Decoherence suppression of a dissipative qubit by the non-Markovian effect

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Abstract

We evaluate exactly the non-Markovian effect on the decoherence dynamics of a qubit interacting with a dissipative vacuum reservoir and find that the coherence of the qubit can be partially trapped in the steady state when the memory effect of the reservoir is considered. Our analysis shows that it is the formation of a bound state between the qubit and its reservoir that results in this residual coherence in the steady state under the non-Markovian dynamics. A physical condition for the decoherence suppression is given explicitly. Our results suggest a potential way to decoherence control by modifying the system–reservoir interaction and the spectrum of the reservoir to the non-Markovian regime in the scenario of reservoir engineering. (Some figures in this article are in colour only in the electronic version)

1. Introduction

Any realistic quantum system inevitably interacts with its surrounding environment, which leads to the loss of coherence or decoherence of the quantum system [1]. The decoherence of a quantum bit (qubit) is deemed a main obstacle to the realization of quantum computation and quantum information processing [2]. Understanding and suppressing the decoherence are therefore major issues in quantum information science. For a Markovian environment, it is well known that the coherence of a qubit experiences an exponential decrease [1]. To beat this unwanted degradation, many controlling strategies, passive or active, have been proposed [3–7].

In recent years much attention has been paid to the non-Markovian effect on the decoherence dynamics of an open quantum system [8–12]. The significance of non-Markovian dynamics in the study of an open quantum system is twofold. (i) It is of fundamental interest to extend the well-developed methods and concepts of Markonian dynamics to the non-Markovian case [1, 13] for the open quantum system in its own right. (ii) There are many new physical situations in which the usually used Markovian assumption is not fulfilled, and thus non-Markovian dynamics has to be introduced. In particular, many experimental results have evidenced the existence of the non-Markovian effect [14–16], which indicates that one can now approach the non-Markovian regime via tuning the relevant parameters of the system and the reservoir. The non-Markovian effect means that the environment, when its state is changed due to the interaction with the quantum system, in turn exerts its dynamical influence back on the system. Consequently one can expect that the decoherence dynamics of the quantum system could exhibit a dramatic deviation from the exponential decaying behaviour. In 2005, DiVincenzo and Loss studied the decoherence dynamics of the spin-boson model for the Ohmic heat bath in the weak-coupling limit. They used the Born approximation and found that the coherence dynamics has a power-law behaviour at a long time scale [17], which greatly prolongs the coherence time of the quantum system. Such power-law behaviour suggests that the non-Markovian effect may play a constructive role in suppressing the decoherence of the system. Nevertheless, in many cases the finite extension of the coherence time of the system is not sufficient for quantum information processing; a question arises whether the coherence of the system can be preserved in the long-time limit, even partially. Theoretically, the answer is affirmative if the environment has a nontrivial structure. It has been shown that some residual coherence
can be preserved in the long-time steady state when the environment is a periodic band gap material [18–21] or leaky cavity [22]. It is stressed that the residual coherence is due to the confined structured environment. A natural question is: can the coherence of the system be dynamically preserved or not by the non-Markovian effect if the environment has no special structure, e.g. a vacuum reservoir?

In this paper, we study the exact decoherence dynamics of a qubit interacting with a vacuum reservoir and examine the possibility of decoherence suppression using the non-Markovian effect. The main aim of this work is to analyze if and how the coherence present in the initial state can be trapped with a noticeable fraction in the steady state even when the environment consists of a vacuum reservoir with a trivial structure. We show that the non-Markovian effect manifests its action on the qubit not only in the transient dynamical process but also in the asymptotical behavior. Our analysis shows that the physical mechanism behind this dynamical suppression to decoherence is the formation of a bound state between the qubit and the reservoir. The non-decaying character of the bound state leads to the inhibition of the decoherence and the residual coherence trapped in the steady state. A similar vacuum-induced coherence trapping in the continuous variable system has been reported in [23, 24]. Such a coherence trapping phenomenon provides an alternative way to suppress decoherence. This could be realized by controlling and modifying the system-reservoir interaction and the properties of the reservoir [18] by the recently developed reservoir engineering technique [25–27].

Our paper is organized as follows. In section 2, we introduce the model of a qubit interacting with a vacuum reservoir and derive the exact master equation. In section 3, two quantities, i.e. purity and decoherence factor, to characterize the decoherence dynamics are introduced. In section 4 we give the numerical study for the time evolution of decay rate, purity and decoherence factor in terms of a coupling constant and cutoff frequency and the physical mechanism of the dynamical decoherence suppression. Finally, discussions and a summary are given in section 5.

2. The model and exact dynamics of the qubit

We consider a qubit interacting with a reservoir which is consisted of a quantized radiation field. The Hamiltonian of the total system is given by

\[ H = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k a_k^\dagger a_k + \sum_k (g_k \sigma_+ a_k + h.c.), \]

where \( \sigma_\pm \) and \( \omega_0 \) are the inversion operators and transition frequency of the qubit, respectively, and \( a_k^\dagger \) and \( a_k \) are the creation and annihilation operators of the \( k \)-th mode with the frequency \( \omega_k \) of the radiation field. The coupling strength between the qubit and the radiation field has the form [1]

\[ g_k = -\frac{i}{\sqrt{2 \epsilon_0 V}} \hat{e}_k \cdot \mathbf{d}, \]

where \( \hat{e}_k \) and \( V \) are the unit polarization vector and the normalization volume of the radiation field, \( \mathbf{d} \) is the dipole moment of the qubit and \( \epsilon_0 \) is the free space permittivity.

Throughout this paper we assume \( \hbar = 1 \). This model has been well studied under the Born–Markovian approximation in quantum optics [1]. However, the physical condition under which such an approximation is applicable, and how the non-Markovian effect affects the decoherence dynamics in different parameter regimes, have not been quantitatively investigated.

If there is no correlation between the qubit and the radiation field initially, then the initial state of the whole system can be factorized into a product of the states of qubit and the field. If the initial state is \( |\Psi(0)\rangle = |\pm , \{0_k\} \rangle \), with \( |\pm \rangle \) and \( |\{0_k\} \rangle \), respectively, denoting the exited state of the qubit and the vacuum state of the radiation field, then governed by the Hamiltonian (1), the state will evolve into the following form:

\[ |\Psi(t)\rangle = b_0(t)|\pm \rangle \{0_k\} + \sum_k b_k(t)|\pm \{- \kappa \}, \]

(3)

where \( |\kappa \rangle \) is the field state containing one photon only in the \( k \)-th mode. From the Schrödinger equation, we can get the time evolution of the probability amplitudes in equation (3). On substituting the formal solution of \( b_k(t) \) into the equation of motion satisfied by \( b_0(t) \), we obtain

\[ b_0(t) + i \omega_0 b_0(t) + \int_{0}^{t} b_0(t) f(t - \tau) d\tau = 0, \]

(4)

where the kernel function is \( f(x) = \sum_{k=0}^{\infty} |g_k|^2 e^{-i \omega_k x} \). The integro-differential equation (4) renders the dynamics of the qubit non-Markovian, with the memory effect of the reservoir registered in the time-nonlocal kernel function \( f(x) \). In the continuous limit of the environment frequency, one can verify from the coupling strength (2) that the kernel function has the form

\[ f(x) = \int_{0}^{\infty} J(\omega) e^{-i \omega x} d\omega, \]

(5)

where \( J(\omega) = \eta \omega^2 e^{-\omega^2} \) is called the spectral density with the coupling constant \( \eta \equiv \omega_0^2 \int \frac{d^3 k}{2 \pi^2} \frac{1}{\omega_0^2 + \omega_k^2} \). Here \( \omega_0^2 \) in \( J(\omega) \) is introduced to make \( \eta \) dimensionless. To eliminate the infinity in frequency integration, we have introduced the cutoff frequency \( \omega_\text{c} \). Physically, introducing the cutoff frequency means that not all of the infinite modes of the reservoir contribute to the interaction with the qubit, and one always expects the spectral density going to zero for the modes with frequencies higher than certain characteristic frequency. It is just this characteristic frequency which determines the specific behaviour and the properties of the reservoir. One can see that in our model, the spectral density has a super-Ohmic form [28].

From the time evolution of equation (3) and the fact that the ground state \( |-\rangle \) of the qubit is immune to the radiation field, one can get the time evolution of any given initial state of the system readily. For a general state of the qubit described by

\[ \rho_{\text{tot}}(0) = (\rho_{11}|+\rangle\langle+| + \rho_{12}|+\rangle\langle-| + \rho_{21}|-\rangle\langle+| + \rho_{22}|-\rangle\langle-|) \otimes |\{0\}\rangle\langle\{0\}|, \]

the time evolution of the total system can be calculated explicitly. In fact, what is needed is the reduced density matrix
of the qubit, which is obtained by tracing over the reservoir variables

\[ \rho(t) = \begin{pmatrix} \rho_{11}(b_0(t))^2 & \rho_{12}b_0(t) \\ \rho_{21}b_0^*(t) & 1 - \rho_{11}(b_0(t))^2 \end{pmatrix}. \]  

(6)

Differentiating equation (6) with respect to time, we arrive at the equation of motion of the reduced density matrix

\[ \dot{\rho}(t) = -i\frac{\Omega(t)}{2} [\sigma_+ \rho \sigma_- + \rho \sigma_+ \sigma_-] + \frac{\gamma(t)}{2} [2\sigma_- \rho(t) \sigma_+] - \sigma_+ \sigma_- \rho(t) - \rho(t) \sigma_+ \sigma_-]. \]

(7)

where \( \Omega(t) = -2i \text{Im} \left[ \frac{b_0(t)}{b_0^*(t)} \right] \) and \( \gamma(t) = -2 \text{Re} \left[ \frac{b_0(t)}{b_0^*(t)} \right] \). \( \Omega(t) \) plays the role of the time-dependent shifted frequency and \( \gamma(t) \) that of the time-dependent decay rate [1]. It is worth mentioning that in the derivation of the master equation (7), we have not resorted to the Born–Markovian approximation, that is, equation (7) is the exact master equation of the qubit system. Compared with the master equation derived in [1] under the condition that the initial state of the qubit is pure, our derivation shows that equation (7) can describe the dynamics not only for the initial pure state but also for any mixed state of the qubit.

It is interesting to note that one can reproduce the conventional Markovian one from our exact non-Markovian master equation under certain approximations. By redefining the probability amplitude as \( b_0(t) = b_0'(t) e^{-i\delta \omega t} \), one can recast equation (4) into

\[ b_0'(t) + \int_0^\infty \text{d} \omega J(\omega) \int_0^t \text{d} t' e^{i(\omega_0-\omega)(t-t')} b_0'(t') = 0. \]

(8)

Then, we take the Markovian approximation \( b_0'(t) \approx b_0(t) \), namely approximately taking the dynamical variable to the one that depends only on the present time so that any memory effect regarding the earlier time is ignored. The Markovian approximation is mainly based on the physical assumption that the correlation time of the reservoir is very small compared with the typical time scale of system evolution. Also under this assumption we can extend the upper limit of the \( \tau \) integration in equation (8) to infinity and use the equality

\[ \lim_{t \to \infty} \int_0^t \text{d} t' e^{i\delta \omega(t' - t')} = \pi \delta(\omega - \omega_0) \mp iP \left( \frac{1}{\omega - \omega_0} \right). \]

(9)

where \( P \) and the delta-function denote the Cauchy principal value and the singularity, respectively. The integro-differential equation (8) is thus reduced to a linear ordinary differential equation. The solutions of \( b_0'(t) \) as well as \( b_0(t) \) can then be easily obtained as

\[ b_0(t) = e^{-i\delta \omega t} b_0'(t) - \pi J(\omega_0)t', \]

where \( \delta \omega = P \int_0^\infty \frac{J(\omega)\text{d} \omega}{\omega-\omega_0} \). Thus one can verify that

\[ \gamma(t) \equiv \gamma_0 = 2\pi J(\omega_0), \quad \Omega(t) \equiv \Omega_0 = 2(\omega_0 - \delta \omega), \]

(10)

which are just the coefficients in the Markovian master equation of the two-level atom system [1].

3. Purity and decoherence factor

To quantify the decoherence dynamics of the qubit, we introduce the following two quantities. The first one is the purity, which is defined as [2]

\[ p(t) = \text{Tr} \rho^2(t). \]

(11)

Clearly \( p = 1 \) for a pure state and \( p < 1 \) for a mixed state. The second quantity describing the decoherence is the decoherence factor \( c(t) \) of the qubit, which is determined by the off-diagonal elements of the reduced density matrix

\[ |\rho_{12}(t)| = c(t)|\rho_{12}(0)|. \]

(12)

The decoherence factor maintains unity when the reservoir is absent and vanishes for the case of complete decoherence.

For definiteness, we consider the following initial pure state of the qubit:

\[ |\Psi(0)\rangle = |\alpha+\rangle + |\beta-\rangle, \]

(13)

in which \( \alpha \) and \( \beta \) satisfy the normalization condition. Using equation (6), the exact time evolution of the qubit is easily obtained:

\[ \rho(t) = \begin{pmatrix} |\alpha|^2 |b_0(t)|^2 & \alpha \beta^* b_0(t) \\ \alpha^* \beta b_0^*(t) & 1 - |\alpha|^2 |b_0(t)|^2 \end{pmatrix}. \]

(14)

With equation (14), the purity and decoherence factor can be expressed explicitly

\[ p(t) = 2|\alpha|^4 |b_0(t)|^2 - 1 + 1, \]

(15)

and

\[ c(t) = |b_0(t)|. \]

(16)

It is easy to verify, under the Born–Markovian approximation, that the purity and decoherence factor have the following forms:

\[ p(t) = 2|\alpha|^4 e^{-2\gamma t}(e^{-\gamma t} - 1) + 1, \]

(17)

and

\[ c(t) = e^{-\frac{\gamma_0 t}{2}}, \]

(18)

where the time-independent decay rate \( \gamma_0 \) is given in equation (10). Obviously, the system asymptotically loses its quantum coherence \( c(\infty) = 0 \) and approaches a pure steady state \( p(\infty) = 1 \) irrespective of the form of the initial state under the Markovian approximation. One can also find from equations (15)–(18) that the probability amplitude of the excited state plays a key role in the decoherence dynamics.

4. Numerical results and analysis

In this section, by numerically solving equation (4), we study the influence of memory effect of the reservoir on the exact dynamics of the qubit. Noting the fact that the memory effect registered in the kernel function is essentially determined by the spectrum density \( J(\omega) \), one can expect that \( J(\omega) \) plays a major role in the exact dynamics of the qubit. In the following, we show how the decoherence of the qubit can be fully suppressed under the non-Markovian dynamics in terms of the relevant parameters of \( J(\omega) \).
the ground state evolution, which results in any initial qubit state evolving into shown that the decay rate is positive in the full range of a stable decay rate to the qubit. Furthermore, it is also just as the result based on the Markovian approximation but the reservoir does not exert decoherence on the qubit abruptly, the small 'jolt' of \( \gamma(t) \) value. The small 'jolt' of \( \gamma(t) \) rate \( \eta \) shows a distinct difference from its Markovian counterpart over a very short time interval. With time, \( \gamma(t) \) tends to a definite positive value. The small 'jolt' of \( \gamma(t) \) in the short time interval is just evidence of the backaction of the memory effect of the reservoir exerted on the qubit [29, 30]. It manifests that the reservoir does not exert decoherence on the qubit abruptly, just as the result based on the Markovian approximation but dynamically influences the qubit and gradually establishes a stable decay rate to the qubit. Furthermore, it is also shown that the decay rate is positive in the full range of evolution, which results in any initial qubit state evolving into the ground state \( |\psi(\infty)\rangle = |\rangle \) irreversibly. Consequently the decoherence factor monotonically decreases to zero with time, and the purity approaches unity in the long-time limit, which is consistent with the result under Markovian approximation. The result indicates that although the reservoir has a backaction effect on the qubit, it is quite small. And the dissipation effect of the reservoir dominates the dynamics of the qubit. Thus no qualitative difference can be expected between the exact result and the Markovian one with the backaction effect ignored. Therefore the widely used Markovian approximation is applicable in this case. Nevertheless, at the short and immediate time scales the overall behaviour is still quite different from that of the Markovian dynamics. The decoherence factor shown in the right-hand panel of figure 1 shows non-exponential decay, which is in agreement with the result obtained previously in the spin-boson model in the weak-coupling limit [17]. However, the situation is dramatically changed if the coupling is strengthened, as discussed below.

With the same cutoff frequency as in figure 1 but a larger coupling constant, we plot in figure 2 the decay rate, purity and decoherence factor in the strong coupling case. In this case the non-negligible backaction of the reservoir has a great impact on the dynamics of the qubit. Firstly, we can see that the decay rate not only exhibits oscillations but also takes negative values in the short-time scale. Physically, the negative decay rate is a sign of strong backaction induced by the non-Markovian memory effect of the reservoir. And the oscillations of the decay rate between negative and positive values reflect the exchange of excitation back and forth between the qubit and the reservoir [10]. Consequently both the decoherence factor and the purity exhibit oscillations in a short-time scale, which shows a dramatic deviation to the Markovian result. Therefore, entirely different to the weak-coupling case in figure 1, the reservoir in the strong-coupling case here has a strong backaction effect on the qubit. Secondly, we also note that the decay rate approaches zero in the long-time limit. The vanishing decay rate means, after several rounds of oscillation, the qubit ceases decaying asymptotically. The non-Markovian purity maintains a steady value asymptotically, which is less than unity. This indicates that the steady state of the qubit is no longer the ground state but a mixed state. The decoherence factor also tends to a non-zero value, which implies that the coherence of the qubit is preserved with a noticeable fraction in the long-time steady state. These phenomena, which are qualitatively different to the Markovian situation, manifest that the memory effect has a considerable contribution not only to the short-time but also to the long-time behaviour of the decoherence dynamics. The presence of the residual coherence in the steady state also suggests a potential active control way to protect the quantum coherence of the qubit from decoherence via the non-Markovian effect.

4.1 The influence of coupling constant

In the following, we numerically analyse the exact decoherence dynamics of the qubit with respect to the decay rate \( \gamma(t) \), purity \( p(t) \) and decoherence factor \( c(t) \) in terms of the coupling constant \( \eta \).

In figure 1 we plot the time evolution of decay rate \( \gamma(t) \), purity \( p(t) \), decoherence factor \( c(t) \) and their Markovian correspondences in the weak-coupling and low cutoff frequency case. We can see that \( \gamma(t) \) shows a distinct difference from its Markovian counterpart over a very short time interval. With time, \( \gamma(t) \) tends to a definite positive value. The small 'jolt' of \( \gamma(t) \) in the short time interval is just evidence of the backaction of the memory effect of the reservoir exerted on the qubit [29, 30]. It manifests that the reservoir does not exert decoherence on the qubit abruptly, just as the result based on the Markovian approximation but dynamically influences the qubit and gradually establishes a stable decay rate to the qubit. Furthermore, it is also shown that the decay rate is positive in the full range of evolution, which results in any initial qubit state evolving into the ground state \( |\psi(\infty)\rangle = |\rangle \) irreversibly. Consequently the decoherence factor monotonically decreases to zero with time, and the purity approaches unity in the long-time limit, which is consistent with the result under Markovian approximation. The result indicates that although the reservoir has a backaction effect on the qubit, it is quite small. And the dissipation effect of the reservoir dominates the dynamics of the qubit. Thus no qualitative difference can be expected between the exact result and the Markovian one with the backaction effect ignored. Therefore the widely used Markovian approximation is applicable in this case. Nevertheless, at the short and immediate time scales the overall behaviour is still quite different from that of the Markovian dynamics. The decoherence factor shown in the right-hand panel of figure 1 shows non-exponential decay, which is in agreement with the result obtained previously in the spin-boson model in the weak-coupling limit [17]. However, the situation is dramatically changed if the coupling is strengthened, as discussed below.

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4.2. The influence of cutoff frequency

The cutoff frequency $\omega_c$, on the one hand, is introduced to eliminate the infinity in the frequency integration. On the other hand it also determines the frequency range in which the power form is valid \[31\]. In the following, we elucidate the influence of cutoff frequency on the exact decoherence dynamics.

Fixing $\eta$ as the value in figure 1 and increasing the cutoff frequency, we plot in figure 3 the dynamics of the qubit in a high cutoff frequency case. It shows that a similar decoherence behaviour as the strong-coupling case in figure 2 can be obtained. After several rounds of oscillation, the decay rate tends to zero in the long-time limit. The negative decay rate causes the lost coherence to be partially recovered. The vanishing decay rate in the long-time limit results in the decoherence being frozen before the qubit gets to its ground state. Thus there is some residual coherence trapped in the steady state. Similar to the strong-coupling case, it is essentially the interplay between the backaction and the dissipation on the dynamics of qubit which results in the inhibition of decoherence. We argue that in this high cutoff frequency regime, the widely used Markovian approximation is not applicable because of the strong backaction effect of the reservoir.

4.3. The physical mechanism of the decoherence inhibition

From the analysis above we can clearly see that the decoherence can be inhibited in the non-Markovian dynamics. A natural question is: what is the physical mechanism to cause such dynamical decoherence inhibition? To answer this question, let us find the eigensolution of equation (1) in the sector of one-excitation in which we are interested. The eigenfunction reads $H|\psi_E\rangle = E|\psi_E\rangle$, where $|\psi_E\rangle = c_0|+\rangle + \sum_{k=0}^{\infty} c_k|\sim_k\rangle$. After some algebraic calculation, we can obtain a transcendental equation of $E$,

$$y(E) \equiv \omega_0 - \int_{0}^{\infty} \frac{J(\omega)}{\omega - E} d\omega = E. \quad (19)$$

From the fact that $y(E)$ decreases monotonically with the increase of $E$ when $E < 0$ we can say that if the condition $y(0) < 0$, i.e.

$$\omega_0 = \frac{2\eta}{\omega_0} < 0, \quad (20)$$

is satisfied, $y(E)$ always has only one intersection in the regime $E < 0$ with the function on the right-hand side of equation (19). Then the system will have an eigenstate with a real (negative) eigenvalue, which is a bound state \[32\], in the Hilbert space of the qubit plus its reservoir. In the regime of $E > 0$ one can see that $y(E)$ is divergent, which means that no real root $E$ can make equation (19) well defined. Consequently equation (19) does not have a positive real root to support the existence of a further bound state. It is noted that equation (19) may possess a complex root. Physically this means that the corresponding eigenstate experiences decay contributed from the imaginary part of the eigenvalue during the time evolution, which causes the excited-state population approaching zero asymptotically and the decoherence of the reduced qubit system.

The formation of bound state is just the physical mechanism responsible for the inhibition of decoherence. This is because a bound state is actually a stationary state with a vanishing decay rate during the time evolution. Thus the population probability of the atomic excited state in a bound state is constant in time, which is named ‘population trapping’ \[18, 20\]. This claim is fully verified by our numerical results. The parameters in figure 1 do not satisfy condition (20) to support the existence of a bound state; then the dynamics experiences a severe decoherence. With the increase of either $\eta$ (in figure 2) or $\omega_c$ (in figure 3), the bound state is formed. Then the system and its environment are so correlated that it causes the decay rate of the system in the non-Markovian dynamics exhibiting (1) a transient negative value due to the backaction of the environment and (2) a vanishing asymptotic value. Such an interesting phenomenon, i.e. the vanishing asymptotic decay rate in the large cutoff frequency regime for the super-Ohmic spectrum density, was also revealed in \[33\]. This effect of course is missing in the conventional Born–Markovian decoherence theory, where the reservoir is memoryless.

In order to understand the exact decoherence dynamics more completely, we plot in figure 4 the crossover from coherence destroying to coherence trapping via increasing either the coupling constant or the cutoff frequency. Coherence trapping can be achieved as long as the bound state is formed. Therefore, one can preserve coherence via tuning the relevant parameters of the system and the reservoir, e.g. the qubit–reservoir coupling constant and the property of the reservoir so that condition (20) is satisfied.
Figure 4. Time evolution of $c(t)$ in the non-Markovian dynamics with different $\eta$ when $\omega_c/\omega = 1.0$ (upper panel) and with different $\omega_c$ when $\eta = 0.08$ (lower panel).

5. Summary and discussions

In summary, we have investigated the exact decoherence dynamics of a qubit in a dissipative vacuum reservoir. We have found that even in a vacuum environment without any nontrivial structure, we can still get the decoherence suppression of the qubit owing to the dynamical mechanism of the non-Markovian effect. From our analytic and numerical results, we find that the non-Markovian reservoir has dual effects on the qubit: dissipation and backaction. The dissipation effect exhausts the coherence of the qubit, whereas the backaction one revives it. In the strong-coupling and/or high cutoff frequency regimes, a bound state between the qubit and its reservoir is formed. It induces a strong backaction effect in the dynamics because the reservoir is strongly correlated with the qubit in the bound state. Furthermore, because of the non-decay character of the bound state, the decay rate in this situation approaches zero asymptotically. The vanishing of the decay rate causes the decoherence to cease before the qubit decays to its ground state. Thus the qubit in the non-Markovian dynamics would evolve into a non-ground steady state, and there is some residual coherence preserved in the long-time limit. Our results make it clear how the non-Markovian effect shows its effects on the decoherence dynamics in different parameter regimes.

The presence of such a coherence trapping phenomenon actually gives us an active way to suppress decoherence via the non-Markovian effect. This could be achieved by modifying the properties of the reservoir to approach the non-Markovian regime via the potential usage of the reservoir engineering technique [25–27, 34]. Many experimental platforms, e.g. mesoscopic ion traps [25, 26], cold atom BEC [27] and photonic crystal materials [18], have exhibited the controllability of the decoherence behaviour of the relevant quantum system via good design of the size (i.e. modifying the spectrum) of the reservoir and/or the coupling strength between the system and the reservoir. It is also worth mentioning that a proposal aimed at simulating the spin-boson model, which is relevant to the one considered in this paper, has been reported in the trapped ion system [35]. On the other hand many practical systems can now be engineered to show the novel non-Markovian effect [14–16, 36]. All these achievements show that the recent advances have paved the way to experimentally simulate the paradigmatic models of an open quantum system, which is one aspect of the newly emergent field of quantum simulators [37]. Our work sheds new light on the way to indirectly control and manipulate the dynamics of a quantum system in this experimental platform.

A final remark is that our results can be generalized to a system consisting of two qubits, each of which interacts with a local reservoir. Because of coherence trapping, we expect that the non-Markovian effect plays a constructive role in the entanglement preservation [21, 24, 38].

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