Scissors Modes and Spin Excitations in Light Nuclei including $\Delta N=2$ excitations:

Behaviour of $^8$Be and $^{10}$Be

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Abstract

Shell model calculations are performed for magnetic dipole excitations in $^8$Be and $^{10}$Be in which all valence configurations plus $2\hbar\omega$ excitations are allowed (large space). We study both the orbital and spin excitations. The results are compared with the ‘valence space only’ calculations (small space). The cumulative energy weighted sums are calculated and compared for the $J = 0^+ T=0$ to $J = 1^+ T=1$ excitations in $^8$Be and for $J = 0^+ T=1$ to both $J = 1^+ T=1$ and $J=1^+ T=2$ excitations in $^{10}$Be. We find for the $J = 0^+ T=1$ to $J = 1^+ T=1$ isovector spin transitions in $^{10}$Be that the summed strength in the large space is less than in the small space. We find that the high energy energy-weighted isovector orbital strength is smaller than the low energy strength for transitions in which the isospin is changed, but for $J = 0^+ T=1$ to $J = 1^+ T=1$ in $^{10}$Be the high energy strength is larger. We find that the low lying orbital strength in $^{10}$Be is anomalously small, when an attempt is made to correlate it with the $B(E2)$ strength to the lowest $2^+$ states. On the other hand a sum rule of Zheng and Zamick which concerns the total $B(E2)$ strength is rea-
sonably satisfied in both $^8$Be and $^{10}$Be. The Wigner supermultiplet scheme is a useful guide in analyzing shell model results. In $^{10}$Be and with a $Q \cdot Q$ interaction the $T = 1$ and $T = 2$ scissors modes are degenerate, with the latter carrying $\frac{5}{3}$ of the $T = 1$ strength.
1 The Experimental Situation

From our perspective, much experimental information is lacking in the nuclei $^8$Be and $^{10}$Be. For example, no $J = 1^+$ states have been identified in $^{10}$Be. The $B(E2)$ from the $2^+_1$ state of $^8$Be to the $J = 0^+$ ground state is not known - this is understandable because of the large decay width to two alpha particles.

The following states and their properties are of interest to us:

(a) $^8$Be

The $J = 2^+_1$ state has an excitation energy of 3.04 MeV. The $J = 4^+_1$ state is at 11.4 MeV. This is consistent with an $J(J+1)$ spectrum of a rotational band, but it should be recalled that any spin-independent interaction gives an $J(J+1)$ spectrum in the $p$ shell. The $J = 1^+_1 T = 1$ state, which we discussed extensively in a previous publication [1] is at 17.64 MeV and the $J = 1^+_1 T = 0$ state is at 18.15 MeV.

The $B(M1)$ from the 17.64 MeV state to the ground state has a strength of 0.15 W.u. or $B(M1)\uparrow = 0.27\mu_N^2$. The $B(M1)$ of this state to the $2^+_1$ state is 0.12 W.u. or $B(M1)\downarrow = 0.21\mu_N^2$ [2]. Of course $B(M1)\uparrow = 3B(M1)\downarrow$.

(b) $^{10}$Be

The $J = 2^+_1$ state is at 3.368 MeV and the $J = 2^+_2$ state at 5.960 MeV. We recall that with a spin independent interaction the $2^+_1$ and $2^+_2$ would be degenerate. The experimental spectrum looks more vibrational. However, the values of $B(E2)$ from the $J = 0^+$ ground state to the $2^+_1$ state is very strong: $B(E2)\uparrow = 52 e^2fm^4$. Raman et. al. deduce from this a deformation parameter $\beta = 1.13$ [3]. As mentioned above, there are no $J = 1^+$ states mentioned in the compilation of Ajzenberg-Selove [2]. Also the $J = 4^+$ state has not been found.
2 The Interactions

We have chosen two types of interactions to do the calculations. First we use a short range ‘simplified realistic’ \((x, y)\) interaction previously used by Zheng and Zamick \[4\], and then we use a long-range quadrupole-quadrupole interaction. By choosing these two extremes, we make sure that the results we obtain are not too dependent on the specifics of the model.

In more detail, the \((x, y)\) Hamiltonian is:

\[
H = \sum_i T_i + \sum_{i<j} V(ij)
\]

where

\[
V = V_c + xV_{so} + yV_t
\]

with \(c \equiv \text{central}, \ s.o. \equiv \text{spin-orbit}, \) and \(t \equiv \text{tensor}.
\]

For \((x, y) = (1,1)\) the matrix elements of this interaction are close to those of realistic \(G\) matrices such as Bonn A. We can study the effects the spin-orbit and tensor interactions by varying \(x\) and \(y\).

Note that we do not add any single-particle energies to the above Hamiltonian. Rather, we let the single-particle energies be implicitly generated by \(H\). Hence, if we set \(x=0\) i.e. turn off the two-body spin-orbit interaction, we will also be turning off the one-body spin-orbit splitting coming from this interaction.

As a counterpoint, we repeat all the calculations with the \(Q \cdot Q\) Hamiltonian

\[
H_Q = \sum_i \frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2 - \chi \sum_{i<j} Q \cdot Q
\]

Note that we have added the term \(\frac{1}{2}m\omega^2 r^2\) which is not present for the \((x, y)\) interaction. The reason for this is that \(Q \cdot Q\) cannot generate any single-particle potential energy splitting whereas the \((x, y)\) interaction can.
Whereas the \((x,y)\) interaction like all realistic interactions is of short range, the \(Q \cdot Q\) interaction is long range. Yet, as we shall see some of the results (but not all) are rather similar for the two interactions. Since the best milieu for the existence of scissors mode excitations (orbital magnetic dipole excitations) are strongly deformed systems, one would expect the \(Q \cdot Q\) Hamiltonian to yield strong scissors modes. But is this also true for the realistic interaction? We will address this question. Another motivation for introducing the \(Q \cdot Q\) Hamiltonian is that it is easy to establish a connection via energy weighted sum rule techniques between isovector orbital \(B(M1)'s\) and isoscalar and isovector \(B(E2)'s\).

We shall be performing the calculations, not only in the \(0\hbar\omega\) space (small space) but also in a space which allows \(2\hbar\omega\) admixtures (large space). For the \(Q \cdot Q\) Hamiltonian in the small space the energy matrix is proportional to \(\chi\). Hence the energy eigenvalues depend linearly on \(\chi\), but the eigenfunctions (and \(B(M1)'s\) and \(B(E2)'s\)) are independent of the interaction strength. In a large space calculation there is one more parameter: the energy splitting induced by \(\frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2\) i.e. \(2\hbar\omega\). Thus the wave function and the corresponding \(B(M1)'s\) and \(B(E2)'s\) will also depend on \(\chi\).

We have chosen values of \(\chi\) appropriate for the large space calculation. We also use these same values in the small space. One can argue that in the small space one should use a renormalized value \(\chi'\) which is close to twice \(\chi\). However, as mentioned above, the wave function and hence \(B(M1)\) and \(B(E2)\) will not change, only the energies. By choosing the same \(\chi\) in the two spaces it is easier to see what the differences in the two calculations are. The values of \(\chi\) are \(0.5762\ \frac{MeV}{fm^4}\) for \(^8Be\) and \(0.3615\ \frac{MeV}{fm^4}\) for \(^{10}Be\).
3 The summed magnetic dipole strength

In Table I we give the summed magnetic dipole strength ($\sum_i B(M1: 0^+_1, T = 1 \rightarrow 1^+_1, T = 1)$ and $\sum_i B(M1: 0^+_1, T = 1 \rightarrow 1^+_1, T = 2)$ ) broken up into isoscalar and isovector and spin and orbit and where we use the $(x,y)$ interaction with $x = 1$, $y = 1$. We first discuss the behaviour as a function of the size of the model space. Later we will make a comparison of the behaviour in $^8$Be and $^{10}$Be. There are striking differences for the two nuclei.

Our small space calculation consists of all configurations of the form $(0s)^4(0p)^4$ for $^8$Be and $(0s)^4(0p)^6$ for $^{10}$Be. The large space consists of those configurations plus $2\hbar\omega$ excitations. Thus one can either excite two particles to the next major shell or excite one particle through two major shells. We also give results for the summed strength in the low-large space -this is the low energy part of the large space covering an energy range more or less equal to that of the small space. It is easy to identify the low energy sector because there is a fairly wide plateau in the summed strength which separates the low energy rise from the high energy rise.

Usually the large space summed strength is somewhat larger than the small space strength e.g. for the isovector orbital strength in $^8$Be the values shown in Table I are 0.6701 $\mu^2_N$ and 0.7283 $\mu^2_N$ respectively. But there is one glaring exception. For the case of $J^\pi = 0^+ T = 1 \rightarrow J^\pi = 1^+ T = 1$ transitions in $^{10}$Be, the summed isovector spin strength in the large space is $2.08 \times 10^{-2} \mu^2_N$ but in the small space it is bigger $2.34 \times 10^{-2} \mu^2_N$. For the orbital strength it is the other way around but for the physical case ($g_{l\pi} = 1, g_{l\nu} = 0, g_{s\pi} = 5.586, g_{s\nu} = -3.826$) the spin prevails and the summed strength in the large space $1.952 \mu^2_N$ is less than in the small space $2.09 \mu^2_N$.

Thus it is not always true that the net result of higher shell admixtures is to rob strength from the low energy sector and move it to higher energies.
In some cases the total strength gets depleted.

We next compare the low energy sum in the large space with the small space sum. In all cases the latter is larger than the low energy sum, thus indicating that there is a quenching of the low energy part due to higher shell admixtures. The hindrance factor \([(low\ large)/small]\) is 0.88 for the isovector orbital in \(^8\text{Be}\), 0.73 for the isovector spin in \(^8\text{Be}\), 0.77 for the total \(M1\) in \(^{10}\text{Be}\) etc.

Note that the total \(M1\)'s for \(^{10}\text{Be}\) are somewhat larger than for \(^8\text{Be}\). However there is a dramatic drop in the orbital strength in \(^{10}\text{Be}\) relative to that in \(^8\text{Be}\). The large space summed orbital strength for \(^8\text{Be}\) is 0.73 \(\mu_N^2\) whereas for \(^{10}\text{Be}\) (to \(J = 1^+ T = 1\) and \(T = 2\)) the value is \((0.196 + 0.183) = 0.38 \mu_N^2\). In the low energy sector the \(^8\text{Be}\) value is 0.59 \(\mu_N^2\) whereas for \(^{10}\text{Be}\) it is \(0.10 + 0.13 = 0.23 \mu_N^2\), less than half the value for \(^8\text{Be}\).

From the systematics of orbital transitions in heavy nuclei one concludes that the proper milieu for isovector orbital transitions is strongly deformed nuclei. Can one conclude that \(^{10}\text{Be}\) is not strongly deformed? The answer, by examining the tables of Raman et. al. is no! There is a strong \(E2\) connecting the \(0_1^+\) and \(2_1^+\) states in \(^{10}\text{Be}\). From this the authors conclude that the deformation parameter \(\beta\) is about 1.13 - quite enormous. Of course \(^8\text{Be}\) might have an even stronger \(E2\) transition - there is no data on this in the Raman paper, probably because of the rapid decay of the \(2_1^+\) state into two alpha particles.

4 The cumulative energy weighted strength for orbital transitions in \(^8\text{Be}\) and \(^{10}\text{Be}\)

In this section we present results and figures for the cumulative energy weighted sum of magnetic dipole strength.
We are motivated in so doing by various energy-weighted sum rules that have been developed e.g. by Zheng and Zamick [5], Heyde and de Coster [6], Moya de Guerra and Zamick [7], Nojarov [8], Hamamoto and Nazarewicz [9] and Fayache and Zamick [1]. We will focus in particular on the orbital strength for which the operator is \((\vec{L}_\pi - \vec{L}_\nu)/2\). In a previous publication we presented results for the \((x,y)\) interaction with \(x=1, y=1\) for \(^8\text{Be}\) [1]. In this work the quadrupole-quadrupole interaction results are compared with the \((x,y)\) interaction results, and furthermore we extend the calculation to \(^{10}\text{Be}\). In the latter nucleus one does not have \(N = Z\) and this leads to big differences.

Whereas in \(^8\text{Be}\) there is only one isospin channel for isovector transitions \(J = 0^+_1 T = 0 \rightarrow J = 1^+ T = 1\), in \(^{10}\text{Be}\) there are two: \(J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 1\) and \(J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 2\). The low lying \(J = 1^+ T = 1\) states in \(^{10}\text{Be}\) are expected to have much smaller excitation energies than the \(J = 1^+ T = 1\) states in \(^8\text{Be}\). This makes it easier to look for such states experimentally.

In Table II we present the results for the summed energy weighted strengths for the \((x,y)\) interaction. As a crude orientation it should be noted that simple models e.g. the Nilsson model used by de Guerra and Zamick [7] and a model by Nojarov [8] would have the ‘large’ result be twice the ‘low large’ result. On the other hand Hamamoto and Nazarewicz [9] have argued that the ‘large’ result should be much more than twice the ‘low large’ result. The actual ratios for the \((x,y)\) and \(Q \cdot Q\) interactions for this calculation (all \(0\hbar\omega\) configurations plus \(2\hbar\omega\) excitations) are
For $^{10}$Be we should actually compare the theoretical models with the combined result (‘$T = 1’ + ‘T = 2’$).

These results indicate that the simple models are not too bad as a first orientation but there are fluctuations -sometimes the ratio is less than two, sometimes greater. We will discuss these matters in more detail in the context of the figures.

In Figs 1 and 2 we show the cumulative energy weighted isovector orbital strength distributions in $^{8}$Be for the $(x, y)$ interaction and for the Hamiltonian $H_Q$ i.e. the quadrupole-quadrupole interaction. The results for the two interactions are quite similar. The outstanding feature is that there are two rises separated by a rather wide plateau. For the $(x, y)$ interaction the first rise is to a plateau at about 12 $\mu_N^2 MeV$ followed by a second rise to about 20.8 $\mu_N^2 MeV$. For the $Q \cdot Q$ interaction the first plateau is at 10.25 $\mu_N^2 MeV$ and the second at 14 $\mu_N^2 MeV$. A simple self-consistent Nilsson model was shown to give the second plateau at twice the energy of the first plateau [5][6]. That is to say the high energy rise was equal to the low energy rise. In the more detailed calculations performed here the high energy rise is less than the low energy rise.

We next turn to $^{10}$Be. Here there are two channels: $J = 0^+ T = 1 \rightarrow J = 1^+ T = 1$ and $J = 0^+ T = 1 \rightarrow J = 1^+ T = 2$. Let us discuss the latter channel first. The behaviour for $T = 2$ in $^{10}$Be is similar to that for $T = 1$ in $^{8}$Be. As shown in Figs 3 and 4 for the $(x, y)$ and $Q \cdot Q$ interactions
respectively, there are two rises separated by a plateau and here the second rise is about twice the first rise for the \((x,y)\) interaction. For the \(Q \cdot Q\) interaction (with \(\chi = 0.3615\)) the low energy rise is to \(1.7 \mu^2_N \text{MeV}\) and the next rise is to \(2.6 \mu^2_N \text{MeV}\) -only 1.5 to one.

There is a big difference in the cumulative energy weighted distributions, shown in Figs 5 and 6, for the \(J = 0^+ T = 1 \rightarrow J = 1^+ T = 1\) channel. For the \((x,y)\) interaction the first plateau (at about \(2.5 \mu^2_N \text{MeV}\)) is not very flat, but the most outstanding feature in the curve is that the high energy rise is much larger than the low energy rise. The energy weighted sum reaches up to about \(8 \mu^2_N \text{MeV}\). Thus the high energy rise is over three time the low energy rise. For the \(Q \cdot Q\) interaction, the first plateau is better defined -it is located at \(1 \mu^2_N \text{MeV}\) and the cumulative sum extends to about \(3.8 \mu^2_N \text{MeV}\).

## 5 The Zheng-Zamick Sum Rule

Energy weighted sum rules for magnetic dipole transitions, be they spin or orbital, are highly model dependent. An energy weighted sum rule for isovector orbital magnetic dipole transitions for the quadrupole-quadrupole interaction \(Q \cdot Q\) was developed by Zheng and Zamick [5]. This was motivated by the work of the Darmstadt group [10] [11] showing a linear relationship between summed orbital \(B(M1)\) strength and the square of the deformation parameter i.e. \(\delta^2\).

The result was

\[
\sum_n (E_n - E_0) B(M1)_o = \frac{9\chi}{16\pi} \left\{ \sum_i [B(E2, 0_1 \rightarrow 2_i)_{\text{isoscalar}} - B(E2, 0_1 \rightarrow 2_i)_{\text{isovector}}] \right\} \quad (\text{EWSR})
\]

where \(B(M1)_o\) is the value for the isovector orbital \(M1\) operator \((g_{\pi\pi} = 0.5 \ g_{\nu\nu} = -0.5 \ g_{s\pi} = 0 \ g_{s\nu} = 0)\) and the operator for the \(E2\) transitions
is $\sum_{\text{protons}} e_p r^2 Y_2 + \sum_{\text{neutrons}} e_n r^2 Y_2$ with $e_p = 1$, $e_n = 1$ for the isoscalar transition, and $e_p = 1$, $e_n = -1$ for the isovector transition.

Let us now describe in detail how this sum rule works. The sum rule should work for single-shell as well mixed-shell space.

We first consider the case of $^8\text{Be}$. We have the following values in a large space calculation for the $H_Q$ interaction corresponding to orbital $M1$ excitations from the $J = 0^+ T = 0$ ground state to all $1^+ T = 1$ states:

1. Energy weighted isovector orbital $M1$ strength: $14.04 \mu_N^2 \text{MeV}$
2. The isoscalar summed strength $B(E2; 1, 1)$: $237.46 e^2 \text{fm}^4$
3. The isovector summed strength $B(E2; 1, -1)$: $89.611 e^2 \text{fm}^4$
4. The right hand side ($\frac{g\chi}{16\pi} = 0.1032$): $15.25 \mu_N^2 \text{MeV}$.

We don’t get exact agreement ($14.04 \mu_N^2 \text{MeV}$ vs. $15.25 \mu_N^2 \text{MeV}$) but it is reasonably close. One possible reason for the disagreement is that spurious states have been removed and/or that only $2\hbar\omega$ excitations to the $\Delta N = 2$ shell are taken into account [6] [10].

There have been other approaches, especially in the context of the Interacting Boson Model [3] which relate the energy weighted orbital magnetic sum to the $B(E2)$ of the lowest $2^+$ state. As a matter of curiosity we shall examine in our calculation what happens if we take only the lowest $2^+$ state in the right hand side of the sum rule ($EWSR$).

For the $2_1^+$ state in $^8\text{Be}$ we obtain (in our large space calculation) $B(E2; 1, 1) = 196.76$ and $B(E2; 1, -1) = 0$ (because the $2_1^+$ state has $T = 0$). The right hand side becomes $20.30 \mu_N^2 \text{MeV}$. We get a larger answer using the lowest $2^+$ state than we do if we use all $2^+$ states in the $0\hbar\omega$ and $2\hbar\omega$ space. The reason for this is that when the lowest $2^+$ state is excluded, the isovector $B(E2)$ is larger than the isoscalar $B(E2)$. 

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Can we also get agreement for the sum rule in the small space \((0s)^4(0p)^4\) for \(^8\)Be? Now the numbers are:

1. Energy weighted isovector orbital \(M1\) strength: \(6.411 \mu^2_N MeV\)
2. The isoscalar summed strength \(B(E2; 1, 1)\): \(72.54 e^2 fm^4\)
3. The isovector summed strength \(B(E2; 1, -1)\): \(10.37 e^2 fm^4\)
4. The right hand side \((\chi_{16} = 0.1032)\): \(6.414 \mu^2_N MeV\).

We get perfect agreement.

We next consider \(^{10}\)Be. The relevant numbers for the large space calculation are:

1. Energy weighted \(M1\) strength: \(J = 0^+, T = 1 \rightarrow J = 1^+, T = 1\) \(3.811 \mu^2_N MeV\)
   \(J = 0^+, T = 1 \rightarrow J = 1^+, T = 2\) \(2.602 \mu^2_N MeV\)

   **Left Hand Side**

2. \(\sum B(E2; 1, 1)\) \(J = 0^+, T = 1 \rightarrow J = 2^+, T = 1\) \(251.4 e^2 fm^4\)
3. (a) \(\sum B(E2; 1, -1)\) \(J = 0^+, T = 1 \rightarrow J = 2^+, T = 1\) \(94.02 e^2 fm^4\)
   (b) \(\sum B(E2; 1, -1)\) \(J = 0^+, T = 1 \rightarrow J = 2^+, T = 2\) \(48.78 e^2 fm^4\)
   (c) \(\sum B(E2; 1, -1)\) Total \(142.8 e^2 fm^4\)
4. Right hand Side \((\chi_{16} = 0.0647)\): \(7.029 \mu^2_N MeV\)

For \(^{10}\)Be we are also curious to see what happens if we use only the lowest \(2^+\) state in the right hand side of the sum rule. But we have to be careful! It turns out that there is substantial \(B(E2)\) strength to the two lowest \(J = 2^+\) states. This can be understood from the fact that with a \(Q \cdot Q\) interaction in a small space calculation \(((0s)^4(0p)^6)\) the two lowest \(2^+\) states are exactly degenerate. The states belong to the \([f] = [42]\) representation. The \(Q \cdot Q\) interaction fails to remove the degeneracy of these states. Another way of
stating this is that the $(\lambda\mu)$ values for both states are (22), and the allowed values of the $K$ quantum number in the Nilsson scheme are $K = \mu, \mu - 2$, etc. Thus the $2^+$ states have $K = 0$ and $K = 2$.

When we go to the large space calculation with a $Q\cdot Q$ interaction, limiting the excitations to $2\hbar\omega$, the degeneracy is removed but the states are still fairly close together. The calculated values are:

|     | $E2(1, 1)$          | $E2(1, -1)$          |
|-----|---------------------|---------------------|
| $2^+_1$ | $E = 2.08\ MeV$ | 64.94               | 12.32               |
| $2^+_2$ | $E = 2.92\ MeV$ | 93.38               | 10.11               |

Thus, using the calculated values of $B(E2)$ for the lowest two $2^+ T = 1$ states in $^{10}$Be, we get for the right hand side of the sum rule a value of 8.80 $\mu^2_N\text{MeV}$. Again, as in the case of $^8$Be, this is larger than the value 7.03 $\mu^2_N\text{MeV}$ that is obtained by using all $2^+ T = 1$ and all $2^+ T = 2$ states.

The corresponding numbers in small space for $^{10}$Be are:

1. Energy weighted $M1$ strength: $J = 0^+, T = 1 \rightarrow J = 1^+, T = 1$ $J = 0^+, T = 1 \rightarrow J = 1^+, T = 2$ $0.7597\ \mu^2_N\text{MeV}$ $1.266\ \mu^2_N\text{MeV}$

2. $\sum B(E2; 1, 1)$ $J = 0^+, T = 1 \rightarrow J = 2^+, T = 1$ $68.31\ e^2\text{fm}^4$

3. (a) $\sum B(E2; 1, -1)$ $J = 0^+, T = 1 \rightarrow J = 2^+, T = 1$ $33.80\ e^2\text{fm}^4$

(b) $\sum B(E2; 1, -1)$ $J = 0^+, T = 1 \rightarrow J = 2^+, T = 2$ $3.203\ e^2\text{fm}^4$

(c) $\sum B(E2; 1, -1)$ Total $37.003\ e^2\text{fm}^4$

4. Right hand Side ($\frac{9\chi_1}{16\pi} = 0.0647$): $2.026\ \mu^2_N\text{MeV}$

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6 A discussion of the calculated B(E2) values

Although the main thrust of this work is on $B(M1)$ values, we have established a connection with $B(E2)$ for the orbital case. It is therefore appropriate to discuss the calculated $B(E2)$ values — comparing the behaviours in $^8$Be and $^{10}$Be, and comparing the different interactions that have been used (see tables III and IV.)

In making the comparison between $^8$Be and $^{10}$Be we should lump together the $B(E2)$'s of the first two $2^+$ states in $^{10}$Be because with the interactions used here — especially $Q \cdot Q$ — these states are nearly degenerate. (However, experimentally the states are well separated $E_{2_1^+} = 3.368$ MeV and $E_{2_2^+} = 5.958$ MeV). When this is done we find that the $B(E2)$ values in the two nuclei are comparable.

For the $(x,y)$ interaction, the calculated (large space) value of $B(E2)$ to the lowest two $2^+$ states in $^{10}$Be is 22.90 $e^2 fm^4$, whereas it is 25.97 $e^2 fm^4$ to the lowest $2^+$ state in $^8$Be. With the $Q \cdot Q$ interaction the two values are respectively 46.40 and 49.16 $e^2 fm^4$. One big difference between the two interactions is the ratio of large to small space values for corresponding $B(E2)$ values. In $^8$Be the ratio of the large sum to the small sum is 1.98 for the $(x,y)$ interaction whereas it is much larger 3.28 for the $Q \cdot Q$ interaction. There is much more core polarization with the $Q \cdot Q$ interaction than with the $(x,y)$ interaction.

There have been many discussions concerning the correlation of summed orbital $M1$ strength and the $B(E2)$ from the $J = 0^+$ ground state to the first $2^+$ state. The latter is an indication of the nuclear deformation. We have noted that the calculated values of $B(E2)$ are about the same in $^8$Be and $^{10}$Be. Thus we would expect the orbital $M1$ strengths in the two nuclei to be about the same.

There is a certain 'vagueness' in what is meant by 'strength'. It is clear
that the experiments thus far sample only low energy strengths up to about 6 MeV in heavy deformed nuclei [10] [11]. Also some of the theories involve summed strength per se and others involve the energy weighted strength. Rather than enter into deep philosophical discussions about what is meant by strength, we will give a variety of ratios of strengths $^{10}\text{Be} / ^8\text{Be}$ in Table V. We see that all the ratios, be they non-energy weighted or energy weighted, be they in small spaces or in large spaces, are substantially less than one. In forming the ratios, we added for the numerator ($^{10}\text{Be}$) the $J = 0^+ \ T = 1$ to $J = 1^+ \ T = 1$ and $J = 0^+ \ T = 1$ to $J = 1^+ \ T = 2$ strengths.

7 A comparison of the $J = 1^+ \to 0_1^+$ and $J = 1^+ \to 2_1^+$ Magnetic Dipole Transitions

Let us assume that the $0_1^+$ and $2_1^+$ states are members of a $K = 0$ rotational band and that the $1^+$ states have $K = 1$. We can then use the rotational formula of Bohr and Mottelson (Eq. 4-92) in their book [12] ($K_1 = 0, \ K_2 = 1$) (We use the notation \[ \begin{pmatrix} J_1 & J_2 & J \\ M_1 & M_2 & M \end{pmatrix} \] for a Clebsch-Gordan coefficient):

$$\langle K_2I_2|\mathcal{M}(\lambda)|K_1 = 0I_1\rangle = \sqrt{2(2I_1 + 1)} \begin{pmatrix} I_1 & \lambda & I_2 \\ 0 & K_2 & K_2 \end{pmatrix} \langle K_2|\mathcal{M}(\lambda, \nu = K_2)|K_1 = 0\rangle$$

From this we can easily deduce

$$r = \frac{B(M1)_{J = 1^+, K = 1 \to J = 2^+, K = 0}}{B(M1)_{J = 1^+, K = 1 \to J = 0^+, K = 0}} = \frac{1}{2}$$

Note, however, that the experimental ratio for $^8\text{Be}$ from the $J = 1^+ \ T = 1$ state at 17.64 MeV (see section 1) is $0.12 / 0.15 = 0.8$. Bohr and Mottelson later discuss corrections to the above simple formula.
We can obtain a value for the above ratio by forming an intrinsic state and projecting out states of angular momentum $J = 0$ and $J = 2$. We assume an extreme prolate shape for $^8$Be and put the four valence nucleons ($N \uparrow, N \downarrow, P \uparrow, P \downarrow$) in the lowest Nilsson orbit with $\Lambda = 0$. The asymptotic wave function is $N r Y_{10}$.

The $J = 0$ wave function is

$$N' \sum_L \begin{pmatrix} 1 & 1 & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & L & 0 \\ 0 & 0 & 0 \end{pmatrix} [L L]^0 = 0.74536[0 0]^0 + 0.66666[2 2]^0$$

where the notation $[L_\pi L_\nu]^J$ is used.

The $J = 2$ wave function is

$$N'' \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} [0 2]^2 + \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} [2 0]^2$$

$$+ \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} [2 2]^2$$

i.e.

$$\psi^{J''=2} = 0.62361[0 2]^2 + 0.62361[2 0]^2 + 0.47141[2 2]^2$$

We don’t actually have to specify the $J = 1^+$ state in detail to get the ratio. We note that only the component $[2 2]^1$ of the $J = 1$ wave function can contribute to the $M1$ transition. The probability of this component factors out in the ratio. With some additional Racah algebra, we can show that $r = \frac{7}{8} = 0.875$. This should be compared with the value $r = \frac{1}{2}$ of the simple rotational formula and with the experimental value $r_{exp} = 0.8$. We see that we get better agreement by using this projection method.

To complete the story, using results described in the next section, we are able to deduce that for $^8$Be $B(M1)_{1^+ \rightarrow 2^+_1} = \frac{7}{4\pi} \mu_N^2$. 

16
8 Supermultiplet Scheme with a $Q \cdot Q$ interaction

8.1 A. Supermultiplet Scheme in $^8Be$

The $Q \cdot Q$ interaction that we have been using fits in nicely with the $L - S$ supermultiplet scheme of Wigner [13]. For the $p$ shell the unitary group $U(3)$ is relevant since there are three states: $L = 1, M = 1, 0$ and $-1$. A very useful reference for this section is the book by Hammermesh [14].

If the Hamiltonian were a Casimir operator of $U(3)$ all states of a given special symmetry $[f] = [f_1, f_2, f_3]$ would be degenerate. For the case of $1p$ shell a state with a given particle symmetry $[f_1, f_2, f_3]$ is identical to a quantum oscillator symmetry state $[\lambda, \mu] = (f_1 - f_2, f_2 - f_3)$. The states $(\lambda, \mu, L)$ are eigenstates of our $Q \cdot Q$ interaction which is a linear combination of the Casimir operator of $SU(3)$ and an $L \cdot L$ interaction. The latter gives rise to a terminating rotational $L(L + 1)$ spectrum for states of different $L$ but with the same $[f]$. Amusingly, as has been pointed out by many, one gets identical bands in all $p$ shell nuclei with this model provided the coefficient of $L \cdot L$ is fixed.

Unlike in the $s, d$ shell, nothing new is added by using the quantum numbers $(\lambda, \mu)$ instead of $[f_1, f_2, f_3]$ for $1p$ shell states. This is because the number of different $M$ states available for particles (3) coincides with the number of possible directions for oscillator quanta ($a_{x}^{\dagger}$, $a_{y}^{\dagger}$, $a_{z}^{\dagger}$) and a single creation operator corresponds to each particle.

In more detail, the Casimir operator is $\tilde{C}_2 = Q \cdot Q - 3\vec{L} \cdot \vec{L}$. Hence,

$$\langle -\chi Q \cdot Q \rangle_{\lambda \mu L} = \bar{\chi} \left[ -\langle \tilde{C}_2 \rangle_{\lambda \mu} + 3L(L + 1) \right]$$
$$= \bar{\chi} \left[ -4(\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu) + 3L(L + 1) \right]$$
where $\bar{\chi} = \chi \frac{5b^4}{32\pi}$ with $b$ the harmonic oscillator length parameter ($b^2 = \frac{\hbar}{m\omega}$).

The magnetic dipole modes in the $LST$ representation are:

$L = 1 \quad S = 0 \quad T = 0$  \quad $L = 0 \quad S = 1 \quad T = 1$ (isovector spin mode)
$L = 1 \quad S = 0 \quad T = 1$ (scissors mode) \quad $L = 1 \quad S = 1 \quad T = 0$
$L = 0 \quad S = 1 \quad T = 0$ \quad $L = 1 \quad S = 1 \quad T = 1$

With the $Q \cdot Q$ interaction that we have chosen, transitions from the $L = 0 \quad S = 0$ ground state in $^8Be$ to all of these modes except one will vanish. The only surviving mode is the $L = 1 \quad S = 0 \quad T = 1$ scissors mode.

Let us give briefly the energies and some properties of the states in $^8Be$ ($\bar{\chi} = 0.1865$):

(a) $[f]=[4,0]$ ($\lambda,\mu=(4,0)$ Ground State Band

| $L$ | $S$ | $T$ | $\frac{E}{\bar{\chi}}$ | $E^*(MeV)$ |
|-----|-----|-----|----------------|------------|
| 0   | 0   | 0   | $-112$         | 0          |
| 2   | 0   | 0   | $-94$          | 3.36       |
| 4   | 0   | 0   | $-52$          | 11.19      |

(b) $[f]=[3,1]$ ($\lambda,\mu=(2,1)$ contains the scissors mode ($L = 1, S = 0, T = 1$).

Note that the $(S,T)$ combinations $(0,1), (1,0)$ and $(1,1)$ are allowed.

| $L$ | $\frac{E}{\bar{\chi}}$ | $E^*(MeV)$ |
|-----|----------------|------------|
| 1   | $-58$         | 10.07      |
| 2   | $-46$         | 12.31      |
| 3   | $-28$         | 15.67      |
(c) $[f]=\{2,2\} \ (\lambda, \mu)=(0,2)$ The $(S, T)$ combinations $(0,0), (0,2), (2,0)$ and $(1,1)$ are allowed.

\[
\begin{array}{|c|c|c|}
\hline
L & \frac{E}{\bar{\chi}} & E^*(MeV) \\
\hline
0 & -40 & 13.43 \\
2 & -22 & 16.78 \\
\hline
\end{array}
\]

(d) $[f]=\{2,1,1\} \ (\lambda, \mu)=(1,0)$ The $(S, T)$ combinations $(0,1), (1,0), (1,1), (1,2)$ and $(2,1)$ are allowed.

\[
\begin{array}{|c|c|c|}
\hline
L & \frac{E}{\bar{\chi}} & E^*(MeV) \\
\hline
1 & -10 & 19.02 \\
\hline
\end{array}
\]

Note that this supermultiplet also has a state with the quantum numbers of the scissors mode $L = 1 \ S = 0 \ T = 1$.

Some further comments are in order. The scissors mode state in $^{156}Gd$, as a single band state originally discovered in electron scattering [L7], was found when finer resolution ($\gamma, \gamma'$) experiments were performed to consist of many states [L8, L9]. This was a beautiful example of intermediate structure. The supermultiplet scheme here affords a concrete example of the origin of the intermediate structure. Our scissors mode state at an energy of $-58 \bar{\chi}$ is degenerate with an $L = 1 \ S = 1 \ T = 1$ state. If we introduce spin-dependent interactions the two states will admix and the degeneracy will be removed. We will get intermediate structure.

### 8.2 B. Supermultiplet Scheme in $^{10}Be$

We now give the energies and some properties of the states in $^{10}Be$ ($\bar{\chi} = 0.1286$): (a) $[f]=\{4,2\} \ (\lambda, \mu)=(2,2)$ (includes ground state).
Allowed states:

| $L$ | $S$ | $T$ | $\frac{E}{\chi}$ | $E^*(MeV)$ |
|-----|-----|-----|------------------|-------------|
| 0   | 0   | 1   | -96              | 0           |
| $2_1$ | 0   | 1   | -78              | 2.32        |
| $2_2$ | 0   | 1   | -78              | 2.32        |
| 3   | 0   | 1   | -60              | 4.63        |
| 4   | 0   | 1   | -36              | 7.72        |

Note that the $2_1^+$ and $2_2^+$ states are degenerate in this scheme. This arises from the fact that, in a rotational picture, the $K$ values that are allowed are $\mu$, $\mu - 2$, .... Thus we have a degeneracy of $J = 2_1^+ K = 0$ and $J = 2_2^+ K = 2$. This degeneracy does not correspond to the experimental situation -the $2_1^+$ and $2_2^+$ states are at 3.368 $MeV$ and 5.960 $MeV$ respectively -well separated.

As a practical matter this degeneracy gives problems for the shell model code $OXBASH$ [20]. Shell model routines often give wrong answers for transition rates when the states involved are degenerate. To overcome this difficulty, we have introduced weak additional terms in the Hamiltonian to remove the degeneracy e.g. we use a weak one-body spin-orbit interaction $-\xi \vec{l} \cdot \vec{s}$ with $\xi = 0.1$ MeV.

(b) Two degenerate bands $[f]=[4,1,1]$ ($\lambda, \mu$)=($3,0$), $[f]=[3,3]$ ($\lambda, \mu$)=($0,3$)

Allowed states:

| $L$ | $S$ | $T$ | $\frac{E}{\chi}$ | $E^*(MeV)$ |
|-----|-----|-----|------------------|-------------|
| 1   | 1   | 1   | -66              | 3.853       |
| 3   | 1   | 1   | -36              | 7.716       |
We get the low-lying $1^+$ states (one from [4,1,1] and one from [3,3]). Note however that we have $L = 1$, $S = 1$, hence the states cannot be excited by either the orbital operator or the spin operator.

(c) Band which contains the scissors mode $[f]=[3,2,1]$ ($\lambda, \mu$)=($1,1$)

| $L$ | $S$ | $T$ | $\bar{E}/\chi$ | $E^*(MeV)$ |
|-----|-----|-----|----------------|------------|
| scissors mode 1 | 0 | 1 | $-30$ | 8.49 |
| scissors mode 1 | 0 | 2 | " | " |
| 1 | 1 | 1 | " | " |
| 1 | 0 | 2 | " | " |
| 1 | 2 | 1 | " | " |

There are also several $L = 2$ states with $\bar{E}/\chi = -18$ and $E^* = 10.03$ MeV.

Note that the $L = 1$, $S = 0$ scissors modes finally make their appearance. There are two branches -one with isospin $T = 1$ and one with isospin $T = 2$. These two scissors mode states are degenerate in energy in the supermultiplet scheme $^{10}Be$.

(d) $[f]=[2,2,2]$ ($\lambda, \nu$)=(0,0)

| $L$ | $\bar{E}/\chi$ | $E^*(MeV)$ |
|-----|----------------|------------|
| 0 | 0 | 12.34 |

The (S T) values are (0 1), (1 0), (1 2), (2 1), (0 4) and (4 0).

8.3 B(M1) Transitions in the $SU(3)$ Scheme

For $^8Be$ and $^{10}Be$ the strength for orbital M1 transitions from $J^\pi = 0^+$ to scissors mode states can be obtained in the $SU(3)$ scheme by observing
that in the process the ground state intrinsic state is transformed to the corresponding intrinsic state of $1^+$ state. For example in $^{8}\text{Be}$, the orbital isovector M1 operator, $\mu = \sum_i l_i \tau^i_z = (L^\pi - L^\nu)$, transforms the four nucleon intrinsic state $(4, 0)$ into the intrinsic state of $1^+\,1$ state that is $(21)$. The operator $(L^\pi - L^\nu)$ is the generator of the scissors mode. The orbital part of ground state and $1^+\,1$ state of $^{8}\text{Be}$ in SU(3) scheme can be projected out from the corresponding maximum weight intrinsic states $|[f](\lambda, \mu, \epsilon, \Lambda, \rho >)$ where $\epsilon = 2\lambda + \mu$ and $\Lambda = \rho = \frac{1}{2}\mu$, by using the following projection[15],

$$|[f](\lambda, \mu)K, L, M> = \frac{(2L + 1)}{a(\lambda \mu L K)} \int d\Omega D^{L}_{M,K}(\Omega)R(\Omega)|[f](\lambda, \mu, \epsilon, \Lambda, \rho >).$$  
(1)

Eq.(1) is a general equation for projecting out orbital part of the wave function for a given L from a $(\lambda, \mu)$ state. In particular for $^{8}\text{Be}$ we have,

$$|[4](40)0, 0, 0> = \frac{1}{a(4000)} \int d\Omega D^{0}_{00}(\Omega)R(\Omega)|[4](40)8, 0, 0 >$$  
(2)

and

$$|[31](21)1, 1, 0> = \frac{3}{a(2111)} \int d\Omega D^{1}_{01}(\Omega)R(\Omega)|[31](21)5, \frac{1}{2}, -\frac{1}{2} >.$$  
(3)

Knowing the $[f]$ representation of the $(\lambda, \mu)$ states one can look up the corresponding conjugate charge spin states to get definite JT states.

The orbital magnetic transition strength $B(M1)$ between these states is calculated by a method similar to that outlined in Appendix A of ref.[21] and is found to be

$$B(M1) = \frac{9}{16\pi} |<[31](21)110, S = 0, T = 1|(L^\pi_0 - L^\nu_0)|[4](40)000, S = 0, T = 0|^2$$

$$= \frac{2}{\pi} \mu^2_n = 0.637 \mu^2_n$$

(4)

In an analogous fashion one can calculate the scissors mode M1 transition strengths for the nucleus $^{10}\text{Be}$. We find the following results for SU(3) limit transition strengths in $^{10}\text{Be}$:
\[
B(M1)(0^+1 \to 1^+) = \frac{9}{32\pi} \mu_n^2 = 0.0895\mu_n^2
\]
\[
B(M1)(0^+1 \to 1^+2) = \frac{15}{32\pi} \mu_n^2 = 0.1492\mu_n^2.
\]

8.4 Realistic Spin-Orbit Interaction and Restoration of SU(3) Symmetry

As pointed out before, an important role of spin dependent part of interaction is to remove the degeneracies present in the SU(3) limit by mixing up the same final angular momentum states arising due to a given intrinsic state as well as from different intrinsic states. In a realistic interaction, the relative strengths of spin dependent and spin independent part of interaction determine whether the wavefunctions are close to SU(3) scheme or a (j-j) coupling scheme is a better description of the system.

To understand further the part played by spin independent part of the full realistic interaction in the restoration of SU(3) symmetry, we consider a small space calculation with the full spin-orbit part of the \((x,y)\) interaction plus a variable \(Q.Q\) interaction. Figure (7) is a plot of isovector orbital, spin and total strength for M1 transitions from \(J = 0^+_1 T = 0 \to J = 1^+_1 T = 1\) states versus \(t\), the parameter multiplying the full \(Q.Q\) interaction matrix elements for \(^8\)Be. For the spin part we use the operator \(9.412 \sum \sigma_t z\) i.e. we include the large isovector factor. In \(^8\)Be, with increasing \(t\) the orbital isovector strength is seen to approach the SU(3) limit value of \(0.637\mu_n^2\). The contribution of isovector spin transition, on the other hand to total B(M1) decreases as \(t\) becomes large. This is because the SU(4) limit is being approached and in this limit the spin contribution vanishes.

In Fig.(7) with only spin-orbit part of realistic interaction in play, the calculated M1 transition strength for \(0^+0\) to \(1^+1\) transitions has a large spin flip
contribution and is found to be as large as $9.7 \mu_n^2$. In a small space calculation with full realistic interaction $(x, y)$ (Table I), a total $B(M1)$ value of $1.0547 \mu_n^2$ is obtained with an isovector orbital transition strength of $0.67 \mu_n^2$ and an isovector spin contribution of $0.38 \mu_n^2$. Of course to get the physical $B(M1)$ we add the spin and orbital amplitudes and square. The spin $B(M1)$ is a factor of 25 lower here than the $t = 0$ value in Fig 7. It still has some effect because of the factor 9.412. We may note that the orbital transition strength arising due to full realistic interaction is very close to the $SU(3)$ limit indicating that the realistic interaction favors a restoration of $SU(3)$ symmetry.

The large space realistic interaction calculation inspite of the correlations induced by shell mixings results in a $B(M1)$ value $1.2866 \mu_n^2$ and isovector orbital transition strength of $0.728 \mu_n^2$ indicating that the wavefunctions are still very close to $SU(3)$ wave functions.

In $^{10}$Be the situation is more interesting due to splitting of scissors mode strength into two degenerate states in $SU(3)$ limit. Figures (8) and (9) show the orbital and spin part respectively of M1 transition strength for transitions from ground state to $J = 1^+ T = 1$ states, $J = 1^+ T = 2$ states and all $J = 1^+$ states. For ground state to $J = 1^+ T = 1$ transitions the orbital $B(M1)$ is seen to dip to a minimum for $t = 0.3$ before it starts increasing so as to approach its $SU(3)$ limit. An opposite trend is observed in the corresponding spin strength that shows some increase, reaches a maximum and then tends to the $SU(4)$ limit of zero. The characteristic behaviour at $t = 0.3$ is possibly a manifestation of a shape change at small deformation before the system stabilizes by acquiring a permanent deformation. The M1 transition sums for ground state to $J = 1^+ T = 2$ states, show a behaviour similar to that observed for ground state to $J = 1^+ T = 1$ transitions in $^8$Be.
9 Magnetic Dipole Transitions To Individual States

We here present several tables of magnetic dipole transitions from the $J = 0^+$ ground states of $^8Be$ and $^{10}Be$ to individual $J = 1^+$ states. We use both the $(x, y)$ interaction with $x = 1$, $y = 1$ and the $Q \cdot Q$ interaction.

Concerning the latter, we learned in the previous section that there are many degenerate states in the $0\hbar\omega$ calculation when a $Q \cdot Q$ interaction is used. Unfortunately, most shell model routines, including the one used here, give erroneous results for transition rates when there are degeneracies. In all our small space ($0\hbar\omega$) calculations using $Q \cdot Q$ we have added a small spin-orbit interaction $-\xi \vec{l} \cdot \vec{s}$ with $\xi = 0.1$ MeV. This works but it introduces an artificial complexity in our tables. However, it is easy to see by eye what states would be degenerate if the spin-orbit interaction is removed. Alternatively, one can use the analytic expressions for the energies in the previous section.

The columns in Tables VI through XIII are defined as in Table I:

|                | $g_{l\pi}$ | $g_{l\nu}$ | $g_{s\pi}$ | $g_{s\nu}$ |
|----------------|------------|------------|------------|------------|
| (a) Isovector Orbital | 0.5        | -0.5       | 0          | 0          |
| (b) Isovector Spin | 0          | 0          | 0.5        | -0.5       |
| (c) Physical      | 1          | 0          | 5.586      | -3.826     |

9.1 Calculated Magnetic Dipole Transitions in $^8Be$

In Tables VI and VII we present the details of the $0\hbar\omega$ calculated $B(M1)$ values from the $J = 0^+$, $T = 0$ ground state of $^8Be$ to the $J = 1^+$, $T = 1$ excited states.
For the realistic \((x, y)\) interaction, the isovector orbital strength is concentrated in three states at 13.7, 16.6 and 18.0 MeV. The sum of the orbital strengths is \(\sum B(M1) \uparrow = 0.67 \mu_N^2\). This is in fair agreement with the experimental value \(B(M1) \uparrow = 0.81 \mu_N^2\) (which is actually deduced from the downward \(\gamma\) decay of the 17.64 MeV \(J = 1^+, T = 1\) state to the ground state). However, in the experiment all the strength is concentrated in one state whereas in our calculation we have considerable fragmentation. On the other hand, if we look at the physical transitions, there is much more concentration in one state at 13.73 MeV with \(B(M1) \uparrow = 0.72 \mu_N^2\). We will discuss this more soon.

With the \(Q \cdot Q\) interaction, all the orbital strength is concentrated in the (2-fold degenerate) state at 10.1 MeV with a summed strength \(B(M1) \uparrow = 0.64 \mu_N^2\), very similar to that of the \((x, y)\) interaction. The energy is too low compared with experiment, but we must remember that we did not renormalize the strength \(\chi\) to allow for \(\Delta N = 2\) admixtures. Note that the isovector spin transitions are zero with the \(Q \cdot Q\) interaction. This is because we are at the \(SU(4)\) limit since \(Q \cdot Q\) is a spin-independent interaction.

Note that with the \(Q \cdot Q\) interaction the summed orbital strength is \(\frac{2}{3} \mu_N^2\), confirming the expressions that were derived in the previous section. Coming back to the \((x, y)\) interaction, we see that here also the isovector spin transitions are very weak. But note that for the 13.73 MeV state whereas the orbital value of \(B(M1) \uparrow\) is 0.2569 \(\mu_N^2\) and the spin value is 0.0013 \(\mu_N^2\), the physical value is 0.7155 \(\mu_N^2\). The reason is that the spin and orbit amplitudes add coherently and that the spin amplitude is multiplied by the factor 9.412.

For other states there is destructive interference between spin and orbit. For example, for the 16.64 MeV state, the value of \(B(M1) \uparrow\) is 0.234 \(\mu_N^2\) for the orbital case but only 0.065 \(\mu_N^2\) for the physical case.
9.2 Calculated Magnetic Dipole Transitions in $^{10}Be$

In Tables VIII and IX we present the details of the $0\hbar\omega$ calculated $B(M1)$ values from the $J = 0^+ T = 1$ ground state of $^{10}Be$ to the $J = 1^+ T = 1$ and to $J = 1^+ T = 2$ excited states. We caution the reader that whereas for $^{8}Be$ we presented the results in units of $\mu_N^2$ (Tables VI and VII), for $^{10}Be$ we use $10^{-2} \mu_N^2$ as the unit. The reason for this is that the orbital transitions to individual states in $^{10}Be$ are considerably smaller than those in $^{8}Be$.

Let us look at the $Q \cdot Q$ interaction (Table IX) first. There are several outstanding features which are explained in the previous section on $L S$ coupling and supermultiplet symmetry.

The first two $J = 1^+ T = 1$ states are degenerate at $E^* = 3.86$ MeV. They carry no spin or orbital strength from the from the ground state. The [f] symmetries are [4 1 1] and [3 3]. They have additional quantum numbers $L = 1 \ S = 1 \ T = 1$. Since the isovector orbital operator ($\vec{L}_\pi - \vec{L}_\nu$) cannot change both $L$ and $S$ from zero to one, the orbital $B(M1)$ vanishes. A similar argument holds for the isovector spin operator. These lowest two states are therefore not scissors mode states.

Then we have a four fold set of degenerate states with [3 2 1] symmetry at about 8.5 MeV excitation which does include the $L = 1 \ S = 0 \ T = 1$ scissors mode. We note that for $^{10}Be$, the $T = 1$ scissors mode is degenerate with the $T = 2$ scissors mode also at 8.5 MeV excitation. This again is a prediction of the supermultiplet theory.

The summed isovector orbital strength is $\frac{9}{32\pi} \mu_N^2$ from $J = 0^+ T = 1$ to the $J = 1^+ T = 1$ states, and it is $\frac{15}{32\pi} \mu_N^2$ to the $J = 1^+ T = 2$ states. We have in effect a $(2T+1)$ rule:

$$\frac{(2T+1)_{T=2}}{(2T+1)_{T=1}} = \frac{5}{3}$$
This coincides with the ratio of $T = 2$ to $T = 1$ strength.

Recalling that the $^8\text{Be}$ strength was $\frac{2}{\pi} \mu_N^2$, we see that the ratio of total strength $\frac{^{10}\text{Be}}{^{8}\text{Be}}$ is $\frac{3}{2}$.

We now come back to Table VIII which shows the same calculational results with the ‘realistic’ $(x,y)$ interaction. There are several similarities but also some differences with the $Q \cdot Q$ results. Just as with the $Q \cdot Q$ interaction, the orbital transitions to the lowest two $J = 1^+ T = 1$ states at 6.14 and 7.68 $MeV$ are very weak 0.16 and $0.17 \times (10^{-2} \mu_N^2)$ respectively. However, the spin transitions, which with $Q \cdot Q$ were also zero, are now sufficiently strong so as to have a visible effect. For example, the physical $B(M1) \uparrow$ to the 7.68 $MeV$ state is calculated to be $1.85 \mu_N^2$. This is certainly measurable.

As with the $Q \cdot Q$ interaction, the scissors mode states with the $(x,y)$ interaction are at a much higher energy than the lowest two $1^+ T = 1$ states 19 $MeV$. Also, the $J = 1^+ T = 1$ and $T = 2$ excitations are roughly in the same energy range -the $Q \cdot Q$ interaction has them degenerate. The ratio of $T = 2$ to $T = 1$ orbital strength is about the same for the $(x,y)$ interaction as for $Q \cdot Q$ 1.44 vs. $\frac{5}{3}$.

One major difference is that the energy scale is larger for the $(x,y)$ interaction than for $Q \cdot Q$. The lowest and higher energies in Table VIII are 6.14 $MeV$ and 30.96 $MeV$ whereas in Table IX they are 3.85 $MeV$ and 12.35 $MeV$.

In Tables X and XI we present results of large space calculations for $^8\text{Be}$ to be compared with the corresponding small space Tables VI and VII. Likewise in Tables XII and XIII we present the large space results for $^{10}\text{Be}$ to be compared with tables VIII and IX. We do not show all the states here, only the low energy sector. The excitation energies are in general larger in the large space calculations. The major changes occur when one has nearly degenerate levels sharing some strength. For example, the lowest two $1^+$
states in the large space calculations, which are at 7.38 MeV and 9.62 MeV have almost equal $M1$ strengths $0.64 \mu^2_N$ and $0.79 \mu^2_N$. In the small space, the lowest state has only $0.011 \mu^2_N$ and the second one $1.8 \mu^2_N$.

A sensible attitude is to assume that neither calculation is accurate enough to give the detailed distribution of strength between the two states - only the summed strength for the two states should be compared with experiment.

With the $Q \cdot Q$ interaction in the large space, the degeneracy encountered in the small space calculation is removed. In part, this is due to the fact that we do not include the single-particle terms $\sum_{i=j} Q(i) \cdot Q(j)$. These will induce a single-particle splitting between $1s$ and $0d$ in the $N = 2$ shell. But since degeneracies give us trouble in our shell model diagonalizations, we are happy to leave the calculation as is.

With $Q \cdot Q$ the scissors mode strength in $^8Be$ gets pushed up from the small space value of 10.1 MeV to the large space value of 15.5 MeV. For $^{10}Be$ the corresponding numbers are 8.5 MeV and 11.3 MeV for the $J = 1^+$, $T = 1$ states and essentially the same for $J = 1^+$, $T = 2$ states. That is, the near degeneracy of the $T = 1$ and $T = 2$ scissors modes in $^{10}Be$ is maintained in the large space calculation.

Lastly, we reiterate the fact that the shell model calculations here yield not only collective magnetic states but also show intermediate structure. That this structure is a natural occurrence is shown by the supermultiplet model, where for a given $[f]_{L=1}$ there are several $S$ and $T$ values possible. It is of course very difficult to get the details of the intermediate structure to come out right, but it is good to be able to explain the origin of this structure.
10 Additional Comments and Closing Remarks

We can gain further insight into the nature of $^8Be$ and $^{10}Be$ by evaluating the quadrupole moments of the $J = 2^+$ states. A small space calculation gives the following values:

| Nucleus | $J$   | $T$  | Quadrupole Moments                                                                 |
|---------|-------|------|-----------------------------------------------------------------------------------|
| $^8Be$  | $2^+$ | 0    | $Q = -8.02 \, e \, fm^2$ | $Q = -7.86 \, e \, fm^2$ |
| $^{10}Be$ | $2_1^+$ | 1    | $Q = -2.52 \, e \, fm^2$ | $Q = -7.68 \, e \, fm^2$ |
| $^{10}Be$ | $2_2^+$ | 1    | $Q = +2.06 \, e \, fm^2$ | $Q = +6.91 \, e \, fm^2$ |

In the rotational model the quadrupole moment of the $2^+$ of a $K = 0$ band is $-\frac{2}{7}Q_0$ where $Q_0$ is the intrinsic quadrupole moment. Thus a negative $Q$ corresponds to a prolate shape and a positive $Q$ to an oblate shape. From the above, $^8Be$ acts as a normal deformed nucleus of the prolate shape.

It has been pointed out by Harvey that in the $SU(3)$ scheme, whenever $\mu$ is less than or equal to $\lambda$ the nucleus becomes oblate [13]. For the ground state band in $^8Be$ $\lambda$ is bigger than $\mu$ but for $^{10}Be$ $\lambda$ and $\mu$ are equal. The situation with $^{10}Be$ is somewhat confusing. With the $Q \cdot Q$ interaction, which one might think would favor deformation, the quadrupole moment of the $2_1^+$ state drops to $-2.52 \, e \, fm^2$. Recall that for a perfect vibrator, the value of $Q$ is zero, so it would appear that $^{10}Be$ is headed in that direction. However with the realistic interaction, which contains a large spin-orbit interaction that one might think would oppose deformation, the quadrupole moment of $^{10}Be$ becomes more negative - almost the same as that of $^8Be$. Note also that the calculated values of $Q$ for the $2_1^+$ and $2_2^+$ states are nearly equal but opposite to both interactions.

We have learned many interesting things by considering scissors modes in light nuclei. First of all there is evidence for their existence. This evidence
comes strangely from a nucleus whose ground state is unstable -$^8$Be. We learn of the existence from the inverse process i.e. $\gamma$ decay from the $J = 1^+, T = 1$ state at 17.64 $MeV$ to the ground state $^8$Be. The decay to the $2^+_1$ state, presumably a member of a $K = 0$ rotational band, is also observed and this suggests that theoretical studies (and experimental ones as well whenever possible) should be made not only between between $J = 1^+ K = 1$ and $J = 0^+ K = 0$ states but also between $J = 1^+ K = 1$ and $J = 2^+ K = 0$ states. This will make the picture of scissors modes more complete. In this work we considered but one example and showed that the ratio of $J = 1^+$ decay rate to $J = 2^+$ vs $J = 0^+$ deviates from the simple rotational formula result of 0.5. Further studies along these lines are being planned.

To make the picture even more complete, one can also study the decay of $J = 1^+ K = 1$ to $J = 2^+ K = 2$. We were almost forced into such a study by the fact that in the $SU(3)$ scheme there is a two-fold degeneracy of the lowest $J = 2^+$ states in $^{10}$Be [14, 16]. Presumably, these two states are admixtures of $2^+ K = 0$ and $2^+ K = 2$.

The shell model approach used here [20] enables us to study fragmentation or intermediate structure. We find for example that with an electromagnetic probe there are, besides the strong scissors mode states, some almost ‘invisible’ states. These have separately substantial orbital contributions and substantial spin contributions to the magnetic dipole excitations but the physical $B(M1)$ is very small because of the destructive interference of the spin and orbital amplitudes. For example, as seen in Table X, in $^8$Be we calculate that the low lying orbital strength is shared almost equally between two states (at 18.0 $MeV$ and 20.8 $MeV$) -the strengths being 0.22 $\mu^2_N$ and 0.30 $\mu^2_N$ respectively. However, the physical $B(M1)$’s are 0.62 $\mu^2_N$ and 0.12 $\mu^2_N$. Thus one can miss considerable orbital strength into these ‘invisible’ states if one uses only an electromagnetic probe.
Another thing we learn is that although spin excitations are strongly
suppressed they cannot be ignored. In the \( SU(4) \) limit, the spin matrix
elements vanish and there is a clear tendency in our calculations in that
direction. However, since the isovector spin operator is multiplied by a factor
of 9.412, the spin and orbital contributions tend to be on the same footing.

In the example of the above paragraph in the decay of the (calculated)
18.0 \( MeV \) state, the orbital \( B(M1) \) is only 0.22 \( \mu^2_N \) but the physical one which
induces the spin contribution is 0.62 \( \mu^2_N \). In \(^{10}Be\) the first two \( J = 1^+ \) states
are calculated to be excited mainly by the spin operator and the \( B(M1) \)’s
should be substantial 0.5 \( \mu^2_N \). Yet in the \( U(3) - SU(4) \) limit, these lowest
two states \([f]= [4,1,1] \) and \([3,3] \) should not be excited at all either by the spin
or by the orbital operators.

We have found the Wigner supermultiplet scheme \([13]\) combined with the
\( SU(3) \) scheme \([15, 16]\) a very useful guide to the more complicated shell model
calculations that we have performed. There is the added simplicity in the \( p \)
shell that there is a one-to-one correspondence between a given \([f]\) symmetry
and the \( (\lambda, \mu) \) symmetries. Many interesting properties about scissors modes
can be literally read off the pages of the book by Hammermesh \([14]\). For
example, there is the fact that the \( T = 1 \) and \( T = 2 \) scissors mode states
in \(^{10}Be\) are degenerate in energy. This is an exact result with the \( Q \cdot Q \)
interaction in a \( p \) shell calculation. Results very close to this are obtained
with a realistic interaction, but we frankly didn’t notice this until we made
an \( SU(3) \) analysis. Also the non-obvious fact that the lowest two \( 1^+ \) states
in \(^{10}Be\) are not scissors mode states is made clear by such an analysis.

Also the fact that scissors mode states everywhere, including \(^{156}Gd\), have
intermediate structure \([18, 19]\) is made clear by the supermultiplet scheme.
For a given \( L = 1 \) state there are often several \( S, T \) combinations which are
degenerate. The removal of the degeneracy and the mixing of these states e.g.

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by a spin-orbit interaction leads to fragmentation and intermediate structure.

By extending the shell model calculations to ‘large space’ i.e. by including $2\hbar\omega$ excitations, we were able to calculate the cumulative energy weighted strength distribution for isovector orbital excitations. The results which are shown in several figures are characterized by a low energy rise followed by a second plateau. The shapes of the distributions were similar for the two contrasting interactions used here - the ‘realistic’ short range interaction and the schematic quadrupole-quadrupole interaction. The results were compared with the simple Nilsson model [7, 8] which predicts that the energy-weighted sum at high energy (beyond the first plateau) should equal the low energy rise i.e. the ratio $\frac{\text{total}}{\text{low energy}}$ should be 2:1. The actual calculated ratios with the $(x, y)$ interaction are 1.75 for $^8\text{Be}$ and 2.52 for $^{10}\text{Be}$. The corresponding numbers for the $Q \cdot Q$ interaction were 1.37 and 2.33. We see that the Nilsson model is not bad as a first orientation but there are fluctuations. The fact that the above ratios are larger for the $(x, y)$ interaction than for the $Q \cdot Q$ interaction may support the idea of Hamamoto and Nazarewicz [9] that the symmetry energy will cause the ratio to increase. Our calculations however do not support their claim that the high energy part of the energy weighted orbital strength should always be much larger than the low energy part - certainly not for a ‘normal’ rotational nucleus like $^8\text{Be}$. For $^{10}\text{Be}$ the ratio is however somewhat larger than the Nilsson model prediction.

Whether this is due to the atypical properties of $^{10}\text{Be}$ mentioned in the text or is a harbinger of what will happen for most other nuclei remains to be seen. From an experimental point of view, it would be helpful to have more data on $^{10}\text{Be}$. Not only have no $J = 1^+$ states been identified but neither has the $J = 4^+_1$ state been seen. The location of this state might help us decide whether $^{10}\text{Be}$ is rotational or vibrational.
At any rate, we should examine a larger range of nuclei and look into more
detail about the symmetry energy in order to be able to make more definitive
statements about the systematics of the cumulative energy weighted distribu-
tions. We note that the Zheng-Zamick sum rule [4] is able to handle the
divergent behaviour between $^8\text{Be}$ and $^{10}\text{Be}$. This sum rule involves the dif-
ference between isoscalar and isovector summed $B(E2)$ strength, whereas
corresponding expressions by Heyde and de Coster [5] based on the I.B.A.
model \cite{22, 23} as well as empirical analyses \cite{10} involve only $B(E2)$ to the
lowest $2^+$ states. Even here more sharpening up is in order.

Our initial reason for studying lighter nuclei is that they would give us
insight into the behaviour of heavier nuclei, and we could carry out more
complete calculations in the low $A$ region. But then we found many results
which made light nuclei studies fascinating for their own sake. One rather
broad lesson we have learned in the light nucleus study is that there can
be considerable change in going from one nucleus to the next, and perhaps
in heavier nuclei too much effort has been made to make the nuclei fit into
a smooth pattern. For example, we find that whereas in $^8\text{Be}$ the lowest
$1^+$ state is dominantly a scissors mode state, in $^{10}\text{Be}$ the lowest $1^+$ states
are not scissors mode states at all -they can only be reached by the spin
operator. We should perhaps be looking for more variety of behaviour in
heavier nuclei. Lastly the supermultiplet scheme which we found extremely
useful was originally thought to be of interest only in light nuclei where the
spin-orbit interaction is small relative to the residual interaction. However, it
is now being realized that even in heavy nuclei -especially for superdeformed
states this scheme may once again be very relevant. This would make our
light nuclear studies all the more important.
Acknowledgement

This work was supported by the Department of Energy Grant No. DE-FG05-86ER-40299. S.S. Sharma would like to thank the Department of Physics at Rutgers University for its hospitality and to acknowledge financial support from CNPq, Brazil. We thank E. Moya de Guerra for familiarizing us with her work in collaboration with J. Retamosa, J.M. Udias and A. Poves. We thank I. Hamamoto for her interest. We also thank N. Sharma for useful comments about matrix diagonalization.
Table I. Summed Magnetic Dipole Strengths (in $\mu_N^2$).

| Space      | Isovector Orbital \(^{(d)}\) | Isovector Spin \(^{(e)}\) | Isoscalar Orbital \(^{(f)}\) | Physical \(^{(g)}\) |
|------------|-------------------------------|---------------------------|-----------------------------|-------------------|
| \(8^\text{Be} J = 0^+_1 T = 0 \rightarrow J = 1^+ T = 1\) |                               |                           |                             |                   |
| Small\(^{(a)}\) | 0.6701                        | 0.00427                   | 1.0547                      |                   |
| Large\(^{(b)}\) | 0.7283                        | 0.00622                   | 1.2866                      |                   |
| Low Large\(^{(c)}\) | 0.5890                        | 0.00322                   | 0.8999                      |                   |
| \(10^\text{Be} J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 1\) |                               |                           |                             |                   |
| Small      | 0.1112                        | 0.0234                    | 0.0245                      | 2.0930            |
| Large      | 0.1963                        | 0.0208                    | 0.0270                      | 1.9517            |
| Low Large  | 0.1002                        | 0.0187                    | 0.0200                      | 1.6070            |
| \(10^\text{Be} J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 2\) |                               |                           |                             |                   |
| Small      | 0.1508                        | 0.00105                   |                             | 0.0597            |
| Large      | 0.1830                        | 0.00222                   |                             | 0.2276            |
| Low Large  | 0.1339                        | 0.000928                  |                             | 0.0754            |
| The \(Q \cdot Q\) interaction |                               |                           |                             |                   |
| \(8^\text{Be} J = 0^+_1 T = 0 \rightarrow J = 1^+ T = 1\) |                               |                           |                             |                   |
| Small      | 0.6364                        | 0.0000                    | 0.6364                      |                   |
| Large      | 0.7408                        | 0.0005                    | 0.7858                      |                   |
| Low Large  | 0.6593                        | 0.0002                    | 0.6775                      |                   |
| \(10^\text{Be} J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 1\) |                               |                           |                             |                   |
| Small      | 0.0895                        | 0.0001                    | 0.0986                      |                   |
| Large      | 0.1788                        | 0.0004                    | 0.2044                      |                   |
| Low Large  | 0.0922                        | 0.0000                    | 0.0881                      |                   |
| \(10^\text{Be} J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 2\) |                               |                           |                             |                   |
| Small      | 0.1492                        | 0.0000                    | 0.1486                      |                   |
| Large      | 0.1744                        | 0.0002                    | 0.1950                      |                   |
| Low Large  | 0.1513                        | 0.0000                    | 0.1617                      |                   |
Table I Captions

(a) Small Space \((0s)^4(0p)^6\)

(b) Large Space \((0s)^4(0p)^6 + \text{all } 2\hbar\omega \text{ excitations}\)

(c) Low energy part of Large Space (up to the first plateau)

(d) \(g_{l\pi} = 0.5 \quad g_{l\nu} = -0.5 \quad g_{s\pi} = 0 \quad g_{s\nu} = 0\)

(e) \(g_{l\pi} = 0 \quad g_{l\nu} = 0 \quad g_{s\pi} = 0.5 \quad g_{s\nu} = -0.5\)

(f) \(g_{l\pi} = 0.5 \quad g_{l\nu} = 0.5 \quad g_{s\pi} = 0 \quad g_{s\nu} = 0\). The value for isoscalar spin is the same as for isoscalar orbital in the case of \(^{10}\text{Be} \ J = 0^+, \ T = 1 \rightarrow J = 1^+, \ T = 1\).

For \(\Delta T = 1\) the isoscalar case gives zero.

(g) \(g_{l\pi} = 1 \quad g_{l\nu} = 0 \quad g_{s\pi} = 5.586 \quad g_{s\nu} = -3.826\)
Table II. Summed Energy Weighted Magnetic Dipole Strengths (in $\mu^2_N\text{MeV}$)

| Space       | Isovector Orbital | Isovector Spin | Isoscalar Orbital | Physical |
|-------------|-------------------|----------------|-------------------|----------|
| **$^8\text{Be} J = 0^+_1 T = 0 \rightarrow J = 1^+ T = 1$** |                   |                 |                   |          |
| Small       | 10.689            | 0.08359        |                   | 16.878 |
| Large       | 20.744            | 0.2789         |                   | 44.092 |
| Low Large   | 11.854            | 0.0778         |                   | 18.349 |
| **$^{10}\text{Be} J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 1$** |                   |                 |                   |          |
| Small       | 2.0750            | 0.1954         | 0.2328            | 18.852  |
| Large       | 7.9511            | 0.3119         | 0.6929            | 35.909  |
| Low Large   | 2.4680            | 0.1814         | 0.2369            | 18.429  |
| **$^{10}\text{Be} J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 2$** |                   |                 |                   |          |
| Small       | 2.984             | 0.0229         |                   | 1.4783  |
| Large       | 6.720             | 0.1167         |                   | 12.83   |
| Low Large   | 3.3590            | 0.0267         |                   | 2.2843  |

The $Q \cdot Q$ interaction

| Space       | Isovector Orbital | Isovector Spin | Isoscalar Orbital | Physical |
|-------------|-------------------|----------------|-------------------|----------|
| **$^8\text{Be} J = 0^+_1 T = 0 \rightarrow J = 1^+ T = 1$** |                   |                 |                   |          |
| Small       | 6.411             | 0.0000         |                   | 6.411    |
| Large       | 14.040            | 0.0229         |                   | 15.861   |
| Low Large   | 10.282            | 0.0037         |                   | 10.619   |
| **$^{10}\text{Be} J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 1$** |                   |                 |                   |          |
| Small       | 0.7597            | 0.0004         |                   | 0.7942   |
| Large       | 3.8108            | 0.0138         |                   | 4.8627   |
| Low Large   | 1.041             | 0.0029         |                   | 0.9904   |
| **$^{10}\text{Be} J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 2$** |                   |                 |                   |          |
| Small       | 1.2661            | 0.0000         |                   | 1.2619   |
| Large       | 2.6019            | 0.0068         |                   | 3.1687   |
| Low Large   | 1.712             | 0.0005         |                   | 1.8335   |

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Table III. The $B(E2)$ values in $e^2 fm^4$ in $^8Be$ with (a) The $(x, y)$ interaction with $x = 1, y = 1$ and (b) the $Q \cdot Q$ interaction.

| Interaction      | $x = 1, y = 1$ | $\frac{1}{2}m\omega^2r^2 - \chi Q \cdot Q$ |
|------------------|----------------|-------------------------------------------|
| $J = 0^+_1 T = 0 \rightarrow J = 2^+ T = 0$ |
| $2^+_1$ Small    | 17.59          | 18.13                                     |
| $2^+_1$ Large    | 25.97          | 49.16                                     |
| Sum Small        | 17.82          | 18.13                                     |
| Sum Large        | 35.21          | 59.38                                     |
| Sum Low Large    | 33.28          | 49.22                                     |
| $J = 0^+_1 T = 1 \rightarrow J = 2^+ T = 1$ |
| Sum Small        | 2.31           | 2.59                                      |
| Sum Large        | 13.47          | 22.44                                     |
| Sum Low Large    | 2.10           | 13.51                                     |
Table IV. The $B(E2)$ values in $e^2 fm^4$ in $^{10}Be$ (large space) with (a) The $(x, y)$ interaction with $x = 1, y = 1$ and (b) the $Q \cdot Q$ interaction.

| Interaction  | $x = 1, y = 1$ | $\frac{1}{2} m\omega^2 r^2 - \chi Q \cdot Q$ |
|--------------|----------------|---------------------------------|
| $J = 0^+_1 \ T = 1 \rightarrow J = 2^+ \ T = 1$ |                 |                                 |
| $2^+_1 \ Small$      | 22.98          | 17.31                           |
| $2^+_2 \ Small$      | 0.30           | 7.67                            |
| $Sum$               | 23.28          | 24.98                           |
| $2^+_1 \ Large$      | 19.68          | 5.17                            |
| $2^+_2 \ Large$      | 3.22           | 41.23                           |
| $Sum$               | 22.90          | 46.40                           |
| $Sum \ Small$        | 24.21          | 25.50                           |
| $Sum \ Large$        | 39.74          | 74.70                           |
| $Sum \ Low \ Large$  | 24.55          | 72.14                           |
| $J = 0^+_1 \ T = 1 \rightarrow J = 2^+ \ T = 2$ |                 |                                 |
| $Sum \ Small$        | 0.7858         | 0.8016                          |
| $Sum \ Large$        | 5.851          | 12.19                           |
| $Sum \ Low \ Large$  | 0.5669         | 10.57                           |
Table V. Ratio of orbital $M1$ strength and orbital energy weighted $M1$ strength $^{10}\text{Be} / ^{8}\text{Be}$ for the $(x,y)$ interaction with $x = 1, y = 1$.

| Space     | Ratio of orbital strength | Ratio of energy weighted orbital strength |
|-----------|---------------------------|------------------------------------------|
| Small     | 0.3910                    | 0.4730                                   |
| Large     | 0.5208                    | 0.7072                                   |
| Low Large | 0.3974                    | 0.4920                                   |

Table VI. Calculated Magnetic Dipole Strength in $^8\text{Be}$ with the $(x,y)$ interaction $(x = 1, y = 1)$ in small space (in units of $\mu_N^2$).

| $E_x(1^+)$ MeV | Isovector Orbital | Isovector Spin | Physical |
|----------------|-------------------|----------------|----------|
|                | $J^\pi = 0^+_1$ T = 1 $\rightarrow J^\pi = 1^+ T = 1$ |                |          |
| 13.73          | 0.2569            | 0.0013         | 0.7155   |
| 16.64          | 0.2344            | 0.0006         | 0.0651   |
| 18.05          | 0.1748            | 0.0002         | 0.0778   |
| 20.73          | 0.0013            | 0.0012         | 0.1307   |
| 26.80          | 0.0007            | 0.0005         | 0.0319   |
| 28.46          | 0.0007            | 0.0004         | 0.0272   |
| 29.66          | 0.0014            | 0.0000         | 0.0001   |
| 34.76          | 0.0000            | 0.0001         | 0.0065   |
| SUM            | 0.6701            | 0.0043         | 1.0547   |
Table VII. Calculated Magnetic Dipole Strength in $^8Be$ with the Hamiltonian

\[ H = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\omega^2r^2 - \chi \mathbf{Q} \cdot \mathbf{Q} \]
with $\chi = 0.5762$ in small space ($\mu^2_\omega$).

| $E_x(1^+)\ MeV$ | Isovector Orbital | Isovector Spin | Physical |
|------------------|-------------------|----------------|----------|
|                  | $J^\pi = 0^+_1\ T = 1 \rightarrow J^\pi = 1^+\ T = 1$            |                |          |
| 10.06            | 0.4599            | 0.0000         | 0.4851   |
| 10.11            | 0.1765            | 0.0000         | 0.1524   |
| 12.33            | 0.0000            | 0.0000         | 0.0001   |
| 13.43            | 0.0000            | 0.0000         | 0.0000   |
| 16.84            | 0.0000            | 0.0000         | 0.0000   |
| 18.97            | 0.0000            | 0.0000         | 0.0000   |
| 19.05            | 0.0000            | 0.0000         | 0.0000   |
| 19.11            | 0.0000            | 0.0000         | 0.0000   |
| SUM              | 0.6364$^a$        | 0.0000         | 0.6376   |

(a) The summed isovector orbital strength is $\frac{2}{\pi} \mu^2_N$.
Table VIII. Calculated Magnetic Dipole Strength in $^{16}\text{Be}$ with the $(x, y)$ interaction $(x = 1, \; y = 1)$ in small space (in units of $10^{-2} \mu_N^2$).

| $E_x(1^+) \text{ MeV}$ | Isovector Orbital | Isovector Spin | Physical |
|--------------------------|-------------------|----------------|----------|
| **A. $J = 0_1^+ \; T = 1 \rightarrow J = 1^+ \; T = 1$** | | | |
| 6.14                     | 0.1641            | 0.0578         | 1.0740   |
| 7.68                     | 0.1699            | 2.1690         | 184.90   |
| 14.31                    | 0.1000            | 0.0048         | 1.2020   |
| 16.48                    | 0.0000            | 0.0105         | 1.1050   |
| 18.14                    | 7.2840            | 0.0020         | 10.530   |
| 19.19                    | 1.7540            | 0.0152         | 4.8420   |
| 22.62                    | 1.1930            | 0.0064         | 0.4793   |
| 23.93                    | 0.3904            | 0.0513         | 2.6420   |
| 25.72                    | 0.0606            | 0.0231         | 2.5720   |
| **SUM**                  | 11.120            | 2.3409         | 209.30   |
| **B. $J = 0_1^+ \; T = 1 \rightarrow J = 1^+ \; T = 2$** | | | |
| 17.91                    | 6.4260            | 0.0331         | 0.6757   |
| 20.67                    | 7.1290            | 0.0494         | 0.3339   |
| 23.50                    | 1.5280            | 0.0039         | 3.3170   |
| 30.96                    | 0.0001            | 0.0188         | 1.6430   |
| **SUM**                  | 15.083            | 0.1052         | 5.9696   |
Table IX. Calculated Magnetic Dipole Strength in $^{10}$Be with the Hamiltonian
\[ H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2 - \chi Q \cdot Q \] with $\chi = 0.3615$ in small space ($10^{-2} \mu_N^2$).

| $E_x(1^+)$ MeV | Isovector Orbital | Isovector Spin | Physical |
|-----------------|------------------|---------------|----------|
|                 | A. $J = 0^+_1$ $T = 1 \rightarrow J = 1^+ T = 1$ |
| 3.848           | 0.0006           | 0.0051        | 0.4173   |
| 3.870           | 0.0006           | 0.0046        | 0.5161   |
| 8.444           | 3.0350           | 0.0001        | 2.7460   |
| 8.486           | 0.0256           | 0.0000        | 0.0206   |
| 8.491           | 3.3300           | 0.0000        | 3.3190   |
| 8.533           | 2.5590           | 0.0001        | 2.8440   |
| 9.988           | 0.0002           | 0.0000        | 0.0001   |
| 10.035          | 0.0003           | 0.0000        | 0.0003   |
| 10.082          | 0.0001           | 0.0000        | 0.0002   |
| SUM             | 8.9555$^a$       | 0.0099        | 9.8636   |

|                 | B. $J = 0^+_1$ $T = 1 \rightarrow J = 1^+ T = 2$ |
| 8.441           | 7.6680           | 0.0003        | 6.8270   |
| 8.536           | 7.2500           | 0.0002        | 8.0320   |
| 10.034          | 0.0000           | 0.0000        | 0.0000   |
| 12.349          | 0.0000           | 0.0000        | 0.0000   |
| SUM             | 14.918$^a$       | 0.0005        | 14.859   |

(a) The summed isovector orbital strength to $T = 1$ states is $\frac{9}{32\pi} \mu_N^2$, and to $T = 2$ states it is $\frac{15}{32\pi} \mu_N^2$. 

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Table X. Calculated Magnetic Dipole Strength in $^8Be$ with the $(x, y)$ interaction $(x = 1, y = 1)$ in large space$^{(a)}$ (in units of $\mu^2_N$).

| $E_x (1^+)$ MeV | Isovector Orbital | Isovector Spin | Physical |
|------------------|-------------------|---------------|----------|
| $J = 0^+_1 \rightarrow J = 1^+_1$ |
| 17.96            | 0.2208            | 0.0011        | 0.6160   |
| 20.80            | 0.2970            | 0.0004        | 0.1213   |
| 22.89            | 0.0670            | 0.0001        | 0.0213   |
| 27.04            | 0.0000            | 0.0010        | 0.0935   |
| 33.51            | 0.0007            | 0.0002        | 0.0081   |
| 34.57            | 0.0000            | 0.0003        | 0.0250   |
| 35.73            | 0.0007            | 0.0000        | 0.0000   |
| 40.22            | 0.0001            | 0.0000        | 0.0036   |
| 46.07            | 0.0003            | 0.0000        | 0.0110   |
| 46.41            | 0.0000            | 0.0000        | 0.0000   |
| 48.46            | 0.0060            | 0.0001        | 0.0001   |
| 49.61            | 0.0019            | 0.0000        | 0.0023   |
| 49.92            | 0.0012            | 0.0000        | 0.0004   |
| 50.32            | 0.0004            | 0.0000        | 0.0036   |
| SUM              | 0.5986            | 0.0033        | 0.9063   |

(a) Large Space: all $0\hbar\omega$ configurations plus $2\hbar\omega$ excitations.
Table XI. Calculated Magnetic Dipole Strength in $^8Be$ with the Hamiltonian
\[ H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2r^2 - \chi Q \cdot Q \] with $\chi = 0.5762$ in large space ($\mu_N^2$).

| $E_x (1^+)$ (MeV) | Isovector Orbital | Isovector Spin | Physical |
|-------------------|-------------------|----------------|----------|
|                   | $J = 0^+ T = 0 \rightarrow J = 1^+ T = 1$                   |                |          |
| 15.49             | 0.6012            | 0.0000         | 0.5513   |
| 15.70             | 0.0552            | 0.0001         | 0.1189   |
| 18.42             | 0.0003            | 0.0000         | 0.0005   |
| 22.69             | 0.0000            | 0.0000         | 0.0020   |
| 25.43             | 0.0000            | 0.0000         | 0.0004   |
| 27.82             | 0.0000            | 0.0000         | 0.0014   |
| 28.14             | 0.0000            | 0.0000         | 0.0002   |
| 28.24             | 0.0000            | 0.0000         | 0.0003   |
| 35.24             | 0.0009            | 0.0000         | 0.0013   |
| 37.88             | 0.0003            | 0.0000         | 0.0000   |
| 39.30             | 0.0001            | 0.0000         | 0.0001   |
| 39.51             | 0.0003            | 0.0000         | 0.0003   |
| 40.17             | 0.0010            | 0.0000         | 0.0010   |
| SUM               | 0.6593            | 0.0002         | 0.6775   |
Table XII. Calculated Magnetic Dipole Strength in $^{10}$Be with the $(x, y)$ interaction ($x = 1, y = 1$) in large space\(^{(a)}\) (in units of $10^{-2} \mu_N^2$).

| $E_x(1^+)$ MeV | Isovector Orbital | Isovector Spin | Physical |
|---------------|------------------|---------------|----------|
| **A. $J = 0^+_1 T = 1 \rightarrow J^+ = 1$** | | | |
| 7.38 | 0.006 | 0.909 | 63.78 |
| 9.62 | 0.223 | 0.847 | 79.47 |
| 18.29 | 0.015 | 0.000 | 0.026 |
| 20.65 | 0.047 | 0.010 | 0.768 |
| 22.73 | 6.087 | 0.004 | 10.40 |
| 23.99 | 1.620 | 0.013 | 4.401 |
| 27.15 | 1.411 | 0.004 | 0.788 |
| 28.90 | 0.086 | 0.047 | 3.528 |
| 30.86 | 0.017 | 0.031 | 2.831 |
| 37.20 | 0.450 | 0.001 | 0.849 |
| 39.96 | 0.002 | 0.003 | 0.150 |
| 40.39 | 0.058 | 0.002 | 0.028 |
| **SUM** | 10.02 | 1.870 | 160.7 |
| **B. $J = 0^+_1 T = 1 \rightarrow J^+ = 1$** | | | |
| 23.27 | 4.733 | 0.015 | 1.058 |
| 25.99 | 7.560 | 0.046 | 0.524 |
| 29.11 | 1.004 | 0.013 | 4.311 |
| 38.33 | 0.000 | 0.017 | 1.524 |
| 49.42 | 0.017 | 0.000 | 0.098 |
| 49.67 | 0.003 | 0.000 | 0.028 |
| 50.84 | 0.075 | 0.000 | 0.000 |
| **SUM** | 13.39 | 0.093 | 7.540 |

\(\text{(a) Large Space: all } 0\hbar\omega \text{ configurations plus } 2\hbar\omega \text{ excitations.}\)
Table XIII. Calculated Magnetic Dipole Strength in $^{10}\text{Be}$ with the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2 - \chi Q \cdot Q$$

with $\chi = 0.3615$ in large space ($10^{-2} \mu_N^2$).

| $E_x(1^+)$ MeV | Isovector Orbital | Isovector Spin | Physical |
|----------------|------------------|----------------|----------|
| A. $J = 0_1^+ T = 1 \rightarrow J = 1^+ T = 1$ |
| 3.60           | 0.002            | 0.000          | 0.019    |
| 6.45           | 0.005            | 0.001          | 0.058    |
| 11.19          | 2.548            | 0.000          | 2.393    |
| 11.31          | 1.685            | 0.000          | 2.229    |
| 11.32          | 4.563            | 0.001          | 3.382    |
| 11.35          | 0.395            | 0.001          | 0.702    |
| 12.92          | 0.019            | 0.000          | 0.019    |
| 12.98          | 0.004            | 0.000          | 0.003    |
| 13.02          | 0.002            | 0.000          | 0.003    |
| 22.69          | 0.011            | 0.001          | 0.154    |
| 23.12          | 0.019            | 0.000          | 0.001    |
| 25.42          | 1.579            | 0.000          | 1.220    |
| 25.94          | 0.043            | 0.003          | 0.497    |
| 26.51          | 0.090            | 0.000          | 0.012    |
| 26.65          | 0.004            | 0.000          | 0.005    |
| SUM            | 10.97            | 0.007          | 10.70    |
| B. $J = 0_1^+ T = 1 \rightarrow J = 1^+ T = 2$ |
| 11.31          | 13.82            | 0.001          | 15.79    |
| 11.34          | 1.301            | 0.003          | 0.348    |
| 12.98          | 0.000            | 0.000          | 0.002    |
| 16.76          | 0.000            | 0.000          | 0.007    |
| 26.58          | 0.002            | 0.000          | 0.001    |
| 26.67          | 0.002            | 0.000          | 0.010    |
| 28.25          | 0.001            | 0.000          | 0.003    |
| SUM            | 15.13            | 0.004          | 16.16    |
Figure Captions

Figure (1): The cumulative sum of the energy-weighted isovector orbital $B(M1)$ strength for the $0^+_1, 0 \rightarrow 1^+, 1$ transitions in $^8Be$ with the realistic interaction ($x = 1, y = 1$).

Figure (2): Same as Figure 1 but with the $Q \cdot Q$ interaction.

Figure (3): The cumulative sum of the energy-weighted isovector orbital $B(M1)$ strength for the $0^+_1, 1 \rightarrow 1^+, 2$ transitions in $^{10}Be$ with the realistic interaction ($x = 1, y = 1$).

Figure (4): Same as Figure 3 but with the $Q \cdot Q$ interaction.

Figure (5): The cumulative sum of the energy-weighted isovector orbital $B(M1)$ strength for the $0^+_1, 1 \rightarrow 1^+, 1$ transitions in $^{10}Be$ with the realistic interaction ($x = 1, y = 1$).

Figure (6): Same as Figure 5 but with the $Q \cdot Q$ interaction.

Figure (7): $\sum B(M1)(J = 0^+_1 T = 0 \rightarrow J = 1^+ T = 1$ states) versus $t$ for $^8Be$. The solid line, dashed line and dot-dash line are the total, spin and orbital parts of $\sum B(M1)$.

Figure (8): The orbital part of $\sum B(M1)$ versus $t$ for $^{10}Be$. The solid line, dashed line and dot-dash line are the total, $\sum B(M1)(J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 1$ states) and $\sum B(M1)(J = 0^+_1 T = 1 \rightarrow J = 1^+ T = 2$ states) respectively.

Figure (9): Same as in Fig.(8) for spin part of $\sum B(M1)$. 
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