Controlling light in Airy and higher-order caustic photonic structures

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Abstract. Caustics form geometrically stable structures in light and are hierarchically categorized by the catastrophe theory. We embed higher-order cusp and swallowtail catastrophes in paraxial beams and investigate their dynamics. Utilizing high-intensity caustics that propagate on curved trajectories, we realize photonic caustic lattices in photosensitive media, and demonstrate waveguiding with a rich diversity of light guiding paths.

1. Introduction
Light reflected by curved surfaces or refracted by refractive index modulations shaped according to nonlinear potential functions forms caustics as geometrically stable structures whose characteristics are predicted by an analysis of these potentials in terms of bifurcation and catastrophe theory [1, 2].

The description of catastrophes within a general bifurcation theory is valid for all areas of nonlinear dynamic systems and describes their abrupt changes due to perturbations when changing external control parameters. In optics, the manifestation of catastrophes as caustics is defined by the envelope of rays, i.e. a discontinuous change of the number of rays that cross in each point of space. Optical catastrophes are classified by the number of state and control parameters, and can be found in nature or created artificially. For a single control parameter, the only structurally stable catastrophe is the fold catastrophe, which exhibits the Airy function shown as a diffraction pattern [3] in Fig. 1 (a1) and as ray optics picture in (a2). Whereas in the fold catastrophe two trajectories coalesce in a singular point, higher-order catastrophes, as the cusp catastrophe, emerge when three merging trajectories lead to the caustic Pearcey beam [4], shown in Fig. 1 as transverse intensity distribution (b1) and as corresponding ray optics picture (b2).

2. Analyzing the propagation of higher-order optical catastrophes
In recent years, caustic beams have experienced a renaissance in optics fostered by spatial light modulator techniques. With these, both the fold catastrophe-based Airy beam and the cusp catastrophe-based Pearcey beam have been realized as paraxial beams. Striking properties of these beams include acceleration in the transverse direction, auto-focusing, and form-invariant propagation [4]. The experimentally obtained propagation of the Pearcey beam is depicted in Fig. 1 (d).
Figure 1. Catastrophes form caustics in light. (a) fold, (b) cusp, (c) swallowtail catastrophe as (1) transverse intensity patterns and (2) corresponding ray pictures. Experimentally observed propagation of a (d) Pearcey beam and (e) swallowtail beam.

We demonstrate our investigations on embedding higher-order catastrophes as e.g. the swallowtail, butterfly, and hyperbolic umbilic catastrophes in spatially structured light fields [5]. Fig. 1 (c) shows a transverse swallowtail intensity pattern (c1) and the corresponding rays (c2). We examine experimentally and theoretically the propagation of these caustic beams [6]. Exemplary, Fig. 1 (e) shows the decay of the unstable swallowtail caustic to a cusp caustic that is stable in the two-dimensional transverse plane.

3. **Nonlinear photonic caustic lattices**

Mapping catastrophes to light to tailor these spatial caustics experienced explosive growth during the past decade and became a vital branch in modern photonics. However, linking nonlinear photonics, especially artificial nonlinear refractive index changes, with these artificial caustic light fields is still a challenge. Combining both topics opens path breaking new aspects in information storage, guiding, and processing, and enables light localization.

In this spirit, we utilize these higher-order cusp and swallowtail catastrophes in paraxial light to transform them into photonic refractive index caustic lattices in nonlinear media [7] as shown in Fig. 4. We demonstrate waveguiding with a rich diversity of light guiding paths along 2D quasi cubic Fig. 4 (a) and 3D curved caustic structures Fig. 4 (b), and present their functionality as e.g. optical splitters Fig. 4 (c). Finally, taking advantage of the strong auto-focusing of Pearcey beams, we demonstrate the formation of a Pearcey solitary wave Fig. 4 (d) in focusing nonlinearity [7].
Figure 2. Waveguiding in caustic lattices and soliton formation. (a) A Pearcey lattice (gray) features light guiding (red) on 2D quasi cubic paths. (b) A Swallowtail lattice provides 3D curved waveguides. (c) Combining caustic beams enables the fabrication of a functional Pearcey-Y-splitter. (d) Nonlinear formation of a Pearcey soliton (gray & red) by exploiting the strong auto-focusing of two Pearcey beams.

4. Conclusion
To summarize, our work represents the first realization of higher-order photonic caustic lattices, and paves the way to apply this approach to a manifold of applications ranging from linear waveguiding to nonlinear soliton formation. Within the class of caustic beams diverse light structures exist that each exhibit unique propagation properties which can be combined to more complex light fields. Thus, our work provides a toolbox for versatile applications in micro-manipulation, material and signal processing, and microscopy.

References
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