In a model recently proposed by Albrecht and Skordis it was suggested that the observed accelerated expansion of the universe could be caused by a scalar field which is trapped in a local minimum of an exponential potential modified by a polynomial prefactor. We show that scalar field cosmologies with this kind of local minimum in the potential are stable and do not decay to the true vacuum if they fulfill the observational constraints from the Type Ia Supernovae experiments. Further we briefly sketch how this potential could be related to a potential of interacting D-branes.

1 Introduction

Recent observations by the Supernovae Cosmology Project (SCP) and the High-Z Supernovae Search Team have revealed that there is an energy component in the universe which is dark and has negative pressure. This is confirmed if one combines observations of the anisotropy in the cosmic microwave background (CMB) radiation and clusters. The simplest way to achieve accelerated expansion of the universe is by introducing an ad hoc cosmological constant. The findings of the SCP are that the probability for a non-vanishing cosmological constant is 99%. If one assumes a flat universe, which seems to be confirmed by recent CMB observations, the universe consists of 30% matter and 70% cosmological constant or dark energy component. A more general way to obtain accelerated expansion of the universe is by introducing a scalar field \( \phi \) which either slowly rolls down a potential or is trapped in a local minimum. In this way the universe eventually becomes vacuum dominated and therefore the expansion accelerates. During recent years dark energy or quintessence models have been proposed with some of these models needing tuning of the initial conditions to fulfill the observational constraints, but the majority requiring only a tuning of the parameters of the model. The novel feature of the model by Albrecht and Skordis is that the parameters involved are roughly of order one in Planck units (\( M_{\text{Pl}} \approx 2.44 \times 10^{18} \text{GeV} \)) and only moderate tuning is required to be within the observational constraints.
Throughout this paper we use natural units ($\hbar = c = 1$) and set the Planck mass $M_{Pl} = (\hbar c^5/8\pi G)^{1/2} = 1$.

2 The dark energy potential and tunneling

The dark energy field in the Albrecht and Skordis model rolls down an exponential potential which is modified by a polynomial prefactor,

$$V(\phi) = V_p(\phi)e^{-\lambda\phi}, \quad V_p(\phi) = (\phi - \beta)^\alpha + \delta.$$ (1)

This prefactor leads to a local minimum in the potential in which the scalar field gets trapped after rolling down the exponential branch. The initial potential energy of the field is “redshifted away” due to the friction term in the field equations, caused by the expansion. Therefore the field gets trapped in the minimum and the expansion starts accelerating. The model was tested for a range of parameters. To illustrate our findings with numbers we took the parameters from Albrecht and Skordis with $\alpha = 2$, $\lambda = 8$ and $\delta = 0.01$. The observational constraints are that the universe today consists of 30% matter, 70% dark energy and we choose the Hubble constant to be $65$ km s$^{-1}$ Mpc$^{-1}$. To fulfill the observational constraints it is necessary to adjust the position of the local minimum by tuning $\beta$ to $33.989$. The potential shown in Fig. 1 is for $\beta = 34.8$. To calculate the transition rate from the false vacuum through

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure1.png}
\caption{An example of an exponential potential with a polynomial prefactor. The parameters are $\alpha = 2$, $\beta = 34.8$, $\lambda = 8$ and $\delta = 0.01$.}
\end{figure}
the barrier we follow the prescription of Coleman and De Luccia. The tunneling rate \( \Gamma \) per volume element \( V \) in the semi-classical approximation is \( \Gamma/V = Ae^{-B} \). In order to calculate \( B \) in the vacuum decay amplitude we need to compute

\[
B = S_{E}(\phi_{\text{cl}}) - S_{E}(\phi_{+}) ,
\]

(2)

with \( S_{E} \) being the analytic continuation of the Minkowskian action

\[
S = \int d^{4}x \sqrt{-g} \left[ \frac{1}{2} g^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) - \frac{R^{2}}{2} \right] ,
\]

(3)

where \( g_{\mu \nu} \) is the metric, \( R \) the corresponding curvature scalar and \( \phi_{+} \) is the field in the local minimum of the potential. In order to obtain the non-trivial classical solution \( \phi_{\text{cl}} \) to the Euclidean field equations we assume that an \( O(4) \) symmetric ansatz minimizes the Euclidean action \( S_{E} \), with the corresponding Euclidean metric of the form \( ds^{2} = d\xi^{2} + \rho(\xi)^{2}d\Omega^{2} \), \( \xi \) the radial coordinate and \( \rho = \sqrt{t^{2} + |x|^{2}} \). The boundary conditions for the tunneling solution in this coordinates are \( \phi(\infty) = \phi_{+} \), \( \phi'(\infty) = 0 \), \( \phi'(0) = 0 \) and \( \rho(0) = 0 \). We solved the Euclidean field equations under these boundary conditions numerically and computed the tunneling suppression factor \( B \). With the parameters above we got

\[ B \approx 10^{123} , \]

where the field reappearing after the barrier at \( \phi \approx 35.5 \) with \( \phi' = 0 \), from where it rolls towards \( \phi = \infty \), the true vacuum state as sketched in Fig. 1. The probability for a tunneling event per unit time, per unit volume is \( \Gamma/V \approx \exp(-10^{123}) \) which is negligible. Therefore no bubbles of true vacuum are formed. In the Coleman and De Luccia prescription the true vacuum state is reached for finite field values. One might wonder if their result is also true if the vacuum state is only reached asymptotically. However we are only interested in the tunneling rate through the barrier and not actually when the field ends up in the true vacuum, so our calculation is valid. We cross checked this result by considering a double Mexican Hat potential, \( V(\phi) \propto \phi^{2}(\phi^{2} - \alpha')(\phi^{2} - \beta') \), with the true vacuum state at a finite field value. If we choose the parameters \( \alpha' \) and \( \beta' \) to get a similar barrier width and height as we get for the polynomial exponential, the tunneling rate is of the same order of magnitude.

3 Discussion

The considerations above show that the Albrecht and Skordis model is a stable solution and the probability that the field decays to the true vacuum state
is negligible. We also observed that if we move the feature in the potential to smaller field values \( \phi \leq 1 \) the tunneling rate is significant and \( B \sim O(1) \) in Planck units. However a feature in the potential at such small field values could not fulfill the observational constraints and resolve the dark energy problem, since the field gets trapped in the local minimum too early. Since the false vacuum state does not decay for the relevant range of parameters, a classical description is sufficient and the field stays trapped in the local minimum. The dark energy model with a polynomial exponential is therefore a possible solution to the problem of the missing dark energy. One might wonder if it is possible to connect such a potential to fundamental physics. A potential with similar features to the one discussed above is an inverse polynomial exponential of the form \( V(\phi) = [(\phi - \beta)^2 + \delta]^{-1} \exp(-\lambda\phi) \). It may be possible to relate this potential to the massive bulk modes of interacting D3-branes, which involves similar contributions. In this case the false vacuum state is also very stable with \( B \approx 10^{120} \). The question of how realistic a rational exponential potential which resolves the dark energy problem is remains open to this point but is under current investigation.

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