Massive Neutrinos as Probe of Higher Unification

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Abstract

There are strong indications for neutrino masses and mixings in the data on solar neutrinos as well as in the observed deficit of muon neutrinos from the atmosphere. The COBE data and other analysis of the large scale structure in the Universe also seems to require a hot component in the universe’s dark matter, which can be interpreted as a massive neutrino with mass in the few eV range. Implications of a non-vanishing neutrino mass for physics beyond the standard model is discussed. It is argued that a non-zero neutrino mass is a strong indication of a new local $B - L$ symmetry of electro-weak interactions. In particular, it is noted that the simplest picture based on the left-right symmetric unification with a high scale for the $B - L$ symmetry (as well as the scale of the right-handed gauge boson $W_R$) as required by the constraints of grandunification and present LEP data in a minimal $SO(10)$ model, can accomodate the solar neutrino data and a weak hot dark matter neutrino but not the atmospheric neutrino puzzle. A model with low scale $W_R$ (in the TeV range) where the Dirac mass of the neutrino arises at the one loop level can also do the same job. The low $W_R$ picture can be tested in many rare decay experiments whereas the minimal $SO(10)$ model can be tested by the $\nu_\mu$ to $\nu_\tau$ oscillation experiments to be carried out soon. Some non-minimal scenerios to accomodate all three data are also discussed.

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I. Introduction:

The standard model of electro-weak interactions is a highly successful, yet a very unsatisfactory theory and it is an absolute certainty that there is new physics beyond it which is waiting to be discovered. One of the tell-tale signs of this new physics is in the arena of neutrinos where there are quite strong indications for neutrino masses and mixings from several different experiments. They are: i) deficit of solar neutrinos now observed in four different experiments[1]; ii) the depletion of atmospheric muon neutrinos observed in three different experiments[2]; and iii) the apparent need for some hot dark matter in the Universe[3]. Let us briefly summarize the experimental results in these areas and state their implications for neutrino masses and mixings.

1.1 Solar Neutrino Deficit:

At present four different experiments involving three different targets (Cl$^{37}$, electrons in water, and Ga$^{71}$)[1] exposed to solar neutrinos have reported measuring a flux of neutrinos from the sun which seem to be roughly $\frac{1}{2}$ to $\frac{1}{3}$ times the expected number now calculated by several groups using the standard solar model. Defining $\Phi$ as the flux of neutrinos and denoting by a subscript the observed and the predicted values for this, the results of the various experiments are:

$$\Phi_{obs}(Cl) = (2.19 \pm 0.25)SNU$$  \hspace{1cm} (1)

$$\Phi_{STD}(Cl) = (8.0 \pm 3.0)SNU \quad [BP, ref.4]$$  \hspace{1cm} (2)

$$\Phi_{STD}(Cl) = (6.4 \pm 1.4)SNU \quad [T, ref.5]$$  \hspace{1cm} (3)

$$\frac{\Phi_{obs}(KAM)}{\Phi_{STD}(KAM)} = (.48 \pm .05 \pm .06) \quad [ref.1]$$  \hspace{1cm} (4)

Turning now to the Gallium experiments, the SAGE experiment reports the result[1]:

$$\Phi_{obs}(Ga^{71}) = (58^{+17}_{-24} \pm 14)$$  \hspace{1cm} (5)

The Gallex results are[1]:

$$\Phi_{obs}(Ga^{71}) = (83 \pm 19 \pm 8)$$  \hspace{1cm} (6)

The theoretical predictions for Gallium are:

$$\Phi_{STD}(Ga^{71}) = (122 to 132 \pm 5 to 7)$$  \hspace{1cm} (7)

This discrepancy between theory and experiment is called the solar neutrino problem.
fit all four pieces of data given below simultaneously if the neutrino is assumed to be massless[6]. Furthermore, the chlorine data also exhibits a time variation which is apparently anti-correlated with the number of sunspots. In this talk, I will ignore this possibly exciting feature of the experiments and concentrate on the remainder of the puzzle which seems to require a massive neutrino, assuming of course that there is no flaw in our understanding of the sun. It is worth pointing out that detailed study of the problem keeping the core temperature of the sun as a free parameter[6] has been made and it has been concluded that all the four results cannot be accomodated without invoking new properties of the neutrino . Qualitatively, the point is that the $^8\text{Be}$ neutrino flux varies like $T_e^{18}$ whereas the $^7\text{Be}$ neutrino flux goes like $T_e^8$ and the pp neutrino flux is almost temperature independent . Therefore by lowering the core temperature of the Sun, $T_e$ by about 4%, one can fit the Kamiokande data ; but this predicts for Chlorine experiment a neutrino signal of over 4SNU’s, which is inconsistent with data at the 2σ level. The exciting implication of this is that this may be the first indication of new physics beyond the standard model ; however, a final conclusion must wait for further data not only from the above on-going experiments but also from experiments under preparation such as the SNO experiment in Canada, the Borexino experiment in Gran Sasso , Italy.

The most straightforward interpretation of the present solar neutrino data taken literally is that massive neutrinos of different generations mix among themselves so that a neutrino emitted in a weak interaction process not being an eigenstate of the Hamiltonian evolves into another weak eigenstate (such as $\nu_\mu$ or $\nu_\tau$) as it travels through space. Since the latter do not interact with the target material, they remain undetected and the original $\nu_e$ flux gets depleted in the process explaining the puzzle. Assuming mixing only between two species of neutrinos, it has been determined that there are two kinds of oscillation solutions: one, where the $\nu_e$ oscillates on its way to the earth ( called the vacuum oscillation [7]) and another, where the oscillation takes place in the solar core by a mechanism similar to the refraction of light in a medium [8] ( called the Mikheyev-Smirnov-Wolfenstein matter oscillation ). For our purpose,what is important is the values of the mass difference squares of $\nu_e$ and $\nu_\mu$ ($\Delta m^2 \equiv |m_{\nu_e}^2 - m_{\nu_\mu}^2|$) and mixing angle $\sin^22\theta$ that fit the four pieces of data simultaneously. They are as follows:

i) The small angle non-adiabatic MSW solution:

$$\Delta m^2_{\nu_e\nu_i} \simeq (.3 - 1.2) \times 10^{-5} eV^2$$
$$\sin^22\theta \simeq (.4 - 1.5) \times 10^{-2}$$

(8)
ii) Large angle MSW solution:

\[ \Delta m^2_{\nu_e\nu_i} \simeq (3 - 3) \times 10^{-5} eV^2 \]
\[ \sin^2 2\theta \simeq 0.6 - 0.9 \]  

(9)

iii) Vacuum oscillation solution:

\[ \Delta m^2_{\nu_e\nu_i} \simeq (0.5 - 1.1) \times 10^{-10} eV^2 \]
\[ \sin^2 2\theta \simeq (0.8 - 1) \]  

(10)

1.2 Atmospheric Neutrino Puzzle:

In the upper atmosphere, hadronic collisions produce \( \pi^\pm \), which decay to \( \mu^\pm \) and muon neutrino (or anti-neutrino) and the \( \mu^\pm \) subsequently decay to \( e^\pm \) and a electron neutrino (or \( \bar{\nu}_e \)) and a \( \nu_\mu \) (or a \( \bar{\nu}_\mu \)). Thus each pion in its decay produces two muon neutrinos for every electron neutrino (or their anti-particles). Several underground experiments have looked for the electrons and muons that would result from the interactions of these atmospheric neutrinos[2] and in three of the experiments what has been discovered is that the number of muon events is far less than the number expected on the basis of the above discussion. In fact, the observed flux ratio of muon to electron events normalized to the flux ratio based on the standard model, denoted by \( R(\mu/e) \) is reported to have values:

\[ R(\mu/e) = 0.60 \pm 0.07 \pm 0.05, \quad [KAMII] \]
\[ = 0.54 \pm 0.05 \pm 0.12, \quad [IMB] \]
\[ = 0.55 \pm 0.27 \pm 0.10, \quad [SoudanII] \]  

(11)

The theoretically expected value of \( R(\mu/e) \) is of course one.

Again, a straightforward explanation of this can be given if one assumes that \( \nu_\mu \) oscillates to another light neutrino. The data can be fitted with[9] the values of \( \Delta m^2_{\nu_\mu\nu_j} \simeq 0.5 - 0.005 eV^2 \) and \( \sin^2 2\theta \simeq 0.5 \). It is important to point out that there are two other experiments one by the NUSEX group and one by the Baksan group, which do not see this effect; they however have lower statistics at this time and their present results are consistent with the above estimates of the mass differences and mixing angles.

1.3 Hot Dark Matter Neutrinos:
Data on the extent of structure in the universe is now available on a wide range of distance scales. Evidence from the COBE results on the anisotropy of the cosmic microwave background radiation, galaxy-galaxy angular correlation, large scale velocity fields, and correlations of galactic clusters can all be fit by a model of the universe containing 70% cold dark matter and 30% hot dark matter. (But perhaps an admixture in the ratio 90% to 10% of CDM to HDM may not be inconsistent). It has also been recently remarked that a warm dark matter alone may be able to fit the long and short distance parts of the power spectrum.

A prime candidate for the hot dark matter is a neutrino with mass in the appropriate range of 7eV to 2eV. Such a model not only provides a consistent explanation of the shape of the density fluctuation spectrum but also the observed estimates of the absolute density on small and large scales.

1.4 Other constraints on Neutrinos:

Astrophysical and cosmological observations have established many constraints on the properties of the various types of neutrinos; in this section, we present only those constraints that are of immediate relevance for the discussion in this paper. The first is the non-observation of neutrinoless double beta decay (\(\beta\beta_0\)), which provides an important limit on an effective Majorana mass, \(m_\nu \leq 1 - 2eV\) \[12\]. Second, if the possibility of light sterile neutrino is considered, its possible oscillation to one of the active neutrino species must be severely limited \[13\] from considerations of nucleosynthesis \[14\]; which implies that at the 95% confidence level, the number of effective neutrino-like relativistic degrees of freedom consistent with present data on Helium abundance is \(\leq 3.3\). This constrains the product \(\Delta m^2_{\nu_s\nu_i} \sin^4 2\theta \leq 3.6 \times 10^{-6} eV^2\).

II. Neutrino Mass Matrices Suggested by Data:

Let us start with a brief reminder about possible structure of neutrino masses in unified theories. Because of the fact that the neutrinos do not have electric charge, their mass matrices enjoy a much richer texture than the charged fermions. It is most convenient to describe neutrino masses in the language of two component neutrinos (denoted by \(\nu_i\), which transform under an \(SL(2,C)\) transformation \(Z\) as

\[\nu_i \rightarrow Z \nu_i\] (12)

Recalling that \(Z\) has determinant one in order to qualify as a Lorentz transformation, a Lorentz scalar can be written as \(i\nu_i^T \sigma_2 \nu_j\) where \(i,j\) go over all two component
chiral fermions. This statement holds of course for all fermions with the important exception that if a fermion has a nonzero "charge", then in order for the mass term to respect invariance under the corresponding group of transformations, the mass term above must necessarily connect two different spinors. Thus the charged fermions have half the dimensionality of neutral fermions such as a neutrino. Denoting the elements of the neutrino mass matrix as $m_{ij}$, it is obvious that for arbitrary mass matrix, there will be mixings between all neutrinos and therefore neutrino oscillations.

Let us now proceed to discuss the kind of neutrino spectrum and their mass matrices that would be required to fit the above observations. This discussion follows a recent paper by D. Caldwell and this author [15]. We have isolated three different scenarios for this purpose (we denote them by A, B, C respectively). In case A, one can make do only with the three known neutrinos with a fine-tuned mass matrix whereas cases B and C postulate the existence of one or more light sterile neutrino species in addition to the three known weak-interaction-active neutrinos. Let us discuss them separately.

**case IIA:**
In this case, we will assume that $\nu_e, \nu_\mu$ and $\nu_\tau$ are all nearly degenerate with mass around 2 to 2.5 eV. The mass differences are appropriately arranged so that $\nu_\mu - \nu_\tau$ oscillations explain the atmospheric neutrino problem and similarly $\nu_e - \nu_\mu$ mass differences as well as mixings are so arranged that they can explain the solar neutrino deficit via the MSW mechanism using the small angle non-adiabatic solution. The simplest mass matrix, which can achieve this is:

$$M = \begin{pmatrix}
m & \delta s_2 s_1 & -\delta s_1 s_2 \\
\delta s_2 s_1 & m + \delta s_2^2 & -\delta s_2 \\
-\delta s_1 s_2 & -\delta s_2 & m + \delta
\end{pmatrix}$$  \hspace{1cm} (13)

In eq.(13), $m \simeq 2-2.5eV; \delta \simeq .08eV; s_1 \simeq .05$ and $s_2 \simeq .35$. It is worth pointing out that a value of the Majorana mass for $\nu_e$ of this magnitude is barely consistent with the present upper limits from neutrinoless double beta decay [9] and will be tested in near future by the high precision $\beta\beta_0$ searches using enriched Germanium now under progress [12]. A very simple way to avoid the double beta decay constraint would be to assume the three neutrinos to be Dirac type so that overall lepton number is conserved. Obviously, hot dark matter in thus case is, distributed between the three active species of neutrinos almost equally.
In this case, the hot dark matter is identified with one or more light sterile neutrinos, $\nu_s$, whereas the known neutrinos are assumed to be extremely light with a mass matrix similar in form to the one in eq.(13) with the important exception that the value of $m$ is no more in the eV range. As a result, the naturalness problem in the theoretical derivation of this matrix is not as severe as in case IIA. The nucleosynthesis constraints then require the masses of $\nu_s$ to be higher for them to acquire the mantle of "dark-matter-dom" [15]. The argument goes as follows: Any light particle present at the epoch of nucleosynthesis will contribute to the energy density at that epoch and will therefore affect the Helium abundance. As mentioned before, present data seems to allow this additional contribution to density to upper bounded in such a way that its effective contribution is less than that of 0.3 neutrino species. Below, we will denote this parameter by $\delta N_{\nu}$ as is conventionally done. Since number density scales like the three quarter power of the energy density, the number density of these particles at the era of structure formation is given by:

$$n_{\nu_s} \simeq n_{\nu_e} \times (\delta N_{\nu})^\frac{3}{4}$$  \hspace{1cm} (14)

Its mass (i.e. $m_{\nu_s}$) will therefore be bigger by a factor $(\delta N_{\nu})^{-\frac{4}{3}}$. For $\delta N_{\nu} = .3$, this leads to $m_{\nu_s} = 17 eV$ for only one species of sterile neutrino. In actual models with sterile neutrinos, this constraint is generally implemented by letting the sterile neutrino decouple much earlier than the epoch of nucleosynthesis. In many practical situations, the sterile neutrino decouples above the quark-hadron phase transition temperature ($\simeq 200 - 400 MeV$). Its contribution to $\delta N_{\nu}$ then is roughly $\simeq 0.1$. In such a situation, the required value of the mass, $m_{\nu_s} \simeq 39 eV$. At the present state of our understanding of the structure formation, there is enough model dependence so that such high values of $m_{\nu_s}$ cannot perhaps strictly be ruled out; however, they are large enough so as to be unlikely candidates for hot dark matter.

Another constraint on such models arise from the fact that, at the epoch of nucleosynthesis, oscillations to the sterile component from the active components must also be suppressed for the same reason as above. It will therefore be extremely difficult test this scenario.

**Case IIC:**

In this scenario, $\nu_\mu$ and $\nu_\tau$ are assumed to have a mass of about 3-4 eV each and constitute the hot dark matter of the universe and also resolve the atmospheric neutrino puzzle via their mutual oscillation. The solar neutrino deficit is explained via the $\nu_e$ to $\nu_s$ oscillation with both of them having mass in the range of $10 - 3$ eV.
Such a picture was advocated for the 17 keV neutrino by Caldwell and Langacker[17]. The simplest mass matrix for this case is:

$$M = \begin{pmatrix} 
\mu_1 & \mu_3 & 0 & 0 \\
\mu_3 & \mu_2 & 0 & 0 \\
0 & 0 & m & \delta/2 \\
0 & 0 & \delta/2 & m + \delta 
\end{pmatrix} \tag{15}$$

where columns refer to $\nu_s, \nu_e, \nu_\mu$ and $\nu_\tau$; $\mu_{1,2}$ are of order $10^{-2} - 10^{-3}$ eV; $\mu_3$ is of order $10^{-4} - 10^{-5}$ eV, $m = 3 - 4 eV$ and $\delta = 0.07$ to 0.0007 eV in order to accommodate all observations.

Before ending this section, we like to caution the reader that the mass matrices displayed in this section are required only if one takes all three indications of nonvanishing neutrino mass described above quite seriously. This is by no means absolutely compelling. In view of this, below we explore the theoretical implications of only the possible existence of a non-vanishing neutrino mass rather than any detailed texture for these masses as for instance envisioned in the above mass matrices. The goal of such an analysis is not only to expose these plausible models to tests via neutrino experiments but also to provide guidance to experimental explorations.

III. Massive Neutrinos, Local B-L Symmetry, and Spontaneous Parity Violation:

Let us start by reminding the reader that in the standard model, the neutrinos are massless because of the fact that only the lefthanded chirality state of the neutrino appears in the the fermion spectrum and the fact that $B - L$ is an exact symmetry of the Lagrangian. In order to obtain massive neutrinos, one must therefore include the right-handed neutrino in the spectrum. It however turns out that as soon as this is done, in the theory there appears a completely triangle anomaly free generator, the $B - L$. This symmetry is then a gaugeable symmetry and it would be rather peculiar if nature chooses not to gauge a symmetry which is gaugeable. If following this line of reasoning, we use $B - L$ as a gauge symmetry, the most natural gauge group turns out to be the Left-Right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [18], which breaks at some high scale to the standard model group. Let us briefly remind the reader about some of the features of the model.
Since the gauge group is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, this model has in addition to the usual left-handed $W^\pm_L$ and $Z$, an additional set of gauge bosons, the $W^\pm_R$ and $Z'$. The quarks and leptons in this model are assigned in a completely left-right symmetric manner, i.e. if we define $Q \equiv (u,d)$ and $\psi \equiv (\nu,e)$, then $Q_{L}(2,1,1/3)$ and $Q_{R}(1,2,1/3)$ are assigned in a left-right symmetric manner and similarly, $\psi_{L}(2,1,-1)$ and $\psi_{R}(1,2,-1)$ (where the numbers in the parenthesis represent the quantum numbers under the gauge group of the corresponding fields).

The Higgs sector of the model that leads naturally to small neutrino masses in this model consistss of the bi-doublet field $\phi \equiv (2,2,0)$ and the triplet fields $\Delta_{L} \equiv (3,1,+2)$ and $\Delta_{R} \equiv (1,3,+2)$. In order to see how the small neutrino masses arise in this model, let us write down the Yukawa couplings of the model:

$$L_Y = h_1 \bar{Q}_L \phi Q_R + h'_1 \bar{Q}_L \tilde{\phi} Q_R h_\ell \bar{\psi}_L \phi \psi_R + h'_\ell \bar{\psi}_L \tilde{\phi} \psi_R + f \psi^T L \tau_2 \Delta_L \bar{\psi}_L + L \to R + h.c.$$ (16)

The gauge symmetry breaking is achieved in two stages: in the first stage, the neutral component of $\Delta_R$ multiplet acquires a vev $<\Delta^0_R> = v_R$, thereby breaking the gauge symmetry down to the $SU(2)_L \times U(1)_Y$ group of the standard model; in the second stage, the neutral components of the multiplet $\phi$ acquire vev breaking the standard model symmetry down to $U(1)_{em}$. At the first stage of symmetry breaking, $W_R$ and $Z'$ acquire masses of order $g v_R$ and in the second stage the familiar $W_L$ and $Z_L$ acquire masses. As mentioned earlier, in order to understand the near maximality of parity violation at low energies, the masses of $W_R$ and $Z'$ must be bigger than those of the $W_L$ and the $Z$ bosons. The existing data have been extensively analysed by various groups[20] and one finds that for the minimal model, the most model independent limit is provided by the existing LEP data[21] and corresponds to $M_{Z'} \geq 800 GeV$ and using the relation between the $W_R$ and $Z'$ masses present in this model, we get $M_{W_R} \geq 475 GeV$.

Now turning to the fermion sector, at the first stage of symmetry breaking, the f-terms in the Yukawa coupling give nonvanishing masses to the three right-handed neutrinos of order $f v_R$ keeping all other fermions massless. At the second stage, quarks, charged leptons as well as the neutrinos acquire Dirac masses. The $\nu_L-\nu_R$ mass mass matrix at this stage is a $6 \times 6$ mass matrix of the following see-saw form[22]:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m^T_D & M_R \end{pmatrix}$$ (17)

In eq.(17), $m_D$ and $M_R$ are $3 \times 3$ matrices. As is well known this see-saw form
leads to three light eigen-values generically of order

$$m_{\nu_i} \simeq \left( \frac{m_D^2}{M_R} \right)$$ (18)

The typical values of $m_D$ are expected to be of order of the charged fermion masses in the theory whereas the $M_R$ corresponds to the scale of $B-L$ breaking which is a very high scale, thereby explaining the smallness of the neutrino masses. The specific value of $m_D$ is however model-dependent and depending on what the value of $m_D$ is, the spectrum of the light left-handed Majorana neutrinos will be of the eV:keV : MeV type or of the micro-milli- eV type. The former type of spectrum can be tested in the double beta decay as well as the conventional beta decay end-point experiments whereas the second spectrum can be tested in the solar neutrino as well as the long base-line neutrino experiments. In view of the discussion of the previous section, the micro-milli-eV spectrum is of great current interest. I would therefore like to explore in the context of left-right symmetric models ways to get such a spectrum. As mentioned, in the simplest realizations of these models, the $m_D \approx m_f$, where $f =$ leptons or quarks. If we therefore want $m_{\nu_\mu} \approx 10^{-3}$ eV, then the mass $M_R$ must be of order $10^{10} - 10^{12}$ GeV. This would suggest grand-unification models of type $SO(10)$ or some higher group containing it. The $SO(10)$ possibility has recently been analysed by Babu and this author[23] and I present this model in the subsequent section.

Another possibility is however that the neutrino Dirac mass may be radiative in origin in which case it will roughly be of the magnitude $m_D \approx (h^2/16\pi^2)m_f$. If we then use the see-saw formula then for $h \approx .1$ and $M_R \approx$ few TeV, one can also get a micro-milli-eV type spectrum relevant in the discussion of solar neutrino puzzle. Such an example was worked out in ref.[24] and I briefly outline the salient features of this model. The main lesson to learn from such a model would be that a neutrino mass in the milli-electron volt range need not necessarily mean a superheavy scale for $B-L$ symmetry breaking.

**IV. A Low $W_R$ Scenerio for the Solar Neutrino Puzzle:**

The particle content and the gauge group of the model is same the left-right symmetric models with a few additional features: i) we demand that the model obey a softly broken Peccei-Quinn symmetry with quarks having PQ charge +1 whereas the leptons are required to have this charge -1. As a result of this, the $\phi$ field has PQ
charge +2 and we must set $h_1' = 0$. On the other hand in the leptonic sector, we must have $h_\ell = 0$. The Higgs potential in this case is such that, the vev of $\phi$ has the form:

$$<\phi> = \begin{pmatrix} \kappa & 0 \\ 0 & 0 \end{pmatrix}$$

(19)

It is then clear that, at the tree level, eq (19) only allows the up quarks and the charged leptons of the three generations to pick up mass. The down type quark masses and the neutrino Dirac masses remain zero at the tree level as do all the CKM angles. We therefore have to extend the model somewhat to get the down quark and the neutrino Dirac masses at the one-loop level. It was shown in ref.[24] that this can be achieved by adding to the model only three extra color triplet scalar fields denoted by $\omega_L, \omega_{R and \omega}$. To the Yukawa Lagrangian described earlier, one can then add the following PQ invariant piece:

$$L'_Y = f' Q_L^T C^{-1} \tau_2 Q_L \omega_L + L \rightarrow R$$

(20)

In eq.(20),the $f'$ is a three by three matrix in generation space. Note that $\omega_L$ and $\omega_R$ have opposite PQ charges of $-2$ and $+2$ respectively. Similarly, to the Higgs potential,we add the term $\omega^*_L \omega + L \rightarrow R$ (which breaks the PQ invariance softly) in addition to the usual PQ invariant renormalizable terms. It was shown in ref.[24] that inclusion of the $\omega$ fields and in particular the above soft PQ breaking terms generate the down quark masses at the one loop level and also a small but finite vev $\kappa'$ that generates small radiative neutrino Dirac masses as claimed.

Let us present a brief outline of down quark and neutrino Dirac masses that result in this model:

$$M_d = \varepsilon f' M_u f' + r M_u$$

(21)

Since the up quark mass matrix at the tree level is diagonal, eqn (21) should generate not only the down quark masses but also the CKM angles. Since $f'$ is a symmetric matrix, there are 8 parameters in eqn.(21) and we fit them to reproduce the six unknowns in the quark sector. In the neutrino sector, it is the parameter $r$ that determines the Dirac mass and in addition, there will be the Yukawa couplings associated with the $\Delta$ ( i.e. the coupling matrix $f$ ) that enter via the see-saw
mechanism: The neutrino mass matrix is then given by

\[ M_\nu = f v_L + \left( \frac{v_R^2}{v_R} \right) M_\ell f^{-1} M_\ell \]  \hspace{1cm} (22)

The value of \( \kappa' \) is estimated to be in the range of 10 to 50 MeV and the value of \( v_L \) is of the order of \( \lambda (\kappa')^2 / v_R \) where \( \lambda \) is a typical scalar self coupling (assumed to be of order \( 10^{-2} \)). This leads to an estimate of \( v_L \) of order \( \approx 10^{-1} \text{eV} \) or so. Therefore with an appropriate choice of the parameters \( f \), an MSW type spectrum can be obtained.

Thus, while the grand-unified type structure may be the simplest way to obtain a micro-milli-eV type neutrino spectrum, a low \( B-L \) scale alternative cannot be completely ruled out. As already mentioned, such low scale \( B-L \) theories can be tested via the rare decay experiments to be carried out or under way[25].

V. Minimal SO(10) GUT and Predictions for Neutrino Masses and Mixings:

The grandunified theories [20](GUTs)provide an elegant extension of physics beyond the standard model. The requirement that the gauge couplings constants in these theories become equal at the GUT scale \( (M_U) \) lends them a predictive power which makes it possible to test them in experiments such as those looking for the decay of the proton. The most predictive such theory is the minimal SU(5) model of Georgi and Glashow, where the SU(5) symmetry breaks in one step to the standard model. The only new mass scale in this model is the \( M_U \) which can be determined by the unification requirement using the low energy values of any two gauge couplings from the standard model. One then predicts not only \( M_U \), but also the remaining low energy gauge coupling constant (say \( \sin^2 \theta_W \)). It is well known that for the minimal SU(5) model, they lead to predictions for the proton life-time as well as \( \sin^2 \theta_W \) both of which are in contradiction with experiments.

This however does not invalidate the idea of grandunification and attention has rightly been focussed on SO(10) [26] GUT models which can accommodate more than one new mass scale. Supersymmetric SU(5) [27] models also belong to this class. In this class of two mass scale theories, the values of low energy gauge coupling constants can determine both the mass scales again making these theories experimentally
testable. The determination of the values of the new mass scales become more precise as the low energy values of the gauge coupling constants become better known. It is therefore not surprising that the recent high precision measurement of $\alpha_{\text{strong}}$ and $\sin^2 \theta_W$ at LEP[28] once again revived interest in grandunified theories[29].

Supersymmetric SU(5) theories have been studied with the goal of predicting the scale of supersymmetry breaking. These models however donot have any room for a nonzero neutrino mass nor natural generation of adequate baryon asymmetry, whereas, the SO(10) model is the minimal GUT scheme that provides a frame-work for a proper understanding both these problems. In recent papers, SO(10) models with a two step breaking to the standard model have been studied[30] and predictions have been given for the two new mass scales i.e. $M_U$ and $M_I$ making it possible to predict the proton life-time as well as the order of magnitude of the neutrino masses. The predictions depend on the nature of the intermediate symmetry $SU(2)_L \times SU(2)_R \times G_c$ where $G_c$ is $SU(4)_C$ (denoted as case (A)) or $SU(3)_c \times U(1)_{B-L}$ (denoted as case (B)). Let us note that in both the above cases we have broken the discrete local $Z_2$ subgroup of SO(10) (called D-parity in ref.[31]) at the GUT scale in order to make the see-saw formula natural[32]. Use of Higgs multiplets belonging to 45 and 210 representations to break SO(10) leads to such a scenario[31] automatically.

The predictions for the GUT and Intermediate scale for the non-SUSY version of this model have been studied including two-loop and threshold corrections in ref[30] and the results are:

$$Model(A) : \quad M_U = 10^{15.8_{-1.7}^{+2.2}} GeV \quad M_I = 10^{11.5_{-1.5}^{+2.8}+0.02} GeV \quad (23)$$

$$Model(B) : \quad M_U = 10^{15.8_{-1.7}^{+2.2}+0.8} GeV \quad M_I = 10^{9_{-3}^{+6}+18} GeV \quad (24)$$

This leads to a prediction for the proton life-time in non-SUSY SO(10) models for the two chains to be:

$$\tau_p = 1.6 \times 10^{35_{-7}^{+0.1}+3.2} years \quad Model(A)$$

$$\tau_p = 10^{35_{-7}^{+0.1}+8} years \quad Model(B) \quad (25)$$

These predictions are within the reach of the Super-Kamiokande experiment[33], which should therefore throw light on the non-SUSY version of the SO(10) model.

Let us now discuss the test of the SO(10) models from neutrino data. This would require making precise predictions of the neutrino masses and mixing angles.
This necessitates detailed knowledge of the Dirac neutrino mass matrix as well as the Majorana neutrino mass matrix. Luckily, it turns out that in $SO(10)$ models, the charge $-1/3$ quark mass matrix is related to the charged lepton matrix and the neutrino Dirac mass matrix is related to the charge $2/3$ quark matrix at the unification scale. However, prior to the work of ref.23, no simple way was known to relate the heavy Majorana matrix to the charged fermion observables. This stood in the way of predicting the light neutrino spectrum. It was however shown in ref.[23] that in a class of minimal $SO(10)$ models, in fact, not only the Dirac neutrino matrix, but the Majorana matrix also gets related to observables in the charged fermion sector. This leads to a very predictive neutrino spectrum, which we analyze. We use a simple Higgs system with one (complex) $10$ and one $126$ that have Yukawa couplings to fermions. The $10$ is needed for quark and lepton masses, the $126$ is needed for the see–saw mechanism. Crucial to the predictivity of the neutrino spectrum is the observation that the standard model doublet contained in the $126$ receives an induced vacuum expectation value (vev) at tree–level. In its absence, one would have the asymptotic mass relations $m_b = m_\tau$, $m_s = m_\mu$, $m_d = m_e$. While the first relation would lead to a successful prediction of $m_b$ at low energies, the last two are in disagreement with observations. The induced vev of the standard doublet of $126$ corrects these bad relations and at the same time also relates the Majorana neutrino mass matrix to observables in the charged fermion sector, leading to a predictive neutrino spectrum.

We shall consider non–Susy $SO(10)$ breaking to the standard model via the $SU(2)_L \times SU(2)_R \times SU(4)_C \equiv G_{224}$ chain as well as Susy-$SO(10)$ breaking directly to the standard model. The breaking of $SO(10)$ via $G_{224}$ is achieved by a $210$ of Higgs which breaks the discrete $D$–parity[31]. The second stage of symmetry breaking goes via the $126$. Finally, the electro–weak symmetry breaking proceeds via the $10$. In Susy-$SO(10)$, the first two symmetry breaking scales coalesce into one.

In the fermion sector, denoting the three families belonging to $16$–dimensional spinor representation of $SO(10)$ by $\psi_a$, $a = 1 − 3$, the complex $10$–plet of Higgs by $H$, and the $126$–plet of Higgs by $\Delta$, the Yukawa couplings can be written down as

$$L_Y = h_{ab} \psi_a \psi_b H + f_{ab} \psi_a \psi_b \overline{\Delta} + H.C. \quad (26)$$

Note that since the $10$–plet is complex, one other coupling $\psi_a \psi_b \overline{H}$ is allowed in general. In Susy–$SO(10)$, the requirement of supersymmetry prevents such a term. In the non–Susy case, we forbid this term by imposing a $U(1)_{PQ}$ symmetry, which may anyway be needed in order to solve the strong CP problem.

The $10$ and $126$ of Higgs have the following decomposition under $G_{224}$: $126 \rightarrow (1, 1, 6) + (1, 3, 10) + (3, 1, \overline{10}) + (2, 2, 15)$, $10 \rightarrow (1, 1, 6) + (2, 2, 1)$. Denote the $(1, 3, 10)$
and (2, 2, 15) components of $\Delta(126)$ by $\Delta_R$ and $\Sigma$ respectively and the (2, 2, 1) component of $H(10)$ by $\Phi$. The vev $<\Delta_R^3> \equiv v_R \sim 10^{12} \text{ GeV}$ breaks the intermediate symmetry down to the standard model and generates Majorana neutrino masses given by $f v_R$. $\Phi$ contains two standard model doublets which acquire vev’s denoted by $\kappa_u$ and $\kappa_d$ with $\kappa_{u,d} \sim 10^2 \text{ GeV}$. $\kappa_u$ generates charge 2/3 quark as well as Dirac neutrino masses, while $\kappa_d$ gives rise to $-1/3$ quark and charged lepton masses.

Within this minimal picture, if $\kappa_u$, $\kappa_d$ and $v_R$ are the only vev’s contributing to fermion masses, in addition to the $SU(5)$ relations $m_b = m_\tau$, $m_s = m_\mu$, $m_d = m_e$, eq. (1) will also lead to the unacceptable relations $m_u : m_c : m_t = m_d : m_s : m_b$. Moreover, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix will be identity. Fortunately, within this minimal scheme, we have found new contributions to the fermion mass matrices which are of the right order of magnitude to correct these bad relations. To see this, note that the scalar potential contains, among other terms, a crucial term

$$V_1 = \lambda \Delta \Sigma \Delta H + H.C. \tag{27}$$

Such a term is invariant under the $U(1)_{PQ}$ symmetry. It will be present in the Susy $SO(10)$ as well, arising from the $210 F$–term. This term induces vev’s for the standard doublets contained in the $\Sigma$ multiplet of $126$. The vev arises through a term $\Delta R \Delta R \Sigma \Phi$ contained in $V_1[34]$.

We can estimate the magnitudes of the induced vev’s of $\Sigma$ (denoted by $v_u$ and $v_d$ along the up and down directions) assuming the survival hypothesis to hold:

$$v_{u,d} \sim \lambda \left( \frac{v_R^2}{M_\Sigma^2} \right) \kappa_{u,d}. \tag{28}$$

Suppose $M_U \sim 10^{15} \text{ GeV}$, $M_I \sim 3 \times 10^{12} \text{ GeV}$ and $M_\Sigma \sim 10^{14} \text{ GeV}$, consistent with survival hypothesis, then $v_u$ and $v_d$ are of order 100 MeV, in the right range for correcting the bad mass relations. We emphasize that there is no need for a second fine–tuning to generate such induced vev’s. In the Susy version with no intermediate scale, the factor $(v_R^2/M_\Sigma^2)$ is not a suppression, so the induced vev’s can be as large as $\kappa_{u,d}$.

We are now in a position to write down the quark and lepton mass matrices of the model:

$$M_u = h\kappa_u + f v_u$$
$$M_d = h\kappa_d + f v_d$$
$$M_\nu = h\kappa_u - 3fv_u$$
\[
M_t = h\kappa_d - 3f v_d \\
M_{\nu}^M = fv_R.
\] (29)

Here \(M_{\nu}^D\) is the Dirac neutrino matrix and \(M_{\nu}^M\) is the Majorana mass matrix. Let us ignore CP-violation, which has been taken into account in [23].

To see the predictive power of the model as regards the neutrino spectrum, note that we can choose a basis where one of the coupling matrices, say \(h\), is real and diagonal. Then there are 13 parameters in all, not counting the superheavy scale \(v_R\): 3 diagonal elements of the matrix \(h\kappa_u\), 6 elements of \(fv_u\), 2 ratios of vev’s \(r_1 = \kappa_d/\kappa_u\) and \(r_2 = \frac{v_d}{v_u}\), and the two phases \(\alpha\) and \(\beta\). These 13 parameters are related to the 13 observables in the charged fermion sector, viz., 9 fermion masses, 3 quark mixing angles and one CP violating phase. The light neutrino mass matrix will then be completely specified in terms of other physical observables and the overall scale \(v_R\). That would lead to 8 predictions in the lepton sector: 3 leptonic mixing angles, 2 neutrino mass ratios and 3 leptonic CP violating phases.

The relations of eq. (4) hold at the intermediate scale \(M_I\) where quark–lepton symmetry and left–right symmetry are intact. There are calculable renormalization corrections to these relations below \(M_I\). The quark and charged lepton masses as well as the CKM matrix elements run between \(M_I\) and low energies. The neutrino masses and mixing angles, however, do not run below \(M_I\), since the right-handed neutrinos have masses of order \(M_I\) and decouple below that scale. The predictions in the neutrino sector should then be arrived at by first extrapolating the charged fermion observables to \(M_I\).

We shall present results for the non–Susy \(SO(10)\) model with the \(G_{224}\) intermediate symmetry. We fix the intermediate scale at \(M_I = 10^{12}\) GeV and use the one–loop standard model renormalization group equations to track the running of the gauge couplings between \(M_Z\) and \(M_I\). For Susy–\(SO(10)\), the results are similar.

To compute the renormalization factors, we choose as low energy inputs the gauge couplings at \(M_Z\) to be \(\alpha_1(M_Z) = 0.01688\), \(\alpha_2(M_Z) = 0.03322\), \(\alpha_3(M_Z) = 0.11\). For the light quark (running) masses, we choose values listed in Ref. (9). The top–quark mass will be allowed to vary between 100 and 200 GeV. Between 1 GeV and \(M_Z\), we use two–loop QCD renormalization group equations for the running of the quark masses and the \(SU(3)_C\) gauge coupling,[35] treating particle thresholds as step functions. From \(M_Z\) to \(M_I\), the running factors are computed semi–analytically both for the fermion masses and for the CKM angles by using the one–loop renormalization group equations for the Yukawa couplings and keeping the heavy top–quark contribution.[36] The running factors, defined as \(\eta_i = m_i(M_I)/m_i(m)\)
[\eta_i = m_i(M_t)/m_i(1 \text{ GeV})] for light quarks \((u, d, s)\) are \(\eta(u,c,t) = (0.273, 0.286, 0.506)\), 
\(\eta(d, s, b) = (0.279, 0.279, 0.327)\), \(\eta(e, \mu, \tau) = 0.960\) for the case of \(m_t = 150 \text{ GeV}\). The
(common) running factors for the CKM angles (we follow the parameterization advocated by the Particle Data Group) \(S_{23}\) and \(S_{13}\) is 1.081 for \(m_t = 150 \text{ GeV}\). The Cabibbo angle \(S_{12}\) and the KM phase \(\delta_{KM}\) are essentially unaltered.

We can rewrite the mass matrices \(M_t, M^D_\nu\) and \(M^M_\nu\) of eq. (29) in terms of the quark mass matrices and three ratios of vev’s – \(r_1 = \kappa_d/\kappa_u\), \(r_2 = v_d/v_u\), \(R = v_u/v_R\):

\[
M_t = \frac{4r_1 r_2}{r_2 - r_1} M_u - \frac{r_1 + 3r_2}{r_2 - r_1} M_d ,
\]

\[
M^D_\nu = \frac{3r_1 + r_2}{r_2 - r_1} M_u - \frac{4}{r_2 - r_1} M_d ,
\]

\[
M^M_\nu = \frac{1}{R} \frac{r_1}{r_2 - r_1} M_u - \frac{1}{R} \frac{1}{r_2 - r_1} M_d .
\]

(30)

It is convenient to go to a basis where \(M_u\) is diagonal. In that basis, \(M_d\) is given by \(M_d = V M^\text{diagonal}_d V^T\), where \(M^\text{diagonal}_d = \text{diagonal}(m_d, m_s, m_b)\) and \(V\) is the CKM matrix. One sees that \(M_t\) of eq. (5) contains only physical observables from the quark sector and two parameters \(r_1\) and \(r_2\). In the CP–conserving limit then, the three eigen–values of \(M_t\) will lead to one mass prediction for the charged fermions. To see this prediction, \(M_t\) needs to be diagonalized. Note first that by taking the Trace of \(M_t\) of eq. (5), one obtains a relation for \(r_1\) in terms of \(r_2\) and the charged fermion masses. This is approximately \(r_1 \simeq (m_\tau + 3m_b)/4m_t\) (as long as \(r_2\) is larger than \(m_b/m_t\)). Since \(|m_b| \simeq |m_\tau|\) at the intermediate scale to within 30% or so, depending on the relative sign of \(m_b\) and \(m_\tau\), \(r_1\) will be close to either \(m_b/m_t\) or to \((m_b/2m_t)\). Note also that if \(r_2 \gg r_1\), \(M_t\) becomes independent of \(r_2\), while \(M^D_\nu\) retains some dependence: \(M_t \simeq 4r_1 M_u - 3M_d\), \(M^D_\nu \simeq M_u - \frac{4}{r_2} M_d\). This means that the parameter \(r_2\) will only be loosely constrained from the charged fermion sector.

We do the fitting as follows. For a fixed value of \(r_2\), we determine \(r_1\) from the \(\text{Tr}(M_t)\) using the input values of the masses and the renormalization factors discussed above. \(M_t\) is then diagonalized numerically. There will be two mass relations among charged fermions. Since the charged lepton masses are precisely known at low energies, we invert these relations to predict the \(d\)–quark and \(s\)–quark masses. The \(s\)–quark mass is sensitive to the muon mass, the \(d\)–mass is related to the electron mass. This procedure is repeated for other values of \(r_2\). For each choice, the light neutrino masses and the leptonic CKM matrix elements are then computed using the see–saw formula.
We find that there are essentially three different solutions. A two-fold ambiguity arises from the unknown relative sign of $m_b$ and $m_\tau$ at $M_I$. Although solutions exist for both signs, we have found that a relative minus sign tends to result in somewhat large value of $m_s/m_d$. Our numerical fit shows that the loosely constrained parameter $r_2$ cannot be smaller than 0.1 or so, otherwise the $d$–quark mass comes out too small. Now, the light neutrino spectrum is sensitive to $r_2$ only when $r_2 \sim 4m_s/m_c \sim \pm 0.4$, since the two terms in $M_D^\nu$ become comparable (for the second family) then. Two qualitatively different solutions are obtained depending on whether $r_2$ is near $\pm 0.4$ or not.

Numerical results for the three different cases are presented below. The input values of the CKM mixing angles are chosen for all cases to be $S_{12} = -0.22$, $S_{23} = 0.052$, $S_{13} = 6.24 \times 10^{-3}$. Since $\delta_{KM}$ has been set to zero for now, we have allowed for the mixing angles to have either sign. Not all signs result in acceptable quark masses though. Similarly, the fermion masses can have either sign, but these are also restricted. The most stringent constraint comes from the $d$–quark mass, which has a tendency to come out too small. Acceptable solutions are obtained when $\theta_{23}$, $\theta_{13}$ are in the first quadrant and $\theta_{12}$ in the fourth quadrant.

Solution 1:

Input : $m_u(1 \text{ GeV}) = 3 \text{ MeV}$, $m_c(m_c) = 1.22 \text{ GeV}$, $m_t = 150 \text{ GeV}$

$m_b(m_b) = -4.35 \text{ GeV}$, $r_1 = -1/51.2$, $r_2 = 2.0$

Output : $m_d(1 \text{ GeV}) = 6.5 \text{ MeV}$, $m_s(1 \text{ GeV}) = 146 \text{ MeV}$

$\left( m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau} \right) = R \left( 2.0 \times 10^{-2}, 9.9, -2.3 \times 10^4 \right) \text{ GeV}$

$$V_{KM}^{\text{lepton}} = \begin{pmatrix} 0.9488 & 0.3157 & 0.0136 \\ -0.3086 & 0.9349 & -0.1755 \\ -0.0681 & 0.1623 & 0.9844 \end{pmatrix} \quad (31)$$

Solution 2:

Input : $m_u(1 \text{ GeV}) = 3 \text{ MeV}$, $m_c(m_c) = 1.22 \text{ GeV}$, $m_t = 150 \text{ GeV}$

$m_b(m_b) = -4.35 \text{ GeV}$, $r_1 = -1/51$, $r_2 = 0.2$

Output : $m_d(1 \text{ GeV}) = 5.6 \text{ MeV}$, $m_s(1 \text{ GeV}) = 156 \text{ MeV}$
\[
(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}) = R \left(7.5 \times 10^{-3}, 2.0, -2.8 \times 10^3\right) \text{ GeV}
\]

\[
V_{\text{lepton}}^\text{KM} = \begin{pmatrix}
0.9961 & 0.0572 & -0.0676 \\
-0.0665 & 0.9873 & -0.1446 \\
0.0584 & 0.1485 & 0.9872
\end{pmatrix}
\]

(32)

Solution 3:

Input : 
\(m_u(1 \text{ GeV}) = 3 \text{ MeV}, m_c(m_c) = 1.27 \text{ GeV}, m_t = 150 \text{ GeV}\)
\(m_b(m_b) = -4.35 \text{ GeV}, r_1 = -1/51.1, r_2 = 0.4\)

Output : 
\(m_d(1 \text{ GeV}) = 6.1 \text{ MeV}, m_s(1 \text{ GeV}) = 150 \text{ MeV}\)
\[(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}) = R \left(4.7 \times 10^{-2}, 1.4, -5.0 \times 10^3\right) \text{ GeV}\]

\[
V_{\text{lepton}}^\text{KM} = \begin{pmatrix}
0.9966 & 0.0627 & -0.0541 \\
-0.0534 & 0.9858 & 0.1589 \\
0.0633 & -0.1555 & 0.9858
\end{pmatrix}
\]

(33)

Solution 1 corresponds to choosing \(r_1 \sim m_b/m_t\). All the charged lepton masses are negative in this case. Since \(r_2\) is large, the Dirac neutrino matrix is essentially \(M_u\), which is diagonal; so is the Majorana matrix. All the leptonic mixing angles arise from the charged lepton sector. Note that the predictions for \(m_d\) and \(m_s\) are within the range quoted in Ref. 37. The mixing angle \(\sin \theta_{\nu_e-\nu_\mu}\) relevant for solar neutrinos is 0.30, close to the Cabibbo angle. Such a value may already be excluded by a combination of all solar neutrino data taken at the 90% CL (but not at the 95% CL)[6]. Actually, within the model, there is a more stringent constraint. Note that the \(\nu_\mu - \nu_\tau\) mixing angle is large, it is approximately \(3|V_{cb}| \simeq 0.16\). For that large a mixing, constraints from \(\nu_\mu - \nu_\tau\) oscillation experiments imply[38] that \(|m_{\nu_\mu}^2 - m_{\nu_\tau}^2| \leq 4 \text{ eV}^2\). Solution 1 also has \(m_{\nu_e}/m_{\nu_\mu} \simeq 2.3 \times 10^3\), requiring that \(m_{\nu_\mu} \leq 0.9 \times 10^{-3} \text{ eV}\). This is a factor of 2 too small for \(\nu_e - \nu_\mu\) MSW oscillation for the solar puzzle (at the 90% CL), but perhaps is not excluded completely, once astrophysical uncertainties are folded in. If \(\nu_\tau\) mass is around \(2 \times 10^{-3} \text{ eV}\), \(\nu_e - \nu_\tau\) oscillation may be relevant, that mixing angle is
\[ \simeq 3|V_{td}| \simeq 6\%. \] It would require the parameter \( R = v_u/v_R \sim 10^{-16}\) or \( v_R \sim 10^{16} \text{ GeV} \) for \( v_u \sim 1 \text{ GeV} \). Such a scenario fits very well within Susy–SO(10).

Solution 2 differs from 1 in that \( r_2 \) is smaller, \( r_2 = 0.2 \). The ratio \( m_s/m_d = 27.8 \) is slightly above the limit in Ref. 37. The \( 1 \rightarrow 2 \) mixing in the neutrino sector is large in this case, so it can cancel the Cabibbo like mixing arising from the charged lepton sector. As we vary \( r_2 \) from around 0.2 to 0.6, this cancellation becomes stronger, the \( \nu_e - \nu_\mu \) mixing angle becoming zero for a critical value of \( r_2 \). For larger \( r_2 \), the solution will approach Solution 1. The \( \nu_\mu - \nu_\tau \) mixing angle is still near \( 3|V_{cb}| \), so as before, \( m_{\nu_e} \leq 2 \text{ eV} \). From the \( \nu_\tau/\nu_\mu \) mass ratio, which is \( 1.4 \times 10^3 \) in this case, we see that \( m_{\nu_\mu} \leq 1.5 \times 10^{-3} \text{ eV} \). This is just within the allowed range[6] (at 95% CL) for small angle non–adiabatic \( \nu_e - \nu_\mu \) MSW oscillation, with a predicted count rate of about 50 SNU for the Gallium experiment. Note that there is a lower limit of about 1 eV for the \( \nu_\tau \) mass in this case. Forthcoming experiments (CHORUS and NOMAD[39] at CERN and the Fermilab expt.) should then be able to observe \( \nu_\mu - \nu_\tau \) oscillations. A \( \nu_\tau \) mass in the (1 to 2) eV range can also be cosmologically significant, it can be at least part of the hot dark matter. In Susy \( SO(10) \), \( \nu_e - \nu_\tau \) oscillation (the relevant mixing is about \( 3|V_{td}| \simeq 5\% \)), could account for the solar neutrino puzzle.

Solution 3 corresponds to choosing \( r_1 = 0.4 \). \( m_s/m_d = 24.6 \) is within the allowed range. However, the mass ratio \( \nu_\tau/\nu_\mu \) is \( \sim 3.6 \times 10^3 \), and \( \sin \theta_{\mu\tau} \simeq 3|V_{cb}| \) so \( \nu_e - \nu_\mu \) oscillation cannot be responsible for solar MSW. As in other cases, \( \nu_e - \nu_\tau \) MSW oscillation with a 6% mixing is a viable possibility.

We have therefore found that contrary to commonly held belief, a class of minimal \( SO(10) \) grand unified models can lead to precise predictions for light neutrino masses and mixing angles in terms of observables in the charged fermion sector. We have been able to make these predictions within the framework of the minimal \( SO(10) \) model without invoking any family symmetries.

VI. Summary and Conclusions:

In this report, I have tried to present a brief overview of the present experimental and theoretical situation with regard to the neutrino masses. I have taken a dual approach in which one line of research tries to take all existing evidences for the neutrino mass seriously and study its implications for the neutrino spectra and the structure of their mass matrices and their possible theoretical origin. The other approach is to analyse existing ideas about new physics beyond the standard model.
motivated by aesthetic considerations (such as fundamental left-right symmetry of nature) that also imply a non-vanishing neutrino mass to isolate their experimentally testable predictions. In the later approach, we have discussed studies of left-right symmetric models with low scale for the $W_R$ gauge boson, which can not only explain the solar neutrino puzzle in an appropriate version but also lead to observable signals in rare decay processes such as muonium-anti-muonium transition, anomalous muon decay, rare kaon decays etc. (for a recent review, see [25]). On the other hand, right-handed scale could be in the super-heavy range of $10^{11}$ GeV or so (as would be indicated by the simple see-saw mechanism with a tree level neutrino Dirac mass and the MSW solution to the solar neutrino puzzle). Such high scale theories would arise naturally in grand-unified models such as those based on the $SO(10)$ group. Such models can only be tested by the neutrino oscillation experiments and proton decay searches. In fact, the present atmospheric neutrino data cannot be accommodated by the minimal $SO(10)$ model; therefore if this data stands the test of time, a second minimal grandunified model will be ruled out by experiments.

While we have not addressed any GUT theories beyond $SO(10)$, a perfectly plausible candidate group is the $E_6$ group by Gursey and collaborators [40]. The fermion spectrum of this model contains a vector-like $SU(2)$ singlet and color triplet fermion, two vector-like $SU(2)$ doublet leptons and a chiral sterile neutral lepton. It was advocated in [15], that this sterile lepton can have a very small mass in simple versions of the model and may serve the role of a warm dark matter if the MSW mechanism operates between the $\nu_e$ and $\nu_\tau$ species as is expected in the case of SUSY $SO(10)$. Thus, the detailed pattern of neutrino spectrum is indeed a strong clue to the kind of higher symmetries that are likely to be operative at shorter distances.

We have not addressed issues arising from recent reports of anomalous events in double beta decay experiments involving $^{76}Ge$, $^{82}Se$ etc.[41]. These reports of unexpected events near the end point of electron energy could be the first signal of a spontaneously broken global $B-L$ symmetry [42] as has been discussed in several recent papers[43]. All these possibilities promise to keep the area of neutrino physics an extremely exciting venue for particle physics research in the nineties and a very optimistic prognosis for the discovery of new physics beyond the standard model.

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