Linear conductance through a quantum dot is calculated under a finite magnetic field using
the modified perturbation theory. The method is based on the second-order perturbation theory
with respect to the Coulomb repulsion, but the self-energy is modified to reproduce the correct
atomic limit and to fulfill the Friedel sum rule exactly. Although this method is applicable only
to zero temperature in a strict sense, it is approximately extended to finite temperatures. It
is found that the conductance near electron-hole symmetry is suppressed by the application
of the magnetic field at low temperatures. Positive magnetoconductance is observed in the case of
large electron-hole asymmetry.

KEYWORDS: quantum dot, Anderson model, Kondo effect, perturbation theory, conductance, magnetic field

To date, numerous theoretical as well as experimental studies have been carried out on the tunneling trans-
port of an electron through a quantum dot connected to the leads. The combination of single electron charging
and energy level quantization causes the Coulomb blockade or Coulomb oscillation phenomena, which re-
fects the particle character of an electron. Furthermore, a dot with an odd number of electrons resembles a
magnetic impurity coupled to the conduction electrons in a metal, and hence, the Kondo-type phenomenon,
which reflects the wave character of electrons, has been predicted by several researchers and several ob-
servations of Kondo-assisted tunneling were reported recently. The Kondo effect in a quantum dot system has
brought up new and interesting issues for physics, e.g., the tunable Kondo effect or the nonequilibrium Kondo
effect.

Several theoretical methods have been devised to explain Kondo-type transport through a quantum dot
using the impurity Anderson model: second-order perturbation theory (SOPT), modified perturbation theory,
slash boson method, noncrossing approximation (NCA), quantum Monte Carlo (QMC), and numerical renormalization
group (NRG). Yeyati et al. proposed an interpolative scheme that reproduces the correct atomic limit in addition to the weak
correlation limit, and applied it to transport through a dot. We have reinvestigated their method and modified it
to create a more natural scheme by introducing the effective energy level within a dot to fulfill the Friedel
sum rule exactly. The main features of a dot were successfully calculated by this method. Our scheme, however,
was applicable only to zero temperature in the strict sense of the Friedel sum formula.

In this letter, we approximately extend our previous study to finite temperatures, and show the temperature
and external magnetic field dependence of the linear conductance through a quantum dot in the Kondo regime.
We also investigate the spin susceptibility and specific heat of the dot, and compare them with the Bethe Ansatz
solution of the Anderson model to confirm the accuracy of our method. As a result of numerical calculation,
the linear conductance through a dot decreases largely with increasing temperature near electron-hole symme-
try. This result is consistent with those of previous theoretical calculations and agrees with experimental results. As regards the magnetic field dependence, we demonstrate the reduction of the conductance near electron-hole symmetry which is similar to the tempera-
ture dependence. Moreover, our calculation predicts the positive magnetoconductance in the electron-hole asym-
metric case. Although it was already pointed out in our previous paper for a limited case, the behavior of the
conductance is clarified in the present letter for a wider range of parameters.

The Hamiltonian for a quantum dot connected to the leads is written as

\[ H = \sum_{\nu, k, \sigma} \varepsilon_k \hat{n}_{k, \sigma} + \sum_\sigma \varepsilon_0 n_{\sigma} + U n_\uparrow n_\downarrow + \sum_{\nu, k, \sigma} V_k \nu^\dagger (c_{k, \sigma}^\dagger c_{0, \sigma} + c_{0, \sigma}^\dagger c_{k, \sigma}^\dagger), \tag{1} \]

where \( \varepsilon_0 \) and \( U \) represent an energy level and the Coulomb repulsion in a dot. The change of \( \varepsilon_0 \) is equivalent to the change of the gate voltage in measurements. \( \varepsilon_k^\nu \) denotes the conduction electron energy in the lead \( \nu (= R \text{ and } L) \). \( V_k^\nu \) denotes the coupling between leads and a dot. We neglect orbital degeneracy and the \( k \) dependence of \( V_k^\nu \). We assume the quantum limit and strong coupling, therefore, the relation \( \Delta \varepsilon \gg U > \Gamma \gg T \) should be satisfied, where \( \Delta \varepsilon \) is the spacing of the discrete level, \( \Gamma \) is the resonant level width and \( T \) is the temperature.
Green’s function for the electron in the dot is given by

\[ G_{\sigma}(\omega) = \frac{1}{\omega - \varepsilon_{0} - Un_{-\sigma} - \Sigma_{\sigma}(\omega) + i\Gamma}, \]  

(2)

where \( n_{\sigma} \) denotes the electron number in the dot and \( \Sigma_{\sigma}(\omega) \) is the self-energy to be calculated in terms of the zeroth-order Green’s function \( G_{\sigma}^{(0)}(\omega) = (\omega - \varepsilon_{0} + i\Gamma)^{-1} \), \( \Gamma/2 = \Gamma_{L} = \Gamma_{R} = \pi\rho_{s}(0)V^{2} \). \( \rho_{s}(0) \) denotes the density of states (DOS) of conduction electrons at the Fermi level. In our modified perturbation theory, we introduce an effective energy level \( \tilde{\varepsilon}_{0,\sigma} \) in place of the Hartree-Fock level \( \varepsilon_{0} + Un_{-\sigma} \) and determine \( \tilde{\varepsilon}_{0,\sigma} \) to satisfy the Friedel sum rule exactly. We term this method the self-consistent energy level second-order perturbation theory (SCEL-SOPT). Thus, the effective first-order Green’s function is given by

\[ G_{\sigma}^{(1)}(\omega) = \frac{1}{\omega - \tilde{\varepsilon}_{0,\sigma} + i\Gamma}. \]  

(3)

Using this Green’s function, we first calculate the ordinary second-order self-energy \( \Sigma_{\sigma}^{(2)}(\omega) \). Then, we introduce the modified self-energy which is correct in the atomic limit ( \( \Gamma \to 0 \), \( \rho_{s}(\omega) \to \delta(\omega - \tilde{\varepsilon}_{0,\sigma}) \)) and

\[ \Sigma_{\sigma}(\omega) = \frac{\Sigma_{\sigma}^{(2)}(\omega)}{1 - \rho_{s}^{2}(\omega)}, \]  

(4)

where

\[ B = \frac{U(1 - n_{-\sigma}^{(1)}) + \varepsilon_{0} - \tilde{\varepsilon}_{0,\sigma}}{U^{2}n_{-\sigma}^{(1)}(1 - n_{-\sigma}^{(1)})}, \]  

(5)

\[ n_{\sigma}^{(1)} = n[G_{\sigma}^{(1)}] \]  

and

\[ n[G_{\sigma}] \equiv \int d\omega f(\omega)(-1/\pi)\text{Im}G_{\sigma}(\omega). \]  

(6)

Note that \( B \) vanishes in the electron-hole symmetric case ( \( \varepsilon_{0} = -U/2 \)).

Next, we construct the second-order Green’s function as

\[ G_{\sigma}^{(2)}(\omega) = \frac{1}{\omega - \varepsilon_{0} - Un_{-\sigma}^{(1)} - \Sigma_{\sigma}(\omega) + i\Gamma}. \]  

(7)

Using this \( G_{\sigma}^{(2)}(\omega) \), we calculate the second-order electron number \( n_{\sigma}^{(2)} = n[G_{\sigma}^{(2)}] \). Furthermore, \( Un_{-\sigma}^{(1)} \) in the denominator of eq. (7) is replaced by \( Un_{-\sigma}^{(2)} \) to obtain the solutions for \( \tilde{\varepsilon}_{0,\sigma} \) over a wide range. We calculate \( n_{\sigma}^{(2)} = n[G_{\sigma}^{(2)}] \) again and \( n_{\sigma}^{(2)FS} = n^{FS}[G_{\sigma}^{(2)}] \) from the Friedel sum formula

\[ n^{FS}[G_{\sigma}] \equiv \frac{1}{2} + \frac{1}{\pi} \tan^{-1} \frac{\text{Re}(G_{\sigma}(0)\text{Im}(G_{\sigma}(0))^{-1})}{\text{Re}(G_{\sigma}(0)\text{Im}(G_{\sigma}(0))^{-1})}. \]  

(8)

Then, we determine \( \tilde{\varepsilon}_{0,\sigma} \) so as to satisfy the relation

\[ n_{\sigma}^{(2)} = n_{\sigma}^{(2)FS} \]  

(Friedel sum rule).

The present scheme (SCEL-SOPT) is applicable only to zero temperature, since the Friedel sum formula is only valid in that case. However, we extend this scheme approximately to investigate the properties of a dot at finite temperatures by using \( \tilde{\varepsilon}_{0,\sigma} \) determined at zero temperature.

To confirm the accuracy of our approximation, we compare several properties with those of the Bethe Ansatz solution to the impurity Anderson model. Figure 1 shows \( T\chi_{m} \) where \( \chi_{m} \) is the spin susceptibility for \( U=4, \Gamma=1 \) and \( \varepsilon_{0}=-2 \). The unit of energy here and throughout the present letter should be meV for typical quantum dots. The full line represents the results calculated by the Bethe Ansatz solution and the circles represent our calculated results.

Fig. 1. The spin susceptibility multiplied by the temperature is shown for \( U=4, \Gamma=1 \) and \( \varepsilon_{0}=-2 \). The full line represents the results calculated by the Bethe Ansatz solution and the circles represent our calculated results.

Fig. 2. The specific heat is shown for \( U=4, \Gamma=1, h=0 \) and \( \varepsilon_{0}=-2 \). The full line represents the results calculated by the Bethe Ansatz solution and the circles represent our calculated results.
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where \( \Gamma' = \Gamma_L \Gamma_R / (\Gamma_L + \Gamma_R) \) and \( f_\nu(\omega) \) is the Fermi function of the lead \( \nu \). We utilize eq. (10) to obtain the differential conductance \( (dI/dV) \) at zero bias voltage which is equivalent to the linear conductance, where the bias voltage is given by the difference of the chemical potential between the left and right leads as \( V = \mu_L - \mu_R \).

Figure 3 shows the temperature dependence of the linear conductance through a quantum dot for \( U = 4, h = 0 \) and \( \Gamma = 1 \) (a), 0.5 (b).

\[ I = \frac{2e}{h} \sum_\sigma \int d\omega \{ f_L(\omega) - f_R(\omega) \} \Gamma'_\sigma(\omega), \tag{10} \]

where \( \Gamma'_\sigma(\omega) \) is given by eq. (10) to calculate the transport properties in a linear response regime.

Figure 3 shows the temperature dependence of the linear conductance through a quantum dot for \( U = 4, h = 0 \) and \( \Gamma = 1 \) (a), 0.5 (b) as a function of the gate voltage, where \( h = \frac{g}{2} \mu_B H \). The full, broken, dotted and dash-dotted lines represent the results for \( T = 0, 0.05, 0.1 \) and 0.2, respectively. At low temperatures, the Kondo peak of DOS emerges at the Fermi level in the symmetric case, and hence, the conductance takes the value \( 2e^2/h \) due to the Kondo-assisted tunneling in that case. In the case of electron-hole asymmetry, the Kondo peak shifts from the Fermi level and the conductance is reduced. As the Kondo effect does not appear in the extreme asymmetric case, \( \varepsilon_0 < -4 \) or \( \varepsilon_0 > 0 \) for example, the conductance does not depend on the temperature. In Fig. 3(b), the conductance exhibits two peaks for \( T = 0.2 \) which is nothing but the Coulomb oscillation. The peaks appear when the discrete energy level of a quantum dot \( \varepsilon_0 \) or \( \varepsilon_0 + U \) agrees with the Fermi level. These two peaks are not seen clearly in Fig. 3(a) since the width of the resonant level \( \Gamma \) is large. These temperature dependencies of the conductance were already obtained theoretically by other groups \( 5, 21, 14 \) and our numerical calculation is consistent with their results. Recent experiments \( 7, 8, 9 \) have revealed Kondo-assisted tunneling at low temperatures that demonstrates qualitative agreement with our results.

Figure 4 shows the magnetic field dependence of the conductance for \( U = 4, \Gamma = 1 \) and \( T = 0 \) (a), 0.05 (b) and 0.1 (c).

\[ I = \frac{2e}{h} \sum_\sigma \int d\omega \{ f_L(\omega) - f_R(\omega) \} \Gamma'_\sigma(\omega), \tag{10} \]
site feature emerges which enhances the conductance by the application of the magnetic field, namely, positive magnetoconductance. As regards the sign of the magnetoconductance, we assume that, in the range of negative magnetoconductance, the dot contains an odd number of electrons and the Kondo-like effect occurs. Otherwise, there are an even number of electrons and the Kondo effect does not occur. Furthermore, we observe that increasing the temperature weakens the magnetic field effect.

In conclusion, we have studied the transport properties through a quantum dot in the Kondo regime by the modified perturbation theory (SCEL-SOPT), which modifies the self-energy so as to reproduce the correct atomic limit and to fulfill the Friedel sum rule exactly. We extended this theory to study the finite temperature properties of a quantum dot. The accuracy of our scheme was confirmed by comparing the spin susceptibility and the specific heat with the exact solution. It has been demonstrated that our scheme can be used for a wide range of temperatures and magnetic field strengths. As regards the transport properties through a dot, we noted the reduction of the conductance near electron-hole symmetry under magnetic field at low temperatures, which explains the recent experimental results. We also showed the positive magnetoconductance in the case of asymmetry. Such properties may also be observed in the experimental results. During the preparation of the present manuscript, we discovered a paper which also calculated the magnetic field dependence of the conductance through a dot by QMC. The result is consistent with that of our calculation near the electron-hole symmetric case. However, because of the use of QMC, their calculation does not cover sufficiently low temperatures or a wide range of gate voltages. Our approach is much simpler with sufficient accuracy and may have wide potential applicability to various situations in quantum dot systems.

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