SU(5) unification for Yukawas through SUSY threshold effects

Ts. Enkhbat

The Abdus Salam ICTP,
Strada Costiera 11, 34014 Trieste, Italy

Abstract

It is well known that the supersymmetric finite threshold effects can induce substantial corrections to the Standard Model fermion masses. This opens an alternative possibility to correct the problematic mass ratios of the lighter generations within the minimal SU(5) GUT. We show that with large soft A–terms, one can achieve simple unification for lighter generations without additional Higgs multiplet, while having sfermions lighter than 1 TeV. The presence of such large A–terms will distort the sfermion mass spectrum upon running from GUT scale down to the electroweak scale making it distinct from the universal SUSY breaking sector, especially in the first two generations. The implications of these splittings are studied in K and D meson oscillations and in rare processes $D^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$, and in the latter case the effect is found to be important.

E-mail address: enkhbat@ictp.it
1 Introduction

The supersymmetric (SUSY) version of the Standard Model (SM), while having a stabilized Higgs mass, displays a much better unification of gauge couplings than the non-SUSY version. The Yukawa couplings of $b$-quark and $\tau$-lepton unify at a reasonably good level as well: a slight discrepancy of $\sim 20\%$ can be remedied by various corrections. These successes fail to extend to the lighter two generations. In particular, the experimentally determined mass ratio between the down and strange quarks is an order of magnitude larger than the electron to muon ratio, which are predicted to be equal to each other if the minimal unification is assumed at the scale of the grand unification theory (GUT) for all generations.

In most GUT models this shortcoming is accounted for by adding either Yukawa interactions of a new Higgs multiplet or multiplets such as $45$ in $SU(5)$ [1] or higher dimensional operators [2]. In SUSY theories another possibility of correcting these wrong relations opens up due to the threshold effects from SUSY breaking which was reported first time in Ref. [3], and applied for GUT in Ref. [4]. Since then, there have been many studies on these effects of SUSY breaking on the fermion masses. An incomplete list is given in Refs. [5]–[22]. In Refs. [8] [10] the importance of these corrections for $b$–quark in the large $\tan\beta$ limit has been pointed out for unified models based on $SO(10)$. In particular, Hall et al. in Ref. [8] showed that the loop induced $Qd^cH_u^*$ interaction contributes a large effect to the down-type quark masses due to $\tan\beta$ enhancement. By now, these SUSY threshold corrections are an integral part of phenomenological studies [23] and every popular code for SUSY spectrum includes them at least for the third generation.

In parallel to these studies there have been many theoretical models which took the advantage of these corrections to explain the observed fermion mass hierarchies. In Ref. [12] a model with 4th family and horizontal gauge symmetry has been considered where it was shown that one can achieve unification for all Yukawas without additional Higgs representation or higher dimensional operators. Along the similar line, in Ref. [13] models were proposed where the masses of the lightest charged fermions are induced purely by the SUSY threshold corrections through flavor violating soft masses. In Ref. [14] Babu et al. showed that the CKM mixing can be induced in a similar manner in left–right models.

All these studies could be classified in the following two categories: models which (i) explain the fermion mass pattern by soft parameters or instead (ii) use them to achieve Yukawa unifications for certain GUT models. In both cases one common trend that has
been observed in a number of studies was to move away from a minimal choice for the SUSY breaking parameters for achieving acceptable fermion masses and mixings [16–22]. Diaz Cruz et al. [16] has considered the effect of large flavor violating A–terms for down type quarks in the first two family. They start with the minimally unified Yukawa couplings then correct the wrong GUT relation via the large A–term effect for the down–type quarks. With the form they have chosen for the A–terms the correct effective Yukawa couplings and the Cabibbo mixing were obtained. To do so, on the other hand, they have concluded that the GUT relation can be corrected with sfermions heavier than 4.4 TeV and $\tan \beta \sim 2$ while gluino is lighter than TeV to be consistent with the FCNC constraints mostly from $\mu \rightarrow e\gamma$.

Soon the LHC will start and probe a new energy frontier around TeV. These corrections become much more interesting if the sfermions are lighter and hopefully reachable at the LHC. Then, presence of large A–terms could be related to the observed sfermion masses. This will be complimentary to any new FCNC signals associated with such corrections. If breakthroughs happen experimentally on both sides, eventually these corrections can be tested or ruled out by the experiments. Also very low $\tan \beta \lesssim 3$ seems to be excluded by LEP II Higgs search analysis [24] except for very small window for $\tan \beta \lesssim 1$ in the case of no left–right scalar mixings which gives no radiatively induced correction to the fermion masses. Thus it is desirable to study the issue of the minimal unification in the range of moderate and large $\tan \beta$ range.

In this paper we study large A–terms for the down–type quarks, which are not proportional to the corresponding Yukawa couplings, nevertheless have flavor diagonal form, that corrects the wrong GUT ratios. This choice, as we demonstrate, will escape the FCNC constraints even for sub TeV sfermion masses. Such large A–terms split the masses of down type sfermions in the first two generations, making the spectrum distinct from usual universal SUSY parameters which could be probed at the LHC and/or ILC. We study the $D$ and $K$ meson oscillations, rare processes $D \rightarrow \pi \nu \bar{\nu}$ and $K \rightarrow \pi \nu \bar{\nu}$. Although the rate for the $D$–meson is found to be four order of magnitude larger than its SM prediction, is still far from the reach of ongoing BESIII experiment. On the other hand we have found a large effect in the Kaon case in some of our solutions of the SUSY threshold induced unification. When the precision of the branching ratio improves [25], it could be used for determining whether there is a trace of such new physics through the SM global fit.

The structure of the paper is as follows. In section 2, I give a brief review of the finite corrections to the fermion masses, and explain based on qualitative arguments why we...
need non minimal soft parameter for the minimal unification. The section 3 is devoted for the details of our numerical calculations and main results. The conclusions is given in section 4. The relevant formulae for the SUSY threshold corrections to the Yukawa couplings are given in the Appendix.

2 Radiative Corrections to Fermion Masses in the MSSM

In this section, we review the SUSY threshold corrections to the fermion masses and highlight the qualitative features of such corrections. This will enable us to see what choices of soft parameters would induce the correct level of threshold effects that are needed for changing the wrong GUT ratio.

2.1 The SUSY threshold corrections

Whenever a heavy field or fields decouple from a theory they induce finite shifts in the parameters of the theory. In the SUSY extension of the SM, these threshold corrections are induced as the SUSY partners of the SM fields decouple. The decoupling scale is believed to be around or not much higher than the electroweak symmetry breaking scale if SUSY is to stabilize the gauge hierarchy. In particular, the Yukawa couplings receive finite threshold corrections from gaugino–sfermion, Higgsino–sfermion loops. The corresponding diagrams are depicted in Figure 1 in the case of quarks. These corrections are especially important due to the fact that the down–type quarks and charged leptons obtain a new Yukawa interaction to the up–type Higgs doublet, which, upon electroweak symmetry breaking, results in tan β enhanced corrections.

Now we elaborate on the details of these corrections in the case of quarks. The full expressions for these corrections are given in the Appendix, which we have used in our numerical analysis. The total correction for the quarks are given as follows:

\[(\delta m_q) = (\delta m_q^G) + (\delta m_q^N) + (\delta m_q^C),\] (1)

where the gluino–squark loop induced correction is given by

\[ (\delta m_q^G)_{ij} \approx -\frac{2\alpha_s}{3\pi} (m_{LR}^q) \tilde{m}_i \tilde{m}_j \left( m_{\tilde{Q}}^2, m_{\tilde{c}}^2, m_{\tilde{g}}^2 \right), \] (2)
The definitions of the mass parameters in the above formula are given in the Appendix along with the details of the remaining two corrections. The loop function behaves approximately as $I(m^2, m^2, m^2) \simeq 1/(2m^2)$. To see the qualitative features, here we concentrate on the finite correction from the gluino–squark loop which is usually the dominant one. For instance, the Bino–squark induced correction has $8\alpha_s/(3\alpha') \simeq 0.03$ factor compared to the above correction. Here we approximate the mass eigenvalues of the squarks by their soft–mass parameters as $m_{\tilde{g}}^2 \simeq m_{\tilde{Q}}^2 \simeq m_{\tilde{u}}^2$, ignoring the mixings as well. If we assume the A–terms proportional to the corresponding Yukawa couplings the correction takes the following form

$$\frac{\delta m_{\tilde{d}_i}}{m_{\tilde{d}_i}^0} \simeq \frac{\alpha_s (a_0 - \mu \tan \beta)}{3\pi} \frac{m_{\tilde{g}}}{m_{\tilde{d}_i}^2}. \quad (3)$$

From this formula we observe that the effect does not decouple in the limit of large SUSY breaking parameters. Also, concentrating on the term proportional to $\tan \beta$, it is easy to realize that the induced effect could be as large as the tree level term in the large $\tan \beta$ limit. For example, for $\tan \beta \sim 50$, we see that the enhancement overcomes the loop suppression factor:

$$\frac{\delta m_{\tilde{d}_i}}{m_{\tilde{d}_i}^0} \simeq \left[10^{-2} \tan \beta\right] \left(\frac{\alpha_s}{0.1}\right) \frac{\mu m_{\tilde{g}}}{m_{\tilde{d}_i}^2}. \quad (4)$$

If the soft masses are universal, we see that the induced changes are flavor universal. On the other hand, the needed corrections to the down and strange quark masses that fix the wrong GUT ratio are far from universal: If we want to make these corrections in the quark sector, we must increase the down quark mass while decreasing the strange mass. Therefore we seem to come to an inevitable situation that we should depart from the universal soft parameters. Indeed, by scanning the flavor universal soft SUSY parameter space, several groups have found no solution to the wrong GUT ratio \[16, 19, 22\]. In the next subsection, we elaborate on this issue.

Although numerically less significant due to weaker interactions, the same $\tan \beta$ enhancement occurs to the contributions from the neutralino– and chargino– squark loops. While they are numerically irrelevant for the lighter two generations, the chargino–stop loop induced correction to the bottom mass could be substantial due to large top Yukawa coupling. In particular, they give sizeable contributions to the (13) and (23) quark mixings \[9\]. In a certain part of parameter space, such corrections lead to $\sim \tan \beta^6$ enhancement for the process $B_s^0 \rightarrow \mu^+\mu^-$ \[11\], which could bring it to the present upper bounds. Furthermore, if a slight discrepancy of $\sim 13\text{–}24\%$ in the $b$–$\tau$ unification is accounted by these
corrections, the branching fraction of the light Higgs decay to $b\bar{b}$ is altered substantially compared to the SM or general two–Higgs doublet models.

2.2 Non–minimal soft A–terms for minimal unification

Inevitable fact about SUSY extension of the SM is that it inflates the number of free parameters in the theory from two dozens to over a hundred upon SUSY breaking in its full generality. They introduce new sources for FCNCs and CP–violations which have been subject of intensive research for over two decades. Experimental constraints from $K^0 – \bar{K}^0$ mixing, flavor violating $\mu \rightarrow e\gamma$ and many other processes suggest that SUSY breaking parameters are either flavor blind or aligned with the Standard Model flavor structure at an extremely high degree. Indeed most of the phenomenological studies, in particular collider analysis, concentrate on one of the universal SUSY breaking scenarios. This is certainly understandable considering the enormous size of the SUSY parameter space, which makes any attempt of generic study impractical. Secondly there is no compelling theoretical reason that points to a certain part of parameter space which differs from those universal ones. On the other hand, as we mentioned earlier, if one wishes to correct the wrong GUT ratios for the light fermions using the SUSY threshold corrections, one should most likely depart from a universal choice for soft terms. Such flavor non–universality in the first two generation could be phenomenologically quite distinct from the traditional universal scenarios and could lead to interesting rare decays. If sfermions are discovered at LHC, their spectrum could confirm/rule out this scenario.

Here first we quantify what amount of threshold corrections are needed for the minimal $SU(5)$ unification using approximate expressions. Then discuss the choices of the soft
SUSY breaking parameters which would induce such corrections.

If one sets the strange quark mass equal to the muon mass at GUT scale, without taking into account the threshold effects, its low energy value would be around $\sim 200$ MeV which is greater by factor of $\sim 4$ than its experimentally measured value. To get the correct mass value we need corrections of order $\sim 150$ MeV. The approximate estimate of the gluino–squark induced term for $s$–quark, for TeV soft masses can be written as follows:

$$\delta m_s \simeq -\frac{2\alpha_s}{3\pi} v (A_s \cos \beta - y_s \mu \sin \beta) \frac{m_{\tilde{g}}}{2m_{\tilde{s}}^2} \simeq \pm 25.6 \text{ MeV} \left(\frac{\alpha_s}{0.1}\right) \left(\frac{m_{\tilde{g}}}{700 \text{ GeV}}\right) \left(\frac{1 \text{ TeV}}{m_{\tilde{s}}}\right)^2 \left[5.0 \times \left(\frac{A_s}{-500 \text{ GeV}}\right) \left(\frac{10}{\tan \beta}\right) + 1.2 \times \left(\frac{\mu}{1 \text{ TeV}}\right) \left(\frac{y_s}{1.2 \times 10^{-2}}\right)\right].$$

The plus (minus) sign belongs to a positive (negative) value for the gluino mass parameter. Here we made an approximation $I(m_{\tilde{s}L}^2, m_{\tilde{g}}^2, m_{\tilde{q}}^2) \simeq -1/(2m_{\tilde{s}}^2)$, where $m_{\tilde{s}}^2 \sim 1 \text{ TeV}^2$. This crude estimate shows that the $A$-term, if chosen to have a large value, in spite of $\tan \beta$ suppression, can be quite important and could even become the dominant source of the threshold correction. Such large values are subject to the metastability condition, which will be discuss shortly. To contribute constructively with the $\mu$–term part one should choose the sign for $A_s$ to be opposite to that of $\mu$.

If $\tan \beta \gtrsim 30$ one approaches the stability with limit of $|A_s| \lesssim 1.75 m_s \text{ TeV}$ (See Eq. (5)). Such a large value can be easily accommodated by slightly increasing one of the soft masses in the condition of metastability given by Eq. (5). To reduce the too large value of the $s$ quark mass due to the unification condition, the net effect must give negative contribution which can be accommodated by the following choices: (i) positive $m_{\tilde{g}}$ and $A_s$ with preferably negative $\mu$–term, (ii) negative $m_{\tilde{g}}$ and $A_s$ with preferably positive $\mu$–term. This choice lowers the down quark mass which has to be increased to $m_{d,\text{exp}}$. Fortunately one can see from the discussion of the $s$–quark case, it is much easier to alter the $d$–quark mass via $A$–term due to its tiny Yukawa coupling. For example, with similar choice of parameter, $A_d \sim 20$–30 GeV is sufficient to induce the needed correction, which is well within the metastability limit of Eq (5). If we try to remedy the lighter two generations only using $\mu$–term, without relying on a large $A$–term, we must induce $\sim -0.35$ MeV change for the $d$–quark. Since everything is specified by the unification, the only freedom left is the choice of the soft masses which translates to the ratio for their loop functions to be $I_d/I_s \simeq 4.7$ ($I_q \equiv I(m_{\tilde{q}L}^2, m_{\tilde{q}}^2, m_{\tilde{g}}^2)$). Such a large mass splitting in
the first two generation is unlikely to survive severe FCNC constraints for sub TeV scalar masses. In any case, at low and moderate values of tan $\beta$, the $\mu$–term part cannot induce enough effects. This leaves us with the choice of a non–minimal $A$–term for either $d$ or $s$–quark.

Let us consider the case where we correct the $d$–quark mass by a large $A$–term while use $\mu \tan \beta$ for correcting the $s$–quark mass. To have a substantial correction for the $s$–quark one must choose a larger value for the $\mu \tan \beta$–term. Such a choice would, at the same time, reduce the $b$–quark mass by a potentially large amount. This reduction can not be too large, otherwise it would jeopardize the already somewhat good $b$–$\tau$ unification. In the MSSM, the RG runnings of the $\tau$ and $b$ Yukawas from $M_Z$ do not give a unified value at GUT. Instead one obtains $m_\tau(M_{GUT})/m_b(M_{GUT}) \simeq 1.13 \div 1.24$ depending on $\tan \beta$. Since we will impose the unification condition $m_\tau(M_{GUT})/m_b(M_{GUT}) = 1$ by equating the Yukawa coupling of the $b$–quark to that of the $\tau$–lepton, its low energy value before the threshold correction will be higher than the experimentally determined value. Thus, to bring to an agreement one must choose the value of the SUSY breaking parameters in such a way that the net threshold correction is around $-24\%$ to $-13\%$ depending on the value of $\tan \beta$. Nevertheless this is a much smaller percentage change compared to the lighter generations: With the same mass as muon at the GUT scale one gets a factor of $\sim 4$ bigger strange quark mass, which requires 75% reduction from the radiative corrections. Therefore, without the effect from large $A$–terms, we have to choose quite heavier sbottom mass as can be seen from

$$
\frac{\delta m_s/m_s^0}{\delta m_b/m_b^0} \simeq \frac{I_s}{I_b} \simeq \frac{m_s^2}{m_b^2} \simeq 3 \div 5. \quad (6)
$$

Here we again come to the conclusion that the squark masses have to be very different from each other. To summarize, our heuristic arguments show that when the wrong GUT ratio of the fermion masses are corrected by the SUSY threshold effects in the down–quark sector, the following options are available on their soft parameters:

(i) One of the lighter generations has a large $A$–term, while the sbottom soft masses are heavier than the remaining generations by a factor of $\sim 1.5$ to 2.4.

(ii) The $A$–terms are proportional to their Yukawa couplings, while the soft masses differ from each other by large amount. In this case the FCNC constraints require them to be very heavy in the range of tens of TeVs.

(iii) The $s$ and $d$ quarks both have a large $A$–term, where soft masses can be chosen to be degenerate and not very heavy.
Certainly any of these options are viable. We find the last one interesting due to its potential implication on LHC phenomenology, and choose it for our numerical study.

2.2.1 The metastability condition for $A$–terms

Beside phenomenological constraints on the soft SUSY breaking parameters there are indirect constraints coming from the requirement of the stability of our vacuum \cite{26}–\cite{28}. In Ref. \cite{15} Borzumati et. al. have discussed the implication of this condition for models of radiative fermion masses. To be absolutely stable against decaying into color/charge breaking vacua along the $D$–flat direction where $|f_i| = |f_c| = |H|$, the following condition must be met the for Yukawa and the trilinear $A$–term, $Y_f \tilde{f} c \tilde{f} H$ and $A_f \tilde{f} c \tilde{f} H$:

$$|A_f| \lesssim |y_f| \tilde{m},$$

$$\tilde{m}^2 \equiv \frac{1}{3} (m_f^2 + m_f^2 c + m_H^2 + \mu^2)$$

where $m_f^2$, $m_f^2 c$ and $m_H^2$ are the soft masses of the fields connected by the $A$–term. This choice of absolute stability, which has no physical justification, severely restricts the size of $A$–term. When applied to the off–diagonal entries it is more constraining than FCNC processes \cite{29}.

On the other hand if we require only metastability, namely the age of the unstable vacuum is longer than the age of the Universe, the constraint from numerical analysis gives \cite{30}–\cite{32}:

$$\frac{|A_f|}{\tilde{m}} \lesssim 1.75.$$ (8)

This metastability condition allows much more relaxed parameter space compared to the condition of absolute stability. With a large $A$–term at one’s disposal it is now much easier to correct light fermion masses by the SUSY threshold effects. Even for large $\tan \beta$ such a large $A$–term can easily overcome $1/ \tan \beta$ suppression compared to the $\mu$–term part for the down type fermions and compete with or even dominate over $\tan \beta$–enhanced contributions. This is an appealing scenario for any GUT model where one does not need to introduce additional Higgs representations.
3 Implications of unification through SUSY radiative corrections

In this section, we present our numerical study of the SUSY radiative corrections to the fermion masses. In the first subsection we describe the choice of the soft parameters and the numerical procedures. In the last part of the section, we discuss the electroweak scale sfermion spectrum and their experimental implications.

3.1 The numerical procedures and the results

Here we describe our numerical calculations and the results. For the input values of the fermion masses at $M_{SUSY} = 1$ TeV we have used the results of a recent update on the running fermion masses [33] (see [34] for earlier analysis), where the running masses are calculated in the case of SM and MSSM at 1 TeV. Their results at 1 TeV for the SM are:

\begin{align*}
    m_d &= 2.50 \pm 1.0 \text{ MeV}, \quad m_s = 47 \pm 14 \text{ MeV}, \quad m_b = 2.43 \pm 0.08 \text{ GeV} \\
    m_u &= 1.10 \pm 0.4 \text{ MeV}, \quad m_c = 0.532 \pm 0.074 \text{ GeV}, \quad m_t = 150.7 \pm 3.4 \text{GeV}. \quad (9)
\end{align*}

We take the central values of these results for our analysis. Here we note that the errors in the estimates given by Xing. et al. in Ref. [33] are from the Particle Data Group [35] which are notably larger compared to lattice QCD analysis [36]. Keeping this in mind we seek results that are as close to the central values as possible. As for the values at $M_{SUSY} = 500$ GeV we have used

\begin{align*}
    m_d &= 2.63 \pm 1.05 \text{ MeV}, \quad m_s = 51 \pm 14 \text{ MeV}, \quad m_b = 2.53 \pm 0.08 \text{ GeV} \\
    m_u &= 1.17 \pm 0.4 \text{ MeV}, \quad m_c = 0.553 \pm 0.074 \text{ GeV}, \quad m_t = 153.6 \pm 3.4 \text{GeV}. \quad (10)
\end{align*}

We have used extensively SOFTSUSY [37], a C++ based publicly available code which calculates the MSSM spectrum. Initially soft masses are chosen with some universal values. First we perform MSSM running without the threshold corrections at SUSY breaking scale $\sim 1$ TeV, to the GUT scale, $\mu_{GUT} \simeq 2 \times 10^{16}$ GeV, and impose minimal $SU(5)$ unification:

\begin{align*}
    \alpha_i &= \alpha_U, \quad (i = 1, 2, 3) \quad (11) \\
    Y_5 &= Y_d = Y_e^T, \quad (12) \\
    A_5 &= A_e = A_d. \quad (13)
\end{align*}
\begin{align}
(m^2_L)_{ij} &= \left(m^2_d\right)_{ij} = \left(m^2_S\right)_{ij} \delta_{ij}, \\
(m^2_{\tilde{e}c})_{ij} &= \left(m^2_{\tilde{d}c}\right)_{ij} = \left(m^2_{\tilde{u}c}\right)_{ij} = \left(m^2_{\tilde{Q}c}\right)_{ij} = \left(m^2_{10}\right) \delta_{ij},
\end{align}

(14)
\begin{align}
(m^2_{\tilde{e}c})_{ij} &= \left(m^2_{\tilde{d}c}\right)_{ij} = \left(m^2_{\tilde{u}c}\right)_{ij} = \left(m^2_{\tilde{Q}c}\right)_{ij} = \left(m^2_{10}\right) \delta_{ij},
\end{align}

(15)

After the GUT conditions in Eq. (11) are imposed we run down back to the SUSY breaking scale. We repeat this enough until we reach stable low scale values for the Yukawa couplings of the down–type quarks. These values will tell us then how much corrections we need from the threshold effects.

We have done the two–loop RGE running to GUT scale and back to \(M_{SUSY}\) to determine the discrepancy between the experimentally determined values and the values derived from the unification \(y_u = y_{\tilde{u}}|_{GUT}\). Since the effects on the charged lepton masses are minor, the initial choice for the leptonic Yukawa couplings before including the threshold corrections would be quite close to the full effective low energy values. Thus, the numerical entries for the unified Yukawa coupling matrix \(Y_5\), defined in Eq. (12), are chosen by the leptonic Yukawa coupling matrix after running them to GUT scale. Therefore the tree level values of the light down–type quark Yukawa couplings will differ from their observed values significantly leading to the wrong GUT ratio. The objective of our numerical study is to tackle this problem by identifying the soft parameters which give the needed corrections. For the \(d\) and \(s\)–quarks the discrepancies are practically independent of \(\tan \beta\) and we have

\[\delta m_d \simeq 1.5 \text{ MeV and } \delta m_s \simeq -156 \text{ MeV for } \tan \beta = 5 \div 50, \quad M_{SUSY} = 1 \text{ TeV}.\]
\[\delta m_d \simeq 1.6 \text{ MeV and } \delta m_s \simeq -163 \text{ MeV for } \tan \beta = 5 \div 50, \quad M_{SUSY} = 500 \text{ GeV}.\]

(16)

As for the \(b\)–quark mass, the needed correction mildly depends on \(\tan \beta\). The result is summarized in Table I. Since the contribution from the \(A\)–term is subleading for \(b\)–quark,

| \(\tan \beta\) | 5   | 10  | 15  | 20  |
|----------------|-----|-----|-----|-----|
| \(\delta m_b\) (GeV), \(M_{SUSY} = 1 \text{ TeV}\) | -0.690 | -0.699 | -0.687 | -0.666 |
| \(\delta m_b\) (GeV), \(M_{SUSY} = 500 \text{ GeV}\) | -0.710 | -0.720 | -0.705 | -0.693 |

Table 1: The needed corrections for the mass of \(b\)–quark.

most of the corrections should come from the \(\mu \tan \beta\) part. Then, the similar change of \(\sim 20\%\) are automatically induced for the other generations. For a very large choice of \(A_b\) at low \(\tan \beta\) one can still get large effect.
3.1.1 Parameter choices and the induced corrections

Now we give the details of our choice for the numerical values of the soft parameters which would yield the needed threshold effects to give the correct effective masses.

The choices of the initial values of the soft terms, although highly dependent on the SUSY breaking mechanism and the scale at which it is mediated, we choose universal scalar masses at GUT scale. We choose the following parameters as the input of the calculation:

(i) At GUT scale: The trilinear $A$–term for the $5$–plet is chosen to be simultaneously diagonalize with its corresponding Yukawa coupling, $Y_5$, but not proportional to the corresponding eigenvalues upon digitalization:

$$(A_5)_{ij} = a_i \delta_{ij} \neq a_0 y_5.$$  \hspace{1cm} (17)

The sfermion soft masses are chosen to be flavor–universal

$$m_{10}^2 = m_Q^2 \times I_{3 \times 3},$$  \hspace{1cm} (18)

For minimal $SU(5)$ unification, we do not have an immediate concern to change the up sector Yukawa. Therefore, we keep the trilinear $A$–term for the $10$ to be minimal at GUT scale $A_{10} = a_{10}^0 Y_{10}$. The gaugino masses are also chosen to be universal.

(ii) The trilinear terms $A_d$, $\mu$–term, the soft highs masses $m_{H_u}^2$ and $m_{H_d}^2$ are chosen at SUSY breaking scale. We work in the basis the Yukawa matrices of the charged leptons and down type–quarks are diagonal. For a chosen set of $m_{10}^2$ and $m_5^2$, first we have determined $\mu$ and $m_{1/2}$ that give the correction given in Table 1. Samples of these are shown in Table 2 for various choices of $\tan \beta$. After this, we have scanned over $A$–terms

| $\tan \beta$ | 5   | 10  | 15  | 20  |
|--------------|-----|-----|-----|-----|
| $m_{1/2}$    | -210| -210| -230| -230|
| $m_{Q_i}^2$  | 0.314| 0.314| 0.336| 0.336|
| $m_{d_i}^2$  | 0.274| 0.274| 0.294| 0.294|

Table 2: The choices for the soft parameters masses at GUT scale for various choices of $\tan \beta$. The units of $m_{1/2}$ is GeV while that of the soft masses is TeV$^2$
| tan $\beta$ | 5   | 10  | 15  | 20  |
|-------------|-----|-----|-----|-----|
| $\mu$ (GeV) | 500 | 550 | 580 | 850 |
| $A_d$ (GeV) | 3.5 | 6.4 | 9.2 | 16.6|
| $A_s$ (GeV) | -280| -460| -760| -900|
| $A_b$ (GeV) | -900| -950| -800| -228|
| $\delta m_d$ (MeV) | 1.50 | 1.43 | 1.55 | 1.69 |
| $\delta m_s$ (GeV) | -0.170| -0.167| -0.158| -0.156| |
| $\delta m_b$ (GeV) | -0.730| -0.732| -0.697| -1.0 |

Table 3: The choices for the $\mu$–term and relevant soft trilinear $A$–terms at low energy and the induced change to the down–type quark masses.

corrections that are close to the ones given in Eq. (16). Such values of $A$–terms are given in Table 3 with the corresponding corrections. As we can see the desired corrections are obtained. Here we have tried to use as low values for the $\mu$–term as possible, such that the fine tuning is minimal. For this reason, $A_b$–term has been chosen to be large, except in the case of $\tan \beta = 20$ for which the $A$–term does not contribute significantly without exceeding the stability condition of Eq. (5).

### 3.1.2 The spectrum and its implications

Due to the large $A$–terms in the second generation, $A_s$ and $A_\mu$, that we found for the Yukawa unification, the mass degeneracy in the sfermions of the first two families is lost during the RG running from the GUT scale down to the electroweak scale. If sparticles are eventually discovered at LHC or ILC, this feature of the mass spectra makes our approach experimentally distinguishable from other scenarios. At the same time, the induced splitting cannot be too large, otherwise could exceed the experimental constraints from the meson oscillations and other rare processes. The sfermion masses for the above choices of parameters are given in Table 4 and we can see a clear splitting in the masses of the first two generations. On the other hand, in the present case such a splitting would be absent in the right up–squark sector, since $A$–terms in the up sector are chosen to be proportional to the corresponding Yukawas. This would change if there were large corrections in the up sector as well.

The masses in Table 4 are calculated in the basis where the down–type Yukawa cou-
Table 4: The soft sfermion mass parameters at low energy in units of TeV$^2$.

| $\tan \beta$ | 5   | 10  | 15  | 20  | $\tan \beta$ | 5   | 10  | 15  | 20  |
|--------------|-----|-----|-----|-----|--------------|-----|-----|-----|-----|
| $m_{Q_1}^2$  | 0.505 | 0.507 | 0.569 | 0.565 | $m_{L_1}^2$  | 0.280 | 0.375 | 0.295 | 0.303 |
| $m_{Q_2}^2$  | 0.493 | 0.475 | 0.481 | 0.440 | $m_{L_2}^2$  | 0.275 | 0.260 | 0.253 | 0.243 |
| $m_{Q_3}^2$  | 0.233 | 0.206 | 0.276 | 0.395 | $m_{L_3}^2$  | 0.202 | 0.175 | 0.198 | 0.290 |
| $m_{\tilde{d}_1}^2$ | 0.454 | 0.456 | 0.515 | 0.508 | $m_{\tilde{e}_1}^2$ | 0.339 | 0.348 | 0.378 | 0.362 |
| $m_{\tilde{d}_2}^2$ | 0.431 | 0.394 | 0.338 | 0.257 | $m_{\tilde{e}_2}^2$ | 0.328 | 0.318 | 0.294 | 0.241 |
| $m_{\tilde{d}_3}^2$ | 0.177 | 0.114 | 0.207 | 0.448 | $m_{\tilde{e}_3}^2$ | 0.184 | 0.147 | 0.184 | 0.337 |

Table 5: The mass splittings in $K$ and $B$ meson systems due to the SUSY effects.

| $\tan \beta$ | 5| 10| 15| 20 |
|--------------|---|---|---|----|
| $\Delta M_D \times 10^{14}$ GeV$^{-1}$ | $1.44 \times 10^{-2}$ | 0.120 | 0.59 | 1.42 |
| $\Delta M_K \times 10^{15}$ GeV$^{-1}$ | $1.59 \times 10^{-2}$ | 0.105 | 0.706 | 1.55 |
| $\Delta M_B \times 10^{15}$ GeV$^{-1}$ | 1.11 | 6.15 | 4.53 | 2.93 |
| $\Delta M_{B_s} \times 10^{14}$ GeV$^{-1}$ | 1.82 | 4.56 | 10.1 | 45.3 |

The coupling matrix is diagonal. Therefore, upon the electroweak symmetry breaking one must rotate the left–handed up–squark mass matrix by the CKM matrix. Then, because of the induced splitting $\delta_{12}m_Q^2 = m_{Q_1}^2 - m_{Q_2}^2$, the Cabibbo part of the rotation will induce non zero (12) entry of order $\simeq \lambda \delta_{12}m_Q^2$ in the left up–squark mass matrix. The immediate consequences are the appearance of non zero mass splittings in neutral $D$–meson and Kaon systems. The $D_0$–$\overline{D}_0$ oscillation is induced by gluino–up squark box diagrams while the $K_0$–$\overline{K}_0$ oscillation is by that of chargino–down squark box. Observe that there is no gluino box diagram for down sector at the leading order, since we have chosen the matrices for the $A$–terms and Yukawa couplings for the down sector to be simultaneously diagonalize exactly for this reason to avoid this correction. We have calculated these oscillation rates and found them at the safe level when checked against the latest experimental results shown in Table 6. We have included here also the results for $B$ and $B_s$ meson and are found to be negligible due to the smallness of the (13), (23) CKM mixings and the chargino–stop contribution at low and moderate $\tan \beta$’s. As we can see the effects are large and, in the example of $\tan \beta = 20$, there is already a tension with the experimental result of $K_0$–$\overline{K}_0$. 

13
| $\Delta M_D$ | $(1.57^{0.438}_{0.471}) \times 10^{-14}$ GeV $[38]$ |
|-------------|-------------------------------------------------|
| $\Delta M_K$ | $(3.483 \pm 0.033) \times 10^{-15}$ GeV $[35]$ |
| $\Delta M_B$ | $(3.337 \pm 0.006) \times 10^{-13}$ GeV $[39]$ |
| $\Delta M_{B_s}$ | $(1.17 \pm 0.008) \times 10^{-11}$ GeV $[38]$ |

Table 6: The experimental values of the mass splittings of the neutral meson.

| $\tan \beta$ | 5   | 10  | 15  | 20  |
|---------------|-----|-----|-----|-----|
| $Br(D^+ \to \pi^+ \bar{\nu} \nu) \times 10^{11}$ | 0.0259 | 0.128 | 8.39  | 0.699 |
| $Br(K^+ \to \pi^+ \bar{\nu} \nu) \times 10^{11}$ | 0.141 | 3.73  | 37.8  | 19.3 |

Table 7: The branching ratios for processes $D^+ \to \pi^+ \bar{\nu} \nu$ and $K^+ \to \pi^+ \bar{\nu} \nu$.

The low energy experimental searches of rare processes in the charm and strange sectors – as they are affected most by the non universality – could shed light on the spectrum we have obtained. We have examined the rates of $D$ and $K$ meson decays into pion, neutrino and antineutrino. The process $D^+ \to \pi^+ \bar{\nu} \nu$ has a very tiny branching ratio in the SM for both short and long distance at the level $\sim 10^{-15}$. Although current upper bound is still very poor, this will improve soon once BESIII experiment starts collecting data soon, which would reach to the level of $\sim 10^{-8}$. For our scenario the level has been calculated and found to be $10^{-11}$ as shown in Table 7. Even though four order of magnitude large than the SM prediction, the rate is beyond the reach of BESIII. Therefore, further experimental advances are needed regarding this channel.

The processes $K^+ \to \pi^+ \bar{\nu} \nu$ and $K^0_L \to \pi^0 \nu \bar{\nu}$ can be estimated reliably in the SM. The branching ratio of the process $K^+ \to \pi^+ \bar{\nu} \nu$ has been measured by E787 and E949 collaborations and found to be $(15.7^{17.5}_{8.2}) \times 10^{-11}$ while for the latter there is only upper bound $6.7 \times 10^{-8}$. If the data is improved, they provide a clean probe to possible new physics. Our results for the $K^+$ decay are shown in Table 7. Indeed in some of our fits we find quite a large effects from our $A$–term induced splitting. For example, for $\tan \beta = 15$, the result is somewhat larger than the experimental result. These are calculated taking into account only the SUSY contributions to see the effect other than that of the SM. Since we do not need a complex phase in any of the $A$–terms, the effect cannot be significant for the neutral Kaon case. Thus in this case, we expect the rate to be far from the model independent Grossman-Nir bound $[40]$. 

14
Although we do not have an explicit model which explains the neutrino oscillation phenomena, most GUT models accommodate the neutrino masses through the see–saw mechanism, wherein one assumes three standard-model singlet right–handed neutrinos with masses in the range $\sim 10^9 \div 10^{14}$ GeV. If one of them has a large Yukawa coupling $y_{\nu_\tau}$ for $y_{\nu_\tau}L_3 \nu^c H_u$ type of interaction, it could alter $\tau$–lepton Yukawa coupling significantly during the running between its mass and the GUT scale [41, 42]. The $\beta$–function of the $\tau$ Yukawa is given by:

$$\mu \frac{dy_\tau}{d\mu} = \beta (y_\tau)_{\text{MSSM}} + \frac{1}{16\pi^2} y_\tau y_{\nu_\tau}^2 \quad \text{for} \quad m_{\nu_\tau} \leq \mu \leq M_{\text{GUT}}.$$  \hspace{1cm} (19)

The additional term increases the value of $y_\tau$ at GUT scale, therefore, increases $y_b$ compared to $y_\tau$ at low energy scale. Although we have not include this possibility in our analysis, we would like to point out that if we get a little lower value for $y_b$ it might be still compatible with the experiment in some specific models with large $y_{\nu_\tau}$. The result for $\tan \beta = 20$, shown in Table [7] is one possible example at hand. In this case if the right–handed neutrino effect causes 8–10% upward deflection on the tau Yukawa coupling running, the resulting low energy $b$–quark mass would come out correct.

4 Conclusions

In this paper, we have studied the possibilities of Yukawa unification for all generations in SUSY $SU(5)$ through the finite radiative SUSY threshold corrections in the presence of flavor non–universal soft parameters. In particular, we have concentrated on the flavor non–universal $A$–terms and their effect on the Yukawa couplings of lighter generation fermions. It is well known that these corrections are important and they can substantially affect the tree level ratios between the third generation Yukawa couplings. Choosing the $A$–terms of diagonal form with large values, especially in the down quark sector, we have shown that the $SU(5)$ unification with minimal Higgs content for the Yukawa interactions for all generation is possible for a relatively light sub TeV sfermion spectrum. This is a welcome scenario at the dawn of the LHC. We have examined neutral $D$ and $K$ meson mass splittings and rare processes $D^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. While the $D^+$–meson decay has been found to be much more enhanced compared to the SM, it is still far from the next generation experiments. On the other hand, we find the latter decay could have a sizable SUSY contribution and could be probed soon.

In the present scenario, due to the large $A$–term for the second generation, the down
sector squark masses are altered from its universal value substantially making them distinct from the most widely studied scenarios. This fact could soon be checked at the LHC/ILC. When the unification is assumed at the GUT scale for the soft terms, the slepton spectrum will display a similar distortions as well. A collider analysis of these type of spectra are needed to be done thoroughly.

One of the most important questions we have not addressed in the present work is the problem of the proton decay, which is known to be a challenge in the minimal $SU(5)$ \cite{43, 44}. When the SUSY corrections are used in a specific GUT models, one will face the question of the proton decay. One could potentially evade it by the mass splitting between the triplet and octet which alters the unification scale with a non–renormalizable operator \cite{45}. Our primary motivation was to avoid exactly these type of operators. Although this seems to be a setback, at least the flavor part is not necessarily affected by such operator if used only for raising the unification scale. These are currently under investigation.

If experimental breakthroughs happen in both the collider and rare decay experiments, the sfermion spectrum and rare Kaon decays could point in the direction of the large $A$–term scenario we have advocated in the present work. We conclude by noting the fact that in the absence of any theoretical prejudice for any particular SUSY breaking, all the phenomenologically consistent parameter space cannot be studied in full generality due to its huge size. Instead, the ones favored by the unification, such as the minimal Yukawa unification, could direct us to the parts other than the universal ones. If SUSY is the solution chosen by the Nature for stabilizing the gauge hierarchy we may find out soon which one is the correct one.

**Acknowledgements**

The author would like to thank Goran Senjanovic and Borut Bajc for many fruitful discussions, Kerim Suruliz for helping with the numerical calculations, and prof. Kaladi S. Babu at Oklahoma Sate University, OK, USA and the theoretical physics group of Jozef Stefan Institute, Ljubljana, Slovenia, for giving the opportunity to present part of the work.
A Appendix

Here we compile the formulae we have used from various sources for our numerical calculations. These are based on the results of Refs. [46, 47]. The superpotential of the MSSM is given by

\[
W = Y_u^i u_i^c H_u Q_j + Y_d^i d_i^c H_d Q_j + Y_l^i l_i^c H_d L_j + \mu H_u H_d.
\]

The soft SUSY breaking part of the Lagrangian is given by:

\[
-\mathcal{L} = \left( m_{L_f}^2 \right)_{ij} \tilde{f}_{L_i}^c \tilde{f}_{L_j} + \left( m_{R_f}^2 \right)_{ij} \tilde{f}_{R_i}^c \tilde{f}_{R_j} + \left( m_{L_R}^2 \right)_{ij} \tilde{f}_{L_i} \tilde{f}_{L_j}^c + \left( m_{R_R}^2 \right)_{ij} \tilde{f}_{R_i} \tilde{f}_{R_j}^c + \left( A_i^a \bar{u}_i^c H_u \tilde{Q}_j + A_d^a \bar{d}_i^c H_d \tilde{Q}_j + A_l^a \bar{l}_i^c H_d \tilde{L}_j + B \mu H_u H_d \right) + \frac{1}{2} m_{\tilde{g}} \tilde{g} \tilde{g} + \frac{1}{2} m_{\tilde{\chi}^0} \tilde{\chi}^0 \tilde{\chi}^0 + \frac{1}{2} m_{\tilde{\chi}^+} \tilde{\chi}^+ \tilde{\chi}^- + (\text{h.c.}) \right).
\]

With the above conventions, the interactions essential to our analysis are given as:

\[
-\mathcal{L} = \tilde{f}_i \left( N_{1ax}^L P_L + N_{1ax}^R P_R \right) \tilde{\chi}_a \tilde{f}_x + \left( \bar{u}_i \left( C_{1ax}^L + C_{1ax}^R \right) \tilde{\chi}_a \tilde{d}_x + \bar{d}_i \left( C_{1ax}^L + C_{1ax}^R \right) \tilde{\chi}_a \tilde{u}_x + \bar{l}_i \left( C_{1ax}^L + C_{1ax}^R \right) \tilde{\chi}_a \tilde{c}_x \right) + \tilde{\chi}_a \tilde{u}_x + \tilde{\chi}_a \tilde{d}_x + \tilde{\chi}_a \tilde{c}_x + \tilde{\chi}_a \tilde{b}_x + \left( \text{h.c.} \right).
\]

Here, the neutralino–fermion–sfermion and chargino–fermion–sfermion couplings are given by:

\[
N_{1ax}^{L(f)} = -\sqrt{2} g_2 \tan \theta_W \left( O_N \right)_{1a} U^f_{x,i+3} - Y_f^i \left( O_N \right)_{a' a} U^f_{x,j} \]
\[
N_{1ax}^{R(f)} = -\sqrt{2} g_2 \left\{ \left( \tan \theta_W \left( Q_f - T_3^f \right) \right) \left( O_N \right)_{a_1} + T_3^f \left( O_N \right)_{a_2} \right\} U^f_{x,i} + Y_f^i \left( O_N \right)_{a' a} U^f_{x,j+3},
\]
\[
C_{1xb}^{L(l)} = Y_{f}^{ij} \left( O_{R} \right)_{b_2} U^f_{x,j},
\]
\[
C_{1xb}^{R(l)} = - g_2 \left( O_{C} \right)_{b_1} U^f_{x,j},
\]
\[
C_{1xb}^{L(d)} = Y_{d}^{ij} \left( O_{C} \right)_{b_2} U^f_{x,j},
\]
\[
C_{1xb}^{R(d)} = - g_2 \left( O_{C} \right)_{b_1} U^f_{x,j} + Y_{u}^{ij} \left( O_{C} \right)_{b_2} U^f_{x,j},
\]
\[
C_{1xb}^{U(u)} = Y_{u}^{ij} \left( O_{C} \right)_{b_2} U^f_{x,j},
\]
\[
C_{1xb}^{F(f)} = - g_2 \left( O_{C} \right)_{b_1} U^f_{x,j} + Y_{d}^{ij} \left( O_{C} \right)_{b_2} U^f_{x,j},
\]

17
where the sfermion mixing matrices and the mass eigenstates are defined as follows:

\[ \text{diag}(m_f^2) = U_f^T M_f^2 U_f, \quad \tilde{f}_x = U^T_{f,i} \tilde{f}_{L_i} + U^T_{f,i+3} \tilde{f}_{R_i}, \quad x = (1 \div 6), \quad f = u, d, l \]  

\[ \text{diag}(m_\nu^2) = U_\nu^T M_\nu^2 U_\nu, \quad \tilde{\nu}_x = U^T_{\nu,i} \tilde{\nu}_{L_i}, \quad x = (1 \div 3). \]  

Here the charged sfermion $6 \times 6$ and sneutrino $3 \times 3$ mass matrices are:

\[ M_f^2 = \begin{pmatrix} m_L^2 & m_{LR}^2 \\ m_{LR}^2 & m_R^2 \end{pmatrix} \quad \text{and} \quad M_\nu^2 = m_L^2 + M_Z^2 \cos 2\beta/2, \]

\[ m_L^2 = m_{f_L}^2 + Y_f^2 v^2 \{ \cos^2 \beta, \sin^2 \beta \}/2 + M_Z^2 \cos 2\beta \left( T_f^3 - Q_f \sin^2 \theta_W \right), \]

\[ m_R^2 = m_{f_R}^2 + Y_f^2 v^2 \{ \cos^2 \beta, \sin^2 \beta \}/2 - M_Z^2 \cos 2\beta \left( T_f^3 - Q_f \sin^2 \theta_W \right), \]

\[ m_{LR}^2 = A_f v \{ \cos \beta, -\sin \beta \}/2 - \mu^* Y_f v \{ \sin \beta, \cos \beta \}/2. \]

The matrices $O^N$ and $(O^C_L, O^C_R)$ diagonalize the neutralino and chargino mass matrices respectively as follows:

\[ \text{diag}(m_{\chi_0}) = O^N M_N (O^N)^T, \quad \text{diag}(m_{\tilde{\psi}_0}) = O^C_L M_C (O^C_L)^T, \]

\[ (\tilde{B}, \tilde{W}_3, H_d^0, H_u^0) = (O^N)^T \chi_0^0, \quad (\tilde{W}^-, H_d^-) = (O^C_L)^T \chi_L^-, \quad (\tilde{W}^-, H_u^-) = (O^C_R)^T \chi_R^-. \]

The index $a'$ of $O^N$ in the neutralino contribution formula in Eq. (23) takes value of $3(4)$ for $T_f^3 = -\frac{1}{2}(\frac{1}{2})$. The Higgs–sfermion couplings are given as

\[ X_{xy}^{1(f)} = D_{xy}^f \cos \beta + F_{xy}^f \cos \beta + A_{ij}^f U_{x,i+3}^f U_{y,j}^{f*} \quad \text{for } f = d, l, \]

\[ X_{xy}^{2(f)} = -D_{xy}^f \sin \beta - \mu Y_f^i U_{x,i+3}^f U_{y,j}^{f*} \quad \text{for } f = d, l, \]

\[ X_{xy}^{1(u)} = D_{xy}^u \cos \beta - \mu Y_u^i U_{x,i+3}^u U_{y,j}^{u*}, \]

\[ X_{xy}^{2(u)} = -D_{xy}^u \sin \beta + F_{xy}^u \sin \beta - A_{ij}^u U_{x,i+3}^u U_{y,j}^{f*}, \]

\[ X_{xy}^{1(\nu)} = D_{xy}^\nu \cos \beta + F_{xy}^\nu \cos \beta, \]

\[ X_{xy}^{2(\nu)} = -D_{xy}^\nu \sin \beta. \]

where the $D$ and $F$–term induced couplings are given in terms of the following expressions:

\[ D_{xy}^f = \frac{g_2 M_Z}{\sqrt{2} \cos \theta_W} \left( \left( T_f^3 - Q_f \sin^2 \theta_W \right) U_{x,i}^f U_{y,i}^{f*} + Q_f \sin^2 \theta_W U_{x,i+3}^f U_{y,i+3}^{f*} \right), \]

\[ D_{xy}^\nu = \frac{g_2 M_Z}{\sqrt{2} \cos \theta_W} U_{i,x}^\nu U_{i,y}^{\nu*}, \]

\[ F_{xy}^f = \frac{\sqrt{2} M_W}{g_2} \left( Y_f^f Y_f^i U_{i,x}^f U_{y,i+3}^{f*} + Y_f^f Y_f^i U_{x,i+3}^f U_{y,i+3}^{f*} \right), \]

\[ F_{xy}^\nu = 0. \]
The neutralino–Higgs and the chargino–Higgs couplings are given by

\[
\lambda_{ab}^{1(N)} = \frac{g_2}{\sqrt{2}} (-\tan \theta_W \left( O^N \right)_{a1} \left( O^N \right)_{b1} + \left( O^N \right)_{a2} \left( O^N \right)_{b3}), \quad (49)
\]

\[
\lambda_{ab}^{2(N)} = \frac{g_2}{\sqrt{2}} (\tan \theta_W \left( O^N \right)_{a1} \left( O^N \right)_{b4} - \left( O^N \right)_{a2} \left( O^N \right)_{b4}), \quad (50)
\]

\[
\lambda_{ab}^{1(C)} = g_2 \left( O^C \right)_{a1} \left( O^C \right)_{b2}, \quad (51)
\]

\[
\lambda_{ab}^{2(C)} = g_2 \left( O^C \right)_{a2} \left( O^C \right)_{b1}, \quad (52)
\]

Finally, the effective Lagrangian for the Yukawa interactions are given by

\[
- \mathcal{L} = \sum_{f_d,f_u} \left[ (Y_f + \delta Y_f')_{ij} \bar{f}_i f_j H^0_d + (\delta Y_u')_{ij} \bar{u}_i u_j H^0_u \right] + (Y_u + \delta Y_u')_{ij} \bar{u}_i u_j H^0_u, \quad (53)
\]

where the corrections, \( \delta Y_f \) and \( \delta Y_f' \), have contributions from the gluino–, neutralino– and chargino–sfermion loops. These are written as follows:

\[
\left( \delta Y_q^G \right)_{ij} = -\frac{2\alpha_s}{3\pi} X_{xy}^{(q)} m_q^2 U^{q*}_{x,y} U^{q}_{y,j} I \left( m_{f_x}^2, m_{f_y}^2, m_{q}^2 \right), \quad (54)
\]

\[
\left( \delta Y_f^N \right)_{ij} = -\frac{N^{L(f)}_{iax} N^{R(f)*}_{jax}}{16\pi^2} X_{xy}^{(f)} m_{\chi^0} I \left( m_{f_x}^2, m_{f_y}^2, m_{\chi^0}^2 \right) \lambda_{ab}^{1(N)} m_{\chi_a} m_{\chi_b} I \left( m_{f_x}^2, m_{f_y}^2, m_{\chi_0}^2 \right), \quad (55)
\]

\[
\left( \delta Y_u^C \right)_{ij} = -\frac{C^{L(u)}_{iax} C^{R(u)*}_{jax}}{16\pi^2} X_{xy}^{(u)} m_{\chi_a} I \left( m_{f_x}^2, m_{f_y}^2, m_{\chi_x}^2 \right) \lambda_{ab}^{1(C)} m_{\chi_a} m_{\chi_b} I \left( m_{f_x}^2, m_{f_y}^2, m_{\chi_b}^2 \right), \quad (56)
\]

\[
\left( \delta Y_d^C \right)_{ij} = -\frac{C^{L(d)}_{iax} C^{R(d)*}_{jax}}{16\pi^2} X_{xy}^{(d)} m_{\chi_a} I \left( m_{f_x}^2, m_{f_y}^2, m_{\chi_x}^2 \right) \lambda_{ab}^{1(C)} m_{\chi_a} m_{\chi_b} I \left( m_{f_x}^2, m_{f_y}^2, m_{\chi_b}^2 \right), \quad (57)
\]

\[
\left( \delta Y_l^C \right)_{ij} = -\frac{C^{L(l)}_{iax} C^{R(l)*}_{jax}}{16\pi^2} X_{xy}^{(l)} m_{\chi_a} I \left( m_{f_x}^2, m_{f_y}^2, m_{\chi_x}^2 \right) \lambda_{ab}^{1(C)} m_{\chi_a} m_{\chi_b} I \left( m_{f_x}^2, m_{f_y}^2, m_{\chi_x}^2 \right), \quad (58)
\]

To obtain the corrections \( \delta Y_f' \) one should replace \((X^1, \lambda^1)\) by \((X^2, \lambda^2)\) everywhere. In the above formulae, if one works in the basis where the down (up) type quark Yukawa matrix \( Y_d \) (\( Y_u \)) to be diagonal then the up (down) type Yukawa matrix must be chosen as \( Y^{\text{diag}}_d V_{c_{km}} \) (\( Y^{\text{diag}}_u V^\dagger_{c_{km}} \)). The loop function, \( I \), is given by

\[
I \left( x, y, z \right) = -\frac{xy \ln(x/y) + yz \ln(y/z) + zx \ln(z/x)}{(x-y)(y-z)(z-x)}, \quad (59)
\]
which has the following behaviors in various limits of its arguments as follows

\[
I(m_{1}^{2}, m_{2}^{2}, m_{3}^{2}) = \begin{cases} 
  1/(2m^2) & \text{for } m_i \to m, \\
  \frac{1}{m^2} \ln \frac{\beta}{\beta-1} & \text{for } m_1 = 0, \beta \equiv m_2^2/m_3^2, \\
  1/m^2 \frac{1}{2} \left( 1 - \beta + \beta \ln \beta \right) & \text{for } m = m_1 = m_2.
\end{cases}
\]  

(60)

References

[1] H. Georgi and C. Jarlskog, Phys. Lett. B 86 (1979) 297.
[2] D. Emmanuel-Costa and S. Wiesenfeldt, Nucl. Phys. B 661, 62 (2003).
[3] W. Buchmuller and D. Wyler, Phys. Lett. B 121, 321 (1983).
[4] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267, 415 (1986).
[5] T. Banks, Nucl. Phys. B 303, 172 (1988).
[6] E. Ma, Phys. Rev. D 39, 1922 (1989).
[7] N. V. Krasnikov, Phys. Lett. B 302, 59 (1993).
[8] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D 50, 7048 (1994).
[9] T. Blazek, S. Raby and S. Pokorski, Phys. Rev. D 52, 4151 (1995).
[10] M. S. Carena, M. Olechowski, S. Pokorski and C. E. M. Wagner, Nucl. Phys. B 426, 269 (1994).
[11] K. S. Babu and C. F. Kolda, Phys. Lett. B 451, 77 (1999).
[12] R. Hempfling, Phys. Rev. D 49, 6168 (1994).
[13] N. Arkani-Hamed, H. C. Cheng and L. J. Hall, Phys. Rev. D 54, 2242 (1996).
[14] K. S. Babu, B. Dutta and R. N. Mohapatra, Phys. Rev. D 60, 095004 (1999).
[15] F. Borzumati, G. R. Farrar, N. Polonsky and S. D. Thomas, Nucl. Phys. B 555, 53 (1999).
[16] J. L. Diaz-Cruz, H. Murayama and A. Pierce, Phys. Rev. D 65, 075011 (2002).
[17] T. Ibrahim and P. Nath, Phys. Rev. D 67, 095003 (2003) [Erratum-ibid. D 68, 019901 (2003)].

[18] J. Ferrandis and N. Haba, Phys. Rev. D 70, 055003 (2004) [arXiv:hep-ph/0404077].

[19] J. L. Díaz-Cruz, M. Gomez-Bock, R. Noriega-Papaqui and A. Rosado, arXiv:hep-ph/0512168.

[20] G. Ross and M. Serna, Phys. Lett. B 664, 97 (2008).

[21] C. M. Maekawa and M. C. Rodriguez, JHEP 0801, 072 (2008).

[22] S. Antusch and M. Spinrath, Phys. Rev. D 78, 075020 (2008).

[23] D. M. Pierce, J. A. Bagger, K. T. Matchev and R. J. Zhang, Nucl. Phys. B 491 (1997) 3.

[24] The LEP Higgs Working group, R. Bock et al., CERN-EP-2000-055 and LEP experiments, ALEPH 2000-28, DELPHI 2000-050, L3-Note 2525, OPAL TN646.

[25] G. Anelli et al., “Proposal to measure the rare decay $K^+ \to \pi^+ \nu \bar{\nu} u$ at the CERN SPS,”, CERN-SPSC-P-326.

[26] J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B 222, 11 (1983).

[27] J. P. Derendinger and C. A. Savoy, Nucl. Phys. B 237, 307 (1984).

[28] C. Kounnas, A. B. Lahanas, D. V. Nanopoulos and M. Quiros, Nucl. Phys. B 236, 438 (1984).

[29] J. A. Casas and S. Dimopoulos, Phys. Lett. B 387, 107 (1996).

[30] A. Kusenko, P. Langacker and G. Segre, Phys. Rev. D 54, 5824 (1996).

[31] A. Kusenko and P. Langacker, Phys. Lett. B 391, 29 (1997).

[32] U. Sarid, Phys. Rev. D 58, 085017 (1998).

[33] Z. z. Xing, H. Zhang and S. Zhou, Phys. Rev. D 77, 113016 (2008).

[34] H. Fusaoka and Y. Koide, Phys. Rev. D 57, 3986 (1998).

[35] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).
[36] B. Blossier et al. [European Twisted Mass Collaboration], JHEP 0804, 020 (2008). M. Gockeler, R. Horsley, A. C. Irving, D. Pleiter, P. E. L. Rakow, G. Schierholz and H. Stuben [QCDSF Collaboration and UKQCD Collaboration], Phys. Lett. B 639, 307 (2006). C. Aubin et al. [MILC Collaboration], Phys. Rev. D 70, 114501 (2004). C. Aubin et al. [HPQCD Collaboration and MILC Collaboration and UKQCD Collaboration], Phys. Rev. D 70, 031504 (2004).

[37] B. C. Allanach, Comput. Phys. Commun. 143, 305 (2002).

[38] E. Barberio et al. [Heavy Flavor Averaging Group], arXiv:hep-ex/0808.1297.

[39] The CDF collaboration, Phys. Rev. Lett. 97, 242003 (2006).

[40] Y. Grossman and Y. Nir, Phys. Lett. B 398, 163 (1997).

[41] F. Vissani and A. Y. Smirnov, Phys. Lett. B 341, 173 (1994).

[42] A. Brignole, H. Murayama and R. Rattazzi, Phys. Lett. B 335, 345 (1994).

[43] T. Goto and T. Nihei, Phys. Rev. D 59, 115009 (1999).

[44] H. Murayama and A. Pierce, Phys. Rev. D 65, 055009 (2002).

[45] B. Bajc, P. Fileviez Perez and G. Senjanovic, Phys. Rev. D 66, 075005 (2002); arXiv:hep-ph/0210374.

[46] J. Rosiek, Phys. Rev. D 41, 3464 (1990); arXiv:hep-ph/9511250.

[47] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B 659, 3 (2003).