Isobaric analog state in $^{96}$Ag

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Previously, in a single-$j$-shell calculation ($j = g_9/2$), we obtained the excitation energy of the $J = 0^+, T = 2$ isobaric analog state in $^{96}$Ag to be a bit below 1 MeV relative to the $J = 8^+, T = 1$ ground state. We here use binding energy data and Coulomb energy estimates to obtain this same excitation energy and to see if the two approaches are consistent.

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I. RESULTS

If there were no violation of charge independence, the binding energy of $^{96}$Pd ground state ($J = 0^+, T = 2$) would be identical to the binding energy of the analog state, also $J = 0^+, T = 2$, in $^{96}$Ag. But, since that is not the case in real life, the excitation energy of the $J = 0^+, T = 2$ state in $^{96}$Ag is then given by

$$E^*(J = 0^+, T = 2) = BE(^{96}\text{Ag}) - BE(^{96}\text{Pd}) + V_C,$$

where the $BE$s are the binding energies and $V_C$ includes all charge-independence violating effects. We here assume that $V_C$ arises from the Coulomb interaction and use the formula of Anderson et al. [1]:

$$V_C = E_1Z/A^{(1/3)} + E_2,$$

where $Z = (Z_1 + Z_2)/2$. Anderson et al. [1] list four sets of values of $E_1$ and $E_2$. We here use the average values $E_1 = 1.441$ MeV and $E_2 = -1.06$ MeV.

We show in Table I results for various nuclei, some for which the excitation energy of the analog state is known and some for which it is not. The binding energy differences are taken from Ref. [2]

The fact that the analog state and Coulomb arguments work well in known cases gives us confidence that we can use these for the unknown case of $^{96}$Ag. Turning things around, if the isobaric analog state were found, then we might have a better constraint on what the binding energy is.

We can compare the results of the calculated excitation energies with selected calculations in the literature. For $^{44}$Sc and $^{46}$Sc, single-$j$-shell results ($f_7/2$) [4] are respectively 3.047 and 4.949 MeV, as compared with Table I's results of 2.873 and 5.024 MeV. The large space results are also shown. In $^{52}$Mn there is reasonable agreement between calculated, single $j$, large space and experiment.

| NUCLEUS | Binding Energy Difference | Coulomb Energy | Excitation Energy | Single $j$ | Large space | Experiment |
|---------|--------------------------|----------------|-------------------|------------|-------------|------------|
| $^{44}$Sc | 4.435 | 7.308 | 2.873 | 3.047<sup>a</sup> | 3.418<sup>b</sup> | 2.779 |
| $^{46}$Sc | 2.160 | 7.184 | 5.024 | 4.949<sup>a</sup> | 5.250<sup>b</sup> | 5.022 |
| $^{52}$Mn | 5.494 | 8.399 | 2.905 | 2.774 | 2.7307 | 2.926 |
| $^{60}$Cu | 6.910 | 9.430 | 2.520 | 2.235 | 2.536 |
| $^{94}$Rh | 10.386 | 13.043 | 2.657 | 1.990<sup>c</sup> | 3.2664<sup>d</sup> 2.87943<sup>f</sup> | 2.048<sup>c</sup> |
| $^{96}$Ag | 12.432 | 13.574 | 1.142 | 0.900<sup>c</sup> | 1.91667<sup>d</sup> 1.64017<sup>f</sup> | 0.842<sup>c</sup> |

<sup>a</sup>Escuderos, Zamick, Bayman (2005) [4].
<sup>b</sup>GXPF1 interaction [9].
<sup>c</sup>Zamick and Escuderos (2012) [5].
<sup>d</sup>jj44b interaction [7].
<sup>e</sup>CCGI interaction [5, 6].
<sup>f</sup>JUN45 interaction [10].
For the small space for $^{60}\text{Cu}$ ($p_{3/2}$) we can use a particle-hole transformation to get the spectrum of this nucleus from the spectrum of $^{58}\text{Cu}$ since 3 $p_{3/2}$ neutrons can be regarded as a single neutron hole. This gives a value of 2.235 MeV as compared with experiment—2.536 MeV.

For $^{96}\text{Ag}$ single-$j$-shell results [5] are 0.900 MeV with INTd and 0.842 MeV with the CCGI interaction [6]. These are lower than the value in Table I of 1.142 MeV. There are also large scale calculations with the jj44b [7] interaction for $^{96}\text{Ag}$—the result is 1.996 MeV, significantly larger than the calculated value. In $^{94}\text{Rh}$ the jj44b interaction yields 3.052 MeV, larger than the Table I's value of 2.657 MeV. The large space calculations with June45 are qualitatively similar. The single- $j$ INTd and CCGI results are lower, 1.990 MeV and 2.048 MeV respectively.

We can also examine this problem using various mass formulas that abound in the literature. To this end we refer to the work of Kirson[17] which contains not only the parameters of the semiempirical mass formula of Bethe and Weisacker [18] but also a more elaborate formula that he developed. Also to be considered is the mass formula of Dulfo and Zuker [19] which is generally considered to be the best on the market.

We here present the results of the excitation energies in the format Nucleus (semiempirical, Zuker, KirsonA, KirsonB, repeat of table 1). In semiempirical and KirsonA we use the Coulomb energies contained the respective mass formulas. In Zuker and KirsonB we use the Coulomb energies from Table 1 [1].

\[
\begin{align*}
44\text{Sc} & \quad (2.526, 2.374, 1.947, 2.592, 2.873) \\
46\text{Sc} & \quad (6.250, 4.744, 4.532, 5.060, 5.024) \\
52\text{Mn} & \quad (1.875, 1.927, 1.418, 1.911, 2.905) \\
60\text{Cu} & \quad (1.408, 2.420, 1.013, 1.514, 2.520) \\
94\text{Rh} & \quad (2.316, 2.205, 1.503, 1.734, 2.657) \\
96\text{Ag} & \quad (0.368, 0.689, -0.036, 0.173, 1.142) \\
\end{align*}
\]

We next list the Coulomb energy difference for (semiempirical, Kirson, Table )

\[
\begin{align*}
44\text{Sc} & \quad (8.025, 6.663, 7.308) \\
46\text{Sc} & \quad (7.906, 6.652, 7.184) \\
52\text{Mn} & \quad (9.071, 7.827, 8.399) \\
60\text{Cu} & \quad (10.061, 8.928, 9.430) \\
94\text{Rh} & \quad (13.526, 12.812, 13.043) \\
96\text{Ag} & \quad (14.035, 13.364, 13.547) \\
\end{align*}
\]

The Kirson value is smaller than the semiempirical one because it includes an exchange term. In the future it would be useful to get a better handle on the Coulomb energies.

In view of the differing results of shell model calculations and mass formulas it would be of great interest to measure the excitation energies of isotopic analog states in the $g_{9/2}$ region. We hope that this work will encourage experimentalists to look not only for the surprisingly neglected $J = 0^+$ isotopic analog states in $^{94}\text{Rh}$ and $^{96}\text{Ag}$, but also for other such states throughout this region.

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