Giant Traversable Wormholes in Massive Gravity

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In this work, we construct traversable wormholes in charged and neutral massive gravity. The shape function $b(r) = a_0 - Q^2 (1/r - 1/a_0) - \gamma (r^2 - a_0^2) + \frac{1}{3} (r^3 - a_0^3)$ inherently obtained from the underlying massive theory with $a_0, Q, \gamma$ and $\Lambda$ being a wormhole throat, a charge, an arbitrary constant and a cosmological constant, respectively, while the variable redshift function $\Phi = \frac{1}{2} \ln \left(1 + p^2 / r^2 \right)$ is opted with $p$ being an arbitrary constant. Contrary to the asymptotically flat space, this work features the asymptotically dS space with a positive cosmological constant. We find that the shape function $b(r)$ is satisfied all required properties. More importantly, we observe that the null, weak, and strong energy conditions for the constructed traversable wormholes in massive gravity both charged case and neutral case are satisfied when the traversable wormholes are big enough. Moreover, we compute an amount of exotic matter by considering a volume integral and find that in all cases the volume integral vanishes at the wormhole throat meaning that one can construct a traversable wormhole without ANEC violating matter. We also examine the wormhole geometry via embedding diagrams and analytically compute the highest charge of which the system is violated all required energy condition if the charges exceed such a highest value. Furthermore, we also confirm for the other two types of the red shift functions, $\Phi = p$ constant and $\Phi = p/r$ that the profiles of energy conditions are found to be very similar.

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Introduction.—Two-relevant passages written by C. Sagan [1] drive us to search for a new perspective on interstellar traveling. The passages dissent an interstellar traveling through black holes and Schwarzschild wormholes. The later is due to the collapse too quickly for anything to cross from one end to the other and it would be possible only if some exotic matters with negative energy density could be used to stabilize them. Wormholes that could be crossed in both directions are known as traversable wormholes [2]. In the literature, many authors have intensively studied various aspects of traversable wormhole (TW) geometries, with and without invoking exotic matters [3]. Very recently, Casimir energy is successfully used as a potential source to generate various types of traversable wormholes, see e.g., [4, 5]. Invoking exotic matters, one expects that traversable wormhole solutions violate the energy conditions at the wormhole throat.

Among various modified theories of gravity, the de Rham-Gabadadze-Tolley (dRGT) massive theory is targeted as one of the compelling scenarios when studying the universe in the cosmic scale. The applications of dRGT massive gravity to study the exotic objects, e.g., black holes, already appeared in the literature [6–8].

In the present work, we extend the original massive gravity by adding the Maxwell field and study new traversable wormhole solutions. We derive the wormhole shape function to obtain $b(r) = a_0 - Q^2 (1/r - 1/a_0) - \gamma (r^2 - a_0^2) + \frac{1}{3} (r^3 - a_0^3)$ that is directly derived from the proposed model with $a_0, Q, \gamma$ and $\Lambda$ being a wormhole throat, a charge, an arbitrary constant and a cosmological constant, respectively and consider the variable redshift function $\Phi = \frac{1}{2} \ln \left(1 + p^2 / r^2 \right)$ with $p$ being also an arbitrary constant. We check the null, weak, and strong condition at the wormhole throat and compute the total amount of averaged null energy condition (ANEC) violating matter in the space-time. We also examine the wormhole geometry via embedding diagrams and analytically compute the highest charge of which the system is violated all required energy condition if the charges exceed such a highest value.

Massive gravity with Maxwell field.—The gravitational action of the dRGT massive gravity with the Maxwell
strength tensor $F_{\mu\nu}$ and anisotropic fluid matter reads
\[ S = \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G} \left( R + m_g^2 U(g,\phi^\alpha) \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{\text{matter}} \right\}, \tag{1} \]
where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, $A_\mu = (-Q/r, 0, 0, 0)$ is the vector potential with the electric charge $Q$ and $\sqrt{-g}$ is the volume element in 4-dimensional spacetime. The potential term of the massive gravity, $U$ is defined by
\[ U = U_2 + \alpha_3 U_3 + \beta \alpha_4 U_4. \tag{2} \]
The $U_2$, $U_3$ and $U_4$ are given by
\[ U_2 = |K|^2 - |K|^2, \quad U_3 = |K|^3 - 3|K||K|^2 + 2|K|^4, \quad U_4 = |K|^4 - 6|K|^2|K|^2 + 8|K|^4 - 3|K|^2 - 6|K|^4, \]
\[ K^{\mu\nu} = \delta^{\mu\nu} - \sqrt{\det K} F_{\alpha\beta} \partial_\alpha \phi^\alpha \partial_\beta \phi^\beta, \tag{3} \]
where a bracket $[ ]$ represents the trace of the rank two tensor, $K^\mu_\mu$. Moreover, the fiducial metric, $F_{\alpha\beta}$ is chosen by the following the diagonal matrix ansatz $F_{\alpha\beta} = (0, 0, k^2, k^2 \sin^2 \theta)$ where $k$ is a positive constant and the unitary gauge is used as $\phi^\alpha = x^\alpha \delta^\alpha_\mu$. In addition, the parameters $\alpha_3, \alpha_4$ are the parameters of the dRGT theory and we will relate these parameters with the graviton mass in the latter.

The Einstein field equation of the dRGT massive gravity with the Maxwell field is obtained by varying the gravitational action in Eq. (1) with respect to the metric, $g^{\mu\nu}$. One finds,
\[ G_{\mu\nu} = 8\pi G \left( T^{(g)}_{\mu\nu} + T^{(c)}_{\mu\nu} + T^{(m)}_{\mu\nu} \right), \tag{4} \]
where $T^{(g)}_{\mu\nu}$, $T^{(c)}_{\mu\nu}$ and $T^{(m)}_{\mu\nu}$ are the energy-momentum tensors of the massive gravity, Maxwell field and matter fluid, respectively. One may write those terms in the following form,
\[ T^{(g)}_{\mu\nu} = \left( \rho^{(g)} + P_t^{(g)} \right) u_\mu u_\nu + P_t^{(g)} g_{\mu\nu} + \left( P_r^{(g)} - P_t^{(g)} \right) \chi_\mu \chi_\nu, \tag{5} \]
\[ T^{(c)}_{\mu\nu} = F_{\mu\sigma} F^{\sigma\nu} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}, \tag{6} \]
\[ T^{(m)}_{\mu\nu} = \left( \rho + P_t \right) u_\mu u_\nu + P_t g_{\mu\nu} + \left( P_r - P_t \right) \chi_\mu \chi_\nu, \tag{7} \]
where the $u_\mu$ is timelike unit vector and the $\chi_\mu$ is the spacelike unit vector orthogonal to the $u_\mu$ with the normalization $u_\mu u^\mu = -1$ and $\chi_\mu \chi^\mu = 1$. The $\rho$, $P_r$ and $P_t$ are the energy density, radial and tangential pressure of matter fluid where the the same variables with the superscript (g) are represented for the energy-momentum tensor of massive gravity. Having used the static spherical symmetric spacetime, we obtain the explicit form of the line element of the dRGT massive gravity which reads
\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \tag{8} \]
where the function $f(r)$ can be written as
\[ f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3} + \gamma r + \zeta, \tag{9} \]
where $M$ is the mass parameter, $\Lambda$ is the effective cosmological constant, and $\gamma$ and $\zeta$ are new parameters and they are linear combinations of the parameters in the dRGT massive gravity via the following relations,
\[ \Lambda = -3m_g^2(1 + \alpha + \beta), \quad \gamma = -m_g^2 k(1 + 2\alpha + 3\beta), \quad \zeta = m_g^2 k^2(\alpha + 3\beta). \tag{10} \]
In this work, we focus our study for the positive cosmological constant. It is worth to mention that the static spherical solution of the Maxwell-dRGT gravity is the generalized Reissner-Nordstrorm de-Sitter solution which leads to the asymptotically de-Sitter for $r \to \infty$. While the unique feature of the massive graviton the dRGT theory are characerized by the parameters $\gamma$ and $\zeta$.

According to Refs. [11–13], one can rewrite the $\rho^{(g)}$ and $P_t^{(g)}$ from the massive gravity potential in Eq. (2). They are given by
\[ \rho^{(g)} = -\frac{m_g^2}{8\pi G} \left( \frac{3r - 2k}{r} + \alpha(3r - k)(r - k) \right) \frac{3(r - k)^2}{r^2} = -P_r^{(g)}, \tag{11} \]
\[ P_t^{(g)} = \frac{m_g^2}{8\pi G} \left( \frac{3r - k}{r} + \alpha(3r - 2k) \right) + \frac{3(k - r)}{r} \tag{12} \]
In addition, we note that the energy-momentum tensor of the massive gravity, $T^{(g)}_{\mu\nu}$ exhibits its behavior as an anisotropic dark energy i.e., $P_r^{(g)} = -\rho^{(g)}$ see Refs. [11–13] for detail discussions and applications.

On the other hand, the components of the Maxwell energy-momentum tensor in Eq. (6) are written by Ref. [13].
\[ T^{(c)}_{\mu\nu} = T^{(c)}_{\mu\nu} = -\frac{Q^2}{2r^4}. \tag{13} \]
Moreover, the conservation of the total energy momentum tensor is hold and the massive gravity, the Maxwell
field and the anisotropic fluid energy-momentum tensors also conserve separately,
\[ \nabla^\mu T_{\mu \nu}^{(g)} = 0, \quad \nabla^\mu T_{\mu \nu}^{(c)} = 0, \quad \nabla^\mu T_{\mu \nu}^{(m)} = 0. \] (15)

**Constructing Traversable Wormholes.**—In order to construct the traversable wormholes, we consider a static and spherically symmetric Morris-Thorne traversable wormhole in the Schwarzschild coordinates given by the following metric tensor [2]
\[ ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \] (16)

The characteristic of the wormhole is determined by the wormhole shape function \( b(r) \) and the red shift function \( \Phi(r) \). There are various choices of the red shift function. In order to formulate the traversable wormhole in charged dRGT spacetime, we match the metric tensor in the radial components of the dRGT metric in Eq. (15) and the metric of the Morris-Thorne traversable wormhole in Eq. (16) as follows:
\[ b(r) = a_0 - Q^2 \left( \frac{1}{r} - \frac{1}{a_0} \right) - \gamma (r^2 - a_0^2) + \frac{\Lambda}{3} (r^3 - a_0^3), \]
where we have replaced 2\( M \) with \( a_0 + Q^2/a_0 + \gamma a_0^2 - a_0^3 \Lambda/3 \) using the condition \( b(a_0) = a_0 \) at the wormhole throat. One notes that the wormhole shape function \( b(r) \) is a generalized asymptotically de-Sitter with the additional linear term in \( r \) from the \( \gamma \) parameter and this leads to an existence of the cosmic horizon in this scenario see [15] for the horizon analysis. In principle, the characteristic of the wormhole is determined by the wormhole shape function \( b(r) \) and the red shift function \( \Phi(r) \).

More precisely, the shape function \( b(r) \) must satisfy the following properties, (i.) \( \frac{b(r)}{r} < 1 \) for \( r > a_0 \), (ii.) \( b(a_0) = a_0 \), (iii.) \( \frac{b(r)}{r} \to \frac{\Lambda r^2}{2} - \gamma r \) as \( r \to \infty \), (iv.) \( b(r) - b(r)/r > 0 \) for \( 3r > a_0 \), (v.) \( b'(a_0) < 1 \), where \( a_0 \) is the throat radius of the wormhole. In addition, we note that the property (iii.) is the generalized asymptotically de-Sitter and it leads to a connecting geometry of two asymptotically spacetimes with the positive cosmological constant, contrary to the asymptotically flat case when the cosmological constant is set to zero. The properties (iv., v.) are the flaring-out conditions. For the traversable wormhole solutions, the throat must be minimum implying that the spacetime has to flare out at and near the throat [16] [17]. With the following set of parameters \( M = 2, \gamma = -0.02, \Lambda = 0.005, Q = 0.25, \zeta = 0.01 \) and \( m_g = 0.00001 \), the characteristics of \( b(r), b(r)/r, (b(r) - r b'(r))/b(r)^2 \), and \( b'(r) - 1 \) are illustrated in Fig[1] for the charged case and in Fig[2] for the neutral one, \( Q = 0.0 \). As shown in the results, we found that the shape function \( b(r) \) from the static spherical solutions derived from the dRGT theory in Eq. (17) is satisfactorily fulfilled all required wormhole properties, both the charged and neutral cases.

**Energy Conditions.**—According to the line element in Eq. (16), the energy density, the pressure in radial part and the pressure in the tangential part are obtained by calculating the Einstein field equation in Eq. (4) with the Planck unit \( G = 1 \). They are given by
\[ \rho = \frac{1}{8\pi r^2} b'(r) - \frac{Q^2}{2r^4} - \frac{\gamma}{4\pi r} - \frac{\zeta}{8\pi r^2} + \frac{\Lambda}{8\pi}, \]
\[ P_r = \frac{1}{8\pi} \left[ 2 + \frac{\Phi'(r)}{\Phi(r)} - \frac{b'(r)}{b(r)} \right], \]
\[ P_t = \frac{1}{8\pi} \left[ 1 - \frac{b(r)}{r} \left( \Phi'(r) + \Phi(r)^2 \right) - \frac{b'(r) - 1}{2r(r - b(r))} \Phi'(r) - \frac{b'(r)r - b(r)}{2r^2(r - b(r))} + \frac{\Phi'(r)}{r} \right] \]
\[ - \frac{Q^2}{2r^4} + \frac{\gamma}{4\pi r} - \frac{\Lambda}{8\pi}. \] (20)

I. Null energy condition (NEC) is expressed in terms.
of energy density and pressure as follows:

$$\rho(r) + P_r(r) \geq 0, \quad (21)$$

II. Weak energy condition (WEC) is given by

$$\rho(r) \geq 0, \quad \rho(r) + P_r(r) \geq 0, \quad (22)$$

III. Strong energy condition (SEC) is governed by

$$\rho(r) + P_l(r) \geq 0, \quad \rho(r) + P_r(r) + 2P_l(r) \geq 0. \quad (23)$$

The characteristic of the wormhole is determined by the wormhole shape function $b(r)$ and the red shift function $\Phi(r)$. There are various choices of the red shift function. In this article, we focus only on a model using a red shift function given by $\Phi = \frac{1}{2} \ln \left(1 + \frac{p^2}{r^2}\right)$ with $p$ being some positive parameter and $r \geq a_0$. Substituting $b(r)$ and $\Phi$ into Eqs. (18-20) and using the energy conditions (I,II,III), we end up with the results illustrated in Fig.3 for a charged case and Fig.4 for the neutral case. A set of parameters we used is $M = 2, \gamma = -0.02, \Lambda = 0.005, Q = 0.25, \zeta = 0.01$ and $m_g = 0.00001$. Invoking this set of parameters, the null, weak, and strong energy conditions (NEC, WEC, and SEC) for the traversable wormhole in massive gravity both charged case and neutral case are satisfied at the wormhole throat or even away from the throat, except regions nearby its core. Having negative values of $\gamma$ and positive values of $\zeta$ feature the traversable wormhole without invoking any exotic matter.

**Amount of exotic matter.**—In this section, we briefly discuss the volume integral, which basically provides information about the total amount of averaged null energy condition (ANEC) violating matter in the spacetime. This quantity is related only to $\rho$ and $P^r$, not to the transverse components. It is defined in terms of the following definite integral

$$I_V = \int_0^\infty \left(\rho + P_r(r) \right) r^2 dr. \quad (24)$$

Here we are going to evaluate this integral for our shape function $b(r)$. Having introduced a cut-off $a$ such that the wormhole extends form $a_0$ to $a$ with $a \geq a_0$, we have instead $I_V = 8 \pi \int_{a_0}^a \left(\rho + P_r(r) \right) r^2 dr$. We now consider the following shift function, $\Phi(r) = \frac{1}{2} \ln (1 + p^2/r^2)$, and observe that in the limit $a \rightarrow 0$ at the wormhole throat, the volume integral vanishes in all cases illustrated in Fig.5 including the charged wormhole and the neutral one.

$$I_V = \frac{1}{3} \left[3 (2M + Q^2) \log(a/a_0) \right. \right.$$  

$$- 3 (2M + \gamma p^2) \log \left[\left(p^2 + a^2\right)/\left(p^2 + a_0^2\right)\right] \right.$$  

$$- A(p, \Lambda, \zeta, Q) \left(\tan^{-1} \left(\frac{a}{p}\right) - \tan^{-1} \left(\frac{a_0}{p}\right)\right) \right.$$  

$$+ \frac{3A}{4} (a^4 - a_0^4) + \frac{1}{3} (5\Lambda - 6\gamma) (a^3 - a_0^3)$$

$$\left. + 3Q^2 \left(\frac{1}{a} - \frac{1}{a_0}\right)\right], \quad (25)$$

where we have defined a new parameter $A(p, \Lambda, \zeta, Q) = \frac{2}{p} (A p^4 + 3(\zeta + 1)p^2 - 3Q^2)$. According to Eq. (25), we discover that the volume integral vanishes in all cases when $a \rightarrow 0$ which means that one can construct a traversable wormhole without ANEC violating matter, a.k.a. exotic matter.

**Embedding diagram.**—To visualize the wormhole geometry embedded in the spacetime manifold, we can take the equator slice $\theta = \pi/2$ at time $t$ constant. The wormhole line element then reads

$$ds^2 = \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\theta^2. \quad (26)$$

Figure 3: The plots show the variation of $\rho(r) + P_r(r)$ (NEC), $\rho(r)$ (NEC, WEC), $\rho(r) + P_l(r)$ (SEC) and $\rho(r) + P_r(r) + 2P_l(r)$ (SEC) as functions of radius, $r$, in the charged dRGWT wormhole with the red shift function $\Phi(r) = \frac{1}{2} \ln \left(1 + \frac{p^2}{r^2}\right)$. We have used a set of parameter as $M = 2, \gamma = -0.02, \Lambda = 0.005, Q = 0.25, \zeta = 0.01$ and $m_g = 0.00001$. The throat of the wormhole in the charged case is located at $a_0 = 4.50$.

Figure 4: The plots show the variation of $\rho(r) + P_r(r)$ (NEC), $\rho(r)$ (NEC, WEC), $\rho(r) + P_l(r)$ (SEC) and $\rho(r) + P_r(r) + 2P_l(r)$ (SEC) as functions of radius, $r$, in the charged dRGWT wormhole with the red shift function $\Phi(r) = \frac{1}{2} \ln \left(1 + \frac{p^2}{r^2}\right)$. We have used a set of parameter as $M = 2, \gamma = -0.02, \Lambda = 0.005, Q = 0.00, \zeta = 0.01$ and $m_g = 0.00001$. The throat of the wormhole in the neutral case is located at $a_0 = 4.52$. 

$$- \frac{3}{2} (\gamma + \zeta) (a^2 - a_0^2) + \left(2AP^2 - 3\zeta\right) (a - a_0)$$

$$+ 3Q^2 \left(\frac{1}{a} - \frac{1}{a_0}\right)\right], \quad (25)$$

where
By matching variables from Eq. (26) and Eq. (27), situated at 16 cases of parameters are shown in Fig. 6. The throat located at 4.

\begin{equation}
\rho^2 \frac{d\psi^2}{r^2} = r^2 d\phi^2, \quad d\rho^2 + dz^2 = \frac{dr^2}{1 - \frac{b(r)}{r}},
\end{equation}

it yields

\begin{equation}
\rho = r, \quad \psi = \phi, \quad \frac{dr}{dz} = \pm \left(r - \frac{b(r)}{r}\right)^{-1/2}.
\end{equation}

We plot the embedding diagrams for the two cases by using a set of parameters in the charged case given by \(M = 2, Q = 0.9, \gamma = -0.02, \Lambda = 0.005, \text{ and } \zeta = 0.01\). The throat of the charged wormhole is located at 4.31, and the cosmological horizon is situated at 16.27, while the neutral wormhole has the throat located at 4.52 and the cosmological horizon is situated at 16.23. The embedding diagrams of the two cases of parameters are shown in Fig. 6.

The largest charge of the wormhole that the dRGT wormhole still satisfies the strong energy condition is

\begin{equation}
Q_{\text{SEC}}^{\text{Max}} = \frac{1}{\sqrt{3B}} \left\{ r \left( -6M \left( p^4 + 3p^2r^2 \right) \right) \right.
+ \left. p^4r \left( -3\zeta + 3\Lambda r^3 - r^2(6\gamma + \Lambda) - 3r(\gamma + \zeta) \right) \right.
+ \left. 3p^2r^3 \left( 2\Lambda r^3 - 2r^2(2\gamma + \Lambda) + r(\gamma - 2\zeta) + 2 \right) \right.
+ \left. 3r^5 \left( -\zeta + \Lambda r^3 - r^2(2\gamma + \Lambda) - \zeta r \right) \right\}^{1/2},
\end{equation}

where we have defined new functions \(\delta(r) \equiv 8\pi G - r - 1\) and \(B(r) \equiv (p^2 + r^2)^2\delta(r) + 2r^4\). It is worth noting that if the values of the charge exceed \(Q_{\text{SEC}}^{\text{Max}}\), the solutions violate not only the strong energy condition but also all energy conditions. Interestingly, we observe that with the present of the charge the wormhole throat gets modified. To be more precise, compared with the neutral case, the throat radius is getting smaller, but is lengthened when having the charge.

**Discussion.**—Let us highlight our present work. Apparently, there have been a multitude of achievements in the existing literature following a framework of dRGT gravity. Many of them ignore a \(\zeta\) term in constructing some particular models. However, our implementation is manipulated by invoking all parameters present in the dRGT theory. In this work, we constructed a charged and neutral traversable wormholes in dRGT massive gravity. The shape function \(b(r)\) is inherently obtained from the underlying massive theory, while the variable redshift function \(\Phi = \frac{1}{2} \ln \left(1 + p^2/r^2\right)\) is opted. Contrary to the asymptotically flat space, this work featured the asymptotically dS space with a positive cosmological constant. We found that the shape function \(b(r)\) is satisfied all required properties. We observed that the null, weak, and strong energy conditions for the constructed traversable wormholes in massive gravity both charged case and neutral case are satisfied when the traversable wormholes are big enough. More importantly, the parameters \(\gamma\) and \(\zeta\) from the massive graviton and fiducial
metric in dRGT theory made all requirements of the energy conditions fulfilled without invoking any exotic matter. Moreover, we computed an amount of exotic matter by considering a volume integral and found that in all cases the volume integral vanished at the wormhole throat. We also examined the wormhole geometry via embedding diagrams. We concluded that the graviton mass might support traversable wormhole and enlarged embedding diagrams. We also examined the wormhole geometry via all cases the volume integral vanished at the wormhole throat. We also confirmed for other two types of the redshift functions, $\Phi = p = \text{constant}$ and $\Phi = p/r$ that the profiles of energy conditions are found to be very similar. However, our present work features asymptotically non-flat (dS) metric and then very little in known how to study gravitational lensing in such a non-flat case. In the meanwhile, the work of [18] on the gravitational lensing and particle motions around non-asymptotically flat black hole spacetime in dRGT gravity could be very useful when one wants to tackle this interesting issue. Additionally, a study of linear stability of the constructed giant wormhole is also worth investigating.

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