Analysis of peristaltic pumping patterns in closed elastic tubes

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Abstract

Balloon dilation catheters are often used to quantify the physiological state of peristaltic activity in tubular organs and comment on its ability to propel fluid which is important for healthy human function. To fully understand this system’s behavior, we analyze the effect of a solitary peristaltic wave on a fluid-filled elastic tube with closed ends. A reduced order model that predicts the resulting tube wall deformations and internal flow, pressure variations is presented. This simplified model is compared with detailed fluid-structure 3D immersed boundary simulations of peristaltic pumping in tube walls made of hyperelastic material. The major dynamics observed in the 3D simulations are also displayed by our 1D model under laminar flow conditions. Using the 1D model, several pumping regimes have been investigated and presented in the form of a regime map. Following that, we present a simple mechanistic explanation on the existence of these regimes and use it to predict the boundaries between the various pumping regimes. Finally, the amount of work done during a peristaltic event in this configuration is defined and quantified. An extension of the 1D model is applied to enhance patient data and find the work done for a typical esophageal swallow. This detailed characterization of the system’s behavior aids in better interpreting the data obtained from dilation catheters used in humans.

Keywords: elastic pumping, elastic tube flow, distensible tubes, one-dimensional reduced-order model, fluid-structure interaction, EndoFLIP, regime (or) performance map, immersed boundary method.

1 Introduction

Peristaltic flow in cylindrical and plane channel geometry has been studied extensively since the mid 1960s [1–4]. The investigations have ranged from analysis of prescribed wall motion on Newtonian [1,5], non-Newtonian [6,7] and particulate fluids [8–10] to the effects of a prescribed forcing on the coupled fluid-structure system [11,13]. Both infinite and finite geometries [13,14] have been considered and the quantitative effects of the
channel geometry (cylindrical ducts vs. rectangular channels) on pumping characteristics have been established [3,15]. But one configuration that has received little attention is the effect of peristalsis in fluid-filled elastic tubes that are closed at both ends. In such a setting, fluid inside the tube can neither enter nor leave the system for the entire duration of peristalsis. This configuration is commonly found in long, slender balloon-type catheters that are used for angioplasty (Gruentzig catheter), characterizing anatomical sphincters and studying motility in tubular organs like the esophagus.

A major assumption utilized to facilitate the mathematical analysis of peristaltic tube flow is that the problem is identical in the fixed (lab) frame of reference and in the reference frame attached to the peristaltic wave. In the latter, the wave is stationary and the shape of the tube walls does not change with time. When the tube length is finite, this assumption is not valid [14]. Thus, in order to build a simplified model for our problem, we turn to the approach taken to analyze flow in collapsible tubes and valveless pumping, both of which involve finite domains, and modify the forcing method and boundary conditions to reflect the operating conditions for our problem. Interestingly, the configuration we aim to study first appears in [1] but is consequently ignored for the rest of the analysis. To the best of our ability, we have not found this configuration appear again in any other studies involving flow in deformable tubes.

The device we intend to focus our analysis on is the EndoFLIP; short for Endolumenal Functional Lumen Imaging Probe [16]. It consists of a long flexible, hollow wire (commonly called a catheter), at the end of which is mounted a polyurethane bag [17]. When deployed during clinical procedures, the device is positioned in such a way that the bag rests within the esophageal lumen. The bag is inflated using saline and the walls’ response is evaluated by monitoring the internal fluid pressure. Various versions of the device exist where the bag length is either 8 or 16 cm. Our focus is on the latter version. When fully inflated with saline, the diameter of the bag is 22 mm. The section of the catheter enclosed by the bag consists of several planimetry sensors that can measure the area along the entire bag length and the bottom tip of the catheter houses a pressure sensor that captures the fluid pressure within it at any given time. The area measured by the sensors is based on the assumption that the cross section of the tube at every point along the axis is circular. The other end of the catheter is connected to a computer that stores the data collected by these sensors. The bag can be filled or drained by pumping fluid through the hollow channel within the catheter and fluid enters or leaves the bag through a small hole on the catheter wall. Fig. 1 is a simplified representation of the bag-catheter assembly and shows the locations of the various sensors on the catheter. For additional details on the device’s construction, data resolution, accuracy and frequency of data collection, see [16,18].

Operating details and simplifying assumptions

The data collected during the procedure can be visualized as shown in Fig. 2. The graphs on the left show the pressure and volume inside the tube as a function of time. It should be noted that the volume change is controlled by the physician conducting the procedure and that the pressure change is a consequence of peristaltic activity and the wall’s response to distension. The right panel of the figure shows the axisymmetric profile of the tube at a certain instant of time; represented by the black vertical line on
the left panel. The dotted lines superimposed on the tube profile show the maximum and minimum area that can be accurately measured by the planimetry sensors. The contraction (highlighted with the three black bands) is detected by finding the location along the tube length where the area is the smallest. From the pressure curve, we see four peaks which indicates that four peristaltic waves have passed over the tube length, one after the other, for the duration of this plot. The plot of the tube profile is oriented in such a way that peristalsis begins at the top and travels downwards. The measurement of area is not continuous along the tube length. Seventeen sensors span the tube length and the area is polled at 1 cm intervals along the tube length in between each pair of sensors. The dots along the tube profile curves represent the location where the area was captured and interpolation between these locations completes the construction of the tube profile for visualization.

As explained earlier, the device is primarily located in the esophageal lumen during operation. The passage of a peristaltic wave causes the (axisymmetric) profile of the bag to change and is accompanied by a change in fluid pressure within. The polyurethane
bag walls have a fixed perimeter at each point along the tube length. As such, when partially filled, the walls remain unstretched, folded and are said to be within the ‘infinite compliance limit’ [19]. In this state, the bag walls do not resist the introduction of additional fluid and any resultant pressure rise within the bag is solely due to stretching of the esophageal walls. After a sufficient amount of fluid is introduced into the bag, the walls finish unfolding and begin to get taut. At this stage, the pressure inside begins to sharply rise. The bag is not in the infinite compliance limit anymore and strongly resists further introduction of fluid. During clinical practice, crossing the infinite compliance limit is avoided and so we will ignore this region of operation as well. Within the infinite compliance limit, the pressure in the bag is a function of the esophageal wall’s mechanical properties but outside of it, the pressure also depends on the stiffness of the polyurethane bag wall material. Fig. 3 further illustrates the shape of the bag wall within and outside the infinite compliance limit.

Figure 3: Cross-sectional view of the device inside the esophagus. (Left) Device within the infinite compliance limit. (Right) Bag walls fully taut. The perimeter of the blue curve is equal in both figures.

Another detail that is important to consider is the presence of the catheter within the device. Its diameter is around 4 mm. During peristalsis, the walls approach the surface of the catheter and occasional contact is possible. Fig. 4 shows the the geometry and possible flow directions within the contraction zone. The flow through this narrow zone occurs in the space between the catheter and the bag surface. Combined with the increased and irregular surface area due to the folds, it is difficult to predict the relationship between flowrate and pressure drop across the contraction zone. The Reynolds number of the system is estimated to be 660 when using the following values for density $\rho = 1000 \text{ kg/m}^3$, viscosity $\mu = 0.001 \text{ Pa \cdot s}$, tube diameter $D = 22 \text{ mm}$ and peristaltic wave speed $c = 3 \text{ cm/s}$. If the flow inside is assumed to be similar to pipe flow at every location then the flow is laminar. But the Reynolds number inside the contraction zone is difficult to estimate so the nature of the flow at the neck is unknown. As such, we will analyze two flow types i.e. we first assume that a simple friction factor can be used to relate flow rate and pressure drop in the entire domain and also study the response of the system when the flow is parabolic (laminar) everywhere. In both cases, we do not account for the catheter’s presence and the flow cross section is circular everywhere instead of annular.
Our goal is to build a simple model to analyze this system and understand the relationship between the tube profile, internal pressure, esophageal wall stiffness and intensity of peristaltic contraction. With sound mathematical foundations, the effort (work) put in by the walls to pump fluid within the bag can be defined. Armed with this knowledge, we can comment on the health of peristaltic activity in the esophagus and the mechanical state of the muscle wall during swallowing which is invaluable for diagnosing dysphagia.

2 Mathematical details of the 1-D model

Flow in a tube with deforming walls can be modeled using the system of equations given by (1) and (2). Here, \( A(x,t) \) is the tube area, \( u(x,t) \) is the fluid velocity. The axial coordinate along the tube length is denoted by \( x \). These equations have been widely used to describe valveless pumping [20][21], and flow in collapsible tubes [22][23]. A detailed derivation of these equations can be found in [24].

\[
\frac{\partial A}{\partial t} + \frac{\partial (Au)}{\partial x} = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{2\tau_R}{\rho R} \tag{2}
\]

To close the system of equations, we assume that a direct relationship between pressure \( p \) and area \( A \) exists. This relationship is commonly referred to as a “tube law”. For our application, we assume that the change in pressure is proportional to the change in area. Experiments carried out with the EndoFLIP show that the distal esophageal walls follow this behavior [19] and a similar form of the tube law is also used in [21]. The form of the tube law is given by Eqn. (3). It should be clear from the form of this tube law that we have assumed the external pressure to be zero for all time and thus the transmural pressure is always equal to the pressure inside the tube.
p = K \left( \frac{A}{A_0} - 1 \right) + Y \frac{\partial A}{\partial t} \quad (3)

Here, $K$ is a measure of the wall stiffness and has the units pascal (Pa). Its exact value depends on the Young’s modulus of the muscle wall and the ratio of its thickness to the undeformed radius. A damping term with coefficient $Y$ is also introduced. By a simple manipulation using the continuity equation, we can see that the introduction of this term leads to a diffusion term in the momentum equation. Thus, the addition of this term helps stabilize the numerical solution.

\[
p = K \left( \frac{A}{A_0} - 1 \right) + Y \frac{\partial A}{\partial t} = K \left( \frac{A}{A_0} - 1 \right) - Y \frac{\partial (Au)}{\partial x} \quad (4)
\]

\[
\frac{\partial p}{\partial x} = K \frac{\partial A}{\partial x} - Y \frac{\partial^2 (Au)}{\partial x^2} \quad (5)
\]

Before we proceed, it is important to make note of the viscous shear stress term in the momentum equation. Several approximations can be made to write $\tau_R$ as a function of tube area and local fluid velocity. We have the following expressions to choose from

\[
\frac{2\tau}{\rho R} = \begin{cases} 
\frac{f u |u|}{2D} & \text{based on the Darcy friction factor} \\
\frac{8\pi \mu u}{\rho A} & \text{assuming parabolic flow everywhere} \\
2\pi \mu \frac{R_u}{\rho A} \left( \frac{Ru}{\delta_{BLT}} \right) & \text{flat profile with a linear boundary layer}
\end{cases}
\]

Here, $\delta_{BLT}$ is the boundary layer thickness (see [24] for more details) and $D$, $R$ are the diameter and radius of the tube and $A = \pi R^2$. Each of these terms can have a quantitative effect on the final solution but the qualitative effect is similar. As mentioned earlier, we choose the expression that uses a friction factor to estimate the viscous flow stress due to the uncertainty introduced by the presence of folds and irregularities in the bag wall during normal operation and also examine the system if the flow is assumed to be parabolic everywhere.

### 2.1 Non-dimensional version of the governing equations

We choose the following expressions for each of the relevant dimensional variables to obtain the nondimensional versions of the continuity, momentum and the tube law equations:

\[
A = \alpha A_0 \quad t = \tau \frac{L_{\text{tube}}}{c} \quad u = Uc \quad p = PK \quad x = \chi L_{\text{tube}}.
\]

The speed of the peristaltic wave is denoted by $c$ and $L_{\text{tube}}$ is the length of the EndoFLIP bag. Substituting these into Eqns. (1), (2) and (3) gives

\[
\frac{\partial \alpha}{\partial \tau} + \frac{\partial (\alpha U)}{\partial \chi} = 0, \quad (6)
\]

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial \chi} + \psi \frac{\partial P}{\partial \chi} + \beta \frac{U |U|}{\sqrt{\alpha}} = 0 \quad \text{and} \quad (7)
\]
\[ P = (\alpha - 1) - \eta \frac{\partial (\alpha U)}{\partial \chi}, \quad (8) \]

where the following nondimensional numbers \( \eta = Y/K(cA_0/L_{\text{tube}}), \psi = K/(\rho c^2) \) and \( \beta = \left( fL_{\text{tube}}/4 \right) \sqrt{\pi/A_0} \) emerge. Eventually, the non-dimensional pressure \( P \) in the momentum equation (7) will be replaced with the right hand side of Eqn. (8). Thus the total number of coupled equations to be solved is two with \( \alpha \) and \( U \) as the unknowns.

When working with the laminar, parabolic flow version of the momentum equation, the nondimensional number for flow resistance \( \beta \) becomes \( 8\pi \mu L_{\text{tube}}/(\rho A_0 c) \) and \( \psi \) remains unchanged.

### 2.2 Boundary conditions and peristaltic wave input

During normal operation, the fluid volume inside the bag is constant because saline pumping is halted and the bag ends are sealed. Thus the fluid velocity at both ends is zero. The bag is attached to the catheter at both ends and tapers off to a point. In this scenario, the velocity boundary conditions,

\[ U (\chi = 0, \tau) = 0 \quad \text{and} \quad U (\chi = 1, \tau) = 0 \quad (9) \]

are straightforward but the boundary conditions for area \( A \) are unclear. Fixing the area to some nonzero constant value is equivalent to fixing the pressure and leads to an inconsistent problem definition. Thus, we choose the following boundary conditions for area which is equivalent to specifying zero pressure gradients at either ends of the tube.

\[ \left. \frac{\partial \alpha}{\partial \chi} \right|_{\chi=0,\tau} = 0 \quad \left. \frac{\partial \alpha}{\partial \chi} \right|_{\chi=1,\tau} = 0 \quad (10) \]

It should be noted that more complicated forms of the tube law can be used which account for longitudinal curvature, bending and tension \[25, 26\] in the tube wall. Such a tube law can include a double derivative, \( A_{xx} \) or a fourth order derivative term \( A_{xxxx} \) of the tube area. Under such a setting, additional boundary conditions for area can be applied in a straightforward manner without leading to inconsistencies in the problem specification. The inclusion of these terms will lead to higher order (sixth degree) spatial derivatives into the governing equations. However, in our current analysis, we choose to work with the simplest version of the tube law and study the effect of nonlinear, higher-order, viscoelastic tube laws on pumping regimes in a future work.

The last ingredient required to complete the model is specifying an activation input to mimic peristaltic contraction. We wish to induce a reduction in the tube area at some location \( \chi \) and at some time \( \tau \) in a manner that resembles a traveling wave. The most common approach to induce contractions is seen in the aforementioned valveless pumping models where an external activation pressure at a specific location is varied sinusoidally with time \[20, 21, 24\]. This is an appropriate method of activation for valveless pumping scenarios due to the fact that the extra pressure is generated due to respiration. But in the esophagus, contractions in area are generated due to a contraction of muscle fibers in the esophageal wall. As such, we add an activation term \( \theta \) to the tube law which changes the reference area of the tube when activated. The modified form of the tube law with an activation term is
\[ P = \left( \frac{\alpha}{\theta} - 1 \right) - \eta \frac{\partial (\alpha U)}{\partial \chi}, \]  

(11)

with the activation term given by

\[ \theta = \begin{cases} 
1 - \left( \frac{1 - \theta_0}{2} \right) \left( 1 + \sin \left( \frac{2 \pi}{w} (\chi - \tau) + \frac{3 \pi}{2} \right) \right), & \tau - w \leq \chi \leq \tau \\
1, & \text{otherwise} 
\end{cases} \]

The nondimensional width of the peristaltic wave is denoted by \( w = W/L_{\text{tube}} \) with \( W \) denoting the actual, dimensional width of the contraction. As mentioned earlier, the area at the tube ends in the device tapers off, so a small correction is made to the activation term to account for the reduced area at the ends. The final form of the activation wave is plotted in Fig. (5).

The value of \( \theta_0 \) specified represents the reference area of the tube at the strongest part of the contraction. The smaller the value of \( \theta_0 \), the smaller is the reference area and higher is the contraction intensity. When \( \theta_0 = 1 \), there is no contraction and the entire system is at rest. Since the velocity scale was chosen to be the same as the speed of the peristaltic wave, in the nondimensional framework, the speed of activation is 1.0.

### 2.3 Numerical solution for the system of PDEs

The nondimensional pressure term \( P \) in the momentum equation is eliminated using the tube law given by Eqn. (11). The final system of equations in matrix form is

\[
\begin{bmatrix}
\alpha_U \\
U_U
\end{bmatrix} = \begin{bmatrix}
0 & \eta \psi (\alpha U) \chi \chi \\
-\eta U \chi \chi - \beta \left( \frac{U \chi}{\sqrt{\alpha}} \right) - \psi \left( \frac{1}{2} \theta_0 \chi - \alpha \theta_0 \chi \chi \chi \chi \right)
\end{bmatrix}
\]

with boundary conditions as given in Eqs. (9) and (10) and zero velocity initial conditions and some initial condition for area which indicates the amount of volume inside the tube.

\[ U (\chi, \tau = 0) = 0 \quad \alpha (\chi, \tau = 0) = \alpha_{IC} \]  

(12)

This set of equations can be solved using the traditional two-step schemes as outlined in [27]. Another approach that is simpler to implement is to add an artificial diffusion
term, $\epsilon (\alpha_{xx})$, to the continuity equation and use MATLAB’s PDEPE routine to obtain the numerical solution. The numerical details of this approach can be found in [28]. When $\eta = 0$, this system becomes strictly hyperbolic. Adding an artificial diffusion term is common practice for obtaining the solution for a hyperbolic system of equations [29,30]. In all the simulations that utilize the 1-D model, we set the value of $\epsilon$ to be $10^{-4}$ and $\eta$ to be $10^{-3}$. The volume of the tube is monitored every timestep and the maximum volume change across all the runs was found to be 2% and the ratio of the residual of the unmodified continuity equation to the magnitude of the largest term did not exceed $10^{-3}$. For the sake of clarity, we rewrite the governing equations and the boundary conditions in the form required by PDEPE and clearly show the flux terms as well.

\[
\left[ \frac{\alpha_{\tau}}{U_{\tau}} \right] = \frac{\partial}{\partial \chi} \left[ \frac{\epsilon \alpha_{\chi}}{\eta \psi (\alpha U)_{\chi}} \right] + \left[ -U \frac{\partial \alpha}{\partial \chi} - \frac{\alpha U}{\eta \psi (\alpha U)_{\chi}} \right] - \frac{\beta U |U|}{\sqrt{\alpha}} \right] \quad (13)
\]

\[
\left[ \begin{array}{c}
0 \\
U \\
1 \\
0 \\
\end{array} \right] + \left[ \begin{array}{c}
0 \\
1 \\
0 \\
0 \\
\end{array} \right] \left[ \begin{array}{c}
\epsilon \alpha_{\chi} \\
\eta \psi (\alpha U)_{\chi} \\
\end{array} \right] = \left[ \begin{array}{c}
0 \\
0 \\
\end{array} \right] \quad \text{for} \quad \chi = 0, 1 \quad (14)
\]

3 Dynamics displayed by the 1-D model

With all the parts of the 1-D model in place, we can now investigate the system’s response to the applied activation as a function of the operating parameters, $\theta_0$, $\psi$ and $\beta$. Changing these values is equivalent to investigating the effect of tube wall stiffness, wave speed, contraction strength, fluid density and flow resistance on the tube wall deformation and internal flow patterns during peristalsis. After studying the system’s behavior for a broad range of operating values, we found four distinct, physiologically relevant patterns of peristaltic pumping based on the way the tube walls and the fluid inside respond to the applied activation. These regimes are shown in Fig. (6). The progression of these regimes from numbers 1 to 4 can be interpreted simply as the response of the system as fluid viscosity is continually increasing. It should be noted that the fourth regime displays very little deformation of tube area. The deformation dynamics for this regime are unremarkable and the tube shape remains quite similar to the shape of the initial condition for the entire duration of wave travel.

The occurrence of these regimes is dictated by a competition between elastic forces generated due to deformation of the tube wall and resistance to flow through the narrowest part of the contraction. A relatively stiff tube wall resists deformation and forces the fluid that was displaced due to the advancing wave to flow back through the contraction. In this scenario, we see the walls deform as shown in Regime 1. An equivalent way of looking at this is to assume a low resistance to flow through the contraction. The energy required to expand the tube walls is significantly higher than the viscous loss through the contraction and the pumping is almost quasi-steady with the tube walls having the same diameter on either side of the contraction. On the other hand, if the resistance to flow is high, it is favorable for the system to expand the tube walls to accommodate the fluid that is being displaced by the advancing peristaltic wave. At moderate flow resistances, this exact situation occurs when the stiffness of the tube walls is extremely low. The tube deformation pattern in this scenario is denoted as Regime 2.
Regime 3 has been investigated in great detail by Takagi and Balmforth [11]. In their work, the lubrication approximation is used in an infinitely long domain with open ends. In our configuration, we see the formation of a “blister” as predicted by their model when the resistance to flow is high and the tube walls are fairly compliant. Regime 4 occurs when the resistance to flow is much higher than that in Regime 3. In this case, the amount of time it takes for the fluid to ‘move’ and respond to the peristaltic contraction is much higher than the amount of time it takes for the wave to travel over a section of the tube. The fluid velocity in this case is extremely low leading to a small value in \( \frac{\partial A}{\partial t} \) as well.

As mentioned earlier, when the dissipative terms are removed, the governing equations are hyperbolic in nature. Thus, when the value of \( \psi \) (which is the inverse of the Mach number squared) is small, the wave-like nature of the system begins to emerge. In this region the speed of the peristaltic wave is comparable to the speed at which a disturbance in the system travels at. For the esophageal wall, this speed is of the order of 10 m/s but the peristaltic wave speed rarely exceeds 5 cm/s. As such, we will not spend too much time investigating the dynamics of the system at these high wave speeds. But
for the sake of completeness, we show some of the tube shapes predicted by our model for this region of operation in Fig. 7. In the first panel, Fig. (7a), we see the wave generate a tube shape that involves two inflated ends connected by a thin channel. The development of this regime is due to the high flow rate generated by the wave pumping through the neck. The closed end forces fluid to flow back through the contraction and due to the high inertia, the fluid stops only upon reaching the other end leading to a long section of the tube with low area and a majority of the fluid mass confined towards the ends. The flow in the second panel Fig. (7b), is observed when the flow resistance in significantly increased. In such a scenario, a large amount of fluid travels as a single blob across the tube length, reflecting from end-to-end till the energy is dissipated. For Figs. (7a) and (7b), the last two of the 5 depicted instances show the shape of tube after the wave has finished traveling over the tube length. To visualize these two regimes, the numerical damping parameters were increased by a factor of 10 to obtain a stable numerical solution. It must be reiterated that these pumping dynamics would never be observed in any physiological setting. It must also be emphasized that these are not all the possible solutions in this region of operation. Additional solutions might exist but no effort was made to identify or discover all pumping patterns for this unphysiological range of operating conditions.

3.1 Comparison with 3D immersed boundary simulations

Before we proceed with our discussion on the regimes and predicting the transition between them as a function of the operating parameters, we attempt to validate our 1D model with detailed fluid-structure interaction simulations using the immersed boundary (IB) method. It has already been stated that the nature of the flow within the device is not known to be definitively either laminar or fully-developed turbulent flow. Preliminary estimates of the Reynolds number indicate that the flow might be laminar. As such we continue with this assumption and setup our 3D simulations such that they develop parabolic flows everywhere within the tube. We obtain the various values of \( \mu, K, c, \rho \) and activated rest length from the 3D simulations and find the corresponding values for
ψ, β and contraction strength and compare the tube shapes and fluid velocity variations for both the models.

The geometry of the structure used in the immersed boundary simulations consists of a long cylindrical tube with open ends. The two ends are closed using ‘caps’ that are simply short flat cylinders of certain thickness. The entire structure is then meshed using hexahedral elements to take advantage of the Adaptive Anisotropic Quadrature that was developed by Kou in [31] and implemented in the open-source immersed boundary framework IBAMR [32]. The structure is represented using finite elements and the fluid is solved on a Cartesian grid using the Finite Volume Method. Mathematical details on the interaction equations, discretization of the Lagrangian and Eulerian domains and temporal schemes for solving the coupled system can be found in [33].

The mechanical properties of the tube consists of two materials. Just like the esophagus, the structure in our IB simulations consists of fibers embedded in an isotropic matrix material. The matrix component is represented by a simple Neo-Hookean strain energy function and the fiber’s effect is implemented using a bilinear strain energy function that computes the stress based on strain in the circumferential direction. Similar to the activation method used in [31], the peristaltic wave is applied as a controlled reduction in the fiber rest length along the tube. This reduction leads to high generation of circumferential stresses in the tube at the activation location, the consequence of which is a reduction of the local tube area.

The results of our simulations are summarized in Fig. 8. The top panel of each subfigure shows the cross section of the tube and the velocity fields within obtained from the 3D IB simulations. The bottom panel shows the shape predicted by the 1D model (blue curves) and superimposed on this is the calculated axial velocity \( U \) (red curve). For the graphs in the bottom panels, the \( y \)-axis on the left corresponds to the non-dimensional radius values and the \( y \)-axis on the right corresponds to the values of the non-dimensional fluid velocity. The figures show the tube shapes generated by the peristaltic wave as it travels from left to right along the tube length for both models. The last two figures (8e) and (8f) show the tube shapes after the wave has passed. For every instant, we see excellent agreement between the velocity variations observed in the immersed boundary calculations and the non-dimensional velocity profiles predicted by the 1D model. The tube shapes are also observed to match well with the shape of the structure computed by the 3D simulations before and after the wave has passed over the tube length. It should be noted that fluid velocity increases rapidly when the wave approaches the end and ‘opens’ up to let the fluid accumulated at end to flow back into the deflated part of the tube. This high flow rate generates a significant amount of shear stress which leads to a noticeable deformation of the tube mesh at that location. As a result of this velocity spike, the scales for \( U \) have been appropriately changed for the later instances to maintain clarity between the flow profiles and tube shapes.

Encouraged by this agreement between the 1D model and the 3D IB simulations, we continue the analysis of the system by probing the 1D model for a wide range of operating conditions. Although the agreement was found only for parabolic flow, we also investigate the system’s response when the viscous term is friction-factor based. We present the results for both these scenarios in a “unified” regime map. One of the primary goals for this comparison with 3D IB simulations was to check if the introduction of the tube damping and artificial diffusion terms significantly affected the response of the
Additional investigation of the system using the 3D IB model revealed that as the value of viscosity is reduced, while simultaneously keeping all the other operating parameters constant, the fluid velocity through the contraction increases. The system is then observed to transition from Regime 2 to Regime 1. However, when the viscosity falls below a certain value, the fluid velocity through the neck is high enough to transition into turbulent flow. This leads to a sudden increase in resistance to flow through the neck even as the viscosity is lowered and the system can then display tube deformations that correspond to Regime 2. Another interesting phenomena occurs when the fluid viscosity falls within a particular range of values – the flow through the neck starts off as purely laminar but as the wave displaces additional fluid and the pressure difference between the “deflated” and “inflated” part of the tube increases, flow through the neck becomes turbulent. Unfortunately, we found no simple way to capture this transition to turbulence to implement in our 1D model. Implementing this effect using empirical friction-factor based models is the focus for our future work.
4 System behavior represented as a regime map

In Section 3, we showed the various patterns of tube wall deformation that can be observed depending on the operating conditions of the system. Our goal is to combine the various operating conditions in a sensible manner to study their cumulative effect on the system and predict the transition between regimes as a function of these collapsed variables. To that effect, we utilize the wave-frame approach that has often been employed by other workers to analyze peristaltic flow. We start with the configuration shown in Fig. 9. The contraction will appear stationary in a reference frame that is attached to the peristaltic wave and the walls of the tube will appear to move with the speed of the peristaltic wave, but in the opposite direction. The fluid velocities in this wave reference frame are represented by black arrows and the velocity of the wall is represented with blue arrows. We have assumed that the velocity of the fluid ahead of the contraction zone, in the lab frame, is zero (this assumption is incorrect if the flow is laminar and parabolic but is valid when the flow through the neck is turbulent).

\[
V_1 = V_{\text{wave}}
\]

\[
\frac{A_1}{A_2} V_1 = \frac{A_c}{L_c} V_{c} = \frac{A_2}{A_1} V_2.
\]

Figure 9: Peristaltic contraction as observed in the wave’s frame of reference

From the Bernoulli equation we have

\[
\frac{p_1}{\rho} + \frac{1}{2} V_1^2 = \frac{p_c}{\rho} + \frac{1}{2} V_{c}^2 + f \frac{L_c}{D_c} \frac{V_{c}^2}{2},
\]

(15)

where the last term on the RHS accounts for viscous losses in the contraction. Note that the kinetic energy is calculated using the velocity in the wave’s reference frame but the loss term uses the lab frame velocity because the motion of the walls must be taken into account to estimate viscous losses. So we use \( V_r = V_c - V_1 \). From conservation of volume, we have

\[
A_1 V_1 = A_c V_c = A_2 V_2.
\]

The friction factor \( f \), depends on the Reynolds number of the flow through the neck. For low \( Re \) laminar flows, \( f = 64/Re \) and in the inertial flow regime (\( Re \gg 1 \)), \( f \) is assumed to be constant. The relationship between pressure and area (tube law) is the same as before and has the form,

\[
p = K \frac{\Delta A}{A_0} = K \left( \frac{A - A_0}{A_0} \right).
\]
4.1 Low Reynolds number flow through the neck

Starting with the Bernoulli equation given in Eqn. (15), we replace the friction factor \( f \) with the low \( Re \) approximation to get

\[
\frac{p_1}{\rho} + \frac{1}{2}V_1^2 = \frac{p_c}{\rho} + \frac{1}{2}V_c^2 + \left( \frac{64\mu}{\rho V_c D_c} \right) \frac{L_c V_r^2}{D_c^2} \frac{1}{2}
\]

We then simplify the terms using the tube law and volume conservation via the following series of steps:

\[
\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + 1 = \frac{p_c}{\rho} + \frac{1}{2}V_c^2 + \left( \frac{64\mu}{\rho V_c D_c} \right) \frac{L_c V_r^2}{D_c V_c} f_w
\]

\[
K \frac{1}{2\rho V_1^2} \left( \frac{A_1 - A_0}{A_0} \right) + 1 = K \frac{1}{2\rho V_1^2} \left( \frac{A_c - A_{c0}}{A_{c0}} \right) + \left( \frac{A_1}{A_c} \right)^2 + f_w \frac{L_c V_r}{D_c} \left( \frac{V_c - V_1}{V_1} \right)
\]

\[
K \frac{1}{2\rho V_1^2} \left( \frac{A_1 - A_0}{A_0} \right) + 1 = K \frac{1}{2\rho V_1^2} \left( \frac{A_c - A_{c0}}{A_{c0}} \right) + \left( \frac{A_1}{A_c} \right)^2 + f_w \frac{L_c}{D_c} \left( \frac{A_1}{A_c} - 1 \right)
\]

The term \( A_{c0} \) represents the reference area of contraction and is much smaller than \( A_0 \). Thus, a reduction in the reference area at some location along the tube length, indicates that the tube segment belongs to the contraction zone. We introduce the following variables to simplify the notation:

\[
\psi_k = \frac{K}{2\rho V_1^2} \quad \psi_f = f_w \frac{L}{D_0} \quad L'_c = \frac{L_c}{L} \quad D'_c = \frac{D_c}{D_0}
\]

The equation thus simplifies into

\[
\psi_k \left( \frac{A_1 - A_0}{A_0} \right) + 1 = \psi_k \left( \frac{A_c - A_{c0}}{A_{c0}} \right) + \left( \frac{A_1}{A_c} \right)^2 + \psi_f \frac{L'_c}{D'_c} \left( \frac{A_1}{A_c} - 1 \right)
\]

4.2 High Reynolds number flow through the neck

Starting from Eqn. (15) again and repeating the same steps as before, we get

\[
\frac{p_1}{\rho} + \frac{1}{2}V_1^2 = \frac{p_c}{\rho} + \frac{1}{2}V_c^2 + f \frac{L_c V_r^2}{D_c} \frac{1}{2}
\]

\[
\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + 1 = \frac{p_c}{\rho} + \frac{1}{2}V_c^2 + f \frac{V_r^2 L_c}{V_1^2 D_c}
\]
\[
\frac{K}{\frac{1}{2}V_1^2\rho} \left( \frac{A_1 - A_0}{A_0} \right) + 1 = \frac{K}{\frac{1}{2}V_1^2\rho} \left( \frac{A_c - A_{c0}}{A_{c0}} \right) + \left( \frac{A_1}{A_c} \right)^2 + f \left( \frac{V_c}{V_1} - 1 \right)^2 \frac{L_c}{D_c}
\]

\[
\psi_k \left( \frac{A_1 - A_0}{A_0} \right) + 1 = \psi_k \left( \frac{A_c - A_{c0}}{A_{c0}} \right) + \left( \frac{A_1}{A_c} \right)^2 + f \left( \frac{L_{D0}}{\Psi_f} \right) \left( \frac{A_1}{A_c} - 1 \right)^2 \frac{L_c'}{D_c'}
\]

Here, \( f \) is assumed to be a constant and does not depend on the local Reynolds number in the tube. If we assume that there is no viscous loss as the fluid enters chamber 2, then we can write

\[
\frac{p_1}{\rho} + \frac{1}{2}V_1^2 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + f \frac{L_c}{D_c} \frac{V_2^2}{V_1^2}
\]

\[
\frac{K}{\frac{1}{2}V_1^2\rho} \left( \frac{A_1 - A_0}{A_0} \right) + 1 = \frac{K}{\frac{1}{2}V_1^2\rho} \left( \frac{A_2 - A_0}{A_0} \right) + \frac{V_2^2}{V_1^2} + f \frac{L_c}{D_c} \frac{V_2^2}{V_1^2}
\]

\[
\psi_k \left( \frac{A_1 - A_0}{A_0} \right) + 1 = \psi_k \left( \frac{A_2 - A_0}{A_0} \right) + \left( \frac{A_1}{A_2} \right)^2 + f \frac{L_c}{D_c} \frac{V_2^2}{V_1^2}
\]

The exact form of the last term depends on the expression chosen for \( f \). We have following two possibilities:

\[
f \frac{L_c}{D_c} \frac{V_2^2}{V_1^2} = \begin{cases} 
\psi_f \frac{L_c'}{D_c} \left( \frac{A_1}{A_c} - 1 \right) & \text{for low Re} \\
\Psi_f \frac{L_c'}{D_c} \left( \frac{A_1}{A_c} - 1 \right)^2 & \text{for high Re}
\end{cases}
\]

**4.3 Prediction of the boundary between regimes**

As the peristaltic wave travels over the tube, it naturally leads to a separation of the tube into two parts. One is the length of the tube over which the wave has already traveled and the other is the section of the tube that lies ahead of the contraction. In our analysis, we assume that the tube areas in each of these sections is constant and the ratio of these areas is \( A_2/A_1 \) from Fig. 9. We wish to solve for \( A_2/A_1 \triangleq x \).

Let \( A_c = \theta A_1 \) and then from Eqn. (16), we can write

\[
\psi_k \left( \frac{\Delta A_1 - \Delta A_2}{A_0} \right) + 1 = \left( \frac{A_1}{A_2} \right)^2 + f \frac{L_c}{D_c} \frac{V_2^2}{V_1^2}
\]

\[
\psi_k \left( \frac{\Delta A_1 - \Delta A_2}{A_0} \right) + 1 = \left( \frac{A_1}{A_2} \right)^2 + \begin{cases} 
\psi_f \frac{L_c'}{D_c} \left( \frac{A_1}{A_c} - 1 \right) & \text{low Re} \\
\Psi_f \frac{L_c'}{D_c} \left( \frac{A_1}{A_c} - 1 \right)^2 & \text{high Re}
\end{cases}
\]

\[
\psi_k \left( \frac{\Delta A_1 - \Delta A_2}{A_0} \right) + 1 = \left( \frac{A_1}{A_2} \right)^2 + \begin{cases} 
\psi_{fc} \left( \frac{1-\theta}{\theta} \right) & \text{if low Re} \\
\Psi_{fc} \left( \frac{1-\theta}{\theta} \right)^2 & \text{if high Re}
\end{cases}
\]

The final version of the equation for each type of flow is given below.
ψ_k A_1 A_0 x^3 + x^2 \left[ \psi_{fc} \left( \frac{1 - \theta}{\theta} \right) - 1 - \psi_k A_1 A_0 \right] + 1 = 0 \quad \text{Low Re flow}

\psi_k A_1 A_0 x^3 + x^2 \left[ \Psi_{fc} \left( \frac{1 - \theta}{\theta} \right)^2 - 1 - \psi_k A_1 A_0 \right] + 1 = 0 \quad \text{High Re flow}

With the introduction of the new variables ψ_f' and Ψ_f', we combine the effect of the contraction strength and the viscous flow resistance term. The equation for low Re flow can now be written as

ψ'_k x^3 + x^2 \left[ ψ'_f - ψ'_k - 1 \right] + 1 = 0 \quad (17)

and for high Re, we have a similar equation

ψ'_k x^3 + x^2 \left[ Ψ'_f - ψ'_k - 1 \right] + 1 = 0 \quad (18)

As we can see, the equations are identical. The only difference between them is the way ψ_f' and Ψ_f' are calculated from the input parameters. For the sake of clarity, we summarize all the variables used in this analysis in the table below

| Symbol | Description |
|--------|-------------|
| V_1    | wave velocity |
| V_c    | Flow speed in the neck (wave frame) |
| V_r = V_c - V_1 | Flow speed in the neck (lab frame) |
| f_W = (64 \mu) / (\rho V_1 D_c) | Low Re friction factor |
| ψ_f = f_W (L/D_0) | Low Re resistance |
| ψ_k = K / (\rho V_1^2 / 2) | inverse of Mach number squared |
| L'_c = L_c / L | ratio of neck length to tube length |
| D'_c = D_c / D_0 | Ratio of neck diameter to the reference diameter |
| \theta = A_c / A_1 | area reduction factor for the contraction |
| ψ_{fc} = ψ_f (L'_c / D'_c) | Combined flow resistance term for low Re flow |
| ψ'_f = ψ_{fc} (1 - \theta) / \theta | All encompassing flow resistance term |
| ψ'_k = ψ_k (A_1 / A_0) | Tube stiffness and fill volume combination |
| Ψ_f = f (L/D_0) | Flow resistance term for high Re flow |
| Ψ_f' and ψ_f' | Cumulative flow resistance terms |

The two terms ψ'_k and (ψ'_f, Ψ_f') together, account for all the relevant parameters of the system. The effect of the tube stiffness, fluid density, wave velocity and the amount of fill volume of the tube is accounted for in ψ'_k and ψ'_f accounts for the fluid’s resistance to flow in the tube, the length of the contraction zone and its intensity as well. All
these effects contribute to the increase or decrease of the area ratio. The utility of our analysis is now clear because if we wanted to visualize the effect of these variables on pumping patterns, we would either need to show dozens of plots or need a plot that has multiple axes, each corresponding to wave velocity, tube stiffness, fluid density, etc. By combining these in a sensible manner, which was made possible due to the above analysis, the visualization of observed regimes can be achieved in a coherent and clear way.

Figure 10: Regime maps obtained from the 1D model

Armed with this simplification, we can present the occurrence of each regime as a set of points on a $\psi'_f$ vs. $\psi'_k$ plot. Fig. 10 shows the occurrence of the various regimes at different values of the two $\psi$'s. In Fig. (11a), we combine all the regime points from the two different flow types and present them in a single plot. We see that the regime data points from both flow types fall within the general vicinity of each other. The set of points belonging to each regime is represented by a specific marker. All combinations of $\psi'_k$ and $\psi'_f$ for regime 1 are represented by red triangles (▲), regime 2 points are represented by purple crosses (×). Regimes 3 and 4 are represented by green circles (●). The wave frame approach that we took for combining the various parameters cannot differentiate between these two regimes and as they both occur in the region of high fluid viscosity, they are logically combined into a single entity for plotting purposes.

From these two plots, we clearly observe the presence of a general boundary between each set of regime points. The boundary represents the change of the system from one solution to another. As such, we hypothesize that the discriminant of the cubic equations given by (17) and (18) will predict the slope of this boundary. Regime 1 corresponds to $x \approx 1$ and Regime 2 corresponds to $x \approx 0$. When the discriminant of this equation is zero, we get the following relationship between $\psi'_f$ and $\psi'_k$:

$$\psi'_f = \psi'_k - 3 \left( \frac{\psi'_k}{2} \right)^{2/3} + 1.$$  \hspace{1cm} (19)

Plotting this equation on the regime map given in Fig. 11a gives us the right slope.
We then add an offset (by a factor of 10) to match the exact boundary as seen in the regime map with the curve plotted by Eqn. [19]. The boundary between regimes 2 and 3 can also be demarcated by another offset using a factor of 10. The combined regime map with these boundaries is shown in Fig. [11b].

(a) Combined regime map for both high and low Re flows  
(b) Regime boundary as predicted by the zero discriminant equation

Figure 11: Regime maps obtained from the 1D model

With the help of these regime maps, we can predict the shape assumed by the tube during peristalsis by simply computing the relevant \( \psi'_f \) and \( \psi'_k \) from the given operating conditions and finding the region which these points belong to. The regime maps also help us understand the change in tube shape that would be observed as one of the operating conditions is changed. For instance, as the tube stiffness increases, the system will deform in such a way that the area ratio \( x \) tends to 1.0. Increasing the fill volume or decreasing the peristaltic wave velocity leads to a similar outcome. When it comes to increasing the fluid’s viscosity, we see a transition from Regime 1 to Regime 2 as predicted by the regime map when the value of \( \psi'_f \) is increased. When the area contraction factor \( \theta \) is reduced i.e. the amount of wave ‘squeezing’ is increased, we again observe the system transition from Regime 1 to Regime 2.

Each of these regimes correspond to a specific deformation pattern observed in the EndoFLIP device in different patients and/or diseases. For instance, assume that the device is calibrated and the operating conditions are benchmarked in such a way that it shows pumping patterns corresponding to Regime 2 in a healthy individual. Under the same operating conditions, a patient with a diseased esophagus will display pumping patterns corresponding to either Regime 1 or Regime 3/4. Depending on the specific behavior displayed, the regime map helps us identify the exact cause of the abnormality. For instance, a patient displaying regime 1 might have a stiffer esophagus due to fibrosis and a patient with Regime 4 is can be said to be suffering from dysphagia due to ineffective peristalsis. Thus the quantification of the device’s behavior in the form of these regime maps directly assists in better interpreting the various shapes seen in patients during the EndoFLIP procedure.
Before we proceed, we need to make note of an obvious fact. When the value of $\theta$ approaches 1, the value of $\psi'$ goes to zero. As such, it should be clear that the regimes maps shown above do not display this region of operation. The complete regime map would show that the regions occupied by regimes 1 and 2 would be flanked by points from Regime 4 not only at the top (as is indeed shown in the plots above) but on the bottom as well. The inclusion of these points for the lower values of $\psi'$ would unnecessarily clutter the regime plot and were thus excluded for clarity.

5 Defining and quantifying pumping effort

In this section, we aim to understand and differentiate between an ‘effective’ and an ‘ineffective’ peristaltic contraction wave. Finding the work done by a peristaltic wave and observing its variation during pumping can offer further insights on the pumping process. The configuration that is being analyzed in this work is a closed system. As such, there is no “net flow rate” or non-zero displacement of fluid volume over one peristaltic event which can be used to quantify efficacy. At the end of peristalsis, all the work done by the peristaltic wave is dissipated due to fluid viscosity. So the amount of energy dissipated is the spent pumping work. During peristalsis however, the stretching of the tube walls leads to some of the spent energy being stored as elastic potential energy. But after the passage of the peristaltic wave, the tube relaxes and releases all the energy back into the fluid which in turn is lost via viscous dissipation. To understand the quantitative relationship between these three agents, we turn to the parabolic version of the momentum equation (2) and multiply both sides with $Au$ to obtain

$$
\rho Au \frac{\partial u}{\partial t} + \rho (Au) u \frac{\partial u}{\partial x} = -Au \frac{\partial p}{\partial x} - \frac{8\pi \mu Q u}{A}.
$$

(20)

Noting that $Q = Au$, we rewrite the equation to form terms that involve derivatives of the kinetic energy of the fluid per unit volume. We end up with

$$
A \frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 \right) + Q \frac{\partial}{\partial x} \left( \frac{1}{2} \rho u^2 \right) = -Au \frac{\partial p}{\partial x} - \frac{8\pi \mu u Q}{A},
$$

(21)

to which we add the continuity equation (multiplied with $\rho u^2/2$) on the LHS and after combining terms using the product rule, we end up with the equation

$$
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho Au^2 \right) + \frac{\partial}{\partial x} \left( \frac{1}{2} \rho Qu^2 \right) = -Au \frac{\partial p}{\partial x} - \frac{8\pi \mu u Q}{A},
$$

(22)

the terms of which can be interpreted as follows,

$$
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho Au^2 \right) + \frac{\partial}{\partial x} \left( \frac{1}{2} \rho Qu^2 \right) = -\frac{\partial (Au p)}{\partial x} + \frac{\partial (Au)}{\partial x} - \frac{8\pi \mu u Q}{A}.
$$

(23)

Note that conservation of volume can be used to replace the $\frac{\partial (Au)}{\partial x}$ term with $-\frac{\partial A}{\partial t}$ to obtain
\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho A u^2 \right) + \frac{\partial}{\partial x} \left( \frac{1}{2} \rho Qu^2 \right) = -\frac{\partial (A u p)}{\partial x} - p \frac{\partial A}{\partial t} - \frac{8\pi \mu u Q}{A}. \tag{24}
\]

Now we integrate this equation over the length of the tube from \( x = 0 \) to \( L \) and apply the zero velocity condition \( Q = u = 0 \), at the tube ends. We obtain

\[
\frac{\partial}{\partial t} \int_0^L \left( \frac{1}{2} \rho A u^2 \right) \, dx + \int_0^L \left( \frac{1}{2} \rho Qu^2 \right) \, dx = -(A u p)_0^L - \int_0^L p \frac{\partial A}{\partial t} \, dx - \int_0^L 8\pi \mu u^2 \, dx, \tag{25}
\]

which simplifies to

\[
\frac{\partial}{\partial t} \int_0^L \left( \frac{1}{2} \rho A u^2 \right) \, dx + \int_0^L p \frac{\partial A}{\partial t} \, dx + \int_0^L 8\pi \mu u^2 \, dx = 0 \tag{26}
\]

This equation succinctly shows how the power is distributed in the system at each time instant. When the pressure term is put on the RHS, it gets a negative sign and then represents the work done by the walls on the fluid. The LHS then shows that part of that energy goes into changing the kinetic energy of the fluid and the rest is lost via viscous dissipation. It is important to realize that the pressure term has contributions from both the passive elastic part of the tube and the rise due to active contraction. When separated, we get a better understanding of the power breakdown,

\[
\frac{\partial}{\partial t} \int_0^L \left( \frac{1}{2} \rho A u^2 \right) \, dx + \int_0^L p \frac{\partial A}{\partial t} \, dx + \int_0^L 8\pi \mu u^2 \, dx = -\int_0^L p_{\text{pass}} \frac{\partial A}{\partial t} \, dx. \tag{27}
\]

Simply put, the RHS is the rate of work done by the active part of the tube wall (the peristaltic contraction) on the confined fluid and the terms on the LHS show the consumers of this spent power. Some of it goes into increasing the kinetic energy of the fluid, some of it is stored in the tube walls when they stretch due to an increase in local pressure and the rest is lost via dissipation. It is important to realize that the pressure term has contributions from both the passive elastic part of the tube and the rise due to active contraction. When separated, we get a better understanding of the power breakdown,

\[
\frac{\partial}{\partial t} \int_0^L \left( \frac{1}{2} \rho A u^2 \right) \, dx + \int_0^L p \frac{\partial A}{\partial t} \, dx + \int_0^L 8\pi \mu u^2 \, dx = -\int_0^L p_{\text{pass}} \frac{\partial A}{\partial t} \, dx. \tag{27}
\]

Armed with this breakdown of fluid pressure, we can compute the values of each of the terms in Eqn. \([27]\) and visualize the variation of work done or energy dissipated over a single peristaltic event for each of the reported regimes. The above analysis can be repeated for the momentum equation with the friction factor stress term but the
resulting work computations show no qualitative differences between the two scenarios. The following results include cases where both forms were utilized for calculating work values.

5.1 Work curves from the reduced order model

As explained earlier, Eqn. (27) gives the balance of power at every instant of time. Integrating this equation over time gives the balance of work done or energy lost as the peristaltic wave advances. In Fig. (12), we show the energy contribution from each of these terms across all the observed regimes. This figure summarizes the energy transfer pathways between the various sinks over the entire range of observed pumping patterns. At time $\tau = 0$, the peristaltic wave begins traveling over the tube length and the active work done by it is zero. As the wave advances, the work done by it i.e. the active work is split into either increasing the potential energy stored in the tube walls or generating flow fields that then lose energy via dissipation. For the parameter space that is being analyzed in this work, the rate of change of fluid kinetic energy is negligible and has not been plotted.

The work curves for each of these regimes show unique identifying features that confirm what is visually observed through the tube deformation patterns. For Regime 1, once the wave has created a zone of reduced area, the active work goes into overcoming the viscous resistance and the potential energy of the walls remains unchanged. Upon approaching the right boundary, the tube relaxes and the stored elastic energy is recovered. For Regime 2 however, we observe a gradual rise in stored elastic energy with time indicating that the activation wave is continuously doing work on the tube wall. Unlike Regime 1, when the wave approaches the end and allows the tube walls to relax, the stored energy is lost via fluid dissipation. This is observed by following the viscous work curve which sharply rises around the $\tau = 0.8$ mark to meet the active work curve. It is also interesting to note that in spite of large tube wall deformations associated with this regime, the majority of active work done is still lost via viscous dissipation. Regimes 3 and 4 reflect the low wall deformations observed from the tube shape plots. The passive work done is extremely small compared to the active work done which is almost entirely lost. It should be noted that the magnitude of active work should always be the largest among the three curves but due to numerical errors, the viscous work appears to be larger. This can be easily understood by non-dimensionalizing Eqn. (27),

$$\frac{\partial}{\partial \tau} \int_0^1 \left( \frac{1}{2} \alpha U^2 \right) d\chi + \psi \int_0^1 P_{\text{pass}} \frac{\partial \alpha}{\partial \tau} d\chi + \beta \int_0^1 U^2 d\chi = -\psi \int_0^1 P_{\text{actv}} \frac{\partial \alpha}{\partial \tau} d\chi. \quad (29)$$

For Regimes 3 and 4, the value of $\beta$ is quite large compared to $\psi$ so the computation of viscous work involves the product of a large and small number which leads to the discrepancy mentioned above. In the above plots, Case-551 is the only example that shows the work curves for Regime 4, the variation is quite unremarkable and the values of work done are quite small compared to curves from the other regimes. The extremely high value of $\beta$ impedes any change in $A$ which directly leads to almost no fluid flow or tube deformation.
This detailed look into the variation of the active, passive and viscous work done during a peristaltic event gives us the tools to identify what a healthy pumping wave looks like and the conditions under which one might observe reasonable tube wall deformations. A significant change in the tube area for this configuration due to peristalsis indicates that the wave has some ability to move fluid forward in a normal setting which involves bolus transport following healthy deglutition.

5.2 Work curves from EndoFLIP patient data

In the previous subsection, we have defined and provided a strong mathematical foundation for computing the value of work done during peristalsis in the current configuration and the various sinks that consume the energy generated by muscular activation. The work calculations were performed in the context of the reduced order model presented in
Section 2. In this section, we wish to extend the utility of the model to compute work curves using data obtained from the FLIP device when used in human subjects. The goal here is to get a sense of the magnitude of peristaltic work done in healthy subjects and to see what characteristics are observed in work curves obtained from patient data.

It is clear that the 1D model in Section 2 takes as input the activation wave and predicts the resultant tube wall deformation and the associated fluid velocity and pressure fields. The FLIP data on the other hand contains the variation of tube wall deformation as a function of time and the value of pressure at the (approximate) location \( x = \chi L = 16 \, \text{cm} \). The values of fluid velocity and pressure at all the other locations along the tube are unknown. To tackle this issue, we first use the continuity equation to obtain the gradient of velocity field and then we turn to the the momentum equation and differentiate it with respect to \( \chi \) to obtain a second order derivative of velocity \( U \). This allows us to apply the zero-velocity boundary conditions at both ends. The pressure reading obtained from the patient data is applied as a pinning condition at the \( \chi = 1 \) boundary. Following that, this equation can be used to determine the velocity and pressure fields as a function of \( \chi \) and \( \tau \). In the following paragraphs, we present the mathematical details of this process.

First, we use the same nondimensional numbers as defined in Section 2 except for pressure which is nondimensionalized using \( \rho c^2 \). We get a similar set of nondimensional governing equations but end up with a single nondimensional number \( \beta = (8\pi \mu L)/(\rho A_0 c) \).

\[
\frac{\partial \alpha}{\partial \tau} + \frac{\partial q}{\partial \chi} = 0 \tag{30}
\]

\[
\frac{\partial q}{\partial \tau} + \frac{\partial}{\partial \chi} \left( q^2 \frac{\alpha}{\alpha} \right) + \alpha \frac{\partial p}{\partial \chi} + \beta q \frac{q}{\alpha} = 0 \tag{31}
\]

In its current form, this system of equations does not allow for specification of boundary conditions at both ends of the domain. We proceed by taking the spatial derivative of Eqn. 30 to get

\[
\frac{\partial^2 \alpha}{\partial \tau \partial \chi} + \frac{\partial^2 q}{\partial \chi^2} = 0 \tag{32}
\]

The nondimensional area \( \alpha \) is obtained from the EndoFLIP data and \( q \) is solved for implicitly (Poisson equation) assuming the boundary condition \( q = 0 \) at \( \chi = 0, 1 \). The momentum equation is then used to calculate \( p \) using the given values of \( \alpha \) and the newly computed \( q \) values. The boundary condition for \( p \) at \( \chi = 1 \) is obtained from the EndoFLIP’s distal pressure sensor and a zero pressure gradient condition is applied at the \( \chi = 0 \) boundary. Just like the continuity equation, the momentum equation is differentiated with respect to \( \chi \) and this equation is used to compute \( p \) implicitly.

\[
\frac{\partial^2 q}{\partial \chi \partial \tau} + \frac{\partial^2}{\partial \chi^2} \left( \frac{q^2}{\alpha} \right) + \frac{\partial}{\partial \chi} \left( \alpha \frac{\partial p}{\partial \chi} \right) + \beta \frac{\partial}{\partial \chi} \left( \frac{q}{\alpha} \right) = 0 \tag{33}
\]

One complication that needs to be addressed during this calculation is the presence of the catheter. The diameter readings reported by the planimetry sensors do not correspond to the area through which fluid flows. The fluid flows between the reported diameter values and the diameter of the catheter. Several possible approaches can be taken to
address this issue. We proceed by simply accounting for the catheter’s presence by subtracting the catheter’s diameter from the reported diameter values. A better approach would be to consider the flow to be annular and improve the mathematical formulation of the model by changing the viscous flow stress term to account for the inner (catheter) diameter. We aim to implement this change in a future version of this analysis tool.

![Fluid pressure variation at \( \chi = 0 \) and \( \chi = 1 \)](image1)

(a) Swallow 1 - pressure variations

![Work done by the eso vs. swallow time](image2)

(b) Swallow 1 - work vs. time curves

![Fluid pressure variation at \( \chi = 0 \) and \( \chi = 1 \)](image3)

(c) Swallow 2 - pressure variations

![Work done by the eso vs. swallow time](image4)

(d) Swallow 2 - work vs. time curves

Figure 13: Pressure and work computations from FLIP data

With all these steps in place, we first isolate a particular contraction wave that we wish to study and generate work curves for. An example of this is already shown in Fig. [2]. We choose a window of readings that cover a single pressure peak, for instance, readings from 50 to 150 are a suitable candidate. At the first reading (#50), the wave begins to travel over the tube length and results in a pressure rise. The wave is traveling as time goes on and by the last reading (#150) the wave has finished traveling over the entire tube length. In Fig. [13], we show the work curves and the pressures at \( \chi = 0 \).
(referred to as the proximal pressure) and $\chi = 1$ (referred to as the distal pressure) as a function of time for two typical swallows. Again, we emphasize that the former is predicted by our model and the latter is applied to the model to fix the pressure levels inside the tube.

The first thing to note from the analysis of the patient swallows is the pressure predicted by the model at the proximal (left) end of the tube at $\chi = 0$. When the wave is incoming, the pressure in the entire system rises, but once it passes over the left end, there is a sharp drop in pressure. This is due to the temporary displacement of fluid due to the peristaltic pumping action of the wave leading to the tube to be locally deflated at the left end. The displaced fluid then stretches the walls of the distal (right) end of the tube at $\chi = 1$ and this is marked by the continuous rise in pressure at that location. Once the wave has passed, the fluid accumulated at the end rushes back and the tube attains a uniform shape and the pressures at both locations become equal at the end of wave travel. This behavior of a proximal pressure trough in the middle of a swallow has also been observed in FLIP prototype devices where there is an additional pressure sensor at the proximal end.

The work curves shown in the right column of Fig. (13) are computed from the estimated fluid pressure and velocity fields. The curves show the same behavior observed in the curves derived from the 1D model i.e. the work done to change the kinetic energy of the fluid is minimal and the active work done is mostly lost via viscous dissipation. This is seen by the near overlap of the viscous curve and the curve showing the negative of active work done. The sign of each of the work curves simply indicates if the term is a source or a sink. The energy sinks are given a positive sign and the sources have a negative sign. Their sum should equal to zero (approximately) and that is what is observed in the work curves for both swallows.

The calculation of active and passive work done from the 1D model in Section 5.1 depends on the value of $\theta$ which is completely known. However, the activation strength is not available from real-world patient data, as such estimating the breakdown of the pressure work term into an active and passive component is difficult. To get around this hurdle, we use the FLIP data to get the tube’s reference area and the tube stiffness constant when the esophagus is fully relaxed. Once this information is obtained, the passive power can be estimated as follows,

$$P_{pass} = \int_0^L p_{pass} \frac{\partial A}{\partial t} \, dx = \int_0^L K \left( \frac{A}{A_0} - 1 \right) \frac{\partial A}{\partial t} \, dx.$$  \hspace{1cm} (34)

The active work is then simply estimated by subtracting the passive work from the total fluid pressure work. These estimated values of passive and active work are plotted in the work curves using dotted lines. The passive work is again seen to be smaller than the magnitudes of active and viscous work. One key observation from both these swallows is that the total magnitude of work done for a single healthy swallow ranges between $10^{-3}$ to $10^{-2}$ joules.

5.3 EndoFLIP work curves vs. axial coordinate

An alternate approach to analyzing work done by the esophagus involves computing each of the work terms as a function of the axial coordinate $x$, integrated over the entire
peristaltic pumping cycle. We begin with Eqn. 24 and integrate from time 0 to \( t \). After some simplifications, we obtain

\[
\frac{1}{2} \rho A u^2 \left|_t^t \right. + \int_0^t \frac{\partial}{\partial x} \left( \frac{1}{2} \rho Q u^2 \right) dt + \int_0^t \frac{\partial}{\partial t} (A u p) dt + \int_0^t \frac{\partial A}{\partial t} dt + \int_0^t \frac{8 \pi \mu u Q}{A} dt = 0. \tag{35}
\]

Using the continuity equation, this can be further simplified to give

\[
\frac{1}{2} \rho A u^2 \left|_t^t \right. + \int_0^t \frac{\partial}{\partial x} \left( \frac{1}{2} \rho Q u^2 \right) dt + \int_0^t A u \frac{\partial p}{\partial x} dt + \int_0^t \frac{8 \pi \mu u Q}{A} dt = 0. \tag{36}
\]

Unlike the previous approach where the zero velocity boundary conditions at each end significantly simplified the energy equation, in this approach, the boundary conditions cannot be applied in a straightforward manner for additional simplification. But initial conditions can be applied along with the fact that at the end of peristalsis, at time \( t \), the fluid velocity drops to zero so the first term in Eqn. 36 is zero for all locations. In Fig. 14, we plot the value of each of these terms for a typical contraction in a healthy individual. For the most part, the gradient in pressure is balanced by the gradient in flow rate. But when the contraction is significantly reducing the tube area, the viscous effects are prominent and the energy spent is substantial. This is evident by the peak observed for Term 4 along the midsection of the tube. The contraction reaches its full extent of occlusion at this point in the domain. At the tube ends, the variation in fluid velocity and associated pressure is quite low and this is clearly visible from the curves which show no activity at regions far away from the midsection. Although this approach of computing work does not reveal anything new, depending on the time interval chosen for analysis, one can estimate how much energy is stored per unit length of the esophagus.
6 Concluding remarks

In this work, we have analyzed a hitherto unstudied configuration of fluid flow in an elastic tube. A simple reduced-order model is presented that is able to predict fluid flow and the resultant tube wall deformation when a peristaltic wave passes over a closed cylindrical tube. The model has been compared to detailed three dimensional simulations and is shown to have satisfactory agreement with them. The system’s response under this set of operating conditions has been thoroughly quantified and visualized as a regime map.

Finally, the system’s utility is demonstrated by applying it to enhance the data collected by balloon dilation catheters. Paired with this tool, the device can now be thought of as having multiple pressure and velocity sensors housed on the catheter. With the help of these quantities, an appropriate mathematical foundation was laid to quantify the amount of peristaltic work done. This step has led us to estimate and confidently state the amount of work done by the esophagus during a healthy swallow under these operating conditions. The device can now be used to find work done in other swallows and comment on its capability to propel fluid.

One of the main drawbacks of the model is the simplistic treatment of the catheter’s presence on fluid flow. Another avenue of exploration is the effect of nonlinear tube laws on the value of work done during peristalsis. The esophagus’ material properties vary along its length as it approaches the stomach. The exact variation of these properties is unknown, so as a first step, we assumed a simple linear tube law to estimate work. Finally, the peristaltic activation wave does not have a constant velocity. The wave can speed up or even completely stop depending on the amount of obstruction it senses. In our configuration, this can become quite important as the ends are closed and continuous pumping will lead to a larger obstruction which can change the speed and intensity of contraction in patients which directly affects the work done by the wave. But as a first approximation, the model presented in this paper lays a great foundation on which these subsequent studies can be based and used to further our understanding on esophageal pumping processes.

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