From nuclear multifragmentation reactions
to supernova explosions

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Abstract

In this talk I discuss properties of hot stellar matter at sub-nuclear densities which is formed in supernova explosions. I emphasize that thermodynamic conditions there are rather similar to those created in the laboratory by intermediate-energy heavy-ion collisions. Theoretical methods developed for the description of multi-fragment final states in such reactions can be used also for description of the stellar matter. I present main steps of the statistical approach to the equation of state and nuclear composition, dealing with an ensemble of nuclear species instead of one “average” nucleus.

1 Introduction

A type II supernova explosion is one of the most spectacular events in astrophysics, with huge energy release of about $10^{53}$ erg or several tens of MeV per nucleon [1]. When the core of a massive star collapses, it reaches densities several times larger than the normal nuclear density $\rho_0 = 0.15 \text{ fm}^{-3}$. The repulsive nucleon-nucleon interaction gives rise to a bounce-off and formation of a shock wave propagating through the in-falling stellar material, predominantly Fe. Hydrodynamical simulations (see e.g. refs. [2, 3]) show that during the collapse and subsequent explosion the temperatures $T \approx (0.5 \div 10) \text{ MeV}$ and baryon densities $\rho_B \approx (10^{-5} \div 2)\rho_0$ can be reached. A schematic view of the post-collapse star core is presented in Fig. 1.

For the realistic description of supernova physics one should certainly use experience accumulated in recent years by studying intermediate-energy nuclear reactions. In particular, multifragmentation reactions provide valuable information about hot nuclei in dense environment. According to present understanding, based on numerous theoretical and experimental studies of multifragmentation reactions, prior to the break-up a transient state of nuclear matter is formed, where hot nuclear fragments exist in equilibrium with free nucleons. This state is characterized by a certain temperature $T \sim 3 - 6 \text{ MeV}$ and
Figure 1: Schematic view of the post-collapse stellar core 230 ms after the bounce-off, as predicted by the hydrodynamical simulations [2]. The neutrino heating and convection processes help to revive the shock. Region between the protoneutron star (PNS) and the shock front is called the Hot Bubble. In-falling matter is represented by thick arrows labelled by $\dot{M}$.

a density which is typically 3-5 times smaller than the nuclear saturation density $\rho_0$. A very good description of such systems is achieved with the Statistical Multifragmentation model (SMM), for a review see ref. [4]. The statistical nature of multifragmentation is confirmed by numerous experimental observations, e.g. "rise and fall" of intermediate-mass fragment production [5, 6], evolution of the fragment mass and multiplicity distributions with excitation energy [7, 8], fragment correlations revealing the critical behavior [7, 8], confirmation of anomaly in the caloric curve [9], isoscaling [10]. Recent experiments [11] directly confirm the basic assumption of the SMM, namely, that the primary fragments are hot, their internal excitation energy may reach up to 3 MeV per nucleon. Therefore, properties of these hot nuclei can be extracted from multifragmentation reactions and used for the description of matter under stellar conditions. The first steps in this direction were made in our papers [12, 13]. A similar model was also used in ref. [14] where, however, only cold nuclei in long-lived states were considered.

2 Statistical description of supernova matter

2.1 General remarks

In the supernova environment, as compared to the multifragmentation reactions, several new important ingredients should be taken into consideration. First, the matter at stellar scales must be electrically neutral, and therefore electrons should be included to balance positive nuclear charge. Second, energetic photons present in hot matter may change nuclear composition via photonuclear reactions. And third, the matter is irradiated by a
strong neutrino wind from the protoneutron star.

Below we consider macroscopic volumes of matter consisting of various nuclear species \((A, Z)\), nucleons \((n = (1, 0)\) and \(p = (1, 1)\)), electrons \((e^-)\) and positrons \((e^+)\) under the condition of electric neutrality. We expect that in this situation an equilibrium ensemble of various nuclear species will be generated like in a liquid-gas coexistence region, as observed in the multifragmentation reactions. Now our system is characterized by the temperature \(T\), baryon density \(\rho_B\) and electron fraction \(Y_e\) (i.e. the ratio of the net electron density to the baryon density). One may expect that the new nuclear effects come into force in this environment. For example, the liquid-drop properties in hot nuclei may be different from those observed in cold nuclei (see discussion e.g. in refs. [10, 15, 16]).

2.2 Equilibrium conditions

Composition of stellar matter can safely be studied within the Grand Canonical Ensemble dealing with chemical potentials of the constituents. Generally, the chemical potential of a species \(i\) with baryon number \(B_i\), charge \(Q_i\) and lepton number \(L_i\), which participates in chemical equilibrium, can be found from the general expression:

\[
\mu_i = B_i \mu_B + Q_i \mu_Q + L_i \mu_L \tag{1}
\]

where \(\mu_B\), \(\mu_Q\) and \(\mu_L\) are three independent chemical potentials which are determined from the conservation of total baryon number \(B = \sum_i B_i\) electric charge \(Q = \sum_i Q_i\) and lepton number \(L = \sum_i L_i\) of the system. This gives

\[
\begin{align*}
\mu_{AZ} &= A \mu_B + Z \mu_Q , \\
\mu_{e^-} &= -\mu_{e^+} = -\mu_Q + \mu_L , \\
\mu_\nu &= -\bar{\nu} = \mu_L .
\end{align*}
\tag{2}
\]

These relations are also valid for nucleons, \(\mu_n = \mu_B\) and \(\mu_p = \mu_B + \mu_Q\). If \(\nu\) and \(\bar{\nu}\) escape freely from the system, the lepton number conservation is irrelevant and \(\mu_L = 0\). In this case two remaining chemical potentials are determined from the conditions of baryon number conservation and electro-neutrality:

\[
\begin{align*}
\rho_B &= \frac{B}{V} = \sum_{AZ} A \rho_{AZ} , \\
\rho_Q &= \frac{Q}{V} = \sum_{AZ} Z \rho_{AZ} - \rho_e = 0 .
\end{align*}
\tag{3}
\]

Here \(\rho_{AZ}\) is the number density of nuclear species \((A, Z)\), \(\rho_e = \rho_{e^-} - \rho_{e^+}\) is the net electron density. The pressure of the relativistic electron-positron gas can be written as

\[
P_e = \frac{\mu_e^4}{12\pi^2} \left[ 1 + 2 \left( \frac{\pi T}{\mu_e} \right)^2 + \frac{7}{15} \left( \frac{\pi T}{\mu_e} \right)^4 - \frac{m_e^2}{\mu_e^2} \left( 3 + \frac{\pi T}{\mu_e} \right)^2 \right] ,
\tag{4}
\]

where the first order correction due to the finite electron mass is included. The net number density \(\rho_e\) and entropy density \(s_e\) can be obtained now from standard thermodynamic relations as \(\rho_e = \partial P_e / \partial \mu_e\) and \(s_e = \partial P_e / \partial T\). Neutrinos are taken into account in the same way, but as massless particles, and with the spin factor twice smaller than the electron one. The photon pressure is \(P_\gamma = (\pi^2/45)T^4\).
2.3 Nuclear statistical ensemble

For describing an ensemble of nuclear species in thermodynamical equilibrium we use the Grand Canonical version of the SMM [4, 17], properly modified for supernova conditions. After integrating out translational degrees of freedom the density of nuclear species with mass $A$ and charge $Z$ is calculated as

$$\rho_{AZ} = \frac{N_{AZ}}{V} = g_{AZ} \frac{V_f A^{3/2}}{V} \lambda_T^3 \exp \left[ -\frac{1}{T} (F_{AZ} - \mu_{AZ}) \right],$$

(5)

were $g_{AZ}$ is the g.s. degeneracy factor of species $(A, Z)$, $\lambda_T = \left( \frac{2\pi\hbar^2}{m_N T} \right)^{1/2}$ is the nucleon thermal wavelength, $m_N \approx 939$ MeV is the average nucleon mass. $V$ is the actual volume of the system and $V_f$ is so called free volume, which accounts for the finite size of nuclear species. We assume that all nuclei have normal nuclear density $\rho_0$, so that the proper volume of a nucleus with mass $A$ is $A/\rho_0$. At low densities the finite-size correction can be taken into account within the excluded volume approximation $V_f/V \approx (1 - \rho_B/\rho_0)$.

The internal excitations of nuclear species $(A, Z)$ play an important role in regulating their abundance. Sometimes they are included through the population of nuclear levels (see e.g. [14]). However, in the supernova environment not only the excited states but also the binding energies of nuclei will be strongly affected by the surrounding matter. By this reason, we find it more justified to use another approach which can easily be generalized to include in-medium modifications of nuclear properties. Namely, the internal free energy of species $(A, Z)$ with $A > 4$ is parameterized in the spirit of the liquid drop model

$$F_{AZ}(T, \rho_e) = F^B_{AZ} + F^S_{AZ} + F^{\text{sym}}_{AZ} + F^C_{AZ},$$

(6)

where the right hand side contains, respectively, the bulk, the surface, the symmetry and the Coulomb terms. The first three terms are written in the standard form [4],

$$F^B_{AZ}(T) = \left( -w_0 - \frac{T^2}{\varepsilon_0} \right) A, \quad F^S_{AZ}(T) = \beta_0 \left( \frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{5/4} A^{2/3}, \quad F^{\text{sym}}_{AZ} = \gamma \frac{(A - 2Z)^2}{A}.$$ 

Here $w_0 = 16$ MeV, $\varepsilon_0 = 16$ MeV, $\beta_0 = 18$ MeV, $T_c = 18$ MeV and $\gamma = 25$ MeV are the model parameters which are extracted from nuclear phenomenology and provide a good description of multifragmentation data [4, 5, 7, 8, 11]. However, some parameters, especially $\gamma$, can be different in hot neutron-rich nuclei, and they need more precise determination in nuclear experiments (see e. g. ref. [18]). In the Coulomb term we include the modification due to the screening effect of electrons. By using the Wigner-Seitz approximation it can be expressed as [19]

$$F^C_{AZ}(\rho_e) = \frac{3}{5} c(\rho_e) \frac{(eZ)^2}{r_0 A^{1/3}}, \quad c(\rho_e) = \left[ 1 - \frac{3}{2} \left( \frac{\rho_e}{\rho_{0p}} \right)^{1/3} + \frac{1}{2} \left( \frac{\rho_e}{\rho_{0p}} \right) \right],$$

(7)

where $r_0 = 1.17$ fm and $\rho_{0p} = (Z/A)\rho_0$ is the proton density inside the nuclei. The screening function $c(\rho_e)$ is 1 at $\rho_e = 0$ and 0 at $\rho_e = \rho_{0p}$. We want to stress that both...
Figure 2: Mean charge-to-mass ratios (left top panel), and mass distributions of hot nuclei (other panels) calculated with the SMM generalized for supernova conditions. Left panels present calculations for temperature $T = 3$ MeV and fixed lepton (electrons+neutrinos) fraction $Y_L = 0.2$ per nucleon. Right panels are calculations for temperature $T = 1$ MeV and fixed electron fractions $Y_e = 0.4$ (top) and 0.2 (bottom). Lines show the fragment mass distributions at different baryon densities (in units of the normal nuclear density $\rho_0 = 0.15 \text{ fm}^{-3}$), indicated in the figure.

the reduction of the surface energy due to the finite temperature and the reduction of the Coulomb energy due to the finite electron density favor the formation of heavy nuclei. Nucleons and light clusters ($A \leq 4$) are considered as structureless particles characterized only by mass and proper volume.

The pressure associated with nuclear species is calculated as for the mixture of ideal gases,

$$P_{\text{nuc}} = T \sum_{AZ} \rho_{AZ} \equiv T \sum_{AZ} g_{AZ} \frac{V_f A^{3/2}}{V} \frac{A^3}{\lambda_T^3} \exp \left[ -\frac{1}{T} (F_{AZ} - \mu_{AZ}) \right],$$

(8)

As follows from eq. (5), the fate of heavy nuclei depends sensitively on the relationship between $F_{AZ}$ and $\mu_{AZ}$. In order to avoid an exponentially divergent contribution to the baryon density, at least in the thermodynamic limit ($A \to \infty$), inequality $F_{AZ} > \mu_{AZ}$ must
Figure 3: Mass fractions of different nuclear species as functions of temperature for $Y_e = 0.4$ calculated for different baryon densities (indicated in the panels). Neutrons, protons, $\alpha$-particles and heavier nuclei ($A > 4$) are shown by dotted, dash-dotted, dashed and solid lines, respectively.

hold. The equality sign here corresponds to the situation when a big, ultimately infinite, nuclear fragment coexists with the gas of smaller clusters \[20\]. When $F_{AZ} > \mu_{AZ}$ only small clusters with nearly exponentially falling mass spectrum are present. However, there exist thermodynamic conditions corresponding to $F_{AZ} \approx \mu_{AZ}$ when the mass distribution of nuclear species is broadest. The advantage of our approach is that we consider all the fragments present in this transition region, contrary to the previous calculations \[21\] \[22\], which consider only one “average” nucleus characterizing the liquid phase.

3 Numerical results

3.1 Nuclear composition

In numerical calculations we first fix temperature $T$, baryon density $\rho_B$ and electron fraction $Y_e$. Then we consider a box containing the baryon number $B = 1000$ and proton number $Z = Y_e \cdot B$. The box volume is fixed by the average baryon density, $V = B/\rho_B$. We use an iterative procedure to find chemical potentials $\mu_B$ and $\mu_Q$. Finally, relative yields
of all nuclei with $1 \leq A \leq 1000$ and $0 \leq Z \leq A$ are calculated from eq. (5). Nuclei with larger masses ($A > 1000$) can be produced only at relatively high densities, $\rho_B > 0.1 \rho_0$, which are relevant for the regions deep inside the protoneutron star, and which are not considered here. First we consider the case when lepton fraction is fixed as expected inside a neutrinosphere. Figure 2 (left panels) shows the results for lepton fraction $Y_L = 0.2$ and typical temperature $T = 3$ MeV. Mass distributions are shown in the lower left panel. One can see that the islands of heavy nuclei, $200 < A < 400$, appear at relatively high baryon density, $\rho_B = 0.1 \rho_0$, corresponding to the vicinity of a protoneutron star. These nuclei are very neutron-rich, $Z/A \approx 0.27$. The $Z/A$ ratios are decreasing with $A$ less rapidly than in the nuclear multifragmentation case \[23\]. This can be explained by the screening effect of electrons. The width of the charge distribution at given $A$ is determined by $T$ and $\gamma$: $\sigma_Z \approx \sqrt{AT/8\gamma} \[17\] \[23\]. At lower density, $\rho_B = 0.01 \rho_0$, the mass distribution is rather flat up to $A \approx 80$ and then decreases rapidly for larger $A$. For $\rho_B = 10^{-3} \rho_0$ only light clusters are present and the mass distribution drops exponentially.
Figure 5: Pressure isotherms as functions of relative baryon density for $Y_e=0.4$. Solid lines show the total pressure including the electron, photon and nuclear contributions. Dotted lines show only the nuclear contribution. Results are presented for temperatures 6, 4, 2, 1 and 0.6 MeV (from top to bottom), as indicated at the corresponding lines.

Let us consider now the situation more appropriate for a hot bubble at early times of a supernova explosion, when the electron fraction of matter did not change significantly by the electron capture reactions. In this case the electron fraction is fixed to the initial value, and the electron and proton chemical potentials are determined independently, without using the equilibrium relation $\mu_e = -\mu_Q$. Corresponding results for $Y_e = 0.4$ and $Y_e = 0.2$ at $T = 1$ MeV and several baryon densities are presented in Fig. 2 (left top and bottom panels).

One can see that heavy and even superheavy nuclei, $50 < A < 400$, can be formed in this case too. They exist in a very broad range of densities, $0.1\rho_0 > \rho_B > 10^{-5}\rho_0$. At given density the mass distribution of heavy nuclei has a Gaussian shape. In the $Y_e = 0.4$ case the most probable nuclei, corresponding to the maxima of distributions, have $Z/A$ ratios 0.400, 0.406, and 0.439, for densities $0.1\rho_0$, $10^{-3}\rho_0$, and $10^{-5}\rho_0$, respectively. The Gaussian mass distributions may in some cases justify earlier calculations [21][22], when only one kind of nuclear species was considered at each density. As seen from the bottom panel, changing the electron fraction from 0.4 to 0.2 leads to a significant increase of
nuclear masses. Also, the nuclei become more neutron rich: the corresponding $Z/A$ ratios are 0.280, 0.359, and 0.420. Our calculations show that even larger effect can be caused by the reduction of the symmetry energy of hot fragments (see ref. [23]).

Figure 3 displays the mass fractions of different nuclear species as functions of temperature for several baryon densities and fixed $Y_e = 0.4$. One can see several interesting trends. First, nuclei with $A > 4$ survive at high temperatures only if the baryon density is large enough, $\rho_B > 10^{-2} \rho_0$. At lower densities they are destroyed by hard photons already at $T > 2$ MeV. On the other hand, the neutron and proton fractions increase gradually and dominate at $\rho_B \leq 10^{-2} \rho_0$. A significant change in the trend is observed at $T > 3$ MeV which can be related to the liquid-gas transition in such a matter. It is interesting to note that $\alpha$-particles may exist abundantly only in a narrow range of temperatures, $2 < T < 4$ MeV (see two lower panels).

### 3.2 Isentropic trajectories

Let us consider now how the composition of matter changes along the isentropic trajectories. Fig. 4 displays the mass distributions of nuclear species along two isentropes, $S/B = 1.0$ and 4.0. One can clearly see the different trends in these two cases. In the first case the widest distribution corresponds to the highest temperature and density state, $T = 3.39$ MeV, $\rho_B = 10^{-1} \rho_0$. The mass distribution extends up to about $A = 230$ in this case. At lower densities the mass distributions are peaked at $A \approx 70$. However, at $S/B = 4.0$ the nuclei are generally much lighter, and the widest distribution corresponds to the lowest density state, $\rho_B = 10^{-3} \rho_0$, $T = 1.03$ MeV. It is remarkable and somewhat unexpected that relatively heavy nuclei with $20 < A < 80$ can survive at such a high specific entropy.

One should bear in mind that the mass distributions which are presented here correspond to hot primary nuclei. After ejection these nuclei will undergo de-excitation. At typical temperatures considered here ($T \leq 3$ MeV) the internal excitation energies are relatively low, less than 1.0 MeV/nucleon. As well known from calculations [4] and nuclear experiments [7, 8, 11], de-excitation of nuclei with $A \leq 200$ will go mainly by means of the nucleon emission. Then the resulting distributions of cold nuclei are not very different from the primary ones, they are shifted to lower masses by several units. One should expect that shell effects (which, however, may be modified by surrounding electrons) will play an important role at the de-excitation stage leading to the fine structure of the mass distribution. We believe that after the de-excitation of hot nuclei, corresponding to the time when the ejected matter reaches very low densities, the $r$-process may be responsible for the final redistribution of the element abundances, leading to the pronounced peaks around $A \approx 80, 130$ and 200 [24].

As well known, nuclear composition is extremely important for the physics of supernova explosions. For example, the electron capture on nuclei plays an important role in supernova dynamics [25]. But the electron capture rates are sensitive to the nuclear composition and details of nuclear structure (see e.g. [26]). The neutrino-induced reactions are very sensitive to the nuclear structure effects and properties of weak interactions in nuclei (see e.g. [27]). It is also important that the presence of nuclei favors the explosion via the energy balance in the bubble [1], since more energy can be used for the explo-
sion. All these considerations show importance of the nuclear physics input in supernova phenomenon.

3.3 Equation of state

Finally, we present results concerning thermodynamical properties of supernova matter. Figure 6 shows the isothermic equation of state on the pressure—density plane. One can clearly see that the pressure is dominated by the relativistic electrons at high baryon densities and by thermal photons at low baryon densities. The nuclear contribution is always small compared to these two contributions.

On the other hand, the nuclear pressure shows the tendency to saturation at higher densities. This is consequence of the liquid-gas phase transition in nuclear subsystem, which in thermodynamic limit will manifest itself by a constant pressure in the coexistence region. This behavior will significantly influence the thermodynamical properties of matter, in particular, its heat capacity [20].

4 Conclusions

- The statistical equilibrium approach, which was successfully used for describing multifragmentation reactions, can be applied also for calculating the equation of state and nuclear composition of supernova matter.

- Survival of (hot) heavy nuclei may significantly influence the explosion dynamics through both the energy balance and modification of the weak reaction rates.

- Statistical mechanism may provide "seed" nuclei for further nuclear transformations in r-, rp, and s- processes.

- Due to the screening effect of electrons, the alpha-decay and spontaneous fission may be suppressed in supernova environments, that opens the pathway to the production of heavy and superheavy elements.

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