Ferromagnetic Film on a Superconducting Substrate

L.N. Bulaevskii\textsuperscript{1,2} and E.M. Chudnovsky\textsuperscript{1}

\textsuperscript{1} Department of Physics and Astronomy, CUNY Lehman College
250 Bedford Park Boulevard West, Bronx, NY 10468-1589

\textsuperscript{2} Los Alamos National Laboratory, Los Alamos, NM 87545

Abstract

We study the equilibrium domain structure and magnetic flux around a ferromagnetic (FM) film with perpendicular magnetization $M_0$ on a superconducting (SC) substrate. At $4\pi M_0 < H_{c1}$ the SC is in the Meissner state and the equilibrium domain width in the film, $l$, scales as $(l/4\pi\lambda_L) = (l_N/4\pi\lambda_L)^{2/3}$ with the domain width on a normal (non-superconducting) substrate, $l_N/4\pi\lambda_L \gg 1$. Here $\lambda_L$ is the London penetration length. For $4\pi M_0 > H_{c1}$ and $l_N$ in excess of about $35\lambda_L$, the domains are connected by SC vortices. We argue that pinning of vortices by magnetic domains in FM/SC multilayers can provide high critical currents.
The interaction between magnetism and superconductivity has been intensively studied in the past, see, e.g., the review\(^1\). The discovery of high-temperature superconductors and advances in manufacturing of nanoscale multilayered systems have added new dimensions to these studies. In this paper we investigate equilibrium magnetic and superconducting phases in a system consisting of a ferromagnetic (FM) film with perpendicular magnetic anisotropy\(^2\) on the surface of a superconductor (SC). The interest to such systems is two-fold. Firstly, the needs of magneto-optic technology have produced a large variety of magnetic films with perpendicular magnetic anisotropy. Some are synthesized on metallic substrates (e.g., Nb) and are well-characterized at room temperature, including their domain structure\(^3\),\(^4\),\(^5\). We will show that, as the temperature of such a system is lowered, its magnetic state must be affected, in a non-trivial manner, by the superconducting transition. Secondly, in a multilayered SC/FM system, the domain structure in the FM layers should produce the pinning of superconducting vortices which may be significantly stronger than the pinning by magnetic dots\(^6\),\(^7\),\(^8\),\(^9\).

The system under consideration is shown in Fig.1. We are assuming no exchange of electrons between the FM film and the superconductor. This will be true either when the ferromagnet is an insulator or when it is separated from the superconductor by a thin insulating buffer layer. Then the FM film and the superconductor are coupled only by the magnetic field. (The systems with the exchange of electrons between the FM and SC layers have been discussed in Ref.\(^10\).) In this case the superconductivity makes a profound effect on the domain structure in the FM layer, which must be easy to detect in experiment. The physics behind this effect is explained below. Consider a FM film of thickness \(d_M\), with perpendicular magnetic anisotropy. In the absence of the superconductor adjacent to the film, its domain structure is determined by the balance of the energy of the magnetic field surrounding the film and the energy of domain walls. The positive energy of the magnetic field favors small domains, so that the field does not spread too far from the film. On the contrary, the positive energy of domain walls favors less walls, that is, large domains. The minimization of the total magnetic energy gives a well-known result\(^1\) for the equilibrium...
domain width, \( l \propto \sqrt{\delta d_M} \), with \( \delta \) being the domain wall thickness. Domains typically observed in magneto-optic films have thickness of a few micron. In the presence of a superconductor adjacent to the FM film, the balance of the magnetic energy changes drastically. This is because the magnetic field must be either expelled from the superconductor due to the Meissner effect or it should penetrate into the superconductor in the form of vortices. In the first case, the superconductor favors FM domains of width below the London penetration depth, \( \lambda_L \). If the room temperature domains are significantly greater then \( \lambda_L \), the effect of the SC phase transition on the domain structure will be dramatic. As we shall see, the new equilibrium will be achieved at \( l \propto (\delta d_M \lambda_L)^{1/3} \). Consequently, on lowering the temperature below the SC critical temperature, the domains in the FM film can shrink by an appreciable factor.

We are assuming the stripe domain structure in the FM film. The width of the FM domain, \( l \), is presumed large compared with the domain wall thickness \( \delta \). The latter is the smallest length in our consideration. Two other characteristic length are the thickness of the FM film, \( d_M \), and the London penetration depth, \( \lambda_L \), of the SC. In the case of \( l < \lambda_L \) the magnetic flux penetrates into the SC as it would penetrate into a normal non-magnetic metal, making superconductivity irrelevant. The case of interest is, therefore, \( l > \lambda_L \). We shall begin with the study of the Meissner state, that is, the state where equilibrium vortices are absent.

The free energy functional for the magnetic field, \( \mathbf{B} = [B_x(x, z), 0, B_z(x, z)] \), is

\[
\mathcal{F}(\mathbf{B}, \mathbf{M}) = \mathcal{F}_S(\mathbf{B}, \mathbf{M}) + \mathcal{F}_M(\mathbf{B}, \mathbf{M}) ,
\]  

(1)

where

\[
\mathcal{F}_S(\mathbf{B}) = \frac{1}{8\pi} \int dV [\lambda_L^2 (\nabla \times \mathbf{B})^2 + \mathbf{B}^2] \\
\mathcal{F}_M(\mathbf{B}, \mathbf{M}) = \int dV \left[ \frac{\mathbf{B}^2}{8\pi} - \mathbf{B} \cdot \mathbf{M} \right] + \mathcal{F}_D .
\]

(2)

Here \( \mathcal{F}_S \) is the free energy due to the magnetic field in the superconductor, \( \mathcal{F}_M \) is the free energy of the magnetic film and the empty space above the film, \( \mathbf{M}(x) \) is the magnetization
inside the magnetic film, and $F_D$ is the energy of domain walls. At $l \gg \delta$, a good approximation for $M(x)$ (see Fig.1) is the step-like function along the X-axis, $M(x) = \pm M_0$ inside the domains. Its Fourier expansion is

$$M(x) = \frac{4M_0}{\pi} \sum_{k=0}^{\infty} \frac{\sin[(2k+1)Qx]}{2k+1},$$

where $M_0$ is the magnetization and $Q = 2\pi/l$. For this domain structure $F_D(l) = \sigma d_M/l$. Here $\sigma = \sqrt{2}\beta M_0^2\delta/\pi$ is the energy of the unit area of the domain wall and $\beta M_0^2$ is the energy density of the perpendicular magnetic anisotropy.

The equilibrium distribution of the magnetic field should be obtained by the minimization of $F(B,M)$ at a given configurations of magnetic domains $M(x)$. Introducing $H = B - 4\pi M$, one obtains in terms of $H$:

$$F_M(H,M) = \int dV \left[ \frac{H^2}{8\pi} - 2\pi M^2 \right] + F_D, \quad F_S(H) = F_S(B = H).$$

The field $H$ is induced by alternating magnetic charges, $\nabla \cdot M$, on the two surfaces of the magnetic film. With account of the Maxwell equation, $\nabla \cdot B = 0$, it satisfies

$$\nabla \cdot H = -4\pi \nabla \cdot M = -4\pi [\delta(z) - \delta(z + d_M)]M(x), \quad \nabla \times H = 0$$

outside the superconductor, that is, inside the magnetic film, in the buffer layer, and in the empty space above the film. Here $\delta(z)$ is the delta-function, $z = 0$ and $z = -d_M$ are coordinates of the film surfaces. Inside the superconductor $H$ satisfies the London equation,

$$\nabla^2 H - \lambda_L^{-2} H = 0,$$

and the boundary condition that $H$ is continuous across the interface between the buffer layer and the superconductor. Equation (6) is valid if the magnetic field changes on the scale greater than the correlation length $\xi$. In our case the smallest relevant scale of spatial variations of the field is the width of FM domains $l$. We shall assume that $l \gg \xi$, which is relevant to most situations of practical interest.
Solving the above equations we obtain that, due to the domain structure, $H(x, z)$ decays exponentially away from the surfaces. Taking into account that $d_M \gg l$ and neglecting exponentially small terms of order $\propto \exp[-d_M(4\pi^2l^{-2} + \lambda_L^{-2})^{1/2}]$, we get

$$H(x, z) = \sum_q H_q \exp(-q|\bar{z}| + iqx)$$

(7)

for the magnetic field inside the superconductor, the film, and in the empty space. Here $\bar{z}$ is the distance along the $z$-axis from the nearest film surface. This gives the Fourier components

$$H_{z, q} = \sum_{k=0}^{\infty} \frac{4M_0}{2k + 1} \delta[q - Q(2k + 1)]$$

$$H_{z, -q} = -\sum_{k=0}^{\infty} \frac{4M_0}{2k + 1} \delta[q + Q(2k + 1)]$$

(8)

and $H_{x, q} = -iqzH_{z,q}/q$ with $q^2 = q^2 + \lambda_L^{-2}$ inside the superconductor and $q_z = q$ elsewhere.

Substituting this equilibrium magnetic field $H$ at a given $l$ into the free energy functional, Eq. (4), we obtain the following expressions for $F_S(l)$ and $F_M(l)$ per unit area:

$$F_S(l) = 4M_0^2 \frac{\lambda_L}{\pi Q^2} \sum_{k=0}^{\infty} \frac{1 + (2k + 1)^2Q^2\lambda_L^2}{(2k + 1)^4}$$

$$F_M(l) = 3F_S(l, \lambda_L^{-1} = 0) + F_D(l)$$

(9)

(10)

Above $T_c$ the free energy of the system as a function of $l$ is given by

$$F_N(l) = 4F_S(l, \lambda_L^{-1} = 0) + F_D$$

(11)

The minimization of (11) gives the well known result for the equilibrium width of the domains when the superconductor is in the normal state[11]

$$l_N = \left(\frac{\sqrt{2\pi}}{7\zeta(3)} \right)^{1/2} (\beta \delta d_M)^{1/2}$$

(12)

For the superconducting state of the substrate it is convenient to introduce $\bar{l} = l/4\pi\lambda_L$ and $\bar{l}_N = l_N/4\pi\lambda_L$. Then

$$F(\bar{l}) = \frac{8M_0^2}{\pi} \sum_{k=0}^{\infty} \frac{\bar{l}}{(2k + 1)^3} \left\{ 3 + \left[ 1 + \frac{4\bar{l}^2}{(2k + 1)^2} \right]^{1/2} + \frac{4\bar{l}_N^2}{\bar{l}^2} \right\}$$

(13)
The minimization of $F$ with respect to $\bar{l}$ produces the dependence of $\bar{l}$ on $\bar{l}_N$ shown in Fig. 2. At $\bar{l}_N \ll 1$ the field penetrates into the SC same way as it penetrates into the normal metal and $\bar{l} \approx \bar{l}_N$ (see insert to Fig. 2). In the opposite case of $\bar{l}_N \gg 1$, a rather accurate approximation is

$$\bar{l} \approx \bar{l}_N^{2/3}. \quad (14)$$

We, therefore, conclude that the SC phase transition in the substrate can result in a significant shrinkage of the equilibrium domain width in the FM film if the substrate is in the Meissner state.

The Meissner state studied above should always be the case when $4\pi M_0 < H_{c1}$. At $4\pi M_0 > H_{c1}$ the equilibrium energy of the Meissner state, $F_M$, should be compared with the energy of the vortex state. Vortex lines connecting domains with opposite magnetization can form only if $\bar{l}_N \gg 1$. The energy of the SC in the vortex state may be easily estimated in the limit of $M_0 \gg 4\pi H_{c1}$. In that case the average distance between vortices is small compared to their magnetic radius $\lambda_L$ and the average field in the SC is close to that in a normal metal. The corresponding total free energy of the system is then close to $F_N$. Some small corrections to that energy arise from the vortex line tension and from the repulsion of vortices, which is of order $k l (\Phi_0 M_0 / 8\pi \lambda_L^2) \ln(H_{c2} / 4\pi M_0)$ ($k \sim 1$ accounting for the curvature of vortex lines and for their additional energy near the FM surface). These corrections are small in the limit of large magnetization. Consequently, for the vortex state of the SC substrate, the equilibrium domain width in the FM film should be close to that on the normal substrate. For the ratio of the equilibrium free energies one obtains $F / F_N = [6/7\zeta(3)]\bar{l}_N^{1/3} \approx 0.713\bar{l}_N^{1/3}$, where $F$ is computed at $\bar{l} = \bar{l}_N^{2/3}$ and $F_N$ is computed at $\bar{l} = \bar{l}_N$. Thus, at $4\pi M_0 > H_{c1}$ and $\bar{l}_N \geq 2.8$ (that is for $l_N$ in excess of about $35\lambda_L$) the vortex state should be energetically more favorable than the Meissner state.

It should be emphasized that all conditions, obtained in this paper, depend on temperature through the temperature dependence of $\lambda_L$ and $l_N$. Nevertheless, the factor that determines the maximum shrinkage of equilibrium FM domains in the Meissner phase is
a universal number, \( \max(l_N/l) = \max(l_N^{1/3}) = 7\zeta(3)/6 \approx 1.4 \). This should be easy to detect in
experiment.

The effects described above fall within common experimental range of parameters. They
will be noticeable if the room-temperature domains in the FM film are wider than
\( l_N \sim 0.5 \) micron for the Nb substrate \( (\lambda_L \sim 40 \) nm) or wider than
\( l_N \sim 1.6 \) micron for a high-temperature SC \( (\lambda_L \sim 130 \) nm). Because equilibrium
\( l_N \) depends on the thickness of the FM film, \( d_M \), the
above condition on \( l_N \) translates, through Eq. (12), into the lower bound on \( d_M \). For, e.g.,
a TbFe film \( (\beta \sim 10^2 \) and \( \delta \sim 15 \) nm) the equilibrium domain width should decrease below
\( T_c \) in films of thickness greater than 0.3 micron on a Nb substrate or in films thicker than 3
micron on a high-\( T_c \) substrate. For TbFe film on a Nb substrate the Meissner state should
occur for \( l_N < 1.4 \) micron \( (d_M < 2.4 \) micron) and the vortex state should occur at \( l_N > 1.4 \)
micron \( (d_M > 2.4 \) micron). In the case of a high-\( T_c \) substrate the Meissner state should occur
for \( l_N < 4.5 \) micron \( (d_M < 25 \) micron), while at \( l_N > 4.5 \) micron \( (d_M > 25 \) micron) the vortex
state should occur.

In a real FM film the stripe domains are curved due to thermal fluctuations
and due to the pinning of domain walls. This, however, should not affect our conclusions as long as the
corresponding radius of curvature of domains is large compared with other characteristic
lengths. Since we are interested in the equilibrium magnetic structure due to the FM-SC
interactions, it is important to acknowledge that strong pinning of domain walls by
the imperfections may prevent the system from reaching that equilibrium. Possible ways to
study the equilibrium magnetic structures include choosing systems with low coercivity (that
is, weak pinning of domain walls), or low Curie temperature (below the critical temperature
of the SC), or rotating the system in a slowly decaying magnetic field. It should be also
possible to extract changes in the magnetic equilibrium from the study of the magnetic
hysteresis in the FM film above and below \( T_c \). A large variety of magnetic materials should
allow experiments in all interesting ranges of temperature and coercivity.

Finally, we would like to comment on the magnetic pinning of vortices in SC/FM mul-
tilayers. In the past the enhancement of pinning in conventional superconductors was done through manufacturing samples with microscopic defects. Several new approaches have been developed in recent years. They include manufacturing films with micrometer-size holes and depositing magnetic particles or magnetic dots onto SC films. For high temperature superconductors a remarkable effect has been achieved in samples with columnar defects produced by heavy ion irradiation. All the above methods are based upon introducing defects that suppress superconductivity. The pinning then arises from the tendency of the vortex normal core to match with the region where the superconductivity is suppressed. The maximal energy per unit length of the vortex, available for such pinning, is the condensation energy of Cooper pairs in the volume of the vortex core, \( (H^2_c/8\pi)\pi\xi^2 \approx (\Phi_0/8\pi\lambda L)^2 \). It is easy to see that the pinning of the magnetic flux of vortices in SC/FM multilayers can be significantly stronger than the pinning of vortex cores. As the problem of statistical mechanics it will be reported elsewhere. Here we shall just estimate the amplitude of the one-vortex pinning potential due to magnetic domains. This estimate can be obtained by considering the energy of an extra vortex, created by an external magnetic field in the presence of the domain structure in FM layers. The upper bound on the magnetic pinning energy is \( \Phi_0 M_0 \) per unit length of the vortex. For \( M_0=500 \text{ emu/cm}^3 \) it is one hundred times the energy of the pinning by columnar defects. The magnetic pinning of SC vortices will be effective if the field in the FM layers does not exceed the coercive field. Above that field the pinning of domain walls disappears and FM domains, together with SC vortices, become mobile. Thus, strong pinning of vortices favors large coercivity of the FM film, the condition opposite to the one required to observe the shrinkage of FM domains in the Meissner phase. Some evidence that magnetic dots produce more pronounced pinning than non-magnetic dots has been recently demonstrated in Nb films. Another study indicated that the irreversibility line of an YBCO film is pushed up when a barium ferrite film is deposited on the surface of the SC film. Thus, the study of SC/FM multilayers, besides being a fascinating "multidimensional" problem, can also be promising in developing SC systems with high critical currents.
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FIGURE CAPTIONS

Fig. 1.
FM film with stripe domains on a SC substrate.

Fig. 2.
Dependence of $\bar{l}$ (normalized equilibrium domain width on the SC substrate) on $\bar{l}_N$ (normalized domain width on the normal substrate).
