Increase of compliance of shock absorbers with cut shells

I Shatskyi1* and A Velychkovych2

1 Ivano-Frankivsk Branch of Pidstryhach-Institute for Applied Problems in Mechanics and Mathematics, NAS of Ukraine, Department of modelling of damping systems, Mykytynetska 3, Ivano-Frankivsk, Ukraine
2 Ivano-Frankivsk National Technical University of Oil and Gas, Department of Construction, Karpatska 15; Ivano-Frankivsk, Ukraine

E-mail: ipshatsky@gmail.com

Abstract. The problem of the protection of equipment and personnel from harmful vibrations is among the most burning issues of current machine building. The aim of the paper is to model prefabricated structures of shelled shock absorbers with the basic component as the cut cylindrical shell with low-compressible elastomeric filler. The conducted analysis showed that as a result of dry friction of the filler against the shell, the contact pressure unevenly distributes along the cylinder, and that is why extensive increase in the length of the shell is not a productive measure to increase the compliance of the damper. We have offered an aggregated shock absorber in the form of a set of consecutively installed shorter cut shells with filler. We have formulated and solved the optimization problem to determine the length of the link of the multi-section shock absorber with fixed sizes of the intermediate piston. We have described the hysteresis characteristics of the multilink shell-shaped absorbers under cyclical loading.

1. Introduction

Vibration processes that occur during the operation of modern machines, mechanisms and buildings lead to undesirable consequences. Usually intensive vibrations decrease the strength, reliability and durability of industrial, civilian and other structures and have a harmful impact on the health of the personnel [1-3]. One of the fundamental ways to solve the problem of vibration insulation is founded on the application of anti-vibration devices – shock absorbers [4], dampers [5], elastic compensators [6], dynamic inertial absorbers [7, 8] etc. That is the reason why research and development [9, 10], as well as theoretical investigations in the development of anti-vibration devices and creating the theory of their calculation [11, 12] are becoming more and more important.

The authors have offered a number of vibration insulation structures [13, 14], among which we should distinguish elastic component constructed on the basis of cut shells with deformed filler (figure 1, a). The working principle of the elastic component is described below. The external load (figure 1, b) affects the hard pistons 3 and makes them move towards each other, compressing the filler 2. The latter changes its shape and makes a contact interaction with the open thin-walled shell. As a result of this, the shell deforms, accumulating potential energy of the elastic deformation. When the external load disappears or decreases, the movable parts of the system by means of the energy accumulated, return to the starting or interim position. Part of the energy of external impact supplied to the system is dispersed mainly due to mutual sliding with the friction of filler 2 and shell 1. Thus, shell elastic components possess both amortisation and damping characteristics.
The aim of the paper is to model prefabricated structures of shelled shock absorbers with the basic component as the cut cylindrical shell with low-compressible elastomeric filler.

2. Analyze of compliance of multi-link shock absorber

To describe the deformation of the elastic element we developed a mechanical mathematical model of the shell with a cut along the generatrix [14]. We took into consideration the main feature of the construction – the rigidity of the cut shell in the tangential direction is lower than along the generatrix. The cut isotropic shell which bends in the conditions of non-axisymmetric contact loading was contrasted against a tightly orthotropic closed cylindrical shell, which is affected by the axisymmetric contact loading. The thickness and the radii of the shells were kept the same. The elasticity module and the admissible stress for the equivalent orthotropic shell were chosen, so that they would on average be identical to the characteristics of the cut shell and its solid model.

The adequacy of the described one-dimensional model was verified through comparing a number of results received by the method of finite elements [15, 16] and the laboratory experiment [17], with separate results of analytical solutions.

As a result of the 1D-analysis [14] we have determined the dependency between the piston $\delta$ movement and the external force $Q$ during active loading:

$$\delta = \Lambda Q,$$

(1)

where $\Lambda$ is compliance coefficient which is calculated by the formula

$$\Lambda(l) = \Lambda_0 \frac{1 - \exp(-\lambda l)}{\lambda},$$

(2)

and $\Lambda_0 = \frac{1}{\pi RE(1 + \varepsilon)}$, $\lambda = \frac{2f}{1 + \varepsilon}$, $\varepsilon = 36 \left( \frac{R}{h} \right)^3 \frac{E}{E_0}$, $l = \frac{a}{R}$; $R$, $h$ are shell radius and thickness, $a$ is half the length of the filler; $E$, $E_0$ are Young's modulus of the incompressible filler and shell respectively; $f$ is friction coefficient of the pair "shell – filler".

**Figure 1.** Cut shell elastic component: (a) – full-scale specimens; (b) – analytic model.
Figure 2 shows the dependencies of the elastic element compliance on the relative length of the filler at various coefficients of friction between the filler and the shell. The system in Figure 1, b was taken as a model with the parameters $h/R = 0.1$, $E/E_0 = 0.0001$.

When the length of the shell grows, the speed of increase in compliance decreases, the more so, the higher the $\lambda$ parameter is.

![Figure 2.](image-url)

Figure 2. Influence of the relative length of the filler on the compliance of the elastic element.

This happens because as a result of friction the contact pressure decreases when it moves away from the piston, and the middle of the shell works inefficiently. That is why extensive build-up of the length of the shell and the filler is not an efficient way to increase the compliance of the damper.

We suggest that the efficiency of the use of the absorber’s size be increased by means of a multi-link construction of the damper which consists of consecutively installed short cut shells with filler. On the one hand, the contact pressure in short shells does not have time to significantly reduce in the median area, and this is a positive factor. On the other hand, we receive the addition of rigid interim pistons that are inefficient for the compliance, which is a negative factor. The presence of competing tendencies urges the authors to look for the point of extremum.

If $2H$ is the total working length of the whole elastic element, $2b$ is the constructive fixed length of the interim piston, $2a$ is the working length of the filler (figure 3).

Then the non-dimensional half-length of the $N$-sectoral elastic element is \( L = Nl + (N - 1)d \), where \( L = H/R \), \( d = b/R \). The compliance of the whole absorber is received as a sum of compliances of the consecutively installed links:

\[
\Lambda_\Sigma(l) = NLl(l) = \frac{L + d}{l + d} \Lambda(l).
\]  

(3)

Let us find the optimal length $l_*$, at which the compliance $\Lambda_\Sigma$ reaches maximum, i.e.

\[
\Lambda_\Sigma(l_*) = \max_{l \in [0, l]} \Lambda_\Sigma(l).
\]  

(4)

The extremum condition $\Lambda'_\Sigma(l) = 0$ signifies that the only point of the function maximum $\Lambda_\Sigma(l)$ is the root $l_*$ of the transcendent equation:

\[
\exp(\lambda l) = 1 + \lambda l + \lambda d,
\]  

(5)

which at given $d$ and $\lambda$ can be solved numerically.

At small $\lambda d$ we found the root asymptotic: $\lambda l \approx \sqrt{2\lambda d}$, hence,
The results of calculating the compliance of the multi-link element by the formula (3) at \( d = 0.5 \), \( \varepsilon = 3.6 \) for different values \( f \) are shown in figure 4. The abscissas of the marked points correspond to the stationary points \( l^* \). As it appears from the shown charts and the formula (6), if the friction coefficient increases, then the optimal length of the shell decreases, and the compliance extremum is more obvious.

This dependency of the shell’s optimal length on the friction coefficient is also confirmed by fragments of the numerical analysis: if \( f = 0.1 \), then \( l^* \approx 4.7 \), if \( f = 0.4 \), then \( l^* \approx 2.3 \), if \( f = 0.7 \), then \( l^* \approx 1.8 \).
For the investigation of the hysteretic qualities of the multi-link absorber at cyclical load, we have used the methodology designated for the calculation of dissipated energy in single-link shell dampers, started in the works [18–20] and developed in publications [21, 22].

In particular, the value of dissipated energy was defined for zero-to-compression load cycle (0 ≤ Q ≤ Q_{\text{max}}):

\[
\Psi = \Lambda_0 Q_{\text{max}}^2 \frac{L + d}{l + d} \frac{(2 + \exp(-\lambda l))(1 - \exp(-\lambda l))^2}{\lambda l}.
\]  

Thus, the received results (5)–(7) allow calculating the optimal size of construction shells and assessing the damping characteristics of highly complying multilink dampers with cut shells.

### 3. Conclusion

We have analyzed the distribution of stresses in the elastic component, which contains the contact pair: cut cylindrical shell – elastic filler. This conducted analysis showed that the impact of the friction forces results in the condition when a significant part of the shell is underloaded, while the extensive build-up of the length of the shell does not lead to an obvious increase in the compliance of the damper.

We have suggested a structure of a multilink absorber that consists of consecutively installed cut cylindrical shells with incompressible filler, which are separated with rigid pistons. The structure provides an increased compliance of the elastic component overall, without changing its strength.

The problem of the optimal design of the multilink absorber size according to the criterion of its maximum compliance is formulated. The existence of the link’s optimal length is proven, and the procedure of its calculation is described. We have shown that with the increase in the friction coefficient, if other parameters of the problem are invariable, the optimal length decreases.

We have received an expression to determine the amount of power that is absorbed by dapper at non-monotonous loading in the zero-to-compression cycle.

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