The vortex state which arises from a projection of SU(2) to $\mathbb{Z}_2$ gauge theory is studied at finite temperatures with a special emphasis on the deconfinement phase transition.

1 Introduction

Since the time that the universality of the parton model, designed for describing deep inelastic nucleon scattering processes, was understood by means of perturbative QCD, the non-liberation of the partons, i.e. confinement, even at high energies is still seeking an explanation. Subsequently, it was observed that a static quark anti-quark potential which linearly rises at large distances successfully describes the charmonium spectra. With the advent of the numerical approach to lattice gauge theories, the assumption of a linear confining potential was put onto solid grounds. The role of these simulations was not only to establish quark confinement from first principles Yang-Mills theory, but also to reveal the origin of the linear rise: the color-electric flux of the quark anti-quark pair is squeezed into a flux tube. In the last decade of intense investigations of Yang-Mills theories, it has turned out crucial for understanding confinement to reveal the so-far hidden roots of the color-electric flux tube formation. A substantial progress in this direction was made by realizing that projection techniques are a convenient tool for these purposes: the link variables which represent an actual configuration of the SU(N) lattice gauge theory are projected onto elements of a group $\mathcal{G}$ the number of degrees of freedom of which counts less than that of the SU(N) theory. The choice of $\mathcal{G}$ is constrained by demanding that the string tension is (almost) unchanged by projection. Preserving confinement while reducing the number of degrees of freedom stirs the hope for clearing up those degrees relevant for confinement. Choosing $\mathcal{G} = U(1)_{\text{compact}}$ uncovers color-magnetic monopoles and a (color) photon. In this setting, numerical evidence is found that color electric flux tube formation is due to a dual Meissner effect which is generated by the condensation of the color magnetic monopoles. Here, we will not follow these lines. We will study the case of a SU(2) gauge theory and will reduce the number of degrees of freedoms by employing $\mathcal{G} = \mathbb{Z}_2$. This case reveals $\mathbb{Z}_2$ vortices as the only configurations of the projected theory. I will discuss the
role of the vortices for confinement at zero as well as finite temperature, and will address the drastic change of the vortex properties at the deconfinement phase transition.

2 The SU(2) vortex vacuum

2.1 Construction of the theory of vortices

The so-called center projection of SU(2) gauge theory on a $Z_2$ gauge theory is accomplished by successive steps of gauge fixing and projection. In a first step one exploits the gauge degrees of freedom for minimizing the off-diagonal elements of the SU(2) link variables $U_\mu(x) = U(X)$, i.e.

$$U(X) = \begin{pmatrix} a_0 + ia_3 & a_2 + ia_1 \\ -a_2 + ia_1 & a_0 - ia_3 \end{pmatrix}, \quad \sum_X (a_2^2(X) + a_3^2(X)) \to \min,$$

where the constraint $a_2^2(X) + a_3^2(X) + a_1^2(X) = 1$ must be obeyed for all $X$. Abelian projection $U(X) \to \bar{U}(X) \in U(1)$ is defined by setting the off-diagonal elements to zero and normalize $\bar{U}$ to ensure $\bar{U}\bar{U}^\dagger = 1$. After this first step of projection the reduced theory is a compact U(1) gauge theory with (color) photons and magnetic monopoles as degrees of freedom. In a second step, one exploits the residual U(1) gauge degree of freedom for a minimization of $\sum_X a_3^2(x)$. Center projection $\bar{U}(X) \to \tilde{U}(X) \in Z_2$ is performed by setting $a_3(X) = 0$ and by normalizing $\tilde{U}\tilde{U}^\dagger = 1$. After this second step the link elements of full SU(2) gauge theory are reduced to elements $\{\pm 1\}$, i.e. $U \in SU(2) \to U' \in Z_2$. The reduced theory possesses a residual $Z_2$ gauge degree of freedom.

A $Z_2$ gauge theory can be phrased as a theory of vortices. For this purpose, we choose a 3-dimensional hypercube of space-time. We define that a vortex pierces a plaquette $p$ if the product of the center projected links which are part of $p$ yield $-1$, i.e. $v_p(x) := \prod_{l \in p} U_l^\dagger(x) = -1$. The plaquette which is pierced by a vortex is part of two hypercubes, and we consider the $Z_2$ flux line which connects the center of the corresponding hypercubes as part of the vortex. By construction, the vortex lines fall on top of the links of the 3-dimensional dual lattice. For revealing the string type nature of these vortex links, we introduce $n$ as the number of vortices which pierce the plaquettes $p$ of 3-dimensional hypercube $c$. Resorting to the identity $1 = \prod_{p \in c} v_p(x) = (-1)^n$ one finds that the number $n$ is even. A vortex never ends inside a cube implying that the vortices form closed lines within the 3-dimensional hypercube. Since one is free in choosing the 3-dimensional hypercube out of the 4-dimensional space
time, we conclude that the vortices from closed world sheets in 4-dimensional space.

Let me finally comment on the ambiguity in defining the $\mathbb{Z}_2$ gauge theory by projection. $\mathbb{Z}_2$ projected theories which are constructed from gauge invariant variables and which do not involve gauge fixing before projection have been discussed in the literature for more than twenty years.\textsuperscript{5,6} These projection techniques preserve the value of the string tension\textsuperscript{7} while the properties of the corresponding vortices do not have a physical interpretation in the continuum limit\textsuperscript{8}. It was first noticed in\textsuperscript{10} that the vortices which emerge from the construction outlined below (1) are physical objects (see also\textsuperscript{11}).

### 2.2 Vortex properties

In the pioneering work\textsuperscript{3}, it was obtained that the $\mathbb{Z}_2$ projection of SU(2) gauge theory as discussed below (1) does not change the string tension. Figure 1 illustrates this fact using our results. At small quark anti-quark distances $r$, the Creutz ratios of the full SU(2) gauge theory show a $1/r^2$ behavior due to Coulomb interactions while they approach the string tension at large distances. The Creutz ratios of the $\mathbb{Z}_2$ projected theory are (almost) constant. The Coulomb part is missing, whereas the value of the string tension is reproduced within statistical errors. By the projection $SU(2) \rightarrow \mathbb{Z}_2$ along the lines described above the number of degrees of freedom is strongly reduced while the confining properties are preserved. Further numerical evidence that confinement is intrinsically related to the center vortices was accumulated in\textsuperscript{9}.

It was pointed out for the first time in\textsuperscript{10,11}, and subsequently confirmed in\textsuperscript{9}, that the vortices which arise from the $\mathbb{Z}_2$ projected theory acquire physical
relevance. In particular, it was observed that the density $\rho$ of vortices which pierce a hyperplane of space-time extrapolates to the continuum limit (see also figure 1). Choosing the string tension $\sigma = (440 \text{ MeV})^2$ as reference scale, one finds $\rho \approx 3.5/\text{fm}^{-2}$. Moreover, a significant correlation between the vortices survive the continuum limit. This correlation amounts for an attraction up to a distance scale of $\approx 0.4 \text{ fm}$.

3 Vortices at finite temperatures

Finite temperatures are introduced in field theory by imposing periodic boundary conditions to Bose fields in time direction. The length of periodicity $L_t = 1/T$ refers to the temperature $T$. It was argued in [12] that 4-dimensional Yang-Mills theory at asymptotic high temperatures effectively behaves like the 3-dimensional counterpart the coupling constant of which is related to temperature [3]. Since 3-dimensional Yang-Mills theory does confine quarks, a theory which is supposed to describe the deconfinement phase transition must cover the following phenomenon: above the critical temperature the string tension which is extracted from Polyakov lines must vanish, while correlations between Polyakov lines which are embedded in the spatial hypercube signal a with temperature increasing ”spatial” string tension.

3.1 Vortex polarization and pairing

In order to reveal a possible connection between vortex properties and the deconfinement phase transition at finite temperatures, we first showed that the
data mechanism of dimensional reduction was nicely confirmed by direct lattice calculations [4].
$Z_2$ projected theory correctly describes the temperature dependence of string tension. In particular, we find that the $Z_2$ theory correctly reproduces the critical temperature $T_c \approx 300$ MeV.

Keeping in mind that the expectation value of a Wilson loop is given in the vortex state by $\langle W \rangle = \sum_n (-1)^n P(n)$, where $P(n)$ is the probability that $n$ vortices pierce the minimal area of the loop, the vortex state might account for dimensional reduction if the vortices are polarized at high temperatures. If the vortices are aligned along the time axis direction by temperature effects, they would not pierce Wilson areas which are spanned by the time axis and one spatial direction. This would account for a vanishing string tension. On the other hand, there would be plenty of vortices randomly piercing spatial Wilson loops thus supporting a non-vanishing spatial string tension. In order to test this idea of the deconfinement phase transition, we investigated the density of vortices piercing time-like and space-like oriented planes. We found that the spatial vortex density indeed increases at large temperatures and that the corresponding spatial string tension is indeed compatible with ideas of dimensional reduction. However, the time-like vortex density has only dropped by a factor of two at twice the critical temperature implying that the sharp decrease of the string tension at the critical temperature is not reflected by the (time-like) vortex density.

3.2 Vortex percolation

All the numerical results which have been obtained so far suggest an intimate relation of confinement and vortex percolation: for $T < T_c$ the vortices percolate and generically form a cluster which fills the (lattice) universe. For $T > T_c$ the vortices cease to percolate. The large vortex cluster decays into smaller ones. If the average cluster size is smaller than the length scale of the Wilson loop, the vortices which are located off the Wilson loop boundaries must pierce the loop an even number of times. Hence, they do not contribute to the Wilson expectation value. Only vortices close to the periphery of the loop provide non-trivial contributions, and the Wilson loop expectation value exhibits a perimeter law signaling deconfinement. In order to underline this picture of the deconfinement phase transition, we have calculated the probability $p(x)$ that a vortex link is part of vortex cluster of size $x$. Thereby, $x = 1$ corresponds to the maximum size of the lattice universe. Figure 2 shows $p(x)$ for several values of the temperature. A clear signal for vortex percolation is found for small temperatures whereas for $T > T_c$ small vortex cluster sizes dominate. Also shown in figure 2 is the percolation probability, i.e. $p(x > 0.9)$, as function of temperature. It sharply decreases at $T = T_c$. 

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