SOFT COMPUTING IN DECISION MAKING AND IN MODELING IN ECONOMICS

Multicriteria q-Rung orthopair fuzzy decision analysis: a novel approach based on Archimedean aggregation operators with the confidence levels

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Abstract
The confidence levels can reduce the influence of the unreasonable evaluation value that was given by the decision-maker on the decision-making results. The Archimedean t-norm and t-conorm (ATS) also have many advantages for the processing of uncertain data. Under this environment, the confidence q-rung orthopair fuzzy aggregation operator based on ATS is one of the most successful extensions of confidence q-rung orthopair fuzzy numbers in which we decrease the deviation caused by the subjective perspective of the decision-maker in the multicriteria group decision-making problems. In this paper, we propose weighted, ordered weighted averaging aggregation operators and weighted, ordered weighted geometric aggregation operators based on ATS, respectively. Moreover, the properties and four specific forms associated with aggregation operators are also investigated. In this study, a novel MCGDM approach is introduced by using the proposed operator. A reasonable example is proposed and compared the results that are obtained by our operators and that in the existing literature, so as to verify the rationality and flexible of our method. From the study, we concluded that the proposed method can reduce the impact of extreme data, and make decision-making results more reasonable by considering the attitudes of decision-makers.

Keywords Nonstandard fuzzy sets · Confidence q-rung orthopair fuzzy sets · Archimedean t-norm and t-conorm · Aggregation operator · Multicriteria group decision-making

1 Introduction
In recent years, due to the complexity of objective things, uncertainty and the fuzziness of cognitive-based human thinking, the research on multi-attribute decision-making (MCGDM) problems in uncertain environment has attracted great attention of scholars. The MCGDM is generally a process in which the decision-makers concentrate on selecting the best option among all the alternatives to be selected and some related theories are presented (Wang and Wang 2020; Liu et al. 2019; Bai et al. 2019; Ju et al. 2019). Its prominent advantage is that multi-person decision-making reflects the fairness and rationality, which can greatly reduce the influence of individual decision-making by its own professional backgrounds and subjective factors. For the real decision-making process, the pivotal problem is whether the evaluating value can be expressed reasonably. However, because the decision-making body is generally human, the evaluation value is usually uncertain and vague, which requires a reasonable tool to express it. On the other hand, most extant classical mathematical theory is used to process accurate data, whereas the fuzziness of decision data leads to the inability of traditional mathematical tools to meet the demand. For this, Zadeh (1965) extended the classical sets of the characteristic functions with value of 0 or 1 and presented fuzzy set (FS) theory whose membership function is within [0, 1]. However, we have to consider the degree of hesitancy on some issues, such as the voting model. In order to solve this problem,
Atanassov (1986, 1999) proposed intuitionistic fuzzy sets (IFSs), which have investigated from three aspects: membership degree (MD), nonmembership degree (NMD) and hesitancy degree (HD). Therefore, the theory of IFSs has more practicality and comprehensiveness when it is portraying uncertainty models. Based on which, many researches with regard to IFSs have emerged and applied in medical diagnosis (Shi et al. 2012), pattern recognition (Garg 2017) and group decision-making (GDM) (Li et al. 2018). These studies were mainly carried out from five different fields.

1. **The basic method research:** For instance, operation laws (De et al. 2000), entropy measure (Chen and Li 2010), fuzzy measure (Tan et al. 2009), divergence measure (Joshi and Kumar 2019), distance and similarity measure (Garg 2017; Shi et al. 2012), etc.

2. **The extended MCGDM methods:** Such as Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method (Wan et al. 2013), Dempster–Shafer theory (DST) (Liu and Gao 2019), an acronym in Portuguese of interactive and multi-criteria decision-making (TODIM) (Qin et al. 2017), etc.

3. **The extended aggregation operators (AOs):** It is well known that the AOs can effectively aggregate information; some related methods have been investigated. For example, Xu (2007) first proposed several intuitionistic fuzzy AOs. Xu and Yager (2006) presented some intuitionistic fuzzy geometric AOs.

4. **Combining the IFSs with other methods:** For example, intuitionistic fuzzy rough sets (Zhan et al. 2019; Zhan and Sun 2018; Zhang et al. 2020), interval-valued intuitionistic uncertain linguistic variables (Li et al. 2018), hesitant-intuitionistic fuzzy information (Zhou et al. 2015), etc.

5. **The extended evaluating value range:** For example, interval-valued IFSs (Atanassov 1999), trapezoidal intuitionistic fuzzy numbers (IFNs) (Wan et al. 2013), triangular IFNs (Wan et al. 2013; Qin et al. 2017), etc.

Nevertheless, it is noted that MD ($\mu_Q$) and NMD ($\nu_Q$) satisfy $\mu_Q + \nu_Q > 1$ under certain conditions. For example, a expert provides an evaluation value with MD being 0.9, while NMD is 0.4; this severely exceeds the range of application of IFSs. To handle it, Yager (2013) presented Pythagorean fuzzy sets (PFSs) whose remarkable characteristic is that MD and NMD satisfy $\mu_Q^2 + \nu_Q^2 \leq 1$. Therefore, Yager successfully extended fuzzy theory so that it can more accurately and reasonably denote uncertainty information. Hereafter, some research results of PFSs are gradually commence. For instance, Yager (2014) defined several AOs based on Pythagorean fuzzy environment. Garg (2016, 2017) presented some generalized AOs based on Einstein operations in the Pythagorean fuzzy environment and applied to realistic MCDM problems. Garg (2019) proposed a novel Pythagorean fuzzy geometric AO based on neutral multiplication and power operational rules by considering neutrally treated MD and NMD.

As the MCGDM problem becomes more complex, Yager (2017) proposed q-rung orthopair fuzzy sets (q-ROFSs), which $\mu_Q, \nu_Q$ satisfies $\mu_Q^q + \nu_Q^q \leq 1$. The q-ROFSs can be regarded as the extension and supplement of the IFSs and the PFSs, so it can convey and handle more uncertain information. After that, many scholars have done a lot of research with respect to related operations and applications based on q-ROFSs. For example, Liu and Wang (2018) showed that q-rung orthopair fuzzy information AOs and their applications on MCDM, such as q-rung orthopair fuzzy weighted averaging (q-ROFWA) operator and the q-rung orthopair fuzzy weighted geometric (q-ROFWG) operator. Joshi et al. (2018) proposed interval-valued q-rung orthopair fuzzy sets and their complement operation and aggregation. Liu et al. (2018) proposed q-rung orthopair fuzzy partitioned Heronian mean operators (q-ROFPHM) to solve MCGDM problems, which considers the division of similar attributes in all attributes into one class, further optimizing the interaction between different attributes.

We usually use AOs and other extension MCGDM theory such as DST and TODIM to solve the problem. However, the difference between these two types is that AOs can get the sorting results and specific values, whereas the extended MCGDM only gets the sorting results. Therefore, the AOs can aggregate information more comprehensively. The study of AOs can be divided into operational law and aggregation function, where operational laws have Archimedean t-norm and t-conorm (ATS) and its special cases; aggregation function has Heronian mean (Liu et al. 2018), Bonferroni mean (Liu and Liu 2018), etc.

In the decision-making problems, the evaluation value was given by the expert and should first be ensured to be fair and reasonable, and then, the results obtained by data fusion should not be too inaccurate. However, the above AOs do not take into account the different confidence levels of each expert on the attributes of the evaluation object. To overcome this shortcoming, Xia et al. (2011) proposed a series of confidence-induced weighted aggregation operators in the context of fuzzy sets and hesitant fuzzy sets. Yu (2014) further studied confidence intuitionistic fuzzy information AOs based on the algebraic operations and the Einstein operations, including the confidence intuitionistic fuzzy weighted averaging (CIFWA) operator, confidence intuitionistic fuzzy Einstein-weighted geometric (CIFEWG) operator, etc. After that, Garg (2017) proposed confidence Pythagorean fuzzy information AOs. Based on the existing theories, Joshi and Gegov (2020) proposed confidence q-rung orthopair fuzzy AOs, such as confidence q-rung orthopair fuzzy weighted averaging (Cq-ROFWA) operator, confidence q-rung orthopair fuzzy weighted geometric
(Cq-ROFWG) operator and so forth. For more applications, see (Muhammad et al. 2020; Hamid et al. 2021; Riaz et al.)

As discussed above, we can find that the methods in the existing literature have both advantages and disadvantages, as follows:

1. The confidence level is an indispensable criterion to measure whether the evaluating value was given by experts, which is fair and reasonable. By changing its value, the influence of unreasonable or extreme data on the result can be balanced.
2. Introducing the confidence levels into various fuzzy AOs has significant advantages, which can reduce the influence of decision makers’ subjective factors.
3. Although the confidence levels have been considered in the fuzzy decision-making, it is not sufficient to solve most decision-making problems based on certain kinds of operational rules. The ATS is a more generalized form of several operational rules, which can provide more operational rules for fuzzy AOs and make them adapt to different decision environments.

According to the aforementioned analysis, in order to overcome these shortcomings, we have to develop some new methods to solve MCGDM problem more optimally. For this, we propose several novel AOs by combining the Cq-ROFNs into the ATS and achieve the following goals.

1. The novel AOs can accommodate to various complex decision environments.
2. The novel AOs are easy to understand and calculate, and it has strong practical significance and application.
3. The proposed method can reduce the influence of extreme data and make it more reasonable for the decision results of practical problems.

In this paper, we present some confidence q-rung orthopair fuzzy information AOs based on ATS, including the confidence q-rung orthopair fuzzy weighted averaging operator based on ATS (ATS-Cq-ROFWA), the confidence q-rung orthopair fuzzy geometric operator based on ATS (ATS-Cq-ROFWG), the q-rung orthopair fuzzy ordered weighted averaging operator based on ATS (ATS-Cq-ROFOWA), and the confidence q-rung orthopair fuzzy ordered weighted geometric operator based on ATS (ATS-Cq-ROFOWG). Furthermore, we define a novel MCGDM method based on the proposed AOs. Finally, we illustrate an example to demonstrate the rationality and superiority of the proposed method, and we verify that the proposed method is superior to the existing literatures (Liu and Wang 2018; Liu et al. 2018) for solving real MCGDM problems.

The rest of this paper is composed of the following parts: In Sect. 2, some basic concepts and related operational laws of q-ROFSs and ATS are briefly introduced. In Sect. 3, we present some novel aggregation operators and discuss the properties associated with aggregation operators. Besides, we also give four special forms of each aggregation operator, such as the confidence q-rung orthopair fuzzy Hammer weighted averaging operator (Cq-ROFHW A) and the confidence q-rung orthopair fuzzy Frank weighted geometric operator (Cq-ROFFWG). In Sect. 4, we discuss a novel MCGDM method based on the proposed AOs. In Sect. 5, an example is provided to illustrate the feasibility and advantages of the proposed method. Then, by comparing with the existing integrate method, the correctness and advantages of the proposed method are verified. In Sect. 6, we give some conclusion remarking and future research.

2 Basic definitions and theorems

In the following, we briefly give some relevant basic concepts.

Definition 1 Yager (2017) Let X be a fixed set; a q-ROFS $\mathcal{Q}$ in X can be described as:

$$\mathcal{Q} = \{(x, \mu_Q(x), \nu_Q(x))|x \in X\}$$

where $\mu_Q : X \rightarrow [0, 1]$ represent the MD, $\nu_Q : X \rightarrow [0, 1]$ represent the NMD, and $\mu_Q(x), \nu_Q(x)$ satisfy the condition of $0 \leq (\mu_Q(x))^q + (\nu_Q(x))^q \leq 1 (q \geq 1)$. In addition, $\pi_Q(x) = (1 - (\mu_Q(x))^q - (\nu_Q(x))^q)^{1/2}$ represent the degree of hesitancy, for all $x \in X$.

For convenience, $\phi = (\mu, \nu)$ is called q-rung orthopair fuzzy number (q-ROFN) by Yager (2017).

In this section, we introduce score function proposed by Liu and Wang (2018) to solve the MCGDM problem.

Definition 2 Liu and Wang (2018) If $\phi = (\mu, \nu)$ be a q-ROFN, then its score function is given by:

$$S(\phi) = \mu^q - \nu^q.$$  \hspace{1cm} (2)

Definition 3 Liu and Wang (2018) If $\phi = (\mu, \nu)$ be a q-ROFN, then its accuracy function is given by:

$$H(\phi) = \mu^q + \nu^q.$$  \hspace{1cm} (3)

Definition 4 Liu and Wang (2018) Let $\phi_1 = (\mu_1, \nu_1)$ and $\phi_2 = (\mu_2, \nu_2)$ be two q-ROFNs, then

1. If $S(\phi_1) > S(\phi_2)$, then $\phi_1 > \phi_2$;
2. If $S(\phi_1) = S(\phi_2)$, then
Definition 5 Xia et al. (2012) If a function $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfies the following conditions:

1. $T(1, x) = x$, for all $x$;
2. $T(x, y) = T(y, x)$, for all $x$ and $y$;
3. $T(x, y) = T(T(x, y), z)$, for all $x$, $y$, and $z$;
4. If $x \leq x'$ and $y \leq y'$, then $T(x, y) \leq T(x', y')$.

Then it is called a t-norm.

Definition 6 Xia et al. (2012) If a function $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfies the following conditions:

1. $S(0, x) = x$, for all $x$;
2. $S(x, y) = S(y, x)$, for all $x$ and $y$;
3. $S(x, y) = S(S(x, y), z)$, for all $x$, $y$, and $z$;
4. If $x \leq x'$ and $y \leq y'$, then $S(x, y) \leq S(x', y')$.

Then, it is called a t-conorm.

Definition 7 Xia et al. (2012) Suppose a t-norm function $T(x, y)$ is continuous and $T(x, x) < x$ for all $x \in [0, 1]$; then, it is called Archimedean t-norm(ATS). If the ATS is strictly decreasing in each variable for $x, y \in (0, 1)$, it is called the strict ATS.

Definition 8 Xia et al. (2012) Suppose a t-conorm function $S(x, y)$ is continuous and $S(x, x) < x$ for all $x \in [0, 1]$, then it is called AS. If the AS is strictly increasing in each variable for $x, y \in (0, 1)$, it is called the strict AS.

According to Klement and Mesiar (2005), we can know that $T(x, y)$ is denoted as $T(x, y) = g^{-1}(g(x) + g(y))$ by its additive generator $g$, and $S(x, y)$ is denoted as $S(x, y) = h^{-1}(h(x) + h(y))$ by its additive generator $h$, where $h(t) = g(1 - t)$.

3 q-Rung orthopair fuzzy information AOs with the confidence levels based on ATS

For real MCGDM problems, we need not only the decision-maker to give the evaluation value of the evaluation object, but also to give the confidence levels of the evaluation value. In this section, we propose the confidence q-rung orthopair fuzzy AOs based on ATS (ATS-Cq-ROFAs) along with their special cases. In addition, we also put forward their corresponding properties.
(2) Given that Eq. (6) holds for \( n = k \), we can get

\[
\text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle \right) = \bigoplus_{i=1}^{k} \omega_i (\lambda_i, \alpha_i)
\]

\[
= \left( h^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i h(\mu_i) \right), g^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i g(v_i) \right) \right).
\]

(3) When \( n = k + 1 \), we have

\[
\text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle \right) = \bigoplus_{i=1}^{k} \omega_i (\lambda_i, \alpha_i) \oplus \omega_{k+1} (\lambda_{k+1}, \alpha_{k+1})
\]

\[
= \left( h^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i h(\mu_i) \right), g^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i g(v_i) \right) \right) + h \left( h^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i h(\mu_i) \right) \right),
\]

\[
g^{-1} \left( g^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i g(v_i) \right) \right) + g \left( g^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i g(v_i) + \omega_{k+1} \omega_{k+1} (v_{k+1}) \right) \right)
\]

\[
= \left( h^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i h(\mu_i) + \omega_{k+1} \lambda_{k+1} h(\mu_{k+1}) \right),
\]

\[
g^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i g(v_i) + \omega_{k+1} \lambda_{k+1} g(v_{k+1}) \right) \right).
\]

That is Eq. (6) holds for \( n = k + 1 \).

Now, we prove that ATS-Cq-ROFWA operator is an q-ROFNs.

As can be seen from above, AT function \( g(t) : [0, 1] \rightarrow [0, \infty] \) is strictly decreasing, and AS function \( h(t) \) is strictly increasing, simultaneously, \( h(t) \) and \( g(t) \) are satisfied \( h(t) = g(1 - t) \), and hence,

\[
0 \leq h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right), g^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i g(v_i)) \right) \leq 1
\]

and

\[
h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right) + g^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i g(v_i)) \right)
\]

\[
\leq h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right) + g^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i g(1 - \mu_i)) \right)
\]

\[
= h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right) + 1 - h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right) = 1.
\]

Therefore, this proof is completed. \( \square \)

Next, we present some basic properties of the ATS-Cq-ROFWA operator.

**Theorem 2** (Idempotency) Let \( \alpha_i = (\mu_i, v_i) \) be a collection of q-ROFNs and suppose \( (\alpha, \alpha_1, \alpha_2, \ldots, \alpha_n) \) are equal, which is \( \alpha = \alpha_i = (\mu, v) \), for \( i = 1, 2, \ldots, n \), then

\[
\text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle \right) = \lambda \alpha.
\]

**Proof** When \( \alpha = \alpha_1 = \alpha_2 = \ldots = \alpha_n = (\mu, v) \), we can get

\[
\text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle \right)
\]

\[
= \text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \alpha \rangle, \langle \lambda_2, \alpha \rangle, \ldots, \langle \lambda_n, \alpha \rangle \right)
\]

\[
= \bigoplus_{i=1}^{n} \omega_i (\lambda_i, \alpha_i) = \left( h^{-1} \left( \sum_{i=1}^{n} \omega_i \lambda_i h(\mu) \right),
\]

\[
g^{-1} \left( \sum_{i=1}^{n} \omega_i \lambda_i g(v_i) \right) \right) = \left( h^{-1} (\lambda h(\mu)), g^{-1} (\lambda g(v)) \right) = \lambda \alpha.
\]

\( \square \)

**Theorem 3** (Monotonicity) Suppose \( \alpha_i = (\alpha_1, \alpha_2, \ldots, \alpha_n) \), \( \tilde{\alpha}_i = (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) \) be two collections of q-ROFNs, when \( \mu_i \leq \tilde{\mu}_i \) and \( v_i \geq \tilde{v}_i \), we have

\[
\text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle \right) \leq \text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \tilde{\alpha}_1 \rangle, \langle \lambda_2, \tilde{\alpha}_2 \rangle, \ldots, \langle \lambda_n, \tilde{\alpha}_n \rangle \right).
\]

**Proof** We have learnt that \( h(t) \) and \( g(t) \) are strictly increasing function and strictly decreasing function, respectively. Since \( \mu_i \leq \tilde{\mu}_i \) and \( v_i \geq \tilde{v}_i \), then we have

\[
h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right) \leq h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\tilde{\mu}_i)) \right)
\]

\[
g^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i g(v_i)) \right) \geq g^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i g(\tilde{v}_i)) \right).
\]

Therefore,

\[
\text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle \right) \leq \text{ATS} - Cq - ROFWA \left( \langle \lambda_1, \tilde{\alpha}_1 \rangle, \langle \lambda_2, \tilde{\alpha}_2 \rangle, \ldots, \langle \lambda_n, \tilde{\alpha}_n \rangle \right).
\]

\( \square \)
Theorem 4 (Boundedness) Suppose \( \hat{a}_i = (\hat{a}_i, \hat{a}_2, \ldots, \hat{a}_n) \) be a collection of q-ROFNs, \( \hat{a}_{\min} = \left( \min_i \{\lambda_i, \mu_i\}, \max_i \{\nu_i, \bar{\nu}_i\} \right) \), \( \hat{a}_{\max} = \left( \max_i \{\lambda_i, \mu_i\}, \min_i \{\nu_i, \bar{\nu}_i\} \right) \), then we have \( \hat{a}_{\min} \preceq \text{ATS}\!-\!Cq\!-\!ROFWA ((\lambda_1, \hat{a}_1), (\lambda_2, \hat{a}_2), \ldots, (\lambda_n, \hat{a}_n)) \preceq \hat{a}_{\max} \).

Proof The proof is similar to Theorem 3; consequently, it is omitted.

In the following, we give some series of special AOs for different additive generator \( g(t) \).

1. If \( g(t) = -\log(t^q) \), we can get confidence q-rung orthopair fuzzy Tung algebraic weighted averaging (Cq-ROFWA) (Joshi and Gegov 2020) operator, that is

\[
\text{Cq-ROFWA} ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \left( \prod_{i=1}^{n} (1 + \mu_i^q \lambda_i^{\omega q}) - (1 - \mu_i^q \lambda_i^{\omega q}) \right)^{\frac{1}{q}},
\]

2. If \( g(t) = \log \left( \frac{2 - \rho t}{\rho t} \right) \), we can get confidence q-rung orthopair fuzzy Einstein weighted averaging (Cq-ROFEWA) operator, that is

\[
\text{Cq-ROFEWA} ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \left( \prod_{i=1}^{n} (1 + \mu_i^q \lambda_i^{\omega q}) - (1 - \mu_i^q \lambda_i^{\omega q}) \right)^{\frac{1}{q}},
\]

3. If \( g(t) = -\log \left( \frac{2 + (1-\rho)t}{\rho t} \right) \), we can get confidence q-rung orthopair fuzzy Frank weighted averaging (Cq-ROFWA) operator, that is

\[
\text{Cq-ROFWA} ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \left( \prod_{i=1}^{n} (1 + \mu_i^q \lambda_i^{\omega q}) - (1 - \mu_i^q \lambda_i^{\omega q}) \right)^{\frac{1}{q}}, \]

4. If \( g(t) = \log \left( \frac{\rho - 1}{\rho t^q - 1} \right), \rho > 1 \), we can get confidence q-rung orthopair fuzzy Frank weighted averaging (Cq-ROFWA) operator, that is

\[
\text{Cq-ROFWA} ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \left( \prod_{i=1}^{n} \left( \frac{\rho - 1 - \mu_i^q \lambda_i^{\omega q}}{(\rho - 1)} \right) \right)^{\frac{1}{q}}.
\]

Remark 3

1. If \( \rho \to 1 \), then the Cq-ROFWA operator reduces to the Cq-ROFWA operator (Joshi and Gegov 2020);
2. If \( \rho \to 1 \) and \( \lambda_i = 1 \), then the Cq-ROFWA operator reduces to the q-ROFWA operator (Liu and Wang 2018).

3.2 ATS-Cq-ROFWG operator

Definition 10 Let \( \alpha_i = (\mu_i, v_i)(i = 1, 2, \ldots, n) \) be a collection of q-ROFNs. The ATS-Cq-ROFWG operator is denoted as

\[
\text{ATS-Cq-ROFWG} ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \bigotimes_{i=1}^{n} (\alpha_i^{t^q})^{\omega_i}
\]

where \( a = 1 + (\rho - 1)\mu_i^q, b = 1 - \mu_i^q, c = v_i^q, d = 1 + (\rho - 1) \left( 1 - v_i^q \right) \).

Remark 4 If \( \lambda_1 = \lambda_2 = \ldots = \lambda_n = 1 \), then the ATS-Cq-ROFWG operator reduces to the q-rung orthopair
fuzzy weighted geometric operator based on ATS(ATS-q-ROFWG)

\[
ATS-q-ROFWG(\langle \alpha_1 \rangle, \langle \alpha_2 \rangle, \ldots, \langle \alpha_n \rangle) = \bigotimes_{i=1}^{n} (\alpha_i)^{\omega_i}
\]

(12)

**Theorem 5** Let \( \alpha_i = (\mu_i, v_i) (i = 1, 2, \ldots, n) \) be a collection of q-ROFNs; and \( \lambda_i \) and \( \omega_i \) are the confidence levels and weight vector of \( \alpha_i \), respectively. Then, the aggregated value of \( \alpha_i \) obtained by ATS-Cq-ROFWG operator is an q-ROFNs and

\[
ATS-Cq-ROFWG (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle) = \bigotimes_{i=1}^{n} (\omega_i^{\lambda_i})^{\omega_i} = \left( g^{-1} \left( \sum_{i=1}^{n} \omega_i \lambda_i g(\mu_i) \right) \right) \cdot h^{-1} \left( \sum_{i=1}^{n} \omega_i \lambda_i h(v_i) \right)
\]

(13)

where \( 0 \leq \lambda_i \leq 1, \sum_{i=1}^{n} \omega_i = 1 \) and \( \omega_i \in (0, 1) \).

**Proof** Equation (13) can be proved by mathematical induction.

1. For \( n = 2 \), we can get

\[
ATS-Cq-ROFWG (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle) = \bigotimes_{i=1}^{2} (\omega_i^{\lambda_i})^{\omega_i} = (\omega_1^{\lambda_1})^{\omega_1} \bigotimes (\omega_2^{\lambda_2})^{\omega_2}
\]

\[
= \left( g^{-1} \left( g \left( g^{-1} (\omega_1 g (g^{-1} (\lambda_1 g(\mu_1)))) \right) \right) \right) \cdot h^{-1} \left( h \left( h^{-1} (\omega_1 h (h^{-1} (\lambda_1 h(v_1)))) \right) \right)
\]

\[
= g^{-1} \left( (\omega_1 \lambda_1 g(\mu_1) + \omega_2 \lambda_2 g(\mu_2)) \right) \cdot h^{-1} \left( (\omega_1 \lambda_1 h(v_1) + \omega_2 \lambda_2 h(v_2)) \right).
\]

2. Given that Eq. (13) holds for \( n = k \), we can get

\[
ATS-Cq-ROFWG (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle) = \bigotimes_{i=1}^{n} (\omega_i^{\lambda_i})^{\omega_i} = \left( g^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i g(\mu_i) \right) \right) \cdot h^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i h(v_i) \right)
\]

3. When \( n = k + 1 \), we have

\[
ATS-Cq-ROFWG (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle) = \bigotimes_{i=1}^{k} (\omega_i^{\lambda_i})^{\omega_i} \bigotimes (\omega_{k+1}^{\lambda_{k+1}})^{\omega_{k+1}}
\]

\[
= \left( g^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i g(\mu_i) \right) \right) \cdot h^{-1} \left( \sum_{i=1}^{k} \omega_i \lambda_i h(v_i) \right) \cdot g^{-1} \left( \omega_{k+1} \lambda_{k+1} g(\mu_{k+1}) \right) \cdot h^{-1} \left( \omega_{k+1} \lambda_{k+1} h(v_{k+1}) \right)
\]

That is Eq. (13) holds for \( n = k + 1 \).

Now, we prove that ATS-Cq-ROFWG operator is an q-ROFNs.

As can be seen from above, AT function \( g(t) : [0, 1] \rightarrow [0, \infty] \) is strictly decreasing, and AS function \( h(t) \) is strictly increasing; simultaneously, \( h(t) \) and \( g(t) \) are satisfied \( h(t) = g(1-t) \), and hence,

\[
0 \leq h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right) \cdot g^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i g(\mu_i)) \right) \leq 1
\]

and

\[
\sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \leq h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right)
\]

\[
+ g^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i g(1-\mu_i)) \right)
\]

\[
= h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right) + 1 - h^{-1} \left( \sum_{i=1}^{n} \omega_i (\lambda_i h(\mu_i)) \right)
\]

\[
= 1.
\]

Therefore, the proof is completed.  \( \square \)
Next, similar to ATS-Cq-ROFWA, we present some basic properties of the ATS-Cq-ROFWG operator.

**Theorem 6** (Idempotency) Let $\alpha_i = (\mu_i, v_i)$ be a collection of q-ROFNs and suppose $(\alpha, \alpha_1, \alpha_2, \ldots, \alpha_n)$ are equal, which is $\alpha = \alpha_i = (\mu, v)$, for $i = 1, 2, \ldots, n$, then

$$\text{ATS-Cq-ROFWG}((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \alpha.$$

**Theorem 7** (Monotonicity) Suppose $\alpha_i = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, $\bar{\alpha}_i = (\bar{\alpha_1}, \bar{\alpha_2}, \ldots, \bar{\alpha_n})$ be two collections of q-ROFNs; when $\mu_i \leq \bar{\mu}_i$ and $v_i \geq \bar{v}_i$, we have

$$\text{ATS-Cq-ROFWG}((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) \leq \text{ATS-Cq-ROFWG}((\lambda_1, \bar{\alpha}_1), (\lambda_2, \bar{\alpha}_2), \ldots, (\lambda_n, \bar{\alpha}_n)).$$

**Theorem 8** (Boundedness) Suppose $\bar{\alpha}_i = (\bar{\alpha_1}, \bar{\alpha_2}, \ldots, \bar{\alpha_n})$ be a collection of q-ROFNs, $\bar{\alpha}_{\min} = \left(\min_i \mu_{\bar{\alpha}_i}^{\lambda_i}\right)$, and $\bar{\alpha}_{\max} = \left(\max_i \mu_{\bar{\alpha}_i}^{\lambda_i}, \min_i \mu_{\bar{\alpha}_i}^{\lambda_i}\right)$, then we have

$$\bar{\alpha}_{\min} \leq \text{ATS-Cq-ROFWG}((\lambda_1, \bar{\alpha}_1), (\lambda_2, \bar{\alpha}_2), \ldots, (\lambda_n, \bar{\alpha}_n)) \leq \bar{\alpha}_{\max}.$$

Next, we give some series of special AOs for different additive generator $g(t)$.

1. If $g(t) = -\log(t^q)$, we can get confidence q-rung orthopair fuzzy algebraic weighted geometric (Cq-ROFWG) (Joshi and Gegov 2020) operator, that is

$$Cq-ROFWG((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n))$$

$$= \left(\prod_{i=1}^{n} (\mu_i)^{\lambda_i\alpha_i}, \left(1 - \prod_{i=1}^{n} (1 - v_i^q)^{\lambda_i\alpha_i}\right)^{\frac{1}{q}}\right).$$

(14)

2. If $g(t) = \log\left(\frac{2 \rho t^q}{q^q}\right)$, we can get confidence q-rung orthopair fuzzy Einstein weighted geometric (Cq-ROFEWG) operator, that is

$$Cq-ROFEWG((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n))$$

$$= \left(\left(\prod_{i=1}^{n} (\mu_i^q)^{\lambda_i\alpha_i}\right)^{\frac{1}{q}}, \left(\prod_{i=1}^{n} \left(2 - \mu_i^q\right)^{\lambda_i\alpha_i} + \prod_{i=1}^{n} (\mu_i^q)^{\lambda_i\alpha_i}\right)^{\frac{1}{q}}\right).$$

(15)

(3) If $g(t) = \log\left(\frac{\rho (1 - \rho)^q}{\rho^q - 1}\right)$, we can get confidence q-rung orthopair fuzzy Hammer weighted geometric (Cq-ROFHWG) operator, that is

$$Cq-ROFHWG((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n))$$

$$= \left(\left(\prod_{i=1}^{n} (b)^{\lambda_i\alpha_i} + (\rho - 1) \prod_{i=1}^{n} (a)^{\lambda_i\alpha_i}\right)^{\frac{1}{q}}, \left(\prod_{i=1}^{n} (c)^{\lambda_i\alpha_i} - (\rho - 1) \prod_{i=1}^{n} (d)^{\lambda_i\alpha_i}\right)^{\frac{1}{q}}\right).$$

(16)

Remark 5

1. If $\rho = 1$, then the Cq-ROFHWG operator reduces to the Cq-ROFWG operator (Joshi and Gegov 2020);
2. If $\rho = 2$, then the Cq-ROFHWG operator reduces to the Cq-ROFEWG operator;
3. If $\rho = 1$ and $\lambda_i = 1$, then the Cq-ROFHWG operator reduces to the q-ROFWG operator (Liu and Wang 2018);
4. If $q = 2$, $\rho = 2$ and $\lambda_i = 1$, then the Cq-ROFHWG operator reduces to the PF EW operator (Garg 2017).

(4) If $g(t) = \log\left(\frac{\rho - 1}{\rho^q - 1}\right)$, we can get confidence q-rung orthopair fuzzy Frank weighted geometric (Cq-ROFFWG) operator, that is

$$Cq-ROFFWG((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n))$$

$$= \left(\left(\prod_{i=1}^{n} \left(1 + \rho \frac{a_i q - 1}{(\rho - 1)}\right)^{\lambda_i\alpha_i}\right)^{\frac{1}{q}}, \left(1 - \prod_{i=1}^{n} \left(1 + \rho \frac{a_i q - 1}{(\rho - 1)}\right)^{\lambda_i\alpha_i}\right)^{\frac{1}{q}}\right).$$

(17)

**Remark 6**

1. If $\rho \to 1$, then the Cq-ROFFWG operator reduces to the Cq-ROFWG operator (Joshi and Gegov 2020);
2. If $\rho \to 1$ and $\lambda_i = 1$, then the Cq-ROFFWG operator reduces to the q-ROFWG operator (Liu and Wang 2018).
3.3 ATS-Cq-ROFOWA operator and ATS-Cq-ROFOWG operator

**Definition 11** Let \( \alpha_i = (\mu_i, v_i) (i = 1, 2, \ldots, n) \) be a collection of q-ROFNs. The ATS-Cq-ROFOWA operator is denoted as:

\[
ATS-C_q-ROFOWA (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle) = \bigoplus_{i=1}^{n} \omega_i (\lambda_{\sigma(i)}, \alpha_{\sigma(i)})
\]

(18)

and the ATS-Cq-ROFOWG operator is denoted as:

\[
ATS-C_q-ROFOWG (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle)
= \bigotimes_{i=1}^{n} (\alpha_{\sigma(i)}^{\lambda_{\sigma(i)}})^{\omega_i}
\]

(19)

where \( \lambda_i \) and \( \omega_i \) are the confidence levels and associated weight vector of \( \alpha_i \), respectively, and taking \( 0 \leq \lambda_i \leq 1 \), \( \sum_{i=1}^{n} \omega_i = 1 \) and \( \omega_i \in (0, 1) \). In addition, \( \lambda_{\sigma(i)} \alpha_{\sigma(i)} \) and \( \alpha_{\sigma(i)}^{\lambda_{\sigma(i)}} \) are the \( i \)-th largest of \( \lambda_i \alpha_i \) and \( \alpha_i^{\lambda_i} \), respectively, and then, the aggregated value of \( \alpha_i \) obtained by ATS-Cq-ROFOWA operator is an q-ROFNs and

\[
ATS-C_q-ROFOWA (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle) = \Big( h^{-1} \Big( \sum_{i=1}^{n} \omega_i (\lambda_{\sigma(i)} h(\mu_{\sigma(i)})) \Big) \Big)
\]

\[\cdot \quad \Bigg( g^{-1} \Big( \sum_{i=1}^{n} \omega_i (\lambda_{\sigma(i)} g(v_{\sigma(i)})) \Big) \Bigg) \Bigg)
\]

(20)

In addition, the aggregated value of \( \alpha_i \) obtained by ATS-Cq-ROFOWG operator is also an q-ROFNs and

\[
ATS-C_q-ROFOWG (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle) = \Bigg( g^{-1} \Bigg( \sum_{i=1}^{n} \omega_i (\lambda_{\sigma(i)} g(\mu_{\sigma(i)})) \Bigg) \Bigg)
\]

\[\cdot \quad \Bigg( h^{-1} \Bigg( \sum_{i=1}^{n} \omega_i (\lambda_{\sigma(i)} h(v_{\sigma(i)})) \Bigg) \Bigg) \Bigg)
\]

(21)

**Proof** Similar to Theorem 1 and Theorem 5, so we omit it.

Similarly, we can get the same properties as ATS-Cq-ROFOWA or ATS-Cq-ROFOWG, so we omit it. In addition, we also give some series of special AOs for different additive generator \( g(t) \).

(1) If \( g(t) = -\log(t^q) \), we can get confidence q-rung orthopair fuzzy algebraic ordered weighted averaging (Cq-ROFOWA) operator (Joshi and Gegov 2020), that is

\[
C_q-ROFOWA (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle)
= \left( \left( 1 - \prod_{i=1}^{n} (1 - \mu_{\sigma(i)}^{q})^{\lambda_{\sigma(i)}} \right)^{\frac{1}{q}}, \prod_{i=1}^{n} (v_{\sigma(i)})^{\frac{1}{q}} \right).
\]

(22)

And we can get confidence q-rung orthopair fuzzy algebraic ordered weighted geometric (Cq-ROFOWG) operator (Joshi and Gegov 2020), that is

\[
C_q-ROFOWG (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle)
= \left( \left( \prod_{i=1}^{n} (1 + \mu_{\sigma(i)}^{q})^{\lambda_{\sigma(i)}} \right)^{\frac{1}{q}}, \prod_{i=1}^{n} (v_{\sigma(i)})^{\frac{1}{q}} \right).
\]

(23)

(2) If \( g(t) = \log \left( \frac{2}{t} \right) \), we can get confidence q-rung orthopair fuzzy Einstein ordered weighted averaging (Cq-ROFEOWA) operator, that is

\[
C_q-ROFEOWA (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle)
= \left( \left( \prod_{i=1}^{n} \left( 1 + \mu_{\sigma(i)}^{q} \right)^{\lambda_{\sigma(i)}^{\alpha_{\sigma(i)}}} \right)^{\frac{1}{q}}, \prod_{i=1}^{n} (v_{\sigma(i)})^{\frac{1}{q}} \right).
\]

(24)

And we can get confidence q-rung orthopair fuzzy Einstein ordered weighted geometric (Cq-ROFEOWG) operator, that is

\[
C_q-ROFEOWG (\langle \lambda_1, \alpha_1 \rangle, \langle \lambda_2, \alpha_2 \rangle, \ldots, \langle \lambda_n, \alpha_n \rangle)
= \left( \left( \prod_{i=1}^{n} \left( 2 - \mu_{\sigma(i)}^{q} \right)^{\lambda_{\sigma(i)}^{\alpha_{\sigma(i)}}} \right)^{\frac{1}{q}}, \prod_{i=1}^{n} (v_{\sigma(i)})^{\frac{1}{q}} \right).
\]
\[
\left( \prod_{i=1}^{n} \left( 1 + v_{\sigma(i)}^q \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} + \left( 1 + v_{\sigma(i)}^q \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} \right) \right)^{\frac{1}{q}}.
\]

(25)

(3) If \( g(t) = \log \left( \frac{t+1-(\rho-1)t^\sqrt{q}}{t} \right) \), we can get confidence q-rung orthopair fuzzy Hammer ordered weighted averaging (Cq-ROFHOWA) operator, that is

\[
Cq-ROFHOWA ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \left( \prod_{i=1}^{n} \left( a \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} - \prod_{i=1}^{n} \left( b \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} \right)^{\frac{1}{q}}.
\]

(26)

where \( a_1 = 1 + (\rho - 1) \mu_{\sigma(i)}^q, b_1 = 1 - \mu_{\sigma(i)}^q, c_1 = v_{\sigma(i)}^q, d_1 = 1 + (\rho - 1) (1 - v_{\sigma(i)}^q) \).

Remark 7

1. If \( \rho = 1 \), then the Cq-ROFHOWA operator reduces to the Cq-ROFFOW A operator (Joshi and Gegov 2020);
2. If \( \rho = 2 \), then the Cq-ROFHOWA operator reduces to the Cq-ROFEOFW A operator;
3. If \( q = 2 \), \( \rho = 1 \) and \( \lambda_i = 1 \), then the Cq-ROFHOWA operator reduces to the PFOWG operator (Garg 2017);
4. If \( q = 2 \), \( \rho = 2 \) and \( \lambda_i = 1 \), then the Cq-ROFHOWA operator reduces to the PFEOFW G operator (Garg 2017).

(4) If \( g(t) = \log \left( \frac{t-1}{t^\sqrt{q}-1} \right) \), we can get confidence q-rung orthopair fuzzy Frank ordered weighted averaged geometric (Cq-ROFFOWG) operator, that is

\[
Cq-ROFFOWG ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \left( \prod_{i=1}^{n} \left( a \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} - \prod_{i=1}^{n} \left( b \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} \right)^{\frac{1}{q}}.
\]

(27)

\[
\left. \begin{array}{l}
a_1 = \mu_{\sigma(i)}^q, b_1 = 1 + (\rho - 1) (1 - \mu_{\sigma(i)}^q), c_1 = \rho_{\sigma(i)}^q, d_1 = 1 + (\rho - 1) (1 - \rho_{\sigma(i)}^q). \\
\end{array} \right\}
\]

Remark 8

1. If \( \rho = 1 \), then the Cq-ROFHOWG operator reduces to the Cq-ROFOW A operator (Joshi and Gegov 2020);
2. If \( \rho = 2 \), then the Cq-ROFHOWG operator reduces to the Cq-ROFEOFW G operator;
3. If \( q = 2 \), \( \rho = 1 \) and \( \lambda_i = 1 \), then the Cq-ROFHOWG operator reduces to the PFOWA operator (Garg 2016);
4. If \( q = 2 \), \( \rho = 2 \) and \( \lambda_i = 1 \), then the Cq-ROFHOWG operator reduces to the PFEOFW A operator (Garg 2016).

(4) If \( g(t) = \log \left( \frac{t-1}{t^\sqrt{q}-1} \right) \), we can get confidence q-rung orthopair fuzzy Frank ordered weighted averaged geometric (Cq-ROFFOWG) operator, that is

\[
Cq-ROFFOWG ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \left( \prod_{i=1}^{n} \left( a \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} - \prod_{i=1}^{n} \left( b \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} \right)^{\frac{1}{q}}.
\]

(28)

\[
\left. \begin{array}{l}
a_1 = \mu_{\sigma(i)}^q, b_1 = 1 + (\rho - 1) (1 - \mu_{\sigma(i)}^q), c_1 = \rho_{\sigma(i)}^q, d_1 = 1 + (\rho - 1) (1 - \rho_{\sigma(i)}^q). \\
\end{array} \right\}
\]

Remark 9

1. If \( \rho \to 1 \), then the Cq-ROFOW A operator reduces to the Cq-ROFOW A operator (Joshi and Gegov 2020);
2. If \( q \to 1 \), then the Cq-ROFOW A operator reduces to the Cq-ROFOW A operator (Garg 2016).

(4) If \( g(t) = \log \left( \frac{t-1}{t^\sqrt{q}-1} \right) \), we can get confidence q-rung orthopair fuzzy Frank ordered weighted averaged geometric (Cq-ROFFOWG) operator, that is

\[
Cq-ROFFOWG ((\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \ldots, (\lambda_n, \alpha_n)) = \left( \prod_{i=1}^{n} \left( a \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} - \prod_{i=1}^{n} \left( b \right)^{\lambda_{\sigma(i)} a_{\sigma(i)}} \right)^{\frac{1}{q}}.
\]

(29)
Remark 10

1. If $\rho \to 1$, then the Cq-ROFFOWG operator reduces to the Cq-ROFOFWG operator (Joshi and Gegov 2020); 
2. If $q = 2, \rho \to 1$ and $\lambda_i = 1$, then the Cq-ROFFOWG operator reduces to the PFOWG operator (Garg 2017).

4 MCGDM approach based on ATS-Cq-ROF information aggregation operators

MCGDM is widely applied in many areas of real life, and the correct and reasonable decision-making result is pursued by the decision-maker. Therefore, decision-making method is particularly important, and different decision-making methods should be selected in different decision situations. For this, in this section, we propose an approach to solve the MCGDM problem based on the proposed AOs theory.

Let $e = \{e_1, e_2, \ldots, e_e\}$ be a set of experts, whose weight vector is $\omega_k (k = 1, 2, \ldots, t)$. Let $A_i (i = 1, 2, \ldots, m)$ are $m$ alternatives, and $G_j (j = 1, 2, \ldots, n)$ are $n$ criteria whose weight vector is $w_j (j = 1, 2, \ldots, n)$, satisfying $w_j > 0, \sum_{j=1}^{n} w_j = 1$. Each expert gives the decision matrix $Q^p = \{(\lambda_{ij}^p, \mu_{ij}^p, \nu_{ij}^p)\}_{m \times n}$ for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$ and $p = 1, 2, \ldots, l$, which contains the confidence level values that take into account their familiarity with the evaluation fields and the evaluation values of $m$ alternatives under $n$ criteria simultaneously. In the following, specific decision steps are given.

Step 1. From the outset, acquire the normalized q-rung orthopair fuzzy decision matrix. As is known to all, the attribute set is generally presented in two opposite sorts. On the one hand, the beneficial type has a positive effect on the outcomes. On the other hand, the cost type has a negative effect on the outcomes. If there are cost criteria values, then we need to convert them to benefit criteria values by the following formula. Otherwise, this step can be neglected.

$$\tilde{\xi}_{ij} = \begin{cases} (\mu_{ij}^b, \nu_{ij}^b), & \text{for benefit criteria } G_j \\ (\mu_{ij}^c, \nu_{ij}^c)^c, & \text{for cost criteria } G_j. \end{cases}$$

Step 2. Compose all the q-rung orthopair fuzzy decision matrices to the integrated decision matrix $Q$ by using the four kinds of the proposed AOs:

$$\begin{align*}
\text{ATS-Cq-ROFWA} (\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \\
\ldots, (\lambda_n, \alpha_n) = \bigoplus_{i=1}^{n} \omega_i (\lambda_{\sigma_i}, \alpha_{\sigma_i}) \\
\text{or}
\end{align*}$$

$$\begin{align*}
\text{ATS-Cq-ROFWG} (\lambda_1, \alpha_1), (\lambda_2, \alpha_2), \\
\ldots, (\lambda_n, \alpha_n) = \bigotimes_{i=1}^{n} \lambda_{\sigma_i} (\lambda_{\sigma_i} \alpha_{\sigma_i})^{\omega_i}.
\end{align*}$$

Among them, the ordered weighted AO is the evaluation value of the priority aggregation with high confidence levels.

Step 3. Based on the collective decision matrix $Q$ of Step 2, the evaluation values $\phi_i (i = 1, \ldots, m)$ of each alternative under various criteria are aggregated by using two kinds of information AOs:

$$\begin{align*}
\text{ATS-q-ROFWA} (\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigoplus_{i=1}^{n} \omega_i (\alpha_i) \\
\text{or}
\end{align*}$$

$$\begin{align*}
\text{ATS-q-ROFWG} (\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigotimes_{i=1}^{n} (\alpha_i)^{\omega_i}.
\end{align*}$$

Step 4. Based on the evaluation values $\phi_i$ obtained in Step 3, we can take advantage of the score function Eq. (2) and accuracy function Eq. (3) to compute the score values and accuracy values $S(\phi_i) (i = 1, \ldots, m)$ of each alternative.

Step 5. According to the score value or accuracy value, rank the alternatives and compare them to select the best alternative.

5 Application example on MCGDMs

5.1 Application of the defined MCGDM approach

India’s shipbuilding industry plans to buy a batch of welding robots for shipbuilding, ship repair, offshore engineering repair, and electromechanical equipment repair. At present, there are five robot companies $A = \{A_1, A_2, A_3, A_4, A_5\}$ to be selected, and their criteria have been investigated from six aspects, namely, load capacity ($G_1$), welding quality ($G_2$), life ($G_4$), brand ($G_4$), welding efficiency ($G_5$), after-sale service ($G_6$). Suppose that $\omega = (0.36, 0.35, 0.29)$ is the weight vector of a set of experts $e = \{e_1, e_2, e_3\}$, and attribute weight is $w = (0.15, 0.24, 0.14, 0.17, 0.19, 0.11)$. Three experts gave the evaluation value for each attribute value of the three alternatives and gave the confidence levels of the corresponding evaluation value, which was shown by using the decision
matrices $Q_1$, $Q_2$, $Q_3$ (see Tables 1, 2, 3). Further, we demand to select the best alternative and take full advantage of the novel method in the previous section to solve this problem. In the following, in order to show the MCGDM process based on the Cq-ROFHWA operator, we make use of the operator to solve the example, where $q=3$, $\rho=3$.

Step 1. Take into account that the six criteria are the benefit type, so it is not necessary to normalize with respect to the evaluation value.

Step 2. Compose all the q-rung orthopair fuzzy decision matrices $Q_1$, $Q_2$, $Q_3$ to the integrated decision matrix $Q$ by using the Cq-ROFHWA operator, as shown in Table 4.

Step 3. According to decision matrix $Q$, the corresponding evaluation values $\phi_i$ ($i=1, \ldots, m$) of each alternative under various criteria are aggregated by using q-ROFHWA, shown as follows:

$$
\phi_1 = (0.592, 0.394), \quad \phi_2 = (0.557, 0.349), \\
\phi_3 = (0.657, 0.283), \quad \phi_4 = (0.571, 0.424), \quad \phi_5 = (0.619, 0.402).
$$

Step 4. Based on the score function Eq. (2), we can obtain

$$
S(\phi_1) = 0.1463, \quad S(\phi_2) = 0.1303, \quad S(\phi_3) = 0.2609, \\
S(\phi_4) = 0.1099, \quad S(\phi_5) = 0.1722.
$$
Step 5. According to the results of score function, we can choose the best alternative in descending order. Because

$$A_3 > A_5 > A_1 > A_2 > A_4.$$ 

So, the best robot company is $A_3$.

5.2 Discussion about the influence of different aggregation operators and parameter values $\rho$ and $q$ on the results

In the following, we explore the influence of different AOs and parameter $\rho$ on the results, as shown in Figs. 1, 2, 4, 5, 6 and Tables 5, 6, 7, 8. It is the ranking result obtained by adopting weighted, ordered weighted geometric operators, respectively, based on four operations of ATS when $q = 3$. From Figs. 1, 2, 4, 5, 6 and Tables 5, 6, 7, 8, we can summarize some important rules, shown as follows.

1. With the change of the parameter $\rho$, the algebraic operations and the Einstein operations can be served as specific forms of the Hamacher operations. Therefore, we can find that if weighted, ordered weighted geometric operators are used to calculate, the score values and the accuracy values decrease with the decrease of parameter value $\rho$, whereas if weighted, ordered weighted averaging operators are used to calculate, the result is exactly the opposite. Furthermore, because the algebraic operation is the specific form of the Frank operation when $\rho = 1$, we can
find that the ranking result of Frank operation and the Hamacher operation AOs changes in the same way.

2. We know that each parameter value \( \rho \) determines a kind of operation law based on ATS. So, by using different AOs when the parameter value \( \rho \) is the same, it has been observed that the ranking results are slightly different. However, we can know that the best alternative is \( A_3 \).

3. As discussed above, the parameter value \( \rho \) can be served as the expert’s attitude; we can find that there is a negative correlation between the value of the parameter \( \rho \) and the score value of the averaging operator. As the parameter value \( \rho \) becomes larger, the pessimism of expert attitude will increase. Therefore, the expert can choose the parameter value \( \rho \) flexibly according to his attitude and the aggregation operator.

Furthermore, in order to analyze the influence of parameter value \( q \) on the result, we list the results of different \( q \) values based on the proposed Cq-ROFHW A operator in Table 9.

From Table 9, on the one hand, it has been observed that if \( q = 3 \), the worst alternative is \( A_4 \), whereas if \( q > 3 \) we get the different result that the worst alternative is \( A_2 \). In spite of this, the best alternative is \( A_3 \). On the other hand, we can find that the score value \( S(A_i) \) decreases with the increase of parameter value \( q \). When selecting the value of \( q \), we should select the smallest \( q \) value in case of satisfying \( \mu_Q + \nu_Q \leq 1 \).

From Figs. 7, 8, we can find that the score values and the accuracy values of ATS-Cq-ROFHOWG increase with the increase in parameter value \( \rho \) and the decrease of parameter value \( q \).

From Figs. 9, 10, we can find that the score values and the accuracy values of ATS-Cq-ROFHW A decrease with the increase in parameter value \( \rho \) and \( q \) value.

5.3 Comparing with the existing literature (Peng et al. 2018; Liu and Chen 2017; Liu and Liu 2018)

In the following, we demand to investigate the rationality and effectiveness of the proposed MCGDM method. To this end, three existing different methods were chosen to handle this example and compare it with the ranking results. And we compare them with our proposed method concerning the results, which is shown in Table 10 (assume that the confidence level of all evaluation value is equal to 1, \( q = 3 \)).

(1) Comparing Analysis with Peng et al.’s Method Peng et al. (2018) Based on q-Rung Orthopair Fuzzy Weighted Exponential Aggregation (q-ROFWEA) Operator

From Peng et al. (2018), we can find that this method cannot calculate the above example. In the following, we analyze the causes and defects.

1. In terms of information data, both of these methods are based on q-ROFNs, so their common advantage is that they can deal with the situation that the evaluation value satisfies \( \mu_Q + \nu_Q \leq 1 \).

2. In terms of operational laws, Peng et al.’s method (Peng et al. 2018) only considers the algebraic operation rules and cannot flexibly adjust the operation rules according to the attitude of the decision-maker. Whereas the proposed method is based on ATS operation rules, thus the appro-
Table 5  Ranking results based on special ATS-Cq-ROFWA aggregation operators

| Methods  | Score values         | Ranking       |
|----------|----------------------|---------------|
| Cq-ROFWA | $S(A_1) = 0.2050, S(A_2) = 0.1645, S(A_3) = 0.3075, S(A_4) = 0.1672, S(A_5) = 0.2244$ | $A_3 > A_5 > A_1 > A_4 > A_2$ |
| Cq-ROFEWA| $S(A_1) = 0.1690, S(A_2) = 0.1433, S(A_3) = 0.2784, S(A_4) = 0.1323, S(A_5) = 0.1922$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFHWA($\rho = 3$) | $S(A_1) = 0.1463, S(A_2) = 0.1303, S(A_3) = 0.2609, S(A_4) = 0.1099, S(A_5) = 0.1722$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFFWA($\rho = 2$) | $S(A_1) = 0.2307, S(A_2) = 0.1911, S(A_3) = 0.3567, S(A_4) = 0.1861, S(A_5) = 0.2565$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |

Table 6  Ranking results based on special ATS-Cq-ROFWWG aggregation operators

| Methods  | Score values         | Ranking       |
|----------|----------------------|---------------|
| Cq-ROFWG | $S(A_1) = 0.1909, S(A_2) = 0.1739, S(A_3) = 0.3116, S(A_4) = 0.1383, S(A_5) = 0.1967$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFEWG| $S(A_1) = 0.2287, S(A_2) = 0.2013, S(A_3) = 0.3458, S(A_4) = 0.1715, S(A_5) = 0.2296$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFHWG($\rho = 3$) | $S(A_1) = 0.2525, S(A_2) = 0.2185, S(A_3) = 0.3657, S(A_4) = 0.1921, S(A_5) = 0.2499$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFFWG($\rho = 2$) | $S(A_1) = 0.2549, S(A_2) = 0.2299, S(A_3) = 0.3957, S(A_4) = 0.1896, S(A_5) = 0.2602$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |

Table 7  Ranking results based on special ATS-Cq-ROFOWA aggregation operators

| Methods  | Score values         | Ranking       |
|----------|----------------------|---------------|
| Cq-ROFOWA | $S(A_1) = 0.2194, S(A_2) = 0.1744, S(A_3) = 0.3208, S(A_4) = 0.1810, S(A_5) = 0.2346$ | $A_3 > A_5 > A_1 > A_4 > A_2$ |
| Cq-ROFEOWA| $S(A_1) = 0.1818, S(A_2) = 0.1532, S(A_3) = 0.2913, S(A_4) = 0.1452, S(A_5) = 0.2022$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFHOWA($\rho = 3$) | $S(A_1) = 0.1600, S(A_2) = 0.1393, S(A_3) = 0.2732, S(A_4) = 0.1236, S(A_5) = 0.1820$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFFOWA($\rho = 2$) | $S(A_1) = 0.2479, S(A_2) = 0.2034, S(A_3) = 0.3744, S(A_4) = 0.2014, S(A_5) = 0.2672$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |

Table 8  Ranking results based on special ATS-Cq-ROFOWG aggregation operators

| Methods  | Score values         | Ranking       |
|----------|----------------------|---------------|
| Cq-ROFOWG | $S(A_1) = 0.1993, S(A_2) = 0.1814, S(A_3) = 0.3187, S(A_4) = 0.1450, S(A_5) = 0.2042$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFEOWG| $S(A_1) = 0.2372, S(A_2) = 0.2094, S(A_3) = 0.3520, S(A_4) = 0.1783, S(A_5) = 0.2370$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFHOWG($\rho = 3$) | $S(A_1) = 0.2595, S(A_2) = 0.2252, S(A_3) = 0.3735, S(A_4) = 0.1988, S(A_5) = 0.2581$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |
| Cq-ROFFOWG($\rho = 2$) | $S(A_1) = 0.2649, S(A_2) = 0.2394, S(A_3) = 0.4063, S(A_4) = 0.1968, S(A_5) = 0.2695$ | $A_3 > A_5 > A_1 > A_2 > A_4$ |

Table 9  Ranking results for different q based on Cq-ROFHWA

| Methods  | Score values         | Ranking       |
|----------|----------------------|---------------|
| $q = 2$ | $S(A_1) = 0.1415, S(A_2) = 0.1461, S(A_3) = 0.3104, S(A_4) = 0.0892, S(A_5) = 0.1741$ | $A_1 > A_4 > A_2 > A_1 > A_4$ |
| $q = 3$ | $S(A_1) = 0.1463, S(A_2) = 0.1303, S(A_3) = 0.2609, S(A_4) = 0.1099, S(A_5) = 0.1722$ | $A_1 > A_4 > A_2 > A_1 > A_4$ |
| $q = 5$ | $S(A_1) = 0.1088, S(A_2) = 0.0772, S(A_3) = 0.1641, S(A_4) = 0.0914, S(A_5) = 0.1229$ | $A_1 > A_4 > A_2 > A_1 > A_4$ |
| $q = 7$ | $S(A_1) = 0.0757, S(A_2) = 0.0445, S(A_3) = 0.1062, S(A_4) = 0.0647, S(A_5) = 0.0827$ | $A_1 > A_4 > A_2 > A_1 > A_4$ |

The appropriate algorithm can be selected according to the attitude of the decision-maker. Therefore, the proposed method is better than Peng et al.’s method (Peng et al. 2018), so it is more comprehensive when solving problems.

(2) Comparing Analysis with Liu and Chen’s Method (Liu and Chen 2017) Based on Intuitionistic Fuzzy Weighted Archimedean Heronian Aggregation (IFWAHA) Operator

We also can easily observe that this method cannot handle this example. In the following, we compare and analyze the advantages and disadvantages of the two methods.

1. In terms of information data, it can be seen from (0.31, 0.72) in Table 1 that $0.31 + 0.72 > 1$, which does not satisfy $\mu_Q + \nu_Q \leq 1$. Therefore, Liu and Chen’s method (Liu and Chen 2017) cannot deal with such data,
whereas the method we proposed is based on the fact that \((\mu_Q, \nu_Q)\) is q-ROFNs, which can flexibly adjust the parameter value \(q\) according to the requires of data in solving various uncertainty problems, so the proposed theory can be more widely applied and more effectively solve such problems.

2. In terms of operational laws, both of these methods are based on more general ATS operation; thus, they are more versatile in solving uncertain problems.

3. In terms of the AOs, the proposed operator takes into account the confidence levels of evaluation value given by the decision-maker, whereas Liu and Chen’s theory takes into account the interrelationships between two evaluation attributes. Therefore, the two methods have different emphases in reducing decision deviation and different applicable scenarios, with their own advantages and disadvantages.

(3) Comparing Analysis With Liu and Liu’s Method (Liu and Liu 2018) Based on q-Rung Orthopair Fuzzy Weighted Bonferroni Mean (q-ROFWBM) Operator

From Liu and Liu (2018), it has been observed that the ranking result is \(A_3 > A_5 > A_1 > A_4 > A_2\), which is consistent with the proposed method. So, this illustrates the rationality of the proposed method. Then, we compare the two methods and find out the merits of the proposed method.

1. In terms of information data, both of these methods are based on q-ROFNs to deal with uncertainty problem more comprehensively and have the same advantages.

2. In terms of operational rules, Liu and Liu’s method (Liu and Liu 2018) only considers the calculation of AOs under the algebraic operation rules, so the ability to deal with problems is limited. However, the proposed method in this paper is based on ATS operations. Therefore, our method
is more flexible and effective in selecting operational rules according to practical problems.

3. In terms of the AOs, Liu and Liu’s method (Liu and Liu 2018) aggregates data taking into account the interrelationships between evaluation attributes, whereas the proposed operator in this paper considers the confidence levels of evaluation value. Therefore, the two methods have different emphases in reducing decision deviation and different applicable scenarios, with their own advantages and disadvantages. However, our method is too complicated to be suitable for big data decision problems. So the proposed method in this paper has obvious advantages in solving the MCGDM problem.

5.4 Further comparing with the existing literature (Liu and Wang 2018; Liu et al. 2018)

We have previously confirmed the rationality and advantages of the proposed method by comparing it with the existing literature. However, by roughly consistent ranking results, we find that the advantages of the proposed method that takes into account the confidence levels cannot be reflected. Based on which, in order to demonstrate the advantages of the proposed method, we can obtain the ranking results by changing some data in the example and comparing them with the original data ranking results. Generally speaking, the evaluation value of alternative in decision data changes from large to small, which may have an impact on the ranking results. Therefore, we will gradually reduce the evaluation value of the best alternative \( A_3 \) and compare the processing ability of the proposed theory for extreme data. For example, we adapt the evaluation values \( \xi_2^s \) and \( \xi_3^s \) from \( (0.89, (0.70, 0.11)), (0.85, (0.90, 0.11)) \) to \( (0.05, (0.01, 0.90)), (0.05, (0.01, 0.90)) \) by gradually increasing NMD and decreasing the confidence levels and MD. From common sense, we can foresee that the ranking result will change and the position of the best alternative \( A_3 \) will gradually move backward with the evaluation value decreases.

To further illustrate the superiority of the proposed method in this paper, we compare and analyze the proposed method with Liu and Wang’s method (Liu and Wang 2018) and Liu et al.’s method (Liu et al. 2018), where they aggregate the decision matrix using \( q \)-ROFWA and \( q \)-ROFPWMSM operators, respectively. While the proposed method aggregates the decision matrix using \( C_q \)-ROFWA operator. Obviously, Liu and Wang’s method (Liu and Wang 2018) simply aggregates data without considering the confidence levels of evaluation value given by the decision-maker or the interrelationships between evaluation attributes. And Liu et al.’s method (Liu et al. 2018) considers the correlation between two or more evaluation attributes, whereas the proposed method considers the confidence levels of the evaluation value. The score values and the ranking results of these three methods are listed in Tables 11 and 12, respectively (suppose the confidence levels of the other two theoretical evaluation values are 1, \( q = 3 \)).

From Table 11, we can observe that the score value of each method also decreases with the decrease of the evaluation value. From Table 12, we can observe that the change of the evaluation value has different effects on the ranking result of each method. When the evaluation value data changes, the results of Liu and Wang’s method (Liu and Wang 2018) and Liu et al.’s method (Liu et al. 2018) will be greatly affected. When the evaluation value becomes smaller and smaller, the best alternative will gradually move backward. But when the evaluation value of the proposed method changes from \( (0.89, (0.70, 0.11)), (0.85, (0.90, 0.11)) \) to \( (0.4, (0.45, 0.52)), (0.4, (0.45, 0.52)) \), the ranking result does not change. And with the evaluation value more and

| Methods | Score values | Ranking |
|---------|--------------|---------|
| Peng et al. MCGDM method Peng et al. (2018) (Based on the \( q \)-ROFWA operator) | Cannot be aggregated | The ranking result cannot be obtained |
| Liu and Chen’s MCGDM method Liu and Chen (2017) (Based on the \( IFWAHA \) operator) | Cannot be aggregated | The ranking result cannot be obtained |
| Liu and Liu’s MCGDM Method Liu and Liu (2018) (Based on the \( q \)-ROFWBM operator, where \( s = 1, t = 1 \)) | \( S(A_1) = -0.181, \) \( S(A_2) = -0.223, \) \( S(A_3) = -0.129, \) \( S(A_4) = -0.207, \) \( S(A_5) = -0.161 \) | \( A_3 > A_5 > A_1 > A_4 > A_2 \) |
| The novel MCGDM method (Based on the \( C_q – ROFWA \) operator) | \( S(A_1) = 0.1463, S(A_2) = 0.1303, \) \( S(A_3) = 0.2609, \) \( S(A_4) = 0.1099, \) \( S(A_5) = 0.1722 \) | \( A_3 > A_5 > A_1 > A_2 > A_4 \) |
more small, the best alternative will only move backward one, the ranking result will be slightly affected.

We can draw some conclusions from the above analysis and compare the advantages of the proposed method with respect to other methods. As we all know, in the decision-making process, some decision-makers will give unreasonable evaluation values for some reasons, which will lead to different evaluation results. Therefore, we need to eliminate the influence of these extreme data as much as possible, whereas the decision-making method determines how much impact these extreme data have on the ranking results. Obviously, because Liu and Wang’s method (Liu and Wang 2018) does not take into account the confidence levels of the evaluation value given by the decision maker or the interrelationships between evaluation attributes, it is most affected by unreasonable extreme data. And although Liu et al.’s method (Liu et al. 2018) considers the correlation between two or more evaluation attributes and reduces the influence of extreme data on decision results. In addition, Liu et al.’s method (Liu et al. 2018) is also vulnerable to the impact of such data from the perspective of ranking results and the processing ability of extreme data is not ideal. But, our operator considers the confidence levels of evaluation value, and according to the ranking results, the alternative position remains unchanged as the evaluation value decreases. When the evaluation value decreases to (0.25, (0.21, 0.60)), the position of alternative A3 remains the second place. Therefore, it is less affected by extreme data and relatively stable and has strong processing ability for extreme data.

In the following, we compare the different aspects of the proposed method with other methods, as shown in Table 13, where 0 stands for “no” and 1 stands for “yes”. Through comparative analysis, it has been concluded that the methods put forward by us and others are different in the aspect of reducing decision-making deviation. The methods of others emphasize that reducing the deviation between attributes, while we tend to reduce the deviation caused by expert decision-making. Two types of the methods have their own advantages; we should choose different theories according to different application scenarios, but the method we propose is less influenced by extreme data and is more suitable for dealing with uncertainty problems.

In word, our method optimizes the existing methods in literatures (Yu 2014; Joshi and Gegov 2020; Garg 2017; Xia et al. 2012) and expands the range of application, which makes decision-making more reasonable in terms of reducing the decision deviation of decision-makers, or considering the attitude of experts by changing parameter value ρ.

### 6 Conclusion

The purpose of this paper is to propose a further novel method to rule out the decision deviation caused by the subjective...
Table 13: Comparison of the characteristics of different MCGDM methods

| Characteristics                              | Liu and Wang’s method (Liu and Wang 2018) | Liu et al. method (Liu et al. 2018) | The Proposed Method |
|---------------------------------------------|------------------------------------------|-----------------------------------|---------------------|
| Whether the confidence levels is considered | 0                                        | 0                                 | 1                   |
| Whether the interrelationship between two or more attributes is considered | 0                                        | 1                                 | 0                   |
| Whether the influences of extreme data can be eliminated | 0                                        | 0                                 | 1                   |
| Whether more complex information can be processed | 0                                        | 0                                 | 1                   |

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