Control Technologies of Deformable Air-Land Amphibious Vehicle Flight System

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Abstract. A deformable air-land amphibious vehicle suitable for complex urban terrain was designed, and its flight control method of area transfer was researched. The flight system structural model and dynamic model of the vehicle are established. Aiming at the problem of system parameter uncertainty caused by the deformation and trajectory tracking with the landing point change, the model reference adaptive compensator based on LQR attitude controller and PID height controller is added to solve it. Finally, the Matlab&Adams co-simulation is used to obtain the limit value of the deformation angle for area transformation. The efficiency of the control method is verified and the vehicle can track the desired trajectory.

1. Introduction
In the densely populated urban areas, air-land amphibious vehicles which have the ability to overcome obstacles and endurance possess many unique advantages. In recent years, based on the four-rotor and coaxial double-rotor UAVs, researchers have developed a number of air-land amphibious vehicles with the above functions [1-2]. While most of them have the following problems: flight and land movement were not integrated as a whole, rotor has no protective measures, discontinuity in mobility of deformable air-land vehicles.

Quad-rotor flight control for a symmetrical airframe or deterministic model has achieved many research results at home and abroad [3-5]. While aiming at a kind of deformable air-land amphibious vehicle, this paper research on the flight control method of air-to-land transfer process to mainly solve the problems of the change of moment of inertia caused by the area transfer deformation and the trajectory tracking with the landing point change. Completes or partially completes deformation for air-to-land transformation in the landing process of air-land amphibious vehicle lays a foundation for the realization of continuous maneuver from air to ground. The attitude controller based on LQR algorithm and the height controller based on PID algorithm are established after linearizing the system by Jacobian matrix. In addition, the model reference adaptive control is joined to compensate the uncertainties of the system model, which guarantees the global asymptotic stability of the closed-loop control system. The joint simulation of Adams and Matlab is used not only for ensuring the effectiveness and reliability of attitude and height control in the process of vehicle area transfer deformation, but also the visibility of the simulation results.
2. Dynamic model

Deformable air-land amphibious vehicle takes the land movement as the chief, switching to flight mode when encounter obstacles and unable to advance, so the robot has three states: flight, deformation and land movement, as shown in Figure 1. The four rotor motors are fixed at the center of the four wheels. The rotor is protected by the rotor guard ring wheel. The deformable mechanism is driven by the rudder to complete the turning deformation and the rotor guard ring wheels will be used as land vehicle.

Figure 1. Structural of deformable air-land amphibious vehicles.

Figure 2 shows a simplified model of the system state for the deformation angle is $\alpha$ (angle with horizontal plane), taking the ground coordinate system as $Ox_iyz$, and the body coordinate system as $Ox_bybz$, the coordinate origin coincides with the center of mass.

In flight state, the X-type Four-rotor mode is adopted, and the coordinate axis $x_b, y_b$ coincides with the diagonal line of the diagonal rotor motor in flight state, $x_b$ is the forward direction; the air-land amphibious vehicles model is regarded as a rigid body as a whole; the rotational speed of the four steering engines is the same in the area transfer deformation process, that is to say, the deformation inclination of the four rotors is the same at any time; the rotation and revolution of the earth are not considered, the effect of propeller swing characteristics does not take into account; the rotor is a thin wireless disk which acts uniformly on the air when it rotates [6].

Figure 2. Area transfer state.

When the deformation angle is $\alpha$, the plane of the propeller inclines accordingly, the lift force and resistance torque will produce forces along the three axes of the airframe coordinate system. As the deformations of the four rotor arms are consistent at all times during the deformation, the forces along the $x_b,y_b$ axis generated by the four rotors are approximately zero, only the lift force $T_f^i$ and resistance
torque $\Omega^z$ along the axis $z_b$ are retained, given by: $T_i^z = T_i \cdot \cos \alpha = \rho AC_i R^2 \omega_i^2 \cdot \cos \alpha = k \omega_i^2 \cdot \cos \alpha$; $\Omega^z = \Omega_i \cdot \cos \alpha = \rho AC_i R^2 \omega_i^2 \cdot \cos \alpha = d \omega_i^2 \cdot \cos \alpha$. Where $C_T$, $C_\alpha$ represent drag coefficient and torque coefficient of rotor; $\rho$ is air density; $A = \pi R^2$ is area of propeller [7];

In the inertial coordinate system:

$$
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = \frac{T}{m} (\sin \varphi \sin \psi + \sin \theta \cos \varphi \cos \psi)
- \frac{T}{m} (\sin \varphi \cos \psi + \sin \theta \cos \varphi \sin \psi)
- \frac{g - \frac{T}{m} (\cos \varphi \cos \theta)}{g - \frac{T}{m} (\cos \varphi \cos \theta)}
$$

(1)

$$
T = \sum_{i=1}^{4} T_i^z
$$

Where $T$ is the total lift, $m$ is the mass of system, $g$ is the gravitational acceleration, $\varphi, \theta, \psi$ represent roll angle, pitch angle and yaw angle, which is given by:

$$
\begin{bmatrix}
\phi \\
\theta \\
\psi
\end{bmatrix} = \begin{bmatrix}
1 & \tan \theta \sin \phi & \tan \theta \cos \phi \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
$$

(2)

Where $p, q, r$ represent angular velocity vector along the three body axis of, given by:

$$
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \frac{1}{J_x} ((J_y - J_z)q r + J_x (\omega_1 - \omega_2 + \omega_3 - \omega_4) q + \tau_\phi)
- \frac{1}{J_y} ((J_z - J_x) p r + J_y (\omega_1 - \omega_2 + \omega_3 - \omega_4) p + \tau_\theta)
- \frac{1}{J_z} ((J_x - J_y) q r + \tau_\psi)
$$

(3)

Where $J_x, J_y, J_z$ are the system's moment of inertia about the axis of the body coordinate system, $\tau_\phi, \tau_\theta, \tau_\psi$ are control torque in the three angular movement directions of roll, pitch and yaw.

$$
\begin{bmatrix}
\tau_\phi \\
\tau_\theta \\
\tau_\psi
\end{bmatrix} = \begin{bmatrix}
\sqrt{2} / 2 l k \cos^2 \alpha (-\omega_1^2 + \omega_2^2 + \omega_3^2 - \omega_4^2) \\
\sqrt{2} / 2 l k \cos^2 \alpha (\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) \\
d \cos \alpha (-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2)
\end{bmatrix}
$$

(4)

Where $l$ is the distance between center of rotor and system centroid.
3. Control model

Model Reference Adaptive Control System (MRACS) needs to design the basic controller before it is designed. The reference model is obtained by using the basic controller and system model, in addition, the model reference adaptive control law is obtained [8-9]. The basic controller method of the air-land amphibious vehicle is divided into two parts: inner loop and outer loop. The inner loop is the LQR attitude controller and the PID height controller which is based on the established system dynamics model after linearized at the equilibrium point by Jacobian matrix. The outer loop is to use PID algorithm to establish the relationship between direction and attitude angle to solve the under-actuation problem of the system. The control block diagram of the vehicle is shown in the figure.

![Control block diagram.](image)

Figure 3. Control block diagram.

3.1. Design of LQR attitude controller

Choose \( u = [\tau_\phi \ \tau_\theta \ \tau_\psi]^T = [u_2 \ u_3 \ u_4]^T \) as the input vector for attitude control, \( x = [\phi \ p \ \theta \ q \ \psi \ r]^T \) as the state vector, The nonlinear dynamic model of the system can be written as a state space form \( x = f(x, u) \). The nonlinear state equation can be linearized by Jacobian matrix[10] as:

\[
\begin{align*}
\dot{x} &= A_a x + B_a u \\
y &= C_a x 
\end{align*}
\]

(5)

Where \( A_a = \frac{\partial f}{\partial x}(x, u), B_a = \frac{\partial f}{\partial u}(x, u), C_a \) as:

\[
C_a = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix},
\]

Considering that the system mostly moves near the hovering and low-speed equilibrium points, it can be linearized near the equilibrium points. Select this time state of the system and put it into the Jacobian matrix, \( A_a \) and \( B_a \) are given as:

\[
A_a = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
B_a = \begin{bmatrix}
0 & 0 & 0 \\
0 & 1/J_x & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1/J_y \\
0 & 0 & 0
\end{bmatrix}
\]

(6)

The block diagram of attitude control based on LQR is constructed as shown in Figure 4.
Figure 4. Diagram of LQR control.

Introduced error vector: $\dot{x}_i = e = r - y = r - C_a x$, the state space model is extended as:

$$
\begin{bmatrix}
\dot{x} \\
\dot{x}_1
\end{bmatrix} =
\begin{bmatrix}
A_a & 0 \\
-C_a & 0
\end{bmatrix}
\begin{bmatrix}
x \\
x_1
\end{bmatrix} +
\begin{bmatrix}
B_a \\
0
\end{bmatrix} u +
\begin{bmatrix}
0 \\
I
\end{bmatrix} r
$$

According to the LQR control theory, the optimal control law is designed as: $u = -K_1 x - K_2 x_1$, where $K_1$ is feedback gain matrix, $K_2$ is feedforward gain matrix, which can minimum the value of quadratic objective function $J = \int_0^\infty x^T Q x + u^T R u \, dt$, where $x = [x \ x_1]^T$; $Q$, $R$ represent positive weighting matrix, whose value will affect the control effect of the system. When $J$ get the minimum value, $\bar{K} = R^{-1} B_a P$. Where $\bar{K} = [K_1 \ K_2]^T$; $P$ is the solution of the Riccati function.

Based on the above analysis, we can get the state space equation of the LQR attitude controller.

$$
\begin{align*}
\dot{x} &= (A_a - B_a K_1)x + B_a K_2 x_1 \\
y &= C_a x
\end{align*}
$$

3.2. Design of PID height controller

The transfer function diagram of the real height control system is shown in Figure 5.

Figure 5. Diagram of real system transfer function.

In this block diagram, $G_i(s)$ represents the transfer function from the motor speed control signal to the pulling force determined by the motor model, $G_z(s)$ represents the transfer function from the input to the state.

$$
G_i(s) = \frac{U(s)}{N(s)} = \frac{k_i}{0.1s + 1}
$$

In this model, a motor with a response time of 0.1s is selected, $k_i$ is the proportional coefficient between the motor speed control signal and the pull force, which can be obtained by the motor pull test.
According to the system model, when system is in the process of height control, the attitude angle of the system changes little, that is to say \( \cos \phi \cos \theta \approx 1 \). The height dynamic equation is simplified as follows:

\[
\dot{z} = g - \frac{T}{m} \cdot (\cos \phi \cos \theta)
\]  

(10)

The PID control law for height controlling is designed as:

\[
n(t) = K_{pz} e(t) + \int_0^t \frac{e(\tau)}{T} d\tau + T_{dz} \frac{de(t)}{dt}
\]  

(11)

The corresponding transfer function is \( G_p = K_{pz} (1 + \frac{1}{T_{iz}s} + T_{dz}s) \).

The closed-loop transfer function of the system's height control is:

\[
G(s) = \frac{Z(s)}{R(s)} = \frac{G_p(s)G_1(s)G_2(s)}{1 + G_p(s)G_1(s)G_2(s)}
\]  

(12)

Select \( x_z = [z \; \dot{z}]^T \) as the state vector. The height control transfer function is transformed into the corresponding state space equation to obtain the coefficients of the state equation \( A_h, B_h, C_h \), as follows:

\[
\begin{cases}
\dot{x}_z = A_h x_z + B_h r_z \\
y_z = C_h x_z
\end{cases}
\]  

(13)

### 3.3. Design of MRACS

When the model parameters are not changed, the above-mentioned attitude and height controllers have small changes in the state equations, which can make the system have good control performance indicators. However, when the vehicle performs the area transfer deformation, the system's moment of inertia will change. For the attitude controller, the equation of state parameters \( B_z \) will be biased, and the LQR control parameters \( K \) will also change. So, the attitude of the closed-loop control system based on the LQR controller will be reduced; when the load changes or the body quality is damaged during the flight and the transformation of the system, the state equation parameters of the system height controller will also change, which will affect the performance of the height control system based on the PID controller. Therefore, in order to ensure the attitude and height control performance of the system in the case of area transfer, load change or damage of the body, model reference adaptive control is considered to add to compensate the model uncertain problem.

Taking attitude control as an example, the height control is similar. The system control block diagram after adding the model reference adaptive control is shown in Figure 6.
Select the reference model as:

\[
\begin{align*}
\dot{x}_m &= A_m x_m + B_m r \\
y_m &= C_m x_m
\end{align*}
\]  
(14)

Where, \( A_m, B_m, C_m \) is the parameter value of the above basic controller state space equation.

The actual model of the system is:

\[
\begin{align*}
\dot{x}_p &= A_p x_p + B_p r_p \\
y_p &= C_p x_p
\end{align*}
\]  
(15)

Where, \( C_p = C_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \), \( r_p = \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} \).

Design adaptive control rate as: \( r_p = K_f r - F_t x_p \), where \( r \) is the expectation input of system, \( K_f \), \( F_t \) represent adaptive control feedforward gain matrix and feedback gain matrix.

Substitute it into equation 15:

\[
\begin{align*}
\dot{x}_p &= (A_p - B_p F_t) x_p + B_p K_t r_p \\
y_p &= C_p x_p
\end{align*}
\]  
(16)

Define the system generalized error as \( e(t) = x_m(t) - x_p(t) \), then:

\[
\begin{align*}
\dot{e}(t) &= \dot{x}_m(t) - \dot{x}_p(t) \\
&= A_m x_m + B_m r - (A_p - B_p F_t) x_p - B_p K_t r_p \\
&= A_m e(t) + (A_m - A_p + B_p F_t) x_p + (B_m - B_p K_t) r
\end{align*}
\]  
(17)
Assume that there is an ideal value $\tilde{K}_t$, $\tilde{F}_t$ for the system gain, which can make the closed-loop system completely match with the reference model, and the equation 28 can be satisfied as: $A_m - A_p + B_p \tilde{F}_t = 0$, $B_m - B_p \tilde{K}_t = 0$, which means: $A_m = A_p + B_p \tilde{F}_t$, $B_m = B_p \tilde{K}_t$.

Defining the parameter deviation matrix: $\Delta K_t = \tilde{K}_t - K_t$, $\Delta F_t = \tilde{F}_t - F_t$, and substitute it into equation 17: $\dot{e}(t) = A_m e(t) + B_m \tilde{K}_t^{-1}(\Delta F_t, x_p + \Delta K_t, r)$

In order to ensure stability of system, choose the Lyapnov function as:

$$V = \frac{1}{2} [e^T P_m e + tr(\Delta F_t^T \Gamma_1 \Delta F_t + \Delta K_t^T \Gamma_2 \Delta K_t)]$$

(18)

Where, $\Gamma_1$, $\Gamma_2$ is positive definite symmetric matrix; $P_m$ satisfy $A_m^T P_m + P_m A_m = -Q_m$, $Q_m$ is positive definite matrix.

Deriving on both sides of equation 18:

$$\dot{V} = \frac{1}{2} e^T (A_m^T P_m + P_m A_m) e$$

$$+ tr(\Delta F_t^T \Gamma_1^{-1} \Delta F_t + x_p e^T P_m B_m \Delta F_t)$$

$$+ tr(\Delta K_t^T \Gamma_2^{-1} \Delta K_t + r e^T P_m B_m \Delta K_t)$$

(19)

The first item of the above formula is negative, so for $\dot{V} < 0$, let the latter two are zero, and the solution is:

$$\Delta \dot{F}_t = \ddot{\tilde{F}}_t - \dot{F}_t = -\Gamma_1 B_m^T P_m e x_p^T$$

$$\Delta \dot{K}_t = \ddot{\tilde{K}}_t - \dot{K}_t = -\Gamma_2 B_m^T P_m e r^T$$

(20)

Where, $\tilde{K}_t$, $\tilde{F}_t$ ideal system gain, the rate of change is zero, so the feedback and feedforward gain matrix corresponding to the adaptive control rate of the system model reference is obtained:

$$F_t = \int_0^t \Gamma_1 B_m^T P_m e x_p^T d\tau + F_t(0)$$

$$K_t = \int_0^t \Gamma_2 B_m^T P_m e r^T d\tau + K_t(0)$$

(21)

For $\dot{V} < 0$, according to the theory of Lyapnov stability, the system generalized error $\lim_{t \to \infty} e(t) = \lim_{t \to \infty} x_m(t) - x_p(t) = 0$, that is, the system is globally asymptotically stable. The height model reference adaptive controller designation is similar to the attitude. Therefore, the actual attitude and height control system performance is closer to the reference model system performance, thus achieving tracking of the reference model.
4. Simulation

4.1. Co-simulation model

The Adams model of the vehicle is established. Its inputs are the rotational speed of four rotor motors and the rotational angle of four deformable steering gears (synchronous deformation means it is one quantity in fact). The output are the current three attitude angles of the model and the space coordinates of the center of mass. The dynamic Module can be obtained as shown in the Figure 6 by using Matlab [11-12].

![Figure 6. Dynamic module of the vehicle.](attachment:image6.png)

In Matlab/Simulink, the input and output of the dynamic module are related to the control system module, that is, the control module of each control system is established in Simulink, and the appropriate custom functions are added to build the co-simulation model. It can be easily identified as shown in Figure 7.

![Figure 7. Co-simulation model.](attachment:image7.png)

4.2. Simulation result analysis

Before the start of the simulation, the parameters of the vehicle system should be initialized. Through the continuous testing of the PID toolbox in Matlab and according to the simulation results, the parameters of the controller are shown in Table 2.
Table 1. Controller parameters.

| Parameters | Value                      |
|------------|----------------------------|
| $K_p(x,y,z)$ | 2.2,2.2,10                |
| $K_i(x,y,z)$  | 0,0,0.01                |
| $K_d(x,y,z)$  | 1.2,1.2,8.5             |
| $Q$         | diag(5,0.02,5,0.02,5,0.02,80,80,80) |
| $R$         | diag(0.5,0.5,0.07)       |
| $\Gamma_1$  | diag(1,1,1)             |
| $\Gamma_2$  | diag(1,1,1)             |
| $Q_m$       | diag(1,1,1,1,1,1)       |

Set the desired height to 10m, and after the vehicle reaches the desired height stability, start to land and set the desired height of the landing to 0m. The whole process simulates the takeoff and landing before deformation. When starting to fall, set the deformation of the area transfer, set the deformation servo to deform to the specified angle (15°, 30°, 45°, and 50°) at 0.5 rad/s. The simulation results are shown in Figure 8.

(a) Flight height response curve of area transfer

(b) Varying curve of area transfer deformable angle

Figure 9. Height and deformable angle curve of area transfer.

The simulation results show that when the deformation angle is not more than 45°, the height of the system is within controllable range, and the control effect is accurate and stable. When the deformation angle is 50°, there is an uncontrollable drop phenomenon, that is, the deformation angle cannot be stable control. Considering that there may be some error in the control of the steering servos in the actual system, the deformation angle $\alpha = 45°$ is taken as the limit value of the deformation angle of the area transfer.

Set up the steering servos start to deform to 15°, 30°, 45° separately at 1 s. The simulation results of the roll angle output response based on LQR control and model reference adaptive control are shown in Figure 10. The simulation results show that after the deformation for the area transfer, the system model parameters are changed. When the model reference adaptive compensator is not added, the attitude control performance will be greatly attenuated, and the overshoot and response time will increase significantly. The comparison shows that the increment increases with the deformable angle. After adding the model reference adaptive compensator, the attitude control performance is greatly improved. The control effects are ideal after the system parameters change caused by the area transfer.
Let the vehicle fly along the desired trajectory shown in Figure 6, the process simulates the takeoff of the vehicle and the landing after the change of the landing point when deformable angle is 30°, keeping the yaw angle unchanged. The simulation results show that the vehicle can complete the flight and land as desired trajectory.

Figure 10. Response curve of $\varphi$, $\theta$, $\psi$ when area transfer.

The desired and simulation curve of x, y and z are shown in the Figure 12. The results show that there is almost no overshoot in x and y direction control and response time is around 2 seconds. The amount of overshoot in z direction is small and response time is around 3 seconds.

Figure 11. Trajectory and simulation curve.
5. Conclusion
This paper proposed a deformable air-land amphibious vehicle suitable for complex urban environments and designed its flight system structure. Based on the research of the area transfer flight control technology of the air-land amphibious vehicles, the model reference adaptive compensator is added based on the LQR attitude controller and the PID height controller to solve the problem of the system model uncertainty caused by the deformation and the trajectory tracking with the landing point change. Combined with the co-simulation to find the limit value of the flight deformation angle for the area transfer, and the simulation results also verified that the attitude and height controller after adding the model reference adaptive control have obvious advantages.

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