This article is devoted to the 100th birthday of my teacher Isaak Markovich Khalatnikov

Low-Temperature Transport in Metals without Inversion Centre

V.P.Mineev$^{1,2,*}$

$^1$Univ. Grenoble Alpes, CEA, IRIG, PHELIQS, F-38000 Grenoble, France
$^2$Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia

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Theory of low temperature kinetic phenomena in metals without inversion center is developed. Kinetic properties of a metal without inversion center are described by four kinetic equations for the diagonal (intra-band) and the off-diagonal (inter-band) elements of matrix distribution function of electrons occupying the states in two bands split by the spin-orbit interaction. The derivation of collision integrals for electron-impurity scatterings and for electron-electron scatterings in a non-centrosymmetric medium is given. Charge, spin and heat transport in the ballisitic and the weak impurity scattering regimes is discussed. It is shown that in 3D media the off-diagonal terms give rise the contribution in charge, spin and heat flows not only due to the interband scattering but also in the collisionless case where they bring to birth the unusual dissipative currents. The zero-temperature residual resistivity and the residual thermal resistivity are determined by scattering on impurities as well as by the electron-electron scattering.

I. INTRODUCTION

During the last decades or so, there has been great interest to the spin-based electronics in application to the systems where spin-orbit coupling plays an important role. In particular there were studied the kinetic properties of two-dimensional semiconductors with broken space parity characterized by both Rashba and Dresselhaus interaction[1–4]. Spin-orbit interaction of electrons with a non-centrosymmetric crystal lattice lifts spin degeneracy of electron states. Each band filled by twice degenerate electron states splits on two bands filled by the electron states with different momenta at the same energy. Usually scalar electron energy and the Fermi distribution function are given now by the matrices $E = \varepsilon_{\sigma\sigma'}$, $\hat{n} = n_{\sigma\sigma'}$ in respect of spin indices. The distribution matrix time variation in the $(k, r)$ space is determined by the quasi-classic kinetic equation that was derived by V.P.Silin[5] and has the following form

$$\frac{\partial \hat{n}}{\partial t} + \frac{1}{2} \left( \frac{\partial \hat{\varepsilon}}{\partial k} \frac{\partial \hat{n}}{\partial r} + \frac{\partial \hat{n}}{\partial k} \frac{\partial \hat{\varepsilon}}{\partial r} \right) - \frac{1}{2} \left( \frac{\partial \hat{\varepsilon}}{\partial r} \frac{\partial \hat{n}}{\partial k} + \frac{\partial \hat{n}}{\partial k} \frac{\partial \hat{\varepsilon}}{\partial r} \right) - i[\hat{\varepsilon}, \hat{n}] = \hat{I}_{st},$$

(1)

where $[\hat{\varepsilon}, \hat{n}]$ is the commutator of $\hat{\varepsilon} = \hat{\varepsilon}(k, r)$ and $\hat{n} = \hat{n}(k, r)$. We put $\hbar = 1$. The collision integral in the rhs determines the relaxation processes.

It is quite natural to rewrite kinetic equation in the band representation where the Hamiltonian is diagonal. It seems that after this transformation we come to kinetic equations for distribution functions in each band which interact each other due to collision integrals including inter-band scattering. So, the theory seems to be similar to the kinetic theory of a two band metal with center of inversion. However, this is not the case. Kinetic processes in a non-centrosymmetric medium are described by four kinetic equations for the diagonal (intra-band) and the off-diagonal (inter-band) elements of matrix distribution function of electrons occupying the states in two bands split by the spin-orbit interaction. The off-diagonal terms give rise the contribution in the transport properties even in the collisionless regime. In the paper is shown that in 3D non-centrosymmetric medium an electric field not only accelerates electrons, but also transfers an energy to the electron gas creating non-equilibrium off-diagonal electron distributions. This unusual phenomenon is completely specific for the 3D materials without inversion center.

The expression for the electron-impurity collision integral in non-centrosymmetric semiconductors or metals one can find in the papers[6, 7]. The authors do not derive the collision integral but write: ”the collision term was derived in many papers” and give the corresponding references. These references, however, do not contain a derivation of collision integral. The derivation of the collision integrals for the electron-impurity collisions as well as for electron-electron collisions in a non-centrosymmetric medium is given in the present article. Along with scattering on impurities the electron-electron collisions in non-centrosymmetric medium leads to the zero-temperature residual resistivity and residual thermal conductivity.

The paper is organized as follows. Section II contains the basic notions of the electron energy spectrum and the equilibrium distribution in metals without inversion. In the Section III there are presented the system of kinetic equations and derived the expressions for electric current, spin current and heat current. For each type of current

*E-mail: vladimir.mineev@cea.fr
the collisionless regime and the weak impurity scattering case are examined. This is shown that in 3D media the off-diagonal terms give rise the contribution in charge, spin and heat transport not only due to interband scattering but also in the collisionless case where they bring to birth the unusual dissipative currents.

The role of electron-electron scattering in formation of zero-temperature residual resistivity and residual thermal conductivity is discussed in the Section IV. In the Conclusion there are enumerated the principal results of the paper. The derivations of collision integral for the electron scattering on scalar impurities as well as for electron-electron scattering are given in the Appendices A and B. The results are derived in application to a medium without inversion center both in two and three dimensional case. The principal distinctions between the kinetic properties of 2D and 3D materials are emphasized.

II. ELECTRONIC STATES IN NON-CENTROSYMMETRIC METALS

The spectrum of noninteracting electrons in a metal without inversion center is:

\[ \tilde{\epsilon}(k) = \epsilon(k) \delta + \gamma(k) \cdot \sigma, \quad (2) \]

where \( \epsilon(k) \) denotes the spin-independent part of the spectrum, \( \delta \) is the unit 2 \( \times \) 2 matrix in the spin space, \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) are the Pauli matrices. The second term in Eq. (2) describes the spin-orbit coupling whose form depends on the specific noncentrosymmetric crystal structure. The pseudovector \( \gamma(k) \) satisfies \( \gamma(-k) = -\gamma(k) \) and \( g \gamma(g^{-1}k) = \gamma(k) \), where \( g \) is any symmetry operation in the point group \( G \) of the crystal. A more detailed theoretical description of noncentrosymmetric metals in normal and in superconducting state is presented in the paper [8]. The tetragonal point group \( G = C_{4v} \), relevant for CePt_3Si, CeRhSi_3 and CeIrSi_3, yields the antisymmetric spin-orbit coupling

\[ \gamma(k) = \gamma(k_x \hat{x} - k_y \hat{y}) + \gamma_{\|} k_x k_y (k_x^2 - k_y^2) \hat{z}. \quad (3) \]

In the purely two-dimensional case, setting \( \gamma_{\|} = 0 \) one recovers the Rashba interaction [4] which is often used to describe the effects of the absence of mirror symmetry in semiconductor quantum wells. The case of isotropic spectrum when \( \epsilon(k) = \frac{k^2}{2m} \) and

\[ \gamma(k) = \gamma k \quad (4) \]

is compatible with the 3D cubic crystal symmetry. Here \( \gamma \) is a constant.

The eigenvalues and eigenfunctions of the matrix (2) are

\[ \epsilon_{\pm}(k) = \epsilon(k) \pm |\gamma(k)|, \quad (5) \]

For the isotropic case near the corresponding Fermi surfaces the dispersion laws have the particular simple form

\[ \xi_{\pm} = \epsilon_{\pm} - \mu \approx v_F (k - k_F \pm) = \epsilon - \mu_{\pm}, \quad (13) \]

with

\[ \epsilon = v_F k, \quad \mu_{\pm} = v_F k_F \pm, \quad \mu_+ - \mu_- = -2mv_F \gamma. \quad (14) \]

Here, \( \gamma_{k_x}, \gamma_{k_y}, \gamma_{k_z} \) are the components of the unit vector \( \gamma(k)/|\gamma(k)| \). The eigen functions obey the orthogonality conditions

\[ \Psi_{\sigma}^{\alpha\beta}(k) \Psi_{\sigma}^{\beta\alpha}(k) = \delta_{\alpha\beta}, \quad \Psi_{\sigma \alpha}(k) \Psi_{\sigma \beta}^{\star}(k) = \delta_{\sigma \alpha \beta}. \quad (7) \]

Here, and in all the subsequent formulas there is implied the summation over the repeating spin \( \sigma = \uparrow, \downarrow \) or band \( \alpha = +, - \) indices.

There are two Fermi surfaces determined by the equations

\[ \epsilon_{\pm}(k) = \mu \quad (8) \]

with different Fermi momenta \( k_{F \pm} \). In the Rashba 2D model and in the 3D isotropic case they are

\[ k_{F \pm} = \mp m \gamma + \sqrt{2m \mu + (m \gamma)^2} \quad (9) \]

and the Fermi velocity has the common value

\[ v_{F \pm} = \frac{\partial(\epsilon \pm \gamma k)}{\partial k}\bigg|_{k=k_{F \pm}} = \hat{k} \sqrt{\frac{2\mu}{m} + \gamma^2}, \quad (10) \]

here \( \hat{k} \) is the unit vector along momentum \( k \). The equivalence of the Fermi velocities at different Fermi momenta is the particular property of the models with isotropic spin-orbital coupling [4] in 3D case and the Rashba interaction in 2D case.

The matrix of equilibrium electron distribution function is

\[ \hat{n} = \frac{n_+ + n_-}{2} \delta + \frac{n_+ - n_-}{2|\gamma|} \gamma \cdot \sigma, \quad (11) \]

where

\[ n_\pm = \frac{1}{\exp \left( \frac{\epsilon_\pm - \mu}{T} \right) + 1} \quad (12) \]

are the Fermi functions. In the isotropic case near the point, the dispersion relation for the Fermi surface is given by

\[ \xi_{\pm} = \epsilon_{\pm} - \mu \approx v_F (k - k_{F \pm}) = \epsilon - \mu_{\pm}, \quad (13) \]

with

\[ \epsilon = v_F k; \quad \mu_{\pm} = v_F k_{F \pm}, \quad \mu_+ - \mu_- = -2mv_F \gamma. \quad (14) \]
III. TRANSPORT PROPERTIES. IMPURITY SCATTERING.

A. Kinetic equation

In presence of time dependent electric field \( \mathbf{E}(t) = \mathbf{E}_0 e^{-i\omega t} \) the linearized kinetic equation \([1]\) is

\[
\frac{\partial \hat{n}}{\partial t} + e \mathbf{E} \frac{\partial \hat{n}}{\partial \mathbf{k}} - i[\mathbf{\hat{\varepsilon}}, \hat{n}] = \hat{I}_{st},
\]

where \( \hat{g} = \hat{n} - \hat{n}^0 \) is the deviation of distribution function from equilibrium distribution \( \hat{n}^0 \).

The hermitian matrices of the nonequilibrium distribution functions in band and spin representations are related as

\[
f_{\alpha\beta}(\mathbf{k}) = \Psi^*_\sigma(\mathbf{k})n_{\sigma_1\sigma_2}\Psi_\beta(\mathbf{k}).
\]

In the band representation the equilibrium distribution function \([11]\) is the diagonal matrix

\[
f^0_{\alpha\beta}(\mathbf{k}) = \Psi^0_\sigma(\mathbf{k})n^0_{\sigma_1\sigma_2}\Psi^0_\beta(\mathbf{k}) = \begin{pmatrix} n_+ & 0 \\ 0 & n_- \end{pmatrix}_{\alpha\beta}.\]

However, the matrix of derivative of the equilibrium distribution in the band representation is not diagonal and given by the following equation

\[
\Psi^*_\sigma(\mathbf{k}) \frac{\partial n^0_{\sigma_1\sigma_2}}{\partial \mathbf{k}} \Psi_\beta(\mathbf{k}) = \frac{\partial f^0_{\alpha\beta}}{\partial \mathbf{k}} + \left[ \Psi^*_\sigma(\mathbf{k}) \frac{\partial \Psi_\beta(\mathbf{k})}{\partial \mathbf{k}}, f^0_{\gamma\beta} \right],
\]

where \([\ldots, \ldots]\) is the commutator. Hence, the matrix kinetic equation for the frequency dependent Fourier amplitudes of non-equilibrium part of distribution function \( g_{\alpha\beta}(\mathbf{k}, t) = g_{\alpha\beta}(\mathbf{k}, \omega) e^{-i\omega t} \) acquires the form

\[
-I_{\alpha\beta}(\mathbf{k}) = \sum_{\gamma\delta} (\mathbf{v}_+ \mathbf{E})_{\alpha\gamma} \frac{\partial \Psi^*_\gamma(\mathbf{k})}{\partial \mathbf{k}} \mathbf{v}_- E_{\beta\delta} (n_+ - n_-) + \begin{pmatrix} 0 & ig_+ (\varepsilon_+ - \varepsilon_-) \\ ig_- (\varepsilon_+ - \varepsilon_-) & 0 \end{pmatrix} = I_{\alpha\beta}.
\]

Here

\[
\mathbf{v}_\alpha = \frac{\partial \varepsilon_\alpha}{\partial \mathbf{k}}, \quad \mathbf{v}_\pm = \Psi^*_\sigma(\mathbf{k}) \frac{\partial \Psi_\sigma(\mathbf{k})}{\partial \mathbf{k}} = \begin{pmatrix} \gamma_{kx} - i\gamma_{ky} \\ 2(\gamma_{kz} + 1) \end{pmatrix}, \quad \mathbf{v}_\mp = -\mathbf{v}_\pm.
\]

The collision integral \( I_{\alpha\beta}(\mathbf{k}) \) for electron scattering on impurities is derived in Appendix A. One can check that it is equal to zero in equilibrium. Hence, it is

\[
I_{\alpha\beta}(\mathbf{k}) = 2\pi n_{imp} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} |V(\mathbf{k} - \mathbf{k}')|^2 \{ O_{\alpha\nu}(\mathbf{k}, \mathbf{k}') [g_{\nu\mu}(\mathbf{k}')O_{\mu\beta}(\mathbf{k}', \mathbf{k}) - O_{\nu\mu}(\mathbf{k}', \mathbf{k})g_{\mu\beta}(\mathbf{k})] \delta(\varepsilon_\nu - \varepsilon_\beta) \\
+ [O_{\alpha\nu}(\mathbf{k}, \mathbf{k}')g_{\nu\mu}(\mathbf{k}') - g_{\alpha\nu}(\mathbf{k})O_{\nu\mu}(\mathbf{k}, \mathbf{k}')] O_{\mu\beta}(\mathbf{k}', \mathbf{k}) \delta(\varepsilon_\mu - \varepsilon_\alpha) \},
\]

\[
O_{\alpha\beta}(\mathbf{k}, \mathbf{k}') = \Psi^*_\sigma(\mathbf{k})\Psi_\beta(\mathbf{k}'), \quad (22)
\]

Here, and in all the subsequent equations when we will discuss 2D case one must substitute the 3D integration over reciprocal space \( \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \) by the corresponding 2D expression \( \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \).

The solution of Eq.\([19]\) has the following form

\[
g_{\alpha\beta} = \begin{pmatrix} g_+ & g_+ \\ g_- & g_- \end{pmatrix} = e \begin{pmatrix} (\mathbf{w}_+ \mathbf{E}) & (\mathbf{w}_+ \mathbf{E}) \\ (\mathbf{w}_- \mathbf{E}) & (\mathbf{w}_- \mathbf{E}) \end{pmatrix}.
\]

After substitution this matrix in the Eq.\([19]\) and in the collision integral Eq.\([21]\) we obtain four scalar equations corresponding to each matrix element of the matrix Eq.\([19]\) for \( \mathbf{k} \) dependent four scalar functions \( (\mathbf{w}_+ \mathbf{E}), (\mathbf{w}_+ \mathbf{E}), (\mathbf{w}_- \mathbf{E}), (\mathbf{w}_- \mathbf{E}) \). These functions, in general, can be determined by solving the equations numerically. The particular solutions for collisionless regime and the weak impurity scattering case are considered in the next sections.

B. Electric current

The electric current density is

\[
\mathbf{j} = e \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\partial \varepsilon_{\sigma\alpha}(\mathbf{k})}{\partial \mathbf{k}} g_{\sigma_1\sigma}(\mathbf{k}, \omega).
\]

Transforming it to the band representation we obtain
where \([\ldots, \ldots]\) is the commutator. Finally we come to

\[
\mathbf{j} = e^2 \int \frac{d^3 k}{(2\pi)^3} \left\{ (\mathbf{v}_+ (\mathbf{w}_+ \mathbf{E}) + \mathbf{v}_- (\mathbf{w}_- \mathbf{E}) \\
+ [\mathbf{v}_\pm (\mathbf{w}_\mp \mathbf{E}) - \mathbf{v}_\mp (\mathbf{w}_\pm \mathbf{E})] (\varepsilon_- - \varepsilon_+) \right\}. \tag{26}
\]

The functions \(\mathbf{w}_+, \mathbf{w}_-, \mathbf{w}_\pm, \mathbf{w}_\mp\) depend from the modulus and the direction of momentum \(\mathbf{k}\). Because of this the direction of electric current does coincide in general with the direction of electric field.

### 1. Ballistic regime

In neglect the scattering terms, that is at \(\omega \tau > 1\) where \(\tau\) is the symbol for the typical times of scattering determined by the different terms in the scattering integral, the Eq.\((19)\) has the following solution

\[
g_+ = e (\mathbf{w}_+ \mathbf{E}) = \frac{e}{i \omega} (\mathbf{v}_+ \mathbf{E}) \frac{\partial n_+}{\partial \xi_+}, \tag{27}
\]

\[
g_- = e (\mathbf{w}_- \mathbf{E}) = \frac{e}{i \omega} (\mathbf{v}_- \mathbf{E}) \frac{\partial n_-}{\partial \xi_-}, \tag{28}
\]

\[
g_\pm = e (\mathbf{w}_\pm \mathbf{E}) = \frac{e (\mathbf{v}_\pm \mathbf{E}) (n_- - n_+)}{i \omega - i (\varepsilon_- - \varepsilon_+)} \tag{29},
\]

\[
g_\mp = e (\mathbf{w}_\mp \mathbf{E}) = \frac{e (\mathbf{v}_\mp \mathbf{E}) (n_+ - n_-)}{i \omega - i (\varepsilon_+ - \varepsilon_-)}. \tag{30}
\]

Substitution these expressions to the Eq.\((26)\) gives

\[
\mathbf{j} = e^2 \int \frac{d^3 k}{(2\pi)^3} \left\{ \left( \mathbf{v}_+ (\mathbf{v}_+ \mathbf{E}) \frac{\partial n_+}{\partial \xi_+} + \mathbf{v}_- (\mathbf{v}_- \mathbf{E}) \frac{\partial n_-}{\partial \xi_-} \right) \right. \\
+ \left. 2 \left( n_+ - n_- \right) (\varepsilon_- - \varepsilon_+) \right\} \frac{i \omega}{\omega^2 - (\varepsilon_- - \varepsilon_+)^2} \left[ i \omega \Im (\mathbf{v}_\pm (\mathbf{v}_\mp \mathbf{E})) \right], \tag{31}
\]

The last term in this formula presents dissipative current. It depends from

\[
\Im (\mathbf{v}_\pm (\mathbf{v}_\pm \mathbf{E})) = \frac{1}{4(\gamma_k z + 1)} \left\{ \nabla_{\mathbf{k}_z} \left[ -\gamma_{kx} \left( \frac{\partial \gamma_{k_y}}{\partial \mathbf{E}} \right) + \gamma_{k_y} \left( \frac{\partial \gamma_{k_x}}{\partial \mathbf{E}} \right) \right] + \left( \frac{\partial \gamma_{k_y}}{\partial \mathbf{k}} \right) \left[ -\gamma_{k_y} \frac{\partial \gamma_{k_x}}{\partial \mathbf{k}} + \gamma_{k_x} \frac{\partial \gamma_{k_y}}{\partial \mathbf{k}} \right] \right\}. \tag{32}
\]

In two dimensions the \(z\)-component of vector \(\gamma_{\mathbf{k}}\) is absent, and \(\Im (\mathbf{v}_\pm (\mathbf{v}_\pm \mathbf{E})) = 0\). Hence, in a 2D non-centrosymmetric media a high frequency electric field in absence of relaxation due to electron scattering on impurities does not produce dissipative current as it is in a media with inversion center.

In contrast, in three dimensional media the last term in Eq.\((31)\) in general is not equal to zero. For instance, one can check by the direct calculation that this term yields non-zero dissipative current directed perpendicular to electric field in metals with symmetry \(C_{4v}\) with vector \(\gamma_{\mathbf{k}}\) given by Eq.\((3)\). Thus, we come to conclusion that in 3D non-centrosymmetric media an electric field not only accelerates electrons, but also transfers an energy to the electron gas creating non-equilibrium states with non-diagonal matrix distribution function. This is unusual phenomenon specific for the 3D materials without inversion center.

The integral in the last two terms of Eq.\((31)\) is taken over the region of reciprocal space in between \(k_{F+}\) and \(k_{F-}\), where \(\varepsilon_+ - \varepsilon_- \approx 2\gamma k_F\). At relatively small frequencies \(\omega < 2\gamma k_F\), but still in collisionless regime \(\omega > \tau^{-1}\),
one can rewrite the Eq.(31) as
\[
j = e^2 \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{v_+(v_+E)}{i\omega} \frac{\partial n_+}{\partial \xi^+} + \frac{v_-(v_-E)}{i\omega} \frac{\partial n_-}{\partial \xi^-} - 2(n_+ - n_-)\Im(v_+(v_+E)) \right\}.
\] (33)

2. Weak impurity scattering

The weak impurity scattering regime is limited by inequality \( \tau^{-1} \ll 2\gamma k_F \). This case one can neglect by

\[
-i\omega g_+ + e(v_+E) \frac{\partial n_+}{\partial \xi^+} = 4\pi n_i \int \frac{d^3k}{2\pi^3} |V(k - k')|^2 \times \nonumber \times \left\{ O_{++}(kk')O_{++}(k'k) [g_+(k') - g_+(k)] \delta(\epsilon'_+ - \epsilon_+) + O_{+-}(kk')O_{+-}(k'k) [g_-(k') - g_-(k)] \delta(\epsilon'_- - \epsilon_-) \right\},
\] (34)

\[
-i\omega g_- + e(v_-E) \frac{\partial n_-}{\partial \xi^-} = 4\pi n_i \int \frac{d^3k}{2\pi^3} |V(k - k')|^2 \times \nonumber \times \left\{ O_{--}(kk')O_{--}(k'k) [g_-(k') - g_-(k)] \delta(\epsilon'_- - \epsilon_-) + O_{-+}(kk')O_{-+}(k'k) [g_+(k') - g_+(k)] \delta(\epsilon'_+ - \epsilon_+) \right\},
\] (35)

obtained neglecting in the scattering integral by the terms with off-diagonal components of the distribution function. These terms are \( \gamma k_F \tau \gg 1 \) times smaller than the terms with diagonal elements. We see that even in the limit of weak impurity scattering the relaxation of diagonal elements of distribution function to equilibrium is determined in general by the four different collision terms.

There were undertaken several attempts \([2,4]\) to solve these equations for the 2D Rashba model in the Born approximation. This case the products

\[
O_{++}(kk')O_{++}(k'k) = O_{--}(kk')O_{--}(k'k) = \cos^2 \frac{\varphi - \varphi'}{2},
\]

\[
O_{-+}(kk')O_{-+}(k'k) = O_{+-}(kk')O_{+-}(k'k) = -\sin^2 \frac{\varphi - \varphi'}{2}
\]

depend from the difference of the azimuthal angles of initial and final vector of momentum. In the Born approach the scattering integrals is expressed through the potential of scattering depending from transferred momentum \( V(k - k') \) that means it also depends from \( \varphi - \varphi' \). This creates possibility to search the solution of Eqs. (34), (35) in the following form

\[
g_+ = -e\alpha_+ \frac{\partial n_+}{\partial \xi^+}(v_+E), \quad g_- = -e\alpha_- \frac{\partial n_-}{\partial \xi^-}(v_-E),
\] (36)

as it was done in Ref.4 where the coefficients \( \alpha_+, \alpha_- \) were found at \( \omega = 0 \). Then the current is

\[
j = e^2 \int \frac{d^3k}{(2\pi)^3} \left\{ a_+ v_+(v_+E) \frac{\partial n_+}{\partial \xi^+} + a_- v_-(v_-E) \frac{\partial n_-}{\partial \xi^-} \right\}.
\] (37)

This approach easily generalized to the finite frequency case.

Similar treatment is possible for the Dresselhaus model \([10]\) where \( \gamma(k) = \gamma_D k_y \hat{y} - k_x \hat{x} \), but not for the model where vector \( \gamma(k) \) is given by the sum of vectors in the Rashba and the Dresselhaus models.

For other 2D or 3D models one can search the solution of Eqs. (34) and (35) by numerical methods. In three dimensional media in weak impurity scattering regime the current is determined as

\[
j = e^2 \int \frac{d^3k}{(2\pi)^3} \left\{ v_+(w_+E) + v_-(w_-E) \right\} - 2(n_+ - n_-)\Im(v_+(v_+E)),
\] (38)

where the first two terms should be found solving Eqs. (34) and (35) and the last term presents an additional dissipative current originating from the off-diagonal terms of the matrix distribution function as in the collisionless regime.

3. Regime of strong scattering

The strong scattering occurs when the typical inverse scattering time is of the order of spin-orbit band splitting \( \tau^{-1} \approx \gamma k_F \). This case at \( \gamma k_F \ll \varepsilon_F \) the quasi-classical kinetic theory is still applicable to description of the kinetic phenomena but one must solve the whole system kinetic equations for the diagonal and the off-diagonal matrix elements of the distribution function.
C. Spin current

An electric field in a crystal without inversion center generates a spin current. The density of spin current arising in an electric field is

\[ j_i = e \int \frac{d^3k}{(2\pi)^3} \sigma^{\sigma \sigma_1} \frac{\partial \varepsilon_{\sigma_1 \sigma_2}(k)}{\partial k_i} g_{\sigma_2 \sigma}(k, \omega). \]  

(39)

\[ j_{xi} = e \int \frac{d^3k}{(2\pi)^3} \left\{ [v_{+i}(w_+E) + v_{-i}(w_-E)] + [v_{+i}(w_-E) - v_{+i}(w_+E)] (\varepsilon_+ - \varepsilon_-) \right\}, \]  

(40)

\[ j_{yi} = ie \int \frac{d^3k}{(2\pi)^3} \left\{ [v_{+i}(w_+E) - v_{-i}(w_-E)] + [v_{+i}(w_-E) + v_{+i}(w_+E)] (\varepsilon_- - \varepsilon_+) \right\}, \]  

(41)

\[ j_{zi} = e \int \frac{d^3k}{(2\pi)^3} \left\{ [v_{+i}(w_+E) - v_{-i}(w_-E)] + [v_{+i}(w_-E) + v_{+i}(w_+E)] (\varepsilon_- - \varepsilon_+) \right\}. \]  

(42)

In collisionless regime the solutions of kinetic equation are given by Eqs. (27)-(30). In two-dimensional case the velocities Eq.[20] are

\[ \mathbf{v}_\alpha = \frac{\partial \varepsilon_\alpha}{\partial k}, \quad \mathbf{v}_\pm = -\frac{1}{2} \frac{\partial \varepsilon_{kr} - i \gamma_{kr}}{\partial k}. \]  

(43)

Obviously, the "diagonal" velocities are odd functions of the wave vector \( \mathbf{v}_\alpha(-\mathbf{k}) = -\mathbf{v}_\alpha(\mathbf{k}) \), and the "off-diagonal" velocities are even functions of the wave vector \( \mathbf{v}_\pm(-\mathbf{k}) = \mathbf{v}_\pm(\mathbf{k}) \). Hence, in a 2D non-centrosymmetric media the ballistic spin currents Eqs. (40), (41) are identically equal to zero

\[ j_{xi} = j_{yi} = 0. \]  

(44)

To calculate the spin current in the case of weak impurity scattering, one can use the off-diagonal matrix elements of distribution function given by Eqs. (20), (30) but for the diagonal elements one should solve the equations (31)-(35). For the 2D Rashba model in the Born approximation the solution is given by the Eq. (36). Thus, in this case the spin current is also equal to zero due to the parity properties of "diagonal" and "off-diagonal" velocities.

In three dimensions the "off-diagonal" velocities Eq.[20] are not even functions any more. The spin current under electric field acquire finite value. Similar to electric current even in ballistic regime it consists from reactive and dissipative additive components.

D. Heat current

In presence of temperature gradient the matrix kinetic equation for non-equilibrium distribution function \( g_{\alpha \beta} \) is

\[ -\frac{1}{T} \left( \begin{array}{c} (\mathbf{v}_+ \nabla T)\xi_+ \frac{\partial \alpha_+}{\partial \xi_+} + \xi_- \frac{\partial \alpha_-}{\partial \xi_-} \\ \frac{1}{2}(\mathbf{v}_- \nabla T)(\varepsilon_+ - \varepsilon_-) \left( \xi_+ \frac{\partial \alpha_+}{\partial \xi_+} + \xi_- \frac{\partial \alpha_-}{\partial \xi_-} \right) + ig_\pm(\varepsilon_- - \varepsilon_+) \end{array} \right) \right) = I_{\alpha \beta}. \]  

(45)

The solution of Eq.[45] has the following form

\[ g_{\alpha \beta} = \left( \begin{array}{c} g_+ \quad g_+ \quad g_- \quad g_- \end{array} \right) = -\frac{1}{T} \left( \begin{array}{c} (\mathbf{u}_+ \nabla T) \quad (\mathbf{u}_+ \nabla T) \\ (\mathbf{u}_+ \nabla T) \quad (\mathbf{u}_+ \nabla T) \\ (\mathbf{u}_- \nabla T) \quad (\mathbf{u}_- \nabla T) \end{array} \right) \right). \]  

(46)

After substitution this matrix in the Eq.[45] and in the collision integral [21] we obtain four equations corresponding to each matrix element of the matrix Eq.[46] for four \( k \) dependent scalar functions \( (\mathbf{u}_+ \nabla T), (\mathbf{u}_- \nabla T), (\mathbf{u}_+ \nabla T), (\mathbf{u}_- \nabla T) \).

The density of heat current is

\[ \mathbf{q} = \int \frac{d^3k}{(2\pi)^3} \xi_{\sigma \sigma_1}(k) \frac{\partial \varepsilon_{\sigma_1 \sigma_2}(k)}{\partial k} g_{\sigma_2 \sigma}(k). \]  

(47)
Transforming it to the band representation we obtain
\[
\begin{align*}
q = & \int \frac{d^3k}{(2\pi)^3} \left\{ \xi_+ v_+ g_+ + \xi_- v_- g_- \\
& + \left[ \xi_+ v_\pm g_\pm - \xi_- v_\mp g_\pm \right] \left( \varepsilon_- - \varepsilon_+ \right) \right\}.
\end{align*}
\] (48)

And substituting \( g_{\alpha\beta} \) from Eq.(46) we come to
\[
\begin{align*}
q = & \frac{1}{T} \int \frac{d^3k}{(2\pi)^3} \left\{ \xi_+ v_+ (u_+ \nabla T) + \xi_- v_- (u_- \nabla T) \\
& + \left[ \xi_+ v_\pm (u_\pm \nabla T) - \xi_- v_\mp (u_\mp \nabla T) \right] \left( \varepsilon_- - \varepsilon_+ \right) \right\}.
\end{align*}
\] (49)

To find the heat current in the weak scattering regime one can use off-diagonal matrix elements obtained in neglect of collisions
\[
\begin{align*}
g_\pm &= -\frac{i}{2T} (v_\pm \nabla T) \left( \xi_+ \frac{\partial n_+}{\partial \xi_+} + \xi_- \frac{\partial n_-}{\partial \xi_-} \right), \quad \text{(50)}
g_+ &= -\frac{i}{2T} (v_+ \nabla T) \left( \xi_+ \frac{\partial n_+}{\partial \xi_+} + \xi_- \frac{\partial n_-}{\partial \xi_-} \right).
\end{align*}
\] (51)

Whereas the diagonal elements should be found from the equations

\[\text{IV. ELECTRON-ELECTRON SCATTERING}\]

The problem of electron-electron scattering in non-centrosymmetric metals has been discussed in the paper [11]. It was done making use the electron-electron collision integral given by Eqs.(B1), (B2) for the spin-matrix distribution function derived by V.P.Silin [12] and J.W.Jeon and W.J.Mullin [13] in application to the quasiparticles scattering in liquid $^3$He. Giving the correct description of relaxation processes for spin-perturbed quasiparticle distributions in Fermi liquid in a centrosymmetric media this integral is not applicable for the description of relaxation in non-centrosymmetric case. Thus, the approach developed in Ref.11 is not valid. The electron-electron collision integral in a media without inversion center given by Eqs.(B1), (B3) has much more cumbersome form. However, the main conclusion of Ref.11 is qualitatively correct: the zero temperature electron-electron scattering time in a non-centrosymmetric medium is finite.

In the equilibrium the integral (B1), (B3) is equal to zero, but a deviation from equilibrium distribution results in non-vanishing collision terms even at zero temperature. The reason for this is that even at zero temperature the scattered quasiparticles can find the non-occupied states in between the two Fermi surfaces with different Fermi momenta corresponding two bands split by the spin-orbital coupling. The situation is in complete analog with spin-polarized liquid $^3$He, where the scattering processes for spin-diffusion in transversal to magnetic field direction involve all the states between Fermi surfaces of spin-up and spin-down quasiparticles, and the relaxation time acquires the finite zero-temperature value [13–15]. Appropriate also to mention the remark made by C.Herring [16] concerning a relaxation in ferromagnetics: “For a ferromagnetic metal... if the spin of quasiparticle at the Fermi surface is reversed, the corresponding quasiparticle state will no longer be closed to the Fermi surface, and it will have a finite, rather than an infinitesimal, decay rate.”

The zero temperature decay rate causes a doubt in validity of the Fermi liquid approach to the description of electrons in metals without inversion center. The estimation made in the Ref.11 and more careful calculations made for the polarised Fermi-gas [17] allow to be sure in the applicability of the Fermi liquid theory so long the splitting of Fermi surfaces in momentum space is small in comparison with the Fermi energy:
\[
v_F (k_{F_+} - k_{F_+}) \ll \varepsilon_F. \quad \text{(54)}
\]

The spin-orbital band splitting \( v_F \Delta k_F \) is directly expressed through the corresponding splitting of the de Haas - van Alphen magnetization oscillation frequencies [18]. Determined experimentally the typical magnitude of band splitting in many non-centrosymmetric metals is of the order of hundreds Kelvin [19–21]. This is much less than the Fermi energy.

Because of finite zero temperature electron-electron scattering relaxation time in a metal without inversion center the total resistivity at zero temperature consists of two parts originating from resistivity due to the electron-electron scattering and due to the electron scattering on impurities
\[
\rho = \rho_{ee}(T = 0) + \rho_{imp}. \quad \text{(55)}
\]
We ignore here the tensorial character of resistivity. The resistivity due impurity scattering is proportional to impurity concentration \( \rho_{\text{imp}} \propto n_{\text{imp}} \). Thus, the zero-temperature resistivity due to electron-electron scattering \( \rho_{ee}(T = 0) \) can be experimentally found by the measuring of low temperature resistivity at several finite impurity concentrations with subsequent taking the formal limit
\[
\rho_{ee}(T = 0) = \rho(n_{\text{imp}} \rightarrow 0).
\] (56)

The corresponding qualitative behaviour is shown in Fig.1. Needless to say, the crystal should be practically perfect because the presence of dislocations, twin boundaries and stacking faults even in an almost ideally pure specimen can completely hide a contribution of electron-electron scattering in the residual resistivity. The saturation of e-e contribution to resistivity has been speculated to be related to the absence of usual \( T^2 \) contribution in recent experiments \[22\]. Similarly, the residual thermal resistivity consists of two parts determined by the scattering on impurities and the electron-electron scattering
\[
\frac{T}{\kappa} = d_{\text{imp}} + d_{ee}(T = 0).
\] (57)

The ratio of thermal conductivity to resistivity at low temperatures in non-centrosymmetric materials is still proportional to temperature according to the Wiedemann-Franz law
\[
\frac{\kappa}{\sigma} = AT.
\] (58)

However, the constant of proportionality is not universal Lorenz number but acquires purity and substance dependent magnitude.

V. CONCLUSION

The spin-orbital coupling in a medium without inversion centre lifts the spin degeneracy of electronic states and splits each conducting band in two bands with different Fermi momenta. The kinetic properties in such metal or semiconductor are described by four kinetic equations for diagonal (intra-band) and off-diagonal (inter-band) matrix elements of distribution function. In contrast with space parity symmetric materials in media without mirror symmetry the external forces not only accelerate electrons but also transfer the energy to the electron gas creating non-equilibrium states with non-diagonal matrix distribution function that bring to birth the dissipative charge, spin and heat currents even in collisionless regime. This pure 3D phenomenon is absent in 2D case. The theory of charge, spin and heat transport is drastically simplified at so called weak impurity scattering when the impurity scattering rate does not exceed the energy of spin-orbit band splitting. This case one can use the collisionless solutions for the off-diagonal matrix elements of the distribution function and work with much simpler system of two kinetic equations for the diagonal elements. The derivations of electron-impurity and electron-electron collision integrals are presented. Along with the scattering on impurities the electron-electron collisions in a medium without inversion center are also responsible for the finite zero temperature residual resistivity and residual thermal resistivity.

Appendix A: Electron-impurity collision integral

The electron-impurity collision integral in the operator form \[23\] for spatially homogeneous system is given by the formula
\[
\hat{I}_{\sigma\sigma'} = n_{\text{imp}} \int \frac{d^3q}{(2\pi)^3} V(q)^2 \int_{-\infty}^{0} d\tau e^{\lambda\tau} \hat{A}_{\sigma\sigma'}(\tau, t) \] (A1)

where \( \lambda \to +0 \) and the integrand in linear approximation in respect to slow varying in time density matrix \( \rho_{\sigma_1,\sigma_2}(r, t) \) is
\[
\hat{A}_{\sigma\sigma'}(\tau, t) = \left[ exp(i\hbar\sigma_1, \tau)[e^{iqr}, \rho_{\sigma_1,\sigma_2}(r, t)] exp(-i\hbar\sigma_2, \tau), e^{-iqr} \right]. (A2)

Square brackets means the commutator, \( \sigma, \sigma_1, \ldots \) are the spin indices,
\[
\hat{h}_{\sigma_1} = \hat{\varepsilon}_{\sigma_1} \left( -i \frac{\partial}{\partial r} \right) \] (A3)

is the hamiltonian of noninteracting electrons given by Eq.\[2\] in coordinate representation. Its eigen functions satisfy the equation
\[
\hat{h}_{\sigma_1} e^{ikr} \Psi_{\sigma_1}^\alpha(k) = \varepsilon_{\alpha} e^{ikr} \Psi_{\sigma}^\alpha(k) \] (A4)

In the Dirac notations they are \( e^{ikr} \Psi_{\sigma}^\alpha(k) = |k\rangle \Psi_{\sigma}^\alpha(k) \). To transform collision integral from coordinate to momentum representation and at the same time from spin to band representation one must calculate the matrix element from the expression Eq.\[A2\]
distribution function as the integral depending from the
in space on the scales bigger than inter-electron distance
The collision integral for a distribution function varying
where
Substituting this formula to Eq.(A1) and performing the integration over
Here
Let us use now the equation (A4) and the orthogonality conditions Eq.(7). For the first term in (A5) we obtain
\[
\int \frac{d^3k}{(2\pi)^3} \left[ \Psi_\alpha^*(k)|k\rho_{\sigma_1\sigma_2}(r,t)|k\right] \Psi_\beta(k)
\]
Substituting this formula to Eq.(A1) and performing the integration over \(d^3q\) and over \(\tau\) we come to
\[
2\pi n_{imp} \int \frac{d^3k'}{(2\pi)^3} |V(k-k')|^2 O_{\alpha\nu}(k,k') f_{\mu\nu}(k') O_{\mu\beta}(k',k) \delta(\epsilon'_{\mu} - \epsilon_{\beta}),
\]  
where
\[
f_{\alpha\beta}(k) = \Psi_{\alpha_1}^*(k)|k\rho_{\sigma_1\sigma_2}(r,t)|k\right] \Psi_{\beta_2}(k),
\]  
\[
\epsilon_\pm = \epsilon_{\pm}(k), \quad \epsilon'_\pm = \epsilon_{\pm}(k') \quad \text{and}
\]
\[
O_{\alpha\beta}(k,k') = \Psi_{\alpha_1}^*(k)\Psi_{\beta_2}(k')
\]  
such that \(O_{\alpha\beta}(k,k') = O_{\beta\alpha}^*(k',k)\). Performing similar transformation with the other terms in Eq.(A5) we come the electron-impurity collision integral in band representation
\[
I_{\alpha\beta}(k) = 2\pi n_{imp} \int \frac{d^3k'}{(2\pi)^3} |V(k-k')|^2 \left[ O_{\alpha\nu}(k,k') f_{\mu\nu}(k') O_{\mu\beta}(k',k) - O_{\nu\mu}(k,k') f_{\mu\beta}(k) \right] \delta(\epsilon'_{\mu} - \epsilon_{\beta})
\]
\[
+ \left[ O_{\alpha\nu}(k,k') f_{\mu\nu}(k') - f_{\alpha\nu}(k) O_{\mu\nu}(k,k') \right] O_{\mu\beta}(k',k) \delta(\epsilon'_{\mu} - \epsilon_{\alpha}) \right] ,
\]  
The collision integral for a distribution function varying
in space on the scales bigger than inter-electron distance has the same form.

One can rewrite the collision integral for the matrix
distribution function as the integral depending from the
vectorial distribution function
\[
f^j(k) = \frac{1}{2} f_{\alpha\beta}(k) \sigma^j_{\beta\alpha},
\]  
where \(\sigma^j_{\beta\alpha} = (\delta_{\beta\alpha}, \sigma^x_{\beta\alpha}, \sigma^y_{\beta\alpha}, \sigma^z_{\beta\alpha})\) is the four component
matrix vector. As result we obtain the scattering integral
amazingly similar to the standard diagonal in the spin
indices expression
\[
I_{\alpha\beta}(k) = 2\pi n_{imp} \int \frac{d^3k'}{(2\pi)^3} |V(k-k')|^2 \left[ O_{\alpha\nu}(k,k') R^j_{\beta\mu}(k',k) \delta(\epsilon'_{\mu} - \epsilon_{\beta})
\]
\[
+ R^j_{\alpha\nu}(k,k') O_{\mu\beta}(k',k) \delta(\epsilon'_{\mu} - \epsilon_{\alpha}) \right] \right] .
\]  
Here
\[
R_{\alpha\beta}(k',k) = O^0(k',k) \delta_{\alpha\beta} + O^x(k',k) \sigma^x_{\alpha\beta} + O^y(k',k) \sigma^y_{\alpha\beta} + O^z(k',k) \sigma^z_{\alpha\beta},
\]
\[
R^j_{\alpha\beta}(k',k) = O^j(k',k) \delta_{\alpha\beta} + O^j(k',k) \sigma^j_{\alpha\beta} + iO^j(k',k) \sigma^j_{\alpha\beta} - iO^j(k',k) \sigma^j_{\alpha\beta},
\]
\[
R_{\nu\beta}(k',k) = O^\nu(k',k) \delta_{\nu\beta} + O^\nu(k',k) \sigma^\nu_{\nu\beta} + iO^\nu(k',k) \sigma^\nu_{\nu\beta} - iO^\nu(k',k) \sigma^\nu_{\nu\beta},
\]
\[
R^j_{\nu\beta}(k',k) = O^j(k',k) \delta_{\nu\beta} + O^j(k',k) \sigma^j_{\nu\beta} + iO^j(k',k) \sigma^j_{\nu\beta} - iO^j(k',k) \sigma^j_{\nu\beta},
\]
and $O^{i}(k', k) = \frac{1}{2} O_{\alpha \beta}(k)\sigma^{i}_{\alpha \beta}$.

**Appendix B: Electron-electron collision integral**

The Fermi particle-particle collisions integral in the Born approximation was derived by V.P.Silin [12] and

$\hat{i}(k_1) = 2\pi \int d^3k' d^3k'' d^3k_2 \frac{1}{(2\pi)^3 (2\pi)^3} \sum_{m} \delta \left( k_1 + k_2 - k' - k'' - \frac{2\pi m}{a} \right) \hat{F}$, \hspace{1cm} (B1)

where $\frac{2\pi m}{a}$ is a vector of reciprocal lattice,

$\hat{F} = \left\{ \frac{1}{2} W_1 \{ [\hat{n}', (\hat{1} - \hat{n}_1)] + Tr((\hat{1} - \hat{n}_2)\hat{n}'') - [(\hat{1} - \hat{n}_1) + Tr(\hat{n}_2(\hat{1} - \hat{n}''))] \right\} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'')$. \hspace{1cm} (B2)

Here $[\hat{A}, \hat{B}]_+$ means the anticommutator of the matrices $A$ and $B$, and the following designations $\hat{n}' = \hat{n}(k')$, $\varepsilon' = \varepsilon(k')$ etc are introduced. In the isotropic Fermi liquid like $^3$He $W_1 = |V([k_1 - k'])|^2$, $W_2 = -V([k_1 - k'] V([k_1 - k''])$ are expressed through the Fourier transform of the quasiparticles potential of interaction. The latter in concrete metal is unknown and due to charge screening one can put them as the constants: $W_1 = W_0/2$, $W_2 = -W_0/2$.

This collision integral can be also obtained from the integral of the electron-electron collisions in the operator representation derived in Ref.23 where was demonstrated that in the case of diagonal in spin indices distribution function the collision integral takes the standard form.

The matrix $\hat{F}$ for the electron-electron collision integral corresponding to Eq. [B2] for the electron matrix distribution function in the band representation in non-centrosymmetric media is

$F_{\alpha \beta} = \frac{1}{2} W_1 \{ [O_{\alpha \nu}(k_1, k') f_{\nu \mu}(k') O_{\mu \lambda}(k', k_1) (\delta_{\lambda \beta} - f_{\lambda \beta}(k_1)) (\delta_{\xi \eta} - f_{\xi \eta}(k_2)) O_{\eta \rho}(k_2, k'') f_{\rho \sigma}(k'') O_{\sigma \delta}(k'', k_2) \}$

$- O_{\alpha \nu}(k_1, k') (\delta_{\nu \mu} - f_{\nu \mu}(k')) O_{\mu \lambda}(k', k_1) f_{\nu \rho}(k_2) O_{\eta \rho}(k_2, k'') (\delta_{\xi \rho} - f_{\xi \rho}(k'')) O_{\rho \sigma}(k'', k_2) \}$

$+ \left\{ [\delta_{\alpha \nu} - f_{\alpha \nu}(k_1)] O_{\nu \mu}(k_1, k') (\delta_{\mu \lambda} - f_{\mu \lambda}(k')) O_{\lambda \sigma}(k', k_1) (\delta_{\xi \beta} - f_{\xi \beta}(k_2)) O_{\beta \rho}(k_2, k'') f_{\rho \sigma}(k'') O_{\sigma \delta}(k'', k_2) \}$

$- f_{\alpha \nu}(k_1) O_{\nu \mu}(k_1, k') f_{\mu \lambda}(k') O_{\lambda \sigma}(k', k_1) f_{\xi \eta}(k_2) O_{\eta \rho}(k_2, k'') (\delta_{\xi \rho} - f_{\xi \rho}(k'')) O_{\rho \sigma}(k'', k_2) \}$

$+ \frac{1}{2} W_2 \{ [O_{\alpha \nu}(k_1, k') f_{\nu \mu}(k') O_{\mu \lambda}(k', k_2) (\delta_{\lambda \beta} - f_{\lambda \beta}(k_2)) O_{\nu \rho}(k_2, k'') f_{\rho \psi}(k'') O_{\psi \sigma}(k'', k_1) (\delta_{\omega \beta} - f_{\omega \beta}(k_1) \}$

$- O_{\alpha \nu}(k_1, k') (\delta_{\nu \mu} - f_{\nu \mu}(k')) O_{\mu \lambda}(k', k_2) f_{\nu \rho}(k_2) O_{\nu \rho}(k_2, k'') (\delta_{\psi \rho} - f_{\psi \rho}(k'')) O_{\rho \sigma}(k'', k_1) f_{\omega \beta}(k_1) \delta(\varepsilon'_{\nu} - \varepsilon_{1 \beta} - \varepsilon_{2 \nu} + \varepsilon''_{\nu}) \}$

$+ \left\{ [\delta_{\alpha \nu} - f_{\alpha \nu}(k_1)] O_{\nu \mu}(k_1, k') O_{\mu \lambda}(k', k_2) (\delta_{\nu \psi} - f_{\nu \psi}(k_2)) O_{\nu \rho}(k_2, k'') f_{\rho \psi}(k'') O_{\psi \sigma}(k'', k_1) \delta(\varepsilon_{\beta} - \varepsilon_{2 \beta} - \varepsilon'_{\nu} + \varepsilon''_{\nu}) \}$

$- f_{\alpha \nu}(k_1) O_{\nu \mu}(k_1, k') (\delta_{\mu \lambda} - f_{\mu \lambda}(k')) O_{\nu \rho}(k_2, k'') f_{\nu \rho}(k_2, k'') O_{\nu \rho}(k_2, k'') \}$

$\left\{ \delta(\varepsilon_{1 \alpha} - \varepsilon_{\mu} + \varepsilon_{2 \psi} - \varepsilon''_{\nu}) \right\} \}. \hspace{1cm} (B3)$

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FIG. 1: (Color online) Residual resistivity