Enhanced Internal Quantum Efficiency of Bandgap-Engineered Green W-Shaped Quantum Well Light-Emitting Diode

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Received: 29 November 2018; Accepted: 20 December 2018; Published: 26 December 2018  

Featured Application: Since ‘green-gap’ is a challenging issue which is of great interest to the community, we present and analyze bandgap-engineered W-shaped quantum well, as a solution, to improve the optoelectronic performance of InGaN-based green light-emitting diodes.

Abstract: To improve the internal quantum efficiency of green light-emitting diodes, we present the numerical design and analysis of bandgap-engineered W-shaped quantum well. The numerical results suggest significant improvement in the internal quantum efficiency of the proposed W-LED. The improvement is associated with significantly improved hole confinement due to the localization of indium in the active region, leading to improved radiative recombination rate. In addition, the proposed device shows reduced defect-assisted Shockley-Read-Hall (SRH) recombination rate as well as Auger recombination rate. Moreover, the efficiency rolloff in the proposed device is associated with increased built-in electromechanical field.

Keywords: quantum well; numerical simulation; internal quantum efficiency

1. Introduction

Enhancing the internal quantum efficiency (IQE) of GaInN-based light-emitting diodes (LEDs) has been discussed in the literature for potential solid-state lighting applications [1–3]. There are multiple reports to improve the IQE of LEDs mainly by using nonpolar substrates [4,5] and bandgap engineering [6–9]. Salient bandgap engineering approaches include triangular-shaped quantum wells [10,11], staggered quantum wells [12], graded quantum wells [8,13], polarization-matched quaternary barriers [14,15], quaternary electron blocking layer (EBL) [16], modification of EBL [17,18], InGaN barriers [19,20], as well as last quantum barrier [21], coupled quantum wells [22], and recently proposed all-quaternary device with and without electron-blocking layer [23,24]. In comparison to the conventional rectangular quantum wells, it has been shown using triangular, staggered, dip-shaped, and trapezoidal wells that the electron-hole wavefunction overlap is significantly improved in the quantum wells (QWs), which leads to improved optoelectronic output of the device [11,25–27]. In addition, indium fluctuations in the GaN-based LEDs are known to influence the optoelectronic properties of the device [28–32]. Most of the reported work on bandgap engineering is based on
LEDs in the blue emission range [11,22,27]. However, because of the ‘green gap’ challenge, it is of great interest to improve the performance of green LEDs [32,33]. Therefore, in this work, we present bandgap-engineered green W-shaped quantum well and discuss its influence on the improved optoelectronic performance of the green light-emitting diode by numerical simulation.

2. Device Structure

Two device structures are numerically employed labeled as R-LED and W-LED. Both the structures have similar configuration, except the quantum well shape. Two quantum wells are used in the active region. 2 nm thick undoped GaN layer is followed by 4.5 nm thick n-GaN layer (doping = \( \times 5 \times 10^{18} \text{ cm}^{-3} \)). 20 nm thick Al\(_{0.1}\)GaN (doping = \( \times 5 \times 10^{17} \text{ cm}^{-3} \)) EBL is followed by 15 nm thick p-GaN layer (doping = \( \times 1 \times 10^{18} \text{ cm}^{-3} \)). The quantum well composition in the two structures is In\(_{0.31}\)GaN and In\(_{0.34}\)GaN/In\(_{0.28}\)GaN/In\(_{0.34}\)GaN for R-LED and W-LED, respectively. The indium composition in the quantum wells of the devices is such that the peak output wavelength of R-LED and W-LED is \( \sim 518 \) and \( \sim 513 \) nm, respectively. The SRH and Auger recombination coefficients are \( 20 \) ns and \( 1.5 \times 10^{30} \), which are similar to the reported literature [34]. Both of the recombination coefficients are reported to be smaller in the graded quantum wells than the regular quantum wells by roughly a factor of half [13,35]. Other simulation parameters, such as mobility, temperature, and background losses are the same as reported in the literature [23]. Figure 1 shows the schematic of the two devices and the respective indium compositions.

Figure 1. Schematic of the (a) device; (b) R-LED and W-LED indium composition in the quantum well.

3. Methodology

To conduct our study, we have used a commercial simulator APSYS to simulate our devices [36]. The energy band is computed as [37]:

\[
E_g(A_{1-x}B_x) = (1 - x)E_g(A) + xE_g(B) - x(1 - x)C
\tag{1}
\]

where \( E_g \) is the bandgap, \( A \) and \( B \) are the two binary elements, the bowing parameter \( C \) account for deviations from the linear behavior and is given in Ref. [38]. The current densities \( \vec{J}_n \) and \( \vec{J}_p \) of both the electrons and holes are originated by the electrostatic field \( \vec{F} \), as well as by the gradient of concentration of electrons \( \nabla n \) and holes \( \nabla p \)

\[
\vec{J}_n = q\mu_n \vec{F} + qD_n \nabla n
\tag{2}
\]

\[
\vec{J}_p = q\mu_p \vec{F} + qD_p \nabla p
\tag{3}
\]
where \( q \) is elementary charge, \( n \) and \( p \) represent carrier densities, and \( \mu_{n,p} \) are their corresponding mobilities, respectively. The heterointerface charge density is estimated using reported nonlinear model [39,40].

\[
P_{sp}^{Al_xGa_{1-x}N} = (0.019 \cdot x \cdot (1 - x)) - (0.034 \cdot (1 - x)) - (0.090 \cdot x)
\]

\[
P_{sp}^{In_xGa_{1-x}N} = (0.038 \cdot x \cdot (1 - x)) - (0.034 \cdot (1 - x)) - (0.042 \cdot x)
\]

The spontaneous, Shockley-Read-Hall, and Auger recombination rates, represented by \( R_{sp} \), \( R_{SRH} \) and \( R_{Aug} \) respectively, are given by [41]:

\[
R_{sp} = B \left( np - n_i^2 \right)
\]

\[
R_{SRH} = \frac{np - n_i^2}{\tau_{SRH}^n (n + n_i) + \tau_{SRH}^p (p + n_i)}
\]

\[
R_{Aug} = \left( C_n n + C_p p \right) \left( np - n_i^2 \right)
\]

where nonradiative SRH lifetimes for electrons (\( n \)), and holes (\( p \)) is given by \( \tau_{SRH}^n, \tau_{SRH}^p \), \( n_i \) is intrinsic density is \( C_n \) and \( C_p \) are Auger coefficients, depending on the type of material. For both the electrons as well as holes, \( \tau_{SRH}^n, \tau_{SRH}^p \) is assumed to be same. Fermi statistics as well as thermionic emission of carriers at heterointerfaces is estimated, as given in [42]. A typical value of offset ratio \( \Delta E_c/\Delta E_v \) is assumed to be 0.7 in our work [43,44]. Other details are given in [45]. The internal quantum efficiency (IQE), \( \eta_i \), is estimated as [46]:

\[
\eta_i = \frac{\int R_{sp} dv}{\int R dv}.
\]

4. Results and Discussion

Energy band diagram is shown in Figure 2. The R-LED and W-LED energy band diagrams are shown. The shaded areas are highlighting the quantum well region. The energyband in both the cases is strongly tilted towards the p-side because of the built-in electromechanical field direction. The W-shape well can utilize the carrier localization within the well to improve the carrier confinement and consequent radiative recombination. Strong band-bending in W-LED is observed because of the increased electrostatic field, owing to the indium fluctuation i.e., 34%–28%–34%, which also hinders carrier transport in the W-LED apart from the indium localization [28,29,31,32,45,47,48]. The influence of the electrostatic field and carrier transport in our proposed device is presented in detail in the following discussion.
Figure 2. Comparison of band energy: (a) Conduction band energy; and, (b) Valence band energy.

The inclined triangular wells in the reference structure are known to reduce the electron-hole wavefunction overlap, therefore reducing the overall device performance [9,32,47]. This overlap is further reduced in green LEDs due to increased built-in electrostatic field arising from the increased indium composition and lattice mismatch [32]. The wells are also inclined in the W-LED but the W-shaped indium composition in the well improves the electron-hole wavefunction overlap, due to its peculiar composition fluctuations, leading to the improved device performance.

Figure 3 shows the estimation of carrier concentrations in the quantum wells of the two devices. The electron concentrations in Figure 3a are quite identical in the two devices, which means that the W-LED has no significant influence on the electron current transport in the quantum wells. The electron concentration peaks towards the p-side of the well. In contrast, the hole concentration in W-LED in Figure 3b shows significant enhancement in comparison to W-LED, specifically in the quantum well towards p-side, which means that significant radiative recombination is expected to occur towards the p-side of the quantum well. The hole concentration tends to decrease in each quantum well from p-side to n-side in the reference structure, which is consistent with the reported results [47]. Since strong localization of holes is known to occur due to the fluctuations in indium composition [28,29,31,32], this results in most of the holes being confined in the last QW of the proposed W-LED in comparison to the first QW, where negligible holes exist, in addition to the effect of increased band-bending coming from the comparatively higher electrostatic field in W-LED.

Figure 3. Comparison of carrier concentrations: (a) Electrons and (b) Holes in the two devices in semilogarithmic scale.
Figure 4a shows the estimation of the radiative recombination rates in the two devices. It can be observed that the radiative recombination rate in W-LED is more than twice than that of R-LED. This is due to the significant accumulation of holes in the well towards the p-side, as discussed in Figure 3. Since the holes are concentrated more towards the n-side in each QW, the radiative recombination in each quantum well is also concentrated towards the n-side because of the better electron-hole wavefunction overlap towards the n-side of the well. Because of the reduction of hole concentration from p-side towards the n-side of the device, owing to the reduced hole transport, the radiative recombination is asymmetrically reduced in the quantum wells of both devices [49]. Figure 4b,c show that W-LED has reduced Shockley-Read-Hall and Auger recombination rates in comparison to R-LED. The defect-assisted Shockley-Read-Hall (SRH) recombination rate is reduced by more than half and the Auger recombination rate is estimated to be negligible in comparison to the R-LED. The comparatively reduced SRH recombination and Auger recombination rates of W-LED is supported by earlier experimental and theoretical reports [13,35]. The rate of Auger recombination has been reported to be higher in rectangular wells than softened or graded wells, hence reducing the device efficiency [35].

![Comparison of recombination rates in the two devices](image)

**Figure 4.** Comparison of recombination rates in the two devices: (a) Radiative; (b) Shockley-Read-Hall (SRH); and, (c) Auger. Insets show the magnified recombination rates.

The internal quantum efficiency comparison is shown in Figure 5. By using W-LED, we are able to increase the internal quantum efficiency of the device significantly. Improved IQE peak and droop ratio in W-LED is observed in comparison to R-LED i.e., droop ratios of R-LED and W-LED are ~65% and ~57%, respectively. The IQE peak of R-LED and W-LED occurs at ~0.18 μA·cm$^{-2}$ and ~0.17 A·cm$^{-2}$ respectively. The IQE peak in W-LED is shifted to higher current density, which is typically a case in graded LEDs [8,13]. This shift of IQE peak or droop onset is desirable in improving the device
performance [2]. Similarly, the light output power of W-LED is improved by ~116% than that of the R-LED at 100 A·cm⁻².

The reason of the droop in W-LED can be understood by the built-in field comparison of the two structures in Figure 6. It can be observed that the average built-in field in W-LED is ~24% higher than the R-LED. Despite the reduced Auger recombination rate, the internal quantum efficiency roll-off in Figure 5 can be associated with the comparatively higher built-in field in the proposed W-LED.

The emission spectra of the two devices is shown in Figure 7. The peak wavelength of R-LED and W-LED is ~518 nm and ~513 nm, respectively. The broad emission spectrum is the characteristic of the indium fluctuations in the quantum well region [50]. The full width half maximum (FWHM) of R-LED and W-LED is 25 nm and 28 nm, respectively at the peak wavelength. With increasing injection current and higher indium fluctuations, FWHM, is reported to be broader in green light-emitting diodes [51]. In addition, indium-rich regions, regardless of high dislocation density, have been attributed to improve radiative recombination because of localization of carriers [51–53]. At peak wavelength, the emission of W-LED is ~93% improved in comparison to the R-LED.
Acknowledgments:

We would also like to thank Higher Education Commission of Pakistan for providing the performance could further be improved.

Author Contributions: The conceptualization M.U. and U.M.; methodology M.U.; software M.U. and D.-G.Z.; validation, M.U., D.-G.Z., and N.M.; data analysis M.U. and D.-P.H.; investigation M.U. and D.-P.H.; resources, U.M.; data curation, N.M. and M.R.; writing—original draft preparation, M.U.; writing—review and editing, N.M.; visualization, N.M.; supervision, M.U.; project administration, M.U.; funding acquisition, M.R.

Funding: This research was funded by HEC, Pakistan, grant number 10496/Balochistan/NRPU/R&D/HEC/2017.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Crawford, M.H. LEDs for solid-state lighting: Performance challenges and recent advances. *IEEE J. Sel. Top. Quantum Electron.* 2009, 15, 1028–1040. [CrossRef]
2. Cho, J.; Schubert, E.F.; Kim, J.K. Efficiency droop in light-emitting diodes: Challenges and countermeasures. *Laser Photonics Rev.* 2013, 7, 408–421. [CrossRef]
3. Weisbuch, C.; Piccardo, M.; Martinelli, L.; Ivanov, J.; Peretti, J.; Speck, J.S. The efficiency challenge of nitride light-emitting diodes for lighting. *Phys. Status Solidi A* 2015, 212, 899–913. [CrossRef]
4. Farrell, R.; Young, E.; Wu, F.; DenBaars, S.; Speck, J. Materials and growth issues for high-performance nonpolar and semipolar light-emitting devices. *Semicond. Sci. Technol.* 2012, 27, 024001. [CrossRef]
5. Ling, S.-C.; Lu, T.-C.; Chang, S.-P.; Chen, J.-R.; Kuo, H.-C.; Wang, S.-C. Low efficiency droop in blue-green m-plane InGaN/GaN light emitting diodes. *Appl. Phys. Lett.* 2010, 96, 231101. [CrossRef]
6. Zhao, H.; Liu, G.; Zhang, J.; Poplawsky, J.D.; Dierolf, V.; Tansu, N. Approaches for high internal quantum efficiency InGaN light-emitting diodes with large overlap quantum wells. *Opt. Express* 2011, 19, A991–A1007. [CrossRef]
7. der Maur, M.A.; Lorenz, K.; Di Carlo, A. Band gap engineering approaches to increase InGaN/GaN LED efficiency. *Opt. Quantum Electron.* 2012, 44, 83–88. [CrossRef]
8. Zhu, L.-H.; Liu, W.; Zeng, F.-M.; Gao, Y.-L.; Liu, B.-L.; Lu, Y.-J.; Chen, Z. Efficiency droop improvement in InGaN/GaN light-emitting diodes by graded-composition multiple quantum wells. IEEE Photonics J. 2013, 5, 8200208.
9. Islam, M.; Usman, M.; Mushtaq, U.; Nawaz, N.; Karimov, K.; Muhammad, N.; Saba, K. Numerical analysis of the indium compositional variation on the efficiency droop of the GaN-based light-emitting diodes. In Proceedings of the SPIE Optical Engineering + Applications, San Diego, CA, USA, 19–23 August 2018; p. 5.
10. Gerber, D.; Droopad, R.; Maracas, G. Comparison of electroabsorption in asymmetric triangular and rectangular GaAs/AlGa1-xAs as multiple quantum wells. Appl. Phys. Lett. 1993, 62, 525–527. [CrossRef]
11. Choi, R.; Hahn, Y.; Shim, H.; Han, M.; Suh, E.; Lee, H. Efficient blue light-emitting diodes with InGaN/GaN triangular shaped multiple quantum wells. Appl. Phys. Lett. 2003, 82, 2764–2766. [CrossRef]
12. Zhao, H.; Arif, R.A.; Tansu, N. Design analysis of staggered InGaN quantum wells light-emitting diodes at 500–540 nm. IEEE J. Sel. Top. Quantum Electron. 2009, 15, 1104–1114. [CrossRef]
13. Chang, C.-Y.; Li, H.; Lu, T.-C. High efficiency InGaN/GaN light emitting diodes with asymmetric triangular multiple quantum wells. Appl. Phys. Lett. 2014, 104, 091111. [CrossRef]
14. Chen, J.-R.; Ling, S.-C.; Huang, H.-M.; Su, P.-Y.; Ko, T.-S.; Lu, T.-C.; Kuo, H.-C.; Kuo, Y.-K.; Wang, S.-C. Numerical study of optical properties of InGaN multi-quantum-well laser diodes with polarization-matched AlGaN barrier layers. Appl. Phys. B 2009, 95, 145–153. [CrossRef]
15. Schubert, M.F.; Xu, J.; Kim, J.K.; Schubert, E.F.; Kim, M.H.; Yoon, S.; Lee, S.M.; Sone, C.; Sakong, T.; Park, Y. Polarization-matched GaN in N/AlGaN in N multi-quantum-well light-emitting diodes with reduced efficiency droop. Appl. Phys. Lett. 2008, 93, 041102. [CrossRef]
16. Wang, T.-H.; Xu, J.-L. Advantage of InGaN-based light-emitting diodes using AlGaN/GaN electron blocking layer coupled with inserting InGaN layer. Opt. Int. J. Light Electron. Opt. 2013, 124, 5866–5870. [CrossRef]
17. Kuo, Y.-K.; Chang, J.-Y.; Tsai, M.-C. Enhancement in hole-injection efficiency of blue InGaN light-emitting diodes from reduced polarization by some specific designs for the electron blocking layer. Opt. Lett. 2010, 35, 3285–3287. [CrossRef] [PubMed]
18. Chung, R.B.; Han, C.; Fan, C.-C.; Pfaff, N.; Speck, J.S.; DenBaars, S.P.; Nakamura, S. The reduction of efficiency droop by Al0.8In0.2N/GaN superlattice electron blocking layer in (0001) oriented GaN-based light-emitting diodes. Appl. Phys. Lett. 2012, 101, 131113. [CrossRef]
19. Kuo, Y.-K.; Chang, J.-Y.; Tsai, M.-C.; Yen, S.-H. Advantages of blue InGaN multiple-quantum well light-emitting diodes with InGaN barriers. Appl. Phys. Lett. 2009, 95, 011116. [CrossRef]
20. Xie, J.; Ni, X.; Fan, Q.; Shimada, R.; Özgür, Ü.; Morkoç, H. On the efficiency droop in InGaN multiple quantum well blue light emitting diodes and its reduction with p-doped quantum well barriers. Appl. Phys. Lett. 2008, 93, 121107. [CrossRef]
21. Xiong, J.-Y.; Xu, Y.-Q.; Zheng, S.-W.; Zhao, F.; Ding, B.-B.; Song, J.-J.; Yu, X.-P.; Zhang, T.; Fan, G.-H. Advantages of GaN based light-emitting diodes with p-AlGaN/InGaN superlattice last quantum barrier. Opt. Commun. 2014, 312, 85–88. [CrossRef]
22. Ni, X.; Fan, Q.; Shimada, R.; Özgür, Ü.; Morkoç, H. Reduction of efficiency droop in InGaN light emitting diodes by coupled quantum wells. Appl. Phys. Lett. 2008, 93, 171113. [CrossRef]
23. Usman, M.; Saba, K.; Han, D.-P.; Muhammad, N. Efficiency improvement of green light-emitting diodes by employing all-quantenary active region and electron-blocking layer. Superlattices Microstruct. 2018, 113, 585–591. [CrossRef]
24. Usman, M.; Mushtaq, U.; Saba, K.; Han, D.-P.; Muhammad, N.; Imtiaz, W.A.; Jahangir, A. Improved optoelectronic performance of green light-emitting diodes by employing GaAllnN quantum wells without electron blocking layer. Phys. E Low-Dimens. Syst. Nanostruct. 2019, 106, 68–72. [CrossRef]
25. Arif, R.A.; Ee, Y.K.; Tansu, N. Nanostructure engineering of staggered InGaN quantum wells light emitting diodes emitting at 420–510 nm. Phys. Status Solidi A 2008, 205, 96–100. [CrossRef]
26. Park, S.-H.; Ahn, D.; Koo, B.-H.; Kim, J.-W. Dip-shaped InGaN/GaN quantum-well light-emitting diodes with high efficiency. Appl. Phys. Lett. 2009, 95, 063507. [CrossRef]
27. Han, S.-H.; Lee, D.-Y.; Shim, H.-W.; Kim, G.-C.; Kim, Y.S.; Kim, S.-T.; Lee, S.-J.; Cho, C.-Y.; Park, S.-J. Improvement of efficiency droop in InGaN/GaN multiple quantum well light-emitting diodes with trapezoidal wells. J. Phys. D Appl. Phys. 2010, 43, 354004. [CrossRef]
28. Bellaiche, L.; Mattila, T.; Wang, L.-W.; Wei, S.-H.; Zunger, A. Resonant hole localization and anomalous optical bowing in InGaN alloys. Appl. Phys. Lett. 1999, 74, 1842–1844. [CrossRef]
29. Chichibu, S.F.; Uedono, A.; Onuma, T.; Haskell, B.A.; Chakraborty, A.; Koyama, T.; Fini, P.T.; Keller, S.; DenBaars, S.P.; Speck, J.S. Origin of defect-insensitive emission probability in In-containing (Al, In, Ga) N alloy semiconductors. *Nat. Mater.* 2006, 5, 810–816. [CrossRef]

30. Wu, Y.-R.; Yeh, S.-T.; Lin, D.-W.; Li, C.-K.; Kuo, H.-C.; Speck, J.S. Influences of indium fluctuation to the carrier transport, auger recombination, and efficiency droop. In Proceedings of the 2013 13th International Conference on Numerical Simulation of Optoelectronic Devices (NUSOD), Vancouver, BC, Canada, 19–22 August 2013; pp. 111–112.

31. Liu, W.; Zhao, D.; Jiang, D.; Chen, P.; Liu, Z.; Zhu, J.; Shi, M.; Zhao, D.; Li, X.; Liu, J. Localization effect in green light emitting InGaN/GaN multiple quantum wells with varying well thickness. *J. Alloys Compd.* 2015, 625, 266–270. [CrossRef]

32. Ding, B. Improving radiative recombination efficiency of green light-emitting diodes. *Mater. Sci. Technol.* 2018, 34, 1615–1630. [CrossRef]

33. Usman, M.; Kim, H.; Shim, J.-I.; Shin, D.-S. Measurement of piezoelectric field in single-and double-quantum-well green LEDs using electroreflectance spectroscopy. *Jpn. J. Appl. Phys.* 2014, 53, 098002. [CrossRef]

34. Meneghini, M.; Trivellin, N.; Meneghesso, G.; Zanoni, E.; Zehnder, U.; Hahn, B. A combined electro-optical method for the determination of the recombination parameters in InGaN-based light-emitting diodes. *J. Appl. Phys.* 2009, 106, 114508. [CrossRef]

35. Vaxenburg, R.; Lifshitz, E.; Efros, A.L. Suppression of Auger-stimulated efficiency droop in nitride-based light emitting diodes. *Appl. Phys. Lett.* 2013, 102, 031120. [CrossRef]

36. *CrossLight®*. APSYS (Advanced Physical Models of Semiconductor Devices). Available online: http://www.crosslight.com/products/apsys/ (accessed on 1 June 2018).

37. Vurgaftman, I.; Meyer, J.; Ram-Mohan, L. Band parameters for III–V compound semiconductors and their alloys. *J. Appl. Phys.* 2001, 89, 5815–5875. [CrossRef]

38. Vurgaftman, I.; Meyer, J.N. Band parameters for nitrogen-containing semiconductors. *J. Appl. Phys.* 2003, 94, 3675–3696. [CrossRef]

39. Bernardini, F.; Fiorentini, V. Nonlinear macroscopic polarization in III-V nitride alloys. *Phys. Rev. B* 2001, 64, 085207. [CrossRef]

40. Bernardini, F.; Fiorentini, V. Nonlinear behavior of spontaneous and piezoelectric polarization in III–V nitride alloys. *Phys. Status Solidi A* 2002, 190, 65–73. [CrossRef]

41. Streiff, M.; Fichtner, W.; Witzig, A. Vertical-cavity surface-emitting lasers: Single-mode control and self-heating effects. In *Optoelectronic Devices*; Springer: New York, NY, USA, 2005; pp. 217–247.

42. Piprek, J. Chapter 7—Edge-Emitting Laser. In *Semiconductor Optoelectronic Devices*; Academic Press: Boston, MA, USA, 2003; pp. 151–169. [CrossRef]

43. Wei, S.H.; Zunger, A. Valence band splittings and band offsets of AlN, GaN, and InN. *Phys. Status Solidi A* 2013, 210, 1103–1106. [CrossRef]

44. Muhammad, U.; Nawaz, N.; Saba, K.; Karimov, K.; Muhammad, N. Electromechanical Fields and Their Influence on the Internal Quantum Efficiency of GaN-Based Light-Emitting Diodes. *Acta Mech. Solida Sin.* 2018, 31, 383–390. [CrossRef]

45. Kim, S.J.; Kim, T.G. Numerical study of enhanced performance in InGaN light-emitting diodes with graded-composition AlGaN/N barriers. *J. Opt. Soc. Korea* 2013, 17, 16–21. [CrossRef]

46. Chang, J.Y.; Kuo, Y.K. Advantages of blue in G a N light-emitting diodes with composition-graded barriers and electron-blocking layer. *Phys. Status Solidi A* 2013, 210, 1103–1106. [CrossRef]

47. Muhammad, U.; Nawaz, N.; Saba, K.; Karimov, K.; Muhammad, N. Experimental and numerical analysis of the indium-content on the internal electromechanical field in GaN-based light-emitting diodes. *Optik* 2018, 172, 1193–1198. [CrossRef]

48. Kang, J.; Li, H.; Li, Z.; Liu, Z.; Ma, P.; Yi, X.; Wang, G. Enhancing the performance of green GaN-based light-emitting diodes with graded superlattice AlGaN/GaN inserting layer. *Appl. Phys. Lett.* 2013, 103, 102104. [CrossRef]

49. Cao, X.; Leboeuf, S.; Rowland, L.; Liu, H. Temperature-dependent electroluminescence in InGaN/GaN multiple-quantum-well light-emitting diodes. *J. Electron. Mater.* 2003, 32, 316–321. [CrossRef]
51. Qi, Y.; Liang, H.; Wang, D.; Lu, Z.; Tang, W.; Lau, K.M. Comparison of blue and green in Ga N/Ga N multiple-quantum-well light-emitting diodes grown by metalorganic vapor phase epitaxy. *Appl. Phys. Lett.* 2005, 86, 101903. [CrossRef]

52. O’donnell, K.; Martin, R.; Middleton, P. Origin of luminescence from InGaN diodes. *Phys. Rev. Lett.* 1999, 82, 237. [CrossRef]

53. Park, I.-K.; Kwon, M.-K.; Kim, J.-O.; Seo, S.-B.; Kim, J.-Y.; Lim, J.-H.; Park, S.-J.; Kim, Y.-S. Green light-emitting diodes with self-assembled In-rich InGaN quantum dots. *Appl. Phys. Lett.* 2007, 91, 133105. [CrossRef]