Parameter Estimation of Multi-Frequency Hopping Signals Based on FastICA Algorithm

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Abstract. In order to solve the parameter estimation problem while multi-frequency hopping signal aliasing, this paper proposes a multi-frequency hopping parameter estimation algorithm based on FastICA (Fast Independent Component Analysis). Firstly, the FastICA algorithm is used to blindly separate the observed signals, and then using the short-time Fourier transform to extract the time-frequency ridges from separated signals. After that, wavelet transform is used to detect its singularities, and to estimate the period of each frequency-hopping signal. The simulation results show that the proposed algorithm can blindly separate multiple hopping signals and estimate their hopping periods, with fast convergence speed and strong robustness.

1. Introduction
Frequency hopping (FH) communication is widely used in modern military and civil communication systems because of its strong anti-multipath, anti-interference, anti-interception and easy networking capabilities [1]. In radio detection, it is often necessary to separate multiple unknown frequency hopping signals and estimate their parameters to intercept useful information. Since it is difficult to obtain a priori knowledge of unknown frequency hopping signals, the blind source separation (BBS) method is usually used to solve such problems, that is, the source signals are recovered by the observations of multiple observers. FastICA is an effective way to solve the BSS problem, and has the advantages of fast convergence and easy implementation [2].

2. Blind separation of multi-hop signals

2.1. Signal model
It is assumed that M frequency hopping signals are observed by N sensors, and the observation signals are independent. Each observed signal is a linear mixture of M independent source signals, the source signal is expressed as $S = [s_1, s_2, ..., s_M]^T$, then there is

$$X = AS$$

(1)

Where $A$ is the mixing matrix, FastICA is the method of estimating the source signal based on the observed signal if the source signal and the mixing matrix are both unknown, and,

$$\hat{S} = BX = BAS$$

(2)

Where, $B = A^+$, "$+$" means generalized inverse. It can be seen that the key problem of BSS is to find the separation matrix. Therefore, the problem is converted from the estimated source signal $S$ to the
solution of separation matrix B. Before solving the separation matrix B, firstly, the observation data X is subjected to the de-averaging operation and whitened, after that we can get the Z.

2.2. *FastICA algorithm based on negative entropy*

ICA is a method for fast estimation of separation matrix B based on the independence criterion between signals. At present, the FastICA algorithm based on negative entropy is widely used. Negative entropy is a function that can measure the non-Gaussian property of a random variable\(^3\). For any random variable \(y\), its information entropy is defined as:

\[
H(y) = -\int p(y) \log p(y) dy
\]

(3)

Where \(p(y)\) represents the probability density function of \(y\). Among the random variables with the same variance, the information entropy of the Gaussian distribution is the largest, the stronger the non-Gaussian, and the smaller the information entropy, the negative entropy is defined as:

\[
J(y) = H(y_{Gauss}) - H(y)
\]

(4)

Where \(J\) is the negative entropy and \(H(y)\) is the entropy of the random variable \(y\). From the central limit theorem, the non-Gaussian of the source signal is stronger than the non-Gaussian of the observed signal. When the non-Gaussian of the separation result is the largest, the best separation results are achieved.

In practical applications, it is usually simplified by the method of following formula,

\[
J(y) = \{E[F(y)] - E[F(v)]\}^2
\]

(5)

Where \(E\) is the mean value operation and \(F\) is the nonlinear function. This article selects \(F = y \cdot \exp(-\frac{y^2}{2})\).

Equation (5) solves the maximum value of \(J(y)\) as the maximum value of \(E[F(v)]\) under constraint conditions \(\|B\|^2 = 1\). Available by Lagrangian multiplier method, we can get following Equation,

\[
E\left\{\frac{\partial f}{\partial b^T z}\right\} - \theta b = 0
\]

(6)

In the formula, \(\theta\) is a constant, and \(f\) is the derivative of \(F\). Solving equation (6) by Newton iteration method, we can get

\[
b_{k+1} = b_k - \frac{E\left\{\frac{\partial f}{\partial (b_k^T z)}\right\} - \lambda b_k}{E\left\{\frac{\partial^2 f}{\partial (b_k^T z)^2}\right\} - \lambda}
\]

(7)

Further simplification and normalization to get the final iteration

\[
\begin{align*}
\{b_{k+1} = E\left\{f(b_k^T z)\right\} - E\left\{f \left( b_k^T z \right) \right\} b_k
\end{align*}
\]

\[
b_{k+1} = \frac{b_{k+1}}{\|b_{k+1}\|_2}
\]

(8)

3. Parameter estimation of frequency hopping signals

3.1. *Time-frequency analysis and ridge extraction*

Time-frequency analysis\(^5\) can realize parameter estimation of frequency hopping signals without prior knowledge, and is an effective tool for processing non-stationary random signals. Common time-frequency analysis includes short-time Fourier transform, Wigner-Ville distribution, and spectral et al. Different time-frequency distributions have different advantages. In this paper, the short-time Fourier transform with strong engineering practicability is selected as the tool for analyzing the frequency hopping signal. The short-time Fourier transform expression of the received signal is given below.
\[
STFT(t, f) = \int_{-\infty}^{\infty} s(\tau) w^*(t - \tau) e^{-j2\pi f \tau} d\tau
\]  
(9)

Where \( s(t) \) represents the source signal obtained after separation, \( w(t) \) represents a window function, \( \tau \) represents the amount of delay, its result represents a two-dimensional distribution of the time and frequency of the source signal. Suppose that the result produced by the short-time Fourier transform is a matrix in which the horizontal direction represents the time dimension and the vertical direction represents the normalized frequency dimension. Each row element of the matrix represents the signal generated by the Fourier transform in the time window.

For the short-time Fourier transform, the time-frequency ridge is the peak frequency at each moment in time-frequency distribution, it is as following

\[
f_r(t) = \text{arg}[\max_{f} X_r(t, f)]
\]  
(10)

The time-frequency ridges\(^6\) are always concentrated around the instantaneous frequency components of the frequency hopping signals, so the periodicity of the frequency hopping signals can be reflected, its results as Figure 1 showing,

![Time-frequency ridges](image)

**Figure 1 Time-frequency ridges**

\subsection*{3.2. Period estimation of frequency hopping signals}

It can be seen from Figure 1 that there is a jump between adjacent frequency components, thereby generating a certain step phenomenon similar to a square wave. The Haar wavelet has a good detection capability for such hopping, and can accurately estimate the parameters such as the hopping period of the signal. So, we can use the wavelet transform to get their period without any prior information. According to the time-frequency ridge, the formula for wavelet transform is

\[
W(a, t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f_r(\tau) \psi^*\left(\frac{\tau - t}{a}\right) d\tau
\]  
(11)

Where \( a \) denotes the scale parameter, \( f_r(\tau) \) denotes the extracted time-frequency ridge, \(^*\) denotes the conjugate, and \( \psi(t) \) denotes the parent function of the Haar wavelet, the expression is

\[
\psi(t) = \begin{cases} 
\frac{1}{\sqrt{a}} & -\frac{a}{2} \leq t < 0 \\
-\frac{1}{\sqrt{a}} & 0 \leq t < \frac{a}{2} \\
0 & \text{else}
\end{cases}
\]  
(12)

In the same period, \( |W(a, t)| \) is always zero. In different hop periods, the value of \( |W(a, t)| \) is not zero. When it is at the position where the frequency jumps, \( |W(a, t)| \) takes the maximum value.
4. Simulation analysis

Simulation Experiment 1: Considering two source signals s1 and s2 to be separated, and the frequency hopping frequency sets are [2.5, 12.5, 17.5, 20, 22.5, 30, 27.5, 25] KHz and [10, 7.5, 15, 5] KHz, the hopping period is 0.5ms and 1ms respectively, the sampling frequency is 1MHz, the sampling point is 4000, the two signals are mixed through the mixing matrix A, and the mixed signals are x1 and x2, the spectrum diagram is shown in Figure 2(a) and Figure 2(b), it is separated by the above algorithm, and the separated spectrum is shown in Figure 3(a) and 3(b). Computer generates a random matrix as mixed matrix A, where,

\[
A = \begin{bmatrix}
0.8285 & 0.2956 \\
0.6291 & 0.6170
\end{bmatrix}
\]

![Figure 2](image2.png)

**Figure 2** Spectrogram of mixed signal

It can be seen from the above Figure 2 that the observation signals x1 and x2 generated mixing of the source signals s1 and s2 through the mixing matrix A, which have amplitudes on their corresponding frequency components, and it is difficult to distinguish the s1 source and s2 source signal. So the received signal must be separated.

As can be seen from Figure 3(a) and 3(b), the FastICA algorithm preferably separates the spectrums of the two frequency hopping signals which are mixed together. However, the separated signals have some uncertainty in amplitude and order, which is acceptable in engineering applications.

![Figure 3](image3.png)

**Figure 3** Separated signal spectrogram

Simulation Experiment 2: Considering the short-time Fourier transform of the mixed signal and the separated signal in Experiment 1, and extracting the time-frequency ridge, using Haar wavelet to detect the mutation point and estimate the frequency hopping period of each signal source. The time-frequency diagram of the mixed signal is shown in Figure 4.
Figure 4 Mixed signal time-frequency diagram

It can be seen from the above figure, the two source signals are mixed together, and the period of the mixed signal is different from the period of the source signal, so the frequency hopping period of the source signal cannot be estimated. After separation by the FastICA algorithm, the separated signal is obtained, and Fourier transform is performed to obtain the results as shown in Figure 5a and Figure 5(b), as shown in the following figures.

Figure 5 Time-frequency diagram of the separated signal

It can be seen from the figures that the separated source signal can represent the frequency hopping period of the signal. After the wavelet transform detects the sudden change of the time-frequency ridge, the frequency hopping period of the source signals s1 and s2 can be well estimated.

Because FastICA is suitable for the noise-free separation method of mixed signals, when the signal-to-noise ratio is low, the algorithm cannot effectively separate the source signals, so that the parameters cannot be correctly estimated. When the relative error of the period estimation is less than 5%, it is a correct estimation. Table 1 shows the correct estimation times of the source signals s1 and s2 after 100 Monte Carlo experiments under different signal-to-noise ratios.

| Signal | SNR | 0 | 5 | 10 | 15 | 20 |
|--------|-----|---|---|----|----|----|
| s1     | 0   | 0 | 55| 66 | 86 | 90 |
| s2     | 0   | 0 | 46| 72 | 79 | 92 |

It can be seen from Table 1 that when the signal-to-noise ratio is lower than 0 dB, the algorithm cannot effectively estimate the frequency hopping period of the source signal. As the SNR increases, the correct number of hopping periods increases gradually. When the SNR is greater than 20 dB, the correct probability reaches more than 90% with high convergence speed.
5. Result
In this paper, the mixed signal is blindly separated by FastICA algorithm, as well as time-frequency analysis of the separated signals is performed by short-time Fourier transform, and used to estimate the frequency hopping periods. The simulation shows that when the signal-to-noise ratio is greater than 20dB, the correct estimation probability of the algorithm for the hopping period is over 90%.

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