Abstract We study the dynamics of Gaussian quantum coherence under the background of a Garfinkle–Horowitz–Strominger dilaton black hole. It is shown that the dilaton field has evident effects on the degree of coherence for all the bipartite subsystems. The bipartite Gaussian coherence is not affected by the frequency of the scalar field for an uncharged or an extreme dilaton black hole. It is found that the initial coherence is not completely destroyed even for an extreme dilaton black hole, which is quite different from the behavior of quantum steering because the latter suffers from a “sudden death” under the same conditions. This is nontrivial because one can employ quantum coherence as a resource for quantum information processing tasks even if quantum correlations have been destroyed by the strong gravitational field. In addition, it is demonstrated that the generation of quantum coherence between the initial separable modes is easier for low-frequency scalar fields. At the same time, quantum coherence is smoothly generated between one pair of partners and exhibits a “sudden birth” behavior between another pairs in the curved spacetime.

1 Introduction

String theory is regarded as one of the most promising candidates for a consistent understanding of quantum mechanics and the theory of gravity. Different from general relativity, it is predicted in string theory that the presence of dilaton fields can change the properties of black holes [1–4]. A solution of static spacetime, i.e., the Garfinkle–Horowitz–Strominger (GHS) dilaton black hole [3,4], can be obtained by choosing the invariant to be the Lagrangian of the electromagnetic field. The Hawking temperature [5] of the GHS black hole not only depends on its mass, but also on the dilaton field because the latter is also a resource of gravity. On the other hand, the behavior of quantum information near the event horizon of black holes has received considerable attention in recent years [6–17]. It is believed that related studies can contribute to a deeper understanding of nonlocality between causally disconnected spacetime regions, as well as to a further understanding of the entropy and the problem of information loss for black holes [18–20].

Quantum coherence [21–24], as a key aspect of quantum physics, is not only an embodiment of the superposition principle of states but also the basis of many unique phenomena in quantum physics, such as quantum entanglement and steering. With the help of quantum coherence, one can implement various quantum information processing tasks which cannot be accomplished classically, such as quantum computing [25,26], quantum information storage [27,28], quantum communication [29,30], and quantum metrology [31–33]. Therefore, understanding quantum coherence in a more general framework is important both for the fundamental research of physics and the development of modern quantum technology. Despite its fundamental role in physics, the study of coherence receives increasing attention until Baumgratz et al. introduced an architecture for the measurement of coherence [21]. In 2016, a quantification of coherence for continuous variables was proposed by Xu [34], which provides a framework of quantum resource theory for quantum superposition in infinite-dimensional quantum systems [35].

In this paper, we study the distribution and generation of continuous-variable coherence in the background of a GHS dilaton black hole. The considered initial state is a two-mode squeezed Gaussian state, which can be employed to define quantum vacuum when the spacetime has at least two asymp-
totically flat regions. This state has a special role in the quantum field theory because the values of squeezing parameter between causally disconnected regions are changed according to the spacetime structure. We find that Gaussian quantum coherence is more robust than the steering-type quantum correlations under the influence of gravity in the GHS spacetime.

The structure of the paper is as follows. In Sect. 2, we discuss the dynamics and second quantization of the scalar field near the GHS dilaton black hole. In Sects. 3 and 4, we discuss the method of measuring the coherence of continuous variables and the behavior of Gaussian quantum coherence in the GHS spacetime, respectively. In the final section, we make a brief summary. Throughout the paper, the units $G = c = \hbar = \kappa_B = 1$ are used.

2 Vacuum structure of the scalar field

In this section, we review the quantum field theory in the GHS black hole spacetime. The equations describing gravity in the context of string theory can be approximated by Einstein’s equations in the regions near the horizon of a black hole. In this scenario, the Schwarzschild solution is a good approximation to describe a static and uncharged black hole within string theory. However, when we consider the solutions of the Einstein–Maxwell equations, the presence of a scalar field called dilaton should be considered. In this case, the added dilaton field couples with the Maxwell field, which changes the spacetime characters of the black hole. In the low energy limit of string theory, the static dilaton black hole solution was provided by Garfinkle, Horowitz, and Strominger [4]. The dilaton gravity in string theory is important because it is a good candidate for an eventual quantum theory of gravity [1,2]. Therefore, it is meaningful to study the behavior of Gaussian quantum coherence in the dilaton spacetime.

The line element for a GHS dilaton black hole can be written as [4]

$$ds^2 = -\left(\frac{r - 2M}{r - 2D}\right)dt^2 + \left(\frac{r - 2M}{r - 2D}\right)^{-1}dr^2 + r(r - 2D)d\Omega^2,$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$, $M$ is the mass of the black hole and $D$ is the dilaton charge. The event horizon of the GHS black hole is located at $r_+ = 2M$.

The dynamics of massless scalar field in a general background is given by the Klein–Gordon equation [36]

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = 0.$$  

Solving the Klein–Gordon equation near the event horizon of the black hole, one obtains the outgoing modes

$$\phi_{\text{out,odm}}(r < r_+) = e^{i\omega u}Y_{lm}(\theta, \phi),$$

$$\phi_{\text{out,odm}}(r > r_+) = e^{-i\omega u}Y_{lm}(\theta, \phi),$$

where $\omega = t + r^*$, $u = t - r^*$, and $r^*$ is the tortoise coordinates.

Employing the Schwarzschild modes given in Eqs. (3) and (4), the scalar field $\Phi$ near the event horizon can be expanded as [36]

$$\Phi = \sum_{lm} \int d\omega [b_{\text{in,odm}}^{\dagger}\phi_{\text{in,odm}}(r < r_+) + b_{\text{in,odm}}\phi_{\text{out,odm}}(r > r_+)] + \sum_{in} b_{\text{out,odm}}^{\dagger}\phi_{\text{in,odm}}(r > r_+),$$

where $b_{\text{in,odm}}$ and $b_{\text{out,odm}}^{\dagger}$ are the annihilation and creation operators acting on the states of the interior region of the dilaton black hole. Similarly, $b_{\text{out,odm}}$ and $b_{\text{out,odm}}^{\dagger}$ are the operators acting on the vacuum of the exterior region, respectively. The Schwarzschild vacuum state for the scalar field can be defined as [36]

$$b_{\text{in,odm}}^{\dagger}[\Phi_{\text{in,odm}}]_{\text{in}} = b_{\text{out,odm}}^{\dagger}[\Phi_{\text{in,odm}}]_{\text{out}} = 0.$$  

On the other hand, the light-like Kruskal coordinates $U$ and $V$ are defined by [37,38],

$$U = -4(M - D)e^{-u/\sqrt{(4M - 4D)}},$$

$$V = 4(M - D)e^{u/\sqrt{(4M - 4D)}}, \quad \text{if} \quad r > r_+;$$

$$U = 4(M - D)e^{-u/\sqrt{(4M - 4D)}},$$

$$V = 4(M - D)e^{u/\sqrt{(4M - 4D)}}, \quad \text{if} \quad r < r_+.$$  

Then we can rewrite the field modes to

$$\phi_{\text{out,odm}}(r < r_+) = e^{i\omega U}\phi_{\text{in,odm}}(\theta, \phi),$$

$$\phi_{\text{out,odm}}(r > r_+) = e^{i\omega V}\phi_{\text{in,odm}}(\theta, \phi).$$

Making an analytic continuation for Eqs. (8) and (9), we obtain [37]

$$\phi_{\text{I,odm}} = e^{2\pi i\omega(M - D)}\phi_{\text{out,odm}}(r > r_+) + e^{-2\pi i\omega(M - D)}\phi_{\text{out,odm}}^*(r < r_+),$$

$$\phi_{\text{II,odm}} = e^{2\pi i\omega(M - D)}\phi_{\text{out,odm}}^*(r > r_+) + e^{-2\pi i\omega(M - D)}\phi_{\text{out,odm}}^*(r < r_+),$$

which shows that one can use the Kruskal coordinates to introduce new orthogonal basis for the scalar field.

By expanding the scalar field $\Phi$ in terms of $\phi_{\text{I,odm}}$ and $\phi_{\text{II,odm}}$ in the GHS spacetime, one obtains

$$\Phi = \sum_{lm} \int d\omega [a_{\text{I,odm}}\phi_{\text{I,odm}} + a_{\text{I,odm}}^{\dagger}\phi_{\text{I,odm}}],$$

$$+ a_{\text{II,odm}}\phi_{\text{II,odm}} + a_{\text{II,odm}}^{\dagger}\phi_{\text{II,odm}}].$$
where the annihilation operator $a_{I,colm}$ can be used to define the Kruskal vacuum outside the event horizon [37]

$$a_{I,colm}|0\rangle_K = 0. \quad (13)$$

It is seen that Eq. (5) is the expansion of the scalar field in Schwarzschild modes, while Eq. (12) corresponds to the decomposition of the field in Kruskal modes [37,38]. Then we can calculate the Bogoliubov transformations between the particle annihilation and creation operators which act on the Schwarzschild vacuum and Kruskal vacuum, respectively. After some calculation, the Bogoliubov transformations are found to be [38]

$$a_{I,colm} = b_{out,colm} \cosh u - b_{in,colm}^\dagger \sinh u,$$
$$a_{I,colm}^\dagger = b_{out,colm}^\dagger \cosh u - b_{in,colm} \sinh u, \quad (14)$$

where $\cosh u = \sqrt{1 - e^{-2\pi u(M - D)}}$, $a_{I,colm}$ and $a_{I,colm}^\dagger$ are the annihilation and creation operators acting on the Kruskal vacuum of the exterior region, respectively. For the observer outside the black hole, the modes in the interior region should be traced over because a local observer has no access to the information in the causally disconnected region.

After normalizing the state vector, it is found that the Kruskal vacuum can be expressed as an entangled two-mode squeezed state

$$|0\rangle_K = \frac{1}{\cosh u} \sum_{n=0}^\infty \tanh^n u |n\rangle_{in} \otimes |n\rangle_{out}, \quad (15)$$

where $|n\rangle_{in}$ and $|n\rangle_{out}$ are excited-states for Schwarzschild modes inside and outside the event horizon. This means that the observers in different coordinates will not agree on the particle content of each of these states. It is then interesting to investigate to what degree the coherence of the quantum state for continuous variables is changed when the state is described by the observers in different coordinates.

### 3 Measurement of quantum coherence for continuous variables

In this section, we review the measurement of quantum coherence for continuous variables [34]. A quantum system is called a continuous-variable system because it has an infinite-dimensional Hilbert space described by observables with continuous eigenspectra [39]. The prototype of a continuous-variable system is represented by $N$ bosonic modes corresponding to the field operators $\{\hat{a}_k, \hat{a}_k^\dagger\}_{k=1}^N$. These annihilation and creation operators can be arranged in a vectorial operator $\hat{b} := (\hat{a}_1, \hat{a}_1^\dagger, \ldots, \hat{a}_N, \hat{a}_N^\dagger)^T$, which must satisfy the bosonic commutation relations

$$[\hat{b}_i, \hat{b}_j] = \Omega_{ij}, \quad (16)$$

where $\Omega_{ij}$ are generic elements of

$$\Omega := \bigoplus_{k=1}^N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (17)$$

known as the symplectic form. The Hilbert space of this system is infinite-dimensional because the single-mode Hilbert space $\mathcal{H}$ is spanned by a countable Fock basis $\{|n\rangle\}_{n=0}^\infty$. Besides the bosonic field operators, the bosonic system may be described by the quadrature field operators, formally arranged in the vector $\hat{R} := (\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_N, \hat{p}_N)^T$ [39], which are related to the annihilation $\hat{a}_i$ and creation $\hat{a}_i^\dagger$ operators of each mode, by the relations $\hat{q}_i = (\hat{a}_i + \hat{a}_i^\dagger) / \sqrt{2}$ and $\hat{p}_i = (\hat{a}_i - \hat{a}_i^\dagger) / \sqrt{2}$. The quadrature operators $\hat{q}_i$ and $\hat{p}_i$ represent the canonical observables of the system. Similarly, the vector operator should satisfy the commutation relationship $[\hat{R}_i, \hat{R}_j] = i\Omega_{ij}$, which takes symplectic form. The most relevant quantities that characterize the nature of a two-mode Gaussian state $\rho_{AB}$ are the statistical moments. The first moment is the mean value $\hat{R} := \langle \hat{R} \rangle = Tr(\hat{R}\rho)$, and the second moment is the covariance matrix $\sigma$, whose arbitrary element is defined by $\sigma_{ij} = \langle (\hat{R}_i - \hat{R}) (\hat{R}_j - \hat{R}) \rangle$. The covariance matrix is a symmetric matrix which must satisfy the uncertainty principle $\sigma + i\Omega \geq 0$ [40], which implies the positive definiteness $\sigma > 0$.

For a two-mode Gaussian state, we can write its covariance matrix in the block form

$$\sigma = \begin{pmatrix} \alpha & \gamma \\ \gamma^T & \beta \end{pmatrix}, \quad (18)$$

where $\alpha = \sigma^T$, $\beta = \beta^T$ and $\gamma$ are $2 \times 2$ real matrices. Then, the Williamson form is simply $\sigma^{\text{WD}} = (\nu_- I) \oplus (\nu_+ I)$, where symplectic spectrum $\{\nu_-, \nu_+\}$ is provided by $\nu_{\pm} = \sqrt{\Delta \pm \sqrt{\Delta^2 - 4 \det \Delta}}$, with $\Delta := \det \alpha + \det \beta + 2 \det \gamma$ and $\det$ is the determinant [39].

As shown in [34], the definition of the continuous variable quantum coherence of a Gaussian state is

$$C(\rho) = \inf S(\rho||\delta), \quad (19)$$

where $S(\rho||\delta) = tr(\rho \log_2 \rho) - tr(\rho \log_2 \delta)$ is the relative entropy, $\delta$ is an incoherent Gaussian state and the minimization runs over all incoherent Gaussian states. In addition, the entropy of $\rho$ is defined by [41]

$$S(\rho) = -tr(\rho \log_2 \rho) = \sum_{i=1}^m f(v_i), \quad (20)$$

where $f(v_i) = \frac{v_i+1}{2} \log_2 \frac{v_i+1}{2} - \frac{v_i-1}{2} \log_2 \frac{v_i-1}{2}$, and $v_i$ are symplectic eigenvalues of each modes. The mean occupation value can be expressed as

$$\bar{\pi}_i = \frac{1}{4} (\sigma_{11}^i + \sigma_{22}^i + [d_{12}^i]^2 + [d_{21}^i]^2 - 2). \quad (21)$$
In this equation, $\sigma'$ are elements of the subsystem of mode $i$ in a continuous variable matrix, and $[d^i]^2$ is the $i$-th first statistical moment of the mode. Then the measurement of quantum coherence of Gaussian states can be expressed as

$$C(\rho) = -S(\rho) + \sum_{i=1}^{m} \left[ (\bar{n}_i + 1) \log_2(\bar{n}_i + 1) - \bar{n}_i \log_2 \bar{n}_i \right].$$

(22)

### 4 The dynamics of quantum coherence in GHS dilaton black hole

#### 4.1 The dynamics of quantum coherence between the modes observed by Alice and Bob

In this subsection, we seek for a phase-space description for the dynamics of Gaussian quantum coherence under the influence of the GHS dilaton black hole. We assume that an observer Alice stays at the asymptotically flat region, while Bob observing subsystem $B$ hovers near the event horizon of the black hole. The initial state shared between them is a two-mode squeezed Gaussian state

$$\sigma_{AB}^{(G)}(s) = \begin{pmatrix} I_2 \cosh 2s & Z_2 \sinh 2s \\ Z_2 \sinh 2s & I_2 \cosh 2s \end{pmatrix},$$

(23)

where $Z_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $s$ is the squeezing parameter.

It has been shown in Eq. (15) that the Kruskal vacuum is an entangled two-mode squeezed state in terms of Schwarzschild modes. The two mode squeezed transformation can be expressed by a symplectic operator in the phase-space

$$S_{B,\bar{B}}(D) = \begin{pmatrix} I_2 \cosh u & Z_2 \sinh u \\ Z_2 \sinh u & I_2 \cosh u \end{pmatrix}. $$

(24)

After the action of the two-mode squeezed transformation, the entire system involves three subsystems: the subsystem $A$ described by the global observer Alice, the subsystem $B$ described by Bob, and the subsystem $\bar{B}$ described by the virtual observer anti-Bob. The covariance matrix $\sigma_{AB\bar{B}}$ of the tripartite quantum system is given by [42]

$$\sigma_{AB\bar{B}}(s, D) = \left[ I_A \oplus S_{B,\bar{B}}(D) \right] \left[ \sigma_{AB}^{(G)}(s) \oplus I_{\bar{B}} \right] \times \left[ I_A \oplus S_{B,\bar{B}}(D) \right].$$

(25)

where $S_{B,\bar{B}}(D)$ is the phase-space representation of the two-mode squeezing operation given in Eq. (24). In Eq. (25), the matrix $\left[ \sigma_{AB}^{(G)}(s) \oplus I_{\bar{B}} \right]$ describes the initial state for the entire system, and $\left[ I_A \oplus S_{B,\bar{B}}(D) \right]$ denotes that the two-mode squeezed transformation is only performed on the bipartite subsystem between Bob and anti-Bob.

Because the exterior region of the black hole is causally disconnected to the inner region, Alice and Bob cannot approach the mode $\bar{B}$. Then one obtains the covariance matrix $\sigma_{AB}(s, D)$ for Alice and Bob by tracing out the mode $\bar{B}$

$$\sigma_{AB}(s, D) = \begin{pmatrix} A_{AB} & C_{AB} \\ C_{AB}^T & B_{AB} \end{pmatrix},$$

(26)

where

$$A_{AB} = \cosh 2s I_2,$$

$$C_{AB} = \cosh u \sinh 2s Z_2,$$

and

$$B_{AB} = [\cosh^2 u \cosh 2s + \sinh^2 u] I_2.$$  

We know that the symplectic matrix $\sigma_{AB}(s, D)$ is a case of the smallest mixed Gaussian state according to the partially transposed symplectic matrix. From the formula above, we obtain

$$\Delta^{(AB)} = 1 + [\cosh^2 u + \sinh^2 u \cosh 2s]^2.$$  

(27)

The mean occupation numbers operator for each mode from the covariance matrix are $\bar{n}_A = \sinh^2 s$ and $\bar{n}_B = \cosh^2 u \cosh^2 s - 1$. Inserting them into Eq. (22), we obtain the quantum coherence of the Gaussian state Eq. (31). Then we can see that the mean occupation numbers as well as the Gaussian coherence depend not only on the dilaton charge $D$ and the mass $M$ of the black hole but also on the frequency $\omega$ and the squeezing parameter $s$.

In Fig. 1, we plot the accessible quantum coherence between Alice and Bob as a function of the initial squeezing parameter $s$ and the frequency $\omega$ of the scalar field. It is
illustrated that the Gaussian quantum coherence is a monotonic increasing function of the frequency. This indicates that one can obtain more coherence by choosing a field with a higher frequency. It is worth noting that the coherence is almost unaffected to the change of frequency if the squeezing parameter is very small. On the other hand, it becomes more sensitive to the change of the frequency when the squeezing parameter becomes larger. That is, the frequency of the field produces positive effects on the storage of quantum coherence in the GHS black hole.

In Fig. 2a, we plot the quantum coherence between the initial correlated Alice and Bob as a function of the ratio between the dilaton charge and the mass of the black hole. We find that Gaussian quantum coherence in the state $\rho_{AB}$ is not affected by the frequency $\omega$ when the ratio is $D/M = 0$. In addition, the coherence is independent of the frequency $\omega$ in the limit of $D \to M$, which corresponds to an extreme dilaton black hole. That is to say, the coherence is not affected by the frequency of the scalar field for an uncharged black hole or an extreme dilaton black hole.

It is found that the Gaussian coherence between Alice and Bob decreases with the growth of the ratio $D/M$, which means that the gravitational field induced by dilaton will destroy the quantum coherence between the initially correlated modes. This verifies the fact that the gravitational field induced by dilaton plays a key role in the dynamics of Gaussian coherence in the dilaton spacetime. It is interesting to note that the Gaussian coherence between Alice and Bob is not completely destroyed even in the limit of $D \to M$, which corresponds to an extreme black hole. This is quite different from the behavior of quantum steering in the GHS spacetime because the latter suffers from a “sudden death” [17]. This is also different from the behavior of quantum entanglement in the GHS spacetime because the entanglement decays to zero only in the limit of $D \to M$ [38]. That is to say, quantum coherence is more robust than entanglement and steering under the influence of spacetime effects near the event horizon of the dilaton black hole.

This is reasonable because quantum resources are hierarchical. It is known that quantum coherence can be defined for the integral system, while quantum correlations characterize the quantum features of a bipartite or a multipartite system [23]. The results in the present paper, as well as those in Refs. [17,38], verify the fact that the quantum resources are hierarchical and quantum coherence is more robust than quantum correlations in the dilaton spacetime. On the other hand, this result indicates the possibility of Bob being capable of performing quantum information tasks even in the case of an extreme dilaton black hole because quantum coherence is a usable resource for the tasks. This is nontrivial because one can employ quantum coherence as the resource of quantum information processing tasks even if the quantum correlations have been destroyed by strong gravitational effects.

Figure 2b shows a contour diagram of Gaussian coherence versus the squeezing parameter $s$ and the ratio $D/M$. We can see that, if the initial squeezing parameter is close to 1, the gravitational field of the spacetime has significant influence on the Gaussian quantum coherence only near the $D \to M$ limit. The coherence between Alice and Bob is nonzero even in the limit of an extreme dilaton black hole. This means that one can perform quantum information processing tasks in the GHS spacetime if sufficient resource is prepared in the initial state. In addition, the coherence becomes more sensitive with the change of the squeezing parameter $s$ for larger dilaton parameters.

![Fig. 2 a Plots of the quantum coherence between Alice and Bob as a function of the ratio $D/M$ between the dilaton charge and the mass of the black hole. The initial squeezing parameter is fixed as $s = 1$ and the frequency of the field are $\omega = 1$ (red line), $\omega = 0.5$ (purple dotted line), $\omega = 0.3$ (green dotted line), respectively. b The contour diagram of Gaussian quantum coherence between Alice and Bob versus the squeezing parameter $s$ and the ratio $D/M$. The frequency of the scalar field is fixed as $\omega = 1$](image-url)
4.2 Generating quantum coherence between the initially uncorrelated modes

In this subsection, we study the dynamics of quantum coherence among the initially uncorrelated modes. The covariance matrix between Alice and the observer anti-Bob is obtained by tracing over the outside mode $B$

$$\sigma_{AB}(s, r) = \begin{pmatrix} A_{AB} & C_{AB} \\ C_{AB}^T & B_{AB} \end{pmatrix},$$

(28)

where

$$A_{AB} = [\cosh(2s)]I_2,$$

$$C_{AB} = [\sinh u \sinh 2s]I_2,$$

and

$$B_{AB} = [\sinh^2 u \cosh 2s + \cosh^2 u]I_2.$$

The corresponding symplectic value of the covariance matrix $\sigma_{AB}$ is found to be $\nu_+ = \cosh^2 u \cosh 2s + \sinh^2 u$ and $\nu_- = 1$. Similarly, we find $\Delta^{(AB)} = 1 + [\cosh^2 u \cosh 2s + \sinh^2 u]^2$. Then we calculate the quantum coherence between the Alice and anti-Bob and plot it in Fig. 3.

Figure 3a shows the Gaussian quantum coherence between Alice and anti-Bob as a function of the ratio $D/M$ for different frequencies. Again, it is found that the quantum coherence is independent of the frequency $\omega$ for the non-dilaton and extreme dilaton black hole. As the increase of the ratio $D/M$, the Gaussian coherence is generated between Alice and anti-Bob, which is quite different from the quantum steering between Alice and Bob for the same state [17], where the Gaussian quantum steering is zero for any dilaton charge. In addition, the lower the frequency of the mode in the state $\rho_{AB}$, the stronger the Gaussian quantum coherence. This indicates the generation of quantum coherence between Alice and Bob is easier for low-frequency fields. In Fig. 3b, it is shown that the Gaussian coherence between Alice and anti-Bob exhibits a “sudden birth” behavior under the influence of the dilaton field.

The covariance matrix between Bob and anti-Bob inside the event horizon is obtained by tracing over the mode $A$

$$\sigma_{B\bar{B}}(s, r) = \begin{pmatrix} A_{B\bar{B}} & C_{B\bar{B}} \\ C_{B\bar{B}}^T & B_{B\bar{B}} \end{pmatrix},$$

(29)

where

$$A_{B\bar{B}} = [\cosh^2 u \cosh 2s + \sinh^2 u]I_2,$$

$$C_{B\bar{B}} = [\sinh 2u \cosh^2 s]Z_2,$$

and

$$B_{B\bar{B}} = [\sinh^2 u \cosh 2s + \cosh^2 u]I_2.$$

We can also calculate the Gaussian quantum coherence between Bob and anti-Bob.

In Fig. 4, we show that the quantum coherence between Bob and anti-Bob is a monotonically increasing function of $D/M$. It changes slowly at first and becomes more sensitive with the increase of $D/M$. It is also found that the quantum coherence is independent of the frequency $\omega$ for the non-dilaton and extreme dilaton black hole. As the increase of the ratio $D/M$, the quantum coherence is smoothly generated between Bob and anti-Bob. This is different from the generation of Gaussian coherence between Alice and anti-Bob because the latter exhibits a “sudden birth” behavior. It is shown that the gravitational field induced by dilaton generates Gaussian quantum coherence between the causally

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Fig. 3 a Plots of the quantum coherence between Alice and anti-Bob as a function of the ratio $D/M$. The initial squeezing parameter is fixed as $s = 1$ and the frequency of the field are $\omega = 1$ (red line), $\omega = 0.5$ (purple dotted line), $\omega = 0.3$ (green dotted line), respectively. b The contour diagram of Gaussian quantum coherence between Alice and anti-Bob versus the squeezing parameter $s$ and the ratio $D/M$. The frequency of the scalar field is fixed as $\omega = 1$.
disconnected regions. In other words, Bob and anti-Bob can perform quantum information processing tasks by local measurements even though they are separated by the event horizon.

5 Summary

In this work, we study the behavior of quantum coherence for Gaussian states in the background of a GHS dilaton black hole. It is shown that the dilaton field has evident effect on the degree of coherence for all the bipartite subsystems. However, the Gaussian coherence is not affected by the frequency of the scalar field for an uncharged or an extreme dilaton black hole. This verifies the fact that the gravity induced by dilaton field plays a key role in the dynamics of Gaussian coherence in the GHS spacetime. The Gaussian coherence between Alice and Bob is not completely destroyed even for an extreme dilaton black hole. This is quite different from the behavior of quantum steering because the latter suffers from a “sudden death” under the same condition. The coherence between Alice and Bob is nonzero even in the limit of an extreme dilaton black hole. This means that one can perform quantum information processing tasks in the GHS spacetime if sufficient resources are prepared in the initial state.

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References

1. G.W. Gibbons, Nucl. Phys. B 207, 337 (1982)
2. A. Gareia, D. Galloso, O. Kechkin, Phys. Rev. Lett. 74, 1276 (1995)
3. G.W. Gibbons, K. Maeda, Nucl. Phys. B 298, 741 (1988)
4. D. Garfinkle, G.T. Horowitz, A. Strominger, Phys. Rev. D 43, 3140 (1991)
5. S.W. Hawking, Phys. Rev. D 14, 2460 (1976)
6. I. Fuentes-Schuller, R.B. Mann, Phys. Rev. Lett. 95, 120404 (2005)
7. J.L. Ball, I. Fuentes-Schuller, F.P. Schuller, Phys. Lett. A 359, 550–554 (2006)
8. J. Wang, Z. Tian, J. Jing, H. Fan, Phys. Rev. A 93, 062105 (2016)
9. E. Martin-Martinecz, L.J. Garay, J. Leon, Phys. Rev. D 82, 064028 (2010)
10. D.E. Bruschi, J. Louko, E. Martin-Martinecz, A. Dragan, I. Fuentes, Phys. Rev. A 82, 42332 (2010)
11. J. Wang, J. Jing, Phys. Rev. A 83, 022314 (2011)
12. D.E. Bruschi, A. Dragan, I. Fuentes, J. Louko, Phys. Rev. D 86, 025026 (2012)
13. D. Sa, T.C. Ralph, Phys. Rev. D 90, 084022 (2014)
14. X. Liu, Z. Tian, J. Wang, J. Jing, Phys. Rev. D 97, 105030 (2018)
15. J. Wang, C. Wen, S. Chen, J. Jing, Phys. Lett. B 800, 135109 (2020)
16. K. Gallock-Yoshimura, E. Tjoa, R.B. Mann, Phys. Rev. D 104, 025001 (2021)
17. B. Hu, C. Wen, J. Wang, J. Jing, Eur. Phys. J. C 81, 925 (2021)
18. D. Harlow, Rev. Mod. Phys. 88, 015002 (2014)

Fig. 4 a Plots of the quantum coherence between Bob and anti-Bob as a function of the ratio $D/M$. The initial squeezing parameter is fixed as $s = 1$. b The contour diagram of Gaussian quantum coherence between Bob and anti-Bob versus the squeezing parameter $s$ and the ratio $D/M$. The frequency of the scalar field is fixed as $\omega = 1$. 

[Diagram of quantum coherence plots and contour diagram]
19. W.G. Unruh, Phys. Rev. D 14, 870 (1976)
20. S.W. Hawking, Nature 30, 248 (1974)
21. T. Baumgratz, M. Cramer, M.B. Plenio, Phys. Rev. Lett. 113, 140401 (2014)
22. A. Streltsov, G. Adesso, M.B. Plenio, Rev. Mod. Phys. 89, 041003 (2017)
23. M. Hu, X. Hu, J. Wang, Y. Peng, Y. Zhang, H. Fan, Phys. Rep. 762, 1–100 (2018)
24. H. Zhou, T. Gao, F. Yan, Phys. Rev. Research 4, 013200 (2022)
25. L.K. Grover, Phys. Rev. Lett. 79, 325 (1997)
26. Y. Wu et al., Phys. Rev. Lett. 127, 180501 (2021)
27. M. Gndoan, P.M. Ledingham, A. Almasi, M. Cristiani, H. Riedmatten, Phys. Rev. Lett. 108, 190504 (2012)
28. C. Li et al., Phys. Rev. Lett. 124, 240504 (2020)
29. Y.F. Hsiao et al., Phys. Rev. Lett. 120, 18360 (2018)
30. C. Harney, S. Pirandola, PRX Quantum 3, 010311 (2022)
31. V. Giovannetti, S. Lloyd, L. Maccone, Science 306, 1330 (1997)
32. V. Giovannetti, S. Lloyd, L. Maccone, Nat. Photonics 5, 222 (2011)
33. Y. Yang, Phys. Rev. Lett. 123, 110501 (2019)
34. J. Xu, Phys. Rev. A 93, 032111 (2016)
35. S.L. Braunstein, P. van Loock, Rev. Mod. Phys. 77, 513 (2015)
36. N.D. Birrell, P.C.W. Davies, Quantum fields in Curved Space (Cambridge University Press, Cambridge, 1982)
37. T. Damoar, R. Ruffini, Phys. Rev. D 14, 332 (1976)
38. J. Wang, S. Chen, Q. Pan, J. Jing, Phys. Lett. B 677, 186 (2009)
39. C. Weedbrook et al., Rev. Mod. Phys. 84, 621 (2012)
40. R. Simon, N. Mukunda, B. Dutta, Phys. Rev. A 49, 1567 (1994)
41. A.S. Holevo, M. Sohma, O. Hirota, Phys. Rev. A 59, 1820 (1999)
42. G. Adesso, I. Fuentes-Schuller, M. Ericsson, Phys. Rev. A 76, 062112 (2007)