The influence of a TM wave on liquid flow in a capillary

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Abstract. Electromagnetic wave propagation along capillary with flowing liquid is considered. It is shown that there are frequencies at which the electromagnetic wave has a longitudinal component of the electric field hitting the double electric layer and causing an increase in the liquid flow rate caused by vibration effects. The increase of the velocity is related to the decrease of the adhesion force of the liquid to the inner wall of the capillary. It is shown that if vibration is switched on, the outer layer of liquid starts to move with some speed, hence the flow rate of liquid will increase. The vibration is caused by electric component of electromagnetic wave. Calculations have shown that the considered effect should also be observed at low intensities of electromagnetic wave.

1. Introduction

In [1] a model experiment with a glass capillary placed in the non-radiating holes of a rectangular waveguide is described. The holes are cut in the middle lines of the wide walls of the waveguide (with the working wave type TM10 the capillary is parallel to the vector of the electric component of the wave). If liquid is passed through the capillary under pressure and the wavelength is changed, the existence of a resonance wavelength can be verified, at which the absorption increases sharply. Additional experiments showed that at resonance frequency the speed of the liquid flowing through the capillary increases while at the same time the adhesion of the liquid to the inner wall of the capillary decreases. This fact can be used to explain the treatment mechanism of obliterating endarteritis, in which the blood capillaries of extremities of the patient are narrowed.

A similar effect should be observed when a capillary is placed in a waveguide, as well as during the propagation of an electromagnetic wave along the waveguide. In this case a TM wave must be considered as it has an electric field component directed along the liquid flow.

2. Methodology

Electromagnetic wave propagation is considered in the following areas:

1. \( r \leq r_1 \) – inner layer – layer of liquid inside the capillary, \( r_1 \leq r \leq r_2 \) – middle layer – capillary tube, \( r \geq r_2 \) – outer area (figure 1).

For the longitudinal and transverse components of electromagnetic waves of the transverse-magnetic type (TM) in selected areas the following expressions are valid [2]:

\[
E_{\eta} = AJ_\eta(k_1 r) \quad H_{\eta\phi} = \frac{i \omega \varepsilon_0 \varepsilon_1}{k_1} AJ_\eta(k_1 r),
\]

\[
E_{\phi} = \frac{i B}{k_1} AJ_\phi(k_1 r) \quad \text{if } r \leq r_1 ;
\]  

(1)
Figure 1. Capillary cross-sectional area.

\[ E_{z1} = BI_0(k_r r) + CK_0(k_r r), \quad H_{\phi_1} = \frac{i \omega \varepsilon_1}{k_2} \left[ BI'_0(k_r r) + CK'_0(k_r r) \right] \]
\[ E_{z2} = \frac{i \omega \varepsilon_2}{k_3} DK'_0(k_r r), \quad H_{\phi_2} = \frac{i \omega \varepsilon_3}{k_3} DK'_0(2k_r r) \]
\[ E_{z3} = \frac{i \omega \varepsilon_3}{k_3} \]

if \( r_1 \leq r \leq r_2 \);

where \( A, B, C, D \) are the unknown coefficients, \( k_n \) and \( \varepsilon_n \) are the transverse wave numbers and permittivities in different areas (layers), \( n \) is the layer number.

Expressions for the interrelation between the longitudinal wave number \( \beta \) and transverse wave numbers \( k_1, k_2, k_3 \) are presented below:

\[ k_1^2 = \mu_1 \varepsilon_1 \frac{\omega^2}{c^2} - \beta^2, \quad k_2^2 = \beta^2 - \mu_2 \varepsilon_2 \frac{\omega^2}{c^2}, \quad k_3^2 = \beta^2 - \mu_3 \varepsilon_3 \frac{\omega^2}{c^2} \]

Using the continuity conditions for the tangential components of the electric and magnetic fields at the layer boundaries (\( H_{\phi_1}(r_1) = H_{\phi_2}(r_1), E_{z1}(r_1) = E_{z2}(r_1), H_{\phi_2}(r_2) = H_{\phi_3}(r_2), E_{z2}(r_2) = E_{z3}(r_2) \)) and using mathematical transformations, a dispersion relation of the following form can be obtained [3]:

\[ \frac{\varepsilon_1}{k_1} J_1(k_1 r_1) + \frac{\varepsilon_2}{k_2} I_1(k_2 r_1) \cdot I_0(k_2 r_1) = \frac{\varepsilon_2}{k_2} K_1(k_2 r_1) - \frac{\varepsilon_1}{k_1} J_1(k_1 r_1) \frac{k_2 J_0(k_2 r_1)}{K_0(k_2 r_1)} - \frac{k_1 J_0(k_1 r_1)}{K_0(k_1 r_1)} \]

For the values of the cross section radii \( r_1 = 1 \text{ mm}, \ r_2 = 2 \text{ mm} \) and at the values of the relative permittivities \( \varepsilon_1 = 60, \varepsilon_2 = 5, \varepsilon_3 = 1 \) at a frequency of \( \omega = 1.4 \times 10^8 \text{ (} \nu = 22.28 \text{ GHz} \text{)} \) the calculations were carried out. The corresponding plot of the field component distribution \(|E_z|\) from the radial component is shown in figure 2.

As it can be seen from figure 2, there is a \( z \)-component of the electric field at the surface of the capillary which will act on the electric double layer occurring at the liquid-capillary boundary. The resulting varying force will produce vibrational effects.

Many vibration effects are known in mechanics: disappearance of initial and appearance of new equilibrium positions and types of system motions, change of equilibrium positions, vibration transport, change under the influence of vibration of rheological properties of objects in relation to slow effects, separation of particles by their properties and others [4, 5]. All these effects are associated with seeming violations of the laws of mechanics (e.g., a heavy metal ball floats in a layer of sand under the influence of vibration).
Figure 2. Distribution of the field component $|E_z|$ of the TM$_{01}$ wave in the capillary as a function of the radial component.

All mentioned effects are characterized by the fact that the motion of the system caused by vibration consists of two components - "slow", which changes slightly during one period of vibration, and "fast" vibration. Assume two observers monitoring the experiments: "fast" - who notices all forces and movements and "slow" - who notices neither fast (usually small) movements, nor fast forces (or, what is the same, experimental data are taken from the device, which does not have time to react to fast changes of the measured value, and therefore shows average values). Such an observer, unlike the "fast" one, will notice only the slow component of motion and therefore he must explain all vibrational effects by introducing additional slow forces, which are called vibrational forces. The motion of a system with one degree of freedom is described by the equation:

$$ m\ddot{x} = F(\dot{x}, x, t) + f(x, x, t, \omega t), $$

where $m$ – mass; $x$ – coordinate; $F$ – «slow» force; $f$ – «fast» force; the point denotes the derivation in time $t$. The "fast" force in the elementary case is a periodic function $\omega t$ with period $2\pi$. Hereafter, assume that the motion is represented in the form:

$$ x = X(t) + \chi(t, \omega t), $$

where $X$ – «slow» component and $\chi$ – «fast» component. A slow observer who does not notice the fast motion $\chi$ and the fast force $f$ will think that the slow motion is described by the equation:

$$ V(\dot{X}, X, t) = \left\{ f(\dot{X} + \dot{\chi}, X + \chi, t, \omega t) \right\} + \left\{ f(\dot{X}, X, \chi, t) \right\}, $$

where $F(\dot{X}, X, \chi, t) = F(\dot{X} + \dot{\chi}, X + \chi, t) - F(\dot{X}, X, t)$.

All the effects discussed above are explained by the occurrence of the vibrational force $V$.

Assume that a horizontal force $f_0 \sin \omega t$ with amplitude $f_0$ and frequency $\omega$ affects an object of mass $m$ lying on a horizontal surface. If the coefficients of dry friction when the object slides forward and backward along the plane $k_\perp$ and $k_\parallel$ are the same and equal to $k$, then it is clear that in case of $f_0 < kmg$ the object is stationary, and in case of $f_0 > kmg$ it vibrates symmetrically relatively to some middle position. Assuming, for example, that $k_\perp > k_\parallel$, the symmetry will be changed, and if $f_0 > k_\perp mg$ the object will be moved in the positive direction.

It is possible to explain the mechanism of water adhesion change by using the above data. Consider a viscous liquid flowing along a pipe of radius $R$ under the influence of a pressure difference at the edges.
of the pipe. If the liquid flows through a vertical pipe, this pressure difference can only be explained by the action of gravity. Distribution of velocities of liquid layers depending on the distance from the pipe axis has the following form:

\[ U(r) = \frac{P_1 - P_2}{4\eta l} \left( R^2 - r^2 \right) \]

where \( P_1 - P_2 \) is the pressure difference at the edges of the pipe, \( \eta \) is the viscosity of the liquid, \( l \) is the length of the pipe. This formula is derived from the assumption that the velocity of the liquid layer adjacent to the pipe wall is equal to zero. If the adhesion force becomes so small that this liquid layer gets a certain velocity \( U_0 \) relatively to the pipe walls, then the velocity of liquid flowing out of the pipe at the same pressure difference will also increase (figure 3).

The new formula for the velocity distribution can be written as follows:

\[ U(r) = \frac{P_1 - P_2}{4\eta l} \left( R^2 - r^2 \right) + U_0. \]

\[ \text{Figure 3. Liquid velocity distribution in the capillary cross section.} \]

3. Results and discussion

Now assume that the liquid flows through a vertical pipe under the action of gravity. The liquid layer is located directly at the pipe wall. This layer is affected by gravity, adhesion force from the pipe wall and viscous friction force from the adjacent layer of liquid.

The adhesive force per unit surface for water \( (\sigma = 73 \text{ mN/m}) \) is equals to \( 50 \text{ N/m}^2 \) [6].

Estimate the forces of gravity and viscosity, which are opposite to the adhesion force for the layer of liquid adhering to the pipe wall. Usually, the radius of capillaries does not exceed 1 mm, and the thickness of the liquid layer adhering to the pipe wall is nearly 200 – 800 Å (it is the thickness of the layer of water involved in the molecular interaction). This force of \( F_A \approx 5 \text{ N/m}^2 \) is one order of magnitude smaller than the adhesion force, i.e. it cannot move the liquid layer relatively to the pipe.

As it can be seen from figure 2 the electric field strength in the vicinity of the double layer electric field has a value of the order of 100 V/m.

Thus, in order for the amplitude of the high-frequency force per unit surface to exceed the adhesion force per unit surface, the effective surface charge density of water must be of the order of

\[ \rho_S = \frac{f_0}{SE_0} = 1 \text{ C/m}^2. \]

Taking into account that there are \( 2 \cdot 10^{22} \text{ m}^2 \text{ water molecules in a liquid layer of } 800 \text{ Å thickness about } 0.1\% \) of water molecules must be ionized to satisfy this condition (experiments have shown that even strongly deionized water interacts with an external electric field[7]). In this case, the Coulomb force acting on the layer of water adjacent to the pipe wall will correspond to the high-frequency force, and
\[ U_0 = \frac{\rho_S E_0}{h\rho_0} \cos \frac{F_{A/S} - F_{B}}{2F_{A/S}} \pi. \]

In this case, the cosine is equal to 0.16, and \( U_0 = \frac{\rho_S}{\rho_0} \cdot 1.6 \cdot 10^5 \) m/sec. Whereas the flow rate is calculated using the formula \( W = \int_0^R 2\pi r U(r) dr \), the liquid flow rate will be increased by \( \Delta W = U_0 \pi R^2 \).

4. Conclusion

As it can be seen, the propagation of electromagnetic waves along a capillary with liquid flowing in it, can increase its flow. It can be usable for both medical and technical purposes. It is also important to note that this effect will depend on the liquid, as for different dielectric permittivities it is necessary to select the frequency at which the field affects the double electric layer. Thus, this circuit could potentially be used to determine the dielectric permittivity of a liquid.

References

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