Consistency of inflation and preheating in $F(\mathcal{R})$ supergravity

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We study inflation and preheating in $F(\mathcal{R})$ supergravity characterized by two mass scales of a scalar degree of freedom (scalaron): $M$ (associated with the inflationary era) and $m$ (associated with the preheating era). The allowed values of the masses $M$ and $m$ are derived from the amplitude of the CMB temperature anisotropies. We show that our model is consistent with the joint observational constraints of WMAP and other measurements in the regime where a sufficient amount of inflation (with the number of e-foldings larger than 50) is realized. In the low-energy regime relevant to preheating, we derive the effective scalar potential in the presence of a pseudo-scalar field $\chi$ coupled to the inflaton (scalaron) field $\phi$. If $m$ is much larger than $M$, we find that there exists the preheating stage in which the field perturbations $\delta \chi$ and $\delta \phi$ rapidly grow by a broad parametric resonance.

I. INTRODUCTION

The gravitational theory dubbed $f(R)$ gravity, where the Lagrangian $f$ is a function of the Ricci scalar $R$, is the viable phenomenological framework to describe inflation and reheating in the early Universe [1] (see also Refs. [2, 3] for early works). For the simple model described by the Lagrangian $f(R) = R - R^2/(6M^2)$, where $M$ corresponds to the mass of a scalar degree of freedom (“scalaron”), the presence of the quadratic term $R^2/(6M^2)$ leads to cosmic acceleration followed by a gravitational reheating [4]. Moreover the predicted power spectra of scalar and tensor perturbations in this model are compatible with the observations of the CMB temperature anisotropies [5] (see Refs. [6, 7] for reviews).

An embedding (or derivation) of a viable $f(R)$ gravity model from a more fundamental physical theory naturally leads to supergravity (the theory of local supersymmetry) as the first step, because supersymmetry is the leading and well motivated proposal for new physics beyond the Standard Model of elementary particles. Supergravity is also the low-energy effective field theory of superstrings.

A manifestly $\mathcal{N} = 1$ locally supersymmetric (minimal) extension of $f(R)$ gravity was recently constructed in a curved superspace [8]. This was dubbed $F(\mathcal{R})$ supergravity, where $F(\mathcal{R})$ is a holomorphic function of the covariantly-chiral scalar curvature superfield $\mathcal{R}$. The basic features of $F(\mathcal{R})$ supergravity and some of its non-trivial models were systematically studied in Refs. [10–13].

The first phenomenologically viable inflationary model based on $F(\mathcal{R})$ supergravity was proposed in Ref. [14]. In the high-energy regime relevant to inflation this is similar to the Starobinsky’s $f(R)$ model with the quadratic term $-R^2/(6M^2)$ [1], but the correction of the form $(-R)^{3/2}$ is also present. The viability of this model was proven in certain limit of its parameter space, but phenomenological and observational bounds on the parameter values were not found yet.

In the low-energy regime relevant to reheating the proposed model of $F(\mathcal{R})$ supergravity [14] is approximately described by the Lagrangian $f(R) = R - R^2/(6m^2)$, where the mass scale $m$ is not identical to the mass $M$ in the high-energy regime. In this case the dynamics of reheating should be different from that studied in Refs. [4] for the original Starobinsky’s $f(R)$ model. It was conjectured in Ref. [14] that the model [14] gives rise to efficient preheating after inflation, but it was not proven or verified by concrete calculations.

In this paper we extend the analysis of Ref. [14] by providing physical interpretation to the parameters of the model [14], confirm its consistency and viability, and find observational bounds on its parameters. In the low-curvature regime we also derive the effective scalar potential of a scalaron field $\phi$ coupled to a pseudo-scalar field $\chi$. This potential is employed for numerical analysis of the preheating stage after inflation. We show the existence of a broad parametric resonance, by which both the field perturbations $\delta \chi$ and $\delta \phi$ are amplified in the parameter region where $m$ is much larger than $M$.

Our paper is organized as follows. In Sec. III we describe how $f(R)$ gravity arises from $F(\mathcal{R})$ supergravity and review the model [14] by adding more details. In Sec. III we study the inflationary dynamics in the model [14] under the slow-roll approximation and provide phenomenological bounds on the model parameters. In Sec. IV our model [14] is confronted with the observational tests of CMB and other measurements. In Sec. V we compute the effective scalar potential of two physical...
scalars in the low-energy regime. In Sec. VII we numerically study the dynamics of preheating of the multi-field system. Our conclusion is given in Sec. VII.

II. MODEL

The action of $F(\mathcal{R})$ supergravity in the chiral (curved) $N = 1$ superspace of $(1 + 3)$-dimensional spacetime is given by \[ S = \int d^4 x d^2 \theta \mathcal{E} F(\mathcal{R}) + \text{H.c.}, \] \[ (1) \]
where $F(\mathcal{R})$ is a holomorphic function of the covariantly-chiral scalar curvature superfield $\mathcal{R}$, and $\mathcal{E}$ is the chiral superspace density [12]. The scalar curvature $\mathcal{R}$ appears as the field coefficient of the $\theta^2$ term in the superfield $\mathcal{R}$. We use the metric signature $(+, -, -, -)$, so that the sign of $\mathcal{R}$ is opposite to that of Ref. [7]. See Ref. [12] for more details about our notation and $F(\mathcal{R})$ supergravity.

The chiral superspace density (in a Wess-Zumino type gauge) reads

\[ \mathcal{E} = \epsilon(x) \left[ 1 - 2i \sigma_a \bar{\psi}^a(x) + \theta^2 B(x) \right], \]
\[ (2) \]
where $\epsilon = \sqrt{-\det g_{\mu \nu}}$, $g_{\mu \nu}$ is a spacetime metric, $\psi^a = \epsilon^{a \mu} \bar{\psi}^\mu$ is a chiral gravitino, and $B = S - iP$ is the complex scalar auxiliary field. When dropping the contribution of gravitinos ($\bar{\psi} = 0$), there is the following formula for the superfield Lagrangian $\mathcal{L}(x, \theta)$:

\[ S = \int d^4 x d^2 \theta \mathcal{E} L = \int d^4 x e \left[ B \mathcal{L}_1(x) + \mathcal{L}_2(x) \right], \]
\[ (3) \]
where

\[ \mathcal{L}_1(x) = | \mathcal{L} |, \quad \mathcal{L}_2(x) = \nabla^2 | \mathcal{L} |. \]
\[ (4) \]
Here the vertical bars denote the leading field components of superfields. In particular one has

\[ | \mathcal{R} | = \frac{\tilde{B}}{3 M_{\text{Pl}}} = \frac{1}{3 M_{\text{Pl}}}(S + iP), \]
\[ (5) \]
\[ \nabla^2 | \mathcal{R} | = \frac{1}{3} R + \frac{4 \tilde{B} B}{9 M_{\text{Pl}}^2}, \]
\[ (6) \]
where $M_{\text{Pl}} = 2.435 \times 10^{18}$ GeV is the reduced Planck mass.

Applying the formula [3] to the action [1], we obtain the bosonic part of the supersymmetric Lagrangian of Eq. [1] in the form [10]

\[ L = 3 X F(\bar{X}) + \left( \frac{1}{3} R + 4 \bar{X} X \right) F'(\bar{X}) + \text{H.c.}, \]
\[ (7) \]
where $X \equiv B/(3 M_{\text{Pl}})$. Here and in the next Secs. III and IV we focus on the further reduced case in which the auxiliary field $B$ is real, i.e. $X = \bar{X}$, by ignoring the complex nature of the supergravity superfields. In physical terms, it amounts to dropping the bosonic degree of freedom which is the pseudo-scalar superpartner of scalaron (see Sec. V), in addition to dropping all fermionic degrees of freedom. Then the Lagrangian [7] reduces to

\[ L = 6 X F(\bar{X}) + 2 \left( \frac{1}{3} R + 4 X^2 \right) F'(\bar{X}). \]
\[ (8) \]

Variation of the action [8] with respect to $X$ gives rise to the following algebraic relation between $X$ and $R$:

\[ 3 F(\bar{X}) + 11 X F'(\bar{X}) + \left( \frac{1}{3} R + 4 X^2 \right) F''(\bar{X}) = 0. \]
\[ (9) \]

It is natural to expand the function $F(\mathcal{R})$ into power series of $\mathcal{R}$, i.e. $F(\mathcal{R}) = \sum_{i=0}^{n} c_i \mathcal{R}^i$, with some (generically complex) coefficients $c_i$. Since we are not interested in the parity-violating terms in this paper, we assume that all the coefficients $c_i$ are real. For example, the choice $F(\mathcal{R}) = f_0 - f_1 \mathcal{R}^2/2$ gives rise to the standard $N = 1$ supergravity with a negative cosmological constant. Slow-roll inflation can be realized by the following function [14]:

\[ F(\mathcal{R}) = -\frac{1}{2} f_1 \mathcal{R} + \frac{1}{2} f_2 \mathcal{R}^2 - \frac{1}{6} f_3 \mathcal{R}^3, \]
\[ (10) \]
where $f_{1,2,3}$ are positive constants having dimensions of [mass]$^2$, [mass]$^1$, and [mass]$^0$ respectively. In this case the Lagrangian [8] yields

\[ L = -\frac{1}{3} f_1 R + \frac{2}{3} f_2 RX - \left( \frac{7}{3} f_1 + \frac{1}{3} f_3 R \right) X^2 + 11 f_2 X^3 - 5 f_3 X^4. \]
\[ (11) \]
It follows from Eq. [9] that

\[ X^3 - \frac{33 f_2}{20 f_3} X^2 + \frac{1}{30} (R + R_0) X - \frac{f_2}{30 f_3} R = 0, \]
\[ (12) \]
where

\[ R_0 \equiv \frac{21 f_1}{f_3}. \]
\[ (13) \]

For the stability of the bosonic embedding given above we require the condition $F'(\bar{X}) < 0$, which translates into $f_3 X^2 - 2 f_2 X + f_1 > 0$ [14]. In order for this relation to hold for any real value of $X$, we have to require the condition

\[ f_2^2 < f_1 f_3. \]
\[ (14) \]
For the successful inflation the second term on the r.h.s. of Eq. [10] needs to be suppressed relative to

\[ 1 \text{ The stronger condition } f_2^2 \ll f_1 f_3 \text{ was used in Ref. [14].} \]
the third term. From the amplitude of the temperature anisotropies observed in CMB we require \[ f_3 \gg 1. \] (15)

In order to avoid large quantum corrections, the scalaron mass in the low-curvature regime [defined in Eq. (25)] needs to be smaller than the Planck mass, so that \[ f_2^2 \gg f_1. \] (16)

There are two asymptotic regimes characterized by (A) \(|R| \gg R_0\) and (B) \(|R| \ll R_0\). The former one is the high-curvature regime related to the generation of large-scale temperature anisotropies observed in CMB. The latter one is the low-curvature regime related to reheating after inflation. In the following we derive the approximate expression of \(X\), as well as the Lagrangian \(L\) in those two different regimes, improving the results of Ref. [14].

- (A) \(|R| \gg R_0\)
  
  In this case the first and third terms on the l.h.s. of Eq. (12) correspond to the dominant contributions, so that the 0-th order solution satisfies \(X_0^2 + (R + R_0)X_0/30 = 0\). In order to connect this solution to the one in the regime \(|R| \ll R_0\) we require \(X_0 < 0\), so that
  \[
  X_0 = -\sqrt{-\frac{R + R_0}{30}}. \] (17)

  We regard the second and fourth terms in Eq. (12) as the perturbations to the leading-order solution (17). Setting \(X = X_0 + \delta X\) (\(|\delta X| \ll |X_0|\)) and expanding Eq. (12) up to the linear order in \(\delta X\), the perturbation \(\delta X\) can be expressed in terms of \(f_2\). This process leads to the following solution:
  \[
  X = -\sqrt{-\frac{R + R_0}{30}} + \frac{f_2}{2 \frac{13R + 33R_0}{20 f_3 (R + R_0) - 33 f_2 \sqrt{-30 (R + R_0)}}}. \] (18)

  Substituting this relation into Eq. (11) and expanding it at linear order in \(f_2\) yields
  \[
  L = -\frac{1}{10} f_1 R + \frac{1}{180} f_3 (R^2 + R_0^2) - \frac{f_2^2}{900} (9R - 11R_0) \sqrt{-30 (R + R_0)}. \] (19)

  In the limit \(|R| \gg R_0\) this Lagrangian reduces to
  \[
  L \approx -\frac{1}{10} f_1 R + \frac{1}{180} f_3 R^2 + \frac{\sqrt{30}}{100} f_2 (-R)^{3/2}. \] (20)

- (B) \(|R| \ll R_0\)
  
  In this regime the dominant contributions in Eq. (12) correspond to the third and fourth terms, so that 0-th order solution is given by
  \[
  X_0 = \frac{f_2 R}{f_3 (R + R_0)}. \] (21)

  which is negative. Taking into account the first and second terms as the perturbations to \(X_0\), we obtain the following solution:
  \[
  X = \frac{f_2 R}{f_3 (R + R_0)} \times \left[ 1 + \frac{3}{2} \frac{f_2^2 R (13R + 33R_0)}{f_3^2 (R + R_0)^3 - 9 f_2^2 R (R + 11R_0)} \right]. \] (22)

  Substituting this solution into Eq. (11) and expanding it in terms of \(f_2\) up to the order of \(f_2^2\) yields
  \[
  L = -\frac{1}{3} f_1 R + \frac{f_2^2 R^2}{3 f_3 (R + R_0)} + \frac{f_2^4 (6R + 11R_0) R^3}{f_3^2 (R + R_0)^4}. \] (23)

  Note that there is also the intermediate regime characterized by \(|R| \gg R_0| \ll R_0\), i.e. when \(R\) is close to \(-R_0\) [14]. In this case the first and fourth terms in Eq. (12) are the dominant contributions. Picking up the next-order contributions as well, we have
  \[
  X = \left( \frac{f_2}{30 f_3} \right)^{1/3} (-R_0)^{1/3} + \frac{f_2^2}{20 f_3}. \] (24)

  In this regime the Lagrangian (11) includes the term proportional to \(R\) alone. This transient era is followed by the reheating epoch characterized by the Lagrangian (23).

  In order to recover the standard behaviour of General Relativity in the low-energy regime we require that \(f_1 = 3 M_{pl}^2/2\). The mass squared of the scalar degree of freedom is given by \(m^2 = 1/(3 f''(R))\), where \(f(R)\) is related with \(L(R)\) as \(L(R) = -M_{pl}^2 f(R)/2\). In the limit \(|R| \ll R_0\) we have
  \[
  m^2 = \frac{21 f_1 M_{pl}^2}{4 f_2^2} = \frac{63 M_{pl}^4}{8 f_2^4}. \] (25)

  In the high-curvature regime given by the Lagrangian (20) the scalaron mass squared is
  \[
  M^2 = \frac{15 M_{pl}^2}{f_3}. \] (26)

  Then the constants \(f_{1,2,3}\) can be expressed by using the three mass scales \(M_{pl}, m, \) and \(M\), as
  \[
  f_1 = \frac{3}{2} M_{pl}^2, \quad f_2 = \sqrt{\frac{63 M_{pl}^2}{8 m}}, \quad f_3 = \frac{15 M_{pl}^2}{M^2}. \] (27)

  The conditions (14), (15), and (16) translate into
  \[
  m > \sqrt{\frac{7}{20}} M, \quad M \ll M_{pl}, \quad m \ll M_{pl}, \] (28)

  respectively.
III. INFLATIONARY DYNAMICS

Let us study the dynamics of inflation for the $f(R)$ model introduced in Sec. II. Here we are interested in the high-energy regime (A) satisfying the condition $|R| \gg R_0$.

We consider the flat Friedmann-Lemaître-Robertson-Walker (FLRW) background described by the line element $ds^2 = dt^2 - a^2(t)dx^2$, where $a(t)$ is the scale factor with the cosmic time $t$. The Ricci scalar is given by $R = -6(2H^2 + \dot{H})$, where $H = \dot{a}/a$ is the Hubble parameter (the dots stand for the derivatives with respect to $t$).

In order to study the dynamics of inflation, it is convenient to introduce the following dimensionless functions:

$$\alpha = \frac{M^2}{mH}, \quad \beta = \frac{M^2}{H^2}. \quad (29)$$

By using Eqs. (13) and (27), $R_0$ can be expressed as $R_0 = 21M^2/10$. During inflation the functions (29) should satisfy the conditions $\alpha \ll 1$ and $\beta \ll 1$ (see below). In Eq. (13) the term $f(R)/M^2_{pl}$ is the dominant contribution during inflation. Hence, we neglect the higher-order terms beyond that of the first (linear) order in $\alpha$ and $\beta$. Then the Lagrangian $f(R) = -2L(R)/M^2_{pl}$ following from (19) is given by

$$f(R) \simeq \frac{3}{10} R - \frac{R^2}{6M^2} - \frac{3\sqrt{105}(-R)^{3/2}}{100 m}. \quad (30)$$

We assume that the Lagrangian (30) is valid by the end of inflation.

In the flat FLRW spacetime the field equations of motion are:

$$3FH^2 = (f - RF)/2 - 3H\dot{F}, \quad (31)$$

$$-2FH = \ddot{F} - H\dot{F}, \quad (32)$$

where $F \equiv f'(R)$. We introduce the slow-roll parameters

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{F}}{2H\dot{F}}, \quad \epsilon_3 = \frac{\dddot{F}}{H\dot{F}}, \quad (33)$$

which satisfy $|\epsilon_i| \ll 1$ ($i = 1, 2, 3$). From Eq. (32) it follows that

$$\epsilon_1 = -\epsilon_2(1 - \epsilon_3). \quad (34)$$

In the following we carry out the linear expansion in terms of the variables $\epsilon_i$ ($i = 1, 2, 3$), $\alpha$, $\beta$, and $s \equiv \dot{H}/(H^2)$.

For the Lagrangian (30) we have

$$F = \frac{4H^2}{M^2} \left(1 + \frac{27\sqrt{35}}{400} \alpha + \frac{3\beta}{4} - \frac{1}{2} \epsilon_1 \right), \quad (35)$$

$$\dot{F} = -\frac{8H^3}{M^2} \epsilon_1 \left(1 + \frac{27\sqrt{35}}{800} \alpha + \frac{1}{4} \epsilon_1 \right). \quad (36)$$

Then the variable $\epsilon_2$ is given by

$$\epsilon_2 = -\epsilon_1 \left(1 - \frac{27\sqrt{35}}{800} \alpha - \frac{3}{4} \beta + \frac{1}{2} \epsilon_1 + \frac{1}{4} \epsilon_3 \right). \quad (37)$$

Comparing this with Eq. (34), we obtain

$$\epsilon_3 = \frac{27\sqrt{35}}{800} \alpha - \frac{3}{4} \beta + \frac{1}{2} \epsilon_1 + \frac{1}{4} \epsilon_3. \quad (38)$$

Similarly, Eq. (31) gives the following relations:

$$\epsilon_1 = \frac{3\sqrt{35}}{200} \alpha + \frac{1}{20} \beta, \quad (39)$$

and

$$\epsilon_2 = -\frac{3\sqrt{35}}{200} \alpha - \frac{1}{20} \beta. \quad (40)$$

Equation (39) is equivalent to

$$H = -\frac{3\sqrt{35} M^2}{200 m} \left(H + \frac{10m}{3\sqrt{35}} \right). \quad (41)$$

This differential equation can be easily integrated. It yields

$$H(t) = \left(H_i + \frac{10m}{3\sqrt{35}} \right) \exp \left[3\sqrt{35} M^2 \frac{1}{200 m}(t_i - t) \right] - \frac{10m}{3\sqrt{35}} \left[ \right], \quad (42)$$

where $H_i$ is the initial value of $H$ at $t = t_i$. So we find

$$s = \frac{3\sqrt{35}}{200} \alpha. \quad (43)$$

Substituting Eqs. (39) and (41) into Eq. (38), we obtain

$$\epsilon_3 = -\frac{3\sqrt{35}}{100} \alpha - \frac{1}{20} \beta. \quad (44)$$

The end of inflation ($t = t_f$) is identified by the condition $\epsilon_1 = 1$. By using the solution (42), we have

$$t_i - t_f = \frac{200 m}{3\sqrt{35} M^2} \ln \left( \frac{63 M^2}{80m(3\sqrt{35}H_i + 10m)} \times \left( 1 + \frac{800}{63} M^2 + \sqrt{1 + \frac{1600}{63} \left( \frac{m}{M} \right)^2 } \right) \right). \quad (45)$$

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2 There are also some other approaches that give rise to the inflationary solution similar to that in the Starobinsky’s f(R) model. Those include a large non-minimal coupling to the inflaton field [15] and a macroscopic theory of the quantum vacuum in terms of conserved relativistic charges [17].

3 Compared to the field equations given in Ref. [6] we only need to change $R \rightarrow -R$ and $f \rightarrow -f$, while $F$ is unchanged.
We define the number of e-foldings from the onset of inflation \( t = t_i \) to the end of inflation \( t = t_f \) as
\[
N(t_i) = \int_{t_i}^{t_f} \frac{dt}{H(t)}.
\]
From Eqs. (12) and (13), we can express \( N(t_i) \) in terms of \( H_i, M, \) and \( m \). The number of e-foldings \( N \) corresponding to the time \( t \) can be derived by replacing \( H_i \) in the expression of \( N(t_i) \) for \( H \). It follows that
\[
N = \frac{1}{126\alpha} \left[ 3\alpha(80\sqrt{35} - 21\alpha - \sqrt{7}(63\alpha^2 + 1600\beta)) 
- 40\beta(8\ln 2 + 3\ln 5) + 800\beta 
\times \ln \left( \frac{\sqrt{7}(63\alpha^2 + 800\beta) + 21\alpha\sqrt{63\alpha^2 + 1600\beta}}{21\alpha + 2\sqrt{35}\beta} \right) \right].
\]
(46)

In the limit \( \alpha \to 0 \) one has \( N \to 10/\beta - 1/2 \), i.e. \( \beta \to 20/(2N + 1) \). In this case the \( R^2/(6M^2) \) term in the Lagrangian \([30]\) dominates over the dynamics of inflation, which corresponds to the Starobinsky’s \( f(R) \) model \([1]\). In another limit \( \beta \to 0 \) it follows that
\[
N \to 40\sqrt{35}/(21\alpha - 1), \text{ i.e. } \alpha \to 40\sqrt{35}/21(N + 1).
\]
Then we obtain the following bounds on \( \alpha \) and \( \beta \):
\[
0 < \alpha < \frac{40\sqrt{35}}{21(N + 1)}, \quad 0 < \beta < \frac{20}{2N + 1}.
\]
(47)

In order to realize inflation with \( N = 60 \), for example, the two variables need to be in the range \( 0 < \alpha < 0.185 \) and \( 0 < \beta < 0.165 \). For the number of e-foldings relevant to the CMB temperature anisotropies \((50 \leq N \leq 60)\) the slow-roll parameters given in Eqs. (39), (40), (44) are much smaller than unity, so that the slow-roll approximation employed above is justified.

### IV. OBSERVATIONAL TESTS

Let us study whether the \( f(R) \) model \([30]\) can satisfy observational constraints of the CMB temperature anisotropies. The power spectra of scalar and tensor perturbations generated during inflation based on \( f(R) \) theories have been calculated in Ref. \([2]\) (see also Ref. \([2]\)).

The scalar power spectrum of the curvature perturbation is given by \([2]\)
\[
\mathcal{P}_s = \frac{1}{24\pi^2 F} \left( \frac{H}{M_{pl}} \right)^2 \frac{1}{\epsilon_2^2}.
\]
(48)

Using Eqs. (35), (39), and (40), it follows that
\[
\mathcal{P}_s \simeq \frac{1250}{3\pi^2} \left( \frac{M}{M_{pl}} \right)^2 \left( 3\sqrt{35}\alpha + 10\beta \right)^{-2},
\]
(49)

where, in the expression of \( F \), we have neglected the terms \( \alpha \) and \( \beta \) relative to 1. Using the WMAP normalization \( \mathcal{P}_s = 2.4 \times 10^{-9} \) at the pivot wave number \( k_0 = 0.002 \ Mpc^{-1} \) \([18]\), the mass \( M \) is constrained to be
\[
M \simeq 7.5 \times 10^{-6} \left( 3\sqrt{35}\alpha + 10\beta \right) M_{pl}.
\]
(50)

In the limit that \( \alpha \to 0 \) and \( \beta \to 20/(2N + 1) \) we have \( M/M_{pl} = 7.5 \times 10^{-4}/(N + 1/2) \). In another limit \( \alpha \to 40\sqrt{35}/[21(N + 1)] \) and \( \beta \to 0 \) it follows that \( M/M_{pl} = 1.5 \times 10^{-3}/(N + 1) \). In the intermediate regime characterized by \([17]\) we can numerically find the values of \( \alpha \) and \( \beta \) for given \( N \) satisfying the constraint \([16]\), which allows us to evaluate \( M \) from Eq. (49). From Eq. (20) the mass scale \( m \) is also known by the relation
\[
m = \left( \frac{\sqrt{3}}{\alpha} \right) M.
\]

In Fig. 1 we plot \( M \) and \( m \) versus \( \alpha \) in the regime \( 10^{-4} \leq \alpha \leq 0.18 \) for the number of e-foldings \( N = 55 \). We also show the upper bound \( \alpha_{\text{max}} = 0.201 \) determined by Eq. (47). \( M \) is weakly dependent on \( \alpha \) with the order of \( 10^{-5} M_{pl} \), whereas \( m \) strongly depends on \( \alpha \). The condition \( m > \sqrt{7}/20 M \) is satisfied for \( \alpha < 0.178 \).

The scalar spectral index \( n_s \) is defined by \( n_s = 1 + d\ln \mathcal{P}_s/d\ln k \), which is evaluated at the Hubble radius crossing \( k = aH \) (where \( k \) is a comoving wave number) \([15, 20]\). In \( f(R) \) gravity this is given by \([7]\)
\[
n_s = 1 - 4\epsilon_1 + 2\epsilon_2 - 2\epsilon_3.
\]
(51)

On using Eqs. (39), (40) and (44), we obtain
\[
n_s = 1 - \frac{3\sqrt{35}}{100} \alpha - \frac{1}{5} \beta.
\]
(52)
The tensor power spectrum is given by \cite{7}
\[ P_t = \frac{2}{\pi^2} F \left( \frac{H}{M_{pl}} \right)^2. \] (53)

From Eqs. (53) and (55), the tensor-to-scalar ratio is
\[ r = \frac{P_t}{P_s} = \frac{48 \epsilon^2}{2500 (3\sqrt{35} + \alpha)^2}. \] (54)

In the limit \( \alpha \to 0 \) and \( \beta \to 20/(2N + 1) \) the observables (52) and (53) reduce to
\[ n_s(\alpha \to 0) = 1 - \frac{4}{2N + 1}, \] (55)
\[ r(\alpha \to 0) = \frac{48}{(2N + 1)^2}, \] (56)
which agree with those in the Starobinsky’s \( f(R) \) model \cite{3}. For \( N = 55 \) one has \( n_s(\alpha \to 0) = 0.964 \) and \( r(\alpha \to 0) = 3.896 \times 10^{-3} \). In another limit \( \alpha \to 40\sqrt{35}/[21(N + 1)] \) and \( \beta \to 0 \) it follows that
\[ n_s(\beta \to 0) = 1 - \frac{2}{N + 1}, \] (57)
\[ r(\beta \to 0) = \frac{48}{(N + 1)^2}. \] (58)

For \( N = 55 \) one has \( n_s(\beta \to 0) = 0.964 \) and \( r(\beta \to 0) = 1.531 \times 10^{-2} \). While the scalar spectral indices (55) and (57) are practically identical for \( N \gg 1 \), \( r(\beta \to 0) \) is about four times as large as \( r(\alpha \to 0) \). For the intermediate values of \( \alpha \) between 0 and \( 40\sqrt{35}/[21(N + 1)] \) we need to numerically derive \( \beta \) satisfying Eq. (50) for given \( N \). Then \( n_s \) and \( r \) are known from Eqs. (52) and (54).

In Fig. 2 we plot the theoretical values of \( n_s \) and \( r \) in the \((n_s, r)\) plane for \( N = 50, 60, 70 \) together with the 1σ and 2σ observational contours constrained by the joint data analysis of WMAP7, BAO, and HST. For \( \alpha \to 0 \), \( n_s \) and \( r \) are given by Eqs. (55) and (57). In the limit \( \beta \to 0 \), \( n_s \) and \( r \) approach the values given in Eqs. (57) and (58).

Figure 2: The three thick lines show the theoretical values of \( n_s \) and \( r \) for \( N = 50, 60, 70 \) with \( \alpha \) ranging in the region (47). The thin solid curves are the 1σ (inside) and 2σ (outside) observational contours constrained by the joint data analysis of WMAP7, BAO, and HST. For \( \alpha \to 0 \), \( n_s \) and \( r \) are given by Eqs. (55) and (57). In the limit \( \beta \to 0 \), \( n_s \) and \( r \) approach the values given in Eqs. (57) and (58).

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The Starobinsky’s \( f(R) \) model, which corresponds to the limit \( \alpha \to 0 \) with the observables given in Eqs. (55) and (56), is well within the current observational bound. In the regime \( \alpha \ll \beta \) one has \( \beta \approx 20/(2N + 1) - \sqrt{35} \alpha/5 \), so that
\[ n_s(\alpha \ll \beta) = 1 - \frac{4}{2N + 1} + \frac{\sqrt{35}}{100} \alpha, \] (59)
\[ r(\alpha \ll \beta) = \frac{48}{(2N + 1)^2} \left[ 1 + \frac{\sqrt{35}(2N + 1)}{200} \alpha \right]^2. \] (60)

This shows that both \( n_s \) and \( r \) increase for larger \( \alpha \) satisfying the condition \( \alpha \ll \beta \). As we see in Fig. 2, \( n_s \) switches to decrease at some value of \( \alpha \), whereas \( r \) continuously grows toward the asymptotic value given in Eq. (58).

From Fig. 2 we find that the \( f(R) \) model (30) in which \( \alpha \) is in the range (47) is inside the 1σ observational contour. The condition \( m > \sqrt{7/20} M \) provides the constraints \( \alpha < 0.194, \alpha < 0.165, \alpha < 0.143 \) for \( N = 50, 60, 70 \) respectively, while the bound (47) in each case corresponds to \( \alpha < 0.221, \alpha < 0.185, \alpha < 0.159 \). When \( N = 60 \) the scalar spectral index and the tensor-to-scalar ratio are \( n_s = 0.969, r = 0.0110 \) for \( \alpha = 0.165 \) and \( n_s = 0.967, r = 0.0129 \) for \( \alpha = 0.185 \), which are not very different from each other. For the background in which inflation is sustained with the number of e-foldings \( N > 50 \) the model is consistent with the current observations.

Note that the nonlinear parameter \( f_{NL} \) of the scalar non-Gaussianities is of the order of the slow-roll parameters in \( f(R) \) gravity \cite{24}. Hence, in current observations, this does not provide additional constraints to those studied above.

V. EFFECTIVE SCALAR POTENTIAL

In this section we derive the effective scalar potential and the kinetic terms of a complex scalaron field in the low-energy regime (B) characterized by \(|R| \ll R_0 \). In
doing so, let us return to the original $F(R)$ supergravity action \(^1\) and perform the superfield Legendre transformation \(^2\). We temporarily set $M_{pl} = 1$ to simplify our calculations. It yields the equivalent action

$$S = \int d^4x \, d^2\theta \, E \left[ -\mathcal{Y} R + Z(\mathcal{Y}) \right] + \text{H.c.}, \quad (61)$$

where we have introduced the new covariantly chiral superfield $\mathcal{Y}$ and the new holomorphic function $Z(\mathcal{Y})$ related to the function $F$ as

$$F(R) = -\mathcal{R} Y(R) + Z(\mathcal{Y}(R)). \quad (62)$$

The equation of motion of the superfield $\mathcal{Y}$, which follows from the variation of the action \(^6\) with respect to $\mathcal{Y}$, has the algebraic form

$$\mathcal{R} = Z'(\mathcal{Y}), \quad (63)$$

so that the function $\mathcal{Y}(R)$ is obtained by inverting the function $Z'$. Substituting the solution $\mathcal{Y}(R)$ back into the action \(^1\) yields the original action \(^1\) because of Eq. \(^{62}\). We also find

$$\mathcal{Y} = -F'(R). \quad (64)$$

The inverse function $\mathcal{R}(\mathcal{Y})$ always exists under the physical condition $F'(R) \neq 0$. As regards the F-function \(^1\), Eq. \(^{62}\) yields a quadratic equation with respect to $\mathcal{R}$, whose solution is

$$\mathcal{R}(\mathcal{Y}) = \frac{\sqrt{14} M^2}{20m} \left[ 1 - \sqrt{1 + \frac{80m^2}{21M^2}(Y - 3/4)} \right], \quad (65)$$

where we have used the parametrization \(^2\). Equation \(^{65}\) is also valid for the leading complex scalar field components $\mathcal{R} = B/3 = \bar{X}$ and $\mathcal{Y} = Y$, where $Y$ is the complex scalar field.

The kinetic terms of $\mathcal{Y}$ are obtained by using the identity

$$\int d^4x \, d^2\theta \, E \mathcal{Y} R + \text{H.c.} = \int d^4x \, d^4\theta \, E^{-1}(\mathcal{Y} + \bar{Y}), \quad (66)$$

where $E^{-1}$ is the full curved superspace density \(^1\). Therefore, the Kähler potential reads

$$K = -3 \ln(\mathcal{Y} + \bar{Y}). \quad (67)$$

It gives rise to the kinetic terms

$$\mathcal{L}_{\text{kin}} = \frac{\partial^2 K}{\partial \mathcal{Y} \partial \bar{Y}} |_{\mathcal{Y} = Y} = \frac{3(\partial_\mu Y)^2 + (\partial_\mu \bar{Y})^2}{4Y^2}, \quad (68)$$

where we have used the notation $Y = y + iz$ in terms of the two real fields $y$ and $z$. The imaginary component $z$ corresponds to a pseudo-scalar field. The kinetic terms \(^{68}\) represent the non-linear sigma model with the hyperbolic target space of (real) dimension two, whose metric is known as the standard Poincaré metric. The kinetic terms are invariant under arbitrary rescalings $Y \rightarrow aY$ with constant parameter $a \neq 0$.

The effective scalar potential $V(\mathcal{Y}, \bar{Y})$ of a complex scalaron $Y$ in the regime (B), where supergravity decouples (it corresponds to rigid supersymmetry), is easily derived from Eq. \(^{64}\) when keeping only scalars (i.e. ignoring their spacetime derivatives together with all fermionic contributions) and eliminating the auxiliary fields, near the minimum of the scalar potential. We find

$$V = \frac{21}{2} |Z'(\mathcal{Y})|^2 = \frac{21}{2} |\mathcal{R}(\mathcal{Y})|^2, \quad (69)$$

which gives rise to the chiral superpotential

$$W(\mathcal{Y}) = \sqrt{\frac{21}{2} Z(\mathcal{Y})}. \quad (70)$$

The superfield equations \(^{67}\) and \(^{60}\) are model-independent, i.e. they apply to any function $F(R)$ in the large $M_{pl}$ limit, near the minimum of the scalar potential with the vanishing cosmological constant.

There is no field redefinition that would bring all the kinetic terms \(^{65}\) to the free form. The canonical (free) kinetic term of a real scalaron $y$ alone can be obtained via the field redefinition

$$y = A \exp(-\sqrt{2/3} \phi). \quad (71)$$

The scalaron potential vanishes at $y = 3/4$. Demanding that this minimum corresponds to $\phi = 0$, we have $A = 3/4$ and hence $y = (3/4) \exp(-\sqrt{2/3} \phi)$. Defining a rescaled field $\chi$ as $\chi = \sqrt{8/3} y$, the kinetic term \(^{65}\) can be written as

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} e^{2\sqrt{2/3} \phi/M_{pl}} (\partial_\mu \chi)^2. \quad (72)$$

Here and in what follows we restore the reduced Planck mass $M_{pl}$.

The total potential \(^{69}\) including both the fields $\phi$ and $\chi$ is given by

$$V(\phi, \chi) = \frac{147 M^4 M_{pl}^2}{400 m^2} \sqrt{B(\phi) + iC(\chi)} - 1 \right|^2, \quad (73)$$

where

$$B(\phi) = 1 + \frac{20m^2}{7 M^2} \left( e^{-2\sqrt{2/3} \phi/M_{pl}} - 1 \right), \quad (74)$$

$$C(\chi) = \frac{80m^2}{21M^2} \sqrt{\frac{3}{8} \chi/M_{pl}}. \quad (75)$$

In order to express \(^{69}\) in a more convenient form we write $\sqrt{B(\phi) + iC(\chi)} = p + iq$, where $p$ and $q$ are real. This gives the relations $p^2 - q^2 = B(\phi)$ and $2pq = C(\chi)$. Solving these equations for $p$, we find

$$p = \frac{1}{\sqrt{2}} \left[ B(\phi) + \sqrt{B^2(\phi) + C^2(\chi)} \right]^{1/2}, \quad (76)$$
where we have chosen the solution $p > 0$ to recover $p = \sqrt{B(\phi)}$ for $B(\phi) > 0$ in the limit $C(\chi) \to 0$. Then the field potential (77) reads

$$V(\phi, \chi) = \frac{147M^4M_{pl}^2}{400m^2} \left[ 1 + \sqrt{B^2(\phi) + C^2(\chi)} - \sqrt{2} \left\{ B(\phi) + \sqrt{B^2(\phi) + C^2(\chi)} \right\} \right]^{1/2}. \quad (77)$$

In the absence of the pseudo-scalar $\chi$ the potential (77) reduces to

$$V(\phi) = \frac{147M^4M_{pl}^2}{400m^2} \left[ 1 + \sqrt{B(\phi)} \right] \left\{ B(\phi) + \sqrt{B^2(\phi) + C^2(\chi)} \right\}^{1/2}. \quad (78)$$

For the field $\phi$ satisfying the condition $B(\phi) < 0$ it follows that

$$V(\phi) = \frac{21}{20}M^2M_{pl}^2 \left( 1 - e^{-\sqrt{2/3}\phi/M_{pl}} \right), \quad (79)$$

which approaches the constant $V(\phi) \to 21M^2M_{pl}^2/20$ in the limit $\phi \to \infty$. Defining the slow-roll parameter $\epsilon_v = (M_{pl}^2/2)(V,\phi/V)^2$, we have

$$\epsilon_v = \frac{x^2}{3(1-x)^2}, \quad x = e^{-\sqrt{2/3}\phi/M_{pl}}. \quad (80)$$

The scalaron $\phi$ is coupled to the pseudo-scalar $\chi$ through the interaction given in the second line of Eq. (62).

### VI. PREHEATING AFTER INFLATION

We study the dynamics of preheating for the two-field system described by the kinetic term (72) and the effective potential (77). The background equations of motion on the flat FLRW background are

$$3M_{pl}^2H^2 = \frac{\dot{\phi}^2}{2} + e^{2b}\chi^2/2 + V, \quad (83)$$

$$\ddot{\phi} + 3H\dot{\phi} + V,\phi - b,\phi e^{2b}\chi^2 = 0, \quad (84)$$

$$\ddot{\chi} + (3H + 2b,\phi)\chi + e^{-2b}V,\chi = 0, \quad (85)$$

where $b(\phi) = \sqrt{2/3}\phi/M_{pl}$ and ".$.,.$" represents a partial derivative with respect to $\phi$.

In Fourier space the field perturbations $\delta\phi_k$ and $\delta\chi_k$ with the comoving wave number $k$ obey the following equations (see Refs. [27] for related works):

$$\ddot{\delta}\phi_k + 3H\dot{\delta}\phi_k + (k^2/a^2 + V,\phi,\phi - (2b,\phi + b,\phi,\phi)e^{2b}\chi^2)\delta\phi_k = -V,\phi,\chi\delta\chi_k + 2b,\phi e^{2b}\chi,\delta\chi_k, \quad (86)$$

$$\ddot{\delta}\chi_k + (3H + 2b,\phi)\delta\chi_k + (k^2/a^2 + e^{-2b}V,\chi)\delta\chi_k = -e^{-2b}(V,\phi,\chi - 2b,\phi V,\chi + 2b,\phi,\phi e^{2b}\chi)\delta\phi_k - 2b,\phi,\phi\delta\phi_k. \quad (87)$$

The derivative $V,\chi$ of the potential (77) vanishes at $\chi = \pm \chi_c$, where

$$\chi_c = \frac{\sqrt{210}M_{pl}}{20m} \left[ 1 - e^{-\sqrt{2/3}\phi/M_{pl}} - \frac{21}{80}\left( \frac{M_{pl}}{m} \right)^2 \right]^{1/2}. \quad (88)$$

The local minima exist in the $\chi$ direction provided that

$$\phi > \sqrt{\frac{3}{2}} \ln \left[ 1 - \frac{21}{80} \left( \frac{M_{pl}}{m} \right)^2 \right]^{-1} M_{pl} \equiv \phi_c, \quad (89)$$

The end of inflation is characterized by the criterion $\epsilon_v = 1$. This gives $x_f = e^{-3/2\phi_f/M_{pl}} = (3 - \sqrt{3})/2$ and hence $\phi_f = 0.558 M_{pl}$. For $m > M$ the condition $B(\phi_f) < 0$ is satisfied, so that the potential (79) is valid at the end of inflation. If $m$ is close to the border value $\sqrt{7/20}M$ — see Eq. (28) —, then the potential (79) is already invalid at the end of inflation.

For small $\phi$ satisfying the condition $B(\phi) > 0$ the potential (78) reads

$$V(\phi) = \frac{147M^4M_{pl}^2}{400m^2} \left[ 1 - \sqrt{1 + \frac{20m^2}{M_{pl}^2} \left( e^{-\sqrt{2/3}\phi/M_{pl}} - 1 \right)^2 \right] \quad (81)$$

In this case Taylor expansion around $\phi = 0$ gives rise to the leading-order contribution $V(\phi) = m^2\phi^2/2$. Reheating occurs around the potential minimum through the oscillation of the canonical field $\phi$.

The total effective potential involving the interaction between the fields $\phi$ and $\chi$ is given by Eq. (77). Expanding the potential (77) around $\phi = \chi = 0$ and picking up the terms up to fourth-order in the fields, we obtain

$$V(\phi, \chi) \approx \frac{1}{2} m^2 \phi^2 + \frac{\sqrt{6m^2}(10m^2 - 7M^2)}{42M^2M_{pl}} \phi^3 + \frac{1500m^4 - 1260m^2M^2 + 343M^4m^2}{1764M^4M_{pl}} \phi^4 + \frac{1}{2} m^2 \chi^2 - \frac{25m^6}{49M^4M_{pl}^4} \phi^2 \chi^2. \quad (82)$$
whereas they disappear for $\phi < \phi_c$. In Fig. 3 we plot the potential (77) with respect to $\phi$ and $\chi$ for $m = 1.14 \times 10^{-4} M_{\text{pl}}$ and $M = 1.62 \times 10^{-5} M_{\text{pl}}$ in this case, the potential has the local minima in the $\chi$ direction for $\phi > 6.5 \times 10^{-3} M_{\text{pl}}$. From Eq. (88), the field value $\chi_c$ increases for larger $\phi$. For the model parameters used in Fig. 3, for example, one has $\chi_c = 0.028 M_{\text{pl}}$ at $\phi = 0.1 M_{\text{pl}}$ and $\chi_c = 0.059 M_{\text{pl}}$ at $\phi = 0.5 M_{\text{pl}}$.

If the initial conditions of the fields are $0 < \chi < \chi_c$ and $\phi > \phi_c$, the field $\chi$ grows toward the local minimum at $\chi = \chi_c$. After $\phi$ drops below $\phi_c$, the field $\chi$ approaches the global minimum at $\chi = 0$. In Fig. 4 we show one example for the evolution of the background fields $\phi$ and $\chi$ with the same values of $m$ and $M$ as those in Fig. 3. The energy density of the field $\chi$ catches up to that of the inflaton around the onset of reheating.

As we see in Eq. (59), the critical field value $\phi_c$ gets smaller for increasing $m/M$. Hence, for larger $m/M$, the potential (77) possesses the local minima at $\chi = \pm \chi_c$ for a wider range of $\phi$. The potential in the region $|\chi| < \chi_c$ can be flat enough to lead to inflation by the slow-roll evolution of the field $\chi$, even if $\phi$ is smaller than $\phi_f = 0.558 M_{\text{pl}}$. For larger ratio $m/M$ inflation ends with the field value much smaller than $\phi_f$. If $m/M = 20$ and $m/M = 83$, for example, the amplitudes of the field $\phi$ at the onset of oscillations are $\phi_i = 1.5 \times 10^{-2} M_{\text{pl}}$ and $\phi_i = 5.0 \times 10^{-3} M_{\text{pl}}$, respectively.

Let us consider the regime where the condition

$$
\left( \frac{m}{M} \right)^2 \frac{\phi}{M_{\text{pl}}} \ll 1
$$

is satisfied. Then the potential (52) is approximately given by $V(\phi, \chi) \simeq m^2 \delta \phi^2 / 2 + m^2 \chi^2 / 2$, in which case both $\phi$ and $\chi$ have the same mass $m$. This gives rise to the matter-dominated epoch (where $H = 2/(3 t)$) driven by the oscillations of two massive scalar fields. From Eq. (54) we have that $\phi + (2/\pi) \phi \phi_0 \approx 0$, whose solution is

$$
\phi(t) \simeq \frac{\pi}{2 m t} \phi_0 \sin(\pi t).
$$

Here the initial field value $\phi_0$ corresponds to the time $t_i = \pi / (2 m)$.

In order to discuss the dynamics of the field perturbations in Eqs. (56) and (77) we define the two frequencies $\omega_\phi$ and $\omega_\chi$, as $\omega_\phi^2 = k^2 / a^2 + V,_{\phi\phi} - (2 b_\phi^2 + b,_{\phi\phi}) e^{2 i b} \chi^2$ and $\omega_\chi^2 = k^2 / a^2 + c e^{-2 b} V,_{\chi\chi}$. As long as the condition (53) is satisfied, it is sufficient to pick up the terms up to cubic order in fields. It then follows that

$$
\omega_\phi^2 \simeq \frac{k^2}{a^2} + m^2 + \sqrt{6 m^2(10 m^2 - 7 M^2)} / M^2 \phi,
$$

$$
\omega_\chi^2 \simeq \frac{k^2}{a^2} + m^2 e^{-2 b} + 10 \sqrt{6 m^4} / M^2 e^{-2 b} \phi,
$$

where, in Eq. (92), we have neglected the contribution of the term $-(2 b_\phi^2 + b,_{\phi\phi}) e^{2 i b} \chi^2$.

We introduce the rescaled fields $\delta \phi_k = a^{3/2} \delta \phi_k$ and $\delta X_k = a^{3/2} e^{b} \delta X_k$ to estimate the growth of perturbations in the regime (81). Neglecting the contributions of the r.h.s. of Eqs. (56) and (77) and also using the approximation $e^{-2 b} \simeq 1$ in the regime $H \ll m$, the field perturbations $\delta \phi_k$ and $\delta X_k$ obey the following equations

$$
\frac{d^2}{dz^2} \delta \phi_k + [A_k - 2 q_\phi \cos(2 z)] \delta \phi_k \simeq 0,
$$

$$
\frac{d^2}{dz^2} \delta X_k + [A_k - 2 q_\chi \cos(2 z)] \delta X_k \simeq 0,
$$

where

\begin{align*}
A_k &= \frac{(2 b_\phi^2 + b,_{\phi\phi})}{6 m^2 (10 m^2 - 7 M^2)} - \frac{k^2}{a^2} + m^2 + \sqrt{6 m^2 (10 m^2 - 7 M^2)}, \\
q_\phi &= \frac{1}{2} - 2 b_\phi^2 - b,_{\phi\phi} + \sqrt{6 m^2 (10 m^2 - 7 M^2)}, \\
q_\chi &= \frac{1}{2} - 2 b_\chi^2 - b,_{\chi\chi} + \sqrt{6 m^2 (10 m^2 - 7 M^2)}. 
\end{align*}
where $2z = mt + \pi/2$. The quantities $A_k$, $q_\phi$, and $q_\chi$ are given by

$$A_k = 4 + 4 \frac{k^2}{m^2 a^2},$$  

$$q_\phi = \frac{20\sqrt{6}}{7} \left(1 - 7M^2/10m^2\right) \left(\frac{m}{M}\right)^2 \frac{\phi_i \pi/2}{M_{pl} \, mt},$$  

$$q_\chi = \frac{20\sqrt{6}}{21} \left(\frac{m}{M}\right)^2 \frac{\phi_i \pi/2}{M_{pl} \, mt},$$

which are time-dependent.

Equations (94) and (95) are the so-called Mathieu equations describing parametric resonance caused by the oscillation of the field $\phi$. (see also the review [20]). In the regime [28] both $q_\phi$ and $q_\chi$ are smaller than 1 for $t \geq t_i = \pi/(2m)$. In this case the resonance occurs in narrow bands near $A_k = l^2$, where $l = 1, 2, \ldots$ [28] [30].

As the physical momentum $k/a$ redshifts away, the field perturbations approach the instability band at $A_k = 4$. Although $\delta \varphi_k$ and $\delta X_k$ can be amplified for $A_k \simeq 4$ and $q_\phi \lesssim 1$, $q_\chi \lesssim 1$, this narrow parametric resonance is not efficient enough to lead to the growth of $\delta \varphi_k$ and $\delta \chi_k$ against the Hubble friction [29].

If the initial field $\phi_i$ satisfies the condition $(m/M)^2 |\phi_i|/M_{pl} \gg 1$, the quantities $q_\phi$ and $q_\chi$ are much larger than 1 at the onset of reheating. This corresponds to the so-called broad resonance regime [28] in which the perturbations $\delta \varphi_k$ and $\delta \chi_k$ can grow even against the Hubble friction. We caution, however, that Eqs. (94) and (95) are no longer valid because the background solution [31] is subject to change due to the effect of higher-order terms in the potential (77). Still, the non-adiabatic particle production occurs around the potential minimum ($\phi = 0$) [29]. In this region the dominant contribution to the potential is the quadratic term $m^2 \phi^2/2$. Hence it is expected that preheating can be efficient for the values of $q_\phi$ and $q_\chi$ much larger than 1 at the onset of the field oscillations.

Numerically we solve the perturbations equations [83] and [57] together with the background equations [85], [83], and [55] for the full potential (77) without using the approximate expression [52]. In Figs. 5 and 6 we plot the evolution of the field perturbations $\delta \varphi_k$ and $\delta \chi_k$ with the wave number $k = m$ for two different choices of the parameters $m$ and $M$ (which are constrained by the WMAP normalization in Fig. 1). The initial conditions of the perturbations are chosen to recover the vacuum state characterized by $\delta \varphi_k(t_i) = e^{-i\omega_\phi t_i}/\sqrt{2\omega_\phi}$ and $\delta X_k(t_i) = e^{-i\omega_\chi t_i}/\sqrt{2\omega_\chi}$.

Figure 5 corresponds to the mass scales $m = 1.16 \times 10^{-3} M_{pl}$ and $M = 1.39 \times 10^{-5} M_{pl}$, i.e., the ratio $m/M = 83$. The field value at the onset of oscillations is found to be $\phi_i = 5.0 \times 10^{-5} M_{pl}$, in which case $q_\phi(t_i) = 244$ and $q_\chi(t_i) = 81$. Figure 5 shows that both $\delta \varphi_k$ and $\delta \chi_k$ grow rapidly by the broad parametric resonance. The growth of the field perturbations ends when $q_\phi$ and $q_\chi$ drop below 1.

Figure 6 corresponds to the ratio $m/M = 20$, in which case $\phi_i = 1.5 \times 10^{-2} M_{pl}$, $q_\phi(t_i) = 41$, and $q_\chi(t_i) = 4.6$. Compared to the evolution in Fig. 5 preheating is less efficient because of the smaller values of $q_\phi(t_i)$ and $q_\chi(t_i)$. 

Figure 5: Evolution of the field perturbations $\delta \varphi_k = k^{3/2} \delta \varphi_k/M_{pl}$ and $\delta \chi_k = k^{3/2} \delta \chi_k/M_{pl}$ with the wave number $k = m$ for $m = 1.16 \times 10^{-3} M_{pl}$ and $M = 1.39 \times 10^{-5} M_{pl}$. We choose the background initial conditions $\phi = 0.1 M_{pl}$, $\chi = 1.0 \times 10^{-3} M_{pl}$, $\dot{\phi} = -8.48 \times 10^{-4} m M_{pl}$, and $\dot{\chi} = 1.18 \times 10^{-5} m M_{pl}$.

Figure 6: Evolution of the field perturbations with the wave number $k = m$ for $m = 2.89 \times 10^{-4} M_{pl}$ and $M = 1.46 \times 10^{-5} M_{pl}$. We choose the background initial conditions $\phi = 0.1 M_{pl}$, $\chi = 1.0 \times 10^{-3} M_{pl}$, $\dot{\phi} = -7.35 \times 10^{-3} m M_{pl}$, and $\dot{\chi} = 6.85 \times 10^{-4} m M_{pl}$. 
The parameter to control the efficiency of preheating is the mass ratio \( m/M \). For larger \( m/M \) the creation of particles tends to be more significant. For the mass \( m \) smaller than \( 10^{-4} M_p \) the field perturbations \( \delta \phi_k \) and \( \delta \chi_k \) hardly grow against the Hubble friction because they are not in the broad resonance regime.

In our numerical simulations we did not take into account the rescattering effect between different modes of the particles. The lattice simulation \([31, 32]\) is required to deal with this problem. It will be of interest to see how the non-linear effect can affect the evolution of perturbations at the final stage of preheating.

VII. CONCLUSIONS

We have studied the viability of the \( f(R) \) inflationary scenario in the context of \( F(R) \) supergravity. In the high-energy regime characterized by the condition \(|R| \gg R_0\) there is a correction of the form \((-R)^{3/2}/m\) to the function \( f(R) = 3R/10 - R^2/(6M^2)\). Introducing the dimensionless functions \( \alpha \) and \( \beta \) in Eqs. \( \text{(29)} \), we showed that these are constrained to be in the range \([-177] \) to realize inflation with the number of e-foldings \( N \).

The masses of the scalaron field in the regimes \(|R| \gg R_0\) and \(|R| \ll R_0\) are approximately given by \( M \) and \( m \), respectively. From the WMAP normalization of the CMB temperature anisotropies we derived \( M \) and \( m \) as a function of \( \alpha \) in Fig. 1. The weak dependence of \( M \) with respect to \( \alpha \) means that the term \(-R^2/(6M^2)\) needs to dominate over the correction \((-R)^{3/2}/m\) during inflation. We also showed that the model is within the 1\( \sigma \) observational contour constrained from the joint data analysis of WMAP7, BAO, and HST, by evaluating the scalar spectral index \( n_s \) and the tensor-to-scalar ratio \( r \).

In the presence of the pseudo-scalar field \( \chi \) coupled to the scalaron field \( \phi \) we derived the effective potential \([177]\) and their kinetic energies \([172]\) in the low-energy regime \((|R| \ll R_0)\). Provided that the condition \([89]\) is satisfied, the effective potential has two local minima at \( \chi = \pm \chi_c \). Around the global minimum at \( \phi = \chi = 0 \) the system is described by two massive scalar fields with other interaction terms given in Eq. \( \text{(52)} \). Even if \( \chi \) is initially close to 0, \( \chi \) typically catches up to \( \phi \) around the onset of the field oscillations (see Fig. 3).

In the regime where the field \( \phi \) is in the range \([171]\) we showed that both the field perturbations \( \delta \phi_k = a^{3/2} \delta \phi_k \) and \( \delta \chi_k = a^{3/2} e^b \delta \chi_k \) obey the Mathieu equations \([94]\) and \([93]\). This corresponds to the narrow resonance regime in which \( q_0 \) and \( q_\chi \) are smaller than the order of unity. The broad resonance regime is characterized by the condition \( (m/M)^2 |\phi|/M_{pl} \gg 1 \), but in this case the expansion \([82]\) of the effective potential around the minimum is no longer valid. In order to confirm the presence of the broad resonance we numerically solved the perturbation equations \([50]\) and \([57]\) for the full potential \([177]\). Indeed we found that preheating of both the perturbations \( \delta \phi_k \) and \( \delta \chi \) is efficient in this regime. As we see in Figs. 5 and 6 the broad parametric resonance is more significant for larger values of \( m/M \).

Our results lend compelling support to the phenomenological viability of the bosonic sector of \( F(R) \) supergravity, in addition to its formal consistency. It is also worthwhile to recall that supergravity unifies bosons and fermions with General Relativity, highly constrains particle spectrum and interactions, has the ideal candidate for a dark matter particle such as the lightest super-particle \([33]\), and can be deduced from quantum gravity such as superstring theory. The \( F(R) \) supergravity action \( \text{(1)} \) is truly chiral in superspace, so that it is expected to be protected against quantum corrections \([12]\), which is important for stabilizing the masses \( M \) and \( m \) in quantum theory.

After the broad parametric resonance and the subsequent rescattering at the final stage of preheating, decay of super-scalaron produces new particles including the visible sector. It can be studied perturbatively, when adding a supersymmetric matter action to the \( F(R) \) supergravity action in Eq. \( \text{(1)} \), along the standard lines, see e.g., Section 13 of Ref. \([12]\) for a review. As is well known, the Starobinsky \( f(R) \) gravity model has the universal reheating mechanism due to the coupling of scalaron to all matter \([1, 4]\). The same applies to super-scalaron in matter coupled \( F(R) \) supergravity \([13]\).

Of course, our analysis of \( F(R) \) supergravity is still incomplete since it does not include fermions such as gravitinos and inflatinos (gravitino is the fermionic superpartner of graviton, and inflatio is the fermionic superpartner of scalaron). We did not study in this paper further particle production to complete the reheating process. A calculation of decay rates and particle abundances after preheating requires an extension of the action \( \text{(1)} \) by some hidden sector to be responsible for supersymmetry breaking. It should be the subject of a separate investigation.

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