Pressure fluctuation prediction of a model pump turbine at no load opening by a nonlinear $k$-$\varepsilon$ turbulence model

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Abstract. In this paper, a new nonlinear $k$-$\varepsilon$ turbulence model based on RNG $k$-$\varepsilon$ turbulence model and Wilcox's $k$-$\omega$ turbulence model was proposed to simulate the unsteady flow and to predict the pressure fluctuation through a model pump turbine for engineering application. Calculations on a curved rectangular duct proved that the nonlinear $k$-$\varepsilon$ turbulence model is applicable for high pressure gradient flows and large curvature flows. The numerically predicted relative pressure amplitude (peak to peak) in time domain to the pump turbine head at no load condition is very close to the experimental data. It is indicated that the prediction of the pressure fluctuation is valid by the present nonlinear $k$-$\varepsilon$ method. The high pressure fluctuation in this area is the main issue for pump turbine design, especially at high head condition.

1. Introduction

Pump storage power plants are key components for the development of renewable energy and for the security enhancement of electricity supply. In pump turbines, rotor-stator interaction may be considered as the source of unsteady phenomena and dynamic force fluctuations, especially at high head operation condition. The interaction between impeller blades and guide vanes is one of the main causes of vibration in pump turbines. The phenomenology of the rotor-stator interaction may be considered as a combination of potential and viscous flow, or wake, interactions. These phenomena are of interest to improve turbine design and to reach better efficiency at off-peak conditions [1].

Zobeiri et al [2] investigated the rotor-stator interactions of a model pump turbine in turbine mode at the maximum discharge operating condition by numerical simulation of the unsteady flow. Although Nicolet et al [3] studied the one-dimensional modeling of rotor-stator interaction in pump-turbine, the three dimensional flow simulation based on the unsteady Reynolds Averaged Navier-Stokes equations (URANS)
is the main solution for prediction of pressure fluctuation in hydraulic machinery for engineering applications [4], even for prediction of cavitation induced pressure fluctuations [5].

Boussinesq hypothesis is a first-order-closure model of turbulence which assumes an explicit algebraic relationship between Reynolds stresses and mean-velocity gradients. The Boussinesq hypothesis is a popular linear eddy viscosity model, which is used in most Reynolds averaged Navies-Stocks equations to predict the shear stress. However, practical engineering flows exhibit complex mean strain associated, for example, with high pressure gradients, separation, impingement, streamline curvature, and swirl. The linear hypothesis is not suitable to capture the flow of anisotropy.

One way to model the anisotropy is to use the Reynolds stress transport equations. Craft and Launder [6] discussed different Reynolds stress turbulence models and found out that higher order pressure strain models which consider the stress redistribution near walls can predict the lateral spreading rate well. Lübecke et al. [7] used another way to simulate the anisotropy. They presented an explicit Reynolds-stress closure which offers a physically sound extension of the most prominent linear Boussinesq viscosity models with modest computational effort.

In this paper, a new $k$-$\varepsilon$ turbulence model was proposed to simulate the unsteady flow and to predict the pressure fluctuation through a model pump turbine for engineering application.

2. Model Development

In Wilcox's $k$-$\omega$ turbulence model, the turbulence kinetic energy transport equation is shown as follows,

$$\rho \frac{D k}{D t} = \frac{\partial}{\partial x_j} \left( \mu + \sigma \mu \right) \frac{\partial k}{\partial x_j} + \rho \tau_{ij} \frac{\partial u_i}{\partial x_j} - \rho \beta' k \omega$$

(1)

and the turbulence specific dissipation rate equation is,

$$\rho \frac{D \omega}{D t} = \frac{\partial}{\partial x_j} \left( \mu + \sigma \mu \right) \frac{\partial \omega}{\partial x_j} + \rho \alpha \omega \tau_{ij} \frac{\partial u_i}{\partial x_j} - \rho \beta \omega^2$$

(2)

The relationship between turbulence dissipation and turbulence specific dissipation rate is,

$$\varepsilon = \beta' k \omega$$

(3)

It can be calculated that,

$$\frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\varepsilon}{\beta' k} \right) = \frac{1}{\beta' k} \frac{\partial \varepsilon}{\partial x_j} - \frac{\varepsilon}{\beta' k^2} \frac{\partial k}{\partial x_j}$$

(4)

$$\frac{D \varepsilon}{D t} = \frac{D}{D t} \left( \beta' k \omega \right) = \beta' \omega \frac{D k}{D t} + \beta' k \frac{D \omega}{D t}$$

(5)

Then, the new equation for turbulence dissipation can be derived,

$$\rho \frac{D \varepsilon}{D t} = \rho \beta' \omega \frac{D k}{D t} + \rho \beta' k \frac{D \omega}{D t}$$

$$= \left( \mu + \sigma \mu \right) \frac{\partial}{\partial x_j} \left( \frac{\partial \varepsilon}{\partial x_j} \right) - 2 \mu \frac{1}{k} \frac{\partial k}{\partial x_j} \frac{\partial \varepsilon}{\partial x_j} + \left( \frac{2 \sigma' \beta' \rho + \beta' \rho \frac{\mu}{\mu} \left( \frac{\partial k}{\partial x_j} \frac{\partial \varepsilon}{\partial x_j} \right)^2 \right) - \sigma \mu \frac{1}{\varepsilon} \left( \frac{\partial \varepsilon}{\partial x_j} \right)^2$$

(6)

$$+ (\alpha + 1) \rho \frac{\varepsilon}{k} \frac{\partial}{\partial x_j} \frac{\partial \varepsilon}{\partial x_j} - (1 + \beta') \rho \frac{\varepsilon^2}{k}$$

For $k$-$\varepsilon$ turbulence model, the equation for turbulence kinetic energy is shown as follows,
\[
\frac{Dk}{Dt} = -\frac{\partial}{\partial x_j} \left( \mu + \sigma \mu \right) \frac{\partial k}{\partial x_j} + \rho \tau_{ij} \frac{\partial u_i}{\partial x_j} - \rho \varepsilon \tag{7}
\]

where the kinematic eddy viscosity,
\[
\mu = \rho C_{\mu \varepsilon} \frac{k^2}{\varepsilon} \tag{8}
\]
The closure coefficients,\[
\alpha = \frac{5}{9}, \quad \beta = \frac{3}{40}, \quad \beta' = \frac{9}{100}, \quad \sigma = \frac{1}{2}, \quad \sigma^* = \frac{1}{2}, \quad C_{\mu \varepsilon} = \frac{9}{100}
\]
The turbulence kinetic energy equation (7) and turbulence dissipation equation (6) form the new \(k-\varepsilon\) turbulence model.
The combination of RNG \(k-\varepsilon\) turbulence model and Wilcox's \(k-\omega\) turbulence model does not need to make use of wall functions, because it is valid up to solid walls.
The Reynolds stress anisotropy tensor proposed by Ehrhard[8] was combined with the new \(k-\varepsilon\) turbulence model. The final form of the nonlinear \(k-\varepsilon\) turbulence model is given by the nonlinear solution.
The Reynolds stress can be calculated by equation (9).
\[
\tau_{ij} = -\rho u_i u_j \tag{9}
\]
The nonlinear solution of Reynolds stress is,
\[
\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} - 2C_v \nu^2 \tau_{ij} + C_\mu \nu^2 T^2 \left( S_{ij} S_{kl} - \frac{1}{3} S_{ik} S_{kj} \delta_{ij} \right) + C_{\mu \rho} \nu^2 T^2 \left( \Omega_{ij} \Omega_{kl} - \frac{1}{3} \Omega_{ik} \Omega_{kj} \delta_{ij} \right) + C_{\mu \rho} \nu^2 T^2 \left( S_{ij} \Omega_{kl} - S_{ik} \Omega_{ij} \right) S_{kl} + C_\rho C_v \nu^2 T^2 \frac{S_{ij}}{\Omega_{kl} \Omega_{kl} S_{ij} + C_\rho C_v \nu^2 T^3 S_{ij} \Omega_{kl}} \tag{10}
\]
where,
\[
C_\mu = \min \left\{ \frac{1}{0.9S^{1.4} + 0.4\Omega^{1.4} + 3.5}, 0.15 \right\} \tag{11}
\]
\[
S = \frac{k}{\nu} \sqrt{2S_{ij} S_{ij}} \tag{12}
\]
\[
\Omega = \frac{k}{\nu} \sqrt{2\Omega_{ij} \Omega_{ij}} \tag{13}
\]
\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{14}
\]
\[
\Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \tag{15}
\]
The closure coefficients, \[
C_1 = -0.2, \quad C_2 = 0.4, \quad C_3 = 2.0 - \exp \left( -\left( S - \Omega \right)^3 \right), \quad C_4 = -32.0C_\mu^2, \quad C_5 = -16.0C_\mu^2, \quad C_6 = 16.0C_\mu^2.
\]
\(\nu\) is the turbulence velocity scale, \(T\) is the turbulence time scale.

3. Pump turbine geometry and calculation methods
3.1 Pump-turbine geometry
The study was performed with the model pump–turbine. Parameters of the model pump–turbine are shown in Table 1. $D_1$ denotes the runner inlet diameter in pump mode; $Z_s$, $Z_g$ and $Z$ are the numbers of stay vanes, guide vanes and runner blades, respectively; $H_d$ denotes the rated head; $n$ denotes the rotational speed of the runner; $Q_d$ denotes the rated discharge. The pump–turbine’s structure is shown in Figure 1. The relative opening $\gamma$ of guide vanes is 20%, which is used for the calculation of the pump–turbine. The no load condition indicates that the moment of the pump-turbine is zero.

![Figure 1 Profile of pump–turbine](image)

| Parameter                        | Value (Unit) |
|----------------------------------|--------------|
| Rated head $H_d$ (m)             | 52.4         |
| Rated floe rate $Q_d$ (m$^3$/s) | 0.45         |
| Rotational speed $n$ (rpm)       | 1200         |
| Runner outlet diameter $D_1$ (m) | 0.30         |

3.2 Calculation methods
The model’s grids, which were composed of an unstructured hexahedron and tetrahedron, were developed using ICEM. Hexahedral grids were used for the runner and draft tube, and mixed grids were used for the other components. A mesh with about 9 million cells in total was chosen for the simulations. The velocity inlet at the spiral casing inlet and the pressure value at draft tube outlet were given for the turbine mode. The runner’s hydraulic region was set as a moving mesh model. The SIMPLEC algorithm was used to enforce mass conservation. A second-order centered difference was used for the pressure interpolation to obtain better results. For present unsteady flow calculation, the time step was 0.0001389 s. For all calculations, simulations were run until convergence, which was determined by a reduction in the residual error to less than 0.0001.

4. Results and discussions
4.1 Curved rectangular duct
To testify the ability of the nonlinear $k$-$\varepsilon$ turbulence model in capturing complex flow characteristics, flow through a three-dimensional (3-D) curved rectangular duct was studied and the results of the internal flow were compared with experimental data. The nonlinear $k$-$\varepsilon$ turbulence model was realized by Fluent software, using user define function (UDF). The computational results of the nonlinear $k$-$\varepsilon$ turbulence
model were also compared with the results performed by RNG $k$-$\varepsilon$ turbulence model and SST $k$-$\omega$ turbulence model. Experimental data of the internal flow in the curved rectangular duct were performed by Kim and Patel [9].

The structure of the curved rectangular duct was shown in Fig. 2. $H=0.203$ m, the radius of the inner circle was 0.608 m. Velocity at the inlet of the tunnel was $u_o=16$ m/s, and the Reynolds number was $Re=u_oH/v=224,000$. The transverse velocity (perpendicular to the measuring line and along the flow direction), axial velocity (perpendicular to the measuring line and flow direction) and longitudinal velocity (along the measuring line) on the measuring curve were calculated.

Figure 2 A curved rectangular duct

Figure 3 shows results of the transverse velocity on the measuring line. The result of the nonlinear $k$-$\varepsilon$ turbulence model is in good agreement with experimental result. The transverse velocity of experimental result on the measuring line has a hump characteristic, and it can be captured by the nonlinear $k$-$\varepsilon$ turbulence model. The results of RNG $k$-$\varepsilon$ model and SST $k$-$\omega$ model couldn’t obtain the nonlinear phenomena.
The longitudinal velocity on the measuring line is shown in the Figure 4. The result of RNG $k$-$\varepsilon$ model and SST $k$-$\omega$ model is smaller than the result of nonlinear $k$-$\varepsilon$ turbulence model at the region of dominant flow. Longitudinal velocity on the measuring line agrees well with the experiment result. The result of longitudinal velocity by nonlinear $k$-$\varepsilon$ turbulence model can also capture the nonlinear phenomena when $0<h/H<0.3$.

The axial velocity on the measuring line is shown in the Figure 5. The maximum point of the axial velocity by the nonlinear $k$-$\varepsilon$ model is very close to the result of experimental data, while the results based on RNG $k$-$\varepsilon$ model and SST $k$-$\omega$ model have large error at this point. The nonlinear characteristic of the axial velocity when $0<h/H<0.3$ can also be captured by the nonlinear $k$-$\varepsilon$ model.

### 4.2 Pressure fluctuation in the vaneless space

The amplitude and dominant frequency of pressure fluctuation at vaneless space between the runner and guide vane of the pump turbine at no load opening has been investigated in this paper. Pressure fluctuations in the vaneless space at time domain is shown in Figure 6. The comparison between the calculated amplitudes and the test ones is shown in Table 2. It is indicated that the prediction of the pressure fluctuation is valid by the present nonlinear method. The amplitude is measured in the time domain from peak to peak values with 97% reliability.

The pressure fluctuation data in time domain is transformed into frequency spectrum in frequency domain with Fast Fourier Transform (FFT), and the result can be seen in Figure 7. And the pressure fluctuation only contains high frequency component, as shown in the picture of frequency domain, in which the dominate
frequency is the runner blade passing frequency and the second dominate frequency is double of the runner blade passing frequency. The second dominate frequency is nearly equal to guide vane passing frequency. Figure 8 are the relative streamlines on the S1 stream surface of runner. Figure 9 only shows the turbulent kinetic energy distribution on S1 surface. At the no load condition the flow separations occur in the blade passage. The turbulent kinetic energy is very large near the pressure surface and close to the blade leading edge, which results in the high pressure oscillation in the vaneless space of the pump turbine.

![Figure 6](image1.png) **Figure 6** Pressure fluctuations on the measured points G1 to G6 at time domain

![Figure 7](image2.png) **Figure 7** Pressure fluctuations at frequency domain

| Table 2 Comparison of predicted amplitude with test data |
|---------------------------------------------------------|
| Working conditions                                      |                                                     |
| Dominate frequency ($f/f_n$)                            | 9                                                   |
| Amplitude                                               |                                                     |
| $\Delta H/H$ calculation                                | 8.1                                                 |
| test                                                    | 8.3                                                 |
| Head(m)                                                 |                                                     |
| calculation                                            | 45.2                                                |
| test                                                    | 45.4                                                |
5. Conclusions
In order to study the pressure fluctuation of the model pump turbine at the turbine mode, the 3D unsteady flow simulation was carried out using the nonlinear RNG $k-\varepsilon$ turbulence model under 20% relative opening of the guide vanes. In the nonlinear method, the shear stress was solved by nonlinear turbulence model which was proposed by Ehrhard.

The numerical predicted performance data and relative amplitude from peak to peak of pressure fluctuation at time domain in the vaneless space of the pump turbine are both very close to the tested date. It is indicated that the prediction of the pressure fluctuation is valid by the present Nonlinear RANS method.

The pressure fluctuation only contains rather high frequency components, in frequency domain the dominate frequency is the runner blade passing frequency and the second dominate frequency is double of the runner blade passing frequency.

At the no load opening condition, the flow separations occur in the blade passage. The turbulent kinetic
energy is very large near the pressure surface and close to the blade leading edge.

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