Lepton pair production in a charged quark gluon plasma

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Abstract. We investigate the effects of a charge asymmetry on the spectrum of dileptons radiating from a quark gluon plasma. We demonstrate the existence of a new set of processes in this regime. The dilepton production rate from the corresponding diagrams is shown to be as important as that obtained from the Born-term quark-antiquark annihilation.

INTRODUCTION

The aim of this talk is to show that, when in a medium there is a finite charge density (i.e., a finite chemical potential), a new set of lepton pair-producing processes actually arises \footnote{Note that a net charge density in a quark gluon plasma does not necessarily imply a net baryon density and vice-versa.}. We then calculate a new contribution to the 3-loop photon self-energy. The various cuts of this self-energy contain higher loop contributions to the usual processes of $q\bar{q} \rightarrow e^+e^-$, $qg \rightarrow qe^+e^-$, $qq \rightarrow qqe^+e^-$, and an entirely new channel: $gg \rightarrow e^+e^-$. We calculate the contribution of this new reaction to the differential production rate of back-to-back dileptons. It is finally shown that, within reasonable values of parameters, this process may become larger than the differential rate from the bare tree level $q\bar{q} \rightarrow e^+e^-$. At zero temperature, and at finite temperature and zero charge density (note: henceforth, a finite density will imply a finite charge density), diagrams in QED that contain a fermion loop with an odd number of photon vertices (e.g. Fig. \ref{fig:diagram}) are cancelled by an equal and opposite contribution coming from the same diagram with fermion lines running in the opposite direction (Furry’s theorem \cite{2,3,4}). This statement can also be generalized to QCD for processes with two gluons and an odd number of photon vertices.

A physical perspective is obtained by noting that all these diagrams are encountered in the perturbative evaluation of Green’s functions with an odd number of gauge field operators. At zero (finite) temperature, in the well defined case of QED we observe quantities like $\langle 0|A_{\mu_1}A_{\mu_2}\cdots A_{\mu_{2n+1}}|0 \rangle \ ( Tr[\rho(\mu,\beta)A_{\mu_1}A_{\mu_2}\cdots A_{\mu_{2n+1}}] )$ under the action of the charge conjugation operator $C$. In QED we know that $CA_{\mu}C^{-1} = -A_{\mu}$. In the case of the vacuum $|0\rangle$, we note that $C|0\rangle = |0\rangle$, as the vacuum is uncharged. As a result

\begin{equation}
\langle 0|A_{\mu_1}A_{\mu_2}\cdots A_{\mu_{2n+1}}|0 \rangle = \langle 0|C^{-1}CA_{\mu_1}C^{-1}CA_{\mu_2}\cdots A_{\mu_{2n+1}}C^{-1}C|0 \rangle = \langle 0|A_{\mu_1}A_{\mu_2}\cdots A_{\mu_{2n+1}}|0 \rangle (-1)^{2n+1} = -\langle 0|A_{\mu_1}A_{\mu_2}\cdots A_{\mu_{2n+1}}|0 \rangle = 0. \quad (1)
\end{equation}
At a temperature $T$, the corresponding quantity to consider is
\[ \sum_{n} \langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta(E_n - \mu Q_n)}, \]
where $\beta = 1/T$ and $\mu$ is a chemical potential. Here, however, $C|n\rangle = e^{i\phi}|-n\rangle$, where $|-n\rangle$ is a state in the ensemble with the same number of antiparticles as there are particles in $|n\rangle$ and vice-versa. If $\mu = 0$ i.e., the ensemble average displays zero density then inserting the operator $C^{-1}C$ as before, we get
\[ \langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta E_n} = -\langle -n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | -n \rangle e^{-\beta E_n}, \]
(2)
The sum over all states will contain the mirror term $\langle -n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | -n \rangle e^{-\beta E_n}$, with the same thermal weight
\[ \Rightarrow \sum_{n} \langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta E_n} = 0, \]
(3)
and Furry’s theorem still holds. However, if $\mu \neq 0$ ($\Rightarrow$ unequal number of particles and antiparticles) then
\[ \langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta (E_n - \mu Q_n)} = -\langle -n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | -n \rangle e^{-\beta (E_n - \mu Q_n)}, \]
(4)
the mirror term this time is $\langle -n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | -n \rangle e^{-\beta (E_n + \mu Q_n)}$, with a different thermal weight, thus
\[ \sum_{n} \langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta (E_n - \mu Q_n)} \neq 0, \]
(5)
and Furry’s theorem will now break down. One may say that the medium, being charged, manifestly breaks charge conjugation invariance and these Green’s functions are thus finite, and will lead to the appearance of new processes in a perturbative expansion. The appearance of processes that can be related to symmetry-breaking in a medium has been noted elsewhere [5].

Let us, now, focus our attention on the diagrams of Fig. 2 for the case of two gluons and a photon attached to a quark loop (the analysis is the same even for QED i.e., for three photons connected to an electron loop). Such a process does not exist at zero temperature, or even at finite temperature and zero density. At finite density this leads to a new source of dilepton or photon production ($gg \rightarrow l^+l^-$). In order to obtain the full
FIGURE 2. The two gluon photon effective vertex as the sum of two diagrams with quark number running in opposite directions.

matrix element of a process containing the above as a sub-diagram one must coherently sum contributions from both diagrams which have fermion number running in opposite directions. The amplitude for $T^{\mu \rho \nu}(-T^{\mu \rho \nu} + T^{\nu \rho \mu})$ is:

$$T^{\mu \rho \nu} = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d^3q}{(2\pi)^3} Tr[t^a t^b] \frac{(q+p-k)^\alpha q_\beta (q+p)\delta}{(q+p-k)^2 q^2 (q+p)^2},$$

$$T^{\nu \rho \mu} = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d^3q}{(2\pi)^3} Tr[t^a t^b] \frac{(q+k-p)^\alpha q_\beta (q-p)\delta}{(q+k-p)^2 q^2 (q-p)^2}. \quad (6)$$

Again, the extension of Furry’s theorem to finite temperature does not hold at finite density: as, if we set $n \rightarrow -n-1$, we note that $q_0 \rightarrow -q_0$, and, as a result, $T^{\mu \rho \nu}(\mu, T) \neq -T^{\nu \rho \mu}(\mu, T)$. Of course, if we now let the chemical potential go to zero ($\mu \rightarrow 0$), we note that for the transformation $n \rightarrow -n-1$, we obtain $q_0 \rightarrow -q_0$, and, thus, $T^{\mu \rho \nu}(0, T) \rightarrow -T^{\nu \rho \mu}(0, T)$. The analysis for fermion loops with larger number of vertices is essentially the same.

A Realistic Calculation

To calculate the contribution made by the diagram of Fig. 2 to the dilepton spectrum emanating from a quark gluon plasma, we calculate the imaginary part of the photon self-energy containing the above diagram as an effective vertex (Fig 3, see reference [1] for details).

$$\Pi^\rho_p = \frac{1}{\beta} \sum_{k,0} \int \frac{d^3k}{(2\pi)^3} \mathcal{D}(k) T^{\rho \mu \nu}(k-p, k; p) D(\xi(k-p)) T^{\xi \rho \eta}(k, k-p; -p), \quad (7)$$

where, the $\mathcal{D}$’s represent bare gluon propagators, and the $T$’s represent the effective vertices. We calculate in the limit of photon three momentum $\vec{p} = 0$. The imaginary part of the considered self-energy contains various cuts. We concentrate, solely, on the cut that represents the process of gluon-gluon to $e^+ e^-$ (see Fig. 3). This is a process, which, to our knowledge, has not been discussed before. The other possible cuts represent finite density contributions to other known processes of dilepton production.
The differential production rate for pairs of massless leptons, with total energy $E$ and total momentum $\vec{p} = 0$, is given in terms of the discontinuity in the photon self-energy, as

$$
\frac{dW}{dE d^3 p}(\vec{p} = 0) = \frac{\alpha}{12\pi^3} \frac{1}{E^2} \frac{1}{1 - e^{E/T}} \frac{1}{2\pi i} \text{Disc}\Pi^\rho_\rho(0),
$$

where $\alpha$ is the electromagnetic coupling constant. The rate of production of a hard lepton pair with total momentum $\vec{p} = 0$, at one-loop order in the photon self-energy (i.e., the Born term), is given for three flavours as

$$
\frac{dW}{dE d^3 p}(\vec{p} = 0) = \frac{5\alpha^2}{6\pi^3} \tilde{n}(E/2 - \mu)\tilde{n}(E/2 + \mu) + \frac{\alpha^2}{6\pi^3} \tilde{n}^2(E/2).
$$

In the above equation, the first term on the r. h. s. is the contribution from the up and down quark sector; and the second part is the contribution from the strange sector. In a realistic plasma, the net charge and baryon imbalance is caused by the valence quarks brought in by the incoming charged baryon-rich nuclei. The baryon and charge imbalance is, thus, manifested solely in the up and down quark sector; hence, the chemical potential influences only the distribution function of the up and down quarks. The strange and anti-strange quarks are produced in equal numbers in the plasma; resulting in a vanishing strange quark chemical potential. Thus, dilepton production, from the channel indicated by Fig. 3, will only receive contributions from the up and down quark flavours.

The initial temperatures of the plasma, formed at RHIC and LHC, have been predicted to lie in the range from 300-800 MeV [7, 8]. For this calculation we use estimates of $T = 400$ MeV (Fig. 4) and $T = 600$ MeV (Fig. 5). To evaluate the effect of a finite chemical potential, we perform the calculation with two extreme values of chemical potential $\mu = 0.1T$ (left plots) and $\mu = 0.5T$ (right plots) [9]. The calculation, is performed for three massless flavours of quarks. In this case, the strong coupling constant is (see [10])

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**FIGURE 3.** The photon self-energy at three loops and the cut that is evaluated.
\[ \alpha_s(T) = \frac{6\pi}{27\ln(T/50\text{MeV})}. \] (10)

The differential rate for the production of dileptons with an invariant mass from 0.5 to 2.5 GeV is presented. On purpose, we avoid regions where the gluons become very soft. In the plots, the dashed line is the rate from tree level \( q\bar{q} \) (Eq. (9)); the solid line is that from the process \( gg \to e^+e^- \). We note that in both cases the gluon-gluon process dominates at low energy and dies out at higher energy leaving the \( q\bar{q} \) process dominant at higher energy.

**FIGURE 4.** The differential production rate of back-to-back dileptons from two processes plotted against dilepton invariant mass. The dashed line represents the contribution from Born term \( q\bar{q} \to e^+e^- \). The solid line corresponds to the process \( gg \to e^+e^- \). Temperature is 400 MeV. Quark chemical potential, in the first figure, is 0.1\( T \). The second figure is the same as the first, but, with \( \mu = 0.5T \).

**FIGURE 5.** Same as above but with \( T=600\text{MeV} \).

**High Temperature Limit**

As the reader may have noted, in the above calculation, \( \alpha \simeq 0.3, g \simeq 2 \), thus, we are not unequivocally in the perturbative regime. At asymptotically high temperatures (\( T \to \infty \)), however, \( g \to 0 \), in this limit one may make the Hard Thermal Loop (HTL) approximation [11, 12] (note: only the main results will be quoted here; the details will appear elsewhere [15]). At very high temperature, contributions from loop diagrams are dominated by loop momenta \( (q) \) of the order of the temperature \( T \). If the momentum flowing in the external legs is much smaller (of the order of \( gT \), i.e., \( q \gg k, p \)), then loop
corrections are of the same order in $g$ as the bare diagrams, and, thus, must be resummed into the tree amplitudes.

In this limit $(q + p - k)_{\alpha} \simeq q_{\alpha}$, and $E_{q+p} \simeq E_q + \hat{p} \cdot \hat{q} + \frac{|\hat{p}|^2}{2E_{\hat{q}}} + \frac{|\hat{p} \cdot \hat{q}|^2}{2E_{\hat{q}}}$. On performing the full Matsubara sum and taking the HTL limit for the numerator, we get

$$
q_{\mu \nu} = \int \frac{d^3q}{(2\pi)^3} \frac{\delta^{ab}}{2} g^2 Tr[\gamma^\mu \gamma^\nu \gamma^b \gamma^a] \frac{\hat{q}_{1,\alpha} \hat{q}_{-s_3,\delta}}{p^0 - s_2E_2 + s_3E_3} \left[ s_1 (\tilde{n}(E_3 - s_3 \mu) - \tilde{n}(E_3 + s_3 \mu)) + s_3 (\tilde{n}(E_1 + s_1 \mu) - \tilde{n}(E_1 - s_1 \mu)) \right] (11)
$$

where, $\hat{q}_{+} = (1, \hat{q}_1, \hat{q}_2, \hat{q}_3)$, and $\hat{q}_{-} = (-1, \hat{q}_1, \hat{q}_2, \hat{q}_3)$. Note that if $\mu$ is set to zero this amplitude vanishes identically. The conventional HTL term (i.e., terms proportional to $(gT)^2$) from triangle graphs such as these would come from the terms with $s_1 = +, s_2 = -, s_3 = -$ and $s_1 = -, s_2 = +, s_3 = +$. The term proportional to $(gT)^2$ is

$$
q_{\mu \nu}^{(gT)^2} = \int \frac{d^3q}{(2\pi)^3} \frac{\delta^{ab}}{2} g^2 Tr[\gamma^\mu \gamma^\nu \gamma^b \gamma^a] \left\{ \frac{\hat{q}_{+\alpha} \hat{q}_{+\beta}}{p^0 - \hat{p} \cdot \hat{q}} \left[ \frac{\partial \tilde{n}(E_1 + \mu)}{\partial E_3} (\tilde{k} \cdot \hat{q}) - \frac{\partial \tilde{n}(E_3 - \mu)}{\partial E_3} (\tilde{k} \cdot \hat{q}) \right] + \frac{\hat{q}_{-\alpha} \hat{q}_{-\beta}}{p^0 + \hat{p} \cdot \hat{q}} \left[ \frac{\partial \tilde{n}(E_3 - \mu)}{\partial E_3} (\tilde{k} \cdot \hat{q}) - \frac{\partial \tilde{n}(E_1 + \mu)}{\partial E_3} (\tilde{k} \cdot \hat{q}) \right] \right\} + \frac{\hat{q}_{-\alpha} \hat{q}_{-\beta}}{p^0 + \hat{p} \cdot \hat{q}} \left[ \frac{\partial \tilde{n}(E_3 - \mu)}{\partial E_3} (\tilde{k} \cdot \hat{q}) - \frac{\partial \tilde{n}(E_1 + \mu)}{\partial E_3} (\tilde{k} \cdot \hat{q}) \right] + \frac{\hat{q}_{+\alpha} \hat{q}_{+\beta}}{p^0 - \hat{p} \cdot \hat{q}} \left[ \frac{\partial \tilde{n}(E_3 - \mu)}{\partial E_3} (\tilde{k} \cdot \hat{q}) - \frac{\partial \tilde{n}(E_1 + \mu)}{\partial E_3} (\tilde{k} \cdot \hat{q}) \right] \right\}.
$$

Note that if we change $\hat{q} \rightarrow -\hat{q}$ in the last two terms then $\hat{q}_{+\alpha} \rightarrow -\hat{q}_{-\alpha}$ and the whole term becomes identically zero. Thus there is no term proportional to $(gT)^2$ in this diagram. The next term is of order $g^2 T$ and is nonzero [15]. However, as is well known, the HTL approximation of the $q\bar{q}\gamma$ vertex is of the order of $(gT)^2$ [14]. Thus, in the high temperature approximation, the $g\bar{q}\gamma$ HTL vertex is suppressed as compared to the $q\bar{q}\gamma$ HTL-resummed vertex. As a result the dilepton production rate from resummed two-gluon-fusion process will also be suppressed compared to the rate from the resummed Born term, in this limit.

On summing up all the HTL contributions to the self energy of the gluon we get two dispersion relations for the transverse and longitudinal modes of the gluon. A plot of the two modes [15] is as shown in Fig. (6). The upper branch is the dispersion relation for the transverse quasi-particle excitation, the lower one is the longitudinal excitation. The straight line, corresponding to free gluons is shown for reference. Note that there is no minima in either branch (except at $k = 0$), unlike in the case of quarks [14]. Hence, we will not see any sharp Van Hove peaks [14] in the resulting dilepton spectra emanating from this process. One may thus expect that the high temperature (and, perhaps, as a result, the very low invariant mass) dilepton production spectra emanating from in-medium gluon-gluon fusion will be suppressed compared to that from in-medium quark anti-quark fusion. However, as we have noted in the previous section the intermediate
invariant mass rate from bare gluon-gluon fusion is comparable and may be larger than that from bare quark anti-quark fusion.

The entire calculation above is at full chemical and thermal equilibrium. In the early part of an ultrarelativistic heavy ion collision, the gluon number has been predicted to be much higher than at full chemical equilibrium. In such a scenario, the contributions to dilepton spectrum from processes such as those presented here will probably out-shine those from other channels.

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REFERENCES

1. A. Majumder and C. Gale, Phys. Rev. D, 63, 114008 (2001); and erratum, in press.
2. W. H. Furry, Phys. Rev., 51, 125 (1937).
3. C. Itzykson, J. B. Zuber, Quantum Field Theory, McGraw Hill, New York, (1980).
4. S. Weinberg, The Quantum Theory of Fields, Vol. 1, Cambridge University Press, (1995).
5. See for example, S. A. Chin, Ann. Phys. 108, 301 (1977); H. A. Weldon, Phys. Lett. B, 274, 133 (1992); G. Wolf, B. Friman and M. Soyeur, Nucl. Phys. A640, 129 (1998); O. Teodorescu, A.K. Dutt-Mazumder and C. Gale, Phys. Rev. C 63, 034903 (2001).
6. C. Gale, and J. I. Kapusta, Nucl. Phys. B357, 65 (1991).
7. X. N. Wang, Phys. Rep. 280, 287 (1997).
8. R. Rapp, [hep-ph/0010101].
9. K. Geiger, and J. I. Kapusta, Phys. Rev. D, 47, 4905 (1993); N. George, for the PHOBOS collaboration, Proceedings of Quark Matter 2001.
10. J. I. Kapusta, and S. M. H. Wong, Phys. Rev. C, 62, 027901 (2000).
11. E. Braaten, and R. D. Pisarski, Nucl. Phys. B337 569 (1990).
12. M. Le Bellac, Thermal Field Theory, Cambridge University Press, (1996).
13. U. Heinz, K. Kajantie, and T. Toimela, Ann. Phys. (N.Y.) 176 218 (1987).
14. E. Braaten, R. D. Pisarski, and T. C. Yuan, Phys. Rev. Lett. 64 2242 (1990).
15. A. Majumder, and C. Gale, in preparation.