EXPERIMENTAL ANALYSIS OF ENCLOSURE ASPECT RATIO INFLUENCE ON THERMO-MAGNETIC CONVECTION HEAT TRANSFER

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Abstract. Experimental analysis of strong magnetic field influence on paramagnetic fluid’s convection in rectangular enclosures with aspect ratios (AR=height/width) 0.5 and 2.0 was conducted. For this purpose, enclosure filled with paramagnetic fluid was placed inside superconducting magnet in Rayleigh–Bénard configuration and temperature measurements of the fluid were performed, with and without magnetic field induction. On the basis of performed research, analysis of heat transfer and character of the flow was conducted. Obtained results allowed characterization of fluid behavior and demonstrated that the aspect ratio has an immense influence on quantitative heat transfer in the system.

1. Introduction
Natural convection in closed systems has been studied with a wide range of configurations since it occurs in numerous natural and industrial processes, especially as heat exchangers. Buoyancy-driven convection is a common phenomenon in many applications, and means to enhance heat exchange are often sought. A strong magnetic field can be an alternative in controlling natural convection process. One way to achieve that is using non-electrically conducting paramagnetic fluid and applying strong magnetic field gradient to the system. Both suppression and enhancement of convective motion could be achieved. In 1991 Braithwaite [1] used magnetic field to both enhance and suppress convection of paramagnetic fluid and Tagawa [2] developed a model equation for thermo-magnetic convection. Bednarz [3] studied thermo-magnetic convection in configuration with one side wall heated and the opposite one cooled, and Pyrda et al. investigated various aspects of thermo-magnetic convection in transient and turbulent flow regimes in cubical enclosures, i.e: heat transfer enhancement [4], non-dimensional analysis [5], oscillatory states and regimes [6], Rayleigh–Bénard–Kelvin Convection [7]. Pleskacz [8] researched magnetic field impact on high and low Reynolds number flows. New studies are carried out in the field concerning usage of nanofluids and magnetic gradients to enhance heat transfer [9].

In 2012 Turan [10] published research concerning laminar convection in systems with different aspect ratios (AR=height/width). He concluded that AR has crucial impact on heat transfer in investigated configurations. Due to lack of experimental data available in literature concerning influence of strong magnetic field on paramagnetic fluid’s convection in systems with different aspect ratios, Authors performed experimental analysis of thermo-magnetic convection in enclosures with aspect ratios 0.5 and 2.0.
2. Experiment

2.1. Experimental apparatus
The experimental setup is presented in Figure 1. It consisted of two experimental enclosures placed in the bore of a superconducting magnet, a heater control system, a water thermo-stated bath with a constant temperature and a data acquisition system connected to a computer. To calculate magnetic field distribution and its gradient, besides real dimensions of system presented on Figure 1(right), current density in internal and external coil should be taken into account, which were equal to $140.9 \times 10^{-6} \, [\text{A/m}^2]$ and $167.3 \times 10^{-6} \, [\text{A/m}^2]$ respectively at maximum value of magnetic field inside the centre of the magnet: $|b_{0,\text{max}}|=10 \, [\text{T}]$.

Experimental enclosures are shown schematically in Figure 2. Plexiglass rectangular vessels, of size $0.032 \, [\text{m}] \times 0.032 \, [\text{m}]$ in base and of $0.016 \, [\text{m}]$ (Enclosure A) and $0.064 \, [\text{m}]$ (Enclosure B) in height were heated with a constant heat flux from the bottom and isothermally cooled from the top, while four remaining walls were insulated. The bottom wall consisted of a copper plate heated with nichrome wire connected to a DC power supply. The electric voltage and current were measured with multi-meter.

Top wall was also composed of copper plate with built in cooling chamber, which was cooled by cold water running from a thermostat. Temperature of heated and cooled walls was measured with six T-type thermocouples inserted into small holes in each plate. Five (Enclosure A) and six (Enclosure B) thermocouples, colour-coded (Figure 2) were inserted into small holes in one wall of the experimental enclosure at 6 [mm] deep and measured temperature of the fluid during tests.

![Figure 1](image.png)

Figure 1. Experimental setup and location of experimental enclosure in the magnet.

2.2. Working fluids
The working fluids were 50% volume glycerol aqueous solutions. Since both water and glycerol are diamagnetic, an addition of 0.8 mol/(kg of solution) of gadolinium nitrate hexahydrate Gd(NO$_3$)$_3$∙6H$_2$O crystals was made to make them paramagnetic. Experimental analyses for both geometries were performed about a year apart, therefore two slightly different fluids were used: P50-A for Enclosure A, and P50-B for Enclosure B. The main goal was to maintain similar magnetic properties of the fluids (mass magnetic susceptibility), which was achieved during preparation of the
second fluid P50-A, but viscosity, density and therefore, thermal expansion coefficient, for fluid P50-A are slightly different than for fluid P50-B. Magnetic susceptibility of the fluids was measured with a magnetic susceptibility balance by Evan’s method. The densities of the working fluids were measured with a pycnometer and thermal expansion coefficient was calculated. Viscosities were measured using Ubbelohde viscometer and other properties were taken from [7]. Fluids properties are listed in Table 1.

![Figure 2. Experimental enclosures: A with AR=0.5 and B with AR=2.0.](image)

**Table 1.** Properties of the working fluid at 298 K.

| Property                              | Symbol | Value           | Unit          |
|---------------------------------------|--------|-----------------|---------------|
| Heat capacity                         | \(c_p\) | 2.92 \times 10^3 | [J/kg·K]      |
| Thermal diffusivity                   | \(\alpha\) | 9.13 \times 10^{-8} | [m²/s]       |
| Thermal expansion coefficient         | \(\beta\) | 1.21 \times 10^{-5} | [1/K]         |
| Dynamic viscosity                     | \(\mu\) | 1.30 \times 10^{-2} | [kg/m·s]     |
| Thermal conductivity                  | \(\lambda\) | 0.376 | [W/m·K]     |
| Kinematic viscosity                   | \(\nu\) | 9.25 \times 10^{-6} | [m²/s]       |
| Density                               | \(\rho\) | 1411 | [kg/m³]      |
| Mass magnetic susceptibility          | \(\chi_m\) | 2.39 \times 10^{-7} | [m³/kg]      |

2.3. Experimental procedure
For both analyzed enclosures procedure was the same. First step was connected with estimating the heat losses from the systems. Both enclosures (separately) were filled with water and placed in the predestined position, but the vessels were rotated 180 degrees, so the cooled wall was at the bottom and the heated one on top. This configuration allowed to reach conduction state in the fluid with temperature stratification. After setting the temperature at bottom and top walls and waiting for the fluid and temperature fields to stabilize, linear temperature distribution was achieved and heating power was measured. With the assumption of one-dimensional conductive heat flow, the heat flux can be calculated from the Fourier’s law of conduction. Therefore, the difference between directly
measured heat flux on the heated wall and heat flux calculated from Fourier’s law, is the heat loss from the measuring system. Heat losses from enclosures A and B are shown in Figure 3. Second step of experimental analysis was natural convection and thermo-magnetic convection measurements. Experimental enclosures where therefore rotated, back to Rayleigh-Bénard configuration and temperature difference between thermally active walls was set. After achieving thermal stabilization, 15-minute recording of temperature signal was obtained. Subsequently magnetic induction was set at desired value, experimental system was left to stabilize and then measurement of temperature took place. Those steps were repeated for values of magnetic induction up to 10 [T].

3. Experiment

Temperature series recorded during experiments enabled investigation of two aspects: the heat transfer rate and flow behaviour. The heat transfer rate was established by calculation of Nusselt number.

![Figure 3. Heat losses in the measurement setup versus temperature difference.](image)

3.1. Heat transfer rate

Nusselt number is a dimensionless criterion speaking of heat transfer in the system, and can be written as follows:

$$\text{Nu} = \frac{Q_{\text{net, conv}}}{Q_{\text{net, cond}}}$$  \hfill (1)

The net conduction ($Q_{\text{net, cond}}$) and net convection ($Q_{\text{net, conv}}$) heat fluxes were estimated by the method proposed by Churchill and Ozoe [11], which is based on following formulas:

$$Q_{\text{net, cond}} = Q_{\text{cond}} - Q_{\text{loss}}$$  \hfill (2)

$$Q_{\text{net, conv}} = Q_{\text{conv}} - Q_{\text{loss}}$$  \hfill (3)

As said in section 2.3, it was assumed that the heat loss depends only on the temperature of the heated wall. As a first step to determine the Nusselt number, conduction measurements were made and the heat losses were estimated from:

$$Q_{\text{loss}} = Q_{\text{cond}} - Q_{\text{Fourier's law}}$$  \hfill (4)

where

$$Q_{\text{theor, cond}} = \frac{a^2 \lambda \Delta T}{d}$$  \hfill (5)

$d$ - enclosure height (Enclosure A – 0.016 [m], Enclosure B – 0.064 [m]); $a$ – enclosure width 0.032 [m]; $\lambda$ – thermal conductivity of the fluid [W/m·K]; $\Delta T$ – temperature difference between heated and cooled walls [K].

Heat flux was calculated for conduction area of 0.032 [m] x 0.032 [m]. The estimated heat loss was approximated linearly for enclosure A:
and enclosure B:

\[ Q_{\text{loss}} \ A = 0.0831 \cdot \Delta T \]  

(7)

Applying equations 2, 4 and 5 to equation 1, Nusselt number can be expressed as:

\[ \text{Nu} = \frac{Q_{\text{conv}} - Q_{\text{loss}}}{a^2 \lambda \Delta T / d} \]  

(8)

The convection heat flux \( Q_{\text{conv}} \) is given by the product of current and voltage of the heater supply.

Results

The results of heat transfer rate analysis are presented in Figure 4 and 5, and the results of fluid flow behaviour are presented in Figure 6 and 7. Thermo-magnetic Rayleigh number is defined as follows:

\[ \text{Ra}_{\text{TM}} = \text{Ra}_T + \text{Ra}_M \]  

(9)

where: \( \text{Ra}_T \) is thermal Rayleigh number:

\[ \text{Ra}_T = \frac{g \beta (T_h - T_c) d^3}{\alpha v} \]  

(10)

\( \text{Ra}_M \) is magnetic Rayleigh number:

\[ \text{Ra}_M = \left[ \frac{1}{\beta T_0} \right] \left[ \frac{g \beta (T_h - T_c) d^3}{2 \alpha v} \right] \]  

(11)

where: \( \gamma \) is magnetization number:

\[ \gamma = \frac{\chi |b_0|^2_{\text{max}}}{\rho \mu_m \gamma d} \]  

(12)

and: \( g \) – gravitation acceleration, \( \beta \) – thermal expansion coefficient, \( \mu_m \) – vacuum magnetic permeability, \( v \) – kinematic viscosity, \( \rho \) – density, \( \chi \) – magnetic susceptibility, \( d \) – characteristic dimension, \( \alpha \)– thermal diffusivity, \( T_h \) – temperature at the heated wall, \( T_c \) – temperature at the cooled wall, \( T_0 \) – reference temperature, \( |b_0|^2_{\text{max}} \) – magnetic induction in the center of the magnet.

Figure 4 and 5 show that aspect ratio has tremendous impact on heat transfer rate. For Enclosure A, which has aspect ratio 0.5, Nusselt number is significantly lower than for Enclosure B with aspect...
ratio 2.0. In Enclosure A Nusselt number values start from 2.95 for $\Delta T=5\, ^\circ C$, 3.46 for $\Delta T=3\, ^\circ C$ and 3.62 for $\Delta T=11\, ^\circ C$ while in Enclosure B it is 13.45, 14.50 and 18.82 for $\Delta T=3$, 5 and 11 $^\circ C$ respectively for case without magnetic induction, that is where magnetic Rayleigh numbers is equal to 0 and therefore $Ra_{TM}=Ra_T$ according to equation (9). When magnetic field is applied to the system, in both cases, Nusselt number increased significantly. For magnetic field of 10 [T], increase of heat rate transfer in Enclosure A for $\Delta T=3\, ^\circ C$ is over 250% to $Nu=9.21$, while in every other case improvement in Nusselt number is over 300%, where $Nu$ reaches values: 11.02 for $\Delta T=5\, ^\circ C$ and 12.66 for $\Delta T=11\, ^\circ C$ and in Enclosure B to 40.34 , 48.91 and 59.33 for $\Delta T=3$, 5 and 11 $^\circ C$ respectively.

Results of temperature time-series obtained for Enclosure A and Enclosure B are shown on Figure 6 and 7 respectively. Colours of the lines in the presented graphs represent thermocouples placed in the experimental enclosure in particular locations, as shown in Figure 2.

![Figure 6](image)

**Figure 6.** Temperature versus time series at $|b_0|_{max}=0\, [T]$ (left) and $|b_0|_{max}=10\, [T]$ (right) for Enclosure A.

For cases of natural convection measurements for Enclosure A temperature lines during tests were horizontal and stable (left side on Figure 6). Thermal Rayleigh number for Enclosure A was $1.84 \cdot 10^3$, $3.12 \cdot 10^3$ and $7.64 \cdot 10^3$ for $\Delta T=3$, 5, 11 $^\circ C$ respectively and for Enclosure B, $Ra_T=3.14 \cdot 10^4$; $4.05 \cdot 10^4$ and $9.19 \cdot 10^4$ for $\Delta T=3$, 5, 11 $^\circ C$. Initial pattern of temperature signals are correlated with results obtained by other researchers focused on pure natural convection phenomenon in closed systems, i.e: Heslot [12].
During stepwise increasing of magnetic induction up to 10 [T], temperature signals from thermocouples tend to unify temperature values at all thermocouple positions and start to oscillate over time. This suggest change of fluid flow pattern during this process.

Figure 7. Temperature versus time series at \( |b_0|_{\text{max}} = 0 \) [T] (left) and \( |b_0|_{\text{max}} = 10 \) [T] (right) for Enclosure B.

5. Summary and conclusions

In this paper experimental analysis of aspect ratio influence on thermo-magnetic convection of paramagnetic fluid was presented. Two enclosures were investigated: Enclosure A with aspect ratio 0.5 and Enclosure B with \( \text{AR}=2.0 \). The influence of various magnetic field inductions on analyzed systems were performed. Estimation of Nusselt number and behaviour of the fluid were performed.

Obtained results showed that aspect ratio of the measurement vessel has an immense impact on heat transfer rate. While for Enclosure A Nusselt number starts from 2.95 to 3.61, for Enclosure B heat transfer is almost at least four times greater for natural convection cases. Performed analyses also demonstrated that magnetic field strongly enhance heat transfer. For every aspect ratio and all temperature differences, changing magnetic field induction from 0 [T] to 10 [T] caused an increase in heat transfer for at least 250%.
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References
[1] Braithwaite D, Beaugnon E and Tournier R 1991 *Nature* 354 134–6
[2] Tagawa T, Shigemitsu R and Ozoe H 2002 *Int. J. Heat Mass Transf.* 45 267–77
[3] Bednarz T, Fornalik E, Tagawa T and Ozoe H 2006 Comparison between Experiment and Numerical Computations- 14 107–14
[4] Pyrda L, Kenjeres S, Fornalik-Wajs E and Szmyd J S 2012 *J. Phys. Conf. Ser.* 395 012125
[5] Pyrda L 2014 *J. Phys. Conf. Ser.* 530 012060
[6] Kenjereš S, Pyrda L, Wrobel W, Fornalik-Wajs E and Szmyd J 2012 *Phys. Rev. E* 85 1–8
[7] Kenjeres S, Pyrda L, Fornalik-Wajs E and Szmyd J S 2014 *Flow, Turbul. Combust.* 92 371–93
[8] Pleskacz L and Fornalik-Wajs E 2014 *J. Phys. Conf. Ser.* 530 012062
[9] Roszko A, Fornalik-Wajs E, Donizak J, Wajs J, Kraszewska A, Pleskacz L and Kenjeres S 2014 *MATEC Web Conf.* 18 03006
[10] Turan O, Poole R J and Chakraborty N 2012 *Int. J. Heat Fluid Flow* 33 131–46
[11] Churchill S W and Ozoe H 1973 *AIChE Symp. Ser.* 69 126-133
[12] Heslot F, Castaing B and Libchaber A 1987 *Phys. Rev. A* 36 5870–3