Page charge of D-branes and its behavior in topologically nontrivial B-fields

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Abstract

The RR Page charges for the D(2p+1)-branes with B-field in type IIB supergravity are constructed consistently from brane source currents. The resulting Page charges are B-independent in nontrivial and intricate way. It is found that in topologically trivial B-field the Page charge is conserved, but in the topologically nontrivial B-field it is no longer to be conserved, instead there is a jump between two Page charges defined in each patch, and we interpret this jump as Hanany-Witten effect.
1. Introduction

Supergravity theories contain two types of Chern-Simons terms: one involves the wedge product of one potential with any number of field strengths, and the other appears in the kinetic term for the modified field strengths, which result in that the equations of motion for gauge fields are nonlinear in the gauge fields and the standard Bianchi identities turn into modified ones \[. \] These peculiar features make the definition of charge for gauge fields in supergravity theories much more subtle, actually, this is central issue raised by Bachas, Douglas and Schweigert in \[2\] where they found that the RR charges of the D2-branes in group manifold are irrational due to the integral \( \int B \). In \[3\], it was argued that the D0-brane charge arising from the integral over the D2-brane of the pullback of the B field is cancelled by the bulk contributions\[1\], so only \( \int F \) should be quantized. In \[5\], three notions of charge for D-brane in type IIA supergravity were proposed, and of particular interest are brane source charge and Page charge, the first is gauge invariant and localized, but not conserved, the second one is conserved, localized and invariant under small gauge transformation\[2\]. In \[6\], the B-independent RR charges of D2p-branes with B fields in type IIA supergravity were constructed explicitly from brane source charges by exploiting the equations of motion and the nonvanishing (modified) Bianchi identities, and were identified with Page charge from their properties — conserved and localized.

In type IIB supergravity there is no covariant action due to the self-dual field strength \( \tilde{F}_5 = \ast \tilde{F}_5 \), also because of the \( SL(2, R) \) symmetry of the type IIB theory, the \( \tilde{F}_5 \) is defined as \( F_5 + \frac{1}{2} C_2 \wedge H - \frac{1}{2} B \wedge F_3 \), but in type IIA supergravity the physical gauge invariant field strength is rather defined as \( \tilde{F}_{2p+2} = F_{2p+2} - C_{2p-1} \wedge H \). When 11D \( N = 1 \) supergravity is dimensionally reduced by compactification on a small circle in the \( X^{11} \) direction, the

\[1\] In \[4\], it rephrased the reasoning of \[3\] and showed that if one writes the Wess-Zumino action in terms of the gauge invariant field strength, the resulting RR brane charges are independent of B fields, but in their construction it is not clear what properties these RR charges possess.

\[2\] Since brane source charge and Page charge were defined separately in \[5\], it is unclear how to relate one to other which is essential to the question whether it is possible to consistently define the general RR Page charges for a D2p-brane from brane source charges.
resulting theory is type IIA supergravity, and 11-momentum turns to the D0-brane charge, which is much simpler than the relation between 11D \( N = 1 \) supergravity and type IIB supergravity. Due to these special features in type IIB supergravity, the approaches in [3], [4], [5] cannot be applied to type IIB case\(^3\), so it would be interesting to check whether it is possible to consistently construct Page charges for D(2p+1)-branes in B fields along the line of [6] and whether the resulting RR Page charges are B-independent or not. In [6], it was shown that the conserved Page charge can be defined consistently only if \( H = dB \) holds in the whole region. When it does not hold, for instance, in the background of NS5-branes, the NS B-field is topologically nontrivial and we have to cover \( S^3 \) with two patches, in each patch we can construct conserved Page charge, then one may ask how these two RR Page charges are connected and how to interpret this physical phenomenon. Because the simple example which encodes brane creation phenomenon [7]-[15] is the D3-brane probe in the background of \( \kappa \) coinciding NS5-branes [18], the further motivation of studying RR Page charges in type IIB supergravity is to survey which charge nonconservation is responsible for the Hanany-Witten effect.

Motivated by the above, in the present paper we discuss in type IIB supergravity how to construct B-independent RR charges for D(2p+1)-branes in B-field from brane source charges. Starting from IIB supergravity plus D-brane sources, we derive the equations of motion and the nonvanishing (modified) Bianchi identities that define the duals of brane source currents for D(2p+1)-branes. We insert the equations for the duals of brane source currents for D(2p+1+2n)-branes into that for the D(2p+1)-brane iteratively, and find that the resulting equations can be recast into the form whose left sides of equations are exterior derivative and the right sides are localized objects which indicates that the right side localized objects can be identified as Page charges because of their conservation and locality. Plugging the brane source charges into the expression of the Page charges, we find that all the Page charges are independent of the background B fields, actually it is highly nontrivial and intricate that all the B-dependent terms from different sources

\(^3\)For instance, Marolf’s argument that Page charges are independent of B-field [6] does not hold in type IIB case.
are exactly cancelled with each other. However in the construction of the Page charges, we assume that in the whole region the relation \( H = dB \) holds, and resulting Page charges are conserved in this region. To see how Page charges behave in topologically nontrivial B-field, we consider D3-brane probe in the background of \( \kappa \) coincident NS5-branes where in the spherical coordinates we have \( dH = 0 \), but \( \int H \neq 0 \). With this particular example, we show that the nonconservation of brane source charge does not describe the Hanany-Witten effect, it only reflects the fact that the brane source charge depends on the background nonconstant B-field, thus we clear up the puzzle raised.

In the topological nontrivial B-field, we show that the Page charge is no longer conserved, instead there is a jump between two Page charges defined in each patch. We interpret this jump as Hanany-Witten effect, in other words, we find a new way to describe brane creation phenomenon.

The paper is organized as follows. In section 2, the Page charges for D\((2p+1)\)-branes in topologically trivial B-field are constructed explicitly from brane source charges by exploiting the equations of motion and the nonvanishing (modified) Bianchi identities. It is shown that all Page charges are independent of B fields. In section 3, the D3-brane probe in the background of \( \kappa \) coincident NS5-branes is discussed, the Page charge of D1-branes is calculated. We find there is a jump between the Page charges constructed in each patch, and interpret it as Hanany-Witten effect. The nonconservation of brane source charge is attributed to the fact \( dB \neq 0 \). In section 4, we present our summary and conclusion.

### 2. Page charges for D\((2p+1)\)-branes in B fields

For IIB supergravity in ten dimensions there is no covariant action due to the self-dual field strength \( \tilde{F}_5 = \ast \tilde{F}_5 \), but the field equations from the following action are consistent with \( \tilde{F}_5 = \ast \tilde{F}_5 \) \[1\]

\[
S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\Phi} \left( R + 4 \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} |H|^2 \right) 
- \frac{1}{2} (|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2} |\tilde{F}_5|^2) \right\} 
- \frac{1}{4\kappa_{10}^2} \int C_4 \wedge H \wedge F_3 \tag{1}
\]
where $\kappa^2_{10}$ is the gravitational coupling in ten dimensions, and the field strengths $H, F_1, F_3, F_5, \tilde{F}_3$, and $\tilde{F}_5$ are defined as:

$$H = dB, \quad F_1 = dC_0, \quad F_3 = dC_2, \quad F_5 = dC_4, \quad \tilde{F}_3 = dC_2 - C_0 \wedge H, \quad \tilde{F}_5 = dC_4 + \frac{1}{2} C_2 \wedge H - \frac{1}{2} B \wedge F_3$$

(2)

The field equations from the above action are consistent with $\tilde{F}_5 = \ast \tilde{F}_5$, but they do not imply it, this must be imposed as an added constraint on the solutions, and it cannot be imposed on the action or else the incorrect equations of motion result. This formulation is satisfactory for a classical treatment, however it is not simple to impose the constraint in the quantum theory but this will not be important for our following purposes. The above action can be recast into

$$S_{IIA} = \int d^{10}x \sqrt{-G} e^{-2\Phi} (R + 4\partial_\mu \Phi \partial^\mu \Phi)$$

$$- \frac{1}{2} \int \left\{ - e^{-2\Phi} H \wedge \ast H + F_1 \wedge \ast F_1 + \tilde{F}_3 \wedge \ast \tilde{F}_3 + \frac{1}{2} \tilde{F}_5 \wedge \ast \tilde{F}_5 + C_4 \wedge H \wedge F_3 \right\}$$

(3)

where we have chosen $2\kappa^2_{10} = 1$ so that the kinetic term is canonical.

A D(2p+1)-brane in the type IIB supergravity background has a world-volume action given by a sum of Born-Infeld and Wess-Zumino terms

$$S_{D-brane} = S_{BI} + S_{WZ}$$

(4)

The Born-Infeld part of the action is

$$S_{BI} = -T_{2p+1} \int e^{-\Phi} \sqrt{-det(G_{ab} + B_{ab} + F_{ab})}$$

(5)

where $G_{ab}, \Phi, B_{ab}$ are the pullback of spacetime metric, dilaton and Neveu-Schwarz two form B to the D(2p+1)-brane world-volume, $F_{ab}$ is the field strength of U(1) gauge field living on the brane. The Wess-Zumino (WZ) terms couple the brane to the spacetime RR gauge field through

$$S_{WZ} = \int \left( \sum \ C^{(i)} \right) \wedge e^{B+F}$$

(6)

\footnote{There is minus sign difference for the field B between our notation and that in \cite{1}.}
2.1 D1-brane in B fields

In this subsection we discuss D1-brane in B fields, the Wess-Zumino term for D1-brane is

\[ S_{WZ}^{D1} = \int \left\{ C_2 + C_0 \wedge (B + F) \right\} \]  

(7)

which indicates that there are D-instantons living on the D-string. Since D-branes are sources of RR gauge field, we consider IIB supergravity + the coupled D-brane source, and see how D-string, D-instantons induce \( C_2, \ C_0 \) gauge field, which can be described by the equations of motion. As the duals of brane source currents arises from varying the brane action with respect to the gauge field, from the total action \( S_T = S_{IIB} + S_{D1} \) we get the equations of motion for \( C_0 \) and \( C_2 \)

\[ \begin{align*}
  d * F_1 - H \wedge * F_3 &= - * j_{D(-1)}^{bs} \\
  d * F_3 - H \wedge F_5 &= - * j^{bs}_{D1} 
\end{align*} \]  

(8)

where in the derivation of the second equation of (8), we have used the fact \( H \wedge H = 0 \). Since the brane source current is gauge invariant the left side of the second equation should be written in terms of the physical field strengths \( \tilde{F}_3 \) and \( \tilde{F}_5 \). Generally speaking, we should write each equation of motion for the gauge fields as a polynomial in the gauge invariant improved field strengths, their hodge duals, and exterior derivatives of these and then let the right hand side be some \(*j\) which are associated with some D-branes, NS5-branes or fundamental strings. The brane source currents for D1-brane and D-instantons are

\[ (j^{bs}_{D1})^{\mu \nu}(x) = \int dX^\mu(\xi) \wedge dX^\nu(\xi) d^{10}(x - X(\xi)) \]  

(9)

\[ (j^{bs}_{D(-1)})(x) = \int d^2 \xi \epsilon^{ab} [B_{\mu \nu}(X(\xi)) \partial_a X^\mu(\xi) \partial_b X^\nu(\xi) + F_{ab}(\xi)] \delta^{10}(x - X(\xi)) \]  

(10)

where \( x^\mu \) are ten-dimensional coordinates, \( X^\nu(\xi) \) are the embedding coordinates of D-string in ten dimensions, and \( \xi^a \) are two-dimensional brane worldvolume coordinates. The brane source charge for D-instantons is

\[ Q^{bs}_{D(-1)} = \int * j^{bs}_{D(-1)} = \int_{V_2} (B + F) \]  

(11)
where $V_2$ is two-dimensional Euclidean worldsheet of D-string.

For the convenience of the following discussion, we write down the equation of motion (or modified Bianchi identity due to self-duality of $\tilde{F}_5$) for $C_4$ and the (modified) Bianchi identity for $\tilde{F}_3$ and $F_1$ without the brane sources for D3-, D5-, D7-branes

$$d \ast \tilde{F}_5 + H \wedge \tilde{F}_3 = d\tilde{F}_5 + H \wedge \tilde{F}_3 = 0$$ (12)

$$d\tilde{F}_3 + H \wedge F_1 = 0$$ (13)

$$dF_1 = 0$$ (14)

where we have impose the self-dual condition $\ast \tilde{F}_5 = \tilde{F}_5$. If we rewrite IIB supergravity in terms of $C_6$ and $C_8$ due to $C_2$ and $C_0$, the Bianchi identities (13) and (14) turn into the equation of motion for $C_6$ and $C_8$. Since there are only D-string and D-instantons on the background of B-field without any other brane sources, so we have the equations (12)–(14). It is easy to see $d \ast j^{bs}_{D1} = 0$, $d \ast j^{bs}_{D(-1)} \neq 0$, that is, $Q^{bs}_{D1}$ is conserved but $Q^{bs}_{D(-1)}$ nonconserved.

Since the brane source charge $Q^{bs}_{D(-1)}$ is not conserved, we explore whether it is possible to find some conserved charge for D-instantons from brane source currents defined by (8), and (12)-(14). In doing so, we first rewrite the second term in the first equation of (8) $H \wedge \ast \tilde{F}_3$ as $d(B \wedge \ast \tilde{F}_3) - B \wedge d \ast \tilde{F}_3$, then insert the second equation of (8) into the term $B \wedge d \ast \tilde{F}_3$, besides the exterior derivative and localized term (in the following description, we do not mention such terms), we have $B \wedge H \wedge \ast \tilde{F}_5$ which can be written as $\frac{1}{2}d(B \wedge B \wedge \tilde{F}_5) - \frac{1}{2}B \wedge B \wedge d\tilde{F}_5$. Plugging (12) into $\frac{1}{2}B \wedge B \wedge d\tilde{F}_5$, we leave with $\frac{1}{2}B \wedge B \wedge H \wedge \tilde{F}_3$ which is recast into $\frac{1}{6}d(B \wedge B \wedge B \wedge \tilde{F}_3) - \frac{1}{6}B \wedge B \wedge B \wedge d\tilde{F}_3$. We further insert (13) into $\frac{1}{6}B \wedge B \wedge B \wedge d\tilde{F}_3$ and have the term $\frac{1}{24}B \wedge B \wedge B \wedge d\tilde{F}_1$. Exploiting (14) and shifting the exterior derivative terms on the left side of the equation and the localized terms on the right side, then we arrive at

$$d\Delta_{D(-1)} = \ast j^{bs}_{D(-1)} - B \wedge \ast j^{bs}_{D1}$$ (15)

This is our strategy to derive RR Page charge for D-brane, in the following subsections we will exploit it.
with
\[ \Delta_{D(-1)} = - * F_1 + B \wedge * \tilde{F}_3 - \frac{1}{2} B \wedge B \wedge \tilde{F}_5 - \frac{1}{6} B \wedge B \wedge B \wedge \tilde{F}_3 - \frac{1}{24} B \wedge B \wedge B \wedge B \wedge F_1 \] (16)

In the above derivation, we have assumed that the D1-brane stays in the region where the relation \( H = dB \) holds. Since Page charge is conserved and localized [5], [6], Eq. (15) shows that the dual of the Page current for D-instantons are
\[ * j^{Page}_{D(-1)} = * j^{bs}_{D(-1)} - B \wedge * j^{bs}_{D1} \] (17)

Here we should mention that in [5], the brane source charge and Page charge were defined separately, the Page charge was given just by definition, they did not discuss how to define Page charge based on the definition for brane source charge, but here (17) is derived from the brane source currents. The RR Page charge is
\[ Q^{Page}_{D(-1)} = \int * j^{Page}_{D(-1)} = \int_{V_2} F \] (18)

which is independent of the background NS B-field. If we do T-duality along other extra four dimensions, the D(-1)/D1 bound state turns into D3/D5 one, and Eq.(17) becomes
\[ * j^{Page}_{D3} = * j^{bs}_{D3} - B \wedge * j^{bs}_{D5} \] (19)

which means we define the RR Page current for D3-branes living on the D5-brane in the way of compatible with T-duality.

### 2.2 D3-brane in B fields

The Wess-Zumino term for D3-brane in the background of B fields is
\[ S^{D3}_{WZ} = \int \left\{ C_4 + C_2 \wedge (B + F) + \frac{1}{2} C_0 \wedge (B + F) \wedge (B + F) \right\} \] (20)

which induces D3-, D1-brane and D-instanton sources, thus Eq.(12) is broken and the nonvanishing of modified Bianchi identity which describes the hodge dual of the D3-brane source current is
\[ d * \tilde{F}_5 + H \wedge \tilde{F}_3 = d \tilde{F}_5 + H \wedge \tilde{F}_3 = - * j^{bs}_{D3} \] (21)
where the brane source current can be derived by variation of the Wess-Zumino term for D3-brane (20) with respect to $C_4$, and we have taken the self-dual condition for $\tilde{F}_5$ into account. Eq.(20) shows that the brane source charges for D-strings and D-instantons in the D3-brane are

$$Q_{bs}^{D_1} = \int \ast j_{bs}^{D_1} = \int (B + F)$$

$$Q_{bs}^{D_{(-1)}} = \int \ast j_{bs}^{D_{(-1)}} = \int \frac{1}{2} (B + F) \wedge (B + F)$$

(22)

From the equations of motion (8), the nonvanishing Bianchi identity (21) and two (modified) Bianchi identities (13) and (14), we see that $Q_{bs}^{D_3}$ is conserved, but $Q_{bs}^{D_1}$ and $Q_{bs}^{D_{(-1)}}$ are not, so we have to look for some conserved charges for D-strings and D-instantons. Making use of the technique mentioned above Eq.(15), from the equations of motion (8), the nonvanishing Bianchi identity (21) and two (modified) Bianchi identities (13) and (14), we reach two equations whose left sides are total derivative and right sides as localized term

$$d \Delta_{D_1} = \ast j_{bs}^{D_1} - B \wedge \ast j_{bs}^{D_3}$$

$$d \Delta_{D_{(-1)}} = \ast j_{bs}^{D_{(-1)}} - B \wedge \ast j_{bs}^{D_1} + \frac{1}{2} B \wedge B \wedge \ast j_{bs}^{D_3}$$

(23)

(24)

where $\Delta_{D_{(-1)}}$ is given in (16) and $\Delta_{D_1}$ is defined as

$$\Delta_{D_1} = - \ast F_3 + B \wedge \tilde{F}_5 + \frac{1}{2} B \wedge B \wedge \tilde{F}_3 + \frac{1}{6} B \wedge B \wedge B \wedge F_1$$

(25)

Eqs.(23) and (24) show that the hodge duals of Page currents for D-strings and D-instantons can be consistently defined

$$\ast j_{Page}^{D_1} = \ast j_{bs}^{D_1} - B \wedge \ast j_{bs}^{D_3}$$

$$\ast j_{Page}^{D_{(-1)}} = \ast j_{bs}^{D_{(-1)}} - B \wedge \ast j_{bs}^{D_1} + \frac{1}{2} B \wedge B \wedge \ast j_{bs}^{D_3}$$

(26)

(27)

From Eqs.(24 and 26 and 27) the RR Page charges for D-strings and D-instantons are

$$Q_{Page}^{D_1} = \int \ast j_{Page}^{D_1} = \int F$$

$$Q_{Page}^{D_{(-1)}} = \int \ast j_{Page}^{D_{(-1)}} = \int \left[ \frac{1}{2} (B + F) \wedge (B + F) - B \wedge (B + F) + \frac{1}{2} B \wedge B \right]$$

$$= \int \frac{1}{2} F \wedge F$$

(28)
where we have seen the B-dependent terms $B \wedge B$, $B \wedge F$ have been cancelled with each other. In [2] and [3], it was argued that the RR Page charge for D1-branes in D3-brane defined by $\int F$ should be quantized [13]. In the context of WZW model, it has been shown that the integral $\int F$ is quantized indeed [10].

### 2.3 D5-brane in B fields

We turn to a D5-brane in B fields, the Wess-Zumino terms for D5-brane is

$$S_{WZ}^{D5} = \int \left\{ C_6 + C_4 \wedge (B + F) + \frac{1}{2} C_2 \wedge (B + F) \wedge (B + F) \right. + \left. \frac{1}{6} C_0 \wedge (B + F) \wedge (B + F) \wedge (B + F) \right\} \tag{29}$$

which describes D5/D3/D1/D(-1) bound state. In the presence of D5-brane source, the modified Bianchi identity (13) turn to be nonzero

$$d\tilde{F}_3 + H \wedge F_1 = *j_{D5}^{bs} \tag{30}$$

Rewriting the term $H \wedge \tilde{F}_3$ in (21) as $d(B \wedge \tilde{F}_3) - B \wedge d\tilde{F}_3$ and inserting (30) into $B \wedge d\tilde{F}_3$, besides the exterior derivative and localized terms, we have the term $\frac{1}{2} d(B \wedge B) \wedge F_1$ which can written as $\frac{1}{2} d(B \wedge B \wedge F_1) - \frac{1}{2} B \wedge B \wedge dF_1$. Exploiting the Bianchi identity for $F_1$ (14), we get the new equation whose left side is exterior derivative and the right side is localized term

$$d\Delta_{D3} = *j_{D5}^{bs} - B \wedge *j_{D5}^{bs} \tag{31}$$

with

$$\Delta_{D3} = -*\tilde{F}_5 - B \wedge \tilde{F}_3 - \frac{1}{2} B \wedge B \wedge F_1 \tag{32}$$

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6In IIB supergravity, the RR gauge field $C_{2p}$ is dual to $C_{(8-2p)}$. Because of $**A_{2p+1} = -A_{2p}$ in 10D Euclidean spacetime [1], there are two ways to performing the electromagnetic duality. For example, consider D5-brane RR gauge field $C_6$ (for illustration, we assume B fields vanish), we can define either $*dC_6 = dC_2$, i.e., $*dC_2 = -dC_6$, or $*dC_2 = dC_6$, i.e., $*dC_6 = -dC_2$. In case 1, the D5-brane source current is described by $dF_3 = *j_{D5}^{bs}$, and in case 2, it is $dF_3 = -*j_{D5}^{bs}$. The WZ term does not contain any information about which definition one should use [17]. In order to make the definition for RR Page current compatible with T-duality, we have to choose $dF_3 = *j_{D5}^{bs}$. 

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which shows that the properly defined RR Page current is

\[ *J_{D3}^{Page} = *j_{D3}^{bs} - B \wedge *j_{D5}^{bs} \]  

(33)

Here we should point out that Eq.(33) is derived from the nonvanishing modified Bianchi identity (21) for D3-brane, the other nonvanishing Bianchi identity (30) for D5-brane, and the Bianchi identity for \( F_1 \) (14). On the other hand, Eq.(19) is obtained from T-duality, which means the ambiguity in the definition of \( C_6 \) can be fixed by T-duality. By exploiting of the equations of motion (8), two nonvanishing modified Bianchi identities (21) and (30), the Bianchi identity (14), and in the similar way used in subsection 2.1, we get the relations

\[ d\Delta_{D1} = *j_{D1}^{bs} - B \wedge *j_{D3}^{bs} + \frac{1}{2}B \wedge B \wedge *j_{D5}^{bs} \]

\[ d\Delta_{D(-1)} = *j_{D(-1)}^{bs} - B \wedge *j_{D1}^{bs} + \frac{1}{2}B \wedge B \wedge *j_{D3}^{bs} - \frac{1}{6}B \wedge B \wedge B \wedge *j_{D5}^{bs} \]  

(34)

where \( \Delta_{D1} \) and \( \Delta_{D(-1)} \) are defined in (23) and (16). Eq.(34) indicates that the hodge dual of the RR Page currents for D-strings and D-instantons should be defined as

\[ *J_{D1}^{Page} = *j_{D1}^{bs} - B \wedge *j_{D3}^{bs} + \frac{1}{2}B \wedge B \wedge *j_{D5}^{bs} \]  

(35)

\[ *J_{D(-1)}^{Page} = *j_{D(-1)}^{bs} - B \wedge *j_{D1}^{bs} + \frac{1}{2}B \wedge B \wedge *j_{D3}^{bs} \]

\[ -\frac{1}{6}B \wedge B \wedge B \wedge *j_{D5}^{bs} \]  

(36)

Recall that for D5/D3/D1/D(-1) bound state, the brane sources can be read off from the WZ term for D5-brane (23), the corresponding brane source charges are

\[ Q_{D3}^{bs} = \int *j_{D3}^{bs} = \int (B + F) \]

\[ Q_{D1}^{bs} = \int *j_{D1}^{bs} = \int \frac{1}{2}(B + F) \wedge (B + F) \]

\[ Q_{D(-1)}^{bs} = \int *j_{D(-1)}^{bs} = \int \frac{1}{6}(B + F) \wedge (B + F) \wedge (B + F) \]  

(37)

\(^7\)If we choose \( dF_3 + H \wedge F_1 = -*j_{D5}^{bs} \) instead of Eq.(30), then we would have \( *J_{D3}^{Page} = *j_{D3}^{bs} + B \wedge *j_{D5}^{bs} \), which is incompatible with T-duality.
Inserting (37) into (33), (35) and (36), we obtain the Page charges for D3-, D1-branes

\[ Q_{Page}^{D3} = \int *j_{Page}^{D3} = \int F \]  

\[ Q_{Page}^{D1} = \int \left( *j_{D1}^{bs} - B \wedge *j_{D3}^{bs} + \frac{1}{2} B \wedge B \wedge *j_{D5}^{bs} \right) \]
\[ = \int \left\{ \frac{1}{2} (B + F) \wedge (B + F) - B \wedge (B + F) + \frac{1}{2} B \wedge B \right\} \]
\[ = \int \frac{1}{2} F \wedge F \]  

and for D-instantons

\[ Q_{Page}^{D\left(-1\right)} = \int \left( *j_{D\left(-1\right)}^{bs} - B \wedge *j_{D1}^{bs} + \frac{1}{2} B \wedge B \wedge *j_{D3}^{bs} - \frac{1}{6} B \wedge B \wedge *j_{D5}^{bs} \right) \]
\[ = \int \left\{ \frac{1}{6} (B + F) \wedge (B + F) \wedge (B + F) - \frac{1}{2} B \wedge (B + F) \wedge (B + F) \right. \]
\[ + \frac{1}{2} B \wedge B \wedge (B + F) - \frac{1}{6} B \wedge B \wedge B \}
\[ = \int \frac{1}{6} F \wedge F \wedge F \]  

Eq.(40) shows that the terms \( B \wedge B \wedge B, B \wedge B \wedge F, B \wedge F \wedge F \) in the RR Page charge of D-instantons living in D5-brane are cancelled with each other nontrivially.

**2.4 D7-brane in B fields**

The Wess-Zumino term for D7-brane in B fields is

\[ S_{WZ}^{D7} = \int \left\{ C_8 + C_6 \wedge (B + F) + \frac{1}{2} C_4 \wedge (B + F) \wedge (B + F) \right. \]
\[ + \frac{1}{6} C_2 \wedge (B + F) \wedge (B + F) \wedge (B + F) \]
\[ + \frac{1}{24} C_0 \wedge (B + F) \wedge (B + F) \wedge (B + F) \wedge (B + F) \} \]  

which shows that the brane source charges for D5-, D3-, D1-branes, and D-instantons in D7-brane, are

\[ Q_{bs}^{D5} = \int *j_{D5}^{bs} = \int (B + F) \]
\[ Q_{D3}^{bs} = \int j_{D3}^{bs} = \int \frac{1}{2} (B + F) \wedge (B + F) \]

\[ Q_{D1}^{bs} = \int j_{D1}^{bs} = \int \frac{1}{6} (B + F) \wedge (B + F) \wedge (B + F) \]

\[ Q_{D(-1)}^{bs} = \int j_{D(-1)}^{bs} = \int \frac{1}{24} (B + F) \wedge (B + F) \wedge (B + F) \wedge (B + F) \quad (42) \]

In the presence of the D7-brane source, the Bianchi identity (14) becomes nonvanishing one which describes the hodge dual of the D7-brane source current

\[ dF_1 = - * j_{D7}^{bs} \quad (43) \]

In the same principle as in the subsection 2.1, from the equations of motion (8) and the nonvanishing (modified) Bianchi identity (21), (30) and (43), we have

\[ d\Delta_{D5} = * j_{D5}^{bs} - B \wedge * j_{D7}^{bs} \]

\[ d\Delta_{D3} = * j_{D3}^{bs} - B \wedge * j_{D5}^{bs} + \frac{1}{2} B \wedge B \wedge * j_{D7}^{bs} \]

\[ d\Delta_{D1} = * j_{D1}^{bs} - B \wedge * j_{D3}^{bs} + \frac{1}{2} B \wedge B \wedge * j_{D5}^{bs} \]

\[ - \frac{1}{6} B \wedge B \wedge * j_{D7}^{bs} \]

\[ d\Delta_{D(-1)} = * j_{D(-1)}^{bs} - B \wedge * j_{D1}^{bs} + \frac{1}{2} B \wedge B \wedge * j_{D3}^{bs} \]

\[ - \frac{1}{6} B \wedge B \wedge * j_{D5}^{bs} + \frac{1}{24} B \wedge B \wedge B \wedge * j_{D7}^{bs} \quad (44) \]

with

\[ \Delta_{D5} = \tilde{F}_3 + B \wedge F_1 \quad (45) \]

Eq. (44) shows that the hodge duals of the RR Page currents for the D5-, D3-, D1-branes, and D-instantons in the D7-brane can be defined as

\[ * j_{Page}^{D5} = * j_{D5}^{bs} - B \wedge * j_{D7}^{bs} \]

\[ j_{Page}^{D3} = * j_{D3}^{bs} - B \wedge * j_{D5}^{bs} + \frac{1}{2} B \wedge B \wedge * j_{D7}^{bs} \]

\[ j_{Page}^{D1} = * j_{D1}^{bs} - B \wedge * j_{D3}^{bs} + \frac{1}{2} B \wedge B \wedge * j_{D5}^{bs} \]
\[
\begin{align*}
\dot{j}_{D(-1)}^{Page} &= \ast j_{D(-1)}^{bs} - B \wedge \ast j_{D1}^{bs} + \frac{1}{2} B \wedge \ast j_{D3}^{bs} \\
&\quad - \frac{1}{6} B \wedge B \wedge \ast j_{D7}^{bs} + \frac{1}{24} B \wedge B \wedge B \wedge \ast j_{D1}^{bs} + \frac{1}{24} B \wedge B \wedge B \wedge \ast j_{D5}^{bs} \\
\end{align*}
\]

Plugging Eq. (42) into (46), the Page charges are given by

\[
Q_{D5}^{Page} = \int \ast j_{D5}^{Page} = \int F
\]

\[
Q_{D3}^{Page} = \int \left( \ast j_{D3}^{bs} - B \wedge \ast j_{D5}^{bs} + \frac{1}{2} B \wedge B \wedge \ast j_{D7}^{bs} \right) \\
\quad \int \left\{ \frac{1}{2} (B + F) \wedge (B + F) - B \wedge (B + F) + \frac{1}{2} B \wedge B \right\} \\
= \int \frac{1}{2} F \wedge F
\]

\[
Q_{D1}^{Page} = \int \left( \ast j_{D1}^{bs} - B \wedge \ast j_{D3}^{bs} + \frac{1}{2} B \wedge B \wedge \ast j_{D5}^{bs} - \frac{1}{6} B \wedge B \wedge \ast j_{D7}^{bs} \right) \\
= \int \left\{ \frac{1}{6} (B + F) \wedge (B + F) \wedge (B + F) - \frac{1}{2} B \wedge (B + F) \wedge (B + F) \right. \\
\quad \left. + \frac{1}{2} B \wedge B \wedge (B + F) - \frac{1}{6} B \wedge B \wedge B \right\} \\
= \int \frac{1}{6} F \wedge F \wedge F
\]

\[
Q_{D(-1)}^{Page} = \int \left( \ast j_{D(-1)}^{bs} - B \wedge \ast j_{D1}^{bs} + \frac{1}{2} B \wedge B \wedge \ast j_{D3}^{bs} - \frac{1}{6} B \wedge B \wedge \ast j_{D5}^{bs} \\
\quad + \frac{1}{24} B \wedge B \wedge B \wedge \ast j_{D7}^{bs} \right) \\
= \int \left\{ \frac{1}{24} (B + F) \wedge (B + F) \wedge (B + F) \wedge (B + F) \right. \\
\quad - \frac{1}{6} B \wedge (B + F) \wedge (B + F) \wedge (B + F) + \frac{1}{4} B \wedge B \wedge (B + F) \wedge (B + F) \\
\quad - \frac{1}{6} B \wedge B \wedge B \wedge (B + F) + \frac{1}{24} B \wedge B \wedge B \wedge B \right\} \\
= \int \frac{1}{24} F \wedge F \wedge F \wedge F
\]
Eq. (50) shows that all the B-dependent terms like $B \wedge B \wedge B \wedge B$, $B \wedge B \wedge B \wedge F$, $B \wedge B \wedge F \wedge F$, and $B \wedge F \wedge F \wedge F$ in the RR Page charge of D-instantons living in the D7-brane are cancelled with each other in intricate way.

3. D3-brane probe in topologically nontrivial B fields

In section 2, we have constructed the conserved RR Page charges for D-branes in B fields, where we have exploited the properties of Page charges – conserved and localized. However, in the construction we have used the assumption that in the whole region in which D-branes move, the relation $H = dB$ holds. In this section, we study how the RR Page charge of D-brane moving in topologically nontrivial B fields behaves. In the context of supergravity, the simple case for brane creation effect [7]-[15] is D3-brane probe in the background of $\kappa$ coinciding NS5-branes [18]-[19] where there is topologically nontrivial B field.

The background fields around a stack of $\kappa$ coinciding flat NS5-branes are given by [20]

$$
\begin{align*}
 ds^2 &= dx^2 + f dy^2, \\
 e^{2\Phi} &= g_s^2 f, \\
 H_{klm} &= -\epsilon_{klmn} \partial_n f
\end{align*}
$$

(51)

where $\{x^\mu\} = (x^0, x^1, \cdots, x^5)$ parameterize the directions along the NS5-branes, $\{y^m\} = (y^6, y^7, y^8, y^9)$ are locations of the NS5-branes, and $g_s$ is the string coupling far from the branes. The harmonic function $f$ depends on the transverse space

$$
\begin{align*}
 f &= 1 + \frac{kl_s^2}{r^2} \\
 r &= |\vec{y}| = \sqrt{kl_s e^\phi}.
\end{align*}
$$

(52)

In the spherical coordinates, the metric and $H$ turn into

$$
\begin{align*}
 ds^2 &= dx^2 + fr^2 (d\phi^2 + d\Omega_3^2), \\
 H &= 2kl_s^2 \omega_3,
\end{align*}
$$

(53)
where $d\Omega^2$ and $\omega_3$ are the metric and volume form on the unit 3-sphere $S^3_{6789}$. The NS 3-form field strength $H$ satisfies $dH = 0$ and $\int_{S^3} H \neq 0$. For the following discussions, we choose the cylindrical coordinates $(z, \rho, \theta, \varphi)$

$$(y^6, y^7, y^8, y^9) = (z, \rho \cos \theta, \rho \sin \theta \cos \varphi, \rho \sin \theta \sin \varphi) \quad (54)$$

and spherical coordinates to replace $(z, \rho)$ by

$$(z, \rho) = (r \cos \psi, r \sin \psi) \quad (55)$$

where $\theta \in [0, \pi]$, $\varphi \sim \varphi + 2\pi$, $\psi \in [0, \pi]$.

Consider a D3-brane probe in the background (51), the D3-brane action is

$$S_{D3} = -T_3 \int e^{-\Phi} \sqrt{-\det(G_{ab} + B_{ab} + F_{ab}) + \int (C_4 + C_2 \wedge (B + F))} \quad (56)$$

Here we assume $(B + F) \wedge (B + F) = 0$, so there are only D-strings living in the D3-brane, but no D-instantons. The equations of motion of the D3-brane in the background (51) can be derived by the variation of the action (56). The BPS equation is [18]

$$\frac{dz}{d\rho} = -\frac{1}{1 + e^{2\phi}} \frac{\psi - \psi_0 - \frac{1}{2} \sin 2\psi}{\sin^2 \psi} \quad (57)$$

The solution of the BPS equation (57) gives the shape of the D3-brane. The typical feature for this BPS D3-brane configuration is that it includes an infinite tube which can be interpreted as D1-brane. The angle $\psi_0$ has a simple geometrical meaning: opening angle as shown in figure 1. Especially when $z_{\text{max}} \to \infty$, the infinite tube can be identified with D-strings [18] and [19].

The 2-form $B + F$ on the D3-brane is given [2], [18], [19]

$$\mathcal{F} = B + F = \kappa l^2_s (\psi - \frac{1}{2} \sin 2\psi - \psi_0) \sin \theta d\theta \wedge d\varphi \quad (58)$$

from the discussion in section 2, we know $\int_{S^2} \mathcal{F}$ is D1-brane source charge and not conserved.
When the observer stays at the same side of D3-brane, i.e., $z > 0$, one can choose the gauge in which the NS B-field is proportional to the volume form of the two-sphere spanned by $(\theta, \phi)$ \[2\] and \[18\]
\[
B = \kappa l_s^2 (\psi - \frac{1}{2} \sin 2\psi) \sin \theta d\theta \wedge d\phi
\]
which is the smooth choice everywhere except at the point $\psi = \pi$ ($z < 0$). In general, if the observer is at one side of the NS5-branes, the proper gauge choice for NS B-field is that its singular point should be at the other side of NS5-branes. Eq. (59) shows that the singular point $\psi = \pi$ is indeed at the opposite side of the observer relative to NS5-branes $z < 0$. From Eq. (58) and (59), the corresponding $U(1)$ gauge field strength $F$ is
\[
F = -\kappa l_s^2 \psi_0
\]
Here we should mention that the parameter $\psi_0$ should be quantized not only from WZW model \[16\] but also from supergravity consideration \[21\]
\[
\psi_0 = \frac{n\pi}{\kappa}
\]
The RR Page charge of D1-branes on the D3-brane in the gauge (59) is
\[
Q_{Page}^{D1} = \frac{1}{(2\pi l_s)^2} \int_{S^2} F = -\frac{\kappa}{(2\pi)^2} \int_{S^2} \psi_0 = -n
\]
For definiteness, we assign the positive charge of D1-branes to represent that D1-branes emanate from the D3-brane to the NS5-branes, then Eq.(52) can be described by figure 2, which means if the observer stays at the same side of the D3-brane \((z > 0)\) relative to the NS5-branes, he/she measures \(n\) D1-branes directing away from the NS5-branes and ending to the flat D3-brane.

Figure 2: When the observer is at the same side of the D3-brane, he/she measures \(n\) D1-branes emanate from the NS5-branes to the D3-brane.

If the observer is at the opposite side of the D3-brane, i.e., \(z < 0\), one has to choose the other gauge

\[
B = \kappa l_s^2 (\psi - \pi - \frac{1}{2} \sin 2\psi) \sin \theta d\theta \wedge d\phi
\]

which is singular at \(\psi = 0\) but smooth at \(\psi = \pi\). Because \(B + F\) is gauge invariant, two different choice of the NS B-field result in different \(F\). From Eqs.(58) and (53), the other \(F\) is given

\[
F = \kappa l_s^2 (\pi - \psi_0)
\]

Then the RR Page charge of D1-branes in the other gauge is

\[
Q_{\text{Page}}^{D1} = \frac{1}{(2\pi l_s)^2} \int_{S^2} F = -\frac{\kappa}{(2\pi)^2} \int_{S^2} (\pi - \psi_0) = \kappa - n
\]

which indicates that if the observer stays at the opposite side of the D3-brane \((z < 0)\), he/she measures \(\kappa - n\) D1-branes direct away from NS5-branes and end to the flat D3-brane which is illustrated in figure 3.
Figure 3: When the observer is at the opposite side of the D3-brane ($z < 0$), he/she measures $\kappa - n$ D1-branes emanate from the NS5-branes to the D3-brane.

Under the reversal $z \rightarrow -z$, that is, if we reassign the observer stays at the side of $z > 0$ but the asymptotically flat D3-brane is at $z_{\text{max}} \rightarrow -\infty$, we have figure 4 which is equivalent to figure 3. In the context of WZW model, Fig.2 corresponds to the Cardy boundary state $|n >_c$, the reversal $z \rightarrow -z$ is realized by the rotation operator $exp\{i\pi(J_0^3 - \bar{J}_0^3)\}$. Acting the rotation operator on the Cardy boundary state $|n >_c$, we have $exp\{i\pi(J_0^3 - \bar{J}_0^3)\}|n >_c = |\kappa - n >_c$ which is described by Fig.4 [15].

Figure 4: The other equivalent description of figure 3, if we assume the observer stays at the side of $z > 0$ but the flat D3-brane is at $z_{\text{max}} \rightarrow -\infty$.

Combining figure 4 and figure 2, the observer (staying in the region $z > 0$) has the
The following physical picture: when $t \to -\infty$, the D3-brane initially locates at $z_{\text{max}} \to -\infty$ where there are $n - \kappa$ D1-branes emanating from the NS5-branes to the D3-brane, late the lower D3-brane passes through the $\kappa$ coinciding NS5-branes and finally stays at $z_{\text{max}} \to \infty$ but there are $n$ D1-branes directing away the NS5-branes and ending on the D3-brane, so in the whole physical process, $\kappa$ D1-branes have been created which is nothing but the Hanany-Witten effect [7]-[15]. In the above discussion, we find that in the topologically nontrivial B-field, the RR Page charge is no longer to be conserved, instead there is a jump in the RR Page charge of the D1-branes when the lower D3-brane crosses the NS5-branes, and it is this jump which describes the brane creation phenomenon.

In the above discussion, we see that due to the topological nontriviality of the NS B-field, there are two different gauge choices for it, one is singular at the south pole $\psi = \pi$ and the other is singular at the north pole $\psi = 0$. Each gauge choice for NS B-field induces different $U(1)$ gauge field strength $F$ which is related to the number of the D1-branes suspending between the NS5-branes and asymptotically flat D3-brane. The jump between two different $F$ can be interpreted as brane creation effect. It is reminiscent of the string creation in the system of the D2-brane probe and the D6-brane background which can be obtained by dimensional reduction from 11D Taub-NUT solution [14], its potential is

$$A_S^\psi = \frac{R}{2} (1 + \cos \theta) \quad (66)$$

where $R$ is the radius of the circle of the eleventh dimension, and the superscript $S$ denotes the coordinates regular at the south pole $\theta = \pi$. The metric of Taub-NUT solution can be chosen to be nonsingular at the south pole but singular at the north pole $\theta = 0$. The singularity at the north pole can be shifted to the south pole by the gauge transformation

$$x_{10}^N = x_{10}^S + R\varphi \quad (67)$$

thus the direction of Dirac string is changed and the potential turns into

$$A_N^\psi = \frac{R}{2} (-1 + \cos \theta) \quad (68)$$

By proper embedding of an M2-brane in the 11D Taub-NUT configuration, it was found that the M2-brane has no winding around eleventh-direction in the coordinate system.
regular at the south pole, but it can wind around this direction indeed in the coordinate system regular at the north pole obtained by the gauge transformation (67), which implies string charge creation from the ten-dimensional point of view [14].

In section 2, we have seen that when the NS B-field is not constant, i.e., \( H = dB \neq 0 \), the RR brane source charge is not conserved. Now we survey whether the nonconservation of the brane source charge encodes any information about Hanany-Witten effect or not. The above D3-brane probe in the NS5-brane background is an ideal laboratory to implement this idea. The equations of motion and (modified) Bianchi identities which describe the D3-brane in the background of the NS5-branes are given by

\[
\begin{align*}
d^* F_1 - H \wedge * \tilde{F}_3 &= 0 \\
d^* \tilde{F}_3 - H \wedge \tilde{F}_5 &= - * j^{bs}_{D1} \\
d^* \tilde{F}_5 + H \wedge \tilde{F}_3 &= d\tilde{F}_5 + H \wedge \tilde{F}_3 = - * j^{bs}_{D3} \\
d\tilde{F}_3 + H \wedge F_1 &= 0 \\
dF_1 &= 0
\end{align*}
\] (69)

In the spherical coordinates, the NS H field strength for the \( \kappa \) coincident NS5-branes satisfies

\[
dH = 0, \quad \int_{S^4} H \neq 0
\] (70)

By exploiting of Eqs.(69) and (70), we find that

\[
d \ast j^{bs}_{D3} = 0
\] (71)

\[
d \ast j^{bs}_{D1} = H \wedge * j^{bs}_{D3}
\] (72)

which indicates that the brane source charge for the D3-brane is conserved, but for D1-branes it is not. In order to see what the physical meaning of Eq.(72) is and whether it encodes the Hanany-Witten effect, we integrate (72) over nine dimensional manifold \( M_9 \) and have

\[
\int_{M_9} d \ast j^{bs}_{D1} = \int_{M_9 \cap D3} H = \int_{S^4} H
\] (73)
Since \( (j_{D1}^{bs}(x))^{\mu
u} \sim \int d^4\xi \epsilon^{abcd} \partial_a X^\mu \partial_b X^\nu (B_{\rho\sigma} \partial_c X^\rho \partial_d X^\sigma + F_{cd}) \delta^{10}(x - X) \), it is easy to check \( d \star j_{D1}^{bs} \sim d(B + F) = H \), and Eq.(73) is a sort of identity \( d(B + F) = H \). Then we find that the nonconservation of the brane source charge of the D1-branes (72) reflects the fact that \( dB = H \neq 0 \), which has nothing to do with Hanany-Witten effect.

4. Summary and Conclusion

In the above, we have derived the RR Page charges for D(2p+1)-branes with topologically trivial B-field from brane source charges in type IIB supergravity. We have considered IIB supergravity plus D-brane sources, from which we have obtained the equations of motion and the nonvanishing (modified) Bianchi identities that define the duals of brane source currents for D(2p+1)-branes. By inserting the equations for the duals of brane source currents for D(2p+1+2n)-branes \((n > 1)\) into that for the D(2p+1)-brane iteratively, we have found that the resulting equations can be recast into the form whose left sides of equations are exterior derivative and the right sides are localized objects, which indicates that the right side localized objects can be identified as Page charges because of their conservation and locality. Since there are two types of Chern-Simons terms in supergravity theories, one is the wedge product of one potential with two field strengths which induces the equation of motion for gauge fields with the universal form \( df^{8-2p} \pm H \wedge f_{5-2p} = - \star j_{2p+2} \) the other appears in the kinetic term for the modified field strengths which results in nonvanishing of modified Bianchi identities with the same forms as the equations of motion. The universal forms \( df^{8-2p} \pm H \wedge f_{5-2p} = - \star j_{2p+2} \) (for the equations of motion and nonvanishing of modified Bianchi identities) make it feasible to rewrite the equations in the form whose left sides are exterior derivative and right side are localized terms, which shows that the Page charges for D(2p+1)-branes can be consistently defined from the brane source charges indeed. Plugging the brane source charges into the expression of the Page charges, we have shown that all the Page charges are independent of the background B fields. In our explicit construction, it is highly non-

\footnote{For D7-brane source, it is \( dF_2 = - \star j_{D7} \), but in our case it is the last equation for the dual of brane source current.}
trivial that the B-dependent terms like $B \wedge B \wedge B \wedge B$, $B \wedge B \wedge B \wedge F$, $B \wedge B \wedge F \wedge F$ and $B \wedge F \wedge F \wedge F$ from different sources are exactly cancelled with each other. After discussing the RR Page charges in topologically trivial B-field, we have turned to the topologically nontrivial case. In order to study how Page charges behave in topologically nontrivial B-field, we have considered D3-brane probe in the background of $\kappa$ coincident NS5-branes. With this example, we have shown that the nonconservation of brane source charge does not describe to the Hanany-Witten effect, it only reflects the fact that the brane source charge depends on the background nonconstant B-field. In the topological nontrivial B-field, we have found that the Page charge is no longer conserved, instead there is a jump between two Page charges defined in each patch, and we have interpreted this jump as Hanany-Witten effect.

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