Destriping Polarised data

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ABSTRACT We show how to take advantage of a circle scanning strategy, such as that planned for the Planck mission, to get rid of low frequency noises for polarised data.

KEYWORDS:

1. INTRODUCTION

Polarisation measurements on the Cosmological Microwave Background (CMB) will be very difficult as the expected polarisation is not expected to exceed 10% of the signal. Therefore, noises of all origins must be searched for and carefully extracted.

Low frequency drifts are expected to arise from mechanic, thermal and electronic origin. If not suitably treated they induce stripes on sky maps such as those shown on figure 1. The elimination of low frequency noise has been already studied in the

FIGURE 1. Stripes on the sky map of the $Q$ Stokes parameter (no signal).
framework of differential measurements as was done in COBE DMR and as planned for the MAP mission (Janssen and Gulkis, 1992; Wright et al. 1996; Tegmark 1997). A solution to this problem in the case of circle scanning strategies has been proposed by Delabrouille (1998a) for temperature measurements. We discuss here how to extend this procedure to polarised data.

2. PRINCIPLE OF THE METHOD

The destriping method uses the two kinds of redundancies provided by a circle scanning strategy:

1) The telescope scans some 60 times a fixed circle in the sky with an angular radius of $\sim 85^\circ$, at a frequency $f_{\text{spin}} \sim 1$ rpm. The signal along one circle can be obtained in a nearly optimal way by averaging all 60 scans. As a result, all non-synchronous noises with frequencies smaller than $f_{\text{spin}}$ reduce to one offset per circle in a first approximation. These offsets are the main source of the stripes in figure 1.

2) Every hour, the spin axis of the mirror is moved by a few arcminutes. The different scanning circles have many intersections in the sky. At any point where two circles cross each other, the physical signal must be the same along both of them, and the resulting constraints allow to determine and subtract the offsets. However at intersection points, a given polarimeter has different directions along the two circles, as shown in figure 2, and therefore records different signals if the polarisation is non-zero. The solution is to compare the Stokes parameters $I$, $Q$ and $U$ measured

\[ \{l,m,1\} = \{m,l,-1\} \]

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FIGURE 2 The intersections of two scanning circle. Here, the polarimeter is assumed to be tangent to the scanning circle.
along the two circle, but defined in a fixed global reference frame, for instance the
ecliptic latitude-longitude frame \((e_1, e_2)\) drawn in figure 2.

3. APPLICATION TO POLARISED DATA

3.1 The relation between polarimeter outputs and Stokes parameters

If the focal plane involves \(n\) polarimeters in a given wavelength, their outputs can

\[
M = \begin{pmatrix} m_1 \\ \vdots \\ m_p \\ \vdots \\ m_n \end{pmatrix}, \quad \text{and} \quad S = \begin{pmatrix} I \\ Q \\ U \end{pmatrix}, \quad \text{related by} \quad M = A \ R(\psi) \ S
\]

be described by a vector \(M\) with \(n\) components, and the Stokes parameters in the
fixed global reference frame by another vector \(S\) with 3 components:

![Figure 3](image-url)

FIGURE 3. The setup of the polarimeters in the focal plane and the angle of rotation \(\psi\) with respect to the fixed reference frame \((e_1, e_2)\)

The matrix \(A\) characterises the setup of the polarimeters in the focal plane, whereas
the rotation \(R(\psi)\) depends on the orientation of the focal plane with respect to the
the fixed global reference frame:

\[
A = \frac{1}{2} \begin{pmatrix}
1 & 1 & 0 \\
\vdots & \vdots & \vdots \\
1 & \cos 2\Delta_p & \sin 2\Delta_p \\
\vdots & \vdots & \vdots \\
1 & \cos 2\Delta_{n-1} & \sin 2\Delta_{n-1}
\end{pmatrix}, \quad \text{and } R(\psi) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos 2\psi & \sin 2\psi \\
0 & -\sin 2\psi & \cos 2\psi
\end{pmatrix}.
\]

The meaning of angles \(\Delta_p\) and \(\psi\) is shown in figure 3

3.2 Determination of the offsets

As shown in figure 2, each scanning circle \(l\) has two intersections with circle \(m\), denoted by \(\{l, m, \delta = \pm 1\}\). Intersections \(\{l, m, \delta\}\) and \(\{m, l, -\delta\}\) correspond to the same point in the sky. To determine the offsets, one uses the fact that the Stokes parameters are uniquely defined in the fixed global reference frame:

\[
S_{l,m,\delta} = S_{m,l,-\delta}
\]  

(1)

From the vector of polarimeter outputs \(\bf{M}\), one gets the Stokes parameters at the intersections and the offsets by minimizing a \(\chi^2\), taking relation (1) into account. After eliminating the Stokes parameters, the minimisation conditions reduce to the following equation:

\[
\sum_{m,\delta} \left[ 1 + R(\psi(l, m, \delta)) \bf{X}_m^{-1} R(\psi(l, m, \delta))^{-1} \bf{X}_l \right]^{-1} \times \\
\left[ \bf{O}_l - R(\psi(l, m, \delta)) \bf{O}_m - \tilde{S}_{l,m,\delta} + R(\psi(l, m, \delta)) \tilde{S}_{m,l,-\delta} \right] = 0.
\]

(2)

In Equation (2) \(\tilde{S}_{l,m,\delta} = \bf{X}_l^{-1} A^T \bf{N}_l^{-1} \bf{M}_{l,m,\delta}\) is known in terms of the vectors of polarimeters outputs \(\bf{M}_l\) along circle \(l\) and their noise matrix \(\bf{N}_l\) (assumed to be white after the averaging procedure). \(\tilde{S}_{l,m,\delta}\) can be interpreted as the vector of the Stokes parameters in the focal reference frame, and \(\bf{X}_l = A^T \bf{N}_l^{-1} A\) is the inverse of the associated noise matrix. Also known is the rotation matrix \(R(\psi(l, m, \delta))\) which relates the focal frame Stokes parameters along circle \(m\) to those along circle \(l\).

The unknown offsets appear through the combination \(\tilde{\bf{O}}_l = \bf{X}_l^{-1} A^T \bf{N}_l^{-1} \bf{O}_l\), which can be viewed as the offsets on the focal frame Stokes parameters.

Once the focal frame offsets \(\tilde{\bf{O}}_l\) are determined by solving equation (2), one can reconstruct the fixed global frame Stokes parameters along all circles as:

\[
S_{l,k} = R_{l,k}(\tilde{S}_{l,k} - \tilde{\bf{O}}_l),
\]

where rotation \(R_{l,k}\) transforms the Stokes Parameters from the focal to the fixed global reference frame at pixel \(k\) along circle \(l\).
4. DISCUSSION

Figure 4 shows the same map as figure 1, after application of the destriping pro-

cedure described above. The stripes have totally disappeared. The barely visible radial structures around the poles are not stripes but regions where the level of noise is smaller because the density of intersection points is larger. Figure 5 shows the density of intersection points, which corresponds to a scanning strategy where

the spin axis moves along a sinusoid around the ecliptic plane.

The algorithm developed above to destripe polarised data is already as efficient as for unpolarised data, even if several points have still to be settled, of which we name a few:

1) Synchronous noises have to be eliminated and this has been treated at this
meeting by Delabrouille et al. (1998b).
2) The quality of the destriping has to be tested in a more sophisticated way, and we are currently developing methods to do so.
3) Measurements along two different circles will in reality be different for other reasons, such as the time response of the electronics and the asymmetry of the field of view. We are studying ways to modelise this phenomenon.
4) Sidelobes might cause difficulties in polarisation measurements because light rays with a large incidence angle could carry a large instrumental polarisation.

REFERENCES
Delabrouille, J. 1998a, A & A Supplement Series 127, 555
Delabrouille, J. et al. 1998b, these proceedings.
Janssen, M. A. and Gulkis, S. 1992, The Infrared and Submillimeter Sky after COBE, in Proceedings of the NATO Advanced Study Institute, M. Signore and C. Dupraz (eds.), Dordrecht: Kluwer, p. 391,
Tegmark, M. 1997, ApJ Letters 480, L87
Wright, E. L., Hinshaw, G., and Bennett, C. L. 1996, ApJ 458, L53