Feedforward and repetitive control of a servo piezo-mechanical hydraulic actuator

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Abstract. In this paper a hybrid actuator is proposed. The hybrid actuator consists of a piezo-mechanical structure and a hydraulic ratio displacement. Particular attention is paid to a liquid spring model of the displacement ratio which represents the hydraulic part of the mechanism to control an intake and exhaust valve of a combustion engine. The whole mechanism is controlled using a cascade repetitive control to track a periodic signal combined with a proportional derivative (PD) regulator and a feedforward action. Measured results are shown.

1. Model of the hybrid actuator
In this part of the paper the model of the actuator is presented. The actuator consists of different technical parts: a piezo-electric structure, a servo-piston and a hydraulic structure.

1.1. Piezo-electric structure
The mathematical model is the same which is adopted in [1], [2] and in [3]. The proposed sandwich model and the corresponding circuit are presented in Fig. 1. The details on this model can be found in [4]. Considering the whole system described in Fig. 1 with the assumption of compressibility of the oil, the whole mechanical system can be represented by a spring mass structure as shown in the conceptual scheme of Fig. 1. Concerning the piezo actuator, observing Fig. 1, \( K \) and \( D \) represent the elasticity and the friction constant of the spring which is antagonist to the piezo effect and is incorporated in the PEA. In the technical literature, factor \( D_x K_x = T_{em} \) is known with the name "transformer ratio" and states that the most important characteristic of the electromechanical transducer in which \( K_x \) is the elasticity constant factor of the PEA and \( D_x \) is the parameter which is responsible to transform voltage into movement. In fact, another well-known physical relation is \( F_z(t) = D_x K_x V_z(t) \) which represents the piezo force in which \( V_z \) is the internal voltage. In the ideal case, we have that \( V_z(t) = V_{in}(t) \) where \( V_{in}(t) \) states the input voltage. According to [4], in Fig. 1 a possible model representation of a piezo actuator is reported.
1.2. Servo piston structure
The displacement ratio is calculated considering the surface quotient between the piezo (radius = 40mm) and the servo piston (radius = 4mm):

\[ i_{Weg} = \frac{A_1}{A_2}. \]  

(1)

The oil compressibility is comparable with Hook’s law of the material technique [5]. In [5] the concept of a liquid spring is introduced and the fluid compressibility is modelled using an elasticity factor. Considering [5], the coefficient of the liquid spring coefficient $K_{OF_L}$ in a pressure form is calculated using the following expression:

\[ K_{OF_L} = \frac{V_0}{\Delta V(t)} \Delta p(t), \]  

(2)

in which $V_0$ represents the total volume in the chamber. $\Delta V(t)$ is the compressed volume because of the acting force which generates a pressure difference equal to $\Delta p(t)$, see [5]. As shown in Fig. 1, two surfaces $A_1$ and $A_2$ play a role in the hydraulic ratio displacement. This ratio states the steady-state gain position factor of this part of the actuator. From Fig. 1 it is possible to observe that the model consists of two hydraulic cylinders. The forces at the connecting surfaces of both cylinders are calculated through the following product:

\[ F_{A1}(t) = A_{F1}K_{OF_L1}x_1(t) = \frac{A_1}{A_1 + A_2} K_{OF_L1}x_1(t), \]  

(3)

and

\[ F_{A2}(t) = A_{F2}K_{OF_L2}x_2(t) = \frac{A_2}{A_1 + A_2} K_{OF_L2}x_2(t). \]  

(4)

1.3. Hydraulic structure
For constant pressures, the volumetric flow $Q_{th}(t)$ of the valve drive is proportional to the length of the opening slit that equals $x_2(t) - \bar{x}_2$. Considering

\[ Q_{th}(t) = (x_2(t) - \bar{x}_2(t)) K_{SP} \]  

(5)
with $K_{SP}$ which represents a parameter depending on the pressure and $\bar{x}_2(t)$ represents the initial servo piston position at which corresponds $Q_{th}(t) = 0$. In Fig. 2 the whole actuator and a possible model of its hydraulic part are shown as proposed in [5]. The model was presented in [5] but in a linear approximation form which is very often used in industrial applications.

![Figure 2. Scheme of the whole actuator. Block diagram structure of the hydraulic part of the actuator](image)

\[ \dot{x}_V(t) = K_H Q_{th}(t), \]  
\[ Q_{th}(t) = K_H^{-1} \dot{x}_V(t), \]  
\[ x_2(t) = K_{SP}^{-1} Q_{th}(t) + \bar{x}_2. \]  

2. Control strategy

In Fig. 3 the adopted control scheme is shown. The control scheme presents a feedforward action to compensate the steady state error because of the absence of the integral action in the controller. Together with the feedforward action a repetitive control algorithm is used because of the presence of a periodic signal to be tracked. It is known that the control loop can need to be stabilised and therefore, a stabilising PD controller is considered in the loop.

2.1. Feed-forward control

Thanks to modelling approximations described above, the inversion of the system described in Eq. (6) is as follows:

\[ Q_{th}(t) = K_H^{-1} \dot{x}_V(t), \]  
\[ x_2(t) = K_{SP}^{-1} Q_{th}(t) + \bar{x}_2. \]
Figure 3. Block diagram of the control structure

The steady-state feedforward control can be summarized as follows: being \( F_z(t) = F_{A1}(t) = D_x K_x V_{inff}(t) \) and \( p(t) = \frac{F_{A2}(t)}{A_2} = \frac{F_{A1}(t)}{A_1} \), then \( F_{A2}(t) = \frac{A_2}{A_1} F_z(t) \) and considering Eq. (4), then \( A_{F2} K_{FL2} x_2(t) = \frac{A_2}{A_1} D_x K_x V_{inff}(t) \). It is straightforward to obtain the following relation:

\[
V_{inff}(t) = \frac{A_{F2} K_{FL2} (K_{SP}^{-1} K_{H}^{-1} x_{Vd}(t) + \bar{x}_2)}{\frac{A_2}{A_1} D_x K_x}.
\]  

(10)

3. Experimental results using repetitive control

According to the standard control scheme represented in Fig. 3 some measurements were done. It is known that a repetitive controller is characterised by the following transfer function:

\[
G_r(s) = \frac{1}{1 - e^{-T_p s}}.
\]  

(11)

Parameter \( T_p \) represents the period of the periodical signal to be tracked. Figure 3 shows also another controller in the loop which is necessary to stabilise the control loop. Normally, the repetitive controller is realised using a discrete technique and Eq. (11) becomes as follows:

\[
G_r(z) = \frac{1}{1 - z^{-T_p}}.
\]  

(12)

Figure 4 shows a detail of the piezo position and a corresponding detail of the servo piston position with 5000 cycles per minute. Figure 5 shows the measured results of pressure inside the stroke ratio and the position measurements. A detailed scheme of the repetitive control idea is shown in Fig. 3. Measured results in an experimental setup using a dSPACE system to implement the control structure confirm that the control scheme described in Fig. 3 can be used as an effective feedforward control for the presented hybrid actuator.

4. Conclusion and outlook

The paper deals with modelling and control of a hybrid actuator. Particular attention is paid to a hydraulic spring model of the ratio displacement which represents the hydraulic part of the mechanism. A repetitive controller is applied to track a periodical signal together with a PD-controller which is devoted to stabilise the feedback control loop. Measured results are presented to demonstrate the effectiveness of the proposed method. Future advancements of this work can include a detailed friction model of the mechanical system and its control using Sliding Mode Control as proposed in [6] and in [7]. Moreover, in order to reduce the number of sensors an implementation of a Kalman Filter can be considered as proposed in [8].
Figure 4. Detail of the piezo position and corresponding detail of the servo piston position with 5000 cycles per minute

Figure 5. Oil chamber pressure and valve position with 5000 cycles per minute

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