Pulse-driven non-linear Alfvén waves and their role in the spectral line broadening

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Accepted 2012 September 14. Received 2012 September 9; in original form 2012 July 25

ABSTRACT
We study the impulsively generated non-linear Alfvén waves in the solar atmosphere and describe their most likely role in the observed non-thermal broadening of some spectral lines in solar coronal holes. We solve numerically the time-dependent magnetohydrodynamic equations to find temporal signatures of large-amplitude Alfvén waves in the solar atmosphere model of open and expanding magnetic field configuration, with a realistic temperature distribution. We calculate the temporally and spatially averaged, instantaneous transversal velocity of non-linear Alfvén waves at different heights of the model atmosphere and estimate its contribution to the unresolved non-thermal motions caused by the waves. We find that the pulse-driven non-linear Alfvén waves with the amplitude $A_v = 50$ km s$^{-1}$ are the most likely candidates for the non-thermal broadening of Si VIII $\lambda 1445.75$ Å line profiles in the polar coronal hole as reported by Banerjee et al. We also demonstrate that the Alfvén waves driven by comparatively smaller velocity pulse with amplitude $A_v = 25$ km s$^{-1}$ may contribute to the spectral line width of the same line at various heights in coronal hole broadening. We conclude that the non-linear Alfvén waves excited impulsively in the lower solar atmosphere may be responsible for the observed spectral line broadening in polar coronal holes. This is an important result as it allows us to conclude that such large amplitude and pulse-driven Alfvén waves may indeed exist in solar coronal holes. The existence of these waves may impart the required momentum to accelerate the solar wind.

Key words: MHD – Sun: atmosphere.

1 INTRODUCTION
There are three fundamental magnetohydrodynamic (MHD) waves, the so-called fast, slow magnetoacoustic and Alfvén waves, with the latter being transverse and purely incompressible in linear regime. Typically, the MHD waves are defined in a background medium that is uniform with homogeneous magnetic fields. Nevertheless, the waves can also exist in the solar atmosphere, which show gradients of its physical parameters. Actually, there are also magnetic structures in the solar atmosphere, like small-scale magnetic flux tubes, loops and others, and these structures may support different types of MHD waves. In this paper, we investigate Alfvén waves and assume that these waves are non-linear and impulsively driven, and that they propagate in the solar atmosphere model of open and expanding magnetic field configuration, with a realistic temperature gradient.

In the last several years, there have been reports of detecting Alfvén waves and also kink and torsional waves in the solar atmosphere. The reports must be taken with caution because interpretation of observational results is not always unique. Keeping this in mind, we now briefly review the available observational evidence. Several indications of the presence of Alfvén waves in the solar atmosphere were found by SOHO using spectral observations. Later, the TRACE mission discovered fast magnetoacoustic kink waves, which were affected by weak perturbations of mass density and were excited in coronal loops (Aschwanden et al. 1999; Nakariakov et al. 1999). Such observations open the new era of seismology of the solar atmosphere as they allow performing measurements of the localized plasma and magnetic field conditions (Nakariakov & Ofman 2001). The spectrometer observations from SOHO/CDS also revealed the presence of kink wave harmonics in the flaring loops (O’Shea et al. 2007). Moreover, high-resolution observations performed by the Hinode, STEREO and the Solar Dynamics Observatory detected kink waves in the solar loops (Van Doorsselaere et al. 2008; Verdini, Velli & Buchlin 2009; Aschwanden &

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Schrijver 2011). Then, Okamoto & De Pontieu (2011), De Pontieu et al. (2007) and Curtain et al. (2007) used Hinode to collect the data and identified signatures of Alfvén waves in the data. Interpretations of these observational results were given by Erdélyi & Fedun (2007) and Antolin et al. (2009), and more recently by Van Doorsselaere, Nakariakov & Verwichte (2008), who concluded that the reported signatures rather correspond to kink waves than to Alfvén waves.

The most likely evidence for the outwardly propagating Alfvén waves in the quiet solar atmosphere was reported by McIntosh et al. (2011). The observations show that amplitudes of these waves are of the order of 20 km s\(^{-1}\) and their periods are within 100–500 s. The authors concluded that the waves may carry enough energy to accelerate the fast solar wind and heat the quiet corona. There is also observational evidence for the existence of torsional Alfvén waves in the solar atmosphere as reported by Jess et al. (2009), who analysed Hz observations obtained with high spatial resolution by the ground-based Swedish Solar Telescope. They interpreted the data in terms of torsional Alfvén waves in the solar chromosphere, with periods from 12 min down to the sampling limit of the observations near 2 min, with maximum power near 6–7 min. The authors concluded that the amount of energy carried by such waves could be sufficient to heat the solar corona above the magnetic bright points.

Several attempts were made to detect MHD waves in the strongly magnetized regions of the solar atmosphere (Hassler et al. 1990; McClements, Harrison & Alexander 1991; Erdélyi et al. 1998; Zaqarashvili 2003; Zaqarashvili, Oliver & Ballester 2006; Zaqarashvili et al. 2007). Possible signatures of dissipation of Alfvén waves were reported by Dwivedi & Srivastava (2006) and Srivastava & Dwivedi (2007), and recently by Bemporad & Abbo (2012). There are also claims of detecting dissipation of Alfvén waves resulting from phase mixing and resonant absorption (Erdélyi & Goossens 1995; Nakariakov, Roberts & Murawski 1997).

The observational results described above well justified extensive studies of Alfvén waves that have been carried out by numerous investigators in the last four decades (e.g. Hollweg 1985; Roberts 1991; Musielak & Moore 1995; Nakariakov, Ofman & Arber 2000; Antolin & Shibata 2010; Fedun, Shelyag & Erdélyi 2011; Vigeesh et al. 2012; Wedemeyer-Böhm et al. 2012, and references cited there). These studies have covered both linear (e.g. Roberts 2004; Hollweg & Isenberg 2007; Murawski & Musielak 2010) and non-linear (e.g. Verdini & Velli 2007; Verwichte et al. 2009; Matsumoto & Shibata 2010; Vasheghani Farahani et al. 2011) Alfvén waves, and different aspects of their generation, propagation and dissipation have been investigated. The specific objectives of these studies were to understand the role of Alfvén waves in the atmospheric heating and in the acceleration of supersonic solar wind. The fast component of the solar wind originates in solar polar coronal holes (e.g. Hassler et al. 1999; Tu et al. 2005, and references cited there), which are the regions where the non-thermal broadening of spectral lines has been observed (e.g. Hassler et al. 1990; Banerjee et al. 1998; Moran 2003; O’Shea, Banerjee & Doyle 2005; Dolla & Solomon 2008). Similar observations have also been done in the equatorial corona (e.g. Harrison, Hood & Pike 2002), and the authors proposed that the radially propagating Alfvén waves may result in the non-thermal broadening of spectral line widths. The results of these observations were investigated analytically by Pekinlü et al. (2002) and Dwivedi & Srivastava (2006).

Moreover, Zaqarashvili et al. (2006) reported that the resonant energy conversion from Alfvén to acoustic waves occurs in the region where plasma β approaches unity in the solar atmosphere. This conversion can be responsible for the spectral line-width variation. However, this theory only explains the most probable cause of the line-width reduction, which was observed only by O’Shea et al. (2005) in solar coronal holes. Additional problem is the fact that there is not enough observational evidence for the resonant energy conversion in the solar corona (e.g. McAteer et al. 2003; Srivastava & Dwivedi 2010). Hence, new studies of the role played by Alfvén waves in the observed spectral line broadening are necessary, and such studies are undertaken in this paper.

We numerically study the behaviour of large-amplitude (non-linear) Alfvén waves in a model that resembles a solar coronal hole. Our main objective is to determine the role played by these waves in the spectral line broadening as observed in the coronal holes (e.g. Banerjee et al. 1998; Dolla & Solomon 2008). We find agreement between our numerical results of non-linear Alfvén waves and the computed line broadening of constituted synthetic spectra at different heights in the model coronal hole and the observational data. This allows us to conclude that large-amplitude Alfvén waves may be most likely responsible for the observed non-thermal broadening of the spectral lines in the coronal holes. Our result is important because it may be the indirect evidence for the existence of non-linear Alfvén waves in solar coronal holes.

The outline of the paper is as follows: our numerical model is described in Section 2; a brief description of the used numerical code and the form of initial perturbations are given in Section 3; the results of our numerical simulations are presented in Section 4; comparison of our results to the observational data is given in Section 5; the obtained results are discussed in Section 6; and our conclusions are given in Section 7.

2 NUMERICAL MODEL OF ALFVÉN WAVES

2.1 MHD equations

Our model of the solar atmosphere contains a gravitationally stratified magnetoplasma, which is described by the following set of ideal MHD equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad (1)
\]

\[
\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla p + \frac{1}{\mu}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mathbf{g}, \quad (2)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \quad (3)
\]

\[
\nabla \cdot \mathbf{B} = 0, \quad (4)
\]

\[
\frac{\partial p}{\partial t} + \mathbf{V} \cdot \nabla p = -\gamma \rho \nabla \cdot \mathbf{V}, \quad (5)
\]

\[
p = \frac{k_B}{m} \rho T \quad (6)
\]

Here \(\rho\) is mass density, \(\mathbf{V}\) and \(\mathbf{B}\) are vectors of, respectively, the flow velocity and the magnetic field, \(p\) is gas pressure, \(\gamma = 5/3\) is the adiabatic index, \(\mathbf{g} = (0, -g, 0)\) is a vector of gravitational acceleration with its value \(g = 274 \text{ m s}^{-2}\), \(T\) is the temperature, \(m\) is the mean particle mass and \(k_B\) is Boltzmann’s constant.
2.2 A model of the solar atmosphere

We consider a model of the solar atmosphere with an invariant coordinate (\(\partial / \partial z = 0\)) and allow the \(z\)-components of velocity (\(V_z\)) and magnetic field (\(B_z\)) to vary with \(x\) and \(y\). In such a 2.5D model, the solar atmosphere is in static equilibrium (\(V_e = 0\)) with force- and current-free magnetic field, i.e.

\[(\nabla \times B_e) \times B_e = 0, \quad \nabla \times B_e = 0.\]  

(7)

Henceforth, the subscript ‘e’ corresponds to equilibrium quantities.

In our model of the atmosphere, a curved magnetic field is given by

\[B_e = \nabla \times A_e,\]  

(8)

where the magnetic flux function (\(A_e\)) has the form

\[A_e = \Lambda_B B_0 \cos \left( \frac{x}{\Lambda_B} \right) \exp \left( -\frac{y - y_1}{\Lambda_B} \right) \hat{z}.\]  

(9)

Here \(\hat{z}\) is a unit vector along the \(z\)-direction, and \(B_0\) is the magnetic field at the reference level, \(y = y_1\), that is chosen at \(y_1 = 10\) Mm. We set and hold fixed \(B_0\) in such a way that the Alfvén speed,

\[c_A = \frac{B_0}{\sqrt{\mu_0 \rho_e(y = y_1)}},\]

is 10 times higher than the sound speed,

\[c_s = \sqrt{\gamma \rho_e(y = y_1) / \rho_e}.\]

Such a choice of \(B_e\) results in equation (7) being satisfied. Here \(\Lambda_B = 2L / \pi\) denotes the magnetic scale-height, and \(L\) is half of the magnetic arcade width. As we aim to model a polar region, we take \(L = 75\) Mm and keep it fixed in our calculations. For such a choice, the magnetic field lines are weakly curved and represent the open and expanding field lines similar to the coronal holes. The equilibrium magnetic field vectors \(B_e\) described by equations (8) and (9) are presented in Fig. 1.

As a result of equation (7), the pressure gradient is balanced by the gravity force

\[-\nabla p_e + \varrho_e g = 0.\]  

(10)

Using the ideal gas law of equation (6) and the \(y\) component of the hydrostatic pressure balance indicated by equation (10), we express the equilibrium gas pressure and the mass density as

\[p_e(y) = p_0 \exp \left( -\int_{y_1}^y \frac{dy'}{\Lambda(y')} \right), \quad \varrho_e(y) = \frac{p_e(y)}{g \Lambda(y)}.\]  

(11)

Here

\[\Lambda(y) = \frac{k_B T_e(y)}{mg}\]  

(12)

is the pressure scale-height, and \(p_0\) denotes the gas pressure at the reference level.

We adopt a realistic model of the plasma temperature profile (Vernazza et al. 1976), displayed in Fig. 2 (top panel). Temperature attains a value of about \(6 \times 10^3\) K at \(y = 1.5\) Mm, it raises abruptly at the transition region that is located at \(y \approx 2.7\) Mm above the temperature minimum that is located around \(y = 0.9\) Mm (not shown), and it grows up to about \(1.5 \times 10^6\) K in the solar corona at \(y = 10\) Mm. Higher up temperature is assumed to be constant; note that this profile is more realistic than the one used by Murawski & Musielak (2010). The temperature profile determines uniquely the equilibrium mass density and gas pressure profiles. Both \(\varrho_e(y)\) and \(p_e(y)\) experience a sudden drop.

Figure 1. Vectors of the equilibrium magnetic field \(B_e\).

Figure 2. Equilibrium profile of the temperature (top) and the Alfvén speed (bottom).
In this model the Alfvén speed, \( c_A \), varies only with \( y \) and is expressed as follows:

\[
c_A(y) = \frac{B_0 c}{\sqrt{\mu \varrho(y)}}.
\]  

(13)

Its profile is displayed in Fig. 2 (bottom panel). Note that the Alfvén speed in the chromosphere, \( c_A(y = 1.75 \text{ Mm}) \), is about 25 km s\(^{-1}\)\). The Alfvén speed rises abruptly through the transition region reaching a value of \( c_A(y = 10 \text{ Mm}) = 10^3 \text{ km s}^{-1} \) (Fig. 2, bottom). The growth of \( c_A(y) \) with height results from a faster decrement of \( \varrho(y) \) than \( B_0(y) \) with the height.

The realistic solar atmosphere above the solar corona reveals complexity of its plasma and magnetic field structure. The magnetic field configuration in the polar coronal hole can be approximated by expanding coronal funnels in the lower part of the atmosphere and comparatively weak open field lines in its upper part (Banerjee et al. 1998; Hackenberg, Marsch & Mann 2000). This implies that the field structure and magnetic scale-height vary above the polar corona. Moreover, the field configuration changes from dipolar to multipolar during the transition from the solar minimum to its maximum. Despite these well-known variations, we assume that the field structure and magnetic scale-height vary above the polar coronal hole for the quiet minimum of the Sun. We want to point out that this assumption does not affect the validity of our numerical results.

3 NUMERICAL SOLUTIONS OF MHD EQUATIONS

Equations (1)–(6) are solved numerically with the use of the FLASH code (Fryxell et al. 2000; Lee & Deane 2009; Lee et al. 2009). This code implements a second-order upwind Godunov solver with various slope limiters and Riemann solvers as well as the adaptive mesh refinement (AMR) (MacNeice et al. 2000). We use the min-mod slope limiter and the Roe Riemann solver (e.g. Toro 2006). We set the simulation box as \((-5 \text{ Mm}, 5 \text{ Mm}) \times (-1 \text{ Mm}, 84 \text{ Mm})\) and impose fixed-in-time boundary conditions for all plasma quantities in the \( x \)- and \( y \)-directions, while all plasma quantities remain invariant along the \( z \)-direction. In our present work, we use an AMR grid with a minimum (maximum) level of refinement set to 3 (8). The refinement strategy is based on controlling the numerical errors in mass density. Blocks are denser below 3 Mm and vertically along the region of Alfvén wave propagation. Every numerical block consists of \( 8 \times 8 \) identical numerical cells. This results in an excellent resolution of vital spatial profiles and greatly reduces the numerical diffusion at these locations.

3.1 Initial perturbations

We perturb initially (at \( t = 0 \text{ s} \)) the model equilibrium, described in Section 2.2, by a Gaussian pulse in the \( z \) component of velocity given by

\[
V_z(x, y, t = 0) = A_v \exp \left[ -\frac{(x-x_0)^2 + (y-y_0)^2}{w^2} \right],
\]

(14)

where \( A_v \) is the amplitude of the pulse, \((x_0, y_0)\) is its initial position and \( w \) denotes its width. We set \( w = 1 \text{ Mm}, (x_0 = 0, y_0 = 1.75) \text{ Mm} \) and consider two cases: (a) \( A_v = 25 \text{ km s}^{-1} \) and (b) \( A_v = 50 \text{ km s}^{-1} \).

Note that in the 2.5D model, we developed, the Alfvén wave decouples from magnetoacoustic waves, and it can be described by \( V_z(x, y, t) \). As a result, the initial pulse triggers Alfvén waves that in the linear limit are described by the wave equation

\[
\frac{\partial^2 V_z}{\partial t^2} = c_A^2(y) \frac{\partial^2 V_z}{\partial y^2}.
\]

(15)

4 RESULTS OF NUMERICAL SIMULATIONS

We simulate impulsively excited non-linear Alfvén waves and investigate their propagation along the open magnetic field lines of a coronal hole model in the outward direction. It should be noted that the effect of the inhomogeneities across the magnetic field lines is not included in our approach. The waves are generated by the transversal velocity pulse perpendicular to the magnetic isosurface (\( X-Y \)) in the \( z \)-direction. This pulse is described by equation (14).

First, we consider the pulse amplitude equal to 25 km s\(^{-1}\). Spatial profile of \( V_z \) for \( A_v = 25 \text{ km s}^{-1} \) and the corresponding time signatures obtained in our numerical simulations of Alfvén waves are shown, respectively, in Figs 3 and 4 (top-right panel). Perturbations of \( V_z \) propagate essentially along magnetic field lines, which is displayed on the spatial profile of \( V_z \) at \( t = 30 \text{ s} \) in Fig. 3, where a part of the whole simulation region is illustrated. Note that the Alfvén wave experiences an acceleration with the height as \( c_A(y) \) increases up to 0.6 Mm s\(^{-1}\) (Fig. 2, bottom). This acceleration of the Alfvén wave is caused by significant fall in mass density at essentially constant magnetic field. Above the Alfvén wave propagates with slightly increased amplitude and almost constant velocity. The spatial resolution of the transition region is set to be \( \Delta x \approx \Delta y \approx 0.04 \text{ Mm} \) in the simulation domain.
We also simulate the Alfvén wave using the large transversal pulse of $A_z = 50 \text{ km s}^{-1}$ that generates the large-amplitude, non-linear Alfvén waves. Time signatures of $V_y$ for $A_z = 50 \text{ km s}^{-1}$ collected at $(x = 0, y = 49) \text{ Mm}$ are shown in the right-bottom panel of Fig. 4. Since the wave is non-linear, it generates a vertical flow (Fig. 4, middle-bottom panel) and mass density perturbations (Fig. 4, left-bottom panel), which are driven by the ponderomotive force.

We collect the temporally and spatially averaged transversal velocity component $V_z$ at each height of the simulation domain generated by the pulses of different wave amplitudes, $A_z = 25$ and $50 \text{ km s}^{-1}$ (see Fig. 4). If we assume that the $y$-direction of the simulation box is placed along the outward open magnetic field of a polar coronal hole, then at each height the transversal amplitude of Alfvénic perturbations may contribute to the otherwise unresolved non-thermal motions by affecting the line width of the observed spectral lines. Finite-amplitude Alfvén waves propagating along the open magnetic field lines of the polar coronal holes can perturb the plasma velocity, which causes positive and negative Doppler shifts that can be detected as a line-width broadening or line-width variation (Banerjee et al. 1998; Harrison et al. 2002; Moran 2003; O’Shea et al. 2005; Dwivedi & Srivastava 2008). We adopt the same scaling throughout the paper, although different authors use different scalings based on their assumption of the degree of freedom of wave motion (Dolla et al. 1997).

Figure 4. Results of the numerical simulation of pulse-excited Alfvén waves: time signatures of $\rho$, $V_y$, and $V_z$ for $A_z = 25 \text{ km s}^{-1}$ (top panels) and $A_z = 50 \text{ km s}^{-1}$ (bottom panels) collected at $(x = 0, y = 49) \text{ Mm}$.

We have estimated the equivalent FWHM ($\sigma$) of its 1445.75 Å line at different heights using the relation (Mariska 1992)

$$\sigma^2 = 4 \ln 2 \left( \frac{\lambda}{c} \right)^2 \left( \frac{2 k_B T}{m_i} + \xi^2 \right) + \sigma_i^2.$$  (16)

It should be noted that the averaged wave velocity amplitude can be scaled appropriately in terms of non-thermal speed as $\xi^2 = 0.5 V_t^2$ by taking into the consideration of polarization and the direction of the propagation of wave with respect to the line of sight (LOS; Banerjee et al. 1998). We adopt the same scaling throughout the paper, although different authors use different scalings based on their assumption of the degree of freedom of wave motion (Dolla & Solomon 2008). At any particular height in the model coronal hole, we average the wave velocity amplitude of transversal oscillation in temporal domain between $t_x = 250$ and $t_x + 250$ s, while in the spatial domain over the entire pulse width, where $t_x$ is the arrival time of a wave signal to the detection point. Therefore, the resultant transversal motion contributes to the non-thermal unresolved motion at each height in the model coronal hole plasma as very few in the available literature. Using SOHO/CDS observations, O’Shea et al. (2005) have shown the line-width increment in the inner corona and then its decrement beyond radial distance 1.21 $R_\odot$ in the coronal hole. Here, $R_\odot$ is the radius of the Sun. However, due to the unavailability of the instrumental width of SOHO/CDS slits, the exact information about the non-thermal velocity could not be derived, so the estimate gave only the uncorrected line-width variation with the height. Moreover, Banerjee, Pérez-Suárez & Doyle (2009) have approximately reproduced the trend of the spatial variation of non-thermal velocity in the polar coronal hole similar to Banerjee et al. (1998) using a 2 arcsec slit of Hinode/EIS. Note that the exact instrumental widths of EIS slits were variables with the wavelength of the observed spectral lines.

By using the temperature averaged over the simulation domain, the averaged density generated from the simulations, and the temporally and spatially averaged transversal speed ($V_y$) of the Alfvén wave generated by the velocity pulse $A_z = 50 \text{ km s}^{-1}$, we have estimated the equivalent FWHM ($\sigma$) of its 1445.75 Å line at different heights using the relation (Mariska 1992)

$$\sigma^2 = 4 \ln 2 \left( \frac{\lambda}{c} \right)^2 \left( \frac{2 k_B T}{m_i} + \xi^2 \right) + \sigma_i^2.$$  (16)

5 COMPARISON TO OBSERVATIONAL DATA

5.1 Line width broadening

The non-thermal motion is a prominent candidate that may modify the observed line profiles by its contribution to the full width at half-maximum (FWHM) of the spectral lines. Banerjee et al. (1998) have estimated the radial variation of the non-thermal velocity in the polar coronal hole by deducing its contribution to the observed line profiles of Si VIII $\lambda 1445$ Å. The sophisticated line-width measurements and thus the observed variation of non-thermal velocity are
5.2 Line profiles obtained with A spectral line broadening (Banerjee et al. 1998). wards and away the LOS leads to the unresolved contribution to the However, in the case of the off-limb polar coronal holes, the strong line-width variation when we consider the projected centre to negligible. Moreover, the Alfvén and slow magnetoacoustic waves emissions from off-limb polar coronal holes where such effects are move through the disc centre to the limb as reported extensively for the range of ions and their EUV/ultraviolet emissions (Erdelyi et al. 1998), based on the inclusion of the non-thermal contribution of the Alfvén wave excited by pulse amplitude

Figure 5. Profile of FWHM (σ) for line 1445.75 Å as a function of the amplitude of initial pulse, \(A_v\). The plus marks denote values of \(σ\) obtained at \(y = 20\) Mm, the stars denote those obtained at \(y = 40\) Mm, while the rhombuses denote those obtained at \(y = 60\) Mm. observed in the form of spectral line broadening by various authors, e.g. Banerjee et al. (1998) and Dolla & Solomon (2008).

We present the FWHM (σ) of line 1445.75 Å as a function of an initial pulse amplitude in Fig. 5 for the following heights: \(y = 20, 40\) and 60 Mm. It is evident that \(σ\) increases and differentiates while a pulse amplitude grows up.

Although the line width of a particular line varies also when we move through the disc centre to the limb as reported extensively for the range of ions and their EUV/ultraviolet emissions (Erdelyi et al. 1998), in the present case we are considering the modelling of emissions from off-limb polar coronal holes where such effects are negligible. Moreover, the Alfvén and slow magnetoacoustic waves both may contribute to the formation of the LOS component of the line-width variation when we consider the projected centre to limb variation (Erdelyi et al. 1998; Dwivedi & Srivastava 2008). However, in the case of the off-limb polar coronal holes, the strong polarization of the magnetic and incompressible Alfvén waves towards and away the LOS leads to the unresolved contribution to the spectral line broadening (Banerjee et al. 1998).

5.2 Line profiles obtained with \(A_v = 50\) km s\(^{-1}\)

We computed the synthetic line profiles (cf., Fig. 6) of Si VIII \(\lambda 1445.75\) Å at three different heights \(y = 20, 40\) and 70 Mm based on the inclusion of the non-thermal contribution of the Alfvén wave excited by a pulse with its amplitude \(A_v = 50\) km s\(^{-1}\). The CHIANTI atomic data base (Dere et al. 1997) is used to produce the synthetic line profiles by computing the averaged density (\(\rho\)), temperature (\(T\)) and FWHM estimated by the averaged velocity (\(V_r\)). The CHIANTI ssw routine ‘synthetic.pro’ is used to produce the line profiles; note that the routine takes into account the ionic equilibrium as reported by Mazzotta et al. (1998), the coronal hole differential emission measure (DEM) values and the coronal abundances as available in the CHIANTI atomic data base. Although the major input parameters are the averaged density (\(\rho\)) (or pressure), temperature (\(T\)) and FWHM (see equation 16) derived from the simulation, while other parameters mentioned above are considered as optional input parameters in the calculation of synthetic spectrum.

In the present case, we compare the deduced theoretical line width with the corrected observed one as reported by Banerjee et al. (1998) using SOHO/SUMER. Therefore, we neglected the implication of the instrumental width of SOHO/SUMER slits in equation (16). We synthesized the line profiles of the Si VIII 1445.75 Å line but did not calculate the continuum as we wanted to only show the shape of the line profiles at various heights and their broadening that compares well with the observations of Banerjee et al. (1998). The line widths of synthetic line profiles of Si VIII 1445.75 Å at 20, 40 and 70 Mm (equation 16) are, respectively, 353, 358 and 363 mÅ when we consider the non-thermal contribution of the impulsively excited Alfvén wave by a pulse with its amplitude \(A_v = 50\) km s\(^{-1}\). It is clear from table 2 of Banerjee et al. (1998) that the estimated corrected line widths of Si VIII 1445.75 Å at heights 27 arcsec (∼19.6 Mm), 57 arcsec (∼41.3 Mm) and 98 arcsec (∼71 Mm) are, respectively, ∼292 mÅ, ∼334 mÅ and ∼369 mÅ.

We also adopted and tested a different way of calculating the line profiles at a particular height (e.g. 15 Mm) in the model coronal hole. Using the results of our numerical simulation, we derived the plasma parameters (\(\rho, T\)) as well as the transversal speed \(V_r\) for each instant (\(X, Y = \text{fixed}, t\)) in which the X space is averaged over 1.0 Mm spatial scale. The time instant was considered with each 10 s resolution between \(t = 250\) s and \(t + 250\) s. We collected the parameters at each instant and then derived the averaged FWHM, which was found to be 284 mÅ for a pulse of \(A_v = 50\) km s\(^{-1}\). The result closely matches the broadened line width of 291 mÅ as reported by Banerjee et al. (1998) at a height of 23 arcsec. Therefore, the alternative estimations are also consistent with our findings.

Although our theoretically estimated line width also shows increment with height in our model coronal hole, the spatial gradient of increment is rather flat compared to the observations by Banerjee et al. (1998). The most likely reason is that we considered the impulsive excitation of the pulse-driven Alfvén wave that

Figure 6. The synthetic line profiles of the Si VIII 1445.75 Å line at three different heights \(y = 20, 40\) and 70 Mm to compare with the observations of Banerjee et al. (1998), based on the inclusion of the non-thermal contribution of the Alfvén wave excited by pulse amplitude \(A_v = 50\) km s\(^{-1}\). The line profile is deduced by considering the ionic equilibrium as reported by Mazzotta et al. (1998), the coronal hole DEM values and the coronal abundances as available in the CHIANTI atomic data base. The simulated temperature and density are also used as input parameters.
also contributes more to the unresolved non-thermal motion near the wave source region in the lower solar atmosphere. Therefore, the computed line width, for instance at 27 arcsec (~19.6 Mm), is higher than the observed line width given by Banerjee et al. (1998) at the same height. However, the theoretically estimated and the observed line widths at coronal heights, e.g. at 40 and 70 Mm, closely match each other.

The results presented in Fig. 6 clearly show that the synthetic line profiles computed with the wave amplitude $A_v = 50 \text{ km s}^{-1}$ closely approximately mimic the observed spectrum of Si VIII $\lambda1445.75$ Å obtained by the detector B on SOHO/SUMER and reported by Banerjee et al. (1998). This provides evidence that the large-amplitude, pulse-excited Alfvén wave can contribute up to some extent to the broadening of the spectral lines observed by Banerjee et al. (1998).

5.3 Line profiles obtained with $A_v = 25 \text{ km s}^{-1}$

We again derived the synthetic line profiles (cf., Fig. 7) of Si VIII $\lambda1445.75$ Å (Dolla & Solomon 2008) at three different heights $y = 43.5, 65.25$ and 72.5 Mm, based on the inclusion of the non-thermal contribution of the Alfvén wave excited by the pulse with its amplitude $A_v = 25 \text{ km s}^{-1}$. The CHIANTI atomic data base and the method described above was again used to produce the synthetic line profiles. The obtained results show that there is not very significant broadening of line profiles in our model coronal hole for these smaller amplitude Alfvén waves; for the observational data see Dolla & Solomon (2008).

The line widths of Si VIII $\lambda1445.75$ Å synthetic line profiles at 43.5 Mm (~60 arcsec), 65.25 Mm (~90 arcsec) and 72.5 Mm (~100 arcsec) (Fig. 7) above the limb in our coronal hole model are estimated, respectively, as ~289, ~290 and 291 mÅ when we consider the non-thermal contribution of the impulsively excited Alfvén wave by a pulse strength of $A_v = 25 \text{ km s}^{-1}$. Dolla & Solomon (2008) have reported the Gaussian half line widths of various coronal spectral lines (e.g. Si VIII $\lambda1445.75$ Å, Fe XII $\lambda1242$ Å, Mg X $\lambda624$ Å) between ~42 Mm (57 arcsec) and ~100 Mm (140 arcsec) in the polar corona (cf., their fig. 8). This shows that our theoretical estimations are below the observationally estimated line width as reported by Dolla & Solomon (2008) for Si VIII $\lambda1445.75$ Å between 40 and 75 Mm in the coronal hole. For example, the estimated observed half line widths at $1/\sqrt{e}$, as reported by Dolla & Solomon (2008, cf. their fig. 8), after conversion in FWHM of Si VIII $\lambda1445.75$ Å at heights ~43.5 (~60 arcsec), ~65.25 (~90 arcsec) and ~72.5 Mm (~100 arcsec), respectively, give ~362 mÅ, ~368 mÅ and ~376 mÅ.

Our theoretically estimated line width shows very small increment with respect to height in the coronal hole model between 40 and 75 Mm height. Now, the observed width of Si VIII $\lambda1445.75$ Å also varies very little but with comparatively higher gradient than theoretical values. The most probable reason is that the plasma and magnetic field properties of the observed coronal hole differ from the assumed conditions in our model, in which the Alfvén wave of smaller amplitude propagates without much growth as well as its contribution to the line broadening.

Our numerical results obtained for non-linear Alfvén waves with the amplitude $A_v = 25 \text{ km s}^{-1}$ show that the resulting contribution of these waves to non-thermal motions approximately qualitatively matches the observational data of spectral line width as observed by Dolla & Solomon (2008) for the Si VIII $\lambda1445$ Å line of SOHO/SUMER, as both show the lesser spectral line broadening. However, the theoretically estimated widths are comparatively smaller than the observed ones at various heights in the coronal hole. Although the theoretically derived line widths only approximately match the observed ones at various heights in the coronal hole, there is no clear evidence of significant line broadening in these cases.

6 DISCUSSION

As already mentioned in Section 1, there are several analytical models that attempted to explain the observed non-thermal broadening of spectral lines in the solar corona by using linear Alfvén waves (e.g. Harrison et al. 2002; Pekünlü et al. 2002; Dwivedi & Srivastava 2006). According to the linear theory of propagating Alfvén waves from the solar photosphere upwards into the solar corona along the magnetic field lines, it is found that the non-thermal velocity is inversely proportional to the quadratic root of the observationally estimated electron density that matches well the characteristics of such waves (e.g. Banerjee et al. 1998; Moran 2003; Banerjee et al. 2009). The falloff in the density with height can amplify the propagating Alfvén wave and increase its group velocity that can contribute to the rms wave amplitude and thus to the non-thermal spectral line-width broadening with height in the solar atmosphere.

The most relevant numerical work has been done by Kudoh & Shibata (1999), who used numerical simulations to investigate the
role of torsional Alfvén waves in both the formation of spicules and the observed non-thermal broadening of spectral lines in the solar corona. They considered Alfvén waves to be generated by perturbations of 1 km s\(^{-1}\) in the solar photosphere. According to these authors, the waves can lift the spicules up to 5000 km, provide the energy flux of \(3.0 \times 10^6\) ergs s\(^{-1}\) cm\(^{-2}\) and produce the non-thermal broadening of emission lines in the solar corona with approximately 20 km s\(^{-1}\).

It is clear from the above discussion that neither previous analytical methods nor previous numerical simulations have been able to explain the full profile of the observed spectral line-width variation with height in the polar coronal holes. Nevertheless, it is noteworthy here that previous observational results have been matched with linear propagating waves coming from photosphere (Banerjee et al. 1998, 2009). They have showed, in linear regime, that the rms wave amplitude is related to some powers of mass density and magnetic field variations. In this paper, we are not attempting this indirect analytical model explanations but instead we directly compute the temporally and spatially averaged transversal velocity component, \(V_z\), from our pulse-excited Alfvén wave model and determine its contribution to the rms wave velocity amplitude. It should be noted then that the non-thermal velocity motion caused by such waves is inherent in the computed line width variation of the synthetic Si VIII 1445.75 Å line with height in the model coronal hole and that it is consistent with the observations reported by Banerjee et al. (1998). Hence, we conclude that the computed spectral line broadening with height in the coronal hole implies the increment of non-thermal velocity as provided by the Alfvén waves. The results of our numerical simulations of non-linear Alfvén waves show that the non-thermal spectral line-width broadening in polar coronal holes can be explained by these waves.

In our pulse-excited model, the pulse of a large amplitude is launched above the solar photosphere and this pulse splits into the upward- and backward-propagating wave trains in the overlying solar atmosphere. The upward-moving pulse train causes the instantaneous displacement of the field lines in a perpendicular plane away and towards the LOS. This effect generates the transversal velocity and thus the Doppler shift as well as line broadening. Therefore, in our case of the pulse-excited non-linear Alfvén wave model, the more stringent contribution of the averaged transversal velocity \(V_z\) component at a particular height causes the non-thermal broadening of the spectral line there.

The theoretically estimated line width of the Si VIII 1445.75 Å synthetic line increased with height in the model coronal hole due to the pulse-excited, large-amplitude Alfvén waves, which is consistent with the observations of the same phenomenon in the polar coronal hole (Banerjee et al. 1998). We have also generated synthetic line profiles of the Si VIII 1445.75 Å line (Fig. 6) that match well the sample observations of Banerjee et al. (1998) showing the spectral line broadening. These measurements may also be the signature of the undamped and growing transversal pulse train that is the pulse-excited Alfvén wave. However, the region of our interest is below 80 Mm in the solar atmosphere, where the observations show the line-width increment consistent with the excitation of the undamped Alfvén waves. It should be noted that the region of decreasing line width as reported by O’Shea et al. (2005) is quite high that was interpreted in terms of Alfvén wave dissipation. We do not consider such regime of observations in our model, which instead is focused on the Alfvénic dynamics in the lower solar atmosphere of the coronal hole that is also the source region of the fast solar wind.

It is well known that the fast solar wind starts accelerating near 20 Mm in the magnetic funnels in the solar coronal regions (Tu et al. 2005). Therefore, the undamped and growing pulse-excited, large-amplitude Alfvén waves can provide the momentum to the solar wind plasma in the inner corona like the classical Alfvén waves, which are typically considered to be generated in the solar photosphere. On the other hand, the generation of large-amplitude Alfvénic pulses above the solar surface considered in this paper is still a puzzle. The reconnection events between the emerging magnetic fields with the existing open magnetic field lines may be one of the most plausible mechanisms that can trigger such strong transversal velocity pulses and thus drive Alfvén waves along the open field lines of the coronal hole. However, such waves and their physical properties may be highly dependent on the local magnetic field configuration of the reconnection region (Kigure et al. 2010).

Recently, Okamoto & De Pontieu (2011) used the high-resolution observations obtained by the Hinode instruments and reported signatures of the propagating Alfvén waves along solar spicules with a velocity amplitude of ~7 km s\(^{-1}\); however, as discussed by Van Doorsselaere et al. (2008) the observed signatures may actually correspond to fast magnetoacoustic kink waves instead. The most likely evidence for the outward-propagating Alfvén waves with amplitudes of the order of 20 km s\(^{-1}\) and periods of the order of 100–500 s throughout the quiescent solar atmosphere was reported by McIntosh et al. (2011), who also concluded that these waves may carry enough energy to accelerate the fast solar wind and heat the quiet corona. It must be noted that the observed wave amplitudes are smaller than those used in our study. Nevertheless, we hope that our results presented in this paper will initiate a search for Alfvén waves with larger amplitudes.

The energy carried by Alfvén waves has been considered to be an important candidate for the heating of the coronal hole and the acceleration of the solar wind by many authors (e.g. Ofman & Davila 1995, 1998; Ofman 2005, references cited there). The dissipation of the energy carried by these waves occurs either in the distant part of the corona (Parker 1991) or by some unique processes, such as phase mixing, taking place in the solar atmosphere (Heyvaerts & Priest 1983; Hood, Ireland & Priest 1997; Nakariakov et al. 1997). Apart from the phase mixing of such waves, the resonant absorption is also found to be another efficient mechanism that may most likely cause the damping of the Alfvén waves (Erdelyi & Goossens 1995; Ofman & Davila 1995; Erdelyi 1998; Nakariakov & Verwichte 2005). The large-amplitude non-linear Alfvén waves can also be dissipated by coupling to longitudinal wave motions in the outer part of the magnetized solar corona (e.g. Boynton & Torkelsson 1996). This shows that the solar atmosphere above the polar corona may be the ideal place for the growth of linear and non-linear Alfvén waves without much damping, and that the waves may carry their energy to the outer part of the corona and can heat the solar wind ions (e.g. Ofman & Davila 2001).

The main objective of the present study was to compute the averaged amplitude of the pulse-excited Alfvén waves at different atmospheric heights and explore its effects on the broadening of the observed spectral lines. Our computational domain covered both the lower part of the solar atmosphere and the inner solar corona, and we did not take into account any dissipative processes in our numerical simulations because it was suggested that such processes may contribute to the observed narrowing of the line profiles (O’Shea et al. 2005). However, this work is out of the scope of this paper and will be considered as a future project.
Alfvén waves are the most likely candidates for the non-thermal broadening of \( \text{Si VIII} \) only approximately contributing to the observed spectral line width. We compared the obtained numerical results to the spectral line broadenings observed by Banerjee et al. (1998) and Dolla & Solomon (2008). We found that the large-amplitude, non-linear, pulse-driven \((A_v = 50 \text{ km s}^{-1})\) Alfvén waves are the most likely candidates for the non-thermal broadening of \( \lambda 1445.75 \text{ Å} \) line profiles in the polar coronal hole (Banerjee et al. 1998). Our results also show that Alfvén waves driven by comparatively smaller velocity pulse, \( A_v = 25 \text{ km s}^{-1} \), only approximately contribute to the observed spectral line width of \( \lambda 1445 \text{ Å} \) at various heights in the coronal hole as reported by Dolla & Solomon (2008), however, without significant evidence of broadening. The results of our numerical simulation and their comparison to the observations of line broadening are important as they become indirect evidence for the existence of larger amplitude, pulse-driven Alfvén waves in solar coronal holes.

Finally, we would like to suggest that more spectroscopic observations should be carried out in future using high-resolution observations (e.g. Hinode/EIS, and also with upcoming Solar-C instruments) to search for direct evidence of such pulse-excited non-linear Alfvén waves in the solar atmosphere. Obviously, more detailed theoretical studies of a variety of pulse-driven non-linear Alfvén waves excited by a range of pulse amplitudes are also needed in order to better understand the role played by these waves in the observed line broadening and the solar wind acceleration.

ACKNOWLEDGMENTS

We thank the referee for valuable suggestions that allowed us to considerably improve our manuscript. The software used in this work was in part developed by the DOE-supported ASC/Alliance Center for Astrophysical Thermonuclear Flashes at the University of Chicago. This work has been supported by the Alexander von Humboldt Foundation (ZEM) and by a Marie Curie International Research Staff Exchange Scheme Fellowship within the seventh European Community Framework Programme (PCh and KM) as well as by the ‘HPC Infrastructure for Grand Challenges of Science and Engineering’ Project, co-financed by the European Regional Development Fund under the Innovative Economy Operational Programme (PCh and KM). We also acknowledge the CHIANTI, which is a collaborative project involving researchers at NRL (USA), RAL (UK), and the Universities of Cambridge (UK), George Mason (USA) and Florence (Italy). AKS thanks Shobhna Srivastava for patient encouragement.

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