Advantages of Multiple Detectors for the Neutrino Mass Hierarchy
Determination at Reactor Experiments

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Abstract

We study the advantages of a second identical detector at a medium baseline reactor neutrino experiment. A major obstruction to the determination of the neutrino mass hierarchy is the detector’s unknown nonlinear energy response, which even under optimistic assumptions reduces the confidence in a hierarchy determination by about 1\(\sigma\) at a single detector experiment. Various energy response models are considered at one and two detector experiments with the same total target mass. A second detector at a sufficiently different baseline eliminates this 1\(\sigma\) reduction. Considering the unknown energy response, we find the confidence in a hierarchy determination at various candidate detector locations for JUNO and RENO 50. The best site for JUNO’s near detector is under ZiLuoShan, 17 km and 66 km from the Yangjiang and Taishan reactor complexes respectively. We briefly describe other advantages, including a more precise determination of \(\theta_{12}\) and the possibility of a DAE\(\delta\)ALUS inspired program to measure the CP-violating phase \(\delta\) using a single pion source about 10 km from one detector and 20 km from the other. Two identical detectors provide a better energy resolution than a single detector, further increasing the confidence in a hierarchy determination.

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1 Introduction

In 2002 Petcov and Piai [1] proposed that the shape of the oscillated $\nu_e$ spectrum observed at a medium baseline from a nuclear power plant can in principle be used to reconstruct the $\nu_e$ survival probability whose fine structure in turn can determine the neutrino mass hierarchy. This fine structure consists of two beating modes of atmospheric oscillations with amplitude $\sin^2(2\theta_{13})$. Last year’s discovery that $\theta_{13}$ is large [2, 3] implies that these oscillations are large enough to be observed. As a result at least two experiments, JUNO [4] and RENO 50 [5], have been proposed to measure the reactor neutrino spectrum using inverse $\beta$ decay and so determine the hierarchy.

These experiments will have to face some challenges which have never before been faced. For example, as the largest and most transparent liquid scintillator detectors in history, the optical properties of the scintillator will need to be understood as never before. Repeated Rayleigh scattering means that photons can scatter multiple times before being detected, smearing the pulse shapes which are usually used to discriminate $\alpha$ particle backgrounds. This background rejection will be all the more difficult because the liquid scintillators will not contain gadolinium and neither experiment will be very deep.

1.1 Nonlinear Energy Response

The present paper concerns a different challenge. The fine structure of the spectra in the case of the two hierarchies differ in the relative energies of the oscillation peaks at high (about 6 MeV) and low (about 3 MeV) energies. This difference, which must be observed to determine the hierarchy, is only about 1%. This means that the statistical fluctuations in the number of photoelectrons observed, which characterize the detector’s energy resolution, must be at least a factor of two smaller than has ever been achieved. Furthermore the systematic shifts in the relative energy response of the detector, which we call the nonlinear energy response, must be understood to an unprecedented level.

The nonlinear energy response arises from a variety of sources. For example the electronics determines the sensitivity to multiple photoelectrons in close proximity. This provides a nonlinear energy dependence. The optical properties of the scintillator determine the number of photons that eventually arrive at the photomultiplier tubes from various locations in the detector, meaning that the nonlinear response also is position dependent. Furthermore Cherenkov radiation by the positrons created in the inverse $\beta$ decay provides an energy dependence and a position dependence. Liquid organic scintillator detectors in reactor neutrino experiments have historically had to deal with time dependence in the optical properties of the scintillators, which degrades the energy resolution but also further complicates this nonlinear response.

The light yield of the scintillator, which determines the energy response, depends on the particle
being detected. As the particles of interest are positrons, one would ideally calibrate this nonlinear response using monochromatic positron sources. The trouble is that lepton number conservation implies that radioactive nuclei emit positrons together with neutrinos, resulting in a continuous positron spectrum. Monochromatic positron sources are thus difficult to come by, and so calibration will be done largely using other particles. While such a calibration can in principle determine the position dependence of the energy response of the detector with reasonable precision, the overall nonlinear shape of the response to positrons is not precisely determined by the response to any other particle. Therefore in this note we will not consider the position dependence of the nonlinearity, assuming that it has been determined perfectly via calibration.

The best yet determination of this nonlinear energy response has been made by the KamLAND experiment. The precision is about 2%. The JUNO experiment hopes to double this, determining the energy response at the 1% level.

This leads one to ask, just how well does the nonlinearity need to be understood to determine the hierarchy? Similarly one can ask, for a certain level of nonlinearity, just how strongly is the confidence in a hierarchy determination degraded? In this note we will answer these questions for various models of the nonlinear response. In particular we will find that, just as the use of identical detectors at distinct baselines appreciably reduces systematic errors due to flux uncertainties in Daya Bay [2] and RENO [3], identical detectors at sufficiently distinct baselines also appreciably reduce correlated systematic errors due to energy uncertainties at JUNO and RENO 50 as had been suggested in Refs. [6, 7]. Evidence for this effect has recently been obtained in Ref. [8] which used only the ratio of the fluxes at the two detectors. Our work instead analyses the full flux spectrum obtained at both detectors.

1.2 Multiple Detectors

While the construction of a second identical detector is likely to be beyond the funding limits of such experiments right now, the second detector can be constructed later. This of course will make the second detector less identical, one will need to make the scintillator in the two detectors as identical as possible. However, unlike $\theta_{13}$ experiments where it is necessary that the two detectors run simultaneously because of the time-dependent flux coming from the reactors, hierarchy determinations are independent of the flux normalization but only depend on the relatively time-independent shape of the fine structure of their energy spectra. Therefore it is less essential that the near and far detectors run at the same time, the only important point is that they have the same energy response.

There are several other motivations for considering multiple identical detectors. First of all, the uncertainty in the background flux is an obstruction to the secondary goals of these experiments of measuring $\theta_{12}$ and geoneutrinos. A second detector breaks the degeneracy between the strengths of
these signals and the backgrounds, allowing them to be more accurately measured.

Multiple detectors are also likely to each be smaller. In our study we have compared a single detector to two half-size detectors, however in practice to balance the cost the pairs of detectors may be even smaller. Smaller detectors have a number of advantages. First of all they are easier to build. In particular the construction of the enormous spherical detector for JUNO promises difficult engineering. More importantly, light does not need to travel so far to reach the photomultiplier tubes. This means that more photons will be received from each event, improving the energy resolution. Even a modest improvement in the energy resolution yields a large improvement in the confidence of the hierarchy determination. Also the photons will arrive in a shorter timescale, facilitating the identification of $\alpha$ particle backgrounds via the 2% of photons emitted in the scintillator’s long decay modes [9].

In the longer term, one is interested in the neutrino mass hierarchy to a large extent because a determination of the hierarchy allows the breaking of a degeneracy at NO$\nu$A and T2K which could then allow a determination of $\sin(\delta)$ where $\delta$ is the CP-violating leptonic phase. However with 2 detectors $\delta$ could be determined directly using a strategy similar to that of LBNE’s DAE$\delta$ALUS program [10]. This program requires multiple cyclotrons which serve as high intensity sources of stationary pions at various baselines which decay to $\bar{\nu}_\mu$, which in turn oscillate to $\bar{\nu}_e$ which is observed at the detector. The uncertainty in the relative intensity of the pion sources translates to an uncertainty in $\delta$.

On the other hand, if JUNO or RENO 50 has multiple detectors then only a single pion source is needed, which then will be at a different baseline from the two detectors. Ref. [11] suggests that a single such cyclotron facility would cost between 25 and 100 million dollars. Thus with a single cyclotron facility, both the experiment is cheaper and also the uncertainty from the relative calibration of the sources is removed, although of course it is important to calibrate the relative cross sections and efficiencies of the detectors. This can in principle yield one of the most precise determinations of $\delta$ of any proposed experiment. Another advantage of JUNO or RENO 50 detectors for such a program is that, like LENA [12], they are liquid scintillator detectors and so efficiently detect $\bar{\nu}_e$ via inverse $\beta$ decay. The antineutrino energy from the pion decay is above the reactor $\bar{\nu}_e$ energy range, so the hierarchy and CP-violation studies can occur simultaneously.

In Sec. 2 we will provide a general description of our assumptions and methods. Then in Sec. 3 we will provide the results for a general study of the effect of the nonlinear energy response upon the confidence of the hierarchy determination from abstract detectors and pairs of detectors at various distances from a single reactor neutrino source. Finally in Sec. 4 we will consider the relevant sites for the JUNO and RENO 50 experiments.
2 Methods

JUNO and RENO 50 both will detect reactor $\nu_e$ using inverse $\beta$ decay (IBD) in a liquid scintillator. The scintillator used by JUNO will be based on LAB, which is 12-13% hydrogen by mass. We will assume that RENO 50 has the same IBD cross section per target mass.

The total flux of $\nu_e$‘s detected is normalized to be 50,000 per 20 kton of target mass per 6 years at a baseline of 58 km from 17.4 GW of thermal capacity of reactors. This includes oscillations, the IBD cross section and the reactor flux normalization. It is chosen for easy comparison with the literature, but also is in reasonable agreement with both a scaled up value from Daya Bay and also with a first principles calculation [13]. We also assume a fractional energy resolution of $3\%/\sqrt{E\text{(MeV)}}$ where $E = E - 0.8\text{ MeV}$ is the prompt energy corresponding to a neutrino of energy $E$.

We use the value of $\Delta M_{21}^2$ from Ref. [14], $\sin^2(2\theta_{12})$ from Ref. [15], $|\Delta M_{32}^2|$ determined by combining $\nu$ and $\bar{\nu}$ mass differences from Ref. [16] and $\sin^2(2\theta_{13})$ from [17]

$$\Delta M_{21}^2 = 7.5 \times 10^{-5}\text{ eV}^2, \quad \sin^2(2\theta_{12}) = 0.857$$
$$|\Delta M_{32}^2| = 2.41 \times 10^{-3}\text{ eV}^2, \quad \sin^2(2\theta_{13}) = 0.089.$$ 

We fix all of the parameters except for $|\Delta M_{32}^2|$, which we fit as described below.

For each configuration we determine the statistic

$$\Delta \chi^2 = \chi^2_I - \chi^2_N.$$  

(1)

Here $\chi^2_N$ ($\chi^2_I$) is the minimal $\chi^2$ value obtained by fitting a given dataset to the theoretical spectrum corresponding the normal (inverted) hierarchy with a penalty term corresponding to the pull parameter treatment for the nuisance parameters. In particularly the pull parameters are minimized separately for the two choices of hierarchies.

We have performed this procedure on both theoretical spectra and on simulated data, finding compatible results. However as our statistical sample is small, in this note we will only report the fits to theoretical data corresponding to the Asimov data set. In Ref. [18] it was shown that, in such experiments, the Asimov $\Delta \chi^2$ agrees with the mean simulated $\Delta \chi^2$ to within 5 to 10 percent. Furthermore formulae were provided relating $\Delta \chi^2$ to the confidence in the determination of the experiment corresponding to each quantile of statistical fluctuations with any given Bayesian prior\(^1\). Until recently there has been no experimental preference for either hierarchy, suggesting a symmetric

\(^1\)An alternate frequentist approach to this confidence was introduced in Ref. [19, 8] reflecting the chance that an experiment yields the correct hierarchy. Our Bayesian approach answers a different question, it provides the probability that a fit to the correct hierarchy yields the lowest $\chi^2$ given that the measured $|\Delta \chi^2|$ assumes its median value or more generally lies in any given quantile.
50% prior for each hierarchy. However the most recent \( \sin^2(2\theta_{13})-\delta \) fit by the T2K collaboration [20] is incompatible with Daya Bay’s result for \( \theta_{13} \) in the case of the inverted hierarchy at a 2\( \sigma \) level, and even 2\( \sigma \) compatibility is only obtained with maximal CP violation \( \delta \sim -\pi/2 \). On the other hand \( \delta \sim -\pi/2 \) allows T2K and Daya Bay to be compatible at the 1\( \sigma \) level in the case of the normal hierarchy. This suggests that a roughly 1\( \sigma \) prior in favor of the normal hierarchy may be reasonable. Nonetheless, in our studies we have used a symmetric 50% prior.

The determination of the hierarchy is difficult because of the degeneracy between the hierarchy and \( |\Delta M^2_{32}| \), therefore in an evaluation of the sensitivity of an experiment to the hierarchy it is essential that \( |\Delta M^2_{32}| \) not be fixed. As a result our code always chooses \( |\Delta M^2_{32}| \) so as to minimize \( \chi^2 \) for each hierarchy separately. We do not introduce a penalty term in \( \chi^2 \) for \( |\Delta M^2_{32}| \), we simply choose the value which minimizes each \( \chi^2 \). We have checked that fitting the other neutrino mass matrix elements and allowing them to vary in simulations only slightly affects the confidence in a hierarchy determination.

To elucidate the effect of the nonlinearity, we perform studies with and without nonlinearity. In each case we fit the nonlinearity using a 3-parameter model

\[
E_{\text{observed}} = a + (1 + b)E_{\text{true}} + cE_{\text{true}}^2. \tag{2}
\]

\( E_{\text{true}} \) is the true energy while \( E_{\text{observed}} \) is the energy that the experimentalist concludes that the \( \nu \) must have had based on the number of photoelectrons observed, the results of his calibration campaign and the results of Monte Carlo simulations. We do not assume that \( E_{\text{observed}} \) is proportional to the number of photoelectrons observed, on the contrary not only will this dependence be nonlinear after the completion of the calibration campaign but also it will depend upon the reconstructed location of the event. Here \( a, b \) and \( c \) are pull parameters for which quadratic penalty terms are added in the definition of \( \chi^2 \) corresponding to their assumed standard errors. These errors reflect the degree to which the experimentalist is confident in his calibrations. In the results that we will present below we have taken the 1\( \sigma \) uncertainty on these three parameters to be \( 2 \times 10^{-2} \) MeV, \( 2 \times 10^{-2} \) and \( 2 \times 10^{-3} \) MeV\(^{-1} \) respectively. As a consistency check on our results, we have found that when analyzing theoretical spectra that were generated with a perfect energy response, the best fit values of \( a, b \) and \( c \) are generally smaller than \( 10^{-5} \).

This fitting of the nonlinearity reflects the fact that the energy response of the detector affects the locations of known structure in the reactor neutrino spectrum, and that therefore this known structure can be used to calibrate the detector by choosing \( a, b \) and \( c \) so as to separately minimize \( \chi^2_N \) and \( \chi^2_I \). Realistically, the observed spectrum at JUNO and RENO 50 will include significant backgrounds, as have recently been estimated in Ref. [8]. The large uncertainties in these backgrounds lead to an uncertainty in the expected flux which can be degenerate with the effects of the nonlinear response. As a result, backgrounds make this kind of calibration less effective. In the current study
we ignore backgrounds, and so our analysis of the consequences of the nonlinear energy response on
the confidence in the hierarchy determination will to some extent be overoptimistic.

As the true energy response model is not known to the experimenter, we fit the unknown energy
response using a fixed parametrization (2) which in two of the three cases considered below does not
contain the model used to generate the spectrum. Ref. [4] considered a form of the nonlinearity which
is nearly constant in the critical regime between 3% and 6%. The unknown energy response in this
model is much milder than the target 1% expected at JUNO and therefore yields a very optimistic
prediction for the effects of nonlinearity.

The energy response of the detector is the number of photoelectrons that are observed as a
function of the energy of the event and its location. It can be determined by a simulation to within
some precision. But this precision is far short of the requirements to determine the hierarchy. This is
why such simulations need to be supplemented with intensive calibration campaigns, which determine
$E_{\text{observed}}$ in Eq. (2). The unknown nonlinearity is the difference between $E_{\text{true}}$ and $E_{\text{observed}}$. It cannot
be determined by any Monte Carlo or calibration, as these are already considered in $E_{\text{observed}}$. The
result is that the form of the unknown nonlinear detector response is totally unknown.

So then what model of nonlinearity do we use? The space of possible functions $E_{\text{observed}}(E_{\text{true}})$ is
infinite-dimensional, it would be futile and useless to try them all. Given any model of nonlinearity,
the space of small perturbations of the nonlinearity generates a vector space. A basis can be chosen
for this vector space such that all of the directions except for one, $v$, leave $\Delta \chi^2$ fixed, although of
course they will not leave the individual $\chi^2$ values fixed which is what allows the pull parameters
to be determined by $\chi^2$ fits. The vector $v$ is important because a general small perturbation in
the nonlinearity, corresponding to a vector $w$, will affect $\Delta \chi^2$ by a quantity proportional to $v \cdot w$.
Therefore instead of considering an infinite-dimensional space of perturbations, it suffices to identify
$v$ and study its effects on $\Delta \chi^2$, and then any other model of nonlinearity will affect $\Delta \chi^2$ in proportion
to its similarity to $v$. Of course, since the effect on $\chi^2$ itself is model-dependent, other nonlinearities
may be easier or harder to calibrate depending on their effect on known features of the spectrum,
such as the 1.8 MeV minimum energy for inverse $\beta$ decay.

Which is the direction, $v$, in deformation space which varies $\Delta \chi^2$? It is the deformation which
interpolates between the spectra of the normal and inverted hierarchies. As the distinction between
the normal and inverted hierarchies is the energy difference between the high and low energy peaks
[13], this means that the most dangerous nonlinearity is one which contracts or expands the spectrum
in the critical region between about 3 MeV and 6 MeV. In Ref. [21] an approximate form of this most
dangerous nonlinearity was found, which we will call the worst case model. This model is defined by

$$E_{\text{observed}} = \frac{2|\Delta M_{32}^2|(eV^2) + 4\cos^2(\theta_{12})\Delta M_{21}^2(eV^2) - \frac{\phi}{1.27} \frac{E_{\text{true}}(\text{MeV})}{L(\text{m})}}{2|\Delta M_{32}^2|(eV^2) + \frac{\phi}{1.27} \frac{E_{\text{true}}(\text{MeV})}{L(\text{m})}} E_{\text{true}}$$  \hspace{1cm} (3)
\[\sin(\phi) = \frac{\cos^2(\theta_{12}) \sin \left( \frac{2.54 \Delta M^2_{31} (eV^2) L (m)}{E_{\text{true}} (MeV)} \right)}{\sqrt{1 - 4\sin^2(\theta_{12}) \cos^2(\theta_{12}) \sin^2 \left( \frac{2.54 \Delta M^2_{31} (eV^2) L (m)}{E_{\text{true}} (MeV)} \right)}}. \]  

(4)

Notice that \(E_{\text{observed}}\) is always less than \(E_{\text{true}}\) with a maximum fractional difference of 3\% near 3 MeV which is reduced to 1.5\% by 6 MeV. As a result the best fit \(|\Delta M^2_{31}|\), which is determined from the energy near 3 MeV [13], is reduced by about 3\%. On the other hand, the spectrum near 6 MeV determines the effective mass [22]

\[|\Delta M^2_{21}| \mp \sin^2(\theta_{12}) |\Delta M^2_{21}| + \sin^2(\theta_{12}) |\Delta M^2_{32}|. \]  

(5)

Therefore the best fit \(\Delta M^2_{\text{eff}}\) is reduced by about 1.5\%. Medium baseline reactor neutrino experiments determine the hierarchy by using the fact that \(|\Delta M^2_{31}| - \Delta M^2_{\text{eff}}\) is positive (negative) if and only if the hierarchy is normal (inverted). The absolute value of this difference is \(\sin^2(\theta_{12}) \Delta M^2_{21}\) which is about 1\% of \(|\Delta M^2_{31}|\). The model (3) reduces \(|\Delta M^2_{31}| - \Delta M^2_{\text{eff}}\) by about 1.5\% of \(|\Delta M^2_{31}|\), changing the best fit hierarchy from normal to inverted.

Such an energy response will not only shift the fine structure of atmospheric oscillations, but will shift all of the expected features in the observed spectrum. This can result in a large \(\chi^2\) value, which allows the experimenter to calibrate to some extent for the nonlinearity even without adding additional sources to the detector. There is no form of the nonlinearity capable of transforming the spectrum of one hierarchy into the other at all baselines. The form (3) has this property at a particular baseline \(L\), but at other baselines the transformation is imperfect, which is the reason that a second detector breaks the degeneracy between the energy response and the hierarchy. \(L\) is a parameter of the nonlinearity model which needs to be chosen in the simulation. We will always choose \(L=55\) km in what follows.

In our general study on nonlinearity in Sec. 3 we will consider 3 models of the unknown energy response. First we will consider a quadratic shift in the energy

\[\tilde{E}_{\text{observed}} = \tilde{E}_{\text{true}} - \alpha \tilde{E}_{\text{true}}^2 \]  

(6)

where \(\alpha = 0.0015\) MeV\(^{-1}\) and again \(\tilde{E} = E - 0.8\) MeV is the prompt energy. We will refer to this model as the quadratic model. As this is an example of an energy response described by Eq. (2), \(\chi^2\) will be minimized when it is eliminated entirely. We will see that therefore our best \(\chi^2\) fit yields an observed spectrum and so a \(\Delta \chi^2\) which is virtually identical to the case in which the energy response of the detector is known perfectly.

Next we will consider a model similar to that of Ref. [4]

\[\tilde{E}_{\text{observed}} = \left( \frac{\alpha + \delta \tilde{E}_{\text{true}}}{1 + \beta e^{-\gamma \tilde{E}_{\text{true}}} + (1 - \alpha)} \right) \tilde{E}_{\text{true}} \]  

(7)
with parameters
\[ \alpha = 1.1057, \quad \beta = 0.23, \quad \gamma = 2.35 \text{ MeV}^{-1}, \quad \delta = 10^{-4} \text{ MeV}^{-1}. \] (8)

We will call this the exponential model. The corresponding unknown nonlinear response varies by only 0.13% between 3 MeV and 6 MeV, therefore this model is quite optimistic. Its main impact is below 3 MeV, where atmospheric oscillations in the spectrum are obscured by the finite energy resolution.

Finally we will consider the worst case model with
\[ \Delta E = E_{\text{observed}} - E_{\text{true}} \] (9)
scaled down by a factor of 3 so that
\[ E_{\text{observed}} = \frac{2|\Delta M_{32}^2|(eV^2) + \frac{4}{3}\cos^2(\theta_{12})\Delta M_{21}^2(eV^2) + \frac{\phi}{3.8} \frac{E_{\text{true(MeV)}}}{L(m)}}{2|\Delta M_{32}^2|(eV^2) + \frac{\phi}{1.27} \frac{E_{\text{true(MeV)}}}{L(m)}} E_{\text{true}}. \] (10)

We will call this the one third worst model. The relative difference in the energy response between 3 MeV and 6 MeV is about one half of a percent, therefore the normalization of the nonlinear response in this model is about twice as good as JUNO’s target 1%. This means that the results of our simulations employing the one third worst model are somewhat optimistic.

3 General Study of the Nonlinear Energy Response

We have performed a systematic study of the effect of the nonlinearity on \( \Delta \chi^2 \). We assumed that all of the reactor neutrinos are emitted by a single 36 GW thermal capacity reactor neutrino point source. Then we considered configurations with one detector at various baselines with 240 kton years of exposure. We also considered two detectors each with 120 kton years of exposure, one at 55 km and the other at various baselines. We considered the generated theoretical spectra and also ran simulations using the three nonlinearity models above. The statistics of the simulations were not sufficient to draw conclusions, but merely to test the consistency of our theoretical results. We then performed similar studies with one half and one quarter of the events in the first study.

We chose \(|\Delta M_{32}^2|\) and also the pull parameters \( a, b \) and \( c \) so as to separately minimize \( \chi^2_N \) and \( \chi^2_I \) for fits to spectra obtained using the corresponding hierarchies. Subtracting these yields \( \Delta \chi^2 \) which is plotted in Figs. 1, 2 and 3 for all of the nonlinearity models with a total exposure of 240 kton years, 120 kton years and 60 kton years respectively.

The most obvious feature of these figures is that in the case of one detector the confidence \( \Delta \chi^2 \) goes to zero on the left whereas in the case of 2 detectors it does not. The reason that the single
Figure 1: In the upper panel $\Delta \chi^2$ is evaluated for the Asimov data assuming a perfectly understood energy response (black), a quadratic nonlinearity model (blue) and an exponential model (red) and 240 kton years of target exposure. The dashed curves correspond to single detectors at various baselines. The solid curves correspond to one half size detector at 55 km and another at the given baseline. Curves on the upper (lower) half of each panel correspond to theoretical spectra generated using the normal (inverted) hierarchy. The quadratic nonlinearity yields the same confidence as no nonlinearity at all as it is fit perfectly by the assumed fitting function. The exponential nonlinearity appreciably reduces the confidence in a hierarchy determination if the true hierarchy is normal in the case of 1 detector or 2 detectors at similar baselines. However this effect is canceled entirely if 1 detector is placed at 55 km and another beneath about 30 km. The lower panel is the same except the no nonunderstood nonlinearity case (black) is compared with the third worst model (blue).
detector confidences tend to zero is that for baselines shorter than 30 km [13] the reactor spectrum is only sensitive to $\Delta M_{\text{eff}}^2$ and not to the hierarchy. The hierarchy can only be determined by comparing $\Delta M_{\text{eff}}^2$ with a different combination of the atmospheric splittings. In the two detector case on the other hand, one detector is always at 55 km. While the near detector measures $\Delta M_{\text{eff}}^2$, the far detector measures other splittings, such as $|\Delta M_{31}^2|$, whose comparison with $\Delta M_{\text{eff}}^2$ yields the hierarchy whatever the location of the near detector. This is why the two detector confidence does not go to zero at the left end of the figures.

One easy consistency check of the figures is that the dashed curves, corresponding to 1-detector configurations, and the solid curves, corresponding to 2-detector configurations, agree at 55 km. This is because the far detector is always at 55 km and the total target volume is held fixed. Thus 2 detectors both at 55 km are equivalent to a single detector at 55 km. Of course in practice these situations are somewhat different because in the case of 2 detectors the photons must travel less distance to the photomultiplier tubes so more will arrive and the energy resolution will be improved and also in the 2 detector case there will be more cosmogenic muons detected, although the total number of cosmogenic muons that pass through any given volume will be the same. However at the level of our calculations 1 detector at 55 km is indistinguishable from 2 half-sized detectors at 55 km and so the dashed and solid curves agree.

As can be seen, the quadratic nonlinearity model is indistinguishable from no nonlinearity at all. This is because the nonlinearity model (6) is a special case of our fitting function (2). On the other hand the exponential nonlinearity has a noticeable effect on the confidence of the hierarchy determination, at 55 km and 240 kton years reducing $\Delta \chi^2$ from -44 to -30 in the case in which the true hierarchy is inverted. Using the statistical interpretation of Ref. [18] this reflects a drop from over 6$\sigma$ of confidence in the median experiment to 5$\sigma$. On the other hand, if one detector is placed at 55 km and another at 30 km then in the case of exponential nonlinearity one still obtains a $\Delta \chi^2$ value of -45.

One may be tempted to claim that the unknown energy response is not so bad, since it reduces the confidence by 1$\sigma$ in the case of the inverted hierarchy but increases the confidence by a similar amount in the case of the normal hierarchy. The trouble is that this preference is the result of a sign choice in the model parameters (8). For the other choice, there would be a 1$\sigma$ reduction in the confidence of the normal hierarchy. As the true parameters in the unknown energy response are of course unknown, it will not be known to the experimenters just which confidence is correct. The resulting confidence intuitively is the average of the $p$-values corresponding to the confidences with both signs, which is essentially just the confidence of the worst case. In other words, when interpreting Figs. 1, 2 and 3 the relevant information at each energy is the curve whose absolute value is smallest, reflecting the most pessimistic of the two hierarchies. As a result, in the analysis that follows we will identify the confidence with the lowest $|\Delta \chi^2|$ statistic of the two hierarchies.
Figure 2: As in Fig. 1 but now the total exposure is 120 kton years.
Figure 3: As in Fig. 1 but now the total exposure is 60 kton years.
Nonetheless this 1σ degradation of the signal due to the nonlinearity may seem small. However the exponential nonlinearity model is extremely optimistic. While below 2.2 MeV it represents a nonlinearity above 1%, by 2.5 MeV that has been reduced to 0.5% and about 3 MeV it is about than 0.13%. Due to the limited energy resolution, the fine structure of atmospheric oscillations are strongly suppressed below about 3 MeV.

The third worst model, while still optimistic, seriously affects the confidence in the hierarchy determination. Consider for example Fig. 1 reflecting 240 kton years. In the case of a single detector the unknown energy response causes the confidence Δχ² to drop from 36 to 24, losing more than 1σ. On the other hand, a near detector at 30 km leads to a confidence of Δχ² = 39, even better than the 1 detector case with no nonlinearity at all.

What about the case of shorter exposures? Consider 120 kton years as shown in Fig. 2, corresponding to 6 years of running of the current JUNO proposal, although without the interference effects between baselines which will be included in Sec. 4. Now with no nonlinearity at 55 km one may expect a confidence of Δχ² = 18 (3.7σ), but the third worse model reduces this to Δχ² = 12 (2.8σ), a reduction of nearly 1σ. On the other hand two half-sized detectors, one at 55 km and one at 30 km, yield Δχ² = 18, again as good as a single detector before the correction for the nonlinearity was applied.

Finally in Fig. 3, 60 kton years of running is considered, corresponding to 3 years of JUNO, 3.3 years of RENO 50 at Munmyeongsan or 6.6 years of RENO 50 at Guemsong. Now the third worst nonlinearity model makes Δχ² fall from 9.5 (2.4σ) to 6 (1.7σ), while a second detector at 30 km leads to a partially recovered confidence of 8.5 (2.2σ). A yet lower baseline for the near detector restores the confidence even more, with about 25 km necessary to fully restore the original confidence. More generally we find that the advantage of a second detector is reduced for smaller numbers of kton years of exposure. The reason for this is that in such cases the hierarchy determination is limited by statistics, and not by systematics such as the nonlinear energy response. For example in the case of JUNO and 60 kton years of total exposure, the 2 detector scenario corresponds to 3 years of a 10 kton detector at 55 km and a 10 kton near detector. Only the far detector is sensitive to |ΔM²_{31}|, in fact only the low energy (3 MeV) part of its spectrum is sensitive to |ΔM²_{31}|. An accurate determination of |ΔM²_{31}| is necessary to determine the hierarchy at a reactor experiment, but 30 kton years of exposure at 55 km presents too few neutrinos in the required range to robustly measure this quantity. For this reason, the second detector proposal is most advantageous for long running experiments, which are limited by systematic errors, like those of Fig. 1.
4 JUNO and RENO 50

4.1 The Sites Considered

We have also considered the effect of the one third worst model of nonlinearity on candidate sites for the JUNO and RENO 50 experiments. The far detector of the JUNO experiment will be in Jiangmen county, at the DongKeng site of Ref. [6]. It will use flux from two reactor complexes, Yangjiang and Taishan. Currently no reactors are operational in either complex. However 20.8 GW of thermal capacity of reactors is under construction and another 15 GW are planned, although it remains to be seen how the recent cancellation of a reactor fuel production plant in Jiangmen will affect these plans. We have identified two sites for a near detector. First, it may be built under the 350m hill LuGuJing about 40 km from each reactor complex. At this baseline it will require about 600 meters of overhead rock to block atmospheric muons responsible for various backgrounds. Second, it may be built under the 750 meter mountain ZiLuoShan which is 17 km from Yangjiang but 66 km from Taishan. Therefore Taishan would provide a background. However the proximity to Yangjiang means that it will enjoy even more reactor neutrino flux than a detector under LuGuJing. These sites are summarized in Table 1.

The preferred site for the far detector of RENO 50 is deep under the 450 m hill GuemSeong. As was observed in Ref. [6], the perpendicular location of GuemSeong with respect to the Hanbit reactor complex means that it is the same 47.4 km from all of the reactors and so there is negligible interference between baselines. A corresponding near detector may be placed under Jangamsan which is 23 km from Hanbit. Alternately a far detector could be placed under Mummyongsan or Munsusan receiving flux from the Hanul complex. While this complex currently has the same power has Hanbit, more reactors are under construction. When these reactors are completed, in 2018 when RENO 50 is scheduled to begin taking data, the total flux will be almost 50% greater. Even more reactors are planned, perhaps doubling the thermal capacity of the site by 2022. In addition these mountains are appreciably higher than GuemSeong, and so such a site would not require vertical digging. The disadvantages are that it could not receive neutrinos from the J-PARC beam and also there would be significant interference from reactors 150 km away. This interference is less at Munsusan, but in exchange there is interference between the various Hanul reactors at this location. Both Hanul sites could use a near detector at a location that we have called Buncheon-ri. These sites are summarized in Table 2.

We consider single detectors of 20 kton for JUNO and 18 kton for RENO 50, as well as two detector scenarios with 10 kton and 9 kton of target mass per detector respectively. Our results are summarized in two tables. In Table 3 we present the expected $\Delta\chi^2$ assuming no nonlinearity and also the third worst model assuming that all reactors which are under construction or planned have
Table 1: JUNO sites

| Reactor      | Status    | Th. Cap | DongKeng(东坑) | LuGuJing(碌古径) | ZiLuoShan(紫罗山) |
|--------------|-----------|---------|----------------|----------------|------------------|
| Latitude     |           |         | 22.12°N        | 21.90°N        | 21.84°N          |
| Longitude    |           |         | 112.52°E       | 112.59°E       | 112.35°E         |
| Elevation    |           |         | 450 m          | 350 m          | 750 m            |
| Relief       |           |         | 450 m          | 350 m          | 750 m            |
| YangJiang 1  | Under Cons| 2905 MW | 52.75 km       | 40.55 km       | 17.43 km         |
| YangJiang 2  | Under Cons| 2905 MW | 52.84 km       | 40.6 km        | 17.52 km         |
| YangJiang 3  | Under Cons| 2905 MW | 52.42 km       | 40.2 km        | 17.09 km         |
| YangJiang 4  | Under Cons| 2905 MW | 52.51 km       | 40.3 km        | 17.18 km         |
| YangJiang 5  | Planned   | 2905 MW | 52.12 km       | 39.9 km        | 16.79 km         |
| YangJiang 6  | Planned   | 2905 MW | 52.21 km       | 40.0 km        | 16.88 km         |
| Taishan 1    | Under Cons| 4590 MW | 52.76 km       | 40.55 km       | 65.84 km         |
| Taishan 2    | Under Cons| 4590 MW | 52.63 km       | 40.4 km        | 65.7 km          |
| Taishan 3    | Planned   | 4590 MW | 52.32 km       | 40.1 km        | 65.5 km          |
| Taishan 4    | Planned   | 4590 MW | 52.20 km       | 39.9 km        | 65.4 km          |
| DB-LA        | Operational| 17430 MW | 215 km   | 215 km        | 242 km            |

As in our general study, the fact that the model of the unknown energy response chosen here (10) increases the confidence in a determination of the inverted hierarchy and decreases that in the normal hierarchy reflects an arbitrary choice. The sign of the systematic bias in energy will never be known. Therefore the correct confidence in a hierarchy determination is better approximated by the entry of a given row of Table 3 and 4 which is most pessimistic for a given model. For example, for the single detector JUNO site DongKeng, the third worst hierarchy leads to a $\Delta \chi^2 = 8.2$ confidence in the case of the normal hierarchy but $-21.5$ in the case of the inverted hierarchy. Since the true nonlinear response could just as well have the opposite bias, no result should be given more than $|\Delta \chi^2| = 8.2$ of confidence, reflecting about $2.1\sigma$. In particular, we claim that given an unknown nonlinear response about as large as the third worst model, JUNO will be able to determine the mass hierarchy with a confidence of about $2\sigma$ with a single 20 kton detector and 6 years of running. This is consistent with the conclusions of Ref. [8], whose statistical interpretation of $\chi^2$ agrees with ours in this regime.
| Reactor    | Status       | Th. Cap | GuemSeong | Jangam | Mumnmyeong | Munsusan | Buncheon |
|------------|--------------|---------|-----------|--------|------------|----------|----------|
| Latitude   |              | 35.05°N | 35.26°N   | 36.82°N | 36.98°N    | 36.93°N  |
| Longitude  |              | 126.70°E| 126.58°E  | 128.92°E| 128.81°E   | 129.09°E |
| Elevation  |              | 450 m   | 480 m     | 870 m   | 1180 m     | 900 m    | 520 m    |
| Relief     |              | 420 m   | 440 m     | 690 m   | 900 m      | 520 m    |
| Hanbit 1   | Operational  | 2787 MW | 47.4 km   | 22.9 km | 274.6 km   | 277.8 km | 294.2 km |
| Hanbit 2   | Operational  | 2787 MW | 47.4 km   | 22.9 km | 274.4 km   | 277.6 km | 293.9 km |
| Hanbit 3   | Operational  | 2825 MW | 47.4 km   | 22.8 km | 274.1 km   | 277.3 km | 293.7 km |
| Hanbit 4   | Operational  | 2825 MW | 47.4 km   | 22.8 km | 273.8 km   | 277.1 km | 293.4 km |
| Hanbit 5   | Operational  | 2825 MW | 47.5 km   | 22.8 km | 273.6 km   | 276.8 km | 293.2 km |
| Hanbit 6   | Operational  | 2825 MW | 47.5 km   | 22.8 km | 273.3 km   | 276.6 km | 292.9 km |
| Hanul 1    | Operational  | 2785 MW | 331.6 km  | 323.9 km| 51.5 km    | 52.0 km  | 31.9 km  |
| Hanul 2    | Operational  | 2775 MW | 331.6 km  | 323.9 km| 51.5 km    | 52.1 km  | 31.9 km  |
| Hanul 3    | Operational  | 2825 MW | 331.6 km  | 323.9 km| 51.5 km    | 52.2 km  | 31.9 km  |
| Hanul 4    | Operational  | 2825 MW | 331.6 km  | 324.0 km| 51.5 km    | 52.3 km  | 31.9 km  |
| Hanul 5    | Operational  | 2815 MW | 331.6 km  | 324.0 km| 51.6 km    | 52.4 km  | 32.0 km  |
| Hanul 6    | Operational  | 2825 MW | 331.6 km  | 324.0 km| 51.6 km    | 52.5 km  | 32.0 km  |
| Wolseong 1 | Operational  | 2061 MW | 262.7 km  | 267.0 km| 133.0 km   | 153.2 km | 139.7 km |
| Wolseong 2 | Operational  | 2061 MW | 262.6 km  | 266.9 km| 133.1 km   | 153.3 km | 139.8 km |
| Wolseong 3 | Operational  | 2061 MW | 262.5 km  | 266.8 km| 133.2 km   | 153.4 km | 140.0 km |
| Wolseong 4 | Operational  | 2061 MW | 262.3 km  | 266.7 km| 133.3 km   | 153.5 km | 140.1 km |
| Shin Wol. 1| Operational  | 2825 MW | 263.1 km  | 267.3 km| 132.2 km   | 152.4 km | 138.9 km |
| Kori 1     | Operational  | 1729 MW | 237.8 km  | 246.0 km| 170.2 km   | 189.9 km | 180.0 km |
| Kori 2     | Operational  | 1882 MW | 237.9 km  | 246.1 km| 170.2 km   | 189.9 km | 180.0 km |
| Kori 3     | Operational  | 2912 MW | 238.2 km  | 246.4 km| 170.3 km   | 189.9 km | 180.0 km |
| Kori 4     | Operational  | 2912 MW | 238.4 km  | 246.6 km| 170.3 km   | 190.0 km | 180.1 km |
| Shin Kori 1| Operational  | 2825 MW | 238.8 km  | 246.9 km| 169.7 km   | 189.3 km | 179.3 km |
| Shin Kori 2| Operational  | 2825 MW | 238.8 km  | 246.9 km| 169.6 km   | 189.2 km | 179.2 km |
| Shin Kori 3| Under Cons   | 3983 MW | 239.8 km  | 247.9 km| 168.7 km   | 188.4 km | 178.3 km |
| Shin Kori 4| Under Cons   | 3938 MW | 239.9 km  | 247.9 km| 168.6 km   | 188.3 km | 178.1 km |
| Shin Wol. 2| Under Cons   | 3938 MW | 239.9 km  | 247.9 km| 168.6 km   | 188.3 km | 178.1 km |
| Shin Hanul 1| Under Cons  | 3938 MW | 331.4 km  | 323.8 km| 51.5 km    | 52.6 km  | 31.9 km  |
| Shin Hanul 2| Under Cons  | 3938 MW | 331.4 km  | 323.8 km| 51.5 km    | 52.7 km  | 31.9 km  |
| Shin Hanul 3| Planned     | 3938 MW | 331.4 km  | 323.8 km| 51.5 km    | 52.9 km  | 32.0 km  |
| Shin Hanul 4| Planned     | 3938 MW | 331.3 km  | 323.8 km| 51.5 km    | 52.9 km  | 32.0 km  |

Table 2: Reno 50 sites
4.2 Greater Sensitivity to the Inverted Hierarchy

Perhaps the most obvious trend in Tables 3 and 4 is that both experiments are more sensitive to the inverse hierarchy than the normal hierarchy. The reason for this is as follows. While the determination of $\Delta M^2_{\text{eff}}$ at a reactor experiment is relatively easy, to determine the hierarchy, one needs to observe the peaks in the spectrum corresponding to neutrinos that have oscillated at least 13 times [13]. The more times they have oscillated, the stronger the hierarchy dependence and so the stronger the signal. For example, the energy of $|\Delta M^2_{31}|/(2\Delta M^2_{21})$th oscillation determines $|\Delta M^2_{31}|$, and the difference between $|\Delta M^2_{31}|$ and $\Delta M^2_{\text{eff}}$ is of order 1% with a sign that determines the hierarchy. If $|\Delta M^2_{31}|$ is increased with $\Delta M^2_{21}$ held fixed then $|\Delta M^2_{31}|/(2\Delta M^2_{21})$ will increase and so one needs to observe more oscillations to measure $|\Delta M^2_{31}|$ and so determine the hierarchy. This means that higher values of $|\Delta M^2_{31}|$ make the hierarchy more difficult to determine [23].

So how do we know $|\Delta M^2_{31}|$? In our study, we have fixed $|\Delta M^2_{32}|$ to the value given by MINOS in Ref. [16]. So $|\Delta M^2_{32}|$ is fixed. If the hierarchy is normal then, for the same value of $|\Delta M^2_{32}|$, $|\Delta M^2_{31}|$ will be higher than it would be for the inverted hierarchy. Then, as a result of the argument in the previous paragraph, fixing $|\Delta M^2_{32}|$, if the hierarchy is normal then JUNO and RENO 50 will determine it with less confidence, reproducing the results of the tables.

Actually these results exaggerate the dependence of the sensitivity on the true hierarchy for two reasons. First of all, MINOS does not really measure $|\Delta M^2_{32}|$. It combines accelerator data, which measures a combination of $|\Delta M^2_{31}|$ and $|\Delta M^2_{32}|$ called the atmospheric mass difference in [22], with atmospheric neutrino data which measures different combinations for different angles. In general these combinations are closer to $|\Delta M^2_{32}|$ than is $\Delta M^2_{\text{eff}}$, and therefore fixing the mass measured by MINOS, medium baseline reactor experiments will nonetheless be more sensitive to the inverted hierarchy than the normal hierarchy. The mass differences measured by MINOS lie between $|\Delta M^2_{31}|$ and $|\Delta M^2_{32}|$, and so by naively identifying them with $|\Delta M^2_{32}|$, as we have done, one exaggerates the effect.

The second reason that the higher sensitivity of the inverted hierarchy is overstated here is that while presently MINOS provides the most precise atmospheric mass difference, already preliminary results by Daya Bay [24] rival this precision. However reactor experiments like Daya Bay measure $\Delta M^2_{\text{eff}}$, which is closer to $|\Delta M^2_{31}|$ than to $|\Delta M^2_{32}|$. Fixing $\Delta M^2_{\text{eff}}$ instead of the atmospheric mass splitting, the medium baseline reactor experiments have similar sensitivities to the two hierarchies. In the future Daya Bay and RENO will continue to improve measurements of $\Delta M^2_{\text{eff}}$ while accelerator experiments like NO$\nu$A and T2K will improve the measurement of the atmospheric mass difference. Thus the sensitivity to the hierarchy will continue to be greater in the case of the inverse hierarchy, but by less than is reported here.
Table 3: $\Delta \chi^2$ obtained with a perfect energy response and with the one third worst model at various JUNO and RENO 50 sites after 6 years of running. Single detectors of 20 kton (18 kton) and pairs of 10 kton (9 kton) detectors are used for JUNO (RENO 50). Flux is considered from all reactors which are operational, under construction and planned.

| Site                        | NH:No Nonlin | IH: No Nonlin | NH: Worst/3 | IH: Worst/3 |
|-----------------------------|--------------|---------------|-------------|-------------|
| DongKeng                    | 14.1         | -17.0         | 8.2         | -21.5       |
| DongKeng+LuGuJing           | 13.2         | -16.2         | 7.8         | -21.4       |
| DongKeng+ZiLuoShan          | 13.5         | -16.1         | 13.9        | -15.3       |
| GuemSeong                   | 6.2          | -7.7          | 3.3         | -10.0       |
| GuemSeong+Jangamsan         | 5.6          | -6.6          | 5.3         | -7.0        |
| Munmyeong                   | 11.8         | -13.6         | 6.7         | -18.3       |
| Munmyeong+Buncheon-ri       | 11.5         | -13.6         | 9.4         | -16.4       |
| Munsusan                    | 9.4          | -11.7         | 5.9         | -16.1       |
| Munsusan+Buncheon-ri        | 10.3         | -12.0         | 8.6         | -14.6       |

Table 4: As in Table 3 but flux is considered from all reactors which are operational or under construction, not from those that are only planned, reflecting the flux expected when these experiments begin taking data.

| Site                        | NH:No Nonlin | IH: No Nonlin | NH: Worst/3 | IH: Worst/3 |
|-----------------------------|--------------|---------------|-------------|-------------|
| DongKeng                    | 8.4          | -11.2         | 5.4         | -13.4       |
| DongKeng+LuGuJing           | 9.2          | -10.6         | 5.1         | -14.0       |
| DongKeng+ZiLuoShan          | 9.2          | -10.4         | 9.2         | -8.8        |
| GuemSeong                   | 6.5          | -7.2          | 2.8         | -9.6        |
| GuemSeong+Jangamsan         | 5.8          | -6.7          | 5.5         | -8.3        |
| Munmyeong                   | 8.2          | -9.7          | 5.3         | -12.1       |
| Munmyeong+Buncheon-ri       | 8.2          | -10.0         | 6.8         | -11.3       |
| Munsusan                    | 6.9          | -8.9          | 4.2         | -11.1       |
| Munsusan+Buncheon-ri        | 7.6          | -9.0          | 7.2         | -10.8       |

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4.3 Consequences of the Nonlinear Energy Response

It is clear from these tables that the nonlinear energy response will be a serious challenge both at JUNO and at RENO 50. Even the optimistic unknown energy response model considered here decreases the expected sensitivity by about 1σ. In particular in the case of JUNO it decreases the confidence from ∆χ² = 14.1 (3.1σ) to 8.2 (2.1σ) whereas for the three locations considered for RENO 50 it decreases the confidence from ∆χ² = 6.2 (1.7σ), 11.8 (2.8σ) and 9.4 (2.4σ) to 3.3 (1σ), 6.7 (1.8σ) and 5.9 (1.6σ) respectively.

In some cases all of this decrease can be eliminated by dividing the target mass into two detectors. However this strategy is only successful if the baselines of the two detectors differ significantly. For example, the LuGuJing near site for JUNO has a baseline which is too close to that of DongKeng to break the degeneracy between the hierarchy and the unknown energy response of the detector. We do not observe any advantage for a second detector placed at LuGuJing if the total target mass is fixed. On the other hand the near site ZiLuoShan does have a sufficiently different baseline and so breaks this degeneracy quite well, restoring the confidence in the hierarchy to ∆χ² = 13.9 (3.1σ), essentially equal to the one detector value before the effects of nonlinearity were considered. Therefore we conclude that a two 10 kton (20 kton) detector proposal for JUNO can yield a more than 3σ confidence determination of the hierarchy in 6 (3) years.

What about RENO 50? Despite the backgrounds from the reactor complexes at Kori and Wolseong, the highest ∆χ² can be achieved at Mummyeonsan. Without considering the unknown energy response it can achieve ∆χ² = 11.8 corresponding to about 2.8σ in 6 years as compared with ∆χ² = 6.2 and 9.4 at GuemSeong and Munsusan respectively. The unknown energy response reduces the confidence at all three sites drop by between 0.7σ and 1σ. More than half of this drop in confidence is recovered with the addition of a second detector. As a result the best confidence achievable at RENO 50 with a total target mass of 18 kton and 6 years of running appears to be ∆χ² = 9.4 corresponding to 2.4σ at Mummyeonsan with an identical near detector at Buncheon-ri. While these confidences are appreciably lower than what may be achieved at JUNO, at all three locations it seems as though RENO 50 is planning to have more overhead burden than the 700 meters foreseen by JUNO, therefore the backgrounds will be lower. In this study we have not included the effects of the backgrounds on our results.

In Table 4 we have considered the case in which reactors which have been planned at the Yangjiang, Taishan and Hanul complexes are not built. In the case of JUNO this results in a reduction of ∆χ² to 5.4 (1.5σ) and 8.8 (2.3σ) in the 1 detector and 2 detector cases. In the case of RENO 50 this has no effect on the preferred GuemSeong site, but it does reduce the confidence at the other two sites. In particular the confidence at Mummyeong is reduced to ∆χ² = 5.3 (1.5σ) and 6.8 (1.9σ) in the 1 and 2 detector cases. Nonetheless, the confidence at both Hanul sites remains
higher than that at the Hanbit site GuemSeong.

5 Conclusions

In this note we have investigated the effect of the systematic unknown nonlinear energy response of the detectors upon the sensitivity to the mass hierarchy which can be expected at medium baseline reactor experiments like JUNO and RENO 50. The size of this unknown depends on details of the calibration scheme which these experiments eventually adopt and so cannot be reliably determined at this time. However a target of 1% has been set for JUNO which is twice as small as the current record holder, KamLAND.

It is clear that this nonlinear response will pose one of the greatest challenges at these experiments and it will reduce the confidence with which the hierarchy has been determined. As had been suggested in Refs. [6, 7] and further confirmed in Ref. [8], a second identical detector at a significantly different baseline greatly reduces this effect as the relative energies of various features of the spectrum seen at the two detectors is independent of the energy response and is sufficient to determine the hierarchy.

In this work we considered three models of detector energy response. All three were optimistic. In particular the first was a subset of our nonlinearity fitting function and so was removed entirely by the fitting procedure. The second, the exponential model, is only appreciable in the very low energy regions of the spectrum where the finite resolution means that atmospheric oscillations are invisible. This part of the spectrum is not useful in a hierarchy determination. Above about 3 MeV, where the spectrum is sensitive to the neutrino mass hierarchy, this energy response model only shifts the energy by about 0.1% and so is an order of magnitude better than JUNO’s target. Despite this we observed that the effect on the confidence of the hierarchy determination is already noticeable.

Our final model of the nonlinear energy response is the worst case model of Ref. [21] scaled down by a factor of three. Once it is scaled down by a factor of three, its effect on the relative energies is about 0.5% and so it is still about twice as optimistic as JUNO’s target, providing a four times more precise determination of the neutrino energies than has ever been provided in such an experiment. Thus this third worst model is still quite optimistic, although less optimistic than the first two models. This optimistic model leads to a reduction in the confidence of the hierarchy determination of roughly 1σ in the case of our one detector configurations. However, two detectors with the same target mass and sufficiently different baselines are much less strongly affected by the unknown energy response, losing between 0 and 0.4σ of confidence.

While our study shows a reasonable advantage for two detectors over one detector with the same total target mass, it nonetheless underestimates this advantage in several key ways. First of all, the
energy response models considered were all very optimistic in that the unknown energy response at useful energies was in general limited to 0.5%. A larger unknown energy response is more problematic for the one detector scenarios, and therefore increases the advantage of a second detector.

Second of all, in the two detector scenario, the second detector is either the same size as in the one detector scenario or else it is smaller. If it is the same size, this means that although engineering considerations limit the mass of a spherical detector of this kind to about 20 kton, it would be possible to have 40 kton of target mass. As seen in our general study, the greatest advantage of the 2 detector setup is in this high statistics range, and indeed it seems to provide the only way to obtain a robust 3σ or more confidence in the hierarchy determination.

Alternately, one may choose a two detector configuration in which each detector is smaller. This also leads to several advantages. First of all, a smaller detector means a smaller distance between the events and the photomultiplier tubes. This means more photoelectrons will arrive per MeV, improving the energy resolution. The confidence in a hierarchy determination is extremely sensitive to small changes in the energy resolution [8]. The smaller distance that a photon must travel also makes the pulse shape narrower, which helps in the rejection of background and also in the determination of the location of the event. As the response of the detector strongly depends on the location already at Daya Bay [25], the determination of this location is necessary to control statistical errors in the number of photoelectrons so as to improve the energy resolution.

Finally, our study has underestimated the advantages of a second detector because we have considered individual experiments, which are statistically limited. Indeed our general study reveals that the greater the target mass, the better the statistics, the greater the advantage in splitting a detector into two at distinct baselines. In practice this statistical limitation can somewhat be overcome by coming data from RENO 50 and JUNO, and indeed even some data from 1 km baseline reactor experiments.

In summary, while the results of this paper show a clear advantage for the two detector proposal, this study underestimates this advantage in several key ways. It may well be, if the unknown nonlinear response is truly of order 1%, that a single detector proposal will not determine the hierarchy with even a single σ of confidence. In such a case a second detector would be necessary to make the resulting hierarchy determination credible.

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References

[1] S. T. Petcov and M. Piai, “The LMA MSW solution of the solar neutrino problem, inverted neutrino mass hierarchy and reactor neutrino experiments,” Phys. Lett. B 533 (2002) 94 [hep-ph/0112074]. S. Choubey, S. T. Petcov and M. Piai, “Precision neutrino oscillation physics with an intermediate baseline reactor neutrino experiment,” Phys. Rev. D 68 (2003) 113006 [hep-ph/0306017].

[2] F. P. An et al. [DAYA-BAY Collaboration], “Observation of electron-antineutrino disappearance at Daya Bay,” Phys. Rev. Lett. 108 (2012) 171803 [arXiv:1203.1669 [hep-ex]].

[3] J. K. Ahn et al. [RENO Collaboration], “Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment,” Phys. Rev. Lett. 108 (2012) 191802 [arXiv:1204.0626 [hep-ex]].

[4] Y. -F. Li, J. Cao, Y. Wang and L. Zhan, “Unambiguous Determination of the Neutrino Mass Hierarchy Using Reactor Neutrinos,” arXiv:1303.6733 [hep-ex].

[5] S. B. Kim, “Proposal for RENO 50: Detector Design and Goals,” presented at the International Workshop on RENO 50.

[6] E. Ciuffoli, J. Evslin and X. Zhang, “Mass Hierarchy Determination Using Neutrinos from Multiple Reactors,” JHEP 1212 (2012) 004 [arXiv:1209.2227 [hep-ph]].

[7] E. Ciuffoli, J. Evslin, Z. Wang, C. Yang, X. Zhang and W. Zhong, “Medium Baseline Reactor Neutrino Experiments with 2 Identical Detectors,” arXiv:1211.6818 [hep-ph].

[8] S. Kettell, J. Ling, X. Qian, M. Yeh, C. Zhang, C. -J. Lin, K. -B. Luk and R. Johnson et al., “Neutrino mass hierarchy determination and other physics potential of medium-baseline reactor neutrino oscillation experiments,” arXiv:1307.7419 [hep-ex].

[9] H. M. O’Keeffe, E. O’Sullivan and M. C. Chen, “Scintillation decay time and pulse shape discrimination in oxygenated and deoxygenated solutions of linear alkylbenzene for the SNO+ experiment,” Nucl. Instrum. Meth. A 640 (2011) 119 [arXiv:1102.0797 [physics.ins-det]].

[10] J. Alonso, F. T. Avignone, W. A. Barletta, R. Barlow, H. T. Baumgartner, A. Bernstein, E. Blucher and L. Bugel et al., “Expression of Interest for a Novel Search for CP Violation in the Neutrino Sector: DAEdALUS,” arXiv:1006.0260 [physics.ins-det]. C. Aberle, A. Adelmann, J. Alonso, W. A. Barletta, R. Barlow, L. Bartoszek,

[11] A. Bungan and A. Calanna et al., “Whitepaper on the DAEdALUS Program,” arXiv:1307.2949 [physics.acc-ph].

[12] M. Wurm et al. [LENA Collaboration], “The next-generation liquid scintillator neutrino observatory LENA,” Astropart. Phys. 35 (2012) 685 [arXiv:1104.5620 [astro-ph.IM]].

[13] E. Ciuffoli, J. Evslin and X. Zhang, “The Neutrino Mass Hierarchy at Reactor Experiments now that theta13 is Large,” JHEP 1303 (2013) 016 [arXiv:1208.1991 [hep-ex]].

[14] K. Nakamura et al. [Particle Data Group Collaboration], “Review of particle physics,” J. Phys. G 37 (2010) 075021.
[15] A. Gando et al. [KamLAND Collaboration], “Constraints on $\theta_{13}$ from A Three-Flavor Oscillation Analysis of Reactor Antineutrinos at KamLAND,” Phys. Rev. D 83 (2011) 052002 [arXiv:1009.4771 [hep-ex]].

[16] R. Nichol, “Final MINOS Results,” presented at Neutrino 2012.

[17] F. P. An et al. [Daya Bay Collaboration], “Improved Measurement of Electron Antineutrino Disappearance at Daya Bay,” Chin. Phys. C 37 (2013) 011001 [arXiv:1210.6327 [hep-ex]].

[18] E. Ciuffoli, J. Evslin and X. Zhang, “Confidence in a Neutrino Mass Hierarchy Determination,” arXiv:1305.5150 [hep-ph].

[19] X. Qian, A. Tan, W. Wang, J. J. Ling, R. D. McKeown and C. Zhang, “Statistical Evaluation of Experimental Determinations of Neutrino Mass Hierarchy,” Phys. Rev. D 86 (2012) 113011 [arXiv:1210.3651 [hep-ph]].

[20] M. Wilking, “New Results from the T2K Experiment: Observation of $\nu_e$ Appearance from a $\nu_\mu$ Beam,” presented at the EPS 2013 Conference.

[21] X. Qian, D. A. Dwyer, R. D. McKeown, P. Vogel, W. Wang and C. Zhang, “Mass Hierarchy Resolution in Reactor Anti-neutrino Experiments: Parameter Degeneracies and Detector Energy Response,” PRD, 87, 033005 (2013) [arXiv:1208.1551 [physics.ins-det]].

[22] H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, “Another possible way to determine the neutrino mass hierarchy,” Phys. Rev. D 72 (2005) 013009 [hep-ph/0503283].

[23] E. Ciuffoli, J. Evslin and X. Zhang, “Optimizing Medium Baseline Reactor Neutrino Experiments,” arXiv:1302.0624 [hep-ph].

[24] Y. Wang, “Neutrino Experiments at Reactors,” presented at the XXVI International Symposium on Lepton Photon Interactions.

[25] H. Huang, X. Ruan, J. Ren, C. Fan, Y. Chen, Y. Lu, Z. Wang and Z. Zhou et al., “Manual Calibration System for Daya Bay Reactor Neutrino Experiment,” arXiv:1305.2343 [physics.ins-det].