Analysis and research on wheel steering motion of four-wheel locomotive

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Abstract: Main purpose of this paper is to analyze the kinematics characteristics of four-wheeled vehicles. Based on the kinematic discrete recursion formula and the kinematic relationship between the wheel and the fuselage, the kinematic discrete recursive model of the system is established, and the theoretical formulas of the front wheel steering and four wheel steering vehicle models are derived respectively. At the same time, Simulink was used in Matlab to simulate the driving process of the four-wheeled vehicle, and the steering angle of the wheel relative to the vehicle body and the trajectory of the front and rear wheels during the driving of the four-wheeled vehicle were obtained. The derived model data is used to simulate the satellite positioning wheel trajectory, and the variation law of the wheel steering angle during the driving process of the four-wheeled vehicle is obtained. The correctness of the analysis process and method is verified by theoretical derivation and numerical simulation, which provides a theoretical basis for the study of kinematics characteristics of smart cars in the future.

1. Introduction
Since the 1970s, developed countries such as the United States and Europe began to study smart cars. Automatic vehicle technology can be divided into automatic vehicle and collaborative intelligent transport system (C - ITS) (Europe), also known as networking technologies (USA) [1]. C - ITS connected vehicle technology to a large extent depends on the vehicular Ad Hoc network (VANET) to transmit C - ITS collaboration in the sense basic safety messages in the message or connected vehicle technology [2]. On the other hand, the autonomous vehicle is usually combined with different techniques to realize the expected level of autonomy. For example, Zhang F et al. [3] combine light detection and ranging systems (LIDAR) with stereo cameras to detect nearby vehicles. In addition, researchers such as Gallardo N et al. [4] suggest new approaches such as deep learning and tensor flow frameworks to navigate unmanned vehicles. A system developed by Qudus M.A [5], uses a digital road map, which is a machine vision of the road that detects the road boundaries and curb using a Light Image Detecting and Ranging (LIDAR) sensor, to keep the vehicle centered between road limits by using the by-wire controls according to Davis L.C[6]. Haddad A[7] et al. proposed an Active Fault Tolerant Control (AFTC) strategy for maintaining path tracking of an autonomous 2-wheel steering 4-wheel drive (2WS4WD) electric vehicle in the event of an actuator failure.

Four wheel steering (4WS) is a kind of vehicle that can not only improve the driving performance, but also improve the vehicle's maneuverability in confined space. The navigation controller designed by Bak T. et al.[8] use two independent control points (front and back) to minimize the distance required by the reference trajectory. Ye Y. et al.[9] used GPS-based navigation method to study the
navigation performance of different steering strategies when tracking different paths. The results show that satisfactory track performance can be obtained by selecting appropriate steering strategy when tracking curves. Raksincharoensak P et al.[10] Use a simplified linear 2-DOF bicycle model to conducted a simulation study and theoretical analysis of lane tracking control performance of an active four-wheel steering vehicle with different steering response requirements.

2. Theoretical Formulations

Basic assumptions

- During the movement of the vehicle, the left and right wheels have similar tracks.it is simplified to the most simple bicycle model to study. It can be approximately considered that the wheels on both sides have the same steering angle and speed, the front and rear wheels can be described by one tire respectively.
- This paper is used to analyze the relationship between the wheel steering angle and the wheel tracks, so the vehicle motion can be regarded as two-dimensional plane motion without considering the influence of vehicle height.
- In the kinematics analysis of the wheel steering angle and the vehicle body, ignoring the tire deformation and other factors, the vehicle body and the tire are equivalent to the rigid body, and the vehicle body is regarded as a rigid bar.

2.1. Front wheel steering model

In the earth coordinate XOY, the length of the vehicle body is \( l \). At any moment, the angle between the vehicle body and the x-axis is \( \alpha \). The vehicle body moves to different positions at different times under the action of the front wheel speed and steering angle. The schematic is shown in Figure 1.

![Schematic diagram of mathematical model of front steering vehicle](image)

**Figure 1.** Schematic diagram of mathematical model of front steering vehicle

In the direction of the vehicle body, according to the velocity projection theorem, \( \phi_b \) is the steering angle of the front wheel. In each tiny sampling time:

\[
V_b \Delta t \cos(\phi_b) = V_c \Delta t
\]

(1)

Where \( V_B \) and \( V_A \) are the speed of the front and rear wheel, respectively. According to the established model, it can be concluded that in the triangle formed by the vehicle body at two adjacent moments, the variable of azimuth can be obtained by the sine theorem:

\[
\frac{l}{\sin(\pi - \phi_b)} = \frac{V_b \cdot \Delta t}{\sin(\alpha)}
\]

(2)

Where \( \alpha \) is azimuth of the variable. Taking the position of \( A_iB_i \) as arbitrary position, the position coordinates of the front and rear wheels at any time can be obtained according to the discrete recursive formula.

The coordinates of the front wheel B at any position:
\[ x_{B1} = x_{B1-1} + V_B \cdot \Delta t \cos (\phi_B + \alpha_{i-1}) \quad , \quad y_{B1} = y_{B1-1} + V_B \cdot \Delta t \sin (\phi_B + \alpha_{i-1}) \] (3)

The coordinates of the rear wheel A at any position:
\[ x_{A1} = x_{A1-1} + V_A \cdot \Delta t \cos \alpha_{i-1} \quad , \quad y_{A1} = y_{A1-1} + V_A \cdot \Delta t \sin \alpha_{i-1} \] (4)

Where \( \alpha_i = \alpha_{i-1} + \alpha \), \( t_i = t_{i-1} + \Delta t \), where \( \alpha_i \) is the angle between the vehicle body and the x-axis is recorded as the azimuth.

2.2. Four wheel steering model

The difference between the four wheel steering vehicle and the front wheel steering vehicle is that the vehicle body moves to different positions at different moments with different steering angles at the front and rear wheels. The schematic is shown in Figure 2.

**Figure 2.** Schematic diagram of mathematical model of four steering vehicle

The speed is analyzed by the base-point method:
\[ V_B = V_A + V_{BA} \] (5)

Where \( V_{BA} \) is the speed of the vehicle body and \( V_{BA} \) is the angular velocity of the vehicle body, the speed of the vehicle and the variable in azimuth can be obtained from the following calculation:
\[ V_{BA} = w_{BA} \cdot \Delta t \quad , \quad \alpha = w_{BA} \cdot \Delta t \] (6)

The coordinates of the front wheel B at any position:
\[ x_{B1} = x_{B1-1} + V_B \cdot \Delta t \cos (\phi_B + \alpha_{i-1}) \quad , \quad y_{B1} = y_{B1-1} + V_B \cdot \Delta t \sin (\phi_B + \alpha_{i-1}) \] (7)

The coordinates of the rear wheel A at any position:
\[ x_{A1} = x_{A1-1} + V_A \cdot \Delta t \cos (\phi_A + \alpha_{i-1}) \quad , \quad y_{A1} = y_{A1-1} + V_A \cdot \Delta t \sin (\phi_A + \alpha_{i-1}) \] (8)

2.3. The kinematics formula of simulated satellite positioning

The real-time coordinate position and azimuth angle in the center of the vehicle body \((x_c, y_c, \alpha)\) can be located when satellite positioning is used, the real-time coordinate position of the front and rear wheel tracks can be obtained by calculation, and finally the wheel steering angles of the front and rear wheels can be obtained. The schematic is shown in Figure 3.

**Figure 3.** Simulated front and rear wheel trajectory
Azimuth of the vehicle body:
\[
\alpha_i = \arctan \frac{y_{Bi} - y_{Ai}}{x_{Bi} - x_{Ai}} \tag{9}
\]

The angle between the tangent lines of the front and rear wheel tracks and the x-axis:
\[
\theta_{Bi} = \arctan \frac{y_{Bi+1} - y_{Bi}}{x_{Bi+1} - x_{Bi}}, \quad \theta_{Ai} = \arctan \frac{y_{Ai+1} - y_{Ai}}{x_{Ai+1} - x_{Ai}} \tag{10}
\]

Steering angle of the front and rear wheels:
\[
\phi_B = \theta_{Bi} - \alpha_i, \quad \phi_A = \theta_{Ai} - \alpha_i \tag{11}
\]

3. Numerical results

3.1. Numerical results for front wheel steering model analysis

Influence of azimuth angle and the speed of vehicle: under the condition that the steering angle of the front wheel is 0 degree, the length of the vehicle body is 4 meters, and the speed of the vehicle is 1 m/s, the initial azimuth angle is 0 degree and 30 degrees respectively, the motion track of the front and rear wheels under different azimuth angles can be obtained; the other conditions unchanged, the vehicle forward speed is set as 2 m/s, and the wheel track at different speeds can be obtained. The simulation results are shown in Fig. 4(a)(b)(c), respectively.

![Figure 4](image)

**Figure 4.** The influence of different speed and azimuth on wheel track
(a) $\alpha_i = 0^\circ$, $V_a = 1$ m/s; (b) $\alpha_i = 30^\circ$, $V_a = 1$ m/s; (c) $\alpha_i = 0^\circ$, $V_a = 2$ m/s

Influence of wheel steering angle: under the condition of the same vehicle speed and azimuth angle, the vehicle front wheel steering angle is set as 10 degrees, 30 degrees and 90 degrees, respectively, the wheel trajectory under different front wheel steering angles can be obtained. The simulation results are shown in Fig. 5(a)(b)(c), respectively.

![Figure 5](image)

**Figure 5.** The influence of different front wheel steering Angle on wheel track
(a) $\phi_B = 10^\circ$; (b) $\phi_B = 30^\circ$; (c) $\phi_B = 90^\circ$
3.2 Numerical results for four wheel steering model analysis

The steering angles of the front and rear wheels of four-wheel steering vehicles are set to be equal and the motion trajectories of the same direction and opposite direction of the front and rear wheels are simulated; when the steering angle of the rear wheel is set to 0 degrees, the same trajectory as the vehicle with the front wheel steering is obtained. When the steering angle of the rear wheel is 0 degree and the front wheel runs with a sinusoidal variation law with an amplitude of 30 degrees, the wheel track will be obtained. The simulation results are shown in Fig.6 (a)(b)(c), respectively.

![Figure 6](image)

**Figure 6.** The influence of different front wheel and rear wheel steering Angle on wheel track
(a) $\phi_a = 30^\circ$, $\phi_b = 30^\circ$; (b) $\phi_a = 30^\circ$, $\phi_b = -30^\circ$ (c) $\phi_a = \sin 30^\circ$, $\phi_b = 0^\circ$

3.3. Numerical results for satellite positioning simulation analysis

The wheel track coordinates that have been simulated before are read by using data file as input when simulating satellite positioning. It is assumed that the wheel track is shown in the Figure 6(c), and the wheel steering angle changes of the front and rear wheel simulation results are shown in the Figure 7.

![Figure 7](image)

**Figure 7.** Simulation diagram of steering angle control of front and rear wheel

Because the sampling time is 0.001 which is not small enough, there is an error is within 0.8% in the simulation results.

4. Conclusions

The following conclusions can be obtained through Simulink simulation in Matlab.

1) For the front wheel steering model analysis, the change of speed and initial azimuth in the driving process has no effect on the driving path. The change of the driving path is due to the change of the steering angle of the wheel, and the radius of curvature of the driving path is inversely related to the steering angle of the wheel.

2) For the four-wheel steering model analysis, the vehicle will do translation movement and the wheel trajectory coincides when the steering angles of the front and rear wheel are equal to the same direction and the opposite direction, respectively. The front wheel steering model is a special case of
the four-wheel steering model when the wheel steering angle is 0 degree.

(3) Simulation results of simulated satellite positioning verifies the correctness of the change in steering angle of the front and rear wheels when the vehicle is traveling along the specified path.

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