Time evolution of the two-jet events
\( e^+ - e^- \rightarrow \text{hadrons} \) in the Dynamical String Model

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Abstract

The time dependent pion emission rate and the mean life time of the pion source in the center of mass frame of the two-jet events have been determined for \( e^+ - e^- \rightarrow \text{hadrons} \) at \( \sqrt{s} = 20 - 50 \) GeV by the Dynamical String Model. It was established that the time needed for the creation of the pion source is 5 fm/c, whereas its life time is \( \tau_0 \approx 7 - 13 \) fm/c for energies \( \sqrt{s} = 20 - 50 \) GeV, respectively.

Introduction

The purpose of this work is to obtain the reliable time evolution of the hadronization process for two-jet events \( e^+e^- \rightarrow \text{hadrons} \) at the energies \( \sqrt{s} \approx 20 - 50 \) GeV in the framework of the Dynamical String Model. This is achieved by making use of the particular sensitivity of the simulated Bose-Einstein correlation to the decay constant of the hadronic string. We show that it is possible to arrive at an agreeable quantitative description of both the Bose-Einstein correlation and the single particle data by an appropriate choice of the string decay constant, being close to its value predicted in Ref.\(^{[10]}\). Having settled the time scale of the hadronization process in this way, we determine the time dependent emission rate and the mean life time of the pion source.

The Dynamical String Model was developed in order to describe high energy hadronic processes\(^{[1]}\). In the model the hadrons are represented by classical open strings and their free motion is governed by the Nambu-Goto action. Two types of string interactions are introduced. The excited hadronic strings decay due to the quark-antiquark pair creation in the chromoelectric field of the flux tube represented by the string. The collision of hadronic
strings is modelled by the arm-exchange mechanism called rearrangement. It is assumed that the strings move freely between two interactions. In addition to, hadronic strings have a finite transverse size ensuring the right order of magnitude of the hadronic cross sections.

The parameters of the model are as follows. The string tension \( \kappa \approx 1 \) GeV/fm is determined from the slope of the leading Regge trajectory of hadron resonances. The radius \( R \) of hadronic strings is fitted to the proton-proton total cross section. The third parameter of the model, the decay constant of the hadronic strings can be expressed in terms of the string radius and the string tension making use of the analogy of the hadronic string with the chromoelectric flux tube \[2\]. Recently, there have been given theoretical arguments in favour of this analogy \[3\]. Nevertheless, such an analogy does not lead to an unambiguous relationship between the string tension \( \kappa \) and the chromoelectric field strength \( E \) as discussed in \[4\]. Changing this relationship modifies the value of the decay constant for a given radius \( R \). The limitations for the above mentioned relationship and for the possible radius values, and their optimal choice \( (eE = 1.5\kappa, R = 0.5 \text{ fm}) \) were given in \[4\] comparing simulated and experimental single-particle distributions for elementary hadronic processes.

In this work we introduce the discrete final state resonances in the Dynamical String Model and investigate their effect on the single-particle distributions for 2-jet events \( e^+e^- \rightarrow \text{hadrons} \) at \( \sqrt{s} = 20 - 50 \) GeV. Then we redetermine the string radius via the decay constant obtained by comparing the results of numerical simulations with experimental data for the Bose-Einstein correlation of identical pions. Finally we present the time dependence of the pion emission.

**Final state resonances**

At the very beginning of any kind of high-energy hadronic processes highly excited hadrons are produced. So it is plausible to describe these processes in terms of classical strings possessing a continuous mass spectrum. However, the experimentally observed particles are low energy hadrons which definitely have a discrete mass spectrum. Therefore the Dynamical String Model dealing originally only with strings belonging to the continuous mass spectrum, has been improved by introducing the discrete final state hadron resonances below the mass threshold 1 GeV. These final hadron states are described by strings in the rotating rod mode. Their properties (rest mass, decay width, the average momentum of their decay products) are chosen according to the phenomenology\[5\]. Thus all the mesons from \( \pi \) to \( a_0 \) are included except of those containing strange (anti)quarks.

The treatment of decays needs a special care when the parent particles belong to the continuous mass spectrum and the daughters to the discrete one. It is assumed that the probability of choosing any species of hadron
resonances is proportional to the degree of degeneracy of this state and the inverse of its rest mass squared \[6\].

The improved model described above was tested using the same fit parameters as for the test of its original version in \[4\]. We concluded that the single-particle distributions are in quantitative agreement with the experimental data for the 2-jet events $e^+e^- \to \text{hadrons}$ at $\sqrt{s} \approx 20 - 50 \text{ GeV}$\[7\]. The effect of the discrete hadron resonances turned out to be significant only for the transverse momentum distribution (Fig. 1). The width of the distribution increased due to the decay of discrete hadron resonances.

### Bose-Einstein correlations

The Bose-Einstein correlation has also been investigated in the framework of the Dynamical String Model to extract information about the detailed phase space structure of the simulated events $e^+e^- \to \text{hadrons}$. We show that the correlation function is rather sensitive to the value of the decay constant. Thus it provides a better tool to fix the parameters of the model than the single-particle distributions.

The Bose-Einstein correlation of identical bosons is the consequence of the symmetrization of their wave function. It is reflected on the enhancement of the number of like sign pion pairs at low momentum differences in hadronic processes.

The correlation function is defined by

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)}$$

(1)

where $P(p_1, p_2)$ is the two-particle momentum distribution and $P(p_i)$ is the single-particle momentum distribution. The product in the denominator of (1) can be replaced by $P_0(p_1, p_2)$ which is the two-particle distribution in the lack of quantum interference effects. If the source function $g(x, p)$ of the emitted particles is known, the correlation function in the plane wave approximation will read

$$C(p_1, p_2) = \frac{\int d^4x d^4x' g(x, p_1)g(x', p_2)[1 + \cos(p_1 - p_2)\mu(x - x')_\mu]}{\int d^4x d^4x' g(x, p_1)g(x', p_2)}$$

(2)

(with the Lorentz index $\mu$). For a homogeneous source, if the momentum of the emitted particle is independent of its space-time coordinate $x$, the source function is a product $g(x, p) = \rho(x)g(p)$. Then the correlation function is given by

$$C(p_1, p_2) = 1 + |\tilde{\rho}(p_1 - p_2)|^2$$

(3)

where the $\tilde{\rho}(p)$ is the Fourier transform of $\rho(x)$, and the width of the correlation function is proportional to the inverse of the size of the source in space.
The source function of the produced pions has been obtained by numerical simulation in the Dynamical String Model. The test for the homogeneity of the source function shows that only the direction of the momentum of the produced particle is independent of its space-time coordinate. The magnitude of the momentum increases as the spatial distance increases between the creation point of the particle and the point of the initial quark-antiquark pair creation. This can be expected from the picture of inside-outside cascade of the hadronization in the string model, where the low momentum states are populated first in time and consequently in space\([9]\).

Making use of the simulated source function the correlation function has been determined according to Eq. (2). Similarly to the presentation of the TPC data\([11]\), the correlation function was calculated as the function of the following single variables: \(q = |(p_1 - p_2)^\mu(p_1 - p_2)_\mu|^{1/2}\), \(q_T\) (the component of momentum difference \(p_1 - p_2\) perpendicular to the momentum sum \(p_1 + p_2\)) and \(q_0\) (the energy difference of the pion pair). In the latter two cases the restrictions \(q_0 < 0.2\) GeV and \(q_T < 0.2\) GeV/c were applied, respectively. Furthermore the same cuts were used as in the experiment to get an appropriate sample of events, namely only the pions with momentum \(0.15\) GeV/c < \(|p| < 1.45\) GeV/c were chosen.

The calculation was carried out at first for the parameters (taken from \([4]\)) giving the best fit to the single-particle distributions and practically no Bose-Einstein effect was found (Fig. 2). The width of the fitted Gaussian function gave for the radius of the pion source an order of magnitude larger value than the experimental source size in space-time. This was basically due to the rather large decay constant slowing down the pion emission tremendously.

At this point one can ask whether the input parameters can be fixed to reproduce the experimental data for the Bose-Einstein correlation function and the single-particle distributions simultaneously. Hence the correlation function is more sensitive to the variation of the decay constant than the single-particle spectra, we tried to fit the correlation function by an appropriate choice of the decay constant and then to find out the string radius and the proper relationship between the string tension \(\kappa\) and the chromoelectric field strength \(\mathcal{E}\). We found a qualitatively good description of the Bose-Einstein correlation for \(\epsilon\mathcal{E} = 2\kappa\) and \(R = 0.6\) fm (Figs. 3-5). The ‘new’ relationship and radius value are still in the range allowed by the single-particle spectra \([4]\). The value of the decay constant is now \(\Lambda = 1.25\) fm\(^{-2}\). This implies that the average ‘life time’ of the expanding quark-antiquark pair in the center of mass system is \(0.9\) fm/c, which is comparable to the ‘life time’ \((1.2 \pm 0.1\) fm/c\) of the string between two coloured nucleons extracted from the rapidity distribution of protons in the collisions of proton on proton\([10]\).

By adjusting the decay constant the qualitative agreement is only achieved for the \(q_0\) dependence of the correlation function (Fig. 5). This agreement guarantees that the time scale of the hadronization process is correct in our model.
The single-particle distributions like the transverse momentum distributions, the Feynman \(x_F\) distributions and the rapidity distributions are only slightly modified by the new decay constant (Figs. 6-8). The calculated average charged particle multiplicities show a softer energy dependence than the experimental data (Fig. 9).

It is the main advantage of the Dynamical String Model that it follows up the space-time evolution of the hadronization process directly. So having fixed the time scale of the process as described above, we determined the time dependence of the pion emission in the center of mass system of the colliding \(e^+e^-\) pair (Fig. 10). It can be seen that the pion emission rate has a maximum at about 5 fm/c independently of the c.m. energy.

For times \(t > 5\) fm/c the pion emission rate decreases exponentially \(dN/dt \propto e^{-t/T_0}\), where \(T_0\) is the mean life time of the pion source in the c.m. system of the 2-jet events (Table 1). These values of \(T_0\) cannot be compared directly to the mean life time \(\tau_0 = (0.62 \pm 0.25)\) fm/c of the pion source extracted from the dependence of the Bose-Einstein correlation function on \(q_0\) and \(q_T\) [11], because \(\tau_0\) measures the life time of the pion source in its rest frame. On the other hand from the ratio of the mean life time in the laboratory and to that in the rest frame the speed of the pion source can be estimated, \(\gamma = (1 - v^2/c^2)\) and \(\gamma = T_0/\tau_0\). Considering the very simplified picture that the pion emission takes place nearby the leading (anti)quark a kind of effective mass of the pion source can be estimated using the speed of the source (Table 2). This estimation leads to the rest mass \(\sqrt{s}/2\gamma \approx 1.0\) GeV of the pion source.

The maximum of the pion emission rate is found at around 2 fm/c proper time (Fig. 11). The slope of the decay of the pion source is determined by the characteristic proper time \(2.2\) fm/c.

**Conclusions**

It was established that the Dynamical String Model improved by introducing the discrete final state hadron resonances provides one of better quality single-particle spectra for the process \(e^+e^- \rightarrow \text{hadrons}\) at \(\sqrt{s} \approx 20 - 50\) GeV than the original version of the model. As the consequence of the decay of the discrete resonances the transverse momentum distributions are now in quantitative agreement with the experimental data for small transverse momenta up to \(p_T < 2\) GeV/c.

The simulations in the framework of the Dynamical String Model have shown that the Bose-Einstein correlation for the input parameters representing the best fit to the single-particle distributions (obtained in [1]) is in contradiction with the TPC experiment. It has also been established that the simulated Bose-Einstein correlation is rather sensitive to the value of the decay constant. Making use of the ambiguities of the hadronic string - flux tube analogy, it was possible to choose the string radius \(R = 0.6\) fm
| c.m. energy (GeV) | 22   | 35   | 50   |
|------------------|------|------|------|
| $T_0$ (fm/c)     | 7.35 ± 0.13 | 10.1 ± 0.1 | 13.2 ± 0.15 |

Table 1: $T_0$ parameters of the exponential decay of the pion emission rate (see text) for different c.m. energies.

| c.m. energy (GeV) | 22   | 35   | 50   |
|------------------|------|------|------|
| $m$ (GeV)        | 0.93 ± 0.02 | 1.07 ± 0.01 | 1.17 ± 0.01 |

Table 2: The "effective mass" of the pion source (see text) for different c.m. energies.

and achieve a decay constant providing a rather good agreement of both the simulated Bose-Einstein correlation and the single particle distributions with the experimental data. The appropriate string decay constant implies the life time 0.9 fm/c of the hadronic string which is close to the value obtained in Ref. [10].

Working with this decay constant that allows to recover the observed $q_0$ dependence of the Bose-Einstein correlation, the time scale of the hadronization process was fixed in a reliable way. Then the pion emission rate and the mean life time of the pion source were determined. It was established that the pion source builds up nearly in the first 5 fm/c in the laboratory frame (2 fm/c in proper time), independently of the c.m. energy. Its mean life time is linearly rising with the c.m. energy. Comparing the mean life time determined by the pion emission rate to the mean life time extracted from the Bose-Einstein correlation an effective rest mass (1.0 GeV) of the pion source nearby the leading (anti)quark can be found.

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Figure 1: Transverse momentum distributions. Lines (full, dotted, dashed) represent the simulations of the model with discrete resonances and using the original input parameters. Dotted-dashed line indicates the data obtained by the model without discrete resonances[1]. Markers are the experimental data taken from Ref. [12].
Figure 2: Correlation function $C$ of like sign pion pairs as a function of the four momentum difference ($q = |p_1 - p_2|$). The simulated data of the model with the original input parameters are represented by the full line. Markers are for the experimental data at 29 GeV of the TPC collaboration[11].
Figure 3: Correlation function $C$ of like sign pion pairs as a function of the four momentum difference ($q = |p_1 - p_2|$). The simulated data of the model with the input parameters giving the best fit are represented by the full line. Markers are for the experimental data of the TPC collaboration[11].
Figure 4: Correlation function $C$ of like sign pion pairs as a function of $q_T$, which is the component of the three momentum difference $\mathbf{p}_1 - \mathbf{p}_2$ perpendicular to the momentum sum $\mathbf{p}_1 + \mathbf{p}_2$, for the energy difference $q_0 < 0.2$ GeV/c (see Fig. 3. for notation).
Figure 5: Correlation function $C$ of like sign pion pairs as a function of the energy difference $q_0$, for $q_T < 0.2$ GeV/c (see Fig. 3. for notation).
Figure 6: Transverse momentum distributions. Lines represent the results of simulations with discrete resonances and using the input parameters fitted to the correlation functions. Markers are for the experimental data taken from Ref. [12].
Figure 7: The Feynmann $x_F \left( x_F = \frac{2p_t}{E_{cm}} \right)$ distributions. (Lines and markers are the same as in Fig. 6.)
Figure 8: Rapidity distributions. (Lines and markers are the same as in Fig. 6.)
Figure 9: The average multiplicity as the function of the c.m. energy.

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Figure 10: The time distribution of the pion emission in the c.m. system of the two-jet events (Lines are the same as in Fig. 6.)
Figure 11: The proper time distribution of the pion emission at c.m. energy 29 GeV.