Limits on the low energy antinucleon-nucleus annihilations from the Heisenberg principle.

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Abstract

We show that the quantum uncertainty principle puts some limits on the effectiveness of the antinucleon-nucleus annihilation at very low energies. This is caused by the fact that the realization a very effective short-distance reaction process implies information on the relative distance of the reacting particles. Some quantitative predictions are possible on this ground, including the approximate $A$-independence of $\bar{N}$-nucleus annihilation rates.

1 Introduction

Recently several experimental data\cite{1–6} have shown that at projectile momenta below 200 MeV/c the behavior of antinucleon-nucleus annihilations is quite different from what could be naively expected.

For $k$ (incident momentum of the antinucleon in the laboratory) below 70 MeV/c there are no evident signs of an increase of the $\bar{p}$—nucleus total annihilation cross section at increasing mass number $A$ of the target\cite{2,4,5}. At 30-50 MeV/c the $\bar{p}p$ total annihilation rate is larger than the corresponding rate for $\bar{p}D$ and $\bar{p}^4$He. The width and shift of the ground level of antiprotonic atom of Hydrogen are larger than the corresponding observables in antiprotonic Deuterium\cite{3}. For the $\bar{p}p$ scattering length $\alpha \equiv \alpha_R + i\alpha_I$ we have\cite{4,6,7} $\alpha_R \approx -\alpha_I \approx 0.7 \div 0.8$ fm, and the $\rho$-parameter (i.e. the ratio between the real and the imaginary part of the forward scattering amplitude) is $\sim -1$ at zero energy. These values mean\cite{6} that at small momenta the elastic interaction is repulsive (i.e. negative phase shifts: the outgoing scattered wave is in advance with respect to the free motion wave) and as much important as the annihilation.
Elastic and annihilation data for $\bar{p}p$ at laboratory momenta $k$ below 600 MeV/c, scattering length data and $\rho-$parameter data from 0 to 600 MeV/c can be well fitted by energy-independent optical potentials[6]. They present some very curious features: (i) an increase of the strength of the imaginary part leads to a decrease of the consequent reaction rate, and an increase in the radius of the imaginary part does not lead to a consistent increase of the reaction rate; (ii) a repulsive elastic amplitude is produced despite the real part of the potential is attractive; (iii) the annihilation rate is much more sensitive to the diffuseness parameter than to the strength or the radius. All this happens for $k < 200$ MeV/c. We could suspect that strange phenomena start from $k \approx 200$ MeV/c, although they become experimentally evident at smaller momenta.

The synthesis of the previous facts can be: stronger and attractive in principle → weaker and repulsive in effect. We and other authors[8–11] have presented explanations of these phenomena that for simplicity we regroup under the name “inversion”. In particular, in [8] it has been shown that, within a multiple scattering framework, double interaction terms interfere destructively with single interaction terms in $\bar{p}$–D interaction. In [9] it has been shown that in the simplified optical model potential $V(r) = -iW$ for $r < R$ (the “black sphere model”) the zero-energy reaction cross section is an increasing function of $W$ for small $W$ only, and it decreases to zero for $W \to \infty$.

In a previous work[10] we have generalized the black sphere analysis showing that the inversion is associated with the formation of a sharp “hole” (i.e. a vacuum region with sharp boundaries, due to the annihilation) in the projectile wavefunction at small momenta. The underlying argument was not related with any specific model for the annihilation. It was stressed that this phenomenon was related to the transition from a semiclassical to a pure quantum, S-wave dominated, regime. We examined also more specific explanations for the inversion, however in the following we would like to further develop that general argument, relating it with the Heisenberg principle $\delta k \delta x > 1$ (in natural units).

Both in Hydrogen and in heavier nucleus targets, the great bulk of the annihilations is supposed to take place within a region of thickness $\Delta \approx 1$ fm placed just out of the nuclear surface[12–16]. Since the realized annihilation implies the statement “$\bar{N}$ and nucleus at relative distance $r \approx R_{nucleus}$ defined within uncertainty $\Delta$”, we expect strong deviations from semiclassical intuition at $k < 1/\Delta \sim 200$ MeV/c.
We begin by spending a few words on the so-called “black disk model”, that assumes complete flux remotion from the lowest partial waves and gives the unitarity limit for the total reaction probability. At very low energies, this model is a nonsense, for a well known limiting property of the phase shifts at zero energy. Indeed, the black disk model assumes $|\exp(2i\delta_o)| = 0$ for the S-wave phase shift $\delta_o$. But in the limit $k \to 0$ any requirement of the kind $|\exp(2i\delta_o)| < B$, where $B$ is a constant smaller than 1, means $\text{Im}(\delta_o) \approx -k\text{Im}(\alpha) \to \text{constant}$, i.e. $\text{Im}(\alpha) \sim -1/k \to -\infty$. This shows that the idea of complete flux remotion and the black disk model are ill-defined concepts at low energies. Of course, one can artificially put $|\exp(2i\delta_o)| = 0$, but at small $k$ one will never be able to obtain this condition starting from a model with confined interactions.

So, in presence of a very effective reaction mechanism, as annihilation is, we expect that a scale $k \sim k_b$ exists for the projectile momentum $k$ such that: (i) for $k >> k_b$ the reaction cross section assumes values which are close to the unitarity limit; (ii) for $k << k_b$ we assist to a breaking of the saturation of the unitarity limit, i.e. the reaction cross section is much smaller that its unitarity limit value.

Assuming that the main distortions in the entrance channel wavefunction are caused by the absorption, the uncertainty principle suggests $k_b \sim 1/\delta r$, where $\delta r$ is the characteristic projectile path in nuclear matter. The consequent physics is very different depending whether this path is peculiar of a nucleus-projectile or of a nucleon-projectile underlying process. In this respect neutron induced nuclear reactions, and reactions like $\bar{N}$-nucleus annihilation or $K^-$-nucleus absorption, are the exact opposite. In the former case the underlying projectile-nucleon interactions are elastic, although their effect is destructive on the full nuclear structure. The reaction process contains the piece of information “the projectile and the nuclear center of mass are at relative distance $< R_{\text{nucleus}}$, i.e. $\delta r = R_{\text{nucleus}}$”. In the latter case the nucleon-projectile interaction is so inelastic that the path of the projectile in nuclear matter is $\Delta \sim R_{\text{nucleon}}$, and the reaction process contains the piece of information “relative distance $= R_{\text{nucleus}} \pm \Delta$”, i.e. $\delta r \sim \Delta$.

In both cases the information implicitly contained in the fact that the reaction has happened is incompatible with the statement “the momentum of their relative motion was smaller than $1/\delta r$”. So, either the reaction can’t happen or we must pay a price, in terms of large-momentum distortions of the projectile wavefunction. These distortions produce a large flux reflection, as we show below, that is the reason for the departure from the saturation of the unitarity limit.
3 The general mechanism.

We assume that the antinucleon-nucleus annihilation reaction is such a violent and effective process to make it necessary for the $\bar{N}$ wavefunction to be zero in all places where the value of the density of the nucleons is close to the nuclear matter value. In other words, as soon as the overlap between the distributions of probability for the antinucleon and for the target nucleons overcomes a certain threshold $<<1$ the annihilation is supposed to take place, with the practical consequence that any consistent overlap of the projectile and target wavefunctions is forbidden. Most models[12,13,16] or phenomenological optical potential analyses[14,15,6] agree on this property.

This produces a thin spherical shell of thickness $\Delta \sim 1$ fm (the exact size depends on the specific model) where the largest part of the annihilations is supposed to take place. We name it “annihilation shell”. The internal surface of the annihilation shell roughly coincides with the surface of the target nucleus or proton, in agreement with the idea that $\bar{N}$ and nuclear matter densities can’t overlap consistently. Depending on the model, the position of the external surface of the annihilation shell is related either with a minimum amount of overlap between antinucleon and nucleon densities required for annihilations, or with the range of a meson/baryon exchange between the annihilating particles. The target independence of $\Delta$, together with the Heisenberg principle, produces a target-independent annihilation cross section. To understand how it realizes, we start with some easy 1-dimensional examples.

We consider a $\bar{N}$ plane wave with momentum $\vec{k} = (0, 0, -k)$ parallel to the $z$-axis. There is no interaction for $z > 0$, while for $z < 0$ absorption of the $\bar{N}$ flux is possible, according to some unknown mechanism. We don’t know how it happens, but we know that most of the flux that enters the absorption region disappears within a range $\Delta$: $|\Psi(-\Delta)| << |\Psi(0)|$. The uncertainty principle implies that in this region the wavefunction has relevant components associated to a single particle momentum $k_z \sim 1/\Delta$. A consequence of this is the obvious geometrical fact that for the absolute value of the logarithmic derivative of $\Psi$ we have $|\Psi'/\Psi| \sim 1/\Delta$ in the damping range $-\Delta < z < 0$, and consequently also in $z = 0 - \epsilon$.

For matching this value with the value of the logarithmic derivative on the positive $z$ side, we need both an incoming and a reflected wave. The general form of $\Psi(z)$ for $z > 0$ is $\Psi = \Psi_{in} sin[k(r - \alpha)] = \Psi_{in} + \Psi_{out}$, with $\alpha$ complex to give account of the reactions. In general $\alpha$ is a function of $k$, however we can identify it with the $k$-independent scattering length since we are interested in the region of small $k$, and we assume that no resonances are present in the $k$-range that we consider.
Below we report standard calculations, but it is easy to understand the relevant points in advance. For \( z > 0 \), \( |\Psi_{in}|^2 \approx |\Psi(z_p)|^2 / 4 \), where \( z_p \) is the lowest positive \( z \) value where the periodical \( \Psi \) attains an oscillation peak. \( |\Psi(0)|^2 \ll |\Psi(z_p)|^2 \) if \( |\Psi'(0)/\Psi(0)| \gg k \). As a consequence, for \( |\Psi'(0)/\Psi(0)| \gg k \) we have also \( |\Psi(0)|^2 \ll |\Psi_{in}|^2 \). In magnitude, \( |\Psi(0)|^2 / |\Psi_{in}|^2 \sim k^2 |\Psi'/\Psi|^2 \sim (k\Delta)^2 \) at small \( k \).

The ratio between the value of \( |\Psi(0)|^2 \) and \( |\Psi_{in}|^2 \) roughly coincides with the ratio between the absorbed and the incoming flux, or at least it represents an upper limit for this ratio. Indeed, only for \( z < 0 \) we may have flux absorption.

The ratio of the absorbed to the incoming flux will be a number of magnitude \( \sim 1 \) only in the case where the condition \( k \gg 1/\Delta \) is realized, because in this case the position \( z_p \) will be close enough to the origin to have \( |\Psi(0)|^2 \approx |\Psi(z_p)|^2 \). Then we are close to the saturation of the unitarity limit for the reaction: full flux absorption, possibly accompanied by elastically scattered diffractive flux (which originates in the interference between absorbed and incident waves). At \( k \sim 1/\Delta \) we start departing from the saturation of the unitarity limit, and for \( k \ll 1/\Delta \) we will be far from it. In the latter case the matching conditions associate a large \( |\Psi'/\Psi|_0 \) to a small flux absorption. As a by-product, elastic cross sections can be large, but they are refractive, not diffractive.

If one wants to check the previous estimates with some calculations, one can normalize \( \Psi \) for \( z > 0 \) so to have \( \Psi_{in} = e^{-ikz} \). Then \( \Psi_o = e^{-2ik\alpha} \), and \( \Psi_{out} = e^{ik(z-2\alpha)} \). Since the flux cannot be created, \( Im(\alpha) < 0 \). Then for \( z > 0 \)

\[
|\Psi|^2 = 1 + e^{4kIm(\alpha)} - 2e^{2kIm(\alpha)}\cos\{2k[z - Re(\alpha)]\}. \tag{1}
\]

In particular, when \( k|Im(\alpha)| \ll 1 \) \( |\Psi|^2 \) becomes \( 2 - 2\cos\{2k[z - Re(\alpha)]\} \), so that also in presence of absorption \( |\Psi(z_p)|^2 / |\Psi_{in}|^2 \approx 4 \) for \( k \) small enough. When both \( k|Im(\alpha)| \ll 1 \) and \( k|Re(\alpha)| \ll 1 \) are satisfied we have \( |\Psi(0)|^2 \approx (4k^2)|\alpha|^2 \ll 1 \), thus confirming that for \( k \) small enough \( |\Psi(0)|^2 \propto (k|\alpha|)^2 \).

The logarithmic derivative of \( \Psi \) in \( z = 0 \) is \( k \cdot cotg(-k\alpha) \approx -1/\alpha \) at small \( k \), so that “\( k \) small enough” means \( k \ll |\Psi'(0)/\Psi(0)| \).

The conclusions of the examined example may change if we consider a reaction region which is limited to \( -z_o < z < 0 \), i.e. for \( z < -z_o \) no particle absorption is possible. We remark that \( z_o \) represents the size of the region where reactions are possible, while \( \Delta \) is the range needed for the projectile wavefunction to pass from \( \Psi \approx \Psi_o \sin[-k(z - \alpha)] \) to \( \Psi \approx 0 \). We must distinguish the two cases where \( z_o \) is smaller or larger than \( \Delta \). In the former case the absorption is proportional to the thickness \( z_o \) of the reaction region. But a saturation condition is reached when \( z_o \) becomes larger than \( \Delta \), and for any \( z_o \gg \Delta \) the conclusions will be the same as in the case \( z_o = \infty \). It is now useful to
notice that for $z_o \gg \Delta$ nothing would be changed by the introduction of
the additional boundary condition $\Psi(-z_o) = 0$. This constraint obliges one
to take into account the reflected wavefunction inside the reaction region, i.e.
that component of $\Psi$ whose absolute value increases at increasing negative $z$
inside the reaction region. But for $z_o \gg \Delta$ this component is very small and
can be neglected.

The latter situation with the additional “reflection” condition in $-z_o$ corre-
sponds to the 1-dimensional reduction of the 3-dimensional problem of $\bar{N}N$
and $\bar{N}$–nucleus annihilation, because the damping of the projectile wavefunc-
tion takes place on a space scale which is short enough to prevent antinucleons
from reaching the origin with any target. From a mathematical point of view
the situation is identical in the two cases, after substituting $\Psi(z + z_o)$ with
$r\Psi(r)$.

In treating the problem, initially we neglect the role of a real strong attracting
potential. The modifications that it introduces will be considered in a further
section. We define $R_m$ and $\Delta$ such that practically all of the annihilations
are supposed to take place at $r$ values comprised between $r = R_m$ and $r = R_m - \Delta$.
We assume $R_m$ as a reasonable matching radius, satisfying the two conditions:
(i) for $r > R_m$ the oscillations of $\chi(r) \equiv r\Psi(r)$ are mainly controlled by the
sum of kinetic and Coulomb potential energy, and the distortions of $\chi$ due
to the absorption are negligible; (ii) at smaller radii the situation becomes
the opposite within a range $<< 1/k$. The interactions directly responsible for
the annihilation have range $R$ and decay exponentially for $r > R$ according to
some $\exp(-r/r_o)$ law (e.g., for a Woods-Saxon potential $R$ is the radius and $r_o$
the diffuseness). Depending on the model, $R_m$ is normally 0.5-1 fm larger than
$R$, suggesting that the relevant processes take place in the exponential tail of
the annihilating forces. Clearly $1/k$ defines the “scale of space resolution” in
the problem, and the following considerations can be applied for $k << 1/r_o$
only.

Summarizing, in our problem we assume both the range $r_o$ characterizing the
exponential damping of the inelastic interaction and the thickness $\Delta$ of the
annihilation shell to be much smaller than $1/k$, and assume the reasonable
matching radius $R_m$ to be larger than $\Delta$. With the previous definitions and
assumptions, all the things that we have written about the “$z$–problem with
reflection condition” can be repeated word by word after substituting $z + z_o$
with $r$, $\Psi(z + z_o)$ with $\chi(r) \equiv r\Psi(r)$, while $r = R_m$ corresponds to $z = 0$ and
$r = 0$ to $z = -z_o$. More properly however, $k$ is the wavenumber produced at
$r = R_m$ by both the kinetic and the Coulomb potential energy.

The saturation condition is expressed by $R_m > \Delta \approx 1$ fm, and seems to be
realized, as above discussed, in antinucleon annihilation on all possible tar-
gets, from proton to heavy nuclei. It implies that the reflected flux is negligi-
ble \textit{inside} the proton/nucleus target. The uncertainty principle assures that the dominating momentum components inside the reaction range are \( \sim 1/\Delta \). When this is transferred to the \( \bar{N} \) wave it means \( |\chi'/\chi|_{R_m} \sim 1/\Delta \), with large flux reflection for \( k\Delta << 1 \).

In the S-wave 1-dimensional reduction of the 3-dimensional scattering problem the reflected wave is a composition of both the scattered and of the untouched initial wave. The disappeared flux corresponds to inelastic reactions, and the ratio of this flux to the incoming one is \( \sim (k\Delta)^2 \) for \( k\Delta << 1 \), in agreement with the previous example. A part of the reflected flux will correspond to elastic reactions, which are not diffractive because we are very far from the unitarity limit. The fact that the above ratio of the absorbed to the incoming flux tends to zero for \( k \to 0 \) is not in contradiction with a finite reaction rate, but target details are lost once \( k < 1/\Delta \).

4 Predictions.

It is easy to estimate upper limits for the complex scattering length \( \alpha \) with the condition

\[
|\chi'/\chi|_{R_m-\epsilon} \approx 1/\Delta. \tag{2}
\]

Using \( |\chi'/\chi|_{R_m+\epsilon} = k \cdot cotg[k(R_m - \alpha)] \) one finds, in the limit \( k \to 0 \),

\[
|\Delta|^2 \approx [R_m - Re(\alpha)]^2 + [Im(\alpha)]^2; \tag{3}
\]

that implies:

\[|Im(\alpha)| \approx \Delta \text{ or smaller},\]

\[Re(\alpha) \text{ is positive and comprised in the range } R_m \pm \Delta.\]

The consequence of this are:

1) \( \Delta \) (rather than \( R_{\text{nucleus}} \)) is the relevant parameter for the low energy reaction probability, which is proportional to \( |Im(\alpha)| \). For \( k << 100 \text{ MeV}/c \) and for \( \bar{n} \) projectiles the reaction probability should be roughly the same for any target nucleus radius, as far as the reaction is S-wave dominated (so, for \( k << 100/A^{1/3} \text{ MeV}/c \)), with magnitude \( \pi \Delta/k_{cm} \approx 6000\Delta/k_{cm} \text{ mb} \) (with \( \Delta \) in fm and \( k \) in MeV/c). For \( \bar{p} \) projectiles the differences will be mostly due to the Coulomb effects, which have been estimated elsewhere\[11,18\]. Both with \( \bar{n} \) and with \( \bar{p} \), \( Im(\alpha) \sim 1 \text{ fm} \) (or smaller) for all nuclear targets.
2) \( \text{Re}(\alpha) \sim +R_m \sim +R_{\text{nucleus}} \) means an \( \bar{n} \)-nucleus total elastic cross section \( \sim 4\pi R_{\text{nucleus}}^2 \), and its positive sign is characteristic of a repulsive interaction. Accordingly, the zero energy \( \rho \)-parameter = \( \text{Re}(\alpha)/\text{Im}(\alpha) \) is negative. We can estimate \( \text{Re}(\alpha) \sim 1 \text{ fm} \) and \( \rho \approx -1 \) for light nuclei, \( \text{Re}(\alpha) \sim 1.3 \ A^{1/3} \) fm and \( \rho \approx -A^{1/3} \) for heavy nuclei. Again, Coulomb effects enhance the total elastic cross section in the \( \bar{p} \) case[6].

3) If one can identify subsets of \( \bar{N}N \) annihilation events which are supposed to be characterized by different \( \Delta \)-parameters, the consequent low-energy cross sections should scale accordingly. E.g., \( \bar{p}p \to 2\pi \) and \( \bar{p}p \to 2K \) have been demonstrated to be characterized by different space scales, because of the different mass of the final states[19]. If the characteristic annihilation distances, measured at \( k >> 200 \text{ MeV/c} \) or estimated by some model, are \( \Delta_1 \) and \( \Delta_2 \), the ratio between the corresponding annihilation rates should be of magnitude \( (\Delta_1/\Delta_2)^2 \) at very small momenta.

For all those reactions (e.g. \( K^- \) absorption on nuclear targets), where the absorption range inside nuclear matter is \( \sim 1 \text{ fm} \), the same considerations apply. Relevant deviations from the previous predictions should be attributed to peculiarities of the external tail of the nuclear density (e.g. a longer tail in deuteron or \( ^3\text{He} \), or a different proton/neutron composition at the surface). In the special case of neutron-halo nuclei the presence of a very long range tail in the nuclear matter distribution removes the basic assumptions of this work.

5 The role of elastic attracting potentials

In presence of a real attracting potential surrounding the annihilation shell the actual zero-energy momentum at \( R_m \) is determined by the potential energy. We must consider two very different cases, i.e. strong or Coulomb interactions.

A strong elastic potential has nuclear characteristic range, so it does not escape the previous general considerations. Now the external surface of the annihilation shell should be displaced to include the region where the distortions of the projectile wavefunction of elastic origin are relevant. This may increase \( \Delta \) up to 2 fm[6,12,15,20]. However, the convergence of the \( \bar{p}p, \bar{p}D, \bar{p}^4\text{He} \) and \( \bar{p}^2\text{0Ne} \) annihilation cross sections to similar values at small momenta, all corresponding to scattering lengths \( < 1 \text{ fm} \) (after subtracting Coulomb effects) suggests that \( \Delta \) is smaller than 1 fm.

The Coulomb potential has atomic range and so escapes all the previous considerations. In \( \bar{p}p \) annihilations, Coulomb forces fix a minimum \( \bar{p} \) kinetic energy of magnitude 1 MeV at the proton surface, corresponding to a momentum 40 MeV/c, that represents a scale for the true zero energy momentum we have to
consider. At much smaller momenta all the modifications that we observe are
due to electromagnetic or atomic effects. With nuclear targets, the Coulomb
energy at the nuclear surface increases proportionally to \( Z/Z^{1/3} = Z^{2/3} \), so
the corresponding zero-energy momentum increases proportionally to \( Z^{1/3} \).
With very heavy nuclei the Coulomb momentum starts becoming compara-
tible in magnitude to the Fermi momentum, introducing a completely different
physics. Apart from this, Coulomb forces produce a large enhancement of
the reaction and elastic cross sections by focusing the \( \bar{p} \) wavefunction on the
nucleus. This effect is widely discussed in other works[6,11,18].

6 Conclusions

We have shown that, within those models where the annihilation probability
is large enough to prevent a consistent overlap between the projectile and
the target wavefunctions, the antinucleon-nucleus annihilation cross section
is largely target-independent, apart for Coulomb effects. The cause of this
behavior is the quantum uncertainty principle, together with the fact that on
most of the nuclear targets the process is characterized by the same value of
the parameter \( \Delta \sim 1 \) fm. \( \Delta \) is the thickness of the spherical shell surrounding
the nucleus where the bulk of the annihilations are supposed to take place.
For the scattering length \( \alpha \) we have estimated \( Im(\alpha) \approx \Delta \), while \( Re(\alpha) \) is
positive and roughly coincides with the larger between the nuclear radius and
\( \Delta \). We have also suggested that the ratio between the low energy annihilation
rates relative to selected final states with different characteristical annihilation
distances \( \Delta_1 \) and \( \Delta_2 \) should be \( \Delta_1/\Delta_2 \).

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