Hydromagnetic 3D Williamson slip flow of nanofluid over a slendering sheet with Cattaneo-Christov heat flux

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Abstract

A theoretical and computational study of a magnetohydrodynamic and three-dimensional flow over a variable thickness sheet (slendering sheet) with Cattaneo-Christov heat flux is presented. The Williamson slip is considered at the modified boundary conditions. The Williamson slip model is employed which is representative of certain industrial polymers. The non-dimensional, transformed boundary layer equations for momentum, energy and species diffusion are transformed with appropriate boundary conditions. The non-linear ordinary differential equations (ODEs) are solved using the Runge-Kutta-Fehlberg integration method. Validation of the numerical solutions is achieved via bench marking with earlier published work. The influence of Williamson slip suppresses the momentum boundary layers thickness and enhances the thermal solutal boundary layer thickness. Graphically studied thermal relaxation parameter, wall thickness parameter, porosity parameter, Brownian motion and thermophoresis parameters.

Key words: Magnetohydrodynamics, porosity, 3D, incompressible viscous flows,

AMS 2010 Classifications: 76Wxx, 76Sxx, 36T30, 76Dxx.

Nomenclature:

\( a \) : Thermal accommodation coefficient
\( A \) : Coefficient related to stretching sheet
\( A^* \) : Dimensional stretching sheet coefficient
\( b \) : Physical parameter related to stretching sheet
\( B(x) \) : Dimensional magnetic field parameter
\( C \) : Concentration of the fluid

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$C_f$: Skin friction coefficient

$C_p$: Specific heat capacity at constant pressure

$C_s$: Concentration susceptibility

$C_\infty$: Concentration of the fluid in the free stream

$d$: Concentration accommodation coefficient

$D_m$: Molecular diffusivity of the species concentration

$e$: Diffusion of organism’s accommodation coefficient

$f_i$: Maxwell’s reflection coefficient

$f^*, g^*$: Dimensionless velocities

$h_1^*, h_2^*, h_3^*, h_4^*$: Dimensional velocity slips parameter

$h_1^+, h_2^+, h_3^+, h_4^+*: Dimensional temperature jump parameter

$h_5^*: Dimensional concentration jump parameter

$h_6^*: Dimensional diffusion jump

h_7: Dimensionless velocity slips parameter

$h_8: Dimensionless temperature jump parameter

h_9: Dimensionless concentration jump parameter

$h_{10}: Dimensionless diffusion jump parameter

k$: Thermal conductivity

$k_T$: Thermal diffusion ratio

$k_r$: Chemical reaction parameter

$Le$: Lewis number

$m$: Velocity power index parameter

$M$: Magnetic field interaction parameter

$n$: Power-law index parameter

$Nb$: Brownian motion

$Nt$: Thermophoresis parameter

$Nu_x$: Local Nusselt number

$Pr$: Prandtl number

$Pe$: Peclet number

$Re_x$: Local Reynolds number
Sh_x : Local Sherwood number
T : Temperature of the fluid
T_m : Mean fluid temperature
T_∞ : Temperature of the fluid in the free stream parameter
u, v, w : Velocity components in x, y and z directions
x : Direction along the surface
y : Direction normal to the surface

Greek Symbols :
β : Casson fluid parameter
β_1 : Fluid parameter
η : Similarity variable
ϕ : Dimensionless concentration
χ : Density of motile organisms
σ : Electrical conductivity of the fluid
τ : Ratio of specific heats
γ : Thermal relaxation parameter
θ : Dimensionless temperature
ρ : Density of the fluid
μ : Dynamic viscosity
ν : Kinematic viscosity
δ : Wall thickness parameter
ξ : Mean free path (constant)
Γ : Positive characteristic time
Λ : Williamson fluid parameter

Introduction

Magnetohydrodynamic (MHD) has found ever-increasing applications in modern smart technologies. The application of magnetic fields has been shown to employ successfully the material characteristics of electro-conductive polymers which are finding new applications in naval industries, offshore and aerospace. Interesting studies in this regard addressing various systems engaging magnetic polymers include thin film fabrication processes and environmental engineering. Coating applications and energy systems enhancement with smart magnetic polymers have also grown substantially in current years. Relevant technologies in this regard are medical engineering, nuclear engineering and hydromagnetic energy generation. In the context of coating applications, it is critical to regulate heat transfer conditions which lead to improved bonding and homogeneity in engineered polymeric surfaces. Many studies have therefore studied the transport phenomena
from different geometrical configurations like pipes, cones, disks, truncated bodies and spheres. The spherical geometry is particularly pertinent to chemical engineering processes.

Many researchers have applied a variety of different material models for the coatings and also numerical methods to solve the associated boundary value problems. Bég et al. used the homotopy analysis method to examine the flow from a sphere in a porous medium. The magnetohydrodynamic (MHD) flow from a vertical semi-infinite plate with isotropic porous medium is computationally studied by Chamkha by using splitting finite difference method. Rossow examined the flow of electrically conducting fluid over a flat plate in the presence of a transverse magnetic field. The existence of electromagnetic hydrodynamic waves is reported by Alfvén. From these all researchers are studies were confined to Newtonian fluids. The unsteady free convection boundary layer flow, heat and mass transfer from a vertical plate with Newtonian heating effect examined by Raju et al. Prasad et al. discussed on isothermal sphere from Casson fluid flow with slip effects. They concluded that the behaviour of fluid on temperature and velocity profiles when thermal and velocity slips are considered. Gorla and Chamkha presented the numerical solutions for natural convection flow over a vertical surface in the presence of porous medium saturated by a nanofluid. These studies, however, did not consider William slip model. Williamson first reported the flow of pseudo plastic materials. Prasannakumara et al. studied the reactive-radiative flow of Williamson visco-elastic nanofluid from a stretching surface in a permeable material by using RKF (Runge-Kutta-Felhberg shooting algorithm). Khan and Khan portrayed the stretching, Blasius and Sakiadis and stagnation point flows of Williamson fluid using Homotopy analysis method (HAM). Abegunrin, Rao and Rao are studied the transport phenomena in Williamson fluid. Recently, Amanulla et al. investigated the numerical solution of thermal radiation and Biot number effects on the flow of a non-Newtonian MHD Williamson fluid over a vertical convective surface. Aziz et al. worked on magnetohydrodynamic flow of nanofluid over a porous non-linear stretching or shrinking sheet with effects of slip and convective conditions.

The Fourier’s law heat flux model can be adapted by adding the relaxation time for flux. It permits the transportation of heat over a propagation of thermal wave with finite velocity. Such types of heat flux model have stimulating real time practical applications such as controlling heating transport systems, solar plant systems and biomedical applications etc. Practically, the stretching sheet need not be flat. The variable thickness of sheet can be encountered more often in real world applications. Plates with variable thickness are often used in machine design, nuclear reactor technology, architecture, acoustical components and naval structures. The variable thickness is one of the significant properties in the analysis of vibration of orthotropic plates. Historically the concepts of variable thickness sheets originate through linearly deforming substance such as needles and nozzles. Crane is the first to introduce the boundary layer flow of viscous fluid through a stretching sheet. Due to this importance initially Cattaneo proposed a heat flux model. Later on, Christov modified the time derivative in Maxwell-Cattaneo model with material invariant formulation. This can be treated as Cattaneo-Christov heat flux model. The solutions and uniqueness of Cattaneo-Christov equation was proved by Ciarletta and Straughan. Hayat et al. analyzed the stagnation point flow with Cattaneo-Christov heat flux and homogeneous-heterogeneous reactions and concluded that the thermal relaxation parameter reduces the temperature field. Flow between two stretchable rotating disks with Cattaneo-Christov heat flux model reported by Hayat et al. He observed that the temperature distribution has decreasing the behavior for Prandtl number. On Cattaneo-Christov heat flux model for Carreau fluid flow over a slendering sheet investigated by Hashim et al. by employing effective shooting algorithm along with the Runge-Kutta Felhberg Scheme. Ramzan et al. presented the effects of magnetohydrodynamic (MHD) homogeneous-heterogeneous reactions on third grade fluid flow with Cattaneo-Christov heat flux model. It is observed that the homogeneous-
heterogeneous reactions depict conflicting the behavior on concentration fields. It is also noted that the increase values of thermal relaxation parameter decelerates the temperature distributions. Williamson fluid is characteristic of a non-Newtonian fluid model with shear thinning property. This model was proposed by Williamson. Gorla et al. addressed the stagnation point flow and heat transfer of a Williamson nanofluid on a linear stretching or shrinking sheet with convective boundary condition. He found that in shrinking sheet case the dual solutions exist, further he can observed that the thermal boundary layer thickness increases with Williamson parameter profiles. Gireesha et al. analyzed the radiating viscoelastic fluid flow and the heat transfer with convective boundary condition in non-uniform channel by using the perturbation method. He explained the effects of hall parameter and Biot number on velocity distribution. The expressions for various fluid flow parameters which satisfy the boundary conditions this implies an accuracy of the solution. Aziz illustrated that a laminar thermal boundary layer flow over a plate with convective boundary condition by using similarity transformation. A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Later on, Makinde reported the hydromagnetic fluid on heat and mass transfer over a vertical plate with a convective surface boundary condition by using a similarity solution. Alsaedi et al. studied that the stagnation point flow of nanofluid over a surface with boundary conditions with the effects of heat generation or absorption.

Most of the above mentioned studies are investigated the boundary layer flow and heat transfer analysis restricted for only flat stretching sheet. Study of flow and heat transfer of viscous fluids over stretching sheet with a variable thickness can be more relevant to the situation in practical applications. For the first time Fang et al. obtained an elegant analytical and numerical solution to the two-dimensional boundary layer flow due to a non-flatness stretching sheet. Further, this problem Subhashini et al. extended the work and including the energy equation and found that thermal boundary layer thicknesses for the first solution were thinner than those of the second solution. The hydromagnetic flow over a stretching sheet with variable thickness and variable surface temperature are analyzed by Anjali et al. The Cattaneo-Christov heat flux model for rotating flow and heat transfer of upper-convected Maxwell fluid was investigated by Mustafa and highlighted that thermal relaxation parameter improves the temperature field. Khader and Meghad studied numerical solution for the flow of a Newtonian fluid over an impermeable stretching sheet embedded in a porous medium with the power law surface velocity and variable thickness in the presence of thermal radiation. Numerical solution for the flow of a Newtonian fluid over a stretching sheet with a power law surface velocity, slip velocity and variable thickness was studied by Khader et al. Anjali Devi et al. reported the slip flow effects of hydromagnetic forced convective flow over a slendering stretching sheet.

The phenomenon of heat transfer happens if there is a difference in temperature between the bodies or between the components of the similar body. These phenomena have a wide range of technological and industrial use, for example, in cooling of atomic reactors, microelectronics, fuel cells, pasteurization of food, power generation, energy production etc. Haddad considered the thermal instability in the Brinkman porous medium by employing heat flux with Cattaneo-Christov expression. The heat flux through Cattaneo-Christov expression in order to explore heat transfer for flow of Maxwell material used by Mustafa. He provided numerical and analytic solutions of governing flow systems. Khan and Khan proposed the three-dimensional flow and heat transfer to burgers fluid using Cattaneo-Christov heat flux model. Jayachandra Babu and Sandeep discussed a three dimensional (3D) magnetohydrodynamic (MHD) slip flow of a nanofluid over a slendering stretching sheet with thermophoresis and Brownian motion effects. Later on many authors are reported the Williamson fluid flow over various geometries (Channel or cone or filled with various type of nanoparticles). Nadeem et al. analyzed the impact of Cattaneo-Christov heat flux model in flow of variable thermal conductivity fluid over a variable thicked surface.
In the present investigation, we considered Cattaneo-Christov heat flux on MHD three-dimensional flow over a Williamson slip flow over a variable thickness sheet with variable thickness sheet. For emerging the temperature and concentration fields and non-uniform heat source or sink also considered into account. The x-axis is considered as the sheet motion and the y-axis is perpendicular to it as depicted in Fig.1.

Assuming \( z = A \left( x + y + b \right)^{1-m} \), \( u_n(x) = U_0 \left( x + y + b \right)^m \), \( v_w = 0, m \neq 1 \), and external electric field as negligible.

![Fig.1 Flow configuration of the problem](image)

With these suppositions, the governing equations for continuity, momentum, thermal and diffusion equations Hayat et al.\(^{47}\); given as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  

(1)

\[
\left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \nu \frac{\partial^2 u}{\partial z^2} - \sqrt{2} \nu \frac{\partial^2 u}{\partial z \partial \xi} - \sigma B(x)^2 u - \frac{v}{k} u
\]  

(2)

\[
\left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \nu \frac{\partial^2 v}{\partial z^2} - \sqrt{2} \nu \frac{\partial^2 v}{\partial z \partial \xi} - \sigma B(x)^2 v - \frac{v}{k} u
\]  

(3)

\[
\left( \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) + \frac{\partial T}{\partial T} + \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \right) \frac{\partial T}{\partial x} + \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} \right) \frac{\partial T}{\partial y} + \left( \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} \right) \frac{\partial T}{\partial z} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} 
\]  

(4)

\[ \frac{\alpha}{\xi} \frac{\partial^2 T}{\partial \xi^2} + \frac{\partial T}{\partial \xi} \left[ D_h \frac{\partial T}{\partial \xi} + \frac{D_l}{T_c} (\frac{\partial T}{\partial \xi})^2 \right] - q_b (T - T_s) \]
\[
\begin{align*}
\frac{u}{\partial x} + \frac{v}{\partial y} + \frac{w}{\partial z} &= D_m \frac{\partial^2 C}{\partial z^2} + \frac{D_x}{T_x} \frac{\partial^2 T}{\partial z^2} - k_0 (C - C_\infty) \\
\end{align*}
\]

The corresponding boundary conditions are

\[
\begin{align*}
\begin{aligned}
&u(x, y, z) = U_w(x, y) + h_1^* \left( \frac{\partial u}{\partial z} \right), \\
&v(x, y, z) = V_w(x, y) + h_1^* \left( \frac{\partial v}{\partial z} \right), \\
&T(x, y, z) = T_w(x, y) + h_1^* \left( \frac{\partial T}{\partial z} \right), \\
&C(x, y, z) = C_w(x, y) + h_1^* \left( \frac{\partial C}{\partial z} \right), \\
\end{aligned}
\end{align*}
\]

and

\[
\begin{align*}
&u = 0, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty, \quad \text{at} \quad y = \infty
\end{align*}
\]

where

\[
\begin{align*}
h_1^* &= \left[ \frac{2 - f_1}{f_1} \right] \xi_1 (x + y + c)^{\frac{1-n}{2}}, \\
\xi_2 &= \left( \frac{2\gamma}{\gamma + 1} \right) \xi_1, \\
h_2^* &= \left[ \frac{2 - b}{b} \right] \xi_2 (x + y + c)^{\frac{1-n}{2}}, \\
\xi_3 &= \left( \frac{2\gamma}{\gamma + 1} \right) \xi_2, \\
h_3^* &= \left[ \frac{2 - d}{d} \right] \xi_3 (x + y + c)^{\frac{1-n}{2}}, \\
\xi_4 &= \left( \frac{2\gamma}{\gamma + 1} \right) \xi_3, \\
h_4^* &= \left[ \frac{2 - e}{e} \right] \xi_4 (x + y + c)^{\frac{1-n}{2}} B(x) = B_0 (x + y + c)^{\frac{n-1}{2}}, \\
U_w(x) &= a (x + y + c)^{\frac{n-1}{2}}, \quad V_w(x) = a (x + y + c)^n, \\
T_w(x) &= T_\infty + T_0 (x + y + c)^{\frac{1-n}{2}}, \quad C_w(x) = C_\infty + C_0 (x + y + c)^{\frac{1-n}{2}}
\end{align*}
\]

The irregular heat source/sink parameter \( q^m \) is described as

\[
q^m = \frac{k f_w U_w(x)}{(x + y + c)^{\frac{n-1}{2}}} \left( A^* (T_w - T_\infty) f' + B^* (T - T_w) \right)
\]

From the equation above, \( A^* > 0, B^* > 0 \) represents the internal heat generation while \( A^* < 0, B^* < 0 \) denotes the heat absorption coefficients respectively. Now we transforming the partial equations into ordinary differential equations we introduce the similarity transformations as

\[
\eta = z \sqrt{\frac{(n+1)a}{2v}} (x + y + c)^{\frac{n-1}{2}}, \quad \theta = \frac{T - T_\infty}{T_w(x) - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w(x) - C_\infty}
\]
\[ u = a (x + y + c)^n f'(\eta), v = a (x + y + c)^n g'(\eta) \]
\[ w = -\frac{2av}{n+1} (x + y + c) \left[ \frac{n+1}{2} (f + g) + \eta \left( \frac{n-1}{2} (f' + g') \right) \right] \]  \hspace{1cm} (13)

with the help of (7)-(13), equations (2)-(5) converted as the below differential equations:

\[ (1+Af^n) \frac{n+1}{2} f^{\prime\prime} + \frac{n+1}{2} (f + g) f^{\prime\prime} - nf^{\prime\prime^2} - nf'g' - (M + K) f' = 0 \]  \hspace{1cm} (14)

\[ (1+Ag^n) \frac{n+1}{2} g^{\prime\prime} + \frac{n+1}{2} (f + g) g^{\prime\prime} - ng^{\prime\prime^2} - nf'g' - (M + K) g' = 0 \]  \hspace{1cm} (15)

\[ \frac{n+1}{2} \theta^\prime - Pr \left[ \frac{1-n}{2} (f' + g') \phi - \frac{n+1}{2} (f + g) \phi' + K, \phi \right] + \frac{Nt}{Nb} \theta^n = 0 \]  \hspace{1cm} (16)

\[ \frac{n+1}{2} \phi' - Le \left[ \frac{1-n}{2} (f' + g') \phi - \frac{n+1}{2} (f + g) \phi' + K, \phi \right] + \frac{Nt}{Nb} \theta^n = 0 \]  \hspace{1cm} (17)

With the corresponding boundary conditions are

\[ f(0) = \delta \left( \frac{1-n}{n+1} \right) [1 + h_1 f^{\prime\prime}(0)], f'(0) = [1 + h_1 f^{\prime\prime}(0)], \]

\[ g(0) = \delta \left( \frac{1-n}{n+1} \right) [1 + h_1 g^{\prime\prime}(0)], g'(0) = [1 + h_1 g^{\prime\prime}(0)], \]

\[ \theta(0) = [1 + h_2 \theta'(0)], \phi(0) = [1 + h_3 \phi'(0)], \]

\[ f'(\infty) = 0, g'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0, \]  \hspace{1cm} (18)

The dimensional parameters are given by

\[ \Lambda = \Gamma U_0^{3/2} (x + y + b)^{3m-1/2} \sqrt{m+1/\nu}, M = \frac{\sigma B_n^2}{\rho a}, Pr = \frac{\mu C_p}{k}, Le = \frac{\nu}{D_b}, Kr = \frac{K_0}{\alpha p(m+1)}, Nt = \frac{\tau D_n T_0}{T_n k_f} \]

\[ \delta = A \sqrt{\frac{(n+1)a}{2 \nu}}, h_1 = \xi_1 \left( \frac{2-f_1}{f_1} \right) \frac{U_0 (m+1)}{2 \nu}, \gamma = \lambda U_0, Q_0 = \frac{q_0}{(m+1) \rho C_p}, Nt = \frac{\tau D_n C_0}{k_f}, \]

\[ h_2 = \xi_2 \left( \frac{2-f_1}{f_1} \right) \frac{U_0 (m+1)}{2 \nu}, h_3 = \xi_3 \left( \frac{2-f_1}{f_1} \right) \frac{U_0 (m+1)}{2 \nu}. \]  \hspace{1cm} (19)
The skin-friction coefficient \( C_f \), local Nusselt number \( Nu_x \) and local Sherwood number \( Sh_x \) are defined as

\[
C_f = \frac{\frac{\partial u}{\partial z}}{\frac{1}{2} \rho U_w^2} \quad \text{Nu}_x = \frac{(x + y + b) \frac{\partial T}{\partial z}}{T_w(x) - T_\infty} \quad \text{Sh}_x = \frac{(x + y + d) \frac{\partial C}{\partial z}}{C_w(x) - C_\infty}
\]

By using (11) the equation (18) becomes

\[
\begin{align*}
C_f \left( \text{Re}_x \right)^{0.5} &= 2 \sqrt{\frac{m+1}{2}} \left( 1 + \Lambda \right) f''(0), \\
\text{Nu}_x \left( \text{Re}_x \right)^{-0.5} &= -\sqrt{\frac{m+1}{2}} \theta'(0), \\
\text{Sh}_x \left( \text{Re}_x \right)^{-0.5} &= -\sqrt{\frac{m+1}{2}} \phi'(0)
\end{align*}
\]

(21)

where \( \text{Re}_x = \frac{U_w X}{\nu} \) and \( X = (x + b) \)

3. Results and Discussion

The set of non-linear ordinary differential equations (14)-(17) have been solved mathematically by utilizing Runge-Kutta-Fehlberg integration method alongside boundary conditions (18). The effects of the non-dimensional parameters such as \( Nb, Nt, \Lambda, \gamma, K_r, Q_\mu, M, \delta \) are shown in graphically.

Brownian motion of the nanoparticles is a significant parameter for revising the effect of nanoparticles on flow fields temperature and concentration profiles. Hence, Fig. 2 and 3 denotes the effect of Brownian motion \( Nb \) on temperature and concentration of the nanofluids. It is witnessed that the temperature of the nanofluid increases with the increase of \( Nb \). In nanofluid systems, due to the size of the nanoparticles, Brownian motion in the presence of magnetic field takes place which can affect the heat transfer properties. As the particle size scale approaches to the nano-meter scale, the particle Brownian motion and its effect on the surrounding liquids play an important role in heat transfer. But the contradictory trend is observed on concentration field. Figs. 4 and 5 display the effect of thermophoresis parameter \( Nt \) on temperature and concentration distributions. Increasing values of \( Nt \) improves the temperature and concentration profiles. We notice that, positive \( Nt \) indicates a cold surface while negative to a hot surface. For hot surfaces, thermophoresis tends to blow the nanoparticle volume fraction boundary layer away from the surface since a hot surface repels the sub-micron sized particles from it, thereby forming a relatively particle-free layer near the surface. In particular, the effect of increasing the
thermophoresis parameter $Nt$ is increases slightly the wall slope of the nanoparticle volume fraction profiles, but decreasing the nanoparticle volume fraction.

Figs. 6-9 illustrate the effect of $\Lambda$ on velocity, temperature and concentration fields. It evident velocity in both the direction is reduced and temperature as well as concentration field is improved. Generally, the rising values of $\Lambda$ generate higher pressure on the opposite to flow, so velocity fields are reduced, this can lead to encourages the temperature and concentration profiles of the flow. The variations of thermal relaxation parameter are plotted in Figs. 10 and 11. The thermal relaxation parameter depreciates the thermal boundary and improves the concentration boundary. This may happen due to dominance of slips influence in the flow. The rising values of chemical reaction on temperature and concentration is plotted in Figs. 12 and 13. The chemical reaction is depreciates the temperature and concentration fields. It is interesting to mention that the $K=0.5$ case has higher temperature and concentration fields compared to $K=0$ case. When increasing values of chemical reaction the interfacial mass transport phenomena is increase, this leads to reduction in concentration boundary.

Figs. 14 and 15 explains the effect of heat source/sink $Q_H$ on temperature and concentration field for both the $K=0$ and $K=0.5$ cases. It is evident that, increasing values of $Q_H$ improves the temperature and mixed performance was seen in concentration profiles. Figs.16-18 illustrates the velocity profiles $f'(\zeta)$ and $g'(\zeta), \theta(\zeta)$ for various values of $M$. The expanding values of $M$ denigrates the velocity profiles for both the cases. This magnetic field parameter progresses the resistive type drag force contradictory to the flow direction which declines the velocity field and enriches the temperature fields (See Fig.16-18). This is due to additional work exhausted in dragging the nanofluid in the boundary layer against the action of the Lorentz force. Figs.19-22 displays the influence of wall thickness parameter $\delta$ on velocities, temperature and concentration fields of both $K=0$ and $K=0.5$ cases. It is obvious that, growing of $\delta$ suppresses the velocities, temperature and concentration boundary layer. From this we can able to say that the slip have propensity to control the flow behavior. From Table 1, the present numerical results are validated with the solution of the Khader and Megahed and found worthy agreement with published paper.

Fig. 2 The effect of $Nt$ on temperature field

Fig. 3 The effect of $Nb$ on concentration field
Fig. 4 The effect of $Nt$ on temperature field

Fig. 5 The effect of $Nt$ on concentration field

Fig. 6 The effect of $\Lambda$ on velocity field

Fig. 7 The effect of $\Lambda$ on velocity field

Fig. 8 The effect of $\Lambda$ on temperature field

Fig. 9 The effect of $\Lambda$ on concentration field
Fig. 10 The effect of $\gamma$ on temperature field

Fig. 11 The effect of $\gamma$ on concentration field

Fig. 12 The effect of $K_r$ on temperature field

Fig. 13 The effect of $K_r$ on concentration field

Fig. 14 The effect of $Q_H$ on temperature field

Fig. 15 The effect of $Q_H$ on concentration field
Fig. 16 The effect of $M$ on velocity field

Fig. 17 The effect of $M$ on velocity field

Fig. 18 The effect of $M$ on temperature field

Fig. 19 The effect of $\delta$ on velocity field

Fig. 20 The effect of $\delta$ on velocity field

Fig. 21 The effect of $\delta$ on temperature field


3. Conclusions

In the present study, the analysis of the three-dimensional flow of Williamson slip flow over variable thickness sheet with Cattaneo-Christov heat flux has been performed. The influence of various flow parameters on the fluid velocity fields, temperature, concentration, surface shear stress, dimensionless heat and mass transfer rates are portrayed and illuminated with the help of graphs and tables. The transformed set of differential equations is numerically solved via Runge-Kutta-Fehlberg integration method.

1. The thermal relaxation parameter depreciates the thermal boundary and improves the concentration boundary layer.
2. The chemical reaction is depreciates the temperature and concentration fields.
3. The Williamson fluid parameter enhances in velocity friction and temperature distributions.
4. The thermophoresis and Brownian motion parameters decreases in heat transfer rate.

Competing Interests:

The authors declare that they have no competing interests.

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