THE EFFECT OF THE ENVIRONMENT ON THE FABER–JACKSON RELATION

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ABSTRACT

We investigate the effect of the environment on the Faber–Jackson (FJ) relation, using a sample of 384 nearby elliptical galaxies and estimating objectively their environment on the typical scale of galaxy clusters. We show that the intrinsic scatter of the FJ relation is significantly reduced when ellipticals in high-density environments are compared to ellipticals in low-density ones. This result, which holds in a limited range of overdensities, is likely to provide an important observational link between scaling relations and formation mechanisms in galaxies.

Key words: galaxies: elliptical and lenticular, cD – galaxies: fundamental parameters

Online-only material: color figures, machine-readable table

1. INTRODUCTION

The Faber–Jackson (FJ) relation is the first scaling relation discovered for elliptical galaxies. Already described by Morgan & Mayall (1957), although in a qualitative way, it was given its first quantified form 20 years later by Faber & Jackson (1976) who, on the basis of a handful of nearby early-type galaxies, were able to prove the existence of a power-law relation linking luminosity ($L_B$) to central velocity dispersion ($\sigma_0$). Soon after Kormendy (1977) found a second scaling relation which holds for elliptical galaxies, relating the effective surface brightness ($\mu_e$) to the effective radius ($R_e$). The Kormendy relation was refined 10 years later for ellipticals by Hamabe & Kormendy (1987) and in that same year Dressler et al. (1987) and Djorgovski & Davis (1987) discovered a more general relation (the fundamental plane, FP) linking log $R_e$, log $\sigma_0$, and $\mu_e$.

The scaling relations are powerful tools that can be used to derive galaxy distances and, even more importantly, constitute an invaluable observational benchmark for theoretical models. It is especially for this latter reason that they have been the object of much interest since their discovery. Understanding the origin and nature of the scaling relations is a fundamental quest for any successful theory of galaxy formation which is expected to be able to predict the observed slope, scatter, possible variation (as a function of luminosity, wavelength, environment), and evolution (with $z$). The much narrower scatter displayed by the FP with respect to the FJ and Kormendy relations rapidly made the former more attractive than the latter two, which were easily interpreted to be partial representations (projections) of the FP onto a lower dimensional space (Dressler et al. 1987; Djorgovski & Davis 1987; Faber et al. 1987; de Zeeuw & Franx 1991).

Very recently the FJ relation appears to have again captured attention from the theoretical point of view, as Sanders (2010) has claimed that it is more fundamental and universal than the FP within the context of modified Newtonian dynamics. This finding is expected to motivate renewed interest in the FJ relation, which so far has not been largely investigated. With the exceptions of studies which have provided evidence for a decrease in its steepness at low luminosity (Tonry 1981; Davies et al. 1983; Held et al. 1992; Fritz et al. 2005; Matković & Guzmán 2005; Bernardi et al. 2006; Desroches et al. 2007; Lauer et al. 2007; Von der Linden et al. 2007; Kourkchi et al. 2012) and studies devoted to investigations of the effect of luminosity, mass, and redshift on it (Fritz et al. 2005; Bernardi et al. 2006; Desroches et al. 2007; Nigoche-Netro et al. 2010, 2011), little work has been carried out on the FJ relation. At variance with the FP, for which the effect of the environment has been largely investigated, although with conflicting results (de Carvalho & Djorgovski 1992; Marquez & Moles 1996, 1999; de la Rosa et al. 2001; Treu et al. 2001; Bernardi et al. 2003; Evstigneeva et al. 2002; González-García & van Albada 2003; Reda et al 2004; Reda et al. 2005; Denicoló et al. 2005; D’Onofrio et al. 2008; La Barbera et al. 2010), so far only four studies exist (Ziegler 2005; Fritz et al. 2005, 2009; Fritz & Ziegler 2009) which have looked for possible effects induced by the environment on the FJ relation of rather distant ($z \in [0.2–0.7]$) early-type galaxies; however, no strong evidence has been found for such effects.

According to the standard cosmological paradigm, structures in the present-day universe have formed through a hierarchical scenario process predicting rather different assembling timescales and evolutionary paths for galaxies in the high- and low-density regions (Baugh et al. 1996; Kauffmann & Charlot 1998; Somerville & Primack 1999; Kauffmann et al. 2004). The environment is thus expected to play a relevant role in shaping galaxy properties and is likely to leave its imprint in the scaling relations also. This is the reason which has motivated the above-mentioned studies (mostly concentrated on the FP) and the present work devoted to investigate the effect of the environment on the FJ relation, using a sample of 384 nearby ellipticals and estimating their environment on the typical scale of galaxy clusters.

The structure of the paper is as follows: in Section 2, we present the sample; in Section 3, we derive the FJ relation for the whole sample and for its bright and faint components, and test the robustness of our results accounting for errors in both $\sigma_0$ and $m_{B}^{0}$; in Section 4, we illustrate the method that we have used to estimate the environment; in Section 5, we show that the scatter of the FJ relation gets largely reduced in high-density environments and increased in low-density ones, and that this difference is neither induced by errors on $\sigma_0$ nor by luminosity differences between the samples; in Section 6, we show that the scatter of the FJ relation increases with decreasing density in overdense environments and decreases with increasing density in underdense environments; finally, in Section 7, we draw the conclusions.

A Hubble constant of $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ is adopted throughout.
The Effect of Distance on Luminosity of Our Sample

| Bin     | $\Delta \sigma$ (km/s) | $N_{\text{ell}}$ | $(M_B)$ | Med $(M_B)$ |
|---------|------------------------|------------------|---------|-------------|
| I       | $2000 \leq v_r < 4000$ | 61               | $-20.25$ | $-20.21$    |
| II      | $4000 \leq v_r < 6000$ | 136              | $-20.59$ | $-20.67$    |
| III     | $6000 \leq v_r < 8000$ | 120              | $-20.98$ | $-21.02$    |
| IV      | $8000 \leq v_r < 10000$| 67               | $-21.39$ | $-21.41$    |

Figure 1 shows the distribution of our sample ellipticals (filled circles) in the $v_r$, $M_B$ plane together with the curve corresponding to the faintest observable $M_B$ in a sample limited to $m_B = 15.5$ (which is the limit that we have imposed on our sample). The curve allows one to visualize the well-known effect induced by distance on luminosity in a flux-limited sample: since the minimum observable luminosity increases with distance, the farthest galaxies will be, on average, also the brightest ones. From Figure 1 we see, however, that the distance effect constrains only the minimum observable luminosity and that its real entity depends quite strongly on the galaxy distribution (i.e., the presence of clusters and groups) and in the present case also to the possible lack of some data which we cannot exclude as we are dealing with a sample drawn from a compilation of available data.

The effect of distance on the luminosity can be perceived better from Figure 2, which shows the absolute magnitude ($M_B$) distribution of ellipticals in the whole sample (dotted histogram) superimposed on the $M_B$ distribution of ellipticals in four bins of increasing radial velocity, each spanning a 2000 km s$^{-1}$ range. The bins and their characteristics are given in Table 2 for each bin (Column 1) gives the radial velocity range (Column 2), total number of ellipticals (Column 3), mean (Column 4), and median (Column 5) value of the $M_B$ distribution (these latter to be compared with $-20.80$ and $-20.87$, which
are the corresponding values of the whole sample). Figure 2 shows rather clearly the progressive shift of the $M_B$ distribution of ellipticals as a function of increasing distance, and from Columns 4 and 5 of Table 2 we see that this distance effect induces an average increase of 1.2 mag between the farthest and the nearest ellipticals in our sample.

3. THE FJ RELATION FOR THE WHOLE SAMPLE

The FJ relation for the total sample is shown in Figure 3. Superimposed on the data (open circles) is the best-fit (green continuous line) derived taking into account errors in $\sigma_0$ (also displayed in Figure 3) and corresponding to $L_B \propto \sigma_0^{1.0}$. The red dashed ($L_B \propto \sigma_0^{5.6}$) and blue dotted ($L_B \propto \sigma_0^{3.2}$) lines represent the best (weighted for errors in $\sigma_0$) fits for the 206 bright ($\log(L_B/L_{B\odot}) \geq 10.5$) and the 178 faint ($\log(L_B/L_{B\odot}) < 10.5$) ellipticals in the sample, with $\log(L_B/L_{B\odot}) = 10.5$ being the value below which data in Figure 3 start to deviate progressively from the fit and to extend toward low values of $\sigma_0$.

From Figure 3, we see that the dispersion of the data around the fits is quite large with the average scatter (rms), $\sigma(\sigma_0)$, amounting to $51.1 \pm 1.2$ km s$^{-1}$ for the whole sample and to $49.3 \pm 1.8$ km s$^{-1}$ and $52.3 \pm 1.7$ km s$^{-1}$, respectively, for the bright and faint subsamples.

The less steep increase of $L_B$ with $\sigma_0$ displayed by faint ellipticals in our sample confirms previous evidence obtained by several authors (Tonry 1981; Davies et al. 1983; Held et al. 1992; Fritz et al. 2005; Matković & Guzmán 2005; Bernardi et al. 2006; Desroches et al. 2007; Lauer et al. 2007; Von der Linden et al. 2007; Kourkchi et al. 2012) on different samples, which finds theoretical justification (Dekel & Silk 1986) in the expected link between the slope of the FJ relation and the amount of dark matter in elliptical galaxies.

Figure 2. Relative contribution to the absolute blue magnitude ($M_B$) distribution of the whole sample (dotted) from ellipticals belonging to four bins of increasing distance (see Table 2).

Figure 3. FJ relation for the whole sample. The green continuous line is the best fit for the whole sample ($L_B \propto \sigma_0^{1.0}$). The red dashed ($L_B \propto \sigma_0^{5.6}$) and the blue dotted ($L_B \propto \sigma_0^{3.2}$) lines are the best fits for the high ($\log(L_B/L_{B\odot}) \geq 10.5$) and low ($\log(L_B/L_{B\odot}) < 10.5$) luminosity ellipticals. All fits have been derived accounting for errors in $\sigma_0$.

(A color version of this figure is available in the online journal.)
unweighted fit we would have obtained the slope of the FJ relation. Had we derived the slopes with an weighted fit on the slopes. The distributions of the relative $\sigma_0$ errors ($\Delta \sigma_0/\sigma_0$) of data lying below (continuous) and above (dotted) the best-fit lines for the unweighted (left) and the weighted (right) fit of the whole sample (upper panels) and of the bright and faint subsamples (middle and lower panels) presented in Figure 4 show that the effect produced by the weighted fit is to reduce the numerical dominance of ellipticals with small $\Delta \sigma_0/\sigma_0$ above the fit lines, increasing, as a consequence, the slope of the FJ relation. Had we derived the slopes with an unweighted fit we would have obtained $L_B \propto \sigma_0^{3.4}$ for the whole sample, $L_B \propto \sigma_0^{4.7}$ for the bright, and $L_B \propto \sigma_0^{2.7}$ for the faint subsample (with average scatters $\sigma(\sigma_0)$, respectively, equal to $51.9 \pm 1.2$ km s$^{-1}$, $49.5 \pm 1.8$ km s$^{-1}$, and $52.9 \pm 1.7$ km s$^{-1}$).

It is worthwhile to stress that a weighted fit will always produce an artificial steepening of the slope (whatever the size of the errors) if data with large errors are preferentially found below the fit. To prove that we have derived the FJ relation for the 314 ellipticals having $\Delta \sigma_0/\sigma_0 \leq 0.1$, we find that the weighted fit induces an increase of the slope from 3.8 to 4.1 for the whole sample, and from 5.4 to 5.8 and from 3.0 to 3.3, respectively, for the bright and faint subsamples. The average dispersion of the data diminishes a little and settles around 48 km s$^{-1}$, the exact value depending on the kind of fit and sample. The slight reduction of $\sigma(\sigma_0)$ is expected as we have excluded data with large relative errors in $\sigma_0$ (i.e., $\Delta \sigma_0/\sigma_0 > 0.1$) which, being less accurate, are more likely to deviate more strongly from the fit.

Table 3 summarizes all the results described above. In Column 1, we list the sample kind (whole, bright, or faint); in Column 2, the number of ellipticals in each sample; in Column 3, the kind of fit (either unweighted or weighted); in Column 4, the

| Sample          | $N_{\text{el}}$ | Fit   | $\alpha$ | $\sigma(\sigma_0)$ (km s$^{-1}$) |
|-----------------|-----------------|-------|----------|----------------------------------|
| Whole           | 384             | Unweighted | $3.4^{+0.3}_{-0.1}$ | $51.9 \pm 1.2$ |
| Bright          | 206             | Unweighted | $4.7^{+0.8}_{-0.6}$ | $49.5 \pm 1.8$ |
| Faint           | 178             | Unweighted | $2.7^{+0.3}_{-0.2}$ | $52.9 \pm 1.7$ |
| Whole (Faint)   | 384             | Weighted  | $4.0^{+0.2}_{-0.2}$ | $51.1 \pm 1.2$ |
| Bright (Faint)  | 206             | Weighted  | $5.6^{+0.9}_{-0.7}$ | $49.3 \pm 1.8$ |
| Faint (Faint)   | 178             | Weighted  | $3.2^{+0.5}_{-0.4}$ | $52.3 \pm 1.7$ |
| Whole (Bright)  | 314             | Unweighted | $3.8^{+0.2}_{-0.2}$ | $48.4 \pm 1.0$ |
| Bright (Bright) | 174             | Unweighted | $5.4^{+1.0}_{-0.8}$ | $47.9 \pm 1.5$ |
| Faint (Bright)  | 140             | Unweighted | $3.0^{+0.3}_{-0.3}$ | $47.2 \pm 1.1$ |
| Whole (Bright)  | 314             | Weighted  | $4.1^{+0.3}_{-0.2}$ | $48.1 \pm 1.0$ |
| Bright (Bright) | 174             | Weighted  | $5.8^{+1.1}_{-0.8}$ | $48.0 \pm 1.5$ |
| Faint (Bright)  | 140             | Weighted  | $3.3^{+0.6}_{-0.4}$ | $47.0 \pm 1.1$ |

Figure 4. Distribution of relative errors in $\sigma_0$ ($\Delta \sigma_0/\sigma_0$) for ellipticals lying above (dotted) and below (continuous) the best-fit line in the whole sample (upper panels) and in the high- and low-luminosity subsamples (middle and lower panels). Left panels refer to unweighted fits, right panels to fits weighted for errors in $\sigma_0$. 

Table 3: FJ Relation Parameters for Ellipticals in the Whole Sample and in the Bright ($\log(L_B/L_B^\odot) \geq 10.5$) and Faint ($\log(L_B/L_B^\odot) < 10.5$) Subsamples

Some caution should be given, however, to slope values derived by means of fits in which errors in $\sigma_0$ (Column 5 of Table 1) have been taken into account, as they might be somewhat artificially increased if data with large $\sigma_0$ errors (which weight less) are mostly found below the fit lines. This seems actually to be the case in our sample (cf. Figure 3), and from Figure 4 we can visualize better the effect induced by the weighted fit on the slopes. The distributions of the relative $\sigma_0$ errors ($\Delta \sigma_0/\sigma_0$) if data with large errors are preferentially found below the fit. To prove that we have derived the FJ relation for the 314 ellipticals having $\Delta \sigma_0/\sigma_0 \leq 0.1$, we find that the weighted fit induces an increase of the slope from 3.8 to 4.1 for the whole sample, and from 5.4 to 5.8 and from 3.0 to 3.3, respectively, for the bright and faint subsamples. The average dispersion of the data diminishes a little and settles around 48 km s$^{-1}$, the...
FJ relation slope ($\alpha$) with related uncertainty; and in Column 5, the average scatter of data around the best-fit line ($\sigma(\sigma_0)$) with related uncertainty. The values in Table 3 do not allow us to establish an exact value for the slope of the FJ relation, either for the whole sample or for its bright and faint components, but allow us to confirm the presence of two distinct (luminosity-dependent) components in the FJ relation characterized by a slope which is steeper for bright ($\log(L_B/L_{B,\odot}) \geq 10.5$) than for faint ($\log(L_B/L_{B,\odot}) < 10.5$) ellipticals, as the difference between the slopes holds (and is larger than the errors) whatever the kind of sample (either whole or whole with small $\Delta\sigma_0/\sigma_0$) and of fit (either unweighted or weighted).

Finally, to check the effect of possible errors on $m_B$ on the derived FJ relation, we have randomly added or subtracted to each $m_B$ (Column 6 in Table 1) either its real (Column 7 in Table 1) or average error (computed on the whole sample). We have repeated this operation 300 times for both cases, thus obtaining two sets of data each including 300 simulated samples. We have subsequently derived the FJ relation (weighted fit) for each simulated sample in each set and grouped the results together to obtain the total distribution of $\sigma(\sigma_0)$, $\alpha$, and $\alpha_{\text{max}} - \alpha_{\text{min}}$ (i.e., twice the maximum uncertainty in the slope $\alpha$). These distributions (normalized to the total number of simulated samples) are shown in Figure 5, where continuous and dotted histograms refer, respectively, to samples obtained by random addition or subtraction of the average or of the real error in $m_B$. The upper panels refer to the whole-simulated samples, while the middle and lower panels refer to the bright and faint subsamples, extracted from the previous ones. Since the artificial random increase or decrease of $m_B$ implies a decrease or increase in $L_B$, the number of ellipticals in the high- or low-luminosity subsamples is no longer constant but varies as a consequence of the variation in $L_B$. We find median values of 204 and 180 ellipticals and of 210 and 174 ellipticals (to be compared with the real value of 206 and 178) for the high- and low-luminosity subsamples when the average or real error on $m_B$ is, respectively, randomly added or subtracted.

The arrow placed on each distribution in Figure 5 indicates the corresponding values obtained in the real case (i.e., the weighted fit for the whole sample or for the bright or for the faint subsample, cf. values in Table 3, lines 4, 5, and 6) and allows us to state that the effect of the error in $m_B$ is stronger for the whole sample than for the bright and faint subsamples, which is not surprising as we have shown (in this Section) that the FJ relation for the total sample can be interpreted as due to the combination of two separate (luminosity-dependent) components. Variations in $L_B$ of each elliptical in the sample are thus expected to produce a stronger (amplified) effect on the FJ relation for the whole sample than on the FJ relations for the separate (luminosity-dependent) components. From Figure 5 we also see that the effect of the error in $m_B$ is stronger for the faint (lower panels) than for the bright (middle panels) subsample. This is also not surprising, since variations in $L_B$ are expected to influence more strongly fits which have a less steep slope.
Table 4
Average Values for the Parameters of the FJ Relation Derived Accounting for Possible Errors in \( m_B \)

| Sample | \( \Delta m_B \) | \( \sigma (\sigma_\alpha) \) (km s\(^{-1}\)) | \( \sigma (\sigma_\alpha)\text{RMS} \) (km s\(^{-1}\)) | \( \alpha \) | \( \alpha\text{RMS} \) | \( \Delta \alpha \) | \( \Delta \alpha\text{RMS} \) |
|--------|-----------------|---------------------------------|---------------------------------|--------|-----------------|-----------------|-----------------|
| Whole  | 0.25            | 52.6                            | 0.6                             | 4.3    | 0.1             | 0.52            | 0.03            |
| Whole  | [0.03–0.72]     | 52.8                            | 0.7                             | 4.3    | 0.1             | 0.52            | 0.03            |
| Bright | 0.25            | 49.8                            | 1.2                             | 5.9    | 0.5             | 1.7             | 0.3             |
| Bright | [0.03–0.72]     | 50.1                            | 1.3                             | 5.7    | 0.5             | 1.6             | 0.3             |
| Faint  | 0.25            | 54.5                            | 1.5                             | 3.5    | 0.3             | 1.1             | 0.2             |
| Faint  | [0.05–0.71]     | 54.2                            | 1.8                             | 3.7    | 0.3             | 1.2             | 0.3             |

In Table 4 we list, for each sample (Column 1), the error \( \Delta m_B \) (either the average value for the whole sample or the range within which the true value is found) which has been randomly added or subtracted to \( m_B \) (Column 2), the mean value of the average scatter of the data around the fit \( \langle \sigma \rangle \) and its rms (Columns 3 and 4), the mean value of the slope \( \langle \alpha \rangle \) and its rms (Columns 5 and 6), and the mean value of \( \sigma_{\text{max}} - \sigma_{\text{min}} \) (here indicated as \( \langle \Delta \alpha \rangle \)) and its rms (Columns 7 and 8). Comparing the mean values listed in Table 4 (Columns 3, 5, and 7) with the values obtained for the real sample and subsamples (arrows in Figure 5 corresponding to the values listed in Table 3, lines 4, 5, and 6) we see that on average the effect of the error in \( m_B \) can be considered moderate. The data in Table 3 and Figure 5 allow us to conclude that even if the \( m_B \) of each elliptical varies by a quantity equal to the average or real error, respectively, which is surely an overestimate of what may happen in the real case, we can confirm the value of the average dispersion of the data \( \sigma (\sigma_\alpha) \) around the best fit which would only increase slightly. The presence of two distinct (luminosity-dependent) components would also be confirmed, since both slopes would increase somewhat but still remain well distinguished (their difference being larger than their errors).

4. THE ENVIRONMENT

To evaluate the environment of each elliptical galaxy in our sample, we have applied the neighbor search code of Focardi & Kelm (2002) to the Updated Zwicky Catalog (UZC; Falco et al. 1999).

UZC is a wide-angle three-dimensional catalog of nearby galaxies that covers the entire northern sky down to a declination of \(-2^\circ.5\) and is claimed (Falco et al. 1999) to be 96% complete for galaxies brighter than \( m_B = 15.5 \).

The neighbor search code is a versatile tool which can be applied to three-dimensional catalogs either to produce galaxy samples characterized by different environment or to estimate galaxy environment on different scales and depth.

A detailed description of the code can be found in Focardi & Kelm (2002), together with the results of the first application of the code to UZC which has produced a large homogeneous sample of compact groups (UZC-CGs; Focardi & Kelm 2002). The code has been subsequently applied to UZC to produce a sample of bright isolated galaxy pairs (UZC-BPGs; Focardi et al. 2006) and a small sample of very isolated bright ellipticals (Memola et al. 2009). It has also been applied to the 2dFGRS (Colles et al. 2001) in order to perform a detailed analysis on the luminosity/environment/spectral type relation for galaxies (Kelm et al. 2005).

When used simply to detect neighbors, as in the present case, the code needs only two input parameters which are the maximum projected distance \( (\Delta R) \) and radial velocity difference \( (\Delta v) \) between each elliptical in the sample and its possible neighbors (from UZC) and includes, obviously, a cross-check on coordinates, \( v_r \) and \( m_B \), to avoid spurious detection of the elliptical as a possible neighbor of itself.

We have set \( \Delta R = 1.5 \) Mpc and \( \Delta v = 1000 \) km s\(^{-1}\), a choice which has allowed us to estimate the environment on the typical scale of galaxy clusters.

Figure 6 shows the distribution of the number of neighbors \( (N_{\text{neigh}}) \) when the whole sample is divided into the four bins of increasing radial velocity that we have defined in Section 2 and whose characteristics are summarized in Table 2. The distributions appear rather different, which is not unexpected as we have already shown (cf. Figures 1 and 2, Table 2: Columns 4 and 5) how the distance effect produces an increase of luminosity with increasing distance. Thus, as the number of galaxies decreases with increasing luminosity we would expect, on average, a decreasing number of neighbors with increasing distance. Figure 6 shows that this is not exactly the case as the expected decrease in \( N_{\text{neigh}} \) is evident only in the fourth bin, while both bins II and III show an anomalous tail (large values of \( N_{\text{neigh}} \), which is due to the presence of several galaxy clusters and groups (belonging, respectively, to the Perseus–Pisces and Coma supercluster) of which some ellipticals in bins II and III are members.

5. THE EFFECT OF THE ENVIRONMENT ON THE FJ RELATION

To look for possible effects induced by the environment on the FJ relation one must compare ellipticals in high- and low-density environments, with density being as large and as small as possible but leaving however a number of ellipticals in each environment that is not too small.

An objective way to define these extreme environments can be obtained by relating the number of neighbors required to enter each sample to the median value of the \( N_{\text{neigh}} \) distribution, the latter computed separately for each bin to account for effects related to distance and non-uniformity in the galaxy distribution.

We find that defining as high or low density an environment characterized by a number of neighbors equal or larger or equal or smaller than 5 times or 0.2 times the median value of \( N_{\text{neigh}} \) provides us with two subsamples of 26 and 36 ellipticals (respectively including 7% and 9% of the whole sample). Since the median value of \( N_{\text{neigh}} \) is equal to 8 in the first bin, to 9 in the second and third bins, and to 4 in the fourth bin, our definition of high-density environment implies \( N_{\text{neigh}} \geq 40 \) (in bin I), \( N_{\text{neigh}} \geq 45 \) (in bins II and III), and \( N_{\text{neigh}} \geq 20 \) (in bin IV), while our definition of low-density environment implies \( N_{\text{neigh}} \leq 1 \) (in bins I, II, and III) and \( N_{\text{neigh}} = 0 \) (in bin IV).

Figure 7 shows the FJ relation for ellipticals in the high- (red filled circles) and low- (blue open squares) density environments defined above. Each datum is displayed with its own error (both in \( m_B \) and in \( \sigma_\alpha \)), while the red continuous and blue dotted lines represent the best weighted fits, respectively, giving \( L_B \propto \sigma_\alpha^{3.79} \) and \( L_B \propto \sigma_0^{3.75} \). It is evident that ellipticals in low-density regions are distributed in a much more dispersed way than ellipticals in high-density ones. The \( \sigma_{\text{max}} \) of the FJ relation in low-density environments \( (67.0 \text{ km s}^{-1}) \) is in fact almost twice that found in high-density ones \( (33.7 \text{ km s}^{-1}) \) and even larger than that found for the whole sample \( (51.1 \text{ km s}^{-1}) \).

Weighted and unweighted fits give exactly the same slope \( (\alpha \approx 3.8) \) in high-density environments, while in low-density environments the slope is steeper in the weighted fit \( (\alpha \approx 3.8) \)
than in the unweighted one ($\alpha \simeq 3.3$), due to the effect produced by the dominance of data with larger $\sigma_0$ errors below the fit line (as discussed in Section 3).

One might then argue that the large $\sigma_0$ errors displayed by ellipticals in low-density environments could be induced from their large $\sigma_0$ errors, but if we exclude from the sample the six ellipticals with the largest relative errors ($\Delta \sigma_0/\sigma_0 \geq 0.15$) we still get large values for $\sigma(\sigma_0)$ (67.9 km s$^{-1}$ and 68.5 km s$^{-1}$, respectively, for the weighted and unweighted fits). Moreover, from Figure 7 we see that the luminosity distribution of ellipticals in high- and low-density environments is rather similar, implying that the larger value of $\sigma(\sigma_0)$ displayed by ellipticals in the latter sample cannot be attributed to a luminosity effect linking intrinsic scatter to luminosity, as already evidenced for the FP (see, e.g., Bender et al. 1992; Hyde & Bernardi 2009). The fraction of faint (log($L_B/L_{B,0}) < 10.5$) ellipticals is in fact almost the same in high- ($\sigma(\sigma_0) \approx 58\%$) and low- ($\sigma(\sigma_0) \approx 56\%$) density environments, implying that the larger $\sigma(\sigma_0)$ displayed by ellipticals in a low-density environment cannot be attributed to a larger content of low-luminosity ellipticals.

An accurate comparison of the luminosity distribution of ellipticals in high- and low-density environments can be obtained from Figure 8, which shows the normalized $M_B$ distribution of ellipticals for the two samples. From Figure 8 we see that ellipticals in high-density environments (shaded histogram) cover a somewhat larger luminosity range, extending at both sides of the distribution, while ellipticals in low-density environments dominate at $M_B \sim -21$. The median value of the distributions is exactly the same ($M_B = -20.63$) and is indicated by an arrow in Figure 8 and the difference between the distributions is not at all significant, as confirmed by the K-S test which gives a probability of $p = 0.77$ that the two distributions are similar. However, if we eliminate five ellipticals (the four brightest and the faintest one) in the high-density sample and the two faintest ellipticals in the low-density one, so as to make both samples cover exactly the same range in luminosity, we find $\alpha = 3.7$, $\sigma(\sigma_0) = 32.1$ km s$^{-1}$ for the unweighted fit and $\alpha = 3.9$, $\sigma(\sigma_0) = 32.1$ km s$^{-1}$ for the weighted fit.
Figure 8. $M_B$ distribution of ellipticals in low- and high- (shaded) density environments. Both distributions are normalized to the total number of ellipticals in each sample. The arrow indicates the median value of $M_B$, which is the same for both samples.

$\sigma(\sigma_0) = 32.2$ km s$^{-1}$ for the weighted fit in the high-density sample, and $\alpha = 2.7$, $\sigma(\sigma_0) = 69.9$ km s$^{-1}$ for the unweighted fit and $\alpha = 3.6$, $\sigma(\sigma_0) = 68.6$ km s$^{-1}$ for the weighted fit in the low-density sample, confirming what we have obtained for the whole luminosity range.

Finally to check how solid our result can be considered we have inspected the SDSS-III (DR 8) database (York et al. 2000; Aihara et al. 2011), looking for $\sigma_0$ measures for ellipticals in the low- and high-density environments. Unfortunately those data are available only for 14 ellipticals in the high-density environment and for 12 ellipticals in the low-density one; they are reported in Tables 5 and 6, in which we list for each elliptical in each sample, its identifier (Column 1), $\sigma_0$ value from the SDSS (when available) with related uncertainty (Column 2), and the difference ($\Delta\sigma_0$) between the Sloan Digital Sky Survey (SDSS) and Hyperleda values for $\sigma_0$ (Column 3). Inspection of the data in Column 3 reveals a $\Delta\sigma_0$ which is in general small, but almost always negative in the high-density environment ($\langle\Delta\sigma_0\rangle = -13.2$ km s$^{-1}$, $\langle\Delta\sigma_0\rangle_{\text{rms}} = 9.8$ km s$^{-1}$) and that is larger and more spread around the zero in the low-density environment ($\langle\Delta\sigma_0\rangle = 5.9$ km s$^{-1}$, $\langle\Delta\sigma_0\rangle_{\text{rms}} = 24.2$ km s$^{-1}$).

Replacing SDSS $\sigma_0$ (when available) by Hyperleda ones gives $L_B \propto \sigma_0^{3.8}$ for the high-density environment (in both weighted and unweighted fits) and $L_B \propto \sigma_0^{4.3}$, $L_B \propto \sigma_0^{3.4}$ for the low-density environment (respectively for the weighted and unweighted fits). The average scatter of the data becomes somewhat worse, especially in high-density environments. We find $\sigma(\sigma_0) = 36.5$ km s$^{-1}$ for the high-density environments and $\sigma(\sigma_0) = 68.1$ km s$^{-1}$ for the low-density ones (to be compared with 33.7 km s$^{-1}$ and 67.0 km s$^{-1}$, which are the corresponding

| Identifier | $\sigma_0$(SDSS) (km s$^{-1}$) | $\Delta\sigma_0$ (km s$^{-1}$) |
|------------|-------------------------------|-----------------------------|
| PGC 6945   | ...                           | ...                         |
| NGC 704    | ...                           | ...                         |
| NGC 3837   | ...                           | ...                         |
| NGC 3842   | 291 ± 5                       | -24                         |
| NGC 3862   | 260 ± 5                       | -11                         |
| NGC 4261   | ...                           | ...                         |
| NGC 4473   | ...                           | ...                         |
| NGC 4816   | ...                           | ...                         |
| PGC 44137  | ...                           | ...                         |
| NGC 4839   | 269 ± 5                       | -16                         |
| NGC 4842A  | 208 ± 4                       | -8                          |
| PGC 44367  | 144 ± 3                       | -12                         |
| PGC 44467  | ...                           | ...                         |
| NGC 4859   | 205 ± 4                       | -3                          |
| NGC 4864   | ...                           | ...                         |
| NGC 4869   | ...                           | ...                         |
| NGC 4874   | ...                           | ...                         |
| NGC 4881   | 193 ± 3                       | -7                          |
| NGC 4882   | 149 ± 3                       | -14                         |
| NGC 4884   | ...                           | ...                         |
| NGC 4906   | 173 ± 3                       | 1                           |
| IC 4041    | 119 ± 3                       | -17                         |
| IC 4045    | 208 ± 3                       | -9                          |
| PGC 44484  | 173 ± 3                       | -40                         |
| NGC 4926   | 264 ± 4                       | -12                         |
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Table 6
Ellipticals in the Underdense Environment

| Identifier | $\sigma_0$(SDSS) (km s\(^{-1}\)) | $\Delta\sigma_0$ (km s\(^{-1}\)) |
|------------|---------------------------------|-------------------------------|
| NGC 631    | ...                             | ...                           |
| NGC 810    | 292 ± 5                         | 34                            |
| NGC 1226   | ...                             | ...                           |
| UGC 3549   | ...                             | ...                           |
| UGC 3844   | 189 ± 3                         | 39                            |
| NGC 2474   | ...                             | ...                           |
| NGC 2800   | ...                             | ...                           |
| NGC 2954   | 188 ± 3                         | −28                           |
| UGC 5313   | 63 ± 4                          | −11                           |
| IC 590     | 296 ± 5                         | 23                            |
| NGC 3392   | 165 ± 2                         | 6                             |
| NGC 3731   | 154 ± 3                         | −19                           |
| NGC 4187   | 277 ± 5                         | −23                           |
| NGC 4272   | ...                             | ...                           |
| UGC 7813   | 256 ± 4                         | −16                           |
| NGC 5583   | ...                             | ...                           |
| NGC 5628   | 254 ± 4                         | 29                            |
| NGC 5771   | 122 ± 2                         | 22                            |
| IC 1101    | ...                             | ...                           |
| NGC 6020   | 205 ± 3                         | 15                            |
| NGC 6051   | ...                             | ...                           |
| IC 1211    | ...                             | ...                           |
| NGC 6442   | ...                             | ...                           |
| NGC 6515   | ...                             | ...                           |
| NGC 6575   | ...                             | ...                           |
| NGC 6697   | ...                             | ...                           |
| NGC 6702   | ...                             | ...                           |
| IC 1317    | ...                             | ...                           |
| NGC 7052   | ...                             | ...                           |
| NGC 7360   | ...                             | ...                           |
| NGC 7512   | ...                             | ...                           |
| PGC 71599  | ...                             | ...                           |
| NGC 7735   | ...                             | ...                           |
| NGC 7751   | ...                             | ...                           |
| NGC 7785   | ...                             | ...                           |
| NGC 7786   | ...                             | ...                           |

Table 7
The FJ Relation in Overdense and Underdense Environments

| Sample                  | $N_{\text{cl}}$ | Fit          | $\alpha$ | $\sigma(\sigma_0)$ (km s\(^{-1}\)) |
|-------------------------|-----------------|--------------|-----------|-------------------------------------|
| Overdense               | 26              | Unweighted   | 3.8_{-0.4}^{+0.5} | 33.7 ± 1.4                           |
| Overdense (with 14 SDSS $\sigma_0$) | 26 | Unweighted | 3.8_{-0.4}^{+0.5} | 33.7 ± 1.4                           |
| Overdense (with 14 SDSS $\sigma_0$) | 26 | Weighted     | 3.8_{-0.5}^{+0.6} | 36.6 ± 0.9                           |
| Underdense              | 36              | Unweighted   | 3.3_{-0.6}^{+1.3} | 67.7 ± 3.7                           |
| Underdense (with 14 SDSS $\sigma_0$) | 36 | Unweighted | 3.4_{-1.1}^{+1.2} | 67.0 ± 3.7                           |
| Underdense (with 14 SDSS $\sigma_0$) | 36 | Weighted     | 3.4_{-0.5}^{+0.7} | 68.5 ± 2.9                           |
| Underdense (with 12 SDSS $\sigma_0$) | 36 | Unweighted | 3.4_{-0.8}^{+1.5} | 69.0 ± 2.7                           |
| Underdense (with 12 SDSS $\sigma_0$) | 36 | Weighted     | 4.3_{-1.0}^{+2.1} | 68.1 ± 2.6                           |

values obtained for the weighted fits when using only data from Hyperleda. Despite the slight increase in $\sigma(\sigma_0)$, which is likely due to the non-homogeneity between the two distinct sets of data (as proved by the shift in $\sigma_0$ reported in Tables 5 and 6, Column 3), the effect of the environment on $\sigma(\sigma_0)$ is confirmed.

Table 7 summarizes all the results described above; in Column 1 we indicate the sample kind (overdense and under-

dense stand for high density and low density) and within parentheses its characteristics related to $\sigma_0$ (either small error or data from the SDSS), in Column 2 the sample size, in Column 3 the kind of fit (either unweighted or weighted), in Column 4 the FJ relation slope and related uncertainty, and in Column 5 the average dispersion of the data around the fit with related uncertainty.

Finally, in analogy with what we have done for the whole sample (see Section 3, Figure 5, and Table 4) we have checked for possible maximum effects due to $m_B$ errors on the FJ relation in the high- and low-density environments. The procedure is exactly the same but in this case we have generated only 30 simulated samples by random addition or subtraction of the real error in $m_B$. The results are shown in Figure 9, which shows the normalized distribution of $\sigma(\sigma_0)$, $\alpha$, and $\Delta\alpha$ for the high-density (upper panels) and low-density (lower panels) simulated samples. The arrow on each plot indicates the value obtained in the real case. From Figure 9, we see that errors in luminosity would produce a general degradation of the fit quality (particularly evident in the possible large increase of $\alpha$ and $\Delta\alpha$ for the low-density sample), but that the difference in $\sigma(\sigma_0)$ would be maintained.

6. HOW OVERDENSE AND UNDERDENSE THE ENVIRONMENTS DO HAVE TO BE?

In the previous section, we have shown that ellipticals in high-density environments display a significant reduction in the FJ relation scatter when compared to ellipticals in low-density ones. Both kinds of environment have been selected objectively by requiring a number of neighbors ($N_{\text{neigh}}$) equal or larger or equal or smaller than 5 times or 0.2 times the median value of the $N_{\text{neigh}}$ distribution (computed separately for each distance bin).

We now progressively reduce and increase the overdensity and underdensity values (i.e., the multiplicative factor that we have applied to the median value of $N_{\text{neigh}}$) to check the level of densities at which the difference in the FJ relation scatter holds.

The results of this test are shown in Tables 8 and 9 where for each value of the overdensity or underdensity (Column 1) we indicate the number of ellipticals in each sample (Column 2), the

Table 8
Relaxing the Overdensity

| Overdensity | $N_{\text{cl}}$ | Fit          | $\alpha$ | $\sigma(\sigma_0)$ (km s\(^{-1}\)) |
|-------------|-----------------|--------------|-----------|-------------------------------------|
| 4.5         | 37              | Unweighted   | 3.6_{-0.3}^{+0.4} | 32.0 ± 1.4                           |
| 4.5         | 37              | Weighted     | 3.7_{-0.3}^{+0.4} | 32.0 ± 1.3                           |
| 3.5         | 45              | Unweighted   | 3.4_{-0.3}^{+0.4} | 34.1 ± 1.2                           |
| 4.0         | 45              | Weighted     | 3.8_{-0.3}^{+0.5} | 34.0 ± 1.2                           |
| 3.5         | 56              | Unweighted   | 3.6_{-0.3}^{+0.4} | 33.2 ± 1.1                           |
| 3.5         | 56              | Weighted     | 3.8_{-0.3}^{+0.4} | 33.0 ± 1.1                           |
| 2.5         | 88              | Unweighted   | 3.5_{-0.3}^{+0.4} | 35.6 ± 1.2                           |
| 2.5         | 88              | Weighted     | 3.7_{-0.3}^{+0.4} | 35.0 ± 1.2                           |
| 2.0         | 108             | Unweighted   | 3.9_{-0.4}^{+0.4} | 45.6 ± 1.9                           |
| 2.0         | 108             | Weighted     | 4.1_{-0.4}^{+0.5} | 45.2 ± 1.9                           |
| 1.5         | 138             | Unweighted   | 3.8_{-0.3}^{+0.4} | 51.4 ± 1.8                           |
| 1.5         | 138             | Weighted     | 4.1_{-0.3}^{+0.4} | 50.9 ± 1.8                           |
| 1.0         | 198             | Unweighted   | 3.6_{-0.2}^{+0.4} | 51.1 ± 1.5                           |
| 1.0         | 198             | Weighted     | 4.1_{-0.2}^{+0.4} | 50.2 ± 1.5                           |
Figure 9. Effect of the error in $m_B$ on the FJ relation for the high-density (upper panels) and low-density (lower panels) environments. Each distribution refers and is normalized to a set of 30 simulated samples that we have obtained by randomly adding or subtracting its true error to each $m_B$. The arrow on each panel indicates the corresponding value obtained for the real sample.

Table 9

| Underdensity | N_{ell} | Fit      | $\alpha$  | $\sigma(\sigma_0)$ (km s$^{-1}$) |
|--------------|--------|----------|-----------|----------------------------------|
| 0.25         | 78     | Unweighted | $3.4^{+0.7}_{-0.5}$ | $59.3 \pm 2.6$                 |
| 0.25         | 78     | Weighted  | $3.9^{+0.9}_{-0.7}$ | $58.8 \pm 2.6$                 |
| 0.40         | 99     | Unweighted | $3.5^{+0.7}_{-0.4}$ | $58.1 \pm 2.4$                 |
| 0.40         | 99     | Weighted  | $3.9^{+0.8}_{-0.5}$ | $57.6 \pm 2.4$                 |
| 0.50         | 127    | Unweighted | $3.1^{+0.4}_{-0.3}$ | $55.9 \pm 2.6$                 |
| 0.50         | 127    | Weighted  | $3.6^{+0.5}_{-0.4}$ | $55.2 \pm 2.5$                 |
| 0.75         | 158    | Unweighted | $2.9^{+0.3}_{-0.2}$ | $53.8 \pm 2.2$                 |
| 0.75         | 158    | Weighted  | $3.4^{+0.4}_{-0.3}$ | $53.2 \pm 2.2$                 |

kind of fit (Column 3), the slope with its uncertainty (Column 4), and the average scatter $\sigma(\sigma_0)$ of the FJ relation with related uncertainty (Column 5). Tables 8 and 9 allow one to follow the increase of $\sigma(\sigma_0)$, as the overdense environment becomes less and less dense and, conversely, the decrease of $\sigma(\sigma_0)$, as the underdense environment gets more and more dense. From Table 8, we see that $\sigma(\sigma_0)$ maintains its small value for overdensities down to a value of 3.5, that it is still small, even if somewhat increased, when the overdensity is equal to 3, and that it then starts to increase more rapidly to reach the characteristic value displayed by the whole sample at an overdensity of 1.5. From Table 9, in contrast, we see that the dispersion is already below 60 km s$^{-1}$ at an underdensity factor of 0.25 and that it decreases progressively, remaining just above the $\sigma(\sigma_0)$ of the whole sample when the underdensity factor is equal to 0.75.

This progressive increase/decrease of $\sigma(\sigma_0)$ with decreasing/increasing density in overdense/underdense environments gives more strength to our result confirming an effect relating environment to the FJ relation scatter.

7. CONCLUSIONS

Using a sample of 384 nearby elliptical galaxies and objectively estimating their environment on the basis of the number of neighbors within the typical galaxy cluster scale, we have provided evidence for an effect relating the intrinsic scatter of the FJ relation to the environment. We have shown that the scatter of the FJ relation is reduced to almost half of its value when ellipticals in the highest overdensities are compared to ellipticals in less dense environments, that the effect is not induced by luminosity differences between the samples, and that it holds for overdensities ranging between 3.5 and 5 for the median value of the number of neighbors’ distribution. Besides indicating a rather simple and quite natural way to reduce the large scatter affecting the FJ relation, our result, if confirmed on larger samples, is very likely to open an interesting perspective for models of galaxy formation.

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