Electromagnetically induced absorption in Rb atoms with circular polarization of laser beams: Effects of neighboring transitions

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1. Introduction

Since the discovery of electromagnetically induced absorption (EIA) and electromagnetically induced transparency (EIT) in the 1990s, various studies using Hanle [1, 2, 3, 4, 5, 6, 7, 8] and coupling-probe configurations [9, 10, 11, 12] have been performed. However, novel neighboring state effects have not been investigated extensively. These effects could govern unexpected EIA or EIT as we describe below.

Unresolved atomic systems under Doppler broadening effects present significant differences in observed EIA and EIT spectra due to the neighboring state contributions in a degenerate multilevel system [13]. In [13], we experimentally and theoretically confirmed that among the adjacent hyperfine transitions, the dominant cycling transition amplitude governs the observed spectral shapes at $^{85}$Rb D2 transitions in a linear orthogonal polarization configuration. In a linear parallel polarization configuration of coupling and probe lasers, even in the case of the closed system of $F_g = 3 \rightarrow F_e = 4$ of $^{85}$Rb, EIA was observed instead of EIA due to the neighboring effect of the dominant $F_g = 3 \rightarrow F_e = 3$ transition in the low-power regime [14].

In this work, EIA and EIT at $F_g = 2 \rightarrow F_e = 1, 2$ and 3 of $^{87}$Rb and $F_g = 3 \rightarrow F_e = 2, 3,$ and 4 of $^{85}$Rb D2 transitions, as shown in Fig. 1(a) and (b) with relevant atomic energy level diagrams of $^{87}$Rb and $^{85}$Rb D2 transition line, respectively are investigated to understand the mechanisms of neighboring effects with the same circular polarization configuration ($\sigma^+ - \sigma^-$) of coupling and probe lasers. EIAs at all transitions of $F_g = 2 \rightarrow F_e = 2, 3$ of $^{87}$Rb and $F_g = 3 \rightarrow F_e = 2, 3$, and 4 of $^{85}$Rb D2 lines are observed. However, only the $F_g = 2 \rightarrow F_e = 1$ transition of $^{87}$Rb transforms to EIT instead of EIA when the same circular polarizations of coupling and probe laser beams are used.

We realize that such differences result from different hyperfine energy splittings of $^{85}$Rb and $^{87}$Rb. We theoretically confirm that EIT for the $F_g = 2 \rightarrow F_e = 1$ transition of $^{87}$Rb under the same circular polarization configuration transforms into EIA for the $F_g = 3 \rightarrow F_e = 2$ transition of $^{85}$Rb owing to the increase in the neighboring effects with a decrease in the hyperfine energy splittings.

The simulated absorption profile considering the neighboring effect matches well with the experimental results. Coupling and probe fields produced by a single laser, combined with two acousto-optic modulators (AOMs) in a coupling-probe configuration, are used to experimentally investigate the effects on EIA (EIT) resonances in a weak coupling-probe power regime.

2. Theoretical calculation

EIA spectra were simulated via the same method used in our previous study [13] for the relevant atomic energy level diagrams shown in...
Fig. 1. Relevant atomic hyperfine energy level diagrams of (a) $^{87}$Rb and (b) $^{85}$Rb D2 transition lines, where the blue and red lines indicate the transitions by the coupling and probe beams, respectively.

Fig. 1(a) and (b). The only difference between this study and Ref. [13] was the polarization configuration: instead of an orthogonal linear polarization, the same circular polarization configuration ($\sigma^+ - \sigma^-$) of coupling and probe lasers was investigated. Here, we describe the method of calculation for $^{87}$Rb briefly. The calculation for $^{85}$Rb can be performed in an analogous method.

The density matrix equation governing the internal dynamics of the atom is given by

$$\dot{\rho} = -(i/\hbar) [H_0 + V, \rho] + \dot{\rho}_{\text{relax}},$$

where the atomic ($H_0$) and interaction ($V$) Hamiltonians are given

$$H_0 = - \sum_{m=-3}^{3} \hbar \delta_m \left| F_m = 3, m \right\rangle \langle F_m = 3, m \right|$$

$$- \sum_{m=-2}^{2} \hbar \left( \delta_m + \Delta_{\text{D2}} \right) \left| F_m = 2, m \right\rangle \langle F_m = 2, m \right|$$

$$- \sum_{m=-1}^{1} \hbar \left( \delta_m + \Delta_{\text{D1}} \right) \left| F_m = 1, m \right\rangle \langle F_m = 1, m \right|,$$

$$V = \frac{\hbar}{2} \sum_{k=1}^{2} \sum_{m=-2}^{2} \left( \Omega_k + \Omega_k e^{i\delta_k} \right) C_{F,m+1}^{k,m+1}$$

$$\times \left| F_m, m + 1 \right\rangle \langle F_m = 2, m \right| + \text{h.c.},$$

respectively. In Eq. (1), $\dot{\rho}_{\text{relax}}$ represents the relaxation terms owing to the spontaneous emission and transit time decay, and is defined in Eq. (18) of Ref. [15]. In Eq. (2), $\delta_k(= \delta_d - kv)$ is the effective detuning of the coupling beam where $d$ denotes the detuning of the coupling beam, $k(=2\pi/x)$ is the wave vector, $x$ is the resonance wavelength of the laser, and $v$ is the velocity of the atom. $\Delta_{\text{D2}}(\Delta_{\text{D1}})$ is the frequency spacing between the hyperfine states $|F_m = 3\rangle$ and $|F_m = 2(1)\rangle$. In Eq. (3), $\Omega_k(\Omega_k)$ is the Rabi frequency of the coupling (probe) beam, $C_{F,m+1}^{k,m+1}$ is the normalized transition strength between $|F_m,m\rangle$ and $|F_m,m\rangle$, and h.c. denotes the Hermitian conjugate. $\delta_k(= \delta_d - kv)$ is the effective frequency of the probe beam relative to the frequency of the coupling beam, and $\delta_d(= \delta_d - kv)$ is the effective detuning of the probe beam where $d$ denotes the detuning of the probe beam.

As reported in Ref. [13], the Zeeman coherences vanish. In addition, the populations are decomposed as $p_j = p_j^{(1)} e^{-i\delta_d t} + p_j^{(2)} e^{-i\delta_d t} + p_j^{(3)} e^{i\delta_d t}$ where $j$ runs for all the magnetic sublevels [17]. The optical coherences between the sublevels with $m_l - m_s = 1$ are decomposed as follows [17, 18]:

$$p_{F_m,n+1}^{(1)} s_n = p_{F_m,n+1}^{(2)} s_n + p_{F_m,n+1}^{(3)} e^{-i\delta_d t} + p_{F_m,n+1}^{(4)} e^{i\delta_d t},$$

where $p_{F_m,n+1}^{(1)} s_n \equiv \langle F_m, m + 1 | \rho | F_m = 2, m \rangle$. The absorption coefficient of the probe beam is given by

$$\alpha = \frac{3\lambda^2 N_a \Gamma}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-i\nu^2/\sigma^2}}{\sqrt{\nu}} \sum_{m=-2}^{2} \sum_{s_m}^{s_m+1} C_{F,m+1}^{s_m+1,m+1} e^{i\nu \delta_d t}.$$

where $\Gamma$ is the decay rate of the excited, $N_a$ is the atomic number density, and $\nu$ is the most probable speed in the vapor cell.

3. Experimental methods

The experimental setup is shown in Fig. 2. A beam from a single tunable external cavity diode laser (DLPro, Toptica Inc.) with the wavelength of 780 nm is separated by a polarizing beam splitter (PBS). The probe beam and the pump beam were generated from a single laser with two AOMs. A half-wave plate (HWP) in front of the PBS controlled the beam powers of coupling and probe beams. The powers of the probe and coupling beams were 15 $\mu$W and 50 $\mu$W, respectively. The BS is used to overlap the co-propagating coupling and probe beams inside a vapor cell. A HWP in front of the PBS controlled the beam powers of coupling and probe beams. The powers of the probe and coupling beams were 15 $\mu$W and 50 $\mu$W, respectively.

The scanning of the coupling beam was scanned by transmitting the beam through two AOMs in a single-pass configuration. The modulation frequency for one of AOMs was fixed at 80 MHz and the other was scanned by $\pm$1 MHz at the center of $-80$ MHz. We used Tektronix RF spectrum analyzer to calibrate the time axis (frequency) of the oscilloscope. The reason why coupling beam was scanned is to remove the Doppler broadening. It is worth noting that the spectra with the scanning of coupling detuning and probe detuning exhibit no discernable difference except for the existence of Doppler background in the probe detuning scanning case [19]. This is because ET and EIA are coherent phenomena. Both the initial coupling and probe beams were expanded five times to have homogeneous beam of a diameter of 4 mm. A beam splitter was used to overlap the co-propagating coupling and probe beams inside a vapor cell. A vapor cell with the length and a diameter of 5 cm and 2.5 cm, respectively was shielded with five layers of $\mu$-metal sheets that prevented stray and the Earth's magnetic fields from affecting the inside of the cell. The stray magnetic field was ap-
proximately several mG, which may not cause significant variation in the EIT and EIA spectra [9].

4. Results and discussion

Spectrally resolved groups of the $F_g = 2 \rightarrow F_e = 1, 2,$ and 3 transitions of $^{87}$Rb and $F_g = 3 \rightarrow F_e = 2, 3,$ and 4 transitions of $^{85}$Rb were observed and simulated with respect to circular polarizations of probe and coupling fields in the low power regime. The results were analyzed based on the $\sigma^+ - \sigma^-$ polarization configuration for the $^{87}$Rb and $^{85}$Rb D2 lines and are shown in Figs. 3 and 4, respectively. Because we concentrated on the conversion between EIA and EIT which can be observed efficiently at the weak intensity of the coupling beam, we performed experiments and calculations in the low power regime.

A comparison of the simulated and observed spectra of the $^{87}$Rb D2 line is shown in Fig. 3 considering (a) degenerate two-level system (DTLS) with resonant transitions of $F_g = 2 \rightarrow F_e = 1, F_g = 2 \rightarrow F_e = 2,$ and $F_g = 2 \rightarrow F_e = 3$; (b) resonant transition of $F_g = 2 \rightarrow F_e = 1$ with neighboring hyperfine transitions of $F_g = 2 \rightarrow F_e = 2$ and 3; resonant transition of $F_g = 2 \rightarrow F_e = 2$ with neighboring transitions of $F_g = 2 \rightarrow F_e = 1$ and 3; resonant transition of $F_g = 2 \rightarrow F_e = 3$ with neighboring transitions of $F_g = 2 \rightarrow F_e = 1$ and 2; and (c) experimentally observed spectra for the resonant transitions of $F_g = 2 \rightarrow F_e = 1, F_g = 2 \rightarrow F_e = 2$ and $F_g = 2 \rightarrow F_e = 3$.

A relatively strong EIA amplitude with a narrow linewidth resolution of 58 kHz at the $F_g = 2 \rightarrow F_e = 3$ closed transition and weak EITs with linewidths of 57 kHz and 73 kHz at $F_g = 2 \rightarrow F_e = 1$ and $F_g = 2 \rightarrow F_e = 2$ open transitions, respectively, were obtained, using a DTLS calculation as shown in Fig. 3(a). Here, the linewidth is defined as the full-width at the half maximum (FWHM). However, the spectra obtained considering all the neighboring transitions and experiments showed a striking similarity in the trends presented in Figs. 3(b) and 3(c). The EIT at the $F_g = 2 \rightarrow F_e = 1$ open transition and EIAs at the $F_g = 2 \rightarrow F_e = 2$ open and $F_g = 2 \rightarrow F_e = 3$ closed transitions were resolved with linewidths of 69 kHz, 47 kHz, and 54 kHz (91 kHz, 41 kHz, and 117 kHz), respectively, considering all the neighboring transitions calculations (experiments). We note that there are substantial deviations in the linewidths between the experimental and theoretical spectra. The discrepancy may result from the effect of the ambient magnetic field, imperfect alignment of coupling and probe laser beams, and other effects that are not included in the calculation of the spectra.

The strong EIA amplitude owing to the same polarization between the coupling and probe fields resulted from a transfer of population [20], producing a coherence between the same magnetic sublevels at the $F_g = 2 \rightarrow F_e = 3$ closed transition, which dominated the nearby neighboring $F_g = 2 \rightarrow F_e = 2$ open transition to transform into EIA, as shown in the middle traces of Figs. 3(b) and 3(c). However, the $F_g = 2 \rightarrow F_e = 1$ open transition was strongly influenced by the near neighboring $F_g = 2 \rightarrow F_e = 2$ open transition, which resulted in relatively strong amplitudes of EIT, as shown in the bottom traces of Figs. 3(b) and 3(c) compared to the amplitudes obtained considering DTLS, as indicated in the bottom trace of Fig. 3(a).

A comparison of the simulated and observed spectra of the $^{85}$Rb D2 line with the same circular polarization of the coupling and probe fields is shown in Fig. 4 considering (a) DTLS with resonant transitions of $F_g = 3 \rightarrow F_e = 2, F_g = 3 \rightarrow F_e = 3,$ and $F_g = 3 \rightarrow F_e = 4$; (b) resonant transition of $F_g = 3 \rightarrow F_e = 2$ with neighboring hyperfine transitions of $F_g = 3 \rightarrow F_e = 3$ and 4, resonant transition of $F_g = 3 \rightarrow F_e = 3$ with neighboring transitions of $F_g = 3 \rightarrow F_e = 4$ and 2, resonant transition of $F_g = 3 \rightarrow F_e = 4$ with neighboring transitions of $F_g = 3 \rightarrow F_e = 2$ and 3; and (c) experimentally observed spectra for the resonant transitions of $F_g = 3 \rightarrow F_e = 2, F_g = 3 \rightarrow F_e = 3,$ and $F_g = 3 \rightarrow F_e = 4$.

For the DTLS simulation shown in Fig. 4(a), strong amplitude EIA dip with a 59 kHz linewidth at the $F_g = 3 \rightarrow F_e = 4$ transition and EIAs with 79 kHz and 69 kHz linewidths at the $F_g = 3 \rightarrow F_e = 2$ and $F_g = 3 \rightarrow F_e = 3$ transitions, respectively, were well resolved as expected. Spectral features at the $F_g = 3 \rightarrow F_e = 2, F_g = 3 \rightarrow F_e = 3,$ and $F_g = 3 \rightarrow F_e = 4$ transitions of $^{85}$Rb D2 line exhibited similar trends to...
those simulated for the $^{87}\text{Rb}$ D2 line presented in Fig. 3(a). However, compared to the spectra obtained for the $^{87}\text{Rb}$ D2 line, the simulations considering all the neighboring states and experiments showed EIA at all the resonant transitions of $F_g = 3 \rightarrow F_e = 2$, $F_g = 3 \rightarrow F_e = 3$, and $F_g = 3 \rightarrow F_e = 4$ of $^{85}\text{Rb}$ atoms owing to strong Doppler overlapping between excited states hyperfine levels, as indicated in Figs. 4(b) and (c). EIs were resolved with 49 kHz, 49 kHz, and 50 kHz (62 kHz, 85 kHz, and 137 kHz) linewidths, considering all neighboring state simulations (experiments) at the $F_g = 3 \rightarrow F_e = 2$, $F_g = 3 \rightarrow F_e = 3$, and $F_g = 3 \rightarrow F_e = 4$ transitions, respectively.

To understand the opposite behavior of the coherent resonances for the $F_g = 2 \rightarrow F_e = 1$ transition in $^{87}\text{Rb}$ and the $F_g = 3 \rightarrow F_e = 2$ transition in $^{85}\text{Rb}$ shown in Figs. 3(c) and 4(c), respectively, we simulated spectral variations for the $F_g = 3 \rightarrow F_e = 2$ transition in $^{85}\text{Rb}$ by varying the hyperfine splittings of $^{85}\text{Rb}$ artificially. The results are presented in Fig. 5. The ratio in Fig. 5 implies the artificial enhancement factor of the hyperfine splittings of $^{85}\text{Rb}$. We note that a factor of 2.3 approximately corresponds to the hyperfine splittings of $^{87}\text{Rb}$. The trace at the unity ratio in Fig. 5, exhibiting EIA, is identical to that in the lowest panel of Fig. 4(b). As the ratio increases, the spectral lineshape changes and the EIA is transformed to EIT at a ratio approximately between 1.5 and 1.8. When a ratio is 2.3 corresponding to $^{87}\text{Rb}$ hyperfine splitting, we observe EIA as shown in Fig. 3(b). We note that there exist no significant differences between the spectra of the $^{87}\text{Rb}$ and $^{85}\text{Rb}$ atoms as long as the hyperfine splittings are artificially set to similar values. In Fig. 5, we can clearly see that the opposite behavior of the resonances for the $F_g = 2 \rightarrow F_e = 1$ transition in $^{87}\text{Rb}$ and the $F_g = 3 \rightarrow F_e = 2$ transition in $^{85}\text{Rb}$ results from the weak influence of the $F_g = 2 \rightarrow F_e = 3$ transition in $^{85}\text{Rb}$ owing to the relatively large hyperfine splitting compared to $^{87}\text{Rb}$. Qualitatively, as the hyperfine splitting increases, the neighboring effects come from the atoms with higher velocities. Accordingly, the effect on EIA and EIT spectra from neighboring transition is weakened.

![Fig. 4. Comparison of simulated and observed spectra at $F_g = 3 \rightarrow F_e = 2$, 3, and 4 of $^{85}\text{Rb}$ D2 line considering (a) DTLS with resonant transitions of only $F_g = 3 \rightarrow F_e = 2$, $F_g = 3 \rightarrow F_e = 3$, and $F_g = 3 \rightarrow F_e = 4$ shown vertically in (a); (b) all neighboring hyperfine transitions near the resonant (neighboring) transitions of $F_g = 3 \rightarrow F_e = 2$ (3 and 4), $F_g = 3 \rightarrow F_e = 3$ (2 and 4), and $F_g = 3 \rightarrow F_e = 4$ (2 and 3); and (c) experimentally observed spectra for the resonant transitions of $F_g = 3 \rightarrow F_e = 2$, $F_g = 3 \rightarrow F_e = 3$, and $F_g = 3 \rightarrow F_e = 4$.](image)

![Fig. 5. Spectral variations with respect to the artificial variation in the hyperfine splittings of $^{87}\text{Rb}$. The unity ratio corresponds to the hyperfine splitting of $^{87}\text{Rb}$ and a ratio of 2.3 corresponds to $^{87}\text{Rb}$ hyperfine splitting.](image)

5. Conclusions

In this work, EIA and EIT at the $F_g = 2 \rightarrow F_e = 1$, 2, and 3 transitions of $^{87}\text{Rb}$ and $F_g = 3 \rightarrow F_e = 2$, 3, and 4 transitions of $^{85}\text{Rb}$ D2 lines were investigated to understand the mechanisms of the neighboring effects with the same circular polarization configurations ($\sigma^+ - \sigma^-$) of coupling and probe lasers. EIA and EIT were observed when $F_g > F_e$ for $^{87}\text{Rb}$ and $^{87}\text{Rb}$, respectively, for the first time under the same circular
polarization configuration of the coupling and probe beams, although all cases where $F_g \leq F_e$ for both $^{85}$Rb and $^{87}$Rb exhibited EIAs.

The reason why spectrum remained an ET for the open $F_g = 2 \rightarrow F_e = 1$ transition of $^{85}$Rb in all neighboring transition simulations despite the dominance of EIA was that the DTLS was far away from the dominant EIA transition line; thus, it remained in all neighboring transition simulations as it behaved in DTLS. By contrast, the spectrum for the $F_g = 3 \rightarrow F_e = 2$ transition of $^{85}$Rb continued to be EIA in all neighboring transition simulations, because the DTLS was close to the dominant EIA transition line. The opposite behavior of the coherent resonances for the $F_g = 2 \rightarrow F_e = 1$ transition in $^{85}$Rb and the $F_g = 3 \rightarrow F_e = 2$ transition in $^{85}$Rb was confirmed by calculating the spectral variations for the $F_g = 3 \rightarrow F_e = 2$ transition in $^{85}$Rb by varying the hyperfine splittings of $^{85}$Rb artificially.

**Declarations**

**Author contribution statement**

Zeeshan Ali Safdar Jadoon: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper. Aisar Ul Hassan: Performed the experiments. Heung-Ryoul Noh & Jin-Tae Kim: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

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**Data availability statement**

No data was used for the research described in the article.

**Declaration of interests statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

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