On the Koide-like Relations for the Running Masses of Charged Leptons, Neutrinos and Quarks

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Abstract

Current experimental data indicate that the Koide relation for the pole masses of charged leptons, which can be parametrized as $Q_{l}^{\text{pole}} = 2/3$, is valid up to the accuracy of $O(10^{-5})$. We show that the running masses of charged leptons fail in satisfying the Koide relation (i.e., $Q_{l}(\mu) \neq 2/3$), but the discrepancy between $Q_{l}(\mu)$ and $Q_{l}^{\text{pole}}$ is only about 0.2% at $\mu = M_Z$. The Koide-like relations for the running masses of neutrinos ($1/3 < Q_{\nu}(M_Z) < 0.6$), up-type quarks ($Q_{U}(M_Z) \sim 0.89$) and down-type quarks ($Q_{D}(M_Z) \sim 0.74$) are also examined from $M_Z$ up to the typical seesaw scale $M_R \sim 10^{14}$ GeV, and they are found to be nearly stable against radiative corrections. The approximate stability of $Q_{U}(\mu)$ and $Q_{D}(\mu)$ is mainly attributed to the strong mass hierarchy of quarks, while that of $Q_{l}(\mu)$ and $Q_{\nu}(\mu)$ is essentially for the reason that the lepton mass ratios are rather insensitive to radiative corrections.

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Thanks to the precise measurements of three charged lepton masses [1],

\[
\begin{align*}
m_e &= (0.510998918 \pm 0.000000044) \text{ MeV}, \\
m_\mu &= (105.6583692 \pm 0.0000094) \text{ MeV}, \\
m_\tau &= (1776.99^{+0.29}_{-0.26}) \text{ MeV},
\end{align*}
\]

the empirical Koide mass relation [2]

\[
Q_{l}^{\text{pole}} = \left( \frac{m_e + m_\mu + m_\tau}{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}} \right)^2 = \frac{2}{3} \tag{2}
\]

has been testified up to the accuracy of \(O(10^{-5})\). Namely, the above experimental data yield \(-0.00001 \leq Q_{l}^{\text{pole}} - 2/3 \leq +0.00002\). This precision is so amazing that there might exist some underlying physics behind the Koide relation. A number of authors have tried to understand Eq. (2) and extend it to neutrino masses or quark masses based on possible flavor symmetries and phenomenological conjectures [3–6]. It is certainly impossible to get a universal Koide relation for both charged fermions and neutrinos by using the experimental data at low energy scales, nor is it likely to achieve such a mass relation at any high energy scales. In this category of exercises, one has to take care of the concepts of fermion masses and clarify the running mass behaviors from one energy scale to another.

Note that Eq. (2) holds for the pole masses of three charged leptons, which have been denoted as \(m_e, m_\mu\), and \(m_\tau\). Is it also applicable to the running masses \(m_e(\mu), m_\mu(\mu)\), and \(m_\tau(\mu)\) at a given energy scale (e.g., \(\mu = M_Z\))? We are going to answer this question both analytically and numerically.

If the Koide relation is not a universal relation for fermion masses at low energy scales, one may wonder whether a universal Koide-like relation is achievable at a superhigh energy scale. We shall show that the answer to this question is negative for both charged fermions and neutrinos, but the dynamical reasons are somehow different in these two cases.

Let us define the following Koide-like parameters for the running masses of charged leptons \((f = l)\), up-type quarks \((f = U)\), down-type quarks \((f = D)\) and neutrinos \((f = \nu)\):

\[
Q_f(\mu) = \left( \frac{m_x(\mu) + m_y(\mu) + m_z(\mu)}{\sqrt{m_x(\mu)} + \sqrt{m_y(\mu)} + \sqrt{m_z(\mu)}} \right)^2 , \tag{3}
\]

where

\[
(x, y, z) = \begin{cases} 
(e, \mu, \tau), & \text{for } f = l; \\
(1, 2, 3), & \text{for } f = \nu; \\
(u, c, t), & \text{for } f = U; \\
(d, s, b), & \text{for } f = D. 
\end{cases}
\]

As pointed out by Gérard et al [6], \(Q_f = 1\) (maximum) would hold when three fermion masses had the extremely hierarchical spectrum (e.g., \(m_x \ll m_y \ll m_z\)), while \(Q_f = 1/3\) (minimum) would occur if three fermion masses were exactly degenerate (i.e., \(m_x = m_y = m_z\)). It might be a puzzle that \(Q_{l}^{\text{pole}}\) happens to take the mean value of \(Q_f^{\text{min}}\) and \(Q_f^{\text{max}}\). The main purpose
of this paper is just to examine the deviations of realistic $Q_l$, $Q_\nu$, $Q_U$ and $Q_D$ from $Q_l^{\text{pole}}$ at and above the electroweak scale.

First of all, let us calculate the discrepancy between $Q_l(M_Z)$ and $Q_l^{\text{pole}}$. The running masses of three charged leptons $m_l(\mu)$ are related with their corresponding pole masses $m_l$ in a very simple way [8],

$$m_l(\mu) = m_l(1 - \Delta_l) ,$$

where

$$\Delta_l = \frac{\alpha(\mu)}{\pi} \left[ \frac{3}{2} \ln \frac{\mu}{m_l(\mu)} + 1 \right]$$

with $\alpha(\mu)$ being the fine-structure constant at the energy scale $\mu$. It is then straightforward to establish the relationship between $Q_l(\mu)$ and $Q_l^{\text{pole}}$ by combining Eqs. (3) and (5). Taking account of the strong mass hierarchy $m_e \ll m_\mu \ll m_\tau$ (more explicitly, $m_e/m_\mu \approx 0.00484$ and $m_\mu/m_\tau \approx 0.0595$ [1]), we find that it is more instructive to make the following analytical approximation:

$$Q_l(\mu) \approx Q_l^{\text{pole}} \left[ 1 + \sqrt{m_\mu/m_\tau} (\Delta_\mu - \Delta_\tau) \right] .$$

One can clearly see that the deviation of $Q_l(\mu)$ from $Q_l^{\text{pole}}$ is strongly suppressed, due to the smallness of $\sqrt{m_\mu/m_\tau}$ and that of $\Delta_l$.

At $\mu = M_Z$ with $\alpha(M_Z) = (128.89)^{-1}$ [9], the running masses of three charged leptons can be evaluated by solving Eq. (5) with the inputs of the pole masses given in Eq. (1). For our purpose, we only need to make use of the central values of $m_e$, $m_\mu$ and $m_\tau$ in the calculation. The results are

$$m_e(M_Z) = 0.486755106 \text{ MeV} ,$$
$$m_\mu(M_Z) = 102.740394 \text{ MeV} ,$$
$$m_\tau(M_Z) = 1746.56 \text{ MeV} .$$

With the help of Eq. (6), we obtain $\Delta_\mu = 0.0276$ and $\Delta_\tau = 0.0171$ at $M_Z$. Therefore,

$$Q_l(M_Z) = 1.00188 \times Q_l^{\text{pole}} = 0.66792 ,$$

achieved from Eqs. (3) and (5). If the analytical approximation made in Eq. (7) is used to estimate $Q_l(M_Z)$, one can get $Q_l(M_Z) \approx 1.00256 \times Q_l^{\text{pole}} = 0.66837$. This result is compatible with Eq. (9) and implies that Eq. (7) is actually a reasonable approximation.

Obviously, the discrepancy between $Q_l(M_Z)$ and $Q_l^{\text{pole}}$ is only about 0.2%. This tiny difference makes sense in physics, because it is much larger than the accuracy of Eq. (2). In other words, the possibility of $Q_l(M_Z) = 2/3$, which inversely leads to $Q_l^{\text{pole}} - 2/3 = -0.00188$, has been ruled out by the experimental data. We are therefore left with the conclusion that the running and pole masses of three charged leptons cannot simultaneously satisfy the Koide relation.
Now let us turn to the Koide-like relations of quark masses. We concentrate on the running masses of six quarks at $\mu = M_Z$ [10],

$$
\begin{align*}
m_u(M_Z) &= (1.7 \pm 0.4) \text{ MeV}, \\
m_c(M_Z) &= (0.62 \pm 0.03) \text{ GeV}, \\
m_t(M_Z) &= (171 \pm 3) \text{ GeV};
\end{align*}
$$

and

$$
\begin{align*}
m_d(M_Z) &= (3.0 \pm 0.6) \text{ MeV}, \\
m_s(M_Z) &= (54 \pm 11) \text{ MeV}, \\
m_b(M_Z) &= (2.87 \pm 0.03) \text{ GeV}.
\end{align*}
$$

The Koide-like parameters $Q_U$ and $Q_D$ defined in Eq. (3) can then be calculated at $M_Z$ with the help of Eqs. (10) and (11). For simplicity, only the central values of those quark masses are taken into account. The results are

$$
Q_U(M_Z) \approx 0.89, \quad Q_D(M_Z) \approx 0.74.
$$

We see that both numbers significantly deviate from $2/3$ at the electroweak scale.

To examine whether the Koide-like relation $Q_U(M_X) \approx Q_D(M_X) \approx 2/3$ could hold at a superhigh energy scale $M_X$ (e.g., $M_X \sim 10^{14-16}$ GeV), one may make use of the one-loop renormalization-group equations (RGEs) of quark Yukawa couplings [12] in the standard model (SM) or in the minimal supersymmetric standard model (MSSM). Note that the strong hierarchy of charged fermion masses and that of the quark mixing angles allow us to simplify those RGEs to a great extent [13]. In particular, the RGE evolution of $m_u/m_c$, $m_d/m_s$ and $m_c/m_t$ from $M_Z$ to $M_X$ are negligibly small in both the SM and the MSSM. Radiative corrections to the mass ratios $m_c/m_t$, $m_s/m_b$ and $m_\mu/m_\tau$ can be written as

$$
\begin{align*}
\frac{m_c(M_X)}{m_t(M_X)} &\approx \frac{m_c(M_Z)}{m_t(M_Z)} \chi_U, \\
\frac{m_s(M_X)}{m_b(M_X)} &\approx \frac{m_s(M_Z)}{m_b(M_Z)} \chi_D, \\
\frac{m_\mu(M_X)}{m_\tau(M_X)} &\approx \frac{m_\mu(M_Z)}{m_\tau(M_Z)} \chi_l,
\end{align*}
$$

where the evolution functions $\chi_U$, $\chi_D$ and $\chi_l$ are defined by

$$
\chi_f \equiv \exp \left[ \frac{1}{16\pi^2} \int_{\ln M_Z}^{\ln M_X} \left( a_f y_t^2 + b_f y_b^2 + c_f y_\tau^2 \right) dt \right].
$$

We do not consider the pole masses of six quarks, because they are neither directly measurable nor relevant for model building. In particular, the pole masses of three light quarks ($u, d, s$) involve large uncertainties, since they can only be evaluated in the region with a large $\alpha_s(\mu)$ [11].
Finally, we consider the Koide-like relation in the neutrino sector. Although the pole mass hierarchies of charged fermions and (ii) the small departure of radiative corrections in both the SM and the MSSM. In contrast, \( \chi_U \) may significantly depart from 1 when \( M_X \gg M_Z \) holds, especially for sizable \( \tan \beta \) in the MSSM. Note that \( \chi_D \gg 1 \) holds in the SM, as a consequence of \( a_D > 0 \) in Eq. (14).

With the help of \( \chi_f \) (for \( f = U, D, l \)) given above, it is easy to derive the Koide-like parameter \( Q_f(M_X) \) in terms of \( Q_f(M_Z) \). We approximately obtain

\[
Q_{U}(M_X) \approx Q_{U}(M_Z) \left[ 1 + 2 \left( 1 - \sqrt{\chi_U} \right) \frac{m_c(M_Z)}{m_t(M_Z)} \right],
\]

\[
Q_{D}(M_X) \approx Q_{D}(M_Z) \left[ 1 + 2 \left( 1 - \sqrt{\chi_D} \right) \frac{m_s(M_Z)}{m_b(M_Z)} \right],
\]

\[
Q_{l}(M_X) \approx Q_{l}(M_Z) \left[ 1 + 2 \left( 1 - \sqrt{\chi_l} \right) \frac{m_{\mu}(M_Z)}{m_{\tau}(M_Z)} \right].
\]

One can see that the deviation of \( Q_f(M_X) \) from \( Q_f(M_Z) \) is significantly suppressed, because of (i) the strong mass hierarchies of charged fermions and (ii) the small departure of \( \chi_f \) from 1. Indeed, \( \sqrt{m_c/m_t} \approx 0.06, \sqrt{m_s/m_b} \approx 0.14 \) and \( \sqrt{m_{\mu}/m_{\tau}} \approx 0.24 \) at \( M_Z \). Taking \( M_X = M_R \sim 10^{14} \) GeV (the typical scale of heavy right-handed Majorana neutrinos in the seesaw models [14]), for example, we get \( \sqrt{\chi_U} \approx 0.97, \sqrt{\chi_D} \approx 1.03 \) and \( \sqrt{\chi_l} \approx 1.00 \) in the SM; or \( \sqrt{\chi_U} \approx 0.86, \sqrt{\chi_D} \approx 0.91 \) and \( \sqrt{\chi_l} \approx 0.98 \) in the MSSM with \( \tan \beta = 50 \). Therefore,

\[
Q_f(M_R) - Q_f(M_Z) \approx \begin{cases} 
3.3 \times 10^{-3}, & \text{for } f = U, \\
-6.5 \times 10^{-3}, & \text{for } f = D, \\
0, & \text{for } f = l
\end{cases}
\]

(16)
in the SM; or \(^2\)

\[
Q_f(M_R) - Q_f(M_Z) \approx \begin{cases} 
1.4 \times 10^{-2}, & \text{for } f = U, \\
1.8 \times 10^{-2}, & \text{for } f = D, \\
4.8 \times 10^{-3}, & \text{for } f = l
\end{cases}
\]

(17)
in the MSSM with \( \tan \beta = 50 \). It is obvious that radiative corrections to the Koide-like parameters are very small.

\(^4\)Finally, we consider the Koide-like relation in the neutrino sector. Although the pole masses of three neutrinos are different from their running masses at \( \mu = M_Z \), this difference

\(^2\)The deviation of \( Q_f(M_R) \) from \( Q_f(M_Z) \) in the MSSM will become much milder, if \( \tan \beta \) takes smaller values (\( \tan \beta = 10 \) shown in Fig. 1(b), for example).
is negligibly tiny because it is strongly suppressed by the Fermi coupling constant. For simplicity, we mainly calculate the Koide-like parameter $Q_\nu(M_Z)$ and examine its sensitivity to radiative corrections from $M_Z$ up to the seesaw scale $M_R$.

Note that the absolute values of three neutrino masses remain unknown, but their upper bound is expected to be $m_i < 0.23$ eV (for $i = 1, 2, 3$) [15]. A global analysis of current experimental data on neutrino oscillations yields [16]: $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (8.0 \pm 0.3) \times 10^{-5}$ eV$^2$ and $\Delta m_{32}^2 \equiv m_3^2 - m_2^2 = \pm (2.5 \pm 0.3) \times 10^{-3}$ eV$^2$ at the 99% confidence level. Since the sign of $\Delta m_{32}^2$ has not been fixed, the neutrino mass spectrum can be classified into two general categories:

- Normal hierarchy: $m_1 < m_2 < m_3$, including the possibility that three neutrino masses are nearly degenerate ($\Delta m_{32}^2 > 0$).
- Inverted hierarchy: $m_3 < m_1 < m_2$, including the possibility that three neutrino masses are nearly degenerate ($\Delta m_{32}^2 < 0$).

Allowing $m_1$ or $m_3$ to vary up to its upper bound and inputting the experimental values of $\Delta m_{21}^2$ and $\Delta m_{32}^2$, we may use Eq. (3) to calculate $Q_\nu(M_Z)$. Our numerical results are illustrated in Fig. 2. One can see that the upper limit of $Q_\nu(M_Z)$ is achieved at $m_1 = 0$ for the normal neutrino mass hierarchy or at $m_3 = 0$ for the inverted neutrino mass hierarchy. Indeed,

$$
\frac{1}{Q_\nu(M_Z)} = \begin{cases} 
1 + 2 \frac{\sqrt{\Delta m_{21}^2} \sqrt{\Delta m_{21}^2 + |\Delta m_{32}^2|}}{\sqrt{\Delta m_{21}^2} + \sqrt{\Delta m_{21}^2 + |\Delta m_{32}^2|}}, & (m_1 \to 0), \\
1 + 2 \frac{\sqrt{|\Delta m_{32}^2|} \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2}}{\sqrt{|\Delta m_{32}^2|} + \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2}}, & (m_3 \to 0). 
\end{cases}
$$

It is then straightforward to understand $Q_\nu(M_Z) \sim 0.6$ for $m_1 \to 0$ and $Q_\nu(M_Z) \approx 0.5$ for $m_3 \to 0$ (as shown in Fig. 2) in the approximation of $|\Delta m_{32}^2| \gg \Delta m_{21}^2$. When three neutrino masses are nearly degenerate, $Q_\nu(M_Z)$ approaches its minimal value 1/3 no matter whether the sign of $\Delta m_{32}^2$ is positive or negative.

The running masses of three neutrinos at $M_R$ can be evaluated by using the one-loop RGEs given in Ref. [12]. In order to understand the sensitivity of $Q_\nu(M_R)$ to radiative corrections, it is more convenient to consider the RGE running behaviors of $m_i/m_j$ (for $i \neq j$). We obtain

$$
m_i(M_X) \approx m_i(M_Z) \chi_{ij}^{(ij)}
$$

in the approximation of $\tau$-lepton dominance [17], where $^3$

$^3$This formula is valid for Majorana neutrinos. If neutrinos were Dirac particles, the coefficient $C/(8\pi^2)$ on the right-hand side of Eq. (20) should be replaced by $C/(16\pi^2)$ [17].
\[ \chi^{(ij)}_\nu \equiv \exp \left[ \frac{C}{8\pi^2} \int_{\ln M_Z}^{\ln M_X} \frac{y'^2_r}{t} \left( |U_{3i}|^2 - |U_{3j}|^2 \right) dt \right] \]  

(20)

with \( C = -1.5 \) in the SM or \( C = 1 \) in the MSSM and \( U_{3i} \) (for \( i = 1, 2, 3 \)) being the elements of the 3 \( \times \) 3 lepton flavor mixing matrix. Because of \( y'^2_r/(8\pi^2) \approx 1.3 \times 10^{-6} \) (SM) or \( y'^2_r/(8\pi^2) \approx 1.3 \times 10^{-6} (1 + \tan^2 \beta) \) (MSSM) at \( M_Z \), together with \( |U_{3i}|^2 < 1 \), the departure of \( \chi^{(ij)}_\nu \) from 1 is expected to be tiny. This observation is illustrated in Fig. 3, where \( \theta_\nu \approx 33.8^\circ \) and \( \theta \approx 45^\circ \) [17] have typically been input for the matrix elements \( |U_{31}| = \sin \theta_\nu \sin \theta \), \( |U_{32}| = \cos \theta_\nu \sin \theta \) and \( |U_{33}| = \cos \theta \).

Note that \( \chi^{(ij)}_\nu \approx 1 \) is essentially independent of the possible mass hierarchies of three neutrinos [18]. It turns out that \( Q_\nu(M_X) \approx Q_\nu(M_Z) \) is a very good approximation. In view of Fig. 2, we conclude that there is no hope to achieve the Koide-like relation \( Q_\nu(\mu) \approx 2/3 \) for neutrino masses.

[5] In summary, the updated Koide relation for the pole masses of charged leptons satisfies \( Q_t^{\text{pole}} = 2/3 \) at the precision level of \( \mathcal{O}(10^{-5}) \). This amazing accuracy motivates us to examine whether the running masses of charged fermions and neutrinos have the similar Koide relation at a given energy scale. We have shown that the running masses of charged leptons cannot satisfy the Koide relation (i.e., \( Q_t(\mu) \neq 2/3 \)), but its discrepancy from \( Q_t^{\text{pole}} \) is only about 0.2% at \( \mu = M_Z \). The Koide-like relations for the running masses of neutrinos (\( 1/3 < Q_\nu(M_Z) < 0.6 \)), up-type quarks (\( Q_\nu(M_Z) \approx 0.89 \)) and down-type quarks (\( Q_\nu(M_Z) \approx 0.74 \)) are analyzed from \( M_Z \) up to the typical seesaw scale \( M_R \approx 10^{14} \) GeV. We find that they are nearly stable against radiative corrections, just like \( Q_t(\mu) \).

The approximate stability of \( Q_\nu(\mu) \) and \( Q_d(\mu) \) are mainly attributed to the strong mass hierarchy of quarks. In contrast, the approximate stability of \( Q_t(\mu) \) and \( Q_u(\mu) \) is essentially for the reason that the lepton mass ratios are very insensitive to radiative corrections.

Although it is impossible to get a universal Koide relation for both charged fermions and neutrinos at a given energy scale, our work remains useful for model building in order to explore the underlying similarities and differences between lepton and quark masses. For example,

\[ Q_U(M_Z) > Q_D(M_Z) > Q_t(M_Z) > Q_\nu(M_Z) \]  

(21)

is an interesting result of our numerical analysis. Once the absolute scale of neutrino masses is measured and the value of \( Q_\nu(M_Z) \) is fixed, one may then speculate whether these four parameters could be related with one another in a phenomenological way. On the other hand, we remark that more theoretical efforts are needed to look for possible flavor symmetries and their breaking effects behind the Koide-like relations.

**ACKNOWLEDGMENTS**

We would like to thank X.D. Ji, S. Zhou, S.H. Zhu and S.L. Zhu for useful discussions. This work is supported in part by the National Natural Science Foundation of China.
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FIG. 1. Illustration of $\chi_U$, $\chi_D$ and $\chi_I$ changing with the energy scale from $M_Z$ to $M_R$. 
FIG. 2. The Koide-like parameter $Q_{\nu}(M_Z)$ evaluated by using current neutrino oscillation data.
FIG. 3. Illustration of $\chi^{(ij)}$ changing with the energy scale from $M_Z$ to $M_R$. The solid and dashed lines denote the Majorana and Dirac cases, respectively. We have typically input $\tan \beta = 50$ and $m_1 = 0.2$ eV with $m^2_{32} > 0$ in our calculation. The almost identical result can be obtained when $m^2_{32} < 0$ is taken. Note that the deviation of $\chi^{(ij)}$ from 1 will be much milder, if tan $\beta$ takes smaller values.