A novel distribution-free hybrid regression model for manufacturing process efficiency improvement

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Abstract

This work is motivated by a particular problem of a modern paper manufacturing industry, in which maximum efficiency of the fiber-filler recovery process is desired. A lot of unwanted materials along with valuable fibers and fillers come out as a by-product of the paper manufacturing process and mostly goes as waste. The job of an efficient Krofta supracell is to separate the unwanted materials from the valuable ones so that fibers and fillers can be collected from the waste materials and reused in the manufacturing process. The efficiency of Krofta depends on several crucial process parameters and monitoring them is a difficult proposition. To solve this problem, we propose a novel hybridization of regression trees (RT) and artificial neural networks (ANN), hybrid RT-ANN model, to solve the problem of low recovery percentage of the supracell. This model is used to achieve the goal of improving supracell efficiency, viz., gain in percentage recovery. In addition, theoretical results for the universal consistency of the proposed model are given with the optimal value of a vital model parameter. Experimental findings show that the proposed hybrid RT-ANN model achieves higher accuracy in predicting Krofta recovery percentage than other conventional regression models for solving the Krofta efficiency problem. This work will help the paper manufacturing company to become environmentally friendly with minimal ecological damage and improved waste recovery.

Keywords: Manufacturing process; Krofta efficiency; Hybrid model; Regression tree; Artificial neural network

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1. Introduction

Regression problems arise in many practical situations where a specific response variable can be expressed through a relationship with the so-called causal variables. In practical applications, it becomes quite challenging to identify the right set of causal variables. This article is motivated while we were dealing with a problem in the modern paper industry. It is well known that paper is produced from pulp, fibers, fillers, and other chemicals and it needs a considerable water resource as well. As it is, a lot of fibers and fillers obtained from the base material are drained out along with outlet water that comes from the paper machine. To save these valuable materials from being drained out, a system called Krofta supracell [1] is used by modern paper manufacturing industries and many other similar types of industries. An efficient recovery system of fibers and fillers from the outlet flow will naturally be cost saving and help the industry to remain competitive in the market. This is also known as the fiber-filler recovery process which is practically a dissolved air flotation cum sedimentation process [2]. An input to this process is the lean backwater which comes out of the main papermaking process and hence not only contains a lot of unwanted materials but also contains fibers and fillers which forms the basis of making any paper. Collection of such fibers and fillers efficiently will save cost and protects the environment where finally these materials are thrown in as wastage. By addressing this problem, one can make the paper manufacturing process an environment-friendly and sustainable manufacturing process with minimal ecological damage also. In manufacturing processes, particularly processes of chemical types, knowledge of the causal variables which affect the Krofta efficiency, is very limited and one has to start from scratch to gather such a knowledge [3].

A lot of preliminary discussions and analysis helped to determine a set of possible variables, some of which are expected to be critical causal variables for Krofta efficiency improvement. Details of these variables are given in Section 4.2. The focus of the problem was to judge and decide statistically which variables need to be controlled to improve the Krofta efficiency to at least 80% from the present level of 60%. While trying to develop a methodology to ensure the targeted improvement, one needs to think about the robustness of the method developed. To ensure robustness, we took recourse to distribution-free regression models. Nonparametric statistical and machine learning models like decision trees, random forest, support vector regressions (SVR) and artificial neural networks, etc have been applied to solve several
problems in analyzing water quality of river [4, 5], surface water planning [6, 7] and urban water demand forecasting [8, 9]. It was found that decision trees and neural networks (NN) can model arbitrary decision boundaries. To utilize the positiveness of these two models, theoretical frameworks for combining both the models are often used jointly to make decisions. The idea of mapping tree-based models into ANN are presented by several papers to solve many supervised learning problems [10, 11]. Similar designs in the interface area of decision trees and neural networks can be found in some recent literature as well [12, 13]. In spite of the use of neural tree models in practical problems of classification, regression, and forecasting, very little is known about the statistical properties of these models. Thus, the significant disadvantages of these algorithms are having poor robustness and having many free tuning parameters.

Motivated by the above discussion, we propose in the present paper a hybrid RT-ANN model and discuss its statistical properties with the optimal values of the model parameters. In this model, a consistent RT algorithm is used as an important causal variable selection algorithm. Further, a neural network with the input variables chosen from RT along with RT algorithm outputs is trained to minimize the empirical risk on the training set which results in a universally consistent hybrid model. We have then introduced the consistency results of hybrid RT-ANN model which assures a basic theoretical guarantee of robustness of the proposed algorithm. The proposed hybrid RT-ANN model has the advantages of significant accuracy, converges much faster than complex hybrid models and easy interpretability as compared to more “black-box-like” advanced neural networks. Also, no parametric assumptions are made on the distribution of the input and output variables in our proposed model. Our model has the advantages of less number of tuning parameters and is useful for high-dimensional small data sets and complex data structures. Unlike other regression models, hybrid RT-ANN model doesn’t have any strong assumption about the normality of the data and homoscedasticity of the noise terms. This model is applied to solve the Krofta efficiency problem of the paper manufacturing company to achieve a specific target for improving supracell efficiency. Through regression modeling, we obtained the necessary control parameters and this model also explains the Krofta recovery percentage with higher accuracy than other conventional regression models. Based on the recommendation of the regression analysis, an experimental design is run to find optimum levels of control parameters which are then implemented in the process and results in better
recovery than the targeted retention of valuable materials. Our recommendations not only results in an economic gain and wastage reduction but also improved production process and environmental benefits.

The paper is organized as follows. In Section 2, we introduce hybrid RT-ANN model and its statistical properties are discussed in Section 3. Usefulness of the proposed model in improving manufacturing process efficiency is shown in Section 4. Finally, the concluding remarks are given in Section 5.

2. Proposed Hybrid model

One of the ultimate goals of designing a regression model is to select the best possible regressors which can predict the response variable accurately. Regression trees (RT) and artificial neural networks (ANN) are competitive techniques for modeling regression problems. RT is a hierarchical nonparametric model [14] relatively superior to ANN in the readability of knowledge. But, ANNs are better in implementing comprehensive inference over the inputs [15]. General sufficient condition for the consistency of data-driven histogram-based regression estimates is when the size of the tree grows with the number of samples at an appropriate rate [16]. And it was theoretically proven that if a one-layered neural network is trained with an appropriately chosen number of neurons to minimize the empirical risk on the training data, then it results in a universally consistent neural network estimates [17].

In the proposed model, we have used RT as a feature selection algorithm [18] which has a built-in mechanism to perform feature selection [19]. RT as a Feature selection algorithm has many advantages such as its ability to select important features from high dimensional feature sets [20]. Feature selection using RT algorithm can be characterized by the following:

- With the set of available features as an input to the algorithm; a tree is created.
- RT has leaf nodes, which represent predicted regression output value.
- The tree branches represent each possible value of the feature node from which it originates.
- RT is used to choose feature vectors by starting from its root and moving through it until a leaf node is identified.
• At each decision node in the RT, one can select the most useful feature using appropriate estimation criteria. The criterion used to identify the best possible features involves mean squared error for regression problems.

In our proposed methodology, we first split the feature space into several areas by RT algorithm. Most important features are chosen using RT and redundant features are eliminated. Further, we build the ANN model using the important variables obtained through RT algorithm along with prediction results made by RT method which are used as another input information in the input layer of neural networks. We then run the ANN model with one hidden layer and the optimum value of the number of neurons in the hidden layer is proposed in Section 3. Since we have taken RT output as an input feature in ANN model, the number of hidden layers are chosen to be one for further modeling with ANN algorithm, and it is also shown to be universally consistent with some regularity conditions in Section 3. The effectiveness of the proposed hybrid RT-ANN model lies in the selection of important features and using regression outputs of the RT model followed by the ANN model. The inclusion of RT output as an input feature increases the dimensionality of feature space and will also increase class separability [21]. The informal work-flow of our proposed hybrid model, shown in Figure 1, is as follows.

• First, we apply a regression tree algorithm to train and build a decision tree model that calculates important features as a feature selection algorithm.

• The prediction result of RT algorithm is used as an additional feature in the input layer of the ANN model.

• We use important input variables obtained from RT along with an additional input variable (RT output) to develop an appropriate ANN model, and the network is generated.

• Run the ANN algorithm with sigmoid activation function and the optimal number of hidden layer neurons as shown in Section 3 and record the regression outputs.

This algorithm is a two-step problem-solving approach such as feature selection using RT and then using all the outputs of feature selection algorithm in the following regression analysis to get a better model in terms of accuracy.
Our proposed model can be used for identifying important features which will satisfy a specific goal to solve Krofta efficiency problem and also can be employed for modeling Krofta recovery percentage in terms of important causal process parameters. Though the model was developed primarily to solve the Krofta efficiency problem, can be used in other similar situations as well.

On the theoretical side, it is necessary to show the universal consistency of the proposed model and other statistical properties for its robustness. We will now introduce a set of regularity conditions to prove the risk consistency of RT as feature selection algorithm and the importance of RT output in a further model building with the ANN algorithm in the proposed hybrid model.

3. Statistical Properties of Hybrid RT-ANN model

RT has a built-in mechanism to perform feature selection and it has the ability to select important features from high dimensional feature sets. To explore its statistical properties, we are going to investigate the sufficient condition(s) for consistency of RT. For a wide range of data-dependent partitioning schemes, the consistency of histogram-based regression estimates was shown in the literature. It requires a set of regularity conditions to be satisfied to show the consistency of histogram regression estimates. Also, it is assumed to have regression variables to be bounded throughout. But in our case, we will represent regression trees where the partitions are chosen to have rectangular cells, i.e., regression trees employing axis-parallel splits and response variable can take values within a certain range. There is no assumption made on the distributions of predictor and response variables.

Let $X$ be the space of all possible values of $p$ features, i.e. $X = (X_1, X_2, ..., X_p)$ and $Y$ be set of all possible values that the response variable can take and $Y \in [-K, K]$. We wish to estimate regression function $r(x) = E(Y|X = x) \in [-K, K]$ based on a training sample $L_n = \{(X_1, Y_1), (X_2, Y_2), ..., (X_n, Y_n)\}$, where $X_i = (X_{i1}, X_{i2}, ..., X_{ip}) \in X$ with $n$ observations and let $\Omega = \{\omega_1, \omega_2, ..., \omega_k\}$ be a partition of the feature space $X$. We denote $\tilde{\Omega}$ as one such partition of $\Omega$. Define $(L_n)_{\omega_i} = \{(X_i, Y_i) \in L_n : X_i \in \omega_i, Y_i \in [-K, K]\}$ to be the subset of $L_n$ induced by $\omega_i$ and let $(L_n)_{\tilde{\Omega}}$ denote the partition of $L_n$ induced by $\tilde{\Omega}$.

The criterion used to identify the best features involves mean squared error for regression problems. We don’t make any assumption on the distri-
Figure 1: An example of hybrid RT-ANN model with $X_i; i = 1, 2, 3, 4, 5$ as Important features obtained by RT, $L_i$ as leaf nodes and OP as RT output.

bution of the pair $(X, Y) \in \mathbb{R}^p \times [-K, K]$. Mean squared error is used to partition feature space into a set $\Omega$ of nodes. There exists a partitioning regression function $d : \Omega \to Y$ such that $d$ is constant on every node of $\Omega$. Now let us define $\hat{L}_n$ be the space of all learning samples and $\mathcal{D}$ be the space of all partitioning regression function then, binary partitioning and regression tree based rule $\Phi : \hat{L}_n \to \mathcal{D}$ such that $\Phi(L_n) = (\psi \circ \phi)(L_n)$, where $\phi$ maps $L_n$ to some induced partition $(L_n)_{\tilde{\Omega}}$ and $\psi$ is an assigning rule which maps $(L_n)_{\tilde{\Omega}}$ to a partitioning regression function $d$ on the partition $\tilde{\Omega}$. The most basic reasonable assigning rule $\psi$ is the plurality rule $\psi_{pl}(L_n)_{\tilde{\Omega}} = d$ such that if
Given $x \in \omega_i$, then

$$d(x) = \arg\min_{Y_i \in [-K, K]} \left\{ \frac{1}{n} \sum_{i=1}^{n} (\Phi(L_n) - Y_i)^2 \right\}$$

The stopping rule in RT is decided based on the minimum number of split in the posterior sample called minsplits. If minsplits $\geq \alpha$ then $\omega_i$ will split into two child nodes and if minsplits $< \alpha$ then $\omega_i$ is a leaf node and no more split is required. Here $\alpha$ is determined by the user, usually it is taken as 10% of the training sample size. Now let $\mathcal{T} = (\hat{\Omega}_1, \hat{\Omega}_2, \ldots)$ be a finite collection of partitions of a measurement space $X$. Let us define maximal node count of $\mathcal{T}$ as the maximum number of nodes in any partition $\hat{\Omega}$ in $\mathcal{T}$ which can be written as

$$\lambda(\mathcal{T}) = \sup_{\hat{\Omega} \in \mathcal{T}} |\hat{\Omega}|$$

Also let, $\Delta(\mathcal{T}, L_n) = |\{ (L_n)_{\hat{\Omega}} \hat{\Omega} \in \mathcal{T} \}|$ be the number of distinct partitions of a training sample of size $n$ induced by partitions in $\mathcal{T}$. Let $\Delta_n(\mathcal{T})$ be the growth function of $\mathcal{T}$ defined as

$$\Delta_n(\mathcal{T}) = \sup_{\{L_n \mid L_n = n\}} \Delta(\mathcal{T}, L_n).$$

Growth function of $\mathcal{T}$ is the maximum number of distinct partitions $(L_n)_{\hat{\Omega}}$ which partition $\hat{\Omega}$ in $\mathcal{T}$ can induce in any training sample with $n$ observations. For a partition $\hat{\Omega}$ of $X$, $\hat{\Omega}[x \in X] = \{ \omega_i \in \hat{\Omega} : x \in \omega \}$ be the node $\omega_i$ in $\hat{\Omega}$ which contains $x$.

For consistency of any histogram based regression estimates, the sub-linear growth of restricted cell counts (see in Eqn. 1), sub-exponential growth of a combinatorial complexity measure (see in Eqn. 2) and shrinking cell (see in Eqn. 3) conditions are to be satisfied [16,22] and these are as follows:

$$\frac{\lambda(\mathcal{T}_n)}{n} \rightarrow 0 \quad \text{as} \quad n \rightarrow \infty.$$ (1)

$$\frac{\log(\Delta_n(\mathcal{T}_n))}{n} \rightarrow 0 \quad \text{as} \quad n \rightarrow \infty.$$ (2)

and for every $\gamma > 0$ and $\delta \in (0, 1)$,

$$\inf_{S \subseteq \mathbb{R}^p : P(S) \geq 1 - \delta} P(x : \text{diam}(\hat{\Omega}_n[x] \cap S) > \gamma) \rightarrow 0 \quad \text{w.p. 1} \quad (3)$$
Now we are going to evolve a sufficient regularity condition for the binary partitioning and regression tree based rule to be universally consistent. It is to be shown that optimal regression trees are consistent when the size of the tree grows as \( o\left(\frac{n}{\log(n)}\right) \), where \( n \) is the number of training samples and it implies the regularity conditions of Eqn. (1), (2) and (3).

**Theorem 1.** Suppose \((X, Y)\) be a random vector in \( \mathbb{R}^p \times [-K, K] \) and \( L_n \) be the training set of \( n \) outcomes of \((X, Y)\). Finally if for every \( n \) and \( w_i \in \tilde{\Omega}_n \), the induced subset \((L_n)_{w_i}\) contains at least \( k_n \) of the vectors of \( X_1, X_2, \ldots, X_n \), then empirically optimal regression trees strategy employing axis parallel splits are consistent when the size \( k_n \) of the tree grows as \( o\left(\frac{n}{\log(n)}\right) \).

**Proof.** Since \( T(k_n) \) contain all the binary RT partitions having \( k_n \) leaves, so \( \lambda(T(k_n)) = k_n \). Therefore, \( \frac{\lambda(T(k_n))}{n} = k_n^n \) tends to zero as \( n \to \infty \). Hence, \( \frac{\lambda(T(k_n))}{n} \to 0 \). Thus condition (1) holds.

Now regression trees having \( k_n \) leaves has \( k_n - 1 \) internal nodes and therefore each partition is based on at most \( k_n - 1 \) intersecting half-spaces. And by Cover’s theorem \[23\], any binary split of \( \mathbb{R}^p \) can divide \( n \) points in at most \( n^p \) ways. So, their intersection will partition \( n \) points in at most \( n^{(k_n-1)p} \) ways. Thus we write,

\[
\Delta_n(T(k_n)) \leq n^{(k_n-1)p}
\]

and consequently,

\[
\log(\Delta_n(T(k_n))) \leq \frac{p(k_n - 1)}{n} \log(n) \tag{4}
\]

As, \( n \to \infty \), RHS of equation (4) goes to zero. So condition (2) holds.

Now \( k_n = o\left(\frac{n}{\log(n)}\right) \), then for every \( n \) and \( \omega \in \tilde{\Omega}_n \), the induced subset \((L_n)_{\omega} \in T(k_n)\) such that for every compact set \( V \subseteq \mathbb{R}^d \) we can write \( \max_{A \in (L_n)_{\omega}} \text{diam}(A \cap V) \to 0 \). This implies condition (3) is satisfied and hence the theorem.

**Remark.** Thus, it may be noted that regression trees are empirically consistent when the size \( k_n \) of the tree grows as \( o\left(\frac{n}{\log(n)}\right) \). It is interesting to know that without employing sufficient conditions as mentioned in equation (1), (2) and (3), we can have a universally consistent regression tree provided the regularity condition of Theorem 1 is satisfied. It is also noted that
with sufficiently large $n$, the optimal regression tree will not divide the regions of the feature space on which the regression function is constant. We further conclude that feature selection using RT algorithm is justified and RT output will also play an important role in designing the regression model for increasing the predictive accuracy of the model. It should also be mentioned that incorporating RT output as an input feature in ANN, the dimensionality gets increased, thus the performance of the ANN model will be improved at a significant rate \[2,4\].

Our proposed hybrid model has two parts: extracting important features from the feature space using RT algorithm and building one hidden layered the ANN model with the important features extracted using RT along with RT output as another input vector in the ANN model. In our base model, the dimension of the input layer, denoted by $d_m(\leq p)$, is the number of important features obtained by RT + 1. It is also noted that one-hidden layered neural networks yield strong universal consistency and there is a little theoretical gain in considering two or more hidden layered neural network \[25\]. In hybrid RT-ANN model, we have used one hidden layer with $k$ neurons. This makes our hybrid model less complex and less time consuming while running the model. Our objective is to state the sufficient condition for universal consistency and then to find out the optimal value of $k$. Before stating the sufficient conditions for the consistency of the algorithm and the optimal number of nodes in the hidden layer, let us define the following:

Let the random variables $Z$ and $Y$ take their values from $\mathbb{R}^{d_m}$ and $[-K,K]$ respectively. Denote the measure of $Z$ over $\mathbb{R}^{d_m}$ by $\mu$ and $m : \mathbb{R}^{d_m} \rightarrow [-K,K]$ be a measurable function such that $m(Z)$ approximates $Y$. Given the training sequence $\xi_n = \{(Z_1,Y_1),(Z_2,Y_2),..., (Z_n,Y_n)\}$ of $n$ iid copies of $(X,Y)$, the parameters of the neural network regression function estimators are chosen such that it minimizes the empirical $L_2$ risk $= \frac{1}{n} \sum_{j=1}^{n} |f(Z_j) - Y_j|^2$. We have used logistic squasher as sigmoid function in neural network which is defined as follows:

**Definition:** A sigmoid function $\sigma(x)$ is called a logistic squasher if it is non-decreasing, $\lim_{x \to -\infty} \sigma(x) = 0$ and $\lim_{x \to \infty} \sigma(x) = 1$ with $\sigma(x) = \frac{1}{1 + \exp(-x)}$.

Now consider the class of neural networks having logistic squasher with
$k$ neurons in the hidden layer with bounded output weights

$$\mathcal{F}_{n,k} = \left\{ \sum_{i=1}^{k} c_i \sigma(a_i^T z + b_i) + c_0 : k \in \mathbb{N}, a_i \in \mathbb{R}^{d_m}, b_i, c_i \in \mathbb{R}, \sum_{i=0}^{k} |c_i| \leq \beta_n \right\}$$

and obtain $m_n \in \mathcal{F}_{n,k}$ satisfying

$$\frac{1}{n} \sum_{i=1}^{n} |m_n(Z_i) - Y_i|^2 \leq \frac{1}{n} \sum_{i=1}^{n} |f(X_i) - Y_i|^2, \text{ if } f \in \mathcal{F}_{n,k}$$

where, $m_n$ be a function that minimizes the empirical $L_2$ risk in $\mathcal{F}_{n,k}$. It was shown that if $k$ and $\beta_n$ are chosen to satisfy:

$$k \to \infty, \beta_n \to \infty, \frac{k \delta_n^4 \log(k \delta_n^2)}{n} \to 0, \text{ and there exists } \delta(>0) \text{ such that } \frac{\delta_n^4}{n^{1+\delta}} \to 0,$$

then

$$E \int (m_n(z) - m(z))^2 \mu(dz) \to 0 \text{ as } n \to \infty$$

and consequently,

$$\int (m_n(z) - m(z))^2 \mu(dz) \to 0 \text{ as } n \to \infty$$

for all distributions of $(Z, Y)$ with $E|Z^2| < \infty$, i.e., $m_n$ is strongly universally consistent (For proof see [17, Theorem 3]). The rate of convergence of the neural network with $k$ neurons with bounded output weights has been pointed out by [26]. This made us conclude that our proposed model is strongly consistent under the mentioned regularity condition and our job is to find an optimal choice of $k$ for our algorithm. To obtain the optimal value of the number of hidden neuron in the hidden layer of the hybrid RT-ANN model having $d_m$ number of input layers, we state the following proposition:

**Proposition 1.** For any fixed $d_m$ and training sequence $\xi_n$, let $Y \in [-K, K]$, and $m, f \in \mathcal{F}_{n,k}$, if the neural network estimate $m_n$ satisfies the above-mentioned regularity conditions of strong universal consistency and $f$ satisfying $\int_{S_r} f^2(z) \mu(dz) < \infty$ where, $S_r$ is a ball with radius $r$ centered at 0, then the optimal choice of $k$ is $O\left(\sqrt{\frac{n}{d_m \log(n)}}\right)$. 

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Proof. To prove Proposition 1, we are going to recall the results of the proofs of universal consistency and rate of convergence [26, Page 301-326].

The expression for rate of convergence of the neural network estimates as defined in $F_{n,k}$ is as follows:

$$E \int_{S_r} |m_n(z) - m(z)|^2 \mu(dz) = O \left( \beta_n^2 \left( \frac{d_m \log(\beta_n n)}{n} \right)^{1/2} \right) \quad \text{since} \quad d_m > 1$$

(5)

Since $\beta_n < \text{constant} < \infty$ and $d_m(>1)$ is a fixed constant. We can write (5) as

$$E \int_{S_r} |m_n(z) - m(z)|^2 \mu(dz) = O \left( \sqrt{\frac{d_m \log(n)}{n}} \right)$$

(6)

It is noted that average estimation error (LHS of (6)) approaches to 0 as $n \to \infty$.

To obtain upper bound on the rate of convergence we will have to impose some more regularity conditions as follows: For every continuous function $f : \mathbb{R}^{d_m} \to [-K, K]$ satisfying the properties of fourier transformation, as mentioned in Proposition 1, we can write an expression of the rate of approximation of neural networks as follows:

$$\inf_{f \in F_{n,k}} \int_{S_r} |f(z) - m(z)|^2 \mu(dz) = O \left( \frac{1}{k} \right)$$

(7)

It is noted that approximation error (LHS of (7)) approaches to 0 as $k \to \infty$.

Since the conditions of strong universal consistency of neural networks and an additional restriction on $f(z)$ are assumed, we can now put an upper bound to the estimation error using the approximation error.

Bringing (6) and (7) together, we have the following:

$$E \int_{S_r} |m_n(z) - m(z)|^2 \mu(dz) - \inf_{f \in F_{k,n}} \int_{S_r} |f(z) - m(z)|^2 \mu(dz) = O \left( \sqrt{\frac{d_m \log(n)}{n}} + \frac{1}{k} \right)$$

For optimal choice of $k$, the problem reduces to equating $\sqrt{\frac{d_m \log(n)}{n}}$ with $\frac{1}{k}$ which gives $k = O \left( \sqrt{\frac{n}{d_m \log(n)}} \right)$.

□
Remark. The optimal choice of $k$ is found to be $O\left(\frac{n}{d_m \log(n)}\right)$ and accordingly we have to choose the number of hidden neurons in the RT-ANN model. For practical use, one can use $k = \sqrt{\frac{n}{d_m \log(n)}}$ for small data sets to get the desired accuracy of the hybrid RT-ANN model. The application of this model and its implementation is shown in Section 4. It is found that the proposed model is highly competitive than the other conventional regression models in terms of accuracy. The universal consistency of the proposed hybrid RT-ANN model is justified, and it also gives the upper bound for the number of hidden neurons in the model.

In our proposed model, the selected important features using RT and RT outputs as input features to the neural network with the number of hidden neurons to be chosen as $O\left(\sqrt{\frac{n}{d_m \log(n)}}\right)$ make a strong universally consistent regression model. It is then applied to solve Krofta efficiency problem and proved to be highly effective through experimental analysis as shown in Section 4.

4. Application

The current problem of Krofta Efficiency aims to improve the Krofta fiber-filler recovery by reducing the fiber, filler losses and improving the paper qualities in terms of opacity, formation and machine runnability. Krofta is fiber-filler recovery equipment having floatation cum sedimentation cell. The inlet to Krofta is the paper machine lean backwater which undergoes froth flotation when treated with chemical and air [27, 28]. The chemical forms the flocks from the fines present in the lean backwater and air helps to form the cake over Krofta supracell for recovery. Supracell removes solids through air floatation and sedimentation process. Turbulence caused by water movement is a significant factor in floatation and greatly reduces the efficiency of the other types of floatation units. In conventional, there must always be water movement for the water to flow from inlet to outlet. With the supracell, the inlet and outlet are not stationary but are rotating about the center. The rotation is synchronized so that the water in the tank achieves zero velocity during floatation [29]. This means that the efficiency of floatation is significantly increased to nearly maximum theoretical limits. In practical terms, this allows better clarification in smaller surface areas and a much shallower tank. Water is processed from the inlet to outlet in
2-3 minutes. Air is dissolved into the water using Krofta air dissolving tube (ADT) \[30\] and unclarified water is released through a valve. The water flows in at the exact center, through a rotary joint, and into the distribution duct. Coarse air is released through a vent pipe in the duct. The flow is directed to eliminate turbulence. Since the inlet distribution is moving forward at the same speed that the water is flowing out, the water stays in one spot in the tank without any movement during floatation. The floated material is recovered from the top surface by means of the Krofta spiral scoop \[31\]. The scoop is designed to remove the floated material at the highest possible consistency, with a minimum surface disturbance. The level of water determines the consistency of the floated material removed. The Krofta floatation system removes the solid content in the water by floating them to the surface for removal. The reason why fiber will float, even if they are heavier than water, is that small air bubbles attach themselves to the particles or flocks and make them buoyant \[32\]. The process flow diagram of Krofta supracell is shown in Figure 2, and the mechanism for forming the air bubbles is as follows:

- The water is pressurized to feed to the ADT.
- Air is added to the pressurized water and gets dissolved at this pressure.
- The pressure is released after the water passes through a valve. The water can no longer hold the extra air which was absorbed, so that small bubbles form spontaneously throughout the liquid. The bubbles formed are tiny.
- A small mastering pulp feeds the chemical flocculants. Chemicals are used to increase the clarifier efficiency by flocking out small and colloidal particles, that otherwise would not float or settle in the clarifier.

4.1. Data Collection Plan

Initially, the efficiency of Krofta performance which needs improvement was measured. The efficiency of Krofta performance\[1\] is quantified in terms of Inlet parts per million (Inlet PPM) and Outlet PPM. To identify the

\[\text{Efficiency} = \left(\frac{\text{Inlet PPM} - \text{Outlet PPM}}{\text{Inlet PPM}}\right) \times 100\]
causal parameters affecting the Krofta efficiency, a failure mode effect analysis (FMEA) was done with the help of process experts. FMEA is a useful analytical tool used in the process industry for understanding possible causes of failures with their impacts and frequencies. The information collected through FMEA are namely; potential failure modes, potential failure effects, potential causes, mechanism of failure and their severity were analyzed. The potential areas of failures were identified where risk priority number (RPN) were found to be high. For the potential failure model where RPN is high, the process experts carried out some minor experiments to understand the exact reasons for the low efficiency of Krofta. However, subsequently considering all such potential areas, a cause and effect diagram was made. The cause and effect diagram revealing the key relationship among causal variables with Krofta efficiency is presented in Figure 3. Since not all the parameters are controllable by the users of the process, with the help of process experts and further brainstorming, a list of controllable parameters is prepared. The data were then collected for two types of tissues, namely tissue 1 and tissue 2, taking Krofta efficiency percentage as the response and other parameters
4.2. **Description of Datasets**

The dataset collected has the following parameters possibly affecting the Krofta performance for both the tissues: Inlet Flow, Water Pressure (water inlet pressure to ADT), Air Pressure, Pressure of Air-Left, Pressure of Air-Right, Pressure of ADT-D Left, Pressure of ADT-D Right and Amount of chemical lubricants. A sample dataset for both the tissues are given in Tables 1 and 2.
Table 1: Sample Dataset for Tissue 1

| Sl. No. | Inlet Flow | Water Pressure | Air Pressure | Air-Left | Air-Right | ADT-D Left | ADT-D Right | Amount of chemical | Recovery Percentage |
|---------|------------|----------------|--------------|----------|-----------|------------|-------------|-------------------|-------------------|
| 1       | 2804       | 5.8            | 3.0          | 2.6      | 2.4       | 2.3        | 2.7         | 2.0               | 49.74             |
| 2       | 3200       | 6.0            | 5.0          | 2.4      | 2.2       | 2.2        | 2.7         | 2.5               | 87.13             |
| 3       | 2600       | 6.0            | 5.0          | 2.0      | 1.2       | 2.1        | 3.1         | 4.2               | 96.99             |
| 4       | 3002       | 6.2            | 5.0          | 2.1      | 2.0       | 1.5        | 3.1         | 4.0               | 96.99             |
| 5       | 2899       | 6.2            | 5.7          | 2.0      | 4.5       | 2.2        | 3.1         | 4.0               | 97.91             |
| 6       | 2995       | 6.0            | 6.0          | 1.5      | 4.0       | 4.0        | 4.0         | 28.87             |                   |

Table 2: Sample Dataset for Tissue 2

| Sl. No. | Inlet Flow | Water Pressure | Air Pressure | Air-Left | Air-Right | ADT-D Left | ADT-D Right | Amount of chemical | Recovery Percentage |
|---------|------------|----------------|--------------|----------|-----------|------------|-------------|-------------------|-------------------|
| 1       | 1794       | 5.2            | 5.6          | 2.4      | 1.0       | 3.0        | 4.0         | 3.0               | 97.47             |
| 2       | 1703       | 6.2            | 6.0          | 2.0      | 1.0       | 3.0        | 4.0         | 2.0               | 26.94             |
| 3       | 1139       | 6.5            | 5.8          | 1.0      | 2.0       | 3.0        | 4.0         | 2.0               | 96.80             |
| 4       | 1448       | 5.5            | 5.0          | 1.0      | 2.0       | 3.0        | 4.0         | 2.0               | 97.01             |
| 5       | 1614       | 6.6            | 5.8          | 3.7      | 5.2       | 4.0        | 4.0         | 4.0               | 97.77             |
| 6       | 1472       | 6.6            | 6.0          | 1.5      | 4.0       | 4.0        | 4.0         | 4.0               |                   |

4.3. Performance Evaluation

To date, a large number of performance metrics have been proposed and employed to evaluate the accuracy of the regression model, but no single performance metric has been recognized as the universal standard. As a result of this, we need to assess the performance based on multiple metrics, and it will be interesting to see if different metrics will give the same ranking for the different regression models to be tested. The metrics used in this study are: mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE), the coefficient of multiple determination ($R^2$) and adjusted $R^2$ (Adj($R^2$)). The use of these measures represents different angles to evaluate regression models. The first two are absolute performance measures while the third one is a relative measure and the last two are measures of “goodness of fit” for the regression models. The formulae used for the performance metrics are as follows:

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|;\quad RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2};\quad MAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|y_i - \hat{y}_i|}{y_i};$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2};\quad Adj(R^2) = 1 - \left(\frac{(1 - R^2)(n - 1)}{n - k - 1}\right)$$
where, $y_i$, $\bar{y}$, $\hat{y}_i$ denote the actual value, average value and predicted value of the dependent variable, respectively for the \(i^{th}\) instant. Here $n$, $k$ denote the number of data used for performance evaluation and the number of independent variables, respectively. The lower the value of MAE, RMSE, and MAPE and the higher the value of $R^2$ and adjusted ($R^2$), the better the model is.

4.4. Analysis of Results

We have shuffled the observations of the Krofta efficiency dataset randomly and split it into training and testing data sets in a ratio of 70 : 30. Each experiment is repeated 5 times with different randomly assigned training and test sets and we will finally report the averages of the performance metrics observed over 5 times validations in Table 3.

With the data sets described in Section 4.2, a few popular regression models such as multiple linear regression (MLR), Stepwise Regression, partial least square (PLS) Regression, SVR, multiple adaptive regression spline (MARS) and neural tree model were first applied but resulted in very low $R^2$ and Adj($R^2$) and very high MAE, RMSE and MAPE. Then we started exploring conventional nonparametric models such as RT and ANN which perform better than the parametric models. Then we applied our proposed hybrid RT-ANN model. All the methods were implemented in the R Statistical package on a PC with 2.1 GHz processor and 8 GB memory. We first select important features using RT algorithm which indicates that Krofta efficiency is significantly dependent on Water Pressure, Air Pressure, Inlet Flow and ADT-D left for Tissue 1. And for Tissue 2, the important features are Water Pressure, Air Pressure, ADT-D Left and ADT-D Right. Then we run our base model with the selected important features along with RT output as shown in Figures 4 and 5. The number of hidden layers was chosen to be $\sqrt{\frac{n}{d_m \log(n)}}$ where $d_m = 5$ for this case. Performance metrics as defined in Section 4.3 are computed for each of the competitive models. Table 3 shows that the proposed hybrid RT-ANN model outperforms the others in terms of the performance metrics.

Based on the model we further created an experimental design to obtain the optimal level of the tuning parameters. Final recommendations based on the results of the design of experiments were implemented in the process to monitor the Krofta efficiency. However, we have discussed here only the
proposed model and its accuracy level compared to other relevant state-of-the-art models. Our model helped the manufacturing process industry to achieve an efficiency level of about 85% from the existing level of about 60% to improve the Krofta supracell recovery percentage.

Table 3: Quantitative measures of performance for different regression models.

| Regression Models       | Tissue Type | MAE  | RMSE | MAPE | $R^2$ | Adj($R^2$) |
|-------------------------|-------------|------|------|------|-------|------------|
| Multiple Linear Regression | Tissue 1    | 17.92| 23.32| 48.40| 26.95 | 17.30      |
|                         | Tissue 2    | 11.67| 16.94| 25.21| 56.70 | 37.83      |
| Stepwise Regression     | Tissue 1    | 18.31| 23.51| 48.57| 27.00 | 09.10      |
|                         | Tissue 2    | 12.04| 17.09| 26.01| 55.97 | 42.65      |
| PLS Regression          | Tissue 1    | 17.50| 22.40| 42.59| 26.95 | 17.49      |
|                         | Tissue 2    | 14.47| 20.16| 30.19| 25.56 | 20.19      |
| MARS                    | Tissue 1    | 16.10| 20.26| 38.86| 45.39 | 29.54      |
|                         | Tissue 2    | 15.71| 19.86| 34.16| 40.49 | 30.57      |
| RT                      | Tissue 1    | 06.31| 10.54| 16.35| 85.07 | 81.21      |
|                         | Tissue 2    | 07.06| 10.29| 15.03| 84.73 | 81.56      |
| ANN                     | Tissue 1    | 05.15| 08.55| 08.88| 90.18 | 87.65      |
|                         | Tissue 2    | 08.27| 11.54| 17.45| 80.25 | 76.05      |
| SVR                     | Tissue 1    | 05.47| 08.97| 08.89| 86.78 | 82.13      |
|                         | Tissue 2    | 08.46| 12.17| 17.31| 81.23 | 78.72      |
| Neural tree             | Tissue 1    | 04.95| 06.44| 06.12| 92.36 | 90.53      |
|                         | Tissue 2    | 06.78| 09.89| 15.93| 85.79 | 83.09      |
| Hybrid RT-ANN           | Tissue 1    | 03.45| 04.89| 06.87| 96.79 | 95.32      |
|                         | Tissue 2    | 05.91| 08.60| 12.67| 88.84 | 87.75      |
5. Conclusions and Discussions

The purpose of this article is to develop a model for selection of critical causal variable(s) of a manufacturing process. Our study presented a hybrid RT-ANN model that combines both the neural network and RT which gives more accuracy than all other traditional models as shown in Table 3 to address the complicated problem of Krofta efficiency improvement. We have
found RT to be the optimal technique for the selection of tuning parameters and found hybrid RT-ANN model to be the optimal regression model for accurately predicting the Krofta recovery percentage. Consequently, the hybrid RT-ANN successfully demonstrated the best performance and offered a practical solution to the problem of finding optimal levels for selecting the tuning parameters to improve the Krofta efficiency. Though the hybrid model developed was mainly to improve the efficiency of Krofta, its proven theoretical consistency makes it robust and hence can generally be applied to other similar situations. We have also provided an upper bound for the number of hidden neurons during the application of ANN. The proven model, when applied to a complex chemical process for efficiency improvement, provided the required causal variables. These variables were then optimized using the design of experiments to achieve an efficiency level of about 85% from the existing efficiency level of about 60%. Since the accuracy of the proposed model is quite high and it is consistent, the model may be applied to other complex problems as well. In the application of the Krofta efficiency problem, not only the efficiency is enhanced, but it also resulted in lesser wastage and provided a significant economic gain for the organization. It also helped the organization to be more environmentally friendly as well.

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