Microwave Quantum Radar using a Josephson Traveling Wave Parametric Amplifier

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Abstract—Detection of low-reflectivity objects can be improved by the so-called Quantum Illumination (QI) procedure. However, quantum detection error probability exponentially decays with the source bandwidth. The Josephson Parametric Amplifiers (JPAs) technology utilized as a source, generating pairs of entangled signals called two-mode squeezed vacuum states, shows a very narrow bandwidth limiting the operation of the Microwave Quantum Illumination (MQI) systems. In this paper, for the first time, a microwave quantum radar setup based on quantum illumination protocol and using a Josephson Traveling Wave Parametric Amplifier (JTWPA) is proposed. We experimentally demonstrate the generation and control of three-wave mixing modes produced by a JTWPA, key enabler radiation source for MQI schemes. Measurement results of the developed JTWPA, pumped at 12 GHz, show the capability to generate entangled modes in the X-band, making our MQI system a promising candidate for the detection of stealth objects.

Introduction

The concept of quantum illumination, initially proposed by S. Lloyd [1] and then improved and extended via Gaussian states [2], is a subject of intense research activity, representing one of the very few examples in which “quantum advantage” is achieved in noise-dominated experimental contexts. Quantum lighting, even if based on entangled radiation sources, is capable to exponentially reduce the error probability $P_e$ in the detection of a target compared to the case of traditional sources, precisely in the limit of low Signal to Noise Ratio (SNR ≪ 0.01), where the input signal is no longer entangled. Quantum illumination schematizes the problem in a binary way, i.e., the discrimination between the presence or absence of a target, and considers the limiting case where the signal at the receiver is dominated by noise, meaning that the probability of receiving radiation reflected from the target is low. In general, both the radiation source and the measurement on the receiver need to be optimized and traditional measurements is limited. In the quantum case, the ideal source generates entangled states such as Two-Mode Squeezed Vacuum states (TMSV) [3], [4], and the ideal receiver is the so-called Feed Forward Sum Frequency Generation (FF-SFG) receiver [5], [6]. The quantum strategy provides an exponential reduction of the error probability in discrimination, namely a ~ 6 dB decrease in the error probability exponent compared to the optimal classical benchmark, when the target is immersed in a background of thermal radiation with high luminosity. Preliminary experimental demonstrations recently carried out using entangled microwave sources based on Josephson Parametric Amplifiers (JPA) revealed a target about 1 meter away from the setup in the laboratory room and outside the cryogenic environment [7], [8]. Although the detection is far from being optimal, experiments showed an improvement in SNR compared to a classic two-beam source of chaotic type (radar noise). A further increase in SNR of about 1 dB compared to the classic optimal case was then achieved by a similar entangled source [9] and a digital phase conjugator receiver, avoiding the need for a quantum memory [10], [11]. This experimental proof of principle indicates that alternative detection schemes are possible and that the ultimate gain limit of 6 dB in the error probability exponent can be reached in different ways.
It has been demonstrated that one-dimensional metamaterials embedding Josephson junctions (JJ) promote strong photon-photon-on-chip interactions, allowing experimentalists to control nonlinear wave mixing phenomena in waveguides [12]–[14]. For example, a weak signal within a metamaterial can interact with a strong pump tone at a different frequency, activating the so-called parametric amplification [15], hence the spontaneous energy transport from the pump to the signal mode. The class of devices based on Josephson elements where this phenomenon is promoted is commonly known as Josephson Traveling Wave Parametric Amplifiers (JTWPA). It has been experimentally shown that JTWPAs are amplifiers with high gain, wide bandwidth and that can nearly reach the so-called quantum limit, hence adding to the signal the least amount of noise allowed by the Heisenberg principle. The ability to overcome the quantum limit is related to the so-called phase-sensitive amplification process, where the metamaterial can operate in degenerate mode (Degenerate Parametric Amplifier, DPA), operating on two waves (signal and idler) at the same frequency amplifying and de-amplifying their quadratures. A broadband squeezed light source such as a JTWPA also supplies great advantage towards scalability, because operations can be parallelized by many pairs of strongly separated two-way compressed frequencies exploiting a single device.

In this work, starting from the operating scheme of a Quantum Illumination (QI) protocol, an experimental set-up in view of microwave quantum radar realization using the superconducting parametric amplifier with Josephson traveling-wave junctions operating in an ultra-cryogenic environment, is proposed. We experimentally show a cryogenic operating setup for the generation of entangled modes via three-wave mixing using a JTWPA. The JTWPA design and measurement results are here reported and discussed.

II. BROADBAND QUANTUM ILLUMINATION

Quantum illumination is a photonic quantum remote sensing protocol in which shared entanglement between a signal, probing a target region, and an idler, stored locally in the emitting station, is used to detect the presence of a weakly reflective object with higher efficiency than a strategy that uses a single signal with the same energy, i.e. with the same average number of photons. The more general quantum illumination protocol foresees the preparation of the signal mode $S$ and the idler mode $I$ in an entangled state, which can be expressed as:

$$|\Psi\rangle_{IS} = \sum_n \sqrt{p_n} |\chi_n\rangle_1 |n\rangle_S$$  \hspace{1cm} (1)

where, $|\chi_n\rangle_1$ are the base states of the idler, and $|n\rangle_S$ are the base numbers (Fock states) of the signal.

The quantum correlation associated with the entanglement of this state is ensured by the linear superposition of the states of the bipartite system composed of signal and idler. In an illumination protocol, the receiving station performs a joint measurement on the reflected mode and the idler mode that minimizes the average probability of error $P_e$. It consists of the convex combination of the probability $P_F$ of incorrect identification of the target (false positive) and the probability $P_M$ of non-identification of the target (miss probability):

$$P_e = \lambda P_F + (1 - \lambda) P_M$$  \hspace{1cm} (2)

In particular, classical radar with a number of signal photons $N_S$ can be associated with a probability of error lower than the value:

$$P_{ecl} = \exp \left[ -\eta N_S \left( \sqrt{N_B + 1} - \sqrt{N_B} \right)^2 \right]$$  \hspace{1cm} (3)

where, $\eta$ is the reflectance of the signal transmitted on the path of the return beam when the target is present and $N_B$ is the number of background thermal photons at the same frequency of the signal ones.

On the contrary, the probability of error of the quantum illumination protocol using a TMSV state is shown to be reduced well below the classical minimum value, up to the limit:

$$P_{eTMSV} = \exp \left[ -\eta N_S / N_B \right]$$  \hspace{1cm} (4)

thus, in the presence of many background photons, a quantum illumination protocol allows a significant reduction in the probability of error both of non-identification and of incorrect identification of the target. In Fig. 1, obtained from [18], the basic protocol is schematically represented, in which the transmitting station prepares the entangled idler-signal state, sends the signal to the target region, holds the idler mode, and then carries out a joint measurement of the reflected signal and the idler mode.

The bandwidth of state-of-art JPAs, used for proof-of-principle QI experiments, is limited to tens of megahertz, but much a higher bandwidth can be obtained using traveling wave amplifiers such as JTWPA. In this view, one can note that the number of independent pulses $M$ emitted by a generic light source depends on its bandwidth $B$ and the total integration time $T$ via the simple relation $M = T \cdot B$.

It is then trivial showing that quantum detection probabilities...
depend exponentially by the bandwidth of the light-source, indeed the transmitted power is approximately $P = E \cdot B$, assuming one photon per mode, where $E = h\nu$ is the single-photon energy with $h$ the Planck’s constant and $\nu$ the photon frequency. It follows that a wider bandwidth allows a higher transmitted power due to the reachable larger number of contemporary pulses. Examples where narrow band sources like JPA are used as radiation sources are treated in [9]–[11], while the case of replacing these devices with a broadband source JTWPA is described in [18].

III. RF-SQUID BASED JOSEPHSON TRAVELING WAVE PARAMETRIC AMPLIFIER

The radio frequency Superconducting QUantum Interference Device (rf-SQUID [19]) based JTWPA is a nanostructured superconducting device composed of a repetition of hundreds of rf-SQUIDs (i.e., superconducting loops with a given geometric inductance $L_g$ interrupted by a Josephson junction with a given capacitance $C_J$ and critical current $I_c$) embedded in a Coplanar Waveguide (CPW), as shown in Fig.2(a). The central conductor of the CPW is coupled to ground by means of a repetition of capacitance $C_g$. This capacitive coupling ensure the correct impedance matching of the transmission line with the external wiring. The non-linear behaviour of the JTWPA is ensured by the JJs, which constitute a uniqueness in the solid state physics, being the only known non-dissipative nonlinear passive elements. Furthermore, the nonlinearity of the device can be tuned by means of a DC current bias, promote or suppressing the three-wave mixing (3WM) interaction ($\nu_P = \nu_S + \nu_I$). In the context of MQI, the JTWPA represent the source of non-classical light, playing the same role of a nonlinear crystals in the QI. Microwave tones propagate through these engineered artificial structures and exchange energy with other tones thanks to the inter-modulation energy-preserving phenomena, generating entangled quantum states. It has been experimentally demonstrated that the gain $G$ of a JTWPA can reach values of about 20 dB over an unprecedentedly wide bandwidth [8]. It has been shown that quantum correlations survive when JTWPA are located in a realistic environment characterized by non-zero thermal noise only when the amplifier source expresses a high gain figure of merit [20]. For this reason this feature turns out to be an important indicator of the radiation source performance. A photograph of a first prototype of rf-SQUID based JTWPA can be seen in Fig. 2(b). The whole chip, realized with a standard micro-fabrication aluminum technology, has an area of 1 cm$^2$ and is composed of 15 sections of CPW, each embedding 66 rf-SQUIDs, connected by curved sections of CPW. The transmission line contains a total amount of 990 non-hysteretic rf-SQUIDs, characterised by the following target values for the circuit parameters: ground capacitance $C_g = 13.0 \text{fF}$, geometrical inductance $L_g = 45 \text{pH}$, Josephson capacitance $C_J = 25.8 \text{fF}$ and Josephson critical current $I_c = 1.5 \mu\text{A}$. To determine the physical layout for $L_g$ (realized as a planar meander inductor), and $C_g$ (realized as a planar interdigitated capacitor) that corresponds to the target values, a finite element simulation exploitig Sonnet have been performed. The layout of the JJs, realized with the Niemeyer-Dolan technique [21], was instead determined selecting a proper junction area.
The JTWPA is excited with a signal tone at frequency $\nu_S = 3.3\, \text{GHz}$, for three different values of the driving pump tone at frequency $\nu_P = 6.75\, \text{GHz}$.

($\approx 0.4\, \mu\text{m}^2$) and a critical current density. This latter quantity can be tuned changing the oxide thickness of the insulating barrier of the JJ, acting on the product $\sqrt{p_{O_2}} \cdot t$, where $p_{O_2}$ is the partial pressure of oxygen in the oxidation chamber, and $t$ is the oxidation time [22].

IV. MEASUREMENT SETUP AND RESULTS

A preliminary cryogenic characterization of the aforementioned device, in terms of 3WM capabilities and gain, was performed exploiting the measurement setup schematized in Fig. 3, hosted into a dry dilution refrigerator capable of reaching a base temperature of 15 mK. The cryostat contains several stages with different cooling powers. The JTWPA is positioned at the bottom of the cryostat, in the coldest stage. This level of cooling is not only necessary because of the superconducting nature of the JTWPA device, but it guarantees state-of-the-art thermal noise background radiation that otherwise would destroy the entanglement generated by the parametric amplification. In order to quantify nonlinear effects, a two-tones measurement was performed by supplying in input a weak signal tone (supplied by port 1 of an Agilent E5071C 300 kHz-20 GHz VNA$^1$) and a driving pump tone (coming from a Rohde&Schwarz SMA100B 8 kHz-20 GHz signal generator). Microwave signals enter the dilution refrigerator and pass through several attenuation stages, getting to the JTWPA after passing a directional coupler (Mini-Circuits ZUDC10-02183-S+) and a first isolation stage provided by a circulator (LNF-CIC412A). The microwave tones are then detected at room temperature after passing through a High Electron Mobility Transistor amplifier (HEMT, LNF-LNC620C) placed on the 4 K stage, which provides more than 30 dB of amplification. This latter has a negligible contribution in terms of noise of the readout setup according to the well known Friis formula for a multistage amplifier. The room temperature switch $SW_1$ allows choosing as a receiver, a spectrum analyzer (Signal Hound SM200B 100kHz-20 GHz) on port 2 or a VNA on port 1, allowing to perform power-spectra or scattering parameters measurements, respectively. The cryogenic electromechanical switch $SW_2$ allows to adapt the setup for transmission or reflection measurements. In both configurations, the output microwave passes through an isolation stage realized by means of two circulators (LNF-CIC412A). A current generator (Keithley 6221) connected to the JTWPA via a couple of bias tees (Marki BT-0018) provides the DC current bias to the device. The entire experimental setup is designed in order to guarantee optimal working conditions in the range 4-12 GHz, essentially limited by the bandwidth of the commercially available cryogenic circulators introduced to prevent the back-action of the HEMT on the JTWPA.

A first characterization of the JTWPA, presented in Fig. 4, was performed measuring the power of the output idler tone $P_3$ generated via 3WM, as a function of the DC bias current $I_{DC}$. The device was fed with a $\nu_P = 6.75\, \text{GHz}$ driving pump tone, with three different power values $P_P (-60, -55, \text{and} - 50\, \text{dBm})$, and a $\nu_S = 3.3\, \text{GHz}$ signal tone with $P_S = -64\, \text{dBm}$ power. For this mixing process the idler is expected to be generated at $\nu_I = \nu_P - \nu_S = 3.45\, \text{GHz}$. The power values are here considered at the input and output ports of the device.

Due to the natural Kerr-nonlinearity [18], [20] of a rf-SQUID with no current bias, one would expect that the 3WM idler would present a minimum at zero $I_{DC}$. Nonetheless Fig. 4 reports a shift of the minima that can be attributed to magnetic field trapping during the cooling phase of the dilution refrigerator. Moreover, the suppression of the 3WM idler tone

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Parameter & Value \\
\hline
3WM power & $P_3$ \\
\hline
DC bias & $I_{DC}$ \\
\hline
\end{tabular}
\caption{Summary of measurement results.}
\end{table}

$^1$The commercial instruments identified in this paper are intended in order to specify the experimental procedure as adequately as possible. Such identification doesn’t imply a recommendation or an endorsement, nor implies that the identified equipment are the best available for the presented purpose.
via $I_{DC}$ is not complete since data reported in Fig. 4 does not reach the noise floor of the setup represented by the dashed horizontal line for every $I_{DC}$ value. It has to be noticed that the modulation of the 3WM process here reported is limited to around 10 dB, since it is reasonably affected by non-idealities of the JTWPA and the surrounding environment.

Even in this preliminary configuration, Fig. 5 reports the crucial figure of merit of the JTWPA under study, being the gain, evaluated by means of pump-on-pump-off technique. There, the effect of the pump power ($P_p$) on the signal gain for the two relevant regimes (degenerate and non-degenerate) of the amplifier is reported. The power gain induced by Josephson non-linearities reaches values around 25 dB, showing that the JTWPA acts as a metamaterial efficiently promoting 3WM processes, the key enabler for the preparation of TMSV states [18].

V. CONCLUSION

In this paper, a novel cryogenic setup toward MQR based on Quantum Illumination Protocol using a JTWPA is proposed. Measurement results of the designed and developed JTWPA pumped at $\nu_p = 12$ GHz show 3WM tunability and gain up to 25 dB with potential ultra-wide signal bandwidth. As quantum detection probability exponentially decays with the source bandwidth, the proposed ultra-wide-band MQR setup represents a good candidate to detect stealth objects.

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