Abstract

It is shown that a certain sum rule for soft supersymmetry-breaking scalar masses, which has been recently found in a certain class of superstring models, is universal for gauge-Yukawa unified models. To explain this coincidence, we argue that the low-energy remnant of the target-space duality invariance in the effective supergravity of compactified superstrings can be identified with the (broken) scale invariance in gauge-Yukawa unified models, and that gauge-Yukawa unification which is indispensable for the sum rule to be satisfied follows from the matching of anomalies.

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Gauge-Yukawa Unification (GYU) \cite{1, 2} is an attempt to relate the gauge and Yukawa couplings, which is a gradual extension of the Grand Unification idea. It has turned out to be a successful scheme to predict the top and bottom quarks masses \cite{3}. Supersymmetry seems to be essential for GYU, but, as it is for any realistic supersymmetric model, the breaking of supersymmetry has to be understood. If a model couples to supergravity, or in the case of gauge–mediated supersymmetry breaking \cite{4}, one can compute in principle the soft supersymmetry–breaking (SSB) terms. Unlike this usual path chosen to reduce the number of the independent parameters, the GYU idea of \cite{1, 2} relies not only on a symmetry principle, but also on the principle of reduction of couplings \cite{6, 7}. This principle is based on the existence of renormalization group (RG) invariant relations among couplings, which do not necessarily result from a symmetry, but nevertheless preserve perturbative renormalizability. Dimensional couplings can also be treated along this line of thought \cite{8}–\cite{11}. When applied to the finite \cite{9, 10} or the minimal \cite{11} supersymmetric $SU(5)$ GUT, one finds that the SSB sector of the model is completely fixed by the gaugino mass $M$.

One of main observations of this letter is that the soft scalar masses in GYU models satisfy a simple universal sum rule eq. (9). This sum rule is derived without using any symmetry of GYU models. Therefore, within the framework of GYU, the sum rule is an accidental byproduct, but its simplicity suggests that it could be understood as a consequence of some symmetry property of a more fundamental theory such as superstrings. Superstring theory, though intensive studies and recent interesting developments \cite{12}, is still in a phase in which one needs various assumptions, especially those on non-perturbative effects, to relate it to low energy physics and then to predict its parameters. These assumptions include also those on supersymmetry breaking. Nevertheless, it is possible to do systematic investigations of SSB terms \cite{13} and to parametrize them in a simple way \cite{13}–\cite{19} (see ref. \cite{14} for a review). Along this line, we re-investigate the Kähler potential under general assumptions to find out its general form that yields the sum rule which coincides with the one in GYU models. As expected, the sum rule results from a certain type of duality invariance (see refs. \cite{20}, \cite{21}, \cite{22} and references therein) in the effective supergravity. In fact, the same sum rule has been independently obtained in various superstring models \cite{15}, \cite{17}, \cite{19}.

As we will also see, the unification of the gauge and Yukawa couplings is indispensable for

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\footnote{See refs. \cite{1}, \cite{2} and references therein.}
the sum rule to be satisfied. This appears mysterious, because one can derive the sum rule in superstrings without explicitly knowing the superpotential. To give a possible answer to this problem, we notice that the duality transformation acts as field-dependent scale transformations on the chiral matter superfields. Given that, we may identify the low-energy remnant of the duality invariance with the (in general broken) scale invariance of the effective renormalizable field theory. We will argue that this interpretation might offer a possibility to understand why in GYU models and in the certain class of superstring models [15, 17, 19] the same soft scalar-mass sum rule is satisfied, at least in one-loop order, to which we will be restricting ourselves throughout this letter.

Let us first derive the announced soft scalar-mass sum rule. To this end, we use the notation and result of ref. [23]. The superpotential (the gauge group is assumed to be a simple group) is given by

\[ W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j , \]

along with the Lagrangian for SSB terms,

\[ - L_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} h^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)_{ij} \phi_i^* \phi_j + \frac{1}{2} M \lambda \lambda + \text{H.c.} \]

The RG functions we need for our purpose are:

\[ \frac{d}{dt} g = \beta_g = \frac{1}{16\pi^2} \beta_g^{(1)} + \ldots , \frac{d}{dt} M = \beta_M = \frac{1}{16\pi^2} \beta_M^{(1)} + \ldots , \]

\[ \frac{d}{dt} Y^{ijk} = \beta_Y^{ijk} = Y^{ijp} \left\{ \frac{1}{16\pi^2} \gamma_p^{(1)k} + \ldots \right\} + (k \leftrightarrow i) + (k \leftrightarrow j) , \]

\[ \frac{d}{dt} h^{ijk} = \beta_h^{ijk} = \frac{1}{16\pi^2} [\beta_h^{(1)}]^{ijk} + \ldots , \frac{d}{dt} (m^2)^{ij} = [\beta_{m^2}^{(1)}]^{ij} + \ldots , \]

where \ldots stands for higher order terms (see ref. [23] and references therein)

\[ \beta_g^{(1)} = g^3 [S(R) - 3C(G)] , \beta_M^{(1)} = 2M \frac{\beta_g^{(1)}}{g} , \gamma_i^{(1)j} = (1/2)Y_{ipq}Y^{jpq} - 2\delta_i^j g^2 C(i) , \]

\[ [\beta_h^{(1)}]^{ijk} = (1/2)h^{ijl}Y_{lmn}Y^{mnk} + Y^{ijl}Y_{lmn}h^{mnk} - 2(h^{ijk} - 2MY^{ijk}) g^2 C(k) + (k \leftrightarrow i) + (k \leftrightarrow j) , \]

\[ [\beta_{m^2}^{(1)}]_{ij} = (1/2)Y_{ipq}Y^{pqn}(m^2)^{nj} + (1/2)Y^{ipq}Y_{pqn}(m^2)^{ni} + 2Y_{ipq}Y^{jpr}(m^2)^{qr} + h_{ipq}Y^{jpr} - 8\delta_i^j MM^g g^2 C(i) , \]
The soft scalar-mass sum rule which we would like to derive is given by

$$m_i^2 + m_j^2 + m_k^2 = MM^\dagger$$ for \(i, j, k\) with \(\rho^{ijk} \neq 0\), \hspace{1cm} (9)

where \(\rho^{ijk}\) is defined in eq. (10) below. (Each set of the subscripts \(\{i, j, k\}\) appearing in the sum rule exactly refers to a non-vanishing cubic term in the superpotential.) The sum rule (9) is satisfied under two conditions which we will specify at the corresponding points below. These conditions are by no means strong, and indeed they are satisfied in all the known GYU models so far. Note that the sum rule (9) is a one-loop result, and so we expect it will be modified in higher orders in perturbation theory.

We proceed from the starting assumption that the Yukawa couplings \(Y^{ijk}\) are expressed in terms of the gauge coupling \(g\):

$$Y^{ijk} = \rho^{ijk} g + \ldots , \hspace{1cm} (10)$$

where \(\rho^{ijk}\) are constants independent of \(g\) and \(\ldots\) stands for higher order terms. Eq. (10) is the one-loop solution to the reduction equation \(\beta^{(1)} Y^{ijk} = \beta_g dY^{ijk}/dg\).

(11)

In the case of a finite theory, the \(\beta\) functions vanish, of course. Nevertheless, the reduction equation keeps its meaning \([6, 24]\), and as we will see later on, the sum rule (9) can be derived for that case, too. We however assume for a while that the \(\beta\) functions do not vanish.

**Condition I**

The coefficients \(\rho^{ijk}\) satisfy the diagonality relation

$$\rho_{ipq} \rho^{jpq} \propto \delta^j_i . \hspace{1cm} (12)$$

This condition implies that the one-loop anomalous dimensions for \(\Phi_i\)'s become diagonal if the reduction solution (11) is inserted, i.e., \(\gamma_i^{(1)} j = \gamma_i \delta^j_i g^2\), where \(\gamma_i\) are also constants independent of \(g\). Therefore, the one-loop \(\beta\) function for \(Y^{ijk}\) in the reduced theory takes the form

$$[\beta^{(1)} Y]^{ijk}/16\pi^2 = \rho^{ijk} (\gamma_i + \gamma_j + \gamma_k) g^3/16\pi^2 . \hspace{1cm} (13)$$

The reduction equation (11), furthermore, requires that

$$\sum_i \rho_i \gamma_i \equiv \gamma_i + \gamma_j + \gamma_k = \beta^{(1)} g^3 = S(R) - 3C(G) \hspace{1cm} (14)$$
for \( i, j, k \) with \( \rho^{ijk} \neq 0 \). (This implies that all the allowed cubic coupling terms \( Y^{ijk}\Phi_i\Phi_j\Phi_k \) transform in the same way under the scale transformation.) Note further that

\[
h^{ijk} = -MY^{ijk} + \ldots = -M\rho^{ijk}g + \ldots
\]

solves the reduction equation for \( h^{ijk} \).

\[
\beta_h^{ijk} = \beta_M \partial h^{ijk} / \partial M + \beta_{M^1} \partial h^{ijk} / \partial M^1 + \beta_g \partial h^{ijk} / \partial g ,
\]

in one-loop order. This can be shown from

\[
h^{ijk} = -MY^{ijk} + \ldots = -M\rho^{ijk}g + \ldots
\]

\[(15)\]

Condition II

The one-loop reduction solution for the scalar masses is diagonal, i.e.,

\[
(m^2)_i^j = m_i^2 \delta_i^j , \quad m_i^2 = \kappa_i M M^\dagger
\]

\[(17)\]

where \( \kappa_i \) are constants to be determined below.

If the soft scalar-mass matrix is diagonal, the one-loop \( \beta \) functions \( [\beta^{(1)}_{m^2}]_i^j \) can be written as

\[
[\beta^{(1)}_{m^2}]_i^j = \rho_{ipq}\rho^{jpq} ( m_i^2 / 2 + m_j^2 / 2 + m_p^2 + m_q^2 )g^2 + h_{ipq}h^{jpq} - 8\delta_i^j M M^\dagger g^2 C(i)
\]

\[
\times \delta_i^j .
\]

Using eqs. \[(12)\] and \[(13)\] we then see that \( \rho_{ipq}\rho^{jpq} ( m_p^2 + m_q^2 ) \) also has to be proportional to \( \delta_i^j \).

This implies that, if the sum rule \[(1)\] is satisfied, the r.h.s of eq. \[(13)\] becomes

\[
\{ 2\rho_{ipq}\rho^{jpq} - 8\delta_i^j C(i) \} M M^\dagger g^2 = 4\gamma_i \delta_i^j M M^\dagger g^2 ,
\]

\[(19)\]

where we have used eqs. \[(1)\] and \[(13)\]. From the reduction equation for \( (m^2)_i^j \),

\[
[\beta_{m^2}]_i^j = \beta_M \partial (m^2)_i^j / \partial M + \beta_{M^1} \partial (m^2)_i^j / \partial M^1 + \beta_g \partial (m^2)_i^j / \partial g ,
\]

\[(20)\]

we finally obtain

\[
\kappa_i = \gamma_i [S(R) - 3C(G)]^{-1},
\]

\[(21)\]

which is consistent with the explicit result \[(14)\] in the minimal supersymmetric \( SU(5) \) GUT.

Eq. \[(21)\] together with eqs. \[(14)\] and \[(17)\] implies the sum rule \[(1)\] so that the use of the sum

\[ Similarly, we can obtain the reduction solution for the B-term (under a certain assumption), \( b^{ij} = - (\gamma_i + \gamma_j) / (S(R) - 3C(G))\mu^{ij} M \), which leads to another type of sum rule, \( m_i^2 + m_j^2 + (b^{ij} / \mu^{ij}) M^\dagger = 0 \).
rule in the intermediate steps of its derivation is self-consistent. If one does not use the sum rule (9), one finds
\[
\frac{d}{dt} (m_i^2 + m_j^2 + m_k^2 - M^\dagger M) = \sum_{r=i,j,k} \rho_{r\rho p} \rho_{\rho q} (m_r^2 + m_p^2 + m_q^2 - M^\dagger M) \frac{g^2}{16\pi^2} \tag{22}
\]
for \(\{i, j, k\}\) with \(\rho_{ijk} \neq 0\).

Therefore, if the sum rule is satisfied at some scale, e.g. at the superstring scale, it remains for other scales. If \(S(R) - 3C(G) = 0\) (which means that \(\beta_g^{(1)} = 0\)), eq. (21) has no meaning. In this finite case, the other \(\beta\) functions also have to vanish as the consequence of the reduction equations (11), (16) and (20). It is easy to see that the reduction solutions (10) and (15) keep their form \([8]\) and \([\beta_m^i]^j_i = 0\) if the sum rule is satisfied. But the constant \(\kappa_i\) remains undetermined. In refs. [9, 10], the symmetric choice \(m_i^2 = m_j^2 = m_k^2 = (1/3)MM^\dagger\) (see also ref. [25]) has been made to preserve higher-order finiteness of the SSB terms.

During the course of the derivation of the sum rule (9), we have made an interesting observation: The reduction solution for \(h^{ijk}\) \([13]\) means that the so-called \(A\) term is proportional to the gaugino mass \(M\), implying that the unwelcome superpartner contribution to the electric dipole moment (EDM) is suppressed \([20, 27]\). Similarly, the reduced \(B\)-term is proportional to \(\mu M\) (see the footnote 2). If \(\mu\) is real, the \(B\)-term and the gaugino mass \(M\) have the same CP phase, which leads to another suppression of EDM.

Let us next analyze how the relations (9) and (15) within the framework of supergravity can come about, where we do not necessarily think of supergravity as an effective theory of superstrings for a while. We begin by considering a non-canonical Kähler potential of the general form
\[
K = \tilde{K}(\Phi_a, \Phi^* a) + \sum_i K_i^i(\Phi_a, \Phi^* a)|\Phi_i|^2 \tag{23}
\]
where \(\Phi_a\)'s and \(\Phi_i\)'s are chiral superfields in the hidden and visible sectors, respectively \([3]\). The basic assumptions to be made are:

1. Supersymmetry is broken by the \(F\)-term condensations \((\langle F_a \rangle \neq 0)\) of the hidden sector fields \(\Phi_a\).

\(^3\)In the following discussions, we adopt the lazy notation that both the chiral superfields and their scalar components are denoted by the same symbol.
2. The gaugino mass $M$ stems from the gauge kinetic function $f$ which depends only on $\Phi_a$, i.e. $f = f(\Phi_a)$.

3. We consider only those Yukawa couplings that have no field dependence.

4. The vacuum energy $V_0$ vanishes, i.e.,

$$ V_0 = \langle F_a F_b \tilde{K}_b^a \rangle - 3m_{3/2}^2 = 0 \ . $$

(24)

Under these assumptions, the SSB parameters can be written as [13]

$$ h^{ijk} = \langle F_a \rangle \langle (\tilde{K} - \ln(K_i^i K_j^j K_k^k)) \rangle Y^{ijk} \ , $$

(25)

$$ M = \langle F_a \rangle \langle (\ln \text{Re} f)^a \rangle \ , $$

(26)

$$ m_i^2 = m_{3/2}^2 - \langle F_a \rangle \langle F_b \rangle \langle (\ln K_i^i)^a \rangle \ , $$

(27)

in the obvious notation. From eq. (13), upon using eqs. (25) and (26), we obtain the relation

$$ \langle F_a \rangle \langle (\ln \text{Re} f)^a \rangle + (\tilde{K} - \ln(K_i^i K_j^j K_k^k)) = 0 \ , $$

(28)

from which we deduce that, if $\langle F_a \rangle$’s are linear-independent, the constraint

$$ K_{(T)}(\Phi_a, \Phi'^a) \equiv \ln(K_i^i K_j^j K_k^k) $$

$$ = \tilde{K} + \ln \text{Re} f + \text{const.} \ \text{for all} \ \{i, j, k\} \ \text{with} \ \rho_{ijk} \neq 0, $$

(29)

has to be satisfied. Therefore, there has to exist a definite relation between the Kähler potential $\tilde{K}$ in the hidden sector, the gauge kinetic function $f$ and the Kähler metric. It is then straightforward to show that the constraint (29) leads to the soft scalar-mass sum rule (9):

$$ \sum_i \mu_i m_i^2 = 3m_{3/2}^2 - \langle F_a \rangle \langle F_b \rangle \langle K_{(T)}^a \rangle \langle (\ln \text{Re} f)^a \rangle = MM^\dagger, $$

(30)

where the use has been made of eqs. (27), (26), (29) and the fact that $\text{Re} f$ is a direct sum of holomorphic functions of $\Phi_a$ and $\Phi'^a$. We thus have arrived at the following generalized Kähler potential which leads to (9) and (13):

$$ G = K + \ln |W|^2, \ K_{(S)}(\Phi_a, \Phi'^a) = -\ln(f(\Phi_a) + \bar{f}(\Phi'^a)) \ , $$

$$ K = K_{(S)}(\Phi_a, \Phi'^a) + K_{(T)}(\Phi_a, \Phi'^a) + \sum_i K_i^i(\Phi_a, \Phi'^a)|\Phi_i|^2. $$

(31)
Given this Kähler potential, we next discuss symmetries behind. One finds that there exist two types of symmetries: The first one corresponds to the Kähler transformation together with the chiral rotation of the matter multiplets,

\[
\Phi_i \rightarrow e^{M_i} \Phi_i, \quad \Phi^*_i \rightarrow e^{\tilde{M}_i} \Phi^*_i,
\]

\[
K^i_i \rightarrow K^i_i e^{-(M_i + \tilde{M}_i)}, \quad K(T) \rightarrow K(T) - M - \tilde{M},
\]

\[
f(\Phi_a) \rightarrow f(\Phi_a), \quad W \rightarrow e^M W,
\]

where \( M_i \) is a function of \( \Phi_a \) and has to satisfy the constraint \( \sum \rho \rho M_i = M \) for all possible \( \rho \)’s (the sum \( \sum \rho \) is defined in eq. (14)). The second one is the invariance of the Kähler metric \( K^a_{(S)b} \) under the transformation

\[
f(\Phi_a) \rightarrow (af(\Phi_a) - ib)/(icf(\Phi_a) + d),
\]

where \( a, b, c \) and \( d \) are integers satisfying \( ad - bc = 1 \). These symmetry properties are exactly what we are searching, because we would like to understand the mass relations (9) and (15) in terms of symmetries. From the transformation rules (32) and (33) we see that it is likely for the symmetries to be realized that the hidden sector fields are divided into two types such that they enter either \( K(S) \) or \( K(T) \). For 4D string models, the symmetries (32) and (33) indeed appear as the so-called target-space duality invariance (see refs. [20, 21] and references therein) and S-duality [22], respectively. Therefore, the sum rule (9) and (15) might be naturally understood in superstring theories.

In the following discussion, we restrict ourselves to a certain class of the orbifold compactification and assume the existence of a non-perturbative superpotential which breaks supersymmetry and that the dilation \( S \) and the overall modulus \( T \) play the dominant role for supersymmetry breaking. So, \( S \) and \( T \) belong to the hidden sector. It is known that the Kähler potential and the gauge kinetic function in this case assumes the form

\[
K = -\ln(S + S^*) - 3\ln(T + T^*) + \sum_i (T + T^*)^{n_i} |\Phi_i|^2, \quad f = kS,
\]

where \( n_i \) are (usually negative) integers and stand for modular weights, and \( k \) is the Kac-Moody level [28–30]. The model possesses the \( SL(2, Z) \) target-space duality invariance, and under the duality transformation, the overall modulus \( T \) transforms like

\[
T \rightarrow (aT - ib)/(icT + d),
\]
where \(a, b, c\) and \(d\) are integers satisfying \(ad - bc = 1\). Chiral matter superfield \(\Phi_i\) with the modular weight \(n_i\) transforms like

\[
\Phi_i \rightarrow (icT + d)^{n_i} \Phi_i
\]

(36)

so that the last term of \(K\) remains invariant. Comparing this with the transformation rule (32), we see that \(\mathcal{M}_i = n_i \ln(icT + d)\), implying that the constraint \(\sum_t \rho^t \mathcal{M}_i = \mathcal{M}\) is satisfied only if \(\sum_t \rho n_l = \text{const}\). Since \(K_i^i = (T + T^*)^{n_i}\), i.e., \(K_{(T)} = \sum_t \rho n_l \ln(T + T^*)\), the const. defined above has to be equal to \(-3\) for the Kähler potential (34) to belong to the class of (31). The superpotential \(W\), therefore, transforms like

\[
W \rightarrow (icT + d)^{-3}W
\]

(37)

implying that \(W\) should have modular weight \(-3\). Since the modular weights are (usually) negative integers, the matter chiral superfields \(\Phi_i\)'s appearing in non-vanishing cubic terms in \(W (\rho^{ijk} \neq 0)\), have to have modular weight \(-1:\)

\[
n_i = n_j = n_k = -1 \text{ for } \{i, j, k\} \text{ with } \rho^{ijk} \neq 0,
\]

(38)

where we have assumed that the reduced Yukawa couplings \(Y^{ijk} = \rho^{ijk} g + \ldots\) have vanishing modular weight. Actually, within the framework of the orbifold models corresponding to the Kähler potential (34), we have [15]

\[
M = \sqrt{3m_3/2} \sin \theta, \quad m_i^2 = m_{3/2}^2 (1 + n_i \cos^2 \theta),
\]

(39)

\[
h^{ijk} = -\sqrt{3} Y^{ijk} m_{3/2} [\sin \theta + \cos \theta (3 + n_i + n_j + n_k)]
\]

(40)

where \(\theta\) is the goldstino angle defined as \(\tan \theta = \sqrt{K_S F^S/(\sqrt{K_T F^T})}\). Using the expressions above, one can explicitly check that the sum rule (9) and (15) are satisfied [13, 17, 19].

So far we have discussed only the soft scalar-mass sum rule (9) and (15). Let us next briefly discuss on the individual reduction solution (17) itself. Comparing eqs. (17) and (21) with (39), there has to exist a relation between \(n_i\) and \(\gamma_i\). However, the allowed values of \(n_i\) for the matter superfields are generally restricted [21], [32] so that the relation can not always be satisfied for a given GYU model. To overcome this problem, we may consider multi-moduli cases, where we have various modular weights and several goldstino angles as free parameters [16, 17]. In these cases, too, one can obtain the relation (13) and the sum rule (9) [19]. Another possibility is
the D-term contribution to soft scalar-masses \[33\]. This contribution can be written as \[34\]

\[
D_i = \sum_A q_i^A D^A, \quad D^A = 2g_A g_B (M^{-2})^{AB} \langle F^I \rangle \langle F_J \rangle \langle (D_B)^I \rangle, \tag{41}
\]

where \(q_i^A\) is the quantum number of \(\Phi_i\) under the diagonal, broken gauge symmetries, \(g_A\)'s are gauge coupling constants, \((M^{-2})^{AB}\) is an inverse mass matrix of massive gauge bosons and \(D^A\) is the corresponding D-term. The indices \(I, J\) run over all the chiral superfields. Gauge invariance of the superpotential \(W\) gives the constraint \(\sum_l \rho q_l^A = 0\), which implies \(\sum_l \rho D_l = 0\). Therefore, the D-term contribution does not affect the sum rule (9), but contributes to the individual soft scalar-masses.

Finally, we would like to emphasize that it is essential for the sum rule (9) and the relation (15) to be satisfied that the Yukawa couplings are reduced in favor of the gauge coupling \(g\). This is in sharp contrast to the case in the effective supergravity; one can derive them solely from the Kähler potential without explicitly knowing the superpotential. So, we can ask ourselves why in GYU models and in the certain superstring models the same type of the soft-mass relations are satisfied. Ibáñez in ref. \[14\] gives an interpretation of the coincidence for the case of finite GYU models in terms of the dilaton dominance and its relation to \(N = 4\) supersymmetry \[4\]. Here we argue that not only in finite GYU models, but also in non-finite ones the question could be answered in terms of a sort of anomaly matching.

Given that the hidden sector fields \(\Phi_a\)'s are supposed to decouple at low energies (which is an absolutely non-trivial assumption), and that the duality transformation \(32\) or \(36\) acts as \(\Phi_a\)-dependent scale transformations on the chiral matter superfields, we may identify the low-energy remnant of the duality invariance with the scale invariance of the effective renormalizable field theory. Needless to say that the transformation (36) with \(c = 0\) defines a global, common scaling for the matter superfields in the untwisted sector, and that the scaling of \(W\) (see eq. (37)) can be canceled by an appropriate scaling of the superspace coordinates (which, so to say, replaces the transformation of the hidden sector fields). At this stage, we would like to recall that the duality invariance has an anomaly (absent at the string level), which is canceled by the Green-Schwarz mechanism and the one-loop threshold corrections to gauge coupling \(g\) \[20, 21\] in the effective supergravity. The hidden sector fields \(\Phi_a\)'s play the basic role for this cancellation.

\[4\] As he points out, the Kähler potential \[34\] indeed retains the \(N = 4\) structure, quite apart from the fact that the matter fields generally are not in the adjoint representation of the gauge group. He argues that the coincidence of the soft scalar-sum rule in finite GYU models should be traced back to its finiteness.
and in their absence at low energies, a definite amount of the uncancelled anomaly remains. In the class of the orbifold models, which we have considered above, it is proportional to the one-loop coefficient $\beta^{(1)}_g$ of the $\beta$ function of $g$ \cite{20, 21}. So, if the scale invariance should be identified with the low-energy remnant of the duality invariance, its anomaly, too, should be controlled by the $\beta$ function of the gauge coupling $g$. However, the $\beta$ functions of the Yukawa couplings also contribute to the scale invariance anomaly at the renormalizable level; unless they are reduced in favor of the gauge coupling $g$ so that there exists only one $\beta$ function in a theory which dictates the anomalous scaling behavior. Note that the reduction equation (11) defines a set of ordinary differential equations of first order, and so the general solutions contain a set of integration constants. But the requirement that the solution is consistent with perturbative renormalizability is usually sufficient to obtain a unique solution \cite{10}. Moreover, only this power series solution (10) is regular in the sense that it has the well-defined $\beta^{(1)}_g \to 0$ limit, the scale invariance limit \cite{24} (where we assume we could vary $\beta^{(1)}_g$ smoothly). The other solutions have an essential singularity in this limit. Since we do not expect such a singularity at the effective supergravity level, the power series solution is singled out: Shortly, GYU is a consequence of the anomaly matching.

We believe that the simplicity and the universality of the soft mass relations in GYU models and the coincidence with their corresponding relations in superstring models have a deep meaning, and hope that our interpretation of it will put us toward a more complete understanding of the relation between gauge-Yukawa unified and superstring theories.

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