Fragmentation of bottom quarks in top quark decay

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Abstract — We review the main aspects of the fragmentation of bottom quarks in top quark decay. The NLO b-quark energy spectrum presents large mass logarithms \( \ln(m_t^2/m_b^2) \), which can be resummed by the use of the approach of perturbative fragmentation functions. Large soft contributions in both coefficient function and initial condition of the perturbative fragmentation function have been resummed as well. Results on the energy distribution of b quarks and b-flavoured hadrons are finally presented in both \( x \) and moment spaces.

1 Introduction

A reliable understanding of bottom quark fragmentation in top quark decay (\( t \to bW \)) will be fundamental to accurately measure the top properties, such as its mass \( m_t \), at present and future high-energy colliders. In fact, the uncertainty on bottom quark fragmentation is one of the sources of systematic error on \( m_t \) at the Tevatron accelerator [1] and will play a crucial role in the reconstruction of \( m_t \) from final states with leptons and \( J/\psi \) at the LHC [2].

In this paper we investigate bottom fragmentation in top decay within the framework of perturbative fragmentation functions [3]. We shall resum collinear logarithms \( \sim \ln(m_t^2/m_b^2) \) and soft terms that appear in the next-to-leading order (NLO) b-quark energy distribution. We shall present results on the b-quark energy spectrum in top quark decay and investigate the impact of collinear and soft resummation. Hadron-level results on b-flavoured hadrons will be shown in \( x_B \) and moment spaces.

2 Collinear and soft resummation

In Ref. [4] NLO corrections to top decay \( t(p_t) \to b(p_b)W(p_W)(g(p_g)) \) have been computed for a massive b quark, and the differential width \( d\Gamma/dx_b \), with \( x_b \) being the normalized b-quark energy fraction in the top rest frame, has been calculated. The differential rate obtained in [4] exhibits large mass logarithms \( \sim \alpha_S \ln(m_t^2/m_b^2) \) that need to be resummed in order to improve the prediction.

Such contributions can be resummed by using the perturbative fragmentation approach [3], which, up to power corrections, factorizes the rate of heavy-quark production into the convolution of a coefficient function, describing the emission of a massless parton, and a perturbative fragmentation function \( D(\mu_F, m) \), where \( \mu_F \) is the factorization scale. In the \( \overline{\text{MS}} \) factorization scheme we have:

\[
\frac{1}{\Gamma_0} \frac{d\Gamma^b}{dx_b}(x_b, m_t, m_b) = \sum_i \int_{x_b}^{1} \frac{dz}{z} \left[ \frac{1}{\Gamma_0} \frac{d\Gamma_i}{dz}(z, m_t, \mu_F) \right]_{\overline{\text{MS}}} D_i \left( \frac{x_b}{z}, \mu_F, m_b \right)_{\overline{\text{MS}}} + \mathcal{O}\left((m_b/m_t)^p\right),
\]

In Eq. (1) \( \Gamma_0 \) is the Born width of the process \( t \to bW \). The \( \mathcal{O}(\alpha_S) \) top decay coefficient function has been computed in [4].
The perturbative fragmentation function expresses the transition of the massless parton into the massive quark, and its value at any scale $\mu_F$ can be obtained by solving the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations $[5, 6]$ once an initial condition at a scale $\mu_0F$ is given.

In $[3]$ the NLO expression for $D(\mu_0F, m)$, which was argued to be process independent, was given. The process independence has been lately established in a more general way in Ref. $[7]$.

The initial condition of the perturbative fragmentation function reads $[3]$: 

$$D_p(x_b, \mu_0F, m_b) = \delta(1 - x_b) + \frac{\alpha_S(\mu_0)C_F}{2\pi} \left[ \frac{1 + x_b^2}{1 - x_b} \left( \ln \frac{\mu_0^2}{m_b^2} - 2 \ln(1 - x_b) - 1 \right) \right]_+,$$ 

(2)

As discussed in $[4]$, solving the DGLAP equations for the evolution $\mu_0F \rightarrow \mu_F$, with a NLO kernel, allows one to resum leading logarithms (LL) $\sim \alpha_S^n \ln^n \left( \frac{\mu_F^2}{\mu_0^2} \right)$ and next-to-leading logarithms (NLL) $\sim \alpha_S^n \ln^{n-1} \left( \frac{\mu_F^2}{\mu_0^2} \right)$ (collinear resummation). If we set $\mu_0F \simeq m_b$ and $\mu_F \simeq m_t$, we resum large logarithms $\sim \ln \left( \frac{m_t^2}{m_b^2} \right)$, which are indeed the terms appearing in the massive, unevolved, NLO $d\Gamma/dx_b$.

Moreover, both the MS coefficient function $[4]$ and the initial condition of the perturbative fragmentation function (2) present, at $\mathcal{O}(\alpha_S)$, terms that behave like $1/(1 - x_b)_+$ or $[\ln(1 - x_b)/(1 - x_b)]_+$, which become large for $x_b \rightarrow 1$, i.e. for soft-gluon radiation. In Mellin moment space, such contributions correspond to behaviours $\sim \ln N$ and $\sim \ln^2 N$ respectively.

Soft contributions in the perturbative fragmentation function are process-independent and have been resummed in $[7]$ with NLL accuracy. Soft terms in the coefficient function are instead process-dependent. Resummation of LL $\sim \alpha_S^n \ln^{n+1} N$ and NLL $\sim \alpha_S^n \ln^n N$ contributions to the top-decay coefficient function has been performed in $[8]$ and we do not report here the formulae for the sake of brevity.

### 3 Parton-level results

We would like to present results on the $b$-quark energy distribution in top decay and investigate the effect of collinear and soft resummation. In Fig. 1 we show the $b$-quark energy spectrum. The NLO calculation lies below the two resummed predictions and is divergent as $x_b \rightarrow 1$. After the resummation of collinear terms $\sim \ln \left( \frac{m_t^2}{m_b^2} \right)$ the distribution exhibits a sharp peak at $x_b$ close to 1. Finally, the inclusion of soft-gluon resummation smoothens out the distribution, which exhibits the so-called Sudakov peak.

As discussed in $[8]$, the implementation of collinear and soft resummation leads to a milder dependence of observables on the factorization and renormalization scales entering the calculation, which corresponds to a reduction

![Figure 1: b-quark energy distribution in top decay, according to the unresummed fixed-order calculation (dotted line), and after inclusion of collinear resummation (dashed) and of both collinear and soft resummations (solid). We have set $m_t = 175$ GeV, $m_b = 5$ GeV, $m_W = 80$ GeV, $\Lambda_{\text{QCD}} = 200$ MeV. In the inset figure, we show the same curves on a logarithmic scale, for $x_b > 0.8$.](image-url)
of the theoretical uncertainty. As an example, Fig. 2 shows the results on the dependence of the $x_b$ spectrum on the factorization scale $\mu_F$ in Eq. (1), which is taken equal to $m_t/2$, $m_t$ and $2m_t$, and the effect of soft resummation. We note that while the unresummed prediction still exhibits a dependence on the value chosen for $\mu_F$, the implementation of soft resummation yields three almost undistinguishable distributions.

Figure 2: $b$-quark energy spectrum for different values of the factorization scale $\mu_F$, with (solid) and without (dashes) NLL soft-gluon resummation.

### 4 Hadron-level results

We would like to make predictions for the spectrum of $b$-flavoured hadrons in top decay. We write the normalized rate for the production of $B$-hadrons $B$ as a convolution of the rate for the production of $b$ quarks and a non-perturbative fragmentation function $D_{np}(x)$:

$$\frac{1}{\Gamma} \frac{d\Gamma^B}{dx_B}(x_B, m_t, m_b) = \frac{1}{\Gamma} \int_{x_B}^{1} dz \frac{d\Gamma^b}{dz}(z, m_t, m_b) D_{np}\left(\frac{x_B}{z}\right),$$

where $x_B$ is the $B$ normalized energy fraction. The parton-level rate $d\Gamma^b/dz$ can be computed following the method which has been discussed in the previous section.

As for the non-perturbative fragmentation function, one can use some phenomenological models with tunable parameters, which are to be fitted to experimental data. We consider a power law with two parameters:

$$D_{np}(x; \alpha, \beta) = \frac{1}{B(\beta + 1, \alpha + 1)} (1 - x)^\alpha x^\beta,$$

the model of Kartvelishvili et al. [9]

$$D_{np}(x; \delta) = (1 + \delta)(2 + \delta)(1 - x)^\delta$$

and the non-perturbative fragmentation function of Peterson et al. [10]:

$$D_{np}(x; \epsilon) = \frac{A}{x[1 - 1/x - \epsilon/(1 - x)]^2},$$

In Eq. 4, $B(x, y)$ is the Euler beta function; in 5 $A$ is a normalization constant. We tune such models to $e^+e^-$ data from the ALEPH [11] and SLD [12] Collaboration. The ALEPH data refer to $b$-flavoured mesons, the SLD data to baryons and mesons. When we do the fits, we must describe the $e^+e^- \rightarrow b\bar{b}$ process within the same framework as we did for top decay, i.e. we use the perturbative fragmentation method, NLL DGLAP evolution and NLL soft resummation. As in 4-8, we shall consider $x_B$ values within the range $0.18 \leq x_B \leq 0.94$.

In Table 1 we show the results of our fits, along with the corresponding values of $\chi^2$ per degree of freedom. One can see that the power law with two parameters 4 and the Kartvelishvili model 5 fit both ALEPH and SLD
Table 1: Results of fits of hadronization models to ALEPH and SLD data on b-flavoured hadron production in $e^+e^-$ annihilation.

|       | ALEPH          | SLD            |
|-------|----------------|----------------|
| $\alpha$ | $0.51 \pm 0.15$ | $2.04 \pm 0.38$ |
| $\beta$  | $13.35 \pm 1.46$ | $25.18 \pm 3.27$ |
| $\chi^2(\alpha, \beta)/\text{dof}$ | $2.56/14$ | $11.50/16$ |
| $\delta$ | $17.76 \pm 0.62$ | $16.59 \pm 0.49$ |
| $\chi^2(\delta)/\text{dof}$ | $10.54/15$ | $22.19/17$ |
| $\epsilon$ | $(1.77 \pm 0.16) \times 10^{-3}$ | $(1.61 \pm 0.14) \times 10^{-3}$ |
| $\chi^2(\epsilon)/\text{dof}$ | $29.83/15$ | $158.15/17$ |

data rather well, while the Peterson fragmentation function is marginally consistent with ALEPH and unable to reproduce the SLD data. Moreover, the values of the best-fit parameters $\delta$ and $\epsilon$, fitted to ALEPH and SLD, are in agreement within two standard deviations.

In Fig. 3 we show our prediction for the $B$-hadron spectrum in top decay, using all three hadronization models fitted to the ALEPH data. In order to account for the uncertainties on the best-fit parameters, for each model we plot a band corresponding to a prediction at one-standard-deviation confidence level. From Fig. 3 we learn that the predictions based on the models (4) and (5) are consistent, while the Peterson model yields a distribution that lies quite far from the other two and is peaked at larger values of $x_B$.

In Fig. 4 we plot the $x_B$ spectra yielded by models (4) and (5), but fitted to SLD. Such distributions statistically agree at the confidence level of two standard deviations.

In Fig. 5 we compare the predictions obtained using the power law with two parameters, but fitted to ALEPH and SLD data. We observe that the spectra are distinguishable; this difference may be related to the different hadron types that the two experiments have reconstructed.

Figure 3: $B$-hadron energy spectrum in top decay, according to the power law (solid line), the Kartvelishvili (dashed) and the Peterson model (dotted), fitted to the $e^+e^- \rightarrow b\bar{b}$ data from ALEPH. The plotted curves are the edges of bands at one-standard-deviation confidence level.

We finally wish to present results on the moments of the $B$-hadron spectrum in moment space $\Gamma_N^{B_N}$. Such moments can be written as the product of a perturbative and a non-perturbative contribution $\Gamma_N^{B_N} = \Gamma_N^{b_N} D_N^{np}$. The advantage of working in moment space is that one can extract $D_N^{np}$ from $e^+e^-$ data without relying on any specific hadronization model.

Predictions for the moments $\Gamma_N^{B_N}$ of $B$-meson spectra in top decay are given in Table 2 where data from the DELPHI Collaboration [13] are used to obtain the non-perturbative information $D_N^{np}$. Two sets of perturbative results ([A]
and [B]) are shown, the first using $\Lambda_{\text{QCD}} = 0.226$ GeV and $m_b = 4.75$ GeV, the second $\Lambda_{\text{QCD}} = 0.2$ GeV and $m_b = 5$ GeV, the default values of this analysis. As expected, the perturbative calculations and the corresponding non-perturbative components differ at the level of few per cent, according to whether one uses set [A] or [B]. However, the final hadron-level predictions for the physical results $\Gamma_{N}^{b}$ differ only at the level of per mille.

5 Conclusions

We have considered bottom quark fragmentation in top quark decay $t \rightarrow bW$. We have pointed out that the fixed-order result on the $b$-quark energy spectrum exhibits large mass logarithms $\sim \ln(m_t^2/m_b^2)$, which can be resummed to NLL accuracy using the approach of perturbative fragmentation functions and DGLAP evolution equations. Moreover, NLL soft contributions to the coefficient function and to the initial condition of the perturbative fragmentation function have been resummed as well.

We have presented results on the $b$-quark spectrum in top decay, which displays a remarkable impact of the inclusion of soft and collinear resummation. In particular, the distributions exhibit very little dependence on factorization and renormalization scales.

Predictions on $b$-flavoured hadron energy distributions in top decay have been obtained using ALEPH and SLD.
data to parametrize some hadronization models in $x_B$ space and DELPHI data to get non-perturbative information in moment space.

The considered approach can now be applied to study several observables, which are relevant to top quark phenomenology at the Tevatron and ultimately at the LHC and compare the obtained results with the ones given by Monte Carlo event generators.

Table 2: DELPHI data for the moments $\sigma_N^B$, the resummed $e^+e^-$ perturbative calculations for $\sigma_N^b$ [7], and the extracted non-perturbative contribution $D_{np}^N$. Using the perturbative results $\Gamma_{bN}^b$, a prediction for the physical observable moments $\Gamma_{bN}^b$ is given. Set [A]: $\Lambda_{QCD} = 0.226$ GeV and $m_b = 4.75$ GeV; set [B]: $\Lambda_{QCD} = 0.2$ GeV and $m_b = 5$ GeV.

|                  | $\langle x \rangle$ | $\langle x^2 \rangle$ | $\langle x^3 \rangle$ | $\langle x^4 \rangle$ |
|------------------|---------------------|------------------------|------------------------|------------------------|
| $e^+e^-$ data $\sigma_N^b$ | 0.7153±0.0052       | 0.6239±0.0064          | 0.5246±0.0065          | 0.4502±0.0064          |
| $e^+e^-$ NLL $\sigma_N^b$ [A] | 0.7666              | 0.6391±0.0064          | 0.5479±0.0065          | 0.4755±0.0064          |
| $e^+e^-$ NLL $\sigma_N^b$ [B] | 0.9331              | 0.8657±0.0064          | 0.8075±0.0064          | 0.7566±0.0064          |
| $D_{np}^N$ [A] | 0.9169              | 0.8392±0.0064          | 0.7731±0.0064          | 0.7163±0.0064          |
| $D_{np}^N$ [B] | 0.9169              | 0.8392±0.0064          | 0.7731±0.0064          | 0.7163±0.0064          |
| t-decay NLL $\Gamma_{bN}^b$ [A] | 0.7750              | 0.6417±0.0064          | 0.5498±0.0064          | 0.4807±0.0064          |
| t-decay NLL $\Gamma_{bN}^b$ [B] | 0.7884              | 0.6617±0.0064          | 0.5737±0.0064          | 0.5072±0.0064          |
| t-decay $\Gamma_{bN}^b$ [A] | 0.7231              | 0.5555±0.0064          | 0.4440±0.0064          | 0.3637±0.0064          |
| t-decay $\Gamma_{bN}^b$ [B] | 0.7228              | 0.5553±0.0064          | 0.4435±0.0064          | 0.3633±0.0064          |

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