On a distinctive feature of problems of calculating time-average characteristics of nuclear reactor optimal control sets

A V Trifonenkov and V P Trifonenkov

National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), 31 Kashirskoe shosse, 115409 Moscow, Russia

E-mail: AVTrifonenkov@mephi.ru

Abstract. This article deals with a feature of problems of calculating time-average characteristics of nuclear reactor optimal control sets. The operation of a nuclear reactor during threatened period is considered. The optimal control search problem is analysed. The xenon poisoning causes limitations on the variety of statements of the problem of calculating time-average characteristics of a set of optimal reactor power off controls. The level of xenon poisoning is limited. There is a problem of choosing an appropriate segment of the time axis to ensure that optimal control problem is consistent. Two procedures of estimation of the duration of this segment are considered. Two estimations as functions of the xenon limitation were plot. Boundaries of the interval of averaging are defined more precisely.

1. Introduction

The present advancement of nuclear power allows building nuclear power plants in areas of unstable climatic conditions or with an increased risk of seismic activity, as well as regions of political instability and terroristic threats [1, 2]. Such power plants may undergo force majeure circumstances, which cause abrupt shutdown of nuclear reactors and their consequent idleness. The consumers of energy produced by the reactor may suffer damages induced by an unexpected shutdown and outage of the chief source of power.

The time when force majeure situation occurs can not be predicted precisely. There is usually a threatened period, which is declared beforehand, given by its duration and probability distribution for the event time. When the threatened event comes, nuclear reactors are shut down. Shutdowns at certain states of a reactor may cause long-term outages due to very high levels of xenon poisoning [2]. Taking the above into consideration, the problem of the search of the best reactor power control set during the threatened period is analyzed in this article.

Consider the time interval between the threatened period’s start and the time of the reactor shutdown. The problem is to find the optimal control function, defined on this interval, which satisfies the conditions and minimizes the cost functional.

The cost functional is defined by some parameter, which depends on the state of the system and the control function. For example, the power generation loss before the reactor shutdown, as well as the reactivity margin or the concentration of xenon-135 in the reactor core at the time of the shutdown, etc. may be used as such parameters.

As the reactor control is impossible in case of exceeding the marginal xenon-135 concentration level inside the reactor core [3, 4], the limitation of xenon-135 concentration before the reactor...
shutdown is introduced. Also, as it was mentioned above, the reactor shutdown at some set of states may cause a long-term reactor outage. The long-term reactor outage due to xenon poisoning may cause damage to the infrastructure and industry. That is why the condition for the values of both iodine-135 and xenon-135 at the end of process is introduced in order to limit the concentration of xenon-135 after the reactor shutdown. Finally, it is stated, that after the reactor shutdown occurs, there is an additional time span, reserved for the elimination of the accident’s consequences, when the reactor can’t operate.

Thus, the optimization problem statement conforms to the problem of optimal control on a limited interval with the limitation of xenon-135 concentration before shutdown and both iodine-135 and xenon-135 at the time of shutdown.

The threatened event time is random; it defines the duration of the process and the control function, which corresponds to the duration value. In practice, for the problem stated the optimal process can’t be determined unless the period of control is given. Nevertheless, if the optimal control search problem is resolved in advance for any process duration value, the functions derived and their characteristics may be useful for applied control modes development.

However, there is a set of process duration values, for which the solution to the problem given doesn’t exist.

2. Statement and proof

That is why issues occur, that are related to problems, which require existence of optimal control for any value of process duration. Problems of calculating time-averaged characteristics of optimal control are of that type. In the time axis, originating simultaneously with the start of the reactor control process, there exists the time interval, containing instants, when the reactor shutdown doesn’t satisfy the conditions. Further this interval is denoted by $\Delta t$. The origin of this interval coincides with the origin of the time axis, the ending is defined by the parameters of the system and the limitations stated.

One-group point model of a nuclear reactor [5] was used in the research. The process of xenon poisoning is described by the system of differential equations [6]

$$\begin{align}
\frac{dx(t)}{dt} &= \lambda_i(t) - \lambda_x x(t) - \sigma_x \varphi x(t) \\
\frac{di(t)}{dt} &= \sigma_x \varphi - \lambda_i(t)
\end{align}$$

(1)

Then for the case of the nuclear reactor after having been shut down, the system takes the following form

$$\begin{align}
\frac{dx(t)}{dt} &= \lambda_i(t) - \lambda_x x(t) \\
\frac{di(t)}{dt} &= -\lambda_i(t)
\end{align}$$

(2)

Two ways of estimating the length of the interval were considered. The first way corresponds with the lower estimation.

Let $t_0$ be the origin of the threatened period and $t_1$ be the instant of the reactor shutdown. In these terms, if $t > t_1$ then $\varphi(t) = 0$ and the transients are described by (2).

Let $\Phi$ be the set of admissible control functions $\varphi(t)$, such that $0 \leq \varphi(t) \leq \varphi_{\text{max}}$, where $t_0 \leq t \leq t_1$ is the time span of the control process. The set of states of the model (1) is represented by the quadrant $Q = \{(i,x) \mid i \geq 0, x \geq 0\}$ of the phase plane $Oix$.

Let $A$ be the set of probable model states at time $\tau$, given $i(t_0) = i_0, x(t_0) = x_0$ for any $\varphi(t) \in \Phi$. Let $B$ be the set of acceptable states at $t = t_1$ that is the end of the control process. In these terms, if
$A \cap B = \{\emptyset\}$ at some instant $\tau$, then, if $t_1 = \tau$, the problem is inconsistent. This is to be proved by the following.

To investigate the properties of the set $B$, consider the function $F(i, x)$ over the quadrant $Q$; the function is defined by (2) solution properties as follows. Let $i(t), x(t)$ be the solution of (2) given initial conditions $i(t_1) = i_1, x(t_1) = x_1$, where $(i_1, x_1) \in Q$ and $t \geq t_1$. By construction, $F(i_1, x_1) = \max_{i \in [i_1, \infty)} x(t)$. Consequently, $B = \{(i, x) \in Q | F(i, x) \leq x_{\max}\}$.

The equation describing the phase curve for the system (2) has the form:

$$x = \frac{\lambda_i}{\lambda_i - \lambda_x} \left( i \frac{\lambda_x}{i_m} - i \right), \text{ with } 0 \leq i \leq i_m.$$

Let $f(i)$ be the right-hand member of this equation; note also that $f(i)$ is convex upwards at $0 < i < i_m$. Then the equiscalar lines of $F(i, x)$ have the form of $x = L(i)$, where

$$L(i) = \begin{cases} f(i), & i \leq i \leq i_m, \\ f(i^*), & 0 \leq i < i^* \end{cases} \text{ and } i^* = i_m \left( \frac{\lambda_x}{\lambda_i} \right)^{\lambda_i^{-1}} - \text{ the maximum point of } f(i) \text{ (see figure 1 below).}$$

It is clear that the value of $F(i, x)$ over the equiscalar line equals the maximal value of $f(i^*)$.

![Figure 1. The form of the equiscalar line $x = L(i)$](image)

Consequently, the function $L(i)$ is also convex upwards at $0 < i < i_m$; furthermore, the set $B$ is also convex. The boundary of the set $B$ at $i > 0$ and $x > 0$ is defined by the equiscalar line $x = L(i)$, corresponding to the value $F(i, x) = x_{\max}$.

The set $B$ is closed, and, if $(i_0, x_0) \notin B$, then there exists a circular area $U_0 = \{(i, x) | \sqrt{(i - i_0)^2 + (x - x_0)^2} \leq a \} (a > 0)$, such that $U_0 \cap B = \{\emptyset\}$. So, the estimation [7]

$$(i(t), x(t)) \in U_0 \text{ where } t_0 \leq t \leq a / M \text{ for any } \phi(t) \in \Phi \text{, given }$$

$$M = \sup_{D} \sqrt{(\lambda_i i - (\lambda_x + \sigma_x \phi) x)^2 + (\sigma_x \phi - \lambda_i i)^2}$$

and $D = \{(i, x, \phi) \in R^3 | (i, x) \in U_0, \ 0 \leq \phi \leq 1\}$,

which follows from the boundedness of right-hand members of (1) over the set $D$, results in $\Delta_t > a / M > 0$, because for any $t_0 \leq t \leq a / M$ the set $A \subset U_0$ and, consequently, $A \cap B = \{\emptyset\}$.

More precise lower estimation of $\Delta_t$ may be obtained by adjusting the shape of sets $A$ and $B$. To simplify the computation procedure convex polygons are considered; they are denoted by $A_\infty$ and $B_\infty$. 


and contain sets $A$ and $B$ respectively. Thus, if $A_c \cap B_c = \{\emptyset\}$, then sets $A$ and $B$ do not intersect either.

The conditions, considered in this article, implying that the initial state $(i_0, x_0)$ coincides with the equilibrium point of the system (1) given $\varphi = \varphi_{\text{max}}$ and $(i_0, x_0) \notin B_c$, then the construction of the set $B_c$ may be based on the following reasoning. In the coordinate system Oixh the tangential plane defined by the equation $h = F(i, x)$ at the point $(i_0, x_0, F(i_0, x_0))$ is considered. The Oix phase plane projection of the intersection line between the tangential plane and the plane $h = x_{\text{lim}}$ may be used as a part of the boundary of the set $B_c$ (line $\Gamma$ in figure 2).

The sides of the polygon $A_c$, estimating the set $A$, are formed by $K$ estimations for the solutions of (1) denoted by $i(t), x(t)$ given $i(t_0) = i_0, x(t_0) = x_0$ and any $\varphi(t) \in \Phi$

$$s_k(t) \leq \alpha_k i(t) + \beta_k x(t), \ |\alpha_k| + |\beta_k| > 0, \text{ for } t_0 \leq t \leq t_1, \ k = 1, \ldots, K.$$ 

The estimations of such type are obtained by the integration of differential inequalities, which follow from (1) given the limitation $0 \leq \varphi(t) \leq \varphi_{\text{max}}$, signs of summands in right-hand members of equations, as well as signs of coefficients $\alpha_k$ and $\beta_k$ for the specific estimation line. For example,

1) $s_1(t) = i_0 \exp(-\lambda_i(t - t_0))$ 

$$A = \sigma_x \varphi_{\text{max}}, \text{ при } \alpha_1 = 1, \beta_1 = 0;$$

2) $s_2(t) = (x_0 - c_0) \exp(-A(t - t_0)) + c_0 \exp(-\lambda_i(t - t_0)), \text{ где } c_0 = \frac{\lambda_i i_0}{\lambda_i + A - \lambda_i},$

$$A = \sigma_x \varphi_{\text{max}}, \text{ при } \alpha_2 = 0, \beta_2 = 1;$$

3) $s_3(t) = i_0 + x_0 - (\lambda_i + A)((x_0 - d_0 - A/\lambda_i) (1 - \exp(-\lambda_i(t - t_0))) / \lambda_i + A(t - t_0) / \lambda_i + d_0 (1 - \exp(-\lambda_i(t - t_0)) / \lambda_i), \text{ где } d_0 = \frac{\lambda_i i_0 - A}{\lambda_i - \lambda_i}, \text{ при } \alpha_3 = 1, \beta_3 = 1;$$

etc. The choice of the count $K$ of estimations and their coefficients $\alpha_k$ and $\beta_k$ may be varied to adjust the set $A_c$ (see figure 2 below).

Then the relative position of $A_c$ and $B_c$ is analyzed for various values of $\tau$ and the point of time is estimated when the condition $A_c \cap B_c = \{\emptyset\}$ is violated. The maximal duration of the time interval $t_0 \leq \tau < t_0 + \Delta_\tau$, where each point $\tau$ satisfies $A_c \cap B_c = \{\emptyset\}$, defines the lower estimation for $\Delta_\tau$.

The upper estimation of the length of the interval was based on the solution of the problem of optimal speed reactor power decrease with limited xenon concentration. The statement of this problem and analytical principles of the solution construction are stated in [8]. The duration of the numerically calculated process is the upper estimation for the interval. The duration of the process as the function of the xenon limitation value was plot.

![Figure 2. Approximations of sets A and B; the intersection line Γ](image)
Figure 3. (a) The example of a transient, representing the optimal speed reactor power decrease, over the phase plane, (b) optimal control

Figure 4. The estimations of the duration of segment $\Delta_t$ as the functions of the limitation value for the concentration of xenon: solid and dashed lines for upper and lower estimations respectively

As results of the research carried two ways of estimation of the time interval observed were developed and applied and the dependencies of the estimations as functions of xenon limitation value were plot. The results obtained may be used in further research of the improvements to the problem statement considered in this article.

In particular, the following researches may include the problems of averaging the reactor control characteristics over the threatened period, which point of origin is known in advance.

Acknowledgments
We would like to express our sincere gratitude to our advisor Ph.D. Prof. Zagrebayev A. M. for sharing his ideas and knowledge, for his guidance, support and interest in our research.

References
[1] Zagrebayev A M, Ovsyannikova N V and Sadchikov S M 2012 Optimization of a mode of change of a nuclear reactor capacity during the threatened period under force-majeur circumstances Modern problems of science and education 4 1–7

[2] Zagrebayev A M, Ovsyannikova N V and Sadchikov S M 2012 The average loss of energy production at a random stop of a reactor with a limited operational reserve of reactivity Natural and technical sciences 58 418–22

[3] Dollezhal’ N A and Emel’yanov I Ya 1980 Channel Nuclear Power Reactor (Moscow: Atomizdat)

[4] Ovchinnikov F Ya, Golybev L I, Dobrynin V D, Klochkov D I, Semenov V V and Tsybenko V I 1979 Operating conditions of water-moderated-water-cooled power reactors (Moscow: Atomizdat)

[5] Zweifel P 1997 Reactor Physics (Moscow: Atomizdat)

[6] Rudik A P 1979 Optimization of reactor physical characteristics (Moscow: Atomizdat)

[7] Pontryagin L S 1974 Ordinary differential equations (Moscow: Nauka)

[8] Rudik A P 1971 Nuclear reactors and Pontryagin’s maximum principle (Moscow: Atomizdat)