Evolution and Merging of Binaries with Compact Objects

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Abstract

In the light of recent observations in which short $\gamma$-ray bursts are interpreted as arising from black-hole (BH), neutron-star (NS) or NS-NS mergings we would like to review our research on the evolution of compact binaries, especially those containing NS's. These were carried out with predictions for LIGO in mind, but are directly applicable to short $\gamma$-ray bursts in the interpretation above.

Most important in our review is that we show that the standard scenario for evolving NS-NS binaries always ends up with a low-mass BH (LMBH), NS binary. Bethe and Brown (1998) showed that this fate could be avoided if the two giants in the progenitor binary burned He at the same time, and that in this way the binary could avoid the common envelope evolution of the NS with red giant companion which sends the first born NS into a BH in the standard scenario. The burning of He at the same time requires, for the more massive giants such as the progenitors of the Hulse-Taylor binary NS that the two giants be within 4\% of each other in ZAMS mass. Applying this criterion to all binaries results in a factor $\sim 5$ of LMBH-NS binaries as compared with NS-NS binaries.

Although this factor is substantially less than the originally claimed factor of 20 which Bethe and Brown (1998) estimated, largely because a careful evolution has been carried through here, our factor 5 is augmented by a factor of $\sim 8$ arising from the higher rate of star formation in the earlier Galaxy from which the BH-NS binaries came from. Furthermore, here we calculate the mergers for short-hard gamma-ray bursts, whereas Bethe and Brown’s factor 20 included a factor of 2 for the higher chirp masses in a BH-NS binary as compared with NS-NS one. In short, we end up with an estimate of factor $\sim 40$ over that calculated with NS-NS binary mergers in our Galaxy alone. Our total rate is estimated to be about one merging of compact objects per year.

Our scenario of NS-NS binaries as having been preceded by a double He-star
binary is collecting observational support in terms of the nearly equal NS masses within a given close binary.

We review our work on population synthesis of compact binaries, pointing out that it is in excellent agreement with the much more detailed synthesis carried out by Portegies Zwart. This is currently of interest because the recent discovery of the double pulsar has substantially increased the number of binary NS’s that will merge gravitationally, giving signals to LIGO. This discovery brings in the low ZAMS mass main sequence progenitors that can evolve into a NS binary, adding importantly to the “visible” binaries that can merge. However it does not affect the factor $\gtrsim 40$ increase, mostly from the much greater number of LMBH-NS binaries, which have only a small probability of being observed before they merge.

We develop the phenomenology which suggests that NS’s evolve from ZAMS mass $\sim 10 – 18M_\odot$ star, LMBH’s from $18 – 20M_\odot$, and high-mass BH’s from $20 – 30M_\odot$. These brackets follow from Woosley’s $\text{^{12}C(\alpha, \gamma)^{16}O}$ rate of 170 MeV barns at 300 keV.

We discuss the observed violation of our previous maximum NS mass $M_{\text{max}}^{\text{NS}} = 1.5M_\odot$, raising our $M_{\text{max}}^{\text{NS}}$ to $1.7M_\odot$ and comment on how our scenario would change if the maximum NS mass is greater than $1.7M_\odot$.

1 Introduction

It is now 27 years since Bethe et al. 1979 (denoted as BBAL) was published. At that time we thought the supernova problem to be solved. Although a tremendous amount of work, much of it numerical, has gone on and even more is going on now, the better the physics input, the further the explosion is from success. However, see the paper by Adam Burrows in this volume.

This sorry situation has led to all sorts of opinions about the nature and evolution of compact objects, the authors feeling free to extrapolate in all directions, since the basic mechanism for producing them does not work. It is not generally known, or if it is known, not generally believed that there is considerable phenomenology developed which connects different phenomena. This was developed chiefly by Woosley and his students in the study of numerical NS formation. On the evolutionary side and the connection of various types of compact stars, much of it was developed by van den Heuvel and his students.

We used their techniques to make many connections between compact stars in the “Formation and Evolution of Black Holes in the Galaxy” (Bethe, Brown, & Lee 2003). In this paper we wish to expose in a simple way and summarize

* Deceased (July 2, 1906 – March 6, 2005).
some of the connections. At this stage our connections mostly have to be considered in a pragmatic sense; they are as good as they work. We hope that we show that they work well, in that one can understand a lot through them.

We adopt the method of population synthesis in evolving binaries with compact companions (Bethe and Brown 1998). We update this work in terms of the much more extensive and more accurate work of Belczynski et al. (denoted as BKB) (2002). These authors performed detailed parameter studies. Many more investigators have tried to estimate the number of binaries and their mergers from those we observe, and then extrapolating to the entire Galaxy and then from our Galaxy to many other galaxies. There are two main problems with this latter method. (i) It is not easy to see systems that emit radio emission only weakly; large corrections have to be made for those we don’t see. This difficulty will become clear with our discussion in Sec. 3 of the newly discovered double pulsar which increases the number of observable gravitational mergers estimated from observed systems by at least an order of magnitude (Kalogera et al. 2004). (ii) Mergers of BH-NS binaries are much more probable than mergers of binary NS’s. Indeed, the signal from the former will tend to be greater because of the larger chirp mass implied by the BH mass being greater than the NS mass. Yet there is of yet little probability of seeing the BH-NS binaries, the number of which we estimate to be 5 times greater than binary NSs, because the latter are observable for ~ 100 times longer than the former, due to the fact that the magnetic field of the first born NS (which turns into a BH in the BH-NS binaries) in a double NS binary gets recycled, by mass accreted from its companion, which brings its magnetic field down a factor ~ 100, and, as we outline, increases the time it can be observed by about the same factor (See our later discussion).

In population synthesis one estimates the frequency of supernova explosions in our Galaxy from those in similar spiral galaxies. (Many in our Galaxy are thought to be obscured by the milky way.) Then from an estimate of binarity, say 50%, one has the number of binaries in which both stars are massive enough to go supernova. In fact, the calculations of Bethe and Brown (1998) proceeded in parallel with those of Portegies Zwart and Yungelson (1998). The latter assumed a Galactic supernova rate of 0.015 yr$^{-1}$, Bethe and Brown (1998), 0.0225 yr$^{-1}$.

## 2 Connection of Fe core and compact star masses

Most troublesome in the study of compact stars is the lack of connection between the Fe core, which can be and has been calculated, most recently by Alex Heger (Brown et al. 2001a) with Woosley’s Kepler, and the mass of the compact object. Woosley chooses the outer edge of the Fe core to be at the
location of a large discontinuous change in $Y_e$ which marked the outer extent of the last stage of convective silicon shell burning.

Bethe and Pizzochero (1990) used for SN 1987A a schematic but realistic treatment of the radiative transfer problem, which allowed them to follow the position in mass of the photosphere as a function of time. They showed that the observations determine uniquely the kinetic energy of the envelope once its mass is known. They obtained a kinetic energy of the ejecta $\geq 1$ foe ($= 10^{51}$ erg), the energy scaling linearly with $M_{\text{env}}$. The main point is that this was done without any input from the numerical models used to describe 1987A. From the envelope masses considered, the range of energies was $\sim 1 - 1.4$ foe.

Using the fact that following the supernova explosion as the shock moves outwards the pressure is mainly in radiation, Bethe and Brown (1995) showed from the known value of $\sim 0.075M_\odot$ of $^{56}\text{Ni}$ production in 1987A that an upper limit on the gravitational mass of $\sim 1.56M_\odot$ could be obtained for the progenitor. The main point was that the matter is very dilute in the bifurcation region so that the amount of fallback depends only weakly on the precise separation distance chosen, also that the amount of fallback is roughly equal in magnitude to the binding energy of the compact object. Thus, the Fe core mass is a good estimate of the mass of the compact object. (See also Table 3 of Brown, Weingartner & Wijers (1996) where the amounts of fallback material from distances of 3500 and 4500 km determined by Woosley are plotted.)

Our conclusion is that we can use calculated Fe core masses as an estimate for the masses of the compact cores which will result.

Woosley outlined in several publications the important role of the magnitude of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction in determining the ZAMS (zero age main sequence) mass at which stars would go into BHs. Whereas $^{12}\text{C}$ is formed essentially by the triple $\alpha$-reaction which goes as the square of the density, the reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ goes with the density, being a binary reaction. With increasing ZAMS mass, the central density of stars decreases, the mass going roughly as the square root of the radius. Thus, there comes a ZAMS mass at which the carbon is removed by the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction as fast as it is formed. At that mass, there is essentially no $^{12}\text{C}$ to be burned.

Now as long as $^{12}\text{C}$ has to be burned, it does so at a temperature of $\sim 80$ keV, $\sim 4$ times greater than that at which the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ removes carbon. In burning at such a high temperature a lot of entropy is carried away, decreasing the entropy substantially.

BBAL (Bethe et al. 1979) showed that the final entropy per nucleon is $\sim 1$ (in units where $k_B = 1$) so that the way in which the higher entropy achieved when convective carbon burning is skipped, is that the Fe core increases substantially...
Fig. 1. Reproduction of Fig. 2 of Brown et al. (2001a). Comparison of the iron core masses resulting from the evolution of “clothed” and “naked” He cores. Filled circles and crosses correspond to the core masses of “clothed” stars at the time of iron core implosion for a finely spaced grid of stellar masses (Heger, Woosley, Martinez-Pinedo, & Langanke 2001). The circular black dots were calculated with the Woosley & Weaver 1995 code, whereas the crosses employ the vastly improved Langanke, Martinez-Pinedo (2000) rates for electron capture and beta decay. Open circles (square) correspond to the naked He stars in case A+AB (B) mass transfer of Fryer et al. (2002), with reduced WR mass loss rate. If the assembled core mass is greater than $M_{PC} = 1.8M_\odot$, where $M_{PC}$ is the proto-compact star mass as defined by Brown & Bethe (1994), there is no stability and no bounce; the core collapses into a high mass BH. $M_{NS} = 1.5M_\odot$ denotes the maximum mass of NS (Brown & Bethe 1994). The mass of the heaviest known well-measured pulsar, PSR B1913+16, is also indicated with dashed horizontal line (Thorsett & Chakrabarty 1999).

in mass. This is just the mass at which stars begin evolving into BHs. A more complete argument of this is given in Brown et al. (2001a).
Alexander Heger, using Woosley and Weaver’s carbon burning rate of 170 keV barns, and the best current physics, reevolved the main sequence stars with the results shown in Fig. 1. The input physics and the results given here were summarized in Heger et al. (2001). The experimental determination of the carbon burning rate is the Stuttgart one (Kunz et al. 2001):

\[ S^{300}_{\text{tot}} = (165 \pm 50) \text{ keV barns}. \]  

(1)

A recent paper summarizing all of the Stuttgart work to date (Hammer et al. 2005), which also consider the data from elastic scattering and from the decay of \(^{16}\text{N}\), adds up to:

\[ S^{300}_{\text{tot}} = (162 \pm 39) \text{ keV barns}. \]  

(2)

The \(^{12}\text{C}(\alpha, \gamma)\) experiment was so time-consuming that it required far more time than the usual 3-5 years generally devoted to an experiment. For the project, the Stuttgart team spent a total of 262 days of beam time, not counting all the days of preparation. In the Stuttgart experiment all up to date technical achievements were combined into a single experiment.

One of the authors of the present paper (G.E.B.) tried a number of times to make a shell-model calculation of this rate. But the unperturbed one-particle, one-hole state and (deformed) three-particle, three hole state mix destructively, giving a small net contribution to the 7.12 MeV \(1^-\) state in \(^{16}\text{O}\), so the matrix element to this state, which then decays to the \(^{16}\text{O}\) ground state by emitting the \(\gamma\)-ray, could not be accurately calculated.

Willy Fowler once said that no nuclear reaction that can be measured in the laboratory should be determined by astronomical observations. However, Woosley’s 170 keV barns, near the central value of the Stuttgart measurements, fits our phenomenology so well, especially the LMBH in SN1987A which had progenitor ZAMS mass of \(18 - 20M_\odot\), combines the laboratory measurement and astronomical phenomenology. To within the accuracy of the former, we believe the question to be settled.

The much used compilation by Schaller et al. (1992) uses an S-factor of \(\sim 100\) keV barns. They bring their central abundance of He down to 0.16 at the end of core He burning only for a \(25M_\odot\) star, below the point for convective He burning, so we believe that they would start evolving high-mass BHs at this mass, \(\sim 5M_\odot\) higher than with the Woosley rate of 170 keV barns.

From Fig. 1 we see that the Fe cores, which we identify with the final compact objects, as outlined in the last section, increase rapidly in mass in the region around \(20M_\odot\) going above our \(1.5M_\odot\) maximum NS mass at \(\sim 18M_\odot\), just the ZAMS mass of 1987A which we believe to have gone into a LMBH. We
reiterate here that we have estimated the compact object mass to be the same as that of the Fe core, the fallback in the latter case compensating for the additional gravitational attraction in the former case. Then at $\sim 20M_\odot$ the Fe cores climb above $1.7 - 1.8M_\odot$ which is our limit for high-mass BHs; i.e., those in which the He envelope is not exploded outwards as in 1987A, but collapses inwards. In Lee, Brown, and Wijers (2002) we find the high-mass BHs in the Galaxy can be made from $20 - 30M_\odot$ stars. Even though the Fe core masses come down somewhat above $23M_\odot$, the envelopes are so massive that they will collapse inwards.

We believe that the key to the different regions in which NS’s, LMBHs (in the very narrow region of $18 - 20M_\odot$) and high-mass BHs can be evolved is the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate of $\sim 170$ keV barns introduced by Woosley. In any case, taking the Fe core masses to be those of the compact object, we do well on the phenomenology of the different regions of ZAMS masses which give NS’s and LMBHs. Whereas a large number of high-mass BHs have been found by now, 17 in the transient sources (Lee, Brown, Wijers 2002) no LMBH in a binary has been identified. But the possible range of $\sim 18 - 20M_\odot$ is very narrow, with only the progenitor of SN 1987A from that range.

3 Evolution of Binary Neutron Stars

In Table 1 is the compilation by Lattimer and Prakash (2004) of the compact objects in binaries.

When we say “evolution” we do not mean a complete one with calculation of kick velocities, etc. in NS formation. Rather, we chiefly discuss the difference in masses of the pulsar and its companion.

Such a situation occurs in the scenario for making binary pulsars, in which the spiral-in of the NS through the companion supergiant expels the envelope, which is hydrodynamically coupled to the drag from the NS, leaving a He star as companion. Hypercritical accretion rates are encountered and Chevalier worked out that for $\dot{M} \gtrsim 10^4 \dot{M}_{\text{Edd}}$ radiation pressure is unable to limit the accretion the photons are simply carried in by the adiabatic inflow. Chevalier estimated that sufficient mass would be accreted by the NS during the common envelope evolution to send it into a BH. Said more simply for such a high $\dot{M}$ the drift (random walk) velocity is inwards onto the NS.

Although not generally accepted by astronomers, partly because it destroyed

\[\text{In fact, this happens at a much lower rate, but the } 10^4 \text{ Eddington limit is needed to build up sufficient density in the accretion shock so that the energy can be carried off by neutrinos.}\]
Table 1
Compilation of the compact objects in binaries by Lattimer and Prakash (2004). References are given in their paper. *Brown et al. (1996) argue that 4U 1700−37, which does not pulse, is a LMBH. †We have added, following the comma, the recent measurement of Van der Meer et al. (2004). ††Results for J0751+1807 is from Nice et al. (2005).

| Object          | Mass ($M_\odot$) | Object          | Mass ($M_\odot$) |
|-----------------|------------------|-----------------|------------------|
|                 |                  |                 |                  |
| **X-ray Binaries** |                  |                 |                  |
| 4U1700−37*      | 2.44$^{+0.27}_{-0.27}$ | Vela X-1        | 1.86$^{+0.16}_{-0.16}$ |
| Cyg X-1         | 1.78$^{+0.23}_{-0.23}$ | 4U1538−52       | 0.96$^{+0.19}_{-0.16}$ |
| SMC X-1†        | 1.17$^{+0.16}_{-0.16}$, 1.05±0.09 | XTE J2123−058  | 1.53$^{+0.30}_{-0.42}$ |
| LMC X-4†        | 1.47$^{+0.22}_{-0.19}$, 1.31±0.14 | Her X-1        | 1.47$^{+0.12}_{-0.18}$ |
| Cen X-3†        | 1.09$^{+0.30}_{-0.26}$, 1.24±0.24 | 2A 1822−371    | > 0.73            |
|                 |                  |                 |                  |
| **Neutron Star - Neutron Star Binaries** |                  |                 |                  |
| 1518+49         | 1.56$^{+0.13}_{-0.44}$ | 1518+49 companion | 1.05$^{+0.45}_{-0.11}$ |
| 1534+12         | 1.3332$^{+0.0010}_{-0.0010}$ | 1534+12 companion | 1.3452$^{+0.0010}_{-0.0010}$ |
| 1913+16         | 1.4408$^{+0.0003}_{-0.0003}$ | 1913+16 companion | 1.3873$^{+0.0003}_{-0.0003}$ |
| 2127+11C        | 1.3491$^{+0.0400}_{-0.0400}$ | 2127+11C companion | 1.3632$^{+0.0400}_{-0.0400}$ |
| J0737−3039A     | 1.3373$^{+0.0050}_{-0.0005}$ | J0737−3039B    | 1.250$^{+0.0050}_{-0.0005}$ |
| J1756−2251      | 1.40$^{+0.02}_{-0.03}$ | J1756−2251 companion | 1.18$^{+0.03}_{-0.02}$ |
|                 |                  |                 |                  |
| **Neutron Star - White Dwarf Binaries** |                  |                 |                  |
| B2303+46        | 1.38$^{+0.06}_{-0.10}$ | J1012+5307      | 1.68$^{+0.22}_{-0.22}$ |
| J1713+0747      | 1.54$^{+0.007}_{-0.008}$ | B1802−07        | 1.26$^{+0.08}_{-0.17}$ |
| B1855+09        | 1.57$^{+0.12}_{-0.11}$ | J0621+1002      | 1.70$^{+0.32}_{-0.29}$ |
| J0751+1807††    | 2.10$^{+0.20}_{-0.20}$ | J0437−4715      | 1.58$^{+0.18}_{-0.18}$ |
| J1141−6545      | 1.30$^{+0.02}_{-0.02}$ | J1045−4509      | < 1.48            |
| J1804−2718      | < 1.70           | J2019+2425      | < 1.51            |
|                 |                  |                 |                  |
| **Neutron Star - Main Sequence Binaries** |                  |                 |                  |
| J0045−7319      | 1.58$^{+0.34}_{-0.34}$ |                 |                  |

the usual scenario for binary pulsar evolution, the trapping of neutrinos and their being carried in by the adiabatic inflow in the collapse of large stars as in BBAL[7] involved the same mechanism, but of course with different parameters. Without the pressure from trapped neutrinos, supernova explosions wouldn’t have any chance of succeeding.
The Brown scenario was worked out in detail by Bethe and Brown (1998) who showed that the first born NS in the conventional scenario would accrete $\sim 1M_\odot$ in the about one year of common envelope evolution, taking it into a BH. A simple analytical treatment was given of the common envelope evolution.

Belczynski et al. (2002) removed the approximation of neglecting the compact object mass made by Bethe and Brown, so that the common envelope problem could be accurately handled by solving a series of partial differential equations. This lowered the Bethe and Brown accretion by about 25%. In the rest of this paper we shall use the Belczynski et al. way.

The Hulse-Taylor pulsar 1913+16 of mass $1.442M_\odot$ with companion mass $1.387M_\odot$ is the most massive of the NS’s in double NS binaries. Burrows and Woosley (1986) evolved it from ZAMS $\sim 20M_\odot$ giants. It can be seen from Fig. 1 that this is just where the Fe core masses increase rapidly with change in mass, possibly giving an explanation of why the difference in pulsar and companion masses is nearly 4%, close to our upper limit for overlapping He burning.

Going down in NS mass we reach 1534+12 and 2127+11C and its companion although the latter is in a globular cluster and often said to have been formed only later by exchange of other stars in binaries. In both of these binaries the two masses are within $\sim 1\%$ of each other. Note that in 1534 the companion mass is greater than that of the pulsar. It can be seen from Fig. 1 that right around ZAMS $15M_\odot$, there are fluctuations where the Fe core mass from the more massive ZAMS mass is lighter than from the less massive one. In any case, we believe that 1534 and 2127 come from ZAMS masses more or less in the middle of our range from $10 - 20M_\odot$. A ZAMS mass $15M_\odot$ corresponds to a He core of $\sim 4M_\odot$, so this is approximately the limit below which the He stars evolve in red giant.

At the bottom of our interval comes the double pulsar J0737A, B and 1756−2251, which we take to have evolved from giants of ZAMS mass $10 - 12M_\odot$. In both of these the pulsars will have accreted some mass in the He-star red giant evolution, so called Case BB mass transfer. As we discuss later, this mass transfer is difficult to calculate quantitatively, because He-star winds are large and unpredictable. We assign an uncertainty of $\lesssim 0.2M_\odot$ to the amount of He accreted in Case BB mass transfer, which we return to later in our discussion.

We believe that we have well-measured binaries at the top and bottom end of our interval ZAMS $10 - 20M_\odot$ and two well-measured binaries 1534 and 2127 in the middle.

Using evolutionary calculations of S.E. Woosley of He stars of mass $< 4M_\odot$ (roughly corresponding to $15M_\odot$ ZAMS) in which wind loss is included, Fryer and Kalogera (1997) show that only special conditions in terms of kick veloc-
ities allow the pulsar in close NS-He binaries to avoid the envelope of these low-mass He stars. In detail, Fryer and Kalogera (1997) found that if they worked backwards from the system of 1913+16 and assume there is no kick, then the preexplosion separation is less than the Roche lobe radius of the helium-star, NS system, leading to a pulsar, helium-star common envelope. Because in the standard scenario (not double helium-star) scenario of binary pulsar evolution, the first mass exchange of the two giant progenitors is usually during the red giant phase (Case B mass transfer) of the more massive progenitor, then the later red giant evolution of the helium star in the helium-star, NS phase, which implies a second common envelope, is called in the literature Case BB mass transfer.

In any case, from Fig. 1 with our approximation of the Fe core mass plus fallback being equal to the NS mass, we see that the NS’s in 1534+12 probably came from the middle of the range $\sim 10 \sim 20M_\odot$.

The small $\sim 1\%$ difference in masses in 1534+12 looks like a striking confirmation of the double He-star scenario. The companion is $\sim 1\%$ more massive than the pulsar. To bring its magnetic field down to $10^{12}$ G we estimate that with the Eddington rate and He burning time possibly as large as double that for the companion in the Hulse-Taylor pulsar, then the pulsar would accrete $\sim 0.03fM_\odot$. We have motivated (Francischelli et al. 2002) $f \sim 0.1$ from the propeller effect, so this need not be much.

We put off a discussion of the evolution of the double pulsar J0737−3039 and of J1756−2251 where the pulsar must have accreted matter during the helium star red giant phase of the companion, Case BB mass transfer, until after we have discussed the 4 more massive binaries.

We now discuss the standard scenario for binary NS formation (van den Heuvel and van Paradijs 1993) and why it does not work. The Bethe and Brown (1998) work was analytical, and the approximation of neglecting the mass of the compact object in comparison with the companion, while in transition from main sequence to He star, mass was made. As noted above, BKB (2002) removed this approximation in their numerical calculations and we now adopt their method.

We follow Pinsonneault and Stanek (2006) in evolving NS binaries, but require the two massive progenitors to be within 4% in mass in order to burn He at the same time, rather than the 5% they use. Using a flat distribution and the fact that the IMF for the second star is not independent of the first star, because it must be of lower mass to evolve later than the first star, one finds that NS binaries should be formed 16% of the time, but 44% of the time if $M_1 = 11M_\odot$. That’s where the twin is most likely formed. We show this in Table 2.
Table 2
Flat Distribution: Proximity probability to evolve NS binaries. $M_1$ is the giant mass, $\Delta M$ is the 4\% mass difference within which the binaries evolve into NS-NS binaries, and $P$ is the probability of having mass difference within 4\%.

| $M_1 [M_\odot]$ | $\Delta M = 0.04$ | $M_1 [M_\odot]$ | $P = \Delta M/(M_1 - 10M_\odot)$ |
|----------------|------------------|------------------|---------------------------------|
| 20             | 0.80             | 0.08             |
| 19             | 0.76             | 0.08             |
| 18             | 0.72             | 0.09             |
| 17             | 0.68             | 0.10             |
| 16             | 0.64             | 0.11             |
| 15             | 0.60             | 0.12             |
| 14             | 0.56             | 0.14             |
| 13             | 0.52             | 0.17             |
| 12             | 0.48             | 0.24             |
| 11             | 0.44             | 0.44             |
| 10             | 0.40             | –                |

Pinsonneault and Stanek (2006) assembled evidence that “Binaries like to be Twins”. They showed that a recently published sample of 21 detached eclipsing binaries in the Small Magellanic Cloud can be evolved in terms of a flat mass function containing 55\% of the systems and a “twins” population with $q > 0.95$ containing the remainder. All of the binaries had orbital period $P < 5$ days, with primary masses $6.9M_\odot < M_1 < 27.3M_\odot$.

Historically large selection effects have been identified (Goldberg et al. 2003; Hogeveen 1992). These will lower the number of twins found by Pinsonneault and Stanek.

The important role of twins is that the two giants are close enough in mass that in Brown’s (1995) scenario they can evolve into NS-NS binaries, whereas if they are further apart in mass they will evolve into a LMBH-NS binary (Chevalier 1993; Bethe and Brown 1998).

Thus the twins may increase the number of NS-NS binaries. We suggest that the resulting number of short hard gamma-ray bursts, which result from the merging of the binaries, which to date are unable to differentiate between the two species, may not be changed much, some of the predicted large excess of LMBH-NS binaries appearing rather as NS-NS binaries. However, because the

\footnotetext{2} Pinsonneault and Stanek used 5\% whereas we prefer 4\% as will be discussed.
latter are so much more easier to observe, the role between what we see and what is present will be tightened.

We point out that Belczyński et al. (2002) in their simulation D2 in which the maximum NS mass is $1.5M_\odot$ and the mass proximity in the progenitor binaries (to evolve NS’s) is taken, like Pinsonneault & Stanek to be 5%, obtain a ratio of 4 for $(\text{BH+NS})/(\text{NS+NS})$ and would obtain the ratio of 5 had they used our 4% proximity in masses.

In their Case D2 BKB (2002) find a total gravitational merging rate of $0.45 \times 10^{-4}\text{ yr}^{-1}$ for the sum of their double NS and BH-NS mergings to compare with the Bethe & Brown $0.70 \times 10^{-4}\text{ yr}^{-1}$ once the Bethe & Brown supernova rate of $0.0225\text{ yr}^{-1}$ is lowered to the BKB $0.0172\text{ yr}^{-1}$.

In short, there is general agreement amongst the authors quoted above, except that it is not clear how many twins will be left once selection effects are taken into account. For simplicity we shall use a total gravitational merging rate in our Galaxy of $10^{-4}\text{ yr}^{-1}$.

Whereas we call the standard scenario of binary NS evolution that of van den Heuvel and van Paradijs (1993), BKB include hypercritical accretion in what they call their standard scenario. We believe that their case D2 with $M_{\text{NS}}^{\text{max}} = 1.5M_\odot$ is strongly favored by the closeness in mass of the double NS binaries.

The Bethe & Brown (1998) work did not cover the Case BB mass transfers in the binaries from the less massive ZAMS masses $\lesssim 15M_\odot$, which we discuss in Sec. 7.

4 Observability Premium

The behavior of the pulsar magnetic field is crucial. Van den Heuvel (1994b) has pointed out that NS’s formed with strong magnetic fields $10^{12} - 5 \times 10^{12}$ G, spin down in a time

$$\tau_{\text{sd}} \sim 5 \times 10^6 \text{ yrs}$$

and then disappear into the graveyard of NS’s. (The pulsation mechanism requires a minimum voltage from the polar cap, which can be obtained from $B_{12}/P^2 \gtrsim 0.2$ with $B_{12} = B/10^{12}$ G and $P$ in seconds.) The relativistic binary pulsar 1913+16 has a weaker field $B \simeq 2.5 \times 10^{10}$ G, and therefore emits less energy in magnetic dipole radiation. Van den Heuvel estimates its spin-down time as $10^8$ yrs. There is thus a premium in observational time for lower
magnetic fields, in that the pulsars can be seen for longer times.

Wettig and Brown (1996) used van den Heuvel’s idea to invent the Observability Premium

$$\Pi = \frac{10^{12} G}{B}$$

(4)

where $B$ is the magnetic field of the pulsar. $\Pi$ gives the time relative to that of a $10^{12}$ G pulsar, that the pulsar can be observed. Taam and van den Heuvel (1986) found empirically that the magnetic field of a pulsar dropped roughly linearly with accreted mass. Thus, the Observability Premium is high, given a large amount of such mass.

Wettig and Brown (1996) brought the Observability Premium $\Pi$ into the weighting in their evolution of binary pulsars, assuming because of the high winds during He burning that accretion occurred only in the NS, He-star stage. Since the maximal accretion of $\dot{M} = 3 \times 10^{-8} M_\odot$ yr$^{-1}$ adds up to make the pulsar observable for a longer time, this gave an explanation of the relatively large number of narrow, short period binary pulsars. As noted earlier, the actual accretion rate may be an order of magnitude smaller because of the propeller effect (Francischelli et al. 2002).

Whereas we have considered LMBH-NS binaries above, the formation of NS binaries at redshift $z \sim 1$ due to the higher star formation rate means that they should play a role increasing Kalogera et al.’s rate. The Hulse-Taylor pulsar and 1534+12 have magnetic fields $B \sim 10^{10}$ G so their observability premium; i.e., the longer time that they can be observed, is $\sim 100$ times that of the Crab pulsar, or $\sim 500$ megayears. In fact, the Hulse-Taylor pulsar is estimated to merge in $\sim 300$ Myr. One would expect the same observability premiums for NS binaries that follow from the $\gtrsim 15 M_\odot$ giant progenitors to be similarly recycled and have similar lifetimes because Wettig and Brown (1996) have pointed out that their recycling occurs mostly in the He star, pulsar stage of their formation, and the observational premium favors the formation of binaries sufficiently close so that the accretion from the He-star winds is at or near the Eddington limit. Thus, by a Gyr the pulsar will have run down, and the binary will be invisible. Because of the larger rate of star formation at redshift $z \sim 1$, there should be $\sim 8$ times more of these than the binaries such as Hulse-Taylor where one still sees the pulsar in operation. We shall develop this point in more detail in the next section.

It is not clear that the matter accreted in the Case BB mass transfer, that in the He red giant stage, as in the double pulsar plays a similar role in bringing the magnetic field of the first born pulsar down, but here $B = 6.3 \times 10^9$ G so that the pulsar would also run down and stop pulsing in Gyr. Thus, we
would expect a factor $\sim 8$ increase in Kalogera et al.’s rate from the double NS binaries born some Gyr ago because of the higher star formation rate.

5 Short Hard Gamma-Ray Bursts

The exciting new development which occurred while writing this review is the observation of the short-hard gamma-ray bursts (SHBs), made possible by the satellites Swift and Hete-2, which are able to detect the $\sim 1$ second bursts and radio their positions to the telescopes which can observe their afterglows. The four afterglows of short-hard gamma-ray bursts and the progenitors inferred for these bursts — as well as for another four SHBs with known or constrained redshift form the basis for the analysis of Nakar et al. (2005) which we follow here. (See the Physics Reports by Nakar in this volume.)

We will discuss these SHBs as chiefly resulting from the LMBH-NS binaries of Bethe and Brown (1998) since we believe that they predominantly result from mergers of these binaries.

There is some evidence of beaming in the SHBs, as in the longer GRBs. Two SHBs (050709 and 050724) have shown a steepening that can be interpreted as a hint of a jet. This interpretation would indicate a beaming factor of $\sim 50$ (Fox et al. 2005). Such a beaming would reduce substantially the average isotropic energy and, therefore, make it more difficult for LIGO to observe.

The Nakar et al. (2005) “best guess” at SHBs is $R_{SHB} \approx 10^5 \text{ Gpc}^{-3} \text{ yr}^{-1}$ with the assumption of beaming factor of 50. The rate of Bethe-Brown mergers in our Galaxy was estimated at $\sim 10^{-4} \text{ yr}^{-1}$. Given $10^5$ galaxies within 200 Mpc, this amounts to $1.25 \times 10^3 \text{ Gpc}^{-3} \text{ yr}^{-1}$, to be increased by factor 8 to $10^4 \text{ Gpc}^{-3} \text{ yr}^{-1}$ by the greater rate of star formation at the time of binary formation (at $z \sim 1$). Note that the Nakar et al. “best guess” includes a factor of 50 for beaming, so there is a factor of 500 between the “best guess” and the Bethe/Brown mergers for LIGO, which would not have a beaming. Without the beaming the factor would be only 10.

As we shall come back to later, the final Nakar et al. estimate for initial LIGO if the merging binary is one of two NS’s is 0.3 mergers per year, just 10 times the central Kalogera et al. (2004) value, or 3 mergers per year in the case the binaries are NS-BH in nature, the 10 times larger value coming from the large chirp mass (from $10M_\odot$ BHs).

We have somewhat different predictions as developed earlier in this paper. Going back to our factor of $\sim 5$ times more LMBH-BH mergings than those of binary NS’s, we would begin with the Kalogera et al. 0.003 binary NS
mergers in our Galaxy and multiply this number by factor 8 for the increased rate of star formation at redshift \( z = 1 \). Our prediction is about 1 merging per year for LIGO I, mostly from BH-NS binaries.

In the Nakar et al. (2005) the SHBs are found to result from binaries with long lifetimes as we discussed in our Sec. 3. They suggest that these binaries are either old, invisible double NS or NS-BH binaries. For the latter case, they chose a BH mass of \( \sim 10M_\odot \), which would be wonderful for LIGO because it would imply a large chirp mass and the gravitational waves from the merger of such a BH-NS binary should be observed rather soon in the LIGO observations. In the Bethe and Brown (1998) scenario the BHs in these binaries would be more like \( \sim 2M_\odot \).

A very important new point of the Nakar et al. work is that the invisible binaries come from a very old population \( \sim 6 \) Gyr old; in other words they were formed when the Universe was only \( \sim \frac{1}{2}t_{\text{Hubble}} \). The NS in a BH-NS binary will not be recycled so that it will be observable for \( \sim 5 \times 10^6 \) yrs (see eq. (3)), or \( \sim 1 \) part in a million of the binary lifetime.

Another important point of Nakar et al. is that if the binaries were born so long ago (We choose redshift \( \sim 1 \)) then the star formation rate was substantially higher then.

Nakar et al. (2005) base their redshift distribution model of star formation history on the Porciani and Madau (2001)

\[
SFR_2(z) \propto \frac{\exp(3.4z)}{\exp(3.4z) + 22} \frac{[\Omega_m(1+z)^3 + \Omega_\Lambda]^{1/2}}{(1+z)^{3/2}} \tag{5}
\]

with \( \Omega_m = 0.3, \Omega_\Lambda = 0.7 \) and \( H_0 = 70 \text{ km}^{-1} \text{ Mpc}^{-1} \) in standard cosmology. We obtain a factor of 8 higher star formation at redshift 1 as compared with \( z = 0 \). Whereas such a large factor should not be applied to the double NS merging rate because these binaries were not formed so long ago, it should be applied to the Bethe and Brown (1998) LMBH-NS binaries.

We, of course, believe that we predicted the short-hard bursts. Our theory would explain the approximate uniformity in them, as arising from compact objects with essentially NS masses, increased \( \sim 0.7M_\odot \) in the LMBHs by accretion.

None the less we believe the Nakar et al. analysis of the SHBs to be very useful, in that they establish without model restrictions that LIGO should observe an order of magnitude mergings for the binary compact objects.
Brown and Bethe (1994) claimed, based on the numerical calculations of Thorsson et al. (1994), that because of kaon condensation which sets in at densities $\rho \sim 3n_0$, where $n_0$ is nuclear matter density, the maximum NS mass is $\sim 1.5M_\odot$. We discussed our determination of the gravitational mass of SN1987A, which we believe went into a BH, in Sec. 2. We found it to be $\lesssim 1.56M_\odot$. The maximum NS mass is decreased by kaon condensation because the electrons, which have Fermi energies change with kaon condensation into a Bose condensate of zero momentum kaons. This substantially decreases the pressure.

There have been many theoretical calculations of kaon condensation in the past 10 years, none of them getting kaon condensation at as low a density as $3n_0$. However, the driving force in kaon condensation is the strangeness chiral symmetry breaking parameter $a_3m_s$ for which we took results for the Thorsson et al. (1994) value of $a_3m_s = -222$ MeV. Lattice gauge calculations have now determined this parameter to be $-231$ MeV to within a quoted accuracy of 3 to 4% (Dong, Lagae, and Liu 1996). None of them used an $a_3m_s$ this large in magnitude.

Recently Brown, Lee, Park and Rho (2006) have greatly simplified the calculation of kaon condensation by calculating about the fixed point, the density for chiral restoration, in the Harada and Yamawaki (2003) Vector Manifestation of the hidden local symmetry theory. The kaon condensation is run completely by the vector meson degrees of freedom as their masses and coupling constants approach zero at the fixed point. We move back to the somewhat lower density for kaon condensation by increasing the Harada and Yamawaki parameter $a$, which is unity at the fixed point, to $a \lesssim 1.3$ obtained by renormalization group analyses. The value of $\Sigma_{KN}$, and the behavior of strange hyperons are irrelevant in the new analysis. The Thorsson et al. (1994) result is again arrived at.

Since Table 1 contains three masses, those of 4U 1700−37, Vela X-1 and J0751+1807 which exceed our $1.5M_\odot$ maximum NS mass, we should comment briefly.

**4U 1700−37**: Although this compact object has the same accretion history as the other high-mass X-ray binaries, it doesn’t pulse like the others. Brown, Weingartner and Wijers [19] evolve the compact object as a LMBH.

**Vela X-1**: J. van Paradijs et al. [77] pointed out that in this binary with floppy B-star companion, the apparent velocity can in some cases increase by up to 30% (from the surface elements of the companion swinging around faster than the center of mass) “thereby increasing the apparent mass of the
compact object by approximately the same amount”. In any case, Barziv et al.[1] from which the Vela X-1 NS mass in our table comes, say “The best value of the mass of Vela X-1 is 1.86\(M_\odot\). Unfortunately, no firm constraints on the equation of state are possible, since systematic deviations in the radial-velocity curve do not allow us to exclude a mass around 1.4\(M_\odot\) as found for the other NS’s.”

\textbf{J0751+1807}: We consider the measurement of a 2.1\(M_\odot\) NS mass in this NS white dwarf binary a serious challenge to our maximum NS mass (Nice et al. 2005). It will be clear in our Section 10 that in the evolution of NS, white dwarf binaries, sufficient mass is furnished during the red giant evolution of the white dwarf progenitor, often in conservative mass transfer so that if accepted by the NS, then most of them would have masses in the vicinity of the quoted mass in J0751+1807 or higher, as found by Tauris and Savonije (1999). These authors did not introduce the propeller effect, whereas Francischelli et al. (2002) found that in evolution of double NS binaries this effect often cuts the accretion down by an order of magnitude.

We have given our reasons earlier \(^3\) that the maximum NS mass cannot be far above 1.5\(M_\odot\).

J0751+1807 has a short orbital period of \(P_b = 6.3\) hours. The short orbital period allows the detection of the effect of gravitational radiation emission. According to general relativity the time rate of change of the orbital period is

\[
\dot{P}_b = -\frac{192}{5} \left( \frac{P_b}{2\pi} \right)^{-5/3} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \\
\times (1 - e^2)^{-7/2} T_5^{5/3} m_{WD} M_{\text{NS}} (m_{WD} + M_{\text{NS}})^{-1/3}.
\]  \(6\)

Since \(m_{WD} \ll M_{\text{NS}}\), the dependence on masses of \(\dot{P}_b\) is proportional to \(m_{WD} M_{\text{NS}}^{2/3}\) or for a given \(\dot{P}_b\), \(M_{\text{NS}} \propto m_{WD}^{-3/2}\). Thus, for \(m_{WD} \sim 0.24M_\odot\), \(M_{\text{NS}} \sim 1.5M_\odot\). The NS mass of Nice et al. (2005) has mass \(2.1^{+0.4}_{-0.5} M_\odot\) at 95% confidence level; i.e., at this level \(M_{\text{NS}}\) could be as low as 1.61\(M_\odot\).

Brown et al. (2006b) have indicated that there is a small correction to Brown et al. (2006a) who calculated the maximum stable NS mass to be 1.5\(M_\odot\) by fluctuating about the fixed point in the Harada-Yamawaki (2003) renormalization group formalism. This arises because the \(K^-\)-mesons in the kaon condensate which causes the NS to collapse into a BH have a fermion substructure; i.e., \(K^- = |\bar{u}s\rangle\) and the \(\bar{u}\) and \(s\) quarks are fermions. Thus, there is a repulsion

\(^3\) Bethe and Brown (1995) estimated the Ni production in 1987A to have come from a NS of maximum mass 1.56\(M_\odot\), whereas these authors believed that the NS had later evolved into a BH.
Calculated accretion onto the pulsar during H and possibly He red giant stage. $M_i$ is the initial pulsar mass, taken to be that of the companion, and therefore a lower limit, $\Delta M$ is the calculated mass accretion onto the first born NS, $M_f$ is the final pulsar mass following accretion, and $\hat{P}$ is the probability of unequal masses of compact objects in NS binaries, therefore, $1 - \hat{P}$ of Table 2. The He core mass of giant star is assumed to be $M_{\text{He}} = 0.08(M_{\text{Giant}}/M_\odot)^{1.45}M_\odot$. The error in the companion masses less than $14M_\odot$ come from our $\sim 0.2M_\odot$ uncertainty in accretion in the He red giant evolution if a NS is to remain.

| Giant Mass [M_\odot] | $M_i$ [M_\odot] | $\Delta M$ [M_\odot] | $M_f$ [M_\odot] | $\hat{P}$ |
|----------------------|-----------------|---------------------|-----------------|---------|
| 20                   | 1.39 (B1913+16) | 0.87                | 2.26            | 0.92    |
| 19                   | 1.38            | 0.84                | 2.22            | 0.92    |
| 18                   | 1.37            | 0.82                | 2.19            | 0.91    |
| 17                   | 1.36            | 0.79                | 2.15            | 0.90    |
| 16                   | 1.35            | 0.76                | 2.11            | 0.89    |
| 15                   | 1.34 (B1534+12) | 0.74                | 2.08            | 0.88    |
| 14                   | 1.33            | 0.71−0.91           | 2.04−2.24       | 0.86    |
| 13                   | 1.31            | 0.67−0.87           | 1.98−2.18       | 0.83    |
| 12                   | 1.28            | 0.63−0.83           | 1.91−2.11       | 0.76    |
| 11                   | 1.25 (J0737−3039B) | 0.60−0.80       | 1.85−2.05       | 0.56    |
| 10                   | 1.18 (J1756−2251) | 0.55−0.75       | 1.73−1.93       |         |

between $K^-$-mesons when brought together from the Pauli exclusion principle. The quarks are, however, not very far from being current quarks, which they become as $n \rightarrow n_{\chi SR}$, the latter being the chiral symmetry restoration density, since the critical density for kaon condensation is

$$n_c \sim \frac{3}{4}n_{\chi SR}.$$

Current quarks are thought to be very small in extent.

Astronomers are not easily convinced by formal arguments, so we take the approach of Lee et al. (2006). Using the extensive and accurate calculation of BKB (2002) of hypercritical accretion, which removed the approximation of neglecting the compact object mass of Bethe and Brown (1998), Lee et al. (2006) calculated a table of masses for the relativistic NS binaries, which we show in Table 3. $M_f$ is the final pulsar mass after having accreted matter from the giant companion, while in common envelope with the latter while it is in red giant stage. The $0.2M_\odot$ uncertainty for the pulsars from giants of masses
< 15M⊙ comes from accretion in the He star red giant stage when additional mass can be transferred.

The higher probability $P$ for the more massive giants such as ZAMS 20M⊙ results because their companion must come from a giant of lower ZAMS mass, as in Table 2. We have given reasons earlier for our placement in the Table 3 of known pulsars B1913+16, etc.

From the Table 3 we see that the overwhelming probability would be for the pulsars in B1913+16 and B1534+12 (and 2127+11C if not evolved from NS exchanges in the globular cluster), J0737−3039B and J1756−2251 to be much more massive than the companion. However, if the maximum NS mass is 1.7M⊙ or less, all of the binaries calculated in the Table 3 with hypercritical accretion will be BH-NS binaries.

At 95% confidence, the errors of $^{+0.4}_{-0.5}$M⊙ are large, although they will be made smaller with longer observing time, and the central value of 2.1M⊙ for J0751+1807 is arrived at through a Bayesian analysis. We feel that it is fair to use the 95% confidence limits, since it sticks up well above the other masses in Table 1.

7 Hypercritical Accretion in Case BB mass transfer

We now consider in more detail the two lowest mass binary NS systems, the double pulsar with mass 1.337M⊙ for J0737−3039A and 1.250M⊙ for J0737−3039B, and J1756−2251 with pulsar mass of 1.40M⊙ and companion mass of 1.18M⊙, which would have come from 10M⊙ or 11M⊙ ZAMS giants. The He stars in the He-star, NS binary which precedes the double NS binary will have been the least massive in the binary NS evolution, so it is reasonable that they evolve into a He red giant (Dewi and van den Heuvel 2004). Dewi and van den Heuvel (2004) did not, however, consider that mass can be accreted onto the NS during the He-star common envelope evolution. Dewi and van den Heuvel inadvertently restricted the directions of the supernova kick imparted to PSR J0737-3039B to the presupernova plane. In the papers of Willems and Kalogera (2004) and Willems et al. (2005) the investigation was significantly extended by incorporating proper motion constraints and the kinematic history of the system in the Galaxy into the analysis.

Hypercritical accretion requires $\dot{M} > 10^4\dot{M}_{\text{Eddington}}$ (Chevalier 1993). In the NS common envelope evolution in hydrogen red giant, $\dot{M}$ is typically $10^8\dot{M}_{\text{Eddington}}$. The typical hydrogen envelope has a mass of $\sim 10M_\odot$. The He star red giant evolution involves only an $\sim 1M_\odot$ evolved envelope, which is sufficient for hypercritical accretion. Using the BKB (2002) equations, which are now ab-
olutely necessary because the Bethe and Brown approximation of neglecting the mass of the compact object would now mean neglecting the pulsar mass as compared with an $\sim 2M_\odot$ He star mass, we find the following results:

For an initial $M_{\text{He},i} = 2.5M_\odot$, final $M_{\text{He},f} = 1.5M_\odot$ and $M_{\text{NS}} = 1.25M_\odot$, the latter being the mass of the first-born pulsar in the double pulsar system, we find that the pulsar accretes $0.2M_\odot$ and the orbit tightens by a factor of 6.7. For $M_{\text{He},i} = 2.5M_\odot$, $M_{\text{He},f} = 1.25M_\odot$, the pulsar accretes $0.19M_\odot$ with tightening by factor 7.4. Since we want a first-born pulsar mass of $1.25M_\odot$, $2.5M_\odot < M_{\text{He},i} < 2.5M_\odot$ seems appropriate.\footnote{\textit{M}_{\text{He},f} corrected downward by the greater gravitational binding energy when it evolves into a NS.} We are unable to lower the accreted mass much below $0.2M_\odot$, which is twice too much, but would fit the difference in mass between pulsar and companion in J1756$-$2251. It does not seem easy to lower the accretion by kick velocities, as Fryer & Kalogera did for 1913+16 and 1534+12 because the double NS eccentricity is only 0.09, the smallest in all double NS’s. It is possible that there has been substantial wind loss, which would diminish the envelope mass and lower the accreted mass. We would have expected the latter to be larger than it is. The point we wish to make is that the amount of mass accreted by the low-mass pulsars in the He envelope evolution of their companions is likely to be $\ll 0.2M_\odot$. This is not enough to send the pulsar into a BH.

We believe that we can explain why the pulsar, the first born NS in the double pulsar and in J1756$-$2251 are substantially ($\ll 0.2M_\odot$) more massive than their companion stars. Certainly they have not gone through the van den Heuvel and van Paradijs (1993) scenario, or they would have accreted $\sim 0.5M_\odot$ from their hydrogen giant companion.

But of course there must have been a double NS formation scenario because otherwise the first-born NS would have accreted $\sim 0.5M_\odot$ from the hydrogen envelope of its companion and then later another $\ll 0.2M_\odot$ from its He companion as the latter evolved. Thus, put together, the amount of matter accreted by the first-born NS would have been comparable to that accreted from the hydrogen red giant stage of the companion in the more massive $\gtrsim 1.5M_\odot$ first born NS’s. These would turn into BHs.

However, if the BH is formed in the lower mass stars (ZAMS mass $\ll 15M_\odot$) then when the companion evolves into a He red giant, the BH will be in a common envelope and the binary will tighten just as effectively as if it were a NS. Thus, the double pulsar and J1756$-$2251, which must have formed, at least in our scenario, through the double He star scenario, should have an order of magnitude more BH-NS counterparts, with similar tight orbits.

Thus, the double helium star scenario necessitated if the maximum NS mass is
as small as we say, $1.7M_\odot$, to produce binary NS systems, results in an order of magnitude more BH-NS binaries, which will merge at about the same rate as binary NS’s.

We are, however, not finished yet in increasing the prediction for compact object mergers. To date these are mostly based on the NS binaries observed in the Galaxy, with extrapolation to include the number of similar systems that should exist in the Galaxy.

The observation of the double pulsar increased the estimate by an order of magnitude. The radio signal in the double pulsar is much weaker than that in the Hulse-Taylor 1913+16 and it must be generally true that the radio signals from the pulsars from the lower mass ZAMS stars are generally much weaker than those from the pulsars already discovered in binaries. So we may be missing many of these.

We end this section by referring to yet another channel suggested by Belczynski and Kalogera (2001). They develop the scenario following the double He star evolution. For the lower mass stars there is second stage of common envelope evolution, now in He red giant envelopes, similar to that for the nearly equal mass evolving hydrogen stars. The survival probability of the two (Type Ic SN) is larger, given the tight orbit before the explosions. The end product is a close NS-NS with very short merger time.

Delgado and Thomas (1981) show that case BB mass transfer is similar to the double He star scenario for He stars of mass greater than the Chandrasekhar mass of white dwarfs, in that Case BB mass transfer can only start after the ignition of carbon, in the sense that helium core burning has already begun when the hydrogen red giant phase takes place in Case B mass transfer in the stars massive enough to evolve into NS’s. However, the chance that the two He stars burn carbon at the same time is rather small, the ratio of carbon burning time to He burning time being only $\sim 0.03$ in ZAMS $12M_\odot$ stars (Schaller et al. 1992). We thus believe that only $\sim 1\%$ of the binaries will go through the double common envelope, the others involving Case BB mass transfer.

BKB (2002) and Belczynski & Kalogera (2001) have motivated a large increase in gravitational mergings by taking the effects of He star red giants; i.e. Case BB mass transfers, into account. Our more schematic estimates suggest that overall they were unduly conservative in their estimates.

Be that as it may, we wish to add the order of magnitude more BH-NS binaries which can be inferred from the presently measured binary NS masses. In these the BHs will be more massive than the NS’s because of accretion and their large chirp masses will make them observable to larger distances.
High mass X-ray binaries give the least reliable information about NS masses. In general the giant is quite “floppy” with various pulsational excitations, the center of mass of the system is inside of the binary and the NS heats up the giant by radiation. Over the years the determination of the NS masses have changed substantially, but with two exceptions 4U 1700−37 and Vela X-1, the NS masses are at an average of $\sim 1.4M_\odot$ and are consistent with a limit of $< 1.5M_\odot$.

Brown, Weingartner, and Wijers (1996) considered 4U 1700−37, which does not pulse in X-rays and has a harder spectrum than other X-ray binaries. The accretion history is, however, similar to that of others. The indication is that the compact object is a BH.

Wellstein and Langer (1999) argue that 4U 1700−37, composed of a $30M_\odot$ O star with $2.6^{+2.3}_{-1.4}M_\odot$ compact companion (Heap and Corcoran 1993, Rubin et al. 1996) must have gone through a nonconservative mass transfer in late Case B or Case C. In this way the system loses substantial amounts of mass and angular momentum and thus becomes a short period binary. (1700−37 has a period of 3.412 days.)

In fact, in Brown et al. (2001a) had the first mass transfer from the primary to the secondary been case A, AB or B, the primary of $\sim 30M_\odot$ would have gone into a NS.\(^5\) However, with mass transfer from the evolving secondary, it would have gone into a LMBH.

From an evolutionary point of view 4U 1700−37 is very interesting because it is the only binary with low-mass compact object, albeit BH, for which one can make the argument that it comes from such a massive region of $> 30M_\odot$. (Brown et al. 1996 estimate $40 \pm 10M_\odot$.)

Vela X-1 may well be the worst system in which to measure the mass of a NS.

“Another systematic effect, due to the distortion of the primary may be quite important in the case of X-ray binaries with a small mass ratio\(^6\) as the Vela X-1 system. In such a system the radial velocities of certain individual surface elements of the primary are much greater than the orbital velocity of the center of mass of this star, in the case of synchronous rotation. When the primary

\(^5\) Rate of wind loss during He burning is not sufficiently well known to say that the primary would have gone into a NS. It might very well have immediately gone into a LMBH.

\(^6\) Authors: Vela X-1 is made up of an $18M_\odot$ B-star and NS. The center of mass is well within the giant.
is tidally distorted and has a variation of effective temperature and gravity across its surface, it is by no means clear that the observed radial velocity, which is given by some spectrophotometric average over the surface, can be identified with the motion of the center of mass of the object.” (J. van Paradijs et al. 1977a)

“In a previous paper we presented a numerical study of the effect of the deformation of the primary on its apparent radial velocity (van Paradijs et al. 1977a). The apparent velocity amplitude can in some cases increase by up to 30 %, thereby increasing the apparent mass of the compact object by approximately the same amount.” (J. van Paradijs et al. 1977b)

It is known that the light curve varies substantially from night to night. Indeed, in Barziv et al. (2001) from which the large NS mass is taken, the authors say “The best estimate of the mass of Vela X-1 is $1.86 M_\odot$. Unfortunately, no firm constraints on the equation of state are possible, since systematic deviations in the radial-velocity curve do not allow us to exclude a mass around $1.4 M_\odot$ as found for other neutron stars.”

9 Carbon-Oxygen White-Dwarf, Neutron Star Binary

The high-field eccentric binaries B2303+46, long thought to be a wide NS binary and J1141−65 have not gone through common envelope evolution which would have circularized them and brought their magnetic fields down. The magnetic field of B2303+46 is $7.9 \times 10^{11}$ G, that of J1141−65, $1.3 \times 10^{12}$ G. The relative time that such a binary can be seen, before it goes into the “graveyard” goes inversely with $B$. Thus we have “observability premium”, Eq. (4), equal to essentially unity as an average for the two unrecycled binaries above.

Tauris et al. (2000) proposed five binaries which they evolved through common envelope. Two of these recycled pulsars in relativistic orbits PSR 1157−5112 and J1757−5322 are discussed by Edwards and Bailes (2001). Two others J1435−6100 and J1454−5846 are discussed by Camilo et al. (2001).

The fifth of the systems favored for common envelope evolution by Tauris, van den Heuvel and Savonije (2000) is J1022+1001. This closely resembles PSR J2145−0750, aside from a more massive white dwarf companion, as remarked by van den Heuvel (1994a), who evolved the latter through common envelope when the white dwarf progenitor was on the AGB (Case C mass transfer). Van den Heuvel suggested for J2145−0750 that there was considerable mass loss because of possible instabilities on the AGB caused by the presence of the NS. This is one possibility of saving our general theme; i.e., that most of the NS, carbon-oxygen white dwarf binaries would end up as LMBH carbon-oxygen
Table 4
Inferred magnetic fields $B$ and the observability premium $\Pi$ for recycled pulsars

| Pulsars      | $B$       | $\Pi$ |
|--------------|-----------|-------|
| J2145+0750   | $6 \times 10^8$ G | 1667  |
| J1022+1001   | $8.4 \times 10^8$ G | 1190  |
| J1157−5112   | $< 6.3 \times 10^8$ G | $> 159$ |
| J1453−58     | $6.1 \times 10^9$ G | 164   |
| J1435−60     | $4.7 \times 10^8$ G | 2127  |

white dwarf binaries, although some might be saved with NS’s because of the possible instabilities caused by the NS while the white dwarf progenitor is on the AGB. Van den Heuvel chose $\lambda = 1/2$ for the parameter that characterizes the structure of the hydrogen envelope of the massive star that is removed in common envelope evolution. Dewi & Tauris (2001) have since carried out detailed calculations that in some cases, “particularly on the asymptotic giant branch of lower-mass stars, it is possible that $\lambda > 4$.” This lowers the binding energy of the envelope by a large factor, so that it can be removed in common envelope evolution and still leave a reasonably wide orbit, as remarked by Dewi & Tauris. We believe that this may be the reason that some binaries have survived common envelope evolution.

In four of the six recycled pulsars (assumed to have been evolved through common envelope evolution) the magnetic field have been inferred as in Table 4. The observability premium $\Pi$ is high for most of these pulsars. Since we see two unrecycled pulsars with high magnetic fields $B \sim 10^{12}$ G; therefore, $\Pi \sim 1$, we should see $\sim 20,000$ recycled pulsars.

The above argument does not take into account the greater difficulty of observing pulsars with low magnetic fields $\sim 10^8 - 10^9$ G, which may remove some of the large predicted numbers of recycled pulsars in case they did not go into BHs during common envelope evolution.

On the other hand, the NS in these cases goes through common envelope with a star of main sequence less than $\sim 10 M_\odot$ since it must end up as a white dwarf, albeit a relatively massive carbon-oxygen one. As discussed by van den Heuvel (1994a) only part of the energy to remove the envelope will be connected with the accretion, the rest coming from wind losses, etc. Thus, our observation that most of these binaries must end up as white-dwarf, low-mass X-ray binaries gives credence to essentially all of the NS’s which go through common envelope evolution in the evolution of binary NS’s ending up as LMBHs.

As noted earlier, there are hopes that during our lifetime - at least that of one
or two of the authors - this can be tested, because the Bethe-Brown prediction of the factor 20 greater contribution of NS-LMBH to binary NS mergers should be robust, essentially independent of the number of the latter.

10 White Dwarf-Neutron Star Binaries

This class of 12 (See Table 1) is the most numerous class. They mostly would be expected to come from a NS with main sequence star of mass between \(1M_{\odot}\) and \(2M_{\odot}\), the \(1M_{\odot}\) because they must evolve in a Hubble time. The main sequence star evolves, either transferring matter to the NS or matter is lost by wind, because it ends up as a typically quite low-mass white dwarf.

In Tab. 5 we have collected the helium white dwarf masses we could find. Note that in particular the binaries B2303+46 and J1141−6545, which we discussed in Sec. 9, do not appear in our table. They have quite massive carbon-oxygen white dwarf companions, and were discussed in the last section.

In Tab. 6 we show the statistics of the mass distribution of white dwarfs. We note that all of our tabulated white dwarfs have masses \(\gtrsim 0.35M_{\odot}\), whereas single white dwarf tend to peak up at \(\sim 0.6M_{\odot}\). This indicates to us immediately that the companion NS is strongly influential in increasing the wind loss. (See the suggestion of van den Heuvel about J1245−0750 in the last section that there was considerable mass loss because of possible instabilities on the AGB (asymptotic giant branch) caused by the presence of the NS.)

The white dwarf, NS binaries have been evolved by Tauris and Savonije (1999). For evolution with stable mass transfer, i.e., for main sequence masses less than \(M_{\text{MS}}\), the evolution was basically conservative, matter accreting below or at the Eddington limit, onto the accretion disc of the white dwarf. In the case of the main sequence mass \(M_{\text{MS}} > M_{\text{NS}}\), for \(M_{\text{MS}}\) up to \(2M_{\odot}\), the evolution was still conservative from the standpoint of the accretion disk, but the amount above the Eddington limit for the white dwarf was expelled with the angular velocity of the NS.

The mass distribution of NS’s of Tauris and Savonije do not give NS masses that look anything like those shown in Table 1. Even though Tauris and Savonije began with the somewhat small NS mass of \(1.3M_{\odot}\), they have copious numbers up to \(2M_{\odot}\). Only J0751+1807 in Tab. 1 really comes this high. We believe the reason for their high NS masses is explained in the last sentence of the caption to their Fig. 4: “The post-accretion \(M_{\text{NS}}\) curves (bottom) assume no mass loss from accretion disk instabilities of propeller effects.” As we discussed in Sec. 3, Francischelli et al. (2002) found that the propeller effects decreased the mass accretion from He star wind onto the pulsar by an order
Table 5
White Dwarf Companion masses \((m_2)\) and orbital period \((P)\) in NS, He-White-Dwarf Binaries. Refs; 1) Thorsett & Chakrabarty 1999, 2) Hansen & Phinney 1997, 3) Tauris, van den Heuvel, & Savonije 2000, 4) Lundgren, Zepka, & Cordes 1995, 5) Navarro et al. 1995, 6) Thorsett, Arzoumanian, & Taylor 1993, 7) van Kerkwijk et al. 2000, 8) Lyne et al. 1990, 9) Phinney & Kulkarni 1994.

| Pulsar           | \(m_2\) \((M_\odot)\) | \(P\) (days) | References |
|------------------|------------------------|--------------|------------|
| \(J0034 - 0534\) | 0.15 - 0.32            | 1.589        | 1, 2       |
| \(J0218 + 4232\) | 0.2                    | 2.029        | 1, 5       |
| \(J0751 + 1807\) | 0.15                   | 0.263        | 1, 2, 4    |
| \(J1012 + 5307\) | 0.165 - 0.215          | 0.605        | 1, 2       |
| \(J1045 - 4509\) | < 0.168                | 4.084        | 1          |
| \(J1232 - 6501\) | 0.175                  | 1.863        | 3          |
| \(J1713 + 0747\) | 0.15 - 0.31            | 67.825       | 1, 2       |
| \(B1744 - 24A\) (\(J1748 - 2464\)) | 0.15 | 0.076 | 1, 8 |
| \(B1800 - 27\) (\(J1803 - 2712\)) | 0.17 | 406.781 | 1, 9 |
| \(J1804 - 2718\) | 0.185 - 0.253          | 11.129       | 1          |
| \(B1855 + 09\) (\(J1857 + 0943\)) | 0.19 - 0.26 | 12.327 | 1, 2 |
| \(J2129 - 5721\) | 0.176                  | 6.625        | 1          |
| \(J2317 + 1439\) | 0.21                   | 2.459        | 1, 9       |
| \(J0437 - 4715\) | 0.15 - 0.375           | 5.7          | 1, 2       |
| \(J1455 - 3330\) | 0.305                  | 76.174       | 1          |
| \(B1620 - 26\) (\(J1623 - 2631\)) | 0.3 | 191.443 | 1, 6 |
| \(J1640 + 2224\) | 0.25 - 0.45            | 175.460      | 1, 2       |
| \(J1643 - 1224\) | 0.341                  | 147.017      | 1          |
| \(B1718 - 19\) (\(J1721 - 1936\)) | 0.3 | 0.258 | 1, 7 |
| \(B1802 - 07\) (\(J1804 - 0735\)) | > 0.29 | 2.617 | 1 |
| \(J1904 + 04\) | 0.27                   | 15.75        | 3          |
| \(B1953 + 29\) (\(J1955 + 2908\)) | 0.328 | 117.349 | 1 |
| \(J2019 + 2425\) | 0.264 - 0.354          | 76.512       | 1, 2       |
| \(J2033 + 17\) | 0.290                  | 56.2         | 1          |
| \(J2229 + 2643\) | 0.315                  | 93.015       | 1          |
Table 6
Statistics of the mass distribution of white dwarfs. Logarithmic distribution of the initial NS, main sequence star is assumed. Last column is the number of observed systems summarized in Table 5.

| $m_2$ (M$_\odot$) | $R_g$ (R$_\odot$) | ln($R_u/R_l$) | Observations |
|------------------|------------------|----------------|--------------|
| 0.15 – 0.25      | 1.47 – 10.0      | 1.92           | 10           |
| 0.25 – 0.35      | 10.0 – 42.0      | 1.44           | 12           |
| 0.35 – 0.46      | 42.0 – 128       | 1.11           | 0            |

of magnitude. In other words, the accretion may be scaled down from the $\sim 1M_\odot$ difference between progenitor main sequence and white dwarf masses, to $\sim 0.1M_\odot$ actually accepted, the remainder being lost through the agency of the propeller effect. Of course, because of different angular momenta in the different binaries we can only give order of magnitude estimates.

Since Brown and Bethe (1994) “A scenario for a large number of low-mass black holes in the Galaxy”, at which time NS masses were spread rather widely, we have got used to seeing their masses fall below our projected maximum $1.5M_\odot$. Thus, at that time, the NS in J2019+2425 had an upper limit on its mass of $\sim 1.64M_\odot$ from the white dwarf mass-period relation. Nice et al. (2001) could constrain the inclination angle of the binary to $i < 72^\circ$ from the proper motion of the binary, with a median likelihood value of $63^\circ$. A similar limit on inclination angle arose from the lack of a detestable Shapiro delay signal. The NS mass was determined to be at most $1.51M_\odot$.

11 Summary Conclusion

Our Selected Papers with Commentary is entitled “Formation and Evolution of Black Holes in the Galaxy”. We were shifted from NSs to BHs by SN 1987A which showed that a relatively low-mass compact core could evolve into a LMBH.

Most important is our double helium star scenario for the evolution of binary NS’s. The motivation of this began with Chevalier (1993) who estimated that a NS would accrete $\sim 1M_\odot$ in common envelope evolution. We confirmed and made more quantitative his estimate. With addition of 0.75$M_\odot$, the NS will certainly go into a BH. The double helium star scenario avoids the NS having to go through common envelope evolution.

The near equality of the masses of pulsar and companion in the binary NS’s is observational support for our double NS scenario. Where the masses of pulsar and companion are substantially different, in the Hulse-Taylor binary, this
difference tells us the range of ZAMS masses that the binary came from. In this case, from the most massive possible range $\sim 20M_\odot$ where the Fe core masses change most rapidly with main sequence mass, because the companion mass is almost 4% lower than that of the pulsar.

The importance of our double He star scenario is that one must then look for the fate of the NS’s which do go through common envelope evolution in the envelope of the evolving giant. Bethe and Brown (1998) show that these do, indeed, produce LMBH fresh NS binaries. There are a factor of 5 more of these than double NS binaries and because of the higher mass of the BH, they give a larger factor in expected mergings.

While the recently discovered double pulsar is very interesting (and was very improbable to be observed) it simply brings the binary NS contribution to LIGO up to snuff, and does not change our additional factor of 5, although it does greatly increase the merging rate of the binaries one sees.

We believe that the many recently observed short hard gamma-ray bursts give credence to the large number of LMBH-NS binaries we predicted and even our factor 40, which includes the higher star formation rate in the early Universe, is easily subsumed in the number of SHBs. However, the gravitational waves from these mergings may still not be sufficient for LIGO observations in the next few years. What LIGO needs is a merger with large chirp mass, as predicted by Portegies Zwart and McMillan (2000) which overwhelms the background. Of course in time, possibly a few years, the LIGO sensitivities should observe the merger of the lower chirp mass binaries discussed here.

12 Discussion

Kip Thorne asked Hans Bethe and Gerry Brown to work out the mergings of NS-BH binaries while they were in Caltech in 1996. This was a new activity for Hans, who was 90 at the time, and he attacked it with gusto. From the paper of Brown (1995) it was clear that there were an order of magnitude more of these than of binary NS’s because the standard scenario for making the latter always ended up with the first NS going into a BH during its common envelope with the companion while in red giant stage.

The authors returned to this problem in 2003, after publishing their joint works (Bethe, Brown and Lee 2003) and Hans was engaged with this problem right until his death. In fact, he had a discussion of it on the telephone with Gerry Brown the morning of the day of his death.

Crucial to our work on evolving the binary objects was Hans’ analytic common
envelope evolution. This is reproduced in his obituary by Brown (2005). It was carried through with Kepler’s and Newton’s Laws using elementary calculus.

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