Noncommutativity and Its Freedoms Due to the PP-Wave and Background Gauge Fields

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Abstract

In this article we consider an open string attached to a D_p-brane, in the presence of the pp-wave and background gauge fields. The effects of the string mass on the open string propagator, open string metric and the noncommutativity parameter are studied. Some free matrices appear in the propagator and in the open string variables. The symmetries of the string propagator and consistency with the zero mass case put some restrictions on these matrices.

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1 Introduction

It is known that the plane wave metric supported by a Ramond-Ramond 5-form background [1] provides examples of exactly solvable string models [2]. Many properties of string theory in such plane wave backgrounds have been deeply investigated [2, 3]. In the other side, the noncommutativity of a D-brane worldvolume has been studied through the open strings in the presence of the background gauge fields [4]. In this article we shall study it by computing the propagator of a massive open string.

We shall consider an open string in the presence of both pp-wave and background gauge fields. This produces a generalized noncommutativity on the D-brane worldvolume that the open string is attached on it. This generalization comes from the propagator equation which is a second order differential equation with source term. The solution of this equation has expression in terms of the (modified) Bessel functions. Beside the background fields, the string mass also appears in the noncommutativity parameter, in the open string metric and in the string propagator.

In addition, some free matrices also arise in these variables. Symmetries of the open string propagator and consistency with the known cases put some restrictions on these matrices. For example, imposing the Hermitian condition on the string propagator leads to a restriction on these matrices and also gives a noncommutativity that for the zero gauge fields is not zero. In the motion of a $D_p$-brane along itself such free matrices also arise in the open string variables and in the propagator [5].

Since the bosonic string theory is sufficient to discuss the noncommutativity of the spacetime, we shall consider only the bosonic open strings.

The paper is organized as follows. In section 2, from the open string action we obtain the open string variables and the equation of the string propagator. In section 3, the propagator equation will be solved. In section 4, the freedoms of the matrices that appear in the open string variables will be restricted.

2 Open string variables

In the background gauge fields and pp-wave the action of open string can be written as in the following

$$S = \frac{1}{4 \pi \alpha'} \int d\tau \int_0^{2\pi \alpha' \rho^+} d\sigma \left( g_{IJ} (\partial_X X^I \partial_X X^J + \partial_{\sigma^0} X^I \partial_{\sigma^0} X^J + \mu^2 X^I X^J) \right)$$

$$- \frac{i}{2} \int d\tau (F_{\alpha\beta} X^\alpha \partial_X X^\beta)_{\sigma^0},$$

(1)
where $g_{IJ}$ is a constant metric and $F_{\alpha\beta}$ is total field strength, i.e., $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + B_{\alpha\beta}$. The field $A_\alpha$ is a $U(1)$ gauge field with constant field strength. Therefore, in the action (1) it has been expressed in the form $A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta$. Furthermore, we assumed that the NS$\otimes$NS field $B_{\alpha\beta}$ is constant and the metric of the worldsheet is Euclidean. The parameter $\sigma_0 = 0$ indicates the end of the open string on the brane. Let us consider the decomposition $\{X^I\} = \{X^\alpha\} \cup \{X^i\}$. The set $\{X^\alpha\}$ shows the brane directions and $\{X^i\}$ are coordinates perpendicular to the brane.

The variation of the action (1) gives the equation of motion and the boundary conditions

\[
(\partial_\tau^2 + \partial_\sigma^2 - \mu^2) X^I(\sigma, \tau) = 0 , \quad \text{boundary conditions} \quad (\delta X^i)_{\sigma_0} = 0 .
\]

As we see the mass "$\mu$" does not affect the boundary conditions.

According to the equations (2) and (3) the open string propagator has the equations

\[
\left( \partial \bar{\partial} - \frac{\mu^2}{4} \right) G^{\alpha\beta}(z, z') = -\frac{\pi}{2} \alpha' G^{\alpha\beta} \delta(2)(z - z') , \quad \text{boundary condition} \quad G^{\alpha\beta}(z, z') \bigg|_{z = \bar{z}} = 0 ,
\]

where $z = \tau + i\sigma$ and $G^{\alpha\beta}$ is the open string metric. The solution of the propagator equations can be written in the form

\[
\begin{align*}
G^{\alpha\beta}(z, z') &= -\alpha' \left[ \frac{1}{2} Q^{\alpha\beta} \ln(z - z') + \frac{1}{2} (Q^T)^{\alpha\beta} \ln(\bar{z} - \bar{z}') \right] \\
&\quad + \left( -\frac{1}{2} Q^{\alpha\beta} + G^{\alpha\beta} + \frac{\theta^{\alpha\beta}}{2\pi\alpha'} \right) \ln(z - z') \\
&\quad + \left( -\frac{1}{2} (Q^T)^{\alpha\beta} + G^{\alpha\beta} - \frac{\theta^{\alpha\beta}}{2\pi\alpha'} \right) \ln(\bar{z} - \bar{z}') - i \frac{\alpha'}{2\alpha} D^{\alpha\beta} ,
\end{align*}
\]

where $\theta^{\alpha\beta}$ is the noncommutativity parameter. For the next purposes let the matrix $D^{\alpha\beta}$ be anti-Hermitian

\[
D^\dagger = -D .
\]

In addition, we assume that $D^{\alpha\beta}$ is constant but the matrix $Q^{\alpha\beta}$ depends on the variables $z, \bar{z}, z'$ and $\bar{z}'$. 

3
Combining the boundary equation (6) and the propagator (7), we obtain the open string metric and the noncommutativity parameter

$$G^{\alpha\beta} = \left((g + 2\pi\alpha'F)^{-1}g - 2\pi\alpha'F\right)^{\alpha\beta}, \quad (9)$$

$$G_{\alpha\beta} = \left(R^{-1}(g - 2\pi\alpha'F)g^{-1}(g + 2\pi\alpha'F)\right)_{\alpha\beta}, \quad (10)$$

$$\theta^{\alpha\beta} = -(2\pi\alpha')^2\left((g + 2\pi\alpha'F)^{-1}F(g - 2\pi\alpha'F)^{-1}R\right)^{\alpha\beta}. \quad (11)$$

In these variables the matrix $R$ has the definition

$$R \equiv \frac{1}{2}(g + 2\pi\alpha'F)Q + (g - 2\pi\alpha'F)Q^T)_{z=\bar{z}}. \quad (12)$$

The open string variables $G$ and $\theta$ depend on the mass $\mu$ through the matrix $Q$. According to the propagator (7) for the zero mass $\mu$ we should have $Q = g^{-1}$. In this case there is $R = 1$, as expected.

The matrix $Q$ should satisfy the boundary condition

$$\left((g + 2\pi\alpha'F)\partial(Q - Q^T) - (g - 2\pi\alpha'F)\bar{\partial}(Q - Q^T)\right)_{z=\bar{z}} = 0. \quad (13)$$

In fact, the equation (6) leads to the relations (9)-(11) and the equation (13). Without lose of generality, we assume that the matrix $Q$ for all coordinates $z$ and $\bar{z}$ to be symmetric, i.e.,

$$Q^T = Q. \quad (14)$$

Therefore, the equation (13) becomes trivial, and the matrix $R$ takes the form $R = gQ|_{z=\bar{z}}$.

## 3 Solution of the propagator equation

According to the propagator (7), the equation (5) reduces to the matrix equation

$$\Omega + \Omega^\dagger = 0, \quad (15)$$

where the matrix $\Omega$ is defined by

$$\Omega(z, \bar{z}) = \left(\bar{\partial}\partial - \frac{\mu^2}{4}\right)\left(\lambda Q - \frac{\mu^2}{4}\left(-\frac{i}{2\alpha'}D + 2(g + 2\pi\alpha'F)^{-1}R \ln(z - \bar{a})\right)\right). \quad (16)$$
In this definition the relation \( \left( G + \frac{\theta}{2\pi\alpha'} \right)^{\alpha\beta} = \left( (g + 2\pi\alpha'F)^{-1}R \right)^{\alpha\beta} \) has been used. The function \( \lambda(z) \) is

\[
\lambda(z) = \ln \left( \frac{z - a}{z - \bar{a}} \right). \tag{17}
\]

Since in the equation (15) \( z' \) and \( \bar{z}' \) are not variables, we demonstrated them as the parameters \( a \) and \( \bar{a} \), respectively.

The general solution of the equation (15) can be written as in the following

\[
\Omega^{\alpha\beta} = \frac{\mu^2}{4} E^{\alpha\beta}_{\mu} (z, \bar{z}), \tag{18}
\]

where \( E^{\alpha\beta}_{\mu} (z, \bar{z}) \) is any anti-Hermitian matrix that depends on the mass \( "\mu" \). The symmetric condition (14) and reduction to the massless case will restrict this matrix. The restrictions on \( E^{\alpha\beta}_{\mu} \) will be discussed in the next section. In fact, \( \mathcal{G}^{\alpha\beta}, G^{\alpha\beta} \) and \( \theta^{\alpha\beta} \) are depended on the mass \( \mu \) through the equation (18).

Note that the equation (18) also can be obtained as the equation of motion from the following action

\[
S_h = \frac{1}{4\pi\alpha'} \int d^2z (\partial h^{\alpha\beta}_{\alpha\beta} \partial h^{\alpha\beta} + \frac{\mu^2}{4} h^{\alpha\beta} h^{\alpha\beta} + h^{\alpha\beta} J^{\alpha\beta}), \tag{19}
\]

where \( h^{\alpha\beta} \) and \( J^{\alpha\beta} \) are

\[
h^{\alpha\beta} = \lambda Q^{\alpha\beta}, \quad J^{\alpha\beta} = \frac{\mu^2}{2} \left[ - \frac{i}{2\alpha'} D^{\alpha\beta} + 2 \left( (g + 2\pi\alpha'F)^{-1}R \right)^{\alpha\beta} \ln(z - \bar{a}) + E^{\alpha\beta}_{\mu} (z, \bar{z}) \right]. \tag{20}
\]

Also we have \( h_{\alpha\beta} = g_{\alpha\gamma} g_{\beta\lambda} h^{\gamma\lambda} \). In this action the elements of the matrix \( Q^{\alpha\beta} \) are degrees of freedom. Therefore, in the equation (18) the matrix \( J^{\alpha\beta} \) can be interpreted as a source for the field \( h^{\alpha\beta} \).

Define the worldsheet coordinates \( \zeta \) and \( \bar{\zeta} \) as

\[
\zeta = z - \bar{a}, \quad \bar{\zeta} = \bar{z} - a. \tag{21}
\]

In terms of these variables the equation (18) takes the form

\[
\left( \frac{\partial}{\partial \zeta} \frac{\partial}{\partial \bar{\zeta}} - \frac{\mu^2}{4} \right) h^{\alpha\beta}(\zeta, \bar{\zeta}) = \frac{\mu^2}{4} \left[ - \frac{i}{2\alpha'} D^{\alpha\beta} + 2 \left( (g + 2\pi\alpha'F)^{-1}R \right)^{\alpha\beta} \ln \zeta + E^{\alpha\beta}_{\mu}(\zeta, \bar{\zeta}) \right]. \tag{22}
\]

In other words, in terms of the polar forms of \( \zeta \) and \( \bar{\zeta} \), i.e.,

\[
\zeta = re^{i\phi}, \quad \bar{\zeta} = re^{-i\phi}, \tag{23}
\]

5
the equation (22) changes to
\[
\left( \partial_r^2 + \frac{1}{r} \partial_r + \frac{1}{r^2} \partial_\phi^2 - \mu^2 \right) h^{\alpha\beta}(r, \phi) = \mu^2 \left[ -\frac{i}{2\alpha'} D^{\alpha\beta} \right] \\
+ 2 \left( (g + 2\pi\alpha' F)^{-1} \right)^{\alpha\beta} (\ln r + i\phi) + E^{\alpha\beta}_\mu(r, \phi) .
\]
(24)

The solution of the equation (24) can be written as
\[
h^{\alpha\beta}(r, \phi) = h_0^{\alpha\beta}(r, \phi) + h_1^{\alpha\beta}(r, \phi) ,
\]
(25)
where \(h_0^{\alpha\beta}(r, \phi)\) is the solution of the homogeneous equation
\[
\left( \partial_r^2 + \frac{1}{r} \partial_r + \frac{1}{r^2} \partial_\phi^2 - \mu^2 \right) h_0^{\alpha\beta}(r, \phi) = 0 .
\]
(26)

The inhomogeneous part \(h_1^{\alpha\beta}(r, \phi)\) completely depends on the right hand side of the equation (24). On the other hand, it has the integral form
\[
h_1^{\alpha\beta}(r, \phi) = \frac{1}{2\pi} \int d^2 r' G(r, r') J^{\alpha\beta}_1(r') ,
\]
(27)
where \(r = (r, \phi)\) denotes a vector. In this integral the vector \(r'\) sweeps all the plane. Also \(G(r, r')\) is Green’s function which satisfies the equation
\[
\left( \partial_r^2 + \frac{1}{r} \partial_r + \frac{1}{r^2} \partial_\phi^2 - \mu^2 \right) G(r, r') = 2\pi \delta^{(2)}(r - r') .
\]
(28)

The source \(J^{\alpha\beta}_1(r, \phi)\) is defined by the right hand side of the equation (24). In other words, we have \(J^{\alpha\beta}_1 = 2J^{\alpha\beta}\). From now on, in \(J^{\alpha\beta}_1\) we apply the approximation \(R \approx 1\).

The Green’s function equation (28) has the solution
\[
G(r, r') = -K_0(\mu|r - r'|) ,
\]
(29)
where \(K_0(x)\) is a modified Bessel function. This function has the integral representation
\[
K_0(x) = \int_0^\infty dt \cos(x \sinh t) , \quad \text{for} \quad x > 0 .
\]
(30)

Note that for \(\mu|r - r'| \to \infty\), the Green’s function (29) goes to zero.

The equation (26) can be solved by the method of separation of the variables. The final result is
\[
h_0^{\alpha\beta}(r, \phi) = \sum_{n=0}^\infty \left[ \left( A_n^{\alpha\beta} \cos(n\phi) + B_n^{\alpha\beta} \sin(n\phi) \right) I_n(\mu r) \right] .
\]
(31)
Some restrictions on the matrices should hold for all $z$ are modified Bessel functions. They have the series form

$$I_n(x) = \sum_{l=0}^{\infty} \frac{1}{l!(l+n)!} \left( \frac{x}{2} \right)^{2l+n}. \quad (32)$$

Since for the integer “$n$” we have $I_n(x) = I_{-n}(x)$, there is no need of adding $I_{-n}(x)$ to the solution (31). Also $\{A_n^{\alpha\beta}, B_n^{\alpha\beta}\}$ are arbitrary constant matrices. In the next section, some restrictions will be put on them. For the massless case the solution (31) reduces to $h_0^{\alpha\beta} = A_0^{\alpha\beta}$.

The homogeneous solution (31) describes the modification of the open string variables due to the string mass. While the inhomogeneous solution (27) gives the effects of the combination of the string mass and gauge fields to these variables.

The (modified) Bessel functions $K_0(x)$ and $I_n(x)$ for the small and large $x$ have opposite behaviors. For $x \to 0$ we have $K_0(x) \to +\infty$, while $I_n(x) \to 0$ (for $n \neq 0$) and $I_0(x) \to 1$. Also for $x \to +\infty$ there are $K_0(x) \to 0$ and $I_n(x) \to +\infty$.

4 Some restrictions on the matrices $A_n^{\alpha\beta}, B_n^{\alpha\beta}$ and $E_\mu^{\alpha\beta}$

In the limit $\mu \to 0$ we should have $Q^\alpha_\beta(z, \bar{z}) = g^\alpha_\beta$. According to the propagator (7) this should hold for all $z$ and $\bar{z}$. It is not restricted to the $z = \bar{z}$ case that appeared in the matrix $R^\alpha_\beta$. This imposes the following restriction on the matrices $A_n^{\alpha\beta}, B_n^{\alpha\beta}$ and $E_\mu^{\alpha\beta}(r, \phi)$,

$$\int d^2r' \left( \mu^2 K_0(\mu|r - r'|) E_\mu^{\alpha\beta}(r') \right)_{\mu=0} = 2\pi(A_0^{\alpha\beta} - \lambda g^{\alpha\beta})$$

$$- \int d^2r' \left[ \mu^2 K_0(\mu|r - r'|) \right]_{\mu=0} \left( -\frac{i}{2\alpha'} D^{\alpha\beta} + 2 \left( (g + 2\pi\alpha' \mathcal{F})^{-1} \right)^{\alpha\beta} \left( \ln r' + i\phi' \right) \right) \right) \right) \quad (33)$$

Note that $|r - r'|$ can be infinite. This implies that for $\mu \to 0$ the argument $\mu|r - r'|$ in general is not small. Therefore, in this equation and also in the next equations, we cannot use the expansion form of the function $K_0(\mu|r - r'|)$ for the small argument.

Another restriction on these matrices comes from the symmetry of the matrix $Q^\alpha_\beta$. On the other hand, we have

$$\mu^2 \int d^2r' \left( K_0(\mu|r - r'|) [E_\mu(r') - E_\mu^T(r')]^{\alpha\beta} \right) = 2\pi \left( h_0^{\alpha\beta}(r, \phi) - (h_0^T)^{\alpha\beta}(r, \phi) \right)$$

$$- \mu^2 \int d^2r' \left[ K_0(\mu|r - r'|) \left( 2\beta_0^{\alpha\beta} \pi\alpha' (\ln r' + i\phi') + \frac{i}{2\alpha'} (D^T - D)^{\alpha\beta} \right) \right], \quad (34)$$
where $\theta_0^{\alpha\beta}$ is the noncommutativity parameter corresponding to the massless string. From this integral equation we obtain
\[
\mu^2 K_0(\mu |r - r'|) \left( E_\mu(r) - E_\mu^T(r) + \frac{2\theta_0}{\pi\alpha'} (\ln r + i\phi) + \frac{i}{2\alpha'} (D^T - D) \right)^{\alpha\beta}
\]
\[
= 2\pi \left( h_0(r, \phi) - h_0^T(r, \phi) \right)^{\alpha\beta} \delta^{(2)}(r - r') + \nabla \cdot W^{\alpha\beta}(r, r'),
\]  
where the components of the vector $W(r, r')$ are antisymmetric matrices. This vector, up to the following equation, is arbitrary
\[
\int d^2 r \nabla \cdot W(r, r') = 0.
\]  
Therefore, the freedom of $E_\mu(r)$ is saved by the vector $W(r, r')$. When the vector $r'$ approaches to $r$, both sides of the equation (35) go to infinity. The equations (35) and (36) are consistent with (33). That is, subtract the transpose of the equation (33) from itself. The resulted equation is satisfied by the equations (35) and (36).

Since the matrix $R^{\alpha\beta}$ should be independent of the worldsheet coordinate $\tau$, we obtain another restriction on the free matrices. Note that we have $R = gQ|_{\sigma = 0}$. For $\sigma = 0$ the polar coordinates are
\[
r(\tau) = \sqrt{(\tau - a)(\tau - \bar{a})},
\]
\[
\phi(\tau) = \frac{i}{2} \ln \left( \frac{\tau - a}{\tau - \bar{a}} \right) = \frac{i}{2} \lambda(\tau).
\]  
These equations show the parametric form of a curve in the $r - \phi$ plane. In other words, $r$ versus $\phi$ is
\[
r(\phi) = \frac{b}{\sin \phi},
\]  
where $b = \frac{a - \bar{a}}{2i}$ is a real number. In fact, motion of the end of the open string on the brane draws a curve. Mapping of this curve in the $r - \phi$ plane leads to the equation (38). This equation gives
\[
Q^{\alpha\beta}(\phi) = \frac{i}{2\phi} \left( h_{00}^{\alpha\beta}[r(\phi), \phi] + h_{10}^{\alpha\beta}[r(\phi), \phi] \right).
\]  
Since $Q^{\alpha\beta}(\phi)$ should be independent of the coordinate $\phi$, we obtain the condition
\[
\int d^2 r' \left( J^{\alpha\beta}(r')(1 - \phi \frac{d}{d\phi})K_0[\mu |r(\phi) - r'|] \right) = \pi (1 - \phi \frac{d}{d\phi})h_0^{\alpha\beta}[r(\phi), \phi].
\]  
This integral equation implies that
\[
J^{\alpha\beta}(r)(1 - \phi' \frac{d}{d\phi'})K_0[\mu |r'(\phi') - r|] = \pi (1 - \phi' \frac{d}{d\phi'})h_0^{\alpha\beta}[r'(\phi), \phi] \delta^{(2)}(r - r') + \nabla \cdot U^{\alpha\beta}(r, r').
\]
The vector $U(r, r')$ should obey the equation (36). Subtract the transpose of the equation (41) from itself. Now eliminate $E_\mu(r) - E_\mu^T(r)$ between the resulted equation and the equation (35). Therefore, the matrices $U(r, r')$ and $W(r, r')$ are related to each other.

In fact, the equations (33), (35) and (41) are restrictions on the free matrices. Up to these equations, these matrices remain arbitrary.

The Hermitian propagator

In the propagator (7) if we use $Q^\dagger$ instead of $Q^T$, the matrix $G^{\alpha\beta}$ will be Hermitian. This change also will appear in the equations (12)-(14). Therefore, the matrix $Q$ should be Hermitian. This condition restricts the free matrices by the equation

$$
\mu^2 \int d^2 r' \left( K_0(\mu |r - r'|) E^{\alpha\beta}_\mu(r') \right) = 2\pi \frac{\tilde{\lambda} h_0^{\alpha\beta}(r, \phi) - \lambda (h_0^\dagger)^{\alpha\beta}(r, \phi)}{\lambda + \tilde{\lambda}}
$$

$$
- \mu^2 \int d^2 r' \left[ K_0(\mu |r - r'|) \left( 2i G_1^{\alpha\beta} \phi' + \frac{\theta_1^{\alpha\beta}}{2\alpha'} \ln r' + \frac{i}{2\alpha'} \frac{\lambda - \lambda'}{\lambda + \lambda} D^{\alpha\beta} \right) \right] .
$$

The matrices $G_1^{\alpha\beta}$ and $\theta_1^{\alpha\beta}$ are the following combinations of $G_0^{\alpha\beta}$ and $\theta_0^{\alpha\beta}$,

$$
G_1^{\alpha\beta} = \left( G_0 - \frac{\lambda - \tilde{\lambda}}{\lambda + \lambda} \frac{\theta_0}{2\pi \alpha'} \right)^{\alpha\beta},
$$

$$
\theta_1^{\alpha\beta} = \left( \theta_0 - 2\pi \alpha' \frac{\lambda - \tilde{\lambda}}{\lambda + \lambda} G_0 \right)^{\alpha\beta},
$$

where $G_0^{\alpha\beta}$ denotes the open string metric for the massless case. These imply that $G_1$ is Hermitian and $\theta_1$ is anti-Hermitian. The matrices $G_1$ and $\theta_1$ may be interpreted as combined open string metric and combined noncommutativity parameter. In the absence of the gauge fields we have $G_0^{\alpha\beta} = g^{\alpha\beta}$ and $\theta_0^{\alpha\beta} = 0$, while $\theta_1^{\alpha\beta}$ is not zero. Therefore, effectively there is a noncommutativity.

The integral equation (42) gives the explicit form of the matrix $E_\mu(r)$,

$$
\mu^2 K_0(\mu |r - r'|) \left( E_\mu(r) + 2i G_1 \phi + \frac{\theta_1}{2\alpha'} \ln r' + \frac{i}{2\alpha'} \frac{\lambda' - \tilde{\lambda}'}{\lambda' + \lambda'} D \right)
$$

$$
= 2\pi \frac{\tilde{\lambda} h_0(r) - \lambda h_0^\dagger(r)}{\lambda + \tilde{\lambda}} \delta^{(2)}(r - r') + \nabla \cdot \mathbf{V}(r, r'),
$$

where $\lambda' = \lambda(r', \phi')$. The vector $\mathbf{V}(r, r')$, that its components are anti-Hermitian matrices, also should satisfy the equation (36).

5 Conclusions

Propagation of an open string which its ends are on a brane, in the presence of the pp-wave and background gauge fields, generalizes the noncommutativity of the brane worldvolume.
The same generalization also occurs in the open string metric and in the propagator. This generalization has two parts. One part completely depends on the string mass. The other part is due to the string mass and the background gauge fields.

As we saw the propagator equation also can be obtained from an action with the source term. Solving the equation of motion of the string propagator, we obtained some free matrices in $G^{\alpha\beta}$, $G^{\alpha\beta}$ and $\theta^{\alpha\beta}$. The symmetries of the string propagator and matching it with the zero mass case put some restrictions on these matrices. Up to some equations, these matrices remain arbitrary. In other words, for the given background fields $g_{\alpha\beta}$, $B_{\alpha\beta}$, $A_{\alpha}$ and $\mu$, there is a family of the noncommutativities. Each element of this family is distinguished by its corresponding matrices $\{A_{n}^{\alpha\beta}, B_{n}^{\alpha\beta}, E_{n}^{\alpha\beta}\}$.

Imposing the Hermitian condition on the string propagator, we obtained a modified open string metric and noncommutativity parameter. In this case for the vanishing gauge fields there is a noncommutativity, proportional to the closed string metric.

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