Torque scaling in Taylor-Couette-Flow — an experimental investigation

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Abstract. Within this work we investigate experimentally the turbulent flow between two independently rotating concentric cylinders with radius ratio \( \eta = 0.5 \). The angular velocity of the fluid \( \omega \) across the gap is measured by Particle Image Velocimetry and Laser Doppler Anemometry for different rotation rates of the inner and outer cylinder. The aim is to analyse the angular momentum transport \( J \omega \). In relation to this we measure, in addition to the optical methods, the torque effecting at the inner cylinder. For the case that only the outer cylinder rotates and the inner cylinder is at rest, we show, that for this linear stable flow, the torque is comparatively high and its dependency a non-trivial problem.

1. Introduction

The flow between two concentric cylinders of radii \( R_1, R_2 \) and length \( L \), which are rotating independently with the angular velocities \( \Omega_1 \) and \( \Omega_2 \) is called Taylor-Couette-flow (TC). Important parameters of this geometry are the radius ratio \( \eta = R_1/R_2 \), the aspect ratio \( \Gamma = L/(R_2 - R_1) = L/d \) and the ratio of rotation rates \( \mu = \Omega_2/\Omega_1 \). To indicate the flow in the gap, one can derive two Reynolds numbers related to the inner and outer rotation , \( Re_{1,2} = R_{1,2}\Omega_{1,2}d/\nu \), with the kinematic viscosity \( \nu \) of the fluid. The flow resulting from relatively slow rotating rates were investigated by Couette (1890) and Taylor (1923) in the early 20th Century. In 1986 C. D. Andereck (1986) did a systematically overview on TC-flow for \( Re_1 < 2250 \) and \(-4000 < Re_2 < 1200 \) at radius ratio \( \eta = 0.88 \). In newer decades B. Eckhardt (2007b,a) derived a theory on the problem of TC-flow. This theory describes an angular momentum transport, which is constant over all radii with

\[
J_\omega = r^3 \left( < u_r \omega >_{A,t} - \nu \partial_r < \omega >_{A,t} \right),
\]

where the brackets \(< >_{A,t}\) describe a mean over a cylindrical surface at radius \( r \) and over time. At the inner boundary \((r = R_1)\), the first part of this equation equals zero \((u_r = 0)\). The second term corresponds to a linear function of the torque \( T \), which the fluid effects at the boundary. Hence, measuring the dimensionless torque \( G = T/(2\pi L \rho \nu^2) \) at the rotating inner cylinder will deliver a quantity of the angular momentum flux by \( J_\omega = \nu^2 G \). This torque has been measured in the past and could be scaled with the rotation Reynolds number with an exponential function. Wendt (1933) measured the torque at the rotating inner cylinder for three different radius ratios \( \eta = 0.68, 0.85, 0.935 \) and Reynolds numbers \( Re_{1,2} = 10^4 - 10^5 \). Wendt found a scaling of the
dimensionless torque $G = Re^\alpha$ with $\alpha = 1.5$ to 1.7. After this work, there were no torque measurements on Taylor-Couette apparatus until 1992. Then, D. P. Lathrop (1992) measured, at a TC apparatus with radius ratio $\eta = 0.724$ for Reynolds number range of $10^3$ to $10^6$, a scaling of the torque with $\alpha = 1.8$, if the apparatus rotates beyond a critical Reynolds number of $1.3 \times 10^8$. G.S. Lewis (1999) repeated these measurements with higher accuracy and verified the results. Today, there are several experiments investigating the torque scaling for co- and counter rotating Taylor-Couette systems (M.S. Paoletti, 2011; G.P.M. van Gils, 2011).

2. Experimental Apparatus

In our initial investigations we use the Taylor-Couette apparatus as illustrated in Fig. 1. The outer cylinder is made of high precision duran glas with an inner radius of $R_2 = 70 \pm 0.02 \text{mm}$. The inner cylinder is made of black anodised aluminium with radius $R_1 = 35 \text{mm}$, finally resulting into a radius ratio $\eta = 0.5$. The end plates are adjustable up to a length of $735 \text{mm}$. In the present experiments we choose this to be fixed at $700 \text{mm}$, resulting in an aspect ratio of $\Gamma = 20$. The inner cylinder is able to rotate up to a rotation rate of $n_1 = 80 \text{Hz}$, the outer one up to $n_2 = 40 \text{Hz}$. Additional the both end plates are rotatable independently with the same speed as the cylinders or inbetween. As working fluid, we use Wacker silicone oil AK065 with a kinematic viscosity of $\nu = 0.65 \times 10^{-6} \text{m}^2\text{s}^{-1}$. Therewith we reach Reynolds numbers of the inner and outer rotation of $Re_{1,2} = 9.5 \times 10^6$. The temperature of the experiment is controlled through a water bath surrounding the outer cylinder. This allows to control the heat, which is produced by the bearings, sealings and inside the experiment. The inner cylinder is subdivided into three parts of lengths $99.4 \text{mm}, 500 \text{mm}$ and $99.4 \text{mm}$ with a gap of $0.6 \text{mm}$ separating the parts (see Fig. 1 right). The both end parts are directly connected to the axial driving shaft. The middle part is mounted rotatable against the shaft by two low friction bearings and attached additional to the shaft by two strain gauges. If the inner cylinder is driven by the $1.1 \text{kW}$ motor, the inner part can distort slightly versus the shaft. The torque working on the middle part of the inner cylinder is quantifiable by the strain gauges. The objective of the separation is to reduce the end effects of the solid end plates onto the torque measurement as in previous experiments (D. P. Lathrop, 1992; G.S. Lewis, 1999; M.S. Paoletti, 2011; G.P.M. van Gils, 2011). Slip rings transfer the signal out of the rotating cylinder into the converter WZSG of the company Wachendorff sampling at a rate of 100 Hertz. This torque measurement allows us to quantify the angular momentum transport $J_\omega$ of equation (1) at the inner boundary. Fluid flow measurements by means of Particle Image Velocimetry (PIV) and Laser Doppler Anometry (LDA) are done through the transparent water bath tank and outer cylinder.

In addition to this experiment we built up another Taylor-Couette experiment (see Fig. 2) with the same geometry as the first. Here the end plates are transparent, but attached to the outer cylinder. This gives us the possibility to measure the azimuthal and radial velocity simultaneously from above without the light refraction at the cylindrical surface. The Reynolds numbers of this experiment are in the size of $10^4$, due to the gearings and driving motors.

3. Angular velocity profiles

Inside the second described Taylor-Couette system with transparent end plate, we applied a PIV measurement. The lightsheet illuminates an azimuthal-radial plane in the mid-height of the experiment, we observe the plane through the upper end. The field of view is about more than one fifth of the whole gap (see Fig. 3). The measured two dimensional, time averaged cartesian flow field is interpolated into a cylindrical coordinate system. The shown profiles of the angular velocities $\omega$ (Fig. 3, 4) are averaged over $2\pi/5$ for different radii and represented in comparison to the analytic laminar Couette profile ($\omega(r) = Ar + B/r$, where A and B are geometric factors). The flow in all these cases is turbulent. Let the inner cylinder be at rest, the
lowest Reynolds numbers of the outer cylinder \((Re_2 = 7700)\) shows the same angular velocity profile as the theoretical laminar case. If the outer cylinder is rotating at higher Reynolds numbers the experimental profile obviously differs from the laminar Couette profile, even this is a linear stable flow. For the case of inner rotation one expects a big derivation between the real flow and the Couette profile (see Fig. 4c). However, for the case of a co-rotation near the Rayleigh criterion \((\eta = \mu^2)\), the experimentally determined profiles again match with the Couette flow. A further investigation on that apparatus is going to observe the onset of instabilities by crossing the Rayleigh line.

For the cases of counter rotation (see Fig. 4f) where \(\mu = -0.5\) the angular velocity profiles show a flat radius dependency in the bulk flow and a strong gradient at the boundaries. The boundary layer is not well resolved in this case. The big distinction to the analytic Couette profile shows, that the transport of angular momentum across the gap is increased. In contrast to the flat bulk flow profile for \(\mu = -0.5\), we observe a bigger influence of the outer boundary for \(\mu = -0.8\) (Fig. 4e). At half of the gap width the impact of the gradient at the boundary is strong.

Profiles of the angular velocity are also measured inside the other experiment with transparency only in radial direction (Fig. 1). A LDA probe is traversed over the whole radius in mid-height of the experiment for several rotation rates of the inner cylinder. The temporal average of the azimuthal velocity profiles is shown in Fig. 5a. The boundary layer at the outer cylinder is resolved in this measurement with several positions, while the layer at the inner boundary is too thin to resolve here. The angular velocity \(\omega = u_\varphi/r\) is non-dimensionalized by the rotation speed of the inner cylinder \(\Omega_1\). Hence, the dimensionless angular velocity profiles show an identical behavior over the entire range of \(Re_1 = 9000\) up to \(61000\) (Fig. 5b). In comparison to the profiles measured with PIV (Fig. 4c), this is in good agreement. Further experiments are going to investigate the behavior of angular velocity for even smaller as well as much bigger Reynolds number of the inner rotation and the influence of the outer rotation on the angular velocity.

4. Verification of Torque measurements

With our Taylor-Couette apparatus we measure the torque acting on the inner cylinder. This is of great interest for our present investigations to quantify the angular momentum transport (Eq. (1)). A precise measured scaling of the torque with flow parameters \((Re_1, Re_2, \eta, \Gamma)\) is connected to the scaling of this flux. Hence, we also analyse the torque at the inner cylinder for \(Re_1 = 0, Re_2 \neq 0\). The scaling of the dimensionless torque of 25800 < \(Re_2\) < 75300 is given in Fig. 6(top). Analytically seen, the Couette profile gives a dimensionless torque of \(G_{lam} = 2\nu^{-1}B\) resulting in \(G_{lam}(Re_2) \approx 3 \times 10^4\) up to \(10^5\). Obviously the measured torque is surprisingly high, even based on the check, that the system does not superimpose a technical error. The reasons for this big difference will be investigated. As we showed before, the flow is not described very well by the Couette profil for \(Re_1 = 0, Re_2 = 1.5 \times 10^4\) and higher (Fig. 4b). The energy dissipation rate of the turbulent wind (B. Eckhardt, 2007b) is also calculated from the torque at the inner cylinder by \(\varepsilon_w = 2\nu^2(\Omega_1 - \Omega_2)(G - G_{lam})/(R_2^2 - R_1^2)\). We obtain unexpected steps in the Reynolds number dependency at \(Re_2 \sim 35700\) and \(\sim 50000\). The process behind this behavior requires further investigations.

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Figure 1. Left: Drawing of the Taylor-Couette system, \( R_1 = 35\text{mm}, R_2 = 70\text{mm}, \eta = 0.5, \Gamma = 20 \) with surrounding water bath. Right: The inner cylinder seperated into three sections. The middle section measures the torque. The upper and lower sections are beared inside the end plates as shown in the left.

Figure 2. Bottom: Drawing of the transparent Taylor-Couette system, \( R_1 = 35\text{mm}, R_2 = 70\text{mm}, \eta = 0.5, \Gamma = 20 \). Top: view through the transparent upper plate into the measurement volume.
Figure 3. Left: Results from PIV measurements inside the transparent TC for Reynolds-numbers \( Re_1 = 60000, Re_2 = 31000 \). The light sheet is adapted in azimuthal-radial plane. The field of view is about \( 2\pi/5 \). The shown measurement is time-averaged over 6.33s. Right: Radial angular velocity profile out of the PIV measurement averaged over \( 2\pi/5 \).

Figure 4. Angular velocity profiles for different pairs of inner and outer rotation Reynolds numbers. The solid line shows the theoretical profile of laminar Couette flow \( \omega(r) = Ar + B/r \), with \( A \) and \( B \) geometrical factors, where the crosses indicate the measured angular velocities \( \omega = u_\phi/r \). Negative Reynolds numbers indicate clockwise rotation of the corresponding cylinder. a,b) Inner cylinder at rest while outer cylinder rotates with different rotation rates (linear stable flow); c) Inner cylinder rotation only (linear unstable flow); d) Inner and outer cylinder co-rotate at the Rayleigh-criterion \( (\eta = \mu^2) \); e+f) Counter rotation cases with different inner rotation number (linear unstable flow).
Figure 5. a) Time averaged azimuthal velocity in dependency on the dimensionless gap width for different Reynolds numbers of the inner cylinder rotation taken by LDA measurement. b) Dimensionless time averaged angular velocity for different Reynolds numbers of the inner cylinder rotation. The values are made dimensionless by the specific angular velocity of the cylinder.

Figure 6. Dimensionless torque $G$ and energy dissipation rate for increasing outer rotation Reynolds number while inner cylinder is at rest.