A discrete differential evolution for fuel loading optimization of the DNRR research reactor and comparison with genetic algorithm

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Abstract. A discrete differential evolution (DE) method has been applied to the problem of fuel loading pattern optimization of the Dalat Nuclear Research Reactor (DNRR). A classic strategy $DE/rand/1/bin$ was chosen for the mutation of the DE method. Numerical calculations have been performed based on the core configuration of 100 highly enriched uranium (HEU) fuel bundles with various burnup levels. Comparison of the performance between the DE method and a genetic algorithm (GA) was also carried out. The optimal LPs obtained from the two methods are significantly better than the reference core. DE is more advantageous in exploring search space and approaching a global optimum than GA.

1. Introduction
In-core fuel management (ICFM) problem of a nuclear reactor core is to find an optimal loading pattern (LP) of fresh and spent fuel bundles to satisfy several design objectives. The problem is known as a multi-objective problem, in which several objectives have different variation trends. Two common objectives are maximizing cycle reactivity and minimizing power peaking factor. Several other objectives can also be considered such as discharged fuel burnup, fuel enrichment, burnable poison inventory and so on [1]. The result of the optimization process is not a single best solution but a set of equally good solutions.

In recent years, advanced meta-heuristic approaches have been extensively applied to many optimization problems due to their diversity in exploring the global search space and avoiding a local optimum. Evolutionary algorithms (EAs) are classified under a family of algorithms based on the evolution of populations of individuals [2]. Genetic algorithm (GA) [3] and differential evolution (DE) [4, 5] are among popular EAs, which are based on three evolutionary operators: crossover, mutation and selection. GA has been well developed and applied to the problem of ICFM [6, 7, 8]. Whereas, only several recent efforts have been conducted to apply DE to the problem of ICFM [9, 10, 11]. DE is originally developed for continuous variable problems. Since the ICFM problem handles integer variables, it needs a strategy to convert real variables to integer ones.
In the present work, a discrete DE algorithm has been applied to the fuel loading optimization of the Dalat nuclear research reactor (DNRR). The 3D core physics calculations were performed using the CITATION code [12]. The group constants were generated by the reactor lattice cell calculations from WIMSD-5B code [13] with ENDF/B-VI.8 nuclear data library [14]. Comparison of the performance between the DE method and the GA with the same convergence criteria was conducted and presented.

2. Methodology

2.1. The DNRR reactor

The DNRR reactor operates with the nominal power of 500 kW and is loaded with the Russian fuel type VVR-M2. The core consists of 121 hexagonal cells of fuel bundles, control rods, irradiation channel, beryllium blocks, and aluminum chocks. More description of the DNRR can be found in Refs. [15, 16, 17, 18]. In the present work, calculations have been conducted based on a reference core consisting of 100 fuel bundles, including 11 fresh fuel bundles and 89 spent bundles with burnup levels in the range of 0 – 12.3% (percent loss of $^{235}$U) as depicted in Figure 1.

In the optimization process using DE and GA, the fitness functions were chosen to maximize the effective multiplication factor ($k_{\text{eff}}$) and to minimize the power peaking factor ($PPF$). The fitness function to be maximized is written as:

$$\text{Fitness} = \alpha \times (k_{\text{eff}} - 1) + \beta \times (PPF_0 - PPF)$$

where, $PPF_0$ is a constant chosen as a constraint of $PPF$, $\alpha$ and $\beta$ are the weighting factors for $k_{\text{eff}}$ and $PPF$, respectively.

In the DE method, $PPF_0 = 2$, $\alpha = 100$ and $\beta = 1000$ were chosen [19, 11]. In the GA, a binary mixed integer coded GA was applied to improve the efficiency in finding the global optimum, in which $\alpha$ and $\beta$ were not fixed but determined by a search scheme.
2.2. Discrete DE method
In the DE method, a parameter vector representing a fuel LP has \( D \) integer variables with the values from 1 to \( D \) (\( D = 100 \) in case of the DNRR). An \( NP \)-size population at generation \( G \) in the DE optimization problem consists of \( NP \) parameter vectors, each of which is a \( D \)-dimensional vector:

\[
X_{i,G} = [x_{1,i,G}, x_{2,i,G}, ..., x_{D,i,G}], \ i = 1, 2, ..., NP
\]

where, \( G \) is the generation in the DE search process, \( x_{i,j} \) is an integer value in range [1, \( D \)]:

\[
x_{j,i} = \text{int} \left[ D \times \text{rand}(0, 1) + 1 \right]
\]

To initiate the DE search process, an initial \( NP \)-size population is randomly generated. Each of \( D \) variables of \( NP \) vectors in the population is randomly assigned by an integer number from 1 to \( D \).

2.2.1. Mutation
An individual of the population is set as a target vector \( X_{i,G} \). The strategy \( DE/rand/1/bin \) was used to create a mutant vector for next generation \( G + 1 \) as follows [4]:

\[
V_{i,G+1} = X_{r1,G} + F \times (X_{r2,G} - X_{r3,G}),
\]

where, \( X_{r1,G} \), \( X_{r2,G} \) and \( X_{r3,G} \) are randomly chosen from \( NP \) individuals of generation \( G \). \( F \) is the mutation scale factor, a real value in the range of [0, 1]. Variable \( v_{j,i,G+1} \) of the mutant vector \( V_{i,G+1} = \{v_{j,i,G+1}\} \) is defined as:

\[
v_{j,i,G+1} = x_{j,r1,G} + F \times (x_{j,r2,G} - x_{j,r3,G}), \ j = 1, ..., D
\]

The variables of the mutant vector in Eq. (4) may have real values or out of range [1, \( D \)]. Thus, a strategy has been implemented for converting the real variables into integer ones and preserving the number of fuel bundles in the core. The real variables are converted to integer ones by keeping their integer parts. Then, if any variable is out of the range [1, \( D \)], it is reassigned by an integer number which has not been assigned to any variable in the range [1, \( D \)]. If two or more variables obtain the same integer, one of them remains the current value, and the others are reassigned for free integer numbers in the range [1, \( D \)].

2.2.2. Crossover
The mutant and target vectors go through the crossover to generate a trial vector \( U_{i,G+1} \). \( P_j \), \( j = [1, ..., D] \), is produced randomly within [0, 1] to compare with a crossover rate (\( CR \)). The trial vector is generated as follows:

\[
u_{j,i,G+1} = \begin{cases} v_{j,i,G+1}, & \text{if } P_j \leq CR \\ x_{j,i,G}, & \text{otherwise} \end{cases}
\]

If several variables of the trial vector have the same value, one of them remains the current value, and the others are reassigned for integer numbers have not been assigned in the range [1, \( D \)].

2.2.3. Selection
The fitness of the trial vector \( U_{i,G+1} \) and target vector \( X_{i,G} \) are calculated and compared to each other. The vector with a higher fitness value is selected as a target vector \( X_{i,G+1} \) in the next generation.
2.3. Genetic algorithm

In this work, the GA was applied for optimizing fuel LPs, and a method based on GA was also developed to determine the weighting factors ($\alpha$, $\beta$) in the fitness function during the search process. The two search schemes of the weighting factors and the optimal LPs were implemented, so that the GA works with two types of chromosomes: integer chromosome and binary chromosome. The integer chromosome represents fuel LPs and the binary chromosome represents weighting factors. This method was called binary mixed integer coded GA [20].

2.3.1. Genetic operators for integer chromosomes

In the search scheme for the optimal LPs, the elitism strategy was used in the selection to preserve the best solutions during the search process. The population can be ranked by sorting through to identify all non-dominated solutions in the archive. The solutions in the archive were transferred to a breeding pool for the next generation.

The one-point method was used to perform the crossover in two steps: 1) two members of the breeding pool be mated are randomly selected with a crossover probability, 2) the two chromosomes undergo crossing over as follows: an integer position $k$ along the chromosome is selected uniformly at random between 1 and the chromosome length less one $D - 1$. Two new strings were created by swapping all characters between positions $k + 1$ and $D$ inclusively. In case two parents have some genes with the same number, crossing the two parents over may create two off-springs which had some identical genes. If this case occurs, small random numbers between 0 and 1 were added to the genes with the same value to make a difference between them, and then rank these genes again to make two new off-springs.

Mutation was conducted by a binary shuffle of two genes in the chromosome. A chromosome to be mutated was randomly selected with a mutation probability. Then, the two genes selected from the chromosome are exchanged their positions.

2.3.2. Genetic operators for binary chromosomes

This search scheme was created to determine the weighting factors ($\alpha$ and $\beta$) of the fitness function in Eq. (1) to fulfill a condition of $\alpha + \beta = 1$. $\alpha$ is encoded into a binary chromosome that is a string with bits of 1 or 0. A chromosome is represented symbolically by the string of $m_i$: $(m_1,m_2,...,m_l)$ where $m_i$ may take on a value of 1 or 0. The length $l$ of the string is the minimum integer which satisfies the following formula:

$$ (\alpha_{\text{max}} - \alpha_{\text{min}}) \times 10^n \leq 2^l - 1, $$

where, $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$ are the minimum and maximum values of $\alpha$, $n$ is the number of digits following the decimal point in the number that represents the value of $\alpha$. The binary string is decoded into the real value of $\alpha$ as follows:

$$ \alpha = \frac{1}{2^l - 1} \sum_{i=1}^{l} m_i \times 2^{l-i}. $$

Genetic operators for the binary chromosomes are similar to that for integer chromosomes. Selection is performed by the roulette wheel spin method to create a breeding pool for the next generation. Crossover is performed by the one-point method to create two off-springs. First, a pair of strings in the breeding pool is randomly chosen with a crossover probability. Second, a crossing site $k$ between positions $l$ and $l1$ was generated uniformly at random. Finally, two new strings are created by swapping all the characters between positions $k + 1$ and $l$ inclusively. Mutation is performed by randomly altering the value of a bit between 1 and 0 with a mutation probability.
3. Results and discussion

Since the performance of the DE algorithm depends on the control parameters such as population size $NP$, mutant scale factor $F$ and crossover rate $CR$. A preliminary survey has been conducted to determine the parameters to be used in the optimization process. In this survey, the values of $NP = 30$, $F = 0.4$ and $CR = 0.2$ have been selected in the DE search process [11].

The population of 30 individuals was evolved for 700 generations, so that 21000 LPs were examined in the application using the DE. The GA was also performed for the same number of generations. The crossover and mutation probability for the GA were selected as 0.5 and 0.001, respectively.

Figure 2 shows the evolution of the populations over 700 generations obtained with DE. The tendency of exploring better solutions over generations in the DE search process. The $k_{eff}$ values improved significantly after the process and more generous than the average $k_{eff}$ of the initial generation value by about 280 pcm. The $PPF$ of the optimal LP is 1.32, which is smaller by about 3% than the average value of the initial population (1.36). Moreover, the optimal solutions obtained from the DE search process are significantly better than the reference core. The $k_{eff}$ and $PPF$ values of the optimal solutions is greater by about 470–495 pcm, while the $PPF$ values are smaller by about 3.7–4.0%.

Table 1 shows the optimal parameters of the best solutions obtained from the DE and GA. The first case corresponds to the optimal LPs selected with the largest $k_{eff}$. The second one corresponds to the optimal LPs selected with the smallest $PPF$. The optimal LPs selected from DE are better than that obtained from GA by about 33–62 pcm in $k_{eff}$ and 1.5–1.7% in $PPF$.

Comparison of the evolution of the objective parameters obtained from the two methods is presented in Figures 3 and 4. It is found that GA rapidly approaches the optimal solutions after about 200 generations. On the other hand, the access of DE to the optimal solutions is slower but more stable than GA. The stable $k_{eff}$ value is reached after 250 generations, and the $PPF$ achieves a minimum value after 150 generations.

The results show that DE is advantageous over GA in finding optimal LPs of the DNRR. DE has a strong possibility to explore the search space because the mutation operator acts on every members of the population. Thus, it controls the search process by increasing the diversity to
Figure 3. Comparison between GA and DE in the maximum (a) and average (b) effective multiplication factor of population versus generation.

Figure 4. Comparison between GA and DE in the minimum (a) and average (b) PPF of population versus generation.

Table 1. Comparison of the optimal LPs of the DNRR obtained with DE and GA.

| No. | Parameter | DE | GA | $\Delta_{DE-GA}$ |
|-----|-----------|----|----|------------------|
| 1   | $k_{eff}$ | 1.06603 | 1.06541 | 62 pcm          |
|     | PPF       | 1.321   | 1.343   | -1.5 %          |
| 2   | $k_{eff}$ | 1.06571 | 1.06536 | 33 pcm          |
|     | PPF       | 1.319   | 1.342   | -1.7 %          |

find more promising solutions. Whereas, the mutation in GA affects only a few members of a generation to guarantee the convergence rate.

4. Conclusion
A discrete DE algorithm was developed and applied to the problem of fuel loading optimization of the DNRR reactor. Numerical calculations were performed based on the core loaded with 100
Figure 5. Optimal LPs of the DNRR obtained from (a) the DE ($k_{eff} = 1.06603$, $PPF = 1.321$) and (b) the GA ($k_{eff} = 1.06541$, $PPF = 1.343$) search processes.

fuel bundles with various burnup levels. The fitness function was chosen to maximize the $k_{eff}$ and minimize the $PPF$. The results show that the two methods exhibit good performance in the problem of fuel LP optimization of the DNRR reactor. The optimal LPs obtained from the search processes using the two methods are significantly better than the reference one. Comparing the performance of DE and GA in the same problem, DE is more advantageous than GA in exploring search space and approaching a global optimum.

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