The method for studying the drag coefficient of spherical particles under non-isothermal conditions

S A Basalaev V A Arkhipov K GPerfilieva and A S Usanina
Tomsk State University, 36 Lenin Avenue, Tomsk 634050 Russian Federation
E-mail: tarm@niipmm.tsu.ru

Abstract. A method and results of an experimental studying the sedimentation of solid spherical particles in a viscous liquid under non-isothermal conditions at low Reynolds numbers (Re<1) are presented. To determine the dependence of the drag coefficient on the temperature difference between the particles and liquid, a special setup has been developed. The experimental setup for determining the effect of non-isothermal conditions on the gravitational sedimentation of solid spherical particles in a viscous liquid consists of a prismatic cell with a viscous liquid, a particle heating device, a device for incorporating the reference and heated particles into the liquid, and a system for visualization of the particle sedimentation. In the experimental setup, two particles of the same diameter, made of the same material, are introduced into the cell at a zero initial velocity. Immediately before introduction into the liquid, one of the particles is heated (or cooled) to a temperature different from the temperature of the other (reference) particle, equal to the temperature of the liquid. The deposition rate of each of the particles is measured by the time-of-flight method using video of the deposition process through the transparent walls of the cell. As a result of a series of experiments, a decrease in the drag coefficient by 38% was obtained at a maximum temperature difference between the particles and liquid of 280°C.

1. Introduction
The movement of particles in the field of gravity is of great practical importance in environmental problems, the coal industry, solving the consequences of catastrophic man-made or natural disasters, power engineering, chemical engineering processes, and a number of other engineering and technology industries [1]. One of the main characteristics determining the motion laws of particles in a two-phase flow is the coefficient of resistance of the medium to the movement of particles \( C_x \) [2], which is included in the motion equation. The common resistance curve and the overwhelming majority of the dependences for \( C_x \) found in the literature for the complicated conditions of flowing around particles are obtained under isothermal conditions (equality of temperatures of the particles and carrier medium) [3-5].

In a number of technical systems and technological processes, the movement of particles in a carrier medium occurs under non-isothermal conditions [6-9]. Wherein, the temperature of particles can be significantly higher or lower than the medium temperature. Under these conditions, the use of the common resistance curve leads to significant errors in calculating the motion velocity of particles. This situation is related with changes in the physical properties (primarily viscosity) of the medium in the boundary layer near the particle, which are included in the Reynolds number [10].

In the present work, a new method and experimental setup for determining the drag coefficient of solid spherical particles upon their gravitational sedimentation in a viscous liquid under non-isothermal conditions are considered [11].
2. Experimental setup

The experimental setup (figure 1) for determining the effect of non-isothermal conditions on the gravitational sedimentation of solid spherical particles in a viscous liquid consists of a prismatic cell 1 with a viscous liquid 2, a particle heating device, a device for incorporating the reference and heated particles into the liquid, and a system for visualization of the particle sedimentation.

![Figure 1. Block-diagram of the experimental setup.](image)

The cell 1 is made of optical glass in the form of a regular prism with the dimensions of 30×30×90 cm. The particle heating device 3 consists of a cylindrical container 4 with an incandescent spiral 5. The device for incorporating the reference 6 and heated 3 particles consists of a fixed 7 and movable 8 plates, which have combined round holes 9. The visualization of the particle sedimentation in the liquid is performed with a Citius C100 high-speed digital video camera 10 with a shooting rate of (50÷200) frames per second. The video is processed using a computer 11.

In the experiments, one of the particles 3 in the container 4 was heated to the set temperature, which was measured by a thermocouple 12 connected through an amplifier 13 with an oscilloscope 11. After heating the particle 3, it was inserted into the hole 9 of the fixed plate 7 due to free fall when the stopper 15 was removed by the electromagnetic driven 16. Then the movable plate 8 was displaced in the horizontal direction using an electromagnetic actuator 14 until matching the holes 9 in the movable 8 and fixed 7 plates. Wherein, the reference 6 and heated 3 particles precipitated in a viscous liquid 2 with a zero initial velocity. The data obtained from the video camera 10 were processed on the computer 11 in order to determine the sedimentation rate of each of the particles by the time-of-flight method.

3. Technique for determining the drag coefficient

The effect of non-isothermal conditions on the gravitational sedimentation of solid spherical particles in a viscous liquid was studied experimentally for steel balls (made of AISI 440C steel) with a
diameter in the range (3÷18) mm. The silicone PMS-1000 oil was used as a viscous liquid. The temperature of the silicone oil was 17°C, and the balls were heated to the temperature from the range (40÷300)°С. Basic characteristics and properties of the steel balls and silicone PMS-1000 oil are presented in Tables 1 and 2 [12].

**Table 1. Basic characteristics of particles.**

| Properties                              | Parameter | Value |
|-----------------------------------------|-----------|-------|
| Particle diameter                       | $D_p$ (mm) | 3÷18  |
| Density                                 | $\rho_p$ (kg/m$^3$) | 7748  |
| Thermal conductivity coefficient        | $\lambda_p$ (W/(m·°C)) | 24    |
| Specific heat of the material           | $c$ (J/(kg·deg)) | 483   |
| Normal elastic modulus                  | $E$ (GPa)  | 2.04  |

**Table 2. Characteristics of the silicone PMS-1000 oil.**

| Properties                              | Parameter | Value |
|-----------------------------------------|-----------|-------|
| Odorless, tasteless, colorless, and transparent liquid | –– | –– |
| Has a good resistance to high and low temperatures | –– | –– |
| Dynamic viscosity of the liquid (at a temperature of 17°C) | $\mu_l$ (Pa·s) | 10.84 |
| Pour point                              | $T_f$ (°C)  | 48    |
| Density (at a temperature of 17°C)      | $\rho_l$ (kg/m$^3$) | 979   |
| Refractive index                        | $n$       | 1.40  |
| Surface tension                         | $\sigma$ (N/m) | 21.3  |
| Flash point                             | $T_{fl}$ (°C) | 315   |

The density of the silicone PMS-1000 oil was measured with a hydrometer with a relative error $\delta\rho_l = 0.1\%$. The dynamic viscosity coefficient $\mu_l$ was determined from the measured stationary sedimentation rate $u$ of the steel ball with a diameter $D_p$ in the Stokes regime [13, 14].

$$\mu_l = \frac{gD_p^2(\rho_p - \rho_l)}{18u}$$

where $\rho_p$ is the ball material density, $\rho_l$ is the liquid density, and $u$ is the sedimentation rate of the steel ball.

The equation for gravitational sedimentation of a solid particle in a liquid has the following form [12]:

$$m \frac{du}{dt} = (\rho_p - \rho_l) V_p g - C_x S_m \rho_l u^2$$

where $m$ is the particle mass, $S_m$ is the area of the middle cross-section, and $C_x$ is the drag coefficient. In steady-state sedimentation regime ($du/dt = 0$), the formula defining the drag coefficient of the spherical particle follows from Eq. (1) [15]:

$$C_x = 4 \frac{gD_p}{3 u^2} \left( \frac{\rho_p}{\rho_l} - 1 \right)$$

When a particle is heated (or cooled), its gravitational sedimentation rate changes due to the heating (or cooling) of the liquid boundary layer adjacent to the particle. When the boundary layer is heated, the liquid viscosity decreases, which leads to a decrease in the drag coefficient and an increase in the particle sedimentation rate. When the boundary layer is cooled, a decrease in the particle sedimentation rate is observed.
The change in the liquid density with increasing the temperature is much less than the change in the dynamic viscosity coefficient ($\Delta \rho_l \ll \Delta \mu$). Assuming $\rho_l = \text{const}$, we can write formula (2) for the reference and heated (or cooled) particles in the following form:

$$C_{x_0} = \frac{A}{u_0^2},$$

(3)

$$C_x(\Delta T) = \frac{A}{[u(\Delta T)]^2},$$

(4)

where $u_0$ is the sedimentation rate of the reference (at a temperature of 17°C) particle and $A = \frac{4}{3} g D_p \left( \frac{\rho_p}{\rho_l} - 1 \right) = \text{const}$. Wherein, the relation defining the drag coefficient of the heated (or cooled) particle follows from (3) and (4):

$$C_x(\Delta T) = C_{x_0} \left[ \frac{u_0}{u(\Delta T)} \right]^2.$$  

(5)

4. Experimental results

Video sequence of frames of the sedimentation of the heated and reference particles with a diameter $D_p = 17.47$ mm is shown in figure 2. From the shown frames in can be seen that the sedimentation rate of the heated particle 6 substantially exceeds the sedimentation rate of the reference particle 3.

![Image of sedimentation video sequence](image-url)

Figure 2. Video sequence of frames of the sedimentation of the heated (300°C) and reference (17°C) particles with a diameter $D_p = 17.47$ mm
The launch of the system for recording the spatial location of particles [16], showed a slight deviation of particles in the horizontal plane.

The dependences of the distance $x$ traveled by the particles heated to $300^\circ$C and reference particles ($D_p = 8.87$ mm and $D_p = 17.47$ mm, respectively) on time $t$ are shown in figure 3. From the plots it follows that particle velocities correspond to the steady-state sedimentation regime.

![Figure 3](image)

**Figure 3.** Dependences of the distance traveled by the heated ($300^\circ$C) and reference ($17^\circ$C) particles on time $t$

Table 3 presents the measured sedimentation rate of the reference particle averaged over 5 duplicate experiments, the calculated drag coefficients $C_{xo}$ (3), and the Reynolds number

$$Re = \frac{\rho \mu u_0 D_p}{\mu_j}.$$ 

| $D_p$, mm | $u_0$, mm/s | Re·10^3 | $C_{xo}$·10^{-3} | $C_{x}^{(appr)}$·10^{-3} |
|-----|--------|--------|-----------------|-----------------|
| 3   | 3.065  | 0.83   | 28.87           | 28.90           |
| 8.87| 26.89  | 21.54  | 1.109           | 1.114           |
| 17.47| 96.02  | 151.50 | 0.171           | 0.166           |

The drag coefficients $C_{x}^{(appr)}$ calculated according to the approximation dependence [5]:

$$C_{x}^{(appr)} = \frac{24}{Re} + \frac{4}{\sqrt{Re}}.$$ (6)
From the results presented in Table 3 it follows that in isothermal conditions, the measured drag coefficients $C_{xo}$ correspond to dependence (6). The discrepancy is equal to 0.1% (for $D_p = 3$ mm), 0.4% (for $D_p = 8.87$ mm), and 2.9% (for $D_p = 17.47$ mm). The increase in the discrepancy for larger particles is apparently related with the error in the approximation dependence (6).

The measured sedimentation rates $u(\Delta T)$ and calculated drag coefficients (5) for the heated particles (at $T_h = 300^\circ$C and $T_l = 17^\circ$C) are presented in Table 4. In addition, the table also presents the relative reduction of the drag coefficient of particles upon their heating to 300°C calculated by the following formula:

$$
\Delta C_x = \frac{C_{xo} - C_x(\Delta T)}{C_{xo}} \cdot 100%.
$$

| $D_p$, mm | $u(\Delta T)$, mm/s | $C_x(\Delta T) \cdot 10^{-3}$ | $C_{xo} \cdot 10^{-3}$ | $\Delta C_x$, % |
|---|---|---|---|---|
| 8.87 | 34.08 | 0.690 | 1.109 | 37.8 |
| 17.47 | 122.05 | 0.106 | 0.171 | 38.0 |

From the results presented in Table 4, it follows that upon heating the balls with a diameter $D_p = 8.87$ mm and $D_p = 17.47$ mm, the relative decrease in the drag coefficient is the same and equals to 38%.

5. Conclusion
- The new method and experimental setup for determining the drag coefficient of solid particles upon their gravitational sedimentation in the viscous liquid under non-isothermal conditions are presented. The method allows using visualization of the deposition of particles in a viscous fluid to obtain results on the effect of non-isothermally on the drag coefficient with high accuracy.
- The new experimental data on the drag coefficient of solid spherical particles under non-isothermal conditions in the range of small Reynolds number ($Re<1$) are obtained. According to the results of the experiments, approximation formulas were obtained for calculating the drag coefficient of a solid spherical particle depending on the temperature difference between the particle and the dispersion medium.
- When a particle is heated (or cooled), its gravitational sedimentation rate changes due to the heating (or cooling) of the liquid boundary layer adjacent to the particle. When the boundary layer is heated, the liquid viscosity decreases, which leads to a decrease in the drag coefficient and an increase in the particle sedimentation rate. When the boundary layer is cooled, a decrease in the particle sedimentation rate is observed. It is shown that the drag coefficient at maximum temperature difference between the particles and liquid (283°C) is decreased by 38%.

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