Gradient expansion formalism for nonlinear superhorizon perturbations

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We develop a theory of nonlinear cosmological perturbations on superhorizon scales where a characteristic length scale of perturbations is longer than the Hubble radius, in general theoretical frameworks. Our formalism is based on the spatial gradient expansion approach by adopting the ADM decomposition. Nonlinear superhorizon perturbation including both scalar (curvature perturbation) and tensor (gravitational waves) modes can be dealt with valid up to a second-order in the expansion. First we will review the formalism for a standard general relativity (GR) gravity plus a general kinetic single scalar (k-inflation) with a general form of the potential in the context of inflationary cosmology. That is the basic overview of our procedure. Then it can be extended to more general framework, that is (1) beyond k-inflation (Galileon inflation), (2) a multi-component scalar field with a general kinetic term and a general form of the potential and also (3) beyond Einstein gravity (general scalar-tensor theory), which can lead to several kinds of modified gravity. These theories are motivated not only inflation, but also the topic of dark energy. We provide a formalism to obtain the solution and construct nonlinear curvature perturbation in such general theoretical situation and it can be applied to the calculation of the superhorizon evolution of a primordial non-Gaussianity beyond the so-called $\delta N$ formalism, showing fully nonlinear interaction of both scalar and tensor modes.

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I. INTRODUCTION

Cosmological nonlinear perturbation on superhorizon scale plays a key role to investigate evolution of primordial perturbation including scalar (curvature) perturbation and tensor (gravitational waves) perturbation. Especially, non-Gaussianity of curvature perturbations is one of the most powerful tool to distinguish different models of inflation (see Ref. [1] and references therein). And also, in the superhorizon region where the characteristic length scale is longer than the Hubble radius, one may consider a classification of quantum fluctuations stretched over from subhorizon scale, however, this fundamental process is unknown. In order to make clear this physics, it is important to investigate evolution of superhorizon perturbation, especially, the classical evolution equation followed by such superhorizon perturbation in the view point of link to a quantum equation. We develop nonlinear cosmological perturbation by adopting a gradient spatial expansion [2], different from the standard second-order perturbation theory [2]. On superhorizon scale, such approach is a powerful tool to allow us to calculate full nonlinear effect in terms of standard perturbation language. The zeroth-order in gradient expansion is equivalent to the formalism called $\delta N$ formalism [4–7], so our next-leading order in expansion can be called beyond $\delta N$ formalism [8–13]. It can contain higher order contributions, related to a violation of slow-roll condition.

The recent observational data of PLANCK satellite in 2015 [16] gave us detailed observational data of Cosmic Microwave Background and the fact that non-Gaussianity of primordial curvature perturbation is very small at the local type, which is predicted by using $\delta N$ formalism. Indeed $\delta N$ formalism just leads to constant contribution which only related to the local type of non-Gaussianity, but our beyond $\delta N$ obtain a time evolution and general kind of non-Gaussianity. We hope that the future precision detection of non-Gaussianity may actually be expected a frequency dependence. Moreover, primordial gravitational waves can be expected to be detected in the near future and general prediction of tensor perturbation in nonlinear cosmological perturbation in a general theoretical setup, such as a modified gravity needs. Thus, to evaluate such superhorizon perturbations, it is necessary to develop a nonlinear theory of cosmological perturbations valid up through the next-leading order in the gradient expansion.

In this paper, we will overview our basic procedure in Sec. II as a proto type of formalism for a single general kinetic inflation (k-inflation) [10]. Then we discuss the extension of our formalism for beyond k-inflation, that is Generalized Galileon (G-inflation) [13] in Sec. III. In Sec. IV and Sec. V, we develop extensions of our formalism for multi-scalar case [12] and beyond GR gravity plus a single scalar, namely the most general scalar-tensor theory (beyond Horndeski theory), respectively. Section VI is devoted to the conclusion.

II. SINGLE-FIELD CASE

In this section, the model of non-canonical single scalar field is a good example as a basic review of our formalism following [3, 10]. Then we consider GR gravity plus a general kinetic single scalar field described by the La-
the nonlinear curvature perturbation:

\[ R_{\text{NL}} \]

and the order in expansion can be expressed as

\[ \frac{1}{12} \]

spatial metric \( \tau \) adopts units such that \( 8\pi G = 1 \).

We introduce a small expansion parameter: \( \epsilon \equiv H L / \text{Hubble scale} \), which is the ratio of the Hubble length scale to the characteristic length scale of perturbations \( L \) and the order in expansion can be expressed as \( O(\epsilon^2) \).

First of all, we show the main result in our formula for the nonlinear curvature perturbation: \( R_{\text{cNL}} \),

\[
\partial_t^2 R_{\text{cNL}} + 2 \frac{\partial_z z}{z} \partial_z R_{\text{cNL}} + \frac{c_z^2}{4} K(2) [R_{\text{cNL}}] = O(\epsilon^4),
\]

which shows two full-nonlinear effects:

1. Nonlinear variable: \( R_{\text{cNL}} \) including full-nonlinear curvature perturbation, \( \delta N \)
2. Source term: \( K(2) [R_{\text{cNL}}] \) is a nonlinear function of curvature perturbations.

In (2.2), \( \tau \) denotes a conformal time and \( z \) is a well-known Mukhanov-Sasaki variable:

\[
z = \frac{a}{H} \sqrt{\rho + P \frac{1}{c_z^2}},
\]

where \( \rho \) and \( P \) denote energy density and pressure of a scalar field, respectively, with a speed of sound for perturbation: \( c_z^2 \) whose explicit definition will be shown later in (2.4). The definition of \( R_{\text{cNL}} \) will be also seen later, in (2.4.1) and the source term \( K(2) [\gamma] \) is the Ricci scalar of the metric \( \gamma \), respectively, whose explicit form will be shown in (2.2). Of course, in the linear limit, it can be reduced to the well-known equation for the curvature perturbation on comoving hypersurfaces; \( \partial_t^2 R_{\text{cLin}} + 2 \frac{\partial_z z}{z} \partial_z R_{\text{cLin}} - c_z^2 K(2) R_{\text{cLin}} = 0 \), with the Laplacian \( \Delta = \nabla^2 \).

We will briefly summarize our formula and show the above results in the following. We adopt the Arnowitt-Deser-Misner (ADM) decomposition and employ the gradient expansion. In the ADM decomposition, the metric is expressed as

\[
ds^2 = -a^2 dt^2 + g_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt),
\]

where \( a \) is the lapse function, \( \beta^i \) is the shift vector and Latin indices run over 1, 2, 3. We introduce the extrinsic curvature \( K_{ij} \) defined by

\[
K_{ij} = \frac{1}{2a} (\partial_t g_{ij} - \nabla_i \beta_j - \nabla_j \beta_i),
\]

where \( \nabla \) is the covariant derivative compatible with the spatial metric \( g_{ij} \). As a result, the basic equations are reduced to the first-order equations for the dynamical variables \((g_{ij}, K_{ij})\), with the two constraint equations. We further decompose them as

\[
\begin{align*}
g_{ij} &= a^2(t)e^{2\gamma} \gamma_{ij}, \\
K_{ij} &= a^2(t)e^{2\gamma} \left( \frac{1}{3} K \gamma_{ij} + A_{ij} \right),
\end{align*}
\]

where \( a(t) \) is the scale factor of the Universe for the background spacetime. \( \gamma_{ij} \) is an unit-determinant metric \( \det[\gamma] = 1 \) and \( A_{ij} \) is the traceless part of the extrinsic curvature. And also, \( K \) is defined by \( K \equiv \gamma^{ij} K_{ij} \). We choose a spatial gauge choice as

\[
\beta^i = 0.
\]

That simplifies the basic equations because it means naively ignoring any vector modes. Of course, one can take into account the condition \( \beta^i \neq 0 \). In that case, one can obtain vector modes as referring [9]. Hereafter we will take this simple spatial gauge choice throughout this paper. In this gauge choice, we obtain evolution equations for curvature perturbation \( \gamma \) and tensor perturbation \( \gamma_{ij} \) as

\[
\begin{align*}
\partial_\tau \gamma &= -\frac{H}{a} \frac{K}{3}, \\
\partial_\tau \gamma_{ij} &= 2 A_{ij},
\end{align*}
\]

where \( \partial_\tau \equiv \partial_t / a \) and \( H \) is the Hubble parameter defined by \( H(t) \equiv \dot{a}(t)/a(t) \) for the background Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime. Hereafter a dot denotes represents differentiation with respect to \( t \). They were derived from the definitions of \( K \) and \( K_{ij} \) given above. And also, two dynamical equations for \((K, A_{ij})\) can be obtained by varying the Lagrangian with respect to \( \gamma_{ij} \), that corresponds to trace part and traceless part as

\[
\begin{align*}
\partial_\tau K &= -\frac{1}{3} K^2 - A_{ij} A^{ij} + \frac{1}{a^2 e^{2\gamma} a} \left( D^2 a + D a D a \right) \\
&= -\frac{1}{2} (S + E), \\
\partial_\tau A_{ij} &= -KA_{ij} + 2 A_{ij} A_{ij} \\
&= -\frac{1}{a^2 e^{2\gamma}} \left( R_{ij} + D_i \psi \partial_j \gamma - D_i \partial_j \gamma \right) \\
&= -\frac{1}{a} \left( D_i D_j a - D_i \partial_a D_j a - \partial_a D_j a \right) + S_{ij},
\end{align*}
\]

where \( D \) is the covariant derivative compatible with \( \gamma_{ij} \), \( D^2 \equiv \gamma^{ij} D_i D_j \), \( R_{ij} \equiv R_{ij} [\gamma] \), that is the Ricci tensor of the spatial metric \( \gamma_{ij} \), and \([\cdot]_{TF} \) means the trace-free operator, which is defined by \( Q_{ij}^{TF} \equiv Q_{ij} - \gamma_{ij} \gamma^{kl} Q_{kl} / 3 \). \( \gamma^{ij} \) is the inverse matrix of \( \gamma_{ij} \) and the index of \( A_{ij} \) is raised by \( \gamma^{ij} \). And also, the matter field part can be given by the energy-momentum tensor \( T_{\mu\nu} \) as \( E \equiv T_{00} / a^2 \) and \( T_{ij} = a^2(t)e^{2\gamma} (S_{ij}/3 + S_{ij}) \) with \( S \equiv \gamma^{ij} T_{ij} \).
Varying $\alpha$ and $\beta^i$ gives two constraints called Hamiltonian and Momentum constraint equations, respectively, which are

$$\frac{1}{a^2 e^{2\zeta}} \left[ R - (4D^2 \zeta + 2D^i \zeta D_i \zeta) \right] + \frac{2}{3} R^2 - A_{ij} A^{ij} = 2E, \quad (2.12)$$

$$\frac{2}{3} \partial_t K - e^{-3\zeta} D_j \left( e^{3\zeta} A^j \right) = J_i, \quad (2.13)$$

where $R \equiv R[\gamma]$ is the Ricci scalar of the normalized spatial metric $\gamma_{ij}$ and $J_i = -P_X \partial_i \phi$. The equation of motion for $\phi$ is given by

$$\frac{2}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} P_X \partial^\mu \phi \right) + P\phi = 0, \quad (2.14)$$

where the subscripts $X$ and $\phi$ represent derivative with respect to $X$ and $\phi$, respectively.

### A. Gradient expansion approach

Next, we will employ the gradient expansion. In this approach we introduce a flat FLRW universe ($a(t), \phi(t)$) as a background. As discussed, we consider the perturbations on superhorizon scales, therefore we consider $\epsilon = 1/(H\lambda) = k/(aH)$ as a small expansion parameter and systematically expand equations by $\epsilon$. Spatial derivative acting on a perturbation raises the order by $\epsilon$ as $\partial_\epsilon Q = O(\epsilon^2)Q$. We attach the superscript ($m$) to a quantity of $O(\epsilon^m)$ throughout this paper.

It is natural to assume the condition for the gradient expansion: $\partial_\epsilon \gamma_{ij} = O(\epsilon)$ since the FLRW universe is recovered as background for $\epsilon \to 0$. Moreover, we assume the stronger condition:

$$\partial_\epsilon \gamma_{ij} = O(\epsilon^2). \quad (2.15)$$

This corresponds to assuming the absence of any decaying modes at the leading-order in the expansion. This is justified in taking inflationary acceleration of the universe. In fact, under the condition: $\partial_\epsilon \gamma_{ij} = O(\epsilon)$, we obtain the general solution $\propto a^{-3}$ in [13] and also in [17], so inflation can wash away such decaying mode soon. In order to solve the above basic equations, one has to fix the gauge condition. For the case of single scalar, the most convenient choice of the temporal coordinate is such that the expansion $K$ is uniform and takes the form:

$$K(t, x^i) = 3H(t), \quad (2.16)$$

called uniform expansion gauge. Adopting this gauge choice, the basic equation reduces simply to

$$\partial_\epsilon \zeta = H(\alpha - 1) = H\delta \alpha(t, x^i). \quad (2.17)$$

It means that the time evolution of the curvature perturbation caused by the inhomogeneous part of the lapse function $\delta \alpha$ only, relating to the non-adiabatic perturbation on superhorizon scale. Hereafter $\delta$ denotes a fluctuation for any quantity as $\delta Q \equiv Q - Q_b$, where the subscript $0$ denotes the background. With the above choice of gauge, the general solution valid up to $O(\epsilon^2)$ can be obtained in [3].

When we focus on a contribution arising from the scalar-type perturbations, we may choose the gauge in which $\gamma_{ij}$ approaches the flat metric as $\gamma_{ij} \to \delta_{ij}$ for $t \to \infty$, when the epoch close to the end of inflation. For the purpose of construct nonlinear curvature perturbation, we take the comoving slicing: $\delta \phi_c(t, x^i) = 0$. Let us use the subscript $K$ and $c$ to indicate the quantity in the uniform expansion($K$) and comoving gauge. So we had obtained the general solution, that is attached to the subscript $K$. By using a transformation from uniform $K$ gauge to the comoving gauge, we obtain the curvature perturbation in the comoving gauge: $\zeta_c = \zeta_K - H \delta \phi_K / \phi + O(\epsilon^3)$.

Now we turn to the problem of properly defining a nonlinear curvature perturbation to $O(\epsilon^2)$ accuracy by using the general solution above. Hereafter we will use the expression $R_c$ on comoving slices to denote it. Let us consider the linear curvature perturbation which is given as $R^{Lin} = (H^{Lin} + \frac{2}{3}\frac{\partial}{\partial t})Y$, where, following the notation in [18], the spatial metric in the linear limit is expressed as $\gamma_{ij} = a^2(\delta_{ij} + 2H_L^{Lin}Y_{ij} + 2H_R^{Lin}Y_{ij})$. These expressions in the linear theory correspond to the Ricci components in our notation as $\zeta = H_R^{Lin}Y$ and $\gamma_{ij} = \delta_{ij} + 2H_L^{Lin}Y_{ij}$. Notice that the variable $\zeta_c$ reduces to $R^{Lin}_c$ at leading-order in the gradient expansion, but not at second-order and it will be also similar to the nonlinear theory. Thus to define a nonlinear generalization of the linear curvature perturbation, we need nonlinear generalizations of $H_L Y$ and $H_T Y$. Our nonlinear $\zeta$ is an apparent natural generalization of $H_L^{Lin}Y$ as $H_L Y = \zeta$. As for $H_T Y$, however, the generalization is non-trivial. It corresponds to the $O(\epsilon^2)$ part of $\gamma_{ij}$. As shown in [10], it can be done by introducing the inverse Laplacian operator $\Delta^{-1}$ on the flat background and we defined the nonlinear generalization of $H_T Y$ as

$$H_T Y = \chi \equiv -\frac{3}{4} \Delta^{-1} \left[ \partial_t e^{-3\zeta} \partial^i e^{3\zeta} (\ln \gamma)_{ij} \right]. \quad (2.18)$$

With these definitions of $H_L Y$ and $H_T Y$, we can define the nonlinear curvature perturbation valid up through $O(\epsilon^2)$ as

$$R^{NL}_c \equiv \zeta_c + \frac{\chi}{3}. \quad (2.19)$$

It is easy to show that this nonlinear quantity can be reduced to $R^{Lin}_c$ in the linear limit. As clear from (2.18), finding $H_T Y$ generally requires a spatially non-local operation, however, in the comoving slicing with the asymptotic condition on the spatial coordinates, we find it is possible to obtain the explicit form of $H_T Y$ without any non-local operation as seen in [10]. Finally, we can derive a nonlinear second-order differential equation that $R^{NL}_c$ (2.19) satisfies at $O(\epsilon^2)$ accuracy by introducing
the conformal time \( \tau \), defined by \( d\tau = dt/a(t) \) and the Mukhanov-Sasaki variable \( \zeta \) with a speed of sound \( c_s^2 \):

\[
\zeta^2 = \frac{P_X}{P_X + 2P_{XX}X}.
\]

The result can be reduced to a simple equation of the form \( \tau \) as a natural extension of the linear version:

\[
\partial^2 \zeta_{\text{NL}} + 2\frac{\partial z}{z} \partial \zeta_{\text{NL}} + \frac{c_s^2}{4} K^2(\zeta_{\text{NL}}) = O(\epsilon^4),
\]

with

\[
K^2(\zeta(0)) = -2(2\Delta \zeta(0) + 3 \dot{\zeta} \zeta(0) + \dot{\zeta} \dot{\zeta}(0))e^{-2\zeta(0)}.
\]

Through up to the second-order in expansion, it can be evaluated as \( K^2(\zeta_{\text{NL}}) = K^2(\zeta(0)) + O(\epsilon^2) \) since \( \zeta_{\text{NL}} = \zeta + O(\epsilon^2) \) with \( \chi_c = O(\epsilon^2) \).

\[\text{B. Linear Theory valid up to } O(\epsilon^2)\]

To obtain the power spectrum, we will use the linear theory of the curvature perturbation in this subsection.

The master equation:

\[
\partial^2 R_{c,\text{NL}} + 2\frac{\partial z}{z} \partial R_{c,\text{NL}} - c_s^2 \Delta R_{c,\text{NL}} = 0,
\]

has two independent solutions; conventionally called a growing mode and a decaying mode. We assume that the growing mode is constant in time at leading order in the spatial gradient expansion as the assumption: \( (2.15) \), justified in an inflationary universe. As shown in \( [10] \), the linear solution valid up to \( O(\epsilon^2) \) can be obtained as

\[
R_{c,k}(\tau) = \left[ \alpha_{k}^{\text{Lin}} + (1 - \alpha_{k}^{\text{Lin}}) \frac{\dot{D}(\tau)}{D_{*}} \right] \left( \tau - \tau_{s} \right) \left. \right|_{\tau = \tau_{s}},
\]

where \( U_k^{(0)} \) denotes an integration constant and the integrals \( \dot{D}(\tau) \) and \( \dot{F}(\tau) \) have been given as

\[
\dot{D}(\tau) = 3\mathcal{H}(\tau_{s}) \int_{\tau_{s}}^{\tau} \frac{d\tau'}{z^2(\tau')} z^2(\tau') \frac{\dot{z}^2(\tau_{s})}{z^2(\tau')},
\]

\[
\dot{F}(\tau) = \int_{\tau_{s}}^{\tau} \frac{d\tau'}{z^2(\tau')} \int_{\tau_{s}}^{\tau'} z^2(\tau'') \epsilon(\tau'') d\tau''.
\]

Here \( \dot{D}_{*} = \dot{D}(\tau_{s}) \), \( \dot{F}_{*} = \dot{F}(\tau_{s}) \), \( \tau_{s} \) and \( \mathcal{H} \) denote an initial time of gradient expansion and the conformal Hubble parameter \( \mathcal{H} = \frac{d\ln a}{dt} \), respectively. The integrals in \( \dot{D} \) and \( \dot{F} \) represent a decaying and growing mode solution, respectively.

Note that \( R_{c,k}(\tau_{s}) = U_k^{(0)} \) that is just a constant solution, while \( R_{c,k}(0) = \alpha_{k}^{\text{Lin}}U_k^{(0)} \). Thus if the factor \( |\alpha_{k}^{\text{Lin}}| \) is large, it represents an enhancement of the curvature perturbation on superhorizon scales due the \( O(\epsilon^2) \) effect.

Here it is useful to consider an explicit expression for \( \alpha_{k}^{\text{Lin}} \) in terms of \( R_{c,k}^{\text{Lin}} \) and its derivative at \( \tau = \tau_{s} \). The result is

\[
\alpha_{k}^{\text{Lin}} = 1 + \frac{\dot{D}_{*}}{3\mathcal{H}} \frac{\partial R_{c,k}^{\text{Lin}}(\tau_{s})}{R_{c,k}^{\text{Lin}}(\tau_{s})} - k^2 F_{*} + O(k^4).
\]

In order to relate our calculation with the standard formula for the curvature perturbation in linear theory, we introduce \( \tau_k \) (or \( t_k \)) which denotes the time at which the comoving wavenumber has crossed the Hubble horizon,

\[
\tau_k = -\frac{r}{k}; \quad 0 < r \ll 1.
\]

The power spectrum at the horizon crossing time is given by

\[
\langle R_{c,k}^{\text{Lin}}(\tau)R_{c,k'}^{\text{Lin}}(\tau) \rangle = (2\pi)^3 P_{R}(k) \delta^3(k + k'),
\]

\[
P_{R}(k) = \left| R_{c,k}^{\text{Lin}}(\tau_k) \right|^2.
\]

By inverting \( R_{c,k}^{\text{Lin}} \) in terms of \( U_k^{(0)} \) as shown in \( [10] \), we can show the final value of the linear curvature perturbation as

\[
R_{c,k}^{\text{Lin}}(0) = \alpha_{k}^{\text{Lin}} U_k^{(0)} = \alpha_{k}^{\text{Lin}} R_{c,k}^{\text{Lin}}(\tau_k) + O(k^4),
\]

where

\[
\alpha_{k}^{\text{Lin}} = 1 + \alpha R_{D} D_{*} - k^2 F_{*},
\]

and

\[
\alpha R_{D} = \frac{1}{3\mathcal{H}(\tau_k)} \left. \frac{\partial R_{c,k}^{\text{Lin}}}{R_{c,k}^{\text{Lin}}} \right|_{\tau = \tau_k}.
\]

\[
D_{*} = 3\mathcal{H}(\tau_k) \int_{\tau_{s}}^{\tau} \frac{d\tau'}{z^2(\tau')} z^2(\tau_{s}) \frac{\dot{z}^2(\tau_{s})}{z^2(\tau')}.
\]

\[
F_{*} = \int_{\tau_{s}}^{\tau} \frac{dt'}{z^2(t')} \int_{\tau_{s}}^{t'} z^2(t'') \epsilon(t'') dt''.
\]

The formula \( \alpha R_{D} \) will be used in the next subsection.

The power spectrum at the final time is thus enhanced by the factor \( |\alpha_{k}^{\text{Lin}}| \) as

\[
\langle R_{c,k}^{\text{Lin}}(0)R_{c,k'}^{\text{Lin}}(0) \rangle = (2\pi)^3 |\alpha_{k}^{\text{Lin}}|^2 P_{R}(k) \delta^3(k + k').
\]

\[\text{C. Nonlinear theory valid up to } O(\epsilon^2)\]

Using the linear solution of the curvature perturbation given by \( \dot{D} = \frac{D(\tau)}{D_{*}} \), here we can derive the nonlinear solution by matching the two at \( \tau = \tau_{s} \). The main purpose of the matching is to make it possible to analyze superhorizon nonlinear evolution valid up to the second-order
in gradient expansion, starting from a solution in the linear theory. In particular, we would like to evaluate the bispectrum induced by the superhorizon nonlinear evolution. For this purpose, we need to have full control over terms up not only to $O(\epsilon^2)$ but also to $O(\delta^2)$, where we suppose that the linear solution is of order $O(\delta)$. Therefore, the matching condition at $\tau = \tau_*$ should be of the form

$$R_{c, k}^{NL}(\tau_*) = R_{c, k}^{Lin}(\tau_*) + s_1(\tau_*) + O(\epsilon^4, \delta^3),$$

$$\partial_\tau R_{c, k}^{NL}(\tau_*) = \partial_\tau R_{c, k}^{Lin}(\tau_*) + s_2(\tau_*) + O(\epsilon^4, \delta^3).$$

(2.33)

where $s_1(\tau_*) = O(\delta^2)$ and $s_2(\tau_*) = O(\delta^2)$ are functions of $\tau_*$ and spatial coordinates. While the linear solution $R_{c, k}^{Lin}(\tau)$ is considered as an input, i.e., initial condition, the additional terms, $s_1(\tau_*)$ and $s_2(\tau_*)$, are to be determined by the following condition. The terms of order $O(\delta^2)$ in $R_{c, k}^{NL}$ and $\partial_\tau R_{c, k}^{NL}$ should vanish at the horizon crossing when $\tau = \tau_*$. Note that $\tau_k < \tau_*$. In other words, $s_1(\tau_*)$ and $s_2(\tau_*)$ represent the $O(\delta^2)$ part of $R_{c, k}^{NL}$ and $\partial_\tau R_{c, k}^{NL}$, respectively, generated during the period between the horizon crossing time and the matching time.

We have to omit the explicit way to determine the terms $s_1$ and $s_2$ for want of space, that was shown in [10]. As a result, using the linear solution of the curvature perturbation given by (2.24) we have the nonlinear comoving curvature perturbation at the final time $\tau = 0$ (or $t = \infty$) given by

$$R_{c, k}^{NL}(0) = R_{c, k}^{Lin}(\tau_k) - (1 - \alpha_{k}^{Lin})R_{c, k}^{Lin}(\tau_k)$$

$$- \frac{1}{4} F_k \tilde{R}^{(2)}[R_{c, k}^{Lin}(\tau_k)] + O(\epsilon^4, \delta^3),$$

(2.34)

where

$$\tilde{R}^{(2)}[\zeta^{(0)}] = 4 \Delta \zeta^{(0)} + K^{(2)}[\zeta^{(0)}]$$

$$= - 2(\delta^2 \partial_\tau \zeta^{(0)} \partial_\tau \zeta^{(0)} - 4 \zeta^{(0)} \Delta \zeta^{(0)}) + O((\zeta^{(0)})^3),$$

(2.35)

which denotes the nonlinear term derived from Ricci scalar $K^{(2)} = \mathcal{K}^{(2)}$. The first term in (2.34) corresponds to the result of the $\delta N$ formalism, that is a constant since we considered the system for a single scalar field, the second term is related to an enhancement on superhorizon scales in linear theory, and the last term is the nonlinear effect which may become important if $F_k$ is large.

Here we can notice that in order to the final values of curvature perturbation both in linear (2.29) and in nonlinear theory (2.34), all one have to do is to estimate the same integrals shown in both theories as $D_k$ and $F_k$ in $\alpha_{k}^{Lin}$. The reason why is that the master equations (2.21) and (2.22) for both theories have the same structures of evolution equation as described before.

In [14], we calculated non-Gaussianity of curvature perturbations for a model of sudden slope change of inflaton’s potential. We also had studied several applications of our formalism in [11, 14], where we considered a model of temporal stopping inflaton and varying sound speed, respectively. Such effect is sensitive to the temporal violation of some kind of slow-roll conditions. So our formalism is good to evaluate time evolution of curvature perturbation in the physics for violation of slow-roll conditions.

### III. BEYOND KINETIC-INFLATION

In this section, we will review [13] and we go beyond kinetic inflation whose the Lagrangian density given as

$$\mathcal{L} = \sqrt{-g} \left( -\frac{(4) R}{2} + (X, \phi) - G(X, \phi) \right),$$

(3.1)

where $G$ is also an arbitrary function of $\phi$ and $X$. Although the above action depends upon the second derivative of $\phi$ through $\Box \phi = g^{\mu \nu} \nabla_\mu \nabla_\nu \phi$, the resulting field equations for $\phi$ and $g_{\mu \nu}$ remain second order. In this sense the above action gives rise to a more general single-field inflation model than k-inflation, i.e., Generalized Galileon inflation(G-inflation) [20]. The same scalar-field Lagrangian is used in the context of dark energy and called kinetic gravity braiding [21]. In fact, the most general inflation model with second-order field equations was proposed in [20] based on Horndeski’s scalar-tensor theory [22, 23]. However, here we focus on the action (3.1) which belongs to a subclass of the most general single-field inflation model, because it involves sufficiently new and interesting ingredients while avoiding unwanted complexity.

We can replace the following equation of motion with

$$\nabla_\mu \left[ (P_X - G_0 - G_X \Box \phi) \nabla^\mu \phi - G_X \nabla^\mu X \right] + P_0 - G_0 \Box \phi = 0.$$

(3.2)

Taking same procedure in the previous section can allow us to obtain the nonlinear curvature perturbation in the comoving gauge satisfying the similar form of evolution equation as a second-order differential equation:

$$\partial_\tau^2 R_{c}^{NL} + \frac{2z}{z} \partial_\tau R_{c}^{NL} + \frac{z^2}{4} \mathcal{K}^{(2)}[R_{c}^{NL}] = O(\epsilon^4),$$

(3.3)

where $\mathcal{K}^{(2)}[R_{c}^{NL}]$ is the Ricci scalar of the metric $\delta_{ij} \exp (2R_{c}^{NL})$ in (2.22) and

$$z := \frac{\Phi \sqrt{g_{GG}}}{\Theta_G}.$$

(3.4)

with

$$G_G(t) := E_X - 3\Theta_G(G_X \dot{\phi})_0,$$

(3.5)

$$\Theta_G(t) := H - (G_X \dot{X} \phi)_0,$$

(3.6)

$$E_X(t) := \left[ P_X + 2X P_{XX} + 9H G_X \dot{\phi} + 6H G_{XX} \dot{\dot{\phi}} - 2G_0 - 2X G_{\phi X} \right]_0.$$
Here $c_s^2$ is the sound speed squared of the scalar fluctuations defined as

$$c_s^2 := \frac{\mathcal{F}_G(t)}{G(t)}$$

This is a generalization of familiar “$z$” in the Mukhanov-Sasaki equation [24], and reduces indeed to $a\sqrt{(\rho + P)/Hc_s}$ in the case of k-inflation as shown in [23].

We have introduced an appropriately defined variable for the nonlinear curvature perturbation in the comoving gauge ($\delta \phi = 0$), $\mathcal{R}_{\text{cNL}}$. It is a combination of $\zeta$ and scalar mode $\chi$ from an unit spatial metric $\gamma_{ij}$, which is given by

$$\mathcal{R}_{\text{NL}} \equiv \psi + \frac{\chi}{3}. \tag{3.9}$$

with

$$\chi \equiv -\frac{3}{4} \Delta^{-1} \left[ \partial^{i}e^{-3\psi} \partial_{i}e^{3\psi} (\gamma_{ij} - \delta_{ij}) \right]. \tag{3.10}$$

The definition of (3.10) is same as (2.18). The evolution equation for $\gamma_{ij}$ is obtained as

$$\partial_{\tau}^{2} \gamma_{ij} + 2H \partial_{\tau} \gamma_{ij} + 2 \mathcal{F}_{ij}^{(2)}[\gamma] = \mathcal{O}(\epsilon^4), \tag{3.11}$$

where the explicit form of $\mathcal{F}_{ij}^{(2)}$ will be shown in (2.23). That is worth comparing with the later result of (3.10) for the beyond GR theory shown in Sec. V, where it is noticed that this equation is a key basic equation for gravitational waves.

Upon linearization, the variable: $\mathcal{R}_{\text{NL}}$ reduces to the previously defined linear curvature perturbation $\mathcal{R}_{\text{cLin}}$ on uniform $\phi$ hypersurfaces. Then, it has been shown that $\mathcal{R}_{\text{NL}}$ satisfies a nonlinear second-order differential equation (3.3), which is a natural extension of the linear perturbation equation for $\mathcal{R}_{\text{cLin}}$ in (2.23) and $\mathcal{R}_{\text{cNL}}$ in (2.21).

IV. MULTI-FIELD CASE

In this section, we will review multi-field case following [12]. We consider Einstein gravity plus a multi-component scalar field described by Lagrangian density of the form

$$\mathcal{L} = \sqrt{-g} \left[ -\frac{\epsilon^{(4)} R}{2} + \mathcal{P}(X^{1J}, \phi^K) \right],$$

$$X^{1J} \equiv -g^{\mu\nu} \partial_{\mu} \phi^{I} \partial_{\nu} \phi^{I}/2, \tag{4.1}$$

where $I, J$ and $K$ run over $1, 2, \ldots, \mathcal{M}$ denoting $\mathcal{M}$-components scalar field. Note that we do not assume the explicit form of both kinetic terms and their potentials, that can be given as arbitrary function of $\mathcal{P}(X^{1J}, \phi^K)$. Notice that it can not be a perfect fluid form, that is different from a single scalar system and we can replace the following equation of motion with (2.14) for multi-scalar field as

$$\partial_{\tau} \left( P_{(IJ)} \partial_{\tau} \phi^{I} \right) + K P_{(IJ)} \partial_{\tau} \phi^{I}$$

$$- \frac{1}{\alpha a e^{3\phi}} \partial_{\tau} \left( a e^{\phi} P_{(IJ)} \gamma^{(ij)} \partial_{\tau} \phi^{I} \right) - \frac{1}{2} P_{I} = 0. \tag{4.2}$$

This is $\mathcal{M}$-coupled second order differential equations.

We introduce the proper time $\eta$ and its definition is

$$\eta(t, x^{i}) \equiv \int_{t^{i}=\text{const.}}^{t} dt \alpha(t, x^{i}). \tag{4.3}$$

In terms of $\eta$, the expression of $K$ at leading order in gradient expansion with a spatial gauge choice: $\beta^{I} = 0$,

$$K = \frac{1}{\alpha} \partial_{\tau} \left( a^{3} \zeta^{(0)} \right) = 3 \frac{\partial_{\nu}(a \zeta^{2})}{a \zeta^{2}}. \tag{4.4}$$

If we associate $a \zeta^{2}$ and $\eta$ with $a$ and $t$ respectively, we can check the correspondence between $K$ and $3H$. Based on these facts, the structure of the above equations is same as that of background equations. Namely, given a background solution $\phi^{I}(t)|_{\text{background}} = \phi_{0}^{I}(t)$, one can construct the solution at leading order in gradient expansion as $\phi^{I}(t, x^{i})|_{\text{gradient}} = \phi_{0}^{I}(\eta)$.

In the standard cosmological perturbation, the e-folding number $N$ is often used as a time coordinate, which is defined by

$$N = \int_{t}^{\infty} dt' H(t'). \tag{4.5}$$

Note that it is the number of e-folding counted backward in time from a fixed final time $t = \infty$. By replacing with $t$ with $\eta$ and $H$ with $K/3$, we can generalize the e-folding number $\mathcal{N}$ as

$$\mathcal{N} \equiv \frac{1}{3} \int_{t}^{t_{0}} \alpha(t, x^{i}) K(t', x^{i})_{\text{gradient}} dt'. \tag{4.6}$$

If we also rewrite basic equation using $\mathcal{N}$ as a time coordinate, we can again easily check that the structure of equations exactly coincide with that of background equations using $N$ as a time coordinate.

As for a gauge choice, we consider the uniform $K$ slicing which is taken in multi field case. At the leading order in the gradient expansion, the evolution equation for the extrinsic curvature can be used in order to obtain the solution of lapse as

$$\alpha^{(0)} = -\frac{2H(t)}{E^{(0)}(t) + P^{(0)}(t, x^{i}) + \mathcal{O}(\epsilon^3)}. \tag{4.7}$$

We can express $\alpha^{(0)}$ as a function of $\eta$ using $E^{(0)}$ and $P^{(0)}$. On the other hands, in a single-scalar system, it reads $\alpha^{(0)} = 1$ and it is the main difference between single and multi case. Since we have assumed that we can solve the scalar field equation and express the scalar field as a function of $\eta$, we know the expressions of $E^{(0)}$ and $P^{(0)}$ as
a function of \( \eta \). This equation means the inhomogeneity of \( \alpha \) is related with that of \( P^{(0)} \). Therefore (4.7) can be schematically expressed as

\[
\alpha = f \left[ t, P(\eta) \right] = f \left[ t, P \left( \int \alpha dt \right) \right]. \tag{4.8}
\]

It is clearly shown that it is almost impossible to solve this equation, at least in an analytical way.

Then we have to solve the Einstein equations on another gauge, that is a uniform \( N \) slice. From Eq (4.10), we can reread the condition of this slice as

\[
\alpha(t, x^i)K(t, x^i) = 3H(t) \quad \Leftrightarrow \quad \partial_t \zeta (t, x^i) = 0. \tag{4.9}
\]

It shows on this slice, curvature perturbation becomes constant which is expressed by a function of \( x^i \) only. It is easy to obtain the solution in this gauge. First, we solve the Einstein equations on uniform \( e \)-folding number slice, the solution of scalar fields are given by the function of cosmic time or background \( e \)-folding number. Next, we consider the gauge transformation from above slice to uniform expansion slice. Applying the derived gauge transformation rules to the solution on above slice, we can find out the solution on uniform expansion slice.

**A. Nonlinear gauge transformation**

We derive the gauge transformation rules for the metric, its derivative (\( K, A_{ij} \)) and the scalar field. We consider a nonlinear gauge transformation from a coordinate system with vanishing shift vector \( \beta^i = 0 \), to another coordinate system in which the new shift vector also vanishes, \( \beta^i = 0 \). We note that once the time slicing is changed, the shift vector appears in the new slicing in general. So the spatial coordinates also need to be changed to eliminate thus appeared shift vector. We use the background \( e \)-folding number \( N \) as the time coordinate and define the temporal and spatial shift, \( n \) and \( L^1 \), respectively, \( \tilde{N} + \tilde{a}(\tilde{N}, \tilde{x}^i) = N, \ \tilde{x}^i + \tilde{L}^i(\tilde{N}, \tilde{x}^i) = x^i \). Under the change of the coordinates, the line element should remain invariant, \( ds^2 = -\frac{\alpha^2}{f_2(N)}dN^2 + a^2(N)e^{2\zeta}dx^idx^j = -\frac{\alpha^2}{f_2(N)}d\tilde{N}^2 + a^2(\tilde{N})e^{2\zeta}dx^idx^j \). Equating the coefficients of \( dN^2, d\tilde{N}dx^i, \) and \( dx^idx^j \) on both sides of the above, we obtain the nonlinear gauge transformation rules in Appendix of [12].

**B. Beyond \( \delta N \) formalism**

Let us briefly summarise the five steps in the Beyond \( \delta N \) formalism.

1. Write down the basic equations (the Einstein equations and scalar field equation) in the uniform \( N \) slicing with \( \beta^i = 0 \) of [27]. For convenience let us call the choice of the coordinates in which one adopts the uniform \( X \) slicing with \( \beta^i = 0 \), the \( X \) gauge. So the above choice is the \( N \) gauge. In this gauge the metric components at leading order are trivial since both \( \zeta \) and \( \gamma_{ij} \) are independent of time.

2. First solve the leading order scalar field equation under an appropriate initial condition and then the next-to-leading order scalar field equation which involves spatial gradients of the leading order solution.

3. Solve the next-to-leading order Einstein equations for the metric components and their derivatives.

4. Determine the gauge transformation from the \( N \) gauge to the \( K \) gauge and apply the gauge transformation rules to obtained the solution obtained in the \( K \) gauge.

5. Evaluate the curvature perturbation \( \mathfrak{R} = \zeta + \chi/3 \) in the \( K \) gauge, where \( \chi \) is to be extracted from \( \gamma_{ij} \).

**C. Solvable example**

In this subsection, we demonstrate how to obtain the solution up to next-to-leading order in gradient expansion by applying our formalism to a specific, analytically solvable model. We consider a canonical scalar field with exponential potential [27],

\[
P = \frac{1}{2}X_{IJ} - V(\phi_I), \quad V(\phi_I) = W \exp \left[ \sum J m_J \phi_J \right], \tag{4.10}
\]

where \( W \) is a constant. The solution is obtained under the assumption of slow-roll conditions by

\[
\phi_I^{(0)}(N) = C_I^\phi + m_I(N - N_0), \tag{4.11}
\]

and the next-leading order solution is also obtained as

\[
\phi_I^{(2)} = \frac{1}{3} D_I^\phi \left[ e^{(3N - N_0)} - 1 \right]
+ \int_{N_0}^{N} dN' e^{3N'} \int_{N_0}^{N'} dN'' e^{-3N''} S_I^\phi \left[ \frac{e^{C^\phi}}{K^{(0)}} D_I^\phi \right] \partial_N \phi_I^{(0)}, \tag{4.12}
\]

where

\[
S_I^\phi = \frac{3K^{(0)}}{e^{C^\phi}} \partial_i \left( \frac{e^{C^\phi}}{K^{(0)}} D^i \phi_I^{(0)} \right)
+ K^{(0)} \left[ D^2 \left( \frac{1}{K^{(0)}} \right) + D^i \left( \frac{1}{K^{(0)}} \right) D_i C^\phi \right] \partial_N \phi_I^{(0)}
- \left[ R - (4D^2 C^\phi + 2D^i C^\phi D_i C^\phi) \right]
+ 2D^i \phi_J^{(0)} D_i \phi_J^{(0)} \partial_N \phi_I^{(0)}, \tag{4.13}
\]
\( N_0 \) is an initial time and \( C^\phi \) and \( D^\phi \) represent the initial values of the scalar field and its time derivative.

We obtain analytic solutions for all variables on the uniform \( N \) gauge. We derive the solution on the \( K \) gauge by applying a gauge transformation to the solution on the \( N \) gauge. To do so, we first need to determine the generator of the gauge transformation between the two slices, \( N \rightarrow \tilde{N} = N + n(N, x^i) \) or conversely \( \tilde{N} + n(\tilde{N}, \tilde{x}^i) = N \).

What we need to know is the final value of \( \tilde{N} \) at sufficiently late times, \( N \to 0 \) \( (a \to a_0 e^{\bar{N}_0}) \). We take \( N_0 \) to be a time around which the scales relevant to cosmological late times, \( \delta N \sim \delta x \rightarrow \tilde{x} \) in the gauge. To do so, we first need to determine the generator of the gauge transformation between the two slices, \( N \rightarrow \tilde{N} = N + n(N, x^i) \) or conversely \( \tilde{N} + n(\tilde{N}, \tilde{x}^i) = N \).

In this case, \( N = 0 \), the curvature perturbation reduces to

\[
\mathcal{R}_K(N = 0) \approx (0)C^\phi + (2)C^\phi - \frac{m^2}{3M^2}D^\phi_i. \quad (4.14)
\]

where we have defined \( M^2 = \sum_I m_I^2 \) and assumed small mass: \( M^2 \ll 1 \). The first term, \( (0)C^\phi \) represents the leading order curvature perturbation obtainable in the usual \( 3N \) formalism, and the remaining terms represent \( O(\epsilon^2) \) contributions, the calculation of which is the main purpose of the beyond \( 3N \) formalism. The small mass \( M \) can give a contribution to large effect on the last term, corresponding to initial time derivative of scalar field \( \phi^\prime \).

### V. BEYOND EINSTEIN GRAVITY

In this section, we will consider gradient expansion formalism for a general single field in beyond GR gravity, namely modified gravity. Such theory is related to a general scalar-tensor theory. We take the Lagrangian density of beyond Horndeski (GLPV) theory \(^{[26]}\) as a boarder generalization of the Galileons to curved space-time, which is of the form \( \mathcal{L} = \sqrt{-g} \sum_a L^a_\phi \), with

\[
L^\phi_2 = G_2(\phi, X), \quad L^\phi_3 = G_3(\phi, X) \partial_\phi, \quad L^4_\phi = G_4(\phi, X)(4) R - 2G_{4,X}(\phi, X)(\Box \phi^2 - \phi^\mu \phi^\nu),
+ F_4(\phi, X)\epsilon^{\mu\nu\rho\sigma} \phi_\mu \phi_\nu \phi_\rho \phi_\sigma, \quad L^5_\phi = G_5(\phi, X)(4) G_{\mu\nu} \phi^{\mu\nu}
+ \frac{1}{3} G_{5,X}(\phi, X)(\Box \phi^3 - 3 \Box \phi \phi^\mu \phi_\mu + 2 \phi \phi^\mu \phi^{\mu\sigma} \phi_\sigma)
+ F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma\delta} \phi_\mu \phi_\nu \phi_\rho \phi_\sigma \phi_\delta, \quad (5.1)
\]

where \( \phi_\mu \equiv \nabla_\mu \phi, \phi_\nu \equiv \nabla_\nu \phi, c_{\mu\nu\rho\sigma} \) is the totally antisymmetric Levi-Civita tensor and \( (4) G_{\mu\nu} \) is the four-dimensional Einstein tensor. Note \( F_4 = F_5 = 0 \) equals to Horndeski theory \(^{[22]}\). It ensures that the equation of motion are second order. Choosing the uniform scalar field \( (\phi = \text{const}) \) hypersurfaces leads to a ADM Lagrangian of the form \( \mathcal{L} = \sqrt{-\gamma} \sum_a L^a_\alpha \), with

\[
L_2 = A_2(t, \alpha), \quad L_3 = A_3(t, \alpha)K,
L_4 = A_4(t, \alpha)(K^2 - K_{ij}K^{ij}) + B_4(t, \alpha)\tilde{R},
L_5 = A_5(t, \alpha)(K^3 - 3K K_{ij}K^{ij} + 2K_{ij}K^{ik}K^{kj})
+ B_5(t, \alpha)K^{ij} \left( \tilde{R}_{ij} - \frac{1}{2} g_{ij} \tilde{R} \right), \quad (5.2)
\]

where \( K_{ij} \) and \( \tilde{R}_{ij} \) are the extrinsic and intrinsic curvature tensors for \( g_{ij} \) on the constant time hypersurface. We set \( K = g^{ij}K_{ij} \) and \( \tilde{R} = g^{ij}\tilde{R}_{ij} \). The coefficients in \((5.2)\) are related to the original functions in \((5.1)\) as shown explicitly in \([26]\). The linear perturbation theory was already studied in \([27]\). We will adopt gradient expansion approach to this theoretical setup (see also our recent work \([15]\) in the same point of view).

In the setup of the Lagrangian \([22]\), the Hamiltonian constraint is obtained by

\[
(A_2 \alpha)' + A_3 \alpha K + (A_4 / \alpha) \alpha^2 \left( \frac{2}{3} K^2 - A_{ij} A^{ij} \right) + (A_5 / \alpha^2) \alpha^3 \left( \frac{2}{9} K^3 - K A_{ij} A^{ij} + 2 A_{i\alpha} A_{j}^{\alpha} A^{ij} \right) + (B_4 \alpha)' \tilde{R} - \frac{\alpha}{6} B_5' K_{ij} \tilde{R} + \alpha B_5' \tilde{R}_{ij} A^{ij} = 0, \quad (5.3)
\]

where a prime represents differentiation with respect to \( \alpha \). The Momentum constraint is obtained by

\[
D^i A_3 + \frac{4}{3} D^i (A_4 K) - 2 e^{-3\xi} D_j (A_4 e^{-3\xi} A^{ij})
+ D^i \left( A_5 \left( \frac{2}{9} K^2 - 3 A_{ab} A_{ab} \right) \right)
- 2 e^{-3\xi} D_j \left( A_5 e^{-3\xi} (K A^{ij} - 3 A^{ia} A_{aj}) \right)
+ \tilde{R}^{ij} D_j B_5 - \frac{1}{2} D^i B_5 \tilde{R} = 0, \quad (5.4)
\]

where \( D \) represents covariant derivative for \( \gamma_{ij} \).

Two dynamical equations for \( (K, A_{ij}) \) can be obtained by varying the Lagrangian with respect to \( g_{ij} \), that corresponds to trace part and traceless part as
(2A_4 + 2A_5K) \vartheta_{\perp K} - 3A_5 A_i^j \vartheta_{\perp A_{ij}} - \frac{3}{2} A_2 + \frac{3}{2} \vartheta_{\perp A_3} + 2K \vartheta_{\perp A_4} + \left( K^2 - \frac{3}{2} A_{ij} A^{ij} \right) \vartheta_{\perp A_5}
+ \left( K^2 + \frac{3}{2} A_{ij} A^{ij} \right) A_4 + \left( \frac{2}{3} K^3 + 3A_{ij} A_i^j A^j \right) A_5
- B_2 \tilde{R} + \frac{2}{\alpha} D^2(\alpha B_4) - \frac{1}{4} \vartheta_{\perp B_5} \tilde{R} + \frac{1}{2} D_i D_j B_5 A_{ij} - \frac{1}{3} K(D^2 B_5) - \frac{2}{3} D^i K D_i B_5
+ D_i A_i^j D_j B_5 - \frac{1}{3} \tilde{R} D_i^a D_i B_5 K + \frac{1}{\alpha} D_i A_i^j D_j B_5 = 0 ,
\begin{equation}
(5.5)
\end{equation}

\begin{equation}
(5.6)
\end{equation}

We will solve a general solution valid up to \mathcal{O}(\epsilon^2) later.

A. Background equations

We obtain the background part of the Lagrangian as

\begin{equation}
\mathcal{L}^{(0)} = \alpha^3 (A_2 \alpha + 3A_4 H + 6A_4 H^2 / \alpha + 6A_5 H^3 / \alpha^2) ,
\end{equation}

by using \( K = 3H / \alpha \), where a bar means a background quantity. Varying Eq. \([5.7]\) with respect to \( \alpha \) and \( a \), we obtain, respectively,

\begin{equation}
- \mathcal{E} := (A_2 \alpha)' + 3A_4' H + 6(A_4 / \alpha)' H^2
+ 6(A_5 / \alpha^2)' H^2 = 0 ,
\end{equation}

\begin{equation}
P := A_2 \alpha - 6A_4 H^2 / \alpha - 12A_5 H^3 / \alpha^2
- \frac{d}{dt}(A_3 + 4A_4 H / \alpha + 6A_5 H^2 / \alpha^2) = 0 .
\end{equation}

B. Leading-order

The leading-order in the gradient expansion can be interpreted as \( \delta N \) for curvature perturbation. In this representation, inhomogeneous parts can be interpreted as perturbed quantities \( \zeta \)

\begin{equation}
\zeta(t, x) = \delta N := N - N(t) ,
\end{equation}

where \( N \) represent a background e-folding number. If one takes FLRW background, i.e. \( \alpha = e^N(t) \), \( \zeta \) quantifies the curvature perturbation as \( \delta N \) in the view of the separate Universe approach. That is the so-called \( \delta N \) formalism.

In this approach, trace and traceless parts of the extrinsic curvature can be given by

\begin{equation}
K = \frac{3}{2} \frac{dN}{dt} + \mathcal{O}(\epsilon^2) , \quad A_{ij} = \frac{1}{2} \frac{d\gamma_{ij}}{dt} + \mathcal{O}(\epsilon^2) .
\end{equation}

Note that \( A_{ij} \) represents a cosmic shear rate.

One can obtain ADM equations \([5.10]\) and \([5.11]\) in the leading-order as

\begin{equation}
2 \Xi \partial_t K = \mathcal{G}_K K - 3A_5 A_i^j \partial_t A_{ij}
+ \frac{3}{2} (A_4 \alpha - A_5) A_{ij} A^{ij}
+ 3A_{ik} A_j^k A_{ij} A_{ij} + \mathcal{O}(\epsilon^2) ,
\end{equation}

\begin{equation}
\Xi \partial_t A_{ij} = - (3H \Xi + \partial_t \Xi) A_{ij} - 6A_5 \partial_t A_{ij} A_{ij}
+ 2(4A_4 + 15A_5 H + 3A_5) A_{ik} A_j^k
+ 6A_5 \partial_t A_{ij} A_k^k + \mathcal{O}(\epsilon^2) .
\end{equation}

where a dot represents differentiation with respect to \( t \) and we defined

\begin{equation}
\Xi(t) = -(A_4 + 3A_5 H / \alpha) ,
\end{equation}

\begin{equation}
\mathcal{G}_K(t) = 3H \Xi + A_4 + 3A_5 H / \alpha .
\end{equation}

If one consider the GR case, it takes \( A_4 = -B_4 = -1 / 2 \), \( A_2 = P(\phi, X) \) and all others vanishing. So the definition of \( \Xi \) is one as being a positive value. Here we assume that cosmic shear is weak, i.e. \( A_{ij} \ll 1 \). It can allow us to ignore the quadratic and cubic terms: \( A_{ij}^2, A_{ij}^3 \).

Then \([5.13]\) reads

\begin{equation}
\partial_t A_{ij} \simeq -(H + \partial_t (\ln \Xi)) A_{ij} + \mathcal{O}(A_{ij}^2, \epsilon^2) ,
\end{equation}

and it shows a leading order solution of \( A_{ij} \) can be ignored since it is just a decaying mode, i.e. \( A_{ij}^0 \propto a^{-3} \Xi^{-1} \).
in the context of inflationary universe with $a \to \infty$. So we can set for general situation

$$A_{ij} = O(e^2), \quad (5.17)$$

that is same condition for (2.15), which we have used in our procedure. Then (5.12) also can be simplified as

$$\partial_t K \simeq \{G_K/(2\Xi)\} K + O(A_{ij}^2, e^2), \quad (5.18)$$

If it shows $G_K(t)/\Xi < 0$, the perturbation of $K$ also can be ignored since it just decays in time as $K \propto \exp \left[ \int G_K/(2\Xi)dt \right]$ in an inflationary cosmology.\(^1\)

\[ \text{C. Next-leading order} \]

In this subsection, we will adopt the condition (5.17). We can introduce the perturbations of $K$ and $\alpha$ as

$$K = \frac{3H(t)}{\alpha(t)} \left[ 1 + \delta K(t, x') \right]$$

$$\alpha = \alpha(t) \left[ 1 + \delta \alpha(t, x') \right]. \quad (5.19)$$

The condition (5.17) can read

$$\partial_t \gamma_{ij} = O(e^2), \quad (5.20)$$

that is equivalent to recovering FLRW Universe in the limit of taking $\epsilon \to 0$ because a shear of the Universe vanishes at the leading order. As shown in the last subsection, we can also take

$$\delta K = \delta \alpha = O(e^2). \quad (5.21)$$

Up to the order of $O(e^2)$, we can obtain ADM equations at the next-leading order, that are Hamiltonian constraint and two dynamical equations (5.5) and (5.6) as

$$\delta \alpha = - \frac{G_A}{\Lambda} \delta K - \frac{F_A}{\Lambda} \hat{R} + O(e^4), \quad (5.22)$$

$$\Xi \partial_t (\delta K) - \lambda \partial_t (\delta \alpha) = G_B \delta K + G_C \delta \alpha$$

$$- \frac{\alpha^2 F_B}{12H} \hat{R} + O(e^4), \quad (5.23)$$

$$\partial_t A_{ij} = \frac{F_C}{\Xi} A_{ij} - \frac{\alpha F_B}{a^2 e^2 \Xi} [\hat{R}]^{ij} + O(e^4), \quad (5.24)$$

where the coefficients are defined as

$$\Lambda = \alpha (A_2 \alpha)' + 3H (A_3 \alpha)' + 6H^2 (A_4 \alpha)' + 6H^3 (A_5 \alpha)' + \frac{\alpha}{2} (\alpha)^2, \quad (5.25)$$

$$G_A = 3H A_3' + 12H^2 (A_4' \alpha) + 18H^3 (A_5' \alpha^2), \quad (5.26)$$

$$F_A = (B_4' \alpha)' / \alpha - B_5' H/2, \quad (5.27)$$

$$G_B = - 3H \Xi - \partial_t \ln (H/\alpha) \Xi$$

$$+ (A_4 + 3H A_5'), \quad (5.28)$$

$$\lambda = \alpha^2 A_3'/4H + A_4' \alpha + 3H A_5'/2, \quad (5.29)$$

$$G_C = - \frac{\alpha (A_2 \alpha)'}{4H} + \frac{3H^2 (A_3 \alpha)'}{2\alpha} + \frac{3H^2 (A_4 \alpha)'}{2\alpha}$$

$$+ \frac{3H}{2\alpha} \partial_t (A_3 \alpha) + \partial_t (A_4 \alpha) + \frac{3H}{2\alpha} \partial_t (A_5 \alpha), \quad (5.30)$$

$$F_B = B_4 + B_5/2(\alpha), \quad (5.31)$$

$$F_C = - 3H \Xi - \partial_t \Xi, \quad (5.32)$$

and we have used the fact that any spatial derivative terms of $B_4$ and $B_5$ can be estimated as higher expansion order, that is same order of $D_4 D_5 \delta \alpha = O(e^4)$ via $B_4(t, \alpha(t, x'))$ with the covariant derivative $D$ for the metric $g_{ij}$. Note that (5.24) is a linearized equation for $A_{ij}$ since we assumed cosmic shear is weak with (5.17).

Ricci scalar and Ricci tensor can be rewritten as

$$\hat{R} = a^2 e^{-2\Xi} \left[ R - (4D^2 \zeta + 2D_4 \zeta D^4 \zeta) \right], \quad (5.33)$$

$$\hat{R}^{TF} = [R_{ij} + D_4 \zeta D_4 \zeta - D_4 D_4 \zeta]^{TF}. \quad (5.34)$$

If one focus on a scalar mode, it needs to solve two eqs (5.22) and (5.23) for $\delta \alpha$ and $\delta K$, respectively. When the coefficient $\lambda$ is non-zero, their time evolution equations are coupled, but they are reduced to one equation by substituting (5.22) into (5.23) as

$$\partial_t (\delta K) = G_D \delta K - F_D (a^2 \hat{R}) + O(e^4) \quad (5.35)$$

The solution of $\delta K$ can be obtained by

$$\delta K = G(t) \left( C^{(2)} - (a^2 \hat{R}) \int D(D')/(G(D')dt') \right), \quad (5.36)$$

where $C^{(2)}$ is an integration constant and we defined

$$G(t) = \exp \left[ \int G_D(D)dt \right], \quad (5.37)$$

$$G_D(t) = \frac{G_B - G_A G_C}{\Lambda^2} - \lambda \partial_t \left( \frac{G_A}{\Lambda} \right) / L, \quad (5.38)$$

$$F_D(t) = \left[ \frac{F_A G_C}{\Lambda a^2} + \frac{\alpha^2 F_B}{12H a^2} + \lambda \partial_t \left( \frac{F_A}{\Lambda a^2} \right) \right] / L, \quad (5.39)$$

$$L = \Xi + \lambda G_A / \Lambda. \quad (5.40)$$

Note that the term $a^2 \hat{R}$ is a just spatial function as shown in (5.33). The term $G_D$ can be rewritten by using back-
ground equations \(5.38\) and \(5.39\) as
\[
G_C = -\frac{\alpha}{4}(3A_3' - 6A_3' (H/\alpha)^2))
+ \frac{\alpha}{4H}(\partial_t(A_3') + 4H/\alpha \partial_t(A_3') + 6(H/\alpha)^2 \partial_t(A_5')).
\]

In the GR limit, it can show easily that the term \(G_C = 0\) and also \(\lambda = 0\). In this case, one can also show \(G_D = G_B/\Xi = -3H - \partial_t \ln (H/\alpha)\) and it can simplify the solution of \(G(t)\) as \(a^{-3}H/\alpha\), that is a decaying mode with inflation. Note that this simple solution is related to a discussion in the previous subsection about leading-order. Therefore a curvature perturbation can be finally obtained as
\[
\partial_t \zeta = H(\delta K + \delta \alpha) + \mathcal{O}(\epsilon^4).
\]

The final solution of \(\zeta\) can be obtained by
\[
\zeta = \left[ \int t' H G(t') \left( 1 - \frac{G_A}{\Lambda} \right) dt' \right] C^{(2)} - F_R(t) (a^2 \dot{R}),
\]
with
\[
F_R(t) = \int_t^\tau (H F_A / \Lambda) dt' + \int \left[ H G(t') \left( 1 - \frac{G_A}{\Lambda} \right) \int t' (F_D / G) dt'' \right] dt'.
\]

This is our main result for curvature perturbation in general scalar-tensor theory. However, it is impossible to construct one master evolution equation since this solution is related to two contributions: \(\delta \alpha\) and \(\delta K\), which is different from the case of \(2.24\). So let us see tensor mode in detail in the next subsection.

### D. Gravitational waves via nonlinear interactions

We focus on the tensor mode in this subsection. Tensor perturbation can be easily dealt with because of no coupling with \(\delta K\) and \(\delta \alpha\) up to the next-leading order in the gradient expansion. First, we obtain the general solution under the condition: \(\ref{2.40}\) by integrating a basic equation for \(A_{ij}\): \(\ref{5.24}\)

\[
A_{ij} = e^{\int \theta dt} \left\{ C^{(2)}_{ij} - F^{(2)}_{ij} \int \left( \frac{\alpha^2}{a^2} e^{-\int \theta dt} \right) dt \right\},
\]

with a sound speed squared: \(c_T^2\);
\[
c_T^2 = \frac{F_B}{\Xi} = \frac{2 B_2 \alpha + \partial_t B_5}{(2 A_4 \alpha + 6 H A_5)},
\]

and we defined two new quantities:
\[
\Theta = \frac{F_C}{\Xi} = -3H - \partial_t (\ln \Xi),
\]
\[
F^{(2)}_{ij} (x) = \frac{\hat{R}_{ij}(\gamma, \zeta)}{e^{2 \zeta}},
\]

and integral constant: \(C^{(2)}_{ij}\). When one can use this solution, it is easy to integrate \(\ref{2.40}\) and reads
\[
\gamma_{ij} = \gamma_{ij}^{(0)} + D(t) C^{(2)}_{ij} + F(t) \frac{D(t)}{\Xi} + \mathcal{O}(\epsilon^4),
\]

where
\[
D(\tau) = 2 \int \tau \frac{\alpha(\tau')}{z^2} d\tau',
\]
\[
F(t) = -2 \int \tau \frac{\alpha(\tau')}{z^2} \left( \int \tau' c_T^2 z^2 (\tau'') d\tau'' \right) d\tau',
\]

with
\[
z := a \sqrt{\Xi} = a \sqrt{-(A_4 + 3 A_5 H/\alpha)}.
\]

where we used \(e^{\int \theta dt} = 1/(a z^2)\) from \(\ref{5.47}\). Here \(\gamma_{ij}^{(0)}\) represents some spatial function as an integral constant at the leading-order. The integral \(D(t)\) and \(F(t)\) are related to decaying and growing modes, respectively. In the GR limit, we have \(z = a\). So the integral results \(D(t) \propto \int a^{-3} dt\), that shows decaying mode. In this case, the integral \(\int z^2 d\tau\) can be reduced to \(\int d\tau\), corresponding to a growing mode. Therefore, the integrals tell us to give a possibility that tensor perturbation can be enhanced when \(z\) decreases or \(z^2 c_T^2\) increases in \(\tau\).

Next, we can easily obtain one master evolution equation for tensor perturbation \(\gamma_{ij}\) by using the solution of \(\ref{5.49}\) as
\[
\frac{\partial^2 \gamma_{ij}}{\partial \tau^2} + 2 \frac{\partial}{z} \frac{\partial \gamma_{ij}}{\partial \tau} + 2 c_T^2 F^{(2)}_{ij} [\gamma] = \mathcal{O}(\epsilon^4).
\]

Here the meaning of the term \(c_T^2\) defined in \(\ref{5.40}\) denotes some propagation speed of gravitational waves. Note that the GR limit always gives constant sound speed \(c_T = 1\) with \(A_4 = -B_4 = -1/2\) and the functions \(A_5\) and \(B_5\) can allow us to change \(c_T^2\). The value of \(\ref{5.40}\) is consistent with the linear perturbation theory of GLPV theory in \(\ref{27}\). If one consider the time varying \(B_5\), the speed of gravitational waves can change in time. It is related to enhancement of gravitational waves on superhorizon scale via time evolution on the effect of \(\mathcal{O}(\epsilon^2)\). And also the variable \(z\) of \(\ref{5.52}\) can lead to the condition of the enhancement of \(\ref{5.53}\) is compatible with linear equation \(\ref{27}\):
\[
\frac{\partial^2 \gamma_{ij}}{\partial \tau^2} + 2 \frac{\partial}{z} \frac{\partial \gamma_{ij}}{\partial \tau} - c_T^2 \Delta \gamma_{ij} = 0.
\]
The source term (5.48) is a given function only depending on spatial coordinate via (5.54). This term is source term for a time evolution of $\gamma_{ij}$ and includes all full-nonlinear interaction over scalar and tensor modes. Actually, when you expand (5.48) as any interaction via $(\gamma_{ij} \times \gamma_{ij})$, $(\gamma_{ij} \times \zeta)$, $(\zeta \times \zeta)$, etc. That gives a main difference between linear and nonlinear theory. It maybe affect non-Gaussianity of tensor modes. Of course, (5.55) can be reduced to a usual GR linear theory as

$$\frac{\partial^2 \gamma_{ij}}{\partial \tau^2} + 2H \frac{\partial \gamma_{ij}}{\partial \tau} - \Delta \gamma_{ij} = 0,$$  

(5.55)

where the last source term can be derived from $\tilde{R}_{ij}[\gamma] = -\Delta \gamma_{ij}/2 + O(\gamma^2)$ in (5.33) with $c_s^2 = 1$ and $z = a$ in the GR limit.

This basic equation (5.55) is a main result for gravitational mode since it contains tensor mode plus scalar mode $\chi$ defined by (3.10). The quantity $\chi$ is important for construct nonlinear curvature perturbation: $\mathcal{R}_c^{NL}$ in (5.30), whose contribution of the next-leading order in expansion is given by $\chi = O(\epsilon^2)$. In the GR limit, the evolution equation (5.55) can be reduced to (3.11) via $2\partial_\tau \ln z = 2H$ and $c_s^2 = 1$.

On the other hand, the tensor perturbation can be extracted from $\gamma_{ij}$ by taking a perturbation for $h_{ij} = \gamma_{ij} - \delta_{ij}$ in the weak limit of $h_{ij}$ and using the following decomposition; where all symmetric traceless tensors: $X_{ij}$ can be decomposed as

$$X_{ij} = \frac{3}{2}(k_i \tilde{k}_j - \frac{1}{3} \gamma_{ij})X_{\parallel} + \sum_{a=2,3} \tilde{k}_a (e_{ij}^a)X_a$$

$$+ \sum_{\lambda = +, \times} c_{ij\lambda}^a \tau \lambda X_{\lambda},$$  

(5.56)

where $k_i$ denotes a constant comoving co-vector since standard plane waves are used as basis at each constant time hypersurface, while the direction of wave vector: $k^i$ changes with time. Here we defined the unit vector $k = k^i/\sqrt{k_i k^i}$, orthogonal basis set spanning the constant time hypersurface: $(k_1, e_1^i, e_2^i)$, and a polarization tensor $c_{ij\lambda}^a$ defined as $c_{ij\lambda}^a = (e_1^i e_j^a - e_1^j e_i^a)\delta_{\lambda a} / \sqrt{2} + (e_2^i e_j^a + e_2^j e_i^a)\delta_{\lambda a} / \sqrt{2}$. By using this method, the tensor perturbation $h_{ij}$ also can be decomposed into $(h_{ij}, h_a, h_{+, \times})$. The physical tensor perturbations, namely gravitational wave modes are $h_{+, \times}$ in the decomposition.

VI. SUMMARY AND DISCUSSION

In this paper, we developed a theory of nonlinear cosmological perturbations on superhorizon scales in the context of inflationary cosmology. First, we followed GR gravity plus a general kinetic single inflaton. In this case, the energy-momentum tensor for the scalar field is equivalent to that of a perfect fluid. We have solved the field equations using spatial gradient expansion in terms of a small parameter $\epsilon = k/(aH)$, where $k$ is a wavenumber, and obtained a general solution for the metric and the scalar field up to $O(\epsilon^2)$. Then we show a matching condition between nonlinear solution and linear solution, but including $k^2$ effect. The master evolution equation is a key result to characterize this system compatible to similar evolution equation in linear perturbation theory.

Second, we extend this formalism to apply Galileon-inflation, for which the inflaton Lagrangian is added by $G(X, \phi)\Box \phi$. In the case of G-inflation, it can no longer be recast into a perfect fluid form, and hence its imperfect nature shows up when the inhomogeneity of the Universe is considered. We have solved the field equations using spatial gradient expansion and also obtained a general solution up to $O(\epsilon^2)$. Then we introduce an appropriately defined variable for the nonlinear curvature perturbation in the uniform $\phi$ (comoving) gauge, $\mathcal{R}_c^{NL}$. Upon linearization, this variable reduces to the previously defined linear curvature perturbation $\mathcal{R}_c^{Lin}$ on a comoving hypersurfaces. It has been also shown that $\mathcal{R}_c^{NL}$ satisfies a nonlinear second-order differential equation (2.21), which is a natural extension of both linear perturbation equation for $\mathcal{R}_c^{Lin}$ and nonlinear for $\mathcal{R}_c^{NL}$ shown in (2.22).

We have shown some applications of our formalism and the effect is sensitive to the temporal violation of some kind of slow-roll conditions. So our formalism is good to evaluate time evolution of curvature perturbation in the physics for violation of slow-roll conditions.

We considered a multi-component scalar field with a general kinetic term and a general form of the potential. To discuss the superhorizon dynamics, we employed the ADM formalism and the spatial gradient expansion approach. Different from the single-field case, there is a difficulty in solving the equations in the multi-field case. At leading-order, the equations take the same form as those for the homogeneous and isotropic FLRW background with suitable identifications of variables.

In cosmological perturbation theory, the most important quantity to be evaluated is the curvature perturbation on the comoving slices which is conserved on superhorizon scales after the universe has reached the adiabatic limit. This quantity accurate to next-to-leading order may be relatively easily obtained in the single-field case because of the above mentioned coincidence among several temporal slicings. On the other hand, in the multi-field case, such a coincidence between different slicings does not hold. We first solve the field equations in a slicing in which the lapse function is trivial. We adopt the uniform $e$-folding number slicing in which the time slices are chosen in such a way that the number of $e$-folds along each orbit orthogonal to the time slices, $X$, is spatially homogeneous on each time slice. Then we can solve the equations to next-to-leading order without encountering the above mentioned problem. After the solution to next-to-leading order is obtained, we transform it to the one in the uniform expansion slicing which is known to be identical to the comoving slicing on superhorizon scales.
in the adiabatic limit. Thus the gauge transformation laws play an essential role in our formalism. We derived them which are accurate to next-to-leading order. Note that they are fully nonlinear in nature in the language of the standard perturbation approach.

Finally, we show an extension of our formalism to beyond Einstein gravity, that is general scalar-tensor theory which can lead to several kinds of modified gravity. These theories are motivated not only inflation, but also the topic of dark energy. We used beyond Horndeski (GLPV) theory at the uniform topic of dark energy. We used beyond Horndeski theory, equivalently the most general second-order in the equation of motions. We construct a dynamical equation for superhorizon tensor perturbation with a full nonlinear interaction between scalar and tensor perturbation in (5.53) with \( z = a / \sqrt{-(A_4 + 3A_5 H / \alpha)} \) as (5.52). The GR case of \( A_4 = -1/2 \), \( A_5 = 0 \) gives us \( z = a \), but in general case of modified gravity n this case, \( z \) can change depending on the models, namely a time varying of \( A_4 \) and \( A_5 \). The integral \( \int z^2 d\tau \) in (5.50) and \( \int z^2 c_s^2 d\tau \) in (5.51) corresponding to a decaying and growing mode, respectively. Therefore, the integrals tell us to give a possibility that tensor perturbation can be enhanced when \( z \) decreases or \( z^2 c_s^2 \) increases in \( \tau \). More application to calculate non-Gaussianity of gravitational waves will be a future task.

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