Comment on “How the No–Cloning Theorem Got its Name”

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Abstract

A review by A. Peres [1] appears recently. It is difficult to add something to such kind of fundamental themes, but here is briefly presented some ideas about challenges of the no-cloning theorem and imaginary modifications of quantum mechanics, that could make precise cloning possible.

Introduction

The no-cloning theorem [2] based on idea of nonlinearity is very straightforward and already hint like “quantum cloning is impossible, because it is not linear” is usually enough for a quantum physicist to recover at least outline of the proof [1]. Why it was really not issued already 75 years ago?

I think, together with some new details uncovered in [1] there is omnipresent “meta-physical” problem — correspondence between a pure mathematical model and the real physical system, and here no-cloning theorem provides some challenges, possibly reflected already in first replies on the article [1]. I would try to draw this trouble in most general terms — simple and clear mathematical model used in [2] stimulates some questions about physical principle of covariance, even more general, than linearity of quantum mechanics.

Sure, it is possible to produce modification of no-cloning proof to take into account this more general principle for particular model with photons and lasers, but in such a case it loses some charm of universality and simplicity of mathematical arguments.

So, here I am briefly discussing yet another idea — considering imaginary modifications of quantum evolution, like nonlinear model briefly discussed in [1], there perfect cloning could be acceptable.

1 Covariance vs. linearity?

Let us use simplified formulations of no-cloning theorem without state of measurement device often used nowadays. There are two basic schemes, those could be found in literature:

\[ |\psi\rangle \rightarrow |\psi\rangle|\psi\rangle, \]

and, more accurate:

\[ |a\rangle|\psi\rangle \rightarrow |\psi\rangle|\psi\rangle, \]

where \( |a\rangle \) is some fixed, known state of ancillary system, sometime denoted simply as \( |0\rangle \) due to “fashionable” applications in quantum information science.

Maybe second expression Eq. (2) is not “more accurate” and in [2] was used rather first setup [2], but more universal and understanding arguments are related rather with second one.

Anyway, let us instead of proper and difficult model of first setup Eq. (2) with Fock spaces, continuous variables, etc., consider naïve nonstandard, “modified” quantum evolution between two finite-dimensional Hilbert spaces with different dimensions:

\[ \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H}. \]
In general, relations between such spaces with different dimensions are not always functions at all (i.e. multi-functions). It is possible to ask, that kind of usual functions, maybe even nonlinear, for such non-standard evolution are appropriate as generalization of unitary linear maps between spaces with equal dimension.

Maybe natural example is unitary linear map between first space and subspace of second space with same dimension (image of linear map). Really it is not very interesting example, because it does not differ much from standard case with two equal spaces — if second one contains subspace that never could be reached, why simply does not cut this “nonphysical junk”? But it is not the only problem.

Let us consider usual linear cloning of two orthogonal states allowed by unitary quantum evolution:

\[ |0\rangle \rightarrow |0\rangle |0\rangle; \quad |1\rangle \rightarrow |1\rangle |1\rangle, \quad (4) \]

with arguments used in [2] we have result, that linear, unitary evolution may clone only this two states.

Let us consider now situation, when modified (imaginary, nonstandard) evolution Eq. (3) describes decay of some hypothetical free spin-1/2 particle into two spin-1/2 particles. For this rather unrealistic process with initial particle and products of decay are staying in rest we get paradoxical situation, that linear cloning Eq. (3) contradicts to principle of covariance, or more simply, uniformity of space, because we have chosen axis in space, \textit{i.e.} the two opposite directions of spin corresponding basis \( |0\rangle \) and \( |1\rangle \) that only could be cloned and so this “space axis” could be found after numerous repeating of same experiment with different states.

Contrary, the nonlinear evolution like Eq. (1) does not have such a problem. It is clear also that such arguments may not be applied to apparently equal scheme Eq. (4), because there is ancillary system in state \( |0\rangle \) corresponding to certain direction of spin and so here is not necessary to introduce some non-homogeneity in model of physical interaction, because a preferred direction presents in initial conditions.

The principle of covariance is very significant and universal, because it is related with inevitable conditions, like independence of physical laws from coordinate system used by us for its description and example above demonstrates such kind of relations. On the other hand, the linearity and covariance is particular case of the same mathematical idea: let us consider some object with operation \( \ast \), then \textit{homomorphism} is map \( H \) to other object with property:

\[ H(a \ast b) = H(a) \ast H(b). \quad (5) \]

It is clear, that if \( \ast \) is addition, then “homomorphism” is “linearity”, but if \( \ast \) is operation of composition in group of transformation of space-time, then “homomorphism” is “covariance”.

Using formal mathematical language, principle of covariance can be expressed as homomorphism between transformations of wave vector and symmetries of space [4]. So, if quantum evolution could accept some fundamental processes between Hilbert spaces with different dimensions like Eq. (3), then general physical principles rather would forget linear evolution, than cloning.

But here again the problem [3] with instantaneous (“superluminal”) communications should be considered with necessary care, but it is not the subject of present note.

\[ ^{3}\text{Main problem here is conservation of angular momentum (if do not ask about less fundamental quantum numbers). It presents also in initial paper [4] and was discussed in replies [5]. But for our hypothetical process Eq. (3) the difficulty with momentum is rather inherited, because already in classics conservation laws due to Noether’s theorem can be considered as consequence of space-time symmetries and homogeneity, it has important influence also in quantum theory [5], but process Eq. (3) may not be described with continuous time, because dimension of phase space spasmodically increases. So consideration below could be considered as tries to introduce reasonable laws for this discontinuous process and seems the violation is really minimal — using classical analogue in discussed hypothetical model is changed only absolute value of angular momentum, not direction, and such a principle is close related with possibility of cloning in quantum case.} \]

\[ ^{4}\text{These arguments do not have much with real physics and used to emphasize problems of given abstract mathematical model.} \]

\[ ^{5}\text{In example with laser it corresponds to a question: “How does it possible to clone only two fixed state of polarization — it breaks rotational symmetry?” and suggestion: “This symmetry may be broken due to spins and nonzero angular momenta of atoms and electrons involved in stimulated emission”.} \]
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References

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[4] N. N. Bogoliubov, D. V. Shirkov, Introduction to the Theory of Quantized Fields, (Wiley, NY 1980; Nauka, Moscow 1984).