Theory of Orbital Kondo Effect with Assisted Hopping in Strongly Correlated Electron Systems: Parquet Equations, Superconductivity and Mass Enhancement

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Abstract

Orbital Kondo effect is treated in a model, where additional to the conduction band there are localized orbitals close to the Fermi energy. If the hopping between the conduction band and the localized heavy orbitals depends on the occupation of the atomic orbitals in the conduction band then orbital Kondo correlation occurs. The noncommutative nature of the coupling required for
the Kondo effect is formally due to the form factors associated with the assisted hopping which in the momentum representation depends on the momenta of the conduction electrons involved. The leading logarithmic vertex corrections are due to the local Coulomb interaction between the electrons on the heavy orbital and in the conduction band. The renormalized vertex functions are obtained as a solution of a closed set of differential equations and they show power behavior. The amplitude of large renormalization is determined by an infrared cutoff due to finite energy and dispersion of the heavy particles. The enhanced assisted hopping rate results in mass enhancement and attractive interaction in the conduction band. The superconductivity transition temperature calculated is largest for intermediate mass enhancement, $m^*/m \approx 2 - 3$. For larger mass enhancement the small one particle weight $(Z)$ in the Green’s function reduces the transition temperature which may be characteristic for other models as well. The theory is developed for different one–dimensional and square lattice models, but the applicability is not limited to them. In the one–dimensional case charge– and spin–density susceptibilities are also discussed. Good candidates for the heavy orbital are $f$–bands in the heavy fermionic systems and non–bonding oxygen orbitals in high temperature superconductors and different flat bands in the quasi–one dimensional organic conductors.
I. INTRODUCTION

Recently the different phase transitions in strongly correlated electronic systems has became the most intensively discussed issue in the theory of solids.

There is a common feature in the theories of these phase transitions, namely logarithmic corrections characteristic for infrared divergences occur in the perturbative expansion. That genuine character exists even for the phase transitions in electronic systems with the logarithmic expression $\ln(x/D)$, where $x = \max(\omega, T)$ is the largest of the energy variables $\omega$ and the temperature $T$, furthermore, the high energy cutoff $D$ is the bandwidth of the conduction electrons. In particular cases an additional infrared, low energy cutoff appears which is an inherent feature of some models. That cutoff smears out the infrared divergences mentioned earlier.

Such logarithmic corrections occur in the theory of a one-dimensional electron gas and also e.g. in two–dimensional. In the first case that is due to the simple Fermi surface, while in the second case either the Fermi surface is of one–dimensional character (nested Fermi surface) or that is originated by the corners of the Fermi surface for a nearly half–filled electron band. These special properties are required to get logarithmic terms in the electron hole channel (zero sound channel), on the other hand, it is well known from the theory of the superconductivity, that the electron–electron channel with total zero momentum (Cooper channel) is always divergent. The appearance of the logarithmic contributions are generally controlled by the conservation of momentum. The importance of that conservation is essentially reduced in those two–band models of arbitrary dimension, where one of the bands is almost dispersionless thus the heavy particle can absorb an arbitrary momentum with almost the same energy transfer. If the heavy band is close to the Fermi energy then the diagrams containing only one light electron (hole) in the intermediate state contribute by logarithmic terms arising from the integration with respect to the energy the light electron (hole). The finite, but small energy and dispersion of the heavy band introduce an inherent smearing of the logarithmic terms, thus it represents a small infrared cutoff mentioned
earlier.

Furthermore, in order to get relevant leading order logarithmic corrections in the absence of special features of the Fermi surface, it is not enough to have an almost dispersionless branch in the excitation spectrum like localized spin excitations, two–level systems etc., as for the most simple models the leading two logarithmic vertex corrections due to intermediate states with a single light electron and hole accompanying the heavy electron cancel each other. Such cancellation occur in the theory of the X–ray absorption by a deep electron level, which phenomena was worked out by Mahan, Nozières and de Dominicis. The logarithmic character of single loop approximation, however, sustains in those cases where the couplings between the light and heavy electrons show non-trivial structure exhibiting noncommutative behavior which may be due to the

(i) spin dependence (Kondo effect) or dependence on the atomic orbital indices of a single atomic site,

(ii) dependence on the momenta of the light electrons, which might appear as a structure factor involving two sites in the coupling

The present paper is devoted to the second case, where the structure factor is associated with the electron assisted hopping (correlated hopping) in a localized orbital picture. In those models the electron hops between two different sites, but the hopping rate depends on the occupation of an other site by electrons. The hopping considered may be between two localized heavy electron states or between a heavy and light electrons mixing the two bands. In solids the mixing cases have usually larger amplitudes as the hopping between two heavy orbitals of smaller radius are essentially weaker.

The general form of the electron assisted interaction in the real space contains the following combination of creation and annihilation operators

\[ a_{\alpha,n_1}^\dagger a_{\beta,n_2} a_{\gamma,n}^\dagger a_{\gamma,n} \]

where \( n_1, n_2 \) are sites involved by the hopping which is affected by the occupation of site \( n \) and \( \alpha, \beta, \gamma \) are band indices. In the following only such models are considered where either
That interaction in the Bloch wave form is associated with a form factors just like in the tight binding approximation.

There are two simple sources for such assisted hopping processes:

(i) the hopping between two sites must depend on the occupation of these sites by other electrons as additional electron or hole on the site modifies the size of the orbital in the real space due to the Coulomb interaction, thus the hopping matrix element is changed as well.

(ii) in the site representation of the two particle Coulomb interaction which contains two creation and two annihilation operators there are always such terms, in which one site appears three times and another site only once. These terms represent the off–diagonal Coulomb interaction.

The form factor appearing in the Fourier transform of the operator product given by the expression (1.1) for e.g. \( n_1 = n \) is

\[
e^{i(k_2+k_4-k_1-k_3)R_1} e^{ik_2\delta R} a_{\alpha,k_1} a_{\beta,k_2} a_{\gamma,k_3} a_{\gamma,k_4}
\]

where the locations of sites \( n_1 \) and \( n_2 \) are \( R_1 \) and \( R_2 \), furthermore \( \delta R = R_2 - R_1 \). The first factor in this expression drops out as it ensures the momentum conservation which is of limited relevance where one of the bands is flat. The important form factor which is responsible for the noncommutative nature of the interaction is the factor \( e^{ik_2\delta R} \). Considering the light electron assisted mixing term between the heavy and light bands one of \( \alpha \) and \( \beta \) belongs to the heavy band and all the others to the light band.

The large renormalizations of the different quantities as the strength of the interaction, the mass of the light electrons and the electron–electron interaction relevant for superconductivity are due to the Coulomb interaction between the electrons in the light and heavy band which screens the charge of the heavy electrons by the light ones. The structure of these interaction is not crucial, thus it can be taken as a local on–site interaction with strength \( U \), except for those cases where the model itself requires more complicated struc-
tures. Such interactions itself without assisted hopping does not lead to leading logarithmic corrections. The theories belonging to that general class exhibit the following common features:

(i) The vertices are renormalized and they are power functions of the largest of the variables $\omega/D$, $T/D$ and $E_0/D$ with a low energy infrared cutoff $E_0$ discussed earlier. In the models where there is only a structureless screening interaction $U$ between the heavy and light electrons the leading term of the exponent is at least quadratic in the interaction. In the case of electron assisted band mixing treated in the present work the exponent is linear in $U$ as that mixing interaction is coupled to three light electrons, in contrary to the renormalization of the screening interaction. The coefficient of $U$ is different from zero even if the structure factor is absent in the interaction, but its presence can enhance it by a factor of two. Such enhancement can be very important.

(ii) The light electron mass corrections exist always due to the assisted mixing, but the large vertex corrections can lead to a large enhancement which may reach several orders of magnitude (heavy fermionic behavior).

(iii) The electron–electron interaction induced by electron assisted mixing, where the heavy electron occurs in the intermediate state, is different from zero in the presence of the form factor. The vertex corrections are important and the interaction may depend strongly on the momenta of the electrons. Without form factors such interactions are not generated.

(iv) Both the induced electron–electron interaction and the mass enhancement increase with the strength of coupling in the weak coupling limit. For stronger coupling the density of the light electron increases, but the single particle weights $Z$ in their spectral functions are drastically reduced. In the induced electron–electron interaction the square of that weights, $Z^2$ occurs, which dominates over the increase in the electron density which is proportional to $Z^{-1}$. Thus in these models the strength of the superconductivity is the strongest for moderate mass enhancement 2–3 and decreasing very fast for larger mass enhancement ($Z \ll 1$). The largest available dimensionless attractive coupling is close to unity. This
feature must be quite general for models where the electron–electron interaction is induced and competes with the mass enhancement.

In all of the behaviors listed above the energy and the dispersion of the heavy band play determining role by limiting the enhancement of the vertex (infrared cutoff).

It is interesting to note, that in most of the cases of logarithmic problems new couplings are generated by solving the parquet or renormalization group equations for the vertex functions. That is the case for the two dimensional electron gas as well. In the present case the finite number of the generated form factors reduces the problem to a closed set of differential equations, and their solution will be presented in analytical form.

The models where the present theory can be relevant are those, where there is a heavy flat band. Those bands can be formed by the atomic orbitals of very small size like $4f$ electrons in heavy fermionic systems and the mixing is between $s$– and $f$–electrons. Another possibility is the case of non–bonding orbitals in complicated crystal structure, where these orbitals are only weakly hybridized with the conducting electrons, e.g. non–bonding $\pi$–orbital of oxygen in the high temperature superconductors or some appropriate orbitals in the quasi one–dimensional conducting molecular crystals where there might be several flat bands. The present paper is not devoted to study particular special cases, but to provide several methods which can be applied to different concrete cases.

The paper is organized as follows: In chapter II. we are presenting the Hamiltonian of a general model with features described above. In chapter III. we are calculating the vertex corrections for the general model. In chapters IV. and V. the calculations of the self–energy and the electron–electron interaction are presented and in the next chapter (VI.) the superconducting transition temperature is determined. That discussion of the relation of the mass enhancement to the generated electron–electron interaction contains quite general considerations which are valid much beyond the model treated here. The formalism developed in the previous chapters is applied to special cases: to the one dimensional electron gas (chapter VII., see also Ref. [11]) and to two models which has a resemblance to the $CuO_2$ plane of the high temperature superconductors (chapter VIII.). In the latter case two sim-
plified models are presented where the solution of vertex equations are essentially different. Finally, in chapter IX. we give a brief comprehensive conclusion with some hint concerning the applicability of the model. The chapters VII. and VIII. can be read independently.

II. THE MODEL

The model proposed consists of two electron bands in the tight binding approximation: the broad light \((l)\) band with bandwidth \(D\) and the narrow heavy \((h)\) one with energy \(\varepsilon_h\) \((|\varepsilon_h| \ll D)\) measured from the Fermi energy \(\varepsilon_F\). The \(l\)-orbitals are on each atom. The heavy orbitals are located at some of the atoms in the cell and they overlap only with \(l\)-orbitals of the nearest atoms. In general theory they may be more than one heavy orbital at one site, which corresponds to index \(\gamma\). In Fig. 1 we are showing such a model in one dimensions, where the light orbitals are \(s\) orbitals and and the heavy orbitals are of \(p\) and \(d\) type. In Fig. 2 as an example of a two dimensional model, the \(CuO_2\) plane and the apical oxygens below the \(Cu\) are shown. The heavy orbitals are associated with the \(p_x\) and \(p_y\) orbitals of the apical oxygen.

The hopping Hamiltonian can be given in the terms of the annihilation operators \(c_{n\delta\sigma}\), and \(h_{n\gamma\sigma}\), where \(n\) stands for the position of the atom with heavy orbitals, the spin is \(\sigma = \pm 1\). The \((n, \delta)\) labels those light atoms, which are the first neighbors of the site \(n\) at \((n+\delta/2)a\), or at site \(n\) for \(\delta = 0\), where \(a\) is the lattice constant. The labels of the \(c\)-operators are not defined in a unique way. For example in the case of square lattice both \((n, \delta)\) and \((n+\delta, -\delta)\) labels the same atom on the positions \((n+\delta/2)a\). Thus the one-particle Hamiltonian \(H_0\) is

\[
H_0 = \sum_{n,\sigma}(\varepsilon_n + \varepsilon_F)n_{n\delta\sigma} + (\varepsilon_h + \varepsilon_F)\sum_{n,\sigma,\gamma}h_{n\gamma\sigma}^\dagger h_{n\gamma\sigma} + \sum_{n,\sigma,\delta,\gamma}t_{\delta\gamma}^h h_{n\gamma\sigma}^\dagger c_{n\delta\sigma} + \sum_{n,\sigma,\delta,\delta'}t_{\delta\delta'} c_{n\delta\sigma}^\dagger c_{n\delta'\sigma} + \text{h.c.},
\]

(2.1)

where \(\varepsilon_n\) is an energy splitting, \(t_{\delta\delta'}\) and \(t_{\delta\gamma}^h\) are hopping parameters \((t_{\delta\gamma}^h \ll t_{\delta\delta'})\) and the definition \(n_{n\delta\sigma} = c_{n\delta\sigma}^\dagger c_{n\delta\sigma}\) is used. \(t_{h,\delta}\) must hold because of the different symmetry of the orbitals. The hopping \(t_{\delta\delta'}\) in the light band may include the site \(\delta = 0\) as well. The
direct weak hoppings between the light and heavy bands will be taken into account as a broadening of the $h$-orbitals ($\Gamma$), which may serve as a low energy cutoff in the logarithmic integrals. The part of Eq. (2.1) due to the $l$-orbitals can be diagonalized and only the band crossing the Fermi energy is kept. In the new band the annihilation operator is denoted by $d_{\mathbf{k}\sigma}$ where $\mathbf{k}$ is the momentum. The contributions to the Fourier transforms of the $c_{\mathbf{k}\delta\sigma}$ of the $d$-band crossing the Fermi surface are

$$c_{\mathbf{k}\delta\sigma} = \phi_\delta(\mathbf{k}) d_{\mathbf{k}\sigma}, \quad (2.2)$$

where $\phi_\delta(\mathbf{k})$ are the amplitudes of the electron on the orbital denoted by $\delta$ in the state $\mathbf{k}$ and the contributions of the other bands are dropped.

The interaction Hamiltonian consists of two parts, $H_{\text{int}} = H_U + H_i$. The Hamiltonian describing the Coulomb repulsion is given by

$$H_U = \sum_{n,\delta} \sum_{\gamma,\sigma,\sigma'} U^\gamma_{\delta} h_{n\gamma\sigma}^\dagger c_{n\delta\sigma'}^\dagger c_{n\delta\sigma}' h_{n\gamma\sigma} + \text{h.c.}, \quad (2.3)$$

where $U^\gamma_{\delta}$ is the Coulomb integral between the heavy orbital $\gamma$ and light orbitals at site $\delta$ (see e.g. Fig. 1c). The Hamiltonian due to assisted hopping is

$$H_i = \sum_{n,\delta} \sum_{\gamma,\sigma} \tilde{t}^\gamma_{\delta} h_{n\delta-\sigma}^\dagger c_{n\delta\sigma}^\dagger c_{n\delta-\sigma} h_{n\gamma\sigma} + \text{h.c.} \quad (2.4)$$

where $\tilde{t}^\gamma_{\delta}$ is the amplitude of the assisted hopping between the heavy orbital $\gamma$ and light orbitals at site $\delta$. The possible role of the on-site correlations between the heavy electrons are taken into account in the renormalized parameter $\varepsilon_h$ which can be used if only the single excitation of smallest energy is considered at each $h$-site and the other excitations shifted in energy due to the large on–site Coulomb interaction are dropped.

The form factor responsible for the orbital Kondo-effect arises from the Fourier transform of the Hamiltonian $H_i$ describing the assisted hopping,

$$H_i = \frac{1}{N^2} \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} \sum_{\delta,\gamma,\sigma} \tilde{t}^\gamma_{\delta} e^{-i(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3)\delta a/2} \phi_\delta^*(\mathbf{k}_1) \phi_\delta^*(\mathbf{k}_2) \phi_\delta(\mathbf{k}_3)$$

$$\times d_{\mathbf{k}_1\sigma}^\dagger d_{\mathbf{k}_2-\sigma}^\dagger d_{\mathbf{k}_3-\sigma} h_{\gamma\mathbf{k}_3,\sigma} + \text{h.c.}, \quad (2.5)$$
where $k_h = k_1 + k_2 - k_3$, the notation given by Eq. (2.2) is introduced and $N$ is the number of unit cells in the sample. This form factors prevents the cancellation of the logarithmically divergent loops which usually cancel if the interaction terms are diagonal. In Fig. 3 the diagrammatic representation of the bare interaction is shown.

Along the calculation several approximations will be applied. The momentum integrals are split to an energy integral perpendicular to the Fermi surface and a momentum integral on the Fermi surface. Thus e.g. in $d$-dimensions

$$
\frac{1}{N} \sum_k = a^d \int \frac{d^d k}{(2\pi)^d} \rightarrow \frac{a^d}{(2\pi)^d} \int_S \frac{dS_\kappa}{v_F(\kappa)} \int_{-D}^D d\varepsilon ,
$$

where $dS_\kappa$ is a surface element of the Fermi surface, $\kappa$ is the unit vector directed from the center of the Brillouin zone to the surface element $dS$, and $k_F(\kappa)$ is the Fermi wave vector in the direction $\kappa$. Furthermore, the form factor appearing in Eq. (2.5) and the wave functions $\phi_\delta(k)$ are replaced by their values taken at the Fermi surface like

$$
e^{-i(k_1 + k_2 - k_3)\delta a/2} \rightarrow e^{-i[k_F(\kappa_1) + k_F(\kappa_2) - k_F(\kappa_3)]\delta a/2} ,
$$

and

$$
\phi_\delta(k) \rightarrow \phi_\delta(k)\bigg|_{k=k_F(\kappa)} = \phi_\delta(\kappa) .
$$

The Fermi velocity $v_F(\kappa)$ is defined as

$$
\left| \frac{\partial \varepsilon}{\partial k} \right|_{k=k_F(\kappa)} = v_F(\kappa) .
$$

III. VERTEX CORRECTIONS

The renormalization scheme consists of two steps due to:

(i) Coulomb interaction $H_U$ which in the leading logarithmic approximation results in the corrections $iU^n \ln^n |D/\varepsilon|$, ($n=1,2,3,...$) to the vertex $\tilde{t}$ (see Fig. 4), where the energy cutoff $\varepsilon$ is the largest of the energy variables, $\varepsilon = \text{Max}(|\omega|, |\varepsilon_h|, \Gamma)$;
(ii) assisted hopping $H_t$ in the next to leading logarithmic approximation by calculating the self–energies proportional to $\omega \tilde{t}^2 \ln D/\varepsilon$ and the $l$-particle four vertex function $\sim \tilde{t}^2 \ln D/\varepsilon$. As a consequence an identity of Ward type, the self–energy depicted on Fig. 4 is connected with the vertex correction giving the first contribution to the induced interaction $V$ in the electron-hole channel (see Sec. V).

In both steps the $h$-orbitals at different sites contribute independently. Joint contributions of different sites are relevant only in higher logarithmic orders which are neglected. The intermediate states with two $l$-particles or two holes do not contribute to the renormalization as logarithmic contribution arises from the particle-particle channel only if their total momentum is zero. The latter is the case of the superconducting gap equation, but not in the processes described above.

The strong renormalization of $\tilde{t}$ is generated by three diagrams which are depicted in Fig. 4b. The summation of these diagrams are performed in the leading logarithmic approximation considering the “parquet” diagrams. As $\tilde{t}$ is a small variable compared with $U (|\tilde{t}| \ll |U|)$, the equation are linearized in $\tilde{t}$; thus two or more $h$-particle intermediate states are not included. This approximation is consistent as the parquet scheme does not contain these diagrams. The parquet equation for vertex $\tilde{t}$ can be written in a schematic form as

$$\tilde{t}(\omega) = \tilde{t} + \cdots \int_{\max(|\omega|,|\varepsilon_h|)}^{D} \tilde{t}(\varepsilon) \frac{1}{(-\varepsilon)} U(\varepsilon) d\varepsilon ,$$

where the momentum integrals and the coefficient are not presented and $U(\omega) = U$ must be taken as it will be discussed later. The variable $\varepsilon$ represents the smallest energy denominator in the one $h$-particle and one $d$-particle (hole) channel.

The differential form of the schematic Eq. (3.1) is the scaling equation

$$\omega \frac{\partial \tilde{t}(\omega)}{\partial \omega} = \cdots \tilde{t}(\omega) U .$$

The renormalization procedure generates an effective assisted hopping, which can be given as

$$\frac{1}{N^2} \sum_{\gamma, \sigma} \sum_{k_1, k_2, k_3} \tilde{t}(k_1; \varepsilon) d_{k_1 \sigma}^\dagger d_{k_2 - \sigma}^\dagger d_{k_3 - \sigma} h_{k_h \gamma \sigma} ,$$

(3.3)
where \( k_h = k_1 + k_2 - k_3 \). The unrenormalized hopping [see Eq. (2.5)] is
\[
\tilde{\gamma}^{(0)}(\kappa_1, \kappa_2, \kappa_3) = \sum_\delta \tilde{\gamma}_\delta e^{-i[k_F(\kappa_1)+k_F(\kappa_2)-k_F(\kappa_3)]\delta a/2}\phi^*_\delta(\kappa_1)\phi^*_\delta(\kappa_2)\phi^*_\delta(\kappa_3) .
\] (3.4)

Similarly, the momentum representation of the effective Coulomb repulsion is
\[
\frac{1}{N^2} \sum_{\gamma,\sigma} \sum_{k_1, k_2, k_3} U^\gamma(\kappa_1, \kappa_2)h_{k_1\gamma\sigma}^d d_{k_1-\sigma}^d h_{k_2\gamma\sigma}^d .
\] (3.5)

where \( k_{h2} = k_{h1} + k_1 - k_2 \).

The present treatment is essentially simplified by the fact, that logarithmic vertex correction of the Coulomb vertices shown in Fig. [a] cancel out as they describe static potential represented by the \( h \)-particle, thus the scattering strength \( U^\gamma(\kappa, \kappa') \) is not renormalized. Thus the effective Coulomb interaction \( U^\gamma(\kappa, \kappa') \) remains diagonal and can be given as
\[
U^\gamma(\kappa, \kappa') = \sum_\delta U^\gamma_\delta e^{-i[k_F(\kappa)-k_F(\kappa')]\delta a/2}\phi^*_\delta(\kappa)\phi^*_\delta(\kappa') .
\] (3.6)

In the case of the assisted hopping such cancelations does not occur, since the form factors in front of the loop contributions are not the same, and we can get the following scaling equation in leading logarithmic order:
\[
\omega \frac{\partial}{\partial \omega} \tilde{\gamma}(\kappa_1, \kappa_2, \kappa_3, \omega) = \rho \langle U^\gamma(\kappa_1, \kappa)\tilde{\gamma}(\kappa, \kappa_2, \kappa_3, \omega) \rangle_{\kappa}
\]
\[
+ \rho \langle U^\gamma(\kappa_2, \kappa)\tilde{\gamma}(\kappa_1, \kappa, \kappa_3, \omega) \rangle_{\kappa}
\]
\[
- \rho \langle \tilde{\gamma}(\kappa_1, \kappa_2, \kappa, \omega)U^\gamma(\kappa, \kappa_3) \rangle_{\kappa} ,
\] (3.7)

where the three terms on the right hand side corresponds to diagrams shown in Fig. [b].

Here we introduced the notation of the average of a function \( f(\kappa) \) over the Fermi surface as
\[
\langle f(\kappa) \rangle_{\kappa} = \frac{a^d}{(2\pi)^d \rho} \int dS_{\kappa} f(\kappa) ,
\] (3.8)
in dimension \( d \), and \( \rho \) is the density of states,
\[
\rho = \frac{a^d}{(2\pi)^d} \int dS_{\kappa} / v_F(\kappa) .
\] (3.9)

The scaling Eq. (3.7) can be transformed to a real space representation by assuming the Ansatz
\[ \tilde{t}^\gamma(\kappa_1, \kappa_2, \kappa_3, \omega) = \sum_{\delta_1, \delta_2, \delta_3} \tilde{t}_{\delta_1, \delta_2, \delta_3}(\omega) \phi^\ast_{\delta_1}(\kappa_1) \phi^\ast_{\delta_2}(\kappa_2) \phi_{\delta_3}(\kappa_3) \times e^{-i[k_p(\kappa_1)\delta_1+k_p(\kappa_2)\delta_2-k_p(\kappa_3)\delta_3]a/2}, \] (3.10)

(where \( \delta \) points to nearest neighbor or \( \delta = 0 \)), thus the following closed set of equations is obtained:

\[ \frac{\partial \tilde{t}^\gamma_{\delta, \delta_1, \delta_2, \delta_3}(\omega)}{\partial \omega} = \rho \sum_{\delta'} \left( U^\gamma_{\delta_1} F_{\delta, \delta_1, \delta_2, \delta_3}(\omega) + U^\gamma_{\delta_2} F_{\delta, \delta_2, \delta_1, \delta_3}(\omega) - \tilde{t}^\gamma_{\delta_1, \delta_2, \delta_3}(\omega) F_{\delta_1, \delta_2, \delta_3} \right). \] (3.11)

Here the \( F_{\delta, \delta'} \) incorporates the form factors by the definition

\[ F_{\delta, \delta'} = \langle e^{i k_p(\kappa)(\delta-\delta') a/2} \phi_{\delta}(\kappa) \phi^\ast_{\delta'}(\kappa) \rangle \] (3.12)

which appears in the transition amplitude of an \( l \)-electron between two sites, \( \delta \) and \( \delta' \), in the tight binding approximation. As a special case, the \( \rho_{\delta} = \rho F_{\delta, \delta} \) gives the partial density of states at a site \( \delta \).

The solution of the differential Eq. (3.11) can be given immediately as:

\[ \tilde{t}^\gamma_{\delta, \delta_1, \delta_2, \delta_3}(\omega) = \sum_{\delta_1', \delta_2', \delta_3'} \left[ \left( \frac{D}{\omega} \right)^{\rho U^\gamma F} \right]_{\delta, \delta_1'} \left[ \left( \frac{D}{\omega} \right)^{\rho U^\gamma F} \right]_{\delta_2', \delta_3'} \tilde{t}^\gamma_{\delta', \delta_1', \delta_2', \delta_3'} \left[ \left( \frac{D}{\omega} \right)^{-\rho U^\gamma F} \right]_{\delta_3', \delta_1, \delta_2}, \] (3.13)

where we used the notation \( F \) and \( U^\gamma \) for the matrix \( F_{\delta, \delta'} \) and the diagonal matrix \( U^\gamma_{\delta, \delta} = U^\gamma_{\delta, \delta'} \). Unfortunately this form of the solution is not suitable for further calculations. In the actual computation let us consider the eigenvalue problem of the matrix \( U^\gamma F \) appearing in the exponent:

\[ \sum_{\delta'} U^\gamma_{\delta, \delta'} s^\gamma_{\delta'} = \lambda^\gamma j s^\gamma_{\delta}, \] (3.14)

where \( j \) labels the eigenvalues thus the spectral decomposition of the matrix \( U^\gamma F \), which is not necessarily uniter, can be given as

\[ U^\gamma_{\delta, \delta'} = \sum_j \lambda^\gamma j s^\gamma_{\delta} \delta_{\delta'}^j, \] (3.15)

with the row vectors \( \mathbf{r} \) orthogonal to the column vectors \( \mathbf{s} \), thus
\[
\sum_{\delta} r_\delta^{\gamma j} s_\delta^{\gamma j'} = \delta_{jj'} .
\] (3.16)

In general the matrix \( U^\gamma F \) is not invertable, that is why the left and the right eigenvectors can be different, such an example is presented in Sec. VIII. (case A).

Using the identity \((U^\gamma F)_{\delta\delta'} = (FU^\gamma)^{\delta\delta'}\), which follows form Eq. (3.12) and that \( U^\gamma \) is diagonal and real, thus the left eigenvectors of \( FU^\gamma \) are the \( s^{\gamma j*} \) and writing \( \tilde{t}_{\delta_1\delta_2\delta_3} \) as a linear combination of the eigenvectors \( s \) as

\[
\tilde{t}_{\delta_1\delta_2\delta_3} = \sum_{ijk} \tilde{t}_{ijk}^{\gamma} (\omega) s_{\delta_1}^{\gamma j} s_{\delta_2}^{\gamma j'} s_{\delta_3}^{\gamma j''} ,
\] (3.17)

the scaling Eq. (3.11) can be diagonalized and we obtain a set of decoupled linear differential equations for the \( \tilde{t}_{ijk}^{\gamma} (\omega) \)

\[
\frac{\partial \tilde{t}_{ijk}^{\gamma} (\omega)}{\partial \omega} = (\lambda_i + \lambda_j - \lambda_k) \rho \tilde{t}_{ijk}^{\gamma} (\omega) ,
\] (3.18)

whose solution is

\[
\tilde{t}_{ijk}^{\gamma} (\omega) = \tilde{t}_{ijk}^{\gamma(0)} \left( \frac{D}{\omega} \right) (\lambda_i + \lambda_j - \lambda_k) \rho .
\] (3.19)

Putting Eqs. (3.10), (3.17) and (3.19) together, we get the fully factorized form of the effective assisted hopping:

\[
\tilde{t}^{\gamma}(\kappa_1, \kappa_2, \kappa_3, \omega) = \sum_{ijk} \xi_i^{\gamma}(\kappa_1) \xi_j^{\gamma}(\kappa_2) \xi_k^{\gamma*}(\kappa_3) \left( \frac{D}{\omega} \right) (\lambda_i + \lambda_j - \lambda_k) \rho \tilde{t}_{ijk}^{\gamma(0)} ,
\] (3.20)

where we have introduced

\[
\xi_j^{\gamma}(\kappa) = \sum_{\delta} \phi_{\delta j}^{\gamma*}(\kappa) s_{\delta}^{\gamma j} e^{-ik_\delta(\kappa)\delta a/2} .
\] (3.21)

The exponents are of the order of \( \rho U \), since in the eigenvalue Eq. (3.14) \( F \) is of the order of unity, thus the magnitude of \( \lambda \) is determined by \( U \).

The couplings \( \tilde{t}_{ijk}^{\gamma(0)} \) can be determined from the initial (unrenormalized) couplings (with \( D \) being the bandwidth) \( \tilde{t}^{\gamma(0)}(\kappa_1, \kappa_2, \kappa_3) = \tilde{t}^{\gamma}(\kappa_1, \kappa_2, \kappa_3, \omega = D) \) [see Eqs. (3.4) and (3.20)], since in that case
\[ \tilde{\gamma}^{(0)}(\kappa_1, \kappa_2, \kappa_3, D) = \sum_{ijk} \xi_i^\gamma(\kappa_1) \xi_j^\gamma(\kappa_2) \xi_k^\gamma(\kappa_3) \tilde{t}_{ijk}^{0} . \]  

(3.22)

After some algebraic manipulations we get

\[ \tilde{t}_{ijk}^{0} = \sum_{\delta} \tilde{t}_{\delta i j}^{0} \tilde{t}_{\delta i k}^{0} \tilde{t}_{\delta j k}^{0} . \]  

(3.23)

The latter equation tells us that \( \tilde{t}_{ijk}^{0} \) is symmetrical in the first two indices (\( \tilde{t}_{ijk}^{0} = \tilde{t}_{jik}^{0} \)) and as a consequence [see Eq. (3.20)] the \( \tilde{\gamma}(\kappa_1, \kappa_2, \kappa_3, \omega) \) is also symmetrical in the first two \( \kappa \) variables:

\[ \tilde{\gamma}(\kappa_1, \kappa_2, \kappa_3, \omega) = \tilde{\gamma}(\kappa_2, \kappa_1, \kappa_3, \omega) . \]  

(3.24)

For later use we mention, that the average of \( \xi^{*} \xi \) can be expressed as

\[ \langle \xi_{j}^{*} \gamma(\kappa) \xi_{j}^{\gamma}(\kappa) \rangle_{\kappa} = \sum_{\delta \delta'} s_{\delta}^{* \gamma} s_{\delta'}^{\gamma} F_{\delta \delta'} s_{\delta'}^{\gamma} \].  

(3.25)

A similar approach was used by Dzyaloshinski\( ^{2} \) in case of the two dimensional nearly nested electron gas. However, unlike in our case, those scaling equations do not form a closed set.

**IV. LIGHT PARTICLE SELF-ENERGY**

Unlike to usual renormalization procedure, where the self–energy corrections become important only in the next to leading logarithmic order, in our case we expect a large mass renormalization for the light electrons as the logarithmic corrections in the self–energy and in the generated electron–electron interaction occur on the same level.

The \( l \)-electron self–energy diagrams are shown in Fig. 5. We are going to evaluate them for those electrons which are close to the Fermi level and we assume that \(|\epsilon_{h}| \ll \epsilon_{F} \) or for energies close to \( \epsilon_{h} \). The latter case is of no importance for later calculation and we are presenting it just for being interesting.

The heavy particle self–energy of the lowest order contains three \( l \)-electron lines in the intermediate state, thus it is nonlogarithmic except in one dimension.
A. Self–energy near the Fermi-level

In the logarithmic approximation the intermediate state with the two smallest \( l \)-electron energy variables \( \xi_1 \) and \( \xi_2 \) must be picked up and then both vertices must be fully renormalized, but their lower cutoff must be replaced by \( |\varepsilon_h| \). A typical integral to be calculated for the self–energy with energy \( \omega \ll \varepsilon_h \) is

\[
\int_{\varepsilon_h}^{D} d\xi_1 \int_{\varepsilon_h}^{D} d\xi_2 \left( \frac{D}{\xi_1 + \xi_2} \right)^{\alpha} \frac{\pm \text{sign} (\varepsilon_h)}{\pm \text{sign} (\varepsilon_h) \omega - (\xi_1 + \xi_2 + |\varepsilon_h|)} = - (\omega \mp \varepsilon_h) A(D/|\varepsilon_h|), \tag{4.1}
\]

where the sign is appropriately chosen to give the correct contributions of different diagrams: the upper sign stands for the first two diagrams and lower sign for the last in Fig. 3. In the actual calculation we pick up the most divergent term, so that the largest exponent is taken as \( \alpha \). The vertex contributions contains also \( \omega \) variables, but its sign is not well defined. It is easy to demonstrate that the appearance of such an \( \omega \) changes the right hand side of Eq. (4.1), by factors like \((1 \pm \alpha)^{-1}\). As the logarithmic approximation is applied, therefore, the corrections proportional to \( U \) must be neglected. Furthermore, the shift of chemical potential \( \varepsilon_h A \) is also only an approximate value, since in this approximation it is difficult to determine exactly\(^\text{II}\). The coefficient \( A \) can be calculated and

\[
A(D/|\varepsilon_h|) = \frac{1}{\alpha} \left[ \left( \frac{D}{|\varepsilon_h|} \right)^{\alpha} - 1 \right] \tag{4.2}
\]

is obtained.

The complete expression for the self–energy is

\[
\Sigma_l(\mathbf{k}, \omega) = \sum_{\gamma} \int_{\text{Max}(\omega, \varepsilon_h)}^{D} d\varepsilon \rho^2 \times \\
\times \left( \left\langle \tilde{t}^\gamma (\mathbf{k}, \mathbf{k}'; \varepsilon) \frac{1}{\varepsilon} \tilde{t}^{\gamma*} (\mathbf{k}, \mathbf{k}', \mathbf{k}''; \varepsilon) \right\rangle_{\mathbf{k}' \mathbf{k}''} \right)_{\mathbf{k}' \mathbf{k}''} \\
+ \left( \left\langle \tilde{t}^\gamma (\mathbf{k}', \mathbf{k}; \varepsilon) \frac{1}{\varepsilon} \tilde{t}^{\gamma*} (\mathbf{k}, \mathbf{k}, \mathbf{k}''; \varepsilon) \right\rangle_{\mathbf{k} \mathbf{k}''} \right)_{\mathbf{k}' \mathbf{k}''} \\
- \left( \left\langle \tilde{t}^\gamma (\mathbf{k}', \mathbf{k}''; \varepsilon) \frac{1}{\varepsilon} \tilde{t}^{\gamma*} (\mathbf{k}, \mathbf{k}', \mathbf{k}; \varepsilon) \right\rangle_{\mathbf{k} \mathbf{k}''} \right)_{\mathbf{k}' \mathbf{k}''}, \tag{4.3}
\]
where the different terms on the right hand side comes from the three diagrams in Fig. [3], respectively. Performing the integration, we get

\[ \Sigma_l(\kappa; \omega) = -\omega [\chi_1(\kappa) + \chi_2(\kappa) + \chi_3(\kappa)] \rho^2 A(D/\varepsilon_h) \]
\[ + \varepsilon_h [\chi_1(\kappa) + \chi_2(\kappa) - \chi_3(\kappa)] \rho^2 A(D/\varepsilon_h), \]

(4.4)

where

\[ \chi_1(\kappa) \]
\[ \chi_2(\kappa) \]
\[ \chi_3(\kappa) \]

and, furthermore, the most divergent term of \( \tilde{\gamma} \) [see Eq. (3.20)] is used (the exponent \( \alpha \) is the largest of the eigenvalue combination \( \lambda_i + \lambda_j - \lambda_k \)), which is a good approximation for most of the cases.

The symmetry of \( \tilde{\gamma} \) [see Eq. (3.24)] implies \( \chi_1(\kappa) = \chi_2(\kappa) \), furthermore, the average over the Fermi surface of every \( \chi_j(\kappa) \) is equal and will be denoted by \( \chi \).

Knowing the self–energy given by Eq. (4.4), the renormalized one particle Green’s function is obtained as

\[ G_l(k, \omega) = \frac{Z_\kappa}{\omega - v_{F,\text{ren}}(\kappa) [k - k_F(\kappa)]}, \]

(4.6)

where we have neglected the renormalization of the chemical potential, which comes from the second term in Eq. (4.4) and it is hard to calculate in leading logarithmic order approximation. The renormalization constant is defined as

\[ Z_\kappa^{-1} = 1 - \frac{\partial \text{Re} \Sigma_l(\omega, k)}{\partial \omega} \bigg|_{\omega=0} \]
\[ = 1 + [2 \chi_1(\kappa) + \chi_3(\kappa)] \rho^2 A(D/|\varepsilon_h|) > 1, \]

(4.7)

and the Fermi velocity is renormalized nearby the Fermi surface as

\[ v_{F,\text{ren}}(\kappa) = v_F(\kappa) Z_\kappa < v_F(\kappa), \]

(4.8)
thus it is suppressed, leading to mass enhancement. The dispersion curve is schematically plotted in Fig. 7. In the case of the large renormalization described by $Z_\kappa \ll 1$ the large modification of the dispersion curve is expected for $|\omega| < |\varepsilon_h|$ and the renormalization gradually disappears as approaching larger energies, $|\omega| > |\varepsilon_h|$. Thus, the large enhancement of the density of states is restricted to a small energy region $|\omega| < |\varepsilon_h|Z_\kappa$.

The average strength of the mass enhancement can be given by

$$\langle Z_\kappa^{-1} \rangle = 1 + 3\chi\rho^2 A(D/|\varepsilon_h|). \quad (4.9)$$

The mass renormalization can be very large, thus the scaling may result in heavy fermionic behavior. The calculation presented are justified only in the region $\alpha \ll 1$, but there is no indication for nonanalytical behavior, thus in order to get qualitative result the extrapolation for intermediate coupling $\alpha < 1/2$ is adequate. As it is known from the X-ray absorption problem, for large $U\rho$ that quantity must be replaced by $\delta/\pi$ where $\delta$ is the phase shift ($\delta \leq \pi/2$).

In Eq. (4.6) there is also a shift of the energy proportional to $\chi_1(\kappa) + \chi_2(\kappa) - \chi_3(\kappa)$ which is sensitive on the direction $\kappa$. That shift changes the number of the electrons inside the Fermi surface, thus the Fermi energy must be corrected in order of $\rho^2\varepsilon_h A(D/\varepsilon_h)Z_\kappa$ to keep the number of particles inside the Fermi surface unchanged and that is associated also with the deformation of shape of the Fermi surface.

Finally it must be mentioned, that even a large wave function renormalization does not play an important role in the vertex equation for $\tilde{t}$, as the Green’s functions are taken between two points in the real space not further apart than the lattice constant. In that case the Fourier transform of the Green’s function multiplied by a slowly varying function like $\exp(ik_xa/2)$ must be integrated with respect to the momentum. The result is almost independent of the self–energy and the correction is $O(\Sigma/D)$. 


B. Self–energy near the singularity

If the energy of the electron is large enough, than the scattering of an electron from the filled Fermi sea to the unoccupied heavy level becomes a real process, and we are faced with a problem very similar to that of the X–ray edge\(^6\), where a power–law behavior is observed in the absorption function. In our case the role of the X–ray is taken over by the incoming light electron, and power–like behavior is expected in the imaginary part of the self–energy and so in the spectral functions. For simplicity we are going to consider the \(\varepsilon_h > 0\) case here. Similar considerations are valid if \(\varepsilon_h < 0\).

It is easy to see, that in Fig. 5 for \(\omega \approx \varepsilon_h\) the first diagram \(\Sigma_{l,1}(\omega, \kappa)\) and second diagram \(\Sigma_{l,2}(\omega, \kappa)\), while for \(\omega \approx -\varepsilon_h\) the third diagram \(\Sigma_{l,3}(\omega, \kappa)\) is singular. In this case the integral in (4.1) is

\[
\int_{-\varepsilon_h}^D d\xi_1 \int_{-\varepsilon_h}^D d\xi_2 \left( \frac{D}{\xi_1 + \xi_2} \right)^\alpha \frac{\pm 1}{\pm \omega - (\xi_1 + \xi_2 + |\varepsilon_h|)} = \nonumber
\]

\[
= -(\omega \mp \varepsilon_h) A(D/|\omega \mp \varepsilon_h|)[1 \mp i\alpha \pi \Theta(\pm \omega \varepsilon_h)] , \quad (4.10)
\]

where the \(\Theta(x)\) is the step function and it ensures the proper analytical behavior. The signs are the same as they were in Eq. (4.1).

For this special choice of energies the renormalization of the assisted hopping [Eq. (3.7)] should be considered more carefully. It turns out that the energies of the internal electron lines are such, that for \(\Sigma_{l,1}(\omega, \kappa)\) the first term in Eq. (3.11) is not singular, similarly for \(\Sigma_{l,2}(\omega, \kappa)\) the second term and for the \(\Sigma_{l,3}(\omega, \kappa)\) the third term can be neglected, so that the exponent \(\lambda_i + \lambda_j - \lambda_k\) is reduced to the values \(\lambda_j - \lambda_k\), \(\lambda_i - \lambda_k\) and \(\lambda_i + \lambda_j\) respectively.

The exponent \(\alpha\) should be associated with the largest exponent for each case separately and will be denoted by \(\alpha_1 = \alpha_2\) and \(\alpha_3\), and \(A_i\) is the function defined by Eq. (4.2) with \(\alpha = \alpha_i\). Putting everything together, the self–energy for energies near \(\omega = \varepsilon_h\) is

\[
\Sigma_l(\kappa; \omega) = -(\omega - \varepsilon_h)[\chi_1(\kappa) + \chi_2(\kappa)]\rho^2 A_1(D/|\omega - \varepsilon_h|)[1 + i\alpha_1 \pi \Theta(\omega - \varepsilon_h)] \quad (4.11)
\]

and

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\[ \Sigma_i(\kappa; \omega) = -(\omega + \varepsilon_h)\chi_3(\kappa)\rho^2 A_3(D/|\omega + \varepsilon_h|)[1 - i\alpha_3 \pi \Theta(-\omega - \varepsilon_h)] \quad (4.12) \]

for \( \omega \approx -\varepsilon_h \). The \( \chi(\kappa) \)-s are the ones defined by Eq. (3.21), however the terms to be averaged are not the same and, therefore, the averages are no more equal.

The imaginary parts of the self–energies \( (4.11) \) contribute to the spectral densities of the particles and it emerges as soon as the energy of the electron is larger then \( \approx \varepsilon_h \), while the self–energy \( (4.12) \) gives similar contribution for the spectral density of the holes having energy below the threshold \( -\varepsilon_h \).

V. GENERATED INTERACTION BETWEEN THE LIGHT ELECTRONS

To address the possibility of superconductivity in light band in these models, we need to calculate the generated interaction in this band for parallel– and anti–parallel spin. The typical diagrams are shown in Fig. 6. In the logarithmic approximation the intermediate state with one \( h \)-electron and the \( l \)-electron with the smallest energy must be picked up and in vertices that energy must must be used as the lower cutoff.

The schematic form of the generated interaction is

\[ \int_{\omega}^{D} t(\varepsilon) \frac{1}{-\varepsilon} t(\varepsilon) d\varepsilon \sim -t^2(\varepsilon) \quad (5.1) \]

without numerical factors containing the strength of the Coulomb interaction.

The calculation is straightforward in the time-ordered diagram technique. For the interaction in the anti-parallel spin channel the contribution of the four diagrams in Fig. 6(a) is

\[ V^\perp(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = -\rho \sum_\gamma \int_{\text{Max}(\omega, \varepsilon_h)}^{D} d\varepsilon \]

\[ \times \left( \langle \tilde{t}^\gamma(\kappa_1, \kappa_2, \kappa; \varepsilon) \tilde{t}^\gamma(\kappa_3, \kappa_4, \kappa; \varepsilon) \rangle_{\kappa} + \langle \tilde{t}^\gamma(\kappa_2, \kappa_1, \kappa; \varepsilon) \tilde{t}^\gamma(\kappa_3, \kappa_4, \kappa; \varepsilon) \rangle_{\kappa} \right. \]

\[ - \langle \tilde{t}^\gamma(\kappa_1, \kappa, \kappa_3; \varepsilon) \tilde{t}^\gamma(\kappa_4, \kappa, \kappa_2; \varepsilon) \rangle_{\kappa} - \langle \tilde{t}^\gamma(\kappa_2, \kappa, \kappa_4; \varepsilon) \tilde{t}^\gamma(\kappa_3, \kappa, \kappa_1; \varepsilon) \rangle_{\kappa} \right) \quad (5.2) \]

The evaluation of diagrams shown in Fig. 6(b) gives in the parallel spin channel
\[ V_{\parallel}(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = -\frac{\rho}{2} \sum_{\gamma} \int_{\max(\omega, \varepsilon_h)}^{D} d\varepsilon \]
\[ \times \left( \left\langle \tilde{\gamma}(\kappa_1, \kappa_2, \kappa_3; \varepsilon) \frac{1}{\varepsilon} \tilde{\gamma}^{\ast}(\kappa_1, \kappa_2; \varepsilon) \right\rangle_{\kappa} + \left\langle \tilde{\gamma}(\kappa_2, \kappa_3, \kappa_4; \varepsilon) \frac{1}{\varepsilon} \tilde{\gamma}^{\ast}(\kappa_2, \kappa_4; \varepsilon) \right\rangle_{\kappa} \right. \]
\[ - \left. \left( \left\langle \tilde{\gamma}(\kappa_1, \kappa_3; \varepsilon) \frac{1}{\varepsilon} \tilde{\gamma}^{\ast}(\kappa_2, \kappa_3; \varepsilon) \right\rangle_{\kappa} - \left\langle \tilde{\gamma}(\kappa_2, \kappa_4; \varepsilon) \frac{1}{\varepsilon} \tilde{\gamma}^{\ast}(\kappa_3, \kappa_1; \varepsilon) \right\rangle_{\kappa} \right) \right), \quad (5.3) \]

where we have antisymmetrized the interaction. These expressions can be evaluated by using Eq. (3.20) for the scattering amplitude. The induced interaction is constant in the energy range \(|\omega| < Z|\varepsilon_h|\) and drops rapidly for higher energies.

The interaction can be split into two terms: the interaction \(V^S\) between electrons forming a singlet and \(V^T\) between electrons in a triplet state. In the singlet state the spin part of the wave function is antisymmetrized, so the \(V^S\) is symmetrical in the \(\kappa_1\) and \(\kappa_2\) variables

\[ V^S(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = V^S(\kappa_2, \kappa_1, \kappa_3, \kappa_4). \quad (5.4) \]

For the triplet case the symmetrical spin wavefunction implies antisymmetrical property of the \(V^T\), so

\[ V^T(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = -V^T(\kappa_2, \kappa_1, \kappa_3, \kappa_4), \quad (5.5) \]

and similar holds for the last two \(\kappa\) variables. Furthermore, it is evident that for both interactions

\[ V^{S,T}(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = V^{S,T}(\kappa_2, \kappa_1, \kappa_3, \kappa_4). \quad (5.6) \]

The \(V_{\parallel}\) is by definition the interaction between electrons in triplet state. To \(V^\perp\), however, both the singlet and triplet states contributes. We can split them by using the symmetry of the \(V^S\) and \(V^T:\)

\[ V^S(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = \frac{1}{2}[V^\perp(\kappa_1, \kappa_2, \kappa_3, \kappa_4) + V^\perp(\kappa_2, \kappa_1, \kappa_3, \kappa_4)], \quad (5.7) \]

and

\[ V^T(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = \frac{1}{2}[V^\perp(\kappa_1, \kappa_2, \kappa_3, \kappa_4) - V^\perp(\kappa_2, \kappa_1, \kappa_3, \kappa_4)]. \quad (5.8) \]
The latter must be exactly $V^\parallel(\kappa_1, \kappa_2, \kappa_3, \kappa_4)$, as it can be seen if we take into account the symmetries of $\tilde{\gamma}$ [Eq. (3.24)]. This separation is possible because of the absence of the spin-orbit coupling in this model.

Performing the energy integral, we get for the interaction in the singlet channel

$$V^S(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = -\frac{\rho}{2} \sum_\gamma A(D/\varepsilon_h) \left\langle 4 \left[ \tilde{\gamma}(\kappa_1, \kappa_2, \kappa) \tilde{\gamma}^*(\kappa_4, \kappa_3, \kappa) \right] \right\rangle_\kappa$$

$$-\left\langle \tilde{\gamma}(\kappa_1, \kappa, \kappa_3) \tilde{\gamma}^*(\kappa_4, \kappa_2) \right\rangle_\kappa - \left\langle \tilde{\gamma}(\kappa_2, \kappa, \kappa_4) \tilde{\gamma}^*(\kappa_3, \kappa, \kappa_1) \right\rangle_\kappa$$

$$-\left\langle \tilde{\gamma}(\kappa_2, \kappa, \kappa_3) \tilde{\gamma}^*(\kappa_4, \kappa, \kappa_1) \right\rangle_\kappa - \left\langle \tilde{\gamma}(\kappa_1, \kappa, \kappa_4) \tilde{\gamma}^*(\kappa_3, \kappa, \kappa_2) \right\rangle_\kappa$$

(5.9)

and for the triplet channel the result is identical with Eq. (5.3) where the energy integral must be replaced by $A(D/\varepsilon_h)$.

We can see that if the assisted hopping $\tilde{\gamma}$ is structureless, than the generated interaction disappears both in singlet and triplet channel. However, a small $\kappa$ dependence can lead to finite interactions which may be enhanced due to large $A(D/|\varepsilon_h|)$.

VI. SUPERCONDUCTING TRANSITION TEMPERATURE

As it has been mentioned in Sec.II the induced interaction between the light particles (Fig. 6) and the self–energy (Fig. 5) are connected by identities similar to the Ward identities. The Ward identities are valid only in the electron-hole channel with zero total spin and not in the Cooper channel. These relations are due to the fact, that both occurs first in the second order of the perturbation theory on $\tilde{t}$. That is in contrast to most of the other logarithmic theories, where the vertex correction exists already in the first order. Thus in the present case, the induced vertex and the self–energy must be treated simultaneously in determinations of the superconducting order parameter $\Delta$ and of the transition temperature, which are based on the selfconsistent equation shown by diagrams in Fig. 8.

The order parameter is defined as $\Delta_{\sigma\sigma'}(k) = \langle c_{k,\sigma} c_{-k,\sigma'} \rangle$, where $\sigma$ and $\sigma'$ stand for spins. Separating the spin wave function, $\Delta$ is either even or odd function of $k$, depending whether
we are looking at singlet ($\Delta^S$) or triplet ($\Delta^T$) Cooper-pairs. Using the interaction $V^{S(T)}$, the self-consistent equation for $\Delta^{S(T)}$ is

$$
\Delta^{S(T)}(k) = -T_c \frac{1}{N} \sum_{k'} \sum_{\omega_m} V^{S(T)}(k, -k, -k', k') \\
\times G(k'; \omega_m) G(-k'; \omega_n - \omega_m) \Delta^{S(T)}(k') ,
$$

(6.1)

with $T_c$ being the transition temperature and $\omega_m = (2m + 1)\pi T$ are the Matsubara frequencies. After summing over the internal energy, we arrive at

$$
\Delta^{S(T)}(\kappa) = -T_c \rho \sum_{\omega_m < \omega_c} \frac{\pi}{[\omega_m]} (V^{S(T)}(\kappa, -\kappa, -\kappa', \kappa') Z(\kappa') \Delta^{S(T)}(\kappa'))_{\kappa'} ,
$$

(6.2)

where we introduced the frequency cutoff $\omega_c$ as we have replaced the $\omega_m$ dependent quantities by their low frequency values, assuming that $T_c$ is much smaller than the characteristic energy $\varepsilon_h$. In our case the $\omega_c$ is determined by the fact, that the contributions for the integral over the internal energy comes from energies smaller than $Z|\varepsilon_h|$, as it is determined by the linear part of the dispersion curve where the density of states is enhanced (Fig. 7), so the summation over the frequencies yields

$$
\Delta^{S(T)}(\kappa) = -\ln \left( \frac{1.13|\varepsilon_h|Z}{T_c} \right) \rho (V^{S(T)}(\kappa, -\kappa, -\kappa', \kappa') Z(\kappa') \Delta^{S(T)}(\kappa'))_{\kappa'} .
$$

(6.3)

In other words, in the selfconsistent equation for the order parameter each interaction $V$ is associated with two Greens’s function. Only the contribution of electrons with energy $\omega < \varepsilon_h$ is kept as the vertex drops rapidly for larger energy. For the dominating low energy electron the strength of the poles is $Z_{\kappa}$, and its energy $|\omega| < \varepsilon_h Z_{\kappa}$ (see Fig. 7). Furthermore, the density of states $\rho$ for such electrons is also enhanced by $Z_{\kappa}^{-1}$ [see Eq. (4.8)].

In the singlet channel, where it can be assumed that the $\kappa$-dependence of the superconducting order parameter $\Delta(\kappa)$ is weak, the approximate strength of the dimensionless effective coupling is obtained by evaluating Eq. (6.3) so that

$$
g_{\text{eff}}^{(S)} \sim \rho (V^{S}(\kappa, -\kappa, -\kappa', \kappa'))_{\kappa,\kappa'} (Z(\kappa'))_{\kappa'}
$$

$$
\sim (Z^{-1})^{-1} \langle V^{S} \rho \rangle ,
$$

(6.4)
where in the last approximation \( Z_\kappa \) is replaced by its average over the Fermi surface [see Eq. (4.9)]. If the \( \kappa \)-dependence of the \( V \) and \( Z \) is not large, then the average of \( V^S \) is of the order of \( \rho \chi A \). Inserting the calculated value of \( Z_\kappa \) given by Eq. (4.9), we can see that the dimensionless coupling is

\[
g_{\text{eff}}^{(S)} = q \frac{\chi \rho^2 A(D/\varepsilon_h)}{1 + 3\chi \rho^2 A(D/\varepsilon_h)}, \tag{6.5}
\]

where \( q \) is of the order of unity and depends on the details on the \( \kappa \)-dependencies, which is approximated in Eq. (6.4). If the mass renormalization is large, \( 1 + 3\chi \rho^2 A(D/\varepsilon_h) \gg 1 \), then the denominator is important and \( g_{\text{eff}}^{(S)} \) saturates.

In this way in the BCS theory the transition temperature is

\[
T_c = |\varepsilon_h| \langle Z_\kappa \rangle e^{-1/g_{\text{eff}}^{(S)}} \tag{6.6}
\]

which can be expressed as a function of the averaged mass enhancement \( \langle Z_\kappa^{-1} \rangle \) given by Eq. (4.9) and of the parameter \( q \). That function is shown in Fig. 9. The transition temperature \( T_c \) is suppressed by the decreasing \( \langle Z_\kappa \rangle \) for strong renormalization. The critical temperature is the largest in an intermediate region of moderate renormalization where the mass enhancement \( \langle Z_\kappa^{-1} \rangle \sim 2 - 5 \). As we have seen, in general it is essential to include the renormalization of the Fermi-velocity due to self-energy to get a consistent treatment of the superconductivity.

VII. THE 1D MODEL

To demonstrate the procedure described in the previous sections, let us apply it to a 1-dimensional case introduced by one of the authors. In this model the light orbitals are on every site \( n \) and the heavy orbitals are on every second site, as shown in Fig. 11. The degeneracy of the heavy orbitals is neglected thus the index \( \gamma \) will be dropped. The hopping to heavy orbitals is \( t^\gamma_{h,\delta} = t_h \) and the hoppings between the neighboring light orbitals are \( t \), the Coulomb repulsion between the light and heavy orbitals is
\[
U_\delta = \begin{cases} U, & \text{if } \delta = 0 \\ U', & \text{if } \delta = \pm 1 \end{cases}, \quad (7.1)
\]

where the on-site and nearest neighbor interaction is kept. Finally, the assisted hopping is

\[
\tilde{t}_\delta = \begin{cases} 0, & \text{if } \delta = 0 \\ \tilde{t}, & \text{if } \delta = \pm 1 \end{cases}, \quad (7.2)
\]
in case of light s and heavy d orbitals, where the wave functions have the same sign in the regions of the overlap. In the previous work\[1] \(U' = 0\) was taken.

The dispersion relation of the fermions in the light band, shown in Fig. 10, is given by

\[
\omega_{\pm}(k) = \frac{\Delta \varepsilon}{2} \pm \sqrt{\left(\frac{\Delta \varepsilon}{2}\right)^2 + 4t^2 \cos^2 \frac{ka}{2}} \quad (7.3)
\]
in the tight binding approximation, where the \(+(--)\) stands for upper(lower) band, the Brillouin zone is \(-\pi/a < k < \pi/a\), and \(\Delta \varepsilon = \varepsilon_\delta - \varepsilon_0\) is the relative shift of the atomic levels.

As a next step, we have to determine the amplitude \(\phi(\kappa)\) for \(\delta = \pm 1, 0\) [see Eq. (2.2)]. In the case of even \(t_\delta\) the \(\phi\)-s are not \(\kappa\) dependent and \(\phi_{+1} = \phi_{-1}\). Since the probability is normalized, \(\phi_{+1}^2 + \phi_{0}^2 = 1\), we can parametrize \(\phi\)-s by \(\phi_{+1} = \phi_{-1} = \cos \varphi\) and \(\phi_{0} = \sin \varphi\). Here the subscript \(+1\) \((-1)\) stands for the right (left) neighbors for the heavy orbital. If the energy levels of the light orbitals are the same \((\varepsilon_\delta = 0)\), then all the \(\phi\)-s are equal to \(1/\sqrt{2}\) \((\varphi = \pi/4)\). In the general case the \(\varphi\) can be estimated as

\[
\tan \varphi = \frac{\Delta \varepsilon}{2t \cos \vartheta} + \sqrt{\left(\frac{\Delta \varepsilon}{2t \cos \vartheta}\right)^2 + 1}, \quad (7.4)
\]

where \(\vartheta = k_Fa/2\) is introduced in accordance with Ref. [1]. It depends on the filling \(\nu\), \(\vartheta = \pi \nu/2\) for the lower band \((\nu < 1)\) and \(\vartheta = \pi(1 - \nu/2)\) for the upper band \((\nu > 1)\).

Specially, in one dimension, Eq. (2.9) reduces to

\[
\frac{1}{N} \sum_k = a \int \frac{dk}{2\pi} \rightarrow \frac{a}{2\pi v_F} \sum_{\kappa=\pm 1} \int_{-D}^{D} d\varepsilon, \quad (7.5)
\]

where \(\kappa = 1\) and \(-1\) denotes right and left moving electrons, with \(k_F(1) = k_F\) and \(k_F(-1) = -k_F\), similarly for the velocities \(v_F(\pm 1) = \pm v_F\). The form factor appearing in Eq. (2.7) is also simplified significantly, since there are only four combinations \(\exp(\pm i\vartheta)\) and \(\exp(\pm 3i\vartheta)\).
The density of states \( \rho \) [Eqs. (3.9)] is the usual one for the tight binding approximation:

\[
\rho = \frac{1}{2\pi} \frac{2a}{v_F},
\]

(7.6)

and the calculation of the amplitudes \( F_{\delta\delta'} \) [Eq. (3.12)] gives

\[
F_{00} = \sin^2 \varphi
\]

\[
F_{++} = F_{--} = \cos^2 \varphi
\]

\[
F_{0+} = F_{0-} = \cos \vartheta \cos \varphi \sin \varphi
\]

\[
F_{+-} = \cos 2\vartheta \cos^2 \varphi.
\]

(7.7)

The \( UF \) matrix is then

\[
UF = \begin{pmatrix}
U' \cos^2 \varphi & U' \cos \vartheta \cos \varphi \sin \varphi & U' \cos 2\vartheta \cos^2 \varphi \\
U \cos \vartheta \cos \varphi \sin \varphi & U \sin^2 \varphi & U \cos \vartheta \cos \varphi \sin \varphi \\
U' \cos 2\vartheta \cos^2 \varphi & U' \cos \vartheta \cos \varphi \sin \varphi & U' \cos^2 \varphi
\end{pmatrix},
\]

(7.8)

The matrix \( UF \) is not symmetric, thus the left and right eigenvectors are different, and one obtains for right and left eigenvectors

\[
s^{(1)} = \begin{pmatrix}
U' \cos \varphi \cos \vartheta \\
U \sin \varphi \\
U' \cos \varphi \cos \vartheta
\end{pmatrix}, \quad s^{(2)} = \begin{pmatrix}
\sin \varphi \\
-2 \cos \varphi \cos \vartheta \\
\sin \varphi
\end{pmatrix}, \quad s^{(3)} = \begin{pmatrix}
1 \\
0 \\
-1
\end{pmatrix}
\]

(7.9)

and

\[
r^{(1)} = \frac{1}{\lambda_1} (\cos \varphi \cos \vartheta, \sin \varphi, \cos \varphi \cos \vartheta)
\]

\[
r^{(2)} = \frac{1}{2\lambda_1} (U \sin \varphi, -2U' \cos \varphi \cos \vartheta, U \sin \varphi)
\]

\[
r^{(3)} = (1/2, 0, -1/2)
\]

(7.10)

with eigenvalues \( \lambda_1 = U \sin^2 \varphi + 2U' \cos^2 \varphi \cos^2 \vartheta \), \( \lambda_2 = 0 \) and \( \lambda_3 = 2U' \cos^2 \varphi \sin^2 \vartheta \).

The functions \( \xi \) [see Eq. (3.21)] are

\[
\xi_1(\kappa) = \lambda_1 \\
\xi_2(\kappa) = 0 \\
\xi_3(\kappa) = -i2 \sin \kappa \vartheta \cos \varphi.
\]

(7.11)
The nonvanishing averages are

\[
\langle |\xi_1(\kappa)|^2 \rangle = \lambda_1^2 \\
\langle |\xi_3(\kappa)|^2 \rangle = 4 \cos^2 \varphi \sin^2 \vartheta .
\]  

(7.12)

From Eq. (3.23), we get

\[
\tilde{t}_{111}^{(0)} = 2 \tilde{t} \frac{1}{\lambda_1^3} \cos^3 \vartheta \cos^3 \varphi \\
\tilde{t}_{133}^{(0)} = \tilde{t}_{313}^{(0)} = \tilde{t}_{331}^{(0)} = \frac{\tilde{t}}{2 \lambda_1} \cos \vartheta \cos \varphi ,
\]  

(7.13)

so that for the effective assisted hopping we get

\[
\tilde{t}(\kappa_1, \kappa_2, \kappa_3; \omega) = 2 \tilde{t} \cos^3 \varphi \left\{ [\cos^3 \vartheta + \cos \vartheta (\sin \kappa_1 \vartheta + \sin \kappa_2 \vartheta)] \sin \kappa_3 \vartheta ] (D/\omega)^{\rho \lambda_1} \\
- \cos \vartheta \sin \kappa_1 \vartheta \sin \kappa_2 \vartheta (D/\omega)^{\rho (2 \lambda_3 - \lambda_1)} \right\} .
\]  

(7.14)

Since in the assisted hopping only the transitions of an electron from one of the neighboring sites to the heavy orbital play role, the amplitude \( \cos^3 \varphi \) is easy to understand being associated with the three light electron lines. Furthermore, we can see that the Coulomb interactions appear in the exponent only as the combinations \( U' \cos^2 \varphi \) and \( U \sin^2 \varphi \), what means that the Coulomb repulsions enters only as effective repulsion normalized by the single site fermion densities.

As \( \kappa \) can have the values \( \pm 1 \), the assisted hopping can be given by the four amplitudes \( \tilde{t}_1, \tilde{t}_2, \tilde{t}_3 \) and \( \tilde{t}_4 \), introduced in Ref. [1]. They can be expressed by \( \tilde{t}(\kappa_1, \kappa_2, \kappa_3; \omega) \) as

\[
\tilde{t}_1 = \tilde{t}(1, -1, 1; \omega) = \tilde{t}^{\ast}(-1, 1, -1; \omega) \\
\tilde{t}_2 = \tilde{t}(1, -1, -1; \omega) = \tilde{t}^{\ast}(-1, 1, 1; \omega) \\
\tilde{t}_3 = \tilde{t}(1, 1, -1; \omega) = \tilde{t}^{\ast}(-1, -1, 1; \omega) \\
\tilde{t}_4 = \tilde{t}(1, 1, 1; \omega) = \tilde{t}^{\ast}(-1, -1, -1; \omega)
\]  

(7.15)

so that
\( \tilde{t}_1 = \tilde{t}_2 = 2\tilde{t} \cos^3 \varphi \left[ \cos^3 \varphi (D/\omega)^\rho t_{\lambda_1} + \cos \varphi \sin^2 \varphi (D/\omega)^\rho (2\lambda_3 - \lambda_1) \right] \)

\( \tilde{t}_3 = 2\tilde{t} \cos^3 \varphi \left[ (\cos^3 \varphi - 2 \cos \varphi \sin^2 \varphi)(D/\omega)^\rho t_{\lambda_1} - \cos \varphi \sin^2 \varphi (D/\omega)^\rho (2\lambda_3 - \lambda_1) \right] \)

\( \tilde{t}_4 = 2\tilde{t} \cos^3 \varphi \left[ (\cos^3 \varphi + 2 \cos \varphi \sin^2 \varphi)(D/\omega)^\rho t_{\lambda_1} - \cos \varphi \sin^2 \varphi (D/\omega)^\rho (2\lambda_3 - \lambda_1) \right] . \)  \((7.16)\)

These equation are exactly the solutions of the Eq. (7) in Ref. 11 with initial couplings \( \tilde{t}_1(0) = \tilde{t}_2(0) = \tilde{t} \cos^3 \varphi \cos \varphi, \tilde{t}_3(0) = \tilde{t} \cos^3 \varphi \cos 3\varphi, U' = 0 \) and \( \varphi = \pi/4. \)

As a next step we calculate the generated interactions between the fermions in the light band. We are using the notations common in the theory of the one–dimensional Fermi gas. For a review, we refer to the paper of Sólyom\(^1\).

In the antiparallel channel the forward scattering, denoted by \( g_2, \) is

\[ g_2 = V^\perp(1, -1, -1, 1) = 2|\tilde{t}_1|^2 - 2|\tilde{t}_3|^2 . \]  \((7.17)\)

Replacing the assisted hopping by its most divergent part, determined by the largest exponent, we get

\[ g_2 = (2\tilde{t} \cos^3 \varphi)^2 \begin{cases} 2 \sin^2 2\varphi \cos 2\varphi (D/\varepsilon_h)^{2\lambda_1}, & \text{if } \lambda_1 > \lambda_3 \\ 0, & \text{if } \lambda_1 < \lambda_3 . \end{cases} \]  \((7.18)\)

If \( \lambda_1 > \lambda_3, \) than \( g_2 \) is positive for small fillings and became negative for larger fillings.

For the backward scattering, where the momentum transfer is large \((2k_F), \) we get

\[ g_{1\perp} = V^\perp(1, -1, 1, -1) = 2\tilde{t}_1(\tilde{t}_2 - \tilde{t}_4) + 2\tilde{t}_1^*(\tilde{t}_2^* - \tilde{t}_4^*) , \]

\[ g_{1\parallel} = V^\parallel(1, -1, 1, -1) - V^\parallel(1, -1, -1, 1) + g_2 = 4|\tilde{t}_1|^2 - 2\tilde{t}_2^*\tilde{t}_4 - 2\tilde{t}_2\tilde{t}_4^* . \]  \((7.19)\)

Since \( t_1 = t_2^*, \) the \( g_{1\perp} = g_{1\parallel} = g_1 \) holds, which means that the model is isotropic, and

\[ g_1 = (2\tilde{t} \cos^3 \varphi)^2 \begin{cases} -\sin^2 2\varphi (1 + \cos 2\varphi)(D/\varepsilon_h)^{2\lambda_1}, & \text{if } \lambda_1 > \lambda_3 \\ \sin^2 2\varphi (1 - \cos 2\varphi)(D/\varepsilon_h)^{4\lambda_3 - 2\lambda_1}, & \text{if } \lambda_1 < \lambda_3 . \end{cases} \]  \((7.20)\)

They are presented in Fig. \((11).\)

In one dimension the values of these couplings determine the nature of the ground state.\(^3\)

We analyse the \( \lambda_1 > \lambda_3 \) and the \( \lambda_3 > \lambda_1 \) region separately. In the first case the \( g_1 \) is
negative, and depending whether the sign of the combination $g_1 - 2g_2$ is positive or negative, the expected ground state is singlet superconductivity or a CDW. A simple calculation shows that for $\vartheta < \vartheta_C$ the ground state is CDW and for larger filling ($\vartheta > \vartheta_C$) we expect superconductivity, where $\cos 2\vartheta_C = -1/5$ (this corresponds to filling $n_C = 0.564$). On the other hand, if the exponent $2\lambda_3 - \lambda_1$ is larger, then $g_1 - 2g_2$ is always positive. Since $g_1$ is positive, we get triplet superconductivity. The two exponents are equal if $U \sin^2 \varphi = -2U' \cos^2 \varphi \cos 2\vartheta$. The phase diagram for this model is then shown in Fig. 12(a), where we can see that the nature of superconductivity changes as the Coulomb interaction increases between the heavy orbital and light orbital on the neighboring site.

There is also a strong mass renormalization in this model. It is easy to see, that $\chi_1 = \chi_2 = \chi_3 = \chi$ are equal [see Eq. (4.5)], and

$$\chi \sim \tilde{t}_1^2 + \tilde{t}_2^2 + \tilde{t}_3^2 + \tilde{t}_4^2. \quad (7.21)$$

Putting in the actual expression for $\tilde{t}$-s, we get

$$\chi = (2\tilde{t} \cos^3 \varphi)^2 \begin{cases} 
4 \cos^2 \vartheta (\cos^4 \vartheta + 2 \sin^4 \vartheta), & \text{if } \lambda_1 > \lambda_3 \\
\sin^4 2\vartheta \cos^2 \vartheta), & \text{if } \lambda_1 < \lambda_3
\end{cases} \quad (7.22)$$

which become large for intermediate fillings.

If the parity of the light and heavy orbitals is not the same (e.g. light $s-$ or $d-$ orbitals and heavy $f$ orbitals, see Fig. 1(b)), then the assisted hopping is odd and

$$\tilde{t}_\delta = \begin{cases} 
0, & \text{if } \delta = 0 \\
\delta \tilde{t}, & \text{if } \delta = \pm 1
\end{cases} \quad (7.23)$$

The parity of the assisted hopping has no effect to the eigenvalues and eigenvectors of the $UF$. Using Eq. (3.23), the $\tilde{t}_{ijk}$'s are in this case

$$\tilde{t}_{333}^{(0)} = \frac{\tilde{t}}{4}, \quad \tilde{t}_{113}^{(0)} = \tilde{t}_{131}^{(0)} = \tilde{t}_{113}^{(0)} = \frac{\tilde{t}}{\lambda_1^2} \cos^2 \vartheta \cos^2 \varphi, \quad (7.24)$$

and for the effective assisted hopping we get

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\( \tilde{t}(\kappa_1, \kappa_2, \kappa_3; \omega) = -2i \tilde{t} \cos^3 \varphi \left\{ \sin \kappa_1 \vartheta \sin \kappa_2 \vartheta \sin \kappa_3 \vartheta + \cos^2 \vartheta (\sin \kappa_1 \vartheta + \sin \kappa_2 \vartheta) \right\} (D/\omega)^{\rho \lambda_3} - \cos^2 \vartheta \sin \kappa_3 \vartheta (D/\omega)^{\rho (2\lambda_1-\lambda_3)} \right\} . \) (7.25)

In this case the \( \tilde{t}_1, \tilde{t}_2, \tilde{t}_3 \) and \( \tilde{t}_4 \) are

\[ \tilde{t}_1^* = \tilde{t}_2 = -2i \tilde{t} \cos^3 \varphi \left\{ \sin^3 \vartheta (D/\omega)^{\rho \lambda_3} + \cos^2 \vartheta \sin \vartheta (D/\omega)^{\rho (2\lambda_1-\lambda_3)} \right\} \]
\[ \tilde{t}_3 = -2i \tilde{t} \cos^3 \varphi \left\{ (\sin^3 \vartheta + 2 \cos^2 \vartheta \sin \vartheta) (D/\omega)^{\rho \lambda_3} + \cos^2 \vartheta \sin \vartheta (D/\omega)^{\rho (2\lambda_1-\lambda_3)} \right\} \]
\[ \tilde{t}_4 = -2i \tilde{t} \cos^3 \varphi \left\{ (\sin^3 \vartheta + 2 \cos^2 \vartheta \sin \vartheta) (D/\omega)^{\rho \lambda_3} - \cos^2 \vartheta \sin \vartheta (D/\omega)^{\rho (2\lambda_1-\lambda_3)} \right\} \] (7.26)

Comparing them to those of the even assisted hopping [Eqs. 7.16], we can see, that disregarding the prefactor of \( i \) and the exponents, the only difference is the interchange of \( \sin \vartheta \) and \( \cos \vartheta \).

Keeping only the most divergent parts in the assisted hopping, for the induced interaction we get

\[ g_2 = (2\tilde{t} \cos^3 \varphi)^2 \left\{ \begin{array}{ll}
0, & \text{if } \lambda_1 > \lambda_3 \\
-2 \sin^2 2\vartheta \cos 2\vartheta (D/\varepsilon_h)^{2\lambda_1}, & \text{if } \lambda_1 < \lambda_3
\end{array} \right. \] (7.27)

If \( \lambda_1 < \lambda_3 \), then \( g_2 \) is negative for small fillings and became positive for larger fillings.

For the backward scattering

\[ g_1 = (2\tilde{t} \cos^3 \varphi)^2 \left\{ \begin{array}{ll}
\sin^2 2\vartheta (1 + \cos 2\vartheta) (D/\varepsilon_h)^{2\lambda_1}, & \text{if } \lambda_1 > \lambda_3 \\
-\sin^2 2\vartheta (1 - \cos 2\vartheta) (D/\varepsilon_h)^{4\lambda_1-2\lambda_3}, & \text{if } \lambda_1 < \lambda_3
\end{array} \right. \] (7.28)

The boundary between the two region with different exponent is the same as it was in the case of even \( \tilde{t}_\delta \). If \( \lambda_1 > \lambda_3 \), both \( g_1 \) and \( g_1 - 2g_2 \) are positive, so the ground state is TS. In the region where \( \lambda_1 < \lambda_3 \) holds, the \( g_1 \) is negative, and for \( \vartheta < \vartheta_C \) we get triplet superconductivity and for \( \vartheta > \vartheta_C \) the ground state is CDW. Here \( \vartheta \) is defined as \( \cos 2\vartheta_C = 1/5 \) and the corresponding filling is \( \nu_C = 0.436 \). The phase diagram is shown in Fig. [2,b].
We can conclude that the main effect of changing the parity leads to exchange of the CDW and superconducting ground state.

In the present calculation we have neglected the interaction between the light electrons. That interaction changes the $g$-s, so that the phase diagram changes also and new phases appear near $\nu = 0$ and $\nu = 1$.

**VIII. TWO-DIMENSIONAL SQUARE LATTICE: CuO$_2$ PLANE**

In the compounds characterized by CuO$_2$ planes a possible representation of the $h$-orbitals are the two non-bonding $p$-orbitals on the apical oxygens located below or above the Cu sites in the CuO$_2$ plane (the O$^{2-}$ ions are in the tetrahedral position around Cu$^{2+}$ ion). For details see Ref. [14],[12]. The CuO$_2$ plane forms a two dimensional square lattice depicted on Fig. 2, where the $h$-orbitals are at the corners and the $l$-band is formed by the orbitals $a_x$, $a_y$ and $b$ at the middle of the sides and at corners, respectively. One possible choice of the symmetry of the orbitals corresponds to the CuO$_2$ plane with $p_x (a_x)$, $p_y (a_y)$ and $d_{x^2−y^2} (b)$, while the two heavy labelled by $\gamma = x, y$ are of $p_x$- and $p_y$-type. For this model the hopping to heavy orbitals is

$$t^\gamma_{h,\delta} = t_h P^\gamma_{\delta}$$

and the hoppings between light orbitals are

$$t_{\delta\delta'} = \begin{cases} t p_{\delta}, & \text{if } \delta' = (0, 0) \\ t' p_{\delta\delta'}, & \text{otherwise} \end{cases}$$

(8.2)

furthermore,

$$\varepsilon_{\delta} = \begin{cases} \varepsilon_b, & \text{if } \delta' = (0, 0) \\ 0, & \text{otherwise} \end{cases}$$

(8.3)

The $p_\delta$, $p_{\delta\delta'}$ and $P^\gamma_\delta$ stand for the relative signs of the real wave functions in the overlap regions. For the nearest neighbor hopping between oxygen and copper in the CuO$_2$ plane
\( p_\delta = -p_{-\delta} \) holds and for the next to the nearest neighbor hopping between oxygens \( p_{\delta,\delta'} = \pm 1 \) with \( +(-) \) sign if \( \delta - \delta' \) is parallel (perpendicular) to the (1,1) direction and for the apical oxygen \( P_{\delta}^\gamma = P_{-\delta}^\gamma \) if \( \delta \) is parallel with \( \gamma = x, y \) and zero otherwise.

The Coulomb repulsion is

\[
U_{\delta}^\gamma = \begin{cases} 
U, & \text{if } \delta = 0 \\
U', & \text{otherwise}
\end{cases}
\]  

(8.4)

where \( U \) is the repulsion between electrons on the apex-\( O \) and \( Cu \), while \( U' \) is between apex-\( O \) and the next oxygens in the plane.

The assisted hopping is given by

\[
\tilde{t}_\delta^\gamma = \tilde{t}P_{\delta}^\gamma .
\]  

(8.5)

In the following we will consider two limiting cases.

**A. Case of** \( U' = t' = 0 \)

Here we are considering the extreme case where both \( t' \) and \( U' \) are zero. The dispersion relation is given by

\[
\omega_{\pm} = \frac{1}{2} \left( \varepsilon_b \pm \sqrt{\varepsilon_b^2 + 4\Omega^2} \right) ,
\]  

(8.6)

where the \(+(-)\) stands for upper(lower) band, \( \Omega^2 = t_{xk}^2 + t_{yk}^2 \) and \( t_{ak} = 2t \sin k_\alpha a/2 \) (\( \alpha = x, y \)). The \( \phi \)-s [see Eq. (2.2)] are given by

\[
\phi_0 = \frac{\omega_{\pm}}{\sqrt{\omega_{\pm}^2 + \Omega^2}} , \quad \text{and} \quad \phi_\delta = \frac{t_{\delta k}}{\sqrt{\omega_{\pm}^2 + \Omega^2}} .
\]  

(8.7)

The calculations of \( \rho \) and \( F \)-s, defined by Eqs. (3.9) and (3.12), are straightforward:

\[
\rho = \frac{1}{(2\pi)^2} \frac{\sqrt{\varepsilon_b^2 + 4\Omega^2}}{t^2} K'(c) ,
\]  

(8.8)

and

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\[ F_{00} = |\phi_0|^2 \]
\[ F_{x0} = F_{y0} = F_{-x0}^* = F_{-y0}^* = \frac{i}{2} \left( \frac{2t}{\omega_\pm} \right) (1 - c)|\phi_0|^2 \]
\[ F_{xx} = F_{-x,-x} = F_{yy} = F_{-y,-y} = \frac{1}{2} \left( \frac{2t}{\omega_\pm} \right)^2 (1 - c)|\phi_0|^2 \]
\[ F_{xy} = -F_{x,-y} = -F_{-x,y} = F_{-x,-y} = \frac{1}{2} \left( \frac{2t}{\omega_\pm} \right)^2 \left[ \frac{E'(c)}{K'(c)} - c \right] |\phi_0|^2 \]
\[ F_{x,-x} = F_{x,x} = F_{y,-y} = F_{y,y} = \frac{1}{2} \left( \frac{2t}{\omega_\pm} \right)^2 \left[ c - 1 - 2c^2 + 2 \frac{E'(c)}{K'(c)} \right] |\phi_0|^2, \quad (8.9) \]

where \( c = 1 - \Omega^2/(2t)^2 \) and the elliptic function are defined as

\[ K(k) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}} \quad \text{and} \quad E(k) = \int_0^{\pi/2} d\varphi \sqrt{1 - k^2 \sin^2 \varphi} \quad (8.10) \]

and \( K'(k) = K(k'), \ E'(k) = E(k') \) where \( k' = \sqrt{1 - k^2} \).

The vectors \( s \) and \( r \) are [see Eq. (3.13)]

\[
\begin{align*}
\mathbf{s}^{(1)} &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{s}^{(2)} = \begin{bmatrix} i \\ -i \\ 0 \\ -2f \end{bmatrix}, \quad \mathbf{s}^{(3)} = \begin{bmatrix} 0 \\ 0 \\ i \\ -2f \end{bmatrix}, \quad \mathbf{s}^{(4)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{s}^{(5)} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad (8.11)
\end{align*}
\]

and

\[
\begin{align*}
\mathbf{r}^{(1)} &= (-if, if, -if, if, 1), \\
\mathbf{r}^{(2)} &= (-i/2, i/2, 0, 0, 0), \\
\mathbf{r}^{(3)} &= (0, 0, -i/2, i/2, 0), \\
\mathbf{r}^{(4)} &= (1/2, 1/2, 0, 0, 0), \\
\mathbf{r}^{(5)} &= (0, 0, 1/2, 1/2, 0), \quad (8.12)
\end{align*}
\]

with eigenvalues \( \lambda_1 = U|\phi_0|^2 \) and \( \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0, \) and

\[ f = (1 - c)t/\omega_\pm, \quad (8.13) \]
Furthermore the $\delta$ indices are ordered as $(x, -x, y, -y, 0)$. The $\xi$ functions [see Eq. (3.21)] are

\[
\begin{align*}
\xi_1(\kappa) &= \phi_0 , \\
\xi_2(\kappa) &= 2\frac{2t}{\omega_\pm} \phi_0 \sin^2(k_{Fx}(\kappa)a/2) - 2f \phi_0 , \\
\xi_3(\kappa) &= 2\frac{2t}{\omega_\pm} \phi_0 \sin^2(k_{Fy}(\kappa)a/2) - 2f \phi_0 , \\
\xi_4(\kappa) &= \frac{2t}{\omega_\pm} \phi_0 \sin(k_{Fx}(\kappa)a) , \\
\xi_5(\kappa) &= \frac{2t}{\omega_\pm} \phi_0 \sin(k_{Fy}(\kappa)a) .
\end{align*}
\] (8.14)

The nonvanishing averages of $\xi \xi^*$ functions are from Eq. (3.23)

\[
\begin{align*}
\langle |\xi_1(\kappa)|^2 \rangle &= |\phi_0|^2 , \\
\langle |\xi_4(\kappa)|^2 \rangle &= \langle |\xi_5(\kappa)|^2 \rangle = 2 \left( \frac{2t}{\omega_\pm} \right)^2 \left( \frac{E'(c)}{K'(c)} - c^2 \right) |\phi_0|^2 .
\end{align*}
\] (8.15)

Calculating the renormalized assisted hopping, we pick up the most divergent term in Eq. (3.20). The largest value of the exponent $\lambda_i + \lambda_j - \lambda_k$ corresponds to the choice of $i = j = 1$ and, since the remaining four eigenvalues are degenerate, with $k = 2, 3, 4$ or 5.

We can determine the initial values $\tilde{t}^\gamma(0)$ from Eq. (3.23), and we get $t^x_{114} = -\tilde{t}f^2$ and $t^y_{115} = -\tilde{t}f^2$ with other $t^\gamma_{11k}$'s being equal to 0. So the assisted hopping is

\[
t^x(\kappa_1, \kappa_2, \kappa_3; \omega) = t^x_{114} \xi_1(\kappa_1) \xi_1(\kappa_2) \xi_3^*(\kappa_3) \left( \frac{D}{\omega} \right)^{2\lambda_1 - \lambda_4} (8.16)
\]

and for $t^y(\kappa_1, \kappa_2, \kappa_3; \omega)$ the $\xi_4^*(\kappa_3)$ is replaced by $\xi_5^*(\kappa_3)$, so that

\[
t^y(\kappa_1, \kappa_2, \kappa_3; \omega) = -\tilde{t}f^2 |\phi_0|^2 \phi_0 \frac{2t}{\omega_\pm} \sin(k_{F\gamma}(\kappa_3)a) \left( \frac{D}{\omega} \right)^{2U|\phi_0|^2 \rho} ,
\] (8.17)

where $k_F = (k_{Fx}, k_{Fy})$ is the Fermi vector parallel to the $\kappa$.

The expressions for $\chi$-s, according to Eq. (4.3), are

\[
\begin{align*}
\chi_1 &= \chi_2 = t^x_{114} t^x_{114} |\xi_1|^2 \langle |\xi_1|^2 \rangle \langle |\xi_3^*(\kappa)|^2 \rangle + t^y_{115} t^y_{115} |\xi_1|^2 \langle |\xi_1|^2 \rangle \langle |\xi_5^*(\kappa)|^2 \rangle \\
\chi_3(\kappa) &= t^x_{114} t^x_{114} \langle |\xi_1|^2 \rangle \langle |\xi_1|^2 \rangle \langle |\xi_4(\kappa)|^2 \rangle + t^y_{115} t^y_{115} \langle |\xi_1|^2 \rangle \langle |\xi_1|^2 \rangle \langle |\xi_5^*(\kappa)|^2 \rangle .
\end{align*}
\] (8.18)
or, inserting the $\xi$-s and $t_{ijk}^{(0)}$-s,

$$\chi_1 = \chi_2 = \frac{1}{16} \phi_0^6 \Omega^8 \left[ \frac{E'(c)}{K'(c)} - c^2 \right] \tilde{t}^2$$

$$\chi_3(\kappa) = \frac{1}{64} \phi_0^6 \Omega^8 \left[ \sin^2(k_{Fx}(\kappa)a) + \sin^2(k_{Fy}(\kappa)a) \right] \tilde{t}^2 . \tag{8.19}$$

A straightforward calculation gives the interaction in the $S$-wave channel,

$$V^S(\kappa, -\kappa, -\kappa', \kappa') = -2 \rho \chi A(D/\varepsilon_h) . \tag{8.20}$$

Since the interaction is independent of momentum, the $\kappa$ dependence of the superconducting order parameter $\Delta^S$ is small [due to small $\kappa$ dependence of $Z(\kappa)$, see Eq. (6.3)]. The approximate strength of the dimensionless effective coupling in the singlet channel [see Eq. (6.4)] is

$$g_{eff}^{(s)} \approx -\frac{2 \rho^2 \chi A(D/\varepsilon_h)}{1 + 3 \chi \rho A(D/\varepsilon_h)} , \tag{8.21}$$

where we can see immediately that

$$-g_{eff}^{(s)} < \frac{2}{3} . \tag{8.22}$$

The relation of the transition temperature $T_c$ and the mass enhancement is given by the curve $q = 2$ in Fig. 3. The highest $T_c$ is obtained for mass enhancement about 2.5–3.5.

Similar calculation for the triplet channel gives

$$V^T(\kappa, -\kappa, -\kappa', \kappa') = \frac{1}{32} \phi_0^6 \rho \frac{\Omega}{\tilde{t}^2 \omega_0^6} A(D/\varepsilon_h)$$

$$\times \left[ \sin(k_{Fx}(\kappa)a) \sin(k_{Fx}(\kappa')a) + \sin(k_{Fy}(\kappa)a) \sin(k_{Fy}(\kappa')a) \right] , \tag{8.23}$$

which results in repulsion.

B. The case of $U = t = 0$

The tight binding Hamiltonian (2.1) with the choice of parameters corresponding to this case has the following form in the $\kappa$ representation:
\[ H = 4t' \sum_{k,\sigma} \sin(k_x a/2) \sin(k_y a/2) \left( a_{x,k,\sigma}^\dagger a_{y,k,\sigma} + \text{h.c.} \right) \]

(8.24)

and can be diagonalized by introducing the

\[
a_{x,k} = \phi_x(k) (d_{k,+} + d_{k,-})
\]

\[
a_{y,k} = \phi_y(k) (d_{k,+} - d_{k,-})
\]

(8.25)

operators, where \( \phi_\alpha(k) = \text{sign}(\sin k_\alpha a/2)/\sqrt{2} \) and the diagonalized Hamiltonian is

\[
H = 4t' \sum_{k,\sigma} |\sin(k_x a/2) \sin(k_y a/2)| \left( d_{k,\sigma,+}^\dagger d_{k,\sigma,+} - d_{k,\sigma,-}^\dagger d_{k,\sigma,-} \right). \]

(8.26)

With this choice of \( \phi_\delta \)-s, the operators \( d_{k,\sigma,+}^\dagger \) and \( d_{k,\sigma,+} \) are associated with the upper band which is combined from the upper parts of the two bands labelled by \( x \) and \( y \). In the following, only the upper band is kept and the index + will be dropped.

For convenience, the momentum is shifted as \( k_x \rightarrow k_x - \pi/a \) and \( k_y \rightarrow k_y - \pi/a \) in all of the following formulas. For example, the dispersion relation changes to \( \varepsilon = 4t' \cos k_x a/2 \cos k_y a/2 \). That shift, however, can be completely incorporated by the amplitudes \( \phi_\delta(k) \):

\[
\phi_{\pm} = \phi_{\pm y} = \pm i/\sqrt{2}
\]

(8.27)

in Eqs. (3.4) and (3.10).

Introducing the notation \( F_{\delta \delta} = \bar{F}_0, F_{\delta,-\delta} = \bar{F}_2 \) and \( F_{xy} = F_{-x-y} = -F_{-x-y} = \bar{F}_1 \), the matrix \( \bar{F}_{\delta \delta'} \) takes the form

\[
\bar{F} = \begin{pmatrix}
\bar{F}_0 & \bar{F}_2 & \bar{F}_1 & -\bar{F}_1 \\
\bar{F}_2 & \bar{F}_0 & -\bar{F}_1 & \bar{F}_1 \\
\bar{F}_1 & -\bar{F}_1 & \bar{F}_0 & \bar{F}_2 \\
-\bar{F}_1 & \bar{F}_1 & \bar{F}_2 & \bar{F}_0 \\
\end{pmatrix}, \]

(8.28)

which eigenvectors and eigenvalues are easy to obtain:
and
\[
\mathbf{r}^{(1)} = \mathbf{s}^{(1)T}/4, \quad \mathbf{r}^{(2)} = \mathbf{s}^{(2)T}/2, \quad \mathbf{r}^{(3)} = \mathbf{s}^{(3)T}/2, \quad \mathbf{r}^{(4)} = \mathbf{s}^{(4)T}/4, \tag{8.30}
\]
where the indices $\delta$ are ordered as $(x, -x, y, -y)$ and the superscript denotes a transposed vector.

The corresponding eigenvalues are
\[
\lambda_1 = U'(\tilde{F}_0 - \tilde{F}_2 + 2\tilde{F}_1),
\]
\[
\lambda_2 = \lambda_3 = U'(\tilde{F}_0 + \tilde{F}_2),
\]
\[
\lambda_4 = U'(\tilde{F}_0 - \tilde{F}_2 - 2\tilde{F}_1). \tag{8.31}
\]

The integrals in $\tilde{F}_0$, $\tilde{F}_1$ and $\tilde{F}_2$ leads to elliptic functions
\[
\rho = K'((\varepsilon_F/4t')/\pi^2 t')
\]
\[
\tilde{F}_0 = \langle |\phi_x|^2 \rangle = 1/2
\]
\[
\tilde{F}_1 = \langle \phi_x^* \phi_y \cos(k_x a/2) \cos(k_y a/2) \rangle = \varepsilon_F/8t'
\]
\[
\tilde{F}_2 = \langle |\phi_x|^2 \cos k_x a \rangle = 1/2 - E'(\varepsilon_F/4t')/K'(\varepsilon_F/4t'). \tag{8.32}
\]
Furthermore, using Eq. (3.21) we get the functions $\xi$:
\[
\xi_1(\kappa) = -i\sqrt{2} \left[ \cos(k_{Fx}(\kappa)a/2) + \cos(k_{Fy}(\kappa)a/2) \right]
\]
\[
\xi_2(\kappa) = -\sqrt{2} \sin(k_{Fx}(\kappa)a/2)
\]
\[
\xi_3(\kappa) = -\sqrt{2} \sin(k_{Fy}(\kappa)a/2)
\]
\[
\xi_4(\kappa) = -i\sqrt{2} \left[ \cos(k_{Fx}(\kappa)a/2) - \cos(k_{Fy}(\kappa)a/2) \right], \tag{8.33}
\]
and the nonvanishing averages of $\xi \xi^*$ are:
\[ \langle |\xi_1(\kappa)|^2 \rangle = 4(\bar{F}_0 - \bar{F}_2 + 2\bar{F}_1) = 4\lambda_1 \]
\[ \langle |\xi_2(\kappa)|^2 \rangle = \langle |\xi_3(\kappa)|^2 \rangle = 2(\bar{F}_0 + \bar{F}_2) = 2\lambda_2 \]
\[ \langle |\xi_4(\kappa)|^2 \rangle = 4(\bar{F}_0 - \bar{F}_2 - 2\bar{F}_1) = 4\lambda_4 . \] (8.34)

The leading terms in Eq. (3.20) with proper symmetry are those with the exponents
\[ 2\lambda_1 - \lambda_2 \] or \[ \lambda_1 + \lambda_2 - \lambda_4 , \]
and the corresponding \( \tilde{t}_{ijk}^{(0)} \) are
\[ \tilde{t}_{112}^{(0)} = \tilde{t}_{113}^{(0)} = \bar{\rho} / 16 , \] (8.35)
or
\[ \tilde{t}_{124}^{(0)} = \tilde{t}_{214}^{(0)} = -\tilde{t}_{134}^{(0)} = -\tilde{t}_{314}^{(0)} = \bar{\rho} / 16 , \] (8.36)
depending on whether \( \varepsilon_F / 4\bar{t} > 0.41 \) or \( < 0.41 \), respectively, so that
\[ \tilde{t}^x(\kappa_1, \kappa_2, \kappa_3) = \frac{\bar{\rho}}{16} \xi_1(\kappa_1)\xi_1(\kappa_2)\xi_2(\kappa_3) \left( \frac{D}{\omega} \right)^{(2\lambda_1 - \lambda_2)\rho} \] (8.37)
for \( \varepsilon_F / 4\bar{t}' < 0.41 \) and
\[ t^x(\kappa_1, \kappa_2, \kappa_3) = \frac{\bar{\rho}}{16} [\xi_1(\kappa_1)\xi_2(\kappa_2) + \xi_2(\kappa_1)\xi_1(\kappa_2)] \xi_3^*(\kappa_3) \left( \frac{D}{\omega} \right)^{(\lambda_1 + \lambda_2 - \lambda_4)\rho} \] (8.38)
for \( \varepsilon_F / 4\bar{t}' > 0.41 \) and for \( t^y(\kappa_1, \kappa_2, \kappa_3) \) the \( \xi_2(\kappa_2) \) should be replaced by \( \xi_3(\kappa_2) \) and multiplied by a minus sign for \( \varepsilon_F / 4\bar{t}' > 0.41 \).

The \( \chi \)-s, defined by Eq. (4.3), calculated by using Eqs. (3.20) and (8.37-8.38) are \( \kappa \)-dependent with
\[ \tilde{\chi}_1(\kappa) = \tilde{\chi}_2(\kappa) = \frac{\bar{\rho}^2}{16}\lambda_1\lambda_2|\xi_1(\kappa)|^2 \]
\[ \tilde{\chi}_3(\kappa) = \frac{\bar{\rho}^2}{16}\lambda_1^2(|\xi_2(\kappa)|^2 + |\xi_3(\kappa)|^2) \]
for \( \varepsilon_F / 4\bar{t}' > 0.41 \) and
\[ \tilde{\chi}_1(\kappa) = \tilde{\chi}_2(\kappa) = \frac{\bar{\rho}^2}{16}[4\lambda_1(|\xi_2(\kappa)|^2 + |\xi_3(\kappa)|^2) + 4\lambda_2|\xi_1(\kappa)|^2] \]
\[ \tilde{\chi}_3(\kappa) = \frac{\bar{\rho}^2}{8}|\xi_4(\kappa)|^2 \] (8.39)
for $\varepsilon_F/4t' < 0.41$. The average of $\tilde{\chi}_j(\kappa)$-s over the Fermi surface is

$$
\tilde{\chi} = \begin{cases} 
\lambda_2 \lambda_1^2 / 4 , & \text{if } \varepsilon_F / 4t' > 0.41 \\
\lambda_1 \lambda_2 \lambda_4 / 2 , & \text{if } \varepsilon_F / 4t' < 0.41 
\end{cases}
$$

(8.40)

In the case $\varepsilon_F / 4t' < 0.41$, for the singlet superconductivity we get

$$
V^S(\kappa, -\kappa, -\kappa', \kappa') = -\frac{t^2}{32} \rho \lambda_2 |\xi_1(\kappa)|^2 |\xi_1(\kappa')|^2 A(D/|\varepsilon_h|),
$$

(8.41)

which gives rise to an $S$-type superconductivity. The triplet spin channel contribution a repulsive interaction of a $p$-type.

For the case $\varepsilon_F / 4t' > 0.41$ we get a $d$-wave repulsive interaction in the singlet channel. (In Ref. 13 we got $d$-wave attraction, which was due to a sign error.)

**IX. DISCUSSION**

The role of the assisted hopping is demonstrated in models where additional to the conduction band there are orbitals near the Fermi surface. The occupations of these orbitals fluctuate between two values. The state of higher occupation is obtained by adding a heavy electron. All the other states are ignored. The energies of these two states include all of the intratomic Coulomb interaction, thus these states are fully renormalized in the atomic sense. We call the attention to two physical realizations:

(i) **Heavy $f$-electrons.** In this case it must be assumed that one of the renormalized $f$-levels is near to the Fermi energy on the scale of the Fermi energy. If for one of the $f$-levels that condition is satisfied, then the model can be applied by considering the conduction electron assisted hybridization of the $4f$-electrons with the conduction band. The assisted process is induced by the change in the occupation of conduction band in the atomic orbital (tight-binding) representation. It is a striking feature, that the very large mass enhancement in the conduction band practically eliminates the superconducting state. The moderate mass enhancement can be correlated with the superconductivity (see chapter VI.).
role of the Coulomb interaction in conduction band has been considered also in Ref. 13 by the slave–boson technique and it makes the superconducting state even more favorable.

(ii) $CuO_2$ plane. The strikingly flat parts of the electronic band structure are due to the non–bonding oxygen orbitals. E.g. in the $YBa_2Cu_3O_{7-\delta}$ compound it is either due to the apex oxygens taking the tetrahedral positions above or below the $Cu$ site or it is generated by the non–bonding $\pi$–orbitals of the oxygens in the $CuO_2$ plane, which are oriented perpendicular to the plane in the $z$-direction. These states can hybridize with the orbitals on $Cu$ only if the $CuO_2$ plane is distorted and the $Cu$ and $O$ atoms form separate planes. Such a flat band has been recently observed by experiment and also reproduced by band structure calculation. The previous case might be related to the influence of the distance of the apex oxygen from the $CuO_2$ plane on the superconducting transition temperature. See for more detailed discussion Refs. 12–14. The calculated induced electron–electron interaction is momentum dependent due to the form factors in the electron assisted hybridization, but the models treated in Ref. 13 the transition leads to always $s$–type of superconductors.

The Hamiltonian for a definite system must be constructed in the atomic orbital picture, thus the tight binding approximation is used. For the sake of simplicity only the conduction band crossing the Fermi energy is kept for the light particle, even if the other broad bands could contribute also to those integrals which have logarithmic character in the most simpler approximation.

The vertex corrections to the conduction electron assisted hopping between the light and heavy bands are determined. The dependence on the occupations of the heavy orbitals (i.e. interaction like $c^\dagger h^\dagger hh$) does not contribute in the logarithmic approximation, thus it is not taken into account. The assisted hopping vertices $\tilde{t}$ are strongly renormalized by the local Coulomb interaction $U$ between the heavy and light–particles. The general formulation is presented in Chapter III., where the number of different couplings are finite as the momentum dependence appears only in the form factors which belong to some certain class. The solution of these vertex equations for $\tilde{t}$ can be very different depending on the
model. In the case of onsite Coulomb interaction $U$, the renormalization by $U$ can reduce the structure in the formfactor by integrating out the dependence on the momenta of the scattered electron. That happens in case A in Chapter VIII. [see Eqs. (8.16)-(8.17)]. On the other hand, if the interaction $U'$ is the next neighbor Coulomb interaction, then the form factors of the vertex corrections are altering in a certain class, but the level of the structure is never reduced, see case B in Chapter VIII. In the one–dimensional model new coupling is generated, and as it is discussed in Chapter VII the structure of the vertex equations is mapped to those in a one–dimensional interacting electron gas with the two couplings $g_1$, $g_2$, $g_3$ and $g_4$ known as the $g$–ology. The results obtained are generalization of those in Ref. [1] and may be relevant in the quasi–one dimensional organic conductors. The functional form of the vertex corrections are always power functions of $\omega/D$, but the exponents and this way the strengths of the enhancement are very sensitive on the actual structure of the formfactors.

The large mass enhancement described in Chapter IV. and calculated in Chapter VII. and VIII. is a quite general consequence of the theory. That can be very large, its value, however, is limited by the low energy infrared cutoff due to the dispersion and energy of the heavy band.

The superconductivity is determined by very similar expressions as the mass enhancement [see e.g. Eqs. (4.7) and (6.5)]. Usually in other theories the similar expressions are related by Ward identities, but here those can not be exploited as the relevant quantities appear in different channels (zero sound and Cooper). The single particle weight $Z$ in the Green’s function are playing crucial role in the strength of the electron–electron interaction and the mass enhancement. The related expressions of Chapter VI. and Fig. [4] represents quite general relations and they may be relevant in other models as well.

In the case of one–dimensional models a much richer class of susceptibilities are discussed and the phase diagrams in Fig. [12] contains spin density wave (SDW) and charge density wave (CDW) and triplet and singlet superconductivity.

Finally, it is worth to point out, that in the actual calculation the electrons on the
different heavy sites are not correlated which is the consequence of our systemic logarithmic approximation schema, but that can be lost beyond the approximation applied. As far as the number of the excited heavy particle levels at a given time is small, i.e. the dynamics occurs on a dilute set of orbitals, the approximation applied is justified.

For any certain problem with conduction electron assisted hopping between a heavy and a light orbital the model can be treated in the general schema presented. The large vertex corrections make it promising, that the weaker assisted hoppings can play a determining role in some systems, even if their bare amplitude is weaker then the Coulomb interaction.\footnote{13}

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FIGURES

FIG. 1. The model for orbitals in one dimensions are shown. The circles represents the light $s$ orbitals, and inside them are the heavy $d$ (a) or $p$ (b) orbitals (the $f$ orbitals are not presented as the $p$ orbitals have the same odd overlap as the deep $f$ orbitals). The energy levels, the hoppings and interactions between them are shown in (c). The clear and shadowed areas indicate the opposite signs of the wave functions.

FIG. 2. The $CuO$ plane of the $YBCO$ compounds is shown as an example of the model in two dimensions. The light $O$ and $Cu$ orbitals are found on the sides and on the corners of the squares, respectively. The two orbitals of the apex oxygen below the $Cu$ sites can play the role of the heavy orbitals.

FIG. 3. (a) The bare assisted hopping vertex is shown where the double line and light lines stand for heavy and light particles, respectively. The wavy line denotes the assisted hopping. (b) The bare Coulomb interaction is indicated by dashed lines.

FIG. 4. (a) The Coulomb corrections are shown in second order. The diagrams contribute by logarithms, but they cancel. (b) The corrections appearing in vertex equation are shown by time ordered diagrams where the assisted hopping is renormalized by the Coulomb interaction.

FIG. 5. The contributions to the light particle self-energy are shown by time ordered skeleton diagrams.

FIG. 6. The interaction between the light particles induced by the assisted hopping is shown separately for the different channels: (a) the spin parallel and (b) antiparallel channels. The diagrams are time ordered.

FIG. 7. The renormalization of the light electron dispersion curve is shown in the neighborhood of the Fermi energy $\varepsilon_F$. The renormalization is essential in the range around the Fermi energy characterized by the low energy cutoff $\varepsilon_h$. The large mass enhancement occurs in an energy range $Z\varepsilon_h$. 

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FIG. 8. The selfconsistent equation for the gap $\Delta$ is illustrated.

FIG. 9. The critical temperature as a function of mass enhancement for different values of $q$ [see Eq. (6.5)]

FIG. 10. The band dispersion of the 1D model.

FIG. 11. The prefactor of $(2\tilde{t}\cos^3 \varphi)^2(D/\varepsilon_h)^{2\alpha}$ of the effective interaction $g_1$, $g_2$ and the $\tilde{\chi}(\kappa)$ as a function of different filling for the even case: (a) $\lambda_1 > \lambda_3$ and (b) $\lambda_1 < \lambda_3$.

FIG. 12. The phase diagram of the 1D model in case of (a) even and (b) odd assisted hopping.