Addendum to “High order U-spin breaking: A precise amplitude relation in $D^0$ decays”
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In Eqs. (29) and (30) we calculated numerical values for the right and left-hand sides on Eq. (23) without including errors in the latter. Here we wish to include all experimental errors consistently, thus providing an ultimate test of flavor symmetries in $D$ decays.

Eq. (24) and the discussion in Sec. VI may be summarized by one important prediction:

$$R_3 - R_4 + \frac{1}{8} \left[ (\sqrt{2}R_1 - 1 - 1)^2 - (\sqrt{2}R_2 - 1 - 1)^2 \right] = \mathcal{O}(\epsilon^3, \delta \epsilon) .$$  \hspace{1cm} (1)

Here $R_i$ ($i = 1, 2, 3, 4$) are ratios of amplitudes defined in Eqs. (17)–(18), (21)–(22), while $\epsilon = \epsilon_1^{(1)}, \epsilon_1^{(1)}$ and $\delta$ are U-spin and isospin breaking parameters. Using values of $\epsilon$ and $\delta$, the term on the right-hand-side of Eq. (1) has been estimated to be a few times $10^{-3}$.

Taking the measured amplitudes we calculate

$$R_3 - R_4 + \frac{1}{8} \left[ (\sqrt{2}R_1 - 1 - 1)^2 - (\sqrt{2}R_2 - 1 - 1)^2 \right] = -0.003 \pm 0.002 .$$  \hspace{1cm} (2)

This small value, a fraction of a percent, is consistent with the prediction in Eq. (1). It provides the first high-precision test of flavor symmetries in $D$ meson decays.
HIGH ORDER U-SPIN BREAKING:
A PRECISE AMPLITUDE RELATION IN $D^0$ DECAYS

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U-spin breaking corrections up to third order are studied in $D^0$ decays to pairs involving a charged pion or kaon. The ratios $|A(D^0 \to K^+\pi^-)|/|A(D^0 \to \pi^+K^-)|$ and $|A(D^0 \to K^+K^-)|/|A(D^0 \to \pi^+\pi^-)|$ determine values of 0.05 and 0.30 for real parts of two distinct first order U-spin breaking parameters of different origins. We show that first and third order corrections vanish in the quantity $\sqrt{|A(D^0 \to K^+K^-)A(D^0 \to \pi^+\pi^-)|/\sqrt{|A(D^0 \to K^+\pi^-)A(D^0 \to \pi^+K^-)|}} = 1$, while second order corrections cancel each other experimentally at a one percent level. We compare this ratio with the above two ratios and a third ratio involving these same four amplitudes, for which expansions up to and including second order are obtained. A nonlinear relation between these four ratios is shown to hold excluding third order U-spin breaking at a fraction of a percent. Isospin breaking in this relation and in the above equality is suppressed by both isospin and U-spin breaking parameters.

I Introduction

U-spin symmetry, an SU(2) subgroup of flavor SU(3) under which the quark pair $(d,s)$ transforms like a doublet, has been shown to have powerful consequences in $D$ meson decays and in $D^0$-$\bar{D}^0$ mixing. Shortly after the discovery of charm in November 1974 a simple U-spin relation has been noted to hold among amplitudes for Cabibbo-favored (CF), singly Cabibbo-suppressed (SCS) and doubly Cabibbo-suppressed (DCS) $D^0$ decays [1, 2],

$$A(\pi^+K^-) : A(\pi^+\pi^-) : A(K^+K^-) : A(K^+\pi^-) = 1 : -\tan\theta_C : \tan\theta_C : -\tan^2\theta_C,$$

(1)

where $\theta_C$ is the Cabibbo angle. Early measurements observed that while $R_1 \equiv |A(D^0 \to K^+\pi^-)|/|A(D^0 \to \pi^+K^-)|\tan^2\theta_C = 1$ holds within a reasonable approximation of order ten or twenty percent, the relation $R_2 \equiv |A(D^0 \to K^+K^-)|/|A(D^0 \to \pi^+\pi^-)| = 1$ is badly broken by about 80%. It has been recently suggested [3, 4] that the large discrepancy of...
this ratio with respect to the U-spin symmetry value may be due to constructive interference between symmetry breaking in $\Delta U = 1$ “tree” and $\Delta U = 0$ “penguin” operators contributing to SCS decays, in contrast to the ratio of DCS and CF amplitudes which involves purely $\Delta U = 1$ transitions [5]. Understanding U-spin breaking in these decays may shed light on the relative strong phase $\delta$ between CF and DCS amplitudes, which vanishes in the U-spin symmetry limit [1, 2, 3]. The phase $\delta$ plays a crucial role in determining $D^0$-$\bar{D}^0$ mixing parameters [7, 8], which formally vanish in the U-spin symmetry limit and also when including first order U-spin breaking [9].

The purpose of this Letter is to examine the amplitude relations (1) with the Cabibbo-Kobayashi-Maskawa (CKM) framework when including first, second and third order U-spin breaking corrections. One of our motivations is searching for signals of new physics, which may be indicated by relations among amplitudes failing at some high order flavor symmetry breaking. Our study is also motivated by a very recent report of the LHCb collaboration [10], measuring the ratio of DCS and CF amplitudes $|A(D^0 \to K^+\pi^-)|/|A(D^0 \to \pi^+K^-)|$ at an impressive high precision of less than a percent. Using this measurement we will update the status of second order corrections in a ratio $R_3$ of a sum of magnitudes of suitably normalized CF and DCS amplitudes and a sum of magnitudes of the two SCS amplitudes, in which first order U-spin breaking corrections have been suggested to cancel [4].

A new ratio $R_4$ involving products of amplitudes will be examined, in which first and third order U-spin breaking corrections will be shown to vanish while second order corrections cancel experimentally at a one percent level. We will prove a nonlinear relation between $R_3 - R_4$ and $R_1$ and $R_2$, violated by a tiny third order correction - at most a fraction of a percent. Isospin breaking corrections in this relation and in $R_3$ and $R_4$ will be shown to be suppressed by both isospin and U-spin breaking parameters. Values of $R_1$ an $R_2$ will be used to calculate real parts of two distinct first order U-spin breaking parameters, Re $\epsilon^{(1)} = 0.3$ and Re $\epsilon_1^{(1)} = 0.05$. The imaginary part of the second parameter determines $\delta$.

Studies assuming flavor SU(3) symmetry for $D$ and $D_s$ decays into all pairs of light pseudoscalar mesons have been presented in Refs. [11, 12, 13]. First order SU(3) breaking corrections were included in the latter two papers, identifying linear relations between amplitudes which hold in the presence of these corrections. Testing these linear relations involving at least three amplitudes requires knowledge of relative strong phases between amplitudes. One of these relations following from U-spin, involving the four amplitudes in (1), was shown in Ref. [12] to imply a corresponding relation among suitably normalized magnitudes of amplitudes as suggested in [4]. Three other well-known amplitude relations involving also a neutral pion or kaon follow from isospin symmetry [14].

II U-spin symmetry limit

A formal proof of (1) follows by considering U-spin properties of states and operators denoted $|U, U_3\rangle$ and $(U, U_3)$, respectively. Initial $|D^0\rangle$ and final $|K^+K^- + \pi^+\pi^-\rangle/\sqrt{2}$ states are U-spin singlets $|0, 0\rangle$, while the three states, $-|\pi^+K^-\rangle, |K^+K^- - \pi^+\pi^-\rangle/\sqrt{2}, |K^+\pi^-\rangle$,
are members of a triplet, $|1, -1\rangle, |1, 0\rangle, |1, +1\rangle$. The three pieces of the Hamiltonian operator responsible for CF, SCS and DCS decays behaving like $(\bar{s}d), (\bar{s}s - \bar{d}d)$ and $(\bar{d}s)$ transform like a triplet:

$$H_{\text{CF}} = -\cos^2 \theta_C (1, -1), \quad H_{\text{SCS}} = \sqrt{2} \cos \theta_C \sin \theta_C (1, 0), \quad H_{\text{DCS}} = -\sin^2 \theta_C (1, +1). \quad (2)$$

We use $V_{ud} = V_{cs} = \cos \theta_C, V_{us} = -V_{cd} = \sin \theta_C$, neglecting in $H_{\text{SCS}}$ tiny contributions proportional to $V_{cb}^* V_{ub}$, which may lead to small CP asymmetries at the level of $10^{-3}$ but contribute negligibly to CP-averaged decay rates. The vanishing matrix element of a triplet operator for the singlet final state $|K^+K^- + \pi^+\pi^-\rangle$ implies $A(\pi^+\pi^-) = -A(K^+K^-)$. Thus the four amplitudes in (1) are given in terms of a common U-spin triplet amplitude $A \equiv \langle 1, U_3 | (1, U_3) | 0, 0 \rangle$,

$$A(\pi^+K^-) = \cos^2 \theta_C A, \quad -A(\pi^+\pi^-) = A(K^+K^-) = \frac{1}{2} \sin 2\theta_C A, \quad A(K^+\pi^-) = -\sin^2 \theta_C A,$$

leading immediately to (1).

### III First, second and third order U-spin breaking

We will introduce U-spin breaking corrections in (1) assuming, as has been done in the past [4, 5, 12, 13, 15, 16, 17, 18] that these corrections may be treated perturbatively. Corrections of arbitrary order to decay amplitudes $\langle f | H_{\text{eff}} | D^0 \rangle$ are obtained by introducing in the Hamiltonian or in the final state powers of an $s-d$ spurion mass operator, $M_{\text{Ubrk}} \propto (\bar{s}s) - (\bar{d}d) = \sqrt{2}(1, 0)$. For SCS decays the effective Hamiltonian obtains at first order an additional $s + d$ penguin term $P_{s+d}$ due to an $s - d$ mass difference [3]. That is,

$$H_{\text{eff}} M_{\text{Ubrk}} = H_{\text{SCS}} M_{\text{Ubrk}} + P_{s+d}, \quad (4)$$

where the first term is a mixture of $(0, 0)$ and $(2, 0)$ while the second term behaves like a pure U-spin singlet. We will now show that corrections of given order have equal magnitudes in pairs of processes ($D^0 \to \pi^+K^-$, $D^0 \to K^+\pi^-$) and ($D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$), while their relative signs within these pairs are positive for even order and negative for odd order. We will make a clear distinction between U-spin breaking parameters in CF or DCS decays and in SCS decays.

Starting with first order corrections,

$$\langle f | H_{\text{eff}} | D^0 \rangle^{(1)} = \langle f | H_{\text{eff}} M_{\text{Ubrk}} | D^0 \rangle + \langle M_{\text{Ubrk}} f | H_{\text{eff}} | D^0 \rangle, \quad (5)$$

we note that since the $D^0$ is a U-spin singlet only the triplet operators in the products $H_{\text{eff}} M_{\text{Ubrk}} \propto (1, \pm 1)(1, 0)$ contribute to the triplet final states $|f\rangle = |K^\pm\pi^\mp\rangle$, and only the triplet states in $M_{\text{Ubrk}} |K^\pm\pi^\mp\rangle \propto (1, 0)|K^\pm\pi^\mp\rangle$ obtain contributions from the triplet Hamiltonian operator. First order U-spin breaking corrections for $K^-\pi^+$ and $K^+\pi^-$ states, obtained by combining the two terms in (5), are equal in magnitude and have opposite signs.
when leaving out prefactors $\cos^2 \theta_C$ and $-\sin^2 \theta_C$. This sign change follows from an identity for Clebsch-Gordan coefficients,

$$(n, 0; 1, -1|1, -1) = (-1)^n(n, 0; 1, 1|1, 1) = (1, 1; n, 0|1, 1) = (-1)^n(1, -1; n, 0|1, -1),$$

applied to $n = 1$. Denoting the correction parameter by $\epsilon_1^{(1)}$, where the superscript represents the order in perturbation and the subscript marks the triplet nature of the transition operator, one has

$$\langle \pi^+ K^-|H_{\text{eff}}|D^0\rangle^{(1)} = -\cos^2 \theta_C A\epsilon_1^{(1)}, \quad \langle K^+\pi^-|H_{\text{eff}}|D^0\rangle^{(1)} = -\sin^2 \theta_C A\epsilon_1^{(1)}.$$

(7)

First order U-spin breaking corrections for the triplet state $|K^+K^- - \pi^+\pi^-\rangle$ vanish because of a vanishing Clebsch-Gordan coefficient, $(1, 0; 1, 0|1, 0) = 0$. This implies that corrections for $K^+K^-$ and $\pi^+\pi^-$ have opposite signs when leaving out prefactors $\cos \theta_C \sin \theta_C$ and $-\cos \theta_C \sin \theta_C$, respectively. For the singlet state $|K^+K^- + \pi^+\pi^-\rangle$ one obtains two contributions for the first term in (5) originating in the two terms of $H_{\text{eff}}M_{\text{Ubrk}}$ in (4). The two terms correspond to a current-current ("tree") operator and an $s + d$ penguin operator occurring in the U-spin breaking phase. (The second term in (5) obtains only a tree contribution.) This distinguishes first order U-spin breaking in decays to $K^+K^-$ and $\pi^+\pi^-$ from that occurring in decays to $K^0\pi^0$ which, as mentioned, involves only a triplet tree operator. Denoting by $\epsilon^{(1)}$ the correction parameter in the former decays, one has

$$\langle K^+K^-|H_{\text{eff}}|D^0\rangle^{(1)} = \langle \pi^+\pi^-|H_{\text{eff}}|D^0\rangle^{(1)} = \cos \theta_C \sin \theta_C A\epsilon^{(1)}.$$

(8)

Second order U-spin breaking corrections are given by

$$\langle f|H_{\text{eff}}|D^0\rangle^{(2)} = \langle f|H_{\text{eff}}M_{\text{Ubrk}}^2|D^0\rangle + \langle f|H_{\text{eff}}M_{\text{Ubrk}}^2|D^0\rangle + \langle M_{\text{Ubrk}}^2|H_{\text{eff}}M_{\text{Ubrk}}|D^0\rangle,$$

(9)

where $M_{\text{Ubrk}}^2 \propto (1, 0)^2 = -\sqrt{1/3}(0, 0) + \sqrt{2/3}(2, 0)$. For final states $|f\rangle = |K^0\pi^0\rangle$ we apply to the first two terms the Clebsch-Gordan identity (6) with $n = 0$ and $n = 2$. Since in this case the identity involves no sign change, contributions of these two terms to $\pi^+K^-$ and $K^+\pi^-$ are equal in magnitude and have equal signs when leaving out the prefactors $\cos^2 \theta_C$ and $-\sin^2 \theta_C$. This is true also for the third term which involves squares of Clebsch-Gordan coefficients, as in this term the states $|M_{\text{Ubrk}}\rangle$ and $H_{\text{eff}}M_{\text{Ubrk}}|D^0\rangle$ must belong to the same U-spin representation. Consequently,

$$\langle \pi^+K^-|H_{\text{eff}}|D^0\rangle^{(2)} = \cos^2 \theta_C A\epsilon_1^{(2)}, \quad \langle K^+\pi^-|H_{\text{eff}}|D^0\rangle^{(2)} = -\sin^2 \theta_C A\epsilon_1^{(2)}.$$

(10)

Second order corrections for the singlet state involving $K^+K^-$ and $\pi^+\pi^-$ vanish, $\langle K^+K^- + \pi^+\pi^-|H_{\text{eff}}|D^0\rangle^{(2)} = 0$, because neither $U = 0$ nor $U = 2$ couples with $U = 1$ to $U = 0$ (or vice versa) and $(1, 0; 1, 0|1, 0) = 0$. Thus,

$$\langle K^+K^-|H_{\text{eff}}|D^0\rangle^{(2)} = \langle \pi^+\pi^-|H_{\text{eff}}|D^0\rangle^{(2)} = \cos \theta_C \sin \theta_C A\epsilon^{(2)}.$$

(11)

For the triplet state $|K^+K^- - \pi^+\pi^-angle$ the first and last terms in (9) involve two contributions from tree and $s + d$ penguin operators in $H_{\text{eff}}M_{\text{Ubrk}}$. As in the case of first order corrections,
this distinguishes the second order parameter \( \epsilon^{(2)} \) in decays to \( K^+K^- \) and \( \pi^+\pi^- \) from \( \epsilon_1^{(2)} \) in \( D^0 \to K^{\pm}\pi^\pm \) which is due to only triplet tree operators.

Third order corrections involve four terms,

\[
\langle f | H_{\text{eff}} | D^0 \rangle^{(3)} = \langle f | H_{\text{eff}} M_{\text{Ubrk}}^3 | D^0 \rangle + \langle M_{\text{Ubrk}}^3 f | H_{\text{eff}} | D^0 \rangle + \langle M_{\text{Ubrk}}^2 f | H_{\text{eff}} M_{\text{Ubrk}}^3 | D^0 \rangle + \langle M_{\text{Ubrk}}^2 f | H_{\text{eff}} M_{\text{Ubrk}}^3 | D^0 \rangle,
\]

where \( M_{\text{Ubrk}}^3 \) is a mixture of \( (1,0) \) and \( (3,0) \). Applying an argument similar to the one used for first order corrections and using the identity (12), one may show that each of these four terms changes sign between \( D^0 \to \pi^+K^- \) and \( D^0 \to \pi^+\pi^- \) when leaving out \( \theta_C \)-dependent prefactors. Consequently, as in first order, the third order correction vanishes for the triplet state \( |K^+K^- - \pi^+\pi^-\rangle \). For the singlet state \( |K^+K^- + \pi^+\pi^-\rangle \), all four terms in (12) but the second term involve contributions due to the two operators in (13), a current-current operator and an \( s+d \) penguin operator.

Combining these properties of third order corrections with Eqs. (5), (7), (8), (10) and (11) one obtains expressions for amplitudes including first, second and third order U-spin breaking corrections:

\[
\begin{align*}
A(D^0 \to \pi^+K^-) &= \cos^2 \theta_C A(1 - \epsilon^{(1)}_1 + \epsilon^{(2)}_1 - \epsilon^{(3)}_1), \\
A(D^0 \to K^+\pi^-) &= -\sin^2 \theta_C A(1 + \epsilon^{(1)}_1 + \epsilon^{(2)}_1 + \epsilon^{(3)}_1), \\
A(D^0 \to K^+K^-) &= \cos \theta_C \sin \theta_C A(1 + \epsilon^{(1)} + \epsilon^{(2)} + \epsilon^{(3)}), \\
A(D^0 \to \pi^+\pi^-) &= -\cos \theta_C \sin \theta_C A(1 - \epsilon^{(1)} + \epsilon^{(2)} - \epsilon^{(3)}).
\end{align*}
\]

In each one of the two pairs of processes first and third order corrections occur with equal signs and may be combined into a single parameter, while the zeroth order term and the second order correction may be combined in the first pair, changing the common factor by a second order correction. This would provide expressions for the four complex amplitudes in terms of four complex parameters, which could be used for investigating U-spin breaking up to second order. We keep separately all six U-spin breaking parameters in (13) in order to study up to third order one particular ratio involving these four amplitudes in which third order corrections vanish.

We stress again that the two distinct U-spin breaking sets of parameters \( \epsilon_1^{(n)} \) and \( \epsilon^{(n)} \) \( (n = 1, 2, 3) \) have different origins. While \( \epsilon_1^{(n)} \) occur in CF and DCS decays, which are due to pure \( \Delta U = 1 \) tree operators in \( H_{\text{eff}} \), \( \epsilon^{(n)} \) in SCS decays combine U-spin breaking in \( \Delta U = 1 \) tree amplitudes with U-spin breaking in \( \Delta U = 0 \) penguin operators with intermediate \( s \) and \( d \) quarks. Consequently one naively expects \( |\epsilon^{(1)}_1| \sim 0.2 \) while \( |\epsilon^{(1)}| \) may be considerably larger if these two U-spin breaking effects add up constructively (3). Higher order U-spin breaking parameters are expected to obey

\[
|\epsilon_1^{(n)}| \sim |\epsilon^{(1)}_1|^n, \quad |\epsilon^{(n)}| \sim |\epsilon^{(1)}|^n, \quad n = 2, 3.
\]

An alternative notation for the U-spin breaking pattern (13) could be in terms of two parameters \( \epsilon_1 \equiv \epsilon^{(1)}_1, \epsilon_2 \equiv \epsilon^{(1)} \) and four coefficients \( a_i^n(i = 1, 2) \) of order one:

\[
A(D^0 \to \pi^+K^-) = \cos^2 \theta_C A[1 - \epsilon_1 + a_i^1(\epsilon_1)^2 - a_i^3(\epsilon_1)^3],
\]
Two often discussed ratios of amplitudes are:

\[
\begin{align*}
A(D^0 \to K^+\pi^-) &= -\sin^2 \theta_A A[1 + \epsilon_1 + a_1^2(\epsilon_1)^2 + a_1^3(\epsilon_1)^3], \\
A(D^0 \to K^+K^-) &= \cos \theta_A \sin \theta_C A[1 + \epsilon_2 + a_2^2(\epsilon_2)^2 + a_2^3(\epsilon_2)^3], \\
A(D^0 \to \pi^+\pi^-) &= -\cos \theta_A \sin \theta_C A[1 - \epsilon_2 + a_2^2(\epsilon_2)^2 - a_2^3(\epsilon_2)^3].
\end{align*}
\] (15)

We will use the shorter notation (13).

IV Four ratios of amplitudes

Magnitudes of amplitudes in (13) may be expanded up to second order using

\[
|1 \pm \epsilon^{(1)} + \epsilon^{(2)}| = 1 \pm \text{Re} \epsilon^{(1)} + \frac{1}{2}(\text{Im} \epsilon^{(1)})^2 + \text{Re} \epsilon^{(2)} + \mathcal{O}[(\epsilon^{(1)})^3].
\] (16)

Two often discussed ratios of amplitudes are:

\[
R_1 \equiv \frac{|A(D^0 \to K^+\pi^-)|}{|A(D^0 \to \pi^+\pi^-)|} \tan^2 \theta_C
\] (17)

and

\[
R_2 \equiv \frac{|A(D^0 \to K^+K^-)|}{|A(D^0 \to \pi^+\pi^-)|}.
\] (18)

Using (13) and (16) one obtains

\[
\begin{align*}
R_1 &= 1 + 2[\text{Re} \epsilon_1^{(1)} + (\text{Re} \epsilon_1^{(1)})^2] + \mathcal{O}[(\epsilon_1^{(1)})^3], \\
R_2 &= 1 + 2[\text{Re} \epsilon^{(1)} + (\text{Re} \epsilon^{(1)})^2] + \mathcal{O}[(\epsilon^{(1)})^3].
\end{align*}
\] (19)

These two ratios involve first order corrections given by 2Re \epsilon^{(1)} and 2Re \epsilon^{(1)}. Interestingly, second order corrections in these ratios are given by squares of these same real parts with no dependence on the second order parameters \epsilon_1^{(2)} and \epsilon^{(2)}. Thus measurements of \( R_1 \) and \( R_2 \) provide ways for calculating \text{Re} \epsilon_1^{(1)} and \text{Re} \epsilon^{(1)} up to third order corrections. Eqs. (19) should include the U-spin symmetry limit, requiring solutions Re \epsilon_1^{(1)} = 0 and Re \epsilon^{(1)} = 0 (rather than Re \epsilon_1^{(1)} = -1 and Re \epsilon^{(1)} = -1) for \( R_1 = 1 \) and \( R_2 = 1 \), respectively. This implies

\[
\begin{align*}
\text{Re} \epsilon_1^{(1)} &= \frac{1}{2} \left( \sqrt{2R_1 - 1} - 1 \right) + \mathcal{O}[(\epsilon_1^{(1)})^3], \\
\text{Re} \epsilon^{(1)} &= \frac{1}{2} \left( \sqrt{2R_2 - 1} - 1 \right) + \mathcal{O}[(\epsilon^{(1)})^3].
\end{align*}
\] (20)

A third ratio \( R_3 \) involving sums of amplitudes has been pointed out in Refs. [4, 12] to differ from one by second order U-spin breaking corrections. Indeed, we find

\[
R_3 \equiv \frac{|A(D^0 \to K^+K^-)| + |A(D^0 \to \pi^+\pi^-)|}{|A(D^0 \to \pi^+K^-)| \tan \theta_C + |A(D^0 \to K^+\pi^-)| \tan^{-1} \theta_C} = 1 + \frac{1}{2}[2(\text{Im} \epsilon^{(1)})^2 + \text{Re} \epsilon^{(2)} - \epsilon_1^{(1)}] + \mathcal{O}[(\epsilon^{(1)})^3, (\epsilon_1^{(1)})^3].
\] (21)
We now propose to consider another ratio involving products of amplitudes,

\[ R_4 \equiv \left| \frac{A(D^0 \to K^+K^-)}{A(D^0 \to \pi^+\pi^-)} \right| \]

\[ = 1 - \frac{1}{2} \text{Re} \left[ (\epsilon^{(1)})^2 - (\epsilon_1^{(1)})^2 \right] + \text{Re} \left[ (\epsilon^{(2)})^2 \right] + \mathcal{O}[(\epsilon^{(1)})^4, (\epsilon_1^{(1)})^4] \]

\[ = 1 - \frac{1}{2}[(\text{Re} \epsilon^{(1)})^2 - (\text{Re} \epsilon_1^{(1)})^2] + \frac{1}{2}[(\text{Im} \epsilon^{(1)})^2 - (\text{Im} \epsilon_1^{(1)})^2] + \text{Re} (\epsilon^{(2)} - \epsilon_1^{(2)}) \]

+ \mathcal{O}[(\epsilon^{(1)})^4, (\epsilon_1^{(1)})^4]. \quad (22) \]

Third order U-spin breaking corrections vanish in \( R_4 \) whereas they contribute in \( R_3 \). These two ratios differ by a second order quantity,

\[ R_3 - R_4 = \frac{1}{2}[(\text{Re} \epsilon^{(1)})^2 - (\text{Re} \epsilon_1^{(1)})^2] + \mathcal{O}[(\epsilon^{(1)})^3, (\epsilon_1^{(1)})^3], \quad (23) \]

This and Eqs. (20) lead to a relation between the four ratios of amplitude which holds up to and including second order U-spin breaking corrections,

\[ R_4 = R_3 - \frac{1}{8} \left[ (\sqrt{2R_2 - 1} - 1)^2 - (\sqrt{2R_1 - 1} - 1)^2 \right] + \mathcal{O}[(\epsilon^{(1)})^3, (\epsilon_1^{(1)})^3]. \quad (24) \]

This relation \textit{which is not an identity} has an interesting consequence. \( R_3 \) involves a positive second order correction of about five percent. (A correction of \( 4.0 \pm 1.6\% \), calculated in Ref. [4] using earlier data, will be updated below to \( 5.6 \pm 0.8\% \) using more recent data.) The positive second order quantity \( [(\sqrt{2R_2 - 1} - 1)^2 - (\sqrt{2R_1 - 1} - 1)^2]/8 \) is only around five percent in spite of the large U-spin breaking in \( R_2 \) because \( (\sqrt{2R_2 - 1} - 1)^2/8 \) involves a strong suppression of this correction while the contribution of the \( R_1 \) term is much smaller. Thus \( R_4 \) is very close to one; namely second order corrections in \( R_4 \) cancel each other.

**V Numerical calculation of \( R_i \) and \( \text{Re} \epsilon_1^{(1)}, \text{Re} \epsilon^{(1)} \)**

Table I: CP-averaged branching fractions [19] and amplitudes in units of \( 10^{-1}(\text{GeV}/c)^{-1/2} \) for \( D^0 \) decays to pairs involving a charged pion and kaon.

| Decay mode | Branching fraction (\( \mathcal{B} \)) [19] | \( p^* \) (GeV/c) | \( |A| = \sqrt{\mathcal{B}/p^*} \) |
|------------|----------------------------------------|----------------|----------------------------------|
| \( D^0 \to \pi^+K^- \) | \( 3.88 \pm 0.05 \) \( \times 10^{-2} \) | 0.861 | 2.123 \( \pm 0.014 \) |
| \( D^0 \to K^+\pi^- \) | \( 3.88 \pm 0.05 \) \( \times 10^{-2}R_D \) | 0.861 | 0.1268 \( \pm 0.0014 \) |
| \( D^0 \to K^+K^- \) | \( 3.96 \pm 0.08 \) \( \times 10^{-3} \) | 0.791 | 0.708 \( \pm 0.007 \) |
| \( D^0 \to \pi^+\pi^- \) | \( 1.402 \pm 0.026 \) \( \times 10^{-3} \) | 0.922 | 0.3899 \( \pm 0.0036 \) |
We now proceed to calculate the four ratios $R_i (i = 1...4)$ using experimental data. Table II quotes CP-averaged branching fractions $\mathcal{B}$ for the four relevant decay processes, and magnitudes of amplitudes defined by $|A| \equiv \sqrt{\mathcal{B}/p^*}$. Since we are only concerned with ratios of amplitudes we disregard common phase space factors which cancel in these ratios. The current precision of the four amplitudes is about one percent. Three of the four branching fractions are taken from Ref. [19] while the fourth one is calculated using a very recent precise measurement of the ratio of DCS and CF branching fractions [10],

$$R_D \equiv \frac{\mathcal{B}(D^0 \to K^+\pi^-)}{\mathcal{B}(D^0 \to \pi^+K^-)} = (3.568 \pm 0.066) \times 10^{-3} . \quad (25)$$

Using Cabibbo-Kobayashi-Maskawa parameters [19], $\cos \theta_C = |V_{ud}| = 0.97425 \pm 0.00022$, $\sin \theta_C = |V_{us}| = 0.2252 \pm 0.0009$ which imply $\tan \theta_C = 0.2312 \pm 0.0009$, we calculate the following values for the four ratios:

$$R_1 = 1.118 \pm 0.014 ,$$
$$R_2 = 1.814 \pm 0.018 ,$$
$$R_3 = 1.056 \pm 0.008 ,$$
$$R_4 = 1.012 \pm 0.007 . \quad (26)$$

It is remarkable that second order U-spin breaking corrections in $R_4$ given in Eq. (22) cancel each other at an accuracy of about one percent.

We note that the absolute branching fraction of $D^0 \to \pi^+K^-$ and its error, for which a new value was reported after completion of this work [20], do not affect the central values and errors in $R_i$ because the other three branching fractions in Table II have been measured by their ratios relative to this reference branching fraction [19]. The latter three ratios determine $R_i$. Thus the errors in $R_2$, $R_3$ ans $R_4$ calculated in (26) are somewhat smaller than those which would have been obtained from errors in amplitudes given in Table II.

Using Eqs. (20) we find

$$\text{Re} \epsilon^{(1)} = 0.056 \pm 0.006 + \mathcal{O}[(\epsilon^{(1)})^3] ,$$
$$\text{Re} \epsilon^{(1)} = 0.311 \pm 0.006 + \mathcal{O}[(\epsilon^{(1)})^3] . \quad (27) (28)$$

The vastly different magnitudes of the real parts of the two U-spin breaking parameters follow from the different origins of these parameters as explained in Section III.

Eq. (23) implies

$$R_3 - R_4 = \frac{1}{2}[(\text{Re} \epsilon^{(1)})^2 - (\text{Re} \epsilon^{(1)})^2] = 0.047 \pm 0.002 , \quad (29)$$

where the the right-hand side is obtained from measured values of $R_1$ and $R_2$. This agrees extremely well with the central value of this difference calculated directly,

$$R_3 - R_4 = 0.044 . \quad (30)$$
VI  Isospin breaking

We have observed a cancellation of second order U-spin breaking at a level of one percent in $R_4$, and third order U-spin breaking at a fraction of a percent in a nonlinear relation (24) between the four ratios $R_i$. At this high precision one should also consider isospin breaking which is expected to be about one percent.

Isospin breaking is introduced in the Hamiltonian $H_{\text{eff}}$ through a $d - u$ spurion mass operator, $M_{\text{brk}} \propto (\bar{d}d - \bar{u}u)$, transforming like a combination of a U-spin singlet and triplet. Isospin breaking contributions of the U-spin singlet operator for the four final states in (13) may be absorbed into the U-spin symmetric amplitude $A$. Contributions of the triplet operator follow the signs of first order U-spin breaking corrections in (13), and are represented by two distinct parameters, $\delta_1$ - for U-spin triplet states $\pi^+K^-$ and $K^+\pi^-$, and $\delta_0$ - for $K^+K^-$ and $\pi^+\pi^-$, the two components of a U-spin singlet state.

Instead of (16) we now expand:

$$|1 \pm \epsilon^{(1)} + \epsilon^{(2)} \pm \delta_0| = 1 \pm \text{Re} \epsilon^{(1)} + \frac{1}{2}(\text{Im} \epsilon^{(1)})^2 + \text{Re} \epsilon^{(2)} \pm \text{Re} \delta_0 + \text{Im} \epsilon^{(1)} \text{Im} \delta_0 + O[(\epsilon^{(1)})^3].$$

Consequently, $R_1$ and $R_2$ in (19) obtain additional isospin breaking terms, $2 \text{Re} \delta_1$ and $2 \text{Re} \delta_0$, respectively, while $R_3$ and $R_4$ receive an identical term, $\text{Im} \epsilon^{(1)} \text{Im} \delta_0 - \text{Im} \epsilon^{(1)} \text{Im} \delta_1$, involving both isospin and U-spin breaking. Although this term cancels on the left-hand side of (23), the right-hand side now obtains isospin breaking corrections of order $\epsilon^{(1)} \delta_0$ and $\epsilon^{(1)} \delta_1$. Thus the nonlinear relation (24) involves new terms of this order which are suppressed by both isospin and U-spin breaking parameters. This correction, expected to be about a fraction of a percent, is consistent with the tiny difference between the values calculated in (29) and (30).

VII  Conclusion

We have calculated first order U-spin breaking parameters $\text{Re} \epsilon^{(1)}$ and $\text{Re} \epsilon_1^{(1)}$ around 0.30 and 0.05 from $R_2$ and $R_1$ and small second order corrections in $R_3$ and $R_4$, at levels of five and one percent. The excellent agreement between (29) and (30) confirms the nonlinear relation (24), implying that third order U-spin breaking corrections in this relation are very tiny - at most a fraction of a percent. These numbers and their hierarchy provide first evidence ever justifying high order (i.e. up to and including third order) perturbative studies of U-spin breaking (or flavor SU(3) breaking) in D meson decay amplitudes.

Isospin breaking corrections in $R_3$, $R_4$ and in (24) have been shown to be suppressed by both isospin and U-spin breaking parameters and are expected to be at a level of a fraction of a percent, consistent with our numerical calculations of $R_4$ and (24). No flavor symmetry breaking effect down to this tiny level has been found which would indicate physics beyond the standard model.

We wish to conclude with two remarks concerning open questions:
• The remarkable cancellation of second order U-spin breaking corrections in $R_4$ given in Eq. (22), and the vastly different magnitudes of $\text{Re} \epsilon^{(1)}_1$ and $\text{Re} \epsilon^{(1)}$, seem to suggest a possible relation between first and second order U-spin breaking parameters, $\text{Re} \epsilon^{(2)}_1 = \frac{1}{2} \text{Re} (\epsilon^{(1)}_1)^2$. $[\text{Re} \epsilon^{(2)}_1$ and $\frac{1}{2} \text{Re} (\epsilon^{(1)}_1)^2$ are expected to be very small in view of Eq. (27).] This could imply $a_2^2 = 1/2$ in the notation (13). Although this may be a purely accidental cancellation, one may seek an explanation for this relation.

• The result $\text{Re} \epsilon^{(1)}_1 = 0.056 \pm 0.006$ determined by the ratio of amplitudes $|A(D^0 \to K^+\pi^-)|/|A(D^0 \to \pi^+K^-)|$ is considerably smaller than typical U-spin breaking which is expected to be around $0.2 - 0.3$. What does this imply for $\delta$, the relative strong phase between these CF and DCS decay amplitudes, a knowledge of which is required for determining $D^0$-$\bar{D}^0$ mixing parameters from time dependence in these decays? The phase $\delta$ vanishes in the U-spin symmetry limit and is given in the linear approximation by $\tan \delta = -2 \text{Im} \epsilon^{(1)}_1$, which affects $R_3$ and $R_4$ quadratically but cannot be extracted from these observables. We note however that in case the phase of $\epsilon^{(1)}_1$ is not very large or not very far from $180^\circ$, for instance making a modest assumption $|\text{Arg} \epsilon^{(1)}_1| < 45^\circ$ or $|\text{Arg} \epsilon^{(1)}_1 - 180^\circ| < 45^\circ$, the small value of $\text{Re} \epsilon^{(1)}_1$ implies $|\delta| < 7^\circ$. This would determine $\delta$ at a much higher accuracy than achieved experimentally [21] using a method based on correlated production of $D^0$ and $\bar{D}^0$ in $e^+e^-$ collisions [8]. This point demonstrates the importance of understanding at least qualitatively or semi-quantitatively the phase of this U-spin breaking parameter.

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