Non-Singular String Cosmology
in a 2d Hybrid Model

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Abstract
The existence of non-singular string cosmologies is established in a class of two-dimensional supersymmetric Hybrid models at finite temperature. The left-moving sector of the Hybrid models gives rise to 16 real ($\mathcal{N}_4 = 4$) spacetime supercharges as in the usual superstring models. The right-moving sector is non-supersymmetric at the massless level, but is characterized by $\text{MSDS}$ symmetry, which ensures boson/fermion degeneracy of the right-moving massive levels. Finite temperature configurations, which are free of Hagedorn instabilities, are constructed in the presence of non-trivial “gravito-magnetic” fluxes. These fluxes inject non-trivial winding charge into the thermal vacuum and restore the thermal $T$-duality symmetry associated with the Euclidean time circle. Thanks to the unbroken right-moving $\text{MSDS}$ symmetry, the one-loop string partition function is exactly calculable beyond any $\alpha'$-approximation. At the self-dual point new massless thermal states appear, sourcing localized spacelike branes, which can be used to connect a contracting thermal Universe to an expanding one. The resulting bouncing cosmology is free of any curvature singularities and the string coupling remains perturbative throughout the cosmological evolution.
1 Introduction

One of the most acute open problems of Modern Cosmology is that of the initial singularity. Assuming that the matter content of the Universe satisfies the weak or the null energy condition, and extrapolating the cosmological evolution arbitrarily back in time by using the equations of quantum field theory and Einstein’s theory of general relativity, one is driven to a curvature singularity, the “Big Bang”, signifying a break-down of our description of the underlying physics. Moreover, the singularity theorems [1] strongly suggest that it is unlikely to overcome this problem within the framework of quantum field theory and general relativity.

In string theory, however, new degrees of freedom arise and various non-geometrical phenomena take place, opening up new possibilities to address the initial singularity problem [2–10]. Indeed, the full set of stringy degrees of freedom cannot be incorporated within a single effective field theory description. Rather, there exist distinct effective descriptions valid in specific restricted regions of the moduli space. Typically in such regions, most stringy states carry large masses of the order of the string scale or higher. At low enough energies, these can be integrated out giving rise to an effective field theory in terms of a finite number of light fields. However, as the moduli fields vary adiabatically, we encounter points where new states become relevant, forcing us to switch from the initial description into another, using the duality symmetries of string theory. An apparently singular geometry may then be mapped into a non-singular, smooth geometry in the dual description. Furthermore, around the transition points a conventional field theory description may be absent, as the full string theory dynamics becomes relevant. For comprehensive discussions, see e.g. [4].

In a cosmological context, where a number of moduli fields and the background metric acquire time dependence, we expect the various dual descriptions to be connected dynamically via non-trivial phase transitions [2,4,8,9,11,13]. The cosmological evolution will be marked by transition eras, where new stringy states become light and dominate the dynamics. In general, the transitions among the various dual phases take place at finite values of the moduli, where the underlying target space background has finite size and/or curvature. Thus, one expects these transitions to occur before the appearance of cosmological singularities. The hope is that, once the underlying string dynamics is properly taken into account, the various non-singular cosmological phases would be connected consistently, successfully
covering the whole history of the cosmological evolution. Previous work on string dualities and cosmology includes [2–10].

The greatest obstacle towards a concrete realization of this attractive cosmological scenario is the appearance of Hagedorn instabilities in string theory at finite temperature and in the presence of other supersymmetry-breaking sources. Due to the exponential growth in the number of single-particle states as a function of mass, the partition function of superstrings at finite temperature diverges once the temperature exceeds the Hagedorn temperature ($T_H \sim M_s$). This divergence, however, is not a pathology of string theory. Rather, it signals a phase transition towards a new thermal vacuum [11, 13–15]. In the Euclidean description of the thermal system, where the time direction is compactified on a circle of radius $R_0$, $T = 1/(2\pi R_0)$, certain stringy states winding the Euclidean time circle become tachyonic precisely when the thermal modulus $R_0$ exceeds its Hagedorn value. Thus, the Hagedorn divergence can be interpreted as an IR-instability of the underlying Euclidean background and the phase transition is driven by tachyon condensation. The crucial point is that, in the presence of such condensates, the thermal vacuum acquires non-trivial winding charge [11, 13]. The question that we would like to address is whether it is possible to identify stable thermal vacua, characterized by non-zero winding charge, whose dynamics could be quantitatively treated by utilizing e.g. the powerful 2d worldsheet CFT techniques of (weakly coupled) string theory.

Considerable progress towards this direction has been achieved in the context of $(4,0)$-supersymmetric type II vacua in arbitrary dimension, where it was found that the introduction into the thermal system of certain discrete “gravito-magnetic” fluxes, associated with the graviphoton ($G_{10}$) and the axial vector ($B_{10}$) backgrounds, lifts the tachyonic instabilities [16,17]. Essentially, these fluxes inject non-trivial winding charge into the thermal vacuum, and restore the stringy $T$-duality symmetry $R_0 \to R_0^2/R_c$ of the finite-temperature system\footnote{In the absence of the gravito-magnetic fluxes the type II thermal partition function does not enjoy thermal duality symmetry. In the heterotic case, however, this duality symmetry is present.}. The self-dual point occurs at the fermionic point $R_c = 1/\sqrt{2}$, where new massless thermal states appear. These states correspond to vortices carrying non-trivial momentum and winding charges along the Euclidean time circle, and lead to an enhancement of the local $U(1)_L$ gauge symmetry to a non-Abelian $[SU(2)_L]_{k=2}$ gauge symmetry. The one-loop string partition function is finite for all values of the thermal modulus $R_0$ and reduces to the
conventional thermal partition function in the large $R_0$ limit.

In this paper we will study the induced cosmological evolution associated with such thermal configurations in a class of two dimensional $(4,0)$-vacua, the so called Hybrid vacua, introduced in [17]. These are very special vacua: the right-moving supersymmetries are broken spontaneously at the string level, but are replaced by the recently-discovered Massive (boson/fermion) Spectrum Degeneracy Symmetries (MSDS) of [9, 18]. The presence of particular discrete “gravito-magnetic” fluxes renders the finite temperature configurations free of tachyonic/Hagedorn divergences and, in addition, the thermal vacuum is characterized by unbroken right-moving MSDS symmetry. As a result of the latter symmetry, the one-loop string partition function can be computed exactly in $\alpha'$, with the contributions of the massive bosonic and fermionic towers of string states exactly cancelling each other. As the value of the thermal modulus $R_0$ is varied, the thermal system exhibits an interesting phase transition, occurring precisely at the extended symmetry point $R_0 = 1/\sqrt{2}$. At this point additional massless states appear, driving the phase transition. The two phases are related by thermal duality symmetry $R_0 \to 1/(2R_0)$ (with the extended symmetry point being the self-dual point). In both phases the temperature, the energy density and pressure are bounded. They achieve their maximal critical values at the extended symmetry point. Thanks to the unbroken MSDS symmetry, the equation of state characterizing each phase is effectively that of massless thermal radiation in two dimensions.

The resulting cosmological evolution is non-singular, and it is marked by a phase transition between the two dual thermal phases. We will show that the phase transition can be described in terms of a space-like brane, corresponding to a localized pressure that is sourced by the additional massless thermal modes that become relevant at the critical temperature. The role of the brane is to glue the two dual thermal phases in a consistent way. As we will see, both the dilaton and the scale factor bounce across the brane. For the scale factor, the bounce corresponds to a smooth reversal of contraction to expansion, with the Big Bang curvature singularity of general relativity being absent. There is a discontinuity in the first time derivative of the dilaton, determined by the tension of the brane. This discontinuity is naturally resolved by the presence of extra massless thermal states localized at the brane.

\footnote{Notice that, at the fermionic point, one may treat lattice states as fermionic oscillator states in a uniform manner.}

\footnote{Various aspects of thermal duality symmetry have been also discussed in [19, 21].}
The system remains weakly coupled throughout the cosmological evolution, with the dilaton attaining its maximal value at the phase transition. This maximal value is determined in terms of the brane tension, the critical temperature and the number of the thermally excited massless degrees of freedom in the system.

We argue that the gluing mechanism provided by such spacelike branes is a fundamental dynamical phenomenon in string theory, capable of connecting consistently the space of “momenta” with the dual space of “windings”. Such branes will typically appear at the self-dual points, which are characterized by enhanced gauge symmetry. The presence of additional marginal operators with non-trivial momentum and winding charges is crucial in order for a dynamical transition between the two dual spaces to occur.

We would like to stress here that our stringy, singularity-free cosmological solution differs from various incarnations of the pre- Big Bang scenario [5], which require the application of a non-perturbative S-duality symmetry in order to resolve the very early, pre- Big Bang phase. In fact, the occurrence of strong coupling dynamics is often another great obstacle in realizing the most general cosmological scenario outlined above. It is an open and interesting question to find branes that can dynamically glue cosmological phases that are related by S-duality symmetry. It is also interesting that the two-dimensional cosmological Hybrid models are amenable to perturbative treatment throughout their entire history.

The plan of this paper is as follows:

In section 2 we summarize the main features concerning tachyon-free, thermal configurations of (4,0)-vacua at finite temperature. In section 3 we focus on the two dimensional Hybrid models and analyze in detail the resulting phase structure of the thermal system. We proceed in section 4 to study the backreaction of the thermal Hybrid state on the initially flat metric and dilaton background and exhibit the resulting non-singular cosmology. In section 5 we present more general non-singular cosmological solutions associated to distributions of spacelike branes. We end up with our conclusions and directions for future research.

2 Thermal (4,0)-type II string vacua

Before describing the two dimensional Hybrid models, we review some general results concerning thermal states of (4,0)-type II models [16,17]. Such models correspond to freely-
acting asymmetric orbifold compactifications, which preserve the 16 real supersymmetries ($N_4 = 4$) arising from the left-moving sector of the worldsheet. The right-moving supersymmetries are broken spontaneously at the string level, via the coupling of an internal cycle, e.g. the $X^9$ direction compactified on a circle, to the right-moving spacetime fermion number $F_R \equiv \bar{a}$. At special values of the internal compactification moduli, we encounter extended symmetry points with a non-Abelian enhancement of the local gauge symmetry group [16,17]. In the following discussion, we take the values of the internal moduli to be frozen at such an extended symmetry point and the string coupling to be sufficiently weak.

The finite temperature partition function can be computed via a Euclidean path integral on the $S^1(R_0) \times \mathcal{M}$ manifold, where the Euclidean time direction is compactified on a circle of radius $R_0$. Here $\mathcal{M}$ denotes the spatial manifold. At least one spatial direction is taken to be very large (or non-compact). The conventional thermal ensemble corresponds to a freely acting orbifold, obtained by modding out with $(-1)^F \delta_0$, where $\delta_0$ is an order-two shift along the Euclidean time circle. The coupling to the spacetime fermion number $F \equiv a + \bar{a}$ is dictated by the spin-statistics connection. The resulting thermal system develops instabilities, the Hagedorn instabilities, when $R_0 < R_H = \sqrt{2}$, where certain stringy states winding the Euclidean time circle become tachyonic [11,12,14]. These instabilities signal the onset of a non-trivial phase transition around the Hagedorn temperature, see e.g. [11,13,15].

Generically, one expects the tachyon condensates involved in the transition to lead to large backreaction, driving the system outside the perturbative domain. A precise quantitative analysis of the dynamics is still lacking. At the very least, we can identify a crucial property of the would-be stable high-temperature phase: since certain winding modes acquire non-trivial expectation values, $\langle O_n \rangle \neq 0$, the thermal vacuum must be characterized by non-zero winding charge.

In this paper, we will consider tachyon-free thermal configurations where, in addition to the temperature deformation, we turn on certain discrete “gravito-magnetic” fluxes associated with the graviphoton ($G_{10}$) and the axial vector ($B_{10}$) gauge fields. These are geometrical fluxes threading the Euclidean time circle together with the internal cycle responsible for the breaking of the right-moving supersymmetries [16,17]. Effectively, they describe gauge field “condensates” which have locally vanishing field strength but a non-zero value of the
Wilson line around the Euclidean time circle\(^4\). They correspond to non-trivial, topological vacuum parameters, defining (at least perturbatively) super-selection sectors of the thermal system. The presence of such fluxes modifies the thermal frequencies and the contribution to the free energy of various states charged under the graviphoton and the axial vector gauge fields. As the would-be winding tachyons are charged under these gauge fields, for large enough values of the “gravito-magnetic fluxes”, the tachyonic instabilities are lifted \([16,17]\).

As an illustrative example, consider the case where the \(X^9\)-cycle factorizes from the rest of the internal compact manifold \([16]\). In the presence of non-zero \(G_{90}/R_9^2 \equiv G\) and \(B_{90}/R_9^2 \equiv B\) background fields and finite temperature, the contribution to the one-loop partition function of the \((X^0, X^9)\) sub-lattice is given by

\[
\frac{R_0}{\tau_2} \sum_{\tilde{m},n} e^{-\frac{\pi R_9^2}{\tau_2} [R_{90}^2 |\tilde{m}^0 + \tau n^0|^2 + R_9^2 |\tilde{m}^9 + G\tilde{n}^0 - \tau (n^9 + Gn^0)|^2]} \times e^{2\pi B (\tilde{m}^9 a^0 - \tilde{n}^9 n^9)}
\]

\[
\times (-1)^{\tilde{m}^0 (a + \tilde{a}) + n^0 (b + \tilde{b}) (-1)^{\tilde{m}^9 \tilde{a} + n^9 \tilde{b} + \tilde{n}^9 n^9}}. \tag{2.1}
\]

The Euclidean-time \(X^0\)-cycle couples to the total spacetime fermion number \(F\) and the internal spatial \(X^9\)-cycle couples to the right-moving fermion number \(F_R\). The conventional finite-temperature system (suffering from the well-known Hagedorn pathologies) corresponds to \(G = B = 0\), where the \(X^0\)-lattice coupling to the total fermion number factorizes. The deformed \textit{tachyon-free} configuration arises at the point \(G = 2B = 1\) \([16]\). This point is very special since it leads to an asymmetric factorization of the original lattice into two parts, one of which couples only to the left-moving \(R\)-symmetry charges, while the second one couples only to the right-moving \(R\)-symmetries. To see this, shift \(\tilde{m}^9 \to \tilde{m}^9 - \tilde{m}^0\) and \(n^9 \to n^9 - n^0\) to define the frame in which the \((2, 2)\)-lattice factorizes into two \((1, 1)\)-lattices, asymmetrically coupled to the left- and right-moving spacetime fermion numbers \(F_L \equiv a\), \(F_R \equiv \tilde{a}\), respectively:

\[
\frac{R_0}{\sqrt{\tau_2}} \sum_{\tilde{m}^0, n^0} e^{-\frac{\pi R^2}{\tau_2} |\tilde{m}^0 + n^0|^2} (-1)^{\tilde{m}^0 a + n^0 b + \tilde{n}^0 n^0} \times \frac{R_0}{\sqrt{\tau_2}} \sum_{\tilde{m}^9, n^9} e^{-\frac{\pi R^2}{\tau_2} |\tilde{m}^9 + n^9|^2} (-1)^{\tilde{m}^9 \tilde{a} + n^9 \tilde{b} + \tilde{n}^9 n^9}. \tag{2.2}
\]

In this representation, the \(X^0\) circle couples to the left-moving spacetime fermion number \(F_L \equiv a\), and so the configuration can be described as a freely-acting asymmetric orbifold \((-1)^{F_L \delta_0}\) of the initially supersymmetric \((4, 0)\) model.

\(^4\)See e.g. \([22]\) for discussions of such configurations in the context of local gauge theories at finite temperature and \([20,23]\) in the context of string theory.
In [17], general classes of (4, 0)-vacua were studied, including cases where the $X^9$-cycle does not factorize from the rest of the compact spatial manifold. It was found that the point $G = 2B = 1$ leads to the unique tachyon-free, thermal configuration associated with any such initially supersymmetric (4, 0)-vacuum. Moreover, it was established that there exists a frame in which the full compactification lattice admits an asymmetric factorization, where the (1, 1) sub-lattice associated to the Euclidean time cycle couples only to $F_L$, while the internal, spatial sub-lattice couples only to $F_R$:

$$\Gamma_{(d,d)}[a, \bar{a}][b, \bar{b}] = \Gamma_{(1,1)}[a,b](R_0) \otimes \Gamma_{(d-1,d-1)}[\bar{a}, \bar{b}](G_{IJ}, B_{IJ}).$$

(2.3)

This result generalizes (2.2). The thermal configurations remain tachyon-free under all possible deformations of the dynamical, transverse moduli associated with the internal spatial sub-lattice, and of the thermal modulus $R_0$ [17]. Indeed when $G = 2B = 1$, the $O_8\bar{O}_8$ sector, which appears in the spectrum, necessarily carries non-trivial momentum and winding charges, and its lowest mass satisfies [17]

$$m^2_{O\bar{O}} \geq \left(\frac{1}{2R_0} - R_0\right)^2,$$

(2.4)

for any deformation of the transverse, internal spatial lattice. As a result the lowest-lying thermal states coming from the dangerous $O_8\bar{O}_8$-sector are at least massless (for details see [17]).

Thus, the lowest mass in the $O_8\bar{O}_8$ sector is never tachyonic but becomes massless when $R_0 = 1/\sqrt{2}$ and for particular values of the transverse moduli in the internal spatial lattice, giving rise to enhanced symmetry points. The one-loop string partition function is finite for all values of the thermal modulus $R_0$, and it enjoys the $T$-duality symmetry: $R_0 \rightarrow 1/(2R_0)$. The self-dual fermionic point $R_0 = 1/\sqrt{2}$ corresponds to the extended symmetry point.

Let us focus on a special point in moduli where the local Abelian gauge symmetry associated to the $X^0$ and $X^9$ cycles is extended to:

$$U(1)^2_L \times U(1)^2_R \times \cdots \rightarrow ([SU(2)_L]_{k=2} \times U(1)_L) \times (U(1)_R \times [SU(2)_R]_{k=2}) \times \cdots.$$  

(2.5)

The first factor and the second factor in each parenthesis on the r.h.s. correspond to the $X^0$, $X^9$-cycles respectively. This enhanced gauge symmetry allows us to re-interpret the deformation in terms of the “gravito-magnetic” fluxes as condensates of vortices carrying
non-trivial momentum and winding charges along the Euclidean-time and the $X^9$-cycles. Indeed, the marginal deformation switching on the “gravito-magnetic” fluxes is:

$$R^2_9 (G + 2B) \left( \partial X^0 \bar{\partial} X^9 \right) + R^2_9 (G - 2B) \left( \partial X^9 \bar{\partial} X^0 \right). \quad (2.6)$$

When $G = 2B = 1$, the second term vanishes. At the enhanced symmetry point, we are free to perform an $SU(2)_L \times SU(2)_R$ rotation\footnote{The three $[SU(2)_L]_{k=2}$-currents are $\partial X^0$, $\psi^0 \cos X^0_L$ and $\psi^0 \sin X^0_L$. At the enhanced symmetry point one may choose any one of the three. Notice that the $SU(2)$-symmetry currents under consideration also utilize the fermionic worldsheet super-partner $\psi^0$ [12].} in order to re-express the current-current deformation as:

$$\partial X^0 \bar{\partial} X^9 \rightarrow (\psi^0 \cos X^0_L) \times (\bar{\psi}^9 \cos X^0_R), \quad (2.7)$$

where $\psi^0$ ($\bar{\psi}^9$) is the worldsheet super-partner of $X^0_L$ ($X^0_R$). In other words, the deformation injects into the thermal vacuum non-trivial momentum and winding charges along the Euclidean-time and the $X^9$-cycles. The contribution to the free energy of the massive string states is effectively regulated, resolving the would-be divergences. This observation illustrates how the Hagedorn instabilities can be lifted once the Euclidean thermal vacuum acquires non-zero winding charge.

As we will see, the (Hagedorn-free) deformed thermal vacua exhibit interesting phase transitions. In the cosmological context, such tachyon-free thermal configurations will be considered in full detail in a class of two-dimensional $(4,0)$-vacua, the so-called Hybrid models, introduced recently in [17]. In this class of models, the modified free energy in each phase satisfies all the usual thermodynamical properties as a function of the temperature. Namely, as the temperature increases, the free energy decreases monotonically up to the phase transition and the specific heat is positive. At the phase transition, the temperature achieves a maximal critical value. As we will see, the resulting phase structure is not only free of the Hagedorn instabilities, but also the induced cosmology turns out to be free of the initial “Big Bang” curvature singularity.

3 The tachyon-free thermal Hybrid model

In this section we study thermal configurations of the two-dimensional Hybrid models introduced in [17]. The Hybrid models describe very special $(4,0)$-supersymmetric vacua: the
right-moving supersymmetries are broken spontaneously at the string level via the coupling of an internal cycle to $F_R$, and are replaced by the Massive Spectrum Degeneracy Symmetries (MSDS) of [9][18]. As in type II superstrings, we can define two versions of the model, Hybrid A and Hybrid B, depending on the relative chirality of the left- and right-moving Ramond sectors. In the maximally symmetric case, the partition functions are given by

$$Z = \frac{V_2}{(2\pi)^2} \int d^2 \tau \frac{4(\text{Im} \tau)^2}{\eta^4} \left[ \frac{1}{2} \sum_{a,b=0,1} (-)^{a+b+\mu ab} \frac{\theta_{[a \ b]}^4}{\eta^4} \right] \left[ \frac{1}{2} \sum_{\gamma,\delta=0,1} \frac{\theta_{[\gamma \delta]}^8}{\eta^8} \right] \left[ \frac{1}{2} \sum_{\bar{a},\bar{b}=0,1} (-)^{\bar{a}+\bar{b}} \frac{\theta_{[\bar{a} \bar{b}]}^{12}}{\bar{\eta}^{12}} \right],$$

where $\mu = 0$ in Hybrid B ($\mu = 1$ in Hybrid A) and $V_2$ stands for the volume of the two large (Euclidean) directions $X^0, X^1$. In terms of $SO(n)$ left- and right-moving characters, the partition function takes the form

$$Z = \frac{V_2}{(2\pi)^2} \int d^2 \tau \frac{4(\text{Im} \tau)^2}{\eta^8} \left( \bar{V}_{24} - \bar{S}_{24} \right) \frac{1}{\eta^8} \Gamma_{E_8}(\tau) \left\{ \begin{array}{c} (V_8 - S_8) \\ (V_8 - C_8) \end{array} \right\},$$

where the upper (lower) entry refers to the Hybrid B (Hybrid A) case. Thus, the left-moving sectors are described in terms of the $SO(8)$-characters, as in conventional superstring models, and in terms of the chiral $E_8$-lattice. The right-moving sectors are described in terms of the MSDS $SO(24)$-characters. In particular, the right-moving characters satisfy the identity

$$\bar{V}_{24} - \bar{S}_{24} = 24,$$

exhibiting the Massive Spectrum Degeneracy Symmetry of the right-moving sector.

The massless content of the Hybrid B (Hybrid A) model consists of $24 \times 8$ bosons and $24 \times 8$ fermions, arising from the $V_8 \ V_{24}$ and $S_8 \ \bar{V}_{24}$ ($C_8 \ \bar{V}_{24}$) sectors, respectively. There are no massless fermions from the right-moving R sector and the RR fields are massive. The model can also be described in terms of a freely-acting asymmetric orbifold compactification of the type II superstring to two dimensions. The relevant half-shifted $(8,8)$-lattice is given by

$$\Gamma_{(8,8)}[\bar{a} \ b] = \Gamma_{E_8} \times \bar{\theta}_{[\bar{a} \ b]}^8 \equiv \sum_{\hat{m}^I,n^J,G=2,...,9} \frac{\sqrt{\det G_{I,J}}}{(\sqrt{\tau_2})^8} \ e^{-\frac{1}{\tau_2} (G+B)(\hat{m}^I + \tau n^J)(\hat{m}^I + \tau n^J)} e^{i\pi (\hat{a} n^b + n^9 \hat{a} + \hat{m}^9 n^9)},$$

where all internal radii are at the fermionic point. The modular covariant cocycle describes

\[\text{Note that in this representation the fermionic point does not simply correspond to } R_i = 1/\sqrt{2}, \text{ as would be the case in simpler compactifications. The reason is the non-trivial coupling to } F_R \text{ which asymmetrically deforms the lattice. The specific values of the } (8,8)\text{-moduli corresponding to the fermionic/MSDS point are given in [17].}\]
the coupling of a single internal cycle of the lattice, which we will henceforth refer to as the $X^9$-cycle, to $F_R$ and leads to the spontaneous breaking of the right-moving supersymmetries. At the MSDS point the $G_{IJ}$ and $B_{IJ}$ tensors take very special values. In fact, the MSDS structure characterizing the right-moving sector gives rise to an enhanced local non-Abelian gauge group: $U(1)^8 \times [SU(2)_R]^8_{k=2}$. A detailed study of the classical moduli space of the MSDS and Hybrid vacua, as well as the study of marginal deformations connecting these vacua to four dimensional supersymmetric vacua, can be found in [17].

At finite temperature, the backreaction on the initially flat metric and dilaton background induces a cosmological evolution. As we saw in section 2, thermal tachyon-free configurations, able to bypass the Hagedorn instabilities, can be constructed in terms of freely-acting asymmetric orbifolds along the Euclidean time circle. In particular, these are obtained by modding out with $(-1)^{F_L}\delta_0$, where $\delta_0$ is a $Z_2$-shift along the Euclidean time circle, and can be interpreted as discrete deformations of the conventional finite temperature models. An important result is that they remain tachyon-free under arbitrary marginal deformations of the transverse dynamical moduli associated with the compact internal eight-manifold [17]. We will describe below some of their main features. Our notations concerning the wrapping numbers $(\tilde{m}, n)$, as well as the left- and right-moving momenta $(p_L, p_R)$ of the $\Gamma(1,1)$-lattice are as follows:

$$
\Gamma_{(1,1)}(R_0) = \frac{R}{\sqrt{\tau^2}} \sum_{\tilde{m}, n} e^{-\pi \frac{q^2}{\tau^2} |\tilde{m} + n\tau|} = \sum_{m, n} \Gamma_{m, n},
\Gamma_{m, n} = q^{\frac{1}{2}} p_L^{\frac{1}{2}} \bar{q}^{\frac{1}{2}} p_R^{\frac{1}{2}}, \quad q = e^{2\pi i \tau}, \quad \text{and} \quad p_{L,R} = \frac{1}{\sqrt{2}} \left( \frac{m}{R_0} \pm nR_0 \right).
$$

(3.5)

For sufficiently weak string coupling, the thermodynamic behavior of the system is captured by the one-loop partition function. In the Hybrid B case, this is given by [16,17]

$$
\frac{Z}{V_1} = \int_F \frac{d^2 \tau}{8\pi (\text{Im} \tau)^{3/2}} \frac{\Gamma_{E_8}}{\eta^8} \sum_{m, n} \left( V_8 \Gamma_{m,2n} + O_8 \Gamma_{m+\frac{1}{2},2n+1} - S_8 \Gamma_{m+\frac{1}{2},2n} - C_8 \Gamma_{m,2n+1} \right) (V_{24} - S_{24}),
$$

(3.6)

where $V_1 = 2\pi R_1$, whereas the Hybrid A case is obtained by the chirality exchange $S_8 \rightarrow C_8$. The $(-1)^{F_L}\delta_0$ orbifold acts thermally on the left-movers and breaks the $(4,0)$-supersymmetries spontaneously. On the other hand, the right-moving MSDS structure remains unbroken. In the odd-winding sector, the left-moving GSO projection is reversed, and so the $O_8$- and $C_8$-sectors appear in the spectrum. As the right-moving sector, however, begins at the massless
level, the model remains tachyon-free for all values of the thermal modulus $R_0$ (as follows by level-matching and the properties of the $\Gamma_{m+\frac{1}{2},2n+1}$-lattice). The Hybrid B and Hybrid A models are mapped into each other under the $T$-duality transformation $R_0 \rightarrow 1/(2R_0)$. The thermal quanta of Hybrid A are mapped into vortices, carrying non-trivial winding number, in the Hybrid B model and vice-versa.

As we already stressed, the one-loop partition function is finite for all values of the thermal modulus $R_0$. In the Hybrid model, the partition function can be computed explicitly thanks to the $MSDS$ identity $\bar{\mathcal{V}}_{24} - \bar{S}_{24} = 24$. The result is exact in $\alpha'$, and it is given in terms of a surprisingly simple expression \[17\]:

$$
\frac{Z}{V_1} = 24 \times \left( R_0 + \frac{1}{2R_0} \right) - 24 \times \left| R_0 - \frac{1}{2R_0} \right|.
$$

(3.7)

The essential feature is a discontinuity in the first derivative of $Z$ as a function of $R_0$, signaling a phase transition. The discontinuity occurs at the self-dual point $R_0 = 1/\sqrt{2}$. It is sourced by the lowest mass states in the $O_8\bar{\mathcal{V}}_{24}$-sector, which become massless precisely at the fermionic point. Indeed, the lowest mass in the $O_8\bar{\mathcal{V}}_{24}$ sector is given by

$$
m^2_{O\bar{V}} = \left( R_0 - \frac{1}{2R_0} \right)^2,
$$

(3.8)

and it is everywhere positive except for the fermionic point, where it vanishes. Similar behavior is exhibited in thermal configurations associated to non-critical heterotic strings in 2d \[24\].

At this point, several comments are in order:

- For $R_0 > 1/\sqrt{2}$, the partition function is given by

$$
\frac{Z}{V_1} = \frac{24}{R_0},
$$

(3.9)

coinciding with the conventional thermal partition function of $24 \times 8$ massless bosons and $24 \times 8$ massless fermions in two dimensions\[7\]. The corresponding temperature is $T = 1/\beta$, where $\beta = 2\pi R_0$. This result can be obtained by Poisson re-summing over the momentum quantum number $m$, and then mapping the integral over the fundamental domain to an integral over the strip \[25\], involving the zero winding ($n = 0$) orbits\[8\]. The latter are

\[7\]The contribution of each massless boson/fermion pair to the one-loop thermal partition function in 2 dimensions is $1/(8R_0)$.

\[8\]Due to the initial $(4,0)$-supersymmetry, the contribution of the modular invariant $(\tilde{m}, n) = (0,0)$ orbit vanishes.
contained in the even winding sectors $V_8 \bar{V}_{24}$, $S_8 \bar{V}_{24}$, $V_8 \bar{S}_{24}$ and $S_8 \bar{S}_{24}$. The splitting of the integral $(3.6)$ into its orbit by orbit contributions, and the subsequent mapping to the strip, is only valid for $R_0 > 1/\sqrt{2}$, since then the individual $(\tilde{m}, n) \neq (0, 0)$ orbit contributions in the Lagrangian lattice are exponentially suppressed as $\tau_2 \to \infty$ and we can safely exchange the orders of summation and integration\footnote{For $R_0 < 1/\sqrt{2}$, we can obtain the result either by applying $T$-duality or by first Poisson re-summing over the winding number $n$, which effectively maps the system on a large radius. See the discussion below.}. For $R_0 = 1/\sqrt{2}$, the lack of this absolute convergence due to the presence of additional massless states, results in the non-analytic part of $(3.7)$ \cite{17, 24}. So for $R_0 > 1/\sqrt{2}$ we obtain \cite{17}:

\[
\frac{Z}{V_1} = R_0 \sum_{\tilde{m} \neq 0}^{\infty} \int \frac{d^2 \tau}{8\pi(\text{Im}\tau)^2} e^{-\pi(\tilde{m}R_0)^2} \frac{\Gamma_{E_8}}{\eta^2} \left( V_8 - (-)^{\tilde{m}} S_8 \right) \left[ \bar{V}_{24} - \bar{S}_{24} \right].
\]

(3.10)

The sectors $V_8 \bar{V}_{24}$ and $S_8 \bar{V}_{24}$, which contain the initially massless bosons and fermions, are thermally excited as in the conventional thermal deformation. The deformation deviates from the conventional one in the sectors $V_8 \bar{S}_{24}$ and $S_8 \bar{S}_{24}$. These two sectors are massive, with masses of the order of the string scale, even at zero temperature. Due to the unbroken right-moving $MSDS$ symmetry, $\bar{V}_{24} - \bar{S}_{24} = 24$, and level-matching, the contributions of all massive boson and fermion string oscillator states cancel. Only the thermally excited massless states give non-vanishing net contributions, leading to the result $(3.9)$:

\[
\frac{Z}{V_1} = (24 \times 8) \frac{1}{\pi^2 R_0} \sum_{\tilde{m}=0}^{\infty} \frac{1}{(2\tilde{m} + 1)^2} = \frac{24}{R_0}.
\]

(3.11)

The integral over the strip allows one to rewrite the partition function $Z$ in terms of (the logarithm of) a (spacetime) “dressed” thermal trace over the 2nd quantized Hilbert space of the initially supersymmetric Hybrid B model. Because of the asymmetric nature of the deformation, this is given by the right-moving fermion index:

\[
Z = \ln \text{tr} \left[ e^{-\beta H} (-1)^{F_R} \right].
\]

(3.12)

This expression is strictly valid for $\beta = 2\pi R_0 > 2\pi/\sqrt{2}$, as indicated by the non-analyticity of the exact result for the partition function $(3.7)$. The right-moving fermion index differs from the canonical thermal trace in the odd-$F_R$ sector ($\bar{a} = 1$). Such states originate from the massive sectors $V_8 \bar{S}_{24}$ and $S_8 \bar{S}_{24}$ (with masses of the order of the string scale). As we have seen, in the presence of unbroken right-moving $MSDS$ symmetry, the contributions
of all states involving massive string oscillators cancel and so the right-moving fermion index reduces to a canonical thermal ensemble over the initially massless fields.\footnote{It would be interesting to see if these cancellations persist at the higher genus levels.} We can also understand the cancellations as follows. The contribution to $Z$ from a single bosonic (fermionic) field of mass $M$ and right-moving fermion number $\bar{a}$ is given by

$$\mp \sum_{k} \ln \left( 1 \mp (-1)^{\bar{a}} e^{-\beta \omega_k} \right), \quad \omega_k = \sqrt{k^2 + M^2}. \quad (3.13)$$

This shows that the contribution of a bosonic field with right-moving fermion number $\bar{a}$ cancels against the contribution of a fermionic field with the same mass and right-moving fermion number $1 - \bar{a}$. The right-moving MSDS symmetry requires that such massive bosons and fermions come in pairs.

- Consider now the canonical ensemble, defined in terms of the conventional (undressed) thermal trace. For sufficiently large $\beta$, the contributions of all states involving massive string oscillators are suppressed and, effectively, the thermal configuration reduces to a system of thermal radiation associated with the initially massless states in the sectors $V_8\bar{V}_{24}$ and $S_8\bar{V}_{24}$. In this sense, the deformed model does not differ appreciably from the conventional thermal system for large $\beta$. For values of $\beta$ close to the string length, massive string oscillators become relevant. The canonical ensemble fails to converge for $\beta < \beta_H = (2\pi)\sqrt{2}$, due to the exponential growth of the number of single-particle oscillator states. The introduction of the discrete gravito-magnetic fluxes effectively regulates this behavior. In their presence, the temperature can be further increased up to the fermionic point $\beta = (2\pi)/\sqrt{2}$, until one encounters a phase transition, which, in the present model, can be followed perturbatively.

- For $R_0 < 1/\sqrt{2}$, the partition function is given by

$$\frac{Z}{V_1} = 24 \times (2R_0). \quad (3.14)$$

This result can be obtained by applying the $R_0 \to 1/(2R_0)$ duality to equation \eqref{3.9}. Alternatively, it can be derived by Poisson-resumming over the winding number $n$ in equation \eqref{3.6} and unfolding the integral over the fundamental domain to an integral over the strip, as before. Essentially, $T$-duality maps the Hybrid B system at small radius $R_0$ to a thermal Hybrid A system at a large radius $1/(2R_0)$. The light states occur in the $V_8\bar{V}_{24}$ and $C_8\bar{V}_{24}$ sectors. In both of them, these are purely winding states (or vortices). They can be interpreted as ordinary thermal excitations, with temperature given by the inverse period of the
$T$-dual circle: $T = 1/\beta$, $\beta = 2\pi/(2R_0)$. This definition of the temperature in the small radius regime is in accordance with our expectation from conventional thermodynamics: typically the one-loop thermal partition function decreases as the temperature decreases. We see that the Hybrid B system at small radii is again effectively cold and better understood as a Hybrid A thermal system at large radii.

- At the self-dual point $R_0 = 1/\sqrt{2}$, there are 24 complex (or 48 real) additional massless states arising from the $O_8\overline{V}_{24}$-sector (see Eq. (3.8)). These states carry simultaneously non-trivial momentum and winding quantum numbers along the Euclidean time circle. Notice that, at the fermionic point, the corresponding values of the left- and right-moving momenta are given by

$$p_L = \pm 1, \quad p_R = 0,$$

and there is a further enhancement of the local gauge symmetry group associated with the compact Euclidean time circle [17]:

$$U(1)_L \times U(1)_R \rightarrow [SU(2)_L]_{k=2} \times U(1)_R.$$

The effect of these states in the partition function (3.7) is to induce the non-analytic part. The latter can also be expressed as $-24|m_{O\overline{V}}|$, which can, in turn, be understood in the following way. Near the extended symmetry point the contribution of the lowest mass states in the $O_8\overline{V}_{24}$-sector is given by

$$48 \times \int_1^\infty \frac{dt}{4\pi t} t^{-1/2} e^{-\pi m_{O\overline{V}}^2 t} = 48 |m_{O\overline{V}}| \frac{1}{4\sqrt{\pi}} \int_{\pi m_{O\overline{V}}^2}^\infty \frac{dy}{y^{1/2}} e^{-y}.$$  

(3.17)

In the limit $m_{O\overline{V}} \to 0$, the leading $m_{O\overline{V}}$-dependent contribution is

$$48 |m_{O\overline{V}}| \frac{1}{4\sqrt{\pi}} \Gamma\left(-\frac{1}{2}\right) = -24 |m_{O\overline{V}}|.$$

(3.18)

In summary, the system defined in Eq.(3.6) has three characteristic regimes, where different light states dominate the underlying effective field theory dynamics as the value of the thermal modulus is varied. There are distinct Hybrid B and Hybrid A phases related by “thermal duality symmetry”, $R_0 \rightarrow 1/(2R_0)$ together with $S_8 \leftrightarrow C_8$, and a phase transition between them at the self-dual point $R_0 = 1/\sqrt{2}$. In the Hybrid B phase $R_0 > 1/\sqrt{2}$, the light thermal states belong to the space of momenta, whereas in the Hybrid A phase $R_0 < 1/\sqrt{2}$, the light thermal states belong to the space of windings. The third intermediate
regime corresponds to the neighborhood of the fermionic point $R_0 = 1/\sqrt{2}$ and governs the transition between the two dual phases.

Thus it is necessary to utilize three distinct local effective field theories in order to describe fully the stringy thermal system. Alternatively, one can use a single non-local description in which the spacetime variables are doubled, utilizing constraint fields that depend simultaneously on variables conjugate to the momentum and winding modes \[26\]. In our specific model, the transition between the momentum and winding spaces can be exactly resolved in terms of localized branes, as we will discuss later.

At the fermionic point, the massless states form a localized $[SU(2)_L]_{k=2}$ symmetric space. Operators associated with the extra massless states, which are localized at the fermionic point $R_0 = 1/\sqrt{2}$, induce transitions between the purely momentum and winding modes. To see this, recall that the corresponding left- and right-moving momentum charges are given by equation (3.15). We denote the corresponding vertex operators, integrated over the worldsheet super-coordinates, $O_+$ and $O_-$ respectively. Next, consider a level-matched, purely momentum state in the $S_8 \bar{V}_{24}$ sector. At the critical point, such states with lowest mass have $p_L = p_R = \pm 1/2$. Let us focus on the state $|\frac{1}{2}, \frac{1}{2}\rangle$. Acting on it with $O_-$ lowers $p_L$ by one unit and leaves $p_R$ unchanged. The new state has $p_R = -p_L = \frac{1}{2}$ and it is a purely winding state. It corresponds to a level-matched state in the $C_8 \bar{V}_{24}$-sector. Pictorially, the action of the $O_-$ marginal operator can be described as

$$O_- \left| S_8; \frac{1}{2}, \frac{1}{2}\right\rangle \rightarrow \left| C_8; -\frac{1}{2}, \frac{1}{2}\right\rangle.$$  \(3.19\)

The explicit mapping can be understood as follows. In the 0-ghost picture, $O_-$ corresponds to the zero-mode of the holomorphic current $J_- = \psi^0 e^{-iX_0^L}$. Moreover, the $(-\frac{1}{2})$-picture vertex operator associated to the massive spacetime fermion state $|S_8; \frac{1}{2}, \frac{1}{2}\rangle$ takes the form $e^{-\phi/2} S_{10,\alpha} e^{\frac{i}{2}X_0^L + \frac{i}{2}X_0^R} \bar{V}_{24}$, where $S_{10,\alpha}$ is the spin-field transforming in the spinorial representation of $SO(10)$ (of definite chirality) and $\bar{V}_{24}$ is the right-moving oscillator contribution from the vectorial representation of $SO(24)$. In our conventions, the spinorial representation comes with odd chirality, so that the overall vertex operator be consistent with the GSO-projection. The action of $J_-$ on $|S_8; \frac{1}{2}, \frac{1}{2}\rangle$ yields:

$$J_-(z) e^{-\phi/2} S_{10,\alpha} e^{\frac{i}{2}X_0^L + \frac{i}{2}X_0^R} \bar{V}_{24}(w) = \frac{1}{z - w} \gamma^0_{\alpha\beta} e^{-\phi/2} C_{10,\beta} e^{\frac{i}{2}X_0^L + \frac{i}{2}X_0^R} \bar{V}_{24}(w) + \text{reg.},$$  \(3.20\)

where $\gamma^0$ is the familiar Dirac $\gamma$-matrix in 10 dimensions. As a result of this action, the
chirality of the spinorial representation changes. The new vertex operator then corresponds
to a physical state in the odd winding sector of the theory. Taking the contour integral picks
up the simple pole contribution from the OPE and we readily verify that $O_-$ indeed maps
the purely momentum state, $|S_8; \frac{1}{2}, \frac{1}{2}\rangle$, into the purely winding state, $|C_8; -\frac{1}{2}, \frac{1}{2}\rangle$, as stated
above.

To make the duality properties of the system more transparent, it is convenient to express
the thermal modulus in terms of the variable $\sigma \in (-\infty, +\infty)$:

$$R_0 = \frac{1}{\sqrt{2}} e^\sigma. \quad (3.21)$$

Then the physical, duality-invariant temperature can be written neatly as follows:

$$T = T_c e^{-|\sigma|}, \quad T_c = \frac{\sqrt{2}}{2\pi}. \quad (3.22)$$

This expression makes manifest the fact that the temperature of the system is bounded
from above, never exceeding a critical value: $T \leq T_c$. As we vary the thermal modulus
$\sigma$ adiabatically from $-\infty$ to $+\infty$, the system heats up to the critical temperature; it then
undergoes a stringy phase transition and, subsequently, it cools down. In terms of the
modulus $\sigma$ and the physical temperature, the partition function (3.7) can be written as

$$\frac{Z}{V_1} = (24\sqrt{2}) e^{-|\sigma|} = \Lambda T, \quad \Lambda = \frac{24\sqrt{2}}{T_c} = 48\pi, \quad (3.23)$$

where we introduced the thermal parameter $\Lambda$ being proportional to the number of thermally
excited light states. The two dual thermal phases occur at $\sigma > 0$ and $\sigma < 0$, respectively.
In each phase the pressure and energy density are given by

$$P = \Lambda T^2,$$

$$\rho = -P + T \frac{\partial P}{\partial T} = \Lambda T^2. \quad (3.24)$$

Therefore, in each phase the equation of state is, effectively, that of thermal massless radia-
tion in two dimensions: $\rho = P$. At low temperature, the two dual phases are distinguished
by the light thermally excited massless spinors: in the Hybrid B phase these transform under
the $S_8$-spinor of the internal $SO(8)$-symmetry group and in the Hybrid A phase in terms
of the conjugate $C_8$-spinor. In addition, the stringy thermal system is characterized by a
critical temperature $T_c$ and, consequently, by a critical energy density given by:

$$\rho_c = \Lambda T_c^2 = 24/\pi. \quad (3.25)$$
In the stringy thermal system, new light modes appear at $T_c$, which are responsible for the transition between the two dual phases. This transition is implied by the underlying $T$-duality of the model and the existence of an extended symmetry point. In particular, it should persist to all orders in the string genus expansion. As we will show in the following section, this phase transition admits a brane interpretation that glues together the space of “momenta” and the dual space of “windings”. This connection is absolutely transparent in the two-dimensional Hybrid model under consideration. In higher dimensional thermal models, we also expect similar gluing connections between the spaces of “momenta” and “windings”, described via localized branes at $T_c$. The simple conical structure of equation (3.7) will be replaced by a more involved structure according to the dimensionality of the Euclidean spacetime.

## 4 Non-singular cosmology

In this section, we will analyze the backreaction of the tachyon-free thermal Hybrid configuration on the initially flat metric and dilaton background. In several models the induced cosmological evolution in the intermediate regime $T \ll T_H$ has been extensively analyzed [27, 28], where an interesting attractor solution corresponding to a radiation-like era was found [28]. Other work in the context of hot string gas cosmology includes [29]. In this work, however, the important high temperature regime and the structure of the stringy phase transition at the Hagedorn temperature will be analyzed. As we have seen in section 3 the thermal stringy system has three characteristic regimes associated to the space of light momenta ($\sigma > 0$), the space of light windings ($\sigma < 0$) and the extended symmetry point which is localized at $\sigma = 0$ and connects the two. At sufficiently low temperature, the first two regimes are effectively described by local field theories involving the light states of the Hybrid B and A models, respectively. At the extended symmetry point ($\sigma = 0$), the effective description is in terms of a one-dimensional non-Abelian gauge theory, based on the gauge symmetry group

$$H_L \times H_R \equiv (SU(2) \times U(1)^8)_L \times (U(1) \times SU(2)^8)_R,$$

in contrast to the other two regimes where $H_L$ is broken down to the Cartan subgroup $H_L \to (U(1) \times U(1)^8)_L$. In all regimes the massless states are scalars transforming in the
The adjoint, adjoint representation of $H_L \times H_R$. 

The Hybrid A and Hybrid B phases are mapped into each other by thermal duality, implying the presence of a maximal critical temperature, $T = T_c \, e^{-|\sigma|}$. The non-vanishing energy density and pressure induce a cosmological evolution, during which the thermal modulus $\sigma$ becomes a monotonic function of time: $\sigma \to \sigma(t)$. Since $\sigma$ is a monotonic function, it can be used as a parametric time variable, scanning all three effective field theory regimes of the stringy thermal system. The cosmological evolution will be marked by a transition from the Hybrid A phase to the Hybrid B phase which, at finite coupling, is well localized at $\sigma = 0$. The phase transition is driven by the extra thermal states that become massless at this point and can be effectively described in terms of space-like branes, localized in time. Stringy space-like brane configurations in different context have been discussed in [30]. As we will see below, the induced cosmological evolution turns out to be non-singular: thanks to the presence of the space-like branes, the “Big Bang” singularity of general relativity is absent.

The scope of this section is to establish the existence of an effective Lorentzian action, able to describe the three stringy regimes simultaneously, which goes beyond the leading $\alpha'$-approximation and is valid up to the genus-1 level. Our guidelines are the symmetry properties and behavior of the thermal Hybrid model and its partition function, namely:

- The exact expression (3.7), beyond any $\alpha'$-approximation, of the one-loop partition function, exhibiting a specific conical structure that is sourced by the additional massless states at $\sigma = 0$.

- The effective equation of state in both Hybrid A and B phases, which is that of thermal massless radiation in 2d. Thanks to the unbroken right-moving $MSDS$ symmetry, the contributions to the partition function of all massive string oscillator states cancel.

- The thermal duality relation between the Hybrid A and Hybrid B phases, $\sigma \to -\sigma$, interchanging simultaneously the left-moving spinors: $S_8 \leftrightarrow C_8$. This symmetry, together with CPT invariance (always present in an effective field theory), imply that below the critical temperature the corresponding effective actions are related by time reversal, namely $\sigma(t) \to -\sigma(-t)$.

- The existence of the extended symmetry point at $\sigma = 0$ and its brane interpretation,
gluing the two phases A and B.

The above ingredients lead to an effective two-dimensional dilaton-gravity action, able to describe simultaneously and in a consistent way the three regimes of the stringy Hybrid model. The effective action is naturally separated into three parts:

\[
S = S_0 + S_1 + S_{\text{brane}},
\]  

where

\[
S_0 = \int d^2x \ e^{-2\phi} \sqrt{-g} \left( \frac{1}{2} R + 2(\nabla \phi)^2 \right),
\]

\[
S_1 = \int d^2x \sqrt{-g} \ P,
\]

\[
S_{\text{brane}} = -\kappa \int dx^1 d\sigma \ e^{-2\phi} \sqrt{g_{11}} \delta(\sigma).
\]

- \( S_0 \) is the familiar genus-0 dilaton-gravity action (written in the string frame).
- \( S_1 \) is the genus-1 contribution of the thermal effective potential \(-P\). The absence of a dilaton factor in this term indicates its origin as a 1-loop effect. The energy density and pressure are given by:

\[
\rho = P = \frac{\Lambda}{\beta^2}, \quad \beta = \beta_c e^{\sigma},
\]

where \( \beta \) is the inverse proper temperature, \( \beta_c = (2\pi)/\sqrt{2} \) the inverse critical temperature and \( \Lambda = 48\pi \) is the thermal parameter computed previously in Eq.(3.23). We emphasize once more that our one-loop computation of the energy density and pressure is exact in \( \alpha' \) as the contributions of the massive string oscillator states cancel due to the underlying right-moving \( MSDS \) symmetry.

- \( S_{\text{brane}} \) is the spacelike brane contribution at the phase transition. It gives rise to localized negative pressure and is sourced by the additional 24 complex massless bosons, \( \chi_i \) (\( i = 1, \ldots, 24 \)), at the extended symmetry point \( \sigma = 0 \). Indeed, the microscopic origin of this term follows from the underlying description of the system at the extended symmetry point, giving rise to a localized action:

\[
S_{\text{brane}} = \int dx^1 \sqrt{g_{11}} \ e^{-2\phi} \left( -g^{11} \frac{\partial \chi_i}{\partial x^1} \frac{\partial \bar{\chi}_i}{\partial x^1} \right) \bigg|_{\sigma(x^0)=0}.
\]

\footnote{In principle, higher-derivative terms have to be included in this action together with the contribution coming from the higher genus levels. As we will see later, such terms are suppressed during most of the cosmological evolution. In the region close to the phase transition their suppression is controlled by the value of the brane tension. The existence of an exact CFT description at the extended symmetry point allows us to have quantitative control even in this region.}
We will be interested in spatially homogeneous solutions for $g_{11}$ and $\phi$. Then the $\bar{\chi}_i$ equation of motion yields:

$$\frac{\partial^2 \chi_i}{\partial x^{12}} = 0 \quad \Rightarrow \quad \chi_i = \alpha_i + \gamma_i \sqrt{g_{11}} x^1,$$

(4.8)

where $\alpha_i$ and $\gamma_i$ are integration constants (invariant under $x^1$-reparametrization). Substituting the above solution back into the action, determines the value and sign of the brane tension in terms of the “rapidity” coefficients $\gamma_i$:

$$\kappa = \sum_{i=1}^{24} |\gamma_i|^2 = 24 |\gamma_s|^2.$$

(4.9)

In the most symmetric configuration all the rapidity coefficients are equal, $(\gamma_i = \gamma_s, i = 1, ..., 24)$, and so the brane tension is proportional to the number of the extra massless states.

Generically, in the effective action, several other light fields appear; namely, there are in total 64 moduli fields parametrizing the coset manifold

$$\mathcal{M} = \frac{SO(8,8)}{SO(8) \times SO(8)}.$$  

(4.10)

In all field theory phases of the thermal Hybrid model, the one-loop effective potential lifts these flat directions [28,31] and the above moduli are attracted to the enhanced $MSDS$ symmetry points. This justifies our inclusion in the action of only the dilaton-gravity background fields and the contribution of the brane. Thus, in all subsequent analysis, we will work in the case where all remaining dynamical (transverse) moduli are frozen to their $MSDS$ values so that the right-moving $MSDS$ symmetry of the theory is preserved.

We are now in a position to determine the induced stringy cosmological evolution by deriving the equations of motion in the presence of the brane term. We parametrize the metric as follows ($x^0 \equiv t$):

$$ds^2 = -N^2 dt^2 + a^2 dx^{12},$$

(4.11)

where the lapse function $N$ and scale factor $a = e^\lambda$ are functions of the time coordinate only. Without loss of generality, we choose the phase transition at $\sigma = 0$ to occur at $t = 0$. In terms of these variables, the action, valid for all times, is given by:

$$S = \int dt \, dx \, e^{-2\phi+\lambda} \left( \frac{1}{N} \left[ 2\dot{\phi} \dot{\lambda} - 2\dot{\phi}^2 \right] - \kappa \delta(t) \right) + \int dt \, dx \, Ne^\lambda \, P.$$  

(4.12)
Varying with respect to $N, \phi, \lambda$, we obtain the following system of equations:

\begin{align*}
\ddot{\phi}^2 - \dot{\phi} \dot{\lambda} &= \frac{1}{2} N^2 e^{2\phi} \rho, \quad \text{(4.13)} \\
\ddot{\lambda} - \dot{\lambda} \left( 2 \dot{\phi} - \dot{\lambda} + \frac{\dot{N}}{N} \right) &= N^2 e^{2\phi} P, \quad \text{(4.14)} \\
\ddot{\phi} - 2 \dot{\phi}^2 + \dot{\phi} \left( \dot{\lambda} - \frac{\dot{N}}{N} \right) &= \frac{1}{2} N^2 e^{2\phi} (P - \rho) - \frac{1}{2} N \kappa \delta(t) = -\frac{1}{2} N \kappa \delta(t). \quad \text{(4.15)}
\end{align*}

In deriving the first equation, we use the thermodynamical identity

\begin{align*}
-P - N \frac{\partial P}{\partial N} = -P - \beta \frac{\partial P}{\partial \beta} = \rho, \quad \text{(4.16)}
\end{align*}

following from the fact that in any frame the inverse proper temperature, or the period of the Euclidean time direction, is proportional to the lapse function $N$. The r.h.s. of equation (4.15) was simplified further by using the equation of state $\rho = P$. The crucial effect of the brane is to induce a discontinuity in the first time derivative of the dilaton at $t = 0$.

The thermal entropy (per co-moving unit cell) is given by

\begin{align*}
S = a \beta (\rho + P) = 2 \Lambda a \frac{\alpha}{\beta}. \quad \text{(4.17)}
\end{align*}

On each side of the brane this quantity is conserved, as can be derived by using the equations of motion. At the brane, the temperature is fixed attaining its maximal critical value, $\beta = \beta_c$. Continuity in the scale factor across the brane implies the continuity of the entropy and vice-versa. Equivalently, the stringy phase transition does not produce latent heat and is similar to a second order transition. More precisely, by utilizing the equations of motion, we derive the following equation:

\begin{align*}
\dot{\rho} + \dot{\lambda} (\rho + P) &= \frac{2 \kappa}{N} e^{-2\phi} \left( \dot{\lambda} - 2 \dot{\phi} \right) \delta(t), \quad \text{(4.18)}
\end{align*}

where the r.h.s. represents the brane contribution. Integrating (4.18), we find that the thermal entropy is given by

\begin{align*}
S(t) = S(t_0) + \kappa \beta \left[ a(0) \right. &\left. \frac{\alpha}{N(0)} e^{-2\phi(0)} \left( \dot{\lambda}(0^+) + \dot{\lambda}(0^-) - 2 \left( \dot{\phi}(0^+) + \dot{\phi}(0^-) \right) \right] \delta(t), \quad \text{(4.19)}
\end{align*}

\[12\] This equation exemplifies a deep connection between gravity and thermodynamics, since in the Euclidean description, where $N = e^{i\sigma}$ (with $x^0 \sim x^0 + 2\pi$), the thermodynamical identity is simply derived by the Euclidean gravity equations of motion. This non-trivial connection of gravity and thermodynamics will be explored in future work.
where \( \vartheta(t) \) is the step function and \( t_0 < 0 \). Continuity of the thermal entropy\(^{13}\) then imposes the following condition for the discontinuities in the first time derivatives of the scale factor \( \lambda \) and the dilaton \( \phi \):

\[
\lambda_+ + \lambda_- = 2 \left( \dot{\phi}_+ + \dot{\phi}_- \right),
\]

where from now on we simplify the notation to \( \dot{\lambda}(0^\pm) = \dot{\lambda}_\pm, \phi(0^\pm) = \phi_\pm \). When the thermal entropy is conserved, we obtain the following relation between the scale factor and the thermal modulus \( \sigma \):

\[
\lambda = |\sigma| + \ln \frac{\beta_c S}{2 \Lambda},
\]

Therefore, the scale factor \( a \) at any time must be greater than (or equal to) a certain critical size

\[
a \geq \beta_c \frac{S}{2 \Lambda},
\]

hinting at the absence of the conventional Big Bang singularity of general relativity.

The equations of motion simplify considerably by choosing an appropriate gauge. To this end, we utilize time-reparametrization invariance and work in the conformal gauge \( N = a \).

The equations of motion then become:

\[
\begin{align*}
(i) & : 2\dot{\phi}^2 - 2\dot{\phi}\dot{\lambda} = Ce^{2\phi}, \\
(ii) & : \ddot{\lambda} - 2\dot{\phi}\dot{\lambda} = Ce^{2\phi}, \\
(iii) & : 2\ddot{\phi} - 4\dot{\phi}^2 = -ke^{\lambda_0}\delta(t),
\end{align*}
\]

where \( a_0 \equiv e^{\lambda_0} \equiv e^{\lambda(0)} = \beta_c S/2\Lambda \) is the value of the scale factor when the system reaches its critical temperature and

\[
C \equiv \frac{S^2}{4\Lambda}
\]

is a constant being proportional to the number of light degrees of freedom.

Subtracting Eq.(4.23) from (4.24), we obtain \( \ddot{\lambda} - 2\dot{\phi}^2 = 0 \). Further subtracting the latter from Eq.(4.25), we obtain \( 2\ddot{\phi} - 2\dot{\phi}^2 - \ddot{\lambda} = -ke^{\lambda_0}\delta(t) \). Observe that both of these equations as well as Eq.(4.25) are independent of the constant \( C \). Their structure clearly shows that only the first time derivative of the dilaton is discontinuous across the phase transition, while its second derivative gives rise to the delta function brane source. The first time derivative

\(^{13}\)Some generalized solutions will be discussed in Section 5.
of the scale factor is continuous across the brane: \( \dot{\lambda}_+ = \dot{\lambda}_- \). The discontinuity in \( \dot{\phi} \) is given in terms of the brane tension:

\[
\dot{\phi}_+ - \dot{\phi}_- = -\frac{1}{2} e^{\lambda_0} \kappa. \tag{4.27}
\]

Using condition (4.20) together with the continuity of \( \dot{\lambda} \), we obtain \( \dot{\lambda}(0) = \dot{\phi}_+ + \dot{\phi}_- \). The conservation of the entropy (4.17) then implies

\[
\dot{\lambda} = \frac{\dot{\beta}}{\beta}, \tag{4.28}
\]

showing that the first time derivative of the temperature is also continuous across the critical point. The temperature attains its maximal (critical) value \( T_c \) at \( t = 0 \) and, thus, its derivative and the derivative of the scale factor must both vanish at this point: \( \dot{\beta}(0) = \dot{\lambda}(0) = 0 \). Eq. (4.20) then implies that the first time derivative of the dilaton flips sign \( \dot{\phi}_+ = -\dot{\phi}_- \) across the brane. The net result shows that the dilaton bounces during the transition.

Solving Eq. (4.25) we obtain the following expression for the time dependent dilaton:

\[
e^{2\phi(t)} = \frac{e^{2\phi_0}}{1 + 2\dot{\phi}_{-}|t|} \equiv g^2_{\text{str}}(t), \tag{4.29}
\]

where \( \dot{\phi}_- \) is a positive constant determined in terms of the brane tension \( \kappa \)

\[
\dot{\phi}_- = -\dot{\phi}_+ = \frac{1}{4} e^{\lambda_0} \kappa. \tag{4.30}
\]

The above expression shows that the string coupling \( g^2_{\text{str}}(t) \) remains smaller than a maximal value, \( g^2_{\text{str}}(0) = e^{2\phi_0} \), for all times and, therefore, the perturbative validity of the model is guaranteed, provided that \( g^2_{\text{str}}(0) \) is sufficiently small. The dilaton reaches its maximal value at the critical point and then decreases monotonically.

It turns out that the string coupling \( g_{\text{str}}(0) \) is not an arbitrary parameter. Indeed, from Eq. (4.23) evaluated at the critical point we obtain a relation between \( C \) and the derivative \( \dot{\phi}_- \) of the dilaton

\[
\dot{\phi}_-^2 = \frac{C}{2} e^{2\phi_0}. \tag{4.31}
\]

Combined with (4.30), it yields:

\[
e^{2\phi_0} = \frac{\kappa^2 \beta^2}{8 \Lambda} = 3\pi |\gamma_s|^4, \tag{4.32}
\]
where the last expression in terms of the rapidity coefficient $γ_s$ is obtained by utilizing Eq. (4.9), relating the brane tension $κ$ with the rapidity, the value of the critical temperature $β_c^{-1}$ and the numerical value of the thermal parameter $Λ$. The validity of the perturbative regime is ensured by the smallness of the rapidity coefficient $γ_s$.

The evolution of the scale factor is obtained by solving Eq. (4.24). The result is:

$$a^2(t) = e^{2λ(t)} = e^{2ϕ - |t|}. \quad (4.33)$$

It is convenient to perform a coordinate transformation $τ = 2ϕ - t, x = 2ϕ - x^1$. In the new coordinates, the cosmological solution for the metric and dilaton is:

$$ds^2 = \frac{4}{κ^2} \frac{e^{|τ|}}{1 + |τ|} \left( -dτ^2 + dx^2 \right), \quad (4.34)$$

$$g^2_{str}(τ) \equiv e^{2ϕ(τ)} = \frac{κ^2}{192} \frac{1}{1 + |τ|} \quad (4.35)$$

The metric has no essential singularities; both the Ricci scalar

$$R = \frac{κ^2}{4} \frac{e^{-|τ|}}{1 + |τ|}, \quad (4.36)$$

and the string coupling are bounded from above. Their maximal values occur at the critical point and are both proportional to the square of the brane tension, $O(κ^2)$. As a result, all higher-derivative terms, as well as higher-genus corrections, are controllable by a perturbative expansion in terms of $κ$, provided that $κ = 24|γ_s|^2 \ll 1$.

The above solution, even for large times, differs from the naive radiation-dominated cosmology, mainly because of the presence of a running dilaton. To see this, we switch to the cosmological frame, where the metric takes the form

$$ds^2 = -dξ^2 + a^2(ξ)d\tilde{x}^2. \quad (4.37)$$

The cosmological time $ξ$ is given in terms of the conformal parametric time $τ$ as:

$$ξ(τ) = \frac{2|τ|}{κ} \int_0^1 \frac{e^{|τ| u/2}}{\sqrt{1 + |τ| u}} du = \sqrt{\frac{8π}{eκ^2}} \left[ \text{erfi} \left( \frac{1 + |τ|}{2} \right) - \text{erfi} \left( \frac{1}{\sqrt{2}} \right) \right], \quad (4.38)$$

with \( \text{erfi}(x) \equiv \frac{1}{4} \text{erf}(ix) \) being the imaginary error function and $\tilde{x} = x/2$. For large cosmological times $ξ$, the asymptotic properties of the error function imply:

$$ξ = \frac{4}{κ} \frac{e^{|τ|/2}}{\sqrt{|τ|}} \left[ 1 + O \left( \frac{1}{|τ|} \right) \right] \implies e^{|τ|} = \left( \frac{κξ}{4} \right)^2 \ln(κξ)^2 + \cdots \quad (4.39)$$
In the very late cosmological time regime, the asymptotic expansions for the scale factor and string coupling are given by
\[ a(\xi) = |\xi| \left( 1 - \frac{1}{\ln(\kappa\xi)^2} + \cdots \right), \quad \frac{1}{g^2_{\text{str}}} = \frac{192}{\pi^2\kappa^2} \left( \ln(\kappa\xi)^2 + \ln(\kappa\xi)^2 + \text{const.} + \cdots \right). \] (4.40)

For large \(|\kappa\xi| \gg 1\), the cosmology asymptotes to the flat thermal Milne Universe. There are logarithmic corrections due to the running dilaton, giving rise to the above behavior. For small cosmological times \(|\kappa\xi| \ll 1\), the scale factor and the string coupling have the following expansion:
\[ a(\xi) = \frac{4}{\kappa} \left( 1 + \frac{1}{16}(\kappa\xi)^2 + \mathcal{O}(\kappa\xi^3) \right), \quad \frac{1}{g^2_{\text{str}}} = \frac{192}{\pi^2\kappa^2} \left[ 1 + \frac{|\kappa\xi|}{2} - \frac{1}{8}(\kappa\xi)^2 + \mathcal{O}(\kappa\xi^3) \right], \] (4.41)
illustrating very clearly that the cosmological scale factor bounces at the location of the brane. The dilaton, on the other hand, exhibits a conical structure, which results in a discontinuity in its first time derivative.

## 5 Generalized solutions

There are two linear combinations of the equations of motion which are independent of the genus-1 thermal contribution, namely
\[ 2\ddot{\phi} - 4\dot{\phi}^2 = -\kappa e^{\lambda_0}\delta(t), \quad 2\ddot{\phi} - 2\ddot{\lambda} = -\kappa e^{\lambda_0}\delta(t). \] (5.1)

The third equation
\[ 2\dot{\phi}^2 - 2\dot{\phi}\dot{\lambda} = Ce^{2\phi}, \]
depends on the thermal contribution through the conserved quantity \(C\), which is determined by the thermal entropy; see Eq.(4.26). The most general solution of the first two \(C\)-independent equations is the following:
\[ e^{2\phi} = \frac{e^{2\phi_0}}{1 + A|t| + Bt}, \quad e^{2\lambda} = e^{2\lambda_0} \frac{e^{A|t| + \Gamma t}}{1 + A|t| + Bt} \quad \text{with} \quad A = \frac{\kappa}{2} e^{\lambda_0}. \] (5.2)

It is very important that the integration constant \(A\), multiplying the conical part \(|t|\), is fixed by the brane tension \(\kappa\) and is, therefore, positive. All solutions satisfying \(|B| \leq A\) avoid the infinite coupling constant singularity and string perturbation theory remains
valid throughout the cosmological evolution. Substituting the general solution to the third equation, yields the following relations among the integration constants:

\[(A + B)(A + \Gamma) = 2C_+ e^{2\phi_0}, \quad \text{for} \quad t > 0,\]
\[(A - B)(A - \Gamma) = 2C_- e^{2\phi_0}, \quad \text{for} \quad t < 0,\]  
(5.3)

where we allow for the possibility of having a change in thermal entropy across the brane at \(t = 0\), giving rise to different conserved quantities \(C_+\) and \(C_-\) at the two sides. Continuity of the thermal entropy, \(C_+ = C_- = C \geq 0\), imposes \(B + \Gamma = 0\). In this case the weak coupling constraint \(|B| \leq A\) is automatically satisfied. The restriction of not exceeding the maximal temperature at the critical point, which is equivalent to \(\lambda(0) = 0\), further imposes \(B = \Gamma = 0\) and we obtain the non-singular, time-reversal invariant solution of section 4.

We proceed to explore other solutions with \(C_+ \neq C_-\), preserving, however, the maximal temperature constraint, which requires \(B = \Gamma\). A very intriguing non-singular solution arises when \(C_- = 0\), which also saturates the weak coupling constraint, \(A = B = \Gamma\):

\[e^{2\phi} = \frac{e^{2\phi_0}}{1 + A \left[|t| + t\right]}, \quad e^{2\lambda} = \frac{e^{2\lambda_0}}{1 + A \left[|t| + t\right]} \]  
(5.4)

For \(t < 0\), the spacetime metric is flat, \(\lambda = \text{const.}\), and the dilaton is constant. For \(t > 0\), the Universe is filled with thermal radiation. The physical relevance of this solution can be understood as follows. The flat \(t \leq 0\) region, including the brane at \(t = 0\), admits a Euclidean interpretation. The Euclidean path integral over it can be used to define the wave-function for the cosmology at \(t > 0\), which summarizes the very early-time history of the Universe, while for later times the Euclidean brane creates the radiation.

To understand this, we consider the Euclidean solutions, whose integration constants satisfy Eq.(5.3) after flipping the signs of the right hand sides. For \(C_- = 0, C_+ > 0\), the solution maintaining \(A = B\) satisfies \(2A(A + \Gamma) = -2C_+ e^{2\phi_0} = -(2A)^2\). The last equality follows by identifying the brane tensions in the Euclidean and Lorentizian solutions and yields \(\Gamma = -3A\). In half of this solution, for Euclidean time \(t_E = z < 0\), the dilaton is constant and the metric takes the form:

\[ds^2 = e^{2\lambda_0 - 4A z} \left[dz^2 + dx^2\right] = d\rho^2 + \rho^2 dy^2 \quad \text{where} \quad 2A \rho = e^{\lambda_0 - Az}, \quad y = 2Ax .\]  
(5.5)

In terms of the polar-like coordinates \(\rho\) and \(y\), the metric is locally flat, with the radial coordinate \(\rho\) being bounded from below: \(\rho \geq e^{\lambda_0}/(2A)\). When \(y \to i\omega\), the metric takes
the static Rindler form, where $\omega$ plays the role of time. Although this space is locally flat, it is also intrinsically thermal possessing an effective “geometrical” (entanglement) entropy. Thus, even though $C_-$ is zero in this system, we can associate to the half-solution a non-trivial entropy. The Rindler geometrical entropy is transformed via the brane into the thermal entropy of the late-time cosmological evolution. The half-Euclidean solution $z \in (-\infty, 0]$, which contains the brane, can be used to define the wave-function of the Universe described by Eq.\( \text{(5.4)} \) at $t = 0$. This mechanism will be investigated further in future work.

Another interesting Euclidean solution, with $C_+ = C_- = 0$, corresponds to $A = B = -\Gamma$, including the brane localized at Euclidean time $t_E = z = 0$:

$$
e^{2\phi} = \frac{e^{2\phi_0}}{1 + A |z|} \quad , \quad e^{2\lambda} = \frac{e^{4[|z| - z]}}{1 + A |z| + z} .$$

(5.6)

As before, when $t_E = z < 0$, the dilaton is constant and the metric is flat, taking the form given in Eq.\( \text{(5.5)} \) under the replacement $A \to A/2$. On the other side of the brane, $t_E = z > 0$, both the metric and the dilaton become non-trivial

$$
e^{2\phi} = \frac{e^{2\phi_0}}{1 + 2A z} \quad , \quad ds^2 = e^{2\lambda_0} \frac{dz^2 + dx^2}{1 + 2A z} .$$

(5.7)

After an appropriate coordinate change, this solution becomes:

$$
e^{2\phi} = \frac{e^{2\phi_0 + 2\lambda_0}}{A^2} \frac{1}{\rho^2} \quad , \quad ds^2 = d\rho^2 + \frac{1}{\rho^2} dy^2 \quad \text{where} \quad A \rho = e^{\lambda_0} \sqrt{1 + 2Az} \quad , \quad y = \frac{e^{2\lambda_0}}{A} x .$$

(5.8)

In this parametrization, we recognize the well-known $T$-dual geometry of the two-dimensional flat space. Here, again, the coordinate $\rho$ is bounded from below, $\rho \geq e^{\lambda_0}/A$, thus avoiding the well-known singularity of the dual-to-flat metric at $\rho = 0$. The string coupling is bounded from above by its value at the position of the brane, $g_{str}^2 \leq e^{2\phi_0}$. The dual-to-Rindler space is obtained by $y \to \mathrm{i}\omega$, and so in this sense, it is also thermal possessing a non-zero “geometrical” entropy. On either side of the brane, the Euclidean solution is supersymmetric\[32\], as is also implied by the choice $C_\pm = 0$. The brane, however, breaks supersymmetry locally and creates the geometrical entropy in the Euclidean. The Euclidean solution clarifies further the importance of the brane. Namely, the brane glues the two dual geometries together in a consistent way.

The time-reversed solution of Eq.\( \text{(5.4)} \) has $C_+ = 0$ and $C_- = C$; it can be obtained when $B = -A < 0$. In this case the spacetime is flat and the dilaton is constant for positive
The Universe is filled with thermal radiation at negative times. As a result, the Universe is static with a thermal Rindler interpretation at later times. In this cosmological solution, the thermal entropy at early times is transformed via the brane at $t = 0$ to the geometrical entropy associated to the late-times Rindler space.

Another interesting solution is obtained by combining these two solutions, assuming branes localized at two different times, namely at $t = \pm \alpha$. A time-reversal invariant solution can be easily derived,

$$e^{2\phi} = \frac{e^{2\phi_0}}{1 + A [ |t + \alpha| + |t - \alpha| - 2\alpha]}, \quad e^{2\lambda} = \frac{e^{2\lambda_0}}{1 + A [ |t + \alpha| + |t - \alpha| - 2\alpha]},$$

with the following behavior:

i) for $t < -\alpha$, the solution for the dilaton and the metric is non-trivial, describing a contracting thermal Universe, with entropy coefficient $Ce^{2\phi_0} = 2A^2$.

ii) for $t > \alpha$, the dilaton and the metric have non-trivial time-dependence, describing an expanding thermal Universe with the same entropy coefficient as in i), $Ce^{2\phi_0} = 2A^2$.

iii) in the intermediate region $-\alpha < t < \alpha$, the metric looks locally flat and the dilaton is constant.

The intermediate flat region $t \in [-\alpha, \alpha]$, including the branes, admits a Euclidean interpretation, and as we explained before in the half-solutions it possesses a geometrical entropy and temperature in the Rindler cosmological frame. Thus the thermal entropy of region i) is transformed via the first brane at $t = -\alpha$ into the geometrical entropy of the intermediate region, which in turn is transformed back to thermal entropy in region ii) via the second brane at $t = \alpha$. In the limit $2\alpha \to 0$, we recover the solution presented in section 4. Thus, the parameter $\alpha$, describing the separation of the branes, provides a new relaxation time scale for the contracting-to-expanding cosmological phase transition. The whole picture can be understood via Euclidean instanton transitions between contracting and expanding thermal Universes. The pure Euclidean as well as the Rindler and dual-to-Rindler solutions will be investigated further in future work.

6 Conclusions

In this work we identified string-theoretic ingredients which can be used to resolve both the Hagedorn instabilities of thermal string theory, as well as the initial “Big Bang” singularity
of the induced cosmological evolution. In order to incorporate these ingredients, it was necessary to work beyond the low energy effective field theory approximation, examining purely stringy phenomena. For technical reasons, we focused on simple \((4,0)\)-supersymmetric string models, where adequate quantitative control over the dynamics is available. More specifically, we considered the class of two-dimensional Hybrid models, which are characterized by a very special asymmetry between the left- and right-moving sectors. As in the usual superstring models, the left-moving sector is supersymmetric, giving rise to the 16 real spacetime supercharges. The right-moving sector is non-supersymmetric at the massless level, but it enjoys the \(MSDS\) symmetry structure, which guarantees boson/fermion degeneracy in all of the right-moving massive levels.

We considered tachyon-free, thermal configurations of the Hybrid models, where in addition to the temperature, we turn on certain discrete “gravito-magnetic” fluxes threading the compact Euclidean time circle. These fluxes inject non-trivial momentum and winding charges into the thermal vacuum, which effectively regulate the contributions to the thermal partition function of the massive string oscillator states and restore the thermal \(T\)-duality symmetry of the finite temperature stringy system. As we have seen, the tachyon-free thermal configurations can be described in terms of freely acting asymmetric orbifolds, which act thermally on the left-movers but leave the right-moving \(MSDS\) symmetry unbroken. Thanks to the latter symmetry, the one-loop thermal partition function was explicitly calculated, beyond any \(\alpha'\)-approximation.

The thermal Hybrid systems have three characteristic regimes, associated to the space of light thermal momenta, the space of light thermal windings and the extended symmetry point, which arises at the self-dual value of the thermal modulus and connects the two. At the extended symmetry point, extra massless thermal states appear, with a clear brane interpretation in the Euclidean. These localized states are responsible for a phase transition between the two dual thermal phases. The physical temperature of the system is bounded from above, attaining its maximal critical value at the phase transition.

An important result of this work is that we succeeded in writing down a stringy effective Lorentzian action able to incorporate the three stringy regimes simultaneously, which is exact in \(\alpha'\) at the two derivatives level and is valid up to genus-1. The essential feature in this action is the spacelike brane contribution that glues together the spaces of light
thermal windings and light thermal momenta. This brane is sourced by the additional massless thermal states, which appear at the extended symmetry point. We explicitly showed how the sign and the value of the brane tension are determined in terms of the “rapidity” coefficients characterizing the classical backgrounds of the corresponding localized thermal fields. The spacelike brane, together with the bulk thermal corrections, induce a non-singular cosmological evolution, describing a bouncing thermal Universe. The bounce coincides with the phase transition between the two dual Hybrid thermal phases. The cosmology remains in the weak coupling and low-curvature regimes, provided that the rapidity coefficients which determine the tension of the brane are small. Continuity of the thermal entropy across the brane uniquely determines the cosmological solution, which is time-reversal invariant.

Separated spacelike branes, localized at different times, transform the thermal entropy into a geometrical entropy associated with a quasi-static thermal Rindler space (or its dual) and vise-versa. Utilizing this property of the branes, we obtained more general, time-reversal invariant solutions, which are also free of the initial Big Bang singularity. The obtained non-singular solutions describe transitions between a contracting thermal Universe and an expanding one. The effect of the spacelike branes is to induce an intermediate relaxation-time period during which the scale factor of the Universe is quasi-static, and the entropy appears in a geometrical Rindler-like form. The intermediate locally flat region, including the branes, admits a Euclidean interpretation describing instanton transitions among contracting and expanding thermal Universes. We would like to investigate this connection further in the near future.

To our knowledge, the two-dimensional thermal Hybrid model presented in this work is the first example in the literature where both the stringy Hagedorn singularities as well as the classical Big Bang singularity are both successfully resolved.

We believe that phenomenologically relevant, higher dimensional string models exist, where the essential ingredients, used in the thermal Hybrid model to overcome the initial singularity problem, are also present. Namely: i) the restoration of thermal duality symmetry in the presence of non-trivial discrete “gravito-magnetic” fluxes, ii) the appearance of new light thermal states at the self-dual point, giving rise to a localized higher-dimensional brane interpretation and iii) the gluing of dual thermal phases via higher dimensional spacelike branes, before the occurrence of the Big Bang singularity that is always present in the
naive field theory approximation. We conjecture that in the higher dimensional cases, the resulting duality structure which encompasses all these ingredients will be much more intricate, involving other supersymmetry breaking scales and perhaps non-perturbative string phenomena as well. Although the technical challenges in the higher dimensional cases are expected to be more delicate, the key property, which is the joint effect of string dualities and branes “protecting” the stringy system from the occurrence of spacelike cosmological singularities, could still be realized. Obviously, more work needs to be dedicated to this ambitious direction in order to obtain non-singular cosmological models that will be phenomenologically viable.

Acknowledgements

We are grateful to C. Bachas, M. Bianchi, R. Brandenberger, C. Callan, J. Estes, M. Green, W. Lerche, D. Luest, S. Patil, T. Tomaras and J. Troost for fruitful discussions. I.F., C.K. and H.P. would like to thank the University of Cyprus for hospitality. N.T. and H.P. acknowledge the Laboratoire de Physique Théorique of Ecole Normale Supérieure for hospitality. C.K and H.P. thank C.E.R.N Theory Division where part of this work was discussed. N.T. would like to thank the Centre de Physique Théorique of Ecole Polytechnique for hospitality. The work of I.F., C.K. and H.P. is partially supported by the ANR 05-BLAN-NT09-573739, and a PEPS contract. The work of I.F., C.K., H.P. and N.T. is also supported by the CEFIPRA/IFCPAR 4104-2 project. The work of H.P. is partially supported by the EU contracts PITN GA-2009-237920 and ERC-AG-226371, and PICS contracts France/Greece and France/USA.

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