Vacuum structure and effective potential at finite temperature:
a variational approach

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Abstract

We compute the effective potential for $\phi^4$ theory with a squeezed coherent state type of construct for the ground state. The method essentially consists in optimising the basis at zero and finite temperatures. The gap equation becomes identical to resumming the infinite series of daisy and super daisy graphs while the effective potential includes multiloop effects and agrees with that obtained through composite operator formalism at finite temperature.
I. INTRODUCTION

There has been a considerable interest in the ground state structure of interacting quantum field theories since a long time. Variational methods like gaussian effective potential (GEP) method appear to be a powerful technique to study the same [1]. The equivalence between GEP and Bogoliubov transformations for $\phi^4$ theory in 1+1 dimensions was demonstrated earlier through an explicit construction of the nonperturbative vacuum as a squeezed coherent state [2] and was applied to study the vacuum structure of O(N) symmetric Thirring model. We shall test here such a nonperturbative variational method to obtain the effective potential and the gap equation in $\phi^4$ theory in 3+1 dimensions. These have been extensively studied by many authors at zero [3] as well as at finite temperatures [4–6] by summing over daisy and superdaisy diagrams. Leading and subleading contributions from multiloop diagrams were also included to obtain the effective potential at finite temperatures [7, 11], where, it became often necessary to drop various finite and divergent contributions. This was partially circumvented in [12, 13] while developing a self-consistent loop expansion of the effective potential at finite temperatures. Here the authors used composite operators [14] and the renormalisation prescriptions of Coleman et al [15]. These extensive discussions make the $\phi^4$ theory a good testing ground for examining a variational approximation scheme [16] developed essentially for quantum chromodynamics (QCD).

The conventional calculations as stated above basically consist of summing over perturbative diagrams, and then considering leading and subleading contributions arising from multiloop approximations. In contrast, the approximation scheme here shall use squeezed coherent state type construction [16] for the ground state which amounts to an explicit vacuum realignment. The input here is equal time quantum algebra with a variational ansatz for the vacuum structure, and has no reference to perturbative expansion or Feynman diagrams. We had earlier seen that this correctly yields the results of Gross-Neveu model [17] as obtained by summing an infinite series of one loop diagrams. We had also seen that it reproduces [18] the gap equation of Nambu-Jona Lasinio model without using Schwinger
Dyson equations. We shall here apply the method to $\phi^4$ theory at zero and finite temperatures and show that it also yields the self-consistent multiloop effects obtained earlier [13]. The $\phi^4$ model has extensive applications for early universe and cosmology [19], and thus an understanding of the same with alternative physical pictures is desirable.

The plan of the paper is as follows. In section II we compute the effective potential at zero temperature using squeezed coherent states. In section III we compute this at finite temperature using [20,21] thermofield dynamics (TFD) and minimise the free energy density [16]. We shall see here that the gap equation is identical to the one obtained by resumming the daisy and superdaisy graphs with appropriate renormalisation [4,13] and the effective potential includes multiloop effects as through self-consistent approach with composite operators [13]. In section IV we summarise the results and compare the same with earlier calculations.

II. EFFECTIVE POTENTIAL AT ZERO TEMPERATURE

We shall compute here the effective potential for $\phi^4$ theory. The Lagrangian is given as

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \lambda \phi^4,$$

(1)

where, $m$ and $\lambda$ are the bare (unrenormalised) mass and coupling constant respectively. The fields satisfy the equal time quantum algebra

$$[\phi(\vec{x}), \dot{\phi}(\vec{y})] = i \delta(\vec{x} - \vec{y}).$$

(2)

We may expand the field operators in terms of creation and annihilation operators at time $t=0$ as

$$\phi(\vec{x}, 0) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\vec{k}}{\sqrt{2\omega(\vec{k})}} \left( a(\vec{k}) + a(-\vec{k})^\dagger \right) e^{i\vec{k} \cdot \vec{x}},$$

(3a)

$$\dot{\phi}(\vec{x}, 0) = \frac{1}{(2\pi)^{3/2}} \times i \int d\vec{k} \sqrt{\frac{\omega(\vec{k})}{2}} \left( -a(\vec{k}) + a(-\vec{k})^\dagger \right) e^{i\vec{k} \cdot \vec{x}}.$$  

(3b)
In the above, \( \omega(\vec{k}) \) is an arbitrary function which for free fields is given by \( \omega(\vec{k}) = \sqrt{\vec{k}^2 + m^2} \) and the perturbative vacuum is defined corresponding to this basis through \( a \mid vac >= 0 \). Further the expansions (3) and the quantum algebra (2) yield the commutation relation for the operators \( a \)'s as

\[
[a(\vec{k}), a(\vec{k}')^\dagger] = \delta(\vec{k} - \vec{k}').
\] (4)

Earlier a redefinition of \( \mid vac > \) to \( \mid vac' > \) through coherent \([22,23]\) or squeezed coherent states \([16,17]\) was seen as equivalent to a Bogoliubov transformation. We shall adopt a similar procedure here to calculate the effective potential. We shall take the ansatz for the trial ground state as \([2]\)

\[
\mid vac' >= U \mid vac >= U_{II}U_I \mid vac >,
\] (5)

\( U_i = \exp(B_i^\dagger - B_i) \), \( (i = I, II) \). Explicitly \( B_i \)'s are given as

\[
B_I^\dagger = \int d\vec{k} \sqrt{\frac{\omega(\vec{k})}{2}} f(\vec{k}) a(\vec{k})^\dagger,
\] (6a)

and

\[
B_{II}^\dagger = \frac{1}{2} \int d\vec{k} g(\vec{k}) a'(\vec{k})^\dagger a'(-\vec{k})^\dagger.
\] (6b)

In the above, \( a'(\vec{k}) = U_I a(\vec{k}) U_I^{-1} = a(\vec{k}) - \sqrt{\frac{\omega(\vec{k})}{2}} f(\vec{k}) \) corresponds to a shifted field operator associated with the coherent state \([24]\) and satisfies the same quantum algebra as given in equation (4). Thus in this construct for the ground state we have two functions \( f(\vec{k}) \) and \( g(\vec{k}) \) which will get determined through minimisation of energy density. Further, since \( \mid vac' > \) contains arbitrary number of \( a'^\dagger \) quanta, \( a' \mid vac' > \neq 0 \). However, we can define the basis \( b(\vec{k}), b(\vec{k})^\dagger \) corresponding to \( \mid vac' > \) through the Bogoliubov transformation as

\[
\begin{pmatrix}
  b(\vec{k}) \\
  b(-\vec{k})^\dagger
\end{pmatrix} = U_{II} \begin{pmatrix}
  a'(\vec{k}) \\
  a'(-\vec{k})^\dagger
\end{pmatrix} \begin{pmatrix}
  \cosh g & -\sinh g \\
  -\sinh g & \cosh g
\end{pmatrix} \begin{pmatrix}
  a'(\vec{k}) \\
  a'(-\vec{k})^\dagger
\end{pmatrix}.
\] (7)

It is easy to check that \( b(\vec{k}) \mid vac' >= 0 \). Further, to preserve translational invariance \( f(\vec{k}) \) has to be proportional to \( \delta(\vec{k}) \) and we shall take \( f(\vec{k}) = \phi_0 \delta(\vec{k}) \times (2\pi)^{3/2} \). \( \phi_0 \) will correspond
to classical field of the conventional approach [24]. We next calculate the expectation value of the Hamiltonian density given as

$$\mathcal{T}^{00} = \left[ \frac{1}{2} \{ (\dot{\phi})^2 + (\nabla \phi)^2 + m^2 \phi^2 \} + \lambda \phi^4 \right]. \quad (8)$$

Using the transformations (7) it is easy to evaluate that

$$< \text{vac}' | \phi | \text{vac}' > = \phi_0, \quad (9a)$$

but,

$$< \text{vac}' | \phi(\vec{z})^2 | \text{vac}' > = \phi_0^2 + I, \quad (9b)$$

where $I$ is the integral given as, with $k = |\vec{k}|$,

$$I = \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2\omega(\vec{k})} (\cosh 2g + \sinh 2g). \quad (9c)$$

Using equations (8) and (9) the energy density of the trial state becomes [2]

$$< \text{vac}' | \mathcal{T}^{00} | \text{vac}' > = \frac{1}{2} \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2\omega(\vec{k})} k^2 (\sinh 2g + \cosh 2g)$$

$$+ \frac{1}{2} \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2\omega(\vec{k})} \omega(\vec{k}) (\cosh 2g - \sinh 2g)$$

$$+ \frac{1}{2} m^2 I + 6\lambda \phi_0^2 + 3\lambda I^2 + \frac{1}{2} m^2 \phi_0^2 + \lambda \phi_0^4, \quad (10)$$

where $I$ is as given in equation (9c). Extremising the above energy density with respect to the function $g(\vec{k})$ now yields for extremised $g(\vec{k})$ as

$$\tanh 2g(\vec{k}) = -\frac{6\lambda I + 6\lambda \phi_0^2}{\omega(\vec{k})^2 + 6\lambda I + 6\lambda \phi_0^2}. \quad (11)$$

Substituting this value of $g(\vec{k})$ in the expression for energy density yields the effective potential as

$$\epsilon_0 = V(\phi_0) = \frac{1}{2} m^2 \phi_0^2 + \lambda \phi_0^4 + \frac{1}{2} \frac{1}{(2\pi)^3} \int d\vec{k} (k^2 + M^2)^{1/2} - 3\lambda I^2 \quad (12)$$

where

5
\[ M^2 = m^2 + 3\lambda I + 12\lambda\phi_0^2 \]  

(13)

with

\[ I = \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2} \frac{1}{(\vec{k}^2 + M^2)^{1/2}} \]  

(14)

obtained from equation (12) after substituting for the condensate function \( g(\vec{k}) \) as in equation (11). The integrals in the equations (12) and (14) are divergent, and require renormalisation. We use the renormalisation prescription as in ref. [15] and thus obtain the renormalised mass \( m_R \) and coupling \( \lambda_R \) through

\[ \frac{m_R^2}{\lambda_R} = \frac{m^2}{\lambda} + 12I_1, \]  

(15a)

\[ \frac{1}{\lambda_R} = \frac{1}{\lambda} + 12I_2(\mu), \]  

(15b)

where \( I_1 \) and \( I_2 \) are divergent integrals given as,

\[ I_1 = \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2k}, \]  

(16a)

\[ I_2(\mu) = \frac{1}{\mu^2} \int \frac{d\vec{k}}{(2\pi)^3} \left( \frac{1}{2\sqrt{k^2 + \mu^2}} - \frac{1}{2} \right) \]  

(16b)

with \( \mu \) as the renormalisation scale. Using equations (15a) and (15b) in equation (13), we have the gap equation for \( M^2 \) in terms of the renormalised parameters as

\[ M^2 = m_R^2 + 12\lambda_R\phi_0^2 + 12\lambda_RI_f(M), \]  

(17)

where,

\[ I_f(M) = \frac{M^2}{16\pi^2} \ln \left( \frac{M^2}{\mu^2} \right). \]  

(18)

Using the above equations we simplify equation (12) to obtain the effective potential in terms of \( \phi_0 \) as

\[ V(\phi_0) = 3\lambda_R\left( \phi_0^2 + \frac{m_R^2}{12\lambda_R} \right)^2 + \frac{M^4}{64\pi^2} \ln \left( \frac{M^2}{\mu^2} - \frac{1}{2} \right) - 3\lambda_RI_f^2 - 2\lambda\phi_0^4. \]  

(19)
This potential is identical to that as obtained through composite operator formalism as in Ref. [12] and is derived through a purely nonperturbative approach through vacuum realignment. We now consider the generalisation of the above to finite temperatures.

III. EFFECTIVE POTENTIAL AT FINITE TEMPERATURE

We shall calculate the effective potential at finite temperature for \( \phi^4 \) theory using thermo-field dynamics [20]. Here the statistical average of an operator is written as an expectation value with respect to a “thermal vacuum” constructed from operators defined on an extended Hilbert space [20]. The “thermal vacuum” is obtained from the zero temperature ground state through a “thermal” Bogoliubov transformation. Thus at finite temperatures, \( |vac' > \) of equation (5), with \( \beta \) as inverse of temperature, goes over to

\[
|vac', \beta > = U(\beta)|vac' >,
\]

where [20]

\[
U(\beta) = \exp(B(\beta)\dagger - B(\beta)),
\]

with

\[
B(\beta)\dagger = \int d\vec{k}\theta(\vec{k}, \beta)b(\vec{k})\dagger \tilde{b}(-\vec{k})\dagger.
\]

In the above, \( \tilde{b} \) is the annihilation operator for thermal modes associated with the doubling of the Hilbert space [20]. The function \( \theta(k, \beta) \) for the corresponding Bogoliubov transformation is related to the distribution function and shall be obtained by minimising free energy density. We first note that the temperature dependant annihilation operators \( b(\vec{k}, \beta) \) and \( \tilde{b}(\vec{k}, \beta) \) corresponding to the thermal vacuum are given as

\[
\begin{pmatrix}
    b(\vec{k}, \beta) \\
    \tilde{b}(\vec{k}, \beta)\dagger
\end{pmatrix} = U(\beta) \begin{pmatrix}
    b(\vec{k}) \\
    \tilde{b}(\vec{k})\dagger
\end{pmatrix} U(\beta)^{-1} = \begin{pmatrix}
    \cosh\theta & -\sinh\theta \\
    -\sinh\theta & \cosh\theta
\end{pmatrix} \begin{pmatrix}
    b(\vec{k}) \\
    \tilde{b}(\vec{k})\dagger
\end{pmatrix}.
\]

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Using the transformations given by (7) and (23), we can calculate the expectation values of the operators in the thermal vacuum. For example, the expectation value of $\phi^2$ of equation (9b) in thermal vacuum becomes

$$< \text{vac}', \beta | \phi(z)^2 | \text{vac}', \beta >= \phi_0^2 + I(\beta),$$

where $I(\beta)$ is the integral

$$I(\beta) = \frac{1}{(2\pi)^3} \int \frac{dk}{2\omega(k)} (\cosh2g + \sinh2g) \cosh2\theta(k, \beta),$$

parallel to equation (10c) at zero temperature. The energy density then becomes the generalisation of equation (10), given as

$$\epsilon \equiv < \text{vac}', \beta | \mathcal{T}^{00} | \text{vac}', \beta > = \frac{1}{2 \omega(k)} k^2 (\sinh 2g + \cosh 2g) \cosh 2\theta$$

$$+ \frac{1}{2 \omega(k)} \frac{1}{2} \int d\vec{k} \omega(k) (\cosh 2g - \sinh 2g) \cosh 2\theta$$

$$+ \frac{1}{2} m^2 I(\beta) + 6\lambda \phi_0^2 I(\beta) + 3\lambda I(\beta)^2 + \frac{1}{2} m^2 \phi_0^2 + \lambda \phi_0^4.$$ 

(26)

At finite temperature the relevant quantity to be extremised is free energy density given as

$$\mathcal{F}(\phi_0, g, \theta) = \epsilon - \frac{1}{\beta} S,$$ 

(27)

with the entropy density $S$ as

$$S = \frac{1}{(2\pi)^3} \int d\vec{k} (\cosh^2 \theta \ln(\cosh^2 \theta) - \sinh^2 \theta \ln(\sinh^2 \theta))$$

(28)

Minimising $\mathcal{F}(\phi_0, g, \theta)$ with respect to $g(\vec{k})$ we obtain that

$$\tanh 2g(\vec{k}, \beta) = -\frac{6\lambda I(\beta) + 6\lambda \phi_0^2}{\omega(k)^2 + 6\lambda I(\beta) + 6\lambda \phi_0^2}.$$ 

(29)

Further, extremising the free energy density with respect to $\theta(\vec{k}, \beta)$ we obtain that

$$\sinh^2 \theta(\vec{k}, \beta) = \frac{1}{e^{\beta \omega(k, \beta)} - 1}.$$ 

(30)

where,
\[ \omega'(\vec{k}, \beta) = (\vec{k}^2 + M(\beta)^2)^{1/2} \]  

with
\[ M(\beta)^2 = m^2 + 3\lambda I(\beta) + 12\lambda \phi_0^2. \]  

The above equation is the finite temperature generalisation of (13) and corresponds to the gap equation after summing over daisy and superdaisy graphs [4,13]. Substituting the optimised expressions for the functions \( \theta(\vec{k}, \beta) \) and \( g(\vec{k}, \beta) \), the free energy density or the effective potential at finite temperature now becomes
\[ V(\phi_0, \beta) \equiv F = V_0 + V_I + V_{II} \]  

with,
\[ V_0 = \frac{1}{2} m^2 \phi_0^2 + \lambda \phi_0^4, \]  

\[ V_I = \frac{1}{2} \frac{1}{(2\pi)^3} \int d\vec{k} (k^2 + M(\beta)^2)^{1/2} \cosh^2 \theta - \frac{1}{\beta} S \]  

and,
\[ V_{II} = -3\lambda I(\beta)^2, \]  

where
\[ I(\beta) = \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2} \frac{\cosh^2 \theta}{(k^2 + M(\beta)^2)^{1/2}}. \]  

We may note that in equation (33) \( V_0 \) is the tree level potential, and, \( V_I \) and \( V_{II} \) can be identified with the one loop and two loop contributions of Ref. [13] after summing over the discrete frequencies corresponding to imaginary time formulation of finite temperature field theory. The gap equation and the effective potential here have been obtained by minimising free energy with a nontrivial vacuum structure as in equation (20).

We note that the finite temperature contributions do not bring in any fresh divergences apart from those encountered at zero temperature. We thus use the renormalisation conditions as in equation (15). In that case, the divergences from \( V_0 \) and \( V_{II} \) combine to cancel.
the same arising from $V_I$ when equation (30) for the distribution function is used \[13\]. The 
renormalised effective potential then becomes

$$V(\phi_0, \beta) = V_0 + V_I + V_{II}$$

(36)

where,

$$V_0 + V_{II} = 3\lambda_R\left(\phi_0^2 + \frac{m_R^2}{12\lambda_R}\right)^2 - 3\lambda_R I_f(\beta)^2 - 2\lambda\phi_0^4$$

(37)

and,

$$V_I = \frac{M(\beta)^4}{64\pi^2} \left( \ln \left( \frac{M(\beta)^2}{\mu^2} \right) - \frac{1}{2} \right) + \frac{1}{\beta (2\pi)^3} \int d\vec{k} \ln \left( 1 - \exp(-\beta \omega') \right).$$

(38)

In the above $M(\beta)$ satisfies the renormalised gap equation

$$M(\beta)^2 = m_R^2 + 12\lambda_R\phi_0^2 + 12\lambda_R I_f(M(\beta)),$$

(39)

where,

$$I_f(M(\beta)) = \frac{M(\beta)^2}{16\pi^2} \ln \left( \frac{M(\beta)^2}{\mu^2} \right) + \int \frac{d\vec{k}}{(2\pi)^3} \frac{\sinh^2 \theta(\vec{k}, \beta)}{(\vec{k}^2 + M(\beta)^2)^{1/2}}.$$ 

(40)

We note that the above expressions for the effective potential as well as the gap equation

(39) are identical respectively to equations (3.19), (3.20) and (3.16) of Ref. \[13\]. Thus the
realignment of the ground state with condensates naturally generates the nonperturbative
features beyond one loop as demonstrated above.

**IV. DISCUSSIONS**

We first note that equation (36) is the same as the earlier equations of Ref. \[13\], and hence
the same conclusions regarding high temperature limits as well as discussions on the nature
of the phase transitions continue to hold good with the present picture of phase transition
through explicit vacuum realignment. In particular we may also note that the functions
arising from composite operators of Ref. \[13\] on summing over discrete frequencies corre-
spond to $I(\beta)$ of equation (34) arising from the variation over the condensate function. The
thermal distribution functions *including interactions* as in equation (30) has been derived through an *extremisation* of the free energy density. Coherent states [21] as well as squeezed states for dissipative systems [25] along with thermofield dynamics have been dealt with for quantum mechanical problems. Here we have applied the techniques to quantum field theory to include nonperturbative effects. Vacuum effects have been known to be relevant for low energy nonperturbative physics [26,27]. We have also seen that a nontrivial vacuum structure with condensates is not only conceptually different, but can have phenomenological implications [16,18,28,29]. Thus the present work with an explicit structure for thermal vacuum may become relevant for phenomenology in cosmology.

It is interesting to note that the present variational ansatz with squeezed vacuum structure leads to daisy- super daisy resummed self consistent two loop effective potential as obtained in Ref. [13]. The reason for such a result lies on the fact that the $\phi^4$ interaction leads to a functional for vacuum energy which is effectively quadratic and we could solve for the ansatz functions explicitly. With a more complicated structure for the ansatz state involving nonlinear canonical transformations one can improve upon the effective potential as calculated here [30]. The inputs have been (i) equal time algebra of interacting fields, (ii) an ansatz for vacuum realignment, and, (iii) use of renormalisation prescriptions of Ref. [15]. We did not have to go through the perturbative route anywhere, and no summation of diagrams was needed. The results thus obtained went beyond the one loop approximation. However, since variational methods are ansatz dependant, it is nice to see that the present picture more easily reproduces the final features of a well studied problem, and adds conceptual ingredients that were absent in the earlier calculations which could be relevant for phenomenology. The analysis thus adds to the confidence with which the present method can be applied, as well as gives fresh understanding of summation of infinite diagrams of perturbation series.
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