Holography and Compactification

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Abstract

Following a recent suggestion by Randall and Sundrum, we consider string compactification scenarios in which a compact slice of AdS-space arises as a subspace of the compactification manifold. A specific example is provided by the type II orientifold equivalent to type I theory on (orbifolds of) $T^6$, upon taking into account the gravitational backreaction of the D3-branes localized inside the $T^6$. The conformal factor of the four-dimensional metric depends exponentially on one of the compact directions, which, via the holographic correspondence, becomes identified with the renormalization group scale in the uncompactified world. This set-up can be viewed as a generalization of the AdS/CFT correspondence to boundary theories that include gravitational dynamics. A striking consequence is that, in this scenario, the fundamental Planck size string and the large N QCD string appear as (two different wavefunctions of) one and the same object.
1 Introduction

In string theory, when considered as a framework for unifying gravity and quantum mechanics, the fundamental strings are naturally thought of as Planck size objects. At much lower energies, such as the typical weak or strong interaction scales, the strings have lost all their internal structure and behave just as ordinary point-particles. The physics in this regime is therefore accurately described in terms of ordinary local quantum field theory, decoupled from the planckian realm of all string and quantum gravitational physics. The existence of this large separation of scales has often been used as an argument against the potential predictive power or even the potential reality of string theory. However, as we will argue in this letter, there are a number of recent insights and ideas that suggest a rather more optimistic scenario, in which string physics may have visible consequences at much lower energies scales than assumed thus far. The two particular recent developments that we wish to combine are:

(i) The AdS/CFT correspondence

The basic example here is the proposed duality between four-dimensional $\mathcal{N} = 4$ SYM theory and type II B string theory on $AdS_5 \times S^5$ geometry

$$ds^2 = e^{-2y/R} \; ds_4^2 + dy^2 + R^2 d\Omega_5^2$$  \hspace{1cm} (1)

Here $ds_4^2 = dx^2$ denotes the flat four-dimensional metric and

$$R^2 = \alpha' \sqrt{4\pi Ng_s}$$  \hspace{1cm} (2)

with $g_s = g_{\text{ym}}^2$ the type IIB string coupling [1] [2]. In this AdS/CFT dictionary, the coordinate $y$ needs to be thought of as parametrizing the four-dimensional scale: two SYM excitations related by a scale transformation

$$x_{ij} \to e^{\lambda} x_{ij}$$  \hspace{1cm} (3)

translate on the AdS-side into two excitations concentrated around different locations in the $y$-direction related by a translation

$$y \to y + \lambda R.$$  \hspace{1cm} (4)

This map thus provides a “holographic” projection of the physics of the gauge theory (which can be thought of as living on the AdS-boundary) onto the one higher-dimensional AdS space.

Truncating the AdS theory to $y$-values larger (or smaller) than some finite value $y = y_0$ amounts to introducing an UV (or IR) cut-off in the gauge theory [3]. In the strict continuum limit, however, the range of $y$-values extends over the full real axis. A consequence of this is that, while the type II theory on the AdS-space contains gravity, the dual conformal gauge theory on the boundary does not. Modes of the AdS gravitational field that extend all the way to the boundary $y \to -\infty$ are not normalizable, and therefore do not fluctuate.
The metric $ds^2$ has the rather striking property that for sufficiently large $y$-values it differs from the four-dimensional brane-metric $ds_4^2$ by an exponentially large “red-shift factor” $e^{-2y/R}$, that is, distances along the $x_7$-directions are measured very differently by both metrics. A most dramatic consequence of this that the fundamental IIB strings, while still of Planck size when viewed by the full metric $ds^2$, can become arbitrarily large when measured in terms world-brane metric $ds_4^2$, simply by moving to larger $y$-values. The SYM interpretation of these large IIB strings is that they represent the color-electric flux lines [2].

In principle this type of holographic correspondence can be generalized to less symmetric gauge theories with non-zero beta-functions and more complicated phase diagrams. The idea remains the same, namely that $y$-translations in effect amount to renormalization group transformations on the gauge theory side. Beta-functions or symmetry breaking or other type of phase transitions thus translate into a non-trivial $y$-dependence (e.g. in the form of domain wall structures) of the metric, dilaton and other fields relative to the AdS$_5$-background.

(ii) **Compactifications with a D3-brane world**

In a large class of type I string compactifications, the gauge theory degrees of freedom in our four-dimensional world may be thought of as bound to a (collection of) D3-brane(s). These 3-branes wrap our world but are otherwise localized as point-like objects somewhere inside the compactification manifold. Scenarios of this type have recently been studied from various points of view, in particular they seem to offer some promising new avenues for addressing the gauge hierarchy problem [4,5]. Besides (perhaps) allowing for the possibility of large extra dimensions [4], a second new aspect of these type of compactifications is that (due to the backreaction of the D3-branes) various fields, such as the dilaton and in particular the conformal factor of the four-dimensional metric, may acquire non-trivial dependence on the compact coordinates.

An interesting example of this type of geometry was recently considered by Randall and Sundrum in [5]. In this set-up, the four-dimensional conformal factor is found to depend exponentially on the extra fifth coordinate $y$, precisely as in the AdS-geometry [1]. A new ingredient, relative to the standard AdS situation, is that in [5] the extra coordinate $y$ is chosen to run over a finite (or semi-infinite) range. As a consequence, there exists a normalizable gravitational collective mode, given by those fluctuations of the metric $ds^2$ that preserve the form (1) but with $ds_4^2$ replaced by a (sufficiently slowly varying but otherwise) general four-dimensional metric

$$ ds_4^2 = g_\mu^\nu_{(4)}(x_\parallel) dx^\mu dx^\nu. $$

Hence this collective mode behaves just as the ordinary graviton of our four-dimensional world. In addition, due to the exponential $y$-behavior of the graviton wave-function, a given object with five-dimensional mass $m$ has an effective four-dimensional mass $m_4$, that depends of its $y$-location via

$$ m_4(y) = me^{-y/R}. $$
In [5] the presence of this exponential red-shift factor (or “warp factor”) was argued to provide a natural explanation of the large mass hierarchies, such as between the weak and Planck scale.

**Putting (i) and (ii) together:**

It seems natural to look for a way of combining these two ideas. Concretely, one could ask the following two (probably equivalent) questions:

- Can the AdS/CFT type of dualities be extended to situations where the radial AdS-coordinate is effectively compactified, and does this compactification indeed automatically imply the presence of four-dimensional gravity on the boundary?

- Are there string compactification scenarios where a compact slice of AdS-space appears as part of the compactification geometry, and does the radial AdS coordinate then again have a “holographic” interpretation as parametrizing the RG scale of the four-dimensional theory?

In the following we will argue that both questions have a positive answer, though the first one only for a (large but) finite set of D3-brane charges $N$. Consequently, the compactified AdS-geometry will be a good approximation provided the string coupling $g_s$ is not too small compared to $1/N$. In the concluding section we will address some of the possible consequences of this new view on string compactification.

## 2 AdS Compactification

Let us start with considering the toy example of a four-dimensional $\mathcal{N}=4$ supersymmetric world described by the $T^6$ compactification of the type I superstring. Its low energy effective description is given by $\mathcal{N}=4$ super-Yang-Mills theory, with as maximal unbroken gauge symmetry group $\mathcal{G} = SO(32)$, coupled to supergravity. In the following we will consider the situation where this symmetry is broken down to an $U(N)$ sub-group with $N \leq 16$.

By applying $T$-duality this type I string theory can be equivalently described as an orientifold of type II string theory on $T^6$. In this representation, there are $2^6 = 64$ orientifold planes located at all the half-way points of the $T^6$. In addition, there are 32 D3-branes inside the $T^6$ which are pairwise identified under the orientifold $\mathbb{Z}_2$-action [4]. Hence it is possible for $N \leq 16$ of these D3-branes to form a small cluster inside the $T^6$, and in the limit where all $N$ coincide at the same point, the unbroken $U(N)$ gauge symmetry appears.

At low energies in the uncompactified world, gravity effectively decouples from the $U(N)$ SYM dynamics. Thus we can look for a regime of parameters in which we can apply the AdS/CFT duality map and obtain type II theory on $AdS_5 \times S^5$ as a good dual low energy description. The main restriction (in order to be able to trust the sigma-model approximation) is that the $AdS_5$ radius of curvature $R$, given in [2], needs to be large compared to the
At the $U(N)$ symmetric point, when $N$ D3-branes coincide, their backreaction produces a (compact) $AdS_5 \times S^5$ sub-region, that can be glued into the $R^4 \times T^6$ geometry.

Fig. At the $U(N)$ symmetric point, when $N$ D3-branes coincide, their backreaction produces a (compact) $AdS_5 \times S^5$ sub-region, that can be glued into the $R^4 \times T^6$ geometry.

string scale,

$$g_s \gg \frac{1}{4\pi N}.$$  \hspace{1cm} (7)

Hence, relative to the usual large $N$ context, we now have a somewhat more limited control over the approximations involved in the duality map.

Around the string scale $L_s$, open string effects will start to modify the SYM dynamics. Let us choose the radial AdS-coordinate $y$ such that this string scale $L_s$ gets mapped to the region around $y \simeq 0$. Then the $AdS_5 \times S^5$-description is a valid approximation for the low-energy regime $y \gg R$. Around $y \simeq 0$, on the other hand, we reach the transition region where the low energy regime meets with the high energy regime described by orientifold of type II on $T^6$. Our main new proposal is that these two regimes can be consistently glued together into one single dual type II string background. In the following we will assume that the size of the $T^6$ is bigger or of the order of the AdS-radius $R$ in (2).

As a first approximation, we can visualize the total target space of this new type II theory as follows. We first cut out a small ball around the D3-branes inside the $T^6$, such that its outside radius coincides with the $S^5$ radius $R$ in (2). Then we replace the inside of the ball by the $y \gtrsim 0$ region of $AdS_5 \times S^5$. (See fig. 1). It seems reasonable to assume that the resulting
total space can be smoothed out to obtain an exact consistent type II background, and the end result of this procedure should be equivalent to taking into account the gravitational backreaction of the D3-branes.

In a more complete treatment, we must also include the backreaction of the 64 orientifold planes. These have a negative tension equal to $-1/4$ times the D3-brane tension, which need to be taken into account. We can write an explicit form for the background metric, by starting from the general form

$$ds^2 = \frac{1}{H(x_\perp)^{1/2}} ds_4^2 + H(x_\perp)^{1/2} dx_\perp^2,$$

(8)

with $ds_4^2 = dx^2$, and where $x_\perp$ denote the coordinates inside the $T^6$. For simplicity, let us consider the most symmetric example of the $SO(32)$ invariant point, where all 16 D3-branes coincide with their 16 $\mathbb{Z}_2$ images at one of the orientifold fixed points, say at $x_\perp = 0$. Taking the $T^6$ to be an exact cube with period $R_c$, the harmonic function $H(x_\perp)$ then takes the form

$$H(x_\perp) = 1 + 4\pi g_s(\alpha')^2 \left[ f_D(x_\perp) - f_O(x_\perp) \right],$$

(9)

where

$$f_D(x_\perp) = 2 \sum_{\vec{n} \in \mathbb{Z}_6} \frac{16}{|\vec{x}_\perp + \vec{n}R_c|^4}$$

(10)

denotes the contribution of the 16 D3-branes and their $\mathbb{Z}_2$-image, and

$$f_O = 2 \sum_{\vec{n} \in \mathbb{Z}_6} \frac{1/4}{|\vec{1} + \vec{n}R_c|^4}$$

(11)

denotes the contribution of the 64 orientifold planes located at all the half-way points inside the $T^6$. In the region close to the D3-branes at $x_\perp = 0$, the above geometry indeed reduces to the $AdS_5 \times S^5$ metric (1). The present minimal example (of the simple $T^6$ orientifold compactification) can straightforwardly be generalized to less symmetric theories obtained e.g. by applying further orbifolds. In this way one can find holographic duals to a large class of four-dimensional type I string theories, typically with a reduced number of (or even zero) supersymmetries. The total target space of the type II duals should all describe consistent compactifications of the $AdS_5 \times S^5$ geometry. The number of such consistent AdS-compactifications, however, is constrained by the usual tadpole cancelation conditions. In particular, one needs to make sure that the RR flux $N$ that escapes the AdS-region gets absorbed by the appropriate number

\footnote{For the $SO(32)$ symmetric example at hand, the $S^5$ must in fact be replaced by its orientifold $S^5/\mathbb{Z}_2$. We further notice that close to the other orientifold planes the function $H$ goes through 0; in this horizon region, however, $\alpha'$ and non-perturbative string-corrections are expected to play an important role.}
of branes and orientifold planes present in the compact outside region. Consistent solutions to this condition can be found only for a finite set of $N$-values. Still, relative to our minimal set-up, one can significantly extend the range of allowed values for $N$, up to values of the order of $N \approx 10^3$ or larger, by considering more general orbifold spaces. Hence in this more general setting, the restriction (7) in effect just amounts to $g_s > 10^{-4}$, which leaves a quite sufficient parameter range for which we can trust our approximations.

3 Gravitational coupling

This geometric set-up is now indeed very similar to that of [5], where our $T^6$ region with the orientifold planes plays the role as the positive tension brane in their scenario. Several of the same conclusions apply. In particular, we must find a collective graviton mode of the exact same type as described in the Introduction, that couples precisely as the four-dimensional graviton to the boundary field theory. In our case this result was of course expected from the start, since the boundary theory is in fact the type I string theory on $T^6 \times \mathbb{R}^4$, that also contains a gravitational closed string sector.

The relation between the four-dimensional Planck length $L_{pl}$ and the 10-dimensional one $L_{10}$ takes the usual form

$$ (L_{10})^8 = (L_{pl})^2 V_6 $$

where $V_6$ is the appropriately measured volume of the $T^6$. Plugging the form (8) into the ten-dimensional action gives

$$ \frac{1}{(L_{10})^8} \int d^{10}x \sqrt{-g_{10}} R_{10} = \frac{V_6}{(L_{10})^8} \int d^4x_\parallel \sqrt{-g_4} R_4 $$

with

$$ V_6 = \int_{T^6} d^6x_\perp H(x_\perp). $$

Notice that, in spite of the fourth order pole in $H(x_\perp)$ near the branes, this integral indeed yields a finite answer.

The collective mode of the 10-dimensional metric that produces the four-dimensional graviton again has the same shape as the full metric, as given in eqn (8), with $ds_4^2$ a general four-metric (though sufficiently slowly varying with $x_\parallel$). The shape of the graviton wavefunction the AdS-region decays with $y$ as $e^{-2y/R}$. This exponential factor is reflects the fact that the coupling of matter to the graviton becomes weaker and weaker at lower and lower scales. Indeed, we can extract the strength of the coupling to matter by inserting the variation $\delta g^{(4)}_{\mu\nu} = h_{\mu\nu}$ of the four-metric $ds_4^2$ into the the ten-metric (8). The corresponding variation of the 10-d matter action reads

$$ \int d^{10}x \sqrt{-g_{10}} \delta g_{10}^{\mu\nu} T_{\mu\nu} = \int d^4x_\parallel h_{\mu\nu} T_{\mu\nu} $$

(15)
with
\[ T_{\mu\nu}(y) = \int d^6x_\perp H(x_\perp) T_{\mu\nu}. \] (16)

Inserting on the right-hand side the energy-momentum tensor (here all indices contracted with the ten-metric \( ds^2 \))
\[ T_{\mu\nu} = \frac{m}{\sqrt{-g_{10}}} \int d\tau \frac{\dot{x}_\mu \dot{x}_\nu}{\sqrt{\dot{x}^2}} \delta_{10}(x-x(\tau)) \] (17)

of a 10-d point-particle, with mass \( m \) located at a given location \( y_\perp \), gives for the effective 4-d energy-momentum tensor (with all indices contracted with the four-metric \( ds^2_4 \))
\[ T_{\mu\nu} = \frac{m}{H(x_\perp)^{1/4}} \int d\tau \frac{\dot{x}_\mu \dot{x}_\nu}{\sqrt{\dot{x}^2}} \delta_4(x_\parallel-x_\parallel(\tau)) \] (18)

So the four-dimensional mass depends on \( x_\perp \) as
\[ m_4(x_\perp) = \frac{m}{H(x_\perp)^{1/4}}. \] (19)

This is the expected red-shift effect; close to the D3 branes it reduces to [3].

4 RG scale as a real extra dimension

Though familiar and standard, the appearance of both open and closed string dynamics in type I string theory is still a deep fact. The two are indeed intertwined in an intricate way, since when one starts including the open string quantum effects, there is no obviously unique way of separating the open string loop diagrams from closed string tadpoles. This very subtlety of course lies at the heart of the D-brane correspondence between gauge theory and gravity. A microscopic reconstruction of the reasoning of [1] indeed shows that the appearance of the curved AdS-metric (1), as describing the near-horizon region close to the D3-branes, must be thought of as a cumulative consequence of open string quantum effects.

Concretely, one can imagine setting up a renormalization group flow, where via a Fischler-Susskind type mechanism, the effect of integrating out successive momentum shells in the open string channel gets absorbed into an appropriate redefinition of the world-sheet sigma-model couplings. Roughly speaking, in this procedure the slice of the AdS target space in between two values \( y_1 \) and \( y_2 \) gets created from the open string dynamics in between the corresponding energy scales. Eventually, after all open strings are integrated out, its planar diagrams have been replaced by a stretched-out closed string worldsheet, moving inside the semi-infinite AdS-throat geometry. Thus, in a quite precise sense, the AdS closed strings provide the long sought after dual representation of the gauge theory planar graphs [1] [2].
If instead one runs this renormalization group flow from the string scale down to a certain finite length scale $L$ (for example the weak scale), the resulting target space consists of a compact slice of AdS bounded by the Planck region $y \simeq 0$ on one side and by an effective D3-brane located at $y \simeq R \log L/L_S$ on the other. The D3-brane hosts the remaining sector of low energy open strings, describing the physics at distance scales larger than $L$. Sub-planckian physics thus happens far inside the AdS-tube, and from this perspective the $T^6$-orientifold region merely acts as a kind of sounding board, providing some appropriate boundary condition in the far away planckian regime. Via high energy experiments on the D3-brane world-volume, however, one can still generate quantum fluctuations that extend to smaller values of $y$ and thereby probe the corresponding planckian geometry.

What new lesson can we extract from this? It is clear that, relative to the standard notion of an extra dimension, the $y$-direction is rather unusual. Normally one would expect that space-like separated events are independent, while here we are learning that $y$-translations are in effect scale transformations. Therefore, to avoid over-counting of the number of degrees of freedom, we thus indeed need to adopt the hypothesis that physics that happens far out in the $y$-direction is not really independent from that, say, at the boundary region $y \simeq 0$, but rather a “holographic image” of physics at $y \simeq 0$ happening at a corresponding scale $L(y) = L_S e^{y/R}$. This is the familiar dictionary of the AdS/CFT correspondence.

Though quite miraculous, in itself this holographic equivalence doesn’t teach us anything new yet, as it simply points to some redundancy in the description. We could for example insist on describing all the physics as happening just at $y \simeq 0$, and consider the holographic map as some purely mathematical equivalence. Relative to the standard AdS/CFT situation, however, we now have four-dimensional gravity as an essential new ingredient. In first approximation the gravity coupling seems quite consistent with holography, since (as seen from the previous section) it looks like to a relatively standard Kaluza-Klein dimensional reduction. The holographic reconstruction does, however, strictly amount to a non-trivial redistribution of matter, and since gravity is still in the game, in the end we do need to specify which energy-momentum distribution acts as a source for the gravitational field.

The new consequence of this holographic view of gravity, therefore, is that the renormalization group scale is promoted to a real physical extra direction. Concretely, this means that for type I string theory the gravitational energy-momentum in our four-dimensional world really spreads out into the extra $y$-direction, indeed similar to a holographic, or in a perhaps more accurate analogy, chromatographic image.

\[ \text{Due to the redshift, these open strings have acquired a renormalized tension } \alpha'_{\text{eff}} \simeq \sqrt{N g_s} L^2. \text{ The energy of a string with this tension stretched over a distance } L \text{ reproduces a force comparable to the } 1/L \text{ Coulombic force of the SYM model.} \]

\[ \text{Because of this holographic identification, the distinction made in [8] between a “hidden” and “visible” brane is no longer applicable in our context. Also the (continuum) KK tower of gravitational excitations must be reinterpreted as actually representing low energy SYM degrees freedom in the boundary theory.} \]
5 Conclusion remarks

Finally, let us try to draw a few obvious and less obvious conclusions. We begin with

The Planck vs large N QCD string

A first striking property of this new scenario is that the planckian type II string and the low energy electric flux strings are in essence made up from one and the same object: the two are simply related by a translation in the $y$-direction. Similarly, in more realistic type I compactifications with QCD-like confining gauge sectors, one expects to find glueball excitations that on the type II side correspond to bound state solutions to the graviton wave-equation, localized (possibly around some domain wall structure) far inside the AdS-region. This is just like in the standard AdS/CFT correspondence. However, the new ingredient here is that the bulk gravity (used for representing the glueballs) and the gravity in the physical “boundary world” are directly linked and produced by the same type II string.

Low energy string phenomenology

The above set-up seems to open up the possibility of finding type II string compactification scenarios that in effect bypass the quark and gluon stage and immediately connect with the infra-red degrees of freedom in terms of mesons, baryons, etc. More generally, instead of the usual “top-down” approach to string phenomenology, one may contemplate a rather different “bottom-up” philosophy. Ideally, one could first try to connect string theory with low-energy physics (e.g. via standard non-compact AdS/CFT type technology and variations thereof) and then afterwards introduce gravity by trying to look for all possible consistent compactifications of this AdS-type space. Of course, there are still important technical as well as conceptual obstacles to deal with, such as supersymmetry breaking, unification, massless moduli fields, and allowing $N$ to be small rather than large, just to name a few.

The gauge hierarchy problem

Perhaps the deepest consequence of the above picture is that the renormalization group scale parameter of the four-dimensional world gets promoted to a real physical extra dimension. In this way all relative mass hierarchies, such as those between the weak scale and the Planck scale, are translated into relative separations in this extra direction. Since the conformal factor decays exponentially with $y$ with a planckian decay length, separations of say 50 or 100 times the Planck length generate scale factors as large as $10^{15}$ or $10^{30}$ [5]. While perhaps one could argue that this gives a natural explanation of the large mass separations in our world, without giving a good reason of why string theory would necessarily choose this type of compactification geometry, one cannot claim it really solves the hierarchy problem. Instead, it seems more sensible to turn the conclusion around, and consider the existence of a large mass hierarchy as evidence supporting this type of compactification scenario as its most natural geometric realization.
The cosmological constant problem

The most vexing hierarchy problem is of course that of the cosmological constant. In essence it arises as a clash between experiment and the theoretical knowledge of general relativity and the renormalization group. There are several ingredients in the present set-up, however, such as (i) the reinterpretation of the RG scale as a holographic extra dimension, (ii) the intimate connection between the RG flow and the AdS-gravity equations of motion, (iii) the presence of a planckian size negative cosmological constant inside the AdS-region, that suggest a rather new perspective on this problem.

In this connection, it seems interesting to note that a naive AdS holographic description of the universe (as a boundary theory with finite temperature equal to that of the microwave background) identifies the total geometry with that of an AdS black hole \[\text{[11]}\] with a horizon radius of about a millimeter, and thus with associated entropy of the order of \(10^{90}\). A suggestive coincidence is that this horizon size is of similar magnitude as the thermal wavelength of the radiation itself, which perhaps indicates that holography may provide a link between the size and the total entropy of the universe \[\text{[12]}\].

Finally, we notice that this view of the universe as an AdS black hole has the striking consequence that any pair of points can be connected via a path of at most a couple of millimeter length. Indeed, one can simply first travel close to the black hole horizon, traverse the required distance in the \(x_{\parallel}\)-direction, and then go back. This does not mean, however, that one can travel at super-luminous speeds, since the travel time (as measured in our world) is delayed correspondingly by the red-shift factor.

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