Baryon Magnetic Moments
in the $1/N_c$ Expansion

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Abstract

Relations among the baryon magnetic and transition magnetic moments are derived in the $1/N_c$ expansion. Relations which hold to all orders in $SU(3)$ breaking and to leading and first subleading orders in the $1/N_c$ expansion are obtained. Additional relations are found which are valid up to $SU(3)$ breaking at first subleading order in the $1/N_c$ expansion. The experimental accuracy of these relations fits the pattern predicted by the $1/N_c$ expansion. The predictions of the $1/N_c$ expansion are compared in detail with those of the non-relativistic quark model. The $1/N_c$ expansion explains why certain quark model relations work to greater accuracy than others.
The $1/N_c$ expansion of 't Hooft is one of the few calculational techniques for obtaining rigorous nonperturbative information about hadrons in QCD. The implications of the $1/N_c$ expansion of QCD for baryons were initially worked out by Witten. Recently, progress has been made in obtaining quantitative results for baryons using the $1/N_c$ expansion. The baryon sector of QCD possesses has a contracted spin-flavor symmetry in the large $N_c$ limit. Deviations from exact spin-flavor symmetry can be studied systematically in the $1/N_c$ expansion by computing $1/N_c$ corrections to the large $N_c$ limit. It has been proven that the first non-trivial correction to ratios of baryon axial vector couplings and of isovector magnetic moments arises at order $1/N_c^2$, which accounts for the success of the $N_c \to \infty$ predictions for these quantities at the 10% level. In ref. the implications of the $1/N_c$ expansion for baryons containing strange quarks are analyzed without assuming $SU(3)$ symmetry. The results obtained are therefore valid to all orders in $SU(3)$ breaking, and can be used to constrain the form of $SU(3)$ breaking for baryons. The $1/N_c$ expansion justifies a number of results found phenomenologically in baryon chiral perturbation theory, and provides insight into the successes of phenomenological models, such as strong coupling theory, the Skyrme model, and the non-relativistic quark model.

Recent work related to refs. can be found in refs. Carone, Georgi and Osofsky realized that spin-flavor symmetry for the low lying baryon multiplets follows from the spin independence of the baryon wavefunctions, and they extended Witten’s Hartree-Fock analysis to baryons containing light quarks. They also noted that $SU(6)$ symmetry relations should hold for the isoscalar axial vector currents. Luty and March-Russell have rederived some of the results in ref. by a different method. Broniowski has proven that the $1/N_c$ correction to the isovector magnetic moments vanishes for two flavors using low energy QCD sum rules.

In this paper, we use the results of refs. to obtain relations among the baryon magnetic and transition magnetic moments in the $1/N_c$ expansion. Our formulæ for the isovector magnetic moments to leading and first subleading order in the $1/N_c$ expansion, and for the isoscalar magnetic moments to leading order in the $1/N_c$ expansion, were derived in ref. The $1/N_c$ corrections to the isoscalar magnetic moments are determined in this paper. The predictions we obtain in the $1/N_c$ expansion are compared in detail with those of the non-relativistic quark model. The $1/N_c$ expansion gives as many relations among the baryon magnetic moments as the non-relativistic quark model. Many of the relations are identical to those of the quark model, but a few differ. The $1/N_c$ relations
which differ from the quark model are in better agreement with experiment than the corresponding quark model relations. In addition, there are some very important advantages to the $1/N_c$ expansion: it is an expansion in QCD, which does not make use of any model description of baryons, and it provides an explanation of why some relations work better than others. Relations which are true up to corrections of order $1/N_c$ work to about 30%, whereas relations which are true up to order $1/N_c^2$ work to about 10%.

The $1/N_c$ analysis of this work is complementary to recent calculations of the baryon magnetic moments in chiral perturbation theory [11][12][13]. The $1/N_c$ results are an expansion in $1/N_c$ valid to all orders in the strange quark mass $m_s$, whereas the chiral perturbation theory results are an expansion in $m_s$ (including non-analytic terms) valid to all orders in $1/N_c$. The relations we derive to second order in the $1/N_c$ expansion are satisfied to all orders in $SU(3)$ breaking. Additional relations are derived which are valid to all orders in $SU(3)$ breaking at leading order in the $1/N_c$ expansion, and at first subleading order in the $1/N_c$ expansion in the $SU(3)$ limit.

In the large $N_c$ limit, baryons are infinitely heavy and can be treated as static fermions. For static baryons, the axial vector and magnetic moment operators $\overline{B}T^a\gamma^\mu\gamma_5 B$ and $\overline{B}T^a\sigma_{\mu\nu}B$ are both proportional to the spin-flavor operator $\overline{B}T^a\hat{\sigma}B$, where $T^a$ is a flavor matrix. As a result, the large $N_c$ consistency conditions for the baryon magnetic moments have the same form as the consistency conditions for the axial couplings [3][5]. The solution of the consistency conditions for two flavors is given in ref. [3] and the solution for three flavors is given in ref. [5]. One subtlety of the three-flavor analysis is that baryon flavor representations change with $N_c$. This complication leads to an ambiguity in identifying baryons for $N_c = 3$ with large $N_c$ baryon states. The flavor $SU(3)$ weight diagram for spin-1/2 baryons in the $N_c \to \infty$ limit is given in fig. [4]. In ref. [3], the $N_c \to \infty$ limit is taken with the isospin, spin and strangeness of the baryon held fixed. Thus, the baryon states of physical interest are located at a fixed distance from the top of the weight diagram as $N_c \to \infty$. With this limiting procedure, one can show that baryon matrix elements are given in terms of an expansion in $I/N_c$, $J/N_c$, $J_s/N_c$ and $K/N_c$, where $I$ is the isospin, $J$ is the spin, $J_s$ is the strange quark spin, and $K = -S/2$, where $S$ is the strangeness. The results of the $1/N_c$ expansion for arbitrary $SU(3)$ breaking follow from a $SU(4) \times SU(2) \times U(1)$ spin-flavor symmetry[6] for baryons in the $N_c \to \infty$ limit.

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1 $SU(4)$ is the spin-flavor symmetry for two flavors, $SU(2)$ is the strange quark spin symmetry, and $U(1)$ is the strangeness.
The baryon magnetic moments are proportional to the quark charge matrix

\[ Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}. \]

\( Q \) is a linear combination of the flavor generators \( T^3 \) and \( T^8 \), which transform as an isovector and an isoscalar, respectively. In the large \( N_c \) limit, the isovector magnetic moments are of order \( N_c \), and the isoscalar magnetic moments are of order one (provided one takes the large-\( N_c \) limit with \( I, J \) and \( K \) held fixed). The isovector magnetic moments \( (\vec{\mu}_V) \) have the same form as the pion-baryon couplings \[ \mu^i_V = N_c \mu(K) X_0^{i3} \left[ 1 + \mathcal{O} \left( \frac{1}{N^2_c} \right) \right], \tag{1} \]

where \( X_0^{ia} \) is the generator of the contracted spin-flavor algebra for baryons defined in refs. \[ \text{[3][5]}, \]

and \( \mu(K) \) is an unknown coefficient which is a constant to leading order in \( 1/N_c \) and is at most linear in \( K \) at order \( 1/N_c \),

\[ \mu(K) = \mu_0 + \frac{1}{N_c} \mu_1 K + \mathcal{O} \left( \frac{1}{N^2_c} \right), \tag{2} \]

where \( \mu_0 \) has both order one and \( 1/N_c \) terms. The matrix elements of \( \mu^i_V \) between baryons labeled by isospin, spin and strangeness quantum numbers \( (I, I_3), (J, J_3) \) and \( K \) were worked out explicitly in ref. \[ \text{[3]}, \]

\[ \langle I' I_3', J' J_3'; K | \mu^i_V | I I_3; J J_3; K \rangle = N_c \mu(K) (-1)^{2J'+J-I'-K} \times \]

\[ \sqrt{(2I+1)(2J'+1)} \left\{ \begin{array}{ccc} 1 & I & I' \\ K & J' & J \end{array} \right\} \left( \begin{array}{ccc} I & 1 & I' \\ I_3 & 3 & I'_3 \end{array} \right) \left( \begin{array}{ccc} J & 1 & J' \\ J_3 & i & J'_3 \end{array} \right) \]

\[ \times \left[ 1 + \mathcal{O} \left( \frac{1}{N^2_c} \right) \right]. \tag{3} \]

Similarly, the isoscalar magnetic moments \( (\vec{\mu}_S) \) have the same form as the \( \eta \)-baryon coupling \[ \text{[3]}, \]

\[ \mu^i_S = a(K) J^i + b(K) J^i_s + \mathcal{O} \left( \frac{1}{N^2_c} \right). \tag{4} \]
where $J^i$ is the total baryon spin, and $J^i_s$ is the “spin of the strange quarks.” The coefficients $a(K)$ and $b(K)$ are constants to leading order in $1/N_c$ and are at most linear in $K$ at order $1/N_c$,

\[
\begin{align*}
a(K) &= a_0 + \frac{1}{N_c} a_1 K + \mathcal{O} \left( \frac{1}{N_c^2} \right), \\
b(K) &= b_0 + \frac{1}{N_c} b_1 K + \mathcal{O} \left( \frac{1}{N_c^2} \right),
\end{align*}
\]

where $a_0$ and $b_0$ contain both order one and $1/N_c$ terms. The order one result for the isoscalar magnetic moments was derived in ref. [5]. Eq. (5) also contains the new result that the $1/N_c$ corrections to the isoscalar magnetic moments are obtained simply by including a linear in $K$ term in the coefficients $a(K)$ and $b(K)$ at order $1/N_c$. No other new operator structures appear at this order. It is important to emphasize that eqs. (1)–(5) were derived without assuming $SU(3)$ flavor symmetry, and are therefore valid to all orders in $SU(3)$ breaking. Thus, to leading and first subleading order in the $1/N_c$ expansion, the isovector magnetic moments are parametrized in terms of two constants $\mu_0$ and $\mu_1$ whereas the isoscalar magnetic moments are parametrized by four constants $a_0$, $a_1$, $b_0$ and $b_1$. In the limit of exact $SU(3)$ flavor symmetry, $\mu_1/\mu_0 = -2$, $a_0/\mu_0 = -4(\alpha + \beta/N_c)/3\sqrt{3}$, $a_1/\mu_0 = 4\sqrt{3} + 8\alpha/\sqrt{3}$, $b_0/\mu_0 = -2\sqrt{3} + 4\sqrt{3}/N_c$, and $b_1 = 0$. These limiting values can be derived using the $SU(3)$ tensor analysis method of ref. [5], and the $1/N_c$ expansion of the $SU(3)$ invariant amplitudes $\mathcal{M}$ and $\mathcal{N}$ defined there, $\mathcal{N}/\mathcal{M} = 1/2 + \alpha/N_c + \beta/N_c^2$.

It is useful to compare the predictions of the $1/N_c$ expansion with those of the non-relativistic quark model. The quark model predictions for the baryon magnetic moments are well-known. The magnetic moment operator for the quarks is given by

\[
\mu = \mu_u J_u + \mu_d J_d + \mu_s J_s,
\]

where $J_q$ is the spin operator of quark $q$ and $\mu_q$ is its magnetic moment. The baryon magnetic moments are obtained by taking the matrix elements of the magnetic moment operator $\mu$ between baryon states with $SU(6)$ symmetric quark model wavefunctions. $SU(3)$ breaking in the quark model arises from explicit $SU(3)$ breaking in the quark magnetic moments. In the isospin limit, $\mu_u = -2\mu_d$, but $\mu_s$ is unrelated to $\mu_d$.

\footnote{The operator $J_s$ has a precise meaning in terms of the induced representations discussed in ref. [3]. It reduces to the spin of the strange quarks in a non-relativistic quark model description for the baryons.}
The matrix elements of $X^i_0$, $J^i$ and $J^i_s$ times the unknown coefficient functions $\mu(K)$, $a(K)$ and $b(K)$, respectively, describe the isovector and isoscalar magnetic moments to leading and first subleading orders in the $1/N_c$ expansion. The matrix elements of the quark spin operators $J^i_q$ times the unknown quark magnetic moments $\mu_q$ describe the baryon magnetic moments in the non-relativistic quark model. The matrix elements of the above operators are given in Table 1 for all of the octet, decuplet and decuplet-octet transition magnetic moments. (In Table 1, the magnetic moment of baryon $B$ is denoted by $B$ and the $B_1 \to B_2$ transition magnetic moment is denoted by $B_2B_1$.) The matrix elements listed in Table 1 are the matrix elements of the $i = 3$ (or $\hat{z}$) components of the operators between $J_3 = 1/2$ states for the octet magnetic moments, between $J_3 = 3/2$ states for the decuplet magnetic moments, and between $J_3 = 1/2$ states of both the spin-1/2 and spin-3/2 baryons for the transition magnetic moments. The matrix elements of $X^i_0$ are given in eq. (3). The operator $J^3$ is the total angular momentum, and has only diagonal matrix elements. The matrix elements of the operator $J^3_s$ can be computed using its definition in terms of the induced baryon representations given in ref. [5]. The matrix elements of the quark model operators are obtained using $SU(6)$ quark model wavefunctions for the octet and decuplet baryons.

There are 27 magnetic moments listed in Table 1. Isospin invariance gives six linear relations $I_1$–$I_6$, as listed in Table 2, leaving 21 independent magnetic moments. These 21 magnetic moments can be divided into 11 isovector combinations such as $(p - n)$ and 10 isoscalar combinations such as $(p + n)$. Linear relations among the magnetic moments can be derived in the $1/N_c$ expansion. These relations are listed in Table 3. The accuracy with which the relations are satisfied is also tabulated for those relations involving magnetic moments which have been measured [14]. Since the dominant uncertainty in the magnetic moment relations is theoretical, not experimental, the relations in Table 3 are written so that all terms on one side of an equation have the same sign in order to avoid misleading cancellations. The accuracy listed in the last column of Table 3 is the difference between the two sides of each equation, i.e. $|\text{lhs} - \text{rhs}|/(\text{lhs} + \text{rhs})/2$.

The isovector magnetic moments depend on the two unknown parameters $\mu_0$ and $\mu_1$ in $\mu(K)$ up to corrections of order $1/N_c^2$ relative to the leading (order $N_c$) term. Thus, there are nine relations among the 11 isovector magnetic moments ($V_1$–$V_7$, $V_8_1$, $V_9_1$) which are valid to relative order $1/N_c^2$ and to all orders in $SU(3)$ breaking. The parameter $\mu_1$ arises at first subleading order in the $1/N_c$ expansion, so there is one additional relation among the isovector magnetic moments ($V_{10_1}$) which is true at leading order, but which is not
satisfied at first subleading order. In the $SU(3)$ limit, $\mu_1$ is related to $\mu_0$, so this relation can be improved by including the $SU(3)$-symmetric $1/N_c$ correction. There are two equivalent forms for this improved relation ($V10_2$ and $V10_3$), each of which is satisfied by the leading term to all orders in $SU(3)$ breaking and by the subleading term in the $SU(3)$ limit. The two forms, $V10_2$ and $V10_3$, differ at order $1/N^2_c$, so it is not possible to determine which is more accurate without computing the $1/N^2_c$ corrections to the isovector magnetic moments. 

The $1/N_c$ relations which are tested by presently available experimental data are accurate at the level predicted by the $1/N_c$ expansion. Relations $V1$, $V8_1$ and $V9_1$ which hold to relative order $1/N^2_c$ work at the 10% level or better. Relation $V10_1$ only holds at the 30% level, which is in keeping with a correction of order $1/N_c$. The $SU(3)$ improved versions of this relation, $V10_2$ and $V10_3$, are accurate at the 10% level or better. This accuracy is as expected since deviations from the relations are suppressed either by one power of $1/N_c$ and $SU(3)$ breaking, or by $1/N^2_c$. The $1/N_c$ expansion predicts that the experimentally untested relations $V2$–$V7$ will hold at the 10% level when the relevant magnetic moments are measured.

The isovector magnetic moment relations of the $1/N_c$ expansion can be contrasted with the predictions of the non-relativistic quark model. Seven of the relations ($V1$–$V7$) are true in the quark model. There are two additional relations, $V8_1$ and $V9_1$, which are valid to relative order $1/N^2_c$ in the $1/N_c$ expansion. The quark model has two very similar predictions, $V8_2$ and $V9_2$. The quark model relation $V8_2$ works as well its $1/N_c$ counterpart $V8_1$. The quark model relation $V9_2$, however, is much less accurate than the $1/N_c$ relation $V9_1$. The failure of the quark model prediction for the $p\Delta^+$ transition magnetic moment is resolved by the $1/N_c$ expansion, which gives a different prediction for the transition magnetic moment. Finally, there is one additional relation in the quark model, $V10_4$, which works well, and is the counterpart of the $1/N_c$ relations $V10_{1,2,3}$.

The isoscalar magnetic moments (up to corrections of order $1/N^2_c$) depend on the four unknown parameters $a_0$, $a_1$, $b_0$ and $b_1$ in $a(K)$ and $b(K)$. Thus, there are six relations among the 10 isoscalar magnetic moments ($S1$–$S6$) which are valid to order $1/N^2_c$ and to all orders in $SU(3)$ breaking. The parameters $a_1$ and $b_1$ arise at first subleading order in the $1/N_c$ expansion, so there are two additional isoscalar relations ($S7$ and $S8$), which are

\[3\] The unmeasured $n\Delta^0$ transition magnetic moment is eliminated from relation $V9$ using the isospin relation $n\Delta^0 = p\Delta^+$. It is known that the $p\Delta^+$ transition moment is not in good agreement with the quark model prediction [13].
satisfied by the leading order term to all orders in $SU(3)$ breaking but which are broken at order $1/N_c$. Since $b_1 = 0$ in the $SU(3)$ limit, one relation (S8) also holds at order $1/N_c$ in the $SU(3)$ limit. The experimental accuracies of the isoscalar relations agree with the predictions of the $1/N_c$ expansion. Of the first six relations, only relation S1 is tested by experimental data. The accuracy of this relation is consistent with a correction of order $1/N_c^2$, as predicted by the $1/N_c$ expansion. Relation S7 which is corrected at order $1/N_c$ (even in the $SU(3)$ limit) is expected to work at the 30% level; it works to 22%. Relation S8 is expected to be satisfied at the 10% level since its correction is order $SU(3)$ breaking/$N_c$. The relation holds to 7% accuracy. The $1/N_c$ expansion also predicts that relations S2–S6 will be satisfied at the 10% level when the relevant magnetic moments are measured.

All eight isoscalar relations are true in the quark model. The quark model, however, makes no prediction for the relative accuracies of the different relations. The $1/N_c$ expansion explains why one of the quark model predictions works much worse than the others.

The isovector and isoscalar relations V2 and S2 cannot be tested because the $\Delta^-$ magnetic moment has not been measured. The two relations can be combined to obtain the prediction for the $\Delta^{++}$ magnetic moment given in the last line of Table 3. This prediction is valid to two orders in the $1/N_c$ expansion in the isovector (order $N_c$ and one) and isoscalar (order one and $1/N_c$) contributions. The experimental measurement of the $\Delta^{++}$ magnetic moment has a large error so the relation cannot be tested precisely. However, the present experimental value is consistent with the theoretical prediction from the $1/N_c$ expansion.

Eighteen relations among the 21 isovector and isoscalar magnetic moments have been obtained in both the $1/N_c$ expansion and the quark model. In the $SU(3)$ limit, the isoscalar parameter $b_0$ is related to the isovector parameter $\mu_0$, so there one additional relation (S/V1) which normalizes the isoscalar magnetic moments relative to the isovector magnetic moments. Relation S/V1 is satisfied by both the order one and $1/N_c$ terms in the $SU(3)$ limit. The $1/N_c$ expansion predicts that this relation is violated by $SU(3)$ breaking at order one in the $1/N_c$ expansion, i.e. at the $\sim 30\%$ level. The relation works a factor of

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4 The isovector contribution in S/V1 has a coefficient of order $1/N_c$, which makes the order $N_c$ isovector term of the same order as the order one isoscalar terms. In deriving S/V1, we have used the expression for the total magnetic moment $\mu = -\mu_V + \mu_S/\sqrt{3}$. The minus sign in front of $\mu_3$ is present if one uses the Condon-Shortley phase convention for the isospin representations.
three better than this prediction. Although this level of accuracy is not prohibited by the $1/N_c$ expansion, it would be interesting if this level of accuracy followed from some other considerations. We leave this issue as an open question. A relation between the isoscalar and isovector magnetic moments can be obtained in chiral perturbation theory (but not in the $1/N_c$ expansion), without relying on any models. The relation obtained in ref. [11], $6\Lambda + \Sigma^- + 4\sqrt{3}\Lambda\Sigma^0 = 4n - \Sigma^+ + 4\Xi^0$ is valid including all $SU(3)$ breaking corrections of order $m_s^{1/2}$, $m_s \ln m_s$, and $m_s$, and works to $6 \pm 4\%$. There is also one additional relation which relates the isoscalar and isovector magnetic moments in the quark model if one imposes the isospin constraint $\mu_u = -2\mu_d$. This relation $(S/V_2)$: $(p - n) = 5(p + n)$, is equivalent to the famous $SU(6)$ prediction $p/n = -3/2$ [16], and is satisfied to $7\%$.

In summary, we have derived relations among the baryon magnetic and transition magnetic moments in the $1/N_c$ expansion. The $1/N_c$ analysis makes definite predictions for the accuracies with which these relations are satisfied. With the notable exception of $S/V_1$, these predictions are in complete accord with experiment. The structure of the $1/N_c$ expansion is much richer than that of the non-relativistic quark model. The quark model predicts all the baryon magnetic moments in terms of three input parameters. Some of the predictions work better than others. The $1/N_c$ expansion naturally predicts the hierarchy of relations given in Table 3, and explains which relations work better than others. There is no particular reason to analyze the magnetic moments in terms of the relations given in Table 3 in the quark model.

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Figure Captions

Fig. 1. The weight diagram for the spin-1/2 baryons for large $N_c$ QCD. The representation reduces to the familiar baryon octet for $N_c = 3$. 
TABLE 1

The matrix elements of the $1/N_c$ and quark model operators for the baryon magnetic moments.

|     | $a(K)$ | $b(K)$ | $N_c \mu(K)$ | $\mu_u$ | $\mu_d$ | $\mu_s$ |
|-----|--------|--------|---------------|---------|---------|---------|
| p   | $1/2$  | 0      | $-1/3$        | $4/3$   | $-1/3$  | 0       |
| n   | $1/2$  | 0      | $1/3$         | $-1/3$  | $4/3$   | 0       |
| $\Lambda$ | $1/2$ | 1/2 | 0             | 0       | 0       | 1       |
| $\Lambda \Sigma^0$ | 0    | 0    | 1/3           | $-\sqrt{1/3}$ | $\sqrt{1/3}$ | 0       |
| $\Sigma^+$ | $1/2$ | $-1/6$ | $-1/3$        | 4/3     | 0       | $-1/3$  |
| $\Sigma^0$ | $1/2$ | $-1/6$ | 0             | 2/3     | 2/3     | $-1/3$  |
| $\Sigma^-$ | $1/2$ | $-1/6$ | 1/3           | 0       | 4/3     | $-1/3$  |
| $\Xi^0$ | $1/2$ | 2/3   | 1/9           | $-1/3$  | 0       | 4/3     |
| $\Xi^-$ | $1/2$ | 2/3   | $-1/9$        | 0       | $-1/3$  | 4/3     |
| $\Delta^{++}$ | 3/2  | 0     | $-3/5$        | 3       | 0       | 0       |
| $\Delta^+$ | 3/2  | 0     | $-1/5$        | 2       | 1       | 0       |
| $\Delta^0$ | 3/2  | 0     | 1/5           | 1       | 2       | 0       |
| $\Delta^-$ | 3/2  | 0     | 3/5           | 0       | 3       | 0       |
| $\Sigma^{*+}$ | 3/2 | 1/2 | $-1/2$        | 2       | 0       | 1       |
| $\Sigma^{*0}$ | 3/2 | 1/2 | 0             | 1       | 1       | 1       |
| $\Sigma^{*-}$ | 3/2 | 1/2 | 1/2           | 0       | 2       | 1       |
| $\Xi^{*0}$ | 3/2 | 1    | $-1/3$        | 1       | 0       | 2       |
| $\Xi^{*-}$ | 3/2 | 1    | 1/3           | 0       | 1       | 2       |
| $\Omega^-$ | 3/2 | 3/2 | 0             | 0       | 0       | 3       |
| p$\Delta^+$ | 0    | 0    | $-\sqrt{2}/3$ | $2\sqrt{2}/3$ | $-2\sqrt{2}/3$ | 0       |
| n$\Delta^0$ | 0    | 0    | $-\sqrt{2}/3$ | $2\sqrt{2}/3$ | $-2\sqrt{2}/3$ | 0       |
| $\Lambda \Sigma^{*0}$ | 0    | 0    | $-\sqrt{2}/3$ | $\sqrt{2}/3$ | $-\sqrt{2}/3$ | 0       |
| $\Sigma \Sigma^{*+}$ | 0    | $-\sqrt{2}/3$ | $-1/3\sqrt{2}$ | $2\sqrt{2}/3$ | 0 | $-2\sqrt{2}/3$ |
| $\Sigma \Sigma^{*0}$ | 0    | $-\sqrt{2}/3$ | 0 | $\sqrt{2}/3$ | $\sqrt{2}/3$ | $-2\sqrt{2}/3$ |
| $\Sigma \Sigma^{*-}$ | 0    | $-\sqrt{2}/3$ | 1/3$\sqrt{2}$ | $2\sqrt{2}/3$ | 0 | $-2\sqrt{2}/3$ |
| $\Xi \Xi^{*0}$ | 0    | $-\sqrt{2}/3$ | $-2\sqrt{2}/3$ | $2\sqrt{2}/3$ | 0 | $-2\sqrt{2}/3$ |
| $\Xi \Xi^{*-}$ | 0    | $-\sqrt{2}/3$ | $2\sqrt{2}/9$ | $2\sqrt{2}/3$ | 0 | $-2\sqrt{2}/3$ |
TABLE 2

Isospin relations among the baryon magnetic moments.

| Isospin Relations |
|-------------------|
| I1 | $\Sigma^+ + \Sigma^- = 2\Sigma^0$ |
| I2 | $\Sigma^{++} + \Sigma^{*-} = 2\Sigma^*^0$ |
| I3 | $\Sigma^0 \Sigma^{**} + \Sigma^0 \Sigma^{*-} = 2\Sigma \Sigma^*^0$ |
| I4 | $\Delta^{++} - \Delta^- = 3(\Delta^+ - \Delta^-)$ |
| I5 | $\Delta^{++} + \Delta^- = \Delta^+ + \Delta^0$ |
| I6 | $p\Delta^+ = n\Delta^0$ |

TABLE 3

Relations among the baryon magnetic moments in the $1/N_c$ expansion, and in the non-relativistic quark model. The isovector magnetic moments are of order $N_c$, and the isoscalar magnetic moments are of order one. A $\sqrt{\ }$ implies that the relation is satisfied to that order in $1/N_c$ to all orders in $SU(3)$ breaking. A $SU(3)$ implies that the relation is satisfied to that order in $1/N_c$ only in the $SU(3)$ limit. The experimental accuracies are given in the last column for the relations whose magnetic moments have been measured.
| Isovector Relations          | N<sub>c</sub> | 1 | QM   |
|------------------------------|---------------|---|------|
| V1   \((p - n) - 3(\Xi^0 - \Xi^-) = 2(\Sigma^+ - \Sigma^-)\) | √  | √ | √ | 10 ± 2% |
| V2   \(\Delta^{++} - \Delta^- = \frac{9}{8}(p - n)\) | √  | √ | √ |
| V3   \(\Lambda_{\Sigma^{*0}} = -\sqrt{2}\Lambda_{\Sigma^0}\) | √  | √ | √ |
| V4   \(\Sigma^{*+} - \Sigma^{*-} = \frac{3}{2}(\Sigma^+ - \Sigma^-)\) | √  | √ | √ |
| V5   \(\Xi^{*0} - \Xi^{*-} = -3(\Xi^0 - \Xi^-)\) | √  | √ | √ |
| V6   \(\sqrt{2}(\Sigma\Sigma^{*+} - \Sigma\Sigma^{*-}) = (\Sigma^+ - \Sigma^-)\) | √  | √ | √ |
| V7   \(\Xi\Xi^{*0} - \Xi\Xi^{*-} = -2\sqrt{2}(\Xi^0 - \Xi^-)\) | √  | √ | √ |
| V8<sub>1</sub> \(-2\Lambda_{\Sigma^0} = (\Sigma^+ - \Sigma^-)\) | √  | √ | No | 11 ± 5% |
| V8<sub>2</sub> \(-2\Lambda_{\Sigma^0} = \frac{\sqrt{3}}{2}(\Sigma^+ - \Sigma^-)\) | No | No | √ | 4 ± 5% |
| V9<sub>1</sub> \(p\Delta^+ + n\Delta^0 = \sqrt{2}(p - n)\) | √  | √ | No | 3 ± 3% |
| V9<sub>2</sub> \(p\Delta^+ + n\Delta^0 = \frac{4\sqrt{3}}{5}(p - n)\) | No | No | √ | 26 ± 4% |
| V10<sub>1</sub> \((\Sigma^+ - \Sigma^-) = (p - n)\) | √  | No | No | 27 ± 1% |
| V10<sub>2</sub> \((\Sigma^+ - \Sigma^-) = \left(1 - \frac{1}{N_c}\right)(p - n)\) | √  | SU(3) | No | 13 ± 2% |
| V10<sub>3</sub> \(\left(1 + \frac{1}{N_c}\right)(\Sigma^+ - \Sigma^-) = (p - n)\) | √  | SU(3) | No | 1 ± 2% |
| V10<sub>4</sub> \((\Sigma^+ - \Sigma^-) = \frac{4}{5}(p - n)\) | No | No | √ | 5 ± 2% |

| Isoscalar Relations          | 1 | 1/N<sub>c</sub> | QM   |
|------------------------------|---|----------------|------|
| S1   \((p + n) - 3(\Xi^0 + \Xi^-) = -3\Lambda + \frac{3}{2}(\Sigma^+ + \Sigma^-) - \frac{4}{3}\Omega^-\) | √  | √ | √ | 4 ± 5% |
| S2   \(\Delta^{++} + \Delta^- = 3(p + n)\) | √  | √ | √ |
| S3   \(\frac{3}{8}(\Xi^{*0} + \Xi^{*-}) = \Lambda + \frac{3}{4}(\Sigma^+ + \Sigma^-) - (p + n) + (\Xi^0 + \Xi^-)\) | √  | √ | √ |
| S4   \(\Sigma^{*+} + \Sigma^{*-} = \frac{3}{2}(\Sigma^+ + \Sigma^-) + 3\Lambda\) | √  | √ | √ |
| S5   \(\frac{3}{2}(\Sigma\Sigma^{*+} + \Sigma\Sigma^{*-}) = 3(\Sigma^+ + \Sigma^-) - (\Sigma^{*+} + \Sigma^{*-})\) | √  | √ | √ |
| S6   \(\Xi\Xi^{*0} + \Xi\Xi^{*-} = -3(\Xi^0 + \Xi^-) + (\Xi^{*0} + \Xi^{*-})\) | √  | √ | √ |
| S7   \(5(p + n) - (\Xi^0 + \Xi^-) = 4(\Sigma^+ + \Sigma^-)\) | √  | No | √ | 22 ± 4% |
| S8   \((p + n) - 3\Lambda = \frac{5}{8}(\Sigma^+ + \Sigma^-) - (\Xi^0 + \Xi^-)\) | √  | SU(3) | √ | 7 ± 1% |

| Isoscalar/Isovector Relations | 1 | 1/N<sub>c</sub> | QM   |
|------------------------------|---|----------------|------|
| S/V<sub>1</sub> \((\Sigma^+ + \Sigma^-) - \frac{1}{2}(\Xi^0 + \Xi^-) = \frac{1}{2}(p + n) + 3\left(\frac{1}{N_c} - \frac{2}{N_c^2}\right)(p - n)\) | SU(3) | SU(3) | No | 10 ± 3% |
| S/V<sub>2</sub> \(p - n = 5(p + n)\) | No | No | √ | 7% |

\(\Delta^{++} = \frac{3}{2}(p + n) + \frac{9}{10}(p - n)\) | √  | √ | √ | 21 ± 10% |
Figure 1
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9405431v1