On the nature of dark energy. Is the Casimir force a manifestation of this exotic kind of energy?

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Summary - Topics concerning dark energy and the accelerating expansion of universe are briefly reported. Arguments about the quantum fluctuations of vacuum are examined in order to decide if the Casimir force really is a laboratory evidence of dark energy.

Key words: quantum electrodynamics, cosmology, Casimir effect.

1 - The rate of the universe expansion

In the last decade of the past century, intensive investigations were undertaken in order to probe the slowing down of the universe expansion. The most important tool was the measurement of magnitudes of type Ia supernovae placed in remote galaxies. Since these objects are characterized by a standard peak brightness, this allows determination of their distances from earth and consequently their antiquity. By measuring their Hubble red-shifts as well, the expansion rate of universe was gauged versus time, starting from the Big Bang event dating about fourteen billion years ago. Expansion rate was found decreasing in the course of the first nine billions, but speeding up over the last five billions. To account for this surprising result, cosmologists proposed a modified form of Einstein general theory of relativity by introducing in the field equation an additional term depending on the vacuum energy, that is,

\[ G_{\mu\nu} = 8\pi G (T_{\mu\nu} + \rho_{\text{VAC}} g_{\mu\nu}), \]

where \( G_{\mu\nu} \) is the space curvature tensor, \( G \) the Newton gravity constant, \( T_{\mu\nu} \) the stress-energy tensor, \( g_{\mu\nu} \) the metric tensor and \( \rho_{\text{VAC}} \) the vacuum energy density usually referred to as "dark energy" [1, 2]. Term \( \rho_{\text{VAC}} g_{\mu\nu} \), opposite to \( T_{\mu\nu} \), represents a "negative-form-of-gravity" due to the pressure of vacuum quantum fluctuations. Since it is an intrinsic property of space, it should remain constant even when the universe expands.

2 - The vacuum quantum fluctuations

Quantum fluctuations of vacuum is a topic which require some specifications. Indeed, owing to the energy-time indeterminacy principle \( \delta w \delta t \simeq \hbar \), fluctuations \( \delta w = 2m_e c^2 \) originate in vacuum virtual electron-positron pairs which join again after a very short time \( \delta t \simeq 4 \cdot 10^{-21} \text{s} \). It follows that the electron-positron separation keeps small with respect to Compton wavelength \( \hbar/m_e c = 2.4 \cdot 10^{-12} \text{m} \). Consequently, pairs act as dipoles and vacuum behaves like a polarizable dielectric which has the effect of reducing all charges by a constant amount [3]. In the vacuum diagrammatic picture, one electron and one positron line starting from a vertex \( A \) rejoin in a vertex \( B \) while a wavy
line, representing a photon, comes back from $B$ to $A$. Six first-order diagrams of this kind are possible, all involving one photon line. Among these, four involve interaction with matter [4]. The arguments just outlined show that vacuum is filled up with electromagnetic energy. This entails the existence of an internal pressure related to energy density, like occurs for the black-body radiation [5].

3 - Does the dark energy really exist?

The previous interpretation of the accelerating rate of the universe expansion is not free from difficulties. Indeed, all the attempts of evaluate $\rho_{vac}$, the dark energy density, led to absurdly large values, giving rise to pressures so high that all matter in the universe would instantly fly apart. Actually, the density magnitude acceptable on the ground of cosmological arguments is somewhat small, which presumably precludes its identification in the laboratory [1]. For this reason, a different interpretation was recently proposed [6]. It assumes that our galaxy, the milky way, lies in an emptier-than-average region of space, a sort of huge void. Consequently, the reduced presence of gravitational matter, that is, barionic and dark matter, originates a reduced slowing down of the space expansion which could be mistaken for an acceleration. Probably, future observations, improving the statistic bases of these researches, will differentiate between the two interpretations.

4 - The Casimir effect

Owing to this disappointing state of affairs, it seems right to consider a different argument in principle useful for understanding the nature of dark energy. In year 1948, the Dutch physicist Hendrik Casimir proposed a peculiar experiment for detecting the quantum fluctuations of vacuum [7]. The Casimir's experimental set-up is as follows. Two plane conducting plates of area $A$ are positioned parallel at a small distance $d$ on the order of one $\mu$m. In the space span between the plates, the wavelength $\lambda$ of virtual photons propagating in direction orthogonal to the plates must satisfy the condition $d = n \lambda/2$ ($n = 1, 2, 3...$), like real photons in a Fabry-Perot cavity. In this way, most of the propagation modes is turned off, while in the external space all modes remain allowed. Consequently, an imbalance in the quantum vacuum pressure follows which originates between the plates the attractive force

$$F_C = \frac{\pi \hbar c A}{480 d^4}$$

that is, just the Casimir force. It is expected to be rather small, indeed by assuming $A = 1 \text{ cm}^2$ and $d = 1 \mu$m, equation (2) yields $F_C = 1.3 \cdot 10^{-7} \text{ N}$, which corresponds to pressure $F_C/A$ of $1.3 \cdot 10^{-8}$ b (1 b = 0.987 Atm). In the course of the last half-century, theory of Casimir effect was intensively studied and various experiments for measuring $F_C$ were carried out, but with poor success [8]. Since this is due mainly to the difficulty of achieving a perfect parallelism between the plates, in year 1997, a conclusive experiment was performed utilizing a sphere-flat-plate geometry. The experimental data were compared with a modified form of equation (2) showing an agreement of 5% with theory [9]. Recently, utilizing the parallel-plate configuration, equation (2) was checked at the 15% precision level in the $0.5 - 3 \mu$m range [10].
These results are acknowledged as a direct evidence of vacuum pressure. Consequently, by letting $P_V$ and $P_P$ be the pressures in the external space and in space between the plates, respectively, and considering that pressures $P_P$ and $F_C/A$ depend on plate separation $d$, we have

$$P_V - P_P(d) = F_C(d)/A.$$  \hspace{1cm} (3)

Actually, when $d \to \infty$ pressure $P_P$ attains its maximum value $P_V$. When $d$ decreases, $P_P$ decreases too in such a way that for $d \to 0$ we can let $P_P = 0$. This merely because no pressure can be accounted when space goes to zero. On this ground, by omitting contribution of $P_P$ and choosing $d = 1 \mu m$, it follows from equation (3) $P_V > 1.3 \cdot 10^{-8}$ b, a figure not manifestly unreliable. But, this choice has no meaning in connection with vacuum fluctuations. It was pointed out in Section 2 that, owing to quickness of vacuum fluctuations, the electron-positron separation is small with respect to Compton wavelength. By choosing tentatively $d$ equal to this length, that is, $2.4 \cdot 10^{-12}$ m, we get $P_V > 3.9 \cdot 10^{14}$ b. This absurdly large figure is due to the diverging character of equation (2) for small values of $d$. This equation is fit for representing macroscopic phenomena, but it appears inadequate as for vacuum fluctuations and the dark energy problem.

In our opinion and also in order to avoid possible confusion with van der Waals force, it seems right to devise alternative experiments fit for detecting vacuum fluctuations in laboratory. For this purpose, consider, for instance, a light emitter placed between conducting plates, like those in Casimir experiment. It is expected to show a modified spectrum, owing to reduced vacuum fluctuations which, as pointed out in Section 2, control the particle charges [2].

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\^ An interesting experiment of this kind has been recently proposed based on measure of current fluctuations in Josephson junctions [11].