We study the spectrum of confining strings in SU(3) pure gauge theory, in different representations of the gauge group. Our results provide direct evidence that the string spectrum agrees with predictions based on n-ality. We also investigate the large-N behavior of the topological susceptibility $\chi$ in four-dimensional SU($N$) gauge theories at finite temperature, and in particular across the finite-temperature transition at $T_c$. The results indicate that $\chi$ has a nonvanishing large-N limit for $T < T_c$, as at $T = 0$, and that the topological properties remain substantially unchanged in the low-temperature phase. On the other hand, above the deconfinement phase transition, $\chi$ shows a large suppression. The comparison between the data for $N = 4$ and $N = 6$ hints at a vanishing large-N limit for $T > T_c$.

1. N-ality

The spectrum of confining strings in $4 - d$ SU($N$) gauge theories has been much investigated recently. Several numerical studies on the lattice have provided results for colour sources associated with representations higher than the fundamental (see [12] for extended lists of references). By general arguments, the string tension must depend only on the n-ality, $k = \text{mod}(l, N)$, of a representation built out of the (anti-)symmetrized tensor product of $l$ copies of the fundamental representation. The confining string with n-ality $k$ is usually called k-string, and $\sigma_k$ is its string tension. Using charge conjugation, $\sigma_k = \sigma_{N-k}$. As a consequence, SU(3) has only one independent string tension determining the large distance behavior of the potential for $k \neq 0$. One must consider larger values of $N$ to look for distinct k-strings.

Model-independent results predict that, in the $N \to \infty$ limit, the k-string ratio $\sigma_k/\sigma \to k$ with corrections that are parametrically of order $1/N^2$. This excludes the Casimir scaling law as an exact formula. Another interesting hypothesis which has been put forward in the context of supersymmetric theories, the sine scaling law, is consistent with this constraint. Lattice results for $N = 4, 5, 6$ show a nontrivial spectrum for the k-strings [15], which is well approximated by the sine formula (see [6] and references therein).

On the other hand, numerical results for different representations with the same n-ality apparently contradict the picture that n-ality is what really matters. For example, MC data in SU(3) for Wilson loops show apparently area laws up to rather large distances, approximately 1 fm, also for representations with zero n-ality, and the extracted string tensions turn out to be consistent with Casimir scaling [7,8].

In the lattice study of Ref. [5,4], considering larger values of $N$, the k-string tensions were extracted from torelon masses, i.e. from the exponential decay of correlations of characters of Polyakov lines. In Ref. [4], while antisymmetric representations provided rather clean measurements of $\sigma_k$ reproducing the sine formula, the numerical results for the symmetric representations suggested different values of the corresponding string tensions. For example, in the case of rank 2, $\sigma_{\text{sym}}/\sigma \gtrsim 2$, which is approximately the value suggested by Casimir scaling or by the propagation of two noninteracting fundamental strings.

These results that apparently contradict n-ality may be explained by arguing that standard colour
sources, associated with representations different from the antisymmetric ones, have very small overlap with the stable $k$-string states, being suppressed by powers of $1/N^2$ in the large-$N$ limit, and in some cases also exponentially $\mathbb{E}$. Since $N = 3$ is supposed to be already large, these arguments may explain why the predictions of $n$-ality have not been directly observed in numerical simulations, which are limited in accuracy. This situation worsens for larger $N$.

We study this issue in SU(3) gauge theory, by MC simulations. We measure “wall-wall” correlators of Polyakov lines in the representations of rank $k = 1$ (fundamental) and $k = 2$ (symmetric) of SU(3), in order to check whether their string tensions $\sigma_k$ are consistent with $n$-ality.

The correlators decay exponentially as $\exp(-m_k t)$, where $m_k$ is the mass of the lightest state in the corresponding representation. For a line of size $L$, $\sigma_k$ is obtained through:

\[ m_k = \sigma_k L - \pi/(3L). \]

We obtained results at $\beta = 5.9$ and for two lattices: $12^3 \times 24$ and $16^3 \times 24$. Use of smearing and blocking leads to a construction of new operators with a better overlap with the lightest string state. In Fig. 1 we show the correlators as a function of the distance $t$ in the cases of $k = 1$ and $k = 2$ representations, from the runs with $L = 16$: a similar figure is provided in $\mathbb{F}$ for $L = 12$. The data for the correlator at $k = 1$ allow us to accurately determine the fundamental string tension, and the two lattices give consistent results, i.e. $\sigma = 0.0664(5)$ and $\sigma = 0.0668(3)$.

On the other hand, such an agreement is not observed for $k = 2$. However, the $L = 16$ data for $k = 2$ shows a clear evidence that its asymptotic behavior is controlled by the fundamental string: indeed, fitting the data for $t \geq 3$ we obtain $\sigma_{\text{sym}} = 0.070(4)$, in agreement with $n$-ality. Although data at small $t$, $t < 3$, show a clear contamination by heavier states, in the $k = 2$ case the overlap with the fundamental string state of the source operator, obtained by performing four blocking steps after smearing, appears to be sufficient to show the eventual asymptotic behavior. This is not observed in the $L = 16$ data using the source operator with three blocking steps (one less) and in the $L = 12$ data (two blocking steps). Up to the distances that we can observe, the correlators are dominated by the propagation of a much heavier state, which would suggest $\sigma_{\text{sym}} \approx 0.16$. Note that $\sigma_{\text{sym}}/\sigma \approx 2.4$ is rather close to the Casimir ratio $5/2$.

In conclusion, our results provide direct evidence that the spectrum of confining strings is according to predictions based on $n$-ality. Torelon correlations in the rank-2 symmetric channel appear to be well reproduced by a two-exponential picture, in which the heavier string state has $\sigma_2/\sigma_1 \sim C_{\text{sym}}/C_l = 5/2$, and the torelon has a much smaller overlap with the lighter $k$-string.

2. Topological susceptibility across $T_c$

At $T = 0$ the anomalous breaking of the axial $U(1)$ flavour symmetry explains the heavier singlet state $\eta'$, whose mass is related to the pure gauge topological susceptibility $\chi$ through the Witten-Veneziano (WV) formula $m_{\eta'}^2 = 4N\chi/f_\pi^2$, up to $O(1/N)$ corrections. In order to clarify the pattern of chiral symmetry breaking, we extend our study of the dependence on the $\theta$ angle in SU($N$) gauge theories $\mathbb{G}$ to the case of finite temperature.

At $T > 0$, chiral symmetry is restored at a crit-
The nature of this phase transition is relevant to understanding the behaviour of hadronic matter under extreme conditions. Below $T_{\text{ch}}$, we expect the WV formula to hold. In order to test the WV mechanism at finite $T$, we study $\chi$ as $T_{\text{ch}}$ is approached from below. Above $T_{\text{ch}}$, the picture is rather different. Here instanton calculus gives contributions for $\chi$ that are exponentially suppressed, $\chi \sim \exp(-N)$. Since the transition between high- and low-$T$ is not fully understood, we also present results above $T_{\text{ch}}$.

Our study is carried out for $N = 4, 6$, on asymmetric lattices with different time extensions $L_t = 6, 8$ and constant aspect ratio $L_t/L_x = 1/4$. The physical temperature is a function of lattice spacing and $L_t$, $T = 1/a(\gamma)L_t$, where $\gamma = \beta/2N^2$. For each $L_t$, $\gamma$ is chosen around $\gamma_c$ which corresponds to the critical temperature $T_{\text{ch}}$. We define absolute and reduced temperatures through:

$$T(L_t, \gamma) \equiv T/\sqrt{\sigma} = 1/(L_t\sqrt{\sigma(\gamma)}), \quad (1)$$
$$t(L_t, \gamma) \equiv T(L_t, \gamma)/T(L_t, \gamma_c(L_t)) - 1, \quad (2)$$

($\sigma$ is computed on symmetric lattices, $T = 0$).

$\chi$ is defined using cooling; we compute:

$$R(L_t, \gamma) \equiv \chi_t(L_t, \gamma)/\chi_t(\infty, \gamma), \quad (3)$$

where $\chi_t(\infty, \gamma)$ is computed at $T = 0$.

Data for the scaling ratio $R$ are displayed in Fig. 2. Its behavior is drastically different in the low- and high-$T$ phases. At low-$T$, all data for $N = 4, L_t = 6, 8$ and $N = 6, L_t = 6$ appear to lie on the same curve, showing both that scaling corrections are small and the large-$N$ limit is approached fast. $R$ remains constant, compatible with 1.0. Only close to $T_{\text{ch}}$, i.e., for $T \approx 0.98T_{\text{ch}}$, it appears to decrease. Thus the physical picture does not change as long as the system remains in the confined phase. On the other hand, above the transition, we observe a clear suppression of $\chi_t$, which appears much stronger at $N = 6$.

Thus, in the confined phase, topological properties remain substantially unchanged up to $T_{\text{ch}}$. In the high-$T$ phase, there is a sharp change of regime where $\chi$ is largely suppressed. Similar results have been recently presented in [10]. The MC data suggests that such a drop grows with increasing $N$. This suppression at large $N$ supports the hypothesis that topological properties in the high-$T$ phase are essentially determined by instantons [11].

REFERENCES

1. L. Del Debbio, H. Panagopoulos, E. Vicari, JHEP 0309 (2003) 034.
2. L. Del Debbio, H. Panagopoulos, E. Vicari, hep-th/0407068, to appear in JHEP.
3. A. Armoni and M. Shifman, Nucl. Phys. B 671 (2003) 67.
4. L. Del Debbio, H. Panagopoulos, P. Rossi, E. Vicari, Phys. Rev. D65 (2002) 021501; JHEP 0201 (2002) 009.
5. B. Lucini, M. Teper, Phys. Rev. D64 (2001) 105019; Phys. Lett. B 501 (2001) 128.
6. M. J. Strassler, Nucl. Phys. Proc. Suppl. 73 (1999) 120 [arXiv:hep-lat/9810059].
7. G. S. Bali, Phys. Rev. D 62 (2000) 114503.
8. S. Deldar, Phys. Rev. D 62 (2000) 034509.
9. L. Del Debbio, H. Panagopoulos and E. Vicari, JHEP 0208 (2002) 044.
10. B. Lucini, M. Teper and U. Wenger, arXiv:hep-lat/0401028.
11. D. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, Phys. Rev. Lett. 81 (1998) 512.