The Homogeneity of the Star-forming Environment of the Milky Way Disk over Time

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Abstract

Stellar abundances and ages afford the means to link chemical enrichment to galactic formation. In the Milky Way, individual element abundances show tight correlations with age, which vary in slope across ([Fe/H]–[α/Fe]). Here, we step from characterizing abundances as measures of age, to understanding how abundances trace properties of stellar birth environment in the disk over time. Using measurements from ~27,000 APOGEE stars (R = 22,500, signal-to-noise ratio > 200), we build simple local linear models to predict a sample of elements (X = Si, O, Ca, Ti, Ni, Al, Mn, Cr) using (Fe, Mg) abundances alone, as fiducial tracers of supernovae production channels. Given [Fe/H] and [Mg/H], we predict these elements, [X/H], to about double the uncertainty of their measurements. The intrinsic dispersion, after subtracting measurement errors in quadrature is ≈0.015–0.04 dex. The residuals of the prediction (measurement – model) for each element demonstrate that each element has an individual link to birth properties at fixed (Fe, Mg). Residuals from primarily massive-star supernovae (i.e., Si, O, Al) partially correlate with guiding radius. Residuals from primarily supernovae Ia (i.e., Mn, Ni) partially correlate with age. A fraction of the intrinsic scatter that persists at fixed (Fe, Mg), however, after accounting for correlations, does not appear to further discriminate between birth properties that can be traced with present-day measurements. Presumably, this is because the residuals are also, in part, a measure of the typical (in-)homogeneity of the disk’s stellar birth environments, previously inferred only using open cluster systems. Our study implies at fixed birth radius and time that there is a median scatter of ≈0.01–0.015 dex in elements generated in supernovae sources.

Unified Astronomy Thesaurus concepts: Milky Way disk (1050); Red giant stars (1372); Milky Way formation (1053); Star formation (1569); Chemical enrichment (225); Chemical abundances (224)

1. Introduction

Chemical abundances from surveys like APOGEE (Majewski et al. 2017), GALAH (Buder et al. 2018), Gaia–eoso (Gilmore et al. 2012) and LAMOST (Zhao et al. 2012), combined with a reference set of ages from Kepler (Borucki et al. 2010) provide the means to link chemical enrichment to galactic formation over time. Temporally, the disk formed inside-out (e.g., Frankel et al. 2019): we observe that there are, on average, older stars in the inner galaxy and younger stars in the outer regions (e.g., Ness et al. 2016; Lu et al. 2021). Chemically, the disk shows a negative metallicity gradient across Galactic radius, that has presumably weakened over time due to radial migration (e.g., Roškar et al. 2008; Frankel et al. 2018; Minchev et al. 2018). The observed gradients of the older stellar populations not only track the chemical evolution of the Galaxy, but carry information about the stellar migration that weakened it over time.

For stars in the disk, studies of individual element distributions, [X/Fe], have revealed that at fixed metallicity, [Fe/H], there are very small intrinsic scatters (≈0.03 dex) around weak age–individual abundance relations (e.g., Nissen 2015; Bedell et al. 2018). Recently, this has been shown for stars across a large radial extent of the disk, and at different metallicities [Fe/H] (e.g., Ness et al. 2019; Delgado Mena et al. 2019; Hayden et al. 2020; Casali et al. 2020; Casamiquela et al. 2021; Espinoza-Rojas et al. 2021). The age–abundance relations vary in their slopes at different values in the chemical plane defined by type Ia supernova (SN Ia) and type II supernova (SN II) sources (Fe, Mg); although the intrinsic scatter in the disk population is comparably small across chemical cells (or bins) in the [Fe/H]–[Mg/Fe] plane (Lu et al. 2021). Previously, we have addressed the question of how individual abundances at fixed [Fe/H] and stellar ages can link to birth radius (e.g., Ness et al. 2019; Ratcliffe et al. 2022). Here, we turn this question around to ask; at fixed birth radius and age, what is the intrinsic scatter in individual elements? That is, what is the abundance scatter set by the time and place of the star-forming environment?

To address this question we first measure how well a set of elements that are produced by supernova sources are predicted by two fiducial supernova sources. For the disk, a two-process model defined by (Fe, Mg) can well describe the smooth individual abundance variation across the disk (Weinberg et al. 2019; Griffith et al. 2021a; Weinberg et al. 2021). Here we use Fe and Mg as fiducial SN Ia and SN II sources, respectively, to quantify how well we can predict an ensemble of eight other supernovae produced elements (Si, O, Ca, Ti, Ni, Al, Mn, Cr), given these two elements alone. We then examine the residual scatter around these predictions and estimate how much of this scatter is not correlated with astrophysical sources. That is, how
much scatter is intrinsic to the birth place of stars in the disk. Section 2 introduces our data, Section 3, our method, Section 4 our results, and in Sections 5 and 6 we include a brief discussion and summary of future prospects.

2. Data

We assemble a high fidelity sample of APOGEE DR16 red giant stars with ten abundance measurements from the ASPCAP pipeline (García Pérez et al. 2016). We use $X =$ Fe, Mg, Si, O, Ca, Ti, Ni, Mn, Cr, Al (we take the reported abundances with respect to Fe and convert them to $[X/H]$ by $+[Fe/H]$). These ten elements are understood to be produced in supernovae channels, specifically via massive-star and white-dwarf explosions (e.g., Kobayashi et al. 2020). We perform the following quality cuts to obtain our sample of ~27,000 stars:

- $T_{\text{eff}} = 4500 - 5500$ K
- $\log g = 1.5 - 3.5$ dex
- $[Fe/H] > -1$ dex
- $[X/Fe]_{\text{error}} < 0.1$ dex
- $\text{SNR} > 200$
- $|V_{\text{helio}}| < 50$ km/s

Flags ASPCAPBAD and ROTATION not set

Most of these stars are located near the Sun, with a mean Galactic radius of $R_{\text{gal}} = 8.6$ and 1σ (standard deviation) of 1 kpc, but range from $R_{\text{gal}} = 2$–15 kpc. In total, the stars span all ages, from stars as young as $\geq 0.3$ Gyr, with a mean distribution of age $= 6.2 \pm 3.6$ Gyr. The stars are concentrated in evolutionary state around a mean log $g$ of 2.6 dex with a 1σ of 0.3 dex. At signal-to-noise ratio (S/N) $> 200$, these ten elements are measured to a precision of between 0.01 and 0.04 dex, on average. We restrict our sample to heliocentric velocities of $-50$ km/s $< V_{\text{helio}} < 50$ km/s.

The ages we use are derived from spectra using data-driven modeling and the Kepler reference objects (Lu et al. 2021). These have individual uncertainties of $\sigma_{\text{age}} \approx 3$ Gyr. The orbital actions and spatial coordinates are from the ASTRO-NN catalog (Leung & Bovy 2019).

Additionally, we use the open cluster members from the Open Cluster Chemical Abundance and Mapping (OCCAM) Survey catalog of open clusters (Donor et al. 2020) for a reference sample of stars known to be born together. In examining the open cluster stars, we relax our S/N limit to S/N $> 100$ and remove our radial velocity restriction. This increases the number of field stars that we work with to $\approx 80,000$ and increases our open cluster sample by a factor of two (to $\approx 100$ open cluster stars in nine clusters). We note that our less restrictive cuts negligibly change our overall results and conclusions for the full ensemble analyses. However, using the higher fidelity sample of stars is a more precise examination of the data, with a lesser contribution from nuisance signals, i.e., artifacts that we find correlate with radial velocity.

3. Method

We want to determine, given stellar evolutionary state and two abundances (Fe, Mg), how well we can predict other elements generated via supernovae channels. To do this, we use linear regression. A local linear regression model is like basic regression (Hastie et al. 2001), but an individual model is built for each object on a subset of data, i.e., a local model. Each local model is constructed by defining a neighborhood around each object in some data space (e.g., see Saeideh et al. 2021, for an example of this using Kepler data). This approach takes on the following steps for our $N = 27,000$ stars, where the parameters that we select as predictors are $Y = (T_{\text{eff}}, \log g, [Mg/H], [Fe/H])$ and our goal is to predict eight abundances $X = (\text{Si}, \text{O}, \text{Ca}, \text{Ti}, \text{Ni}, \text{Al}, \text{Mn}, \text{Cr})$:

1. Standardization of parameters: we first standardize the parameters that we wish to use for calculating a distance between each star and every other star. We use these distance measurements to define a neighborhood for every star. The parameters we select are evolutionary state and the elements from two fiducial supernovae channels: $Y = (T_{\text{eff}}, \log g, [Mg/H], [Fe/H])$. To standardize each parameter, we subtract the mean and divide by the standard deviation, calculated for the N stars, for each: $y \approx (Y - \bar{Y}) / \sigma_Y$.

2. Distance metric: for each star $n$ in our set of N objects, we determine, via a $\chi^2$ comparison, the distance to each other star in the sample using the four standardized parameters, $d_n \approx (T_{\text{eff}}, \log g, [Fe/H]/[Mg/H])$. For any two stars, the $\chi^2$ value is given by the following equation:

$$\chi^2_{nn'} = \sum_{i=1}^{4} \frac{(y_{ni} - y_{ni'})^2}{\sigma_{ni}^2 + \sigma_{ni'}^2},$$

where the indices $n$ and $n'$ denote the two stars, $i$ is index of each of the standardized parameters, and $y_{ni}$ is the standardized parameter measurements with standardized uncertainty $\sigma_{ni}$.

3. Define the local model: For each star, $n$, we then take the $k$ nearest neighbors with the smallest $\chi^2$ values (where $k$ sets the size of the neighborhood). In practice the model is fairly insensitive to the choice of $k$ and between $k = 50$–300 produces comparable results. We select $k = 100$.

4. Regression: using the local model for each star of $k$ nearest neighbors, and excluding the star used to define the neighborhood, we use the linear regression function in python’s sklearn package (Pedregosa et al. 2011). This modeling step learns the relationship between the parameters of $Y = (T_{\text{eff}}, \log g, [Fe/H], [Mg/Fe])$ and each of the eight abundances $[X/H]$ (separately), parameterized for each $[X/H]$ with five coefficients for each local model (the intercept and one for each $X$). Then abundances $[X/H]$ are predicted from the five parameters.

5. Predict: for each star, we then (separately) predict each of the eight abundances defined above $[X/H]$ using the four parameters of $Y = (T_{\text{eff}}, \log g, [Fe/H], [Mg/Fe])$ and the coefficients determined in step (4). This gives our prediction for $[X/H]$, that we can compare to the measured abundance from APSCAP.

6. Repeat: this procedure for every star 1 to N.

This method thereby enables us to undertake an assessment of how precisely two fiducial channels (Fe, Mg) can predict the other abundances $[X/H]$. We can quantify this prediction using the measure of intrinsic dispersion, which is the scatter of each measured versus predicted abundance, accounting for the measurement uncertainties. We find that abundance residuals themselves (measured abundance-predicted abundance) that we

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6 We test using both $[X/H]$ and $[X/Fe]$ and for local linear regression, and all of the results and inferences are the same.
calculate show strong correlations with velocity. This is presumably not related to the star, but nuisance imprints in the spectra that are a significant fraction of the residual amplitude, from sky emission and tellurics and diffuse interstellar bands (Holtzman et al. 2018, and K. McKinnon et al. 2022, in preparation). This motivates our selection of $|V_{\text{helio}}| < 50$ km s$^{-1}$ for the main part of our analysis.

We examine the correlations between individual element abundance residuals and orbital actions and ages. We subsequently introduce additional predictors into our regression model in order to assess the intrinsic dispersion once age and orbital parameters are accounted for. This provides an estimate of the birth scatter in the elements conditioned on (Fe,Mg) alone.

4. Results

4.1. Local Linear Model Predictions

The mean summary of our element predictions compared to their measurements using local linear modeling is shown in Figure 1, for our N = 27,000 stars. The x-axis shows the ASPCAP abundances for the stars and the y-axis shows the prediction using a linear regression of the local neighborhood abundances for the stars and the Figure 1, for our N measurements using local linear modeling is shown in Figure 1. The intrinsic scatter of the prediction is indicated, as is the rms difference between reference (measured) and predicted abundance, and bias. The stars are colored by [Fe/H] to demonstrate that there is no metallicity bias. The mean error on the measurements is indicated at the bottom right of each subpanel. Elements are predicted with a precision of between 0.01 and 0.04 dex.

Predicted versus the measured abundance [X/H] is also included at top left (bias = 0 as expected from linear regression). We note that the predicted abundances are unbiased for [Fe/H] > −1. However, this is not the case for the halo, which is a far more chemically diverse population at fixed (Fe, Mg). The scatter around the 1:1 line is also relatively metallicity independent. The mean measurement uncertainty is reported in the bottom-right corner of each subpanel. We note that the measurement uncertainty on the predictors in our model ($T_{\text{eff}}$, log $g$, [Fe/H], [Mg/Fe]) propagate negligibly to the intrinsic scatter that we measure. We note that the systematic uncertainty (accuracy) of the individual element abundance measurements is much larger than the intrinsic dispersion values. Including $T_{\text{eff}}$ and log $g$ as predictors is done so as to remove systematic biases imputed by stellar model approximations (e.g., Jofré et al. 2019). We note that while these will remove systematics in the model from model atmospheres, they will not remove biases from anything that is inherited at the level of the spectra itself (see Figure 6).

We examine the correlation with intrinsic dispersion as a function of [Fe/H] as shown in Figure 2. The 1σ intrinsic dispersion of the [X/H] measurement compared to the prediction is shown for each element. There are unique trends in each element; some on the order of a few percent and some on the order of a factor of two, across [Fe/H]. The elements Cr and O show the most dramatic increase with decreasing [Fe/H], and the element Mn shows the most significant increase at higher [Fe/H]. Beyond noting that the intrinsic dispersion varies across metallicity [Fe/H], we do not draw strong astrophysical conclusions from these trends. This is because the total scatter of the [X/H] prediction and subsequently the intrinsic dispersion values that we calculate are sensitive to the
accuracy of the abundance measurement errors, which vary across the [Fe/H] plane. For all elements, the measurement errors increase with decreasing metallicity, with the mean error for elements Cr and O increasing the most dramatically by a factor of more than two. We also explored correlations of the intrinsic dispersion with $T_{\text{eff}}$ and $\log g$. We find interesting deviations from smooth trends in the $\log g - \sigma_{\text{intrinsic}}$ relations around $\log g = 2.4$ for all elements, and marginal increases in intrinsic dispersion with increasing $\log g$. There are small gradients in the intrinsic dispersion with $T_{\text{eff}}$ which for some elements including O, Ti, and Al persist even when examining only narrow metallicity ranges. This indicates that the driver of the correlations reported in Figure 2 for some elements may be a consequence of the $T_{\text{eff}}$ of the stars, which is correlated with [Fe/H]; the hotter stars are marginally more metal-poor. The overall extent of the trends is however captured in Figure 2. We do not find different results or draw different conclusions if we repeat our analysis using a narrower range in $T_{\text{eff}}$ or $\log g$, or over a smaller spatial extent of the disk.

Each element’s mean intrinsic dispersion across all [Fe/H] (or intrinsic scatter) from Figure 1 is compared in Figure 3. The dashed line indicates the median intrinsic dispersion for these eight elements, which is $\sigma_{\text{intrinsic}} \approx 0.021$ dex. That is, using (Fe, Mg) alone, the other elements produced in supernovae can be predicted on average to this precision, but with a range of 0.014 dex (for Ni) to 0.04 dex (for Cr).

The intrinsic scatter for each element [X/H] is a quantification of the effective individuality of the element. That is, how much additional information it captures beyond (Fe, Mg) alone. The intrinsic scatter also represents the diversity of the star-forming environment. For example, given the (Fe, Mg) of a star alone, to learn something additional from Si, from Figure 1, we see that we need to measure the Si abundance to < 0.015 dex.

4.2. Residuals: What Do These Mean?

The chemical elements, even those produced in the same progenitor, have unique details in their production, and therefore, in principle, capture individual information about their genesis (e.g., Freeman & Bland-Hawthorn 2002; Buder et al. 2018; Blancato 2019; Ting & Weinberg 2021; Kobayashi et al. 2020). However, they are summarized in Figure 3, only marginally (compared to typical measurement uncertainties), for supernovae sources. While the ten elements considered, Fe, Mg, Si, O, Ca, Ti, Ni, Al, Mn, and Cr, are produced in supernovae, this is (i) via different SN II compared to SN Ia fractions, (ii) with different [Fe/H] dependencies of their yields, (iii) with different mass...
dependencies (e.g., Woosley & Weaver 1995; Kobayashi et al. 2020; Matteucci 2021, and references therein), as well as (iv) subject to possible progenitor rotation impacts on the yields (e.g., Marassi et al. 2019; Prantzos et al. 2018). For each element shown in Figure 1, we calculate the residual of that element—measured abundance from ASPCAP—model’s prediction. A positive residual means the measured abundance is higher than predicted by the local neighborhood, and a negative residual means the abundance is less than that expected in the local neighborhood. As the residuals quantify how far each star deviates from the model’s prediction, in principle these may be valuable tracers of birth environment, which is independent of (Fe, Mg). That is, because the element abundances ([Fe/H], [Mg/H]) are the predictors, the metallicity dependence of the yield has been removed, so scatter is caused by other conditions of the star-forming environment.

Figure 4 shows a matrix of the correlations of the residuals of the predictions. This Figure shows that intra-family elements (e.g., Si and O, produced predominately in SN II) appear to be correlated, and inter-family elements (e.g., Cr and Si, produced predominantly in SN Ia and SN II, respectively) appear anticorrelated. Note, all correlations are fairly weak (a Pearson correlation coefficient of 1 would be a perfectly correlated set of two error-free distributions).

We now investigate correlations of the residuals with age and present-day orbital properties (time and place). The correlations between the element residuals, and their trends with age and orbit are revealing as to the difference in the star-forming environment that has given rise to their abundances. Furthermore such correlations of the residuals would indicate additional resolving power in the eight elements in Figure 1 to identify birth environment beyond Fe and Mg alone (Ting & Weinberg 2021).

4.3. Correlations of Residuals with Age and Orbit

Figure 5 shows the correlation between the element abundance residuals and the angular momentum (at left) and vertical action (at right), \( J_z \) and \( L_z \), respectively. We do not include the radial action \( J_r \), as the residual trends are flat and the range is limited, given as these are disk stars. The (smoothed) running mean of action-residual trends is shown, and the shaded regions represent the 1σ confidence on the mean. The axes are scaled to be the amplitude of the intrinsic dispersion measured as reported in Figure 1. The axis scaling highlights that the spatial/orbital correlations explain are only (a very small) part of the total magnitude of this scatter.

Similarly to Figure 5, Figure 6, at left, shows the residual of each element versus age. There are ~15,000 stars with ages (Lu et al. 2021) that are used to make the panel showing the age correlation. The right panel of Figure 6 shows a larger sample of 80,000 stars with an S/N cut of >100 (and no velocity restriction imposed). Figure 6 at right shows the correlation between residual amplitude for these 80,000 stars and heliocentric velocity. This is the most significant variable accounting for correlations in the abundance residuals—and likely due to nuisance signals in the spectra near the limits of the precision. Initial results suggest that residual telluric and sky line and other nonstellar features remain in the APOGEE spectra at the level of 1%–2% across many wavelengths. Because these features are in the (approximate) observer frame, the stellar rest frame spectra have contamination (which impacts measured abundances) that correlates with stellar heliocentric velocity (see K. McKinnon et al. 2022, in preparation).

The relations are non-monotonic and are nontrivial to remove, and therefore separate from the abundance measurements themselves. We note that if we repeat the exercise of calculating the residuals of the [X/H] predictions and subsequently examining the orbit and age correlations with a lower fidelity sample, e.g., S/N > 100 and remove any velocity limit, we find effectively the same results to those shown for our higher fidelity sample here. However, we measure higher intrinsic scatters for a few of the elements (shown in Figure 9).

In summary, these correlations in Figures 5 and 6 demonstrate that the astrophysical origins of the intrinsic scatter of each element [X/H] around the model’s prediction from (Fe, Mg) can only partially be attributed to birth time and present orbit/location.

4.4. Intrinsic Variance Not Explained by Astrophysical Properties

We see from Figures 5 and 6 that orbital parameters and age correlations explain only part of the intrinsic scatter around the abundance predictions shown in Figure 1 (residual amplitude). To test how much the intrinsic scatter decreases once we remove known correlations, we add to the regression model the additional parameters shown in Figures 5 and 6, of \( J_z \), \( L_z \), age, and \( V_{\text{helio}} \). We do this to try to assess how much of the variance remains once we account for other sources of correlation. We notice that additional small correlations are seen in the residuals with mean fiber number and S/N, and subsequently add these as additional parameters to our model as well. Furthermore, as we are interested in finding a model with a minimum level of intrinsic dispersion of the measurement minus the predicted [X/H], we explore additional parameter cuts. We find that introducing a restricted vertical extent of the stellar orbits, \( J_z \), gives another ~5% reduction in \( \sigma_{\text{intrinsic}} \). We therefore restrict this sample to disk stars with \( J_z < 30\ \text{kms}^{-1}\text{kpc} \), which leaves a total of ~13,000 stars.

In Figure 7, we show the summary of the intrinsic dispersion for each element from the model with the additional parameters included as predictors. Including this additional set of parameters
in the model reduces the intrinsic dispersion of the eight elements to a median level of \( \approx 0.015 \) dex. This provides an estimate of the residual variance that is not able to be explained with age or orbit up to \( \approx 0.015 \) dex for these eight elements. The local linear model does not, however, entirely remove the correlations of these variables. Specifically, for Ni and Mn, partial trends with age persist at the < 0.005 dex level. Therefore, the median intrinsic dispersion of \( \approx 0.015 \) dex represents an upper limit. We note that for this model we found that a larger neighborhood, with \( k = 300 \), gave a marginally lower intrinsic dispersion for the element Cr in particular, by about 15%. This is not surprising that a larger \( k \) might be favored given the larger number of variables used in the model.

We verify that the smaller intrinsic dispersion we obtain is a consequence of including additional predictors in the model by using our initial four-parameter model applied to this subset of 13,000 stars. The intrinsic dispersion obtained with the smaller subset of 13,000 stars and four-parameter model is nearly identical to that obtained with the 27,000 fiducial sample, with a median \( \sigma_{\text{intrinsic}} = 0.20 \) dex compared to \( \sigma_{\text{intrinsic}} = 0.21 \) dex for the full sample. The subsequent reduction in the scatter of the elements around the model with additional parameters is therefore a consequence of including these additional correlated variables in the model regression and not the different samples.

To validate these results, we do an additional calculation as follows, to estimate how much of the overall intrinsic scatter we measure for each element is not a consequence of correlation with physical parameters and velocity: \( L_z, J_z, \) age, and \( V_{\text{helio}} \). For each of these four parameters, we subtract in quadrature the average 1\( \sigma \) dispersion around the mean trends reported in Figures 5 and 6, from the rms dispersion as measured around the prediction of the model in Figure 1, for each element. This gives an overall measure of the amplitude of the scatter that can be explained by each of \( L_z, J_z, \) age, and \( V_{\text{helio}} \) (combined with the measurement errors) for each element. The quadrature sum of these for each element represents the total scatter (combined with the measurement errors) accounted for by these joint parameters (ignoring any correlation between them, so as to obtain a lower limit), \( \sigma_{\text{corr}} \). We then take the quadrature difference for each element between the intrinsic dispersion of the model’s prediction summarized in Figure 3 and the quadrature sum of the scatter explained by the four parameters: \( \sqrt{\sigma_{\text{intrinsic}}^2 - \sigma_{\text{corr}}^2} \). This gives us how much intrinsic variance is not explained by these parameters. On average, we find this to be \( \approx 0.01 \) dex, for the eight elements.

Is it possible that there are correlations with birth radius that would explain the remaining intrinsic scatter, but as stars
migrate over time, this is simply lost (e.g., Sellwood & Binney 2002; Roškar et al. 2008). We examine the $L_e$ and $J_z$ trends for only the youngest populations. Although the residual-action gradients are steeper for the youngest populations, this is not always the case. For example, Sellwood & Binney (1989) show that for clusters with a mean age of 0.5 Gyr, the residual-action gradients are steeper for the youngest clusters. However, this is not always the case, and some clusters may have flatter residual-action gradients. Therefore, we conclude that the residual-action gradients are not a robust indicator of age for all populations.

4.5. The Residual Scatter of Open Clusters

We test the intrinsic scatter of residual abundances for the eight individual elements, [X/H], using known sites of common birth origin—open clusters. Cluster members are selected from our set of ~27,000 stars as those with a membership probability of $>0.7$ as reported in the APOGEE OCCAM open cluster catalog (Donor et al. 2020). This gives us a total of 46 stars distributed among the following four clusters: NGC 188, NGC 2682, NGC 6705, and NGC 6819. For each element, in each cluster, we calculate the mean measurement error for the set of stars, and the 1σ dispersion of the residuals in each [X/H]. The quadratic difference of the scatter of the residuals and the measurement uncertainties gives an average intrinsic dispersion of $≈0.01$ dex for the eight elements and four clusters (with a range of 0.005 dex for Ti to 0.02 dex for Al). This mean value of $≈0.01$ dex is consistent with our inference from the age distribution of the cluster stars, and the observed dispersion of S/N.

We now use a more lenient selection of stars to study the residuals in more open clusters. We use the larger set of 80,000 stars with S/N > 100. From this sample, we obtain 106 stars from nine different clusters with a membership probability of $>0.7$ and at least eight stars per cluster. Figure 9 shows a summary of the residual amplitude away from the model’s prediction, for each element, in each cluster. These are ordered from the youngest to oldest cluster. For reference, in the context of Figures 5 and 6, these clusters have the following distributions in parameters: age = $3±1.8$ Gyr (0.25–6.5 Gyr), $L_e = 10±9$ kms$^{-1}$kpc (0.26–26 kms$^{-1}$kpc), $J_z = 2135±390$ kms$^{-1}$kpc (1468–2812 kms$^{-1}$kpc), and $V_{helio} = 29±48$ kms$^{-1}$ (−54–94 kms$^{-1}$). The properties of these clusters are listed individually in Table 1. Although selected with a minimum S/N > 100, 70% of the open cluster stars have S/N > 200.

In Figure 9, the mean residuals are colored by mean [Fe/H] of the cluster stars (which range from $−0.5$ dex < [Fe/H] < 0.1 dex). The thin error bars around the mean report the 1σ dispersion of the residual distributions. The thick, colored error bars show the intrinsic dispersion of the residuals (from the quadratic difference of the 1σ dispersion and the mean error of the cluster stars). The red dashed lines in Figure 9 show the intrinsic dispersion measurements calculated for each element using the set of stars with the model from Figure 7. As described in Section 4.3, this comprises a sample of 13,000 field stars of the disk (S/N > 200), with age and orbital actions (and also velocity) included in the model as predictors for each
Figure 9. The distribution of the residuals (of the measurement — prediction) for each element \([X/H]\) in the nine clusters (each with eight or more members). These are ordered from the youngest to oldest cluster (as indicated in vertical annotation; see the text). The points show the mean of the residual distribution, colored by \([Fe/H]\). The thin error bars show the 1σ dispersion around the mean. The wider colored error bars show the intrinsic dispersion of the residual distribution (from the quadrature difference of the dispersion of the 1σ residual distribution and the mean error of the cluster stars, for each \([X/H]\)). The red dashed lines indicate the intrinsic dispersion from the field model for 13,000 stars shown in Figure 7. The clusters have a range of intrinsic dispersion values, and overall are not dramatically dissimilar to the field comparison shown in the dashed red lines. The median intrinsic dispersion of the clusters for these eight elements is \(\sim 0.015\) dex (correspondingly, \(\sim 50\%\) of the cluster intrinsic dispersion measurements are smaller than the field stars and \(\sim 50\%\) are larger). Note the bias around the zero-line in the mean residual amplitude for many elements in all clusters. This is in part explained by the residual trends documented in Figures 5 and 6, with exceptions.

Table 1

| Cluster Name | Number of Stars | Age (Gyr) | \(L_z\) (kms\(^{-1}\)kpc) | \(J_z\) (kms\(^{-1}\)kpc) |
|--------------|-----------------|-----------|--------------------------|-----------------------------|
| NGC 6705     | 9               | 0.25      | 1469 ± 43.4              | 0.4 ± 0.08                  |
| NCG 7789     | 8               | 1.7       | 1885 ± 10.2              | 0.9 ± 0.06                  |
| NCG 2204     | 13              | 2.2       | 2396 ± 119               | 19.8 ± 2.9                  |
| NCG 2420     | 8               | 2.8       | 2142 ± 11.3              | 14 ± 0.7                    |
| NCG 2682     | 10              | 2.81      | 1938 ± 4.4               | 7.7 ± 0.6                   |
| Trumpler 5   | 8               | 2.91      | 2585 ± 11.3              | 0.3 ± 0.04                  |
| NCG 6819     | 28              | 3.1       | 1882 ± 9.5               | 6.4 ± 0.5                   |
| NCG 2243     | 10              | 5.6       | 2812 ± 112.4             | 26.4 ± 3.1                  |
| NCG 188      | 12              | 6.5       | 2107 ± 18                | 14.2 ± 0.8                  |

Table 1: Mean Cluster Properties for the Open Clusters in Figures 9 and 10

of the residuals for the eight elements and nine clusters is \(\sim 0.015\) dex (with a variation of \(\pm 0.02\) dex on average for the distribution of all clusters and all elements, with individual elements measuring between 0 and 0.1 dex). The smaller variance compared to the field (red dashed lines in Figure 9) for some clusters in some elements indicates that the cluster comprises a more homogeneous environment, where elements are more precisely predicted by (Fe, Mg), compared to the field. However, on average, the intrinsic dispersion of the clusters is not markedly different from the field.

Looking at the distribution of the mean residual values in Figure 9, it is clear that the cluster stars have biased mean residuals with respect to the field. On average, the intrinsic dispersion of each measurement is smaller when the bias is larger. The mean of the residuals in the field sample is shown with the black line at zero, for reference. For individual elements, the set of clusters is approximately distributed around zero-mean, with the exception of Al and Cr. Individual clusters themselves, however, show bias away from zero-mean. This must be due to some difference between the clusters and the field distribution. Part of the bias in each element for the clusters can be explained by the correlations reported in Figures 5 and 6, and the interplay between these correlations.

Figure 10 shows the residuals of the four elements with the largest scatter in the cluster means (Ti, Al, Mn, and Cr) ordered in increasing vertical action, \(J_z\) (for Ti and Cr), angular momentum, \(L_z\) (for Al), as well as dispersion in heliocentric velocity, \(\sigma_V\) (for Mn). We look at the correlation with velocity dispersion due to its association with tracing cluster mass (e.g., Poovelil et al. 2020). Figure 10 reveals that the correlations with the orbital actions appear stronger than in the field. The residuals for Al show a relationship across \(J_z\) that echoes some of the structured variation in the mean seen in the young field stars of the disk, in Figure A1. However, NGC 6705 has an
anomalously low Al residual, and the mean residual value for three of the clusters exceeds the 1σ intrinsic dispersion of the field. The residuals for the elements Ti and, in particular, Cr, show a correlation with $J_z$. The correlation between the residual of Cr and $J_z$, in particular, is not similarly seen in the field sample. The relationship between the residual in Cr and $J_z$ suggests that the amplitude of Cr at fixed (Mg,Fe) is a tag of birth height from the plane of the disk, which is erased with cluster dissolution and heating. The element residuals for Cr and Al have the largest intrinsic dispersion values, by a factor of about two, compared to the other elements (with $\sigma_{\text{intrinsic}} = 0.03$ dex). The correlations with the (presumably birth) orbital actions seen here for these elements likely account for part of that scatter measured in the field.

The residuals of the element Mn show a correlation with age in Figure 9 that is overall consistent with the trends seen in Figure 6, with the exception of NGC 188, which is the oldest cluster with age = 6.5 Gyr. From Figure 6, old stars are biased to have negative residual values for Mn. Yet, the mean Mn residual for NGC 188 is $\approx 0.05$ dex (and nearly identical to that of the youngest cluster, NGC 6705). Interestingly, the two clusters, NGC 188 and NGC 6705, are the most metal-rich clusters and show similar velocity dispersion values. These two clusters also have anomalously low residuals in Ca.

We examine the cluster with the highest overall intrinsic dispersion in the elements, NGC 2243. This cluster shows a dramatically larger intrinsic dispersion in the residual of Cr, compared to the other clusters. Interestingly, this cluster has a very high radial action compared to the other clusters, with $J_r = 200 \pm 54$ km s$^{-1}$ kpc. The other eight clusters have $J_r = 5 – 35$ km s$^{-1}$ kpc. NGC 2243 is also the most metal-poor cluster, and Figure 2 shows that for the field sample, the residual scatter for six of the eight elements increases with decreasing metallicity, most substantially for the element Cr (by a factor of about two between [Fe/H] = 0 and [Fe/H] = −0.5).

In summary, the analysis of the open clusters shows that there is, on average, an intrinsic scatter of $\approx 0.01 – 0.015$ dex in the individual element residuals [X/H] predicted from (Fe,Mg) in groups of stars known to be born together. This is the same order of magnitude as the intrinsic dispersion in the field, measured after accounting for orbital and age correlations. However, the clusters also show mean residuals that are biased away from zero. Part of this is explained by the expected correlations between the residuals and age and orbital actions. However, the correlations appear to be more substantial in the open clusters compared to the field, for some elements (and most notably perhaps for Cr). This implies that the residuals for these elements are correlated with, and thereby discriminative of, mean birth location in the disk.

### 4.6. The Holistic View of Residual Scatter in the Milky Way Disk and Halo

We now demonstrate the potential of the information expressed in the element residuals from the prediction using (Fe,Mg) elements alone, across the Milky Way, more broadly. We proceed by comparing the residual scatter in the set of eight elements for disk stars to halo stars. The stars are selected to have S/N > 100, and disk and halo are separated only via a metallicity criteria. Stars more metal-rich than [Fe/H] > −1.0 are assigned as disk (∼80,000 stars), and stars more metal-poor than [Fe/H] < −1.0 dex are assigned as halo (∼3000 stars). Figure 11 shows the spatial plane of Galactocentric radius and height $R$−$z$, for stars of the disk at left, and halo at right, with at least five stars per bin. Each bin in the $R$−$z$ plane is colored by mean residual amplitude, as shown in each panel’s color bar. The color bar in each panel is scaled to 1σ of the element’s residual. The disk is smooth in residual trends, with some gradients seen for the elements radially, as explored in Figure 5, in particular seen in Si. In the halo, the residual scatter is two or more times larger than in the disk, and groups of stars in the spatial plane have common and distinguishing mean residual amplitudes (but also in some cases significant scatter within individual substructure). Figure 11 at right shows the stars that are grouped in spatial location have different mean residual amplitudes. The targeted APOGEE halo fields are labeled, and these correspond to the groups seen on the sky with similar residual amplitudes. Note that the modeling of the halo and the residual amplitude is also sensitive to the neighborhood $k$; for Figure 11, $k$ is the same as that of the disk, $k = 100$. A full exploration of this is beyond the scope of the paper, but different $k$ have different utility in examining halo substructure.

Figure 12 shows ∼36,000 stars selected with S/N > 200, including ∼500 stars with [Fe/H] < −1.0 dex (with the same
The $T_{eff}$ and $g$ cuts as in Section 2. The stars are colored in the [Fe/H]–[Mg/Fe] plane by their residuals (the measured $[X/H]$–the predicted $[X/H]$). This demonstrates the striking structure in the residuals in the population that is not disk material. Presumably, the halo shows this interesting residual variability and structure as it comprises stars from many different star formation environments (e.g., Bonaca et al. 2020; Naidu et al. 2020; Scannapieco et al. 2021; Naidu et al. 2021; Hallakoun & Maoz 2021; Ishigaki et al. 2021; Buder et al. 2022). Note that where some elements show large positive residuals, others show large negative residuals. The elements Mn and Al are particularly powerful diagnostics of and within the halo subsystems, compared to the disk; stars with negative residuals in [Al/H] look to be a potential additional chemical tag of the Gaia-Enceladus-Sausage system (Helmi et al. 2018; Belokurov et al. 2018). This is not necessarily surprising, given that Al and Mn...
have been used as diagnostics of halo substructure previously (e.g., Horta et al. 2021; Das et al. 2020). Using the residuals effectively maps individual [X/H] measurements to a common chemical reference frame. Additionally, stars with low-[Mg/Fe] at higher metallicities, [Fe/H] > −1.0 dex, show high residual amplitudes: a probable signature of these stars having an origin ex situ the Milky Way disk.

5. Discussion

We demonstrated that using two abundances (Fe, Mg) for disk stars, we can predict an ensemble of other elements produced in supernovae to high precision, with \( \sigma_{\text{intrinsic}} \approx 0.021 \) dex. This reveals the (small) amplitude of the individuality of these elements, near the limits of the precision of many large surveys (e.g., García Pérez et al. 2015; Kollmeier et al. 2017; Buder et al. 2018; Jofré et al. 2019). The stars we examine comprise different metallicities, ages, and birth radius, having migrated across the disk over time (e.g., Bensby et al. 2014; Minchev et al. 2018; Frankel et al. 2018, 2019; Ratcliffe et al. 2022; Sharma et al. 2021). From a galactic evolutionary perspective, the extent to which (Fe, Mg) do not predict the other eight elements we study is, in part, due to environmental differences at birth in the radial and temporal extent of the disk, as told by the level of individuality of the elements (see also Ting & Weinberg 2021; Weinberg et al. 2021). However, we note that the correlations with present-day radius and stellar age are small and that the most dominant correlations for some elements are imputed by artifacts in the spectra. We propose that the residual scatter we measure, after accounting for measurement uncertainties and astrophysical correlations, affords a way of assessing the intrinsic scatter of elements at fixed birth radius and time in the disk. We note however, that the amplitude of the intrinsic scatter that we report for the elements is sensitive to the measurement errors, and this is the biggest source of uncertainty in our estimate.

This result complements recent work using open clusters, which finds that hard chemical tagging is prohibitive due to the significant overlap in the chemical signatures of the clusters.

We explore correlations between the residuals of the measured [X/H] compared to the model’s prediction from (Fe, Mg), as shown in Figures 5 and 6. Looking in detail at these correlations, we see different behaviors for each element. We note that as the correlations between elements themselves are contaminated by systematic imprints (e.g., seen in Figure 6 and K. McKinnon et al. 2002, in preparation), we focus on only these age and orbital correlations, and not inter-element correlations. A positive residual means that there is excess [X/H] compared to the model’s prediction, and a negative value means there is less than the prediction from (Fe, Mg) alone. The iron-peak elements of Mn and Ni show the largest age correlations, with positive residuals at the youngest ages and negative residuals for old stars. This is presumably because younger stars are born in environments that have relatively higher contributions of SNe Ia. The primarily massive-star, SN II elements, show (very small) correlations with mean guiding radius, measured with \( L_\odot \). Presumably this may be due to the different star formation histories at different radii across the disk. The elements Ca and Si correlate in opposite directions with \( L_\odot \). The Si (explosive element) abundance is higher than the Ca (hydrostatic) element abundance at smaller guiding radius (see also Blanco et al. 2019), yet their age relations are relatively flat. Regardless of age, this hints at a very small difference in the initial mass distribution of stars in the inner and outer parts of the Galaxy (e.g., Krumholz 2014; Griffith et al. 2021a; Matteucci 2021). We find that the intrinsic dispersion of the elements is slightly age dependent. In general, the dispersion increases at earlier times (for older stars). This is shown in Figure A2 in the Appendix.

Quantitative modeling, however, is needed to understand and explain the underlying parameters of the environment that give rise to the systematic changes in individual abundance distributions over time and radius at fixed (Fe, Mg), as seen in the distributions of the residuals of these elements (e.g., Johnson et al. 2021; Spitoni et al. 2021; Matteucci 2021; Philcox & Rybizki 2019; Rybizki et al. 2017, and references therein).

Overall, Figures 5 and 6 reveal that present-day orbits and age explain only a fraction of the intrinsic scatter. After removing the astrophysical correlations of orbit and age with individual element residuals and accounting for errors, a level of \( \sigma_{\text{intrinsic}} \approx 0.01−0.015 \) dex in residual scatter around the model’s prediction remains. This is likely a measure of the scatter in the elements that are produced in supernovae, at fixed birth radius and age. Correspondingly, this is a complementary result to studies that use open clusters—stars known to be born together, which report an intrinsic scatter of \( \approx 0.02−0.05 \) dex for these elements within clusters (e.g., Bovy 2016; Ness 2018; Price-Jones & Bovy 2018; Liu et al. 2019; Souto et al. 2019; Cheng et al. 2021; Kos et al. 2021). Studies of the distribution of H II gas in the plane of the disk reveal deviations from a smooth abundance gradient azimuthally (Wenger et al. 2019). Similar azimuthal inhomogeneity is seen in external galaxies, with a typical global scatter of 0.03–0.05 dex in the [O/H] abundance across radius (Kreckel et al. 2020). Across spatial scales of \( \sim 600 \) pc, the scatter in [O/H] in other spiral galaxies is on the order of that measured in this study, of \( \sim 0.02−0.03 \) dex (and at the limitation of the precision of the measurement).

We examine the residual distributions of the eight elements, predicted using (Fe, Mg, \( \log g \), and \( T_{\text{eff}} \)) in open cluster systems. We find that these sites, comprising stars of common birth origin, have similar residual intrinsic dispersion measurements to the field, on average. Furthermore, there are trends between the amplitude of the intrinsic dispersion of the residuals and [Fe/H], as seen in the field, and reported in Figure 2. However, the clusters show bias with respect to the field, i.e., nonzero mean residual values. Some of this bias is consistent with correlations between the residual amplitudes and the orbital actions and age, as seen in the field. However, some correlations in the cluster population are not observed in the field population. The significant trend of the residual of Cr and the orbital action \( J_z \), which is seen for the cluster stars and not in the field, implies that the cluster dissolution process, heating, and/or radial migration removes this relationship. Using the cluster sample, the residual for Cr is revealed to be a likely tag of mean birth \( J_z \). Presumably, in general, we see stronger correlations with orbital actions and the cluster stars compared to the field, as the open clusters have not migrated significantly nor have they dissolved (Spina et al. 2021).

We highlight that it is perhaps not surprising that the elements produced in two supernova sources are predictive of one another. However, the quantification of the level to which these eight elements can be predicted by two elements alone, is important, for a number of reasons. First, an effective method
to quantify the residual information in some elements after conditioning on others, which we introduce here, is important to develop and validate. Specifically, the residuals of the measured abundances compared to the predicted must be unbiased, along the predictors. We found this could only be achieved with a local linear implementation of a regression model; we found that a global model, even regressing using a higher-order polynomial, gives biased residuals across the predictors. At the level of the residuals that we are seeing in this analysis, it is particularly critical to avoid bias inherited from the model. Furthermore, the local linear modeling approach that we employ is both simple and interpretable.

Second, it is somewhat remarkable that for an integrated population of stars born over the entire disk, plus subsequent events in a stellar lifetime (planet accretion, binary interaction), the scatter of the elements is as small as it is (as is the finding that open cluster environments are not particularly dissimilar to that of the field). This must put constraints on mixing, as well as the possible contribution of evolutionary events at the red giant phase in lending scatter to the abundance distribution at fixed (Fe,Mg). Third, these results show that if surveys do not measure these eight elements to the precision of the intrinsic scatter, for the stars of the disk, then for most cases, it is worthwhile to use only two numbers that quantify the iron-peak and α-element contributions (i.e., Fe, Mg) alone in pursuits in galactic archaeology. Finally, this paper serves as the basis to now apply this modeling and analysis to other element families. Of particular interest are the r- and s-process element channels, available in the GALAH and Gaia-eso survey data. We expect that that a similar line of empirical analysis for additional nucleosynthetic channels may chart the relationship between the origin of these elements over time and metallicity, and quantify how much additional information they comprise once supernova sources are measured (see Spina et al. 2018). One hard criterion to employ this modeling approach, however, is that the abundances be measured precisely, with accurate uncertainties (e.g., see Griffith et al. 2021b).

Although the residuals of the abundances at fixed (Fe, Mg) have very small amplitudes in the disk, the variance in the halo is substantially larger. Therefore in the halo, residual abundances may be useful to constrain and differentiate star formation environments and progenitors. Figure 11 also reveals the large residual variance at very low [Mg/Fe] ([Fe/H] > −1.0 dex, [Mg/Fe] < −0.2 dex), which is presumably ex situ material. In the disk, where the intrinsic variance is very small, stars that are significant outliers from the model’s prediction may be interesting objects to study. These may have had their abundances modified over the course of their evolution, due to dynamical effects like rotation, binarity, and planet accretion.

6. Conclusions

The detailed star formation environment across the Galactic disk over time is traced by the variance in element abundances, [X/H], at fixed fiducial supernova contributions of (Fe, Mg). We quantify this variance by building local linear models to predict eight abundance ratios [X/H] (for X = Si, O, Ca, Ti, Ni, Al, Mn, and Cr) that are produced in supernovae, given the chemical abundances of [Fe/H] and [Mg/H] and evolutionary state parameters of log g and T eff. Investigation of correlations between the residual of the model (measured [X/H]− predicted [X/H]) and astrophysical parameters reveals that the residuals in part correlate with orbit and age. However, the residual measurement, which is near the measurement precision limits, is also hampered by nuisance signals in the spectra, traced by radial velocity correlations. These signals will be important to model and remove in order to fully exploit the information content in stellar spectra (e.g., Price-Jones & Bovy 2018; de Mijolla et al. 2021; Feeney et al. 2021; Wheeler et al. 2021). Overall, we find that (Fe, Mg) predict the other abundances with a median intrinsic dispersion of ≈0.021 dex, which decreases to ≈0.15 dex once additional nonchemical parameters are added to the model.

We claim, based on this study, that disk stars in the Milky Way have ≈0.01–0.015 dex, on average, scatter in elements synthesized in supernovae, at fixed birth radius and time. This corresponds to ~20%–50% of the individual element variance at fixed (Fe, Mg), for elements produced in supernovae. We report some weak dependence of the intrinsic scatter of the residuals of each element on [Fe/H]. The intrinsic scatter of individual elements, at fixed birth radius and time, plus any [Fe/H] dependence, has implications for initial cluster mass distributions and timescales of formation (which are presumably similar regardless of metallicity and age), as well as the mechanics of element mixing (e.g., Bland-Hawthorn et al. 2010; Arnold et al. 2018; Krumholz et al. 2019). Chemical evolution modeling to fit these data will inform the environmental parameters that give rise to the trends we see in individual elements across both present-day radius and age (see Matteucci 2021, and references therein). We expect new survey data will enable us to expand this assessment of the intrinsic scatter at fixed radius and time in the disk, to examine different elements and nucleosynthetic families (e.g., De Silva et al. 2015; Buder et al. 2021; Kollmeier et al. 2017).

We note that we selected giant stars only for this analysis. In clusters, processes of stellar diffusion can change the abundances of elements to the level of the overall intrinsic scatter (Liu et al. 2019; Souto et al. 2019; Bertelli Motta et al. 2018). However, the regression model approach, which removes the correlation between abundances and evolutionary state, via log g and T eff can, in principle, calibrate these effects out; thus allowing for a wider set of evolutionary state to be considered in future analyses.

Finally, we highlight that this study has focused on the disk population ([Fe/H] > −1.0 dex), where the joint element abundance variance is marginal (Ness et al. 2019; Weinberg et al. 2019; Feeney et al. 2021; Lu et al. 2021; Rampalli et al. 2021). This is not the case in the halo ([Fe/H] < −1.0 dex) where residual abundances calculated here are likely much stronger tags of common birth environment.

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Appendix

Figure A1 shows the ∼3300 stars with ages <3.5 Gyr and the running mean of their residuals in the eight elements and Lz and J∗. These trends between residuals of the elements and
orographic actions are stronger for the younger population compared to the full field distribution.

Figure A2 shows the correlation between the intrinsic dispersion of each element and age, for the 15,000 stars in our sample with ages.

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Figure A1. The running mean of the angular momentum \( L_z \) at top, and the log of the vertical action \( J_z \) at bottom, vs. the residual amplitude for each element for stars with ages < 3.5 Gyr; the y-axis shows each scaled to the intrinsic dispersion. The astrophysical correlation with \( L_z \) represents only a fraction of the residual amplitude, but the trends are stronger than those reported in Figures 5 and 6. The shaded region represents the confidence on the mean.

Figure A2. Stellar age vs. the running mean of the intrinsic dispersion for each element for the 15,000 high fidelity stars in our sample with stellar ages. Note that the intrinsic dispersion is on average higher at earlier times (for older stars).
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