Preliminary study of bamboo-like tree structure based on granular particle-spring model: Relaxation and tortuosity

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Abstract. Bamboo-like tree structure is modelled using granular particles connected with linear and bending springs. The first type of spring is responsible for contraction and relaxation in axial direction, while the second one is in angular direction. For simplicity the system is first considered only two-dimension in this work. Previous work showed that for two branches tree, wind influenced only one branch which has resonance with the wind velocity. Only relaxation stage is reported in this work, that can not yet achieved stable state.

Keywords: granular particle, spring, bamboo tree, relaxation stage

1. Introduction

It is interesting to study bamboo and its structure, since its die-back could influence tree regeneration [1] as important component of ecosystems [2], it can be used as functionally graded but affordable materials [3], its-like structure is observed in nanotubes growth [4], and also its interactions with environment that influences its morphogenesis [5]. Bamboo-like trees can be considered as a cylindrical beam or a wire, which has certain Youngs’ modulus \( E \) and shear modulus \( G \). For a wire a simple model using only \( G \) as parameter has been proposed [6], and also for two-branches-tree with using linear and bending springs [7]. In this work, bending spring is replaced by linear spring and to achieve similar effect the linear springs must have certain configuration explained in the model.

2. Theory

2.1. Model

In this part linear springs refers to push and pull spring, which is differed from bending spring. The first type of spring is related to Young’s modulus \( E \), while the second type to shear modulus \( G \). A configuration of springs and masses proposed here uses only linear spring, but the system can hopefully can also accommodate shear modulus \( G \) in term of only linear spring.
Figure 1. Several models for bending wire due to its own weight or bamboo-like tree system: (a) it fails when using only linear spring $k_L$ between two adjacent particle masses, (b) using linear spring $k_L$ and bending spring $k_B$ [7], and (c) using two types of linear spring $k_{L1}$ and $k_{L2}$, where connection between particles with odd and event indices are separated for clearer illustration.

Supposed that there are $N$ particle masses with mass of each is

$$ m = \frac{M}{N}, $$

(1)

with total mass of the system is $M$. Particle index is $i = 1, \ldots, N$, where first particle is fixed on the ground. In the model proposed in this work there are two types of spring, one with spring constant $k_{L1}$ and the other with $k_{L2}$. Spring $k_{L1}$ connects two adjacent particles with index $i$ and $i+1$, while spring $k_{L2}$ connects odd and even indices only, e.g. $i$ and $i + 2$. If length of the system $L$ then initial distance between particle mass is

$$ \Delta l = \frac{L}{N-1}, $$

(2)

with initial position of particle $i$ is

$$ \vec{r}_i = (i - 1)\Delta l \left( \hat{e}_x \cos \theta_0 + \hat{e}_y \sin \theta_0 \right) $$

(3)
and $\theta_0$ is initial angle measured from negative direction of gravity $\ddot{g}$. This initial angle is the same as in model of wire curvature [6]. Gravitation force acted upon particle $i$ is simple defined as

$$ F^G_i = m\ddot{g} $$

(4)

and the spring force due to interaction other particle mass $j$ is

$$ F^l_{ij} = -k_L(r_{ij} - l_{ij})\ddot{r}_{ij}, $$

(5)

where $k_L$ could be $k_{L1}$ or $k_{L2}$, depends on the interacting particles. Using Newton’s second law of motion

$$ \sum F = m \ddot{r} $$

(6)

following differential equation can be obtained

$$ \frac{d^2\ddot{r}}{dt^2} + \frac{1}{m} \sum_{j=i}^{i+2} \sum_{k=1}^{2} \left[ u(j-1)(1-u(j-N-1))\delta_{k+1,j}k_L(r_{ij} - \Delta t)\ddot{r}_{ij} \right] - \ddot{g} = 0 $$

(7)

for particle $i$. Boundary conditions sourced from particles index $i$, which hold only from 1 to $N$, requires step function

$$ u(x - x_0) = \begin{cases} 1, & x \geq x_0, \\ 0, & x < x_0. \end{cases} $$

(8)

Equation (7) must be modified using Equation (8) into

$$ \frac{d^2\ddot{r}}{dt^2} + \frac{1}{m} \sum_{j=i}^{i+2} \sum_{k=1}^{2} \left[ u(j-1)(1-u(j-N-1))\delta_{k+1,j}k_L(r_{ij} - \Delta t)\ddot{r}_{ij} \right] - \ddot{g} = 0 $$

(9)

to assure the boundary conditions. In the condition where there is wind, another force must be added to Equation (6)

$$ \ddot{F}^D_i = -3\pi\eta D(v_i - \bar{v}_w) $$

(10)

which is the drag force for spherical particle with diameter $D$. Wind velocity is represented by $\bar{v}_w$. Here, air viscosity must be also considered in the term of $\eta$.

2.2. Numerical method

Equation (9) will be solved using numerical method, e.g. Euler algorithm, which gives

$$ \frac{d^2\ddot{r}}{dt^2} = \ddot{g} - \frac{1}{m} \sum_{j=i}^{i+2} \sum_{k=1}^{2} \left[ u(j-1)(1-u(j-N-1))\delta_{k+1,j}k_L(r_{ij} - \Delta t)\ddot{r}_{ij} \right] $$

(11)

and

$$ \frac{d^n\ddot{r}}{dt^n} = \frac{d^n\ddot{r}}{dt^n} + \Delta t \frac{d^{n+1}\ddot{r}}{dt^{n+1}}, \quad n = 0, 1. $$

(12)

Value of $\Delta t$ must be chosen carefully, that is smaller than any physical time that could characterize the system, e.g. 1/100 smaller than contact time as in granular simulation [8].

Final condition is achieved then position of every particle mass does not change, or the change
\[ \varepsilon(t) = \sum_{i=1}^{N} \left| \dot{\mathbf{r}}_i(t + \Delta t) - \dot{\mathbf{r}}_i(t) \right|. \]  

is smaller than a certain value. This value should be comparable any physical length that could characterize the system, e.g. 1/100 smaller than \( \Delta l \).

2.3. Calculation steps

There are two steps in implementing the numerical. The first is relaxation step and the second is resonance step. Both step will require following equations

\[ \frac{d^2 \dot{\mathbf{r}}_i(t)}{dt^2} = -\frac{3\pi \eta D}{m} (\mathbf{v}_i - \mathbf{v}_w) - \frac{1}{m} \sum_{j=1}^{N} \sum_{k=i}^{N} \left( u(\omega(j-1)[1-u(j-N-1)]) \delta_{j|k-l} \mathbf{k}_{il} (\mathbf{r}_i - \mathbf{r}_j) \right), \]  

\[ \frac{d\mathbf{r}_i(t + \Delta t)}{dt} = \frac{d\mathbf{r}_i(t)}{dt} + \Delta t \frac{d^2 \mathbf{r}_i(t)}{dt^2}, \]  

\[ \mathbf{r}_i(t + \Delta t) = \mathbf{r}_i(t) + \Delta t \frac{d\mathbf{r}_i(t)}{dt}. \]  

**i. Relaxation step.** This step is required to set \( \mathbf{v}_w = 0 \) and then iteratively performs Equations (14.a) – (14.c) until time \( t \) where condition of \( \varepsilon(t) < \varepsilon_{\text{calc}} \) is reached.

**ii. Resonance step.** Using the relaxed particles configuration wind velocity can be varied e.g. \( \mathbf{v}_w = \mathbf{v}_w(t) \). If there is a period or duration of the wind, value of time step \( \Delta t \) must be chosen to be smaller than that.

3. Results and Discussion

Following parameters are used in the simulation if not further specified.

| Parameters | Value | Unit |
|------------|-------|------|
| \( \rho \)  | 500   | kg/m³|
| \( L \)    | 30    | m    |
| \( D \)    | 0.25  | m    |
| \( E \)    | 20    | GPa  |
| \( k \)    | \( 1.722 \times 10^{18} \) | N/m |
| \( N \)    | 20    | -    |
| \( \theta_0 \) | 10   | °    |

Using parameters in Table 1 a typical relaxation process is given in Figure 2, which unfortunately does not reach a stable expected final state. As previously investigated [6], even not yet a stable final state, it is interesting to see the evolution of tortuosity \( T \), which is given in Figure 3. It can be seen that gravitation potential energy \( U \sim y \) is decreasing with time and also tortuosity. The first aspect indicates that it relaxes and but the second need further analysis for interpretation, such as that reduction of tortuosity indicates that the structure of bamboo-like trees becomes more curved than its initial structure, which could be addressed to value of \( k_L \) (\( k_{L1} = k_{L2} = k_L \)) that has not yet represent the right value of bending spring \( k_L \) or shear modulus \( G \) [6]. Not constant value of \( k_L \) could be proposed for next work and also some damping to vanish the oscillation.
Figure 2. Relaxation process of bamboo-like system at time $t$: (a) 0, (b) 1, (c) 1.5, (d) 2, (e) 2.5, and (f) 2.8.
Figure 3. Change of tortuosity $T(\bigcirc)$ and vertical position $y(\square)$ in relaxation process.

4. Conclusion
Structure of bamboo-like tree based on granular particle-spring model has been proposed, but unfortunately it can reach final stable state in the relaxation stage. It could be addressed to the same value of spring constant along the structure.

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