Does the Feigel effect violate the first law?

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Abstract

A recent theory by Feigel posits that the quantum vacuum can transfer momentum to magnetoelectric media (the Feigel effect). The theory’s predictions of this effect have yet to be observed experimentally. Theoretically, the existence of the phenomenon remains contentious. Many related theories of vacuum momentum transfer have been proposed. Some investigations predict a measurable effect, others conclude the momentum transfer is not physically possible. Some of these, including Feigel’s original analysis, do not model an experimentally realistic geometry. I recently derived an alternative derivation of Feigel’s original theory and applied it to realistic, experimentally testable geometries. I here show here that in such geometries the existence of a steady Feigel effect is equivalent to a violation of the first law of thermodynamics: the momentum extracted from the vacuum lead macroscopically to a stress, which could be used to carry out work with no energy input. This is a very strong argument that a steady Feigel effect should not be observable. I will show how indeed a semi-classical analysis, with proper consideration of boundaries, predicts there should be no net vacuum contribution on a magnetoelectric medium.

Introduction

Since Einstein and Stern added a zero-point energy correction to Planck’s law to guarantee the correct high temperature limit of radiator energy (Einstein & Stern 1913; Milonni, 1994), our knowledge of the quantum vacuum has increased considerably. That we can know anything at all about ‘empty space’ is remarkable. Indeed we have discovered that space is not empty at all but teems with vacuum fluctuations. These can be thought as virtual photons with energy $\hbar \omega_k/2$ and momentum $\hbar k/2$, per mode $k$. Initially it was doubted that these fluctuations should have physically measurable consequences, Wolfgang Pauli was famously skeptical (Milonni, 1994). However, vacuum effects predicted by quantum electrodynamics have been experimentally observed, notably the Lamb shift of atomic levels and the Casimir effect (Milonni, 1994). The latter, only recently measured conclusively (Lamoreaux, 1997), is a particularly spectacular macroscopic manifestation of vacuum fluctuations. Parallel metal plates are driven together due the imbalance between the
vacuum pressure on their outside and between them. This pressure can be considered as due to the momentum transfer of vacuum modes on the metal surfaces (Milonni et al. 1988). In the Casimir effect, statistically the momentum transfer is symmetric. In contrast, a recent untested theory by Feigel posits that the vacuum can transfer momentum to magnetoelectric materials asymmetrically (Feigel, 2004). The molecular structure of these materials causes both the time and space symmetries of EM fields to be broken leading to optical anisotropy: virtual photons travel faster in one direction than in the opposite, resulting in a transfer of momentum to the material.

Feigel’s idea is a very interesting one and has stimulated several theoretical investigations (e.g. van Tiggelen et al 2005, van Tiggelen et al 2006, Shen et al 2006, Birkeland & Brevik 2007, Obukhov & Hehl 2008, Kawka et al 2010, Croze 2012, Rikken & van Tiggelen 2011, Silveirinha & Maslovski 2012). It should be remarked that Feigel’s original proposal amounts to considering a magnetoelectric (ME) liquid that fills all of space away from boundaries. The magnetoelectric response of the fluid is induced by applying strong perpendicular electric and magnetic fields, which also need to permeate all space if one is to avoid introducing field gradients and boundaries. This unbounded scenario is not experimentally realistic, but some theoretical analyses have considered it concluding that the Feigel effect vanishes when the theory is properly regularised (van Tiggelen et al 2005, van Tiggelen et al 2006). In alternative to effectively ME liquids, one can instead consider solid slabs whose magnetoelectricity can be induced by application of cooling and a smaller magnetic field (Jung et al., 2004). Obukhov & Hehl (2008) considered the abstraction of such magnetoelectric systems, evaluating the net force on an infinite slab of finite thickness it semi-classically. They predict a nonzero force for light on a magnetoelectric (real photons, e.g. due to counter propagating laser beams hitting the slab from both sides), but that this force vanishes for slabs in a vacuum. Other theoretical proposals consider magnetoelectric media sandwiched between parallel metal plates (Casimir geometries) (Birkeland & Brevik 2007, van Tiggelen et al 2006, Silveirinha & Maslovski 2012). This allows a regularisation of the theory without introducing a cut-off, but any predictions that have been made for momenta remain unmeasurably small (van Tiggelen et al 2006). Only two investigations support the existence of a Feigel effect, on top of Feigel’s original proposal. The first is my own analysis of realistic bounded steady Feigel effect. I have provided an alternative derivation Feigel’s result, and a new expression for the vacuum stress on an ME liquid which I use to make an experimentally observable prediction (Croze, 2012). The second is microscopic theory by Kawka et al (2010) who predicted a finite momentum transfer in a gas. The only experimental test of a Feigel effect is by Rikken & van Tiggelen (2011) who considered ME media in oscillating fields. They rule out my correction of the effect originally predicted by Feigel, and state that QED prediction by Kawka et al (2010) was too small to be measured.

I this paper I will show how if a realistic Feigel effect exists, the first law of thermodynamics will be violated. I will then show how an incomplete analysis of the boundary conditions led me to consider only half the Feigel problem and when the full problem is
considered the steady prediction of the ME fluid vanishes. I will then briefly consider transient effects and conclude by proposing a new Casimir-Feigel effect.

**Thermodynamic considerations**

In my recent analysis I considered two experimental reasonable scenarios to test the Feigel effect:

- (i) an organometallic dielectric liquid rendered magnetoelectric by the application of large perpendicular electric and magnetic fields
- (ii) a vacuum radiometer with paddles made of a magnetoelectric solid such as a polar ferrimagnet.

For case (i) I corrected Feigel’s derivation of the momentum density in the ME fluid caused by the anisotropic propagation of vacuum modes. From one can derive either semiclassically, or through a kinetic theory argument a new expression for the stress these modes cause on the fluid in the ME region: $T_{\text{vac}} \approx \Delta \chi \hbar c/\lambda_c^4$, where $\Delta \chi \equiv \chi_{xy} - \chi_{yx}$ is the difference between the nonzero components of the magnetoelectric susceptibility tensor, $\hbar$ is Planck’s angular constant, $c$ is the speed of light in vacuo, and $\lambda_c$ is the cut-off wavelength below which vacuum modes do not experience a ME medium. I used this new vacuum stress expression in the Navier-Stokes equation to evaluate the maximum speed and corresponding flowrate of an organometallic fluid in a tube, a short portion of which is exposed to strong orthogonal electric and magnetic fields inducing ME response. Ignoring gradient effects at the region boundaries I obtained a flow vacuum flow speed $U_{\text{vac}} = T_0 d^2/(4\eta L) \approx 100 \mu m/s$ (where $a, L$ are the tube radius and length, respectively, and $\eta$ dynamic viscosity of fluid) and corresponding flow rate $\Phi_{\text{vac}} = \pi a^2 U_{\text{vac}}/2$ driven by a vacuum pressure of $T_{\text{vac}} = 0.03$ Pa. I based these estimates on reasonable experimental ME fluid parameters considering a tube with $a = 1\text{mm}$ and $L = 2\text{m}$. Arranging this tube into a closed loop it is clear that, with $\mathbf{E}$ and $\mathbf{B}$ fields steady, a non-zero Feigel effect in the ME region implies the vacuum can continuously drive flow in a circle. This flow will dissipate energy at a rate $P = T_{\text{vac}} \Phi \sim 1\text{nW}$. Where is this energy coming from? The steady fields making the fluid magnetoelectric cannot be putting any energy into the system (charges and fields can do no work), with the implication that either the vacuum transfers no momentum to the ME medium or energy (albeit in tiny amounts) can be extracted from the vacuum without other energy input. This is a blatant violation of conservation of energy the first law of thermodynamics. A similar argument applies to the vacuum radiometer I considered: the rotation of its magnetoelectric vanes caused by the vacuum would be dissipated by friction at the pivot at a rate $\sim \gamma \omega_{\text{vac}}^2$. In the Casimir effect difference in the vacuum energy between the plates and outside (due to the change in the mode spectrum by the boundaries) is converted into kinetic energy of the plates until they touch and stop moving. An equal amount of work needs to be performed to move the plates apart.
Revised quantum electrodynamic analysis

I here re-analyse the QED prediction for the momentum transfer of the vacuum to an organometallic fluid contained in a tube (length $L$, radius $a$), a section of which is exposed to the fields, case (i) above. The region of the tube where the fields act is denoted as region 1. In this region the field induce magnetoelectric (ME) susceptibilities $\mu_{ij}$. The rest of the tube is denoted as region 2. The situation is depicted in figure 1 below (not to scale). We evaluate the electromagnetic force density in regions 1 and 2. In general this is given by:

$$f = \frac{\partial g}{\partial t} - \nabla \cdot T$$

(1)

where $g$ is the EM momentum density and $T$ is the EM stress tensor. In region 1, the fields cause the dielectric fluid to behave like a ME material with constitutive equations (note: I use Gaussian units throughout):

$$D = \varepsilon E + \hat{\chi} H,$$

(2)

$$B = \mu H + \hat{\chi}^T E,$$

(3)

Figure 1: (a) an organometallic ME fluid in crossed fields contained in a tube, as envisaged by Croze (2011). The tube is divided into two regions: region 1 where the fields act (so that the fluid behaves like a magnetoelectric) and region 2 where there are no fields (and no additional stresses). (b) If the tube is a closed loop, a non-zero steady Feigel effect implies vacuum momentum transfer could be used to generate heat without performing work; (c) similarly the vacuum momentum transfer on a ME slab would cause vacuum radiometer to turn and the dissipation at the pivot would generate useable heat. These are fragrant violations of the first law of thermodynamics.
where the $\hat{\chi}$ is the magnetoelectric susceptibility tensor, defined, in matrix form, by:

$$
\hat{\chi} \equiv \begin{pmatrix}
0 & \chi_{xy} & 0 \\
\chi_{yx} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

(4)

We can choose the momentum density $g = (\mathbf{D} \times \mathbf{B} - \mathbf{E} \times \mathbf{H})/(4\pi c)$ [?] and (2), (3) to find:

$$
g = \frac{1}{4\pi c} \left[ \left( \epsilon - \frac{1}{\mu} \right) \mathbf{E} \times \mathbf{B} + \frac{1}{\mu} \mathbf{E} \times (\hat{\chi}^T \mathbf{E}) + \frac{1}{\mu} (\hat{\chi} \mathbf{B}) \times \mathbf{B} \right]
$$

(5)

The field modes and refractive indices for electromagnetic waves in a magnetoelectric are given by (taking the optical axis along the z-direction, $e_3$),

$$
[E_{\pm k_1}, B_{\pm k_1}] = E_{\pm k_1} [e_1, n_{\pm k_1} e_2], \quad [E_{\pm k_2}, B_{\pm k_2}] = E_{\pm k_2} [e_2, -n_{\pm k_2} e_1]
$$

(6)

where $E_{\pm k\lambda} = E_{0k} \cos (k_\lambda z - \omega t)$, for each polarisation $\lambda = 1, 2$, and

$$
n_{\pm k,1} = \pm n_0 + \chi_{xy}, \quad n_{\pm k,2} = \pm n_0 - \chi_{yx}
$$

(7)

Evaluating (5) for the counterpropagating EM modes in the z-direction using (6) (see Croze (2011) and Feigel (2004)) gives the time-averaged momentum density per wavevector $k$:

$$
\overline{g_k} = \Delta \chi \frac{\epsilon E_{0k}^2}{c}; \quad \frac{\partial g_k}{\partial t} = 0
$$

(8)

where $\Delta \chi = \chi_{xy} - \chi_{yx}$. The time-average $\partial g_k/\partial t$ is zero because in the evaluation of $\partial g_k/\partial t$ all terms are multiplied by factors $\sim \sin [2(k_\lambda z - \omega t)]$, whose time average is zero. It is thus correct to state that $\partial g_k/\partial t$ for steady state external fields, since then the $\chi_{ij}$ are constant in time and the time-average of the fields vanishes. When the fields are not steady and $\chi_{ij} = \chi_{ij}(t)$, however, $\partial g_k/\partial t$ are nonzero, see below.

One can similarly evaluate the EM stress tensor. Since for field modes propagating along $z$, $\nabla \cdot T = \partial_z T^{zz}$, we need only calculate $T^{zz}$ (the Abraham and Minkowski expressions coincide for this term):

$$
T^{zz} = -\frac{1}{4\pi c} \left[ \frac{1}{2} \epsilon E^2 - \frac{1}{2\mu} B^2 \right]
$$

(9)

Note when calculating the contribution of modes we need to evaluate the net stress:

$$
T_k^{zz} = \sum_\lambda (T_{+k\lambda}^{zz} - T_{-k\lambda}^{zz})
$$

(10)

As before using (6) in (9) and (10), we find:

$$
\overline{T_k^{zz}} = \Delta \chi \frac{\epsilon E_{0k}^2}{4\pi}; \quad \frac{\partial T_k^{zz}}{\partial z} \left( = \frac{\partial T_k^{zz}}{\partial z} \right) = 0
$$

(11)
We see that the time-averaged stress is constant. Quantisation of the EM energy density and summing over all modes leads to the vacuum momentum density $\langle 0| g | 0 \rangle = g_0$ and stress $\langle 0| T^{zz} | 0 \rangle = T_0 = g_0/c$. It is this stress that could give rise to a Feigel effect. It is true to say that in region 1 at steady state the EM force density vanishes:

$$\bar{f}_1 = \frac{\partial g_1}{\partial t} - \nabla \cdot \bar{T}_1 = 0 \quad (12)$$

In region 2 there are no external fields to cause anisotropy so we might expect $f_2 = 0$ in the ME medium. However, due to the ME anisotropy of region 1, we have in region 2 a non-zero stress at the left boundary given by $T^{zz}(z = 0) = T_0$. At the tube exit no stress acts, so $T^{zz}(z = L) = 0$, where $L$ is the length of the tube. Assuming a constant stress gradient along the tube the stress gradient (divergence) has the form $\partial_z T^{zz} = \text{const.}$; integrating, we find $\text{const.} = T_0/L$. Thus

$$\langle 0| f_{12} | 0 \rangle = \frac{\partial}{\partial z} \langle 0| T^{zz} | 0 \rangle k = \frac{T_0}{L} k. \quad (13)$$

Thus, the net force density between regions 1 and 2 is $f_{\text{vac}} \equiv \langle 0| f | 0 \rangle = \langle 0| \bar{f}_1 + \bar{f}_2 + \bar{f}_{12} | 0 \rangle = \frac{T_0}{L} k$, and it appears that the vacuum should indeed exert a force on the ME medium. This analysis, however, does not fully treat the EM boundary conditions of the problem. Let us consider a looped tube as in figure ?? Then there are still two bulk regions: the ME region (1), the region outside it (2), but there also two surfaces between the regions, denoted by (12) and (21). An identical calculation of the EM modes for the (21) region gives

$$\langle 0| f_{21} | 0 \rangle = -\langle 0| f_{12} | 0 \rangle = -\frac{T_0}{L} k. \quad (14)$$

The net force density on the ME region is thus given by

$$f_{\text{vac}} = \langle 0| \bar{f}_1 + \bar{f}_2 + \bar{f}_{12} + \bar{f}_{21} | 0 \rangle = 0, \quad (15)$$

That is, when boundary conditions are fully considered, the net force density on the ME fluid is zero for steady ME susceptibility (steady fields) is zero: the first law is safe! This agrees with Obukhov & Hehl analysis for a ME slab and has the same physical origin. That the steady Feigel effect should be zero is clear from consideration of the change of momentum of the vacuum EM modes as they enter and exit the ME region via its boundaries, as shown in figure (2) (Bongs, 2012). Right (left) travelling modes gain (lose) momentum on entry across the region $2 \rightarrow 1$ ($1 \rightarrow 2$) surface, but lose (gain) an equal and opposite amount on exit through the region $1 \rightarrow 2$ ($2 \rightarrow 1$) surface. The net momentum transfer is zero, so the fluid is not set into motion for steady values of the ME susceptibility (steady fields)
Discussion

I have shown that a steady Feigel effect is incompatible with the first law of thermodynamics. A revision of my recent analysis, properly accounting for the momentum transfer at the boundaries of a magnetoelectric fluid, shows that QED does not predict a net force from the quantum vacuum on a magnetoelectric medium with a magnetoelectric susceptibility constant (steady applied fields inducing the ME response in a fluid). Thus the steady Feigel effect is a null effect, and does not break the first law. Interestingly, Obukhov & Hehl (2008) claim that a ‘classical steady Feigel effect’, such as ME slab placed in counter propagating laser beams, should instead lead to a nonzero net force. The analysis in this paper is semiclassical, so, aside from the final replacement of the EM energy with its quantum operator form, the argument for the force density holds true and for the ME susceptibility of equation (4), a steady Feigel effect is not expected in for light. A transient effect, however, should exist without violating the first law: the energy of time-dependent $E$ and $B$ is transferred to the system. Simple estimates on the magnitude of this transient effect reveal it might be too small to measure using current experimental techniques (Forgan, 2012).

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