Model of Torque in Disc-Type Magnetorheological Brake Driven by Shape Memory Alloy

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Abstract—A magnetorheological brake driven by shape memory alloy is presented in this paper. The shape memory alloy spring is used to push the magnetorheological fluid into the working gap under thermal effect. Herschel-Bulkley model is used to describe the constitutive characteristics of magnetorheological fluids subject to an applied magnetic field. The braking torque is produced by the axial elongation of SMA spring. When the temperature is equal to 70°C, the MR fluid is filled with the working gap because of the axial elongation of SMA spring. \( \phi = \pi / (A_1 + A_2) \). In the process of cooling, \( T_{m}(M_r + M_t)/2, \phi = \pi / (A_1 + A_2) \). In the process of cooling, when \( A_1 = A_2 \), the start and finish transformation temperatures of martensite and austenite, respectively.

\[ G(T) = G_M + (G_A - G_M) / [2(1 + \sin \phi(T - T_m))] \]

where \( G_M \) and \( G_A \) are the shear modulus of martensite and austenite, respectively. In the process of heating, \( T_m = (A_1 + A_2)/2 \), \( T_{m_0} = (M_r + M_t)/2 \), \( M_r, M_t, A_1, \) and \( A_2 \) are the start and finish transformation temperatures of martensite and austenite, respectively. The actual volume \( V_a \) which the SMA spring pushes the MR fluid into working gap is

\[ V_a = S(T) \cdot A \]

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MR fluids exhibit a controllable yield stress-like behavior in shear, whereby the application of a magnetic field transverse to the flow creates a resistance to flow which increases with an increasing magnetic field. To accommodate the shear thinning observed in MR fluids, the Herschel-Bulkley model \[12\] can be used to describe the flow behavior of MR fluid:

\[
\tau = \tau_y(H) + K |\dot{\gamma}|^{m} \text{sgn}(\dot{\gamma}) \quad \tau \geq \tau_y(H) \\
\dot{\gamma} = 0 \quad \tau < \tau_y(H)
\]  

(4)

where, \(\tau\) is the total shear stress, \(\tau_y(H)\) is the yield strength caused by the applied magnetic field, \(\dot{\gamma}\) is the shear rate, \(m, K\) are constants. In the Herschel-Bulkley model, the constants \(m, K\) and the function \(\tau_y(H)\) are empirically determined from experiments.

IV. MATHEMATICAL MODEL OF TORQUE

A. Flow Equation

The diagram of the operational mode of a disc-type MR brake is shown in Figure 2. In order to derive the equation of the fluid flow in the gap between two parallel circular plates, the following assumptions are given: the fluid is incompressible. There is no flow in radial direction and axial direction, but only tangential flow. The flow velocity of MR fluid is a function of radius. The pressure in the thickness direction of MR fluid is constant. The strength of magnetic field in the gap of the activation region is well-distributed. In cylindrical coordinates \((r, \theta, z)\), the distribution of the flow velocity is

\[V_r = 0, V_\theta = r \omega(z), V_z = 0\]  

(5)

where \(V_r, V_\theta,\) and \(V_z\) are the flow velocity of the fluid in the \(r\)-direction, the \(\theta\)-direction and \(z\)-direction, respectively;

\(\omega(z)\) is the rotation angular velocity of the fluid in the \(\theta\)-direction. The angular velocity \(\omega(z)\) is the function of \(z\)-coordinate. The flow equation of the MR fluid in the \(\theta\)-direction may be approximated by

\[
d^2\omega(z)/dz^2 = (1/\eta)d\sigma_{\theta\theta}/d\theta
\]  

(6)

where \(d\sigma_{\theta\theta}/d\theta\) is the gradient of the pressure in the \(\theta\)-direction. It is assumed that the fluid between two parallel circular plates is a steady-state flow, \(d\sigma_{\theta\theta}/d\theta = \text{const}\).

Integrating the (6), the rotation angular velocity \(\omega(z)\) can be indicated as:

\[
\omega(z) = \left(z^2/2\eta\right)d\sigma_{\theta\theta}/d\theta + c_1z + c_2
\]  

(7)

where \(c_1\) and \(c_2\) are the two integrating constant. It is assumed that the fluid in touch with the surface of the turning plate has the same velocity as the turning plate. The angular velocity of the MR fluid increases with \(z\)-direction. It is assumed that there is no magnetic flux line at the upper \((r = R_2)\) and the lower \((r = R_1)\) portions of the domain. So, applying the boundary conditions of \(\omega(z) = 0\) at \(z=0\) and \(\omega(z) = \omega\) at \(z=h\), the flow velocity \(\omega(z)\) can be mathematically manipulated to yield the flow as follows:

\[
\omega(z) = \omega z/h + \left(1/2\eta\right)\left(z^2 - zh\right)d\sigma_{\theta\theta}/d\theta
\]  

(8)

where \(h\) is the gap between two parallel circular plates. \(\omega\) is the rotational velocities of the rotor-plate.
the θ- direction for the cylindrical coordinates may be approximated by

$$\frac{1}{r} \frac{\partial}{\partial \theta} (r \sigma_{\theta\theta}) + \frac{1}{r} \frac{\partial}{\partial \theta} (r \tau_{r\theta}) + 2 \frac{\partial}{\partial \theta} \tau_{\theta\theta} = 0 \quad (9)$$

It is assumed that the MR fluid between two parallel circular plates is continuous flow. The gradient of the pressure in the θ-direction is

$$\frac{d \sigma_{\theta\theta}}{d \theta} = (6 \eta \omega / h^3) (h - h_0) \quad (10)$$

where \( h_0 \) is the thickness of the MR fluid at maximum pressure. Because the two plates in the clutch are parallel each other (\( h = h_0 \)), so the gradient's value of the pressure in the θ-direction is zero (\( d \sigma_{\theta\theta} / d \theta = 0 \)). The flow angular velocity \( \omega(z) \) in the θ-direction is constant (\( \tau_{r\theta} = 0 \)). Equation (9) is simplified to

$$d \tau_{r\theta} / dz = 0 \quad (11)$$

The fluid shear strain rate of (4) may be approximated by

$$\dot{\gamma} = r \left(\frac{d \omega(z)}{dz}\right) \quad (12)$$

C. Torque Equation

The torque transmitted by the disk-type brake is calculated by integrating the shear stress of the MR fluid. In Figure 2, \( R_1 \) and \( R_2 \) are the inner and outer radius of two circular plates, respectively. Take a micro-unit at distance \( r \) location in a circle, micro shear torque of the unit imposed on disc is as follows:

$$dJ = dF \cdot r = (\tau \cdot dS) \cdot r \quad (13)$$

The total transmission torque is:

$$J = N \int_r dJ = 2 \pi N \int_r \tau_{r\theta} r^2 dr \quad (14)$$

where \( N \) is the number of working gap of MR fluid. Equations (4), (8), (12) and (14) can be mathematically manipulated to yield the torque as follows:

$$J = \frac{2 \pi N}{3} (R_2^3 - R_1^3) \tau_j (H) + \frac{2 \pi KN}{m+3} (R_2^{m+3} - R_1^{m+3})(\frac{\omega}{h})^n \quad (15)$$

V. COMPUTATIONAL RESULTS AND DISCUSSIONS

Figure 3 shows the effect of the temperature on the MR fluid volume pushed by the SMA spring into the working gap. In this calculation, \( A_s = 50^\circ C, A_f = 70^\circ C, G_M = 7.5 \text{GPa}, G_A = 25 \text{GPa}, d=3.7 \text{mm}, D=22 \text{mm}, n=3, \Delta P=0.743 \text{MPa}, A=500 \text{mm}^2, \Delta h=16 \text{mm}. \) When the temperature is higher than 50°C, SMA spring begins to push the MR fluid into the working gap. When the temperature is equal to 70°C, the MR fluid can be entirely pushed by the SMA spring into the working gap.

For this example, we choose the commercial MR fluid (MRF-132DG) manufactured by Lord Corporation [13]. Figure 4 shows the yield strength of MRF-132DG under different magnetic field strength, measured by experiment.
kPa for the applied magnetic field strength of 0~200 kAmp/m. The ultimate strength of MR fluid is limited by magnetic saturation. The result shows that with the increase of the applied magnetic field strength, the dynamic yield stress goes up rapidly.

According to (15), the effect of magnetic field strength in transmission torque of the brake be analyzed, show in Figure 5. In this study, a commercially available MR fluid, LORD MRF-132DG, is used. The viscosity of MRF-132DG, $\eta=0.0925$ Pa·s. It is assumed that the constants $m=1$, $K=\eta$ in the Herschel-Bulkley model. Geometric parameters of the brake are: inner radius $R_1=20$ mm, outer radius $R_2=50$ mm, working gap $h=1$ mm. The input angular velocity is, $\omega=100$ rad/s. When the magnetic field is applied, the torque is 7.7 N.m, 14.8 N.m and 19.3 N.m at the strength of magnetic field of 50kAmp/m, 100kAmp/m and 150kAmp/m, respectively. When MR fluid is saturated at the strength of magnetic field of 200kAmp/m, the torque is 21.9 N.m. The results indicate that with the increase of the applied magnetic field, the torque developed by MR fluid goes up rapidly.

VI. CONCLUSIONS

The braking torque of a MR brake driven by SMA is investigated theoretically in this paper. The mathematical model of braking torque developed by the MR fluid is derived. The torque of the brake under different magnetic field strength is analyzed. With the increase of the applied magnetic field strength, the braking torque of the brake is increased rapidly.

![Figure V. Braking Torque Versus Magnetic Field Strength](image-url)

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