B_s \to \mu^+\mu^- \text{ in Supersymmetric Grand Unified Theories}

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Abstract

We investigate the recent CDF measurement of the \( \text{Br}(B_s \to \mu^+\mu^-) \) which shows excess over the Standard Model. We consider minimal supergravity motivated models (mSUGRA)/CMSSM and grand unified models, SU(5) and SO(10). In the grand unified models, the neutrino mixings provide an additional source of squark flavor violation through the quark-lepton unification. In the context of minimal SU(5) model, we find that the new CDF measurement has imposed a lower bound on the branching ratio of \( \tau \to \mu \gamma \) for a large CP phase in the \( B_s \bar{B}_s \) mixing. Recall that there have been indication for a large CP phase in \( B_s \) mixing from \( B_s \to J/\psi \phi \) (Tevatron and LHCb) and dimuon asymmetry (D0). We also predict \( \text{Br}(\tau \to \mu \eta) \) for the possible range of values of \( \text{Br}(\tau \to \mu \gamma) \).
1 Introduction

Recently, the CDF collaboration has reported the measurement of the rare decay process of $B_s$ meson to $\mu^+\mu^-$ [1], and the branching ratio of the decay is

$$\text{Br}(B_s \rightarrow \mu^+\mu^-)^{CDF} = (1.8^{+1.1}_{-0.9}) \times 10^{-8}. \quad (1)$$

The 90% confidence level (CL) range is

$$4.6 \times 10^{-9} < \text{Br}(B_s \rightarrow \mu^+\mu^-)^{CDF} < 3.9 \times 10^{-8}. \quad (2)$$

The measured branching ratio at the 90% CL is deviated from the standard model (SM) prediction [2, 3]:

$$\text{Br}(B_s \rightarrow \mu^+\mu^-)^{SM} = (3.19 \pm 0.19) \times 10^{-9}. \quad (3)$$

The SM prediction above is calculated from the ratio of the branching ratios and the $B_s$ mass difference, $\Delta M_s$. The discrepancy cannot be explained by the hadronic uncertainties, which comes from the uncertainty on the bag parameter $B_{B_s} = 1.33 \pm 0.06$ [3]. Thus, this discrepancy implies the existence of a new physics (NP) beyond the standard model. The excess can be soon verified at the LHC [4].

The excess of the $B_s \rightarrow \mu^+\mu^-$ rare decay can be reproduced in many NP models. Supersymmetry (SUSY) is one of the promising candidates for the new physics. In the minimal SUSY extension of the standard model (MSSM), the rare decay of $B_s \rightarrow \mu^+\mu^-$ is induced by the neutral Higgs mediated flavor changing operator [5]. The operator is proportional to $\tan^3 \beta/m_A^2$, where $\tan \beta$ is the ratio of vacuum expectation values of the up-type and down-type Higgs fields, and $m_A$ is the mass of the CP odd Higgs field. Therefore, the branching ratio is proportional to $\tan^6 \beta$, and the excess of this rare decay process implies a large value of $\tan \beta$ for the experimentally allowed parameter region of the model, where the masses of SUSY particles (especially for stops) are bounded from below. The flavor changing Higgs coupling is induced by finite corrections in the down-type quark mass matrix, and it can be generated even in the minimal flavor violating models where the CKM (Cabibbo-Kobayashi-Maskawa) quark mixing is the only source of the flavor violation. In this case, the operator is generated by a chargino-stop loop.

In the SUSY grand unified theories (GUTs), flavor mixing in the lepton sector (i.e. neutrino mixing) can cause a squark flavor violation due to the quark and lepton unification [6, 7]. The squark flavor violation can induce an additional contribution to the Higgs mediated flavor changing coupling via a gluino-sbottom loop. In the minimal type of SU(5) GUT, the right-handed down-type quark and the left-handed lepton doublet are unified in a multiplet, the
right-handed down-type squarks have the flavor violating source [8], and the left-handed squarks (i.e. both up- and down-type) can have the flavor violating sources in SO(10) GUTs, where the size of the squark/slepton flavor violation can be related to the enhancement of the proton life time [9]. The detail investigation of the flavor violating processes are important to find the footprint of the SUSY GUTs [10, 11, 12, 13].

The right-handed squark flavor violation with a large \(\tan \beta\) can induce a sizable \(B_s - \bar{B}_s\) mixing amplitude by a flavor changing Higgs mediation rather than the box diagram contribution [14, 15, 16, 17]. The \(b-s\) flavor violation, which is suggested by \(B\) decay experiments (the CP violation in \(B_s \to J/\psi\phi\) decay [18, 19] and dimuon asymmetry in semileptonic \(B\) decays [20]), can be related to the \(\tau-\mu\) flavor violation, such as \(\tau \to \mu \gamma\), \(\tau \to 3\mu\) and \(\tau \to \mu \eta\), in SUSY GUTs. Due to the bounds of the flavor violating \(\tau\) decays, especially for \(\tau \to \mu \gamma\), the size of the \(b-s\) flavor violation and the CP odd Higgs mass can be bounded to obtain a large CP phase. As a result, the existence of a large CP phase for \(B_s - \bar{B}_s\) mixing can constrain \(\text{Br}(B_s \to \mu^+ \mu^-)\) in the case where the quark and lepton unification is manifested. In fact, we obtain that the \(\text{Br}(B_s \to \mu^+ \mu^-)\) should be larger than \(O(10^{-8})\) [21]. Inversely speaking, the branching ratio measured by CDF gives a bound on the flavor violating \(\tau\) decays. In the paper, we will investigate the bound of \(\tau\) decays in SUSY GUTs.

This paper is organized as follows: in section II, we discuss \(B_s \to \mu^+ \mu^-\) in minimal supergravity (mSUGRA), in section III, we discuss large \(B_s\) mixing phase in SUSY GUT models, in section IV, we discuss the flavor violating \(\tau\) decays and we conclude in section V.

### 2 \(B_s \to \mu^+ \mu^-\) in minimal supergravity (mSUGRA)

The rare decay of \(B_s \to \mu^+ \mu^-\) can be generated even if the CKM quark mixing is the only source of flavor violation in MSSM. We first study the parameter space in minimal supergravity model (mSUGRA) (constrained MSSM (CMSSM)) [22] to realize the CDF measured \(B_s \to \mu^+ \mu^-\).

In mSUGRA, the parameters are the universal scalar mass (\(m_0\)), the unified gaugino mass (\(m_{1/2}\)), the universal trilinear scalar coupling (\(A_0\)), the ratio of the Higgs vev (\(\tan \beta\)), and the sign of the Higgs mixing parameter \(\mu\). The current experimental constraints, \(b \to s \gamma\) and the lightest Higgs mass bounds raise the squark masses. As a result, lower \(\tan \beta\) is dis favored to obtain \(\text{Br}(B_s \to \mu^+ \mu^-) \gtrsim 10^{-8}\) [23].

Recent results from ATLAS with 165 \(\text{pb}^{-1}\) of luminosity show that the maximum ruled out value of \(m_{1/2}\) is \(\sim 450\) GeV [24], which corresponds to squark, gluino masses \(\sim 1\) TeV.

In Fig.1 we show a typical parameter space for mSUGRA, with \(\tan \beta = 40\) and 50. The model parameters are already significantly constrained by different experimental results. The most important constraints for limiting the parameter space are: (i) the light Higgs mass bound
of $m_{h^0} > 114.4$ GeV from LEP [25] (red dotted line shows the contours of $m_{h^0} = 114.4$ GeV, as calculated using FeynHiggs-2.6.5 [26]); (ii) the $b \rightarrow s\gamma$ branching ratio [27] (95% CL excluded in the yellow shaded region in Fig. 1); (iii) the $2\sigma$ bound on the dark matter relic density: $0.106 < \Omega_{CDM} h^2 < 0.121$ from WMAP [28] (blue region in Fig. 1); (iv) the bound on the lightest chargino mass of $m_{\tilde{\chi}_1^\pm} > 103.5$ GeV from LEP [29] (region left to the black dashed line is excluded) and (v) the muon magnetic moment anomaly $a_\mu = (g_\mu - 2)/2$ (pink shaded region in Fig. 1) is within $2\sigma$ of where one gets a 4$\sigma$ deviation from the SM as suggested by the experimental results [30] and the analysis in [31]). We also show the $2\sigma$ contours from $3.2\sigma$ deviation based on [32] by slanted dashed purple lines. These two references use recent changes in the hadronic contribution to calculate the leading order hadronic contribution. Assuming that the future data confirms the $a_\mu$ anomaly, the combined effects of $g_\mu - 2$ and $m_{\tilde{\chi}_1^\pm} > 103.5$ GeV then only allows $\mu > 0$. The grey shaded region in Fig. 1 is excluded for not satisfying the electroweak symmetry breaking condition, while the brick-colored region is excluded because the stau is lighter than the neutralino hence neutralino cannot be the dark matter candidate. The red solid line shows the neutralino-proton elastic scattering cross-section contour of $9 \times 10^{-9}$ pb which approximately is the bound from XENON100 [33] for 70 GeV neutralino mass. Note, however, that the theoretical cross-section can easily have large uncertainties due to the hadronic factor determinations, the dark matter profile and the galactic velocity distribution. We show the CDF 90% CL contour of $\text{Br}(B_s \rightarrow \mu^+\mu^-)$, as in Eq. (2), as a green shaded region. The maximum allowed values of $m_{1/2}$ and $m_0$ go up to
Table 1: Sparticle mass ranges for \( \tan \beta = 50 \) and 40 for 90\% CL and 1\( \sigma \) allowed values of the \( \text{Br}(B_s \to \mu^+\mu^-) \) in the dark matter allowed regions. The lower bound for \( \tan \beta = 40 \) is fixed by the \( b \to s\gamma \) constraint, hence the same for both 90\% CL and 1\( \sigma \).

| \( \tan \beta \) | \( m_{1/2} \) (TeV) | \( \tilde{g} \) (GeV) | \( \tilde{u}_1 \) | \( \tilde{d}_1 \) | \( \tilde{t}_1 \) | \( \tilde{b}_1 \) | \( \tilde{e}_1 \) | \( \tilde{\tau}_1 \) | \( \tilde{\chi}^0 \) | \( \tilde{\chi}^\pm \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 50              | 0.46            | 1088            | 950             | 947             | 743             | 833             | 352             | 204             | 193             | 374             |
|                 | 1.61            | 3418            | 2917            | 2906            | 2379            | 2646            | 1054            | 740             | 722             | 1395            |
| (1\( \sigma \)) | 0.52            | 1211            | 1057            | 1053            | 832             | 931             | 387             | 232             | 220             | 424             |
|                 | 0.88            | 1952            | 1674            | 1669            | 1351            | 1503            | 581             | 384             | 381             | 738             |
| 40              | 0.36            | 879             | 758             | 756             | 588             | 680             | 246             | 155             | 150             | 287             |
|                 | 1.05            | 2314            | 1954            | 1944            | 1593            | 1804            | 595             | 464             | 461             | 891             |
| (1\( \sigma \)) | 0.58            | 1341            | 1143            | 1141            | 913             | 1042            | 351             | 252             | 246             | 474             |

\( \sim 1100 \) GeV for \( A_0 = 0 \) and \( \tan \beta = 40 \). This covers the whole neutralino-stau coannihilation band favored by dark matter. If we increase \( \tan \beta \) to 50, the region allowed by the 90\% CL range of \( \text{Br}(B_s \to \mu^+\mu^-) \) increases and the maximum allowed value of \( m_{1/2} \), that satisfies the dark matter constraint, goes up to 1.6 TeV. In addition, we also show the 1\( \sigma \) contour, Eq.(1), as dashed light-green lines in both figures. The lowest \( \tan \beta \) value, for \( A_0 = 0 \), allowed by the limits on the Higgs mass and the \( b \to s\gamma \) branching ratio, and within the CDF 1\( \sigma \) region of \( \text{Br}(B_s \to \mu^+\mu^-) \) is about 30, but using the 90\% CL range \( \tan \beta \) can be as low as about 20.

In Table 1 we list the upper and lower spectrums of the sparticle masses in the mSUGRA model for \( \tan \beta = 50 \) and 40 for the 90\% CL and 1\( \sigma \) allowed values of the \( \text{Br}(B_s \to \mu^+\mu^-) \), when we satisfy the dark matter and other constraints.

Because \( \text{Br}(B_s \to \mu^+\mu^-) \) depends on the CP odd Higgs mass \( m_A \) and the Higgsino mass \( \mu \), the constraints of the SUSY parameters from the CDF measurement of the branching ratio is different in the case of the non-universal Higgs mass (NUHM) boundary condition. The recent analysis of \( \text{Br}(B_s \to \mu^+\mu^-) \) in NUHM can be found in [34] (See also [35] for earlier analysis.).

### 3 Large \( B_s \) mixing in SUSY GUTs

In SUSY GUT theories, it is often assumed that the SUSY breaking sfermion masses are flavor-universal, but the off-diagonal elements of the mass matrices are generated by the loop effects. The FCNC sources are the Dirac/Majorana neutrino Yukawa couplings, which are responsible for the large neutrino mixings [6]. Since the left-handed leptons \( L \) and the right-handed down-type quarks \( D^c \) are unified in \( 5 \), the Dirac neutrino Yukawa couplings can be written as \( Y_{\nu ij}\tilde{5}_i N^c_j H_5 \), where \( N^c \) is the right-handed neutrino. The flavor non-universality of the SUSY
breaking $\tilde{D}^c$ masses is generated by the colored Higgs and the $N^c$ loop diagram [8], and the non-universal part of the mass matrix is $\delta M^2_{\tilde{D}^c} \simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)Y_\nu Y_\nu^\dagger \ln(M_*/M_{HC})$, where $M_*$ is a cut-off scale (e.g. the Planck scale), $M_{HC}$ is a colored Higgs mass, $m_0$ is the universal scalar mass and $A_0$ is the universal scalar trilinear coupling. The left-handed Majorana neutrino coupling $LL\Delta_L$ ($\Delta_L$ is an SU(2)$_L$ triplet) can also provide contributions to the light neutrino mass (type II seesaw [36]), and can generate the FCNC in the sfermion masses when the fermions are unified.

As a convention in this paper, we will call the model with the FCNC source arising from the Dirac neutrino Yukawa coupling as the minimal type of SU(5). In this case, the off-diagonal elements of $10$ ($Q,U^c,E^c$) representations are small because they originate from the CKM mixings. In a competitive model which we call the minimal type of SO(10), the Majorana couplings, which contribute to the neutrino mass, generate the off-diagonal elements for all sfermion species since the Majorana couplings $f_{ij}L_iL_j\Delta_L$ can be unified to the $f_{ij}16_i16_j126$ coupling [37]. The detail can be found in Ref.[11, 21].

The NP contribution of the $B_s-\bar{B}_s$ mixing amplitude can be described as

$$M^{s}_{12} = M^{s}_{12,SM} + M^{s}_{12,NP} = C_s M^{s}_{12,SM} e^{2i\phi_{B_s}},$$

(4)

where $C_s$ is a real positive number. From the measurement of the mass difference, $\Delta M_s = 2|M^{s}_{12}|$, the experimental result is consistent with $C_s = 1$. Even if $C_s = 1$, there is room for new physics because of the phase freedom, $\phi_{B_s}$. The phase can be measured by $B_s \to J/\psi\phi$ decay and the dimuon asymmetry from semileptonic $B$ decays. In the SM, the CP violation of $B_s \to J/\psi\phi$ is tiny ($2\beta_{s}^{SM} \sim 0.04$). The CP phase $2\beta_s = 2(\beta_{s}^{SM} + \phi_{B_s})$ and the decay width difference $\Delta \Gamma_s$ have been measured at the Tevatron [18] and the LHCb [19], and a large phase is allowed. The dimuon asymmetry reported by D0 implies a large CP phase in $B_s-B\bar{s}$ mixing [20].

By definition in the Eq.(4), we obtain

$$\sin^2 \phi_{B_s} = \frac{(A^{s}_{NP}/A^{s}_{SM})^2 - (1 - C_s)^2}{4C_s},$$

(5)

where $A^{s}_{NP} = |M^{s}_{12,NP}|$ and $A^{s}_{SM} = |M^{s}_{12,SM}|$. In the case of $C_s \simeq 1$, we obtain $2\sin \phi_{B_s} \simeq A^{s}_{NP}/A^{s}_{SM}$. The large dimuon asymmetry reported by D0 requires $A^{s}_{NP}/A^{s}_{SM} \sim O(1)$.

In SUSY GUTs, where quark and lepton unification is manifested, such large values of $A^{s}_{NP}/A^{s}_{SM}$ indicate a large lepton flavor violation such as $\tau \to \mu\gamma$. Therefore, in order to satisfy the current experimental bound [38]:

$$\text{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8},$$

(6)
SU(5), $\tan \beta = 40$, $m_{1/2} = 500$ GeV, $m_0 = 1$ TeV, $A_{s\text{NP}}/A_{s\text{SM}} = 0.5$ in the case of the minimal type of SU(5) for universal boundary condition.

the parameters in the model are constrained. Since $\text{Br}(\tau \to \mu \gamma)$ is proportional to $\tan^2 \beta$, the constraint is more severe for a larger $\tan \beta$. The CDF reported $\text{Br}(B_s \to \mu^+ \mu^-)$ actually prefers a large $\tan \beta$.

In the MSSM, the $B_s$-$\bar{B}_s$ mixing amplitude is induced by the box diagram contribution and the Higgs mediated contribution. The box contribution does not depend on $\tan \beta$ explicitly, while the Higgs mediated contribution is proportional to $\tan^4 \beta/m_A^2$. Therefore, for a large $\tan \beta$, the Higgs mediated contribution can dominate over the box contribution, and it can induce a sizable value of $A_{s\text{NP}}/A_{s\text{SM}}$ satisfying the experimental bound of $\text{Br}(\tau \to \mu \gamma)$ in SUSY GUTs. The flavor violating Higgs coupling can induce not only a large contribution to the $B_s$-$\bar{B}_s$ mixing amplitude but also to $B_s \to \mu^+ \mu^-$. As a result, for a given size of $A_{s\text{NP}}/A_{s\text{SM}}$, $\text{Br}(\tau \to \mu \gamma)$ and $\text{Br}(B_s \to \mu^+ \mu^-)$ are bounded.

In Fig. 2, we show the correlation of $\text{Br}(\tau \to \mu \gamma)$ and $\text{Br}(B_s \to \mu^+ \mu^-)$ for $A_{s\text{NP}}/A_{s\text{SM}} = 0.5$ in the case of the minimal type of SU(5). In the plot, the Higgsino mass $\mu$ and the CP odd Higgs mass $m_A$ are varied assuming that the SUSY breaking Higgs masses $m_{H_u}$ and $m_{H_d}$ are different from the universal sfermion mass ($m_0 = 1$ TeV). We choose the gaugino mass $m_{1/2} = 500$ GeV and $\tan \beta = 40$. For a given value of $\text{Br}(\tau \to \mu \gamma)$, $\text{Br}(B_s \to \mu^+ \mu^-)$ is bounded from below, and vice versa. In this example, in order to satisfy the CDF result $\text{Br}(B_s \to \mu^+ \mu^-) < 3.5 \times 10^{-8}$, the branching ratio of $\tau \to \mu \gamma$ has to be larger than $10^{-9}$. $\text{Br}(B_s \to \mu^+ \mu^-)$ is also bounded from below due to the $\text{Br}(\tau \to \mu \gamma)$ constraint.
In Fig[3] we plot the boundaries of $\text{Br}(\tau \to \mu \gamma)$ and $\text{Br}(B_s \to \mu^+ \mu^-)$ correlation regions in the same manner as the plot in Fig[2] while changing the input parameters $m_0$ and $m_{1/2}$. We plot two cases $A^{NP}_s/A^{SM}_s = 0.5$ and 1. Surely, a larger $A^{NP}_s/A^{SM}_s = 1$ provides more stringent constraints. The squark mass is less dependent on $m_0$ due to the gluino loop compared to the slepton mass. Therefore, larger $m_0$ gives a smaller $\text{Br}(\tau \to \mu \gamma)$. In the case of $A^{NP}_s/A^{SM}_s = 1$, $m_0 = 500$ GeV is already excluded by $\text{Br}(B_s \to \mu^+ \mu^-)$ and $\text{Br}(\tau \to \mu \gamma)$ bounds.

These bounds are significant when the quark and lepton unification is manifested. In the minimal type of SU(5) GUT where the squark flavor violation is generated from Dirac neutrino Yukawa coupling, the unification is manifested. In the case of SO(10) GUT, the bounds can be relaxed by a choice of symmetry breaking vacua [9]. Therefore, the accurate measurements of $\text{Br}(B_s \to \mu^+ \mu^-)$ and the $B_s$-$\bar{B}_s$ mixing phase are informative to distinguish the symmetry breaking vacua.

4 Flavor violating $\tau$ decays

The rare decay process $B_s \to \mu^+ \mu^-$ is dominantly generated by the Higgs mediated diagram. The leptonic version of the Higgs mediated diagram gives $\tau^+ \to \mu^+ \mu^- \mu^+ [39]$ or $\tau \to \mu \eta [40]$, and they are also proportional to $\tan^6 \beta/m_A^2$. Therefore, the correlation between the Higgs mediated $\tau$ decay and $B_s \to \mu^+ \mu^-$ can be important to investigate the quark and lepton unification.
Figure 4: $\text{Br}(\tau \rightarrow \mu \gamma)$ is shown as a function of $\text{Br}(\tau \rightarrow \mu \eta)$ for universal boundary condition (given in the text). The red dotted points satisfy the CDF allowed range of $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$.

The current experimental bounds are $\text{Br}(\tau \rightarrow \mu \eta) < 2.3 \times 10^{-8}$ [41] and $\text{Br}(\tau \rightarrow 3 \mu) < 2.1 \times 10^{-8}$ [42]. Because the Higgs mediated contribution to these processes generates a fixed ratio $\text{Br}(\tau \rightarrow \mu \eta)/\text{Br}(\tau \rightarrow 3 \mu) = 8.4$ [40], the $\tau \rightarrow \mu \eta$ decay gives more important bounds to study the Higgs mediated $\tau$-$\mu$ flavor violation [43].

The measured $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ has an impact on the $\text{Br}(\tau \rightarrow \mu \eta)$ for given boundary conditions. In Fig.4 we plot $\text{Br}(\tau \rightarrow \mu \eta)$ and $\text{Br}(\tau \rightarrow \mu \gamma)$ for $m_{1/2} = 500$ GeV, $A_0 = 0$ and $\tan \beta = 50$. We vary the SUSY breaking sfermion mass $m_0 < 1.5$ TeV. We take the non-universal SUSY breaking Higgs masses to vary $\mu$ and $m_A$. We assume that the left-handed slepton mass matrix can have off-diagonal elements due to the right-handed neutrino loop. Since the $\tau$-$\mu$ Higgs coupling is induced by the finite correction, the $\tau \rightarrow \mu \gamma$ operator is correlated and they are proportional to each other for given mass spectrum. The red points in the plot satisfy the 1 sigma range of $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ reported by the CDF. The CDF result constrains $\text{Br}(\tau \rightarrow \mu \eta)$ for a given value of $\text{Br}(\tau \rightarrow \mu \gamma)$.

The Higgs mediated $b_R-s_L$ coupling is generated by stop-chargino loop diagram, and the Higgs mediated $\tau_R-\mu_L$ coupling is generated by stau-chargino loop diagram. As a result, the stop and stau mass spectrum is important for the correlation between $\text{Br}(\tau \rightarrow \mu \eta)$ and $\text{Br}(\tau \rightarrow \mu \gamma)$ for a given $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$. Therefore, the correlation can be a probe of the SUSY breaking sfermion mass universality and the gaugino mass unification. For example, if the sfermion masses are not universal, the correlation between $\text{Br}(\tau \rightarrow \mu \eta)$ and $\text{Br}(\tau \rightarrow \mu \gamma)$ will be broken. In order to illustrate it, we show the plot for the case of sfermion mass non-universality. In Fig.5 we plot $\text{Br}(\tau \rightarrow \mu \eta)$ and $\text{Br}(\tau \rightarrow \mu \gamma)$ for $m_{1/2} = 500$ GeV, $A_0 = 0$ and $\tan \beta = 50$. [43]
Figure 5: $\text{Br}(\tau \rightarrow \mu \gamma)$ vs $\text{Br}(\tau \rightarrow \mu \eta)$ is plotted for non-universal boundary condition (given in the text). The points satisfy the CDF allowed range of $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$. The red dotted points satisfy the universal boundary conditions (as in Fig.4).

We choose the SUSY breaking sfermion mass $m_{10} \neq m_5$ where $m_{10} = m_{\tilde{Q}} = m_{\tilde{U}} = m_{\tilde{E}}$ and $m_5 = m_{\tilde{D}} = m_{\tilde{L}}$. The points satisfies the 1 sigma $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ range reported by CDF. The red points correspond to the case of sfermion universality and thus, they are same as the red points in Fig[4]. The green points correspond to the case of non-universal sfermion mass.

The correlation between $\text{Br}(\tau \rightarrow \mu \eta)$ and $\text{Br}(\tau \rightarrow \mu \gamma)$ can be also broken if $b$-$s$ Higgs coupling is accidentally canceled by a new left-handed squark flavor violating source (which is absent in the minimal type of SU(5)). If the sizes of the off-diagonal elements are same for all sfermion species, the $\tau \rightarrow \mu \gamma$ bound can be easily saturated and the $B_s \rightarrow \mu^+ \mu^-$ amplitude will not be canceled. Therefore, the flavor violation in the quark sector has to be larger than one in the lepton sector choosing the SO(10) breaking vacua to allow the cancellation. However, if the $b_{R}$-$s_{L}$ Higgs coupling is canceled (namely the new FCNC contribution is destructive), then the new FCNC provides constructive SUSY contribution to the $b \rightarrow s \gamma$ operator ($C_{7L}$) [15], which is disfavored from the experimental constraint.

It is expected that the upper bounds of the branching ratios of flavor violating $\tau$ decays can be roughly one order below at the super B factory [44]. It may be hard to achieve the allowed region of $\text{Br}(\tau \rightarrow \mu \eta)$. If, however, $\text{Br}(\tau \rightarrow \mu \eta)$ is measured to be $\gtrsim 10^{-9}$, the slepton and squark mass universality or gaugino mass unification may need to be broken to explain.
5 Conclusion

The $b$-$s$ flavor changing transition is one of the important probes of new physics. The purely leptonic $B_s$ meson decays are helicity suppressed in SM, and the prediction of the branching ratio is very small. Therefore, the measurement of the rare decay process of $B_s \rightarrow \mu^+\mu^-$ plays an important role in finding the signature of the new physics beyond SM. In fact, in the MSSM, the decay rate has a strong dependence on $\tan \beta$, and it can be large even if the colored SUSY particles are heavy, as preferred by the current experimental constraints. In other words, a large $\tan \beta$ is preferred to reproduce the large branching ratio of $B_s \rightarrow \mu^+\mu^-$ for heavy colored SUSY particles. In this way, the CDF measurement of the $B_s \rightarrow \mu^+\mu^-$ rare decay process has a tremendous impact to bound the SUSY parameters, and can provide a crucial upper bound.

Even in the minimal models of the SUSY breaking parameters in which flavor universality is assumed, the $B_s \rightarrow \mu^+\mu^-$ process can be large due to the flavor violation originated from CKM mixings. In GUT models, the flavor violation can be related to the symmetry breaking and the GUT particle spectrum, and the branching ratio of $B_s \rightarrow \mu^+\mu^-$ process can give a constraint on the GUT models. Especially for the minimal type of SU(5) GUT where the flavor violation comes from the Dirac neutrino Yukawa coupling, the parameter space is really restricted and the branching ratios $\text{Br}(B_s \rightarrow \mu^+\mu^-)$ and $\text{BR}(\tau \rightarrow \mu\gamma)$ are bounded from below if the CP violation in $B_s$ mixing is large, which is indicated by the $B_s \rightarrow J/\psi\phi$ decay reported by the Tevatron and the LHCb, and the dimuon asymmetry in the semileptonic $B$ decays reported by D0. The flavor violating lepton decays are also important to find a footprint of the GUT scale physics.

The excess of the $B_s \rightarrow \mu^+\mu^-$ decay deviation from the SM prediction at more than 90% CL can be soon verified at the LHC, and the squarks and the gluino can be soon found if the true value of $\text{Br}(B_s \rightarrow \mu^+\mu^-)$ lies in the 1 sigma range of CDF measurements. The large $B_s$ CP phase can be also soon verified at the LHC. A surge of experimental results will appear soon to test the whole structure of the models presented.

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References

[1] CDF Collaboration, arXiv:1107.2304 [hep-ex].

[2] G. Buchalla and A. J. Buras, Nucl. Phys. B 400, 225 (1993); A. J. Buras, Phys. Lett. B 566, 115 (2003) [hep-ph/0303060].

[3] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Phys. Rev. D 80, 014503 (2009) [arXiv:0902.1815 [hep-lat]].

[4] R. Aaij et al. [the LHCb Collaboration], Phys. Lett. B 699, 330 (2011) [arXiv:1103.2465 [hep-ex]].

[5] S. R. Choudhury and N. Gaur, Phys. Lett. B 451, 86 (1999) [hep-ph/9810307]; K. S. Babu and C. F. Kolda, Phys. Rev. Lett. 84, 228 (2000) [hep-ph/9909476]; C. S. Huang, W. Liao, Q. S. Yan and S. H. Zhu, Phys. Rev. D 63, 114021 (2001) [Erratum-ibid. D 64, 059902 (2001)] [hep-ph/0006250]; P. H. Chankowski and L. Slawianowska, Phys. Rev. D 63, 054012 (2001) [hep-ph/0008046]; C. Bobeth, T. Ewerth, F. Kruger and J. Urban, Phys. Rev. D 64, 074014 (2001) [hep-ph/0104284]; Phys. Rev. D 66, 074021 (2002) [hep-ph/0204225].

[6] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57, 961 (1986); J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B 357, 579 (1995) [hep-ph/9501407].

[7] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267, 415 (1986); R. Barbieri and L. J. Hall, Phys. Lett. B 338, 212 (1994) [hep-ph/9408406]; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Lett. B 391, 341 (1997) [hep-ph/9605296].

[8] T. Moroi, Phys. Lett. B 493, 366 (2000) [hep-ph/0007328]; D. Chang, A. Masiero and H. Murayama, Phys. Rev. D 67, 075013 (2003) [hep-ph/0205111]; R. Harnik, D. T. Larson, H. Murayama and A. Pierce, Phys. Rev. D 69, 094024 (2004) [hep-ph/0212180].

[9] B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Rev. Lett. 100, 181801 (2008) [arXiv:0712.1206 [hep-ph]].

[10] J. K. Parry, Nucl. Phys. B 760, 38 (2007) [hep-ph/0510305]; Mod. Phys. Lett. A 21, 2853 (2006) [hep-ph/0608192]; J. K. Parry and H. h. Zhang, Nucl. Phys. B 802, 63 (2008) [arXiv:0710.5443 [hep-ph]]; J. K. Parry, Phys. Lett. B 694, 363 (2011) [arXiv:1006.5331 [hep-ph]].
[11] B. Dutta and Y. Mimura, Phys. Rev. Lett. 97, 241802 (2006) [hep-ph/0607147]; Phys. Rev. D 75, 015006 (2007) [hep-ph/0611268]; Phys. Rev. D 77, 051701 (2008) [arXiv:0708.3080 [hep-ph]]; Phys. Rev. D 78, 071702 (2008) [arXiv:0805.2988 [hep-ph]].

[12] T. Goto, Y. Okada, T. Shindou and M. Tanaka, Phys. Rev. D 77, 095010 (2008) [arXiv:0711.2935 [hep-ph]].

[13] J. Hisano and Y. Shimizu, Phys. Lett. B 669, 301 (2008) [arXiv:0805.3327 [hep-ph]]; J. H. Park and M. Yamaguchi, Phys. Lett. B 670, 356 (2009) [arXiv:0809.2614 [hep-ph]].

[14] C. Hamzaoui, M. Pospelov and M. Toharia, Phys. Rev. D 59, 095005 (1999) [hep-ph/9807350]; M. Gorbahn, S. Jager, U. Nierste and S. Trine, arXiv:0901.2065 [hep-ph].

[15] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B 619, 434 (2001) [hep-ph/0107048]; Phys. Lett. B 546, 96 (2002) [hep-ph/0207241]; Nucl. Phys. B 659, 3 (2003) [hep-ph/0210145].

[16] J. Foster, K. Okumura and L. Roszkowski, Phys. Lett. B 609, 102 (2005) [hep-ph/0410323]; JHEP 0508, 094 (2005) [hep-ph/0506146]; Phys. Lett. B 641, 452 (2006) [hep-ph/0604121].

[17] B. Dutta and Y. Mimura, Phys. Lett. B 677, 164 (2009) [arXiv:0902.0016 [hep-ph]].

[18] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 100, 161802 (2008) [arXiv:0712.2397 [hep-ex]]; V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 101, 241801 (2008) [arXiv:0802.2255 [hep-ex]].

[19] The LHCb Collaboration, LHCb-CONF-2011-006.

[20] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. D 82, 032001 (2010) [arXiv:1005.2757 [hep-ex]]; Phys. Rev. Lett. 105, 081801 (2010) [arXiv:1007.0395 [hep-ex]]; arXiv:1106.6308 [hep-ex].

[21] B. Dutta, Y. Mimura and Y. Santoso, Phys. Rev. D 80, 095005 (2009) [arXiv:0907.4946 [hep-ph]]; Phys. Rev. D 82, 055017 (2010) [arXiv:1007.3696 [hep-ph]].

[22] A. H. Chamseddine, R. L. Arnowitt, P. Nath, Phys. Rev. Lett. 49, 970 (1982); R. Barbieri, S. Ferrara, C. A. Savoy, Phys. Lett. B119, 343 (1982); L. J. Hall, J. D. Lykken, S. Weinberg, Phys. Rev. D27, 2359-2378 (1983); P. Nath, R. L. Arnowitt, A. H. Chamseddine,
Nucl. Phys. B\textbf{227}, 121 (1983); For a review, see H. P. Nilles, Phys. Rept. \textbf{110}, 1-162 (1984).

[23] R. L. Arnowitt, B. Dutta, T. Kamon and M. Tanaka, Phys. Lett. B \textbf{538}, 121 (2002) [hep-ph/0203069]; A. Dedes, H. K. Dreiner, U. Nierste, Phys. Rev. Lett. \textbf{87}, 251804 (2001). [hep-ph/0108037]; H. Baer, C. Balazs, A. Belyaev, J. K. Mizukoshi, X. Tata, Y. Wang, JHEP \textbf{0207}, 050 (2002). [hep-ph/0205325]; J. R. Ellis, K. A. Olive, V. C. Spanos, Phys. Lett. B\textbf{624}, 47-59 (2005). [hep-ph/0504196].

[24] Talk presented at PPC 2011, CERN, by Beate Hein, ATLAS-conf-2011-086.

[25] ALEPH, DELPHI, L3, OPAL Collaborations, G. Abbiendi, \textit{et al.}, (The LEP Working Group for Higgs Boson Searches), Phys. Lett. B \textbf{565}, 61 (2003).

[26] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, Comput. Phys. Commun. \textbf{180} (2009) 1426.

[27] S. Chen \textit{et al.} [CLEO Collaboration], Phys. Rev. Lett. \textbf{87} (2001) 251807 [hep-ex/0108032]; P. Koppenburg \textit{et al.} [Belle Collaboration], Phys. Rev. Lett. \textbf{93} (2004) 061803 [hep-ex/0403004]; B. Aubert \textit{et al.} [BaBar Collaboration], [hep-ex/0207076]; M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B \textbf{527} (1998) 21 [hep-ph/9710335]; Nucl. Phys. B \textbf{534} (1998) 3 [hep-ph/9806308]; C. Degrassi, P. Gambino and G. F. Giudice, JHEP \textbf{0012} (2000) 009 [hep-ph/0009337]; P. Gambino and M. Misiak, Nucl. Phys. B \textbf{611} (2001) 338.

[28] E. Komatsu \textit{et al.} [WMAP Collaboration], Astrophys. J. Suppl. \textbf{192}, 18 (2011). [arXiv:1001.438][astro-ph.CO].

[29] K. Nakamura \textit{et al.} [Particle Data Group], J. Phys. G \textbf{37}, 075021 (2010).

[30] Muon \textit{g} – 2 Collaboration, G. Bennett \textit{et al.}, Phys. Rev. Lett. \textbf{74}, 161802 (2004).

[31] T. Teubner, K. Hagiwara, R. Liao, A. D. Martin and D. Nomura, arXiv:1001.5401 [hep-ph].

[32] M. Davier, A. Hoecker, B. Malaescu, C. Z. Yuan and Z. Zhang, Eur. Phys. J. C \textbf{66}, 1 (2010) arXiv:0908.4300 [hep-ph].

[33] E. Aprile \textit{et al.} [XENON100 Collaboration], arXiv:1104.2549 [astro-ph.CO].

[34] I. Gogoladze, R. Khalid, Y. Mimura and Q. Shafi, Phys. Rev. D \textbf{83}, 095007 (2011) arXiv:1012.1613 [hep-ph].
[35] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, JHEP **0605**, 063 (2006) [arXiv:hep-ph/0603136].

[36] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981); G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, 287 (1981).

[37] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993) [hep-ph/9209215].

[38] K. Hayasaka *et al.* [Belle Collaboration], Phys. Lett. B **666**, 16 (2008) [arXiv:0705.0650 [hep-ex]]; B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **104**, 021802 (2010) [arXiv:0908.2381 [hep-ex]].

[39] K. S. Babu and C. Kolda, Phys. Rev. Lett. **89**, 241802 (2002) [hep-ph/0206310].

[40] M. Sher, Phys. Rev. D **66**, 057301 (2002) [hep-ph/0207136].

[41] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **98**, 061803 (2007) [hep-ex/0610067]; Y. Miyazaki *et al.* [BELLE Collaboration], Phys. Lett. B **648**, 341 (2007) [hep-ex/0703009]; K. Hayasaka, arXiv:1103.0094 [hep-ex].

[42] G. Marchiori [BABAR Collaboration], AIP Conf. Proc. **1200**, 857 (2010) [arXiv:0909.3870 [hep-ex]]; K. Hayasaka *et al.* [Belle Collaboration], Phys. Lett. B **687**, 139 (2010) [arXiv:1001.3221 [hep-ex]].

[43] A. Dedes, J. R. Ellis and M. Raidal, Phys. Lett. B **549**, 159 (2002) [hep-ph/0209207]; A. Brignole and A. Rossi, Nucl. Phys. B **701**, 3 (2004) [hep-ph/0404211]; S. Kanemura, T. Ota and K. Tsumura, Phys. Rev. D **73**, 016006 (2006) [hep-ph/0505191]; P. Paradisi, JHEP **0602**, 050 (2006) [hep-ph/0508054]; E. Arganda, M. J. Herrero and J. Portoles, JHEP **0806**, 079 (2008) [arXiv:0803.2039 [hep-ph]]; M. J. Herrero, J. Portoles and A. M. Rodriguez-Sanchez, Phys. Rev. D **80**, 015023 (2009) [arXiv:0903.5151 [hep-ph]]; M. Cannoni and O. Panella, Phys. Rev. D **81**, 036009 (2010) [arXiv:0910.3316 [hep-ph]].

[44] T. Aushev *et al.*, arXiv:1002.5012 [hep-ex].