Tetra-axial metamaterial

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Abstract. In this work we theoretically and numerically demonstrate that triple non-connected wire medium has four optical axes. The axes coincide with diagonals of the unit cell.

1. Introduction

In the general case, any homogeneous local dielectric medium can be described by symmetric effective permittivity tensor which can be diagonalized in some coordinate system. The principal elements \(\varepsilon_{ii}\) \((i = x, y, z)\) of the diagonal tensor are called principal permittivities [2] and the relations between them determine the shape of dispersion surfaces of the medium. There are three types of dielectric media \((\varepsilon_{ii} > 0)\): 1) isotropic media \((\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz})\) with isofrequency surfaces in the form of a sphere; 2) uniaxial media \((\varepsilon_{ii} \neq \varepsilon_{jj}, \text{ for some } i \neq j \neq l)\) with spherical isofrequency surface for ordinary waves and ellipsoid of revolution isofrequency surface for extraordinary waves which touch each other in direction of optical axis directed along \(i\)-axis; 3) biaxial media \((\varepsilon_{xx} \neq \varepsilon_{yy} \neq \varepsilon_{zz})\) with complex-shaped isofrequency surfaces with two axes in the plane \(il\) such that \(\varepsilon_{ii} > \varepsilon_{jj} > \varepsilon_{ll}\) [3]. The presence of optical axes in biaxial media results in the effect of conical refraction which finds many applications in optics [4]. If not all principal permittivities of an uniaxial medium are positive then the extraordinary waves in the medium have hyperbolic isofrequency contours and such medium is called hyperbolic metamaterial [5].

In this paper we study triple non-connected wire medium [1] and demonstrate that it has four optical axes.

2. Main part

In this work, we will consider a triple non-connected wire medium [1] with a unit cell shown in Fig. 1(a). The metamaterial consists of three two-dimensional arrays of parallel infinite straight
metal wires (along x, along y, or along z). In each of the arrays, the axes of the wires form a square lattice in a plane perpendicular to their direction. We will analyze a configuration of the medium in which the wires have the same radii \( r_0 \), the periodicity in all directions is the same and equal to \( a \), and the distances between axes of nearest perpendicular wires is equal to half of the period \( a/2 \) (see Fig. 1(a)). Mathematically, the geometry can be described by defining coordinates of the wire axes by the following way [1]:

(i) the \( x \)-directed wires: \( y = an + a/2 \) and \( z = al + a/2 \),

(ii) the \( y \)-directed wires: \( x = am + a/2 \) and \( z = al \),

(iii) the \( z \)-directed wires: \( x = am \) and \( y = an \).

The triple wire medium is a medium with strong spatial dispersion at low frequencies. Earlier it was shown that the effects of spatial dispersion (in quasi-static case, \( ka << \pi \) and \( k_0a << \pi \)) in this material are described by the permittivity tensor [1]:

\[
\varepsilon_{ii} = 1 - \frac{k_p^2}{k_0^2 - k_i^2}, \quad i, \in \{x, y, z\}; \quad k_p^2 = \frac{2\pi/a^2}{\ln(a/2\pi r_0) + \pi/6} \]  

(1)

where \( \vec{k} = (k_x, k_y, k_z)^T \) is the wave vector in the medium of wires, \( k_0 \) is the wave vector in the host medium, \( k_p \) is the wave vector corresponding to the effective plasma frequency of the wire medium.

For an arbitrary anisotropic medium, the dispersion equation can be written in the following form [1, 2]:

\[
(k_y^2 + k_z^2 - k_0^2\varepsilon_{xx})(k_x^2 + k_z^2 - k_0^2\varepsilon_{yy})(k_x^2 + k_y^2 - k_0^2\varepsilon_{zz}) - \\
-(k_y^2 + k_z^2 - k_0^2\varepsilon_{xx})k_yk_z^2 - (k_x^2 + k_z^2 - k_0^2\varepsilon_{yy})k_xk_z^2 - (k_x^2 + k_y^2 - k_0^2\varepsilon_{zz})k_xk_y^2 - 2k_x^2k_y^2k_z^2 = 0. 
\]  

(2)

Now we can substitute the components of the tensor \( \varepsilon \) from Eq. 1 into the dispersion equation (Eq. 2) and calculate the isofrequency surfaces in the space of wave vectors. In the previous work [1], the hyperbolic isofrequency contours in the \( k_z = 0 \) plane were mainly considered and the complete picture of the dispersion of the metamaterial under consideration was not discussed.
Figure 2. Isofrequency contour for $\omega a/2\pi c = 0.1$ in $\Gamma MR$ section of the Brillouine zone (shown on the right side of the plot). Black lines are numerically confirmed isofrequency contours, pink dashed curves are determined by the Eq. 2, blue and red arrows show the direction of $E^{av}$ and $H^{av}$ averaged over the unit cell, diagonal $\Gamma R$ gray line is an optical axis.

The figure 1(b) shows an isofrequency surface for the triple wire medium at $\omega a/2\pi c = 0.1$. It can be noticed that this surface has a rather unusual: namely, it has eight points of self-intersection (one in each of the eighth parts of the zone). These points are located at the on the main diagonals of the cubic Brillouin zone. This observation allows us to manifest the presence of four optical axes for the the wire medium.

We have performed a series of numerical simulations using commercial software package CST MWS to compute isofrequency surfaces for a triple wire medium (applying the periodic boundary conditions to the unit cell and using the eigenmode solver) in the $\Gamma MR$ plane (plane $x = y$ in Fig. 1(b); according to the theory one of four optical axes lies in this plane) and to determine the polarization of the corresponding modes.

The simulation results are shown in Fig. 2 (the fourth part of the diagonal plane of the $\Gamma MR$ zone section): black lines are numerically confirmed isofrequency contours, pink dashed curves – theoretical isofrequency contours according the dispersion equation in a quasistatic case (Eq. 2) where $ka << \pi$ and $k_0a << \pi$, blue and red arrows show the direction of electric and magnetic fields averaged over the unit cell, respectively, at certain points of the contour [9]. Mode crossing at the boundary of the Brillouin zone is described by explicit dispersion equation [1].

This is a numerical confirmation of the presence of a self-intersection point of the isofrequency surface on the main diagonal of the cubic Brillouin zone ($k_{sip} = (k_d, k_d, k_d)^T$), which coordinate in $k$-space depends on the frequency and changes according to the law (which can be deducted by applying to Eq. 2 a condition $k_x = k_y = k_z = k_d$):

$$k_d^2 = \frac{k_0}{3}\left(2k_0 + \sqrt{k_0^2 + 3k_{pl}^2}\right), \quad i \in \{x, y, z\}$$

(3)

Figure 3 shows this dependence, and also the points obtained from the simulation, which are in good agreement with Eq. 3 at low frequencies.
Figure 3. The diagonal self-intersection point (circled on the one eighth of the isofrequency surface (Fig. 1(b)) on the right side of the plot) coordinate $k_d$ versus frequency $\omega$ plot. Pink line is given by the Eq. 3, black x-markers are obtained from numerical simulation.

3. Conclusion

The picture of mode polarization near the point of intersection of isofrequency surfaces convinces us of the presence of optical axes predicted by the theory for a given medium and, as a consequence, of the possibility of observing the effect of conical refraction for a given medium [4]. The latter effect has a very wide range of applications, from the creation of optical tweezers to trapping of Bose-Einstein condensates [4]. Thus, the triple wire medium is an easy-to-manufacture alternative to biaxial crystals with adjustable geometry parameters [1] and stable optical axes in a wide frequency range.

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