Adaptive Integral Sliding Mode Guidance Law with Impact Angle Constraint Considering Autopilot Lag

X B Li\(^1\,*\), G R Zhao\(^1\), S Liu\(^1\) and X Han\(^1\)

\(^1\)Naval Aviation University, 188 Er-ma Road, Yantai, China

\(*\)Corresponding author’s e-mail: lixiaobaohjhy@163.com

Abstract. For the interception problem of maneuvering targets, considering impact angle constraint and first-order autopilot lag, an adaptive integral sliding mode guidance law is designed. A new nonsingular finite-time integral sliding mode surface is constructed, which consists of LOS angle, LOS angular rate and LOS angular acceleration, and ensures that the states of the guidance system converge to zero on the sliding mode surface in a finite time. Moreover, an adaptive law is designed to estimate the upper bound of the target’s unknown acceleration. The Lyapunov stability theory proves that the guidance system can converge to zero strictly in a finite time with the proposed guidance law. Finally, the simulation results verify the effectiveness of the designed guidance law.

1. Introduction

The autopilot dynamics of the missile during the terminal guidance process is not ideal and has a delay effect. When designing the guidance law without considering autopilot lag, the actual miss distance, impact angle and other guidance indicators will not have a good performance. For the interception of a maneuvering target, the guidance law will be significantly weakened, and may even fail to intercept the target [1]. Therefore, it is of great practical significance to consider the dynamic characteristics of the autopilot when designing the terminal guidance law.

The dynamic characteristics of the missile's autopilot can be regarded as a first-order inertia link. The related guidance law research methods include optimal control [2], sliding mode control [3] and Lyapunov stability theory [4]. However, the research on the combination of missile autopilot lag and finite time convergence is still rare. In [5], a nonsingular terminal sliding mode surface is designed by using LOS angular rate and LOS angular acceleration, and a finite time convergent guidance law with a first-order autopilot lag is proposed. But it does not consider the constraint of impact angle because the terminal guidance process will become more complicated when considering the dynamic characteristics of the autopilot, and the guidance system becomes high-order equation with three state variables when considering impact angle constraint. In [6], a finite time convergent guidance law based on two terminal sliding mode surfaces solve the problem of impact angle constraint, but the solution of two terminal sliding mode surfaces is complex and the amount of calculation is large.

In order to solve the terminal guidance problem of intercepting maneuvering targets, this paper studies a guidance law with impact angle constraint considering the dynamics of autopilot. When considering a first-order autopilot lag, a new type of terminal sliding mode surface is constructed. By introducing the integral term to avoid the singularity problem and using the characteristics of a class of nonlinear finite-time stable systems, the proposed guidance law ensures the guidance system converge to zero in a finite time, the design of a single sliding mode is not only simple and effective, but also...
meets the requirements of impact angle constraint.

2. Problem description and related lemma

The terminal guidance process can be simplified to be performed in a two-dimensional plane as shown in Figure 1. \( r \) and \( q \) represent the distance between the missile \( M \) and target \( T \) and the LOS angle. \( \gamma_M \) and \( \gamma_T \) represent the flight path angles of the missile and target, respectively. \( a_M \) and \( a_T \) represent the normal accelerations of the missile and target. \( V_M \) and \( V_T \) represent the speeds and are constant.

![Figure 1. Missile and target engagement geometry of terminal guidance](image)

The engagement kinematics of the guidance system can be written as

\[
\dot{r} = V_r \cos(\gamma_T - q) - V_M \cos(\gamma_M - q) \tag{1a}
\]

\[
r \dot{q} = V_r \sin(\gamma_T - q) - V_M \sin(\gamma_M - q) \tag{1b}
\]

Differentiating eqs. (1) with respect to time yields

\[
\ddot{r} = r \dot{q}^2 - a_{M_r} + a_{T_r} \tag{2a}
\]

\[
\ddot{q} = \left( -2r \dot{q} - a_{M_q} + a_{T_q} \right) / r \tag{2b}
\]

where \( a_{M_r} = a_M \sin(q - \gamma_M) \) and \( a_{T_r} = a_T \sin(q - \gamma_T) \) denote the accelerations of the missile and target along the LOS, \( a_{M_q} = a_M \cos(q - \gamma_M) \) and \( a_{T_q} = a_T \cos(q - \gamma_T) \) denote the accelerations of the missile and target normal to the LOS.

The missile's autopilot dynamics are usually described by a first-order term

\[
\dot{a}_{M_q} = \left( u - a_{M_q} \right) / \tau \tag{3}
\]

where \( \tau \) is the time constant of the autopilot and \( u \) is the guidance command to the autopilot.

The missile's impact angle \( \theta_{imp} \) constraint can be transformed into the terminal LOS angle \( q_f \) constraint. Assume \( q_d \) is the desired terminal LOS angle, \( x_1 = q - q_d \) is the LOS angle tracking error, \( x_2 = \dot{x}_1 \) is the LOS angular rate, \( x_3 = \dot{x}_2 \) is the LOS angular acceleration. Eqs. (2) can be written as

\[
\dot{x}_1 = x_2 \tag{4a}
\]

\[
\dot{x}_2 = x_3 \tag{4b}
\]

\[
\dot{x}_3 = f + bu + d / r \tag{4c}
\]

where \( f = \left( -2r \dot{x}_2 - 3 \dot{x}_1 + a_{M_q} / \tau \right) / r \), \( b = -1 / \tau r \), \( d = a_{T_q} \).

Assumption 1. The target’s acceleration \( a_T \) and its derivative \( \dot{a}_T \) satisfy \(|a_T| \leq \Delta_1\), \(|\dot{a}_T| \leq \Delta_2\), where \( \Delta_1 \) and \( \Delta_2 \) are two positive constants.
In Eq. (4), $d$ can be regarded as the unknown disturbance caused by the target acceleration. It is known from Assumption 1 that there is an upper bound $\Delta$ that satisfies $|d| \leq \Delta$. By designing the guidance command $u$ to ensure $x_1$ and $x_2$ converge to the origin in a finite time, so that the missile can hit the target with the desired terminal LOS angle $\phi_0$ considering the first-order autopilot lag.

**Lemma 1** [7]. For a continuous positive definite function $V(t)$, if $\dot{V}(t) \leq -\alpha V(t) - \beta V(t)^{\gamma}$, $\forall t > t_0$, where $\alpha > 0$, $\beta > 0$ and $0 < \gamma < 1$, $V(t)$ will converge to the equilibrium point in finite time $t_f$ given by

$$t_f \leq t_0 + \frac{1}{\alpha(1-\gamma)} \ln\frac{\alpha V(t_0)^{1-\gamma} + \beta}{\beta}$$

(5)

**Lemma 2** [8]. For a chain system

$$\dot{y}_1 = y_2, \dot{y}_2 = y_3, \ldots, \dot{y}_{m-1} = y_m, \dot{y}_m = u$$

(6)

Suppose $D(x) = x^m + k_m x^{m-1} + \ldots + k_2 x + k_1$ is a Hurwitz polynomial and $k_1, k_2, \ldots, k_m > 0$, then the system (6) is finite time stable with the feedback control:

$$u = -k_1 \text{sign}^{\alpha_1} y_1 - k_2 \text{sign}^{\alpha_2} y_2 - \ldots - k_m \text{sign}^{\alpha_m} y_m$$

(7)

where $\alpha_{i-1} = \alpha_i \alpha_{i+1} / (2 \alpha_{i+1} - \alpha_i)$, $\alpha_m = \alpha$, $\alpha_{m+1} = 1$, $\alpha \in (0, 1)$, $i = 2, 3, \ldots, m$.

3. Adaptive integral sliding mode guidance law design

3.1. Nonsingular integral sliding mode surface

The guidance system (4) with impact angle constraint and first-order autopilot lag contains three state variables. However, the traditional terminal sliding mode surface is usually composed of two state variables to achieve the finite time convergence on the sliding mode surface. In order to avoid designing two sliding modes surfaces as in [6], the LOS angle tracking error, LOS angular rate and LOS angular acceleration are considered together. A new type of terminal sliding mode surface containing three state variables is proposed in [9] and these state variables eventually converge to zero in a finite time. The sliding mode surface is as follows:

$$s = x_3 + k_2 \text{sign}^{\alpha_2} x_2 + k_1 \text{sign}^{\alpha_1} x_1$$

(8)

where $k_1, k_2 > 0$, $0 < \alpha_1 < 1$, $\alpha_2 = 2 \alpha_1 / (1 + \alpha_1)$. Because $0 < \alpha_1, \alpha_2 < 1$, there is a problem of singularity in the sliding mode surface (8), and no further research is implemented on this problem in [9]. To solve the singularity problem, an integrated sliding mode without autopilot lag is proposed in [10]. Inspired by these, a new type of nonsingular integral sliding surface is constructed:

$$s = x_3 + \int_0^t k_i \text{sign}^{\alpha_i} x_1 + k_2 \text{sign}^{\alpha_2} x_2 + k_3 \text{sign}^{\alpha_3} x_3 \, dt$$

(9)

where $\alpha_1 = \alpha_2 \alpha_3 / (2 \alpha_3 - \alpha_2)$, $\alpha_2 = \alpha_3 / (2 - \alpha_3)$, $0 < \alpha_3 < 1$ and $k_1, k_2, k_3 > 0$ satisfy Hurwitz conditions.

3.2. Guidance law design and stability analysis

**Theorem 1.** For the guidance system (4) with the sliding mode surface (9), if the guidance law $u$ is designed as

$$u = -\frac{1}{b} \left( f + \frac{c \text{sign}(s)}{r} + k_1 \text{sign}^{\alpha_1} x_1 + k_2 \text{sign}^{\alpha_2} x_2 + k_3 \text{sign}^{\alpha_3} x_3 + \alpha s + \beta \text{sign}^{\gamma} s \right)$$

(10)

where $\alpha, \beta > 0$, $\sigma > 1$, $0 < \gamma < 1$, $\dot{\Delta}$ is an adaptive estimate of $\Delta$, and the adaptive law is

$$\dot{\Delta} = \sigma |s| / r, \; \Delta(0) > 0$$

(11)
Then the following conclusions are established:
1) The sliding variable $s$ and estimation error $\hat{\Delta} = \Delta - \hat{\Delta}$ are bounded;
2) The sliding variable $s$ converge to zero in a finite time;
3) The guidance states $x_1, x_2$ converge to zero in a finite time.

**Proof.** The proving process is divided into the following three steps.

**Step 1.** Define a Lyapunov function $V = \frac{1}{2} s^2 + \frac{1}{2} \Delta^2$, and according to Eqs. (9)-(11), the derivative of $V$ can be written as follows:

$$\dot{V} = ss + \Delta \hat{\Delta}$$

$$= s \left( f + bu + \frac{d}{r} + k_1 \text{sign} \alpha x_1 + k_2 \text{sign} \alpha x_2 + k_3 \text{sign} \alpha x_3 \right) - \Delta \hat{\Delta}$$

$$= \frac{ds}{r} - \frac{\sigma \alpha s}{r} - \beta |s|^{r+1} - \sigma |s| (\Delta - \hat{\Delta})$$

$$\leq \frac{\Delta |s| (1 - \sigma)}{r} - \alpha s^2 - \beta |s|^{r+1}$$

$$\leq -\alpha s^2 - \beta |s|^{r+1}$$

Since $\dot{V} \leq 0$, it’s clear that the sliding variable $s$ and the estimation error $\hat{\Delta}$ are bounded.

**Step 2.** Define a Lyapunov function $V_i = \frac{1}{2} s^2$. Differentiating $V_i$ and substituting Eqs. (9)(10) gives

$$\dot{V}_i = s \left( f + bu + \frac{d}{r} + k_1 \text{sign} \alpha x_1 + k_2 \text{sign} \alpha x_2 + k_3 \text{sign} \alpha x_3 \right)$$

$$= \frac{ds}{r} - \frac{\sigma \alpha s}{r} - \beta |s|^{r+1}$$

$$\leq \frac{|s|(\Delta - \sigma \hat{\Delta})}{r} - \alpha s^2 - \beta |s|^{r+1}$$

Since $\hat{\Delta}(0) \geq 0$ and $\dot{\hat{\Delta}} \geq 0$, there is $\dot{\hat{\Delta}}(t) \geq \hat{\Delta}(0) \geq 0$. Choose $\hat{\Delta}(0)$ large enough and $\sigma$ satisfying $\sigma \geq 1 + \sqrt{\Delta^2(0) + \eta} / \hat{\Delta}(0)$, where $\eta \geq 0$. Then

$$\Delta - \sigma \hat{\Delta} \leq \Delta - \hat{\Delta}(0) - \sqrt{\hat{\Delta}^2(0) + \eta}$$

$$\leq |\Delta(0)| - \sqrt{\hat{\Delta}^2(0) + \eta}$$

$$\leq \hat{\Delta}(0) - \sqrt{\hat{\Delta}^2(0) + \eta}$$

$$\leq 0$$

Therefore, it can be obtained that $\dot{V}_i \leq -\alpha s^2 - \beta |s|^{r+1} \leq -2\alpha V - \frac{\beta V^{r+1}}{2}$. According to Lemma 1, the integral sliding variable $s$ will converge to zero in finite time.

**Step 3.** It is known that $s = \dot{s} = 0$ on the sliding mode surface. Differentiating $s$ gives

$$\dot{x}_3 = -k_1 \text{sign} \alpha x_1 - k_2 \text{sign} \alpha x_2 - k_3 \text{sign} \alpha x_3$$

According to Lemma 2, the states $x_1, x_2, x_3$ of the guidance system on the sliding surface (9) eventually converge to zero in finite time. This completes the proof.
The guidance law (10) is discontinuous which may cause the chattering problem because of the signum function $\text{sign}(s)$. So $\text{sign}(s)$ can be replaced with a continuous sigmoid function approximately:

$$\text{sgmf}(s) = \begin{cases} 
\text{sign}(s), & |s| > \varepsilon \\
\left(1 - e^{-\varepsilon s}\right)/\left(1 + e^{-\varepsilon s}\right), & |s| \leq \varepsilon
\end{cases}$$

(16)

Hence, the guidance law in (10) is modified as:

$$u = -\frac{1}{b}\left(f + \frac{\sigma \text{sgmf}(s)}{r} + k_{1}\text{sign}(s_{1}) + k_{2}\text{sign}(s_{2})x_{2} + k_{3}\text{sign}(s_{3})x_{3} + \alpha s + \beta \text{sgm}'(s)\right)$$

(17)

The guidance law (17) is called a finite time convergent adaptive integral sliding mode guidance law with autopilot lag (ALAISMG).

4. Numerical simulations

In this section, the effectiveness of the proposed guidance law is demonstrated through numerical simulations. The initial positions for the missile and the target are $x_{M0} = -3\text{km}$, $y_{M0} = 10\text{km}$ and $x_{T0} = 0\text{m}$, $y_{T0} = 0\text{m}$. The initial flight path angles are $\gamma_{M0} = -30\text{deg}$ and $\gamma_{T0} = 135\text{deg}$. The initial velocities are $V_{Ma} = 600\text{m/s}$, $V_{Ta} = 200\text{m/s}$. The desired terminal LOS angle of $-80\text{deg}$ ($u_{max} = -40\text{g}$), $g$ is the acceleration of gravity ($g = 9.8\text{m/s}^2$). The parameters of ALAISMG are chosen as $\alpha = 20$, $\beta = 0.5$, $\alpha_{1} = 0.57$, $\alpha_{2} = 0.67$, $\alpha_{3} = 0.8$, $k_{1} = 0.4$, $k_{2} = 0.8$, $k_{3} = 1.2$, $\gamma = 0.8$, $\tau = 0.5$, $\sigma = 2$, $\dot{\Lambda}(0) = 50$.

4.1 Performance under different maneuvering targets

In order to fully analyze the performance of ALAISMG, considering three different maneuvering targets: 1) cosine maneuvering $a_{T} = 5g \cos (4\pi t)$; 2) constant maneuvering $a_{T} = 5g$; 3) step maneuvering $a_{T} = 5g$, $t < 5s$; $a_{T} = -5g$, $t \geq 5s$. Simulation results are shown in Figure 2.

Figure 2(a) shows that the missile can intercept different maneuvering targets with ALAISMG successfully. Figure 2(b)(c) shows that LOS angle $q$ and LOS angular rate $\dot{q}$ can converge to $q_d$ and zero in a finite time respectively and short peaks occur in the early stage which cause the normal accelerations of the missile to peak. It can be seen from Figure 2(d) that in the case of the limiting of the guidance command $u$, all of $a_{Mq}$ rapidly increase to near 30g in the early stage and then decrease rapidly as $\dot{q}$ converge to zero. Besides, $a_{Mq}$ curves are smooth and there are no chattering problems.
4.2 Comparison with other guidance laws

In order to demonstrate the superiority of ALAISMG, comparisons with an adaptive integral sliding mode guidance law (AISMG) in [10] and a sliding mode guidance with first-order autopilot lag (ALSMG) in [11] are made in this subsection. The acceleration of target is selected as \( a_I = 5g \cos(4\pi t) \).

Figure 3(a) shows that when considering autopilot lag, both ALAISMG and ALSMG can enable the missile to intercept the target successfully, while AISMG fails to intercept the target which demonstrates that autopilot lag is a key factor affecting the terminal guidance performance. Figure
(b)(c) show that AISMG fails to satisfy the finite time convergence characteristics of $q$ and $\dot{q}$. Although $q$ and $\dot{q}$ in ALSMG has a faster convergence rate than ALAISMG, $q$ in ALSMG can’t converge to the desired LOS angle $q_d$ in finite time because ALSMG does not consider the impact angle constraint. Figure 3(d) shows the normal acceleration $a_{nq}$ curves of the missile. It can be seen that the curve of AISMG changes sharply and is always saturated. The curves of ALAISMG and ALSMG fluctuate in the early stage and decrease to near zero in the later stage. Besides, the ALAISMG curve is gentler and smoother.

5. Conclusions
A terminal guidance law with first-order autopilot lag is proposed, which enables the interception of maneuvering targets with a specified impact angle. A new type of integral sliding mode composed of LOS angle tracking error, LOS angular rate and LOS angular acceleration is constructed. The guidance proposed can ensure that the LOS angle tracking error and the LOS angular rate converge to zero in a finite time, which solves the problem of impact angle constraint and autopilot lag simultaneously on the basis of a single sliding surface. The simulation results show that the guidance law designed in this paper can intercept different maneuvering targets with the desired impact angle. Compared with the existing guidance laws, the feasibility of the guidance law proposed is further verified.

Acknowledgments
This paper is supported by the Project for the National Natural Science Foundation of China under Grant No. 61473306.

Reference
[1] Golestani M, Mohammadzaman I and Vali A R 2014 Finite-time convergent guidance law based on integral backstepping control Aerospace Science and Technology 39 370
[2] Aggrawal R K 1998 Terminal guidance algorithm for ramjet-powered missiles Journal of Guidance, Control, and Dynamics 21 862
[3] Koren A, Idan M and Golan O M 2008 Integrated sliding mode guidance and control for a missile with on-off actuators Journal of Guidance, Control, and Dynamics 31 204
[4] Chaw D Y, Chol J Y 2003 Adaptive nonlinear guidance law considering control loop dynamics IEEE Transactions on Aerospace and Electronic Systems 39 1134
[5] Sun S, Zhou D and Hou W T 2013 A guidance law with finite time convergence accounting for autopilot lag Aerospace Science and Technology 25 132
[6] Zhou H B, Song J H and Song S M 2018 Sliding mode guidance law considering missile dynamic characteristics and impact angle constraints International Journal of Automation and Computing 15 218
[7] Yu S H, Yu X H and Shirinzadeh B 2005 Continuous finite time control for robotic manipulators with terminal sliding mode Automatica 41 1957
[8] Bhat S, Bernstein D 2005 Geometric homogeneity with applications to finite time stability Mathematics of Control Signals and Systems 17 101
[9] Zhang Y X, Liu J and Zou K S 2016 Sliding-mode guidance law with impact angle constraint accounting for autopilot lag 28th Chinese Control and Decision Conference Yinchuan 637
[10] Zhao B, Zhou J and Lu X D 2017 Adaptive integral sliding mode guidance law considering impact angle constraint Control and Decision 32 1695
[11] Song J H, Song S M 2015 Adaptive sliding mode guidance law with input constraints and autopilot lag Journal of Chinese Inertial Technology 23 339