Heterogeneous Prestressed Precast Beams Multiperiod Production Planning Problem: Modeling and Solution Methods

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Abstract
A prestressed precast beam is a type of beam that is stretched with traction elements. A common task in a factory of prestressed precast beams involves fulfilling, within a time horizon, the demand ordered by clients. A typical order includes beams of different lengths and types, with distinct beams potentially requiring different curing periods. We refer to the problem of planning such production as Heterogeneous Prestressed Precast Beams Multiperiod Production Planning (HPPBMPP). We formally define the HPPBMPP, argue its NP-hardness, and introduce four novel integer programming models for its solution and a size reduction heuristic (SRH). We propose six priority rules to produce feasible solutions. We perform computational tests on a set of synthetic instances that are based on data from a real-world scenario and discuss a case study. Our experiments suggest that the models can optimally solve small instances, while the SRH can produce high-quality solutions for most instances.

Keywords: prestressed concrete; precast beams; modular construction; integer linear programming; size reduction heuristics.

1. Introduction

Unlike conventional precast beams, prestressed precast beams have a different production process, in which they are tensioned using traction elements prior to supporting any actual load. The aim of this process is to improve the resistance and behavior of the beams in service. For their production, a factory uses concrete, along with the traction elements and a set of reusable molds. Traction elements are positioned and tensioned within the molds, after which concrete is cast. This is followed by a curing period, during which the concrete bonds to the traction elements. Those elements are then released and, as their material attempts to resume its original (untensioned) length, the concrete is compressed due to static friction. Prestressed beams are common in factories of civil construction materials, since they are often used in a variety of construction types. They are preferred over steel beams, since concrete has a low price and requires less maintenance when compared to steel. For the purpose of this paper, prestressed precast beams can vary with respect to curing time, length, and the number of traction elements used.

A common task in this type of factory involves fulfilling the demand of a set of clients, within a given time horizon. A typical order includes beams of different lengths and types, with different types of beams potentially requiring different curing times. A mold can be used to produce several beams simultaneously,
with the total length of the beams being limited by the mold’s capacity. While a given mold can be used to produce different types of beams in different periods, only one type of beam can be produced at a given mold at any given time. The problem of planning such production while minimizing the time to completion will be referred to as the Heterogeneous Prestressed Precast Beams Multi-period Production Planning (HPPMBPP).

To the best of the authors’ knowledge, the HPPMBPP is novel, despite its similarity with existing cutting problems. Indeed, we argue that the problem includes a known NP-hard cutting problem as a particular case. This fact justifies the investigation of mathematical programming models for producing optimal solutions. The combinatorial nature of the problem makes it hard for managers to generate good schedules in practice, which results in inefficiencies and delays in production. The practical importance of the problem also derives from the high-performance, durability, and versatility of prestressed precast beams. Those factors are responsible for the frequent use of such beams in a number of building types and civil structures, ranging from houses and office buildings to bridges and dams. Optimizing the production of prestressed beams has the potential effect of speeding up overall construction time, while improving the usage of molds, allowing factories to accept additional orders due to shorter lead times.

In this paper, four integer linear programming models for the multi-period production planning of prestressed precast concrete beams of multiple types are presented. The first model maximizes the usage of molds, while the second model minimizes the makespan of the entire production, the third model consists in an alternative model to minimize the makespan, and the fourth model and last one minimizes the total completion time. A set of benchmark instances for the problem are also introduced, being derived from data of a real-life application. The remainder of this paper is organized as follows. In Section 2, the related literature is reviewed. In Section 3, the HPPMBPP problem is formally defined, we argue that it is NP-hard, and the four mathematical programming models and specific sets of patterns are described. In Section 4 a total of 6 priority rules are proposed. In Section 5 the instances used in this paper are characterized, an auxiliary constraint programming model for the generation of production patterns is outlined and the computational performance of the models using different sets of patterns is evaluated. In Section 6 a case study is solved and its solutions are analyzed. In Section 7 some conclusions are derived and opportunities for further research are discussed.

2. Related Work

To the best of our knowledge, the HPPMBPP problem has not been previously studied in the revised literature. A special case of the problem has been introduced in [1] and an integer programming model has been proposed for its solution. There, it was argued that the problem is closely related to cutting stock and sequencing problems, both of which have been extensively investigated. We bring attention to the following similarities between those problems and the HPPMBPP:

1. In the HPPMBPP setting, a mold can represent a large beam of a certain type that must be cut into smaller pieces, with each piece corresponding to the beams that are produced in the mold. In this interpretation, the leftover part of the large beam corresponds to the mold’s unused capacity, rather than actual wasted material;

2. In HPPMBPP, the production might require several periods before the entire demand has been met, i.e., before all beams have been produced. Different beam types may require different curing times. The usage of the molds must be scheduled in such a way as to avoid overlapping (the same mold being used to simultaneously produce different types of beams), while respecting the maximum time allowed, or while minimizing some notion of tardiness.

Cutting stock and sequencing are among the most studied problems in the operations research literature. The one-dimensional cutting problem, in particular, bears close resemblance to the HPPMBPP, in the sense that the production in each mold can be planned (equivalently, the mold can be “cut”) independently of other molds. In what follows, we highlight some studies that tackle scheduling and cutting problems, and that we consider relevant to our study.

A variety of heuristic methods have been successfully applied to those two classes of problems. [2] suggested two heuristics for sequencing cutting patterns in the Australian glass industry and reported
substantial savings and low computing times. [3] studied the computational performance of heuristics for the one-dimensional cutting stock problem that work by exploring the neighborhood of an optimal solution to the linear relaxation of a model. The heuristics were reported to find optimal solutions for the majority of the instances tested. [4] presented a genetic algorithm (GA) for solving the one-dimensional cutting stock problem. The authors also studied three real-life scenarios arising from a steel workshop and compared the solutions (cutting schedules) obtained by their algorithm with the actual workshop cutting schedules. [5] presented three heuristic approaches to deal with an integrated pattern generating and sequencing problem. The authors considered the trade-off between the different objective functions involved and compared them in the one-dimensional cutting case. [6] proposed a multi-objective flow shop scheduling model for bespoke precast concrete production planning and used a genetic algorithm for its solution. [7] developed new solution procedures for finding efficient cutting plans while minimizing trim loss and the number of stocks used for the cutting stock problem of construction steel bars.

Studies that are solely based on exact methods as a solution procedure have also been reported. [8] proposed a new mathematical model for the cutting stock/leftover problem (CSLP). Due to the exceedingly large size of the model, the authors proposed to solve its linear relaxation via column generation and to use heuristics for constructing feasible solutions based on the relaxed solution. [9] explored an exact and compact assignment formulation for the combined cutting stock and scheduling, along with valid inequalities that are used with a cutting-plane algorithm.

Another fruitful line of work involves the use of both heuristic and exact methods in a combined solution approach. For instance, [10] solved to optimality an integrated problem that involved a cutting stock problem under particular pattern sequencing constraints. Their approach included an integer linear programming (ILP) model, a proposed decomposition scheme to solve the model, a modified subgradient method to solve the dual problem, and several heuristic algorithms. [11] formulated a mixed-integer mathematical model for solving the combined cutting stock and lot-sizing problem in a multi-period planning scenario. The authors proposed a heuristic method based on a shortest path algorithm to minimize trim loss. [12] proposed a new column generating solution procedure for the combined cutting-stock and lot-sizing problem, combined with tree-like and sequential heuristics. [13] presented three approaches for solving the one-dimensional cutting stock problem: a genetic algorithm, a linear programming model, and an ILP model. The authors studied three real-life case studies from a steel workshop. [14] proposed an exact ILP formulation for the cutting stock problem with due dates with the aim of minimizing a combination of the number of objects cut and weighted tardiness. The authors developed primal heuristics, upper bounds, and an implicit enumeration scheme.

The production of precast items has also been previously considered from the optimization viewpoint. As an example, [15] optimized a production project of precast items via a mixed integer linear programming model based on grouping concepts and a recursive procedure. [16] approached the problem of scheduling precast production considering six steps: mold assembly, placement of reinforcement and all embedded parts, concrete casting, curing, mold stripping, and product finishing. The authors developed a mathematical model and a multi-objective genetic algorithm to solve it. [17] dealt with the optimization of resources and costs for the precast production of complex configurations by means of a mixed ILP model based on prefabrication configuration and component grouping ideas. [18] made a study in precast production proposing a model for the Flowshop Problem of Multiple Production Lines and developed a genetic algorithm for the problem optimization. The authors identify several objective functions and optimization constraints, although only the optimization objective of makespan minimization was used to simplify the comparisons of the proposed approach. [19] proposed an ILP model for optimizing precast production planning, allocation of component storage, and transportation, as well as for making timely adjustments for contracted projects, with the aim of minimizing production costs.

Additionally, some studies have tackled the cutting stock problem considering due dates or multiple periods. For example, [20] developed heuristics and two two-dimensional cutting stock models with due date and release date constraints, in which meeting orders’ due dates are more important than minimizing the waste of materials. [21] proposed a non-linear optimization model for the combined cutting-stock and lot-sizing problem and suggested several heuristics for finding feasible solutions. [22] proposed new optimization models for solving the cutting stock problem when orders have due dates. The authors solved the models
via column generation, with the corresponding pricing problems solved with shortest path algorithms. \[1\] proposed an integer programming model for the multi-period production planning of precast concrete beams. The proposed model, however, handled the simplest case, in which all beams are of the same type, or, equivalently, have unitary curing time.

3. Problem Statement

The HPPMBPP consists in finding a feasible production planning to cast certain quantities of prestressed precast concrete beams, possibly of different types, while minimizing the total unused capacity of the molds, i.e., the total idle capacity. We also consider alternative objective functions that accounts for the number of periods required to fulfill the demand and total completion time. In order to formulate the problem, we define a set of parameters as follows:

- \( M \): number of molds in which the beams are produced;
- \( T \): number of available periods to complete the production;
- \( C \): number of beam types;
- \( q_c \): number of distinct lengths of beams of type \( c \), with \( c = 1, \ldots, C \);
- \( l(c,1), \ldots, l(c,q_c) \): real numbers corresponding to the actual lengths of beams of type \( c \), with \( c = 1, \ldots, C \);
- \( d(c,k) \): demand for beams of type \( c \) and length \( l(c,k) \), with \( c = 1, \ldots, C \) and \( k = 1, \ldots, q_c \);
- \( t_c \): integer number corresponding to the curing time (in terms of periods) of beams of type \( c \), for \( c = 1, \ldots, C \);
- \( L_m \): real number corresponding to the capacity of the \( m \)-th mold, with \( m = 1, \ldots, M \).

Each mold can only be used to cast one type of beam at a time. It is possible, however, to simultaneously cast beams of different lengths in the same mold, as long as they are of the same type. The total length of the beams produced during a given period in the \( m \)-th mold cannot be greater than \( L_m \) and the total number of days required to complete the production cannot be greater than \( T \). An example of a feasible production plan is shown in Figure 1. The optimization task consists of computing a production plan that minimizes the sum of unused capacities over all molds and periods.

![Figure 1 about here.]

The HPPMBPP is a combinatorial problem that arises in practical scenarios. Finding an optimal solution to the problem can become a challenging task, as soon as parameters such as the numbers of beam types, lengths and demands increase beyond trivial values. Nevertheless, and despite the similarities between the HPPMBPP and cutting problems, the problem does not precisely fit any existing formulation in combinatorial optimization. We can, however, establish the hardness of the problem:

**Proposition 3.1.** HPPMBPP is NP-hard.

This assertion can be made due to the fact that HPPMBPP includes, as a particular case, the classical one-dimensional cutting stock problem. Indeed, the case in which there exists only one beam type (i.e., all beams have the same number of cables and the same curing time) turns out to be precisely an instance of the one-dimensional cutting stock problem: the items to be cut correspond to the molds, while the waste of material is equivalent to the unused capacity of each mold. The 1-dimensional cutting stock problem has been known to be NP-hard from the fact that the knapsack problem is reducible to it \[23\].

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In this section, four models that extend the model proposed by \cite{1} are described for the case of multiple beam types. In this scenario, different beam types can demand different curing times, unlike the problem treated by \cite{1}. Moreover, a mold cannot be used to simultaneously produce beams of different types.

If beams of type $c$ are produced in the $m$-th mold at a given period, it is possible to describe the current state of the mold as a non-negative integer tuple $(a_1, a_2, \ldots, a_{q_c})$, with each $a_k$ ($1 \leq k \leq q_c$) specifying the quantity of beams of length $l(c,k)$ that are currently being produced in the mold. The information of the beam type and the tuple that describes the quantity of each beam length produced — i.e., the pair $(c, (a_1, a_2, \ldots, a_{q_c}))$ — will be called a pattern. Naturally, only patterns that do not exceed the $m$-th mold capacity can be produced in that mold. Thus, as a practical matter, we can limit ourselves to taking into consideration only patterns that do not exceed the largest capacity among the molds in the problem’s data.

A solution for the HPPMBPP requires fully specifying the pattern that is used in each mold during each of the $T$ periods, with the same pattern being potentially used more than once. The existence of a special pattern $P_0$ is assumed, which is used to denote that a mold is currently being used for the casting of a pattern that began in a previous period and whose production extends at least up to the current period. Since the curing time of each beam type can be different, it is necessary to include constraints in the model that identify the patterns associated with the consecutive periods during which a particular pattern is under production. When our model selects pattern $P_i = (c, (a_1, a_2, \ldots, a_{q_c}))$ to be initiated in the $m$-th mold at period $t$, it will accordingly select pattern $P_0$ to be used in that mold during the subsequent periods $t + 1, \ldots, t + t_c - 1$.

For instance, consider that a mold $m$ is used to initiate the production of beams of curing time 3 at period 5. Then, the pattern corresponding to the production of those beams must be assigned to period 5 while $P_0$ must be assigned to the subsequent periods in $m$: 6 and 7. This fully describes the state of the mold during periods 5, 6, and 7.

In order to refer to specific information on a given pattern $P_i = (c, (a_1, \ldots, a_{q_c}))$, we define the following notation:

- $P_i(c,k)$: number of beams of type $c$ and length $l(c,k)$ that pattern $P_i$ includes. If $c = \bar{c}$, then $P_i(c,k) = \bar{a}_k$, with $k \in \{1, \ldots, q_c\}$; otherwise, $P_i(c,k) = 0$, for any $k$.

- $u(P_i)$: capacity used by $P_i$, i.e. $u(P_i) = \sum_{k=1}^{q_c} l(\bar{c}, k) \cdot P_i(\bar{c}, k)$.

- $E_i$: number of periods required to produce the beams in $P_i$. This number equals the quantity of consecutive periods in which $P_i$ remains occupying a mold and is precisely the curing time of beams of type $\bar{c}$, given by $t_c$.

- $F_i^m$: idle capacity of the $m$-th mold when pattern $P_i$ is used in that mold. Note that this quantity depends on the lengths specified in the pattern, the mold capacity, and the value of $E_i$. $F_i^m$ can be computed as follows: $E_i \cdot (L_m - u(P_i))$. For instance, if the capacity of the $m$-th mold is 10, the capacity used by pattern $P_i$ is 6, and $E_i = 3$, then we have $F_i^m = 3 \cdot (10 - 6) = 12$.

Both $E_i$ and $F_i^m$ can be directly calculated from the problem’s data. The value of $F_i^m$, associated to the empty pattern $P_0$, is defined as zero. However, it is possible to envision variants of the formulation proposed here, in which alternative values for $F_0^m$ are used, depending on the particular objective function to be optimized.

A remark concerning the use of the $P_0$ pattern is in order. Note that an idle mold (in other words, a mold that is not being used during a specific period) is not assigned the pattern $P_0$. In fact, it has no pattern assigned to it. Moreover, this type of situation is not regarded as a loss. On the other hand, when a mold is used to initiate the production of pattern $P_i$ at period $t$, the subsequent $E_i - 1$ periods are assigned the empty pattern $P_0$. This situation results in a total loss of $F_i^m$, which corresponds to the unused capacity of the mold, multiplied by the number of days required for the production of $P_i$.

Given a set of patterns $\{P_1, \ldots, P_r\}$, not including the empty pattern $P_0$, we define the following sets:
• $Q(m)$: set containing the indices of the patterns whose capacity does not exceed the capacity of the $m$-th mold: $Q(m) = \{ i \in \{1, \ldots, r \} : u(P_i) \leq L_m \}$, for $m = 1, \ldots, M$. The same pattern can be used in different molds of potentially distinct lengths.

• $S(j)$: set of indices of the patterns that have curing time $j \in \{1, \ldots, R\}$, with $R$ being the largest curing time of all beam types present in the problem instance.

• $Q^*(m)$: set $Q(m)$ including pattern $P_0$, i.e. $Q^*(m) = Q(m) \cup \{0\}$.

Our models involve the binary decision variables $x_{i,m,t}$, for $i = 1, \ldots, r$, $m = 1, \ldots, M$, $t = 1, \ldots, T$, each of which is associated with the use of pattern $P_i$ in the $m$-th mold during period $t$, as follows:

$$x_{i,m,t} = \begin{cases} 1, & \text{if pattern } P_i \text{ is initiated in the } m\text{-th mold at period } t; \\ 0, & \text{otherwise.} \end{cases}$$

In a scenario of uninterrupted production, exceeding the prescribed demand is typically not a problem, since it is possible to keep a stock of spare beams. In view of that, the model presented next satisfies the demand with the possibility of surplus. In addition, in a real-life scenario, it might be desirable to use only patterns that have a minimal percentage of occupation of the molds. If we limit ourselves to using those types of patterns, it may become impossible to satisfy the demands at equality (it could be necessary to use extremely simple patterns to achieve equality). Therefore, the choice of satisfying demands with the possibility of excess seems to be of practical value.

3.1. Model for minimizing idle capacity

We now introduce our main model for the HPPMBPP as follows:

$$(M1) \quad \begin{align*}
& \text{minimize} \\
& \sum_{m=1}^{M} \sum_{i \in Q(m)} \sum_{t=1}^{T} x_{i,m,t} P_{i}^{m} \tag{1}
\end{align*}$$

subject to

$$\sum_{i \in Q^*(m)} x_{i,m,t} \leq 1, \quad m = 1, \ldots, M, \quad t = 1, \ldots, T \tag{2}$$

$$\sum_{m=1}^{M} \sum_{i \in Q(m)} \sum_{t=1}^{T-E_i+1} P_{i}(c,k) x_{i,m,t} \geq d(c,k), \quad k = 1, \ldots, q_c, \quad c = 1, \ldots, C \tag{3}$$

$$(E_i - 1) x_{i,m,t} \leq \sum_{\alpha=1}^{E_i-1} x_{0,m,t+\alpha}, \quad m = 1, \ldots, M, \quad t = 1, \ldots, T - E_i + 1, \quad i \in Q(m) \tag{4}$$

$$x_{0,m,t} \leq \sum_{\beta=2}^{R} \sum_{j=\beta}^{R} \sum_{i \in Q(m) \cap S_j} x_{i,m-t+\beta+1}, \quad m = 1, \ldots, M, \quad t = 1, \ldots, T \tag{5}$$

$$x_{i,m,t} \in \{0, 1\}, \quad m = 1, \ldots, M, \quad t = 1, \ldots, T, \quad i \in Q(m) \cup \{0\}. \tag{6}$$
The minimization of objective function (1) has the intent of reducing the total idle capacity of molds and, consequently, concrete waste in used molds. Constraints (2) ensure that at most one pattern shall be assigned to mold \( m \) at period \( t \), with the possibility of this pattern being the empty one, \( P_0 \). Constraint set (3) requires that all demands be satisfied. Constraints (4) force that, if pattern \( P_i \) is initiated at period \( t \), then the next \( E_i - 1 \) periods shall have the pattern \( P_0 \) assigned to them (the right-hand side of the constraint remains unconstrained, in case \( x_{i,m,t} = 0 \)). Constraints (5) establish that the empty pattern shall only be used in the \( m \)-th mold if there is some pattern \( P_i \) associated with a previous period in the same mold, such that \( P_i \)'s production has not yet been completed. Constraints (6) define the domain of the decision variables.

3.2. Model for minimizing the makespan

In model (M1), minimizing the objective function (1) could cause some molds to be unnecessarily filled, particularly in the final periods of production, since the goal was to reduce waste. The next model switches the focus from waste to the time required to fulfill the demand. It makes use of an alternative objective function that captures the number of uninterrupted periods during which at least one mold is used before the production of all the demanded beams is completed. This corresponds to the criterion often used in scheduling problems that measures the time of completion of all jobs, or makespan.

In order to express the minimization of makespan in the model, we introduce another type of decision variable associated with the fact that there is at least one mold used at period \( t \):

\[
z_t = \begin{cases} 
1, & \text{if at least one mold is used at period } t, \text{ for } t = 1, \ldots, T; \\
0, & \text{otherwise.}
\end{cases}
\]

The following model is a specialization of model (M1) and requires the minimization of the makespan:

\[
\text{(M2) minimize} \quad \sum_{t=1}^{T} z_t \quad \text{subject to} \quad \begin{align*} 
\text{(2)} - \text{(6)} \\
M z_t & \geq \sum_{m=1}^{M} \left( \sum_{i \in Q^*(m)} x_{i,m,t} \right), & t = 1, \ldots, T \\
\sum_{i \in Q^*(m)} x_{i,m,t} & \geq \sum_{i \in Q^*(m)} x_{i,m,t+1}, & m = 1, \ldots, M, \\
z_t & \in \{0,1\}, & t = 1, \ldots, T.
\end{align*}
\]

Model (M2) includes two additional sets of constraints. Constraint set (8) ensures that \( z_t = 1 \) when period \( t \) is used for the production of any beam, for each period \( t \). Note that the periods in which only the empty pattern is used are taken into account by the correspond constraint from (8), since those patterns correspond to actual production of beams. If no pattern is assigned to any mold during period \( t \), then the corresponding variable \( z_t \) is not constrained. Since the model minimizes (7) then \( z_t \) will be set to zero whenever (8) does not impose \( z_t = 1 \).

Minimizing (7) subject to (2)-(6) and (8)-(10) effectively minimizes the number of days in which production takes place. However, a solution satisfying those constraints might still involve periods of inactivity (i.e., periods in which the production is interrupted), followed by periods of activity. This means that the time of production of the latest beam produced is not necessarily as early as possible. In order to properly capture the makespan of the production plan and accomplish its minimization, we use constraint set (8):
once the production is interrupted in the $m$-th mold at period $t$, it never resumes. Thus, the complete model minimizes the number of days in which production takes place, while ensuring that those days are contiguous.

A desirable property of model (M2) in the scenario of continuous production is that, once a mold becomes idle, it can be used to start the production of beams to satisfy a demand that was not yet available during the scheduling of the current production plan.

Model (M2) also can be formulated in an alternative way using variable $z$ as an integer variable that defines the makespan in model (aM2):

**(aM2)** minimize

$$ z $$

subject to

$$ (2) - (9) $$

$$ z \geq t \sum_{i \in Q^*(m)} x_{i,m,t}^t, \quad t = 1, \ldots, T, $$

$$ m = 1, \ldots, M $$

$$ z \in \{1, \ldots, T\}. $$

Constraints (12) with objective function (11) minimization state that $z$ is equal to the index of the last period used to produce beams.

3.3. Model for minimizing the total completion time

Although model (M2) generates solutions with less molds unnecessarily filled than model (M1), it still might return solutions with a great amount of unnecessary beams, i.e. more beams than the demand, which have to be stocked. Then, one way of avoiding unnecessary beams is minimizing total completion time to achieve the demand. In order to do that, we define model (M3) as follows:

**(M3)** minimize

$$ T \sum_{t=1}^{T} \sum_{m=1}^{M} \left( \sum_{i \in Q^*(m)} x_{i,m,t}^t \right) $$

subject to

$$ (2) - (9), (9). $$

Objective function (14) aims at minimizing the total completion time of beam production.

3.4. Overview of the models

Model (M1) involves $O(MTr)$ decision variables and $O(q + MTr)$ constraints, with $q = \sum_{c=1}^{C} q_c$, while model (M2), (aM2) and (M3) have $O(MTr)$ variables and $O(q + MTr)$ constraints, as well. Depending on the total number of possible patterns, there may be an excessive number of variables in all models. In a practical scenario, the unavailability of certain lengths for given beam types might limit the quantity of patterns. Differently, this number can be limited by the exclusive usage of sets of patterns with specific properties.

Models (M1), (M2), and (M3) are linear and have only binary decision variables, a fact that allows for their solution via standard integer linear programming software. A competitive disadvantage of model (aM2) is that it includes one general integer variable. It is interesting to note that all models are also amenable to solution via an iterative scheme of column generation, in which the set of patterns available are generated on demand. This might prove useful when dealing with problems that admit very large numbers of patterns.
3.5. Maximal patterns

A pattern \( P_i = (c, (a_1, a_2, \ldots, a_q)) \) is defined as maximal with respect to the \( m \)-th mold if it is not possible to add any beam of type \( c \) to \( P_i \) without violating the capacity of the mold. In our models, we only used variables associated with maximal patterns in their respective molds: that is, a variable \( x_{i}^{m,t} \) will only exist in the model if \( P_i \) is a maximal pattern in mold \( m \).

After excluding variables that are associated with non-maximal patterns, there is typically a substantial reduction in the number of variables of all models. This, in turn, can improve their solution times considerably.

It is important to remark that, by excluding those variables, no optimal solution of model (M1) is lost, as the following theorem shows.

**Theorem 3.2.** Restricting model (M1) to use only maximal patterns does not modify its sets of optimal solutions.

**Proof.** Let us first note that a pattern \( P_i = (c, (a_1, a_2, \ldots, a_q)) \) that is not maximal with respect to the \( m \)-th mold can always be transformed into a maximal pattern with respect to that mold via the addition of more beams of its own type.

Without loss of generality, let us assume that \( l(c,1) < l(c,2) < \cdots < l(c, q_c) \) holds for each beam type \( c \).

Let \( \mathcal{P} \) be the set of all patterns, and let us define \( I : \mathcal{P} \to \mathcal{P} \), such that

\[
I : (c, (a_1, a_2, \ldots, a_q)) \mapsto (c, (a_1 + \delta, a_2, \ldots, a_q)),
\]

with

\[
\delta = \left[ \frac{L_m - \sum_{k=1}^{q_c} l(c,k)a_k}{l(c,1)} \right].
\]

If \( P_i \) is not maximal with respect to the \( m \)-th mold, then we have \( \delta \geq 1 \) and, therefore, \( u(I(P_i)) > u(P_i) \).

Moreover, \( I(P_i) \) is maximal with respect to the \( m \)-th mold: the leftover capacity in \( I(P_i) \) is not enough to support the addition of yet another beam to the pattern.

Let us assume, for the sake of contradiction, that \( \hat{x} \) is an optimal solution of model (M1) that uses a non-maximal pattern, i.e., \( x_{i}^{m,t} = 1 \), for some mold \( m \), period \( t \) and pattern \( P_i \) that is non-maximal with respect to \( m \). Now, let \( \hat{x} \) be a solution obtained as follows:

\[
\hat{x}_{i}^{m,t} = \begin{cases} 
0, & \text{if } \ell = s; \\
1, & \text{if } P_{\ell} = I(P_s); \\
\hat{x}_{i}^{m,t}, & \text{otherwise,}
\end{cases}
\]

for all \( t = 1, \ldots, T \). In other words, whenever pattern \( P_s \) is used in the \( m \)-th mold, we replace it with \( P_j = I(P_s) \), the maximal pattern (with respect to mold \( m \)) obtained as shown above.

Note that \( \hat{x} \) is a feasible solution to model (M1). Indeed, constraints (2) are satisfied because we only set \( \hat{x}_{i}^{m,t} = 1 \) when the corresponding variable \( x_{i}^{m,t} \) was set to 0, thereby maintaining the left-hand side of (2) unchanged with respect to \( \hat{x} \). Additionally, (3) is satisfied because \( P_j(c,k) \geq P_i(c,k) \), for all \( k \). Constraints (4) are satisfied because if \( \hat{x}_{i}^{m,t} = 1 \), then \( \hat{x}_{j}^{m,t} = 1 \) forces the same set of \( x_{j}^{0,t+\alpha} \) variables to be set to 1.

Finally, (5) is satisfied because patterns \( P_s \) and \( P_j \) have the same curing time.

If we denote by \( v(\cdot) \) the objective function value of a solution to (M1), we can write

\[
v(\hat{x}) = v(\hat{x}) = F_{s}^{m} \sum_{t=1}^{T} \hat{x}_{i}^{m,t} + F_{j}^{m} \sum_{t=1}^{T} \hat{x}_{j}^{m,t}(1 - \hat{x}_{j}^{m,t}),
\]

where the first summation concerns the periods in which pattern \( P_s \) is used in mold \( m \) in solution \( \hat{x} \), but not in \( \hat{x} \), while the second summation only takes into account the periods in which pattern \( P_j \) is used in mold \( m \) in solution \( \hat{x} \) but not in \( \hat{x} \).
Now, since $F_{m}^{s} = E_{s}(L_{m} - u(P_{s}))$, $F_{j}^{m} = E_{j}(L_{m} - u(P_{j}))$, $E_{s} = E_{j}$, and $u(P_{j}) > u(P_{s})$, we have $F_{m}^{s} > F_{j}^{m}$. Moreover, $\sum_{t=1}^{T}\hat{y}_{m,t} = \sum_{t=1}^{T}\bar{y}_{j,t}(1 - \hat{y}_{j,t})$ by construction of $\hat{x}$. Therefore, $v(\hat{x}) < v(\check{x})$, contradicting the optimality of $\hat{x}$. Thus, since $\hat{x}$ cannot use a non-maximal pattern, we have shown that restricting (M1) to use only maximal patterns does not change its set of optimal solutions.

In fact, we have just shown the following result:

**Corollary 3.3.** An optimal solution of model (M1) only uses maximal patterns.

A result that is similar to Theorem 3.2 holds for model (M2). We state it here without proof, since the argument is very similar to the one used in the proof of Theorem 3.2.

**Theorem 3.4.** Restricting model (M2) and (aM2) to using only maximal patterns does not modify their sets of optimal solutions.

Indeed, replacing a non-maximal pattern with a maximal one in an optimal solution to model (M2) or (aM2) will not have an impact on the makespan because the actual number of days used to fulfill the demand will remain unaffected, given that all patterns of a given type have the same curing time.

**Theorem 3.5.** Restricting model (M3) to using only maximal patterns does not modify its sets of optimal solutions.

As in the case of model (M2), we can see that the optimal solution value of the total completion time will not be affected after replacing a non-maximal pattern with a maximal one. Thus, we can restrict models (M1), (M2), (aM2) and (M3) to maximal patterns without affecting the optimal values of their objective functions (1), (7), (11) and (14).

### 3.6. Size-reduction heuristic

Size-reduction heuristics are solution methods that consist in solving a reduced version of the MILP model in which only a subset of variables is considered. This means that we can drastically reduce the size of the MILP model and depending on the choice of the subset we may be led to promising sub-optimal solutions in shorter execution times and less memory usage. To cite some examples, [24] proposed several size-reduction heuristics for the unrelated parallel machines scheduling problem, reducing the number of machines to only a subset of promising ones taking several criteria into consideration. [25] introduced some matheuristics to the unrelated parallel machines scheduling problem with additional resources. One of which consists in a size-reduction method named as *job-machine reduction*. Such method involves selecting only variables in which the jobs are associated to the $\ell$ “best” machines, otherwise they are removed from the MILP model.

Regarding the HPPMBPP, since the number of maximal patterns still can be too large for state-of-the-art solvers to handle the corresponding MILP model, we can select a subset of maximal patterns to solve the problem, thus not necessarily leading us to an optimal solution, or even a feasible one, for the global problem. Since the number of patterns in the problem is smaller, the number variables and constraints in the MILP model will be smaller.

We define $q_{c}$-maximal patterns as a subset of patterns that are maximal on the shortest mold from a specific instance that covers the largest number of distinct lengths of its beam type. For example, a set $q_{c}$-maximal patterns in which $q_{c} = 2$ is a set that has patterns that contain at least 2 beams of distinct lengths. If there is no pattern that covers all beam lengths of a certain type, the set $q_{c}$-maximal patterns will be composed of by patterns that covers $q_{c} - 1$ distinct beam lengths, and so on until the set of patterns covers each beam length. Since one characteristic of the problem in practice is that usually there are molds large enough to accommodate a large quantity of beams, it is highly expected that there are patterns that covers all $q_{c}$ beam lengths for each beam type.

[Table 1 about here.]
In Figure 2, we can see a Venn diagram that illustrates the distribution of patterns in instance hbp1_30_1, described in Table 1, which is used in the computational tests in this work. The sets P, MP, and QMP, represent the set of patterns, maximal patterns, and \( q_c \)-maximal patterns generated for such instance, which have cardinalities 128674, 12078, and 1732, respectively. We can see that a large quantity of patterns is discarded when we consider only \( q_c \)-maximal patterns. The methods to generate such patterns are described afterwards, in Section 5.

[Figure 2 about here.]

4. Priority Rules

In this section we propose six constructive heuristics, which we refer to as priority rules, to obtain feasible solutions for the problems under study. Each priority rule that we propose consists in, whenever a mold is freed, selecting a beam type, according to some priority measure regarding the curing time, whose demand has not been attended, and associating it to the current freed mold. Then, we fulfill the current mold with beams of the selected beam type following a second priority measure regarding beam lengths until the demand of the current beam type is achieved or the pattern associated to the current mold is maximal in such mold.

Note that each heuristic described in this section will return solutions that satisfy the demand with no beam surplus, which may lead us to solutions that are composed of patterns that are not necessarily maximal in their respective molds. Regarding this, each of the priority rules that we propose have two phases: the first phase consists in generating a solution producing all demanded beams; the second phase consists in converting each of the patterns used in such solution into maximal patterns. This phase involves filling the patterns with beams of its type from the largest one to the shortest one in matter of length, until each of the generated pattern is maximal in its respective mold.

The priority measures proposed for curing time are:

- **Shortest curing time first:** consists in selecting the beam type with the shortest curing time first among beam types that did not achieved their respective demands;
- **Longest curing time first:** consists in selecting the beam type with the longest curing time first among beam types that did not achieved their respective demands.

The priority measures proposed for beam lengths are:

- **Shortest length first:** to select the beam with the shortest length first among beam length from a given type whose demands have not yet been achieved;
- **Largest length first:** to select the beam with the largest length first among beam length from a given type whose demands have not yet been achieved;
- **Alternate lengths:** to select alternately the beam with the shortest length and the beam with the longest length whose demands have not yet been achieved;

Based on the measures described above, we name the proposed priority rules as follows:

- **Shortest curing time shortest length first** (SCTSL);
- **Shortest curing time largest length first** (SCTLL);
- **Shortest curing time alternate length first** (SCTAL);
- **Longest curing time shortest length first** (LCTSL);
- **Longest curing time largest length first** (LCTLL);
• Longest curing time alternate length first (LCTAL).

In Table 2 we describe on which priority measure the priority rules are based.

[Table 2 about here.]

5. Computational Tests

In this section, we evaluate the performance of the models proposed in Section 3 on a set of benchmark instances. We introduce a set of instances that are based on data arising from a real-life scenario. The different instances represent a sample of the variability of the problem’s parameters, such as demand, number of molds, and mold lengths. We also discuss some practical aspects of the implementation of the four models.

5.1. Pattern generation

Instead of carrying out an exhaustive enumeration, we generated the desirable patterns for a given instance using a constraint programming model. Consider the following notation:

• $K$: the largest number of different lengths among beam types, i.e. $K = \max\{q_c : c = 1, \ldots, C\}$. For example, in an instance with 2 beam types, in which type 1 has 6 distinct beam lengths and type 2 has 4 distinct beam lengths, we have $K = 6$.

• $L$: the largest capacity among the molds, i.e. $L = \max\{L_m : m = 1, \ldots, M\}$.

• $V \in \{1, \ldots, C\}$ is a decision variable that corresponds to the type of beam used by the pattern.

• $A \in \mathbb{Z}^K$: a vector of decision variables, with $A_j$ representing the number of beams of the length $\ell_j$, for all $j \in \{1, \ldots, K\}$. Note that, for a type $c$ with $q_c < k$, the components $A_{q_c+1}, \ldots, A_k$ are necessarily zero. Given a pattern $P_i$ of type $c$, the possibly nonzero components of vector $A$ correspond to $[P_i(c,j)]_{j=1}^{q_c}$.

• $P = P(V, A) = (V, (A_1, \ldots, A_{q_c}))$: the generated pattern.

• $\epsilon$: a parameter $(0 < \epsilon < L)$ that establishes a tolerance for avoiding the generation of an empty pattern, or patterns with very low capacity.

We present the model as follows:

$$1 \leq V \leq C$$
$$V = c \iff A_j = 0 \quad c = 1, \ldots, C, \quad j = q_c + 1, \ldots, K$$
$$V = c \iff \epsilon < \sum_{j=1}^{q_c} l(c,j) \cdot A_j \leq L \quad c = 1, \ldots, C.$$  \hfill (15) \hfill (16) \hfill (17)

Constraint (15) implies that the pattern type has domain $\in \{1, \ldots, C\}$. Constraint set (16) implies that if the generated pattern is of type $c$ then it includes no beam of size $l(c,j)$, such that $j > q_c$. Constraint set (17) imposes that the capacity used by the generated pattern is simultaneously larger than $\epsilon$ and no larger than the length of the longest mold. The empty pattern is, therefore, not generated and has to be manually included in the final set of patterns. We utilized the Gecode solver [26] to enumerate all the solutions of model (15) - (17).

Finally, due to the size of certain instances, we decided to generate only patterns satisfying:

$$\epsilon = \alpha - \omega.$$  \hfill (18)
with $\alpha$ equal to the capacity of the shortest mold and $\omega$ equals the length of the longest beam among all the beam types.

The pattern generator is flexible and can be conveniently calibrated to restrict the model to use only a certain subset of patterns that meet specific criteria of the decision maker. The use of patterns with a maximum number of beams to be cut (for instance, a maximum of 5 pieces in each pattern) is an example of such a criterion.

5.2. Instance generation

The instances used in this paper were randomly generated based on an existing order arising from a real-world production plant. For privacy reasons, we are not allowed to use or provide here the actual data coming from the aforementioned instance.

We implemented an instance generator using the MATLAB programming language, with parameters that allow us to vary the number of beams, the number of molds, the maximum value for the demands, as well as the number of distinct curing times, and their maximum value.

In our experiments, the mold length is fixed at 60 and the possible beam lengths are: 1.15, 2.5, 2.9, 3.05, 3.1, 3.2, 3.65, 3.8, 3.95, 4.05, 4.35, 4.6, 5.05, 5.6, 5.7, 5.95, 6, 6.45, 6.65, 6.9 and 7.15. From this set, we uniformly select a sample of 7 lengths for each type of beam and afterwards we remove the duplicate lengths. The demand of each beam length, for each type, is independently and uniformly selected from the set $\{12, \ldots, 35\}$.

The value of $T$ is initially calculated for each instance in the following way:

$$
T = \left[ \sum_{i=1}^{C} \ell_c \cdot \left( \sum_{k=1}^{q_c} l(c,k) \cdot d(c,k) \right) \right] \left( \sum_{m=1}^{M} L_m \right)^{-1}.
$$

(19)

Considering that the calculation of $T$ this way is a lower bound on the maximum number of periods necessary for the total production, the problem may well become infeasible. For this reason, if $T$ was less than or equal to 10, we increased its value in one unit; otherwise, we increased the value of $T$ by 10 percent.

We avoided the calculation of a guaranteed upper bound, because in case such a bound was not particularly tight, its value could significantly increase the number of decision variables in our models, hindering their computational solution via standard software for integer linear programming.

5.3. Experimental evaluation

All computational tests carried out on this subsection were performed on an Intel (R) Core i5-3470 CPU @ 2 x 3.20GHz processor, with 8 GB RAM, running 64-bit Windows 7 Professional. The software used for implementing the models were Gecode 5.0.0 (for pattern generation) and IBM ILOG CPLEX Optimization Studio 12.7.1, using the C++ programming language with IDE Microsoft Visual Studio 2015 (for solving the integer linear programming models). Note that constraints \[2\] can be implemented in Concert technology as SOS1 constraints. Carrying out preliminary tests we concluded that there were no significant gains on such implementation for our problem as compared to the linear formulation. Therefore we decided to implement such constraints as classical linear constraints in every experimental evaluation described in this work.

For the computational tests in this subsection we used four groups of instances, each containing 10 instances, classified as follows:

1. Instances with 1 beam type, with $E_1 = 1$;
2. Instances with 2 beam types, with $E_1 = 1$ and $E_2 = 2$;
3. Instances with 3 beam types, with $E_1 = 1$, $E_2 = 2$ and $E_3 = 3$;
4. Instances with 4 beam types, with $E_1 = 1$, $E_2 = 2$, $E_3 = 3$ and $E_4 = 4$;
Instances of group 1 have names starting with “hbp1”, and instances of groups 2, 3, and 4 are named “hbp2”, “hbp3”, and “hbp4”, respectively. Each group is divided into two subgroups, each one containing 5 instances. The first subgroup contains instances with 15 molds while the second includes instances with 30 molds. The full name of each instance reflects its group, number of molds, and a sequential number (from 1 to 10) that identifies its subgroup.

All models were solved using only variables corresponding to maximal patterns in their respective molds. The results obtained with each model are presented in terms of best value of the objective function and total running time in seconds, for each instance. The number of molds, the maximum number of production periods, and the number of patterns for each instance are also shown in the table. The time limit used for solving each instance with CPLEX was 3,600 seconds. We did not impose a limit on the time taken to generate patterns with Gecode, since this preprocessing phase requires very short times as compared to solving the integer programming model. The running times shown in the tables correspond to the sum of the CPLEX running times.

We define the notation used in the test tables in this subsection as follows:

- LPv: Value of objective function of the linear relaxation problem;
- LPt: Running time of linear relaxation problem in seconds;
- IPv: Value of objective function of the integer linear problem;
- IPt: Running time of the integer linear problem in seconds;
- Gap: Gap between the best integer objective and the objective of the best node remaining (0% means an optimal solution);
- BCn: Number of branch-and-cut nodes used by CPLEX.

The objective function values corresponding to the entries with “3,600” in the “IPt” columns are not necessarily optimal. In those cases where the value of gap is nonzero, the instance was not solved to optimality within the prescribed maximum solution time. For this reason, the objective function values reflect the best solution found within 3,600 seconds of running time.

Some instances were not solved by some of the models. Those cases are marked with “–”. Two possible scenarios were documented: (i) the amount of memory available was not enough to support CPLEX’s solution procedure, and the optimization process was interrupted before finding a feasible solution; or (ii) the time limit of 3,600 seconds was exceed before a feasible solution was found. It is important to note that patterns were successfully generated for all instances in our tests. Thus, any memory-related interruption occurred as part of CPLEX’s solution process.

The number of patterns vary widely, and there seems to be no directly relation between that number and $M$ or $T$, although the larger the molds are the more patterns we have since there are more combinations of beam lengths that can fit into the molds. The number of patterns can also be affected by the randomly chosen lengths of beams.

We can see in Table 3, for each instance, the number of molds in column $M$, the number of time periods in column $T$, the number of patterns generated that are maximal in molds of length 60 plus pattern $P_0$ in column Maximal patterns, and the number of patterns which covers all lengths for their type that are maximal in molds of length 60 plus pattern $P_0$ in column $q_e$-maximal patterns.

The means of number of patterns generated for type 1, type 2, type 3 and type 4 are 4451.30, 8734.60, 14925.70 and 15979.70, respectively, and the standard deviations are 4451.30, 8734.60, 14925.70 and 15979.70. We can see that the number of patterns reduction when we adopt exclusively $q_e$-maximal patterns is enormous, with an average reduction of 84.45%. Thus we can see that, for the instances that we generated for this work, the more types we have the greater will be the number of patterns, but high standard deviations lead us to deduce that not only the number of types influences the number of patterns, but the number of beams for each, as well as their lengths (which are randomly generated).
5.3.1. Computational tests with maximal patterns

In these experiments we carry out computational tests for the models (M1), (M2), (aM2) and (M3) proposed in this paper considering all maximal patterns plus pattern $P_0$ for the generated instances.

In Table 4 we can see that almost all instances were solved to optimality within the time limit of 3,600 seconds. With instance hbp4_30_3 we could only obtain the linear relaxation solution. Note that the only integer solution found with a value different than zero was with instance hbp3_30_4, what can be explained by the fact that since the molds are much bigger than the beam lengths and usually there are many different beam lengths, this situation can result in many combinations between them leading to a high probability to have patterns with an associated waste zero. It is also interesting to observe that only instance hbp4_15_3 was not solved in the root of the branch-and-cut tree by the solver. We can also see that there is a trend that the more patterns there are in an instance the more difficult it is to solve it.

5.3.2. Computational tests with size-reduction heuristic

In these experiments we carry out computational tests for the models (M1), (M2) and (M3) using the size reduction approach (SRH) proposed in this paper, that consists in considering only $q_c$-maximal patterns plus pattern $P_0$ on the group of generated instances. We observe the behavior of solutions using this method and compare to the models considering all maximal patterns plus pattern $P_0$.

In Table 5 we can see that 12 instances could not be solved to optimality within the time limit of 3,600 seconds by model (M2); 3 of them could not even be solved to an integer solution. The largest gap among the instances was 100% and for the vast majority of instances CPLEX did not use more than the root node in the branch-and-cut procedure. Also in Table 5 we can see that 15 instances could not be solved to optimality within the time limit of 3,600 seconds by model (aM2), 14 of them could not even be solved to a feasible integer solution. No integer solutions were found for none of instances of subgroup 4. These results led us to infer that model (M2) is better for solving the problem considering makespan minimization for this group of instances than model (aM2).

In Table 6 we can see that 28 instances could not be solved to optimality within the time limit of 3,600 seconds by model (M3) and only 2 of them could not even be solved to a feasible integer solution. The largest gap was 56.52% with instance hbp4_15_5. Unlike the other models, for the majority of instances the solver used more branch-and-cut nodes than only the root.

The general outlook revealed by the experiments suggests that all models can be used for the exact solution of instances with a few types of beams and a limited number of patterns. However, as those parameters increase, solving the models can become quite difficult. The size of the models increases relatively fast with the parameters, demanding the use of substantially more memory, or slowing down the solution process in a significant way.

5.3.2. Computational tests with size-reduction heuristic

In these experiments we carry out computational tests for the models (M1), (M2) and (M3) using the size reduction approach (SRH) proposed in this paper, that consists in considering only $q_c$-maximal patterns plus pattern $P_0$ on the group of generated instances. We observe the behavior of solutions using this method and compare to the models considering all maximal patterns plus pattern $P_0$.

We can see in Table 7 that from the 39 instances solved to optimality with all maximal patterns by model (M1) we got the optimal solution for the 25 instances with SRH. It is important to see that we were also able to find the optimal solution for instance hbp4_30_3, the instance for which we could not even find a feasible integer solution solving by model (M1) with all maximal patterns. We can infer that such a solution is a global optimum for the whole problem since the idle capacity cannot be negative.

We can see in Table 8 that from the 31 instances that were solved to optimality by model (M2) with all maximal patterns we could find the optimal solution for 24 of them using the SRH. For the 9 instances
that could not be solved to optimality by model (M2) with all maximal patterns we were able to find better objective functions values for 6 instances and for the other 3 we found equal makespans using the SRH. Also, using the SRH, we were able to obtain feasible solutions for the 3 instances that could not be solved to a feasible solution by model (M2) with all maximal patterns.

[Table 8 about here.]

We can see in Table 9 that from the 12 instances that were solved to optimality by model (M3) with all maximal patterns we could find the optimal solution for 11 instances only using the SRH. From the 26 instances that could not be solved to optimality by model (M3) with all maximal patterns we found equal or better objective function values for all them, with strictly better objective function values for 14 of them. Using the SRH, we were also able to find feasible solutions for the 2 instances that could not be solved to a feasible solution by model (M3).

[Table 9 about here.]

In general we can see that using the SRH with only \( q_c \)-maximal patterns is a promising way of exploring the problem leading to good feasible solutions and much less running time and/or memory usage. This set of patterns could be also used as an initial set of columns in a column decomposition approach to get an optimal solution to the problem as a whole.

5.3.3. Comparing Solutions Obtained via Mathematical Models and Priority Rules

Regarding solutions obtained by priority rules in matter of idle capacities, we can see in Table 10 that all priority rules had significantly inferior solution quality compared to the ones obtained via mathematical model, even though the elapsed time to get to these solutions was insignificant, < 0.00 seconds. In none of the instances the priority rules found neither an equal nor a best value of idle capacities than those found by models (M1) or the SRH version of model (M1). Note that in the tables in this subsection \( nBV \) means the number of instances for which the solution method found the best objective function value, not necessarily optimal.

[Table 10 about here.]

With respect to solutions obtained by priority rules in matter of makespan, we can see in Table 11 that some of priority rules had makespans more similar to the ones obtained via mathematical model. The heuristic LCTAL was the best priority rule for these computational tests, identifying the best makespan among solution methods for 36 instances, being inferior only to model (M2) using the SRH, which found 37 best solutions regarding the makespan.

[Table 11 about here.]

Considering solutions obtained by priority rules in matter of total completion times, we can see in Table 12 that the priority rules often have had higher total completion time values than models (M3) and the SRH version of model (M3). The heuristics which found the best solution regarding total completion time were SCTAL and LCTAL, each identifying the best value for 19 instances. Model (M3) using SRH was by far the best solution method for this purpose, finding the best value for 38 instances. Note that for instance hbp4_30_4, the heuristic methods found better solutions than the mathematical model. However, after an analysis of variance we saw that \( H_0 \) was not rejected, thus there is no significant difference among the priority rules with \( p\)-values 0.99, 0.77 and 1 for idle capacities, makespans, and total completion times, respectively.

[Table 12 about here.]
6. Case Study

In this section, we present a case study with an instance that was generated based on real life data that for reasons of industrial secret could not be provided for tests. It is important to remark that this instance is not an actual client order, but all its data are very similar to the data that we could find in practice. The instance generated, termed Instance 1, is defined in Table 13.

Rasterized table: Table 13

Instance 1 admits 1047 maximal patterns on forms of length 60m were generated plus pattern $P_0$, resulting in a total of 1048 patterns. Regarding only $q_c$-maximal pattern, there were 381 patterns generated plus pattern $P_0$, 382 totally. In the Gantt charts in this section, each component represents a pattern characterized by a label that represents its index and a color which states its type.

To evaluate the performance of the solutions obtained we used the following indicators:

- Total capacity: the sum of capacity of all molds available along the time horizon, measured in meters;
- Idle capacity: the sum of idle capacity of all molds along the time horizon, measured in meters;
- Productive capacity loss: the percentage of idle capacity along the time horizon;
- Concrete waste: concrete in the molds that where not used for beam production, measured in meters;
- Beam surplus: number of beams that were produced in addition to the quantity demanded.

Note that the total concrete waste multiplied by the number of periods in which such concrete were in the molds is equal to the total idle capacity.

6.1. Solving the case study with all maximal patterns

After solving Instance 1 by model (M1) we get the feasible solution in Figure 3, with objective function value of 1.05m total idle capacity, in 3,600 seconds with a gap of 14.19% and 0.35m of real waste of concrete. We can see in Table 14 that all molds were well used, 99.97% of their capacity was used along all time periods available. A large quantity of beam surplus, i.e. more beams than the ordered demand, were fabricated. Details can be seen in Table 15.

Rasterized tables: Table 14, Table 15

When solving Instance 1 with model (M2) we get the optimal solution shown in Figure 4, with objective function value of 3 periods, in 5.7 seconds. The idle capacities in the molds on the periods 1, 2, 3 and 4, respectively, were 20.15m, 19.6m, 18.75m and 0m. As we can see in Table 16, the molds were not so efficiently used as in the solution of model (M1), but we got a reduction of 1 time period for the demand production. 97.83% of mold capacity was used along 3 time periods out of the 4 time periods available. We got 30.1m waste of concrete for this solution and 58.5m of idle capacity. Unlike model (M1), the solution of model (M2) produced a small quantity of beam surplus, as we can see in Table 17.

Rasterized figures: Figure 3, Figure 4

Rasterized tables: Table 16, Table 17
When solving Instance 1 with model (M3) we get the optimal solution shown in Figure 5 with objective function value equal to 45 periods, in 1990.8 seconds. As we can see in Table 18 the molds in the solution of model (M3) were almost so efficiently used as in the solution of model (M1). 99.86% of mold capacity was used along the 4 time periods available. We got 1.9m of waste of concrete for this solution and 5.2m of idle capacity, but the solution was unbalanced compared to the solutions of models (M1) and (M2). Like model (M2), model (M3) solution produced a small quantity of beam surplus, 7 beams more were produced in comparison to model (M2), as we can see in Table 19.

6.2. Solving the case study with size-reduction heuristic

Regarding the solution for the case study obtained with using model (M1) with the SRH, which we can see in Figure 6 we got an objective function value of 1.35m total idle capacity, after 3,600 seconds of running time with final gap of 18.52%. It involved 0.45m of waste of concrete. In Table 20 the molds were efficiently used, 99.96% of their capacity was used along all time periods available. A large amount of beam surplus was produced, see Table 21.

In Figure 7 we can see the solution returned by model (M2) using the SRH for the case study. The solution has a makespan value of 3 periods, having required 2.32 seconds of running time and showing a final gap of 0%. It involved 27.85m of waste of concrete and 53.85m of idle capacity along the time horizon of 3 periods. In Table 22 the molds were not as efficiently used as model (M1) using the SRH, 98.01% of their capacity was used along all time periods available. A small quantity of beam surplus was produced, see Table 23 only 1 unit of beam of length 4 from type 1, and 1 unit of beam of length 4 from type 3.

In Figure 8 we can see the solution returned by model (M3) using the SRH for the case study. The solution has a total completion time value of 45 periods, after 18.01 seconds of running time with final gap of 0%. It involved 7.20m of waste of concrete and 13.90m idle capacity along the 4 periods time horizon. In Table 24 the molds were almost as efficiently used as model (M1) using the SRH, 99.61% of their capacity was used along all 4 time periods available. A small quantity of beam surplus was produced, detailed in Table 25.
6.3. Comparing solutions obtained with models and priority rules

As we can see in Table 26, there was a significant reduction on execution time when we solved the models using the SRH. We also observed empirically a large reduction in memory usage: since the number of patterns is less numerous, the number of variables and constraints are drastically reduced making the model much smaller than the one with all maximal patterns. Regarding solution quality of model using SRH, there was not a significant difference as compared to the complete model. The makespans and total completion times were all the same for each model using SRH or not, changes were only detected on idle capacities and concrete waste.

Regarding the priority rules, we can see that their solutions were very much alike among each other. Although they spend an insignificant execution time, the quality of solution in terms of idle capacities and concrete waste are too low when compared to those of models (M1) and (M3). We can see that the quality of solutions obtained regarding makespan and total completion time are not so low when compared to those of the mathematical models. Nevertheless, none of the priority rules could find neither the optimal makespan nor the best total completion time found by the models for Instance 1.

6.4. Symmetry breaking constraints

As we can see from the solutions of all proposed models, there may be many symmetric solutions. For example, if we change the order of production of the patterns in mold 13 of the solution shown in Figure 3 we would get the same solution with a different representation. We call this a period-based symmetry. The same occurs when we move the patterns produced in a mold to another one, for example, if we move the produced patterns on mold 14 to the mold 15 and vice versa of the solution shown in Figure 3 we would get the same solution with a different representation. We call this a mold-based symmetry. In order to circumvent this problem and and, consequently, improve the running times of the models we define two sets of constraints for breaking both period and mold-based symmetries:

\[
\sum_{i \in Q^*(m)} i x_{i}^{m,t} \leq \sum_{i \in Q'(m)} i x_{i}^{m,t+1} + P \cdot x_{0}^{m,t+1}, \quad m = 1, \ldots, M, \ t = 1, \ldots, T - 1
\]  

\[
\sum_{t=1}^{T} \sum_{i \in Q^*(m)} i t x_{i}^{m,t} \leq \sum_{t=1}^{T} \sum_{i \in Q'(m+1)} i t x_{i}^{m+1,t}, \quad m = 1, \ldots, M - 1.
\]

Constraints (20) state that patterns will be fabricated in a crescent order of indexes in the molds. Constraints (21) define that molds will be used for patterns production in a crescent order defined by the value of \(\sum_{t=1}^{T} \sum_{i \in Q^*(m)} i t x_{i}^{m,t}\) for each mold \(m\). Thus, it is easy to see that Proposition 6.1 is true.

**Proposition 6.1.** Addition of symmetry breaking constraints does not modify the optimal solution value of any of the proposed models.

Although considering symmetric solutions increases drastically the problem’s search space and symmetry breaking constraints reduce the number of solutions greatly, we noted, during preliminary computational tests, that the usage of these constraints made the models much bigger and hard to solve. The models required greatly more memory and execution time than before and their insufficient solution performance led us to not consider such constraints in our larger computational tests.

7. Conclusions

In this paper, we introduced a novel variant of cutting sequencing problems, called HPPMBPP, along with four integer linear programming models for its solution, one for idle capacity minimization, two for makespan minimization, and one the total completion time minimization.
We proposed a size reduction heuristic consisting in the reduction of the number of patterns, thus cutting down, drastically, the number of constraints and variables, which led us to lighter and easier to solve problems. We also created 6 priority rules based on classic scheduling constructive heuristics as a way of finding feasible solutions really fast. With big-size instances they showed to be a good method compared even to the mathematical models, which sometimes spend too much time and/or memory to get to feasible solutions. Such constructive heuristics can be a promising way to find initial solutions for some metaheuristic method.

We proposed a set of randomly generated benchmark instances for the HPPMBPP, all of which are based on data arising from a real-life application. Computational tests were performed using off-the-shelf optimization software, in order to assess the effectiveness of the mathematical models. The results suggest that model (M1) was relativity easier to solve compared to the other models, since as the diversity of beam lengths increases, so does the likelihood of the model finding a solution with zero idle capacity. Thus, in practice, simply filling the molds to the maximum capacity is not so attractive since a lot of beams produced will be stocked and the production line will not be so flexible with respect to possible changes in the plan.

As regards the models that minimize makespan and total completion time, in general, they were able to solve the great majority of instances with up to two types of beams to optimality within short computing times. When solving instances with 3 or 4 types of beams, the models tended to require excessive memory and/or running time. Tests carried out using the size reduction heuristic with a specific type of patterns showed to be an interesting way of exploring the proposed models presenting good solutions with less memory and running time requirements.

In the case study we were able to find good solutions for the instance explored with mathematical models using SRH or not. Model (M1) tends to find solutions with extremely low idle capacity and concrete waste, thus with a high makespan an total completion time. Model (M2) found solutions with an optimal makespan and optimal total completion time, and higher idle capacities than model (M1), around 2% of total capacity loss, which is still low compared to industry loss that is around 5%-10%. Model (M3) found the optimal total completion and provided good solutions in terms of molds usage. The production line tended to be nevertheless excessively unbalanced aggravating the makespan.

Natural directions for future work, in the context of exact models, entail the use of more sophisticated solution approaches (such as column generation), or different modeling strategies. Alternatively, the development of heuristic algorithms for producing high-quality solutions can be useful when handling large instances. The practical nature of the problem also suggests the study of variants of the HPPMBPP, possibly including scheduling constraints, or the reuse of leftover material.

8. Data Availability Statement

Data generated or analyzed during the study are available from the corresponding author upon request.

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Figure 1: Example of feasible solution. Three beam types, of various lengths, produced in three molds.
Figure 2: Patterns distribution for instance hbp1_30_1
Figure 3: Case Study: solution obtained with model (M1)
Figure 4: Case Study: solution obtained with model (M2)
Figure 5: Case Study: solution obtained with model (M3)

| Time | Type 1 | Type 2 | Type 3 |
|------|--------|--------|--------|
| 1    | 511    | 781    | 781    |
| 2    | 511    | 781    | 781    |
| 3    | 511    | 781    | 888    |
| 4    | 511    | 781    | 888    |
| 5    | 511    | 1022   | 888    |
| 6    | 511    | 1022   | 888    |
| 7    | 511    | 819    | 818    |
| 8    | 511    | 819    | 864    |
| 9    | 781    | 781    | 781    |
| 10   | 781    | 781    | 781    |
| 11   | 781    | 781    | 781    |
| 12   | 781    | 781    | 781    |
| 13   | 781    | 781    | 781    |
| 14   | 781    | 781    | 781    |
| 15   | 781    | 781    | 781    |

Legend:
- Type 1
- Type 2
- Type 3
Figure 6: Case Study: solution obtained with model (M1) with SRH
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Table 1: Instance hbp1_30_1

|                                |       |
|--------------------------------|-------|
| Number of beam types           | 1     |
| Number of molds                | 30    |
| Number of periods              | 2     |
| Molds length (m)               | 60    |

| Type 1                      |       |
|-----------------------------|-------|
| Cure time                   | 1     |
| Number of beams             | 6     |
| Lengths (m)                 | 1.15  3.10  3.20  3.80  5.60  5.70 |
| Demands                     | 26    26    27    13    20    22 |
Table 2: Description of priority rules proposed

| Heuristic | Priority measure  | curing time | Beam length |
|-----------|-------------------|-------------|-------------|
| SCTSL     | Shortest          | Shortest    |             |
| SCTLL     | Shortest          | Largest     |             |
| SCTAL     | Shortest          | Alternate   |             |
| LCTSL     | Longest           | Shortest    |             |
| LCTLL     | Longest           | Largest     |             |
| LCTAL     | Longest           | Alternate   |             |
### Table 3: Instance information

| Instance  | T  | M  | Maximal patterns | \(q_r\)-Maximal patterns | Patterns set reduction |
|-----------|----|----|------------------|--------------------------|-----------------------|
| hbp1_15_1 | 2  | 15 | 3,348            | 831                      | 75.18%                |
| hbp1_15_2 | 2  | 15 | 7,944            | 762                      | 90.41%                |
| hbp1_15_3 | 2  | 15 | 1,322            | 235                      | 82.22%                |
| hbp1_15_4 | 2  | 15 | 256              | 83                       | 67.58%                |
| hbp1_15_5 | 2  | 15 | 7,200            | 640                      | 91.11%                |
| hbp1_30_1 | 2  | 30 | 12,078           | 1,732                    | 85.66%                |
| hbp1_30_2 | 2  | 30 | 420              | 162                      | 61.43%                |
| hbp1_30_3 | 2  | 30 | 1,452            | 254                      | 82.51%                |
| hbp1_30_4 | 2  | 30 | 4,365            | 392                      | 91.02%                |
| hbp1_30_5 | 2  | 30 | 6,128            | 1,953                    | 68.13%                |
| hbp2_15_1 | 4  | 15 | 6,595            | 386                      | 94.15%                |
| hbp2_15_2 | 4  | 15 | 13,389           | 1,281                    | 90.43%                |
| hbp2_15_3 | 4  | 15 | 7,867            | 669                      | 91.50%                |
| hbp2_15_4 | 4  | 15 | 4,287            | 333                      | 92.23%                |
| hbp2_15_5 | 3  | 15 | 3,638            | 743                      | 79.58%                |
| hbp2_30_1 | 2  | 30 | 15,588           | 1,693                    | 89.14%                |
| hbp2_30_2 | 3  | 30 | 6,024            | 578                      | 90.41%                |
| hbp2_30_3 | 3  | 30 | 5,211            | 604                      | 88.41%                |
| hbp2_30_4 | 2  | 30 | 20,192           | 3,190                    | 84.20%                |
| hbp2_30_5 | 2  | 30 | 4,555            | 1,103                    | 75.78%                |
| hbp3_15_1 | 6  | 15 | 14,771           | 1,489                    | 89.92%                |
| hbp3_15_2 | 4  | 15 | 32,897           | 6,309                    | 80.82%                |
| hbp3_15_3 | 5  | 15 | 15,049           | 1,587                    | 89.45%                |
| hbp3_15_4 | 5  | 15 | 17,221           | 2,207                    | 87.18%                |
| hbp3_15_5 | 5  | 15 | 6,788            | 1,066                    | 84.30%                |
| hbp3_30_1 | 4  | 30 | 28,874           | 3,468                    | 87.99%                |
| hbp3_30_2 | 4  | 30 | 9,798            | 1,343                    | 86.29%                |
| hbp3_30_3 | 3  | 30 | 8,744            | 1,277                    | 85.40%                |
| hbp3_30_4 | 3  | 30 | 10,723           | 1,287                    | 88.00%                |
| hbp3_30_5 | 3  | 30 | 4,392            | 1,023                    | 76.71%                |
| hbp4_15_1 | 7  | 15 | 19,271           | 4,368                    | 77.33%                |
| hbp4_15_2 | 8  | 15 | 12,150           | 1,573                    | 87.05%                |
| hbp4_15_3 | 7  | 15 | 7,528            | 1,410                    | 81.27%                |
| hbp4_15_4 | 8  | 15 | 15,007           | 1,791                    | 88.07%                |
| hbp4_15_5 | 8  | 15 | 7,745            | 859                      | 88.91%                |
| hbp4_30_1 | 5  | 30 | 6,706            | 823                      | 87.73%                |
| hbp4_30_2 | 6  | 30 | 12,063           | 1,124                    | 90.68%                |
| hbp4_30_3 | 4  | 30 | 41,005           | 6,475                    | 84.21%                |
| hbp4_30_4 | 5  | 30 | 23,252           | 3,219                    | 86.16%                |
| hbp4_30_5 | 4  | 30 | 15,070           | 3,073                    | 79.61%                |
### Table 4: Results of computational tests with model (M1)

| Instance | LPv | LPt | IPv | BCn | Gap | IPt |
|----------|-----|-----|-----|-----|-----|-----|
| hbp1_15_1 | 0.00 | 1.21 | 0.00 | 0 | 0.00% | 2.22 |
| hbp1_15_2 | 0.00 | 2.97 | 0.00 | 0 | 0.00% | 5.34 |
| hbp1_15_3 | 0.00 | 0.39 | 0.00 | 0 | 0.00% | 0.70 |
| hbp1_15_4 | 0.00 | 0.09 | 0.00 | 0 | 0.00% | 0.17 |
| hbp1_15_5 | 0.00 | 2.25 | 0.00 | 0 | 0.00% | 5.06 |
| hbp1_30_1 | 0.00 | 7.88 | 0.00 | 0 | 0.00% | 15.50 |
| hbp1_30_2 | 0.00 | 0.25 | 0.00 | 0 | 0.00% | 0.47 |
| hbp1_30_3 | 0.00 | 0.85 | 0.00 | 0 | 0.00% | 1.50 |
| hbp1_30_4 | 0.00 | 2.85 | 0.00 | 0 | 0.00% | 6.39 |
| hbp1_30_5 | 0.00 | 3.77 | 0.00 | 0 | 0.00% | 8.68 |
| hbp2_15_1 | 0.00 | 7.97 | 0.00 | 0 | 0.00% | 14.04 |
| hbp2_15_2 | 0.00 | 30.14 | 0.00 | 0 | 0.00% | 39.86 |
| hbp2_15_3 | 0.00 | 9.08 | 0.00 | 0 | 0.00% | 22.90 |
| hbp2_15_4 | 0.00 | 4.20 | 0.00 | 0 | 0.00% | 8.83 |
| hbp2_15_5 | 0.00 | 2.08 | 0.00 | 0 | 0.00% | 6.67 |
| hbp2_30_1 | 0.00 | 12.48 | 0.00 | 0 | 0.00% | 28.20 |
| hbp2_30_2 | 0.00 | 6.95 | 0.00 | 0 | 0.00% | 17.03 |
| hbp2_30_3 | 0.00 | 7.12 | 0.00 | 0 | 0.00% | 12.29 |
| hbp2_30_4 | 0.00 | 23.25 | 0.00 | 0 | 0.00% | 48.95 |
| hbp2_30_5 | 0.00 | 3.13 | 0.00 | 0 | 0.00% | 8.83 |
| hbp3_15_1 | 0.00 | 23.27 | 0.00 | 0 | 0.00% | 194.30 |
| hbp3_15_2 | 0.00 | 96.69 | 0.00 | 0 | 0.00% | 292.76 |
| hbp3_15_3 | 0.00 | 45.72 | 0.00 | 0 | 0.00% | 126.93 |
| hbp3_15_4 | 0.00 | 52.12 | 0.00 | 0 | 0.00% | 119.55 |
| hbp3_15_5 | 0.00 | 9.51 | 0.00 | 0 | 0.00% | 30.44 |
| hbp3_30_1 | 0.00 | 85.37 | 0.00 | 0 | 0.00% | 714.98 |
| hbp3_30_2 | 0.00 | 17.73 | 0.00 | 0 | 0.00% | 54.53 |
| hbp3_30_3 | 0.00 | 11.15 | 0.00 | 0 | 0.00% | 34.60 |
| hbp3_30_4 | 2.55 | 25.34 | 2.55 | 0 | 0.00% | 49.84 |
| hbp3_30_5 | 0.00 | 4.24 | 0.00 | 0 | 0.00% | 15.23 |
| hbp4_15_1 | 0.00 | 87.60 | 0.00 | 0 | 0.00% | 1035.95 |
| hbp4_15_2 | 0.00 | 75.15 | 0.00 | 0 | 0.00% | 206.15 |
| hbp4_15_3 | 0.00 | 11.73 | 0.00 | 1312 | 0.00% | 532.82 |
| hbp4_15_4 | 0.00 | 34.48 | 0.00 | 0 | 0.00% | 313.79 |
| hbp4_15_5 | 0.00 | 18.60 | 0.00 | 0 | 0.00% | 135.39 |
| hbp4_30_1 | 0.00 | 16.97 | 0.00 | 0 | 0.00% | 70.77 |
| hbp4_30_2 | 0.00 | 51.36 | 0.00 | 0 | 0.00% | 266.83 |
| hbp4_30_3 | 0.00 | 577.39 | – | – | – | – |
| hbp4_30_4 | 0.00 | 320.97 | 0.00 | 0 | 0.00% | 1556.48 |
| hbp4_30_5 | 0.00 | 26.37 | 0.00 | 0 | 0.00% | 108.52 |
| Instance       | LPv | LPt | IPv | BCn | Gap  | LPv | LPt | IPv | BCn | Gap  |
|----------------|-----|-----|-----|-----|------|-----|-----|-----|-----|------|-----|
| **hbp1_15_1** | 0.44| 1.74| 1   | 0   | 0.00%| 3.29| 0.29| 1.01| 1   | 0    | 0.00%|
| **hbp1_15_2** | 0.78| 3.24| 1   | 0   | 0.00%| 8.25| 0.52| 2.70| 1   | 0    | 0.00%|
| **hbp1_15_3** | 0.66| 0.49| 1   | 0   | 0.00%| 2.09| 0.44| 0.41| 1   | 0    | 0.00%|
| **hbp1_15_4** | 0.65| 0.13| 1   | 0   | 0.00%| 0.36| 0.43| 0.15| 1   | 0    | 0.00%|
| **hbp1_15_5** | 0.90| 2.84| 1   | 0   | 0.00%| 10.50| 0.60| 2.55| 1   | 0    | 0.00%|
| **hbp1_30_1** | 0.27| 10.63| 1 | 0 | 0.00%| 32.57| 0.18| 9.02| 1 | 0 | 0.00%|
| **hbp1_30_2** | 0.24| 0.30| 1 | 0 | 0.00%| 0.67| 0.16| 0.28| 1 | 0 | 0.00%|
| **hbp1_30_3** | 0.34| 1.09| 1 | 0 | 0.00%| 2.63| 0.23| 0.85| 1 | 0 | 0.00%|
| **hbp1_30_4** | 0.33| 3.58| 1 | 0 | 0.00%| 8.75| 0.22| 2.88| 1 | 0 | 0.00%|
| **hbp1_30_5** | 0.21| 5.02| 1 | 0 | 0.00%| 13.72| 0.14| 4.12| 1 | 0 | 0.00%|
| **hbp2_15_1** | 1.68| 8.74| 3 | 111 | 0.00%| 51.62| 0.81| 7.85| 3 | 0 | 0.00%|
| **hbp2_15_2** | 1.69| 18.88| 3 | 0 | 0.00%| 89.72| 0.81| 33.87| 3 | 0 | 0.00%|
| **hbp2_15_3** | 1.77| 9.15| 3 | 0 | 0.00%| 36.89| 0.85| 10.41| 3 | 0 | 0.00%|
| **hbp2_15_4** | 1.59| 5.86| 3 | 0 | 0.00%| 13.48| 0.76| 5.93| 3 | 0 | 0.00%|
| **hbp2_15_5** | 1.31| 2.63| 2 | 30,333 | 0.00%| 537.26| 0.71| 2.26| 2 | 262,116 | 0.00%|
| **hbp2_30_1** | 0.58| 17.38| 2 | 0 | 0.00%| 30.12| 0.39| 22.30| 2 | 0 | 0.00%|
| **hbp2_30_2** | 0.81| 9.38| 2 | 0 | 0.00%| 31.12| 0.44| 8.94| 2 | 0 | 0.00%|
| **hbp2_30_3** | 0.74| 8.81| 2 | 0 | 0.00%| 23.45| 0.40| 8.44| 2 | 0 | 0.00%|
| **hbp2_30_4** | 0.66| 23.10| 2 | 0 | 0.00%| 46.52| 0.44| 34.47| 2 | 0 | 0.00%|
| **hbp2_30_5** | 0.53| 4.18| 2 | 0 | 0.00%| 7.44| 0.35| 3.90| 2 | 0 | 0.00%|
| **hbp3_15_1** | 2.18| 5.75| 6 | 0 | 50.00%| 3,600.00| 0.89| 56.79| 6 | 0 | 66.67%| 3,600.00|
| **hbp3_15_2** | 1.30| 84.88| 4 | 0 | 25.00%| 3,600.00| 0.62| 101.92| – | – | – | – |
| **hbp3_15_3** | 2.05| 53.56| 5 | 0 | 40.00%| 3,600.00| 0.90| 50.31| – | – | – | – |
| **hbp3_15_4** | 1.83| 54.40| 5 | 13 | 40.00%| 3,600.00| 0.80| 60.61| – | – | – | – |
| **hbp3_15_5** | 1.62| 19.80| 4 | 4,796 | 0.00%| 1,068.38| 0.71| 27.05| 4 | 4,759 | 0.00%| 959.68|
| **hbp3_30_1** | 1.08| 157.98| 3 | 0 | 0.00%| 1,841.38| 0.52| 105.39| – | – | – | – |
| **hbp3_30_2** | 1.03| 30.50| 3 | 0 | 0.00%| 229.34| 0.49| 28.34| 3 | 0 | 0.00%| 452.44|
| **hbp3_30_3** | 0.92| 13.26| 3 | 0 | 0.00%| 26.32| 0.50| 13.47| 3 | 0 | 0.00%| 241.84|
| **hbp3_30_4** | 1.04| 28.60| 3 | 0 | 0.00%| 43.97| 0.57| 32.15| 3 | 0 | 0.00%| 440.86|
| **hbp3_30_5** | 0.80| 5.70| 3 | 0 | 0.00%| 14.60| 0.43| 5.52| 3 | 0 | 0.00%| 111.38|
| **hbp4_15_1** | 2.31| 96.85| 7 | 0 | 42.86%| 3,600.00| 0.89| 117.00| – | – | – | – |
| **hbp4_15_2** | 2.71| 61.34| – | – | – | – | 1.00| 56.93| – | – | – | – |
| **hbp4_15_3** | 2.53| 20.59| – | – | – | – | 0.98| 19.69| – | – | – | – |
| **hbp4_15_4** | 2.61| 60.55| 8 | 0 | 50.00%| 3,600.00| 0.96| 57.24| – | – | – | – |
| **hbp4_15_5** | 2.55| 37.39| – | – | – | – | 0.94| 31.79| – | – | – | – |
| **hbp4_30_1** | 1.44| 29.42| 4 | 0 | 0.00%| 633.72| 0.63| 28.19| – | – | – | – |
| **hbp4_30_2** | 1.66| 108.30| 6 | 0 | 33.33%| 3,600.00| 0.68| 91.54| – | – | – | – |
| **hbp4_30_3** | 1.21| 288.02| 4 | 0 | 100.00%| 288.02| 0.58| 652.91| – | – | – | – |
| **hbp4_30_4** | 1.32| 503.35| 5 | 0 | 20.00%| 3,600.00| 0.58| 334.07| – | – | – | – |
| **hbp4_30_5** | 1.27| 35.51| 4 | 0 | 0.00%| 566.33| 0.61| 31.96| – | – | – | – |
| Instance    | LPv  | LPt  | IPv | BCn | Gap   | IPt   |
|-------------|------|------|-----|-----|-------|-------|
| hbp1_15_1   | 6.60 | 0.91 | 7   | 0   | 0.00% | 2.82  |
| hbp1_15_2   | 11.63| 2.32 | 12  | 0   | 0.00% | 9.41  |
| hbp1_15_3   | 9.94 | 0.32 | 10  | 0   | 0.00% | 2.27  |
| hbp1_15_4   | 9.75 | 0.08 | 10  | 0   | 0.00% | 0.24  |
| hbp1_15_5   | 13.57| 2.03 | 14  | 0   | 0.00% | 8.69  |
| hbp1_30_1   | 8.06 | 7.10 | 9   | 0   | 0.00% | 23.79 |
| hbp1_30_2   | 7.08 | 0.20 | 8   | 0   | 0.00% | 0.58  |
| hbp1_30_3   | 10.34| 0.73 | 11  | 0   | 0.00% | 2.06  |
| hbp1_30_4   | 9.84 | 2.40 | 10  | 885 | 0.00% | 62.95 |
| hbp1_30_5   | 6.22 | 3.35 | 7   | 0   | 0.00% | 3,600.00 |
| hbp2_15_1   | 25.19| 6.54 | 40  | 0   | 0.00% | 24.06 |
| hbp2_15_2   | 25.29| 13.92| 40  | 33,610| 2.73% | 3,600.00 |
| hbp2_15_3   | 26.49| 7.13 | 42  | 84,398| 3.04% | 3,600.00 |
| hbp2_15_4   | 23.81| 5.12 | 39  | 161,369| 4.96% | 3,600.00 |
| hbp2_15_5   | 19.66| 2.04 | 30  | 0   | 0.00% | 12.40 |
| hbp2_30_1   | 17.51| 13.61| 28  | 21,089| 8.18% | 3,600.00 |
| hbp2_30_2   | 24.21| 7.35 | 38  | 49,235| 4.58% | 3,600.00 |
| hbp2_30_3   | 22.10| 7.41 | 37  | 71,651| 3.65% | 3,600.00 |
| hbp2_30_4   | 19.67| 18.87| 30  | 27,081| 6.72% | 3,600.00 |
| hbp2_30_5   | 15.94| 3.36 | 27  | 97,008| 7.60% | 3,600.00 |
| hbp3_15_1   | 32.66| 43.82| 67  | 186  | 35.23%| 3,600.00 |
| hbp3_15_2   | 19.50| 70.84| 46  | 176  | 46.28%| 3,600.00 |
| hbp3_15_3   | 30.82| 52.57| 64  | 579  | 32.03%| 3,600.00 |
| hbp3_15_4   | 27.41| 47.65| 63  | 611  | 40.85%| 3,600.00 |
| hbp3_15_5   | 24.23| 14.30| 53  | 1,769| 21.82%| 3,600.00 |
| hbp3_30_1   | 32.41| 113.63| 71  | 1    | 36.92%| 3,600.00 |
| hbp3_30_2   | 30.80| 22.15| 67  | 516  | 40.04%| 3,600.00 |
| hbp3_30_3   | 27.62| 11.25| 60  | 5,654| 4.51% | 3,600.00 |
| hbp3_30_4   | 31.19| 36.95| 62  | 26,639| 4.36% | 3,600.00 |
| hbp3_30_5   | 23.88| 4.70 | 50  | 27,622| 5.06% | 3,600.00 |
| hbp4_15_1   | 34.65| 87.65| 104 | 0    | 56.04%| 3,600.00 |
| hbp4_15_2   | 40.68| 54.61| 104 | 126  | 48.81%| 3,600.00 |
| hbp4_15_3   | 37.94| 19.46| 91  | 429  | 46.35%| 3,600.00 |
| hbp4_15_4   | 39.10| 52.42| 101 | 5    | 50.89%| 3,600.00 |
| hbp4_15_5   | 38.29| 33.30| 106 | 0    | 56.52%| 3,600.00 |
| hbp4_30_1   | 43.06| 21.85| 110 | 270  | 48.91%| 3,600.00 |
| hbp4_30_2   | 49.91| 85.16| 138 | 51   | 54.03%| 3,600.00 |
| hbp4_30_3   | 36.18| 347.03|  –   |  –   | –    | 3,600.00 |
| hbp4_30_4   | 39.54| 390.46|  –   |  –   | –    | 3,600.00 |
| hbp4_30_5   | 38.23| 27.03| 94  | 0    | 48.74%| 3,600.00 |
| Instance    | LPv  | LPt  | IPv  | BCn | Gap     | IPt (s) |
|-------------|------|------|------|-----|---------|---------|
| hbp1_15_1   | 0.00 | 0.20 | 0.00 | 0   | 0.00%   | 0.14    |
| hbp1_15_2   | 0.00 | 0.19 | 0.00 | 0   | 0.00%   | 0.18    |
| hbp1_15_3   | 0.00 | 0.06 | 0.00 | 0   | 0.00%   | 0.07    |
| hbp1_15_4   | 1.75 | 0.03 | 1.80 | 0   | 0.00%   | 0.12    |
| hbp1_15_5   | 0.00 | 0.16 | 0.00 | 0   | 0.00%   | 0.15    |
| hbp1_30_1   | 0.00 | 0.84 | 0.00 | 0   | 0.00%   | 0.70    |
| hbp1_30_2   | 0.00 | 0.08 | 0.00 | 0   | 0.00%   | 0.07    |
| hbp1_30_3   | 0.00 | 0.12 | 0.00 | 0   | 0.00%   | 0.10    |
| hbp1_30_4   | 0.00 | 0.18 | 0.00 | 0   | 0.00%   | 0.19    |
| hbp1_30_5   | 0.00 | 0.91 | 0.00 | 0   | 0.00%   | 0.71    |
| hbp2_15_1   | 1.04 | 0.20 | 1.05 | 0   | 0.00%   | 1.10    |
| hbp2_15_2   | 0.00 | 0.73 | 0.00 | 0   | 0.00%   | 3.52    |
| hbp2_15_3   | 0.00 | 0.32 | 0.90 | 0   | 0.00%   | 2.59    |
| hbp2_15_4   | 0.00 | 0.18 | 0.00 | 0   | 0.00%   | 0.69    |
| hbp2_15_5   | 1.10 | 0.28 | 1.20 | 0   | 0.00%   | 1.28    |
| hbp2_30_1   | 0.00 | 0.82 | 0.00 | 0   | 0.00%   | 3.49    |
| hbp2_30_2   | 0.00 | 0.43 | 0.00 | 0   | 0.00%   | 1.24    |
| hbp2_30_3   | 0.00 | 0.44 | 0.10 | 0   | 0.00%   | 1.47    |
| hbp2_30_4   | 0.00 | 2.08 | 0.00 | 0   | 0.00%   | 5.27    |
| hbp2_30_5   | 0.00 | 0.57 | 0.20 | 0   | 0.00%   | 2.60    |
| hbp3_15_1   | 0.00 | 1.42 | 0.00 | 0   | 0.00%   | 12.95   |
| hbp3_15_2   | 0.00 | 5.19 | 0.30 | 50,199 | 0.00% | 705.52  |
| hbp3_15_3   | 0.00 | 1.36 | 0.00 | 0   | 0.00%   | 8.94    |
| hbp3_15_4   | 0.00 | 1.89 | 0.00 | 0   | 0.00%   | 10.59   |
| hbp3_15_5   | 0.00 | 0.67 | 0.15 | 2,475 | 0.00% | 7.74    |
| hbp3_30_1   | 0.00 | 3.68 | 0.00 | 0   | 0.00%   | 17.91   |
| hbp3_30_2   | 0.00 | 1.33 | 0.00 | 0   | 0.00%   | 6.50    |
| hbp3_30_3   | 0.00 | 1.07 | 0.00 | 0   | 0.00%   | 5.07    |
| hbp3_30_4   | 11.55 | 1.29 | 11.65 | 64,777 | 0.00% | 45.42   |
| hbp3_30_5   | 0.00 | 0.68 | 0.00 | 0   | 0.00%   | 2.39    |
| hbp4_15_1   | 0.00 | 5.21 | 0.00 | 0   | 0.00%   | 49.84   |
| hbp4_15_2   | 0.00 | 2.35 | 1.60 | 29,941 | 41.49% | 3,600.00 |
| hbp4_15_3   | 0.00 | 1.47 | 0.40 | 1,005,780 | 100.00% | 3,600.00 |
| hbp4_15_4   | 0.00 | 2.24 | 0.00 | 0   | 0.00%   | 18.00   |
| hbp4_15_5   | 0.00 | 0.97 | 0.55 | 1,999,090 | 100.00% | 3,600.00 |
| hbp4_30_1   | 2.00 | 1.30 | 2.65 | 282,061 | 0.00% | 355.24  |
| hbp4_30_2   | 0.00 | 2.16 | 1.15 | 1,602,117 | 18.60% | 3,600.00 |
| hbp4_30_3   | 0.00 | 10.18 | 0.00 | 0   | 0.00%   | 41.00   |
| hbp4_30_4   | 0.00 | 6.75 | 0.00 | 0   | 0.00%   | 31.31   |
| hbp4_30_5   | 0.00 | 2.93 | 0.00 | 0   | 0.00%   | 16.23   |
Table 8: Results of computational tests with model (M2) using the SRH

| Instance     | LVp | LPt | IPv | BCn | Gap  | IIPt (s) |
|--------------|-----|-----|-----|-----|------|----------|
| hbp1_15_1    | 0.44| 0.24| 1   | 0   | 0.00%| 0.55     |
| hbp1_15_2    | 0.78| 0.23| 1   | 0   | 0.00%| 0.72     |
| hbp1_15_3    | 0.66| 0.08| 1   | 0   | 0.00%| 0.28     |
| hbp1_15_4    | 0.65| 0.03| 1   | 0   | 0.00%| 0.13     |
| hbp1_15_5    | 0.90| 0.20| 1   | 0   | 0.00%| 0.95     |
| hbp1_30_1    | 0.27| 1.20| 1   | 0   | 0.00%| 2.49     |
| hbp1_30_2    | 0.24| 0.09| 1   | 0   | 0.00%| 0.23     |
| hbp1_30_3    | 0.35| 0.14| 1   | 0   | 0.00%| 0.36     |
| hbp1_30_4    | 0.33| 0.26| 1   | 0   | 0.00%| 0.56     |
| hbp1_30_5    | 0.21| 1.24| 1   | 0   | 0.00%| 3.32     |
| hbp2_15_1    | 1.71| 0.54| 3   | 0   | 0.00%| 1.50     |
| hbp2_15_2    | 1.70| 1.12| 3   | 0   | 0.00%| 4.48     |
| hbp2_15_3    | 1.83| 0.47| 3   | 0   | 0.00%| 2.66     |
| hbp2_15_4    | 1.62| 0.38| 3   | 0   | 0.00%| 1.12     |
| hbp2_15_5    | 1.32| 0.38| 3   | 1,841,543 | 33.33%| 3,600.00 |
| hbp2_30_1    | 0.59| 1.21| 2   | 0   | 0.00%| 3.61     |
| hbp2_30_2    | 0.82| 0.66| 2   | 0   | 0.00%| 1.78     |
| hbp2_30_3    | 0.76| 0.64| 2   | 0   | 0.00%| 1.87     |
| hbp2_30_4    | 0.66| 2.50| 2   | 0   | 0.00%| 5.31     |
| hbp2_30_5    | 0.54| 0.71| 2   | 0   | 0.00%| 2.58     |
| hbp3_15_1    | 2.19| 2.62| 5   | 28,252 | 20.00%| 3,600.00 |
| hbp3_15_2    | 1.31| 7.57| 3   | 781   | 0.00%| 158.47   |
| hbp3_15_3    | 2.07| 2.40| 5   | 116,809 | 20.00%| 3,600.00 |
| hbp3_15_4    | 1.84| 3.27| 4   | 2,609  | 0.00%| 395.11   |
| hbp3_15_5    | 1.63| 1.36| 4   | 0   | 0.00%| 22.16    |
| hbp3_30_1    | 1.09| 6.16| 3   | 0   | 0.00%| 353.33   |
| hbp3_30_2    | 1.03| 2.50| 3   | 0   | 0.00%| 13.93    |
| hbp3_30_3    | 0.93| 1.51| 3   | 0   | 0.00%| 4.59     |
| hbp3_30_4    | 1.06| 1.55| 3   | 0   | 0.00%| 9.21     |
| hbp3_30_5    | 0.80| 0.89| 3   | 0   | 0.00%| 9.10     |
| hbp4_15_1    | 2.32| 12.74| 7   | 15   | 42.86%| 3,600.00 |
| hbp4_15_2    | 2.75| 3.70| 8   | 3,836 | 50.00%| 3,600.00 |
| hbp4_15_3    | 2.55| 2.39| 6   | 27,395| 16.67%| 3,600.00 |
| hbp4_15_4    | 2.62| 4.22| 7   | 9,003 | 14.29%| 3,600.00 |
| hbp4_15_5    | 2.59| 1.84| 7   | 14,909| 14.29%| 3,600.00 |
| hbp4_30_1    | 1.45| 2.07| 4   | 0   | 0.00%| 53.24    |
| hbp4_30_2    | 1.70| 3.89| 5   | 4,843 | 0.00%| 533.59   |
| hbp4_30_3    | 1.21| 16.28| 4  | 0   | 0.00%| 40.79    |
| hbp4_30_4    | 1.34| 8.91| 4   | 0   | 0.00%| 113.59   |
| hbp4_30_5    | 1.28| 4.52| 4   | 0   | 0.00%| 16.12    |
| Instance     | LPv | LPlt | IPv | BCn | Gap  | IPt (s) |
|--------------|-----|------|-----|-----|------|--------|
| hbp1_15_1    | 6.60| 0.22 | 7   | 0   | 0.00%| 0.55   |
| hbp1_15_2    | 11.65| 0.24 | 12  | 0   | 0.00%| 1.05   |
| hbp1_15_3    | 9.96 | 0.08 | 10  | 0   | 0.00%| 0.38   |
| hbp1_15_4    | 9.79 | 0.02 | 10  | 0   | 0.00%| 0.12   |
| hbp1_15_5    | 13.57| 0.21 | 14  | 0   | 0.00%| 0.71   |
| hbp1_30_1    | 8.06 | 1.05 | 9   | 0   | 0.00%| 3.04   |
| hbp1_30_2    | 7.08 | 0.09 | 8   | 0   | 0.00%| 0.22   |
| hbp1_30_3    | 10.35| 0.15 | 11  | 0   | 0.00%| 0.46   |
| hbp1_30_4    | 9.85 | 0.24 | 10  | 0   | 0.00%| 1.17   |
| hbp1_30_5    | 6.23 | 1.20 | 7   | 0   | 0.00%| 2.40   |
| hbp2_15_1    | 25.63| 0.57 | 40  | 0   | 0.00%| 1.15   |
| hbp2_15_2    | 25.57| 1.06 | 40  | 2.438| 0.00%| 44.27  |
| hbp2_15_3    | 27.45| 0.49 | 42  | 0   | 0.00%| 6.08   |
| hbp2_15_4    | 24.29| 0.37 | 39  | 1,789,265| 4.45%| 3,600.00|
| hbp2_15_5    | 19.87| 0.36 | 32  | 869,712| 7.43%| 3,600.00|
| hbp2_30_1    | 17.78| 1.17 | 28  | 174,674| 7.97%| 3,600.00|
| hbp2_30_2    | 24.48| 0.67 | 38  | 332,998| 4.14%| 3,600.00|
| hbp2_30_3    | 22.74| 0.69 | 37  | 2,519  | 0.00%| 87.05  |
| hbp2_30_4    | 19.81| 2.65 | 30  | 143,757| 6.45%| 3,600.00|
| hbp2_30_5    | 16.26| 0.71 | 27  | 223,157| 7.13%| 3,600.00|
| hbp3_15_1    | 32.90| 2.61 | 65  | 15,782| 10.97%|3,600.00|
| hbp3_15_2    | 19.64| 7.64 | 45  | 47,173| 8.16% | 3,600.00|
| hbp3_15_3    | 31.02| 2.81 | 64  | 38,337| 19.29%| 3,600.00|
| hbp3_15_4    | 27.65| 3.38 | 58  | 28,532| 16.53%| 3,600.00|
| hbp3_15_5    | 24.47| 1.38 | 53  | 87,367| 3.59% | 3,600.00|
| hbp3_30_1    | 32.73| 6.08 | 71  | 1,239 | 11.64%| 3,600.00|
| hbp3_30_2    | 31.00| 2.54 | 64  | 27,627| 19.52%| 3,600.00|
| hbp3_30_3    | 27.96| 1.53 | 60  | 134,206| 3.00%| 3,600.00|
| hbp3_30_4    | 31.85| 1.54 | 62  | 47,824| 0.00% | 1,878.17|
| hbp3_30_5    | 24.10| 0.91 | 50  | 163,957| 7.38%| 3,600.00|
| hbp4_15_1    | 34.85| 11.61| 84  | 501   | 45.56%| 3,600.00|
| hbp4_15_2    | 41.18| 3.82 | 99  | 1,583 | 30.23%| 3,600.00|
| hbp4_15_3    | 38.29| 2.28 | 87  | 39,966| 23.40%| 3,600.00|
| hbp4_15_4    | 39.36| 4.10 | 97  | 10,343| 25.82%| 3,600.00|
| hbp4_15_5    | 38.87| 1.83 | 103 | 32,940| 16.75%| 3,600.00|
| hbp4_30_1    | 43.57| 2.10 | 109 | 38,958| 12.65%| 3,600.00|
| hbp4_30_2    | 50.92| 3.71 | 131 | 10,734| 9.70% | 3,600.00|
| hbp4_30_3    | 36.32| 16.38| 91  | 0     | 40.00%| 3,600.00|
| hbp4_30_4    | 40.13| 8.82 | 109 | 596   | 53.15%| 3,600.00|
| hbp4_30_5    | 38.46| 4.39 | 91  | 672   | 38.77%| 3,600.00|
Table 10: Idle capacities obtained by solution methods

| Instance | SCTSL | SCTLL | SCTAL | LCTSL | LCTLL | LCTAL | M1 | M1-SRH |
|----------|-------|-------|-------|-------|-------|-------|----|--------|
| hbp1_15_1 | 7.60  | 4.70  | 7.60  | 7.60  | 4.70  | 7.60  | 0.00| 0.00  |
| hbp1_15_2 | 8.35  | 6.35  | 13.30 | 8.35  | 6.35  | 13.30 | 0.00| 0.00  |
| hbp1_15_3 | 9.05  | 6.55  | 11.95 | 9.05  | 6.55  | 11.95 | 0.00| 0.00  |
| hbp1_15_4 | 15.95 | 15.95 | 15.95 | 15.95 | 15.95 | 15.95 | 0.00| 1.80  |
| hbp1_15_5 | 20.05 | 20.45 | 19.90 | 20.05 | 20.45 | 19.90 | 0.00| 0.00  |
| hbp1_30_1 | 4.60  | 3.85  | 4.95  | 4.60  | 3.85  | 4.95  | 0.00| 4.95  |
| hbp1_30_2 | 13.60 | 10.40 | 10.35 | 13.60 | 10.40 | 10.35 | 0.00| 0.00  |
| hbp1_30_3 | 15.90 | 17.90 | 15.60 | 15.90 | 17.90 | 15.60 | 0.00| 0.00  |
| hbp1_30_4 | 6.60  | 8.10  | 7.75  | 6.60  | 8.10  | 7.75  | 0.00| 7.75  |
| hbp1_30_5 | 6.45  | 9.95  | 9.35  | 6.45  | 9.95  | 9.35  | 0.00| 9.35  |
| hbp2_15_1 | 26.05 | 32.70 | 38.90 | 26.05 | 32.70 | 38.90 | 0.00| 1.05  |
| hbp2_15_2 | 21.40 | 24.70 | 27.40 | 21.40 | 24.70 | 27.40 | 0.00| 0.00  |
| hbp2_15_3 | 46.20 | 45.75 | 51.50 | 46.20 | 45.75 | 51.50 | 0.00| 0.90  |
| hbp2_15_4 | 34.15 | 42.95 | 44.40 | 34.15 | 42.95 | 44.40 | 0.00| 0.00  |
| hbp2_15_5 | 50.10 | 60.20 | 58.30 | 50.10 | 60.20 | 58.30 | 0.00| 1.20  |
| hbp2_30_1 | 39.55 | 33.45 | 26.95 | 39.55 | 33.45 | 26.95 | 0.00| 0.00  |
| hbp2_30_2 | 46.40 | 39.75 | 39.25 | 46.40 | 39.75 | 39.25 | 0.00| 0.00  |
| hbp2_30_3 | 18.10 | 22.00 | 25.80 | 18.10 | 22.00 | 25.80 | 0.00| 0.20  |
| hbp3_15_1 | 63.40 | 62.00 | 76.75 | 63.40 | 62.00 | 76.75 | 0.00| 0.00  |
| hbp3_15_2 | 47.95 | 50.35 | 59.65 | 47.95 | 50.35 | 59.65 | 0.00| 0.00  |
| hbp3_15_3 | 49.10 | 46.00 | 49.70 | 49.10 | 46.00 | 49.70 | 0.00| 0.00  |
| hbp3_15_4 | 59.75 | 55.45 | 61.80 | 59.75 | 55.45 | 61.80 | 0.00| 0.15  |
| hbp3_15_5 | 69.50 | 74.90 | 63.55 | 69.50 | 74.90 | 63.55 | 0.00| 0.00  |
| hbp3_30_1 | 104.15 | 100.70 | 100.90 | 104.15 | 100.70 | 100.90 | 2.55| 11.65 |
| hbp3_30_2 | 50.05 | 58.05 | 70.40 | 50.05 | 58.05 | 70.40 | 0.00| 0.00  |
| hbp3_30_3 | 138.40 | 143.40 | 146.95 | 138.40 | 143.40 | 146.95 | 0.00| 2.65  |
| hbp3_30_4 | 194.50 | 166.85 | 206.75 | 194.50 | 166.85 | 206.75 | 0.00| 1.15  |
| hbp3_30_5 | 64.15 | 80.50 | 82.05 | 64.15 | 80.50 | 82.05 | –  | 0.00  |
| hbp4_15_1 | 68.05 | 82.35 | 70.15 | 68.05 | 82.35 | 70.15 | 0.00| 0.00  |
| hbp4_15_2 | 117.35 | 103.75 | 104.50 | 117.35 | 103.75 | 104.50 | 0.00| 1.60  |
| hbp4_15_3 | 70.35 | 74.40 | 95.00 | 70.35 | 74.40 | 95.00 | 0.00| 0.40  |
| hbp4_15_4 | 97.70 | 88.50 | 113.65 | 97.70 | 88.50 | 113.65 | 0.00| 0.00  |
| hbp4_15_5 | 142.60 | 147.95 | 151.65 | 142.60 | 147.95 | 151.65 | 0.00| 0.55  |
| hbp4_30_1 | 138.40 | 143.40 | 146.95 | 138.40 | 143.40 | 146.95 | 0.00| 2.65  |
| hbp4_30_2 | 194.50 | 166.85 | 206.75 | 194.50 | 166.85 | 206.75 | 0.00| 1.15  |
| hbp4_30_3 | 64.15 | 80.50 | 82.05 | 64.15 | 80.50 | 82.05 | –  | 0.00  |
| hbp4_30_4 | 128.50 | 154.35 | 123.45 | 128.50 | 154.35 | 123.45 | 0.00| 0.00  |
| hbp4_30_5 | 109.85 | 105.40 | 124.10 | 109.85 | 105.40 | 124.10 | 0.00| 0.00  |

| nBV | 0 | 0 | 0 | 0 | 0 | 0 | 39 | 26 |

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Table 11: Makespans obtained by solution methods

| Instance     | SCTSL | SCTLL | SCTAL | LCTSL | LCTLL | LCTAL | M2  | M2-SRH |
|--------------|-------|-------|-------|-------|-------|-------|-----|--------|
| hbp1_15_1    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_15_2    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_15_3    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_15_4    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_15_5    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_30_1    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_30_2    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_30_3    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_30_4    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp1_30_5    | 1     | 1     | 1     | 1     | 1     | 1     | 1   | 1      |
| hbp2_15_1    | 3     | 3     | 3     | 3     | 3     | 3     | 3   | 3      |
| hbp2_15_2    | 3     | 3     | 3     | 3     | 3     | 3     | 3   | 3      |
| hbp2_15_3    | 4     | 4     | 3     | 4     | 3     | 3     | 3   | 3      |
| hbp2_15_4    | 3     | 3     | 3     | 3     | 3     | 3     | 3   | 3      |
| hbp2_15_5    | 3     | 3     | 3     | 3     | 3     | 3     | 3   | 2      |
| hbp2_30_1    | 2     | 2     | 2     | 2     | 2     | 2     | 2   | 2      |
| hbp2_30_2    | 2     | 2     | 2     | 2     | 2     | 2     | 2   | 2      |
| hbp2_30_3    | 2     | 2     | 2     | 2     | 2     | 2     | 2   | 2      |
| hbp2_30_4    | 2     | 2     | 2     | 2     | 2     | 2     | 2   | 2      |
| hbp2_30_5    | 2     | 2     | 2     | 2     | 2     | 2     | 2   | 2      |
| hbp3_15_1    | 6     | 6     | 6     | 5     | 5     | 5     | 6   | 5      |
| hbp3_15_2    | 4     | 4     | 4     | 4     | 4     | 4     | 4   | 3      |
| hbp3_15_3    | 6     | 6     | 6     | 5     | 5     | 5     | 5   | 5      |
| hbp3_15_4    | 5     | 5     | 5     | 5     | 5     | 5     | 5   | 4      |
| hbp3_15_5    | 5     | 5     | 5     | 4     | 4     | 4     | 4   | 4      |
| hbp3_30_1    | 4     | 4     | 4     | 3     | 3     | 3     | 3   | 3      |
| hbp3_30_2    | 4     | 4     | 4     | 3     | 3     | 3     | 3   | 3      |
| hbp3_30_3    | 3     | 3     | 3     | 3     | 3     | 3     | 3   | 3      |
| hbp3_30_4    | 4     | 4     | 4     | 3     | 3     | 3     | 3   | 3      |
| hbp3_30_5    | 3     | 3     | 3     | 3     | 3     | 3     | 3   | 3      |
| hbp4_15_1    | 7     | 7     | 7     | 6     | 6     | 6     | 7   | 7      |
| hbp4_15_2    | 9     | 9     | 9     | 7     | 7     | 7     | 7   | 8      |
| hbp4_15_3    | 7     | 7     | 7     | 7     | 6     | 6     | 6   | 6      |
| hbp4_15_4    | 8     | 8     | 8     | 7     | 7     | 7     | 7   | 7      |
| hbp4_15_5    | 8     | 8     | 8     | 8     | 8     | 8     | 8   | 7      |
| hbp4_30_1    | 6     | 6     | 6     | 4     | 4     | 4     | 4   | 4      |
| hbp4_30_2    | 6     | 6     | 6     | 5     | 5     | 5     | 6   | 5      |
| hbp4_30_3    | 5     | 5     | 5     | 4     | 4     | 4     | 4   | 4      |
| hbp4_30_4    | 6     | 6     | 6     | 4     | 4     | 4     | 4   | 4      |
| hbp4_30_5    | 5     | 5     | 5     | 4     | 4     | 4     | 4   | 4      |
| nBV          | 20    | 20    | 21    | 34    | 34    | 36    | 30  | 37     |

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Table 12: Total completion times obtained by solution methods

| Instance   | SCTSL | SCTLL | SCTAL | LCTSL | LCTLL | LCTAL | M3   | M3-SRH |
|------------|-------|-------|-------|-------|-------|-------|------|--------|
| hbp1_15_1  | 7     | 7     | 7     | 7     | 7     | 7     | 7    | 7      |
| hbp1_15_2  | 13    | 13    | 12    | 13    | 13    | 12    | 12   | 12     |
| hbp1_15_3  | 11    | 11    | 11    | 11    | 11    | 11    | 10   | 10     |
| hbp1_15_4  | 11    | 11    | 11    | 11    | 11    | 11    | 10   | 10     |
| hbp1_15_5  | 15    | 15    | 14    | 15    | 15    | 14    | 14   | 14     |
| hbp1_30_1  | 9     | 9     | 9     | 9     | 9     | 9     | 9    | 9      |
| hbp1_30_2  | 8     | 8     | 8     | 8     | 8     | 8     | 8    | 8      |
| hbp1_30_3  | 11    | 11    | 11    | 11    | 11    | 11    | 11   | 11     |
| hbp1_30_4  | 11    | 11    | 11    | 11    | 11    | 11    | 10   | 10     |
| hbp1_30_5  | 7     | 7     | 7     | 7     | 7     | 7     | 7    | 7      |
| hbp2_15_1  | 42    | 42    | 42    | 42    | 42    | 42    | 40   | 40     |
| hbp2_15_2  | 42    | 42    | 40    | 42    | 42    | 40    | 40   | 40     |
| hbp2_15_3  | 45    | 45    | 43    | 45    | 45    | 43    | 42   | 42     |
| hbp2_15_4  | 41    | 41    | 39    | 41    | 41    | 39    | 39   | 39     |
| hbp2_15_5  | 32    | 32    | 32    | 32    | 32    | 32    | 30   | 30     |
| hbp2_30_1  | 28    | 28    | 28    | 28    | 28    | 28    | 28   | 28     |
| hbp2_30_2  | 39    | 39    | 39    | 39    | 39    | 39    | 38   | 38     |
| hbp2_30_3  | 39    | 39    | 39    | 39    | 39    | 39    | 37   | 37     |
| hbp2_30_4  | 30    | 30    | 30    | 30    | 30    | 30    | 30   | 30     |
| hbp2_30_5  | 27    | 27    | 27    | 27    | 27    | 27    | 27   | 27     |
| hbp3_15_1  | 67    | 67    | 67    | 67    | 67    | 67    | 65   | 65     |
| hbp3_15_2  | 46    | 46    | 46    | 46    | 46    | 46    | 45   | 45     |
| hbp3_15_3  | 64    | 64    | 66    | 64    | 64    | 66    | 64   | 64     |
| hbp3_15_4  | 63    | 63    | 63    | 63    | 63    | 63    | 58   | 58     |
| hbp3_15_5  | 54    | 54    | 53    | 54    | 54    | 53    | 53   | 53     |
| hbp3_30_1  | 71    | 71    | 71    | 71    | 71    | 71    | 71   | 71     |
| hbp3_30_2  | 67    | 67    | 67    | 67    | 67    | 67    | 64   | 64     |
| hbp3_30_3  | 63    | 63    | 63    | 63    | 63    | 63    | 60   | 60     |
| hbp3_30_4  | 64    | 64    | 64    | 64    | 64    | 64    | 62   | 62     |
| hbp3_30_5  | 50    | 50    | 50    | 50    | 50    | 50    | 50   | 50     |
| hbp4_15_1  | 84    | 84    | 84    | 84    | 84    | 84    | 104  | 104    |
| hbp4_15_2  | 101   | 101   | 101   | 101   | 101   | 101   | 104  | 104    |
| hbp4_15_3  | 91    | 91    | 90    | 91    | 91    | 90    | 91   | 87     |
| hbp4_15_4  | 97    | 97    | 97    | 97    | 97    | 97    | 101  | 97     |
| hbp4_15_5  | 106   | 106   | 106   | 106   | 106   | 106   | 104  | 104    |
| hbp4_30_1  | 109   | 110   | 109   | 109   | 109   | 109   | 110  | 109    |
| hbp4_30_2  | 140   | 140   | 140   | 140   | 140   | 140   | 138  | 131    |
| hbp4_30_3  | 98    | 98    | 95    | 98    | 98    | 95    | –    | 91     |
| hbp4_30_4  | 109   | 109   | 107   | 109   | 109   | 107   | –    | 109    |
| hbp4_30_5  | 98    | 98    | 97    | 98    | 98    | 97    | 94   | 91     |
| nBV        | 14    | 13    | 19    | 14    | 13    | 19    | 26   | 38     |
| Type  | Cure time | Number of Beams | Lengths (m) | Demands  |
|-------|-----------|-----------------|-------------|----------|
| Type 1| 1         | 4               | 2.9 3.2 4.6 7.15 | 13 35 34 22 |
| Type 2| 2         | 4               | 2.9 3.95 6 6.9   | 34 26 9 31  |
| Type 3| 3         | 4               | 3.95 5.7 5.95 6.9| 26 13 28 9  |
Table 14: Case Study: mold usage of solution of model (M1) along the time horizon

| Period | Total capacity | Idle capacity | Productive capacity loss |
|--------|----------------|---------------|-------------------------|
| 1      | 900            | 0.10          | 0.01%                   |
| 2      | 900            | 0.35          | 0.04%                   |
| 3      | 900            | 0.35          | 0.04%                   |
| 4      | 900            | 0.25          | 0.03%                   |
| Total  | 3600           | 1.05          | 0.03%                   |
Table 15: Case Study: beam surplus from solution for instance by model (M1)

| Type | Length 1 | Length 2 | Length 3 | Length 4 |
|------|----------|----------|----------|----------|
| 1    | 48       | 5        | 9        | 0        |
| 2    | 1        | 2        | 0        | 0        |
| 3    | 2        | 1        | 1        | 34       |
Table 16: Case Study: mold usage of solution of model (M2) along the time horizon

| Period | Total capacity | Idle capacity | Productive capacity loss |
|--------|----------------|---------------|--------------------------|
| 1      | 900            | 20.15         | 2.24%                    |
| 2      | 900            | 19.6          | 2.18%                    |
| 3      | 900            | 18.75         | 2.08%                    |
| 4      | –              | –             | –                        |
| Total  | 2700           | 58.5          | 2.17%                    |
Table 17: Case Study: beam surplus from solution obtained with model (M2)

| Type | Length 1 | Length 2 | Length 3 | Length 4 |
|------|----------|----------|----------|----------|
| 1    | 1        | 1        | 0        | 0        |
| 2    | 0        | 0        | 0        | 0        |
| 3    | 0        | 1        | 0        | 0        |
Table 18: Case Study: mold usage of solution of model (M3) along the time horizon

| Period | Total capacity | Idle capacity | Productive capacity loss |
|--------|----------------|---------------|--------------------------|
| 1      | 900            | 1.05          | 0.12%                    |
| 2      | 900            | 1.65          | 0.18%                    |
| 3      | 900            | 1.65          | 0.18%                    |
| 4      | 900            | 0.85          | 0.09%                    |
| Total  | 3600           | 5.2           | 0.14%                    |
Table 19: Case Study: beam surplus from solution obtained with model (M3)

| Type | Length 1 | Length 2 | Length 3 | Length 4 |
|------|----------|----------|----------|----------|
| 1    | 0        | 2        | 1        | 0        |
| 2    | 1        | 2        | 0        | 0        |
| 3    | 0        | 0        | 1        | 1        |
Table 20: Case Study: mold usage of solution of model (M1) with SRH along the time horizon

| Period | Total capacity | Idle capacity | Productive capacity loss |
|--------|----------------|---------------|--------------------------|
| 1      | 900            | 0.35          | 0.04%                    |
| 2      | 900            | 0.45          | 0.05%                    |
| 3      | 900            | 0.45          | 0.05%                    |
| 4      | 900            | 0.10          | 0.01%                    |
| Total  | 3600           | 1.35          | 0.04%                    |
Table 21: Case Study: beam surplus from solution obtained with model (M1) with SRH

| Type | Length 1 | Length 2 | Length 3 | Length 4 |
|------|----------|----------|----------|----------|
| 1    | 47       | 13       | 14       | 2        |
| 2    | 8        | 4        | 3        | 2        |
| 3    | 1        | 1        | 2        | 25       |
Table 22: Case Study: mold usage of solution of model (M2) with SRH along the time horizon

| Period | Total capacity | Idle capacity | Productive capacity loss |
|--------|----------------|---------------|-------------------------|
| 1      | 900            | 17.55         | 1.95%                   |
| 2      | 900            | 18.40         | 2.04%                   |
| 3      | 900            | 17.90         | 1.99%                   |
| 4      | –              | –             | –                       |
| Total  | 2700           | 53.85         | 1.99%                   |
Table 23: Case Study: beam surplus from solution obtained with model (M2) with SRH

| Type | Length 1 | Length 2 | Length 3 | Length 4 |
|------|----------|----------|----------|----------|
| 1    | 0        | 0        | 0        | 1        |
| 2    | 0        | 0        | 0        | 0        |
| 3    | 0        | 0        | 0        | 1        |
Table 24: Case Study: mold usage of solution of model (M3) with SRH along the time horizon

| Period | Total capacity | Idle capacity | Productive capacity loss |
|--------|----------------|---------------|--------------------------|
| 1      | 900            | 3.95          | 0.44%                    |
| 2      | 900            | 5.70          | 0.63%                    |
| 3      | 900            | 3.80          | 0.42%                    |
| 4      | 900            | 0.45          | 0.05%                    |
| Total  | 3600           | 13.90         | 0.39%                    |
Table 25: Case Study: beam surplus from solution obtained with model (M3) with SRH

| Type | Length 1 | Length 2 | Length 3 | Length 4 |
|------|----------|----------|----------|----------|
| 1    | 1        | 1        | 0        | 1        |
| 2    | 1        | 0        | 0        | 1        |
| 3    | 0        | 1        | 1        | 0        |
Table 26: Comparisons among solutions for the Case Study

|       | Idle capacity | Concrete waste | Makespan | Total completion time | Execution time (s) |
|-------|---------------|----------------|----------|-----------------------|--------------------|
| M1    | 1.05          | 0.35           | 4        | 60                    | 3600.00            |
| M1-SRH| 1.35          | 0.45           | 4        | 60                    | 3600.00            |
| M2    | 58.50         | 30.10          | 3        | 45                    | 5.70               |
| M2-SRH| 53.85         | 27.85          | 3        | 45                    | 2.32               |
| M3    | 5.20          | 1.90           | 4        | 45                    | 1990.80            |
| M3-SRH| 13.90         | 7.20           | 4        | 45                    | 18.01              |
| SCTSL | 57.10         | 29.60          | 4        | 50                    | 0.00               |
| SCTLL | 64.00         | 31.55          | 5        | 50                    | 0.00               |
| SCTAL | 60.20         | 32.30          | 5        | 47                    | 0.00               |
| LCTSL | 57.10         | 29.60          | 4        | 50                    | 0.00               |
| LCTLL | 64.00         | 31.55          | 4        | 50                    | 0.00               |
| LCTAL | 60.20         | 32.30          | 4        | 47                    | 0.00               |