Analysis of The Efficiency SA (Simulated Annealing) Model And Fuzzy Time Series (Case Study: Inventory System PT Y)

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Abstract. Forecasting sales and minimizing storage costs is the company’s strategy to maximize inventory in anticipation of sales spikes in stores so that it can be fulfilled even though storage warehouses are limited, on the other hand there is competition among distributors in ordering goods, which is trying to get the maximum level of service from suppliers so warehouse needs can be fulfilled. By using the forecasting method, Chen Fuzzy Time Series to predict future sales combined with the simulation annealing method (simulated annealing algorithm) as an optimization method in order to obtain the lowest possible storage costs. This combination of methods can improve shipping services from warehouse to store as a result of loss sales cause inventory shortages can be controlled so that inventory costs are more minimal and warehouse storage of goods can be maximized, the combination of these methods is more effective than conventional methods available at PT Y.

Keywords: Forecasting, Fuzzy Time Series, Simulated Annealing Algorithm, Inventory Control, inventory system.

1. introduction

Predicting trends and patterns of consumer behavior in meeting daily needs is a difficult one. This relates to the environmental conditions and income of each different region and the location or strategic marketing area of a particular product, in addition there is a policy of promos for new output goods, goods that have been agreed between suppliers with retail / operations to do promos, and goods with the purpose of increasing sales.

Accuracy in managing inventory has an important role for a company to achieve service targets for consumers. large target value determines the needs of goods that must be provided by the company, because the amount of value needs with the number of targeted values is directly proportional, so that a large impact on storage costs, if more inventory is stored, resulting in significant costs and requires a volume of storage space (storage area ) on a large scale. However, if the inventory does not result in out of stock (empty goods) in retail or warehouse. This is because the sales target exceeds the targeted number, and the compliance of the supplier is less than the maximum. The impact of this shortage is a serious inventory problem in the company's operations. Then the problem of over stock is caused by the large value of the buffer stock which is affected by the amount of demand, the promo and sales targets, and the waiting time for shipping by the supplier, but physically sold is smaller than the supply that has been provided. the impact of over stock results in high costs for storing and maintaining goods during the storage process in the warehouse, even though the inventory can be used for other things that are
more profitable. Every company basically has a target to maximize profits, not reduce or increase inventory. Therefore, every company needs to calculate carefully so that stock can be kept as minimal as possible, but can guarantee smooth operations. To get high technical and economic efficiency, certainly the right method must be sought so that when procuring the amount of goods for new stock is neither more nor less. This method is known as controlling stock of goods [1].

From the problems above, the researcher offers several methods to get the best and efficient results. The use of methods in accordance with these conditions is the methods of forecasting and optimization with the aim of predicting goods that sell in the future and then added to the buffer stock for inventory until the supplier makes a return shipment. Many studies study and develop forecasting and optimization methods. In this paper the authors use the forecasting method, fuzzy forecast with a new concept proposed by [3] is the fuzzy time series method based on the concept of linguistic variables and the fuzzy set theory and its application by Zadeh. Fuzzy Time Series is used to solve forecasting problems in which trend historical data. Then the simulated annealing (SA) algorithm method simulates the annealing process in the manufacture of material consisting of crystal (glassy) or metal grains. The purpose of this process is to produce a good crystal structure using minimal energy [2]. In 1983, Kirkpatrick and colleagues [KIR 83] used the idea of the algorithm Metropolis and applied it to optimization problems. The idea is how to use SA to find variable and converging solutions to the optimal solution [2].

2. Methods

2.1. Fuzzy Time Series

The basic concept was first introduced by [3]. The main difference between conventional forecasting and fuzzy time series is the value used in forecasting, which is the fuzzy set of real numbers for a given set can be interpreted as a number class with vague boundaries. If \( U \) is the universe of discourse, \( U = \{U_1, U_2, \ldots, U_n\} \), then a fuzzy set \( A \) from \( U \) is defined as [4]:

\[
A = \frac{f_A(U_1)}{u_1} + \frac{f_A(U_2)}{u_2} + \cdots + \frac{f_A(U_n)}{u_n}
\]

(1)

Where \( f_A \) is the membership function of the fuzzy set \( A \), \( f_A: U \rightarrow [0,1] \). Notated as a degree of membership from \( U_i \) in a fuzzy set \( A \), and \( 1 \leq i \leq n \). While the definition of fuzzy time series:

Definition 1

Suppose a subset of \( R \) is \( Y(t)(t = \ldots, 0,1,2,\ldots) \), universe of discourse in a fuzzy set \( f_i(t)(i = 1,2,\ldots) \) which is defined and given \( F(t) \) which is a collection of \( f_1(t), f_2(t), \ldots \)

\( F(t) \) is called fuzzy time series which is defined in \( (t)(t = \ldots, 0,1,2,\ldots) \) [5]

if \( F(t) \) is caused by \( F(t-1) \) if and only if \( F(t-1) \rightarrow F(t) \), then it can be expressed as \( F(t) = F(t-1) \times R(t, t-1) \), where \( R(t, t-1) \) is a fuzzy relation between \( F(t-1) \) and \( F(t) \). And \( F(t) \) is called the first order forecasting model by:

\[
F(t) = F(t-1) \times R(t, t-1)
\]

(2)

Where \( F(t-1) \rightarrow F(t) \) is called a fuzzy logic relation, \( F(t-1) \) is the current condition and \( F(t) \) is the next condition [5]

Definition 2

If there is a fuzzy relation \( R(t, t-1) \), such that \( F(t) = F(t-1) \times R(t-1, t) \) with \( \times \) is an operator, then \( F(t) \) is said to be dependent on \( F(t-1) \). Provided that the intended operator can be a max-min, min-max, or arithmetic operator [6]. If \( F(t-1) = A_i \) and \( F(t) = A_j \) the relation between a \( F(t-1) \) dan \( F(t) \) is called the fuzzy logic relation which is denoted as follows:

\[
A_i \rightarrow A_j
\]

(3)

Definition 3

Fuzzy logic relations with the same fuzzy set on the other hand can be categorized into fuzzy logic relations groups [6]. Based on fuzzy logic relations so;

\[
A_i \rightarrow A_{j1},
A_i \rightarrow A_{j2},
\]

(4)
Can be grouped into fuzzy logic relations groups
\[ A_i \rightarrow A_{j1}, A_{j2}, \ldots \] (5)

**Definition 4**
Given \( F(t) \) is a fuzzy logic relation that results from \( F(t-1) \), and \( F(t) = F(t-1) \times R(t, t-1) \), for each \( t \). If \( R(t, t-1) \) is dependent on \( t \), then \( F(t) \) fuzzy time series time-invariant, but if it is independent of \( t \), then \( F(t) \) fuzzy time series time-variant [5]

### 2.2 Mathematical model
The following is a mathematical model that has been built:

#### 2.2.1 Parameter:
- \( i \) = Type of item
- \( t \) = Planning time
- \( o \) = Ordering
- \( s \) = Stock
- \( d \) = Demand forecasting
- \( LT \) = Lead Time
- \( P_i \) = Price item \( i \)
- \( G \) = Storage capacity
- \( r \) = Minimum order
- \( C_{0i} \) = Ordering cost item \( i \)
- \( C_{Si} \) = Stock cost item \( i \)
- \( C_{Di} \) = Demand cost item \( i \)

#### 2.2.2 Decision variables:
- \( y_t \) = Number of transportation in the \( t \)
- \( I_{it} \) = Inventory level item \( i \) at \( t \) period
- \( x_{it} \) = Number of order quantity item \( i \) at \( t \) period
- \( r_i \) = Reorder level item \( i \)
- \( c_{ij} \) = Can order level item \( j \) when item \( i \) order
- \( u_{ij} \) = Order up to level item \( j \) when item \( i \) order

#### 2.2.3 Purpose function:
\[
\begin{align*}
\text{Min } Z_1 &= \sum_{t=1}^{T} y_t \\
\text{Min } Z_2 &= \sum_{t=1}^{T} \sum_{i=1}^{I} x_{it} P_i \\
x_{it} &= LT + d - s, \text{ by maximizing the value of storage capacity and minimum order}
\end{align*}
\]

### 2.3 Simulated Annealing Algorithm
The SA idea came from a paper published by Metropolis in 1953 [MET53]. If we heat a hard material until it melts and then cools it, then the structural properties of the material depend on the degree of cooling. If the material is slowly cooled, good quality crystals will be produced. Conversely, if the liquid material is cooled rapidly, the crystals that are formed aren’t perfect[2].
3. **The Proposed Methods**

The proposed method for forecasting and optimizing are present as follows:

The following is forecasting data with time series:

**Step 1:** Determined $D_{\text{min}}, D_{\text{max}}$ of known historical data, and $D_1, D_2$ are positive numbers, so we can determined the universal of discourse $U$ as $[D_{\text{min}} - D_1, D_{\text{max}} + D_2]$. So $U$ can be partitioned into the same interval length $u_1, u_2, u_3, ..., u_t$. With the midpoints of each interval $m_1, m_2, m_3, ..., m_t$.

**Step 2:** Define fuzzy sets $A_i$ and fuzzification data. Each fuzzy set $A_i$ is shown for linguistic terms and can be defined with intervals $u_1, u_2, u_3, ..., u_t$.

$$A_i = f_{A_i}(u_i)/u_i + f_{A_i}(u_2)/u_2 + ... + f_{A_i}(u_t)/u_t$$ (6)

After getting the interval then the data is defuzzified

**Step 3:** Form the relation between fuzzy logic and fuzzy logic group in the following way:

$$A_j \rightarrow A_q,$$

$$A_j \rightarrow A_r,$$

$$...$$

$$A_m \rightarrow A_s,$$

$$A_m \rightarrow A_t,$$

$$...$$

Rearrange fuzzy logic relations into fuzzy logic relations groups based on the same fuzzy set on the left side of the fuzzy logic relations:

$$A_j \rightarrow A_q, A_r, ..., (7)$$

$$A_m \rightarrow A_s, A_t, ..., (8)$$

**Step 4:** Forecasting:

If $F(t-1) = A_j$, forecasting $F(t)$ then use the following rules:

1. If the fuzzy logic relation group of $A_j$ is empty (zero output results), then the forecast variation is 0 ($F(t)$ is $m_j$, the midpoint of $u_j$), so:

$$\text{Forecasting} = m_j$$ (9)

2. If the fuzzy logic relation group of $A_j$ is a one-to-one relation, such as $A_j \rightarrow A_{p1}, A_{p2}, ..., A_{pk}$, forecast of $F(t)$ is the same as the arithmetic average of $m_{p1}, m_{p2}, ..., m_{pk}$, each midpoint of $u_{p1}, u_{p2}, ..., u_{pk}$.

$$\text{Forecasting} = \frac{\sum_{i=1}^{k} m_{pi}}{k}$$ (10)

**Step 5:** Calculate the root mean square error (RMSE) [7]

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (\text{forecasted value}_i - \text{actual value}_i)^2}{n}}$$ (11)

The conceptual model that has been created is intended for algorithm development. The step scanning in the SA algorithm is used as follows[8]:

**Step 1:** Initialization

Examples of cases of temperature reducing factor (cr) and number of cycles (n) using the SA algorithm parameter

**Step 2:** Solution generation

Some of the initial solution variables are reorder level ($r_i$), can order level ($c_{ij}$), order up to level ($u_{ij}$), and forecast sales ($d_{it}$). Then with the SA algorithm the solution is generated randomly

**Step 3:** Calculation of the objective function

Completion of multi-purpose cases 1 and objective functions 2

**Step 4:** Generating new solutions

A new solution can be obtained from its closest value or commonly called a neighbourhood search, replacing the initial solution that has been generated. Changes to this solution are carried out by
combining changes to the addition of 1 unit, subtracting 1 unit, or no change (0 units added) to the variable \( r, c, u \).

**Step 5:** Comparing old solutions with new solutions
After getting a new solution then the calculation is calculated \( \Delta f \) (the difference between the objective function of the new solution and the old solution). If the value of \( \Delta f < 0 \) the new solution is better than the previous solution, so the new solution will be accepted. However, if the new solution is not better than the old solution, then the metropolis criteria will be calculated to determine whether the new solution is accepted or not.

**Step 6:** Update iteration. Cycle and temperature
Temperature will be reduced by using the temperature reducing factor (cr) after reaching the n cycle. Additionally, the return cycle value is set to be equal to one

**Step 7:** Stopping criteria
Stopping criteria stopping criteria used are temperature values that are close to 0

4. Result and Discussion
In this section, using proposed method to forecast the historical data, sales from the year 2017 to the year 2020:
Let \( n \) be the number of clusters, and we use \( n = 110 \). The method for forecasting is presented as follows

**Step 1:** Define \( U \). From the historical data that, \( D_{\text{min}} = 432, D_{\text{max}} = 1732, D_1 = 2, \) and \( D_2 = 8 \). Thus, \( U = [430,1740] \).

**Step 2:** Partition \( U \) into even lengthy intervals \( u_1, u_2, \ldots, u_t \). In this paper, we partition \( U = [430,1740] \) into 110 intervals.

**Step 3:** Let \( A_1, A_2, \ldots, A_t \) be the fuzzy sets which are linguistic values of the linguistic variables sales PT Y. In this paper, there are 110 linguistic values, as formula (12)

\[
A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \cdots + \frac{0}{u_{110}} \\
A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \cdots + \frac{0}{u_{110}} \\
\vdots \\
A_{110} = \frac{0}{u_1} + \frac{0}{u_2} + \cdots + \frac{1}{u_{110}}
\]

The fuzzification of sales PT Y is shown in table 1. Then, divide the derived fuzzy logical relationships into groups based on the current state of the fuzzy logical relationships, in table 2.

| Year | Month | Actual Data | Fuzzification |
|------|-------|-------------|---------------|
| 2017 | Jan   | 1355        | A78           |
| 2017 | Feb   | 987         | A47           |
| 2017 | Mar   | 1288        | A72           |
| \vdots | \vdots | \vdots | \vdots |
| 2020 | Jan   | 500         | A6            |
| \vdots | \vdots | \vdots | \vdots |
| Jul  | 739   | A26         |               |

**Step 4:** The forecasting results are shown in Table 3

| FLRG of the Data |
|------------------|
| G1 A1 -> A14     |
| G2               |
| \vdots           |
G6 A6 -> A23
: 
G48 A48 -> A48 A48 -> A79
: 
G109 G109 -> A67
G110

**Table 3. Forecasting Result**

| Year | Month | Actual Data | Proposed Method |
|------|-------|-------------|-----------------|
| 2017 | Jan   | 1355        |                 |
| 2017 | Feb   | 987         | 987,5           |
| 2017 | Mar   | 1288        | 1287,5          |
|      |       |             |                 |
| 2020 | Jan   | 500         | 495,5           |
|      |       |             |                 |
|      | Jul   | 739         | 735,3           |

**Figure 1.** Forecasting result for \( n = 110 \)

**Step 5:** Calculate of the value RMSE, shown as Table 4

**Table 4.** The root mean square error (RMSE)

| \( n \) | Chen Method |
|---------|-------------|
| 110     | 109.63      |

**Step 6:** Simulated annealing process

Example the value of parameter and temperature are shown as Table 5 and 6, process inner and outer iteration are shown as Table 7, compare conventional calculations with propose method is shown as Table 8

**Table 5.** Parameter

| Parameter | Value |
|-----------|-------|
| \( T_0 \) | 100   |
| \( T_{end} \) | 1     |
| \( N \)     | 2     |
| \( \alpha \) | 0.5   |
| Best solution | 1.3   |
### Table 6. Temperatur

| $T_{i+1}$ | $\alpha T_i$ |
|-----------|--------------|
| $T_1$     | 50           |
| $T_2$     | 25           |
| $T_3$     | 12.5         |
| $T_4$     | 6.25         |
| $T_5$     | 3.12         |
| $T_6$     | 1.56         |
| $T_7$     | 0.78         |

### Table 7. inner and outer iteration process

| Temperature | $n_1$   | $n_2$   |
|-------------|---------|---------|
| 100         | 2 – 1.6 | 1.6 – 2.24 |
| 50          | 2.24 – 1.8 | 1.8 – 2.51 |
| 25          | 2.51 – 2  | 2 – 2.81  |
| 12.5        | 2.81 – 2.2 | 2.2 – 2.15 |
| 6.25        | 2.15 – 2.5 | 2.5 – 2.52 |
| 3.125       | 2.52 – 2  | 2 – 2.01  |
| 1.5625      | 2.01 – 1.6 | 1.6 – 1.6 |
| 0.78125     | 1.6 – 1.3 | 1.3 – 1.3 |

### Table 8. konvensional and proposed method

| Kode   | Vendor | Volume (m$^3$) | Demand | Cost  |
|--------|--------|----------------|--------|-------|
|        | Konv   | Proposed       | Konv   | Proposed |
| A1003  | 50     | 26             | 192    | 96     |
|        | 10.291,200 | 7,833,600       |

5. Conclusion

This combination of methods, we have presented a Chen Fuzzy Time Series with cluster $n = 110$, can produce a forecast value that is close to the actual value, then with the SA algorithm we can minimizing the order cost. It can improve shipping services from warehouse to store as a result of loss sales cause inventory shortages can be controlled so that inventory costs are more minimal and warehouse storage of goods can be maximized. The combination of these methods is more effective than conventional methods available at PT Y

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