The theoretical analysis and simulation of double cladding Er/Yb Co doped optical fiber laser

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Abstract—Rare earth ion fiber laser has the characteristics of compact structure and excellent beam quality. Cladding pumped fiber laser has become the best choice to obtain high-power laser. This paper introduces the characteristics of Er/Yb Co doped fiber laser. The theoretical model of Er/Yb Co doped fiber laser is established from the atomic energy level and energy transfer in erbium ytterbium Co doped fiber. By changing the initial pumping conditions and the reflectivity of the rear grating, the output power of the laser with different pumping power and different output mirror reflectivity can be calculated and numerically simulated. In this paper, the theoretical model of Er / Yb Co doped fiber laser is established, the numerical simulation is carried out, and the gain characteristics of the laser are analyzed.

1. Introduction
Fiber lasers have reached the power level of traditional lamp pumped and LD pumped solid-state lasers, and there is a great trend to replace them. Due to the introduction of cladding pumping technology, the pulse peak power and single pulse energy of Q-switched fiber laser are greatly improved. Q-switched fiber laser with compact structure, natural cooling, high peak power, high pulse energy and high optical conversion efficiency has many advantages in many application fields such as medicine, industrial processing, communication, military, nonlinear frequency conversion, ranging, remote sensing and so on.

In the 1990s, the research on erbium-doped fiber amplifier was carried out in many aspects[1-3]. In order to further improve the power, the research on erbium-doped fiber was gradually increased. However, if the Er ion concentration is too high, the first problem is that the concentration quenching and amplification efficiency will be reduced, and the second problem is crystallization. Co doping ytterbium (Yb) ions in Er doped fiber is a solution. It can not only provide a broadband absorption from 800 nm to 1100 nm, provide great flexibility for the selection of pumping light sources, but also improve the pumping rate and pumping efficiency and suppress the self-pulsation effect caused by ion pairs. Therefore, Er / Yb Co doped fiber active devices have attracted great interest[4].

Using ytterbium erbium Co doped fiber can effectively suppress the self-pulse effect in erbium-doped fiber caused by erbium ion pair, improve the effective pumping rate and provide stable laser operation. When the concentration of erbium ion pair in erbium-doped fiber reaches a certain level, it can play a role of saturable absorption and cause self-pulse operation. This is verified experimentally and theoretically, and satisfactory results are obtained. Although this does not mean that the existence of ion pair is the only physical factor leading to self-pulse in erbium-doped fiber laser, it does play a very important role.
2. The characteristics of Er / Yb Co doped Optical fiber

The so-called concentration quenching is because the distance between Er ions is short, any two adjacent Er ions in $^4I_{13/2}$ excited state will interact, conduct energy conversion and ion up conversion. One ion transfers the energy to another ion and returns to the ground state $^4I_{15/2}$, while the ion that obtains the energy transitions to a higher energy level $^4I_{15/2}$. The ion on $^4I_{15/2}$ relaxes rapidly to the energy level $^4I_{13/2}$ through phonon emission, resulting in the conversion of the energy obtained by an excited ion into heat energy, which reduces the quantum conversion efficiency (QCE). Obviously, in order to obtain the best amplification performance, there is an optimal doping ratio. At present, the doping weight ratio in pure silicon host is generally hundreds of PPM, so increasing the ion concentration to improve the amplification gain has certain limitations. Although the addition of aluminum can inhibit the concentration quenching and can be increased to the doping weight ratio of 1000ppm, it is also limited. Recently, it has been reported that changing the core glass matrix can also improve the solubility of Er.

In order to increase the doping concentration and expand the wavelength, erbium ions are Co doped with other rare earth ions, including the preparation process of Er / Yb, Er / Tm and Er / La Co doped fibers, and their spectral properties are studied. Taking Er/Yb Co doped fiber as an example, Yb ions, like Er ions, have less solubility in silicon. They have similar ion radius and can form ion clusters. Because these ions alternate with Er, the probability of ion up conversion due to interaction between Er ions and Er ions can be reduced, resulting in high ion conversion efficiency. This is because when a large number of Yb ions gather around Er ions to form ion clusters, the spacing between Er ions and Er ions is increased, which can inhibit the occurrence of QCE between Er ions. Figure 1 shows the energy level diagram of Er Yb Co doped system. In Er /Yb Co doped fiber, Yb ion absorption pump light is excited to $^4I_{15/2}$ level. After Er ion is excited to $^4I_{11/2}$ level, Yb ion transfers energy to ER and returns to ground state. Er ion absorbing energy quickly returns from $^4I_{11/2}$ level to $^4I_{13/2}$ level through relaxation, which forms ion number inversion between $^4I_{13/2}$ and $^4I_{15/2}$ levels. Therefore, Co doping ytterbium ion in erbium-doped fiber is a very effective method, which can improve the pumping rate and pumping efficiency at the same time [5].

3. The theoretical model of Er / Yb Co doped fiber

The energy level and energy transfer process of atoms in erbium ytterbium Co doped fiber are shown in Fig. 2, the particle number densities on $^4I_{15/2}$, $^4I_{13/2}$, $^4I_{11/2}$ and $^4I_{9/2}$ of Er$^{3+}$ are represented by $N_1$, $N_2$, $N_3$ and $N_4$ respectively, and the particle number densities on $^2F_{5/2}$ and $^2F_{7/2}$ of Yb$^{3+}$ are represented by $N_6$ and $N_5$ respectively. In ER / Yb Co doped fiber, Yb ion absorption pump light is excited to $^4F_{5/2}$ level. After Er ion is excited to $^4I_{11/2}$ level, Yb ion transfers energy to Er and returns to ground state. Er ion absorbing energy quickly returns from $^4I_{11/2}$ level to $^4I_{13/2}$ level through relaxation, which forms particle number inversion between $^4I_{13/2}$ and $^4I_{15/2}$ levels. By analyzing the transition of Er$^{3+}$ and Yb$^{3+}$ particles, the following rate equation of particle number in steady state can be obtained.
\[ \frac{dN_4}{dt} = -\frac{N_4}{\tau_{43}} + C_{up}N_2^2 = 0 \]  
\[ \frac{dN_3}{dt} = W_{11}N_1 - \frac{N_3}{\tau_{43}} + \frac{N_4}{\tau_{43}} + C_{cr}N_1N_6 = 0 \]  
\[ \frac{dN_2}{dt} = W_{12}N_1 - W_{21}N_2 + \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - 2C_{up}N_2^2 = 0 \]  
\[ N_{N_3} = N_1 + N_2 + N_3 + N_4 \]  
\[ \frac{dN_6}{dt} = W_{56}N_5 + \frac{N_6}{\tau_{65}} - W_{56}N_6 - C_{cr}N_1N_6 = 0 \]  
\[ N_{N_6} = N_5 + N_6 \]

Where \( W_{ij} \) represents the probability of stimulated absorption or stimulated radiation transition between energy levels \( I \) and \( j \), \( \tau_{21} \) and \( \tau_{65} \) represents the lifetime of spontaneous emission of particles at \( ^4I_{13/2} \) and \( ^2F_{5/2} \) levels, \( \tau_{43} \) and \( \tau_{32} \) represents the lifetime of spontaneous emission of particles at \( ^4I_{9/2} \) and \( ^2I_{11/2} \) levels, the quadratic term coefficient \( cup \) represents the energy up conversion process from \( ^4I_{13/2} \) state to \( ^4I_{15/2} \) state and \( ^4I_{9/2} \) state respectively, and the cross relaxation coefficient \( C_{cr} \) represents the energy transfer process from \( \text{Yb}^{3+} \) to \( \text{Er}^{3+} \). \( W_{12} \) and \( W_{21} \) are the stimulated absorption and stimulated radiation transition probabilities of signal light respectively, \( w_{13} \) is the stimulated absorption probability of \( \text{Er}^{3+} \) ion to pump light, \( w_{56} \) and \( w_{65} \) are the stimulated absorption and stimulated radiation transition probabilities of \( \text{Yb}^{3+} \) ion to pump light respectively, which are given by the following formula

\[ W_{12}(r, \theta, z) = \frac{\sigma_{12}(\nu_p)}{h\nu_s} I_s(r, \theta, z, \nu_s) + \int_{\nu} \frac{\sigma_{12}(\nu)}{h\nu_s} I_{ASE}^s(r, \theta, z, \nu) + \frac{I_{ASE}^p(r, \theta, z, \nu)}{h\nu_s} d\nu \]  
\[ W_{21}(r, \theta, z) = \frac{\sigma_{21}(\nu_p)}{h\nu_s} I_s(r, \theta, z, \nu_s) + \int_{\nu} \frac{\sigma_{21}(\nu)}{h\nu_s} I_{ASE}^s(r, \theta, z, \nu) + \frac{I_{ASE}^p(r, \theta, z, \nu)}{h\nu_s} d\nu \]

\[ W_{13}(r, \theta, z) = \frac{\sigma_{13}(\nu_p)}{h\nu_p} I_p(r, \theta, z, \nu_p) \]  
\[ W_{56}(r, \theta, z) = \frac{\sigma_{56}(\nu_p)}{h\nu_p} I_p(r, \theta, z, \nu_p) \]  
\[ W_{65}(r, \theta, z) = \frac{\sigma_{65}(\nu_p)}{h\nu_p} I_p(r, \theta, z, \nu_p) \]
Among them, $\sigma_{12}(\nu)$, $\sigma_{21}(\nu)$, $\sigma_{13}(\nu)$, $\sigma_{56}(\nu)$ and $\sigma_{65}(\nu)$ are the absorption and emission cross-sectional areas between the energy levels of Er$^{3+}$ and Yb$^{3+}$ related to frequency, respectively, and $H$ is the Planck constant, $\nu_s, \nu_p$ is the wavelength of laser and pump light respectively, and the integral is the integral of the whole spontaneous emission interval $\nu$.

$I_p(r, \theta, z, \nu_p)$, $I_s(r, \theta, z, \nu_s)$ are the light intensity of pump light and signal light, $I_{aYb}(r, \theta, z, \nu)$, $I_{eYb}(r, \theta, z, \nu)$ is the light intensity of forward and reverse transmitted spontaneous emission light respectively.

In Er$^{3+}$/Yb$^{3+}$ Co doped fiber, the energy up conversion process between high concentration erbium ions is effectively suppressed due to the addition of ytterbium ions. Therefore, the energy up conversion process of erbium ions cannot be considered in the actual calculation process [6-7]. Because the particle lifetimes at $^4I_{11/2}$ and $^4I_{13/2}$ levels of Er$^{3+}$ are much shorter than those at $^4I_{13/2}$ levels ($\tau_{43} \approx 1\text{ns}$, $\tau_{32} \approx 0.1\mu$s, $\tau_{21} \approx 1\text{ms}$).

The solution of the rate equation is as follows

$$N_{e} = \frac{\Gamma_{e} \lambda_{p} P_{p}}{h c A} \sigma_{aYb}(\lambda_{p}) N_{fb}$$

$$N_{s} = \frac{\Gamma_{s} \lambda_{p} P_{p}}{h c A} \sigma_{aYb}(\lambda_{p}) N_{fb} + \frac{1}{\tau_{es}} + C_{cr} \left( N_{es} - N_{s} \right)$$

$$N_{e} N_{s} = \frac{1}{\tau_{p} + \Gamma_{e} \lambda_{p} P_{p}} \sigma_{aYb}(\lambda_{p}) N_{fb} + \frac{\Gamma_{s} \lambda_{p} P_{p}}{h c A} \sigma_{aYb}(\lambda_{p}) N_{fb} + C_{cr} N_{s}$$

Where $P_p$ is the pump power and $P_s$ is the signal optical power. The transport equation of optical power can be written as

$$\frac{\partial P_{s}}{\partial z} = -\Gamma_{s} [\sigma_{aYb}(\lambda_{s}) N_{s}(z) \lambda_{i} - \sigma_{aYb}(\lambda_{s}) N_{s}(z) - \sigma_{aEr}(\lambda_{s}) N_{s}(z)] \cdot P_{p} - \alpha(\lambda_{s}) P_{s}$$

$$\pm \frac{\partial P_{s}}{\partial z} = \pm \Gamma_{s} [\sigma_{aEr}(\lambda_{s}) N_{s}(z) \lambda_{i} - \sigma_{aEr}(\lambda_{s}) N_{s}(z)] P_{e} \mp \alpha(\lambda_{s}) P_{s}$$

$$P_{s} = P_{s}^{+} + P_{s}^{-}$$

Where $h$ is the Planck constant, $C$ is the speed of light, and $a$ is the doped area of the core, $\sigma_{aYb}(\lambda)$ and $\sigma_{aEr}(\lambda)$ is the absorption cross section of Yb and Er at different wavelengths, $\sigma_{aYb}(\lambda)$ and $\sigma_{aEr}(\lambda)$ is the emission cross section of Yb and Er at different wavelengths. $P_{s}$ is the signal optical power and $P_{p}$ is the pump optical power. $\Gamma_{s}$ and $\Gamma_{p}$ is the overlap factor of signal light and pump light respectively. It is assumed that the normalized doping diameter of optical fiber is core diameter $D$, mode field diameter $w$ of signal light and signal light overlap factor

$$\Gamma_{s} = 1 - e^{-2D_{core}^2/w^2}$$

Due to the existence of mutual coupling terms in equations above, it is difficult to obtain the analytical solution. The above equations can be solved numerically by MATLAB. The pump wavelength is 980nm and the lasing wavelength is 1550nm. The boundary conditions of equations are:

$$\left\{ \begin{array}{l} P_{s}^{+}(0) = R_1 P_{s}^{-}(0) \\ P_{s}^{-}(L) = R_2 P_{s}^{+}(L) \end{array} \right.$$  

Where $R$ is the reflectivity of the back cavity mirror and $l$ is the length of erbium ytterbium Co doped fiber. The initial pumping conditions are:

$$P_{p}(0) = P_{p0}$$

4. Numerical simulation of Er / Yb Co doped fiber laser

Since the reflectivity of the front grating is very high, $R_1 = 1$ can be taken. By changing the initial pumping conditions and the reflectivity of the rear grating, the output power of the laser with different pumping power and different output mirror reflectivity can be obtained. The parameters used for numerical calculation are as follows:

4.
\[ \sigma_{\text{Yb}}(\lambda_p) = 2.5 \times 10^{-24} \text{m}^2, \quad \sigma_{\text{Er}}(\lambda_p) = 2.0 \times 10^{-24} \text{m}^2, \quad \sigma_{\text{Er}}(\lambda_s) = 6.0 \times 10^{-25} \text{m}^2, \]
\[ \sigma_{\text{Er}}(\lambda_p) = 5.5 \times 10^{-25} \text{m}^2, \quad \sigma_{\text{Er}}(\lambda_s) = 7.5 \times 10^{-25} \text{m}^2, \quad \sigma_{\text{Yb}}(\lambda_s) = 7.5 \times 10^{-25} \text{m}^2, \]
\[ C_{\text{cr}} = 2.52 \times 10^{-22} \text{m}^3 \text{s}^{-1}, \quad D_{\text{core}} = 7.5 \mu m \text{, } \alpha = 3.95 \times 10^{-3} \text{m}^{-1}, \quad N_{\text{Yb}} = 4.85 \times 10^{25}, \quad N_{\text{Yb}} = 3.75 \times 10^{26}, \quad \alpha_s = 3.50 \times 10^{-3} \text{m}^{-1}, \quad \tau_{21} = 12 \text{ms}, \quad \tau_{65} = 2.0 \text{ms}, \quad \Gamma_p = 0.0032 \]

5. Study on axial power distribution characteristics

As shown in Figure 3, for the forward pumped fiber laser, the forward transmitted laser in the fiber increases exponentially at the beginning and tends to be saturated at a certain length. On the contrary, the reverse transmitted laser decreases exponentially at the beginning and tends to be stable at a certain length. For a given pump power and fiber length, the higher the reflectivity of the rear cavity mirror, the higher the laser power in the fiber, even higher than the pump power (as shown in Figure 3a and 3b), so this should be taken into account in the design of fiber laser. Especially for high-power fiber laser, the output cavity mirror with low reflectivity should be used to avoid excessive power in the fiber.

6. Study on gain characteristics

The gain coefficient of fiber laser is:
\[ g = \sigma_{\text{Er}}(\lambda_s) N_2 - \sigma_{\text{Er}}(\lambda_s) N_1. \]

According to the calculated number of upper-level particles, the gain coefficient of fiber laser is very high on the side close to the pump end, decreases rapidly with the change of fiber length, and finally tends to be stable and close to 0, which just shows the change characteristics of power. The optical power of forward transmission increases rapidly at the beginning, reaches stability after reaching a certain fiber length, and basically does not change, at this time, the gain coefficient in the optical fiber is 0. The gain of fiber laser is high and the distribution is uneven. When designing the structure of fiber laser, if the technical conditions permit, the scheme of double end pumping or distributed pumping should be adopted as far as possible. Moreover, from the calculation results, the number of Yb ions in the upper level of Er / Yb Co doped fiber is one order of magnitude larger than that of Er ions, which indicates that Yb ions store a lot of energy, and ER / Yb fiber is at 1 μ M-band also has strong gain, as shown in Figure 4.
7. Conclusion

The characteristics of Er/Yb Co doped fiber laser are introduced. The theoretical model of Er/Yb Co doped fiber laser is established from the atomic energy level and energy transfer in erbium ytterbium Co doped fiber. By changing the initial pumping conditions and the reflectivity of the rear grating, the output power of the laser with different pumping power and different output mirror reflectivity can be calculated and numerically simulated, the laser power distribution is analyzed. It is concluded that for high-power fiber laser, the output cavity mirror with low reflectivity should be used to avoid too high power in the fiber. From the calculation results, the number of Yb ions in the upper level in Er/Yb Co doped fiber is one order of magnitude larger than that of Er ions, which indicates that Yb ions store a lot of energy. After the simulation calculation, we will study the influence of the output power of the laser on the pump power with the increase of the reflectivity of the output mirror by limiting the pump power and fiber length, and find the best fiber length through experiments.

Acknowledgments

The author would like to thank her collaborators from and support of Tianhua College of Shanghai Normal University, Section of Practice and Training and Lab. of Excited State Processes, Chinese Academy of Sciences, Changchun Institute of Optics, Fine mechanics and Physics, CAS.

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