia
E-mail: visanta@ngs.ru

Abstract. On the basis of a system of singular integral equations connecting the boundary values of all stress and displacement components for an arbitrary simply connected region, relations are obtained for the stress and displacement components at the boundary of a rock block with a rectangular cavity. Examples of numerical implementation are given.

Numerical calculations in mechanics are mostly based on the theory of elasticity. The mathematical modeling permitting the theoretical analysis of the solution allows a more detailed estimation of the behavior of a specimen under loading.

Knowing stress state is required to calculate strength, rigidity and stability, including all components of stresses and displacements at the boundary and contacts in order to detect initiation of failure. In calculation of stresses and strains in the finite domains with an arbitrary hole, no analytical solution exists.

This paper addresses the issue of stress state determination at the boundary of a rectangular domain with a hole for the main boundary-value problems without factual finding of complex potentials $\varphi(z)$ and $\psi(z)$ in a computation domain. The author attempts to analyze simultaneously and uniformly all three basic problems of elasticity and to propose their implementation methods in the context of rock blocks with a weakening rectangular hole. The model is based on the system of singular integral equations connecting all boundary values of stresses and displacement [3].

Consider a rock block with a rectangular hole as schematically shown in Figure 1a. Given the geometrical and force symmetry, the problem is greatly simplified. In this case, thanks to the symmetry, it is sufficient to analyze a half or a quarter block which is a simply connected region [4, 5].

Figure 1. Schematic layouts of rock blocks with the rectangular holes.
Let us discuss the case of the asymmetrical weakening. This is a biconnected region, which induces certain complexity in the problem solving. For this reason, it is suggested to divide the block into four parts. Figs. 1b and 1c illustrate such division: \( l_1, l_2, l_3 \) and \( l_4 \) are the dividing lines and simultaneously the boundaries of the domains which compose the block and are the simply connected region each. With such division of the computation domain, it is possible to find the values of the normal and shear stresses and displacements at \( l_1, l_2, l_3 \) and \( l_4 \), depending on which boundary information is required, without determining stress state inside the whole region under analysis.

The boundary conditions can be set in different combinations in the framework of the basic problems of elasticity.

As an example, we suggest the variant of the boundary conditions below:

\[
\begin{align*}
\sigma_n &= \sigma_0 = -1, \quad u = 0 \quad \text{at the boundaries } -b \leq x \leq b, \quad y = -h \text{ and } y = h, \\
\sigma_n &= 0, \quad \tau_n = 0 \quad \text{at the boundaries } -h \leq y \leq h, \quad x = -b \text{ and } x = b, \\
\sigma_n &= 0, \quad \tau_n = 0 \quad \text{at the weakening hole boundary: } h2 \leq y \leq h1, \quad x = a2 \text{ and } x = a1, \\
a2 \leq x \leq a1, \quad y = h2 \quad \text{at } y = h1,
\end{align*}
\]

where \( \sigma_n, \tau_n \) — normal and shear stresses; \( u \) — horizontal displacement.

In the general form for an arbitrary simply connected region, the system of singular integral equations connecting boundary values of all stresses and displacements has the form [3] and contains the function \( f(t) \):

\[
f(t) = i\int_0^t (X_n + iY_n)ds,
\]

where \( X_n, Y_n \) — forces at the boundary in the line of the axes \( x \) and \( y \), respectively; \( t \) — point at the boundary; \( i \) — unit imaginary number.

It is assumed that cohesion exists at the dividing lines—boundaries of the block parts:

\[
\sigma_n^+ = \sigma_n^-, \quad \tau_n^+ = \tau_n^-, \quad u^+ = u^-, \quad \nu^+ = \nu^-,
\]

which means continuity of the normal and shear stresses (the upper symbol identifies the corresponding part).

Based on [3], using the earlier obtained equations for all displacements and the function \( f(t) \) at the boundary of a rectangular domain [4] and the condition (3), the relations connecting stresses and displacements at the boundaries and interfaces of the block parts have been written.

For the numerical implementation of the resultant system of equation, we use the dimensionless values and assume the dimensions of length as the half length of the weakening and the dimensions of stresses as the characteristic values of the boundary stresses. The calculations involved \( h = 4, \ b = 3, \ \nu = 0.25, \ E = 10^4 \) in the combinations below:

\[
1) \ h1 = 3, \ h2 = 2, \ a2 = -2, \ a1 = 0; \quad 2) \ h1 = -1, \ h2 = -3, \ a2 = 1, \ a1 = 2.
\]

Figure 2 demonstrates the calculated deformation of the boundary of the analyzed block in different variants of geometry and location of the weakening (4).
Figure 2. Deformation of the rock block boundary: (a) $h_1 = 3$, $h_2 = 2$, $a_1 = 0$; (b) $h_1 = -1$, $h_2 = -3$, $a_1 = 1$, $a_1 = 2$.

Figure 3 illustrates the calculated results for the horizontal and vertical displacements $u(x)$, $v(x)$ and the real and imaginary parts of the function $f$ at the contact line $l_1$: $y = h_1$, $-b \leq x \leq b$ (see Figure 1b) fitting the first combination in (4). The analysis of the function (2) allows estimating the respective stresses both at the contact lines and at other boundaries of the rock blocks. To this effect, it is sufficient to differentiate the obtained results for the real and imaginary parts of the function $f$.

Figure 3. Displacements and the function $f$ at the line $l_1$. 
In this manner, the obtained system of singular integral equations and its implementation offers all necessary information on normal and shear stresses and displacement at the whole boundary of a rock block with a rectangular hole. The solution in the integral form makes it possible to analyze the solution itself and to vary the input parameters of the problem with a view to reaching the wanted results.

**Conclusion**

The author has obtained the relations connecting boundary stresses and displacements in the domains composing a rock block with a rectangular weakening. When the computational domain is divided into parts, these relations allow additional information on the stresses and displacements at the dividing lines without actual determination of stress state inside the whole domain. The research findings enable calculation of stress state in composite blocks with rectangular weakening.

**References**

[1] Muskhelishvili NI 1968 *Some Basic Problems of the Mathematical Theory of Elasticity* Moscow: Nauka (in Russian)

[2] Muskhelishvili NI 1968 *Singular Integral Equations* Moscow: Nauka (in Russian)

[3] Mirenkov VE 1978 Relation between stresses and shifts at the periphery of a working *J. Min. Sci.* Vol 14 No 3 pp 251–254

[4] Mirenkov VE 2007 Contact problems in rock mechanics *J. Min. Sci.* Vol 43 No 4 pp 370–381

[5] Krasnovsky AA 2017 Deformation of Rock Specimens with a Rectangular Opening *J. Fundament. Appl. Min. Sci.* Vol 4 No 1 pp 131–135 (in Russian)