Radiative corrections to $e^+e^- \rightarrow WW \rightarrow 4$ fermions

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Abstract:
The structure of the double-pole approximation for the $O(\alpha)$ corrections to $e^+e^- \rightarrow WW \rightarrow 4$ fermions is described, and some results are presented. Moreover, results on full tree-level predictions for $e^+e^- \rightarrow 4$ fermions+$\gamma$ are given.

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1 Introduction

The investigation of the reactions $e^+e^- \rightarrow WW \rightarrow 4\text{fermions} (+\gamma)$ at future high-energy and high-luminosity linear colliders is very important, since it provides us with precise information about the W-boson mass $M_W$ and the gauge-boson self-interactions. The most promising methods for the determination of $M_W$ are the cross-section measurement at the W-pair threshold and the reconstruction of the invariant masses of the W bosons at any energies. The experimental accuracies of these two measurements are expected to be of the order of 6 MeV [1] and 15 MeV [2] for TESLA, respectively, which should be compared with the expected accuracy of 30 MeV at LEP2. While the triple gauge-boson couplings are probed at LEP2 at the level of 10%, future $e^+e^-$ linear colliders can even exceed the per-cent level [3]. At future colliders it will also be possible to derive significant bounds on quartic gauge-boson couplings by inspecting W-pair production in association with a hard photon [4]; even at LEP2, where the statistics for such events is poor, first bounds on quartic couplings can be derived [5].

The described physical goals can only be reached if precise predictions for the processes $e^+e^- \rightarrow WW \rightarrow 4\text{f} (+\gamma)$ are known. Assuming an integrated luminosity of the order of $10^2 \text{fb}^{-1}$ leads to about $10^6$ pairs of W bosons. This means that physical observables should be known at the level of some 0.1%. High-precision calculations for four-fermion production are, however, complicated for various reasons. At the aimed accuracy, a pure on-shell approximation for the W bosons is not acceptable, i.e. the W bosons have to be treated as resonances. Since the description of resonances necessarily goes beyond a fixed-order calculation in perturbation theory, problems with gauge invariance occur. Discussions of this issue can be found in Refs. [6, 7]. A second complication arises from the need to take into account electroweak radiative corrections of $O(\alpha)$ beyond the universal corrections. The full treatment of the processes $e^+e^- \rightarrow 4\text{f} (+\gamma)$ at the one-loop level is of enormous complexity and involves severe theoretical problems with gauge invariance; up to now such results do not exist.

Here we summarize recent progress concerning an approximate approach to include $O(\alpha)$ corrections to $e^+e^- \rightarrow WW \rightarrow 4\text{f}$. The approximation is based on the idea to correct only those pieces of the transition matrix elements that are enhanced by two W-boson resonances and is therefore called double-pole approximation (DPA). Corrections of $O(\alpha)$ to contributions that involve at most one resonant W boson are of the order of $(\alpha/\pi) \times (\Gamma_W/M_W) \times \log(\cdot \cdot) \lesssim 0.1\%$, which thus is a measure for the intrinsic uncertainty of the DPA. Corrections induced by real photon emission may be treated accordingly, but full tree-level predictions for $e^+e^- \rightarrow 4\text{f} + \gamma$ have already been presented for selected final states in Refs. [8] and for all final states in Ref. [9]. The most important results of Ref. [9] are reviewed below.

2 Full tree-level predictions for $e^+e^- \rightarrow 4\text{f} + \gamma$

The processes $e^+e^- \rightarrow 4\text{f} + \gamma$ do not only yield important corrections to $e^+e^- \rightarrow 4\text{f}$, they are also interesting in their own right, since they involve both triple and quartic gauge-boson couplings.

Most of the existing work on hard-photon radiation in W-pair production is based on the approximation of stable W bosons (see Ref. [11] and references in Refs. [8, 4]). A
Table 1: Comparison of different width schemes for several processes (taken from Ref. [9])

| Process                  | $\sigma/\text{fb}$ | $\sqrt{s}$ = 189 GeV | $500$ GeV | $2$ TeV | $10$ TeV |
|--------------------------|---------------------|-----------------------|-----------|---------|---------|
| $e^+e^- \rightarrow u\bar{d}\mu^+\bar{\nu}_\mu\gamma$ | constant width     | 224.0(4)              | 83.4(3)   | 6.98(5) | 0.457(6) |
|                          | running width       | 224.6(4)              | 84.2(3)   | 19.2(1) | 368(6)  |
|                          | complex mass        | 223.9(4)              | 83.3(3)   | 6.98(5) | 0.460(6) |
| $e^+e^- \rightarrow u\bar{d}e^-\bar{\nu}_e\gamma$    | constant width     | 230.0(4)              | 136.5(5)  | 84.0(7) | 16.8(5)  |
|                          | running width       | 230.6(4)              | 137.3(5)  | 95.7(7) | 379(6)   |
|                          | complex mass        | 229.9(4)              | 136.4(5)  | 84.1(6) | 16.8(5)  |

first step of including the off-shellness of W bosons in $e^+e^- \rightarrow WW \rightarrow 4f + \gamma$ was done in Ref. [11], where only photon emission from diagrams with two resonant W bosons was taken into account. However, it is desirable to have a full lowest-order calculation for $e^+e^- \rightarrow 4f + \gamma$ for two reasons. As described in Section 3.3, the definition of the DPA for $e^+e^- \rightarrow WW \rightarrow 4f + \gamma$ is non-trivial so that possible versions of the DPA should be carefully compared to the full result. Secondly, one expects a similar impact of off-shell effects as in the case without photon, where so-called background diagrams (diagrams with at most one resonant W boson) can reach a significant fraction of the full cross section. Therefore, they should be included at least in predictions for detectable photons. In the following we briefly summarize some results of Ref. [9], where an event generator for all final states $4f + \gamma$ with massless fermions is described.

In the event generator of Ref. [9] different schemes for treating gauge-boson widths are implemented. A comparison of results obtained by different ways of introducing these decay widths is useful in order to get information about the size of gauge-invariance-breaking effects, which are present in some finite-width schemes. Table 1 contains some results on the total cross section for two semi-leptonic four-fermion final states and a photon, evaluated with different finite-width treatments. Similar to the case without photon emission, the SU(2)-breaking effects induced by a running width render the predictions totally wrong in the TeV range. For a constant width such effects are suppressed, as can be seen from a comparison with the results of the complex-mass scheme, which exactly preserves gauge invariance.

Figure 1 shows the photon-energy spectra for some typical four-fermion final states that correspond to WW$\gamma$ production. Apart from the usual soft-photon pole, the spectra contain several threshold and peaking structures that are caused by photon emission from the initial state. The two relevant classes of diagrams are illustrated in Figure 2. Diagrams with the structure of Figure 2a correspond to triple-gauge-boson-production subprocesses and yield dominant contributions as long as the two virtual gauge bosons can become simultaneously resonant. For instance, WW$\gamma$ production is dominant for $E_\gamma < 26.3$ GeV (224 GeV) for a CM energy of 189 GeV (500 GeV). The diagrams of Figure 2b correspond to $\gamma Z$ production with a subsequent four-particle decay of the resonant Z boson mediated by a soft photon or gluon $V_3$. Owing to the two-particle kinematics of $\gamma Z$ production such contributions lead to peak structures around a fixed value of $E_\gamma$, which is located at 72.5 GeV (242 GeV) for a CM energy of 189 GeV (500 GeV).
Table 1 and Figure 1 illustrate the effect of background diagrams, since final states that are related by the interchange of muon and electrons differ only by background diagrams. While the impact of background diagrams is of the order of some per cent for CM energies around 200 GeV, there is a large effect of background contributions already at 500 GeV. The main effect is due to forward-scattered $e^{\pm}$, which is familiar from the results on $e^{+}e^{-} \rightarrow 4f$. More numerical results for $e^{+}e^{-} \rightarrow 4f + \gamma$ can be found in Ref. [9].

3 Electroweak radiative corrections

3.1 Relevance of electroweak corrections

Present-day Monte Carlo generators for off-shell W-pair production (see e.g. Ref. [12]) typically include only universal electroweak $O(\alpha)$ corrections\(^1\), such as the running of the electromagnetic coupling, $\alpha(q^2)$, leading corrections entering via the $\rho$-parameter, the Coulomb singularity \([14]\), which is important near threshold, and mass-singular logarithms $\alpha \ln(m_e^2/Q^2)$ from initial-state radiation. In leading logarithmic approximation, the scale $Q^2$ is not determined and has to be set to a typical scale for the process; in the following we take $Q^2 = s$.

\(^1\)The QCD corrections for hadronic final states are discussed in Ref. [13].
The size of the neglected \( O(\alpha) \) contributions is estimated by inspecting on-shell W-pair production, for which the exact \( O(\alpha) \) correction and the leading contributions were given in Refs. [15] and [16], respectively. Table 2 shows the difference between an “improved Born approximation” \( \delta_{\text{IBA}} \), which is based on the above-mentioned universal corrections, and the corresponding full \( O(\alpha) \) correction \( \delta \) to the Born cross-section integrated over the W-production angle \( \theta \) for some centre-of-mass (CM) energies \( \sqrt{s} \).

Table 2: Size of “non-leading” corrections to on-shell W-pair production (\( \delta_{\text{IBA}} \) and \( \delta \) include only soft-photon emission)

| \( \theta \) range | \( \sqrt{s} \)/GeV | 161 | 175 | 200 | 500 | 1000 | 2000 |
|---------------------|------------------|-----|-----|-----|-----|------|------|
| \( 0^\circ < \theta < 180^\circ \) | \((\delta_{\text{IBA}} - \delta)/\%\) | 1.5 | 1.3 | 1.5 | 3.7 | 6.0 | 9.3 |
| \( 10^\circ < \theta < 170^\circ \) | \( 1.5 \) | 1.3 | 1.5 | 4.7 | 11 | 22 |

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### 3.2 Photon radiation and W line shape

A thorough description of real-photon emission is of particular importance for the realistic prediction of the W-line shape, which is the basic observable for the reconstruction of the W-boson mass from the W-decay products. This fact can be easily understood by comparing the impact of photon radiation on the line shape of the W boson with the one of the Z boson, observed in \( e^+e^- \rightarrow Z \rightarrow f\bar{f} \) at LEP1 and the SLC (see also Ref. [13]).

The Z line shape is defined as a function of \( s \), which is fully determined by the initial state, by the cross section \( \sigma(s) \). Photon radiation from the initial state effectively reduces the value of \( s \) available for the production of the Z boson so that \( \sigma(s) \) also receives resonant contributions for \( s > M_Z^2 \), induced by this radiative return to the Z resonance and known as radiative tail. Final-state radiation is not enhanced by such kinematical effects, thus yielding moderate corrections.

The W line shape is reconstructed from the kinematical variables in the final state. More precisely, it is defined by the distributions \( d\sigma/dM_W^2 \), where \( M_W^2 \) are the reconstructed invariant masses of the W\( ^\pm \) bosons. We now consider the fermion pair \( f_1(k_1)\bar{f}_2(k_2) \) produced by a nearly resonant W boson with momentum \( k_+ \), i.e. \( k_+^2 \sim M_W^2 \). In this case, photon radiation from the final state decreases the invariant mass of this fermion pair, i.e. \( (k_1 + k_2)^2 < k_+^2 = (k_1 + k_2 + k_\gamma)^2 \), while initial-state radiation leads to \( (k_1 + k_2)^2 = k_+^2 < (k_1 + k_2 + k_\gamma)^2 \). Thus, a consistent identification of \( M_W^2 = (k_1 + k_2)^2 \) also leads to a radiative tail, but now induced by final-state radiation and for \( M_W^2 < M_Z^2 \). However, such an identification is experimentally not possible for almost all cases\(^2\) since nearly collinear

\(^2\)Semi-leptonic final states with a muon may be an exception, where \( (k_\mu + k_{\nu_\mu})^2 \) could be determined from all detected final-state particles other than the muon.
On-shell production

On-shell decays

Figure 3: Diagrammatic structure of factorizable corrections to $e^+e^- \to WW \to 4f$

and soft photons in the final state cannot be separated from the outgoing fermions (except
for muons). A realistic definition of $M_2^\pm$ necessarily depends on the details of the exper-
imental treatment of photons in the final state, underlining the importance of a careful
investigation of the W line shape in the presence of photon radiation.

3.3 Features of the double-pole approximation

Fortunately, the full off-shell calculation for the processes $e^+e^- \to WW \to 4f$ in
$O(\alpha)$ is not needed for most applications. Sufficiently above the W-pair thresh-
old a good approximation can be obtained by taking into account only the doubly-resonant part of
the amplitude

$$M = \frac{R_+(k_+^2, k_-^2)}{(k_+^2 - M_W^2)(k_-^2 - M_W^2)} + \frac{R_-(k_+^2, k_-^2)}{k_+^2 - M_W^2} + \frac{R_-(k_+^2, k_-^2)}{k_-^2 - M_W^2} + N(k_+^2, k_-^2), \quad (3.1)$$

doubly-resonant

singly-resonant

non-resonant

as explained in the introduction. The DPA amounts to the replacement

$$M \rightarrow \frac{R_+(M_W^2, M_W^2)}{(k_+^2 - M_W^2 + iM_W\Gamma_W)(k_-^2 - M_W^2 + iM_W\Gamma_W)}.$$

Note that the numerator $R_+(k_+^2, k_-^2)$ is replaced by the gauge-independent residue
$R_+(M_W^2, M_W^2)$ [20, 21].

Doubly-resonant corrections to $e^+e^- \to WW \to 4f$ can be classified into two types [3, 21, 22]: factorizable and non-factorizable corrections. The former are those that corre-
spond either to W-pair production or to W decay. They are represented by the schematic
diagram of Figure 3 in which the shaded blobs contain all one-loop corrections to the pro-
duction and decay processes, and the open blobs include the corrections to the W prop-
agators. The remaining corrections are called non-factorizable, since they do not contain
the product of two independent Breit–Wigner-type resonances for the W bosons, i.e. the
production and decay subprocesses are not independent in this case. Non-factorizable
corrections include all diagrams involving particle exchange between these subprocesses.
Simple power-counting arguments reveal that such diagrams only lead to doubly-resonant
contributions if the exchanged particle is a photon with energy $E_\gamma \lesssim \Gamma_W$; all other non-
factorizable diagrams are negligible in DPA. Two relevant diagrams are shown in Figure 4,
where the full blobs represent tree-level subgraphs. We note that diagrams involving pho-
ton exchange between the W bosons contribute both to factorizable and non-factorizable
corrections; otherwise the splitting into those parts is not gauge-invariant. The non-factorizable corrections to \( e^+e^- \rightarrow WW \rightarrow 4f \) are discussed in Section 3.4 in more detail.

The factorizable corrections consist of contributions from virtual corrections and real-photon bremsstrahlung. The known results on the virtual corrections to the pair production \([15]\) and the decay \([23]\) of on-shell W bosons can be used as building blocks for the DPA. The formulation of a consistent DPA for the real corrections is, however, non-trivial. The main complication originates from the emission of photons from the resonant W bosons. A diagram with a radiating W boson involves two propagators the momenta of which differ by the momentum of the emitted photon. If the photon momentum is large (\( E_\gamma \gg \Gamma_W \)), the resonances of these two propagators are well separated in phase space, and their contributions can be associated with photon radiation from exactly one of the production or decay subprocesses. For soft photons (\( E_\gamma \ll \Gamma_W \)) a similar splitting is possible. However, for \( E_\gamma \sim \Gamma_W \) the two resonance factors for the radiating W boson overlap so that a simple decomposition into contributions associated with the subprocesses is not obvious.

### 3.4 Non-factorizable corrections

Non-factorizable corrections account for the exchange of photons with \( E_\gamma \lesssim \Gamma_W \) between the W-pair production and W decay subprocesses (see Figure 4). Already before their explicit calculation, it was shown \([24]\) that such corrections vanish if the invariant masses of both W bosons are integrated over. Thus, they do not influence pure angular distributions, which are of particular importance for the analysis of gauge-boson couplings. For exclusive quantities the non-factorizable corrections are non-vanishing. A first hint on their actual size was obtained by investigating the non-factorizable correction that is contained in the Coulomb singularity \([24]\).

The explicit analytical calculation of the non-factorizable corrections was performed by different groups \([26, 27, 28]\). In these studies, the photon momentum was integrated over, resulting in a correction factor to the differential Born cross section for the process without photon emission. This correction factor is non-universal \([28]\) in the sense that it depends on the parametrization of phase space. The analytical results show that all effects from the initial \( e^+e^- \) state cancel so that the correction factor does not depend on the W-production angle. Fermion-mass singularities appear in individual contributions, but cancel in the sum. Moreover, the correction factor vanishes like \((M_+^2 - M_W^2)/(\Gamma_W M_W)\)

\[\text{The original result of the older calculation} \quad (M_+^2 - M_W^2)/(\Gamma_W M_W)\]

\[\text{does not agree with the two more recent results} \quad (M_+^2 - M_W^2)/(\Gamma_W M_W)\]

\[\text{which are in mutual agreement. As known from the authors of Ref.} \quad (M_+^2 - M_W^2)/(\Gamma_W M_W)\]

\[\text{their corrected results also agree with the ones of Refs.} \quad (M_+^2 - M_W^2)/(\Gamma_W M_W)\].
on resonance and tends to zero in the high-energy limit, both leading to a suppression of the non-factorizable corrections with respect to the factorizable ones.

The non-factorizable corrections to \(e^+e^- \rightarrow WW \rightarrow 4f\) with purely leptonic and hadronic final states were numerically evaluated in Ref. [27] and for all final states in Ref. [28]. The corrections to invariant-mass distributions (see Figure 5) turn out to be qualitatively similar for all final states and are of the order of \(\sim 1\%\) for LEP2 energies, shifting the maximum of the distributions by 1–2 MeV. Multiple distributions in angular or energy variables and in at least one of the invariant masses of the W bosons receive larger corrections of a few per cent.

Although non-factorizable corrections to four-fermion production turn out to be small with respect to LEP2 accuracy, they can be of relevance at future \(e^+e^-\) colliders with higher luminosity.

### 3.5 Results for \(\mathcal{O}(\alpha)\) corrections in double-pole approximation

In Ref. [29] the \(\mathcal{O}(\alpha)\) corrections to four-lepton production were treated in DPA, following a semi-analytical approach. The DPA is applied both to the virtual and real corrections, and the off-shellness of the W bosons is kept only in the W propagators, but nowhere else. In particular, the phase space is factorized into on-shell phase spaces and independent invariant masses \(M_{\pm}\) for the W bosons. The corresponding W-boson momenta \(k_{\pm}\) were strictly identified with the sum of the momenta of the corresponding decay fermions, i.e. no photon recombination was considered. For the total cross section the approach of Ref. [29] is closely related to taking the cross section for on-shell W-pair production multiplied by a branching ratio. Ref. [29] also contains results for the \(M_{\pm}\) distributions and various angular distributions for a CM energy of 184 GeV. In particular, the authors of Ref. [29] find relatively large shifts in the peak position of the W line shape, namely \(-20\) MeV, \(-39\) MeV, and \(-77\) MeV for \(\tau^+\nu_\tau, \mu^+\nu_\mu,\) and \(e^+\nu_e\) final states respectively. These results have been qualitatively confirmed by YFSWW in Ref. [30], where the \(\mathcal{O}(\alpha)\) corrections to W-pair production were supplemented by final-state radiation in a leading-log approach. Note, however, that the large shifts are due to mass-singular logarithms like \(\alpha \ln(m_t/M_W)\), since no photon recombination of collinear photons is performed. More realistic definitions of \(k^2_{\pm}\), which have to include photon recombination, effectively replace the mass-singular logarithms by logarithms of a
minimum opening angle for collinear photon emission. This expectation is also confirmed by the leading-log study of Ref. [30].

Moreover, a DPA for the factorizable corrections to $e^+e^- \rightarrow WW \rightarrow \tau^+\nu_\tau\mu^-\bar{\nu}_\mu$ was used in Ref. [32] in order to estimate the quality of a high-energy approximation \[18, 33\] for the virtual corrections at energies $\sqrt{s} \gtrsim 500$ GeV. Ref. [32] contains results on total cross sections and distributions in the W production angle.

Very recently, we succeeded in constructing the first Monte Carlo generator for off-shell W-pair production that includes the complete $O(\alpha)$ corrections in DPA. This generator, called RACOONWW, contains the full lowest-order matrix elements for $e^+e^- \rightarrow 4f$ for any four-fermion final state. The complete virtual corrections to W-pair production and W decay, and the virtual non-factorizable corrections are included in the DPA. The exact four-fermion phase space is used throughout. For the real corrections the matrix elements for the minimal gauge-invariant subset comprising all doubly-resonant contributions of the processes $e^+e^- \rightarrow WW \rightarrow 4f\gamma$, i.e. the photon radiation from the CC11 subset, are included. By using these matrix elements for the real radiation, we avoid the problems in defining a DPA for semi-soft photons ($E_\gamma \sim \Gamma_W$). The real and virtual corrections are carefully combined in the soft and collinear regions, in order to avoid mismatch between IR and mass singularities.

As a first result of this generator, we show the $O(\alpha)$-corrected total cross section and the distribution in the W-production angle $\theta$ for four-lepton production in Figure 3.5. A detailed presentation of numerical results as well as a comparison to existing results will appear elsewhere.

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