RESONANT STATES IN $^3P$ CHANNELS OF CHARMONIUM

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Abstract

We employ QCD sum rules, implemented with two numerical algorithms already tested in two different channels of Charmonium, in order to predict masses of resonances just above the ground states in the $^3P$ channels. We find that such masses are above the threshold of open charm. We calculate also the partial decay widths of the ground states into light hadrons and, for even spins, into two gammas; we find consistency with data.

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A lot of experimental and theoretical efforts have been performed in order to study hadrons formed by one or more heavy quarks. In particular, as regards spectroscopy predictions, essentially three different approaches have been adopted in the literature: 

i) potential models\cite{1, 2, 3};

ii) lattice calculations\cite{4};

iii) QCD sum rules\cite{5, 6, 7, 8}. Concerning heavy quarkonia, a systematic study of spectra has been done on Bottomonium\cite{3, 10}, but not on Charmonium. We think the gap can be partially filled in by means of power moment QCD sum rules\cite{5, 6, 8, 11, 12}, which, incidentally, in the case of Charmonium appear more suitable than exponential sum rules\cite{7, 9, 10, 13}. These have been already used successfully, with the help of two rather complex numerical algorithms, in the $^1S_0$ and $^1P_1$ channels\cite{11, 12} of Charmonium, in order to determine the mass of the first excited state - be it a bound state ($\eta'_c$, $^1S_0$ channel) or a resonant state ($h'_c$, $^1P_1$ channel) - and, as a byproduct, some partial decay widths of ground states. In this letter we extend our investigation to the $^3P$-channels of Charmonium; as a result we find that they have no bound states besides the ground one. Moreover we calculate the partial decay widths of the ground states into light hadrons and (for even spin resonances) into gammas, finding consistency with data.

We recall shortly the power moment QCD sum rules, together with the two numerical algorithms, which we have elaborated in preceding papers\cite{11, 12}. We assume a dispersion relation for the Fourier transform of the two-point function of the current (or density) of the charmed quarks with the quantum numbers of the channel considered. We consider spacelike overall momenta, $q^2$, therefore we may develop the two-point function according to a Wilson expansion - a nontrivial one, including the contribution of the lowest-dimensional gluon condensate. Then for a given channel $\Gamma$ and for any positive integer $n$ we get

\begin{equation}
\end{equation}
\[ A_n^\Gamma [1 + \alpha_s a_n^\Gamma(\xi) + \Phi b_n^\Gamma(\xi)] = \frac{1}{\pi} \int_0^\infty \frac{\text{Im}\Pi^\Gamma(s)}{(s + Q^2)^{n+1}} ds, \tag{1} \]

where \( Q^2 = -q^2, \alpha_s = \alpha_s[4(m_c^0)^2 + Q^2] \) is the running coupling constant \( (\alpha_s[4(m_c^0)^2] = \bar{\alpha}_s \sim 0.3), \xi = \frac{Q^2}{4(m_c^0)^2}, m_c^0 = m_c(p^2 = -m_c^2), \)

\[ \Phi = \frac{\pi^2}{36(m_c^0)^4} < 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} | 0 > \tag{2} \]

and \( G^a_{\mu\nu} \) is the QCD strength tensor field. The l.h.s. of eq. (1) coincides, up to an \( n \)-dependent factor, with the \( n \)-th moment of the Wilson expansion. Concerning the spectral function, which appears at the r.h.s. of eq. (1), for heavy quarkonia it may be parametrized as follows:

\[ \text{Im}\Pi^\Gamma(s) = \frac{9\pi}{4} \sum_{i=1}^{N} \frac{m_i^2}{g_i^2} \delta(m_i^2 - s) + \frac{\sigma_0^\Gamma}{8\pi}(1 + \frac{\bar{\alpha}_s}{\pi}) \theta(s - s_0), \tag{3} \]

where \( m_i \) is the mass of the \( i \)-th resonance of the channel and \( g_i \) the coupling constant of that resonance to the current (or density) relative to the channel, while \( s_0 \) is the threshold energy squared of continuum; lastly \( \sigma_0^\Gamma \) is a positive integer, which we set equal to 1 for \( P \)-channels\([12]\). Owing to duality, we are free to assume \( N = 1 \) or 2 in the sum that appears at the l.h.s. of eq. (3), including the other resonances in the continuum contribution. If we take \( N = 1 \), from eqs. (1) and (3) we deduce the mass of the ground state, i. e.,

\[ m_1^2 = \frac{\bar{M}_n^\Gamma}{\bar{M}_n^\Gamma} - Q^2, \quad \bar{M}_n^\Gamma = M_n^\Gamma - \frac{\sigma_0^\Gamma}{8\pi n(s_0 + Q^2)^n}, \tag{4} \]

where \( M_n^\Gamma = A_n^\Gamma[1 + \alpha_s a_n^\Gamma(\xi) + \Phi b_n^\Gamma(\xi)] \) is the l.h.s. of eq. (1). If, instead, we take \( N = 2 \), we get the mass of the first excited state, i. e.,

\[ m_2^2 = \frac{N_n^\Gamma}{N_n^\Gamma} - Q^2, \quad N_n^\Gamma = \bar{M}_n^\Gamma - \frac{9m_1^2}{4g_1^2(m_1^2 + Q^2)^{n+1}}. \tag{5} \]

Equations (4) and (5) yield respectively \( m_1 \) and \( m_2 \) as functions of \( n, Q^2, \bar{\alpha}_s, \Phi, m_c^0 \) and \( s_0 \), the last parameter varying from channel to channel. \( m_2 \) depends also on \( g_1 \), which is not known for most channels. Preceding analyses yield \( \Phi = 1.35 \times 10^{-3} [13, 14], \bar{\alpha}_s = 0.3 [15] \) and \( m_c^0 = 1.268 \text{ GeV} [11] \). Moreover, as we have already shown\([11]\), we can fix criteria for determining the other parameters.

In principle the value of \( s_0 \) that appears in formula (4) could be different from the one used in formula (3), since in the latter case we exclude the contribution of \( m_2 \) from the continuum of the spectral function. However eq. (3) is a simplified parametrization of the...
real spectral function, which presents narrow resonances at low energies and therefore has a greater weight at low $s$ in the integral at the r.h.s. of eq. (1). Then $s_0$ results to be a decreasing function of $n$ and $Q^2$. We can set $s_0^{(1)} = s_0^{(2)}$, provided $n_1 \geq n_2$ and $Q^2_{(1)} > Q^2_{(2)}$, the index 1 (2) referring to the option $N = 1$ (2). This self-consistency condition is always respected, both in the present analysis (see below) and in preceding ones[11, 12, 14, 15].

In order to fix $n$ and $Q^2$, first of all we consider the dependence of $m_i(n)$ on $n$ at a fixed $Q^2$. $m_i(n)$ present minima at given $n_i$, $i = 1, 2$, and therefore plateaux around such values (see e.g. fig. 1). Furthermore the amplitude of each plateau, which can be measured by the inverse of the quantity

$$D_i = m_i(\bar{n}_i - k_i) + m_i(\bar{n}_i + k_i) - 2m_i(\bar{n}_i),$$

$k_1 = 2$, $k_2 = 1)$, varies with $Q^2$. We choose $\xi_i$ so as to coincide with locations of minima of $D_i$ (see figs. 2). In particular we pick up for $\xi_1$ the lowest local minimum (at least for not too large $\xi$) of $D_1$, provided the corresponding $m_1$ is stable with respect to the parameters $m^0_c$ and $s_0$; as we shall see below, the stability condition in the $^3P_1$ channel requires a particular care. As to $\xi_2$, we pick up the lowest local minimum of $D_2$ which does not coincide with any minima of $D_1$.

We determine $s_0$ by equalling $m_1$ to the mass of the ground state, which is generally known experimentally. In order to determine $g_1$, we proceed as in the vector channel[11], considering the graph of $D_2$ (that is, of the value of $D_2$ obtained by selecting the minimum in the way just described) versus $g_1$. The value corresponding to an oblique inflexion (fig. 3) is assumed to be the correct one, provided it is stable with respect to small variations of the parameters. This in turn allows to determine $m_2$ by the formulae exposed above.

Table 1: Results of the analysis in different channels of Charmonium

| $P$  | $m_2$(MeV)  | $g_1$          | $s_0$(GeV)$^2$ | $n$       | $Q^2$ (GeV)$^2$ |
|------|-------------|----------------|----------------|-----------|----------------|
| $^3P_0$ | $4097^{+32}_{-44}$ | $13.38^{+0.34}_{-0.07}$ | $23.7^{+0.12}_{-0.16}$ | $9(6)$   | $15.4(8.4)$    |
| $^3P_1$ | $4327^{+40}_{-42}$ | $16.0^{+0.5}_{-0.7}$ | $19.7^{+1.2}_{-0.9}$ | $8(6)$   | $18.7(13.4)$   |
| $^3P_2$ | $4466^{+34}_{-41}$ | $11.82^{+0.6}_{-0.32}$ | $20.8^{+1.3}_{-0.8}$ | $8(6)$   | $21.5(15.8)$   |

Table 1 resumes the results of our analysis in the $^3P$-channels. In particular values of $n$ and $Q^2$ outside (inside) parentheses refer to the option $N = 1$ (2) in eq. (3). The values of $m_2$ are above the threshold of open charm (3727 MeV), therefore according to our predictions they are resonant states, which decay into charmed particles. We observe that the above mentioned self-consistency condition is satisfied for all $^3P$-channels. Another condition, necessary in order to check stability of minima of $D_1$ with respect to parameters, consists in examining the two graphs corresponding to eqs. $m_1(m^0_c, s_0) = m_{GS}$, $m_2(m^0_c, s_0) = m_{FR}$,
where GS means "ground state" and FR "first resonant state". Such graphs, represented in figs. 4 and 5, are quite similar to those drawn for the vector channel\[1\], to the difference that in the vector channel both \(m_{\text{GS}}\) and \(m_{\text{FR}}\) are known experimentally, whereas in the other channels \(m_{\text{FR}}\) is deduced from our numerical procedure. The analogous behaviour witnesses the self-consistency of our method. This is why in the \(^3P_1\)-channel we have not chosen the lowest minimum of \(D_1\). Indeed this minimum - which yields erratic values of some parameters, like \(\xi_i\) and \(s_0\), and a mass \(m_2\) greater than the one of the first resonance of the \(^3P_2\)-channel - corresponds to a pair of graphs quite different from the others, as one can see in fig. 5b.

Our mass predictions differ from others deduced on the basis of potential models\[16\]; on the contrary they fulfill, within errors, the center-of-gravity rule derived for \(P\)-channels of Charmonium (see\[3\] and refs. therein).

We have already shown\[11, 12\] that \(g_1\) contains useful information for predicting some partial decay widths. The formulae for decay widths of \(\chi_{c0(2)}\) into two gammas (gluons) read

\[
\Gamma_{\chi \rightarrow 2\gamma} = \frac{4}{3} \frac{2\pi g^2}{(2J + 1)m} \left(\frac{2}{3}\right)^4 \alpha^2, \quad \Gamma_{\chi \rightarrow 2g} = \frac{9}{8} \left(\frac{\alpha_s}{\alpha}\right)^2 \Gamma_{\chi \rightarrow 2\gamma}
\]

where \(m = m_{\text{GS}}\), \(J\) (0 or 2) the spin of the resonance,

\[
g^2 = \frac{9m^2}{4g_1^2fT}, \quad T = \frac{1}{4m_c^2} Tr[(k - \hat{q} + m_c)\tilde{O}(k + m_c)O],
\]

\(k\) is the four-momentum of the quark, \(l\) an integer, \(O\) an operator and \(f\) an \(O\)-dependent normalization factor. For \(^3P_{0(2)}\) channels \(T\) turns out to depend critically on the modulus squared of the relative four-momentum \(p = 2k - q\) of the quark with respect to the antiquark. The average of \(p^2\) over the hadronic state results to be \(m^2 - 4m_c^2\). According to Novikov et al.\[3\] (see also ref.\[12\]), we assume \(p^2 = -m_c^2\), therefore \(m_c^2 = \frac{1}{3}m^2\). In the \(^3P_0\)-channel \(l_{\chi_{c0}} = 1\), moreover \(O_{\chi_{c0}}\) is the identity operator, yielding \(f = 1\) and \(T_{\chi_{c0}} \simeq 0.268\). As to the \(^3P_2\)-channel, \(l_{\chi_{c2}} = 2\) and

\[
O_{\chi_{c2}} = \gamma_\mu p_\nu + \gamma_\nu p_\mu + \frac{2}{3} \eta_{\mu\nu} \not{q}, \quad \eta_{\mu\nu} = \frac{g_\mu g_\nu}{q^2} - g_{\mu\nu},
\]

Hence \(T_{\chi_{c2}} = \frac{46}{3}m_c^2\) and \(f = 1/5\). Calculations yield

\[
\Gamma_{\chi_{c0} \rightarrow 2\gamma} = 14.1^{+0.14}_{-0.07} \ (5.6^{+6.4}_{-4.1}) \ \text{keV}, \quad (10)
\]
\[
\Gamma_{\chi_{c0} \rightarrow 2g} = 26.8^{+0.3}_{-1.3} \ (14.0 \pm 5) \ \text{MeV}, \quad (11)
\]
\[
\Gamma_{\chi_{c2} \rightarrow 2\gamma} = 0.99 \pm 0.09 \ (0.32^{+0.14}_{-0.08}) \ \text{keV}, \quad (12)
\]
\[
\Gamma_{\chi_{c2} \rightarrow 2g} = 1.88^{+0.11}_{-0.13} \ (1.73 \pm 0.16) \ \text{MeV}, \quad (13)
\]
parentheses reporting experimental values.\[7\]

As regards $\chi c_1$, we assume, according to Novikov et al.\[5\], the decay into light hadrons to be dominated by the two-step process $\chi c_1 \rightarrow g g^*, g^* \rightarrow q\bar{q}$, i.e.,

$$\Gamma_{\chi c_1 \rightarrow g q\bar{q}} = \frac{1}{2m_c} \frac{\alpha_s^2}{3} \left(4\pi\alpha_s\right)^3 \int d\Phi_3 \frac{1}{q^4} \frac{1}{4} h_{\mu\nu} H^{\mu\nu},$$  \hspace{1cm} (14)

where $d\Phi_3$ is the phase space of the $g q\bar{q}$ system, $h_{\mu\nu}$ ($H_{\mu\nu}$) the hadronic tensor of light (charmed) quarks and $\tilde{q}^2$ the effective mass squared of the $q\bar{q}$ system. Furthermore $\tilde{g}_{\chi c_1}^2$ is given by eqs. (8), with $l_{\chi c_1} = 2$ and $O_{\chi c_1} = \gamma_5\gamma_\mu$, whence $T_{\chi c_1} = 1$ and $f = 1$. Then eq. (14) yields

$$\Gamma_{\chi c_1 \rightarrow g q\bar{q}} = \frac{4}{3} \frac{12m^6\alpha_s^3}{\tilde{q}^2 m_c^2} \int_{-1}^1 d\cos\theta \int_0^{0.5} dx \omega x^2 \frac{1 - \cos\theta\cos\phi}{\tilde{q}^2},$$  \hspace{1cm} (15)

where $\epsilon = \delta/[1 + \delta - \cos\theta(1 - \delta)]$, $\delta = q_0^2/m^2$, $q_0^2$ is a lower limit to $\tilde{q}^2$, $\omega$ the energy of the "real" gluon, $\theta$ ($\phi$) the angle between the direction of the real gluon and the one of the (anti-) quark in the overall cms. $\tilde{q}^2$, $\omega$ and $\phi$ depend on $\theta$ and $x$ through four-momentum conservation constraints. Taking into account the value of $g_1$ (see table 1), and assuming, according to the lower bound of perturbative QCD scales, $q_0^2 = 4 - 5 \text{ GeV}^2$, we get $\Gamma_{\chi c_1 \rightarrow g q\bar{q}} = (658^{+131}_{-207}) \text{ keV}$, quite consistent with the experimental data, $(640 \pm 100) \text{ keV}$.

Other authors\[18, 19, 20\] calculate partial decay widths of Charmonium $P$-wave resonances by means of the factorization theorem, including colour octet contributions in order to eliminate infrared divergences. They do not calculate the nonperturbative factors involved, rather they determine them either from some of the partial decay widths or from low-energy $e^+ - e^-$ data. On the contrary, we do not take into account the octet contribution (relatively small in percentage); however, thanks to QCD sum rules, we can calculate nonperturbative factors.

References

[1] M.Baker, J.S.Ball and F.Zachariasen: Phys. Rev. D 47, 3021 (1992)
[2] T.Appelquist et al.: Ann. Rev. Nucl. Part. Sci. 28 ,387 (1978)
[3] D.B.Lichtenberg and R.Potting: Phys. Rev. D 46, 2150 (1992)
[4] A.El-Khadra: Talk given at Tau Charm Factory Workshop, Argonne, Il, June 21-23 (1995), [hep-ph/9509381]
[5] V.A.Novikov, L.B.Okun, M.A.Shifman, A.I.Vainshtein, M.B.Voloshin and V.I.Zakharov: Phys.Rep. 41, 1 (1978)
[6] M.A.Shifman, A.I.Vainshtein, M.B. Voloshin and V.I.Zakharov: Phys.Lett. B 77, 80 (1978)

[7] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov: Nucl.Phys. B 147, 385, 448, 519 (1979)

[8] L.J.Reinders, H.R.Rubinstein and S.Yazaki: Nucl.Phys. B 186, 109 (1981)

[9] S.Narison: Phys. Lett. B 387, 162 (1996); Nucl. Phys. B Proc. Suppl. 54 A, 238 (1997)

[10] C.A.Dominguez and N.Paver: Phys. Lett. B 293, 197 (1992)

[11] E.DiSalvo and M.Pallavicini: Nucl.Phys. B 427, 22 (1994)

[12] E.DiSalvo, M.Pallavicini, E.Robutti and S.Marsano: Phys. Lett. B 387, 395 (1996)

[13] J.S.Bell and R.A.Bertlmann: Nucl. Phys. B 177, 218 (1981)

[14] E.DiSalvo, M.Pallavicini and E.Robutti: Nucl. Phys. B Proc. Suppl. 54 A, 233 (1997)

[15] E.DiSalvo: Proc. Int. School of Physics "E.Fermi", Course CXXX, ed. A.DiGiacomo and D.Diakonov, IOS Press, Amsterdam 1996, p. 469

[16] V.Yu.Borue and S.B.Khokhlachev: Modern Phys. Lett. 3, 1499 (1988); JETP Lett. 47, 440 (1988)

[17] Particle Data Group, R.M.Barnett et al., Review of Particle Physics: Phys. Rev. D 54, 1 (1996)

[18] G.T.Bodwin, E.Braaten and G.P.Lepage: Phys. Rev. D 46, R1914 (1992); Phys. Rev. D 51, 1125 (1995)

[19] H.W.Huang and K.T.Chao: Phys. Rev. D 54, 6850 (1996)

[20] E.Braaten and Yu-Qi Chen: Phys. Rev. D 55, 7152 (1997)

[21] R.Barbieri, M.Caffo, R.Gatto and E.Remiddi: Nucl. Phys. B 192, 61 (1981)

[22] F.Yuan, C.-F.Qiao and K.-T.Chao: Phys. Rev. D 56 1663 (1997)
Figure 1: \(^3P_0\) channel: behaviour of \(m_1\) vs \(n\) for \(\xi = 2.5\)

Figure 2: \(^3P_1\) channel: behaviour of \(a) D_1\) and \(b) D_2\) vs \(\xi\)
Figure 3: $^3P_2$ channel: $D_2$ vs $g_1$. The arrow indicates the oblique inflexion.

Figure 4: $^3P_0$ and $^3P_2$ channels: $m_0^c$ vs $s_0$. The full lines refer to eq. $m_1 = m_{GS}$, the dashed ones to eq. $m_2 = m_{FR}$, where $GS(FR)$ means ”ground state” (“first resonant state”).
Figure 5: $^3P_1$ channel: $m_c^0$ vs $s_0$. See caption of fig. 4. Graphs b) refer to the last minimum of $D_1$ and present a clear anomaly with respect to the others.