Addressing the Hubble and $S_8$ tensions with a kinetically mixed dark sector

Stephon Alexander, a Heliudson Bernardo b and Michael W. Toomey a

aBrown Theoretical Physics Center and Department of Physics, Brown University, Providence, RI 02912, U.S.A.
bDepartment of Physics, McGill University, Montreal, QC, H3A 2T8, Canada
E-mail: stephon_alexander@brown.edu, heliudson@hep.physics.mcgill.ca, michael_toomey@brown.edu

Received October 8, 2022
Revised February 11, 2023
Accepted February 17, 2023
Published March 14, 2023

Abstract. We present a kinetically mixed dark sector (KMIX) model to address the Hubble and $S_8$ tensions. Inspired from string theory, our model includes two fields: an axion, which plays a role similar to the scalar field in early dark energy models, and a dilaton. This theory differs from other axio-dilaton models aimed at the Hubble tension in that there is necessarily kinetic mixing between the two fields which allows for efficient energy transfer from the axion into the dilaton which has $w \approx 1$. As a direct consequence of these dynamics, we find the model does not need to resort to a fine-tuned potential to solve the Hubble tension and naturally accommodates a standard axion potential. Furthermore, the axion will necessarily makeup a small (fuzzy) fraction of $\Omega_{\text{cdm}}$ once it begins to oscillate at the bottom of its potential and will suppress the growth of perturbations on scales sensitive to $S_8$. Interestingly, the scale of the potential for the dilaton has to be small, $\lesssim \mathcal{O}(10 \text{ meV})^4$, suggesting the possibility for a connection to dark energy. Implementing the dynamics for the background and perturbations in a modified Boltzmann code we calculate the CMB and matter power spectra for our theory. Exploring the parameter space of our model, we find regions which can accommodate a $\sim 10\%$ increase in $H_0$ from the Planck inferred value and $S_8$ values that are consistent with large-scale structure constraints.

Keywords: axions, cosmological applications of theories with extra dimensions, cosmology of theories beyond the SM, string theory and cosmology

ArXiv ePrint: 2207.13086
1 Introduction and motivation

The tension between local and global measurements of \( H_0 \) — the first ever measured parameter of the ΛCDM model — has arrived at the 4–6σ range, depending on the combined data sets [1–5]. In fact, the \( H_0 \) value inferred from the Planck 2018 cosmic microwave background (CMB) data disagrees with local measurements by the SH0ES collaboration at a statistical significance of 5σ [6]. While the SH0ES result relies on cepheid calibration of the cosmic distance ladder [7], its is not clear that changing the calibration method alleviates the tension [8, 9] due to a loss in precision.

If not due to non-obvious systematic effects in the measurements, this “Hubble tension” requires modifying the standard cosmological model, a challenging task for the precision fits of other parameters and the cosmological concordance between data across different scales [10]. In particular, the angular scale \( \theta_s \) of the comoving sound horizon \( r_s \) at decoupling is measured from the acoustic peaks of the CMB power spectrum at a precision of 0.03% [2] and its value is fixed by the ΛCDM parameters through its dependence on the sound horizon and the angular diameter distance,

\[
\theta_s = \frac{r_s(z_s)}{D_A(z_s)}, \quad r_s(z_s) = \int_{z_s}^{\infty} \frac{c_s}{H(z)} \, dz, \quad D_A(z_s) = \int_0^{z_s} \frac{1}{H(z)} \, dz, \quad (1.1)
\]

where \( z_s \) is the redshift at photon-baryon decoupling. Hence, shifts in the CMB-inferred \( H_0 \) towards local-measured values should be followed by a change in \( r_s \) so as to keep \( \theta_s \) unchanged. Comparing Planck 2018 and SH0ES central values for \( H_0 \), we need an increase of \( \sim 10\% \) to solve the Hubble tension. This entails a decrease in \( r_s(z_s) \) by the same factor. But since the integral defining \( r_s(z_s) \) is dominated by the cosmological evolution just prior to recombination, the most efficient way to shrink it is by increasing the background energy density around \( z_s \).
There are currently several proposals for increasing $H(z_0)$ to solve the Hubble tension (see [5, 11] and references therein). “Early Dark Energy” (EDE) models are a subclass of those that postulates the existence of a subdominant energy density component that peaks before $z_0$ but that quickly redshifts shortly afterwards [12–23]. This idea is often implemented after introducing a canonically normalized scalar field $\phi$ with a potential $V(\phi)$ that has a curvature scale $\sqrt{V_{\phi\phi}}$ of the order of $H(z_0)$ such that it is held constant by Hubble friction at early times. Around before $z_0$, typically closer to matter-radiation equality, the field rolls down its potential and the subsequent late time evolution depends on $V(\phi)$. For instance, coherent (an)harmonic oscillations around $\phi^{2n}$ minima makes the energy density in the field redshift as $a^{-\frac{6n}{n+1}}$ [24], so if $n > 2$ the energy density in the field redshifts faster than radiation. Moreover, since $H(z_0) \sim 10^{-28} \text{eV}$, an ultra-light field is needed.

From a theoretical perspective, the EDE scalar field should be an ultralight axion whose potential is generated by non-perturbative effects, $V \sim m_\phi^2 f_\phi^2 (1 - \cos (\phi/f_\phi))$, where $m_\phi$ and $f_\phi$ are the axion mass and decay constant, respectively. However, if there is no energy transfer from $\phi$ to another component that redshifts faster than radiation, higher harmonics need to dominate the instanton expansion to recover the $\Lambda$CDM cosmological evolution after recombination. Potentials of this form, $V \sim m_\phi^2 f_\phi^2 (1 - \cos (\phi/f_\phi))^n$, were studied in [25] and constitute 3 extra parameters relative to $\Lambda$CDM for a choice of $n$: the initial displacement of the field $\theta_0 := \phi_0/f_\phi$, the redshift $z_c$ at which the energy density fraction in $\phi$ is maximum, and its value $f_{\phi EDE}(z_c)$ at the maximum. It was shown in [26] that data has a preference for $n = 3$ models. However, having a controlled instanton expansion dominated by a higher harmonic term requires fine-tuning the expansion coefficients. These are fixed by the details of the non-perturbative physics giving rise to the expansion. However, this a theoretical challenge for EDE models with $n \geq 2$ and currently there are no concrete derivations for such a potential. Phenomenologically, this is a mild issue as we should let data constrain $V(\phi)$.

Combining CMB and large-scale structure (LSS) data yields stringent constraints on EDE models. The $\Lambda$CDM parameter shift necessary to accommodate the EDE phase exacerbates the tension between the amplitude of density fluctuations at late times as inferred from CMB and LSS data [10, 27–31]. Although for a short period, the EDE component slightly reduces the growth of perturbations such that the $\Lambda$CDM parameters are shifted to compensate for that. In particular, the physical cold dark matter density increases leading to a slightly larger $S_8$ as compared with $\Lambda$CDM [27]. Hence, generically, any model that works on these grounds will increase the $S_8$ tension between early and late-time data sets [32]. This tension is even present in $\Lambda$CDM with the Planck inferred value at $S_8 = 0.834^{+0.016}_{-0.016}$ [2] and current constraints from large-scale structure at $S_8 = 0.800^{+0.029}_{-0.029}$ for HSC Y1 [33], $S_8 = 0.750_{-0.021}^{+0.024}$ for KiDS-1000 [34], and $S_8 = 0.772^{+0.016}_{-0.015}$ for DES Y3 [35] — this is the “$S_8$ tension”.

The challenges described above motivate searching for alternatives to single-field EDE models. Indeed, many phenomenological extensions of the standard EDE dynamics have been studied in the literature. These include a portion of dark matter decaying at late times [37], the addition of a second, light axion contribution to dark matter [38, 39], massive neutrinos [40], and a coupling of dark matter to the EDE scalar motivated by the swampland distance conjecture [41], all with the aim to suppress structure formation. In a similar spirit, proposals for dissipating the EDE density into other components, which naturally redshift faster than radiation, have also been studied [15, 16, 18, 42, 43]. In particular, in [42] a dissipative axion model was proposed in which the energy gets transferred to dark radiation,

\footnote{It should be pointed out that this tension is less clear and could be a statistical fluctuation [36].}
the latter acting as an extra friction in the $\phi$ equation of motion. Thermal friction was also shown to alleviate the Hubble and LSS tensions [43], but the data fit is such that the energy transfer happens at redshifts much higher than matter-radiation equality, deep into radiation domination where data has lost sensitivity to the transition and the model asymptotes to an extra radiation solution. One can of course also consider abandoning EDE-based models altogether. Indeed, models inspired by supersymmetry which modify $N_{\text{eff}}$ have seen success in not only addressing the Hubble tension [44] but also the $S_8$ tension [45]. While the latter model is a step in the right direction, there is a clear need for more well-motivated models that have the capacity to naturally address both tensions.

Motivated by string theory effective actions, we propose a two-field model that shares some similarities with usual EDE and thermal friction models: on one hand, there is still a phase of “early dark energy” which modifies the size of the sound horizon and on the other hand there is energy transfer from one component to another, which also contributes as friction to the evolution of the energy injecting field. However, no thermal effects are needed, the energy transfer is due to a necessary kinetic coupling between the fields, something that to the best of our knowledge has not been considered in the literature. Such a kinetic coupling is a generic feature of the effective theory for complex moduli fields in string compactification. More specifically, our starting action is

\[
S \supset \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} f(\chi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\chi, \phi) \right],
\]  

(1.2)

where the kinetic mixing function $f(\chi)$ is assumed to be exponential — the natural case when $\chi$ is the modulus associated to extra dimensions — and the potential $V(\chi, \phi)$ does not need to be fine tuned (see appendix A for string theoretic derivation). As we shall discuss in the next section, the kinetic coupling between $\phi$ and $\chi$ produces a source term in the equation of motion for $\chi$ and a friction term in the equation of motion for $\phi$, making the energy transfer possible but damped by friction. Furthermore, an unavoidable consequence of our model is a small (fuzzy) axion contribution to dark matter which naturally suppresses structure on scales sensitive to $S_8$.

Promisingly, the string axio-dilaton modulus has the kinetic structure in eq. (1.2) regardless of the model considered. In fact, in [15], a string theory inspired axio-dilaton model was shown to resolve the Hubble tension. A major difference being that the dilaton, and not the axion, played the role of dark energy. The dilaton is instead stabilized by the axion until the mass of the latter falls below $H$. At that point the dilaton becomes destabilized and enters fast-roll, thus diluting its energy density as $a^{-6}$. However, since the kinetic coupling was not considered in [15], the model relied on the choice of an interaction potential to dilute the dilaton’s energy density. This is in contrast to our proposed model,\(^2\) which works for natural potentials such as mass terms or the first harmonic of an instanton expansion.

In section 2 we discuss the background dynamics of the scalar field sector where the action of the form in eq. (1.2) is studied. We show that, in a homogeneous and isotropic background, the energy transfer from $\phi$ to $\chi$ is generic and that the $\chi$-component redshifts as stiff matter as long as its potential is negligible. Using a modified Boltzmann code we then study the background dynamics for our theory numerically. In section 3 we derive the linearized, perturbed Klein-Gordon equations for our theory and compare with those for

\(^2\)Note that although we call $\chi$ a “dilaton”, it is not necessarily the string dilaton which sets the string coupling. In string theory, a pair like $(\phi, \chi)$ might also correspond to the size of cycles in the compact internal manifold and fluxes on those cycles, respectively.
EDE. We show that $\phi$ will inevitably contribute a small fraction to the dark matter density, naturally suppressing the growth of structure on scales sensitive to $S_8$. Using our modified Boltzmann code we then calculate the CMB and matter power spectra to show that there are regions of parameter space in our model which have the right dynamics to resolve the Hubble and $S_8$ tensions. In section 4 we discuss the implications of our theory and directions for future work.

**Notation and conventions.** If not stated otherwise, we use Planckian units $M_{\text{Pl}} = 8\pi G \equiv 1$ and work in the mostly plus metric signature. We denote $f_\phi$ the axion decay constant and $f_{\text{KMIX}}(z)$ the fraction of energy density in the dark sector fields at the redshift $z$.

## 2 Background dynamics

Consider the scalar-sector action

$$S[\chi, \phi] = -\int d^4x\sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + \frac{1}{2} f(\chi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\chi, \phi) \right],$$

with $f(\chi) > 0$. In an arbitrary background, its equations of motion are

$$\nabla^2 \chi - \frac{1}{2} f_\chi (\chi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_\chi = 0,$$

$$\nabla_\mu [f(\chi)g^{\mu\nu} \partial_\nu \phi] - V_\phi = 0,$$

and for homogeneous fields in a flat FLRW metric in comoving coordinates, we have

$$\ddot{\phi} + \left( 3H + \frac{f_\chi}{f} \dot{\chi} \right) \dot{\phi} + \frac{1}{f} V_\phi = 0,$$

$$\ddot{\chi} + 3H \dot{\chi} - \frac{1}{2} f_\chi \dot{\phi}^2 + V_\chi = 0,$$

where we denote partial derivatives with respect to the fields by a subscript. We see that for a non-trivial kinetic coupling function, $f(\chi)$, there is a friction term for $\dot{\phi}$ that is proportional to $\dot{\chi}$ and a source term for $\dot{\chi}$ which is proportional to $\dot{\phi}^2$. So, if $\dot{\phi} \neq 0$, the energy in the $\phi$-component of the system gets transferred to the $\chi$-component.

### 2.1 Analytics

To understand the dynamics of the fields with the kinetic coupling, let us consider the system in flat space and $V = 0$, but still assume homogeneous fields. With those assumptions, the system is integrable since there are two conserved quantities associated to translations in time and in $\phi$,

$$f(\chi)\dot{\phi} = p, \quad f(\chi)\dot{\phi}^2 + \frac{1}{2} \dot{\chi}^2 = E,$$

where $p$ and $E$ are constants. These equations imply that

$$\frac{1}{2} \dot{\chi}^2 + \frac{p^2}{2f(\chi)} = E.$$
and so the dynamics for \( \chi(t) \) resembles that of an one-dimensional system with an effective potential \( p^2 f^{-1}/2 \). If \( \phi \) has no initial velocity, \( p = 0 \) and the solution for \( \chi \) is a linear function of \( t \), while \( \phi \) is constant. However, if \( \phi \) has a non-trivial initial kinetic energy then \( p \neq 0 \) and the motion for \( \chi \) is determined from

\[
\dot{\chi} = \pm \sqrt{2E - p^2 f(\chi)^{-1}}.
\]

So, if \( f_\chi > 0 \) (\( f_\chi < 0 \)) for all \( \chi \), then the physical \( \chi(t) \) motion is restricted to values \( \chi(t) \geq \chi_0 \) (\( \chi(t) \leq \chi_0 \)), where \( \chi_0 \) satisfies \( 2Ef(\chi_0) = p^2 \). In both cases, \( |\dot{\chi}| \to \sqrt{2E} \) and \( \dot{\phi} \to 0 \) asymptotically, with a small period of energy transfer from \( \chi \) to \( \phi \) if \( f_\chi < 0 \). If \( f(\chi) \) has a minimum, the motion can be periodic, with a cyclic energy exchange between \( \phi \) and \( \chi \).

For the case we are interested in, \( f(\chi) = \e^{\lambda \chi} \), the analysis above show us that the energy in \( \phi \) gets transferred to \( \chi \). In fact, assuming this functional form for \( f(\chi) \) and \( p \neq 0 \), we can integrate the equations of motion exactly,

\[
\begin{align*}
\phi(t) &= \phi_0 + \frac{2\sqrt{2E}}{\lambda p} \left\{ \tanh \left[ \arctanh A + \sqrt{\frac{E}{2} \lambda (t - t_0)} \right] - A \right\}, \\
\chi(t) &= -\frac{1}{\lambda} \ln \left\{ \frac{2E}{p^2} \left[ 1 - \tanh^2 \left[ \arctanh A + \sqrt{\frac{E}{2} \lambda (t - t_0)} \right] \right] \right\},
\end{align*}
\]

where \( A = \sqrt{1 - \frac{p^2}{2E}} \e^{-\lambda \chi_0} \) and we denoted \( \phi_0 \) and \( \chi_0 \) the value of the fields at \( t_0 \). From this result, one can check that \( \dot{\phi} \to 0 \) and \( |\dot{\chi}| \to \sqrt{2E} \) as \( |\chi(t)| \) increases.

From the discussion so far, we conclude that the energy transfer is a generic feature of kinetic coupling. For the rest of this section, we study whether this can dissipate the early dark energy density and provide a solution to the Hubble tension. To accomplish that, it is necessary to modify the integrable limit for eq. (2.4) in two ways: firstly, we need to turn on the potential, and secondly we need to consider the dynamics in an FLRW background. Generically, these modifications break the symmetry under \( \phi \) and \( t \)-translations such that the system is no longer integrable and we need to solve it numerically.

Notwithstanding, we can talk about the qualitative asymptotic behaviour after some assumptions. Consider for instance the case when \( V_\chi = 0 \) and negligible Hubble friction. In this case,

\[
\begin{align*}
\dot{\phi} &= -V_\phi, \\
\dot{\chi} &= \pm \sqrt{2E - p^2 f(\chi)^{-1} - V(\phi)},
\end{align*}
\]

and the \( \phi \) evolution does not allow us to reduce the system to one-dimensional motion. If \( \phi(t) \) is such that \( V(\phi(t)) \) decreases, we still get a constant \( \chi \) velocity asymptotically. If \( V(\phi) \) has a minimum, then even if \( \phi(t) \) gets at the minimum with significant kinetic energy, the \( \dot{\chi} \) dependent damping term will deplete its evolution (note that for \( f = \e^{\lambda \chi} \), \( f_\chi \chi/f = \lambda \chi \) tends to a constant). So, the presence of a mass term or a cosine potential for \( \phi \) does not prevent the energy-transfer mechanism from working.

Finally, let’s consider how the Hubble friction term affects the dynamics of the two-field system. This term acts as a time-dependent friction term for both fields and its time dependence breaks the time-translation symmetry. As long as \( H(t) \) is large enough for preventing \( \phi \) from “rolling down” its potential, there can not be efficient energy transfer.
However, as soon as the friction is small enough for $\phi$ to start evolving, the energy transfer commences as described before. Then, after most of the kinetic energy is transferred, $\chi$ will stop increasing and $\phi$ will again be subjected to the Hubble friction and its potential. On the other hand, in the absence of a potential for $\chi$, $\chi$ will asymptote to zero, since the Hubble friction will act to halt the $\chi$ evolution. The potential for $\chi$ may change this late behaviour, but as long as it does not have enough amplitude to play a role during the energy transfer phase, the dynamics during that period goes as described before. Indeed, after sequestering the energy from $\phi$, the energy density in $\chi$ redshifts as a stiff component, so that its potential energy might only be relevant once the energy density is completely negligible. We discuss numerical solutions of the full system in section 2.2.

In this paper we shall consider a model with $f(\chi) = e^{\lambda \chi}$ and a cosine potential for $\phi$, $V = m_\phi^2 (1 - \cos \phi)$. Strictly speaking $\lambda$ is an $O(1)$ number which depends on the choice of internal manifolds for string compactification, however we will henceforth treat it as a free parameter of our theory. Let us now estimate $f_{\text{Kmix}}(z_c)$, the fraction of the total energy density in the fields $\phi$ and $\chi$ at the redshift $z_c$ where it reaches its maximum. Assuming the $\chi$-potential does not play a role in the dynamics of our model, we have the following parameters: the initial values of the fields $\phi_i$ and $\chi_i$, the mass $m_\phi$, and the kinetic coupling parameter $\lambda$. Our goal is to now find $f_{\text{Kmix}}(z_c)$ and $z_c$ in terms of these parameters.

Similar to EDE, $\phi$ only becomes dynamical when $3H \sim \sqrt{V_{\phi\phi}/f}$. So, $z_c$ is fixed by

$$3H(z_c) \approx e^{-\lambda \chi_i}/2m_\phi \cos \phi_i. \quad (2.9)$$

Before $z_c$, the fraction of energy in $\chi$ is much smaller than in $\phi$. Moreover, since $\phi$ is essentially constant until approximately $z_c$, it will not source $\chi$ leading up to this point. Thus, $\chi(z_c) = \chi_i$ and we can approximate (neglecting the $\chi$ contribution to $V$)

$$f_{\text{Kmix}}(z_c) \approx e^{\lambda \chi_i} \phi(z_c)/2 + V(\phi(z_c)) \rho_{\text{tot}}(z_c). \quad (2.10)$$

Neglecting the kinetic energy in $\phi$ and approximating $\rho_{\text{tot}}(z_c)$ by $3H^2(z_c) \sim |V_{\phi\phi}(\phi_i)|/3f(\chi_i)$, we find that

$$f_{\text{Kmix}}(z_c) \propto f(\chi_i) V(\phi_i) / |V_{\phi\phi}(\phi_i)|. \quad (2.11)$$

Hence, $f_{\text{Kmix}}(z_c)$ is essentially set by the initial values of the fields and $\lambda$, in particular it does not depend on $m_\phi$

$$f_{\text{Kmix}}(z_c) \propto e^{\lambda \chi_i}/M_{\text{Pl}} \left(1 - \cos(\phi_i/M_{\text{Pl}}) / \cos(\phi_i/M_{\text{Pl}}) \right), \quad (2.12)$$

where we restored the units. The main difference from the standard EDE results is the presence of the $e^{\lambda \chi_i}$ factors in the equations above.

### 2.2 Numerics

To fully understand the background dynamics we have modified the publicly available Boltzmann code CLASS$^3$ [46, 47] to realize our model. For $\phi$ we use a standard axion potential and we supply $\chi$ with a small, constant potential that plays the role of a cosmological constant,

$$V(\chi, \phi) = m_\phi^2 f_\phi^2 \left(1 - \cos \phi / f_\phi \right) + \Lambda. \quad (2.13)$$

[^3]: http://class-code.net.
The axion mass and decay constant are free parameters, along with the initial value of $\phi$, but $\Lambda$ is chosen such that our cosmology fulfills the budget equation, $\sum_i \Omega_i = 1$. We display the evolution of the fraction of the total energy density in figure 1 for our model with example $H_0$ and $S_8$ resolving parameters (see eq. (3.5)). For comparison, we also overlay the evolution for a $n = 3$ EDE model with the same peak energy density, $f_{EDE} = 0.097$, and corresponding redshift, $z_c = 3350$, which we calculate using CLASS_EDE. Generically, EDE models with $n = 1$ potentials are ruled out as $w \approx 0$ once the field begins to oscillate at the bottom of its potential, preventing the energy density from depleting fast enough. This results in a significant addition to $\Omega_{cdm}$ and negatively alters the expansion history [26]. Figure 1 makes clear that this is simply not the case for this model: with natural values for $\lambda$ the increase to $\Omega_{cdm}$ is $\sim 0.1$ to 1%. As outlined in the last section, the ability for $\phi$ to redshift sufficiently stems from it being kinetically coupled to $\chi$. This can best be understood by looking at the evolution of the fields and their equations of state, which is presented in figure 2. We see that after $\phi$ starts to roll down its potential $\chi$ grows rapidly, which is intuitive as $\dot{\phi}^2$ sources $\chi$ (see eq. (2.3b)). This realizes a transfer of energy from $\phi$ into $\chi$, with $w \approx 1$, that allows the total energy density to redshift from $\approx 1/a^3$ to $1/a^6$ depending on the choice of $\lambda$. To demonstrate this, we plot in the left panel of figure 3 the combined equation of state for both fields, including the cycle average, for the example presented in figure 1. This shows that the combined energy density in this case redshifts (approximately) as radiation initially, which is qualitatively similar to $n = 2$ EDE models. The efficiency of this transfer is controlled primarily by the choice of $\lambda$: larger $\lambda$’s result in faster depletion of the energy density, which can also be understood from the source term in eq. (2.3b). We show this explicitly in the right panel of figure 3 where the cycle averaged equation of state for different values of $\lambda$ is plotted.

\footnote{https://github.com/mwt5345/class_ede_v3.2.0.}
3 Perturbations

To understand the role perturbations play in our model we derive the perturbation equations by varying the action, eq. (2.1), to quadratic order. Thus we will not take a fluid approximation and instead solve the full linearized, coupled Klein-Gordon equations along with the usual equations for the metric perturbations. In synchronous gauge we find that the equation of motion for the perturbation in $\chi$ is given by,

$$\delta \chi'' + 2aH \delta \chi' + \left[ k^2 + a^2 V_{\chi\chi} - \frac{1}{2} \lambda^2 (\phi')^2 \right] \delta \chi = \lambda f \phi' \delta \phi' - \frac{h' \chi'}{2},$$

(3.1)

and for $\phi$,

$$\delta \phi'' + \left[ 2aH + \lambda \chi' \right] \delta \phi' + \left[ k^2 + \frac{a^2}{f} V_{\phi\phi} \right] \delta \phi = \lambda \left( \frac{a^2}{f} V_{\phi} \delta \chi - \phi' \delta \chi' \right) - \frac{h' \phi'}{2},$$

(3.2)

where $'$ denotes a derivative with respect to conformal time, $H$ is the Hubble parameter in cosmic time, and $h$ is the trace of the spatial metric perturbation. In the limit that the initial field value for $\chi$ vanishes, which we generally assume, and the absence of kinetic coupling, $\lambda = 0$, the dynamics for perturbations (and background) reduce to that of EDE. Arguably, this is a desirable feature given that EDE can achieve great fits to the CMB with non-negligible parameter shifts relative to $\Lambda$CDM. Indeed, as pointed out in [16], any new dark species will modify the phase and amplitude of the CMB peaks from gravitationally driving the photon-baryon acoustic oscillations. In the case of EDE-like models, the scalar field perturbations oscillate leading up to recombination, rather than grow, which results in a suppression of the Weyl potential. While the physical dark matter density can be increased to maintain a good fit to the CMB, this affects the perturbations in the late-time universe by enhancing the growth of structure as quantified by a noticeable increase in the $S_8$ parameter.
Figure 3. Evolution of the combined equation of state for both fields (green) and the corresponding cycle average (red) for the cosmology used in figure 1 (left panel). We also plot the cycle average equations of state for differing values of \( \lambda \) (right panel). For reference we plot the equation of state for radiation (blue).

relative to \( \Lambda \)CDM. Clearly perturbations are what will make or break any model aimed at addressing the \( H_0 \) tension, so the new dynamics in our model are welcome to confront the challenges faced in the late-time universe.

3.1 Resolving the large-scale structure tension

One important feature of our model is that a fraction of \( \Omega_{\text{cdm}} \) will necessarily be an \( O(10^{-27} \text{ eV}) \) axion. This is unavoidable as the contribution is from \( \phi \) after its has fallen down its potential — a direct byproduct of a true axion potential. The fraction of dark matter in the light axion, \( F = \Omega_\phi/\Omega_{\text{cdm}} \), is controlled by \( \lambda \) — larger values result in smaller \( F \). As is well known, the presence of such a light axion can have a major impact on the growth of large-scale structure [38, 48, 49] as the sound speed for scalar dark matter is scale dependent [50, 51]. Below the Jeans scale, \( k_J/a = 6^{1/4}\sqrt{Hm} \), scalar dark matter density perturbations do not grow, but oscillate, resulting in a suppression of large-scale structure. However, during radiation domination the growth of dark matter density perturbations are already strongly suppressed due to the Mészáros effect [52], so deviations from \( F = 0 \) do not become relevant until after matter-radiation equality. We can get a handle on the expected impact by first calculating the Jeans wave number for \( \delta \phi \) at equality, \( k_J^{\phi} \sim 10^m^{1/2} \text{ Mpc}^{-1} \), which for a typical axion in our theory has \( k_J^{\phi} \sim 0.03 \text{ Mpc}^{-1} \). The impact of \( F \neq 0 \) will be a modification of the typical linear growth of dark matter perturbations post-equality for scales below \( k_J^{\phi} \):

\[
\delta_{\text{cdm}} \propto a \rightarrow a^q.
\]

The deviation can be approximated as [48, 49],

\[
q = \left( \sqrt{25 - 24F} - 1 \right)/4, \tag{3.3}
\]
and the suppression of the matter power spectrum for $k > k^\text{eq}_J$ is given by,

$$\frac{P_{\phi+\text{cdm}}}{P_{\text{cdm}}} \approx \left( \frac{k^\text{eq}_J}{k} \right)^{8(1-q)} ,$$

which manifests as a noticeable step in the matter power spectrum at $k^\text{eq}_J$ [48]. For a typical axion in our theory with $F = 0.01$, well outside current constraints from the CMB and LSS [53], there will be a $\sim 10\%$ suppression in power over scales sensitive to $S_8$, more than enough to resolve the tension.

While it is less influential, another feature of our model that helps alleviate the tension is that perturbations should not be suppressed as effectively pre-recombination. We can understand this by looking at the perturbed Klein-Gordon equations, eqs. (3.1) and (3.2). The most obvious effect is the presence of the extra source terms which should help to alleviate the decay of the Weyl potential. Note that in the thermal friction model presented in [43] there is also a source term in the perturbation equations between the scalar field and dark radiation which, in that case, has the consequence of degrading the fit to the CMB. This stems primarily from the extra source term (which is $\propto \phi^2$) dominating over the sourcing of the metric perturbations, resulting in the suppression of the radiation perturbations relative to other species. However, the source term in our model will be a much weaker effect as the transfer of energy between fields has less influence on the background dynamics compared to thermal friction models. Thus, our source terms should serve as a small correction to metric perturbations and, by extension, reduce the required physical dark matter density to fit the CMB.

### 3.2 Avoiding Planckian field excursions

A less prominent, but still important, theoretical hurdle of best-fit EDE models with $n = 3$ potentials is the preference for large initial field displacements, i.e. $\theta_i \sim \pi$. This is problematic as best-fit values for the decay constant are near the Planck scale [27] meaning the EDE scalar undergoes Planckian field excursions, $\Delta \phi \sim M_{\text{pl}}$, which is a major theoretical issue from the perspective of the Distance and the Weak Gravity Conjectures [54, 55]. What forces EDE models to prefer large field values is the impact on perturbations, particularly from constraints by Planck EE and TE power spectra.\(^5\) Specifically, as pointed out in [16, 26], for a good fit to the CMB it is important to maximize the number of modes around the time of energy injection with a sound speed $c_s^2 \approx w$, provided $w > 1/3$. The range of modes inside the horizon with the right sound speed is maximized (see [26] for more details) for $\theta_i^n - \sqrt{\frac{E_n}{E_{n,\theta}}} \gg 1$, where $E_n = (1 - \cos \theta)^n$ is the effective potential. For $n = 3$ this requires $\theta_i \sim \pi$, whereas this ratio becomes large for $\theta_i \sim \pi/2$ in $n = 1$ models. This can be understood as the $n = 1$ potential becoming convex for smaller $\theta_i$ relative to $n = 3$ potentials. With this insight in mind, our model should accommodate the same energy injection as $n = 3$ EDE but with $\Delta \phi \sim 0.35 M_{\text{pl}}$, alleviating the concern for Planckian field excursions.

### 3.3 Parameter space of the model

To see if our theory does indeed exhibit the right properties to address the Hubble and LSS tensions, we have explored the parameters space of our model. In figure 4 we show the CMB

---

\(^5\)While [31] are not able to well constrain $\theta_i$, for a CMB analysis independent of Planck with ACT data there appears to be a preference for much smaller values of $\theta_i$. 
Figure 4. Difference in CMB temperature (top), polarization (middle), and cross-correlation (bottom) power spectra between best-fit ΛCDM model from the Planck 2018 analysis and KMIX with parameters given in eq. (3.5) in units of cosmic variance per ℓ-mode of the best-fit ΛCDM model.

temperature, polarization, and cross-correlation power spectra for our model with example $H_0$ and $S_8$ resolving parameters,

$$H_0 = 72.75 \text{ km/s/Mpc} \quad 100\omega_b = 2.283, \quad \omega_{\text{cdm}} = 0.1285 \quad 10^9 A_s = 2.250,$$

$$n_s = 0.970 \quad \tau_{\text{reio}} = 0.055, \quad \lambda = \sqrt{1.6} \quad \theta_1 = 1.60,$$

$$m_\phi = 1.00 \times 10^{-27} \text{ eV} \quad f_\phi = 4.50 \times 10^{26} \text{ eV},$$

relative to ΛCDM with best-fit parameters from the Planck 2018 analysis [2],

$$H_0 = 67.32 \text{ km/s/Mpc} \quad 100\omega_b = 2.283, \quad \omega_{\text{cdm}} = 0.1201 \quad 10^9 A_s = 2.210,$$

$$n_s = 0.9661 \quad \tau_{\text{reio}} = 0.0543,$$

in units of cosmic variance per ℓ-mode of the best-fit ΛCDM model. The residuals between the two models are well within cosmic variance, so the parameters given by eq. (3.5) can be understood to represent a good realization of the CMB. Interestingly, considering the values used in eq. (3.5), we can already see the impact of the source term in the perturbations. The best-fit Planck EDE model from [26] necessitated $\omega_{\text{cdm}} = 0.1306$, an increase of $\sim 9\%$ over ΛCDM, to achieve an $H_0 = 72.19$ km/s/Mpc. In contrast, our example accommodates a larger $H_0$ value but with a lower physical dark matter density.
Figure 5. Linear matter power spectra (left panel) and ratio of spectra (right panel) at $z = 0$ for $\Lambda$CDM with the DES-Y3 + BAO + BBN best-fit values $\Omega_m = 0.296$, $S_8 = 0.809$, $\Omega_b = 0.048$, $H_0 = 67.3$ km/s/Mpc from [56] (black), the best-fit $\Lambda$CDM from the Planck 2018 analysis [2] (green), EDE best-fit from [27] (blue), and for the KMIX model with parameters given in eq. (3.5) (red). Ratios in the right panel are taken with respect to the DES-Y3 $\Lambda$CDM spectrum, thus giving an indication of how well the other models' predictions match the DES-Y3 constraints. The blue shaded region shows the approximate range of comoving wavenumbers probed in the DES-Y3 analysis.

To see how perturbations for this example evolve into the late-time universe we also calculate the linear matter power spectrum in CLASS, figure 5. For comparison we also plot the power spectra for the best-fit $\Lambda$CDM model from the Planck 2018 analysis [2], best-fit Planck EDE for CMB with parameters from [27], and for $\Lambda$CDM with the best-fit DES-Y3 + BAO + BBN values from [56]. The blue region corresponds to the approximate comoving wave numbers probed by DES. What is clear from this plot is that our model does not suffer from the same excess of power present in the best-fit Planck $n = 3$ EDE model and, instead, performs similarly to the best-fit Planck $\Lambda$CDM model. In fact, this is exactly what we expect to see for our model. Specifically, notice that the matter power spectrum has a clear step in the residuals plot starting $\sim 0.03$ Mpc$^{-1}$ and realizes a suppression of $\sim 10\%$. This is exactly the effect we anticipate for the axion in our theory if $F = 0.01$ (see figure 1). For this example we calculate $S_8 = 0.807$, whereas the best-fit Planck $\Lambda$CDM model has $S_8 = 0.831$ and the best-fit Planck EDE model has $S_8 = 0.847$. Note that the $S_8$ value for our model is lower than Planck, despite having more power in figure 5, as $\Omega_m$ is smaller. While eq. (3.5) are not the best-fit parameters for our model, this would of course necessitate a full MCMC analysis, these results corroborate our expectation from the theory that there should be regions of parameter space that have the right dynamics to solve both the Hubble and $S_8$ tensions. Concretely, for values that produce CMB power spectra close to the best-fit $\Lambda$CDM result but with large $H_0$ values, we see in the late-time universe the matter power spectrum does not have a significant excess of power at scales sensitive to $S_8$. 

– 12 –
4 Discussion and conclusion

In this work we have presented a new string theory inspired model which has the ability to resolve both the Hubble and $S_8$ tensions. The dynamics are governed by two fields, an axion and a dilaton which are kinetically coupled. Our axion plays a similar role to the scalar field in EDE in that its purpose is to decrease the size of the sound horizon. The dilaton plays a critical role through its coupling to the axion: it siphons off energy density as the axion falls down its potential allowing it to redshift faster. These dynamics allow use of a standard axion cosine potential viable for solving the Hubble tension, unlike in EDE. From a theoretical perspective, this a significant improvement over EDE models where one must choose a potential with fine-tuned coefficients for leading terms in the instanton expansion to get the correct background dynamics.

More still, the axion in our model plays a double role. Given that it must have a mass $\mathcal{O}(10^{-27}$ eV) to resolve the Hubble tension and contribute a small fraction to dark matter post-recombination, it will naturally suppress power on scales sensitive to $S_8$. Concretely, if it contributes 1% to $\Omega_{\text{cdm}}$, the matter power spectrum will be suppressed for relevant $k$ by $\sim 10\%$, more than enough to resolve the $S_8$ tension. While such a small fraction is not ruled out by the CMB and LSS [53], future measurements of the 21 cm signal [57] will be able to constrain small fractions of very light axion dark matter. Interestingly, in [38] a phenomenological model was studied with a period of early dark energy but supplemented by a small contribution from a separate very light scalar. Their results were found to alleviate both the $H_0$ and $S_8$ tensions. Since our model can be seen as a concrete and physically well motivated realization of their toy model, this makes the prospect for viability of our model promising. Furthermore, a similar action to eq. (2.1) was studied in [58], but in the context of axion dark matter relic abundances. The dynamics explored in that work are similar to how $F$ can be controlled with a choice of $\lambda$ in our model.

From the theory side, it is worthwhile to recall that the amplitude of the axion potential should be a fraction of the energy scale of the background close to recombination. Since our model is likely to be embedded into string theory, this implies that we should be able to reproduce $\mathcal{O}(1)$ eV-scale physics from the string scale. Assuming the axionic Weak Gravity Conjecture, we need an ultralight axion for that to be possible. So, one might think that there is some sort of “coincidence” or “why now” issue with our construction, like in all EDE models. However, this is likely to be solved by the UV completion in our model while, on the other hand, there are no proposals for such completions in the $n=3$ EDE case. In fact, semi-realistic string models containing ultralight axions were recently discussed in [59, 60].

Going beyond analytics, we have also used a modified version of the Boltzmann code CLASS to explore the parameter space of our model. Calculating the background dynamics, CMB power spectra and linear matter power spectrum we have identified regions that can accommodate larger values for $H_0$ without increasing $S_8$ — a $\sim 10\%$ increase in $H_0$ from the Planck inferred value results in $S_8$ values consistent with LSS constraints. However, we again stress that we have not reported the best-fit parameters for our theory but have instead displayed a representative cosmology that exhibits the features of a Hubble and $S_8$ tension resolving model. We leave the detailed investigation of the perturbations and a full MCMC analysis for future work.

Besides what we have already discussed, one of the appealing properties of our model is that we only have one additional free parameter compared to EDE, this is $\lambda$. What makes this parameter particularly interesting is that it is related to internal manifolds of the string
compactification, realizing a potential avenue to constrain string theory. From another point
of view, similar to how one makes a choice of \( n = 3 \) in EDE models, one could also select
a well informed \( \lambda \) and reduce the model parameters by one. Furthermore, a simple \( m^2 \phi^2 \)
potential will suffice to resolve the Hubble tension at the background level, i.e. just the leading
term in the limit \( \phi/f_\phi \) is small, with the added benefit of further reducing the number of
parameters. However, its not clear that data will favor this choice as power law potentials
generally do not fit CMB data well.

To conclude on a more speculative note, it is interesting that the dynamics of our model
prefer that the potential for the dilaton be small. While our choice to implement (late-time)
dark energy as the constant potential of \( \chi \) was done to fulfill the budget equation in \textit{CLASS},
it would be interesting if a connection between a period of early dark energy and dark energy
could be made. Indeed, it is well known that a dilaton can explain late-time dark energy (see
for example [61]) and motivates exploring a true dilaton potential, \( V(\chi) = V_0 e^{\beta \chi} \).

Acknowledgments

We would like to thank Kim Berghaus, Robert Brandenberger, Colin Hill, and Evan McDonough
for helpful comments on an early draft of this work. We additionally thank Steven Clark,
Mikhail Ivanov, Tanvi Karwal, Oliver Philcox, and David Spergel for helpful discussions.
H.B. would like to thank the CCA/Flatiron Institute, the Symmetry and Cosmology group of
the Humboldt University of Berlin and ETH-Zurich for hospitality during the execution
of this work. H.B. research is supported by the Fonds de Resercher du Québec (PBEEE/303549)
and partially by funds from NSERC.

A String theoretic embedding of KMIX

We now review how scalar sectors of the form in eq. (1.2) arise in string theory. It is customary
to compactify string theory on internal spaces that preserve \( N = 1 \) at low energies [62, 63].
After dimensional reduction, the four-dimensional action contains many fields associated
to the shape and size of the internal manifold, but since the effective theory should be a
four-dimensional \( N = 1 \) supergravity, these should come in supersymmetric multiplets whose
action is fixed by a few functions of the multiplets: a Kähler potential \( K \) that fixes the
kinetic term of the scalar moduli, a gauge kinetic function that fixes the kinetic term of gauge
fields and a superpotential that determines the interactions among the fields [64, 65]. An
important issue in string phenomenology is to explain how to obtain these functions from
ten-dimensional physics, and the literature on moduli stabilization is very extensive. For
us, the important feature of these effective actions is the presence of the kinetic coupling
mentioned above, so we will focus on the Kähler potentials that originates from string theory.

For both heterotic and type II superstring theories, the Kähler potential for certain
chiral multiplets has a “no-scale” form [66], generically given by [67]
\[
K = -\ln(S + \bar{S}) - \ln V, \tag{A.1}
\]
where the so called axio-dilaton moduli is a complex field whose real part is related to the
dilaton \( \Phi \) and its imaginary part is an axion \( a, S = e^{-\Phi} + ia \). The axion appears after
dualizing the four-dimensional components of the Kalb-Rammond 2-form field that is present
in any perturbative limit of the theory. For that reason, \( a \) is also referred to as the model-
independent axion (because it does not depend on the details of the compactification) [68].
The quantity $V$ is the volume of the internal manifold in string-scale units and, for Calabi-Yau (CY) three-folds,

$$V = \frac{1}{6} d_{ijk} t^i t^j t^k,$$

where $t^i = (T^i + \bar{T}^i)/2$ with $T^i$ being the (complexified) size of 2-cycles in the internal geometry, and $d_{ijk}$ is the triple intersection number of the CY manifold.

The Kähler metric $K_{i\bar{j}} = \partial_i \partial_\bar{j} K$ is the field space metric that sets the kinetic terms for the complex fields $S$ and $T^i$ (the indices $I$ and $J$ are associated to axio-dilaton and Kähler fields). The moduli fields are canonically normalized only if $K_{i\bar{j}}$ can be diagonalized everywhere in the field space. Generically, for Kähler potentials of the form in eq. (A.1), there will be a kinetic coupling between pairs of moduli fields. As an illustrative example, for a straight toroidal orbifold where all direction have the same radius, there is only one modulus, setting the overall volume of the torus. In this case, we might write $V = (T + \bar{T})^3$ [69]. Then, the kinetic term for the axio-dilaton and Kähler moduli will be

$$K_{i\bar{j}} \partial_\mu T^i \partial_\nu \bar{T}^j g^{\mu\nu} = \frac{1}{(S + \bar{S})^2} \partial_\mu S \partial^\mu S + \frac{3}{(T + \bar{T})^2} \partial_\mu T \partial^\mu T$$

$$= \frac{1}{4} (\partial \Phi)^2 + \frac{e^{2\Phi}}{4} (\partial a)^2 + \frac{3}{4} (\partial \Psi)^2 + \frac{3 e^{-2\Psi}}{4} (\partial b)^2,$$

where we decomposed $S = e^{-\Phi} + ia$ and $T = e^{\Psi} + ib$ in the second equality. Hence, the kinetic coupling function has an exponential form. We should stress that such mixing is mainly due to the existence of extra dimensions and the kinetic term for $p$-form fields in higher dimensions, so it might also appear in other extra dimensional theories whether they come from string theory or not. For more complicated internal manifolds, the kinetic coupling might be even more involved, but since $K_{ij}$ is hermitian, it will not depend on the imaginary part of $T^i$ and so its kinetic term will always be non-canonical and couple with $(T + \bar{T})^i$. That is, string theory axions generically exhibit the kinetic coupling with other moduli. Of course, if the latter are assumed to be stabilized, the axion can be canonically normalized and its decay constant will depend on these stabilized values (see [70, 71] for early studies on string axions in cosmology). But if that is not the case, the axion will have a non-trivial kinetic coupling or, in other words, a spacetime-dependent decay constant. Varying decay constants may also appear for throat-axions, which are axions that descend from fluxes along warped throats in the internal space [59, 72–75].

## B Perturbation equations

In this appendix we derive the perturbations for our theory and discuss the implementation of their dynamics in CLASS.\textsuperscript{7} We start by decomposing the two scalar fields into smooth time dependant background and spatially varying perturbations,

$$\phi(\vec{x}, \tau) = \phi(\tau) + \delta \phi(\vec{x}, \tau),$$

$$\chi(\vec{x}, \tau) = \chi(\tau) + \delta \chi(\vec{x}, \tau).$$

\textsuperscript{6}This is specific to hetoritic strings, for type II strings one usually writes $V$ in terms of 4-cycles moduli. This technical point is irrelevant to our discussion.

\textsuperscript{7}Note that until CLASS v2.10 differential equations were solved with respect to conformal time. This forced one to use a fluid approach due to the increasing frequency of scalar field oscillations with $\tau$ for such light scalar fields with $n = 1$ potentials. This problem is now alleviated as new versions of CLASS solve the dynamics with respect to $\log a$. 

- 15 -
In synchronous gauge the perturbed metric is given by,

\[ ds^2 = a^2(\tau) \left(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j \right). \]  

(B.3)

The metric perturbation \( h_{ij} \) can be decomposed into trace and traceless components,

\[ h_{ij} = \frac{1}{3} h \delta_{ij} + \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) \mu + \partial_i A_j + h_{ij}^T, \]  

(B.4)

where \( h = h_{ii} \) and \( \mu \) are scalar, \( A_i \) vector, and \( h_{ij}^T \) tensor perturbations, respectively. Considering only scalar perturbations, the real space degrees of freedom are related to those in \( k \)-space as,

\[ h_{ij}(\vec{x}, \tau) = \int d^3k e^{i \vec{k} \cdot \vec{x}} \left[ \hat{k}_i \hat{k}_j h(\vec{k}, \tau) + \left( \hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) 6 \eta(\vec{k}, \tau) \right], \]  

(B.5)

where \( \vec{k} = k \hat{\vec{k}} \). In Fourier transforming we use \( \partial_i \rightarrow ik \) and \( \partial_i \partial_i \rightarrow -k^2 \). The equations of motion are found from varying the action, eq. (2.1), to quadratic order in \( \delta \chi \) and \( \delta \phi \). Doing this we find,

\[ \delta \chi'' + 2aH \delta \chi' + \left[ k^2 + a^2 \chi'' - \frac{1}{2} \lambda^2 f(\phi')^2 \right] \delta \chi = \lambda f \phi' \delta \phi' - \frac{h' \chi'}{2}, \]  

(B.6)

\[ \delta \phi'' + [2aH + \lambda \chi] \delta \phi' + \left[ k^2 + \frac{a^2}{f} V_{\phi\phi} \right] \delta \phi = \lambda \left( \frac{a^2}{f} V_{\phi \delta \chi} - \phi' \delta \chi' \right) - \frac{h' \phi'}{2}. \]  

(B.7)

To include our model in CLASS we need to translate our second order perturbed Klein-Gordon equations into the equivalent set of first order fluid equations. The energy-momentum tensor for our model is just the sum of the contributions from both fields,

\[ T_{\mu\nu} = T_{\mu\nu}^{\phi} + T_{\mu\nu}^{\chi}, \]  

(B.8)

where the energy-momentum tensor for \( \chi \) is the same as for a typical scalar field,

\[ T^{(\chi)\mu}_{\nu} = \partial^\mu \chi \partial^\nu \chi - \delta^\mu_{\nu} \left( \frac{1}{2} \partial^\alpha \chi \partial^\alpha \chi + V(\chi) \right). \]  

(B.9)

The energy-momentum tensor for \( \phi \) is modified due to the kinetic coupling and becomes,

\[ T^{(\phi)\mu}_{\nu} = f \partial^\mu \phi \partial^\nu \phi - \delta^\mu_{\nu} \left( \frac{1}{2} f \partial^\alpha \phi \partial^\alpha \phi + V(\phi) \right). \]  

(B.10)

Importantly, the Bianchi identities demand that the combined energy-momentum tensor be conserved,

\[ \nabla_\mu \left( T^{(\phi)\mu}_{\nu} + T^{(\chi)\mu}_{\nu} \right) = 0. \]  

(B.11)

Furthermore, we can read off the density and pressure from the energy-momentum tensors,

\[ \rho_{\chi} = \frac{1}{2a^2} \chi'^2 + V(\chi), \]  

(B.12)

\[ p_{\chi} = \frac{1}{2a^2} \chi'^2 - V(\chi), \]  

(B.13)
which resemble that for a standard scalar field. \( \phi \) is again modified due to the kinetic coupling,

\[
\rho_\phi = \frac{1}{2a^2} f \dot{\phi}^2 + V(\phi),
\]

\[
p_\phi = \frac{1}{2a^2} f \dot{\phi}^2 - V(\phi).
\]  

We now want to calculate the perturbed density and pressure for both fields. Starting first with the \( \chi \) field,

\[
p_\chi = -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi - V(\chi),
\]

\[
\rho_\chi = -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi + V(\chi),
\]

which we perturb to find,

\[
\delta p_\chi = -\frac{1}{2} \lambda \delta g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi - \frac{1}{2} \lambda \delta g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \delta \chi - V_\chi \delta \chi,
\]

\[
\delta \rho_\chi = -\frac{1}{2} \lambda \delta g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi - \frac{1}{2} \lambda \delta g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \delta \chi + V_\chi \delta \chi.
\]

Now plugging in for our metric we have,

\[
\delta p_\chi = \frac{1}{a^2} \dot{\chi}^2 - V_\chi \delta \chi,
\]

\[
\delta \rho_\chi = \frac{1}{a^2} \dot{\chi}^2 + V_\chi \delta \chi,
\]

which agree with the general result for a scalar field as derived in eq. A4 of [50]. Repeating the same procedure but for \( \phi \),

\[
p_\phi = -\frac{1}{2} f g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi),
\]

\[
\rho_\phi = -\frac{1}{2} f g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi).
\]

We now consider perturbations to the above,

\[
\delta p_\phi = -\frac{1}{2} f' \lambda \dot{\chi} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} f \delta g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} f g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \delta \phi + V_\phi \delta \phi,
\]

\[
\delta \rho_\phi = -\frac{1}{2} f' \lambda \dot{\chi} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} f \delta g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} f g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \delta \phi + V_\phi \delta \phi.
\]

These reduce to,

\[
\delta p_\phi = \frac{1}{a^2} f \dot{\phi} \delta \phi - V_\phi \delta \phi + \frac{\lambda}{2a^2} f \dot{\phi}^2 \delta \chi,
\]

\[
\delta \rho_\phi = \frac{1}{a^2} f \dot{\phi} \delta \phi + V_\phi \delta \phi + \frac{\lambda}{2a^2} f \dot{\phi}^2 \delta \chi,
\]

and both reduce to the same form as the dilaton in absence of kinetic coupling, i.e. \( f = 1 \).

We now calculate the scalar field velocity perturbations. A fluid with small coordinate velocity \( v^i \) has, to linear order in perturbations, a stress tensor component defined as,

\[
(\rho + p) v_j = \delta T^0_j.
\]
Using this equation, we will be able to calculate the velocity divergence, $\theta_A \equiv \partial_t v^i_A$, which is needed to solve the linearized Einstein equations in CLASS (see eq. 22 in [76]). Starting with $\chi$, we perturb the energy-momentum tensor,

$$
\delta T^{(\chi)}_{\mu \nu} = \delta \left[ \partial^\mu \chi \partial^\nu \chi - \delta^\mu_{\nu} \left( \frac{1}{2} \partial^\alpha \chi \partial_\alpha \chi + V(\chi) \right) \right],
$$

(B.29)

notice that since we want an off diagonal component, we can ignore terms proportional to $\delta^\mu_{\nu}$ completely. Now we have,

$$
\delta T^{(\chi)}_{\mu \nu} = \delta g^{\rho \mu} \partial^\rho \chi \partial^\nu \chi + g^{\rho \mu} \left( \partial_\rho \delta \chi \partial^\nu \chi + \partial^\rho \chi \partial_\nu \delta \chi \right).
$$

(B.30)

Since we need the $\delta T^0_j$ component, this leaves us with,

$$
\delta T^{(\chi)}_{0 j} = -\frac{1}{a^2} \dot{\chi} \partial_j \delta \chi.
$$

(B.31)

We can now plug this into our defining equation for the velocity perturbations,

$$
(\rho_\chi + p_\chi) v_j = -\frac{1}{a^2} \dot{\chi} \partial_j \delta \chi,
$$

(B.32)

and take the gradient,

$$
(\rho_\chi + p_\chi) \partial^j v_j = -\frac{1}{a^2} \dot{\chi} \partial^j \partial_j \delta \chi.
$$

(B.33)

Since the velocity divergence is defined as, $\theta_A \equiv \partial_t v^i_A$, and we use the convention that $\partial_j \partial^j = -k^2$, we arrive at our desired result,

$$
(\rho_\chi + p_\chi) \theta_\chi = \frac{k^2}{a^2} \dot{\chi} \delta \chi,
$$

(B.34)

which is the standard result for the velocity divergence of a scalar field, see eq. A6 of [50] and eq. 16 of [53].

Let us now consider the contribution for $\phi$. We begin in the same fashion by perturbing the energy-momentum tensor,

$$
\delta T^{(\phi)}_{\mu \nu} = \delta \left[ f \partial^\mu \phi \partial^\nu \phi - \delta^\mu_{\nu} \left( \frac{1}{2} f \partial^\alpha \phi \partial_\alpha \phi + V(\phi) \right) \right],
$$

(B.35)

which reduces to,

$$
\delta T^{(\phi)}_{0 j} = -\frac{f}{a^2} \dot{\phi} \partial_j \delta \phi,
$$

(B.36)

such that we can now write,

$$
(\rho_\phi + p_\phi) v_j = -\frac{f}{a^2} \dot{\phi} \partial_j \delta \phi.
$$

(B.37)

Taking the divergence of both sides yields,

$$
(\rho_\phi + p_\phi) \partial^j v_j = -\frac{f}{a^2} \dot{\phi} \partial^j \partial_j \delta \phi,
$$

(B.38)

which gives us,

$$
(\rho_\phi + p_\phi) \theta_\phi = \frac{k^2 f}{a^2} \dot{\phi} \delta \phi,
$$

(B.39)

which again reduces to the standard result for $f = 1$. 

– 18 –
References

[1] L. Verde, T. Treu and A.G. Riess, Tensions between the Early and the Late Universe, Nature Astron. 3 (2019) 891 [arXiv:1907.10625] [nSPIRE].

[2] PLANCK collaboration, Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6 [Erratum ibid. 652 (2021) C4] [arXiv:1807.06209] [nSPIRE].

[3] A.G. Riess et al., Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics beyond ΛCDM, Astrophys. J. 876 (2019) 85 [arXiv:1903.07603] [nSPIRE].

[4] E. Abdalla et al., Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies, JHEAp 34 (2022) 49 [arXiv:2203.06142] [nSPIRE].

[5] E. Di Valentino et al., In the realm of the Hubble tension — a review of solutions, Class. Quant. Grav. 38 (2021) 153001 [arXiv:2103.01183] [nSPIRE].

[6] A.G. Riess et al., A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s$^{-1}$ Mpc$^{-1}$ Uncertainty from the Hubble Space Telescope and the SH0ES Team, Astrophys. J. Lett. 934 (2022) L7 [arXiv:2112.04510] [nSPIRE].

[7] A. Sandage et al., The Hubble Constant: A Summary of the HST Program for the Luminosity Calibration of Type Ia Supernovae by Means of Cepheids, Astrophys. J. 653 (2006) 843 [astro-ph/0603647] [nSPIRE].

[8] W.L. Freedman et al., Calibration of the Tip of the Red Giant Branch (TRGB), arXiv:2002.01550 [DOI:10.3847/1538-4357/ab7339] [nSPIRE].

[9] C.D. Huang et al., Hubble Space Telescope Observations of Mira Variables in the Type Ia Supernova Host NGC 1559: An Alternative Candle to Measure the Hubble Constant, arXiv:1908.10883 [DOI:10.3847/1538-4357/ab5dbd] [nSPIRE].

[10] M.M. Ivanov et al., Constraining Early Dark Energy with Large-Scale Structure, Phys. Rev. D 102 (2020) 103502 [arXiv:2006.11235] [nSPIRE].

[11] N. Schöneberg et al., The H0 Olympics: A fair ranking of proposed models, Phys. Rept. 984 (2022) 1 [arXiv:2107.10291] [nSPIRE].

[12] T. Karwal and M. Kamionkowski, Dark energy at early times, the Hubble parameter, and the string axiverse, Phys. Rev. D 94 (2016) 103523 [arXiv:1608.01309] [nSPIRE].

[13] V. Poulin, T.L. Smith, T. Karwal and M. Kamionkowski, Early Dark Energy Can Resolve The Hubble Tension, Phys. Rev. Lett. 122 (2019) 221301 [arXiv:1811.04083] [nSPIRE].

[14] P. Agrawal, F.-Y. Cyr-Racine, D. Pinner and L. Randall, Rock ‘n’ Roll Solutions to the Hubble Tension, arXiv:1904.01016 [nSPIRE].

[15] S. Alexander and E. McDonough, Axion-Dilaton Destabilization and the Hubble Tension, Phys. Lett. B 797 (2019) 134830 [arXiv:1904.08912] [nSPIRE].

[16] M.-X. Lin, G. Benevento, W. Hu and M. Raveri, Acoustic Dark Energy: Potential Conversion of the Hubble Tension, Phys. Rev. D 100 (2019) 063542 [arXiv:1905.12618] [nSPIRE].

[17] A. Perez, D. Sudarsky and E. Wilson-Ewing, Resolving the $H_0$ tension with diffusion, Gen. Rel. Grav. 53 (2021) 7 [arXiv:2001.07536] [nSPIRE].

[18] F. Niedermann and M.S. Sloth, New early dark energy, Phys. Rev. D 103 (2021) L041303 [arXiv:1910.10739] [nSPIRE].

[19] J. Sakstein and M. Trodden, Early Dark Energy from Massive Neutrinos as a Natural Resolution of the Hubble Tension, Phys. Rev. Lett. 124 (2020) 161301 [arXiv:1911.11760] [nSPIRE].
[20] G. Ye and Y.-S. Piao, *Is the Hubble tension a hint of AdS phase around recombination?*, *Phys. Rev. D* **101** (2020) 083507 [arXiv:2001.02451] [inSPIRE].

[21] K. Freese and M.W. Winkler, *Chain early dark energy: A Proposal for solving the Hubble tension and explaining today’s dark energy*, *Phys. Rev. D* **104** (2021) 083533 [arXiv:2102.13655] [inSPIRE].

[22] M. Braglia et al., *Unified framework for early dark energy from $\alpha$-attractors*, *Phys. Rev. D* **102** (2020) 083513 [arXiv:2005.14053] [inSPIRE].

[23] A. Gogoi, R.K. Sharma, P. Chanda and S. Das, *Early Mass-varying Neutrino Dark Energy: Nugget Formation and Hubble Anomaly*, *Astrophys. J.* **915** (2021) 132 [arXiv:2005.11889] [inSPIRE].

[24] M.S. Turner, *Coherent Scalar Field Oscillations in an Expanding Universe*, *Phys. Rev. D* **28** (1983) 1243 [inSPIRE].

[25] V. Poulin et al., *Cosmological implications of ultralight axionlike fields*, *Phys. Rev. D* **98** (2018) 083525 [arXiv:1806.10608] [inSPIRE].

[26] T.L. Smith, V. Poulin and M.A. Amin, *Oscillating scalar fields and the Hubble tension: a resolution with novel signatures*, *Phys. Rev. D* **101** (2020) 063523 [arXiv:1908.06995] [inSPIRE].

[27] J.C. Hill, E. McDonough, M.W. Toomey and S. Alexander, *Early dark energy does not restore cosmological concordance*, *Phys. Rev. D* **102** (2020) 043507 [arXiv:2003.07350] [inSPIRE].

[28] G. D’Amico, L. Senatore, P. Zhang and H. Zheng, *The Hubble Tension in Light of the Full-Shape Analysis of Large-Scale Structure Data*, *JCAP* **05** (2021) 072 [arXiv:2006.12420] [inSPIRE].

[29] L. Herold, E.G.M. Ferreira and E. Komatsu, *New Constraint on Early Dark Energy from Planck and BOSS Data Using the Profile Likelihood*, *Astrophys. J. Lett.* **929** (2022) L16 [arXiv:2112.12140] [inSPIRE].

[30] T.L. Smith et al., *Early dark energy is not excluded by current large-scale structure data*, *Phys. Rev. D* **103** (2021) 123542 [arXiv:2009.10740] [inSPIRE].

[31] J.C. Hill et al., *Atacama Cosmology Telescope: Constraints on prerecombination early dark energy*, *Phys. Rev. D* **105** (2022) 123536 [arXiv:2109.04451] [inSPIRE].

[32] S. Vagnozzi, *Consistency tests of $\Lambda$CDM from the early integrated Sachs-Wolfe effect: Implications for early-time new physics and the Hubble tension*, *Phys. Rev. D* **104** (2021) 063524 [arXiv:2105.10425] [inSPIRE].

[33] HSC collaboration, *Cosmology from cosmic shear power spectra with Subaru Hyper Suprime-Cam first-year data*, *Publ. Astron. Soc. Jap.* **71** (2019) 43 [arXiv:1809.09148] [inSPIRE].

[34] KiDS collaboration, *KiDS-1000 Cosmology: Cosmic shear constraints and comparison between two point statistics*, *Astron. Astrophys.* **645** (2021) A104 [arXiv:2007.15633] [inSPIRE].

[35] DES collaboration, *Dark Energy Survey Year 3 results: Cosmology from cosmic shear and robustness to modeling uncertainty*, *Phys. Rev. D* **106** (2022) 023515 [arXiv:2105.13544] [inSPIRE].

[36] R.C. Nunes and S. Vagnozzi, *Arbitrating the S8 discrepancy with growth rate measurements from redshift-space distortions*, *Mon. Not. Roy. Astron. Soc.* **505** (2021) 5427 [arXiv:2106.01208] [inSPIRE].

[37] S.J. Clark, K. Vattis, J.J. Fan and S.M. Koushiappas, *The $H_0$ and $S_8$ tensions necessitate early and late time changes to $\Lambda$CDM*, arXiv:2110.09562 [inSPIRE].

[38] I.J. Allali, M.P. Hertzberg and F. Rompineve, *Dark sector to restore cosmological concordance*, *Phys. Rev. D* **104** (2021) L081303 [arXiv:2104.12798] [inSPIRE].
[39] G. Ye, J. Zhang and Y.-S. Piao, Alleviating both $H_0$ and $S_8$ tensions: Early dark energy lifts the CMB-lockdown on ultralight axion, *Phys. Lett. B* **839** (2023) 137770 [arXiv:2107.13391] [insSPIRE].

[40] A. Reeves et al., Restoring cosmological concordance with early dark energy and massive neutrinos?, *Mon. Not. Roy. Astron. Soc.* **520** (2023) 3688 [arXiv:2207.01501] [insSPIRE].

[41] E. McDonough et al., Early dark sector, the Hubble tension, and the swampland, *Phys. Rev. D* **106** (2022) 043525 [arXiv:2112.09128] [insSPIRE].

[42] K.V. Berghaus and T. Karwal, Thermal Friction as a Solution to the Hubble Tension, *Phys. Rev. D* **101** (2020) 083537 [arXiv:1911.06281] [insSPIRE].

[43] K.V. Berghaus and T. Karwal, Thermal Friction as a Solution to the Hubble and Large-Scale Structure Tensions, arXiv:2204.09133 [insSPIRE].

[44] D. Aloni et al., A Step in understanding the Hubble tension, *Phys. Rev. D* **105** (2022) 123516 [arXiv:2111.00014] [insSPIRE].

[45] M. Joseph et al., A Step in Understanding the $S_8$ Tension, arXiv:2207.03500 [insSPIRE].

[46] J. Lesgourgues, The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview, arXiv:1104.2932 [insSPIRE].

[47] D. Blas, J. Lesgourgues and T. Tram, The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes, *JCAP* **07** (2011) 034 [arXiv:1104.2933] [insSPIRE].

[48] D.J.E. Marsh and P.G. Ferreira, Ultra-Light Scalar Fields and the Growth of Structure in the Universe, *Phys. Rev. D* **82** (2010) 103528 [arXiv:1009.3501] [insSPIRE].

[49] L. Amendola and R. Barbieri, Dark matter from an ultra-light pseudo-Goldstone-boson, *Phys. Lett. B* **642** (2006) 192 [hep-ph/0509257] [insSPIRE].

[50] W. Hu, Structure formation with generalized dark matter, *Astrophys. J.* **506** (1998) 485 [astro-ph/9801234] [insSPIRE].

[51] J.-C. Hwang and H. Noh, Axion as a Cold Dark Matter candidate, *Phys. Lett. B* **680** (2009) 1 [arXiv:0902.4738] [insSPIRE].

[52] P. Mészáros, The behaviour of point masses in an expanding cosmological substratum, *Astron. Astrophys.* **37** (1974) 225 [insSPIRE].

[53] R. Hlozek, D. Grin, D.J.E. Marsh and P.G. Ferreira, A search for ultralight axions using precision cosmological data, *Phys. Rev. D* **91** (2015) 103512 [arXiv:1410.2896] [insSPIRE].

[54] H. Ooguri and C. Vafa, On the Geometry of the String Landscape and the Swampland, *Nucl. Phys. B* **766** (2007) 21 [hep-th/0605264] [insSPIRE].

[55] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, The String landscape, black holes and gravity as the weakest force, *JHEP* **06** (2007) 060 [hep-th/0601001] [insSPIRE].

[56] DES collaboration, Dark Energy Survey Year 3 results: Cosmological constraints from galaxy clustering and weak lensing, *Phys. Rev. D* **105** (2022) 023520 [arXiv:2105.13549] [insSPIRE].

[57] J. Flitter and E.D. Kovetz, Closing the window on fuzzy dark matter with the 21-cm signal, *Phys. Rev. D* **106** (2022) 063504 [arXiv:2207.05083] [insSPIRE].

[58] I.J. Allali, M.P. Hertzberg and Y. Lyu, Altered axion abundance from a dynamical Peccei-Quinn scale, *Phys. Rev. D* **105** (2022) 123517 [arXiv:2203.15817] [insSPIRE].

[59] M. Cicoli, V. Guidetti, N. Righi and A. Westphal, Fuzzy Dark Matter candidates from string theory, *JHEP* **05** (2022) 107 [arXiv:2110.02964] [insSPIRE].

[60] H. Bernardo, R. Brandenberger and J. Fröhlich, Towards a dark sector model from string theory, *JCAP* **09** (2022) 040 [arXiv:2201.04668] [insSPIRE].
[61] C.P. Burgess, D. Dineen and F. Quevedo, *Yoga Dark Energy: natural relaxation and other dark implications of a supersymmetric gravity sector*, *JCAP* 03 (2022) 064 [arXiv:2111.07286] [inSPIRE].

[62] M. Grana, *Flux compactifications in string theory: A Comprehensive review*, *Phys. Rept.* 423 (2006) 91 [hep-th/0509003] [inSPIRE].

[63] R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, *Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes*, *Phys. Rept.* 445 (2007) 1 [hep-th/0610327] [inSPIRE].

[64] J. Wess and J. Bagger, *Supersymmetry and Supergravity: Revised Edition*, vol. 103, Princeton University Press (2020).

[65] D.Z. Freedman and A. Van Proeyen, *Supergravity*, Cambridge University Press (2012) [DOI:10.1017/CBO9781139026833] [inSPIRE].

[66] A.B. Lahanas and D.V. Nanopoulos, *The Road to No Scale Supergravity*, *Phys. Rept.* 145 (1987) 1 [inSPIRE].

[67] M. Cicoli, S. de Alwis and A. Westphal, *Heterotic Moduli Stabilisation*, *JHEP* 10 (2013) 199 [arXiv:1304.1809] [inSPIRE].

[68] P. Svrcek and E. Witten, *Axions In String Theory*, *JHEP* 06 (2006) 051 [hep-th/0605206] [inSPIRE].

[69] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (2007) [DOI:10.1017/CBO9780511618123] [inSPIRE].

[70] A. Arvanitaki et al., *String Axiverse*, *Phys. Rev. D* 81 (2010) 123530 [arXiv:0905.4720] [inSPIRE].

[71] M. Cicoli, M. Goodsell and A. Ringwald, *The type IIB string axiverse and its low-energy phenomenology*, *JHEP* 10 (2012) 146 [arXiv:1206.0819] [inSPIRE].

[72] K. Dasgupta, H. Firouzjahi and R. Gwyn, *On The Warped Heterotic Axion*, *JHEP* 06 (2008) 056 [arXiv:0803.3828] [inSPIRE].

[73] S. Franco, D. Galloni, A. Retolaza and A. Uranga, *On axion monodromy inflation in warped throats*, *JHEP* 02 (2015) 086 [arXiv:1405.7044] [inSPIRE].

[74] E. McDonough and S. Alexander, *Observable Chiral Gravitational Waves from Inflation in String Theory*, *JCAP* 11 (2018) 030 [arXiv:1806.05684] [inSPIRE].

[75] A. Hebecker, S. Leonhardt, J. Moritz and A. Westphal, *Thraxions: Ultralight Throat Axions*, *JHEP* 04 (2019) 158 [arXiv:1812.03999] [inSPIRE].

[76] C.-P. Ma and E. Bertschinger, *Cosmological perturbation theory in the synchronous and conformal Newtonian gauges*, *Astrophys. J.* 455 (1995) 7 [astro-ph/9506072] [inSPIRE].