Gravitational Waves from Sub-lunar Mass Primordial Black Hole Binaries
- A New Probe of Extradimensions -

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In many braneworld models, gravity is largely modified at the electro-weak scale $\sim 1$TeV. In such models, primordial black holes (PBHs) with a lunar mass $M \sim 10^{-7}M_\odot$ might have been produced when the temperature of the universe was at $\sim 1$TeV. If a significant fraction of the dark halo of our galaxy consists of these lunar mass PBHs, a huge number of BH binaries will exist in our neighborhood. Third generation detectors such as EURO can detect gravitational waves from these binaries, and can also determine their chirp mass. With a new detector designed to be sensitive at high frequency bands $\gtrsim 1$kHz, the existence of extradimensions could be confirmed.

Introduction In recent years, there has been a great interest in braneworld scenarios in which the ordinary matter fields are confined in a four dimensional object (“brane”), while the gravitational field propagates in the higher dimensional spacetime (“bulk”). One motivation for the brane world scenario is solving the hierarchy problem between the Planck scale and the electro-weak scale ($\sim 1$TeV). The possibility of relating the Planck scale to the TeV scale by considering large sized extradimensions was pointed out [1]. In the model proposed by Randall and Sundrum (RSII) [2] one of the extradimensions extends infinitely, but compactification of the extradimension is effectively achieved by the warped geometry. In this model the four dimensional Planck mass $M_p$ is related to the brane tension $\sigma$ and the bulk curvature length $l$ as $M_p^2 = 4\pi\sigma l^2/3$. Hence, keeping the scale of the brane tension at the TeV scale, the Planck scale can be derived by setting $l \approx 0.1$mm, although this fact does not mean that the RSII model solves the hierarchy problem.

In such braneworld scenarios, the evolution of the universe at a temperature above $\sim 1$TeV can be dramatically altered. For instance, the brane universe may have had an era of violent “mesoscopic” activity at scale as large as $l \approx 1$mm at a temperature $T \gtrsim 1$TeV [3]. Such a violent activity may generate large-amplitude fluctuations at the horizon scale $H^{-1}$. Then a large number of primordial black holes (PBHs) with lunar masses $M_{BH} \sim H^{-1}M_p^2 \sim 10^{-7}M_\odot$ could be produced. To date, PBH physics in the context of braneworld scenarios has been studied [4,5], but none of the literature has shed any light on the astrophysical implication of considering PBHs as cold dark matter.

Currently, there is no stringent observational constraint on Sub-Lunar mass Compact Objects (SULCOs) with mass $10^{-12}M_\odot \lesssim M \lesssim 10^{-7}M_\odot$ as the dark matter candidates, which can be either molecular clouds, small planets, or PBHs. Microlensing tests such as the EROS and MACHO collaborations ruled out the possibility that compact objects with mass $M \sim 10^{-7}M_\odot$ make up the halo dark matter in the Milky Way [6].

On the other hand, femtosecond tests of Gamma-ray bursts (GRBs) ruled out the mass range $10^{-16}M_\odot \lesssim M \lesssim 10^{-13}M_\odot$ in the universe assuming that GRBs are at cosmological distance so that the angular size of the GRB source is sufficiently small [7]. SULCOs also induce picolensing of GRBs, but the observational limit is very weak [7]. Dynamical constraints on the number of SULCOs in the dark halo are also less stringent [8].

In this letter, we consider the PBH SULCOs that have been produced by violent activity at temperatures $\sim$TeV as the dominant constituent of the cold dark matter. Then there must be a huge number of PBHs ($> 10^{18}$) in the dark halo of the Milky Way. As is discussed in the PBH MACHO scenario [9], it is natural to expect that most of the PBHs form binaries and some of them coalesce emitting gravitational waves within the present age of the universe. From the frequency evolution in such gravitational waves, we may be able to probe the deviation from Newtonian gravity typical of scenarios with infinite extradimensions.

Formation and evolution of PBH SULCO binaries) We consider the formation of PBHs in the context of gravity theories which have a critical energy scale at the TeV scale. If PBHs are formed at a temperature $T_c$ $\sim$ TeV, they are of the lunar mass, $M_{BH} \sim H^{-1}M_p^2 \sim 0.1$mm $M_\odot^2 (T_c/\text{TeV})^{-2} \sim 10^{-7}M_\odot (T_c/\text{TeV})^{-2}$. Here we have assumed that the number of effective degrees of freedom at the PBH formation epoch is about 100. Although it has been argued that PBHs could undergo substantial growth by accreting material from the cosmological background in the high-energy regime ($\rho \gtrsim \sigma$) [4,5], such an effect is not relevant in our discussion because here PBHs are formed right after the end of the high-energy regime.

In order to estimate the event rate of coalescence of PBH SULCO binaries, we follow the arguments in [9]. For simplicity, we assume that the PBH SULCOs dominate the dark matter, i.e., $\Omega_m = \Omega_{PBH}$, and the PBH mass spectrum is monochromatic.

At matter-radiation equality, the comoving mean separation is $\bar{x} = 2 \times 10^{10} (M_{BH}/M_\odot)^{1/3} (\Omega_m h^2)^{-4/3}\text{cm}$, where $h$ is the Hubble parameter in units of 100km.
s^{-1}Mpc^{-1}. Henceforth, we adopt $\Omega_m h^2 = 0.15$ as suggested by recent observations. A binary with its separation $x$ decouples from the cosmic expansion when the local energy density becomes a few times larger than the energy density of the background radiation. The tidal force from the nearest PBH at a comoving distance $y$ to the center of mass of the binary adds angular momentum to keep the holes from a head-on collision. For simplicity, we assume that the distribution of PBHs are spatially homogeneous and isotropic. Then the distribution of $x$ and $y$ takes the form $p(x, y) \propto x^2 y^2 \exp(-y^3/x^3)$, which is further approximated by a uniform distribution $\propto x^2 y^2$ in the range $x < y < \bar{x}$.

The coalescence time $t$ due to emission of gravitational waves can be written in terms of the semi-major axis $a$ and the eccentricity $e$ as $t = t_0(a_0/a_0)^4(1 - e^2)^{7/2}$, where $t_0 = 10^{10}\text{yr}$ and $a_0 = 2.4 \times 10^{11}(M_{BH}/M_\odot)^{3/4}\text{cm}$ is the initial separation of a circular binary which coalesce in $t_0$. Thus, writing $x$ and $y$ in terms of $t$, $e$, and $\bar{x}$, one obtains the distribution of the coalescing time:

$$p(t)dt = \frac{3}{29} \left[ \left( \frac{t}{t_{\text{max}}} \right)^{3/37} - \left( \frac{t}{t_{\text{max}}} \right)^{3/8} \right] dt,$$

where $t_{\text{max}} = t_0(\bar{x}/a_0)^4$. Let us denote the remaining time before coalescence by $T$. For small $T$, we can neglect the eccentricity $e$ because the radiation reaction acts to reduce it. Then, $T$ has a one-to-one correspondence with the semimajor axis, and hence with the frequency of the emitted gravitational wave $f_g$. The explicit relation becomes

$$f_g \sim 3 \times 10^3\text{Hz} \left( \frac{M_{BH}}{10^{-7}M_\odot} \right)^{-5/8} \left( \frac{T}{5\text{ yr}} \right)^{-3/8}.$$

Hence, provided that $T \ll t_0$, the probability distribution for $f_g$ is obtained as

$$f_g P(f_g) = p(t_0 + T) \frac{dT}{d\log f_g}$$

$$\sim 2 \times 10^{-12} \left( \frac{T(f_g)}{5\text{yr}} \right)^{5/37} \left( \frac{M_{BH}}{M_\odot} \right)^{5/37}.$$

**Gravitational waves from PBH SULCO binaries** Now we discuss observability of gravitational waves from PBH SULCO binaries. We assume that the local dark halo density in the solar neighborhood is $0.0079 M_\odot \text{pc}^{-3}$ [10]. Let the total number of PBHs within the distance $D_\text{SULCO}$ be $N(D_\text{SULCO})$. Then the distance $D_\text{min}$ such that we can expect at least one coalescence event within $T$ at $D \leq D_\text{min}$ is obtained by solving $N(D_\text{min}) f_g P(f_g) = 1$. Thus we have

$$D_\text{min} \sim 11 \text{pc} \left( \frac{f_g}{100\text{Hz}} \right)^{8/9} \left( \frac{M_{BH}}{10^{-7}M_\odot} \right)^{281/333}.$$

After averaging over the orbital period and the orientations of the binary orbital plane, the amplitude of gravitational waves from a coalescing binary in a circular orbit with distance $D$ at frequency $f_g$ is $h \sim 6 \times 10^{-23} (D/20\text{Mpc})^{-1}(M_{BH}/M_\odot)^{5/3}(f_g/100\text{Hz})^{2/3}$. Substituting $D$ here with $D_\text{min}$, we obtain the characteristic amplitude $h_c = h\sqrt{n}$, where $n = f_g \Delta T$ is the number of cycles during the observation time $\Delta T$. Then the wave strength of gravitational waves from a monochromatic source observed by using an interferometer $\hat{h}_s = h_c/(5f_g)^{1/2}$ is evaluated as

$$\hat{h}_s \sim 1 \times 10^{-24} \left( \frac{M_{BH}}{10^{-7}M_\odot} \right)^{281/333} \left( \frac{f_g}{100\text{Hz}} \right)^{-7/2} \left( \frac{\Delta T}{5\text{yr}} \right)^{1/2},$$

where we have taken into account the antenna pattern of the detector sensitivity averaged over the direction to the binary by multiplying the factor $1/\sqrt{5}$. The rms of the signal to noise ratio averaged over the source directions and orientations is given by $\hat{h}_s \Sigma_s^{1/2}(f_g)$, where $\Sigma_s^{1/2}(f)$ is the strain sensitivity in unit of Hz$^{-1/2}$. Note that the above estimate for $\hat{h}_s$ is valid only for frequencies below $f_s = 3 \times 10^3(M_{BH}/M_\odot)^{-5/8}(T/5\text{yr})^{-3/8}$ for which the source frequency does not change rapidly during the observation time $\Delta T \ll T$.

The planned spectral noise density of the European Gravitational Wave Observatory (EURO) is [11],

$$S_s(f) = 10^{-50} \times \left[ (f/245\text{Hz})^{-4} + (f/360\text{Hz})^{-2} \right.$$

$$\left. + (f_k/770\text{Hz})(1 + f^2/f_k^2) \right] \text{Hz}^{-1},$$

where $f_k$ is the knee-frequency. As seen from Fig.1, gravitational waves from PBH SULCO binaries are detectable by such an interferometer for integration time $\Delta T > 5\text{yr}$. Furthermore, we can measure the chirp mass $M_c = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}$ of each binary, where $M_1$ and $M_2$ denote the masses of respective PBHs. This is because the change in frequency during the observation time $\Delta T$, $\Delta \nu = (df_g/dt)\Delta T$, becomes sufficiently large compared with the frequency resolution $\sim 1/\Delta T$ at $f_0 \gg 2\text{Hz}$ for $\Delta T = 5\text{yr}$. Thus, the existence of SULCO binaries can be confirmed by observing gravitational waves.

**Probing extradimensions from gravitational waves** If PBH SULCO binaries are detected, we will have a chance to probe the existence of extradimensions. Here we focus on the RSII model as a typical example in which the Kaluza-Klein (KK) spectrum is continuous with no mass gap. In our present context, the brane tension is to be set to $\sim (\text{TeV})^4$ since it is expected to be related to the energy scale of the Standard Model fields localized on the brane. Then, the observed four dimensional Planck mass is reproduced by setting the bulk curvature to the observationally allowed maximum value $\sim 0.1\text{mm}$. The cosmic expansion law in this model is given by $H^2 = 8\pi (\rho + 2\rho^2/\sigma)/3M^2_\nu$, where $\rho$ is the energy density of the universe. When $\rho \gtrsim \sigma$, the cosmic expansion law dramatically changes.

Let us consider the non-relativistic gravitational potential $V$ for a binary that consists of two point particles
with mass $M_1$ and $M_2$ on the brane. Let $M = M_1 + M_2$ be the total mass and $\mu = M_1M_2/M$ be the reduced mass. The separation between the two particles is denoted by $r$. Continuous KK modes add a correction to the standard Newtonian term as [12],

$$V(r) = \frac{G\mu M}{r} \left[ 1 + \frac{\alpha}{(GM)^2} \left( \frac{GM}{r} \right)^2 \right], \quad \alpha = \frac{2^2}{3},$$

provided that the separation $r$ is much larger than the AdS radius $l$. Therefore, the correction owing to the continuous KK modes is second order in $\epsilon = GM/r$, i.e., the second post-Newtonian (2PN) order. In our scenario, the coefficient of this 2PN correction is $O(1)$ because $l$ is related to the total mass $M$ as $l/\sqrt{GM} \sim 1$. This correction affects the relation between the energy and the orbital angular velocity of the binary, and hence the frequency evolution of the binary is modified. As for the energy loss rate from the binary, one can show that the correction is of higher order.\(^1\)

Fourier components of gravitational waves from an inspiralling binary in a quasi-circular orbit are given by

$$\tilde{h}(f) = \frac{Q}{D(GM_c)^{5/6}f^{-7/6}} \exp(i\Psi(f)),$$

where $Q$ is the factor depending on the direction and the orientation of the binary and the detector antenna pattern. Keeping the second order term in $\epsilon$, the phase $\Psi(f)$ can be written as

$$\Psi(f) = 2\pi ft_c - \phi_c - \pi/4 + \gamma(f)(\pi GM_c f)^{-5/3},$$

$$\gamma(f) = \frac{3}{128} \left( 1 + \left( \frac{3715}{756} + \frac{55}{9} \eta \right) (\pi GM f)^{2/3} + (4\beta - 16\pi)(\pi GM f) + \left( \frac{1529365}{508032} + \frac{27145}{504} \eta \right) + \frac{3085}{72} \eta^2 - \frac{100\alpha}{(GM)^2} (\pi GM f)^{4/3} \right),$$

where $\eta = \mu/M$, $t_c$ is the coalescence time, $\beta$ denotes the spin parameter, and $\phi_c$ determines the phase. The KK mode correction appears in the last term in $\gamma(f)$. Here we have neglected the spin dependent term at the 2PN order. One may think that the KK modes contribution is indistinguishable from this effect. However, for binaries whose spin parameter at the 1.5PN order $\beta$ is measured to be small, one can neglect the spin effect at the 2PN order. Hence, the effect of extradimensions can be observed as an anomalous “2PN” correction.

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\(^1\)According to the AdS/CFT correspondence, the energy loss owing to geometrical particle creation in the RSII model is $30\pi^2/G$ times as large as that for a single conformally coupled scalar field [13]. Using the result for a scalar field [14], one can easily find that the correction is suppressed by a factor $f^2 \ell^2$, and hence is of 3PN order.

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**FIG. 1.** Sensitivity curves $\tilde{h}_n(f) = S_{n/2}(f)$ for LIGO-II and EURO with different specifications, (A,B,C) $f_k = 10^9$Hz, $f_k = 6.5 \times 10^9$Hz, and the xylophone type, respectively. Disks and circles denote the wave strength $\tilde{h}_n$ for the nearest PBH SULCO binary with mass $M_{BH} = 10^{-7}M_\odot$ for the observational time $\Delta T = 10, 1$yr, respectively. Dotted lines and dashed lines represent the evolution of the amplitude $Q(GM_c)^{5/6}f^{-7/6}/D_{\text{min}}$ for the nearest PBH SULCO binary with $M_{BH} = 10^{-7}M_\odot$, $M_{BH} = 10^{-8}M_\odot$, respectively. The end of each line in the left-upper side corresponds to the remaining time $T = 10$yr before coalescence.

Let us evaluate how accurately we can determine the AdS radius $l$, using third generation detectors such as EURO. We set $f_k = 6.5 \times 10^9$Hz in the following analysis. We assume that a PBH SULCO binary with lunar mass $M_1 = M_2 = 10^{-7}M_\odot$ is in a quasi-circular orbit. With AdS radius $l = M_p^2M = 0.3$mm, a binary that emits gravitational waves at a frequency $f = 2.5 \times 10^9$Hz coalesces in 10 yr. The distance to the nearest binary is estimated as $D_{\text{min}} \sim 3 \times 10^2$pc, which yields the amplitude $\tilde{h} = Q(GM_c)^{5/6}D^{-1}f^{-2/3} \sim 1 \times 10^{-24}$.

In the matched filter analysis up to 2PN order for the phase $\Psi(f)$, we marginalize seven parameters $\{\lambda_i\} = \{\ln A, t_c, \phi_c, \ln M_c, \ln \mu, \beta, \ln f\}$. The error of each parameter can be estimated by a quadratic fitting of the likelihood function assuming uncorrelated Gaussian noise [15]. Using the EURO detector, we find that the nearest PBH SULCO binary is expected to be detected with a signal-to-noise ratio $S/N = 3$. The relative error in $l$ is $\Delta l/l = 3 \times 10^{-1}$ whereas $\Delta t_c = 2 \times 10^{-4}$sec, $\Delta \phi_c = 5 \times 10^2$rad, $\Delta M_c/M_c = 7 \times 10^{-11}$, $\Delta \mu/\mu = 7 \times 10^{-5}$ and $\Delta \beta = 4 \times 10^{-2}$. Although the $S/N$ is not sufficiently large, the AdS radius $l$ can be determined within $\sim 30$
percent error. To gain a large S/N value, we need a detector that is highly sensitive at high frequency bands (>1kHz), say, a “xylophone” type detector [11] that consists of several narrow-band interferometers.

Summary and Discussions) In this letter, we discussed observability of gravitational waves from the nearest coalescing PBH SULCO binary. If PBH SULCOs with mass \( M_{BH} \sim 10^{-7}M_{\odot} \) constitute the cold dark matter, the third generation interferometer, EURO, has sufficient sensitivity to confirm their existence. For the case that the PBH formation is related to the braneworld scenario, we further discussed observability of the imprint of extradimensions. We found that the sensitivity of EURO is marginal to discriminate the RSII model. However, once we discover the PBH SULCO binaries, we would be able to construct a new detector that is suitable for probing the extradimensions.

Although we have investigated the RSII model as an example, our estimate of observability is not very restrictive to this model. In the RSII model, the deviation from general relativity starts at the 2PN order. In other models the deviation may appear at lower PN order. Even in that case, the correlation with other parameters such as \( M_c, \mu \) and \( \beta \) renders new physics unveiled up to the 2PN order. Therefore, it is plausible to conclude that our result is generic to a rather wide class of models.

Because the number of PBH SULCOs in the dark halo is huge, one might expect that the radiation from gas accretion to the PBHs is detectable [16]. However, as we shall see, the chance of detection is hopelessly small. Let us consider that a lunar mass \( \dot{m} \equiv \dot{M}_{BH}/M_{Edd} \sim 5 \times 10^{-4}(M_{BH}/M_{\odot})(n_g/10^2 cm^{-3})(10 km/s/V)^3 \) where \( n_g \) is the number density of the surrounding gas, and the \( M_{Edd} \) is the Eddington accretion rate with unit efficiency. Adopting the thin-disk model [17], the total luminosity from the disk is estimated, at most as \( L \sim GM_{BH}M_{BH}/(2R_{min}) \) where \( R_{min} \) is the innermost accretion radius. Assuming black-body radiation from the disk, \( \dot{m} = 1 \) and \( R_{min} = 3R_{Schw} \) where \( R_{Schw} \) is the Schwarzschild radius, the temperature becomes \( T \sim 1 \times 10^7 K(M_{BH}/M_{\odot})^{-1/4} \). Thus smaller mass BHs have a higher temperature. Now consider lunar mass PBHs moving in the Orion Nebulae at 400 pc away. Assuming that the dark matter density in the solar neighborhood is \( 0.0079 M_{\odot}pc^{-3} \) [10], the total number of lunar mass BHs is just \( \sim 8 \times 10^7 \) in the molecular cloud if its extension is \( 10^3 pc^3 \). Provided that the velocity distribution of the PBHs is Maxwellian, and the number density of the molecular gas is \( n_g = 10^2 cm^{-3} \), we find that the expected total luminosity is \( L \sim 2 \times 10^{24} ergs/s \) and the peak frequency is \( f \sim 10^{18} \text{Hz} \). On the other hand, the sensitivity of the X-ray telescopes such as the Advanced X-Ray Astrophysics Facility (AXAF) is about \( \sim 10^{29} \text{ergs/s} \) for a source at 400pc at \( f \sim 10^{18} \text{Hz} \). Thus we cannot expect any X-ray visible BHs in the Orion molecular cloud.

Based on the AdS/CFT correspondence, some authors have conjectured that the evaporation rate of BHs in the RSII model is greatly increased [18,19]. The lifetime of massive BHs in the RSII model is estimated as \( \tau \sim 10^5(M_{BH}/10M_{\odot})^3(0.1 mm/l)^2 \text{yr} \) for a BH with a mass \( M_{BH} \gg \Lambda M_p^2 \) [18,19]. Thus, most astrophysical BHs with moderate mass \( 10M_{\odot} \lesssim M_{BH} \lesssim 10^3 M_{\odot} \) would have evaporated by now, leaving remnants with a small mass \( \sim \Lambda M_p^2 \). If this should be the case, we would be able to observe gravitational waves from BH SULCO binaries that are not primordial ones, though the observational evidence of stability of stellar mass BHs would give a stringent constraint on the size of the AdS radius \( \Lambda \).

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[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B 429, 263 (1998).
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 85, 4690 (1999).
[3] C. Hogan, Phys. Rev. Lett. 85, 2044 (2000).
[4] R. Guedens, D. Clancy, and A.R. Liddle, Phys. Rev. D 66, 043513 (2002); 083509 (2002).
[5] A.S. Majumdar, Phys. Rev. Lett. 90, 031303 (2003).
[6] C. Alcock, et al., Astrophys. J. 499, L9 (1998).
[7] G.F. Marani, et al., Astrophys. J. 512, L13 (1999).
[8] B.J. Carr and M. Sakellariadou, Astrophys. J. 516, 195 (1999).
[9] T. Nakamura, M. Sasaki, T. Tanaka and K. Thorne, Astrophys.J. 487, L139 (1997).
[10] C. Alcock, et al., Astrophys.J. 542, 281 (2000).
[11] EURO homepage: http://www.eso.org./geo/euro
[12] J.Garriga and T.Tanaka, Phys. Rev. Lett.94, 2778 (2000).
[13] M.J. Duff and J.T. Liu, Phys. Rev. Lett. 85, 2052 (2000).
[14] J. Garriga, D. Harari and E. Verdaguer, Nucl. Phys. B 339, 560 (1990).
[15] C. Cutler, E. E., Flanagan, Phys. Rev. D 49, 2658 (1994).
[16] Y. Fujita et al., Astrophys.J. 495, L85 (1998).
[17] J.R. Ipser and R.H. Price, Astrophys.J. 216, 578 (1977).
[18] T. Tanaka, astro-ph/0203082.
[19] R. Emparan, J. García-Bellido, and N. Kaloper, hep-th/0212132.