SOME STATISTICAL TECHNIQUES APPLIED TO ENGINEERING MECHANICS PROBLEMS

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Abstract. This article deals with statistical techniques normally used in engineering. Variables or parameters in models of engineering mechanics always face data of: a) materials (with technical specification); b) analysing model using specific software; c) measurement using variety of devices and approaches; and d) the technology process of manufacture (outcome). An engineering object to be studied has k variables and each variable has m values or level of status, it will need m^k cases to be solved. This has to conduct a very large number of test cases to be solved for target objective(s). A Taguchi method will be applied for finding solution in which much less effort of computation is paid and other different conditions of noise could be taken into account. Besides, other statistical tools, ANOVA have also proved to be useful in quantifying uncertainties in engineering problems, both in aleatory (nature) and epistemic (knowledge and measurement) categories. A typical example of engineering problem is chosen to study using above-mentioned Taguchi method and statistical tools. This method is very useful for design of experiments, both in traditional laboratory and computer numerical modeling and it can be used to optimize the set of input data for obtaining the best results of outcome product.

Keywords: Taguchi method, orthogonal matrices, noise, degree of freedom, ANOVA.

Classification numbers: 2.10.1, 5.4.3.

1. INTRODUCTION

In recent years, applied mechanics is increasingly developed and closely involved to real world. Several aspects of engineering mechanics have been thoroughly studied to make the real industrial world change. People can understand more the process of manufacture with visual models, both numerical and physical ones and a lot of theoretical research gradually lead to applied studies. So many techniques have been applied, all are to visualize the products from elements size to structures scale. So many practical procedures and approaches have been developed to tackle problems such as: topographical optimization for mechanical element, stress concentration around corners or holes, fracture mechanics subjected to impact loadings, interaction between kinds of materials in difference states, etc. As a first step, computational models always include data of materials, parameters in softwares, from lab tests or measurements using devices and probe and its approach of installation. Finally, the outcome or
final manufacturing products, in turn, must ensure a policy of quality control and quality assurance in some ways.

This study is an author’s effort focusing on a statistical technique that applied successfully in scientific research, in engineering and technology, in the hope of providing some understanding, an aspiration to undergraduate and postgraduate students in having more useful tools in their academic careers.

2. RESEARCH METHODS IN ENGINEERING MECHANICS

2.1. Modeling

There are various ways of modeling in engineering mechanics. Every model needs mathematical equations or expressions in terms of geometrical size and dimensions, mechanical properties, loading regime, especially with assumptions as boundary or initial conditions. However, it’s hard to consider to what extent, results withdrawn from such models are valid as conditions are changing by chance with uncertainties and random variation of input data.

2.1.1. Mathematical modeling

The finite element method was found and well developed from years of the 1940’s by the work of Hrennikoff and Courant [1]. The concept of method is to subdivide the main body of objective into smaller and simpler systems, and then the variational computation is utilized for these systems as per some specified shape functions. This is an approximated algorithm for smaller parts and combined them into a bigger system. Variety of methods that based on mathematical approximation of shape function has been postulated to put Smooth Finite Element Method (SFEM), Expanded Finite Element Method (XFEM) comes into being. All are to reduce the size of the matrix and to solve the problem with easier algorithms and higher accuracy and less time consumption. Fig. 1 is an illustration for the method with an extent boundary.

![Planar strain model with infinite elements and extent boundary](image)

**Figure 1.** Planar strain model with infinite elements and extent boundary [2].

FELA is a finite element software in the limit analysis that uses an optimization algorithm to seek the allowable and ultimate collapse load exerted to the mechanical system of structures. Allowable loading is determined by analyzing static equilibrium, then the maximum for getting equilibrium state is lower bound of failure loading. Some yield conditions or criteria must be applied to the systems. Upper bound aims at finding the ultimate loading by analyzing stress-
strain response within the system under the displacement field, or the kinetic velocity field governed by flow rules. In the upper bound analysis, optimization algorithm for finding the minimum collapse loading is requested (for instance second-order cone programming algorithm, MOSEK, etc. as currently developed in recent research works of computational mechanics).

Beside approaches for minimizing the physical or topographical quantities, there are well-known methods for optimizing the objectives subjected to constraints such as: Strategies of Adaptive meshing [3] (Fig. 2), Monte Carlo Stochastic Optimization [4], Second Order Cone Optimization [5], Different Evolution Optimization [6], Multi-objective optimization NSGA with single or nested loops [7], etc. There were some procedures of choosing the best solution for factors (for instance, mixing proportion of ingredients and admixtures for obtaining the highest strength of products), interaction parameters of engineering problems.

However, reliability for those models is a compulsory requirement in which statistics plays a very important role relating to variables and parameters in models. For quantifying uncertainties and randomness, distribution laws between them, standard deviation (variance), mean and coefficient of variation are essential. With reliability analysis, a robust design is postulated in which every change in input data would not affect much to output [8].

2.1.2. Physical modeling

For validating the mathematical models to some extends, physical models were established with scaled models, with/without scale factors. Information among dimensionless ratios of variables and parameters involved in the model was found by using laws of similarities. Similarities between real models (prototype) and scaled models are always to be suggested first, then small-scaled models were made to measure some response; results are analyzed to make use of them for inferring the response of real structures before implementing them in real industry. With some conditions expressing correlation between variables and parameters (i.e. ratios of quantities in scale models to those of prototype models) but not guarantee similarities, models are often called technical models that were used to clarify qualitatively some quantities of response.

2.2. Experimental models

2.2.1. Why design of experiments?

The question “Which model is the most predictable than others?” should be answered by comparison between the reliability of statistical correlation between predictors and outcome in reality. The unique way to clarify this correlation is experiment. How to overcome errors and difficulties in those methods of modeling? Experimentally based validation is a viable approach.
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for them. There were a lot of approaches for carrying out those needed validations. Besides, back analysis in which mathematical model was calibrated by adjusting sensitive properties of materials using real data of response, it should firstly conduct some design of experiments, both in numerical models and laboratory models. Statistical data hence are very important to use, with support of softwares to treat a giant amount of input data. Among them, there is a very specific method of design of experiments, Taguchi method in design of experiment.

This step is very important to avoid taking unimportant variables and parameters into account, to reduce errors in modeling and measurements. Every model always includes geometrical data, materials, restraints and loadings. On the other hand, design of experiments is helpful in finding out what variables and parameters contributed most and significantly to reflect model.

By observing the relation between output and input data, together with uncontrolled conditions of experiments due to software, repetitive measurements, inherent errors and orthogonality of some specific sets of data, Taguchi suggested a theory for experiment design in which statistical characteristics of annoying variables are concerned. Taguchi method reduces remarkably numbers of experiments when facing independent $n$ variables and parameters with $k$ different levels of state or quantitative values. Steps of the method are presented in literature review [9-12].

2.2.2. Numerical model as the first step of governing the design of experiment

An engineering objective has dimensions of the cross section to be designed, materials including stiffness and allowable stresses to be verified and the state of working subjected to some state of loadings, stress-strain relationship. Hence, variables include geometrical attribution (cross section, volume), mechanical properties (stiffness, strength), compressibility, etc. Schematic diagram with constraints or restraint is to be analyzed with some conditions that we have to measure to assess its compliance to some standards. A specified software may be used to analyze it. After design of cross section and verification for materials, the objective is manufactured as an element or structure or a mechanism with many materials and links between elements.

2.3. Example for illustration

An example based on real data of soil foundation for Taguchi method was considered as follows:

It is necessary to find the best set of control factors that provides the best performance of a group of soil mixed cement columns. Unlike pile that resists 100 % load, soil-cement columns are a semi-rigid solution to support partially load in lieu of perfectly rigid reinforced concrete pile. But when standing together in a group, the frequent question is what factor(s) has the most important influence on the efficiency of the load-bearing capacity of the number of columns. Four factors prescribed in the problem are Diameter, Spacing of columns, Number of columns in a group, and the Length of columns. Noise E, F, G is two-level, relating to condition of Rinter, undrained or drained mode of analysis and updated mesh or not in advanced option (for large deformation only); value that equals to 1 is the case which Rinter =1, undrained and without updating mesh during analysis; value 2 is a case in which Rinter = 0.8, drained and with updating mesh. Orthogonal array (OA) for conducting experiments with finite element model using Plaxis V8.5 package was simplified as in Fig. 3 with a total of 36 runs, that were
conducted and results were tabulated as in Fig. 3 with some conditions that we have to measure to assess its compliance with some standards. A specified software may be used to analyze it. After the design of the cross section and verification for materials, the objective is manufactured as an element or structure or a mechanism with many materials and links between elements.

![Figure 3. Simplified OA of the problem with 4 factors and 2 noises, with data filled.](image)

### 3. RESULTS AND DISCUSSION

3.1. Identifying factors that have the highest and lowest effect on a process of manufacture

Figure 4 shows an example of a specific problem in foundation engineering.

\[
\alpha_s = \frac{\alpha_{\text{column}}}{\alpha_{\text{column}} + \alpha_{\text{soil}}}
\]

![Figure 4. Soil cement columns in groups, \( \alpha_s \) is defined as replacement ratio and Plaxis model.](image)

The assumption is that a group of columns will be less effective in supporting load than that of columns accumulated together, due to interference of soil between them; the bigger diameter, the higher spacing is; the more columns in the group are, the less bearing capacity of columns is. Columns standing together as a group will support the load altogether with soil around them. Soil Cement Columns are constructed at site so they are called Deep Mixed Columns and the method is called Deep Mixing Method [13]. Columns are distinguished specifically with piles by stiffness and stress concentration partly on columns instead of 100% on pile.
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![Graph](image)

**Figure 5.** Results from a numerical model using Plaxis: a) Load-displacement curve of a 6 column group; b) Axial force in columns of group [14].

It is a process of finding out factors which have the highest and the lowest effect on bearing capacity of a group of soil cement mixing columns for soil improvement. Hence, there were 4 factors including: A (diameter of soil-cement column, diameter Φ =400 mm, 600 mm and 800 mm), B (spacing center to center of columns, 3 Φ, 4 Φ and 6 Φ), C (number of columns in a group, equals to 2, 4 and 6 columns respectively) and D (length of columns, equals to 5 Φ, 10 Φ and 15 Φ). The problem under consideration related to material (column inertia, section, and stiffness in the numerical model) and configuration of the group (diameter, spacing, number of columns and length of columns). By using orthogonal array L9(3^4); after carrying out experiments by model Mohr-Coulomb in the Plaxis package, results are filled out in table (yellow cells). S/N ratio stands for the variability of response relative to the outcome value with respect to different conditions of noises. A high value of the S/N ratio of a factor under different conditions means that factor has a high effect on target value. The signal-to-noise ratio measures how the response varies relative to the nominal or target value under different noise conditions. The appropriate formula for S/N of bearing capacity of soil-cement columns is “The bigger is better”, as follows:

$$ S/N = -10 \log \left( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i} \right) $$  \hspace{1cm} (1)

Taguchi’s signal-to-noise of experiment process using Plaxis were tabulated as in Fig. 6.
ANALYSING AND RANKING DEGREE OF IMPORTANCE FOR FACTORS IN BEARING CAPACITY PREDICTION

| Levels of factors | A pile diameter | B spacing | C number in group | D depth of pile | Average | 1/y^2 | S/N |
|-------------------|----------------|-----------|------------------|----------------|---------|-------|-----|
| A= 400, 600, 800 | 374            | 358       | 367              | 366            | 366.25  | 7.149E-06 | 7.408E-06 |
| B=3Φ, 4Φ, 6Φ    | 353            | 579       | 547              | 428            | 476.75  | 8.025E-06 | 2.983E-06 |
| C=numbers of pile | 823           | 747       | 630              | 617            | 704.29  | 1.476E-06 | 1.792E-06 |
| D=5Φ, 10Φ, 15Φ  | 870            | 737       | 679              | 602            | 722.32  | 1.321E-06 | 1.841E-06 |
| A,1              | 620            | 710       | 750              | 870            | 737.75  | 2.601E-06 | 1.984E-06 |
|                  | 265            | 567       | 652              | 796            | 558.5   | 1.428E-06 | 3.111E-06 |
|                  | 800            | 710       | 722              | 667            | 724.75  | 1.560E-06 | 1.984E-06 |
|                  | 613            | 535       | 603              | 402            | 537.75  | 2.661E-06 | 3.494E-06 |
|                  | 850            | 857       | 799              | 705            | 802.75  | 1.384E-06 | 1.562E-06 |
| Level             | Calculation S/N ratios: |
| 1                 | 53.698         | 58.64242803 | 56.2551859 | 59.00439813 |
| 2                 | 59.12368487   | 58.3365286   | 59.522958   | 57.8220976  |
| 3                 | 59.08413775   | 59.3888072    | 60.5496732   | 59.0132132  |
| A=Max.- Min.      | 2.762          | 1.022       | 4.295         | 1.679        |

| Rank= 2 | Rank= 4 | Rank= 1 | Rank= 3 |

Figure 6. Taguchi method indicates the highest and the lowest effect on the target value of experiment.

3.2. Determining optimum values of factors and their values

Based on Taguchi’s orthogonal array (OA) in Fig. 3, total signal $S_{factor X, level m}$ is:

$$S_{A,1}=\eta_1+\eta_2+\eta_3; S_{A,2}=\eta_4+\eta_5+\eta_6; S_{A,3}=\eta_7+\eta_8+\eta_9; S_{B,1}=\eta_{10}+\eta_{11}+\eta_{12}; S_{B,2}=\eta_{13}+\eta_{14}+\eta_{15}; S_{B,3}=\eta_{16}+\eta_{17}+\eta_{18}; S_{C,1}=\eta_{19}+\eta_{20}+\eta_{21}; S_{C,2}=\eta_{22}+\eta_{23}+\eta_{24}; S_{C,3}=\eta_{25}+\eta_{26}+\eta_{27}; S_{D,1}=\eta_{28}+\eta_{29}+\eta_{30}; S_{D,2}=\eta_{31}+\eta_{32}+\eta_{33}; S_{D,3}=\eta_{34}+\eta_{35}+\eta_{36};$$

where $\eta_k$ is the $S/N$ ratio corresponding to trial (run) $k$. For each level $k$ of the target value, the average value of $\eta_k$, that equals to $S_{factor X, level k}$. For example, average S/N corresponding to column diameter _factor A _ at level 1 (i.e $\Phi=400$ mm) is $S_{A,1}/3 = (\eta_1+\eta_2+\eta_3)/3$; average S/N corresponding to column diameter _factor A _ at level 2 is $S_{A,2}/3$, etc.

Table 1. Average S/N ratios for each factor.

| Level | Factor A | Factor B | Factor C | Factor D |
|-------|----------|----------|----------|----------|
|       | Sum (S_{A,level}) | Avg S/N   | Sum (S_{B,level}) | Avg S/N   | Sum (S_{C,level}) | Avg S/N   | Sum (S_{D,level}) | Avg S/N   |
| 1     | 161.0945 | 53.69    | 165.362  | 55.12    | 158.2001  | 52.73    | 166.4477  | 55.48    |
| 2     | 166.8056 | 55.60    | 164.4441 | 54.81    | 168.0034  | 56.00    | 162.9008  | 54.30    |
| 3     | 169.3869 | 56.46    | 167.4809 | 55.82    | 171.0835  | 57.03    | 167.9385  | 55.98    |

The factor levels corresponding to the highest the S/N ratio will govern the optimum value of the target, or give the highest performance. Examining plots in Fig. 7, it is easy to recognize the optimum values of factors are someway like this:

- The more number of columns in the group is, the more efficient is (chart of factor C, ranked 1)
- The bigger diameter is, the more efficient is (chart of factor A, ranked 2);
- The length of columns and the spacing are both auxiliarily important.
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So if a cost-effective design is demanded, the optimum set of factors will be C3A3D1B3. It means that unless other specified on the settlement, a group of 6 columns of 800 mm diameter, with the length of only 4 meters (L=5Φ), and spacing 3Φ will provide both the highest bearing capacity and need not a longer and/or wider group of columns. This optimum values of factors will be used to create soil cement samples and test in small-scale model with many trials before validating by constructing in real condition.

3.3. Analysis of variance (ANOVA) to determine percentage of contribution

For analyzing which variable or factor is sensitive most, which is not influencing the performance of the process, an analysis of variance (ANOVA) is investigated. Besides, based upon experiment analysis (run on numerical model or trials in laboratory), ANOVA can point out the percentage of contribution of each factor/variable on performance value. The concept for the analysis is the higher value of the sum of square of an independent variable (factor), the more it has the influence of outcome performance; and the percentage of contribution is defined as the ratio of individual sum of square of a particular factor to the total sum of square of all factors. The formula for percentage of contribution is as follows:

\[
p\% = \frac{SS_j}{SS_T} = \frac{(y_i - \bar{y}_j)^2}{\sum_{i=1}^{N^V} (y_i - \bar{y}_j)^2}
\]  

A special note here is that p % indicates the response magnitude in (%) of each factor in the orthogonal experiment and that total sums of squares (total variation) SS_T is equal to the sum of squares of all the control factors and error components involved the variations of each factor (SS_k). SS_k is the sum of squares of the deviations over the factor k at level j (k is among A, B, C, D for this example)

\[
SS_k = \sum_{j=1}^{t} \frac{S^2_j}{t} - \frac{G^2}{N}
\]  

\[
SS_E = \sum_{i=1}^{N} (y_i - \bar{y})^2 - \frac{G^2}{N}
\]  

Figure 7. Charts of S/N ratios v/s Level for each factor, result from Minitab [15].
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G is the sum of the resulting data of all trial runs and n is total number of trial run. Because all the experiments in the problem are carried out in a computer, so the second term in expression (3b) is disregarded. Degree of freedom for the total model as per Taguchi method is a combination of degree of freedom for factors and that of interaction treatment. F ratio was defined as in (5):

\[(df)_{\text{model}} = \sum (df)_{\text{factor}} + \sum (df)_{\text{interaction}}\]  

\[F = \frac{\text{Variance between Treatments}}{\text{Variance Within Treatments}} = \frac{MS_{\text{Treatment}}}{MS_{\text{Error}}}\]  

High F value indicates that prediction of outcome response is acceptable. For the problem under studying, four factors will lead to the Taguchi’s degree of freedom is \((df)_{\text{Taguchi}} = \text{No. Factor} \times (\text{No. Level} - 1) = 4(3 - 1) = 8;\) if factors had two-way interaction AB, AC, AD, these three interactions have 3 x 4 degree of freedom each, equal to 12. So the degree of freedom for whole system is 4(3 - 1) + 12 = 20.

![Figure 8. ANOVA output by Minitab [15].](image)

Percentage of contribution for each factor was computed and shown in Table 2.

Table 2. Results on percentage of contribution of factors.

| Factor            | Level | S/N  | Mean S/N of factors | \(\frac{SS_y}{SS_{\text{Total}}} = \frac{1}{N-1}(y - \bar{y})^2\) | \(p \% = \frac{SS_y}{SS_{\text{Total}}} \times 100\) | Importance |
|-------------------|-------|------|--------------------|-------------------------------------------------|-------------------------------------------------|------------|
| A (Diameter)      | 1     | 53.71| 17.90              | 55.25                                           | 0.495                                            | 24.0 %     | 2          |
|                   | 2     | 55.6 | 18.5               |                                                 |                                                 |            |            |
|                   | 3     | 56.46| 18.82              |                                                 |                                                 |            |            |
| B (Spacing)       | 1     | 55.12| 18.37              | 55.25                                           | 0.108                                            | 5.3 %      | 4          |
|                   | 2     | 54.81| 18.27              |                                                 |                                                 |            |            |
|                   | 3     | 55.84| 18.61              |                                                 |                                                 |            |            |
| C (number of columns in group) | 1     | 52.73| 17.57              | 55.25                                           | 1.264                                            | 61.5 %     | 1          |
|                   | 2     | 58.00| 18.67              |                                                 |                                                 |            |            |
|                   | 3     | 60.70| 19.01              |                                                 |                                                 |            |            |
| D (Length)        | 1     | 55.48| 18.49              | 55.25                                           | 0.187                                            | 9.1 %      | 3          |
|                   | 2     | 54.3 | 18.10              |                                                 |                                                 |            |            |
|                   | 3     | 55.99| 18.66              |                                                 |                                                 |            |            |

\(\Sigma SS = SS_y = 2.054\) 100 %
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The percentage of contribution can be computed by manual calculation using worksheet with Excel or some built-in tool in Minitab package. Some output obtained by Minitab R17, were described in Fig. 8.

Result of importance in Table 2 confirms the consistency with that in section 3.2 above. Regression equation was obtained and R-squared (adjusted) indicated that the model is rather statistically reliable. ANOVA also provides results about interaction treatment (not described in details in this paper).

4. CONCLUSION

This study deals with a specific statistic technique in engineering and scientific research. Statistical method always is a companion of Experimental and Mathematical model. As scientific research works always required a model in which a) some assumptions to be validated; b) data of material, stress-strain relationship to be chosen; c) process of analysis to be controlled in different conditions of the environment, intervening factors, etc., statistical methods and techniques can be uniquely used to analyze quantitatively the consistent consideration between input dataset, treatments of process with/without interactions between factors, and output/outcome that relates to uncertainties and randomly varied input parameters in models. It is difficult to harness such the variability of variables and parameters involved in mathematical and/or experimental model for problem. Besides, statistical methods can be helpful in summarization quantitative results and provide more convincing conclusions.

Taguchi method is an efficient tool of statistical technique where experiment is to be designed smartly, results obtained can be analyzed to point out effective variables and parameters subjected to different conditions. Thanks to Taguchi Method will be applied for finding solution in which much less effort of computation is paid and other different conditions of noise could be taken into account; uncertainties and randomness in engineering problems are quantified, considering errors in measurement and defects in knowledge of researchers. Besides, Taguchi method can provide an approach to build a statistically based optimization for engineering mechanical model.

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