LEPTONIC CP-VIOLATION IN SUPERSYMMETRIC
STANDARD MODEL

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Abstract

We point out the possibility of spontaneous and hard CP-violation in the scalar potential of R-parity broken supersymmetric Standard Model. The existence of spontaneous CP-violation depends crucially on the R-parity breaking terms in the superpotential and, in addition, on the choice of the soft supersymmetry breaking terms. Unlike in theories with R-parity conservation, it is natural, in the context of the present model, for the sneutrinos to acquire (complex) vacuum expectation values. In the context of this model we examine here the global implications, like the strength of the CP-violating interactions and the neutrino masses.
The minimal standard electroweak model provides adequate description of CP-violation hitherto seen in laboratory [1]. In addition to this CP-violation, there seems to be a good reason to expect CP-violation in the leptonic sector. The motivation for having such CP-violation comes from the desire to generate the observed baryon asymmetry in the universe at electroweak scale [2]. It is well known that the sphaleron induced baryon number violation tends to erase the baryon asymmetry generated at the GUT scale in theories with exact $B - L$ symmetry. But if lepton number violating interactions generate some lepton asymmetry, this could be transformed to baryon asymmetry by sphaleron induced reactions. Generation of lepton number asymmetry needs both, lepton non-conserving interactions and CP- as well as C-violation in them. Neither of these are present in the Standard Model. Thus it is important to look for models which contain both, L- and CP-violating interactions. Such a study is important in its own right, independent of the arguments given above, since it might be easier to detect additional CP-violating processes in the leptonic sector. In this note we study the nature of CP-violation in lepton non-conserving and R-parity violating ($\bar{R}$) Minimal Supersymmetric Standard Model (MSSM).

The CP-violation in the MSSM has been extensively discussed in the literature. The presence of supersymmetric particles leads to new sources of CP-violation [3]. These have been shown to be insufficient for the explanation of CP-violation in $K^0 - \bar{K}^0$ system [3]. On the other hand they lead to large electric dipole moments (edm) for the neutron and electron [4]. All the discussions in the literature are confined to R-parity conserving MSSM. Introduction of R-violating terms automatically generates lepton ($\bar{L}$) or baryon ($\bar{B}$) number violating interactions. These new interactions change the features of CP-violation in the MSSM in a qualitative manner. They introduce additional parameters which allow both, explicit CP-violation in the Higgs potential as well as the possibility of breaking CP spontaneously. We study here this
CP-violation as well as constraints on its magnitude. Since this kind of CP-violation is exclusively connected to lepton number violating interactions its effects will show up only in the context of $\mathcal{L}$-reactions. Hence its effect on ‘usual’ CP-violation is negligible.

R-parity assigns the quantum number $+1$ to conventional particles and $-1$ to their superpartners. More specifically it can be written as

$$R = (-1)^{3B + L + 2S}$$

where $B$, $L$ and are the baryon and lepton number and $S$ is the spin. Let us then split the superpotential $W$ of the MSSM into a R-parity conserving part ($W_0$) and R-parity violating term ($W_R$) i.e.

$$W = W_0 + W_R$$

In the following we use a symbol with a hat, $\hat{A}$, to indicate a chiral superfield and the same symbol without a hat, $A$, for the spin-zero field content of the chiral supermultiplet. Let then $\hat{L}_i$ ($\hat{E}_i^C$) and $\hat{Q}_i$ ($\hat{U}_i^C, \hat{D}_i^C$) denote the lepton and quark doublets (lepton and quarks $SU(2)$ singlets) with generation index $i$, respectively and let $\hat{H}_1, 2$ be the super-Higgs fields. The standard form for $W_0$ is

$$W_0 = \epsilon_{ab} \left[ h_{ij} \hat{L}_i^a \hat{H}_j^b \hat{E}_j^C + h'_{ij} \hat{Q}_i^a \hat{H}_j^b \hat{D}_j^C + h''_{ij} \hat{Q}_i^a \hat{H}_j^b \hat{U}_j^C + \mu \hat{H}_1^a \hat{H}_2^b \right]$$

where $a, b$ are $SU(2)$ group indices. The $U(1)_Y$ quantum number assignment is as usual: $Y(\hat{L}_i) = -1$, $Y(\hat{E}_i^C) = 2$, $Y(\hat{Q}_i) = 1/3$, $Y(\hat{D}_i^C) = 2/3$, $Y(\hat{U}_i^C) = -4/3$, $Y(\hat{H}_1) = -1$, $Y(\hat{H}_2) = 1$.

In general, the R-parity violating part $W_R$ reads

$$W_R = \epsilon_{ab} \left[ \lambda_{ijk} \hat{L}_i^a \hat{L}_j^b \hat{L}_k^C + \lambda'_{ijk} \hat{L}_i^a \hat{E}_j^b \hat{D}_k^C + \lambda''_{ijk} \hat{L}_i^a \hat{D}_j^b \hat{D}_k^C + \mu_i \hat{L}_i^b \hat{H}_2^b \right] + \chi_{ijk} \hat{U}_i^C \hat{D}_j^C \hat{D}_k^C$$
The terms in eq. (4) proportional to $\lambda_{ijk} = -\lambda_{jik}$, $\lambda'_{ijk}$ and $\mu_i$ violate lepton number whereas the baryon number is explicitly broken by the $\lambda''_{ijk}$-term ($\lambda''_{ijk} = -\lambda''_{ikj}$). It is well known that keeping both these terms in the lagrangian (i.e. $L$ and $B$ interaction terms) leads to difficulties with proton lifetime \cite{6}. Therefore we will set from now on $\lambda''_{ijk} = 0$.

The term $\epsilon_{ab\mu_i} \hat{L}_i^a \hat{H}_2^b$ is also not included conventionally. This is due to the fact that this term can always be rotated away from the superpotential by redefinition of the Higgs $\hat{H}_1$ and the leptonic superfields $\hat{L}_i$. It is worth stressing, however, that such a redefinition does not leave the full lagrangian (including soft breaking terms) invariant. Apart from changing $\lambda_{ijk}$, $\lambda'_{ijk}$ in eq. (4) in a well known way this redefinition also affects the soft supersymmetry breaking terms which are usually induced through supergravity. Given the superpotential in eqs. (3) and (4), the soft terms involving the scalars have the following general form in MSSM

$$V_{soft} = m_1^2 H_1 H_1 + m_2^2 H_2 H_2 + m_{L_i}^2 \hat{L}_i \hat{L}_i$$

$$- (m_{12}^2 \epsilon_{ab} H_1 H_2 + h.c.) + (\kappa'_i \epsilon_{ab} H_2 \hat{L}_i + h.c.)$$

$$+ \text{cubic terms} \quad (5)$$

The parameters $\kappa'_i$ and $m_{12}^2$ would be related to the parameters of the superpotential (3) and (4) at Planck scale in the usual way. The cubic terms are soft breaking terms in correspondence to cubic terms in $W$. While the $\mu_i$ term in eq. (4) can always be rotated away, the corresponding $\kappa'_i$-terms would still be present in the low energy theory. Removal of the $\epsilon_{ab\mu_i} \hat{L}_i^a \hat{H}_2^b$ term in eq. (4) needs a redefinition

$$\mu' \hat{H}_1 = \mu \hat{H}_1 + \mu_i \hat{L}_i \quad (6)$$

Each of the $\hat{L}_i$ fields have to be replaced by a combination orthogonal to (6). It is easy to see that this orthogonal transformation does not leave the soft breaking
terms in Eq. (6) invariant. Hence even if one removes the the \( \mu_i \)-terms from \( W_{\not R} \) (i.e. sets \( \kappa'_i = 0 \)) the term \( \epsilon_{ab} L^a_i H^b_2 \) as well as an additional term \( L^\dagger_i H_1 \) will get generated in \( V_{soft} \). Conversely, if one does not rotate the term proportional to \( \mu_i \) in (4), the \( L^\dagger_i H_1 \) part will arise from the F-term associated with \( H_2 \) and the \( \kappa'_i \) term would come from the general soft breaking expressions. In either case, one would obtain two additional complex parameters. We prefer to retain the \( \epsilon_{ab} L^a_i \hat{H}^b_1 \) in (4) and discuss its implications. The effect of these additional terms in \( V_{soft} \) generated in the process of removing the \( \mu_i \)-dependent terms in \( W_{\not R} \) is not investigated in the literature (see however [7]). These terms play an important role in generating spontaneous CP-violation as we will see.

With the change of notation \( \varphi_i \equiv L_i \), \( \phi_2 \equiv H_2 \) and \( \phi_1 \equiv -i\tau_2 H^*_1 \) (\( \tau_2 \) being the Pauli matrix, \( (i\tau_2)_{ab} = \epsilon_{ab} \)) we derive the Higgs potential

\[
V_{Higgs} = \mu_1^2 |\phi_1|^2 + \mu_2^2 |\phi_2|^2 + \mu_{L_1}^2 (\varphi_1^\dagger \varphi_1)
+ \frac{1}{2} \lambda_1 \left[ |\phi_1|^4 + |\phi_2|^4 + (\varphi_1^\dagger \varphi_1)^2 + 2|\phi_1|^2 (\varphi_1^\dagger \varphi_1) - 2|\phi_2|^2 (\varphi_1^\dagger \varphi_1) \right]
+ \lambda_2 |\phi_1|^2 |\phi_2|^2 - (\lambda_1 + \lambda_2) |\phi_1^\dagger \phi_2|^2 + \left( \lambda_3 (\phi_1^\dagger \phi_2) + h.c. \right)
+ \left( i\kappa_i (\phi_1^T \tau_2 \varphi_i) + h.c. \right) + \left( i\kappa'_i (\phi_2^T \tau_2 \varphi_i) + h.c. \right) + V_{rest} \tag{7}
\]

In the above equation it is assumed that all \( \mu_i \) are the same for all generations, \( \mu_i \equiv \mu_0 \). This is not essential and we have done it to simplify things [8]. The parameters in Eq. (7) like \( \lambda_i \) (\( i = 1, 2, 3 \)), \( \mu_i \) (\( i = 1, 2 \)), \( \mu_{L_j} \) and \( \kappa'_j \) (\( j \) is the generation index) can be expressed as in the standard case in terms of the \( SU(2) \) \( (U(1)_Y) \) coupling constant \( g \) \( (g') \) as well as the parameters entering eqs. (3)-(5).

\[
\begin{align*}
\mu_1^2 &= m_1^2 + |\mu|^2, \quad \mu_2^2 = m_2^2 + |\mu|^2 + \mu_i \mu^*_i, \quad \mu_{L_i}^2 = m_{L_i}^2 + |\mu_i|^2 \\
\lambda_1 &= \frac{1}{4} (g^2 + g'^2), \quad \lambda_2 = \frac{1}{2} g^2 - \lambda_1, \quad \lambda_3 = -m_{12}^2, \quad \kappa_i = \mu^*_i \mu_i 
\end{align*}
\tag{8}
\]

\( V_{rest} \) in (7) contains all terms of the potential which are not relevant for minimization.
The full form of $V_{\text{rest}}$ will be given elsewhere. Here we merely write two terms to display the presence of hard CP-violation in $V_{\text{Higgs}}$

$$V_{\text{rest}} = \kappa_{ij}(\phi_1^\dagger \varphi_j)(\varphi_1^\dagger \phi_1) + \kappa_{nmij}(\varphi_i^T \tau_2 \varphi_j)(\varphi_n^T \tau_2 \varphi_m)^\dagger + ...$$

(9)

with

$$\kappa_{jk} = \kappa_{kj}^* \equiv h_{ji}^* h_{ki}$$

$$\kappa_{nmij} = -\kappa_{mnij} = -\kappa_{nmji} = \kappa_{ijnm}^* \equiv \lambda_{nmk}^* \lambda_{ijk}$$

(10)

where $h_{ij}$ is the leptonic Yukawa coupling in (3) and $\lambda_{ijk}$ enters eq.(4). Note that the leptonic Yukawa coupling need not be diagonal.

The potential in (3) contains two additional (in general complex) parameters $\kappa_i$ and $\kappa'_i$ for every generation index $i$. Their presence gives rise to three important features not present in the R-conserving MSSM. (i) Firstly, when both $\kappa_i$ and $\kappa'_i$ are present, $V_{\text{Higgs}}$ is not invariant under CP. (ii) Even if CP is imposed on $V_{\text{Higgs}}$, the simultaneous presence of $\kappa_i$ and $\kappa'_i$ allows the possibility of spontaneous CP-violation. (iii) These new terms are linear in the sneutrino fields and as consequence $\varphi_i$ acquire vacuum expectation values (vev). This in turn generates masses for neutrinos via neutralino neutrino mixing. We discuss this features in what follows.

It is easy to see that $V_{\text{Higgs}}$ (in the first step without $V_{\text{rest}}$) is CP-invariant only if

$$\Im m(\kappa_i \kappa'_j \lambda_3) \delta_{ij} = 0$$

(11)

Hence, when both $\kappa_i$ and $\kappa'_i$ are present the potential violates CP. Independently of $V_{\text{rest}}$ is CP-invariant only if the following condition holds

$$\Im m(\kappa_{nm'ij'} \kappa_{mj}^* \kappa_{nm'}^*) \delta_{n'n} \delta_{m'm} \delta_{ij} \delta_{ij'} = 0$$

(12)

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Other similar conditions can be derived which signal the presence of hard CP-violation in the potential (i.e. CP-violation which is independent of the possibility of spontaneous CP-violation). In order to demonstrate the latter, let us assume the parameters $\kappa_i$, $\kappa'_i$ and $\lambda_3$ to be real. Let us denote the vacuum expectation values (vev’s) of the fields by $<\phi^T_{1,2}> = (0, v_{1,2})$ and $<\varphi^T_i> = (w_i, 0)$. Then at the minimum

$$v_1^\ast \left[\mu_1^2 + \lambda_1(|v_1|^2 - |v_2|^2 + w_i^* w_i)\right] + \lambda_3 v_2^* - \kappa_i w_i = 0$$

$$v_2^\ast \left[\mu_2^2 - \lambda_1(|v_1|^2 - |v_2|^2 + w_j^* w_j)\right] + \lambda_3 v_1^* - \kappa'_j w_j = 0$$

$$w_j^* \delta_{ij} \left[\mu_{L_i}^2 + \lambda_1(|v_1|^2 - |v_2|^2 + w_k^* w_k)\right] - \kappa_i v_1 - \kappa'_i v_2 = 0 \quad (13)$$

It follows from these conditions that the $w_i$’s are automatically non-zero as long as $\kappa_i$’s, $\kappa'_i$’s and $v_{1,2}$ are non-zero. Setting $w_i$ to zero leads to adjusting the parameters of the potential and hence one must allow vev’s for all three sneutrino fields. It follows namely from (13) by setting $w_i = 0$ that (without loss of generality for real parameters)

$$(\mu_1^2 + \mu_2^2)\kappa_i \kappa'_j \delta_{ij} = \lambda_3 (\kappa_i^2 + \kappa'^2_i) \quad (14)$$

In other words it is natural in the context of the potential (7) for the sneutrinos to acquire vev’s. This situation should be contrasted with MSSM without $R_i$, i.e. putting $\kappa_i = \kappa'_i = 0$ in eq. (13). Even there it is possible to obtain a non-zero vev $w_i$ provided the following equation is satisfied

$$(\mu_1^2 - \mu_{L_k}^2)(\mu_2^2 + \mu_{L_j}^2) = \lambda_3^2 \quad (15)$$

for any two generation indices $i, j$. Here $w_i = 0$ would be the natural choice [8]. If $w_i \neq 0$ then this vev is expected to be large [10], in general, and would conflict with phenomenology (see later) unless parameters are restricted [7]. We shall assume that the parameters satisfy such restriction derived in ref. [7] and that $w_i = 0$ when $\kappa_i$ and
\( \kappa_i' \) are zero. In such a situation, the lepton number is not spontaneously broken and the spectrum does not contain any majoron. This has important phenomenological implications which we will discuss later.

There is yet another, physical motivation why sneutrinos should have non-zero complex vev’s once R-parity is explicitly broken in the lagrangian. Dropping the crucial term \( \epsilon_{ab} \hat{L}_i \hat{H}_2^b \), but retaining the \( \lambda_{ijk} \) and \( \lambda'_{ijk} \) terms in eq. (4) and assuming \( w_i = 0 \) the relevant sneutrino mass terms are simply

\[
\left[ \mu^2_{Li} + \lambda_1 (v_1^2 - v_2^2) \right] (\varphi^R_{0i} \varphi^R_{0i} + \varphi^I_{0i} \varphi^I_{0i})
\]

where \( \varphi^R_{0i} \) and \( \varphi^I_{0i} \) are the real and imaginary parts of the neutral component of \( \varphi_i \), respectively. They correspond to states with \( L = 1 \) and \( L = -1 \) quantum numbers. We see from eq. (16) that in spite of having \( \mathcal{L} \)-terms in the lagrangian these states do not mix. Since such mixing would be natural in lepton number violating theory we need complex vev’s of sneutrinos (note that for instance the terms proportional \( \kappa_i \) mix real and imaginary components of \( \phi_1 \) and \( \varphi_i \)).

Indeed eqs. (13) allow for complex vev’s as long as the crucial parameters \( \kappa_i \) and \( \kappa'_i \) are non-zero. To see this explicitly set \( v_1 \) real, \( v_2 = |v_2| e^{i \alpha} \), \( w_3 = w = |w| e^{i \gamma} \), \( w_{1,2} = 0 \), \( \kappa_{1,2} = \kappa'_{1,2} = 0 \) and \( \kappa_3 = \kappa \) and \( \kappa'_3 = \kappa' \) (say, in one generation case). Then from (13) we get

\[
|v_2| \lambda_3 \sin \alpha + |w| \kappa \sin \gamma = 0
\]
\[
|v_1| \lambda_3 \sin \alpha - |w| \kappa' \sin (\alpha + \gamma) = 0
\]

Solving this for the phases one obtains

\[
\cos \alpha = \frac{A^2(B^2 - 1) + 1}{2AB}
\]
\[
\cos \gamma = \frac{1}{A} \frac{A^2(B^2 + 1) + 1}{2AB}
\]
where $A$ and $B$ are defined through

$$A \equiv -\frac{|w|\kappa}{|v_2|\lambda_3}, \quad B \equiv \frac{v_1\lambda_3}{\kappa'|w|} \quad (19)$$

In general, the amount of CP-violation (eq. (18)) characterized through $\kappa$, $\kappa'$ and $w$ is restricted from phenomenology. The restriction on sneutrino vev come from (a) LEP data around the the $Z^0$ resonance and (b) from restrictions on neutrino masses [12,13]. If $w \neq 0$ when $\kappa = \kappa' = 0$ then the theory contains a majoron. In this case $Z^0$ could decay into a majoron and an associated scalar. The invisible $Z^0$ width strongly constrains this possibility. Even if there is no majoron, as in the present case, the LEP data do imply significant restrictions [12]. However, more important restrictions come from the neutrino masses. The presence of the parameter $\mu_i$ in $W_{R_i}$ leads directly to mixing between neutrino and higgsino and as a consequence, to neutrino masses. In addition, $\kappa_i$ and $\kappa'_i$ induce, as shown before, sneutrino vev’s which mix neutrinos with gauginos $\tilde{\lambda}_a$. Thus neutrino masses constrain the parameters $\kappa_i/\mu$ and $w_i$. We assume only one generation for simplicity. Then the neutralino mass matrix in one generation case and the $(\tilde{B}, \tilde{W}_3, \tilde{H}^0_1, \tilde{H}^0_2, \nu)$ basis takes the following form

$$M_{\tilde{\lambda}_a/\nu} = \begin{pmatrix}
    cM & 0 & -g'v_1/2 & g'v_2/2 & -g'w/2 \\
    0 & M & gv_1/2 & -gv_2/2 & gw/2 \\
    -g'v_1/2 & gv_1/2 & 0 & -\mu & -\kappa/\mu \\
    g'v_2 & -gv_2/2 & -\mu & 0 & 0 \\
    -g'w/2 & gw/2 & -\kappa/\mu & 0 & 0 \\
\end{pmatrix} \quad (20)$$

where we have dropped all possible CP-violating phases. This has been analyzed in ref. [12] in the limit $\kappa \to 0$. The presence of $\kappa$ makes a minor modification. The parameter $c$ has been taken in [12] to be 0.49 with the assumption that the gaugino masses scale like gauge couplings. The mass matrix (20) leads to the following
neutrino mass

\[ m_\nu \simeq \frac{g^2}{4\mu^2 \cos^2 \theta_W} \left( \frac{\mu w + \kappa v_1/\mu}{(M_Z^2/\mu) \sin 2\beta - bM} \right) \]  

(21)

with

\[ b = \frac{c}{c \cos^2 \theta_W + \sin^2 \theta_W}, \quad \tan \beta = \frac{v_1}{v_2} \]  

(22)

The neutrino masses are required to be \( \leq O(10\text{eV}) \). Otherwise they will overclose the universe. For \( M \sim \mu \sim \text{TeV} \) and \( \tan \beta = 1 \) dominant contribution to \( m_\nu \) comes from the \( w \) term in eq. (21) and one obtains

\[ w \leq O(\text{MeV}) \]  

(23)

We note here that due to absence of the majoron, the decay of a heavier neutrino into a lighter one plus majoron is not possible in the present case. But the presence of flavor changing couplings of neutrinos to \( Z^0 \) may allow fast decay into three neutrinos. If this happens then the limit on \( w \) could be relaxed.

Much stronger constraints on \( \kappa_i \) and \( \kappa'_i \) can be derived from considerations based on baryon asymmetry [7], [14]. The lepton number violating interaction as well as sphaleron induced \( B + L \) violating processes, if simultaneously in equilibrium, will wash out the original baryon asymmetry. Demanding that the processes induced by the \( \mu_i \) terms in eq. (4) be out of equilibrium typically requires

\[ \kappa_i \leq 10^{-6}\text{MeV}^2 \]  

(24)

Similar constraint holds for \( \kappa'_i \). However, these are model independent constraints. If there is some unbroken global symmetry associated with family lepton number (e.g. \( \kappa_1 = 0 \)) then the constraint (24) does not apply [13]. But constraint (23) still holds. Assuming \( w \sim \mu_i \sim \kappa/\mu \) one sees from eq. (19) that \( A \sim 1/B \leq \left( \frac{\text{MeV}}{\text{TeV}} \right)^2 \simeq 10^{-12} \).
Hence it follows from eqs. (17,18) that the phase $\alpha$ between the vev’s of $\phi_1$ and $\phi_2$ is extremely small. This phase would be associated with CP-violation in lepton number conserving processes. In contrast, the relative phase $\gamma$ is $O(AB)$ and could therefore be large. But this phase will invariably be accompanied by lepton number violation signified by the sneutrino vev. Hence one would expect the CP-violation in $\bar{L}$-processes to be large.

At this point it might be instructive to compare other efforts to introduce spontaneous CP-violation in MSSM. First note that in a general two Higgs doublet model with softly broken $Z_2$ symmetry [16] the CP-violation comes from the following two terms of the potential

$$\frac{1}{2} \lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_6 (\phi_1^\dagger \phi_2) + \text{h.c.}$$  \hspace{1cm} (25)

In MSSM without R-parity breaking $\lambda_6 = \lambda_3$ form eq. (7), but $\lambda_5 = 0$ at tree level. The idea is then to generate this term radiatively [17]. For real $\lambda_5$ and $\lambda_6$ spontaneous CP-violation is possible modulo a restriction on the parameters which essentially comes from the obvious inequality $|\cos \xi| \leq 1$ where $\xi$ is the CP-violating phase. It then turns out that $\lambda_5$ is very small which together with the parameter constraint leads to a very light Higgs boson inconsistent with LEP data [18]. On the other hand, one can induce radiatively other (complex) couplings like $\lambda_7$ and $\lambda_8$ giving rise to the interaction terms of the form

$$\lambda_7 (\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_1) + \lambda_8 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_2) + \text{h.c.}$$  \hspace{1cm} (26)

This is possible since the CP is violated in other sectors of the MSSM lagrangian. The amount of such CP-violation is then heavily constrained by limits of edm of the neutron and turns out to be too small to be of any significance for phenomenology [18]. Note that the model discussed here evades the limits coming from edm in a natural way.

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Enlarging the superpotential by interaction terms with a singlet field $\hat{N}$ it is possible to have spontaneous CP-violation \cite{19}. These new interaction terms are, in general, linear combination of the following invariants

\begin{equation}
\hat{N} \hat{H}_1 \tau_2 \hat{H}_2, \\
\hat{N}^3, \quad \hat{N}^2, \quad \hat{N}
\end{equation}

Indeed the additional superpotential $W_N$ consisting of the first three terms in eq. (27) has been shown to give rise to spontaneous CP-violation \cite{19}. Furthermore such a model, with real Kobayashi-Maskawa matrix, can explain the CP-violation in $K^0 - \overline{K}^0$ system. In ref. \cite{20} it has been proved that any combination of the first term in eq. (27) with one of the other terms (involving only the singlet field) does not lead to spontaneous CP-violation at tree level. Higher order corrections to the potential can change this result \cite{21} and spontaneous CP-violation becomes possible in such model. This model requires relatively light Higgs bosons due to a sum rule $m_{H_1} + m_{H_2} \leq 100\text{GeV}$ \cite{21}. One should bear in mind that generating spontaneous CP-violation through higher order corrections can be delicate matter due to the Georgi-Pais result. The latter states that provided the loop corrections are small and the true minimum is close to its tree level value spontaneous CP-violation cannot be produced through quantum effects unless a massless particle different from the Goldstone mode appears in the spectrum. With the present limit on the top mass one can, however, argue that the loop corrections to the potential are not small any more \cite{21}.

Finally we mention that spontaneous CP-violation in the context of MSSM at finite temperature has been proposed and discussed in \cite{22}

In summary, we have shown that the MSSM contains additional sources of CP-violation associated with R-parity and lepton number breaking processes. This CP-violation is argued to be constrained by neutrino masses and sphaleron induced
transitions. CP-violation associated with $L$-processes could be large. Such a situation would arise typically in a $R$ transition such as the decay of the lightest supersymmetric particle induced through the type of interaction terms we have considered. This may have no significance as far as laboratory experiments are concerned, but it may have cosmological implications for baryo-genesis via lepto-genesis \cite{23}.

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[9] We note here that eqs. (15) and (16) are scale dependent. Furthermore adding a one loop correction to the tree level potential can convert the equality to an inequality hereby relaxing the fine tuning which would otherwise follow from (15) or (16).

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