Inhomogeneous cylindrically bianchi type I space-time with flat potential

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Abstract. Inhomogeneous cylindrically Bianchi type I space metric with flat potential is investigated in this paper. To finding inflationary solution assume the flat region where potential V (ψ) is constant and condition Y=X^2 where X= α(χ)β(t) and Z= λ(χ)μ(t) on metric scalar. We observed the proper volume increases with time which indicates the eternal inflation of the present universe. Expanding, the non-rotating and shearing universe is investigated by the developed model. The negative deceleration parameter provides the accelerating phase of the universe. Some other physical and structural features of the model are also discussed

Keywords: Inhomogeneous, Cylindrically, Bianchi Type I Space-time, Flat Potential

1. Introduction

In recent years, Einstein’s theory has been subject of interest for its attainment in explaining the accelerated expansion of the universe. Many cosmological problems like homogeneity, isotropy, flatness, and monopole can be explained by inflationary theory. The study of cosmic microwave background also confirmed the inflationary scenario in general relativity. In a recent scenario, it is curious to study the cosmological problems to construct a mathematical model of the physical universe that exactly predicts the result by astronomical facts FRW model describes that universe supposed to be isotropic and homogenous. The studies of inflationary models have great importance in understanding the large scale evolution of the early universe. The basic idea of inflation is firstly given by Guth [1] provided that it is due to the effect of false vacuum energy. The concept of the higg’s field with potential V (ψ) plays an important role in this discussion. Many cosmologists [2-10] have observed various aspects of the inflationary scenario and role of a scalar field (ψ) in the evolution of physical universe. The cylindrically symmetric cosmological models have significant role in the study of the present universe on large scale with anisotropy and inhomogeneous are considered.

The introduction of inhomogeneity in the bianchi model has a precious contribution to the study of behavior of physical universe from the early stage of evolution. The study of the anisotropic model provides solution of nonlinear field equation permits us to obtain cosmological model more general FRW model. Bali and Poonia [11]have constructed Bianchi type IX space metric under flat potential Tripathi et al.[12] have studied Bianchi type I inhomogeneous cosmological model with string
cosmology. Pradhan and Singh [13] have derived cylindrically inhomogeneous cosmological models in presence of time-dependent cosmological constant ($\Lambda$). Rai and Singh [14] have constructed cylindrically inhomogeneous space-time under effect of electromagnetic field. Bali and Jain [15] constructed Bianchi Type I inflationary model Bali and Tyagi [16-17] have investigated cylindrically symmetric inhomogeneous space- time with stiff fluid in the electro-magnetic field. Sharma and Chhajed [18] have constructed inhomogeneous Bianchi type VI0 space- time for distribution of stiff fluid.

In the standard FRW line element universe is supposed to be homogeneous and isotropize but astronomical observation indicates there are homogeneities less than 150Mpc on the scale. Lemaitre-Tolman-Bondi metric or Szekeres metric unable us to understand the influence of homogeneities on accelerated expansion of the universe in which solutions of fields equation are obtained under ignorance of homogeneity. In the absence of homogeneities, the cosmological data may depend on position of the observer and this is important to describe the geometry of the physical parameter.

In recent paper inhomogeneous cylindrically Bianchi type I inflationary model in existence with scalar field and flat potential ($\psi$) is investigated. To obtain the proposed solution, we have considered flat region where potential is taken constant and assuming the relation between coefficients of metric. Also considering the metric coefficients are function of x and t. Expanding, non-rotating and the shearing metric is observed. The proper volume increases with time which represents inflationary scenario of physical universe. The negative deceleration parameter indicates accelerating phase of the universe. Other physical and structural features of the model are also discussed.

2. The metric and field equations

We consider the inhomogeneous cylindrically Bianchi type I is described by line element in the form

$$ds^2 = X^2(dx^2 - dt^2) + Y^2 dy^2 + Z^2$$  \hspace{1cm} (1)

Where X, Y and Z are function of x and t

The action of gravitational field coupled minimally to scalar field with potential $V(\psi)$ is given by

$$l = \int \sqrt{-g} \left[ R - \frac{1}{2} g^{ij} \psi_i \psi_j - V(\psi) \right] dx^4$$  \hspace{1cm} (2)

Which on variation on $l$ w.r.t. dynamical field provided Einstein field equation is given by

$$R^j_i - \frac{1}{2} R g^j_i = -T^j_i$$  \hspace{1cm} (3)

The energy momentum tensor is given by

$$T^j_i = \psi_i \psi_j - \left( \frac{1}{2} \psi_k \psi^k + V(\psi) \right) g_{ij}$$  \hspace{1cm} (4)

And

$$\frac{1}{\sqrt{-g}} \partial_i \left( \sqrt{-g} \psi_i \right) = -\frac{dV(\psi)}{d\psi}$$  \hspace{1cm} (5)

Where $\psi_i = \frac{\partial \psi}{\partial x^i}$, $\psi_j = \frac{\partial \psi}{\partial x^j}$ and $\psi^k = g^{kn} \frac{\partial \psi}{\partial x^n}$  \hspace{1cm} (6)
Einstein field equation (3) for line element (1) with the help of equation (4) leads to

\[
\frac{Y_1 Z_4}{YZ} - \frac{Y_{44}}{Y} - \frac{Z_{44}}{Z} + \frac{X_4}{X} \left( \frac{Y_4}{Y} + \frac{Z_4}{Z} \right) + \frac{X_1}{X} \left( \frac{Y_1}{Y} + \frac{Z_1}{Z} \right) - \frac{Y_4 Z_4}{Y Z} = \frac{\psi_4^2}{\omega} - V(\psi)X^2
\]  

(7)

\[
\left( \frac{X_4}{X} \right)^1 - \left( \frac{X_4}{X} \right)^4 - \frac{Z_{44}}{Z} + \frac{Z_{11}}{Z} = \frac{\psi_4^2}{\omega} - V(\psi)X^2
\]  

(8)

\[
\left( \frac{X_4}{X} \right)^1 - \left( \frac{X_4}{X} \right)^4 - \frac{Y_{44}}{Y} + \frac{Y_{11}}{Y} = \frac{\psi_4^2}{\omega} - V(\psi)X^2
\]  

(9)

\[
\frac{X_4}{X} \left( \frac{Y_4}{Y} + \frac{Z_4}{Z} \right) - \frac{Y_{11}}{Y} - \frac{Z_{11}}{Z} + \frac{X_1}{X} \left( \frac{Y_1}{Y} + \frac{Z_1}{Z} \right) - \frac{Y_4 Z_4}{Y Z} + \frac{Y_4 Z_4}{Z X} = -\frac{\psi_4^2}{\omega} - V(\psi)X^2
\]  

(10)

\[
\frac{Y_{14}}{Y} + \frac{Z_{14}}{Z} = \frac{X_4}{X} \left( \frac{Y_4}{Y} + \frac{Z_4}{Z} \right) - \frac{X_1}{X} \left( \frac{Y_1}{Y} + \frac{Z_1}{Z} \right) = 0
\]  

(11)

Where, the 1 and 4 indices in metric coefficient denotes ordinary derivative w.r.t x and t respectively.

The term rotation ($\omega^2$) vanishes identically. The coefficient of expansion ($\theta$), proper volume ($v$), shear ($\sigma^2$) can be obtain as

\[
\theta = u^I_j = \frac{1}{X} \left( \frac{X_4}{X} + \frac{Y_4}{Y} + \frac{Z_4}{Z} \right)
\]  

(12)

\[
\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{3} g^2 - \frac{1}{x^2} \left[ \frac{X_4 Y_4}{XY} + \frac{Y_4 Z_4}{YZ} + \frac{Z_4 X_4}{ZX} \right]
\]  

(13)

\[
V = \sqrt{-g} = X^2YZ
\]  

(14)

Where g is determinant of line element (1)

Equation (5) for line element (1) leads to

\[
\psi_{44} = \left( \frac{X_4}{X} + \frac{Y_4}{Y} + \frac{Z_4}{Z} \right) \psi_4 = -\frac{dV}{d\psi}
\]  

(15)

3. Solution of field equations

For inflationary solution flat region is assumed where potential $V (\psi)$ is constant

i.e. $V (\psi) = V_0$ (say)

Equation (15) become

\[
\psi_{44} = \left( \frac{X_4}{X} + \frac{Y_4}{Y} + \frac{Z_4}{Z} \right) \psi_4 = 0
\]
which provide \( \psi_4 = \frac{e}{x^2 y z} \) \hspace{1cm} (16)

Where \( e \) is integrating constant.

To get required solutions, we have assumed supplementary condition

\[
Y = X^2
\] \hspace{1cm} (17)

from equation (8) and (9), we get

\[
2 \left( \frac{X_4}{X} \right)^2 - 2 \left( \frac{X_1}{X} \right) + \frac{Z_{44}}{Z} - \frac{Z_{11}}{Z} + \frac{Y_{44}}{Y} - \frac{Y_{11}}{Y} = 0
\] \hspace{1cm} (18)

Equation (17) and (18) give

\[
4 \frac{X_{44}}{X} + \frac{Z_{44}}{Z} = 4 \frac{X_{11}}{X} + \frac{Z_{11}}{Z} = m
\] \hspace{1cm} (19)

we consider the metric coefficient in the form

\[
X = \alpha(x) \beta(t) \text{ and } Z = \lambda(x) \mu(t)
\] \hspace{1cm} (20)

using equation (17) and (11), we have

\[
2 \frac{X_{44}}{X} + \frac{Z_{44}}{Z} - 2 \frac{X_1 X_4}{X^2} - \frac{X_1 Z_4}{X Z} - \frac{X_4 Z_1}{X Z} = 0
\] \hspace{1cm} (21)

Equation (20) and (21) leads to

\[
\frac{\lambda_1}{\lambda} \left[ \frac{\mu_1}{\mu} - \frac{\beta_4}{\beta} \right] = \frac{\alpha_1}{\alpha} \frac{\mu_4}{\mu}
\] \hspace{1cm} (22)

\[
\frac{\lambda_1}{\lambda} \left[ \frac{\mu_4}{\mu} \right] = \frac{\alpha_1}{\alpha} \frac{\mu_4}{\mu}
\] \hspace{1cm} (23)

which provide \( \lambda = r a^n \) and \( \mu = d \beta y \) \hspace{1cm} (24)

where \( r \) and \( d \) are integrating constant and \( y = \frac{n}{n-1} \)

from equation (19) and (24), we obtained

\[
\beta \beta_{44} + \xi \beta_4^2 = m \beta^2
\] \hspace{1cm} (25)

on solving we get

\[
\beta = C_1^{\frac{1}{\xi + 1}} \sinh^{\frac{1}{\xi + 1}}(bt + t_0)
\] \hspace{1cm} (26)
Where, $b = \sqrt{m(\xi + 1)}$ and $C_1$ is integrating constant.

and, $\mu = d \left[ C_1^{\frac{y}{\xi+1}} \sinh^{\frac{y}{\xi+1}}(bt + t_0) \right]$  \hspace{1cm} (27)

From equation (19), (20) and (24)

$a \alpha_{11} + \delta \alpha_2 = \eta \alpha^2$  \hspace{1cm} (28)

we obtain,  \hspace{1cm} $a = C_2^{\frac{1}{\delta+1}} \sinh^{\frac{1}{\delta+1}}(hx + x_0)$  \hspace{1cm} (29)

where, $h = \sqrt{\eta(\delta + 1)}$ and $C_2$ is integrating constant

and, $\lambda = r \left[ C_2^{\frac{n}{\delta+1}} \sinh^{\frac{n}{\delta+1}}(hx + x_0) \right]$  \hspace{1cm} (30)

From equation (20), we obtain

$X = M \sinh^{\frac{1}{\delta+1}}(hx + x_0) \sinh^{\frac{1}{\xi+1}}(bt + t_0)$  \hspace{1cm} (31)

Where, $M = C_1^{\frac{1}{\xi+1}} C_2^{\frac{1}{\delta+1}}$

and, $Z = N \sinh^{\frac{n}{\delta+1}}(hx + x_0) \sinh^{\frac{\nu}{\xi+1}}(bt + t_0)$  \hspace{1cm} (32)

where, $N = r d c_4^{\frac{\nu}{\xi+1}} C_2^{\frac{n}{\delta+1}}$

Equation (17) leads to

$Y = M^2 \sinh^{\frac{2}{\delta+1}}(hx + x_0) \sinh^{\frac{2}{\xi+1}}(bt + t_0)$  \hspace{1cm} (33)

using suitable transformation line element (1) can be obtained

\[ ds^2 = M^2 \sinh^{\frac{2}{\delta+1}}(hx^*) \sinh^{\frac{2}{\xi+1}}(bt^*) \left( dx^2 + dt^2 \right) + M^4 \sinh^{\frac{4}{\delta+1}}(hx^*) \sinh^{\frac{4}{\xi+1}}(bt^*) \left( dy^* \right)^2 + \]
\[ N^2 \sinh^{\frac{2n}{\delta+1}}(hx^*) \sinh^{\frac{2\nu}{\xi+1}}(bt^*) \left( dz^2 \right) \]  \hspace{1cm} (34)

Where, $hx + x_0 = hx^*$, $bt + t_0 = bt^*$, $y = y^*$ and $z = z^*$
4. Physical and geometrical features

Proper volume \((V)\) can be obtain as

\[
V = X^2YZ = X^4Z = M^4N\sinh^4(hx^*)\sinh^n(bt^*)\sinh^{\frac{4}{y}}(bhx^*)\sinh^{\frac{n}{y}}(bt^*) (35)
\]

Expansion scalar can be obtained as

\[
\theta = \frac{1}{x} \left[ \frac{3+\gamma}{(x+1)} \right] bcoth(bt^*) (36)
\]

Shear scalar is given by

\[
\sigma^2 = \frac{1}{3x^2} \left[ \frac{c}{(x+1)^2} \cosh^2(bt^*) \right] c^{-2} (37)
\]

Where, \(c^2 = \gamma^2 + 15\gamma + 15\)

Rate of higg’s field can be obtain as

\[
\psi_h = \frac{\epsilon}{M^4N\sinh^4(hx^*)\sinh^{\frac{n}{y}}(bhx^*)\sinh^{\frac{n}{y}}(bt^*)} (38)
\]

Ratio of shear and expansion scalar can be obtained as

\[
\frac{\sigma}{\theta} = \frac{c}{\sqrt{3(3+\gamma)}} = \text{constant} (39)
\]

Magnitude of rotation is zero i.e. \(\omega = 0\) (40)

Deceleration parameter can be obtain as

\[
q = -\left[ \frac{3(\gamma^2 + 1) + 3(1 + \gamma^2)\tanh^2(bt^*)}{4c} \right] (41)
\]

5. Conclusions

In the present paper, we have obtained the solution of nonlinear field equations for cylindrically symmetric Bianchi Type I space-time with flat potential for the inhomogeneous universe. The constructed model obeys big-bang at \(t^* = 0\) and expansion goes on increasing as time tends to infinity. Proper volume is increasing function of time which agreed to inflationary model. The fraction of shear and expansion is finite represent, for large time model does not approach to isotropy. The rate of higg’s decreases slowly but universe is expanding. The accelerating scenario of universe is provided by negative deceleration parameter. The magnitude of rotation factor is zero. Non rotating, shearing and expanding universe is investigated in the presented model.
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