Granular hypoplastic interface model considering surface roughness

YongGwang Jong¹,², *, YongMin Ma¹,³, Yang Liu¹

¹School of Civil & Resource Engineering, University of Science and Technology Beijing, Beijing 100083, People’s Republic of China
²School of Material Engineering, Kim Chaek University of Technology, Pyongyang 999093, Democratic People’s Republic of Korea
³School of Resource prospecting Engineering, Kim Chaek University of Technology, Pyongyang 999093, Democratic People's Republic of Korea

*Corresponding author: b20180623@xs.ustb.edu.cn

Abstract. The problem of soil-structure interface is an important part in studying of soil structure interaction. The interface between soil and structure is widely present in water conservancy, civil engineering, municipal and transportation projects. Roughness is the main factor affecting the mechanical properties of the contact surface. From this requirement, a new contact model has been developed that takes into account the relative surface roughness. It is based on the granular hypoplasticity. The experimental results are compared with simulations using the Mohr-Coulomb model and the new model, respectively. It is shown that the new models can represent the non-linearity of the mechanical properties of the contact surface and agree well with the experimental data.

1. Introduction
The study of the mechanical properties of the interface between soil and structure has always been one of the important topics in civil engineering and geotechnical engineering. In particular, the modeling of interfaces is an important component in simulating soil-structure contact behavior. Therefore, many researchers have been begun to study suitable constitutive models. From past to present, many interface models are available, using elastic models, elasto-plastic model [1, 2] and damage models [3]. Recently, hypoplastic interface models are proposed to simulate the mechanical property of the contact surface. Hypoplasticity is a powerful tool for simulating the non-linearity of the soil behavior [4, 5]. To provide consistent modeling of the contact surface behavior, the hypoplastic model takes account of the frictional contact between soil and structures with the different roughness. However, the method of considering roughness is different according to the model [6, 7].

In this paper, the new model based on existing granular hypoplastic model was developed, using the new method to consider roughness. The new models have been validated by comparing with Mohr-Coulomb model and experimental data.

2. Hypoplastic interface models
The new model formulations are based on the granular hypoplastic model [8].
2.1. Used constitutive formulation

The framework of the hypoplastic model can be described as:

\[
\hat{T} = f_s \left( L : D + f_d N \| \hat{D} \right) \tag{1}
\]

where \( \hat{T} \) and \( D \) are the stress and strain rate tensors, respectively. \( N \) and \( L \) are the second- and fourth-order constitutive tensors. \( f_s \) is the stiffness factor that controls the influence of the mean stress, and \( f_d \) is the factor to consider the effect of relative density. The fourth order tensor \( L \) is written as:

\[
L = f_s \frac{1}{\hat{T} : \hat{T}} \left( F^2 I + a^2 \hat{T} \otimes \hat{T} \right) \tag{2}
\]

where \( \hat{T} = T / trT \) is a deviator stress and \( I \) is the fourth order unity tensor. The coefficient \( a \) is written as:

\[
a = \frac{\sqrt{3}(3 - \sin \varphi_c)}{2\sqrt{2}\sin \varphi_c} \tag{3}
\]

where \( \varphi_c \) is a critical friction angle. According to the Matsuoka-Nakai condition, the scalar coefficient \( F \) is defined by:

\[
F = \sqrt{\frac{1}{8} \tan^2 \psi + \frac{2 - \tan^2 \psi}{2 + \sqrt{2}\tan \psi \cos 3\theta} - \frac{1}{2\sqrt{2}} \tan \psi} \tag{4}
\]

with the Lode angle \( \theta \),

\[
\cos 3\theta = -\sqrt{6} \sqrt{\frac{tr(\hat{T}^*)}{\hat{T} : \hat{T}}}^{1/2} \tag{5}
\]

where \( \hat{T}^* = \hat{T} - \frac{1}{3} I \) is a deviator stress and \( \tan \psi = \sqrt{3} \| \hat{T}^* \|. \) The second-order constitutive tensor \( N \) is defined as:

\[
N = f_s f_d \frac{a \cdot F}{\hat{T} : \hat{T}} \left( \hat{T} + \hat{T}^* \right) \tag{6}
\]

The stiffness factor \( f_s \) is given as:

\[
f_s = \frac{h_s}{n} \left( \frac{e_i}{e} \right)^\beta \left[ \frac{1 + e_i}{e_i} \left( -tr(T) \right) \right]^{-1 - n} \left[ 3 + a^2 - a\sqrt{3} \left( \frac{e_i e_0 - e_0 e_i}{e_i e_0 - e_0 e_i} \right) \right]^{-1} \tag{7}
\]
The pyknotropy factor $f_d$ is written by:

$$f_d = \left( \frac{e - e_d}{e_c - e_d} \right)^\alpha$$  \hspace{1cm} (8)

2.2. Including the surface roughness into constitutive formulation

According to [9, 10], the following formulation is defined as:

$$\frac{\tan \delta_p}{\tan \phi_p} = 1.373 + 2.402 \log R$$  \hspace{1cm} (9)

where $\tan \delta_p$ and $\tan \phi_p$ are the peak friction coefficient of the sand-structure contact surface and sand under the same normal stress, respectively, and $R$ is the surface roughness. M. D. Bolton [11] based on Rowe’s stress dilatancy theory, proposed the relationship between the peak friction angle, the maximum dilatancy angle and the critical friction angle:

$$\delta_p = \phi_c + 0.8 \psi$$  \hspace{1cm} (10)

where $\psi$ is the peak dilatancy angle and $\phi_c$ is critical friction angle, respectively.

Combining Equation (9) and Equation (10), the critical friction angle of contact surface is described as:

$$\phi_c = \arctan[(1.373 + 2.402 \log R)\tan \phi_p] - 0.8 \psi$$  \hspace{1cm} (11)

Considering Equation (3) and Equation (11), it is found that the surface roughness $R$ was included into granular hypoplastic model formulation.

3. Simulation method

3.1. Outline of the numerical direct shear interface test

The size of the soil sample is $40 \times 10 \times 10 cm$ (Figure 1). The structural part has a dimension of $50 \times 18 \times 5 cm$. The structure has Young’s modulus of $E = 1 GPa$ and Poisson’s ratio of $\nu = 0.25$. The assembly was meshed by a linear interpolation of eight nodal elements (C3D8). The entire loading process is divided into two stages; the first stage applies $78 KPa(\sigma_n)$ pressure on the top surface of the soil and, in the second stage, the soil is horizontally displaced by $u_x = 6 mm$ in the positive direction. The friction characteristics of the contact surface are specified by the frictional subroutine (FRIC). The simulation was compared with the standard Mohr-Coulomb model and the experimental data.
3.2. Simulation scheme
The schematic diagram for the FRIC subroutine is shown in Figure 2.

4. Result
4.1. Comparison with experimental results
H. Kishida and M. Uesugi [12] used a simple and direct shear device to study the effect of surface roughness \( R_{\text{max}} = 46.0, 20.5 \text{ and } 2.4 \mu \text{m} \), respectively) on the relation between shear stress and shear strain under the constant normal stress \( \sigma_n = 78 \text{KPa} \). The sample was Toyoura sand. The tangential displacement increases at \( 1 \text{mm/s} \) to reach \( 6 \text{mm} \). The new model was used to reproduce the experimental data; the results are illustrated in Figure 3. The approach proposed in the hypoplastic model was validated, using the parameters for Toyoura sand (Table 1).
Table 1. Parameters for the evaluation of Hypoplastic contact model (Toyoura sand).

| Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|
| $\varphi_c$ $[^\circ]$ | 30 | $e_{\infty}$ | 1.10 |
| $h_s$ [MPa] | 2600 | $\alpha$ | 0.18 |
| $n$ | 0.27 | $\beta$ | 1.1 |
| $e_{\infty}$ | 0.61 | $\varphi_p$ $[^\circ]$ | 42.4 |
| $e_{\infty}$ | 0.98 | |

As shown in Figure 3, the simulation results agree well with the experimental data according to the different roughness. It is found that the shear stress increases, as the roughness increases. In other words, it shows the dependence of roughness. As a result, it can be seen that the hypoplastic model reproduces the experimental results and has high reliability. As mentioned earlier, the new model formulation includes items on non-linearity and the roughness, which fits well with the experimental results.

Figure 3. Verification using the experimental data of H. Kishida and M. Uesugi [12].

4.2. Comparison with Mohr-Coulomb model

The Mohr-Coulomb model and the new model are compared using the condition of the direct interface shear test (Section 3.1). The contact of the Mohr-Coulomb model is characterized only by the friction
coefficient; it is decided to consider the roughness of the contact surface [12]. The relation between the shear stress-shear displacements simulated by the two models is shown in Figure 4. The default friction model is the Coulomb model of linear elasticity-ideal plasticity, so it can not represent the hardening/softening properties of the contact surface. Whereas the new model is a typical nonlinear model, the subroutine simulation results are in good agreement with the realistic condition of the contact surface and represent the effect of roughness on the shear stress to the shear displacement. As a result, it is proved that the new model has the correctness and feasibility.

**Figure 4.** Shear displacement $u_x$ – shear stress $\tau_x$ for the comparison of Mohr-Coulomb (a) and hypoplastic interface model (b)
5. Conclusions
Based on the analysis of the hypoplastic model that has already presented, a new contact model considering the relative surface roughness was studied. The new model is compared with the simulation of Mohr-Coulomb model and experimental data.

(1) The new model includes the items on the nonlinearity and roughness, so it reflects the realistic property of the sand-structure contact surface and agrees well with experimental data.

(2) As a result of comparison with the Mohr-Coulomb model, the new model can better simulate the shear stress-shear strain of the contact surface.

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