△-N ELECTROMAGNETIC TRANSITION

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Abstract

The EM ratio for a free △ electromagnetic transition is discussed within the frame work of nonrelativistic approach. Such an approach gives a good account of data for a free △ but is less important for an intrinsically relativistic nuclear many body problem.

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I. INTRODUCTION

The free nucleon-nucleon (NN) interaction constitutes the basis of the microscopic approach to the description of the properties of the nuclear system. Apart from the NN potential problem, the conventional nuclear theory, in which nucleus is considered as an assembly of bound nucleons only, however, fails badly to account for the nuclear processes like photo and electro-disintegration of deuteron and in charge and magnetic form factors of three body nuclei, H$_3$ and He$_3^{3}[1,2]$ in particular at higher momentum transfer. Thus it becomes evident that the only nucleon picture does not fully simulate all physical effects of the hadronic fields even if hadronic fields are replaced by the effective potential. The reason for this is that the hadronic field is charged and therefore is subject to the electromagnetic interaction, leading to the presence of meson exchange currents. It is therefore realised that the concept of inert particles has only limited value and that in the certain processes isobar degree of freedom should be considered. Thus a new picture of the nucleus emerged in which the nucleons are not considered as being inert but having internal degrees of freedom (d.o.f.). Such a configuration (called isobar configuration) is effectively described by allowing virtually excited isobar to be present in the nucleus. These isobar configurations are dominated by $\Delta$ d.o.f.$^{[3,4]}$. Nucleonic interactions are clearly influenced by the intrinsic excitation modes of the nucleon. At the center-of-mass excitation energy of roughly 300 Mev, pion-nucleon reactions reveal a striking resonance behaviour in the $(J^p, T) = (3^+, \frac{3}{2})$ channel. This resonance can be understood as the formation of an excited state of the nucleon N(1/2$^+, 1/2$), namely the $\Delta (3/2^+, 3/2)$ and is seen as strong resonance in IIN-Scattering.

The nucleon has many resonant states which are particularly evident in pion-nucleon reactions. These resonant states can also be produced in the electromagnetic processes because most of them are coupled to the photon channel. Some of such isobar states give rise to large enhancements in the total absorption cross section. They are produced through different excitations (i.e., M1, E1, and E2 for $\Delta (1236)$, N(1520) and N(1680) respectively) and decay mainly into the pion-nucleon channel. The study of such transitions is important from the point of view of the internal structure and should give complementary informations with respect to what can be learned from the elastic electron scattering. The quark contribution
to the radiative transitions between the hadrons has been investigated. The standard hypothesis is that a photon is emitted or absorbed by a single quark in the hadron and the transition $\gamma X \rightarrow Y^*$ is thereby trigged. The most relevant transition is the one involving $\Delta$ ($\gamma P \rightarrow \Delta$) state. The process may be described in terms of a direct $\gamma N \Delta$ coupling leading to an intermediate $\Delta$- state, which subsequently decays into the $\Pi$-nucleon channel. There is considerable interest in the helicity amplitudes for the above transition, since these determine the electric quadrupole $E_2$ and magnetic dipole $M_1$ transition amplitude, the former depending on the $D$-state percentage in the shell model quark wave function.

II. $\Delta \rightarrow N \gamma$ ELECTROMAGNETIC TRANSITION

The transition of $3/2^+$ to $1/2^+$ state takes place by interaction with either the $M_1$ or $E_2$ (the magnetic dipole or electric quadrupole respectively) multipoles. It was noted by Becchi and Morpurgo[5] that in constituent quark model the $E_2$ transition is forbidden in line with the data. This is essentially because the electric quadrupole transition is proportional to the charge operator which cannot cause transition between quark isospin $3/2$ and $1/2$ states and hence the matrix element vanishes by orthogonality of the quark wave functions. Also the $E_2$ transition matrix element involves the spherical harmonics $Y_2$ which cannot cause transition between $L = 0$ special wave functions as proton and $\Delta^+$ are both $L = 0$ in the quark model. The magnetic dipole transition involves the quark magnetic moments and hence the spin-operator and this can lead to the transition between $3/2$ to $1/2$ state[6]. The $E_2/M_1$ ratio thus vanishes in the constituent quark model. But from the analysis of the experimental data[7,8] this ratio is,

$$\frac{E_2}{M_1} = -0.013 \pm 0.005$$

$$= -0.015 \pm 0.002$$

However subsequent introduction of the effect of quark-quark tensor and spin-spin forces on the radiative decay of $\Delta$-isobar[9] leads to a finite but rather very small $E_2/M_1$ ratio. In the absence of the tensor force the nucleon and the isobar are both in the state $S$ and their can be no $E_2$ transition.
To calculate the $E_2/M_1$ ratio (EMR) in the radiative transition $\Delta$-isobar by making use of the helicity amplitudes, we start with the current matrix element,

$$<\Delta p's' | J_{\mu\nu} | N_{ps}> = \bar{U}_{\Delta}(p',s')O_{\mu\nu}U_N(p,s)$$  \hspace{1cm} (3)

where $U_N$ is the nucleon spinor and $U_\Delta$ is the Rarita schwinger spinor which describes the $\Delta$-isobar. Taking both the baryons on mass shell, the Lorentz invariance and parity conservation for the current yields the following form of the tensor;

$$O_{\mu\nu} = a_1[(g_{\mu\nu}q^2 + p_{\mu}p_{\nu})]\gamma_5 + a_2[g_{\mu\nu}(M_{\Delta}^2 - M_N^2) + p_{\mu}P_{\nu}]\gamma_5 + a_3[g_{\mu\nu}(M_{\Delta} + M_N) + p_{\mu}P_{\nu}]\gamma_5,$$  \hspace{1cm} (4)

where,

$$q = p' - p$$  

$$P = p' + p$$

The functions $a_i$ are real and depend only on $q^2$. For excitation through real photon $a_1$ does not contribute since $q^2 = 0$ and $\varepsilon.q = 0$, where $\varepsilon'$ is the photon polarization vector. The form factors $a_2$ and $a_3$ are related to the functions $G_i$ of Pilkuhn[10] by;

$$a_2 = G_2/2(M_{\Delta} + M_N)^2$$  

$$a_3 = G_1/(M_{\Delta} + M_N)$$

Using the explicit expressions for the spinors and keeping only the lowest order terms in $P/M_{\Delta}$ and $P/M_N$, the current density in non-relativistic limit becomes,

$$<\Delta p's' | J | N_{ps}> = \Psi_\Delta^\dagger \left[a_\sigma_{\Delta N}\left(\sigma_\mu \left(\frac{q}{\mu} - \frac{P}{\bar{\mu}}\right)\right) + b\left((\sigma_{\Delta N} \times \sigma) \times \left(\frac{q}{\mu} - \frac{P}{\bar{\mu}}\right)\right)\right] \Psi_{Ns},$$  \hspace{1cm} (5)

where,

$$a = \frac{1}{2} \left[a_1(M_{\Delta} - M_N) + a_2(M_{\Delta} + M_N) - a_3\right] (M_{\Delta} - M_N)$$  

$$b = \frac{1}{2} MN a_3$$  

$$\frac{1}{\mu} = \left(\frac{1}{M_N} + \frac{1}{M_{\Delta}}\right)$$  

$$\frac{1}{\bar{\mu}} = \left(\frac{1}{M_N} - \frac{1}{M_{\Delta}}\right)$$  \hspace{1cm} (6)
The explicit results are given in the appendix.

The magnetic dipole and electric quadrupole form factors are given by,

\[ G_{\Delta N}^{M1} = \frac{2}{e(a - 2b)} \]
\[ G_{\Delta N}^{E2} = \frac{4}{ea} \]  \hspace{1cm} (7)

Putting Eq.(7) in Eq.(5) gives the following form of the current density;

\[ J_{\Delta N} = e \left[ -\frac{1}{2} \sqrt{\frac{5}{3}} G_{\Delta N}^{E2} \left\{ \sigma_{\Delta N}^{[2]} \times \left( \frac{q^{[1]}}{2} - \frac{P^{[1]}}{2} \right) \right\}^{[1]} + i \frac{2}{4} G_{\Delta N}^{E2} \sigma_{\Delta N} \times \left( \frac{q}{\mu} - \frac{P}{\mu} \right) \right] \]  \hspace{1cm} (8)

where,

\[ \sigma_{\Delta N}^{[2]} = \left[ \sigma_{\Delta N}^{[1]} \times \sigma^{[1]} \right]^{[2]} \]

The helicity amplitudes are defined as,

\[ A_{3/2} = < \Delta | -e \vec{J}_{\Delta N} \cdot \vec{A} | N > \]  \hspace{1cm} (9)

The relation between the multipole amplitudes and the helicity amplitudes arise solely due to the resonance production \( N \gamma \rightarrow \Delta \).

Substituting Eq.(8) in Eq.(9), we get,

\[ A_{3/2} = -\sqrt{\frac{\pi \omega}{2M}} e^2 \left( G_{M1} + G_{E2} \right) \]  \hspace{1cm} (10)

similarly,

\[ A_{1/2} = -\sqrt{\frac{\pi \omega}{6M}} e^2 \left( G_{M1} - 3G_{E2} \right) \]  \hspace{1cm} (11)

where,

\[ \omega = M_{\Delta} - M \]

is the energy of photon.

The ratio of helicity amplitudes is given by,

\[ \frac{A_{3/2}}{A_{1/2}} = \sqrt{3} \left( \frac{1 + G_{E2}/G_{M1}}{1 - 3G_{E2}/G_{M1}} \right) \]  \hspace{1cm} (12)

The helicity amplitudes, for the transition M1 and E2 nucleon resonance for proton excitation are[11]
\[ A_{3/2} = -179 \]
\[ A_{1/2} = -103 \]

This yields an electric quadrupole to magnetic dipole transition ratio for \( \Delta \to N\gamma \) equal to,

\[ \text{EMR} = E_2/M_1 = -0.001 \]

**III. CONCLUSION**

Although the importance of the \( \Delta \)-states in nuclear physics has been widely studied, it has been only been possible to draw tentative conclusions. The non-relativistic approach used for free \( \Delta \) gives a good account of data but for intrinsically relativistic nuclear many body problem the non-relativistic approximation seems to be less important. The relativistic approach to nucleons and \( \Delta \) in nuclear matter may serve to help in clarifying this problem.

**APPENDIX A: \( \Delta - N \) CURRENT**

The general expression for the current matrix element has the form;

\[
< \Delta_{p's'} | J_\nu | N_{ps} > = \hat{U}_\Delta(p', s')O_{\mu\nu}(p', p)U_N(p, s) \tag{A1}
\]

where,

\[
O_{\mu\nu}(p', p) = \left( \frac{c_3}{M} \gamma^\nu + \frac{c_4}{M^2} p'^\nu + \frac{c_5}{M^2} p^\nu \right) \left( g_{\lambda\mu} g_{\rho\nu} - g_{\lambda\rho} g_{\mu\nu} \right) q^\rho \gamma_5
\]

and \( c_i \) are the Schiff and Tran functions. Define,

\[
a_1 = \frac{c_4 - c_5}{2M^2}, \quad a_2 = \frac{c_4 + c_5}{2M^2}, \quad a_3 = \frac{c_3}{M}
\]

where \( a_i \) are real and depend only on \( q^2 \). The tensor becomes,

\[
O_{\mu\nu} = \left( g_{\lambda\mu} a - q_{\lambda\mu} b_\mu \right) \tag{A2}
\]

with,

\[
a = \left( \frac{M_\Delta + M}{M} \right)c_3 + \left( \frac{c_4 + c_5}{2M^2} \right)(M_\Delta^2 - M^2) + \left( \frac{c_4 - c_5}{2M^2} \right)q^2
\]

\[
b_\mu = a_3 \gamma_{\mu} + a_2 p_{\mu} + a_1 q_{\mu}
\]
Thus $\triangle - N$ current is given by;

$$J_\mu = \bar{U}(p) \left[ g_{\lambda\mu} a - \frac{q_\lambda}{M} b_\mu \right] \gamma_5 U(p) \tag{A3}$$

The leading term is,

$$J_i = c_3 \bar{U}(p) \left[ g_{\lambda i} \frac{M_\triangle + M}{M} - \gamma_5 \frac{q_\lambda}{M} \right] \gamma_5 U(p) \tag{A4}$$

Using the following form of the spinors;

$$U(p) = \sqrt{\frac{E + M}{2M}} \left( \frac{1}{\vec{\sigma} \cdot \vec{p}/(E+M)} \right) \Psi_N$$

$$\bar{U}^\lambda(p') = \sqrt{\frac{E + M_\triangle}{2M_\triangle}} \left( 1 - \frac{\vec{\sigma} \cdot \vec{p}/(E+M_\triangle)}{\vec{\sigma} \cdot \vec{p}/(E+M_\triangle)} \right) g^\lambda \Psi_\triangle^\dagger \gamma_0, \tag{A5}$$

where,

$$g^\lambda = L^\lambda_b(p') S^{\dagger \nu}$$

and,

$$g^i = -L^i_j S^{\dagger j} = \left( -M_\triangle S^{\dagger i} + \frac{p_i S^{\dagger} / M_\triangle}{E + M_\triangle} \right) / M_\triangle$$

$$g^0 = \frac{p_i S^{\dagger i}}{M_\triangle}$$

the vector part of the current simplifies to,

$$J_i = \frac{a_3 M}{2} \left( \vec{S} \times \vec{\sigma} \right) \times \left( \vec{q} / \mu - \frac{\vec{P}}{\bar{\mu}} \right) - \frac{a_3}{4} M_\triangle \vec{S} \cdot \vec{\sigma} \left( \vec{q} / \mu - \frac{\vec{P}}{\bar{\mu}} \right) \tag{A6}$$

The $a_1$ and $a_2$ terms are given by,

$$X \simeq \frac{a_1}{4} \left[ -S \cdot \vec{\sigma} \cdot \vec{A} (M_\triangle - M)^2 \right]$$

$$Y \simeq \frac{a_2}{4} \left[ -S \cdot \vec{\sigma} \cdot \vec{A} (M_\triangle^2 - M^2) \right] \tag{A7}$$

where,

$$\vec{A} = \frac{1}{2} \left( \frac{\vec{P}}{\bar{\mu}} - \frac{\vec{q}}{\mu} \right)$$

Putting everything together gives the following form of $\triangle - N$ current;

$$< \triangle | \vec{J} | N > = e \Psi_\triangle^\dagger \left[ a \vec{S} \cdot (\vec{\sigma} \cdot \vec{A}) + b (\vec{S} \times \vec{\sigma}) \times \vec{A} \right] \Psi_N$$

$$< \triangle | J_0 | N > = e \Psi_\triangle^\dagger \left[ \frac{a}{M_\triangle - M} (\vec{S} \cdot \vec{\sigma} \cdot \vec{A} - \frac{b}{M_\triangle M_N} \left( (\vec{S} \times \vec{\sigma}) \times \vec{P} \right) \cdot \vec{q} \right] \Psi_N \tag{A8}$$
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