I. INTRODUCTION

For almost thirty years it has been known that the velocity \(v_s\) of surface acoustic waves (SAWs) in piezoelectric crystals can be affected by the electrical properties of nearby conductors.\(^1\) If the nearby conductors are dissipative, then they can allow the SAW to attenuate also. Work by Ingebrigtsen\(^2\), and later authors\(^3\), showed that when a piezoelectric is brought next to a thin layer of a conducting medium, the SAW velocity shift \(\Delta v_s\) and the attenuation coefficient \(\kappa\) satisfy the relation

\[
\frac{\Delta v_s}{v_s} = \frac{i\alpha}{q} = \frac{\alpha^2/2}{1 + i\sigma_{xx}(q,\omega)} \sigma_m
\]

where \(\sigma_{xx}(q,\omega)\) is the longitudinal conductivity of the adjoining medium at wavevector \(q\) and frequency \(\omega = \omega_s q\) with \(\omega_s\) the velocity of the SAW. The coefficients \(\alpha\) and \(\sigma_m\) are calculated and it is found that \(\alpha\) has a nontrivial dependence on the product \(qd\).

When a surface acoustic wave (SAW) is coupled piezoelectrically to a two dimensional electron gas (2DEG), a velocity shift and attenuation of the SAW are induced that reflect the conductivity of the 2DEG. This paper considers the case of a AlGaAs heterostructure with a 2DEG a distance \(d\) from a (100) surface of the crystal where the SAWs are propagated in the [011] direction at wavevector \(q\). It is found that the velocity shift \(\Delta v_s\) and the attenuation coefficient \(\kappa\) satisfy the well known equation

\[
(\Delta v_s/v_s) - (i\alpha/q) = (\alpha^2/2) \left(1 + \frac{\sigma_{xx}(q,\omega)}{\sigma_m}\right)
\]

where \(\sigma_{xx}(q,\omega)\) is the complex conductivity at wavevector \(q\) and frequency \(\omega = \omega_s q\) with \(\omega_s\) the velocity of the SAW. The coefficients \(\alpha\) and \(\sigma_m\) are calculated and it is found that \(\alpha\) has a nontrivial dependence on the product \(qd\).
Fig. 1: Model Geometry for Surface Acoustic Wave Experiments. In the experiments the spacing \( d \) is typically between 1000 and 5000 Angstroms. The Al\(_x\)Ga\(_{1-x}\)As typically has a fraction of Al given by \( x \approx 30\% \).

II. RESPONSE FUNCTIONS

The density-density response \( K_{00}(q, \omega) \) is defined by the relation

\[ n(q, \omega) = K_{00}(q, \omega)\phi^{\text{ext}}(q, \omega) \]  

where \( \phi^{\text{ext}} \) is the perturbing externally applied scalar potential applied at a frequency \( \omega \) and wavevector \( q = qx \), and \( n(q, \omega) \) is the induced fluctuation density. As we will see below, the SAW experiments directly measure \( K_{00} \) at finite frequency and wavevector.

Many linear response measurements do not measure, however, the ratio of induced density to the externally applied potential but rather the response to the total potential. A density \( n(q) \) induced by the external vector potential, gives rise to a Coulomb scalar potential

\[ \phi^{\text{ind}}(q, \omega) = -v(q)n(q, \omega) \]

where \( v(q) = \frac{2\pi}{\epsilon_{\text{eff}}q} \) is the Fourier transform of the usual Coulomb interaction \( v(r) = 1/|\epsilon_{\text{eff}}|r| \). (In principle, currents in the sample give rise to an induced vector as well as scalar potential, but in practice these fields are negligible). Here \( \epsilon_{\text{eff}} \) is the effective background dielectric constant. The wavevector dependent form of this dielectric constant for the case of the model geometry of Fig. 1 is derived in the appendix, and is given by

\[ \frac{\epsilon_{\text{eff}}}{\epsilon} = \frac{1}{2} \left( \frac{(\epsilon + \epsilon_0) \exp(qd)}{\epsilon \cosh(qd) + \epsilon_0 \sinh(qd)} \right) \]

where \( \epsilon \) is the dielectric constant of the bulk Al\(_x\)Ga\(_{1-x}\)As and \( \epsilon_0 \) is the dielectric constant of the medium above the surface (\( \approx 1 \)). The dielectric constant \( \epsilon_{\text{eff}} \) for Al\(_x\)Ga\(_{1-x}\)As with \( x \approx .3 \) is approximately 12.5 (which is slightly lower than the dielectric constant for GaAs which is approximately 13.0).

Using Eq. 3, the total scalar potential

\[ \phi^{\text{tot}} = \phi^{\text{ext}} + \phi^{\text{ind}} \]

is now written as

\[ \phi^{\text{tot}} = (1 - v(q)K_{00})\phi^{\text{ext}}. \]

It then becomes useful to define the polarization \( \Pi_{00}(q, \omega) \), which relates the induced density \( n(q, \omega) \) to the total scalar potential via

\[ n(q, \omega) = \Pi_{00}(q, \omega)\phi^{\text{tot}}(q, \omega) \]

Combining this definition with Eqs. 2 and 6 yields the equation

\[ [K_{00}]^{-1} = [\Pi_{00}]^{-1} + v(q). \]

Since the response function \( \Pi_{00} \) relates the density to the total vector potential, it is useful to write this function in terms of the conductivity \( \sigma_{\alpha\beta} \) which relates the two spatial components of the current \((j_x, j_y)\) to the two spatial components of the total electric field \((E_x, E_y)\) via \( \mathbf{j} = \sigma \mathbf{E} \). Using current conservation to give \( j_x = (\omega n/q) \), then yields

\[ \sigma_{xx} = -\frac{i\omega}{q^2} \Pi_{00} \]

In particular, this allows us to write the general relation

\[ [K_{00}]^{-1} = v(q) - \frac{i\omega}{q^2 \sigma_{xx}} \]
Throughout this work, we will assume that $\omega = v_s q$ with $v_s$ the SAW velocity, which then implies

$$K_{00}(q, \omega) = \frac{\epsilon_{\text{eff}} q}{2\pi(1 - i\sigma_m/\sigma_{xx}(q, \omega))}$$

with

$$\sigma_m = \frac{v_s \epsilon_{\text{eff}}}{2\pi}.$$  

The function $\sigma_m(qd)$ is shown in Fig. 2. Here, the experimentally relevant parameters for References 3 and 4 are used. These are $\epsilon = 12.5$ and $v_s = 3010\, m/sec$ (see section IV below).

### III. INDUCED ENERGY SHIFT

Due to the piezoelectric coupling, an external scalar potential $\phi_{\text{ext}}$ is induced in the 2DEG. For now, we will write

$$\phi_{\text{ext}} = C e_{14} F(qd)/\epsilon$$

where $C$ is the amplitude of the SAW, $e_{14}$ the piezoelectric stress constant, and $F$ is a dimensionless function of $qd$ that represents the fact that the SAW decays into the bulk. Clearly $F$ should approach a constant as $qd \to 0$ and should approach zero as $qd \to \infty$. Roughly, one should expect that the function $F$ should decay as $e^{-qd}$.

The induced energy density per unit area due to this external potential is given by

$$\delta U = \frac{1}{2} K_{00} |\phi_{\text{ext}}|^2$$

This expression is obtained from integrating a differential $d\delta U = n(\phi_{\text{ext}}) d\phi_{\text{ext}}$ and using Eq. 3. (Note that using $\phi_{\text{tot}}$ here instead would account for only the electrical energy). Using Eq. 10 we can rewrite this shift as

$$\delta U = \frac{\epsilon_{\text{eff}} q}{4\pi(1 - i\sigma_m/\sigma_{xx}(q, \omega))} |\phi_{\text{ext}}|^2$$

We now want to measure this energy shift with respect to the shift for $\sigma_{xx} \to \infty$. Thus,

$$\Delta U = \delta U - \delta U(\sigma_{xx} = \infty)$$

$$= \frac{\epsilon_{\text{eff}} q}{4\pi} \left[ \frac{1}{1 - i\sigma_m/\sigma_{xx}(q, \omega)} - 1 \right] |\phi_{\text{ext}}|^2$$

$$= \frac{\epsilon_{\text{eff}} q}{4\pi} \left[ \frac{1}{1 + i\sigma_{xx}(q, \omega)/\sigma_m} \right] |\phi_{\text{ext}}|^2$$

It is found below that the surface acoustic wave has an energy density proportional to $C^2 q^2$ where $C$ is the amplitude of the wave and $q$ is the wavevector. Furthermore, the wave decays exponentially into the bulk with a decay constant proportional to $q$. Thus, when integrated in the $\hat{z}$ direction, the energy $U$ per unit surface area is given by

$$U = q C^2 H$$

where $H$ is a factor that depends on material parameters that we will determine below. Combining this with the results of the above section, the fractional energy shift is then given by

$$\frac{\Delta U}{U} = \frac{\alpha^2/2}{1 + i\sigma_{xx}(q, \omega)/\sigma_m}$$

where

$$\frac{\alpha^2}{2} = \frac{\epsilon_{\text{eff}} e_{14}^2}{\epsilon} \frac{1}{4\pi \rho \epsilon H} |F(qd)|^2$$

(Note that the factor of $4\pi$ will vanish when $\epsilon_{\text{eff}}$ is converted into MKSI units). Clearly, this result implies the velocity shift and attenuation relation given by Eq. 21. All that now remains is the tedious job of evaluating the constant $H$ as well as the functional form $F$. It should be noted that in the small $qd$ limit, various experiments have measured the value of the coupling constant and have found $\alpha^2/2 \approx 3.2 \times 10^{-4}$. As is discussed below in section IV, these measurements should be viewed with caution. As discussed above, one expects roughly that $F$ decays as $e^{-qd}$ so that $\alpha^2/2$ decays as $e^{-2qd}$. This, however, contradicts experimental observation. Below, in a more careful analysis, we will see why the decay is actually somewhat slower and shows a nonmonotonic dependence on $qd$.

### IV. NON-PIEZOELECTRIC SAWS

We begin by discussing the solution of the SAW equations with the piezoelectric coupling set to zero. The piezoelectric coupling will then be added at lowest order.

Defining a displacement vector $u_k$, the elastic wave equation is given by

$$c_{ijkl} \partial_i \partial_j u_k + \rho \ddot{u}_j = 0$$

where $\rho$ is the mass density, $c$ is the elastic tensor, we have used the notations $\partial_i f = \frac{\partial f}{\partial x_i}$, $f = \frac{\partial f}{\partial t}$, and repeated indices are summed. For GaAs, AlAs, and other crystals of cubic symmetry there are only 3 independent elastic constants. These constants are conventionally called $c_{11}$, $c_{12}$, and $c_{44}$. For GaAs at low temperatures, the elastic constants $c_{11}$, $c_{12}$, and $c_{44}$ are given by $12.26 \times 10^{10}$, $5.71 \times 10^{10}$, and $6.00 \times 10^{10} \, N/m^2$ respectively. The constants for AlAs are given approximately by $12.2 \times 10^{10}$, $5.5 \times 10^{10}$, and $5.7 \times 10^{10} \, N/m^2$ respectively. It is noted that the elastic constants of the two materials are roughly the same. For $Al_xGa_{1-x}$As it is reasonable to interpolate for any value of $x$. (Experimentally, there may be some uncertainty in $x$). The density of GaAs is $5307 \, kg/m^3$, and the density of AlAs is $3598 \, kg/m^3$. Thus for $Al_xGa_{1-x}$As with $x \approx 0.3$, the
density interpolates to approximately 4794 kg/m³, which differs from that of GaAs by only 10%.

In considering surface waves, the wave equation must be supplemented with the boundary condition at the free surface that there is no total force at the surface. This condition is written as

\[ c_{zjk} \partial_t u_k = 0. \]  \hspace{1cm} (23)

where the subscript \( z \) represents the direction normal to the surface. For certain geometries, analytic solutions of the SAW equations are available. In the present case of a (100) surface with wave propagation in the [011] direction, the velocity of SAW propagation is given by the solution (here we are interested in the lowest velocity solution) of the cubic equation

\[
(1 - \frac{c_{11}}{c_{44}}) \left( \frac{c_{11} c_{11} - c_{12}^2}{c_{11}} - X \right)^2 = X^2 \left( \frac{c_{11}}{c_{11}} - X \right)
\]

where \( c_{11} = \frac{1}{2} (c_{11} + c_{12} + 2c_{44}) \) and \( X = \rho v^2/c_{11} \) gives the SAW velocity \( v_s \).

For Al₁₋ₓGaₓAs with \( x \approx 0.3 \), the velocity is approximately 3010 m/sec. (This differs from that of pure GaAs by only 5%). Once the velocity is determined, one can easily solve analytically for the form of the SAW. For the experimental geometry we are presently considering, the displacements for this wave can be written as

\[
u_x = C \left( e^{-\Omega qz - i\varphi} + e^{-\Omega qz + i\varphi} \right) e^{i(q(x - vx)t)}\]

(25)

\[
u_z = C \left( \gamma e^{-\Omega qz - i\varphi} + \gamma^* e^{-\Omega qz + i\varphi} \right) e^{i(q(x - vx)t)}\]

(26)

with \( \nu_x = 0 \) and \( C \) the amplitude \( (C \) has dimensions of length). Here, the \( x \) direction is the direction normal to the surface (the [011] direction). The parameters \( \Omega, \gamma, \) and \( \varphi \) are determined by

\[
0 = (c_{11}' - X c_{11} - \Omega^2 c_{11}) (c_{44} - X c_{11} - \Omega^2 c_{44}) + \Omega^2 (c_{12} + c_{44})^2
\]

(27)

\[
\gamma = \Omega \left[ \frac{c_{12} + c_{44}}{c_{44} - (X + \Omega^2) c_{11}} \right]
\]

(28)

\[
e^{-2i\varphi} = -\frac{\gamma^* - \Omega^*}{\gamma - \Omega}
\]

(29)

In the case of Al₁₋ₓGaₓAs with \( x = 0.3 \), the values of \( \gamma \) and \( \Omega \) and \( \varphi \) are given in this case by \( \Omega \approx 501 + 472i \), \( \gamma \approx -705 + 1146i \), and \( \varphi \approx 1.06 \).

The local energy density of this wave can be written as

\[
E = \frac{1}{2} c_{ijkl} \partial_i \nu_k + \nu_i \partial_i \nu_k + \rho \dot{\nu}_j = 0
\]

(30)

where \( \nu_{ij} \) is given by

\[
\nu_{ij} = \frac{1}{2} (\partial_i \nu_j + \partial_j \nu_i)
\]

(31)

For the AlGaAs surface wave discussed above, the energy density can be written as

\[
E = \frac{1}{2} (c_{11}' |u_{xx}|^2 + c_{11} |u_{zz}|^2 + 2c_{12} Re[u_{xx}^* u_{zz}] + c_{44} |2u_{xx}|^2)
\]

(32)

Inserting the above described form of the wave, yields the strains

\[
u_{xx} = iq u_x
\]

(33)

\[
u_{zz} = iq C (\gamma \Omega e^{-\Omega qz - i\varphi} + c.c.) e^{iq(x - vx)v}
\]

(34)

\[
2u_{xx} = C q [\gamma - \Omega] e^{-\Omega qz + i\varphi} + c.c. e^{iq(x - vx)v}
\]

(35)

with “c.c.” meaning complex conjugate. Finally, integrating the result of Eq. 32 in the \( z \) direction yields an energy per unit surface area in the form given by Eq. 19 with

\[
H = Re \left[ c_{11}' \left( \frac{e^{-2i\varphi}}{\Omega} + \frac{1}{\alpha} \right)
\]

(36)

\[
+ c_{11} \left( \frac{(\gamma - \Omega)^2 e^{-2i\varphi} + |\gamma - \Omega|^2}{\alpha} \right)
\]

\[
+ c_{44} \left( \gamma e^{-2i\varphi} + \frac{Re(\gamma)}{\alpha} \right)
\]

\[
+ 2c_{12} \left( \gamma e^{-2i\varphi} + \frac{Re(\gamma)}{\alpha} \right) \]

(37)

where \( \alpha = Re(\Omega) \). This yields a numerical value of \( H \approx 28.8 \times 10^{10} \text{N/m}^2 \).

The value for pure GaAs is lower by only about 2%.

V. PIEZOELECTRIC COUPLING

When a piezoelectric coupling is added, the wave equations take the form

\[
c_{ijkl} \partial_i \partial_k u_k + c_{ijkl} \partial_i \nu_j + \rho \dot{\nu}_j = 0
\]

(38)

\[
e_{ijkl} \partial_i \nu_k - 2 \epsilon \nabla^2 \phi = 0
\]

(39)

where \( \epsilon \) is the piezoelectric stress tensor, \( \phi \) is the electric potential, and \( \epsilon \) is the dielectric constant of the medium (here \( \epsilon \) is assumed to be isotropic). For GaAs, AlAs (and other cubic crystals of class 43m), there is only one independent nonzero component of the piezoelectric tensor called \( e_{14} \).

The value of \( e_{14} \) for GaAs has an accepted value of approximately .157 C/m². However, it should be noted that there is a small amount of evidence that the actual value might be somewhat lower (by perhaps as much as 40%). For the present work we will choose to work with the accepted value. For the case of AlAs, it is even more
The proposed form of solution is
\[ \phi = \frac{ic e_{14}}{\epsilon} e^{iq(x-v_0 t)} \left[ A_1 e^{-\Omega q z - i\varphi} + A_2 e^{-\Omega q z + i\varphi} + A_3 e^{-q z} \right] \] (47)

with C the amplitude of the SAW. Eq. 45 immediately yields the conditions
\[ A_1 = A_2^* = \frac{\gamma - 2\Omega}{\Omega^2 - 1} \] (48)

Finally, using Eq.46 yields
\[ A_3 = \frac{-2}{1 + r} \left[ \cos \varphi + r \text{Re}(A_1 e^{-i\varphi}) + \text{Re}(\Omega A_1 e^{-i\varphi}) \right] \] (49)

with \( r = \epsilon_0/\epsilon \approx \frac{1}{12.5} \).

Once the potential \( \phi \) has been determined, this potential is then treated as the external potential \( \phi^{\text{ext}} \) applied to the 2DEG. Note that the scalar potential \( \phi^{\text{ind}} \) then induced by the density fluctuations in the 2DEG does not change the solution to Eqs. 39 and 40 above since \( \nabla^2 \phi^{\text{ind}} = 0 \) everywhere outside of the 2DEG and \( \epsilon_0 q \phi^{\text{ind}} = \epsilon q \phi^{\text{ind}} \) at the surface (the solution of such a boundary condition is discussed in the appendix). The form of the potential is given by Eq. 3 where the functional dependence \( F \) is given by
\[ F(qd) = 2|A_1|e^{-qd} \cos(\beta q z + \phi + \xi) + A_3 e^{-qd} \] (50)
where \( \Omega = \alpha + \beta i \) and \( A_1 = |A_1|e^{-i\xi} \). Here we have the values \( |A_1| \approx 1.59, \phi + \xi \approx 2.41, A_3 \approx -3.10 \). Using these values in Eq. 21 yields a the coupling constant \( \alpha^2/2 \). The functional dependence of \( \alpha^2/2 \) on \( qd \) is shown in Fig. 3. It is clear that the dependence is quite non-trivial. First of all, the exponential decay at large \( qd \) is roughly proportional to \( e^{-qd} \) rather than \( e^{-2qd} \). This is because, due to the precise material parameters, the SAW decays into the bulk as \( e^{-qd} \) with \( \alpha \approx \frac{1}{2} \). More importantly, at small \( qd \) the coupling seems to oscillate. The reason for this is roughly that the boundary condition fixes the strain \( u_{xx} \) to be zero at \( z = 0 \). Thus \( E_x(z = 0) \) is mainly caused by the surface charge (ie, the \( A_3 \) term in Eq. 17). As \( z \) (or \( d \)) increases, the effect of the surface charge term quickly decays, but the strain \( u_{xx} \) becomes nonzero so that the coupling decays first, but then increases. Finally, at large \( qd \), the exponential decay of the SAW damps out the coupling.

VI. CONCLUSION AND FURTHER CONSIDERATIONS

This work has focused on surface acoustic waves in Al-GaAs coupled to a 2DEG a distance \( d \) away from the surface of the sample. The general relation (Eq. 1) between the fractional SAW velocity shift \( \Delta u_s/v_s \), the attenuation \( \kappa \) and the conductivity \( \sigma_{xx} \) of the 2DEG was derived, and the coefficients \( \alpha^2/2 \) and \( \sigma_m \) were explicitly calculated as a function of the product \( qd \) of the wavevector \( q \) and the distance \( d \) to the surface.

Although Eq. 1 is very general, the values of the coefficients \( \sigma_m \) and \( \alpha^2/2 \) are quite dependent on material parameters. As discussed in the text and in the appendix, \( \sigma_m \) is dependent only on the velocity of the SAW and on the effective background dielectric constant in the 2DEG (which is in general wavevector dependent). The coupling constant \( \alpha^2/2 \), on the other hand, is very sensitive to the details of the sample. In this paper we have focused only on a relatively simple model geometry where the sample is assumed to be a homogeneous slab of AlGaAs to simplify the solution of the wave problem. In actual experiments, the samples are often complicated many layer heterostructures. In the relevant experiments, the bulk of the crystal (below the 2DEG) is pure GaAs and most of the crystal between the 2DEG and the surface is Al\(_x\)Ga\(_{1-x}\)As with \( x \approx 0.3 \). However, additional thin layers of GaAs are added in this region, along with Si dopants. In principle, we could solve the wave equations for this complicated geometry and apply similar methods, but in practice such problems can only be solved numerically. However, since the elastic constants, densities, and dielectric constants of AlGaAs and GaAs are so similar we suspect that these heterostructures can be well approximated by the homogeneous system discussed here.

In using Eq. 1 to extract \( \sigma_{xx}(q, \omega) \) from experimental data, there are several complications. To begin with, accurate measurements of the attenuation are extremely difficult, as are absolute measurements of the velocity\( \Omega \). However, measurements of the relative velocity shift can be made quite accurately. Another complication is that the above formula for the velocity shift (Eq. 1) gives the velocity shift \( \Delta v_s \) relative to the velocity of the SAW if the conductivity of the 2DEG were infinite. In practice, the velocity shift is usually measured relative to the velocity of the SAW at zero magnetic field. It is often the case in high mobility samples (particularly at low frequency) that the conductivity at zero magnetic field is sufficiently large that it can be considered infinite and this approximation becomes reasonable. However, more generally, if the conductivity at zero field is well known, the resulting measured shift can be appropriately adjusted.

In References \( \| \| \| \), the parameters \( \sigma_m \) and \( \alpha^2/2 \) are both fit to experiment. To do this, the dc conductivity is measured and put into Eq. 1, the values of \( \sigma_m \) and \( \alpha^2/2 \) are then varied until a good fit is obtained to the experimentally measured values of \( \Delta v_s/v_s \) as a function of magnetic field. There are several possible problems with this procedure. To begin with, the zero frequency (dc) conductivity is expected to be somewhat different from the finite frequency and wavevector conductivity \( \sigma_{xx}(q, \omega = v_s q) \) that must be used in Eq. 1. Furthermore, there are indications that due to large scale inhomogeneities in the sample, the measured dc conductivity may not accurately represent the spatial average of \( \sigma_{xx} \). We thus conclude that these experimental fits of these parameters to the dc conductivity should be viewed with caution. Nonetheless the qualitative features of these experiments are relatively robust and many of the conclusions drawn from these experiments are relatively independent\( \| \) of the precise value of the fit parameters \( \sigma_m \) and \( \alpha^2/2 \). A more careful quantitative analysis of these data is given in Reference \( \| \).

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APPENDIX A: COULOMB INTERACTION NEAR A DIELECTRIC INTERFACE

The Coulomb interaction between electrons in the 2DEG is affected by the presence of the free surface of AlGaAs since the dielectric constant of the medium above the surface (\( \epsilon_0 \approx 1 \)) is much less than the dielectric constant of AlGaAs (\( \epsilon \approx 12.5 \)). In this appendix, we consider the electrostatic problem of a charge in a 2DEG a distance \( d \) from this AlGaAs/air interface. Consider a
charge $e = 1$ placed in the 2DEG at position $r = 0$ such that the AlGaAs surface is at the coordinate $z = d$. It is a standard result of electrostatics that the electrostatic potential in the AlGaAs generated by such a charge is given by

$$\Phi(r) = \frac{e}{\epsilon} \left( \frac{1}{|r|} + \frac{e - \epsilon_0}{\epsilon + \epsilon_0} \frac{1}{|r + 2zd|} \right). \quad (A1)$$

Here, $|r + 2zd|$ is the distance from $r$ to the image charge, a distance $d$ away from the surface on the air side. Restricting $r$ to lie in the plane of the 2DEG, and Fourier transforming, yields

$$\nu(k) = \int d^2r e^{i k \cdot r} \Phi(r) \quad (A2)$$

which can be evaluated using Eqs. 6.564, 8.411, and 8.469.3 from Reference 29 to yield

$$\nu(k) = \frac{2\pi}{\epsilon_0 k} \quad (A3)$$

where the effective dielectric constant is defined by

$$\frac{1}{\epsilon_{\text{eff}}} = \frac{1}{\epsilon} \left( 1 + \frac{e - \epsilon_0}{\epsilon + \epsilon_0} e^{-2kd} \right) \quad (A4)$$

which can be rewritten in the form of Eq. 4. Note that the effective dielectric constant ranges from $\epsilon$ for large $qd \gg 1$ to $(\epsilon + \epsilon_0)/2$ for $qd \ll 1$.

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