Transport in Floquet-Bloch bands

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Floquet band engineering is emerging as a powerful method for nonequilibrium material control and synthesis [11-39]. Quantum control of transport in driven lattices may hold the key to new device types, unexplored techniques for ultrafast transport of energy and information in solids, and dynamical tools for controlling and probing condensed matter [3-11]. Cold atoms in optical lattices offer a near-ideal experimental platform for the study of nonequilibrium quantum transport [12-30]. Here we report Floquet band engineering of long-range transport and direct imaging of Floquet-Bloch bands in an amplitude-modulated optical lattice. In one variety of Floquet-Bloch band we observe tunable rapid long-range high-fidelity transport of a Bose condensate across thousands of lattice sites. Quenching into an opposite-parity Floquet-hybridized band allows Wannier-Stark localization to be controllably turned on and off. A central result of this work is the use of transport dynamics to demonstrate direct imaging of a Floquet-Bloch band structure. These results open a path to unexplored applications of Floquet engineering, quantum emulation of ultrafast multi-band electronic dynamics, and Floquet-enhanced metrology.

Ultracold atomic gases have enabled investigation of a wide variety of transport-related phenomena, including Bloch oscillations [12-14], Anderson localization [15-17], photoconductivity [15], superfluid critical velocity [19-21], spin-orbit coupling [22], and pulsed modulation techniques for wavepacket manipulation [23, 24]. The application of Floquet techniques in optical lattices [21-24] has expanded the control over these systems and enabled the study of phenomena including topological dynamics [25], renormalization of tunneling [26], correlated tunneling [27], tunable mobility [28], and synthetic ferromagnets [29]. While modulation techniques have been used to modify the spatial width of wavepackets [30], the control of center-of-mass transport dynamics in Floquet-Bloch bands remains largely unexplored.

Amplitude modulation of an optical lattice creates quasimomentum-selective band crossings which can be used to stitch together hybridized Floquet-Bloch bands in a variety of ways, allowing robust tunable modification of transport phenomena. We report a series of experiments probing and controlling transport of ultracold bosonic lithium atoms in Floquet-Bloch bands. Floquet hybridization in the presence of an applied force can be used to generate coherent transport over thousands of lattice sites, switch on and off Bloch oscillations, and tune the band dispersion by manipulating drive param-

![Image of rapid long-range transport in a Floquet-Bloch band.](https://example.com/image.png)

**FIG. 1.** Rapid long-range transport in a Floquet-Bloch band. (a) Time sequence of images of a condensate in the ground band with lattice depth $V_0 = 5.4 E_R$. A force of $6 \times 10^{-26}$ N per atom induces Bloch oscillations [14]. (b) Time sequence of images of a condensate in a $(0, 2)$ hybridized Floquet-Bloch band created via amplitude modulation with $\nu = 170$ kHz and $\alpha = 0.2$, with the same initial force as in (a). Note the rapid cyclic high-fidelity transport across the trap. (c) Unmodified band structure. Vertical rippled lines indicate band coupling at the hybridizing quasimomentum for this modulation frequency. (d) Calculated dispersion of the unmodified ground band (solid) and hybridized Floquet-Bloch band (dashed). (e) Position-velocity evolution in the same hybrid band as (b) with an initial force of $5 \times 10^{-26}$ N per atom. Solid line is theory; points are data, taken at equally-spaced times.
eters. As we demonstrate, experimental measurements of dynamical evolution enable direct imaging of Floquet-Bloch band structure.

Our experimental platform for Floquet band engineering is discussed in the Methods section. The system can be described by the 1D time-dependent Hamiltonian

$$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} - V_0 (1 + \alpha \sin(2\pi \nu t)) \cos^2 \left( \frac{\pi x}{d} \right) + \frac{1}{2} m \omega^2 (x - x_0)^2,$$

where $x$ is position along the lattice, $d$ is the lattice spacing, $m$ is the atomic mass of $^7$Li, $V_0$ is the static lattice depth, $\alpha$ is the modulation strength, and $\nu$ is the modulation frequency. External coils generate harmonic confinement with trap frequency $\omega$ centered about $x = x_0$; the resulting force $F(x) = -m \omega^2 (x - x_0)$ drives transport. Transverse degrees of freedom play no role in the dynamics we report. In the absence of the modulation and for weak force, the spectrum of equation (1) consists of Bloch bands with energy $\{ E_j(q) \}$, where $j$ is the band index and $q$ the quasimomentum. Amplitude modulation satisfying an interband resonance of the $i$th and $j$th band $\Delta E = (E_i(q^+) - E_j(q^-))$, where $q^\pm$ is the resonant quasimomentum and $n$ the photon number, hybridizes the static spectrum into quasienergy bands $\{ E_{ij}(q) \}$. While the drive can in principle hybridize any set of bands with arbitrary order $n$, this work focuses on the case of single-photon resonant hybridization of the ground band with the $j$th excited band, which we denote as $(0,j)$ hybridization. This notation, while non-unique, emphasizes the band hybridizations that govern the observed dynamics and is sufficient to specify the relevant hybrid bands that have maximal overlap with the static ground band at the center of the Brillouin zone.

The wavepacket center-of-mass dynamics are dictated by the local force and the group velocity of the hybridized band: $dq/dt = F$ and $dx/dt = dE/dq$. Dynamics are initiated by switching off the confining optical dipole trap and simultaneously turning on the lattice modulation, quenching the atomic ensemble into the Floquet-Bloch band. If the hybridizing quasimomentum $q^*$ is sufficiently different from the initial quasimomentum we do not observe heating from the quench. After variable hold time in the modulated lattice, the atomic position distribution is measured by in-situ absorption imaging.

Fig. 2 demonstrates the dramatic difference between transport in the static ground band and transport in a $(0,2)$ Floquet-Bloch band, a hybrid of the ground and second excited bands. In the absence of modulation, the local force $F$ induces Bloch oscillations (Fig. 2a) whose amplitude in position space is proportional to the static bandwidth $\Delta E$. Amplitude modulation at frequency $\nu = 170$ kHz and amplitude $\alpha = 0.2$ hybridizes the ground and second excited band at quasimomentum $q^* \approx 0.4 \hbar k_L$ as diagrammed in Fig. 1b. In the Floquet-Bloch band, instead of exhibiting Wannier-Stark localization, the atomic ensemble undergoes rapid coherent oscillatory transport across approximately 2000 lattice sites (Fig. 2b).

This striking transport behavior is a direct consequence of the hybridized structure of the Floquet-Bloch band. As the quasimomentum evolves, the group velocity sharply increases at the high-curvature point of the Floquet-Bloch band where $|q(t)| = q^*$. The resulting
rapid transport carries the ensemble thousands of lattice sites in real space. On such large length scales, the force due to the harmonic confinement is no longer approximately constant; the ensemble moves across the entire applied potential, gaining and then losing quasimomentum without reaching the edge of the Brillouin zone. At the position where the potential energy is the same as it was at the first point of high band curvature, energy conservation requires that the ensemble again has quasimomentum \( q^* \), and the group velocity sharply decreases. The transport of the wavepacket in the hybridized band is thus characterized by periods of rapid transfer across the entire trapping region connected by relatively slow Bloch-oscillation-like motion at the turning points. The full position-velocity evolution is shown in Fig. 3. Notable features of the observed dynamics include high-fidelity long-range transport of nearly all atoms in the condensate, coherent Bloch oscillation dynamics at opposite ends of the trap, maximum transport velocities far in excess of those expected for the bare harmonic potential, and a high degree of control attainable by varying drive properties.

To further probe the dynamics we study the dependence of long-range transport on initial applied force, drive frequency, and hybridized band indices. As shown in Fig. 2a and b, the total transport distance \( D \) increases with increasing force while the oscillation period \( T \) decreases. The observed behavior agrees quantitatively with fit-parameter-free numerical calculations for the Floquet-Bloch band shown as solid black lines in Figs. 2a, b, and c. Figs. 2e, f show the results of varying the hybridizing frequency \( \nu \) at constant force. Increasing \( \nu \) hybridizes the bands closer to the edge of the Brillouin zone, which increases the time to reach \( q^* \) such that the oscillation period increases while the transport distance remains fixed. Again, the observed wavepacket evolution agrees quantitatively with fit-parameter-free numerical predictions, demonstrating the effectiveness of the Floquet-Bloch formalism for describing this controllable long-distance transport.

Floquet hybridization of different pairs of static bands gives rise to distinct transport properties. Fig. 2a and 2h compare transport dynamics in (0, 2) and (0, 4) hybrid bands. We observe that evolution in the (0, 4) hybridization leads to dramatically faster long-range transport, due to the increased group velocity and band curvature, while still preserving near-unity fidelity. During this evolution, the ensemble stretches across the entire extent of the trapping potential, but still returns to the static ground band at the far edge of the trap.

Diabatic quenches between static bands and hybridized Floquet bands provide a powerful experimental tool for dynamical control of transport properties. Quenching back to the static lattice after a total modulation time \( t_q \) projects the Floquet-Bloch state back onto the original static spectrum. For \( q(t_q) \) sufficiently far from \( q^* \) we observe no significant heating during the quench. Fig. 3 shows the results of such quenched modulation experiments in which the modulation depth \( \alpha \) is set suddenly to zero after some variable time of evolution in the Floquet-Bloch band. Quenching near the turning points of the position-space oscillation projects the atomic ensemble back onto the Wannier-Stark localized ground band, at a position which can vary by thousands of lattice sites depending on \( t_q \). The identical amplitude and frequency of position-space Bloch oscillations after all quenches in Fig. 3 indicates the non-dissipative nature of quenched Floquet-Bloch transport. The relative phase shifts of the position-space Bloch oscillations are consistent with expectations based on the time spent in the hybridized state and the changing sign of the force during the transport.

Hybridizing bands of opposite parity gives rise to qualitatively different phenomena. While opposite-parity coupling is forbidden at \( q^* = 0 \) due to the even-parity nature of amplitude modulation, hybridization at finite quasimomentum is both allowed and observed. Fig. 4 shows the result of (0, 1) coupling at \( q^* \approx 0.66 \hbar k L \), using amplitude modulation with \( \nu = 56 \text{ kHz} \), \( \alpha = 0.25 \), and \( V_0 = 3.6 \ E_R \). At this reduced lattice depth, atoms...
in the unmodified ground band do not Bloch oscillate but undergo ballistic transport in the trapping potential (Fig. 4a). Quenching into the (0, 1) hybrid band causes Wannier-Stark localization due to Bloch oscillations in the Floquet-hybridized band (Fig. 4b). Here the opposite curvature of the two static bands at \( q = 0 \) results in a hybrid band with smaller bandwidth and local extrema in the dispersion, as shown in Fig. 4.

Strikingly, these oscillations enable direct imaging of the Floquet-Bloch band structure. A recent experiment [13] demonstrated that the position evolution of an atomic ensemble undergoing Bloch oscillations in a static band constitutes a direct image of the energy-momentum dispersion relation according to the mapping:

\[
E = F_x, \quad q = Ft.
\]  

(2)

Fig. 4f experimentally demonstrates that this mapping extends to Floquet-Bloch bands by comparing the center-of-mass motion in a (0, 1) hybrid band to the calculated band dispersion of Fig. 4e, scaled according to equation (2). There are no fit parameters in this plot, as the force is measured independently. The close agreement between the measured atomic position and the band dispersion demonstrates direct imaging of a hybridized Floquet-Bloch band.

This Floquet-Bloch band image is reminiscent of angle-resolved photoemission maps of Floquet bands in laser-driven topological insulators [4]. This analogy suggests potentially fruitful connections between the results we present and topics of current interest in condensed matter, including the effect of Bloch oscillations and interband transitions on high-harmonic generation in crystals and prospects for all-optical band structure reconstruction in solids [5-11]. Using techniques like those we present, cold atom quantum simulation experiments may be able to serve as a complementary tool for exploration of band dynamics, probing and realizing phenomena at the edge of current ultrafast experimental capabilities.

The well-controlled Floquet-Bloch transport dynamics presented here also open the path to the realization and study of more complex Floquet-engineered phenomena, including polychromatic driving for hybridization of multiple bands at multiple quasimomenta, Floquet-based creation of topologically nontrivial bands, and the controlled introduction of disorder. The enhanced control of band structure and transport demonstrated here may also be useful for metrology. One possibility along these lines would be atom interferometry using wavepackets split by a large distance in a (0, 2) or (0, 4) hybrid band.

In summary, we have demonstrated tunable coherent control of long-range quantum transport in hybridized Floquet-Bloch bands. We have used hybridization of various pairs of four separate static bands at varying quasimomenta to realize rapid long-distance transport of a Bose condensate, switchable Wannier-Stark localization, and direct imaging of a hybrid Floquet-Bloch band.

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Our experimental platform for Floquet band engineering is a degenerate quantum gas of $^7$Li in an amplitude-modulated optical lattice and an applied harmonic magnetic potential. Each experiment begins by producing a Bose condensate of approximately $10^5$ $^7$Li atoms in the $|F = 1, m_F = 1\rangle$ hyperfine state in a crossed optical dipole trap. After the final stage of cooling, an applied magnetic field is tuned to the shallow scattering length $\lambda = 1064$ nm is the lattice wavelength, and $m$ is the atomic mass. Amplitude modulation of the lattice creates the hybridized Floquet-Bloch bands. Floquet-Bloch band properties and dynamics are calculated numerically by computing the eigenvalues and eigenstates of the hermitian generator of the single-period time evolution operator [2].

\[ E = \frac{1}{2m} k^2 \left( \frac{2\pi}{\lambda} \right)^2, \]

\[ k = \frac{2\pi}{\lambda}. \]