Scaling in SU(3) theory with a MCRG improved lattice action

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We test various improved gauge actions which are made of linear combinations of Wilson loops. We observe the restoration of rotational symmetry in the static interquark potential already on coarse lattices as small as $6^3 12$. Furthermore, we study scaling and asymptotic scaling of the string tension with a MCRG-improved action on $12^3 24$ lattices. Preliminary results show that scaling sets in at $a \approx 0.3$ fm.

1. INTRODUCTION

There is a great deal of work being done to reduce artifacts in lattice QCD [1]. This is important, if one aims to extract precise continuum physics on the emerging teraflop-range computers.

We will focus here on the improvement of the pure gauge sector [2–5]. The general form of the action that we try is

$$S_{\text{gauge}} = -\beta \sum_{\text{loops}} \left[ \text{Square} + c_1 \cdot \text{Rectangle} + c_2 \cdot \text{Parallelogram} \right]$$

For $c_1 = c_2 = 0$ one recovers the Wilson action. The additional Wilson loops are six-link loops in two and three dimensions (excluding the bent rectangle which is redundant to order $O(a^2)$).

A “good” improved action should reduce lattice artifacts and should be practical for computations. Therefore, one would like to have as little as possible additional terms in the action. Classically, at order $O(a^2)$, a convenient choice is $c_1 = -1/20$ and $c_2 = 0$ [2]. Corrections of these values at leading order in bare coupling amount to a small nonvanishing $c_2$. Recently, it was observed that an expansion in a renormalized coupling [3] produces an impressive improvement of the classical values.

A different strategy to select couplings is to compute them by performing Renormalization Group (RG) transformations. Ideally, one could construct Fixed Point (FP) or “perfect” actions [7]. For computational convenience, we prefer RG improved actions which are less complex [8]. One earlier approach has blocked continuum gauge fields by expanding in the bare coupling [3]. More recently, by blocking lattice gauge fields during a Monte Carlo simulation (MCRG), one has constructed improved gauge actions [9,10].

Since a thorough study of scaling with the MCRG improved gauge actions is missing, we decided to investigate this problem. (The corresponding study with the Iwasaki action can be found in [10,11].) As we will see RG improved actions lead to a better restoration of rotational symmetry than mean field improved actions. Therefore we decided to study also the scaling properties with the MCRG improved action.

2. ROTATIONAL SYMMETRY ON COARSE LATTICES

In the table below we give the simulation parameters with improved gauge actions on small lattices.

| Action | $c_1$  | $c_2$  | $\beta$ | Lattice size |
|--------|--------|--------|---------|-------------|
| MF1    | -0.074 | 0      | 6.25    | $6^3 12$    |
| MF2    | -0.0827| -0.0124| 6.80    | $6^4$       |
| IW     | -0.0907| 0      | 6.50    | $6^3 12$    |
| MCRG   | -0.115 | 0      | 7.20    | $6^3 12$    |

Table 1: Simulation parameters on coarse lattices

The couplings of the mean field improved action are given by tadpole improved one loop results (for their definition see [4]). For all the
actions we generated 100 independent configurations, out of 11000 sweeps in total. More results with increased statistics will be reported elsewhere [12].

In Figure 1 we show the static potential in lattice units for the above actions. The solid line is a 3-parameter fit: \( V_0 + K r - \alpha / r \) with the scale set by the string tension \( \sqrt{\sigma} = 420 \) MeV. In Table 2 we give the quality of the fit and the lattice spacings.

| Action | \( \chi^2 / \text{dof} \) | Lattice spacing (fm) |
|--------|-----------------|---------------------|
| MF1    | 0.24            | 0.37(3)             |
| MF2    | 0.67            | 0.35(6)             |
| IW     | 0.07            | 0.42(1)             |
| MCRG   | 0.07            | 0.42(2)             |

Table 2

Quality of the fit to the potential parametrization and the corresponding lattice spacings on coarse lattices

Our results indicate that the Wilson action should not be used in simulations on coarse lattices. We observe further that violations of rotational symmetry are smaller for the RG improved actions.

Figure 2. Scaling of the static interquark potential on coarse lattices with MF1 (a), MF2 (b), Iwasaki (c) and MCRG improved actions (d)

3. SCALING OF STRING TENSION WITH THE MCRG IMPROVED ACTION

We have performed simulations on \( 12^3 \)24 lattices with the MCRG improved action for \( \beta \in [8, 13] \) with increment \( \Delta \beta = 0.5 \). The main results are tabulated below (see [12] for details).

| \( \beta \) | \( K \)     | \( \chi^2 / \text{dof} \) | \( a \) (fm)  |
|----------|-------------|-----------------|--------------|
| 8.0      | 0.5147(103) | 0.4901          | 0.3416(34)   |
| 8.5      | 0.3772(111) | 0.1356          | 0.2924(43)   |
| 9.0      | 0.2677(59)  | 0.083           | 0.2464(27)   |
| 9.5      | 0.1995(50)  | 0.0521          | 0.2127(27)   |
| 10.0     | 0.1502(40)  | 0.0056          | 0.1824(24)   |
| 10.5     | 0.1138(25)  | 0.0022          | 0.1606(18)   |
| 11.0     | 0.0928(23)  | 0.0011          | 0.1451(18)   |
| 11.5     | 0.0761(13)  | 0.0002          | 0.1314(11)   |
| 12.0     | 0.0649(24)  | 0.0007          | 0.1213(23)   |
| 12.5     | 0.0554(20)  | 0.0004          | 0.1121(20)   |
| 13.0     | 0.0505(9)   | 0.0001          | 0.1070(10)   |

Table 3

Simulation results on a \( 12^3 \)24 lattice and quality of the fit for the static potential

In Fig. 2 we observe scaling of the potential already at \( \beta = 8 \), our starting point, corresponding to a lattice spacing \( a \approx 0.34 \) fm.
Provided the fermion action is improved at the same order, this result shows that one could compute the spectrum in quenched QCD already on a $8^3$ space volume amounting to a lattice size $L \approx 2.7$ fm. This conclusion is consistent with the results of hadron spectroscopy on $8^316$ blocked configurations used to determine the MCRG improved action \[\Lambda_{LQCD}\]. Note that the same accuracy with the Wilson gauge and fermion action has been achieved on $32^4$ lattices.

Therefore, these results together with those in \[\cite{13}\] strongly suggest that one could perform phenomenological lattice calculation runs in the quenched approximation with sufficient accuracy (1\% level) on lattices with 12 – 20 points in each dimension.

We have tested asymptotic scaling too. In Figure 3 we show $\Lambda_{LQCD}$ as a function of $\beta$. It can be seen that a saturation seems to start already at $\beta = 12.5$. In Figure 4 we plot the discrete beta-function based on the string tension results. The results obtained up to now are not yet conclusive for the onset of asymptotic scaling. At present we are running simulations at higher $\beta$s on $18^336$ lattices and at the same time we are increasing the statistics. The results will reveal the behavior of the discrete $\beta$-function deep in the perturbative region \[\cite{12}\].

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