Potential of the neutron Lloyd’s mirror interferometer for the search for new interactions

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Abstract

We discuss the potential of the neutron Lloyd’s mirror interferometer in a search for new interactions at small scales. We consider three hypothetical interactions that may be tested using the interferometer. The chameleon scalar field proposed to solve the enigma of accelerating expansion of the Universe produces interaction between particles and matter. The axion-like spin-dependent coupling between neutron and nuclei or/and electrons may cause P- and T-non-invariant interaction with matter. Hypothetical non-Newtonian gravitational interactions mediates additional short-range potential between neutrons and bulk matter. These interactions between the neutron and the mirror of the Lloyd’s type neutron interferometer cause phase shift of neutron waves. We estimate the sensitivity and systematic effects of possible experiments.

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1. Introduction

It is believed that the Standard model is a low energy approximation of some more fundamental theory. Most popular extensions of the Standard Model: supersymmetry and superstring theories, predict the existence of new particles and hence new interactions. These new particles were not detected up to now because of their too large mass, or because of too weak interaction with ordinary matter. This last case is of interest in our discussion of a search for new hypothetical weak interactions of different nature.

The possible existence of new interactions with macroscopic ranges and weak coupling to matter currently attracts increasing attention. Significant number of experiments has been performed to search for new forces in a wide range of distance scales. Here we consider possibilities of the neutron Lloyd’s mirror interferometer in searching for some of these new interactions.

The Lloyd’s mirror interferometer (see Fig. 1) well known in the light optics has not yet been discussed in the experimental neutron optics.

The geometric phase shift is determined by the difference of path lengths of the reflected and non-reflected beams:

$$\varphi_{\text{geom}} = \varphi_{\text{II,geom}} - \varphi_{\text{I,geom}} = k \left( \sqrt{L^2 + (b + a)^2} - \sqrt{L^2 + (b - a)^2} \right) \approx \frac{2kab}{L}, \quad (1)$$

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where $k$ is the neutron wave vector, $L$, $b$ and $a$ are given in the Fig. 1 caption. The last equation is valid with relative precision better than $ab/L^2$. The geometric phase shift linearly depends on the interference coordinate $b$. It means that the interference pattern $I \sim \sin^2(\pi ab/\lambda_n L)$ in the absence of any potentials is sinusoidal with high precision: $ab/L^2 \sim 10^{-8}$ at $a \sim b \sim 10^{-2} \text{cm}$ and $L = 1 \text{m}$. The period of oscillations in the interference pattern is $\Lambda_{osc} = \lambda_n L/(2a)$, where $\lambda_n$ is the neutron wavelength, and is $\sim 1 \mu\text{m}$ for the thermal neutrons energy range and reasonable parameters of the interferometer. But for very cold neutrons in the $\mu\text{eV}$-energy range the period of the interference oscillations approaches dozens of $\mu\text{m}$, and an interference picture may be registered with a narrow (about $\sim 1 \mu\text{m}$) slit at a detector window or with modern high resolution position sensitive neutron detectors.

The idea of possible application of the Lloyd’s mirror interferometer for the search for new hypothetical interaction between matter and particles consists in measuring the neutron wave phase shift produced by a hypothetical mirror-neutron potential. We here consider three actively discussed hypothetical interactions: the cosmological scalar fields proposed to explain the accelerated expansion of the Universe, the axion-like spin-dependent pseudoscalar nucleon-nucleon and/or nucleon-electron interaction, and hypothetical deviation of the gravitation law from the Newtonian one.

### 2. Chameleon scalar field

There is evidence of the accelerated expansion of the Universe. The nature of this effect is one of the most exciting problems in physics and cosmology. It is not clear yet whether the explanation of the fact that gravity becomes repulsive at large distances should be found within General Relativity or within a new theory of gravitation. One possibility to explain this fact is to modify the General Relativity Theory, and there was a number of proposals of this kind. Among various ideas proposed to explain this astronomical observation in a different way, one of popular variants is a new matter component of the Universe – a cosmological scalar field of the quintessence type [1] dominating the present day density of the Universe (for the recent reviews see [2, 3]).

Acting on cosmological distances the mass of this field should be of the order of the Hubble constant: $\bar{h}H_0/c^2 = 10^{-33} \text{eV}/c^2$.

The massless scalar fields appearing in string and supergravity theories couple to matter with gravitational strength. Because of direct coupling to matter with a strength of gravity, the existence of light scalar fields leads to a violation of the equivalence principle. In the absence of self-interaction of the scalar field, the experimental constraints on such a field are very strict, requiring its coupling to matter to be unnaturally small.

The solution proposed in [4, 5, 6, 7, 8, 9] consists in the introduction of the coupling of the scalar field with matter of such a form that as a result of self-interaction and the interaction of the scalar field with matter, the mass of the scalar field depends on the local matter environment.

In the proposed variant, the coupling to matter is of the order as expected from string theory, but is very small on cosmological scales. In the environment of the high matter density, the mass of the field increases, the interaction range strongly decreases, and the equivalence principle is not violated in laboratory experiments for the search for the long-range fifth force. The field is confined inside the matter screening its existence to the external world. In this way the chameleon fields evade tests of the equivalence principle and the fifth force experiments even if these fields are strongly coupled to matter. As a result of the screening effect the laboratory gravitational experiments are unable to set an upper limit on the strength of the chameleon-matter coupling.

The deviations of results of measurements of gravity forces at macroscopic distances from calculations based on Newtonian physics can be seen in the experiments of Galileo-, Eötvös-or Cavendish-type [10] performed with macro-bodies. At smaller distances ($10^{-7} - 10^{-2}$) cm the effect of these forces can be
observed in measurements of the Casimir force between closely placed macro-bodies (for review see [11]) or in the atomic force microscopy experiments. Casimir force measurements may to some degree evade the screening and probe the interactions of the chameleon field at the micrometer range despite the presence of the screening effect [9, 12, 13].

At even smaller distances such experiments are not sensitive enough, and high precision particle scattering experiments may play their role. In view of absence of electric charge the experiments with neutrons are more sensitive than with charged particles, electromagnetic effects in scattering of neutrons by nuclei are generally known and can be accounted for with high precision [14, 15].

As regards the chameleon interaction of elementary particles with bulk matter, it was mentioned in [16] that neutron should not show a screening effect - the chameleon-induced interaction potential of bulk matter with neutron can be observed. It was also proposed in [16] to search for the chameleon field through energy shift of ultracold neutrons in the vicinity of reflecting horizontal mirror. From the already performed experiments on the observation of gravitational levels of neutrons, the constraints on parameters, characterizing the force of chameleon-matter interaction were obtained in [16].

Chameleons can also couple to photons. It was proposed in [17, 18] to search for in a closed vacuum cavity for the afterglow effect resulting from the chameleon-photon interaction in a magnetic field. The GammeV-CHASE [19, 20] and ADMX [21] experiments based on this approach are intended to measure (constrain) the coupling of chameleon scalar field to matter and photons.

In the approach proposed here only the chameleon-matter interaction is measured irrespective of the existence of the chameleon-photon interaction. The approach is based on the standard method of measurement the phase shift of a neutron wave in the interaction potential.

Testing the interaction of particles with matter at small distances may be interesting irrespective of any particular variant of the theory.

In one of popular variants of the chameleon scalar field theory [4, 5, 6, 7, 8, 9], the chameleon effective potential is

\[ V_{eff}(\phi) = V(\phi) + e^{\beta \phi / M_{Pl}} \rho, \]  

where

\[ V(\phi) = \Lambda^4 + \frac{\Lambda^{4+n}}{\phi^n}. \]  

is the scalar field potential, \( M_{Pl} \) is the Planck mass, \( \rho \) is the local energy density of the environment, \( \Lambda = (\hbar^2 c^3 \rho_{d.e.})^{1/4} = 2.4 \text{ meV} \) is the dark energy scale, \( \rho_{d.e.} \approx 0.7 \times 10^{-8} \text{ erg/cm}^3 \) is the dark energy density, and \( \beta \) is the interaction parameter not predicted by the theory.

The chameleon interaction potential of a neutron with bulk matter (mirror) was calculated in [16]:

\[ V(z) = \frac{\beta}{M_{Pl} \lambda} \left( \frac{2 + n}{\sqrt{2}} \right)^{2/(2+n)} \left( \frac{z}{\lambda} \right)^{2/(2+n)} = \beta \cdot 0.9 \cdot 10^{-21} eV \left( \frac{2 + n}{\sqrt{2}} \right)^{2/(2+n)} \left( \frac{z}{\lambda} \right)^{2/(2+n)} = V_0 \left( \frac{z}{\lambda} \right)^{2/(2+n)}, \]  

\[ V_0 = \beta \cdot 0.9 \cdot 10^{-21} eV \left( \frac{2 + n}{\sqrt{2}} \right)^{2/(2+n)}, \]  

where \( m \) is the neutron mass and \( \lambda = \hbar c / \Lambda = 82 \mu m \).

To reduce the strong effect of Earth’s gravity the mirror of the interferometer is vertical.

The neutron wave vector \( k' \) in the potential \( V \) is

\[ k'^2 = k^2 - \frac{2mV}{\hbar^2}, \quad k' = k - \frac{mV}{k \hbar^2}. \]
The phase shift due to the chameleon-mediated interaction potential of a neutron with the mirror, depending on the distance from the mirror, is obtained by integration along trajectories \( \varphi = \oint k’ds = \varphi_{II} - \varphi_I \), where \( \varphi_I \) and \( \varphi_{II} \) are the phases obtained along trajectories I and II:

\[
\varphi_I = k \sqrt{L^2 + (b-a)^2} - \frac{mV_0}{k^2h^2} \frac{\sqrt{1 + ((b-a)/L)^2}}{\lambda_{n-1}a_{n-1}} \int_0^L \left( a + \frac{b-a}{L}x \right)^{\alpha_n-1} dx = \\
= \varphi_{I, \text{geom}} - \frac{\gamma \sqrt{1 + ((b-a)/L)^2}}{\lambda_{n-1}a_{n-1}(b-a)} \left( b^{\alpha_n} - a^{\alpha_n} \right)
\]

and

\[
\varphi_{II} = k \sqrt{L^2 + (b+a)^2} - \frac{mV_0}{k^2h^2} \frac{\sqrt{1 + ((b+a)/L)^2}}{\lambda_{n-1}a_{n-1}} \left[ \int_0^l \left( a - \frac{b+a}{L}x \right)^{\alpha_n-1} dx + \int_l^L \left( \frac{b+a}{L}x - a \right)^{\alpha_n-1} dx \right] = \\
= \varphi_{II, \text{geom}} + \frac{\gamma \sqrt{1 + ((b+a)/L)^2}}{\lambda_{n-1}a_{n-1}(b+a)} \left( b^{\alpha_n} + a^{\alpha_n} \right).
\]

Here \( l = (aL)/(a+b) \) is the x-coordinate of the beam II reflection point from the mirror, \( \gamma = (mV_0L)/(k^2) \), and \( \alpha_n = (4+n)/(2+n) \).

The phase shift from the chameleon neutron-mirror potential

\[
\varphi_{\text{cham}} = \varphi_{II, \text{cham}} - \varphi_{I, \text{cham}} = \\
= \frac{\gamma}{\lambda_{n-1}a_{n-1}} \frac{\sqrt{1 + ((b-a)/L)^2} - \sqrt{1 + ((b+a)/L)^2}}{\left( b^{\alpha_n} + a^{\alpha_n} \right)} \approx \\
\approx \frac{\gamma}{\lambda_{n-1}a_{n-1}} \frac{2ab}{2} \frac{b^{\alpha_n-1} - a^{\alpha_n-1}}{b^2 - a^2}.
\]

For a non-strictly vertical mirror, the component of the Earth’s gravity normal to the surface of the mirror produces the potential \( V_{gr} = \kappa mgz \), where \( g \) is the acceleration of gravity, and the coefficient \( \kappa \) depends on the angle \( \theta \) between gravity vector and the mirror plane. At \( \theta = 10^\circ \), we have \( \kappa \approx 5 \times 10^{-5} \). This linear potential leads to additional phase shift

\[
\varphi_{gr} = \varphi_{II, gr} - \varphi_{I, gr} = \frac{\kappa gm^2}{2k^2h^2(a+b)} \left[ L^2 + (b-a)^2 - (b^2 + a^2) \sqrt{L^2 + (b+a)^2} \right] \approx \frac{\kappa gm^2}{k^2h^2} \frac{abL}{a+b},
\]

(10)
calculated in analogy with Eqs.(6)-(9).

The Coriolis phase shift due to Earth’s rotation \( \Omega \) is

\[
\varphi_{Cor} = \frac{2m}{\hbar} (\Omega \cdot A),
\]

(11)
where \( \Omega \) is the vector of angular rotation of Earth and \( A \) is the vector of the area enclosed by the interferometer beams.

As \( A = (abL)/(a+b) \), for the location of the Institute Laue-Langevin (where a good very cold neutron source has been constructed \([23]\)) we have \( \varphi_{Cor} = 0.16(abL)/(a+b) \) rad (a,b,L in cm). As expected it is similar to the gravitational phase shift in its dependence both of the slit and the interference coordinates.

We must also calculate the phase shift of the neutron wave along beam II at the point of reflection. Neglecting the imaginary part of the potential of the mirror, the amplitude of the reflected wave is \( r = e^{-i\varphi_{refl}} \), with the phase

\[
\varphi_{refl} = 2 \arccos(k_\perp/k_b) \approx \pi - \delta \varphi_{refl},
\]

(12)
where

$$\delta\varphi_{refl} = 2k_{\|}/k_b \approx \pi - 2k a/b L,$$

(13)

$k_{\|}$ is the neutron wave vector component normal to the mirror’s surface, and $k_b$ is the boundary wave vector of the mirror. This phase shift linearly depends on $b$ similarly to the geometric phase shift $\varphi_{geom}$.

The reflected and non-reflected beams follow slightly different paths in the interferometer. Therefore in the vertical arrangement of the reflecting mirror they spent different times in Earth’s gravitational field with $\Delta t = 2ab/(L v)$, where $v$ is the neutron velocity. The difference in vertical shifts of the reflected and non-reflected beams is $\Delta h = 2gab/v^2$, and the phase shift due to this difference

$$\Delta\varphi_{vert} = k g^2 a b L/v^4.$$

(14)

With our parameters of the interferometer this value is of the order $\sim 10^{-4}$.

The total measured phase shift is

$$\varphi = \varphi_{geom} + \varphi_{cham} + \varphi_{gr} + \varphi_{Cor} + \varphi_{refl}.$$  

(15)

The gravitational phase shift can be suppressed by installing the mirror vertically with the highest possible precision. On the other hand the gravitational phase shift may be used for calibration of the interferometer by rotation around horizontal axis. The phase shifts due to Earth rotation $\varphi_{Cor}$ and reflection $\varphi_{refl}$ may be calculated and taken into account in analysis of the interference curve.

Figure 2 shows the calculated phase shift $\varphi_{cham}$ for an idealized Lloyd’s mirror interferometer (strictly monochromatic neutrons, the width of the slit is zero, the detector resolution is perfect) with parameters: the neutron wave length $\lambda_n=100$ Å, (the neutron velocity 40 m/s), $L=1$ m, $a = 100$ $\mu$m, $\beta = 10^7$, at $n=1$ and $n=6$. Shown also are the gravitational phase shift $\varphi_{gr}$ at $c = 5 \times 10^{-5}$ (deviation of the mirror from verticality is $10^5$), and $\delta\varphi_{refl} = \pi$—phase shift of the ray II at reflection ($k_b = 10^6$ cm$^{-1}$).

It is essential that the sought phase shift due to hypothetical chameleon potential depends on the interference coordinate nonlinearly. Effect of the hypothetical interaction has to be inferred from analysis of the interference pattern after subtracting off the effects of Earth gravity, Coriolis and reflection.

Figure 3 demonstrates the calculated interference pattern for the same parameters of the interferometer as in Fig. 2 for two cases: (1) with the geometrical phase shift $\varphi_{geom}$, the gravitational phase shift $\varphi_{gr}$ at $\kappa = 5 \times 10^{-5}$, and the phase shift of the ray II at reflection $\delta\varphi_{refl} = \pi - \varphi_{refl}$ ($k_b = 10^6$ cm$^{-1}$) taken into account; (2) the same plus the phase shift due to the chameleon field with matter interaction parameters $\beta = 10^7$, $n = 1$.

After subtracting all the phase shifts except purely geometrical the interference pattern should be strongly sinusoidal with the period of oscillations determined by the geometric phase shift: $\Lambda_{osc} = \lambda_n L/(2a)$. The number of oscillations in an interference pattern with the coordinate less than $b$ is $n_{osc} = 2ab/(\lambda_n L)$.

It follows from these calculations that the effect of the chameleon interaction of a neutron with matter may be tested in the range of strong coupling with the parameter of interaction down to $\beta \sim 10^7$ or lower.

Existing constraints on the parameters $\beta$ and $n$ may be found in Fig. 1 of Ref. [16]. For example the allowed range of parameters for the strong coupling regime $\beta \gg 1$ are: $50 < \beta < 5 \times 10^{10}$ for $n=1$, $10 < \beta < 2 \times 10^{10}$ for $n=2$, and $\beta < 10^{10}$ for $n > 2$. It is seen that the Lloyd’s mirror interferometer may be able to constrain the chameleon field in the large coupling area of the theory parameters.

3. Axion-like spin-dependent interaction.

There are general theoretical indications of the existence of interactions coupling mass to particle spin

[26, 27, 28, 29, 30]. Experimental search for these forces is promising way to discover new physics.
On the other hand, a number of concrete proposals were published of new light, scalar or pseudoscalar, vector or pseudovector weakly interacting bosons. The masses and the coupling of these new hypothetical particles to nucleons, leptons, and photons are not predicted by the proposed models.

The popular solution of the strong CP problem is the existence of a light pseudoscalar boson - the axion \[^{[31]}\]. The axion coupling to fermions has general view $g_{\text{aff}} = C_f m_f / f_a$, where $C_f$ is the model dependent factor. Here $f_a$ is the scale of Peccei-Quinn symmetry breaking which is not predicted so that the axion may have a priori mass in a very large range: $(10^{-12} < m_a < 10^6)$ eV. The main part of this mass range from both – low and high mass boundaries – was excluded in result of numerous experiments and constraints from astrophysical considerations \[^{[32]}\]. Astrophysical bounds are based on some assumptions concerning the axion and photon fluxes produced in stellar plasma.

More recent constraints limit the axion mass to $(10^{-5} < m_a < 10^{-3})$ eV with respectively very small coupling constants to quarks and photon \[^{[32]}\].

The axion is one of the best candidates for the cold dark matter of the Universe \[^{[33, 34]}\].

Axions can mediate a P- and T-reversal violating monopole-dipole interaction potential between spin and matter (polarized and unpolarized nucleons or electrons) \[^{[35]}\]:

$$V_{\text{mon-dip}}(r) = g_s g_p \frac{\hbar^2}{8\pi m_n} \left( \frac{1}{r} + \frac{1}{r^2} \right) e^{-r/\lambda},$$  \hspace{1cm} (16)

where $g_s$ and $g_p$ are dimensionless coupling constants of the scalar and pseudoscalar vertices (unpolarized and polarized particles), $m_n$ the nucleon mass at the polarized vertex, the nucleon spin $s = \hbar \sigma / 2$, $r$ is the distance between the nucleons, $\lambda = \hbar / (m_a c)$ is the range of the force, $m_a$ - the axion mass, and $n = r/r$ is the unitary vector.

Several laboratory searches (mostly by the torsion pendulum method) provided constraints on the product of the scalar and pseudoscalar couplings at macroscopic distances $\lambda > 10^{-2}$ cm (see reviews \[^{[36, 37, 38, 39]}\]).

There are also experiments on the search for the monopole-dipole interactions in which the polarized probe is an elementary particle: neutron \[^{[40, 41, 42, 43]}\], or atoms and nuclei \[^{[44, 45]}\], correction in \[^{[46]}\].

For the monopole-monopole interaction due to exchange of the pseudo-scalar boson \[^{[35]}\]

$$V_{\text{mon-mon}}(r) = \frac{g_s^2 \hbar c}{4\pi r} e^{-r/\lambda},$$  \hspace{1cm} (17)

the limit on the scalar coupling constant $g_s$ can be inferred from the experimental search for the "fifth force" in the form of the Yukawa-type gravity potential $U_5(r) = \alpha_5 G M m e^{-r/\lambda}/r$:

$$g_s^2 = \frac{4\pi G m_n^2 \alpha_5}{\hbar c} \approx 10^{-37} \alpha_5,$$  \hspace{1cm} (18)

where $\alpha_5$ is the "fifth force" Yukawa-type interaction constant.

It follows from the experimental tests of gravitation at small distances (see reviews in \[^{[36, 37]}\]) that $g_s^2$ is limited by the value $10^{-40} - 10^{-38}$ in the interaction range $1$ cm $> \lambda > 10^{-4}$ cm. The sensitivity of these experiments falls with decreasing the interaction range below $\sim 0.1$ cm.

The pseudoscalar coupling constant is restricted to $g_p < 10^{-9}$ from astrophysical considerations \[^{[47, 39]}\].

It is seen that the constraints obtained and expected from further laboratory searches are weak compared to the limit on the product $g_s g_p < 10^{-28}$ inferred from the above mentioned separate constraints on $g_s$ and $g_p$. Although laboratory experiments may not lead to bounds that are strongest numerically, measurements
made in terrestrial laboratories produce the most reliable results. The direct experimental constraints on the monopole-dipole interaction may be useful for limiting more general class of low-mass bosons irrespective of any particular theoretical model. In what follows the constraint on product \( g_s g_p \) may be used for the limits on the coupling constant of this more general interaction.

It follows from Eq. (16) that the potential between the layer of substance and the nucleon separated by the distance \( x \) from the surface is:

\[
V_{\text{mon-dip}}(x) = \pm g_s g_p \frac{\hbar^2 N \lambda}{4 m_n} e^{-x/\lambda} - e^{-(x+d)/\lambda} = \pm V_0 e^{-x/\lambda} \quad (d \gg \lambda),
\]

where \( V_0 = g_s g_p \hbar^2 N \lambda / (4 m_n) \), \( N \) is the nucleon density in the layer, \( d \) is the layer’s thickness. The ”+” and ”-” depends on the nucleon spin projection on \( x \)-axis (the surface normal).

Phase shifts of beams I and II due to interaction of Eq. (20) are calculated similarly to Eqs. (7) and (8):

\[
\varphi_I = k \sqrt{L^2 + (b-a)^2} + \frac{m V_0}{\hbar^2 L} \sqrt{L^2 + (b-a)^2} \int_0^L \exp\left[-\left(\frac{a+b-a}{L} x\right)/\lambda\right] dx = \]

\[= \varphi_{\text{geom.I}} + \frac{m V_0}{\hbar^2 L} \sqrt{L^2 + (b-a)^2} \frac{\lambda}{b-a} \left(e^{-a/\lambda} - e^{-b/\lambda}\right) = \varphi_{\text{geom.I}} + \varphi_{\text{pot.I}} \quad (20)
\]

and

\[
\varphi_{II} = k \sqrt{L^2 + (b+a)^2} + \frac{m V_0}{\hbar^2 L} \sqrt{L^2 + (b+a)^2} \int_0^L \exp\left[-\left(\frac{b+a-a}{L} x\right)/\lambda\right] dx = \]

\[= \varphi_{\text{geom.II}} + \frac{m V_0}{\hbar^2 L} \sqrt{L^2 + (b+a)^2} \frac{\lambda}{b+a} \left(2 - e^{-a/\lambda} - e^{-b/\lambda}\right) = \varphi_{\text{geom.II}} + \varphi_{\text{pot.II}}. \quad (21)
\]

For the spin-dependent potential of Eq.(19) the signs of potential \( V_0 \) and, respectively, the phase shifts \( \varphi_{\text{pot}} \) are opposite for two spin orientations in respect to the mirror surface normal.

The difference in these phase shifts, measured in the experiment

\[(\varphi_I^+ - \varphi_I^-) - (\varphi_{II}^+ - \varphi_{II}^-) = 2(\varphi_I - \varphi_{II}) = \delta \varphi. \quad (22)\]

The geometric, gravitational phase shifts and phase shift of the beam II at reflection calculated earlier do not depend on spin.

The phase shift due to axion interaction is

\[
\varphi_{ax} = 2 \gamma \lambda \left[ a (1 - e^{-b/\lambda}) - b (1 - e^{-a/\lambda}) \right] / (b^2 - a^2),
\]

where \( \gamma = g_s g_p N \lambda L / (4 \hbar) \). At \( b = a \), and \( \lambda / a \ll 1 \), \( \varphi_{ax} \rightarrow \gamma \lambda / a \).

Figure 4 shows the neutron wave phase shift \( \varphi_{ax} \) for different interaction range \( \lambda \). Lloyd’s mirror interferometer has the following parameters: neutron wave length 100 Å (neutron velocity 40 m/s), \( L=1 \) m, \( a = 100 \mu m \), and the interaction strength \( g_s g_p = 10^{-18} \).

The possible sensitivity seen from this figure shows that constraints on the monopole-dipole interaction which can be obtained with the method of the neutron Lloyd’s mirror interferometry is competing with best constraints obtained by other methods (see Ref. [39]).
Figure 5 shows the calculated interference pattern due to an axion-like spin-dependent interaction. The gradient of the external magnetic field \( \nabla (\mu B) \) normal to the mirror plane (\( \mu_n \) is the neutron magnetic moment) may produce the phase shift effect on polarized neutrons, similar to the effect of gravitational force \( F_{\text{gr}} = mg \) (see Eq. (10)). Simple calculation gives that magnetic field gradient 0.01 Oe/cm is equivalent to \( \sim 5 \times 10^{-5} \) of the Earth gravitation.

A significant increase in sensitivity may be achieved in the range of small \( \lambda (\lambda / a \ll 1) \) if the geometry shown in Fig. 1(2) is used, where the slit is located in close vicinity to the surface of additional (upper), non-reflecting mirror. The axion-like potential is produced in this case by both mirrors, but with opposite signs in accordance to Eq. (20).

To avoid multiple reflections, the boundary wave vector of the reflecting mirror must satisfy the condition \( k_b \leq 2ka/L \), or the neutron beam incident on the slit should be collimated so the the first half of the reflecting mirror is not illuminated by the neutrons. In this geometry the phase shift due to axion-like monopole-dipole interaction of the neutron with both mirrors is

\[
\varphi_{\text{ax}} = 2\gamma \lambda a (e^{(b-a)/\lambda} - e^{-a/\lambda} + e^{-b/\lambda} - 1)/(b^2 - a^2). \tag{24}
\]

In this case \( \varphi_{\text{ax}} \to \gamma (1 - e^{-a/\lambda}) \to \gamma \) as \( b \to a \), for \( \lambda / a \ll 1 \) compared to \( \gamma \lambda / a \) for the case of one mirror (Eq. (23)).

The gain in sensitivity at \( \lambda / a \ll 1 \) compared to the case of Fig. 4 is illustrated at Fig. 6.

4. Non-Newtonian gravity

New short-distance spin-independent forces are frequently predicted in theories expanding the Standard Model. These interactions can violate the Equivalence Principle if they depend on the composition of bodies, or the sort of particles.

Precision experiments to search for deviations from Newton’s inverse square law and of violation the Weak Equivalence Principle have been performed in a number laboratories (reviews may be found in [10, 36, 37, 38, 39]).

The pioneering ideas of the multi-dimensional models first formulated in the first half of the XX-th century (G. Nordström, T. Kaluza and O. Klein) received renewed interest in [48, 49, 50]. The development of supergravity and superstring theories required for their consistency extra-dimensions. A more recent promising development contained in [51, 52, 53, 54, 55] proposed mechanisms in which the Standard Model fields are located on the 4-dimensional brane while gravity propagates to the (4+n)-bulk with a larger number of dimensions. As a result the gravitational law may be different from the Newtonian one.

The frequently used parametrization of new spin-independent hypothetical short-range interaction potential has the Yukawa-type form

\[
U(r) = \frac{\alpha G M m}{r} e^{-r/\lambda}, \tag{25}
\]

where \( G \) is the Newtonian gravitational constant, \( M \) and \( m \) are the masses of gravitating bodies, \( \alpha \) is the dimensionless parameter characterizing the strength of the new force relative to gravity, and \( \lambda = \hbar/(m_0 c) \) is the Compton wave length of the particle with the mass \( m_0 \). The mass \( m_0 \) can be the mass of the new scalar field responsible for the short-range interaction. In this case \( \alpha \sim g_\nu^2 \) - the product of the scalar coupling constants. Or the mass \( m_0 \) can be the mass of the lightest Kaluza-Klein state (which is the leading order mode) when the short-range interaction comes from the extra-dimensional expansion of the Standard Model.

The strength \( \alpha \) is constrained to be below unity for \( \lambda \geq 100 \mu m \), [39, 56], but for shorter distances the measurements are not as sensitive being complicated by the Casimir and electrostatic forces [11]. The sensitivity reached in the experiments aiming to test spin-independent interactions between elementary
particles and matter are orders of magnitude less sensitive: at $\lambda = 100 \, \mu m$ it is at the level $\alpha > 10^{11}$ \cite{57} with loss of sensitivity at lower distances.

The potential following from the interaction of Eq. (25) between the layer of substance and a neutron separated by the distance $x$ from the surface is:

$$V_{Yuk}(x) = 2\pi \alpha m_n^2 N G \lambda^2 e^{-x/\lambda} = V_0 e^{-x/\lambda},$$  \hspace{1cm} (26)

where $N \approx \rho/m_n$ is the nucleon density in the layer, $\rho$ is density of the mirror, and $V_0 = 2\pi \alpha m_n^2 N G \lambda^2$

The potential of Eq. (26) has the same coordinate dependence as the axion-like interaction potential of Eq. (19), therefore the expressions for the phase shifts are similar to Eq. (23) with $\gamma = 2\pi \alpha \rho \lambda^2 m_n^2 L/(k \hbar^2)$.

Fig. 7 shows the phase shifts due to the non-Newtonian interaction of Eq. (26) at the same parameters of the interferometer of Fig. 2, and $\rho = 10 \, g/cm^3$.

For the "inverted" Lloyd’s mirror geometry when the reflecting mirror has much lower density so that its gravitational effect is insignificant compared to the effect of the upper mirror the phase shift is

$$\varphi_{Yuk} = \frac{2\gamma \lambda}{b^2 - a^2} \left[ a(e^{-a/\lambda} - e^{(b-a)/\lambda}) + b(1 - e^{-a/\lambda}) \right].$$  \hspace{1cm} (27)

At $b \to a$, and $\lambda/a \to 0$, $\varphi_{Yuk} \to \gamma$ with significant gain in sensitivity for $\lambda/a \ll 1$ compared to the geometry of Fig. 1(1) and Eq. (23).

To avoid multiple reflections in this case, the geometry of Fig. 1(3) may be applied, in which the reflecting mirror has only half length compared to Fig. 1(1). The gain in sensitivity is illustrated in Fig. 8.

5. Feasibility

As mentioned above, the interference may be measured step by step shifting a narrow slit with the width $\delta b \ll \lambda_{osc} \sim 1 - 5 \mu m$. A better option is to use the coordinate detector measuring in this way all the interference picture simultaneously. The current spatial resolution of position-sensitive slow neutron detectors is at the level of $5 \, \mu m$ with electronic registration \cite{58} and about $1 \, \mu m$ with the plastic nuclear track detection technique \cite{59}. With a $1 \, \mu m$ thick $^{10}B$ neutron converter the efficiency of registration of the 100 $\AA$–wave-length neutrons may approach 100 %.

From Ref. \cite{23} where the neutron phase density at the PF-2 very cold (VCN) channel at the Institute Laue-Langevin was measured to be $0.25 \, cm^{-3} \, (m/s)^{-3}$ at $v=50 \, m/s$ it is possible to estimate the VCN flux density as $\phi_{VCN} = 1.66 \times 10^5 \, cm^{-2} s^{-1} (m/s)^{-1} \approx 1 \times 10^5 \, cm^{-2} s^{-1} \AA^{-1}$ (at the boundary velocity of the neutron guide $6.5 \, m/s$). On the other hand Ref. \cite{60} gives the larger value $\phi_{VCN} = 4 \times 10^5 \, cm^{-2} s^{-1} (m/s)^{-1}$ for the same channel.

Using the Zernike theorem it is possible to calculate the width $d_{sl}$ of the slit necessary to satisfy good coherence within the coherence aperture $\omega$, i.e. the maximum angle between diverging interfering beams: $x = \pi \omega d_{sl}/\lambda_n \leq 1$, if the slit is irradiated with an incoherent neutron flux. As $\omega = 2b_{max}/L$, the slit width $d_{sl} \leq L\lambda_n/(2\pi b_{max}) \approx 2 \mu m$ at $b_{max} = 1 \, mm$ (20 orders of interference at the period of interference $\lambda_{osc} = \lambda_n L/(2a) = 50 \mu m$ at $a = 100 \mu m$).

At the monochromaticity 5 $\AA$ (the coherence length $l_{coh} = 20 \, \lambda_n$), the slit width $d_{sl} = 2 \mu m$, the length of the slit 3 cm, the divergence of the incident beam determined by the the VCN guide boundary velocity of 6.5 $m/s$: $\Omega = 6.5/100 = 0.065$, the interference aperture $\omega = 2b/L = 0.2/100 = 2 \times 10^{-3}$, and hence $\omega/\Omega = 0.03$, we have the count rate to all interference curve with a width of 1 mm (20 orders of interference) is given by $10^3 \times 5 \times 2 \cdot 10^{-4} \times 3 \times 0.03 \sim 10 \, s^{-1}$. In one day measurement the number of events in one period of interference is $\sim 4 \times 10^4$. It is enough to observe the phase shift of $\sim 0.1$ corresponding to the effect at $\beta = 10^7$. 

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Distinctive feature of the Lloyd’s mirror interferometer is the possibility to register all the interference pattern simultaneously along z-coordinate starting from z=0 (Fig. 1). The measured interference pattern is then analyzed as regards the presence of the sought for effects, after the corrections taking the known gravity, Coriolis, and reflection phase shifts into account.

The LLL-type interferometers [24, 25] may be used in principle to search for new hypothetical interactions placing a piece of matter in the vicinity of interfering beams. But the geometry of these interferometers does not permit probing hypothetical short-range interactions: the axion-like or non-Newtonian gravity in this way.

We may estimate sensitivity of the LLL-type interferometer to the chameleon potential, which is actually not short-range. The phase shift in this case is

\[ \phi_{LLL} = \frac{2\gamma \sqrt{1 + (2a/L)^2}}{\lambda^{\alpha_n-1}} (2a)^{\alpha_n-1}, \tag{28} \]

where \( a \) is the half distance between the beams of the LLL-interferometer. The sensitivity of the Lloyd’s mirror and the LLL-interferometers is determined by the factor \( L a^{\alpha_n-1}/k \), which is much in favor of the Lloyd’s mirror interferometer.

In the interferometers of the LLL-type [24, 25] an interference pattern is obtained point by point by rotation of a phase flag introduced into the beams. In the case of the VCN three grating interferometers (for example [61, 62, 63]) the phase shift between the beams is realized by the same method or by shifting position of the grating.

The neutron Lloyd’s mirror experiments may be performed with monochromatic very cold neutrons, or in the time-of-flight mode using large wave length range, for example 80-120 Å. The pseudo-random modulation [64, 65] is used in the correlation time-of-flight spectrometry. It was realized in the very low neutron energy range [66]. In this case a two-dimensional interference coordinate – time-of-flight registration gives significant statistical gain.

As in the VCN interferometers based on three gratings, in the Lloyd’s mirror neutron interferometer the space between the beams is small (parts of mm). Therefore it hardly can be used in experiments where some devices are introduced in the beams, or between the beams (for example to investigate non-local quantum-mechanical effects). But it may be applicable to search for short-range interactions when they are produced by a reflecting mirror.

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Figure 1: Three possible configurations of the neutron Lloyd’s mirror interferometer. 1 – the standard Lloyd’s mirror geometry, 2 – interferometer with two mirrors, only the bottom one is reflecting, 3 – the length of the reflecting mirror is decreased twice to avoid multiple reflections. The height of the slit above the reflecting plane is $a$, $L$ is the distance from the slit to the detector surface, $b$ is the distance of the detector coordinate from the reflecting plane.
Figure 2: The neutron wave phase shifts $\varphi_{cham}$ in the Lloyd’s mirror interferometer with parameters: the neutron wave length $100 \, \text{Å}$, $L = 1 \, \text{m}$, $a = 100 \, \mu \text{m}$, the interaction parameters of the chameleon field with matter $\beta = 10^7$, $n = 1$ and $n = 6$. Also shown: the gravitational phase shift $\varphi_{gr}$ at $\kappa = 5 \times 10^{-5}$; the Coriolis phase shift $\varphi_{Cor}$, and the effect of reflection $\delta \varphi_{refl} = \pi - \varphi_{refl}$ at $k_b = 10^6 \, \text{cm}^{-1}$. The period of oscillations in the interference pattern is $\Lambda_{osc} = \lambda_n L / (2a) = 50 \, \mu \text{m}$. 
Figure 3: The calculated interference pattern for the neutron-mirror interaction via the chameleon field (the parameters of the interferometer are the same as in Fig. 2). 1 – purely geometrical phase shift $\varphi_{\text{geom}}$, 2 – the geometrical phase shift plus the phase shift due to the chameleon field with matter interaction parameters $\beta = 10^7$, $n = 1$. 
Figure 4: The neutron wave phase shift $\delta_{ax}$ at different interaction range $\lambda_{inter}$ of the axion-like spin-dependent interaction with the product of the coupling constants $g_s g_p = 10^{-18}$. Lloyd’s mirror interferometer has parameters the same as in Fig. 2.
Figure 5: Calculated interference pattern due to the axion-like spin-dependent interaction with $g_s g_p = 10^{-18}$ and interaction range $\lambda = 500 \mu m$. The interferometer has the same parameters as in Fig. 2.
Figure 6: The neutron wave phase shift $\varphi_{ax}$ at different interaction range $\lambda_{\text{inter}}$ of the axion-like spin-dependent interaction with the product of the coupling constants $g_s g_p = 10^{-18}$. Lloyd’s mirror interferometer has the geometry of Fig. 1(2) with the same parameters as in Fig. 2, and the distance between mirrors 100 $\mu$m.
Figure 7: The neutron wave phase shifts $\varphi_{Yuk}^{\text{Yuk}}$ in the Lloyd’s mirror interferometer of geometry of Fig. 1(1) and with the same parameters as in Fig. 2.
Figure 8: The neutron wave phase shifts $\varphi_{Yuk}$ in the Lloyd’s mirror interferometer of the geometry of Fig. 1(3) and with the same parameters as in Fig. 2.