Polygonal Maxwell’s fisheye lens via transformation optics as multimode waveguide crossing

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Abstract
Multimode waveguide crossings are crucial components for novel mode-division-multiplexing systems. One of the challenges of multimode waveguide routing in MDM systems is decreasing the inter-mode crosstalk and mode leakage of waveguide crossings. In this work, we present the intersections of three and four waveguides based on polygonal Maxwell’s fisheye lens via transformation optics. The designed lenses are implemented by mapping their refractive index to the thickness of guiding Si layer. The three-dimensional finite-difference time-domain simulations are used to evaluate the performance of the proposed 3 × 3 and 4 × 4 crossings. The footprint of the 3 × 3 and 4 × 4 waveguide star crossings are 18.6 × 18.6 and 27.5 × 27.5 μm², respectively. For both waveguide crossings, the intermodal crosstalk in the output port is lower than −22 dB while the crosstalk to other ports is lower than −37 dB for TE0, TE1, and TE2 modes. The insertion losses for these modes are lower than 0.5 dB in a bandwidth of 415 nm covering the whole optical telecommunication bands.

Keywords: transformation optics, waveguide intersection, all-dielectric metamaterials, Maxwell’s fish-eye lens

(Some figures may appear in colour only in the online journal)

1. Introduction

The ultimate goal of the nanophotonics is to squeeze a large number of optical components onto a single chip. Hence, crossing of waveguides connecting these components is inevitable. Various methods for designing a broadband waveguide crossing with low insertion loss and low crosstalk levels have been proposed. Most notable photonic crystal waveguide crossings include designs based on resonant cavity [1–3], coupled-cavity waveguide [4], utilizing the symmetric properties of the propagation modes of square-lattice [5], nonidentical coupled resonator waveguides [6], cascading cavities [7, 8], topology optimization [9], Wannier basis design and optimization [10], and self-collimation phenomenon [11]. The intersections based on resonant cavities have inherently narrow bandwidth with crosstalk levels below −30 dB. To increase the bandwidth, the Q-factor is decreased, resulting in weaker mode-matching between the waveguides and resonant cavities and consequently lower transmission. There has been no report of multimode waveguide crossing based on the above methods [1–11] and they only support a single propagating mode. Silicon-on-insulator (SOI) waveguide crossings can be designed based on multimode interference (MMI) [12–18], mode expanders [19, 20], subwavelength grating [21, 22], and wavefront matching [23, 24]. The designs based on MMI typically have bandwidth of 60–100 nm, crosstalk lower than −18 dB, and with footprints larger than 4.8 μm × 4.8 μm. Expanding the MMI designs to support higher order modes is challenging due to the different self-imaging distances for each mode. On the other hand, designs based on mode expander have narrower bandwidth of 20–25 nm, desirable crosstalk levels, i.e. lower...
crossings are theoretically designed and numerically eval-

uated to prove that the aberration-free imaging properties of the MFE

forming the circular MFE lens into the hexagonal and

octagonal MFE lenses. The MFE lens is a gradient index

(GrIN) medium with a refractive index defined as [31]

\[ n_{\text{Lenses}}(r) = \frac{2 \times n_{\text{min}}}{1 + (r/R_{\text{Lens}})^2}, \quad (0 \leq r \leq R_{\text{Lens}}), \] (1)

where \( n_{\text{min}} \) is the minimum refractive index of the lens at its

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eq 

edge, \( r \) is the radial distance from the center of the lens and

\( R_{\text{Lens}} \) is the radius of the lens.

2. Design of polygonal Maxwell’s fisheye lens

TO gives us a tool to transform any optical device with the

given geometry into infinite number of new geometries with

the same optical response. However, transforming a given

coordinate space, virtual domain, into a new arbitrary one,

physical domain, introduces some limitations such that the

required permittivity and permeability may become aniso-

tropic with extremely high or low values [36]. The resonant

metamaterials used in implementation of these extreme

values severely limit the bandwidth of the optical device.

Provided that the transformed device includes only limited

anisotropy and sub-unity refractive index regions, some

simplifications have been proposed [37]. Nevertheless,

designed material complexities can be minimized by

quasi-conformal transformation optics (QCTO). In QCTO

technique, angles between the coordinate lines in virtual and

physical domains are maintained [38, 39]. Inherently two-

dimensional QCTO is applied to design planar photonic

components.

In this work, the circular MFE lens in the virtual domain

is transformed to a polygon in the physical domain [40].

The first step in QCTO is to generate an orthogonal grid, i.e., grid

lines are orthogonal to each other, in the virtual and physical

domains [38]. The orthogonal grid is generated by solving the

Laplace equation. Boundary orthogonality is achieved by

applying Dirichlet–Neumann boundary conditions. Knowing

that inverse of a conformal mapping is conformal, two

domains with the same conformal module, \( M \), can be mapped

onto each other conformally by mapping them onto an

intermediate domain. The intermediate domain is a rectangle

with the same conformal module, \( M \) [41]. Conformal module
is the ratio of the lengths of the two adjacent sides of a domain. Since only the refractive index and wavefront mismatches at the edge of the MFE lens with the waveguides are important, as shown in figure 1, we excluded the inner center of the MFE lens with \( r < 1 \mu m \) from the transformation. For the \( 3 \times 3 \) waveguide crossing, three waveguides intersect the MFE lens in six sides so the lens is divided into six equal parts. The quadrilateral virtual domain formed by this manner is displayed in figure 1(a). The quadrilateral virtual domain is transformed to the quadrilateral physical domain of figure 1(b). The dashed lines specify the sides of the quadrilateral of virtual and physical domains used in QCTO. The generated orthogonal grid and the refractive indices of virtual and physical domains are also shown in this figure. The rectangular intermediate domain is not shown in this figure. Through our transformation, the circular edge is flattened and the refractive index of the lens at its edge is increased in the physical domain. Our simulations reveal that the light wave does not pass from the corners of the hexagonal lens. Therefore, we truncated the corners of the hexagonal MFE lens, to simplify the implementation of the lens, as displayed in figure 1(c). The curves used in the truncation of the lens correspond to the refractive index contour level of 1.7. The material properties of the virtual domain are

\[
\mu' = 1, \quad \varepsilon' = n_{\varepsilon}^2(r') = \begin{cases} n_{\text{trans}}^2(r') & r' \leq R_{\text{trans}} \\ n_{\text{lin}}^2 & r' > R_{\text{trans}} \end{cases},
\]

(2)

where \( \varepsilon' \) and \( \mu' \) are permittivity and permeability of the virtual domain. We have chosen \( n_{\text{lin}} = 1.45 \) and the diameter of the lens is 10 \( \mu m \). The transformation from the virtual domain \((x', y', z')\) to physical domain \((x, y, z)\) is described with

\[
x = x(x', y'), \quad y = y(x', y'), \quad z = z',
\]

(3)

which is mapped to the material properties by

\[
\varepsilon = \frac{A \varepsilon' A^T}{|A|}, \quad \mu = \frac{A \mu' A^T}{|A|},
\]

(4)

where \( \varepsilon \) and \( \mu \) are permittivity and permeability of the physical domain, respectively. \( A \) is the Jacobian transformation matrix between the virtual and physical domains:

\[
A = \begin{bmatrix}
\frac{\partial x}{\partial x'} & \frac{\partial x}{\partial y'} & 0 \\
\frac{\partial y}{\partial x'} & \frac{\partial y}{\partial y'} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(5)

when Cauchy–Riemann conditions are satisfied, i.e.

\[
\frac{\partial x}{\partial x'} = \frac{\partial y}{\partial y'}, \quad \frac{\partial y}{\partial x'} = -\frac{\partial x}{\partial y'}
\]

(6)

then the material properties in transverse electric (TE) mode, where the electric field is parallel to the \( z \)-axis, can be calculated by

\[
\mu = 1, \quad \varepsilon = \frac{n_{\text{trans}}^2(r)}{|\det A|}.
\]

(7)

The method proposed to design a \( 3 \times 3 \) waveguide crossing can be expanded to design a \( 4 \times 4 \) waveguide crossing. To design a \( 4 \times 4 \) waveguide crossing, we need to transform a circular MFE lens into an octagonal one. Similar to figure 1(a), we divide a circle with a radius of 15 \( \mu m \) into eight equal parts. Afterwards, a quadrilateral with curved side created with this method is transformed to a quadrilateral with flat side. The transformed quadrilateral is rotated with \( N \times 45^\circ \) where \( N = 1, 2, 3, \ldots, 7 \). Eventually, the transformed octagonal MFE lens is obtained (figure 2(a)). Due to the negligible effect of the lens’s corners on its performance, the lens is truncated as illustrated in figure 2(b). The curves used in the truncation of the octagonal lens correspond to the refractive index contour level of 1.65.

3. Implementation of the square MFE lens

We have implemented the designed hexagonal and octagonal MFE lenses on SOI platform with varying the thickness of Si slab waveguide. GRIN lenses can also be implemented by graded photonic crystal (GPC) or multilayer structures [42, 43]. The refractive index profile of the MFE lens can be implemented by varying the height of a silicon slab.

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**Figure 2.** (a) The octagonal MFE lens, (b) truncated octagonal MFE lens used as \( 4 \times 4 \) waveguide crossing medium.
waveguide on top of a SiO\textsubscript{2} substrate. Air is considered as the top cladding material \cite{32}. The effective index method was applied to map the refractive index to the thickness of the Si layer. The silicon slab shown in figure 3 with silicon dioxide substrate and air cladding was considered in the effective refractive index calculations. For the TE mode, where light propagates in the z direction, the electric field is \cite{44}

\[
E_y(x) = \begin{cases} 
Ce^{-qy} & x \geq 0 \\
C \left[ \coshx - \frac{q}{h} \sinhx \right] & 0 \geq x \geq -t \\
C \left[ \cosh t + \frac{q}{h} \sinh t \right] e^{p(x+t)} & x \leq -t
\end{cases}
\] (10)

and

\[
q = \sqrt{\beta^2 - k_0^2 n_{Air}^2}, \quad p = \sqrt{\beta^2 - k_0^2 n_{SiO2}^2},
\]

\[
h = \sqrt{k_0^2 n_{Si}^2 - \beta^2}, \quad (11)
\]

where \(k_0 = 2\pi/\lambda_0\) is the free-space wavenumber, \(\beta = k_0 n_{eff}\) is the propagation constant, and C is a constant. The eigenvalue equation for the TE modes of the slab waveguide is

\[
\tan(ht) = \frac{p + q}{h \left( 1 - \frac{pq}{h^2} \right)}, \quad (12)
\]

where the only unknown quantity is \(\beta\). The discrete values of \(\beta\) satisfying equation (12) are the modes of the slab waveguide.

\[\text{Figure 3.} \text{ Silicon slab waveguide used for effective index calculation and mapping of silicon thickness to effective refractive index of slab waveguide are demonstrated.}\]

\[\text{Figure 4.} \text{ The implementation of (a) hexagonal and (b) octagonal MFE lenses based on varying the guiding layer thickness. The SiO}_2 \text{ and air claddings are not shown in the 3D implementation.}\]

\[\text{Figure 5.} \text{ The } 3 \times 3 \text{ waveguide intersection based on the truncated hexagonal MFE lens. The power-streams illustrate the energy flow of the TE}_1 \text{ mode. The silica substrate and air cladding are not shown in this figure.}\]
The calculated values of $\beta$ are used to determine the $n_{\text{eff}}$ of that mode. The calculated $n_{\text{eff}}$ of TE$_0$ mode is shown in figure 3. In the effective refractive index calculations, the thickness of 3 $\mu$m was considered for silica substrate and air cladding.

The truncated hexagonal and octagonal MFE lenses implemented with this method are shown in figure 4. The underlying SiO$_2$ layer and air cladding are not shown in this figure. The corners of the lenses were not implemented since the refractive index of the transformed lenses are lower than unity and their contribution to the performance of the crossing were negligible.

4. Results and discussion

The three-dimensional finite-difference time-domain numerical simulations were carried out to evaluate the performance of the lenses implemented with varying the thickness of guiding layer as waveguide crossing. Figure 5 depicts the three crossing waveguides, the lens, and the power-streams of the TE$_1$ mode. In this figure, the underlying silica substrate and air upper cladding are not shown. The effective refractive index of the waveguides is considered as 2.2 and hence the thickness of the waveguides was chosen as 110 nm. The width of the waveguides was chosen as 3 $\mu$m to support at least three modes. It should be noted that the refractive index of the lens at the middle of its interface with the waveguides is 2.22 but it slightly decreases to 2.07 at the edges of waveguides. This translates into thickness variation in the interface.
of the waveguides and the lens, which is obvious in figure 5. However, our simulation results indicate that the slight refractive index variation of the lens at its interface with the waveguides has negligible effect on the performance of the crossing. Noticeable step-like changes in the thickness of the guiding layer would increase reflection. The gradual thickness variation of the guiding layer in our implementation ensures low reflection of the waveguide crossing. A constant refractive index in the interface of the lens and waveguides can be achieved by increasing the size of the lens. By easing this constraint, we have been able to reduce the footprint without degrading the performance of the crossing. For the $3 \times 3$ crossing, the average insertion losses are 0.14, 0.27, and 0.38 dB for the TE$_0$, TE$_1$, and TE$_2$ modes, respectively. Crosstalk levels at the ports of in2, out2, in3, out3 are below $-53$, $-46$, and $-43$ dB for the TE$_0$, TE$_1$, and TE$_2$ modes, respectively. In addition, the intermodal crosstalk at the output port is below $-23$ dB for these modes. The designed waveguide crossing has an ultra-wide bandwidth covering 1260–1675 nm.

For the $4 \times 4$ waveguide crossing, the thickness of Si layer in the waveguides was 80 nm with a width of 3 µm. The reason for this choice is that the thickness (or refractive index) of the octagonal lens at its edges is smaller than the hexagonal lens. This is apparent in figure 4. The $H_e$ field distribution of the TE$_0$, TE$_1$, and TE$_2$ modes for a light of 1550 nm wavelength are displayed in figure 6. The light signal is injected from in1 port. The reflection, transmission, and crosstalk at 1550 nm are also shown in this figure. The scattering parameters of the TE$_0$, TE$_1$, and TE$_2$ modes are shown in figure 7. The average insertion loss in the bandwidth of 1250–1675 nm is 0.16, 0.21, and 0.33 for the TE$_0$, TE$_1$, and TE$_2$ modes, respectively. The intermodal crosstalk is lower than $-22$ dB in the out1 port while crosstalk levels at other ports are lower than $-37$ dB.

4.1. Comparison with previous works

The characteristics of the designed multimode intersections and [17, 18, 28, 30, 32] are summarized in table 1. The crossing mechanism, insertion loss, central wavelength, bandwidth, crosstalk, footprint, number of supported modes, and number of crossing waveguides are compared in this table. Since the insertion loss usually increases as the order of the modes increases, the insertion loss of the highest-order mode supported by the crossing is reported in the table. The works [17, 18, 28] only reported $2 \times 2$ crossings so we focus on comparing our design with [30, 32]. First of all, we should acknowledge that [30, 32] reported experimental measurements while our results are based on numerical simulations. The insertion loss for this work, as well as [30, 32], are 0.33, 0.3, and 2.68 dB, respectively. On the other hand, the simulation results of [32] predicted the maximum insertion loss of 1.14 dB while our design has the maximum insertion loss of 0.5 dB. Due to the limitations in the measurement setup, [30, 32] reported limited bandwidth but the simulation results of the [32] indicate that it has a bandwidth of 394 nm. We report a 415 nm bandwidth which is in the same range as [32]. However, [30] which is implemented by GPC, only reports the bandwidth of 1500–1600 nm. Its performance may be degraded in lower wavelengths due to the step-wise profile of the GPC structure. The implementation of our work and [32] are based on varying the thickness of slab waveguide which imposes no bandwidth limitation to the device. In the $4 \times 4$ crossings, [30] has the smallest footprint, however, this was achieved by truncating the MFE lens. This may degrade the imaging properties of the lens which consequently increases the intermodal crosstalk. Xu et al [30] did not report the intermodal crosstalk. We took a simpler approach in the transformation of the MFE lens compared to [32] and we also relaxed the constraint of constant refractive index at the sides of the designed lens without introducing significant loss. By this method, we were able to reduce the $4 \times 4$ crossing’s footprint by more than 50% compared to [32]. We have also incorporated the fabrication imperfections in our simulations by introducing random deviations to the designed lens. Our simulations show that the introduction of 15% random Si thickness deviations from the designed thickness has negligible effect on the insertion loss of the device.

5. Conclusion

MDM can increase the bandwidth density in on-chip optical interconnects. Multimode waveguide crossing is one of the important building blocks in MDM systems. We designed the
3 × 3 and 4 × 4 multimode waveguide crossings with 18.6 × 18.6 and 27.5 × 27.5 μm² footprints, respectively. For the 4 × 4 crossing, the average insertion losses of 0.16, 0.20, and 0.33 dB for the TE₀, TE₁, and TE₂ modes are achieved, respectively. The intermodal crosstalk is below −22 dB at the output port while the crosstalk levels at other ports are lower than −43 dB. The proposed waveguide crossing covers the entire O, E, S, C, L, and U bands of optical communication.

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