NANOGrav Hints to Primordial Black Holes as Dark Matter

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The NANOGrav Collaboration has recently published a strong evidence for a stochastic common-spectrum process that may be interpreted as a stochastic gravitational wave background. We show that such a signal can be explained by second-order gravitational waves produced during the formation of primordial black holes from the collapse of sizeable scalar perturbations generated during inflation. This possibility has two predictions: i) the primordial black holes may comprise the totality of the dark matter with the dominant contribution to their mass function falling in the range \((10^{-15} \div 10^{-11}) M_{\odot}\) and ii) the gravitational wave stochastic background will be seen as well by the LISA experiment.

Introduction. The NANOGrav Collaboration has recently published an analysis of 12.5 yrs of pulsar timing data [1] reporting a strong evidence for a stochastic common-spectrum process. The latter may be compatible with a Gravitational Wave (GW) signal with strain amplitude \(\sim 10^{-8}\) at a frequency \(f \sim 3 \times 10^{-8} \text{ Hz}\) with an almost flat GW spectrum, \(\Omega_{gw}(f) \sim f^{-1.5 \div 0.5}\) at 1σ-level. In particular, their analysis shows the presence of a stochastic process across fourty-five pulsars which can be interpreted in terms of a common-spectrum process strongly preferred against independent red-noise signals. Despite being in partial contrast with some other bounds on the stochastic background of GWs, the NANOGrav Collaboration stresses that the detected signal is due to an improved treatment of the intrinsic pulsar red noise. On the other side, it is important to stress that the NANOGrav Collaboration does not claim a detection of GWs since the signal does not possess quadrupole correlations.

The goal of this paper is to show that the NANOGrav signal, if interpreted as a GW background, can be naturally explained by a flat spectrum of GWs inevitably generated at second-order in perturbation theory during the formation of Primordial Black Holes (PBHs) in the case in which the latter form from the collapse of large curvature perturbation generated during inflation upon horizon re-entry (see also Refs. [2, 3] for reviews on PBHs physics and Ref. [4] for a review on the constraints on their abundance).

Two nice by-products of this explanation are that i) the dominant contribution to the PBH mass function falls in the range \((10^{-15} \div 10^{-11}) M_{\odot}\) where the PBHs can comprise the totality of the dark matter in the universe; ii) the GW stochastic spectrum propagates to frequencies testable by future experiments, such as LISA [5].

The PBH abundance. The most common formation scenario for PBHs is through an enhancement of the power spectrum of the comoving curvature perturbation \(\zeta\) during inflation, at scales much smaller than those probed by CMB observations [6–8]. During the radiation-dominated phase, an overdense region collapses to form a PBH at horizon re-entry if the volume-averaged density contrast is larger than a critical value \(\delta_c\), which has been found with dedicated relativistic numerical simulations in Refs. [9–11].

We define the comoving curvature perturbation power spectrum as

\[
\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \rangle' = \frac{2\pi^2}{k_1} P_{\zeta}(k_1),
\]

where we have adopted the standard prime notation indicating that we do not explicitly write down the \((2\pi)^3\) times the Dirac delta of momentum conservation. In comoving slices, the overdensity is usually expressed in terms of the curvature perturbation through the nonlinear relation [11]

\[
\delta(\vec{x}) = -\frac{8}{9a^2 H^2} e^{-5\zeta(\vec{x})/2} \nabla^2 e^{\zeta(\vec{x})/2},
\]

where \(a\) is the scale factor and \(H = \dot{a}/a\) the Hubble rate. Assuming Gaussian curvature perturbations, one can estimate the mass fraction of the Universe that collapses to form PBHs at formation by computing the probability \(P(\delta)\) that the overdensity is larger than the critical threshold following the Press-Schechter formalism as [2]

\[
\beta(M_{PBH}) = \int_{\delta_c}^{\infty} d\delta \frac{M_{PBH}}{M_H} P(\delta),
\]

where one has to keep into account the scaling law relating the PBH mass to the horizon mass \(M_H\) for overdensities close to the critical threshold for collapse as [12–14]

\[
M_{PBH} = \kappa M_H (\delta - \delta_c)^{\gamma_c}
\]

in terms of the constants \(\kappa = 3.3\) and \(\gamma_c = 0.36\) in a radiation-dominated universe [15–20]. The horizon mass \(M_H\) is related to the characteristic comoving frequency of the perturbation as

\[
M_H \simeq 33 \left(\frac{10^{-9} \text{ Hz}}{f}\right)^2 M_{\odot}.
\]
The variance of the density field $\delta(x)$ is given by
\[ \sigma^2 = \int_0^\infty d\ln k T^2(k, R_H) W^2(k, R_H) \mathcal{P}_\delta(k), \] (6)
in terms of the density power spectrum $\mathcal{P}_\delta(k)$. A real space top-hat window function $W(k, R_H)$ is introduced to smooth out the density contrast on the comoving horizon length $R_H \sim 1/aH$, given by
\[ W(k, R_H) = 3 \frac{\sin(kR_H) - (kR_H) \cos(kR_H)}{(kR_H)^3}, \] (7)
and the transfer function during radiation domination with constant degrees of freedom is provided by
\[ T(k, R_H) = 3 \frac{\sin(kR_H / \sqrt{3}) - (kR_H / \sqrt{3}) \cos(kR_H / \sqrt{3})}{(kR_H / \sqrt{3})^3}. \] (8)

Notice that, in our results, we have accounted for the ineludible non-Gaussianity inherited by the non-linear relation between the curvature perturbation and the density contrast as shown in Eq. (2), see Refs. [21, 22].

To assess if PBHs may or not represent the dark matter in the universe, one usually introduces the PBH mass function $f_{\text{PBH}}(M_{\text{PBH}})$ as the fraction of PBHs with $M_{\text{PBH}}$ [2]
\[ f_{\text{PBH}}(M_{\text{PBH}}) = \frac{1}{\Omega_{\text{DM}}} \frac{d\Omega_{\text{PBH}}}{d\ln M_{\text{PBH}}}, \] (9)
such that the total fraction of dark matter in the form of PBHs is given by
\[ f_{\text{PBH}} = \int f_{\text{PBH}}(M_{\text{PBH}}) d\ln M_{\text{PBH}}. \] (10)

After matter-radiation equality, it can be expressed in terms of the mass fraction $\beta$ as (see for example [23])
\[ f_{\text{PBH}}(M_{\text{PBH}}) = \frac{1}{\Omega_{\text{DM}}} \left( \frac{M_{\text{s}}}{M_{\text{PBH}}} \right)^{1/2} \beta(M_{\text{PBH}}), \] (11)
where the overall factor, dependent on the horizon mass at matter-radiation equality $M_{\text{s}} = 2.8 \times 10^{17} M_\odot$, accounts for the energy density evolution during the remaining radiation-dominated phase after formation of PBHs of mass $M_{\text{PBH}}$.

In the following sections, we will show that a signal as the one observed by NANOGrav naturally arises from a class of models with a broad and flat power spectrum of the curvature perturbation of the form [24]
\[ \mathcal{P}_\zeta(k) \approx A_\zeta \Theta(k_s - k) \Theta(k - k_l), \quad k_s \gg k_l \] (12)
where $\Theta$ is the Heaviside step function and $A_\zeta$ is the amplitude of the power spectrum. This shape may be generated naturally for modes which exit the Hubble radius during a non-attractor phase, obtained through an ultra slow-roll regime of the inflaton potential, as a result of a duality transformation which maps the non-attractor phase into a slow-roll phase [25–28].

The power spectrum of GWs. To investigate the GW signal, we first define the linearized line element in tightly-coupled radiation domination as
\[ ds^2 = a^2 \left\{ -(1 + 2\Psi) dt^2 + \left[ 1 - 2\Psi \delta_{ij} + \frac{h_{ij}}{2} \right] dx^i dx^j \right\}, \] (13)
in terms of the Newtonian-gauge scalar metric perturbation $\Psi$ and the transverse-traceless tensor metric perturbation $h_{ij}$. Focusing on the signal sourced at second-order by linear scalar perturbations [29–42], one can expand the Einstein’s equations and determine the equation of motion for the GWs as
\[ h_{ij}'' + 2H h_{ij}' - \nabla^2 h_{ij} = -4\mathcal{T}_{ij}^{\ell m} S_{\ell m}, \] (14)
where $'$ is the derivative with respect to the conformal time $\eta$ and $\mathcal{T}_{ij}^{\ell m}$ projects the source term $S_{\ell m}$ into its transverse and traceless part. In the radiation phase the source is given by [29]
\[ S_{ij} = 2\partial_\tau \partial_j (\Psi^2) - 2\partial_i \partial_\tau \partial_j (\frac{\Psi'}{H} + \Psi) \partial_j (\frac{\Psi'}{H} + \Psi), \] (15)
while the projector in Fourier space using the chiral basis is
\[ \mathcal{T}_{ij}^{\ell m}(k) = e_{ij}^{\ell m}(k) \otimes e^{\ell m}(k) + e_{ij}^{\ell r}(k) \otimes e^{\ell r}(k), \] (16)
where $e_{ij}^{\ell r}$ are the polarisation tensors. In Eq. (15) the scalar perturbation $\Psi(\eta, k)$ can be expressed in terms of the comoving curvature perturbation as [43]
\[ \Psi(\eta, k) \equiv \frac{2}{3} T(k \eta) \zeta(k). \] (17)

The current abundance of GWs is found to be [44]
\[ \frac{\Omega_{\text{GW}}(f)}{\Omega_{\text{0}}} = \frac{c_g}{972} \int_S dx dy \frac{x^2}{y^2} \left[ 1 - \frac{(1 + x^2 - y^2)^2}{4x^2} \right]^2 \times \mathcal{P}_\zeta(k_x) \mathcal{P}_\zeta(k_y) \mathcal{I}_\zeta(x, y), \] (18)
where the integration region $S$ extends to $x > 0$ and to $|1 - x| \leq y \leq 1 + x$, and $k = 2\pi f$. The parameter $c_g$ defined as
\[ c_g \equiv \frac{g_s(M_H)}{g_s^0} \left( \frac{M_{\text{PBH}}}{M_{\text{PBH}}^0} \right)^{4/3} \] (19)
accounts for the change of the effective degrees of freedom of the thermal radiation $g_s$ and $g_{*, S}$ (where the superscript $^0$ indicates the values today) during the evolution (assuming Standard Model physics), and it is of order $c_g = 0.4$ for modes related to the formation of asteroid-mass PBHs. Also, $\Omega_{\text{0}}$ stands for the current radiation density if the neutrinos were massless, $\mathcal{I} = \mathcal{I}_x^2 + \mathcal{I}_y^2$ and
\[ \mathcal{I}_x(x, y) = 4 \int_0^\infty d\tau \tau (-\sin \tau) \left[ 2T(x\tau)T(y\tau) \right. \right.
\[ + \left. \left( T(x\tau) + x\tau \right) T'(y\tau) \right] \left( T(y\tau) + y\tau T'(y\tau) \right), \] (20)
$\mathcal{I}_x(x, y)$ being the same function, but with $(-\sin \tau)$ replaced by $(\cos \tau)$, see Refs. [44, 45].
Results. We have collected our results in Fig. 1. In the left panel, we have plotted the mass function corresponding to the primordial curvature perturbation given in Eq. (12). As described in Ref. [24], the peak of the mass function for a broad flat spectrum (12) corresponds to the mass inside the horizon when the shortest scale $\sim 1/k$, re-enters the horizon. At smaller masses, the mass function goes as $M_{\text{PBH}}^{3.8}$, due to the dynamics of the critical collapse, while at larger masses falls down as $\sim M_{\text{PBH}}^{-3/2}$ and has a sub-dominant peak around $\sim M_0$ due to the change of equation of state during the QCD phase transition [23, 46]. Given the absence of constraints in the mass range of support of the PBH mass function, the integral of the latter can be chosen in such a way that the PBHs contribute to the totality of the dark matter, that is

$$f_{\text{PBH}} = \int f_{\text{PBH}}(M_{\text{PBH}}) \, d\ln M_{\text{PBH}} = 1.$$  

(21)

As a consequence the first prediction of our scenario is that the signal seen by NANOGrav, if interpreted as a stochastic background of GWs produced as second-order within the PBH model, is in agreement with the possibility that all the dark matter is in the form of extremely light PBHs.

On the right panel of Fig. 1, we show the corresponding spectrum of the second-order GW abundance as a function of the frequency which falls within the 95% C.I. from the NANOGrav 12.5 yrs observation. Shown are the constraints coming from experiment EPTA [58], PPTA [59], NANOGrav 11 yrs [60, 61] and future sensitivity curves for planned experiments like SKA [62], LISA [5] (power-law integrated sensitivity curve expected to fall in between the designs named C1 and C2 in Ref. [63]), DECIGO/BBO [64], CE [65], Einstein Telescope [66, 67], Advanced Ligo + Virgo collaboration [68], Magis-AION-space and Magis-100 [69]. Notice that a portion of the 95% C.I. of NANOGrav 12.5 yrs is in tension with NANOGrav 11 yrs and PPTA. However, according to the NANOGrav Collaboration [1] the improved priors for the intrinsic pulsar red noise used in the novel analysis relaxes the NANOGrav 11 yrs bound. Nevertheless, the predicted signal within our scenario falls below all bounds. The GW abundance spectrum propagates flat entering the LISA detectable region and decays rapidly at the frequency corresponding to the shortest scale $1/k$. The second prediction of our scenario is therefore that the second-order GWs seen by NANOGrav should also be detected by the forthcoming experiment LISA, and eventually MS and BBO as well.

Both predictions of the scenario described in this paper depend only on the choice of the shortest scale $1/k$, and the requirement of the PBH abundance being equal to the dark matter one.

Conclusions. The discovery of a primordial stochastic background of GWs would be another fundamental pillar in GW astronomy. In this Letter, we have shown that the recently published stochastic common-spectrum process by the NANOGrav Collaboration, if interpreted as an indication of a GW background, can be naturally linked to the physics of PBHs. Indeed, the formation of PBHs in the early universe due to the collapse of sizeable overdensities generated during inflation is inevitably accompanied by the generation of GWs. Interestingly
enough, the NANOGrav observation is consistent with a mass range of PBHs such that the latter can comprise the totality of the dark matter. Furthermore, the GW signal is characterized by a flat spectrum which will make it visible even at much larger frequencies, most notably by the forthcoming experiment LISA.

Note Added. When completing this work, other interpretations of the NANOGrav signal have appeared other than through super-massive BH coalescences [70]. Refs. [71, 72] point out the possible generation of GWs through cosmic strings. Ref. [73] discusses as well the possibility of explaining the NANOGrav signal with second-order GWs related to the PBH scenario. They identify a mass range explaining the NANOGrav signal with second-order GWs [71, 72] point out the possible generation of GWs through super-massive BH coalescences [70]. Refs. 71 and 72 provide different interpretations of the NANOGrav signal have appeared other than through super-massive BH coalescences [70].

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