Abstract

We describe the results of our recent work on the determination of the value of the parameter $\Lambda_{MS}^{(4)}$ and of the $Q^2$-dependence of the Gross–Llewellyn Smith (GLS) sum rule from the experimental data of the CCFR collaboration on neutrino–nucleon deep-inelastic scattering, using the Jacobi polynomials QCD analysis. The obtained results are compared with the information, available in the literature, information on the previous experimental measurements of the GLS sum rule.

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1On leave of absence at CERN from February 1994
2Email: SIDOROV@THEOR.JINRC.DUBNA.SU
1. The detailed experimental and theoretical studies of the deep-inelastic scattering processes provide the important information on the applicability of perturbative principles for describing the observed $Q^2$-dependence of the nucleon structure functions (SF) $F_i(x, Q^2)$ ($i = 2, 3$) and of the related moments

$$M_n(Q^2) = \int_0^1 x^{n-1}F_i(x, Q^2)dx$$  \hspace{1cm} (1)$$

within the framework of QCD. The non-singlet (NS) SF $xF_3(x, Q^2) = (xF_3^u + xF_3^d)/2$, which characterizes the difference of quark and antiquark distributions, can be measured in the deep-inelastic processes with charged electroweak currents. The most precise experimental data for this quantity were recently obtained by the CCFR group at the Fermilab Tevatron [1]. The comparison of these data with the perturbative QCD predictions for $xF_3$ was originally made with the help of the computer program developed in Ref. [2], based on the solution of the Altarelli–Parisi equation. The fits were made for various $Q^2$ cuts of the data. In particular, fitting the data at $Q^2 > 10$ GeV$^2$, the CCFR collaboration obtained the following value of the parameter $\Lambda_{\overline{MS}}^{(4)}$ [2]:

$$\Lambda_{\overline{MS}}^{(4)} = 171 \pm 32(stat) \pm 54(syst)$ MeV.  \hspace{1cm} (2)$$

This value turns out to be almost non-sensitive to the variation of the $Q^2$ cuts, imposed for allowing one to neglect the effects of the high-twist (HT) contributions at low energies.

Another important result, obtained by the CCFR collaboration, is the accurate measurement of the Gross–Llewellyn Smith sum rule

$$GLS(Q^2) = 1 \int_0^1 \frac{xF_3^{u+p+\overline{p}}(x, Q^2)}{x}dx$$  \hspace{1cm} (3)$$

at the scale $Q^2 = 3$ GeV$^2$ [3]:

$$GLS(Q^2 = 3 \text{ GeV}^2) = 2.50 \pm 0.018(stat.) \pm 0.078(syst.).  \hspace{1cm} (4)$$

It was already shown [1] that this value of the GLS sum rule is consistent with the QCD predictions, provided one takes into account not only the perturbative QCD corrections [1, 3] to the quark-parton prediction $GLS_{As} = 3$, but the non-perturbative three-point function QCD sum rules estimates of the HT contributions [28] as well. However, the interesting question of the possibility of extracting the $Q^2$-dependence of the GLS sum rule from the CCFR data remained non-studied.

In recent work [8] we analysed this problem with the method of the SF reconstruction over their Mellin moments, which is based on the following expansion of the SF over the Jacobi polynomials [31–33]:

$$xF_3^{N_{\text{max}}=12}(x, Q^2) = x^\alpha(1 - x)^\beta \sum_{n=0}^{N_{\text{max}}=12} \Theta_n^{\alpha, \beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_j^{NS}(Q^2),  \hspace{1cm} (5)$$

where $c_j^{(n)}(\alpha, \beta)$ are the coefficients that are expressed through $\Gamma$-functions and the values of the parameters $\alpha, \beta$, namely $\alpha = 0.12$ and $\beta = 2.0$, were determined in Ref. [10]. The $Q^2$ evolution
of the moments $M^N_{j+2}$ can be determined from the solution of the corresponding renormalization-group equation expressed in the form presented in Ref.\[12\]. The basic expansion parameter is of course the QCD coupling constant $\alpha_s$, which can be expressed through the QCD scale parameter $\Lambda_{\text{MS}}$ in the standard way:

$$\alpha_s(Q^2) = 1/\beta_0 \ln(Q^2/\Lambda_{\text{MS}}^2) - \beta_1 \ln \ln(Q^2/\Lambda_{\text{MS}}^2)/\beta_0 \ln^2(Q^2/\Lambda_{\text{MS}}^2)$$

where $\beta_0 = 11 - 2/3 f$, $\beta_1 = 102 - 38/3 f$. The relation of Eq. (5), supplemented by the corresponding solution of the renormalization-group equation for $M^N_{j+2}$, forms the basis of the computer program created by the authors of Ref.\[10\]. It was previously tested and used by the members of the BCDMS collaboration in the course of a detailed QCD analysis of the experimental data for $F_2(x, Q^2)$ SF of the deep-inelastic muon-nucleon scattering \[13\]. We were using in our studies also this program, thus building the bridge between the determination of $\Lambda^{(4)}_{\text{MS}}$ from $F_2(x, Q^2)$ and $xF_3(x, Q^2)$ SFs.

2. In accordance with the original NS fit of the CCFR collaboration \[1, 3\], we have chosen the parametrization of the parton distributions at fixed momentum transfer in the simplest form

$$xF_3(x, Q^2_0) = A(Q^2_0) x^{b(Q^2_0)} (1 - x)^{c(Q^2_0)}.$$  \hspace{1cm} (6)

The constants $A(Q^2_0)$, $b(Q^2_0)$ and $c(Q^2_0)$ in Eq. (6) and the QCD scale parameter $\Lambda^{(4)}_{\text{MS}}$ were considered as free parameters, which were determined for concrete values of $Q^2_0$. In order to avoid the influence of the HT effects and the TM corrections, following the original CCFR analysis we used the experimental points of the concrete CCFR data in the plane $(x, Q^2)$ with $0.015 < x < 0.65$ and $10 \text{ GeV}^2 < Q^2 < 501 \text{ GeV}^2$. Note that we restricted ourselves by taking $f = 4$ throughout the whole work. Moreover, we did not take into account any threshold effects in the process of our analysis.

Using Eq. (5) we reconstructed the theoretical expression for $xF_3^{N_{\text{max}}=12}(x, Q^2, A, b, c, \Lambda)$ in all experimental points $(x_{\text{exp}}, Q_{\text{exp}}^2)$ (for the detailed description see Ref. \[8\]). The determination of the free parameters of the fit (namely $A, b, c, \Lambda$) was made by minimization of $\chi^2$ by the MINUIT program, which also automatically calculated their statistical errors. The numerical value of the GLS sum rule at different values of the reference scale $Q^2_0$ was determined by substituting the concrete values of the parameters $A(Q^2_0)$, $b(Q^2_0)$ and $c(Q^2_0)$ into Eq. (7) and calculating its first moment, which determines the expression for the GLS sum rule. The statistical errors for the sum rule were calculated from the statistical errors of the parameters $A(Q^2_0)$, $b(Q^2_0)$ and $c(Q^2_0)$.

The results of the concrete calculations, made for various $Q^2_0$ points, are presented in Table 1. Figure 1 demonstrates our results for the GLS sum rule obtained in the process of the next-to-leading order (NLO) fit. It is worth emphasizing that putting $n=1$ in the NLO expression for the moments $M_n^{NS}(Q^2)$ one can reconstruct the LO expression for the GLS sum rule only, namely $GLS_{\text{LO}}(Q^2) = 3[1 - \alpha_s/\pi]$. Therefore, in Fig. 1 we compare our result of the NLO fit with the LO perturbative expression of the GLS sum rule.
| $|Q_0^2|$ | NLO | LO |
|-------|-----|-----|
| [GeV^2] | $\Lambda^{(4)}_{\overline{MS}}$ [MeV] | $\chi^2_{d.f.}$ | GLS sum rule | $\Lambda^{(4)}_{\overline{LO}}$ [MeV] | $\chi^2_{d.f.}$ | GLS sum rule |
| 2 | 209 ± 32 | 71.5/62 | 2.401 ± 0.126 | 154 ± 16 | 87.6/62 | 2.515 |
| 3 | 213 ± 31 | 71.5/62 | 2.446 ± 0.081 | 154 ± 29 | 87.7/62 | 2.525 |
| 5 | 215 ± 32 | 71.8/62 | 2.496 ± 0.121 | 154 ± 28 | 88.0/62 | 2.537 |
| 7 | 215 ± 34 | 72.2/62 | 2.525 ± 0.105 | 155 ± 27 | 88.3/62 | 2.549 |
| 10 | 215 ± 35 | 72.6/62 | 2.533 ± 0.107 | 154 ± 29 | 88.5/62 | 2.558 |
| 15 | 215 ± 34 | 73.2/62 | 2.583 ± 0.111 | 155 ± 28 | 88.8/62 | 2.569 |
| 25 | 214 ± 31 | 74.1/62 | 2.618 ± 0.113 | 155 ± 17 | 89.2/62 | 2.583 |
| 50 | 213 ± 33 | 75.4/62 | 2.661 ± 0.119 | 155 ± 27 | 90.2/62 | 2.603 |
| 70 | 212 ± 34 | 76.1/62 | 2.680 ± 0.120 | 155 ± 26 | 90.3/62 | 2.614 |
| 100 | 211 ± 33 | 76.8/62 | 2.699 ± 0.123 | 154 ± 29 | 90.7/62 | 2.623 |
| 150 | 210 ± 34 | 77.6/62 | 2.720 ± 0.126 | 154 ± 29 | 91.2/62 | 2.635 |
| 200 | 209 ± 33 | 78.2/62 | 2.735 ± 0.127 | 154 ± 29 | 91.5/62 | 2.643 |
| 300 | 209 ± 33 | 79.0/62 | 2.755 ± 0.129 | 153 ± 29 | 92.0/62 | 2.655 |
| 500 | 207 ± 35 | 80.1/62 | 2.779 ± 0.155 | 153 ± 29 | 92.7/62 | 2.664 |

Table 1. The results of the LO and NLO QCD fit of the CCFR $x F_3$ SF data for $f = 4$, $Q^2 > 10$ GeV^2, $N_{max} = 12$ with the corresponding statistical errors. The symbol $\chi^2_{d.f.}$ is for the $\chi^2$ parameter normalized to the number of degrees of freedom $d.f.$

![Fig.1](image-url) Fig.1. The comparison of the result of the NLO fit of the $Q^2$ evolution of the GLS sum rule with the statistical error bars taken into account with the LO perurbative QCD prediction.
The value of the parameter $\Lambda_{\text{MS}}^{(4)}$ in the LO perturbative expression for the GLS sum rule in Fig. 1 was taken in accordance with the results of our analysis of the CCFR data for the SF $xF_3$ at the reference point $Q_0^2 = 3 \text{ GeV}^2$ (see Table 1). Notice, that the points presented in Fig. 1 are strongly correlated. The explanation is very simple: they were all obtained from the whole set of data.

3. Taking into account our estimates of the statistical uncertainties and the estimate, determined by the CCFR group, of the systematic uncertainty [3], we obtained the following value of the GLS sum rule at the scale $Q_0^2 = 3 \text{ GeV}^2$ [8]:

$$GLS(Q_0^2 = 3 \text{ GeV}^2) = 2.446 \pm 0.081(\text{stat}) \pm 0.078(\text{syst}).$$ (7)

This in the agreement with the result (4) obtained by the CCFR group. The smaller statistical error of the CCFR result of Eq. (4) comes from their more refined analysis of this type of uncertainties. The results of our NLO determination of the GLS sum rule (see Table 1) do not contradict either the previous, less accurate, measurements of this sum rule at different values of $Q^2$, as presented in Table 2.

| Collaboration | Reference | Typical $Q^2$ [GeV$^2$] | Result        |
|---------------|-----------|-------------------------|---------------|
| CDHS          | [14]      | 1–180                   | $3.20 \pm 0.50$ |
| CHARM         | [15]      | 10                      | $2.56 \pm 0.41 \pm 0.10$ |
| BEBC–         | [16]      | 1–10                    | $2.89 \pm 0.33 \pm 0.23$ |
| Gargamelle    |           | 10–20                   | $3.13 \pm 0.48 \pm 0.28$ |
| CCFRR         | [17]      | 3                       | $2.83 \pm 0.15 \pm 0.10$ |
| WA25          | [18]      | 3                       | $2.70 \pm 0.40$ |
| SKAT          | [19]      | 0.5–10                  | $3.10 \pm 0.60$ |
| CCFR          | [20]      | 3                       | $2.78 \pm 0.08 \pm 0.13$ |
| CCFR          | [3]       | 3                       | $2.50 \pm 0.02 \pm 0.08$ |
| CCFR          | [8]       | 3                       | $2.45 \pm 0.08 \pm 0.08$ |

Table 2. The summary of various determinations of the GLS sum rule with the corresponding statistical and systematical uncertainties.

Note, that the results of the CHARM collaboration [15] were obtained after integrating $xF_3$ SF in the interval $0.0075 < x < 1$. We expect, that after interpolation of the data in the region of small $x$ and integrating them in the whole interval $0 < x < 1$ the corresponding CHARM results for the GLS sum rule will approach the quark-parton prediction $GLS_{\Lambda_{\text{As}}} = 3$ and will be even less accurate than the claimed result [15] presented in Table 2.

The results for $\Lambda_{\text{MS}}^{(4)}$ obtained from the NLO fit of the CCFR data, using the Jacobi polynomial expansion (see Table 1), are in exact agreement with the outcome of the fit of the
BCDMS data for the $F_2(x, Q^2)$ SF with the help of the same computer program \cite{13}, namely \( \Lambda_{MS}^{(4)} = 230 \pm 20(stat) \pm 60(syst) \) MeV.

The result, used at Fig. 1, namely \( \Lambda_{MS}^{(4)} = 213 \pm 31(stat) \) MeV, which was obtained using the expressions for the Mellin moments \( M_n^{NS} \), is somewhat larger than the result of Eq. (2) obtained by the CCFR group with the method based on the solution of the Altarelli-Parisi equation. A similar feature was previously observed in the process of the analogous fits of the $xF_3$ less precise data obtained at Protvino \cite{21}: the Altarelli-Parisi method gave \( \Lambda_{MS}^{(4)} = 170 \pm 60(stat) \pm 120(syst) \) MeV (compare with Eq. (2)), while the fit over the Mellin moments resulted in the value \( \Lambda_{MS}^{(4)} = 230 \pm 40(stat) \pm 100(syst) \) MeV, which should be compared with the results of our fit (see Table 1). This slight discrepancy between the central values of the outcomes of different fits of the same data might be due to the intrinsic features of the two different ways the perturbative QCD corrections are taken into account. One can hope that this difference will be minimized after incorporating higher-order perturbative QCD effects into both methods.

4. The $Q^2$ dependence of the GLS sum rule (see Fig. 1), extracted by us from the CCFR data, does not contradict the previous measurements of the $Q^2$ dependence of this sum rule, made by the BEBC–Gargamelle collaboration \cite{16} (see Fig. 2) and WA25 collaboration \cite{18} (see Fig. 3).

Fig.2. Data on the GLS sum rule from the combined BEBC narrow-band neon and GGM-PS freon neutrino/antineutrino experiments \cite{16}. Errors shown are statistical only.
However, the larger uncertainties of these data did not allow one to reveal the characteristic behaviour of the experimental results in the low-energy region, clearly seen in the analysis of the CCFR data \[3, 8\]. Indeed, the results of Eqs. (4) and (10) lie much lower than the theoretical predictions of the pure perturbative QCD. This feature demonstrates the importance of taking account of the HT contributions in the theoretical expression for the GLS sum rule. Their general structure is known from the results of Ref. \[22\]. The corresponding numerical calculations of these terms were made in Ref. \[28\] and more recently in Ref. \[23\], using the same three-point function QCD sum rules technique. Combining all available information about the GLS sum rule we can write the following theoretical expression:

$$GLS_{QCD}(Q^2) = 3 \left[ 1 - a - (4.583 - 0.333 f) a^2 - (41.441 - 8.02 f + 0.177 f^2) a^3 - \frac{8 \langle (O) \rangle}{27 Q^2} \right]$$

(8)

where $a = \alpha_s/\pi$ is the coupling constant in the $\overline{MS}$ scheme. The NLO and NNLO perturbative QCD corrections were calculated in Refs. \[3, 6\] respectively. Note, that the perturbative expression for the GLS sum rule has one interesting feature, namely it is related to the perturbative expression for the $e^+e^- \rightarrow$ hadrons D-function, calculated at the NLO and NNLO levels in Refs. \[24, 25\] (see also \[26\]), by the non-trivial generalization of the quark-parton connection of Ref. \[27\]. This generalization, discovered in Ref. \[28\], has the perturbative corrections starting from the NLO level. They are proportional to the two-loop QCD $\beta$-function. The theoretical consequences of the appearance of this factor are not yet clear. However, even at the current level of understanding it is possible to conclude, that the results of the work \[28\] provide the strongest argument in favour of the correctness of the results of the NLO and NNLO calculations of the GLS sum rule and the $e^+e^-$-annihilation R-ratio.
Let us return to the discussions of the effects of the HT-contributions in Eq. (12). The original calculation [28] of the matrix element $\langle\langle O \rangle\rangle$ gave the following estimate $\langle\langle O \rangle\rangle = 0.33 \pm 0.16 \text{ GeV}^2$. The QCD prediction of Eq. (12) with this value of the HT term was used in Ref. [4] for the extraction of the value of $\Lambda^{(4)}_{\text{MS}}$ from the experimental result of Ref. [3]. We will not discuss here all details of the work of Ref. [4], but present only final outcomes of the NLO analysis in the $\overline{\text{MS}}$ scheme with HT terms

$$\Lambda^{(4)}_{\overline{\text{MS}}} = 318 \pm 23(\text{stat}) \pm 99(\text{syst}) \pm 62(\text{twist}) \text{ MeV}$$

and without HT terms

$$\Lambda^{(4)}_{\overline{\text{MS}}} = 435 \pm 20(\text{stat}) \pm 87(\text{syst}) \text{ MeV}.$$  

(9)

(10)

It can be seen that the HT terms are decreasing the difference of the extracted values of the parameter $\Lambda^{(4)}_{\overline{\text{MS}}}$ from the results of other NLO fits, say from our results of Table 1. Even better agreement can be obtained after taking into account NNLO corrections in the GLS sum rule (see Eq. (12)), scheme-dependence ambiguities (see Ref. [4]) or the new estimates of the HT-contribution, namely $\langle\langle O \rangle\rangle = 0.53 \pm 0.04 \text{ GeV}^2$ [23], which however have surprisingly small error bars of over 10% (it is known that the typical uncertainties of different three-point function QCD sum predictions lie within error bars of at least 30%).

Since at the scale $Q^2 = 3 \text{ GeV}^2$ the result of our extraction of the GLS sum rule value [8] is in agreement with the original result of the CCFR group [3] (compare Eq. (4) with Eq. (10)), the conclusions of Ref. [4] remain valid in our case also. Moreover, we consider the deviation of the $Q^2$ dependence of the GLS sum rules results that we observed in the low-energy region from the prediction of perturbative QCD (see Fig. 1) as an indication of the necessity for a detailed study of the HT effects in the region of $Q^2 < 10 \text{ GeV}^2$. This conclusion joins the results of the quantitative analysis [4], [29] of the effects of the HT contributions to the GLS sum rule [28] and the Bjorken polarized sum rules [30] correspondingly (see also Ref. [23]) and support the necessity of taking into considerations of these effects in the detailed description of the $Q^2$ dependence of the deep-inelastic scattering sum rules in the low energy region.

In the high-energy region $Q^2 \geq 10 \text{ GeV}^2$ the $Q^2$ behaviour of the GLS sum rule, obtained by us and depicted in Fig. 1, is in qualitative agreement with the perturbative QCD expectations. However, at the quantitative level there are indications of the existence of the deviation between theoretical predictions and the results of our analysis. This phenomenon might be related to the necessity for detailed studies of the effects of the NNLO corrections to the NS anomalous dimensions (which are known at present only for even moments [31]) and of the NNLO coefficients of the NS moments of the $xF_3$ SF [32]. Another important task is an improvement of the understanding of the behaviour of the $xF_3$ SF in the region of small $x$. We hope that a possible future analysis will allow one to study the $Q^2$ dependence of the GLS sum rule in more detail and to understand the status of the non-standard theoretical explanation of the behaviour of the GLS sum rule observed by us at moderate $Q^2$ [33].
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