Magnetic field of DECY-13 from numerical extrapolation of the measurement result for beam trajectory simulations in central region

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Abstract. DECY-13 cyclotron is a radioisotope production cyclotron developed by PSTA. In a cyclotron, it is important to understand magnetic fields and electric field distribution for the proton beam trajectories. This paper describes the distribution of the magnetic field from the experimental measurement. Magnetic field has been converted into 3-dimensional data by extrapolation. Data validation was done by comparing the Opera3D simulation and used in simulations of the beam trajectories. Simulations were carried out by using Scilab 5.4.1 and the Runge-Kutta (RK4) approximation method. The parameters used in DECY-13 cyclotron were 40 kV Dee voltage with a radiofrequency (RF) 77.66 MHz and a fourth-harmonics type. The calculations and simulations result showed the beam could pass through the puller at the distance of the ion source with a puller was 6 mm and the optimum distance of 4 mm. The largest difference in error at z = 6 mm was 0.023 T of the average magnetic field 1.275 T. The phase acceptance in horizontal and vertical motion in the central region was about 38 Degrees from -19° to 19°. This experimental and simulation data could be used as a reference for DeCY-13 cyclotron magnetic and electric field distribution profiles.

1. Introduction
Cancer is one of the leading causes of death worldwide. The incidence of cancer in Indonesia (136.2/100,000 population) ranks 8 in Southeast Asia, whereas in Asia it ranks 23 [1]. One effort that can be done to prevent cancer is by early detection. One of the tools that can demonstrate this process is by detection using Positron Emission Tomography-Computed Tomography (PET-CT) [2]. The use of PET-CT requires radiopharmaceutical that is inserted into the patient's body which radiation results from the interaction will be detected by using PET-CT to produce images of the patient's body tissue [3]. Radiopharmaceutical compounds used in PET-CT are generally produced by cyclotrons.

Under these conditions, the mastery of cyclotron technology needs to be done so that the supply and maintenance of cyclotron for PET can be done in the country with higher local content [4]. A cyclotron is an accelerator of charged particles with the direction of motion of a circular particle beam, from the circular motion of a small radius to the radius of an increasingly larger trajectory [5]. The greater the radius of the path, the greater the particle beam energy. The particle beam motion in the
form of a circular trajectory due to acceleration by an RF voltage, which is then deflected by a magnetic field that is perpendicular to the motion of the particle beam trajectory [4].

CAST-BATAN (Center for Accelerator and Science Technology – National Nuclear Energy Agency) is developing accelerator technology, one of which is the development of a 13 MeV cyclotron for PET named DECY-13 (Development of Experimental Cyclotron in Yogyakarta - 13 MeV). One of the main things in the cyclotron is knowing the trajectory of the beam from the ion source to the target using the magnetic field and the electric field distribution [5]. This will be closely related to the energy output and beam currents that will be obtained from the cyclotron system.

Previously, beam trajectory simulations had been carried out to determine that the design of the magnet and accelerator electrodes could produce ions beams up to 13 MeV energy. But it still uses a simulated magnetic field. For this reason, simulations using data from the measurement results of the DECY-13 cyclotron magnetic field need to be carried out. The purpose of this research was to numerically extrapolate the magnetic field data from the measurement results to transform into three dimensions and simulate with Scilab-5.4.1 software to obtain the shape of the beam trajectory and the energy on the proton cyclotron is 13 MeV.

2. Experimental method

2.1. DECY-13 magnet field measurement

The measurement of the magnetic field is done by mapping. This mapping process is carried out by measuring the magnetic field perpendicular to the plane of the magnetic pole in the center of the magnetic pole gap in the Cartesian coordinates (x, y). The position interval used is 5 mm, with the mapping time needed for 9 hours [7]. The magnetic field mapping system uses 2 hall probes to map the entire magnetic sector (4 sectors) measuring 960 mm × 960 mm. Thus the results of mapping the entire magnetic sector can be obtained by combining magnetic field data from the 2 hall probes. The mapping data is in the form of a magnetic field in the center of the gap between the upper and lower magnets, so the resulting magnetic field data is along the x and y axes 960 mm × 960 mm in the direction of z = 0 (center of the gap).

2.2. Extrapolating magnetic field data

Extrapolation was carried out to convert 2-dimensional magnetic field data into 3–dimensions because the data obtained from the measurement results are in 2–dimensional coordinates (x = 0 – 960 mm, y = 0 – 960 mm, with z = 0), whereas for trajectory simulation is required data at 3–dimensional coordinates (x = 0 – 960 mm, y = 0 – 960 mm, with z = −20 – 20 mm) and only the field value in the z (B) direction. Extrapolation is done by a numerical approach to find the values of $B_x$, $B_y$, and $B_z$ in all areas of the cyclotron magnetic field. Referring to Laplace’s equation in Cartesian coordinates, the magnetic field will satisfy (1)

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

(1)

with the resulting magnetic induction, the cross product will fulfill (2)

$$\nabla \times \vec{B} = \mu \cdot i + D \frac{\partial E}{\partial t}$$

(2)

Assuming the value of $i \approx 0$ then $\mu \cdot i = 0$ , and $\frac{\partial E}{\partial t} \approx 0$, this is because the effect on the value of $B$ is so small that it can be ignored so that $D \frac{\partial E}{\partial t} = 0$ then (2) will change to

$$\left(\frac{\partial B_y}{\partial z} - \frac{\partial B_z}{\partial y}\right) \hat{x} + \left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right) \hat{y} + \left(\frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x}\right) \hat{z} = 0$$

(3)
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$$\left( \frac{\partial B_z}{\partial x} - \frac{\partial B_z}{\partial y} \right) \hat{x} = 0 \quad \text{means} \quad \frac{\partial B_z}{\partial y} = \frac{\partial B_z}{\partial x}$$  \hspace{2cm} (4)

$$\left( \frac{\partial B_x}{\partial x} - \frac{\partial B_z}{\partial z} \right) \hat{y} = 0 \quad \text{means} \quad \frac{\partial B_z}{\partial x} = \frac{\partial B_x}{\partial z}$$  \hspace{2cm} (5)

$$\left( \frac{\partial B_y}{\partial y} - \frac{\partial B_z}{\partial z} \right) \hat{z} = 0 \quad \text{means} \quad \frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z}$$  \hspace{2cm} (6)

By using the Taylor series approach, the formulation is done to find the value of $B_4 B_3 B_2$ and $B$ can be written as

$$B(x,y,z) = B(x,y,z)\vert_{z=0} + z \frac{\partial B_z}{\partial x} \bigg|_{z=0} + z^2 \frac{\partial^2 B_z}{\partial x^2} \bigg|_{z=0} + z^2 \frac{\partial^2 B_z}{\partial x^2} \bigg|_{z=0} + z^3 \frac{\partial^3 B_z}{\partial x^3} \bigg|_{z=0} + z^4 \frac{\partial^4 B_z}{\partial x^4} \bigg|_{z=0} + \cdots$$  \hspace{1cm} (7)

For variations in the value of $B_4 B_3 B_2$ and $B$, because there are anti-symmetry properties on a cyclotron magnet, which is a condition that gives the same amount of value in the opposite direction, so that at the value of $z$ of the odd rank applies a value $B_z(z) = -B_z(-z)$ and $B_y(z) = -B_y(-z)$. So the Taylor series equation to find the value of $B_4 B_3 B_2$ and $B$ will meet (8) and (9)

$$B_4(x,y,z) = B_4(x,y,z)\vert_{z=0} + z \frac{\partial B_z}{\partial x} \bigg|_{z=0} + z^2 \frac{\partial^2 B_z}{\partial x^2} \bigg|_{z=0} - \frac{z^2}{6} \left( \frac{\partial^3 B_z}{\partial x^3} + \frac{\partial^3 B_z}{\partial x^3} \right)_{z=0}$$  \hspace{2cm} (8)

$$B_5(x,y,z) = B_5(x,y,z)\vert_{z=0} + z \frac{\partial B_z}{\partial y} \bigg|_{z=0} + z^2 \frac{\partial^2 B_z}{\partial y^2} \bigg|_{z=0} - \frac{z^2}{6} \left( \frac{\partial^3 B_z}{\partial y^3} + \frac{\partial^3 B_z}{\partial y^3} \right)_{z=0}$$  \hspace{2cm} (9)

while for the variation of $B$ due to the symmetry nature of the cyclotron magnetism, which is a condition where the value of the value $B(z) = B(-z)$ then the variation in the value of $B$ only applies to the even $z$ value so that (7) will become (10)

$$B_4(x,y,z) = B_4(x,y,z)\vert_{z=0} + z^2 \frac{\partial^2 B_z}{\partial x^2} \bigg|_{z=0} + z^2 \frac{\partial^2 B_z}{\partial y^2} \bigg|_{z=0} + \frac{z^4}{24} \left( \frac{\partial^4 B_z}{\partial x^4} + 2 \frac{\partial^4 B_z}{\partial y^4} + \frac{\partial^4 B_z}{\partial y^4} \right)_{z=0}$$  \hspace{1cm} (10)

2.3. Finite difference methods

In using the formula to find the values of $B_4 B_3 B_2$ and $B$, extrapolation of data is done by using the shadow method. The shadow method is applied by reflecting each data in the direction of $-x$ and $-y$ as far as 60 mm. This is used because the symmetrical magnetic properties and the data used in the processing are $\frac{1}{4}$ parts. In addition, for each increase towards $z$, three approaches are used to find the values of $B_4 B_3 B_2$ and $B$ namely; (1) in the central difference in the middle of the data, (2) forward difference and backward difference in the edge of the edge data, and (3) a mixture of forward difference or backward difference with the central difference in the middle edge data, as shown in figure 1.

![Figure 1. Use of numerical methods.](image)

2.4. Beam trajectory calculations

In beam trajectories simulation, magnetic field distribution is obtained from the measurement results of the DECY-13 magnetic field experimentally and the electric field calculation results with OPERA3D. Electrical potential data $V(x,y,z)$ were obtained by numerical simulation using the
Opera3D Tosca module for an area of 960 mm x 960 mm x 40 mm with a mesh size of 6 mm and a data collection resolution of 1 mm [6]. Simulation using Scilab software 5.4.1 using the Runge-Kutta approach. Maximum ion source and puller distances are 6 mm [6] and distance optimization is at 4 mm [8]. The maximum directional beam direction (z axis) that crosses is 6 mm [6]. The parameters used in calculating the file path are presented in table 1.

| Parameter                  | Value                  |
|----------------------------|------------------------|
| Proton kinetic energy      | 13 MeV                 |
| Resonant frequency         | 77.66 MHz              |
| Dee voltage                | 40 kV                  |
| Dee number                 | 2                      |
| Sector number              | 4                      |
| Valley / Hill gap          | 120 / 40 mm            |
| B₀                         | 1.275 T                |
| Maximum B₀                 | 1.942 T                |
| Minimum B₀                 | 0.768 T                |
| Coil current               | 2×44012 Ampere/turn    |
| Extraction radius          | 0.48 m                 |
| Hill Angels                | 35°–44° from magnet center |

2.5. Extrapolation steps

The extrapolation steps to obtain the distribution of the magnetic field are shown in figure 2.

![Extrapolated flow chart](image)

Figure 2. Extrapolated flow chart.

3. Result and discussion
The magnetic field distribution data from measurements are only in the central magnetic region \( (z = 0 \text{ mm}) \) with a data interval of 5 mm and only the \( z \) field \( (B_z) \). Magnetic field distribution for use in beam trajectory simulations it is necessary to in all areas of ions that can cross \((-20 < z < 20 \text{ mm})\) with data intervals of 1 mm and fields \( B_x, B_y, B_z \). Therefore, extrapolating data magnetic field \( B_x, B_y, \) and \( B_z \) needs to be done.

In accordance with equations 8 and 9, the magnetic fields of \( B_x \) and \( B_y \) can be obtained from the surrounding magnetic fields of \( B_z \). Therefore, the magnetic field distribution of \( B_z \) needs to be known first. Using equation 10 for the measured magnetic field data, extrapolating the data towards \( z \) is shown in Figure 3. The distribution of the magnetic field at the center moves away as the field increases in \( z \)-direction (Figure 3a). This is different from the inside data condition (Figure 3b). So it is necessary to apply the shadow method in applying the finite difference method in extrapolating data.

Data extrapolation in three dimensions is shown in Figure 4a at 5 mm intervals and Figure 4b shows the data from the Opera3D simulation results. There are differences in the shape of the field produced, so that data interpolation is necessary. Data interpolation is done by finding the value of data distribution between the two existing data into 1 mm intervals.
Figure 3. (a) extrapolate in the direction of \( z \) in the center of the central region (b) extrapolate in the direction of \( z \) in the area \( x = 120, y = 120 \) mm.

Figure 5 shows the final 3–dimensional shape of the extrapolated \( B \) field. The resulting magnets field distribution has been able to approach the distribution of simulation results.

Referring to the simulation results on DECY-13 by Anggraita [6], the magnetic field direction of the vertical is in the region of −6 to 6 mm, and variations direction in the vertical can be accepted up to a 6 mm. Based on these references, data from the extrapolations are compared to the data obtained from the simulation (Table 2). Data errors are compared to the highest field and the lowest field produced during extrapolation and compared to the field in the simulation. From the data, the resulting error value increases with increasing fields towards in \( z \).

Figure 4. Three-dimensional shape at center \( z \): (a) Field \( B \) measurement (b) Field \( B \) Simulation.

Figure 5. The three-dimensional shape at center \( z \) with Field \( B \) measurement after interpolation.

3.1. Horizontal and vertical ion trajectory

Figures 6a and 6b show the horizontal and vertical trajectories of the ions. The starting RF phase varies from −25° to 25°, and the starting angle is perpendicular to the slit and parallel to the median plane. The distance of the ion source with a puller is 6 mm [6] and the optimum distance of 4 mm [8]. The RF phase acceptance in horizontal and vertical motion is about 38 degrees from 19° to 19° for simulating the beam trajectory in the central region. The maximum vertical displacement from the median plane is about 10 mm. The vertical height of the center Dee from the median plane is 20 mm. The RF Phase acceptance in KIRAM-13 is 55 degrees with maximum vertical displacement is 6.8 mm [5], CYCIAE-14 the RF Phase acceptance is 50 degree, with maximum vertical displacement is 5 mm.
[9], and in CYCHU-10 the RF Phase acceptance is 25 degree, with maximum vertical displacement is 6 mm [10].

Table 2. Comparison of values magnetic field distribution in the extrapolated to opera3d simulation.

| Value of z (mm) | $B_{\text{max extrapolation/simulation}}$ (T) | Error (T) | $B_{\text{min extrapolation/simulation}}$ (T) | Error (T) |
|----------------|-----------------------------------------------|-----------|-----------------------------------------------|-----------|
| 1              | 1.962525 / 1.942168                          | 0.020357  | 0.079919 / 0.078825                            | 0.001094  |
| 2              | 1.9626122 / 1.942199                         | 0.020413  | 0.0798383 / 0.078622                           | 0.001216  |
| 3              | 1.9627088 / 1.942233                         | 0.020476  | 0.0798451 / 0.07842                            | 0.0014251 |
| 4              | 1.9628047 / 1.942312                         | 0.020493  | 0.0799464 / 0.078218                           | 0.0017284 |
| 5              | 1.9629328 / 1.94239                          | 0.020543  | 0.0801661 / 0.078016                           | 0.0021501 |
| 6              | 1.9663689 / 1.942523                         | 0.023846  | 0.0805306 / 0.077814                           | 0.0027166 |

Figure 6. (a) the horizontal beam trajectories (b) the vertical beam trajectories; green = 0 degree, blue = -10 and 10 degree, red = -15 and 15 degree, yellow = -19 and 19 degree.

4. Conclusion

Based on the results of the study, the distribution of the magnetic field can be extrapolated from the 2-dimensional data of the measurement results into 3-dimensional data. The extrapolation results compared with the simulation results show a maximum error value of 0.023 T from the mean 1275 T at the height of $z = 6$ mm. The RF phase acceptance in horizontal and vertical motion is about 38 degrees from 19° to 19° for simulating the beam trajectory in the central region. The maximum vertical displacement from the median plane is about 10 mm.

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References

[1] Kemnentrian Kesehatan, Pusat Data dan Informasi Kementrian Kanker (2015). Retrieved Agustus 5, 2019, from depkes.go.id: http://www.depkes.go.id/folder/view/01/structure-publikasi-pusdatin-info-datin.html

[2] E. Mulyani, Design of Central Region for 13 MeV Proton Cyclotron, Thesis, Gadjah Mada University (2013).

[3] Taufik, Design of 13 MeV Proton Cyclotron Magnet using Opera3D Software, Thesis, Gadjah Mada University (2013).

[4] Taufik, A. Hermanto, P. Anggraita, S. Santosa, Determination of Magnet Specification of 13 MeV Proton Cyclotron Based on Opera 3D, Atom Indonesia 40 (2014) 69-75.

[5] D. H. An et al, Beam Trajectory Simulations for Kirams-13 Cyclotron (2008).

[6] A. Pramudita, E. Mulyani, I.A. Kudus, Simulations of Beam Quality in a 13 MeV PET Cyclotron, Atom Indonesia 41 (2015) 145-149.

[7] I. A. Kudus, Taufik, K. Wibowo, F. S. Permana, Comparation of Construction and Simulation Result from Isochronus of Cyclotron Magnet DECY-13, Ganendra Journal of Nuclear Science and Technology 20 (2017) 83-90.

[8] Silakhuddin and I.A. Kudus, Optimization of Ion Source Head Position in the Central Region of DECY-13 Cyclotron, Atom Indonesia 43 (2017) 81-86.

[9] T. Zhang, M. Li, J. Zhong, S. An, S. Wei, Beam Dynamics study for a small, high current 14 MeV PET Cyclotron, Nuclear Instruments and Methods in Physics Research B 269 (2011) 2955-2958.

[10] Q. Bin, L. K. Feng, F. Y. Zhang, F. M. Wui, Central Region Design for a 10 MeV Internal Ion Source Cyclotron, Chinese Physics C 33 (2009) 682-686.