Conjectures on the Khovanov Homology of Legendrian and Transversely Simple Knots

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Abstract

A theorem of Kronheimer and Mrowka states that Khovanov homology is able to detect the unknot [27]. That is, if a knot has the Khovanov homology of the unknot, then it is equivalent to it. Similar results hold for the trefoils [7] and the figure-eight knot [6]. These are the simplest of the Legendrian simple knots. We conjecture that Khovanov homology detects all Legendrian and Transversely simple knots. Using the torus and twist knots, numerical evidence is provided for all prime knots up to 19 crossings. This includes conjectured Legendrian simple knots from [5].

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1 Legendrian and Transverse Knots and Links

A knot is a smooth embedding of the circle $S^1$ into $\mathbb{R}^3$. A link is a smooth embedding of $N \in \mathbb{N}$ disjoint circles in $\mathbb{R}^3$. We may impose extra structure by considering the standard contact structure of $\mathbb{R}^3$. This is an assignment of a plane to every point in $\mathbb{R}^3$ such that there is no surface $M \subset \mathbb{R}^3$ (even infinitesimally) where for every $p \in M$ the tangent plane $T_pM$ is given by the plane of the contact structure. In $\mathbb{R}^3$ this is described by the one-form $dz - ydx$, the plane at $(x, y, z)$ being spanned by the vectors $\partial_x + y\partial_z$ and $\partial_y$. The hyperplane distribution is shown in Fig. 1. While there are no everywhere tangent surfaces, it is possible for a curve to be everywhere tangent to this distribution of planes. A Legendrian link is a link that is everywhere tangent to the contact structure. A gentle introduction can be found in [33].

Two Legendrian links are considered to be Legendrian equivalent if there is an isotopy $H : L \times [0, 1] \to \mathbb{R}^3$ between them, $L = \sqcup_{k=0}^{N-1} S^1$, such that for all $t \in [0, 1]$ the link $H_t$ is Legendrian. It is possible for two links to be topologically equivalent but not Legendrian equivalent. In the other direction, any link can be made Legendrian by an appropriate isotopy (See the introduction of [37]). The two classical invariants of Legendrian links are the Thurston-Bennequin and rotation numbers [16]. A link type is said to be Legendrian simple if any two Legendrian embeddings of it with the same Thurston-Bennequin and rotation numbers are Legendrian equivalent. That is, if the classical invariants uniquely classify all Legendrian representations of the knot. Certain knot types are known to be Legendrian simple, the first discovered being the unknot in 1998 by Eliashberg and Fraser [19]. Torus knots and the figure eight knot are also known to be Legendrian simple [20], and in [22] Etnyre, Ng, and Vertesi classify which twist knots are Legendrian simple. In particular, the $K_m$ twist knot is Legendrian simple if and only if $m \geq -3$. 

![Figure 1: Hyperplane Distribution for $dz - ydx$]
Not every knot is Legendrian simple, the mirror of the $K_3$ twist knot (also known as the $5_2$ knot, which is isotopic to the $K_{-4}$ twist knot) being an example discovered by Chekanov [14], and independently by Eliashberg [18]. More examples can be found in the Legendrian knot atlas [5].

Transverse links are links that are everywhere transverse to the contact structure. That is, at every point $p$ on the link the velocity vector and the contact hyperplane at $p$ span $\mathbb{R}^3$. Any Legendrian link can be made transverse by a small perturbation in the direction normal to the given plane in the contact structure. Two transverse links are transversely equivalent if there is an isotopy $H : L \times [0, 1] \rightarrow \mathbb{R}^3$ such that $H_t$ is a transverse link for all $t \in [0, 1]$. The Bennequin number of a transverse knot is defined by the algebraic crossing number $e(K)$ and the braid index $n(K)$. It is:

$$\beta(K) = e(K) - n(K) \quad (1)$$

It is not an invariant of topological knots, but is an invariant under transverse equivalence. This is essentially the self-linking number of the corresponding topological knot with respect to the natural framing coming from the trivialization of the contact structure. Similar to Legendrian simple, we define a knot (or link) type to be transversely simple if all of its transverse representations are uniquely determined by their Bennequin number (See [12]) and by whether its velocity vectors point into the half space where the contact structure is positive or not. A paper by Etnyre, Ng, and Vertesi [22] classifies when twist knots are transversely simple. In particular, infinitely many such knots are transversely simple\(^1\) giving us a family of knots to test conjectures with. It is also true that infinite families of non-Legendrian simple and non-transversely simple knots exist. See, for example, the works of Etnyre and Honda [21] and Birman and Menasco [11]. More examples can be found in [23].

There are several common ways of representing topological knots, the three used in our computations are extended Gauss code, planar diagram code (PD code), and Dowker-Thistlewaite code (DT code). Given a knot diagram with $N$ crossings, the (unsigned) Gauss code is a string with $2N$ characters (extended Gauss code is $3N$), PD code is a string that is $4N$ long, and DT code is $N$ characters long. Each has its benefits. Extended Gauss code can distinguish mirrors, DT code cannot (See [17]). PD code is perhaps the easiest to reconstruct the knot diagram, and DT code is the shortest. Because of this we will present our examples in DT code. To obtain the DT code of a knot diagram, place your finger on the knot and walk along the diagram, labelling the crossings. When you get back to your starting point each crossing will have two numbers associated with it. It is not difficult to see that each crossing will have exactly one odd number and one even number. For each even number, if that number was associated with an over crossing (that is, your finger ran over the crossing as you were labelling it), place a minus sign in front. Write out the pairs of integers as

\(^1\)The twist knot $K_m$ is transversely simple for $m \geq -2$ or $m$ odd [22].
(1, a₁), (3, a₂), ..., (2n − 1, aₙ). The DT code is the list a₁, a₂, ..., aₙ. See [1] for several examples.

It is possible to go from DT code to unsigned Gauss code (i.e. the usual Gauss code, and not the extended Gauss code) and back [1]. For certain computations, like the Alexander polynomial which is mirror insensitive, DT code is easiest since it is the shortest. For things like the Jones polynomial and Khovanov homology, invariants that distinguish mirrors, extended Gauss code is a must.

A note on notation. The tables on the following pages identify knots using their DT codes. The Regina library [13] uses a clever trick to efficiently express a knot’s DT code for all knots of up to 26 crossings, and we’ll adopt this notation for the remainder of the paper. The DT code of an n crossing knot is a string of length n of signed even integers up to 2n. Since all entries are even, we can divide by two and obtain a string of length n of signed integers up to n. So long as n is not larger than 26 we can replace integers with letters. We then use lower case letters for positive integers, and upper case for negative. For example, the trefoil T(2, 3) becomes:

4, 6, 2 ⇒ 2, 3, 1 ⇒ bca

The knot 10₁₃₂, which is the 10 crossing knot in Tab. 2 with the same Jones polynomial as T(2, 5), becomes:

8, 6, 18, 2, −12, −16, 20, −10, 4, 14 ⇒ 4, 3, 9, 1, −6, −8, 10, −5, 2, 7
⇒ dciaFHjEbg

2 Khovanov Homology

The Khovanov homology of a link is a powerful, if computationally expensive, invariant first introduced by Mikhail Khovanov [26] (See also [9] for an excellent introduction). It is closely related to the Jones polynomial, but able to distinguish many more knots and links. The homology groups KHₗ(L) of a link (or knot) L are the direct sum of homogeneous components KHᵣₗ(L) and the Khovanov Polynomial (See [4]) is given by:

\[ Kh(L)(q,t) = \sum_{r,t} t^rq^d\dim(KH^r_\ell(L)) \] (5)

The Jones polynomial of L is recovered via:

\[ J(L)(q) = Kh(L)(q,-1) \] (6)

---

2 The naïve algorithm is exponential in the number of crossings. Improvements by Bar-Natan [10] have sped up computations but no polynomial-time algorithm is known at the time of this writing.
Khovanov homology is not a perfect invariant [38]. That is, there are distinct knots with the same Khovanov homology, but it is a powerful invariant and is capable of detecting certain knot types.

**Theorem 1** (Kronheimer and Mrowka, 2011). *If a knot $K$ has the same Khovanov homology as the unknot, then $K$ is equivalent to the unknot.*

The unknotting problem asks one to determine if a given knot diagram is equivalent to the unknot. Khovanov homology is a powerful enough tool to accomplish this task. The Khovanov polynomial is a generalization of the Jones polynomial and it has been conjectured that if a knot has the same Jones polynomial as the unknot, then that knot is equivalent to the unknot. At the time of this writing it has not been proven, but there is evidence for and against the claim. Thistlewaite found links with the same Jones polynomial as the unlink [35], and there is a 3-crossing virtual knot that has the same Jones polynomial as the unknot. For the claim, all knots of up to 24 crossings are either the unknot, or have a Jones polynomial different from the unknot [36].

In recent years it has been discovered that Khovanov homology can detect a few other knot types. In particular, both the left and right handed trefoils, as well as the figure eight.

**Theorem 2** (Baldwin and Sivek, 2022). *If a knot $K$ has the same Khovanov homology as either of the trefoils, then $K$ is equivalent to one of them.*

**Theorem 3** (Baldwin, Dowlin, Levine, Lidman, and Sazdanovic, 2021). *If a knot $K$ has the same Khovanov homology as the figure-eight knot, then $K$ is equivalent to it.*

See [7] and [6], respectively.

## 3 Conjectures on Khovanov Homology

Khovanov homology is capable of detecting the unknot, trefoils, and figure-eight knot. The Khovanov homology with coefficients in $\mathbb{Z}/2\mathbb{Z}$ is also capable of detecting the cinquefoil knot [8], which is the $T(5, 2)$ torus knot. The Jones polynomial, on the other hand, is not capable of detecting the $T(5, 2)$ torus knot since the $10_{132}$ knot yields the same polynomial (See Tab. 1). These are the easiest of the Legendrian simple knots leading us to the following.

**Conjecture 1.** *If a link type $L$ is Legendrian simple, then the Khovanov homology of $L$ distinguishes it. That is, if $\tilde{L}$ is another link with the same Khovanov homology, then $\tilde{L}$ is equivalent to $L$.*

Numerical evidence has been tallied for torus knots, which are Legendrian simple [20], against all prime knots up to 19 crossings, a total of 352,152,252 knots. There are four non-torus knots that have the same Jones polynomial as a torus knot ($T(2, 5)$ matches 10 and 17 crossing knots, $T(2, 7)$ matches a 12 crossing
knot, and \( T(2,11) \) matches a 14 crossing knot) so we cannot generalize the Jones unknot conjecture. Nevertheless, in all cases the Khovanov homologies were different (see Numerical Results). We also performed the computation with twist knots, which are Legendrian simple for \( m \geq -3 \) \cite{22}, and for the 26 conjecturally Legendrian simple knots from the Legendrian knot atlas \cite{5}.

We explore each of these three cases later in the Numerical Results sections.

The computations were done as follows. There are libraries in Python, Sage, and C++ for working with knot polynomials. In particular, we used Regina \cite{13}, SnapPy \cite{15}, the Sage knot library \cite{34}, and our own ever-growing C library \cite{31}.

We first gather the torus knots that could potentially have the same Jones polynomial as some other knot based on the bounds of the degree. The Jones polynomials of torus knots were computed using the formula \cite{25}:

\[
J(T(m,n))(q) = q^{(m-1)(n-1)/2} \frac{1 - q^{m+1} - q^{n+1} + q^{m+n}}{1 - q^2}
\]

(7)

The Kauffman bracket, of which the Jones polynomial is a simple normalization \cite{9}, of a knot \( K \) with \( N \) crossings has the following formula:

\[
\langle K \rangle = \sum_{n=0}^{2^N - 1} (-q)^{w(n)}(q + q^{-1})^{c(n)}
\]

(8)

where \( w(n) \) is the Hamming weight of \( n \), the number of 1’s in the binary representation, and \( c(n) \) is the number of disjoint cycles that result in the complete smoothing of \( K \) corresponding to \( 0 \leq n \leq 2^N - 1 \). With this the degree of the bracket polynomial is bounded by \( 2N + 1 \) \((w(n) \) is bounded by \( N \), and \( c(n) \) is bounded by \( N + 1 \)). The degree of the normalization is bounded by \( 2N \) \cite{9}, so the Jones polynomial of a knot with \( N \) crossings has degree at most \( 4N + 1 \).

Since we are looking at knots up to 19 crossings, we collect the co-prime pairs \((m, n)\) with \( 1 < m < n \) such that the degree is less than \( 4 \cdot 19 + 1 = 77 \). Since the Jones polynomial of a mirror can be computed by substituting \( q \mapsto q^{-1} \) \cite{24}, we need not look at negative values. The Khovanov polynomial makes a similar change, \( (q, t) \mapsto (q^{-1}, t^{-1}) \) \cite{39}.

Using any of the aforementioned libraries, the Jones polynomials of all prime knots up to 19 crossings were computed and compared against this table of torus knot Jones polynomials (Eqn. 7). If a match was found the regina library was used to determine if the knots were actually identical. That is, if the knot whose Jones polynomial was being compared against the torus knots was indeed

\[\text{There are several ways to count distinct knots since a knot, its mirror, its inverse, and its inverse mirror can all be distinct. We do not distinguish a knot from its inverse in our counting, but do treat a knot and its mirror with more care.}\]
a torus knot itself. If the knots were distinct, this knot was saved in a text file for later examination. At the end of the computation 4 non-torus knots had the same Jones polynomial as a torus knot (See Tab. 1). Since the Khovanov polynomial contains the Jones polynomial in it (recall \( J(L)(q) = Kh(L)(q, -1) \)) the only possible non-torus knots with the same Khovanov homology as a torus knot were these 4.

Using the Java library JavaKh\(^4\) we found that these four knots with the same Jones polynomials as some torus knot all had different Khovanov homologies. Thus, we have the following claim:

**Theorem 4.** If a prime knot \( K \) has less than or equal to 19 crossings and has the Khovanov homology of a torus knot \( T \), then \( K \) is equivalent to \( T \).

A similar search through the twist knots yielded more results. The Jones polynomials of the twist knots are known, with the formula:

\[
J(K_n)(q) = \begin{cases} 
(1 + q^{-2} + q^{-n} + q^{-n-3})/(1 + q), & n \text{ odd} \\
(1 + q - q^{3-n} + q^{-n})/(1 + q), & n \text{ even}
\end{cases}
\]

A search through all prime knots up to 19 crossings against twist knots provided eleven more matches for the Jones polynomial (See Tab. 2), but none for Khovanov homology. Infinitely many of the twists knots are transversely simple, making them a good candidate to test the following conjecture on.

**Conjecture 2.** If a link type \( L \) is transversely simple, then the Khovanov homology of \( L \) distinguish it. That is, if \( \tilde{L} \) is another link with the same Khovanov homology, then \( \tilde{L} \) is equivalent to \( L \).

We also performed our computations on the conjecturally transverse simple knots in [5]. Like the twist knots, no matches were found for the Khovanov polynomial.

A similar search for Knot Floer Homology (KFH), a homology theory first introduced by Peter Ozsváth and Zoltán Szabó [32], using the Alexander polynomial was performed, but a bug was found that caused knots with identical Alexander polynomials to give false negatives. This has been corrected, and when performing a new search there are several distinct knots with the same Knot Floer Homology as a Legendrian simple knot. Steven Sivek pointed out that the pretzel knots \( P(-3, 3, 2n + 1) \) all have the same KFH, meaning the \( 6_1 \) twist knot matches the KFH of the \( 9_{46} \) knot in the Rolfsen table. This is what first hinted at a bug in our KFH code. Matthew Hedden also informed us that the \( T(4, 3) \) knot and the \( (2, 3) \) cable of the trefoil also have matching KFH.

\(^4\)Thanks must be paid to Nikolay Pultsin who made edits to JavaKh-v2 so that it may run on a GNU/Linux machine using OpenJDK 17.
4 Numerical Results: Torus Knots

| Torus Knot | Non-Torus Knot | Jones Polynomial |
|------------|----------------|-----------------|
| T(2, 5)    | dciaFHjEbg     | $-q^{13} + q^{12} - 2q^6 - q^4 + q^3$ |
| T(2, 7)    | fJGkHICEABd    | $-q^{20} + q^{18} - 2q^{10} + q^8 - q^4 + q^2$ |
| T(2, 11)   | gHImJnKBDFAce  | $-q^{13} + q^{12} - 2q^6 - q^4 + q^3$ |
| T(2, 5)    | iNHFJqCOKpmABgE | $-q^{13} + q^{12} - 2q^6 - q^4 + q^3$ |

Table 1: Knots whose Jones polynomial matches that of a Torus Knot

In Tab. 1 we have found four non-torus knots with the same Jones polynomial as some torus knot. In all four cases the Khovanov polynomials differ. The tables for these polynomials are given in Appendix A. The coefficient of $q^i t^j$ is given by the corresponding entry. Empty equates to zero.

The cinquefoil $T(2, 5)$ has the same Jones polynomial as two non-torus knots. As shown in Tabs. 7-9, the Khovanov polynomials differ (This had already been known, see [4]). Note the columns for the Euler characteristic $\chi$ are identical, indicating matching Jones polynomials.

The $T(7, 2)$ knot, also the 7_1 knot, and occasionally called the septafoil, has the same Jones polynomial as $fJGkHICEABd$. The Khovanov polynomials are distinct (Tabs. 10-11). Lastly, the $T(11, 2)$ torus knot has the same Jones polynomial as the 14 crossing knot $gHImJnKBDFAce$. Once again the Khovanov polynomials differ.

5 Numerical Results: Twist Knots

| Twist Knot | Non-Twist Knot | Jones Polynomial |
|------------|----------------|-----------------|
| K_2        | eJGkHjFjEbg    | $q^3 - q^2 + 1 - q^2 + q^3$ |
| K_3        | dgjEjHakbJe     | $-q^{13} + q^{12} - 2q^6 + q^4 - q^2$ |
| K_5        | ghKHIaLHBDc     | $-q^{13} + q^{12} - 2q^6 + q^4 - q^2$ |
| K_7        | hJeHjICaERfi    | $-q^{13} + q^{12} - 2q^6 + q^4 - q^2$ |
| K_9        | eKjJjGjJase      | $-q^{13} + q^{12} - 2q^6 + q^4 - q^2$ |
| K_6        | ceJjgbaJbOh      | $q^3 - q^2 + 2q^6 - 2q^4 - q^2$ |
| K_1        | feNbaJfLOHbd     | $q^3 - q^2 + 2q^6 - 2q^4 - q^2$ |
| K_2        | gplFjHkqicjJfbIch | $q^3 - q^2 + 2q^6 - 2q^4 - q^2$ |
| K_7        | cgjFItaJebk      | $-q^3 - q^2 + 2q^6 - 2q^4 - q^2$ |
| K_8        | nEiBjGjJGjGjWajbJgF | $q^3 - q^2 + 2q^6 - 2q^4 - q^2$ |
| K_9        | gpfHmIrjHkEhrch   | $q^3 - q^2 + 2q^6 - 2q^4 - q^2$ |

Table 2: Knots whose Jones polynomial matches that of a Twist Knot

The twist knots have a few more matching non-twists knots for the Jones polynomial (Tab. 2). All tables are provided in Appendix B. The first match is the figure-eight knot, $K_2$, which has the same Jones polynomial as K11n19 from the Hoste-Thistlewaite table. Again, this had already been known (See [2] and [3]). The Khovanov polynomials are given in Tabs. 14-15.
The $K_{-4}$ twist knot, which is 5$_2$ on the Rolfsen table, has the same Jones polynomial as (at least) three other knots. In each case the Khovanov polynomial distinguishes it (Tabs. 16-19). The $K_5$ twist knot, which is 7$_2$ on the Rolfsen table, has the same Jones polynomial as $\text{bhdijckae}$f. The Khovanov polynomials are given in Tabs. 20 and 21.

Our original experiment concluded at 17 crossings. We have expanded this to 19, and in doing so we have added a few new results, the first of which comes with $K_6$. An 18 crossing knot was found that has the same Jones polynomial, but the Khovanov polynomials differ. This is shown in Tabs. 22 and 25. $K_6$ also shares its Jones polynomial with an 11 and a 13 crossing knot. The Khovanov polynomials are given in Tabs. 23 and 24. Something to note is that the the Khovanov polynomials of $\text{cefgbajkd}$ and $\text{femijaJLCGhd}$ are identical, indicating that the Khovanov polynomial is not a perfect invariant.

Lastly, the twist knots $K_7$, $K_8$, and $K_9$ have the same Jones polynomial of at least one other knot. As before, the Khovanov polynomials differ in each case. We may now make the following claim.

**Theorem 5.** If a prime knot $K$ has less than or equal to 19 crossings and has the Khovanov homology of a twist knot $T$, then $K$ is equivalent to $T$.

An interesting thing to note is that not all twist knots are transversely, or Legendrian, simple, yet there are no knots up to 19 crossings with the same Khovanov polynomial as a twist knot, other than twist knots themselves. We ask the following.

**Question 1.** Does Khovanov homology distinguish all twist knots?

### 6 Numerical Results: Conjectured Legendrian Simple Knots

Ng presents several knots in [5] that are possibly Legendrian simple but not confirmed. For completeness we used our scripts on these knots as well and found several knots with identical Jones polynomials, but again all had differing Khovanov polynomials. These are listed in Tab. 3 ($m$ indicates the knots mirror). Note this table also includes knots that are conjectured to be transversely simple. The tables for the Khovanov polynomials are given in Appendix C.

### 7 Jones and Khovanov Polynomial Databases

Using the regina and JavaKh libraries, the Jones polynomials of all knots with less than or equal to 19 crossings were tabulated, and the Khovanov polynomials of all knots up to 17 crossings were computed.$^5$ This allows us to measure how

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$^5$This data is publicly available. See [28] and [29].
common it is for a knot to have a unique Jones or Khovanov polynomial when compared with other low crossing knots. Unsurprisingly, the Jones polynomial does not distinguish nearly as many knots as the Khovanov polynomial. The statistics for these knot invariants are shown in Tabs. 5 and 6, the key for the tables is given in Tab. 4.

Computational limits (temporarily) prohibit us from expanding Tab. 6 to 19 crossings, but this is being worked on. After examining the data one sees that the Khovanov polynomial distinguishes about 32% of knots with up to 17 crossings. Put it another way, about 7 out of 10 knots share their Khovanov polynomial with another knot in the list.

We can use this data to play around with the statistics and ask whether or not the conjectures we’ve made in the previous sections are sound. That is, we can collaborate between the second author and Peter Doyle.  

The smaller the probability of a knot having a unique Khovanov polynomial among all other prime knots up to N crossings, the more likely our findings are statistically significant. We note that the probability of having a unique Jones or Khovanov polynomial appears to be monotonically decreasing with crossing number. Since we do not have the Khovanov data for 18 or 19 crossings, we can use the value for 17 crossings. If the trend continues and the probability for N = 19 is smaller, that will only make our heuristic argument stronger. To avoid mixing the statistics, we also use the 17 crossing probability for the Jones polynomial. Here we know that 19 crossings produces a smaller probability since this data is available to us.

Table 3: Conjectured Legendrian Simple Knots

| Ng Knot | Matching Knot | Jones Polynomial |
|---------|---------------|------------------|
| m(6j1)  | glfjCcbMkhDdLle | \(q^{16} - 2q^6 + 2q^4 - 2q^2 + 2q^{-4} - 1 + q^{-6}\) |
| m(6j2)  | hkmGeDhLlaIf | \(q^{16} - 2q^6 + 2q^4 - 2q^2 + 2q^{-4} - 1 + q^{-6}\) |
| m(6j3)  | gKHmldJCBaHf | \(q^{16} - 2q^6 + 2q^4 - 2q^2 + 2q^{-4} - 1 + q^{-6}\) |
| m(6j4)  | hekJmGilbd | \(q^{16} - 2q^6 + 2q^4 - 2q^2 + 2q^{-4} - 1 + q^{-6}\) |
| m(7j1)  | hgelmibaJFcd | \(-q^{18} + q^{16} - 2q^{14} + 3q^{12} - 2q^{10} + 2q^{8} - q^{6} + q^{4}\) |
| m(7j4)  | gKHbJjDae | \(-q^{16} + q^{14} - 2q^{12} + 3q^{10} - 2q^{8} + 3q^{6} - 2q^{4} + q^{2}\) |
| m(9j18) | gnxqKDJImnpEaHblfc | \(q^2 - 3 + 4q^{-2} - 4q^{-4} + 6q^{-6} - 4q^{-8} - 3q^{-10} - 2q^{-12}\) |
| m(9j49) | 1PKjAEnDcpBhmg | \(q^{-4} - 2q^{-6} + q^{-5} - 4q^{-3} - 4q^{-4} - 3q^{-5} - 3q^{-6} - 2q^{-8}\) |
| m(10j2a) | eRPnGj1BFoiaDckW | \(-q^{10} + q^{9} - 2q^{8} + q^{7} - q^{6} + q^{5} - q^{4} - q^{3} + q^{2} - q + 1\) |
| m(10j2b) | edjGkG1FbCh | \(-q^{10} + q^{9} - 2q^{8} + q^{7} - q^{6} + q^{5} - q^{4} - q^{3} + q^{2} - q + 1\) |
| m(10j10a) | igDRmJasEaFcc | \(-q^{10} + 2q^{9} - 2q^{8} + 3q^{-6} - 2q^{-7} + 2q^{-8} - q^{-9}\) |
| l0j15 | eoaKgJGmupD1baL | \(-q^{10} + q^{9} - q^{8} - q^{7} + q^{6} + q^{5}\) |
| l0j16 | kWhjKHPFEOOngAd | \(-q^{10} + q^{9} - q^{8} + q^{7} + q^{6}\) |
| l0j16 | b0qgrjIammeipFagkbcd | \(-q^{10} + q^{9} - q^{8} + q^{7} + q^{6}\) |  

\(^{6}\)The Khovanov polynomial throws away the torsion component of Khovanov homology. It would be interesting to compare how much stronger the full Khovanov homology is.
A first approach is to say that the probability of a prime knot having a unique Khovanov polynomial among all other prime knots with up to 17 crossings is 0.32 (Tab. 6). Ignoring mirrors, there are 16 twist knots and 14 torus knots with less than or equal to 17 crossings. For twist knots these are the knots $K_m$ for $m = 0, 1, \cdots, 15$, and for torus knots these are given by the co-prime pairs $(2, 3), (2, 5), (2, 7), (2, 9), (2, 11), (2, 13), (2, 15), (2, 17), (3, 4), (3, 5), (3, 7), (3, 8), (4, 5)$, and the unknot. The (naïve) probability of all of these knots having unique Khovanov polynomials is \( \frac{32}{16+14-1} \approx 4.5 \times 10^{-15} \), where the \(-1\) in the exponent comes from double counting the unknot, once as a twist knot and once as a torus knot. For twist knots alone this is \( \frac{32}{16} \approx 1.2 \times 10^{-8} \) and for torus knots this is \( \frac{32}{14} \approx 1.2 \times 10^{-7} \).

These are ridiculously small probabilities, which is good for our conjecture, but this assumes these knots are more-or-less random. Let us use their Jones polynomials to demonstrate that they more-or-less are. The Jones polynomial has a 0.26 probability of distinguishing a knot in our list of prime knots up to 17 crossings. Since having a unique Jones polynomial would imply having a unique Khovanov polynomial (recall $Kh_K(q, -1) = J_K(q)$), the set of knots that are distinguished by their Jones polynomial is a subset of the set of knots distinguished by the Khovanov polynomial. If we label $KH$ as the event that a knot is determined by its Khovanov polynomial (when considered among prime knots with 17 crossings or less), and similarly label $J$ for the Jones polynomial, the conditional probability is:

$$P(\text{not } J \mid KH) = \frac{P(KH) - P(J)}{P(KH)} \approx 0.21 \quad (10)$$

So with a 0.21 probability a knot that is known to be distinguished by its Khovanov polynomial will not be distinguished by its Jones polynomial. We have shown by brute force methods that the torus and twist knots are part of the Khovanov-distinguished family for prime knots up to 17 crossings, so we can apply this probability. There were 4 torus knots that are not distinguished by their Jones polynomial and 9 such twist knots.\(^7\) When accounting for mirrors, this gives a 0.15 probability of a torus knot being distinguished by the Khovanov polynomial and not the Jones polynomial, and a 0.3 probability for twist knots. Roughly, this means torus knots are more likely to be distinguished by the Jones polynomial than the average Khovanov-distinguished knot, and the twist knots are less likely. The total probability for the union of these two families is 0.23, which is close to the general probability of 0.21. It is then fair to say that these families are behaving randomly, as far as their Jones detection rates are concerned.

We can also look at the conditional probability of finding a knot that is distinguished by its Khovanov polynomial given that we know it is not distinguished

---

\(^7\)Tab. 2 has 11 such twist knots, but two of these matches are for knots with more than 17 crossings. To avoid using the wrong probabilities with this data, we stick to 17 crossings.
by its Jones polynomial. First, a quick review of probability. Given events $A, B \subseteq X$, for some probability space $X$, if $P(B)$ is non-zero, then:

$$P(B) = P((A \cap B) \cup (A^C \cap B))$$

$$= P(A \cap B) + P(A^C \cap B)$$

$$\Rightarrow 1 = \frac{P(A \cap B)}{P(B)} + \frac{P(A^C \cap B)}{P(B)}$$

$$= P(A | B) + P(A^C | B)$$

where $A^C = X \setminus A$, the complement of $A$ in $X$, and where $P(A | B)$ denotes the conditional probability. Note that $P(A^C)$ is the same thing as $P(\text{not } A)$. Rearranging we have:

$$P(A | B) = 1 - P(A^C | B)$$

a rather standard and intuitive result, we will now use this in our following argument. We seek the probability $P(\text{KH} | \text{not } J)$. That is, the probability a knot will be distinguished by its Khovanov polynomial given that we know it is not distinguished by its Jones polynomial. Combining our previous derivation with Bayes' theorem, we obtain:

$$P(\text{KH} | \text{not } J) = \frac{P(\text{not } J) P(\text{KH})}{P(\text{not } J) P(\text{KH})}$$

$$= \frac{(1 - P(J | \text{KH})) P(\text{KH})}{P(\text{not } J)}$$

$$= \frac{(1 - \frac{P(\text{KH} | J) P(J)}{P(\text{KH})}) P(\text{KH})}{P(\text{not } J)}$$

$$= \frac{(1 - \frac{P(J)}{P(\text{KH})}) P(\text{KH})}{P(\text{not } J)}$$

$$= \frac{P(\text{KH}) - P(J)}{P(\text{not } J)}$$

$$= 0.092$$

We have used the fact that $P(\text{KH} | J) = 1$ to obtain our result. To be on the conservative side we can round this up (smaller probabilities make our conjecture stronger) to 10%. So with roughly a 10% probability a knot that is not Jones-distinguished will still be distinguished by its Khovanov polynomial. We found 11 twist knots and 4 torus knots that are not Jones-distinguished, but are still Khovanov-distinguished. Hence for the torus knots we may choose a $p$ value of 0.001, and for the twist knots we can go much lower, indicating that it is likely a statistically significant result.
Now one may say that most of the prime knots with 17 or less crossings happen to have 17 crossings, whereas most of the twist knots we’ve examined do not. A similar claim can be made for the torus knots. We can reformulate our probability argument as follows to account for this. Let \( p(k) \) be the fraction of prime knots of up to \( k \) crossings that are distinguished by their Khovanov polynomial. Let \( Cr(K) \) denote the crossing number of \( K \). The product \( \prod_m p(Cr(K_m)) \) over all twist knots with up to 17 crossings then serves as a better probability that all should be Khovanov-distinguished. Using our tables this number is \( 2.8 \times 10^{-5} \), which is still very small. For the torus knots we obtain \( 8.4 \times 10^{-6} \), a tiny probability.

Lastly, one may argue that several of the torus and twist knots are known to be Khovanov-distinguished among all possible knots, like the unknot, and so should not be included in these probabilities. Our previous argument handles this since these knots have crossing number no greater than 5, and \( p(k) = 1 \) for \( k \leq 9 \).

The premise behind these arguments, that knots are random, is a point of contention. Nevertheless the above argument is not entirely meaningless and the authors believe it does motivate further study of these conjectures.

8 Acknowledgements

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| Cr | Unique | Almost | Total | Knots | FracU | FracT | FracK |
|----|--------|--------|-------|-------|-------|-------|-------|
| 03 | 1      | 0      | 1     | 1     | 1.000000 | 1.000000 | 1.000000 |
| 04 | 2      | 0      | 2     | 2     | 1.000000 | 1.000000 | 1.000000 |
| 05 | 4      | 0      | 4     | 4     | 1.000000 | 1.000000 | 1.000000 |
| 06 | 7      | 0      | 7     | 7     | 1.000000 | 1.000000 | 1.000000 |
| 07 | 14     | 0      | 14    | 14    | 1.000000 | 1.000000 | 1.000000 |
| 08 | 35     | 0      | 35    | 35    | 1.000000 | 1.000000 | 1.000000 |
| 09 | 84     | 0      | 84    | 84    | 1.000000 | 1.000000 | 1.000000 |
| 10 | 223    | 13     | 236   | 249   | 0.94915  | 0.947791 | 0.895582 |
| 11 | 626    | 77     | 710   | 801   | 0.881690 | 0.886392 | 0.781523 |
| 12 | 1981   | 345    | 2420  | 2977  | 0.818595 | 0.812899 | 0.665435 |
| 13 | 6855   | 1695   | 9287  | 12965 | 0.738129 | 0.716313 | 0.528731 |
| 14 | 25271  | 7439   | 37578 | 59937 | 0.672495 | 0.626958 | 0.421626 |
| 15 | 105246 | 3571   | 170363| 313230| 0.617775 | 0.543891 | 0.336002 |
| 16 | 487774 | 173677 | 829284| 1701935| 0.588187 | 0.487260 | 0.286600 |
| 17 | 2498968| 89450  | 4342890| 9755328| 0.575416 | 0.445181 | 0.256164 |
| 18 | 13817237| 4863074| 24116048| 58021794| 0.572948 | 0.415638 | 0.238139 |
| 19 | 82712788| 27409120| 141439472| 352152252| 0.584793 | 0.401643 | 0.234878 |

Table 5: Statistics for the Jones Polynomial

| Cr | Unique | Almost | Total | Knots | FracU | FracT | FracK |
|----|--------|--------|-------|-------|-------|-------|-------|
| 03 | 1      | 0      | 1     | 1     | 1.000000 | 1.000000 | 1.000000 |
| 04 | 2      | 0      | 2     | 2     | 1.000000 | 1.000000 | 1.000000 |
| 05 | 4      | 0      | 4     | 4     | 1.000000 | 1.000000 | 1.000000 |
| 06 | 7      | 0      | 7     | 7     | 1.000000 | 1.000000 | 1.000000 |
| 07 | 14     | 0      | 14    | 14    | 1.000000 | 1.000000 | 1.000000 |
| 08 | 35     | 0      | 35    | 35    | 1.000000 | 1.000000 | 1.000000 |
| 09 | 84     | 0      | 84    | 84    | 1.000000 | 1.000000 | 1.000000 |
| 10 | 223    | 12     | 237   | 249   | 0.949367 | 0.951807 | 0.903614 |
| 11 | 641    | 71     | 718   | 801   | 0.892758 | 0.896380 | 0.800250 |
| 12 | 2051   | 326    | 2462  | 2977  | 0.833063 | 0.827007 | 0.68949 |
| 13 | 7223   | 1636   | 9539  | 12965 | 0.757207 | 0.735750 | 0.557115 |
| 14 | 27317  | 7441   | 39222 | 59937 | 0.696471 | 0.654387 | 0.455762 |
| 15 | 118534 | 36867  | 182598| 313230| 0.649153 | 0.582952 | 0.378425 |
| 16 | 578928 | 187639 | 919835| 1701935| 0.629382 | 0.540464 | 0.340159 |
| 17 | 3167028| 1001101| 5033403| 9755328| 0.629202 | 0.515965 | 0.324646 |

Table 6: Statistics for the Khovanov Polynomial
database freely to the public. The algorithm that was devised to compute the Jones polynomial efficiently (the software is publicly available, see [31]) was made possible by contributions from Peter Doyle, and we also thank him for guiding us in our statistical argument. We thank Steven Sivek and Matthew Heddon for alarming us about the bug that existed in our KFH code.
9 Appendix A: Torus Knot Data

| \( q/t \) | -5 | -4 | -3 | -2 | -1 | 0 | \( \chi \) |
| --- | --- | --- | --- | --- | --- | --- | --- |
| -15 | 1 |   |   |   |   |   | -1 |
| -13 |   |   |   |   |   |   |   |
| -11 | 1 | 1 |   |   |   |   |   |
| -9  |   |   |   |   |   |   |   |
| -7  |   |   |   |   | 1 | 1 |   |
| -5  |   |   |   |   | 1 | 1 |   |
| -3  |   |   |   |   | 1 | 1 |   |

Table 7: Khovanov Polynomial for \( T(5, 2) \)

| \( q/t \) | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | \( \chi \) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| -15 | 1   |   |   |   |   |   |   |   | -1 |
| -13 |   |   |   |   |   |   |   |   |   |
| -11 | 1 | 1 |   |   |   |   |   |   |   |
| -9  |   |   |   |   | 1 | 1 |   |   |   |
| -7  |   |   |   |   | 1 |   | 1 |   |   |
| -5  |   |   |   | 1 | 2 |   |   | 1 |   |
| -3  |   |   |   | 1 |   | 1 |   |   |   |
| -1  |   |   |   | 1 |   | 1 |   |   |   |
| 1   |   |   |   |   |   |   | 1 | 1 |   |

Table 8: Khovanov Polynomial for \( dciaFHjEbg \)

| \( q/t \) | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | \( \chi \) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| -15 | 1   |   |   |   |   |   |   |   |   |   | -1 |
| -13 |   |   |   |   |   |   |   |   |   |   |   |
| -11 | 1 | 1 |   |   |   |   |   |   |   |   |   |
| -9  |   |   |   |   | 1 | 1 |   |   |   |   |   |
| -7  |   |   |   |   | 1 |   | 1 |   |   |   |   |
| -5  |   |   |   | 1 | 2 |   |   | 1 |   |   |   |
| -3  |   |   |   | 1 |   | 1 |   |   |   |   |   |
| -1  |   |   |   | 1 |   | 1 |   |   |   |   |   |
| 1   |   |   |   |   |   |   | 1 | 1 |   |   |   |

Table 9: Khovanov Polynomial for \( iNHlPJqCoKFmdABgE \)
Table 10: Khovanov Polynomial for $T(7, 2)$

| $q \backslash t$ | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | $\chi$ |
|-----------------|----|----|----|----|----|----|----|---|-------|
| -21             | 1  |    |    |    |    |    |    |   | -1    |
| -19             |    |    |    |    |    |    |    |   |       |
| -17             |    | 1  | 1  |    |    |    |    |   |       |
| -15             |    |    |    | 1  |    | 1  |    |   |       |
| -13             |    |    |    |    | 1  |    | 1  |   |       |
| -11             |    |    |    |    |    | 1  |    | 1 |       |
| -9              |    | 1  |    |    |    |    | 2  | 1 |       |
| -7              |    |    |    |    |    |    |    | 1 |       |
| -5              | 1  |    |    |    |    |    |    |   |       |

Table 11: Khovanov Polynomial for $f_{\text{JGkHlICEABd}}$

| $q \backslash t$ | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | $\chi$ |
|-----------------|----|----|----|----|----|----|----|----|----|---|-------|
| -21             | 1  |    |    |    |    |    |    |    |    |   | -1    |
| -19             |    |    |    |    |    |    |    |    |    |   |       |
| -17             |    | 1  | 1  |    |    |    |    |    |    |   |       |
| -15             |    |    |    | 1  |    | 1  |    |    |    |   |       |
| -13             |    |    |    |    | 1  |    | 1  |    |    |   |       |
| -11             |    |    |    |    |    | 1  |    | 2 |    | 1 |       |
| -9              |    |    |    |    |    |    |    | 1 |    |   |       |
| -7              |    |    |    |    |    |    |    |    | 1  | 1 |       |
| -5              |    |    |    |    |    |    |    |    |    | 1 |       |
| -3              |    |    |    |    |    |    |    |    |    | 1 |       |

Table 12: Khovanov Polynomial for $T(11, 2)$

| $q \backslash t$ | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | $\chi$ |
|-----------------|----|----|----|----|----|----|----|----|----|-----|----|-----|-------|
| 9               | 1  |    |    |    |    |    |    |    |    |     | 1  |     |       |
| 11              | 1  |    |    |    |    |    |    |    |    |     | 1  |     |       |
| 13              |    | 1  |    |    |    |    |    |    |    |     | 1  |     |       |
| 15              |    |    |    |    |    |    |    |    |    |     | 1  |     |       |
| 17              |    |    |    |    |    |    | 1  |    | 1  |     |    |    |       |
| 19              |    |    |    |    |    |    |    |    |    |     |    |    |       |
| 21              |    |    |    |    |    |    | 1  |    | 1  |     |    |    |       |
| 23              |    |    |    |    |    |    |    |    |    |     |    |    |       |
| 25              |    |    |    |    |    |    |    |    |    |     | 1  |    |       |
| 27              |    |    |    |    |    |    |    |    |    |     |    | 1  |       |
| 29              |    |    |    |    |    |    |    |    |    |     |    |    |       |
| 31              |    |    |    |    |    |    |    |    |    |     |    |    |       |
| 33              |    |    |    |    |    |    |    |    |    |     |    |    |       |
| $q^t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | $\chi$ |
|-------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|------|
| 9     | 1 |   |   |   |   |   |   |   |   |   |    |    |    |    | 1    |
| 11    | 1 |   |   |   |   |   |   |   |   |   |    |    |    |    | 1    |
| 13    | 1 |   |   |   |   |   |   |   |   |   |    |    |    |    | 1    |
| 15    |   | 1 | 1 |   |   |   |   |   |   |   |    |    |    |    |      |
| 17    |   | 1 | 1 |   |   |   |   |   |   |   |    |    |    |    |      |
| 19    |   |   | 1 | 2 | 1 |   |   |   |   |   |    |    |    |    |      |
| 21    |   |   |   | 1 | 1 |   |   |   |   |   |    |    |    |    |      |
| 23    |   |   |   |   | 1 | 2 | 1 |   |   |   |    |    |    |    |      |
| 25    |   |   |   |   |   | 1 | 1 |   |   |   |    |    |    |    |      |
| 27    |   |   |   |   |   |   | 1 | 1 |   |   |    |    |    |    |      |
| 29    |   |   |   |   |   |   |   | 1 | 1 |   |    |    |    |    |      |
| 31    |   |   |   |   |   |   |   |   | 1 | 1 |    |    |    |    |      |
| 33    |   |   |   |   |   |   |   |   |   | 1 | 1 |    |    |    |      |

Table 13: Khovanov Polynomial for $gH1ImJnKBDFAce$
10 Appendix B: Twist Knot Data

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
q/t & -2 & -1 & 0 & 1 & 2 & \chi \\
\hline
-5 & 1 & & & & & 1 \\
-3 & & & & & & \\
-1 & 1 & 1 & & & & \\
1 & & 1 & 1 & & & \\
3 & & & & & & \\
5 & & & & 1 & 1 & \\
\hline
\end{tabular}
\caption{Khovanov Polynomial for the Figure-Eight Knot}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
q/t & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & \chi \\
\hline
-5 & 1 & & & & & & & 1 \\
-3 & & & & & & & & \\
-1 & 1 & 1 & & & & & & \\
1 & & 1 & 1 & & & & & \\
3 & & & 1 & 1 & & & & \\
5 & & & & 1 & 1 & 1 & 1 & \\
7 & & & & & & & & \\
9 & & & & & & & 1 & 1 & \\
\hline
\end{tabular}
\caption{Khovanov Polynomial for eikGbHJCaFd}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
q/t & 0 & 1 & 2 & 3 & 4 & 5 & \chi \\
\hline
1 & 1 & & & & & & 1 \\
3 & 1 & 1 & & & & & \\
5 & & 1 & & & & 1 & \\
7 & & 1 & & & 1 & & \\
9 & & & 1 & 1 & & & \\
11 & & & & & & 1 & -1 & \\
13 & & & & & & & & \\
\hline
\end{tabular}
\caption{Khovanov Polynomial for the 5_2 Knot}
\end{table}
| $q \backslash t$ | $-2$ | $-1$ | 0  | 1  | 2  | 3  | 4  | 5  | 6  | $\chi$ |
|----------------|------|------|----|----|----|----|----|----|----|-------|
| 1              | 1    | 1    |    |    |    |    |    |    |    | 1     |
| 3              |      |      |    |    |    |    |    |    |    | 1     |
| 5              | 1    | 2    |    |    |    |    |    |    |    | 1     |
| 7              | 1    | 1    | 1  |    |    |    |    |    |    | 1     |
| 9              |      | 1    |    |    |    |    |    |    |    | 1     |
| 11             | 1    | 2    | 1  |    |    |    |    |    |    | 1     |
| 13             |      | 1    | 1  |    |    |    |    |    |    | 1     |
| 15             |      |      |    |    |    |    |    |    |    | -1    |
| 17             |      |      |    |    |    |    |    |    |    | 1     |

Table 17: Khovanov Polynomial for dgikFHajbc

| $q \backslash t$ | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | $\chi$ |
|----------------|----|----|----|----|----|----|----|----|-------|
| $-1$           | 1  | 1  |    |    |    |    |    |    | 1     |
| 1              | 1  |    |    |    |    |    |    |    | 1     |
| 3              | 1  |    |    |    |    |    |    |    | 1     |
| 5              |    |    |    |    |    |    |    |    | 1     |
| 7              |    | 1  |    |    |    |    |    |    | 1     |
| 9              |    |    |    |    |    |    |    |    | 1     |
| 11             |    |    |    |    |    |    |    |    | 1     |
| 13             |    |    |    |    |    |    |    |    | -1    |

Table 18: Khovanov Polynomial for gfJKH1aIEBCD

| $q \backslash t$ | $-2$ | $-1$ | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | $\chi$ |
|----------------|------|------|----|----|----|----|----|----|----|----|----|-------|
| 1              | 1    |    |    |    |    |    |    |    |    |    |    | 1     |
| 3              |      |    |    |    |    |    |    |    |    |    |    | 1     |
| 5              | 1    | 2    |    |    |    |    |    |    |    |    |    | 1     |
| 7              | 1    | 1    | 1  |    |    |    |    |    |    |    |    | 1     |
| 9              |      | 1    |    |    |    |    |    |    |    |    |    | 1     |
| 11             |      |      |    |    |    |    |    |    |    |    |    | 1     |
| 13             |      |      |    |    |    |    |    |    |    |    |    | -1    |
| 15             |      |      |    |    |    |    |    |    |    |    |    | 1     |
| 17             |      |      |    |    |    |    |    |    |    |    |    | 1     |
| 19             |      |      |    |    |    |    |    |    |    |    |    | 1     |
| 21             |      |      |    |    |    |    |    |    |    |    |    | 1     |

Table 19: Khovanov Polynomial for hGJaMcEdKBfI
Table 20: Khovanov Polynomial for the $7_2$ Knot

| $q/t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\chi$ |
|-------|---|---|---|---|---|---|---|---|-------|
| 1     | 1 |   |   |   |   |   |   |   | 1     |
| 3     | 1 | 1 |   |   |   |   |   |   |       |
| 5     |   | 1 |   |   |   |   |   |   | 1     |
| 7     |   | 1 | 1 |   |   |   |   |   |       |
| 9     |   | 1 | 1 |   |   |   |   |   |       |
| 11    |   | 1 | 1 |   |   |   |   |   | 1     |
| 13    |   |   | 1 |   |   |   |   |   |       |
| 15    |   |   |   |   |   |   |   |   |       |
| 17    |   |   |   |   |   |   |   | 1 | -1   |

Table 21: Khovanov Polynomial for $bhDGijCkaef$

| $q/t$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | $\chi$ |
|-------|----|----|---|---|---|---|---|---|-------|
| 1     | 1  |    | 1 |   |   |   |   |   |       |
| 3     |    |    | 1 | 2 |   |   |   |   |       |
| 5     |    |    | 1 | 1 |   |   |   |   | 1     |
| 7     |    |    |   | 1 |   |   |   |   |       |
| 9     |    |    |   | 1 |   |   |   |   |       |
| 11    |    |    |   | 1 | 1 | 1 |   |   | 1     |
| 13    |    |    |   | 1 | 1 | 1 |   |   |       |
| 15    |    |    |   |   | 1 | 1 |   |   |       |
| 17    |    |    |   |   |   | 1 | 1 | -1 |       |

Table 22: Khovanov Polynomial for the $8_1$ Knot

| $q/t$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | $\chi$ |
|-------|----|----|---|---|---|---|---|---|---|-------|
| -5    | 1  |    |   |   |   |   |   |   |   | 1     |
| -3    |    |    |   |   |   |   |   |   |   |       |
| -1    |    | 1  | 2 |   |   |   |   |   |   |       |
| 1     |    | 1  | 1 |   |   |   |   |   |   |       |
| 3     |    | 1  | 1 |   |   |   |   |   |   |       |
| 5     |    |    | 1 | 1 |   |   |   |   |   |       |
| 7     |    |    |   | 1 | 1 |   |   |   |   |       |
| 9     |    |    |   | 1 | 1 |   |   |   |   |       |
| 11    |    |    |   |   | 1 | 1 |   |   |   |       |
| 13    |    |    |   |   |   | 1 | 1 |   |   |       |
| 17    |    |    |   |   |   |   | 1 | 1 |   |       |
Table 23: Khovanov Polynomial for cefIgbajkDh

| q't  | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | χ |
|------|----|----|----|----|---|---|---|---|---|---|
| -5   | 1  |    |    |    |   |   |   |   |   | 1 |
| -3   |    |    |    |    |   |   |   |   |   | 1 |
| -1   | 1  | 2  |    |    |   |   |   |   |   | 1 |
| 1    |    |    |    |    |   |   |   |   |   | 1 |
| 3    |    |    |    |    |   |   |   |   |   | 2 |
| 5    |    |    |    |    |   |   |   |   |   | 1 |
| 7    |    |    |    |    |   |   |   |   |   | 1 |
| 9    |    |    |    |    |   |   |   |   |   | 2 |
| 11   |    |    |    |    |   |   |   |   |   | 2 |
| 13   |    |    |    |    |   |   |   |   |   | 1 |

Table 24: Khovanov Polynomial for femIbaJKLCGHd

| q't  | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | χ |
|------|----|----|----|----|---|---|---|---|---|---|
| -5   | 1  |    |    |    |   |   |   |   |   | 1 |
| -3   |    |    |    |    |   |   |   |   |   | 1 |
| -1   | 1  | 2  |    |    |   |   |   |   |   | 1 |
| 1    |    |    |    |    |   |   |   |   |   | 1 |
| 3    |    |    |    |    |   |   |   |   |   | 2 |
| 5    |    |    |    |    |   |   |   |   |   | 1 |
| 7    |    |    |    |    |   |   |   |   |   | 1 |
| 9    |    |    |    |    |   |   |   |   |   | 2 |
| 11   |    |    |    |    |   |   |   |   |   | 2 |
| 13   |    |    |    |    |   |   |   |   |   | 1 |

Table 25: Khovanov Polynomial for jpIFNMrClqOhkEDabg

| q't  | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | χ |
|------|----|----|----|----|---|---|---|---|---|---|---|---|---|---|
| -5   | 1  |    |    |    |   |   |   |   |   |   |   |   |   | 1 |
| -3   |    |    |    |    |   |   |   |   |   |   |   |   |   | 1 |
| -1   | 1  | 2  |    |    |   |   |   |   |   |   |   |   |   | 1 |
| 1    |    |    |    |    |   |   |   |   |   |   |   |   |   | 1 |
| 3    |    |    |    |    |   |   |   |   |   |   |   |   |   | 2 |
| 5    |    |    |    |    |   |   |   |   |   |   |   |   |   | 2 |
| 7    |    |    |    |    |   |   |   |   |   |   |   |   |   | 1 |
| 9    |    |    |    |    |   |   |   |   |   |   |   |   |   | 3 |
| 11   |    |    |    |    |   |   |   |   |   |   |   |   |   | 1 |
| 13   |    |    |    |    |   |   |   |   |   |   |   |   |   | 2 |
| 15   |    |    |    |    |   |   |   |   |   |   |   |   |   | 1 |
| 17   |    |    |    |    |   |   |   |   |   |   |   |   |   | 2 |
| 19   |    |    |    |    |   |   |   |   |   |   |   |   |   | 1 |
| $q \backslash t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | $\chi$ |
|-----------------|---|---|---|---|---|---|---|---|---|---|-------|
| 1               | 1 | 1 |   |   |   |   |   |   |   | 1 |       |
| 3               | 1 | 1 |   |   |   |   |   |   |   |   |       |
| 5               | 1 |   |   |   |   |   |   |   |   | 1 |       |
| 7               | 1 | 1 |   |   |   |   |   |   |   |   |       |
| 9               | 1 | 1 |   |   |   |   |   |   |   |   |       |
| 11              | 1 | 1 |   |   |   |   |   |   |   |   |       |
| 13              | 1 | 1 |   |   |   |   |   |   |   |   |       |
| 15              | 1 | 1 |   |   |   |   |   |   |   | 1 |       |
| 17              | 1 | 1 |   |   |   |   |   |   |   |   |       |
| 19              |   |   |   |   |   |   |   |   |   |   |       |
| 21              |   |   |   |   |   |   |   |   | 1 | −1 |       |

Table 26: Khovanov Polynomial for $K_7$

| $q \backslash t$ | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\chi$ |
|-----------------|----|----|---|---|---|---|---|---|---|---|-------|
| 1               | 1  | 1  |   |   |   |   |   |   |   |   |       |
| 3               |    |    |   |   |   |   |   |   |   |   |       |
| 5               | 1  | 2  |   |   |   |   |   |   |   | 1 |       |
| 7               | 1  | 1  |   |   |   |   |   |   |   |   |       |
| 9               | 1  | 1  |   |   |   |   |   |   |   |   |       |
| 11              | 1  | 1  |   |   |   |   |   |   |   |   |       |
| 13              | 1  | 1  |   |   |   |   |   |   |   |   |       |
| 15              | 1  | 1  |   |   |   |   |   |   |   | 1 |       |
| 17              | 1  | 1  |   |   |   |   |   |   |   |   |       |
| 19              |   |    |   |   |   |   |   |   |   |   |       |
| 21              |   |    |   |   |   |   |   |   | 1 | −1 |       |

Table 27: Khovanov Polynomial for cgjFHIaDEkb
### Table 28: Khovanov Polynomial for $K_8$

| $q \setminus t$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | $\chi$ |
|-----------------|-----|-----|---|---|---|---|---|---|---|---|---|-----|
| $-5$            |     |     | 1 |   |   |   |   |   |   |   |   | 1   |
| $-3$            |     |     |   |   |   |   |   |   |   |   |   | 1   |
| $-1$            | 1   |     | 2 |   |   |   |   |   |   |   |   | 1   |
| $1$             | 1   | 1   |   |   |   |   |   |   |   |   |   |     |
| $3$             | 1   | 1   |   |   |   |   |   |   |   |   |   |     |
| $5$             |     |     | 1 | 1 |   |   |   |   |   |   |   |     |
| $7$             |     |     |   |   | 1 | 1 |   |   |   |   |   |     |
| $9$             |     |     |   |   |   |   | 1 | 1 |   |   |   |     |
| $11$            |     |     |   |   |   |   |   |   | 1 | 1 |   |     |
| $13$            |     |     |   |   |   |   |   |   |   |   | 1 |     |
| $15$            |     |     |   |   |   |   |   |   |   |   |   | 1   |
| $17$            |     |     |   |   |   |   |   |   |   |   |   | 1   |

### Table 29: Khovanov Polynomial for $knIHoBjCDQrMPaeLgF$

| $q \setminus t$ | $-6$ | $-5$ | $-4$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | $\chi$ |
|-----------------|-----|-----|-----|-----|-----|-----|---|---|---|---|---|---|---|-----|
| $-5$            |     |     |     |     |     |     | 1 |   |   |   |   |   |   | 1   |
| $-3$            |     |     |     |     |     |     |   |   |   |   |   |   |   | 1   |
| $-1$            | 1   |     |     |     |     |     |   | 1 |   |   |   |   |   | 1   |
| $1$             | 1   | 1   |     |     |     |     |   |   |   |   |   |   |   |     |
| $3$             |     |     | 2   |     | 1   | 1   |   |   |   |   |   |   |   |     |
| $5$             |     |     | 1   | 3   | 2   |     |   |   |   |   |   |   |   |     |
| $7$             |     |     |     |     | 1   | 1   | 2 |   |   |   |   |   |   |     |
| $9$             |     |     |     |     | 1   |     | 2 | 2 |   |   |   |   |   |     |
| $11$            |     |     |     |     |     |     |   | 1 | 3 | 1 |   |   | $-1$ |
| $13$            |     |     |     |     |     |     |   |   | 1 |   | 1 |   |     |
| $15$            |     |     |     |     |     |     |   |   |   |   | 1 | 1 |     |
| $17$            |     |     |     |     |     |     |   |   |   |   |   |   | 1   |

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### Table 30: Khovanov Polynomial for $K_9$

| $q \backslash t$ | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | $\chi$ |
|-----------------|----|----|----|----|----|----|----|----|----|----|----|----|-------|
| 1               | 1  |    |    |    |    |    |    |    |    |    |    |    | 1     |
| 3               | 1  | 1  |    |    |    |    |    |    |    |    |    |    |       |
| 5               | 1  |    |    |    |    |    |    |    |    |    |    |    | 1     |
| 7               | 1  | 1  |    |    |    |    |    |    |    |    |    |    |       |
| 9               | 1  | 1  |    |    |    |    |    |    |    |    |    |    |       |
| 11              | 1  | 1  |    |    |    |    |    |    |    |    |    |    |       |
| 13              | 1  | 1  |    |    |    |    |    |    |    |    |    |    |       |
| 15              | 1  | 1  |    |    |    |    |    |    |    |    |    |    |       |
| 17              | 1  |    |    |    |    |    |    |    |    |    |    |    | 1     |
| 19              | 1  |    |    |    |    |    |    |    |    |    |    |    |       |
| 21              | 1  | 1  |    |    |    |    |    |    |    |    |    |    |       |
| 23              |    |    |    |    |    |    |    |    |    |    |    |    |       |
| 25              |    |    |    |    |    |    |    |    |    |    | 1  | -1  |       |

### Table 31: Khovanov Polynomial for jopIFMrD1qNhkEabcg

| $q \backslash t$ | -2 | -1 | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | $\chi$ |
|-----------------|----|----|----|----|----|----|----|----|----|----|----|----|-------|
| 1               | 1  |    |    |    |    |    |    |    |    |    |    |    | 1     |
| 3               |    |    |    |    |    |    |    |    |    |    |    |    |       |
| 5               | 1  | 2  |    |    |    |    |    |    |    |    |    |    |       |
| 7               | 1  | 2  | 1  |    |    |    |    |    |    |    |    |    |       |
| 9               | 1  | 1  |    |    |    |    |    |    |    |    |    |    |       |
| 11              | 2  | 3  | 1  |    |    |    |    |    |    |    |    |    |       |
| 13              | 1  | 2  | 1  |    |    |    |    |    |    |    |    |    |       |
| 15              | 2  |    |    |    |    |    |    |    |    |    |    |    | 1     |
| 17              | 2  | 3  | 1  |    |    |    |    |    |    |    |    |    |       |
| 19              | 1  |    |    |    |    |    |    |    |    |    |    |    |       |
| 21              |    |    |    |    |    |    |    |    |    |    | 2  | 2  |       |
| 23              |    |    |    |    |    |    |    |    |    |    |    |    |       |
| 25              |    |    |    |    |    |    |    |    |    |    |    |    | 1     |

Table 31: Khovanov Polynomial for jopIFMrD1qNhkEabcg
Table 32: Khovanov Polynomial for $6_2$

| $q \backslash t$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | $\chi$ |
|----------------|------|-----|---|---|---|---|---|------|
| $-3$           | 1    |     |   |   |   |   |   | 1    |
| $-1$           |      |     |   |   |   |   |   | 1    |
| 1              | 1    | 2   |   |   |   |   |   | 1    |
| 3              | 1    | 1   |   |   |   |   |   | 1    |
| 5              |      | 1   |   |   |   |   |   | 1    |
| 7              |      | 1   |   |   |   |   |   | 1    |
| 9              |      | 1   |   |   |   |   | $-1$| 1    |
| 11             |      |     |   |   |   |   |   | 1    |

Table 33: Khovanov Polynomial for $glf0JcbKMNDaHIe$

| $q \backslash t$ | $-4$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 | $\chi$ |
|----------------|------|------|------|-----|---|---|---|---|---|---|------|
| $-3$           | 1    |      |      |     |   |   |   |   |   |   | 1    |
| $-1$           |      | 1    | 2    |     |   |   |   |   |   |   | 1    |
| 1              |      | 1    | 1    |     |   |   |   |   |   |   | 1    |
| 3              |      | 2    | 2    |     |   |   |   |   |   |   | 1    |
| 5              |      |      | 1    | 2   |   |   |   |   |   |   | 1    |
| 7              |      |      | 1    | 2   | 1 |   |   |   |   |   | 1    |
| 9              |      |      |      | 1   |   |   |   |   |   |   | $-1$|
| 11             |      |      |      | 2   | 2 | 1 |   |   |   |   | 1    |
| 13             |      |      |      |     |   |   |   |   |   |   | 1    |
| 15             |      |      |      |      |   |   |   |   |   |   | 1    |

11 Appendix C: Conjectured Legendrian Simple Knot Data
| q \ t | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | χ |
|-------|----|----|----|----|---|---|---|---|---|---|---|
| -3    | 1  |    |    |    | 1 |   |   |   |   |   |   |
| -1    | 1  | 1  | 2  |    |   |   |   |   |   |   |   |
| 3     | 1  | 1  |    |    |   |   |   |   |   |   |   |
| 5     | 2  | 2  |    |    |   |   |   |   |   |   |   |
| 7     |    | 2  | 2  | 1  |   |   |   |   |   |   |   |
| 9     |    | 1  |    |    | 1 |   |   |   |   |   | -1|
| 11    |    | 2  | 2  | 1  |   |   |   |   |   |   | 1 |
| 13    |    |    |    |    |   |   |   |   |   |   |   |
| 15    |    |    |    |    |   | 1 |   |   |   |   |   |

Table 34: Khovanov Polynomial for hknEGmDbJLaIfc

| q \ t | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | χ |
|-------|----|----|----|----|---|---|---|---|---|---|---|---|
| -9    | 1  | 1  |    |    |   |   |   |   |   |   |   |   |
| -7    |    |    |    |    |   |   |   |   |   |   |   |   |
| -5    | 1  | 2  | 1  |    |   |   |   |   |   |   |   |   |
| -3    |    | 1  | 2  | 1  |   |   |   |   |   |   |   |   |
| -1    |    | 1  | 2  | 2  |   |   |   |   |   |   |   |   |
| 1     |    | 1  |    |    | 1 |   |   |   |   |   |   |   |
| 3     |    |    |    |    |   | 1 |   |   |   |   |   |   |
| 5     |    |    |    | 1  | 2 | 1 |   |   |   |   |   |   |
| 7     |    |    |    |    | 1 | 1 |   |   |   |   |   |   |
| 9     |    |    |    |    | 1 | 1 | -1|   |   |   |   |   |
| 11    |    | 1  |    |    |   |   | 1 |   |   |   |   |   |
| 13    |    |    |    |    |   | 1 | 2 | 1 |   |   |   |   |
| 15    |    |    |    |    |   | 1 | 1 |   |   |   |   |   |
| 17    |    |    |    |    |   |   |   |   | 1 | 1 |   |   |

Table 35: Khovanov Polynomial for gKHlmIdJCEABf

| q \ t | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | χ |
|-------|----|----|----|----|---|---|---|---|---|---|---|---|
| -3    | 1  |    |    |    |   |   |   |   |   |   |   |   |
| -1    |    | 1  | 1  |    |   |   |   |   |   |   |   |   |
| 1     |    | 1  | 2  | 1  |   |   |   |   |   |   |   |   |
| 3     |    | 1  | 1  |    |   |   |   |   |   |   |   |   |
| 5     |    |    |    | 2  | 2 |   |   |   |   |   |   |   |
| 7     |    |    |    | 2  | 3 | 1 |   |   |   |   |   |   |
| 9     |    |    |    | 1  | 1 | 1 | -1|   |   |   |   |   |
| 11    |    |    |    | 2  | 2 | 1 | 1 |   |   |   |   |   |
| 13    |    |    |    | 1  | 2 | 1 |   |   |   |   |   |   |
| 15    |    |    |    | 1  | 1 |   |   |   |   |   |   |   |
| 17    |    |    |    |    |   | 1 | 1 |   |   |   |   |   |

Table 36: Khovanov Polynomial for ehkmGIaFlcbd
| $q/t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\chi$ |
|-------|---|---|---|---|---|---|---|---|-------|
|  3   | 1 |   |   |   |   |   |   |   |  1    |
|  5   | 1 |   |   |   |   |   |   |   |  1    |
|  7   | 1 |   |   |   |   |   |   |   |  1    |
|  9   | 1 |   |   |   |   |   |   |   |  1    |
| 11   | 1 | 2 |   |   |   |   |   |   |  1    |
| 13   |   |   |   |   |   |   |   | 1  |  1    |
| 15   |   | 2 | 1 |   |   |   |   |   |  1    |
| 17   |   |   |   |   |   |   |   |   |  1    |
| 19   |   |   |   |   |   |   |   | 1  |  1    |

Table 37: Khovanov Polynomial for $7_3$

| $q/t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\chi$ |
|-------|---|---|---|---|---|---|---|---|-------|
|  1   | 1 | 1 |   |   |   |   |   |   |  1    |
|  3   | 1 |   |   |   |   |   |   |   |  1    |
|  5   | 2 | 2 |   |   |   |   |   |   |  1    |
|  7   | 1 |   |   |   |   |   |   |   |  1    |
|  9   | 1 | 2 | 1 |   |   |   |   |   |  1    |
| 11   |   | 1 | 2 |   |   |   |   |   |  1    |
| 13   |   |   |   |   |   |   |   | 1  |  1    |
| 15   |   | 2 | 1 |   |   |   |   |   |  1    |
| 17   |   |   |   |   |   |   |   |   |  1    |
| 19   |   |   |   |   |   |   |   | 1  |  1    |

Table 38: Khovanov Polynomial for $hgelkIbaJFcd$

| $q/t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\chi$ |
|-------|---|---|---|---|---|---|---|---|-------|
|  1   | 1 |   |   |   |   |   |   | 1  |  1    |
|  3   | 1 | 2 |   |   |   |   |   |   |  1    |
|  5   | 1 |   |   |   |   |   |   |   |  1    |
|  7   | 2 | 1 |   |   |   |   |   |   |  1    |
|  9   | 1 | 2 |   |   |   |   |   |   |  1    |
| 11   |   |   |   |   |   |   |   | 1  |  1    |
| 13   |   | 2 | 1 |   |   |   |   |   |  1    |
| 15   |   |   |   |   |   |   |   |   |  1    |
| 17   |   |   |   |   |   |   |   | 1  |  1    |

Table 39: Khovanov Polynomial for $7_4$
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
$q \backslash t$ & $-2$ & $-1$ & 0 & 1 & 2 & 3 & 4 & 5 & 6 & $\chi$ \\
\hline
1 & 1 & & & & & & & & & 1 \\
3 & 1 & & & $-1$ & & & & & & \\
5 & 1 & 2 & & & & & & & & 1 \\
7 & 2 & 2 & 1 & & & & & & & 1 \\
9 & 1 & 2 & & & & & & & & 1 \\
11 & & 2 & 2 & 1 & & & & & & 1 \\
13 & & & 2 & 2 & 1 & & & & & $-1$ \\
15 & & & & 1 & 1 & & & & & \\
17 & & & & & 2 & 1 & 1 & & & \\
\hline
\end{tabular}
\caption{Khovanov Polynomial for gfHljIDae}
\end{table}

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
$q \backslash t$ & $-5$ & $-4$ & $-3$ & $-2$ & $-1$ & 0 & 1 & 2 & $\chi$ \\
\hline
$-13$ & 2 & & & & & & & & $-2$ \\
$-11$ & 1 & & & & & & & & 1 \\
$-9$ & 2 & 3 & & & & & & & $-1$ \\
$-7$ & 1 & 3 & & & & & & & 2 \\
$-5$ & 3 & 1 & & & & & & & 2 \\
$-3$ & & 3 & 3 & & & & & & & \\
$-1$ & & & 2 & 1 & 1 & & & & & \\
1 & & & & & & & 2 & $-2$ & & \\
3 & & & & & & & & & & 1 & 1 \\
\hline
\end{tabular}
\caption{Khovanov Polynomial for 948}
\end{table}

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
$q \backslash t$ & $-7$ & $-6$ & $-5$ & $-4$ & $-3$ & $-2$ & $-1$ & 0 & 1 & 2 & 3 & 4 & $\chi$ \\
\hline
$-15$ & 1 & 1 & & & & & & & & & & & \\
$-13$ & & 2 & & & & $-2$ & & & & & & & \\
$-11$ & 1 & 2 & 2 & & & & & & & & & & 1 \\
$-9$ & 1 & 3 & 5 & 1 & & & & & & & & & $-1$ \\
$-7$ & & 1 & 2 & 3 & & & & & & & & & 2 \\
$-5$ & & & 1 & 6 & 4 & 1 & & & & & & & 2 \\
$-3$ & & & & 3 & 4 & 1 & & & & & & & \\
$-1$ & & & & & 1 & 4 & 2 & & & & & & 1 \\
1 & & & & & & & 3 & 2 & 1 & & & $-2$ \\
3 & & & & & & & & & & & 1 & 1 & \\
5 & & & & & & & & & & & & 1 & 1 \\
\hline
\end{tabular}
\caption{Khovanov Polynomial for gnoqKjIMrpEaHblfc}
\end{table}
Table 43: Khovanov Polynomial for $9_{49}$

| $q \setminus t$ | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | $\chi$ |
|-----------------|---|---|---|---|---|---|---|---|------|
| $-19$           | 2 |   |   |   |   |   |   |   | -2   |
| $-17$           | 1 |   |   |   |   |   |   |   | 1    |
| $-15$           | 2 | 3 |   |   |   |   |   |   | -1   |
| $-13$           | 1 | 2 |   |   |   |   |   |   | 1    |
| $-11$           | 3 | 2 |   |   |   |   |   |   | 1    |
| $-9$            |   |   |   |   | 2 |   |   |   | -1   |
| $-7$            |   |   |   |   | 2 |   |   |   | 1    |
| $-5$            |   |   |   |   |   | 2 | 1 | -1 |      |
| $-3$            |   |   |   |   |   |   | 1 | 1 |      |

Table 44: Khovanov Polynomial for $lFKJIOAEnDCpBhmG$

| $q \setminus t$ | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | $\chi$ |
|-----------------|---|---|---|---|---|---|---|---|---|---|------|
| $-19$           | 2 |   |   |   |   |   |   |   |   | -2 |     |
| $-17$           | 1 |   |   |   |   |   |   |   |   | 1  |     |
| $-15$           | 2 | 3 |   |   |   |   |   |   |   | -1 |     |
| $-13$           | 1 | 3 | 1 |   |   |   |   |   |   | 1  |     |
| $-11$           | 3 | 2 |   |   |   |   |   |   |   | 1  |     |
| $-9$            |   | 3 | 4 | 1 |   |   |   |   |   | 2  |     |
| $-7$            |   | 2 | 1 | 1 | 2 |   |   |   |   |    |     |
| $-5$            |   |   | 3 | 2 |   | -1 |   |   |   | 1  |     |
| $-3$            |   |   |   | 1 | 1 | 1 | 1 |   |   | 1  |     |
| $-1$            |   |   |   |   |   | 1 | 1 |   |   | 1  |     |

Table 45: Khovanov Polynomial for $10_{128}$

| $q \setminus t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\chi$ |
|-----------------|---|---|---|---|---|---|---|---|------|
| 5               | 1 |   |   |   |   |   |   |   | 1    |
| 7               | 1 | 1 |   |   |   |   |   |   |      |
| 9               | 1 |   |   |   |   |   |   |   | 1    |
| 11              | 1 | 1 | 1 |   |   |   |   |   | 1    |
| 13              | 1 | 2 |   |   |   |   |   |   | 1    |
| 15              | 1 | 1 |   |   |   |   |   |   |      |
| 17              | 2 | 1 | -1 |   |   |   |   |   |      |
| 19              |   |   |   |   |   |   |   |   |      |
| 21              | 1 | -1 |   |   |   |   |   |   |      |

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Table 46: Khovanov Polynomial for eHPnqGJlBFoiADcKm

| q/t | -1 | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | χ  |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1   | 1  | 1  |    |    |    |    |    |    |    |    |    |    |
| 3   | 1  | 1  |    |    |    |    |    |    |    |    |    |    |
| 5   | 1  | 1  | 1  |    |    |    |    |    |    |    |    |    |
| 7   |    | 2  | 2  |    |    |    |    |    |    |    |    |    |
| 9   |    | 1  | 1  | 1  |    |    |    |    |    |    |    |    |
| 11  |    |    | 1  | 3  | 1  |    |    |    |    |    |    |    |
| 13  |    |    |    | 1  | 2  |    |    |    |    |    |    |    |
| 15  |    |    |    |    | 1  |    |    |    |    |    |    |    |
| 17  |    |    |    |    |    | 2  | 1  |    |    |    |    | -1 |
| 19  |    |    |    |    |    |    |    |    |    |    |    |    |
| 21  |    |    |    |    |    |    |    |    |    |    |    | 1  |

Table 47: Khovanov Polynomial for edjkaIGIlFbch

| q/t | -3 | -2 | -1 | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | χ  |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 3   | 1  |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 5   |    | 1  |    |    |    |    |    |    |    |    |    |    |    |    |
| 7   |    |    | 2  | 2  |    |    |    |    |    |    |    |    |    |    |
| 9   |    |    |    | 1  |    |    |    |    |    |    |    |    |    |    |
| 11  |    |    |    |    | 1  | 3  | 1  |    |    |    |    |    |    |    |
| 13  |    |    |    |    |    | 1  | 2  |    |    |    |    |    |    |    |
| 15  |    |    |    |    |    |    | 1  |    |    |    |    |    |    |    |
| 17  |    |    |    |    |    |    |    | 2  | 1  |    |    |    |    | -1 |
| 19  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 21  |    |    |    |    |    |    |    |    |    |    |    |    |    | 1  |

Table 48: Khovanov Polynomial for 10_{136}

| q/t | -3 | -2 | -1 | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | χ  |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|
| -9  |    |    | 1  |    |    |    |    |    |    |    |    | -1 |
| -7  |    |    | 1  |    |    |    |    |    |    |    |    |    |    |
| -5  |    |    |    | 1  | 1  |    |    |    |    |    |    |    |    |
| -3  |    |    |    |    | 1  | 2  |    |    |    |    |    |    |    |
| -1  |    |    |    |    |    | 2  | 1  |    |    |    |    |    |    |
| 1   |    |    |    |    |    |    | 1  | 2  |    |    |    |    |    |
| 3   |    |    |    |    |    |    |    | 1  |    |    |    |    |    |
| 5   |    |    |    |    |    |    |    |    | 1  | 1  |    | -1 |
| 7   |    |    |    |    |    |    |    |    |    |    | 1  | 1  |    |

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### Table 49: Khovanov Polynomial for \text{igDKHJaEbFC}

| q \ t | −9 | −8 | −7 | −6 | −5 | −4 | −3 | −2 | −1 | 0 | 1 | 2 | χ |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|----|
| −9    | 1  |    |    |    |    |    |    |    |    |    |    |    | −1 |
| −7    | 1  |    |    |    |    |    |    |    |    |    |    |    | 1  |
| −5    | 1  | 1  |    |    |    |    |    |    |    |    |    |    | 1  |
| −3    | 1  | 2  | 2  |    |    |    |    |    |    |    |    |    | 1  |
| −1    | 1  | 1  | 1  | 1  |    |    |    |    |    |    |    |    | 1  |
| 1     | 2  | 2  |    |    |    |    |    |    |    |    |    |    | 1  |
| 3     | 1  | 1  |    |    |    |    |    |    |    |    |    |    | 1  |
| 5     | 1  | −1 |    |    |    |    |    |    |    |    |    |    | 1  |
| 7     |    |    |    |    |    |    |    |    |    |    |    |    | 1  |

### Table 50: Khovanov Polynomial for 10_{145}

| q \ t | −9 | −8 | −7 | −6 | −5 | −4 | −3 | −2 | −1 | 0 | 1 | χ |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|
| −21   | 1  |    |    |    |    |    |    |    |    |    |    | −1 |
| −19   |    |    |    |    |    |    |    |    |    |    |    | 1  |
| −17   | 1  | 1  |    |    |    |    |    |    |    |    |    | 1  |
| −15   |    | 1  | 1  |    |    |    |    |    |    |    |    | 1  |
| −13   |    | 1  |    |    |    |    |    |    |    |    |    | 1  |
| −11   |    | 1  | 2  | 1  |    |    |    |    |    |    |    | 1  |
| −9    |    |    |    |    |    |    |    |    |    |    |    | 1  |
| −7    |    |    |    |    |    |    |    | 1  | 1  |    |    | 1  |
| −5    |    |    |    |    |    |    | 1  | 1  |    |    |    | 1  |
| −3    |    |    |    |    |    | 1  | 1  |    |    |    |    | 1  |

### Table 51: Khovanov Polynomial for \text{eoHKqGJnCFmPDibaL}

| q \ t | −9 | −8 | −7 | −6 | −5 | −4 | −3 | −2 | −1 | 0 | 1 | χ |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|
| −21   | 1  |    |    |    |    |    |    |    |    |    |    | −1 |
| −19   |    |    |    |    |    |    |    |    |    |    |    | 1  |
| −17   | 1  | 1  |    |    |    |    |    |    |    |    |    | 1  |
| −15   |    | 1  | 1  |    |    |    |    |    |    |    |    | 1  |
| −13   |    | 1  |    |    |    |    |    |    |    |    |    | 1  |
| −11   |    | 1  | 2  | 1  |    |    |    |    |    |    |    | 1  |
| −9    |    |    |    |    |    |    |    |    |    |    |    | 1  |
| −7    |    |    |    |    |    |    |    | 1  | 1  |    |    | 1  |
| −5    |    |    |    |    |    |    | 1  | 1  |    |    |    | 1  |
| −3    |    |    |    |    |    | 1  | 1  |    |    |    |    | 1  |

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Table 52: Khovanov Polynomial for kNJIpHLFCoMGABd

| $q \backslash t$ | −11 | −10 | −9 | −8 | −7 | −6 | −5 | −4 | −3 | −2 | −1 | 0 | $\chi$ |
|----------------|-----|-----|----|----|----|----|----|----|----|----|----|----|-----|
| −21            | 1   |     |    |    |    |    |    |    |    |    |    |    | −1  |
| −19            |     | 1   | 1  |    |    |    |    |    |    |    |    |    |     |
| −17            |     | 1   | 1  |    |    |    |    |    |    |    |    |    |     |
| −15            |     |     | 1  | 1  |    |    |    |    |    |    |    |    |     |
| −13            |     |     | 1  |    | 1  |    |    |    |    |    |    |    |     |
| −11            |     |     | 1  | 2  | 1  |    |    |    |    |    |    |    |     |
| −9             |     |     |     |    | 1  | 1  |    |    |    |    |    |    |     |
| −7             |     |     |     |     | 1  |    | 1  |    |    |    |    |    |     |
| −5             |     |     |     |     |     | 1  |    | 1  |    | 1  |    |    |     |
| −3             |     |     |     |     |     |     | 1  |    |    |    |    | 1  |     |
| −1             |     |     |     |     |     |     |     | 1  |    |    |    |    | 1  |

Table 53: Khovanov Polynomial for 10_{161}

| $q \backslash t$ | −9 | −8 | −7 | −6 | −5 | −4 | −3 | −2 | −1 | 0 | $\chi$ |
|----------------|----|----|----|----|----|----|----|----|----|---|-----|
| −23            | 1  |    |    |    |    |    |    |    |    |   | −1  |
| −21            |    | 1  | 1  |    |    |    |    |    |    |   |     |
| −19            |    | 1  | 1  |    |    |    |    |    |    |   |     |
| −17            |    | 1  | 1  |    |    |    |    |    |    |   |     |
| −15            |    | 1  |    | 1  |    |    |    |    |    |   |     |
| −13            |    | 1  | 2  | 1  |    |    |    |    |    |   |     |
| −11            |    |     | 1  |    | 1  |    |    |    |    |   | 1   |
| −9             |    |     | 1  |    |    | 1  |    |    |    |   |     |
| −7             |    |     |     | 1  |    |    | 1  |    |    |   |     |
| −5             |    |     |     |     | 1  |    |    | 1  |    |   |     |
| −3             |    |     |     |     |     | 1  |    |    |    |   | 1   |

Table 54: Khovanov Polynomial for h0qr1jsnMeipFAgkcbd
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