Exploiting the quantum Zeno effect to beat photon loss in linear optical quantum information processors

Federico M. Spedalieri, Hwang Lee, Marian Florescu, Kishore T. Kapale, Ulvi Yurtsever, and Jonathan P. Dowling

Quantum Computing Technologies Group, Jet Propulsion Laboratory, California Institute of Technology, Mail Stop 126-347, 4800 Oak Grove Drive, Pasadena, California 91109-8099

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803-4001

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We devise a new technique to enhance transmission of quantum information through linear optical quantum information processors. The idea is based on applying the Quantum Zeno effect to the process of photon absorption. By frequently monitoring the presence of the photon through a QND (quantum non-demolition) measurement the absorption is suppressed. Quantum information is encoded in the polarization degrees of freedom and is therefore not affected by the measurement. Some implementations of the QND measurement are proposed.

I. INTRODUCTION

Photons are the information carriers in linear optical quantum information processing devices [1,2]. Information is encoded in the state of the photon (e.g., photon number or polarization), and the processing is accomplished by sending the photon through a system of linear optical elements and using photodetectors to measure the outcome. The photons are usually routed through optical fibers or waveguides, that can transmit them from one part of the device to another and act as delay lines or even as primitive memories, by letting the photon go around a loop of fiber. This useful property of a fiber as a transmission line has its origin in the interaction between the photon and the atoms in the fiber. However, this interaction has the undesirable side effect of photon absorption, which introduces errors in the quantum information processing. Thus it has been an active area of research to develop new approaches to suppress photon absorption in optical processors.

Since we cannot amplify a quantum signal, our efforts should be aimed at preventing its degradation due to the loss of photons caused by the interaction between the fiber and the electromagnetic field. For this purpose, we propose to use the well-known Quantum Zeno effect [3], which has already been applied to quantum information processing [4]. To understand the idea we note that this loss of photons can be viewed as the evolution of a quantum state in which the initial state contains one photon and, due to the interaction between the photon and the fiber, is transformed into a state in which the photon has been absorbed. The Quantum Zeno effect tells us that by frequently monitoring the presence of the photon we can inhibit this evolution, and hence prevent the photon from being absorbed by the fiber. It may seem rather strange that we propose to preserve the quantum information encoded in the system by measuring it, since measurement usually destroy quantum coherence. The key idea is to encode the information in the polarization degrees of freedom of the photon, while performing a quantum non-demolition measurement only on the photon number. In this way no information about the polarization state is acquired, but frequently registering the photon presence prevents it from being absorbed.

II. THE MODEL

To simplify our discussion, we will couch our argument in terms of single photon loss in telecom fibers. However, the scheme applies to loss in any type of linear optical quantum information processor.

To start our analysis we first need to identify the physical mechanisms that produce photon loss on a fiber. At the usual telecom wavelength of 1.55µm, there are two main loss mechanisms whose deleterious effect on the photon transport have approximately the same magnitude: (i) the Rayleigh scattering of photons on the inhomogeneities of the refractive index along the fiber; (ii) the absorption of photons due to their interaction with the vibrational excitations (phonons). In principle, the effect of Rayleigh scattering can be lowered by improving construction to minimize the inhomogeneities in the fiber. On the other hand, the interaction with phonons will always be present, and hence it is useful to design a procedure to reduce its effects.

Our goal then is to analyze the interaction between the signal photon and the phonons in the fiber, and show that we can prevent the loss of the photon due to this interaction. For this purpose we consider the following Hamiltonian

\[ H = H_{\text{signal}} + H_{\text{bath}} + H_{\text{int}}, \]

where \( H_{\text{signal}} = \sum \omega_k a_k^\dagger a_k \) is the Hamiltonian of the propagating signal photon, \( H_{\text{bath}} = \sum \hbar \omega_i r_i^\dagger r_i \)

*Electronic address: Federico.Spedalieri@jpl.nasa.gov
is the Hamiltonian of the phonon bath, and $H_{\text{int}} = \sum_k (g_{kk}a_k^\dagger a_k^\dagger + g_{kk}'a_k^\dagger a_k)$, is the interaction Hamiltonian. The precise form of $H_{\text{int}}$ is not essential to understand the idea behind our scheme, although it is important in determining the regime of validity of this proposal.

The signal will consist of a photon pulse of the form $|\psi_{\text{signal}}(t)\rangle = \sum_k f(k)|1_k\rangle(\alpha|H\rangle + \beta|V\rangle)$, where $|H\rangle$ and $|V\rangle$ represent horizontal and vertical polarizations respectively. The quantum information carried by the signal is encoded in the values of the coefficients $\alpha$ and $\beta$. The function $f(k)$ is chosen such that the pulse has a finite spatial extent and the mean number of photons in the pulse is 1.

The initial state of the photon and the fiber can be written as $|\Psi(t=0)\rangle = |\psi_{\text{signal}}(t)\rangle|\phi_{\text{bath}}\rangle$, where $|\phi_{\text{bath}}\rangle$ is the initial state of the phonons in the fiber. To simplify the notation, let us assume that initially the we have no phonons in the fiber, so we have $|\phi_{\text{bath}}\rangle = |00\cdots\rangle$. As the pulse propagates, the interaction part of the Hamiltonian will entangle the photons with the phonon in the fiber. At some later time $t$, the state of the system will have the form

$$|\Psi(t)\rangle = \left(\sum_k f(k,t)|1_k\rangle(\alpha|H\rangle + \beta|V\rangle)\right)|00\cdots\rangle +$$
$$+ \sum_j c_{0j}(t)|0\rangle_{\text{photon}}|0\cdots\rangle_{j^\dagger\text{mode}}. \quad (2)$$

The first sum represents the free propagation of the photon, while the second sum shows that the photon can be absorbed only by creating a phonon in one of the modes of the fiber. To find the time dependent functions $f(k,t)$ and $c_{0j}(t)$ we can insert (2) into the Schrödinger equation and solve the resulting differential equations. However, this will not be necessary for our particular purpose.

Our goal is to use the Quantum Zeno effect to suppress the absorption of the photon due to its interaction with the phonons in the fiber. This is accomplished by performing frequent quantum non-demolition (QND) measurements of the photon number of the pulse. If we measure the photon number at time $t = \tau$ and the outcome is 1, then the state of the system after the measurement is given by

$$|\Psi(\tau)\rangle = \left(\sum_k f(k,\tau)|1_k\rangle(\alpha|H\rangle + \beta|V\rangle)\right)|00\cdots\rangle. \quad (3)$$

Since the photon number QND measurement does not affect the polarization degrees of freedom, the quantum information carried by them is not affected by the measurement. This is the key idea of our proposal. The state in Eq. (3) is the new initial condition to be propagated for another interval $\tau$ until the next QND measurement. Note that this initial condition might be different from the one at $t = 0$, since the functions $f(k,\tau)$ might differ from the functions $f(k)|t=0$. However this is not a problem, because the quantum information carried by the pulse is independent of its form.

The Quantum Zeno effect tells us that these frequent QND photon number measurements can prevent photon absorption. To see how this works, let $P_s(t)$ be the survival probability of the photon pulse at time $t$ as it propagates through the fiber. From Eq. (2) we have that

$$P_s(t) = \sum_k |f(k,t)|^2. \quad \text{Due to the unitarity of quantum evolution, is a well-known fact that this survival probability must follow a quadratic decay for small values of $t$. If the system is interacting with a continuum of modes, it is also well-known that this decay becomes exponential for longer times. The time scale at which the transition occurs between these two regimes strongly depends on the details of the interaction and the physical properties of the interacting systems. We will come back to this point later. For now let us analyze the effects of frequent measurements during the quadratic decay. Let $\tau$ be the time interval between measurements. The survival probability at time $\tau$ will be given by $P_s(\tau) \approx 1 - (\gamma\tau)^2$, for some constant $\gamma$. With probability $P_s(\tau)$ the outcome of the photon number QND measurement is 1, and we know with certainty that the pulse has exactly 1 photon after the measurement. This pulse evolves again for a time $\tau$, and the probability of it containing exactly 1 photon is again given by $P_s(\tau)$. After performing $N$ QND measurements separated by intervals $\tau$, the probability of survival at time $T = N\tau$ is given by

$$P_s(T) = \left(1 - (\gamma\tau)^2\right)^N$$
$$= \left(1 - \frac{T\gamma^2}{N}\right)^N$$
$$\approx e^{-(\gamma^2\tau)T}, \quad (4)$$

where we considered the large $N$ limit on the last line. We see that the decay is still exponential, but the decay rate depends on the parameter $\tau$. By choosing smaller values of $\tau$ (i.e., increasing the frequency of the measurements), we can suppress the decay rate and hence diminish the probability of the photon being absorbed.

To apply this idea to the problem of photon loss, we consider the usual telecom fiber and insert a QND measurement device at regular intervals. Since the successful application of the Quantum Zeno effect requires that successive measurements be made while the decay of the survival probability is quadratic, and this occurs for times shorter than a certain value $T_\eta$, the QND devices must be placed such that the distance between them is less than $v_f T_\eta$, where $v_f$ is the velocity of the pulse in the fiber.

The time scale given by $T_\eta$ is an important parameter in our proposal since it determines how close together the QND devices must be inserted along the fiber. A calculation of this value from first principles appears to be a daunting task, since it will depend crucially on the details of the photon-phonon interaction, and in particular on the exact form of the density of phononic modes on the fiber. A simplified calculation of this time scale is highly dependent on the approximations used to model the density of states, giving answers that differ by several orders of magnitude [3]. This suggests that this type
of calculation would not be reliable unless the density of modes is known with great detail. However this time scale should not be difficult to measure. To that end we propose the following simple experiment: measure the single photon loss for different lengths of fiber, starting with a length for which exponential decay of the output signal has been established, and repeat the experiment for shorter lengths until a departure from the exponential decay is observed. That will happen about the time scale given by \( T_q \), for which the decay becomes quadratic.

It is interesting to note that the same idea we used to enhance the transmission of quantum information through a fiber or other linear optical quantum information processor, can be used to design an improved memory device for optical quantum computation. Instead of inserting many QND devices along a transmission line, we can just insert only one device on a loop of fiber. Again by encoding the quantum information on the polarization degrees of freedom of the signal pulse and performing a QND measurement of the photon number, we can suppress the absorption of the pulse by the fiber, and hence increase the time we can store the quantum information in the fiber. This alternative use could be viable even if the transmission application turns out not to be practical because it requires too many QND devices.

### III. QND DEVICES

We will now present a few ideas for the implementation of the QND device required by our proposal. First, let us note that we only need a QND photon number measuring device that can distinguish between 1 and 0 photons. We do not need it to be able to discriminate for higher photon number, since our signal pulse will be constructed such that it contains only one photon. Then, a simple CNOT gate acting on the \( \{|0\rangle, |1\rangle\} \) subspace and one ancilla mode are sufficient to implement the QND measurement. If we prepare the ancilla in the \( |0\rangle \) state, and use it as the target of a CNOT gate controlled by the presence or absence of a photon in the fiber, by measuring the ancilla we can determine whether the gate has been applied or not, and hence whether there was a photon in the fiber or not. We can even apply the KLM \(^1\) scheme that uses linear optical elements to implement the CNOT gate. This implementation of the gate is non-deterministic but, its probability of success can be boosted arbitrarily close to unity by adding more and more ancillas.

We now proceed to give possible implementations for the QND device in quantum optical systems. Strong nonlinearities are essential to achieve a non-destructive measurement of the photon number state. Such strong nonlinearities can be attained through either atomic coherence effects, for example, electromagnetically induced transparency (EIT) or strong atom-field interaction obtained in the cavity-QED schemes.

An interesting proposal for implementation of the QND device exists that can be adopted for the present scheme. Munro et al. \(^5\) have proposed a high-efficiency QND single photon number resolving detector based on cross-Kerr nonlinearity obtainable through EIT. To illustrate its mechanism, we note that the QND Hamiltonian can be written as \( H_{QND} = \hbar \chi s^d s^p \), where \( s \) and \( p \) are the ladder operators for the signal and probe fields respectively. The input signal in a fock state \( |n_s\rangle \) can be detected non-destructively through the phase acquired by a coherent probe field \( |\alpha_p\rangle \):

\[
|\Psi(t)_{\text{out}}\rangle = e^{i \chi t s^d s^p} |n_s\rangle |\alpha_p\rangle = |n_s\rangle |\alpha_p e^{-i n_s \chi t}\rangle.
\]

Thus, a momentum quadrature measurement on the probe field through an efficient homodyne device ascertains the presence of the signal photon non-destructively.

This Kerr-nonlinearity based QND device can be implemented through a 4-level EIT scheme shown in Fig. II (a). The atomic medium starts in the initial state \( |1\rangle \) and remains in that state after the interaction with the signal except of an overall phase acquired by the probe according to Eq. 5. A momentum-quadrature measurement is performed on the probe field to reveal the presence of the signal photon non-destructively. Munro et al. \(^5\) carry out a detailed analysis of this four level scheme and obtain various conditions necessary for efficient QND measurement on the signal photon.

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**FIG. 1:** EIT based QND device (level schemes): (a) A general 4-level scheme: QND measurement of a green signal photon \( (\Omega_s) \) is achieved through a phase quadrature measurement of the highly-detuned blue probe field \( (\Omega_p) \). The strong red driving field \( (\Omega_d) \) facilitates EIT. (b) Level scheme for the polarization-insensitive QND device: The levels are denoted by a pair of hyperfine quantum numbers as \( |F,m_F\rangle \). The color code for various fields is the same as in (a). Probe and drive fields are linearly polarized to facilitate extraction of the photon number of the signal photon immaterial of its linear or circular polarization state.

It is, however, important to note that the Munro scheme, when applied to an actual atomic system, would depend on and affect the polarization state of the signal photon. For the success of our proposal the QND device should be insensitive to the polarization of the incoming photon. Moreover, the measurement process should neither change this polarization state nor leave its imprint on the atom or the probe.

We note that radiative atomic transitions couple to a specific polarization states of light and devise an implementation of a polarization-insensitive QND device using hyperfine sublevels of a certain atom as shown.
in Fig. (IIb). It can be easily seen that model in Fig. (IIb) has two level schemes of Fig. (IIa) (namely |1⟩ − |2⟩, |3⟩ − |4⟩) embedded in it. We require the drive and probe fields to be linearly polarized. Thus, the drive field couples to both |2⟩ − |3⟩ and |2⟩ − |3⟩ transitions according to the hyperfine-transition selection rules. This is possible because linearly polarized field can be represented as a superposition of equal-magnitude left- and right-circularly polarized components. Similarly, through appropriate choice of hyperfine quantum numbers, the linearly polarized probe field can be set to couple a single |3⟩ − |4⟩ transition. It can be easily seen that the probe field would not carry any information about the polarization of the signal photon as the oscillator strengths of |1⟩ − |2⟩ and |1⟩ − |2⟩ transitions are the same. Thus, immaterial of the polarization state of the incoming photon (linearly or circularly polarized) the level scheme of Fig. (IIb) would respond to it in the same way and a quadrature measurement on the probe would neither disturb nor reveal this state.

It can be noted that the level scheme used in Fig. (IIb) is fairly general, as the values for the quantum number $F_c$, namely $f_1$, $f_2$, $f_3$, and $f_4$, are unspecified and only the specific $m_F$ values are assumed. One can come up with various examples of atoms that have level-scheme matching the one proposed here. A detailed analysis of this scheme will be carried out elsewhere. It is useful to note that such a polarization-insensitive QND device could be realized in a variety of systems including cold atomic clouds, single alkali atom trapped in a high-Q cavity and rare earth doped telecommunication fiber.

Furthermore, we note that the quantum phase gate $(|c, t⟩ \rightarrow \exp(i\phi\delta_{c,t}) |c, t⟩$ with a phase shift ($\phi = \pi$) and single qubit rotations (on |t⟩) could be combined to perform a CNOT operation $|c, t⟩ \rightarrow |c, c \oplus t⟩$, where |c⟩ and |t⟩ are the control and target qubits respectively. This opens up another powerful possibility for the implementation of the QND device, through the CNOT gate, based on well developed tools of cavity-QED. There are both experimental [10] and theoretical [11] proposals for realizing a quantum phase gate in cavity-QED systems for photonic qubits. One can easily adopt these techniques by coupling the signal photon to the cavity to perform the role of the control qubit. Once again the main requirement would be to make these schemes insensitive to the polarization of the signal photon.

The cavity-QED techniques can be easily extended to the regime of fiber optics by replacing the bulky superconducting cavities by either Fabry-Perot cavities, which are much easier to integrated into the telecommunication fiber, or the microresonators, which can be easily coupled to the fiber through the evanescent field of the signal photon. To note, Kimble’s group at caltech has done a lot of work on extending cavity-QED to both the Fabry-Perot cavities and microresonators [12].

In this letter we have presented a new scheme to enhance the transmission of quantum information through linear optical quantum information processing systems by taking advantage of the Quantum Zeno effect. We propose encoding the quantum information in the polarization degrees of freedom of the photon. To overcome the losses due to absorption of the photon by the fiber, we propose to frequently monitor the presence of the photon by performing a QND measurement of the photon number. Due to the Quantum Zeno effect, this procedure can suppress the absorption if the measurements are performed frequently enough. The quantum information carried by the photon is not disturbed since the polarization state is not affected by the QND photon number measurement. We also presented some possible implementations of this QND measurement, and proposed an experimental method to determine how often the measurements should be performed. This scheme could also be used as a memory, by preventing a photon stored in a loop of fiber from being absorbed, hence preserving the quantum information it carries for a longer time.

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