Asynchronous Execution of Python Code on Task-Based Runtime Systems

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Abstract—Despite advancements in the areas of parallel and distributed computing, the complexity of programming on High Performance Computing (HPC) resources has deterred many domain experts, especially in the areas of machine learning and artificial intelligence (AI), from utilizing performance benefits of such systems. Researchers and scientists favor high-productivity languages to avoid the inconvenience of programming in low-level languages and costs of acquiring the necessary skills required for programming at this level. In recent years, Python, with the support of linear algebra libraries like NumPy, has gained popularity despite facing limitations which prevent this code from distributed runs. Here we present a solution which maintains both high level programming abstractions as well as parallel and distributed efficiency. Phylanx, is an asynchronous array processing toolkit which transforms Python and NumPy operations into code which can be executed in parallel on HPC resources by mapping Python and NumPy functions and variables into a dependency tree executed by HPX, a general purpose, parallel, task-based runtime system written in C++. Phylanx additionally provides introspection and visualization capabilities for debugging and performance analysis. We have tested the foundations of our approach by comparing our implementation of widely used machine learning algorithms to accepted NumPy standards.

Index Terms—Array computing, Asynchronous, High Performance Computing, HPX, Python, Runtime systems

I. INTRODUCTION

The ever-increasing size of data sets in recent years have given rise to the term “big data.” The field of big data includes applications that utilize data sets so large that traditional means of processing cannot handle them [1],[2]. The tools that operate on such data sets are often termed as big data platforms. Some prominent examples are Spark, Hadoop, Theano and Tensorflow [3],[4].

One field which benefits form big data technology is Machine learning. Machine learning techniques are used to extract useful data from these large data sets [5],[6]. Theano [7] and Tensorflow [8] are two prominent frameworks that support machine learning as well as deep learning [9] technology. Both frameworks provide a Python interface, that has become the lingua franca for machine learning experts. This is due, in part, to the elegant math-like syntax of Python that has been popular with domain scientists. Furthermore, the existence of frameworks and libraries catering to machine learning in Python such as NumPy, SciPy and Scikit-Learn have made Python the de facto standard for machine learning.

While these solutions work well with mid-sized data sets, larger data sets still pose a big challenge to the field. Phylanx tackles this issue by providing a framework that can execute arbitrary Python code in a distributed setting using an asynchronous many-task runtime system. Phylanx is based on the open source C++ library for parallelism and concurrency (HPX [10],[11]).

This paper introduces the architecture of Phylanx and demonstrates how this solution enables code expressed in Python to run in an HPC environment with minimal changes. While Phylanx provides general distributed array functionalities that are applicable beyond the field of machine learning, the examples in this paper focus on machine learning applications, the main target of our research.

This paper makes the following contributions:

- Describe the futurization technique used to decouple the logical dependencies of the execution tree from its execution.
- Illustrate the software architecture of Phylanx.
- Demonstrate the tooling support which visualizes Phylanx’s performance data to easily find bottlenecks and enhance performance.
- Present initial performance results of the method.

We will describe the background in Section III Phylanx’s architecture in Section IV study the performance of several machine learning algorithms in Section V discuss related work in Section II and present conclusions in Section VI.
II. RELATED WORK

Because of the popularity of Python, there have been many efforts to improve the performance of this language. Some specialized their solutions to machine learning while others provide wider range of support for numerical computations in general. NumPy [12] provides excellent support for numerical computations on CPUs within a single node. Theano [13] provides a syntax similar to NumPy, however, it supports multiple architectures as the backend. Theano uses a symbolic representation to enable a range of optimizations through its compiler. PyTorch [14] makes heavy use of GPUs for high performance execution of deep learning algorithms. Numba [15] is a jit compiler that speeds up Python code by using decorators. It makes use of LLVM compiler to compile and optimize the decorated parts of the Python code. Numba relies on other libraries, like Dask [16] to support distributed computation. Dask is a distributed parallel computation library implemented purely in Python with support for both local and distributed executions of the Python code. Dask works tightly with NumPy and Pandas [17] data objects. The main limitation of Dask is that its scheduler has a per task overhead in the range of few hundred microseconds, which limits its scaling beyond a few thousand of cores. Google’s Tensorflow [8] is a symbolic math library with support for parallel and distributed execution on many architectures and provides many optimizations for operations widely used in machine learning. Tensorflow is a library for dataflow programming which is a programming paradigm not natively supported by Python and, therefore, not widely used.

III. TECHNOLOGIES UTILIZED TO IMPLEMENT PHYLAX

HPX [10], [11] is an asynchronous many-task runtime system capable of running scientific applications both on a single process as well as in a distributed setting on thousands of nodes. HPX achieves a high degree of parallelism via lightweight tasks called HPX threads. These threads are scheduled on top of the Operating System threads via the HPX scheduler, which implements an $M : N$ thread scheduling system. HPX threads can also be executed remotely via a form of active threads. We briefly introduce the technique of futurization, which is utilized within Phylanx. For more details we refer to [11].

The concept of futurization [21] is illustrated in Listing 1. The function in Line 2 is intended to be executed in parallel on one of the lightweight HPX threads. Line 3 shows the usage of the asynchronous return type hpx::future<int>, the so-called Future, of the asynchronous function call hpx::async. Note that hpx::async returns the future immediately even though the computation within convert may not have started yet. In Line 6 the result of the future is accessed via its member function .get(). Listing 1 is just a simple usecase of futurization which does not handle synchronization very efficiently. Consider the call to .get(), if the Future has not become "ready" .get() will cause the current thread to suspend. Each suspension will incur a context switch from the current thread which adds overhead to the execution time. It is very important to avoid these unnecessary suspensions for maximum efficiency.

Fortunately, HPX provides barriers for the synchronization of dependencies. These include: hpx::wait_any, hpx::wait_any, and hpx::wait_all(). These barriers provide the user with a means to wait until a future is ready before attempting to retrieve its value. In HPX we have combined the hpx::wait_all().then() facility and provided the user with the hpx::dataflow API [21] demonstrated in Listing 2.

```cpp
template <typename Func>
future<int> traverse(node & n, Func && f)
{
  // traversal of left and right sub-tree
  future<int> left =
    n.left ? traverse(*n.left, f)
      : make_ready_future(0);
  future<int> right =
    n.right ? traverse(*n.right, f)
      : make_ready_future(0);

  // return overall result for current node
  return dataflow(
    [&n, &f](future<int> l, future<int> r)
      -> int
    {
      // calling .get() does not suspend
      return f(n) + l.get() + r.get();
    },
    std::move(left), std::move(right)
    );
}
```

Listing 2. Example for the concept of hpx::dataflow for the transverse of a tree. Example code was adapted from [20].

Listing 2 uses hpx::dataflow to traverse a tree. In Line 5 and Line 8 the futures for the left and right traversal are returned. Note that these futures may have not been computed yet when they are passed into the dataflow on Line 13. The user could have used an hpx::async here instead of hpx::dataflow, but the Future passed to the called function may have suspended the thread while waiting for its results in the .get() function. The hpx::dataflow will not pass the Future arguments

```cpp
int convert(std::string s){ return std::stoi(s); } //Definition of the function
//Asynchronous execution of the function
hpx::future<int> f = hpx::async(convert, "42"); //Accessing the result of the function
std::cout << f.get() << std::endl;
```

Listing 1. Example for the concept of futurization within HPX. Example code was adapted from [20].
to the function until all of the Futures passed to the `hpx::dataflow` are "ready". This avoids the suspension of the child function call. In Section IV, futurization and the facility `hpx::dataflow` are heavily utilized to construct the asynchronous architecture of Phylanx.

Finally, the last technology we used to guide the development of Phylanx is NumPy. NumPy [12] is a highly optimized numerical computation library for Python. NumPy is used in many scientific applications and supports highly performant, multidimensional array operations for scientific computing. Phylanx uses the library’s API as the interface to the user. In addition, Phylanx supports numerical computation on NumPy data objects through pybind11 [22] without the need for data copies.

IV. Phylanx

In Python, the order of code blocks determines the execution order of a program and implicit parallelism is only available within each block. Therefore, asynchrony and parallelism across code blocks must be explicitly explored by the programmer, a process which is tedious and error prone. In this section we discuss the implementation of our approach in Phylanx for automatic generation of task graphs and the infrastructure used for running them on HPX for parallel, asynchronous, distributed execution. We also discuss a suite of analysis and optimization tools included in the Phylanx toolkit. Figure 1 provides an overview of program flow in Phylanx.

A. Frontend

The Phylanx frontend provides two essential functionalities:

- Transform the python code into a Phylanx internal representation called PhySL (Phylanx Specification Language).
- Copy-free handling of data objects between Python and Phylanx executor (in C++).

In addition, the frontend exposes two main functionalities of Phylanx that are implemented in C++ and required for generation and evaluation of the execution tree in Python. These functions are the compile and eval methods.

1) Code Transformation: Performance benefits of many-task runtime systems, like HPX, are more prominent when the compute load of the system exceeds the available resources. Therefore, using these runtimes for sections of a program which are not computationally intensive may result in little performance benefit. Moreover, the overhead of code transformation and inherent extraneous work imposed by runtime systems may even cause performance degradation. Therefore, we have opted to limit our optimizations to performance critical parts of the code which we call computational kernels. The Phylanx frontend provides a decorator (`@Phylanx`) to trigger transformation of kernels into the execution tree.

We have developed a custom internal representation of Python AST in order to facilitate the analysis of static optimizations and streamline the generation of the execution tree. The human-readable version of the AST, aka PhySL, is automatically generated and compiled into the execution tree by the frontend. More details on this compilation process can be found in IV-B. The benefit of using PhySL as the intermediate representation is twofold: (1) it closely reflects the nodes of the execution tree as each PhySL node represents a function that will be run by an HPX task during evaluation, and (2) it can be used for debugging and analyzing purposes for developers interested in custom optimizations.

The compiled kernel is cached and can be be invoked directly in Python or in other kernels.

2) Data Handling: Phylanx’s data structures rely on the high-performance open-source C++ library Blaze [23], [24]. Blaze already supports HPX as a parallelization library backend and it perfectly maps its data to Python data structures. Each Python list is mapped to a C++ vector and 1-D and 2-D NumPy arrays are mapped to a Blaze vector and Blaze matrix respectively. To avoid data copies between Python and C++, we take advantage of Python buffer protocol through pybind11 library. Figure 1 shows how Phylanx manages interactions with external libraries.

B. Execution Tree

After the transformation phase, the frontend passes the generated AST to the Phylanx compiler to construct the execution tree where nodes are primitives and edges represent dependencies between parents and children pairs. Primitives are the cornerstones of the Phylanx toolkit and building blocks of the Phylanx execution tree. Primitives are C++ objects which contain a single execute function. This function is wrapped in a dataflow and can be
as simple as a single instruction or as complex as a sophisticated algorithm. We have implemented and optimized most Python constructs as well as many NumPy methods as primitives. Futurization and asynchronous execution of tasks are enabled through these constructs. One can consider primitives as lightweight tasks that are mapped to HPX threads. Each primitive accepts a list of futures as its arguments and returns the result of its wrapped function as a future. In this way, the primitive can accept both constant values known at compile time as well as the results of previous primitives known only after being computed.

C. Futurized Execution

Upon the invocation of a kernel, Phylanx triggers the evaluation function of the root node. This node represents the primitive corresponding to the result of the kernel. In the evaluation function, the root node will call the evaluation function of all of its children and those primitives will call the evaluation functions of their children. This process will continue until the the leaf nodes have been reached where the primitives evaluation functions do not depend on other primitives to be resolved (e.g. a primitive which is a constant, a primitive which reads from a file, etc.). It is important to note that it does not matter where each primitive is placed in a distributed system as HPX will resolve its location and properly call its eval function as well as return the primitive’s result to the caller.

As the leaf primitives are reached and their values, held in futures, are returned to their parents the tree will unravel at the speed of the critical path through the tree. The results from each primitive satisfy one of the inputs of its parent node. After the root primitive finishes its execution, the result of the entire tree is then ready to be consumed by the calling function.

D. Instrumentation

Application performance analysis is a critical part of developing a parallel application. Phylanx enables performance analysis by providing performance counters to provide insight into its intrinsics. Time performance counters show the amount of time that is spent executing code in each subtree of the execution tree, and count performance counters show how many times an execution tree node is executed. This data aides in identifying performance hotspots and bottlenecks, which can either be directly used by the users or fed into APEX for adaptive load balancing. The data can also be used by the visualization tools described in the next section.

E. Visualization

Embedding annotations and measurements for visualizations and performance analysis within the runtime provides a way to determine where performance bottlenecks are occurring and to gain insight into the resource management within the machines. We show an example of Phylanx’s visualization capabilities in Figure 2. This tree shows the execution tree from a test run of the factorial algorithm, implemented in Python. In the tree, nodes are
Phylanx primitives and edges show parent/child relationships regarding how the child was called. The nodes are colored purple for the inclusive time, the total time spent executing that primitive and its children. A switch in the toolbox in the upper left corner allows for the user to switch from inclusive time to exclusive time, the time spent executing only that primitive. This allows for identification of hotspots in the tree. Each primitive can be executed asynchronously or synchronously in the parent thread. This distinction is shown in dotted versus solid circles for nodes in the tree. The tree is interactive, allowing users to drill down and focus by expanding or collapsing tree nodes and hover for more details. The visualization is linked with a code view showing the Python source code (the corresponding PhySL is shown as well). Hovering over a node or line in one will highlight the corresponding line or node in the other.

V. Experiments

This section details the performance comparison of PhySL to a corresponding Python implementation utilizing multiple cores on top of NumPy using OpenBlas for BLAS/LAPACK routines. We used reference implementa-

Fig. 3. Phylanx program flow. Phylanx frontend generates AST (PhySL) of the decorated Python code. The AST could be directly passed to the compiler to generate the execution tree or, optionally, fed to the optimizer first and then the compiler. Once the Kernel is invoked, Phylanx triggers the evaluation of the the execution tree on HPX. After finishing the evaluation, the result is returned in Python.

A. Experimental Testbed

We ran our experiments on LSU’s Rostam cluster. These experiments were performed on a node consisting of Intel(R) Xeon(R) CPU E5-2660 v3 clocked at 2.6GHZ, with 10 cores (20 threads), and 128 GB DDR4 Memory. All Experiments were performed on HPX v1.2 commit 9182ac6182, Phylanx v0.1 commit 116c46a8 Python v3.5.1, NumPy v1.15.0, OpenBlas v0.3.2 and Blaze v3.3.

B. LRA

We implemented the Binary Logistic Regression algorithm in Python and used the Phylanx decorator to generate the corresponding PhySL code. In order to test the performance of the two implementations of the Logistic regression algorithm, we created a custom binary classification dataset with 10,000 features and 10,000 observations.

Figure 5 shows the performance of the Python and PhySL codes in terms of the execution time. Our experiments show that on a single thread both PhySL and Python perform on par with each other. However, PhySL scales faster up to eight cores and plateaus afterwards while Python scales at a lower rate but up to 20 cores.

C. Alternating Least Squares

Alternating Least Squares is a method used in collaborative filtering based on matrix factorization [26]. Collaborative filtering as a recommender system is utilized to predict a user’s interest in a set of items based on other users interaction with those items, and also the user’s interactions with other items. In order to test the implementation of the Alternating Least Squares algorithm in Phylanx, we implemented the algorithm in Python using NumPy and generated the corresponding PhySL implementation using the Phylanx decorator. The two implementations of the algorithms were tested on MovieLens-20M dataset [29], which is a collection of 20 million ratings gathered by 138,000 users over 27,000 movies.

Figure 6 shows the execution times of the PhySL and Python versions of Alternating least squares. Both versions were run with number of factors set to 40 while the number of movies were set to 5,000, 10,000 and 20,000. The PhySL version of the ALS algorithm starts to outperform the Numpy/Python version as the number of threads increases. The fastest time for the Phylanx implementation is seen using 16 threads and the number of movies set to 20,000. When the number of movies were set to 10,000, the fastest time was seen using 12 threads. There
is a noticeable difference in execution time between the Python and the PhySL implementation when the number of threads is set to one. In this configuration the Phylanx version is much slower than the Python implementation. Such behavior is not seen with the Logistic Regression example. This behavior is currently under investigation.

Figure 4 shows the speedup of the Phylanx implementation of the Alternating least square using the Python implementation’s performance as the baseline. Both implementations were run with the number of factors set to 10, 20 and 40. While the number of movies were set to 5,000, 10,000, and 20,000. It is seen that the Phylanx implementation outperforms the Python implementation as the number of threads are increased on wide variety of problem sizes.

VI. CONCLUSION

Despite the solutions provided by current machine learning frameworks, better methodologies are needed to process the large amounts of data consumed by cutting edge machine learning applications in a timely manner. These new tools need to be accessible to the domain scientists who currently use them as well as efficient with the computational resources provided to them.

In this paper, we have introduced a novel approach for transformation and execution of high-productivity languages on top of the highly performant, low-level HPX runtime system. We have implemented our approach along with a suite of performance and visualization tools in the Phylanx array processing toolkit. Phylanx enables automatic generation of asynchronous task graphs from regular Python code and facilitates finer grain configurability. Our early experiments on representative applications and datasets demonstrate the performance benefits of our methods on a single node. However, we expect that the real benefits of our approach will manifest in a distributed runtime environment. Here the asynchronous execution and
locality abstractions provided by HPX stand to benefit users immensely by drastically decreasing execution times with little effort from the user.

VII. Future Work

As the Phylanx technology matures we intend focus on two major goals: first, to improve the performance of single node runs and second to extend the framework to automatically run user supplied codes in distributed settings. Improving single node runs will entail implementing more basic algorithms utilized by the machine learning community and using that experience to improve the underlying toolkit. We anticipate that we will be able to uncover performance bugs as well as opportunities to improve the performance of our toolkit from this experience. One such opportunity is the support for hardware accelerators such as GPUs.

Phylanx plans to enable users to execute their existing Python code on clusters. This provides them with the ability to handle large data sets and achieve better scaling. While Phylanx is built with distributed runs in mind (execution trees can span across several nodes and evaluation is done using features from HPX), distributed runs will require extensions to the current toolkit to determine optimal data layout and data tiling given the algorithms provided by the user. We also intend to look at using code transformations to replace slower user-written algorithms with more efficient ones. While these goals present a challenge, we believe the support provided by the HPX runtime system will substantially reduce the barriers to providing distributed execution capabilities.

VIII. Acknowledgments

This work was funded by the NSF Phylanx project award #1737785. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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