Analysis of harmonic gearbox tooth contact pressure

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Abstract. An analysis of a conventional gear wheel shot with MPK methodology has been processed by many authors before. Harmonic gearbox systems are composed of rigid and flexible parts through which the torque is transmitted, thus generating power. The contact pressure is specifically distributed through more teeth whilst against conventional gearbox systems. Firstly, a design description of a dynamic calculation model of a harmonic gearbox system in the planar strain is analysed in this article, as well as the definition of each contact constraint between the rigid and flexible parts of these gear systems in respect of “GLUE” and “TOUCHING” contact types. The last part of this article deals with the force distribution analysis in the teeth with respect to the frictional forces in them. The graphical conclusions of this pressure on each tooth were made. This article mainly focuses on the analysis of the stress in a gear tooth of the harmonic gearbox systems, as well as the research of a special trajectory of the rigid and flexible entity in a solid circular spline of such gear systems.

1. Introduction

The year 1955 is considered a milestone in the development of harmonic gears. In this year, Clarence Walton Musser, an American inventor, first published a new concept of the gear mechanism. He called this mechanism “Strain Wave Gearing”. Later, in 1957, he managed to patent the invention, calling it “Harmonic Drive Gearing, Strain Wave Gearing”. This mechanism was based on a theory which opposed the gear rigid-body mechanics that had been used many years until then. This new concept worked on the principle of a mechanical gear with a flexible spline. In fact, this invention was based on a theory of dynamics of an elastic body. This idea of the gear systems was significantly different from a globally accepted opinion on rigid gear systems. The first prototypes of this mechanism were successfully made in 1958 in America, in the United Machinery Group Corp — the Harmonic Drive Division. Nowadays, harmonic gears have been made in several countries. The greatest companies producing these gear systems include Harmonic Drive LLC (USA), Harmonic Drive Systems (JAP), Harmonic Drive AG (GER). Harmonic drive gearing systems have been widely used in the area of precise control. Precise reduction and positioning have been mainly used in robotics. Its high rate of precision and weight has been used in the aviation industry. The system is also used in the engineering industry for the transmission of high torque, conveyor and turbine drives, and in many other machine applications.
2. The Principle of the Harmonic Gear Functioning
Firstly, it is important to realise that harmonic gears (hereinafter referred to as “HG”) were derived from traditional epicyclic gearing systems. In that case, a sun gear is the main input member and output speed and torque are obtained from planets that diverge from a central axis. HG is, in fact, a differential gear with a cylindrical gear pair in which gearing is achieved by elastic deformation of the flexible spline. This spline is characterised by its unique structure called a “Strain cam wave generator”. The whole gear system is composed of four basic components, see Figure 1.

![Figure 1. The basic components of a harmonic drive gear.](image)

Input torque is applied to a wave generator. This component is specific for its ellipse-shaped geometry. However, this ellipse is not regular — to secure clearance between the generating teeth of a circular spline and a flexible spline when rolling. The shape of the ellipse is flattened in line with the prescribed geometric function characteristic for stretching or flattening the shape. On this component, a flexible ball bearing is pressed and flexibly deformed according to the ellipse shape of the wave generator. When gearing this system, waves are generated, which is the basic driving component of this gear system. Finally, a flexible spline is pressed on the wave generator system and the flexible bearing. The assembled internal part of this drive gear as a whole is then inserted into a circular spline. The circular spline is attached to a gearbox, thus ensuring a rigid embedding of the HG. Output torque and speed are obtained from the flexible spline. In the back part of this component, there are mostly two openings made for a pin and 6–12 openings, depending on the rate of the torque transferred to the screws. We need to realise that the flexible spline rotates counter to the rotation of the wave generator. The advantage of this mechanism is almost a zero-tooth clearance and its ability to transfer a rather huge gear ratio, mostly 1:50, 1:100, 1:200 [1].

3. Development of a Mathematical Calculation Model
To develop a calculation HG model, it is necessary to consider a number of requirements. First, an analysis of the geometric shape of the wave generator and then the dimensions of the flexible ball bearing is required. In the next stage, it is necessary to correctly calculate the tooth geometry of the circular spline and the flexible spline. These teeth must have the same module and pitch circles. It is important to acknowledge that the bodies are flexibly deformed after pressing. For this reason, their geometry must first be calculated for a non-deformed shape, and then for a deformed shape, too. This analysis is vital for the functioning of the whole HG and its fundamental operating characteristics such as backslash; rigidity, the gear ratio and a total vexillary size of the HG. Then, in the second stage, it is necessary to process the geometry to develop a 2D and 3D form in CAD software. This geometry must be correctly meshed in a 2D and 3D meshing appliance. Subsequently, a calculation model with boundary conditions and correct parameters must be created. In the last stage, the analysis results and assessments must be analysed correctly.
3.1. The Analysis of the Wave Generator Geometry in Matlab (software)

Matlab was used to develop a mathematical model. In this programme, equations for calculating the body geometries were mathematically defined. This geometry was exported from Matlab to a text document as a cloud of points with x and y coordinates. These points were later imported to Creo Parametric, a 3D CAD software, where a 2D and 3D model was created [2-3].

\[
R = A + B \cos \left\{ 2, \frac{\pi}{2}, \left( \frac{\phi}{2\pi} \right)^n \right\} \tag{1}
\]

\[
R = \int_0^\pi \frac{\pi}{2} ds = L_k = \frac{\pi D}{4} \tag{2}
\]

\[
ds = \left[ \left( \frac{dx}{d\phi} \right)^2 + \left( \frac{dy}{d\phi} \right)^2 \right]^{1/2} \tag{3}
\]

Based on the equations (1), (2) and (3), forming parameters were calculated to describe the resulting ellipse shape of Figure 3. The coefficients A and B describe the length of the main and the minor semi-axis of the ellipse. The coefficient n impacts the flattening or stretching of the ellipse shape. The ellipse was described as a quarter, which was reflected in Creo Parametric 3D software.

3.2. Geometry Analysis of the Flexible Spline and the Circular Spline in Matlab

A mathematical calculation model was designed in Matlab to generate the geometry of these teeth bodies, as well as for the wave generator. This programme generated the geometry of both bodies simultaneously; see Figure 2. The generating was primarily for the shape geometry of the teeth. The following parameters were defined as programme inputs: teeth module, the number of teeth of both bodies, working addendum and the size of the meshing circles. For this gearing, we considered 200 and 202 teeth.

![Figure 2](image2.png)

**Figure 2.** Detail of the shape of the flexible spline gearing (in red) and lines of action (in yellow).

Established patterns and relations were used to calculate the gearing geometry. This programme calculated gearing pressures according to Hertzian contact stress and a material coefficient in line with the equation [4-5].

4. Designing the 2D and 3D Model for Simulation

The point clouds generated this way were then imported into a Creo Parametric sketchbook and they were used for developing volumetric and planar bodies, see Figure 4 and 5.

![Figure 3](image3.png)

**Figure 3.** The ellipse shape in Matlab.
4.1. Developing the Mesh for 3D Simulation
A 2D and 3D mesh was developed for 2D and 3D models in MSC Apex Iberian Lynx. For the task regarding 2D, the Creo Parametric model was created by surface modelling through surfaces. It was exported in step format as a surface. It was meshed in Apex using linear quadrilateral elements. The 3D model was also exported in step format as a solid body. It was meshed using a linear hexagonal mesh. For a symmetrical mesh, only a section of the gear system was meshed. These bodies were reflected in the Marc Mentant and joined using a “sweep” function. The number of elements for 2D mesh amounted to 392,706 (Figure 7), and for 3D mesh it was 543,336 (Figure 6). The 2D task was meshed with a rather fine mesh - due to the lower number of elements. It is recommended to work with a model of up to 600,000 elements. If the number is exceeded, huge calculation parameters and a long calculation time are needed. At the same time, it leads to a problem with post-processing.

4.2. Developing a Simulation Calculation Model
The calculation model was created for the 2D and 3D tasks in Marc Mentant 2018. The task was broken down into several steps. Firstly, the gear bodies were assigned material characteristics and contact dependence was determined; see Figure 8 and 9. In Marc Mentant, it is possible to set a contact GLUE analysis for a solid bonding of bodies, without considering friction, and TOUCHING analysis for the contact of the bodies with prescribed friction. It is the option of defining the contact between the bodies that allows the whole gear to be set into rotating motion rather easily, thus allowing examination of individual characteristics directly “on the fly” of the harmonic drive gear. It is also possible to let the gear rotate as long as necessary and examine slips and wear on the teeth. This, however, is not a subject of this article [6].
To make the simulation as realistic as possible, the calculation model was broken down into individual minor steps. The first step was to let the entire gear be pressed. After the pressing, the tooth contact was switched from GLUE to TOUCHING as the second step. After having switched the gearing contact dependence, the gear was set into rotating motion using the wave generator by 180°. After having finished the rolling, the wave generator was stopped and load torque $M_k = 100 \text{ Nm}$ was applied to the flexible spline in the opposite direction of the rotation. As a last step, the direction of the torque was switched to the direction of the rotation. Through the last two steps, the gear clearance (stressing the backslash) was discovered. The whole task had 261 increments. The results were analysed in each of them [7-8].

5. Evaluation of the Results
The evaluation of the results was post-processed in Marc Mentant. Contact stresses for 2D and 3D models engaged/meshed with the gear system were evaluated, see Figure 10 - 13.
The distribution of the stress is caused by “attaching” the flexible spline in the crown of the circular spline; see Figure 14 and 15. The stress on gearing is distributed evenly everywhere else.

Figure 14. Displaying the distribution of the contact stress with regard to 2D.

Figure 15. Details of the contact stress with regard to 2D.

Figure 16. Displaying the distribution of the contact stress on flexible spline gearing.

Figure 17. Comparison of the trajectory of maximum stress in the gearing of the harmonic gear system for 2D and 3D analyses.

6. Appendices

The simulation revealed that the flexible spline gearing is deformed in two directions simultaneously. The first depending on the pressing on the wave generator, and the second one depending on the impact of the torque. When transferring the performance from the gear, the flexible spline has been twisted in the circular spline; see Figure 16. This causes the greatest stress in the front part of the gearing. Figure 17 shows the course of the stress when the gear is rolling. The figures on the graph were acquired from the point with the greatest contact stress on a flexible spline tooth. The maximum stress with the 3D model was 1206 MPa and 995 MPa for the 2D model. The measured maximum stress with the 2D and 3D models differed by 17.49%, which was caused by different mesh densities. From the viewpoint of possible calculations, it is not possible to use as fine mesh for a 3D model as for a 3D analysis. Furthermore, this research proved that in the case of harmonic drive gears, several teeth mate simultaneously. With 200 teeth for a circular spline in a 2D task, 12 teeth mate/are engaged at the same time on the upper part of the gear and 12 teeth at the same time on the opposite part of the gear. It means that 24 teeth mate/are engaged at once. With the 3D model, the number amounted to 10 teeth on the upper part and 10 teeth on the opposite part of the gear. Consequently, 20 teeth mated/were engaged at once. This also implies that the rigidity and accuracy of these harmonic gearbox systems with a flexible component is huge in comparison to traditional rigid gear systems.

The main purpose of this article was to describe the basic topic concerning these special harmonic drive gears that have been increasingly popular, mainly in the area of robotics and precise control. Also, the article dealt with the main differences between conventional toothed gear systems. Given the recent exponential growth of the engineering industry, automation and robotization, we will surely come across the term of Harmonic Gearbox more often due to its great advantages and a really small size. The most important features of Harmonic Gearbox definitely include very big rigidity and high accuracy. The
high stiffness is due to multiple teeth engaging on both sides of the gearbox. The geometry of the gearbox and its effect on the stiffness and durability will be the subject of the following research to address this issue.

Acknowledgement
This study was supported by Slovak Research and Development Agency under the contract no. APVV-18-0450 - Research of the influence design parameters of special transmissions with a high gear ratio with respect to kinematic properties.

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