A NEW CODING/DECODING ALGORITHM USING FIBONACCI NUMBERS

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Abstract. In this paper we present a new method of coding/decoding algorithms using Fibonacci $Q$-matrices. This method is based on the blocked message matrices. The main advantage of our model is the encryption of each message matrix with different keys. Our approach will not only increase the security of information but also has high correct ability.

1. Introduction and Background

It is well known that the Fibonacci sequence is defined by

$$F_n = F_{n-1} + F_{n-2} \text{ with } n \geq 2,$$

with the initial terms $F_0 = 0$, $F_1 = 1$. The Fibonacci $Q$-matrix is defined in [2] and [3] as follows:

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

From [5] and [7], we known that the $n$.th power of the Fibonacci $Q$-matrix is of the following form:

$$Q^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}.$$

In recent days information security becomes a more important matter in terms of data transfer over communication channel. So coding/decoding algorithms are of great importance to help in improving information security. Especially, Fibonacci coding theory has been considered in many aspects (see [1], [4], [6], [10].

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\[ \text{[7, 8 and 9 for more details). For example, in [7], it was given a new coding theory using the generalization of the Cassini formula for Fibonacci } p\text{-numbers and } Q_p\text{-matrices.} \]

In this study we present a new coding/decoding algorithm using Fibonacci \(Q\)-matrices. The main idea of our method depend on dividing the message matrix into the block matrices of size \(2 \times 2\). We use different numbered alphabet for each message, so we get a more reliable coding method. The alphabet is determined by the number of block matrices of the message matrix. Our approach will not only increase the security of information but also has high correct ability for data transfer over communication channel.

2. The Blocking Algorithm

At first, we put our message in a matrix of even size adding zero between two words and end of the message until we obtain the size of the message matrix is even. Dividing the message square matrix \(M\) of size \(2m\) into the matrices, named \(B_i\) \((1 \leq i \leq m^2)\) of size \(2 \times 2\), from left to right, we construct a new coding method.

Now we explain the symbols of our coding method. Assume that matrices \(B_i\), \(E_i\) and \(Q^n\) are of the following forms:

\[
B_i = \begin{pmatrix} b_{i1} & b_{i2} \\ b_{i3} & b_{i4} \end{pmatrix}, \quad E_i = \begin{pmatrix} e_{i1} & e_{i2} \\ e_{i3} & e_{i4} \end{pmatrix} \quad \text{and} \quad Q^n = \begin{pmatrix} q_1 & q_2 \\ q_3 & q_4 \end{pmatrix}.
\]

The number of the block matrices \(B_i\) is denoted by \(b\). According to \(b\), we choose the number \(n\) as follows:

\[
n = \begin{cases} 
3 & \text{if } b \leq 3 \\
 b & \text{if } b > 3
\end{cases}.
\]

Using the choosen \(n\), we write the following letter table according to \(mod27\) (this table can be extended according to the used characters in the message matrix). We begin the “\(n\)” for the first character.
Now we explain the following new coding and decoding algorithms.

**Coding Algorithm (Fibonacci Blocking Algorithm)**

**Step 1.** Divide the matrix $M$ into blocks $B_i$ ($1 \leq i \leq m^2$).

**Step 2.** Choose $n$.

**Step 3.** Determine $b_i^j$ ($1 \leq j \leq 4$).

**Step 4.** Compute $\det(B_i) \rightarrow d_i$.

**Step 5.** Construct $F = [d_i, b_i^k]_{k \in \{1, 2, 4\}}$.

**Step 6.** End of algorithm.

**Decoding Algorithm**

**Step 1.** Compute $Q^n$.

**Step 2.** Determine $q_j$ ($1 \leq j \leq 4$).

**Step 3.** Compute $q_1b_i^1 + q_3b_i^2 \rightarrow e_i^1$ ($1 \leq i \leq m^2$).

**Step 4.** Compute $q_2b_i^1 + q_4b_i^2 \rightarrow e_i^2$.

**Step 5.** Solve $(-1)^nd_i = e_i^1(2q_2x_i + q_4b_3) - e_i^2(q_1x_i + q_3b_4)$.

**Step 6.** Substitute for $x_i = b_3^i$.

**Step 7.** Construct $B_i$.

**Step 8.** Construct $M$.

**Step 9.** End of algorithm.

In the following examples we give applications of the above algorithm for $b > 3$ and $b \leq 3$, respectively.

**Example 2.1.** Let us consider the message matrix for the message text “NIHAL HELLO”:

$$M = \begin{pmatrix} N & I & H & A \\ L & 0 & H & E \\ L & L & O & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{4 \times 4}.$$

**Coding Algorithm:**
Step 1. We can divide the message matrix $M$ of size $4 \times 4$ into the matrices, named $B_i$ ($1 \leq i \leq 4$), from left to right, each of size $2 \times 2$:

$$
B_1 = \begin{pmatrix} N & I \\ L & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} H & A \\ H & E \end{pmatrix}, \quad B_3 = \begin{pmatrix} L & L \\ 0 & 0 \end{pmatrix} \text{ and } B_4 = \begin{pmatrix} O & 0 \\ 0 & 0 \end{pmatrix}.
$$

Step 2. Since $b = 4 \geq 3$, we choose $n = 4$. For $n = 4$, we use the following “letter table” for the message matrix $M$:

| N | I | H | A | L | 0 | H | E | L | L | O |
|---|---|---|---|---|---|---|---|---|---|---|
| 17 | 12 | 11 | 4 | 15 | 3 | 11 | 8 | 15 | 15 | 18 |

Step 3. We have the elements of the blocks $B_i$ ($1 \leq i \leq 4$) as follows:

$\begin{align*}
&b_1^1 = 17, \quad b_2^1 = 12, \quad b_3^1 = 15, \quad b_4^1 = 3 \\
&b_1^2 = 11, \quad b_2^2 = 4, \quad b_3^2 = 11, \quad b_4^2 = 8 \\
&b_1^3 = 15, \quad b_2^3 = 15, \quad b_3^3 = 3, \quad b_4^3 = 3 \\
&b_1^4 = 18, \quad b_2^4 = 3, \quad b_3^4 = 3, \quad b_4^4 = 3
\end{align*}$

Step 4. Now we calculate the determinants $d_i$ of the blocks $B_i$:

$\begin{align*}
d_1 &= \det(B_1) = -129 \\
d_2 &= \det(B_2) = 44 \\
d_3 &= \det(B_3) = 0 \\
d_4 &= \det(B_4) = 45
\end{align*}$

Step 5. Using Step 3 and Step 4 we obtain the following matrix $F$:

$$
F = \begin{pmatrix}
-129 & 17 & 12 & 3 \\
44 & 11 & 4 & 8 \\
0 & 15 & 15 & 3 \\
45 & 18 & 3 & 3
\end{pmatrix}.
$$

Step 6. End of algorithm.

Decoding algorithm:

Step 1. It is known that

$$
Q^4 = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}.
$$

Step 2. The elements of $Q^4$ are denoted by

$q_1 = 5, \quad q_2 = 3, \quad q_3 = 3 \text{ and } q_4 = 2.$
Step 3. We compute the elements $e_1^i$ to construct the matrix $E_i$:

$$e_1^1 = 121, e_1^2 = 67, e_1^3 = 120 \text{ and } e_1^4 = 99.$$  

Step 4. We compute the elements $e_2^i$ to construct the matrix $E_i$:

$$e_2^1 = 75, e_2^2 = 41, e_2^3 = 75 \text{ and } e_2^4 = 60.$$  

Step 5. We calculate the elements $x_i$:

$$(-1)^4(-129) = 121(3x_1 + 6) - 75(5x_1 + 9) \Rightarrow x_1 = 15.$$  

$$(-1)^444 = 67(3x_2 + 16) - 41(5x_2 + 24) \Rightarrow x_2 = 11.$$  

$$(-1)^40 = 120(3x_3 + 6) - 75(5x_3 + 9) \Rightarrow x_3 = 3.$$  

$$(-1)^45 = 99(3x_4 + 6) - 60(5x_4 + 9) \Rightarrow x_4 = 3.$$  

Step 6. We rename $x_i$ as follows:

$$x_1 = b_1^3 = 15, x_2 = b_2^3 = 11, x_3 = b_3^3 = 3 \text{ and } x_4 = b_4^3 = 3.$$  

Step 7. We construct the block matrices $B_i$:

$$B_1 = \begin{pmatrix} 17 & 12 \\ 15 & 3 \end{pmatrix}, B_2 = \begin{pmatrix} 11 & 4 \\ 11 & 8 \end{pmatrix}, B_3 = \begin{pmatrix} 15 & 15 \\ 3 & 3 \end{pmatrix} \text{ and } B_4 = \begin{pmatrix} 18 & 3 \\ 3 & 3 \end{pmatrix}.$$  

Step 8. We obtain the message matrix $M$:

$$M = \begin{pmatrix} 17 & 12 & 11 & 4 \\ 15 & 3 & 11 & 8 \\ 15 & 15 & 18 & 3 \\ 3 & 3 & 3 & 3 \end{pmatrix} = \begin{pmatrix} N & I & H & A \\ L & 0 & H & E \\ L & L & O & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$  

Step 9. End of algorithm.
Example 2.2. Let us consider the message matrix for the message text “MATH”:

\[
M = \begin{pmatrix} M & A \\ T & H \end{pmatrix}_{2 \times 2}.
\]

**Coding Algorithm:**

**Step 1.** Since the size of message matrix \(M\) is \(2 \times 2\), we have only one block matrix \(B_1 = M\).

**Step 2.** Since \(b = 1 < 3\), we choose \(n = 3\). For \(n = 3\), we use the following “letter table” for the message matrix \(M\):

|   | A | T | H |
|---|---|---|---|
| 15| 3 | 22| 10|

**Step 3.** We have the elements of the blocks \(B_1\) as follows:

\[
B_1 = \begin{pmatrix} 15 & 3 \\ 22 & 10 \end{pmatrix}.
\]

**Step 4.** Now we calculate the determinant \(d_1\) of the block matrix \(B_1\):

\[
d_1 = \det(B_1) = 84.
\]

**Step 5.** Using Step 3 and Step 4 we obtain the following matrix \(F\):

\[
F = \begin{pmatrix} 84 & 15 & 3 & 10 \end{pmatrix}.
\]

**Step 6.** End of algorithm.

**Decoding algorithm:**

**Step 1.** It is known that

\[
Q^3 = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}.
\]

**Step 2.** The elements of \(Q^3\) are denoted by

\(q_1 = 3, q_2 = 2, q_3 = 2\) and \(q_4 = 1\).

**Step 3.** We compute the element \(e_1^1\) to construct the matrix \(E_1\):

\[
e_1^1 = q_1b_1^1 + q_3b_2^1 = 51.
\]

**Step 4.** We compute the element \(e_2^1\) to construct the matrix \(E_1\):

\[
e_2^1 = q_2b_1^1 + q_4b_2^1 = 33.
\]
Step 5. We calculate the element $x_1$:

$$(-1)^3(84) = 51(2x_1 + 10) - 33(3x_1 + 20)$$

$$\Rightarrow x_1 = 22.$$ 

Step 6. We rename $x_1$ as follows:

$$x_1 = b_3^1 = 22.$$ 

Step 7. We construct the block matrix $B_i$:

$$B_1 = \begin{pmatrix} 15 & 3 \\ 22 & 10 \end{pmatrix}.$$ 

Step 8. We obtain the message matrix $M$:

$$M = \begin{pmatrix} 15 & 3 \\ 22 & 10 \end{pmatrix} = \begin{pmatrix} M & A \\ T & H \end{pmatrix}.$$ 

Step 9. End of algorithm.

3. A Computer Application

To determine the verification of our coding method, in this section we construct a computer algorithm. We create the MATLAB codes for the examples given in the previous section. So our blocking algorithm is checked for $n = 2$ and $n = 4$ for different message texts, respectively. It can be seen that the algorithm works errorless. Moreover, complex message texts are solved correctly thanks to these algorithms. By a similar way, this algorithm can be extended for convenient $n$. At first we define the Fibonacci numbers in the algorithm then we give the following code segments for $n = 2$ (see Table [1]).

The second algorithm seen in Table [2] provides more faster solution for complex text. The readers can verify these algorithms for varied values of $n$.

4. Conclusion

The main idea of our method depends on dividing the message matrix into the block matrices of size $2 \times 2$. We give an algorithm using Fibonacci numbers and the numbers corresponding to each letter in used alphabet changes for each new message matrix. We verify our new algorithm with illustrative examples. Also our method is supported by a code in MATLAB programme for $n = 2$ and $n = 4$. This algorithm can be improved for any $n$ in the light of similar arguments.
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Table 1. The Algorithm for $n = 2$

\[
\begin{align*}
a &= \text{input}(a = '); \\
b &= \text{input}(b = ');
\end{align*}
\]
\[
c &= \text{input}(c = ');
\]
\[
d &= \text{input}(d = ');
\]
\[
A &= [a, b; c, d]; \\
F &= [\text{det}(A), a; c, d]; \text{End of Coding Algorithm}
\]
\[
fibf(1) = 1; \\
fibf(2) = 1; \\
p &= 3;
\]
\[
\text{while } \text{fibf}(p - 1) < 1000 \text{ fibf}(p) = \text{fibf}(1) + \text{fibf}(p - 2); \\
p &= p + 1; \text{ end}
\]
\[
Q &= [\text{fibf}(n + 1), \text{fibf}(n); \text{fibf}(n), \text{fibf}(n - 1)];
\]
\[
e1 &= \text{fibf}(n + 1)*a + \text{fibf}(n)*b; \\
e2 &= \text{fibf}(n)*a + \text{fibf}(n - 1)*b;
\]
\[
x &= \text{sym}('x');
\]
\[
eqn &= (-1)^n*\text{det}(A) == (e1*(\text{fibf}(n)*x + \text{fibf}(n - 1)*d))-(e2*(\text{fibf}(n + 1)*x + \text{fibf}(n)*d));
\]
\[
solx &= \text{solve}(\text{eqn}, 'x');
\]
\[
E &= [a, b; solx, d] \text{ End of Decoding Algorithm}
\]
Table 2. The Algorithm for \( n = 4 \)

\[
a = \text{input}(\text{a = '}); \ b = \text{input}(\text{b = '}); \ c = \text{input}(\text{c = '}); \ d = \text{input}(\text{d = '}); \\
e = \text{input}(\text{e = '}); \ f = \text{input}(\text{f = '}); \ g = \text{input}(\text{g = '}); \ h = \text{input}(\text{h = '}); \\
i = \text{input}(\text{i = '}); \ j = \text{input}(\text{j = '}); \ k = \text{input}(\text{k = '}); \ l = \text{input}(\text{l = '}); \\
m = \text{input}(\text{m = '}); \ n = \text{input}(\text{n = '}); \ p = \text{input}(\text{p = '}); \ r = \text{input}(\text{r = '}); \\
A = [a, b, c, d; e, f, g, h; i, j, k, l; m, n, p, r]; \\
M = [a, b; e, f]; \ N = [c, d; g, h]; \ O = [i, j; m, n]; \ P = [k, l; p, r]; \\
d_1 = \text{det}(M); \ d_2 = \text{det}(N); \ d_3 = \text{det}(O); \ d_4 = \text{det}(P); \\
F = [d_1, a, b; d_2, c, d; d_3, i, j; d_4, k, l, r]; \\
\text{fibf}(1) = 1; \ \text{fibf}(2) = 1; \ p = 3; \\
\text{while fibf}(p - 1) < 1000 \ \text{fibf}(p) = \text{fibf}(p - 1) + \text{fibf}(p - 2); \ ; \\
p = p + 1; \text{end} \\
dc_{11} = \text{fibf}(s + 1) * a + \text{fibf}(s) * b; \\
dc_{12} = \text{fibf}(s + 1) * c + \text{fibf}(s) * d; \\
dc_{13} = \text{fibf}(s + 1) * i + \text{fibf}(s) * j; \\
dc_{14} = \text{fibf}(s + 1) * k + \text{fibf}(s) * l; \\
dc_{21} = \text{fibf}(s) * a + \text{fibf}(s - 1) * b; \\
dc_{22} = \text{fibf}(s) * c + \text{fibf}(s - 1) * d; \\
dc_{23} = \text{fibf}(s) * i + \text{fibf}(s - 1) * j; \\
dc_{24} = \text{fibf}(s) * k + \text{fibf}(s - 1) * l; \\
x = \text{sym}(\text{x}); \\
eqn_1 = (\text{fibf}(s + 1) * x + \text{fibf}(s) * f) - (\text{fibf}(s - 1) * x + \text{fibf}(s) * f); \\
eqn_2 = (\text{fibf}(s + 1) * x + \text{fibf}(s) * h) - (\text{fibf}(s - 1) * x + \text{fibf}(s) * h); \\
eqn_3 = (\text{fibf}(s + 1) * x + \text{fibf}(s) * n) - (\text{fibf}(s - 1) * x + \text{fibf}(s) * n); \\
eqn_4 = (\text{fibf}(s + 1) * x + \text{fibf}(s) * r) - (\text{fibf}(s - 1) * x + \text{fibf}(s) * r); \\
solx_1 = \text{solve}(\text{eqn}_1, x); \ solx_2 = \text{solve}(\text{eqn}_2, x); \\
solx_3 = \text{solve}(\text{eqn}_3, x); \ solx_4 = \text{solve}(\text{eqn}_4, x); \\
B_1 = [a, b; solx_1, f]; \ B_2 = [c, d; solx_2, h]; \ B_3 = [i, j; solx_3, n]; \ B_4 = [k, l; solx_4, r]; \\
M = [a, b, c, d; solx_1, f, solx_2, h; i, j, k, l; solx_3, n, solx_4, r]; \\
\text{End of Decoding Algorithm}