On Coarse Spectral Geometry in Even Dimension

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Abstract

Let $\sigma$ be the involution of the Roe algebra $C^*|\mathbb{R}|$ which is induced from the reflection $\mathbb{R} \to \mathbb{R}; \ x \mapsto -x$. A graded Fredholm module over a separable $C^*$-algebra $A$ gives rise to a homomorphism $\tilde{\rho} : A \to C^*|\mathbb{R}|^\sigma$ to the fixed-point subalgebra. We use this observation to give an even-dimensional analogue of a result of Roe. Namely, we show that the $K$-theory of this symmetric Roe algebra is $K_0(C^*|\mathbb{R}|^\sigma) \cong \mathbb{Z}$, $K_1(C^*|\mathbb{R}|^\sigma) = 0$, and that the induced map $\tilde{\rho}_* : K_0(A) \to \mathbb{Z}$ on $K$-theory gives the index pairing of $K$-homology with $K$-theory.

1 Introduction

In [Roe97], Roe observed that a Dirac operator $D$ on an odd-dimensional closed manifold $M$ gives rise to a $C^*$-algebra homomorphism

$$\tilde{\rho} : C(M) \to C^*|\mathbb{R}|$$

from the continuous functions on $M$ to the Roe algebra of the real line $\mathbb{R}$. The space $\mathbb{R}$ appears because, up to coarse equivalence, it is the spectrum of the self-adjoint operator $D$. The $K$-theory of $C^*|\mathbb{R}|$ is

$$K_n(C^*|\mathbb{R}|) \cong \begin{cases} 0, & n = 0, \\ \mathbb{Z}, & n = 1, \end{cases}$$

and the map

$$\tilde{\rho}_* : K_1(C(M)) \to K_1(C^*|\mathbb{R}|) \cong \mathbb{Z}$$

agrees with the index pairing of $K$-theory with the $K$-homology class $[D] \in K_1(M)$.

This point of view was extensively developed by Luu ([Luu05]), who showed that analytic $K$-homology can be reformulated entirely in the language of coarse spectral geometry. Specifically, let $A$ be a separable $C^*$-algebra. Luu defined groups $KC^n(A, \mathbb{C})$ whose cycles are $\ast$-homomorphisms $\rho : A \to C^*|\mathbb{R}^n|$ and

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1 The most natural coarse structure on $\mathbb{R}^n$ here is the topologically controlled coarse structure associated to the compactification of $\mathbb{R}^n$ by a sphere at infinity. (See [Roe03] for the definition.) If $A$ is separable, it turns out to be equivalent to use the standard metric coarse structure on $\mathbb{R}^n$, although the construction becomes somewhat more technical. The $K$-theory of $C^*|\mathbb{R}^n|$ is the same in either case.
then proved that $KC^n(A, C) \cong KK^n(A, C)$. In fact, Luu worked with an arbitrary ($\sigma$-unital) coefficient algebra $B$, to produce groups $KC^n(A, B)$ isomorphic to $KK^n(A, B)$. We choose not to work in that generality here.

Luu’s picture of $K$-homology is aesthetically very pleasing. The price of this elegance, however, is some computational complexity in even dimensions. The isomorphism of $KK$ and $KC$ in even dimension is achieved via a map $KK^0(A, C) \to KC^2(A, C)$ which requires as input a balanced Fredholm module, i.e. a graded Fredholm module of the form $(H = H_0 \oplus H_0, \rho = \rho_0 \oplus \phi_0, F = (g_0 U_0^*)$ for some Hilbert space $H_0$, representation $\rho_0$ and Fredholm operator $U : H_0 \to H_0$. While every $K^0$-class can be represented by a balanced Fredholm module, the process of “balancing” is quite heavy-handed. For instance, given a Dirac operator on an even dimensional manifold, the Hilbert space of the associated balanced Fredholm module is an infinite direct sum of $L^2$-sections of the spinor bundle. (See [HR00, Proposition 8.3.12].) The relationship between the spectrum of $U$ and that of the original operator $D$ is not obvious.

In this paper, we describe an alternative approach to controlled spectral geometry in even dimension which is more convenient for geometric applications. Let $s : \mathbb{R} \to \mathbb{R}$ denote the reflection through the origin. This induces a $*$-involution $\sigma$ of the Roe algebra $C^*|\mathbb{R}|$ (see Section 3). Given a graded Fredholm module $(H, \rho, D)$ for $A$, Roe’s construction in fact produces a $*$-homomorphism $\tilde{\rho} : A \to C^*|\mathbb{R}|^\sigma$ into the fixed-point algebra of $\sigma$. Our main result is the following.

**Theorem 1.1.** The $K$-theory of the symmetric Roe algebra is

$$K_n(C^*|\mathbb{R}|^\sigma) \cong \begin{cases} \mathbb{Z}, & n = 0, \\ 0, & n = 1, \end{cases}$$

and the induced map

$$\tilde{\rho}_* : K_0(A) \to K_0(C^*|\mathbb{R}|^\sigma) \cong \mathbb{Z}$$

agrees with the index pairing of $[(H, \rho, D)] \in K^0(A)$ with $K$-theory.

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## 2 Preliminaries: The Roe algebra $C^*|\mathbb{R}|$

We shall use $|\mathbb{R}|$ to denote the real line equipped with the topological coarse structure induced from the two-point compactification $\overline{\mathbb{R}} := \mathbb{R} \cup \{\pm \infty\}$. Thus, a set $E \subseteq \mathbb{R} \times \mathbb{R}$ is controlled if for any sequence $(x_n, y_n) \in E$, $x_n \to \infty$ (resp. $-\infty$) if and only if $y_n \to -\infty$ (resp. $-\infty$).

We shall refer to a Hilbert space $H$ equipped with a nondegenerate representation $m : C_0(\mathbb{R}) \to B(H)$ as a geometric $\mathbb{R}$-Hilbert space. By the spectral theorem, $m$ extends naturally to the algebra of Borel functions $B(\mathbb{R})$. We shall
typically suppress mention of $m$ in the notation. We use $\chi_Y$ to denote the characteristic function of a subset $Y \subset \mathbb{R}$.

An operator $T \in \mathcal{B}(H)$ is **locally compact** if $fT, Tf \in \mathcal{K}(H)$ for all $f \in C_0(\mathbb{R})$. It is **controlled** (for the above topological coarse structure) if for all $R \in \mathbb{R}$ there exists $S \in \mathbb{R}$ such that

$$\chi_{(-\infty,R]} T \chi_{[S,\infty]} = 0, \quad \chi_{[S,\infty)} T \chi_{(-\infty,R]} = 0,$$

$$\chi_{(-R,\infty]} T \chi_{(-\infty,-S]} = 0, \quad \chi_{(-\infty,-S]} T \chi_{(-R,\infty]} = 0.$$

One defines $C^*((\mathbb{R}; H))$ as the norm-closure of the locally compact and controlled operators on $H$. This $C^*$-algebra is independent of the choice of $H$ as long as $H$ is **ample**, i.e. $m(f)$ is noncompact for all nonzero $f \in C_0(\mathbb{R})$. In that case, the algebra is referred to as the **Roe algebra** $C^*|\mathbb{R}|$.

The following standard facts are easy consequences of the definitions. The reader familiar with Roe algebras may prefer to recognize them as consequences of the coarsely excisive decomposition $\mathbb{R} = (-\infty,0] \cup [0,\infty)$, where we note that the ideal $C^*|\mathbb{R}|(\{0\}; H)$ associated to the inclusion of a point into $\mathbb{R}$ is just the compact operators. (See [HRY93],[HPR97].)

**Lemma 2.1.** Let $T \in C^*((\mathbb{R}; H))$. For any $R_1, R_2 \in \mathbb{R}$,

(i) $\chi_{(-\infty,R_1]} T \chi_{[R_2,\infty)}$ and $\chi_{[R_2,\infty)} T \chi_{(-\infty,R_1]}$ are compact operators.

(ii) $[T, \chi_{(-\infty,R_1]}]$ and $[T, \chi_{[R_2,\infty)}]$ are compact operators.

### 3 Graded Fredholm modules and the symmetric Roe algebra

In what follows, we shall use the unbounded (‘Baaj-Julg’) picture of $K$-homology. This is a purely aesthetic choice—see Remark 3.3 for the construction using bounded Fredholm modules.

Let $A$ be a $C^*$-algebra, and let $(H,\rho,D)$ be a graded unbounded Fredholm module for $A$, i.e. $H$ is a $\mathbb{Z}/2\mathbb{Z}$-graded Hilbert space, $\rho$ is a representation of $A$ by even operators on $H$, and $D$ is an odd self-adjoint unbounded operator on $H$ such that

(1) for all $a \in A$, $(1 + D^2)^{-\frac{1}{2}} \rho(a)$ extends to a compact operator,

(2) for a dense set of $a \in A$, $[D,\rho(a)]$ is densely defined and extends to a bounded operator.

Let $\gamma_{ev}, \gamma_{od}$ denote the projections onto the even and odd components of $H$, and $\gamma = \gamma_{ev} - \gamma_{od}$ be the grading operator. Let $\sigma$ be the involution of $\mathcal{B}(H)$ defined by $\sigma : T \mapsto \gamma T \gamma$.

Functional calculus on the operator $D$ provides $H$ with a geometric $\mathbb{R}$ structure, namely $m : B(\mathbb{R}) \to \mathcal{B}(H); \ f \mapsto f(D)$. For any $f \in C_0(\mathbb{R})$,

$$\sigma(m(f)) = f(\gamma D \gamma) = f(-D) = m(f \circ s),$$

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where \( s : \mathbb{R} \to \mathbb{R} \) is the reflection in the origin. In coarse language, \( \gamma \) is a covering isometry for \( s \). It follows that \( \sigma \) restricts to an involution of \( C^*([\mathbb{R}]; H) \). The subalgebra fixed by \( \sigma \) will be denoted \( C^*([\mathbb{R}]; H)^\sigma \).

Taking this symmetry into account gives an immediate strengthening of Roe’s construction for ungraded Fredholm modules.

**Proposition 3.1.** The image of \( \rho \) lies in \( C^*([\mathbb{R}]; H)^\sigma \).

**Proof.** The function \( f(x) = x(1+x^2)^{-\frac{1}{2}} \) generates \( C([\mathbb{R}]) \), and the ideal generated by \( g(x) = (1 + x^2)^{-\frac{1}{2}} \) is \( C_0(\mathbb{R}) \). Using [HR00, Theorem 6.5.1], Properties (1) and (2) above imply that \( \rho(a) \in C^*([\mathbb{R}]; H) \) for any \( a \in A \). Since \( \rho(a) \) is even, \( \sigma(a) = \gamma \rho(a) \gamma = \rho(a) \). \( \square \)

This geometric \( \mathbb{R} \)-Hilbert space \( H \) is not typically ample. However, one can always embed \( H \) into an ample geometric \( \mathbb{R} \)-Hilbert space. For specificity, let us put \( \mathcal{H} := H \oplus L^2(\mathbb{R}) \), where \( L^2(\mathbb{R}) \) has its natural geometric \( \mathbb{R} \)-structure. Extension of operators by zero gives an inclusion \( \iota : C^*([\mathbb{R}]; H) \to C^*|\mathcal{H}| \). Put \( \hat{\rho} = \iota \circ \rho : A \to C^*|\mathcal{H}| \).

The symmetry \( g \mapsto g \circ s \) defines a grading operator on \( L^2(\mathbb{R}) \). We shall reuse \( \gamma \) to denote the total grading operator on \( \mathcal{H} \). Likewise, we use \( \sigma \) to denote conjugation by \( \gamma \) in \( \mathcal{B}(\mathcal{H}) \). Then \( \hat{\rho} \) has image in \( C^*|\mathcal{H}|^\sigma \).

**Remark 3.2.** In the above, we have employed a specific choice of symmetry \( \sigma \in \text{Aut}(C^*|\mathbb{R}|) \) associated to the reflection \( s \) of \( \mathbb{R} \). For the expert concerned about the uniqueness of this definition, we supply some brief comments without proof. They shall not be needed in what follows.

Let \( \mathcal{H} \) be any ample geometric \( \mathbb{R} \)-Hilbert space. By [Lau05, Prop. 2.2.11(iii)] (following [HRY93]), there exists a unitary \( \gamma : \mathcal{H} \to \mathcal{H} \) which covers \( s \), in the sense that \( (1 \times s)(\text{Supp}(\gamma)) \subseteq \mathbb{R} \times \mathbb{R} \) is a controlled set. By carrying out the proof of this fact in a way that maintains the reflective symmetry, one can ensure that \( \gamma \) is involutive, \( \gamma^2 = 1 \). Then \( \sigma : T \to \gamma T \gamma \) is an involution of \( C^*|\mathcal{H}| \). If \( \gamma' \) is another involutive covering isometry for \( s \), then there is a controlled unitary \( V \in \mathcal{B}(\mathcal{H}) \) such that \( \gamma' = V \gamma \) ([Lau05, Prop. 2.2.11(iv)] following [HRY93]). If \( \sigma' \) is conjugation by \( \gamma' \), then \( C^*|\mathcal{H}|^\sigma' = VC^*|\mathcal{H}|V^* \). Thus the symmetric Roe algebra \( C^*|\mathcal{H}|^\sigma \) is unique up to controlled unitary equivalence.

**Remark 3.3.** The bounded Fredholm module corresponding to \( (H, \rho, D) \) is \( (H, \rho, F := D(1 + D^2)^{-\frac{1}{2}}) \). The map \( \phi : x \mapsto x(1 + x^2)^{-\frac{1}{2}} \) defines a coarse equivalence from \( |\mathbb{R}| \) to the interval \( |(-1,1)| \), with topological coarse structure associated to its two-point compactification \([-1,1]\). Thus, the bounded picture of \( K \)-homology provides a morphism \( \rho : A \to C^*|(-1,1)| \cong C^*|\mathbb{R}| \).

## 4 K-theory of the symmetric Roe algebra \( C^*|\mathbb{R}|^\sigma \)

**Proposition 4.1.** The \( K \)-theory of \( C^*|\mathbb{R}|^\sigma \) is

\[
K_\bullet(C^*|\mathbb{R}|^\sigma) \cong \begin{cases} 
\mathbb{Z}, & \bullet = 0, \\
0, & \bullet = 1.
\end{cases}
\]
Moreover, $K_0(C^*|R|)^\sigma$ is generated by finite rank projections $p \in M_n(C^*|R|)^\sigma$, and for such projections, the map to $Z$ is given by

$$[p] \mapsto \dim \mathcal{H}_{ev} - \dim \mathcal{H}_{od}.$$  

We use a Mayer-Vietoris type argument (cf. [HRY93]). Put $Y_+ := [1, \infty)$, $Y_- := (-\infty, -1]$, with their coarse structures inherited from $|R|$. We will abbreviate $\chi_{Y\pm}$ as $\chi\pm$. Since $\mathcal{H}_+ := \chi_+ \mathcal{H}$ is an ample geometric $Y_+$-Hilbert space, we can define the Roe algebra $C^*|Y_+|$ as the corner algebra $C^*(|Y_+|; \mathcal{H}_+) = \chi_+ C^*|R|\chi_+$. Likewise for $C^*|Y_-|$.

Note that $\sigma(\chi\pm) = \chi\pm$, so that $\sigma$ interchanges $C^*|Y_+|$ and $C^*|Y_-|$. Since $\chi_+ \chi_- = 0$, the symmetrization map $(I + \sigma) : T \mapsto T + \sigma(T)$ is a $\ast$-homomorphism from $C^*|Y_+|$ into $C^*|R|^{\sigma}$. We obtain a morphism of short-exact sequences,

$$0 \longrightarrow \mathcal{K}(|Y_+|) \longrightarrow C^*|Y_+| \longrightarrow C^*|Y_+|/\mathcal{K}(|Y_+|) \longrightarrow 0$$

for such projections. The map to $Z$ is given by

$$[p] \mapsto \dim \mathcal{H}_{ev} - \dim \mathcal{H}_{od}.$$  

Lemma 4.2. The right-hand map $(I + \sigma) : C^*|Y_+|/\mathcal{K}(|Y_+|) \rightarrow C^*|R|^{\sigma}/\mathcal{K}(|Y_+|)$ is an isomorphism.

Proof. Let $\psi : C^*|R|^{\sigma} \rightarrow C^*|Y_+|$ denote the cut-down map $T \mapsto \chi_+ T \chi_+$. By using Lemma 2.1(ii), $\psi$ is a homomorphism modulo compacts, so it descends to a homomorphism $\psi : C^*|R|^{\sigma}/\mathcal{K}(|Y_+|) \rightarrow C^*|Y_+|/\mathcal{K}(|Y_+|)$. By Lemma 2.1(i), for any $T \in C^*|R|^{\sigma}$ we have

$$T \equiv \chi_+ T \chi_+ + \chi_- T \chi_- \mod \mathcal{K}(|Y_+|),$$

so that $\psi$ is inverse to $(I + \sigma)$. \hfill \square

Put $\mathcal{H}_{ev} := \gamma_{ev} \mathcal{H}$, $\mathcal{H}_{od} := \gamma_{od} \mathcal{H}$.

Lemma 4.3. We have $\mathcal{K}(|Y_+|) \cong \mathcal{K}(|Y_+|) \oplus \mathcal{K}(|Y_+|)$ via $T \mapsto T_{\gamma_{ev}} \oplus T_{\gamma_{od}}$. In particular, $K_0(\mathcal{K}(|Y_+|)) \cong \mathbb{Z} \oplus \mathbb{Z}$ via the map which sends the class of a projection $p$ to $(\dim (p \mathcal{H}_{ev}), \dim (p \mathcal{H}_{od}))$.

Proof. Note that any $T \in C^*|R|^{\sigma}$ commutes with $\gamma$, so $T \mapsto T_{\gamma_{ev}} \oplus T_{\gamma_{od}}$ is indeed a homomorphism. The inverse homomorphism is $T_1 \oplus T_2 \mapsto T_1 + T_2$. \hfill \square

Lemma 4.4. Under the identifications $K_0(\mathcal{K}(|Y_+|)) \cong \mathbb{Z}$ and $K_0(\mathcal{K}(|Y_+|)) \cong \mathbb{Z} \oplus \mathbb{Z}$, the map $(I + \sigma)_*$ is $n \mapsto (n, n)$.

Proof. Let $p$ be a projection in $\mathcal{K}(|Y_+|)$. Then $p = \chi_{Y_+} p \chi_{Y_+}$, so $p^\gamma = \chi_{Y_+} p^\gamma \chi_{Y_+}$, and hence $\text{Tr}(p^\gamma) = 0$. Since $\gamma_{ev/od} = \frac{1}{2}(1 \pm \gamma)$, $\text{Tr}(p \gamma_{ev}) = \text{Tr}(p \gamma_{od}) = \frac{1}{2} \text{Tr}(p)$. Similarly, $\text{Tr}(p \gamma_{ev}) = \frac{1}{2} \text{Tr}(p \gamma_{od}) = \frac{1}{2} \text{Tr}(p)$. Hence, $\text{Tr}((I + \sigma)(p \gamma_{ev})) = \text{Tr}((I + \sigma)(p \gamma_{od})) = \text{Tr}(p)$, and the result follows from the previous lemma. \hfill \square
By [Roe96, Proposition 9.4], \( C^*|Y_+ | \) has trivial \( K \)-theory. The boundary maps in \( K \)-theory induced from the diagram (4.1) give

\[
\begin{array}{ccc}
K_1(C^*|Y_+ |/\mathcal{K}(\mathcal{H}_+)) & \overset{\partial}{\rightarrow} & K_0(\mathcal{K}(\mathcal{H}_+)) \cong \mathbb{Z} \\
& (I + \sigma) & \\
K_1(C^*|\mathbb{R}^\sigma/\mathcal{K}(\mathcal{H})^\sigma) & \overset{\partial}{\rightarrow} & K_0(\mathcal{K}(\mathcal{H})^\sigma) \cong \mathbb{Z} \oplus \mathbb{Z} \\
& (I + \sigma) & \\
\end{array}
\tag{4.2}
\]

We see that \( K_1(C^*|\mathbb{R}^\sigma/\mathcal{K}(\mathcal{H})^\sigma) \cong \mathbb{Z}, \) and the image of its boundary map into \( K_0(\mathcal{K}(\mathcal{H})^\sigma) \) is \( \{(n, n) \mid n \in \mathbb{Z}\}. \) The corresponding diagram in the other degree gives \( K_0(\mathcal{K}(\mathcal{H})^\sigma) \cong 0. \)

Now the six-term exact sequence associated to the bottom row of (4.1) becomes

\[
\begin{array}{cccc}
(n, n) & \mathbb{Z} \oplus \mathbb{Z} & K_0(C^*|\mathbb{R}^\sigma) & 0 \\
\downarrow n & & \downarrow & \\
\mathbb{Z} & K_1(C^*|\mathbb{R}^\sigma) & 0
\end{array}
\]

Thus, \( K_0(C^*|\mathbb{R}^\sigma) \cong \mathbb{Z} \) and \( K_1(C^*|\mathbb{R}^\sigma) \cong 0. \) With an appropriate choice of sign, top-left horizontal map is given by \((m, n) \mapsto m - n.\) Applying Lemma 4.3 this completes the proof of (4.1).

5 The index pairing

Let \( \theta \in K^0(A) \) be the \( K \)-homology class of a graded unbounded Fredholm module \((H, \rho, D), \) and put \( F := D(1 + D^2)^{-\frac{1}{2}}. \) Let \( p \) be a projection in \( M_n(A). \)

The index pairing \( K^0(A) \times K_0(A) \to \mathbb{Z} \) is given by

\[
(\theta, [p]) := \text{Index} \left[ \rho(p)(F \otimes I_n)\rho(p) : \rho(p)\mathcal{H}^n_{ev} \to \rho(p)\mathcal{H}^n_{od} \right],
\]

(where \( I_n \) denotes the identity in \( M_n(\mathbb{C})\).)

Let \( P = \tilde{\rho}(p) \in M_n(C^*|\mathbb{R}^\sigma) \), and let \( f \) denote the function \( f(x) = x(1 + x^2)^{-\frac{1}{2}} \), as represented on the geometric \( |\mathbb{R}| \)-Hilbert space \( \mathcal{H}. \) Then

\[
(\theta, [p]) = \text{Index}(P(f \otimes I_n)P : P\mathcal{H}^n_{ev} \to P\mathcal{H}^n_{od}).
\]

The right-hand side here depends only on the class of \( P \) in \( K_0(C^*|\mathbb{R}^\sigma) \). By Proposition 4.1 we may therefore replace \( P \) by a finite rank projection \( Q, \) and the index is

\[
(\theta, [p]) = \text{Index}(Q(f \otimes I_n)Q : Q\mathcal{H}^n_{ev} \to Q\mathcal{H}^n_{od}) = \dim(Q\mathcal{H}^n_{ev}) - \dim(Q\mathcal{H}^n_{od}) = [Q] = \rho_*[p].
\]

This completes the proof of Theorem 1.1.
Remark 5.1. Given the above results, it is natural to expect a reformulation of $KK^0(A,\mathbb{C})$ in the spirit of Luu. Indeed, one can define a group $KC^0_\sigma(A,\mathbb{C})$ as follows. Cycles are morphisms from $A$ into the symmetric Roe algebra $C^*|\mathbb{R}|^\sigma$. Equivalence of cycles is generated by controlled unitary equivalences (preserving the involution $\gamma$) and weak homotopies (respecting the symmetry $\sigma$). Then $KC^0_\sigma(A,\mathbb{C}) \cong KK^0(A,\mathbb{C})$. We shall not develop this in detail here, as the results follow [Luu05] closely.

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