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CLEO measurement of $B \to \pi^+\pi^-$ and determination of weak phase $\alpha$

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Abstract

The CLEO collaboration has recently reported a (first) measurement of $BR(B \to \pi^+\pi^-) = (4.7^{+1.8}_{-1.5} \pm 0.6) \times 10^{-6}$. We study, using recent results on QCD improved factorization, the implications of this measurement for the determination of the CKM phase $\alpha$ and also for the rate for $B \to \pi^0\pi^0$. If the $B \to \pi^-$ form factor is large ($\sim 0.3$), then we find that the CLEO measurement favors small $|V_{ub}/V_{cb}|$ so that the expected error (due to neglecting the QCD penguin amplitude) in the measurement of $\alpha$ using only the time-dependent analysis of the decay $B \to \pi^+\pi^-$ is large $\sim 15^\circ$. However, if $|V_{ub}/V_{cb}|$ is known, then it is possible to determine the correct value of $\sin 2\alpha$.

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1 Introduction

Recently, the CLEO collaboration has reported the first observation of the decay \( B \to \pi^+\pi^- \) and a limit on the rate for \( B^\pm \to \pi^\pm\pi^0 \) \cite{1}. In this letter, we determine the range of parameters, especially \( |V_{ub}/V_{cb}| \) (entering the calculations of \( B \) decay rates) which is preferred by this measurement/limit. Then, we study, in turn, the implications of these preferred values of parameters for the measurement of the CKM phase \( \alpha \) using time-dependent studies of \( B \to \pi^+\pi^- \) which will be done at the \( e^+e^- \) machines in the next few years and also for the rate for \( B \to \pi^0\pi^0 \).

The effective Hamiltonian for \( B \) decays is \cite{4}:

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{ud}^* (C_1 O_1^u + C_2 O_2^u) + V_{cb}V_{cd}^* (C_1 O_1^c + C_2 O_2^c) - V_{tb}V_{tq}^* \sum_{i=3}^{6} C_i O_i \right], \tag{1}
\]

where \( q = d, s \). The \( C_i \)'s are the Wilson coefficients (WC's) which are scheme- and scale-dependent; these unphysical dependences are cancelled by the corresponding scheme- and scale-dependences of the matrix elements of the operators.

In a recent paper, Beneke et al. found that the matrix elements for the decays \( B \to \pi\pi \), in the large \( m_b \) limit, can be written as \cite{2}:

\[
\langle \pi\pi|O_i|B\rangle = \langle \pi|j_1|B\rangle\langle \pi|j_2|0\rangle \\
\times \left[ 1 + \sum r_n \alpha_s^{n}(m_b) + O(\Lambda_{QCD}/m_b) \right], \tag{2}
\]

where \( j_1 \) and \( j_2 \) are bilinear quark currents. If the radiative corrections in \( \alpha_s \) and \( O(\Lambda_{QCD}/m_b) \) corrections are neglected, then the matrix element on the left-hand side factorizes into a product of a form factor and a meson decay constant so that we recover the “conventional” factorization formula. These

\[\text{We neglect the electroweak penguin operators which are expected to contribute to the } B \to \pi\pi \text{ decays only at the few } \% \text{ level.}\]
authors computed the $O(\alpha_s)$ corrections (in perturbation theory) using the meson light-cone distribution amplitudes \cite{footnote}. In this approach, the strong interaction (final-state rescattering) phases are included in the radiative corrections in $\alpha_s$ and thus the $O(\alpha_s)$ strong interaction phases are determined in \cite{footnote}. The scale- and scheme-dependence of the WC’s are cancelled by these $O(\alpha_s^n)$ corrections.

2 Formulae for $B \rightarrow \pi \pi$

The matrix elements for $B \rightarrow \pi \pi$ are \cite{footnote}:

$$iM(\bar{B}_d \rightarrow \pi^+\pi^-) = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{ud}^*(a_1 + a_4 + a_6 r_\chi) \right. + \left. V_{cb}V_{cd}^*(a_4 + a_6 r_\chi) \right] \times X. \quad (3)$$

Here

$$X = f_\pi \left( m_B^2 - m_\pi^2 \right) F_0^{B \rightarrow \pi^-} \left( m_\pi^2 \right), \quad (4)$$

where $f_\pi = 131$ MeV is the pion decay constant and $F_0^{B \rightarrow \pi^-}$ is a ($q^2$ dependent) form factor.

$$iM(B^- \rightarrow \pi^-\pi^0) = \frac{G_F}{\sqrt{2}} V_{ub}V_{ud}^*(a_1 + a_2) \times Y, \quad (5)$$

where

$$Y = f_\pi \left( m_B^2 - m_\pi^2 \right) F_0^{B \rightarrow \pi^0} \left( m_\pi^2 \right). \quad (6)$$

$$iM(\bar{B}_d \rightarrow \pi^0\pi^0) = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{ud}^*(a_2 - a_4 - a_6 r_\chi) \right. - \left. V_{cb}V_{cd}^*(a_4 + a_6 r_\chi) \right] \times \sqrt{2} \times Y. \quad (7)$$

In the above equations, the $a_i$’s are (combinations of) WC’s with the $O(\alpha_s)$ corrections added so that the $a_i$’s are scheme- and (almost) scale-independent. The values of the $a_i$’s are given in Table 1 \cite{footnote}. The imaginary parts of $a_i$’s are due to final-state rescattering.
For the $CP$ conjugate processes, the CKM elements have to be complex-conjugated.

The branching ratios are given by:

$$BR\left(B^- \to \pi^- \pi^0\right) = \tau_B \frac{1}{8\pi} |M|^2 \frac{|p|}{m_B^2},$$

(8)

where $\tau_B$ is the lifetime of the $B$ meson and $|p|$ is the momentum of the pion in the rest frame of the $B$ meson. There is a factor of $1/2$ for $\pi^0 \pi^0$ due to identical final state particles.

We neglect the $q^2$ dependence of the form factors between $q^2 = 0$ and $q^2 = m_B^2$, i.e., set $F_0^{B \to \pi}(0) = F_0^{B \to \pi}(m_B^2)$. We will use two values of the form factors: $F_0^{B \to \pi} = 0.27$ and 0.33 with $F_0^{B \to \pi^0} = 1/\sqrt{2} F_0^{B \to \pi}$. Model calculations indicate that the $SU(3)$ breaking in the form factors is given by $F_0^{B \to K} \approx 1.13 F_0^{B \to \pi} [3, 4]$. The large measured $BR(B \to K \eta')$ requires $F_0^{B \to K} \approx 0.36 [5]$ which, in turn, implies a larger value of $F_0^{B \to \pi} (\approx 0.33)$. If $F_0^{B \to K} \approx 0.36$, then we require a “new” mechanism to account for $BR(B \to K \eta')$: high charm content of $\eta'$ [6], QCD anomaly [7] or new physics. Also, if $F_0^{B \to \pi} < 0.27$, then the value of $F_0^{B \to K}$ is too small to explain the measured BR’s for $B \to K \pi$ [8].

We use $|V_{cb}| = 0.0395$, $|V_{ud}| = 0.974$, $|V_{cd}| = 0.224$, $m_B = 5.28$ GeV and $\tau_B = 1.6$ ps [9].

3 Constraints on parameters from $B \to \pi^+ \pi^-$, $\Delta m_s$ and $B \to \pi^0 \pi^0$

We first comment briefly on the upper limit on $\gamma$ using the recent limit on $B_s^0 - \bar{B}_s^0$ mass difference, $\Delta m_s > 14.3$ ps$^{-1}$ [10]. In the SM, we have

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s} B_{B_s} f_{B_s}^2 |V_{td}V_{ts}|^2}{m_{B_d} B_{B_d} f_{B_d}^2 |V_{tb}V_{ts}|^2}.$$

(9)
| $a_1$ | $1.047 + 0.033 \, i$ |
|-------|-----------------------|
| $a_2$ | $0.061 - 0.106 \, i$ |
| $a_u^0$ | $-0.030 - 0.019 \, i$ |
| $a_s^0$ | $-0.038 - 0.009 \, i$ |
| $a_{u,c}^{\alpha,c\, r_X}$ | $-0.036$ |

Table 1: The factorization coefficients for the renormalization scale $\mu = m_b/2$.

With $\Delta m_d$ (the $B^0 - \bar{B}^0$ mass difference) = $0.481 \pm 0.017 \, \text{ps}^{-1}$ [8], $m_{B_s} = 5.37 \, \text{GeV}$ [9] and $\sqrt{\frac{B_{B_s} f_{B_s}}{B_{B_d} f_{B_d}}} = 1.15 \pm 0.05$ [11], we get

$$\frac{|V_{td}|}{|V_{ts}|} < 0.214.$$ (10)

In the Wolfenstein parametrization, this constrains $|1 - \rho - i \eta| < 0.96$ which implies $\gamma \lesssim 90^\circ$.

In Fig. 1 we show the CP-averaged BR for $B \rightarrow \pi^+\pi^-$ as a functions of $\gamma$ for $F^{B \rightarrow \pi^-} = 0.33$ and 0.27 and for $|V_{ub}/V_{cb}| = 0.1, 0.08$ and 0.06 [11]. The CLEO measurement is $B \rightarrow \pi^+\pi^- = \left(4.7^{+1.8}_{-1.5} \pm 0.6\right) \times 10^{-6}$ [8]. If $F^{B \rightarrow \pi^-} = 0.33$ and for $\gamma \lesssim 90^\circ$, we see from the figures that smaller values of $|V_{ub}/V_{cb}| \approx 0.06$ are preferred: $|V_{ub}/V_{cb}| = 0.08$ is still allowed at the 2$\sigma$ level for $\gamma \sim 100^\circ$. However, if the smaller value of the form factor (0.27) is used, then the CLEO measurement is consistent with $|V_{ub}/V_{cb}| \approx 0.08$.

We obtain similar results using “effective” WC’s ($C^{\alpha f f}$)'s and $N = 3$ in the earlier factorization framework (neglecting final state rescattering) [8].

The CLEO collaboration also quotes a “value” for $BR(B \rightarrow \pi^\pm \pi^0)$ of $(5.4^{+2.1}_{-2.0} \pm 1.5) \times 10^{-6}$, but they say that the statistical significance of the excess over background is not sufficient for an observation and so they quote

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5 The Particle Data Group quotes $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ [8].
Figure 1: $CP$-averaged $BR(B \rightarrow \pi^+\pi^-)$ as a function of $\gamma$ for $F^{B \rightarrow \pi} = 0.27$ (left) and 0.33 (right) and for $|V_{ub}/V_{cb}| = 0.1$ (solid curves), 0.08 (dashed curves) and 0.06 (dotted curves). The BR measured by the CLEO collaboration lies (at the 1 $\sigma$ level) between the two horizontal (thicker) solid lines. The errors on the CLEO measurement have been added in quadrature to compute the 1 $\sigma$ limits.
Figure 2: $BR(B^\pm \rightarrow \pi^\pm \pi^0)$ as a function of $|V_{ub}/V_{cb}|$ for $F^{B\rightarrow \pi^{-}} = 0.27$ (solid curve) and 0.33 (dashed curve). The 90 % C. L. upper limit from the CLEO collaboration is $12 \times 10^{-6}$.

A 90 % C.L. upper limit of $12 \times 10^{-6}$ [1]. The $BR$ for $B \rightarrow \pi^\pm \pi^0$ is shown in Fig. 2 as a function of $|V_{ub}/V_{cb}|$. The upper limit for $B \rightarrow \pi^\pm \pi^0$ allows $|V_{ub}/V_{cb}|$ up to 0.1. But, assuming an observation at the BR quoted and if $F^{B\rightarrow \pi^{-}} \approx 0.33$, then there is a preference for small $|V_{ub}/V_{cb}|$ from this decay mode consistent with that from $B \rightarrow \pi^+ \pi^-$. 
4 Implications for measurements of $\alpha$ and $B \to \pi^0 \pi^0$

The unitarity triangle is a representation in the complex plane of the relation: $V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0$. The angles of the triangle are: $\alpha \equiv \text{Arg} \left( \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right)$, $\beta \equiv \text{Arg} \left( \frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*} \right)$, and $\gamma \equiv \text{Arg} \left( \frac{V_{ub} V_{ud}^*}{V_{cb} V_{cd}^*} \right)$ with $\alpha + \beta + \gamma = 180^\circ$. Choosing $V_{cb} V_{cd}^* = -|V_{cb} V_{cd}^*|$, we get $V_{ub} V_{ud}^* = |V_{ub} V_{ud}^*| e^{i\gamma}$ and

$$\tan \beta = \frac{|V_{ub} V_{ud}^*| \sin \gamma}{|V_{cb} V_{cd}^*| - |V_{ub} V_{ud}^*| \cos \gamma}.$$  \hspace{1cm} (11)

Fixing $|V_{ud}^*| = 0.974$ and $|V_{cd}^*| = 0.224$ \[9\], $\beta$ (see above equation) and $\alpha$ can be obtained as a function of $\gamma$ and $r \equiv |V_{ub}/V_{cb}|$:

$$\alpha = 180^\circ - \gamma - \tan^{-1} \left( \frac{r |V_{ud}^*| \sin \gamma}{|V_{cd}^*| - r |V_{ud}^*| \cos \gamma} \right).$$ \hspace{1cm} (12)

The time-dependent decay rates for an initial pure $B_d$ or $\bar{B}_d$ to decay into a $CP$ eigenstate final state $f$ are (assuming the total decay widths (denoted by $\Gamma$) of the two mass eigenstates are the same):

$$\Gamma \left( B_d(t) \to f \right) = |\mathcal{M}|^2 e^{-\Gamma t} \left( \frac{1 + |\lambda|^2}{2} \right)$$

$$+ \frac{1}{2} \left( \cos(\Delta m_d t) - \text{Im} \lambda \sin(\Delta m_d t) \right)$$

$$\Gamma \left( \bar{B}_d(t) \to f \right) = |\bar{\mathcal{M}}|^2 e^{-\Gamma t} \left( \frac{1 + |\lambda|^2}{2} \right)$$

$$- \frac{1}{2} \left( \cos(\Delta m_d t) + \text{Im} \lambda \sin(\Delta m_d t) \right)$$ \hspace{1cm} (13)

with

$$\lambda \equiv \frac{q \bar{\mathcal{M}}}{p \mathcal{M}},$$ \hspace{1cm} (14)

where

$$\mathcal{M} \equiv \langle f | \mathcal{H}_{eff} | B_d \rangle, \quad \bar{\mathcal{M}} \equiv \langle f | \mathcal{H}_{eff} | \bar{B}_d \rangle$$ \hspace{1cm} (15)
and the two mass eigenstates are

$$|B_{L,H}\rangle = p|B_d\rangle \pm q\bar{B}_d\rangle.$$  \hspace{1cm} (16)

In the SM,

$$\frac{q}{p} = e^{-i2\beta}$$  \hspace{1cm} (17)

since the $B_d - \bar{B}_d$ mixing phase is $2\beta$. In the case of $f = \pi^+\pi^-$, if we neglect the (QCD) penguin operators, i.e., set $a_{4,6} = 0$ in Eq. (3), we get

$$\frac{\mathcal{M}}{\mathcal{M}} = e^{-i2\gamma}$$  \hspace{1cm} (18)

and

$$\text{Im}\lambda = \sin(-2(\beta + \gamma)) = \sin 2\alpha.$$  \hspace{1cm} (19)

Thus, the parameter $\text{Im}\lambda$ can be obtained from the measurement of the time-dependent asymmetry in $B \rightarrow \pi^+\pi^-$ decays (Eq. (13)) and if the penguin contribution can be neglected, $\sin 2\alpha$ can be determined (Eq. (19)).

In the presence of the penguin contribution, however, $\frac{\mathcal{M}}{M} \neq e^{-i2\gamma}$ so that $\text{Im}\lambda \neq \sin 2\alpha$. We define

$$\text{Im}\lambda = \text{Im}(e^{-i2\beta}\frac{\mathcal{M}}{M}) \equiv \sin 2\alpha_{\text{meas.}}$$  \hspace{1cm} (20)

as the “measured” value of $\sin 2\alpha$, i.e., $\sin 2\alpha_{\text{meas.}} = \sin 2\alpha$ if the penguin operators can be neglected.

In Fig. 3 we plot the error in the measurement of $\alpha$, $\Delta\alpha \equiv \alpha_{\text{meas.}} - \alpha$, where $\alpha_{\text{meas.}}$ is obtained from Eq. (21) (using the amplitudes of Eq. (3) and the value of $\beta$ from Eq. (11)) and $\alpha$ is obtained from Eq. (12). $\Delta\alpha$ depends only on $\gamma$ and $|V_{ub}/V_{cb}|$ and is independent of $F^{B\rightarrow\pi^-}$ since the form factor cancels in the ratio $\mathcal{M}/\mathcal{M}$. We see that for the values of $|V_{ub}/V_{cb}| \approx 0.06$ preferred by the $B \rightarrow \pi^+\pi^-$ measurement (if $F^{B\rightarrow\pi^-} \approx 0.33$), the error in the determination of $\alpha$ is large $\sim 15^\circ$ (for $\gamma \sim 90^\circ$). If $F^{B\rightarrow\pi^-} \approx 0.27$, then $|V_{ub}/V_{cb}| \approx 0.08$ is consistent with the $B \rightarrow \pi^+\pi^-$ measurement which gives $\Delta\alpha \sim 10^\circ$ (for $\gamma \sim 90^\circ$).
The computation of Beneke et al. \cite{2} includes final state rescattering phases, \textit{i.e.}, it is \textit{exact} up to $O(\Lambda_{QCD}/m_b)$ and $O(\alpha_s^2)$ corrections. Thus, the value of $\sin 2\alpha$ “measured” in $B \to \pi^+\pi^-$ decays (Eq. (20)) is a known function of $\gamma$ and $|V_{ub}/V_{cb}|$ only (in particular, there is no dependence on the phenomenological parameter $\xi \sim 1/N$ and strong phases are included unlike in the earlier factorization framework \cite{1}). Since, the “true” value of $\alpha$ can also be expressed in terms of $\gamma$ and $|V_{ub}/V_{cb}|$ (Eq. (12)), we can estimate the “true” value of $\sin 2\alpha$ from the “measured” value of $\sin 2\alpha$ for a given value of $|V_{ub}/V_{cb}|$ (of course, up to $O(\Lambda_{QCD}/m_b)$ and $O(\alpha_s^2)$ corrections); this is shown in Fig. \[ where we have restricted $\gamma$ to be in the range $(40^\circ, 120^\circ)$ as indicated by constraints on the unitarity triangle from present data. If $0^\circ \leq \gamma \leq 180^\circ$ is allowed, then there will be a discrete ambiguity in the determination of $\sin 2\alpha$ from $\sin 2\alpha_{\text{meas.}}$.

Gronau, London \cite{13} showed how to include penguin contributions in the determination of $\alpha$, but their method requires, in addition to the time-dependent decay rates for $B \to \pi^+\pi^-$, the measurement of rates for the (tagged) decays $B_d, \bar{B}_d \to \pi^0\pi^0$ and the rate for the decay $B \to \pi^0\pi^0$. We show $BR( B \to \pi^0\pi^0)$ as a function of $\gamma$ in Fig. \[, again for $F^{B\to\pi^-}_{B\to\pi^0\pi^0} = 0.27$ and 0.33 and for $|V_{ub}/V_{cb}| = 0.1, 0.08$ and 0.06 \[. We see that for $|V_{ub}/V_{cb}| \approx 0.06$ (which is preferred by the $B \to \pi^+\pi^-$ measurement for $F^{B\to\pi^-}_{B\to\pi^0\pi^0} \approx 0.33$), this rate is very small: $BR \approx 3 \times 10^{-7}$. Thus, the measurement of the rates for the (tagged) decays $B_d, \bar{B}_d \to \pi^0\pi^0$ is very difficult in say few years of running of the current $e^+e^-$ machines due to the very small rate. Since time-dependent measurements of $B_d, \bar{B}_d \to \pi^+\pi^-$ will be achieved at these

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6In \cite{12} also a plot of $\sin 2\alpha_{\text{meas.}}$ as a function of $\sin 2\alpha$ is shown, but for a fixed $\sin 2\beta$.

7Since this measurement of the “true” $\alpha$ using $B \to \pi^+\pi^-$ decays relies on “other” information about the CKM matrix, \textit{i.e.}, $|V_{ub}/V_{cb}|$, it is not an “independent” determination of $\alpha$, but it can be used as a consistency check.

8Numerically, the coefficient $a_2$ which determines the tree-level amplitude for $B \to \pi^0\pi^0$ is suppressed (at lowest order) due to a cancellation between the WC’s $C_1$ and $C_2$ and thus (unlike $a_1$) is very sensitive to the $O(\alpha_s)$ corrections. Thus, we obtain slightly different results using “effective” WC’s ($C^{eff}$)’s and $N = 3$ \cite{1}.
Figure 3: The error in the measurement of CKM phase $\alpha$ using (only) time-dependent $B \to \pi^+\pi^-$ decays as a function of $\gamma$ for $|V_{ub}/V_{cb}| = 0.1$ (solid curve), 0.08 (dashed curve) and 0.06 (dotted curve).
Figure 4: The “true” value of $\sin 2\alpha$ as a function of the value of $\sin 2\alpha$ “measured” in $B \to \pi^+ \pi^-$ decays for $|V_{ub}/V_{cb}| = 0.1$ (solid curve), 0.08 (dashed curve) and 0.06 (dotted curve).
machines, it is interesting to see how accurately we can measure $\alpha$ with only $B \to \pi^+\pi^-$. 

5 Summary

To summarize, if $F_{B\to\pi^-} \approx 0.33$, then we have shown that the recent (and first) CLEO measurement of $BR(B \to \pi^+\pi^-) \left(4.7^{+1.8}_{-1.5} \pm 0.6\right) \times 10^{-6}$ favors small $|V_{ub}/V_{cb}| \approx 0.06$. This result is obtained using the recent computation of the matrix elements [2] which includes the strong interaction phases. The small value of $|V_{ub}/V_{cb}|$ enhances the penguin amplitude relative to the tree amplitude which implies that the error (due to neglecting the penguin contribution) in the determination of the CKM phase $\alpha$ using only (time-dependent) $B \to \pi^+\pi^-$ decays is large $\sim 15^\circ$ for $\gamma \sim 90^\circ$. However, if $F_{B\to\pi^-} \approx 0.27$, then $|V_{ub}/V_{cb}| \approx 0.08$ is consistent with the value of $BR(B \to \pi^+\pi^-)$ which implies that the error in the determination of $\alpha$ is $\sim 10^\circ$ for $(\gamma \sim 90^\circ)$. Actually, if $|V_{ub}/V_{cb}|$ is known, then the correct value of $\sin 2\alpha$ can be determined. Also, $|V_{ub}/V_{cb}| \approx 0.06$ implies that $BR(B \to \pi^0\pi^0)$ is expected to be very small $\lesssim 3 \times 10^{-7}$ and $BR(B \to \pi^\pm\pi^0)$ is expected to be $\sim 4 \times 10^{-6}$.

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