Relations between the QCD sum rules for BBV couplings and vector-dominance model

Valeri Zamiralov
Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University Moscow, Russia
E-mail: zamir@depni.sinp.msu.ru

Abstract. Story of QCD sum rules is briefly reviewed. The dispersion superconvergence sum rule is shown to relate \( \pi N \) coupling constant with the 33-resonance width without unitary symmetry arguments. The QCD sum rules instead reveal the unitary symmetry pattern as all the correlation functions can be reduced to 2 ones repeating the F and D couplings for the octet baryons \( 1/2^+ \). Strong relations between the correlators are established. Vector dominance hypothesis allows to put new restrictions to the QCD sum rules for VBB couplings.

1. Introduction

The QCD sum rules begin their story from the dispersion sum rules formulated more than half century ago. I would remind superconvergent sum rules (SCSR) which have been formulated in 1966 [1] for elastic \( \pi N \) scattering. Assuming that spin-flip amplitude \( B \) goes to zero faster than \( 1/E \) the Cauchy integral over the closed contour with the cuts would be zero. Upon disregarding the integral over the large circle one get

\[
\int_{-\infty}^{\infty} \text{Im} B(E, \theta) dE = 0.
\]

With saturation of the integral with the nucleon pole and 33-resonance contribution [1] \( \pi N \) coupling constant was related to the width of the 33-resonance resulting in \( \Gamma_{\Delta N \pi} = 117 \text{MeV} \) with \( g_{\pi N}^2/4\pi = 14.6 \) in nice agreement with the \( \Gamma_{\Delta N \pi}^{\text{exp}} = 116 - 118 \text{MeV} \). It was the beginning. The next important step was the construction of the FESR - Finite Energy Sum Rules [2]. Let \( A \) be some amplitude and \( A_R \) be its high-energy model behavior and let it be, say, Regge - model one: \( A_R = \beta(t)s^{\alpha(t)} \). For their difference one can construct SCSR:

\[
\int (A(s,t) - A_R(s,t)) ds = 0. \tag{1}
\]

If for \( s \leq R \) the \( A \) and \( A_R \) become practically equal one arrives at the FESR;

\[
\int_{\text{thresh}}^{R} A(s,t) ds = \frac{\beta(t)}{\alpha(t) + 1} R^{\alpha(t)+1}. \tag{2}
\]

The LHS can be calculated through some phenomenological quantities while the RHS can be calculated analytically in terms of high-energy model parameters.
2. QCD sum rules

With arrival of QCD as the theory of strong interaction basing on quark and gluons one soon has understood that in the region far from the perturbative expansion it is impossible to advance without some kind of non-perturbative theory. The way was found in a similar fashion to the FESR’s: Shifman, Vainstein and Zakharov in 1979 [3] have proposed the sum rule where the LHS could be calculated by inserting phenomenological states while the RHS could be rather safely calculated in terms of well-defined operators of definite dimensions.

In order to maintain only the ground states and to suppress the high-dimensional contributions they used Borel transformation and arrived at the well-controlled sum rules with a small number of universal parameters, namely, non-perturbative vacuum expectation values (vev’s) of the operators of definite dimensions like $a_q = -(2\pi)^2 \langle 0|\bar{q}q|0\rangle$, $a_q m_0^2 = (2\pi)^2 g_s \langle 0|\bar{q}\sigma \cdot Gq|0\rangle$, $b = (2\pi)^2 \langle 0|GG|0\rangle$, as well as characteristic QCD parameter $\Lambda_{QCD} \approx 100$ MeV and normalization point $\mu \approx 0.5$ GeV to which the used values of condensates are referred.

There are mainly two free parameters left that is the so called Borel parameter $M^2$ and the threshold parameter $s_0$ which effectively controls the suppression of the continuous spectrum and high-excited states. One more parameter was introduced connected with a possibility to choose various so-called interpolating currents.

The basic notion in QCD sum rules is the polarization operator or correlator function [3], [4]. The correlator for baryon-baryon-vector meson vertex would have the form

$$\Pi(p) = \int e^{ipx} \langle 0|T(\eta(x)\bar{\eta}(0))|V\rangle dx,$$

(3)

where $\eta$ is the so-called correlation current and should be constructed in QCD in terms of quark fields. The $\Sigma^0$ interpolating current can be written:

$$\eta^{\Sigma^0} = \sqrt{\frac{1}{2}} \epsilon^{abc} \left[ \left( u^{aT} C s^b \right) \gamma_5 d^c - \left( s^{aT} C d^b \right) \gamma_5 u^c + \beta \left( u^{aT} C s^b \gamma_5 \right) d^c - \beta \left( s^{aT} C \gamma_5 d^b \right) u^c \right],$$

(4)

where $C$ is the charge conjugation operator, $a, b, c, d, e, f$ are the color indices and $\beta$ is an arbitrary parameter, $\beta = -1$ corresponds to the Ioffe current [4]. The currents of the other baryons can be obtained from $\Sigma^0$ current by making appropriate substitutions of quarks. The $\Lambda$ interpolating current can be obtained through $\sqrt{3} \eta^\Lambda = -\eta^{\Sigma^0} + 2\eta^{\Sigma^0(d+s+s)}$.

3. Quark-biquark model and SU(3)

To obtain unitary symmetry results in the framework of quark scheme we invoke quark-biquark model that is assume that photon (or any other boson quantum) distinguishes between the (equal) quarks which form a biquark and a single quark [5]. The final formula for the magnetic moment of any hyperon with the $\Sigma$-like quark wave function would have the form

$$\mu(\Sigma(ud,Q)) = (e_u + e_d) \mu_F + e_Q (\mu_F - \mu_D),$$

(5)

where $\mu_F$ and $\mu_D$ are usual SU(3) quantities. Important result of this model is that formal group-theoretical couplings of the $F$ and $D$ type have in it a well-defined physical meaning.

In a similar way new formulae for the meson-baryon coupling constants can be written in terms of the quark couplings with mesons and SU(3) constants $F$ and $D$. A coupling constant of any $\Sigma$-like hyperon would have the form

$$G(\Sigma(ud,Q)) = (g_{Muu} + g_{Mdd})F + g_{MQQ}(F - D).$$

(6)

It is a little astonishing that these formulas are valid also for the QCD sum rules.
4. Relations between Σ and Λ quantities in quark model and QCD
To establish unitary symmetry relations between Sigma and Lambda quantities we remind that isospin wave functions $|\Sigma^0\rangle$ and $|\Lambda\rangle$ are related to the ones of U- and V- spins as

$$-2|\Lambda_{us}\rangle = \sqrt{3}|\Sigma^0\rangle + |\Lambda\rangle, \quad -2|\Lambda_{ds}\rangle = -\sqrt{3}|\Sigma^0\rangle + |\Lambda\rangle,$$

$$2|\Sigma_{us}\rangle = -|\Sigma^0\rangle + \sqrt{3}|\Lambda\rangle, \quad 2|\Sigma_{ds}\rangle = |\Sigma^0\rangle + \sqrt{3}|\Lambda\rangle,$$

where $ds$ ($us$) means interchange of flavors $d(u)$ and $s$.

For any operator $O$ the following relations are valid between the matrix elements:

$$2\langle \Sigma_{us} | O | \Sigma_{us} \rangle + 2\langle \Sigma_{us} | O | \Sigma_{us} \rangle - \langle \Sigma^0 | O | \Sigma^0 \rangle = 3\langle \Lambda | O | \Lambda \rangle,$$

$$-2\langle \Sigma_{us} | O | \Sigma_{us} \rangle + 2\langle \Sigma_{us} | O | \Sigma_{us} \rangle = \sqrt{3}\langle \Sigma^0 | O | \Lambda \rangle + \langle \Lambda | O | \Sigma^0 \rangle,$$

and similar ones with interchange of the $\Lambda$ and $\Sigma$ symbols. We introduce also a new relation which reads as

$$\langle \Sigma\Lambda \rangle_{us} + \langle \Sigma\Lambda \rangle_{ds} = \langle \Sigma\Lambda \rangle.$$

As we have noted already these relations are simple but in very tedious QCD sum rule calculations they turn to be very useful. We write now the analogous relations for the QCD correlators [6]:

$$2[\Pi^{\Sigma_0}(ds) + \Pi^{\Sigma_0}(us)] - \Pi^{\Sigma_0} = 3\Pi^\Lambda,$$

$$2[\Pi^{\Lambda}(ds) + \Pi^{\Lambda}(us)] - \Pi^{\Lambda} = 3\Pi^{\Sigma_0}.$$

Our new relation for correlation functions reads

$$\Pi^{\Sigma_0(ds)} + \Pi^{\Sigma_0(us)} = \Pi^{\Sigma_0}$$

5. QCD sum rules for the vector meson–baryon coupling constants
The following correlation function is considered [7] (see also, e.g., [8],[9]):

$$\Pi^{B_1 \to B_2 V} = i \int d^4xe^{ipx} \langle V(q) | T \{ \eta_{B_2}(x)\bar{\eta}_{B_1}(0) \} | 0 \rangle$$

(12)

The correlator can be calculated in terms of the hadrons, as well as in the deep Euclidean region $p^2 \to -\infty$, in terms of the quark and gluon degrees of freedom.

To construct the phenomenological part of the correlator we insert a complete set of intermediate states with the quantum numbers of current operators $\eta_B$.

After isolating the ground state baryons we have:

$$\Pi^{B_1 \to B_2 V} (p_1^2, p_2^2) = \frac{\langle 0 | \eta_{B_2} B_2(p_2) \rangle}{p_2^2 - m_2^2} \langle B_2(p_2) V(q) | B_1(p_1) \rangle \frac{(B_1(p_1) | \bar{\eta}_{B_1}(0) \rangle}{p_1^2 - m_1^2} + \cdots,$$

(13)

$p_1 = p_2 + q$, $m_i$ is the mass of baryon $B_i$, and $\cdots$ means contributions of the higher states and the continuum. Matrix elements are defined as [7]:

$$\langle 0 | \eta_{B_1} B_1(p_1) \rangle = \lambda_{B_1} u(p_1), \langle B_2(p_2) V(q) | B_1(p_1) \rangle = \bar{u}(p_2) \left[ f_1 \gamma_\mu - f_2 \frac{i}{m_1 + m_2} \sigma_{\mu\nu} q^\nu \right] u(p_1) e^\mu,$$

(14)
\( \lambda_{B_i} \) is the overlap amplitude for baryon \( B_i \) [4]. We choose in (13) structures \( \phi \phi \phi \) and \( \phi (e \cdot p) \) as they exhibit better convergence. QCD sum rules for vector-meson baryon coupling constants for electric- and magnetic-type couplings are then [7]

\[
f_1 = \frac{\lambda^2}{\lambda_B^2} \Pi f_1 (u, d, s; M^2, s_0, \beta) \quad f_{1+2} = f_1 + f_2 = \frac{\lambda^2}{\lambda_B^2} \Pi f_{1+2} \quad \kappa = e^{-(m_r^2/M^2)-(m_s^2/M^2)}.
\]

(15)

Correlators for \( VBB \) couplings can be written in terms of only two invariant functions for each coupling, of the electric and magnetic type. For \( \Sigma^0 \Sigma^0 V \) it reads similar to (6)

\[
\Pi^{\Sigma^0 \Sigma^0 V}(u, d, s; M^2, s_0, \beta) = g_{Vuu} \Pi_1 (u, d, s; M^2, s_0, \beta) + g_{Vdd} \Pi_2 (u, d, s; M^2, s_0, \beta).
\]

(16)

6. Unitary symmetry and VDM

Now we try to relate baryon charges, magnetic moments and strong coupling constants with light vector mesons. We start from the Lagrangian for \( VBB \) couplings maintaining only unitary indices but taking in mind that there are two kinds of \( VBB \) couplings, electric \( g^{(e)}_{VBB} \) and magnetic \( g^{(m)}_{VBB} \) ones.

\[
\sqrt{2} \mathcal{L}^{VBB} = (F + D) \bar{B}_\gamma B_\gamma V^\beta + (D - F) \bar{B}_\gamma B^\alpha V_\beta - (D - F) \cdot Tr(\bar{B}B) \cdot TrV,
\]

(17)

that is \( g(\rho^0 pp) = F + D, \ g(\omega pp) = 3F - D, \ g(\phi pp) = 0, \) etc. We write the Lagrangian for the direct \( V\gamma \) interaction as

\[
\mathcal{L}^{V\gamma} = \frac{\sqrt{2}}{g_{V\gamma}} (V_1^\gamma - \frac{1}{3} TrV) = \frac{1}{g_{V\gamma}} (\rho^0 + \frac{1}{3} \omega - \frac{\sqrt{2}}{3} \phi),
\]

(18)

that is \( g_{\rho\rho\gamma} = g_{V\gamma}, \ g_{\omega\gamma} = 3g_{\rho\rho\gamma}, \ g_{\phi\gamma} = -3g_{\rho\rho\gamma}/\sqrt{2}. \) Now we construct baryon electric charges and magnetic moments using formula

\[
e_B, \mu_B = \frac{g^{(e,m)}_{VBB}}{g_{\rho\gamma}} + \frac{g^{(e,m)}_{\omega\BB}}{g_{\omega\gamma}} + \frac{g^{(e,m)}_{\phi\BB}}{g_{\phi\gamma}}.
\]

(19)

We write it for magnetic moments for the proton using Eq.(18):

\[
\mu_p = \frac{F^m + D^m}{g_{\rho\gamma}} + \frac{3F^m - D^m}{g_{\omega\gamma}} = \frac{2}{g_{\rho\gamma}} (F^m + \frac{1}{3} D^m) = \mu_F + \frac{1}{3} \mu_D.
\]

(20)

Baryon octet magnetic moments are given by \( SU(3) \) with renormalized \( F \) and \( D \) couplings \( \mu_F = 2F^m/g_{\rho\gamma}, \ \mu_D = 2D^m/g_{\rho\gamma}. \) Electric charges of baryons are obtained by changing \( \mu_F \) to \( e \) and putting \( \mu_D = 0 \) and \( / \) or deduced from the formulas in terms of \( D_e \) and \( F_e \) similar with \( D^e = 0 \) and \( 2F^e = g_{\rho\gamma}. \) This equality gives us criterium to distinguish between the models if we believe in Vector Domiance Model ( VDM). We proceed with the work [10]. Sum rules for the "electric" structure \( p_\mu \gamma (p_\nu \gamma) \mu \) Eqs.(24-27) of [10] for the \( \Sigma^0 \) and \( \Lambda \) in a little simplified form read

\[
(g_{\Sigma^0} V + C_\Sigma) \Lambda^2 e^{-m^2_{\Sigma}/M^2} = M^6 E_1 \Sigma L^{-4/3} (g_u + g_d + g_s) + \frac{4}{3} L^{4/3} [(g_u + g_d)a_u a_d + g_s a_u a_d] + M^2 [-2m_s a_s (g_u + g_d) - 4m_u a_u g_d - 4m_d a_d g_u + 4m_d a_u g_d + 4m_s a_d g_u];
\]

(21)
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