Improving the Accuracy of Magnetic Field Tracing by Velocity Gradients: Principal Component Analysis

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ABSTRACT

Tracing of the magnetic field with Velocity Gradient Technique (VGT) allows observers to probe magnetic field directions with spectroscopic data. In this paper, we employ the method of Principal Component Analysis (PCA) to extract the spectroscopic information most valuable for VGT. By using synthetic observation data from numerical simulations, we show that PCA acts in a way similar to spatial filtering along the velocity axis. We study both subsonic and supersonic simulations and show that with the PCA filtering the tracing of magnetic fields by the VGT is significantly improved. Using 21 cm GALFA data, we demonstrate that the PCA filtering improves the alignment of the velocity gradients and the Planck dust polarization.

Key words: ISM: structure — ISM: turbulence—magnetohydrodynamics (MHD) — methods: numerical

1 INTRODUCTION

Turbulence is ubiquitous in the interstellar medium (ISM) at different scales (Armstrong et al. 1995; Chepurnov & Lazarian 2010) and magnetic field plays an important role for most of the ISM physics. In particular, magnetic fields are essential for the star formation (Mouschovias & Spitzer 1976; Burkhart et al. 2015; Chapman et al. 2013), propagation and acceleration of cosmic ray (Fermi 1949; Schleicher et al. 2010), transport of heat and mass in the galaxy (Lazarian 2006; Narayan & Medvedev 2001). More recently, the importance of studying the structure of magnetic field was motivated by attempts to study elusive B-modes of cosmological origin (Ferreira et al. 2014). The latter produce polarization that is being confused with the foreground polarization arising from interstellar magnetic fields (Jones 1989; Voshchinnikov et al. 2016). However, the study of magnetic fields in the ISM is complicated. Therefore it is extremely interesting in finding alternative ways for magnetic field tracing.

The VGT technique employs either Velocity Centroid Gradients (VCGs) (Gonzalez-Casanova & Lazarian 2017; Yuen & Lazarian 2017a,b) or Reduced Velocity Centroid Gradients (RVCGs) (Lazarian & Yuen 2018a) or Velocity Channel Gradients (VChGs) (Lazarian & Yuen 2018a). In this paper, we use the VCGs, but the approach that we discuss in this paper is also applicable to other realizations of the VGT.

The VGT is founded by the modern understanding of MHD turbulence theory (Goldreich & Sridhar 1995, hereafter GS95) that includes the concept of fast turbulent reconnection (Lazarian & Vishniac 1999, henceforth LV99) and is supported by numerical studies (see Cho & Vishniac 2000; Maron & Goldreich 2001; Cho et al. 2002; Cho & Lazarian 2003; Kowal et al. 2009). Due to turbulent reconnection, motions perpendicular to the local direction of the magnetic field are not constrained, and therefore eddies rotating perpendicular to magnetic field have the Kolmogorov spectrum with eddy velocity $v_l \sim l_\perp^{1/3}$. The index $\perp$ in $l_\perp$ indicate that the motions are perpendicular to the magnetic field of the eddy. As a result, the gradient of velocity scales as $v_l/l_\perp \sim l_\perp^{-1/3}$, means that the smallest resolved eddies induce the largest gradients. These gradients are perpendicular to the local direction of magnetic field. A more detailed explanation of the foundations of the VGT can be found in (Lazarian & Yuen 2018a).

The VGT has been a fast developing branch of research. For instance, the tracing of the direction of magnetic field in diffuse (Yuen & Lazarian 2017a) and self-gravitating media (Yuen & Lazarian 2017b; Lazarian & Yuen 2018a) has been performed, as well as the estimations of the sonic ($M_s$, Yuen et al. 2018a) and Alfvénic ($M_A$, Lazarian et al. 2018) Mach numbers. As a separate development, the approach of studying magnetic fields with gradients has been also applied to...
synchrotron intensities (Lazarian et al. 2017), which resulted in the Synchrotron Intensity Gradients (SIGs) technique, as well as to synchrotron polarization (Lazarian & Yuen 2018b), which resulted in two techniques, the Synchrotron Polarization Gradients (SPGs) and Synchrotron Polarization Derivative Gradients (SPDGs). The theoretical foundations of the procedures employed in the aforementioned techniques mentioned above are similar to those of the VGT, and therefore we expect that the improvements of the data analysis, in particular, the use of the Principal Component Analysis can be also advantageous for improving the accuracy of other gradient techniques, e.g. those dealing with synchrotron.

The practical application of VGT is affected by the quality of the data. The noise suppression method for VGT has been explored in Lazarian et al. (2017) and elaborated in Lazarian & Yuen (2018a), showing that a convolution of the observational map with a small $\sigma$ Gaussian kernel would retrieve the spatial structure of the molecular cloud. Moreover, in Yuen & Lazarian (2017b) they showed that the filtering of non-turbulence contribution in Fourier space could improve the accuracy of VGT in tracing magnetic field. In this paper, we proceed with the work of improving magnetic field tracing with the VGT. For this purpose, we explore the application of the Principal Component Analysis (PCA). The PCA is widely used in image processing and image compression. Regarding astrophysical applications the PCA analysis was used in Brunt & Heyer (2002a,b) for obtaining the turbulence spectrum from observations. Later, in Heyer et al. (2008) the PCA was employed for studying turbulence anisotropies. Our present use of the PCA is different: we use it as a tool to provide the preliminary processing of the spectroscopic data.

The idea of the PCA is that the image of size $N^2$ can be effectively represented by $n < N$ eigen-maps. The physical meaning of the eigenvalues from the PCA analysis is closely related to the value of the turbulence velocity dispersion $\nu^2$. We apply the VCGs to the individual eigen-images and explore for which of them the magnetic field is traced the best.

In what follows, we briefly describe the numerical code and setup for simulation in §1. In §3, we test the implementation of the VCGs with the PCA using numerical simulations. §4 shows the observational example with VCG-PCA technique. In §6, we give our discussion about our technique and conclusion.

| Model Name | $M_S$ | $M_A$ | Resolution |
|------------|-------|-------|------------|
| Ms0.4Ma0.04| 0.41  | 0.04  | 480$^3$    |
| Ms0.8Ma0.08| 0.92  | 0.09  | 480$^3$    |
| Ms1.6Ma0.16| 1.95  | 0.18  | 480$^3$    |
| Ms3.2Ma0.32| 3.88  | 0.35  | 480$^3$    |
| Ms6.4Ma0.64| 7.14  | 0.66  | 480$^3$    |

Table 1. MHD simulations used in the present work. $M_S$ and $M_A$ denote the instantaneous values of the sonic and Alfvén Mach numbers at each of the snapshots.

2 NUMERICAL SETTING FOR SYNERGISTIC USE OF PCA AND VGT

For our studies of gradients with PCA, we use the same numerical cubes as in Yuen et al. (2018) (see Table 1). The simulation parameters with different combinations of Alfvénic Mach numbers $M_A = V_A/V_s$ and sonic Mach numbers $M_S = V_s/V_A$ are listed in Table 1. There $V_s$ is the turbulence injection velocity and $V_A$ and $V_s$ are the Alfvén and sonic velocities, respectively.

For this study, we consider the optically thin case ($\tau \sim \int \kappa(\nu)d\nu \ll 1$) and synthesize observational maps similar to that in Yuen et al. (2018). We assume that the emissivity is proportional to density, but do not consider this as a significant limitation. For instance, the case of emissivity proportional to the density squared is regarded in Kandel et al. (2017) with the change of the results being insignificant.

We denote the intensity within of the Position-Position-Velocity (PPV) cubes as $\rho(x,y,v)$, and the cubes dimensions $n_x \times n_y \times n_v$, where the $n_v$ means the number of velocity channels along the spectral line direction $v$ (line-of-sight direction, LOS), which is $n_v = 400$ for our studies unless specifically mentioned.

The PPV cubes are preprocessed first by the PCA similar to that described in Brunt & Heyer (2002b). After that, the VCG technique is applied to the eigen-images of a different order. The product of PCA would be a set of eigen-images $I_i$ with decreasing order of eigenvalues $\lambda_i \sim \nu^2$, where the latter records the velocity variance along the line of sight.

As we discussed in the §1 that for studying turbulence the velocity variance is related to the eddy size along the line of sight. Heyer et al. (2008) splits the PPV cube into vertical and horizontal Position-Velocity (PV) tires, where every PV tire is a vertical or horizontal slice from the PPV map $\rho(x,y,v)$ averaged over the $x$-direction or $y$-direction, respectively. The eigenvalue is obtained by solving the eigenvalue equation for each PV tire.

Similarly, in our work, we assume the PPV cube is properly normalized 1 and treat the PPV cube $\rho(x,y,v)$ as the probability density function of three random variables $x, y, v$, we can then define the covariance matrix (Brunt & Heyer

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1 Incidentally, our approach of block averaging is also applicable to studies of gradients of column densities. The corresponding Intensity Gradient Technique (IGT) should not be confused with the Histograms of Relative Orientation (HRO) proposed in Soler et al. (2013). The IGT traces both magnetic fields and shocks (see Yuen & Lazarian (2017b), Lazarian & Yuen (2018a)), while HRO provides a statistical relation between the relative orientation of the magnetic field and intensity gradients as a function of the column density. The latter is a measure calculated for the entire image and it cannot be used to trace the spatial variations of magnetic fields. We view the IGT as a part of the gradient technique. Its synergy with the VGT was demonstrated e.g. in Lazarian & Yuen (2018a).

2 In principle one shall use the normalized PPV cube $\rho' = \rho / \int \rho$. However, for the treatment of PCA, the difference of a constant does not alter the result. Therefore we stay with $\rho$ for simplicity.
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Figure 1. The eigen-centroid maps with gradients (red) and magnetic field (blue) plotted with different eigenvalues. The simulation used here is \( M_S = 0.41 \) and \( M_A = 0.04 \). Please note, each figure is in individual color-scale.

The textbook definition of covariance matrix should be:

\[
S(v_1, v_2) = \int dx dy \rho(x, y, v_1) \rho(x, y, v_2)
\]

hence an eigenvalue equation for this covariance matrix is:

\[\Sigma u = \lambda u\]  

where the \( \lambda_i \) are the eigenvalues associated with the eigenvectors \( u_i \) with \( i = 1, 2, ..., n \). One can solve the eigenvalue equation to get the eigenvalue and eigenvector of each channel. The eigenvectors \( u_i \) contain the weight of how one can construct the eigen-maps of rank \( i \) with the channel maps.

We apply eigenvalues \( \lambda_i \) as the weighting coefficients for each channel. Then the eigen-intensity maps \( I_{eigen} \) can be computed by:

\[
C_{eigen}(x, y) = \int dx dy \rho(x, y, v_1) \rho(x, y, v_2) v \cdot \lambda(v)
\]

\[I_{eigen}(x, y) = \int dx dy \rho(x, y, v) \cdot \lambda(v)\]

For the gradient computation, we shall follow the sub-block averaging method developed in Yuen & Lazarian (2017a), which will tell the sub-block averaged orientation of gradients. The resultant gradients will be rotated 90° to correspond to the expected magnetic field directions. The error estimation method (Lazarian & Yuen 2018a) is also employed to signify how accurate the Gaussian fitting function used in sub-block averaging is when computing the average gradient direction within a sub-block. The orientation of gradients from VGT is compared with the synthetic polarization, assuming a constant emissivity in the dust grain alignment (Lazarian 2007).

That means the mock Stokes parameters \( Q(x, y) \) and \( U(x, y) \) (Clark et al. 2015) can be expressed in terms of the angle \( \theta \) between the \( x \) and \( y \) direction magnetic fields by

\[
Q(x, y) \propto \int dz \rho(x, y, z) \cos(2\theta(x, y, z))
\]

\[
U(x, y) \propto \int dz \rho(x, y, z) \sin(2\theta(x, y, z))
\]
The polarization angle $\Phi = 0.5\arctan(2 \frac{U_Q}{U_T})$ is then defined correspondingly, which gives an probe of projected magnetic field in realistic scenarios.

The relative orientations between the $90^\circ$ rotated gradients and project magnetic field directions from polarization angles are measured by the Alignment Measure (AM) used in our previous studies (González-Casanova & Lazarian 2017; Yuen & Lazarian 2017a):

$$AM = 2(\langle \cos^2 \theta_r \rangle - \frac{1}{2})$$

Where $\theta_r$ is the relative angle between the gradients (rotated $90^\circ$) and the direction of the projected magnetic field. The range of AM is [-1,1]. When $AM = 1$, the gradients (rotated $90^\circ$) are parallel to the projected magnetic field. When $AM = -1$, the gradients (rotated $90^\circ$) are perpendicular to the projected magnetic field. We expect to get $AM \sim 1$ in most scenarios.

3 APPLYING VCGS TO EIGEN-IMAGES

The eigen-images produced by the PCA are the product to which we apply the VCGs analysis. For doing the latter, we apply the procedures described in our earlier papers, e.g. (Yuen & Lazarian 2017a), i.e. compute the sub-block averaged VCGs for each eigen-image and compare the obtained gradient directions with the projected magnetic field directions. Fig. 1 illustrates the gradients and the structure of some selected eigen-centroids for the cube Ms0.4Ma0.04. One can see for the eigen-centroids, the first eigen-channel map shows a lower level of alignment, the rest are essentially equally aligned. The structure of the eigen-centroids becomes more filamentary when the rank of eigen-channel map increases.

We analyze the visual patterns in Fig 1 using the AM-eigenvalue plot. The pink curves in Fig. 2 shows how the AM of the gradients from eigen-images and projected magnetic field varies concerning the eigenvalues from PCA analysis for the numerical cubes listed in Table 1. To test the power of PCA on noise reduction, we add white noise with mean amplitude $0.1\sigma_C$ to the centroid maps. The results are shown as the blue curves in Fig. 2. The x-axis in Fig. 2 represents the rank of eigenvalues sorted in decreasing order, i.e. if $\lambda_1 > \lambda_2 > ... > \lambda_n$, then we shall use the number 1 (the rank) to represent $\lambda_1$, rank 2 for $\lambda_2$ etc. We see that for all simulations we tested, the peak rank is at around ~10. As the rank increases (i.e., smaller eigenvalues), the AM of the respective gradients of eigen-centroids to magnetic field decreases significantly. In noisy environments (blue curves in Fig. 2), the AM of the images corresponding to the small ranks are approximately the same as the case without noise (pink curves), but the AM in higher rank cases drop significantly. The experiment in Fig. 2 shows that using the method of PCA before applying VGT, we can retrieve the strong signal part, which has a lower rank in PCA, from the noisy part, which has a higher rank.

We also notice that the AM for the images with ranks in the range ~1 – 5 is generally smaller than that for the
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Figure 3. A plot showing how the eigen-centroid amplitudes varies with the rank of the eigen-values on the synthetic map from the cube Ms0.4Ma0.04.

Figure 4. A plot showing the AM(top) and mean centroid amplitude(bottom) versus the rank of the eigen-values on the PPV cube from observation.

ranks in the range ~ 10 – 15. The reason behind this is that the largest velocity dispersion $v^2$ extracted from PCA corresponds to the largest-scale eddies along the line of sight, that is affected by the energy injection. Taking into account that for Alfvénic strong turbulence that $v^2 \sim l^{2/3}$ (GS95, LV99), the images with range of ranks about 10 – 15 correspond to the turbulent eddies in the inertial range of our numerical cubes ($k_{\text{inertial}} \approx 10 – 30$). In fact, when we refer to the eigen-centroid amplitudes from the cube Ms0.4Ma0.04 (Fig 3), we can see the amplitude becomes insignificant after rank $> 20$. The amplitude of the eigen-centroids with the rank higher than 20 is at least $0.1 – 0.01$ compared to the first few eigen-centroids. Therefore to use VCG$_{PCA}$ for its full potential, it is advantageous to remove the largest eigenvalues together with those having rank $> 20$ to obtain the best result in magnetic field tracing.

4 APPLICATION TO OBSERVATIONS
For testing our recipe, we use the well-studied region from Clark et al. (2015), with further information that can be found in Peek et al. (2018). The region spans right ascension (R.A.) 212.5° to 265° and declination (DEC.) 19.1° to 38.3°, covering a substantial piece of HI region with different physical conditions. The HI-cube has 41 velocity channels with each $\sim 3$ km/s wide. In previous studies (Yuen et al. 2018a; Lazarian et al. 2018) we explored $M_s$ and $M_A$ in the same region, showing the region is super-sonic and sub-Alfvénic ($M_A \sim 0.75$), which is close to the condition we had in Table 1. We then use the same strategy as we did in §3 to analyze the gradient orientation with the PCA.

We apply the PCA to the selected region and choose the 2nd and 10th eigen-centroid maps based on our experience that we had in Fig 2. Since the PCA eigen-rank is similar to the wavenumber in the spectral analysis, the choice we made here should not be affected by the short numerical inertial range in our simulation as we are choosing the first few eigenvalues for analysis. We show the magnetic field tracing with VCGs and compare it to the magnetic field directions traced by the 353GHz Planck polarization data, which we illustrated in Fig.5 with two eigen-centroid maps $A_2$ and $A_{10}$. The corresponding figure showing the AM-eigenvalue variation is in Fig 4, which has the same trend as Fig 2. The 10th eigen-map has an obviously better AM compared to that of 2nd, which is consistent with the study we have in §3.

5 DISCUSSION
5.1 Studying media magnetization
The magnetization of the interstellar media can be characterized by the Alfvén Mach number $M_A$. By itself, $M_A$ is critical parameter the knowledge of which is essential for understanding the vital astrophysical processes, including the transport and acceleration of cosmic rays (see Lazarian & Yan 2014), transport of heat (see Lazarian 2006), etc. With known $M_A$ and known velocity dispersion one can get the value of the interstellar magnetic field (see Lazarian et al. 2018).
Figure 5. The region is from GALFA-HI and spans right ascension 212.5° to 265° and declination 19.1° to 38.3°, stretches from b = 30° above the Galactic plane to b = 81.7°, nearly Galactic zenith. We compare the gradients got from VCGs(red lines) with the Planck polarization data (black lines). Please note, each figure is in individual color-scale.

The technique of studying media $M_A$ using velocity gradients was suggested in Lazarian et al. (2018). This technique was tested with numerical simulations and applied to 21 cm data. The decrease of the noise that we observe applying the PCA technique for the initial filtering of the data is valuable for the studies of $M_A$. We plan to demonstrate this elsewhere.

5.2 Obtaining 3D structure of magnetic field

The employment of eigenvalue decompositions through PCA also provides a way to study three-dimensional (3D) magnetic field. As we can isolate the contribution of turbulent eddies along the line of sight with PCA, we can then stack the prediction from different eigen-centroids and construct the 3D tomography by sorting the eigenvalue axis. In a separate development the gradients of synchrotron intensity (Lazarian et al. 2017) and polarization intensity (Lazarian & Yuen 2018b) have been used to construct the 3D magnetic field morphology with multi-frequency measurements. A similar idea of constructing 3D magnetic field morphology with VGT on spectroscopic data has been tested in González-Casanova & Lazarian (2018) when the galactic rotation curve is available. With these 3D field tracing methods available, the productive application of VGT and synergy with different techniques will then shift the paradigm of studying magnetic fields from polarimetry measurements to studies of gradients on both interferometric and spectroscopic data.

5.3 Application within other gradient techniques

We expect the PCA filtering to be useful when applied with other velocity gradient techniques, e.g., with VChGs. However, the application of the procedure is not limited to the velocity gradients.

It is explained, e.g., in Lazarian & Yuen (2018b), that the VGT is one of the techniques that employ the properties of MHD turbulence to study magnetic fields. Magnetic fluctuations enter Alfvénic turbulence in a symmetric way to velocity fluctuations. Therefore both synchrotron intensity gradients (see Lazarian et al. 2017) and synchrotron polarization gradients (Lazarian & Yuen 2018b) can be used to trace magnetic field and study $M_A$. Naturally, the improvement that we are suggesting here with the pre-filtering the images using the PCA seems an attractive possibility for these synchrotron-based techniques.

We would like to mention that while the statistics of density fluctuations in MHD turbulence (Beresnyak et al. 2005; Kowal et al. 2007) does not follow closely, especially at high sonic Mach numbers, the statistics of velocity and magnetic field fluctuation, the Intensity Gradient Technique (IGT)\(^6\) are also very informative. We also expect to see

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\(^6\) As we mentioned earlier, one should distinguish the IGT technique and the Histogram of Relative Orientation (HRO) (Soler et al. 2013). We would like to stress that the former employs the technology that we developed for the velocity and magnetic gradients and therefore can provide the spatial distribution of magnetic fields, shocks, regions of gravitational collapse, etc. (see Yuen & Lazarian 2017b, Lazarian & Yuen 2018a). It is important that the IGT technique does not require any polarization data to get this information. HRO, on the contrary, compares the relative orien-
the improvements for the IG technique when the PCA pre-filtering is employed.

6 SUMMARY

In the present paper, we have utilized filtering of images using the PCA. We have shown using both synthetic and observational maps, that the extraction of eigen-centroids with the rank number of $\sim 10$ can effectively probe the direction of magnetic field with a very high AM. As a result, for the studies of the projected magnetic field, the improved technique can provide higher accuracy of magnetic field tracing.

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