Towards New Tests of Strong-field Gravity with Measurements of Surface Atomic Line Redshifts from Neutron Stars

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In contrast to gravity in the weak-field regime, which has been subject to numerous experimental tests, gravity in the strong-field regime is largely unconstrained by observations. We show that gravity theories that cannot be rejected by solar system tests but that diverge from general relativity in the strong-field regime predict neutron stars with significantly different properties than their general relativistic counterparts. In particular, the range of redshifts of surface atomic lines predicted by such gravity theories is significantly larger than the uncertainty introduced by our lack of knowledge of the equation of state of ultra-dense matter. Measurements of such redshifted lines with current X-ray observatories such as Chandra and XMM-Newton can thus provide interesting new constraints on strong-field gravity.

The rapid variability and high luminosities of accreting compact objects strongly suggest that their high-energy emission originates very deep in their gravitational potentials. As a result, the properties of their X-ray and γ-ray emission can, in principle, be used to test directly the strong-field regime of a gravity theory. Unfortunately, there are many astrophysical considerations that complicate the modeling of the emission of accreting neutron stars and black holes and it is difficult to disentangle the effects of gravity from those due to astrophysical processes. For this reason tests of gravity involving high-energy phenomena around compact objects have received little attention in the past. This is despite the fact that the behavior of gravity in the strong-field regime is largely unconstrained by observations.

The properties of a neutron star and its external spacetime, in contrast to the high-energy phenomena above its surface, are determined solely by the strong-field behavior of gravity and the equation of state. To date, studies of neutron stars have focused on better determining the equation of state, whose details are still debated. We will argue, however, that the uncertainty in the behavior of strong-field gravity introduces much greater differences in neutron-star properties than do current uncertainties in the neutron-star equation of state. Furthermore, new observational developments allow for the measurement of the gravitational redshift of surface atomic lines, which is the clearest source of information about the mass, radius, and spin of neutron stars. The first such observations, with Chandra and XMM-Newton, have already been reported

Testing rigorously General Relativity (GR) in the strong-field regime requires a general framework of which GR is a subset; this would be equivalent to the Parametrized Post-Newtonian formulation in the weak-field. Such a framework is yet to be constructed. Thus, in order to provide a proof of principle for the efficacy of the proposed tests, we will investigate here the constraints that can be imposed on a parametrized subclass of scalar-tensor theories (see, e.g., Ref. [8]). As an additional example, we will also consider the limits that can be placed on Rosen’s bimetric theory [4]; although binary-pulsar experiments have already excluded the bimetric theory [4], our results demonstrate the potential of the proposed tests for constraining gravity theories.

We consider first scalar-tensor theories. The general class of scalar-tensor theories, where the gravitational force felt by matter is mediated by both a rank-two tensor, $g_{\mu \nu}$, and a scalar field, $\phi$, is one of the most natural extensions of Einstein’s theory. The action is

$$S = \frac{1}{16\pi G_*} \int d^4 x \sqrt{-g}(R_* - 2g^{\mu \nu} \phi_{,\mu} \phi_{,\nu}) + S_m[A^2(\phi) g_{\mu \nu}].$$

Here $\Psi_m$ refers collectively to all matter fields other than $\phi$, $G_*$ is a dimensional constant, and $A(\phi)$ is a function of $\phi$. The tensor $g_{\mu \nu}$ is the metric in the Einstein frame and the Brans-Dicke frame tensor is $g_{\mu \nu} = A^2(\phi) g_{\mu \nu}$. For $A(\phi)$ unity, we recover GR.

Solar system experiments constrain the coupling function $A$ to be very flat at the cosmological value of the scalar field $\phi$. If, however, the scalar field inside a compact object fluctuates far enough away from its cosmological value to discover a steep part of $A$, the system will be able to reach the more energetically favorable configuration with large non-zero $\phi$ near the center of the object. This non-perturbative effect leads to neutron stars that are significantly more massive and larger than their GR counterparts, but it is invisible to solar system experiments that probe the weak-field regime. This is the phenomenon of “spontaneous scalarization” discovered by Damour & Esposito-Farèse [8], who used the Hulse-Taylor pulsar to put constraints on the strong-field regime of such theories.
Following Damour & Esposito-Farèse, we choose
\[ A(\phi) = e^{4\beta \phi^2}, \]
where \( \beta \) is a real number. Spontaneous scalarization occurs only when \( \beta \leq -4.85 \). Throughout this paper we will be interested only in solutions where \( \phi_0 \), the cosmological value of the scalar field, is zero and \( A(\phi_0) = 1 \).

Following Ref. [6], we write the relativistic equations of stellar structure for a non-rotating neutron star in scalar-tensor gravity. We write the metric as
\[ ds^2 = -e^{\nu(r)} dt^2 + \left[ 1 - \frac{2\mu(r)}{r} \right]^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]
and describe the matter fields as a perfect fluid in the physical frame, i.e.,
\[ T_{\mu\nu} = (\rho + p) u_\mu u_\nu + pg_{\mu\nu}. \]
Here \( p \) is the pressure, \( \rho \) is the density, and \( u_\mu \) is the four-velocity of the fluid. The differential equations to be integrated are then:

\[ \mu' = 4\pi G_* r^2 A^4 \rho + \frac{1}{2} r (r - 2\mu) \psi^2, \]
\[ \nu' = 8\pi G_* \frac{r^2 A^4 p}{r - 2\mu} + \frac{2\mu}{r (r - 2\mu)}, \]
\[ \psi' = \frac{4\pi G_* r^4 A^2}{r - 2\mu} \left[ \alpha (\rho - 3p) + r (\rho - p) \psi \right] + \frac{\mu}{r (r - 2\mu)} \psi, \]
\[ N' = 4\pi n A^3 r^2 \left( 1 - \frac{2\mu}{r} \right)^{-1/2}, \]
\[ p' = - (\rho + p) \left[ 4\pi G_* \frac{r^2 A^4 p}{r - 2\mu} + \frac{1}{2} r \psi^2 \right] - (\rho + p) \left[ \frac{\mu}{r (r - 2\mu)} + \alpha \psi \right]. \]

Here \( N \) is the baryon number, \( n \) is the number density, and primes denote derivatives with respect to \( r \). We supplement these equations with a (zero temperature) equation of state, \( \rho = p(\rho) \) and \( n = n(\rho) \).

We choose two commonly used equations of state, which cover a broad subset of the wide range discussed in Cook et al. [8]. In order of increasing stiffness, these are EOS A [10] and EOS UU [11]. Neutron star models computed with these two equations of state bracket the uncertainty introduced by our inability to calculate from first principles the properties of ultra-dense matter, when condensates or unconfined u-d-s quark matter is not taken into account.

Having chosen an equation of state, we then integrate the system of equations (5)–(10) from the center of the star, where we specify \( \mu(0) = \nu(0) = N(0) = 0, \phi(0) = \phi_c, \) and \( \rho(0) = \rho_c \), to the surface, where \( \mu(R) = 0 \). We then integrate equations (3)–(8) from the surface of the star to infinity to determine the form of the metric exterior to the star. We note that the Arnowitt-Deser-Misner (ADM) mass felt by an observer far away is in general different from the mass that contributes to the redshift.

In addition to considering the scalar-tensor theory, we also investigate the bimetric theory, proposed by Rosen [4]. This involves, in addition to the dynamical metric \( g_{\mu\nu} \), a nondynamical flat metric \( \eta_{\mu\nu} \).

The metric \( g_{\mu\nu} \) in Rosen’s theory may be written as
\[ ds^2 = -e^{-2m_{\Phi}(r)/r} dt^2 + e^{2m_{\Lambda}(r)/r} \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]
where \( m_{\Phi}(r) \) and \( m_{\Lambda}(r) \) are given, interior to the star, by
\[ m_{\Phi}(r) = 4\pi \int_0^r e^{\Phi + 3\Lambda}(\rho + 3p)r^2 dr, \]
\[ m_{\Lambda}(r) = 4\pi \int_0^r e^{\Phi + \Lambda}(\rho - p)r^2 dr, \]
and \( \Phi \) and \( \Lambda \) are given by the field equations
\[ \Phi' = G m_{\Phi}(r)/r^2, \]
\[ \Lambda' = -G m_{\Lambda}(r)/r^2. \]

The equation of hydrostatic equilibrium is
\[ p' = -(\rho + p)\Phi'. \]

Outside the star, \( m_{\Phi}(r) = m_{\Phi}(R) \equiv M_{\Phi} \) and \( m_{\Lambda}(r) = m_{\Lambda}(R) \equiv M_{\Lambda} \) are constants. \( M_{\Phi} \) is equal to the Kepler-measured mass at large distances. We specify an equation of state, \( \rho = p(\rho) \), and the boundary conditions \( \rho(0) = \rho_c, \Phi(\infty) = \Lambda(\infty) = 0 \).

Figure 1 shows the relationship between ADM mass and neutron-star radius for the three theories under consideration. For the case of the scalar-tensor theory, we show, as an example, the case \( \beta = -8 \), which is comparable to the most negative value of \( \beta \) not yet ruled out by binary pulsar timing [12]. For each of the three
FIG. 1: Mass-radius relations for neutron stars in the GR, bimetric, and scalar-tensor theories. For the scalar-tensor theory, we plot the relation for only one value of the parameter $\beta$. The shaded regions show the allowed values of $M$ and $R$ for equations of state with stiffness between EOS A and EOS UU.

As can be seen immediately, all three theories allow neutron stars with masses in the currently observed range of $1.35 - 1.8 M_\odot$ to exist [13]. As a result, measurement of neutron star masses alone cannot distinguish between them. This degeneracy is broken, however, by considering the predicted radii. In the astrophysically relevant range $M_{\text{ADM}} \geq 1.3 M_\odot$, the three theories occupy mutually exclusive regions in the $(M_{\text{ADM}}, R)$ space. Observations that can put limits on both the mass and radius of a neutron star can thus constrain, without ambiguity, the permissible set of gravity theories.

One such observation is of the surface redshift factor, $z$, relative to infinity. This is defined by $E_\infty = [1/(1+z)]E_{\text{surf}}$, where $E_{\text{surf}}$ is the energy of a photon emitted from the stellar surface and measured at infinity with energy $E_\infty$. In the GR Schwartzschild metric, this provides a direct measurement of only the ratio $M_{\text{ADM}}/R$. In the case of the scalar-tensor and bimetric theories the relationship between $M_{\text{ADM}}$, $R$, and $z$ is more complicated. In all cases, however, the formula,

$$E_\infty = \left(\frac{g_{00,s}}{g_{00,\infty}}\right)^{1/2} E_{\text{surf}},$$  \hspace{1cm} (17)$$

holds, where $g_{00,s}$ is evaluated at the surface of the star and $g_{00,\infty}$ is evaluated at infinity.

Figures 2 and 3 show the redshift factor $z$ for the scalar-tensor and GR theories as contours on a plot of the mass $M_{\text{ADM}}$ versus the parameter $\beta$, for EOS A and EOS UU, respectively. As discussed above, the phenomenon of spontaneous scalarization occurs only for $\beta \lesssim -4.35$, and so above this value the contours are parallel to the $\beta$ axis and equal to the GR value. The heavy lines on the plot show the upper mass limits for neutron stars as well as the bounding region, in the $(M_{\text{ADM}}, \beta)$ space, where spontaneously scalarized stars are produced. The dashed line shows the value of $M_{\text{ADM}}$ as a function of $\beta$ for which the baryonic mass is equal to $1.4 M_\odot$. Note the $z = 0.35$ contour, the value recently measured by Cottam et al. [1] in the neutron star source EXO 0748–676.
The redshifts found for stars in the scalar-tensor theories differ greatly from their general relativistic values for both equations of state. The trend is for the redshift values to increase (nearly always) monotonically as $\beta$ becomes more negative. If we can place constraints on the neutron star mass, a redshift measurement will put an upper bound on the value of $-\beta$.

Figure 4 shows the redshift factor $z$ as a function of $M_{\text{ADM}}$ for neutron stars produced in the GR and bimetric theories. We plot two shaded regions, one for GR and one for the bimetric theory, that cover the range between EOS A and EOS UU. The dashed line shows the recent $z = 0.35$ measurement [1]. Even when a GR and a bimetric star have the same redshift, the mass serves to break the degeneracy, providing a test of strong field gravity despite the uncertainty in the equation of state.

The recent measured redshift of $z = 0.35$ [1] cannot by itself constrain strong-field gravity in the theories we consider here. However, strong limits can be placed if the mass of the neutron star is measured directly from binary dynamics or constrained by arguments from formation mechanisms (e.g., restricting the baryonic mass to be above the Chandrasekhar limit for a degenerate core.) The neutron star EXO 0748–676, for which this redshift was measured, is a member of an eclipsing binary with an orbital period of 3.8 h [16] and hence is a prime candidate for producing a mass measurement that can provide such a constraint. Simply requiring the baryonic mass of the star to be greater than $1.4M_\odot$ (the Chandrasekhar limit for its degenerate progenitor) places an upper limit on $-\beta$ of 7 (EOS A) to 9 (EOS UU). As expected, the constraint on strong-field gravity depends only weakly on the equation of state.

In addition to the redshift of atomic lines, a number of other observational characteristics of neutron stars depend in more complicated ways on the nature of strong-field gravity. These are the maximum allowed spin frequency, the Eddington limit for accreting stars, bursting behavior, cooling rates, and the frequencies of quasi-periodic oscillations. The constraints imposed by quasi-periodic oscillations will be reported in a forthcoming paper.

The constraints on scalar-tensor discussed here have a wider applicability than the structure of compact objects because varieties of scalar-tensor theories naturally arise as low-energy limits of gravitational theories in higher dimensions [17]. They are also under consideration in cosmology as explanations of evidence for an accelerating universe. To date, little attention has been given to the implications of such theories for the properties of compact objects that are now observable. The uncertainties in the equation of state at ultra-high densities were thought to make such studies fruitless. Our investigation has shown that this is not the case and that measurements of the redshifts of surface atomic lines from neutron stars can provide new tests of strong-field gravity.

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