Oscillations of Pseudo-Dirac Neutrinos and the Solar Neutrino Problem

C. Giunti

Istituto Nazionale di Fisica Nucleare
Sezione di Torino
I-10125 Torino, Italy

C.W. Kim and U.W. Lee

Department of Physics and Astronomy
The Johns Hopkins University
Baltimore, Maryland 21218, USA

Abstract

The oscillations of pseudo-Dirac neutrinos in matter are discussed and applied to the solar neutrino problem. Several scenarios such as both $\nu_e$ and $\nu_\mu$ being pseudo-Dirac and only $\nu_e$ or $\nu_\mu$ being pseudo-Dirac are examined. It is shown that the allowed region in the mass-mixing angle parameter space obtained by comparing the solar neutrino data with the calculations based on the standard solar model and the MSW effect is not unique. The results depend on the nature of neutrinos; for example, if both $\nu_e$ and $\nu_\mu$ are pseudo-Dirac, the allowed region determined by the current solar neutrino data does not overlap with that obtained in the usual case of pure Dirac or Majorana neutrinos.
The observation of the solar neutrinos, combined with the standard solar model and calculations of the MSW effect can provide important clues to understanding the basic properties of neutrinos. Current data from Homestake, Kamioka and SAGE have already narrowed down considerably the allowed region in the mass-mixing angle parameter space.

Recently there has been a proposal to explain the solar neutrino problem by using the MSW effect with only one generation of pseudo-Dirac electron neutrinos with a large transition magnetic moment. Majorana neutrinos emerge naturally in most extensions of the standard model, but some models (see, for example, Ref. ) yield pseudo-Dirac neutrinos, which were used, among others, to explain the solar neutrino puzzle.

In this paper, we generalize the one generation picture to the case of two generations and examine the consequences. In particular, it is shown that in the pseudo-Dirac neutrino case the current data yield entirely different (non-overlapping) allowed regions in the mass-mixing angle parameter space from those in the standard Dirac or Majorana neutrino cases.

The Dirac-Majorana mass matrix for the neutrino states \( \nu^e_L \) and \( \nu^e_R \) is given by

\[
\begin{pmatrix}
\nu^e_L \\
\nu^e_R \\
\end{pmatrix} =
\begin{pmatrix}
\nu^e_L \\
\nu^e_R \\
\end{pmatrix} =
\begin{pmatrix}
m^e_L & M^e_D \\
M^e_D & m^e_R \\
\end{pmatrix}
\]

where \( M^e_D \) and \( m^e_{L,R} \) are Dirac and Majorana masses, respectively. The assumption that \( m^e_L + m^e_R = 0 \) leads to pseudo-Dirac neutrinos generated by a mechanism similar to that originally discussed by Wolfenstein. In the following we make the assumption that \( M^e_D \gg |m^e_{L,R}| \) which also leads to pseudo-Dirac neutrinos, i.e. two almost degenerate (in mass) left-handed neutrino states \( \nu^e_1 \) and \( \nu^e_2 \) (with masses \( m^e_1 \sim m^e_2 \sim M^e_D \)) which are expressed as

\[
\begin{align*}
\nu^e_1 &= i \cos \theta_e \nu^e_L - i \sin \theta_e \nu^e_R \\
\nu^e_2 &= \sin \theta_e \nu^e_L + \cos \theta_e \nu^e_R
\end{align*}
\]

where the factor \( i \) guarantees the positivity of the mass eigenvalues. The mixing angle \( \theta_e \) is given by

\[
\tan(2\theta_e) = \frac{2M^e_D}{(m^e_R - m^e_L)}
\]

In our case \( \theta_e \sim 45^\circ \) because \( M^e_D \gg m^e_{L,R} \). Therefore, our pseudo-Dirac neutrinos are special ones, with almost 45\(^\circ\) mixing angle. In general, however, pseudo-Dirac neutrinos can have any mixing angle and whenever pseudo-Dirac neutrinos are generated with one generation one has to introduce sterile neutrinos.
In the following, we assume that $10^{-11} \text{eV}^2 \lesssim \Delta m^2_{\nu} \equiv (m^2_2 - m^2_1) \lesssim 10^{-7} \text{eV}^2$ where the upper limit comes from a cosmological constraint \cite{10} on the oscillation into sterile neutrinos and the lower limit is necessary in order to have a vacuum oscillation length much shorter than the sun-earth distance. In this case, during the sun-earth propagation, one half of the initial flux of $\nu^e_L$ will be depleted due to the maximal $(45^\circ)$ mixing oscillations between $\nu^e_L$ and $\nu^e_R$ when the time average is taken. Therefore, the ratio of the $\nu^e_L$ flux at the earth and the initial flux is one half for the SAGE (S), Homestake (H) and Kamioka (K) experiments:

$$S = H = K = \frac{1}{2}.$$ (4)

If neutrinos have magnetic moments large enough to induce a spin-flip during their propagation in the magnetic field of the sun \cite{11}, the ratios of the neutrino fluxes are

$$\nu^e_L : \overline{\nu}^e_L : \nu^e_R : \overline{\nu}^e_R = \alpha : \frac{1 - 2\alpha}{2} : \frac{1 - 2\alpha}{2} : \alpha$$ (5)

with $0 \leq \alpha \leq 0.5$. Since $\alpha$ is related to the spin-flip probability $P_{sf}$ as $\alpha = (1 - P_{sf})/2$, any deviation from $\alpha = 0.5$ is an indication that spin-flips actually took place. The detection rates are then

$$S = H = \alpha \quad , \quad K = \alpha + 0.42 \frac{1 - 2\alpha}{2}$$ (6)

where we have used $\sigma(\nu^e_Le^-) \simeq 0.42\sigma(\nu^\mu_Le^-)$. Assuming that the standard solar model \cite{1} gives the correct $\nu^e_L$ flux produced in the core of the sun, the ratios of the observed fluxes and the initial flux are $H_{\text{exp}} = 0.27 \pm 0.04$ \cite{4}, $K_{\text{exp}} = 0.46 \pm 0.08$ \cite{4} and $S_{\text{exp}} = 0.15 \pm 0.27$ \cite{4}. The range of the parameter $\alpha$ for which Eq.(6) explains the observed ratios is $0.29 \lesssim \alpha \lesssim 0.31$.

Now we generalize the above one generation picture to the case of two generations and study its consequences. We assume that, before the mixing between the electron and muon sectors, the muon neutrinos are also pseudo-Dirac particles, i.e. $M_{\mu} \gg |m^\mu_{L,R}|$ so that $\theta_{\mu} \sim 45^\circ$. The two almost degenerate ($m^\mu_1 \sim m^\mu_2 \sim M_{\mu}$) mass eigenstate muon-neutrino states $\nu^\mu_1$ and $\nu^\mu_2$ are given by

$$\nu^\mu_1 = i \cos \theta_{\mu} \nu^e_L - i \sin \theta_{\mu} \overline{\nu}^e_R$$

$$\nu^\mu_2 = \sin \theta_{\mu} \nu^e_L + \cos \theta_{\mu} \overline{\nu}^e_R$$ (7)

In the muon neutrino sector, we assume that $10^{-11} \text{eV}^2 \lesssim \Delta m^2_\mu \equiv (m^2_2 - m^2_1) \lesssim 10^{-2} \text{eV}^2$, where the upper limit is due to the cosmological constraint \cite{11}. However, both
the mixing angles among electron and muon neutrinos and $\Delta m^2_{e\mu} \sim (M^2_\mu - M^2_\nu)^2$ are left unknown. For simplicity, we assume that $\Delta m^2_{e\mu} \gg \Delta m^2_e, \Delta m^2_\mu$ and that the mass eigenstates are given by

$$
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix} = \begin{pmatrix}
c_\theta & 0 & -s_\theta & 0 \\
0 & c_\theta & 0 & -s_\theta \\
s_\theta & 0 & c_\theta & 0 \\
0 & s_\theta & 0 & c_\theta
\end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix}
i & -i & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & i & -i \\
0 & 0 & 1 & 1
\end{pmatrix} \begin{pmatrix}
\nu_e^c \\
\nu_\mu^c
\end{pmatrix}
$$

(8)

where $c_\theta \equiv \cos \theta_{e\mu}$ and $s_\theta \equiv \sin \theta_{e\mu}$. The mixing angle $\theta_{e\mu}$ is practically equivalent to the usual mixing angle between $\nu_e$ and $\nu_\mu$ in the Dirac or Majorana cases. Also, in Eq.(8) the pseudo-Dirac mixing angles $\theta_e$ and $\theta_\mu$ have been approximated to 45°. The mass matrix in the weak basis $M^2_w$ is given by

$$
M^2_w = \begin{pmatrix}
m^2_{12} c_\theta^2 + m^2_{34} s_\theta^2 + A_e & \Delta m^2_{12} c_\theta s_\theta + \Delta m^2_{34} s_\theta & (m^2_{34} - m^2_{12}) c_\theta s_\theta & (\Delta m^2_{34} - \Delta m^2_{12}) c_\theta s_\theta \\
\Delta m^2_{12} c_\theta s_\theta + m^2_{34} s_\theta & m^2_{12} c_\theta^2 + m^2_{34} s_\theta & (\Delta m^2_{34} - \Delta m^2_{12}) c_\theta s_\theta & (m^2_{34} - m^2_{12}) c_\theta s_\theta \\
(m^2_{34} - m^2_{12}) c_\theta s_\theta & (m^2_{34} - m^2_{12}) c_\theta s_\theta & m^2_{12} s_\theta^2 + (m^2_{34} - m^2_{12}) c_\theta s_\theta + A_\mu & \Delta m^2_{12} c_\theta^2 + \Delta m^2_{34} c_\theta^2 \\
(\Delta m^2_{34} - \Delta m^2_{12}) c_\theta s_\theta & (m^2_{34} - m^2_{12}) c_\theta s_\theta & m^2_{12} s_\theta^2 + (m^2_{34} - m^2_{12}) c_\theta s_\theta & m^2_{12} s_\theta^2 + m^2_{34} c_\theta^2
\end{pmatrix}
$$

(9)

with

$$
\begin{align*}
m^2_{12} & = \frac{m^2_1 + m^2_2}{2}, & m^2_{34} & = \frac{m^2_2 + m^2_3}{2} \\
\Delta m^2_{12} & = \frac{m^2_2 - m^2_1}{2}, & \Delta m^2_{34} & = \frac{m^2_1 - m^2_3}{2} \\
A_e & = A_{CC} + A_{NC}, & A_\mu & = A_{NC} \\
A_{CC} & = 2\sqrt{2} G_F E N_e, & A_{NC} & = -\sqrt{2} G_F E N_\mu
\end{align*}
$$

(10)

where $m_1, m_2, m_3$ and $m_4$ are the mass eigenvalues. The values of the effective mass squared in matter are shown in Fig.1 for $\nu_1, \nu_2, \nu_3$ and $\nu_4$ and their antiparticles as functions of the matter density $\rho$. In Fig.1 there are two MSW resonance regions $R1$ and $R2$ and two possible spin-flip resonance regions $R1_m$ and $R2_m$ in addition to the region $R$ discussed in Ref.[3]. In the region $R$ the maximal vacuum oscillations lead to the 1/2 suppression of the $\nu^c_\mu$ flux, as discussed above.

First we discuss the case in which neutrinos have large enough magnetic moments to induce resonant spin-flips and the resonance regions $R1_m$ and $R2_m$ are in the convection
zone. In this case the right-handed neutrinos $\overline{\nu}_L$, $\overline{\nu}_R$, $\overline{\nu}^\mu_R$ and $\overline{\nu}^\alpha_R$ can be generated from the original $\nu_\ell^e$ by the resonant spin-flip processes directly or indirectly, e.g. via $\nu_\ell^e \to \overline{\nu}_R^\mu$, $\nu_\ell^e \to \overline{\nu}_L^e$, $\nu_\ell^e \to \nu_\mu^e \to \overline{\nu}_L^e$.

Regardless of whether the resonance spin-flip processes are adiabatic or not, the flux ratios at the earth are expressed as

$$\frac{\nu^e_L : \overline{\nu}^e_L : \nu^e_R : \overline{\nu}^e_R : \nu^\mu_L : \overline{\nu}^\mu_L : \nu^\mu_R : \overline{\nu}^\mu_R} = a\alpha : a\frac{1 - 2\alpha}{2} : a\alpha : b\beta : b\frac{1 - 2\beta}{2} : b\frac{1 - 2\beta}{2} : b\beta$$

with $a + b = 1$ and $0 \leq \alpha, \beta \leq 0.5$. The detection rates are then

$$S = H = a\alpha$$

(12)

where we have used $\sigma(\nu^\mu_R e^-) \simeq \frac{1}{6}\sigma(\nu^\mu_L e^-)$, and $\sigma(\overline{\nu}^e_R e^-) \simeq \frac{1}{6}\sigma(\overline{\nu}^e_L e^-)$ instead of $\sigma(\overline{\nu}^e_R e^-) \simeq \frac{1}{7}\sigma(\nu^e_L e^-)$ for simplicity. This approximation makes the second equation in Eq.(12) free of the parameter $\beta$. The results in Eq.(12) are consistent with $S_{\text{exp}}$, $H_{\text{exp}}$ and $K_{\text{exp}}$ for the parameter ranges $a \gtrsim 0.93$ and $0.29 \lesssim \alpha \lesssim 0.31$ within $1\sigma$. The fact that $a$ must be very close to unity implies that the Landau-Zener transitions at the resonances $R1$ and $R2$ are extremely non-adiabatic in this model. Since the deviation from $\alpha = 0.5$ indicates the presence of spin-flip, the above range of $\alpha$ requires a large transition magnetic moment between $\nu^e_L$ and $\nu^e_R$ (e.g. $\sim 10^{-10} \mu_B$ for $B \sim 10$ KG in the convection zone). Although there exist many models that can yield such a large magnetic moment, they appear somewhat unnatural and thus we do not consider this scenario further.

In the absence of magnetic moments, no right-handed neutrinos are produced as $\nu^e_L$ ($\simeq \nu_\ell$ in the core) propagates outward from the core. At the resonance $R2$, $\nu_\ell$ is split into $\nu_\ell$ and $\nu_3$ with fractions $(1 - P_{R2})$ and $P_{R2}$, respectively, where $P_{R2}$ is the Landau-Zener transition probability at the resonance $R2$. At the resonance $R1$, the fraction $P_{R2}$ of $\nu_3$ is further split into $\nu_3$ and $\nu_2$ with fractions $(1 - P_{R1})P_{R2}$ and $P_{R1}P_{R2}$, respectively, $P_{R1}$ being the Landau-Zener transition probability at the resonance $R1$.

Let us assume that the two resonance regions do not overlap so that the two resonances can be treated separately. Then one can estimate the respective Landau-Zener transition probabilities as follows.

(1) **First resonance** (R1). This is a resonance between $\nu^e_L$ and $\nu^\mu_R$ and occurs when the first and third diagonal elements of $M^2_\ell$ are equal, i.e. for $A_{CC} = (m^2_{34} - m^2_{12})c_{2\theta}$. In the neighborhood of the resonance the MSW evolution equation is dominated by the $\nu^e_L - \nu^\mu_L$ 2 $\times$ 2 sector:

$$\begin{pmatrix}
    m^2_{12}s^2_\theta + m^2_{34}s^2_\theta + A_e & (m^2_{34} - m^2_{12})c_\theta s_\theta \\
    (m^2_{34} - m^2_{12})c_\theta s_\theta & m^2_{12}s^2_\theta + m^2_{34}c^2_\theta + A_\mu
\end{pmatrix}.$$
The Landau-Zener transition probability at the resonance is given by

\[
P_{R1} = \exp \left[ -\frac{\pi}{4h_{R1}} \frac{s_{2\theta}^2}{c_{2\theta}} \frac{m_{34}^2 - m_{12}^2}{E} \right].
\]

with \( h_{R1} \equiv \frac{1}{\rho} \frac{\partial \rho}{\partial x} \bigg|_{R1} \). The mass factor in the exponent, \((m_{34}^2 - m_{12}^2)\), is equivalent to the usual \(\Delta m_{\nu_{\mu}}^2\).

(2) Second resonance \((R2)\). This is a resonance between \(\nu_e^L\) and \(\nu_{\mu}^R\) and occurs when the first and fourth diagonal elements of \(M_\nu^2\) are equal, i.e. \(A_e = (m_{34}^2 - m_{12}^2) c_{2\theta}\). In the neighborhood of the resonance the MSW evolution equation is dominated by the \(\nu_e^L - \nu_{\mu}^R\) 2 \(\times\) 2 sector:

\[
M_\nu^2 = \begin{pmatrix}
    m_{12}^2 c_\theta^2 + m_{34}^2 s_\theta^2 + A_e (\Delta m_{34}^2 - \Delta m_{12}^2) c_\theta s_\theta \\
    (\Delta m_{34}^2 - \Delta m_{12}^2) c_\theta s_\theta & m_{12}^2 s_\theta^2 + m_{34}^2 c_\theta^2
\end{pmatrix}.
\]

The Landau-Zener transition probability at the resonance is given by

\[
P_{R2} = \exp \left[ -\frac{\pi}{4h_{R2}} \frac{s_{2\theta}^2}{c_{2\theta}} \frac{m_{34}^2 - m_{12}^2}{E} \left( \frac{\Delta m_{34}^2 - \Delta m_{12}^2}{m_{34}^2 - m_{12}^2} \right)^2 \right].
\]

with \( h_{R2} \equiv \frac{1}{\rho} \frac{\partial \rho}{\partial x} \bigg|_{R2} \). The ratio of the mass factors in the exponents of Eqs.(14) and (16) is \([\Delta m_{34}^2 - \Delta m_{12}^2] / (m_{34}^2 - m_{12}^2)^2 \sim (m_{34}^2 / m_{12}^2)^2 \sim (M_D^\mu m_{L,R}^\mu / M_D^\mu)^2 \sim (m_{L,R}^\mu / M_D^\mu)^2\). The numerical value of this ratio is supposed to be small for the pseudo-Dirac neutrinos under consideration. Therefore, we take the non-adiabatic approximation for the resonant transition at \(R2\). In order to see the region of validity for this approximation, let us consider the exponent of Eq.(16) which is written as

\[
Q_{R2} \simeq -\frac{\pi \sin^2(2\theta_{\mu e})}{4 \cos(2\theta_{\mu e})} \frac{R_\odot}{10.45} \frac{\Delta m_{\mu e}^2}{E} \left( \frac{m_{L,R}^\mu}{M_D^\mu} \right)^2 \\
\simeq -2.6 \times 10^3 \frac{\sin^2(2\theta_{\mu e})}{\cos(2\theta_{\mu e})} \frac{\Delta m_{\mu e}^2}{eV^2}
\]

where we have used \(E = 10\ \text{MeV}\) and for definiteness \(m_{L,R}^\mu / M_D^\mu \simeq 0.01\) which corresponds to the mixing angle \(\theta_\mu = 44.86^\circ\). The non-adiabatic region which satisfies \(|Q_{R2}| < 1\) is below the solid line in the upper right-hand corner in Fig.2. It will be shown that the solution of the solar neutrino problem in the pseudo-Dirac neutrino model indeed lies in this region.
Therefore, one has the following ratios of fluxes at the earth

\[ \frac{\nu_e^L}{\nu_e^R} = \frac{1}{2} \left[ P_{\nu_1} \cos^2 \theta_{e\mu} + (1 - P_{\nu_1}) \sin^2 \theta_{e\mu} \right] ; \quad \frac{1}{2} \left[ P_{\nu_1} \sin^2 \theta_{e\mu} + (1 - P_{\nu_1}) \cos^2 \theta_{e\mu} \right] \]  

leading to

\[ S = \mathcal{H} = \frac{1}{2} \left[ \frac{1}{2} - \frac{1}{2} (1 - 2P_{\nu_1}) \cos(2\theta_{e\mu}) \right] , \quad \mathcal{K} = \frac{1}{2} + \frac{5}{12} \left[ \frac{1}{2} - \frac{1}{2} (1 - 2P_{\nu_1}) \cos(2\theta_{e\mu}) \right] \]  

where we have used the fact that, because of the maximal mixings between \( \nu_e^L \leftrightarrow \nu_e^R \) and \( \nu_{\mu}^L \leftrightarrow \nu_{\mu}^R \), only half of the neutrinos can be detected.

Treating \( P_{\nu_1} \) as a constant parameter (neglecting the energy dependence of the transition probability), \( S_{\exp} \), \( H_{\exp} \) and \( K_{\exp} \) are not reproduced by Eq.(19) within 1\( \sigma \) for any values of \( P_{\nu_1} \), but are reproduced within 2\( \sigma \) for

\[ 0.52 \lesssim \frac{1}{2} - \frac{1}{2} (1 - 2P_{\nu_1}) \cos(2\theta_{e\mu}) \lesssim 0.70 \]  

The two-generation pseudo-Dirac neutrinos discussed here are a special case of two-generation sterile neutrinos in which \( \theta_e \) and \( \theta_\mu \) are 45\( ^\circ \). Since the only mixing angle which is relevant in the analysis is \( \theta_{e\mu} \), the above result should be compared with the usual MSW effect with two generations of neutrinos with the same mixing angle. In this case one has

\[ S = \mathcal{H} = \frac{1}{2} - \frac{1}{2} (1 - 2P_{\nu_1}) \cos(2\theta_{e\mu}) , \quad \mathcal{K} = \frac{1}{6} + \frac{5}{6} \left[ \frac{1}{2} - \frac{1}{2} (1 - 2P_{\nu_1}) \cos(2\theta_{e\mu}) \right] \]  

with the Landau-Zener transition probability \( P_{\nu_1} \). Equation (21) can reproduce \( S_{\exp} \), \( H_{\exp} \) and \( K_{\exp} \) within 1\( \sigma \) if 0.26 \( \lesssim \frac{1}{2} - \frac{1}{2} (1 - 2P_{\nu_1}) \cos(2\theta_{e\mu}) \) \( \lesssim 0.31 \), and within 2\( \sigma \) if

\[ 0.19 \lesssim \frac{1}{2} - \frac{1}{2} (1 - 2P_{\nu_1}) \cos(2\theta_{e\mu}) \lesssim 0.35 \]  

There is an important difference between Eqs.(19) and (21): In Eq.(19), there is an additional factor 1/2 due to the pseudo-Dirac nature of neutrinos, i.e. \( \nu_e^L \) and \( \nu_e^R \) oscillate into \( \nu_{\mu}^L \leftrightarrow \nu_{\mu}^R \), respectively, with 45\( ^\circ \) mixing, depleting the active (to detection) neutrinos by one half.

Since \( P_{\nu_1} \) depends on the energy \( E \) and the SAGE, Homestake and Kamioka experiments have different energy thresholds, \( P_{\nu_1} \) can be different for \( S \), \( H \) and \( K \). Using the expression for \( P_{\nu_1} \), we have plotted the \( \Delta m^2_{\nu_2} \sin^2(2\theta_{e\mu})/\cos(2\theta_{e\mu}) \) diagram in Fig.2 for the pseudo-Dirac MSW and the usual MSW effects based on \( S_{\exp} \), \( H_{\exp} \) and \( K_{\exp} \). We have taken \( \langle E \rangle = 2.0 \text{ MeV}, 7.5 \text{ MeV} \) and 10 MeV for \( S \), \( H \) and \( K \) respectively. The
region which satisfies $S_{\exp}$, $H_{\exp}$ and $K_{\exp}$ within $2\sigma$ in the standard two generation MSW effect (with Eq. (21)) is shown as the area inside the dotted lines in Fig. 2. The region which satisfies $S_{\exp}$, $H_{\exp}$ and $K_{\exp}$ within $2\sigma$ in the pseudo-Dirac case (with Eq. (19)) is shown as the shaded area inside the solid lines in Fig. 2. It is important to emphasize here that the two allowed regions in the $\Delta m^2_{e\mu} - \sin^2(2\theta_{e\mu})/\cos(2\theta_{e\mu})$ plot based on the same data, $S_{\exp}$, $H_{\exp}$ and $K_{\exp}$, do not overlap, even within $2\sigma$ errors, i.e. the two generation pseudo-Dirac neutrinos produce different allowed regions from those based on the usual two generation model. As variations of the above pseudo-Dirac neutrino scenario, we consider the following two cases: (I) The electron neutrino is a pseudo-Dirac neutrino but the muon neutrino is an ordinary Majorana or Dirac neutrino; (II) The electron neutrino is a Majorana or Dirac neutrino but the muon neutrino is a pseudo-Dirac neutrino. In case (I) there is one MSW resonance region, and the neutrino fluxes at the earth depend on the corresponding Landau-Zener transition probability $P_{R_1}$. By assuming a mixing between $\nu_{\mu}$ and $\nu^c_e$ with mixing angle $\theta_{e\mu}$, we have

$$\nu^c_e : \nu^c_\mu = \frac{1}{2} \left[ P_{R_1} \cos^2 \theta_{e\mu} + (1 - P_{R_1}) \sin^2 \theta_{e\mu} \right] : \left[ P_{R_1} \sin^2 \theta_{e\mu} + (1 - P_{R_1}) \cos^2 \theta_{e\mu} \right]$$

leading to

$$S = H = \frac{1}{2} \left[ \frac{1}{2} - \frac{1}{2} (1 - 2P_{R_1}) \cos(2\theta_{e\mu}) \right] , \quad K = \frac{1}{6} + \frac{1}{3} \left[ \frac{1}{2} - \frac{1}{2} (1 - 2P_{R_1}) \cos(2\theta_{e\mu}) \right]$$

When we neglect the energy dependence of the detection rates, there is a region which satisfies the three detection rates within $2\sigma$ uncertainties

$$0.40 \lesssim \left[ \frac{1}{2} - \frac{1}{2} (1 - 2P_{R_1}) \cos(2\theta_{e\mu}) \right] \lesssim 0.70.$$  

The allowed region is similar to the case in which both neutrinos are pseudo-Dirac (see Eq. (20)) since in both cases there is only one adiabatic MSW transition region.

In case (II) there are two MSW resonance regions, but in the region $R_2$ the transition is extremely non-adiabatic, i.e. $P_{R_2} \approx 1$. The neutrino fluxes at the earth depend on the Landau-Zener transition probability $P_{R_1}$. By assuming a mixing between $\nu^c_\mu$ and $\nu_e$ with mixing angle $\theta_{e\mu}$, we have

$$\nu^c_\mu : \nu^c_e = \left[ P_{R_1} \cos^2 \theta_{e\mu} + (1 - P_{R_1}) \sin^2 \theta_{e\mu} \right] : \frac{1}{2} \left[ P_{R_1} \sin^2 \theta_{e\mu} + (1 - P_{R_1}) \cos^2 \theta_{e\mu} \right]$$

leading to

$$S = H = \frac{1}{2} \left[ \frac{1}{2} - \frac{1}{2} (1 - 2P_{R_1}) \cos(2\theta_{e\mu}) \right] , \quad K = \frac{1}{12} + \frac{1}{12} \left[ \frac{1}{2} - \frac{1}{2} (1 - 2P_{R_1}) \cos(2\theta_{e\mu}) \right]$$
When we neglect the energy dependence of the detection rates, we find a region which satisfies the three detection rates within $2\sigma$ uncertainty as

$$0.24 \lesssim \left[ \frac{1}{2} - \frac{1}{2}(1 - 2P_{\mu\mu}) \cos(2\theta_{e\mu}) \right] \lesssim 0.35.$$  

(28)

This allowed region is similar to the case of the ordinary MSW result (see Eq. (22)). Note that this case is different from the previous one and the case of both neutrinos being pseudo-Dirac because the electron neutrino flux is not depleted in half.

In summary, the allowed regions in the mass-mixing angle parameter space which are obtained from the SAGE, Homestake and Kamioka experiments are very different depending on the particle content of the neutrino sector. For example, the region allowed when both $\nu_e$ and $\nu_\mu$ are pseudo-Dirac neutrinos is very different from the region allowed by the analysis based on the usual two generation MSW effect. Consequently, in order to pin down the values of $\Delta m^2_{e\mu}$ and $\theta_{e\mu}$ from future solar neutrino experiments, it is necessary to have a complete understanding of the neutrino sector, in particular whether sterile neutrinos actually exist or not; if they do, what their nature would be and so on. Finally, we conclude with short comments on the apparent atmospheric neutrino puzzle and the neutrinoless double beta decay. First, the atmospheric neutrino puzzle is that

$$\frac{\Phi_{\nu_e}^{\text{obs}}}{\Phi_{\nu_e}^{\text{cal}}} = \begin{cases} 
0.65 \pm 0.08 \pm 0.06 & \text{Kamioka} \\
0.64 \pm 0.09 \pm 0.12 & \text{IMB} 
\end{cases}$$

(29)

where $\Phi_{\nu_e,\nu_\mu}^{\text{obs}}$ and $\Phi_{\nu_e,\nu_\mu}^{\text{cal}}$ are the observed and calculated fluxes of atmospheric neutrinos, respectively. This puzzle can easily be solved in the scenario in which both $\nu_e$ and $\nu_\mu$ are pseudo-Dirac, as mentioned in Ref. [12], or in the scenario (II) discussed above as long as the constraints $\Delta m^2_{e\mu} \lesssim 10^{-7}$eV$^2$ and $10^{-4}$eV$^2 \lesssim \Delta m^2_{\mu\mu} \lesssim 10^{-2}$eV$^2$ are met. The upper limits are both due to the cosmological argument [10] and the lower limit for $\Delta m^2_{\mu\mu}$ is necessary in order to have an oscillation length much shorter than the radius of the earth. In these scenarios, $\nu_\mu$ is depleted in half simply because of its pseudo-Dirac nature. Furthermore, a value $\Delta m^2_{\mu\mu} \sim 10^{-4}$eV$^2$, which correspond to an oscillation length equal to the earth diameter for $E \sim 500$ MeV, could explain the observed suppression of the flux of low-energy muon neutrinos and a value between 0.5 and 1 for the ratio given in Eq. (29). Note that, in the case in which both neutrinos are pseudo-Dirac, $\nu_e$ is not depleted because the oscillation length is much longer than the radius of the earth due to the cosmological limit mentioned above. Secondly, as already discussed in the past [13], neutrinoless double beta decay rates are naturally suppressed because they become proportional to $m_L^2$, for the pseudo-Dirac neutrinos that we have discussed. This implies that non-observation
of neutrinoless double beta decay cannot automatically lead to the conclusion that the electron neutrino is a Dirac particle.

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Figure Captions

**Fig.1** Effective masses squared in matter for the energy eigenstates $\nu_1$, $\nu_2$, $\nu_3$ and $\nu_4$ as functions of the matter density $\rho$.

**Fig.2** $\Delta m^2_{e\mu} \cdot \sin^2(2\theta_{e\mu}) / \cos(2\theta_{e\mu})$ plots. The area inside the dotted line is the allowed region in the standard MSW effect, whereas the shaded area is the allowed region in the pseudo-Dirac case. The transition at $R2$ becomes non-adiabatic in the region below the solid line in the upper right corner.