Image Compression Technology Based on Wavelet Transform

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Abstract. Image compression based on wavelet transform is to apply wavelet algorithm to multi-resolution decomposition of the image, and to achieve image compression by encoding the obtained wavelet coefficients. First, multi-level wavelet decomposition is performed on the image to obtain the corresponding wavelet coefficients. Then, each layer of wavelet coefficients is quantized to obtain quantized coefficient objects. Finally, the quantized coefficient objects are encoded to obtain compression results. This paper uses Haar wavelet as the wavelet base, selects the 2-level decomposition scale, performs wavelet transform, compresses the reconstructed image by setting a threshold, and calculates the file size ratio and PSNR value before and after compression. The wavelet reconstruction function uses the coefficient matrix, dimensionality information and wavelet base type obtained by wavelet decomposition to perform wavelet reconstruction. The global threshold setting method is selected, and the wavelet high-frequency coefficients are threshold filtered to achieve compression before the reconstruction operation, and then the compressed image is obtained through wavelet reconstruction. After the image has undergone wavelet transformation with a specified scale, most of the energy is concentrated in the fractional part of the wavelet decomposition coefficients. The coefficients of other parts are set to constants by setting a threshold, and only a few decomposition coefficients are retained to represent the entire image. The experimental results show that the storage space of the compressed image is greatly saved and the compressed image does not change significantly from the original image visually.

1. Introduction
In actual life, while ensuring the quality of image compression, we must also consider the price of the output during the compression process. Only in this way can the research results be truly turned into science and technology that benefits the people and benefits the people. Research on image compression technology first started in Bell Labs in the United States. At that time, researchers from the National Lab began to study how to use data redundancy to compress image information. After decades of development, image compression technology quickly became popular in many scientific fields. Among them, image compression using wavelet analysis theory became a popular research topic at that time.
Image compression is to reduce the amount of data required by encoding the data information contained in the original image in a certain sense, thereby achieving the purpose of saving image storage space and transmission time. Data redundancy-based compression means that there is a large amount of redundant data in the digital image that can be compressed and encoded, and this redundancy can be recovered losslessly after the image is decompressed and compressed. Compression based on visual redundancy is based on human visual images, without affecting the vision of the person in charge, by reducing the data accuracy of the image signal, and compressing the data with certain objective distortion.

2. Wavelet definition
Wavelet refers to a small-scale, decaying wave. The detailed mathematical language can be expressed as that the square of the function $\gamma(t)$ exhibits a fading phenomenon during the integration process [1-3]. That is, if $\gamma(t) \in L^2$ meets:

$$\int_{-\infty}^{\infty} \gamma(t) = 0 \quad (1)$$

Call $\gamma(t)$ a wavelet base. The integrand $\varphi(t)$ is convergent in the entire domain. At infinity, the wavelet has an energy value of zero. It can also be understood that $\gamma(t)$ can gradually reach zero when $t$ tends to $\infty$. Based on the above mathematical expression of (1) integral mathematics, it can be known from the pure mathematical theory that the area of the image above the X axis and the size of the trifoliate orange below the X axis are the same. The waveform of $\gamma(t)$ changes with time $t$, which is the origin of wavelet [4,5].

3. One-dimensional discrete and continuous wavelet
The mathematical formula of continuous wavelet transform is:

$$\int_{-\infty}^{\infty} \left| \frac{\varphi(\omega)}{\omega} \right| d\omega < \infty \quad (2)$$

If the function $\psi(t)$ satisfies the formula (2), $\psi(t)$ is called a basic wavelet. Then using the expansion and translation of the basic wavelet transform can get the mathematical expression of the wavelet function $\psi_{a,b}(t)$ is as follows [6-8].

$$\psi_{a,b} = a^{1/2} \psi\left( \frac{t-b}{a} \right) \quad (3)$$

Assuming all about $f(t) \in L^2$, if any $\psi \in L^2(R)$, the continuous wavelet of $f$ can be defined as:

$$G_f(a,b) \leq \psi_{a,b}(t), f \geq \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi_{a,b}\left( \frac{t-b}{a} \right) f(t) dt \quad (4)$$

The formula of the original function $f(t)$ reconstructed by (4) wavelet transform $G_f(a,b)$ is:

$$f(t) = \frac{1}{D_{\psi}} \int G_f(a,b) \int_{-\infty}^{\infty} \psi_{a,b}\left( \frac{t-b}{a} \right) \frac{1}{a^2} da db \quad (5)$$

$$D_{\psi} = \int_{-\infty}^{\infty} \left| \frac{\varphi(\omega)}{\omega} \right| d\omega < \infty \quad (6)$$
In formula (3), the scale scalar $a$ is to extend or shorten the basic wavelet $\psi(t)$. According to the scalar $b$, you can know the specific time point of the analysis using the function $f(t)$. Here, the time point can be called the center of time. If $\psi(t)$ scales $a$ times and becomes $\psi(\frac{L}{a})$, when $a > 1$, the larger $a$, the wider the time domain of $\psi(\frac{L}{a})$. If $a$ is smaller, $\psi(\frac{L}{a})$ width is narrower.

The computer calculates the wavelet transform of the discontinuous scale and displacement, that is, the wavelet transform of these discontinuous data information and the reconstruction of the image information by the wavelet transform coefficients of these discontinuous points. Its mathematical expression is:

$$\psi_{m,n}(t) = \frac{1}{a_0^m} \psi\left(\frac{t-na_m b_0}{a_0^m}\right) = a_0^n \psi(a_0^{-m} t - n b_0)$$

(7)

### 4. Two-dimensional discrete and continuous wavelet transform

Because computer processing is two-dimensional information, this paper can extend the theory of wavelet analysis to two-dimensional through the above analysis. First, understand the mathematical expression of two-dimensional continuous wavelet as follows:

$$< f, \phi_{a,b} \geq G_f(a,b_1,b_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t_1,t_2) \phi_{a,b}(t_1,t_2) dt_1 dt_2$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t_1,t_2) \frac{1}{a} \phi_{a,b}(\frac{t_1,t_2}{a})(\frac{1}{a})(b_1,b_2) dt_1 dt_2$$

Where $a > 0$ in formula (8), where the mathematical expression of inverse transformation is:

$$f(t_1,t_2) = \frac{1}{D_f} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a^{-3} G_f(a,b_1,b_2) \phi_{a,b}(t_1,t_2) dt_1 dt_2 da$$

(9)

The integrand $\phi_{a,b}(t_1,t_2)$ in formula (9) is defined as a two-dimensional basic wavelet.

Because the signal processed by the computer is not a continuous signal, this article defines the two-dimensional discrete wavelet as follows. In this paper, we first introduce a space $S_{n \cdot n} \in Z$, and let $\otimes$ be the one-dimensional multi-resolution analysis tensor product.

$$S_0 = S_0 \otimes S_0 = \text{span}\{F(t_1,t_2) = f(t_1)y(t_2); y \in S_0\}$$

(10)

$$F \in S_{n} \iff F(2^{-n} \cdot 2^{-n}) \in S_{0} \otimes S_{0}$$

(11)

$S_0$ should meet the following conditions:

$$\cdots S_2 \subset S_1 \subset S_0 \subset S_{-1} \subset S_{-2} \cdots$$

(12)

The two-dimensional scaling function can be defined as:

$$\phi_{0,m_0,n_0}(t_1,t_2) = \varphi(t_1,-m_2), m_1, m_2 \in Z$$

(13)
5. Wavelet packet definition
The definition of wavelet packet is based on multi-resolution analysis. According to the difference scale parameter \( j \), Hilbert space \( L^2(\mathbb{R}) \) can be transformed into the orthogonal sum of all subspaces \( W_j, j \in \mathbb{Z} \). Where \( W_j \) is the closure of the wavelet function \( \Psi(t) \) (wavelet subspace).

Generally, when the wavelet packet is used for analysis, the defined subspace \( U_j \) is used to express the scale space \( V_j \) and the wavelet subspace \( W_j \) in all sets, so that

\[
\begin{align*}
U_j^0 &= V_j, \\
U_j^1 &= W_j, \\
& j \in \mathbb{Z}
\end{align*}
\]  
(14)

Then the orthogonal decomposition \( V_{j+1} = V_j \oplus W_j \) of Hilbert space can be expressed by the mathematical formula of \( U_j \) respectively as:

\[
U_{j+1}^0 = U_j^0 \oplus U_j^1, \quad j \in \mathbb{Z}
\]  
(15)

Subinterval \( U_j \) is the closure space of function \( U_n(t) \), and \( U_n(t) \) is the closure space of function \( U_{2n}(t) \). During the analysis, let \( U_n(t) \) satisfy the dual-scale equation of (16):

\[
\begin{align*}
\phi(t) &= \sum_{k \in \mathbb{Z}} h(k) u_k(2t-k) \\
\psi(t) &= \sum_{k \in \mathbb{Z}} g(k) u_k(2t-k)
\end{align*}
\]  
(16)

When \( n=0 \), the above two formulas can be expressed by formula (17):

\[
\begin{align*}
\phi(t) &= \sum_{k \in \mathbb{Z}} h_k u_0(2t-k) \\
\psi(t) &= \sum_{k \in \mathbb{Z}} g_k u_0(2t-k)
\end{align*}
\]  
(17)

In the multi-resolution analysis process, \( \phi(t) \) and \( \psi(t) \) satisfies the dual-scale equation of (18):

\[
\begin{align*}
\phi(t) &= \sum_{k \in \mathbb{Z}} h_k \phi(2t-k) \\
\psi(t) &= \sum_{k \in \mathbb{Z}} g_k \phi(2t-k)
\end{align*}
\]  
(18)

Through the above comparative analysis, we can see that \( u_n(t) \) and \( u_n(t) \) can be transformed into scale function \( \phi(t) \) and wavelet basis function \( \psi(t) \). Equation (17) is the same expression as Equation (15). The expression of the equivalent formula described in this article can be applied to the special case of \( n \in \mathbb{Z}_+ \) (non-negative integer), and the same formula (17) can be expressed as:

\[
U_{j+1}^0 = U_j^0 \oplus U_{j}^{2n+1}, \quad j \in \mathbb{Z}, n \in \mathbb{Z}_+
\]  
(19)

Definition (Wavelet Packet) The sequence \( \{u_n(t)\} \) (where \( n \in \mathbb{Z}_+ \)) expressed by equation (16) is defined as the orthogonal wavelet packet defined by the wavelet basis function \( u_n(t) = \phi(t) \). When \( n=0 \), it is the case of formula (17).

6. Image compression algorithm simulation and analysis
This article selects the classic legend cameraman.tif in the field of image processing for processing, as
shown in Figure 1. Using Haar wavelet as the wavelet base, select a 2-level decomposition scale, perform wavelet transform, and then set a global threshold to compress and reconstruct, and finally output the resulting image to a png file and compare the storage space size and PSNR value before and after compression. In order to demonstrate the characteristics of wavelet coefficients intuitively, MATLAB is used to draw the sub-graph drawing technology and image matrix merging technology to draw the distribution image of wavelet coefficients as shown in Figure 2, and the tower image is shown in Figure 3.

The wavelet reconstruction function is to carry out wavelet reconstruction through the coefficient matrix, dimension information and wavelet base type obtained by wavelet decomposition. In this paper, we use a global threshold setting method to perform threshold filtering on the wavelet high-frequency coefficients before the reconstruction operation to achieve compression, and then obtain the compressed image through wavelet reconstruction. Set a global threshold of 10, perform Haar wavelet basis to get 2 levels of decomposition and reconstruction, and get a reconstructed image, as shown in Figure 4, there is no obvious distortion in the visual effect. In this experiment, the storage space required for the image before compression is: 162056.00bytes, the storage space required for the compressed image is: 23784.00bytes, the file size ratio is: 6.81, and the PSNR value of the image before and after compression is: 17.92.

![Figure 1. Original image](image1.png)

![Figure 2. Distribution of wavelet coefficients](image2.png)
7. Summary
This paper analyzes the multi-resolution characteristics of wavelet transform and applies it to image compression. After the image undergoes the wavelet transform of the specified scale, most of the energy is concentrated on the fractional part of the wavelet decomposition coefficients. The coefficients of other parts are set as constants by setting thresholds, leaving only a few decomposition coefficients used to represent the entire image. In this way, a higher compression ratio can be obtained. In addition, the compression effect of the algorithm can be improved by setting different adaptive thresholds.

Acknowledgments
This work was supported by the Quality Engineering Project No. 2015zy073, the Scientific Research Project of Anhui Natural Science Research Project No. KJ2017A628, the backbone Teacher Project No. 2018xgg04 and Student Innovation Project No. 201812216022.

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