Top-quark decay into Υ-meson

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Abstract

The calculation of the partial width of the rare $t$-quark decay into $\Upsilon$-meson, $W$-boson and $b$-quark ($t \to \Upsilon Wb$) is presented. The branching ratio equals $\text{Br}(t \to \Upsilon Wb) = 1.3 \times 10^{-5}$ that make possible searches for this rare $t$-quark decay at LHC.

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1. Introduction

In the SM the decay $t \to bW$ is by far the dominant one. The rates for other decay channels are predicted to be smaller by several orders of magnitude in the SM [1].

For example, for semi-exclusive $t$-quark decays the interaction of quarks among the $t$-decay products may lead to final states with one hadron (meson) recoiling against a jet. The decays of the top through an off-shell $W$ with virtual mass $M_{W^*}$ near to some resonance $h$, like $\pi^+, \rho^+, K^+, D_s^+$, leads to the estimate as follows [1]:

$$\Gamma(t \to bh) \approx \frac{G_F^2 m_t^3}{144\pi} f_M^2 |V_{qq'}|^2$$  \hspace{1cm} (1)

where the parameter $f_M$ is same as a well-known coupling $f_\pi$. The typical values of the corresponding branching ratios are too small to be measured [1]:

$$\text{Br}(t \to b\pi) \sim 4 \times 10^{-8}, \quad \text{Br}(t \to bD_s) \sim 2 \times 10^{-7}$$  \hspace{1cm} (2)

There are several two-body $t$-quark decay through flavour changing neutral currents:

$$t \to \gamma q, \quad t \to Z q, \quad t \to g q; \quad q = u, c$$

These processes in the SM can occur due to loop contribution only and are highly suppressed due to GIM mechanism. The estimated branching ratios are as follows [1]:

$$\text{Br}(t \to V q) \sim O(10^{-11} \div 10^{-13}), \quad V = \gamma, Z, g, \quad q = u, c$$

In addition, it worth noting that almost all “interesting” $t$-quark rare decays have very small branching ratios and almost impossible to measured in experiment.

Among rare top-quark decays one can single out the processes with the production of heavy quark $Q\bar{Q}'$-pair (for example, $b\bar{b}$) followed by the formation of a heavy $M(Q\bar{Q}')$-meson. In this case, the description of such mesons production allows the use of the NRQCD-model [2, 3].

Note, that the top quarks production processes with subsequent $t$-quark decays into heavy quark $Q\bar{Q}'$-pair is described within SM with high accuracy. Therefore, the search and study of such $t$-quark rare decays can allow, in particular, to find out in more detail which models formation of quarkonium (Color-Evaporation Model, the Color-Singlet Model or the Color-Octet Mechanism, see [4] for detailed discussion of various mechanisms) describe more correctly such processes.

In this article we calculate the $t$-quark decay widths into $\Upsilon$-meson within NRQCD model [2, 3]. As will be seen below, at least one decay channel has a “relatively” large branching fraction, providing an opportunity for experimental searches.

2. The effective $b \bar{b} \Upsilon$-vertex

Within NRQCD approach the integration on virtual momentum in the loop with two heavy quarks that entered into heavy $M(Q\bar{Q})$ meson (see fig. 1)
effectively produces the following expression (see [2, 3]) for details:

\[
\int \frac{d^4p}{i(2\pi)^4} G \left( -\frac{k}{2} + p \right) \hat{\epsilon} G \left( \frac{k}{2} + p \right) \{\cdots\} \Psi_\Upsilon \\
\Rightarrow \frac{R_s(0)}{\sqrt{4\pi M^3}} \left( m_b - \frac{k}{2} \right) \hat{\epsilon} \left( m_b + \frac{k}{2} \right) \{\cdots\}
\]

(3)

where \( G = (m + \hat{p})/(m^2 - p^2) \) is fermion propagator, \( M = m_\Upsilon \) stands for \( \Upsilon \)-meson mass, \( \Psi_\Upsilon \) is the \( \Upsilon \)-meson wave function, \( \{\cdots\} \) is other terms in the loop; \( \hat{\epsilon}^\mu \) is the polarization vector of the \( \Upsilon \)-meson. The \( \Upsilon \)-meson wave function at the origin of the \( R_s(0) \) is related to the lepton decay width [2, 3] as follows:

\[
\Gamma(\Upsilon \to \ell^+\ell^-) = \frac{4e_b^2\alpha^2}{M^2} |R_s(0)|^2
\]

(4)

here \( e_b \) is \( b \)-quark charge, \( \alpha = e^2/(4\pi) \).

Note, that in the final expression (3) the heavy \( b \)-quarks (entered in the heavy \( \Upsilon \)-meson vertex) are considered to be on-shell with mass equals:

\[
m_b = \frac{M}{2}
\]

Taking into account that \( (k\epsilon) = 0 \) we get the final expression for \( b\bar{b}\Upsilon \) vertex:

\[
V(\bar{b}b\Upsilon) = g_\Upsilon \hat{\epsilon}(M + \hat{k}), \quad g_\Upsilon = \frac{M}{2} \cdot \frac{R_s(0)}{\sqrt{4\pi M^3}} = \frac{R_s(0)}{4\sqrt{\pi M}}
\]

(5)

3. \( t \to \Upsilon c \) decay

In this section we present the evaluation of the two-body \( t \)-quark decay width

\[
t \to \Upsilon c
\]

(6)

within NRQCD Color Singlet model approach. This width was calculated previously in [5, 6]. For the sake of completeness we repeat the evaluation of this quantity. The diagram describing this decay is shown in fig. 2. We set the mass of the light \( c \)-quark equals zero; \( m_t \) is the mass of \( t \)-quark, \( M = m_\Upsilon \) is the \( \Upsilon \) mass\(^1\).

\(^1\)Throughout of this article we follow [7] for the notations, the SM vertices and SM parameters.
The amplitude has the following form (see [2] for details):

\[ A = \left( \frac{g \gamma^2 V_{bc}}{2} \right) D_W^{\alpha\beta} \bar{u}(q)\gamma^\alpha P_L \tilde{u} (M + k) \gamma^\beta P_L u(p), \quad P_L = (1 - \gamma^5)/2 \]  

(7)

where \( g = 2M_W \sqrt{2} G_F \) (\( G_F \) is the Fermi coupling constant); \( D_W^{\alpha\beta} \) is the \( W \)-boson propagator, \( \varepsilon \) is the \( \Upsilon \)-meson polarization vector:

\[ D_W^{\alpha\beta} = g^{\alpha\beta} - \frac{p_W^\alpha p_W^\beta}{M_W^2}, \quad \sum_{pol\Upsilon} \varepsilon^\mu \varepsilon^\nu = g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2}, \quad (\varepsilon k) = 0 \]

Then the decay width equals

\[ \Gamma(t \to c\Upsilon) = \left( \frac{g \gamma^2 V_{bc}}{768 \pi Z^2} \right) \frac{m_t}{m_t^2} \left( 1 - \frac{M^2}{m_t^2} \right) \times U \]

(8)

\[ U = 6m_t^2 F^2 + (m_t^2 - M^2) \left[ 8 \left( 1 + \frac{M^2}{8M_W^2} \right)^2 - \frac{m_t^2 m_W^2}{2M_W^4} \right], \quad F = \frac{p_W^2}{M_W^2} - 2 \]

\[ p_W^2 = \frac{m_t^2}{2} - \frac{M^2}{4}, \quad Z^2 = (M_W^2 - p_W^2)^2 + (\Gamma_W M_W)^2 \]

here \( \Gamma_W \) is the total decay width of the \( W \)-boson. The resulted width (with \( m_t = 172.5 \) GeV, \( |V_{bc}| = 0.04 \) [8]) equals

\[ \Gamma(t \to \Upsilon c) = 6.35 \times 10^{-10} \text{ GeV} \]

(9)

and is very similar to previous result [5]. At the same time this quantity is 2.5 smaller then result from [6]. This difference can be explained by the fact that authors used contributions of both color singlet and color octet to this decay channel (see [6] for details).

For calculation of the branching ratio we use LO \( t \)-quark decay width value of

\[ \Gamma(t \to bW^+) = 1.47 \text{ GeV} \]

(10)

and get the corresponding branching ratio for this decay channel

\[ Br(t \to \Upsilon c) = \frac{\Gamma(t \to \Upsilon c)}{\Gamma(t \to bW^+)}_{LO} = 4.32 \times 10^{-10} \]

(11)
4. Top-quark decay $t \rightarrow \Upsilon W b$

It follows from previous section that two-body $t$-quark decay $t \rightarrow \Upsilon c$ is very small (see (11)). It is explained by very small value of $|V_{tc}| \approx 0.04$ and high virtuality of the $W$-boson ($p_{W}^2 = m_t^2/2 - M^2/4 \gg M_W^2$, see (8)).

To avoid such suppression factors we consider $t$-quark decay width additional $b\bar{b}$-pair production in the final state:

$$t \rightarrow bW^+ \bar{b}$$

This decay process is described by 28 Feynman diagrams. We use the C++ version of the TopReX package [9] for calculation the decay width into this channel. The results equal

$$\Gamma(t \rightarrow bW^+ \bar{b}) = 8.37 \times 10^{-4} \text{ GeV}$$
$$Br(t \rightarrow bW^+ \bar{b}) = 5.7 \times 10^{-4}$$

(13)

However, the diagrams with $b\bar{b}$ pair production due to Higgs, $Z$-boson or $\gamma$ exchange are highly suppressed (due to small couplings). As a result we have 4 diagrams, describing $t \rightarrow \Upsilon Wb$ decay channel. The diagrams with $W$-boson exchange (see fig. 3) are also highly suppressed (due to small couplings and high virtuality of intermediate $u, c, t$-quarks and $W$).

Figure 3: The diagrams describing $t \rightarrow \Upsilon Wb$ decay through $W$-boson exchange.

Therefore, the dominant contribution to the amplitude of $t \rightarrow \Upsilon Wb$ decay comes from two diagrams with gluon exchange (see fig. 4).

Figure 4: The diagrams describing $t \rightarrow \Upsilon Wb$ decay through gluon exchange.

The amplitude $A$ has the form (the particle’s momenta notations are shown in the fig. 4):

$$A = A_1 + A_2, \quad A_1 = \left( \frac{g_\gamma g g^2}{\sqrt{2} z_1 q^2} \right) \rho g^2 \times T_1, \quad A_2 = \left( \frac{g_\gamma g g^2}{\sqrt{2} z_2 q^2} \right) \rho g^2 \times T_2$$

$$T_1 = \rho g^2 \varepsilon W \bar{u}(p_0) \gamma^\alpha \varepsilon_T \left( M + \hat{k} \right) \gamma^\beta \left( \frac{M}{2} + \hat{x}_1 \right) \gamma^\lambda P_L u(p)$$
$$T_2 = \rho g^2 \varepsilon W \bar{u}(p_0) \gamma^\alpha \varepsilon_T \left( M + \hat{k} \right) \gamma^\lambda P_L (m + \hat{x}_2) \gamma^\beta u(p)$$

$$x_1 = q + k/2, \quad x_2 = p - q, \quad q = p_0 + k/2, \quad z_1 = \frac{M^2}{4} - x_1^2, \quad z_2 = m_t^2 - x_2^2$$

(14)
The square of the full amplitude is rather cumbersome and we present it in the Appendix.

As before we use the C++ version of the TopReX package [9] for calculation of the decay width. In the table 1 we present the results for three Υ-meson states (Υ(1S), Υ(2S) and Υ(3S)).

Table 1: The partial top-quark decay widths (Γ) and branching ratios (Br) into three Υ-meson states. The widths are in GeV.

| Υ     | Γ(t → ΥW⁺b)   | Br(t → ΥW⁺b) | Γ(t → Υc)  | Br(t → Υc)  |
|-------|---------------|--------------|-----------|------------|
| Υ(1S) | 1.95 × 10⁻⁵   | 1.33 × 10⁻⁵  | 6.4 × 10⁻¹⁰| 4.35 × 10⁻¹⁰|
| Υ(2S) | 0.83 × 10⁻⁵   | 0.56 × 10⁻⁵  | 3.1 × 10⁻¹⁰| 2.11 × 10⁻¹⁰|
| Υ(3S) | 0.58 × 10⁻⁵   | 0.33 × 10⁻⁵  | 2.3 × 10⁻¹⁰| 1.56 × 10⁻¹⁰|

As mentioned above two-body t-quark decays t → Υ(nS)c (two right columns) have very small branching ratios for experimental study. On the other hand the decay channel t → Υ W⁺b looks much more promising for experimental searches.

In the fig. 5 we present the distributions on invariant masses of the final state particles: M(b, W), M(b, Υ), and M(W, Υ).

Figure 5: dΓ/dM_{inv} distributions. The left (red), central (blue), and right (green) curves correspond to M_{inv} = M(b Υ), M(b W) and M(W Υ) invariant masses, respectively.

As it seen the final W and b-quark are rather well separated, while b and Υ pair (the left
curve, fig. 5) has an invariant mass very close to $m_b + m_\Upsilon$. Therefore, one may expect that the $\Upsilon$-meson will produce dominantly inside final $b$-jet.

Now we present very rough estimates of the expected number of events for this rare $t$-quark decay channel for LHC Run-2 option. We consider the process of $t\bar{t}$-pair production with subsequent $t$-quark (or $\bar{t}$-quark) decay into three $\Upsilon(nS)$ states $t \rightarrow \Upsilon(nS)Wb$, $n = 1, 23$. The total $t\bar{t}$ cross section, extrapolated to the full phase space, is [10]:

$$\sigma_{t\bar{t}} = 803 \pm 2{\text{stat}} \pm 25{\text{syst}} \pm 20{\text{lumi}} \text{ pb}$$ \hspace{1cm} (15)

Then, for estimation the expected number of events we use the following options:
- the LHC Run-2 integrated luminosity equals $L_{\text{tot}} = 100 \text{ fb}^{-1}$,
- $W^+W^-$ decay into lepton and quark pairs $W^+W^- \rightarrow e(\mu)\nu q\bar{q}'$,
- all three $\Upsilon$ states decay into charged leptons $\Upsilon(nS) \rightarrow ee$ or $\mu\mu$.

As a result, at LHC Run-2 the expected number of events for $t\bar{t}$-pair production with subsequent $t \rightarrow \Upsilon W^+b$ decay are as follows:

$$pp \rightarrow t\bar{t}, \hspace{0.5cm} t \rightarrow \Upsilon_{1S+2S+3S}Wb, \hspace{0.5cm} \Upsilon \rightarrow \ell^+\ell^-$$

$$N(\Upsilon_{1S+2S+3S} \rightarrow \ell\ell) bb W^+W^- = 230$$

$$N(\Upsilon_{1S+2S+3S} \rightarrow \ell\ell) bb \ell\nu q\bar{q}') = 80$$ \hspace{1cm} (16)

The total number of events $N = 80$ is not very large. However, this number looks more or less suitable for the experimental study.

Conclusion

In this paper the calculation of the partial width for rare $t$-quark decay into $\Upsilon$-meson ($t \rightarrow \Upsilon Wb$) is presented. The decay width was evaluated within NRQCD-model. The calculated branching ratio equals $\text{Br}(t \rightarrow \Upsilon(1S)Wb) = 1.3 \times 10^{-5}$ that make possible searches for this rare $t$-quark decay at LHC.

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Appendix

Here we present the amplitude square $|T|^2$ from (14). The parameters $m_t$, $M$, and $M_W$ stand for $t$-quark, $\Upsilon$ and $W$-boson masses, respectively. $(a.b)$ is the scalar product of two 4-momenta.

$$|T|^2 = |(T_1 + T_2)^2| = 8 (\chi_1 + \chi_2 + \chi_{\text{int}})$$

$$\chi_1 = 4(p.p_b)(p_b.k)M^2 + 4(p.p_b)(p_b.k)^2 - 2(p.p_b)M^4 + 4(p.k)(p_b.k)M^2 - (p.k)M^4$$

$$+ \frac{2(p.p_W)}{M_W^4} [4(p_b.k)(p_b.p_W)M^2 + 4(p_b.k)(k.p_W)M^2 + 4(p_b.k)^2(p_b.p_W) - 2(p_b.p_W)M^4$$

$$- (k.p_W)M^4]$$

7
\[
\chi_2 = -20(p.p_b)(p.k)(k.q) + 12(p.p_b)(p.k)^2 + 8(p.p_b)(k.q)^2 - (p.p_b)M^2q^2
+ \frac{1}{(p.k)} \left[ 4(p.q)(p_b.k) - 4(p.q)M^2 - 2(p_b.k)q^2 - 6(p.k)(p_b.k)m^2 + 4(p_b.q)(k.q)
- 3M^2m^2 - 4(p.k)(p_b.q) \right]
+ \frac{1}{M_W^2} \left[ 24(p.p_b)(p.k)(p.p_W)(k.p_W) - 24(p.p_b)(k.p_W)(q.p_W) + 16(p.p_b)(p.q)(k.p_W)^2
- 16(p.p_b)(p.p_W)(k.q)(k.p_W) + 4(p.p_b)(p.p_W)(q.p_W)M^2 + 16(p.p_b)(k.q)(k.p_W)(q.p_W)
- 8(p.k)(p.p_W)(p.q)(k.p_W) + 2(p.k)(p.p_W)(q.p_W)M^2 - 8(p.p_b)(k.p_W)^2q^2
- 4(p.p_b)(q.p_W)^2M^2 + 8(p.k)(p.p_b)(k.p_W) + 8(p.k)(p.b.q)(k.p_W)(q.p_W)
- 4(p.k)(p.p_b)(k.p_W)(q.p_W)^2 - 2(p.k)(q.p_W)^2M^2 + 4(p.q)(p.p_W)(p.b.p_W)^2
+ 2(p.q)(p.p_W)(k.p_W)M^2 + 6(p.q)(k.p_W)(q.p_W)M^2 + 4(p.p_W)(p.b.q)(q.p_W)M^2
- 2(p.p_W)(p.p_W)^2q^2 + 2(p.p_W)(q.p_W)M^2 - 4(p.p_W)(k.p_W)M^2q^2
- 4(p.p_W)^2(p.b.q)M^2 - 2(p.p_W)^2(q.k)^2 \right]
+ \frac{2m^2}{M_W^2} \left[ -6(p.p_W)(p.b.q)(k.p_W) - 3(p.p_W)(k.p_W)M^2 + 6(p.b.q)(k.p_W)(q.p_W)
- 2(p.b.q)(k.p_W)^2 + 2(p.p_W)(k.q)(k.p_W) - 2(p.p_W)(q.p_W)M^2 + 2(k.p_W)(q.p_W)M^2 \right]
}\]

\[
\chi_{int} = -8(p.p_b)(p.k)(p.b.q) + 4(p.p_b)(p.k)M^2 - 4(p.p_b)(p.b.q)(k.q) - 8(p.p_b)(p.b.q)M^2
+ 4(p.q)(p.b.k)^2 + 8(p.p_b)^2M^2 - 4(p.p_b)M^2 - 6(p.q)(p.b.k)M^2
+ 12(p.b.k)(p.b.q) + 6(p.b.q)(p.b.k)M^2 + 4(p.k)(k.q)M^2 + 2(p.b.k)^2m^2
+ 4(p.b.k)^2M^2 - (p.q)^4 + 10(p.p_b)(p.b.q)M^2
\]

\[
+ \frac{4(p.p_b)}{M_W^2} \left[ -2(p.p_W)(p.b.k)(k.p_W) + 4(p.p_W)(p.b.p_W)M^2 + 4(p.p_W)(k.p_W)^2
+ 2(p.b.q)(k.p_W)(q.p_W) - 2(p.p_W)(q.p_W)M^2 - (k.p_W)(q.p_W)M^2 \right]
\]

\[
+ \frac{4}{M_W^2} \left[ (p.b.k)(p.p_W)(q.p_W)M^2 - 2(p.b.q)(p.b.p_W)(k.p_W) + (p.q)(p.b.p_W)(k.p_W)M^2
+ 2(p.q)(p.p_W)^2M^2 - (p.q)(p.p_W)^2M^2 \right]
\]

\[
+ \frac{4(p.p_W)}{M_W^2} \left[ -2(p.b.k)(p.b.p_W) - 2(p.b.k)(p.p_W)M^2 - 2(p.k)(k.p_W)M^2 + 2(p.b.k)(p.b.p_W)(k.q)
- (p.b.q)(q.p_W)M^2 - 2(p.b.k)^2(q.p_W) - 2(p.b.q)(p.p_W)M^2 - 3(p.b.q)(k.p_W)M^2
+ 2(p.p_W)(k.q)M^2 + (k.q)(k.p_W)M^2 + (p.p_W)(p.b.k)M^2 + 2(p.p_W)(p.b.k)^2 \right]
\]
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