Instantons in the QCD Vacuum and in Deep Inelastic Scattering*

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Abstract

We give a brief status report on our on-going investigation of the prospects to discover QCD instantons in deep inelastic scattering (DIS) at HERA. A recent high-quality lattice study of the topological structure of the QCD vacuum is exploited to provide crucial support of our predictions for DIS, based on instanton perturbation theory.

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1 INTRODUCTION

The ground state ("vacuum") of non-abelian gauge theories like QCD is known to be very rich. It includes topologically non-trivial fluctuations of the gauge fields, carrying an integer topological charge $Q$. The simplest building blocks of topological structure in the vacuum, localized (i.e. "instantaneous") in (euclidean) time and space are $I$ instantons ($I$) with $Q = +1$ and anti-instantons ($\bar{I}$) with $Q = -1$. While they are believed to play an important role in various long-distance aspects of QCD, there are also important short-distance implications. In QCD with $n_f$ (massless) flavours, instantons induce hard processes violating “chirality” $Q_5$ by an amount $\Delta Q_5 = 2 n_f Q$, in accord with the general ABJ chiral anomaly relation [2]. While in ordinary perturbative QCD ($Q = 0$), these processes are forbidden, their experimental discovery would clearly be of basic significance. The DIS regime is strongly favoured in this respect, since hard $I$-induced processes are both calculable [3, 4] within $I$-pertubation theory and have good prospects for experimental detection at HERA [4, 5, 6, 7].

2 INSTANTONS IN THE QCD VACUUM

Crucial information [8] on the range of validity of our DIS predictions [3, 4] comes from a recent high-quality lattice investigation [9] on the topological structure of the QCD vacuum (for $n_f = 0$). In order to make $I$-effects visible in lattice simulations with given lattice spacing $a$, the raw data have to be “cooled” first. This procedure is to filter out (dominating) fluctuations of short wavelength $O(a)$, while affecting the topological fluctuations of much longer wavelength $\rho \gg a$ comparatively little. After cooling, an ensemble of $I$’s and $\bar{I}$’s can clearly be seen (and studied) as bumps in the topological charge density (e.g. fig. 1 (left)) and in the Lagrange density.

Next, we note that crucial $I$-observables in DIS, like the $I$-induced rate at HERA, are closely related to $I$-observables in the QCD vacuum, as measured in lattice simulations.

The link is provided through two basic quantities of the $I$-calculus, $D(\rho)$, the $I$-size distribution and $\Omega(U, R^2/\rho^2, \bar{\rho}/\rho)$, the $\bar{I}I$-interaction. Here $\rho(\bar{\rho}), R_\mu$, and the matrix $U$ denote the $I$ ($\bar{I}$)-sizes, the $I\bar{I}$-distance 4-vector and the $I\bar{I}$ relative color orientation, respectively. Within $I$-perturbation theory, the functional form of $D$ and $\Omega$ is known for $\alpha(\mu_r) \log(\mu_r \rho) \ll 1$ and $R^2/\rho^2 \gg 1$, respectively, with $\mu_r$ being the renormalization scale. Within the so-called “$\bar{I}I$-valley” approximation [11, 12, 13], $\Omega_{\text{valley}}$ is even analytically known for all $R^2$.

Fig. 1 (middle) illustrates the striking agreement in shape and normalization [8] of $2D(\rho)$ with the continuum limit of the high-quality UKQCD lattice data [10] for $dn_{I+\bar{I}}/d^3x d\rho$. The predicted normalization of $D(\rho)$ is very sensitive to $\Lambda_{\text{MS}, n_f=0}^{\text{pert}}$ for which we took the most accurate (non-perturbative) result from ALPHA [15]. The theoretically favoured choice $\mu_r \rho = \mathcal{O}(1)$ in fig. 1 (middle), optimizes the range of agreement, extending right up to the peak around $\rho \simeq 0.5$ fm. However, due to its two-loop renormalization-group invariance, $D(\rho)$ is almost independent of $\mu_r$ for $\rho \lesssim 0.3$ fm over the large range $2 \lesssim \mu_r \lesssim 20$ GeV. Hence for $\rho \lesssim 0.3$ fm, there is effectively no free parameter involved!
Figure 1: (left): Topological charge density $q(\vec{x}, t)$ on the lattice \([10]\) after “cooling”, displayed as function of $z$ and $t$ with $x, y$ fixed. Three $I$’s ($q(\vec{x}, t) > 0$) and two $\bar{I}$’s ($q(\vec{x}, t) < 0$) are visible as bumps. Continuum limit \([8]\) of “equivalent” UKQCD data \([9, 14]\) for the $(I + \bar{I})$-size distribution (middle) and the normalized $I\bar{I}$-distance distribution (right) along with the respective predictions from $I$-perturbation theory and the valley form of the $I\bar{I}$-interaction \([8]\). The 3-loop form of $\alpha_{\overline{MS}}$ with $\Lambda_{\overline{MS}} n_f = 0$ from ALPHA \([15]\) was used.

Fig. 1 (right) displays the continuum limit \([8]\) of the UKQCD data \([9, 14]\) for the distance distribution of $I\bar{I}$-pairs, $dn_{I\bar{I}}/d^4 x d^4 R$, along with the theoretical prediction \([8]\). The latter involves (numerical) integrations of $\exp(-4\pi/\alpha \cdot \Omega_{\text{valley}})$ over the $I\bar{I}$ relative color orientation $(U)$, as well as $\rho$ and $\bar{\rho}$. For the respective weight $D(\rho)D(\bar{\rho})$, a Gaussian fit to the lattice data was used in order to avoid convergence problems at large $\rho, \bar{\rho}$. We note a good agreement with the lattice data down to $I\bar{I}$-distances $R/\langle \rho \rangle \simeq 1$. These results imply first direct support for the validity of the “valley”-form of the interaction $\Omega$ between $I\bar{I}$-pairs.

In summary: The striking agreement of the UKQCD lattice data with $I$-perturbation theory is a very interesting result by itself. The extracted lattice constraints on the range of validity of $I$-perturbation theory can be directly translated into a “fiducial” kinematical region for our DIS-predictions \([4, 8]\). Our results also suggest a promising proposal \([8]\): One may try and replace the two crucial quantities of the perturbative $I$-calculus $D(\rho)$ and $\Omega(U, R^2/\rho, \bar{\rho}/\rho)$ by their actual form inferred from the lattice data. The present “fiducial” cuts in DIS may then be considerably relaxed, high-$E_T$ photoproduction becomes accessible theoretically, etc.

3 SEARCH STRATEGIES IN DIS

An indispensable tool for investigating the prospects to detect $I$-induced processes at HERA, is our $I$-event generator \([6]\) QCDINS-1.60, which is interfaced (by default) to HERWIG 5.9.

In a recent detailed study \([7]\), based on QCDINS and standard DIS event generators, a number of basic (experimental) questions has been investigated: How to isolate an $I$-enriched data sample
by means of cuts to a set of observables? How large are the dependencies on Monte-Carlo models, both for $I$-induced (INS) and normal DIS events? Can the Bjorken-variables ($Q', x'$) of the $I$-subprocess (to which “fiducial” cuts should be applied) be reconstructed?

Let us briefly summarize the main results. While the “$I$-separation power” = $\text{INS}_{\text{eff}} / \text{DIS}_{\text{eff}}$ typically ranges around $O(20)$ for single observables, a set of six observables (among $\sim 30$) with much improved $I$-separation power = $O(130)$ could be found. The systematics induced by varying the modelling of $I$-induced events remains surprisingly small (fig. 2). In contrast, the modelling of normal DIS events in the relevant region of phase space turns out to depend quite strongly on the used generators and parameters (fig. 3). Despite a relatively high expected rate of $O(100)$ pb for $I$-events in the “fiducial” DIS region $[4]$, a better understanding of the tails of distributions for normal DIS events turns out to be quite important.

Cuts:

\[
\begin{align*}
55 \text{ GeV}^2 &< Q^2_{\text{rec}} < 95 \text{ GeV}^2 \\
(p_T (\text{Jet}) > 4 \text{ GeV}) \quad (E_{T,B}^\text{in} - E_{T,B}^\text{out}) / E_{T,B}^\text{in} &< 0.4 \\
H_{10} &> 0.84 \\
E_{T,B}^\text{r} &> 8 \text{ GeV} \\
N_B &\geq 7
\end{align*}
\]

Figure 2: Dependence of the $I$-separation power for a multi-dimensional cut scenario on the variation of Monte-Carlo models and parameters $[7]$. Corresponding efficiencies (eff) and event numbers for $\int \mathcal{L} dt = 30 \text{ pb}^{-1}$ are also shown. DIS = default, INS varied.
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\[p_T (\text{Jet}) > 4 \text{ GeV}\]

\[H_{10} > 0.84\]

\[E_{T,B} > 8 \text{ GeV}\]

\[(E_{\text{in},B} - E_{\text{out},B}) / E_{\text{in},B} < 0.4\]

\[n'_B \geq 7\]

**INS** = QCDINS + HERWIG tuned (default)

**DIS** = ...

| HERWIG (tuned) | HERWIG (not tuned) | LEPTO (SCI) | LEPTO (no SCI) | ARIADNE (Pomeron, default) | ARIADNE (no Pom.) |
|----------------|---------------------|-------------|----------------|-----------------------------|-------------------|
| 275            | 250                 | 225         | 200            | 175                         | 150               |

**INS** = 0.104

**DIS** = 0.0004

**N**_{INS} = 669

**N**_{DIS} = 930

Figure 3: As in fig. 2, but INS = default, DIS varied.

**References**

[1] A. Belavin, A. Polyakov, A. Schwarz and Yu. Tyupkin, Phys. Lett. B 59 (1975) 85.

[2] G. 't Hooft, Phys. Rev. Lett. 37 (1976) 8; Phys. Rev. D 14 (1976) 3432; Phys. Rev. D 18 (1978) 2199 (Erratum).

[3] S. Moch, A. Ringwald and F. Schrempp, Nucl. Phys. B 507 (1997) 134.

[4] A. Ringwald and F. Schrempp, Phys. Lett. B 438 (1998) 217.

[5] A. Ringwald and F. Schrempp, hep-ph/9411217, in: *Quarks '94*, Proc. 8th Int. Seminar, Vladimir, Russia, 1994, pp. 170-193.
[6] M. Gibbs, A. Ringwald and F. Schrempp, hep-ph/9506392, in: Proc. DIS95, Paris, 1995, pp. 341-344.

[7] T. Carli, J. Gerigk, A. Ringwald and F. Schrempp, to appear.

[8] A. Ringwald and F. Schrempp, hep-lat/9903039.

[9] D.A. Smith and M.J. Teper, (UKQCD), Phys. Rev. D 58 (1998) 014505.

[10] M.-C. Chu, J.M. Grandy, S. Huang and J.W. Negele, Phys. Rev. D 49 (1994) 6039.

[11] A. Yung, Nucl. Phys. B 297 (1988) 47.

[12] V.V. Khoze and A. Ringwald, Phys. Lett. B 259 (1991) 106.

[13] J. Verbaarschot, Nucl. Phys. B 362 (1991) 33.

[14] M. Teper, private communication.

[15] S. Capitani, M. Lüscher, R. Sommer and H. Wittig, Nucl. Phys. B 544 (1999) 669.