Laser-guided relativistic quantum dynamics

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New Journal of Physics 11 (2009) 105045 (21pp)
Received 22 May 2009
Published 30 October 2009
Online at http://www.njp.org/
doi:10.1088/1367-2630/11/10/105045

Abstract. Super-strong short laser pulses are employed to accelerate electron quantum wave packets stemming from atoms and to guide them into recollision in spite of the magnetically induced relativistic drift motion. The recollision-induced processes of production of new particles and generation of high-frequency light are considered. We optimize the collisions by controlling the laser field polarization, the carrier–envelope phase and the beam focusing parameters.

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1. Introduction

Laser fields can drive electrons to relativistic energies [1]–[3]. Thus, in the most powerful laser fields available nowadays (intensities of an order of $10^{22} \text{W cm}^{-2}$ [4]), an electron gains an oscillatory ponderomotive energy of about 10 GeV. Much higher electromagnetic fields are envisaged in the extreme light infrastructure [5]. However, free electrons cannot obtain a permanent energy increase from a plane monochromatic laser field. In fact, due to a phase slippage of the electron with respect to the laser field, there is a symmetry between the accelerating and the decelerating phases of the electron in the field and no net acceleration arises. To effectively gain energy from the laser field, one has to break the symmetry in the energy exchange process between the electron and the laser field. One way to realize this is a phase synchronism between the electron motion and the driving field. This is achieved in free-electron lasers where the additional undulator field causes the transverse velocity of the electron to change the sign synchronously with the laser field [6], or in plasma wakefield accelerators where the laser beam creates a longitudinal plasma wave which can be synchronous with the electrons [7]. Yet, the number of synchronous electrons is always challenging to enhance, and the achievable current densities are limited [8].

Another way to exploit the temporary energy gain of the electron in the laser beam is to initiate high-energy processes \textit{in situ}, i.e. inside the laser beam [9]. In this case the energy gain of the electron during a half cycle of the laser wave is used to trigger some process during which the electron state may change (in particular, the electron may annihilate) and the desired asymmetry will be achieved. In fact, the second method is widely employed in the nonrelativistic regime in modern attoscience [10]. Its basis is formed by the so-called laser-driven recollision of an ionized electron with its parent atom described by the three-step model [11]–[13]. The strong laser field liberates the electron from an atom and accelerates it away during a half cycle. When the laser field changes the sign in the course of its oscillation, the electron returns to the atomic core with a significant net amount of energy absorbed from the laser field due to a sub-cycle motion and is scattered by the core. During the recollision, among other effects, the ponderomotive energy acquired by the electron in the laser field can be transformed to atomic excitations, triggering non-sequential ionization [14], or to high-energy photons when the electron recombines to a bound state (high-order harmonic generation (HHG) [15]). The recollision can be used for probing the atom in the attosecond pump–probe setup [16], or for imaging molecular orbitals [17]. During the recollision, the electron wave packets are engineered by the laser field on the atomic scale with a possibility to create large current densities apart from high energies.

Can the temporary energy gain of the electron in the laser beam also be employed in the relativistic regime when the ponderomotive energy is larger than the electron rest energy? The extension of the recollision scheme straightforwardly into the relativistic regime is known to be hindered by the relativistic drift due to the magnetic component of the Lorentz force [3]. Moreover, at high intensities, the atomic medium is ionized and transformed into plasma, which is why the main attention of the laser–matter interaction at relativistic intensities is directed towards plasma effects. In particular, HHG in a keV photon energy range can be attained by means of strong laser pulse reflection from the surface of an overdense plasma [18]. While the high value of the electron ponderomotive energy is fully employed in the strong laser beam interaction with the plasma, the most important feature of the single-atom attosecond recollision is lost, though, because in plasma, there is no well-controlled coherent collision.
of the atomic constituent particles at microscopic impact parameters. However, the coherent collision enables it to generate very large current densities—a single relativistic electron wave packet of a microscopic 20 Å size carries an enormous current density of $10^{12}$ A cm$^{-2}$—which could be harnessed to enhance the yield of the high-energy process triggered by the recollision, e.g. nuclear excitations as in [19].

Various methods for counteracting the relativistic drift have been proposed. To this end, one may use modified driving laser fields: especially tailored laser pulses [20], a tightly focused laser beam [21], two laser beams in different geometries and polarization [22]–[25], in particular, counterpropagating circularly polarized (CCP) waves of equal handedness [19], two consecutive short laser pulses [25], strong counterpropagating attosecond pulse trains [26], a laser field assisted by a weak attosecond pulse train [27] or by a strong magnetic field [28]. Alternatively, modified atomic systems can be employed: highly charged relativistic ions propagating opposite to the laser beam [29, 30], antisymmetric molecular orbitals [31] or positronium (Ps) atoms [32].

A realization of high-energy relativistic recollisions in a GeV energy range with Ps atoms aiming at particle production has been proposed in [33]. After the instantaneous ionization in a strong laser field, the electron and positron originating from Ps oscillate in the opposite directions along the laser electric field but experience an identical ponderomotive drift motion along the laser propagation direction which leads to periodic $e^+e^-$ collisions [32]. Such an electron–positron microcollider based on the combination of Ps atoms with intense laser fields is suitable to induce elementary particle reactions like heavy lepton-pair production in electron–positron collisions. The corresponding reaction rate and feasibility of the process Ps $\rightarrow \mu^+\mu^-$ in the presence of an intense plane laser wave are investigated in [34]. The electrons and positrons in this process are initially nonrelativistic and their annihilation into a muon pair cannot happen without photon absorption from the external laser field. The highest current density of recolliding particles is achieved in a setup of counterpropagating strong and short laser pulses like the ones anticipated in the Astra Gemini Project of the Rutherford Appleton Laboratory, UK [35].

A related muon production process is considered in [36]. Rather than from Ps atoms, here the initial electron and positron emerge from vacuum polarization in the collision of a superstrong laser pulse with a relativistic ion beam, and the muons are subsequently produced in a laser-driven $e^+e^-$ collision.

In this paper, we investigate the feasibility of the laser-driven collider via a gas of Ps atoms with counterpropagating focused laser beams. The use of CCP as well as counterpropagating linearly polarized (CLP) waves are discussed. We show that with focused laser beams, the CLP setup is more advantageous than the CCP configuration allowing for a substantially larger size of the Ps target. The role of the carrier-envelope phase (CEP) of the pulses, of the Coulomb interaction between the particles and of radiation losses are analyzed in detail. We optimize the recollision energy, the wave packet size of the particles, the coordinate offset at the recollision moment and the excursion distance of the particles, with respect to the muon production reaction in the focused laser beam, in terms of the applied intensities and CEP phase, and determine the maximal operable volume. We also investigate HHG with a usual atomic target in the CCP field in a weakly relativistic regime. The CCP field configuration is efficient to suppress the relativistic drift and to focus the rescattering electron current enhancing the HHG yield from a single atom. However, severe difficulties for attaining phase-matched harmonic emission arise in this setup, which are also discussed.

For the atomic HHG, the CCP field configuration has a clear advantage with respect to the CLP (see section 3).
2. Relativistic recollisions for muon production

2.1. Laser-driven collider setup

A laser-driven collider setup has been proposed in [33]. Counterpropagating strong short laser pulses strike a jet of Ps atoms, see figure 1. The electron and the positron are immediately transferred from the bound Ps state into the continuum by the strong laser field and subsequently guided into a high-energy collision. Since the electron and the positron stem from a spatially well-confined bound state and because the classical trajectories in the laser field bring the particles into a close approach, the impact parameter of the quantum recollision is of a size of the electron wave packet at the recollision moment and has microscopic dimensions, usually not exceeding 20 atomic units (au). One can define the luminosity due to this kind of coherent collision as $L = N_a f / a_{wp}^2$, where $N_a$ is the number of atoms in the laser beam, $a_{wp}$ is the electron wave packet size at the recollision moment, and $f$ is the pulse repetition rate. The luminosity can be large because of the small value of the wave packet size even when the number of atoms in the laser beam is not large (this is usually the case for a Ps beam). The coherent collisions account for the leading contribution in the total luminosity when $N_a \lesssim (w_0/a_{wp})^2$, with the waist size of the laser beam $w_0$ [33]. The coherent collisions will be efficient if the electron wave packet spreading is not large. For this reason, the counterpropagating laser beam setup has an essential advantage in comparison with a single laser pulse because in the former case, the recollision time is shorter and, therefore, the wave packet spreading is reduced, while the achievable energies are the same.
The electrons and positrons in this process are initially nonrelativistic. By absorbing photons from the laser field, they can attain the necessary energy at the recollision moment to induce particle reactions. In the counterpropagating laser waves, the electron and positron energy at the recollision moment arises mainly due to the transversal momentum of the particles which scales as \( p_x \sim mc\xi \), where \( \xi = eE_0/mc\omega \) is the relativistic field parameter, \( E_0 \) and \( \omega \) are the laser field amplitude and frequency, respectively, \( e \) and \( m \) the electron charge and mass, respectively and \( c \) the speed of light. According to the simple-man’s model (see [34]), the rate of the laser-driven process can be expressed via a convolution of the field-free cross-section with the rescattering electron wave packet. We consider the basic particle reaction that could be triggered in the laser-driven collider: electron–positron annihilation with production of a muon–antimuon pair, \( e^+e^- \rightarrow \mu^+\mu^- \). The energy threshold for this process is \( \varepsilon_{\text{th}} = 2Mc^2 = 414mc^2 \), with the muon mass \( M \), which can be achieved with a laser field of \( \xi \sim 200 \) corresponding to realistic near-future laser intensities of an order of \( 8 \times 10^{22} \text{ W cm}^{-2} \). Note that the field-free cross-section of this process \([37]\)

\[
\sigma = \frac{4\pi r_0^2 m^2 c^4}{3e^2} \sqrt{1 - \frac{4M^2c^4}{\varepsilon^2}} \left( 1 + \frac{2M^2c^4}{\varepsilon^2} \right),
\]

with the classical electron radius \( r_0 \) attains the maximal value \( \sigma_m \sim r_0^2m^2/M^2 \), at the electron center-of-mass energy of \( \varepsilon_0 = 506.7mc^2 \). In the next sections, we discuss the realization of the laser collider with focused laser beams. The impact of the laser field polarization and the CEP phase on the operational properties of the laser collider are investigated concentrating on the process of muon production in the coherent collisions. The efficient operation of the laser collider would require, apart from the optimal energy \( \varepsilon_0 \), a rather small wave packet size at the recollision moment \( a\w \cdot p \cdot \ll w_0/\sqrt{N_a} \), a small coordinate offset at the recollision moment and a small electron excursion distance \( l \ll 1/n_1^{1/3} \sim 2000 \text{ au} \), with the density of Ps atoms \( n_a \sim 10^{15} \text{ cm}^{-3} \)[38].

2.2. Linear versus circular polarization. Optimization of the laser intensity and the CEP phase for the muon production reaction

In [33], we employed two laser pulses constituting CCP as a driving field for the laser collider. The choice was based on the fact that this field configuration exhibits the unique feature of parallel electric and magnetic fields in contrast to a propagating laser wave (see figure 2). Therefore, an electron wave packet moving in such a field does not experience a drift along the propagation direction when the relativistic intensity regime is reached [19]. Moreover, the Lorentz force in this case induces the focusing of the electron wave packet, thereby even further increasing the recollision current density. However, there is a significant drawback with this field configuration: when a Ps atom is not reached simultaneously by both counterpropagating laser pulses, an offset arises in the recollision coordinate. This is because the electron and positron from such a Ps atom move in a single laser pulse in the initial stage of the interaction and acquire a longitudinal velocity of the same sign. This velocity gives rise to a Lorentz force with a different sign when particles move within the area of superimposed pulses causing the offset in the direction transversal to the propagation and the local polarization directions. This imposes a very strong limitation on the distance of the Ps atom from the symmetry plane with respect to the pulses, i.e. on the longitudinal size of the Ps jet [33]. In the following, we compare the recollision quality in CCP with that in CLP in focused laser beams.
Figure 2. The field of a standing laser wave consisting of two counterpropagating equal-handed circularly polarized waves. The electric (red) and magnetic (green) field of the laser field are shown. Both parts of the field are parallel in contrast to a standing wave of linear polarization or of opposite-handed circular polarization.

Usually, tightly focusing of laser beams is used to produce strong laser fields, therefore, a highly accurate Gaussian description of the laser beam is necessary. We employ a Gaussian paraxial beam propagating along the z-axis (and another one moving to the opposite direction). For a laser beam of linear polarization (polarization along the x-axis), we use the field expressions from [40] (see also equations (53)–(61) from [3] and [41, 42]) up to the third-order corrections in the small parameter of the diffraction angle \( \epsilon = w_0/z_r \), where \( z_r = \pi w_0^2/\lambda \) is the Rayleigh length and \( \lambda \) the laser wavelength,

\[
E_x = E \left\{ S_0 + \epsilon^2 \left[ \zeta^2 S_2 - \frac{\rho^4 S_3}{4} \right] \right\},
E_y = E \zeta \nu \epsilon^2 S_2,
E_z = E \zeta \left\{ \epsilon C_1 + \epsilon^3 \left[ - \frac{C_2}{2} + \rho^2 C_3 - \frac{\rho^4 C_4}{4} \right] \right\},
B_x = 0,
B_y = E \left\{ S_0 + \epsilon^2 \left[ \frac{\rho^2 S_2}{2} - \frac{\rho^4 S_3}{4} \right] \right\},
B_z = E \nu \left\{ \epsilon C_1 + \epsilon^3 \left[ \frac{C_2}{2} + \rho^2 C_3 - \frac{\rho^4 C_4}{4} \right] \right\},
\] (2)

with

\[
E = E_0 \frac{w_0}{w} \exp \left[ - \left( \frac{r}{w} \right)^2 \right] \cos^2 \left( \frac{\omega t - kz}{n} \right),
S_n = \left( \frac{w_0}{w} \right)^n \sin(\phi + n\phi_G), \quad C_n = \left( \frac{w_0}{w} \right)^n \cos(\phi + n\phi_G),
\] (3)

We have confirmed that the corrections up to 7th order in the parameter \( \epsilon \) do not modify the results noticeably.
where \(E_0\) is the amplitude of the single pulse, \(w(z) = w_0 \sqrt{1 + (z/z_r)^2}\) the beam waist radius at position \(z\) along the propagation axis, \(r^2 = x^2 + y^2\), \(\rho = r/w_0\), \(\zeta = x/w_0\) and \(\nu = y/w_0\). The field phase is \(\phi = \Phi_{\text{CEP}} + \phi_p - \phi_R + \phi_{G}\) with the CEP phase \(\Phi_{\text{CEP}}\), \(\phi_p = o t - k z\), \(k = \omega/c\), the Guoy phase \(\phi_G = \tan^{-1}(z/z_r)\), \(\phi_R = k r^2/(2 R)\) and \(R(z) = z + z_r^2/z\). Additionally, a \(\cos^2\)-shaped temporal profile of the laser pulse is adopted with a pulse duration \(\tau_{\text{FWHM}} = 3 T/4\), with the laser period \(T\). The field of the counterpropagating (along-\(z\)) focused beam is derived with the following transformations:

\[
E'_x = E_x, \quad E'_y = E_y, \quad E'_z = -E_z, \quad B'_y = 0, \quad B'_z = -B_y, \quad B'_z = B_z. \tag{4}
\]

The laser field for a circularly polarized focused beam is straightforwardly derived following Davis \cite{39} and is expressed via a combination of equations (2):

\[
E''_x = E_x + E_y \left( \phi \rightarrow \phi \pm \frac{\pi}{2} \right),
\]

\[
E''_y = E_y + E_x \left( \zeta \leftarrow \nu, \phi \rightarrow \phi \pm \frac{\pi}{2} \right),
\]

\[
E''_z = E_z + E_\zeta \left( \nu \leftarrow \zeta, \phi \rightarrow \phi \pm \frac{\pi}{2} \right),
\]

\[
B''_x = -B_y \left( \phi \rightarrow \phi \pm \frac{\pi}{2} \right),
\]

\[
B''_y = B_y,
\]

\[
B''_z = B_z - B_\zeta \left( \nu \leftarrow \zeta, \phi \rightarrow \phi \pm \frac{\pi}{2} \right). \tag{7}
\]

We apply the Monte Carlo simulation method to investigate the relativistic dynamics of the Ps atom constituents in the laser-driven collider. The classical Monte Carlo methods have confirmed their validity in the description of atomic ionization phenomena in strong laser fields \cite{43, 44}. When spin and interference effects are not important, the classical Monte Carlo method provides an adequate way to mimic the quantum mechanical wave packet spreading instead of numerically extensive quantum mechanical propagation. We simulate the electron wave packet propagation in the laser field via the evolution of a classical ensemble which is initially prepared as a microcanonical one corresponding to the Ps bound state \cite{45}.

First, the case of CCP is considered. We investigate at which laser parameters (intensity, CEP phase) and initial coordinates of Ps atoms in the focus of the laser beam, the constraints for muon production in coherent collisions, mentioned in section 2.1, can be fulfilled. The range of affordable initial coordinates will determine the operable size of the Ps gas in the laser-collider setup. The constraints require a rather small wave packet size and coordinate offset at the recollision moment and a small excursion distance. We know from the plane-wave consideration of \cite{33} that in the CCP field, the electron wave packet spreading is confined and the excursion distance is not large. The main limiting factor for the laser-collider operation in this case is the offset at the recollision moment which arises when the initial \(z_0\)-coordinate of the Ps is shifted from the symmetric point with respect to the laser pulses. This is why at first we optimize the CEP phase to have the largest possible \(z_0\)-coordinate (measured from the symmetric point) with an affordable offset and sufficiently high electron energy. In figure 3, the dependence of the coordinate offset \(\rho = \sqrt{\sum_i (x_i^0 - x_i^z)^2}\) and the total electron–positron

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Figure 3. Contour plots of (a) the offset $\rho$ and (b) momentum $p/mc$ distributions at the recollision moment versus $kz_0$ and the CEP phase ($\Phi_{CEP}$) for a Ps atom in a focused CCP with $w_0 = 10 \mu$m, $\lambda = 0.8 \mu$m and laser intensity $\xi = 210 \ (I = 9.3 \times 10^{22} \text{ W cm}^{-2})$. The Ps atom is initially located at the origin in the transverse plane of the laser beams: $kx_0 = 0$ and $ky_0 = 0$. The optimal values of the total momentum, when the cross-section is the largest, and of the offset are indicated with a dot-dashed line.

momentum $p = \sqrt{\sum_i (p_i^+ - p_i^-)^2}$, $i \in \{x, y, z\}$ at the recollision moment, on the Ps initial coordinate $z_0$ and the laser CEP phase is shown with the electron and positron coordinates $x_i^\pm$ and momenta $p_i^\pm$ at the recollision moment, respectively. The Ps atom is initially located at the origin in the transverse plane of the laser beams: $kx_0 = 0$ and $ky_0 = 0$ (we normalize the coordinates by the wave vector of an infrared wave $k = 2\pi/\lambda$, with $\lambda = 0.8 \mu$m). We see that the total scattering energy is larger than the threshold energy for muon production in a rather large range of the CEP phase: $4.5 < \Phi_{CEP} < 5.6$ ($\xi = 210$). However, the offset essentially depends on the CEP phase and the initial $z_0$-coordinate. An affordable offset $\rho < 4 \text{ au}$ (later, we will see that this value of $\rho$ corresponds to the wave packet size) will not depend on the CEP phase only if the initial Ps is very close to the symmetric point $kz_0 < 0.0012$. If not, the offset is affordable only in very limited regions of the CEP phase. Taking into account that the total scattering energy should be larger than the threshold energy for muon production, an optimal value of the CEP phase of $\Phi_{CEP} \approx 5$ can be deduced from figure 3. At this phase, the largest initial $z_0$-coordinate can be available at a coordinate offset of $\rho < 4 \text{ au}$. The maximum of the initial coordinate is $kz_0 = 0.01$.

We continue to determine the range of a possible variation of the initial coordinates along the laser beam radius in the case of CCP. The variation of the Ps radial position from the axis of the laser beam causes again a coordinate offset at the recollision point. We investigate the latter in figure 4 where the dependence of the coordinate offset and the total electron–positron momentum at the recollision moment on the Ps initial coordinates $x_0$ and $y_0$ is shown. Two limiting cases of the initial $z_0$-coordinate are considered: $kz_0 = 0$ and 0.008. At $\rho = 4 \text{ au}$, the...
Figure 4. Contour plots of (a, b) the offset \( \rho \) and (c, d) the momentum \( p/mc \) distributions versus \( kx_0 \) and \( ky_0 \) for a Ps atom in the focused CCP. For the left column \( kz_0 = 0 \) and for the right column \( kz_0 = 0.008 \). \( \Phi_{\text{CEP}} = 5 \) and the other parameters are the same as those in figure 3. The dot-dashed lines indicate the optimal values of the parameters.

limitation on \( x_0 \) and \( y_0 \) comes from the offset for the Ps with the initial coordinate \( kz_0 = 0.008 \): \( kx_0 = 15 \) and \( ky_0 = 2 \) (see figure 4(b)). In these limits, the requirement on the attainable energy \((\varepsilon \sim \varepsilon_0)\) is fulfilled, see figures 4(c) and (d). An example of the wave packet spreading in the CCP case is shown in figure 5. The wave packet size is rather small, not exceeding 4 au, due to the focusing in this field. Thus, the coordinate offset at the recollision point is the main limiting factor for the coherent recollision in CCP which restricts the initial size of the Ps atom gas to at most \( kx_0 < 15, ky_0 < 2 \) and \( kz_0 < 0.01 \).

Now, let us consider the case of CLP. First, we comment on choosing the laser intensity. In the counterpropagating laser beam setup, usually, the size of the electron wave packet at the recollision moment decreases with increasing laser intensity (see table 1). This is because the electron energy increases with rising laser intensity, which decreases the effective velocity \( \Delta v_{\text{eff}} \sim v_0/\gamma \) responsible for the wave packet spreading, where \( v_0 \) is the velocity spread in the initial wave packet and \( \gamma \) the Lorentz gamma-factor of the laser-driven electron motion. However, the increase of the laser intensity, apart from being technically more difficult for realization, also causes an increase in the electron excursion distance which could cause incoherent collisions with other particles. Therefore, one has to limit the excursion distance and a compromise is required when choosing the laser intensity.
Figure 5. Coordinate-space distributions of (a, c) the electron and (b, d) the positron wave packets at the recollision moment in the focused CCP with a Ps atom initially located at a position: $k_x = 10$, $k_y = 1$, $k_z = 0.008$. $\Phi_{\text{CEP}} = 5$ and the other parameters are the same as those in figure 3.

Table 1. The spreading of the electron wave packet ($\Delta x$, $\Delta y$, $\Delta z$) in au at the recollision moment in a plane-wave CLP.

| $\xi$ | $\Delta x$ | $\Delta y$ | $\Delta z$ |
|-------|------------|------------|------------|
| 0.1   | 40         | 40         | 100        |
| 10    | 8          | 20         | 100        |
| 100   | 4          | 8          | 48         |
| 300   | 4          | 4          | 36         |
| 1000  | 4          | 4          | 20         |

In contrast to the CCP, there is no offset at the recollision point in the plane-wave case. In a focused beam of CLP, a small offset arises when the initial Ps atom coordinate is shifted from the beam axis, but this is not the main limitation for the coherent recollision realization. In fact, in the CLP, the electron excursion distance becomes large when the Ps initial $z_0$-coordinate is shifted from the symmetry point. In this case, the CEP phase is optimal when the largest $z_0$-coordinate is attained with an affordable excursion distance. In figure 6, the dependence of the total electron–positron momentum and the electron excursion distance $|z - z_0|$ at the recollision moment on the Ps initial coordinates $z_0$ and the laser CEP phase is shown. From this figure,
Figure 6. Contour plots of (a, b) the momentum $p/mc$ and (c, d) the excursion distance $|z-z_0|$ distributions versus $kz_0$ and $\Phi_{CEP}$ for a Ps atom in a focused CLP with $w_0 = 10 \mu m$, $\lambda = 0.8 \mu m$. The left column is for the laser intensity $\xi = 150 (I = 4.7 \times 10^{22} W cm^{-2})$ and the right column for $\xi = 259.6 (I = 1.4 \times 10^{23} W cm^{-2})$. The Ps atom is initially located at the origin in the transverse plane of the laser beams: $kx_0 = 0$ and $ky_0 = 0$. The dot-dashed lines indicate the optimal values of the parameters.

one can deduce the optimal phase of the laser pulses. This equals $\Phi_{CEP} = 5.1$ at $\xi = 150$ and $\Phi_{CEP} = 5.4$ at $\xi = 259.6$. However, the latter $\xi = 259.6$ intensity is more advantageous because in this case, the possible variation of the initial $z_0$-coordinate, which still keeps the excursion distance below the imposed limit, is almost twice as large as for $\xi = 150$: $kz_0 = 0.5$.

The variation of the initial Ps coordinates $x_0$, $y_0$ in the transverse direction causes some offset at the recollision moment as one can see in figure 7. The acceptable value for the offset can be inferred from the electron wave packet size at the recollision moment. The wave packet in two representative cases with different initial coordinates of the Ps are shown in figures 8 and 9. The wave packet size in the $y$-direction is about $4$ au in both cases, while in the $z$-direction the largest size is $18$ au, when the Ps is initially at the origin, and $0.6$ au when $kx_0 = 8$, $ky_0 = 15$, $kz_0 = 0.45$. Figure 7(a) shows that an offset of $\rho < 4$ au can be fulfilled up to $kx_0 = 10$. The limitation on the initial $y_0$-coordinate comes from the achievable energies: $ky_0 = 20$ (see figure 7(b)). Further, let us see if the limitation on the excursion distance of the recolliding electron trajectory can impose any restriction on the initial spatial distribution of Ps atoms. In fact, figure 10 shows that this is not the case. Within the limits $kx_0 < 10$, $ky_0 < 20$, the excursion distance is restricted to $\delta x \sim 10$ au, $\delta y \sim 60$ au and $\delta z \sim 1800$ au, which are

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Figure 7. Contour plots of (a) the offset $\rho$ and (b) the momentum $p/mc$ distributions versus $kx_0$ and $ky_0$ for a Ps atom in the focused CLP. The Ps atom is initially located at $kz_0 = 0.45$. $\Phi_{\text{CEP}} = 5.4$, the other parameters are the same as those in figure 6. The dot-dashed line indicates the optimal value of the energy.

Figure 8. Coordinate-space distributions of the electron and positron wave packets at the recollision moment in the focused CLP with a Ps atom initially located at the origin: $kx_0 = 0, ky_0 = 0, kz_0 = 0$. $\Phi_{\text{CEP}} = 5.4$, the other parameters are the same as those in figure 6.
Figure 9. Same as figure 8 but with a Ps atom initially located at \( k_x = 8, k_y = 15, k_z = 0.45 \). \( \Phi_{\text{CEP}} = 5.4 \).
parameters within 10% is not dramatic for the operation of the laser collider in both CCP and CLP cases. The optimal operation regime is still achievable with a slight adjustment of the CEP phase.

Let us estimate the achievable reaction rate in the coherent collision scheme. The coherent collisions have a dominant contribution in the total luminosity when the number of Ps atoms in the interaction region is not too large: \( N_a < S/a_y a_z \sim 10^8 \), with the cross-section area \( S \) of the interaction volume in the \( y - z \) plane, the electron wave packet dimensions \( a_y \) and \( a_z \) in the corresponding directions. The interaction volume does not expand during the recollision and the cross-section area can be estimated as \( S = 4y_{0}^{\text{max}} z_{0}^{\text{max}} \), where \( y_{0}^{\text{max}}, z_{0}^{\text{max}} \) are the maximal available initial coordinates. Assuming \( N_a = 10^7 \), the number of reaction events per laser shot.

**Figure 10.** Contour plots of the excursion distance (a) \( |x - x_0| \), (b) \( |y - y_0| \) and (c) \( |z - z_0| \) distributions versus \( kx_0 \) and \( ky_0 \) at the recollision moment in a focused CLP. The parameters are the same as those in figure 7. \((x_0, y_0, z_0)\) indicates the initial electron positions. The dot-dashed line indicates the optimal values of the energy.
Figure 11. Coordinate-space distributions of electron and positron wave packets at the recollision moment with a Ps atom initially located at a position: $k_x = 8$, $k_y = 15$, $k_z = 0.45$. The parameters are the same as those in figure 9 but taking into account the Coulomb attraction between the electron and the positron.

Can the radiation damping impede the operation of the laser-driven collider? Let us estimate the energy loss of the electron in the laser collider due to radiation damping. We estimate the

$N \sim \sigma_m N_a/\alpha_x \sim 10^{-7}$, with the maximal value of the cross-section $\sigma_m \sim r_0^2 m^2/M^2$. We have investigated the influence of the Coulomb interaction between the electron and the positron during the excursion in the laser field. Calculations of the wave packet dynamics taking into account the Coulomb interaction have been carried out for different laser intensities in an interval of $10 \leq \xi \leq 1000$ and for different initial coordinates of the Ps atom. No significant deviation of the dynamics due to the Coulomb field has been found. Figure 11 shows an example representing the wave packet at the recollision moment corresponding to the case of figure 9 with an additional inclusion of the electron–positron Coulomb interaction via a central soft-core potential $V(r_+ - r_-) = -1/\sqrt{(1/2 I_p)^2 + (r_+ - r_-)^2}$ [46] with the Ps ionization potential $I_p = 0.25$ au. We see that, compared with figure 9, in figure 11 there is little change in the probability distribution of the electron wave packet, which justifies the neglecting of the Coulomb interaction between the electron and the positron in other calculations. The particle motion is completely dominated by the ultrastrong laser fields.

2.3. The energy loss due to radiation damping

Can the radiation damping impede the operation of the laser-driven collider? Let us estimate the energy loss of the electron in the laser collider due to radiation damping. We estimate the
radiated energy of the electron taking into account that on a coherence length $l_{coh}$, the probability of photon emission of the characteristic frequency $\omega_c$ equals the fine structure constant $\alpha$ [47]. Then, the total emitted energy during a recollision is
\[
\Delta \epsilon \sim \alpha \omega_c v_c,
\]
where $v_c$ is the number of coherence lengths during the electron excursion. The coherence length is defined as the path length from which the radiation is emitted within the characteristic angle $1/\gamma$, with the electron $\gamma$-factor. The characteristic frequency is determined by the condition of the phase matching of the emitted radiation on the coherence length: $(\omega_c - \mathbf{k}, \mathbf{v})/l_{coh}/v < \pi$, with the electron velocity $v$, from which $\omega_c \sim 2\pi c\gamma v^2/l_{coh}$ follows. For the purposes of this section, we model the electron motion in the laser collider with the motion in an oscillating electric field. Then, we have $l_{coh} \sim \lambda$ and $\gamma \sim \xi$, which yields the characteristic frequency $\omega_c \sim \xi^2 \omega$ and the ratio of the emitted energy to the electron energy is
\[
\Delta \epsilon/\epsilon \sim r_0 \xi/\lambda.
\]
The latter is negligible for the laser-collider parameters, e.g. $\Delta \epsilon/\epsilon \sim 10^{-6}$ at $\xi \sim 300$ and $\lambda = 1 \mu m$. The conclusion is that the radiation losses do not constitute a limiting factor for the laser-collider operation.

3. High-order harmonic generation

The CCP field configuration suppresses the relativistic drift and one may try to use it for HHG in the relativistic domain. We consider HHG with usual atomic systems but not with Ps atoms because the low density of the latter hinder its use as a generating medium for a HHG in the relativistic domain. We consider HHG with usual atomic systems but not with Ps atoms because the low density of the latter hinder its use as a generating medium for a HHG in the relativistic domain. We consider HHG with usual atomic systems but not with Ps atoms because the low density of the latter hinder its use as a generating medium for a HHG in the relativistic domain.

$$M = -i \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \int d^3 r \int d^3 r' \Phi_0^* (r, t) \tilde{V}_H(r, t) G_{1t}(r, t; r', t') V(r') \Phi_0 (r', t').$$

(10)

Here, $\Phi_0 (r, t) = \phi_0 (r) \exp[i(I_p t - r A(0, t)/c)]$ is the eigenstate of the physical energy operator according to [50], with the ionization potential $I_p$ and $\phi_0 (r)$ the stationary ground-state wave function of a zero-range potential $V(r) = 2\pi c/\kappa \delta(r) \tilde{\delta} r; \tilde{V}_H(r, t) = A(r, t) (-i\nabla + A(0, t)/c)$, where $\kappa = \sqrt{2I_p}, A_H(r, t) = c \sqrt{2\pi \omega_0^2} \hat{e}_H \exp(\omega_0 t - ik_H r) i\mathbf{A}(r, t)$ is the vector potential of the harmonic field and $\mathbf{A}(r, t)$ the vector potential of the CCP waves:

$$\mathbf{A}(r, t) = -\hat{E}_0 \frac{e}{\omega} \sin(\omega t) \left( \cos(kz) \hat{x} - \sin(kz) \hat{y} \right),$$

(11)

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with the total field amplitude $\tilde{E}_0$. The Green function for the electron propagation in the laser field is

$$G_t(r, t; r', t') = -i\Theta(t - t') \int d^3p \Psi_t^* p(r, t)(\Psi_t^r p(r', t'))^*, \quad (12)$$

which is defined via a solution $\Psi_t^r p(r, t)$ of the nondipole Schrödinger equation with the Hamiltonian

$$H_t = \frac{1}{2} \left(-i\nabla + \frac{A(r, t)}{c}\right)^2 - \frac{1}{8c^2} \left(-i\nabla + \frac{A(r, t)}{c}\right)^4. \quad (13)$$

The vector potential of the driving field is expanded into terms up to order $1/c^2$ in the Hamiltonian of equation (13) and, consequently, the relativistic mass shift is taken into account which is of the same order of magnitude. The terms of higher order can be omitted for a relativistic parameter of $\xi = \tilde{E}_0/\omega c < 0.3$. After these approximations, the Hamiltonian of the electron in the laser field reads

$$H_t = -\frac{1}{2} \Delta - i\frac{A(0, t)}{c}\left((1 - \frac{1}{2}k^2z^2) \frac{\partial}{\partial x} - k_z \frac{\partial}{\partial y}\right) + \frac{A^2(0, t)}{2c^2} - \frac{1}{8c^2} \left(-i\nabla + \frac{A(0, t)}{c}\right)^4. \quad (14)$$

The solution of the corresponding Schrödinger equation can be found by the ansatz

$$\Psi_t^r p(r, t) = \frac{1}{(2\pi)^{3/2}} \exp\left(i\mathbf{p}\mathbf{r} + w(t) + g(t)kz + h(t)k^2z^2\right). \quad (15)$$

The first two terms in the exponent of the ansatz are chosen in similarity to the Volkov function within the dipole approximation. The other two terms are chosen in order to match the spatial dependence of the Taylor expanded vector potential. Plugging the ansatz into the Schrödinger equation and solving each order of $\varepsilon^n$ ($n = 0, 1, 2$) independently, leads to

$$w(t) = -\frac{\mathbf{p}^2}{2}t + \frac{\mathbf{p}^4}{8c^2t} - \frac{\tilde{E}_0 p_x \cos(\omega t)}{\omega^2} - \frac{\tilde{E}_0^2 p_y \sin(2\omega t)}{4\omega^2} - \frac{\tilde{E}_0^3 p_z \sin(3\omega t)}{8\omega^3} - \frac{3\tilde{E}_0^2 p_x^2 \sin(2\omega t)}{16c^2\omega^3} - \frac{3\tilde{E}_0^2 p_y^2 \sin(2\omega t)}{16c^2\omega^3} - \frac{\tilde{E}_0^2 p_z^2 \sin(2\omega t)}{8c^2\omega^2} - \frac{3\tilde{E}_0^2 p_x p_y \sin(2\omega t)}{256c^2\omega^5} + \frac{3\tilde{E}_0 p_x \cos(\omega t)}{8c^2\omega^4} - \frac{\tilde{E}_0^3 p_x \cos(3\omega t)}{24c^2\omega^4} + \frac{3\tilde{E}_0^4 t}{2c^2\omega^2} + \frac{\tilde{E}_0^3 p_x p_y \cos(\omega t)}{2c^2\omega^2} - \frac{\tilde{E}_0^4 t}{64c^2\omega^4}, \quad (16)$$

$$g(t) = -\frac{\tilde{E}_0^2 p_x p_z \sin(\omega t)}{c\omega^2} + \frac{\tilde{E}_0 p_x \cos(\omega t)}{\omega^2}, \quad (17)$$

$$h(t) = \frac{\tilde{E}_0 p_z \cos(\omega t)}{2\omega^2}. \quad (18)$$

In deriving the electron wave function in a standing laser field, we impose a condition that the wave function reduces to a free electron plane wave when the field is switched off adiabatically. The infinitely slow application of the field does not alter the normalization integral and the wave functions satisfy the same orthonormality conditions as the free plane waves (see also [51]).

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Figure 12. High-order harmonic spectra are displayed for different laser intensities: (a) $I = 6.6 \times 10^{16}$ W cm$^{-2}$ and (b) $I = 1.5 \times 10^{17}$ W cm$^{-2}$. The red curve (background) is the emission spectrum in a standing laser wave consisting of two CCP laser fields whereas the blue curve is for a linearly polarized traveling wave. Further, the nonrelativistic dipole approximation (gray) is shown. The ionization potential is $I_p = 4.51$ au (Li$^2+$) and the laser frequency $\omega = 0.05$ au.

The expression for the HHG amplitude of equation (11) can be recast as a fivefold integral which is calculated by the saddle point method in the long wavelength regime $\omega \ll I_p \ll U_p$, with the ponderomotive potential $U_p = \tilde{E}_0^2/4\omega^2$. The calculated differential rate $d\omega_n/d\Omega = (|M|^2/T)d^3 k_H/(2\pi)^3$, with the interaction time $T$ and the emission solid angle $\Omega$, is plotted in figure 12. At a laser intensity of $I = 6.6 \times 10^{16}$ W cm$^{-2}$ HHG up to 6 keV photon energies can be attained and with $I = 1.5 \times 10^{17}$ W cm$^{-2}$ the HHG cutoff extends to 13.5 keV. The spectra show that in the weakly relativistic regime the standing wave configuration and the dipole approximation coincide. The usual damping of HHG due to the relativistic drift is absent. For comparison, the spectrum of a linearly polarized propagating wave with the same electric field strength is shown (dashed line). By increasing the intensity, the latter rate becomes suppressed compared to the two other spectra. We note, however, that the absolute value of the HHG emission rate in the relativistic regime, even in the ideal case of the dipole approximation, is significantly less compared to common nonrelativistic HHG rates. In the relativistic regime, the spreading of the electron wave packet is larger because the driving field is much stronger. The spectra indicate as well that the distortion of the wave packet in the standing wave does not decrease the yield at these intensities. Moreover, both long and short trajectories can contribute, which can be seen from the remaining oscillations in the spectrum. Here the focusing of the wave packet is not pronounced because the driving field is not very strong.

Due to the driftless electron motion in the CCP field, the single-atom response with respect to HHG is sizable in the relativistic regime. Nevertheless, the HHG realization with this field configuration is problematic. The bottleneck in this case is connected with the realization of phase-matched emission of harmonics from different atoms. In the CCP field of equation (11), electrons from different atoms oscillate synchronously and emit the harmonic field at the same time but from different spatial points. However, the polarization of the harmonic field circulates along the $z$-coordinate. As a consequence, the coherence length of the emission is restricted only to half of the harmonic wavelength. Nevertheless, due to the rotation of the emission polarization along the $z$-axis, a small net emission amplitude arises from the contribution of the atoms situated along the $z$-axis within a quarter wavelength of the driving field. If the density
of the medium is sharply modulated by the quarter laser wavelength with a precision of the harmonic wavelength, one may avoid the cancellation of the emission amplitude due to the contribution of the atoms in the next quarter laser wavelength. In this way, a rather small net HHG yield could be achieved: the number of coherently emitting atoms can be estimated to be \( \rho L w_0^2 \lambda_H / \lambda \), where \( \rho \) is the density of atoms, \( L \) the interaction length along the \( z \)-axis, \( \lambda_H \) and \( \lambda \) the wavelengths of the harmonic and the laser fields, respectively. In this field configuration, the role of the free electron background due to ionized electrons is not crucial. The coherence conditions \( \omega_H = (2l + 1) \omega \) and \( k_H = (2q + 1) k \) \((l, q \) are natural numbers), i.e. \( \omega_H = (2q + 1) n_0 \omega \), with the refractive index \( n_0 = \sqrt{1 - \omega_p^2 / \omega^2} \) of the laser wave and the plasma frequency \( \omega_p \), can be fulfilled for a specific number of harmonics at a specific choice of the medium density \( n \) \((\omega_p \sim \sqrt{n})\). The bottom line is that the HHG in CCP is very problematic to realize because the number of atoms, which produce phase-matched emission appears to be very restricted. The same kind of phase-matching problem exists for the HHG in CLP.

4. Conclusion

We have investigated the possibility of operation of a laser-driven collider with focused short intense counterpropagating laser pulses. First, we have compared the recollision quality using CCP laser pulses with that of CLP. The advantage of the CCP setup is the focusing of the recolliding electron wave packet. However, this advantage is frustrated by another unfavorable characteristic: a spatial offset arises in the electron–positron collision when the initial coordinate of the Ps atom deviates from the symmetric position between the laser pulses. The latter disadvantage imposes a severe restriction on the Ps beam size along the laser propagation direction. From this point of view, the CLP setup is preferable. Here, the offset at the recollision is much smaller. At the same time, the electron wave packet size at the recollision is not much larger compared with the CCP case. This allows us to increase the longitudinal size of the Ps beam more than 50 times in the laser parameter regime of interest. The main limitation on the Ps beam size in the CLP case comes from the requirement that the size of the excursion trajectory of the recolliding electron should be small enough to avoid incoherent scattering with other particles. The focusing of the laser fields does not have a detrimental impact on the collision parameters. Due to the focusing, some limitations arise on the Ps atom initial coordinate in the laser polarization direction, but the limitation is not severe. The impact of the Coulomb interaction between the electron and positron has negligible influence on the recollision dynamics because the latter is mainly governed by the strong laser fields. The radiative energy loss of the particles in the laser collider is also negligible. In conclusion, we found the scheme of the laser-driven collider to be operable in a realistic focused laser beam configuration of linear polarization.

Concerning HHG with atoms in CCP, we can say that, even though the single-atom yield of HHG is large and approaches the ideal value of the dipole approximation case, the difficulty in realization of phase matching with this field configuration nevertheless decreases the number of atoms which contribute coherently to the emission and negate the possibility to have a significant macroscopic yield of HHG.

Acknowledgments

We thank Michael Klaiber for fruitful discussions and Peter Brunner for producing figure 1.
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