Laboratory Testing the Layer Oriented Wavefront Sensor for the Multi-conjugate Adaptive Optics Demonstrator

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ABSTRACT

The Multi-conjugate Adaptive Optics Demonstrator (MAD) for ESO - Very Large Telescopes (VLT) will demonstrate on sky the Multi-conjugate Adaptive Optics (MCAO) technique. In this paper the laboratory tests relative to the first preliminary acceptance in Europe of the Layer Oriented (LO) Wavefront Sensor (WFS) for MAD will be described: the capabilities of the LO approach have been checked and the ability of the WFS to measure phase screens positioned at different altitudes has been experimented. The LO WFS was opto-mechanically integrated and aligned in INAF - ASTRON (Largo E. Fermi) before the delivering to ESO (Garching) to be installed on the optical bench. The LO WFS looks for up to 8 reference stars on a 2 arcmin Field of View and up to 8 pyramids can be positioned where the focal spot in phases of the reference stars form, splitting the light in four beams. Then two objectives conjugated at different altitudes simultaneously produce a quadruple pupil image of each reference star.

An optical bench setup and transparent plastic screens have been used to simulate telescope and static atmospheric layers at different altitudes and a set of optical beams as (white) light source.

The plastic screens set has been characterized using an interferometer and the wave-front measurement data related to the LO WFS ones have shown correlation up to 95%.

Keywords: Multi-conjugate Adaptive Optics system, Layer oriented MCAO, Wavefront Sensors, MAD

1. INTRODUCTION

The Multi-conjugate Adaptive Optics Demonstrator (MAD) will be the first Multi-conjugate Adaptive Optics (MCAO) instrument working on sky. It will be mounted aboard the Nasmyth platform of the UT-3 Very Large Telescopes (VLT). In this paper the preliminary results of the laboratory tests performed in the Layer Oriented (LO) Wavefront Sensor (WFS) for MAD will be presented: the Wavefront (WFS) reconstruction capabilities of the LO approach have been checked and the ability of the WFS to measure phase screens positioned at different altitudes have been experimented. The Layer Oriented (LO) approach\textsuperscript{1} had been tested in laboratory so far using a mini-prototype of the MAD LO WFS\textsuperscript{2}. The LO WFS for MAD has been aligned and tested at the ASTRON (Largo E. Fermi) where it was mounted on an optical bench and making part of an optical set-up intended to mimic the F/20 MAD focus at the LOWFS baseline. Hereafter this optical setup is called "telescope simulator". It simulates in highly visible stars over the 2 arcmin field of view of the WFS, the telescope pupil and a turbulent static multi-layers atmosphere.

The LO WFS has a technical Field of View of 2 arcmin in and up to 8 pyramids can be positioned where the focal spot in phases of the reference stars form, splitting the light in four beams. Then two objectives conjugated at different altitudes simultaneously produce a quadruple pupil image of each reference star.

Two optical relays compose the so called Ground and High layer WFS, they can be optically conjugated to altitudes between 0 and 18 km by simply positioning the CCD using two as many linear stages, controlled
via software by the Instrument Control Software (ICS). For each reference star four geometrically identical pupil images are imaged on the two CCDs, and the intensity variations due to different WF distortions are measured simultaneously for all the references. Finally, the local tip-tilt can be computed by simply measuring the illumination difference between same portions of different pupils. In the experimental setup installed in Arosti the metapupils (pupil plane projection of the Field of View at the conjugation altitude) are imaged on a couple of temporary CCDs (Electrum 1000N) because the two MARCONICCD39s were not available. Electrum 1000N uses 4.75 3.66 mm sensing area divided in 652 494 pixels. Because the correspondent pixel size is 7.4 μm, the CCD 39s 24 μm, it was possible to achieve a very high precision (better than a tenth of sub-aperture) in the alignment phase. In order to shrink the dimension of the pupils on the CCD the F number from the initial F/20 have been enlarged to F/300 using a simple optical relay composed by two small lenses placed on the same mounting of the pyramids in order to be placed in correspondence of reference star position on the focal plane. In this way the dimension of the spot on the pyramids is enlarged of factor 15 while the pupil dimension is shrank by the same factor, giving finally the 384μm of the pupil dimension. These optical relay will be called hereafter "star enlarger". Several tests have been performed to check the optical quality, the alignment and the WF measurement repeatability. Moreover the LOW FS measurements have been calibrated to translate the slope measurement in nm of the WF using theoretical formula and ray-tracing simulations considering the defocus introduced by moving of the light sources toward and forward with respect to the zero defocus positions along the optical axis direction. A set of plastic screens has been characterized using an interferometer and the WF measurements compared to the LOW FS ones showing very high correlations (Perfect correlation, 1, No correlation 0, Anticorrelation -1). Finally two tests are presented:

1. The verification of the sensor ability to measure phase screen WF at the non-conjugation altitudes;

2. The verification of the sensor ability to measure the WF of conjugated plane inserting, in different orders, two screens at the two conjugation altitudes and one in between.

More and detailed information about the LOW FS for MAD could be found in the paper Vernet et al.4

2. THE TELESCOPE SIMULATOR

![Figure 1. Left: This picture shows the Telescope Simulator optical setup. On a corner of the bench (top-left in this image) the plate for bars positioning. The telescope simulator is composed by two identical 150 mm lenses spaced by twice their focal length (793 mm) and a stop pupil in their common focus. A turbulent static screen holder is arranged to place them on corresponding position between 0 and 18 km from the ground pupil. Right: the LOW FS MAD mounted on the bench. The telescope simulator is positioned to match its F/20 focal plane with the LOW FS entering focal plane. All the system has been aligned using as reference the optical axis defined by the common lenses of the pupil re-imager. Connected to the wires it is possible to identify the X Y linear stages that move the pyramids over the 2 arcmin in FoV. The higher wires are connected to the CCDs.](image-url)
The telescope simulator intends to mimic the F/20 MAD focus at the LOW FS base positioned that, according to the opto-mechanical drawings, falls 10 mm above the mechanical base of the system (by drawings 4.35 mm before the position of the star enlargers). At this plane is projected the 2 arc in FoV seen by the sensor. In the alignment optical setup used the LOW FS base-plate stands on four legs to leave room below it for a mirror at 45 degrees with respect to the optical axis direction to fold this inside the LOW FS.

The experimental optical setup has been designed in a way similar to the system used to test the LO prototype\(^2\). It will allow testing, in open loop conditions, the stand-alone LOW FS by using white light emitted by a set of beams and distorted by a set of perturbing screens to simulate the atmospheric turbulent layers.

Two identical lenses spaced, by twice their focal length, and a stop pupil, in their common focus, compose the telescope simulator. A turbulent static screen mounting is arranged to place them on positions corresponding between 0 and 18 km from the ground pupil.

Two lenses of 150 mm diameter with a focal length of 793 mm are used to mimic the telecentric beam. The optical axis is parallel to the bench plane and it passes in the center of the stop pupil between the L1 and L2. It is a 40 mm diaphragm on the same mounting of the static screens. The 40 mm aperture diameter lens simulates a telescope pupil and the F/20 angle, it is placed in focal plane of both the L1 and L2. The tip-tilt of the pupil is corrected in order to be orthogonal to the optical axis positioning at the mirror on the diaphragm. The positioning of the tip-tilt of the pupil is better than 20 arcsec, while the precision in the height positioning of the pupil is better than 0.5 mm.

The optical beams that mimic the star sources can be inserted on a large number of slots on a plate. This plate can be moved in z (the optical axis of the telescope simulator) using a linear stage. The telescope simulator re-imagines a 1:1 copy of the reference object (the beam plate); the 2 arc in FoV corresponds to a circular region of 94 mm diameter on the plate where the optical beam can be arranged in several different configurations, to mimic different guide stars constellations.

![Diagram](image)

Figure 2.0. Optical layout of the telescope simulator, the different rays correspond to different positions of the reference optical beams on the beam plate.

The optical beams used in the measurements presented in this document have a 50 mm core. They do not transmit an identical amount of light, so they were selected opportunistically, according the test to be performed. Regarding the optical quality tests (see section 2.1), the beam core dimension does not affect the results because the spot dimension on the CCD is produced by the holes size in the mask placed at the pupil plane. Furthermore, the dimension of an ideal source imaged in the LOW FS focal plane by the telescope simulator has 180 mm of
diameter. However, we checked that using fibers with different core sizes works such as using different modulation of the pyramid and that it changes the calibration factor that transforms SHF measurements from arbitrary to real length units.

2.1 Telescope Simulator optical quality
The telescope simulator exhibits a number of potential departures from the optical system composing MAD. These are hereby listed and discussed:

Field curvature. This is taken into account and corrected for by adjusting the fiber positions along the optical axis on the fiber plate, when they are placed off-axis. The defocus measurement is made using the pyramid WF sensor embedded in the instrument and it achieves usually a quality better than about 40nm.

![Defocus vs on-axis distance](image)

Figure 3. In this plot a portion of the field curvature introduced by the telescope simulator lenses is presented. Using fibers more and more off-axis the defocus term is more and more large. Large departure from a flat focal plane had been predicted and taken into account.

Telecentricity and half F/20 have been checked during the alignment procedure through auto-collimation in the pupil plane and it is well within the specifications of the MAD exit relay. According to the optical requirements for the input beam of the LOW FS an F/20 0.2 is requested and with a maximum non-telecentricity angle of 1.9arcmin. The half focal ratio (15 F/20= F/300) has been checked looking to the dimension of the re-imaged pupils on the detectors: we measured 385 m 4 m to be compared with 388 m (by the optical design). Moreover, considering that the star enlargers increase the total focal ratio by a factor 15, it is possible to obtain for the telescope simulator an F/20 0.2. For telecentricity has been reached 1mm precision in the positioning of the lens L2 with respect to the pupil mask on the screen holder, which corresponds to an error of 50micron of non-telecentricity angle.

Chromaticity is, by Zeeman design, very small and negligible with respect to the one introduced by the pyramid. Recall that the telescope simulator is made up by a couple of achromatic doublets that introduce an overall blur of 1/50 of sub-aperture.

The residual aberration is mainly astigmatism of third order. This is not negligible and depends upon the position on the Field of View. However, to perform phase screens analysis a measurement of the static aberrations with for each guide stars constellation is performed and later subtracted from the WF.
measurements. This is somehow similar to the handling of non-common path aberrations as will be done on the MAD experiments, even if in this case we will manage them as reference slope with respect to which the MCAO loop will be closed.

3. CALIBRATION OF THE LOW FS

The LOW FS has been calibrated in order to translate the differential illumination measurements of the 4 meta-pupils in terms of wavefront distortions measured in wavefront displacement units (nanometers).

To achieve this goal, we move of a known quantity the ber-plate along the optical axis using a linear stage, which axis was previously aligned to the optical axis (defined by the laser beam direction).

We use the formula:

$$ PV = \frac{d^2 f}{8 f_0^2} \tag{1} $$

This formula relates the Peak to Valley (PV) of the defocus aberration to the shift applied to the object on the focal plane along the optical axis. In our case, $d$ is the pupil diaphragm diameter (40 mm), $f_0$ is the focal length of the first lens (793 mm), and $f$ is the amount of shift to be read on the linear stage. The validity of this formula was checked using Ray-tracing both giving $PV = 90\pm46\text{nm}$.

Experimentally, we found the calibration coefficients to translate wavefronts from arbitrary to nm units.

4. PUPIL IMAGE OPTICAL QUALITY

In order to check the optical quality of the WFS objectives we used a pupil mask with small holes (Figure 4) distributed on a cross over the 40mm circular aperture defining the telescope simulator pupil. The diameter of each spot has been fixed to 1/50 of the pupil size (0.8mm) to the dimension of the CCD's pixel size corresponding to 1/52 of the pupil diameter. In this way, and in pure geometrical approximation, the holes in ages present a dimension equivalent to one pixel, when the mask is placed on the pupil position. In fact, one pixel is 7.4 m, while the image of the hole should be 7.7 m. We measured the dimension of the spots using only one ber source, one on-axis and later o-axis. The image of the mask has been taken for each star enlarger (2 lenses and 1 pyramid). The ber light source emits white light centered at 0.4 m with a large spectrum. In order to avoid un-realistic chromatic effects due especially to the pyramid, the white light should be modulated in wavelength with a filter as close as possible to the LOW FS spectral characteristics. Such a filter was not available and we checked how the measurements fit the theoretical provisions.

![Figure 4](image-url)

Figure 4. The picture shows the pupil mask with 9 holes described in the text above. Each hole has 0.8 mm size.

The ber's illuminator has a bandwidth of 0.25 m to 0.80 m with a 3250K Kelvin color temperature at the maximum intensity. The Electrim 1000N has a known spectrum response. In this test, the two LO objectives are imaging an object (the 9 holes mask) positioned at the pupil (ground) position. The pupil of the star enlarger + objective system is defined by the dimension of the second lens of the star enlarger (12.7 mm diameter) however it is not completely illuminated (only about 10 mm are effectively used). Considering the LOW FS-MAD
central operation wavelength is 0.55 m and the 115.7 mm objective focal length, the $/D$ corresponds to a 6.4 m FWHM. But the image chromatic elongation produced by the pyramidal prism overcomes the diffraction effect. In fact, the divergence angle, $\theta$, of the 4 beams exiting each pyramid depends on the diffraction index through the equation:

$$\theta = (n - 1)$$

where $\theta$ is the pyramid physical vertex angle (here 1.176 degrees) and $n$ is the refraction index. In particular, the pyramids are made of BK7 glass, which spectral behaviour is presented in Figure 5:

**Figure 5.** This picture shows the refractive index of the BK7 glass used to manufacture the pyramids with respect to the wavelength.

![BK7 refractive index spectral variation](image)

**Figure 6.** This plot shows the elongation of the spot due to the different spectral components of the light of the reference source. In the laboratory case here discussed, the spectral range is larger. One extreme of the spectral range is fixed to 0.9 m, while the smaller one varies between 0.2 m and 0.9 m.
This refractive index variation produces a spot elongation, larger and larger as the bandwidth used increases (Figure 6 and Figure 7). Moreover, the elongation directions are defined by the pyramid faces orthogonal planes, corresponding to the direction of the 4-pupils barycenter if projected on the CCD sensor.

Figure 7. This picture shows the images of the PSFs generated by the high LOW FSI objective conjugated to the ground plane where the 9 holes mask has been placed. The 9 holes generate a pretty identical image and here only the central one is shown. The 4 lines correspond to the 4 quadrants generated by the pyramid faces. The first 8 columns refer to different star enlargers looking the same reference mask placed at center of the 2 arc min field of view. The elongation direction is directed toward the 4-pupils barycenter. The last four columns present the images of the central hole of 4 different star enlargers but in two co-axial positions.

Figure 8. This plot shows the spectrum and the quantum efficiency respectively of the ber light (dotted line) and of the Electrin 1000N CCD (dashed). The solid line represents the combination of the two. A 0.4 m spectral bandwidth (over 20% of the peak value) in the range 0.4 m - 0.8 m has been actually used. The picture on the right shows the elongation spot simulated by using the quantum efficiency curve shown in the left panel. The position of the direction-limited spot was computed for a set of (in the range of interest), and the image summed according the position found, each one intensity-weighted according to the quantum efficiency. In this way was found 7.34 m (considering the spot-sizes of the interpolated gaussian) or 7.91 m (on the simulated image (of course with zero noise such as Read Out photon noise).

The ber illuminator emits between 0.25 m to 0.80 m. To the wavelength range 0.31-0.80 corresponds a 45 m elongation, equivalent to 6.2 Electrin pixels. But the emissivity function has not at spectrum and should be multiplied for the Electrin quantum efficiency (QE) response function to have a realistic idea of the intensity.
distribution over the elongated spot. In order to check the matching of the measurements with the expected results, the latter has been computed with the QE (V S I lambda) we have in the experimental setup. Please note, however, that this is rather at in the range of interest.

However, we have measured spot elongations with FW H M up to 24.9 m that in a first order analysis gives a maximum elongation of about 50 m. All the point spread functions (PSF) measured have been interpolated with 2 dimensional Gaussian functions. The fit took into account both ellipsoidal shape of the PSF and the axis rotation.

The spot m s computed fitting the measurement data of the PSFs is 8.04 m with an m s of 0.58 m equivalent to 0.167 ground sub-apertures (1sub = 48 m) equivalent to 52nm W FE according to the Wavefront Error Budget analysis performed so far6 (Figure 9). In fact the blur of the pupil images due to mis-alignment, chromatism, dirty optics, etc., introduces a loss of information in the WF/slope reconstruction, we quantified this effect through numerical simulation and it can be described such as a linear relation between blur of the pupil images and residual phase m s in closed loop operation that exceeds with respect to the ideal case with no blur.

In the instrument requirements was given for the chromatism effect a 57nm m s W ave-Front-Error to be compared to 52nm we have measured in the setup used. This error corresponds to an equivalent spot m s of 0.15 sub-apertures (compared to 0.207). But we saw that elongation effect depends on the waveband used. In particular we have to compare the 7.9 m spot m s measure to the elongation spot due to the system elongation m mounted on the bench. The quantum efficiency of the system illuminator and Electrim 1000N was computed (using technical data-sheets) and the effective spot elongation simulated.

|            | Average [m] | Average [sub-a-per] |
|------------|-------------|---------------------|
| LOW FS1 \Ground** | 7.99 | 0.39 | 0.166 | 0.008 |
| LOW FS2 \High**  | 8.00 | 0.49 | 0.167 | 0.010 |

Table 1. In this table are presented the average value of the spot m s values for the different star selector on and off -axis. The values are given both in m and ground sub-apertures. These values should be compared to the 7.91 m theoretically expected.

The_m LOW FS system will have a similar chromatic answer and then similar elongation spot sizes. More detail about specifications on the pyramid vertex angle and measurements can be found in the cited paper7.
5. LAYER SMOOTHING AT DIFFERENT ALTITUDES

We verified that the LOW FS is able to measure the turbulence at the conjugated altitude and in an atmospheric volume close to that plane. But the non-conjugated layers are seen more and more smoothed as the distance from the conjugation plane increases. To check this statement, a single plastic screen has been measured with an interferometer and then placed in 16 different positions corresponding to altitude between 0 and 15 km, 1 km spaced, (Figure 10). The two objectives had been previously conjugated to 0 (LOW FS1) and 9 km (LOW FS2). For this test 3 and 8 different reference sources have been used positioned in a restricted FoV of 1 arcmin for the case with 3 stars and over the full 2 arcmin in for the case with 8 stars. The interferometric measurements of the plastic screen aberration refer only to a 50.8 mm pupil, to be compared to the 66.2 mm size of the meta-pupil at 9 km. In the comparison we had not considered the outer region of the LOW FS measurements. For both W FS a couple of 4 meta-pupils in age with slightly different tip-tilt have been taken (introduced de-centring the pyramid optical axis using the XY linear stages to simulate a tip-tilt modulation).

A posteriori, the static W FS have been removed from the phase measurements. Results and comments are in the picture captions.

![Figure 10. This picture shows the plastic screens holder. In the \"smoothing\" test the same screen was moved from the position corresponding to 0 km to 15 km.](image)

6. SCANNING THE ATMOSPHERE

In this test we verified how the ground and high WFS measurements are still correlated when screen at different altitudes are inserted. In particular we first place a screen at 0 km, and then we added a screen at 9 km and finally at 4 km. Then we removed, in this order, the 4 km and the 0 km screen. For each of this configuration ground and high conjugation plane (9 km) have been retrieved.

A posteriori, the static WFS have been removed from the phase measurements. Results and comments are presented in the captions of Figure 12 and 13.

7. CONCLUSIONS

In this paper we show rapidly the more interesting topic related to the alignment of the LOW FS for MAD. We described how this had been reached and a few indicator of the quality achieved, such as the pupil optical quality. Finally we presented several results to show the ability of the LOW FS to measure WF both in classical Adaptive Optics and MCAO modes comparing plastic screen WFs measurements to the ones performed by an interferometer. This comparison gives very high correlations especially with low strength turbulence, a behavior already predicted by pyramid WFS models. In fact it is known how pyramid WFs gives best performance
Figure 11. The plots above show the correlation values between the WF measured both with the LOW FS and the interferometer. The correlation increases more and more the screen is close to the conjugation plane. To neglect high order errors due to processing procedures in the interferometric and LOW FS measurements and to deal with different pixel sizes the interpolation with the first 50 Zernike polynomials has been computed and interpolated. Dashed lines present data without tip-tilt modulation, solid with it. The red/grey case refers to the case with only 3 stars in central arcmin, the black ones to the 8 stars. Using more stars helps to see better high conjugation planes.

Figure 12. These pictures show: on the left the image of the 4 pupils illumination measurements with ground conjugated WF; on the right the same screen seen by the high conjugated (9 km) WF.

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Figure 13. These pictures show W F measurements for: on the left the ground W FS observing the screen at 0km; on the right the high W FS ones for the screen at 9km. From the left to the right (top row) the Interferometer W F data, the LO W FS W F and the relative difference on the bottom the same W F but interpolated with the first 50 Zernike polynomials. The values written on the top are in nm with on the right the correlation values; on the bottom the Zernike interpolated W F values. ($Ptv^\prime$ is W F Peak to Valley, $\mu m^\prime$ the standard deviation over the metapupils). Five stars on the two arc in FoV have been used and static aberration removed numerically.

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been measured also for the high WFS...