Inflation versus collapse in brane matter

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Abstract

Mapping of fundamental branes to their worldsheet (ws) multiplets originating from spontaneous breaking of the Poincare symmetry is studied. The interaction Lagrangian for fields of the Nambu-Goldstone multiplet is shown to encode $R^2$ gravity on the ws. The power law $k_p \sim T_p^{4(p+1)/p}$ for the SO(D-p-1) gauge coupling $k_p$ as the function of the p-brane tension $T_p$ is assumed. It points to the presence of asymptotic freedom and confinement phases in brane matter. Their connection with collapse and inflation of the branes is discussed.

1 Introduction

The geometric approach [1], [2] reduces search for fundamental Nambu-Goto strings and Dirac branes to studying the Plateau problem for hyper-surfaces in pseudo-Riemannian spaces [3], [4]. The problem is investigated by the Cartan method of moving frames used in the theory of Lie groups and symmetric spaces [5], [6]. This method was applied for description of the Nambu-Goldstone (NG) bosons in systems with a spontaneously broken internal symmetry nonlinearly realized by a group $G$ [7-11]. Interpretation of the NG bosons as the constrained multiplets of the gauge symmetry $H$,

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arisen from localizing the global vacuum symmetry $H \in G$, was proposed in Ref. [12]. These ideas were used for investigation of $p$-dimensional minimal hyper-worldsheet (ws) $\Sigma_{p+1}^{min}$ swept by strings ($p = 1$) [13] or $p$-branes [14], [15] embedded into the $D$-dimensional Minkowski space. Thereat, the space-time Poincare group $ISO(1, D − 1)$ was treated as the brane symmetry spontaneously broken to $ISO(1, p) \times SO(D − p − 1)$ initiated by the embedding of $\Sigma_{p+1}^{min}$ [16], [17], [18]. The gauge and diff-invariant field action considered in [14] includes the $SO(D − p − 1)$ gauge multiplet $B_{\mu}^{ab}$ interacting with the NG tensor multiplet $l_{\mu\nu}^a$ and the ws metric $g_{\mu\nu}$ on $\Sigma_{p+1}^{min}$.

Here we find that the action [14] gives an implicit ws realization of $(p + 1)$-dimensional gravity described by quadratic curvature terms similar to those used in Refs. [19-23]. These terms are encoded by the quartic potential for the field $l_{\mu\nu}^a$ identified with the second fundamental form of $\Sigma_{p+1}^{min}$. For the codimension 1, this reveals an alternative formulation of the $R^2$ gravity [20]. The latter also contains a cosmological term represented by an integration constant $\Lambda$.

On the other hand, we understand the field action to map the Dirac brane action into the one which describes the Yang-Mills (YM) and tensor ws multiplets. The map unambiguity demands addition of the Ricci-Codazzi equations as the initial data for the Euler-Lagrange variational equations (see proof in Ref. [14]). If $\Lambda = 0$ the action has only one coupling constant $k_p$ depending of the brane tension $T_p$ by means of the power law $k_p \sim T_p^{\frac{3−p}{2(p+1)}}$. This shows that $k_p$ treated as the function of $T_p$ has three different regimes of behavior corresponding to the cases $p < 3$, $p = 3$ and $p > 3$. Taking into account that the value of tension defines the energy scale, one can estimate how $k_p$ changes with energy. Then we explain the regimes as the ones corresponding to the asymptotic freedom or confinement. Using the w-s curvature dependence on $k_p$ we find that the field regimes describe the inflation or collapse phases of the branes [1]. For $p = 3$ the coupling $k_3$ becomes dimensionless and independent of $T_3$. Then the field action becomes scale invariant in correspondence with the results obtained in [26-29] (see also Ref. [30]).

\footnote{Some exact solutions describing collapsing $p$-branes were found in Refs. [24] and [25].}
2 Branes and quadratic curvature gravity

In string theory, a (p+1)-dimensional hyper-ws embedded into D-dimensional Minkowski space $\mathbb{R}^{1,D-1}$ is described by its worldvector $x(\xi^\rho)$ depending on the internal coordinates $\xi^\mu = (\tau, \sigma^r)$, $r = 1, 2, .., p$. In the Cartan approach to differential geometry, a (p+1)-dimensional hyper-surface $\Sigma_{p+1}$ is described by the Maurer-Cartan or Gauss-Ricci-Codazzi (G-R-C) equations \[4\] that can be rewritten as the covariant field constraints studied in Ref.[14].

The Gauss constraints for the Riemann tensor $R_{\mu\nu\gamma\lambda}(\xi)$ of $\Sigma_{p+1}$

$$R_{\mu\nu\gamma\lambda} = l_{[\mu}^{\gamma} l_{\nu]\lambda}$$

express this tensor through the second fundamental form $l_{\mu\nu}^a(\xi)$. The indexes $a, b = p+1, p+2, ..., D-1$ enumerate the orts $n_a(\xi^\rho)$ of an orthonormal moving frame attached to $\Sigma_{p+1}$ which are orthogonal to it. The symmetric tensor $l_{\mu\nu}^a$ is treated as a constrained multiplet of the local group $SO(D-p-1)^2$.

The Ricci equations are equivalent to the constraints

$$H_{\mu\nu}^{ab} = l_{[\mu}^{\gamma} l_{\nu]\gamma}^b, \quad H_{\mu\nu a}^b : = (\partial_{[\mu} B_{\nu]} + [B_{\mu}, B_{\nu}])^b_{a}$$

for the field strength $H_{\mu\nu a}^b$ of the $SO(D-p-1)$ gauge field $B_{\mu a}^b(\xi) = -B_{b a}^\mu(\xi)$ in the fundamental representation. The brackets $\{\mu\nu\}, \llbracket\mu\nu\rrbracket$ imply the $\mu, \nu$ symmetrization and antisymmetrization, respectively. Eqs. (2) appear only for the codimension $D - (p+1) \geq 2$, otherwise $B_{\mu a}^b \equiv 0$.

The Codazzi equations are equivalent to the field constraints

$$\nabla_{\mu\nu\rho} l^{\mu}_{\nu\rho} = 0,$$

where $\nabla_{\mu\nu\rho}^+ l^{\mu}_{\nu\rho}$ is the metric and YM covariant derivative

$$\nabla_{\mu\nu\rho}^\pm l^{\mu}_{\nu\rho} := \partial_{\mu} l^{\pm}_{\nu\rho} - \Gamma^\lambda_{\mu\nu} l^{\pm}_{\lambda\rho} - \Gamma^\lambda_{\mu\rho} l^{\pm}_{\nu\lambda} + B^{\pm}_{\rho} l^{\pm}_{\nu\lambda}. \quad (4)$$

The corresponding Bianchi identities have the form

$$[\nabla_{\gamma}^+, \nabla_{\mu}^+] l^{\mu\rho a} = R_{\gamma \mu}^\mu \lambda^{\rho a} + R_{\gamma \nu}^\mu \lambda^{\rho a} + H_{\gamma \mu}^a b^{\mu\rho b}. \quad (5)$$

The GRC eqs. (1-3) complemented by the minimality conditions for $\Sigma_{p+1}$

$$Sp l^a := g^{\mu\nu} l_{\nu\mu}^a = 0 \quad (6)$$

\[^2\]All notations and definitions coincide with the ones used in Ref. [14].
form a complete set of the equations of motion (EOM) for the Dirac p-brane embedded into $\mathbb{R}^{1,D-1}$.

The $SO(D - p - 1)$ and diff-invariant action of a p-brane sweeping a minimal hyper-ws $\Sigma_{p+1}^{\text{min}}$ and consistent with (1-3) has the form [14]

$$S_{\text{Dir}} = \frac{1}{k_p^2} \int dB^{p+1} \sqrt{|g|} \left\{ -\frac{1}{4} \text{Sp}(H_{\mu\nu}H^{\nu\mu}) + \frac{1}{2} \nabla^\perp_{\mu} l_{\nu\rho a} \nabla^\perp_{\nu} (\mu^\rho)^a_\mu - \nabla^\perp_{\mu} l_{\rho a} \nabla^\perp_{\nu} l^{\nu\rho a} + V_{\text{Dir}}(l, g) \right\},$$

(7)

where $\text{Sp}(H_{\mu\nu}H^{\nu\mu}) := H_{\mu\nu}^a H^{\nu\mu}_{ba}$ is the trace with respect to the indexes $a, b$. Here we use $\hbar = c = 1$ and the field dimensions $[l_{\mu\nu}^a] = [B_{\mu}^{ab}] = [\nabla_{\nu}^\perp] = [L^{-1}], [\xi^\mu] = [L], [g_{\mu\nu}] = 1$ resulting in the dimension of the coupling $k_p$

$$[k_p] = [L^{\frac{d_p - 4}{2}}], \quad [c_p] = [L^{-4}], \quad d_p := p + 1.$$  

(8)

The diff-invariant potential $V_{\text{Dir}}(l, g)$ encodes self interaction of the NG multiplet $l_{\mu\nu}^a$ in the gravitational background $g_{\mu\nu}(\xi^\rho)$

$$V_{\text{Dir}} = -\frac{1}{2} \text{Sp}(l_{\mu}^a l_{\nu}^b) \text{Sp}(l_{\mu}^a l_{\nu}^b) + \text{Sp}(l_{\mu}^a l_{\nu}^b l_{\rho}^a l_{\nu}^b) - \text{Sp}(l_{\mu}^a l_{\nu}^b l_{\rho}^a l_{\nu}^b) + c_p,$$

(9)

where $c_p$ is an integration constant. The Euler-Lagrange PDEs following from [7] have a unique solution that describes the fundamental branes provided that Eqs. (2-3) are chosen as the Cauchy initial data. The latter considered as the functions of the proper time $\tau$ turned out to be invariants of the evolution equations for of $l_{\mu\nu}^a, B_{\mu}^{ab}$ following from $S_{\text{Dir}}$ (see proof in Ref[14]).

The metric dynamics is described by Eqs. (1) treated as the evolution PDEs for $g_{\mu\nu}$. These equations are consistent with the used variational principle since they have selected $V_{\text{Dir}}$ which can be rewritten in the form

$$V_{\text{Dir}} = -\frac{1}{4} R_{\mu\nu\gamma}^a R^{\mu\nu\gamma} + \frac{1}{2} R_{\mu\nu} R^{\mu\nu} + \frac{1}{4} H_{\mu\nu} H^{\mu\nu} + c_p.$$  

(10)

Relation (10) was derived using Eqs.(1-2) and (6) resulting in the relations

$$\frac{1}{2} R_{\mu\nu\gamma} R^{\mu\nu\gamma} = \text{Sp}(l_{\mu}^a l_{\nu}^b) \text{Sp}(l_{\mu}^a l_{\nu}^b) - \text{Sp}(l_{\mu}^a l_{\nu}^b l_{\rho}^a l_{\nu}^b),$$

(11)

$$\frac{1}{2} H_{\mu\nu} H^{\mu\nu} = \text{Sp}(l_{\mu}^a l_{\nu}^b l_{\rho}^a l_{\nu}^b) - \text{Sp}(l_{\mu}^a l_{\nu}^b l_{\rho}^a l_{\nu}^b) + \text{Sp}(l_{\mu}^a l_{\nu}^b l_{\rho}^a l_{\nu}^b), \quad \text{Sp}(l_{\mu}^a l_{\nu}^b l_{\rho}^a l_{\nu}^b) = 0,$$

(12)
which were combined with the quadratic expressions for the Ricci tensor $R_{\mu\nu}$ and the scalar curvature $R$ of the minimal hypers $\Sigma_{\mu+1}^{\min}$

\[ R_{\mu\nu} = -(l^a l_a)_{\mu\nu}, \quad R = -Sp(l^a l_a). \quad (13) \]

The brane potential (10) contains the curvature squared terms similar to those in the action of string theory [22]. In the codimension 1, i.e. when $D = p + 2$, the field $B_{\mu\nu}^{ab} \equiv 0$ since $a = b = p + 1$ and (7) reduces to the action

\[ S_{D=p+2} = \frac{1}{k_p^2} \int d^{p+1}\xi \sqrt{|g|} \left( \frac{1}{2} \nabla_{\mu} l_{\nu\perp} \nabla_{\nu\perp} l_{\mu\perp} - \frac{1}{2} [Sp(l_{\perp\perp})]^2 + c_p \right), \quad (14) \]

where $p + 1$ is denoted as $\perp$ and the pure metric covariant derivative as $\nabla_{\mu}$

\[ \nabla_{\mu} l_{\nu\rho} := \partial_{\mu} l_{\nu\rho} - \Gamma_{\mu\nu\rho} l_{\lambda\rho} - \Gamma_{\mu\rho\lambda} l_{\nu\lambda}, \quad l_{\lambda\rho} = -l_{\lambda\rho\perp} \equiv -l_{\lambda\rho}. \quad (15) \]

Then, the quadratic terms in (14) reduce to one half of the squared curvature

\[ \frac{1}{2} [Sp(l_{\perp\perp})]^2 = \frac{1}{2} R^2 \quad (16) \]

in view of the relations (10,13). This result extends the statement [31] that the first leading correction to string action is quadratic in $R$, to the Dirac $p$-branes with codimensions equal to 1.

So, the discussed action $S_{\text{Dir}}$ encodes the terms describing $R^2$ gravity on the string/brane ws in the potential $V_{\text{Dir}}$ of the N-G fields $l^a_{\mu\nu}$.

### 3 Phases of brane matter

The considered approach maps the known Dirac brane action [3]

\[ S = T_p \int d^{p+1}\xi \sqrt{\det(\partial_{\mu} x \partial_{\nu} x)} \quad (17) \]

into the field action (17) consistent with Eqs. (13) and containing one coupling constant $k_p$ if $c_p = 0$. Then $k_p$ must be a function of the tension $T_p$.

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3For the open strings the term linear in $R$ contributes to the boundary conditions for the EOM uncovering a hidden topological structure of the action extremals [32].
Choosing the dimension \([x] = [\xi^\mu] = [L]\), we find that

\[
[T_p] = [L^{-d_p}], \quad [k_p] = [L^{\frac{d_p-4}{2d_p}}] = [T_p]^{\frac{4-d_p}{2d_p}}
\] (18)

using (8). Transition to the fields with the canonical dimension \([L^{\frac{2-d_p}{2}}]\)

\[\tilde{l}_\mu^a := k_p^{-1}l_\mu^a, \quad \tilde{B}_{\mu}^{ab} := k_p^{-1}B_{\mu}^{ab}\] (19)

shows that \(k_p\) coincides with the gauge coupling for the group \(SO(D-p-1)\):

\[
\nabla_\mu \tilde{l}_\nu^a = k_p^{-1}\nabla_\mu l_\nu^a \equiv \partial_\mu \tilde{l}_\nu^a - \Gamma_\mu^\lambda \tilde{l}_\lambda^a - \Gamma_\mu^\nu \tilde{l}_\lambda^a + k_p \tilde{B}_{\mu}^{ab} \tilde{l}_{\nu^b},
\]

\[
\tilde{H}_{\mu\nu}^{ab} = k_p^{-1}H_{\mu\nu}^{ab} \equiv (\partial_\mu \tilde{B}_{\nu}^b + k_p \tilde{B}_{[\mu}^{ab} \tilde{B}_{\nu]}^b)_{ab}
\] (20)

for \(D \geq p + 3\) when \(\tilde{B}_{\nu}^{ab} \neq 0\).

In this case, \(k_p\) squared is equal to the interaction coupling \(\lambda_p\) in \(V_{Dir}(\tilde{l})\)

\[
\lambda_p = k_p^2
\] (21)

In terms of the canonical fields, the action (17) takes the form

\[
S_{Dir} = \int d^{p+1}\xi \sqrt{|g|} \left\{ -\frac{1}{4} Sp(\tilde{H}_{\mu\nu} \tilde{H}^{\mu\nu}) + \frac{1}{2} \nabla_\mu \tilde{l}_\nu^a \nabla_\nu (\tilde{l}_{\nu^a}^\rho) - \nabla_\mu \tilde{l}_\nu^a \nabla_\nu (\tilde{l}_{\nu^a}^{\rho}) - k_p^2 \left[ -\frac{1}{2} Sp(\tilde{l}_{\mu\nu} \tilde{l}^\mu \tilde{l}^\nu) + Sp(\tilde{l}_{\mu\nu} \tilde{l}^\mu \tilde{l}^\nu) - Sp(\tilde{l}_{\mu\nu} \tilde{l}^\mu \tilde{l}^\nu) \right] + \Lambda_p \right\},
\] (22)

where \(\Lambda_p := c_p/k_p^2\) is a cosmological constant with the dimension \([\Lambda_p] = [L^{-d_p}] = [T_p]\) in view of (8). When \(\Lambda_p = 0\), dimensional analysis implies that

\[
k_p \sim T_p^{\frac{4-d_p}{2d_p}} \equiv T_p^{\frac{3-p}{2(p+1)}}, \quad \lambda_p \sim T_p^{\frac{3-p}{p+1}}.
\] (23)

By analogy with condensed matter physics, one can treat \(T_p = 0\) and \(\alpha_p\)

\[
\alpha_p := \frac{3-p}{2(p+1)}, \quad p = 1, 2, ..., D-1
\] (24)

as the critical temperature and exponent, respectively, depending on the brane dimension \(p\). The tensionless limit corresponds to super-Planckian energies when the Planck mass becomes negligible (see e.g. Refs. [33-38]).
The power law (23) shows that the behavior of the coupling constants as the functions of brane tensions is characterized by three different regimes defined by the sign of $\alpha_p$. They correspond to different phases of the multiplet fields describing the brane matter in (22). So, we observe the decrease of the constants in cases when $p < 3$ corresponding to strings and membranes
\[ k_1 \sim T_{1}^{\frac{1}{2}}, \quad \lambda_1 \sim T_{1}; \quad k_2 \sim T_{2}^{\frac{1}{2}}, \quad \lambda_2 \sim T_{2}^{\frac{1}{2}} \]
and their increase for the phases corresponding to $p$-branes with $p > 3$
\[ k_4 \sim T_{4}^{-\frac{1}{10}}, \quad \lambda_4 \sim T_{4}^{-\frac{1}{5}}, \quad k_5 \sim T_{5}^{-\frac{1}{6}}, \quad \lambda_5 \sim T_{5}^{-\frac{1}{3}}; \]
when $T_p$ decreases together with $k_\infty \sim T_{\infty}^{-\frac{1}{2}}, \quad \lambda_\infty \sim T_{\infty}^{-1}$ for $D, p \to \infty$.

The dependence of the metric and YM curvatures of the hyper-ws on $k_p$ follows from Eqs. (1-3) rewritten in terms of the canonical fields
\[ R_{\mu\nu}^{\gamma\lambda} = k_p^{\frac{2}{p}} l_{[\mu}^{\gamma} l_{\nu]}^{\lambda} \]
(25)
\[ \tilde{H}_{\mu
u}^{ab} = k_\mu^{\frac{1}{p}} l_{[\mu}^{\gamma} l_{\nu]}^{b} \]
(26)
\[ \nabla_{[\mu} l_{\nu]}^{\perp\rho a} = 0. \]
(27)
These eqs. show that in super-Planckian region, the string and membrane curvatures go to zero that corresponds to their inflation. On the contrary, the curvature of branes with $p > 3$ goes to infinity that could be treated as their collapse if their rotations do not compensate their large tensions.

Thus, at high energies the regimes of confinement ($k_{3,4,\ldots} \to \infty$) or asymptotic freedom ($k_1, k_2 \to 0$) in the field model (22) correspond to brane matter phases describing its collapse or inflation depending on the value of $p$.

In the low-energy limit, when $T_p \to \infty$, the curvatures go to infinity for $p = 1, 2$ and to zero for $p > 3$ that could be treated as the collapse of non-rotating strings, membranes contrary to an inflation of the rotating branes with $p = 4, 5, \ldots, D - 2$.

Finally, the field regime when $\alpha_p = 0$ emerges at $p = 3$. In this exclusive case the coupling constant $k_3$ in (22) is dimensionless and independent of $T_p$. Since in our analysis the constant $\Lambda_3$ with the dimension $[L^{-4}]$ vanishes the action (22) with $p = 3$ becomes invariant under the global Weyl transformations
\[ g'_{\mu\nu}(\xi) = \rho g_{\mu\nu}(\xi), \quad \tilde{l}'_{\mu\nu}(\xi) = \rho^{1/2} \tilde{l}_{\mu\nu}(\xi), \quad \tilde{B}'_\mu^{ab}(\xi) = \tilde{B}_\mu^{ab}(\xi). \]
(28)
So, we come into contact with the well-known conformal invariant theories of gravity (see Ref. [39] and references therein).
4 Summary

The correspondence between fundamental p-branes, sweeping minimal hyper-surfaces in D-dimensional Minkowski space, and their SO($D - p - 1$) gauge ws multiplets is studied. It is shown that the interaction potential of the Nambu-Goldstone multiplet encodes the ws action of $R^2$ gravity. Based on dimensional analysis of the ws action (22) with zero cosmological constant, we obtain the power law $k_p \sim T_p^{3-p/(p+1)}$ for the coupling constant as the function of the p-brane tension $T_p$. This points to possible existence of brane matter phases interpreted as asymptotic freedom and confinement phases. Their connection with collapse and inflation of p-brane hyper-ws is discussed. The phase corresponding to $p = 3$ describes a scale-invariant model of $R^2$ gravity on a three-brane hyper-ws. These observations could be applied in cosmology. Indeed, action (22) proposes a (p+1)-dimensional field model unifying the SO($D - p - 1$) YM and NG multiplets with gravity. Then the extended object - a fundamental p-brane - emerges as the solution of the Cauchy-Kowalewskaya problem for the EOM. Consideration of the brane hyper-ws as a p-braneworld model for the space-time similarly to [40] could explain some of the open questions. On the other hand, one can consider p-branes as dynamical objects filling our universe and forming domains with various dimensions $p = 1, 2, 3, 4, .., D - 2$. Coalescence and fragmentation of the domains during the evolution could change their dimensions and trigger the above discussed phase transitions in brane matter, respectively.

Treatment of these processes demands consideration of the interaction between the branes. This interaction can be realized using the Kalb-Ramond action-at-a-distance Lagrangian and the corresponding invariant action for interacting strings [41] generalized to the case of interacting p-branes [42]. The interaction of p-branes is mediated by the gauge field presented by a completely antisymmetric tensor $B_{m_1m_2...m_p}$ of the rank (p+1) [43] generalizing the Kalb-Ramond gauge boson $B_{mn}$.

The discussed geometric approach can be extended to include fermionic fields using the ideas of supersymmetry and supergravity. This way implies extension of the Minkowski space by the Grassmannian spinor coordinates $\theta^\alpha$ generating fermionic degrees of freedom. The resulting flat superspace formed by the coordinates $(x^m, \theta^\alpha)$ is invariant under the global super-Poincare group. Supersymmetric field theories formulated in the superspace become superfield theories. Localization of the super-Poincare group yields
the gravity theory including the fermionic Rarita-Schwinger gauge field carrying spin $3/2$. General relativity reformulated by Cartan using the exterior differential forms as the theory on a group manifold was extended to arbitrary superspaces. The generalized Maurer-Cartan structure equations were constructed \[44\], \[45\]. The Generalized Action Principle (GAP) together with the rheonomy principle formulated in Ref. \[45\] allowed to extend a component formalism of supergravity on the superspace. Application of the GAP complemented by the new principle, called the rheotropy, allowed to formulate the procedure of minimal embedding of super hyper-surfaces of superstrings and super p-branes into the Minkowski target superspace (see Ref. \[46\] and references therein). This development of the Cartan’s approach to the differential geometry of embedded super hyper-surfaces gives tools for construction of a supersymmetric generalization of the bosonic action $S_{Dir}$. A generalization of the Wheeler-Feynman electromagnetic action-at-a-distance theory onto superspace was considered in Ref. \[47\]. The classical vector and spinor fields belonging to the Maxwell supermultiplet were built from the worldline coordinates of the charged and neutral particles in super-space. We believe that extension of these results to superstrings and super p-branes will make it possible to include the fermionic DOF in the processes of coalescence and fragmentation of super p-branes.

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