Endpoint of the hot electroweak phase transition

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Abstract

We give the nonperturbative phase diagram of the four-dimensional hot electroweak phase transition. The Monte-Carlo analysis is done on lattices with different lattice spacings ($a$). A systematic extrapolation $a \to 0$ is done. Our results show that the finite temperature SU(2)-Higgs phase transition is of first order for Higgs-boson masses $m_H < 66.5 \pm 1.4$ GeV. At this endpoint the phase transition is of second order, whereas above it only a rapid cross-over can be seen. The full four-dimensional result agrees completely with that of the dimensional reduction approximation. This fact is of particular importance, because it indicates that the fermionic sector of the Standard Model can be included perturbatively. We obtain that the Higgs-boson endpoint mass in the Standard Model is $72.4 \pm 1.7$ GeV. Taking into account the LEP Higgs-boson mass lower bound excludes any electroweak phase transition in the Standard Model.

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The observed baryon asymmetry is finally determined at the electroweak phase transition (EWPT) [6]. The understanding of this asymmetry needs a quantitative description of this phase transition. Unfortunately, the perturbative approach breaks down for the physically allowed Higgs-boson masses (e.g. $m_H > 70$ GeV) [7]. In order to understand this nonperturbative phenomenon a systematically controllable technique is used, namely lattice Monte-Carlo (MC) simulations. Since merely the bosonic sector is responsible for the bad perturbative features (due to infrared problems) the simulations are done without the inclusion of fermions. The first results dedicated to this questions were obtained on four-dimensional lattices [8]. Soon after, simulations of the reduced model in three-dimensions were initiated, as another approach [9]. This technique contains two steps. The first is a perturbative reduction of the original four-dimensional model to a three-dimensional one by integrating out the heavy degrees of freedom. The second step is the nonperturbative analysis of the

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three-dimensional model on the lattice, which is less CPU-time consuming than the MC simulation in the four-dimensional model. The comparison of the results obtained by the two techniques is not only a useful cross-check on the perturbative reduction procedure but also a necessity. The reason for that is that the fermions, which behave as the heavy bosonic modes, must be included perturbatively, anyhow.

In the recent years exhaustive studies have been carried out both in the four-dimensional \[4\] and in the three-dimensional \[4\] sectors of the problem. These works determined several cosmologically important quantities such as the critical temperature \(T_c\), interface tension \(\sigma\) and latent heat \(\Delta c\).

Previous works show that the strength of the first order EWPT gets weaker as the mass of the Higgs-boson increases. Actually the line of the first order phase transitions, separating the symmetric and broken phases on the \(m_H - T_c\) plane has an endpoint, \(m_{H,c}\). There are several direct and indirect evidences for that. In four dimension at \(m_H \approx 80\) GeV the EWPT turned out to be extremely weak, even consistent with the no phase transition scenario on the 1.5-\(\sigma\) level \[7\]. Three-dimensional results show that for \(m_H > 95\) GeV no first order phase transition exists \[8\] and more specifically that the endpoint is \(m_{H,c} \approx 67\) GeV \[9\]. In this letter we present the analysis of the endpoint on four dimensional lattices. We study the thermodynamical limit of the first Lee-Yang zeros of the partition function \[10\]. In order to get rid of the finite lattice spacing effects a careful extrapolation to the continuum limit is performed. The endpoint value of the SU(2)-Higgs model is perturbatively transformed to the full Standard Model (SM).

We will study the four-dimensional SU(2)-Higgs lattice model on asymmetric lattices, i.e. lattices with different spacings in temporal \((a_t)\) and spatial \((a_s)\) directions. Equal lattice spacings are used in the three spatial directions \((a_i = a_s, i = 1, 2, 3)\) and another one in the temporal direction \((a_4 = a_t)\). The asymmetry of the lattice spacings is given by the asymmetry factor \(\xi = a_s/a_t\). The different lattice spacings can be ensured by different coupling strengths in the action for time-like and space-like directions. The action reads

\[
S[U, \varphi] = \beta_s \sum_{sp} \left( 1 - \frac{1}{2} \text{Tr} U_{pt} \right) + \beta_t \sum_{tp} \left( 1 - \frac{1}{2} \text{Tr} U_{pl} \right) + \sum_x \left\{ \frac{1}{2} \text{Tr} (\varphi_x ^+ \varphi_x) + \lambda \left[ \frac{1}{2} \text{Tr} (\varphi_x ^+ \varphi_x) - 1 \right]^2 - \kappa_s \sum_{\mu=1}^3 \text{Tr} (\varphi_{x+\mu} ^+ U_{x,\mu} \varphi_x) - \kappa_t \text{Tr} (\varphi_{x+4} ^+ U_{x,4} \varphi_x) \right\},
\]

where \(U_{x,\mu}\) denotes the SU(2) gauge link variable, \(U_{sp}\) and \(U_{tp}\) the path-ordered product of the four \(U_{x,\mu}\) around a space-space or space-time plaquette, respectively; \(\varphi_x\) stands for the Higgs field. It is useful to introduce the hopping parameter \(\kappa^2 = \kappa_s \kappa_t\) and \(\beta^2 = \beta_s \beta_t\). The anisotropies \(\gamma_\beta = \beta_t/\beta_s\) and \(\gamma_\kappa = \kappa_t/\kappa_s\) are functions of the asymmetry \(\xi\). These functions have been determined perturbatively \[11\] and non-perturbatively \[11\] demanding the restoration of the rotational symmetry in different channels. In this paper we use the asymmetry parameter \(\xi = 4.052\), which gives \(\gamma_\kappa = 4\) and \(\gamma_\beta = 3.919\). Details of the simulation techniques can be found in \[4\].

We have performed our simulations on finer and finer lattices, moving along the lines of constant physics (LCP). In our case there are three bare parameters \((\kappa, \beta, \lambda)\). The bare parameters are chosen in a way that the zero temperature renormalized gauge coupling \(g_R\) is held constant and the mass ratio for the Higgs- and W-bosons \(R_{HW} = m_H/m_W\) corresponds to the Higgs mass at the endpoint of first order phase transitions: \(R_{HW,c}\). These two conditions determine a LCP as a one-dimensional subspace in the original space of bare parameters. The position on the LCP gives the lattice spacing \(a\). As the lattice spacing decreases \(R_{HW,c} \rightarrow R_{HW,cont.}\). A schematic illustration is shown in Fig. 1. The LPC (solid line) defined by the endpoint represents the above idea. The short dashed lines give
Figure 1: Schematic view of the phase diagram. The solid line represents the LCP defined by the endpoint condition. The numbers on the line correspond to the temporal extension for which the endpoint is realized (the dashed lines show their projection to the $\kappa - \lambda$ plane). The dotted lines running into these numbered points correspond to first order phase transitions for $g_R^2 = \text{const.}$ but different $R_{HW}$-s. A LCP defined by a constant $R_{HW}$ value is shown by the long dashed line.

The increasing numbers on the LCP show the temporal extensions of the lattice, thus corresponding to smaller and smaller lattice spacings. The dotted lines represent phase transition points of theories with fixed renormalized $g^2$ and $L_t$ but different $R_{HW}$ values. Along the dotted lines one can observe first order phase transitions up to the LCP defined by the endpoint condition. Note, however, that this endpoint LCP is not the same as the LCP defined by the constant $R_{HW} = R_{HW,\text{cont.}}$ value (long dashed line). They merge for decreasing lattice spacings, but at larger $a$ the difference is the result of the ‘poor realization’ of Wilson’s RG transformations with only three terms and parameters in the action. It is worth mentioning that the SU(2)-Higgs model is trivial for small gauge couplings, therefore, the $a \to 0$ limit cannot be performed. Even the points on the endpoint LCP do not define continuum theories. The second order phase transitions on it merely reflect a finite temperature phenomenon, the corresponding zero temperature SU(2)-Higgs theory is still trivial.

Since our theory is a bosonic one we assumed that the finite size corrections are quadratic in the lattice spacings; therefore an $a^2$ fit has been performed for $R_{HW,c}$ in order to determine its continuum value.

The technical implementation of the above LCP idea has been done as follows. By fixing $\beta = 8.0$ in the simulations, we have observed that $g_R$ is essentially constant within our errors. For the small differences in $g_R$ we have performed perturbative corrections. We have carried out $T \neq 0$ simulations on $L_t = 2, 3, 4, 5$ lattices (for the finite temperature case one uses $L_t \ll L_x, L_y, L_z$), and tuned $\kappa$ to the transition point. This condition fixes the lattice spacings: $a_t = a_s/\xi = 1/(T_c L_t)$ in terms of the transition temperature $T_c$ in physical units. The third parameter $\lambda$, finally specifying the physical Higgs mass in lattice units, has been chosen in a way that the transition corresponds to the endpoint of the first order phase transition subspace.

In this paper $V = L_t \cdot L_s^3$ type four-dimensional lattices are used. For each $L_t$ we had 8 different lattices, each of them had approximately twice as large lattice-volume as the previous one. The smallest lattice was $V = 2 \cdot 5^3$ and the largest one was $V = 5 \cdot 50^3$. We collected quite a large statistics and the Ferrenberg-Swendsen reweighting [12] was used to obtain information in the vicinity of a simulation point.
The determination of the endpoint of the finite temperature EWPT, thus a characteristic feature of the phase diagram, is done by the use of the Lee-Yang zeros of the partition function $Z$. Near the first order phase transition point the partition function reads

$$Z = Z_s + Z_b \propto \exp(-Vf_s) + \exp(-Vf_b),$$

where the indices s(b) refer to the symmetric (broken) phase and $f$ stands for the free-energy densities. Near the phase transition point we also have

$$f_b = f_s + \alpha(\kappa - \kappa_c),$$

since the free-energy density is continuous. It follows that

$$Z = 2 \exp[-V(f_s + f_b)/2] \cosh[-V\alpha(\kappa - \kappa_c)]$$

which shows that for complex $\kappa$ $Z$ vanishes at

$$\text{Im}(\kappa) = \pi \cdot (n - 1/2)/(V\alpha)$$

for integer $n$. In case a first order phase transition is present, these Lee-Yang zeros move to the real axis as the volume goes to infinity. In case a phase transition is absent the Lee-Yang zeros stay away from the real $\kappa$ axis. Thus the way the Lee-Yang zeros move in this limit is a good indicator for the presence or absence of a first order phase transition $[13]$. Denoting $\kappa_0$ the lowest zero of $Z$, i.e. the position of the zero closest zero to the real axis, one expects in the vicinity of the endpoint the scaling law $\text{Im}(\kappa_0) = c_1(L_t, \lambda)V\nu + c_2(L_t, \lambda)$. In order to pin down the endpoint we are looking for a $\lambda$ value for which $c_2$ vanish. In practice we analytically continue $Z$ to complex values of $\kappa$ by reweighting the available data. Also small changes in $\lambda$ have been done by reweighting. As an example, the dependence of $c_2$ on $\lambda$ for $L_t = 3$ is shown in fig. 2. To determine the critical value of $\lambda$ i.e. the largest value, where $c_2$ vanish, we have performed fits linear in $\lambda$ to the nonnegative $c_2$ values.

Having determined the endpoint $\lambda_{\text{crit.}}(L_t)$ for each $L_t$ we calculate the $T = 0$ quantities $(R_{HW}, g_R^2)$ on $V = (32L_t) \cdot (8L_t) \cdot (6L_t)^3$ lattices, where $32L_t$ belongs to the temporal extension, and extrapolate to the continuum limit. All the $T = 0$ simulations were performed at $\lambda = 0.000178$ and an extrapolation to the $\lambda_{\text{crit.}}(L_t)$ has been made. The parameters and results of the simulations are collected in table 1, while table 2 shows the $R_{HW}$ values extrapolated to the $\lambda_{\text{crit.}}(L_t)$. Having established the correspondence between $\lambda_{\text{crit.}}(L_t)$ and $R_{HW}$, the $L_t$ dependence of the critical $R_{HW}$ is easily obtained. Fig. 3 shows the dependence of the endpoint $R_{HW}$ values on $1/L_t^2$. A linear extrapolation in $1/L_t^2$ yields the infinite volume (i.e. continuum limit) value of the endpoint $R_{HW}$. We obtain $66.5 \pm 1.4$ GeV, which is our final result.

Comparing our result to those of the 3d analyses $[9]$ one observes complete agreement. Since the error bars on the endpoint determinations are on the few percent level, the uncertainty of the
Dimensional reduction procedure is also in this range. This indicates that the analogous perturbative inclusion of the fermionic sector results also in few percent error on $M_H$.

Based on our published data [5, 11] and the results of this paper we are now able to draw the precise phase diagram of the SU(2)-Higgs model in the $(T_c/m_H - R_{HW})$ plane. This is shown in fig. 4. The continuous line – representing the phase-boundary – is a quadratic fit to the data points.

Finally, we determine what is the endpoint value in the full SM. Our nonperturbative analysis shows that the perturbative integration of the heavy modes is correct within our error bars. Therefore we use perturbation theory [14] to transform the SU(2)-Higgs model endpoint value to the full SM. We obtain $72.4 \pm 1.7$ GeV, where the error includes the measured error of $R_{HW,cont.}$, $g_R^2$ and the estimated uncertainty [15] due to the different definitions of the gauge couplings between this paper and [14]. The dominant error comes from the uncertainty on the position of the endpoint.

In conclusion, we have determined the endpoint of hot EWPT with the technique of Lee-Yang zeros from simulations in four-dimensional SU(2)-Higgs model. The phase diagram has been also presented. The phase transition is first order for Higgs masses less than $66.5 \pm 1.4$ GeV, while for larger Higgs masses only a rapid cross-over is expected. One of the most important results of the present letter is that integrating out the heavy modes perturbatively is precise as shown by a comparison to our nonperturbative results. Thus the above $66.5 \pm 1.4$ GeV value can be perturbatively transformed to the full SM. We obtain $72.4 \pm 1.7$ GeV for the endpoint Higgs mass. As pointed out above the

$$\begin{array}{|c|c|c|c|}
\hline
L_t & \lambda_{\text{crit.}} & \kappa_{\text{crit.}} & R_{HW,c} \\
\hline
2 & 0.0001773(14) & 0.1077292(2) & 0.932(10) \\
3 & 0.0001664(27) & 0.1069581(2) & 0.883(12) \\
4 & 0.0001590(44) & 0.1066316(3) & 0.856(8) \\
5 & 0.0001664(20) & 0.1064948(6) & 0.838(36) \\
\hline
\end{array}$$

Table 2: Critical $\lambda$ corresponding to the endpoint of phase transition as function of $L_t$ and the corresponding value of $R_{HW}$.

Figure 2: Dependence of $c_2$ on $\lambda$ for $L_t = 3$. 


Dependence of $R_{HW,c}$, i.e. $R_{HW}$ corresponding to the endpoint of first order phase transitions on $1/L^2$ and extrapolation to the infinite volume limit.

perturbative inclusion of the fermionic sector of the SM is also correct to a few percent error level.

The present experimental lower limit of the SM Higgs-boson mass is 89.8 GeV \cite{16}. Taking into account all errors (in particular those coming from integrating out the heavy fermionic modes), our endpoint value excludes the possibility of any EWPT in the SM. This also means that the SM baryogenesis in the early Universe is ruled out.

More details of this investigation will be published in a forthcoming publication \cite{17}.

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