Surface Waves and Forced Oscillations in QHE Planar Samples

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Abstract
Dispersion relations and polarizations for surface waves in infinite planar samples in the QHE regime are explicitly determined in the small wavevector limit in which the dielectric tensor can be considered local. The wavelength and frequency regions of applicability of the results extends to the infrared region for typical experimental conditions. Then, standard samples with millimetric sizes seem to be able to support such excitations. Forced oscillations are also determined which should be generated in the 2DEG by external electromagnetic sources. They show an almost frequency independent wavelength which decreases with the magnetic field. A qualitative model based in these solutions is also presented to describe a recently found new class of resonances appearing near the edge of a 2DEG in the QHE regime.

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1 Introduction

In spite of intense research activity to determine precisely the laws for the behavior of charge and current distributions in samples undergoing the QHE, there remains many interesting questions to be answered, and experimental results constantly add new aspects to the subject. See for example [1]-[2].

During early activity in the field, [3],[4],[5] static effective equations for the electric and magnetic fields were obtained. Already, at that time, there were predictions that the fields would be localized on the boundaries. Recently experiments have observed this phenomenon in small mesoscopic samples [6]. Also in [4] In addition, it was pointed out that the magnetic field should contribute to the charge density. This follows simply from charge conservation. These kinds of density distributions are essential ingredients of the Chern-Simons approach proposed early in Refs. [7], [8]. At that time, however, the relevance of the 3D character of the electromagnetic field was not appreciated. Because of this, the merits of the CS approach were not fully realized. To clarify the way in which the CS action is relevant for the QHE it was argued in [7] that discussions in [3]-[5] could also be considered as being described by the CS action. Simply, this action describes in the Euler equations for an electromagnetic field, the response of a medium showing linear Hall conductivity. In addition it was argued that the purely 2D Chern-Simons discussion, with its predictions of Meissner effect and a massive photon, could be realized in semiconductor superlattices. The case of time dependent fields was also considered in that work. More recently, the importance of the Chern-Simons action for the explanation of the electromagnetic response, also emerged from the side of the anyonic and composite fermion approaches for the QHE [10], [11], [12], [13]. Using the microscopic composite fermion approach effective electromagnetic equations were derived in [1], that resembled the previous results of [3]-[5]. This discussion in [1] supported the conclusions of the former approach [3], [4], [9] by again stressing the role CS (Hall) currents. These currents while concentrated
near the border are different from the purely edge currents that flow very much closer
to it. This fact does not invalidate the Buttiker picture because both currents are
closely linked by gauge invariance [13].

In the present work, we investigate the dynamical predictions of the effective Maxwell
equations obtained in [9] in connection with the existence of surface waves in planar
samples. Defining experimental criteria for the detection of these waves or other dy-
amical responses for the 2DEG could be helpful in determining the precise effective
electromagnetic equations for QHE samples.

The plan of the paper is as follows. In Section 2 the set of Maxwell equations given
in [9] are reviewed and a simple argument is given showing that the CS action is all
that is needed for a lagrangean description of a medium having charge conserving Hall
currents.

Section 3 is devoted to solving the equations for surface waves. The dispersion rela-
tions and wave polarizations are explicitly found. Only one of the solutions corresponds
to a proper surface wave. The dispersion relation for the range of wavevectors and fre-
quencies in which the equations were obtained is almost linear, with a propagation
velocity coinciding with the light velocity in the surrounding dielectric medium. For
wavevectors and frequencies in the above mentioned region, that is \( q r_o < 1, w < w_c \)
(\( r_o, w_c \) being the magnetic length and the cyclotronic frequency respectively), the stan-
dard size of present experimental samples (~1 mm) is very much greater than the in-
frared optical wavelength associated with the cyclotronic frequency at the usual values
of the magnetic field. Therefore, it seems possible that the infinite plane approxima-
tion considered here can be satisfied in real experimental situations. Another kind of
solution also follows. It can be interpreted as corresponding to forced oscillations of
the 2DEG due charge and current densities in the near regions. The dispersion relation
in this case shows a gap for the spatial wavevector \( q \) below which real solutions for
the frequency do not exist. For low frequencies, the wavevector turns out to be almost
independent of frequency and varies only with the magnetic field. For typical values of the field, the associated wavelength is of the order of 0.1 µm.

These properties of the forced oscillations motivated us to use these solution to construct a model for describing special resonances recently found in experiments [2]. In Section 4 the model is discussed. In these experiments [2] QHE samples were prepared having gate surfaces that covered a portion of the sample as well as a section of the edge. After imposing a positive gate voltage, a metallic region (which they called the ”Puddle”) was created inside the bulk area. Results show the appearance of well defined and equally spaced conductance resonances when the capacitance is measured as a function of increasing gate voltage. It was estimated that the Puddle region during these resonances was separated from the edge zone by an incompressible strip (IS) in the QHE regime having a width of the order of 0.1 µm. This quantity is of the same order as the wavelength of the above discussed forced oscillations. The varying gate voltage can also be imagined as controlling the width of the incompressible strip. In addition, the capacitance was measured at nonzero frequency by applying an AC voltage to the gate. The proposed model assumes that these resonances are related to the generation of forced oscillations by the AC electric fields at values of the gate voltage where the width of the IS equals an integer number of halves of the characteristic wavelength. After selecting fitting parameters chosen for a specific value of the magnetic field, the dependence of the resonance maxima on the magnetic field value matches qualitatively the experimental data. A closer investigation of the consistence of this model will be discussed elsewhere.

2 The equations: the CS action as implied by Hall conductivity & charge conservation

Maxwell’s equations for the 4-vector potential $a_\mu$ describing small time dependent electromagnetic perturbations of the constant magnetic field $B$ in a planar sample for
the QHE regime were obtained previously \[9\]. For our problem, these equations take the form,

\[
(\nabla^2 + \epsilon_d \partial_4^2)\epsilon_{\mu\nu} a_\nu + i\delta(x_3)\sigma_{\epsilon_{\mu\rho\sigma}} n_\alpha \partial_\sigma a_\nu + \delta(x_3)\epsilon (P_{\mu\nu}(u.\partial)^2 - u.\partial(u_\mu P_{\nu\alpha} + u_\nu P_{\mu\alpha})\partial_\alpha + u_\mu u_\nu P_{\alpha\beta} \partial_\alpha \partial_\beta) a_\nu = 0,
\]

where \(x_\mu \equiv (\vec{x}, x_4), x_4 = ix_\sigma = i ct, \epsilon_{\mu\nu}\) is a kind of dielectric tensor, \(P_{\mu\nu}\) is a projection operator tensor on the 2DEG plane, \(u_\mu\) is the 4-velocity of the same plane and \(n_\mu\), is a unit 4-vector being normal to the plane. These quantities take the explicit forms,

\[
\epsilon_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \epsilon_d \end{pmatrix}, \quad P_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},
\]

\[
u_\mu \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad n_\mu \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},
\]

where \(\epsilon_d\) is the dielectric constant of the medium where the 2DEG is embedded. For usual experimental situations, this quantity is a number of the order of ten. In reference \[9\], these equations were obtained for the vacuum case, that is \(\epsilon_d = 1\). However, here we are trying to describe more realistic situations that appear in mesoscopic systems. This requires inclusion of the dielectric properties of the surrounding medium for the 2DEG. These equations correspond to the following generalization of the Lorentz gauge for the 4-vector potential

\[
\partial_\mu \epsilon_{\mu\nu} a_\nu = \partial_i a_i + \epsilon_d \partial_4 a_4 = 0,
\]

which as usual does not fully fix. This allows us to impose in addition the axial gauge

\[
n_\mu a_\mu = a_3 = 0.
\]

Equations (1) were obtained by performing a one loop calculation of the dielectric tensor of the 2DEG placed in a magnetic field \(B\) for densities corresponding to the
complete filling of $l + 1$ Landau levels. Only a local approximation was considered in which long wavelength $r_{\alpha}|\vec{k}| \ll 1$ and low frequency $w \ll w_c$ conditions were assumed.

The terms in the second line of (1) represent Chern-Simons sources which are equivalent to the Hall conductivity current and density of the system. The third line of (1) is the 4-current associated with the dielectric properties of the electron gas in the presence of a magnetic field. The associated parameters $\sigma$ and $\epsilon$ are given by

$$\sigma = \frac{4(l + 1)e^2}{\hbar c}$$

$$\epsilon = \frac{4(l + 1)emc}{B\hbar}$$

Note that the dielectric polarizability $\epsilon$ is inversely proportional to the magnetic field. In (5), the mass parameter is assumed to take the typical value for the effective mass for electrons in the AsGa samples equal to $\sim 0.07$ of the electron mass. As defined above, $B$ is the external magnetic field which is of the order of 10 Tesla in current experimental conditions.

The static limit of equation (1) reproduce many of the features of the discussion of [3] and [5], but in addition includes a complementary magnetic field dependent charge density. This term was discussed for the first time [4] as arising from the requirement of charge conservation. In the context of our previous work [9], both approaches are contained within equations based on a Chern-Simons action that generates Hall currents as well as magnetic field dependent charges, and where the electromagnetic fields are considered in 3D space.

The set of equations (1) can be translated into a more physically appealing representation by expressing them in terms of the electric and magnetic field intensities

$$E_i = i(\partial_t a_4 - \partial_a a_i)$$

$$B_i = \epsilon^{ijk}\partial_j a_k$$

Using these relations and the gauge conditions (2) and (3) for to eliminate the $a_\mu$ components from the Maxwell equations (1), the equations can be written in the
following form

\[ \vec{\nabla}.(\epsilon_d \vec{E} + \delta(x_3) \epsilon \vec{P} \cdot \vec{E}) = \delta(x_3) \sigma \vec{n} \cdot \vec{B} \]  
(7)

\[ \vec{\nabla} \times \vec{B} = \delta(x_3) \sigma \vec{E} \times \vec{n} + \frac{1}{c} \frac{\partial}{\partial t}(\epsilon_d \vec{E} + \delta(x_3) \epsilon \vec{P} \cdot \vec{E}) \]  
(8)

\[ \vec{\nabla} \times \vec{E} + \frac{1}{c} \frac{\partial}{\partial t} \vec{B} = 0 \]  
(9)

\[ \vec{\nabla} \cdot \vec{B} = 0 \]  
(10)

where \( \vec{P} \) is the projector on the 2DEG diadic 3D-tensor and \( \vec{n} \) is a unit vector normal to the plane.

Equations (9) and (10) are analogues of the vacuum equations for Faraday induction and the absence of magnetic charge. In addition in eqs. (7) and (8), the \( \epsilon \) dependent terms show that the electron gas behaves as a dielectric surface which polarizes itself linearly only due to the tangential components of the electric field. The Hall currents appear now explicitly in the r.h.s. of (8). The only unusual contribution is the magnetic field dependent charge density appearing in the r.h.s. of (7). However, as mentioned above, as discussed in [4] this term arises from the general condition of charge conservation. Therefore, its presence should not be considered as an unusual outcome. Rather, such charge densities are determined by assuming the existence of a Hall conductivity in a planar medium. This fact will be discussed below.

In the set of equations (7)-(10), the magnetic field charge density is substituted by an undefined function \( \rho^{\text{Hall}} \). All the other terms remain unaltered. After taking the divergence of equation (8), and adding the result to equation (7) the following conservation conditions follow for the polarization and for the Hall currents and densities

\[ \frac{\partial}{\partial t}(\rho^{\text{Hall}} + \rho^{\text{Pol}}) + \vec{\nabla} \cdot (\vec{J}^{\text{Hall}} + \vec{J}^{\text{Pol}}) = 0, \]

\[ \frac{\partial}{\partial t} \rho^{\text{Hall}} + \vec{\nabla} \cdot \vec{J}^{\text{Hall}} = 0. \]  
(11)

After substituting in (11) the expression for the Hall currents, it follows that

\[ \frac{\partial}{\partial t} \rho^{\text{Hall}} = -\delta(x_3) \vec{\nabla} \cdot (\vec{E} \times \vec{n}) \]
This relation shows that the unknown quantity $\rho^{Hall}$ should differ by a time independent function from the magnetic field dependent charge density that appears in eq. (7). Then, after assuming that before any perturbations (by, for example, incoming waves) the charge densities vanished, it follows that the magnetic field dependent surface charge densities (and then the whole Chern-Simons structure of the Hall current 4-vector) is implied by the existence of a Hall conductivity.

These are the basic equations to be used to investigate the spectrum of surface waves and forced oscillations for the 2DEG in the integral QHE regime.

3 Surface waves and resonances

We look for solutions of the Maxwell equations (1) that have harmonic behavior corresponding to the propagation of energy along the plane. To be specific, we consider the propagation along the direction $x_2$ of the coordinate axes. Oscillatory solutions showing an exponential decay away from the $x_3$ axis will correspond to surface waves and ones that grow outside the plane should describe forced oscillations driven by currents and charges at some boundary.

The vector potential will take the form

$$a_\mu(x) = a_\mu(k) \exp[i(qx_2 + k_3|x_3| - \sqrt{\epsilon_d}w t)],$$

where the wavevector component along the $x_2$ axis will be designed as

$$k_2 = q,$$

and all the wavevector components $q, k_3$ and $k_4 = ik_0 = iw/c$ are chosen to satisfy the wave equation in the surrounding dielectric medium. That is, they obey

$$q^2 + k_3^2 - \epsilon_d w^2 / c^2 = 0.$$
Relation (14) allows us to write an expression for the wavevector component \( k_3 \), which controls the behavior of the fields outside the plane,

\[
\begin{align*}
   k_3 &= (-q^2 + \epsilon_d w^2/c^2)^{1/2} \\
      &= f i (q^2 - \epsilon_d w^2/c^2)^{1/2},
\end{align*}
\]

(15)

where the positive value for the squared root is taken when its argument is positive. Then, the factor \( f \) can have two values

\[ f = \pm 1. \]

For \( f = 1 \) or \( f = -1 \), the wave amplitude decreases or grows when away from the surface.

Determination of the solution is simplified by considering the first term in the Maxwell equation (1) after substituting the expression (13) for \( a_\mu \). As \( a_\mu \) satisfies the wave equation in a dielectric medium, the result reduces to a surface contribution on the following form

\[
(\nabla^2 + \epsilon_d \partial_4^2)\epsilon_{\mu\nu} a_{\nu} = 2 i k_3(\delta(x_3)\epsilon_{\mu\nu} a_{\nu}).
\]

(16)

Therefore, the set (1) becomes a matrix equation over the 2DEG plane for the nonvanishing polarization components \( a_\mu, \mu = 1, 2, 4 \). It can be written in the form

\[
\begin{pmatrix}
   -2f X + \epsilon k_o^2 \\
   -i \sigma k_o \\
   \sigma q \\
   \sigma q
\end{pmatrix} \begin{pmatrix}
   i \sigma k_o \\
   -2f X + \epsilon k_o^2 \\
   i k_o q \\
   i k_o q
\end{pmatrix} \begin{pmatrix}
   a_1 \\
   a_2 \\
   a_3 \\
   a_4
\end{pmatrix} = 0,
\]

(17)

where \( X \) is

\[
X = (q^2 - \epsilon_d w^2/c^2)^{1/2}.
\]

(18)

After imposing the condition that the determinant of the matrix should vanish, the following dispersion relations arise

\[
f \sqrt{(\epsilon q)^2 - \epsilon_d (\epsilon k_o)^2} = -(\epsilon_d - (\epsilon k_o)^2/4 + \sigma^2/4)
\]

\[
\pm \sqrt{(\epsilon_d - (\epsilon k_o)^2 + \sigma^2)^2 + \epsilon_d (\epsilon k_o)^2}.
\]

(19)
From these, it can be seen that the cases $f = 1$ and $f = -1$ correspond to the positive and negative values of the squared root respectively. Then, two possibilities appear: surface wave propagation given by $f=1$ and forced oscillations given by $f=-1$.

The vector potential polarization vectors can be written in the following way

$$a_\mu = \begin{pmatrix} \frac{1}{2}[-2f(q^2 - \epsilon_d k_o^2)^{1/2} + \epsilon k_o^2 - \epsilon q^2/\epsilon_d] \\ k_o \\ 0 \\ -q/(i \epsilon_d) \end{pmatrix}. \quad (20)$$

Below, physical conditions for the validity of these solutions will be discussed in each of the cases: the surface waves and the forced oscillations

### 3.1 Surface waves

The set of Maxwell equations (1) was obtained using the local approximation where the wavevector of the perturbations $a_\mu$ was assumed to satisfy $r_o q \ll 1$ and $\omega \ll w_c$. The magnetic radius and the cyclotronic frequency have the usual expressions $r_o = \sqrt{\hbar c/eB}$ and $w_c = eB/mc$.

Therefore, even though the dispersion relation is nonlinear, in the region of validity for the approximations, the dependence is nearly linear in behavior. The velocity of the waves in this low momentum zone coincides with the light velocity in the surrounding dielectric medium. The general form of the dispersion curve is shown in Fig. 1 by the branch crossing the origin. The whole curve gives the surface wave dispersion for a classical problem characterized by planar Hall conductivity and dielectric responses. However, application to the case of a realistic 2DEG in the QHE regime reduces to a narrow region where $\epsilon k_o \ll 0.029$ for $B = 6 T$ and $\epsilon_d = 10$.

Let us assume that the wavevectors obey these conditions, and discuss the connection with the experimental situation.

The limitations imposed by $r_o q \ll 1$ are not very strong because the normal values for $r_o$ in present experiments are of the order of $100 \cdot A^c$. Thus, up to optical wavelengths this approximation is well satisfied.
The more restrictive bound comes from the frequency relation \( w \ll w_c \). In this case, after considering typical values for the magnetic field found in experiments \( B \approx 6 \, T \), the cyclotronic frequency is of the order of \( w_c = 1.5 \times 10^{13} \). Hence, the dispersion relations are expected to hold up to the infrared region.

In connection with the possibilities for detection of such waves under present laboratory conditions, it should be noted that the above bounds on the frequency are related to wavelengths \( \lambda \approx 0.0039 \, cm \) for \( \epsilon_d = 10 \). These values are much smaller than the normal sample sizes which can presently be prepared. Therefore, in this sense, the present analysis favors the possibility of the generation and detection of such waves. Clearly, the stability of the present picture under boundary effects present in real mesoscopic samples should be considered in more detail.

### 3.2 Forced oscillations

The case \( f=-1 \) corresponds to oscillations which grow in the \( x_3 \) directions away from the 2DEG plane. Let us consider two planar surfaces parallel to the \( x_3 \) plane and at the same distances, say \( h \) from it. Consider also, choosing the solutions outside these planes to be equal to the corresponding ones inside the planes but substituting \( k_3 \to -k_3 \). The special surface current and charge density distributions on these planes that allow such a global solutions, can be calculated by applying the wave equation. This picture allows to understand the forced nature of the oscillations. It also suggests that neighboring structures in mesoscopic systems can act as generators which force this resonant behavior.

The dispersion curve associated with forced oscillations is illustrated in Fig. 1 by the curve not intersecting the origin of the coordinate axes. The result shows the existence of a critical value for the spatial momentum \( q \) below which real solutions for frequency are absent. In fact, as in the surface wave case, for the 2DEG experimental samples, the curve are only valid for frequencies smaller than \( w_c \). In this region, the wavelength of the waves are almost frequency independent. In terms of the dimensionless units
used in Fig.1, this region corresponds to $\epsilon k_o \ll 0.029$.

The critical wavelength for the forced oscillations is inversely depending on the magnetic field $B$. Its value for normal experimental conditions $B = 6 T$, $\epsilon_d = 10$, and $m = 0.07m_e$ is as small as $\lambda = 0.182\mu m$. The properties for such solutions led us to suspect that they are relevant for describing recent experiments ([2]) in which special tunneling resonances were reported.

In the next section, a model for the explanation of these resonances based on these forced oscillations will be proposed.

4 The model for conductance resonances

In reference [2], the authors reported experiments done on QHE samples in which a gate surface covered part of the bulk as well as part of the edge. After applying a positive gate voltage, a metallic region (the ”Puddle”) was created inside the bulk. It was composed of electrons excited to the partially filled next Landau level by the attracting gate voltage. The capacitance between the gate and the edge contacts was measured as a function of the applied gate voltage for a range of values of the frequency and the magnetic field. The results show the appearance of equally spaced resonances when the gate voltage was increased. It has been estimated that the Puddle region should be separated from the edge by an incompressible strip (IS) in the QHE regime having a width of the order of $0.1\mu m$. A reasonable phenomenological model used by the authors indicates that these resonances are associated with resonant tunneling currents passing across the IS.

Below, we give arguments suggesting that the forced oscillations discussed in last section are candidates for producing these resonances.

The reasoning goes as follows. As noticed before, the low frequency spectrum of forced oscillations is characterized by a constant wavelength which is given by

$$
\lambda = \frac{\pi \epsilon}{\epsilon_d + \sigma^2 / 4}
$$
\[
\frac{4\pi(l + 1)en}{\hbar B(\epsilon_d + \sigma^2/4)}. \tag{21}
\]

In experiments the IS is subject to a time varying voltage which is applied to measure the capacitance. The spatially inhomogeneous form of this potential can be considered as the source for the external charges which necessary for the forced oscillations to be excited.

We consider now the following assumptions:

1). The IS only resonates when its length \( L \) coincides with an integer number of half wavelength of the forced wave.

2) The odd symmetry of the applied potential with respect to the center of the plate restricts the the resonating condition requiring the length \( L \) to correspond to an odd number of halfs wavelength.

3) Increasing the gate voltages reduces the width \( L \) of the IS, with the various resonances being produced according to the previous rules.

The main elements considered within the model are illustrated in Fig. 2.

Under the above suppositions, we can write the condition which links the length of the strip with the magnetic field as follows

\[
\frac{2\pi L}{\lambda} = (2n + 1)\pi. \tag{22}
\]

However, the experimental data are given in terms of the dependence of the resonances on the gate voltage \( U_g \). Thus, in order to compare the consequences of the model with experiment, it is convenient to estimate the functional link between the length \( L \) of the IS and the gate voltage \( U_g \), by fitting some experimental data. For this purpose, we assume a linear relationship between \( L \) and \( U_g \) as

\[
L = \alpha(U_o - U_g), \tag{23}
\]

where \( U_o \) will also be fixed by estimating the voltage \( U_g \) at which the Puddle joints with the edge.
Taking the voltage shift between resonances as equal to $0.040 \text{ mV}$ [2], the parameter $\alpha$ is given by

$$\alpha = \frac{\lambda}{0.040}. \quad (24)$$

Restricting to the first Landau level $l = 0$, the voltage $U_o$ can be evaluated from Fig. 2 in Ref. [2] to be approximately $U_o \approx 350 \text{ mV}$

Next, we defined a function of the variable $x = L - (n + 1/2)\lambda$ to be a sum of six gaussian functions having maxima at each integral value of $x = 0, 1, .., 5$. The variable $x$ is expressed in terms of the magnetic field $B$ and the gate voltage $U_g$ by using (22) and (23) in order to investigate the behavior of this function of $U_g$ as the magnetic field is varied. The result is shown in Fig. 3 and reproduces qualitatively the data of Fig. 2 in [2]. The most interesting fact is that the slopes for the dependence of the maxima on the magnetic field match. The fitting parameters taken from the experiment were the voltage separation between resonances at the value of $B = 6 \text{ T}$ and the voltage $U_o \approx 350 \text{ mV}$ for the disappearance of the incompressible strip at the same field value. The description of the slopes is a prediction of the model which introduces a dependence of the wavelength on the magnetic field that qualitatively agrees with the experiment.

It can also be added that the proposed approach naturally explains the global character of the resonances which appear for various sizes of the gate electrode. In future work, a derivation of the main assumptions adopted above will be studied.

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References

[1] N. Nagaosa and M. Kohmoto, preprint cond-mat/9505134 (1995)

[2] N. Zhitenev, M. Brodski and R.C. Ashoori, preprint cond-mat 9601157 (1996)

[3] A.C. Mac Donald, T.M. Rice and W. Brinkman, Phys. Rev. B28, 3648 (1983)

[4] J. Riess, Phys. Rev. B31, 8265 (1985)

[5] D.J. Thouless, J. Phys. C18, 6211 (1985)

[6] P.F. Fontein, J.A.Kleine, P. Hendriks, F.A.P.Blom, J.H. Wolter, H.G.M. Lochs, F.A.J.M. Driessen, L.J. Giling and C.W.J. Beenaker, Phys. Rev. B43, 12090 (1991)

[7] Y. N. Srivastava and A.W. Widom, Lett. Nuovo Cimento 39, 285 (1984)

[8] K. Ishikawa, Phys. Rev. Lett. 53, 1615 (1984)

[9] A. Cabo and D. Oliva, Phys. Lett. A146, 75 (1990)

[10] S.M. Girvin in: The Quantum Hall Effect, Eds. R. Prange and S. Girvin (Springer, Berlin 1987)

[11] S.C. Zhang, T.H. Hansson and S. Kivelson, Phys. Rev. Lett. 62, 82 (1989)

[12] A. Lopez and E. Fradkin, Phys. Rev. B44, 5246 (1991)

[13] X.G. Wen, Phys. Rev. Lett. 64, 2206 (1990)
Figure Captions

- **Fig. 1**
  The surface wave (starting at the origin) and forced oscillations dispersion relations plotted in terms of the dimensionless wavevector components $\epsilon q$ and $\epsilon k_o$. The parameters chosen were: $B = 6 \, T, \epsilon_d = 10, m = 0.07 m_e$. Only in a small frequency region $\epsilon k_o < 0.029$ do the dispersion relations satisfy the conditions imposed by the local approximation for the dielectric response.

- **Fig. 2**
  A picture of the main elements of the model for conductance resonances.

- **Fig. 3**
  The graphical representation for the dependence on the gate voltage and the external magnetic field of the functions representing the resonance maxima predicted by the model. The fitting parameters were the voltage difference between the maxima at $B = 6 \, T$ and the voltage $U_o$ at which the incompressible strip seems to disappear at the same magnetic field value of $6 \, T$. 
