Proposal of the Use of a Fuzzy Stochastic Network for the Preliminary Evaluation of the Feasibility of the Process of the Adaptation of a Historical Building to a Particular Form of Use

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Abstract. The knowledge of a real estate developer regarding the possibilities of adapting a historical building to a particular form of use and the knowledge of the approximate costs associated with this process are some of the more important pieces of information that can influence the making of the final decision regarding commencing with such a project. The preliminary analysis of the process of adapting a historical building is a difficult task due to the specific character of this type of project. The specific character of such a project is proven by the fact that the often insufficient analysis of the structure and architecture of a building and its historical substance at the stage of carrying out the process of adaptation can generate the necessity to perform previously unforeseen additional actions. An equally important problem is the difficulty in estimating the funds required to conduct research and the analyses associated with developing design documentation, as well as carrying out construction and conservation work. This is why a real estate developer should analyse various scenarios of carrying out a project during the stage of the preliminary analysis of its feasibility, taking into account the fact that some of them can occur in a random manner. The authors of the paper propose the use of one of the planning tools known as stochastic networks, which can be used to model the undetermined structure of these types of projects. Fuzzy logic was used in order to estimate uncertain values of the parameters of a model (the probability of performing work and paying the associated costs). The approach proposed by the authors was used to perform a preliminary analysis of the adaptation of the Arsenal in Gdańsk to a particular form of use along with estimating the costs associated with it. The results that were obtained have confirmed the potential of this method for real-world application.

1. Introduction

The diversity of historical buildings has a significant influence on the ability to adapt them to new forms of use. The type of architectural and structural layout, the type of the load-bearing system, the technical condition of the structure in question, as well as the physical, chemical and mechanical properties of the materials used in the construction of a historical building, as well as their quality and suitability for reuse are of significant importance in the evaluation of the adaptability of a building [1].

The process of adapting a building to a new form of use in and of itself is quite specific, due to:
difficulty in assessing the scope of adaptation work due to the limited identification of the structure of a historical building,

- complicated course of design work and the low predictability of the process of obtaining the legally required opinions and decisions, including several rejections of proposed adaptation solutions,

- possibility of making new discoveries during construction work, which results in additional archaeological research and conservation work,

- uncertainty of assessing the parameters (the probability of occurrence, duration and cost of performance) of each type of work, including construction and conservation work, due to their technical and unique character, as well as the atypical conditions of carrying them out.

The aforementioned problems are the reason behind the fact that attempting to estimate the cost of carrying out such projects requires taking into account the random character of the adaptation process - due to the various situations that can arise during both design work, as well as during the performance of adaptation work. In addition, uncertainty in the assessment of the parameters of each type of adaptation work requires the employment of the knowledge and experience of experts who specialise in the process of the adaptation of historical buildings.

These problems lead to a situation in which practical adaptation projects are badly planned, which, as a consequence, generates unforeseen problems which are often rooted in underestimating their cost and completion time [2,3].

Due to the above, a tool is required that can take into account the specifics of the process of adapting historical buildings, making it possible to plan this process along with assessing the probabilities of the occurrence of the scenarios of its completion, and, as a consequence, estimating the costs of carrying out a given scenario.

One proven manner of the modelling of projects with an undetermined structure is the use of network planning methods based on the GAN convention (Generalized Activity Network). This concept, as a generalisation of traditional networks, had been started by [4], which, in practice, made it possible to generate variants of dependencies within a network's structure. The introduction of the appropriate calculations of graphs in [5, 6] allowed the analysis of such networks. Work [7] features a proposition of the GERT method (Graphical Evaluation and Review Technique), which was later developed in [8] and has, along with its simulation variant - GERTS - constituted the basic method of analysing stochastic networks, in which parameters are described using random variables. Finally, [9] has brought the concept of FGERT (Fuzzy Graphical Evaluation and Review Technique), the parameters of which have a fuzzy character, in accordance with the principles of fuzzy logic [10-12]. This approach, continued in, for instance [13-16], has allowed the alternative inclusion of the uncertainty of the estimation of the aforementioned parameters of a network alongside classical probability theory. The concept of stochastic networks shown above, supplemented with elements of fuzzy logic, can thus be an attractive tool in the planning of the process of adapting a historical building due to the lack of empirical knowledge, which is characteristic of these types of projects.

The goals of this work are:

1. Formulating a proposition of using the capabilities of a stochastic network featuring fuzzy parameters for the purpose of modelling the process of adapting a historical building to a new form of use.

2. Formulating a proposition of the manner of the construction of a triangular type-1 fuzzy membership function for a fuzzy number modelling the uncertainty of the opinions regarding the values of the parameters of a network model.
3. Providing an extension of the method of graph reduction to feature the capability to analyse the network model of the adaptation process not only in the function of time - which has hitherto been possible in the classic graph reduction method [5,6], but also in the function of the costs of its performance.

4. The use of the approach outlined above to analyse the possibility of adapting the building of the Great Armoury in Gdańsk to a new form of use, along with estimating the uncertain costs associated with such a project.

2. The proposed approach

2.1. The construction of a network model of the process of adapting a historical building

The verbal description of the process being discussed - which has an undetermined structure - can be converted into a description featuring the use of a stochastic network based on a graph [4], which will make it possible to model various scenarios of performing the process of adapting a historical building, which are generated in a random manner.

Using this concept [17], a coherent graph has been defined in the form of the ordered pair \( G = (W, T) \), in which: \( W \) - a finite set of \( w_i \) nodes, which means that \( W = \{w_i\} \), where: node \( w_i \) symbolises an event (the state of the process's progress), \( T \) - a finite set of transmittances. Transmittance is determined in the finite set \( U \), which consists of ordered pairs \( \{w_i, w_j\} \), thus \( U = \{(w_i, w_j)\} \). If \( w_i \neq w_j \), then the dependency between such nodes will be called an arc. If \( w_i = w_j \) then the dependency between such nodes will be called a loop of the first order, while the set of ordered pairs \( \{u_i, u_j, u_d, u_l\} \) will be called a loop of the second order - a cycle. The arcs and loops within the graph will symbolise the actions associated with the process of adapting a historical building.

For each change in state from \( w_i \) to \( w_j \) in the graph that is being defined, certain parameters - called the loads of its arcs and loops - are going to be appointed. The use of a stochastic network in order to model the process of adapting a historical building in this work required the defining of three types of loads of the arcs and loops of the network: \( p_{ij} \) - the probability of the change from state \( w_i \) to state \( w_j \), \( t_{ij} \) - the duration of the change from state \( w_i \) to state \( w_j \), \( c_{ij} \) - the cost of the change from state \( w_i \) to \( w_j \).

2.2. Modelling the uncertainty of estimating the parameters of a network model of the process of adapting a historical building

The aforementioned specifics of adaptation projects mean that the abovementioned parameters feature a large degree of uncertainty, which is a considerable problem in their estimation. In order to model the uncertainty of these parameters, the authors of this article decided to use the knowledge of experts and the properties of fuzzy sets [10].

The proposed approach to the construction of a type-1 fuzzy number describing the uncertainty of a given parameter (the probability of performance, cost, duration) for each task of the project that is being modelled, first requires gathering the opinions of experts. For each of the actions associated with the adaptation process, the value of the mean \( \tilde{\sigma} \) and the standard deviation \( \sigma \) for the opinions of experts can allow the construction of a membership function for a triangular type-1 fuzzy number describing the uncertainty regarding the opinions of the range of values for a given parameter.

Assuming that the opinions of experts are described by a normal distribution or one that is close to normal, we could, in accordance with the three-sigma principle, change the spectrum of the occurrence
of the values of a parameter of a given action within the adaptation process that is being discussed [18]. Such an approach will allow the decision-maker to take into account various levels of aversion to risk regarding the spectrum of the values of parameters in reference to each of the actions modelled by the network. For instance, the value of the fuzzy parameter of a given task with the highest possibility is going to reflect the mean $\tilde{\delta}$ while the extreme values of the same parameter are going to be equal to $\tilde{\delta} - \sigma$ and $\tilde{\delta} + \sigma$ [19] (Fig. 1).

![Figure 1. Example of a triangular type-1 fuzzy number for the values of the parameters of a given action within the process of the adaptation of a historical building](image)

2.3. Analysis of the network model of the process of adapting a historical building
The stage of the analysis of the network model is going to feature a reduction of a stochastic network - namely, a simplification of the network until a less complicated substitute network can be defined, which is going to describe the base network in an unambiguous manner [5,6]. The character of the method of graph reduction allows the implementation of calculations on type-1 fuzzy numbers in a direct manner, which is, without a doubt, a great benefit of using it [20]. Table 1 presents the basic schemes of graph reduction, to which, apart from the typical parameters that are being analysed (the probability of the activation of actions and the time of their performance), the authors introduced the possibility of analysing the costs of carrying out the project that is being modelled by the network.

We can obtain the following results from the graph reduction process:

| Table 1. Graph reduction diagram along with the arithmetic of triangular fuzzy numbers |
|--------------------------|--------------------------|
| **Elementary graph reduction diagram** | **The arithmetic triangular of fuzzy numbers** |
| $\tilde{P}_a \times \tilde{P}_b = (P_{a_1}, P_{a_2}, P_{a_3}, P_{b_1}, P_{b_2}, P_{b_3})$ | $\tilde{P}_a \times \tilde{P}_b = (P_{a_1}, P_{a_2}, P_{a_3}, P_{b_1}, P_{b_2}, P_{b_3})$ |
| $\tilde{t}_a + \tilde{t}_b = (t_{a_1} + t_{a_2} + t_{a_3} + t_{b_1} + t_{b_2} + t_{b_3})$ | $\tilde{t}_a + \tilde{t}_b = (t_{a_1} + t_{a_2} + t_{a_3} + t_{b_1} + t_{b_2} + t_{b_3})$ |
| $\tilde{c}_a + \tilde{c}_b = (c_{a_1} + c_{a_2} + c_{a_3} + c_{b_1} + c_{b_2} + c_{b_3})$ | $\tilde{c}_a + \tilde{c}_b = (c_{a_1} + c_{a_2} + c_{a_3} + c_{b_1} + c_{b_2} + c_{b_3})$ |
The time of the carrying out of a given scenario of the process of adapting a historical building that is being modelled by the network:

\[ E(t) = \sum_i \tilde{t}_i, \]

where \( \tilde{t} \) is the time of the carrying out of a given scenario of the process of adapting a historical building that is being modelled by the network.
The cost of carrying out a process of the scenario of the adaptation of a historical building that is being modelled by the network:

\[ E(c) = \tilde{c} = \sum_i P_i \tilde{c}_i \]  

\( i = i \) path.

The resultant fuzzy numbers that describe the uncertainty of the final results (the probability of carrying out a task, cost and duration) of the analysed project can be converted into a crisp form with a chosen method of the defuzzification of type-1 fuzzy sets, additionally taking into account \( \alpha \)-cross-sections [22].

3. Case study

The subject of the analysis is the building of the Great Armoury, which has been introduced into the Gdańsk registry of historical sites by the decision of the Voivodship Historical Site Conservator of 10.07.1967, located in Gdańsk. This building, also called the Grand Arsenal, has become one of the greatest lay buildings of the urban built environment of Gdańsk, and one of the largest buildings of this type in Europe.

This building was built in the years 1602-1605, according to the idea of the well-known architect Antoni van Obbergen. It was built using small Dutch bricks, and the entirety of the facades is decorated with various decorations from sandstone, polychromes and rich gildings (Fig. 2, courtesy of IPG sp. z o.o., W.K. Geoplan Roland Chrobak). The building has three storeys and a cellar. Up to the year 1793, the Grand Arsenal was used in accordance with its intended role, as a weapons stockpile. It would later become the place where the portraits of Polish kings were stored. In 1923, the ground level was adapted into a shopping gallery, while the upper floors housed workshops and a storehouse of theatrical decorations, as well as the backrooms for the merchants. Currently, the two uppermost levels house the Academy of Fine Arts while the ground floor, with a surface area of 1819 m\(^2\), as well as the cellar, with a surface area of 1528 m\(^2\), constitute a space that has not been actively used in recent years [23] (Fig. 3, courtesy of IPG sp. z o.o., W.K. Geoplan Roland Chrobak).

Figure 2. The facade of the Great Armoury in Gdańsk from the side of Targ Węglowy

Figure 3. A 3D model of the visualisation of the ground floor and cellars of the Great Armoury

One of the concepts of adapting the abovementioned levels can be the adaptation of the cellar to a restaurant and the ground floor to an art gallery. The process of adapting the historical building that is discussed in this work requires a series of actions focused on the architecture, structure and infrastructure of the building, not to mention conservation work, as well as archaeological research. The network model of the analysed project is shown in figure 4, while the description of the planned activities is contained in table 2. In order to assess the parameters of the process of the adaptation, a
five-person group of experts expressed their opinions regarding the probability of the occurrence of the planned activities and the costs and duration of their performance. The group of experts chosen by the authors had detailed knowledge of the analysed building and experience in the adaptation of historical buildings. The opinions of the experts were used to construct the membership functions for fuzzy numbers describing the opinions regarding the values of the parameters of the project that were mentioned above.

4. Results and discussions
As a result of the analysis of the fuzzy network model of the adaptation of the building of the Grand Armoury to a new form of use, the authors obtained the distributions of the possible, final values of the parameters of the project (Figures 5,6).

Figure 4. Network model of the process of adapting the ground floor to an arts gallery and the cellar to a small restaurant in the building of the Great Armoury in Gdańsk

Analysing the results above, we can state that the probability of success associated with performing the process of adapting the Grand Armoury to the form of use that is being discussed (achieving node no. 20 in the network model) is the highest in comparison to all the other analysed scenarios of the completion of the project. Although the distribution of the possible values of this probability has the widest spectrum of their occurrence, a value of probability at a level around 0.5 has the highest membership to the fuzzy number modelling the uncertainty of opinions regarding the value of the parameter in comparison with the remaining scenarios. The distribution of the possible values of the probability of abandoning the adaptation project (achieving node no. 22 in the network model) due to the impossibility of placing infrastructure in the fill spaces of the vaults (which is the only method accepted by conservation authorities) is wide enough, that this fact demands serious attention. The costs, as well as the time that needs to be spent on the implementation of the abovementioned scenario are relatively small in comparison to the estimated costs of performing the adaptation process. However, the values that are characterised by a high membership to the fuzzy number of these costs are around 100% higher than the costs of carrying out a scenario associated with abandoning the adaptation process due to the impossibility of adapting the building to fire safety regulations because of the character of the new form of use (achieving node no. 21 in the network model). The possible probability values of the scenario which was mentioned last are small and have a smaller uncertainty than the remaining scenarios.
| Id | Activity | Numbers nodes | Name of Activity                                                                                                                                 |
|----|----------|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| 1  | W1       | 1 2           | Holding a meeting of the tender committee and the preparation of the specifications regarding the important terms of the commission, the hosting of a competition for the selection of a designer |
| 2  | W2       | 2 3           | Tender procedure for the selection of a designer                                                                                                                                                     |
| 3  | W3       | 3 2           | Rejection of proposals and the annulment of the tender                                                                                                                                         |
| 4  | W4       | 3 4           | Signing an agreement with a designer and the performing of hydrological and geological assessments (the local unearthing of the existing foundations in order to assess their technical condition) |
| 5  | W5       | 4 5           | The development of a documentation of the extant state of the building and of a conservation programme.                                                                                             |
| 6  | W6       | 4 7           | Filing a request for the conditions of the allocation of amenities and obtaining them                                                                                                               |
| 7  | W7       | 4 7           | Technical expertise regarding the structure of the building                                                                                                                                  |
| 8  | W8       | 5 5           | Amending the conservation programme due to review by the Conservator                                                                                                                          |
| 9  | W9       | 5 6           | The acceptance of the conservation programme and the preparation of a conceptual design of the adaptation, to be reviewed by the Conservator                                                  |
| 10 | W10      | 6 6           | Amending the conceptual design due to review by the Conservator                                                                                                                              |
| 11 | W11      | 6 7           | The acceptance of the conceptual design and the preparation of basic architectural drawings                                                                                                     |
| 12 | W12      | 7 8           | Preparing fire safety documentation and filing a request with the Voivodship Fire Department Commissioner for the issuance of an ordinance regarding the meeting of the current fire safety regulations in a different manner than stated in the regulations. |
| 13 | W13      | 8 8           | Correcting the fire safety documentation upon review by the Voivodship Fire Department Commissioner                                                                                           |
| 14 | W14      | 8 9           | The development of a technical design, along with the obtaining of approvals and the filing of a request for the issuance of a building warrant.                                                   |
| 15 | W15      | 9 9           | Amending the technical design upon review by the administrative organ issuing the building warrant.                                                                                                 |
| 16 | W16      | 9 10          | Obtaining a building warrant and the development of a detailed construction design of the adaptation, the performance of a stratigraphic analysis of the floors of the ground floor in order to ascertain the possibility of placing infrastructure in the fill spaces of vaults. |
| 17 | W17      | 10 11         | The confirmation of the possibility of placing infrastructure within the fill spaces of vaults. The preparation of specifications regarding the important terms of the commission and organising a tender for the selection of the contractor that will perform the adaptation work |
| 18 | W18      | 11 12         | The tender procedure for the selection of the contractor that will perform the adaptation work                                                                                                                                                   |
| 19 | W19      | 12 11         | Rejection of offers and the annulment of the tender.                                                                                                                                           |
| 20 | W20      | 12 13         | Signing an agreement with the contractor                                                                                                                                                       |
| 21 | W21      | 13 14         | Unearthing the foundations (under archaeological supervision) of the stairwell in order to ascertain their technical condition (slight increase in load).                                         |
| 22 | W22      | 13 15         | Demolition work on the vertical barriers within the cellar                                                                                                                                  |
| 23 | W23      | 13 16         | The uncovering of brickwork patterns in the cellar through removing the plasters and its technical and conservatorial evaluation                                                                |
| 24 | W24      | 14 14         | The performance of additional archaeological research                                                                                                                                   |
| 25 | W25      | 14 17         | Foundations in good technical condition.                                                                                                                                                       |
| 26 | W26      | 14 17         | Foundations in bad technical condition. It is necessary to repair them.                                                                                                                     |
| 27 | W27      | 15 19         | The construction of a glass ceiling in place of the vault destroyed during wartime.                                                                                                            |
| 28 | W28      | 16 18         | The brickwork of the brick vaults is in good condition. The performance of drying work.                                                                                                   |
| 29 | W29      | 16 18         | Vaults in bad technical condition (defects, delamination). The performance of drying work and the repair of their structure.                                                                 |
| 30 | W30      | 17 19         | Redevelopment of the stairwell, along with constructing a ventilation shaft and the fitting of an elevator.                                                                                   |
| 31 | W31      | 15 20         | Erecting fire zone barriers along with infrastructure ducts. The stripping of the floor layers and the fitting of infrastructure within the fill spaces of vaults, the recreation of stone floors. |
| 32 | W32      | 18 20         | The conservation of the exposed brickwork.                                                                                                                                                     |
| 33 | W33      | 19 20         | Remaining finishing work                                                                                                                                                                       |
| 34 | W34      | 8 21          | No possibility of obtaining an approval of meeting fire safety regulations in a different manner than stated in regulations.                                                               |
| 35 | W35      | 10 22         | No possibility of placing infrastructure within the fill spaces of vaults. The project is abandoned for reasons related to conservation.                                                       |
Figure 5. Resultant membership functions for the fuzzy number of the value of the parameter of the probability of carrying out each scenario of the analysed process of the adaptation of a historical building to a new form of use

Figure 6. Resultant membership functions for the fuzzy number of the value of the parameter of the costs and duration of the performance of each scenario of the analysed process of the adaptation of a historical building to a new form of use

In order to comprehensively interpret the results, the authors performed a defuzzification of the obtained fuzzy numbers using the centre of mass method [21], which resulted in the obtainment of crisp values of the abovementioned parameters. However, these values should be treated only as supplemental information (Table 3) in the scope of the interpretation that has been performed above.

| Parameter type | The scenario of completing the adaptation to the proposed form of use (achieving node no. 20 in the network model) | The scenario of the abandonment of the project to a new form of use due to there being no possibility of meeting fire safety requirements (achieving node no. 21 within the network model) | The scenario of the abandonment of the project for reasons related to conservation and the limitation in the fitting of new infrastructure (achieving node no. 22 in the network model) |
|----------------|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| Probability of the occurrence of a scenario | 0.51 | 0.19 | 0.30 |
| Duration of the performance of a scenario [workdays] | 655.90 | 210.05 | 418.76 |
| The costs of the performance of a scenario [PLN] | 5407889.33 | 216118.99 | 544332.68 |
5. Conclusions
Before making a decision regarding the carrying out of a given project involving the adaptation of a historical building to a new form of use, a property developer should analyse various scenarios of its performance and familiarise oneself with the associated costs. In order to obtain this information, the property developer should make initial plans for a project.

The fundamental problem in the planning of an adaptation process is the difficulty of estimating the uncertain values of parameters such as: the probability of performing specific adaptation work and the costs associated with this work.

The authors of this work, in an attempt to address these problems, have used the capabilities of stochastic network models to reproduce the undetermined structure of this type of project, using the properties of type-1 fuzzy sets to model the uncertainty of the analysed parameters of such projects.

The paper proposes a triangular shape of the type-1 membership function for fuzzy numbers that model the uncertainty of opinions regarding the values of the parameters of a network model, providing a concept of its construction.

For the purposes of the analysis of the abovementioned model, the authors used a graph reduction method adjusted to arithmetical calculations on fuzzy numbers [5,6,20], additionally expanding the scope of the analysis of the network model by including the analysis of the estimated costs of the adaptation process of the historical building, which required the expansion of the aforementioned graph reduction method so that this parameter could be taken into account.

It needs to be pointed out that the implemented method of network planning is not a direct tool for the development of a timetable of the performance and control of a project. It is a helpful research tool, which can aid a property developer in the identification and analysis of its scenarios, as well as the possible values of the costs of its performance during the initial phase of the planning of an adaptation project.

Acknowledgment(s)
The authors would like to thank the companies: IPG (inter project group sp. z o.o.) and W.K. Geoplan Roland Chrobak for providing information on the subject of the Great Armoury in Gdansk.

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