Evanescent Gravitational Waves

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We describe the properties of evanescent gravitational waves (EGWs)—wave solutions of Einstein equations which decay exponentially in some direction while propagating in another. Evanescent waves are well known in acoustics and optics and have recently received much attention due to their extraordinary properties such as their transverse spin and spin-momentum locking. We show that EGWs possess similarly remarkable properties, carrying transverse spin angular momenta and driving freely falling test masses along in elliptical trajectories. Hence, test masses on a plane transverse to the direction of propagation exhibit vector and scalar-like deformation, complicating efforts to use detection of such modes as evidence for modified gravity. We demonstrate that EGWs are present and dominant in the vicinity of sub-wavelength sources such as orbiting binaries.

I. INTRODUCTION

Evanescent waves, or fields, are solutions to the wave equation which instead of propagating away from the source, decay exponentially. While evanescent waves have been known for a very long time, only recently have they been intensively studied, following increased interest in small scale physics. In nanophotonics, evanescent waves play a dominant role [1]. Recent awareness of their interesting properties has spurred huge interest: evanescent fields were recently found to have a transverse spin [2–5], and to exhibit spin-momentum locking [2–5], and to exhibit spin-momentum locking [6–9], leading to a myriad of practical applications in light nano-routing, quantum optics, nonreciprocal devices, optical forces and polarimetry [10–17]. Beyond electromagnetism, evanescent waves have now been found to exhibit analogous properties in other wave fields, such as quantum mechanics, evanescent waves have now been found to exhibit analogous properties in other wave fields, such as acoustics [18–20]. This work explores the existence of evanescent waves in the framework of linearised gravity. Inspired by the analogy to other wave fields, we discuss their remarkable properties, which include the transverse spinning of free-falling test masses. Evanescent gravitational waves also imply the excitation of vector and longitudinal components of the wave, which is noteworthy, as the presence of these components in a vacuum is often assumed to signify a deviation from general relativity [21]. We show that evanescent gravitational waves are not a rare occurrence. They are present and even dominant in the vicinity of any sub-wavelength source of gravitational waves, such as compact binary systems.

II. EVANESCENT WAVES

Evanescent waves can be described with a wave-function that is an eigensmode of the momentum and energy operators, and as such are mathematically identical to plane waves, \( \psi(t, x) = \Psi \exp(i k \cdot x - i \omega t) \), where \( \mathbf{x} \) is the position vector, \( t \) is the coordinate time, \( \Psi \) is the complex amplitude of this field, \( \mathbf{k} \) is the wave-vector, and \( \omega \) is the angular frequency. The only difference from travelling plane waves is that the wave-vector, or momentum, will be complex, \( \mathbf{k} = \mathbf{k}' + i \mathbf{k}'' \), with an imaginary component in the direction of the exponential decay. In the case of a vector field, such as the electromagnetic field, the mathematical form of evanescent waves is exactly as above, with \( \Psi \) substituted by the electric field amplitude \( \mathbf{E} \). Maxwell equations of electromagnetism impose two conditions on its wave solutions [6, 22]. Firstly, as every solution to the homogeneous Helmholtz wave-equation, the wave has to be null-like, in other words it satisfies the dispersion relation \( \mathbf{k}^2 = \mathbf{k} \cdot \mathbf{k} = k_x^2 + k_y^2 + k_z^2 \). It is important to stress that for complex-valued wave vector, the quantity \( \mathbf{k}^2 = \mathbf{k} \cdot \mathbf{k} = |k'|^2 - |k''|^2 + 2i k' \cdot k'' \) is not equal to the magnitude of the wave-vector \( |\mathbf{k}|^2 = \mathbf{k} \cdot \mathbf{k} = |k'|^2 + |k''|^2 \) [6, 9, 22]. The dispersion relation shows that a wave may surprisingly have \( |k'| > k_0 \), as long as \( |k''| \neq 0 \), demonstrating the mathematical existence of evanescent waves as valid solutions. In a vacuum, this condition can only be satisfied if \( k' \cdot k'' = 0 \), so the direction of decay is necessarily transverse to the direction of propagation. Secondly, the electric field must fulfil the transversality condition \( \mathbf{k} \cdot \mathbf{E} = 0 \) [6, 22]. This condition restricts the allowed polarisation modes of the wave. It reduces, by one, the three degrees of freedom of vector \( \mathbf{E} \), allowing us to express it as a linear combination of two polarisation basis vectors \( \mathbf{E} = E_1 \hat{\mathbf{e}}_1 + E_2 \hat{\mathbf{e}}_2 \). For plane waves, the condition restricts the electric field to lie on a plane transverse to \( \mathbf{k} \), e.g. two orthogonal linearly polarised waves, or two opposite handedness of circularly polarised waves. For evanescent waves, the same mathematical formulation for the basis vectors can be used, but they become complex-valued [6, 9] and, while still fulfilling the condition \( \mathbf{k} \cdot \mathbf{E} = 0 \), the modes acquire longitudinal components of the field ultimately resulting in the remarkable polarisation properties of evanescent waves.
III. GRAVITATIONAL WAVES

The theory of linearised gravity describes gravitational plane waves in terms of the metric perturbation symmetric second rank tensor,

\[ h_{\mu
u}(t, x) = H_{\mu
u} \exp(ik \cdot x - i\omega t). \]  

The massless wave equation requires the null condition

\[ k_\mu k^\mu = -k_0^2 + k_x^2 + k_y^2 + k_z^2 = 0 \]  

which imposes the dispersion relation. Meanwhile, fixing the gauge to be transverse-traceless implies that, in a vacuum, only the spatial components \( h_{ij} \) of \( h_{\mu\nu} \) are non-vanishing for radiation. In this gauge, the transversality condition is

\[ k^i h_{ij} = 0. \]  

That, together with the trace-less condition \( h^i_i = 0 \) gives a set of four equations which reduce the original six degrees of freedom of the symmetric matrix \( h_{ij} \) down to two, therefore restricting it to two allowed polarisation modes. For propagating gravitational plane waves, the two modes may be chosen as the well-known “plus” (+) and “cross” (×) modes, but, in analogy to electromagnetism, these two modes can be extended to the case of complex \( k \) with the use of a complex basis, as follows.

Consider an energy and momentum eigenmode gravitational wave, Eq. (1), travelling in the z-direction and decaying in \( x \) (without loss of generality due to the fact that \( k^i \cdot k''^i = 0 \)). This implies a complex wave-vector \( k = k_0(\alpha, 0, \kappa) \) where \( \alpha \) and \( \kappa \) are both real. The null condition requires that \( 1 = \kappa^2 - \alpha^2 \). Any propagating mode can then be expressed as a linear combination of two complex polarisation modes (Appendix A):

\[ H_{ij} = h_+ \begin{pmatrix} \kappa^2 & 0 & -i\alpha\kappa \\ 0 & -1 & 0 \\ -i\alpha\kappa & 0 & -\alpha^2 \end{pmatrix} + h_\times \begin{pmatrix} 0 & \kappa & 0 \\ \kappa & 0 & -i\alpha \\ 0 & -i\alpha & 0 \end{pmatrix}. \]  

The complex nature of these amplitudes accounts, as usual, for the amplitude and phase of each component. Note that these two modes reduce to the usual gravitational transverse “plus” and “cross” modes when \( \kappa \to 1 \), and correspondingly \( \alpha \to 0 \) due to the null condition. Crucially, in this basis, components which are not transverse to the direction of propagation given by \( k' \) are

| \( \mathcal{H} \) | \( \mathcal{H}_+ (1 + \alpha^2/2) \) | Plus mode |
|------------------|-----------------------------|----------|
| \( \mathcal{H}_x \) | \( h_\times \sqrt{1 + \alpha^2} \) | Cross mode |
| \( \mathcal{H}_1 \) | \( -h_\chi i\kappa \) | Vector-x mode |
| \( \mathcal{H}_2 \) | \( -h_\times i\alpha \) | Vector-y mode |
| \( \mathcal{H}_3 \) | \( -h_\chi \alpha^2 \) | Longitudinal mode |
| \( \mathcal{H}_0 \) | \( h_\times \alpha^2/2 \) | Breathing mode |
present, even though the transversality condition $k^i h^i_{ab} = 0$ is satisfied. This is analogous to the appearance of longitudinal fields in evanescent electromagnetic waves. To show this clearly, we may decompose our basis in terms of the real polarisation basis $H_{ij} = \sum A e^{ij}_A$ (Appendix A, Table I), summed over $A \in \{+ , \times , 0, 1, 2, 3\}$ as shown in (Table I). Thus, evanescent waves in a vacuum can excite the vector and scalar modes even in general relativity—complicating efforts to use the detection of such modes as a smoking gun evidence for modified gravity theories [21]. Nevertheless, we emphasise that these are not additional modes as there are only two effective propagating degrees of freedom—the key point is that the components of these real modes are correlated.

IV. MOTION OF TEST MASSES AND TRANSVERSE SPIN

To study the effects of the wave, we can consider a cloud of freely falling test masses surrounding a fixed point. As long as the cloud is small compared to the cloud of freely falling test masses surrounding a fixed point. For an energy-momentum eigenstate, the key point is that the components of these real modes are correlated.

$$x^i(t) = x^i_0 + \delta x^i(t) = x^i_0 + \frac{1}{2} \Re(h^i_{ij}(t)) x^j_0,$$

where $\delta x^i$ is the relative displacement of the test mass from its initial position. For an energy-momentum eigenmode wave (Eq. (1)), the displacement can be written as

$$\delta x^i(t) = \frac{1}{2} [\Re (H^i_{ij}) \cos(\omega t) + \Im(H^i_{ij}) \sin(\omega t)] x^j_0,$$

which is the parametric equation of an ellipse. Therefore, each test mass will move along a fixed ellipse with centre at $x^i_0$ and semi-axes defined by two conjugate diameter vectors $\frac{1}{2} \Re(H^i_{ij}) x^j_0$ and $\frac{1}{2} \Im(H^i_{ij}) x^j_0$. If we consider propagating non-evanescent plane waves with $\kappa = 1$ in Eq. (4), the elliptical orbit becomes a line segment perpendicular to the propagation vector $k$. Thus, under the influence of non-evanescent gravitational waves, test masses oscillate within a fixed plane perpendicular to the direction of the wave vector $k$, alternately stretching and squeezing along perpendicular directions in the distinct $+$ and $\times$ pattern. Under the influence of evanescent waves, test masses show the same pattern but acquire an additional movement in the longitudinal direction $k'$. When both movements are combined coherently, the masses follow elliptical trajectories on planes parallel to $k'$. For high values of $\kappa \rightarrow \infty$, corresponding to more confined evanescent waves, some of the trajectories become perfect circles. The net effect of this motion is that the test masses carry out the usual $+$ and $\times$ oscillations, but they do so on a plane that is not perpendicular to $k'$ at all times, and instead pivots along an axis either parallel or perpendicular to the imaginary component of the wave vector—like the rocking motion of a playground see-saw (Fig. 1 and https://youtu.be/DB7mHGqsrLk).

Note that the imaginary component of the wave-vector breaks the rotational symmetry of the two modes. The elliptical movement of the masses is hugely reminiscent of the transverse spin of evanescent electromagnetic and acoustic waves. In fact, as described in Appendix C, one may calculate the spin angular momentum density of an evanescent gravitational wave as

$$S = \frac{W}{\omega} \left[ 2\sigma \frac{k'}{|k'|} + 2 \frac{k' \times k''}{|k'|^2} \right],$$

where $\sigma$ is the normalised third Stokes parameter or helicity parameter, equal to $\pm 1$ for purely circularly polarized waves, defined as

$$\sigma = \frac{2 \Im(h^i_{+} h^i_{\times})}{|h_+|^2 + |h_\times|^2},$$

and $W$ is the time-averaged energy density [24]

$$W = \frac{c^2}{128\pi G} \left[ \partial_i h^i_{ij} \partial_j h^{ijj} + c^2 \epsilon_{jmn} \epsilon^{kji} \partial_l h^i_{lk} \partial^m h^{ijn} \right].$$

From Eq. (6), the expected spin-2 nature of the longitudinal intrinsic angular momentum of gravitational waves appears in the first term—in clear analogy to the spin-1 nature of the electromagnetic wave [5]. However, the novel second term represents an intrinsic transverse spin with a value of $+2\alpha/\kappa$. (Interestingly, this is identical to the transverse spin of acoustic waves [20], and twice that of electromagnetic waves). Spin-momentum locking is manifest because reversing the direction of $k'$ also changes the sign of $S$.

V. GENERATION OF EVANESCENT FIELDS

Having described evanescent gravitational waves as a valid solution to the wave equation, we now discuss their occurrence in nature. A straightforward way to produce evanescent waves is to use the phenomenon of total internal reflection. For example, consider an electromagnetic plane wave incident on an interface between two media with a different index of refraction. If the angle of incidence is greater than the critical angle then it is possible to show using conservation of $\kappa$ parallel to the interface, i.e. Snell’s law, that the wave vector of the refracted wave will be complex [1, 22]. For gravitational waves, even though theoretically conceivable, this possibility seems to be physically unrealistic since scattering by matter is negligible [25]. As a consequence, the medium which could
refract or reflect gravitational waves requires either exotic material with large shear modulus or shear viscosity [26] or an array of tightly packed sufficiently compact objects (like black holes or neutron stars) [25]. However, we need not consider such exotic scenarios to observe evanescent waves. Evanescent gravitational fields, in analogy to any other type of wave, are present in the near-field zone of any sub-wavelength source [1]. This fact can be understood via the position-momentum Fourier properties of any wave $\Delta x \Delta k \geq 1/2$. A localised sub-wavelength source necessarily has a wide range of momentum values, i.e. range of wave-vectors, which extend beyond the wave-number of free space, constituting evanescent components. The decay of these components when far from the source is responsible for the diffraction limit in far-field imaging.

VI. MOMENTUM SPACE REPRESENTATION

To show that there are evanescent components near a sub-wavelength source of gravitational radiation, we will use the angular spectrum representation. This is a standard tool for studying wave-fields in homogeneous media and is widely used in nanophotonics to study scattering, beam propagation, focusing, holography, and many other phenomena [27]. The main idea is that, in general, solutions to the wave equation are not momentum eigenmodes with a well-defined wave-vector $k$ as in Eq. (1), but rather a distribution of them constituting a continuous spectrum. The generalisation of this representation to higher ranked tensors such as vectors and rank-2 tensor gravitational waves can be made, as shown below. Suppose we know a field $h_{ij}(r,t)$ at any point. We may assume it is time-harmonic without loss of generality as we can always perform a temporal Fourier transform. We can consider this field in a plane $z = \text{constant}$ which is transverse to an arbitrary $z$-direction. In this plane, we can write the field as a 2D inverse spatial Fourier transform [27]:

$$h_{ij}(x,y,z,t) = \int_{-\infty}^{\infty} \tilde{h}_{ij}(k_x, k_y; z \alpha(k_x, k_y, x, y \omega t) \, dk_x \, dk_y.$$ 

Since the wave satisfies the wave equation and hence the null condition Eq. (2) it is possible to “propagate” the field from the source plane to any other plane with different $z = \text{constant}$ via a simple multiplicative transfer function [27]:

$$\tilde{h}_{ij}(k_x, k_y, z) = \tilde{h}_{ij}^{(+)}(k_x, k_y, 0)e^{ik_z z} + \tilde{h}_{ij}^{(-)}(k_x, k_y, 0)e^{-ik_z z}.$$ 

where the two terms (+) and (−), not to be confused with the $+$ and $\times$ modes, account for the two possible signs of $k_z = \left(k_0^2 - k_x^2 - k_y^2\right)^{1/2}$. When the fields originate from a localised source at $z = 0$, only the sign of $k_z$ propagating away from the source needs to be considered; hence we use the plus representation for $z > 0$ and the minus representation for $z < 0$. This means that a complete knowledge of the fields in the entirety of space can be gained from one single plane. A key simplification can be made by realising that the integrand becomes

$$\tilde{h}_{ij}^{(±)}(k_x, k_y, 0)e^{ik_z z},$$

which has exactly the same form as Eq. (1), and therefore, for each value of transverse momentum $(k_x, k_y)$, it must fulfill Eqs. (2) and (3) and can thus be reduced to a superposition of only two tensor modes as in Eq. (4):

$$\tilde{h}_{ij}^{(±)}(k_x, k_y; 0) = h_{ij}^{(±)} e_1^{±} + h_{ij}^{(±)} e_2^{±},$$

where $e_1^{±}(k_x, k_y, ±k_z)$ and $e_2^{±}(k_x, k_y, ±k_z)$ are the same complex basis tensors introduced in Eq. (4), but generalised for arbitrary directions (see Appendix A). Therefore, a pair of scalar complex-valued angular spectra $h_{ij}^{(±)}(k_x, k_y)$ and $h_{ij}^{(±)}(k_x, k_y)$ completely describe the source in momentum space, and hence in all of real space via the spectral representation. These two spectra include all information of the amplitude and phase of the two polarisation modes of the propagating far field plane waves in every direction (the momentum representation in the region $k_x^2 + k_y^2 < k_0^2$, corresponding to real $k_z$ and also tell us about all the evanescent near fields components (in the region $k_x^2 + k_y^2 > k_0^2$, corresponding to imaginary $k_z$) whose amplitude decays exponentially as $|z|$ is increased.

As the simplest example, in electromagnetism, we can find evanescent fields near a radiating electric dipole [1, 28, 29]. Due to the quadrupolar nature of gravitational waves, it is natural to expect there will be evanescent fields in the vicinity of a radiating gravitational quadrupole. In close analogy with [30] we present an analytical expression of the angular spectrum of gravitational quadrupole radiation. We consider a binary system of compact objects with same mass $M$ separated by a distance $d$ in a stable circular orbit with frequency $\Omega$ around a common centre of mass (Fig. 2). Furthermore, we assume that the speed of the masses is not relativistic $v \ll c$ (note that this directly implies that this is a sub-wavelength source $k \cdot d \ll 1 \leftrightarrow d \ll \lambda$) and that the observer is at distance $|r - r_0| \gg 2d$ so that one can neglect effects due to the retarded position. For such a system the solution simplifies to the well-known quadrupole formula [23]

$$h_{ij}(t, r) = \frac{2G M}{r^3} \frac{d^2}{dt^2} Q_{ij}(t - \frac{r}{c}),$$

where $Q_{ij}$ is the quadrupole moment of mass density $\rho$ defined as

$$Q_{ij}(t) = \iiint_{\mathbb{R}^3} \rho(t, x) x_i x_j d^3x.$$ 

The source considered is time harmonic with wave frequency $\omega = k_0 c = 2\Omega$. The solution of Eq. (9), in fre-
frequency space is given by (see Appendix E):
\[
\tilde{h}_{ij} (\omega, r) = \frac{G}{4c^4} \frac{\omega^2}{r} e^{i\omega r/c} q_{ij},
\]
where
\[
q_{ij} \equiv M d^2 \frac{1}{2} \begin{pmatrix}
-1 & 0 & -1 \\
0 & 0 & 1 \\
-i & 0 & 1
\end{pmatrix}
\]
is a constant tensor. This form allows us to reproduce identical mathematical steps as taken in [30] for an electromagnetic dipole to find the angular spectra of the gravitational quadrupole (see Appendix E). As a result, we present expressions for the two scalar amplitudes which represent separately the two polarisation modes of the gravitational quadrupole:
\[
\begin{align*}
&h_+^{(\pm)} (k_x, k_y, 0) = \frac{iG}{16\pi c^2} \frac{k_0^2}{k_z} k_x e^{ij} q_{ij} e^{ij} (k_x, k_y, \pm k_z), \quad (10a) \\
&h_\times^{(\pm)} (k_x, k_y, 0) = \frac{iG}{16\pi c^2} \frac{k_0^2}{k_z} q_{ij} e^{ij} (k_x, k_y, \pm k_z). \quad (10b)
\end{align*}
\]
These amplitudes contain all the information necessary to reconstruct the fields of the quadrupole source at every location in space, including its near field. The complex amplitudes of the two spectra $|h_+^{(\pm)}|$ and $|h_\times^{(\pm)}|$ are plotted in Fig. 2 after propagating them to a plane $z = -\lambda/2\pi$ via the transfer function $e^{-ik_z z}$. We see that for the sub-wavelength distance here considered, there is a strong presence of waves with $k$ in the region $k_z^2 + k_y^2 > k_0^2$, implying an imaginary component of $k$ in the $z$ direction due to the null-condition, corresponding to evanescent waves. A similar procedure can be repeated for any other source of gravitational waves, so one can see that in the near field of any source there will be a full spectrum of evanescent waves.

VII. CONCLUSIONS

In recent years, evanescent waves and their properties have raised considerable interest in optics and acoustics. This letter is the first work dedicated to the study of these in the context of gravity. This required extending the formalism from vector to tensor modes. We have found that not only evanescent gravitational waves can exist, but also that they are not exotic phenomena and one can expect them in the near zone of any source of gravitational waves. It turns out that in analogy with electromagnetic and acoustic waves, gravitational waves also possess non-trivial polarisations. This fact confirms the trend seen in other types of waves, that evanescent waves are always associated with a transverse spin, and exhibit similar spin-momentum locking. Another implication of the existence of evanescent gravitational waves is that, even if non-tensorial modes of gravitational waves in a vacuum are detected, this does not necessarily contradict general relativity as they may originate from an evanescent field: one can check whether or not the polarisation modes are correlated. Non-tensorial modes may also originate from the coherent superposition of two propagating plane waves arriving simultaneously at a detector, whose combined polarisation can be locally identical to that of an evanescent wave (see Appendix D), in analogy to the transverse spin that appears in electromagnetic two-wave interference [5]. Direct detection of near field evanescent gravitational waves is technologically challenging—at least until we can launch our detectors near compact objects like black holes, or possess sufficient sensitivities in pulsar timing arrays measurements [31]. In the meantime, due to the effect of evanescent gravitational waves on test particles, there are some potential options for indirect detection. For example, charged particles moving on an elliptical trajectory due to the evanescent fields near the source of gravitational waves should radiate electromagnetic radiation whose polarisation signature we could detect.

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Appendix A: Polarisation basis of evanescent fields

In linearised gravity, gravitational waves are often expressed in the transverse-traceless (TT) gauge in terms of the traceless symmetric metric perturbation

\[ h_{\mu\nu}(t, \mathbf{x}) = H_{\mu\nu} \exp(i \mathbf{k} \cdot \mathbf{x} - i\omega t) \]  

(A1)

where \( \mathbf{k} \) is a three vector pointing in the spatial dimensions and \( \omega \) is the frequency, and the Greek indices run from 0 to 3. In the TT gauge, there always exist an appropriate gauge transformation such that \( h_{0\mu} = 0 \) (even in the presence of evanescent waves), hence in vacuum one can represent the perturbation in its spatial components \( h_{ij} \) \((i, j = 1, 2, 3)\) in the Cartesian coordinate basis.

In vacuum, it is well known that gravitational wave solutions of the Einstein equation possess two polarisation modes,

\[ H_{ij} = h_{+} e^{+}_{ij}(\mathbf{k}) + h_{\times} e^{\times}_{ij}(\mathbf{k}) \]  

(A2)

where \( e^{+}_{ij}(\mathbf{k}) \) and \( e^{\times}_{ij}(\mathbf{k}) \) are the + and \( \times \) polarisation modes, which depend on the wave vector \( \mathbf{k} \). Given any \( \mathbf{k} \) with components \( k_i = (k_x, k_y, k_z) \) and \( k_0 \equiv \sqrt{k_x^2 + k_y^2 + k_z^2} \), one can construct these basis modes via the following construction

\[ e^{+}_{ij}(\mathbf{k}) = e^{i}_{i}(\mathbf{k}) e^{j}_{j}(\mathbf{k}) - e^{j}_{i}(\mathbf{k}) e^{i}_{j}(\mathbf{k}) \],

\[ e^{\times}_{ij}(\mathbf{k}) = e^{i}_{i}(\mathbf{k}) e^{j}_{j}(\mathbf{k}) + e^{j}_{i}(\mathbf{k}) e^{i}_{j}(\mathbf{k}) \]  

(A3)

where

\[ e^{i}_{i}(\mathbf{k}) = \frac{1}{\sqrt{k_x^2 + k_y^2}} \left( -k_y, k_x, 0 \right)^T \]

\[ e^{j}_{j}(\mathbf{k}) = \frac{1}{k_0 \sqrt{k_x^2 + k_y^2}} \left( k_z, k_y k_z, -k_x^2 - k_y^2 \right)^T \]

are two unit vectors transverse to the wave-vector \( (e^{i}_{i}(\mathbf{k})) k^i = e^{j}_{j}(\mathbf{k}) k^j = 0 \) which, when \( \mathbf{k} \) is real, correspond to the usual basis vectors in spherical coordinates. But for any arbitrary vector, including complex valued ones, we can still use Eq. (A3) to construct the polarisation basis as

\[ e^{+}_{ij}(\mathbf{k}) = \frac{1}{k_0} \begin{pmatrix}
\frac{k_x^2 - k_y^2 k_z^2}{k_x^2 + k_y^2} & \frac{k_x^2 - k_y^2 k_z^2}{k_x^2 + k_y^2} & k_x k_y - k_x k_z \\
\frac{k_y^2 - k_x^2 k_z^2}{k_x^2 + k_y^2} & \frac{k_y^2 - k_x^2 k_z^2}{k_x^2 + k_y^2} & k_y k_z - k_x k_y \\
-k_x k_z & -k_y k_z & k_x^2 + k_y^2
\end{pmatrix} \]  

(A4)

and

\[ e^{\times}_{ij}(\mathbf{k}) = \frac{1}{k_0} \begin{pmatrix}
\frac{2k_x k_y k_z}{k_x^2 + k_y^2} & \frac{k_x^2 - k_y^2 k_z^2}{k_x^2 + k_y^2} & k_x k_y \\
\frac{k_y^2 - k_x^2 k_z^2}{k_x^2 + k_y^2} & \frac{2k_x k_y k_z}{k_x^2 + k_y^2} & k_y k_z \\
-k_y & k_x & 0
\end{pmatrix}. \]  

(A5)

When waves are evanescent, the wave-vector \( k^i \) is complex in general, and the transversality condition also implies that the basis vectors \( e^{i}_{i}(\mathbf{k}) \) and \( e^{j}_{j}(\mathbf{k}) \) are also complex. Hence, the polarisation modes Eq. (A4) and Eq. (A5) are also in general complex. However, it is easy to show that both polarisation modes are still solutions of the gravitational wave equation as long as the null condition \( \omega^2/c^2 = k_0^2 = k_x^2 + k_y^2 + k_z^2 \) is satisfied, and therefore form a valid basis in general. In (Table I) in the main text we decomposed the wave into a real polarisation basis \( H_{ij} = \sum A_A e^{A}_{ij}(\mathbf{k} = \mathbf{k}') \), summed over \( A \in \{+, \times, 0, 1, 2, 3\} \), note that it is decomposed along \( \mathbf{k}' \) which is the real part of the wave vector, \( A_A \) is an amplitude of the corresponding mode \( A \) and basis tensors \( e^{A}_{ij} \) are [32]

| TABLE II. Real polarisation basis defined |
|------------------------------------------|
| \( e^{A}_{ij}(\mathbf{k}') = e^{A}_{i}(\mathbf{k}') e^{j}_{j}(\mathbf{k}') - e^{j}_{i}(\mathbf{k}') e^{A}_{i}(\mathbf{k}') \) | Plus mode |
| \( e^{A}_{ij}(\mathbf{k}') = e^{i}_{i}(\mathbf{k}') e^{+}_{j}(\mathbf{k}') + e^{j}_{j}(\mathbf{k}') e^{i}_{i}(\mathbf{k}') \) | Cross mode |
| \( e^{i}_{i}(\mathbf{k}') = e^{i}_{i}(\mathbf{k}') \hat{k}'_i + \hat{k}'_i e^{i}_{i}(\mathbf{k}') \) | Vector-x mode |
| \( e^{j}_{j}(\mathbf{k}') = e^{j}_{j}(\mathbf{k}') \hat{k}'_j + \hat{k}'_j e^{j}_{j}(\mathbf{k}') \) | Vector-y mode |
| \( e^{\times}_{ij}(\mathbf{k}') = e^{i}_{i}(\mathbf{k}') e^{\times}_{j}(\mathbf{k}') + e^{j}_{j}(\mathbf{k}') e^{i}_{i}(\mathbf{k}') \) | Breathing mode |

Appendix B: Motion of test masses under Evanescent Waves

In this section, we calculate the motion of test masses in the presence of a single mode of evanescent gravitational wave, and show that the loci of test masses are ellipses (as opposed to straight lines in plane waves). In vacuum \( k' \cdot k'' = 0 \), so without loss of generality we may orient our axes to consider a wave vector

\[ \mathbf{k} = k_0 \begin{pmatrix}
\kappa_x \\
0 \\
\kappa_z
\end{pmatrix}, \ 1 = \kappa_x^2 + \kappa_z^2, \]  

(B1)

where the second equation imposes the null-like condition. Using Eq. (A4) and Eq. (A5), the polarisation modes for this wave are then

\[ e^{+}_{ij} = \begin{pmatrix}
\kappa_x^2 & 0 & -\kappa_x \kappa_z \\
0 & 0 & -\kappa_z \kappa_x \\
-\kappa_x \kappa_z & 0 & \kappa_z^2
\end{pmatrix}, \]  

(B2)

\[ e^{\times}_{ij} = \begin{pmatrix}
0 & \kappa_z & 0 \\
-\kappa_z & 0 & 0 \\
0 & 0 & \kappa_x
\end{pmatrix}. \]  

(B3)

When \( \kappa_z > 1 \), then \( \kappa_x \) becomes imaginary, and the wave becomes evanescent. In the main text we define \( \kappa_z = \kappa \) and \( \kappa_x = i\alpha \) to keep the variables real.

The effect of the wave on freely falling test masses can
be examined using the geodesic deviation equation
\[
\frac{\partial^2 x^\rho}{\partial t^2} = -R^\rho_{00}(t)x^\nu - \frac{1}{2} \frac{\partial^2}{\partial t^2}(h^\rho_\nu)_{\text{TT}}(t)x^\nu ,
\]
where \(x^\nu\) are vectors representing the proper positions of the test masses with respect to a fixed point. These equations have the solutions
\[
x^\rho(t) = x^\rho_0 + \frac{1}{2} \text{Re}(h^\rho_\nu(t))x^\nu_0 + \frac{1}{2} \text{Im}(h^\rho_\nu(t))x^\nu_0 \sin(\omega t) .
\]
where \(x^\rho_0\) is the initial condition (i.e. initial positions of the masses). If the waves are evanescent, \(h^\rho_\nu_{\text{TT}}\) has complex coefficients, and we can express the solution as a sum of its real and imaginary components
\[
x^\rho = x^\rho_0 + \frac{1}{2} \text{Re}(H^\rho_\nu(t))x^\nu_0 \cos(\omega t) + \frac{1}{2} \text{Im}(H^\rho_\nu(t))x^\nu_0 \sin(\omega t) .
\]
Notice that for plane waves, \(\text{Im}(H^\rho_\nu)_{\text{TT}} = 0\), and we recover the usual solution where test masses oscillate along a straight line with frequency \(\omega\). The presence of the imaginary component which is off-phase to the real component means that the locus of test particles become ellipses as we asserted earlier.

Also, masses which are initially at \(x^i_0 = (x_0, y_0, z_0)\) with \(z_0 = 0\) or \(y_0 = 0\) will move on a trajectory which is confined to a plane parallel to \(k'\). To prove this we find the normal vector of this plane which is \(a \times b\), where \(a^i = \frac{1}{2} \text{Re}(H^i_1)_{\text{TT}}x^j_0\) and \(b^j = \frac{1}{2} \text{Im}(H^j_1)_{\text{TT}}x^i_0\) and take a dot product with \(k'\). Without loss of generality we use \(k = h_0(\omega, 0, \kappa)\) and Eq. (4) from the main text
\[
(k') \cdot (a \times b) = -(h^2_+ + h^2_-)\kappa^2 \alpha y_0 z_0 , \tag{B6}
\]
note that this makes sense only if \(\alpha \neq 0\), because otherwise vector \(a \times b\) does not exist.

**Appendix C: Transverse spin angular momentum**

In this section will briefly discuss the transverse spin of evanescent gravitational waves. Transverse spin is a signature property of evanescent waves. It is extensively studied for electromagnetic waves [4, 5] but it has also been recently discovered in acoustic waves [20].

The most convenient way to study the spin of gravitational waves is using a Maxwellian form of linearised gravity proposed in [24]. This formalism introduces gravitational analogues of the electric and magnetic fields which can be expressed in terms of \(h_{ij}\) in TT gauge as
\[
E_{ij} = -\partial_j h^T_{ij} , \tag{C1}
\]
\[
B_{ij} = \epsilon_{jlm} \partial_l h^T_{im} . \tag{C2}
\]
This treatment of the gravitational field is possible as long as we consider weak field limit in a flat, Minkowski, background. One can then find the expression for spin angular momentum (SAM) density by calculating Noether charge associated with rotations and isolate the spin part. The time averaged SAM is
\[
S^i = \frac{c^2}{64\pi G} \left[ E^*_{jm} E_{km} + c^2 B^*_{jm} B_{km} \right] \epsilon^{ijk} , \tag{C3}
\]
noting that the complex conjugates come from the time average (here \(E_{ij}\) and \(B_{ij}\) are considered to be phasors) and unlike the source material we keep \(c\) and \(G\) explicit.

In analogy with [5, 20] we normalise the spin using time-averaged energy density
\[
W = \frac{c^2}{128\pi G} \left( E^*_{ij} E^{ij} + c^2 B^*_{ij} B^{ij} \right) . \tag{C4}
\]
Now one can examine the transverse spin in evanescent gravitational waves. Assuming evanescent wave with \(k = k' + i k''\) and arbitrary polarisation Eq. (4) in the main text leads to
\[
S = \frac{W}{\omega} \left[ 2 \frac{\sigma}{k} \frac{k'}{|k'|^2} + 2 \frac{\alpha}{k} \frac{k' \times k''}{|k'|^2} \right] = \frac{W}{\omega} \left[ 2 \sigma c |k''|^2 \frac{2 \alpha k' \times k''}{|k'|^2} \right] , \tag{C5}
\]
where \(\sigma\) is normalised third Stokes parameter, helicity parameter, which is defined as
\[
\sigma = \frac{2 \text{Im}(h^*_{+} h_{\times})}{|h_{+}|^2 + |h_{\times}|^2} .
\]
As one might expect, it is manifest that for linearly polarised travelling plane wave the longitudinal spin vanishes (as \(h^*_{ij} = h_{ij}\)). Eq. (C5) shows that evanescent gravitational wave will acquire transverse spin
\[
\frac{\omega S}{W} = 2 \frac{k' \times k''}{|k'|^2} \tag{C6}
\]
which will be present for any polarisation and which is momentum locked (if the direction of \(k'\) is reversed so will be the direction of this spin).

Table III presents comparison between gravitational, electromagnetic and acoustic waves. One can see that the transverse spin for an acoustic field also has the factor of two but in the case of an acoustic field this is due to uneven contribution of the acoustic pressure \(p\) and velocity \(v\) (these fields play the role of \(E\) and \(H\) fields in acoustics) to the SAM [20].

**Appendix D: Local description as a linear combination of travelling waves**

In analogy with electromagnetism, it is possible to locally describe the polarisation of an evanescent wave as that of two interfering plane waves travelling in mutually
TABLE III. The extension of (Table I) published in [20]. This table compares gravity, electromagnetism and acoustics. Here $\tilde{h}_{\mu\nu}$ is a trace-reversed metric perturbation and $h_{ij}^{TT}$ is its transverse traceless part. $A^\perp$ is an electromagnetic four-potential and $A^+$ is its transverse part. For gravitational field we define $\epsilon = \epsilon_0 \epsilon_r$ and $\mu = \mu_0 \mu_r$ same as in analogy to electromagnetism. Quantities $W$ and $S$ are averaged over a period.

| potentials | Linearised Gravity | Electromagnetism | Acoustics |
|------------|-------------------|-----------------|-----------|
| $\tilde{h}^{\mu\nu}$ | $\hat{A}^\nu$ | $\varphi$ |
| wave equation | $\Box \tilde{h}_{\mu\nu} = 0$ | $\Box A_\nu = 0$ | $\Box \varphi = 0$ |
| gauge condition | $\partial_\mu \tilde{h}^\mu_{\nu} = 0$ | $\partial_\nu \hat{A}^\nu = 0$ | $\partial_\nu \varphi = 0$ |
| fields | $E_{ij} = -\partial_t h_{ij}^{TT}$ | $E = -\partial_t A^\perp$ | $p = \rho \partial_\nu \varphi$ |
| | $\mu H_{ij} = \epsilon_{jim} \partial_i (h^{TT})_{mn}$ | $\mu H = \nabla \times A^\perp$ | $\nu = \nabla \varphi$ |
| constraints | $\partial^\nu E_{ij} = \partial^\nu H_{ij} = 0$ | $\nabla \cdot E = \nabla \cdot H = 0$ | $\nabla \times \nu = 0$ |
| medium parameters | $\epsilon_0 = \frac{c^2}{32\pi G} = \frac{1}{c^2 \mu_0}$ | $\epsilon, \mu$ | $\rho, \beta$ |
| energy density $W$ | $\frac{1}{4} (\epsilon E_{ij}^\alpha E^{ij} + \mu H_{ij}^\alpha H^{ij})$ | $\frac{1}{4} (\mu |E|^2 + \rho |H|^2)$ | $\frac{1}{4} (\beta |\rho|^2 + \rho |\nu|^2)$ |
| SAM density $S$ | $\frac{\epsilon_{ijk}}{2\omega} \left[ \epsilon E_{jm} E_{kn} + \mu E_{jm} E_{kn} \right]$ | $\frac{1}{4\omega} \left[ \epsilon E^\perp \times E + \mu H^\perp \times H \right]$ | $\frac{1}{2\omega} \left[ \rho \nu^\perp \times \nu \right]$ |
| transverse spin density | $\frac{\omega S_{ij}}{W} = 2 \frac{k^i \times k^j}{|k|^2}$ | $\frac{\omega S_{ij}}{W} = \frac{k^i \times k^j}{|k|^2}$ | $\frac{\omega S_{ij}}{W} = 2 \frac{k^i \times k^j}{|k|^2}$ |
| longitudinal spin density | $\frac{\omega S_{ij}}{W} = \frac{\sigma k^i}{|k|}$ | $\frac{\omega S_{ij}}{W} = \frac{\sigma k^i}{|k|}$ | $\frac{\omega S_{ij}}{W} = 0$ |

Orthogonal directions. Without loss of generality, for any evanescent wave it is possible to find a frame where

$$e^{+}_{ij}(k' + ik'') = \begin{pmatrix} \kappa^2 & 0 & -i\alpha \kappa \\ 0 & -1 & 0 \\ -i\alpha \kappa & 0 & -\alpha^2 \end{pmatrix},$$

$$e^{-}_{ij}(k' + ik'') = \begin{pmatrix} 0 & \kappa & 0 \\ \kappa & 0 & -i\alpha \\ 0 & -i\alpha & 0 \end{pmatrix},$$

with $\kappa^2 = 1 + \alpha^2$ being the null condition. Now one can see that

$$\begin{pmatrix} \kappa^2 & 0 & -i\alpha \kappa \\ 0 & -1 & 0 \\ -i\alpha \kappa & 0 & -\alpha^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \alpha^2 & 0 & -i\alpha \kappa \\ 0 & 0 & 0 \\ -i\alpha \kappa & 0 & -\alpha^2 \end{pmatrix},$$

which is a linear combination of a plus polarised wave in the direction of $k'$ and an elliptically polarised wave in the direction of $k' \times k''$

$$e^{+}_{ij}(k' + ik'') = e^{+}_{ij}(k') + (\alpha^2 e^{+}_{ij}(k' \times k'') - i\alpha k e^{\perp}_{ij}(k' \times k'')).$$

Similarly for the cross polarisation

$$\begin{pmatrix} 0 & \kappa & 0 \\ \kappa & 0 & -i\alpha \\ 0 & -i\alpha & 0 \end{pmatrix} = \kappa \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} - i\alpha \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

which is a linear combination of a cross polarised wave in the direction of $k'$ and an out-of-phase cross polarised wave in the direction of $k''$

$$e^{-}_{ij}(k' + ik'') = \kappa e^{\perp}_{ij}(k') - i\alpha e^{-}_{ij}(k'').$$

Appendix E: Evanescent gravitational waves near a quadrupolar source

A system with two equal masses $M$ orbiting each other at a distance $d$ in a stable circular orbit with frequency $\Omega$ around a common centre of mass on the $x-z$ plane (Fig. 2) is given by

$$x_s(t) = \frac{d}{2} \left( \cos(\Omega t), 0, \sin(\Omega t) \right)^T,$$

$$\rho(t,x) = M \left[ \delta^3(x - x_s(t)) + \delta^3(x + x_s(t)) \right].$$
Assuming that

- the field is weak $|h| \ll 1$,
- point masses are in stable circular orbit,
- the speed of the source is not relativistic $v \ll c$, which directly implies that the source is sub-wavelength $k \cdot d \ll \lambda/2\pi$,
- The observer is not too close to the source (at distance comparable to $2d$) which does not necessarily mean that the observer is in the far field thanks to the previous assumption. The observer can be at distance $r$ for which $2d \ll r \ll \lambda$.

This system has the well-known solution

$$h_{ij}(t, r) = \frac{2G1}{\epsilon^2} \frac{d^2}{r \, dt^2} Q_{ij}(t - r/c) , \quad (E2)$$

where $Q_{ij}$ is the quadrupole moment of a mass density $\rho$

$$Q_{ij} = \iiint_{\mathbb{R}^3} \rho(t, x_i, x_j) \, dx . \quad (E3)$$

The solution of Eq. (E2), in frequency space is given by

$$h_{ij}(\omega, r) = \frac{G\omega^2}{4\epsilon^2} q_{ij} \frac{e^{i\omega r/c}}{r} , \quad (E4)$$

where

$$q_{ij} = \begin{bmatrix} -1 & 0 & -i \\ 0 & 0 & 0 \\ -i & 0 & 1 \end{bmatrix} . \quad (E5)$$

Eq. (E4) describes a spherical wave, with origin at $r = 0$ or $x = 0$. At $r \gg \lambda$, the wavefront is asymptotic to a plane wave locally. However, in the near zone, this is not true. Our goal hence is to find a decomposition of a spherical wave into a spectrum of plane waves, labelled by $(k_x, k_y, k_z)$. This angular spectrum representation is a standard problem in the study of reflection and refraction of spherical waves. This decomposition can be performed by first choosing a special axis (we choose $z$), and then performing a 2-D Fourier transform on the plane (we choose $(x, y)$) as follows:

$$\frac{e^{i\omega r/c}}{r} = \frac{i}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{k_z} e^{ik_x x + ik_y y + i k_z z} \, dk_x dk_y , \quad (E6)$$

where we choose the plus representation for $z > 0$ and the minus representation for $z < 0$. This representation is often known as the Weyl identity and its derivation can be found on Ref. [27]. This implies that Eq. (E4) possesses the following spectrum of plane waves

$$h_{ij}(\omega, x, y, z) = \int_{-\infty}^{\infty} \tilde{h}_{ij}^{(\pm)}(\omega, k_x, k_y, z) e^{ik_x x + ik_y y} \, dk_x dk_y , \quad (E7)$$

with

$$\tilde{h}_{ij}^{(\pm)}(\omega, k_x, k_y, z) = \frac{G}{8\pi\epsilon^2} \frac{i\omega^2}{k_z} q_{ij} e^{\pm ik_z z} . \quad (E8)$$

Note that this decomposition, and the resulting spectrum Eq. (E8) breaks the spherical symmetry of Eq. (E4). The square root term can be identified directly with the $z$ component of the wave-vector, owing to the null distance $r$. This angular spectrum representation for evanescent components, which in fact dominate in amplitude when sufficiently close to the source.

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