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Mixed convection Jeffrey fluid flow over an exponentially stretching sheet with magnetohydrodynamic effect

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The steady two-dimensional MHD mixed convection boundary layer flow and heat transfer of a Jeffrey fluid over an exponentially stretched plate is investigated. The governing partial differential equations are first reduced to nonlinear ordinary differential equations, before being solved numerically using an implicit finite difference scheme. Local similarity solutions are obtained for some embedded parameters, such as Deborah number $\beta$, mixed convection parameter $\lambda$, Prandtl number $Pr$ and Hartmann number $H$, are analyzed and discussed. © 2016 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). [http://dx.doi.org/10.1063/1.4945401]

I. INTRODUCTION

In very-high-power output devices, the forced convection alone is not enough to dissipate all the heat. As such, combining natural convection with forced convection (mixed convection) will often give desired results.1 The phenomenon of mixed convection mainly occurs in many industrial and technical applications, such as the cooling of nuclear reactors during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors, electronic devices cooled by fans and etc.2 Owing to its great practical applications, it is no wonder the subject has caught researchers’ attention over the last half century, leading to studies in the field involving diverse aspects and physical conditions. Kliegel3 obtained the theoretical solution of heat transfer by mixed convection about a vertical flat plate and Szewczyk4 analyzed the combination of forced and free convection laminar flow. Merkin5 considered the mixed convection problem, taking into account that heat was supplied to the fluid at the wall at a constant rate by using series expansion. Throughout the years, lots of studies have been conducted.6–12

The MHD boundary layer flow over stretching sheet is encountered numerously in many industrial and engineering processes. The quality of the final desired product very much depends on the heat transfer and stretching rate. For example, in metal extrusion processes, the metal sheets need to be “straightened” by stretching the sheets. Drawing the plates in an electrically-conducting fluid subjected to a transverse magnetic field is used to control the cooling rate, which improves the final output.13 The velocity of the stretching sheet may be linear and may not necessarily be linear.14 Nadeem et al.15 studied boundary layer flow of Jeffrey fluid over an exponentially stretching surface with thermal radiation effect, taking into account two cases of heat transfer analysis, i.e prescribed exponential order surface temperature (PEST) and prescribed exponential order heat flux (PEHF). Cortell16 performed numerical analysis of boundary layer flow induced by continuous stretching velocity $u_w(x) \sim x^{1/3}$ with radiation effect. Mustafa et al.17 addressed a steady flow of Maxwell nanofluid induced by an exponentially stretching sheet subjected to convective heating, and Sharada...
The appropriate similarity variables are:

\[ \eta = y \sqrt{\frac{u_o}{2vL}} \exp \left( \frac{x}{2L} \right), \quad u = u_o \exp \left( \frac{x}{L} \right) f'(\eta), \]

\[ v = -\frac{u_o \nu}{2L} \exp \left( \frac{x}{2L} \right) \left[ f(\eta) + \eta f'(\eta) \right], \]

\[ \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}. \]

where \( \eta \) is the similarity variable, \( f \) is the dimensionless stream function and \( \theta \) is the dimensionless temperature. Substituting (5) into (1)–(3), Eq. (1) is automatically satisfied and Eqs. (2)-(3) are reduced to the following nonlinear ordinary differential equations

\[ f''' + \beta \left( f' f''' + \frac{3}{2} f'' f' - \frac{1}{2} f f'' \right) + (1 + \lambda_1) \left[ f f'' - 2(f')^2 + 2\lambda \theta - \nu \theta' - H^2 f' \right] = 0, \]

\[ \theta'' + Pr (f \theta' - 4 f' \theta) = 0, \]

II. ANALYSIS

Consider a steady two-dimensional incompressible Jeffrey fluid flow over an exponentially stretching sheet in the presence of a uniform magnetic field applied normally to the sheet. The stretching sheet is assumed to be stretched vertically with velocity \( u_w = u_o \exp \left( \frac{x}{L} \right) \) and having surface temperature \( T_w = T_{\infty} + T_o \exp \left( \frac{-x}{L} \right) \), where \( u_o \) and \( T_o \) are constant, \( T_{\infty} \) is the temperature far away from the stretching sheet and \( L \) is the reference length. It should be pointed out that \( T_w > T_{\infty} \) and \( T_w < T_{\infty} \) corresponds to the assisting and opposing flow, respectively. Under the Boussinesq and boundary layer approximations, the governing boundary layer equations and heat transfer for the problem are

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{v}{\lambda_1} \left[ \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y} \right] + g \beta T (T - T_{\infty}) - \frac{\sigma B^2}{\rho} u, \]

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha}{\lambda_2} \frac{\partial^2 T}{\partial y^2}, \]

where \( u \) and \( v \) are the velocity components in the \( x \) and \( y \) directions, respectively; \( \lambda_1 \) is the ratio of the relaxation times; \( \lambda_2 \) is the relaxation and retardation times; \( v = \frac{\mu}{\rho} \) is the kinematic viscosity, where \( \mu \) is the coefficient of fluid viscosity and \( \rho \) is the fluid density. \( g, \beta T, \sigma \) and \( \alpha \) are gravitational acceleration, thermal expansion coefficient, electrical conductivity and thermal diffusivity, respectively. Here, \( B \) is the magnetic field which is assumed as \( B = B_o \exp \left( \frac{-x}{L} \right) \) where \( B_o \) is a constant magnetic field. Note that (2) reduced to viscous flow when \( \lambda_1 = \lambda_2 = 0 \). The corresponding boundary conditions for the flow problem are

\[ u = u_w(x), \quad v = 0, \quad T = T_{\infty} \quad \text{at} \quad y = 0, \]

\[ u \to 0, \quad \frac{\partial u}{\partial y} \to 0, \quad T \to T_{\infty} \quad \text{as} \quad y \to \infty. \]
and the transformed boundary conditions can be written as

\[
\begin{align*}
f(0) &= 0, \quad f'(0) = 1, \quad \theta(0) = 1 \quad \text{at} \quad \eta = 0, \\
f'(\eta) &\to 0, \quad f''(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty.
\end{align*}
\]

(8)

Here, prime denotes differentiation with respect to \( \eta \) and \( \beta = \frac{\alpha \eta}{L} \) is the Deborah number. Since \( \beta \) depends on \( x \), the resultant ordinary differential equation are not pure similarity equations for the whole flow domain but only represent local similarity for any given values of \( x \).

\[ \lambda = \frac{Gr}{Re} \]
\[ \lambda_1 \approx \frac{H^2}{\nu} \]
\[ Pr = \frac{\lambda}{8} \]

is the mixed convection parameter with \( Gr = \frac{g \beta T L^3}{v^2} \) and \( Re = \frac{\alpha u L}{\nu} \) are the local Grashof number and local Reynolds number, respectively. \( H^2 = \frac{2 \sigma L Re^2}{\rho u^2} \) is the Hartmann number and \( Pr = \frac{T}{\nu} \) is the Prandtl number. It should be pointed out that \( \lambda \) is a constant, with \( \lambda > 0 \) and \( \lambda < 0 \) represent assisting flow (heated plate) and opposing flow (cooled plate), respectively, while \( \lambda = 0 \) corresponds to forced convection and \( \lambda \neq 0 \) corresponds to mixed convection regime.

From an engineering point of view, there are two important physical quantities of interest; i.e. the skin friction coefficient \( C_f \) and the local Nusselt number \( Nu_x \), which could be obtained once the values of \( f''(0) \) and \( -\theta'(0) \) are found.

### III. RESULTS AND DISCUSSION

Eqs. (6) and (7) subject to boundary condition (8) have been solved numerically using the method described in the book by Cebeci and Bradshaw, for some arbitrary values of Deborah number \( \beta \), the mixed convection parameter \( \lambda \), the Hartmann number \( H \) and the Prandtl number \( Pr \). The ratio of the relaxation and retardation times \( \lambda_1 \) is held fixed (=0) throughout the study. Here, we used a uniform grid of \( \Delta \eta = 0.01 \) and found it to be satisfactory for a convergence criterion of \( 10^{-5} \). The boundary layer thickness \( \eta \) chosen starts from 6 up to 15 to ensure the velocity and the temperature profiles approach zero, which validates the boundary condition (8).

The numerical results obtained for \( f''(0) \) and \( -\theta'(0) \) when \( Pr = 7 \) for both assisting \( (\lambda > 0) \) and opposing flows \( (\lambda < 0) \) are depicted in figures 1 and 2, respectively. These figures suggest that an assisting buoyancy flow creates an increase in \( f''(0) \) and \( -\theta'(0) \), while an opposite phenomena is found to occur for the opposing buoyant flow. This is due to the fact that as \( \lambda \gg 0 \), the flow is assisted by the buoyancy force to increase the velocity, which in turn increases the skin friction at

FIG. 1. The values of \( f''(0) \) for several values of \( H, \beta \) and \( \lambda \) when \( Pr = 7 \).
FIG. 2. The values of $-\theta'(0)$ for several values of $H$, $\beta$ and $\lambda$ when $Pr = 7$.

The effect of the Hartmann number $H$ on the $f''(0)$ and $-\theta'(0)$ is also depicted on the same figures, i.e. figures 1 and 2. It is seen that as $H$ increases, both of $f''(0)$ and $-\theta'(0)$ decrease. This is because of the Lorentz force, which acts against the flow and slows down the fluid velocity, decreasing friction at the surface. This is in line with the velocity profile plotted in figure 3. Even though the local heat transfer rate presented by $-\theta'(0)$ decreases as $H$ increases, it is found that the temperature increases as $H$ increases, which results in a thicker thermal boundary layer, as displayed in figure 4.

Figures 1 and 2 also displayed the effect of the Deborah number $\beta$ towards the flow and thermal fields. An interesting behaviour is found to happen when the flow is assisted by the buoyancy force, i.e when $\lambda < 0$.
force, namely at $\lambda = \lambda_c$. At this particular point, the flow retains similar value of $f''(0)$ and $-\theta'(0)$ respectively, regardless whether the flow is Newtonian or non-Newtonian. For $\lambda < \lambda_c$, the Deborah number is found to increase both $f''(0)$ and $-\theta'(0)$, and beyond this value ($\lambda > \lambda_c$), an opposite trend is observed. On top of that, the value of $\lambda_c$ is found to arise at a higher value of the mixed convection parameter $\lambda$ when $H = 4$ compared with $H = 1$.

The effect of Prandtl number when $\lambda = 5$ on the $f''(0)$ and $-\theta'(0)$ are depicted in figures 5 and 6, respectively. These figures reveal that an increase in $Pr$ is found to decrease both $f''(0)$ and $-\theta'(0)$. However, a significance drop is more pronounced for the $-\theta'(0)$ as compared with the $f''(0)$. The existence of $Pr$ term in Eq. (5) is the main reason behind such a situation. Additionally, the effects of $H$ and $\beta$ are more noticeable for the flow field as compared with the thermal field. This is expected as $H$ and $\beta$ only take place in the flow field given by Eq. (6). As such, the results obtained for $-\theta'(0)$ are almost similar, even with different values of $H$ and $\beta$ when $Pr$ is fixed, as depicted in figure 6. Nonetheless, the close up numerical results obtained shows that the $-\theta'(0)$ is slightly higher when $H = 4$ compared with $H = 1$.
IV. CONCLUSION

The present study considered the local similarity solution for MHD mixed convection Jeffrey fluid flow over an exponentially stretching sheet. The numerical results obtained for the $f''(0)$ and $-\theta'(0)$ are found to exist for any values of assisting flow ($\lambda > 0$) and are limited for the opposing flow ($\lambda < 0$). As the Deborah number $\beta$ increases, an interesting phenomenon occurred at a certain value of $\lambda_c$, mainly when the flow is being assisted.

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