Final state QCD corrections to off-shell single top production in hadron collisions.

R. Pittau*
Paul Scherrer Institute, CH-5232 Villigen-PSI, Switzerland
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Abstract

In this paper, I study final state QCD radiative corrections to off-shell single top production via decaying $W$ at hadron collider energies. The tree level Breit-Wigner distribution of the produced top invariant mass is distorted by final state QCD radiation, while the peak position remains unchanged. The exact one loop QCD calculation and the narrow width approximation agree in predicting the cross section, the hadronic transverse energy distribution and the bottom-lepton invariant mass distribution.

*email address: pittau@psw218.psi.ch
1 Introduction

The discovery of the top quark at CDF and D0 \cite{1}, has opened a new era of measurements in top-quark physics. Now, the properties of the top quark can be directly investigated, not only inferred from their effects in radiative corrections.

At hadron colliders, the dominant production mechanisms are, of course, the $t\bar{t}$ channels

\[ q\bar{q} \rightarrow t\bar{t} \]
\[ gg \rightarrow t\bar{t}, \] (1)

but single top quarks events are also present, such as

\[ q'g(W^+g) \rightarrow qt\bar{b} \]
\[ q'b \rightarrow qt \]
\[ q'\bar{q} \rightarrow W^* \rightarrow t\bar{b} \]
\[ gb \rightarrow W^- t. \] (2)

The first two mechanisms are known as W-gluon processes \cite{4}, the third one as $W^*$ production \cite{5} and the fourth one as $Wt$ production \cite{6}. The cross sections in eq. (2) are ordered according to their magnitude in $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV for $m_t = 176$ GeV \cite{7}.

Even with less expected events, single top production processes are important because they provide a consistency check on the measured parameters of the top quark in $t\bar{t}$ production.

Radiative corrections to the processes in eq. (1) and eq. (2) are well known in the literature \cite{6}, but usually, performed in the narrow width approximation, in which production and decay of the top are treated independently. A check on the validity of this approach is still missing. Since, in the narrow width approximation, diagrams connecting decaying products with the production process are missing, one especially expects deviations due to a non exact treatment of the gluonic radiation, which is an important quantity for the reconstruction of $m_t$ in $t\bar{t}$ events \cite{7}. Therefore a precise study of it, even in a simpler case, can give hints on its relevance in the main production mechanisms of eq. (1).

For those reasons, I decided to perform a complete calculation of the final state QCD radiative corrections to $W^*$ single top production, taking into
account all the subsequent decays. Among all the others, the $W^*$ mechanism is interesting because possible new physics may introduce a high mass state (say particle $V$) to couple strongly with the $t\bar{b}$ system such that the production rate from $q'\bar{q} \rightarrow V \rightarrow t\bar{b}$ can deviate from the standard model $W^*$ rate \[^{5,8}\]. Therefore accurate predictions of the standard properties of the top in this channel are also important by themselves.

The background QCD contribution is known \[^3\], so I shall study here the QCD one loop corrections to the signal diagram of fig. 1. For this process, thanks to the color structure, initial and final state QCD corrections do not interfere and are separately gauge invariant, so that, in order not to obscure the effects I want to study, I decided, in this paper, to concentrate my attention on final state gluonic corrections. Initial state corrections are however very simple and a study including them will be treated elsewhere \[^9\]. I chose the semi leptonic final state $\nu_l l^+ b \bar{b}$ because it is easier to detect experimentally. The extension to $q'\bar{q} b \bar{b}$ is anyway trivial. In fact, diagrams with gluons connecting $b$ or $\bar{b}$ with $q'$ or $\bar{q}$ are killed, at the one loop level, by the color factor, so one is left with simple gluonic corrections for the $W q' \bar{q}$ vertex.

Figure 1: Tree level diagram for $\bar{d} u \rightarrow \nu_l l^+ b \bar{b}$ via single top production. Here, and in the following figures, dashed lines denote $W$'s.

2 The calculation

The tree level diagram for the process is drawn in fig.1, while in fig. 2 and 3 I show the one loop virtual diagrams and the real bremsstrahlung.
I computed the virtual corrections using standard Passarino-Veltman techniques \cite{10}, with the help of the Symbolic Manipulation program FORM \cite{11}. I used dimensional regularization for ultraviolet, collinear and soft divergences. Furthermore, I kept everywhere complex masses for top and $W$, but I systematically neglected the bottom mass.

An analytic approximation in $n$ dimensions for the soft-collinear part of the real emission was built following ref. \cite{12} and the cancellation of all diver-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Final state one loop QCD virtual diagrams.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Real gluon emission.}
\end{figure}
gences performed analytically. Both the virtual and the real contributions have been computed applying helicity amplitudes methods \[13\], and the final expressions implemented in a Monte Carlo program \[9\], that uses the self-optimization techniques of ref. \[14\]. Since those techniques are applied here, for the first time, in loop calculations, it may be useful to briefly discuss the adopted strategy. More details will be found in ref. \[9\].

The problem here is the matching between hard and soft phase space. Schematically, the final result for the cross section $\sigma_{\text{tot}}$ (with any kind of cuts) can be written as a sum of four contributions

$$\sigma_{\text{tot}} = \sigma_0 + \sigma_V + \sigma_S(\delta) + \sigma_H(\delta)$$

where $\sigma_0$ is the lowest order result, $\sigma_V$ the virtual contribution and $\sigma_S, \sigma_H$ the soft and hard real radiation.

The sum, $\sigma_V + \sigma_S(\delta)$ does not contain soft and collinear singularities. On the other hand $\sigma_S(\delta) + \sigma_H(\delta)$ is independent on $\delta$, where $\delta$ is the separation between soft and hard gluons (following again ref. \[12\]), $\delta$ in a cut on the invariant mass of $g + b$ and $g + \bar{b}$). The last statement is true only if an exact computation of $\sigma_S(\delta)$ is performed. Instead, what one usually does is computing $\sigma_S(\delta)$ for small $\delta$. In such a limit, because of factorization properties (\[12, 15\]), very simple expressions are obtainable in terms of the born result multiplied by universal coefficients containing $\log(\delta)$ and $\log^2(\delta)$. At this point, by numerically going to the limit $\delta \to 0$, one gets unbiased results. Of course, if $\delta$ is too small, large numerical cancellations take place between $\sigma_S(\delta)$ and $\sigma_H(\delta)$, resulting in large errors in the Monte Carlo integration. A good value for $\delta$ can be usually found by numerically checking the independence on $\delta$ of the results.

For fixed $\delta$ one would like to know how many Monte Carlo points have to be spent to separately compute all four contributions in eq. (3), mainly because usually the most time consuming part is $\sigma_V$, that contains loop diagrams. This is a typical problem that can be solved using the Multichannel self-optimizing approach of ref. \[14\]. One starts with the same amount of points for all channels and, during the run, the Monte Carlo self-adjusts itself, so that afterwards one usually obtains a smaller percentage of the computational time spent in the evaluation of $\sigma_V$, which means a better estimate of $\sigma_{\text{tot}}$ in a shorter time.
Table 1: Percentage of the Monte Carlo points used for each channel in the evaluation of $\sigma_{\text{tot}}$ before and after the self-optimization. The first three channels take care of the peaking structure of $\sigma_H$ given by the Feynman diagrams in fig. 3, channel 4 refers to $\sigma_0 + \sigma_S$ and channel 5 to $\sigma_V$.

| channel | percentage before opt. | percentage after opt. |
|---------|------------------------|-----------------------|
| 1       | 0.2                    | 0.0996                |
| 2       | 0.2                    | 0.5436                |
| 3       | 0.2                    | 0.1265                |
| 4       | 0.2                    | 0.2083                |
| 5       | 0.2                    | 0.0220                |

In table 1 I show a typical result of the self-optimization procedure. I made several checks on the final result. First of all, I verified that the $CP$ invariance of the tree level current

$$T_{\alpha\mu} = \bar{u}(b) \gamma_\alpha(1 - \gamma_5)(\not{p}_t + \not{p}_\eta + \not{p}_b + m_t)\gamma_\mu(1 - \gamma_5)\not{v}(\bar{b})$$

remains after QCD loop corrections. Then, by numerically rescaling $\Gamma_W$, $\Gamma_t$ and the cross section by the same amount, I checked the agreement between the Monte Carlo estimate of the total $O(\alpha_s)$ $t\bar{b}$ on-shell cross section ($\sigma_{MC}$) and (for example) the analytic result ($\sigma_{AN}$) of ref. [16] (see table 2). I also tested, for small $\delta$, the independence on $\delta$ of the results. All numbers in this paper are obtained with $\delta = 0.2 \text{ GeV}^2$.

A last comment is in order. Taking into account the widths of the decaying particles gives rise to conceptual problems with respect to the gauge invariance. The correct gauge invariant prescription would be to compute the widths as the imaginary part of the one loop renormalized propagators and all set of loop diagrams necessary to restore gauge invariance. Since, in the process at hand, $W$ and $t$ decay via electroweak interactions, this would imply to include terms of the one loop $O(\alpha)$ calculation at the tree level and part of the two loop $O(\alpha\alpha_s)$ corrections in the $O(\alpha_s)$ contribution. Since I am interested here in $O(\alpha_s)$ corrections, the neglected $O(\alpha)$ terms are expected to be small, so I followed the naive prescription of considering everywhere constant complex masses. For the sake of consistency, when computing $\text{Im}(m_t^2)$, I used the lowest order top width value $\Gamma_t = 1.6429 \text{ GeV}$ for
Table 2: Comparison between the Monte Carlo total cross section, in the limit of vanishing widths, and the analytic on-shell calculation. No convolution with the parton densities has been performed. The first entry is the tree level result. In the second entry all final state QCD corrections are included.

| $\sqrt{s}$ (GeV) | $\sigma_{MC}$ (pb) | $\sigma_{AN}$ |
|------------------|-------------------|---------------|
| 300              | 0.09086 $\pm$ 0.00002 | 0.09088       |
|                  | 0.10499 $\pm$ 0.00035 | 0.10547       |
| 600              | 0.03508 $\pm$ 0.00001 | 0.03507       |
|                  | 0.03784 $\pm$ 0.00027 | 0.03774       |
| 900              | 0.01653 $\pm$ 0.00001 | 0.01653       |
|                  | 0.01763 $\pm$ 0.00019 | 0.01748       |
| 1200             | 0.00948 $\pm$ 0.00001 | 0.00948       |
|                  | 0.00999 $\pm$ 0.00014 | 0.00996       |
| 1500             | 0.00612 $\pm$ 0.00001 | 0.00612       |
|                  | 0.00643 $\pm$ 0.00010 | 0.00640       |
| 1800             | 0.00427 $\pm$ 0.00001 | 0.00427       |
|                  | 0.00454 $\pm$ 0.00007 | 0.00446       |

quantities at the tree level and the QCD corrected value $\Gamma_t = 1.5117$ GeV when including QCD corrections.

3 Results

In this section, I present some results for the process $\bar{d} u \rightarrow \nu_t t^+ b \bar{b}$ obtained with the Monte Carlo of ref. [3]. I chose to plot three useful distributions for measuring the top mass in $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV, namely the total hadronic transverse energy ($H_T$), the invariant mass $\sqrt{(p_{t^+} + p_b)^2}$ ($m_{tb}$) and the ”top mass distribution” $\sqrt{(p_{\nu_t} + p_{t^+} + p_b)^2}$ ($m_{t\nu}$). Of course, due to the presence of an undetected neutrino, the last quantity is not going to be easy to reconstruct experimentally. However, $m_{t\nu}$ is of theoretical interest and directly measurable in the channel $q' \bar{q} b \bar{b}$.
I used the following input parameters and cuts

\[
\alpha = \frac{1}{128}, \quad \sin^2 \theta = 0.2224, \quad \alpha_s = 0.1 \\
M_W = 80.41 \text{ GeV}, \quad M_t = 176 \text{ GeV} \\
\Gamma_W = 2.1185 \text{ GeV} \\
E_T(\nu), \ E_T(l^+), \ E_T(b), \ E_T(\bar{b}) > 15 \text{ GeV} \\
|\eta(l^+)|, \ |\eta(b)|, \ |\eta(\bar{b})| < 2, \ \Delta R(b\bar{b}) > 0.7 ,
\]

(5)
together with the cone jet-definition algorithm of ref. [17] (with jet cone size \(R = 0.7\)) and the CTEQ2M parton densities of ref. [18]. Two jets with 

\(b\) content are required to be present in the visible region defined by the above cuts, and no extra (gluonic) jets. In order to compare the full QCD calculation with the narrow width approximation, I also produced histograms for \(H_T\) and \(m_{bl}\), in which \(W\) and top are put on-shell by numerically rescaling \(\Gamma_W, t\) and the cross section by the same amount.

In fig. 4 and 5, I show \(H_T\) and \(m_{bl}\) in the off-shell case and in the on-shell limit, including all final state gluonic corrections. For comparisons, I also show the tree level result. As one can see, the on-shell and off-shell distributions are almost indistinguishable, therefore the narrow width approximation works very well.

In fig 6, the off-shell top invariant mass distribution is shown with and without final state QCD corrections. QCD radiation is responsible for a distortion in the Breit-Wigner distribution: more events are produced in the left tail and less in the right side, but the position of the peak is essentially unchanged. Comparing with the narrow width approximation is difficult for \(m_{bl\nu}\). One would be forced to impose a Breit-Wigner by hand. I did not try that. Fig. 6 already gives a quantitative prediction for the distortion of the one loop distribution with respect to the tree level result. A precise quantitative knowledge of this effect may be useful in fitting the experimental distributions, and estimating the systematic errors.

In on-shell production, the cross section (with the cuts and input parameters of eq. (5), including final state QCD corrections) is \(0.02677 \pm 0.00025 \text{ pb}\), while one gets \(0.02681 \pm 0.00024 \text{ pb}\) in the off-shell case. That means that the narrow width approach can be safely used also in computing the cross

\(^1\)By looking at fig. 4 and 5 one can recognize a similar distortion in \(H_T\) and \(m_{bl}\) as well.
section. The question of the total number of produced events is important when looking at the single top production rate in this channel for new physics searches.

4 Conclusions

I have performed a complete one loop calculation of the final state QCD corrections to the single top production process $\bar{d} u \to \nu_t t^+ b \bar{b}$ in the off-shell case. Final state gluonic radiation is responsible for a distortion in the distributions useful for top mass reconstruction. The validity of the narrow width approximation is confirmed at the level of accuracy one naively expects, namely $O(\Gamma_t/m_t)$. However, one should observe that, no one loop QCD diagrams connecting initial and final states can contribute to the process at hand. On the other hand, such diagrams are present in the main production mechanisms of eq. (1). Therefore, in order to check the narrow width approach in that more general case, a full off-shell QCD loop calculation for the $t\bar{t}$ channels is still needed.

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Figure 4: Total hadronic transverse energy for on-shell (dashed histogram) and off-shell (solid histogram) single top production. Final state QCD corrections included. The dotted line is the off-shell tree level result ($\alpha_s = 0$).
Figure 5: The histograms are the invariant mass distribution of $l^+ + b$ for on-shell (dashed) and off-shell (solid) single top production, including final state gluonic corrections. The dotted line is the off-shell tree level result ($\alpha_s = 0$).
Figure 6: Invariant mass of $l^+ + b + \nu_l$ for off-shell single top production, at the tree level (dotted histogram) and including final state QCD corrections (solid histogram).