Research Article

Pythagorean Fuzzy Digraphs and Its Application in Healthcare Center

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The notion of fuzzy set is introduced to deal with uncertainty, whereas the conventional sets are used for certainty. The extensions of fuzzy set theory such as intuitionistic fuzzy set (IFS) and Pythagorean fuzzy sets (PyFS) were introduced to overcome drawbacks in fuzzy theory. Fuzzy graph structure is used to deal with the uncertainty in a network and describe its relation on the nonempty vertex set. One of the extensions of intuitionistic fuzzy digraph (IFDG) is Pythagorean fuzzy digraph (PyFDG). IFDG cannot handle if the sum of degree of acceptance and degree of rejection for an arc weight exceeds 1. So, we introduced PyFDG to overcome the limitations in IFDG and it deals with the imprecise arc weight involving degree of acceptance and degree of rejection. Pythagorean fuzzy digraph (PyFDG) and its basic operations and score function of PyFDG are defined in this paper. Algorithm is proposed to solve application problem in healthcare center.

1. Introduction

Fuzzy set is unlike ordinary set whose elements of a set have membership value. It is found by Zadeh [1] in the year 1965. Graph theory is a mathematical tool to solve network problem and study the relation between objects (node). Rosenfeld [2] explained the notion of graph theory under fuzzy environment. Later, researchers defined the various operations of fuzzy graph and types of fuzzy graph such as complement of a fuzzy graph and regular fuzzy graph [3, 4].

Definition of IFS is a generalized fuzzy set whose elements of a set have both membership and nonmembership value and it is introduced by Atanassov [5–7]. The concepts of intuitionistic fuzzy relations (IFR) and intuitionistic fuzzy graph (IFG) are the generalizations of fuzzy graphs (FG) and it is developed by Shannon and Atanassov [8]. Operations on IFG and shortest path problem under intuitionistic fuzzy environment were developed by Parvathi et al. [9–11] and Karunambigai et al. [12]. Intuitionistic fuzzy graphs of nth type were developed by Davvaz et al. [13]. Intuitionistic fuzzy graphs of nth type are a generalization of intuitionistic fuzzy graph and intuitionistic fuzzy graph of second type. In research, several extensions of IFG can be seen in recent years [14–24]. However, the use of IFSs is identified in many fields; it has some limitations. Limitation of IFSs confines the truth and false membership value, that is, sum of truth and false membership does not exceed 1. Pythagorean fuzzy set is one of the extensions of intuitionistic fuzzy set. PyFS was developed to overcome the limitations in IFS and this theory is introduced by Yager [25–27]where a membership grade ($\mu$) and a nonmembership grade ($\nu$) with the condition $\mu^2 + \nu^2 \leq 1$. Pythagorean fuzzy number (PyFN) is developed by Zhang and Xu [28] to interpret the dual aspects of an element.

But Zhang and Xu theory failed to address the decision-making problem where the membership grade and the nonmembership grade are, respectively, 0.9 and 0.3; but $0.9^2 + 0.3^2 \leq 1$. PyFG was originally studied by Naz et al. [29] as a generalized notion of IFG and application of the proposed notion was also investigated. Akram et al. [30] recently studied the operations of PyFGs and properties of
PyFGs. The concept of planar graph under Pythagorean fuzzy environment was developed by Akram et al. [31]. Notion of maximal product of two PyFGs, residue product of two PyFGs, and its properties have been introduced and studied by Akram et al. [32]. He et al. [33] combined Pythagorean 2-tuple linguistic fuzzy set and QUALIFLEX method. This combination is used to evaluate the full quality of operation personnel in engineering field. Zhang et al. [17] combined the novel TIDIM along with cumulative prospect theory under 2-tuple linguistic Pythagorean fuzzy sets (2-TLPS) and also basic definitions and operators of 2-TLPS introduced by Zhang et al. [34]. Li et al. [35] proposed a new similarity measure under Pythagorean fuzzy environment and investigated multiple criteria group decision-making problem to prove the feasibility of the proposed method.

The objective of our work in this paper is to introduce some operations on Pythagorean fuzzy digraph, decision-making algorithm for solving problem using the notion of Pythagorean fuzzy digraph. Finally, we explore the proposed algorithm with a real-life example. This paper may motivate to study various real-life problems using the proposed algorithm.

Basic definitions which are used in our work are presented in Section 2. Definition of Pythagorean fuzzy digraph and its operations are presented in Section 3. Algorithm for solving decision-making problem using the proposed concept is developed in Section 4. Also, a decision-making problem is considered and solved using the developed algorithm in Section 5. Comparative study is presented in Section 6. In Section 7, conclusion is presented and also discussed the future work.

2. Basic Definitions

This section contributes to present the basic definitions of [1, 2, 10, 28, 30] which we used in our work. Throughout the paper, let U be the universal set.

Definition 1 (see [1]).

An object $A = \{ (a, \mu_A(a)) : a \in X \}$ is called FS over the universe X, where the mapping $\mu_A : X \rightarrow [0, 1]$ is called the membership function of A for each element $a \in X$.

Definition 2 (see [2]).

Let V be a nonempty vertex set. Fuzzy graph is denoted by $G = (P, E)$ where the mappings of a fuzzy set $P : V \rightarrow [0, 1]$ on V and the relation $E : V \times V \rightarrow [0, 1]$ on V such that $\mu_E(a_1a_2) \leq \mu_p(a_1) \land \mu_p(a_2), \forall a_1, a_2 \in V$.

The notation $\land$ represents the minimum operator.

Definition 3 (see [5]).

An object $A = \{ (a, \mu_A(a), \eta_A(a)) : a \in X \}$ is called IFS over the universe X, where the mapping $\mu_A : X \rightarrow [0, 1]$ and $\eta_A : X \rightarrow [0, 1]$ are called the membership function and nonmembership function of A for each element $a \in X$.

Definition 4 (see [28]).

Pythagorean fuzzy set is an order pair $\mu_A(e), \eta_A(e)$ whose first element is positive grade value and second element is negative grade value for each element $e \in U$. Positive and negative grade mapping, respectively, $\mu_A : U \rightarrow [0, 1], \eta_A : U \rightarrow [0, 1]$, with $0 \leq \mu_A(e) + \eta_A(e) \leq 1$. The value of $r^2_A(e) = 1 - [\mu_A(e) + \eta_A(e)]$ is called the refusal membership.

Definition 5 (see [30]).

Let V be a nonempty vertex set. PyFG is denoted by $G = (P, E)$ where the mappings of Pythagorean fuzzy set $P : V \rightarrow [0, 1]$ on V and the relation $E : V \times V \rightarrow [0, 1]$ on V such that $\mu_E(a_1a_2) \leq \mu_p(a_1) \land \mu_p(a_2), \eta_E(a_1a_2) \geq \eta_p(a_1) \lor \eta_p(a_2), \forall a_1, a_2 \in V, a_1a_2 \in E$.

The notation $\land$ represents the minimum operator; $\lor$ represents the maximum operator.

3. Pythagorean Fuzzy Directed Graphs

Pythagorean fuzzy digraph is defined in this section and also operations of Pythagorean fuzzy number, score function is presented in this section.

Definition 6. The graph $G = (P, E)$ is called PyFDG if the vertex set $P = (V, \mu, \eta)$ is a PyFS on a nonempty PyFS V and edge set $E = (V \times V, \mu_i, \eta_i)$ is defined on a relation V, where the mappings $\mu, \eta : V \rightarrow [0, 1]$ is positive and negative membership values for each vertex $v_i \in V$, and each edge $e_{ij} = (v_i, v_j) \in V \times V$ with the constraint $0 \leq \mu_i(e_{ij}) + \eta_i(e_{ij}) \leq 1$.

Remark 1. As the name implies, PyFDG does not hold a symmetric relation on V, like a PyFG holding on V.

Example 1. PyFDG $G = (V, E)$ with the vertices $V = \{a, b, c, d, e\}$, where the PyFN of each vertex in V are $a = (0.6, 0.4), b = (0.4, 0.8), c = (0.6, 0.2), d = (0.2, 0.1)$, and $e = (0.7, 0.2)$ (see Figure 1). PyFDG and its index matrix are shown in Table 1.

Definition 7. Let $P_1 = (\mu_1, \eta_1), P_2 = (\mu_2, \eta_2)$ be two PyFNs. The operations PyFN are defined as follows:

1. $P_1 \oplus P_2 = (\sqrt{\mu_1 + \mu_2 - (\mu_1)^2 (\mu_2)^2}, \eta_1 \eta_2)$
2. $P_1 \ominus P_2 = (\mu_1 \mu_2, \sqrt{\eta_1 + \eta_2 - (\eta_1)^2 (\eta_2)^2})$
3. $aP_1 = (\sqrt{1 - (1 - \mu_1)^a (\eta_1)^a}$
4. $P_1^n = (\mu_1^n, \sqrt{1 - (1 - \eta_1)^a})$

Definition 8. Let $P = (\mu, \eta)$ be PyFN. Its score function and accuracy function can be derived by using the following formulas:

1. $S(P) = \mu^2 - \eta^2$
2. $A(P) = \mu^2 + \eta^2$

Let $P_1 = (\mu_1, \eta_1), P_2 = (\mu_2, \eta_2)$ be two PyFNs. Comparisons of two PyFNs are defined as follows:
Step 3: identify the minimum \( p_j \) in step 2 and then label \([p_j, i]\) if \( p_j \) is reached from node \( i \)

Step 4: find the shortest Pythagorean fuzzy path from source node to \( j = 2, 3, 4, \ldots, n \) by combining the label \([p_j, i]\) calculated in step 3 and its corresponding \( p_i \)

Step 5: choose the path from source node to destination node and its corresponding \( p_i \) is the shortest Pythagorean fuzzy length

5. Application of Pythagorean Fuzzy Digraph

Healthcare center is a center for maintaining, improving, and helping individual’s health through diagnosis and treatment by professionals. Healthcare centers are depending on medical professionals, psychiatrists, physiotherapists, dentists, and nurses. Healthcare centers are classified into four types, namely, primary healthcare center, secondary healthcare center, tertiary healthcare center, and quaternary healthcare center. Primary healthcare center is a first point of contact by all patients within the region. General practitioner provides treatment to the patients in this center. If the problem is serious then the practitioner in the primary healthcare center recommends visiting secondary healthcare center. Secondary healthcare center is found in emergency unit in the hospital. In this center, professionals provide treatment for severe injury and emergency medical condition and during child birth also. Secondary healthcare center service is for a short-period of time while the primary healthcare center service is for a day. Tertiary healthcare center provides advanced medical treatment. The patients admitted in this center are mostly referred by primary or secondary healthcare center. Professionals working in this center are specialists for cardiac, cancer, plastic surgery, neuro surgery, and more complex illnesses. Quaternary healthcare center is a national health center because these centers are found only in limited regions. This center provides advanced treatment compared to tertiary healthcare center.

Medical practitioner in the primary healthcare center attends the patients; if the health issues of the patients are severe, then he will recommend visiting secondary healthcare center. Also, if the problem is in advanced level, then the medical practitioner recommends the patients to visit tertiary healthcare center. If the health issue of a patient became severe and is in advanced level, then the patient may visit quaternary healthcare center. There are two possibilities to each patient who visits these healthcare centers either he/she gets recovered or is forwarded to next healthcare center. Positive membership refers to the patient if he/she recovers and helping individual’s health through diagnosis and treatment by professionals. Healthcare centers are depending on medical professionals, psychiatrists, physiotherapists, dentists, and nurses. Healthcare centers are classified into four types, namely, primary healthcare center, secondary healthcare center, tertiary healthcare center, and quaternary healthcare center. Primary healthcare center is a first point of contact by all patients within the region. General practitioner provides treatment to the patients in this center. If the problem is serious then the practitioner in the primary healthcare center recommends visiting secondary healthcare center. Secondary healthcare center is found in emergency unit in the hospital. In this center, professionals provide treatment for severe injury and emergency medical condition and during child birth also. Secondary healthcare center service is for a short-period of time while the primary healthcare center service is for a day. Tertiary healthcare center provides advanced medical treatment. The patients admitted in this center are mostly referred by primary or secondary healthcare center. Professionals working in this center are specialists for cardiac, cancer, plastic surgery, neuro surgery, and more complex illnesses. Quaternary healthcare center is a national health center because these centers are found only in limited regions. This center provides advanced treatment compared to tertiary healthcare center.

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that region recommends visiting the another center. After diagnosing the patients, the practitioner recommends to visit another center depending on the severity of the issue. We constructed a Pythagorean fuzzy network for this case which is shown in Figure 2 and arc weight of this network is shown in Table 2. Presume that a patient visits center 1 with an ailment. The proposed algorithm shows the way to visit center 1 to center 8.

The nodes in the above PyFDG (see Figure 2) are the healthcare centers and connection between the healthcare centers is denoted by edges whose weights are PyFN. Weight of each edge is given in Table 2.

We applied the proposed algorithm to find the shortest path from healthcare center 1 to center 8.

Healthcare center 1 is assumed as source node $p_1 = (0, 1)$ and distance is labelled as $p_1 = [(0, 1), 1]$ and center 8 is the terminus node. So, Pythagorean fuzzy value of $p_j$, where $j = 2, 3, 4, 5, 6, 7, 8$, can be obtained in the following iterations.

**Iteration 1:** healthcare center 2 has reached from center 1

\[ p_2 = p_1 \oplus p_{12} = (0, 1) \oplus (0.7, 0.3) = (0.7, 0.3) \]

and label $p_2 = [(0.7, 0.3), 1]$

**Iteration 2:** healthcare center 3 has reached from center 1 and center 2

\[ p_3 = p_2 \oplus p_{13} = (0.7, 0.3) \oplus (0.6, 0.4) = (0.6, 0.4) \]

and $s(0.6, 0.4) = 0.2$ (min)

\[ p_3 = p_2 \oplus p_{23} = (0.7, 0.3) \oplus (0.5, 0.2) = (0.785812, 0.06) \]

and $s(0.785812, 0.06) = 0.6139$

The label $p_3 = [(0.6, 0.4), 1]$

**Iteration 3:** healthcare center 4 has reached from center 3

\[ p_4 = p_3 \oplus p_{34} = (0.6, 0.4) \oplus (0.9, 0.2) = (0.93723, 0.08) \]

The label $p_4 = [(0.93723, 0.08), 3]$

**Iteration 4:** healthcare center 5 has reached from center 2, 3, and 4

\[ p_5 = p_4 \oplus p_{25} = (0.7, 0.3) \oplus (0.8, 0.1) = (0.903549, 0.03) \]

and $s(0.903549, 0.03) = 0.8155$

\[ p_5 = p_4 \oplus p_{35} = (0.6, 0.4) \oplus (0.7, 0.4) = (0.820731, 0.16) \]

and $s(0.820731, 0.16) = 0.648$ (min)

\[ p_5 = p_4 \oplus p_{45} = (0.93723, 0.08) \oplus (0.3, 0.5) = (0.94305, 0.04) \]

and $s(0.94305, 0.04) = 0.887744$

The label $p_5 = [(0.820731, 0.16), 3]$

**Iteration 5:** healthcare center 6 has reached from center 5

\[ p_6 = p_5 \oplus p_{56} = (0.820731, 0.16) \oplus (0.6, 0.1) = (0.88944, 0.016) \]

and $s(0.88944, 0.016) = 0.790848$

The label $p_6 = [(0.88944, 0.016), 5]$

**Iteration 6:** healthcare center 7 has reached from center 4, 5, and 6

\[ p_7 = p_6 \oplus p_{47} = (0.93723, 0.08) \oplus (0.3, 0.3) = (0.94305, 0.024) \]

and $s(0.94305, 0.024) = 0.888768$

**Figure 2:** Network with Pythagorean fuzzy distance.

**Table 2:** Pythagorean fuzzy weights (PyFW) of the network graph G.

| S. No. | Arc name | PyFW |
|-------|----------|------|
| 1     | (1, 2)   | (0.7, 0.3) |
| 2     | (1, 3)   | (0.6, 0.4) |
| 3     | (2, 3)   | (0.5, 0.2) |
| 4     | (2, 5)   | (0.8, 0.1) |
| 5     | (3, 4)   | (0.9, 0.2) |
| 6     | (3, 5)   | (0.7, 0.4) |
| 7     | (4, 5)   | (0.3, 0.5) |

**Table 3:** Shortest Pythagorean fuzzy path and its weights.

| Node $j$ | Shortest Pythagorean fuzzy path from 1 to $j$ | $p_i$ |
|---------|---------------------------------------------|------|
| 2       | 1 $\longrightarrow$ 2                       | (0.7, 0.3) |
| 3       | 1 $\longrightarrow$ 3                       | (0.6, 0.4) |
| 4       | 1 $\longrightarrow$ 3 $\longrightarrow$ 4   | (0.93723, 0.08) |
| 5       | 1 $\longrightarrow$ 3 $\longrightarrow$ 5   | (0.820731, 0.16) |
| 6       | 1 $\longrightarrow$ 3 $\longrightarrow$ 5 $\longrightarrow$ 6 | (0.88944, 0.016) |
| 7       | 1 $\longrightarrow$ 3 $\longrightarrow$ 5 $\longrightarrow$ 7 | (0.869022, 0.032) |
| 8       | 1 $\longrightarrow$ 3 $\longrightarrow$ 5 $\longrightarrow$ 8 | (0.88944, 0.064) |

$P_7 = p_5 \oplus p_{35} = (0.820731, 0.16) \oplus (0.5, 0.2) = (0.869022, 0.032)$ and $s(0.869022, 0.032) = 0.754176$ (min)

$P_7 = p_5 \oplus p_{45} = (0.88944, 0.016) \oplus (0.6, 0.1) = (0.930756, 0.0016)$ and $s(0.930756, 0.0016) = 0.866304$

The label $p_7 = [(0.869022, 0.032), 5]$

**Iteration 7:** healthcare center 8 has reached from center 5, 6, and 7

$P_8 = p_7 \oplus p_{58} = (0.820731, 0.16) \oplus (0.6, 0.4) = (0.88944, 0.064)$ and $s(0.88944, 0.064) = 0.787008$ (min)

$P_8 = p_7 \oplus p_{68} = (0.88944, 0.016) \oplus (0.8, 0.1) = (0.961664, 0.0016)$ and $s(0.961664, 0.0016) = 0.924541$

$P_8 = p_7 \oplus p_{78} = (0.869022, 0.032) \oplus (0.5, 0.4) = (0.903548, 0.0128)$ and $s(0.903548, 0.0128) = 0.816236$

The label $p_8 = [(0.88944, 0.064), 5]$
Shortest path of PyFDG is obtained by working backward from healthcare center 8 and including the permanently labelled healthcare centers from which the subsequent label arose.

The shortest path of PyFDG is 1 → 3 → 5 → 8, with the length (0.88944, 0.064).

SP from the healthcare center 1 to healthcare center \( j \) given in Table 3 and the thick lines in PyFDG indicate the SP from the healthcare center 1 to healthcare center 8 shown in Figure 3.

### 6. Comparative Analysis

Advantages and limitations of existing digraph and also Pythagorean digraph are shown in Table 4.

### 7. Conclusion

Pythagorean fuzzy set has been applied in many fields to deal with uncertainty. This set has been applied in graph structure to find the shortest path. But Pythagorean fuzzy set is not discussed for digraph. So, Pythagorean fuzzy digraph is defined and operations on PyFDG are studied. Crisp values are Pythagorean fuzzified for calculation and score function is used for Pythagorean defuzzification. A real-life problem is investigated with the help of the proposed algorithm. The advantage of this work is to handle imprecise edge weight when sum of membership and nonmembership of an edge exceeds one. Future work will be investigating the various complex problems using Pythagorean fuzzy digraph.

### Abbreviation

| Type of digraph                        | Advantages                                                                 | Limitations                                                                 |
|----------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Classical digraph [33]                 | This is applicable when arc weights are precise                           | This method is not applicable when arc weights are imprecise               |
| Fuzzy digraph [34]                     | This concept can be applied for imprecise arc weights                      | Membership degree in an arc is discussed but the nonmembership degree in the same arc is discussed |
| Intuitionistic fuzzy digraph [35]      | This notion can be applied to the imprecise edge weight involving membership and nonmembership degree | This concept fails when sum of membership and nonmembership degree of an edge weight exceeds 1 |
| Pythagorean fuzzy digraph (proposed model) | This environment can deal with imprecise edge weight when sum of membership and nonmembership degree of an edge weight exceeds 1 | Hesitancy degree of an edge weight is not discussed in this concept |

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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