THE EVOLUTION OF COOL ALGOLS

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ABSTRACT

We apply a model of dynamo-driven mass loss, magnetic braking, and tidal friction to the evolution of stars with cool convective envelopes; in particular, we apply it to binary stars where the combination of magnetic braking and tidal friction can cause angular momentum loss from the orbit. For the present we consider the simplification that only one component of a binary is subject to these nonconservative effects, but we emphasize the need in some circumstances to permit such effects in both components. The model is applied to examples of (1) the Sun, (2) BY Dra binaries, (3) Am binaries, (4) RS CVn binaries, (5) Algols, and (6) post-Algols. A number of problems regarding some of these systems appear to find a natural explanation in our model. There are indications from other systems that some coefficients in our model may vary by a factor of 2 or so from system to system; this may be a result of the chaotic nature of dynamo activity.

Subject headings: binaries: eclipsing — stars: evolution — stars: magnetic fields — stars: mass loss

1. INTRODUCTION

In a previous paper (Nelson & Eggleton 2001, hereafter Paper I) we constructed a large grid of models of case A binary evolution, according to the assumption of conservative evolution. We found that these fitted reasonably the parameters of certain observed "hot Algols," i.e., semidetached binaries in which both components were earlier than about spectral type G0. We also found that agreement was quite poor for some "cool Algols," by which we mean those in which at least one component is later than G0. Since several of the latter appeared to disagree on account of having less angular momentum, and/or less total mass, than the theoretical models, we suggested that the discrepancy is due to dynamo activity in stars with cool convective envelopes. Such activity can reasonably be expected to be substantially greater than in single stars of the same spectral type because components in Algols are typically rotating much faster than single stars, or stars in binaries where the orbital period is quite long. This activity may reasonably be expected to carry off both mass and angular momentum, but whether it can carry off the right amounts is not clear and is the main subject of this paper.

In another paper (Eggleton 2001, hereafter Paper II), one of us (P. P. E.) suggested a simplistic model of dynamo activity, suitable for inclusion in a binary-evolution code. In the present paper we present some results. The model is simplistic in the sense that it gives the mass-loss rate, the overall poloidal magnetic field, and the consequent magnetic braking rate as functions of just four parameters: the mass, luminosity, radius, and rotational period of the star. In order that the magnetic braking should be able to drain angular momentum from the orbit, it is necessary to include also a model of tidal friction. Of course, this is also necessary to get the star to spin faster in a binary than it would if it were single. We follow the prescription of Eggleton & Kiseleva (2001, hereafter Paper III) but specializing to the simple case of only two bodies, and stellar spin parallel to orbital spin.

In §2 we discuss briefly the implementation of these models in a stellar evolution code, noting that there are some considerable approximations that influence particularly low mass systems, where quite probably both components contribute to mass loss and angular momentum loss. In this paper we allow only the initially more massive star to be subject to these processes. This is forced on us by numerical considerations, but we hope to circumvent them in the future. In §3 we discuss our results and consider the further evolution of such systems. In §4 we consider what may be required in future modeling.

2. INCORPORATING NONCONSERVATIVE EFFECTS IN A BINARY STELLAR EVOLUTION CODE: THE SINGLY NONCONSERVATIVE MODEL

Several aspects of the evolution code used here have been described elsewhere: we refer particularly to Paper I. Because of the non-Lagrangian nature of the mesh-point distribution, it is particularly easy to add to the mass transfer rate (due to Roche lobe overflow [RLOF]) an additional mass-loss rate due to stellar wind, provided that it is a function only of surface parameters such as mass, radius, and luminosity, but we expect it to depend also on the stellar rotation rate $\Omega$, presumably through the Rossby number.

We have first to add to the usual structure equations a further (rather trivial) differential equation for the moment of inertia. We assume here uniform rotation in a star, appealing to the argument of Spruit (1998) that internal magnetic field, even if rather weak, is liable to wipe out such differential rotation as might otherwise be expected when the core shrinks and the envelope grows. We have to include $\Omega(t)$ as a variable to be solved for, along with the variables normally determined by a stellar evolution code.

We further have to include two more unknown functions of time only that have to be solved for: the orbital angular momentum ($H_{\text{orb}}$) and the eccentricity ($e$). The solution package in the code allows for the possibility that along with the normal variables that change with position, i.e., pres-

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For the sake of a name, we call such quantities “eigenvalues,” since like eigenvalues they are constant in space but vary in time as the structure varies. To our usual set of 10 functions \( m, r, L, p, T, X_1, X_4, X_{12}, X_{14}, \) and \( X_{16}, \) i.e., mass, radius, luminosity, pressure, temperature, and five composition variables and one eigenvalue (a constant that normalizes the mesh-point distribution), we therefore add one more function (\( I \)) and three more eigenvalues (\( \Omega, H_{\text{orb}}, \) and \( e \)). We have one more differential equation to add—for \( I(r) \)—and four more boundary conditions. One is \( I = 0 \) at the center, and the other three are equations at the surface for the rates of change with time of \( H_{\text{orb}}, e, \) and \( \Omega. \)

In the \textit{conservative} approximation to RLOF it is very convenient that it is possible to split the evolutionary calculation into two separate parts. First, we compute the evolution of star 1, allowing it to lose mass if and when its radius exceeds its Roche lobe radius. We can in principle follow star 1 until it becomes either a white dwarf or a supernova, without reference to the internal structure of star 2. The only information we need about star 2 during this calculation is its mass, which is anyway uniquely determined by the mass of star 1 and the assumption of conservative mass exchange. Subsequently, we can follow the evolution of star 2, giving it a mass-gain rate that is the negative of star 1’s loss rate. This separation becomes invalid only when star 2 reaches its own Roche lobe—as must happen eventually. We then have to throw away the subsequent evolution of star 1, which will no longer be valid; but at least we have a model for the evolution of both stars up to this point.

This conveniently simple procedure unfortunately breaks down for nonconservative evolution, if \textit{both} stars are subject to stellar wind and magnetic braking, since the orbit of star 1 will be varying at a rate partly due to the nonconservative processes on star 2. The best solution to this will be to follow both stars \textit{simultaneously}, with each component subject to boundary conditions that involve parameters at the surface of both components. We are developing such a “doubly nonconservative” \textit{(DNC) code}, but for the present we content ourselves with a “singly nonconservative” \textit{(SNC) model}, in which only star 1 is subject to mass loss and magnetic braking. We can partly justify this on the grounds that in many Algols star 2 (now the hotter and more massive component) is of type A or B and probably not as subject to dynamo activity as star 1, which is typically a red giant or subgiant. We return to this point subsequently.

Assuming, therefore, that only star 1 is subject to dynamo activity, we can continue with the procedure outlined above: follow star 1 first and subsequently star 2. The boundary conditions at the surface of star 1 are written below, more or less directly in the form that they are computed in the code.

Since we are using an implicit technique, we are provided with initial guesses of the following quantities at the surface: \( M_1, R, L, I, P_{\text{rot}}, H_{\text{orb}}, e, \) and \( M_2. \) Note that we omit the subscript 1 from all quantities relating to star 1 except mass, since, in regard to star 2, only the mass, and no other quantity, is involved. We then compute in order, according to equations (1)–(13) below, the following quantities:

1. \( R_{\text{HT}}, \) a semi-empirical approximation to the Hayashi-track radius of a star of mass \( M \) and luminosity \( L; \) the ratio \( R/R_{\text{HT}} \) (0.55 for the Sun) is a convenient measure of how convective a star is, and the fractional depth of the surface convection zone is roughly proportional to its 2.1th power;
2. \( t_{\text{ET}}, \) the envelope turnover timescale for the convective envelope;
3. \( R_0, \) the Rossby number, and \( \Omega, \) the intrinsic spin rate;
4. \( \zeta, \) the mass-loss rate by dynamo-driven stellar wind;
5. \( B_p, \) the mean poloidal magnetic field;
6. \( R_A, \) the Alfvén radius of the stellar wind;
7. The rate of loss of angular momentum from the system;
8. \( a \) and \( \omega, \) the semimajor axis and mean orbital frequency of the binary; and \( M \) and \( q, \) the total mass and the mass ratio;
9. \( t_{\text{Tf}} \), an estimate of the timescale of tidal friction;
10. The rate of change of eccentricity due to tidal friction;
11. The rate of loss of angular momentum from the orbit: not the same as quantity 7 above because angular momentum can be exchanged between the orbit and the stellar spin, through tidal friction, as well as carried away from the system by wind;
12. \( R_L, \) the Roche lobe radius of the star—assuming, as is reasonable, that the orbit will have circularized before \( R_L \) becomes important; and
13. \( M_1, \) the mass-loss rate of star 1, a combination of RLOF if it occurs and of stellar wind \( \zeta \) above; and also \( M_2, \) the mass-loss rate of the binary.

In the following equations, \( M \) is in units of \( 10^{33} \) g, \( R \) in units of \( 10^{11} \) cm, \( L \) in units of \( 10^{33} \) ergs s\(^{-1} \) (time including inverse angular frequencies) in seconds, angular momentum in units of \( 10^{33} \) cm\(^2\) g s\(^{-1} \), and magnetic field in Gauss. These imply that the Newtonian constant \( G \) is \( 10^{11} \) times its usual value.

\[
R_{\text{HT}} = \frac{0.755L^{0.47} + 0.05L^{0.8}}{M_1^{0.31}},
\]

\[
t_{\text{ET}} = 1.04 \times 10^7 \left( \frac{M_1 R_2^2}{L} \right)^{1/3} \frac{R}{R_{\text{HT}}}^{2.7},
\]

\[
R_0 = \frac{P_{\text{rot}}}{t_{\text{ET}}}, \quad \Omega = \frac{2\pi}{P_{\text{rot}}},
\]

\[
\zeta = 1.54 \times 10^{-17} \frac{R L}{M_1} \left( \frac{R}{R_{\text{HT}}} \right)^2 (1 + 2.8R_0^2)^{-3.67},
\]

\[
B_p = 54 \left( \frac{M_1}{R^3} \right)^{1/2} \left( \frac{R L}{M_1} \right)^{1/3} \left( \frac{R}{R_{\text{HT}}} \right)^{3.4} (1 + 2.8R_0^2)^{-1.21},
\]

\[
\left( \frac{R_A}{R} \right)^{3/2} = 2.5 \times 10^{-36} \frac{R^5 B_p^2}{M_1 \zeta^2},
\]
\[
\frac{d}{dt}(H_{\text{orb}} + I\Omega) = -\zeta(R_1^3 + \frac{2}{3} R_2^3)\Omega - \zeta \frac{M_2}{M M_1} H_{\text{orb}},
\]

(7)

\[
M \equiv M_1 + M_2, \quad q \equiv \frac{M_1}{M_2}.
\]

(8)

\[
t_{\text{TF}} = 3 \times 10^6 \left( \frac{M_1 R_1^2}{L} \right)^{1/3} \left( \frac{a}{R} \right)^8 \frac{M_1}{M_2} M.
\]

(9)

\[
de = -\frac{9e}{t_{\text{TF}}} \left\{ \frac{1 + (15/4)e^2 + (15/8)e^4}{(1 - e^2)^{13/2}} - \frac{11\Omega 1 + (15/4)e^2 + (1/8)e^4}{18\omega} \right\}.
\]

(10)

\[
R_L = a \frac{0.49 q^{3/5}}{0.6q^{2/3} + \ln(1 + q^{1/3})},
\]

(12)

\[
\dot{M}_1 = -\zeta - 1.58 \times 10^{-5} \left[ \ln \left( \frac{R}{R_L} \right) \right]^3, \quad M = -\zeta.
\]

(13)

In equation (13) the square brackets have a specific meaning:

\[
[X] \equiv \max(0, X),
\]

(14)

to the fact that even without magnetic linkage the wind will carry off some spin angular momentum, and the third is due to the fact that even if \( R_A = 0 \) and \( R = 0 \), i.e., the star is a point mass, wind (assumed spherically symmetric and fast) will carry off the same angular momentum per unit mass as resides in the orbital motion of star 1.

4. The terms involving tidal friction (those that include \( t_{\text{TF}} \)) come from the equilibrium tide theory (Hut 1981; Eggleton, Kiseleva, & Hut 1998). The timescale, as in \( t_{\text{TF}} \), has as a main factor the quantity \( (M_1 R_1^2/L)^{1/3} \), which has the dimensions of time and can be seen as a turbulence-driven timescale whether or not the star is actually convective.

5. If tidal friction is rather strong, equation (11) implies a transient equilibrium between spin-up and spin-down. In the case that \( \zeta = 0 \) this gives the usual pseudosynchronous period ratio \( \Omega/\omega \) (Hut 1981), but the ratio is modified if the mass-loss timescale \( (M_1/\zeta) \) is comparable to the tidal friction timescale.

6. Equations (7), (10), and (11) are very “stiff” if the tidal friction timescale is short, as it is in close binaries. However, this is no problem numerically because these equations, along with the stellar structure equations, are solved implicitly.

7. The mass-loss rate, emerging along with the magnetic field from an \((\alpha, \omega)\) model, turns out to resemble the empirical law of Reimers (1975), although not by design. It differs in having a factor dependent on the depth of convection and another factor dependent on Rossby number. For \( R = R_{\text{HT}} \), i.e., a fully convective star, and for \( R \sim 3 \), we obtain roughly Reimers’ law.

Equations (7), (10), and (11) are the three extra surface boundary conditions that are necessary to determine the three extra eigenvalues \( H_{\text{orb}}, \Omega, \) and \( e \). Equation (13) is a boundary condition that was already included in the conservative code (with \( \zeta = 0 \), of course).

3. RESULTS

We start by modeling the effects of mass loss, magnetic braking, and tidal friction in a single star and in detached binaries that have not yet evolved off the main sequence. We consider three problems:

1. The Sun;
2. BY Dra and YY Gem, two low-mass close binaries where both magnetic braking and tidal friction are likely to play a role; and
3. Am binaries, where it has been suggested (Abt & Bidelman 1969) that tidal friction in binaries with \( 2.5 \leq P \leq 100 \) days may have slowed the spin of an A star enough to allow the process of selective diffusion to create the observed composition anomalies.

3.1. The Sun

If, following our prescription, we evolve a 1 \( M_\odot \) star that starts with fairly rapid rotation (say 3 days), we do not get very good agreement with the present-day Sun. This turns out to be entirely because our model predicts a loss of mass of \( \sim 1.4\% \) during the early rapidly rotating phase. We found good agreement (Table 1) if we started with 1.014 \( M_\odot \); at age 4.57 Gyr the luminosity and radius agreed with measured values to better than 2% and 0.1%, respectively. The
rotational period was 24.8 days, only about 5% faster than observed. Agreement could no doubt be made exact by varying fractionally the initial period (and consequentially the initial mass), the metallicity \((Z = 0.02)\), and the mixing-length ratio \(\alpha = 2.0\); 63% of the mass loss occurred before age 0.25 Gyr, by which time the rotational period had increased to 6.6 days. The mass-loss rate at this point had dropped by less than a factor of 3 from its initial rate of \(5 \times 10^{-11} \ M_\odot \text{ yr}^{-1}\), but subsequently it dropped rapidly, as expected from our Lawrentian form of dependence on Rossby number, reaching \(3 \times 10^{-14} \ M_\odot \text{ yr}^{-1}\) currently. It is of course no coincidence that we can get the period and mass-loss rate about right, since these (and the current Alfvén radius) were used to normalize our formulae. Our predicted early loss of mass does not seem to contradict any obvious feature of the Sun and solar system, but it may have some relevance to problems of light-element abundances in the Sun.

3.2. BY Dra and YY Gem

BY Dra (Vogt & Fekel 1979) is a 6 day binary of two K dwarfs, with a rather high eccentricity \((e = 0.5)\). At least one component is spotted and rotates with a period of 4 days. This period is rather surprising because it is not the pseudosynchronous period that one might expect (~2 days). It is expected that pseudosynchronism is reached rather quickly, well before the orbit becomes circularized. Of course, the system is young, and perhaps it has not yet reached even pseudosynchronism. We find however that the combination of our tidal friction and magnetic-braking models can allow this system to have reached a fairly steady transient equilibrium in which magnetic braking, slowing the star down, is balanced by tidal friction, speeding it up.

Although there is the possibility that one or both components have not yet reached the zero-age main sequence (ZAMS), we restrict ourselves in this paper to models where both components “start” on the ZAMS. BY Dra’s orbit is well determined spectroscopically, but there are no eclipses. Consequently, although the mass ratio \(q = 1.2\) is known, the masses and radii are indeterminate. We adopt masses of \(0.71 + 0.58 M_\odot\), which assume \(\sin^3 i = 0.1\) and which seem reasonably appropriate for the late K spectral type. We started the system with orbital period and eccentricity both slightly greater than the present values and with a rotational period (for star 1 only) of 2 days. Note that throughout this paper “star 1” is the component that was initially the more massive. For some observed systems there may be scope for argument, but in later discussion we make our preference clear with this convention.

The left panel of Figure 1 shows how we expect the orbital period, rotational period, and eccentricity to develop, as functions of log(age), according to the prescriptions of § 2. We see that star 1 spins down rather rapidly (in ~30 Myr) to a period of about 5 days and then much more slowly, in transient equilibrium between the magnetic-braking torque and the tidal friction torque, as the period and eccentricity diminish from their original values to reach the present values at about ~300 Myr. The right panel of Figure 1 is a similar plot but with \(R_A\) from equation (6) reduced by a factor of 0.55, so that magnetic braking is diminished but not mass loss or tidal friction. The transient equilibrium period is about 3 days. Since the transient equilibrium lasts for a long time, while the initial spin-down (or spin-up, if we start with an initial rotational period that is substantially larger) is rather rapid, we feel that it is likely that the observed system has indeed reached its transient equilibrium value and therefore that the tidal friction timescale and the magnetic-braking timescale are rather comparable in this system, as predicted by our model.

We do not feel, however, that we can confidently renormalize our model on the basis of this one experiment. We have already mentioned some substantial uncertainties, principally in the masses and in the evolutionary state. If one or both components are pre–main-sequence stars, then the radii could be substantially larger and several parameters might be quite different, although there would probably still exist a transient equilibrium that could be at a similar rotational period. However, to investigate such a possibility we would have to have a clearer idea of the origin of this (and other) close binaries. The stars might not be coeval if the binary formed from some capture or exchange mechanism. At present there is no clear understanding of the origin of close binaries.

The two cases plotted in Figure 1 show very different behavior in the long-term future. In our canonical model (Fig. 1, left panel), magnetic braking is strong enough for the orbital period to decrease strongly after about 3 Gyr; the system will probably become a contact binary at ~6 Gyr and then, with further magnetic braking, a merged single star. With the slightly reduced magnetic braking (Fig. 1, right panel) this does not happen. Indeed, the orbital period ultimately increases because the mass of star 1, and so of the system as a whole, decreases significantly. By the end of the run shown in the right panel of Figure 1, \(M_1\) was reduced to 0.44 \(M_\odot\). Of course, this emphasizes the absurdity of not allowing star 2 to be correspondingly nonconservative. Provided that both stars stay near the ZAMS, it should be impossible for star 1 to become less massive than star 2.

YY Gem is another low-mass binary of short period (0.81 days); it is a distant part of the well-known sextuple system \(\alpha\) Gem. Because it eclipses, the masses (0.62 + 0.57 \(M_\odot\); Leung & Schneider 1978) and radii are much better determined and reasonably consistent with ZAMS models. We

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| Age (Gyr) | Mass \((M_\odot)\) | \(\log R\) \((R_\odot)\) | \(\log L\) \((L_\odot)\) | \(P_{\text{rot}}\) (days) | \(M\) \((M_\odot \text{ yr}^{-1})\) | \(\Delta M_{\text{scz}}^*\) |
|----------|---------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.0.......| 1.0140              | -0.0509         | -0.1497         | 2.94            | -5.32 \times 10^{-11} | 0.032           |
| 0.25.......| 1.0052              | -0.0456         | -0.1198         | 6.61            | -1.94 \times 10^{-11} | 0.032           |
| 4.57.......| 1.0005              | 0.0002          | 0.0076          | 24.8            | 3.14 \times 10^{-14}  | 0.022           |

* Mass of the surface convection zone.
started the system with these masses, an eccentricity of 0.3, and orbital and rotational periods of 2.4 days. This system reached the present orbital period after 1 Gyr. Such an outcome is not inconsistent with the fact that \( C11 \) Gem contains two early A main-sequence stars. Their masses are not known but are likely to be in the range 2 to 5 \( M_\odot \), which would give main-sequence lifetimes of 1.2–0.7 Gyr.

YY Gem should be currently decreasing its period on a timescale of 0.5 Gyr. Sowell et al. (2001) found the period of YY Gem to be approximately constant over the last 75 yr. One can estimate crudely from their Fig. 4 that the timing of the eclipse did not depart systematically by more than 5 minutes from their revised ephemeris over this period. This gives a crude lower limit to the timescale \( t_P = \frac{P}{P \log \text{age}} \) of \( jP \) 0.07 Gyr, which is certainly consistent with the fact that \( \alpha \) Gem contains two early A main-sequence stars. Their masses are not known but are likely to be in the range 2 to 2.5 \( M_\odot \), which would give main-sequence lifetimes of 1.2–0.7 Gyr.

3.3. Tidal Friction and Am Stars

The appearance of Am characteristics in A stars is well known to correlate with rotational velocity: for \( V \sin i \approx 80 \) km s\(^{-1}\) A-type spectra are generally "normal," while for slower velocities they often have Am peculiarities. \( V \sin i \approx 40 \) km s\(^{-1}\) is reasonably typical for Am stars (Preston 1974). Abt & Bidelman (1969) found that Am stars are typically in binaries with 2.5 \( \leq P \leq 100 \) days and that normal A stars are in either shorter or longer orbits or else single. They suggested that this is due to tidal friction. In short-period binaries the star is unable to spin slowly enough because of tidal friction, while in long-period binaries its spin remains at its natal rate, which on this hypothesis is typically faster than the critical value above.

We have computed a number of binaries with masses 2.0 + 1.6 \( M_\odot \), a range of period from 10 to 320 days, an initial eccentricity of 0.5, and an initial rotational period of 0.76 days. We were pleasantly surprised to find that tidal friction, according to our recipe of §2, can have a significant effect even at a period of 80 days, although only if the evolution is followed all the way across the main sequence. In Table 2 we give the orbital period and the age at which the rotational velocity decreased to 40 km s\(^{-1}\). The end of the main sequence, defined somewhat arbitrarily as the point to contact (in \( \leq 10 \) Gyr) if the initial period is \( \leq 3 \) days and expands if it is \( \geq 5 \) days.
where central hydrogen had dropped to 0.02, was 1171 Myr. We see that at an orbital period of 40 days the star spends ~70% of its lifetime rotating more slowly than 40 km s\(^{-1}\), whereas at an orbital period of ≥160 days it spends only ~15% of its life rotating this slowly. Only at the shortest orbital period considered (10 days) did the orbit become fully circularized and synchronized, at 6.9 days, before the end of the main sequence, although at 20 days the eccentricity had been reduced to 0.16 at this point.

By contrast, with an initial orbital period of 3 days, which rapidly reduced to 2.4 days by circularization, the rotational velocity of the star dropped below 40 km s\(^{-1}\) only between ages 5 and 386 Myr. It therefore spent ~30% of its life below this rotation velocity, whereas those with \(P \sim 1-40\) days spent ≥70% of their lives rotating this slowly.

### 3.4. Provisional Summary

Our formulation of § 2 appears to give some reasonably satisfactory agreement with a number of features of detached binaries of main-sequence stars. It gives a rather plausible explanation of the rotational period of the spotted component in BY Dra and can account for the relatively slow rotation of Am stars in binaries of period up to ~80 days. We are emboldened to see whether it can give significant results in more evolved binaries, including mass-transferring (Algol) binaries, where mass loss and angular momentum loss can be expected to play a much bigger role.

### 4. MORE EVOLVED BINARIES

We now consider some binaries that have undergone substantial evolution and contain a red subgiant or giant.

#### 4.1. RS CVn Binaries

Red giants or subgiants in close but detached binaries (commonly RS CVn stars) are known to be unusually active compared to single red giants or to those in wide binaries. The interpretation of this is that the giant is forced to rotate faster in a binary than it would if it were single. Single stars should spin down strongly as they evolve, partly because of evolutionary expansion and partly because of magnetic braking. Tidal friction however opposes this spin down.

Table 3 gives some observational data for four RS CVn stars. The uncertainties in the data are discussed in the papers cited; however, see below for a rediscussion of the radii in Z Her. Effective temperatures are either directly from these papers or else from \(B-V\) colors given there and combined with the \(\log T\) versus \(B-V\) Table 2 of Popper (1980). It can be seen that Z Her in particular shows a remarkable anomaly: the cooler and more evolved component has less mass than the companion, despite the fact that it does not fill its Roche lobe. This was interpreted (Eggleton 1986) as evidence that the activity of the giant is so substantially enhanced by rotation that it is losing mass on a nuclear timescale, probably 2 or 3 orders of magnitude faster than one would expect for a single star in the same evolutionary state.

No other RS CVn binary shows quite so marked a mass anomaly, although RZ Eri and RW UMa show a marginal anomaly. However, several RS CVns, including the prototype, have mass ratios remarkably close to unity, and this itself is a little surprising since one component is usually substantially more evolved than the other. It could be that such systems started with, say, a 5%–10% mass difference, which has been whittled down to nearly zero: 2% in the case of RS CVn.

Unlike in the conservative case, it is difficult in the non-conservative case to estimate initial configurations that will lead to currently observed parameters. After some experimentation, we started a Z Her model with parameters \((1.8 + 1.61 M_\odot, P_{\text{orb}} = 3.9\) days, \(P_{\text{rot}} = 2.0\) days, \(e = 0.3)\): row 1 of Table 4. The orbit circularized fairly rapidly (row 2). Star 1 lost little mass (~0.05 \(M_\odot\)) until its radius had increased from ~1.5 \(R_\odot\) on the ZAMS to about 2.8 \(R_\odot\) (row 3). Thereafter mass loss accelerated strongly. When \(M_1\) had dropped to 1.31 \(M_\odot\) (row 4) the period had increased to 3.96 days, roughly the observed value.

An apparent problem with our model for Z Her is that the current radii of both components (row 4 of Table 4) are too large, compared with the entries of Table 3, by about 25%. D. M. Popper (1990, private communication) has suggested that, because the system is only partially eclipsing, its inclination is rather uncertain and may have been overestimated. A slightly lower inclination allows the radii to be larger. Such a readjustment would also account for the apparently low luminosity (5 \(L_\odot\)) of the F5 component as tabulated by Popper (1988a)—as low as expected for a completely unevolved star of the same mass.

Table 4 continues the evolution to the onset of RLOF (row 5) at 1664 Myr, its cessation (row 6) at 1698 Myr, and well into the post-RLOF phase when star 1 is a hot subdwarf (row 7). Because of the fact that the mass ratio is

| Name | Spectra | \(M_1\) \((M_\odot)\) | \(M_2\) \((M_\odot)\) | \(P\) (days) | \(e\) | \(R_1\) \((R_\odot)\) | \(R_2\) \((R_\odot)\) | \(\log T_1\) | \(\log T_2\) | Reference |
|------|---------|----------------|----------------|--------|----|--------------|--------------|------------|------------|----------|
| RS CVn..... | G9 IV+F4 | 1.44 | 1.41 | 4.80 | ... | 4.0 | 2.0 | 3.707 | 3.817 | Popper 1988a |
| Z Her ....... | K0 IV+F5 | 1.21 | 1.61 | 3.99 | ... | 2.7 | 1.9 | 3.697 | 3.806 | Popper 1988a |
| RZ Eri ....... | K2 III+F5m | 1.62 | 1.68 | 39.3 | 0.35 | 7.0 | 2.8 | 3.625 | 3.810 | Popper 1988b |
| RW UMa ....... | K4 IV+V+F8 | 1.45 | 1.5 | 7.33 | ... | 3.8 | 2 | 3.630 | 3.795 | Popper 1980 |
reduced from an initial 1.12 to 0.75 prior to RLOF, the mass transfer is relatively well behaved and not the rapid hydrodynamic transfer (Paczynski 1967) expected because the loser has a deep convective envelope. In the whole of the evolution the total mass decreased by 27% and the orbital angular momentum by 56%. The angular momentum loss was split approximately evenly between the detached pre-RLOF evolution than in the semidetached phase.

Angular momentum by 56%. The angular momentum loss was split approximately evenly between the detached pre-RLOF evolution and the semidetached evolution, while the mass loss was somewhat greater in the immediate pre-RLOF evolution than in the semidetached phase.

The much wider system RZ Eri (Table 3) can also be reasonably comfortably fitted with the same model. We started with parameters \(1.75 + 1.68 \, M_\odot, P_{\text{orb}} = 49 \, \text{days}, P_{\text{rot}} = 2.0 \, \text{days}, e = 0.5\). The results are shown in Table 5. The system parameters are very little altered until star 1 increases its radius from 1.5 to 5.6\( R_\odot \) (row 2), except that pseudosynchronism is reached substantially sooner. Circularization is just beginning. Subsequently, the orbit circularizes and star 1 loses mass on much the same faster timescale. The eccentricity, period, and mass of star 1 all reach approximate their observed values at 1688 Myr (row 3). Star 2 also has about the right radius at this point, but star 1’s radius is modestly too large: 9\( R_\odot \) instead of 7\( R_\odot \). In further evolution, the orbit circularizes once star 1 grows to 12\( R_\odot \) (row 4), and RLOF begins when star 1 reaches \( \sim 21 \, R_\odot \), by which time its mass is already reduced to 0.63\( M_\odot \) (row 5). RLOF continued until star 1 was reduced to 0.34\( M_\odot \) (row 6), after which star 1 began, as our run ended, to shrink toward the hot-subdwarf region (row 7).

Figure 2 illustrates some further aspects of the evolution of RZ Eri. In the H-R diagram (left panel of Fig. 2) we note that the mass loss from star 1 prior to RLOF causes the upward march on the giant branch to be temporarily reversed; however, the luminosity only drops by \( \sim 25\% \) while the mass is almost halved. The upward march is restored, at \( L \sim 100 \, L_\odot \), star 1 fills its Roche lobe, and at about \( \sim 260 \, L_\odot \) star 1 detaches again, shrinking down to the B subdwarf region. The middle panel of Figure 2 is a superposition of four curves: the stellar and Roche lobe radii of both components, as functions of mass. The stars start toward the right, below the middle. By the time star 1 reaches its Roche lobe it is much less massive than star 2, and as a result the RLOF is on a very slow, roughly nuclear, timescale and not on the very short (hydrodynamic) timescale that would normally be expected of a loser with a deeply convective atmosphere.

The right panel of Figure 2 shows the evolution of eccentricity (asterisks) as well as of rotational period (dots) and orbital period (plus signs). Until \( \sim 1.6 \, \text{Gyr} \), \( e \) and \( P_{\text{orb}} \) were essentially constant, while \( P_{\text{rot}} \) increased steadily from 2 days (an arbitrary starting value) to the pseudosynchronous value of 18 days. The evolution of \( P_{\text{rot}} \) between 16 and 17 Gyr is quite complex because several timescales become comparable: nuclear evolution, mass loss, angular momentum loss, and tidal friction. There is a brief drop in \( P_{\text{rot}} \) at the “hook” on the terminal main sequence, then a substantial increase, in two steps, due to evolutionary expansion, and then a decrease as tidal friction reasserts itself, the larger stellar radius countering the fact that the nuclear timescale has shortened. However, tidal friction increases so rapidly that it starts to circularize the orbit, and thus as the star converges back to the pseudosynchronous value the pseudosynchronous value itself converges to the synchronous value, which is larger (34 days).

### Table 4

**Possible Evolutionary History of Z Her**

| Evolutionary Stage | Age (Myr) | \( M_1 \) (M\(_\odot\)) | \( M_2 \) (M\(_\odot\)) | \( P_{\text{orb}} \) (days) | \( P_{\text{rot}} \) (days) | \( e \) | \( R_1 \) (R\(_\odot\)) | \( R_2 \) (R\(_\odot\)) | \( \log T_1 \) | \( \log T_2 \) | \( H_{\text{orb}} \) |
|-------------------|-----------|-------------------------|-------------------------|-------------------------|-------------------------|-----|-------------------------|-------------------------|-------------------|-------------------|------------------|
| Circularized      | 0         | 1.80                    | 1.61                    | 3.90                    | 2.0                     | 0.3 | 1.53                    | 1.48                     | 3.927             | 3.884             | 13.80             |
| ML starts         | 157       | 1.80                    | 1.61                    | 3.39                    | 3.39                    | 0.01| 1.59                    | 1.50                     | 3.924             | 3.885             | 13.81             |
| Present           | 1645      | 1.31                    | 1.61                    | 3.96                    | 3.96                    | 0   | 3.47                    | 2.33                     | 3.709             | 3.818             | 11.15             |
| RLOF starts       | 1664      | 1.20                    | 1.61                    | 3.56                    | 3.56                    | 0   | 4.90                    | 2.38                     | 3.687             | 3.817             | 10.00             |
| RLOF ends         | 1994      | 0.27                    | 2.21                    | 22.9                   | 8.26                    | 0   | 0.030                   | 3.95                     | 3.830             | 3.948             | 6.12              |
| *H_{\text{orb}}* is in arbitrary units.*

### Table 5

**Possible Evolutionary History of RZ Eri**

| Evolutionary Stage | Age (Myr) | \( M_1 \) (M\(_\odot\)) | \( M_2 \) (M\(_\odot\)) | \( P_{\text{orb}} \) (days) | \( P_{\text{rot}} \) (days) | \( e \) | \( R_1 \) (R\(_\odot\)) | \( R_2 \) (R\(_\odot\)) | \( \log T_1 \) | \( \log T_2 \) | \( H_{\text{orb}} \) |
|-------------------|-----------|-------------------------|-------------------------|-------------------------|-------------------------|-----|-------------------------|-------------------------|-------------------|-------------------|------------------|
| ML starts         | 1676      | 1.74                    | 1.68                    | 49.0                    | 2.0                     | 0.5 | 1.50                    | 1.49                     | 3.914             | 3.897             | 29.7              |
| Present           | 1688      | 1.62                    | 1.68                    | 49.0                    | 2.0                     | 0.5 | 1.50                    | 1.49                     | 3.914             | 3.897             | 29.7              |
| Circularized      | 1697      | 1.45                    | 1.68                    | 34.1                    | 34.1                    | 0   | 12.1                    | 2.89                     | 3.663             | 3.799             | 26.0              |
| RLOF starts       | 1726      | 0.63                    | 1.68                    | 43.9                    | 43.9                    | ...| 20.7                    | 2.93                     | 3.603             | 3.797             | 13.5              |
| RLOF ends         | 1741      | 0.34                    | 1.89                    | 129                    | 129                    | ...| 34.3                    | 3.10                     | 3.601             | 3.832             | 12.0              |
| 1743              | 0.34      | 1.89                    | 129                    | 22.6                   | 5.55                   | 3.10| 4.008                   | 3.831                    | 12.0              |                   |                  |

*a H_{\text{orb}}* is in arbitrary units.*
The star remains in synchronism until it starts to shrink rapidly away from its Roche lobe. Although the radius decreases by a large factor, the moment of inertia does not because the shrinkage is confined to the thin envelope surrounding the degenerate core, having only ~1% of the mass. Consequently, the star spins up by only about a factor of 4 as it shrinks to the subdwarf region. We should emphasize that we have assumed uniform rotation throughout star 1, at all times.

**RW UMa** (Table 3) is also moderately well explained within the context of our nonconservative model. Although the mass deficit of star 1 is by no means as well determined as in **Z Her**, it is presumably substantially smaller, and this is accounted for in our model by the fact that the period is substantially larger so that the mass-loss rate is significantly reduced. We start with masses $1.68 + 1.5 \ M_\odot$, i.e., much the same mass ratio as in **Z Her**, on the basis that this is necessary to obtain the considerable evolution required in star 2 that is manifested by its radius. Then we find that star 1’s mass is reduced to its present value when its radius is $\sim 5 \ R_\odot$, a little larger than the observed value but probably within observational uncertainty.

In fact, **RS CVn** itself is a rather harder example to fit with our model than the three other systems of Table 3. This system presumably started with much the same mass ratio as **Z Her** and **RW UMa**, since the present components show much the same degree of evolution and of differential evolution between the two components as manifested by their radii. Probable initial masses would therefore be $\sim 1.6 + 1.41 \ M_\odot$. Since the period is slightly greater than in **Z Her**, we might expect slightly less mass loss from star 1, but actually we seem to have much less mass loss, by a factor of $\sim 3$. We have not been able to obtain these parameters with the present model: our best attempts give about twice as much mass loss as is required.

We do not try here to produce a detailed fit to the observed systems, say by a least-squares approach, for the following three reasons:

1. The observational uncertainties are quite substantial (see the papers cited in Table 3). Only for **Z Her** is the sign of the mass difference clearly significant. Given that the uncertainties are substantial, it would probably not be difficult to find somewhat better fits to **Z Her** (Table 4) and **RZ Eri** (Table 5), but this would not be a very strong confirmation of the model.

2. The theoretical model of § 2 is not put forward as definitive. It might be thought of as containing several coefficients and exponents whose values are unknown a priori and which could be determined by fitting computed models to observed systems; however, it is rather unlikely that there is any unique formulation that will correctly determine the mean behavior of such a chaotic process as dynamo activity. By varying enough coefficients from system to system we could reasonably expect to fit almost any observed system.

3. We should probably not assume that all these stars have the same metallicity as the Sun, but nevertheless we have not varied Z from a value of 0.02.

We content ourselves by noting that the formulation of § 2, without varying any of the coefficients or exponents that it contains, seems to describe reasonably well some observed **RS CVn** systems and may have to be varied by factors of about 2 to account for some more.

### 4.2. Semidetached Binaries

The first four Algols listed in Table 6 show very clearly that they must have lost some angular momentum and/or mass. Although some periods are not especially short, the very small mass ratios ($q \sim 0.075–0.16$) mean that the systems have low angular momentum. Any Algol must have evolved through an equal-mass configuration, and the orbital period then, assuming conservative evolution, is so small for **S Cnc–R CMa** that even unevolved components would have overfilled their outer, let alone their inner, critical potentials. In addition, **R CMa** has such a low total mass
that it is difficult to see how it could have evolved at all in a Hubble time. This would require that the initial mass ratio was almost as extreme in the opposite sense as the current value; and in that case the expectation would be that instead of steady RLOF we would have the hydrodynamic mass transfer of Paczynski (1967), probably leading to a common-envelope phase with spiral-in (Paczynski 1976) and hence, we try to advance in its evolution, with radius nearer to the expected ZAMS radius for the current mass, than in the RS CVn systems. Furthermore, compared with the four Algols we are considering, we therefore explore some shorter initial periods. Attempts to improve the starting parameters however were inadequate. At age 1379 Myr the corresponding parameters (Table 6) that reasonably resembles DN Ori at a late stage presently observed DN Ori. It is also very close to the end of RLOF, and in row 6 star 1 has shrunk to an O-type subdwarf. We did not follow the system further, but in row 6 star 2 is very close to the end of its main-sequence life and will shortly expand rapidly to bring about reverse RLOF. Although our suggested mass for star 1 is substantially less than that Table 6, it is within the error estimate of Etzel & Olson (1995), viz., 0.34 ± 0.10 $M_\odot$. An uncertainty of this magnitude is probably fairly typical of low-mass companions in this kind of system. Etzel & Olson obtained a slightly detached solution, which could easily be consistent with our finding that it is right at the end of RLOF.

S Cnc is rather similar to DN Ori: Olson & Etzel (1993) found this system to be also slightly detached. Starting values (2.24 ± 1.41 $M_\odot$, 1.51 days) appear to be roughly adequate. At age 1379 Myr the corresponding parameters are (0.25 ± 0.53 $M_\odot$, 9.55 days). Star 2 is rather more evolved across the main-sequence band than we require. Attempts to improve the starting parameters however were.

### TABLE 6

| Name          | Spectra        | $M_1$ ($M_\odot$) | $M_2$ ($M_\odot$) | $P$ (days) | $R_1$ ($R_\odot$) | $R_2$ ($R_\odot$) | Reference          |
|---------------|----------------|-------------------|-------------------|------------|-------------------|-------------------|--------------------|
| DN Ori        | G5 III+A0      | 0.34: 2.8         | 13.0              | 6.7        | 2.4               | 4.022             | Etzel & Olson 1995 |
| S Cnc         | G8 III+B9.5 V  | 0.23: 2.4         | 9.49              | 5          | 2.2               | 3.765             | Olson & Etzel 1993 |
| AS Eri        | K0+A3          | 0.2               | 1.9               | 2.66       | 2.2               | 3.871             | Popper 1980        |
| R CMa         | G8 IV+F1       | 0.17: 1.07        | 1.14              | 1.15       | 1.5               | 3.994             | Sarma et al. 1996  |
| RT Lac        | G9 IV+G5 IV    | 0.63: 1.57        | 6.07              | 4.6        | 4.3               | 3.994             | Popper 1980        |
| RZ Cnc        | K4+K1          | 0.54: 3.2         | 21.6              | 12.2       | 10.2              | 3.994             | Popper 1980        |
| AR Mon        | K3+K0          | 0.8               | 2.7               | 21.2       | 14.2              | 3.994             | Popper 1980        |
| DL Vir        | K0-2+AS V      | 1.1: 2.2: 1.32     | 2.4: 1.8: 2.2: 1.8: | Schoeffel 1977 |
| DL Vira       | (K+A)+G8 III   | 3.3: 1.9: 2200:   | ...               | ...        | 3.729             | 3.950             | Schoeffel 1977     |
| $\theta$ Tuc  | F5+A7 IV       | 0.063b 0.7b       | 7.10              | ...        | 3.48              | 4.525             | De May, Daems, & Sterken 1998 |
| V1379 Aql     | SDB+K0 III–IV  | 0.30: 2.27        | 20.7              | 0.05       | 9.0               | 3.994             | Jeffery & Simon 1997 |
| FF Aqr        | SDOB+G8 III    | 0.35: 1.4         | 9.21              | 0.16       | 7.2               | 3.994             | Vaccaro 2002       |
| AY Cet        | WD+G5 IIIe     | 0.55: 2.1         | 56.8              | 0.012      | 6.8               | 3.994             | Simon, Fekel, & Gibson 1985 |
| V651 Mon      | SDOB+G5 V      | ... 0.007b        | 16.1              | ...        | 3.729             | 3.950             | Menez & Niemela 1981 |
| 0957–666      | WDA+WDA        | 0.32: 0.37        | 0.061             | ...        | 3.729             | 3.950             | Moran, Marsh, & Bragaglia 1997 |
| AA Dor        | SDO+4K        | 0.25: 0.05        | 0.26: 016: 0.09: 3.994 | Wlodarczyk 1984 |

### TABLE 7

| Evolutionary Stage (1) | Age (Myr) | $M_1$ ($M_\odot$) | $M_2$ ($M_\odot$) | $P_{orb}$ (days) | $H_{obs}$ (24) | $R_1$ ($R_\odot$) | $R_2$ ($R_\odot$) | log $T_1$ | log $T_2$ |
|------------------------|-----------|-------------------|-------------------|-----------------|----------------|-------------------|-------------------|-----------|-----------|
| RLOF starts............ | 725       | 2.18              | 1.58              | 1.62            | 12.8           | 1.69              | 1.48              | 4.002     | 3.877     |
| RLOF ends............. | 734       | 1.49              | 2.25              | 1.70            | 12.4           | 3.20              | 1.87              | 3.765     | 3.994     |
| RLOF starts............ | 1072      | 0.932             | 2.25              | 1.54            | 7.91           | 2.52              | 2.56              | 3.729     | 3.950     |
| Present; RLOF ends..... | 1201      | 0.263             | 2.68              | 13.0            | 5.55           | 6.85              | 3.36              | 3.749     | 3.970     |
|                        | 1212      | 0.262             | 2.68              | 13.1            | 5.55           | 0.084             | 3.48              | 4.525     | 3.964     |

$H_{obs}$ is in arbitrary units.
handicapped by the fact there are at least five very different types of outcome that seem to arise in quite a limited range of input parameter space. Three of these are fairly similar to outcomes in conservative case A, as described in Paper I. There we identified a total of eight distinct outcomes, sub-cases of case A that we called AD, AR, . . . , AN. The three relevant here are cases AR, AG, and AL; however, the fourth and fifth, with mass loss and angular momentum loss, respectively, being the dominant characteristic, have no analogy, and we call them here cases AM and AA.

1. AR—Rapid evolution to contact: thermal-timescale RLOF, with a fairly large initial mass ratio, causes star 2 to expand rapidly to contact after rather little mass exchange.

2. AG—Giant contact: star 2 just misses the rapid contact of AR, but its substantial growth in mass allows it to catch up and overtake star 1’s evolution so that contact between two red giants is reached at a later stage.

3. AL—Late overtaking: star 2 misses rapid contact by a somewhat wider margin but at a late stage, when star 1 is already a hot subdwarf, expands to fill its Roche lobe, initiating reverse mass transfer. This will no doubt be on a dynamical timescale, leading to common-envelope evolution, spiral-in, and either a merger (of the hot subdwarf with the white dwarf core of star 2) or a close detached pair of white dwarfs.

4. AM—Mass-loss dominated: star 1 loses mass sufficiently copiously that it never reaches its Roche lobe, although it may still evolve to a red subgiant and then a hot subdwarf. Alternatively, there might be two minor episodes of RLOF separated by a substantial detached interval where wind alone causes the binary to modify its period on about the same (nuclear) timescale as star 1 evolves. The evolution of DN Ori in Table 7 is of this character. Later evolution will be the same as in case AL.

5. AA—Angular momentum loss dominated: star 1, and consequently the binary, loses angular momentum sufficiently rapidly that its Roche lobe shrinks rapidly to the stellar radius. Star 1 evolves down or very close to the ZAMS; usually star 2 expands rapidly too, and the result is a contact system much as in the conservative cases AR or AD (Paper I).

In Paper I we investigated the three-dimensional space of initial parameters for conservative evolution with several thousand models, to identify regions where the different cases AD–AN occur. With our nonconservative model here the detailed behavior even in the much more limited domain that probably covers most of the systems in Tables 2 and 5 seems more complex. We therefore content ourselves with very qualitative agreement. It is possible that both DN Ori and S Cnc will require that some coefficients in our nonconservative model be modified, for example, to give more angular momentum loss relative to mass loss, but the case for this does not seem compelling.

AS Eri and R CMa are rather more difficult to account for with our model. AS Eri has substantially less angular momentum than DN Ori or S Cnc; however, if we start with a substantially shorter period, then the stars are so close together that after RLOF begins star 2 tends to expand quickly to a contact configuration (case AR). Even if this is just avoided, our formulation produces more mass loss at shorter period, and as a result the Alfvén radius is reduced and there is less angular momentum loss, relative to the mass loss. We appear to need a formulation with perhaps twice as much magnetic braking relative to mass loss as in our canonical model.

Some attempted models of AS Eri and R CMa had sufficient mass loss that RLOF was avoided altogether (case AM), even though star 1 might be stripped down to a red subgiant and then a hot-subdwarf core at a period not unlike the period of AS Eri. This raises a potentially awkward question: can we be sure that all of these binaries are indeed semidetached? It is normally taken as an assumption that a system is semidetached if the larger, cooler star is much less massive than the smaller, hotter star. However, if it is accepted that mass loss by wind, from a star that does not yet fill its Roche lobe, can be on a nuclear timescale (as it must be at least in the case of Z Her), then the period and separation can increase on a nuclear timescale and so allow the windys star to remain a little smaller than its Roche lobe for a substantial period of time. There may in practice be rather little difference in appearance between a system where star 2 is accreting part of a wind from star 1 and one where star 2 is accreting from RLOF of star 1 while star 1 is also losing mass by wind. However, the measured parameters of such a system, particularly the inclination, will depend on whether star 1’s radius is 90% or 100% of its Roche lobe radius.

We note that our model did not include the possibility of partial accretion by star 2 of the wind from star 1. It would not in principle be very difficult to include it, provided we had a formulation of the process that we believed in. There is no doubt that such a process can take place: for example, many though not all Ba stars must have accreted from the Ba-rich stellar wind of an asymptotic giant branch (AGB) star companion rather than from RLOF (Han et al. 1995). However, this process may be more effective for the copious, slow, cool winds of AGB stars than the more meager, fast and hot winds of red subgiants. A preliminary estimate is that perhaps 10% or less of the wind would be accreted, which we consider small enough to be ignored.

The next three entries in Table 6, RT Lac, RZ Cnc, and AR Mon, are all double-(sub)giant binaries, examples of case AG. The fact that star 2 is almost as evolved as star 1 suggests that they started with more nearly equal masses (say \(q_0 \leq 1.05\), and perhaps \(q_0 \sim 1.01–1.02\)) than even the RS CVn systems, let alone the first four Algols. This presents us with the extra problem that they will almost certainly need a DNC model and be less agreeable to our SNC model. We have therefore not attempted to model them. However, mass loss on a nuclear timescale from star 1 can enlarge the region of case AG since it slows down the evolution of star 1 and so makes it easier for star 2 to catch up.

DL Vir (Table 6) is a particularly interesting system because not only is it triple but the distant third body (star 3, say) is already evolved to the giant branch. This gives us the important information that star 1 of the close Algol pair must have had much the same initial mass as star 3. This in turn poses an upper limit to the amount of mass lost by the system, since star 2 must have had less mass than star 1 initially. However, at the same time there is a lower limit because if the initial mass ratio were close to unity star 2 would be more evolved than it is, as in the RS CVn systems (\(q_0 \sim 1.05–1.12\)) and the three double-giant Algols. Taking the numbers in Table 6 at face value, we have in fact very little scope for mass loss: the starting masses must have been \(\sim 1.9 + 1.6 M_\odot\), and the mass lost \(\sim 0.2 M_\odot\). However, the uncertainties in the observed masses are considerable, so
there is probably also scope for as much mass loss (relatively) as in our tentative model of DN Ori (Table 7), particularly as the modest mass ratio in DL Vir puts it at a much earlier stage of mass transfer.

Although several Algols are in triple systems (Chambliss 1992), DL Vir is the only one where star 3 is evolved to the giant branch and thus affords us a real estimate of the initial mass of star 1, not just a lower limit. We would hope that this important system could be reanalyzed with modern technology.

4.3. Post-Algols

Table 6 contains two binaries, θ Tuc and V1379 Aql, that can be recognized as probable post-Algols. Although the inclination of θ Tuc is not known, a guess of $\sin^3 i \sim 0.3 - 0.4$ makes it rather similar to DN Ori and S Cnc, but presumably at a more advanced state of evolution where star 1 has detached from its Roche lobe. The fact that we find it slightly difficult to get periods as short as in S Cnc and DN Ori, and very difficult to get periods as short as in AS Eri, may mean that θ Tuc requires slightly enhanced angular momentum loss relative to our canonical model.

However, V1379 Aql seems to fit fairly comfortably into the future evolution of Z Her, as indicated in Table 4. The only discrepancy is that the $M_1$ that we end up with is about 10% less than observed. That is rather significant in this unusually accurately determined system. It will not be easy to get rid of this by tinkering with initial parameters, since the mass of the remnant is largely dictated by the size of its current Roche lobe, i.e., by the size of the immediate red giant precursor. The radius of a red giant is very sensitive to the mass of its core, and to get a core 10% more massive should require a lobe almost twice as large. However, C. S. Jeffery (1997, private communication) has suggested that the system may be slightly metal-poor, and this would certainly act in the right direction.

It is possible that the system is in fact losing angular momentum currently, thanks to the activity of star 2, which has strong activity as in RS CVns. As we have emphasized several times, we ignore the activity of star 2. However, it is unlikely to have reduced the orbital angular momentum by as much as the mass discrepancy requires.

A very different anomaly in V1379 Aql is the fact that its orbit is very significantly noncircular: $e = 0.09 \pm 0.01$. A possible explanation, though not our preferred one, is as follows. When RLOF ended, star 2 would probably have been rotating rapidly, since the accretion stream from star 1 acquires angular momentum, because of Coriolis force, as it travels to star 2. Equation (10) shows that if $\Omega > 18 \omega_1/11$ (for $e \sim 0$) then eccentricity increases. Tidal friction would probably have been unimportant at first, but as star 2 grew to its present radius tidal friction may have recently become important. This would require that even after some evolutionary expansion, and also some angular momentum loss by magnetic braking, star 2 was still rotating substantially faster than $\sim 1.65 \omega_1$. However, the eccentricity would have to build up from some small (but nonzero) value. We estimate that the Algol might have finished RLOF with $e \sim 5 \times 10^{-5}$, on the following grounds. Certain radio pulsar plus WD binaries with comparable orbital period show such an eccentricity, which is probably due (Phinney 1992) to small inhomogeneities of density and hence gravity on the scale of turbulent convective elements in the red giant loser before it contracted to a WD. This would have to be amplified by a factor of $\sim 2 \times 10^3$ to reach the present value. It seems unlikely that star 2 would have had enough angular velocity to do this, since the same process that increases $e$ also decreases $\Omega$ to its corotational value.

A more probable cause of the eccentricity, we believe, is the action of a third body in a wide orbit highly inclined to the known orbit. Such a body can drive long-period cycles, of both eccentricity and inclination, in the 21 day orbit (Kozai cycles; Kozai 1962). The amplitude of the eccentricity fluctuation does not depend on either the mass of the third body (which might therefore be a very inconspicuous M dwarf) or its orbital period (which might be several years) but only on the inclination of the outer to the inner orbit. Such third bodies have recently been found for two systems (SS Lac, Torres & Stefanik 2000; V907 Sco, Lacy, Helt, & Vaz 1999), with longer orbits of 679 and 99 days, respectively. These third bodies were needed to account for the fact that the close pairs sometimes eclipse and sometimes do not, as a result of the fluctuation of inclination to the line of sight. Another Algol system, δ Lib, has also recently been found to have a third body (Worek 2001), in a 1008 days orbit.

Whether such third-body orbits are typically highly inclined or not is at present unclear; however, those of SS Lac and V907 Sco must be, in order to cause the variation of inclination of the eclipsing orbit to the line of sight that is observed. The prototype Algol has a third body in an orbit inclined at 100° to the eclipsing orbit (Lestrade et al. 1993). Eggleton & Kiseleva (2001) give equations governing the interaction of tidal friction, tidal distortion, and Kozai cycles in SS Lac and other triples. They found that the inclination of the outer orbit to the inner in SS Lac had to be 29°. If hierarchical triples are typically caused by binary-binary interactions in a dense star-forming cluster, then inclinations higher than 60° are as likely as lower inclinations. An inclination of 60° would cause the eccentricity to oscillate between zero and 0.764. The period of the oscillation is $\sim P_2/P_{in}$, multiplied by a factor of total mass over $M_3$. Although the inclination and eccentricity fluctuate in the course of a Kozai cycle, the period does not.

The next three systems of Table 6, FF Aqr, AY Cet, and V651 Mon, are potential post-Algols because each has a hot-subdwarf star 1, a less evolved star 2, and an orbital period in the expected range. However, although the masses of the hot components are all uncertain, they appear to be on the high side: one at least appears to be more appropriate to an AGB star than to an early first giant branch star. This suggests that they are products of common-envelope evolution. Although the mass of the hot subdwarf in V651 Mon is unknown, the system is the central star of the planetary nebula NGC 2346, which suggests a recent common-envelope origin. However, the systems are somewhat surprising in that context also since the usual expectation is that the common-envelope phase will trigger spiral-in to a short period, 1 day or less, thus allowing the system to evolve subsequently to a cataclysmic variable. In a tentative study of common-envelope evolution, to be published, we conclude that the only common-envelope precursors to suffer substantial orbital shrinkage are those with a mass ratio more severe than 4 : 1, in the sense of (AGB star) : (WD or MS companion). Systems with a less severe mass ratio shrink their orbits only by a modest factor—apparently the envelope of the AGB star is expelled so easily by the companion, if the com-
rapid mass transfer, in the manner of Paczynski (1976), followed by the right mass. This would no doubt have required very say, 30 days, by which time its core would have about all. It may have filled its Roche lobe in an initial orbit of, see how an envelope as massive as 0.75 $M_\odot$ is expected from a single red giant after the helium flash, and may be fairly readily expelled by the low-mass companion without it spiraling in all the way to a merger.

With our canonical model there was generally not enough mass loss, and too much angular momentum loss, for this scenario, but only by modest factors, $\lesssim 2$. While we have little doubt that we could obtain good agreement by adjusting coefficients in the formulae of § 2, we conclude simply that $\sim 75\%$ mass loss prior to RLOF, and spiral-in afterward, is a viable way of producing the unusual system AA Dor, and more attractive than any alternative so far. Although one could hardly call the system a post-Algol, it may be influenced by much the same physical processes as pre-Algols, Algols, and post-Algols.

5. CONCLUSIONS

We have presented a simplistic formulation of mass loss driven by dynamo activity, angular momentum loss driven by magnetic braking, and tidal friction, that in the first instance has no free parameters. It is calibrated to agree with the present-day Sun and is scaled according to the depth of convective envelope and the Rossby number in a realistic way. It appears to account for a fairly wide range of phenomena: the surprisingly slow rotation in BY Dra, the surprisingly low mass in the more evolved component of Z Her, and the surprisingly low angular momentum of Algols such as DN Ori and S Cnc. Variants of this model, in which some coefficients are altered by factors of up to $\sim 2$, may account for a wider range of observed objects, such as AS Eri, R CMa, and AA Dor. Given that the dynamo activity that is the basis of both the mass loss and the magnetic braking is an inherently chaotic process, it is certainly not surprising that factors of 2 or more should be necessary as between one system and another.

The same formulation can of course be applied to other classes of object: contact binaries, cataclysmic variables, pre-cataclysmic systems, and low-mass X-ray binaries. We hope to pursue these in a future paper. Estimates have already been given in Paper II.

An important improvement that will be necessary to understand some other systems, such as cool double-subgiant binaries, is to include star 2 within the formalism; for the present only star 1 is allowed to be subject to these non-conservative processes. This will be quite a major undertaking, since it will be necessary to solve for both components simultaneously. However, this is also necessary if one is to follow the evolution of contact binaries, since an additional model, for heat transport between the two components, is necessary there. We hope to produce this in due course.

A different but very important way of pursuing the same topics is to model these interactions in a fully three-dimensional stellar model or pair of models. For example, it would be desirable to model tidal friction in such a way, to make a better estimate of the viscous timescale that is incorporated in the constant factor of equation (9). If MHD is included, presumably just in the frozen-in approximation, then three-dimensional calculations could also serve to calibrate the other nonconservative processes. Most importantly, they could also give us insight into the poorly understood process of heat transport in contact binaries.

The DJEHUTY project is currently being developed at the Lawrence Livermore National Laboratory, with a view to tackling the three-dimensional structure of stars. At
present, grids of $\sim 10^8$ mesh points are available, and as computer power increases we hope to improve this to $\sim 10^{10}$–$10^{11}$. Of course, one would not evolve such a three-dimensional model for several gigayears but only for modest times such as $\sim 1$ yr. This should allow us to make a one-dimensional average of such processes as tidal friction and incorporate them in a one-dimensional code such as the one used here.

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REFERENCES

Abt, H. A., & Bidelman, W. P. 1969, ApJ, 158, 1091
Brandenburg, A., Saar, S. H., & Turpin, C. R. 1998, ApJ, 498, L51
Chambliss, C. R. 1992, PASP, 104, 663
Conti, P. S., Dearborn, D., & Massey, P. 1981, MNRAS, 195, 165
De May, K., Daems, K., & Sterken, C. 1998, A&A, 336, 527
Eggleton, P. P. 1986, in The Evolution of Galactic X-Ray Binaries, ed. J. Truemper, W. Lewin, & W. Brinkmann (NATO ASI Ser. C, 167; Dordrecht: Reidel), 57
———. 2001, in ASP Conf. Ser. 229, Evolution of Binary and Multiple Systems, ed. Ph. Podsiadlowski, S. Rappaport, A. R. King, F. D’Antona, & L. Burderi (San Francisco: ASP), 157 (Paper II)
Eggleton, P. P., & Kiseleva, L. G. 2001, ApJ, 562, 1012 (Paper III)
Eggleton, P. P., & Kiseleva, L. G., & Hut, P. 1998, ApJ, 499, 853
Etzel, P. B., & Olson, E. C. 1995, AJ, 110, 1809
Han, Zh., Eggleton, P. P., Podsiadlowski, Ph., & Tout, C. A. 1995, MNRAS, 277, 1443
Hut, P. 1981, A&A, 99, 126
Jeffery, C. S., & Simon, T. 1997, MNRAS, 286, 487
Kozai, Y. 1962, AJ, 67, 591
Lacy, C. H. S., Helt, B. E., & Vaz, L. P. R. 1999, AJ, 117, 541
Lebrade, J.-F., Phillips, R. B., Hodges, M. W., & Preston, R. A. 1993, ApJ, 410, 808
Leung, K., & Schneider, D. 1978, AJ, 83, 618
Marsh, T. R., Dhillon, V. S., & Duck, S. R. 1995, MNRAS, 275, 828
Méndez, R. H., & Niemela, V. S. 1981, ApJ, 250, 240
Mestel, L., & Spruit, H. 1987, MNRAS, 226, 57
Moran, C., Marsh, T. R., & Bragaglia, A. 1997, MNRAS, 288, 538
Nelson, C. A., & Eggleton, P. P. 2001, ApJ, 552, 664 (Paper I)
Olson, E. C., & Etzel, P. B. 1993, AJ, 106, 1162
Paczynski, B. 1967, A&A, 15, 89
———. 1976, in IAU Symp. 73, Structure and Evolution of Close Binary Systems, ed. P. P. Eggleton, S. Mitton, & J. Whelan (Dordrecht: Reidel), 75
Phinney, E. S. 1992, Philos. Trans. R. Soc. London, A, 341, 39
Pizzo, V., Schwenn, R., Marsch, E., Rosenbauer, H., Mühlhäuser, K.-H., & Neubauer, F. M. 1983, ApJ, 271, 335
Popper, D. M. 1980, ARA&A, 18, 115
———. 1988a, AJ, 95, 1242
———. 1988b, AJ, 96, 1040
Preston, G. W. 1974, ARA&A, 12, 257
Sarma, M. B. K., Vivekananda Rao, P., & Abhyankar, K. D. 1996, ApJ, 458, 371
Schoeffel, E. 1977, A&A, 61, 107
Simon, T., Fekel, F. C., & Gibson, G. M. 1985, ApJ, 295, 153
Skumanich, A. 1972, ApJ, 171, 565
Sowell, J. R., Hughes, S. B., Hall, D. S., & Howard, B. A. 2001, AJ, 122, 1965
Spruit, H. 1998, A&A, 333, 603
Stepien, K. 1995, MNRAS, 274, 1019
Torres, G., & Stefanik, R. P. 2000, AJ, 119, 1914
Vaccaro, T. 2002, Ph.D. thesis, University of Florida, Gainesville
Vogt, S., & Fekel, F. C. 1979, ApJ, 234, 958
Wodarczyk, K. 1984, A&A, 34, 381
Worek, T. F. 2001, PASP, 113, 964

No. 1, 2002 EVOLUTION OF COOL ALGOLS 473