Research Article

On Constructing Strongly Connected Dominating and Absorbing Set in 3-Dimensional Wireless Ad Hoc Networks

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Received 15 February 2019; Revised 28 December 2019; Accepted 27 January 2020; Published 24 February 2020

Academic Editor: Toshikazu Kuniya

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In a wireless ad hoc network, the size of the virtual backbone (VB) is an important factor for measuring the quality of the VB. The smaller the VB is, the less the overhead caused by the VB. Since ball graphs (BGs) have been used to model 3-dimensional wireless ad hoc networks and since a connected dominating set can be used to represent a VB undertaking routing-related tasks, the problem of finding the smallest VB is transformed into the problem of finding a minimum connected dominating set (MCDS).

Many research results on the MCDS problem have been obtained for unit disk graphs and unit ball graphs, in which the transmission ranges of all nodes are identical. In some situations, the node powers can vary. One can model such a network as a graph with different transmission ranges for different nodes. In this paper, we focus on the problem of minimum strongly connected dominating and absorbing sets (MSCDASs) in a strongly connected directed ball graph with different transmission ranges, which is also NP-hard. We design an algorithm considering the construction of a strongly connected dominating and absorbing set (SCDAS), whose size does not exceed $((319/15)k^3 + (116/5)k^2 + (29/5)k \text{opt} + ((29/3)k^3 + (116/5)k^2 + (87/5)k + (13/15))$, where opt is the size of an MCDAS and $k$ denotes the ratio of $r_{\text{max}}$ to $r_{\text{min}}$ in the ad hoc network with transmission range $[r_{\text{min}}, r_{\text{max}}]$. Our simulations show the feasibility of the algorithm proposed in this paper.

1. Introduction

Along with the rapid development of wireless radio communication technologies, embedded sensors, and VLSI, the cost of establishing a wireless ad hoc network is decreasing and the performance of wireless ad hoc networks is improving. Recently, wireless ad hoc networks have been widely used in many areas, such as disaster rescue, environmental monitoring, military operations, and mobile computing (see [1–4]), where it is difficult to build a physical backbone for the network; it can be expected that wireless ad hoc networks will play an increasingly important role in the communication of future networks.

In wireless ad hoc networks, the power source that each node (sensor) possesses is limited, which results in a limited distance over which the nodes in the wireless ad hoc network can transfer information and the time that the nodes can continue to work. To reduce energy and storage requirements and avoid information conflict and broadcast storms, a backbone-like structure has been proposed [5], called the virtual backbone (VB). A VB is defined as a subset of wireless ad hoc network nodes. Since every operation request between nodes in a wireless ad hoc network can be transformed into a homologous operation on the VB, the routing overhead in the wireless ad hoc network can be significantly reduced by the VB [6]. When the nodes in the VB perform tasks related to routing, they are interfered to some extent by the equipment in the fixed physical backbones. To reduce the interference from the fixed physical backbones, it is natural to attempt to design a VB in which the number of nodes is minimized. In many wireless ad hoc networks [7], a VB can be modeled by a connected dominating set of the wireless ad hoc network. In other words, finding the minimum size VB is equal to determining the minimum connected dominating set in the wireless ad hoc network.
Many results related to the problem of computing the MCDS can be found in references [6, 8–11]. Note that the previous references about MCDS are based on the unit disk graph in the two-dimensional plane.

However, in some situations, such as underwater resource exploration, disaster prevention, offshore exploration, and ocean environmental monitoring (see [12, 13]), it is not suitable to use MCDS of the unit disk graph to describe the VB of the wireless ad hoc networks. For this reason, people have studied the MCDS problem in three-dimensional space. For example, D. Kim et al. studied the MCDS problem in unit ball graphs and obtained an approximation algorithm of MCDS with a performance ratio of 14.937 (see [13]). In [14], Chizari and Schmutz obtained two CDS algorithms on random unit ball graphs and compared them. In [15], Butenko et al. discussed the MCDS problem in unit-ball graphs. In [16], Gao et al. noted that bugs existed in the approximation deduction process of [13] and proposed a new method to correct these bugs; they then used pruning techniques to optimize the algorithms to choose MCDS. UDG can be viewed as a special case of UBG in which all nodes are restrained to be coplanar. In other words, the MCDS problem in UBGs has more generality than that in UDGs.

However, in UDGs, and in UBGs, it is assumed that all nodes in the wireless ad hoc network have the same transmission range at any time; this is incorrect in some situations. When we focus on the powers of nodes in a wireless ad hoc network, differences exist in their functionalities and control technologies for connectivity, and thus, the powers of these nodes may be different. According to the different requirements of the measured frequency in collisions, a node may have to change its transmission range. In such situations, the MCDS problem in a unit disk graph becomes that of a minimum strongly connected dominating and absorbing set (MSCDAS) in a disk graph in reference [17], which was the first paper to study MSCDAS in a network with different transmission ranges. A strongly connected dominating and absorbing set (SCDAS) can usually be used to denote a VB of a wireless ad hoc network in which the nodes have different transmission ranges. This model (SCDAS) guarantees the sharing and interworking of information in the whole network. The research results on SCDAS can be found in the following references. Wu [18] extended the concept of the dominating set in UDG to the dominating and absorbing set (DAS) in disk graphs (DGs) and proposed a localized algorithm to find an SCDAS, which was also extended by the marking process in UDG. In [19], My Thai et al. first presented an approximation algorithm to solve the problem of the strongly connected dominating set (SCDS), and then, based on the obtained SCDS, they proposed a heuristic algorithm to obtain the solution to the SCDS problem with an approximation ratio. Li et al. discussed the problem of constructing an SCDAS for an asymmetric multihop wireless ad hoc network [20]. Results related to the SCDAS problem can also be found in [21–23]. Note that the previously mentioned results related to the SCDAS problem are all based on two-dimensional space. However, to the best of our knowledge, the field of wireless ad hoc networks with nodes that have different transmission ranges in three-dimensional space has not been studied. In this paper, we study the MSCDAS problem of a wireless ad hoc network with different transmission ranges in three-dimensional space. Note that the MSCDAS problem is also NP-hard because the MCDS problem in UBG is NP-hard and because UBG is a special situation of a ball graph (BG).

The remainder of this paper is organized as follows. Section 2 introduces some basic definitions and terms. Section 3 separately calculates an improved upper bound for maximal independent sets in UBGs and BGs. Then, we present an algorithm to construct an SCDS in Section 4. Section 5 provides the simulation results with different parameter settings. Finally, in Section 6, we summarize this paper.

2. Preliminaries

For the sake of convenience, we introduce some background information, including special terms in graph theory, which will be used in the paper. A wireless ad hoc network, in which each node has a transmission range, can be denoted by a directed graph \( G = (V, E) \). For each node \( v \in V \), let \( r_v \) denote the transmission range of node \( v \). The transmission range of the wireless ad hoc network \( N \) in which each node has a transmission range, can be denoted by a directed graph \( G = (V, E) \). For each node \( v \in V \), let \( r_v \) denote the transmission range of node \( v \). The transmission range of the wireless ad hoc network \( N \) is the minimum (maximum) transmission range of the wireless ad hoc network \( N \) and \( i d(v) \) is the unique identification of \( v \). For two nodes \( v_i, v_j \in V \), we use \( d(v_i, v_j) \) to denote the Euclidean distance between \( v_i \) and \( v_j \) and define that \( (v_i, v_j) \in E \) if and only if \( d(v_i, v_j) \leq r_{v_i} \). The edge \( (v_i, v_j) \) is said to be a unidirectional edge from \( v_i \) to \( v_j \) if \( (v_i, v_j) \in E \) and \( (v_j, v_i) \notin E \). The edge \( (v_i, v_j) \) between \( v_i \) and \( v_j \) is said to be a bidirectional edge if \( (v_i, v_j) \in E \) and \( (v_j, v_i) \in E \). According to the above assumptions, we can conclude that the edge \( (v_i, v_j) \) is a bi-directional edge if and only if \( d(v_i, v_j) \leq \min \{ r_{v_i}, r_{v_j} \} \).

For node \( v \in V \), we use \( N^+(v) = \{ u \in V \, | \, (u, v) \in E \} \) and \( N^-(v) = \{ u \in V \, | \, (v, u) \in E \} \) to denote the incoming neighborhood (respectively, the outgoing neighborhood, the closed incoming neighborhood, and the closed outgoing neighborhood) of \( v \). Assume that \( S \) is a dominating set of \( G = (V, E) \); then, \( S \cup N^+(S) - S \), where \( N^+(S) = \cup_{v \in S} N^+(v) \). A subset \( R \subseteq V \) is called an absorbing set of \( G \) if, for any node \( x \in V - R \), there exists a node \( y \in R \) such that \( (x, y) \in E \). It is obvious that if \( R \) is an absorbing set, then for any node \( u \in V - R \), \( N^+(u) \cap R \neq \emptyset \). A subset \( S \) is called an independent set (IS) of \( G \) if and only if, for any two nodes \( u, v \in S \), \( (u, v) \notin E \) or \( (v, u) \notin E \). A directed graph \( G = (V, E) \) is called a strongly connected graph if and only if, for any two nodes \( v_i, v_j \in V \), there exist two directed paths, one of them being from \( v_i \) to \( v_j \) and the other being from \( v_j \) to \( v_i \). A subset \( S \subseteq V \) is called a strongly connected dominating set (SCDS) if and only if \( S \) is a DS, and \( G[S] \), which is a subgraph of \( G \) induced by \( S \), is strongly connected. A subset \( S \subseteq V \) is called a strongly connected dominating and absorbing set (SCDAS) if and only if \( S \) is an SCDS, and for each node \( u \notin S \), \( N^+(u) \cap S \neq \emptyset \).
3. An Improved Upper Bound for Maximal Independent Sets

For an undirected graph $H = (V, E)$, when an approximation algorithm for the MCDS of $H$ is considered to obtain a CDS with a performance ratio with respect to MCDS, generally, there are two steps. The first step is to find an MIS $M$ in $H$, which is a domination set of $H$. The second step is to find some nodes in $V - M$ to connect the nodes in $M$ to obtain a CDS. However, for a directed graph $G = (V, E)$, when we design an approximation algorithm for an SCDAS of $G$ with a performance ratio with respect to MSCDAS, it is not sufficient that there are only two steps as in the above method for a CDS. The main reason for this is that for a directed graph $G$, an MIS is not necessarily a dominating set. In this paper, we use the following three steps to obtain an SCDAS of $G$.

3.1. An Improved Upper Bound for the Size of an MIS in a Unit Ball Graph

In this section, we determine an upper bound for the size of an MIS in a unit ball graph $BallGraph$. The first step is to find a dominating set $M$ for $G$. The second step is to locate some nodes in $V - M$ to obtain a new directed ball graph $\overline{G} = (V, E)$. A similar method to the first and second steps is used to obtain a connected dominating set $\overline{S}_d$ of $\overline{G}$, which is a connected absorbing set for $G$. Then, $S_d \cup \overline{S}_d$, an SCDAS of $G$, is obtained.

For the sake of convenience, let us first discuss the upper bound of the size of MIS for a unit ball graph $G = (V, E)$.

**Lemma 1** (see [24]). Assume that $G = (V, E)$ is a unit ball graph. Then, for any node $v \in V$, the unit ball with the center $v$ has at most 12 independent nodes.

For the bound of an MIS in the unit ball graph $G = (V, E)$, using Lemma 1, Butenko et al. [15] showed that $\alpha(G) \leq 11 \text{opt}_1 + 1$. Remarkably, using Lemma 1, Kim et al. [13] improved the result in [15] and obtained an upper bound of the size of the MIS in UBG as follows.

**Lemma 2** (see [13]). Assume that $G = (V, E)$ is a unit ball graph. Then, $\alpha(G) \leq 10.917 \text{opt}_1 + 1.083$.

To obtain an improved upper bound for maximal independent sets in a connected unit ball graph, we present some important results in the following.

Let $D_i$ with the center $e_i$ and $D_j$ with the center $e_j$ be two connected unit balls (see Figure 1). Let $\overline{D}_i$ and $\overline{D}_j$ be two balls with radius 1.5 and centers $e_i$ and $e_j$, respectively (see Figure 2), where $d(e_i, e_j) = 1$. Let $B_i = D_i \cup \overline{D}_i$ and $B_j = D_j \cup \overline{D}_j$. The following result is obvious.

![Figure 1](image1.png)

**Figure 1:** Two adjacent unit balls with centers $e_i$ and $e_j$, where $d(e_i, e_j) = 1$.

![Figure 2](image2.png)

**Figure 2:** Two adjacent balls with centers $e_i$ and $e_j$, where $d(e_i, e_j) = 1$ and the radius of the balls is 1.5.

**Lemma 3.** The number of independent nodes in $B_i$ does not exceed the number corresponding to the maximum packing of balls with radius 0.5 in $B_2$.

According to Lemma 3, we know that the number of independent nodes in a UDS is related to the upper bounds for the sphere backing problem. It is worth mentioning that there are a lot of studies being related to the bounds for the sphere backing problem (see [25, 26]). Now, we derive the main result in the section.

**Lemma 4.** Assume that $G = (V, E)$ is a connected unit ball graph and $\overline{G} = (V, E)$ is another connected ball graph with $V = V$, for which the radius of each ball is 1.5. Let $I = \{e_1, e_2, \ldots, e_{\text{opt}_1}\}$ be an MCDS of $G$, and $A$ is the covered area with balls in $\overline{G}$ with centers $e_i$ in $I$. Then, the volume of $A$ is at most $14.1372 + 6.8068(\text{opt}_1 - 1)$, where $\text{opt}_1$ is the number of nodes in the MCDS of $G$. 
Proof. Assume that \( \overline{D}_i \) with the center \( e_i \) and \( \overline{D}_j \) with the center \( e_j \) are two connected balls with radius 1.5 in \( G \). Then, \( d(e_i, e_j) \leq 1 \). We first consider the case in which \( d(e_i, e_j) = 1 \). Let \( V_{ij} \) and \( V_{ij}^\prime \) denote the volumes of \( \overline{D}_i \) and \( (\overline{D}_j - \overline{D}_i) \), respectively; then, \( V_{ij} = (4/3)\pi r_{ij}^3 \leq 14.1372 \). From Figure 2, we see that \( V_{ij} = V_{ij} - 2 \times (1/3)\pi r_{ij}^3 \times (3 \times 1.5 - h) = V_{ij} - 2 \times (1/3)\pi r_{ij}^2 \times (3 \times 1.5 - 1) \leq 6.8068 \). It is obvious that for the case \( d(e_i, e_j) < 1 \), the result \( V_{ij} \leq 6.8068 \) is still true. Hence, the volume of \( A \) is at most 14.1372 + 6.8068 (opt1 − 1).

A rhombic dodecahedron has 12 identical rhombic faces with 24 edges and 14 vertices [27]. In these 14 vertices, there are 6 vertices, called Type I vertex, in which each one is exactly the intersection of four rhombic faces, and there are also 8 vertices, called Type II vertex, in which each one is exactly the intersection of three rhombic faces. For example, in Figure 3, \( A, I, K, M, C, \) and \( F \) belong to Type I vertex and \( D, B, E, G, H, N, L, \) and \( J \) belong to Type II.

**Lemma 5.** The volume of a rhombic dodecahedron that surrounds an inscribed ball with radius 0.5 is 0.7071.

Proof. Let \( a \) be the edge length of a rhombic dodecahedron and \( r \) be the radius of its inscribed ball (see Figure 3); then, the volume of the rhombic dodecahedron is \( V_{12} = (16/9) \sqrt{3}a^3 \), and the relationship between \( a \) and \( r \) is \( r = (\sqrt{2}/3)a \). Since \( r = 0.5 \), \( V_{12} = 4\sqrt{2}r^3 = 4 \times 1.4142 \times 0.5^3 = 0.7071 \).

According to Lemma 3, to obtain an upper bound of the size of the maximal independent set in the connected unit ball graph \( G \), we need to number the corresponding to the maximum packing of balls with radius 0.5 in \( G \). Since the volume of a ball with radius 0.5 is \( (4/3)\pi r^3 = 0.5236 \), we obtain an upper bound of the size of the MIS:

\[
\alpha(G) \leq \frac{14.1372 + 6.8068(\text{opt1} - 1)}{0.5236} = 13\text{opt1} + 14.
\]

(1)

Note that for two independent nodes \( u \) and \( v \) in \( G \), there exist two corresponding packing balls with radius 0.5 and centers \( u \) and \( v \), respectively, denoted by \( B_u \) and \( B_v \), in \( G \) and they are not adjacent. In other words, there is space between \( B_u \) and \( B_v \). To derive a better upper bound of the size of the maximal independent set in the connected unit ball graph \( G \), we use a rhombic dodecahedron, which surrounds a ball with radius 0.5 to replace each packing ball with radius 0.5 in \( G \). The reasons are described as follows.

Let \( B_u^{0.5} \) denote a ball with center \( u \) and radius 0.5, \( B_u^{rd} \) denote a rhombic dodecahedron surrounding \( B_u^{0.5} \), \( V_u^{0.5} \) be the volume of the ball \( B_u^{0.5} \), \( V_u^{rd} \) be the volume of \( B_u^{rd} \), \( V_A \) be the volume of \( A \), and the covered area with balls in \( G \) with centers \( e_i \) in \( I \). According to Lemma 3, we conclude that the number of independent nodes in \( G \), denoted by \( I \), does not exceed the number of the maximum packing of balls with radius 0.5 in \( A \), denoted by \( k \). In other words, \( I \leq k \). Since there is some space between these balls, \( V_A/V_{u_i}^{0.5} \) may be not a better upper bound for \( \alpha(G) \). On the other hand, since the area covered by \( G \) can be seamlessly filled by rhombic dodecahedrons [27], it is expectable to obtain \( V_A/V_u^{rd} \) an upper bound of \( \alpha(G) \), which is better than \( V_A/V_{u_i}^{0.5} \).

Note that when one node \( u_i \) in the independent set \( G \) is on the surface of the unit ball with center \( e_i \) in \( I \), the corresponding ball \( B_{u_i}^{rd} \) in \( G \) is inscribed by some balls with center \( e_i \) and radius 1.5 in \( G \). In this case, the rhombic dodecahedrons surrounding \( B_{u_i}^{rd} \) may be incompletely covered by \( A \). In other words, there may be some rhombic dodecahedrons, say \( B_{u_j}^{rd} \) in \( S_{0.5}^{rd} \) such that the part of \( B_{u_j}^{rd} \) is outside \( A \). In the case, let \( \lambda_1 \) be the volume of \( B_{u_j}^{rd} \cap A (1 \leq i \leq k) \). Without loss of generality, suppose that \( \lambda_1 = \max[\lambda_i] \), then \( k = (V_{S_{0.5}^{rd}}/V_u^{rd}) \leq (V_A/V_u^{rd} - \lambda_1) < (V_A/V_u^{rd}) \), which implies that \( (V_A/V_u^{rd} - \lambda_1) \) is better than \( (V_A/V_u^{rd}) \) as a upper bound of \( \alpha(G) \). In this case, it is necessary to analyse the upper bound of \( \lambda_1 \) to evaluate a better upper bound of \( V_A/V_u^{rd} - \lambda_1 \).

It is obvious that the largest \( \lambda_1 \) can take place only if node \( u_j \) is on the surface of some unit ball with center \( e_j \) in \( I \). Under the situation, as it is illustrated in Figure 4, there are at most 4 vertices of \( B_{u_j}^{rd} \) inside the ball with center \( e_j \) and radius 1.5 (\( M, N, K, \) and \( L \) in Figure 4), where two of them belong to Type I and other two belong to Type II, which implies that there are at most 4 vertices of \( B_{u_j}^{rd} \) outside \( A \). Let \( w \) be the
volume of \((B_{|u|}^{id} - B_{|u|}^{id} \cap B_{|u|}^{0.5})\), where \(B_{|u|}^{1.5}\) is the ball with center \(e_i\) and radius 1.5 and \(z\) is the volume of \((B_{|u|}^{id} - B_{|u|}^{id} \cap B_{|u|}^{0.5})\). Consider a partition of \((B_{|u|}^{id} - B_{|u|}^{id} \cap B_{|u|}^{0.5})\), \(\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_{14}\}\), which satisfies the following conditions:

1. Each \(\sigma_i\) in \(\sigma\) contains one and only one vertex of \(B_{|u|}^{id}\)
2. Each \(\sigma_i\) in \(\sigma\) containing one Type I vertex is identical and has the same volume, denoted by \(x\)
3. Each \(\sigma_i\) in \(\sigma\) containing one Type II vertex is identical and has the same volume, denoted by \(y\)
4. \(w \leq (2x + y)\)

It is easily seen that \(z = 6x + 8y\). Next, consider the following cases:

Case 1. \(x \leq y\). Since \(2(x + y) - (2/7)z = (2/7)(x - y) \leq 0, 2(x + y) \leq (2/7)z\).

Case 2. \(x > y\). Since \(z = 6x + 8y, 2(x + y) = (1/4)z + (1/2)x \leq (1/3)z\).

Hence, \(\lambda_1 \leq w \leq 2(x + y) \leq \max((2/7)z, (1/3)z)\), and then, \(\lambda_1 \leq (1/3)z\).

Then, we obtain a new upper bound of MIS in \(G\) as follows:

\[
\alpha(G) \leq k \leq \frac{V_A}{\lambda_1} - \lambda_1 \\
\leq \frac{14.1372 + 6.8068(\text{opt}_1 - 1)}{4\sqrt{2} \times 0.5^3 - (1/3)(4\sqrt{2} \times 0.5^3 - (4/3)\pi \times 0.5^3)} \\
\leq \frac{14.1372 + 6.8068(\text{opt}_1 - 1)}{0.6459} \\
\leq 10.5385 \text{opt}_1 + 11.3491.
\]

Theorem 1. Let \(\text{opt}_1\) be the number of nodes in an MCDS of a connected unit ball graph \(G\) and \(\alpha(G)\) is the number of nodes in an MIS of \(G\); then,

\[
\alpha(G) \leq 10.5385 \text{opt}_1 + 11.3491.
\] (4)

Remark 1. When \(\text{opt}_1 \geq 28\), the result is better than that in [13].

3.2. An Upper Bound of the Size of an MIS in a Directed Ball Graph.

In this section, we use a method similar to that in the previous section to obtain an upper bound of a directed ball graph. Suppose that a directed graph \(G = (V, E)\) represents a wireless ad hoc network with transmission range \([r_{\min}, r_{\max}]\) and is strongly connected. Let OPT be a minimum strongly connected dominating set of \(G\), OPT denote a minimum strongly connected dominating and absorbed set of \(G\), \(\text{opt}_d = |\text{OPT}|\), and \(\text{opt} = |\text{OPT}|\). Since OPT \(\subseteq \text{OPT}_d\), \(\text{opt} \leq \text{opt}_d\). Since OPT \(\subseteq \text{OPT}_d\) is strongly connected, we can always order an array of nodes in OPT as \(\{v_1, v_2, \ldots, v_{\text{opt}_d}\}\) such that for each integer \(k \in \{2, 3, \ldots, \text{opt}_d\}\), there exists at least an integer \(l \in \{1, 2, \ldots, k - 1\}\) satisfying \(d(v_k, v_l) \leq r_{\max}\).

Let \(D_i\) be a ball with center \(e_i\) and radius \(r_{e_i} \in [r_{\min}, r_{\max}]\). \(D_j\) be a ball with center \(e_j\) and radius \(r_{e_j} \in [r_{\min}, r_{\max}]\). \(D_i\) and \(D_j\) be two balls with radius \(r_{\max} + (1/2)r_{\min}\) and centers \(e_i\) and \(e_j\), respectively, where \(e_i, e_j \in V\), and \(d(e_i, e_j) \leq r_{\max}\).

Let \(B_1 = D_i \cup D_j\) and \(B_2 = D_i \cup D_j\).

Lemma 6. The volume of \(\text{OPT}_d\) denoted by \(V_{\text{OPT}_d}\) does not exceed

\[
\frac{11\pi}{12} r_{\max}^3 + \pi r_{\max}^2 r_{\min} + \frac{\pi}{4} r_{\max}^2 r_{\min}^2.
\] (5)

Proof. We first consider the case \(d(e_i, e_j) = r_{\max}\). In Figure 5, we see that the height of a spherical cap is \(h = (1/2)r_{\max} + (1/2)r_{\min}\); then, the volume of the spherical cap \(V_{sc}\) is determined as follows:

\[
V_{sc} = \frac{\pi}{3} r_{\max}^2 (3R - h) = \frac{\pi}{3} \left(\frac{1}{2} r_{\max} + \frac{1}{2} r_{\min}\right)^2 \times \left(3 \times \left(\frac{1}{2} r_{\max} + \frac{1}{2} r_{\min}\right) - \left(\frac{1}{2} r_{\max} + \frac{1}{2} r_{\min}\right)\right)
\]

\[
= \frac{5\pi}{24} r_{\max}^3 + \frac{\pi}{2} r_{\max}^2 r_{\min} + \frac{3\pi}{8} r_{\min}^2 r_{\max} + \frac{\pi}{12} r_{\min}^3.
\] (6)

Then, we have that

\[
V_{\text{OPT}_d} = \frac{4}{3} \pi R^3 - 2V_{sc}
\]

\[
= \frac{11\pi}{12} r_{\max}^3 + \pi r_{\max}^2 r_{\min} + \frac{\pi}{4} r_{\max}^2 r_{\min}^2.
\] (7)

In the case \(d(e_i, e_j) < r_{\max}\), it is obvious that

\[
V_{\text{OPT}_d} \leq \frac{11\pi}{12} r_{\max}^3 + \pi r_{\max}^2 r_{\min} + \frac{\pi}{4} r_{\max}^2 r_{\min}^2.
\] (8)
According to the assumption on the order of \( \text{OPT}_d \), there exists at least one node \( v_i \in \{v_1, v_2, \ldots, v_k\} \) such that \( d(v_i, v_{k+1}) \leq r_{\text{max}} \). According to Lemma 6, we have that the volume of \( D_{k+1} - D_i \), denoted by \( V_{D_{k+1} - D_i} \), exceeds \( V_0 \), where \( D_i(D_{k+1}) \) is a ball with radius \( R \) and center \( v_i(v_{k+1}) \). Hence, the inequality
\[
A_{k+1}^* \leq A_k^* + V_{D_{k+1} - D_i} \\
\leq A_k^* + 4V_0 \leq \frac{4}{3} \pi R^3 + ((k + 1) - 1) \times V_0
\]  

implies that, when \( \text{OPT}_d = k + 1 \), the result is also true.

According to Lemma 8, we have that the number of nodes in an MIS of \( G \) does not exceed the number representing the maximum packing of balls with radius \( (1/2) r_{\text{min}} \) in \( A^* \). Note that for any two independent nodes \( u \) and \( v \) in \( G \), the two corresponding balls with radius \( (1/2) r_{\text{min}} \) and centers \( u \) and \( v \) are contained in an area \( A^* \) and they are disjoint. In addition, since rhombic dodecahedrons can be used to densely fill a space, we can use a rhombic dodecahedron, which surrounds a ball with radius \( (1/2) r_{\text{min}} \), to replace the ball with radius \( (1/2) r_{\text{min}} \) to fill \( A^* \). It is easily determined that the volume of a rhombic dodecahedron surrounding a ball with radius \( (1/2) r_{\text{min}} \) is \( 4 \sqrt{2} ((1/2) r_{\text{min}})^3 \). At the same time, we note that when an independent node \( w \) is on the surface of a ball with a center node in \( \text{OPT}_d \), the corresponding rhombic dodecahedron (surrounding the ball) may not be completely contained in an area \( A^* \) and the volume of the part of the rhombic dodecahedron outside \( A^* \) may attain \((1/7) (4 \sqrt{2} - (4/3) \pi) ((1/2) r_{\text{min}})^3 \). From the above analysis, we can obtain an upper bound of the size of an MIS for \( G \) as follows:
\[
\alpha(G) \leq \frac{4}{3} \pi R^3 + (\text{opt}_d - 1) \times V_0
\]
\[
\leq \frac{4}{3} \pi R^3 + (\text{opt}_d - 1) \times V_0
\]
\[
\leq \frac{4}{3} \pi R^3 + (\text{opt}_d - 1) \times V_0
\]
\[
\leq \frac{4}{3} \pi R^3 + (\text{opt}_d - 1) \times V_0
\]

Since \( \text{opt}_d \leq \text{opt} \),
\[
\alpha(G) \leq \left( \frac{319}{60} k^3 + \frac{29}{5} k^2 + \frac{29}{20} k \right) \text{opt}_d
\]
\[
\leq \left( \frac{319}{60} k^3 + \frac{29}{5} k^2 + \frac{29}{20} k \right) \text{opt}_d
\]

\( \square \)

**Theorem 2.** In a directed connected ball graph \( G \),
\[
\alpha(G) \leq \left( \frac{319}{60} k^3 + \frac{29}{5} k^2 + \frac{29}{20} k \right) \text{opt}_d
\]
\[
\leq \left( \frac{319}{60} k^3 + \frac{29}{5} k^2 + \frac{29}{20} k \right) \text{opt}_d
\]
4. An SCDAS Computation Algorithm in a Directed Ball Graph

In this section, we introduce an efficient heuristic algorithm to construct an SCDAS for a directed ball graph $G = (V, E)$. The idea of this algorithm is described as follows. We choose a node $s$ with the largest degree in $G$ and construct two node sets $S_s$ and $S_y$ by calling a subroutine called UnidTree such that $S = S_s \cup S_y$ is an SCDAS for $G$, where UnidTree is a subroutine, which can generate a rooted tree for $G$.

Let $G = (V, E)$ be a strongly connected ball graph and $L_v \subseteq G$ be a set of independent nodes in $N^+(x)$ and not in $C$. To obtain a DAS, we first use a greedy method to find a DS $S_d$. Second, we consider another strongly connected ball graph $G' = (V, E)$, where $V = V, E = \{(u, v) \mid (v, u) \in E\}$, and then, we use the same method to find a DS $S_d$ in $G'$. It is obvious that $S_a$ is an absorbing set in $G$. In addition, $S = S_d \cup S_y$ is a DAS of $G$, and the details of the computation for the above DAS $S$ can be found in Algorithm 1 LYZLQL.

Lemma 10. Assume that $L_0$ is obtained after executing UnidTree (Algorithm 2). Decompose $L_0$ into two subsets: $L_1, L_2 \subseteq L_0$, $L_1 \neq \emptyset (i = 1, 2), L_1 \cup L_2 = L_0$, and $L_1 \cap L_2 = \emptyset$. Then, the distances $d_1$ and $d_2$ in $G = (V, E)$ do not exceed two hops.

Proof. Without loss of generality, assume that $L_0 = \{v_0, v_1, \ldots, v_k\}$, where $v_0 = s$ and $v_i (1 \leq i \leq k)$ is the node added to $L_0$ after the $i$th iteration (see lines 4–8). By line 6 and line 7, we have that, for integer $1 \leq i \leq k$, $v_i$ has an incoming node in $W$, which is an outgoing node of some node, say $v_i$, in $\{v_0, v_1, \ldots, v_{i-1}\}$. In other words, there exists one node $v_i$ in $\{v_0, v_1, \ldots, v_{i-1}\}$ such that the distance between $v_i$ and $v_i$ is at most two hops. Since $L_1, L_2 \subseteq L_0$, $L_1 \neq \emptyset (i = 1, 2), L_1 \cup L_2 = L_0$, and $L_1 \cap L_2 = \emptyset$, there exist some integers $1 \leq i \leq k$ such that $v_i \in L_1, \{v_0, v_1, \ldots, v_{i-1}\} \subseteq L_2$ or $v_i \in L_2, \{v_0, v_1, \ldots, v_{i-1}\} \subseteq L_1$. Hence, the distance between $L_1$ and $L_2$ does not exceed two hops.

Lemma 11. After UnidTree (Algorithm 2) is executed, the following conditions hold:

1. $L_0 \subseteq C$
2. For any node $u \in C - \{s\}$, there exists a direct path, say, $P_{su}$, from root node $s$ to $u$ such that all nodes of $P_{su}$ belong to $C$.
3. $|C| \leq 2|L_0| - 1$

Proof. Condition (1) is trivial.

Now, we show that condition (2) is true. Let $C_i$ be the set $C$ after the $i$th iteration and $C_{0i} = \{s\}$, and $R_i = C_i - C_0$; it is obvious that $R_i \cap R_j = \emptyset (i \neq j)$ and $R_i \cap C_0 = \emptyset (i \neq 0)$. Assume that the number of iterations is $n$ (lines 10–13 in Algorithm 2). For any node $u \in C$, if $u \in C_0$, the result is trivial. Assuming that $u \notin C_0$, there exists an integer $i$ such that $u \in R_i$. Next, we show that condition (2) is true by induction on $i (1 \leq i \leq n)$. When $i = 1$, if $u$ is the blue node in $R_1$, then $u \in N^+(s)$, and the result is true. If $u$ is a node in $L_v \cap C$, then $s \rightarrow v \rightarrow u$ is a directed path, which implies that the result is true. Assume that the result is true for $i \leq k, k \leq n - 1$. When $i = k + 1$, if $u$ is the blue node in $R_{k+1}$, since $u \in N^+(C_k)$ and $C_k = C_0 \cup R_1 \cup \cdots \cup R_k$, then there exists a node $w \in L_0$ or $w \in R_1 (1 \leq j \leq k)$ such that $(w, u) \notin E$. By the inductive hypothesis, there exists a directed path $P_{sw}$ from $s$ to $w$. Then, $P_{sw} \cup (w, u)$ is a directed path from $s$ to $u$. If $u$ is in $L_{k+1}$, then by the above argument, there exists a directed path $P_{uw}$ from $u$ to $v$. Hence, $P_{uw} \cup (v, u)$ is a directed path from $s$ to $u$. Hence, when $i = k + 1$, the result is true.

The proof of condition (3): after line 9, $C$ contains only a black node $s$. In lines 10–13, after each iteration, one blue node is added to $C$, and the number of black nodes added to $C$ is $|L_v \cap C|$. By the process of producing black nodes (lines 5–8), we have that $|L_v \cap C| \geq 1$. Therefore, after line 14, the number of black nodes contained in $C$ is greater than or equal to the number of blue nodes contained in $C$. Hence, $|C| \leq 2|L_0| - 1$.

Theorem 3. Assume that $G = (V, E)$ is a directed strongly connected ball graph, which possesses transmission ranges $[r_{min}, r_{max}]$. OPT is an MCDAS of size opt for $G$. Then, the following conditions hold:

(1) The set of nodes $S$ output by the algorithm is a strongly connected dominating and absorbing set
(2) $|S| \leq ((319/15)k^2 + (116/5)k^2 + (29/5)k)^{opt} + (29/3)k^3 + (116/5)k^2 + (87/5)k + (13/15))$

Proof. (1) We first show that the set of nodes $S$ output by the algorithm is a dominating and absorbing set. We claim that $L_v$ produced by UnidTree (Algorithm 2) is a DS for the graph run by unidTree (Algorithm 2). In lines 4–8 of Algorithm 2, it is easily seen that each node in $W$ has at least one incoming neighbor in $L_0$, and after line 8, since $V_1 = \emptyset$, all nodes are in $W$ or $L_0$, which implies that $L_0$ is a DS for the graph run by UnidTree (Algorithm 2). By Lemma 11, we have that $C$ is a dominating set for the graph run by unidTree (Algorithm 2). Hence, $S_a(S_v)$ is a DS for $G = (V, E)$ ($G' = (V', E')$). Furthermore, $S_a$ is an absorbing set for $G = (V, E)$, which implies that $S = S_a \cup S_y$ is a DAS for $G = (V, E)$.

Now, we show that $S$ is strongly connected. We claim that for any node $v \in S$, there are two directed paths: one path is from root node $s$ to node $v$ and the other path is from node $v$ to root node $s$. If $v = s$, this claim is trivial. Assume that $v \neq s$. Since $S = S_a \cup S_y$, $v \in S_a$ or $v \in S_y$, consider the following cases:

Case 1. $v \in S_a \cup S_y$. According to Lemma 11, there exists a directed path $P_{sv} = s \rightarrow \cdots \rightarrow v$ from root node $s$ to node $v$ in $G$, and another directed path $Q_{sv}$ exists from root node $s$ to node $v$ in $G$. We reverse all edges in $Q_{sv}$, and we obtain a directed path, denoted by $Q_{sv}$, from node $v$ to root node $s$ in $G$.

Case 2. $v \in S_y - S_a$. According to Lemma 11, there exists a directed path $P_{sv} = s \rightarrow \cdots \rightarrow v$ from root
(1) **Input:** A strongly connected directed ball graph $G = (V, E)$.
(2) **Output:** An SCDAS $S$.
(3) Choose a root $s \in V$ with the largest degree $N^+(s) ∪ N^-(s)$;
(4) $S_s = \text{UnidTree}(V, E, s)$;
(5) Reverse all directed edges in $E$, form a new edge set $\overline{E} = \{(u, v) | u, v \in V, (v, u) \in E\}$;
(6) $S_u = \text{UnidTree}(V, \overline{E}, s)$;
(7) $S = S_s ∪ S_u$;
(8) Return $S$;

**Algorithm 1: LYZLLQ.**

(1) **Procedure** UnidTree $(V, E, s)$
(2) Set $L_0 = \phi, V_1 = V$;
(3) Color $s$ black;
(4) Set $L_0 = \{s\}, W = N^+(s)$, and $V_1 = V_1 - \{s\} ∪ N^-(s)$;
(5) **While** $V_1 \neq \phi$ **do**
(6) Choose $x \in N^+(W)$ such that $|N^+(x) ∩ V_1|$ is maximized, color $x$ back;
(7) Set $L_0 = L_0 ∪ \{x\}, W = W ∪ (N^+(x) ∩ V_1)$, and $V_1 = V_1 - (\{x\} ∪ N^+(x))$;
(8) **End while**
(9) Set $C = \{s\}, L = L_0 - \{s\}$;
(10) **While** $L \neq \phi$ **do**
(11) Choose a node $v \in N^+(C)$ such that $|L_{v,C}| = \max\{|L_{x,C}| : x \in N^+(C)\}$, color $v$ blue;
(12) Set $C = C ∪ \{v\} ∪ L_{v,C}, L = L - L_{v,C}$;
(13) **End while**
(14) Return $C$;
(15) **End procedure**

**Algorithm 2:** Subroutine UnidTree $(V, E, s)$.

node $s$ to node $v$ in $G$. Since $S_s$ is a dominating set for $G$, there exists a node, $u_i$, in $S_s$ such that $(u, v) \in \overline{E}$, which implies that $(v, u) \in E$. Additionally, according to condition (2) in Lemma 11, there exists a directed path, denoted by $P_{aw}$ from root node $s$ to node $u$ for $G$. We reverse all edges in $P_{aw}$, and we obtain another directed path, denoted by $P_{aw}$, from node $u$ to root node $s$. Then, $(v, u) \cup P_{aw}$ is a directed path from node $v$ to root node $s$.

Case 3. $v \in S_s - S_d$. A similar argument in Case 2 can be used here.

Next, we show that $|S| \leq ((319/15)k^3 + (116/5)k^2 + (29/5)k) + ((29/3)k^3 + (116/5)k^2 + (87/5)k + (13/15))$. We first show that the set of black nodes $L_0$ produced by unidTree (Algorithm 2) is an independent set for the graph run by unidTree (Algorithm 2). As the proof of Lemma 10, assume that $L_0 = \{v_0, v_1, \ldots, v_k\}$, where $v_0 = s$, and $v_i (1 \leq i \leq k)$ is the node added to $L_0$ after the $i$th iteration (see lines 5–7). Since there exist two hops from $v_i$ to $v_{i+1}$, $(v_i, v_{i+1}) \notin E$, which implies that $(v_i, v_{i+1})$ is independent. By lines 5–8, we conclude that, for any two nodes $v_i, v_j \in L_0 (j > i + 2)$, $v_i$ is not an incoming neighbor of $v_j$. Hence, $L_0$ is an independent set for the graph run by unidTree (Algorithm 2). By Lemma 11, it is obtained that $|C| = 2|L_0| > 2(2α(G) - 1)$, where $α(G)$ is the bound of the maximum independent set for the strongly connected directed graph $G = (V, E)$. After line 8 of Algorithm 1, we have that $S = S_s ∪ S_u$ and $s \in S_s ∩ S_u$, which means that $S \leq |S_s| + |S_u| - 1 \leq 2|C| - 1 \leq 4α(G) - 3$. By Theorem 2, we have

$$|S| \leq \left(\frac{319}{15}k^3 + \frac{116}{5}k^2 + \frac{29}{5}k\right) \text{opt} + \left(\frac{29}{3}k^3 + \frac{116}{5}k^2 + \frac{87}{5}k + \frac{13}{15}\right).$$

(15)

### 5. Simulation

In this section, we employ simulations to analyse the performance (in terms of the number of nodes in SCDAS) of our algorithm, denoted by LYZLLQ, by making a comparison with the work in [18], which has been used in [28] to compute SCDAS and the work in [29]. We call them EMP and CDS-BFS from now on. More specifically, we investigate how the network density and the ratio of the transmission range impact the performance of each scheme in terms of the size of the SCDAS. First, let us introduce the procedure for a candidate of a strongly connected network for simulation. Let a given number of nodes be randomly distributed in an area with a given size, and we randomly select each node’s transmission range according to a given range of transmission ranges $[r_{min}, r_{max}]$. After such a network has been generated, it is necessary to determine if it is strongly connected. If this network is not strongly
connected, it will be abandoned; otherwise, it is considered a candidate for the simulations. Repeating this procedure, 1000 network candidates can be obtained. For each candidate network, we compute the performance of LYZLLQ, EMP, and CDS-BFS, respectively, and take the average value as the result of the simulations. Figures 6 and 7 are network candidates generated by the above method, which has 20 nodes distributed in a spatial area of size $1000 \times 1000 \times 1000$ m, where Figure 6 shows the connected relationship of nodes in the network and Figure 7 shows the positions of the 20 nodes in the network.

5.1. Impact of Network Density When Varying the Number of Nodes. Suppose that the size of the fixed three-dimensional virtual space is $1000 \times 1000 \times 1000$ m and that the range of network transmission ranges is $[r_{\text{min}}, r_{\text{max}}]$, where $r_{\text{min}} = 200$ m and $r_{\text{max}} = 600$ m. We randomly deploy nodes in such a fixed virtual space. Assume that the number of nodes $n$ changes from 10 to 150, with an increment of 10. Each node chooses a transmission range in $[r_{\text{min}}, r_{\text{max}}]$. For each $n$, we investigate 1000 network candidates and take the average value as the result of the simulations. Figures 6 and 7 are network candidates generated by the above method, which has 20 nodes distributed in a spatial area of size $1000 \times 1000 \times 1000$ m, where Figure 6 shows the connected relationship of nodes in the network and Figure 7 shows the positions of the 20 nodes in the network.

Figure 6: A 20-node strongly connected network with different transmission ranges.

Figure 7: Illustration of the positions of nodes in a 20-node strongly connected network.

Figure 8: Impact of the number of nodes on the performance of the SCDAS.

of nodes $n$ changes from 10 to 150, with an increment of 10. Each node chooses a transmission range in $[r_{\text{min}}, r_{\text{max}}]$. For each $n$, we investigate 1000 network candidates and take the average value as the result of the simulations.

Figure 8 shows the results of the simulations with respect to the performance of LYZLLQ, EMP, and CDS-BFS as the number of nodes increases. In Figure 8, it is easily seen that, as the size of the network increases, the number of nodes in the SCDAS increases for LYZLLQ, EMP, and CDS-BFS, respectively. More specifically, for LYZLLQ, when the size of the network $n$ changes in the range $[10, 40]$, the number of nodes in SCDAS chosen by LYZLLQ is more than 50% of the total number of nodes $n$ in the network. When $n = 80, 110, \text{ and } 130$, the number of nodes in SCDAS chosen by LYZLLQ is approximately
39%, 32%, and 29% of $n$. For EMP, when the size of the network $n$ changes in the range $[10, 40]$, the number of nodes in SCDAS chosen by EMP is also more than 50% of the total number of nodes $n$ in the network. When $n = 80, 110, \text{ and } 130$, the number of nodes in SCDAS chosen by EMP is approximately 64%, 62%, and 60% of $n$. For CDS-BFS, when the size of the network $n$ changes in the range $[10, 40]$, the number of nodes in SCDAS chosen by CDS-BFS is also more than 50% of the total number of nodes $n$ in the network. When $n = 80, 110, \text{ and } 130$, the number of nodes in SCDAS chosen by LYZLLQ is approximately 43%, 36%, and 33% of $n$.

Naturally, the sparser the network is, the larger the number of nodes contained by an SCDAS of the network, and the denser a network is, the smaller the number of nodes contained by an SCDAS of the network. Figure 8 shows that when $n$, the size of the network, changes in $[10, 17]$, the size of the SCDAS in EMP [18] is lightly less than that in LYZLLQ (our algorithm) and the size of the SCDAS in LYZLLQ is lightly less than that in CDS-BFS [29]. However, when $n$ is more than 20, the size of the SCDAS in EMP is larger than that in CDS-BFS and the size of the SCDAS in CDS-BFS is larger than that in LYZLLQ (our algorithm) and the difference of them increases as $n$ increases. This shows that the performance of our algorithm in terms of the size of SCDAS is better than that of [18, 28] and that of [29] in a given set of nodes $n (n \geq 20)$.

5.2. Impact of Network Density When Varying the Spatial Size of the Network. In the previous section, we evaluate the impact of the network density when varying the number of nodes on the performance of the algorithms. In this section, we consider the impact of the network density when the spatial size of the network is varied. In the experiment, let us fix the number of nodes in a network to be 100, and we change the size of the network from $600 \times 600 \times 600$ m to $1300 \times 1300 \times 1300$ m with an edge-length increment of 100. In addition, we randomly arranged 100 nodes in the area with size varying as described above, and we randomly set up a value for the transmission range in the range of $[200\text{, }600\text{ m}]$ for each node. For each case (in terms of the area), we generated 1000 candidates of the network according to the method described in the start of this section, and then, we ran the simulations for each network candidate and averaged the results of 1000 network candidates.

Figure 9 shows the size of the SCDAS output by the algorithms under the impact of network density when varying the area with a fixed number of nodes. As the area of the network increases, the size of the SCDAS increases almost linearly. More specifically, for LYZLLQ, when the edge length of the area $l = 600\text{ m}, 700\text{ m}, \text{ and } 800\text{ m},$ SCDAS contains approximately 13, 18, and 23 nodes (the number of nodes in each candidate network is always 100), and when the edge length of the area is $l = 1100\text{ m}, 1200\text{ m}, \text{ and } 1300\text{ m},$ SCDAS contains approximately 38, 43, and 47 nodes, respectively. For EMP, when the edge length of the area $l = 600\text{ m}, 700\text{ m}, \text{ and } 800\text{ m},$ SCDAS contains approximately 27, 38, and 48 nodes, and when the edge length of the area is $l = 1100\text{ m}, 1200\text{ m}, \text{ and } 1300\text{ m},$ SCDAS contains approximately 67, 71, and 72 nodes, respectively. Now, let us consider the situation of CDS-BFS, when the edge length of the area $l = 600\text{ m}, 700\text{ m}, \text{ and } 800\text{ m},$ SCDAS contains approximately 17, 22, and 27 nodes, and when the edge length of the area is $l = 1100\text{ m}, 1200\text{ m}, \text{ and } 1300\text{ m},$ SCDAS contains approximately 43, 48, and 52 nodes, respectively.

Intuitively, this follows the law mentioned in the previous section: the sparser a network is, the greater the number of nodes contained in an SCDAS of the network. In the case, the size of SCDAS output by our algorithm is at least 14 less than that output by the relative algorithm in [18] or [28] and is at least 4 less than that output by the algorithm in [29], which implies that the performance of our algorithm is better than that of both the algorithms in [18, 28, 29].

5.3. Impact of Varying the Transmission Range Ratio. In this section, to change the ratio of the transmission range $k = r_{\text{max}}/r_{\text{min}}$, we fixed $r_{\text{min}} = 200\text{ m}$ and changed $r_{\text{max}}$ from 200 m to 800 m with an increment of 100. In this experiment, we deployed 100 nodes in a fixed area of size $1000 \times 1000 \text{ m} \times 1000 \text{ m}$. We randomly chose a transmission range in $[r_{\text{min}}, r_{\text{max}}]$ for each node. For each case (in terms of the ratio of transmission ranges $k = r_{\text{max}}/r_{\text{min}}$), we generated 1000 network candidates according to the method described in the first part of this section. Then, we ran the simulations for each network candidate and averaged the results of 1000 network candidates.

Figure 10 shows the change in the number of nodes in SCDAS as the ratio of the transmission range $k$ varies. As the ratio of the transmission range $k$ increases, the number of nodes in the SCDAS decreases, and as $k$ increases with step length 0.5 from $k = 3$, the decrement of the number of nodes
in the SCDAS becomes small. More specifically, for LYZLLQ, when \( k = 2 \) and 2.5, the number of nodes in SCDAS is approximately 48 and 39, respectively, and when \( k = 3.5 \) and 4, the number of nodes in SCDAS is 29 and 25, respectively. For EMP, when \( k = 2 \) and 2.5, the number of nodes in SCDAS is approximately 74 and 69, respectively, and when \( k = 3.5 \) and 4, the number of nodes in SCDAS is 55 and 48, respectively. Similarly, we can find that for CDS-BFS, when \( k = 2 \) and 2.5, the number of nodes in SCDAS is approximately 52 and 44, respectively, and when \( k = 3.5 \) and 4, the number of nodes in SCDAS is 34 and 32, respectively. Intuitively, the larger the \( k \) is, the larger the maximum transmission range \( r_{\text{max}} \). Then, it is possible that the transmission range of the randomly chosen node will be larger, which results in fewer nodes in the SCDAS. In the case, the size of SCDAS output by our algorithm is at least 23 less than that output by the relative algorithm in [18] or [28] and at least 4 less than that output by the relative algorithm in [29]. This can also explain that the performance of our algorithm is better than that of the algorithm, which is better than that of the algorithm in [29] and [18, 28].

6. Conclusion

In this paper, we mainly study the problem of constructing MSCDASs in a directed strongly connected ball graph with different transmission ranges for its nodes, which is NP-hard for obtaining an optimal solution. To obtain a constant factor approximation solution for MSCDASs in a strongly directed connected ball graph, we proposed an algorithm that produces an SCDAS by computing a dominating set and an absorbing set. We proved that the dominating set and absorbing set are independent sets. To obtain the ratio of the SCDAS to MSCDAS, we first proved that the upper bound of the number of nodes in MIS in a directed strongly connected ball graph is \(((319/15)k^3 + (116/5)k^2 + (29/5)k)\text{opt} + ((29/3)k^3 + (116/5)k^2 + (87/5)k + (13/15))\). Using the upper bound, we derived that the size of the SCDAS generated by the algorithm proposed in the paper does not exceed \(((319/15)k^3 + (116/5)k^2 + (29/5)k)\text{opt} + ((29/3)k^3 + (116/5)k^2 + (87/5)k + (13/15))\).

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported in part by the Natural Science Foundation of China under grant nos. 61862003 and 61761006 and in part by the Natural Science Foundation of the Guangxi Zhuang Autonomous Region of China under grant no. 2018GXNSFDA281052.

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