Upper and Lower Limits on Neutralino WIMP Mass and Spin–Independent Scattering Cross Section, and Impact of New $(g - 2)_\mu$ Measurement

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Abstract: We derive the allowed ranges of the spin–independent interaction cross section $\sigma^\text{SI}_p$ for the elastic scattering of neutralinos on proton for wide ranges of parameters of the general Minimal Supersymmetric Standard Model. We investigate the effects of the lower limits on Higgs and superpartner masses from colliders, as well as the impact of constraints from $b \rightarrow s\gamma$ and the new measurement of $(g - 2)_\mu$ on the upper and lower limits on $\sigma^\text{SI}_p$. We further explore the impact of the neutralino relic density, including coannihilation, and of theoretical assumptions about the largest allowed values of the supersymmetric parameters. For $\mu > 0$, requiring the latter to lie below 1 TeV leads to $\sigma^\text{SI}_p \gtrsim 10^{-11}$ pb at $m_\chi \sim 100$ GeV and $\sigma^\text{SI}_p \gtrsim 10^{-8}$ pb at $m_\chi \sim 1$ TeV. When the supersymmetric parameters are allowed above 1 TeV, for $440$ GeV $\lesssim m_\chi \lesssim 1020$ GeV we derive a parameter–independent lower limit of $\sigma^\text{SI}_p \gtrsim 2 \times 10^{-12}$ pb. (No similar lower limits can be set for $\mu < 0$ nor for $1020$ GeV $\lesssim m_\chi \lesssim 2.6$ TeV.) Requiring $\Omega_\chi h^2 < 0.3$ implies a parameter–independent upper limit $m_\chi \lesssim 2.6$ TeV. The new $e^+e^-$–based measurement of $(g - 2)_\mu$ restricts $m_\chi \lesssim 350$ GeV at 1 $\sigma$ CL and $m_\chi \lesssim 515$ GeV at 2 $\sigma$ CL, and implies $\mu > 0$. The largest allowed values of $\sigma^\text{SI}_p$ have already become accessible to recent experimental searches.

Keywords: Supersymmetric Effective Theories, Cosmology of Theories beyond the SM, Dark Matter.
1. Introduction

The hypothesis of the lightest neutralino $\chi$, as the lightest supersymmetric particle (LSP), providing the dominant contribution to cold dark matter (CDM) in the Universe, has inspired much activity in the overlap of today’s particle physics and cosmology. It is well-known that the relic density of the neutralinos is often comparable with the critical density [1, 2]. The expectation that the Galactic dark matter (DM) halo is mostly made of weakly–interacting massive particles (WIMPs) has further led to much experimental activity. In particular, the experiments looking for CDM WIMPs elastically scattering off underground targets have recently set limits on spin–independent (SI), or scalar, cross section of the order of $10^{-6}$ pb [3, 4, 5]. They have also nearly ruled out the region of $(m_\chi, \sigma_\text{SI})$ that has been claimed by the DAMA experiment to be consistent with an annual modulation effect [6]. Initial and early studies [7, 8, 9, 10] were followed by more recent work [11], where it was concluded that current experimental sensitivity is generally comparable with the ranges expected from the neutralino WIMP in the Minimal Supersymmetric Standard Model (MSSM). It is, however, still at least on order of magnitude, or so, above the ranges predicted by recent analyses of the Constrained MSSM (CMSSM) [12, 13, 14, 15, 16].

For comparison, cross sections for spin–dependent (SD) interactions are in the case of the neutralino generally some two or three orders of magnitude larger than the SI ones. On the other hand, at present detectors are still not sensitive enough to explore the parameter space of the MSSM, despite recent progress [17].

In light of the ongoing and planned experimental activities, it is timely to conduct a thorough and careful re–analysis of the predicted cross sections for SI scattering of neutralino WIMPs. Such a study is rather challenging because resulting ranges often strongly depend on a given SUSY model and on related theoretical assumptions. They are further
affected by experimental limits on SUSY, both from colliders and from indirect searches, as well as by cosmological input, where the relic abundance of the CDM has been measured with better accuracy both directly and in CMBR studies [18]. Over the last few years and months there have been also new results for LEP lower bounds on the masses of the lightest Higgs and electroweakly–interacting superpartners, Tevatron lower limits on strongly–interacting superpartners, as well as limits on allowed SUSY contributions to $b \to s\gamma$, and especially to the anomalous magnetic moment of the muon $(g-2)_\mu$ [19]. The new result for $(g-2)_\mu$ indicates a sizable deviation from the Standard Model prediction, whose value is still a subject of much discussion. As we will show, when interpreted in terms of SUSY, the required extra contribution to $(g-2)_\mu$ plays a unique role in implying a stringent upper bound $m_\chi \lesssim 350$ GeV (1 $\sigma$ CL) and $m_\chi \lesssim 512$ GeV (2 $\sigma$), but it does not affect much the allowed ranges of the SI scattering cross section $\sigma_{p}^{SI}$.

In this paper we carefully study the impact of the above constraints. In an attempt to minimize theoretical bias, we work here in the context of the general MSSM, which will be defined below, with an additional assumption of $R$–parity conservation. We focus here on the SI cross section case. Other recent studies of the general MSSM include [20, 21, 22]. The results presented here show that the level of experimental sensitivity that has recently been reached [3, 4, 5] has now indeed allowed one to start exploring cosmologically favored ranges of the neutralino WIMP mass and SI cross section. However, we point out a number of caveats and relations and further discuss the origin of, and robustness of, the upper and lower limits on $\sigma_{p}^{SI}$. In particular, for a big range of the heavy neutralino mass $440$ GeV $\lesssim m_\chi \lesssim 1020$ GeV GeV we are able to derive parameter–independent lower bounds on $\sigma_{p}^{SI}$. Collider lower limits on Higgs and superpartner masses plus requiring $\Omega_\chi h^2 < 0.3$ alone leads to a parameter–independent upper bound $m_\chi \lesssim 2.6$ TeV.

2. The MSSM

We start by reviewing relevant features of the general MSSM [23]. (We follow the convention of [24].) By the MSSM we mean a supersymmetrized version of the Standard Model, with Yukawa and soft SUSY–breaking terms consistent with $R$–parity. We neglect CP–violating phases in the Higgs and SUSY sectors and assume no mixings among different generations of squarks and sleptons, since both are probably small. In the MSSM, all the slepton and squark masses can be considered as a priori free parameters set at the electroweak scale. In the same way we treat the trilinear parameters $A_i$ ($i = t, b, \tau$) of the third generation while neglecting the ones of the first two. The Higgs sector is determined at the tree level by the usual ratio of the neutral Higgs VEV’s $\tan \beta = v_t/v_b$ and the mass of the pseudoscalar $m_A$. In computing full one-loop and leading two-loop radiative corrections to the lightest scalar Higgs we use the package FeynHiggsFast (FHF) [25]. As we will see, Higgs masses will play an important role in the analysis.

The lightest neutralino $\chi$ is lightest of the four mass eigenstates of the linear combinations of the bino $\tilde{B}$, the wino $\tilde{W}_3^0$ and the two higgsinos $\tilde{H}_b^0$ and $\tilde{H}_t^0$

$$\chi \equiv \chi_0^0 = N_{11} \tilde{B} + N_{12} \tilde{W}_3^0 + N_{13} \tilde{H}_b^0 + N_{14} \tilde{H}_t^0.$$  (2.1)
The neutralino mass matrix $M$ is determined by the $U(1)_{Y}$ and $SU(2)_{L}$ gaugino mass parameters $M_{1}$ and $M_{2}$, respectively (and we impose the relation $M_{1} = \frac{5}{3} \tan^{2} \theta_{W} M_{2}$, which comes from assuming gaugino mass unification at GUT scale), the Higgs/higgsino mass parameter $\mu$, as well as $\tan \beta$. In the region $|\mu| \gg M_{1}$, the lightest neutralino is mostly a bino with mass $m_{\chi} \simeq M_{1}$. In the other extreme, it is mostly higgsino-dominated and $m_{\chi} \simeq |\mu|$.

For the purpose of this analysis, we take as independent parameters: $\tan \beta$, $\mu$, $M_{2}$, $m_{A}$, $A_{t,b}$ as well as the soft masses of the sleptons and of the squarks. In order to make our analysis manageable, we make an additional assumption that, at the electroweak scale, the soft mass parameters of the sleptons are all equal to some common value $\tilde{m}_{\tilde{l}}$, and analogously $\tilde{m}_{\tilde{q}}$ for all the squarks. One normally expects certain relations among the physical masses of the sleptons and the squarks since they in addition receive well–defined D–term and F–term contributions to their mass matrices. Assuming common soft mass terms, at either GUT or electroweak scale, this normally leads to the sleptons being lighter than the squarks. Furthermore, for the sfermions of the 3rd generation it is natural to expect large mass splittings. However, we believe that, for our purpose, introducing just two separate common soft mass scales $\tilde{m}_{\tilde{l}}$ and $\tilde{m}_{\tilde{q}}$, while greatly simplifying the analysis, will not play much role in our overall conclusions for the SI cross sections. (This is in contrast to often assumed full degeneracy of soft sfermion masses which leads in our opinion to an unnecessary limitation on the allowed SUSY parameter space.)

What we do find important is to disentangle squark and slepton masses. This is mainly because experimental limits on slepton masses are significantly weaker than in the case of squarks. For the case of the bino–like neutralino, which is the most natural case for providing $\Omega_{\chi} h^{2} \sim 1$ [26], it is therefore the mass of the lightest slepton which often predominantly determines $\Omega_{\chi} h^{2}$. Assuming common soft masses for sleptons and squarks at the electroweak scale would therefore generally lead to overestimating $\Omega_{\chi} h^{2}$ by missing the cases where relatively light sleptons (below current squark mass bounds) would otherwise provide acceptable $\Omega_{\chi} h^{2}$. An additional effect is that of coannihilation. When slepton mass is only somewhat larger than that of the LSP, the neutralino relic abundance is strongly reduced and otherwise forbidden cases become allowed [27].

3. The Spin–Independent Cross Section

For non-relativistic Majorana particles, like the neutralino WIMP, the elastic scattering off constituent quarks and gluons of some nucleon $\frac{A}{Z}X$ is given by an effective differential cross section [7, 8, 10]

$$\frac{d\sigma}{d|\vec{q}|^{2}} = \frac{d\sigma^{SI}}{d|\vec{q}|^{2}} + \frac{d\sigma^{SD}}{d|\vec{q}|^{2}}, \quad (3.1)$$

where the transferred momentum $\vec{q} = \mu_{A} \vec{v}$ depends on the velocity $\vec{v}$ of the incident WIMP, and $\mu_{A} = m_{A} m_{\chi}/(m_{A} + m_{\chi})$ is the reduced mass of the system. The effective WIMP–nucleon cross sections $\sigma^{SI}$ and $\sigma^{SD}$ are computed by evaluating nucleonic matrix elements of corresponding WIMP–quark and WIMP–gluon interaction operators.
In the SI part, contributions from individual nucleons in the nucleus add coherently and the finite size effects are accounted for by including the SI nuclear form factor $F(q)$. The differential cross section for the scalar part then takes the form \[ \frac{d\sigma_{SI}}{|q|^2} = \frac{1}{\pi v^2} |Zf_p + (A - Z)f_n|^2 F^2(q), \] (3.2)

where $f_p$ and $f_n$ are the effective neutralino couplings to protons and neutrons, respectively. Explicit expressions for the case of the supersymmetric neutralino can be found, e.g., in [28]. The formalism we follow has been reviewed in several recent papers [2, 28, 12]. We have re-done the original complete calculation of Drees and Nojiri [10] and agreed with their results.

A convenient quantity which is customarily used in comparing theory and experimental results for SI interactions is the cross section $\sigma_{p}^{SI}$ for WIMP elastic scattering of free proton in the limit of zero momentum transfer:

\[ \sigma_{p}^{SI} = \frac{4}{\pi} \mu_p^2 f_p^2 \] (3.3)

where $\mu_p$ is defined similarly to $\mu_A$ above. The analogous quantity for a target with nuclei with mass number $A$ can then be expressed in terms of $\sigma_p^{SI}$ as

\[ \sigma_{A}^{SI} = \frac{4}{\pi} \mu_A^2 [Zf_p + (A - Z)f_n]^2 = \left( \frac{\mu_A}{\mu_p} \right)^2 A^2 \sigma_p^{SI}. \] (3.4)

One can do so because, for Majorana WIMPs, $f_p \simeq f_n$.

The coefficients $f_{p,n}$ can be expressed as \[f_{p} = \sum_{q=u,d,s} \frac{f_{Tq}^{(p)}}{m_q} f_q + \frac{2}{27} f_{TG}^{(p)} \sum_{c,b,t} \frac{f_q}{m_q} + \ldots\]

where $f_{Tq}^{(p)} = 1 - \sum_{q=u,d,s} f_{Tq}^{(p)}$, and $f_{Tq}^{(p)}$ is given by $<p|m_q \bar{q}q|p> = m_p f_{Tq}^{(p)} (q = u, d, s)$, and analogously for the neutron. The masses and ratios $B_q = <p|\bar{q}q|p>$ of light constituent quarks in a nucleon come with some uncertainties. For definiteness, we follow a recent re-evaluation [12] and assume $m_u/m_d = 0.553 \pm 0.043$, $m_s/m_d = 18.9 \pm 0.8$, and $B_d/B_u = 0.73 \pm 0.02$, as well as

\[ f_{Tq}^{(p)} = 0.020 \pm 0.004, \quad f_{Tq}^{(n)} = 0.026 \pm 0.005, \quad f_{Tq}^{(p)} = 0.118 \pm 0.062 \]

\[ f_{Tu}^{(n)} = 0.014 \pm 0.003, \quad f_{Td}^{(n)} = 0.036 \pm 0.008, \quad f_{Ts}^{(n)} = 0.118 \pm 0.062, \]

which numerically gives SI cross section values very similar to using the set of [2]. Some other recent studies use a new determination of $\sigma_{\pi N}$ to derive a much larger value for $f_{Ts}^{(p)} \simeq 0.37$ [29]. We find such values somewhat questionable since they imply that the strange quark component of the nucleon would be larger than the up and down ones. We have numerically checked that using the set of input parameters of [28] gives typically SI cross section values a factor of six higher than in our case.
It is worth noting that, despite several different diagrams and complicated expressions, it is the exchange of the heavy scalar Higgs that in most cases comes out to be numerically dominant. It is further enhanced when the neutralino is a mixed gaugino–higgsino state [30]. As \( m_H \approx m_A \) increases, \( \sigma_{SI}^p \) drops as \( m_H^{-4} \) because of the \( t \)-channel propagator effect in \( \chi q \to \chi q \) elastic scattering. Eventually, at smaller \( \sigma_{SI}^p \), squark exchange becomes important, and even dominant, instead.

4. Details of the Scan

As outlined above, we use seven parameters \( \tan \beta, \ M_2, \mu, \ m_A, \ \tilde{m}_{\tilde{q}}, \ \tilde{m}_{\tilde{\ell}} \) and \( A_t = A_b \) to conduct a careful scan of the general MSSM parameter space. For their allowed ranges we take:

\[
\begin{align*}
50 \text{ GeV} & \leq M_2 \leq 2 \text{ TeV} \\
50 \text{ GeV} & \leq |\mu| \leq 2 \text{ TeV (4 TeV)} \\
50 \text{ GeV} & \leq \tilde{m}_{\tilde{\ell}} \leq 2 \text{ TeV (4 TeV)} \\
200 \text{ GeV} & \leq \tilde{m}_{\tilde{q}} \leq 2 \text{ TeV (4 TeV)} \\
90 \text{ GeV} & \leq m_A \leq 2 \text{ TeV} \\
0 & \leq |A_{t,b}| \leq 1 \text{ TeV} \\
5 & \leq \tan \beta \leq 65
\end{align*}
\]

while we set \( A_{\tau} = 0 \) since we treat the masses of the slepton as independent parameters anyway. In addition to a general scan of the parameter space, in many cases we do several focused scans and explore the effect of extremely large values of \( \mu, \ \tilde{m}_{\tilde{\ell}} \) and \( \tilde{m}_{\tilde{q}} \) beyond 2 TeV (given in brackets above) in order to derive parameter–independent lower limits on \( \sigma_{SI}^p \) for a big range of large \( m_\chi \), as described below. The minimum values are set so that the resulting physical masses of Higgs and superpartners are limited from below by collider bounds. For the lighter chargino we take \( m_{\chi^\pm} > 104 \text{ GeV} \) [31], for sleptons the lower limit of 90 GeV [32] and for squarks 200 GeV [32]. The lower limit on \( m_\chi \) depends not only on a model but also on a number of additional assumptions [33]. For this reason, in our analysis we conservatively do not impose a direct experimental limit on \( m_\chi \), but instead infer it from the other limits, especially the one on the chargino mass. As regards the lightest Higgs mass \( m_h \), in much of the parameter space \( (m_A > 120 \text{ GeV}) \) the lower limit on the Standard Model Higgs of 114.1 GeV [34] applies. However there are two important points to note. Firstly, theoretical uncertainties in computing \( m_h \) in the MSSM are estimated at 2 – 3 GeV. Conservatively, we thus require only \( m_h > 111 \text{ GeV} \). Secondly, for 90 GeV \( < m_A < 120 \text{ GeV} \), there still remains a sizable range of the \( (m_h, m_A) \)–plane where the lightest Higgs mass given roughly by \( m_h > 0.78(m_A + 21.7 \text{ GeV}) \) is allowed [34]. We will comment below on the effect of this low–mass range on increasing the largest allowed values of \( \sigma_{SI}^p \).

Among indirect limits on SUSY, \( b \to s \gamma \) often places an important additional constraint on the allowed parameter space. We calculate the SM contribution to \( \text{BR}(B \to X_s \gamma) \) at
the full NLO level and include dominant tan β–enhanced NLO SUSY [35], and also include the c-quark mass effect on the SM value [37]. At this level of accuracy the SM prediction is \( \text{BR}(B \to X_s \gamma) = (3.70 \pm 0.30) \times 10^{-4} \) [37]. This range is partially overlapping with the new world–average [39]

\[
\text{BR}(B \to X_s \gamma) = (3.41 \pm 0.36) \times 10^{-4}
\] (4.2)

which has gone up from the previous range of \((3.23 \pm 0.72) \times 10^{-4}\) following the new result from BaBAR [36]. As described in more detail in [14], with an update in [38], we accordingly allow the full SM+SUSY contribution to be in the range \( \text{BR}(B \to X_s \gamma) = (3.41 \pm 0.67) \times 10^{-4}. \) It is important, however, to stress here an important salient point. In computing the SUSY contribution to \( \text{BR}(B \to X_s \gamma) \) one usually makes an implicit assumption of minimal flavor violation in the down–type squark sector, which is theoretically poorly justified. Even a slight modification of the assumption often leads to a significant relaxation of the bound from \( b \to s\gamma \) to the point of even allowing \( \mu < 0 \) and relatively light superpartner masses [39].

A very recent measurement of the anomalous magnetic moment of the muon \( a_\mu = (g_\mu - 2)/2 \) [19] has confirmed a previous value [40] but with twice–increased precision [19]

\[
a_\mu^{\text{expt}} - 11659000 \times 10^{-10} = (203 \pm 8) \times 10^{-10}.
\] (4.3)

The LO hadronic vacuum polarization contribution has recently been re–evaluated in [41, 42] and the light–by–light corrections in [43]. In [41] the updated SM prediction of [44] has been found to be \( a_\mu^{\text{SM}} - 11659000 \times 10^{-10} = (169.1 \pm 7.8) \times 10^{-10} \) when applying data from \( e^+e^- \) annihilation cross sections and \((186.3 \pm 7.1) \times 10^{-10} \) when applying \( \tau \)–decay data. This leads to a 3σ discrepancy

\[
\Delta a_\mu = a_\mu^{\text{expt}} - a_\mu^{\text{SM}} = (33.9 \pm 11.2) \times 10^{-10}
\] (4.4)

when using the \( e^+e^- \)–based data, or to a 1.6σ deviation

\[
\Delta a_\mu = a_\mu^{\text{expt}} - a_\mu^{\text{SM}} = (16.7 \pm 10.7) \times 10^{-10}
\] (4.5)

when applying the \( \tau \)–based data.

If interpreted in terms of SUSY, eq. (4.4) restricts the allowed SUSY contribution to

\[
22.7 \times 10^{-10} < a_\mu^{\text{SUSY}} < 45.1 \times 10^{-10} \quad (1\sigma)
\] (4.6)

\[
11.5 \times 10^{-10} < a_\mu^{\text{SUSY}} < 56.3 \times 10^{-10} \quad (2\sigma).
\] (4.7)

Similar ranges are obtained by using the results of [12, 15]. The eqs. (4.4) and (4.5) further imply that \( \mu > 0 \) (the sign of the SUSY contribution is the same as that of \( \mu \)).

This is clearly an intriguing hint for “new physics”. However, since some other recent evaluations tend to give a larger value of \( a_\mu^{\text{SM}} \) [48] and a larger error bar [47], at this point we will not strictly impose the \((g - 2)_\mu \) constraint on the parameter space that is otherwise allowed by all other constraints. Nevertheless, below we will discuss the important impact it has on the upper bounds on \( m_\chi \).
Figure 1: Ranges of $\sigma_{SI}^{p}$ in the general MSSM vs. $m_\chi$ for $\tan\beta = 35$, $A_t = A_b = 1$ TeV and $\mu > 0$, which are allowed by the bounds from colliders, $b \to s\gamma$ and $0.1 < \Omega_\chi h^2 < 0.2$, but not from $(g - 2)_\mu$. Also marked are some results of recent experimental WIMP searches. The thick black line (and a left-pointing arrow) indicates a parameter–independent lower bound on $\sigma_{SI}^{p}$ for $550 \text{ GeV} < m_\chi < 1020 \text{ GeV}$. No similar bound can be set for lower $m_\chi$ because of the neutralino–slepton coannihilation effect, as explained in the text.

As regards the WIMP relic abundance, a lower limit on the age of the Universe conservatively gives $\Omega_\chi h^2 < 0.3$, while “direct” measurements of the CDM lead to $0.1 < \Omega_\chi h^2 < 0.2$ which we will treat as a preferred range. Recent studies of the CMBR seem to imply even more restrictive ranges; for example, $\Omega_\chi h^2 < 0.12 \pm 0.04$ in [48]. We will not apply this narrower range yet but will comment below on its impact on the upper and/or lower limits on $\sigma_{SI}^{p}$. We compute $\Omega_\chi h^2$ as accurately as one reasonably can, at the level of a few per cent both near and further away from poles and thresholds, by applying our recently derived exact analytic expressions for neutralino pair–annihilation [49] and neutralino–slepton coannihilation [50], and by using an exact procedure for the neutralino coannihilation with chargino and next–to–lightest neutralino [51, 52].

5. Results

The allowed ranges of the SI cross section that result from our scans are illustrated in Fig. 1 for $\tan\beta = 35$, $A_t = A_b = 1$ TeV and $\mu > 0$. Since $\sigma_{SI}^{p}$ generally grows with $\tan\beta$ due to an enhancement in the heavy scalar Higgs coupling to down–type quarks, the above choice is a reasonable compromise between the low and the very large values of $\tan\beta$. In deriving the allowed ranges of $\sigma_{SI}^{p}$ we have imposed the bounds from colliders and from $b \to s\gamma$, and have further required $0.1 < \Omega_\chi h^2 < 0.2$, as described earlier, but not yet the bound from $(g - 2)_\mu$. We also mark with a thick solid line a parameter–independent lower bound on $\sigma_{SI}^{p}$ which will be explained below.

We can see a big spread of $\sigma_{SI}^{p}$ over some seven (four) orders of magnitude at small (large) $m_\chi$. In fact, the upper values of $\sigma_{SI}^{p}$ exceed the latest experimental limits, including
Figure 2: Sensitivity of the upper and lower limits on $\sigma_p^{SI}$ in Fig. 1 to various assumptions and constraints. Upper left window: the light blue region would be allowed if $\Omega_\chi h^2 < 0.1$. The dark–red region would be excluded if one neglected the effect of neutralino–slepton coannihilation. Upper right window: the light blue region would be allowed if one lifted the constraint from $b \rightarrow s\gamma$. The regions to the right of the vertical dotted (dashed) lines are excluded by imposing current 1 $\sigma$ (2 $\sigma$) CL bound from $(g-2)_\mu$. Lower left window: the upper dark–red region would be excluded by assuming $m_{h} > 111$ GeV for all $m_A$ (i.e., by neglecting a window of lighter $m_{h}$ which is still allowed for $m_A < 120$ GeV). Also shown in this window is the effect of restricting $m_A < 1$ TeV. Lower right window: the same as for $m_A$ but for $\mu$.

the recent result from Edelweiss [4] and the new limit from the UKDMC Zeplin I detector [5]. Also shown is the CDMS bound and the so–called DAMA region. It is clear that today’s experiments have already started probing the most favored ranges of $\sigma_p^{SI}$ that come from SUSY predictions for neutralino cold dark matter.
In the four windows of Fig. 2 we show the effect of the most important constraints on the upper and lower limits on the allowed ranges of $\sigma_p^{SI}$. Firstly, in the upper left window we show the effect of relaxing the cosmological bound by allowing $\Omega_\chi h^2 < 0.1$. Obviously, larger ranges of $\sigma_p^{SI}$ now become allowed since an enhancement in the neutralino pair-annihilation cross section often, by crossing symmetry, implies an increase in $\sigma_p^{SI}$. Note, however, that a combination of all the other constraints, most notably a lower limit on $m_h$ from LEP and the constraint from $b \to s\gamma$, prevents $\sigma_p^{SI}$ from rising by more than about one order of magnitude and only for not very large values of $m_\chi$. On the other hand, imposing a narrower range $0.08 < \Omega_\chi h^2 < 0.16$ has almost no effect on the upper and lower limits on $\sigma_p^{SI}$, although it does remove a number of points from the allowed ranges of $\sigma_p^{SI}$.

In the same window we also show an important effect, already pointed out in [21], of including neutralino coannihilation with sleptons (predominantly with the lighter stau) on allowing very low ranges of $\sigma_p^{SI}$ at smaller $m_\chi$. At lowest $m_\chi \lesssim 120$ GeV the relic abundance can be reduced to the favored range by choosing in the scan light enough sleptons, even without coannihilation. By simultaneously choosing large enough heavy Higgs and squark masses, one can reduce $\sigma_p^{SI}$ to very low values of a few $\times 10^{-12}$ pb. As $m_\chi$ increases, $\Omega_\chi h^2$ would normally increase as well, and become too large, but it is there that neutralino–slepton coannihilation kicks in. Since $\sigma_p^{SI}$ is independent of the slepton masses, by carefully scanning the parameter space, one can always find $\tilde{m}_l$ not much above $m_\chi$, in which case $\Omega_\chi h^2$ can be sufficiently reduced again to fall into the favored range. The effect is very strong for smaller $m_\chi$, thus explaining a sharp rise of the left side of the dark–red region allowed by neutralino–slepton coannihilation, but, as the process becomes increasingly inefficient at larger $m_\chi$ [27, 50], it gradually fades away.

In the upper right window of Fig. 2 we present the effect of imposing the constraint from $(g - 2)_\mu$. We can see that, for this case, $m_\chi \lesssim 245$ GeV (1σ CL) and $m_\chi \lesssim 420$ GeV (2σ CL). This upper limit comes from the fact that, as $m_\chi$ increases, the SUSY contribution from the $\chi - \tilde{\mu}$ and $\chi^- - \tilde{\nu}_\mu$ loops become suppressed and at some point becomes too small to explain the apparent discrepancy between the SM and the experimental measurement [53, 54]. On the other hand, the upper and lower limits on $\sigma_p^{SI}$ are not really affected.

In the same window we also show the effect of relaxing the constraints from $b \to s\gamma$. We can see that if they were not imposed, at large $m_\chi$ the upper limit on $\sigma_p^{SI}$ would significantly increase. In this region, the mass of the pseudoscalar Higgs, and therefore also the heavier scalar, is rather small, thus giving larger $\sigma_p^{SI}$. However, the mass of the charged Higgs is then also on the lower side, and a cancellation between a charged Higgs–top quark loop and chargino–stop loop contribution is not sufficient to reduce $\text{BR}(B \to X_s\gamma)$ to agree with the experimental limit.

However, we remind the reader that a slight relaxation of the underlying assumption of minimal flavor violation in the squark sector often leads to a significant weakening of the bound from $b \to s\gamma$ to the point of even allowing $\mu < 0$ [39]. We would therefore be cautious in applying the $b \to s\gamma$ constraint rigidly.

In the lower left window of Fig. 2 we present the sensitivity of the upper limit on $\sigma_p^{SI}$ to the lower limit on the light Higgs mass. We can see that it would sizably decrease
Figure 3: Sensitivity of $\sigma_p^{SI}$ to $m_A$ and $\mu$ for the case of Fig. 1. We concentrate on the region of parameter space where $m_\chi \sim 800$ GeV. The whole marked region is consistent with all the constraints from colliders and $b \to s\gamma$. By further imposing the constraint $0.1 < \Omega_\chi h^2 < 0.2$ one selects only the red region. Near the resonance $m_A \simeq 2 m_\chi$, significantly smaller values of $\sigma_p^{SI}$ become allowed by $0.1 < \Omega_\chi h^2 < 0.2$, (mostly in the increasingly pure bino region), but become eventually limited from below, independently of increasing the maximum allowed value of $\mu$.

if we neglected the region of small $90$ GeV $\lesssim m_h \lesssim 111$ GeV which is still allowed at $m_A \lesssim 120$ GeV, and instead required $m_h > 111$ GeV for all $m_A$. Note also that the new experimental limits on $\sigma_p^{SI}$ are for the most part inconsistent with the possibility of the light Higgs scalar.

In the same window and in the lower right window we explore the existence of the lower limit on $\sigma_p^{SI}$ and its dependence on the assumed upper limit on $m_A$ and $\mu$, respectively. As we can see, the lowest values of $\sigma_p^{SI}$ are often to a large extent determined by a somewhat subjective restrictions from above on these parameters. As one allows either $\mu$ or $m_A$ above 1 TeV the lower limit on $\sigma_p^{SI}$ relaxes considerably.

However, we argue that, by requiring sizable enough $\Omega_\chi h^2$ (e.g., $\Omega_\chi h^2 > 0.1$), it is possible to set a parameter–independent lower bound on $\sigma_p^{SI}$ for a considerable range of large $m_\chi \lesssim 1020$ GeV (marked with a thick solid line and left–pointing arrow in Fig. 3). The limit holds for an arbitrary case of the neutralino (gaugino, higgsino or mixed), independently of how large $\mu$ and other SUSY parameters are taken. Let us first consider the gaugino limit $\mu \gg M_2$. In this case the origin of the parameter–independent bound is displayed in Fig. 3 where we plot $\sigma_p^{SI}$ as a function of the pseudoscalar Higgs $m_A$ for $m_\chi \simeq 800$ GeV and scan over all the other parameters. As $m_A$ and other parameters are varied, large ranges of $\sigma_p^{SI}$ remain allowed by collider and indirect constraints, depending on the maximum allowed value of $\mu$. For each fixed $\mu$, $\sigma_p^{SI}$ decreases proportionally to the fourth power of $m_H \simeq m_A$ because of the (typically dominant) $t$–channel exchange of the heavy Higgs, as the marked cases of $\mu$ in Fig. 3 clearly demonstrate. As $\mu$ increases, at fixed $m_\chi$ one moves deeper into the gaugino region and typically finds large $\Omega_\chi h^2 > 0.2$. By imposing $0.1 < \Omega_\chi h^2 < 0.2$ one selects only a narrow red (dark) range with a large bino component. The bino purity
are inconsistent with $\Omega_\chi h^2$. However, for each $m_\chi$ one can choose $m_A \simeq 2m_\chi$ (roughly $m_A = 1600$ GeV in Fig. 3) in which case $\Omega_\chi h^2$ is reduced to an allowed level by a wide resonance due to $A$-exchange. This leads to allowing a much reduced $\sigma_p^{SI}$, while still being consistent with $0.1 < \Omega_\chi h^2 < 0.2$. However, because of the finite width of the $A$-resonance, at large enough $\mu$ one reaches the lowest value of $\sigma_p^{SI}$ (in this case $2 \times 10^{-11}$ pb) which is parameter-independent. Away from the resonance (for example, if one imposed $m_A < 1$ TeV), one would obtain the lower bound $\sigma_p^{SI} \gtrsim 3 \times 10^{-8}$ pb, basically independently of whether $\mu < 1$ TeV is imposed or not. In fact, one can see this effect in the lower left window of Fig. 3 where imposing $m_A < 1$ TeV causes the lower limit on $\sigma_p^{SI}$ to suddenly jump up at around $m_\chi \simeq m_A/2 \simeq 500$ GeV.

The above parameter-independent lower limit arises in the gaugino case and applies to arbitrarily large $m_\chi$. The case of the gaugino–like LSP can be argued to be more attractive as being less fine–tuned than the higgsino–like one. In the MSSM the LSP is a nearly–pure higgsino for $M_2 \gtrsim 300$ GeV which, by applying the assumption of gaugino mass–unification (see below) to the gluino mass, implies $m_\tilde{g} = \frac{3}{500} M_2 \gtrsim 1$ TeV and therefore large and less “natural” soft SUSY–breaking scale. Nevertheless, in the spirit of generality, we need to extend the analysis to the case of the higgsino–like neutralino.

For the higgsino–like LSP ($M_2 \gg \mu \simeq m_\chi$), if we remain within the ranges of parameters given in (2.4), we find $\sigma_p^{SI}$ some two orders of magnitude larger than in the gaugino case presented above. However, one can in principle reduce $\sigma_p^{SI}$ to arbitrarily small values by going to the limit of pure enough higgsino ($M_2$ in the multi–TeV range), in which case the neutralino–Higgs coupling would be arbitrarily reduced, and by further suppressing the squark contribution by making them extremely heavy. It is therefore reasonable to question the existence of the lower bound on $\sigma_p^{SI}$. However, in the multi–hundred GeV range ($m_f < m_\chi \lesssim 10^{20}$ GeV) $\Omega_\chi h^2$ remains typically very small $\Omega_\chi h^2 \ll 0.1$ although it does increase with $m_\chi$. For $\tan \beta = 35$ it reaches 0.1 for $m_\chi \lesssim 10^{20}$ GeV, 0.2 for $m_\chi \lesssim 1.6$ TeV and 0.3 for $m_\chi \lesssim 2.5$ TeV. (These values decrease somewhat with increasing $\tan \beta$.) Since, as mentioned above, we impose $0.1 < \Omega_\chi h^2$ (or a similar sizable lower limit on $\Omega_\chi h^2$), in the range of $m_\chi < 1$ TeV displayed in Figs. 1 and 3 the parameter–independent lower limit on $\sigma_p^{SI}$ holds. (In order to be clear that the parameter–independent lower bound on $\sigma_p^{SI}$ applies to a general neutralino so long as $m_\chi \lesssim 10^{20}$ GeV, in Figs. 1 and 3 we have put a left–pointing arrow at $m_\chi = 1$ TeV.)

For the higgsino–like LSP in the mass range $10^{20}$ GeV $\lesssim m_\chi \lesssim 1600$ GeV the relic abundance is $0.1 < \Omega_\chi h^2 < 0.2$, basically independently of how large $M_2$ is, and no lower bound on $\sigma_p^{SI}$ can in principle be set. The range of $m_\chi$ increases to 2.5 TeV if we allow $\Omega_\chi h^2 < 0.3$. Larger values of $m_\chi$ for an arbitrary neutralino gaugino/higgsino composition are inconsistent with $\Omega_\chi h^2 < 0.3$. This upper limit on $m_\chi$ relaxes to 2.6 TeV for $\tan \beta = 10$.

On the other side, at low enough $m_\chi$ coannihilation with sleptons prevents one from deriving a firm lower limit on $\sigma_p^{SI}$. Indeed, by suitably choosing the slepton mass not too much above $m_\chi$, we can always reduce $\Omega_\chi h^2$ below 0.2. Thus, it is possible to set firm lower limits on $\sigma_p^{SI}$ but only for large enough $m_\chi$ and even though they correspond to extremely large values of $\mu$ and accordingly involve much fine–tuning.
The above discussion of the parameter–independent lower limit on $\sigma_p^{SI}$ and upper limit on $m_\chi$ has been presented in the case of $\mu > 0$ but it obviously applies also to the case $\mu < 0$.

The dependence of $\sigma_p^{SI}$ on $A_t$ and $A_b$ is rather weak. As the tri–linear terms deviate from zero, the mass of the lightest Higgs generally increases due to somewhat larger mass splittings among the stops and sbottoms. As a result, the normally subdominant contribution to $\sigma_p^{SI}$ from the $t$–channel $h$–exchange is slightly reduced.

A far more important effect is that the number of SUSY configurations satisfying all experimental constraints, especially that from $b \rightarrow s\gamma$, decreases significantly. This is because the cancellation between charged Higgs loop and chargino loop contribution to the $b \rightarrow s\gamma$ decay rate becomes more inefficient as $A_t$ decreases. For example, for $A_t = -1$ TeV only a handful of points remain allowed. Generally, we have concluded that the regions allowed by lower values of $A_t$ and $A_b$ fall into the regions allowed by the choice $A_t = 1$ TeV.

In order to display the dependence on $\tan \beta$, in the left and right window of Fig. 4 we present the cases of $\tan \beta = 10$ and 50, respectively. Note that, for small $\tan \beta = 10$ the largest allowed values of $\sigma_p^{SI}$ are roughly an order of magnitude smaller than at $\tan \beta = 50$ because of the $\tan \beta$–dependence of the heavy scalar coupling to down–type quarks, as mentioned earlier. Notice a significant decrease in the upper ranges of $\sigma_p^{SI}$ at large $m_\chi$ for $\tan \beta = 10$, which is caused by exceeding the upper limit ($4.08 \times 10^{-4}$) of the allowed range of BR($B \rightarrow X_s\gamma$). It is clear that the constraint is more severe in the case $\tan \beta = 10$ rather than at larger $\tan \beta$. This may sound somewhat counter–intuitive since, for example, in the Constrained MSSM, the constraint from $b \rightarrow s\gamma$ on the (CMSSM) parameter space becomes more pronounced at larger $\tan \beta$. This is because, in the CMSSM the pseudoscalar Higgs mass, hence also the charged Higgs mass, is typically large, and the corresponding charged Higgs–top quark loop contribution to BR($B \rightarrow X_s\gamma$) becomes suppressed. At smaller
Figure 5: Ranges of $\sigma_p^{SI}$ in the general MSSM vs. $m_\chi$ for $\mu > 0$, which are allowed by collider bounds, $b \to s\gamma$ and $0.1 < \Omega_\chi h^2 < 0.2$. Also marked are some results of recent experimental WIMP searches. The thick black line and a left–pointing arrow indicate a parameter–independent lower bound. The region below the dashed line is excluded if one imposes the constraint $\mu < 1 \text{ TeV}$. The ranges of $m_\chi$ to the vertical lines are excluded at $1\sigma$ and $2\sigma$ CL by the current discrepancy between the experimental value of $(g - 2)_\mu$ and the Standard Model prediction.

$m_{1/2}$ (thus also $m_\chi$ and $m_{\chi^\pm}$) and $m_0$ (thus also light enough stop) the tan $\beta$–dependent (negative) contribution from the chargino-stop loop gives too small BR($B \to X_s\gamma$), below the lower experimental limit, thus producing a strong lower bound on $m_\chi$ at not too large $m_0$. In contrast, in the general MSSM case, we can choose small values of $m_A$ which is a free parameter. This small $m_A$ implies a big positive charged Higgs loop contribution to BR($B \to X_s\gamma$). For smaller tan $\beta$ and at smaller $m_\chi$ this is reduced to an acceptable range by the chargino-stop loop contribution. However, at large $m_\chi$, the chargino-stop loop cannot cancel the charged Higgs contribution anymore and one exceeds the upper experimental limit on BR($B \to X_s\gamma$). Since the chargino loop contribution is proportional to tan $\beta$, at large tan $\beta$ the cancellation can be achieved even at large $m_\chi$. For this reason, in the right window of Fig. 4 there is no analogous decrease in the largest allowed $\sigma_p^{SI}$ at large $m_\chi$, in contrast to the left window. Clearly, the constraint from $b \to s\gamma$ is more severe for smaller tan $\beta$.

A similar effect of can be observed in the case of the $(g - 2)_\mu$ constraint. By comparing
Fig. 4 with the upper right window of Fig. 2 one can see that it produces as stronger upper bound on $m_\chi$ at smaller $\tan \beta$. For example, imposing a 1 $\sigma$ bound implies $m_\chi \lesssim 120$ GeV for $\tan \beta = 10$, while for $\tan \beta = 50$ the bound moves up to $m_\chi \lesssim 310$ GeV, as denoted in the two windows of Fig. 4. Also marked is the effect of imposing $\mu < 1$ TeV. While not being a firm constraint, it does, in our opinion, indicate the region which may be considered as somewhat less fine–tuned.

As discussed above, for a considerable range of large $m_\chi \lesssim 1020$ GeV (marked with a thick solid line and left–pointing arrow in Fig. 4), we can again set up the parameter–independent lower limit on $\sigma_{SI}^p$, in analogy with the case $\tan \beta = 35$ of Fig. 2. In order to do so, we had to explore extremely large ranges of $\mu$ up to some 4 TeV at smaller $\tan \beta$ in order to saturate the bound $\Omega_\chi h^2 < 0.2$. On the other hand, at smaller $m_\chi \lesssim 440(800)$ GeV for $\tan \beta = 10(50)$ the lower limit on $\sigma_{SI}^p$ remains basically independent of $\tan \beta$ since $\Omega_\chi h^2$ at lower $m_\chi$ is determined mostly by the coannihilation with sleptons, as discussed in detail in the case $\tan \beta = 35$. In this case the lower limit on $\sigma_{SI}^p$ arises from restricting $\mu$ below the value for which the parameter–independent lower limit arises at larger $m_\chi$. Notice that the thick line extends to lower $m_\chi$ at smaller $\tan \beta$ because the efficiency of the neutralino–slepton coannihilation increases with $\tan \beta$.

Finally, in Fig. 5 we summarize the results for the full scan conducted so far for $\tan \beta = 5, 10, 35, 50, 55, 60, 65$ and for $\mu > 0$. We repeat that, in determining the allowed (blue) region we applied the constraints from collider searches, $b \to s\gamma$ and $0.1 < \Omega_\chi h^2 < 0.2$. The lower limit on $\sigma_{SI}^p$ mostly comes from low $\tan \beta$ and, where applicable, we mark with the solid line where it is parameter–independent. The effect of restricting $\mu < 1$ TeV is marked with a dashed line. Also indicated is the impact of the new measurement of $(g−2)_\mu$ on the mass of the neutralino. If confirmed, the $e^+e^−$–based range (4.4) will imply rather stringent upper limits on $m_\chi$

$$m_\chi \lesssim 350 \text{ GeV} \quad (1 \sigma \text{ CL})$$

$$m_\chi \lesssim 510 \text{ GeV} \quad (2 \sigma \text{ CL}).$$

If the $\tau$–based numbers (4.5) are applied instead, one obtains $m_\chi \lesssim 800$ GeV (1 $\sigma$ CL) and no upper bound at 2 $\sigma$ CL (for $\mu > 0$).

The vast ranges of $\sigma_{SI}^p$ predicted in the framework of the general MSSM may be somewhat discouraging to DM WIMP hunters. It is worth noting, however, that it is the region of smaller $m_\chi$, below a few hundred GeV, that not only is implied by the new result for $(g−2)_\mu$, but is also theoretically more favored as corresponding to less fine–tuning. Furthermore, ranges of very small $10^{-12} \text{ pb} \lesssim \sigma_{SI}^p \lesssim 10^{-8} \text{ pb}$ generally correspond either to very large (and therefore perhaps somewhat less natural) values of $\mu$ and/or $m_A$, or become allowed by selecting slepton masses on the light side, and in the $\chi$–slepton coannihilation region, within some 20 GeV of $m_\chi$, which again can be be considered as a finely–tuned case.

At the end, we comment again on the case of the Constrained MSSM. Because the model is much more restrictive, the ranges of $\sigma_{SI}^p$ that one obtains in the parameter space allowed by all constraints, are very much narrower [12, 13, 14, 15, 16]. They are also
typically somewhat lower than the largest ones allowed in the general MSSM. For example, at $\tan \beta = 50$ we find $\sigma_{SI}^p \sim 10^{-7}$ pb at $m_\chi = 100$ GeV and $\sigma_{SI}^p \sim 7 \times 10^{-11}$ pb at the largest (neglecting $(g - 2)_\mu$) allowed value of $m_\chi = 800$ GeV. On the other hand, because the model is defined at the grand-unified, and not electroweak, scale, in the case of large $\tan \beta \gtrsim 50$ and/or large scalar masses, theoretical uncertainties involved in the running of parameters are substantial and have much impact on the resulting ranges of both $m_\chi$ and $\sigma_{SI}^p$. We will explore the case of the Constrained MSSM in an oncoming publication.

6. Conclusions

We have delineated the ranges of the SI cross section $\sigma_{SI}^p$ in the general MSSM, which are consistent with current experimental bounds and for which one finds the expected amount of dark matter. We have further discussed the dependence of our results on the experimental constraints and on the underlying theoretical assumptions. While the ranges which we have obtain extend over more than six orders of magnitude, we find it encouraging that the experimental sensitivity that has recently been reached, now allows one to explore our theoretical predictions for the MSSM. As we have argued above, smaller values of the WIMP mass and also larger values of $\sigma_{SI}^p$ may be considered as more natural, which will hopefully be confirmed by a measuring a positive WIMP detection signal in the near future.

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