Modeling of the content of the physics course based on the percolation coefficient

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Abstract. This article deals with the problem of modeling the assessment of the level of integrity of the content of the physics course learned by students. The need for such modeling is due to the need to bring education courses to online format. The degree of integrity of learning is proposed to be established by modeling the percolation of intradisciplinary connections established in the course structure. The model uses the graph model of intradisciplinary connections (T. N Gnitetskaya) and the principles of percolation theory (P-theory). The article provides a few practical solutions for the use of P-theory. An algorithm has been developed that allows forming the topics of the physics course into the studied structure.

1. Introduction
Recent years have significantly increased the interest in the development of quantitative methods applied for solving the problems of selecting and optimizing the content of academic disciplines due to the need to constantly change the educational content for the curricula in new areas of training. The pace of change is increasing; therefore, intuitive methods have to be changed to quantitative ones in the selection and systematization of content, especially in fundamental and mathematical disciplines. For example, it has to be done for the Course of General Physics (CGP) and the Course of Mathematical Analysis (CMA) because their content has been developed for centuries and is designed for a certain number of hours. Today, however, the content needs to be reduced as the number of hours allocated to both courses in the curricular has been cut significantly. The question is what to remove as all the sections and topics of the courses are interrelated and important. The reductions distort perception of the content of physics and cause a problem of misunderstanding the description of natural phenomena via mathematical methods. The authors of the article suggest addressing the problem by optimizing the intradisciplinary space of physics and mathematics content based on intradisciplinary connections. This goal can be achieved by applying the graph model of intradisciplinary connections (T. N Gnitetskaya) in combination with the principles of percolation theory (P-theory). The task is to determine the percolation coefficient and apply it to find the “trajectory” of intradisciplinary percolation using the example of intradisciplinary space of Mathematical Analysis (basic) and General Physics (connected) courses.

2. Formatting the title, authors and affiliations
Numerous works in pedagogics, psychology and physiology have been devoted to the problem of understanding. However, they suggest only a few solutions for modeling the understanding (see, for example, the works of T.N. Gnitetskaya, I.I. Ankudinov [2]) which could help to organize the academic process to achieve the maximum possible understanding of the studied content. Pedagogical and psychological academic schools in Russia connect the phenomenon of understanding and its conditioning in the learning process with the activity-based approach to education. The activity-based approach to education emerged in Russia in the middle of the 20th century on the basis of the psychological theory of activity suggested by A.N. Leontiev and P.Ya. Galperin [3, 4]. Their ideas
were later developed by N.F. Talyzina in the theory of stage-by-stage formation of mental actions [5]. This theory implemented the idea of classifying the cognitive styles of personality. Despite the obvious scientific attractiveness of those ideas, a model of understanding was not developed. American psychologists viewed the problem of understanding through the prism of cognitive studies. Their research was closely interrelated with and influenced by the new mathematical theories – the information theory [6], cybernetics [7, 8, 9], and synergetics [10]. This research included development of models as well. The concept of a model also emerged in the cognitive sciences. More specifically, Johnson-Laird introduced the concept of mental model [11] that was his main contribution to the development of cognitive sciences. However, despite their close connection to mathematical theories, this and other models had a descriptive character. Therefore, their universality is called into question due to the lack of quantitative characteristics.

The problem of assessing the degree of coherence between the course of General Physics (CGP) and the course of Mathematical Analysis (CMA) has been covered in numerous works in different contexts – historical [12], didactic [13], and methodological [14, 15]. These works emphasize the importance of intradisciplinary connections between physics and mathematics; implementations of these connections into the academic process is suggested through a special organization of classes or by using pedagogical principles, though without involving quantitative methods.

3. Intradisciplinary connections (graph model), theory of percolation, and optimization

According to T.N.Gnitetskaya’s definition [1], the intradisciplinary connections are established between the whole basic course and a structural element of the connected course by means of a linking object – any element of knowledge (concept, law, theory, principle, or model) of the basic course. In the intradisciplinary space (by V.A. Ilyin and E.G. Pozdynyak [16]) the CMA is a basic course and the CGP, compiled by I.V. Savelyev [17], is a connected course. This intradisciplinary space is presented in Table 1. The top section of Table 1 shows the array of elements in the CPG course arranged according to the sequence of their studying. The left column of the table shows the cluster of CPG knowledge elements as an array of mathematical concepts from its content. Table 1 shows a fragment with ten out of the sixty-eight mathematical concepts identified by us in the content of CPG. The fragment contains fundamental concepts which, at least once, are included in the content of academic tasks of each section.

Intersection of arrays is depicted by the cells where rhomboids denote the use of corresponding mathematical concepts in the material of the CPG structural element. The cell is shaded if the concept is formed in CMA during the study of the corresponding element of the CPG structural element. For example, the concept of continuity of function is introduced into CMA during the study of conservation principles in CPG, and the concept of function is introduced during the study of kinematics. The right side of the table shows numerical values of relative characteristics of intradisciplinary connections – length of intradisciplinary connections (L), intradisciplinary connections © and coefficient (h) – that show the degree of significance of the mathematical concept through which the intradisciplinary connection of the whole CMA content is established with the CPG structural element. The ranking and hierarchizing of concepts is carried out in accordance with this coefficient. A computer programs was developed in Python 3.9.1 to automate the calculation of the abovementioned quantitative characteristics of graphs of intradisciplinary connections and consequently to calculate the numerical value of the course connectivity.
This fragment of intradisciplinary space shows that many mathematical concepts are introduced into CMA later than in the content of CGP. This fact creates a problem of students' understanding of a number of topics from CGP that require mathematical knowledge. For example, the concept of derivative is introduced in the physics course at the beginning of the first semester during the study of the first structural element; however, in the course of mathematics it is studied later, during the study of the third structural element of the CGP.

In the previous paper the authors placed the structural elements of the abstract course into the cells of the percolation lattice; however, no characteristic that could be a percolation coefficient [18] was suggested. The present research suggests establishing the probability of intradisciplinary information percolating (a group of mathematical concepts) between two elements of CGP structure that will allow introducing a percolation coefficient the definition of which will be provided below. The probability of percolating from the selected element of structure to any other element depends on the amount of intradisciplinary connections with corresponding objects of relation in the considered elements of CGP structure. The more there are such intradisciplinary connections, the higher their total carrying capacity and probability of percolation will be.

As was mentioned above, a mathematical concept may be introduced into the CGP earlier than in the CMA. Consequently, intradisciplinary connections with CPM can be established through this concept both before and after they are introduced into CMA. The graph model defines $p_1$ – capacity of intradisciplinary relation, the value of which for these cases equals $p_1 = 1$ and $p_2 = 2$ respectively [19]. In the percolation task the direction of intradisciplinary information transfer is deterministic, coinciding with the flow of the academic process, just as in the graph model of intradisciplinary connections. The difference is the property of percolation continuity whereas intradisciplinary connections are discrete. Continuity leads to the necessity of introducing an intermediate numerical value of carrying capacity $p_{cp} = 1.5$. Thus, percolation through the concept of a derivative between the structural elements of CGP “Kinematics” and “Dynamics” is characterized by $p_1 = 1$, between “Kinematics” and “Elastic waves” is $p_{cp} = 1.5$, and between “Thermodynamics” and “Elastic waves” is $p_2 = 2$. Other concepts also contribute to percolation; therefore, the total carrying capacity is considered as the total sum of carrying capacities of all intradisciplinary connections established in two structural elements of CGP through identical objects of connections (mathematical concepts). For example, this total sum is equal to 15 between kinematics and oscillations by the concepts shown in the fragment of intradisciplinary space in Table 1. Thus, it is possible to calculate the total carrying capacity of intradisciplinary connections established in all combinations of pairs of structural elements of CGP out of 20 possible ones. The results of these
calculations are provided in Table 2. Two structural elements of CGP are combined at the intersection of rows and columns of Table 2; their numbering coincides with the numbering of structural elements in Table 1. Thus, number 10 shows is the number of the CGP structural elements – “Solid bodies”. The cells of Table 2 contain the value of total carrying capacity of intradisciplinary connections established in corresponding structural elements of CGP. For example, cell {1.6} is located at the intersection of the first “Kinematics” and sixth “Universal Gravitation” structural elements of CGP. It contains the value of 16.5 which is equal to the value of the maximum carrying capacity of intradisciplinary connections established in these structural elements by coinciding mathematical concepts (the first ten concepts are given in Table 1). Obviously, this combination and the result will be repeated in cell {7.3} which is under the median of the table.

Table 2. Table of values of maximum carrying capacity of intradisciplinary connections established through coinciding mathematical concepts in two structural elements of the Course of General Physics when CGP and CMA are studied simultaneously.

| Element of structure | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|----------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1                    | 14.5 | 16 | 16 | 15 | 16.5 | 17 | 14.5 | 15 | 14.5 | 13 | 14 | 17 | 17 | 17 | 17 | 17 | 17 | 14 | 14 |
| 2                    | 15.5 | 16 | 15 | 15 | 15.5 | 17 | 14.5 | 15 | 14.5 | 13 | 14 | 17 | 17 | 17 | 17 | 17 | 17 | 14 | 14 |
| 3                    | 16 | 16 | 16 | 15 | 16.5 | 17 | 14.5 | 15 | 14.5 | 13 | 14 | 17 | 17 | 17 | 17 | 17 | 17 | 14 | 14 |
| 4                    | 17.5 | 17 | 17 | 17 | 17.5 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 |
| 5                    | 18.5 | 18 | 18 | 18 | 18.5 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 |
| 6                    | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 |

Table 3. Table of values of maximum carrying capacity of intradisciplinary connections established through coinciding mathematical concepts in two structural elements of the Course of General Physics when CMA is studied before CGP.
Obviously, introduction of mathematical concepts into CMA earlier or simultaneously with CGP may lead to an increase in the total carrying capacity and, as a consequence, to an increase of the percolation probability. Let’s calculate the total carrying capacities of intradisciplinary connections for the case when learning of CGP starts, for example, in the second semester after studying CMA in the first semester (see Table 3). The results in Table 3 show an increase in the total carrying capacity of intradisciplinary connections in the majority of cases. Thus, the total carrying capacity of intradisciplinary connections for the considered combination of structural elements of CGP in cell \(1.6\) went up from 16.5 to the value of 20. The principle of filling the cells in Tables 2 and 3 can be compared to the filling of adjacency matrix, in which the graph weight is the maximum carrying capacity of intradisciplinary connections.

The considered fragment of the cluster of knowledge elements by group of concepts contains ten mathematical concepts; therefore, the total carrying capacity of intradisciplinary connections can’t exceed 20. Thus, the probability of intradisciplinary percolation of information between two elements of the CGP structure can be found from the ratio of the total carrying capacity for these elements to its maximum value.

Table 4. Distribution of structural elements of CGP over the percolation lattice.

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 8 | 7 | 6 | 5 |
| 9 | 10 | 11 | 12 |
| 16 | 15 | 14 | 13 |
| 17 | 18 | 19 | 20 |

The percolation theory relies on the model of percolation lattice where percolation occurs between the cells of the lattice [20]. In this paper, the percolation lattice is represented by a rectangle divided into cells; however, a lattice in the shape of a chain of cells is also possible. The number of cells is equal to the number of structural elements of the connected course. Twenty elements of the CGP structure are arranged in cells – from the upper left (the 1st element of the CGP structure – “Kinematics”) to the lower right one (the 20th element) in the sequence of their study so that each successive element of the structure is adjacent to the previous one (see Table 4).

The percolation process of intradisciplinary information in the percolation lattice will start with the upper cell and follow a certain algorithm. The cell is shaded. The direction of movement through the lattice is then chosen randomly. Percolation into this cell may or may not occur depending on the value of its probability. If no percolation occurs, the next available direction is chosen, its probability checked and so on. If percolation occurs in this cell, then the algorithm is repeated with the cell where the process followed and the cell gets shaded, and so on. In case no available direction remains, the algorithm is terminated. A quantitative measure of percolation is known – the percolation coefficient is equal to the ratio of the number of shaded cells in the percolation lattice to their total number [21].

The experiment shows that running this algorithm for 10000 times results in the following: the average percolation coefficient – the ratio of shaded cells averaged over all iterations to their total number (20 for the case under study) is equal to 0.345. If we run this algorithm for the case when the CMA is studied prior to the CGP (see Table 3), the value of the coefficient goes up approximately to 0.401. If, on the contrary, the CMA structure is changed in order to introduce the mathematical concepts into CGP in the tenth structural element, the ratio will go down to 0.17. The values of relation between the CGP and CMA obtained with the graph model of intradisciplinary connections for the given transpositions changed in a similar way [1]. This coincidence of results proves the consistency of ideas that we relied on in our calculations of the percolation coefficient.
Conclusions

1. The algorithm of intradisciplinary information percolation in the percolation lattice, constructed from the elements of the connected course, suggested by the authors is reliable, since the results of its implementation do not contradict the data on the courses connectivity calculated with the graph model of intradisciplinary connections.

2. The method of calculating the percolation coefficient can serve as a prototype for the quantitative model of the percolation process in the content of intradisciplinary information of the academic course. The percolation, described above, is compared with the process of understanding and can be considered a subject of further research.

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