Torsion as a dynamic degree of freedom of quantum gravity

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Abstract
The gauge approach to gravity based on the local Lorentz group with a general independent affine connection $A_{\mu cd}$ is developed. We consider $SO(1, 3)$ gauge theory with a Lagrangian quadratic in curvature as a simple model of quantum gravity. The torsion is proposed to represent a dynamic degree of freedom of quantum gravity at scales above the Planckian energy. The Einstein–Hilbert theory is induced as an effective theory due to quantum corrections of torsion via generating a stable gravito-magnetic condensate. We conjecture that torsion possesses an intrinsic quantum nature and can be confined.

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1. Introduction

The idea that torsion should play an important role in gravity descends from works by Cartan [1] who had deep insight into the nature of spacetime geometry. Cartan was first to realize the tensorial character of torsion and subsequently the approach to gravity including torsion was developed extensively [2–5]. Near that time Weyl invented a gauge invariance principle [6] which was used successfully as a guiding rule in constructing the modern theories of electro-weak and strong interactions. The gauge approach to gravity based on Lorentz and Poincaré group was proposed in [7–10] and later was developed by many physicists [11–14]. The translational gauge formalism of Einstein theory was further developed in [12]. In these approaches, the initial classical Lagrangian includes the Einstein–Hilbert term (in addition to other possible terms quadratic in curvature and torsion) providing the correct limit of Einstein gravity. Note that in higher-derivative quantum gravity models with the Einstein–Hilbert term the unitarity problem of the physical $S$-matrix can be resolved consistently [13]. The Carmeli theory [14] based on the Lorentz gauge group $SO(1, 3) \simeq SL(2, C)$ is an example of a model
which contains only a quadratic curvature term of Maxwell type and, therefore, appears to be a fourth-order theory for the metric tensor. For pure gravity without matter the equations of motion in Carmeli’s model lead to the Newman–Penrose form of the vacuum Einstein–Hilbert equations. This is an interesting hint that the $SO(1, 3)$ connection can be regarded as a dynamical variable just like the metric tensor. Such a point of view was adopted in [15] where authors treated the gauge connection in Carmeli’s model as an independent variable and demonstrated the renormalizability of the theory.

Einstein gravity is an effective theory and can be deduced from some more fundamental theory as discussed by Zel’dovich and Sakharov in 1967 [16, 17]. The possibility of inducing the Einstein theory via quantum corrections was considered in the past by many physicists in various approaches: scale (conformal) invariance breaking schemes [18, 19], nonlinear realizations of the Lorentz group [20, 21], models with spontaneous symmetry breaking [22, 23] where the graviton emerges as a Goldstone boson [24–26] and others [27, 28]. In most of these approaches the Einstein–Hilbert term is induced by quantum corrections due to interaction with a scalar matter field. The disadvantage of this is that we have no any experimental evidence for the existence of fundamental scalar fields (like a dilaton or inflaton). Moreover, the inducing of Einstein gravity from the interaction with matter fields looks unsatisfactory because such a mechanism does not give any answer to the origin of the classical Einstein theory limit in the absence of matter. In contrast to this, we use a minimal formulation of the theory without scalar fields and demonstrate that even in a pure quantum gravity model with torsion the Einstein–Hilbert action can be induced due to quantum dynamics of the torsion via formation of a non-trivial vacuum with a gravito-magnetic condensate.

In the present paper, we are constrained to follow the gauge approach to gravity based on the Lorentz gauge group. We do not consider a more general case which includes the Poincaré gauge group, although such a formalism can be developed as well. Our motivation to follow only the Lorentz gauge group approach is twofold. First of all, the local Lorentz symmetry has its own deep physical meaning since it reflects the presence of a weak equivalence principle which is a basic fundamental principle lying in the nature of the gravitational force. We expect that contortion as a part of Lorentz gauge connection could play a principal role at quantum level. The second reason for our constrained consideration is that we are looking for a possible mechanism of emergent Einstein gravity which can be induced as an effective theory during phase transition at Planckian scale. So we are interested in such a theory where the metric obtains its dynamical content after phase transition at lower energy scale. The contortion could be a good candidate to switch on the mechanism of dynamical symmetry breaking through the quantum corrections. We develop this idea by suggesting that the torsion (contortion) represents exactly the dynamic variable of quantum gravity. Moreover, we conjecture that torsion can be confined and exists intrinsically as a quantum object, and its quantum dynamics manifests itself by inducing the Einstein–Hilbert theory as an effective theory of quantum gravity below the Planckian scale.

The paper is organized as follows. In section 2, we describe the main lines of the model following the Utiyama–Kibble–Sciama gauge approach to gravity [7, 8, 14, 15] based on the Lorentz gauge group. In section 3, an effective quantum action is calculated in one-loop approximation by integrating out torsion degrees of freedom. It has been shown that the corresponding effective potential possesses a non-trivial minimum which provides a vacuum gravito-magnetic condensate. The classical limit of Einstein–Hilbert theory is obtained as an effective theory induced by the quantum dynamics of torsion. In section 4, we outline some parallels between the quantum gravity model with torsion and quantum chromodynamics
(QCD). The last section discusses the non-positiveness of the classical Hamiltonian and our main results.

2. Utiyama–Kibble–Sciama gauge approach to gravity

In early approaches to the formulation of gravity as a gauge theory of the Poincaré group \[7, 8, 11, 12\] the vielbein \(e^a_\mu\) and Lorentz affine connection \(A_{\mu cd}\) were introduced (we use Greek letters \(\mu, \nu, \ldots\) for the spacetime indices and Latin letters \(a, b, c, \ldots\) for the Lorentz indices). For simplicity, we will not consider the Poincaré gauge group and restrict ourselves to the case of the proper Lorentz group \(SO(1, 3)\). The infinitesimal transformation of a Lorentz vector \(V_a\) is given by

\[
\delta V_a = [A, V_a] = A_a^b V_b,
\]

where \(A \equiv A_{\lambda cd} \Omega^d\) is a Lie algebra valued parameter and \(\Omega^d\) is a generator of the Lorentz Lie algebra. The vielbein \(e^a_\mu\) allows us to convert Lorentz indices into spacetime indices and vice versa. We assume that the vielbein is invertible

\[
e_a^\mu e_\mu b = \eta_{ab},
\]

and the signature of the flat metric \(\eta_{ab}\) in the tangent spacetime is Minkowskian, \(\eta_{ab} = \text{diag}(+\ldots-\ldots)\). Throughout the paper we follow Kibble’s approach \[8\] and treat the vielbein as a fixed background field which does not manifest any dynamical content. This is not a principal limitation and one can overcome it by generalization to Poincaré group. The covariant derivative with respect to the Lorentz group is defined in a standard manner:

\[
D_\mu = e^\mu_a (\partial_\mu + g A_\mu),
\]

where \(A_\mu \equiv A_{\mu cd} \Omega^d\) is a general affine connection taking values in the Lorentz Lie algebra and \(g\) is a new gravitational gauge coupling constant corresponding to the Lorentz gauge group. For brevity of notation we will use a redefined connection which absorbs the coupling constant. The general gauge connection \(A_{\mu cd}\) can be written as the sum

\[
A_{\mu cd} = \varphi_{\mu cd} (e) + K_{\mu cd},
\]

where \(K_{\mu cd}\) is a contortion and \(\varphi_{\mu cd} (e)\) is a Levi-Civita spin connection given in terms of the vielbein

\[
\varphi_{\mu a}^\nu (e) = \frac{1}{2} \left( e^d_\nu \partial_\mu e_{da} - e^a_\nu e^d_\mu \partial^b e_{bc} + e^d_\mu \partial_\nu e^a_\mu - e^a_\mu \partial_\nu e^d_\mu + e^{ab} \partial_\nu e_{ac} - \partial^b e_{ac} \right).
\]

The original Lorentz gauge transformation has the form

\[
\delta e^{\mu}_a = A^{\lambda}_a e^{\mu}_\lambda, \quad \delta \varphi_{\mu a}^\nu (e) = -\partial_\nu A_{\lambda a}, \quad \delta K_{\mu a} = -[K_{\mu a}, A] \equiv -[K_{\mu a}, A],
\]

where \(\varphi_{\mu a}^\nu \equiv \varphi_{\mu a}^\nu \Omega^d\) and \(K_{\mu a} \equiv K_{\mu a} \Omega^d\). Let us consider the main lines of Riemann–Cartan geometry (see, e.g., [3]). The torsion and curvature tensors are defined in a standard way

\[
[D_a, D_b] = T_{ab}^\epsilon D_\epsilon + R_{ab},
\]

where \(R_{ab} \equiv R_{abcd} \Omega^d\). The non-holonomicity coefficients \(C_{abc}^d\) are given by the equations

\[
[\Delta_a, \Delta_b] = C_{abc}^d \Delta_c, \quad \Delta_a \equiv e^a_\mu \partial_\mu.
\]

To make the derivative \(D_\mu\) covariant under the general coordinate transformation one includes the Riemann–Cartan connection \(\Gamma_{\mu}^\nu\), as well:

\[
D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu \rho} V^\rho.
\]

The Riemann–Cartan connection is related to the general Lorentz connection \(A_{\mu cd}\) by definition, via

\[
D_\mu e^{\nu a} = \partial_\mu e^{\nu a} + \Gamma^\nu_{\mu \rho} e^{\rho a} - e^{ab} A_{\mu ba} = 0.
\]
The Christoffel symbol $\hat{\Gamma}^{\rho}_{\mu\nu} = \hat{\Gamma}^{\rho}_{\nu\mu}$ is related to the Levi-Civita connection by means of the reduced equation

$$\hat{D}_\mu e^a = \partial_\mu e^a + \hat{\Gamma}^{\rho}_{\mu\nu} e^a - e^b \hat{\varphi}^{\nu b}_a = 0. \quad (11)$$

Solving this equation one can find a standard relationship between $\hat{\Gamma}^{\rho}_{\mu\nu}$ and $\varphi^{\nu a}_b$:

$$\hat{\Gamma}^{\rho}_{\mu\nu} = e_a e^\nu e^\mu + \hat{\Gamma}^{\rho}_{\mu\nu}, \quad (12)$$

An antisymmetric part of the Riemann–Cartan connection defines the torsion components in the holonomic basis:

$$\Gamma^{\rho}_{\mu\nu} = T^{\rho}_{\nu\mu}. \quad (13)$$

The contortion components in the unholonomic basis can be expressed in terms of torsion, and conversely

$$T^{\alpha}_{\mu\nu} = A^{\alpha}_{\mu\nu} - A^{\alpha}_{\nu\mu} + C^{\alpha}_{\mu\nu} - K^{\alpha}_{\nu\mu}, \quad (14)$$

$$K^{\alpha}_{\mu\nu} = \frac{1}{2}(T^{\alpha}_{\mu\nu} - T^{\alpha}_{\nu\mu} + T^{\alpha}_{\mu\nu}), \quad (15)$$

Under decomposition (4) the Riemann–Cartan curvature is split into two parts:

$$R^{\alpha\beta\gamma\delta}_{abcd} = \hat{R}^{\alpha\beta\gamma\delta}_{abcd} + \tilde{R}^{\alpha\beta\gamma\delta}_{abcd},$$

$$\hat{R}^{\alpha\beta\gamma\delta}_{abcd} = D_a \varphi^{\beta\gamma\delta}_{b c d} + \varphi^{\beta\gamma\delta}_{a c d} - \varphi^{\beta\gamma\delta}_{b c d},$$

$$\tilde{R}^{\alpha\beta\gamma\delta}_{abcd} = D_a K^{\beta\gamma\delta}_{b c d} + K^{\beta\gamma\delta}_{a c d} - K^{\beta\gamma\delta}_{b c d}, \quad (16)$$

where the underlined indices stand for indices over which the covariantization has been performed, and we use a short notation for the index antisymmetrization $\alpha\beta \gamma \delta = \alpha \beta - \beta \alpha$. One can define the classical Lagrangian for a pure gravity with torsion which contains only terms quadratic in curvature tensor:

$$L = -\frac{1}{4}(\alpha R^{\alpha\beta\gamma\delta}_{abcd} R^{\alpha\beta\gamma\delta}_{abcd} - 4\beta R^{\alpha\beta} R^{\alpha\beta} + \gamma R^2), \quad (17)$$

where $R^{\alpha\beta}_{ab} \equiv \eta^{\alpha\beta} R^{\alpha\beta}_{abcd}$ and $R \equiv \eta^{\alpha\beta} R^{\alpha\beta}$ and $\alpha, \beta, \gamma$ are real numbers. The case $\alpha = \beta = \gamma = 1$ corresponding to Gauss–Bonnet-type Lagrangian has been considered recently in [29]. In the present paper, we deal with Yang–Mills-type Lagrangian setting $\alpha = 1, \beta = \gamma = 0$. The Yang–Mills-type Lagrangian with the general Lorentz connection constructed from $SL(2, C)$ dyad and vielbein was considered by Carmeli [14]. It had been demonstrated that the corresponding equation of motion after projection with the vielbein produces the vacuum Einstein–Hilbert equation in Newman–Penrose form. Later Martellini and Sodano considered Carmeli’s model treating the Lorentz gauge connection as independent of vielbein and proving its renormalizability [15]. The Lagrangian (17) has also been considered in [30] with studying the problem of the non-compactness of the Lorentz group.

Since the Lorentz group is not compact the Lagrangian $L_0$ leads to a non-positive definite Hamiltonian. For that reason one usually adds to the Lagrangian the Einstein–Hilbert term $\sqrt{-g} R$ which helps to resolve the non-unitarity problem connected with negative energy states. One should stress that we do not introduce the Einstein–Hilbert term as in the Utiyama–Kibble–Sciama approach [7, 8]. Neither do we introduce the $SL(2, C)$ spin frame as Carmeli did in his approach [14] to make the correspondence with Einstein–Hilbert vacuum equations in the Newman–Penrose formalism.

There have been some attempts to consistently quantize the gauge theory with a non-compact structure group [30]. An interesting example is given by the gauge theory for a non-compact Virasoro–Kac–Moody group which can be quantized properly due to spontaneous symmetry breaking [31]. We will show that our model implies the quantum effective action...
with a non-zero vacuum condensate which provides a new scale in the theory. The presence of the new dynamically generated scale can justify the self-consistency of the quantization procedure in our model with the initial non-positive definite classical Hamiltonian.

We will adopt the point of view that even though the classical action (17) does not produce a positive definite Hamiltonian, nevertheless, a consistent quantum theory can be formulated. Since the canonical quantization method fails to handle the quantization problem we will apply the quantization scheme based on the functional integration in Euclidean spacetime. Within this quantization scheme the quantum theory can be constructed since the Lorentz group is locally isomorphic to the product of compact unitary groups $SU(2) \times SU(2)'$ in Euclidean spacetime.

3. Effective action

The general approach to derivation of a low energy effective theory is based on integrating out all high energy (heavy massive) modes in the generating functional of the effective action while keeping light modes (massless or light particles) as a classical background. A simple example of such an effective theory is represented by the well-known Euler–Heisenberg effective Lagrangian in quantum electrodynamics (QED) \([32, 33]\) which includes quantum contributions of electron loops while the massless photon is kept as a classical external field. In a similar way we will integrate out the contortion (which is supposed to gain an effective mass dynamically) keeping the massless gravitational field $e^\mu_a$ as a fixed background.

In general, unlike the QED Euler–Heisenberg effective action, the quantum corrections can induce additional changes in the structure of the initial perturbative vacuum and generate phase transitions to new phases with non-trivial vacua. During the phase transition some non-vanishing vacuum field condensates may be formed. Such a phenomenon occurs in QCD during the transition between deconfinement and confinement phases where non-vanishing quark and gluon condensates are generated. The presence of non-trivial vacuum condensates leads to additional modification of the classical Lagrangian and Green functions.

With this preliminary let us start with a pure Yang–Mills-type classical action:

$$S_{cl} = \int d^4x \sqrt{-g}L_0 = -\frac{1}{4} \int d^4x \sqrt{-g}(\hat{R}_{\mu\nu}^{cd} + \tilde{R}_{\mu\nu}^{cd})^2.$$  \(18\)

A special feature of this action is the presence of an additional local symmetry under the following so-called quantum gauge transformations:

$$\delta e^\mu_a = \delta \varphi^\mu(e) = 0, \quad \delta K_{\mu} = -\hat{D}_{\mu}A - [K_{\mu}, A],$$  \(19\)

where $\hat{A}_{\mu} = \tilde{A}_{\mu} + \Omega^d_{\mu}$ and the restricted covariant derivative $\hat{D}_{\mu} = \partial_{\mu} + \varphi^\mu$ is defined by means of the Levi-Civita connection only. The restricted derivative $\hat{D}_{\mu}$ is covariant only under the original Lorentz gauge transformation (6). Note that the construction of the restricted derivative has a deep analogy with the mathematical structure of Abelian projection in quantum chromodynamics \([35]\). We will consider the physical implications of this analogy in the following section.

To study the problem of whether vacuum condensates can be formed one has to calculate first the effective action $\Gamma[\hat{K}; e]$ which is a generating functional of one-point irreducible Green functions \([34]\). The ‘classical’ field $\tilde{K}_{\mu}^{cd} \equiv \langle K_{\mu}^{cd} \rangle$ represents the vacuum averaged value of the field operator at the presence of a source and $e^\mu_a$ is a fixed background metric. Then, from the structure of the effective potential part of the effective action $\Gamma[\hat{K}; e]$ one can find whether a non-trivial vacuum condensate can be generated.
To calculate the effective action one should split the contortion \( K_{\mu c d} \) into the ‘classical’ field \( \tilde{K}_{\mu c d} \) and the quantum fluctuating part \( Q_{\mu c d} \):

\[
K_{\mu c d} = \tilde{K}_{\mu c d} + Q_{\mu c d}.
\] (20)

Exact calculation of the effective action for an arbitrary field \( \tilde{K}_{\mu c d} \) and given external field \( e_{\mu}^{a} \) is a hard unresolved problem. To simplify the calculation we use the important property of the Yang–Mills-type Lagrangian (18) due to the additional symmetry under the quantum transformation; namely, the vacuum torsion condensate \( \langle 0 | \tilde{R}_{abcd} | 0 \rangle \) can appear only in the form of a gauge covariant additive combination \( \tilde{R}_{abcd} + \langle \tilde{R}_{abcd} \rangle \) that follows directly from the general renormalization properties of the model [15]. So that to find the functional dependence of the effective potential \( V_{\text{eff}}(\tilde{R} + \langle \tilde{R} \rangle) \) on \( \langle \tilde{R} \rangle \) we first calculate the effective potential \( V_{\text{eff}}(\tilde{R}) \) by setting \( \tilde{K}_{\mu c d} = 0 \), i.e., \( K_{\mu c d} = Q_{\mu c d} \). Then, after completing the calculation we will restore the dependence on torsion condensate \( \langle \tilde{R} \rangle \) by simply adding this term to \( \tilde{R} \) in the final expression for \( V_{\text{eff}}(\tilde{R}) \). In this way the calculation becomes technically much simpler and very similar to derivation of the effective action in \( SU(2) \) gauge theory.

We follow the standard formalism of quantization based on the functional integration [34]. The quantization procedure is performed with respect to quantum gauge transformation (19), and the resulting effective action will keep the original Lorentz gauge invariance (6). With the generalized Lorenz gauge fixing condition \( \hat{D}_{\mu} Q^{\mu} = 0 \) one can find the gauge fixing term \( L_{\text{gf}} \) and Faddeev–Popov ghost term \( L_{\text{FP}} \):

\[
L_{\text{gf}} = \frac{1}{2} \text{tr}(\hat{D}_{\mu} Q^{\mu})^{2},
\]

(21)

\[
L_{\text{FP}} = \text{tr}(\bar{c}(\hat{D}_{\mu} c)).
\]

(22)

After taking integration by parts the effective action can be written in the form

\[
\exp(i\Gamma_{\text{eff}}) = \int DQ_{\mu} Dc D\bar{c}
\]

\[
\times \exp \left\{ i \int d^{4}x \sqrt{-g} \text{tr} \left[ \frac{1}{4} \tilde{R}_{\mu \nu}^{2} - \frac{1}{2} Q^{\mu}(g_{\mu \nu} \hat{D}_{\nu} - 4 \hat{R}_{\mu \nu})Q^{\nu} + \bar{c} \tilde{D}_{\mu} c \right] \right\}.
\]

(23)

The formal expression for the one-loop effective action is simplified to

\[
\Gamma_{\text{eff}} = S_{cl} - \frac{i}{2} \text{Tr} \ln \left[ (g_{\mu \nu} \hat{D}_{\nu})_{ab}^{cd} - 2 \hat{R}_{\mu \nu}^{\alpha \beta} (f_{\alpha \beta})_{ab}^{cd} \right] + i \text{ Tr} \ln \left[ (\hat{D}_{\mu} \hat{D}_{\nu})_{ab}^{cd} \right],
\]

(24)

where \( (f_{\alpha \beta})_{ab}^{cd} \) are the structure constants of the Lorentz Lie algebra. It should be stressed that the background curvature tensor \( \hat{R}_{\mu \nu cd} \) in the last equation represents an arbitrary non-constant background. The rigorous way to calculate the full effective action including both the real part and especially the imaginary part must specify the field background. For a constant background the calculation can be carried out in full analogy with the case of \( SU(2) \) QCD [36, 37].

The functional determinants in equation (24) are not well defined in Minkowski spacetime. As is known, adding an infinitesimal number factor \( -i \) to the bare Laplace operator in the full operator \( \hat{D}_{\mu} \hat{D}_{\nu} \) is conditioned by the requirement of causality. This infinitesimal addition uniquely defines the Wick rotation to the Euclidean spacetime. In our case, we should perform the Wick rotation in the base spacetime and in the tangent spacetime as well, so that the Lorentz group in the Euclidean sector turns into the compact orthogonal group \( SO(4) \simeq SU(2) \times SU(2)' \). With this the functional integral becomes well defined. Certainly there remains the problem of analytical continuation of the final expressions from Euclidean spacetime back to Minkowski spacetime.
It is convenient to use the Weyl representation for Dirac matrices $\gamma^\mu$ in Van der Warden notation. In Euclidean spacetime the Weyl representation is defined as follows:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma_\mu^a \\ \bar{\sigma}^\mu_{a\bar{a}} & 0 \end{pmatrix}, \quad \sigma_\mu^a = (I, -\bar{\tau}), \quad \bar{\sigma}^\mu_{a\bar{a}} = (I, \bar{\tau}),$$

(25)

where $\bar{\tau}$ are Pauli matrices. The Cartan algebra of the Dirac matrices is given by

$$\{\gamma^\mu, \gamma^\nu\} = -2\gamma^\rho \epsilon_{\rho\mu\nu}.$$  

(26)

With this notation one can easily convert any real antisymmetric Lorentz tensor $V_{cd}$ into the complex symmetric $SU(2)$ spin tensor $V_{\alpha\beta}$:

$$V_{\alpha\beta} = V_{cd} \sigma_{\alpha\beta}^{cd}, \quad \bar{V}_{\alpha\beta} = V_{cd} \bar{\sigma}_{\alpha\beta}^{cd},$$

(27)

$$\sigma_{\alpha\beta}^{cd} = \frac{1}{2} \epsilon_{\alpha\rho} \sigma^d \rho_{a\beta} + \epsilon_{\beta\rho} \sigma^d \rho_a \bar{\epsilon}_{\alpha\beta}, \quad \bar{\sigma}_{\alpha\beta}^{cd} = \frac{1}{2} \epsilon_{\beta\rho} \bar{\sigma}^d \rho_{a\beta} - \epsilon_{\alpha\rho} \bar{\sigma}^d \rho_a \bar{\epsilon}_{\alpha\beta}.$$  

The spinor indices can be lowered and raised with the help of the spinor metric $\delta_{\alpha\beta}$.

$$V_{\alpha\beta} \delta_{\alpha\beta} = V_{cd} \sigma_{\alpha\beta}^{cd}, \quad \bar{V}_{\alpha\beta} \delta_{\alpha\beta} = V_{cd} \bar{\sigma}_{\alpha\beta}^{cd}.$$  

(28)

In spinor notation the Lorentz algebra can be termed with a complex generator $\omega_{\alpha\beta} = \frac{1}{2}(\sigma_{\alpha\beta} + \epsilon_{\alpha\beta})$ (and its complex conjugate $\bar{\omega}_{\alpha\beta}$):

$$[\omega_{\alpha\beta}, \omega_{\gamma\delta}] = \frac{1}{2}(\epsilon_{\alpha\gamma} \omega_{\beta\delta} + \epsilon_{\beta\delta} \omega_{\alpha\gamma} + \epsilon_{\alpha\delta} \omega_{\beta\gamma} + \epsilon_{\beta\gamma} \omega_{\alpha\delta}).$$  

(29)

Now we can explicitly express the local isomorphism $SO(4) \cong SU(2) \times SU(2)'$ by defining the generators of $SU(2) \times SU(2)'$ Lie algebra:

$$T^i \equiv -2\bar{\tau}^i_{\alpha\beta} \omega_{\alpha\beta}, \quad T'^i \equiv -2\bar{\tau}^i_{\alpha\beta} \omega_{\alpha\beta}.$$  

(30)

Note that the above relations allow us to convert any symmetric second rank spin tensor $T_{\alpha\beta}$ into an $SU(2)$ vector $T^i$, so that we can find the $SU(2) \times SU(2)'$ Lie algebra generated by $T^i, T'^j$:

$$[T^i, T^j] = i\epsilon^{ijk} T^k,$$

$$[T'^i, T'^j] = i\epsilon^{ijk} T'^k, \quad \epsilon^{123} = 1.$$  

(31)

With this one can write any Lorentz Lie algebra valued object in $SU(2)$ notation. For instance, one has the following $SU(2) \times SU(2)'$ decomposition for the gauge Lorentz parameter $\Lambda$:

$$\Lambda_{ij} \Omega_i^d = \Lambda_{ij}^\alpha \Omega_{ij}^\alpha + \Lambda'^{ij}_{\alpha\beta} \Omega_{ij}^\alpha = -i(\Lambda' T^i + \Lambda'^\alpha T'^\alpha).$$  

(32)

In $SU(2)$ notation the gauge Lorentz transformation of the affine connection $A_{\mu}^{\alpha \beta} \Omega_{\mu \nu \rho}$ reveals a ‘chiral’ structure corresponding to the group product $SU(2) \times SU(2)'$:

$$\delta A'_{\mu} T^i = -\partial_\mu \Lambda' T^i - i[A_{\mu}, \Lambda], \quad \delta A'^{\alpha}_{\mu} T'^\alpha = -\partial_\mu \Lambda'^\alpha T'^\alpha - i[A'^{\alpha}_{\mu}, \Lambda'].$$  

(33)

We have a similar factorization property for the curvature tensor

$$R_{\mu\nu\rho\sigma} \Omega_{\mu\nu\rho\sigma} = -i(R_{\mu\nu} T^i + R'^{\alpha}_{\mu\nu} T'^\alpha).$$  

(34)

The functional determinants in (24) are factorized into the direct product of $SU(2)$ determinants, and the effective action takes a simpler form

$$\Gamma_{\text{eff}} = S_3 - \frac{1}{2} \text{Tr} \left[ (g_{\mu\nu}(\hat{D} \hat{D})^{\mu\nu} - 2 \hat{R}^{\mu\nu} \epsilon^{\mu\nu}) \right] - \frac{1}{2} \text{Tr} \left[ (g_{\mu\nu}(\hat{D}' \hat{D}')^{\mu\nu} - 2 \hat{R}'^{\mu\nu} \epsilon^{\mu\nu}) \right] + i \text{Tr} \ln[(\hat{D} \hat{D})^{\mu\nu}] + i \text{Tr} \ln[(\hat{D}' \hat{D}')^{\mu\nu}],$$  

(35)

where all quantities corresponding to the group $SU(2)'$ are marked with a prime.
Note that the $SU(2)$ curvature squared term $(\hat{R}_{\mu\nu})^2$ contains not only the Riemann tensor $\hat{R}_{\mu
u\alpha\beta}$ but also its dual counterpart $\hat{R}^{\mu\nu\alpha\beta}$:

\begin{equation}
(\hat{R}_{\mu\nu})^2 = \frac{1}{8}(\hat{R}_{\mu\nu\alpha\beta} \hat{R}^{\alpha\beta\mu\nu} + \hat{R}^{\mu\nu\alpha\beta} \hat{R}_{\alpha\beta\mu\nu}) \equiv \frac{1}{8}(\hat{R}^2 + \hat{R} \hat{R}^*),
\end{equation}

(36)

In electrodynamics and Abelianized QCD [36] we can define two main gauge invariant variables corresponding to the magnetic $(B)$ and electric $(E)$ fields (in an appropriate Lorentz frame):

\begin{equation}
B = \frac{1}{2} \sqrt{F^2 + (FF^*)^2 + F^2}, \quad E = \frac{1}{2} \sqrt{F^2 + (FF^*)^2 - F^2},
\end{equation}

(37)

where $F_{\mu\nu}$ is the Abelian field strength. For a pure magnetic background one has $FF^* = 0$ and $F^2 = 2B^2$. In a similar way as in Maxwell electrodynamics we will consider the constant gravito-magnetic field without specifying an explicit structure of the corresponding connection (or torsion) assuming that the following conditions are satisfied:

\begin{equation}
\hat{R} \hat{R}^* = 0, \quad \hat{R}^2 > 0.
\end{equation}

(38)

For such a pure gravito-magnetic background one has $(\hat{R}_{\mu\nu})^2 = (\hat{R}^{\mu\nu})^2 \equiv 2H^2 > 0$. With this the functional determinants in (35) can be simplified to a scalar form [36]

\begin{equation}
\Gamma_{eff} = S_\lambda - i Tr \ln[\hat{D} \hat{D} - 2H] - i Tr \ln[\hat{D} \hat{D} + 2H] - i Tr \ln[\hat{D} \hat{D}' - 2H]
\end{equation}

\begin{equation}
- i Tr \ln[\hat{D} \hat{D}' + 2H],
\end{equation}

\begin{equation}
\hat{D}_\mu \equiv \partial_\mu + ig \hat{A}_\mu,
\end{equation}

(39)

where $\hat{A}_\mu, (\hat{A}_\mu)$ is an Abelian projected component of $SU(2)$ $(SU(2'))$ gauge connection. Applying Schwinger’s proper time method of calculation of the effective action [36, 38] and the $\zeta$-function regularization we obtain the one-loop effective Lagrangian:

\begin{equation}
\mathcal{L}_{eff} = -\frac{1}{2}H^2 - \frac{11g^2}{48\pi^2}H^2 \left( \ln \frac{gH}{\mu^2} - c \right) + \frac{ig^2}{8\pi}H^2,
\end{equation}

\begin{equation}
c = 1 - \frac{1}{2} - \frac{24}{11} \zeta \left(-1, \frac{3}{2} \right) = 1.29214 \ldots .
\end{equation}

(40)

Formally, the expression for the effective Lagrangian is identical to that of $SU(2)$ QCD. Note that the logarithmic term corresponds to the nonperturbative contribution of all higher order one-loop Feynman diagrams. Now, using the renormalizability property of our model we can restore the torsion vacuum field in equation (40) by substitution $H^2 \rightarrow (\hat{R} + \langle \hat{R} \rangle)^2/2$. With this the real part of the effective potential $V_{eff} \equiv -Re(\mathcal{L}_{eff})$ takes a final form

\begin{equation}
V_{eff} = \frac{1}{4}(\hat{R} + \langle \hat{R} \rangle)^2 + \frac{11g^2}{96\pi^2}(\hat{R} + \langle \hat{R} \rangle)^2 \left( \ln \frac{g\sqrt{(\hat{R} + \langle \hat{R} \rangle)^2/2}}{\mu^2} - c \right).
\end{equation}

(41)

Obviously, the real part of the effective potential has a new non-trivial minimum at $\hat{R} = 0$ and with a non-zero vacuum torsion condensate $\langle \hat{R}^{\mu\nu\alpha\beta} \rangle$. Note that for the vacuum state corresponding to that minimum we have still an imaginary part of the effective Lagrangian $ig^2(\hat{R}^2)/16\pi^2$ which is nothing but the Nielsen–Olesen imaginary part derived in $SU(2)$ QCD a long time ago [39]. The appearance of the imaginary part is a natural consequence of our constant field approximation which implies that the vacuum is not stable and can be treated only as an approximation to a true vacuum with a lower energy.
To make some estimations let us consider for simplicity the real part of the effective potential with the torsion condensate in a flat metric background, \( \tilde{R}_{\mu\nu cd} = 0 \),

\[
V_{\text{non-ren}} = \frac{1}{2} \tilde{H}^2 + \frac{11g^2}{48\pi^2} \tilde{H}^2 \left( \ln \frac{\tilde{H}}{\mu^2} - c \right), \quad \tilde{H}^2 = \frac{1}{2} \langle \tilde{R}_{\mu\nu cd}^2 \rangle.
\]  

(42)

One can renormalize the effective potential \( V_{\text{non-ren}} \) imposing an appropriate normalization condition

\[
\frac{\partial^2 V_{\text{non-ren}}}{\partial H^2} \bigg|_{H=\tilde{H}} = \frac{g^2}{\bar{g}^2}.
\]  

(43)

The renormalized effective potential includes the running coupling constant \( \bar{g}(\tilde{\mu}) \) which depends on energy scale parameter \( \tilde{\mu} \):

\[
V_{\text{ren}} = \frac{1}{2} \tilde{H}^2 + \frac{11\bar{g}^2}{48\pi^2} \tilde{H}^2 \left( \ln \frac{\bar{g}\tilde{H}}{\bar{\mu}^2} - \frac{3}{2} \right).
\]  

(44)

One can check that the effective potential satisfies the renormalization group equation with the same \( \beta \)-function as that in a pure \( SU(2) \) Yang–Mills theory. The effective potential has a non-trivial minimum \( V_{\text{min}} \) and leads to a gravito-magnetic condensate \( \langle \tilde{H} \rangle \):

\[
V_{\text{min}} = -\frac{11\bar{g}^2}{192\pi^2} \langle \tilde{H} \rangle^2.
\]  

(45)

\[
\langle \tilde{H} \rangle = \frac{\bar{\mu}^2}{\bar{g}} \exp \left[ -\frac{24\pi^2}{11\bar{g}^2} + 1 \right].
\]  

(46)

The presence of the minimum of the effective potential does not guarantee that the corresponding new vacuum is stable. The stability of the vacuum condensate even in a pure \( SU(2) \) model of QCD presents a long-standing problem, and resolving that problem has passed through several controversial results since the early papers by Savvidy, Nielsen and Olesen [39–43]. Without clear evidence or at least a strong indication of vacuum stability one cannot make any serious statement based on the existence of a non-trivial vacuum condensate. Recently, substantial progress in resolving this problem in favor of stability of the magnetic vacuum has been achieved [36, 37]. Moreover, it has been found that a stable classical configuration made of monopole–antimonopole strings does exist in the \( SU(2) \) model of QCD [44], providing a strong argument that a stable magnetic vacuum can exist in QCD and, therefore, in our gauge model of quantum gravity with torsion as well.

The gravito-magnetic condensate \( \langle \tilde{H} \rangle \) generates a new renorminvariant scale \( M \) in the theory. We suppose that the local gauge Lorentz symmetry is not broken, so the lowest torsion condensate must vanish \( \langle T_{abc} \rangle = 0 \). Since one has a non-trivial dynamically generated scale \( M \) one can expect a non-vanishing vacuum averaged value for the curvature tensor corresponding to the torsion

\[
\langle \tilde{R}_{abcd} \rangle = \frac{1}{2} \bar{M}^2 (\eta_{ae} \eta_{bd} - \eta_{ad} \eta_{be}).
\]  

(47)

The factor \( \bar{M}^2 \) needs to be positive since it corresponds to a positive curvature spacetime which can only be created due to quantum fluctuations of torsion through the vacuum transition from the trivial vacuum to the non-trivial one. It is important to stress that the above expression for \( \langle \tilde{R}_{abcd} \rangle \) with a positive scale \( \bar{M}^2 \) satisfies the conditions (38) for the gravito-magnetic field. Moreover, the tensor structure of (47) is unique because adding another term proportional to the antisymmetric tensor \( \epsilon_{abcd} \) is forbidden by (38), and such a term would contain a gravito-electric component implying quantum instability.
Expanding the original classical Lagrangian (18) around the new vacuum by shifting \( \tilde{R}_{abcd} \rightarrow \tilde{R}_{abcd} + \langle \tilde{R}_{abcd} \rangle \) one obtains
\[
L_{\text{Eff}} = -\frac{1}{4} (\tilde{R}_{abcd} + \langle \tilde{R}_{abcd} \rangle)^2 = -\frac{1}{3} \tilde{R}^2 - \frac{1}{16} \tilde{R} M^2 - \frac{3}{2} M^4. \tag{48}
\]
One should emphasize that even though the vacuum averaged value \( \langle \tilde{R}_{abcd} \rangle \) is specified by (47), the classical gravitational field \( \hat{R}_{abcd} \) is not constrained in general. The last term in the equation corresponds to a positive vacuum energy density which is supposed to be formed during the vacuum transition. From this we can estimate the numeric value of \( M \) by setting \( 3M^4/2 = |V_{\text{min}}| \):
\[
M^2 = \frac{\bar{g}}{\pi} \sqrt{\frac{11}{288}} \langle \hat{H} \rangle = \frac{\bar{\mu}^2}{\pi} \sqrt{\frac{11}{288}} \exp \left[ -\frac{24\pi^2}{11\bar{g}^2} + 1 \right]. \tag{49}
\]
Note that the scale \( M \) is renorminvariant, i.e., it does not depend on a particular value of \( \bar{\mu} \).

The last two terms in (48) produce the Einstein–Hilbert-type terms in the effective Lagrangian (in units \( \bar{\hbar} = c = 1 \)):
\[
L_{\text{Eff}} = -\frac{1}{4} \hat{R}_{abcd}^2 - \frac{1}{16\pi G} (\hat{R} + 2\lambda). \tag{50}
\]
So that the Newton constant \( G \) and the cosmological constant \( \lambda \) are determined by only one renorminvariant scale \( M^2 \), in other words by the renormalized running coupling constant \( \bar{g} \) at some scale \( \bar{\mu} \), expected to be of the order of the Planckian energy \( 10^{19} \text{ GeV} \).

Certainly, the assumption (47) leads to a desired induced Einstein–Hilbert term, as has been known before. The most important point is to provide a foundation for that hypothesis. In our approach, we put this assumption on real ground by explicitly calculating the effective potential and having found a stable classical vacuum solution in the \( SU(2) \) gauge model \[44\] which is a part of Lorentz gauge theory in the Euclidean formulation. We have hope that this assumption might be true, at least in the framework of our simple model.

One cannot fit both experimental values of constants \( G \) and \( \lambda \) concurrently by adjusting the scale parameter \( M \), so that the cosmological problem remains unresolved. An additional uncertainty is related to the unknown value of the energy scale \( \bar{\mu} \) which is only supposed to be of order Planckian one, but for possible various phases the scale \( \bar{\mu} \) may be different. We consider two particular cases when the scale parameter \( M \) corresponds to the experimental value of \( G \) and \( \lambda \). In each case we will find that the coupling constant \( \bar{g} \) takes large and small values, respectively. It is possible that there are two phases corresponding to these strong and weak coupling constants. Recently, the existence of two phases in gravity was suggested in \[45\] within a different approach.

To justify the known value for Newton’s constant one should put the following value of the structure constant \( \alpha_g = \bar{g}^2/4\pi \approx 1.52 \), which corresponds to a strong coupling phase. Note that the cosmological constant is of the Planckian order. This value is consistent with cosmological models which elaborate the large value of the cosmological constant in the very early universe to provide the fast initial inflation.

The weak coupling phase can be determined by the known experimental bound for the vacuum energy density:
\[
\rho_v = \frac{\lambda}{8\pi G} \simeq 2 \times 10^{-47} \text{(GeV)}^4. \tag{51}
\]
This implies the following value of the scale \( M \) according to (48), (49):
\[
M^2 = 3.64 \times 10^{-24} \text{(GeV)}^2. \tag{52}
\]
The corresponding value for the structure constant is small:
\[
\alpha_g = \frac{\bar{g}^2}{4\pi} = 0.0123. \tag{53}
\]
This value can be compared with the value $\alpha_{\text{SSGUT}} \simeq 0.04$ of the structure constant in supersymmetric $SO(10)$ GUT model at unification scale $2 \times 10^{16}$ GeV. The same order of the structure constants $\alpha_g$ and $\alpha_{\text{SSGUT}}$ might be a hint as to the origin of supersymmetry and its relation to quantum gravity. It would be natural to suspect such a connection since the algebra of supersymmetry was invented exactly as an extension of the Poincaré Lie algebra which describes the spacetime symmetries intimately related to gravity. The weak coupling phase can be considered as an analog to the quark–gluon plasma phase of QCD where some intriguing results have been obtained recently [46]. Namely, this phase is described by the liquid model with unexpected still non-vanishing gluon condensate. It would be of interest to study the possible relationship of the gravitational weak phase to the hydrodynamic modeling the quantum gravity phenomenology proposed in [47].

4. Parallels to QCD: torsion to be confined

The consistent quantum theory of strong interaction is presented by quantum chromodynamics. One should stress that at a classical level QCD cannot serve merely as a classical theory because single quarks and gluons are not observable in principle as free particles. The classical theory of strong interaction below the confinement scale is described by other phenomenological models where classical states are represented by hadrons, the bound states of quarks and gluons. There is a deep analogy between our gauge model of gravity with torsion and the theory of strong interaction in mathematical structure. It is a very intriguing question up to what extent this analogy lies. We recall the main construction of the gauge invariant Abelian projection in $SU(2)$ QCD [35]. The Abelian decomposition of the full gauge potential of $SU(2)$ gauge theory has been implemented with a scalar triplet $\hat{n}$ which represents pure topological degrees of freedom classified by the homotopy group $\pi_3(S^2) \simeq \mathbb{Z}$:

$$\hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g} \hat{n} \times \partial_\mu \hat{n} + \bar{X}_\mu = \hat{A}_\mu + \bar{X}_\mu, \quad (\hat{n}^2 = 1, \hat{n} \cdot \bar{X}_\mu = 0),$$

(54)

where $\hat{A}_\mu$ is a restricted potential and $\bar{X}_\mu$ represents the off-diagonal component, the so-called valence gluon. The important property of the restricted potential $\hat{A}_\mu$ is that it possesses the transformation properties of the full $SU(2)$ gauge connection even in the absence of the valence gluon. The scalar $\hat{n}$ is a covariant constant

$$\hat{D}_\mu \hat{n} = \partial_\mu \hat{n} + g \hat{A}_\mu \times \hat{n} = 0$$

(55)

and has a clear mathematical origin as an isometry vector corresponding to the Cartan subalgebra $U(1)$ of $SU(2)$ Lie algebra. Under the infinitesimal gauge transformation

$$\delta \hat{n} = -\bar{\alpha} \times \hat{n},$$

$$\delta \hat{A}_\mu = \frac{1}{g} (\partial_\mu \bar{\alpha} + g \bar{A}_\mu \times \bar{\alpha}) \equiv \frac{1}{g} \hat{D}_\mu \bar{\alpha},$$

(56)

one has the following transformation rules for the Abelian and off-diagonal gluon components:

$$\delta A_\mu = \frac{1}{g} \hat{n} \cdot \partial_\mu \bar{\alpha}, \quad \delta \bar{X}_\mu = -\bar{\alpha} \times \bar{X}_\mu.$$

(57)

Note that the valence gluon $\bar{X}_\mu$ transforms covariantly like a vector and plays the role of charged matter. In addition to the initial classical transformation (56), there is a second type
of transformation which originates from the additive structure of the Abelian decomposition (54):

\[
\begin{align*}
\delta \hat{n} &= 0, \\
\delta A_\mu &= \frac{1}{g} \hat{n} \cdot \vec{D}_\mu \vec{a}, \\
\delta X_\mu &= \frac{1}{g} (\vec{D}_\mu \vec{a} - (\hat{n} \cdot \vec{D}_\mu \vec{a}) \hat{n}).
\end{align*}
\] (58)

From the comparison of the SU(2) QCD structure with the Lorentz gauge model introduced in section 2 one can immediately find the analogy between the restricted potential \( \hat{A}_\mu \) and valence gluon \( \vec{X}_\mu \) in QCD on the one hand, and the Levi-Civita connection \( \omega_{\mu c\delta} \) and contortion \( K_{\mu c\delta} \) in the Lorentz gauge model on the other hand.

Obviously, in QCD (as well as in the Weinberg–Salam model which contains the group SU(2) as a subgroup) we cannot treat the off-diagonal component \( \vec{X}_\mu \) as a true covariant vector. The reason is that if we introduce, for instance, a mass term for the off-diagonal gluon into the Lagrangian the renormalizability will be lost. For the same reason, we cannot treat the contortion \( K_{\mu c\delta} \) as a true tensor in gravity models in attempts to formulate a quantum renormalizable theory.

Let us consider the following aspect of the confinement problem in QCD regarding the fact that quarks and gluons are not observable single particles. One heuristic argument why we cannot observe the color single states is the following:\footnote{One of authors (DGP) acknowledges Y M Cho for elucidating this argument.}: quarks and gluons are not gauge invariant and we have no conserved color charge like the electric charge in Maxwell theory. So quarks and gluons cannot be observable as single physical particles unless the color SU(3) symmetry breaks down. The reason why we can observe the vector bosons in the Weinberg–Salam model is due to the spontaneous breaking of the symmetry. Besides, in Maxwell theory the classical electric and magnetic fields are gauge invariant concepts. This is not the case with the non-Abelian theory of QCD where gluons are not gauge invariant objects. This fact along with the color symmetry being unbroken gives a natural explanation of the confinement phenomenon from the symmetry point of view.

If we accept the hypothesis that a Lorentz gauge model of gravity with torsion possesses two types of gauge symmetry (6), (19) then we will be forced to accept the confinement of torsion unless the quantum Lorentz gauge symmetry breaks down. The only classical objects which can be observed are bound states of \( K_{\mu c\delta} \) and torsion vacuum condensates. If torsion is confined this could help to circumvent difficulties related to available experimental limits and some theoretical severe restrictions on propagating torsion [48, 49].

5. Discussion

Let us return to the problem of positive definiteness of the Hamiltonian in the classical theory corresponding to the Lagrangian (17). Note that the existence of the quantum theory when the corresponding classical theory cannot be defined consistently is not a new situation in quantum field theory. The classical field theory of the Dirac electron is not well defined due to the existence of negative energy states (Dirac sea). Upon postulating the anticommutative relations for the creation and annihilation operators the quantum theory of the electron becomes consistent, but still the classical energy remains positive indefinite. The deep origin of that problem was studied in [50–52], and it is related to transformation properties of some operators under time reflection. As was pointed out in [50], there was a principal contradiction between
the facts that the Lagrangian is a scalar function under time reversal; meanwhile the energy–momentum vector $P_\nu = \int d\sigma^\mu T_{\mu\nu}$ is a pseudovector, the electric charge $Q = 1/c \int d\sigma^\mu j_\mu$ and the mass term of the electron $M \tilde{\psi} \psi$ are pseudoscalars. In the path integral approach these facts allow the interpretation of the negative energy states as positrons moving backward in time [52]. The quantization based on the functional integration is more general since it allows the quantization of non-quadratic in momentum Lagrangians and, as in the particular example of quantum electrodynamics, the path integral sums up all paths including those that correspond to time reversed direction. The compact group structure of the Lorentz group in Euclidean spacetime guarantees the consistent quantization of the theory (in the present paper we do not consider the problems of analytical continuation from Euclidean spacetime back to the Minkowski one). Besides this we give a heuristic argument on a possible resolution of the negative energy problem of the classical Hamiltonian. Let us consider the time reflection operation $t \rightarrow -t$. To find the properties of the contortion under time reflection it is convenient to write the linearized equations of motion in the flat spacetime background in the generalized Lorenz gauge $\partial_\mu K^{\mu cd} = 0$:

$$\partial_\mu K^{\mu cd} = \bar{\Psi}(\gamma^c \Sigma_{cd} + \Sigma_{cd} \gamma_0)\Psi.$$ (59)

Under time reversal the transformation rule for a spinor is given by $\Psi \rightarrow \tilde{T} \Psi$ with $\tilde{T} = \gamma_1 \gamma_2 \gamma_3$. This implies $\bar{\Psi} \Psi \rightarrow -\bar{\Psi} \Psi$, i.e., the spinor matter density is a pseudoscalar. From the equations of motion one can find the transformation properties of the contortion under time reflection

$$K_{ijk} \rightarrow -K_{ijk}, \quad K_{i0k} \rightarrow +K_{i0k},$$

$$K_{0jk} \rightarrow +K_{0jk}, \quad K_{00k} \rightarrow -K_{00k},$$ (60)

so that $K_{ijk}$ manifests itself as a pseudovector (like a photon $A_\mu$ or a pseudovector combination $\bar{\psi} \gamma_\mu \gamma_5 \psi$ made of the Dirac spinor) whereas the part of contortion $K_{i0k}$, which gives a negative energy contribution, represents the opposite properties of a vector. Since the contortion is invariant under charge conjugation it represents a neutral particle. By analogy with the path integral formulation of quantum mechanics we can interpret $K_{i0k}$ as a particle moving back in time with a positive energy like a positron in the Feynman theory of positrons [52]. Obviously, such an analysis cannot be performed in the case of gauge models with a non-compact internal group.

In conclusion, we propose a simple gauge model with a local Lorentz group which is supposed to describe the quantum theory of gravity with torsion. One-loop effective action is calculated for a constant curvature spacetime background. We have demonstrated that the Hilbert–Einstein gravity can be induced due to the quantum dynamics of torsion via formation of a stable gravito-magnetic condensate. One should note that in our paper we have treated the metric as a fixed metric of the classical spacetime background while the contortion is supposed to be a quantum field. Such a treatment of the metric is not very satisfactory from the conceptual point of view since one has to assume the pre-existence of spacetime with a metric given a priori. One possible way to resolve that problem is to extend the Lorentz gauge group to the Poincaré one, as mentioned in the introduction. In that case the gauge potential of the Poincaré group, the vielbein, obtains dynamical content on equal footing with torsion. Another interesting possibility is to consider the Gauss–Bonnet-type gravity model with torsion [29]. The model in the absence of torsion reduces to a pure topological theory with arbitrary metric. Surprisingly, within this model the torsion has the same number of physical degrees of freedom for its spin two-field component as the metric tensor. This provides an additional argument supporting our conjecture that torsion can play an important role as a quantum counterpart to the metric. Possible implications of our results in the cosmology of the early universe will be considered elsewhere.
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