Chiral Mass Splitting for $c \bar{s}$ and $c \bar{n}$ Mesons in the $\tilde{U}(12)$-Classification Scheme of Hadrons

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Abstract. We investigate the chiral mass splitting of parity-doubled $J = 0, 1$ states for $c \bar{s}$ and $c \bar{n}$ meson systems in the $\tilde{U}(12)$-classification scheme of hadrons, using the linear sigma model to describe the light-quark pseudoscalar and scalar mesons together with the spontaneous breaking of chiral symmetry, and consequently predict the masses of as-yet-unobserved $(0^+, 1^+)$ $c \bar{n}$ mesons. We also mention some indications of their existence in the recent published data from the Belle and BABAR Collaborations.

INTRODUCTION

Recently, Ishida et al. have proposed the covariant $\tilde{U}(12)_{SF}$-classification scheme of hadrons [1], which gives covariant quark representations for composite hadrons with definite Lorentz and chiral transformation properties. The $\tilde{U}(12)_{SF}$-classification scheme has a unitary symmetry in the hadron rest frame, called “static $U(12)_{SF}$ symmetry” [2], embedded in the covariant $\tilde{U}(12)_{SF}$-representation space, of which tensors can be decomposed into representations of $U(4)_{DS \times SU(3)_F}$, $U(4)_{DS}$ being the pseudounitary homogeneous Lorentz group for Dirac spinors. The static $U(12)_{SF}$ contains the Dirac spin group $U(4)_{DS}$ in its subgroups and $U(4)_{DS}$ contains two SU(2) subgroups as $U(4)_{DS} \supset SU(2)_p \times SU(2)_\sigma$, where $SU(2)_p$ and $SU(2)_\sigma$ are the spin groups concerning the boosting and intrinsic-spin rotation, respectively, of constituent quarks, being connected with decomposition of Dirac $\gamma$-matrices, $\gamma \equiv \rho \otimes \sigma$. Thus the static $U(12)_{SF}$ symmetry includes the chiral $SU(3)_L \times SU(3)_R$ symmetry as $U(12)_{SF} \supset SU(3)_L \times SU(3)_R \times SU(2)_\sigma$. This implies that the $\tilde{U}(12)_{SF}$-classification scheme is able to incorporate effectively the effects of chiral symmetry and its spontaneous breaking, essential for understanding of properties of the low-lying hadrons, into what is called a constituent quark model.

EXPERIMENTAL CANDIDATES FOR THE GROUND-STATE QUARK-ANTIQUARK MESONS

An essential feature of the $\tilde{U}(12)_{SF}$-classification scheme is to have the static $U(4)_{DS}$ symmetry for light $u, d, s$ quarks confined inside hadrons. The degree of freedom on the $\rho$-spin, being indispensable for covariant description of spin $1/2$ particles, offers a
basis to define the rule of chiral transformation for quark-composite hadrons. Since we have the $\rho$-spin degree of freedom, which is discriminated by the eigenvalues of $\rho_3$, $r = \pm$, in addition to the ordinary $\sigma$-spin, the ground states of light-quark $q\bar{q}$ mesons are composed of eight $SU(3)_F$ multiplets with respective $J^{PC}$ quantum numbers, two pseudoscalars $(0^+_N, 0^-_E)$, two scalars $(0^+_N, 0^+_E)$, two vectors $(1^-_N, 1^-_E)$, and two axial-vectors $(1^+_N, 1^+_E)$ (N and E denoting “normal” and “extra”), where each N (E) even-parity multiplet is the chiral partner of the corresponding N (E) odd-parity multiplet and they form linear representations of the chiral symmetry.

Since the eigenstates only with the $\rho_3$-eigenvalue $r = +$ are taken for heavy quarks, we have for heavy-light meson systems two heavy-spin multiplets, $(0^-, 1^-)$ and $(0^+, 1^+)$, which are the chiral partner each other, while for heavy-heavy meson systems we have the same $(0^-, 1^-)$-spin multiplets as in the conventional nonrelativistic quark model.

The $\tilde{U}(12)_{SF}$-scheme assignments for the observed mesons

We try to assign some of the observed mesons to the predicted $q\bar{q}$ multiplets, resorting to their $J^{PC}$ quantum numbers and masses. The observed meson data are taken from the Particle Data Group 2004 edition [3], except for the following mesons:

- $\rho(1250)$. There are several experimental indications of the existence of the $\rho(1250)$ reported by the OBELIX[4] and LASS[5] Collaborations, and others.\(^1\)
- $\omega(1200)$. The existence of $\omega(1200)$ is claimed in the analysis of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section by the SND Collaboration[6].

We accept the existence of these vector mesons as true[7]. The resulting assignments, though some of them are ambiguous, are shown in Table 1. Here we make some comments on these assignments.

1. The light scalar mesons $\{a_0(980), \sigma, f_0(980), \kappa\}$ are assined to the $(0^+_-)$-nonet as a chiral partner of the $\pi$-meson $(0^+_N)$-nonet.
2. The low-mass vector mesons $\{\rho(1250), \omega(1200), K^+(1410)\}$ are assined to the $(1^-_E)$-nonet as a chiral partner of the $(1^+_E)$-nonet $\{b_1(1235), h_1(1170), h_1(1380), K_1(1400)\}$.
3. The axial-vector mesons $\{a_1(1260), f_1(1285), f_1(1420), K_1(1270)\}$ are assined to the $(1^+_N)$-nonet as a chiral partner of the $\rho(770)$-meson $(1^-_N)$-nonet.
4. The recent observed mesons $\{D^*_s(2317), D_{sJ}(2460)\}$ are assined to the $(0^+, 1^+)$ multiplet as a chiral partner of the $(0^-, 1^-)$ multiplet $\{D_s, D^*_s\}$ [8]. These newly observed mesons, together with the $\sigma$-meson nonet, are the best candidates for the hadronic states with $r = -$ whose existence is expected in the $\tilde{U}(12)_{SF}$ scheme.
5. It is noted that the normal (N) and extra (E) states with the same $J^{PC}$ generally mix together due to the spontaneous as well as explicit breaking of chiral symmetry and some other mechanism.

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\(^1\) See the $\rho(1450)$ Particle Listings and the “Note on the $\rho(1700)$” in [3].
TABLE 1. Experimental candidates for ground-state mesons in the \( \bar{U}(12)_{SF} \)-classification scheme.

| \( q\bar{q} \) | \( P \) | \( S_a \) | \( P \) | \( \bar{S}_a \) | \( V \) | \( A \) | \( \bar{V} \) | \( B \) |
|---|---|---|---|---|---|---|---|---|
| \( \pi \) | \( \eta \) | \( \eta(980) \) | \( K \) | \( K^\ast \) | \( \rho(770) \) | \( a_1(1260) \) | \( \rho(1250) \) | \( b_1(1235) \) |
| \( \eta \) | \( \sigma \) | \( \eta(1295) \) | \( 1^+ \) | \( 1^+ \) | \( \alpha(1225) \) | \( f_0(1270) \) | \( \alpha(1200) \) | \( h_0(1170) \) |
| \( \eta(980) \) | \( f_0(980) \) | \( \eta(1475) \) | \( 1^+ \) | \( 1^+ \) | \( \phi(1020) \) | \( f_0(1420) \) | \( \phi(1380) \) |

CHIRAL MASS SPLITTING FOR THE CHARMED AND CHARMED-STRANGE MESON SYSTEMS

In the \( \bar{U}(12)_{SF} \)-classification scheme heavy-light \((c\bar{q})\) meson fields, aside from the internal space-time wave functions, are given by

\[
\Phi(v) = \frac{1}{2\sqrt{2}}(1 - iv \cdot \gamma)(i\gamma_5 D + i\tilde{\gamma}_\mu D^\ast_\mu + D_0 + i\gamma_5 \tilde{\gamma}_\mu D_{1\mu})
\]

(1)

with \( v_\mu \equiv P_\mu/M \), \( \tilde{\gamma}_\mu \equiv \gamma_\mu + v_\mu(v \cdot \gamma) \), where \((D, D^\ast_\mu, D_0, D_{1\mu})\) represent the local fields for the \( c\bar{q} \) mesons with \( J^P = (0^-, 1^-, 0^+, 1^+) \), \( P_\mu \) \( (M) \) is the four-momentum \( (mass) \) of meson fields, and flavor indices are omitted for simplicity. To describe the light-quark pseudoscalar and scalar mesons together with the spontaneous breaking of chiral symmetry, we adopt the \( SU(3) \) linear sigma model, introducing the chiral field \( \Sigma_5 \) defined by

\[
\Sigma_5 = s - i\gamma_5 \phi
\]

(2)

with

\[
s = \frac{1}{\sqrt{2}}s^a \lambda^a, \quad \phi = \frac{1}{\sqrt{2}}\phi^a \lambda^a \quad (a = 0, \ldots, 8),
\]

where \( \lambda^0 = \sqrt{2/3} \mathbf{1} \) and \( s^a (\phi^a) \) are the scalar (pseudoscalar) fields. We now write a chiral-symmetric effective Lagrangian which gives the chiral mass splitting between the heavy-light \((0^-, 1^-)\) and \((0^+, 1^+)\) multiplets through the spontaneous breaking of chiral symmetry\( [11, 8] \):

\[
\mathcal{L}_{ND} = -g_{ND} Tr[\Phi \Sigma_5 \Phi^\dagger],
\]

(3)
where $g_{ND}$ is the dimensionless coupling constant of Yukawa interaction in the non-derivative form and the trace is taken over the spinor and flavor indices.

When the chiral symmetry is spontaneously broken, $s$ has the vacuum expectation value, $\langle s \rangle_0 = \text{diag}(a, a, b)$, where $a$ and $b$ are related to the pion and kaon decay constants by

\[
a = \frac{1}{\sqrt{2}} f_\pi, \quad b = \frac{1}{\sqrt{2}} (2f_K - f_\pi). \tag{4}
\]

Then the mass splitting between the two multiplets is induced and the mass differences $\Delta M_\chi(c\bar{n})$ are given by $\Delta M_\chi(c\bar{n}) = 2g_{ND}a$ and $\Delta M_\chi(c\bar{s}) = 2g_{ND}b$, which leads to the relation

\[
\Delta M_\chi(c\bar{n}) = \Delta M_\chi(c\bar{s}) \frac{a}{b} = \Delta M_\chi(c\bar{s}) \left( \frac{2f_K}{f_\pi} - 1 \right)^{-1}. \tag{5}
\]

From this relation with the experimental values\textsuperscript{[3]}, $\Delta M_\chi(c\bar{s}) = 348.0 \pm 0.8$ MeV and $f_K/f_\pi = 1.223 \pm 0.015$, we obtain $\Delta M_\chi(c\bar{n}) = 240.8 \pm 5.4$ MeV and consequently predict the masses

\[
M(D_0^*) = 2.11 \pm 0.01 \text{ GeV}, \quad M(D_1) = 2.25 \pm 0.01 \text{ GeV} \tag{6}
\]

for the $(0^+, 1^+)$ $c\bar{n}$ mesons, using the measured mass values\textsuperscript{[3]} of the $D(0^-)$ and $D^*(1^-)$ mesons. We hereafter refer to these predicted mesons, respectively, as “$D_0^*(2110)$” and “$D_1(2250)$”.

**POSSIBLE INDICATIONS OF THE EXISTENCE OF LIGHT SCALAR AND AXIAL-VECTOR CHARMED MESONS**

We could ask experimental data whether there was some evidence for the existence of $D_0^*(2110)$ and $D_1(2250)$. Here we check on the recent published data on the $D\pi$ and $D^*\pi$ mass distributions in $B \rightarrow (D\pi)\pi$, $(D^*\pi)\pi$ decays from the Belle\textsuperscript{[9]} and BABAR\textsuperscript{[10]} Collaborations.

- **$D\pi$ mass spectrum**: In the Belle data\textsuperscript{[2]} we see an excess of events, a single data point of 20 MeV bin, at a mass of 2.13 GeV near the predicted mass of the $D_0^*(2110)$, and so might regard it as an indication of that resonance, though it is natural to think that its data point should be within a statistical error. On the other hand, it would seem to us that the BABAR data\textsuperscript{[3]} around a mass of 2.1 GeV might show a typical pattern of interference between two or more resonances.

- **$D^*\pi$ mass spectrum**: In the Belle data\textsuperscript{[4]} there is also an excess of events, a single data point of 10 MeV bin, at a mass of 2.255 GeV near the predicted mass of the $D_1(2250)$, and so it might be an indication of the resonance. Although it is not clear, the BABAR data\textsuperscript{[5]} around a mass of 2.26 GeV might show a typical pattern.

\textsuperscript{2} See the $D\pi$ mass distribution in Figure 3 of [9].
\textsuperscript{3} See the $D\pi$ mass distribution in Figure 3 (right) of [10].
\textsuperscript{4} See the $D^*\pi$ mass distribution in Figure 9 of [9].
\textsuperscript{5} See the $D^*\pi$ mass distribution in Figure 3 (left) of [10].
of interference.

If the $D_0^*(2110)$ and $D_1^*(2250)$ resonances really exist, their widths have to be narrow, $\leq 20$-30 MeV, judging from the data mentioned above. The dominant decay modes of these resonances are $D\pi$ and $D^*\pi$, respectively, and thus we examine their single pion transitions. To estimate the widths of $D_0^* \rightarrow D + \pi$ and $D_1 \rightarrow D^* + \pi$ decays, together with $D^* \rightarrow D + \pi$, we set up, in addition to the nonderivative interaction $\mathcal{L}_{ND}$ in Eq. (3), the chiral-invariant effective interaction with the derivative form:

$$\mathcal{L}_D = g_D Tr[\Phi(\partial_\mu \Sigma_5)\gamma_\mu (F_U \bar{\Phi})],$$

(7)

where $F_U = \gamma \cdot \partial / \sqrt{\partial \cdot \partial}$ and $g_D$ is the coupling constant with a dimension of $(mass)^{-1}$, which is related to the axial coupling constant $g_A$ by $g_D = g_A / 2a = g_A / \sqrt{2}f_\pi$. The pionic decay widths of the $D^*$, $D_0^*$, and $D_1$ states are derived from $\mathcal{L}_{ND}$ and $\mathcal{L}_D$, and the decay widths of $D_0^*$ and $D_1$ are identical. Using the measured value of $\Gamma[D^+ \rightarrow D^0 + \pi^+] = 65$ keV[3] and $g_{ND} = 1.84$ from $\Delta M_\pi(c\bar{n}) = 241$ MeV, the coupling $g_D$ is fixed to 3.96 GeV$^{-1}$ (corresponding to $g_A = 0.521$), and then we obtain

$$\Gamma[D_0^*(2110) \rightarrow D + \pi] = \Gamma[D_1(2250) \rightarrow D^* + \pi] \approx 30 \text{ MeV}.$$  

(8)

This value is consistent with the speculated widths of the $D_0^*(2110)$ and $D_1(2250)$.

**CONCLUDING REMARKS**

We have presented the possible assignments for some of the observed mesons in the covariant $\tilde{U}(12)_{SF}$-classification scheme. It is necessary and important to examine the strong- and radiative-decay[12] properties of the assigned states in order to establish their assignments. On the basis of these assignments we have also predicted the existence of the low-mass $(0^+, 1^+) c\bar{n}$ mesons with narrow width, which might have been seen in the recent published data on the $D\pi$ and $D^*\pi$ mass distributions from the Belle and BABAR Collaborations.

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