No stable dissipative phantom scenario in the framework of a complete cosmological dynamics

Norman Cruz, Samuel Lepe, Yoelsy Leyva, Francisco Peña and Joel Saavedra

Departamento de Física, Universidad de Santiago de Chile, Casilla 307, Santiago, Chile
Instituto de Física, Pontificia Universidad Católica de Valparaíso Casilla 4950, Valparaíso, Chile.
Departamento de Ciencias Físicas, Facultad de Ingeniería y Ciencias, Universidad de la Frontera Casilla 54-D, Temuco, Chile.

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We investigate the phase space dynamics of a bulk viscosity models in the framework of spatially flat Friedmann-Robertson-Walker universe. We have included two barotropic fluids and a dark energy component. One of the barotropic fluids is treated as an imperfect fluid having bulk viscosity whereas the other components are assumed to behave as a perfect fluids. Both barotropic fluids are identified as radiation and dark matter. Considering that the bulk viscosity acts on either radiation or dark matter, we find that viscous phantom solutions with stable behavior are not allowed in the framework of complete cosmological dynamics. Only an almost zero value of the bulk viscosity allow a transition from a radiation dominated era to a matter dominated epoch and then evolve to an accelerated late time expansion, dominated by the dark energy.

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I. INTRODUCTION

Observational evidence indicates that the present speed up of the Universe may be successfully explained by a cosmic fluid with negative pressure which has been baptized as dark energy. This exotic component contributes about 68% of the total energy of the Universe. Evidence for dark energy is provided by several complementary probes such as the high redshift surveys of supernovae [2], the cosmic microwave background (CMB) [1, 3, 4], the integrated Sachs-Wolfe effect [5, 6].

The dark energy is considered as fluid characterized by a negative pressure and usually represented by the EoS for this dark component is most preferred, from the statistical point of view, than phantom and quintessence models, while the phantom dark energy is slightly better, to fits the observational data, than the quintessence does. However, in [18], the best-fit values of the parameters of dark energy models, in which the phantom scalar field initially mimics a Cosmological Constant term to finally evolves slowly to the Big Rip singularity, are determined jointly with all other cosmological parameters by the Markov chain Monte Carlo method [21] using observational data on cosmic microwave background anisotropies and polarization [22], supernovae type Ia [23, 24], BAO [9, 14], Big Bang nucleosynthesis [27, 28] and Hubble constant measurements from HST [29]. Similar computations are carried out for ΛCDM and a quintessence scalar field model of dark energy [18]. It is shown that the current data slightly prefer the phantom model, but the differences in the maximum likelihoods are not statistically significant. So the possibility of phantom behavior for the dark energy fluid can not be discarded.

Furthermore, an important result, and prior to the discovery of present speed up of the Universe, was the fact that a dissipative mechanism like the bulk viscosity may give rise to an accelerated evolution of the Universe [30–32]. At the same time, the bulk viscosity provide the only dissipative mechanism consistent, in a homogeneous and isotropic background, with the Cosmological Princi-
ple. Thus, it has been proposed as one of the possible ways to induce an accelerated phase in the evolution of the Universe, namely: early time inflation \[38-41\] or the present accelerated period \[42-51\].

In a FRW universe, the inclusion of a bulk viscosity allows the possibility of violating the Dominant Energy Condition \[31, 52\] and hence is possible to obtain phantom solutions. In the Eckart approach \[53\], the bulk viscosity introduces dissipation by only redefining the effective pressure of the cosmic fluid, namely:

\[ P_{eff} = p + \Pi = p - 3\zeta H, \]  

(1)

where \(p\) is the kinetic pressure of the cosmic fluid, \(\Pi\) is the bulk viscous pressure, \(H\) is the Hubble parameter and the bulk viscosity coefficient, \(\zeta\), satisfies:

\[ \zeta \geq 0, \]  

(2)

this latter requirement guarantees no violation of the Second Law of Thermodynamics \[55, 56\].

Since the equation of energy balance states that:

\[ \dot{\rho} + 3H(\rho + p + \Pi) = 0, \]  

(3)

the violation of Dominant Energy Condition, i.e., \(\rho + p + \Pi < 0\), implies an increasing energy density of the fluid that fills the universe.

The phantom behavior due to the presence of a bulk viscosity have been investigated in many cosmological contexts. In \[42\], a big rip singularity solution was obtained, in the full causal Israel-Stewart-Hiscock framework theory \[51\], assuming a late time universe filled with only one barotropic fluid. The late time evolution of a Chaplygin gas model with bulk viscosity, in a causal and truncated version of the Israel-Stewart formalism and in the Eckart approach, was studied in \[58\]. In both framework, the authors found new types of future singularities. However, a viable phantom solution was derived only in the truncated approach. In the context of the Eckart approach, the following investigations have been addressed: big rip singularities for various forms of \(w = w(\rho)\) and the bulk viscosity \(\zeta = \zeta(\rho)\) \[59\]; little rip cosmologies \[60, 61\]; phantom crossing in modified gravity \[43, 62\] and unified dark fluid cosmologies \[63, 64\]. In addition, the conditions for the physical viability of a cosmological model in which dark matter has bulk viscosity and also interacts with dark energy was discussed in \[51\]. In this case, the model took into account radiation, baryons, dark matter and dark energy and considered a general interaction term between the dark components. In relation to the viscosity, the authors assumed the Ansatz \(\zeta \propto \zeta_o \sqrt{\rho_T}\), where \(\rho_T\) is the total matter density of the model and \(\zeta_o\) a positive constant. The joint analysis of the phase space of the model and the observational test shown that: a) a complete cosmological dynamics requires either null or negative bulk viscosity. b) observations consistently point to a negative value of the bulk viscous coefficient. c) a phantom nature of the dark energy \((w_{de} < -1)\) was consistently suggested by the cosmological observations. The first two results found in \[51\] rule out any viscous model with the Ansatz \(\zeta \propto \zeta_o \sqrt{p_T}\) despite the last result is in line with recent results \[1\].

More recently, a deep discussion concerning the degeneracy in bulk viscosity models when \(\zeta \equiv \zeta(H)\) was presented in \[50\]. At the same time, the possibility that phantom solutions could be caused by the existence of bulk viscosity pressure in any fluid component (radiation and pressureless matter) was discussed using multiple observational test. However, as a consequence of the degeneracy criterion used, it was not possible to obtain, in the observational analysis performed by the authors, the best fit values/signs of parameters of the proposed viscosity model\[4\], namely eg. the coefficient of the bulk viscosity.

In the present work we are interesting in to study the phase space of the models presented in \[51\]. Our main goal is to explore if the presence of a phantom solution, obtained by adding the bulk viscosity (as a new extra imperfect pressure) to either radiation \((\zeta \equiv \zeta(\rho_r))\), where \(\rho_r\) is the energy density of the radiation) or pressureless matter \((\zeta \equiv \zeta(\rho_m))\), where \(\rho_m\) is the energy density of the pressureless matter \[51\], is compatible with the so called complete cosmological dynamics \[51, 68\] which states that, at early enough times, all physically viable model must allow the existence of radiation and matter domination periods previous to the present accelerated expansion of the Universe.

The organization of the paper is as follows: in Section II we present the field equations for a flat FRW universe filled two barotropic fluids and dark energy obeying a barotropic EoS. We assume that one of the barotropic fluids present bulk viscosity and the effective pressure is treated within the framework of the Eckart theory \[53\]. The bulk viscous coefficient is taken to be proportional to the square root of the dark matter density. In Section III, we analyzed the evolution equations from the perspective of dynamical systems. We studied two models by matching the barotropic fluids with radiation and dark matter. A detailed scheme of the critical points and their conditions upon the parameters of the model is found. Finally in Section IV is devote to conclusions.

\[^3\] For a fluid on a FRW geometry, the local entropy production is defined as \(TV_{\mu \nu} \sigma^\mu \equiv 9H\zeta \sqrt{\zeta} \zeta_o \equiv 9H\zeta \sqrt{\zeta} \zeta_o\), where \(V_{\mu \nu} \sigma^\mu \) is the rate of entropy production in a unit volume and \(T\) is the temperature of the fluid. Since the the second law of thermodynamics provides that \(V_{\mu \nu} \sigma^\mu \geq 0\), then for an expanding Universe \((H > 0)\) \(\zeta \geq 0\).

\[^4\] The Ansatz \(\zeta \propto \zeta_o \sqrt{p_T}\), used in \[51\], is a specific case of \(\xi \equiv \xi(H)\).

\[^5\] Constraints on the dark matter viscosity can be found in \[48\] for two models: a) a constant \(\zeta\), and b) \(\zeta \propto \frac{\zeta_o}{\sqrt{\rho_m}}\).
II. THE MODEL

We study a cosmological model in a spatially flat FRW metric, in which the matter components are two barotropic fluids and dark energy (DE). One of the barotropic fluid and the DE are assumed as perfect fluids, whereas the remaining fluid is treated as an imperfect fluid having bulk viscosity.

The Friedmann constraint and the conservation equations for the matter fluids can be written as:

\begin{align}
H^2 & = \frac{8\pi G}{3} (\rho_1 + \rho_2 + \rho_{de}) , \\
\dot{\rho}_1 & = -3\gamma_1 H \rho_1 + 9H^2 \zeta, \quad \text{(4a)} \\
\dot{\rho}_2 & = -3\gamma_2 H \rho_2 , \quad \text{(4b)} \\
\dot{\rho}_{de} & = -3H\gamma_{de}\rho_{de}, \quad \text{(4d)}
\end{align}

where G is the Newton gravitational constant, H the Hubble parameter, \(\rho_1, \rho_2, \rho_{de}\) are the energy densities of barotropic and DE fluid components respectively, and \(\gamma_{de}\) is the barotropic index of the equation of state (EOS) of DE, which is defined from the relationship

\[ H \equiv \frac{\pi G}{H_0 \Omega_0^{\frac{1}{2}}} , \]

and

\[ \zeta \sim \frac{\pi G}{60} \kappa_0 , \]

where the derivatives are with respect to the e-folding number \(N \equiv \ln a\) and

\[ \zeta \geq 0 . \]

In term of the new variables, the Friedmann constraint (4a) can be written as:

\[ \Omega_2 = \frac{8\pi G}{3H^2} \rho_2 = 1 - x - y , \]

and then we can choose \((x, y)\) as the only independent dynamical variables.

Taking into account that \(0 \leq \Omega_2 \leq 1\), and imposing the conditions that both the barotropic fluids and DE components are both positive definite and bounded at all times, we can define the phase space of Eqs. (8) as:

\[ \Psi = \{(x, y) : 0 \leq 1 - x - y \leq 1, 0 \leq x \leq 1, 0 \leq y \leq 1\} . \]

Other cosmological parameters of interest are the total effective EOS, \(w_{\text{eff}}\), and the deceleration parameter, \(q = -(1 + H/H^2)\), which can be written, respectively, as

\begin{align}
w_{\text{eff}} & = -1 - \sqrt{\frac{7\Omega_{\text{de}}}{3}} + y (\gamma_1 - \gamma_2) + 3\gamma_2 - x (\gamma_2 - \gamma_{\text{de}}) , \quad \text{(13a)} \\
q & = \frac{1}{2} [-2 - \sqrt{\frac{7\Omega_{\text{de}}}{3}} + 3y (\gamma_1 - \gamma_2) + 3\gamma_2 - 3x (\gamma_2 - \gamma_{\text{de}})] . \quad \text{(13b)}
\end{align}

A. The case with \(\gamma_1 = 1\) (dark matter) and \(\gamma_2 = \frac{4}{3}\) (radiation)

This case corresponds to a scenario where dark matter (DM) is treated as an imperfect fluid having bulk viscosity.

\[ \kappa_0 = \frac{\pi G}{60} \]

Recall that, once we set the values of \(\gamma_1\) and \(\gamma_2\) we will able to identify \(\Omega_1\) and \(\Omega_2\) with the dimensionless density parameters of radiation and dark matter, if we set \(\gamma_1 = \frac{4}{3}\) and \(\gamma_2 = 1\), and vice versa, if we set \(\gamma_1 = 1\) and \(\gamma_2 = \frac{4}{3}\).
viscosity with a null hydrodynamical pressure. As the same time is clear from the definition of the dimensionless variables that: $x = \Omega_{de}, y = \Omega_1 = \Omega_m$ and $\Omega_2 = \Omega_r$. The full set of critical points of the autonomous system (8a-8b), the existence conditions and the stability conditions for $\gamma_1 = 1$ and $\gamma_2 = \frac{1}{3}$ are summarized in Table II. Whereas the eigenvalues of the linear perturbation matrix associated to each of the critical points and some important physical parameters are given in Table I.

1. Critical points and stability

Critical point $P_1$ corresponds to a pure radiation domination era, $\Omega_r = 1$, and always exists independently of the value/sign of the viscosity parameter $\xi$. It also represents a decelerating expansion solution with $w_{eff} = -\frac{1}{3}$ and $q = 1$. The stability of this critical point is the following:

- Unstable if $\gamma_{de} < 1$ and $\xi \geq 0$.

$P_2$ is a critical point dominated by the pressureless matter component, $\Omega_m = 1$, and always exists. This critical point exhibits two different stability behaviors, namely:

- Stable if $\gamma_{de} < 1$ and $\xi > 3 - 3\gamma_{de}$,
- Saddle if $\gamma_{de} < 1$ and $0 \leq \xi < 3 - 3\gamma_{de}$.

From Table III we notice that if $\xi^2 < 1$ this point corresponds to the standard matter domination period ($w_{eff} = 0$). An interesting fact of $P_2$ is the value of EOS parameter $w_{eff} = -\frac{2}{3}$. In the stable region ($\gamma_{de} < 1$ and $\xi > 3 - 3\gamma_{de}$), this critical point represents an accelerating phantom solution with $w_{eff} < -1$ if:

$$\xi > 3 \quad \text{and} \quad \frac{3 - \xi}{3} < \gamma_{de} < 1.$$  

Despite this result is in correspondence with those observations that tend to mild favor a present day value of $w_{eff}$ in the phantom region, it will shown in the next subsection that is not possible to associate this behavior with a realistic late time solution.

$P_3$ corresponds to an scaling solution between pressureless matter and dark energy and exists when

$$\gamma_{de} < 1 \quad \text{and} \quad 0 \leq \xi \leq 3(1 - \gamma_{de}).$$

It represents and accelerated solution ($q < 0$) if:

$$\gamma_{de} < \frac{2}{3},$$ (14)

and shown an stable behavior given that:

- $\gamma_{de} < 1$ and $0 < \xi < 3 - 3\gamma_{de}$.

In the particular case $\xi \ll 1$ the strict dark energy domination is recovered ($x = \Omega_{de} = 1$).

Finally, critical point $P_4$ represents a pure dark energy domination solution ($\Omega_{de} = 1$) and always exists. The stability of $P_4$ is the following

- Stable if $\gamma_{de} < 1$ and $\xi = 0$,
- Saddle if $\gamma_{de} < 1$ and $\xi > 0$,

for a dark energy fluid with $\gamma_{de} < 1$ and $\xi > 0$, this critical point is always an accelerated solution with a saddle behavior (see Table III for further details).

2. Cosmology evolution from critical points

According to the complete cosmological dynamics paradigm (see [51, 68] for recent discussion about this topic), our model should describe: (a) a radiation dominated era at early times (RDE), (b) a matter domination era (MDE) at intermediate stages of the evolution, and finally, (c) the present stage of accelerated expansion of the Universe. The required dominance stages can be translated into critical points, and the desire transitions between them into the heteroclinic orbits that connects two critical points [69, 70].

According to the complete cosmological dynamics basis, one of the critical points of the model should correspond to a RDE, and this point should has an unstable nature. The unstable behavior of this critical point assurances that it can be the source of any orbits in the phase space. The only candidate in our model is the critical point $P_1$. In fact, $P_1$ satisfies the condition for a purely radiation dominance ($\Omega_r = 1$) and always exists. Its unstable nature, for all realistic dark energy fluids ($\gamma_{de} < 1$), is clear in the previous section.

As we mention before, the Universe requires the existence of a matter dominated era with the aim to explain the formation of the cosmic structure. This MDE is recovered by a $P_2$ ($\Omega_m = 1$), which always exists. However, as Table III shown, we cannot recover an standard pressureless matter dominated picture ($w_{eff} = 0, q = \frac{1}{2}$) unless $\xi = 0$. For a non null value of $\xi$, $P_2$ represents a decelerating solution if $0 < \xi < 1$ ($q > 0$) or an accelerating expansion solution if $\xi > 1$ ($q < 0$), see Table III for further details. In both regions with $\xi \neq 0$, the presence of the bulk viscosity lead to an undesirable expansion rate of the Universe and makes impossible to associate this critical point with a realistic MDE.

An interesting characteristic of $P_2$ is that, in certain region in the parameter space ($\xi, \gamma_{de}$), its represents not only an accelerated solution but also a phantom one. As Table II shown, the presence of this phantom solution

\footnote{Recall that the Second Law of Thermodynamics requires a non-negative value of $\xi$, see [19].}
(w_{eff} = -\frac{\xi}{3} < -1) in P_2 depends only of the value of the viscosity parameter \(\xi\) and not of the nature of the dark energy component. However, as Table II and Table III shown, its stability behavior depends of the value of the barotropic index. This stable and viscous phantom behavior of P_2 could be important to represent the late time behavior of the Universe and, at the same time, be in line with those cosmological observations that mild favor present date value for the EOS parameter in the phantom region \(w_{eff} < -1\). However, is not possible to select initial conditions that connects a RDE (P_1) to phantom attractor solution (P_2) through a true MDE. As we mention before, only P_2 (with \(\xi = 0\)) could play the role. Furthermore, the conditions for the existence and stability of the viscous phantom solution restricts the critical points of the autonomous system to three, namely P_1, P_2 and P_3, and none of them corresponds to a true MDE. Thus, is not possible to associate this viscous phantom behavior in P_2 with a realistic late time solution. Fig. II shows some example orbits in the plane \((x, y)\) to illustrate this situation.

Another feature of the model is the presence of two more accelerated solutions, described by critical points P_3 and P_4. P_3 is an scaling solution between dark matter and dark energy and can be an attractor for \(\gamma_{de} < 1\) and \(0 < \xi < 3 - 3\gamma_{de}\) whereas P_4 is always a saddle solution for all realistic dark energy with \(\xi > 0\). As Fig. 2 shown, the existence of an attractor phantom solution in P_3 depends on the dark energy component, in other words: it is only possible to obtain an attractor phantom solutions if a phantom dark energy component, \(w_{de} < -1 (\gamma_{de} < 0)\), is present. A favorable scheme would be one in which the initial conditions allow a complete cosmological dynamics, eg.: \(P_1 \Longrightarrow P_2 \Longrightarrow P_4\). In terms of the cosmological evolution of the Universe, the above favorable scenario implies that the Universe started at early times from a
RDE, then evolve into a MDE, to enter in the final phase of accelerated expansion. Recall that in this favorable scenario, it is necessary to select a sufficiently small value for the bulk viscosity $\xi \ll 1$ in order to recover a true MDE ($P_2$) and that is not possible to obtain a realistic phantom solution. Fig. 3 shows some example orbits in the plane $(x, y)$ to illustrate the above situation.

**B. The case with $\gamma_1 = \frac{1}{3}$ (radiation) and $\gamma_2 = 1$ (DM)**

This case corresponds to an scenario where radiation is treated as an imperfect fluid having bulk viscosity. As the same time is clear from the definition of the dimensionless variables that: $x = \Omega_{de}, y = \Omega_1 = \Omega_r$ and $\Omega_2 = \Omega_m$. The full set of critical points of the autonomous system (12) (13), the existence conditions and the stability conditions for $\gamma_1 = \frac{1}{3}$ and $\gamma_2 = 1$ are summarized in Table III. Whereas the eigenvalues of the linear perturbation matrix associated to each of the critical points and some important physical parameters are given in Table IV.

1. **Critical points and stability**

Critical point $P_1$ corresponds to a radiation dominated era ($\Omega_r = 1$). The stability of this critical point is the following:

- Unstable if $\gamma_{de} < 1$ and $0 \leq \xi < 1$,
- Saddle if $\gamma_{de} < 1$ and $1 < \xi < 4 - 3\gamma_{de}$,
- Stable if $\gamma_{de} < 1$ and $\xi > 4 - 3\gamma_{de}$.

As we expect, a background level this point represents to a true decelerating RDE if $\xi \ll 1$ ($w_{eff} \approx \frac{1}{3}$ and $q \approx 1$). If this condition is satisfied, $P_1$ behaves as an unstable solution. However, $P_1$ is also able to mimics, a background level a dark matter ($w_{eff} = 0$) and a phantom fluids ($w_{eff} < -1$) if the bulk viscosity is $\xi = 1$ and $\xi > 4$ respectively. If the phantom solution exists, then $P_1$ will behave as an stable late time solution of the autonomous system (12) (13). We will discuss in more details the cosmological implications of this critical point in the next subsection.

The hyperbolic critical point $P_2$ corresponds to an scaling solution between the radiation and the dark matter and exists if $0 < \xi < 1$, being a decelerated solution $q = \frac{1}{2}$. From the stability point of view, this critical point always has a saddle behavior demanding that:

- $\gamma_{de} < 1$ and $0 < \xi < 1$.

In the limit, when $\xi \ll 1$, this point recovers a true DME ($\Omega_m = 1$).

Critical point $P_3$ corresponds to pure DM dominance solution ($\Omega_m = 1, w_{eff} = 0$) and always exists. A striking aspect of this solution is it dynamical behavior: for a non null bulk viscosity on the radiation fluid, $P_2$ behaves as an unstable solution (see Table III). This means that it is not possible to associate $P_3$ with an intermediate stage of matter domination in the evolution of the Universe.
TABLE III. Location, existence conditions according to the physical phase space \[12\], and stability of the critical points of the autonomous system \[55\] for \(\gamma_1 = \frac{3}{4}\) and \(\gamma_2 = 1\). The eigenvalues of the linear perturbation matrix associated to each of the following critical points are displayed in Table IV.

| \(P_i\) | \(x\) | \(y\) | Existence | Stability |
|--------|------|------|-----------|----------|
| \(P_1\) | 0    | 1    | Always    | Stable if \(\gamma_{de} < 1\) and \(\xi > 4 - 3\gamma_{de}\) |
|         |      |      |           | Unstable if \(\gamma_{de} < 1\) and \(0 \leq \xi < 1\) |
|         |      |      |           | Saddle if \(\gamma_{de} < 1\) and \(1 < \xi < 4 - 3\gamma_{de}\) |
| \(P_2\) | 0    | \(\xi^2\) | \(0 < \xi < 1\) | Saddle if \(\gamma_{de} < 1\) and \(0 < \xi < 1\) |
| \(P_3\) | 0    | 0    | Always    | Unstable if \(\gamma_{de} < 1\) and \(\xi > 0\) |
|         |      |      |           | Saddle if \(\gamma_{de} < 1\) and \(\xi = 0\) |
| \(P_4\) | \(1 - \frac{\xi^2}{(4 - 3\gamma_{de})^2}\) | \(\frac{\xi^2}{(4 - 3\gamma_{de})^2}\) | \(\gamma_{de} < 1\) and \(0 < \xi < 4 - 3\gamma_{de}\) | Stable if \(\gamma_{de} < 1\) and \(0 < \xi < 4 - 3\gamma_{de}\) |
| \(P_5\) | 1    | 0    | Always    | Stable if \(\gamma_{de} < 1\) and \(\xi = 0\) |
|         |      |      |           | Saddle if \(\gamma_{de} < 1\) and \(\xi > 0\) |

TABLE IV. Eigenvalues and some basic physical parameters for the critical points listed in Table III12 see also Eqs. \(7\) and \(13\). We have used the definition \(A = \text{Abs} \left[8 + \xi^2 + 9 \left(-2 + \gamma_{de}\right)\gamma_{de}\right]\).

| \(P_i\) | \(\lambda_1\) | \(\lambda_2\) | \(\Omega_m\) | \(w_{\text{eff}}\) | \(q\) |
|--------|--------------|--------------|--------------|--------------|------|
| \(P_1\) | \(1 - \xi\)  | \(4 - \xi - 3\gamma_{de}\) | 0            | \(\frac{1}{3} - \frac{\xi}{1}\) | \(1 - \xi\) |
| \(P_2\) | \(\frac{\xi}{(1 + \xi^2)}\) | \(3 - 3\gamma_{de}\) | \(1 - \xi^2\) | 0 | \(\frac{1}{3}\) |
| \(P_3\) | \(\text{sgn}(\xi)\infty\) | \(3 - 3\gamma_{de}\) | 1            | 0 | \(\frac{1}{3}\) |
| \(P_4\) | \(\frac{1}{3} \left(-10 + 9\gamma_{de} + \frac{\xi^2 + 4}{4 - 3\gamma_{de}}\right)\) | \(\frac{1}{3} \left(-10 + \frac{\xi^2 + 4}{4 - 3\gamma_{de}} + 9\gamma_{de}\right)\) | 0 | \(-1 + \gamma_{de} - 1 + \frac{3\gamma_{de}}{2}\) |
| \(P_5\) | \(-4 + 3\gamma_{de}\) | \(\text{sgn}(\xi)\infty\) | 0 | \(-1 + \gamma_{de} - 1 + \frac{3\gamma_{de}}{2}\) |

The hyperbolic critical point \(P_4\) represents an scaling solution between the radiation and the DE. In addition, it is an stable solution for:

- \(\gamma_{de} < 1\) and \(0 < \xi < 4 - 3\gamma_{de}\).

As Table IV shown, \(P_4\) is an accelerated solution if:

\[-1 + \frac{3}{2}\gamma_{de} < 0, \quad (15)\]

recovering the strict dark energy domination if \(\xi \ll 1\).

Finally, \(P_3\) corresponds a pure dark energy domination solution (\(\Omega_{de} = 1\)) and always exists. For a non null bulk viscosity, this critical point represent a saddle solution if

- \(\gamma_{de} < 1\) and \(\xi > 0\).

Just like the previous critical point, \(P_3\) represents an accelerated solution if \(13\) is satisfied.

2. Cosmology evolution from critical points

To be in line with the complete cosmological dynamics, it is necessary to associate an unstable critical point with a RDE. In this model, we can identify two critical points with an unstable behavior, namely \(P_1\) and \(P_3\). The first one represents a true RDE with \(\Omega_r = 1\) and \(w_{\text{eff}} = 0\) for \(\xi \ll 1\). And the latter one represents a dark matter solution \(\Omega_m = 1\) and \(w_{\text{eff}} = 0\). The fact that both solutions \((P_1\) and \(P_3\)) are unstable simultaneously, when \(\gamma_{de} < 1\) and \(0 \leq \xi < 1\), implies that we need to choose those initial condition in the neighborhood of \(P_1\) to guarantee that the radiation dominated era (RDE) is the source of any orbits, of cosmological importance, in the phase space. Otherwise, it will be impossible to obtain, in term of \(P_3\), a successful description of the early time evolution of the Universe. This analysis also implies that those scenarios in which, under a proper selection of the bulk viscosity parameter, \(P_1\) mimics a MDE \((\xi = 1)\) or an stable phantom solution \((\xi > 4)\) are ruled out due to the impossibility of reproducing, in a correct way, the dynamics of the early time Universe. Fig. 4 shows some example orbits in the phase space to illustrate the situation in which \(P_1\) mimics a stable phantom solution (late time attractor).

In addition to a RDE, we also need to associate a saddle critical point with a MDE. The saddle nature of this point guarantee that the cosmological solutions will remain a lapse of time around it, before ultimately approaching an stable late time solution. The candidates in this model are \(P_2\) and \(P_3\) \[10\]. As we mention before, \(P_3\) always exhibits an unstable nature for a non-null value of the bulk viscosity \((\xi > 0)\). As a possible past attractor of the model, all the orbits that start at \(P_3\) describe solutions where there is not a radiation dominated era

10 Recall that \(P_3\) is ruled out as a MDE even if \(\xi = 1\) \((w_{\text{eff}} = 0)\).
Recall that the presence of a RDE is necessary to describe process such as the primordial nucleosynthesis. Only $P_2$ is able to reproduce a true MDE. This critical point represents an scaling solution between radiation and DM fluids and its existence is a direct consequence of having considered the viscosity on the radiation fluid. In order to recover a true MDE ($\Omega_m = 1$), the following condition must met: $\xi \ll 1$. In addition, Table III and IV shown that, despite being an scaling solution between radiation and DM, $P_2$ is not able to mimics, a background level, a true RDE even if $\xi \approx 1$ ($\Omega_\gamma \approx 1$) since $w_{\text{eff}} = 0$.

Finally, the model has two more accelerated critical points: $P_4$ and $P_3$ [50]. In the first case, $P_4$ represents an stable scaling solution between radiation and dark energy fluids. As shown in Table IV the presence of a bulk viscosity on the radiation fluid does not induce a crossing of the phantom divide in $P_4$, thus the crossing is only possible if a phantom dark energy fluid is considered in the model ($\gamma_{\text{de}} < 0$). In the other hand, $P_3$ corresponds to a pure dark energy domination period with a saddle behavior for a non-null value of the bulk viscosity. In this model, a favorable scenario will be a transition from $P_1$ to $P_3$.

$\Rightarrow P_2 \Rightarrow P_4$: starting in a RDE at early times, then enter into a MDE to finally evolve to an stable accelerated phase. This positive transition is represented in Fig. 5 through several orbits in the phase space of the model. As we comment before, the existence of a RDE and MDE, in terms of $P_1$ and $P_4$ respectively, demand that $\xi \ll 1$. This latter requirement also implies that the phantom solution with an stable behavior ($P_1$ with $\xi > 4$) is not compatible with a well behaved model from the point of view of the complete cosmological dynamics.

IV. CONCLUSIONS

Late time evolution of a dissipative fluid leading to a universe with phantom behavior have been investigated within different approaches, e.g. [42, 50, 51]. In this work, our aim was to study the dynamics of a universe filled with radiation, dark matter and dark energy, where a bulk viscosity may be present in one of the matter fluids, with emphasis on the viability of phantom solutions. In the first studied model, the dissipation in the dark matter was characterized by the bulk viscosity proportional to the dark matter energy density, i.e., $\zeta = \zeta_0 \sqrt{\rho_{m_0}/\rho_{m_0}}$ [50]. Whereas in the second model, the dissipation in the radiation was described by the bulk viscosity proportional to the radiation energy density, i.e., $\zeta = \zeta_0 \sqrt{\rho_{r_0}/\rho_{r_0}}$ [50]. In both models, the study was re-

FIG. 4. Vector field in the plane $(x, y)$ for the autonomous system $(8a-8b)$ with $\gamma_1 = -\frac{3}{2}$ and $\gamma_2 = 1$. The free parameters have been chosen to be: $(\xi, \gamma_{\text{de}})=(4.1, 0.01)$. In this case, the radiation dominated solution, $P_1$, is the late time attractor of the system, representing a viscous phantom solution ($w_{\text{eff}} = -1.033$). This latter time attractor solution, restricts the critical point of the system to three ($P_1$, $P_3$ and $P_4$) and none of them corresponds to a true early time RDE or MDE.

FIG. 5. Vector field in the plane $(x, y)$ for the autonomous system $(8a-8b)$ with $\gamma_1 = -\frac{3}{2}$ and $\gamma_2 = 1$. The free parameters have been chosen to be: $(\xi, \gamma_{\text{de}})=(0.01, 0.01)$. In this case, the radiation-DE scaling solution, $P_4$, is the late time attractor of the system, representing and accelerated solution. The transition from the RDE ($P_1$) to $P_4$ allow to selects appropriate initial conditions to recover a true MDE ($P_2$) with $w_{\text{eff}} \approx 0$ and $q \approx \frac{1}{3}$.

\textsuperscript{11} The presence of a RDE is necessary to describe process such as the primordial nucleosynthesis.

\textsuperscript{12} Recall that $P_1$ is able to mimics an stable phantom solution if $\xi > 4$ but this possibility is discarded because the model is not able to describe a RDE and MDE as Fig. 4 shown.
stricted to the cases in which $\zeta \geq 0$, a condition that comes from the Local Second Law of Thermodynamics.

By making a dynamical system analysis of the both possible scenarios and imposing that both models must follow the so called complete cosmological dynamics, we found that the viscous phantom solutions with an stable behavior are not allowed. This results from the fact that is not possible to recover a viable MDE in the first model if the viscous phantom solution exists ($P_2$ with $\zeta > 3$). Whereas in the second model, the existence of an stable viscous phantom solution ($P_1$ with $\zeta > 4$) does not allow the existence of a true RDE and MDE. In other words, there is no smooth transition from a radiation dominated epoch to a matter dominated phase and then to an accelerated late time expansion, dominated by dark energy. For purposes of illustration, we have shown some numerical elaboration for several values of the free parameters of the models ($\xi$, $\gamma_{de}$).

Additionally, it was shown that, in both models, is possible to accommodate a complete cosmological dynamics whenever $\xi \approx 0$. However, in this favorable scenario, the late time attractor solution is characterized by $w_{eff} = -1 + \gamma_{de}$, therefore the nature of this solution depends only of the nature of the dark energy fluid namely: if the dark energy fluid is a quintessence (phantom) fluid $0 < \gamma_{de} < \frac{3}{2}$ ($\gamma_{de} < 0$), then the late time stable solution corresponds to quintessence (phantom) solution. If $\gamma_{de} = 0$, then the de Sitter solution will be the late time attractor.

As we mention before, we have focused only to the cases in which $\alpha = \frac{1}{2}$ [see Eq. (5)]. Thus, the question about the viability of the phantom solutions, in term of the complete cosmological dynamical, for $\alpha \neq \frac{1}{2}$ is still an open problem. We hope to address this more general and complex scenario in a forthcoming paper.

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