Maelstrom: A Python package for identifying companions to pulsating stars from their light travel time variations

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Summary

Most stars are members of binary systems. Traditional observations of these systems rely on eclipses, where the secondary body occludes light from the primary star and vice versa. This leads to a strong bias on the orbital parameters – to observe these eclipses the system must have a high inclination with respect to Earth, and the orbital period must be sufficiently short to be observed within the time-span of the data. Other methods for detecting binarity suffer from similar constraints: radial velocity measurements are useful mostly for orbital periods less than tens of days, whilst long-baseline interferometry is suited only to relatively nearby and bright stars.

However, in some binary systems, one or both of the components are pulsating variable stars. Some variable stars make excellent clocks, possessing stable pulsations that do not vary significantly throughout their orbit. One such type are the \textit{δ} Scuti variables, a class of A/F type stars lying along the classical instability strip. As the pulsating star is tugged around by the gravity of its companion the time taken for its light to reach Earth varies, which periodically modulates the pulsation phases. A map of the binary orbit can then be constructed by observing the phase of the pulsations over time, which can be converted into time delays – a measure of the relative time taken for the light to reach Earth (as shown in Fig. 1). This method, phase modulation, is uniquely suited to observing intermediate period binaries (Murphy & Shibahashi, 2015).

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Figure 1: Phase modulation of the binary system, KIC 4471379, which is composed of two δ Scuti pulsating stars. The top panel shows the observed flux of the system (the light curve). The middle panel shows the amplitude spectrum of the light curve, which is the superposition of each star’s pulsation spectrum. The bottom panel shows the time delay derived from the pulsations. Blue and orange points correspond to the first and second stars in the system respectively. As they orbit each other, the time taken for the light to reach us changes over time. Since both stars are pulsating, we can identify which pulsations belong to which star.

Previous work has analysed these variations by splitting the light curve into equally sized subdivisions and calculating the time delay in each division (Murphy et al., 2018). Whilst useful for longer period binaries (>20 d), shorter period and eccentric binaries suffer from a smearing of the orbital signal due to large variations when the stars are at their closest approach (Murphy, Shibahashi, & Bedding, 2016). Since the phase uncertainty is inversely proportional to the size of the subdivision, shorter period binaries can not be accurately determined. We have developed a novel technique for mitigating this problem by forward modelling the time-delay effect directly onto the light curve, which allows for every data point in the light curve to be modelled simultaneously, removing the need to choose a subdivision size. This technique is explained in detail in our corresponding paper.

We have developed a Python package, Maelstrom, which implements this technique. Maelstrom is written using the popular Bayesian inference framework, PyMC3, allowing for the use of gradient based samplers such as No-U-Turn (Hoffman & Gelman, 2011) and Hamiltonian...
Monte Carlo (Duane, Kennedy, Pendleton, & Roweth, 1987). **Maelstrom** features a series of pre-defined PyMC3 models for analysing binary motion within stellar pulsations. These are powered by the `orbit` module, which returns a light curve given the frequencies of pulsation and the classical orbital parameters. Using this light curve, one can compare with photometric data from the *Kepler* and *TESS* space missions to fit for binary motion. For more complex systems outside the pre-defined scope, the `orbit` module can be used to construct custom models with different priors, and combine them with other PyMC3 codes, such as `exoplanet` (Dan Foreman-Mackey et al., 2019). To the best of our knowledge, **Maelstrom** is currently the only available open code for analysing time delay signals.

The documentation of **Maelstrom** consists of pages describing the various available functions, as well as tutorial notebooks.

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