SHARP H I EDGES AT HIGH z: THE GAS DISTRIBUTION FROM DAMPED Lyx TO LYMAN LIMIT ABSORPTION SYSTEMS

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ABSTRACT

We derive the distribution of neutral and ionized gas in high-redshift clouds, which are optically thick to hydrogen ionizing radiation, using published data on Lyman limit and damped Lyx absorption systems in the redshift range 1.75 ≤ z < 3.25. We assume that the distribution of the hydrogen total (H I + H II) column density in the absorbers, N_{H}, follows a power law KN_{H}^{-a}, whereas the observed H I column density distribution deviates from a pure power law as a result of ionization from a background radiation field. We use an accurate radiative transfer code for computing the rapidly varying ratio N_{H}/N_{HI}, as a function of N_{HI}. Comparison of the models and observations gave excellent fits with maximum likelihood solutions for the exponent a and X, the value of log (N_{H}/N_{HI}) when the Lyman limit optical depth along the line of sight is τ_{LL} = 1. The slope of the total gas column density distribution with its relative 3 σ errors is a = 2.7^{+1.0}_{-0.4} and X = 2.75 ± 0.35. This value of X is much lower than what would be obtained for a gaseous distribution in equilibrium under its own gravity. The ratio η_{0} of dark matter to gas density, however, is not well constrained since log (η_{0}) = 1.1 ± 0.8. An extrapolation of our derived power-law distribution toward systems of lower column density, the Lyx forest, tends to favor models with log (η_{0}) ≤ 1.1 and a ~ 2.7 ~ 3.3. With a appreciably larger than 2, Lyman limit systems contain more gas than damped Lyx systems and Lyx forest clouds even more. Estimates of the cosmological gas and dark matter density due to absorbers of different column density at z ~ 2.5 are also given.

Subject headings: dark matter — intergalactic medium — quasars: absorption lines — radiative transfer

1. INTRODUCTION

Lyman limit systems (hereafter LLSs) are detected as intergalactic clouds that absorb the quasar radiation at energies above the ionization edge of neutral atomic hydrogen. The optical depth of the continuum radiation at the Lyman edge is τ_{LL} = N_{H}/1.6 × 10^{17} cm^{-2}, where N_{H}, is the H I column density along the line of sight. For τ_{LL} ~ 1, N_{H}, can be measured accurately regardless of the velocity structure of multiple components; for τ_{LL} ≫ 1, the absorption can be detected, although N_{H}, cannot be measured. We will not analyze in detail the Lyx forest systems, at much smaller N_{H}, but will consider the whole range of column densities from the LLSs to the mostly neutral, damped Lyx systems (hereafter DLSs) with much larger N_{H}. We will then discuss a possible extrapolation of our results toward the Lyx forest region. There have been some suggestions that LLSs, as well as Mg II absorbers with similar H I column densities, may be related to the outskirts of galaxies (Bergeron & Boissé 1991; Rao & Turnshek 2000), while DLSs are more likely to originate from their inner regions (Prochaska & Wolfe 1998 and references therein). We will show that, either this suggestion is true or not, the ratio of the total-to-neutral hydrogen column density for typical LLSs can be estimated assuming only that the total gas column density distribution g(N_{H},) is a smoothly varying function from the LLS to DLS range. In particular, we will derive a value for X, the logarithm of N_{H}/N_{HI}, for a line-of-sight value of N_{H} = 1.6 × 10^{17} cm^{-2} at intermediate redshifts, namely, 1.75 ≤ z ≤ 3.25. From X we will derive a, the exponent in the power law for g.

If the dimensionless parameter X is very large, the ionization fraction changes rapidly, and the value of N_{HI}, can increase from ~10^{17} to ~10^{19} cm^{-2} over a fairly small increase of the total column density. Breaks have been found in the distribution of N_{HI}, of absorbers from the Lyx forest to the DLS (Petitjean et al. 1993; Storrie-Lombardi, Irwin, & McMahon 1996; Storrie-Lombardi & Wolfe 2000). We will show that this behavior of N_{HI}, is compatible with a single power-law distribution for the total hydrogen column density, once the change in the ionization fraction is taken into account. In fact, it is the change in the slope of the N_{HI}, distribution from the LLS to DLS region that enables us to derive a value for X.

A similar phenomenon is found in the outskirts of today’s galaxies where a sharp decline in the H I column density over a narrow range of the galaxy radius occurs. One can explain this occurrence in terms of an H I – H II transition zone that takes place when the column density gets sufficiently low so that the gas becomes optically thin to the extragalactic ionizing flux. For today’s galaxies, the known rotational velocity constrains the dark matter content, which, together with the measured radial decline of the H I column density at the H I – H II transition zone, can be used to estimate the UV ionizing flux at z = 0, otherwise unobservable. Corbelli & Salpeter (1993) in fact have modeled the sharp H I decline observed in M33 by considering a
total gas distribution compressed by the local dark matter (inferred from the observed rotational velocities) and irradiated by the extragalactic UV and soft X-ray background. They found a best fit to the observed H I radial distribution in the outermost disk and to its sharp decline when the intensity of the background radiation at the Lyman edge is $6 \times 10^{-23}$ ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$. For LLSs, the situation is reversed since the metagalactic ionizing flux at high $z$ is better known than the dark matter content of absorbers; from $X$ we will derive the volume gas density $n$, which is larger than self-gravity alone can produce and therefore gives information on the dark matter gravitational potential.

Unfortunately, the difficulty in measuring the residual flux of LLSs at large optical depths limits the determination of the H I column density of the absorber above $10^{18}$ cm$^{-2}$. For this reason, we are forced to introduce a new technique for treating large uncertainties in the $N_{H}$ values. In the construction of a database for LLSs, we include also the high column density systems, known as damped absorbers, whose column density is determined from the scatter of the Ly$\alpha$ radiation. Both the database and the data treatment are described in detail in Bandiera & Corbelli (2001, hereafter Paper II). In § 2 of this paper we derive the number density of LLSs and DLSs from our database, and in § 3 we describe radiative transfer processes and the gas stratification model for the gas in LLSs and DLSs. The power-law distribution for the total hydrogen column density and the H fractional ionizations that fit at best the data at $1.75 \leq z < 3.25$ in the Lyman limit and damped Ly$\alpha$ region will be derived in § 4. Having the H ionization fractions for LLSs, we can then compute the total gas content of LLSs and DLSs and estimate their dark matter potential. Also given in § 4 is an extrapolation of our fit toward absorption systems of even lower H I column density.

2. NUMBER DENSITY OF LYMAN LIMIT SYSTEMS AND DAMPED LY$\alpha$ ABSORBERS

From the available literature we have collected data on LLSs with $\tau_{LL} > 0.4$ and on DLSs with rest-frame equivalent width $W \geq 5$ Å for $z < 4.7$. We have 661 quasars, and for each quasar we have the redshift path covered with a given sensitivity, the redshift of the intervening absorbers, and their estimated H I column density, $N_{HI}$. We disregard absorbers and paths within 5000 km s$^{-1}$ of the quasar redshift. The sensitivity of a search is expressed as a lower limit to the H I column densities searched. A given direction may correspond to more than one path if different redshift paths were searched with different sensitivities. If no estimates of the absorber H I column density are available (if no residual flux is detected shortward of the Lyman break for example), a lower limit to it is given, while upper limits may be derived from searches of damped Ly$\alpha$ absorption lines. The H I column density values obtained from Voigt profile fits, when these are available, are included in our database. A comparison of the H I column densities derived directly from the equivalent width with those determined from the Voigt profile fitted to the same absorption lines shows that the Voigt fit procedure gives a systematically higher value of $N_{HI}$. This might be due to an underestimate of the equivalent width because of an effective absorption of the quasar continuum radiation by intervening forest clouds. Using data where Voigt fits are available, we derive an average correction for $N_{HI}(W)$, the column density derived from $W$, and we correct the data whenever no Voigt fits are available as follows:

$$\log N_{HI} = 6.261 + 0.705 \log N_{HI}(W).$$

(1)

Since the maximum dispersion found in the data used for deriving equation (1) is $\pm 0.4$ in $\log N_{HI}$, we use this value for estimating the errors on $\log N_{HI}$, whenever we apply the above correction. Our compilation of data is described in more detail in Paper II and is available on request from the authors.

The redshift path density $w(z)$ (Lanzetta et al. 1991), defined as the number of lines of sight at each redshift searched for LLSs with any sensitivity or for DLSs with $W \geq 5$, is shown in Figure 1 for all our data. From our data set we extract the LLS sample and the DLS sample. The LLS sample consists of all observations that were sensitive to LLSs with $\tau_{LL} \geq 1$. The DLS sample consists of observations sensitive to absorbers with column density $\log N_{HI} \geq 20.13$. We wish to examine the number density of absorbers per unit redshift, $dn/dz$, in the LLS and DLS sample. As usual we represent the number density as a power law of the form

$$N(z) = N_0(1 + z)^\gamma.$$

(2)

In order to avoid binning the data in redshift intervals, which can generate errors in the estimate of the evolutionary trend, we use the maximum likelihood method described by Storrie-Lombardi et al. (1994) to estimate $N_0$ and $\gamma$. For the LLS sample the maximum value of the likelihood is found for

$$\gamma = 1.66 \pm 0.3,\quad N_0 = 0.23,$$

(3)

where the errors quoted are the 68.3% confidence limits. The 99.7% confidence limits for $\gamma$ are 0.80 and 2.64. These values are similar to the ones obtained by Stengler-Larrea...
et al. (1995). For the DLS sample we find
\[ \gamma = 0.86 \pm 0.3, \quad N_0 = 0.18. \] (4)
The 99.7% confidence limits are 0.11 and 1.72. The value of \( \gamma \) is consistent but slightly lower than what has been found by Storrie-Lombardi & Wolfe (2000) for higher column density systems. If we restrict our DLS sample to only those systems that have a measured \( N_{HI} \geq 20.3 \), we obtain the same values of \( N_0 \) and \( \gamma \) as given by Storrie-Lombardi & Wolfe (2000). Figure 2 shows the cumulative number of absorbers versus \( z \) for both samples. Overplotted is the expected number of LLSs and DLSs from the maximum likelihood estimate. Notice that DLSs at \( z \geq 1.75 \) seem to require a lower value of \( \gamma \) than what is given by the maximum likelihood estimate over the whole \( z \) range. In fact, by considering only \( z \geq 1.75 \) the match between the data and the best-fit cumulative function for DLSs improves: in this case \( \gamma = -0.07 \) with -1.58 and 1.36 as confidence limits larger than 99.7%.

For the present paper, we will use data only in the intermediate redshift range \( 1.75 \leq z < 3.25 \), and for this range the slight difference in evolution implied by equations (3) and (4) is unimportant. We use an average \( \gamma \) value of 1.0, which is consistent with the number density evolution of both DLSs and of LLSs. We exclude from the detections absorbers that were declared as “nondamped” if they were not detected as LLSs or for which no LLS searches have been performed. For nondamped Ly\( \alpha \) absorbers that were detected as LLSs, the \( N_{HI} \) is set to the value derived from \( \tau_{L} \) when this is available; otherwise we will use the lower limit to \( \tau_{L} \) for a lower limit to \( N_{HI} \) and the column density value derived from \( W \) as upper limit.

3. **THE \( N_{HI,I} \)-\( N_{HI,L} \) CURVE**

Our numerical code solves for the ionization, chemical, and hydrostatic equilibrium of a gaseous absorber that receives photons from a background radiation field. It computes the \( \text{H}\alpha \) column density perpendicular to the plane, \( N_{HI,I} \), in a plane parallel geometry when the total hydrogen column density perpendicular to the plane is \( N_{HI,L} \) and the density stratification in the vertical direction \( y \) is in the following exponential form:
\[ \rho(y) = \rho_0 \exp(-y^2/2h^2), \] (5)
where \( \rho_0 = M_g/(2\pi)^{1/2}h \) is the central gas density for a given total mass surface density per cm\(^2\) of the gaseous component, \( M_g \). Equation (5) is the exact solution for the equilibrium of an isothermal gaseous disk in a dark matter halo potential when self-gravity is negligible and the dark matter density does not change within the vertical gaseous extent; (see, Spitzer 1942; Maloney 1993). In this case the scale height is
\[ \frac{1}{h^2} = \frac{4\pi G \rho_{dm}}{c_s^2} = \frac{(2\pi G)^2 M_{dm}^2}{c_s^2}, \] (6)
where \( c_s \) is the sound speed (computed using \( T \), the mass-averaged temperature of the slab), and \( \rho_{dm} \) and \( M_{dm} \) are the dark matter volume density and the dark matter surface density between \( +h \) and \( -h \) in the slab, respectively. In the absence of dark matter, equation (5) is instead only an approximation of the self-gravitating isothermal gas layer solution (see, e.g., Spitzer 1942):
\[ \rho(y) = \rho_0 \sech^2\left(\frac{y}{2h_{y}}\right), \quad \frac{1}{h_{y}^2} = \frac{2\pi^2 G^2 M_g^2}{c_s^4}. \] (7)

In this case the gas central density is \( \rho_0 = M_g/(2\pi)^{1/2}h_{y} \). If we approximate the above exact solution with equation (5), given the same central volume density \( \rho_0 \) and surface density \( M_g \), we should use the following expression for the vertical scale height:
\[ \frac{1}{h^2} = \frac{\pi}{4h_{y}^2} \simeq \frac{16G^2 M_g^2}{c_s^4}. \] (8)
It will be shown in § 4 that equation (5), with the scale height \( h \) given by equation (8), is a good approximation to the exact self-gravitating gas layer solution for the purposes of this paper. We therefore can write a general expression for \( h \) if we use equation (5) to describe the vertical stratification of the gas when both self-gravity and dark matter are taken into account:
\[ \frac{1}{h^2} = \frac{16G^2 M_g^2}{c_s^4} (1 + \eta^2), \] with
\[ \eta \equiv \frac{\pi M_{dm}}{2M_g}. \] (9)
The compression factor \( \eta \) for the volume density is in general considered as the effect of gravity from matter other than the gas (dark matter, stars, or brown dwarfs that coincide with the gas location). We write \( \eta \) as
\[ \eta \equiv \frac{c_s^4}{\tilde{c}_s^4}, \] (10)
with \( \tilde{c}_s \) being the value of \( c_s \) for \( T = 10^4 \) K and for a mean gas mass per particle of \( \mu = 1.3 \). In this paper we will consider a set of models, each corresponding to a different \( \eta_0 \).
value, and in each model we keep $\eta_0$ constant as we vary $M_{\odot}/c_s$; $c_s$ is computed from the radiative transfer and energy equation. A constant $\eta_0$ value for $\eta_0 \gg 1$ implies

$$\rho_{dm} \propto M_{\odot}^2. \quad (11)$$

The meaning of the above proportionality is not immediate, but for spherical isothermal dark matter halos, equation (11) implies a constant ratio between the gaseous and dark total surface densities. The variable $\eta$ instead is proportional to the dark matter surface density between $+h$ and $-h$, and since $h$ scales linearly with $c_s$, $\eta$ will also scale with $c_s$.

The equilibrium fractional ionizations of H, He, and He II are found step by step through the slab taking into account the diffusion of photons from the recombination of H, He, and He II and the secondary electrons produced by the harder photons. The temperature at each step through the slab is given by balancing photoionization heating with the cooling from hydrogen and helium gas (collisional ionizations, recombinations, lines excitation, and free-free processes) from metal lines and from H$_2$ cooling. After each iteration the mass-averaged temperature $T$ is computed consistently and is used to determine the gas vertical dispersion and stratification. The ionization states of metals (C, O, Fe) are computed step by step consistently with the photoionization rates and charge-exchange reactions; very high ionization states (i.e., $E \gg 54$ eV) are not considered. For metal abundances, we use $Z = 0.02 Z_{\odot}$ throughout this paper, but we will discuss briefly its possible variations. H$_2$ fractions, although small, are computed via primordial chemical reactions (Corbelli, Galli, & Palla 1997).

From the ionization-recombination balance we know that for a small neutral fraction the ratio $N_{H_{\perp}}/N_{H_{\perp}}$ depends on the ratio between the ionization and recombination coefficients and varies inversely with the gas volume density. We define $X$ as

$$X \equiv \log \frac{N_{H_{\perp}}}{N_{H_{\perp}}},$$

for

$$N_{H_{\perp}} = 1.6 \times 10^{17} \text{ cm}^{-2}. \quad (12)$$

Notice that $X$ is defined at the line-of-sight value of $N_{H_{\perp}} = 1.6 \times 10^{17} \text{ cm}^{-2}$, and therefore the corresponding $N_{H_{\perp}}$ value depends on the thickness ratio $h/R$. We derive for $\eta_0 \geq 1$ the following relation:

$$X \approx 2.94 + 0.36 \log \frac{J_L}{\eta_0}, \quad (13)$$

where $J_L$ is the intensity of the background flux at 912 Å written in units of $10^{-22}$ ergs cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ sr$^{-1}$. For our redshift range ($1.75 < z < 3.25$), we use a constant $J_L$ and a flux spectrum computed for $z = 2.5$ by Haardt & Madau (1996, hereafter H-M) for emission by quasars and absorption by intervening clouds. A nearly constant ionization rate between $z = 2$ and 4 is also required by the comparison of results from hydrodynamical simulations of structure formation with the measured opacity of the Ly$\alpha$ forest. The H-M flux intensity at $z \approx 2.5, J_L \approx 5.5$, agrees sufficiently well with other estimates (Giallongo et al. 1996: Rauch et al. 1997), and we will use $J_L = 5.5$ throughout. The compression factor $\eta_0$ is unknown a priori and will be determined by trial and error. Hence $\eta_0$ (or equivalently $X$) and $x$ are parameters to be determined.

In Figure 3 we show three curves for $N_{H_{\perp}}$ as a function of $N_{H_{\perp}}$ for $J_L/\eta_0 = -1.9$, $-0.4$, and 0.7, which correspond to $X = 2.3, 2.8$, and 3.2, respectively. All curves in Figure 3 are characterized by three regions: a first region at high column density where $N_{H_{\perp}} \approx N_{H_{\perp}}$, a second region where for a small change in $N_{H_{\perp}}$ the H I column density changes very rapidly, and a third region, toward the bottom of the plot, where the log $N_{H_{\perp}}/\log N_{H_{\perp}}$ relation is again linear with a slope smaller than unity and which in general depends on the variations of $\eta$ with $N_{H_{\perp}}$. For the models in this paper, we assume that $\eta_0$ in equation (9) does not vary with gas surface density $M_{\odot}$ (see above discussion). The factor $J_L/(1 + \eta_0)$ is what determines the critical column density where the H I $\rightarrow$ H II transition occurs, i.e., the column density where the steep slope of the second region starts. The rapidity of the decrease of $N_{H_{\perp}}$ in the second region also depends on metal abundances. When $Z$ increases above 0.05 $Z_{\odot}$, metal line cooling becomes quite important in that region, and the transition gets less sharp as $Z$ increases. The shape of these curves is quite important because it determines the slope of the distribution function around the Lyman limit region.

4. FROM THE OBSERVED $N_{H_{\perp}}$ DISTRIBUTION FUNCTION TO THE TOTAL AMOUNT OF GAS DISTRIBUTED IN LLSS AND DLSS

Several papers on the H I column density distribution of absorbers (Tytler 1987; Lanzetta et al. 1991; Petitjean et al. 1993) have shown that a power law $N_{H_{\perp}}^{-1.5}$ fits the data from $10^{13}$ to $10^{21}$ cm$^{-2}$ approximately but not well enough to satisfy statistical tests such as the Kolmogorov-Smirnov test. In this paper we will use the deviations of $N_{H_{\perp}}$ distribution from a power law to determine $X$, after assuming that the total gas column density $N_{H_{\perp}}$ has a power-law distribution function of the form

$$g(N_{H_{\perp}}) = K(1 + z)^{N_{H_{\perp}}}x^2. \quad (14)$$

In order to derive the $N_{H_{\perp}}$ distribution from $g(N_{H_{\perp}})$ we must orient the absorbers randomly in the plane of the sky, assume an axial ratio $h/R$ for the slab, and apply the $N_{H_{\perp}}/N_{H_{\perp}}$ conversion factor. For our model fitting, we will use $Z = 0.02 Z_{\odot}$, an $X$ independent of redshift, and an axial ratio of $h/R = 0.2$, since clouds are likely to be neither...
spherical nor thin disks. Results essentially do not depend on h/R for h/R ≤ 0.5. We derive χ and X by comparing the resulting f(N_{H_i}) for γ = 1.0 with the data present in our compilation at N_{H_i} ≥ 1.6 × 10^{17} cm^{-2}. The variable K is determined by the normalization condition for f, based on the observed number of absorbers.

We emphasize that it is not possible to present the data relative to the distribution function in a model-independent way because of LLSs with undetermined ε_{LL}. A deterioration of the available data might result from the operation of binning in N_{H_i} in order to render straightforward the comparison with the model distributions. Instead of binning the data, we match the model distribution to the individual detections. The details of the fitting procedure are given in Paper II; we underline here the main characteristics. The procedure is rather similar to that used by Storrie-Lombardi et al. (1996), but implements the algorithm in order to consider also the uncertainty in the determination of any single value of N_{H_i}. Large observational errors are included by leaving undetermined the “real” position of each event. We determine χ and X by a maximum likelihood analysis to the projected H I column density distribution f, fixing the real position of each event to the measured values of N_{H_i} when this is available; otherwise we use in the likelihood the integral of f between the maximum and minimum value of N_{H_i}. We normalize the theoretical distribution to give a number of detections with N_{H_i} ≥ 1.6 × 10^{17} cm^{-2} equal to the observed one. Two maxima for the likelihood are found:

\[
X = 2.82, \quad \alpha = 2.70, \quad K = 1.2 \times 10^{34},
\]

\[
X = 2.74, \quad \alpha = 2.57, \quad K = 1.8 \times 10^{31}.
\]

The 68.3%, 95.5%, and 99% confidence levels in the X-α plane are shown in Figure 4, where the circles indicate the location of the maxima as in equations (15) and (16). The self-gravitating gas solution (η = 0, α = 4.63, X = 3.33) lies well outside of the 99% confidence level (we would need the 99.999% confidence level to include it), and therefore it is not consistent with the data. We also have checked that a similar conclusion holds if we use the exact self-gravitating gas solution, as given by equation (7), in deriving the N_{H_i,1} - N_{H_i,0} relation. For this case, both X and the best-fit α value are within 1% of the values obtained using equation (5) and η = 0 for the vertical gas stratification.

In Figure 5 we compare the observed value of the cumulative function with the theoretical ones derived from the integral of the projected H I column density; we show results for the two best-fit models (the two maxima in Fig. 4) and for two models corresponding to the highest and lowest X values on the 95.5% confidence level of Figure 4. For points that have no defined N_{H_i}, i.e., which have large errors, we then compute their best distribution for a given f by spreading the data in the allowed range of N_{H_i}, according to f weighted with the redshift path. Between all the possible permutations of points with undetermined N_{H_i}, we then choose those that satisfy the C test on the deviations between the observed and the expected cumulative functions over the error interval (R), and over N_{H_i} (C) (see Paper II for more details and for the use of a numerical simulation to proof the validity of this approach). For X-α inside the 99% confidence level of Figure 4, the K-S tests on R and C are satisfied to the 99.9% level.

We have proved that the gas distribution between the LLS and DLS region follows a single power law with index α > 2 and the ionization level is such that less than 1% of the total gas is neutral when N_{H_i} = 1.6 × 10^{17} cm^{-2}. There is no need of a distribution more complicated than a power law once one takes into account ionization effects. Our results on α and X still hold even if we do not include in the data set the damped lines with 5 Å ≤ W ≤ 10 Å or if we exclude a certain percentage of these lines because of possible blending with smaller lines.

4.1. The Total Gas Content of LLSs and DLSs

We will discuss here some results relative to the best-fit values of X and α as given in equations (15) and (16) and to the two most extreme values of X on the 95.5% confidence level, namely, X = 3.07 (η_0 = 2.3) and α = 3.32 (hereafter 2σ-high); and X = 2.53 (η_0 = 75) and α = 2.27 (hereafter 2σ-low). The distribution function for the total column density \( \bar{N}_{H,i} \equiv N_{H,i}/10^{20} \text{cm}^{-2} \) can be written as

\[
g(\bar{N}_{H,i}) = \bar{K}(1 + z)\bar{N}_{H,i}^{-3}.
\]

In Figure 6, in arbitrary scale, the continuous lines show log f(\(N_{H,i}\)), the H I distribution function for the best-fit values of X and α as given in equations (15) and (16) and for the two most extreme values of X on the 95.5% confidence level, namely, X = 3.07 (η_0 = 2.3) and α = 3.32 (hereafter 2σ-high); and X = 2.53 (η_0 = 75) and α = 2.27 (hereafter 2σ-low). The distribution function for the total column density \( \bar{N}_{H,i} \equiv N_{H,i}/10^{20} \text{cm}^{-2} \) can be written as

\[
\bar{N}_{H,i} \equiv \frac{2 \times 10^{-3}\bar{K}}{2 - \alpha}(1 + z)^{\alpha - 2 - \bar{N}_{H,i}} - (1 + z)^\alpha, \quad \Omega_{\text{gas}} \simeq \frac{2 \times 10^{-3}\bar{K}}{2 - \alpha}(1 + z)^{\alpha - 2 - \bar{N}_{H,i}} - (1 + z)^\alpha
\]

where δ depends on the cosmological model and is a function of \(z, \Omega_M, \Omega_{\Lambda}\). For a standard Friedmann universe.
in which \( q_0 = 0 \), we have \( \delta = 1 \), and we will use this value for the rest of this section (for \( \Omega_M = 0.3 \) and \( \Omega_\Lambda = 0.7 \) \( \delta \) instead depends on \( z \) and is close to zero at \( z \sim 2.5 \)).

In Table 1 we give the values of \( \Omega_{\text{gas}}(i) h_{60} \) for \( i = 1, 2, \) and 3, which is the total gas density in the universe at \( z \approx 2.5 \) produced by absorbing clouds whose \( \text{H} \upiota \) column density projected along the line of sight is between \( N_{\text{HI}}^\text{min} \) and \( 10^{22} \) \( \text{cm}^{-2} \). We will consider \( N_{\text{HI},1} = 10^{14} \) (see \( \S \) 4.2), \( N_{\text{HI},2} = 1.6 \times 10^{17} \), and \( N_{\text{HI},3} = 1.3 \times 10^{20} \) \( \text{cm}^{-2} \). For each corresponding value of \( N_{\text{HI},i} \) we give \( X_i \), the log ratio of total to neutral gas column density. Results are given both for the best-fitting models and for the two most extreme values of \( X \) on the 95.5\% confidence level of Figure 4. In the table we also show the gas scale heights for \( N_{\text{HI},2} = 1.6 \times 10^{17} \) and \( N_{\text{HI},3} = 1.3 \times 10^{20} \) \( \text{cm}^{-2} \) and values of \( \Omega_{\text{dark}}(i) \).

For \( \Omega_{\text{dark}} \) in the regions coinciding with the gas, the factor \( \zeta \) to be substituted into Table 1 is 1, independent of assumptions on cloud size and relative distribution of dark matter and gas. For rotating disks embedded in spherical dark halos, one can compute the contribution of the total dark matter surface density to the cosmological matter density. This contribution associated with DLS or LLS systems depends on the rotational velocity \( V \) and is given by \( \Omega_{\text{dark}} \) using \( \zeta \approx V/c_s \). This factor may be close to 1 for dwarf clouds, but \( \zeta \approx 1 \) would hold for giant disk protogalaxies. For the range of uncertainties in Table 1, the value of

| Cloud | 2 \( \sigma \)-High | Best-Fit 1 | Best-Fit 2 | 2 \( \sigma \)-Low |
|-------|-------------------|------------|------------|-----------------|
| \( x \) | 3.32 | 2.70 | 2.57 | 2.27 |
| \( R \) | 3.81 | 0.97 | 0.74 | 0.42 |
| \( \eta_0 \) | 2.3 | 12.5 | 20 | 75 |
| \( X(1) \) | 5.5 | 5.2 | 5.1 | 4.9 |
| \( X(2) \equiv X \) | 3.1 | 2.8 | 2.7 | 2.5 |
| \( X(3) \) | 0.4 | 0.2 | 0.2 | 0.1 |
| \( h(2)/\text{kpc} \) | 2.6 | 0.8 | 0.6 | 0.3 |
| \( h(3)/\text{kpc} \) | 1.0 | 0.3 | 0.2 | 0.05 |
| \( \Omega_{\text{gas}}(1) h_{60} \) | 0.06 | 0.01 | 0.01 | 0.006 |
| \( \Omega_{\text{gas}}(2) h_{60} \) | 0.005 | 0.004 | 0.004 | 0.003 |
| \( \Omega_{\text{gas}}(3) h_{60} \) | 0.002 | 0.002 | 0.002 | 0.002 |
| \( \Omega_{\text{dark}}(1) h_{60} \) | 0.1\( \zeta \) | 0.2\( \zeta \) | 0.2\( \zeta \) | 0.3\( \zeta \) |
| \( \Omega_{\text{dark}}(2) h_{60} \) | 0.01\( \zeta \) | 0.05\( \zeta \) | 0.07\( \zeta \) | 0.3\( \zeta \) |
The excess of systems in the damped region, suggested by the $N_{\text{HI}}$ data, is naturally explained in terms of a sharp transition from a highly ionized to a highly neutral gas distribution, and it does not require any break in the distribution of the total gas column density.

2. The total gas column density distributions, $g(N_{\text{HI}})$, that best fit the data for $1.75 < z < 3.25$ in the LLS and DLS region can be described by a power law of index $-\alpha$ with $2 \leq \alpha \leq 3.7$. This has the important consequence that low column density systems contains more mass than high column density systems.

3. We have tested that our results, which are relative to the redshift bin $1.75 < z < 3.25$, depend weakly on the data selection and on the number redshift evolution. They also do not show a strong dependence on the assumed thickness to diameter ratio of the slab or from its metallicity if $Z \leq 0.05 Z_\odot$.

4. The gas fractional ionizations increase with decreasing column density, and data are best fitted when hydrogen fractional ionizations are of the order $\sim 0.002$ at $\tau_{\text{HI}} = 1$, i.e., $X \sim 2.8$. Gas fractional ionizations for absorbers in a background radiation field depend on the gas volume densities and temperature. The model presented in detail in this paper considers the gas self-gravity and a dark matter potential in which the dark matter surface density inside 1 gas scale height is proportional to the gas surface density (constant $\eta_0$). However, the range of $\alpha$ and $X$ values compatible with the data are similar if one considers instead a model where $\eta_0$ varies with $N_{\text{HI}}$ (e.g., if $\eta_0 N_{\text{HI}}$ is kept constant instead). Table 1 summarizes our main results; the values of $\eta_0$ for more general models refer to LLS column densities that are no longer opaque to the UV ionizing radiation.

Although our likelihood analysis allows power-law indices $\alpha$ in a rather wide range, 2–3.7, other arguments make the upper half of this range more likely: $\alpha \geq 2.7$, $\eta_0 \leq 13$, and $X \geq 2.8$. For larger values of $\eta_0$, the gas scale height $h(3)$ for DLSs is uncomfortably small, and the LLS contribution $\Omega_{\text{dark}}(2)$ to cosmological matter density is uncomfortably large (see Table 1; the $\Omega$ values given for $g_0 = 0$; $\Omega$ values for $\Omega_M \sim 0.3$ and $\Omega_\Lambda = 0.7$ are larger by a factor $\sim 2.5$). For the total dark matter mass, the factor $\zeta$ for $\Omega_{\text{dark}}$ is of the order $V/10$ km s$^{-1}$, which suggests $V < 100$ km s$^{-1}$ on average for LLSs and DLSs at redshifts $z \sim 2.5$. This finding is in agreement with the rotational velocities of absorber halos predicted by Valageas, Schaeffer, & Silk (1999) and with models that propose that a large fraction of Ly$\alpha$ absorption systems originate from small, low-luminosity systems (Abel & Mo 1998; Haehnelt, Steinmetz, & Rauch 2000; Rao & Turnshek 2000). However Steidel, Dickinson, & Persson (1994) have shown that small rotational velocities are not required by low-redshift Mg $\text{II}$ absorption systems with $W > 0.3$ Å. These have the same average density per unit path of redshift as LLSs and seem to be associated with normal galaxies whose halos extends for $\sim 40$ kpc. This opens the issue about possible evolutionary scenarios for LLSs, which can be fully addressed only once we know the redshift evolution of galactic halos and of the galaxy luminosity function. The possible role of faint galaxies as those in pairs close to bright galaxies (Churchill 2000) should also be taken into account. Futhermore, it would be extremely useful to have a larger statistical sample of LLSs and DLSs at $z < 1.75$ and at $z > 3.25$, which,
together with a better knowledge of the evolution of the background ionizing radiation field, allows a determination of \(z\) and \(\eta\) at these redshifts (see Paper II for an attempt with the actual data set).

5. Although Ly\(\alpha\) forest clouds, with their much smaller column density, might be physically different and have different values of \(\alpha\) and \(\eta_0\), extrapolations using constant \(\alpha\) and \(\eta_0\) values as for LLSs and DLSs give surprisingly good fits.

We cannot make definitive statements about the low-density Ly\(\alpha\) forest clouds because of possible dynamic deviations from hydrostatic equilibrium and possible pressure confinement. Nevertheless, the fact that \(\alpha\) is appreciably larger than 2 near LLSs for \(z \sim 2.5\) suggests that forest clouds contained more gas than LLSs, which in turn contained more gas than DLSs. Several papers have shown that the gas content in DLSs at \(z \sim 2.5\) is below the mass content in galaxies in the local universe (Storrie-Lombardi & Wolfe 2000), \(\Omega_{\text{m}} \approx 3 \times 10^{-3} - 7 \times 10^{-3}\), and that the stellar mass density in stellar systems at \(z \sim 2.5\) is very small (Madau, Pozzetti, & Dickinson 1998). It therefore is likely that gas in forest clouds and LLSs at \(z \sim 2.5\) has collapsed and contributed to stars in present-day galaxies.

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