Calculation of francium hyperfine anomaly

E. A. Konovalova, Yu. A. Demidov, M. G. Kozlov, and A. E. Barzakh

1 Petersburg Nuclear Physics Institute of NRC “Kurchatov center”,
Gatchina, Leningrad District 188300, Russia
2 St. Petersburg Electrotechnical University “LETI”, Prof. Popov Str. 5, 197376 St. Petersburg

(Dated: May 28, 2018)

The Dirac-Hartree-Fock plus many-body perturbation theory (DHF+MBPT) method has been used to calculate hyperfine structure constants for Fr. Calculated hyperfine structure anomaly for hydrogen-like ion has been shown to be in good agreement with analytical expressions. It has been shown that the ratio of the anomalies for s and p_{1/2} states is weakly dependent on the principal quantum number. Finally, we estimate Bohr–Weisskopf corrections for several Fr isotopes. Our results may be used to improve experimental accuracy for the nuclear g factors of short-lived isotopes.

I. INTRODUCTION

In recent years, the precision achieved in laser spectroscopy experiments coupled with advances in atomic theory has enabled new atomic physics based tests of nuclear models. The hyperfine structure constants and isotope shifts are highly sensitive to the changes of charge and magnetization distributions inside the nucleus because they depend on the behavior of the electron wave function in this region. The hyperfine structure (HFS) measurements can serve as very useful tool for understanding of shape coexistence phenomena in atomic nuclei [1].

The ratio of magnetic hyperfine constants $A$ for different isotopes is usually assumed to be equal to the ratio of their nuclear $g$ factors $g_I = \mu / I$, where $\mu$ and $I$ are magnetic moment and spin of the nucleus. However, this is true only for the point-like nucleus. For the finite nucleus one should take into account (i) distribution of the magnetization inside the nucleus and (ii) dependence of the electron wave function on the nuclear charge radius. Former correction is called magnetic, or Bohr–Weisskopf (BW) correction [2] and the latter one is called charge, or Breit-Rosenthal (BR) correction [3, 4]). These corrections break proportionality between magnetic hyperfine constants and nuclear $g$ factors. This phenomenon is called hyperfine anomaly (HFA) [2]. Below we discuss how to calculate HFA for many-electron atoms with available atomic package [6], which is based on the original Dirac-Hartree-Fock code [7]. This package was often used to calculate different atomic properties including HFS constants of Ti [8, 9], Yb [10], Mg [11], and Pb [12].

We study francium atom, because there are comprehensive experimental data [13–18] and many theoretical calculations [19–22] for this isotopic chain. In particular, changes of the nuclear charge radii in the Fr isotopic series were calculated from isotope shift measurements [23, 24] and absolute values of the nuclear charge radii were obtained [25].

II. THEORY AND METHODS

It is generally accepted that the observed HFS constant $A$ can be written in the following form:

$$ A = g_I A_0 (1 - \delta)(1 - \epsilon). \quad (1) $$

Here $g_I$ is a nuclear $g$ factor, $g_I A_0$ is a HFS constant for the point-like nucleus, $\delta$ and $\epsilon$ are the nuclear charge distribution (BR) and magnetization distribution (BW) corrections respectively. $A_0$ is independent on the nuclear $g$ factor. In the case of hydrogen-like ions the expression for $A_0$ was obtained in the analytical form by Shabaev [5]:

$$ A_0 = \frac{\alpha (\alpha Z)^3}{j(j+1)} \frac{m}{m_p} \frac{\gamma(2\gamma + n_r - N)}{N^4(4\gamma^2 - 1)} mc^2. \quad (2) $$

Here $\alpha$ is the fine-structure constant, $Z$ is the nuclear charge, $m$ and $m_p$ are electron and proton masses, $j$ is the total electron angular momentum, $\gamma$ is a relativistic quantum number, $N = n_r^2 + 2n_r\gamma + \gamma^2$, $n_r$ is a radial quantum number, $\gamma = \sqrt{\gamma^2 - (\alpha Z)^2}$. We use the model of the homogeneously charged ball of the radius $R = (\frac{3}{4}\langle r^2 \rangle)^{1/2}$. The extended nuclear magnetization leads to a modification of the hyperfine interaction. It was shown in Refs. [26, 27] that the corresponding contribution to the HFS constant may be factorized by “atomic” and “nuclear” factors. Following Refs. [26, 27] the corresponding factor $d_{\text{nuc}}$ depending on the nuclear spin and configuration, was introduced. Then corrections $\delta$ and $\epsilon$ for a given $Z$ and electron state can be written as [28]:

$$ \delta(R) = b_N R^{2\gamma - 1}, \quad \epsilon(R, d_{\text{nuc}}) = b_M d_{\text{nuc}} R^{2\gamma - 1}, \quad (3) $$

where $b_N$ and $b_M$ are factors, which are independent of the nuclear radius and structure.

It follows from Eqs. (1) and (3), that if we calculate HFS constant numerically for different $R$ and $d_{\text{nuc}}$, we should get following dependence on the radius in the first order in $\delta$ and $\epsilon$:

$$ A(g_I, d_{\text{nuc}}, R) = g_I A_0 (1 - (b_N + b_M d_{\text{nuc}}) R^{2\gamma - 1}). \quad (4) $$
Within the point-like magnetic dipole approximation ($d_{\text{nuc}} = 0$) the Bohr–Weisskopf correction $\epsilon$ is equal to zero, and the HFS constant can be fitted by the function:

$$A(1, 0, R) = A_0 (1 - b_N R^{2\gamma - 1}).$$  \hspace{1cm} (5)$$

On the other hand, for $d_{\text{nuc}} = 1$ one obtains:

$$A(1, 1, R) = A_0 (1 - (b_N + b_M) R^{2\gamma - 1}).$$  \hspace{1cm} (6)$$

Let us compare HFS constants for two isotopes with nuclear $g$ factors $g_f(1)$ and $g_f(2)$, slightly different nuclear radii $R(1,2) = R + r$, and nuclear factors $d_{\text{nuc}}^{(1)} = d_{\text{nuc}}^{(2)} = 0$:

$$\frac{A(g_f(1), 0, R + r)}{A(g_f(2), 0, R - r)} \approx 1 + 2r \frac{\partial A(g_f(1), 0, R) / \partial R}{A(g_f(2), 0, R)}.$$

Then the part of the HFS anomaly related to the change of the nuclear charge distribution $^1\Delta_{\text{BR}}^2(R, r)$ is:

$$^1\Delta_{\text{BR}}^2(R, r) \equiv \frac{g_f(2)^{2} A(g_f(1), 0, R + r) - 1}{g_f(1)^{2} A(g_f(2), 0, R - r)} \approx 2(2\gamma - 1)b_N R^{2\gamma - 2}r. \hspace{1cm} (7)$$

Nuclear radii of heavy isotopes are typically very close, then $r \ll R$, and anomaly (7) is therefore small. For isotopes with the same nuclear factors $d_{\text{nuc}}$ similar dependence on the nuclear radii holds for the magnetic part of the HFS anomaly $^1\Delta_{\text{BW}}^2$. However, the nuclear factors may significantly vary from one isotope to another. In this case we can neglect the radial dependence of the magnetic part of the HFS anomaly and write it as $^1\Delta_{\text{BW}}^2(R, d_{\text{nuc}}^{(1)}, d_{\text{nuc}}^{(2)})$. Thus, the HFS anomaly can be divided into two terms related to the nuclear charge and magnetization distributions:

$$^1\Delta^2(R, r, d_{\text{nuc}}^{(1)}, d_{\text{nuc}}^{(2)}) =$$

$$= ^1\Delta_{\text{BR}}^2(R, r) + ^1\Delta_{\text{BW}}^2(R, d_{\text{nuc}}^{(1)}, d_{\text{nuc}}^{(2)}). \hspace{1cm} (8)$$

In this work we calculate the magnetic hyperfine constants and HFS anomalies for low-lying states of Fr atom within the Dirac–Hartree–Fock (DHF) approximation and the DHF plus many-body perturbation theory (DHF+MBPT) method. The effects of the Breit corrections and spin-polarization of the core are also considered.

III. RESULTS AND DISCUSSION

A. HFS anomaly for H-like francium ion

Here we calculate HFS constants of the 1s, 2s, and $2p_{1/2}$ states of Fr$^{86+}$ for the different nuclear radii $R$ and compare our results with analytical expressions from Ref. [5]. Figure 1 shows the dependence of the hyperfine constant $A(1s)$ on the nuclear radius $R$. We see very good agreement with Eqs. (5, 6).

![Figure 1: Dependence of the HFS constant $A(g_f, d_{\text{nuc}}, R)$ for the ground state of H-like Fr ion on the nuclear radius. Dots and circles correspond to the calculated values. Dashed lines correspond to the fits by Eqs. (5, 6).](image_url)

TABLE I: Compilation of the fitting parameters for HFS of the H-like Fr ion: $A_0$ is HFS constant for the point-like nucleus and $g_f = 1$, $\delta$ and $\epsilon$ are the nuclear charge and magnetization distribution corrections parametrized by coefficients $b_N$ and $b_M$. Corrections $\delta$ and $\epsilon$ for $^{210}\text{Fr}^{86+}$ are calculated for $R = 7.1766$ fm [25] and $d_{\text{nuc}} = 1$.

| State | $A_0$ (THz) | $b_N \cdot 10^2/fm^{2\gamma - 1}$ | $b_M \cdot 10^2/fm^{2\gamma - 1}$ | $\delta^{(210}\text{Fr}}$ | $\epsilon^{(210}\text{Fr}, d_{\text{nuc}} = 1)$ |
|-------|-------------|-----------------|-----------------|----------------|----------------|
| 1s    | 292.0       | 4.817           | 0.1411          | 0.0208         | 0.0075         |
| 2s    | 49.5        | 5.161           | 0.1512          | 0.0223         | 0.0075         |
| $2p_{1/2}$ | 15.2 | 1.650 | 0.0483 | 0.257 | 0.0075 |

Table I summarizes our results for H-like Fr ion. For all three states we see good agreement between analytical values of $A_0$ from Eq. (2) and the values obtained from the fit of calculated HFS constants for finite nuclei. According to our calculations the ratios of the parameters $b_N$ and $b_M$ for 1s and 2s states are close to unity: $b_N(1s)/b_N(2s) = 0.933$ and $b_M(1s)/b_M(2s) = 0.933$. This is expected, as in the first approximation the wave functions of the same symmetry should be proportional to each other inside the nucleus. Similar ratios for 2s and $2p_{1/2}$ states are $b_N(2s)/b_N(2p_{1/2}) = 3.128$ and $b_M(2s)/b_M(2p_{1/2}) = 2.961$. Again, one can expect that these ratios weakly depend on the principle quantum number.
TABLE II: Compilation of the fitting parameters for HFS constants of neutral Fr atom: $A_0$ (MHz) is HFS constant for point-like nucleus and $g_I = 1$; coefficients $b_N$ and $b_M$ (fm$^{-1}$–$^{-2}$).

|               | $A_0$     | $\frac{\gamma_{7s/2}}{b_N \cdot 10^2}$ | $b_M \cdot 10^2$ | $\frac{\gamma_{7p_{1/2}}}{b_N \cdot 10^2}$ | $b_M \cdot 10^2$ | $\frac{\gamma_{7p_{1/2}}}{b_N \cdot 10^2}$ | $b_M \cdot 10^2$ |
|---------------|-----------|----------------------------------------|------------------|----------------------------------------|------------------|----------------------------------------|------------------|
| DHF           | 7894.710  | 5.3030                                 | 0.7646           | 746.580                                | 1.8241           | 0.2842                                 | 55.524           |
| DHF+Br        | 7887.813  | 5.2989                                 | 0.7642           | 740.345                                | 1.8204           | 0.2837                                 | 55.151           |
| DHF+MBPT      | 10602.174 | 4.7502                                 | 0.8584           | 1130.031                               | 1.5661           | 0.3223                                 | 77.870           |
| DHF+MBPT+Br   | 10581.950 | 4.7013                                 | 0.8566           | 1120.865                               | 1.5461           | 0.3160                                 | 74.437           |
| DHF+RPA       | 8684.144  | 5.1092                                 | 0.8008           | 865.034                                | 1.6205           | 0.2606                                 | 94.984           |
| DHF+Br+RPA    | 8682.028  | 5.1020                                 | 0.8007           | 861.718                                | 1.6223           | 0.2627                                 | 94.721           |
| DHF+MBPT+RPA  | 11518.484 | 4.6067                                 | 0.8844           | 1308.388                               | 1.4018           | 0.2929                                 | 132.482          |
| DHF+MBPT+Br+RPA| 11507.415 | 4.5516                                 | 0.8738           | 1300.950                               | 1.3879           | 0.2891                                 | 131.988          |

B. HFS anomaly of neutral francium atom

The ground configuration of the neutral francium atom is [Rn]$^7$1s. If we treat francium as an one-electron system with the frozen core, we can do calculation using Dirac–Hartree–Fock (DHF) method. In this case the dependence of the HFS constants on the nuclear radius is similar to the one-electron ion.

In the DHF approximation the HFS constant $A(7p_{3/2}) = 0.56$ GHz is very small and practically does not depend on $R$ (see Table II). At the same time, the HFS constants $A(7s)$ and $A(7p_{1/2})$ are well described by Eqs. (5) and (6). According to our calculations, the ratios of coefficients $b_N$ and $b_M$ for $s$ and $p_{1/2}$ waves are close to the respective ratios in H-like ion $\frac{b_N(1s)}{b_N(7s)} = 0.908$ and $\frac{b_M(1s)}{b_M(7s)} = 0.929$. This result is compatible with assertion that the hyperfine anomaly measured for the $s$ states in Rb is weakly dependent on the principal quantum number [29]. Ratios of the parameters $b_N$ and $b_M$ for 7s and 7p$_{1/2}$ are: $\frac{b_N(7s)}{b_N(7p_{1/2})} = 2.907$ and $\frac{b_M(7s)}{b_M(7p_{1/2})} = 2.690$, while for the H-like ion we had 3.128 and 2.961 respectively.

Situation changes when we include spin-polarization of the core via random phase approximation (RPA) corrections. These corrections lead to effective mixing of different partial waves, thus $A(7p_{3/2})$ constant acquires contributions from the $s$ and $p_{1/2}$ waves. Due to the RPA corrections the value of the constant $A(7p_{3/2})$ is significantly changed. At the same time this constant becomes sensitive to the distributions inside the nucleus. To account for that, we can use Eq. (3) with the same $\gamma$ as for $s$ and $p_{1/2}$ states. The RPA corrections for the 7s and 7p$_{1/2}$ states are smaller than for 7p$_{3/2}$, but they are also significant. Due to the RPA corrections the ratios of the parameters $b_N$ and $b_M$ for 7s and 7p$_{1/2}$ states change by $\sim 15\%$: $\frac{b_N(7s)}{b_N(7p_{1/2})} = 3.153$ and $\frac{b_M(7s)}{b_M(7p_{1/2})} = 3.073$.

Core-valence and core-core electron correlations were taken into consideration within DHF+MBPT method [6]. Electron correlation corrections significantly change $A_0$ values. The parameters $b_N$ and $b_M$ also change, but ratios of these parameters for the 7s and 7p$_{1/2}$ states remain stable. Without RPA corrections these ratios are equal to: $\frac{b_N(7s)}{b_N(7p_{1/2})} = 3.033$ and $\frac{b_M(7s)}{b_M(7p_{1/2})} = 2.663$. Final ratios were obtained in terms of DHF+MBPT approximation with RPA and Breit corrections:

$$\frac{b_N(7s)}{b_N(7p_{1/2})} = 3.280,$$  
$$\frac{b_M(7s)}{b_M(7p_{1/2})} = 3.023.$$  

According to Mårtensson-Pendrill [21] the ratio of $b_N$ parameters obtained by scaling the Breit-Rosenthal corrections for Tl is equal to 3.2 in good agreement with our result. For the ratio of the $b_M$ parameters the value of 3.0 was used in [21] also in agreement with our results.

Information about parameters $b_N$ and $b_M$ can be extracted from the experimentally measured ratio of HFS constants $\rho = A(7s)/A(7p_{1/2})$. This ratio can be written as a function of nuclear radius and nuclear factor:

$$1 - \frac{\rho(R,d_{\text{nuc}})}{\rho_0} \approx \frac{(b_N(7s) - b_N(7p_{1/2}))(R^{2\gamma-1} + d_{\text{nuc}}(b_M(7s) - b_M(7p_{1/2}))R^{2\gamma-1}),}$$

where $\rho_0 = A_0(7s)/A_0(7p_{1/2})$. Several experimentally measured values of $\rho$ for odd-odd and even-odd isotopes [13] and corresponding fits by Eq. (10) are presented in Fig. 2.

Even-odd Fr isotopes with neutron number $N \leq 126 (A \leq 213)$ have spin $I = 9/2$ and nearly constant magnetic moments. When going from $A = 213$ to $A = 207$, the magnetic moment $\mu(A,9/2)$ changes only by 3%. Their ground states are regarded as a pure shell-model $h_9/2$ states, therefore one can assume that $d_{\text{nuc}}$ factor is also constant within 3% limits. Factor $d_{\text{nuc}}$ was calculated by the simple shell-model formula [21]: $d_{\text{nuc}} = 0.3$ for $A = 207 - 213$. Then the one-parameter fit with $\rho_0$ as free parameter gives us the following relation: $\rho = 8.456 (1 - 0.033 R^{2\gamma-1})$, where we used our final results for $b_N$ and $b_M$ from Table II, or $\rho = 8.404 (1 - 0.031 R^{2\gamma-1})$ within two-parameter fit. Comparing these two results we can estimate the error bars for fitting parameters to be: $\rho_0 = 8.43(3)$ and $b_N + d_{\text{nuc}} b_M = 0.032(1)$. Note that the theoretical value of $\rho_0$ obtained within DHF+MBPT+Br+RPA method is equal to 8.85, which is 5% larger. Taking into account the
possible change of the $d_{\text{nuc}}$ factor (3%) and its possible deviation from the shell-model value, the correspondence between fitted and calculated $\rho$ values should be regarded as satisfactorily.

We used formulas from Ref. [26] to calculate $d_{\text{nuc}}$ factor for the odd-odd Fr isotopes. Spins and configurations for odd-odd Fr isotopes with $A = 206 - 212$ are different ($I = 5, 6, 7, 3$). Correspondingly, $d_{\text{nuc}}$ factor is different for different isotopes. However, it can be shown that for all these cases $d_{\text{nuc}} = 0.5(1)$. To check the general applicability of our approach we used the same nuclear factor for the all considered odd-odd Fr isotopes. We fix $\rho_0$ obtained for even-odd Fr isotopes and fit nuclear factor for odd-odd ones which gives us $d_{\text{nuc}} = 0.49$ in agreement with the shell-model estimation. The deviation of the experimental $\rho$ values for $^{206m}\text{Fr}$ and $^{206}\text{Fr}$ from the fit line (see Fig. 2) is obviously connected with the structural changes in these nuclei resulting in the changes of the $d_{\text{nuc}}$ factor (see discussion in Ref. [13]). For $^{221}\text{Fr}$ the fit gives $d_{\text{nuc}} = 0.05$.

The accuracy reached in our calculations of HFS constants for neutral Fr can be estimated in comparison with available experimental and theoretical data presented in Table III. Due to Bohr–Weisskopf correction calculated

\[
A(7s) = A(7p_{1/2})
\]

for even-odd and odd-odd Fr isotopes [13]. The nuclear radii $R$ are taken from Ref. [25]. Lines are the one-parameter fits by Eq. (10), dashed line corresponds to two-parameter fit. For even-odd isotopes we use $d_{\text{nuc}} = 0.3$ [21] and parameters $\rho_0$ (one-parameter fit), or $\rho_0$ and $(b_N + d_{\text{nuc}} b_M)$ (two-parameter fit). Then for odd-odd isotopes we fix the obtained by the one-parameter fit $\rho_0$ value and fit nuclear factor $d_{\text{nuc}}$, with the result $d_{\text{nuc}} = 0.49$. For $^{221}\text{Fr}$ the fit gives $d_{\text{nuc}} = 0.05$.

The accuracy reached in our calculations of HFS constants for neutral Fr can be estimated in comparison with available experimental and theoretical data presented in Table III. Due to Bohr–Weisskopf correction calculated

\[
A(7s) = A(7p_{1/2})
\]

for even-odd and odd-odd Fr isotopes [13]. The nuclear radii $R$ are taken from Ref. [25]. Lines are the one-parameter fits by Eq. (10), dashed line corresponds to two-parameter fit. For even-odd isotopes we use $d_{\text{nuc}} = 0.3$ [21] and parameters $\rho_0$ (one-parameter fit), or $\rho_0$ and $(b_N + d_{\text{nuc}} b_M)$ (two-parameter fit). Then for odd-odd isotopes we fix the obtained by the one-parameter fit $\rho_0$ value and fit nuclear factor $d_{\text{nuc}}$, with the result $d_{\text{nuc}} = 0.49$. For $^{221}\text{Fr}$ the fit gives $d_{\text{nuc}} = 0.05$.

We used formulas from Ref. [26] to calculate $d_{\text{nuc}}$ factor for the odd-odd Fr isotopes. Spins and configurations for odd-odd Fr isotopes with $A = 206 - 212$ are different ($I = 5, 6, 7, 3$). Correspondingly, $d_{\text{nuc}}$ factor is different for different isotopes. However, it can be shown that for all these cases $d_{\text{nuc}} = 0.5(1)$. To check the general applicability of our approach we used the same nuclear factor for the all considered odd-odd Fr isotopes. We fix $\rho_0$ obtained for even-odd Fr isotopes and fit nuclear factor for odd-odd ones which gives us $d_{\text{nuc}} = 0.49$ in agreement with the shell-model estimation. The deviation of the experimental $\rho$ values for $^{206m}\text{Fr}$ and $^{206}\text{Fr}$ from the fit line (see Fig. 2) is obviously connected with the structural changes in these nuclei resulting in the changes of the $d_{\text{nuc}}$ factor (see discussion in Ref. [13]). For $^{221}\text{Fr}$ the fit gives $d_{\text{nuc}} = 0.05$. This result can be of a particular interest for nuclear physics and more detailed analysis will be presented in the forthcoming paper.

The accuracy reached in our calculations of HFS constants for neutral Fr can be estimated in comparison with available experimental and theoretical data presented in Table III. Due to Bohr–Weisskopf correction calculated

\[
A(7s) = A(7p_{1/2})
\]

...
tain more accurate values for the nuclear $g$ factors of the Fr isotopes from the ratios of the HFS constants. The reliability of the applied method enables one to determine the nuclear factor $d_{\text{nuc}}$ which gives important nuclear-structure information and may be compared with the theoretical predictions.

Acknowledgments

Thanks are due to Vladimir Shabaev, Ilya Tupitsyn and Leonid Skripnikov for helpful discussions. The work was supported by the Foundation for the advancement of theoretical physics “BASIS” (grant # 17-11-136).

[1] A. Andreyev, M. Huyse, P. Van Duppen, L. Weissman, D. Ackermann, J. Gerl, F. Hessberger, S. Hofmann, A. Kleinbühl, G. Münzenberg, et al., Nature 405, 430 (2000).
[2] A. Bohr and V. F. Weisskopf, Phys. Rev. 77, 94 (1950).
[3] J. E. Rosenthal and G. Breit, Phys. Rev. 41, 459 (1932).
[4] M. Crawford and A. Schawlow, Phys. Rev. 76, 1310 (1949).
[5] V. M. Shabaev, J. Phys. B 27, 5825 (1994).
[6] M. Kozlov, P. Porosev, M. Safronova, and I. Tupitsyn, Comput. Phys. Commun. 195, 199 (2015), ISSN 0010-4655.
[7] V. F. Bratsev, G. B. Deyneka, and I. I. Tupitsyn, Bull. Acad. Sci. USSR, Phys. Ser. 41, 173 (1977).
[8] V. A. Dzuba, V. V. Flambaum, M. G. Kozlov, and S. G. Porosev, Sov. Phys.–JETP 87, 885 (1998).
[9] M. G. Kozlov, S. G. Porosev, and W. R. Johnson, Phys. Rev. A 64, 052107 (2001), arXiv: physics/0105090.
[10] S. G. Porosev, Y. G. Rakhлина, and M. G. Kozlov, J. Phys. B 32, 1113 (1999), arXiv: physics/9810011.
[11] N. K. Kjøller, S. G. Porosev, P. G. Westergaard, N. Andersen, and J. W. Thomsen, Phys. Rev. A 91, 032515 (2015).
[12] S. G. Porosev, M. G. Kozlov, M. S. Safronova, and I. I. Tupitsyn, Phys. Rev. A 93, 012501 (2016), 1510.06679.
[13] J. Zhang, M. Tandecki, R. Collister, S. Aubin, J. Behr, E. Gomez, G. Gwinner, L. Orozco, M. Pearson, G. Sprouse, et al., Phys. Rev. Lett. 115, 042501 (2015).
[14] J. Grossman, L. Orozco, M. Pearson, J. Simsarian, G. Sprouse, and W. Zhao, Phys. Rev. Lett. 83, 935 (1999).
[15] I. Budinčević, J. Billowes, M. Bissell, T. E. Cocolios, R. De Groote, S. De Schepper, V. N. Fedosseev, K. T. Flanagan, S. Franchoo, R. G. Ruiz, et al., Phys. Rev. C 90, 014317 (2014).
[16] R. De Groote, I. Budinčević, J. Billowes, M. Bissell, T. E. Cocolios, G. J. Farooq-Smith, V. Fedosseev, K. Flanagan, S. Franchoo, R. G. Ruiz, et al., Phys. Rev. Lett. 115, 132501 (2015).
[17] K. Flanagan, K. Lynch, J. Billowes, M. Bissell, I. Budinčević, T. E. Cocolios, R. De Groote, S. De Schepper, V. Fedosseev, S. Franchoo, et al., Phys. Rev. Lett. 111, 212501 (2013).
[18] Z.-T. Lu, K. Corwin, K. Vogel, C. Wieman, T. Dinneen, J. Maddì, and H. Gould, Phys. Rev. Lett. 79, 994 (1997).
[19] E. Gomez, S. Aubin, L. Orozco, G. Sprouse, E. Iskrenova-Tchoukova, and M. Safronova, Phys. Rev. Lett. 100, 172502 (2008).
[20] B. Sahoo, D. Nandy, B. Das, and Y. Sakemi, Phys. Rev. A 91, 042507 (2015).
[21] A. M. Mårtensson-Pendrill, Hyperfine Interact. 127, 41 (2000).
[22] V. Dzuba, V. Flambaum, and O. Sushkov, J. Phys. B 17, 1953 (1984).
[23] V. Dzuba, W. Johnson, and M. Safronova, Phys. Rev. A 72, 022503 (2005).
[24] M. Kalita, J. Behr, A. Gorelov, M. Pearson, A. Dehart, G. Gwinner, M. Kossin, S. Aubin, E. Gomez, L. A. Orozco, et al., in APS Division of Atomic, Molecular and Optical Physics Meeting Abstracts (2017).
[25] A. Voss, F. Buchinger, B. Cheal, J. Crawford, J. Dilling, M. Kortelainen, A. Kwiatkowski, A. Leary, C. Levy, F. Mooshammer, et al., Phys. Rev. C 91, 044307 (2015).
[26] S. Büttenbach, Hyperfine Interact. 20, 1 (1984).
[27] A.-M. Mårtessson-Pendrill, Phys. Rev. Lett. 74, 2184 (1995).
[28] E. Konovalova, M. Kozlov, Y. Demidov, and A. Barzakh, Rad. Applic. 2, 181 (2017), ISSN 2466-4294, arXiv:1703.10048.
[29] A. P. Galván, Y. Zhao, L. Orozco, E. Gómez, A. Lange, F. Baumer, and G. Sprouse, Phys. Lett. B 655, 114 (2007).
[30] A. Coc, C. Thibault, F. Touchard, H. Dương, P. Jun-\ncar, S. Liberman, J. Pinard, J. Lermé, J. Vialle, S. Büttenbach, et al., Phys. Lett. B 163, 66 (1985).