Optomechanical transistor with mechanical gain

X. Z. Zhang,1,2 Lin Tian,3 and Yong Li1,4,5
1Beijing Computational Science Research Center, Beijing, 100193, China
2College of Physics and Materials Science, Tianjin Normal University, Tianjin 300387, China
3University of California, Merced, 5200 North Lake Road, Merced, California 95343, USA
4Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China
5Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, China

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We study an optomechanical transistor, where an input field can be transferred and amplified unidirectionally in a cyclic three-mode optomechanical system. In this system, the mechanical resonator is coupled simultaneously to two cavity modes. We show that it only requires a finite mechanical gain to achieve the nonreciprocal amplification. Here the nonreciprocity is caused by the phase difference between the linearized optomechanical couplings that breaks the time-reversal symmetry of this system. The amplification arises from the mechanical gain, which provides an effective phonon bath that pumps the mechanical mode coherently. This effect is analogous to the stimulated emission of atoms, where the probe field can be amplified when its frequency is in resonance with that of the anti-Stokes transition. We show that by choosing optimal parameters, this optomechanical transistor can reach perfect unidirectionality accompanied with strong amplification. In addition, the presence of the mechanical gain can result in ultra-long delay in the phase of the probe field, which provides an alternative way to controlling light transport in optomechanical systems.

I. INTRODUCTION

The interaction between light and mechanical objects in the low-energy scale has been intensively studied both in theory and in experiment during the past two decades. Given the rapid advance in microfabrication [1–3], cavity optomechanical systems have been exploited for both fundamental questions and various applications. Such systems provide an appealing platform to study the quantum behavior of macroscopic systems [4]. Meanwhile, applications of optomechanical systems, such as ultra-sensitive measurement in the molecular scale [5–10], weak-force detection [11], quantum wavelength conversion between microwave and optical frequencies [12–13], and quantum illumination [14], have been investigated. Furthermore, optomechanical systems have also been used to demonstrate quantum optical effects, such as optomechanically induced transparency and absorption [15–24] and optomechanically induced amplification [25–26].

Among these applications, nonreciprocal transmission and amplification of light fields are of great interest, similar to their analogues in electronic devices. The nonreciprocal devices, which exhibit asymmetric response if the input and output channels are interchanged, can protect unwanted singles from entering into the network, where are essential to signal processing and communications. At the heart of the nonreciprocal devices is an element that breaks the Lorentz reciprocity of the system [44]. Effects that have been used to realize the nonreciprocity include the magneto-optical Faraday effect in ferrite materials [45–46], parametric modulation of system parameters [47–52], optical nonlinearity [53–54], chiral light-matter interaction [55], and the rotation of device in the real space [56]. It has been shown that the nonreciprocal propagation of light can be realized with optical devices [28–51]. Meanwhile, unconventional propagation of light has been demonstrated by engineering effective non-Hermitian Hamiltonians in optical systems [57–64], which can be used to realize on-chip isolators and circulators [65]. Recently, \( \mathcal{PT} \) symmetry breaking in optomechanical systems with coupled cavities, often accompanied by the coalescence of eigenstates at an exceptional point in the discrete spectrum, has been studied [66–67], and low-power phonon emissions [68], \( \mathcal{PT} \) nonreciprocal energy transfer [69], and asymmetric mode switching [70] have been observed. More recently, optomechanical isolators, circulators, and directional amplifiers have been studied in multi-mode systems by modulating the gauge-invariant phases [32–43].

Here we present a scheme for realizing an optomechanical transistor in a cyclic three-mode optomechanical system with finite mechanical gain. In this system, two optical modes are linearly coupled with each other and are also coupled simultaneously to a common mechanical mode. The phase difference between the optomechanical couplings breaks the time-reversal symmetry of this system and ensures nonreciprocity in the state transmission. Meanwhile, amplification arises from the mechanical gain, which induces a phonon-photon parametric process. Compared to our previous work [74], this approach does not require the frequency matching between the pump fields on the cavity and the mechanical modes. Furthermore, we show that within the operational parameter window of the optomechanical transistor, an ultra-long delay in the phase of the probe field occurs due to the finite mechanical gain. These findings provide an alternative way to achieving controlled light
transport in optomechanical systems and can stimulate future works in light amplification with optomechanical devices.

This paper is organized as follows. In Sec. III we introduce the three-mode optomechanical system with finite mechanical gain. The stability of this system is also discussed in this section. We then derive the transmission coefficients of this system in a generic setting in Sec. III. The behavior of the optomechanical transistor and the ultra-long delay in the phase of the pump field are studied in detail in Sec. IV. Finally, conclusions are given in Sec. V.

II. THE MODEL

Consider an optomechanical system that contains a mechanical mode with frequency $\omega_m$ and two cavity modes with frequencies $\omega_1$ and $\omega_2$, respectively, as illustrated in Fig. I. The Hamiltonian of this system has the form ($\hbar = 1$)

$$H = H_0 + H_I + H_d,$$

$$H_0 = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \omega_m b^\dagger b,$$

$$H_I = J(a_1^\dagger a_2 + a_2^\dagger a_1) + \sum_i g_i a_i^\dagger a_i (b + b^\dagger),$$

$$H_d = \sum_i i \varepsilon_i (a_i^\dagger e^{-i \omega_d t} e^{i \theta_i} - \text{H.c.}).$$

Here $H_0$ is the Hamiltonian of the uncoupled cavity and mechanical modes, where $a_i^\dagger$ ($a_i$) for $i = 1, 2$ and $b^\dagger$ ($b$) are the corresponding creation (annihilation) operators of these modes. The Hamiltonian $H_I$ describes the linear interaction between the cavities with coupling strength $J$ and the radiation-pressure interactions between the cavity and the mechanical modes with coupling strength $g_1$ and $g_2$. The Hamiltonian $H_d$ represents the pump fields applied to the cavities with frequency $\omega_d$, amplitudes $\varepsilon_{1,2}$ and phases $\theta_{1,2}$. Without loss of generality, we assume that the parameters $J, g_{1,2},$ and $\varepsilon_{1,2}$ are real numbers.

In the rotating frame of $\omega_d$, the Hamiltonian becomes

$$H_{\text{rot}} = \sum_i \Delta_i a_i^\dagger a_i + \omega_m b^\dagger b + J(a_1^\dagger a_2 + a_2^\dagger a_1) + \sum_i g_i a_i^\dagger a_i (b + b^\dagger) + i \varepsilon_i (a_i^\dagger e^{i \theta_i} - \text{H.c.}),$$

where $\Delta_i = \omega_i - \omega_d$ ($i = 1, 2$) is the detuning of the pump field from the cavity resonance.

We assume the cavity and the mechanical modes are subject to input noise denoted by $f_i^m$ ($i = 1, 2$) for the cavity input operators and $f_b^m$ for the mechanical input with $\langle f_i^m \rangle = \langle f_b^m \rangle = 0$. With Hamiltonian (5), the Quantum Langevin equations (QLEs) for the above optomechanical system are

$$\dot{a}_1 = \{-\gamma_1 - i [\Delta_1 + g_1 (b + b^\dagger)]\} a_1 - i J a_2 + \varepsilon_1 e^{i \theta_1} + \sqrt{2 \gamma_1} f_1^m,$$

$$\dot{a}_2 = \{-\gamma_2 - i [\Delta_2 + g_2 (b + b^\dagger)]\} a_2 - i J a_1 + \varepsilon_2 e^{i \theta_2} + \sqrt{2 \gamma_2} f_2^m,$$

$$\dot{b} = (G_m - i \omega_m) b - i (g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2) + \sqrt{2 G_m} f_b^m,$$

where $\gamma_i$ ($i = 1, 2$) is the decay rate of the corresponding cavity mode and $G_m$ denotes the controllable gain of the mechanical mode. In practical systems, the mechanical gain can be obtained with various methods, e.g., through phonon lasing or by coupling the mechanical mode to another cavity mode and applying blue-detuned driving to the cavity.

With strong pumping, the steady-state solutions of the cavity modes $\langle a_i \rangle$ and of the mechanical mode $\langle b \rangle$ can be obtained as

$$\langle a_1 \rangle = \frac{(\gamma_2 + i \Delta_2) \varepsilon_1 e^{i \theta_1} - i J \varepsilon_2 e^{i \theta_2}}{(\gamma_1 + i \Delta_1)(\gamma_2 + i \Delta_2) + J^2},$$

$$\langle a_2 \rangle = \frac{(\gamma_1 + i \Delta_1) \varepsilon_2 e^{i \theta_2} - i J \varepsilon_1 e^{i \theta_1}}{(\gamma_1 + i \Delta_1)(\gamma_2 + i \Delta_2) + J^2},$$

$$\langle b \rangle = \frac{-i (g_1 |\langle a_1 \rangle|^2 + g_2 |\langle a_2 \rangle|^2)}{G_m + i \omega_m}.$$
With these assumptions, we can apply the rotating-wave approximation to the above QLEs and neglect the fast-oscillating counter-rotating terms. The QLEs become

\begin{align*}
\dot{\delta a}_1 &= (-\gamma_1 - i\Delta'_1)\delta a_1 - iG_1(\delta b + \delta b^\dagger) - iJ\delta a_2 + \sqrt{2\gamma_1}f_{1n}^\dagger, \\
\dot{\delta a}_2 &= (-\gamma_2 - i\Delta'_2)\delta a_2 - iG_2(\delta b + \delta b^\dagger) - iJ\delta a_1 + \sqrt{2\gamma_2}f_{2n}^\dagger, \\
\dot{\delta b} &= (G_m - i\omega_m)\delta b - i(G_1\delta a_1^\dagger + G_1^*\delta a_1) - i(G_2\delta a_2^\dagger + G_2^*\delta a_2) + \sqrt{2G_m}f_{bn}^\dagger,
\end{align*}

where \( G_i = g_i(a_i) \) \((i = 1, 2)\) is the effective linear coupling between the \( i \)th cavity and the mechanical mode. We assume that the system is operated in the resolved sideband regime with \( \gamma_i, G_i, G_m \ll \omega_m \) and \( \Delta'_i \sim \omega_m \). With these assumptions, we can apply the rotating-wave approximation to the above QLEs and neglect the fast-oscillating counter-rotating terms. The QLEs become

\begin{align*}
\dot{\delta a}_1 &= -\Gamma_0\delta a_1 - iG_1\delta b - iJ\delta a_2 + \sqrt{2\gamma_1}f_{1n}^\dagger, \\
\dot{\delta a}_2 &= -\Gamma_0\delta a_2 - iG_2\delta b - iJ\delta a_1 + \sqrt{2\gamma_2}f_{2n}^\dagger, \\
\dot{\delta b} &= -\Gamma_0\delta b - iG_1^*\delta a_1 - iG_2^*\delta a_2 + \sqrt{2G_m}f_{bn}^\dagger,
\end{align*}

where \( \Gamma_0 = \gamma_i + i\Delta'_i \) and \( \Gamma_{m0} = -G_m + i\omega_m \). For simplicity, we rewrite the linearized QLEs in matrix form with

\[
\frac{d}{dt}\lambda = -M\lambda + \Upsilon\lambda^{in},
\]

where the fluctuation vector \( \lambda = (\delta a_1, \delta a_2, \delta b)^T \), the input field \( \lambda^{in} = (f_{1n}^\dagger, f_{2n}^\dagger, f_{bn}^\dagger)^T \), the coupling matrix for the input operators \( \Upsilon = \text{diag}(\sqrt{2\gamma_1}, \sqrt{2\gamma_2}, \sqrt{2G_m}) \), and the dynamic matrix

\[
M = \begin{pmatrix}
\Gamma_0 & iJ & iG_1 \\
iJ & \Gamma_{20} & iG_2 \\
iG_1^* & iG_2^* & \Gamma_{m0}
\end{pmatrix}.
\]

The stability of this optomechanical system can be influenced by the mechanical gain. The stability condition for this system can be derived using the Routh-Hurwitz criterion, which is equivalent to the requirement that the eigenvalues of matrix \( M \) have no positive real part. In Fig. 2 we plot two typical cases that are employed to investigate the optical response of this system in the following sections, where the gray regions are stable and the white regions are unstable. When \( J = \gamma_1 = \gamma_2 = 10G_m \), as shown in Fig. 2(b), the system is stable with all possible values of \( \theta \) except for \( \theta = \pi/2 \). However, when the system parameters are \( J = 11G_m, \gamma_1 = 10G_m \), and \( \gamma_1 = 15G_m \), the stable region covers all values of \( \theta \), which can be seen in Fig. 2(a).

**III. TRANSMISSION COEFFICIENTS**

Apply a probe field to cavity 1 in the form of \( i(\varepsilon_pe^{i\omega_kt} - H.c.) \), as illustrated by the thin solid arrow in Fig. 1. The response to a probe field applied to cavity 2 (the thin dashed arrow in Fig. 1) can be obtained by exchanging the subscripts 1 and 2 in the following results. We assume that the amplitude of the probe field \( \varepsilon_p \) is much smaller than that of the control field \( \varepsilon_{1,2} \), and the steady-state solutions of the operators \( a_1, a_2, b \) will not be affected by the probe field.

**FIG. 2:** (Color online) Numerical calculation of the stability of this system with the parameters (a) \( \gamma_1 = 10G_m, \gamma_2 = 15G_m \) and (b) \( \gamma_1 = 10G_m, \gamma_2 = 10G_m \). Other parameters are \( G_1 = [G_2] \equiv G = \sqrt{G_m/\sin \theta} \), and \( \omega_m/G_m = 10^3 \). Each panel contains two regions. The gray (white) regions represent the stable (unstable) regions of this system. In particular, in (b), when \( J = 10G_m \), the system is stable with all values \( \theta \) except for \( \theta = \pi/2 \).
Hence the only change in the QLEs is that one extra term $\varepsilon_p e^{-i(\omega_p - \omega_d)t}$ is added to Eq. (15). To solve this set of linear QLEs, we use another interaction picture by transforming $\delta a_i \rightarrow \delta a_i e^{-i(\omega_p - \omega_d)t}$, $f_{b}^{in} \rightarrow f_{b}^{in} e^{-i(\omega_p - \omega_d)t}$ ($i = 1, 2$), and $f_{b}^{in} \rightarrow f_{b}^{in} e^{-i(\omega_p - \omega_d)t}$. The corresponding QLEs become

$$
\delta a_1 = -G_1 \delta a_1 - iG_1 \delta b - iJ \delta a_2 + \varepsilon_p + \sqrt{2} \gamma_1 f_{b}^{in}, \tag{20}
$$

$$
\delta a_2 = -G_2 \delta a_2 - iG_2 \delta b - iJ \delta a_1 + \sqrt{2} \gamma_1 f_{b}^{in}, \tag{21}
$$

$$
\delta b = -G_m \delta b - iG_1^* \delta a_1 - iG_2^* \delta a_2 + \sqrt{2} \gamma_1 f_{b}^{in}, \tag{22}
$$

where $\Gamma_i = \gamma_i + i\Delta_i'$ and $\Gamma_m = -\Gamma_m + i\Delta_m$ with $\Delta_i' = \Delta_i - \langle \omega_p - \omega_d \rangle$ and $\Delta_m = \omega_m - \langle \omega_p - \omega_d \rangle$ being the detunings in the new frame.

The optical response of this system to the probe field can be obtained by solving the steady state of Eqs. (20–22). By setting $\delta a_i = \delta b = 0$ and neglecting the noise terms, we obtain

$$
\langle \delta a_1 \rangle = \varepsilon_p (\Gamma_1 \Gamma_m + |G_2|^2)/D, \tag{23}
$$

$$
\langle \delta a_2 \rangle = -\varepsilon_p (G_1^* G_2 + iJ \Gamma_m)/D, \tag{24}
$$

$$
\langle \delta b \rangle = -\varepsilon_p (iG_1^* \Gamma_2 + J G_2^2)/D \tag{25}
$$

with the denominator

$$
D = J^2 \Gamma_m + \Gamma_m \Gamma_1 \Gamma_2 + \left[ \Gamma_1 |G_2|^2 + \Gamma_2 |G_1|^2 \right].
$$

The amplitudes $\langle \delta a_i^{out} \rangle$ of the experimentally accessible cavity output fields are related to the cavity field $\langle \delta a_i \rangle$ by the input-output relation

$$
\langle \delta a_i^{out} \rangle + \langle \delta a_i^{in} \rangle = \sqrt{2} \gamma_i \langle \delta a_i \rangle, (i = 1, 2) \tag{27}
$$

where $\langle \delta a_i^{in} \rangle = \varepsilon_p / \sqrt{2} \gamma_i$, $\langle \delta a_i^{out} \rangle = 0$, and $\gamma_i$ is the external damping rate that describes the coupling between the cavity mode and the input field. We can write $\gamma_i = \eta \gamma_i$ with $\eta$ being the ratio between the external damping rate and the total damping rate. For the coupling parameter $\eta \ll 1$, the cavity is under-coupled; and when $\eta \approx 1$, the cavity is over-coupled. The ratio $\eta$ can be continuously adjusted in experiments. In this work, we consider the cases of over-coupled cavities with $\eta = 1$ and neglect the cavity intrinsic dissipation.

Using Eqs. (23)(24)(27), the transmission coefficients $t_{12} = \partial \langle \delta a_2^{out} \rangle / \partial \langle \delta a_1^{in} \rangle$ can be derived as

$$
t_{12} = -2 \sqrt{\gamma_1 \gamma_2} \left[ G_1^* G_2 + iJ \Gamma_m \right]/D. \tag{28}
$$

By interchanging indices 1 and 2 in (28), we find that

$$
t_{12} = -2 \sqrt{\gamma_1 \gamma_2} \left[ G_2^* G_1 + iJ \Gamma_m \right]/D. \tag{29}
$$

From (28)(29), we find that by manipulating the phase difference between the optomechanical couplings $G_1$ and $G_2$, nonreciprocal propagation of the probe field can be achieved, i.e., $|t_{12}/t_{21}|$ can be adjusted by varying the phase difference. This effect can be understood through the effective Hamiltonian associated with (20–22).

$$
H_{eff} = \sum_i \Delta_i'^2 \delta a_i \delta a_i + \Delta_m \delta b \delta b
$$

$$
+ \sum_i G_i \delta a_i \delta b + J \delta a_i \delta a_2 + H.c., \tag{30}
$$

which describes a typical three-mode cyclic system. The propagation of light fields in such a system depends strongly on the interference between different paths in the loop. A non-zero phase difference between the couplings $G_1$ and $G_2$ can break the time-reversal symmetry of this system and gives rise to nonreciprocal optical response. Compared to our previous work [74], the advantage of this scheme is that it does not require the matching of the pump frequencies between the optical and mechanical fields to achieve nonreciprocal propagation of the probe field. The mechanical gain can be viewed as a coherent bath that converts the beam-splitter operation between the mechanical mode and the cavities into phonon-photon parametric processes. We will discuss these points in detail in the following section.

IV. NONRECIPROCAL AMPLIFICATION AND OPTICAL DELAY

In this section, we will investigate the properties of the transmission coefficients under a special setup, i.e., when the system acts as an optomechanical transistor. We will show the feasibility of achieving signal amplification and nonreciprocity and study the delayed output response in this three-mode optomechanical system.

A. Optomechanical transistor

We first analyze the behavior of the transmission matrix elements $t_{12}$ and $t_{21}$, as given by (28) and (29). For simplicity, we assume $G_1 \equiv G$ with $G > 0$, $G_2 \equiv Ge^{-i\theta}$ with a phase difference $\theta$ from $G_1$, and $\Delta'' = \Delta_m = \Delta$ with $\Delta_i' = \omega_m$ for the pump fields. The transmission coefficients under these conditions can be written as

$$
t_{12} = 2 \sqrt{\gamma_1 \gamma_2} \left[ -iJ (Gm + i\Delta) - G^2 e^{i\theta} \right]/D, \tag{31}
$$

$$
t_{21} = 2 \sqrt{\gamma_1 \gamma_2} \left[ -iJ (Gm + i\Delta) - G^2 e^{-i\theta} \right]/D, \tag{32}
$$

with the denominator

$$
D_r = (\gamma_1 + i\Delta) (\gamma_2 + i\Delta) (\gamma_m + i\Delta)
+ G^2 (\gamma_1 + \gamma_2 + 2i\Delta) + J^2 (\gamma_m + i\Delta)
- 2iJ G^2 \cos \theta. \tag{33}
$$

Using (31) for the coefficient $t_{12}$, we choose the phase difference $\theta$ to satisfy the condition $G = \sqrt{G_m}/\sin \theta$.
As shown in Fig. 3(b), the system becomes unstable for \( G_m/\gamma_1 > 1.325 \).

In Fig. 4, we plot the logarithm of the transmission probability \( \lg(T_{21}) \) at the optimal conditions for unidirectional propagation, i.e., with \( G^2 = J G_m/\sin \theta \) and \( \Delta = G_m \cos \theta/\sin \theta \). It is shown that in the neighborhood of \( \theta = \pi/2 \), the transmission probability reaches its maximum with \( \max(T_{21}) \approx 10^5 \). This strong amplification, together with the nonreciprocity, clearly shows that our system can be used as an optomechanical transistor facilitated by the mechanical gain.

\[ \tau_{ij} = \frac{d\delta_{ij}}{d\omega_p}, \quad (34) \]

where \( \delta_{ij} = \arctan(t_{ij}(\omega_p)) \) is the phase of the output field at the frequency \( \omega_p \). We consider the system operated in the regime of an optomechanical transistor with \( |t_{21}| \gg 1 \) and \( t_{12} = 0 \). To ensure unidirectional amplification and similar to the previous subsection, we let the parameters satisfy the relations: \( J = 10G_m, G_1 = G (G > 0), G_2 \equiv G e^{-i\theta}, \Delta' = \Delta, \gamma_1 = 10G_m, \gamma_2 = 15G_m \), and \( G^2 = J G_m/\sin \theta \). In Fig. 5 we plot the phase \( \delta_{21} \) and the group delay \( \tau_{21} \) as functions of the detuning \( \Delta \). It can be shown that strong group delay occurs in the working window of the optomechanical transistor. As \( \theta \) approaches the value of \( \pi/2 \), the group delay exhibits sharp increase. This indicates that the strengthening of the amplification process gives rise to dramatic increase in the group delay. Note that near \( \theta = \pi/2 \), as shown in Fig. 3(a), the system is close to the boundary between the stable and the unstable regions, and is more fragile to the mechanical and the cavity modes to effective parametric processes between these modes. The parametric processes greatly enhance the photoelastic scattering \([67]\). This effect is in analogues to the stimulated emission process in atomic systems when the frequency of the probe field is resonant with that of the anti-Stokes field, where amplification of the incident photon field can be achieved. This system can work as an optomechanical transistor at strong mechanical gain with \( G_m \sim \gamma_1 \) by choosing appropriate parameters. As shown in Fig. 3(b), strong unidirectional amplification can be achieved at \( G_m = 1.3\gamma_1 \). Meanwhile, the increase of the mechanical gain can induce instability to this system. With the parameters in Fig. 3(b), the system becomes unstable for \( G_m/\gamma_1 > 1.325 \).

In Fig. 3 we plot the logarithm of the transmission probability \( \lg(T_{21}) \) and \( \lg(T_{22}) \) versus the detuning \( \Delta \). Other parameters are \( \gamma_1 = 10G_m, \gamma_2 = 15G_m, J = 11G_m, \theta = \pi/2, \) and \( G_1 = |G_2| \equiv G = \sqrt{G_m} \). In the vicinity of \( \Delta = 0 \), the transmission exhibits unidirectional amplification in agreement with the analytical result. (b) The logarithms of the transmission probabilities \( \lg(T_{12}) \) and \( \lg(T_{21}) \) versus the mechanical gain \( G_m \). Other parameters are \( \gamma_2 = 1.3\gamma_1, J = 1.3G_1, \theta = \pi/2, G_1 = |G_2| \equiv G = J, \) and \( \Delta = 0 \). Here when \( G_m/\gamma_1 > 1.325 \), the system becomes unstable.

\[ t_{ij} = |t_{ij}|^2 \]

The result gives a clear feature of unidirectional amplification of the probe field in the vicinity of \( \Delta = 0 \), which agrees with our theoretical prediction. The physics origin of the amplification arises from the mechanical gain, which can be viewed as a coherent phonon bath that converts the beam-splitter operation between

\[ \tau_{ij} = \frac{d\delta_{ij}}{d\omega_p}, \quad (34) \]

where \( \delta_{ij} = \arctan(t_{ij}(\omega_p)) \) is the phase of the output field at the frequency \( \omega_p \). We consider the system operated in the regime of an optomechanical transistor with \( |t_{21}| \gg 1 \) and \( t_{12} = 0 \). To ensure unidirectional amplification and similar to the previous subsection, we let the parameters satisfy the relations: \( J = 10G_m, G_1 \equiv G (G > 0), G_2 \equiv G e^{-i\theta}, \Delta' = \Delta, \gamma_1 = 10G_m, \gamma_2 = 15G_m \), and \( G^2 = J G_m/\sin \theta \). In Fig. 5 we plot the phase \( \delta_{21} \) and the group delay \( \tau_{21} \) as functions of the detuning \( \Delta \). It can be shown that strong group delay occurs in the working window of the optomechanical transistor. As \( \theta \) approaches the value of \( \pi/2 \), the group delay exhibits sharp increase. This indicates that the strengthening of the amplification process gives rise to dramatic increase in the group delay. Note that near \( \theta = \pi/2 \), as shown in Fig. 3(a), the system is close to the boundary between the stable and the unstable regions, and is more fragile to

\[ \tau_{ij} = \frac{d\delta_{ij}}{d\omega_p}, \quad (34) \]
FIG. 4: (Color online) Contour plot of the logarithm of the transmission probability \( \lg(T_{21}) \) for (a) \( \gamma_1 = 10G_m, \gamma_2 = 15G_m \) and (b) \( \gamma_1 = 10G_m, \gamma_2 = 10G_m \). The optimal unidirectional conditions are used with \( G_1 = |G_2| \equiv G = \sqrt{JG_m}/\sin \theta \) and \( \Delta = G_m \cos \theta/\sin \theta \). It is shown that when \( \theta \to \pi/2 \), the transmission probability reaches its maximum.

environmental disturbance. Therefore, there is a trade-off between the amplification and group delay and the stability of this system. By selecting appropriate parameters, one can realize an optomechanical transistor with significant time delay.

V. CONCLUSIONS

To conclude, we have shown that an optomechanical transistor can be realized in a cyclic optomechanical system with finite mechanical gain. The nonreciprocal behavior of this system arises from the phase difference between the optomechanical couplings \( G_1 \) and \( G_2 \), which breaks the time-reversal symmetry of this system. The mechanical gain does not affect the nonreciprocity of the optical response, but it plays a key role in achieving the amplification for the probe field. The presence of the mechanical gain can generate strong parametric processes between the mechanical mode and the cavities and significantly enhance the photoelastic scattering when the probe field is at an optimal frequency. Combining the phase difference between the couplings with the mechanical gain thus enables the unidirectional amplification of the probe field. Furthermore, the amplification of the probe field is accompanied by an ultra-long group delay in the output field. Our work hence provides an effective approach to control the light propagation in an optomechanical system and could stimulate future studies of nonreciprocal optomechanical interfaces in nonlinear photonic devices.

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FIG. 5: (Color online) The group delay $\tau_{21}$ vs the detuning $\Delta$ for (a) $\theta = 0.3\pi$, (b) $\theta = 0.4\pi$, (c) $\theta = 0.45\pi$, and (d) $\theta = 0.47\pi$. The group delay $\tau_{21}$ is given in units of $1/\mathcal{G}_m$. Other parameters are $J = 10\mathcal{G}_m$, $\gamma_1 = 10\mathcal{G}_m$, $\gamma_2 = 15\mathcal{G}_m$, and $G = \sqrt{J\mathcal{G}_m}/\sin\theta$.
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