Modeling and optimization of the processes of turbulence

I Ya Lvovich¹, Ya I Lvovich², A P Preobrazhenskiy¹ and O N Choporov²

¹ Information Systems and Technologies Department, Voronezh institute of high technologies, 73a Lenin Street, Voronezh 394043, Russia
² Information Security Department, Voronezh state technical university, 14 Moscow Avenue, Voronezh 394026, Russia

e-mail: komkovvivt@yandex.ru

Abstract. In this paper the problem associated with the modeling and simulation of turbulent flows is considered. Turbulent flows are quite common and a complete theory is not built for them at the moment. Among the models that are based on the subgrid scale, the model proposed by Smagorinsky was chosen. On the basis of the created subsystem in the form of a cross-platform dynamic library for modeling turbulent media, the simulation of water flow around a circular obstacle was carried out. The simulation results are presented. Suggestions are given for the optimization procedure, with variations in the implementation of the considered system of medium motion.

1. Introduction
Currently, it is possible to observe the development of integration of CAD (computer aided design, CAD) with modeling systems (Computer-aided engineering, CAE), allowing to test the created object directly during the design. In addition, the means to put computational experiments have been long used in science, when full-scale experiments are too expensive, impossible or necessary to be repeated a large number of times in the same conditions.

Despite the fact that most technical processes are described by known physical laws, it is often almost impossible to imagine in advance how an object will behave in certain conditions.

This fact is due only to the extreme complexity of modern systems. Computational physics is a way of conducting scientific and technical research, in addition to experiment and theory. In this case, the computer plays the role of a device that gives new opportunities to study the properties of various physical models.

2. The analysis of features of turbulent flows
The development of methods for numerical simulation of flows began in the 50s the last century with the advent of computational hydroaerodynamics. After the mid-70s to the 80s, due to the lack of computer power, computational fluid dynamics was used to a greater extent in scientific purposes, for the simulation of relatively simple and elementary cases. It was at this time that its mathematical and physical foundation was laid.

With the advent of the 90s to the present, the development of computer technology has allowed to widely apply the methods of numerical modeling.
Computational fluid dynamics has significantly expanded its mathematical and physical apparatus, improved models of turbulence, radiation, acoustics, two-phase flows, methods of discretization and numerical solution of differential equations, etc.

Thus, computational fluid dynamics is an applied science based on various fields of science and successfully used to solve a wide range of problems.

Currently it continues to develop actively.

With the development of computer technology and improvement of methods of calculation and data analysis, computational fluid dynamics can take a dominant place, surpassing the prevalence of experimental and analytical methods.

The advantages of numerical simulation of flows in engineering practice include:

• reducing development time of technical devices;
• modeling of processes and flows for which experiment is not available to conduct;
• detailed picture of processes;
• reducing the cost of development.

On the other hand, computational fluid dynamics has such disadvantages as the final accuracy of the calculation, which in some cases is capable of cross out the advantages of numerical modeling.

Along with the computational part, analytical and experimental methods are developed. The use of all three methods, mutually complementing each other, allows you to effectively develop complex technical devices such as aviation, space and automobile engines, aircraft, ships, submarines, ventilation systems, etc.

The simulation of turbulent flows is of a particular interest. Primarily due to the fact that such trends are most common.

In addition, there is still no complete theory that would describe similar flows. Research by experimental methods is also hindered. Therefore, the modeling of turbulent takes a leading role in practical calculations and theoretical studies.

3. The mathematical model

One of the largest branches of computational physics can be considered computational fluid dynamics (CFD).

Due to the application of approaches based on numerical modeling in the description of fluid motion, it is possible to significantly accelerate the development of different technical devices. Most movements of liquids, gas and plasma flows encountered in technology are turbulent [1].

A large number of models related to the subgrid scale are based on assumptions related to the existence of vortex viscosity [2].

There is a process of influence of small structures on large structures. This can be accounted for due to the introduced viscosity $\mu_t$:

$$\mu_{eff} = \mu + \mu_t.$$ 

In cases where an unknown subgrid tensor $\tau_{ij}^{SGS}$ of stress $\tau_{ij}$ is considered, it is required to proceed from such a model equation [3]:

$$\tau_{ij}^{SGS} - \frac{1}{3} \tau_{ij}^{SGS} \delta_{ij} \approx \tau_{ij}^{mod} \delta_{ij} = 2 \mu_t S_{ij}.$$ 

The process of calculating the strain rate tensor using the expression:

$$\overline{S_{ij}} = \frac{1}{2} \left( \frac{\partial \overline{u_i}}{\partial x_j} - \frac{\partial \overline{u_j}}{\partial x_i} \right).$$
To date, a large number of models have been developed to determine the turbulent viscosity $\mu_t$. It should be noted the model for subgrid stresses, which was suggested by Smagorinsky [4]. Similar to the Prandtl model, the turbulent viscosity is modeled as follows:

$$\mu_t = \ell^2 |S_{ij}|.$$  

Here $|S_{ij}| = \sqrt{|S_{ij} \cdot S_{ij}|}$, and $\ell$ is the scale of turbulence. In order to determine the scale of turbulence, apply the width of the filter at the coefficient of proportionality $C_S$:

$$\ell = C_S \Delta.$$  

In the above expression, the Smagorinsky constant is the coefficient of proportionality $C_S$. We can calibrate it. The range of its variation at different currents is between 0.01 and 0.24. We used $C_S = 0.1$ in the calculations.

The Smagorinsky model is characterized by simplicity and good convergence due to the fact that it is dissipative.

The considered model was supplemented by the procedure of multi-alternative optimization [5]. When multi-alternative aggregation was carried out by a variation of the embodiments of the system flow.

The vector of alternative variables was used on the basis of which the process of dichotomous reduction was realized.

The variety of sets of parameters was limited due to the fact that they were consistently divided into two parts and there was a choice of one of them as a starting point for the next division to be carried out.

In order to automatically implement the dichotomous reduction procedure, it is necessary to carry out the transition to the equivalent problem associated with a multi-alternative choice of a set of random variables.

They are independent random discrete numbers.

The reduction of the differences of the parameter sets will be performed in the case when the values of the probabilities of the possible realizations will be close to the value 1. For this purpose, it is required to record the optimization problem using alternative variables and to use randomized multi-alternative optimization when searching for solutions.

The general structure of optimization models reflecting the mechanisms [6, 7] under consideration is related to aspect of the duality principle implementation by dividing the efficiency indicators of the system into two groups that meet extreme and boundary requirements [8-10]. Full set of indicators $y_{g_2}, g_2 \geq G$. Some of these indicators $g_1 \geq G_1$ are subject to extreme requirements

$$y_{g_1} \rightarrow \text{extr}, g_1 = \overline{1, G_1},$$  

to the rest – boundary

$$y_{g_2} \leq y_{g_2}^{bd}, g_2 = \overline{1, G_2}, \overline{1, G_1} \cup \overline{1, G_2} = \overline{1, G}.$$  

These indicators depend on the choice of certain characteristics of the implementation of reduction and transformation mechanisms, which are formalized as a vector of optimized variables measured on continuous and discrete scales

$$x = (x_1, \ldots, x_j, \ldots, x_J), \quad y_{g_1} = \Psi_{g_1}(x^1, x^2) \rightarrow \text{extr}, g_1 = \overline{1, G_1},$$  

where $j = \overline{1, J}$ – the numbering set of optimized variables.

This is another aspect of the duality of the two forms of realization of the vector $x$:
\[ x^1 = \left( x^1_1, \ldots, x^1_j, \ldots, x^1_k \right) \] - vector of continuous variables;
\[ x^2 = \left( x^2_1, \ldots, x^2_j, \ldots, x^2_k \right) \] - vector of discrete variables, in particular, Boolean:
\[ x^2_j = \begin{cases} 1, & j = 1, J_2, \\ 0, & \end{cases} \]

The optimization model has the following form:
\[ y_{g_1} = y_{g_1}(x^1, x^2) \rightarrow \text{extr}, g_1 = 1, G_1; \]
\[ y_{g_2} = f_{g_2}(x^1, x^2) \leq y_{g_2}^{bd}, g_2 = 1, G_2; \]
\[ x^1_j^{\text{min}} \leq x^1_j \leq x^1_j^{\text{max}}, j = 1, J_1; \]
\[ x^2_j = \begin{cases} 1, & j = 1, J_2, \\ 0, & \end{cases} \]

Linear models with Boolean variables are of particular importance for system optimization
\[ \sum_{j=1}^{J} c_j x^2_j \rightarrow \text{extr}; \]
\[ \sum_{j=1}^{J} a_{g_2 j} x^2_j \leq b_{g_2}, g_2 = 1, G_2; \]
\[ \sum_{j=1}^{J} a_{g_2 j} x^2_j \leq b_{g_2}, g_2 = 1, G_2; \]
\[ x^2_j = \begin{cases} 1, & j = 1, J_2, \\ 0, & \end{cases} \]

where \( c_j \) - coefficients of the linear objective function,
\( a_{g_2 j} \) - coefficients of the linear system of constraints.

The next aspect is related to two mechanisms of optimization modeling process organization: randomization and smoothing. Randomization is the transition from vectors \( x^1, x^2 \) to their probabilistic analogues. So instead of the coordinates of the vector \( x^1_j, j = 1, J_2 \) in the process of optimization of the search on the \( k \) iteration \( k = 1, 2, 3, \ldots \) random realizations are considered \( x^k_j, j = 1, J \), that meet the following condition for a given distribution type
\[ m(\tilde{x}_j) = x^k_j, \]

where \( m(\cdot) \) - the symbol of mathematical expectation. In the case of a vector of discrete variables and in particular Boolean variables, random variables are introduced \( \tilde{x}^j, j = 1, J \) with distribution
\[ P(\tilde{x}_j^2 = 1) = p_{x_j}, \quad P(\tilde{x}_j^2 = 0) = q_{x_j}, \quad p_{x_j} + q_{x_j} = 1. \]

Introduction-Robin optimized random variables \( \tilde{x}_j^2 \), leads to another aspect of duality - dichotomy of reduction performed in automatic mode. The reduction of the variety of sets characterizing the training system is performed if the probability values of combining perspective subsets are close to 1, and unpromising subsets are close to 0. In the conditions of combinatorial uncertainty of the choice of perspective sets at the first step of dichotomous reduction the uniform distribution is accepted

\[ p_{x_j}^1 = 0.5, \quad q_{x_j}^1 = 0.5. \]

4. Results of modeling

A subsystem was created in the form of a cross-platform dynamic library for modeling turbulent environments. It can be used in the development of CAD, as well as for scientific purposes. To check the correctness of the model, we created a simple geometry in the environment – water flow around a circular obstacle.

Then, based on the geometry, we will create a grid with the element size 0.5.

The figures below show the results of modeling turbulent water flow around an obstacle with Reynolds number equal to 100.

The length of the time step is 0.05, the simulation lasted for 2000 time steps, with a record of the result of the simulation every 10 steps.

Figure 1. Step 80.

Tests of the program at various parameters of modeling have shown that the subsystem of modeling turbulent environments successfully copes with its task – simulates turbulent flow around various solids.
5. Conclusion
The algorithm for modeling turbulent environments on the basis of the Navier-Stokes equations and Smagorinsky model was developed.

The algorithm was implemented as a compact cross-platform subsystem that can be used in the development of CAD, as well as for scientific purposes.

References
[1] Yun A 2005 Development and Analysis of Advanced Explicit Algebraic Turbulence and Scalar Flux Models for Complex Engineering Configurations (Darmstadt)
[2] Menter F R 2002 Methoden, Moglichkeiten und Grezen Numerischer Stromungsberechnungen (Erlangen: Numet)
[3] Wachter E M 2005 Anwendung der Instationaren Flamelet Methode auf Diffusions Flammen im Post-Processing-Modus (VDI)
[4] Smagorinsky J S 1963 General circulation experiments with the primitive equations. 1. The basis element Monthly Weather Review 91 99–164
[5] Lvovich Ya E 2006 Multi-Alternative Optimization: Theory and Applications (Voronezh: Kvarta)
[6] Ogawa N and Shimizu A 2016 Collegewide promotion of e-Learning/active learning and faculty development Proc. of the Int. Conf. e-Learning pp 179–84
[7] Igaki H et al. 2013 Programming process visualization for supporting students in programming exercise J. Inf. Process. Soc. Jpn 54 1
[8] Kato T and Ishikawa T 2012 Design and evaluation of support functions of course management systems for assessing learning conditions in programming practicums Proc. of ICALT-2012 pp 205–7
[9] Reschly A L and Christenson S L 2012 Jingle, Jangle, and Conceptual Haziness: Evolution and Future Directions of the Engagement Construct. Handbook of Research on Student Engagement pp 3–19
[10] Odu G O and Charles-Owaba O E 2013 Review of multi-criteria optimization methods - theory and applications IOSR Journal of Engineering 3 1–14