Double explosions and jet formation in GRB-supernova progenitors

Maxim Lyutikov
Department of Physics, Purdue University,
525 Northwestern Avenue, West Lafayette, IN 47907-2036

ABSTRACT

Progenitors of long GRBs, and core-collapse supernovae in general, may have two separate mechanisms driving the outflows: quasi-isotropic neutrino-driven supernova explosions followed by a highly collimated relativistic outflow driven by the GRB central engine, a black hole or a magnetar. We consider the dynamics of the second GRB-driven explosion propagating through expanding envelope generated by the passage of the primary supernova shock. Beyond the central core, in the region of steep density gradient created by the SN shock breakout, the accelerating secondary quasi-spherical GRB shock become unstable to corrugation and under certain conditions may form a highly collimated jet, a “chimney”, when a flow expands almost exclusively along a nearly cylindrically collimated channel. Thus, weakly non-spherical driving and/or non-spherical initial conditions of the wind cavity may produce highly non-spherical, jetted outflows. For a constant luminosity GRB central engine, this occurs for density gradient in the envelope \( \rho \propto r^{-\omega} \) steeper than \( \omega > 4 \).

1. Double explosion in core collapse supernovae

Long Gamma Ray Bursts (GRBs) are intrinsically linked to core collapse supernovae. This conclusion comes from the detection of Type Ic supernovae nearly coincident with long GRBs (Stanek et al. 2003; Hjorth et al. 2003). It is also confirmed by studies of the host galaxies of long GRBs, which turned out to be actively star-forming (Djorgovski 2001). The leading model of long GRBs is a collapsar model (MacFadyen & Woosley 1999; Barkov & Komissarov 2008), which postulates that a compact central source (a black hole or rapidly rotating neutron star Usov 1992) forms inside the collapsing core. The central engine generates a collimated outflow, which upon breaking out of the star reaches relativistic velocities and eventually produces \( \gamma \)-rays.

Modern models of neutrino-driven SN explosion are not stable, in a sense that different groups do not agree with each other and the role of different ingredient is not settled (e.g. Burrows & Nordhaus 2009). The collapsar model assumes that in addition to the conventional neutrino-driven SN explosion, there is an addition source of energy, the GRB central engine. It is possible that depending on the detailed properties of the pre-collapse core (like angular momentum, initial magnetic field, small differences in composition etc), the two energy sources that may potentially lead to the explosion, neutrino-driven convection and the GRB central engine, may contribute
different amount of energy, resulting in different observed types of SNe and/or GRBs. Neutrino-driven explosions generates quasi-spherical sub-relativistic outflows, while GRB explosion results in a jetted relativistic component. Classical SNe are all neutrino driven, where GRB engine is negligible. As the relative strength of the GRB engine increases, this leads to phenomena of sub-energetic and regular GRBs. The relative contribution of the neutrino-driven and GRB energies are definitely not independent of each other: one expects that in case of a successful neutrino-driven SN, the amount of material accreted on the central source is small, resulting in weak or no relativistic component and a weak GRB: recall that all SN-associated GRBs are subluminous.

Thus, a SN explosion can be viewed as a two-parameter phenomenon, the two parameters being the power of the neutrino-driven and GRB-driven outflows. Most supernova are neutrino-driven quasi-spherical outflows, where the GRB-driven component is weak or non-existent. The recent discovery of the relativistic type Ibc supernova without a detected GRB signal (SN 2009bb \cite{Soderberg2010}) requires both two energy sources: one to generate the relativistic outflow, another to expel the SN envelope. In addition, some of the well studied supernova remnants, like Cas A, do show jet-like features. \cite{Laming2006} indeed suggested that Cas A could have been a failed GRB, generating a jet with a typical energy order of magnitude smaller than a typical long GRB.

As the power of the neutrino-driven outflow decrease, the fall-back material may power the GRB-like central engine. Thus, the SN and GRB explosions are two related, but different events: SN shock expels or nearly expels the envelope, while the GRB outflow is concentrated in a narrow solid angle, presumably along the axis of rotation of the central object. In this picture, the GRB engine does need (but still may) to contribute to overall dynamics of the envelope: most of the heavy lifting (unbinding the envelope) is done by the neutrinos.

For the purpose of this paper, we accept this paradigm, that both GRBs and some non-GRB SNe have two driving mechanisms with the relative energies of the two outflows varying over a large range. We assume that GRB shock follows that of the SN, but not by much, so that the confining ram pressure of the ejecta is important for propagating of the secondary GRB shock.

In passing we note that a two stage model of SN explosions have already been advocated by \cite{Grasberg1986} long before the discovery of jet-like features in SNe. This was based on modeling of line emission, in particular of remarkably narrow emission and absorption lines. They suggest that weak explosion proceeded the SN shock, though. Also, a supranova model of \cite{Dermer2002} postulates a two-staged explosion.

2. Formation of a GRB jet: Kompaneets approximation

How is the GRB jet collimated? Most models of jet formation in a collapsing star employ a highly anisotropic driving, either through neutrino-induced heating \cite{MacFadyen1999} or magnetic collimation \cite{Lyutikov2003, Komissarov2007, Bucciantini2007}.
In this paper we investigate an alternative mechanism that produces a highly anisotropic outflow, while been driven by a weakly anisotropic central source. The collimation mechanism relies on the ram pressure of the external mediums and large scale Raleigh-Taylor (RT) instability of accelerating shocks.

There is an extensive literature on RT instability of point explosions propagating in steep density gradient (e.g. Ryu & Vishniac 1987; Chevalier 1990). The particular case of a driven shock has not been investigated to the best of our knowledge. Another qualitative difference of our approach from the conventional SN theory (Ryu & Vishniac 1987; Chevalier 1990) is that we allow much steeper external density gradients, established by a passage of a preceding SN shock.

The distinction between point explosions propagating in steep density gradient, and driven explosions (sub-sonically or supersonically) is important. Accelerating shocks from point explosions obey the so called second type of self-similarity (Zeldovich & Raizer 2003), whereby the flow passes through critical point so that the flow dynamics is, in some sense, self-determined. In subsonically driven explosions the shock obeys a Sedov-type scaling, where post-shock pressure is related to the average pressure in post-sock cavity. Supersonically driven shocks are qualitatively different. If the wind velocity is higher than the shock velocity, then the shock motion is determined (in the thin shell approximation) by the pressure balance between the wind luminosity at the retarded time and the ram pressure of the external medium. This is the essence of the Kompaneets (1960) approximation (see also Laumbach & Probstein 1969; Zeldovich & Raizer 2003). In application to winds, the Kompaneets approximation assumes that the shock dynamics is given by the pressure balance along the shock normal between the pressure of the radially expanding wind and the ram pressure of the confining medium.

As a model problem, assume that the primary SN shock has passed though the star. As a result, the density distribution in the expanding envelope will consist of a nearly constant density core and an envelope with a steep power law density distribution $\rho \propto r^{-\omega}$ (Chevalier 1982; Nadezhin 1985; Truelove & McKee 1999); we assume $\omega = 9$ for definiteness. Soon after the passing of the SN shock, a central GBR engine turns on, acting as an energy source which with luminosity $L_{iso}(\theta)$ constant in time, but depending on the polar angle $\theta$ (the flow is assumed to be axially symmetric). We assume that luminosity is produced in a form of highly supersonic wind with the velocity close to the speed of light, $v_w \sim c$. The central engine will drive a second shock, which, generally, will be non-spherical. We describe the dynamics of this second non-spherical shock driven by the GBR-type engine into an external medium with spherically symmetric power law density distribution. We treat the GRB shock dynamics in the Kompaneets-Laumbach-Probstein approximation, appropriate to describe accelerating shocks driven by the central source.

Consider a small section of non-spherical non-relativistically expanding contact discontinuity (CD) with radius $R(t, \theta)$. The CD expands under the ram pressure of the wind, so that in the thin shell approximation the normal stress at the bubble surface is balanced by the ram pressure of the surrounding medium. At the spherical polar angle $\theta$ the CD propagates at an angle $\tan \alpha =$
\[ -\partial \ln R/\partial \theta \] to the radius vector. Balancing the pressure inside the bubble, \( \sim L_{\text{iso}}/(4\pi r^2 c) \), with the ram pressure of the shocked plasma, \( \sim \rho (\dot{r} - v_0)^2 \), gives (see also Kompaneets 1960; Icke 1988)

\[
\left( \frac{\partial r}{\partial t} - v_0 \right)^2 = \frac{L_{\text{iso}}(\theta)}{4\pi r^2 \rho c} \left[ 1 + \left( \frac{\partial \ln r}{\partial \theta} \right)^2 \right]
\]

(1)

where \( v_0 \) is the velocity of radial motion of the outside medium. We assume highly supersonic flows and neglect for simplicity the difference between the ram and the post-shock thermal pressures. Below we refer to Eq. (1) as the Kompaneets equation. It describes the evolution of non-spherical shocks with energy supply.

The Kompaneets equation (1) shows that non-sphericity of the expanding shock at a given moment depends both on the anisotropic driving (\( L_{\text{iso}}(\theta) \) term) and collimating effects of the stellar material - the term in parenthesis, which under certain conditions tends to amplify non-sphericity. (We assume that the density distribution is spherically symmetric). Most importantly, Eq. (1) is non-linear and under certain conditions even a small anisotropy driven by either the luminosity of the central source or by the initial non-spherical shape of the shock can be amplified and may lead to formation of highly collimated jets. This is indeed what happens in a steep density part of the post-SN shock profile of a star.

### 3. Dynamics of secondary GRB shock

#### 3.1. Structure of young SNRs

Interaction of the SN ejecta with the stellar envelope have been considered by Chevalier (1982); Nadezhin (1985); Truelove & McKee (1999). After the SN shock passed through the progenitor star, it creates an expanding SNR with expansion velocity linearly increasing with distance,

\[ v = \frac{r}{r_0}v_0 = \frac{r}{t} \]

(2)

where \( r_0 = v_0 t \) is the outer radius of ejecta freely expanding with velocity \( v_0 \). The density structure consists of a nearly constant density core, and an envelope with a steep density profile created by the SN shock breakout from the surface of the progenitor. If we assume self-similar expansion, so that at each moment the relative size of the core remains constant, \( r_c = \eta_c r_0 = \eta_c v_0 t \), and that the envelope density profile is a power law \( \rho \propto r^{-\omega} \), the density at time \( t \) is

\[
\rho = \frac{f_0}{t^3} g(r)
\]

\[ g(r) = \begin{cases} 
1, & r < r_c \\
\left( \frac{r}{r_c} \right)^\omega, & r > r_c
\end{cases}
\]

\[ f_0 = \frac{3(\omega - 3) M_{\text{ej}}}{4\pi \omega \eta_c^2 v_0^3} \]

(3)
The wind has a fraction $3/\omega$ of total mass and a fraction $5/\omega$ of the total energy. Thus, for $\omega = 9$ - the fiducial value we will use below - the wind has approximately 30% of mass of the ejecta.

### 3.2. Spherical secondary shock

Let us discuss first the dynamic of a spherical GRB shock propagating through a newly cerated SNR. In the Kompaneets approximation

\[
\frac{\partial r}{\partial t} = \frac{r}{t} + \frac{R^{1/4}}{\sqrt{g(r)}} \frac{r^{3/2}}{r} \quad \mathcal{R} = \left( \frac{L_{\text{iso}}}{4\pi c f_0} \right)^2 \tag{4}
\]

Assuming that the start of the GRB engine is delayed with respect to the SN explosion by time $\Delta t$ and that luminosity $L_{\text{iso}}$ is constant in time, in the core (where $g(r) = 1$) the GRB shock propagates according to

\[
r = 2\mathcal{R}(t + \Delta t)\sqrt{t + \Delta t - \sqrt{\Delta t}} = \begin{cases} \frac{\sqrt{2}\Delta t^{3/4}}{2\mathcal{R}t^{5/4}}, & t \ll \Delta t \\ \sqrt{\Delta t + \sqrt{t_0}} - \sqrt{\Delta t}, & t \gg \Delta t \end{cases} \tag{5}
\]

(GBR central engine starts operating at $t = 0$). Thus, if there is a delay of the switching on of the GBR engine by time $\Delta t$, the resulting GRB shock slows down for a time $\Delta t$, and only later starts to accelerate.

The GRB shock reaches the edge of the expanding core at time and radius

\[
t_c = \left( \sqrt{\Delta t + \sqrt{t_0}} \right)^2 - \Delta t \approx \begin{cases} 2\sqrt{\Delta t t_0}, & t_0 \ll \Delta t \\ t_0, & t_0 \ll \Delta t \end{cases}
\]

\[
R_c = \eta_c \left( \left( \sqrt{\Delta t + \sqrt{t_0}} \right)^2 - \Delta t \right) v_0 \tag{6}
\]

where

\[
t_0 = \frac{3}{16} \frac{M_{ej} v_0 c}{L_{\text{iso}}} \tag{7}
\]

The time $t_c$ (or $t_0$) and the radius $R_c$ give the typical scales of the problem. In case of time delay between the SN explosion and the activation of the GRB engine, the time it takes for the GRB shock to reach the edge of the expanding core (this time is the main constraint on the model) is the geometrical mean of the dynamical time (7) and the delay time $\Delta t$.

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1. We assume that the GRB shock was initiated when the density structure was given by Eq. (3) with time $t = \Delta t$. Since relations (3) are asymptotic, for times much longer than the SN shock breakout time, the relations below can be used only as order of magnitude estimates for times $\Delta t$ of the order or longer than the SN breakout time.
The intrinsic dynamical time $t_0$ and distance at which the GRB shock reaches the surface of the core (for zero delay, $\Delta t = 0$) is

$$t_0 = 40 \, \text{sec} \left( \frac{M}{2 M_\odot} \right) \left( \frac{\eta_c}{3} \right) \left( \frac{v}{10^4 \text{ kms}^{-1}} \right) \left( \frac{L_{\text{iso}}}{10^{51} \text{ ergs}^{-1}} \right)^{-1}$$

$$R_0 = \eta_c v_0 t_0 = 10^{10} \, \text{cm} \left( \frac{M}{2 M_\odot} \right) \left( \frac{\eta_c}{3} \right)^2 \left( \frac{v}{10^4 \text{ kms}^{-1}} \right)^2 \left( \frac{L_{\text{iso}}}{10^{51} \text{ ergs}^{-1}} \right)^{-1} \quad (8)$$

These typical time and scales are characteristic of the Long GRBs. For a delay of $\Delta t = 50$ seconds, $t_c \sim 200$ sec and $R_c \sim 5 \times 10^{10}$ cm. Variations of the GRB engine power, various delays between the activation of the central engine and other variations of the parameters may change these estimates by an order of magnitude in each direction.

After the GRB shock reached the edge of the core, it moves, approximately for $\omega \gg 1$, according to

$$r \approx \eta_c \left( 1 - \frac{\omega - 4}{\sqrt{3}} \left( \sqrt{\frac{t}{t_c}} - 1 \right) \right)^{-2/(\omega-4)} v_0 t \quad (9)$$

For $\omega > 4$ the GRB shock experiences finite time singularity, when its velocity formally becomes infinite in a finite time. This occurs at times only slightly longer than $t_c$, by a factor $\sim 1 + 1/\omega$, see Fig. 1.

In what follows, we present results of calculations for $\omega = 9$. Results for other power law profiles with $\omega \gg 4$ are similar, while for $\omega \to 4$ the GRB shock break out occurs at very long times.

### 4. Jet formation in the steep density gradient

Accelerating shocks are typical RT unstable (e.g., Zeldovich & Raizer 2003). A particularly strong instability develops for shocks that accelerate to arbitrary large velocities in finite time. In such shocks a small perturbation from the spherical shape gets infinitely amplified on the time scale of the acceleration.

To demonstrate the instability, we numerically solve Eq. (1) with density profile given by Eq. (3). We separate the two effects that may contribute to generation of jets – non-spherical driving and non-spherical initial conditions. To facilitate comparison with the model results, we introduce new radial coordinate $r = 2 \tilde{r} t^{5/4}$. For an isotropic shock inside the core the new coordinate $\tilde{r}$ remains constant, $\tilde{r} = \mathcal{R}$.

First, we consider a shell of spherical form at $t = 0$ located at small distance and driven by non-spherical wind. As a simple exemplary case we chose wind luminosity $f(\theta) = 1 + \cos^2 \theta$, so that the wind power along the polar direction is two times of that along the equator. Numerical integration shows that at times $t \leq t_0$, the evolution proceeds in a linear regime, so that a weakly anisotropic driving results in a weakly anisotropic shock. Soon after the shock reaches the edge
Fig. 1.— Comparison of the numerical integration and analytical solutions for spherical expansion of the GRB shock. Shown are the world lines for the surface of the core \( r_0 = \eta_c v_0 t \) (dashed) and for the GRB-driven shock propagating into expanding SN ejecta. The core boundary is crossed at time \( t_0 \) when its radius is \( r_0 = \eta_c v_0 t_0 \). No time delay \( \Delta t = 0 \). In the core, the numerical solution follow the analytical result for a slightly accelerating shock \( r \propto t^{5/4} \) (dot-dashed line), while in the envelope it quickly reaches finite time singularity.

of the core at \( t_0 \) and enters the steep density gradient in the envelope, it becomes unstable and develops a collimated jet, see Figs. 2, 3. For \( \omega \gg 4 \), the jets develops on the time scale \( t_0 \).

Secondly, we consider an isotropic driver, \( f = 1 \) but starting with a non-spherical initial shape, \( r(t = 0) = 1 + \cos^2 \theta \). In this case the GRB shock dynamics is qualitatively similar to the non-spherical wind: the shock remains weakly anisotropic in the core and develops a jet soon after reaching the envelope, Figs. 2, 3.
Fig. 2.— Shapes of the secondary GRB-driven shock in normalized coordinate \( \tilde{r}(\theta, t) \). Inserts show that deep in the core the shape of the shock, \( \propto 1 + \cos^2 \theta \), remains constant and reflects either the weakly anisotropic driving or initial conditions. After the shock reaches the envelope, it develops a strongly anisotropic shape, producing a highly collimated "chimney" in a short time just before the breakout. **Left panel**: anisotropic driver \( L \propto 1 + \cos^2 \theta \), isotropic initial conditions, \( r(t = 0) = \text{constant} \). **Right panel**: isotropic driver \( L = \text{constant} \), anisotropic initial conditions, \( r(t = 0) \propto 1 + \cos^2 \theta \).

5. Discussion

We suggest that the outflows driven by core-collapse supernova generically have two driving mechanisms: neutrinos and a GRB-like central engine. Most supernovae are dominated by the neutrino-driven quasi-spherical outflow, while long GRBs are dominated by the central engine, producing relativistic collimated jets. The second GRB-driven shock propagates into a density structure created by the primary SN shock. This density structure of early SNRs has a central core with nearly constant density and an envelope with steeply decreasing density. In the envelope the secondary shock is accelerating, is unstable to Raleigh-Taylor instabilities and may form "chimneys", cylindrically collimated outflows. Jet formation occurs even if the central engine produces only weakly anisotropic wind. This is a non-linear processes, by which a non-sphericity of the shock is amplified by the wind. The formation of the "chimney" occurs over extremely short dynamical time, while the time of the "chimney" formation is related to break out time. It is also required that the GRB engine is sufficiently powerful, so it can drive the secondary GRB shock through the constant density core on sufficiently short time scales. Thus, there is a lower power threshold on the GRB engine required for the production of collimated outflows. We suggest that if this threshold is not reached, weakly anisotropic SNe are generated; e.g., Cas A and SN 2009bb may be examples of such failed GRBs.
Fig. 3.— Same as Fig. 2 but in terms of the physical distance $r(t)$.

We presented a numerical model illustrating the formation of collimated outflows in a model expanding star. The model is naturally idealized, we assumed the SN ejecta structure is described by the asymptotic profile, which takes a SN shock break out time to establish, of the order of hundred seconds, which is similar to the dynamical time \( t_\text{dyn} \).
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