Correlation analysis between OD demand and traffic flow in urban traffic network

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Abstract—Traffic congestion has become a serious problem in urban area. It may be alleviated by effectively controlling the external conditions affecting the traffic networks. In this paper, various traffic assignment methods are employed to analyze the correlation between OD demand and traffic flow in urban traffic network. To explore the effects of external conditions, three algorithms including all or nothing algorithm, Dial algorithm and MSA algorithm are applied for traffic networks. We study the sensitivity of traffic assignment methods with variation of external conditions (e.g. the number of road sections, the number of intermediate nodes, the number of OD pairs). The comparison results show that the degree of influences depends on OD demands. Additionally, the traffic congestion can be alleviated by removing the node or sections of traffic networks, which can be used to develop traffic management strategies.

1. Introduction

In recent years, with the development of urbanization and motorization, the rapid growth of motor vehicle ownership has brought unprecedented pressure to ground traffic, resulting in road delays, exhaust emissions and other problems that need to be solved urgently. The large increase in the number of cars caused by the shift from other modes of transportation to cars means that the road network needs to be used more frequently, resulting in more and more serious congestion. In view of the increasingly severe road traffic situation, adding more sections in the traffic network will only increase the total road congestion but not reduce it. In congested traffic network restricted under the condition of urban development, the attention from the newly built road network planning to the management of the existing road network and rational urban traffic planning is an important aspect of solving traffic congestion, and the key is to accurately grasp the formation mechanism of traffic demand and traffic flow, including the traveler path choice behavior, the formation process of network traffic, the changing of the external conditions, etc.

Many theorists and traffic engineers have devoted a great deal of energy to the study of traffic flow assignment. The shortest path algorithm\cite{1} is the most basic algorithm for traffic assignment. Zhan and Noon \cite{2} recommended several algorithms suitable for actual traffic networks. For the equilibrium assignment method, Beckmann \cite{3} et al. proposed Beckmann mathematical programming model for solving Wardrop first principle\cite{4}. LeBlance \cite{5} et al. used the Frank-Wolfe algorithm to solve Beckmann model. Method of Successive Average (MSA) is a better algorithm for Beckmann model. Its core is STOCH algorithm for solving Logit random network loading model. The commonly used non-equilibrium assignment methods include all or nothing method and incremental assignment method.
Dial algorithm[6] is an effective method of Logit multi-path traffic assignment. In recent years, more and more studies and applications have been made on sensitivity analysis of traffic assignment. Dafermos and Nagurney [7] discussed the influence on the equilibrium flow when the parameters in the existing functions change. Tobin[8] systematically analyzed and studied the sensitivity analysis method of variational inequality, put forward the basic definitions, proved the relevant theorems, and drew effective conclusions.

In this paper, traffic network assignment algorithms (all or nothing method, Dial algorithm and MSA algorithm) are used to study the relationship between OD demand and section flow, and obtain the changes of the relationship when external factors change. According to the correlation analysis, the mutual feedback law between OD demand and section flow was obtained through sensitivity analysis, and a series of statistical laws under different circumstances were summarized.

2. Analysis of Urban Traffic Network

The change of scale and shape of urban traffic network will bring significant influence on traffic assignment. Before sensitivity analysis is applied to study the feedback law between traffic flow and OD demand, characteristics, components and relationship of each part of traffic network should be analyzed.

2.1. Transportation System Network Representation

The actual urban traffic network not only has a large number of starting and ending nodes, but also has multiple paths for travelers to choose between each pair of starting and ending nodes. Moreover, the larger the network scale is, the more alternative paths there are between nodes. In this paper, the traffic network is represented by a topology, that is, a set of nodes and arcs. Nodes in a traffic network represent intersections, stations, and regional centers, while arcs represent roads and modes of transportation between connecting nodes.

The Figure 1 shows a simple traffic network with one starting node A, two ending nodes B and C, four nodes and five sections. In a traffic network, a path represents a series of nodes from start to end and a collection of sections connecting these nodes.

![Figure 1. An example of a traffic network](image)

![Figure 1. An example of a traffic network](image)

2.2. Three ways from A to B

![Figure 2. Three ways from A to B](image)
In this traffic network, there are three paths from A to B, namely A-1-2-3-B, A-1-4-3-B, and A-1-2-4-3-B, and two paths from A to C, namely A-1-2-4-C and A-1-4-C.

2.2. Analysis of relationship between traffic network elements

In this paper, the relation between different elements in traffic network is represented quantitatively by using the incidence matrix.

Node and node. Nodes are connected with each other through sections. A traffic network has \( n \) nodes, and its incidence matrix is a matrix with \( n \) rows and \( n \) columns.

\[
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\]  

(1)

\[a_{ij} = \begin{cases} 
1, & \text{if there is a connection between } i \text{ and } j \\
0, & \text{if not}
\end{cases}
\]  

(2)

Section and path. For a traffic network with \( m \) sections, if there are \( p \) feasible paths, then the incidence matrix between sections and paths is as follows.

\[
\Delta = \begin{bmatrix}
\delta_{11} & \delta_{12} & \cdots & \delta_{1p} \\
\delta_{21} & \delta_{22} & \cdots & \delta_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
\delta_{m1} & \delta_{m2} & \cdots & \delta_{mp}
\end{bmatrix}
\]  

(3)

\[
\delta_{u,k} = \begin{cases} 
1, & \text{if section } A \text{ is on path } K \text{ from } R \text{ to } S \\
0, & \text{if not}
\end{cases}
\]  

(4)

OD pair and path. For a traffic network with \( w \) OD pairs and \( p \) paths, the incidence matrix between OD pairs and paths is as follows.

\[
B = \begin{bmatrix}
v_{11} & v_{12} & \cdots & v_{1p} \\
v_{21} & v_{22} & \cdots & v_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
v_{w1} & v_{w2} & \cdots & v_{wp}
\end{bmatrix}
\]  

(5)

\[v_{k} = \begin{cases} 
1, & \text{if path } K \text{ connects } R \text{ and } S \\
0, & \text{if not}
\end{cases}
\]  

(6)
3. Traffic Assignment Method and Resistance Function

In this paper, three traffic network assignment algorithms (all or nothing method, Dial algorithm and MSA algorithm) are adopted, and the mutual feedback law between OD demand and traffic flow is obtained by changing the external factors of traffic network.

3.1. All or Nothing

The all or nothing assignment method is also known as the shortest path assignment method. This method is the simplest flow assignment method. It does not consider that road resistance is affected by traffic load, that is, it assumes that road resistance remains unchanged. Specific steps are as follows.

Step 1: Initialize, set all traffic in the road network to 0, and find out the non-flow resistance of each section.

Step 2: Calculate the shortest path between each start and end node in the network.

Step 3: Allocate the flow between each end node to the corresponding shortest path.

3.2. Dial Algorithm

Dial algorithm is a multi-path assignment method that can effectively implement the Logit model on the road network, but it does not need to consider all selection probabilities and traffic flow of OD pairs. Instead of deciding which path to choose at the starting node, the Dial algorithm recognizes that the traveler makes a choice at each node along the way to determine the path choice. When a traveler selects a section at a node, he does not take all sections into account, but only those sections where the upstream end is closer to the starting node than the downstream end, and the downstream end is closer to the end node than the upstream end. Specific steps are as follows.

Step 1: Initialize and determine the effective section. Calculate the minimum path resistance from the starting node r to all nodes, denoted as \( r(i) \); Calculate the minimum path resistance from all nodes to terminal s, denoted as \( s(i) \); \( Q_i \) is defined as the set of the end nodes of a section whose starting node is i; \( D_i \) is defined as the set of the start nodes of a section whose end node is i. \( t(i, j) \) is defined as resistance between i and j. For each section \( (i, j) \), the section likelihood value \( L(i, j) \) is calculated according to the following formula (the parameter b is assumed to be 1):

\[
L(i, j) = \begin{cases} 
  e^{b(t(i, j) - t(i, j))} & \text{if } r(i) < r(j) \text{ and } s(i) > s(j) \\
  0 & \text{otherwise} 
\end{cases} \tag{7}
\]

Step 2: Calculate the section weight \( W(i, j) \) through the likelihood value \( L(i, j) \) in the ascending order from the starting node. When i = s, the calculation of the weight value stops, and the end node is reached.

\[
W(i, j) = \begin{cases} 
  L(i, j) & \text{if } i = r \\
  L(i, j) \times \sum_{m \in D} W(m, j) & \text{if not} 
\end{cases} \tag{8}
\]

Step 3: Calculate the traffic flow of each node from the end node according to the ascending order of \( s(i) \).

\[
x(i, j) = \begin{cases} 
  q_e \frac{W(i, j)}{\sum_{m \in D} W(m, j)} & \text{if } j = s \\
  \sum_{m \in O} x(j, m) \frac{W(i, j)}{\sum_{m \in D} W(m, j)} & \text{if not} 
\end{cases} \tag{9}
\]
\( q_{rs} \) is the total OD demand from starting node r to ending node s. When j = r, that is, when the starting node is reached, the entire traffic assignment ends.

### 3.3. MSA

By controlling the number of cycles or iterative precision, MSA (Method of Successive Average) can make assignment results close to equilibrium. It can solve Beckmann model, and widely applied in traffic assignment and optimization.

**Step 1** Initialization. Make all traffic flow to 0, and all or nothing method is used to load the OD demand on traffic network, and get section traffic flow \( x_a^0 \) (a denote each section), and set the number of iterations \( n = 1 \).

**Step 2** Calculate the cost of the road. \( t_a^{(n)} = t_a \left( x_a^{(n)} \right) \).

**Step 3** Determine the feasible search direction. According to the new section cost \( t_a^{(n)} \), all or nothing method is used to load the OD demand again to get section traffic flow \( y_a^{(n)} \).

**Step 4** Update section flow. \( x_a^{(n+1)} = x_a^{(n)} + \left( y_a^{(n)} - x_a^{(n)} \right) / n \).

**Step 5** Convergence test. If meet the following convergence precision, the algorithm is terminated, and the optimal solution is \( x_a^{(e+1)} \). Otherwise, set \( n = n+1 \) and go to step 2.

\[
\sqrt{\frac{\sum_a \left( x_a^{(n+1)} - x_a^{(n)} \right)^2}{\sum_a x_a^{(n)}}} \leq \varepsilon
\]  

### 3.4. Resistance Function

In this paper, the road resistance varies with the change of section flow. The road resistance function uses the BPR function (q denotes section flow, C denotes section capacity, and \( t_0 \) denotes non-flow resistance), with the recommended value \( \alpha = 0.15 \) and \( \beta = 4 \).

\[
t = t_0 \times \left[ 1 + 0.15 \left( \frac{q}{C} \right) \right]^{\beta}
\]  

### 4. Analysis of Example

In this paper, all or nothing assignment method, Dial algorithm and MSA algorithm are applied for traffic assignment on a nine-node traffic network, with the starting node 1 and the end node 9.
TABLE I. Basic traffic network data

| Section | Resistance | Capacity | Section | Resistance | Capacity |
|---------|------------|----------|---------|------------|----------|
| (1,2)   | 6.66       | 1000     | (5,6)   | 5.64       | 1000     |
| (2,3)   | 7.68       | 500      | (4,7)   | 7.98       | 800      |
| (1,4)   | 6          | 1500     | (5,8)   | 6.66       | 500      |
| (2,5)   | 6.36       | 500      | (6,9)   | 8.64       | 500      |
| (3,6)   | 5.34       | 500      | (7,8)   | 2.34       | 800      |
| (4,5)   | 4.68       | 1000     | (8,9)   | 6          | 800      |

TABLE II. Resistance of each section

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|---|---|---|---|---|---|---|---|---|
| 1    | ∞ | 6.66 | ∞ | 6 | ∞ | ∞ | ∞ | ∞ | ∞ |
| 2    | ∞ | 0 | 7.68 | ∞ | 6.36 | ∞ | ∞ | ∞ | ∞ |
| 3    | ∞ | ∞ | 0 | ∞ | ∞ | 5.34 | ∞ | ∞ | ∞ |
| 4    | ∞ | ∞ | ∞ | ∞ | 0 | 4.68 | ∞ | 7.98 | ∞ | ∞ |
| 5    | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | 5.64 | ∞ | 6.66 | ∞ |
| 6    | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | ∞ | ∞ | 8.64 |
| 7    | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | 0 | 2.34 | ∞ |
| 8    | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | 6 |
| 9    | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ |

TABLE III. Capacity of each section

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|---|---|---|---|---|---|---|---|---|
| 1    | ∞ | 1000 | 0 | 1500 | 0 | 0 | 0 | 0 | 0 |
| 2    | 0 | ∞ | 500 | 0 | 500 | 0 | 0 | 0 | 0 |
| 3    | 0 | 0 | ∞ | 0 | 0 | 0 | 500 | 0 | 0 |
| 4    | 0 | 0 | ∞ | 1000 | 0 | 0 | 800 | 0 | 0 |
| 5    | 0 | 0 | 0 | ∞ | 0 | 0 | 1000 | 0 | 500 |
| 6    | 0 | 0 | 0 | 0 | ∞ | 0 | 0 | 0 | 500 |
| 7    | 0 | 0 | 0 | 0 | 0 | 0 | ∞ | 800 | 0 |
| 8    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ∞ | 800 |
| 9    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ∞ |

Based on the basic data of the example, all or nothing method, Dial algorithm and MSA algorithm are applied to carry out traffic network assignment, and total road resistance, average road resistance and load degree are used to evaluate traffic network congestion status.

Total road resistance:

\[ T = \sum_{a} t_{a} \cdot x_{a} \]  \hspace{1cm} (12)

Average road resistance:

\[ \bar{T} = T / q \]  \hspace{1cm} (13)

Load degree:

\[ vc = Max \left( \frac{x_{a}}{C_{a}} \right) \]  \hspace{1cm} (14)
In this paper, the change of traffic network evaluation index is obtained by changing the input OD demand to carry out the correlation analysis. Taking all or nothing algorithm as an example, the relationship between OD demand and evaluation indexes is shown in the figure below.

![Figure 5. The relationship between OD demand and total road resistance](image1)

![Figure 6. The relationship between OD demand and average road resistance](image2)

The trend of total road resistance and average road resistance change with OD demand is roughly the same. The curve rises slowly at first and then quickly at a critical point, and the slope is increasing, too.

![Figure 7. The relationship between OD demand and load degree](image3)

Load degree is almost directly proportional to OD demand, and load degree goes up as OD demand goes up.
By changing other external conditions (remove nodes, change OD pairs, remove sections), the possible change of total road resistance, average road resistance and load degree under different conditions are as follows. (↑ denotes rise, ↓ denotes down, - denotes unchanged)

### TABLE IV. Results of sensitivity analysis

| Sensitivity analysis | Total road resistance | Average road resistance | Load degree |
|----------------------|-----------------------|-------------------------|-------------|
| **All or nothing**   |                       |                         |             |
| Remove nodes         | ↑,↓,¬                | ↑,↓,¬                   | ↑,¬         |
| Change OD pairs      | ↑,↓,¬                | ↑,↓,¬                   | ↑,¬         |
| Remove sections      | -                    | -                       | -           |
| **Dial**             |                       |                         |             |
| Remove nodes         | ↑,¬                  | ↑,¬                     | ↑,¬         |
| Change OD pairs      | ↑,¬                  | ↑,¬                     | ↑,¬         |
| Remove sections      | -                    | -                       | -           |
| **MSA**              |                       |                         |             |
| Remove nodes         | ↓                    | ↓                       | ↓,¬         |
| Remove sections      | -                    | -                       | -           |

By analyzing the figures and tables, we get the following results:

1. Correlation analysis of OD demand and various parameters: In the all or nothing method and Dial algorithm, when the demand increases from 0, the total road resistance increases and the slope increases continuously, while the average road resistance first remains unchanged. When the demand reaches a certain value, the average road resistance also starts to increase and the slope increases continuously; In the MSA algorithm, the average and total road resistance increase with the increase of OD demand, and the relationship is approximately linear. In the three methods, the load degree increases linearly with the increase of OD demand, and the slope almost stays the same.

2. Sensitivity analysis of the assignment results: Under different conditions, the relationship between average road resistance, total road resistance and OD demand changes in line with different conditions.

3. Correlation and sensitivity comparison of different assignment methods: When OD demand is small, the changes of total road resistance, average road resistance and OD demand changes in line with different conditions.

5. Summary

In this paper, three traffic assignment methods are employed to analyze the correlation between OD demand and traffic flow in urban traffic network. It can be seen from the results that the degree of influence of different assignment methods on the changes of external conditions depends on OD demand. When OD demand is small, the results of the three methods are less affected by external conditions. When OD demand is large, the sensitivity of different traffic assignment methods depends on the specific conditions of the changes. Removing the nodes and sections, or changing OD pairs can reduce the traffic flows. The results can be used to develop traffic manage strategies to alleviate traffic congestion.
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