Direct capture astrophysical $S$ factors at low energy

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Abstract

We investigate the energy dependence of the astrophysical $S$ factors for the reactions $^7\text{Be}(p, \gamma)^8\text{B}$, the primary source of high-energy solar neutrinos in the solar $pp$ chain, and $^{16}\text{O}(p, \gamma)^{17}\text{F}$, an important reaction in the CNO cycle. Both of these reactions have predicted $S$ factors which rise at low energies; we find the source of this behavior to be a pole in the $S$ factor at a center-of-mass energy $E = -E_B$, the point where the energy of the emitted photon vanishes. The pole arises from a divergence of the radial integrals.
The $^7\text{Be}(p,\gamma)^8\text{B}$ reaction, at center-of-mass energies $E$ near 20 keV, plays an important role in the production of solar neutrinos [1]. The neutrinos from the subsequent decay of $^8\text{B}$ provide the high energy neutrinos to which many solar neutrino detectors are sensitive. The cross section for this reaction is conventionally expressed in terms of the $S_{17}$ factor, where the $S$ factor is defined in terms of the cross section $\sigma$ by

$$S(E) = \sigma(E)E \exp(2\pi\eta(E)),$$

(1)

where $\eta(E) = Z_1Z_2\alpha\sqrt{\mu c^2/2E}$ is the Sommerfeld parameter for nuclei of charges $Z_1, Z_2$ and reduced mass $\mu$. The exponential factor in the definition of $S$ removes the rapid energy dependence of the cross section due to Coulomb repulsion between the two nuclei. In the stellar core the probability of capture of protons by $^7\text{Be}$, obtained by folding the thermal distribution of nuclei with the cross section, peaks at $\sim 20$ keV. Because the cross section diminishes exponentially at low energies, the only method of obtaining information about $S_{17}$ at those energies is to extrapolate data taken at experimentally accessible energies ($E > 100$ keV).

The $^{16}\text{O}(p,\gamma)^{17}\text{F}^\ast (\frac{1}{2}^+, \frac{1}{2}^-, 0.498$ MeV [2]) reaction, which occurs in the CNO cycle, is of little importance for energy production in the sun but of greater importance for hotter stars. As is the case for $^7\text{Be}(p,\gamma)^8\text{B}$, extrapolation of data taken at high energies is necessary to obtain the $S$ factor at energies applicable in the stellar core, $E \sim 25$ keV.

Direct capture calculations [3] of these two reactions [4–6] predict an upturn in the $S$ factor at threshold. As the capture in both reactions is primarily external, the $S$ factors at astrophysical energies are determined by the product of the spectroscopic factor, the asymptotic normalization of the final (bound) state wave functions, and a purely Coulombic term. As the spectroscopic factor is independent of energy, the energy dependence of the $S$ factor, away from resonances, may be studied without detailed knowledge of the nuclear structure.

In each reaction the weakly bound final state ($E_B = 137.5$ keV [4] for $^8\text{B}$ and $E_B = 105.2$ keV for the first excited state of $^{17}\text{F}$ [3]) causes the $S$ factor to rise as the center-of-mass energy approaches 0. In the case of $^{16}\text{O}(p,\gamma)^{17}\text{F}^\ast$ both the data and direct-capture calculations [4,8] show clear evidence of this low-energy rise. The $^{16}\text{O}(p,\gamma)^{17}\text{F}$ capture to the ground state shows no such rise because the final state is more deeply bound and has higher angular momentum. For $^7\text{Be}(p,\gamma)^8\text{B}$ the upturn occurs below the lowest experimental point so it is only observed in the calculations.

Williams and Koonin [5] do an explicit expansion about zero energy for $S_{17}$ and give the first two coefficients in a Taylor series for the logarithmic derivative of $S_{17}$ as $-2.350$ MeV$^{-1}$ and $28.3$ MeV$^{-2}$. Using these in a (1,1) Padé approximant gives:

$$S_{17} = \frac{(1 + 4.85E)}{(1 + 7.20E)},$$

(2)

where $E$ is in MeV. This Padé approximant has a pole at $-139$ keV which is very close to their bound state energy of 136 keV. Thus we see that in the region of the threshold the bound state is important and induces a pole. In fact, a Taylor series expansion would converge only with a radius of the binding energy — barely to the region that is experimentally accessible. Hence any functional form for the extrapolation to zero energy should contain the contribution from the pole.

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Following [3], we write the astrophysical $S$ factor as

$$S = C(I_0^2 + 2I_2^2)E_\gamma^3 \left(J_{11}\beta_{11}^2 + J_{12}\beta_{12}^2\right)\frac{1}{1 - e^{-2\pi\eta}}, \quad (3)$$

where

$$I_L = \int_0^\infty dr \, r^2 \psi_{iL}(r)\psi_f(r)/k \quad (4)$$

$$C = \frac{5\pi}{9} \frac{1}{(\hbar c)^3} (2\pi\eta k)^2 \mu^2 \left(\frac{Z_1}{M_1} - \frac{Z_2}{M_2}\right)^2. \quad (5)$$

In Eq. (3), $J_{LS}$ is the spectroscopic factor for a given angular momentum, $L$, and channel spin, $S$, $\beta_{LS}$ is the asymptotic normalization of the bound state wave function, $E_\gamma$ is the photon energy, and $k$ is the momentum of the incident proton. The final bound state wave function $\psi_f(r)$ is normalized asymptotically to $\psi_f(r) = W_{\alpha l}(kr)/r$ while the initial wave function reduces to the regular Coulomb wave function divided by $\sqrt{2\pi\eta}/(e^{2\pi\eta} - 1)$. The unusual choice of normalizations is just to eliminate uninteresting factors from quantities of interest. Most of those factors have been collected in the coefficient $C$.

To investigate the behavior of the integrals in Eq. (3), we first consider $\psi_f(r) = W_{\alpha l}(kr)/r$ for all radii and take $\psi_{i0}(r) = F_0(kr)/((2\pi\eta)/\sqrt{2\pi\eta}/(e^{2\pi\eta} - 1))$. The integral for the $s$-wave then becomes

$$I_0 = \int_0^\infty dr \, rW_{\alpha l}(kr)F_0(kr)/(k\sqrt{2\pi\eta}/(e^{2\pi\eta} - 1)). \quad (6)$$

At threshold, the integrands are peaked at large $r$: 40 fm for $^7$Be($p, \gamma)^8$B, and 65 fm for $^{16}$O($p, \gamma)^{17}$F*. The tails of both integrands extend well beyond 100 fm, and are, in each case, indicative of halo states. The integral is smooth as $k$ passes through zero and diverges as $k \rightarrow i\kappa$ ($E \rightarrow -E_B$). The nature of the divergence is determined by the asymptotic forms of the Coulomb wave function and Whittaker function for large $r$. For large $r$ the Whittaker function is proportional to $r^{-|\eta k|/\kappa}e^{-\kappa r}$ [3] ($\eta k$ is independent of $k$). While above threshold the Coulomb wave function oscillates at large radii, below threshold it is exponentially growing and is proportional to $r^{|\eta|}e^{k|r|}$. Thus the integrand approaches

$$r^{-|\eta k|(1/k - 1/|k|)} \exp[-(\kappa - |k|)r] \quad (7)$$

for large $r$ and the integral diverges as

$$I_0 \propto 1/(\kappa - |k|)^2 \propto 1/(E_B + E)^2 = 1/E_\gamma^2. \quad (8)$$

Since the integrand diverges as $1/E_\gamma^2$ the leading term and first correction term are both determined purely by the asymptotic behavior of the wave functions. The first correction term is not simply $1/E_\gamma$ but also involves logarithmic terms coming from the $r^{-|\eta k|(1/k - 1/|k|)}$ factor. The second correction term, of order $E_\gamma^0$, is not determined purely by the asymptotic value of wave function alone but also depends on the wave function at finite $r$.

From Eq. (3), we see that the quadratic divergence of $I_0$ gives rise to a simple pole in $S$ at $E_\gamma = 0$. This suggests writing the $S$ factor as a Laurent series:

$$S = d_{-1}E_\gamma^{-1} + d_0 + d_1E_\gamma + \ldots \quad (9)$$
As before the coefficients of the first two terms, \( d_{-1} \) and \( d_0 \), are determined purely by the asymptotic forms of the wave functions while the third coefficient, \( d_1 \), is also dependent on the short range properties of the wave functions.

In Fig. 1, we present the data of Morlock et al. [8] for the \(^{16}\text{O}(p, \gamma)^{17}\text{F}^*\) \( S \) factor (top) and for the product \( E_\gamma S \) (bottom). In the top panel, the energy dependence of the \( S \) factor is well approximated by the form:

\[
S = n \frac{1 + c_1 E}{E_\gamma} = n \frac{1 + c_1 E}{E + E_B}
\]

(10)

where the constants \( c_1 \) and \( n \) are determined by the straight line fit to \( E_\gamma S \) shown in the bottom panel. The numerical values are given in Table I. There is remarkable agreement with the data except near the resonance at 2.504 MeV. Eq. (10) is a convenient form for fitting experimental data and is motivated by both the Padé approximant and Eq. (9).

In Fig. 2, the data of Filippone [9] (circles) and Kavanagh [10] (diamonds) for the \( S \) factor (top panels) and the product \( E_\gamma S \) (bottom panels) for the \(^7\text{Be}(p, \gamma)^8\text{B}\) reaction are presented for energies well below the \( E = 633 \) keV M1 resonance. We take the data as normalized by Johnson et al. [11] to \( \sigma_{dp} = 157 \) mb. The curves are similar in form to Eq. (10), but with a quadratic term added,

\[
S = n \frac{1 + c_1 E + c_2 E^2}{E_\gamma} = n \frac{1 + c_1 E + c_2 E^2}{E + E_B}.
\]

(11)

The values of \( n \) and \( c_i \) for this reaction are also listed in Table I. Different values of the normalization \( n \) are required to reproduce the Filippone and Kavanagh data, but the \( c_i \) are determined from the threshold energy dependence of a direct-capture calculation following Ref. [5]. A cut-off radius of \( r_0 = 2.3 \) fm was chosen to be consistent with the phase shift and energy dependence found by Barker [12]. The upturn at threshold is clearly observed in the results of the calculation. The data are insufficient to determine this behavior or, equivalently, \( c_i \). It will be very difficult to experimentally confirm this upturn since it is only pronounced below 100 keV. Fortunately it is theoretically well understood and both \( n \) and \( c_1 \) depend primarily on the asymptotic normalization, spectroscopic factor and properties of the Coulomb force. Note that the curves presented in Fig. 2 should not be mistaken for a serious attempt at determining the \( S_{17} \) factor at zero energy; rather, they are illustrative of the energy dependence.

The straight line approximation for \( E_\gamma S \) is valid for the \(^{16}\text{O}(p, \gamma)^{17}\text{F}^*\) reaction up to \( \approx 3 \) MeV. However, the quadratic approximation for the \(^7\text{Be}(p, \gamma)^8\text{B}\) reaction is not valid for energies above 0.4 MeV. Initially, the breakdown is caused by the resonance at 0.633 MeV. Above the resonance higher order terms in \( E \), arising predominantly from \( d \)-wave direct capture, become significant.

In conclusion we see that the threshold peak in the \( S \) factor is associated with weakly bound states and arises from a pole at \( E_\gamma = 0 \). Those bound states in \(^8\text{B}\) and \(^{17}\text{F}\) are halo in nature and so the associated radial integrals are, by necessity, long range.

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FIG. 1. Astrophysical $S$ factor (top) and $E_\gamma S$ (bottom) for $^{16}\text{O}(p, \gamma)^{17}\text{F}^*$. The data of Morlock et al. [8] are compared to the fit as described in the text (solid line).
FIG. 2. The low-energy part of the astrophysical $S$ factor and $E_\gamma S$ for $^7\text{Be}(p, \gamma)^8\text{B}$. The data of Filippone [9] (circles) and Kavanagh [10] (diamonds) are compared to the results of the calculations as described in the text (solid line).
TABLES

TABLE I. The numerical constants $c_i$ and $E_B$ used to determine the energy dependences, with the normalizations $n$ used when displaying the curves with the discussed data sets.

| Reaction      | $c_1$ (MeV$^{-1}$) | $c_2$ (MeV$^{-2}$) | $E_B$ (MeV) | $n$ (keV$^2$b) |
|---------------|--------------------|--------------------|-------------|----------------|
| $^{16}$O$(p, \gamma)^{17}$F* | 1.18               | 0                  | 0.1052      | $1.59 \times 10^3$ |
| $^{7}$Be$(p, \gamma)^{8}$B   | 5.36               | 1.80               | 0.1375      | 3.74$^a$       |
|                |                    |                    |             | 2.99$^b$       |

(a) Kavanagh et al. $^{[10]}$
(b) Filippone et al. $^{[9]}$