The long-term memory analysis of industrial indices of the Chinese stock market

Lin Yong
Renmin University of China, Information School,
Department of Mathematics, Beijing 100872, China.
linyong01@ruc.edu.cn

Abstract: The main work of this paper is to apply the fractional market theory and time series analysis for analyzing various industrial indices of the Chinese stock market by rescaling range analysis. Hurst index and the long-term memory of price change in Chinese stock market are studied.

1. Introduction
Chinese stock market saw the founding, the growing, normalizing and several sharp rising and slumping from its foundation in the beginning of 1990s. It becomes increasingly important for Chinese enterprises to finance and for people to invest, and it is the important part of the Chinese economy.

In 1960s, Mandelbrot introduced the R/S analysis to Finance Theory, and found the fractional structure and non-periodic circulation. When he studied economic time series, he found that some financial time series varied independently, but the others like the interest of short sale, the price of cotton have long, definitive positive dependence. Mandelbrot call it as Joseph phenomenon, deriving from prophesy that the Egypt will have seven years harvest and seven years disaster in Bible.

Rescaled range analysis (R/S) is a statistic analysis method first used by hydrologist Hurst to solve the control of reservoir when he worked for Nile river. The variation of data range depends on the length of time. Hurst used his formula to get a new statistic: Hurst exponent (H). Hurst found that most natural phenomenon are the stochastic motion with deviation—a trend plus noise. The strength of noise and the level of noise can be measured by the variation of rescaled range with time or how big the value of H to 0.5. We intend to apply the research on the time series of natural phenomenon to the time series of finance, and we can found whether the time series is stochastic or a fractional time series.

Many scholars have done a lot of empirical study in the long-term memory of the stock market, and most research support that the stock market has long-term memory, such as Bollerslev & Mikkelsen(1996) and Peters(1991) for the American stock market, Wang Chunfeng, Zhang Qingcui (2004), Zhao Zhenquan, Zhou Baicheng, Zhou Jianwen(2005), Li Wei, Lao Chuanqi(2006), for the empirical study on Shanghai and Shenzhen stock market. Peters(1996) has analyzed the monthly data of S&P500 index about American stock market by R/S analysis, and found a 48-month mean fluctuant period. Zhao Zhenquan and his colleague have analyzed the monthly data of Shanghai stock market, and found a 17-month mean fluctuant period in Chinese stock market. Hong Rumin(2005) proposed

1 To whom any correspondence should be addressed.
that the statistic Max H can be used to measure the mean fluctuant period of Chinese security market by R/S analysis, and found a fluctuant period of 96——127 exchange days less than 6 months a little, by analyzing the Shanghai and Shenzhen daily data from 1993 to 2001. He suggests that it is because Chinese listed corporations must publish 6-month yield report and yearly yield reports.

The main work of this paper is to apply the tools of fractional market theory and time series analysis, analyzing various industrial indices of Chinese stock market by rescaling range analysis, studying the Hurst index and the long-term memory of the price change of Chinese stock market. We select 3 industrial indices, and the data period are from 2-19-2002 to 2-19-2006. Those are public affair index, real estate index and finance and insure index. We select weekly data for avoiding the effect of Monday and weekend. We found that there are different fluctuant periods in different industrial indices. The long-term memory period of public affair index is about 50 weeks, real estate index is 95 weeks, finance and insure index is about 170.

2. The classification and calculation of period of index
The main idea of R/S analysis is to analyzing the scaling behavior of accumulative mean deviation with multi-scale. It supposes that a point move on a time axis, so the accumulative mean deviation is the distance of the point from the beginning point. The following formulas are from Peters(1996).

The instruction of variation:
(1) \( M_{t}, t = 1,2,3,\cdots,501 \): the index value, in this paper we choose weekly data;
(2) \( N_{t}, t = 1,2,3,\cdots,501 \): the logarithmic ratio of the first order differential stock index sequence;
(3) \( M \): the number of the selected data, in this paper the number is 501;
(4) \( H \): Hurst index.

Step:
(1) Set the length of time series equal to \( M \), and transfer time series with length \( M \) to that logarithmic ratio with length \( N = M - 1 \):
\[
N_{t} = \log(M_{t+1}/M_{t}), t = 1,2,3,\cdots, M - 1
\]
(2) Divide the time interval with length \( N \) to A subinterval with length \( n \), so \( A \times n = N \). Mark every subinterval with \( I_{a}, a = 1,\cdots, A \). Every point in \( I_{a} \) can be denoted by \( N_{k,a}, k = 1,\cdots, n; a = 1,\cdots, A \). For every subinterval \( I_{a} \) with length \( n \), the mean is:
\[
e_{a} = \frac{1}{n} \sum_{k=1}^{n} N_{k,a}
\]
(3) For every subinterval, the mean accumulative is as follow:
\[
X_{k,a} = \sum_{i=1}^{k} (N_{i,a} - e_{a}), \quad k = 1,2,\cdots, n
\]
(4) Range is defined as the value of maximal \( X_{k,a} \) minus minimal \( X_{k,a} \) in every subinterval \( I_{a} \).
\[
R_{I_{a}} = \max_{k} X_{k,a} - \min_{k} X_{k,a}, \quad k = 1,2,\cdots, n
\]
(5) Calculate the standard deviation \( S_{I_{a}} \) in every subinterval \( I_{a} \):
\[
S_{I_{a}} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (N_{k,a} - e_{a})^{2}}
\]
(6) The scaling range of every subinterval \( I_a \) is equal to \( R_{i_a} / S_{i_a} \). From the second step, there are subintervals with the length \( n \). The mean R/S of the length \( n \) is defined as follow:

\[
(R / S)_n = \frac{1}{A} \sum_{a=1}^{A} \left( \frac{R_{i_a}}{S_{i_a}} \right)
\]

(7) Repeat the calculative process above for different length \( n \), we figure out many mean scaling ranges. We must pay attention to that: the length \( n \) is increasing, and the \( \frac{N}{n} \) is always a integer, and we can get a group of numbers: \{\log(R / S)_n\} and \{\log(n)\}. Hurst found that:

\[
(R / S)_n = c \cdot n^H
\]

So we have the equation:

\[
\log(R / S)_n = \log(c) + H \cdot \log(n)
\]

According to the instruction above, we can make linear regression to \{\log(R / S)_n\} and \{\log(n)\}, the slope is Hurst index \( H \).

We select the weekly closing price of industrial indices (from 2-19-2002 to 2-19-2006) to get 501 data. Then make \( N \) equal 500 for the easy selection of \( n \).

We use Excel to figure out the data by the seven steps above. The first table is the logarithmic ratio result of the price for different \( n \) and logarithmic mean scaling range \( \log(R / S)_n \). We make linear regression to these data. The second table is the linear regression result of the data by SPSS. And we get the value of Hurst index \( H \). So we can find how the Hurst index varies with the variation of \( n \).

The results are as follow:

1. Public affair index:

   Data by R/S analysis:

   \[\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|}
   \hline
   n & 25 & 30 & 35 & 40 & 45 & 50 & 55 & 60 & 65 \\
   \hline
   H & 0.713 & 0.708 & 0.688 & 0.71 & 0.708 & 0.707 & 0.696 & 0.679 & 0.679 \\
   \hline
   \end{array}\]

   \[\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
   \hline
   n & 70 & 75 & 80 & 85 & 90 & 95 & 100 & 105 & 110 \\
   \hline
   H & 0.677 & 0.676 & 0.682 & 0.678 & 0.676 & 0.674 & 0.672 & 0.669 & 0.664 \\
   \hline
   \end{array}\]

   \[\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
   \hline
   n & 115 & 120 & 125 & 130 & 135 & 140 & 145 & 150 & 155 \\
   \hline
   H & 0.662 & 0.656 & 0.647 & 0.639 & 0.633 & 0.629 & 0.625 & 0.622 & 0.619 \\
   \hline
   \end{array}\]

   \[\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
   \hline
   n & 160 & 165 & 170 & 175 & 180 & 185 & 190 & 195 & 200 \\
   \hline
   H & 0.618 & 0.611 & 0.604 & 0.593 & 0.582 & 0.572 & 0.564 & 0.555 & 0.544 \\
   \hline
   \end{array}\]

   We can find that the Hurst index of public affair index reaches its maximum between 40 and 50 weeks, we think the long-term memory to be 50 weeks, or about 1 year.

2. Real estate index:

   R/S analysis data:
We can find that the Hurst index of real estate index reaches its maximum between 90 and 95 weeks, we think the long-term memory to be 95 weeks.

3. Finance and insure index

R/S analysis data:

| n  | 15  | 20  | 25  | 30  | 35  | 40  | 45  | 50  | 55  | 60  |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| H  | 0.698| 0.691| 0.681| 0.691| 0.684| 0.687| 0.692| 0.693| 0.702| 0.7 |
| n  | 65  | 70  | 75  | 80  | 85  | 90  | 95  | 100 | 105 | 110 |
| H  | 0.711| 0.711| 0.714| 0.720| 0.722| 0.722| 0.721| 0.718| 0.717| 0.718 |
| n  | 115 | 120 | 125 | 130 | 135 | 140 | 145 | 150 | 155 | 160 |
| H  | 0.717| 0.717| 0.705| 0.703| 0.703| 0.704| 0.704| 0.705 | 0.706| 0.709 |
| n  | 165 | 170 | 175 | 180 | 185 | 190 | 195 | 200 |  
| H  | 0.709| 0.709| 0.706| 0.705| 0.703| 0.702| 0.698| 0.695|  

We can find that the Hurst index of finance and insure index reaches its maximum at 170 weeks, we think the long-term memory to be 170 weeks.

3. Conclusion

From the result in section 2, we conclude that except for the Hurst index of public affair index is 0.544, the other 2 indices are 0.653 and 0.695. They have the property of long-term memory, coinciding with the studies by other scholars.
We found that the range of Hurst index of finance and insure index is between 0.64 and 0.67, the variation is very small. Hurst index reaches its maximum at 170 weeks, and have about 4-year mean fluctuant period. So we can think that the policy and market information have a large effect to finance and insure industry which have strong consistence as a whole industry and have small risk for investment. The mean fluctuant period of real estate index is between 95 weeks and 115 weeks, or about 2 years, relating to the industrial character of the enterprises. For example, Chinese real estate enterprises need about 2 years to gain profit, these industries have shorter fluctuant period than American market etc that have 4-year period. It indicates that these industrial are immature, and in the future they will have a big improvement but fluctuate drastically, strongly affected by policy and market information, so they are highly risk investment. Public affair index have 50 weeks mean fluctuant period. It contains the city’s water supply, electricity supply, heating supply and so on. It is affected by the price of water and electricity that varies frequently these years. Furthermore these enterprises have little independence, affected strongly by the price of water and electricity controlled by government. It belongs to the industry affected by government policy, so it has short mean fluctuant period. It can be invested according to the adjustment of the price of water and electricity.

In generally, different industries have different long-term memory and mean fluctuant periods according to their industrial character. We can make long-term or short-term investment by comparing with different industrial long-term memory and mean fluctuant periods. The stock market varies quickly and uncertainly. The volatility of price is not only dependence to the past trend of the stock price but also dependence to macroeconomic and microeconomic environment, the government policy, the international economic, the petroleum price, natural disaster and the mental variation of investors etc. So we hope to study to the variation of Hurst index to found the things behind the volatility of the stock price.

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