Modeling of Thermal Process in Polymeric Bearings System on a Common Shaft

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Abstract. We propose a technique for calculating the dynamics of the temperature field in a system of polymeric sliding bearings, taking into account the speed of rotating shaft. A numerical algorithm for calculating the thermal processes in sliding bearings on a common shaft is developed by solving a system of two-dimensional and three-dimensional heat equations. We determined the time step, depending on the speed of rotation of the shaft by computational experiments. This allows us to determine the temperature fields with acceptable accuracy for practical use. The test results of sliding bearings made of composite material based on fluoropolymer are presented, which show the adequacy of the proposed mathematical model to the real thermal process.

1. Introduction
One of the promising ways to improve the new technics, designed to work at low temperatures is to replace the traditional elements of friction units on polymeric solid lubricating elements [1, 2, 3]. The antifriction properties inherent in polymer and composite materials make it possible to maintain the operability of friction units in conditions of limited lubrication or in the absence of it. At the same time, polymeric friction units are not widely used because of low permissible load-speed conditions operating. Simulation of the operation of friction units in various conditions using mathematical models allows us to foresee the achievement of limiting conditions for the temperature of polymer materials at the design stage and to determine the maximum permissible load and slip velocity [4, 5].

2. Model of heat processes

Figure 1. Calculation scheme for a system of sliding bearings: (1) shaft; (2) sleeve; (3) race.
Figure 1 shows a system of sliding bearings made of polymer composite materials. Rotating with angular velocity $\Omega$ steel shaft is supported by N bushes from antifriction material, rigidly coupled to the steel body (race) bearings.

The proposed model is based on supposition that temperature distribution in uniform along the bearing and the body, for heat transfer at their end surfaces is insignificant. Thus, the sleeve and the body can be regarded as plane while the shaft is considered to be three dimensional.

The nonstationary temperature field in the bearings is described by two-dimensional quasilinear heat equations for sleeve and bodies:

$$C_k \frac{\partial \lambda_k}{\partial t} = \frac{1}{r} \int_{r_k}^{R_k} \rho \omega \left( \frac{\partial \lambda_k}{\partial r} \right) r \frac{\partial \lambda_k}{\partial r} + \frac{1}{r^2} \frac{\partial}{\partial \varphi} \left( \lambda_k(T) \frac{\partial \lambda_k}{\partial \varphi} \right) ,$$

$$k = 1, 2, \ldots, N, \quad i = 2, 3,$$

For the shaft, this field is described by the following three-dimensional equation with a convection term to take into account the rotation of the shaft:

$$C(U) \frac{\partial \lambda}{\partial t} r \frac{\partial \lambda}{\partial r} + \frac{\partial}{\partial \varphi} \left( \lambda(U) \frac{\partial \lambda}{\partial \varphi} \right)$$

$$0 < r < R_1, \quad -\pi < \varphi < \pi, \quad 0 < t \leq t_m.$$

In shaft-sleeve friction zones, the conditions of friction heat transfer are set as follows:

$$\lambda_k(U) = \int_{L + 1}^{L} \frac{\partial}{\partial r} \varphi \bigg|_{r=R_1}^{r=R_k} T_{k}(r, \varphi, t) dz + \frac{1}{T_k} \int_{R_{1, k}}^{R_k} \varphi \bigg|_{z=L_k}^{z=L_{k+1}} U(R_1, \varphi, t) dz = \lambda_k(R_{1, k} \varphi, t).$$

Traditional conditions of the first and the third kind are set on the other boundaries. The initial temperature distribution is homogeneous.

3. Numerical solution

Problems (1)–(11) are solved by the finite difference method and are reduced to a set of one-dimensional heat equations [6–12]. The presence of a convection term in heat equation (2) to take into account the rotation of the shaft leads to certain difficulties in solving the problem numerically. Using monotonic and locally onedimensional difference equations to approximate the sum allows one to meet the maximum principle, i.e., at any $\tau$ and $h\varphi$ steps, an approximated solution can be found for the time and angular variable [13, 14, 15].

The algorithm developed for the numerical determination of the temperature field should be used for to solve the boundary problem by solving the inverse heat transfer problem in order to determine friction heat transfer. Thus, in the numerical solution of the direct problem, the machine time needed to solve the inverse problem depends on the time step. If the step is too small, the machine time can be impracticable. So, the maximum possible time step at the preset sticking criterion should be chosen.

Based on computational experiments that use a detailed spatial network [16, 17, 18], a time step that provides a convergence solution was determined at different values of the Courant number

$$\gamma = \tau \Omega / h\varphi, \quad \nu = R_1 \Omega$$

that characterize the relation of time step based on the angular variable with...
the rotational speed of the shaft and time step [19]. Since the rotating shaft is common for all of the sliding bearings, it is enough to consider the case of one bearing. The time step found can be used for temperature calculation for a system of several bearings.

Results of temperature calculations vs. the Courant number are given for the following geometric dimensions: \( R_{1k} = 12 \) mm, \( R_{2k} = 13 \) mm, \( R_{3k} = 16 \) mm, \( R_{4k} = 30 \) mm, and \( k = 1 \) (Fig. 2). The material of the shaft and the race is steel, while the sleeve is made of F4K20 - filled PTFE. The rotational speed of the shaft \( \pi \) rad/s and contact angle is 30°. The intensity of the heat transfer is constant at \( Q = 67 \) kWt/m². The convergence solution is found at \( \gamma < 1 \). For practical calculations, the time step can be determined from the condition \( \gamma = 2 \), since, at \( \gamma < 2 \), the temperature values change within a 1° interval.

![Figure 2](image)

**Figure 2.** Calculated dependences of maximum temperatures in the friction zone at various Courant numbers \( \gamma \): (1) \( \gamma = 36 \); (2) \( \gamma = 12 \); (3) \( \gamma = 2 \); (4) \( \gamma = 1 \); (5) \( \gamma = 1.8 \).

4. Comparison of calculated and experimental temperatures

To establish whether our mathematical model with two-dimensional and three-dimensional heat equations is adequate to the real thermal process in plain bearings, we compared the simulated and experimental temperature (Fig. 3).

We made experiments on a friction machine SMT-1 for one bearing, recording the temperature with copper-constantan thermocouples of diameter 0.1 mm and the use of multichannel “TERMODAT” device at five points of a hub at 0.5 mm away from the friction zone. The angle of contact was 60°.

Repeating the experiment three times, we determined a confidence interval for the dependence of temperature on time. Fig. 3 shows that the calculated dependence of temperature for \( \phi = 0 \) lies within the range of experimental data. We obtained similar results for other points of temperature measurements, which confirms that our description of the thermal process in a plain bearing under consideration is adequate.

The calculations resulted in determining the effective coefficients of the heat equation ensuring that the simulated and experimental temperatures are close. The difference between the effective coefficients and the handbook values of thermophysical characteristics of materials is not greater than
the range of properties of the polymer composition and steel. The effective heat conductivity coefficient \( \lambda_2 \) of the filled fluoropolymer F4K20 is 0.34 W/(m·°C), the spatial heat capacity \( C_2 \) equals 2.2·10^6 J/(m^3·°C), while for steel \( \lambda_1 = 46 \) W/(m·°C) and \( C_1 = 3.48·10^6 \) J/(m^3·°C).

We calculated the heat exchange coefficient of the rotating shaft as [20, 21]:

\[
\alpha_i = \frac{Nu^{2/3}}{2R_1}, \quad Nu = \frac{95(2 Re^2 + Cr)^{0.35}}{95(2 Re^2 + Cr)^{0.35}}
\]  

(5)

The comparison of temperature data indicates that the proposed mathematical model is applicable for determining the temperature field in plain bearings.

![Figure 3](image)

**Figure 3.** Calculated dependences of temperature on time in an inner point of a sleeve at a distance of 0.5 mm from the friction zone. (I is confidence interval for experimental temperature data).

5. **Summary**

We proposed a mathematical model of thermal process in a system of plain bearings for the rotation of the shaft and a technique enabling us to determine, basing on simulations, a temporal step suitable for practical calculations. By comparing the simulated and experimental temperature, we established that this mathematical model is adequate to the real thermal process in a plain bearing.

6. **References**

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