Negative magnetoresistance in viscous flow of two-dimensional electrons

P. S. Alekseev*
Ioffe Physical Technical Institute, 194021 St. Petersburg, Russia

At low temperatures, in very clean two-dimensional (2D) samples the electron mean free path for collisions with static defects and phonons becomes greater than the sample width. Under this condition, the electron transport occurs by formation of a viscous flow of an electron fluid. We study the viscous flow of 2D electrons in a magnetic field perpendicular to the 2D layer. We calculate the viscosity coefficients as the functions of magnetic field and temperature. The off-diagonal viscosity coefficient determines the dispersion of the 2D hydrodynamic waves. The decrease of the diagonal viscosity in magnetic field leads to negative magnetoresistance which is temperature- and size-dependent. Our analysis demonstrates that the viscous mechanism is responsible for the giant negative magnetoresistance recently observed in the ultra-high-mobility GaAs quantum wells. We conclude that 2D electrons in that structures in moderate magnetic fields should be treated as a viscous fluid.

PACS numbers: 72.20.-i, 73.63.Hs, 75.47.De, 75.47.Gk

1. In modern high-quality GaAs heterostructure samples with low-temperature mobilities of 2D electrons of the order of $10^6 - 10^7 \text{cm}^2/\text{V}\cdot\text{s}$ the electron mean free path for collisions with static defects and phonons, $l$, can be greater than the sample width $w$. In this case, the transport properties depend on the character of electron scattering at the sample edges. If the scattering is **specular** and the sample has the form of a long rectangle, then, after several collisions with the edges, an electron will eventually be scattered by a defect or a phonon. These processes will determine the Drude resistivity $\rho_D = m/e^2 n \tau = l/v_F$, similar to the usual case when $l \ll w$. Here $n$ is the electron concentration, $e$ and $m$ are the electron charge and the effective mass, and $v_F$ is the Fermi velocity.

If the scattering on the sample edges is **diffusive**, the electron transport will be controlled by the relation between the mean free path for electron-electron collisions, $l_{ee}$, and the sample width $w$. When $l_{ee}$ is greater than $w$, the scattering at the edges dominates and the transport mean free path will be of the order of $w$. The corresponding “ballistic” resistivity is $\rho_{ball} = m/e^2 n \tau_{ball}$, where $\tau_{ball} \sim w/v_F$. In the opposite case, $l_{ee} \ll w$, the electron transport should resemble the Poiseuille flow in conventional hydrodynamics with the resistance proportional to the electron shear viscosity $\eta \sim v_F l_{ee}$. This idea was put forward (for three-dimensional metals) by R.N. Gurzhi with coauthors a long time ago [1–3], and more recently it was also applied to various aspects of two-dimensional electron transport [4–10]. The equations describing a flow of a viscous electron fluid in a sample have some common features with the magnetohydrodynamic equations of charge-compensated viscous fluids (e.g., plasma in the hydrodynamic limit) [11, 12].

If a sample is placed in magnetic field and the electron cyclotron radius $R_c$ is much smaller than the sample width $w$, the hydrodynamic regime can be realized even when $l_{ee} > w$ (but herewith $l_{ee} \ll l$) [2]. Indeed, an electron moving along the trajectory similar to the circle with the radius $R_c \ll w$ does not scatter on the sample edges, but undergoes all other types of scattering. The electron viscosity, like other kinetic coefficients, becomes a tensor depending on magnetic field [13, 14].

Another type of solid state systems with the hydrodynamic mechanism of electron transport was studied in Ref. [15]. The authors of that paper considered a 2D viscous electron flow bypassing the defects located one from another at the distances of the order of $d \gg l_{ee}$. If the electron-electron scattering dominates, a viscous flow in the regions between the defects is formed and the sample resistance is again proportional to the viscosity $\eta$.

In this Letter we develop the hydrodynamic approach for the 2D electron transport in magnetic field [16]. We calculate the electron viscosity tensor in a shortcut way similar to the textbook derivation of the Drude conductivity. The non-diagonal viscosity $\eta_{xy}$ determines the dispersion law of the 2D hydrodynamic waves in magnetic field. The decrease of the diagonal viscosity $\eta_{xx}$ with magnetic field provides a mechanism for large negative magnetoresistance of 2D electrons which is temperature- and sample width-dependent [17]. We perform detailed calculations of magnetoresistance for the conventional Poiseuille flow in a long rectangular GaAs sample with rough edges. We also qualitatively demonstrate that the hydrodynamic negative magnetoresistance arises in the 2D samples of other types, in particular, in the samples containing large-radius defects.

The temperature-dependent giant negative magnetoresistance of 2D electrons in high-quality GaAs quantum wells at low temperatures and moderate magnetic fields, reported in several recent publications [15, 24], and especially the “colossal” negative magnetoresistance, observed in Ref. [21], are not understood at the present time. Several striking features of these experiments, es-
especially, the temperature dependence of magnetoresistance, are in a fine agreement with the predictions of our model. Our theory explains the existence of a magnetoresistance peak as well as its broadening and disappearing with temperature \( T \) [22, 24]. Thereby we conclude that 2D electrons in the ultra-high-mobility GaAs quantum wells in moderate magnetic fields form a viscous fluid \( \mp \).

2. We recall the simple hydrodynamic approach in the extreme case when the electron mean free path \( l_{ee} \) is much less than the 2D sample width \( w \), while the mean free path for scattering by phonons and static defects is much greater than \( w \). Also the sample length \( L \) is assumed to be much greater than \( w \). The hydrodynamic electron velocity \( V(y) \), directed along \( x \), obeys the equation:

\[
\frac{\partial V}{\partial t} = \eta \frac{\partial^2 V}{\partial y^2} + \frac{e}{m} E,
\]

where \( \eta = v_F l_{ee} / 4 \) is the viscosity of the 2D degenerate electron gas and \( E \) is the electric field directed along \( x \). In the present work we neglect the compressibility and the thermal conductivity effects.

The conventional boundary conditions require \( V = 0 \) at \( y = \pm w/2 \). This implies that the electron scattering at the sample edges is diffusive [28]. In stationary case, the solution of Eq. (1) gives the parabolic velocity profile \( V(y) \). Integrating the current density \( j_x(y) = evV(y) \) over \( y \) one obtains the resistivity:

\[
\rho = \frac{m}{e^2 n \tau^*}, \quad \tau^* = \frac{w^2}{12\eta}.
\]

Here \( \tau^* \) is the “effective” relaxation time which, in the hydrodynamic regime, replaces the normal momentum relaxation time \( \tau \) in the formula \( \eta \rho = m/e^2n\tau \).

Saying precisely, by the electron-electron scattering time \( \tau_{ee} = l_{ee}/v_F \) we have to imply the relaxation time \( \tau_{2,ee} \) of the second moment of the electron distribution function (i.e., its harmonics \( \sim e^{i m \phi} \) with \( m = \pm 2 \), where \( \phi \) is the angle of the single electron velocity). For such the time a calculation was done for an almost ideal Fermi gas and the Debye model for screening of the Coulomb potential. Following the approach of Ref. [29], we obtained:

\[
\frac{\hbar}{\tau_{2,ee}(T)} = A_{ee} T^2 / E_F,
\]

where \( T \) is the temperature, \( E_F = mv_F^2 / 2 \) is the Fermi energy, and \( A_{ee} = A_{ee}(E_F) \) is a dimensionless value of the order of 1 for \( E_F \) corresponding to typical GaAs samples. However, for that samples the electron-electron interaction energy is of the same order of magnitude as the electron kinetic energy. Calculation of the time \( \tau_{2,ee} \) for a system of strongly interacting electrons is very laborious, but it leads to the result [10], which is quite similar to Eq. (3) (see Refs. [30, 31]).

Thus the characteristic features of the ideal viscous electron transport consist in (i) inverse dependence of resistivity on the square of the sample width, \( \rho \sim 1/w^2 \), and (ii) inverse dependence of resistivity on the square of temperature \( T \), \( \rho \sim \eta \sim \tau_{2,ee} \sim 1/T^2 \). A hint on this very unusual decrease of resistivity with increasing temperature was recently reported in Ref. [21] in a limited temperature interval below 5 K.

It should be noted that generally the electron viscosity is not necessarily related to electron-electron collisions. Any process providing the relaxation of the second moment of the electron distribution function (e.g., scattering on static defects or, more generally, on disorder) gives rise to viscosity. So the viscosity coefficient \( \eta \) is proportional to the relaxation time \( \tau_{2} \), for which the reciprocal value, \( 1/\tau_{2} \), contains the contribution [3] from the electron-electron scattering as well as the temperature-independent contribution from electron scattering on disorder:

\[
\eta = \frac{1}{4} v_F^2 \tau_2, \quad \frac{1}{\tau_2(T)} = \frac{1}{\tau_{2,ee}(T)} + \frac{1}{\tau_{2,0}}.
\]

The result given by Eq. (2) is modified if the momentum relaxation time \( \tau \) due to interaction with phonons and static defects is comparable to \( \tau^* \). In this case, the usual bulk friction term \(-V/\tau \) should be added to the right-hand side of Eq. (1). The modified velocity \( V(y) \) profile can be easily found, and integration over \( y \) gives the following expression for the resistivity [2, 3]:

\[
\rho = \frac{m}{e^2 n \tau} \frac{1}{1 - \tanh(\xi)/\xi}, \quad \xi = \sqrt{3\tau^*/\tau}.
\]

For \( \tau \gg \tau^* \), \( \tanh \xi \approx \xi - \xi^3/3 \) and the expression for the resistivity in Eq. (5) reduces to Eq. (2). In the opposite case, when \( \tau \ll \tau^* \), \( \tanh \xi \approx 1 \ll \xi \) and one recovers
the usual Drude resistivity \( \rho_D = \frac{m}{e^2 n \tau} \) defined by the momentum relaxation time \( \tau \).

It turns out that the following simple interpolation formula:

\[
\rho = \frac{m}{e^2 n} \left( \frac{1}{\tau} + \frac{1}{\tau_{ph}} \right), \tag{6}
\]

reproduces the expression \([3]\) for any value of \( \tau^*/\tau \) with an accuracy better than 11%. Thus the effect of the electron viscosity can be regarded as a parallel channel of electron momentum relaxation.

The values of the momentum relaxation time \( \tau_{ph} \) for scattering of 2D electrons in GaAs quantum well were estimated by using the results of Refs. \([32]\). According to those papers, the momentum relaxation rate is proportional to temperature, \( 1/\tau_{ph}(T) = A_{ph} T \), at \( T \gtrsim 4 \) K and to higher powers of temperature at \( T \lesssim 4 \) K (for the structure studied in Ref. \([31]\)). For the total bulk momentum relaxation rate we should use the expression:

\[
\frac{1}{\tau(T)} = \frac{1}{\tau_{ph}(T)} + \frac{1}{\tau_0}, \tag{7}
\]

where the term \( 1/\tau_0 \) does not depend on temperature and is due to electron scattering on disorder.

Fig. 1 shows the temperature dependencies of the mean free paths \( l_{xy} = v_F \tau_0 \) and \( l = v_F \tau \) calculated according to Eqs. \([1],[7],[10]\), and Ref. \([32]\) with the parameters \( \tau_0, A_{ph}, \tau_0, A_{ph}^e \) that will be used further in the text to fit the experimental data from Ref. \([31]\).

3. We now address our main point: the effects resulting from the dependence of the electron viscosity on magnetic field.

The internal friction between two layers of the electron fluid moving with different velocities is provided by the exchange of electrons between these layers (see Fig. 2). In the absence of magnetic field electrons from one layer penetrate into another one on a distance which is of the order of \( l_2 \) and this is what defines the viscosity. However, in the presence of magnetic field this distance is limited by the cyclotron radius \( R_c \). Thus at strong magnetic field the viscosity should tend to zero.

We derived the following expressions for the electron viscosity tensor \( \eta_{ij} \) \([31]\):

\[
\eta_{xx} = \frac{\eta}{1 + (2\omega_c \tau_0)^2}, \quad \eta_{xy} = \frac{2\omega_c \tau_0 \eta}{1 + (2\omega_c \tau_0)^2}, \tag{8}
\]

where \( \omega_c = eB/mc \) is the cyclotron frequency, and \( \eta \) is the viscosity at zero magnetic field introduced above. Dissipation of energy in a viscous flow is related only to the coefficient \( \eta_{xx} \).

The formula for the dissipative viscosity coefficient \( \eta_{xx} \), similar to the expression in Eq. \([8]\), was obtained by M.S. Steinberg for a 3D metal in Ref. \([37]\). The non-diagonal viscosity \( \eta_{xy} \) to our knowledge was not considered for 2D electrons in literature previously.

For the hydrodynamic velocity of a 2D viscous flow in magnetic field we derived the motion equation \([31]\):

\[
\frac{\partial \mathbf{V}}{\partial t} = \eta_{xx} \Delta \mathbf{V} + [(\eta_{xy} \Delta \mathbf{V} + \omega_c \mathbf{V}) \times \mathbf{e}_z] + \frac{e}{m}\mathbf{E} - \frac{\mathbf{V}}{\tau}, \tag{9}
\]

where \( \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 \). Since we neglect compressibility of the electron fluid, we must assume that \( \text{div} \mathbf{V} = 0 \).

In the stationary regime and in the absence of the Hall current, \( V_y \equiv 0 \), Eq. \([9]\) for a long sample reduces to:

\[
\eta_{xx} \frac{d^2 V}{dy^2} + \frac{e}{m} E_x - \frac{V}{\tau} = 0, \tag{10}
\]

\[
\eta_{xy} \frac{d^2 V}{dy^2} + \omega_c V - \frac{e}{m} E_y = 0. \tag{11}
\]

Here \( V = V_x, E_x(y) = \text{const} \) is the electric field due to the applied voltage, and \( E_y(y) \) is the Hall electric field corresponding to the condition \( V_y \equiv 0 \).

For the case of the absence of momentum relaxation in the bulk, \( 1/\tau = 0 \), Eq. \([10]\) coincides with the stationary version of Eq. \([10]\) if one replaces \( \eta \) by \( \eta_{xx} \). Thus the resistivity \( \rho \) will be given by Eq. \([2]\) with the additional factor \( [1+(2\omega_c \tau_0)^2]^{-1} \), describing the giant negative magnetoresistance. For the case of a nonzero bulk momentum relaxation rate, \( 1/\tau \neq 0 \), the resistivity \( \rho \) corresponding to Eq. \([10]\) will be calculated by Eq. \([5]\), where

\[
\xi = \sqrt{\frac{3\tau^*}{\tau}[1 + (2\omega_c \tau_0)^2]}, \tag{12}
\]

or by the approximation formula analogous to Eq. \([4]\):

\[
\rho = \frac{m}{e^2 n} \left( \frac{1}{\tau} + \frac{1}{\tau^*} \frac{1}{1 + (2\omega_c \tau_0)^2} \right). \tag{13}
\]
It is seen from Eq. (13) that the decrease of $\tau_2$ and fastening of the relaxation rate $1/\tau$ with temperature leads to broadening and a shift upwards of magnetoresistance curves (see Fig. 3). The increase of $\tau_2$ with temperature results in vanishing of negative magnetoresistance. At low temperatures and high magnetic fields, $\omega_c \tau_2 \gg 1$, the equations (7) and (13) yield a finite value of the resistance, $m/e^2 n \tau_0$, which is related only to the electron momentum relaxation on disorder in the bulk.

The Hall voltage can be found by integration of Eq. (11) over $y$. The first term in the left-hand side of Eq. (11), proportional to the viscosity coefficient $\eta_{xy}$, is of the order of $\omega_c (l_2/w)^2 V$ at $\omega_c \tau_2 \ll 1$ or $\omega_c (R_c/w)^2 V$ at $\omega_c \tau_2 \gg 1$, while the second term $\omega_c V$ is much greater. Thus in calculation of the Hall voltage one should take into account only the second term, and for the Hall coefficient we obtain the usual result: $R_H = 1/nec$.

The viscosity coefficient $\eta_{xy}$ is essential for non-stationary flows. For example, it is seen from Eq. (9) that the term proportional to $\eta_{xy}$ gives a contribution to dispersion of the hydrodynamic waves, while $\eta_{xx}$ is responsible for their dissipation. Indeed, if we seek the solution of Eq. (9) in the wave form: $V_k(r,t) = A_k \exp(-i \omega_k t + ik \cdot r)$, assuming the absence of electric field and bulk momentum relaxation, we easily obtain:

$$\omega_k = \pm \left( \omega_c - \eta_{xy} k^2 \right) - i \eta_{xx} k^2 . \quad (14)$$

4. We now discuss the recent experimental results on the giant negative magnetoresistance of 2D electrons [18–21] in the light of our theory. Fig. 3(a) demonstrates the experimental magnetoresistance curves obtained in Ref. 21 for an ultra-high-quality GaAs sample at different temperatures. For the same temperatures and magnetic fields we calculated magnetoresistance of that sample within our theory [see Fig. 3(b)]. Herewith we used the disorder relaxation times $\tau_0$, $\tau_2$, $\phi$ and the amplitudes $A_{ph}$, $A_{ph}^e$ in Eqs. (7) and (13) as fitting parameters.

Although by the appropriate choice of the fitting parameters we are able to perfectly reproduce the form of the experimental curves and their evolution with temperature, it is not possible to obtain in such the procedure the absolute values of the sample resistance observed in the experiments. The only way to obtain the measured magnitudes of resistivity within our theory is to replace the sample width $w$ by some effective width $w_{eff} < w$. This can be understood in the following way. The sample contains inhomogeneities which result in formation of the conducting channels in the sample with the widths smaller the sample width $w$.

Indeed, in the samples where the giant negative magnetoresistance effect was observed there often exist large-radius oval defects arising in the process of growth of the heterostructures [38, 39]. The distance $d$ between the defects varies in the range 20-100µm, while their radii are of the order of 20µm [38].

In vicinities of the defects the hydrodynamic velocity $V(r)$ cannot have a component in the direction perpendicular to the defect edge. A slowdown of the flow occurs due to the viscous transfer of the $x$ component of the electron momentum in the $y$ direction from the regions between the defects to the regions which are immediately in front of the defects (in the $x$ direction). So the large-radius defects lead to momentum relaxation by the mechanism, analogous to the diffusive scattering on rough sample boundaries, as well as to formation of the conducting channels with the widths smaller than $w$. At the scales of the order of $d$ the picture of fluid motion is rather similar to the Poiseuille flow in a rectangular sample with the width $w_{eff} \sim d$. The details of the velocity field $V(r)$ are very complicated, but the relationship for

![Figure 3: Temperature-dependent magnetoresistance of high-mobility 2D electrons in the GaAs quantum well experimentally studied in Ref. 21. The panel (a) is taken from Ref. 21. The curves at the panel (b) are drawn according to Eqs. 4 and 12 with the numerical parameters presented in the main text.](image-url)
the averaged resistance:
\[ \rho \sim \frac{1}{\tau} + \frac{\eta_{xx}}{d^2}, \]  
(15)
along with Eq. (13), will sustain (see Ref. 21 for a qualitative derivation of Eq. (15) and Ref. 40 for its rigorous derivation and analysis).

In Fig. 3(b) we drew magnetoresistance calculated with the following fitting parameters: \( \tau_0 = 4.5 \times 10^{-10} \text{s} \), \( \tau_{2,0} = 1.1 \times 10^{-11} \text{s} \), \( A_{ph} = 10^9 \text{s}^{-1} \text{K}^{-1} \), \( A_{ee}^{Fl} = 1.3 \times 10^9 \text{s}^{-1} \text{K}^{-2} \), and \( w_{eff} = 10 \mu \text{m} \). Herewith the condition of applicability of the hydrodynamic approach, \( l_2 \ll w_{eff} \), is fulfilled at all the temperatures (see Fig. 1). The used values of \( A_{ph} \) and \( A_{ee} \) are in agreement with the order of magnitude with the result of our estimations of the parameters \( A_{ph} \) and \( A_{ee} \) for the quantum well studied in Ref. 21.

5. In conclusion, a hydrodynamic mechanism for 2D electron transport in magnetic field has been studied. We have demonstrated that this mechanism is responsible for the giant negative magnetoresistance, recently observed in the ultra-high-mobility 2D electrons in GaAs/AlGaAs heterostructures.

The author wishes to thank Prof. M.I. Dyakonov, under whose guidance this research was undertaken, for his advice and support during the course of the work, for his participating in writing the text of the paper. The author also thanks B. A. Aronzon, A. P. Dmitriev, I. V. Gornyi, V. Yu. Kachorovskii, G. M. Minkov, A. D. Mirlin, and D. G. Polyakov for numerous fruitful discussions. The work was supported by the Russian Fund for Basic Research (Contracts No. 16-02-01166-a, 15-02-04496-a, 14-02-00198-a), by the Russian Ministry of Education and Science (Contract No. 14.Z50.31.0021, Leading scientist: M. Bayer), by the Russian Federation President Grants (Contracts No. NSh-1085.2014.2 and MK-8826.2016.2), and by the Dynasty Foundation.

"Corresponding author: pavel.alekseev@mail.ioffe.ru"

[1] R. N. Gurzhi, Sov. Phys. JETP 17, 521 (1963).
[2] R. N. Gurzhi and S. I. Shevchenko, Sov. Phys. JETP 27, 1019 (1968).
[3] R. N. Gurzhi, Sov. Phys. Uspekhi 11, 255 (1968).
[4] L. W. Molenkamp and M. J. M. de Jong, Phys. Rev. B 49, 5038 (1994).
[5] R. N. Gurzhi, A. N. Kalinenko, and A. I. Kopeliovich, Phys. Rev. Lett. 74, 3872 (1995).
[6] H. Buhmann, et al., Low Temp. Phys. 24, 737 (1998).
[7] H. Predel, et al., Phys. Rev. B 62, 2057 (2000).
[8] A. Tomadin, G. Vignale, and M. Polini, Phys. Rev. Lett. 113, 235901 (2014).
[9] M. Mendoza, H. J. Herrmann, and S.ucci, Scientific reports 3, 1052 (2013).
[10] A. V. Andreev, S. A. Kivelson, and B. Spivak, Phys. Rev. Lett. 106, 256804 (2011).
[11] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media. Pergamon Press, Oxford (1960).
[12] S. Childress, Journal of Fluid Mechanics 15, 429 (1963); A. V. Chechkin, Journ. of Exp. Theor. Phys. 89, 677 (1999); W. A. Manyonge, D. W. Kiema, and C. C. W. Iyaya, Int. Journ. of Pure and Appl. Math. 76, 661 (2012).
[13] E. M. Lifshitz and L.P. Pitaevskii, Physical kinetics. Pergamon Press, Oxford (1981).
[14] Yu. M. Aliiev, J. Appl. Mech. Tech. Phys. 3, 11 (1965).
[15] M. Hruska and B. Spivak, Phys. Rev. B 65, 033315 (2002).
[16] A part of the results of this work was obtained together with Prof. M.I. Dyakonov and reported at the conferences: Workshop: Disorder and its Role in Transport in 2D Systems (DiRT2D 2015), Okinawa, Japan, January 21-23, 2015; Workshop: Quantum transport in 2D systems, Luchon, France, May 23-30, 2015; Russian conference on semiconductor physics, Zvenigorod, Russia, September 21-25, 2015.
[17] For the first time, the hydrodynamic mechanism for negative magnetoresistance of 2D electrons was discussed in Ref. 21. However, the authors of Ref. 21 believed that such a mechanism is unlikely for existing GaAs quantum wells.
[18] A. T. Hatke, M. A. Zudov, J. L. Reno, L. N. Pfeiffer, and K. W. West, Phys. Rev. B 85, 081304 (2012).
[19] R. G. Mani, A. Kriisa, and W. Wegscheider, Scientific reports 3, 2747 (2013).
[20] L. Bockhorn, P. Barthold, D. Schuh, W. Wegscheider, and R. J. Haug Phys. Rev. B 83, 113301 (2011); L. Bockhorn, A. Hodaei, D. Schuh, W. Wegscheider, R. J. Haug, Journal of Physics: Conference Series 456, 012003 (2013).
[21] Q. Shi, P. D. Martin, Q. A. Ehmer, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. B 89, 201301 (2014).
[22] Other existing mechanisms for strong negative magnetoresistance (see Refs. 23, 24) cannot explain the experiments 18, 21. Indeed, the mechanisms proposed in Refs. 23, 24 are based on the memory effects in scattering of 2D electrons on disorder, and therefore the resulting magnetoresistance is independent on temperature in contrast to the experimental data. The work 22 deals with the quasi-two-dimensional systems in quantizing magnetic field, which are significantly different from the GaAs systems studied in Refs. 18, 21.
[23] A. D. Mirlin, D. G. Polyakov, F. Evers, and P. Wolfe, Phys. Rev. Lett. 87, 126805 (2001).
[24] A. Dmitriev, D. M. Dyakonov, and R. Jullien, Phys. Rev. B 64, 233221 (2001).
[25] P. Goswami, J. H. Pixley, and S. Das Sarma, Phys. Rev. B 92, 075205 (2015).
[26] D. T. Son and B. Z. Spivak, Phys. Rev. B 88, 104412 (2013).
[27] During the publication process, I became aware of the work P. J. W. Moll, P. Kushwaha, N. Nandi, B. Schmidt, and A. P. Mackenzie, Science 351, 1061 (2016), in which negative magnetoresistance in the high-mobility 2D metal PdCoO2 was reported and interpreted within the hydrodynamic approach, analogous to one developed in the present Letter.
[28] Electron scattering on the edges can be treated as diffusive when the sample edges are rough or when the conducting subregion of the sample has a complicated geometry; see some discussion of this topic in Section 4 and in...
SUPPLEMENTAL MATERIAL

We discuss the temperature dependencies of the 2D electron relaxation times in the presence of the strong electron-electron interaction and the interaction with disorder. We present a Drude-like derivation of the hydrodynamic equations for a 2D electron fluid in magnetic field as well as a qualitative consideration of a 2D viscous flow in a sample containing large-radius defects.

1. 2D electron relaxation times

For the case of strongly interacting electrons, consideration of viscous transport should be based on the Fermi-liquid theory. The time $\tau_{2,ee}$ should be treated as the relaxation time of the second moment of the quasiparticle distribution function due to quasiparticle-quasiparticle collisions. It should be calculated from the quantum kinetic equation for the excitations in the 2D electron Fermi-liquid. In Ref. [30] it was shown that the dependence $1/\tau_{2,ee}(T)$, apart from the $T^2$ factor related to the Fermi statistics of the quasiparticles, additionally contains the logarithmic factor (related to renormalization of scattering probability for the pairs of quasiparticles with approximately zero total momentum):

$$\frac{1}{\tau_{2,ee}(T)} = A_{\text{FL}}^{ee} \frac{T^2}{\ln(2E_F/T)^2}.$$  \hspace{1cm} (16)

The coefficient $A_{\text{FL}}^{ee}$ is expressed via the Landau interaction parameters, which depend on relative magnitudes of the electron-electron interaction energy $E_C$ and the Fermi energy $E_F$. Unfortunately, the Landau parameters (and, thus, the value of $\tau_{2,ee}$) cannot be found in a closed form as functions of the interaction parameter $r_s = E_C/E_F$.

It should be noted that, even for weakly interacting degenerate 2D electrons, the dependence of the quantum lifetime $\tau_{0,ee}$ due to the electron-electron scattering contains the logarithmic factor [24]:

$$\frac{\hbar}{\tau_{0,ee}(T)} = A_0^{ee} \frac{T^2 \ln(2E_F/T)}{E_F},$$  \hspace{1cm} (17)

where $A_0^{ee}$ is the numeric constant of the order of 1. Such the behavior of $\tau_{0,ee}(T)$ is related to kinematics of electron-electron collisions in the 2D case.

Disorder in a structure containing 2D electrons can modify the physical nature and the probability of electron-electron collisions. In particular, when a collision of two electrons in the presence of a disorder potential occurs, the total momentum of colliding electrons is not conserved. Our estimations show that the corresponding disorder-assisted contribution to the electron-electron relaxation rate of the second moment of the dis-
distribution function is quadratic by the temperature:

\[ \frac{1}{\tau_{ee}^2(T)} = A_{ee}^a T^2. \]  

(18)

The value of the coefficient \( A_{ee}^a \) depends on the strength and the type of disorder.

2. Drude-like derivation of the hydrodynamic equations in magnetic field

Here we present a simple derivation of the formulas (8) for the electron viscosity tensor \( \eta_{ij} \) in magnetic field and the motion equation (9) for the velocity field \( \vec{V}(\vec{r}, t) \).

The standard method, which takes many pages of laborious calculations, is based on the kinetic equation for the electron distribution function in the presence of the external fields \( \vec{E} \) and \( \vec{B} \) and a space-dependent hydrodynamic velocity \( \vec{V}(\vec{r}, t) \) (see, for example, Refs. [13, 14, 67]). We will use a simplifying shortcut similar to the Drude approach to the electron transport where the momentum relaxation time \( \tau \) is considered as a given parameter. In our case this main given parameter will be the time \( \tau_2 \) of relaxation of the second moment of the distribution function due to electron-electron and disorder scattering. The only (but important) advantage of the kinetic equation approach compared to the Drude-like approach is that the relevant relaxation times as well as the numerical coefficients in \( \eta_{ij} \) are calculated on the way.

The viscosity terms in the hydrodynamic equation can generally be expressed through the viscous stress tensor (per one particle) \( \Pi_{ij} = m \langle v_i v_j \rangle \), where \( \vec{v} = (v_x, v_y) \) is the 2D velocity of a single electron and the angular brackets stand for averaging over the electron velocity distribution at a given point \( \vec{r} = (x, y) \). The motion equation for the hydrodynamic velocity \( \vec{V} = \langle \vec{v} \rangle \) in the absence of magnetic field is:

\[ \frac{m}{\tau} \partial V_i / \partial t = - \frac{\partial \Pi_{ij}}{\partial x_j} - \frac{m V_i}{\tau} + e E_i. \]  

(19)

Here and below, summation over repeating indices is assumed. At a time scale much greater than the relaxation time \( \tau_2 \) the expression for \( \Pi_{ij} \) is given by [32]:

\[ \Pi_{ij} = \Pi_{ij}^{(0)} = - m \eta_i V_j, \quad V_j = \frac{\partial V_j}{\partial x_j} + \frac{\partial V_j}{\partial x_i}. \]  

(20)

Using Eqs. (19) and (20) and taking into account that the electron fluid is considered as incompressible in the present study (\( \text{div} \vec{V} = 0 \)), one obtains the basic Eq. (1) with the additional term \(- V / \tau\). The value given by Eq. (20) is attained during the time \( \tau_2 \), as described by the Drude-like equation:

\[ \frac{\partial \Pi_{ij}}{\partial t} = - \frac{1}{\tau_2} (\Pi_{ij} - \Pi_{ij}^{(0)}). \]  

(21)

In the presence of magnetic field, additional terms will appear in the equations for \( \partial V_i / \partial t \) and \( \partial \Pi_{ij} / \partial t \), since now the quantities \( \langle v_i \rangle \) and \( \langle v_i v_j \rangle \) will change in time not only because of collisions and the electric field, but also due to the magnetic part of the Lorenz force: \( (\partial v_i / \partial t)_{\text{mag}} = \omega_c \epsilon_{zik} v_k \). Here \( \epsilon_{zik} \) is the unit antisymmetric tensor and \( z \) is the direction of the magnetic field \( \vec{B} \), which is perpendicular to the 2D electron layer. Thus for the additional terms in the motion equations one obtains:

\[ \left( \frac{\partial \langle v_i \rangle}{\partial t} \right)_{\text{mag}} = \omega_c \epsilon_{zik} \langle v_k \rangle, \]  

(22)

\[ \left( \frac{\partial \langle v_i v_j \rangle}{\partial t} \right)_{\text{mag}} = \omega_c (\epsilon_{zik} \langle v_k v_j \rangle + \epsilon_{zjk} \langle v_i v_k \rangle). \]  

The terms (22) should be added to the right-hand side of Eqs. (19) and (21).

Considering the steady-state solution, we get from Eq. (21) and (22) the following relation:

\[ \Pi_{ij} - \omega_c \tau_2 (\epsilon_{zik} \Pi_{kj} + \epsilon_{zjk} \Pi_{ik}) = \Pi_{ij}^{(0)}, \]  

(23)

allowing to find the components of the tensor \( \Pi_{ij} \):

\[ \Pi_{xx} = - \Pi_{yy} = \frac{1}{1 + \beta^2} \Pi_{xx}^{(0)} + \frac{\beta}{1 + \beta^2} \Pi_{xy}^{(0)}, \]  

(24)

\[ \Pi_{xy} = - \Pi_{yx} = \frac{1}{1 + \beta^2} \Pi_{xy}^{(0)} - \frac{\beta}{1 + \beta^2} \Pi_{xx}^{(0)}, \]

where \( \beta = 2 \omega_c \tau_2 \). Here we used the relationship \( \Pi_{ii} = 0 \), which follows from \( \text{div} \vec{V} = 0 \) and Eq. (23). The components of the viscosity tensor are defined as the coefficients in the linear relationship between the tensors \( \Pi_{ij} \) and \( V_{ij} \) (see Ref. [13]):

\[ \Pi_{ij} = m (\eta_{xx} V_{ij} + \eta_{xy} \epsilon_{zik} V_{kj}), \]  

(25)

which follows from Eqs. (20) and (24).

With the help of Eqs. (19), (22), (21), and (25), and the condition \( \text{div} \vec{V} = 0 \) we arrive to Eqs. (8) and (9) that were used in the main text.

We have checked that the conventional method based on the classical kinetic equation gives exactly the same results as this simple derivation.

We assumed in the main text that the zero-temperature relaxation times \( \tau_0 \) and \( \tau_{2,0} \), related to electron scattering on disorder, do not depend on magnetic field. For example, this is the true for electron scattering on the isolated static defects if the electron collisions with defects are described by classical mechanics and the cyclotron radius is much greater than the distance between neighbor defects [24]. However, for many types of disorder and intervals of magnetic field, 2D electron kinetics can be considered only quantum-mechanically. In this case, for example, the bulk conductivity tensor even in
non-quantizing magnetic fields, $\hbar \omega_c \ll T$, is not given by
the Drude formulas \([33, 34]\). The motion equations, analogous to Eqs. \((19)\) and \((21)\), should be derived from the quantum kinetic equation. In the resulting quantum hydrodynamic equation, the magnetic field dependencies of the kinetic coefficients (analogous to $\eta_{xx}$, $\eta_{xy}$, and $1/\tau_0$) will be determined by the type of the disorder and temperature.

3. 2D viscous flow in a sample with large-radius defects

The hydrodynamic mechanism of electron transport can be realized in the 2D high-mobility samples containing isolated widely spaced defects \([10, 13]\). A viscous flow of electron fluid bypassing the defects will be formed if the mean distance between the defects, $d$, is enough large: $d \gg l_2$. The 2D electron fluid is confined by the sample edges as well as by the edges of the defects (thus the boundary of the conducting subregion of the sample has a complicated geometry).

For the case of the absence of magnetic field such the model was studied in Refs. \([10, 12]\). It was obtained that resistivity of the system is proportional to the viscosity:

$$\rho \sim \eta , \quad (26)$$

when the radius of the defects, $r_0$, is large: $r_0 \gg l_2$; or to the viscosity with a logarithmic factor:

$$\rho \sim \frac{\eta}{\ln (d/l_2)} , \quad l_2 \sim \eta , \quad (27)$$

when the radius of the defects is small: $r_0 \ll l_2$. Up to the logarithmic factor, which cannot be very large for real samples, the results \((26)\) and \((27)\) are similar to the formula \((2)\), obtained for the Poiseuille flow in a long rectangular sample.

The exact consideration of a 2D flow in the sample with the large-radius defects in the presence of magnetic field must be done on base of Eq. \((9)\) with the corresponding boundary conditions on $V(r)$ and $E(r)$ at the sample edges and at the defect edges. We believe that the relationship \((15)\), leading to the giant negative magnetoresistance, will be obtained in a wide range of the parameters $d$, $r_0$, $l_2$. Below we give a qualitative consideration of such the flow for simplest case when the radius of defects is of the same order of magnitude as the mean distance between them: $r_0 \sim d \gg l_2$.

We study the electron flow in a long rectangular sample with the width $w \gg d$ and the length $L \gg w$. Following to Ref. \([35]\), we introduce the mean values of velocity $\overline{V}$ and electric field $\overline{E}$, which are the results of averaging of the values $V(r)$ and $E(r)$ over a volume with the size much greater than the character distance between defects, $d$. The result of such averaging of the value $\Delta V$ can be estimated as follows:

$$\Delta V \sim \frac{\overline{V}}{d^2}. \quad (28)$$

Here we make use of spatial homogeneity of the system on the scales much greater that $d$ and an approximately oscillating character of the dependence $V(r)$ with the characteristic period of the order of $d \sim r_0$.

As $L \gg w$, the component of the averaged electric field along the sample $\overline{E}_x$ is equal to the electric field from the applied voltage and the averaged velocity $\overline{V}$ has only the component $\overline{V}_x$ along the sample direction. The $x$ component of the Navier-Stokes equation \((9)\) takes the form:

$$- \eta_{xx} \frac{\overline{V}_x}{d^2} + \frac{e}{m} \overline{E}_x - \frac{\overline{V}_x}{\tau} = 0 , \quad (29)$$

which immediately leads to the magnetoresistance \((13)\) and \((15)\). Let us remind here that Eq. \((13)\) is just the result of mathematical interpolation of the exact formula \((5)\), obtained for the conventional Poiseuille flow in the flat sample. So Eq. \((13)\) is more general than the assumptions about the geometry of the flow used for its derivation.

In formation of a viscous flow in a sample with the large-radius defects, the character of electron scattering (diffusive or specular) on the sample edges and the defect edges is not essential. The nature of momentum relaxation in such the system is following. In vicinities of the defects the hydrodynamic velocity cannot has a component perpendicular to the defect surface. A slowdown of the flow occurs due to the viscous transfer of the $x$ component of the momentum in the $y$ direction between the regions which are immediately in front of the defects (in the $x$ direction) and the regions between the defects.

Comparing Eqs. \((2)\) and \((29)\), we conclude that the time $\tau^* = d^2/\eta_{xx}$ can be interpreted as the non-local momentum relaxation time due to the viscosity effect. In other words, the time $\tau^*$ describes the momentum relaxation in the systems which are characterized by the two features: (i) spatial inhomogeneity of the electron momentum relaxation rate [related to bypassing the large-radius defects or to diffusive scattering on the rough sample edges]; (ii) viscous transfer of the mean electron momentum due to inhomogeneity of the velocity field $V(r)$.

Besides that, typical GaAs 2D samples often have a complicated geometry of their edges (for example, see the sample image in Ref. \([19]\)). Irregularities of the sample edges may lead to a slowdown of a viscous flow by the way similar as it was described above for the viscous flow in a sample with the large-radius defects.

We believe that the relative contributions from the processes of diffusive scattering on the sample edges and by bypassing the large-radius defects in formation of a viscous flow significantly vary from sample to sample, resulting in a variety of observing features of the giant negative magnetoresistance effect (see Refs. \([18, 21]\) ).