Determination of bulog regional sub-division in east java using connected domination number theory

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Abstract. Bulog is a state-owned public company engaged in food logistics. There are about 18 Bulog warehouses in East Java Province with an uneven distribution. Because the scope of Bulog's business includes logistics or warehousing, surveys and pest eradication, provision of plastic bags, transportation business, food commodity trading and retail business, there must be good coordination between regional sub-divisions and warehouse complexes. In this study, the minimum regional sub-division recommendations in the East Java region were determined using the theory of connected domination number in graphs. The location of the warehouse complex is described as the vertex and adjacent warehouse complexes are connected by edges. In this research, we also determine the connected domination number of unicyclic graph and vertex amalgamation operations for several classes of graphs.

1. Introduction
BULOG is a state-owned public company engaged in food logistics. The company's business scope includes logistics/warehousing, surveys and pest control, supply of plastic bags, transportation business, food commodity trading and retail business. As a company that continues to carry out public duties from the government, BULOG continues to carry out activities to maintain the Basic Purchase Price for grain, stabilize prices, especially basic prices, distribute rice for social assistance (Bansos) and manage food stocks [1].

BULOG in an area is tasked with meeting the rice consumption needs not only for its own region, but BULOG is also tasked with supplying other regions with a rice deficit. At Perum BULOG, the distribution of rice stocks is known as “MoveNas” which stands for Move Nasional. Prior to the Move Nas, Perum BULOG will meet stock needs in all warehouses in each region through procurement from local farmers [1]. If the stock in the area has been met, the rice will be moved to other areas that need it to meet the rice stock in the area. This is done so that the distribution of rice throughout Indonesia is even. In order to optimize this role, it is necessary to prepare a strategy in terms of regulating which Bulogs have excess stock, so that other Bulogs which usually require supplies can be observed in order of priority. The distance and access between Bulogs also need to be considered. To observe this, we can use connected dominating number theory. Perum BULOG has more than a thousand warehouse units spread throughout Indonesia which are managed by 26 Regional Offices. These warehouses are used to store rice reserves that have been absorbed by local farmers. Of all Regional Offices, there are 3 regions that have the largest storage capacity. In the first position, the East Java Regional Office has a total of 378 warehouse units with a total capacity of more than one million tons.
The distribution of Bulog's warehouse units can be described as points on the graph, while the easy and close distribution access is analogous to the edges connecting the points. Furthermore, from the graph formed, it can be determined which main warehouses are considered as supplying warehouses for rice stocks for other areas.

Domination is one of the topics discussed in graph theory which is known from the philosophy of the chess game. Dominating Number is one of the interesting topics in graph theory. The dominating number has existed since 1850 [2]. It appeared among chess fans in Europe, namely the determination of how many queens should be placed on an 8 × 8 chessboard, so that all tiles on the chessboard can be controlled by the queen and the number of queens placed on the board and it must be minimal. The problem of domination is one of the most widely studied topics in graph theory [3]. Several studies on dominating number have been developed, including: On Domination Number of Cartesian Product of Graphs yang diteliti oleh Gravier dan Mollard [4], The Domination Number of The Cartesian Products of Path and Cycles [5]. While in 1997, topics related to dominating number were developed namely connected dominating set [6]. Other research related to the dominating of the graph, such as Split Domination Number of Some Special Graphs [7,10] and Results Connecting Domination, Steiner and Steiner Domination Number of Graphs by Ramalakshmi and K. Palani [8]. The concept of connected dominating set is the suitable concept for Determination of Bulog Regional Sub-Divisions. In 1996 Some inequalities about connected domination number are given by Cheng Bo and Bollian Liu in 1996 [9]. And in 2013, some researchers can determine the Bounds on The Connected Domination Number of a Graph based on the value of girth, minimum degree, and connected order sum number [11].

The formal definition of dominating set and dominating of graph are showed in Definition 1 and Definition 2 as below.

**Definition 1.** [12] Let G be a graph with vertex set V and edge set E. Let S be the subset of vertex set V. If every vertex in V\S is adjacent to minimum one vertex of S then S is said to be a dominating set.

**Definition 2.** [12] The size of a smallest dominating set is referred as the dominating number of a graph G denoted by γ(G).

While the main difference of dominating set and connected dominating set is if the connected dominating set must be in the form of subgraph of the main graph like in Definition 3. Figure 1 gives the difference among dominating set and connected dominating set of a graph.

**Definition 3.** [6] A dominating Set D is said to be connected dominating set, if the induced sub graph D is connected. The connected domination number γ_c (G) is the minimum cardinality of a connected dominating set.

![Figure 1. A graph G with dominating number γ(G) = 3 and connected dominating number γ_c(G) = 4.](image)

2. Results and Discussion

We start this section by giving the observation concerning on the connected domination number of unicyclic graph such as cycle with two pendant, pan graph, subdivision of pan graph, and subdivision of sun graph.
Observation 1. Let $C_n^2$ be a cycle with two pendant graph, for $n \geq 3$, $\gamma_c(C_n^2) = \gamma_c(C_n)$.

In Theorem 4 we know that $\gamma_c(C_n) = n - 2$. For $C_n^2$ graph, it has minimum connected dominating set if two vertices which adjacent to the pendant are elements of connected dominating set, so it can dominate both of the pendants. Thus, we can say that $\gamma_c(C_n^2) = \gamma_c(C_n)$.

Observation 2. Let $P(n)$ be a pan graph, for $n \geq 3$, $\gamma_c(P(n)) = \gamma_c(C_n)$.

This case is equivalent with observation 9, the difference is pan graph just has one pendant. Thus the minimum connected dominating set is $n - 2$ where the vertex whose degree equals three is also element of connected dominating set. Therefore we can say that $\gamma_c(P(n)) = \gamma_c(C_n)$.

\[\text{Figure 2. Connected dominating set of unicyclic graph.}\]

Observation 3. Let $S(P(n))$ be a subdivision of pan graph, for $n \geq 3$, $\gamma_c(S(P(n))) = 2n - 1$.

Subdivision of pan graph can be said as a cycle graph which one of the vertex has degree equals tree and connected to two vertices (it form a path with length is two). Then the connected dominating set will be minimum if we choose $2n - 2$ vertices in cycle are element of connected dominating set and also a vertex that adjacent to the vertex whose the degree equals tree. So, the connected dominating number is $\gamma_c(S(P(n))) = 2n - 2 + 1 = 2n - 1$.

Observation 4. Let $S(S(n))$ be a subdivision of sun graph, for $n \geq 3$, $\gamma_c(S(S(n))) = 3n - 1$.

Subdivision of sun graph has $n$ pendant, so all vertices which adjacent to the pendant must be elements of connected dominating set. To make a connected subgraph, so all vertices in the cycle also must be elements of connected dominating set. For getting the minimum cardinality, we can remove one vertex which the degree is two for not to be elements of connected dominating number. Then, $\gamma_c(S(S(n))) = 3n - 1$.

Next, we show the Connected Dominating Number of vertex amalgamation graph that will be describing on Theorem 1 to Theorem 4.

Theorem 1. Let $G = V_{[v]} \{W_{n_1}, W_{n_2}, ..., W_{n_t}\}$ is amalgamation vetex of wheel graph, then

\[\gamma_c(G) = \begin{cases} 
1, & \text{if } v \text{ is centre vertex of } W_{n_i}, 2 \leq i \leq t \\
 t + 1, & \text{if } v \text{ is outer vertex of } W_{n_i}, 2 \leq i \leq t 
\end{cases}\]

Proof: If the linkage vertex of $G = V_{[v]} \{W_{n_1}, W_{n_2}, ..., W_{n_t}\}$ is centre vertex of $W_{n_i}$ then for any $k \in V(G) \setminus v$ show that $d(v, k) = 1$. It means that $v$ can dominate all vertices of $V(G)$, thus $\gamma_c(G) = 1$. If the linkage vertex of $G = V_{[v]} \{W_{n_1}, W_{n_2}, ..., W_{n_t}\}$ is outer vertex of $W_{n_i}$ then for any $k$ element of outer vertex in every $W_{n_i}$ show that $d(v, k) = 2$. Let $S$ is connected dominating number of $G$, so we
can take \(v_i\) which the centre vertex of every \(W_{n_i}\) with \(d(v, v_i) = 1\) as the element of \(S\). Then the set \(S = \{v, v_i | 2 \leq i \leq t\}\) is subgraph of \(G\) and it can dominate all vertices of \(V(G)\), thus \(\gamma_c(G) = t + 1\).

**Theorem 2.** Let \(G = V^1_{\{v\}} \{F_{n_1}, F_{n_2}, \ldots, F_{n_t}\}\) is amalgamation vertex of friendship graph, then

\[
\gamma_c(G) = \begin{cases} 
1, & \text{if } v \text{ is centre vertex of } F_{n_i}, 2 \leq i \leq t \\
(t + 1), & \text{if } v \text{ is any vertex of degree } 2, 2 \leq i \leq t 
\end{cases}
\]

**Proof:** If the linkage vertex of \(G = V^1_{\{v\}} \{F_{n_1}, F_{n_2}, \ldots, F_{n_t}\}\) is centre vertex of \(F_{n_i}\) then for any \(k \in V(G)\setminus v\) show that \(d(v, k) = 1\). It means that \(v\) can dominate all vertices of \(V(G)\), thus \(\gamma_c(G) = 1\). If the linkage vertex of \(G = V^1_{\{v\}} \{F_{n_1}, F_{n_2}, \ldots, F_{n_t}\}\) is any vertex of degree 2 of \(F_{n_i}\) then for any \(k \in V(G)\setminus v\) show that \(d(v, k) = 2\). Let \(S\) is connected dominating number of \(G\), so we can take \(v_i\) which the centre vertex of every \(F_{n_i}\) with \(d(v, v_i) = 1\) as the element of \(S\). Then the set \(S = \{v, v_i | 2 \leq i \leq t\}\) is subgraph of \(G\) and it can dominate all vertices of \(V(G)\), thus \(\gamma_c(G) = t + 1\).

**Theorem 3.** Let \(G = V^1_{\{v\}} \{C_n, P_m, S_k, B_{x,y}\}\) is amalgamation vertex of cycle, path, star, and complete bipartite graph with the each order is \(n \geq 3, m \geq 2, k \geq 3, x, y \geq 2\), then

\[
\gamma_c(G) = \begin{cases} 
(n + m - 2), & \text{if } v \text{ of } S_k \text{ is one of the pendant}, v \text{ of } P_m \text{ is } v_1 \text{ or } v_m \\
(n + m - 3), & \text{if } v \text{ of } S_k \text{ is one of the pendant}, v \text{ of } P_m \text{ is } v_i \text{ for } 2 \leq i \leq n - 1 
\end{cases}
\]

**Proof:** A cycle graph has a connected dominating number that is the number of vertices minus 2 (Theorem 4). The connected dominating vertex in a cycle graph are all vertices except two neighboring vertices, in this case all vertices can be chosen to be the connected dominating vertex. In a path graph, the connected dominating vertex are all vertices of degree 2. Thus, there are vertices that cannot be a dominating vertex, namely a vertex of degree 1. However, for the first case, one of the vertex whose the degree 1 in path is linkage vertex for vertex amalgamation. The star graph has a connected dominating number equal to one and one of its pendant is a linkage vertex. Meanwhile, in a complete bipartite graph, the connected dominating number is equal to two, i.e. two adjacent vertices. One of these vertex is the linkage vertex. Therefore connected dominating number of \(G = V^1_{\{v\}} \{C_n, P_m, S_k, B_{x,y}\}\) is the sum of each connected dominating number in every graph except a complete bipartite graph because an element of connected dominating set set is a linkage vertex, then

\[
\gamma_c = \gamma_c(C_n) + \gamma_c(P_m) + \gamma_c(S_k) + (\gamma_c(B_{x,y}) - 1)
\]

\[
\gamma_c = (n - 2) + (m - 2) + 1 + (2 - 1) = n + m - 4 + 2
\]

\[
\gamma_c = n + m - 2
\]

While for the second case, the difference is that the linkage vertex of the graph \(P_m\) is a vertex of degree 2, then reduce one vertex of connected dominating number \(P_m\). Therefore

\[
\gamma_c = \gamma_c(C_n) + (\gamma_c(P_m) - 1) + \gamma_c(S_k) + (\gamma_c(B_{x,y}) - 1)
\]

\[
\gamma_c = (n - 2) + ((m - 2) - 1) + 1 + (2 - 1) = n + m - 5 + 2
\]

\[
\gamma_c = n + m - 3
\]

This number is minimum because subtracting just one from the connected dominating vertex will cause the subgraph that is formed to be unconnected.

As an example of using the theorem above, suppose graph \(G\) in Figure 3 is \(G = V^1_{\{v\}} \{C_6, P_6, S_7, B_{2,3}\}\), then it has \((6 + 6 - 2) = 10\) connected dominating number.
Figure 3. Amalgamation vertex of cycle, path, star, and complete bipartite graph.

**Theorem 4.** Let \( G = \bigvee_{[v]} \{ C_n, P_m, S_k, B_{x,y} \} \) is amalgamation vertex of cycle, path, star, and complete bipartite graph with the order is \( n \geq 3, m \geq 2, k \geq 3, x, y \geq 2 \), then

\[
\gamma_c(G) = \begin{cases} 
(n + m - 3), & \text{if } \nu \text{ of } S_k \text{ is a vertex with order } n - 1, \nu \text{ of } P_m \text{ is } \nu_1 \text{ or } \nu_m \\
(n + m - 4), & \text{if } \nu \text{ of } S_k \text{ is a vertex with order } n - 1, \nu \text{ of } P_m \text{ is } \nu_i \text{ for } 2 \leq i \leq n - 1
\end{cases}
\]

**Proof:** By the same procedure to the proof in Theorem 15, we can start by observe that a cycle graph with order \( n \) has a connected dominating number equals \( n - 2 \) (Theorem 4). The connected dominating vertex in a cycle graph are all vertices except two neighboring vertices, in this case all vertices can be chosen to be the connected dominating vertex. In a path graph, the connected dominating vertex are all vertices of degree 2. Thus, there are vertices that cannot be a dominating vertex, namely a vertex of degree 1. However, for the first case, one of the vertex whose the degree 1 in path is linkage vertex for vertex amalgamation. The star graph has a connected dominating number equal to one and it is the centre vertex or vertex whose the degree is \( k - 1 \). It is also a linkage vertex in \( G = \bigvee_{[v]} \{ C_n, P_m, S_k, B_{x,y} \} \). Meanwhile, in a complete bipartite graph, the connected dominating number is equal to two, i.e. two adjacent vertices. One of these vertex is the linkage vertex. Therefore connected dominating number of \( G = \bigvee_{[v]} \{ C_n, P_m, S_k, B_{x,y} \} \) is the sum of each connected dominating number in every graph except star and complete bipartite graph because an element of connected dominating set set is a linkage vertex, then

\[
\gamma_c = \gamma_c(C_n) + \gamma_c(P_m) + (\gamma_c(S_k) - 1) + (\gamma_c(B_{x,y}) - 1) \\
\gamma_c = (n - 2) + (m - 2) + (1 - 1) + (2 - 1) \\
\gamma_c = n + m - 4 + 1 \\
\gamma_c = n + m - 3
\]

While for the second case, the difference is that the linkage vertex of the graph \( P_m \) is a vertex of degree 2, then reduce one vertex of connected dominating number \( P_m \). Therefore

\[
\gamma_c = \gamma_c(C_n) + (\gamma_c(P_m) - 1) + (\gamma_c(S_k) - 1) + (\gamma_c(B_{x,y}) - 1) \\
\gamma_c = (n - 2) + ((m - 2) - 1) + (1 - 1) + (2 - 1) \\
\gamma_c = n + m - 5 + 1 \\
\gamma_c = n + m - 4
\]

This number is minimum because subtracting just one from the connected dominating vertex will cause the subgraph that is formed to be unconnected.

The last result is the case study of determination of bulog regional sub-divisions in east java using connected domination number theory. In this paper we only choose 18 vertex and 20 edes. Based on the location in the Figure 4, we represent the map into Bulog Office Graph or it call BU – graph like in Figure 5. Based on the theory of connected domination number and the result of the observation of BU – graph, we get 6 vetices which make a subgraph of BU – graph and can dominate all vertices in BU – graph. Thus, \( \gamma_c(BU – graph) = 6 \).
3. Conclusion
In this paper, we can determine the connected domination number of unicyclic graph and some vertex amalgamation graphs which consisting of wheel, friendship, cycle, path, star, and complete bipartite graphs with some possibilities for the position of linkage vertex amalgamation. By applying the theory, we also can determine the number of Bulog Regional Sub-divisions in east Java. From 18 location, we get 6 location can be recommended as Bulog Regional Sub-divisions to dominate all location.

Open problem 1. Find the connected domination number for another particular classes of graphs and the graphs obtained from graph operations such as edge amalgamation, join, shackle, etc.

Open problem 2. Find the algorithm to solve the connected domination number for case in real life.

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