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Data-driven approximation of a high fidelity gust-oriented flexible aircraft dynamical model

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Abstract: Computing responses to discrete gusts are sizing steps when designing and optimizing a new aircraft structure and geometry. Indeed, this is part of the imposed clearance certifications requested by the flight authorities. During the aircraft preliminary design phase, this clearance is done by intensive simulations, however, due to the involved models complexity, these latter are time consuming and imply an important computational burden. Especially as these simulations are involved at different steps of the aircraft optimisation process e.g. by aeroelastic, flight and control engineers. In this paper we propose a systematic way to fasten the gust simulation step and simplify the analysis by mean of data-driven model approximation in the Loewner framework. The proposed approach gathers recent advances in aeroelastic modelling and model approximation techniques. As illustrated on a high fidelity long range aircraft model, the drastic reduction of the simulation time does not induce any significant loss of accuracy.

Keywords: Model reduction, data-driven, flexible aircraft

1. INTRODUCTION

Motivations and contribution 1: within the aeronautical industry, the design and optimization of an aircraft structure is both a challenging and time consuming task for structural and aeronautical engineers. One of the major specification derived by the authorities before allowing commercial flight is dictated by its response to discrete gust disturbances, modeled by the so-called 1-cosine family gust signals (see e.g. Tang et al. (1996); Haghighat et al. (2012)). The latter represent a set of specific disturbances exciting the structure around a wide range of frequencies and resulting in an important stress, and varying loads affecting the wingspan. This stress is monitored to control the fatigue and to ensure both passenger safety and aircraft clearance in all flight situations. In addition to the clearance constraint, lightning aircraft structure stands as a major lever to meet aircraft consumption and overall traffic footprint impact reduction objectives. However, the clearance and the lightning objectives are somehow contradictory. Indeed, lightning renders the structure more flexible, leading to different dynamical effects blending 2 and might thus render the aircraft more sensitive to gust disturbances (see Poussot-Vassal et al. (2017)).

A challenge for aeronautical, structural and control engineers is then to reduce the overall weight while both minimizing the gust effect, and, in the same time, maintain the aircraft maneuverability (see Meyer et al. (2016)). To reach these challenges, a large amount of simulations is usually needed, thus simple but representative models are desirable. Nevertheless, since the involved models are usually highly complex, dedicated numerical software have to be used. Consequently, simulation time and computational burden tend to rise and are becoming a limiting factor. These effects are even more amplified when other engineering fields are involved (e.g. in control, analysis, flight mechanics etc.) as they may have even higher requirements concerning the complexity of the considered models.

In order to generate a suitable state-space aeroelastic model in the time-domain 3, several approaches have been used. For instance, Smith et al. (2004) provide a summary of the techniques available to generate an aeroelastic model in a state-space form. The most used rational approaches are the ones proposed by Roger (1977) and the Minimum-State Method proposed by Karpel (1982). They both introduce an approximation error due to the basis used to represent the frequency-domain data. The rational approaches have been commonly combined with the Mode Displacement Method (MDM) in order to recover the dynamic loads acting over the airframe Moulin and Karpel (2007). However, the MDM shows a lower convergence rate with an increasing number of structural modes than the Force Summation Method (FSM), see Pototzky and Perry III (1986). In order to avoid the convergence rate problems for the dynamic loads computation, a rational approach can be done on the FSM formulation (Pototzky and Perry III (1986)). Other authors apply the rational approach over the complete set of panels used for the aerodynamic discretisation (see Castrichini et al. (2017)), leading to a significant number of additional aero-dynamic lag states. Also, additional aspects have to be taken into account when applying the rational approach to a gust input.

1 This work has been funded within the frame of the Joint Technology Initiative JTl Clean Sky 2. AIRFRAME Integrated Technology Demonstrator platform “AIRFRAME ITD” (contract N CSJU-CS2-GAM-AIR-2014-15-01 Annex 1, Issue B04, October 2nd, 2015) being part of the Horizon 2020 research and Innovation framework programme of the European Commission.

2 Usually flight mechanics, loads, vibrations, etc. frequency bands are rather decoupled. When lighter aircraft are designed, these frequency ranges tend to overlap.

3 Note that time-domain formulation allows for the design of controllers for load alleviation or flutter suppression purposes. However, in this work, the control surfaces are not considered. They can be included in a straightforward way though.
due to exponential complex term in the frequency domain for the associated gust mode. The consideration of several gust modes can avoid such problems at the cost of an increasing number of states, see Wang and Chen (2017). Another approach to generate a state-space model of the aeroelastic systems is the Eigensystem Realization Algorithm (ERA), which has been applied to flexible structures (excluding rigid body modes) for the computation of generalized coordinates (see Kim et al. (2005); Silva (2007)). In this work the ERA has been extended to an aeroelastic system including rigid body modes and dynamic loads as system outputs. However and as stated later, special care must be taken when applying the algorithm to dynamic loads quantities.

In order to reduce the number of considered states in the time-domain representation of the aeroelastic system, we propose to approximate the HiFi (High Fidelity) model-based simulation results in gust cases (obtained by a complex and accurate simulation setup), by a reduced order LTI (Linear Time Invariant) dynamical model, which is both accurate and simple, and for which extensive simulation can be performed very efficiently. The approach combines different techniques from model construction, simulation, and data-driven model approximation with a stable low complexity LTI model. Moreover, the use-case involved in this paper, defined in Section 2, as well as some metrics computation functions will be rendered available as a benchmark for model approximation in a MATLAB format on the MOR Wiki.

Notations: in this paper we denote \( \mathbb{N} \), \( \mathbb{R} \), and \( \mathbb{C} \), the natural, real and complex values sets. The \( \mathcal{H}_2(\mathbb{C}^+) \) (or simply \( \mathcal{H}_2 \)) space denotes the set of complex-valued matrix functions \( \mathcal{H}(\cdot) \) analytic over \( \mathbb{C}_+ \) and which inner product integral is bounded along the imaginary axis, the \( \mathcal{H}_\infty(\mathbb{C}^+) \) (or simply \( \mathcal{H}_\infty \)) space, denotes the ones which supremum along the imaginary axis is bounded (the \( \mathcal{H}_2(\mathbb{C}^-) \) and \( \mathcal{H}_\infty(\mathbb{C}^-) \) spaces are similarly defined). In addition, \( RH_i \) denotes the sets of rational functions in \( \mathcal{H}_i \). Then \( \mathcal{L}_\infty(\mathbb{R}) = \mathcal{H}_\infty(\mathbb{C}^+) \oplus \mathcal{H}_\infty(\mathbb{C}^-) \) and \( \mathcal{L}_2(\mathbb{R}) = \mathcal{H}_2(\mathbb{C}^+) \oplus \mathcal{H}_2(\mathbb{C}^-) \). \( \mathcal{H}(S)(s) \) denotes the complex-valued transfer function, where \( s \) is the Laplace variable, and \( S = (E, A, B, C, D) \) its associated realization of dimension \( n \) which eigenvalues \( \lambda_i, i = 1, \ldots, n \), are the pencil or generalized eigenvalues of \( (E, A) \), and we denote \( \rho(E, A) = \mathbb{C} \cup \{ \infty \} \setminus \{ \lambda_i \} \) is the resolvent of this matrix pencil. The operator vec stacks the columns of a matrix underneath each other to form a single vector.

Structure of the paper: in Section 2, the aeroelastic structure and aerodynamical dynamic of the considered flexible aircraft benchmark are detailed. The way the data are collected is also described. Then, Section 3 describes the Loewner-based data-driven \( RH_\infty \) stable model approximation procedure used for this specific problem, allowing interpolating the data collected with a stable dynamical model. Then, Section 4 illustrates the approach on the considered benchmark and quantify the accuracy control and computational gain. Finally, conclusions and perspectives are discussed in Section 5.

2. LONGITUDINAL FLEXIBLE AIRCRAFT MODELING

Let us first describe the considered flexible aircraft model and the simulation setup used to obtain the frequency responses, denoted as data later on, on which the model will be constructed.

Structural, aerodynamic and aeroelastic models: a configuration based on typical long-range aircraft dimensions is used as reference. The lifting surfaces are defined by NACA0012 profiles. Geometrical and mass properties together with the first natural frequencies of the structure are available in Quero (2017). For the structural model six rigid-body and fifty flexible modes are taken into account. The flexible modes are normalized to yield an identity generalized mass matrix. Note that a very low frequency for the first flexible mode has been considered in order to investigate the applicability of the present approach to flexible configurations.

In this work Euler equations discretised over a grid containing approximately \( 1.5 \cdot 10^6 \) nodes are considered. The selected transonic flight point is at Mach number 0.84 and a steady angle of attack \( \alpha_s = 2 \) (deg) corresponding to a trimmed aircraft with total lift coefficient of 0.45 neglecting the effect of the horizontal tail plane on the pitch moment equilibrium. The steady pressure coefficient distribution over the surface as predicted by Computational Fluid Dynamics (CFD) is shown in Figure 1, illustrating a shock wave over the wing component. The corresponding freestream velocity and gust translation speed is \( U_\infty = 257.93 \) (m/s).

Now an aeroelastic model for perturbations around the non-linear steady state is obtained. The method is described by Quero (2017) and is based on the unsteady correction of the Aerodynamic Influence Coefficient (AIC) matrices for a set of correction modes. The modified AIC matrices are obtained by solving the linear least squares problem (1).

\[
(W^T(k) \otimes \bar{A})vec(x) = vec(P_Q(M_\infty, k, \alpha_s)) - vec(P_Q(\tilde{M_\infty}, k)) \tag{1}
\]

where the symbol \( \otimes \) denotes the Kronecker product and

\[
vec(x) = \begin{bmatrix}
\Delta Q_{11} \\
\sqrt{|Q_{11}|} \\
\Delta Q_{N_1,1} \\
\sqrt{|Q_{N_1,1}|} \\
\vdots \\
\Delta Q_{1N_p} \\
\sqrt{|Q_{1N_p}|} \\
\Delta Q_{N_1,N_p} \\
\sqrt{|Q_{N_1,N_p}|} \\
\Delta Q_{N_p,N_p}
\end{bmatrix}^T 	ag{2}
\]

The number of panels used for the aerodynamic surface discretization is denoted by \( N_p \) and the reduced frequency \( k \) is defined by \( k = \omega L_{ref}/U_\infty \), where \( \omega \) is the circular frequency and \( L_{ref} \) the reference length taken as 4.5 (m) for this configuration. The matrix \( W(k) \in \mathbb{C}^{N_p \times M} \) contains in columns the downwash distribution corresponding to a number \( M \) of correction modes, while the matrix \( P_Q(M_\infty, k, \alpha_s) \in \mathbb{C}^{N_p \times M} \) (where \( N_c \) represents the number of constrains imposed) contains the set of CFD data for the corresponding correction modes to be matched by the corrected AIC matrices and the matrix \( \bar{A} \) computes the corresponding aerodynamic quantities. In this application the local lift and local aerodynamic pitch

4 http://morwiki.mpi-magdeburg.mpg.de/morwiki/index.php

Fig. 1. \( M_\infty = 0.84 \) and \( \alpha_s = 2 \) (deg). CFD steady pressure distribution (Euler solution).
moment coefficients acting at the strips of the lifting components have been chosen as shown in Figure 2 and thus the number of constraints $N_y$ is twice the number of aerodynamic strips corresponding to the local pitch and local aerodynamic pitch coefficients. There, interpolation errors in the classical rational approach due to the gust mode vector $\phi$ has been neglected), $q_{\infty}$ the dynamic pressure, $D_{ij,k}$ and $D_{2jk}$ are substantial derivative matrices, $H_{gh}$ includes the structural modes retained. The matrices $Q_{g}$, $Q_{c,gh}$ and $Q_{c,bj}$ represent aerodynamic contributions. For a detailed explanation of these matrices see Quero (2017). The gust mode vector $\phi_0 \in \mathbb{C}^{p_y}$ in (5) is responsible for the interpolation errors in the classical rational approach due to the presence of the complex exponential term. There, $n_l$ is the local normal vector to the panel, $e_j$ the (vertical) gust direction and $x_j$ the panel x-coordinate defined at the 3/4 local chord point. The gust input $\overline{u}(\omega)$ is specified by the (nondimensional) gust profile at the aircraft nose and the set of outputs in $\overline{Y}(\omega) = \begin{bmatrix} u_h(\omega) & e_T(\omega) & c_m(\omega) \end{bmatrix}^T$ includes the time derivative of the rigid body coordinates, the generalized coordinates corresponding to the flexible modes and the local lift and local pitch aerodynamic coefficients acting over the strips. Note that the time derivative has been considered for the rigid body motion in order to avoid the heave motion to be periodic, which would cause the incremental vertical displacement to return to zero at the end of the simulation. Actually, due to the singularity at zero frequency in the matrix $Q_{c,bj}$ for the column corresponding to the heave motion, the vertical displacement of the aircraft at the end of the simulation is not equal to the initial one for a nonzero gust amplitude. Thus the term $i\omega$ in (4) performs the time derivative over the rigid body coordinates. The matrix $T_r$ selects the rows corresponding to the rigid body modes and the matrix $T_f$ selects the flexible ones.

### 3. DATA-DRIVEN MODEL APPROXIMATION IN THE $\mathcal{RH}_\infty$ SPACE

#### 3.1 Preliminaries in the data-driven approach

Due to irrational terms in the model description, simulation is performed. Then, based on the frequency-domain collected data, we are now ready to describe the proposed complete two-steps approximation process: the data-driven approximation and projection onto a stable subspace. Within the data-driven approximation framework, we are given

$$\{s_i, \Phi_i\}_{i=1}^{N_x},$$

a set of frequency-domain (or complex-domain) $u_x$ inputs, $u_y$ outputs data $\Phi_i \in \mathbb{C}^{n_y \times n_u}$ collected at varying frequencies $s_i \in \mathbb{C}$ either from experimental measurements or any numerical simulation. These data satisfy, for $i = 1, \ldots, N_x$, $\overline{Y}(s_i) = G(s_i)\overline{U}(s_i) = \Phi_i\overline{U}(s_i)$, where $\overline{U}(s) \in \mathbb{C}^{n_x}$, $\overline{Y}(s) \in \mathbb{C}^{n_y}$ respectively are the Laplace transform values of the inputs $u(t) \in \mathbb{R}^{n_x}$ and outputs $y(t) \in \mathbb{R}^{n_y}$, evaluated at $s_i$. In this setting, the objective is to find an LTI dynamical model represented by a realization of “complexity” $n \in \mathbb{N}$.

$$\mathcal{S}(t) = \mathcal{A}\mathcal{X}(t) + \mathcal{B}\mathcal{U}(t), \quad \mathcal{Y}(t) = \mathcal{C}\mathcal{X}(t) + \mathcal{D}\mathcal{U}(t)$$

where $\mathcal{X}(t) \in \mathbb{R}^n$ denotes the internal variables and where $E, A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n_u}$, $C \in \mathbb{R}^{n_y \times n}$ and $D \in \mathbb{R}^{n_y \times n_u}$, are constant matrices, for which the frequency response of the model (7) well reproduces the data set $\{s_i, \Phi_i\}_{i=1}^{N_x}$. Obviously, realization (7) is non unique, then for $n_u, n_y, n_y \in \mathbb{N}$, by denoting $S_{n_u, n_y, n_y} \leftrightarrow \begin{bmatrix} \mathbb{R}^{n \times n} \mathbb{R}^{n \times n_u} \mathbb{R}^{n \times n_u} \mathbb{R}^{n_y \times n_y} \mathbb{R}^{n_y \times n} \mathbb{R}^{n_y \times n_y} \end{bmatrix}$, the set of all $n_y \times n_u$ realizations of dimension $n$, if the matrix pencil $(E, A)$ is regular / non-singular for some finite $\lambda \in \mathbb{C}$,

$$\mathcal{H}(\lambda) : \rho(E, A),$$

is the transfer function associated to $\mathcal{S} : (E, A, B, C, D) \in S_{n_u, n_y, n_y}$ and $\rho(E, A)$ the resolvent of the matrix pencil $(E, A)$. The set of all realizations of $\mathcal{H}(\lambda)$ is denoted $\mathcal{S}(\mathcal{H}(\lambda))$. This operator will play an important role in the second part of the paper to ensure model stability.

#### 3.2 Preliminaries on data-driven ERA

In the case the $E$ matrix is invertible, the Eigensystem Realization Algorithm (ERA) can also be applied. Thus the first

5 Note that when the data $\Phi_i$ are collected from experimental or HiFi software simulation, one usually selects $s_i = \omega_i$, where $\omega_i \in \mathbb{R}$, is the pulsation of the experiment. The case where $s_i \in \mathbb{C}$ is involved in some cases where the model is analytically known. This is rarely the case when complex simulator enters in the picture.

6 $x(t) \in \mathbb{R}^n$ are the state variables if $E$ is invertible.

7 One should note that the above objective is at the boundaries with the traditional model identification problem. However here, an interpolatory framework rather than a norm minimization is sought.
equation in (7) can be replaced by \( \dot{x}(t) = Ax(t) + Bu(t) \) after premultiplying by the matrix \( E^{-1} \). The ERA is then applied over the discrete-time version of the system, \( \bar{x}(k + 1) = A_{\bar{x}} \bar{x}(k) + B_{\bar{u}} \bar{u}(k) \) and \( y(k) = C_{\bar{x}} \bar{x}(k) + D_{\bar{u}} \bar{u}(k) \), where \( k = 1, \ldots, N_t \) with \( N_t \) the number of time samples. The ERA algorithm is based on the construction of the generalized Hankel matrix containing the discrete-time impulse responses for all input/output combinations and the later singular value decomposition to obtain the state-space representation, see Silva (2007). A conversion from the discrete to continuous system allows the final state-space formulation.

In this work the aerodynamic coefficients acting over the component strips are considered in the frequency-domain data, imposing a difficulty in the construction of the impulse responses for the ERA, which is believed to be caused by the added-mass aerodynamic term at increasing frequencies. In order to consider aerodynamic forces in a state-space form where the matrix \( E \) is invertible different approaches which explicitly deal with the existence of the added mass term have been presented, see Brunton et al. (2014). In this work, instead of physically identifying the different contributions, a more general approach is intended by dividing the transfer function data provided in the frequency domain by the term \( \omega \). Consequently, the \( \tau \) term has to be multiplied by \( \omega \), causing the input to the state-space model obtained by the ERA to be the time derivative of the original input. This introduces a limitation when applying the ERA algorithm to transfer functions including the added-mass term, as the original input variables for controller design and not the time derivatives are sought. Additionally, different subsystem blocks had to be considered in order for the ERA to provide a proper state-space model, leading to an increasing number of states for the free flexible aircraft model. In Section 4, the performance of the (below) proposed method is compared to that obtained with the ERA. Note that the following Loewner-based approach will allow to get rig of ERA limitations.

3.3 Data-driven approximation in the Loewner framework

Let us now assume that data (6) are splitted into two subsets: more specifically, \( s_i \) into left \( \{\mu_j\}_{j=1}^q \) and right \( \{\lambda_i\}_{i=1}^k \) and \( \Phi_i \) into left \( \{v_j\}_{j=1}^q \) and right \( \{w_i\}_{i=1}^k \). These data are completed with tangential directions \( \{\bar{r}_j\}_{j=1}^q \) and \( \{\bar{r}_i\}_{i=1}^k \) arbitrarily chosen.

The Loewner matrices offer a versatile and compliant framework to deal with frequency-domain data, by seeking for a realization interpolating these data, in the barycentric sense. More specifically, let us be given left interpolation driving frequencies \( \{\mu_j\}_{j=1}^q \in \mathbb{C} \) with left output or tangential directions \( \{l_j\}_{j=1}^q \in \mathbb{C}^{n_u} \), producing the left responses \( \{v_j\}_{j=1}^q \in \mathbb{C}^{n_y} \) and right interpolation driving frequencies \( \{\lambda_i\}_{i=1}^k \in \mathbb{C} \) with right input or tangential directions \( \{r_i\}_{i=1}^k \in \mathbb{C}^{n_y} \), producing the right responses \( \{w_i\}_{i=1}^k \in \mathbb{C}^{n_u} \), one aims at finding a realization \( \mathcal{S} \) such that the resulting transfer function \( \mathcal{H}(\mathcal{S}) \) is a tangential interpolant of the data, i.e. satisfies the following left and right interpolation conditions:

\[
\begin{cases}
\mathcal{H}(\mathcal{S})(\mu_j) = v_j^* l_j^* \Phi_j, & \text{for } j = 1, \ldots, q, \\
\mathcal{H}(\mathcal{S})(\lambda_i) = w_i = \Phi_i r_i, & \text{for } i = 1, \ldots, k.
\end{cases}
\]

(9)

The main ingredient to achieve (9) is the Loewner matrix, which was developed in a series of papers (see e.g. Mayo and Antoulas (2007); Antoulas et al. (2016)). In the sequel, we recall the main steps. Let be given, the left (or row) data and the right (or column) data:

\[
\{\mu_j, l_j^*, v_j^*\}_{j=1}^q \quad \text{and} \quad \{\lambda_i, r_i, w_i\}_{i=1}^k.
\]

(10)

Moreover, let us assume that \( \lambda_i \) and \( \mu_j \) are distinct, then the associated Loewner \( \mathcal{L} \in \mathbb{C}^{q \times k} \) and shifted Loewner \( \mathcal{L}_\sigma \) matrices, also referred to as the Loewner pencil, are constructed as follows, for \( i = 1, \ldots, q \):

\[
[\mathcal{L}]_{i,j} = \frac{v_i^* r_j - \lambda_i^* w_j}{\mu_j - \lambda_i}, \quad [\mathcal{L}_\sigma]_{i,j} = \frac{\mu_j v_i^* r_j - \lambda_i^* w_j}{\mu_j - \lambda_i}.
\]

(11)

Then, by organizing the left and right interpolation data as:

\[
\mathbf{M} = \text{diag}(\{\mu_1, \ldots, \mu_q\}), \quad \mathbf{L}^* = \begin{bmatrix} l_1 & \cdots & l_q \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} r_1 & \cdots & r_k \end{bmatrix}, \quad \mathbf{V}^* = \begin{bmatrix} v_1 & \cdots & v_q \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} w_1 & \cdots & w_k \end{bmatrix}
\]

(12)

the Loewner \( \mathcal{L} \) and shifted Loewner \( \mathcal{L}_\sigma \) matrices (11) satisfy the following Sylvester equations:

\[
\begin{align*}
\mathbf{M} \mathbf{L}^* &= \mathbf{L}^* \mathbf{R}, \\
\mathbf{L}^* \mathbf{R} &= \mathbf{L}^* \mathbf{L} \mathbf{R} = \mathbf{L} \mathbf{L} \mathbf{R} = \mathbf{L} \mathbf{W} \mathbf{\Lambda} \\
\end{align*}
\]

with \( \mathbf{\Sigma} = \mathbf{L} \mathbf{W} \mathbf{\Lambda} \). Consequently, following the main results of Mayo and Antoulas (2007), given the right and left interpolation data as in (7), and assuming that \( k = q \) and \( (\mathbf{L}, \mathbf{L}_\sigma) \) is a regular pencil where \( \lambda_i \) or \( \mu_j \) are not eigenvalues, the rational transfer function \( \mathcal{H}(\mathcal{S})(s) = C(s\mathbf{E} - \mathbf{A})^{-1} \mathbf{B} \) with realization \( \mathcal{S} : (\mathbf{E}, \mathbf{A}, \mathbf{B}, \mathbf{C}, 0) \) constructed as:

\[
\mathbf{E} = -\mathbf{L}, \quad \mathbf{A} = -\mathbf{L}_\sigma, \quad \mathbf{B} = \mathbf{V} \quad \text{and} \quad \mathbf{C} = \mathbf{W},
\]

(13)

is a minimal descriptor realization which interpolates the left and right constraints, i.e. ensures (5).

Now the interpolatory framework, connected to the Loewner one has been reminded, it is obvious that, starting from data (10) either obtained from experiments (see e.g. Poussot-Vassal et al. (2017); Meyer et al. (2016)) or simulations performed on a complex model, it is simple to construct a rational interpolant \( \mathcal{S} \) and its associated transfer function \( \mathcal{H}(\mathcal{S}) \) which exactly interpolates the data. One additional possibility embedded in this framework is that it is possible, at reduced numerical cost, to obtain a \( r \)-th \((r \leq n)\) order approximated model. This is obtained through a rank revealing matrix factorisation such as:

\[
\mathbf{L} = \begin{bmatrix} \mathbf{Y}_1 & \mathbf{Y}_2 \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_2 \end{bmatrix} \begin{bmatrix} \mathbf{X}_1^* \\ \mathbf{X}_2^* \end{bmatrix},
\]

(14)

where \( \mathbf{\Sigma}_1, \mathbf{\Sigma}_2 \in \mathbb{R}^{n \times n} \), \( \mathbf{Y}_1, \mathbf{Y}_2, \mathbf{X}_1, \mathbf{X}_2 \) are of appropriate dimensions. The \( r \)-th order reduced order model with realization \( \mathcal{S} \) is obtained by Petrov-Galerkin projection:

\[
\mathbf{S} = (\mathbf{Y}_1^* \mathbf{L}_\mathbf{X}_1 - \mathbf{Y}_1^* \mathbf{L}_\mathbf{X}_1, \mathbf{Y}_1^* \mathbf{V}, \mathbf{W}_\mathbf{X}_1, 0).
\]

This later reduce-order model will be exploited in Section 4 using the data collected on the HiFi simulator to derive an approximate model with monitored mismatch error.

3.4 Optimal \( \mathcal{R}\mathcal{H}_\infty \) approximation

While the Loewner ensures some interpolatory properties, it does not provide any guarantee concerning the stability of the resulting matrix pencil \( (\mathbf{E}, \mathbf{A}) \). In the perspective of time-domain simulation for aircraft load computations and response to gust disturbance, this is a major issue. Indeed, authorities criterion are based on discrete time-domain gust simulations rather than frequency ones. Therefore, a post-treatment should be applied to approximate the (often unstable) Loewner-based interpolant by a stable one. To this aim, the recent work of Kohler (2014) is used to project a rational unstable model onto its best stable approximant. Mathematically, let the obtained Loewner-based transfer \( \mathcal{H}(\mathcal{S}) \in \mathcal{R}\mathcal{L}_\infty \) or \( \mathcal{R}\mathcal{L}_2 \), the
objective is to construct \( H(\hat{S}) \in \mathcal{RH}_\infty \) or \( \mathcal{RH}_2 \). In the case where a MIMO LTI continuous-time dynamical system can be represented by a first order descriptor realization \( S : (E, A, B, C, D) \) with \( n_u \) inputs, \( n_y \) outputs and \( n \) internal variables, the model is given by a set of DAE as in (7). Similarly to \( S_{n,n,n,a} \), one can define the following sets: 

\[ S_{0,n,n,a}^0 \triangleq \{(E, A, B, C, D) \in S_{n,n,a} | \mathbb{R} \subset \rho(E, A)\}, \]

\[ S_{n,n,a}^+ \triangleq \{(E, A, B, C, D) \in S_{n,n,a} | \mathbb{C}_{\geq 0} \subset \rho(E, A)\} \]

and 

\[ S_{n,n,a}^- \triangleq \{(E, A, B, C, D) \in S_{n,n,a} | \mathbb{C}_{\leq 0} \subset \rho(E, A)\} \]

where \( S_{n,n,a}^+ \) and \( S_{n,n,a}^- \) are sets of stable and anti-stable realization-based systems, respectively. Mathematically, given \( p = [2, \infty] \) and a realization \( S \in S_{0,n,n,a} \), let \( H(S) \in \mathcal{RL}_\infty \) or \( \mathcal{RL}_2 \), our aim is finding \( \hat{S} \in S_{n,n,n,a} \) such that:

\[
\|H(S) - H(\hat{S})\|_{\mathcal{H}_p} \leq \inf_{\hat{G} \in \mathcal{H}_p} \|H(S) - H(\hat{G})\|_{\mathcal{L}_p}.
\]

(15)

Due to the orthogonality of the \( L_2 \) space decomposition from its stable and anti-stable part, the case where \( p = 2 \) is straightforwardly solved by simply taking the stable part of \( H(S) \). However, in that case, the obtained model is no longer of dimension \( n \), but \( \tilde{n} < n \). In Kohler (2014), the case where \( p = \infty \) is solved. By reminding that \( \mathcal{L}_\infty(\mathbb{R}) = \mathcal{H}_\infty(\mathbb{C}_+) \oplus \mathcal{H}_\infty(\mathbb{C}_-) \), one can similarly write \( \hat{S} = S_+ \oplus S_- \), where \( S_+ \in S_{n,n,n,a}^+ \), and \( S_- = (E_-, A_-, B_-, C_-, D_-) \in S_{n,n,n,a}^- \) (see Theorem 1), we are now ready to apply it on the considered data generated by the \texttt{HiFi} simulation of the flexible aircraft structure, presented in Section 2. Based on the data collected in Section 2, the Loewner matrices are constructed. Computing the normalized singular values of \( L \), one obtains the singular value decay (not illustrated here space limitations). The rank revealing factorization shows that a perfect match is obtained with \( n = 321 \), while more than 420 measurements are used. Constructing \( H(S) \) with realization \( S \in S_{n,n,n,a} \) leads to a perfect matching, but still, with an unusable model (more that 40 unstable eigenvalues). Then, based on the unstable realization, the stability enforcement in the \( \mathcal{RH}_\infty \) sense, is applied. Then, Figure 3 illustrates the resulting mean and max relative mismatch errors as a function of the approximation order. These errors are the \( L_\infty \) and \( L_2 \)-norms of the relative mismatch computed between the stable interpolated model \( H(\tilde{S})(s_i) \) for varying order \( n \) and the original data \( \Phi_i \). For \( n = 100 \), a mean relative mismatch of 1% is achieved. This error drops around 0.1% for \( n = 250 \).

4. APPLICATION AND PERFORMANCE EVALUATIONS

Following Section 3, let us now summarize the procedure. It has been clarified that from any frequency-domain measurement set given as (6), one can obtain an exact or approximate interpolant model \( H(S) \) with realization \( S \in S_{n,n,n,a} \) of minimal McMillian degree thanks to the Loewner procedure. Then, in view of time-domain gust simulation, based on the descriptor realization \( S \), the optimal projection onto the \( \mathcal{RH}_\infty \) space, denoted \( \hat{S} \in S_{n,n,n,a} \), can be obtained by reasonable algebraic manipulations computations (see Theorem 1).

\[
\begin{array}{c}
\textbf{Maximal mismatch} \\
\textbf{Mean mismatch}
\end{array}
\]

Fig. 3. Point-wise relative mismatch error in both the \( L_\infty \) (blue rounds) and \( L_2 \) (red triangles) sense as a function of the approximation order \( n \).

Since the considered data have \( n_u = 1 \) inputs and \( n_y = 92 \) outputs, located at different points of the aircraft and wing and representing different measurements (load, moments, etc), it is complicated to illustrate. However, Figure 4 shows the frequency responses along the right wing span of the \texttt{HiFi} collected data \( \Phi \), and the model \( H(\tilde{S}) \), for the exact case where \( n = 321 \). It clearly shows a very good reproduction / interpolation of the data, by the stable model. Finally, Figure 5 illustrates the time response of output \#1, obtained for varying gust input (left) with the \texttt{HiFi} simulator and the approximated one of dimension \( n = 100 \), showing a very good accuracy.

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8 This is the type of model obtained by the Loewner framework.

9 Note that in both \( p = 2 \) and \( p = \infty \) cases one does not consider any eigenvalue on the imaginary axis.

\[ ||S||_{\mathcal{H}_p} = \inf_{\hat{G} \in \mathcal{H}_p} \|H(S) - H(\hat{G})\|_{\mathcal{H}_p} \]
In this paper we have presented an aeroelastic benchmark representing a flexible aircraft model which frequency response data have been obtained from a HiFi simulator. Then, as rooted on the Loewner framework, followed with a dedicated post-processing treatment, ensuring model stability, we obtain a simple but yet accurate model of this complex aircraft use-case, which can be used in place of the original large-scale one for optimisation and intensive simulation. Future works, held in this European project, will include additional inputs of the model (e.g. control surfaces) and a parametrisation of the data as a function of some geometrical data and flight configurations.

5. CONCLUSION

In this paper we have presented an aeroelastic benchmark representing a flexible aircraft model which frequency response data have been obtained from a HiFi simulator. Then, as rooted on the Loewner framework, followed with a dedicated post-processing treatment, ensuring model stability, we obtain a simple but yet accurate model of this complex aircraft use-case, which can be used in place of the original large-scale one for optimisation and intensive simulation. Future works, held in this European project, will include additional inputs of the model (e.g. control surfaces) and a parametrisation of the data as a function of some geometrical data and flight configurations.

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