Large Scale (~25 m²) metal diffraction grating of submicron period as possible optoelectronic detector for short scalar gravitational waves

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ABSTRACT

A method of detecting of short scalar gravitational waves with a wavelength of $\lambda \sim 0.5 \mu$m is proposed, in contrast to LIGO Project, aimed at detecting of long quadrupole gravitational waves ($\lambda \sim 43 \div 10000$ km). The conduction electrons in a metal are proposed to use as gravitational receiving antennas (pendulums) instead of massive mirrors in LIGO. It is shown that using a Large Scale metal diffraction grating with area of 25 m² you can convert the mechanical vibrations of the conduction electrons of metal into a plane electromagnetic wave propagating along the normal to the grating. It is shown that when the amplitude of the scalar gravitational wave in a source (in quasar at the center of our galaxy) is greater than $A_{40} \approx 5 \cdot 10^{20}$ cm/s², you can register it with the help of a large optical telescope equipped with the proposed diffraction grating. It is shown that the special theory of relativity allows the amplitude of the scalar gravitational waves in this source by 5 orders of magnitude greater than the above-mentioned minimum value.

Keywords: strongly correlated electron oscillations, optoelectronics, Large Scale metal diffraction grating, scalar gravitational waves, gravitational telescope

1. INTRODUCTION

Currently in theoretical astrophysics the prevailing view is that in nature there are only two types of gravitational waves: tensor (quadrupole) and scalar¹. In recent years, in the frames of LIGO project, attempts were made, yet unsuccessful, to detect long-wavelength (with $\lambda$ in interval $43\div10000$ km) quadrupole gravitational radiation from the millisecond pulsars², synchronous with their electromagnetic radiation. It should be noted that the proposed hitherto antennas (detectors) of long wave gravitational radiation are massive pendulums, coupled with sensitive displacement sensors. At the same time there are powerful explosive processes associated with relativistic accretion of matter into a supermassive "black hole" (quasar), located in the center of our galaxy³⁻⁶. See Figure 1. Processes of accretion in the quasar are accompanied by intense electromagnetic radiation in all observable spectral regions. The power of these processes is superior on the tens of orders the power of processes associated with the rotation of the non-axisymmetric pulsars⁵,⁶. As it follows from¹, the scalar gravitational waves should also be emitted at such accretion. Moreover, it must be in sync with electromagnetic bursts and in all regions of the spectrum, including the optical frequencies.

2. STATEMENT OF THE PROBLEM

In this paper we propose a detector of scalar gravitational waves with a length of $\lambda \sim 0.5$ microns. In the last stage of registration of gravitational waves the energy of these waves is converted into the energy of electromagnetic waves of the same frequency. Therefore, the wavelength is selected with the expectation that, as a receiver of these oscillations will be used optical telescope (the tool that is the most sensitive for them). The detector - converter, proposed by us, is a flat metal film with the thickness of the order of $\lambda$. We will place this film so that the wave vector of gravitational waves, whose direction is taken as the X-axis of our coordinate system, will lie in the plane of the film. The electrons in the conduction band of the metal are used in this detector as pendulums. By analogy with the Mossbauer Effect we believe that the ion subsystem of metal at temperatures below the Debye temperature will not respond to the impact of short

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gravitational waves. As example, we give Debye temperature for some metals in Table 1. For reasons of symmetry the scalar gravitational wave is a longitudinal wave of vibrations of the gravitational potential gradient, such as a wave of longitudinal oscillations of the particles in an acoustic wave in the gas. Therefore, we believe that the scalar gravitational wave in the linear approximation (weak field) Galileo metric is mathematically described by the formula

![Figure 1. Milky Way Galaxy (front view at the top, side view at the bottom): mutual position of the quasar and the Sun.](image)

| metal | Al | Cu | α-Fe | Ti | W | Ni |
|-------|----|----|------|----|---|----|
| Θ_D(K) | 429 | 344 | 464 | 420 | 405 | 440 |

Table 1. Debye’s Temperature Θ_D in Kelvin for some metals.
\[ A_g(R, t) = \left( \frac{A_{g0} R_g}{R} \right) \cos(\omega t - kR), \]

where \( A_g(R, t) \) is the value of the potential gradient of variable gravitational field at a distance \( R \) from the source (the supermassive "black hole" in the center of the Galaxy \( ^{11, 33} \)) at time \( t \). In our case \( R \approx 8.33 \text{ kpc} \) (or converting to kilometers \( R \approx 2.6 \times 10^{17} \text{ km} \)). \( A_{g0} \) is the peak value of the alternating gradient of the gravitational field in the source at a distance equal to the Schwarzschild radius from the center. This radius, otherwise known as the gravitational radius of the "black hole" \( R_g \) is equal \( R_g = 2\gamma M/c^2 \), where \( \gamma = 6.67 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-3} \) ( \( \gamma = 6.67 \times 10^{-8} \text{ cm}^3 \text{g}^{-1} \text{s}^{-3} \)) is the gravitational constant. For a "black hole" in the center of the galaxy having a mass \( M_* \approx 4.31 \times 10^9 M_{\text{Sun}} \), where \( M_{\text{Sun}} \) is the mass of the Sun, \( R_g \approx 12 \times 10^6 \text{ km} \). On considered distances from the source we can approximately assume that the spherical gravitational wave is flat \( ^6 \) and the formula \( (1) \) takes the form

\[ A_g(x, t) = \left( \frac{A_{g0} R_g}{R} \right) \cos(\omega t - kx), \]

where \( x \)-coordinate is measured along the direction of the wave propagation. In the quantum theory of solids it is assumed that the motion of electrons in the conduction band of the metal is described by a collective wave function composed of individual electron orbitals, the so-called periodic Bloch functions \( ^8 \).

When considering the process of reflection of weak electromagnetic waves from the surface of a metal mirror, it is believed that additional motion under the influence of the electromagnetic wave is linearly superimposed on the orbital motion of electrons. This additional motion of charges, in turn, stimulates the secondary (mirrored) electromagnetic wave \( ^9 \). By analogy with the process of reflection of electromagnetic waves by electrons of the metal we consider the process of interaction with a gravitational wave. The electrons in the conduction band of the metal, by virtue of the principle of equivalence of inertial and gravitational masses, under the influence of gravitational waves, in addition to their motion on Bloch orbitals will oscillate along the direction of wave propagation with an acceleration equal to the gradient of the potential in the wave: \( A_g(x, t) \). Each electron with its nearest metal ion of the crystal lattice can be considered as an elementary harmonic oscillator - Hertz dipole. This is done, for example, when calculating the bremsstrahlung X-ray radiation \( ^{11} \). This dipole can be described by a function of the form

\[ p(x, t) = q_e \cdot l(x, t) = p_0 \cdot f(x, t), \]

where \( p_0 = q_e \cdot l_0 \), \( q_e \) is electron charge, \( l_0 \) is the maximum amplitude of the deviation from the equilibrium position of the electron. (Here and below we follow the presentation \( ^{11, 11} \) and will use a system of units CGSE.) From the formulas \( (1') \) and \( (2) \) and the principle of equivalence of inertial and gravitational masses it follows:

\[ \dot{l}(x, t) = A_g(x, t) = A_{g0} \cos(\omega t - kx) \cdot (R_g / r) \]

Assuming \( l(x, t) = l_0 \cos(\omega t - kx) \) we get \( \ddot{l}(t) = -l_0 \omega^2 \cos(\omega t - kx) \). Hence, in turn, it follows

\[ l_0 = \left| A_{g0}(R_g / r) / \omega^2 \right| \]

The distances between the individual Hertz dipoles (conduction electrons in metals) are equal approximately to the lattice constant, thus it is about of 0.5 nm (5 * 10^4 cm). In result of acceleration transmitted to electrons from gravitational waves these dipoles will radiate electromagnetic waves. The characteristic frequency of the oscillations in the emitted electromagnetic wave remains the same, and the speeds of propagation of electromagnetic and gravitational waves in vacuum coincide. Thus wave number \( k \) for the searched electromagnetic wave is equal to wave number \( k \) for the gravitational wave from the source. According to \( ^{11} \), outside of the metal film, at a distance \( r \) from it, which satisfies the condition of far zone \((kr >> 1)\), these oscillations are described by the formula:

\[ E = k^2 \left[ \hat{n} \times \left[ \hat{n} \times \vec{E}(x, t) \right] \right] / r = k^2 \left[ \hat{n} \times (q_e \cdot l_0 \cos(\omega t - kx) \hat{e}_x + 0 \cdot \hat{e}_y + 0 \cdot \hat{e}_z) \right] / r. \]

It is easy to see that \( \vec{E} = E_x \hat{e}_x + 0 \cdot \hat{e}_y + 0 \cdot \hat{e}_z \), and \( E_x = E_0 \cos(\omega t - kx) = \varphi_0 \cos(\omega t - kx) / r \), where \( E_0 = \varphi_0 / r \) and \( \varphi_0 = k^2 q_e \cdot l_0 \). Simultaneously the fluctuations of the magnetic field will occur \( ^{11} \) that will be described by the formula \( \vec{B} = \left[ \hat{n} \times \vec{B} \right] \). Consider the picture of the electric field distribution at a fixed time \( t \). For each area of the metal film for which, according to formula \( (5) \), the vector \( \vec{E} \) is directed along the direction of the gravitational waves will match the area for which the vector \( \vec{E} \) is directed in the opposite direction. In space above the metal film the values of the electric field vector, for which the sources are in these areas, when averaged give a zero result.
Consider on metal film two adjacent parallel strips that are perpendicular to the direction to the source of gravitational waves, and separated by a distance of $\lambda/2$. See Figure 2.

![Electric fields $E_{\Sigma 1}$ and $E_{\Sigma 2}$ above two dedicated strips on flat metal film.](image)

At a distance $z$, deferred from the film perpendicular to it, the total electric field from the two dedicated strips above the first strip is:

$$E_{\Sigma 1} = \varphi_0 \cos(\omega t - kx) \left( \frac{1}{z} - 1 - \frac{1}{\sqrt{z^2 + \lambda^2}} + \frac{1}{4} \right) \propto \varphi_0 \cos(\omega t - kx) \lambda^2 / 8z^3.$$  

The total electric field above the second stripe is equal to:

$$E_{\Sigma 2} = \varphi_0 \cos[\omega t - k \cdot (x + \lambda/2)] \left( \frac{1}{z} - 1 - \frac{1}{\sqrt{z^2 + \lambda^2}} + \frac{1}{4} \right) \propto -\varphi_0 \cos(\omega t - kx) \lambda^2 / 8z^3. $$  

I.e. field decreases as $1/z^3$. The electromagnetic field above our metal film in the “far” zone (at $kr \gg 1$) for considered electromagnetic vibrators will be absent.

3. METHOD

For arising of this electromagnetic field, you need to make an orderly change on $\pi$ of the phase of the oscillations generated in the metal in the areas in which the electrons oscillate in opposite directions. For this purpose, we will make the surface of our metal film corrugated so that its coordinate along axis $Z$ direction, perpendicular to the film plane will vary in accordance with formula

$$z(x) = \frac{\lambda}{4} (1 + \cos kx) + \frac{3\lambda}{4}. $$  

Here the origin of the $Z$ - coordinate is already in the bottom surface of the metal film. This is reflected in Figure 3.
Figure 3. Cosine shape profile of the metal diffraction grating.

Suppose that for the considered gravitational waves the characteristic coherence time exists during which the wave can be considered as stationary. As is well known\textsuperscript{12,13}, the stationary wave field from sources distributed in the plane XY, at a short distance $z$ from this plane, in the case when diffraction can be neglected is given by the expression

$$E(x, y, z, t) = E(x, y) \cos \omega t \cos k z,$$

where the factor $\cos k z$ characterizes the wave propagation in the direction $Z$, perpendicular to XY plane. In this case, $E(x, y, z, t) = E(x, z, t) = E_0 \cos(\omega t - k x) \cos k z,$

and $E_0 = k^2 q_{c} \cdot l_{0}/r$ according to formula (5). Then, in a plane passing through the top of the folds of our corrugated surface which is predetermined by the formula (6), the expression (7') is transformed into:

$$E_0 \cos(\omega t - k x) \cos k z = E_0 \cos \omega t \cos k x \cos(\pi/2(1 + \cos k x)) - E_0 \sin \omega t \sin k x \cos(\pi/2(1 + \cos k x)).$$

We introduce the notation: $f_1(x) = \cos k x \cos(\pi/2(1 + \cos k x))$ and $f_2(x) = \sin k x \cos(\pi/2(1 + \cos k x))$. Then the formula (8) can be rewritten as:

$$E(x, z, t) = E_0 \cos \omega t \cdot f_1(x) - E_0 \sin \omega t \cdot f_2(x).$$

Figure 4 and Figure 5 show graphs of functions $f_1(x)$ and $f_2(x)$ as well as their simplest approximation functions $1/2 + \cos(2kx)/2$ and $\sin(2kx)/\sqrt{2.3}$, accordingly.

Figure 4. The solid red line is graph of the function $f_1(x)$; for comparison: the blue dashed line in the same Figure is graph of the function $1/2 + \cos(2kx)/2$. 

Figure 5. The solid red line is graph of the function $f_2(x)$; for comparison: the blue dashed line in the same Figure is graph of the function $1/2 + \cos(2kx)/2$. 

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Figure 5. The solid red line is the graph of the function \( f_2(x) \); for comparison, dashed blue line in the same figure is the graph of the function \( \sin(2kx)/\sqrt{2.3} \).

Standard deviations of functions \( f_1(x) \) and \( f_2(x) \) for the relevant comparison functions were calculated:

\[
\sqrt{\int_0^1 \left[ f_1(x) - (1 + \cos 2kx)/2 \right]^2 dx} = 0.134
\]
\[
\sqrt{\int_0^1 \left[ f_2(x) - \sin 2kx/\sqrt{2.3} \right]^2 dx} = 0.114
\]

As can be seen from the graphs in Figure 4 and formula (9), the first term on the right-hand side of (8') is in good agreement with the function \( E_0 \cos \omega t(1/2 + \cos 2kx)/2 \). This function, in turn, can be divided into two components: \((E_0/2)\cos \omega t\) and \((E_0/2)\cos \omega t \cos 2kx\). Comparison with formula (7) shows that first component, the function \((E_0/2)\cos \omega t\) will correspond to a plane electromagnetic wave. This wave has a constant within the wave front amplitude of the electric vector, equal to \(E_0/2\) and propagates normal to the metal film. Consider now the second component in the approximating function that shown in Figure 4 - function \((E_0/2)\cos 2kx\) depending on the variable \(x\).

We get a field \((E_0/2)\cos \omega t \cos 2kx\) taking also into account depending on the time. From the theory of oscillations is known that the sum of two waves propagating in opposite directions, namely \(\cos(\omega t + kx) + \cos(\omega t - kx) = 2\cos \omega t \cos kx\), is a standing wave. In our case it will correspond to the "pseudo-standing" electromagnetic wave "propagating" parallel to the metal film in a fictitious medium with refractive index \(n = 2\). Such waves will not be radiated into the environment. The second term in equation (8') that is approximated by function \(E_0 \sin \omega t \sin 2kx/\sqrt{2.3}\) (see Figure 5), will also match "pseudo standing" wave "propagating" parallel to the film. Thus, only about a third of the energy transmitted by a gravitational wave to electron subsystem of the metal film to be transformed into the energy of the electromagnetic wave, propagating out of the film.

4. RESULTS

A plane light wave obtained in our antenna should be directed to the parabolic mirror of modern large optical telescope parallel to the optical axis of its main mirror and then to a photo detector of telescope, see Figure 6. Figure 7 shows the optical (radio) telescope before (upper panel a)) and after (bottom panel b)) of its conversion into a gravitational telescope. At the bottom panel b) the flat metal membrane (film) is shown, that is stretched perpendicular to the optical axis of the telescope. "Finishing touches" of the diffraction grating are shown on this membrane for clarity conditionally (not to scale). In reality these "touches" should be towards the surface of the telescope mirror. Gravitational wave
$A_g(x,t)$ (see formula (1')) incident on our "antenna" (diffraction grating) must excite in it a light wave with a power greater than the limit sensitivity of the photo detector of the optical telescope $P_{\text{min}}$.

![Image of gravitational telescope scheme](image)

Figure 6. Scheme of gravitational telescope.

We now define the lower limit for the amplitude of the scalar gravitational wave in source $A_{g0}$, which may be registered by means of the obtained gravitational telescope. We use the formula out of (11), which gives the time-averaged power of the electromagnetic energy emitted by the elementary Hertz dipole in all directions

$$P_1 = \frac{4}{3} \frac{\omega^4}{(3c^3)} = \left(\frac{2\pi}{\lambda}\right)^4 \left(\frac{p_0^2 c}{3}\right).$$

(10)

In our case only part of this power will be radiated outside the antenna. Firstly, it will be part attributable to half of the total solid angle $4\pi$ radian. Secondly, it will be one third of that half because of the incomplete conversion of the remaining part of the radiation into the plane wave with the wave surface that is parallel to the plane of the antenna. As a result, in all subsequent formulas for the radiated electromagnetic energy the factor $1/6$ will be needed to enter.

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In the available literature there are no direct data on the limit sensitivity of the optical telescope $P_{\text{min}}$. But we can estimate

$$A_{g0} = \frac{R_g}{eR_0} \sqrt{\frac{18c^3}{\lambda \lambda_n S}} P_{\text{min}}.$$

(12)
it by indirect data about extremely distant stars detected by modern large telescopes. To date\textsuperscript{14}, it is a "red giant" that is separated from the solar system by a distance of $R_{rg} = 400000$ light years. Suppose, according to\textsuperscript{15}, that the luminosity of such star is equal $\sim 10^2 L_\odot$, where $L_\odot = 2.8 \cdot 10^{26}$ Watt is luminosity of the Sun. Then the incident radiation from the "red giant" on the mirror of the telescope with diameter of $D_t = 10\text{ m}$ has power equal to

$$P_{\text{min}} = 10^2 L_\odot D_t^2 / (16 R_{rg}^2).$$

Substituting in this formula the values of all variables, we obtain for the limit sensitivity of the optical telescope the value: $P_{\text{min}} \approx 10^{-14}$ W or in the system CGS: $P_{\text{min}} \approx 10^{-7}$ ergs/sec. For reliable detection of gravitational wave the power of a plane light wave emitted by gravitational antenna on the mirror of the telescope should be greater than this value. Substituting this value $P_{\text{min}}$ into (12), we obtain that the value of the amplitude of the gravitational wave in the source is equal to $A_{g0} \approx 5 \cdot 10^{-20} \text{ cm/s}^2$. We now estimate the maximum value of the amplitude at the source, the allowable accordingly special relativity. To do this, we check does not exceed the peak value of the velocity of the body, which is the source of this wave, the speed of light. We assume that the gravitational wave is the result of a quasi-periodic process with a characteristic angular frequency $\omega = c \cdot 2\pi / \lambda$. Then, amplitude value of the velocity is expressed by the formula $v_{\text{ampl}} = A_{g0} / \omega$ and in this case it is equal to $v_{\text{ampl}} \approx 1.5 \cdot 10^5$ cm/sec at the speed of light in the CGS equal to $c \approx 3 \cdot 10^{10}$ cm/sec. Thus we have the limit value of the amplitude of the gravitational wave in the source equal to $A_{g0} \approx 10^{26}$ cm/s$^2$ and margin of safety allowed by the theory of special relativity in 5 additional decimal orders.

5. CONCLUSION

Suppose now that in the vicinity of the solar system, not exceeding 1 light year, there is a "black hole" of stellar mass. And let in such "black hole" the processes of accretion occur. Let us now state the question whether it is possible to register the gravitational waves emitted by such "black hole" using the proposed antennas? Let the mass of a black hole
is approximately equal to $2.5 \cdot M_{\text{sun}}$, then its Schwarzschild radius $R_g$ is equal to $\sim 5$ km. Consider the case of a gravitational antenna combined with an optical telescope. For this case, the minimum amplitude of the gravitational wave in a source, that is sufficient to detect the signal, is equal: $A_{g_5} \approx 4 \cdot 10^{72}$ cm$^2$/s$^2$, and the maximum value, based on the special theory of relativity, is equal: $A_{g_5, \text{lim}} \approx 0.6 \cdot 10^{76}$ cm$^2$/s$^2$. Thus we have the margin of safety about of 3 decimal orders of magnitude. Now imagine a situation when into the Solar system from the direction opposite to that where we see a star Proxima Centauri, at a distance of $\sim 1$ light year, with a speed of 500 km/sec that is possible for former kernel of supernovae, a wandering black hole is moving. In this case, there is a chance to detect it using the suggested tools and it remains about 600 years for humanity for thinking about what to do with this information. The following method can be proposed for manufacturing of the above-described diffraction grating with area of 25 m$^2$. Take a square polished silicon wafer with size 200x200 mm$^2$. Create the desired metal profile on it by photolithography and plasma-ion etching. And then to put together the 625 such plates on flat ground.

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