Abstract. Cosmological N–body simulations have revealed a remark-
able similarity in the structure of dark matter halos formed in hierarchi-
cally clustering universes. Regardless of halo mass, cosmological param-
ters, and power spectrum of initial density fluctuations, the spherically
averaged density profiles of dark matter halos have a universal shape.
The logarithmic slope of this profile is shallower than isothermal near the
center, and steepens gently outwards, becoming steeper than isothermal
near the halo virial radius. This profile can be well approximated by a
simple formula with only two free parameters: halo mass and “character-
istic” density, e.g., the density at the radius where the logarithmic slope
equals the isothermal value. This characteristic density is proportional
to the mean density of the universe at the time of collapse of each sys-
tem, and decreases systematically with increasing halo mass, reflecting
the later collapse of more massive halos. I use these results to examine
what constraints can be derived for Cold Dark Matter models from the
rotation curves of disk galaxies.

1. Introduction

The structure of dark matter halos formed through gravitational collapse in hi-
erarchically clustering universes has received close attention ever since the work
of Gunn & Gott (1972) showed that the virialized structure of halos may contain
clues to the cosmological parameters. Subsequent analytic work, which focussed
mainly on the density profiles of systems formed from scalefree initial con-
ditions, concluded that the equilibrium mass profiles of dark halos should be well
approximated by power laws, and that the power-law slope should depend sensi-
tively on the cosmological parameters (Fillmore & Goldreich 1984, Bertschinger
1985). These results influenced the interpretation of early numerical studies,
and prompted many authors to fit power-laws to the results of N–body simula-
tions (Quinn et al 1986, Frenk et al 1988, Efstathiou et al 1988, Zurek, Quinn
& Salmon 1988, Crone et al 1994). The general trends predicted by analytic
studies were generally confirmed, although significant deviations from power-
laws were also reported. These deviations were established beyond doubt by
the work of Dubinski & Carlberg (1991) and Navarro, Frenk & White (1995),
who found that halos formed in a Cold Dark Matter (CDM) universe were best
described by a density profile with a gently changing logarithmic slope rather than a single power law.

2. A Universal Density Profile from Hierarchical Clustering

Further simulations confirmed these results and indicated that this structure appears universal: density profiles of halos of different mass, formed in a variety of hierarchically clustering models (CDM and power-law initial density fluctuation spectra, $P(k) \propto k^n$, with different values of $\Omega_0$ and $\Lambda$), can be scaled to look identical (Navarro, Frenk & White 1997, NFW97). This is shown in Figure 1a, where we plot the spherically averaged density profiles of one of the least and one of the most massive halos in each series. These halos span four orders of magnitude in mass in the case of the CDM models and about two orders of magnitude in mass in the case of the power-law runs.

We define the mass of a halo, $M_{200}$, as that of a sphere with mean interior density equal to $200\rho_{\text{crit}}$, where $\rho_{\text{crit}} = 3H_0^2/8\pi G$ is the critical density for closure. We write Hubble’s constant as $H_0 = 100\ h\ \text{km s}^{-1}\ \text{Mpc}^{-1}$ in this contribution. The radius of this sphere, $r_{200}$, is usually called the “virial radius” of the halo. The virial radius and the circular velocity at $r_{200}$, $V_{200} = (r_{200}/h^{-1}\text{kpc})\ \text{km/s}$, are alternative, equivalent measures of halo mass.

The solid lines in Figure 1a are fits of the form proposed by Navarro, Frenk & White (1996) [see also Cole & Lacey (1996), and Tormen, Bouche & White (1997)]

$$\frac{\rho(r)}{\rho_{\text{crit}}} = \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2}. \quad (1)$$

Here, $\delta_c$ is a (dimensionless) characteristic density, and $r_s$ is a scale radius.
This simple formula provides a good fit to the structure of all halos over about two decades in radius, from the gravitational softening radius (indicated by arrows in Figure 1a) to about the virial radius of each halo. The quality of the fit is essentially independent of halo mass or cosmological model, and implies a remarkable similarity of structure between dark matter halos formed in different hierarchically clustering scenarios.

There is a single free parameter in eq. 1 for halos of given mass. This parameter can be expressed either as the characteristic density \( \delta_c \) or as the “concentration” of the halo, defined by the ratio \( c = r_{200}/r_s \). (\( \delta_c \) and \( c \) are related by a simple formula.) Our models show that \( M_{200} \) and \( \delta_c \) (or \( c \)) are strongly correlated. The characteristic density is simply proportional to the mean density of the universe at the time of collapse,

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\delta_c(M) \propto \Omega_0(1 + z_{\text{coll}}(M))^3.
\] (2)
as shown in Figure 1b. Hereafter we shall use the concentration, \( c \), as a measure of the characteristic density of a halo, since it is more easily compared with observations.

Because collapse redshifts depend on the value of the cosmological parameters, and can be computed analytically (see details in NFW97), observational constraints on halo concentrations can be translated directly into cosmological constraints. For example, the shape of the power spectrum regulates the dependence of \( c \) on halo mass. In CDM-like models, where structure grows very fast, different mass scales collapse almost at the same time, and \( c \) depends weakly on mass. Galaxy and galaxy cluster halos are thus expected to have values of \( c \) differing by a factor of about two, a useful prediction which can be tested observationally. On the other hand, the mass dependence of \( c \) in a universe where structure develops slowly, e.g. a model with white-noise initial perturbation spectrum (\( P(k) = \text{constant} \)), would be much stronger. Beyond the shape of the power spectrum, halo concentrations can also be used to gain insight on the density of the universe since, at fixed collapse redshift, the characteristic density of a halo scales directly with \( \Omega_0 \) (eq. 2). We investigate below how rotation curves can be used to constrain the concentration of dark halos surrounding disk galaxies and their consequences for CDM models.

3. Rotation Curves of Disk Galaxies

We have analyzed the rotation curves of more than 100 disk galaxies taken from the literature in order to examine whether the structure of their surrounding dark halos is consistent with eq. 1 (for details, see Navarro 1998). The sample covers

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1 We note that, although the density in eq. 1 diverges like \( r^{-1} \) near the center, the simulations reported here do not prove that this is the correct asymptotic behaviour. They only show that eq. 1 describes well the structure of halos in the radial range indicated above. Recent work by Moore et al. and Kravtsov et al. (see their contributions in this volume) suggests that the inner asymptotic slope may differ from \( r^{-1} \). Moore et al propose a steeper inner slope, \( r^{-1.4} \), for galaxy clusters and Kravtsov et al a shallower slope, \( r^{-0.7} \), for dwarf galaxies formed in CDM universes. Further numerical work is underway to establish conclusively whether the inner slope depends on mass in the way suggested by these authors.
a wide range in galaxy parameters, spanning almost four orders of magnitude in luminosity, two orders of magnitude in surface brightness, and almost a decade in disk rotation speed. Figure 2a shows fits to the rotation curves of two galaxies in the sample, performed using eq. 1 (hereafter called NFW model) and a non-singular isothermal sphere (hereafter called ISO model) for modeling the dark component.

This figure serves to illustrate that rotation curves are generally consistent with either NFW or ISO halo structures, although the contribution of the disk to the circular velocity (dotted lines) can differ dramatically depending on which model is adopted. There are a handful of exceptions: the HI rotation curves of six low-surface brightness galaxies (LSBs) are better fitted with an ISO model. Four of these galaxies are shown in Figure 2b. The rotation speed seems indeed to rise too rapidly with radius (as a “solid body”) to be consistent with the NFW mass profile. However, the differences are small, and the significance of the discrepancy may have been overemphasized by optimistic velocity error bars.

The case of NGC 3109 shows that this is a true possibility. Here, two datasets are available for the same galaxy, one based on HI observations only (open circles, Jobin & Carignan 1990), and one based on independent Hα and HI observations (filled circles, Carignan 1985). The two resulting rotation curves are dramatically different. While the open-circle data excludes the NFW model with high significance, the filled circles are fully compatible with an NFW halo profile. Therefore, the possibility remains that the discrepancies between NFW halo models and the rotation curves of some low-surface brightness galaxies may be less important than previously thought (Moore 1994, Flores & Primack 1994). A thorough reanalysis of the rotation curves of these galaxies is needed to sort out the problem. Other possible resolutions of this problem include

Figure 2. a) Rotation curve fits using the NFW and the ISO halo models shown for a high-surface brightness galaxy (NGC 3198, Bege mann 1987) and a low-surface brightness galaxy (F563-1, de Blok 1997). Note that both halo models produce acceptable fits, although they require different disk mass-to-light ratios. b) same as (a), but for four LSB galaxies where the ISO halo model fits better than NFW.
modifications to the dark matter profile caused during the assembly of the disk (Navarro, Eke & Frenk 1996), or small systematic deviations from a strict NFW shape, as proposed by Kravtsov et al (1997).

With this caveat, let us now explore the consequences for CDM models of fitting NFW halos to disk galaxy rotation curves. The first thing to note is that the overall shape of the rotation curve defines a firm upper bound to the concentration of the halo. This is shown in Figure 3a, where we illustrate the circular velocity profiles of halos of similar mass (ie. similar $V_{200}$) but different concentrations. These curves ignore the contribution of the luminous component. From the figure, it is clear that $c = 26$, the value of the concentration that best fits the rotation curve neglecting the luminous component, represents an upper limit to the concentration of the halo that surrounds NGC 3198. Halos with $c < 26$ could in principle be made consistent with the data by suitable addition of a massive disk component, but $c > 26$ halos result in rotation speeds that are in excess of the data even before allowing for the presence of the disk. This upper limit is quite insensitive to the halo mass adopted (expressed by $V_{200}$), which merely sets the velocity scale of the fit.

A second thing to note in Figure 3a is that the value of c retrieved by fitting halo-only models to the data (referred to hereafter as $c_{fit}$) is a good indicator of the shape of the rotation curve. Values of $c_{fit} \lesssim 10-20$ indicate that the rotation curve rises slowly, while $c_{fit} \gtrsim 10-20$ describe a sharply rising rotation curve that is either flat or declines in the outer regions.

Upper limits to the halo concentration, ie. $c_{fit}$, derived individually for all galaxies in our sample are shown in Figure 3b as a function of $V_{200}$. Overlaid are halo concentrations expected in three CDM cosmogonies. SCDM refers to the standard biased ($\sigma_8 = 0.6$) $\Omega = 1$ CDM model. The two dotted lines correspond to low-density, flat ($\Omega + \Lambda = 1$) CDM cosmogonies normalized to
match the fluctuations in the cosmic microwave background observed by COBE (see, eg., Eke, Cole & Frenk 1996). The scatter around each of these lines is expected to be less than about 30% (Navarro et al, in preparation). As discussed earlier, the lines are almost horizontal, indicating that the concentration is a weak function of mass in CDM models. The data presented in Figure 3b seems to be inconsistent with the SCDM model, and appears to favor the low-density CDM models since, in order to be acceptable, halo concentrations should be below all individual upper limits.

It is instructive to see which galaxies cannot be reconciled with the SCDM model. Figure 4 shows $c_{fit}$ as a function of the effective surface brightness of the galaxy, $\Sigma_{eff} = L_I / \pi r_{disk}^2$, defined as the surface brightness of a galaxy if all its light were concentrated within one exponential disk scalelength. (All luminosities quoted are in the I-band.) There is a strong correlation between surface brightness and $c_{fit}$, indicating that LSBs have slowly rising rotation curves while high-surface brightness galaxies (HSBs) have steeply rising, flat rotation curves. Most galaxies incompatible with SCDM (ie, those with $c_{fit} \lesssim 10$) are LSBs.

What does the correlation between $c_{fit}$ and $\Sigma_{eff}$ mean? If the universe is dominated by cold dark matter, we expect all halos, regardless of mass, to have similar values of $c$ previous to the collapse of the luminous component (see the nearly horizontal lines in Figure 3b). Thus, the $c_{fit}-\Sigma_{eff}$ correlation reflects the relative importance of disks in shaping the rotation curves of galaxies of different surface brightness. In very low surface brightness systems the luminous component is unimportant gravitationally and the rotation curve traces the mass distribution of the halo, ie. $c_{fit} \sim c \approx 3$ or 5, depending on the value of $\Omega_0$ (see Figure 3b). In high surface brightness systems the disk gravity steepens the rotation curve in the inner regions, resulting in higher values of $c_{fit}$.

We can take this analysis one step further and ask what the $c_{fit}-\Sigma_{eff}$ correlation means for the mass-to-light ratio of the disk and for the relationship between the halo circular velocity and the rotation speed of the disk. In other words, assuming that all halos have initially the same concentration, eg. $c \approx 3$ (for $\Omega = 0.2, \Lambda = 0.8$, see Figure 3b), for what combination of halo masses and

Figure 4. The shape of the rotation curve, parameterized by $c_{fit}$, plotted as a function of effective surface brightness.
disk mass-to-light ratios can one recover the observed $c_{fit}$-$\Sigma_{eff}$ relation whilst at the same time satisfying other constraints such as the luminosity-surface brightness relation and the Tully-Fisher relation?

For illustration, let us assume that all disks have the same stellar mass-to-light ratio, $(M/L)_{disk} = 1h(M_\odot/L_\odot)$. Assuming that the initial concentration of the halo is $c = 3$, we compute $c_{fit}$ for disks of different surface brightness. At each surface brightness we take the luminosity of the disk to be that given by the $L_I$-$\Sigma_{eff}$ relation. We then adjust the circular velocity of the halo ($V_{200}$) in order to match the rotation speed within the luminous radius of a galaxy of that luminosity given by the Tully-Fisher relation. Both observational relations are constructed internally using galaxies in the same sample.

The result of this exercise is shown with short-dashed lines in Figure 5. Halos are required to be more massive than expected from the disk rotation speed (ie. $V_{200} > V_{rot}$) in order to satisfy the Tully-Fisher relation (upper-right panel). However, no significant correlation is found between surface brightness and the shape of the rotation curve; irrespective of surface brightness all galaxies have approximately the same value of $c_{fit}$ (upper-left panel).

A second example is provided by the dotted lines in Figure 5, which assume that the circular velocity of the halo is the same as the rotation speed of the disk, ie. $V_{200} = V_{rot}$. In this case the mass-to-light ratio of the disk has to be higher than unity to match the Tully-Fisher relation. As is clear from Figure 5, this assumption also provides a poor fit to the $c_{fit}$-$\Sigma_{eff}$ relation.

Thus, the existence of a correlation between $c_{fit}$ and $\Sigma_{eff}$ implies that the disk mass-to-light ratios and the ratio between $V_{200}$ and $V_{rot}$ cannot remain constant for all galaxies. The solid and long-dashed lines in Figure 5 are constructed to match the observed $c_{fit}$-$\Sigma_{eff}$ relation. (Solid and long-dashed lines refer to halos formed in the $\Omega = 0.2$ and $\Omega = 0.3$ models shown in Figure 3b, respectively.) The resulting disk mass-to-light ratios increase from $\sim 0.5$ in faint, slow-rotating disks to $3-5\, hM_\odot/L_\odot$ in the fastest rotators: $(M/L_I)_{disk} \approx (L_I/10^9L_\odot)^{0.3}h\, M_\odot/L_\odot \approx (V_{rot}/100\, \text{km s}^{-1}) M_\odot/L_\odot$. The color differences between disks of different morphology/surface brightness suggest that systematic trends of this magnitude between disk mass-to-light ratios and luminosity do indeed exist (de Jong 1995).

The relationship between $V_{200}$ and $V_{rot}$ that results is also intriguing. Halos of disks with $V_{rot} < 150\, \text{km s}^{-1}$ have $V_{200} > V_{rot}$, by up to 60% for $V_{rot} \sim 100\, \text{km s}^{-1}$. On the other hand, disks that rotate faster than $\sim 150\, \text{km s}^{-1}$ all have similar halo circular velocities, $V_{200} \approx 200\, \text{km s}^{-1}$. This is reminiscent of a well-known result of dynamical studies of satellite/primary pairs: there is little correlation between the rotation speed of luminous disks and the mass of their surrounding halos (Zaritsky et al 1997). It is comforting that we arrive at a similar conclusion using a completely different approach.

One curious corollary is that few disk galaxies inhabit halos more massive than $V_{200} \approx 200\, \text{km s}^{-1}$, a result which may reflect the onset of disk instabilities in massive galaxies. As discussed by Mo, Mao & White (1997), stable disks embedded in NFW halos are only stable if the disk contributes a small fraction of the total mass. More specifically, their analysis suggests that only in systems where $M_{disk}/M_{200} < \lambda$ can disks avoid being disrupted by global instabilities. Here $\lambda = JE^{1/2}/GM^{5/2}$ is the usual dimensionless rotation parameter which, as
shown by extensive numerical work, seldom exceeds $\sim 0.1$ (Cole & Lacey 1996). In other words, if the amount of baryons that collapse to form the disk is such that $M_{\text{disk}}/M_{200}$ exceeds about 0.1, very few of these systems would survive as disks to the present. The fraction of the total mass that can collect in the disk cannot exceed the universal baryon fraction, $\Omega_b/\Omega_0$ (White et al 1993). For the low density models we are considering here, and adopting the usual Big Bang nucleosynthesis value for $\Omega_b$, $M_{\text{disk}}/M_{200} \lesssim 0.2-0.3$. We see that long-lived disks can only form in systems where fewer than about half of all available baryons have cooled and assembled into the disk.

The lower-right panel in Figure 5 shows that, in models that satisfy the observed $c_{\text{fid}}-\Sigma_{\text{eff}}$ relation, the disk mass fraction increases sharply with halo mass, exceeding the critical value of $\sim 0.1$ at $V_{200} \approx 200 \text{ km s}^{-1}$. Matching the rotation curve shapes thus requires the mechanism regulating the disk mass fraction (e.g. feedback from supernovae and evolving stars) to be highly efficient in low mass halos, but relatively inefficient in halos more massive than about $V_{200} \approx 200 \text{ km s}^{-1}$. Indeed, this rapidly varying “efficiency” of assembly of baryons into galaxies is at the heart of all successful hierarchical galaxy formation models, where it is invoked to reconcile the relative scarcity of dwarf galaxies with the myriad of low-mass halos expected in hierarchically clustering universes (Kauffmann, White & Guiderdoni 1993, Cole et al 1994). It is interesting and suggestive that the same feedback process needed to explain the relative number of dwarf and bright galaxies in hierarchical models is actually required to match the shape of the rotation curves of present-day disk galaxies.

Figure 5. Correlations between different parameters of various different rotation curve models, described in detail in the text.
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