10−11\textsuperscript{Li} and 13−14\textsuperscript{Be} Studied by Projectile Fragmentation and pp-RPA

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Abstract. The level ordering in the unbound nuclei \textsuperscript{10}Li and \textsuperscript{13}Be is established on a firm basis using a time dependent projectile fragmentation model, comparing to experimental data and using structure inputs obtained from a semi-phenomenological core-vibration coupling model of two-neutron halo nuclei. The information on the shell ordering corresponds to the understanding of the neutron-core interaction. This is a building block of any three body model of borromean nuclei. As a consistency test we show that the energy spectra of some Beryllium isotopes obtained by using pp-RPA predict the same shell ordering as extracted from the reaction model vs. data analysis and that the well known structure of \textsuperscript{11}Li is reproduced.

1. Introduction

Projectile fragmentation is used to study two neutron halo projectiles as in Refs.[1]-[17]. The two-neutron emission is supposed to be due to the nuclear interaction with the target. The mechanism is such that one neutron is stripped by the target, and thus taken away from the elastic channel: it cannot therefore be measured in coincidence with the core. The other neutron is left behind, for example in a resonance state, which then decays. For a decaying s-state this mechanism has been described by the sudden approximation in Ref.[2] under the hypothesis that while the first neutron is stripped and not detected, the second neutron is emitted at large impact parameters with no final state interaction with the target. This method leads to a transition probability independent of the impact parameter. The second neutron however re-interacts with the core which, for example, is going to be \textsuperscript{9}Li in the case of the \textsuperscript{11}Li projectile, while it will be \textsuperscript{12}Be in the case of the projectile fragmentation of \textsuperscript{14}Be, since both \textsuperscript{10}Li and \textsuperscript{13}Be are not bound. Experiments with a \textsuperscript{14}B projectile [3] have also been performed, in which the n-\textsuperscript{12}Be relative energy spectra have been reconstructed by coincidence measurements. In such a nucleus the valence neutron is weakly bound, while the valence proton is strongly bound. Thus the neutron will probably be emitted in the first step and then re-scattered by the core minus one proton nucleus. The projectile-target distances at which this kind of mechanism would be relevant are probably not so large to neglect the effect of the neutron-target interaction.
2. Projectile Fragmentation

We calculate breakup by a semiclassical approximation in which the dynamics is controlled by the three potentials describing nucleon-core, nucleon-target, and core-target interactions. The collision is described in terms of only the three-body variables of nucleon coordinate, projectile coordinate, and target coordinate. The projectile-target relative motion is treated semiclassically by using a trajectory of the center of the projectile relative to the center of the target $R(t)$ along with constant velocity $v$ in the $z$ direction and impact parameter $b$, in the $xy$ plane. The formalism is then valid for incident energies above the Coulomb barrier. To first order such inelastic-like excitations can be described along this trajectory by the time dependent perturbation amplitude $|\psi_f\rangle$ for a transition from a nucleon state $\psi_i$ bound in the projectile, to a final continuum state $\psi_f$, such that $A_{fi} = \frac{1}{\hbar k} \int_{-\infty}^{\infty} dt \langle \psi_f(\mathbf{r}, t) | V(\mathbf{r}, \mathbf{R}(t)) | \psi_i(\mathbf{r}, t) \rangle$, where the interaction responsible for the neutron transition $V(\mathbf{r}, \mathbf{R}(t))$, moves past on the constant velocity path $\mathbf{R}(t)$. The radial part $\phi_i(\mathbf{r})$ of the single particle initial state wave function $\psi_i(\mathbf{r}, t)$ is calculated in a potential $V_{WS}(r)$ which is fixed in space and chosen such as to reproduce the experimental valence particle separation energy. In the special case of exotic nuclei the traditional approach to inelastic excitations needs to be modified. For example the final state can be eigenstate of a potential $U(\mathbf{r})$ (cf. Eq. (3)) modified with respect to $V_{WS}$ because some other particle is emitted during the reaction process as discussed in the Introduction. The final state interaction might also have an imaginary part which would take into account the coupling to a final continuum state $\psi$, such that $\delta$ function potential $V(r) = v_2 \delta(x) \delta(y) \delta(z)$. The value of the strength $v_2$ is used in the calculation is taken equal to the volume integral of the appropriate neutron-target interaction. The coordinate system and other details of the calculations can be found in Ref.[4].

The probabilities for different processes can be represented in terms of the amplitude as $dP/d\xi = \sum |A_{fi}|^2 \delta(\xi - \xi_f)$ where $\xi$ can be momentum, energy or any other variable for which a differential cross section is measured. Direct one-particle re-arrangement reactions of the peripheral type in presence of strong core-target absorption can be described by an equation like $|\psi_f\rangle = C^2 S \int d\mathbf{b}_c \frac{dP_{ct}(\mathbf{b}_c)}{d\xi_f} \langle \psi_i(\mathbf{b}_c) |$ in which $S$ is the spectroscopic factor for the initial single particle state. The core survival probability is defined in terms of a $S$-matrix function of the core-target distance of closest approach $b_c$. A simple parameterisation is $P_{ct}(b_c) = |S_{ct}|^2 = e^{-\text{ln}2exp((R_{ct}-b_c)/a)}$. The strong absorption radius $R_s \approx 1.4(A_p^{1/3} + A_t^{1/3})$ fm is defined as the distance of closest approach for a trajectory that is 50% absorbed from the elastic channel and $a=0.6$ fm is a diffuse parameter.

Due to the strong core absorption these calculations are performed using the asymptotic form of the initial and final state wave functions. Introducing the quantization condition[19] the probability spectrum reads

$$\frac{dP_{m}}{d\xi_f} = \frac{2}{\pi} \frac{v_2^2}{\hbar^2 v^2} C_i^2 m \frac{1}{2 l_i + 1} \sum_{m_i, m_f} |\sin(\delta + \nu)|^2 |I_{m_i, m_f}|^2. \tag{1}$$

The generalization including spin is given in Appendix B of Ref.[4] where also the form factor $|I_{m_i, m_f}|$ is given explicitly: $|\sin(\delta + \nu)|^2 = |1 - \hat{S}|^2$ where $\hat{S} = e^{2i(\delta+\nu)}$ is an off-the-energy-shell $S$-matrix representing the final state interaction of the neutron with the projectile core. It depends on a phase which is the sum of $\delta$, the free particle $n$-core phase shift, plus $\nu$ the phase of the matrix element $|I|$. An analytical estimate value of $|I|^2$, for an s to s transition, is...
\[ |I|^2 = \frac{1}{b^4} \frac{2 \pi b_c}{\sqrt{\gamma^2 + k^2}} e^{-2 \gamma b_c} \exp \left( -\gamma b_c \frac{\gamma^2 + k^2}{4k^2} \right) \\
= \frac{1}{b^4} \frac{2 \pi}{\sqrt{\gamma^2 + k^2}} \exp \left( -2 \gamma b_c \left( 1 + \frac{q}{4k} \right) \right), \quad (2) \]

where \( \tilde{k} = mv/\hbar \) and \( q = (\gamma^2 + k^2)/2k \).

The above analytical expression is accurate to within 10\% for impact parameters around the strong absorption radius and for neutron-core energies less that 1.5MeV. The agreement improves for larger impact parameters.

On the other hand, the sudden approximation has been followed in Ref.[2] leading to
\[ \frac{d\sigma}{d\epsilon} \sim \frac{1}{k} \frac{1}{(\gamma + k)^2} \sin(\delta + \beta), \]
with \( \beta = \arctan(k/\gamma) \). It is clear that while in the sudden approach the initial and final state overlap is taken in the whole coordinate space, irrespective of the target and of the beam velocity, in our method the overlap of the initial and final wave functions decreases with the core-target impact parameter according to the form factor \( |I|^2 \).

3. Structure and Reactions of \(^{13-14}\)Be

| Table 1. LHS: Scattering length of the 2s continuum state, energies and widths of the p- and d-resonances in \(^{10}\)Li (top) and \(^{13}\)Be (bottom) and corresponding strength parameters for the \(\delta V\) potential. |
|-----------------|----------|----------|
| \(\varepsilon_{res}\) | \(\Gamma_j\) | \(a_s\) |
| MeV | MeV | fm\(^{-1}\) | MeV |
| 2s\(_{1/2}\) | -17.2 | -10.0 |
| 1p\(_{1/2}\) | 0.63 | 0.35 | 3.3 |
| 1d\(_{5/2}\) | 1.55 | 0.18 | -9.8 |
| 2s\(_{1/2}\) | -0.8 | 8.0 |
| 1p\(_{1/2}\) | 0.67 | 0.28 | 8.34 |
| 1d\(_{5/2}\) | 2.0 | 0.40 | -2.36 |

Uncertainties in the interpretation of experimental results as compared to structure calculations were at the origin of our motivations to try to understand whether the neutron-\(^{12}\)Be relative energy spectra obtained from fragmentation of \(^{14}\)Be or \(^{14}\)B would show differences predictable in a theoretical model. If differences will be found in the experimental results with \(^{14}\)B and \(^{14}\)Be beams they could be due to an interplay between structure and reaction effects. Here we discuss only the \(^{14}\)Be case. Some results are also given for the \(^{10-11}\)Li systems [5], which being already well understood are used as test cases.

The ground state of \(^{14}\)Be has spin \(J^p = 0^+\). In a simple model assuming two neutrons added to a \(^{12}\)Be core in its ground state the wave function is: \(|^{14}\)Be \(\rangle = |b_1(2s\_{1/2})^2 + b_2(1p\_{1/2})^2 + b_3(1d\_{5/2})^2\rangle \otimes |^{12}\)Be, \(0^+\rangle\). Then the bound neutron can be in a 2s, 1p\(_{1/2}\) or 1d\(_{5/2}\) state. However the situation is much more complicated [12]-[15] and in particular the calculations of Ref. [16] show that there is a large component \((2s\_{1/2}, 1d\_{5/2}) \otimes |^{12}\)Be, \(2^+\rangle\) with the core in its low energy \(2^+\) state which can modify the neutron distribution.
Figure 1. LHS: Sum of all transitions from the s and p initial states, weighted by the known C²S, for the reaction \(^{11}\text{Li} + ^{12}\text{C} \rightarrow n + ^{9}\text{Li} + X\). Full thick line is the folding of the calculated spectrum with the experimental resolution curve. RHS: Sum of all transitions from the s initial state for the reaction \(^{14}\text{Be} + ^{12}\text{C} \rightarrow n + ^{12}\text{Be} + X\). Dashed line is the folding of the calculated spectrum with the experimental resolution curve.

Table 2. Theoretical and experimental values of \(S_{2n}\) (MeV) in \(^{10}\text{Be}\) and \(^{12}\text{Be}\), \(\langle r^2 \rangle^{1/2}\) (fm), \(\langle \lambda^2 \rangle^{1/2}\) (fm) and \(\langle \rho^2 \rangle^{1/2}\) (fm) in \(^{12}\text{Be}\).

| Theory     | \(S_{2n}(^{10}\text{Be})\) | \(S_{2n}(^{12}\text{Be})\) | \(\langle r^2 \rangle^{1/2}\) | \(\langle \lambda^2 \rangle^{1/2}\) | \(\langle \rho^2 \rangle^{1/2}\) |
|------------|---------------------------|---------------------------|-----------------|-----------------|-----------------|
| Exp.       | 8.48                      | 3.67±0.01                 | 2.59±0.06       |

Table 3. Theoretical and experimental values of \(S_{2n}\) (MeV) in \(^{12}\text{Be}\) and \(^{14}\text{Be}\), \(\langle r^2 \rangle^{1/2}\) (fm), \(\langle \lambda^2 \rangle^{1/2}\) (fm) and \(\langle \rho^2 \rangle^{1/2}\) (fm) in \(^{14}\text{Be}\) in the two cases of non-inversion (first line) and inversion (second line) of the 2s and 1p\(_{1/2}\) shells.

| Theory | \(\epsilon(1p_{1/2})\) | \(\epsilon(2s)\) | \(S_{2n}(^{12}\text{Be})\) | \(S_{2n}(^{14}\text{Be})\) | \(\langle r^2 \rangle^{1/2}\) | \(\langle \lambda^2 \rangle^{1/2}\) | \(\langle \rho^2 \rangle^{1/2}\) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Exp.   | -               | -               | 3.67±0.01       | 1.34±0.11       | 3.1±0.4         | 4.5±1.0         | 5.4±1.0         |

Table 4. Main RPA amplitudes of \(^{12}\text{Be}\) ground state.

| \(X_{ab}\) | \((1p_{1/2})^2\) | \((1p_{1/2} 2p_{1/2})\) | \((2s)^2\) | \((1d_{5/2})^2\) | \((1p_{3/2})^2\) | \(X_{\alpha\beta}\) |
|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.76        | 0.31            | 0.50            | 0.43            | 0.57            |

To describe the valence neutron in the \(^{13}\text{Be}\) continuum we assume that the single neutron hamiltonian with respect to \(^{12}\text{Be}\) has the form \(h = t + U + iW\) where \(t\) is the kinetic energy and \(U(r) = V_{WS} + \delta V\) is the real part of the neutron-core interaction. For the time being the imaginary part is taken equal to zero. \(V_{WS}\) is again a Woods-Saxon potential plus spin-orbit whose parameters are given in Ref. [4], and \(\delta V\) is a correction:

\[
\delta V(r) = 16ae^{2(r-R)/a}/(1 + e^{(r-R)/a})^4
\]  

(3)

which originates from particle-vibration couplings [13] relevant only for the low energy spectrum. Values for the parameter \(a\) are given in Table 1. The above form is suggested by a calculation of
Table 5. Main RPA amplitudes for $^{14}$Be ground state without (first line) and with (second line) inversion of $2s$-$1p_{1/2}$ shells.

| $\epsilon(1p_{1/2})$ | $\epsilon(2s)$ | $X_{ab}(2s)^2$ | $(1d_{5/2})^2$ | $X_{\alpha\beta}(1p_{3/2})^2$ | $(1p_{1/2})^2$ |
|----------------------|----------------|----------------|----------------|----------------|----------------|
| -3.15                | 0.09           | -0.93          | -0.49          | 0.32           | 0.36           |
|                      |                | $(1d_{5/2})^2$ | $(1p_{1/2})^2$ | $(1p_{1/2})^2(2p_{1/2})$ | $(1p_{3/2})^2(2s)^2$ |
| 0.48                 | -3.15          | 0.69           | -0.73          | 0.52           | -0.61          | 0.45           |

Figure 2. Individual transitions from the bound s,p,d components of the initial wave function to the unbound s,p,d states. Only the contribution from the initial s-orbital are then summed to give the full curve.

such couplings using Bohr and Mottelson collective model of the transition amplitudes between zero and one phonon states. Therefore our structure model is not a simple single-particle in a potential model but contains in it the full complexity of single-particle vs. collective couplings. However the fact that such a complexity can be put in a form like Eq.(3) is an added value to our approach. If simple fittings of experimental data will be obtained, then the parameters of a semi-phenomenological potential can be deduced and linked to a more microscopic model. To give some examples of a possible comparison with available data, we show in Fig. 1 the experimental points from H. Simon et al. [9] for the reactions $^{11}$Li+$^{12}$C $\rightarrow n+^{10}$Li $+X$ and $^{14}$Be+$^{12}$C $\rightarrow n+^{12}$Be+$X$ at 250 A.MeV and our calculations. More details in the legenda and in Refs. [4, 5].

Results obtained including the s, p and d initial states in $^{14}$Be leading to unbound $^{13}$Be states are shown in Fig.2. Each curve corresponds to just one transition as indicated. The solid curve is the sum of all transitions from the s-bound state which gives the dominant contribution to the transition probability due to the long tail of the wave function. To make them visible some curves have been multiplied by a factor of five as indicated in the legenda.

In Fig.3 we compare the s to s cross sections from our model with the results of the sudden formula used in [2]. It is clear that the sudden model is accurate only for the true virtual state of $^{10}$Li. The s-threshold state in $^{13}$Be has a very small scattering length and it is probably a complicated state coupled to an excited core.

On the other hand with the s and p single particle states of the LHS of Table 3, pp-RPA calculations have been performed. The details of the formalism, preliminary results and comparison with experimental data are presented in Tables 2,3,4 from [21] and [22], using two hypothesis for the level ordering. The best results are obtained for the s(bound)-p(unbound) inversion case. From the same tables it is also interesting to look at the RPA amplitudes, given in Table 5 which are closely related to the spectroscopic factors and give a hint on the occupation
probabilities of the orbitals. It is evident that the mixing of s-p-d states is very strong in both $^{12}$Be and $^{14}$Be and that these nuclei have a dynamical nature far from the traditional shell ordering and closure concepts. Furthermore, notice that $^{13}$Be is the first nucleus in which a s-p inversion is understood to happen across threshold.

Finally there is the question of the s-state in the continuum clearly seen in various experiments and necessary to interpret the $^{13}$Be spectrum. As we have discussed in Ref.[4] and also found in [9] this state has a small scattering length thus implying a weak interaction in the $l = 0$ channel between the neutron and $^{12}$Be. It does not appear to be important in the final $^{14}$Be level scheme and in Ref.[22] it is interpreted as a phantom of the s-bound state near threshold in $^{13}$Be.

4. Conclusions and Outlook
The field of Rare Isotopes Studies is very active, growing steadily and rapidly. Some recent achievements in projectile fragmentation theory for elastic breakup have been presented here together with new structure results of $^{13-14}$Be. From the structure point of view, in the search for the dripline position, a very important role is played by the study of nuclei unstable by neutron emission. This is one of the most important subjects which need to be addressed and developed further in the near future and for which some suggestions of possible structure and reaction formalisms have been presented.
References

[1] Marqués F M et al 2001 Phys. Rev. C 64 061301(R); Orr N 2003 Prog. Theor. Phys. Suppl. 146 201
[2] Bertsch G F, Hencken K and Esbensen H 1998 Phys. Rev. C 57 1366
[3] Lecouey J I, 2004 Few Body Syst 34 21
[4] Blanchon G, Bonaccorso A, Brink D M, García-Camacho A and Vinh Mau N 2007 Nucl. Phys. A784 49
[5] Blanchon G, Bonaccorso A, Brink D M and Vinh Mau N 2007 Nucl. Phys. A 791 303
[6] Blanchon G, Bonaccorso A and Vinh Mau N 2004 Nucl. Phys. A 739 259
[7] Thoennessen M et al 1999 Phys. Rev. C 59 111; 1999 Phys. Rev. C 60 027303
[8] Labiche M et al 2001 Phys. Rev. Lett. 86 600
[9] Simon H et al 2007 Nucl. Phys. A 791 267
[10] Korsheninnikov A A et al 1995 Phys. Lett. B 343 53
[11] Kondo Y it et al 2007 RIKEN Accel. Prog. Rep. 40 27
[12] Thompson I J and Zhukov M V 1996 Phys. Rev. C 53 708
[13] Vinh Mau N and Pacheco J C 1996 Nucl. Phys. A 607 163
[14] Pacheco J C and Vinh Mau N 2002 Phys. Rev. C 65 044004
[15] Descouvemont P 1994 Phys. Lett. B 331 71 ; 1995 Phys. Rev. C 52 704
[16] Tarutina T, Thompson I J, Tostevin J A 2004 Nucl. Phys. A 733 53
[17] Labiche M, Marques F M, Sorlin O and Vinh Mau N 1999 Phys. Rev. C 60 027303
[18] Alder K and Winther A 1975 Electromagnetic Excitation North-Holland
[19] Bonaccorso A and Brink D M 1988 Phys. Rev. C 38 1776; 1991 Phys. Rev. C 43 299; 1991 Phys. Rev. C 44 1559
[20] Bonaccorso A 1999 Phys. Rev. C 60 054604
[21] Blanchon G 2008 Nuclear Reactions and Structure For Nuclei Far from Stability Ph.D. thesis, University of Pisa, unpublished. http://www.infn.it/thesis/
[22] Blanchon G, Vinh Mau N and Bonaccorso A 2009 in preparation