Numerical investigation of the effect of net charge injection on the electric field deviation in a TE CO₂ laser

Nahid Jahanian¹,², Majid Aram³, Nader Morshedian⁴ and Ahmad Mehramiz¹

¹Department of Physics, Faculty of Science, Imam Khomeini International University, Qazvin, Iran
²Photonic and Quantum Technologies Research School, Nuclear Science and Technology Research Institute, P.O. Box 14155-1339, Tehran, Iran
³Plasma and Fusion Research School, Nuclear Science and Technology Research Institute, P.O. Box 14399-51113, Tehran, Iran

E-mail: nahid.jahanian@gmail.com

Abstract. In this report, the distribution of and deviation in the electric field were investigated in the active medium of a TE CO₂ laser. The variation in the electric field is due to injection of net electron and proton charges as a plasma generator. The charged-particles beam density is assumed to be Gaussian. The electric potential and electric field distribution were simulated by solving Poisson’s equation using the SOR numerical method. The minimum deviation of the electric field obtained was about 2.2% and 6% for the electrons and protons beams, respectively, for a charged-particles beam-density of 10⁶ cm⁻³. This result was obtained for a system geometry ensuring a mean-free-path of the particles beam of 15 mm. It was also found that the field deviation increases for a the mean-free-path smaller than that or larger than 25 mm. Moreover, the electric field deviation decreases when the electrons beam density exceeds 10⁶ cm⁻³.

1. Introduction

The TE CO₂ laser was introduced in 1968. Initially, the output energy of these lasers was in the order of a joule, but this laser’s technology began developing very quickly and the energy reached kilojoules, especially for the purposes of nuclear fusion [1], as a result of the large research effort concentrated on increasing the laser energy in the last four decades. Producing a high CO₂ laser pulse energy necessitates a large active medium with as uniform as possible excitation of the CO₂ molecules [2]. Apart from the electrical discharge as the main process of creating plasma, excited molecules and, subsequently, population inversion, complementary techniques, such as pre-ionization, net charge injection [3] and special electrodes geometry [4], have been considered in order to produce a uniform glow discharge, thereby achieving as high as possible population inversion in the active medium. In the present work, we focused our attention on the charged-particles injection technique. The

⁴To whom any correspondence should be addressed.
consecutive injection of pulsed electron or proton beams can generate a pre-plasma medium facilitating the development of the main discharge [9]. Generally, the method of charge injection can be considered as pertinent to switching or triggering a discharge system. For instance, when an analog switch is turned on and off, a small amount of charge can be capacitively coupled (injected) from the digital control line to the analog signal path [10].

In the work reported, using the numerical method of successive over relaxation (SOR), we studied the electric field distribution and uniformity in the active laser medium when net charge is supplied by injecting a charged particles beam.

2. Physical conditions
The charged particles beam is assumed to be injected perpendicularly to the laser optical axis in the front side of the discharge electrode. The cross section of the electrode, and the active medium geometry and boundaries are shown in the figure 1. The electrode edges are rounded with a radius of curvature of about 2 mm and all four external sides are grounded.

![Figure 1. Schematic diagram of the system studied.](image)

The distribution function of net charge beam injection is as follows;

\[ \rho = en \exp\left(\frac{-x^2}{L^2}\right) \]

where, the \( \rho \) is the charge density, \( e \) is the electron charge, \( n \) is the charged-particles number density, \( L \) (mm) is the mean-free-path of the charges, or the distance where the charge density decays to 1/e of its initial value. If the value of \( L \) increases, then the space charge density in the active medium also increases. We solved numerically Poisson's equation (2) by using the SOR method for the space charge in the medium.

\[ \nabla^2 V = \frac{\rho}{\varepsilon_0} \]

where \( V \) is the potential and \( \varepsilon_0 \) is the permittivity of vacuum. The parameter \( \varepsilon_0 \) should be redefined for calculating the potential in a medium with different permittivity at the boundary conditions as shown schematically in figure 2. In the figure, \( h \) is the distance scale for the mesh construction. The solution of the potential equation at each point is based on the potential value obtained at that point in the previous stage, plus the average of the four potential values of the points that are in the vicinity of the point considered in the mesh construction.
3. Numerical method

To simulate numerically the electric potential in the discharge medium, we chose to apply the SOR method. Generally, the memory of a personal computer may limit one in processing the large amount of data needed for mesh construction. On the other hand, if no limit is set on the processing time, the SOR method overcomes the memory restriction. Figure 3, illustrates the potential equation solution at each point as based on the potential value obtained according to relation (3) in the iteration process [6].

\[ x^{(k)}_i = \left(1 - w\right)x^{(k-1)}_i + \frac{w}{a_{ij}} \left( b_i - \sum_{j=1}^{i-1} a_{ij}x^{(k)}_j - \sum_{j=i+1}^{n} a_{ij}x^{(k-1)}_j \right), \]

where, \( x^{(k)}_i \) is the response vector or scalar that refers to the potential quantity, \( i = 1, 2, \ldots, n \), \( n \) being the number of iterations, \( w \) is the scaling factor (\( w = 0.2 \)); if \( w = 1 \), the above expression is converted to the Gauss–Seidel equation. Further, \( a_{ij} \) is the matrix array of coefficients; \( b_i \) is the constant value of the equation and \( x^{(k-1)}_i \) is the value of the potential obtained at the previous stage. Thus, the final value of the potential is derived by the iteration process provided that the solution converges by choosing a specific scaling factor (\( w \)) value. Rewriting relation (3) based on the SOR method, one arrives at relation (4) [6].

\[ V_{new}(i, j) = \left(1 - w\right)V(i, j) + \]
\[ \frac{w}{4} \left( \frac{\varepsilon_1 V(i, j + 1) + \varepsilon_3 V_{new}(i, j - 1) + \varepsilon_4 V(i + 1, j) + \varepsilon_2 V_{new}(i - 1, j)}{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4} \right), \]

where, \( \varepsilon_1 \) and \( \varepsilon_2 \) are the permittivity at the boundary of the two media, \( V(i \pm 1, j \pm 1) \) are the four potential values in the vicinity of the point considered, \( V_{new} (i, j-1) \) is the potential value obtained at the previous stage, \( \rho(i,j) \) is the density of the net charge injected and \( h \) is the distance scale of the mesh. The field deviation (FD) is defined as

\[ FD = \left(\frac{\Delta E}{E}\right)\times100; \quad \Delta E = E - E_0, \]
where the $E_0$ is a reference quantity defined as the electric field value at the central point, or the symmetry point of the system, and $\Delta E$ is the difference between the electric field values at an arbitrary point in the mesh and the central point.

4. Results and discussion

The experimental conditions and system parameters were assumed as quoted by Maxwell Company (USA). Thus, the electrode potential is +75 kV and, as mentioned above, all external boundaries are at zero potential (grounded). The electrode voltage value is optimized in order to obtain a realistic reduced electric field ($E/N$) value, which for a CO$_2$ laser is about $4 \times 10^{16}$ V cm$^2$ [1]. The electron energy distribution has then a maximum at 1-2 eV when the mean-free-path is varied. The field deviation is a criterion for the electric field uniformity needed to achieve a proper glow discharge in the active medium, i.e., the generation is prevented of sparks and/or arcs in discharge. Table 1 summarizes the dielectric constant of different gas mixtures and the related field deviation in percents [8].

Typical simulated 3D images of the potential and electric field distributions are shown in figures 4 and 5 for Laplace equation, i.e. without any space charge.

Figures 6 and 7 illustrate the potential and electric field distributions obtained by solving the Poisson’s equation when an electron beam is injected. The expected basic result becomes apparent, namely, the field deviation is reduced due to the injection of a net electron charge (compare with figures 4 and 5). The $FD$ (%) decays rapidly vs. the charged particles density with a threshold of $10^7$ cm$^{-3}$ for electrons.

![Figure 4. 3D image of the potential distribution without beam injection (electrode potential +75 kV).](image1)

![Figure 5. 3D image of the electric field distribution without beam injection (electric potential +75 kV).](image2)

**Table 1.** Permittivity and field deviation values for different gas mixtures.

| CO$_2$:N$_2$:He | K   | $\varepsilon_{\text{mix}} (\times 10^8)$ | $C_{N,\text{min}}$ | FD (%) |
|-----------------|-----|-----------------------------------|------------------|--------|
| 1:1:1           | 1.0007 | 8.8565                          | 0.9970            |
| 1:1:8           | 1.0006 | 8.8557                          | 0.9970            |
| 1:1:3           | 1.0007 | 8.8560                          | 0.9970            |
However, for a beam of protons for a mean-free-path $L = 15$ mm this behavior is inverted, as shown in figure 8. Similarly, in the case of electron beam injection and $L = 25$ mm, $FD$ starts decreasing, the threshold then being $10^6$ cm$^{-3}$ (figure 9). Moreover, $FD$ as a function of the mean-free-path starts increasing at $L = 25$ mm for charged particles density of $10^8$ cm$^{-3}$, as shown in figure 10.

Comparing the cases of electron or proton beam injection, one can clearly see that the minimum $FD$ is obtained using electron beam injection. There is a local minimum in $FD$ vs. charged particles density using a proton beam, as shown in figures 8 and 9. In addition, for a proton beam there is generally an incremental $FD$ dependence on $L$, as seen in figure 10. In contrast, the minimum of $FD$ for an electron beam occurs at larger values of the charged particles density. This situation can be considered as a competition of the two parameters, i) the external electric field initiating the discharge and, ii) the electric field induced by the space charge due to the injection of a charged particles beam.

![Figure 6. 3D image of the potential distribution of beam injection (electric potential +75 kV).](image)

![Figure 7. 3D image of the electric field distribution with beam injection (electric potential +75 kV).](image)

![Figure 8. $FD$ versus charged particles density for mean-free-path 15 mm.](image)

![Figure 9. $FD$ versus charged particles density for mean-free-path 25 mm.](image)
5. Conclusions
Pre-plasma generation is important for creating a uniform discharge medium and preventing arcs and/or sparks in the discharge medium. One of the methods of producing uniform conditions is injection of a charged beam in order to obtain a glow discharge and uniform pumping for optimal population inversion. We concluded that there are two basic parameters i.e., space charge density created by the particles beam and mean-free-path of the beam injected, both of which should be optimized in view of minimizing the electric field deviation in the active medium. We obtained the optimized parameters of the injected charged particles (electrons or protons) for a TE CO$_2$ laser by solving Poisson' equation using the SOR method. The results show that the $FD$ is considerably less when an electron beam, rather than a proton beam.

Acknowledgements
The authors wish to thank the Technical Assistance Group and Heads of the two Research Schools.

References
[1] Witteman W H 1987 The CO$_2$ Lasers ch 6 (Springer-Verlag)
[2] Sourabh and Tripathi S B L 2012 Calculation of Optical Gain as a Function of Pump-Power (Research Scholar Dept of Physics CMJ University)
[3] Mesyats G, Osipov V V and Tarasenko V F 1995 Pulsed Gas Lasers (Nauka Moskow))
[4] Aghayari E, Sade F and Aram M 2015 Investigation of the electrodes cross sections for the TEA CO$_2$ laser Proc. 3-th Engin. Conf. (KAM) (2015 Iran)
[5] http://liberica.org
[6] Nagel J R 2012 Solving the Generalized Poisson Equation Using the Finite-Difference Method (FDM) (University of Utah)
[7] http://eng.utah.edu/~cfurse/NASA/User%20Guide/01%20Zo%20Catalog/Description%20of%20FDFD%20Method.pdf
[8] Schmidt J W and Moldover M R 2003 Int. J. Thermophys. 24/2 375-403
[9] Sim H S, Park S, Kim T H, Choi Y K, Lee J S and Lee S H 2010 Mater. Trans. JIM 51 1156
[10] Holloway M D and Nwaoha Ch 2012 Dictionary of Industrial Terms ISBN: 978-1-118-34457-6