Vulnerability-Aware Poisoning Mechanism for Online RL with Unknown Dynamics

Yanchao Sun
Department of Computer Science
University of Maryland, College Park
ycs@umd.edu

Furong Huang
Department of Computer Science
University of Maryland, College Park
furongh@umd.edu

Abstract

Poisoning attacks, although have been studied extensively in supervised learning, are not well understood in Reinforcement Learning (RL), especially in deep RL. Prior works on poisoning RL usually either assume the attacker knows the underlying Markov Decision Process (MDP), or directly apply the poisoning methods in supervised learning to RL. In this work, we build a generic poisoning framework for online RL via a comprehensive investigation of heterogeneous types/victims of poisoning attacks in RL, considering the unique challenges in RL such as data no longer being i.i.d. Without any prior knowledge of the MDP, we propose a strategic poisoning algorithm called Vulnerability-Aware Adversarial Critic Poison (VA2C-P), which works for most policy-based deep RL agents, using a novel metric, stability radius in RL, that measures the vulnerability of RL algorithms. Experiments on multiple deep RL agents and multiple environments show that our poisoning algorithm successfully prevents agents from learning a good policy, with a limited attacking budget. Our experiment results demonstrate varying vulnerabilities of different deep RL agents in multiple environments, benefiting the understanding and applications of deep RL under security threat scenarios.

1 Introduction

Understanding the security of RL techniques is crucial as reinforcement learning (RL) is widely applied in various real-world scenarios, including high-stakes ones such as autonomous driving vehicles and healthcare systems; one should not rely on the decisions made by a RL agent easily misled by an adversarial attacker. However, most existing researches on adversarial attack/defending focus on supervised learning (SL), and adversarial learning for RL is not yet well understood.

In this paper, we systematically study the adversarial attacks in RL and vulnerability of RL algorithms, with a focus on poisoning. Different from evasion attacks that craft adversarial test examples for a well-trained model, poisoning attacks could cause failure of a learning process or “teach” the learner an attacker-specified model in training. In RL, we categorize an attack as poisoning if it influences the policy learned by the agent. Poisoning in RL is significantly different from poisoning in classic SL, and could be more difficult due to the following challenges.

Challenge I – Future Data Unavailable in Online RL. Online RL is a decision making process where the learner (the agent) learns a policy by interacting with the environment to generate trajectories iteratively. A poisoning attack changes the current learner’s policy, and thus changes the future trajectories generated by the policy. So, at each iteration, the optimal poison should work in the future iterations to successfully poison the learning process, although the future is yet to be generated.

Challenge II – Data Samples No Longer i.i.d.. In RL, data samples (state-action transitions) are no longer i.i.d, due to the Markovian property. Forcing an agent to choose a less-rewarding action in one
We consider the online learning scenario, where the RL agent (the learner) does not know the sequence of states, actions, rewards and the terminal state flags in iteration k from the MDP M. At each iteration k, the learner uses its previous policy πk−1 to roll out observations \( O_k \) from the MDP M. \( O_k \) is a concatenation of multiple trajectories, denoted as \( O_k = (s_k, a_k, r_k, d_k) \), where \( s_k = [s_k^1, s_k^2, \cdots] \), \( a_k = [a_k^1, a_k^2, \cdots] \), \( r_k = [r_k^1, r_k^2, \cdots] \), \( d_k = [d_k^1, d_k^2, \cdots] \) are respectively the sequence of states, actions, rewards and the terminal state flags in iteration k. Then, with the ob-

**Challenge III – Unknown Dynamics of Environment.** Although challenge I and II can be partially addressed by predicting the future trajectories or steps, it requires prior knowledge on the dynamics of the underlying Markov Decision Process (MDP). However, in complex environments such as Atari games, knowing the dynamics of the MDP is difficult. Although the attacker could potentially interact with the environment to build an estimate of the environment model, the cost of interacting with the environment could be unrealistically high; market making [Spooner et al., 2018] for instance.

**Challenge IV – Heterogeneous Poisoning Target Types.** Moreover, poisoning could happen at heterogeneous targets in RL, including states [Behzadan and Munir, 2017], actions [Pinto et al., 2017], rewards [Ma et al., 2019], and even the underlying environment [Rakhsha et al., 2020], different from SL where poisoning only happens at data features or labels. The lack of a uniform framework for poisoning in RL makes it difficult to compare existing literature involving different attacking types.

Previous works either do not address any of the aforementioned challenges or only address some of them. Behzadan and Munir [2017] achieve policy induction attacks for deep Q networks (DQN). However, they view output actions of DQN similarly to labels in SL, and do not consider Challenge II that the current action will influence future interactions. Ma et al. [2019] propose a poisoning attack for model-based RL, but they suppose the agent learns from a batch of given data, not considering Challenge I. Rakhsha et al. [2020] study poisoning for online RL, but they require perfect knowledge of the MDP dynamics, which is unrealistic as stated in Challenge III.

In this paper, we systematically investigate poisoning in RL by considering all the aforementioned RL-specific challenges.

**Summary of Contributions.** (1) We propose a generic poisoning framework for RL, showing their connections to poisoning in SL as well as unique new challenges in solving online RL poisoning problems; specifically we formalize the optimal poisoning process in online RL as a sequential bi-level optimization problem; (2) we propose a practical poisoning algorithm called Vulnerability-Aware Adversarial Critic Poison (VA2C-P) that works for multiple policy-gradient learners without any prior knowledge of the environment, demonstrating RL agents’ vulnerabilities to even weaker attackers with limited knowledge; (3) we introduce a novel metric, stability radius, to characterize the stability of RL algorithms, measuring and comparing the vulnerabilities of RL algorithms in different environments.

In Appendix [A], we summarize some additional related works on evasion attacks in RL and poisoning in supervised machine learning.

## 2 A Poisoning Framework for RL and Related Works

In this section, we establish a generic framework of poisoning in online RL, systematically characterizing its challenges and difficulties from multiple perspectives - objective of poisoning, target type of poisoning and attacker’s knowledge. Our in-depth comparison with the SL allows a thorough understanding of the additional vulnerability of online RL systems compared with the well-understood SL systems. Our framework also provides a clear context to correctly position prior works in the literature as well as to compare our work with existing works. Compared to the poisoning framework described by [Huang and Zhu, 2019], we provide a solution for unifying these target types in one attack model in Section [B].

We consider the online learning scenario, where the RL agent (the learner) does not know the dynamics or rewards of the underlying MDP \( M \) with state space \( S \), action space \( A \), transition dynamics \( P \), rewards \( R \) and discount factor \( \gamma \).

**Settings and Notations** In online RL, the learner interacts with the environment and collects observations. The learner’s algorithm, denoted by \( f \), iteratively searches for a policy \( \pi \) parametrized by \( \theta \), through \( K \) interactions with the environment. Before learning starts, the learner initializes a policy \( \pi_0 \). At each iteration \( k \), the learner uses its previous policy \( \pi_{k-1} \) to roll out observations \( O_k \) from the MDP \( M \). \( O_k \) is a concatenation of multiple trajectories, denoted as \( O_k = (s_k, a_k, r_k, d_k) \), where \( s_k = [s_k^1, s_k^2, \cdots] \), \( a_k = [a_k^1, a_k^2, \cdots] \), \( r_k = [r_k^1, r_k^2, \cdots] \), \( d_k = [d_k^1, d_k^2, \cdots] \) are respectively the sequence of states, actions, rewards and the terminal state flags in iteration k. Then, with the ob-

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servation $O_k$, the learner updates its policy by attempting to solve $\text{argmax}_\pi J (\pi, \pi_{k-1}, O_k)$, where $J$ is the objective function. The generated policy by the learner’s algorithm $\pi_k = f(\pi_{k-1}, O_k)$ does not necessarily achieve the maximization of the objective function.

In this paper, an overhead check sign ‘$\cdot$’ on a variable always denotes that the variable is poisoned. For example, if the attacker changes a reward $r_t$, then the poisoned reward is denoted as $\hat{r}_t$.

**Poison Objective.** We use $L_A$ to denote loss function of the poisoning attack, which the attacker attempts to minimize. The form of $L_A$ is determined by its goal, which falls into one of the two categories, non-targeted and targeted poisoning.

In non-targeted poisoning, the attack poisons a policy $\pi$ to $\hat{\pi}$ to minimize the learner’s expected rewards. Therefore the poison objective $L_A$ is to minimize the learner’s value $\eta(\hat{\pi})$.

In targeted poisoning, the attack “teaches” the agent to learn a pre-defined target policy $\pi^t$. Therefore the poison objective $L_A$ is defined as distance $d(\hat{\pi}, \pi^t)$.

Most existing poison RL researches focus on targeted poisoning [Ma et al. 2019, Rakhsha et al. 2020], and non-targeted poisoning, although discussed by Huang and Zhu [2019], remains relatively untouched.

**Poison Target Types.** To influence the behaviors of the learner, an attacker could inject poison at multiple locations of the learner’s learning process as detailed in Figure 1. Part of the reason why poisoning in RL is more challenging than in SL is that it involves more target types of poisoning, some of which adapt with the environment, increasing the uncertainty.

![Figure 1: Different target types of poisoning in supervised learning and reinforcement learning.](image)

**Target Type I – Poison Observation ($O_\tau, O_o$).** The attacker could manipulate the observation of the learner, i.e., change $O$ into $\hat{O}$. This may happen when the attacker is able to intercept the communication between the learner and the environment, similar to the man-in-the-middle attack in cryptography. The attacker could target the rewards, called $O_\tau$-poisoning, studied by Huang and Zhu [2019]; or the states, called $O_o$-poisoning, investigated by Behzadan and Munir [2017].

**Target Type II – Poison MDP ($M_R, M_P$).** An attack could directly change the MDP (environment) that the learner is interacting with, i.e., change $M$ into $\hat{M}$. For example, a seller could influence the behaviors of customers by changing the prices of products. The poison of MDP could be injected at the reward model $R$ or the transition dynamics $P$, respectively denoted as $M_R$-poisoning (studied by Ma et al. [2019]) and $M_P$-poisoning (studied by Rakhsha et al. [2020]). The analogy of poison MDP in SL is to manipulate the underlying data distribution of the training data.

**Target Type III – Poison Executor $E_\alpha$.** The executor of the learner could be poisoned. For example, an attacker applies a force to the agent, so that the intended action “north” becomes “northeast”. Pinto et al. [2017] train a robust RL agent against the executor poisoner. Denote this type of poisoning as $E_\alpha$-poisoning. We show in the appendix that $E_\alpha$-poisoning is equivalent to directly changing $\alpha$ stored in the observation $O = (s, a, r, d)$.

See Appendix C for more explanations and examples.

**Attacker’s Knowledge** At the $k$-th iteration, what an attacker can do depends on its current knowledge set, denoted by $K_k$. $K_k$ could contain the underlying MDP $M$, the learner’s algorithm $f$, the learner’s previous policy models $\pi_{1:k-1}$ as well as the previous and current observations $O_{1:k}$.

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Footnotes:
1 For algorithms with experience replay, the update can be extended as $\pi_k = f(\pi_{k-1}, O_{1:k})$.
2 There are many ways to define the distance between two policies, for instance KL-divergence for stochastic policies [Schulman et al. 2015a], and average mismatch for deterministic policies [Rakhsha et al. 2020].
An omniscient attacker knows everything, i.e., $\mathcal{K}^{(O)}_k = \{\mathcal{M}, f, \theta_{1:k-1}, \mathcal{O}_{1:k}\}$. Most guaranteed policy teaching literature [Rakhsha et al., 2020; Ma et al., 2019] assume omniscient attacker. However as motivated in the introduction, it is often unrealistic to exactly know the underlying environment. We discuss two more realistic setting where the attacker only has limited knowledge as follows.

A monitoring attacker has some information but does not know the underlying MDP $\mathcal{M}$, i.e., $\mathcal{K}^{(M)}_k = \{f, \theta_{1:k-1}, \mathcal{O}_{1:k}\}$. This is especially relevant in applications where learner’s information is not secure (or even open), or an attacker hacks to steal information from the learner. Monitoring attacker is similar to the white-box attacker in supervised learning.

A tapping attacker has very limited knowledge and knows the observations only, i.e., $\mathcal{K}^{(T)}_k = \{\mathcal{O}_{1:k}\}$. This is widely applicable since the tapping the communication between the learner and the environment is easy. Tapping attacker is analogous to the black-box attacker in supervised learning. Behzadan and Munir [2017] consider a tapping attacker, which observes and manipulates the states of the target type. Therefore, at iteration $k$, the attacker may (a) poison observations after they are generated, (b) poison MDP before the learner generates observations, or (c) poison the policy when it is used to generate observations. Among these three scenarios, $\mathcal{O}_k$ is always influenced by the attack, thus denoted as $\hat{\mathcal{O}}_k$.

### 3 Poisoning Strategy via A Sequential Bilevel Optimization

In this section, we formulate the problem of poisoning in online RL through the lens of optimization. We deliberately formulate the optimization problem to be as generic as possible, covering non-targeted and targeted poison objective and all 3 poison target types.

![Figure 2: The online poisoning vs learning. Blue solid lines denote the learning processes, while red dashed lines denote the poisoning processes. In iteration $k$, the learner uses its previous policy $\pi_{k-1}$ to roll out observations $\mathcal{O}_k$ from current MDP $\mathcal{M}_k$, then updates its model and policy by $\pi_k = f(\pi_{k-1}, \mathcal{O}_k) = \arg\max_{\pi} J(\pi, \pi_{k-1}, \mathcal{O})$. The attacker may (a) poison observations after they are generated, (b) poison MDP before the learner generates observations, or (c) poison the policy when it is used to generate observations. Among these three scenarios, $\mathcal{O}_k$ is always influenced by the attack, thus denoted as $\hat{\mathcal{O}}_k$.](image)

#### Unifed Problem Formulation

We introduce Problem (Q), a generic sequential bilevel optimization for poisoning attacks. As detailed in Section 2, the attacker’s loss is the learner’s value $L_A = \eta(\pi)$ if poisoning is non-targeted, and $L_A = \text{distance}(\hat{\pi}, \pi)$ if poisoning is targeted at $\pi$. We use $\mathcal{D}$-poisoning to collectively denote any target types of the poisoning, where $\mathcal{D} \in \{\mathcal{M}_R, \mathcal{M}_F, \mathcal{O}_R, \mathcal{O}_a, E_a\}$, illustrating the generality of our formulation. The online poisoning process is visualized in Figure 2, which shows $\mathcal{O}_k$ is poisoned to $\hat{\mathcal{O}}_k$, conditioned on $\hat{\mathcal{D}}_k$, regardless of the target type. Therefore, at iteration $k$, the optimal subsequent poisoning $\hat{\mathcal{D}}_k, \hat{\mathcal{D}}_{k+1}, \cdots, \hat{\mathcal{D}}_K$ are given by the optimal solution of Problem (Q).

\[
\begin{align*}
\arg\min_{\hat{\mathcal{D}}_k, \cdots, \hat{\mathcal{D}}_K} & \quad \sum_{j=k}^{K} L_A(\hat{\pi}_j) \\
\text{s.t.} & \quad \hat{\pi}_j = \arg\max_{\pi} J(\pi, \pi_{j-1}, \hat{\mathcal{O}}_j|\hat{\mathcal{D}}_j), \forall k \leq J \leq K \quad \text{(learner’s poisoned learning)} \\
& \quad \sum_{j=k}^{K} 1\{\hat{\mathcal{D}}_j \neq \hat{\mathcal{D}}_j\} \leq C \quad \text{(limited-budget)} \\
& \quad U(\hat{\mathcal{D}}_j, \hat{\mathcal{D}}_j) \leq \epsilon, \forall k \leq j \leq K \quad \text{(limited-power)}
\end{align*}
\]

At each iteration, the learner attempts to obtain a policy that maximizes the objective function $J$ as shown in condition “(learner’s poisoned learning)”. For example, the objective function for TRPO is $\pi_k = \arg\max_{\pi} \mathbb{E}_{t} \{\pi(a_t|s_t) \mid \tilde{A}_t\} = \mathcal{D}^{\max}(\hat{\pi}_{k-1}, \pi)$, where $\tilde{A}$ is related with $\mathcal{O}_k$, and $\mathcal{D}^{\max}$ are defined in the paper [Schulman et al., 2015a].

To truthfully represent the real-world scenarios, we consider attacker’s constraints in two forms: (1) the attack power $\epsilon$ restricts the maximum change $U(\hat{\mathcal{D}}_k, \hat{\mathcal{D}}_k)$ in one iteration, which can be defined by any distance metric (e.g., $\ell_p$-norm), in condition “(limited-power)” and (2) the attack budget $C$ restricts the number of iterations that the attacker could attack, in condition “(limited-budget)”.

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Difficult in Sequential Bilevel Optimization. In supervised learning, the poison attacker solves a \textit{bilevel optimization problem} since the attacker, when making the decision of how to poison, has to solve an optimization problem to predict how the learner reacts to the poison injected [Muñoz-González et al., 2017]. Problem (Q) in online RL is even more challenging as the attacker has to predict how the learner reacts to the poison not only at the current iteration $k$, but also for all future iterations $k+1, \ldots$, since all decisions made at current time will have consequences in the future. As a result, Problem (Q) in online RL is a \textit{sequential bilevel optimization problem}, much more challenging than in the supervised learning setting.

Possible Directions in Addressing Problem (Q)

Case (1) Under the often unrealistic setting of attacker being omniscient, the attacker can exactly compute all observations $\hat{O}_k$ based on environment dynamics. Then, Problem (Q) reduces to a fully-online case of the supervised learning attack studied by [Wang and Chaudhuri, 2018] where the training data stream is known to the attacker. Note that even in this case, solving Problem (Q) is NP-hard if $C \ll K$.

Case (2) For a non-omniscient attacker, future observations are not available. But it might be possible for the attacker to train a prediction model [Oh et al., 2015] of the environment in advance, and use the estimated model to predict future observations of the learner based on its current policy. However, the computational cost could be very high.

Case (3) If the non-omniscient attacker can not study the environment ahead of time, and can only learn along with the learner, then predicting the future becomes prohibitive. In this case, the attacker has to sequentially make two decisions: \textit{when to attack}, and \textit{how to attack}, in order to approximately solve Problem (Q).

We aim to tackle the more challenging but practical Case (3) and propose a poisoning algorithm described in Section 4, with a focus on policy-gradient learners for demonstration purpose.

4 VA2C-P: Online Poisoning Algorithm for Policy Gradient Learners

In this section, we propose a practical and efficient poisoning algorithm called Vulnerability-Aware Adversarial Critic Poison (VA2C-P) for policy gradient learners with on-policy updates. Although the idea applies to the generic Problem (Q), we use the policy gradient learners as an example for a simple demonstration of our algorithm.

Settings. In our algorithm, we focus on a monitoring attacker who knows the learner’s policy model $\theta$ (and thus $\pi(\theta)$), observations $O$, as well as the learner’s algorithm $f$, but does not have access to the environment. Without loss of generality, the attacker conducts $O_r$-poisoning for simplicity of demonstration. Different from most literature [Rakhsha et al., 2020, Ma et al., 2019], we choose non-targeted poisoning, since a target policy is not usually available. Note that the learner is not aware of the existence of the attacker.

We first address the decisions of \textit{“how to attack”} and \textit{“when to attack”} respectively, then introduce the algorithm in Algorithm 1 in Appendix E.

4.1 How to Attack – Adversarial Critic

In the $k$-th iteration, the goal of the attacker is to find the optimal $\hat{r}_k$ that minimizes the expected total rewards of all the learner’s future policies $\pi_{k,K}$, as suggested by Problem (Q). However, as the future observations are not known yet, it is hard to determine which $\hat{r}_k$ is optimal for reducing all future rewards of the learner. Thus we propose to minimize the reward of the immediate next iteration, instead of all future iterations. Then the optimization problem (Q) is relaxed as (P).

$$\hat{r}_k = \arg\min_{r} \quad \eta_{\pi_{k-1}}(\pi_k) = E_{\tau \sim \pi_{k-1}}[\frac{\pi_k(\tau)}{\pi_{k-1}(\tau)} r(\tau)]$$

\text{subject to} \quad \pi_k = f(\pi_{k-1}, \hat{r}) \quad \|\hat{r}_k - r_k\| \leq \epsilon

Adversarial Critic. (P) uses importance sampling to evaluate the new policy $\pi_k$ via the trajectories generated by $\pi_{k-1}$. But in practice, directly using the sampled rewards to compute the policy value may suffer from a high variance [Schulman et al., 2015b]. Thus, inspired by the Actor-Critic method,
we propose to let the attacker fit a value function (network) \( V_\omega \) with the historical trajectories of the learner, and use \( V_\omega \) to direct the learner to the decreasing-value direction, which is called \textit{Adversarial Critic}. Then the advantage function is estimated by \( A(s_t, a_t) = G(s_t, a_t) - V_\omega(s_t) \), where \( G \) denotes the discounted future reward \( r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots \). Then the new objective becomes \( \mathbb{E}_{s,a \sim \pi_{k-1}} [\phi \frac{1}{K} (\pi'_{k-1}(a|s) - \pi_{k-1}(a|s)) A_{\pi_{k-1}}(a|s)] \), which is a first order taylor approximation of \( \eta(\pi_k) \), similarly with [Schulman et al., 2015a].

We use projected gradient descent to search the optimal \( \hat{r} \). And the computation details are illustrated in Appendix \textbf{E}.

**Theoretical Interpretation of the Relaxation.** The solution to Problem \( \text{P} \), namely \( \hat{r}_k^\# \) is always feasible to Problem \( \text{Q} \), although might not be optimal. Problem \( \text{P} \) only considers the immediate next iteration instead of all successive iterations as Problem \( \text{Q} \) does, thus only if \( \hat{r}_k^\# \) is also a minimizer for \( \sum_{j=k+1}^K \eta(\pi_j) \), could \( \hat{r}_k^\# \) be optimal to Problem \( \text{Q} \), as detailed in Appendix \textbf{F}. We also show in Appendix \textbf{F} that one can justify whether a past poisoning decision could be optimal to the future iterations once the future observations are received, which may help the attacker adjust its decisions accordingly.

### 4.2 Decision 2: When to Attack – Vulnerability-Aware

It is important to select appropriate iterations to poison due to the limited attack budget \( C \). The question is then “in which iterations could the attacker achieve relatively stronger poisoning?” A poisoning attack is considered as strong if it depraves the learner’s policy to a large degree.

We now define the vulnerability of a RL learning algorithm, which is not formally defined in prior works. Inspired by the notion of stability in learning theory, which measures how a machine learning algorithm changes due to a small perturbation of the input data, we formally investigate the stability of an RL algorithm, a first attempt in the existing literature to the best of our knowledge.

**Stability of RL Algorithms** We first focus on one single update process of an algorithm \( f \). Intuitively, an update \( \pi' = f(\pi, \mathcal{O}) \) is stable if a poison does not cause any difference on the output policy \( \pi' \). That is, the learning algorithm produces the same result regardless of the presence of the poison.

More formally, we define the concept of **stability radius of one update** in Definition \textbf{1}.

**Definition 1 (Stability Radius of One Update).** For the update of an RL algorithm \( \pi' = f(\pi, \mathcal{O}) \), under \( \mathcal{D} \)-poison of any target types, the \( \delta \)-stability radius of the update is defined as the minimum poison power needed to cause \( \delta \) change in policy (called \( \delta \) policy descrepancy)

\[
\phi_\delta(f, \pi, \mathcal{O}) = \inf_\varepsilon \{ \exists \mathcal{D} \text{ s.t. } U(\mathcal{D}, \mathcal{D}) \leq \varepsilon \text{ and } d_{\max}[\pi'|\|\pi] > \delta, \pi' = f(\pi, \mathcal{O})\}, 
\]

where \( U(\mathcal{D}, \mathcal{D}) \) denotes the poison power. \( Policy \text{ discrepancy } d_{\max}[\pi_1||\pi_2] = \max_s d[\pi_1(\cdot|s)||\pi_2(\cdot|s)] \), where \( d[||] \) could be any measure of distribution distance.

**Remarks.** (1) The one-update stability radius is w.r.t. the algorithm \( f \), the base policy \( \pi \) and the clean observation \( \mathcal{O} \). (2) The attacker takes effort to manipulate \( \mathcal{D} \) and finally let the observation \( \mathcal{O} \) be changed to \( \mathcal{O}' \). For different types of target \( \mathcal{D} \), the stability radius could be different. (3) Poison with power under \( \phi_\delta(f, \pi, \mathcal{O}) \) will not cause the policy distributions to change more than \( \delta \). (4) As shown by Proposition \textbf{2} in Section \textbf{5}, poison with power under \( \phi_\delta(f, \pi, \mathcal{O}) \) will not make the reward drop too much.

**Estimating the Stability Property** Directly computing the stability radius by definition is difficult, as it accounts for all the possible \( \varepsilon \)-powered attacks in the space. So we consider the reversed way: fix an attack power \( \varepsilon \), then estimate how much the output policy will change by computing the policy discrepancy \( \psi_k = d[\pi_k||\pi_k^\#] = \mathbb{E}_{s,a \sim \pi_{k-1}} [d[\pi_k(\cdot|s)||\pi_k^\#(\cdot|s)]] \). By definition, \( \varepsilon \) is greater than or equal to the \( \psi \)-stability radius of the algorithm for the current update. So, with the same \( \varepsilon \), an update with relatively large \( \psi \) is more likely to be vulnerable against poisoning.

**Poisoning Algorithm VA2C-P.** The implementation details are illustrated in Algorithm \textbf{1} in Appendix \textbf{E} and the main steps are summarized as below.

With \( k \) iterations already poisoned, at iteration \( k \),

**Step 1:** compute the policy discrepancy achieved by \( \varepsilon \)-powered poison: \( \hat{\psi}_k \);
We observe that in some experiments (e.g., Hopper-PPO, Walker-ACKTR, etc), the robustness radius. We also present a counterpart of stability radius in test time, namely the vulnerability-aware attack. The proof is in Appendix D.3.

With the one-update stability measure, we are able to formally define the stability radius of an RL algorithm w.r.t. an MDP.

Definition 3 (Stability Radius w.r.t an MDP). The \( \delta \)-stability radius of an algorithm \( f \) in an MDP \( \mathcal{M} \) is defined as the minimal stability radius of all observations drawn from the MDP, and all possible policies in policy space \( \Pi \). If \( f \) is on-policy, then \( \phi(f, \mathcal{M}) = \min_{\pi \in \Pi} \phi(f, \pi, \mathcal{D}) \); if \( f \) is off-policy, then \( \phi(f, \mathcal{M}) = \min_{\pi \in \Pi} \phi(f, \pi, \mathcal{D}) \), where \( \pi \) is the behavior policy.

We also present a counterpart of stability radius in test time, namely robustness radius in Appendix D.1, as well as a comparison of vulnerability between supervised learning and RL in Appendix D.2. Stability radius and robustness radius together provide a principled way to measure vulnerability of RL algorithms, respectively in training time and test time.

6 Experiments

In this section, we testify the performance of our proposed VA2C-P through experiments with multiple algorithms on various environments.

Experiment Setup We choose 4 policy-based learning algorithms, including Vanilla Policy Gradient [Sutton et al., 2000], as well as the state-of-the-art algorithms PPO [Schulman et al., 2017], A2C [Mnih et al., 2016], and ACKTR [Wu et al., 2017]. And we choose 4 gym [Brockman et al., 2016] environments with increasing difficulty levels: CartPole, Hopper, Walker and HalfCheetah.

Results of Poisoning Effects As discussed in Section 4, our VA2C-P aims to reduce the total rewards (non-target poisoning) gained by the learner in online RL. The attacker knows the learner’s algorithm, current model and observations, but does not know the underlying environment. Since there is few methods working on the same setting, we compare the performance of VA2C-P with two baselines: (1) a random attacker which randomly choose \( C \) iterations, and perturbs the reward to an arbitrary direction by \( \epsilon \); and (2) a simplified version of our algorithm, called Adversarial Critic Poison (AC-P), which decides “how to attack” in the same way with VA2C-P, but chooses “when to attack” randomly.

Figure 3 shows the mean rewards each learner gains under different kinds of poisoning methods, with various budget \( C \). Compared with random attack, our proposed VA2C-P, and the simplified version AC-P make the reward drop more significantly. And VA2C-P outperforms AC-P in most cases, which implies that the vulnerability-based attack decision works in practice. As \( C/K \) increases, VA2C-P and AC-P achieve stronger attacks, while random poison method does not show a consistent tendency.

We observe that in some experiments (e.g., Hopper-PPO, Walker-ACKTR, etc), the random poison method not only does not make the learning worse, but also even “facilitates” the learner. For Hopper-ACKTR, the learner with a random attacker obtains 3 times reward than without any poisoning. This phenomenon is rare in SL, because the input data and the output labels have relatively deterministic relations. If the attacker randomly changes the label of one picture, then the changed label should
be different from the true one, and as a result, will mislead the learner to some degree. However, in RL, a reward signal is not the true value of the corresponding state-action pair, and it is very possible that a randomly changed reward will lead the learner to find a rewarding state afterwards. Therefore, poisoning RL is more difficult than poisoning SL, due to the uncertainty of the environment. This also reflects Challenge I and Challenge II as pointed out in Section 1.

Discussion: How Does Vulnerability Vary? As we states by the Decision 2 in Section 4, we approximately measure the vulnerability by computing the policy discrepancy triggered by a fixed poison power $\epsilon$. Figure 4 shows some examples of how policy discrepancy changes with the iteration number $k$, when there is no poisoning.

We see that the policy discrepancy decreases as $k$ gets larger, which implies that the later iterations tend to be more stable than former iterations. It is intuitively reasonable, because as the learner accumulates more experience, it becomes less sensitive to its inputs. In addition, the same algorithm tends to be more vulnerable in more complex Walker environment than the simpler CartPole environment, matching the results discovered by Gleave et al. [2019], which show that the learner is more vulnerable to adversarial policies in higher-dimensional environments in test time. Also, we find that in the same environment, the more complicated algorithm PPO is more vulnerable than the simpler VPG. This phenomenon is analogous to the accuracy-robustness dilemma in SL, i.e., classifiers with higher classification accuracy are usually less robust [Zhang et al. 2019].

We put more experiment results in Appendix C and demo videos in supplementary materials.

7 Conclusion and Discussion

In this paper, we build a generic poisoning framework for online RL with an in-depth comparison with SL, and propose VA2C-P, an attack for online RL. A metric, stability radius, is introduced to measure the vulnerability of RL algorithms. In this paper, we discuss the scenario where the attacker only poisons one type of target. But it could also be extended to a simultaneous poisoning on multiple target types. Ultimately, by studying the attacks in RL, we aim to understand the vulnerability of RL agents and finally design robust RL agents that can combat adversarial attacks.
**Broader Impact**

Despite the rapid advancement on interactive AI and ML systems using RL agents, the learning agent could fail catastrophically in the presence of adversarial attacks, exposing a serious vulnerability in current RL systems such as autonomous driving systems, market making systems and security monitoring systems. Understanding their vulnerability and studying attack models are of vital importance; “if you know yourself and your enemy, you’ll never lose a battle” as they say.

In this work, we define a unified framework for poisoning in online Reinforcement Learning (RL), and formalize the optimal poisoning process as a sequential bi-level optimization. We further propose a practical and generic poisoning algorithm that can poison the learner to learn a bad policy, without any prior knowledge of the underlying environment. The algorithm is based on the relaxation of the original sequential bi-level optimization problem. We provide theoretical analysis of the relaxation tightness.

We also propose a metric, stability radius, to measure the stability of RL algorithms under poisoning attacks, as well as a metric, robustness radius, for measuring the robustness of RL algorithms under evasion attacks (in Appendix [D.1]). These metrics make it possible to characterize and compare the vulnerability of RL algorithms on different environments and different learning stages. In RL-based applications where policy stability is important, or where adversarial attacks exist, our proposed stability radius and robustness radius could help make informed and secure decisions, such as which algorithm to use, when to be more alarmed about possible attackers, etc. As a result, our work has the potential to help combat the threat to high-stakes systems such as autonomous driving systems, healthcare medical systems and national security.

Our work puts an effort to bridge the gap between two communities – security in supervised ML and Reinforcement Learning – with the goal to finally push advancement of security in reinforcement learning by help adapt the well established knowledge of poisoning in Supervised Learning (SL) to poisoning in online RL. Through a thorough comparison with poisoning in SL, we show the connections as well as unique new challenges of solving RL poisoning problems.

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Appendix: Vulnerability-Aware Poisoning Mechanism for Online RL with Unknown Dynamics

A Additional Related Works

Adversarial Attacks in Supervised Learning Adversarial attacking and defending are attracting more and more attention nowadays [Huang et al., 2011, Biggio et al., 2012, Steinhardt et al., 2017]. Poisoning, as one popular type of adversarial attacking, has been well-studied in the field of machine learning [Biggio et al., 2012, Mei and Zhu, 2015], including deep learning [Muñoz-González et al., 2017, Shafahi et al., 2018]. Wang and Chaudhuri [2018] consider online learning, which is similar to ours. However, they focus on supervised learning, where the attacker knows all the data stream, while in our RL setting, the whole training data stream is not available to the attacker (as Challenge I states).

Poisoning in Reinforcement Learning Some related works focus on poisoning stochastic bandits [Jun et al., 2018, Liu and Shroff, 2019] and contextual bandits [Ma et al., 2018]. Policy teaching (targeted poisoning) is a specific type of poisoning for RL, where the attacker could lead the agent to learn a pre-defined target policy by manipulating the rewards [Zhang and Parke, 2008, Zhang et al., 2009] or dynamics [Rakhsha et al., 2020] of the MDP. However, they require the attackers to have prior knowledge of the environments (e.g., the dynamics of the MDP), which is often unrealistic or difficult in practice. Ma et al. [2019] introduce a policy teaching framework for batch-learning model-based agents, while the attacker knows the MDP and the training data batch before the agent learns. Huang and Zhu [2019] propose a reward-poisoning attack model, and provide convergence analysis for Q-learning. However, the introduced attack also requires the attacker has a perfect knowledge of the MDP. Behzadan and Munir [2017] achieve policy poisoning on DQN by crafting perturbed states such that the target action is selected by the learner, which is analogous to supervised learning, and the data-correlation problem of RL is not considered.

Evasion Attacks in Reinforcement Learning As summarized by Chen et al., 2019, evasion attacks at test-time such as adversarial perturbation and defense mechanisms in deep reinforcement learning (DRL) are under rapid development. A line of works consider adversarial perturbations on observations, similar to adversarial examples in supervised learning [Huang et al., 2017] first show that neural network policies are vulnerable to evasion attacks on states, by generating state perturbation with fast gradient sign method (FGSM) [Goodfellow et al., 2014]. Lin et al. [2017] consider the data-correlation problem in RL, and propose a strategical attack method, which perturbs an input state $s$ only if the agent has “a strong preference on action selection”, i.e., $\max_a \pi(a|s) - \min_a \pi(a|s)$ is large. This work and our works both consider the constraint that the attacker is only able to attack for limited times, and provide strategies to decide “when to attack” to maximize the attacker’s profit. However, our work deals with poisoning attacks, which can not be tackled by the method in [Lin et al., 2017] since the agent does not follow a good policy with effective “action preference” in training time.

In addition, Gleave et al. [2019] show that choosing an adversarial policy in multi-agent RL could also negatively affect the victim agent. There is also a line of work focusing on adversarial examples in path-finding problems [Xiang et al., 2018, Bai et al., 2018].

B Notations and Preliminaries

In RL, an agent interacts with the environment by taking actions, observing states and receiving rewards. The environment is modeled by a Markov Decision Process (MDP), which is denoted by a tuple $M = (S, A, P, R, \gamma)$, where $S$ is the state space, $A$ the action space, $P$ the transition kernel, $R$ the reward function, and $\gamma \in (0, 1)$ the discount factor.

At every step, the agent selects an action based on the current policy $\pi$. A stochastic policy $\pi : S \times A \to [0, 1]$ defines the probability of choosing each action in each state. A trajectory $\tau$ generated by $\pi$ is a sequence $s_0, a_1, r_1, s_2, a_2, \ldots$, where $s_0 \sim \mu, a_t \sim \pi(a|s_t)$, $s_{t+1} \sim P(s|s_t, a_t)$ and $r_t = R(s_t, a_t)$. Define an observation sequence as the concatenation of multiple trajectories generated by taking a policy in an environment, denoted as $O = (s, o, r, d)$, where $s = [s_1, s_2, \ldots]$, $a = [a_1, a_2, \ldots]$, $r = [r_1, r_2, \ldots]$, $d = [d_1, d_2, \ldots]$ are respectively the sequence of states, actions, rewards and the terminal state flags.
The goal of an RL agent is to find a policy $\pi^*$ that maximizes the expected total rewards $\eta$, which is defined as $\eta(\pi) = \mathbb{E}_{r \sim \pi} [r(\tau)] = \mathbb{E}_{s_0, a_1, \cdots} [\sum_{t=1}^{\infty} \gamma^{t-1} r_t]$.

The state value function $V^\pi(s)$ is defined as $V^\pi(s) = \mathbb{E}_{a_t \sim \cdot, \cdots} [\sum_{h=0}^{\infty} \gamma^h r_{t+h} | s_t = s]$. Similarly, the state-action value function $Q^\pi(s, a)$ is $Q^\pi(s, a) = \mathbb{E}_{s_{t+1}, \cdots} [\sum_{h=0}^{\infty} \gamma^h r_{t+h} | s_t = s, a_t = a]$.

### C Target Types of Attacking: More Detailed Explanation

![Diagram of poisoning types](image)

**Figure 5:** The online poisoning vs learning. Blue solid lines denote the learning processes, while red dashed lines denote the poisoning processes. In iteration $k$, the learner uses its previous policy $\pi_{k-1}$ to roll out observations $O_k$ from current MDP $M_k$, then updates its model and policy by $\pi_k = f(\pi_{k-1}, O_k) = \text{argmax}_\pi J(\pi, \pi_{k-1}, O)$. The attacker may (a) poison observations after they are generated, (b) poison MDP before the learner generates observations, or (c) poison the policy when it is used to generate observations. Among these three scenarios, $O_k$ is always influenced by the attack, thus denoted as $\tilde{O}_k$.

#### C.1 Poison Observation

Consider a deep reinforcement learning algorithm, which uses its old policy to generate a series of observations and updates its policy with the collected observations at every iteration. The observations are stored in a temporary buffer in the form of $(s, a, r, d)$, where $s, r, d$ are returned by the environment, and $a$ is produced by the policy itself.

An attacker could stay in the middle between the learner and the environment and falsify $s, r$ returned by the environment before the learner receives them. (It is also possible to alter the terminal state flags $d$, but it is relatively easier for the learner to detect, and its influence to the learner is not as large as $s$ and $r$, so we do not discuss this attack type.) On the other hand, the attacker may also hack the observation buffer to change $s, r$. In both cases, poisoning $s$ and $r$ are called $O_s$-poisoning and $O_r$-poisoning respectively.

Note that in the case of hacking observation buffer, the attacker also has the option to manipulate $a$ in $O$. But we do not include this kind of attack in observation poisoning, because not like states and rewards, the actions are taken by the learner, so it is easy for the learner to detect the change of action sequence stored in the buffer. And we will show in Section C.3 that changing $a$ in $O$ is similar to the case of executor poisoning.

#### C.2 Poison MDP

MDP poisoning can be considered as “changing the reality” for the learner. For example, an attacker decides to increase the reward for state-action pair $(s, a)$ by $\Delta$ at iteration $t$, it then changes the parameter $R_k(s, a)$ of the environment to $\tilde{R}_k(s, a) = R_k(s, a) + \Delta$. As a result, whenever the learner visits $(s, a)$ in the current iteration, it receives $\tilde{R}_k(s, a)$ as its reward. A more intuitive example is depicted in Figure 6, where we want to train a mouse to find the cheese in a maze. However, if some bad man adds another piece of cheese in the training environment, the mouse is likely to be misled. Then, in the test time, the “malicious cheese” is removed, but the mouse still wants to find the malicious cheese instead of the original one.

Compared with observation poisoning and executor poisoning, MDP Poisoning is more powerful and more difficult to defend against. An example of MDP poisoning is given by Rakhsha et al. [2020], where the attacker can force the learner to learn a target policy with guarantees.
C.3 Poison Executor

In the online RL process, the learner learns by taking actions and getting feedbacks of the actions. If observation poisoning is viewed as changing the feedbacks, then executor poisoning can be viewed as perturbing the actions taken by the learner. For example, for an auto-driving agent, one can slightly add some force to the steering wheel, so when the agent takes “steer left 90 degrees”, what actually happens is “steer left 100 degrees”. In this way, the trained policy is biased.

We now show that the aforementioned attack of changing $a_i$ in $O$ could be converted to executor poisoning. We describe the following two scenarios respectively for these two poisoning attacks and show they have equal effects. For simplicity, we only consider one step of the interaction, which can be simply extended to multiple steps.

1) Poisoning $a_i$ in $O$. For one-step experience, the learner observes state $s$, takes action $a_i$, receives reward $r_i = R(s, a_i)$ and observes a new state $s'_i \sim P(\cdot \mid s, a_i)$, then the tuple $(s, a_i, r_i, s'_i)$ will be stored in the observation buffer. Now the attacker may change the action $a_i$ to $a_j$, and the tuple becomes $(s, a_j, r_i, s'_i)$. Finally, the learner regard $r_i, s'_i$ and the following observations as caused by $a_j$, instead of $a_i$.

2) Poisoning Executor. In the same environment, suppose the learner observes state $s$, and takes action $a_j$, then the attacker conducts executor poisoning and changes $a_j$ to $a_i$, then the reward and the next state will also change to $r_i = R(s, a_i)$ and $s'_i \sim P(\cdot \mid s, a_i)$. The learner does not know the falsification of action and stores $(s, a_j, r_i, s'_i)$ to the buffer. Finally, the attacker will take $r_i, s'_i$ and the future observations as caused by $a_j$, and updates its policy accordingly.

Based on the descriptions, we can see changing $a_i$ to $a_j$ in the buffer is equivalent to changing $a_j$ to $a_i$ on the executor. If we assume the policy always has non-zero probability on all actions, then for any manipulation on $a$ in $O$, it is possible to achieve the same effects by attacking the executor. Hence, given the fact that the former attack can be trivially defended, we only discuss executor poisoning in our poisoning framework.

D Robustness and Stability of RL Algorithms: Supplementary Materials

D.1 Robustness Radius

Different with poisoning, test-time evasion (adversarial examples) misleads the agent by manipulating the states only, since the agent no longer learns from interactions and feedbacks. Note that although Gleave et al. [2019] propose an attack called “adversarial policy”, the perturbation does not happen in the policy, but still happens in the input states (observations) of the agent.

To study how robust a trained policy is, we define the robustness radius with regard to both a single state and the whole environment.

Definition 4 (Robustness Radius of Policy w.r.t. a State). The robustness radius of a deterministic policy $\pi$ on a state $s$ is defined as the minimal perturbation of $s$ which changes the output action, i.e.,

$$\rho(\pi, s) = \inf_{\varepsilon} \{ \exists \bar{s} \in \mathcal{S} \cap \mathcal{B}_\varepsilon(s) \text{ s.t. } \pi(s) \neq \pi(\bar{s}) \}$$
Similarly, for any $0 < \delta < 1$, the $\delta$-robustness radius of a stochastic policy $\pi$ on a state $s$ is defined as the minimal perturbation of $s$ which makes the output action distribution disagrees with the original $\pi(s)$ with probability more than $\delta$, i.e.,

$$\rho(\pi, s) = \inf_{\epsilon} \{ \exists \tilde{s} \in S \cap \mathcal{B}_\epsilon(s) \text{ s.t. } \delta(\pi(\cdot | s)||\pi(\cdot | \tilde{s})) > \delta \}$$

where $\delta(\cdot | \cdot)$ could be any distance measure between two distributions.

**Remark.** If we regard the policy as a classifier which “classifies” a state to an action, then the robustness radius of policy defined above is analogous to the robustness radius of classifiers defined by Wang et al. [2017], with an extension to stochastic predictions.

**Definition 5 (Robustness Radius w.r.t an MDP).** The ($\delta$-)robustness radius of a policy $\pi$ in an MDP $\mathcal{M}$ is defined as the maximal robustness radius of all states, i.e., $\rho(\pi, \mathcal{M}) = \min_{s \in S} \rho(\pi, s)$

**Remarks.** (1) A deterministic policy is robust against any state perturbation smaller than $\rho(\pi, \mathcal{M})$.

(2) For a stochastic policy, if its $\delta$-robustness radius in an $\mathcal{M}$ is $\epsilon$, then any state perturbation within $\epsilon$ will cause reward drop by no more than

$$\frac{4\delta^2 \gamma \max_{s,a} A_\pi(s, a)}{(1 - \gamma)^2} + 2\delta \mathbb{E}_{\pi \sim \pi^*} [\max_a A_\pi'(s, a)] - \mathbb{E}_{\pi \sim \pi^*} [A_\pi'(s, a)],$$

which is proven in the same way with Proposition 2 (see Appendix D.3).

### D.2 Vulnerability Comparison: Difference Between SL and RL

To shed some light on understanding adversarial attacks in RL, we compare SL and RL in terms of their vulnerability to poisoning and adversarial examples.

At test time, a policy network receives states as input, and returns probabilities of choosing each actions as output; a value network receives states (or state-action pairs) as input, and returns the corresponding value as the output. Thus, test-time RL systems are very similar to SL systems, as one can view the policy networks as classification networks, and value networks as regression networks. However, the key difference between evasion in RL and evasion in SL is, data samples are not independent in RL. A single adversarial example in SL test dataset may cause at most one misclassification instance, whereas an adversarial example in RL may case a drastic change of the gained rewards (e.g., by leading the agent to a “devastating” or “absorbing” state).

At training time, SL systems and RL systems are significantly different, as Figure 1 shows. Even when the supervised learner also learns from data streams in an online manner, the training data are independent with the learner’s classifier. In contrast, the distribution of training data samples changes as the learner updates its policy. Poisoning attacks against an SL system could alter the decision boundary, so that the learner makes wrong decisions for certain data samples. For an RL system, poisoning attacks could (1) alter the decision boundary so that the learner chooses bad actions for certain states, and also (2) change the following observations and interactions due to a different selection of action.

In summary, an adversarial attacker may cause higher damages on RL systems than on SL systems, with the same power and budget. But it does not suggests attacking RL systems is easier than attacking SL systems. As every coin has two sides, the high uncertainty of the environment may help an attack reduce the learner’s reward, but may also lead the learner to gain higher reward in the future (as shown in Section 6). Therefore, it is more challenging to successfully attack RL systems than SL systems with a specific goal.

### D.3 Proof of Proposition 2

**Proof.** According to the definition of $\delta$-stability radius, for any poisoning effort within $\epsilon$, the poisoned policy satisfies $D_{TV}[\pi^*||\pi'] \leq \delta$ (assume total variance $D_{TV}$ is the distance measure between policy distributions). We are interested in the difference of expected rewards $\eta(\pi') - \eta(\pi)$.

Define $L_\pi(\pi') = \eta(\pi') + \sum_{s \in S} \rho_\pi'(s) \sum_{a \in A} \pi'(a|s) A_\pi'(s, a)$.
We upper bound the term \(-\partial \theta \partial \eta\). Algorithm 1 shows the detailed procedure of VA2C-P, where the learner’s policy which can be transformed to 
\[ \partial \theta \]
Combining the above results, we obtain 
\[ E \]
which can be transformed to 
\[ \eta(\pi') - \eta(\pi') \leq \frac{4\delta^2 \gamma \max_{s,a} A_{\pi'}(s,a)}{(1 - \gamma)^2} \]
So we have 
\[ |\eta(\pi') - L_{\pi'}(\pi')| \leq \frac{4 \delta^2 \gamma \max_{s,a} A_{\pi'}(s,a)}{(1 - \gamma)^2} \]
\[ \left(1 \right) \]
We upper bound the term 
\[ -\sum_{s \in S} \rho_{\pi'}(s) \sum_{a \in A} \pi'(a|s) A_{\pi'}(s,a) \]
as below.
\[ = E_{s \sim \pi'}[- \sum_{a \in A} \pi'(a|s) A_{\pi'}(s,a)] \]
\[ = E_{s \sim \pi'}[- \sum_{a \in A} \pi'(a|s) A_{\pi'}(s,a) + \sum_{a \in A} \pi'(a|s) A_{\pi'}(s,a) - \sum_{a \in A} \pi'(a|s) A_{\pi'}(s,a)] \]
\[ = E_{s \sim \pi'}[A_{\pi'}(s,a)] \left( \sum_{a \in A} (\pi'(a|s) - \pi'(a|s)) \right) - \sum_{a \in A} \pi'(a|s) A_{\pi'}(s,a) \]
\[ \leq E_{s \sim \pi'}[2\delta \max_{a \in A} A_{\pi'}(s,a) - \sum_{a \in A} \pi'(a|s) A_{\pi'}(s,a)] \]
\[ = 2\delta E_{s \sim \pi'}[\max_{a \in A} A_{\pi'}(s,a)] - E_{s,a \sim \pi'}[A_{\pi'}(s,a)] \]
Combining the above results, we obtain
\[ \eta(\pi') - \eta(\pi') \leq \frac{4 \delta^2 \gamma \max_{s,a} A_{\pi'}(s,a)}{(1 - \gamma)^2} + 2\delta E_{s \sim \pi'}[\max_{a \in A} A_{\pi'}(s,a)] - E_{s,a \sim \pi'}[A_{\pi'}(s,a)] \]
\[ \left(10\right) \]

**E Algorithm**
Algorithm [1] shows the detailed procedure of VA2C-P, where the learner’s policy \( \pi \) is parametrized by \( \theta \).

To solve \( P \), assume \( \frac{\partial \theta}{\partial \pi} \) exists, one can use projected gradient descent to update \( r \) by using the chain rule:
\[ \frac{\partial \eta_{\pi_{k-1}}(\pi_{\theta_k})}{\partial \pi} = \frac{\partial \eta_{\pi_{k-1}}(\pi_{\theta_k})}{\partial \theta_k} \frac{\partial \theta_k}{\partial \pi}. \]

For Vanilla Policy Gradient (VPG) whose update rule is, \( \theta_k = \theta_{k-1} + \alpha \nabla \theta_{k-1} \tilde{\eta}(\theta_{k-1}, r) \), where
\[ \nabla \theta_{k-1} \tilde{\eta}(\pi_{k-1}, r) = \left( \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta_{k-1}} \log \pi_{\theta_{k-1}}(a_t^{(i)} | s_t^{(i)}) \sum_{t=1}^{T} r_t^{(i)} \right) \right), \]
we can derive
\[ \nabla \theta \eta_{\pi_{k-1}}(\pi_{\theta_k}) \approx \left( \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta_{k-1}} \log \pi_{\theta_{k-1}}(a_t^{(i)} | s_t^{(i)}) \sum_{t=1}^{T} r_t^{(i)} \right) \right) \]
\[ \left(12\right) \]
whose update rule is an argmax function. Therefore, we use the Direct Gradient Method proposed by Yang et al. [2017] to approximate the gradient by
\[ \nabla \theta_k = \sum_{j=1}^{t} \nabla \pi_{\theta_{k-1}}(a_j | s_j) \gamma^{t-j} \] (13)

Although \( \frac{\partial \theta}{\partial \pi} \) has a closed-form expression for simple learners like VPG, analytically computing how the poisoned reward influences the model is challenging for more complicated learners like PPO, whose update rule is an argmax function. Therefore, we use the Direct Gradient Method proposed by Yang et al. [2017] to approximate the gradient by
\[ \frac{\partial \eta_{\pi_{\theta_{k-1}}}}{\partial \hat{r}} = \frac{\eta_{\pi_{\theta_{k-1}}}(f(\pi_{\theta_{k-1}}, r + \Delta)) - \eta_{\pi_{\theta_{k-1}}}(f(\pi_{\theta_{k-1}}, r))}{\Delta} \] (14)

\[ \text{Algorithm 1: VA2C-P with } O_r \text{-Poisoning} \]

**Input:** \( \text{total iterations of learning } K; \) poisoning power \( c; \) poisoning budget \( C; \) attacker’s learning rate \( \beta; \) maximum computing iterations \( J; \) distribution distance measure \( d \)

1. Initialize policy discrepancies as an empty list \( \Psi = \emptyset \)
2. Initialize the number of already poisoned iterations \( c = 0 \)
3. Initialize value network \( V \)
4. for \( k = 1, \ldots, K \) do
   if \( c > C \) then
     Break
   Get the current observation \( O_k \) and the learner’s policy model \( \theta_{k-1} \)
   Fit the value function: \( \omega \leftarrow \arg\min_{\omega} \sum_{i \in O_k} \sum_{t=1}^T (V_{\omega}(s_t^{(i)}) - \sum_{t'=t}^T \gamma^{t'-t} r_t^{(i)})^2 \)
   Imitate learner’s update with the clean rewards \( \theta_k, \eta \leftarrow \text{Update}(\theta_{k-1}, r) \)
   Initialize \( \hat{r} \) as the original \( r \) in \( O_k \)
   Set \( \eta_0 = \eta_c \)
   for \( j = 1, \ldots, J \) do
     for \( i = 1, \ldots, N \) and \( t = 1, \ldots, T \) do
       Copy \( r' \) from \( \hat{r} \) and add a small value \( \Delta \) to \( r_t^{(i)} \)
       Imitate learner’s update with poisoned rewards \( \theta', \eta' \leftarrow \text{Update}(\theta_{k-1}, r') \)
       Compute the direct gradient: \( \frac{\partial \eta}{\partial \omega} \leftarrow \frac{\eta' - \eta_{t-1}}{\Delta} \)
       Update the poisoned reward: \( \hat{r} \leftarrow \Pi_{B_{\eta}}(\hat{r} - \beta \frac{\partial \eta}{\partial \omega}) \)
       Imitate learner’s update with poisoned rewards \( \theta_k, \eta_j \leftarrow \text{Update}(\theta_{k-1}, r') \)
     if \( (\eta_j - \eta_{j-1}) \text{ converges} \) then
       Break
     Compute \( \psi_k = \frac{1}{\mathcal{N}_t} \sum_{t'} d(\pi_{\theta_k} || \pi_{\theta_{k-1}}) \) and add \( \psi_k \) to \( \Psi \)
     if \( \psi_k \) is larger than the \( \lfloor (C - c)/K \rfloor \)-th largest element in \( \Psi \) then
       Attack: replace \( r \) with \( \hat{r} \) in \( O_k \) and send it back to the learner
     \( c \leftarrow c + 1 \)
   Procedure \( \text{Update}(\theta, r) \)
   Perform an update with the learner’s algorithm \( \theta' \leftarrow f(\theta, r) \)
   Compute the attacker’s objective
   \( \eta \leftarrow \frac{1}{\mathcal{N}_t} \sum_{i \in O_k} \sum_{t=1}^T (\frac{\pi_{\theta'}(s_t^{(i)} | s_{t-1}^{(i)})}{\pi_{\theta}(s_t^{(i)} | s_{t-1}^{(i)})}) (\sum_{t'=t}^T \gamma^{t'-t} r_t^{(i)} - V_{\omega}(s_t^{(i)})) \)

F Theoretical Interpretation of the Bi-level Optimization Problem

In this section, we discuss the problem relaxation made in Section 4.1.

F.1 Problem Forms

Suppose the attacker has budget \( C = K \). Then following the format of Problem (Q), we could define the original \( O_r \)-poisoning optimization problem for the poisoning at the \( k \)-th iteration as
We claim that the relaxation does not change the feasibility as stated in Proposition 6.

We call the optimal solution to Problem $(P_\mathcal{A})$ as $(\hat{\theta}_K, \hat{\pi}_K)$ and all the way to $(\hat{\theta}_j, \hat{\pi}_j)$, which makes the relaxation not necessarily optimal.

Note that we only optimize on $\hat{r}_k$ to $\hat{r}_K$, since previous $k - 1$ decisions have already been made at the $k$-th iteration.

Although an omniscient attacker is able to predict all the observations and solve Problem $(P_\mathcal{A})$ directly, for the non-omniscient attacker that we focus on, Problem $(P_\mathcal{A})$ is not solvable because of the unknown observations in iteration $k + 1$ to $K$. Hence, as discussed in Section 4, we relax $(P_\mathcal{A})$, a multi-variable optimization problem, to $(K - k + 1)$ sequential single-variable optimization problems.

**F.2 Tightness of Problem Relaxation**

Problem $(P_\mathcal{A})$ is a $(K - k + 1)$-variable optimization problem with $(K - k + 1)$ equality constraints and $(K - k + 1)$ inequality constraints. And Problem $(P_1), (P_2), \ldots, (P_K)$ are $(K - k + 1)$ single-variable optimization problems respectively with 1 equality constraints and 1 inequality constraints. The two sets of problems are naturally equivalent if $\theta_k, \ldots, \theta_K$, as well as the constraints are independent. However, due to the online learning process (Figure 2), $r_k$ and $r_{k+1}$ are all dependent on $\theta_k$ and $\hat{r}_k$, which makes the relaxation not necessarily optimal.

We call the optimal solution to Problem $(P_\mathcal{A})$ as $(\hat{r}_K^\#, \ldots, \hat{r}_k^\#)$, and the optimal solutions to Problem $(P_1)$ to $(P_K)$ as $(\hat{r}_K, \ldots, \hat{r}_k)$. For simplicity, we assume the environment and the policies are all deterministic. So that the value of observed reward $r_j$ is a deterministic function of $\theta_{j-1}$.

We claim that the relaxation does not change the feasibility as stated in Proposition 6.

**Proposition 6.** $(\hat{r}_K^\#, \ldots, \hat{r}_k^\#)$ is a feasible solution to Problem $(P_\mathcal{A})$.

**Proof.** Note that $r_k$ is known and the same for both $(P_\mathcal{A})$ and $(P_k)$.

(1) $\hat{r}_k^\#$ is feasible to $(P_\mathcal{A})$ because it satisfies $\|\hat{r}_k^\# - r_k\| \leq \epsilon$. 

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Assumption 2
Assumption 1

We first make the following assumptions.

\[ \mathcal{P} \]

We can rewrite \( (\mathcal{P}_1) \) as

\[ \text{At iteration 1, Problem } (\mathcal{P}_1) \]

where \( \theta \) and \( \rho \) are known, and

\[ \theta \]

\[ \rho \]

Since the optimal solution to \( (\mathcal{P}_1) \), \( \hat{\rho}_{k+1} \), satisfies \( \| \hat{\rho}_{k+1} - \rho_k + 1 \| \leq \epsilon \), \( \hat{\rho}_{k+1} \) is also feasible to \( (\mathcal{P}_2) \).

(3) By induction, \( \{ \hat{\rho}_{j} \}_{j=k}^{K} \) all satisfy the constraints in \( (\mathcal{P}_2) \) at the same time, so \( (\hat{\rho}_{k}, \cdots, \hat{\rho}_{K}) \) is a feasible solution to Problem \( (\mathcal{P}_2) \).

Note that if the environment or the policy is stochastic, then \( r_j \) is a random variable sampled from some distribution defined by \( \theta_{j-1} \). In this case, the constraints for Problem \( (\mathcal{P}_2) \) should be \( \Pr(\| r_j - r_j^\ast \| \leq \epsilon) \geq \forall j = k, \cdots, K \), where \( \epsilon \in [0, 1] \) is a threshold probability. Then, with appropriate \( t \), Proposition 6 also holds.

So far we have shown that the relaxation is feasible, so that the attacker will not plan for a non-feasible poisoning with VA2C-P. Next, we discuss the optimality of the relaxation.

We first make the following assumptions.

**Assumption 1:** the learner’s update function \( f(\theta, \hat{\rho}) \) is differentiable w.r.t. \( \hat{\rho} \) and \( \theta \).

**Assumption 2:** the environment and the policies are all deterministic, and the reward \( r \) generated by a policy \( \pi_0 \) is differentiable w.r.t. \( \theta \) (i.e. \( R(s, \pi_0(s)) \) is differentiable w.r.t. \( \theta \)).

**Proposition 7.** If Assumption 1 and 2 hold, the necessary condition for \( (\hat{\rho}_{k}, \cdots, \hat{\rho}_{K}) \) being an optimal solution to Problem \( (\mathcal{P}_2) \) is, for all \( j = k, \cdots, K \),

\[
\frac{\partial \eta(\theta_j)}{\partial \theta_j} \frac{\partial \theta_j}{\partial r_j} \bigg|_{r_j = r_j^\ast} = \sum_{j'=j+1}^{K} \frac{\partial \eta(\theta_{j'})}{\partial \theta_{j'}} \frac{\partial \theta_{j'}}{\partial \theta_{j'-1}} \cdots \frac{\partial \theta_{j+1}}{\partial \theta_j} \frac{\partial \theta_j}{\partial r_j} \bigg|_{r_j = r_j^\ast}
\]

(15)

**Proof.** Consider the case \( K = 2 \) for simplicity, and the results naturally extend to a larger \( K \).

At iteration 1, Problem \( (\mathcal{P}_2) \) becomes

\[
\begin{array}{l}
\text{argmin} \quad \eta(\theta_1) + \eta(\theta_2) \\
\text{s.t.} \quad \theta_1 = f(\theta_0, \hat{\rho}_1) \\
\quad \theta_2 = f(\theta_1, r_2) \\
\quad \| \hat{\rho}_1 - r_1 \| \leq \epsilon \\
\quad \| \hat{\rho}_2 - r_2 \| \leq \epsilon 
\end{array}
\]

(P0)

where \( \theta_0 \) and \( r_1 \) are known.

The relaxed problems are

\[
\begin{array}{l}
\text{argmin} \quad \eta(\theta_1) \\
\text{s.t.} \quad \theta_1 = f(\theta_0, \hat{\rho}_1) \\
\quad \| \hat{\rho}_1 - r_1 \| \leq \epsilon 
\end{array}
\]

(P1)

where \( \theta_0 \) and \( r_1 \) are known, and

\[
\begin{array}{l}
\text{argmin} \quad \eta(\theta_2) \\
\text{s.t.} \quad \theta_2 = f(\theta_1, \hat{\rho}_2) \\
\quad \| \hat{\rho}_2 - r_2 \| \leq \epsilon 
\end{array}
\]

(P2)

where \( \theta_1 \) and \( \hat{\rho}_2 \) are determined by \( \hat{\rho}_1 \), the solution to (P1).

Suppose \( \hat{\rho}_1^\ast \) and \( \hat{\rho}_2^\ast \) are the optimal solutions to (P1) and (P2). And we discuss the necessary condition for \( \hat{\rho}_1^\ast \) and \( \hat{\rho}_2^\ast \) being optimal to (P0).

We can rewrite (P1) as

\[
\begin{array}{l}
\text{argmin} \quad \eta(f(\theta_0, \hat{\rho}_1)) \\
\text{s.t.} \quad \| \hat{\rho}_1 - r_1 \| ^2 - \epsilon^2 + \eta_1^2 = 0 
\end{array}
\]

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And the Lagrange function of the above problem is

\[ L(\dot{r}_1, \lambda_1, y_1) = \eta(f(\theta_0, \dot{r}_1)) + \lambda_1(||\dot{r}_1 - r_1||^2 - \varepsilon^2 + y_1^2) \]  \hspace{1cm} (16)

The necessary conditions for \( \dot{r}_1 \) being optimal are

\[ \frac{\partial \eta(\theta_0, \dot{r}_1)}{\partial \dot{r}_1} + \frac{\partial \lambda_1(||\dot{r}_1 - r_1||^2 - \varepsilon^2 + y_1^2)}{\partial \dot{r}_1} = 0 \]  \hspace{1cm} (17)

\[ ||\dot{r}_1 - r_1||^2 - \varepsilon^2 + y_1^2 = 0 \]  \hspace{1cm} (18)

\[ \lambda_1 y_1 = 0 \]  \hspace{1cm} (19)

for some \( \lambda_1 \) and \( y_1 \).

Similarly, for Problem \( (P2) \), the necessary conditions of optimality are

\[ \frac{\partial \eta(\theta_1, \dot{r}_2)}{\partial \dot{r}_2} + \frac{\partial \lambda_2(||\dot{r}_2 - r_2||^2 - \varepsilon^2 + y_2^2)}{\partial \dot{r}_2} = 0 \]  \hspace{1cm} (20)

\[ ||\dot{r}_2 - r_2||^2 - \varepsilon^2 + y_2^2 = 0 \]  \hspace{1cm} (21)

\[ \lambda_2 y_2 = 0 \]  \hspace{1cm} (22)

for some \( \lambda_2 \) and \( y_2 \).

Since \( \dot{r}_1^\# \) and \( \dot{r}_2^\# \) are the optimal solutions to \( (P1) \) and \( (P2) \), \( \dot{r}_1^\# \) and \( \dot{r}_2^\# \) satisfy Equation \( (17) \sim (22) \).

Expanding \( (17) \) and \( (20) \), we get

\[ \frac{\partial \eta}{\partial \theta_1} \frac{\partial \theta_1}{\partial \dot{r}_1} \bigg|_{\dot{r}_1 = \dot{r}_1^\#} + 2\lambda_1 (\dot{r}_1^\# - r_1) = 0 \]  \hspace{1cm} (23)

\[ \frac{\partial \eta}{\partial \theta_2} \frac{\partial \theta_2}{\partial \dot{r}_2} \bigg|_{\dot{r}_2 = \dot{r}_2^\#} + 2\lambda_2 (\dot{r}_2^\# - r_2) = 0 \]  \hspace{1cm} (24)

For Problem \( (P0) \), the Lagrange is

\[ L(\dot{r}_1, \dot{r}_2, \lambda_1', \lambda_2', y'_1, y'_2) = \eta(f(\theta_0, \dot{r}_1)) + \lambda_1'(||\dot{r}_1 - r_1||^2 - \varepsilon^2 + (y'_1)^2) + \eta(f(\theta_2, \dot{r}_2)) + \lambda_2'(||\dot{r}_2 - r_2||^2 - \varepsilon^2 + (y'_2)^2) \]  \hspace{1cm} (25)

And the necessary conditions for \( \dot{r}_1, \dot{r}_2 \) being optimal are

\[ \frac{\partial \eta(\theta_0, \dot{r}_1)}{\partial \dot{r}_1} + \frac{\partial \lambda_1'||\dot{r}_1 - r_1||^2 - \varepsilon^2 + (y'_1)^2}{\partial \dot{r}_1} + \frac{\partial \lambda_2'||\dot{r}_2 - r_2||^2 - \varepsilon^2 + (y'_2)^2}{\partial \dot{r}_1} = 0 \]  \hspace{1cm} (26)

\[ \frac{\partial \eta(\theta_1, \dot{r}_2)}{\partial \dot{r}_2} + \frac{\partial \lambda_1'||\dot{r}_2 - r_2||^2 - \varepsilon^2 + (y'_2)^2}{\partial \dot{r}_2} + \frac{\partial \lambda_2'||\dot{r}_2 - r_2||^2 - \varepsilon^2 + (y'_2)^2}{\partial \dot{r}_1} = 0 \]  \hspace{1cm} (27)

\[ ||\dot{r}_1 - r_1||^2 - \varepsilon^2 + (y'_1)^2 = 0 \]  \hspace{1cm} (28)

\[ ||\dot{r}_2 - r_2||^2 - \varepsilon^2 + (y'_2)^2 = 0 \]  \hspace{1cm} (29)

\[ \lambda_1' y'_1 = 0 \]  \hspace{1cm} (30)

\[ \lambda_2' y'_2 = 0 \]  \hspace{1cm} (31)

for some \( \lambda_1', \lambda_2', y'_1, y'_2 \) (not the same with \( \lambda_1, \lambda_2, y_1, y_2 \)).

Expanding \( (26) \) and \( (27) \), we get

\[ \frac{\partial \eta}{\partial \theta_1} \frac{\partial \theta_1}{\partial \dot{r}_1} \bigg|_{\dot{r}_1 = \dot{r}_1^\#} + \frac{\partial \eta}{\partial \theta_2} \frac{\partial \theta_2}{\partial \dot{r}_2} \bigg|_{\dot{r}_2 = \dot{r}_2^\#} \frac{\partial \theta_1}{\partial \dot{r}_1} \bigg|_{\dot{r}_1 = \dot{r}_1^\#} + 2\lambda_1'(\dot{r}_1^\# - r_1) + 2\lambda_2'(\dot{r}_2^\# - r_2) \frac{\partial \theta_2}{\partial \dot{r}_2} \bigg|_{\dot{r}_2 = \dot{r}_2^\#} \frac{\partial \theta_1}{\partial \dot{r}_1} \bigg|_{\dot{r}_1 = \dot{r}_1^\#} = 0 \]  \hspace{1cm} (32)

\[ \frac{\partial \lambda_1'}{\partial \theta_2} \frac{\partial \theta_2}{\partial \dot{r}_2} \bigg|_{\dot{r}_2 = \dot{r}_2^\#} + 2\lambda_2'(\dot{r}_2^\# - r_2) = 0 \]  \hspace{1cm} (33)
Combining (25), (24), (32) and (35), we obtain
\[
\frac{\partial \eta}{\partial \theta_2} \frac{\partial \theta_1}{\partial r_1} \bigg|_{r_1 = r_1^\#} = \zeta \frac{\partial \eta}{\partial \theta_1} \frac{\partial \theta_1}{\partial r_1} \bigg|_{r_1 = r_1^\#}
\]
for some \( \zeta \), which is the necessary condition for \( r_1^\# \) and \( r_2^\# \) being optimal to Problem \((P0)\).

Intuitively, this condition implies that the gradient of \( \eta(\theta_2) \) w.r.t. \( r_1^\# \) (the RHS) should be aligned with the gradient of \( \eta(\theta_1) \) w.r.t. \( r_1 \) (the LHS), without considering the influence of \( r_1 \) to \( r_2 \). That is, although \( r_1 \) influence \( \theta_1 \), \( \theta_1 \) influences both \( \theta_2 \) and \( r_2 \) (because \( r_2 \) is generated by \( \pi_{\theta_1} \)). LHS does not include \( \frac{\partial \theta_2}{\partial \theta_2} \frac{\partial \theta_1}{\partial r_1} \), which makes (34) computable in many cases.

However, for the setting of our VA2C-P (monitoring attacker for online RL), the attacker at iteration \( k = 1 \) does not know the observed reward \( r_2 \) of iteration \( k = 2 \), which prevent the agent from conducting the optimal attack. But (34) could help the attacker verify whether a past poison is likely to be optimal for the iterations so far. For example, if the learner is using VPG, then at iteration \( k = 2 \), the attacker can test whether its previous poison \( r_1 \) did a good job in minimizing \( \eta(\theta_1) + \eta(\theta_2) \) by evaluating whether \( \left( I + \nabla_{\theta_1} \eta(\theta_1) \right) \nabla_{\theta_2} \eta(\theta_2) \) is equal to \( \nabla_{\theta_1} \eta(\theta_1) \).

### G Additional Experiment Results

Figure 7 shows additional experiment results of poisoning on multiple algorithms. Note that the selection of \( \epsilon \) depends on the environment and the algorithm. The larger \( \epsilon \) is, the better attackers can do.

![Figure 7](image)

**Figure 7:** Additional Results for comparison of mean rewards of VPG, PPO, A2C, ACKTR on various environments, with no poisoning, random poisoning, AC-P and VA2C-P.

Figure 8 demonstrates the additional results of policy discrepancy for VPG, PPO on Hopper and HalfCheetah environments.

In the supplementary materials we provide the code and instructions, as well as demo videos of poisoning A2C in the Hopper environment, where one can see under the same budget constraints, random poisoning has nearly no influence the agent’s behaviors, while our proposed VA2C-P successfully prevents the agent from hopping forward.
Figure 8: Additional results for comparison of policy discrepancies at every iteration.