Apply a hydrological model to estimate local temperature trends

Masao Igarashi¹, Tatsuya Shinozawa¹

1 College of Bioresource Sciences, Nihon University, 1866 Kameino Fujisawa Kanagawa
252-0880, Japan
E-mail: igarashi.masao@nihon-u.ac.jp

Abstract. Continuous times series \( f(x) \) such as a depth of water is written \( f(x) = T(x) + P(x) + S(x) + C(x) \) in hydrological science where \( T(x) \), \( P(x) \), \( S(x) \) and \( C(x) \) are called the trend, periodic, stochastic and catastrophic components respectively. We simplify this model and apply it to the local temperature data such as given E. Halley (1693), the UK (1853-2010), Germany (1880-2010), Japan (1876-2010). We also apply the model to \( \text{CO}_2 \) data. The model coefficients are evaluated by a symbolic computation by using a standard personal computer. The accuracy of obtained nonlinear curve is evaluated by the arithmetic mean of relative errors between the data and estimations. E. Halley estimated the temperature of Gresham College from 11/1692 to 11/1693. The simplified model shows that the temperature at the time rather cold compared with the recent of London. The UK and Germany data sets show that the maximum and minimum temperatures increased slowly from the 1890s to 1940s, increased rapidly from the 1940s to 1980s and have been decreasing since the 1980s with the exception of a few local stations. The trend of Japan is similar to these results.

1. Introduction

Meteorological offices, including the UK Met Office (UKMO), Deutscher Wetterdienst (DWD) and Japan Meteorological Agency provide the mean of local monthly maximum and minimum temperature values. The mean monthly values of \( \text{CO}_2 \) (ppm) at Manuoloa and Ayari are also provided by, such as United States’ National Aeronautics and Space Administration (NASA). Many agencies show the trend curve of them. Here we provide a simple and robust method to estimate a year trend of these data.

We propose a following nonlinear model (1) to evaluate the trends of temperature and \( \text{CO}_2 \) data at the observed points by ignoring the stochastic and catastrophic components of the hydrological model,

\[
f(x) = T(x) + P(x) = \underbrace{a_1 + a_2 x}_{\text{Trend component}} + \underbrace{a_3 \sin \left( a_4 + \frac{\pi x}{n} \right)}_{\text{Periodical component}}
\]

The unknown coefficients \( a_i, \; i = 1, 2, 3, 4 \) and \( n \) are usually evaluated by the numerical computation based on the least square method and Newton iteration method. For simplicity, we apply a symbolic computation to obtain the coefficients under the fixed integer \( n \). As the temperature periodically changes, so the integer \( n \) is estimated 6 for the month variable \( x \). Hence we fixed the integer \( n \) between \( 1 \leq n \leq 12 \). The best \( n \) among them is determined...
by the arithmetic mean of the sum of the relative errors between the observed and estimated temperatures.

The temperatures and CO$_2$ data sets are obtained from the following paper and URLs, respectively:

(i) E. Halley 1694 *Phil. Trans. Royal Society of London* 18 p 185
(ii) http://www.metoffice.gov.uk/climate/uk/stationdata/
(iii) http://www.dwd.de/bvbw/appmanager/bvbw/
(iv) http://cdiac.ornl.gov/trends/co2/sio-mlo.html/
(v) http://ds.data.jma.go.jp/ghg/kanshi/obs/co2/

Reducing $f(x)$ to the upper $f_u(x)$ and lower $f_l(x)$ linear line respectively, we can estimate the annual temperature trend and also count the number of points included between $f_u(x)$ and $f_l(x)$.

2. Materials and Method

We describe our proposed method on the basis of data from the Oxford meteorological station.

2.1. Data

The UK Met Office provides monthly mean maximum (MAX) and minimum temperatures (MIN) data from January 1853 to December 2011. We classify the data according to decadal periods (e.g. 1860-2010, 1870-2010, · · · · · · , 2000-2010) to allow simple comparison with results from other stations. The monthly mean maximum and minimum temperatures (°C) for the period 01/1860–12/2010 are expressed as follows:

MAX=(6.8, 5.2, 8.4, 10.6, 17, 17, 19.3, 17.9, 15.6, 13.4, 7.4, 4.4, 3.8, · · · · · , 8.2, 2.7)
MIN=(0.8, -1.6, 1.9, 1.7, 7.7, 9.5, 10.3, 10.9, 7.0, 7.3, 2.1, 0.7∗, -0.7, · · · , 2.9, -2.2)

For example, the first record, 6.8 (MAX) and 0.8 (MIN), indicates the monthly mean maximum and minimum temperatures for January 1860. The last record, 2.7 (MAX) and -2.2 (MIN), refers to the monthly mean maximum and minimum temperatures for December 2010. Each data set consists of 151×12 = 1812 data points. Any missing data are interpolated using the neighbouring values. The interpolated MAX and MIN values are indicated by an asterisk. For example, 0.7∗ (MIN) represents a missing datum and is interpolated from the neighboring values as (2.1-0.7)/2=0.7. Meteorological stations for which numerous data were missing (e.g. Chiven) or have been closed (e.g. Ringway) are not considered.

2.2. Model

We apply the following nonlinear curve to the data from each meteorological station:

$$y = f(x) = a_1 + a_2 x + a_3 \sin(a_4 + \pi x/n) \quad (2)$$

where $a_1$, $a_2$, $a_3$, $a_4$ are unknown parameters and the integer $n$ is fixed between $1 \leq n \leq 12$. For simplicity, we use Mathematica FindFit function to evaluate the coefficients. The model is applied to the data of CO$_2$.

2.3. Criterion

To choose the best integer $n$ between $1 \leq n \leq 12$, we introduce the criterion $CR_n$ [1]:

$$CR_n = \frac{\sum_{k=1}^{m} |\hat{y}_k - y_k|}{\max(|\hat{y}_k|, |y_k|)} \quad (3)$$
where \( m \) is the number of data points, and \( \{\tilde{y}_k\} \) and \( \{y_k\} \) \( (k = 1, 2, \ldots, m) \) are the observed and estimated temperatures (CO\(_2\)), respectively. CR\(_n\) is the arithmetic mean of the sum of the relative errors between the observed and estimated temperatures. Hence, if CR\(_n\) is small, \( y_k = f(k) \) in (2) agrees well with the observed temperatures [2]. Conversely, if CR\(_n\) is large, \( f(k) \) does not agree well with the observed temperatures. Thus, an appropriate integer \( n \) is selected to obtain the smallest value of CR\(_n\).

### 3. Results

We show the typical nonlinear curve with related matters. \( f(x) \) and \( g(x) \) are the nonlinear curve of maximum and minimum temperatures. The reduced linear line \( f_u(x) \) and \( f_l(x) \) of \( f(x) \) show the upper and lower trend of maximum temperature. And also the reduced linear line \( g_u(x) \) and \( g_l(x) \) show the upper and lower trend of minimum temperature. In the case of CO\(_2\), these are denoted \( h(x) \), \( h_u(x) \) and \( h_l(x) \).

Figure 1 shows the data (dots), the obtained curve and trend lines of Oxford maximum and minimum temperature. 83.3% of data points are contains between the line of \( f_u(x) \) and \( f_l(x) \). Figure 2 shows the results of CO\(_2\) for Manuaola and Ayari.
Oxford : 01/1860-12/2010

\[ f(x) = 13.33 + 0.0006069x - 7.6198 \sin(0.9854 + \pi x/6), \quad \text{CR}_6 = 0.041 \]
\[ f_u(x) = 20.95 + 0.00728x, \quad f_l(x) = 5.710 + 0.00728x, \quad 83.3\% \text{ data are included} \]
\[ g(x) = 5.53 + 0.0007190x - 5.5151 \sin(0.8349 + \pi x/6), \quad \text{CR}_6 = 0.066 \]
\[ g_u(x) = 11.05 + 0.00862x, \quad g_l(x) = 0.019 + 0.0086, \quad 80.4\% \text{ data are included} \]

Aachen : 01/1900-12/2010

\[ f(x) = 13.11 + 0.0008218x - 8.9991 \sin(1.01189 + \pi x/6), \quad \text{CR}_6 = 0.053 \]
\[ f_u(x) = 4.11 + 0.0008218x, \quad f_l(x) = 22.11 + 0.0008218x, \quad 83.4\% \text{ data are included} \]
\[ g(x) = 5.45 + 0.001209x - 6.7895 \sin(0.8779 + \pi x/6), \quad \text{CR}_6 = 0.067 \]
\[ g_u(x) = -1.33 + 0.001209x, \quad g_l(x) = 12.24 + 0.001209x, \quad 81.1\% \text{ data are included} \]

Tokyo : 01/1880-12/2010

\[ f(x) = 18.19 + 0.0012999x - 10.2015 \sin(0.8175 + \pi x/6), \quad \text{CR}_6 = 0.032 \]
\[ f_u(x) = 28.39 + 0.0012999x, \quad f_l(x) = 7.9943 + 0.0012999x, \quad 82.9\% \text{ included} \]
\[ g(x) = 8.79 + 0.002823x + 11.3499 \sin(3.9396 + \pi x/6), \quad \text{CR}_6 = 0.053 \]
\[ g_u(x) = 20.14 + 0.002823x, \quad g_l(x) = -2.5532 + 0.002823x, \quad 84.2\% \text{ included} \]

Manueloa : 01/1965-12/2008

\[ h(x) = 315.951 + 0.1264x - 2.87594 \sin(44.5759 - \pi x/6), \quad \text{CR}_6 = 0.0017 \]
\[ h_u(x) = 318.827 + 0.1264x, \quad h_l(x) = 313.075 + 0.1264x, \quad 68.3\% \text{ included} \]

Ayari : 01/1987-12/2008

\[ h(x) = 349.489 + 0.1460x + 5.9508 \sin(2.8080 - \pi x/6), \quad \text{CR}_6 = 0.0045 \]
\[ h_u(x) = 355.440 + 0.1460x, \quad h_l(x) = 343.538 + 0.1460x, \quad 77.6\% \text{ included} \]

4. Conclusion

We introduce a nonlinear model and apply it to the data of local temperature of the UK (33-station), Germany (52-station) and Japan (57-station). The nonlinear curves are practically evaluated for decadal year data set of all stations and CR\(_6\) always gives the smallest value. It illustrates the robustness of the proposed method. The annual trends of maximum and minimum temperatures are estimated by \( f_u(x) \) and \( g_u(x) \). For example the annual trend of maximum temperature of Tokyo is estimated 1.5 \(^\circ\)C up per 100-year, because of 0.00129 \( \times 12 \times 100 = 1.5 \).

We also show the trend of CO\(_2\) of Manueloa and Ayari. The value CR\(_6\) is more improved if we introduce \( a_5 x^2 \) term in (2).

References
[1] M. IGARASHI and E. NUNOHIRO and J. G. PARK 2009 I. J. Geology 3 97
[2] S. D. Conte and Carl de Boor 1972 Elementary Numerical Analysis (McGraw-Hill Book Company) p 10