Analysis of Oscillatory Behavior of Heart by Using a Novel Neuroevolutionary Approach

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The authors would like to thank the Deanship of Scientific Research at Umm Al-Qura University for supporting this work by Grant Code: 19-COM-1-01-0022.

ABSTRACT This paper aims at the analysis of the VdP heartbeat mathematical model. We have analysed the conditionality of a mathematical model which represents the oscillatory behaviour of the heart. A novel neuroevolutionary approach is chosen to analyse the mathematical model. The characteristics of the cardiac pulse of the heart are examined by considering two major scenarios with sixteen different cases. Artificial neural networks (ANNs) are constructed to obtain the best solutions for the heartbeat model. Unknown weights are finely tuned by a combination of a global search technique the Harris Hawks Optimizer (HHO) and a local search technique the Interior Point Algorithm (IPA). Stable behaviour of solutions obtained by considering different cases demonstrates that the model under consideration is well-conditioned. The accuracy of our novel procedure is established by getting the lowest residual errors in our solution for all cases. Graphical and statistical analysis are added to further elaborate the accuracy of our approach.

INDEX TERMS Cardiac pulse model, hybridized soft computing, artificial neural networks, non-linear ordinary differential equations, heuristics, interior-point algorithm, Harris Hawks optimizer.

I. INTRODUCTION

The main objective of this work is to examine the efficiency of a novel neuroevolutionary approach consisting of hybridized heuristics. Our stochastic procedure is used to analyse the dynamics of nonlinear Van der Pol (VdP) based heartbeat mathematical model of second-order nonlinear ordinary differential equations (ODEs). The VdP oscillatory system has been used for the accurate, and theoretical insight to understand different behaviours of cardiac pulses [1]–[3], such as periodicity, erratic behavior, relaxation, and bifurcations [4]. In terms of nonlinear oscillator [5], [6], the modified form of VdP heart dynamic model is mathematically represented as following:

\[ \ddot{x} + \alpha (x - v_2)(x - v_1) + \frac{x(x + e)(x + d)}{ed} = F(t), \]

\[ x(0) = c_1 \text{ and } \dot{x}(0) = c_2, \quad (1) \]

in equation (1) the fiber of heart is represented by \( x \), \( \alpha \) is pulse shape modification factor of heartbeat. When the heart model is simulated, the value of \( \alpha \) changes, parameters \( v_1 \) and \( v_2 \) which are asymmetric component that modify damping term that exist in typical VdP ordinary differential equation, \( e \) is the duration of ventricular contraction while the term \( d \) is a factor that is created to replace the harmonic force term in standard VdP equation with the cubic term and the factor \( F(t) \) on right hand side of equation (1) is representing the external force factor. System in equation (1) is a nonlinear, second-order differential equation with two initial conditions representing a well-posed problem. Exact solution for VdP nonlinear oscillatory system is not available. Due to this reason, various numerical and exact methods are designed to find out the approximate solutions. For example, the Adomian Decomposition Method (ADM) [7], [8], He’s parameter expanding method [9], Laplace Decomposition Method (LDM) [10], method of linearization [11] and Homotopy Analysis Method (HAM) [12], etc. All these methods have their own applications, characteristics and limitations, but the stochastic techniques has its own organized potency, because of their strength. Moreover, techniques listed above are rarely used for the solution of Van der Pol dynamic model in the field of bio-informatics.
Artificial intelligence techniques are considered effective, accurate and reliable for the solution of many unconstrained and constrained optimization problems arising in different fields [13]–[15]. Some recent artificial intelligence methodologies based on artificial neural networks (ANNs) appeared with different applications [16], [17]. These include second-kind fredholm integral equations [18], analysis the bending of beam column [19], astrophysics models [20], bilinear programming problems [21] and inverse kinematic problems [22].

Feed-forward ANNs are used as universal function approximation procedures for the development of stochastic numerical solvers. Due to their strength and stability, they are widely used for the solutions of nonlinear systems [23]–[27]. By combining global search and local search optimization approaches, these networks are typically optimized by reducing the residual errors in solutions. Recent implementation of these methods is the solution of VdP oscillatory nonlinear systems [28], [29], fractional optimal control problems [30], [31], fuel ignition model in the theory of combustion, longitudinal heat transformation fins model [32], nano-fluidics problems, the fuel ignition mechanism in the theory of combustion [33], Navier Stoke’s equations [34], the longitudinal heat transfer fins model [35], [36], nonlinear Troesch form equations in the field of plasma physics [37], [38], thin film flow problems in fluid mechanics [39], system of linear Volterra integral equations [40], pantograph form of functional ODE and boundary value problems (BVPs) [41], [42], traveling singularity problems of the nonlinear Painleve form equations [43], magneto-hydro dynamics (MHD) study [44]–[46], electrical conducting solids models [47], electromagnetic theory problems [48], fuzzy differential equations [49], [50], the study of spherical cloud model in thermodynamics [51], nonlinear equations of Lane Emden form [51]–[53] and nonlinear systems of fractional order [54], [55].

These methodologies have encouraged many researchers to scrutinize explicitly the stability and power of stochastic numerical techniques to build an alternative, yet simple, precise, intelligent, efficient, stable, steady computing systems to examine the problems like VdP model of the heartbeat.

In this research article, a stochastic technique is established based on feed-forward ANNs which are optimized with a hybrid of the “Harris Hawks Optimizer” (HHO) and the “Interior Point Algorithm” (IPA). This soft computing paradigm is used to analyse the VdP nonlinear dynamic heartbeat model as in equation (1). Global and local search characteristics of HHO and IPA are combined to optimize the design parameters of the ANNs for solutions of VdP nonlinear dynamic heartbeat model. The results of the proposed method for the model (1) are compared with reference numerical solutions to verify the accuracy of the proposed method. Four main scenarios and sixteen different cases are considered by varying the factor of external forcing, damping coefficients, and pulse shape modification factor, while the value of the ventricular contraction period is kept constant.

The convergence and accuracy of our results obtained by the proposed scheme are statistically analysed in terms of standard deviation, mean square errors, absolute errors, mean absolute deviation (MAD), root-mean-square error (RMSE), and error in Nash–Sutcliffe efficiency (ENSE) by using results of multiple independent runs. Moreover, Nash Sutcliffe efficiency illustrates its reliability, applicability, and effectiveness.

II. HEART BEAT MODELING BASED ON VAN DER POL NONLINEAR OSCILLATOR

In this portion of the paper, we describe the essential background of the Van der Pol (Vdp) model as presented in equation (1). The Van der Pol oscillators which are also known as relaxation oscillators, were originally proposed for modelling of electronic circuits [56] in electrical engineering and are been frequently used in theoretical models of biological sciences like cardiac rhythm. The following nonlinear oscillatory model [56], [57] is a system based mathematical modeling of VdP heart model:

\[ \ddot{x} + \alpha(x^2 - 1)\dot{x} + \omega x = 0, \]

here \( \omega \) and \( \alpha \) in system (2) are constant coefficients, associated to duffing and damping parameters of the system.

In the terms of synchronization, chaos and limited cycles, VdP equation is similar to biological systems and that is why VdP system based differential equations are frequently used in representations of theoretical heart oscillations [1], [2], [4], [5], [7], [9], [12]. Moreover, VdP equation generates the external dynamic frequency of pacemaker, without any variation in amplitude and this is a vital particularity of the cardiac pacemaker. Zebrowski and Grudzinski were first to introduce these classical VdP models of heart [1].

Later on scientists modified the classical model of VdP and the properties of VdP model of heart are dramatically changed by fixing stable and saddle locations as \( x = -2d \) and \( x = -d \) respectively. Voltage related VdP heart model with updated terms of asymmetric damping is as follows:

\[ \ddot{x} + \alpha(x^2 - \mu)\dot{x} + \frac{x(x + d)(x + 2d)}{d^2} = 0. \]

The distance among these fixed points can not be changed, so for further modification in equation (3) and changes in depolarization period another parameter \( e \) is introduced as follows:

\[ \ddot{x} + \alpha(x^2 - \mu)\dot{x} + \frac{x(x + e)(x + d)}{ed} = 0, \]

the damping term \( \alpha(x^2 - \mu) \) is replaced with \( (x - v_2)(x - v_1) \), which is asymmetric with respect to the variable \( x \), for further updates in (4) it is modified as [4], [29], [56], [58]:

\[ \ddot{x} + \alpha(x - v_2)(x - v_1)\dot{x} + \frac{x(x + e)(x + d)}{ed} = 0. \]
The conditions $v_1, v_2 < 0$, must be satisfied to keep up the automatic oscillatory feature of heart. The modified model has the capability mathematically represent the fundamental physical characteristics of heart pulse subject to normal conditions. But in the presence of external forcing factor or external pacemaker $F(t)$, system in equation (5) is given as:

$$\ddot{x} + \alpha(x - v_2)(x - v_1)\dot{x} + \frac{x(x + e)(x + d)}{ed} = F(t),$$

$$x(0) = c_1, \quad \dot{x}(0) = c_2. \quad (6)$$

The given system in equation (6) is a nonlinear VdP oscillations based heart model to examine the characteristics of a cardiac pulse. Extra information about the given model is in [59].

### III. PROPOSED DESIGN OF SOFT COMPUTING

The proposed soft computing scheme for the study of heart dynamics model, consists of two parts, in the very first part of the scheme an unsupervised ANNs model is designed for the system of a differential equation (1) and in the second part of the scheme unknown weights are finely tuned using a hybrid algorithm of the “Harris Hawks Optimizer” (HHO) and the “Interior Point Algorithm” (IPA). The designed methodology is graphically presented in figure (5).

#### A. SERIES SOLUTIONS FOR HEART BEAT

**MATHEMATICAL MODEL**

The mathematical model for solutions of ordinary differential equations is formulated in the form of continuous mapping by manipulating the strength of approximation theory [58]. These networks and their $n^{th}$-derivatives of the solutions $x(t)$ are given as follow:

$$\ddot{x}(t) = \sum_{j=1}^{m} \varphi_j \left( w_j t + \beta_j \right),$$

$$\dot{x}(t) = \sum_{j=1}^{m} \varphi_j \dot{w}_j \left( w_j t + \beta_j \right),$$

$$\dot{x}(t) = \sum_{j=1}^{m} \varphi_j \dot{w}_j \left( w_j t + \beta_j \right),$$

$$\ddot{x}(t) = \sum_{j=1}^{m} \varphi_j \ddot{w}_j \left( w_j t + \beta_j \right),$$

$$\dddot{x}(t) = \sum_{j=1}^{m} \varphi_j \dddot{w}_j \left( w_j t + \beta_j \right),$$

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$$\dddot{x}(t) = \sum_{j=1}^{m} \varphi_j \dddot{w}_j \left( w_j t + \beta_j \right),$$

in above equation (7), $\varphi = [\varphi_1, \varphi_2, \varphi_3, \ldots, \varphi_m], \quad w = [w_1, w_2, w_3, \ldots, w_m]\quad \text{and}\quad \beta = [\beta_1, \beta_2, \beta_3, \ldots, \beta_m]$ are real valued vectors with bounded ranges. Networks in (7) are activated with log-sigmoid function $\xi(t) = 1/(1 + e^{-t})$ and the derivatives of log-sigmoid function in updated form are given in equations (8) below:

$$\hat{x}(t) = \sum_{j=1}^{m} \varphi_j \left( \frac{1}{1 + e^{-(w_j t + \beta_j)}} \right),$$

$$\hat{\dot{x}}(t) = \sum_{j=1}^{m} \varphi_j \dot{w}_j \left( \frac{e^{-(w_j t + \beta_j)}}{(1 + e^{-(w_j t + \beta_j)})^2} \right),$$

$$\hat{\ddot{x}}(t) = \sum_{j=1}^{m} \varphi_j \ddot{w}_j \left( \frac{2e^{-(w_j t + \beta_j)}}{(1 + e^{-(w_j t + \beta_j)})^3} - \frac{e^{-(w_j t + \beta_j)}}{(1 + e^{-(w_j t + \beta_j)})^2} \right).$$

(8)

The mathematical model of (1) can be construct by using appropriate combination of neural networks given in equations (7) or (8). The graphical composition of neural network based solutions of VdP based heart model are presented in the form of input, hidden layers and output as given in figure (1).

1) **FITNESS FUNCTIONS**

Fitness function for finding best solutions to heart dynamics model (1) are constructed as optimization problems. A minimization objective function of mean squared error is formulated as:

$$\text{minimize } \varepsilon = \varepsilon_1 + \varepsilon_2, \quad \varepsilon_1 = \frac{1}{N} \sum_{m=1}^{M} \left( \ddot{\hat{x}}(t) + \alpha(\ddot{\hat{x}}_m - v_2)(\ddot{\hat{x}}_m - v_1)\ddot{\hat{x}}_m \right. \left. + \frac{\ddot{\hat{x}}_m(\ddot{\hat{x}}_m + e)(\ddot{\hat{x}}_m + d)}{e \times d} \right)^2,$$

for $N = \frac{1}{h}, \quad \ddot{\hat{x}}_m = \ddot{x}(t_m) \quad \text{and} \quad t_m = mh. \quad (10)$

on the other hand $\varepsilon_2$ represents mean squared error related to given initial conditions as following:

$$\frac{1}{2}((\varepsilon_1 + \ddot{x}_0)^2 + (-\ddot{x}_0^2)). \quad (11)$$

With the help of a hybrid optimization technique we will tune the solution weights $w = [\varphi, w, \beta]$ for ANNs model, such that, the fitness value $\varepsilon$ of the system (1) minimize solution to zero. In this case solution of heart beat dynamic model (1) will be an ideal or near to exact solution. i.e. if $\varepsilon \rightarrow 0$ then $\ddot{x}(t) \rightarrow x(t)$.

#### B. HYBRID OPTIMIZER HHO-IPA

The unknown parameters of the ANNs model are required to be trained for getting the best solutions of nonlinear VdP dynamic heart model (1). To accomplish this task, we have combined two optimization algorithms to get an intelligence computing technique. This hybrid technique is based on the Harris Hawks Optimizer (HHO) and the Interior point algorithm (IPA). HHO is considered a nimble, accurate, intelligent, potent, and reliable technique in the class.
of particle swarm intelligence paradigms which are used mostly in various fields of numerical and applied sciences. This paradigm is Nature-inspired and was first introduced by Heidari et al. [60]. HHO is a global search technique, which means that it finds suitable or near best candidate solutions of the given optimization problems inside a unified search zone. The flowchart of HHO is in figure (2). HHO is an efficient optimizer to solve accurately unconstrained and constrained nonlinear optimization problems. The global search strength of HHO is hybridized with an effective local search algorithm, namely, IPA to get the best results rather quickly for an optimization problem. The finest individual solution of the global search algorithm HHO is selected as a starting point of local search technique IPA. Thus for quick and further tuning of unknown weights, IPA is used which is a single path following technique with better local search capability. The graphical abstract of IPA is given in figure (3) [61]. Many optimization problems are solved successfully by using IPA appearing in different fields, including hyperbolicity cone problems [62], nonlinear non-convex programming [63], parameter approximation of discrete-time infective disease models [64] and the flow of optimal power with FACTS devices [65], solutions of these problems motivated us to choose IPA, which is an interesting choice for local search.

Keeping in mind the power of HHO as a global search technique and IPA as a local search technique, a hybrid computing scheme HHO-IPA in figure (5) is applied for obtaining suitable design parameters of ANNs model to get solutions to the system of heartbeat model as shown in equation (1). For HHO we will use MATLAB script while IPA is executed in Matlab toolbox built-in function “fmincon”. Proposed scheme HHO-IPA is sensitive to settings in tables (1) and (2), a small change in these settings may cause premature convergence of the algorithm. Parameters settings are prepared with comprehensive experimentations and care.
IV. EXPERIMENTAL SETUP AND RESULTS

In this section we present our results obtained by the hybrid soft computing approach for two major scenarios with sixteen different cases related to the VdP heartbeat model, see figure (4). The problem considered here is a generalized VdP equation with initial conditions. It consists of a second-order, nonlinear ordinary differential equation (ODEs) given as an initial value problem (IVP). Different cases are taken for each scenario based on different values of asymmetric damping terms, i.e. $v_1$ and $v_2$, and pulse shape modification factor $\alpha$. The results of the proposed scheme are compared with the reference numerical solutions obtained from the Adams method (AM). The worth of the proposed scheme is proved through the numerical and graphical interpretation of the results.

 TABLE 1. Setting of parameters used for Harris Hawks Optimization.

| Parameters                       | Settings   | Parameters              | Settings |
|----------------------------------|------------|-------------------------|----------|
| Bounds [lower, upper]            | [-30 30]   | Maximum iterations      | 1000     |
| Search agents                    | 30         | Population creation     | Uniform  |
A. Khan et al.: Analysis of Oscillatory Behavior of Heart by Using a Novel Neuroevolutionary Approach

**FIGURE 3.** Illustration of the IPA.

**TABLE 2.** Setting of parameters used for “fmincon” program for the implementation of “interior point algorithm”.

| Parameters                  | Settings | Parameters              | Settings |
|-----------------------------|----------|-------------------------|----------|
| Upper bounds                | $[10]_{1 \times 30}$ | Maximum iterations       | 5000     |
| Lower bounds                | $[-10]_{1 \times 30}$ | Hessian                 | BFGS     |
| Scaling                     | Objective and Constraints | Max function evaluations | 320,000  |
| Algorithm                   | ‘Interior-point algorithm’ | X-Tolerance ‘TolX’      | $10^{-22}$ |
| Relative tolerance          | 0.1      | ‘TolCon’                | $10^{-18}$ |
| Initial weights             | HHO global best | ‘TolFun’                | $10^{-18}$ |
| Type of finite difference   | ‘Central’ | Other                   | Default  |

**A. PROBLEM-1: VDP DYNAMIC HEARTBEAT MODEL IN THE ABSENCE OF FORCING TERM**

Two different scenarios are taken into this problem. Scenario-1 is taken based on changes in pulse shape modification term $\alpha$ while scenario-2 consider the changes in asymmetric damping terms ($v_1$, $v_2$) which represent terms associated to the voltage of heartbeat dynamic model (1), in the absence of forcing term $F(t)$ that appears in a normal state of a natural pacemaker. While $d$ represents the coefficient of cubic factor which switches to harmonic forcing term and the term $e$ in classical VdP equation is constant and is used to tune period of ventricular contraction [4], [29], [56], [58].

(a) **Scenario-1**: Effects of variations in pulse shape modification factor “($\alpha$)” of heartbeat model. To analyse the effects of changes in value of “($\alpha$)” we considered four cases as follows [58]:

**Case 1**: Consider Dynamic heartbeat model for $v_1 = 0.83$, $e = 6$, $d = 3$, $v_2 = -0.83$, and $\alpha = 3$.

**Case 2**: Consider Dynamic heartbeat model for $v_1 = 0.83$, $e = 6$, $d = 3$, $v_2 = -0.83$, and $\alpha = 2$.

**Case 3**: Consider Dynamic heartbeat model for $v_1 = 0.83$, $e = 6$, $d = 3$, $v_2 = -0.83$, and $\alpha = 1$. 
Case 4: Consider Dynamic heartbeat model for $v_1 = 0.83$, $e = 6$, $d = 3$, $v_2 = -0.83$, and $\alpha = 0.01$.

The VdP nonlinear equation, obtained for the present scenario with corresponding initial conditions can be written as follows:

$$\ddot{x} + \alpha(x - 0.83)(x + 0.83)\dot{x} + \frac{x(x + 6)(x + 3)}{18} = 0,$$

$$x(0) = -0.1 \quad \text{and} \quad \dot{x}(0) = 0.025, \quad (12)$$

Equation (12) represents the cases $C_1$, $C_2$, $C_3$ and $C_4$, for $\alpha$ equal to 3, 2, 1 and 0.01 accordingly.

Exact solution for the system in equation (12) does not exist, while the best numerical solutions of respective cases of the current scenario are determined with state-of-the-art “Runge Kutta Method” (RKM) using Matlab function ode45. The performance of designed scheme is compared with the reference solutions of RKM for solutions with inputs $x$ in [0, 2] and with a step size of $h = 0.1$. The proposed methodology described in the last section is used to solve (12), while the fitness function (FF) for this scenario is given as following:

$$\varepsilon = \frac{1}{N} \sum_{m=1}^{N} \left( \ddot{x}_m + \alpha(\ddot{x}_m - 0.83)(\ddot{x}_m + 0.83)\dot{x}_m + \frac{\ddot{x}_m(\ddot{x}_m + 6)(\ddot{x}_m + 3)}{18} \right)^2 + \frac{1}{2} \left( (\ddot{x}_0 + 0.1)^2 + (\ddot{x}_0 - 0.025)^2 \right). \quad (13)$$

Optimization of the fitness function (FF) (13) is performed with the proposed hybrid scheme HHO-IPA and we got the best set of trained weights with fitness values for the cases $C_1$, $C_2$, $C_3$ and $C_4$ as $6.6486 \times 10^{-12}$, $1.0447 \times 10^{-11}$, $4.7537 \times 10^{-13}$ and $2.0005 \times 10^{-12}$ respectively, see figure (6). Solutions for the four cases, i.e. $\ddot{x}_c_1$, $\ddot{x}_c_2$, $\ddot{x}_c_3$ and $\ddot{x}_c_4$ are derived.
using the weights of figure (7) and are given as follows:

\[
\hat{x}_{c1} = \begin{cases} 
-2.1285 \\
1 + e^{-(0.4053t-0.084)} \\
-0.3400 \\
1 + e^{-(-0.3502t-2.632)} \\
1.8950 \\
1 + e^{-(2.5582t-3.196)} \\
1.7623 \\
1 + e^{-(0.7653t-0.356)} \\
0.1552 \\
1 + e^{-(1.5600t-3.2773)} \\
1.7523 \\
1 + e^{-(4.1185t-7.2004)} \\
-0.6997 \\
1 + e^{-(2.2227t-3.1798)} \\
1.5691 \\
1 + e^{-(1.1985t+0.1017)} \\
1 + e^{-(3.4359t-6.4259)} \\
\end{cases}
\]

\[
\hat{x}_{c2} = \begin{cases} 
2.1116 \\
1 + e^{-(1.4059t-2.6925)} \\
1 + e^{-(0.4214t+1.7213)} \\
1.7800 \\
1 + e^{-(0.3502t-2.6372)} \\
-0.3400 \\
1.8950 \\
1 + e^{-(2.5582t-3.196)} \\
1.7623 \\
1 + e^{-(0.7653t-0.356)} \\
0.1552 \\
1 + e^{-(1.5600t-3.2773)} \\
1.7523 \\
1 + e^{-(4.1185t-7.2004)} \\
-0.6997 \\
1 + e^{-(2.2227t-3.1798)} \\
1.5691 \\
1 + e^{-(1.1985t+0.1017)} \\
1 + e^{-(3.4359t-6.4259)} \\
-2.4066 \\
1 + e^{-(0.9261t-2.4286)} \\
1 + e^{-(0.1838t+0.6096)} \\
-2.6162 \\
1 + e^{-(0.4300t+2.7706)} \\
1 + e^{-(1.3608t-1.1842)} \\
-2.4066 \\
1 + e^{-(0.9261t-2.4286)} \\
1 + e^{-(0.1838t+0.6096)} \\
-2.6162 \\
1 + e^{-(0.4300t+2.7706)} \\
1 + e^{-(1.3608t-1.1842)} \end{cases}
\]

FIGURE 5. Graphical flowchart of designed technique for solution of VdP dynamics heartbeat model.
The VdP nonlinear equation, obtained for the present scenario with corresponding initial conditions can be written as follow:

\[
\begin{aligned}
\ddot{x} + 2(x - v_2(x - v_1))\dot{x} + \frac{x(x + 6)(x + 3)}{6 \times 3} &= 0, \\
(x(0) &= -0.1 \quad \text{and} \quad \dot{x}(0) = 0.025,
\end{aligned}
\]

(18)
equation (18) is analysed for four different cases $C_1, C_2, C_3$ and $C_4$ respectively for $(v_1, v_2)$ chosen as $(0.93, -0.93), (0.83, -0.83), (0.63, -0.63)$ and $(0.43, -0.43)$.

Exact solution for the problem in equation (18) is also not known, for this reason, numerical solutions of equation (18) are calculated by using AM, and these solutions are used as reference points to calculate errors in our outcome. We have used the same experimental settings as in the previous scenario, but the fitness function is given as:

\[
\varepsilon = \frac{1}{N} \sum_{m=1}^{N} \left( \dot{x}_m + 2(\dot{x}_m - v_2(\dot{x}_m - v_1))\dot{x}_m \\
+ \dot{x}_m(\dot{x}_m + 6(\dot{x}_m + 3)) \right)^2 \\
+ \frac{1}{2} (\ddot{x}_0 + 0.1)^2 + (\ddot{x}_0 - 0.025)^2
\]

(19)

Weights for ANNs trained by HHO-IPA, with FF values for cases 1 and 4, are $7.9429 \times 10^{-12}, 2.3429 \times 10^{-13}$ and $3.0588 \times 10^{-13}$ respectively, and are graphically shown in figure (8). Corresponding solutions based on these weights are given as in 20-22:

\[
\begin{aligned}
\ddot{x}_c &= \left\{ \begin{array}{l}
-3.6967 \\
+ 1 + e^{-(-1.7278 \times 1.39495)} \\
+ 1 + e^{-(-0.8181 \times -0.3241)} \\
\end{array} \right. \\
\ddot{x}_c &= \left\{ \begin{array}{l}
0.0043 \\
+ 1 + e^{-(-2.4621 \times -2.1053)} \\
+ 1 + e^{-(-0.8423 \times -0.3244)} \\
\end{array} \right. \\
\ddot{x}_c &= \left\{ \begin{array}{l}
4.1642 \\
+ 1 + e^{-(-1.8931 \times 4.9025)} \\
+ 1 + e^{-(-0.6783 \times -0.3366)} \\
\end{array} \right. \\
\ddot{x}_c &= \left\{ \begin{array}{l}
-1.2122 \\
+ 1 + e^{-(-2.7061 \times 5.9757)} \\
+ 1 + e^{-(-0.2985 \times -0.8385)} \\
\end{array} \right. \\
\ddot{x}_c &= \left\{ \begin{array}{l}
-1.1322 \\
+ 1 + e^{-(-0.8168 \times -0.3253)} \\
+ 1 + e^{-(-0.8254 \times -0.3250)} \\
\end{array} \right.
\]

(20)

\[
\begin{aligned}
\ddot{x}_c &= \left\{ \begin{array}{l}
1.1060 \\
+ 1 + e^{-(-0.5050 \times -0.7056)} \\
+ 1 + e^{-(-0.7550 \times -2.7135)} \\
\end{array} \right. \\
\ddot{x}_c &= \left\{ \begin{array}{l}
-2.9744 \\
+ 1 + e^{-(-1.0749 \times 2.0097)} \\
+ 1 + e^{-(-0.1283 \times -0.5130)} \\
\end{array} \right. \\
\ddot{x}_c &= \left\{ \begin{array}{l}
-0.4305 \\
+ 1 + e^{-(-0.6703 \times -2.6204)} \\
+ 1 + e^{-(-0.0668 \times -2.1888)} \\
\end{array} \right. \\
\ddot{x}_c &= \left\{ \begin{array}{l}
2.2456 \\
+ 1 + e^{-(-0.9271 \times -0.5100)} \\
+ 1 + e^{-(-0.4390 \times -1.7351)} \\
\end{array} \right. \\
\ddot{x}_c &= \left\{ \begin{array}{l}
2.6392 \\
+ 1 + e^{-(-1.1526 \times 2.1596)} \\
+ 1 + e^{-(-1.1039 \times -2.6948)} \\
\end{array} \right.
\]

(21)
A. Khan et al.: Analysis of Oscillatory Behavior of Heart by Using a Novel Neuroevolutionary Approach

FIGURE 6. Solutions obtained by our approach are shown in Fig 6(a) and absolute errors are given in Fig 6(b) for four cases of problem 1, scenario-1.

FIGURE 7. Trained weights for ANNs model optimized through hybrid scheme for cases C_1, C_2, C_3 and C_4 based on variation in α for problem 1, scenario-1.

The solution of \( \hat{x}_{c2} \) of scenario-2 is same as the solution of case_2 of scenario-1. Suggested solutions are presented in figure (8a) and are formed for inputs in interval [0, 2], and using the step size of \( h = 0.1 \). However, the results of absolute error for every case are graphically illustrated in figure (8b). It is observed that the proposed hybrid scheme attained the best accuracy of order \( 10^{-15} - 10^{-11} \).

To obtain the best weights for the ANNs model, fifty independent runs of designed scheme HHO-IPA are performed, and the experimental outcomes are listed in the table (3) in the form of lowest error in solution, worst error in solution, Mean and standard deviation (STD) of errors. It is clear that for these four cases i.e. C_1, C_3 and C_4 the mean values are around \( 10^{-05} - 10^{-04}, 10^{-11} - 10^{-10} \), and \( 10^{-10} - 10^{-09} \). Besides, the accuracy of the designed scheme HHO-IPA is verified by the lower values of STD.

B. PROBLEM-2: VDP DYNAMIC HEARTBEAT MODEL IN THE PRESENCE OF FORCING TERM

Two different scenarios are taken into this problem. Scenario-1 is taken based on changes in pulse shape modification term \( \alpha \) while scenario-2 consider the changes in asymmetric damping terms \( (v_1, v_2) \) which represent terms associated to the voltage of heartbeat dynamic model (1), with the presence of forcing term \( F(t) \) that appears in a normal state of the
natural pacemaker. While $d$ represents the coefficient of cubic factor which switches to harmonic forcing term and the term $e$ in classical VdP equation is constant that is used to control the period of ventricular contraction. Rest of the settings are the same as in problem-1, mathematically forcing term can be represented as follows:

$$F(t) = A \sin(\omega t).$$  \hspace{1cm} (23)

(a) Scenario-1: Effects of variations in pulse shape modification factor “$(\alpha)$” of heartbeat model. To analyse the effects of changes in value of “$(\alpha)$” we considered four cases as follows [58]:

Case 1: Consider Dynamic heartbeat model for $\alpha = 0.5$, $b = 2.5$, $\omega = 1.9$, $e = 6$, $v_1 = 0.97$, $v_2 = -1$ and $d = 3$.

Case 2: Consider heartbeat model for $\alpha = 0.4$, $b = 2.5$, $\omega = 1.9$, $e = 6$, $v_1 = 0.97$, $v_2 = -1$ and $d = 3$. 

(a) Scenario-1: Solutions obtained by our approach are shown in Fig 8(a) and absolute errors are given in Fig 8(b) for four cases of problem 1, scenario-2.

(b) FIGURE 9. Trained weights for ANNs model optimized through hybrid scheme for cases $C_1$, $C_2$, $C_3$ and $C_4$ based on variation in asymmetric damping terms ($v_1$, $v_2$) for problem 1, scenario-2.

TABLE 3. The statistical analysis of errors of four different cases for scenario-1 and scenario-2 of problem-1, of heartbeat model.

| Scenarios | Cases | $\alpha$ | ($v_1$, $v_2$) | Best error in solution | Worst error in solution | Mean | STD |
|-----------|-------|----------|----------------|------------------------|-------------------------|------|-----|
| Scenario-1 | $C_1$ | 3 | (0.83, -0.83) | 6.6486E-12 | 6.5993E-07 | 1.7342E-08 | 9.3016E-08 |
|             | $C_2$ | 2 | (0.83, -0.83) | 1.0447E-11 | 7.1801E-06 | 2.8539E-07 | 1.3008E-06 |
|             | $C_3$ | 1 | (0.83, -0.83) | 4.7537E-13 | 2.2572E-04 | 5.0817E-06 | 3.1994E-05 |
|             | $C_4$ | 0.01 | (0.83, -0.83) | 2.0605E-12 | 8.4920E-09 | 1.0183E-09 | 1.7513E-09 |
| Scenario-2 | $C_1$ | 2 | (0.93, -0.93) | 7.9429E-12 | 3.2799E-04 | 1.3996E-05 | 6.5026E-05 |
|             | $C_2$ | 2 | (0.83, -0.83) | 1.0447E-11 | 7.1801E-06 | 2.8539E-07 | 1.3008E-06 |
|             | $C_3$ | 2 | (0.63, -0.63) | 2.3429E-13 | 2.5385E-09 | 5.2768E-10 | 6.0442E-10 |
|             | $C_4$ | 2 | (0.43, -0.43) | 3.0588E-13 | 1.5198E-08 | 1.4576E-09 | 3.4988E-09 |
A. Khan et al.: Analysis of Oscillatory Behavior of Heart by Using a Novel Neuroevolutionary Approach

\( \omega = 1.9, e = 6, v_1 = 0.97, v_2 = -1 \) and \( d = 3 \).

**Case 4:** Consider heartbeat model for \( \alpha = 0.2, b = 2.5, \omega = 1.9, e = 6, v_1 = 0.97, v_2 = -1 \) and \( d = 3 \).

The VdP equation derived from (1) for this scenario with initial conditions is following:

\[
\begin{align*}
\ddot{x} + \alpha(x+1)(x-0.97)x + \frac{x(x+6)(x+3)}{6 \times 3} &= 2.5 \sin(1.9t), \\
x(0) &= -0.1 \text{ and } \dot{x}(0) = 0.025,
\end{align*}
\]

(24)

for \( \alpha \) equal to 0.5, 0.4, 0.3 and 0.2 we analyse the model by four cases \( C_1, C_2, C_3 \) and \( C_4 \) respectively. Equation (24) is used for this scenario.

Adams numerical technique is used to calculate reference solutions for the second order ODE (24). Exact solutions are not available for this case. We have used same experimental settings as in previous scenarios. The fitness function for this scenario is given as:

\[
\varepsilon = \frac{1}{N} \sum_{m=1}^{N} \left( \frac{\hat{x}_m + \alpha(x_m + 1)(\hat{x}_m - 0.97)\hat{x}_m}{6 \times 3} - 2.5 \times \sin(1.9 \times t) \right) + \frac{1}{2} \left( (\hat{x}_0 + 0.1)^2 + (\hat{x}_0 - 0.025)^2 \right).
\]

(25)

We have obtained the fitness values for problem-2 scenario-1 as \( 2.8836 \times 10^{-10}, 2.7479 \times 10^{-10}, 8.0300 \times 10^{-11} \) and \( 2.7565 \times 10^{-11} \) for a set of best weights respectively, for cases \( C_1, C_2, C_3 \) and \( C_4 \). We have graphically illustrated the ranges of these weights, see figure (11). Series solutions obtained by using best weights which we have obtained for this scenario are given in (26-29):

\[
\hat{x}_c_1 = \begin{bmatrix}
0.7467 \\
0.7248 \\
0.2206 \\
0.2727 \\
-2.7338 \\
-2.4487 \\
-2.3071 \\
-0.3258 \\
-0.5979
\end{bmatrix}

\] + \begin{bmatrix}
-1.0405 \\
1.7577 \\
1.3928 \\
-5.1389 \\
3.2154 \\
2.5692 \\
2.4369 \\
0.2154 \\
0.8287
\end{bmatrix}

(26)

\[
\hat{x}_c_4 = \begin{bmatrix}
1.1808 \\
3.1784 \\
0.4893 \\
4.9540 \\
-2.9827 \\
1.3345 \\
-1.0076 \\
-2.0877 \\
3.5433
\end{bmatrix}

\] + \begin{bmatrix}
-3.9300 \\
2.6626 \\
-0.0228 \\
0.4425 \\
-3.2730 \\
-0.2943 \\
1.9483 \\
-1.0879 \\
-3.2468
\end{bmatrix}

(29)

Solutions for this scenario are achieved by using inputs between [0, 2] with a step size of \( h = 0.1 \). By using our best set of weights we get equations (26 – 29) and are illustrated in figure (10). The values of AE in our solutions from the reference numerical solution for all cases of problem-2 scenario-1 are given in figure (11). It is obvious that AEs for all cases are achieved of order around \( 10^{-09} - 10^{-13} \). The statistical analysis of our results is measured in terms of errors in the best solution, worst solution, mean values of absolute errors (AE) and standard deviation (STD). These results are established based on 50 independent runs and are given in table (4). The mean values for cases i.e. \( C_1, C_2, C_3 \) and \( C_4 \) are around \( 10^{-09} - 10^{-07}, 10^{-04} - 10^{-02}, 10^{-09} - 10^{-07} \) and \( 10^{-10} - 10^{-08} \). It is worth to note, that the accuracy of our designed scheme HHO-IPA is verified by the lower values of standard deviation in errors for solutions of all cases.

(b) Scenario-2: Effects of variations in asymmetric damping terms \( (v_1, v_2) \) on the dynamic heartbeat model.

In this scenario we have studied the effects of variation in asymmetric damping parameters \( (v_1, v_2) \) on the heartbeat dynamic model. We have considered four cases for this purpose as follows:

**Case 1** Consider Dynamic heartbeat model for \( v_2 = -1, b = 2.5, e = 6, \omega = 1.9, \alpha = 0.5 \) and \( v_1 = 0.97, d = 3 \).

**Case 2** Consider Dynamic heartbeat model for \( v_2 = -3, b = 2.5, e = 6, \omega = 1.9, \alpha = 0.5 \) and \( v_1 = 0.87, d = 3 \).

**Case 3** Consider Dynamic heartbeat model for \( v_2 = -4, b = 2.5, e = 6, \omega = 1.9, \alpha = 0.5 \) and \( v_1 = 0.67, d = 3 \).

**Case 3** Consider Dynamic heartbeat model for \( v_2 = -5, b = 2.5, e = 6, \omega = 1.9, \alpha = 0.5 \) and \( v_1 = 0.47, d = 3 \).

The mathematical model of VdP and corresponding initial conditions for this scenario is given as:

\[
\begin{align*}
\ddot{x} + 0.5(x - v_2)(x - v_1)\dot{x} + \frac{x(x + 3)(x + 6)}{3 \times 6} &= 2.5 \times \sin(t \times 1.9), \\
x(0) &= -0.1 \text{ and } \dot{x}(0) = 0.025,
\end{align*}
\]

(30)
replacing \((v_1, v_2) = (0.97, -1), (0.87, -3), (0.67, -4)\) and \((0.47, -5)\), in equation (30) we get cases \(C_1, C_2, C_3\) and \(C_4\) of this scenario, respectively.

Exact solution for the system in equation (24) does not exist in literature; for this reason, approximate solutions of equation (24) are found by using AM, and these solutions are used as reference solutions. We have used the same experimental settings as in previous scenarios. Fitness function for these variations is as follows:

\[
\varepsilon = \frac{1}{N} \sum_{m=1}^{N} \left( \hat{x}_m + 0.5(\hat{x}_m - v_2)(\hat{x}_m - v_1)\hat{x}_m + \frac{\hat{x}_m(\hat{x}_m + 6)(\hat{x}_m + 3)}{6 \times 3} \right)^2 \\
- 2.5 \times \sin(t \times 1.9) \\
+ \frac{1}{2} \left( \hat{x}_0 - 0.025 \right)^2 + (\hat{x}_0 + 0.1)^2
\]

(31)

The set of trained weights optimized through HHO-IPA, respectively for cases \(C_2, C_3\) and \(C_4\) with fitness values \(3.6962 \times 10^{-10}, 1.8650 \times 10^{-10}\) and \(2.0357 \times 10^{-10}\) are graphically represented in figure (13). Using these weights the derived solutions for cases \(C_2, C_3\) and \(C_4\) are mathematically defined as follows:

\[
\hat{x}_c = \begin{cases} 
0.7467 \\
\frac{1}{1+e^{-(-0.4597t-1.4897)}} + \frac{1}{1+e^{-(3.5827t-4.9514)}} - 2.5529 \\
+ \frac{1}{1+e^{-(0.4013t-3.9512)}} + \frac{1}{1+e^{-(2.6355t-7.9241)}} \\
\end{cases}
+ \frac{1}{1+e^{(-3.1418t+3.7057)}} + \frac{1}{1+e^{(-2.7727t+2.5359)}} \\
- 2.4484 + \frac{1}{1+e^{-(1.9932t-0.2220)}} + \frac{1}{1+e^{-(1.9035t+3.7091)}} - 5.1389 \\
+ \frac{1}{1+e^{(-0.4013t-3.9512)}} + \frac{1}{1+e^{-(2.6355t-7.9241)}}
\]

(32)
FIGURE 12. Solutions obtained by our approach are shown in Fig 12(a) and absolute errors are given in Fig 12(b) for four cases of problem 2, scenario-2.

FIGURE 13. Trained weights for ANNs model optimized through hybrid scheme for cases $C_1$, $C_2$, $C_3$ and $C_4$ based on variation in asymmetric damping terms ($v_1$, $v_2$) for problem-2, scenario-2.

TABLE 4. The statistical analysis of errors for four different cases for scenario-1 and scenario-2 of problem-2, of heartbeat model.

| Scenarios | Cases | $\alpha$ | ($v_1$, $v_2$) | Best error in solution | Worst error in solution | Mean | STD |
|-----------|-------|----------|----------------|------------------------|-------------------------|------|-----|
| Scenario-1 | $C_1$ | 0.5 | (0.97, -1) | 2.8836E-10 | 6.6261E-08 | 1.2908E-08 | 1.3785E-08 |
| | $C_2$ | 0.4 | (0.97, -1) | 2.7479E-10 | 2.4219E-08 | 5.8951E-09 | 6.1479E-09 |
| | $C_3$ | 0.3 | (0.97, -1) | 8.0300E-11 | 9.7161E-09 | 2.2891E-09 | 2.7488E-09 |
| | $C_4$ | 0.2 | (0.97, -1) | 2.7565E-11 | 2.2436E-08 | 2.8613E-09 | 5.1755E-09 |
| Scenario-2 | $C_1$ | 0.5 | (0.97, -1) | 2.8836E-10 | 2.5547E-06 | 3.6693E-08 | 2.5406E-07 |
| | $C_2$ | 0.5 | (0.87, -3) | 3.6962E-10 | 8.5945E-08 | 2.2093E-08 | 2.3081E-08 |
| | $C_3$ | 0.5 | (0.67, -4) | 1.8560E-10 | 9.1357E-08 | 1.8537E-08 | 2.0247E-08 |
| | $C_4$ | 0.5 | (0.47, -5) | 2.0358E-10 | 2.8389E-07 | 3.7522E-08 | 4.5560E-08 |

$$\hat{x}_c = \begin{cases} 
0.3091 \\
\frac{1}{1+e^{-(2.6845t+0.0899)}} \\
\frac{1}{1+e^{-(1.9091t-2.0154)}} + 0.2328 \\
\frac{1}{1+e^{-(3.2578t-3.3338)}} - 1.5976 \\
\frac{1}{1+e^{-(0.3611t-1.3396)}} + 1.3492 \\
\frac{1}{1+e^{-(2.3540t+2.3764)}}
\end{cases} + \begin{cases} 
-3.5999 \\
\frac{-0.9953t-1.5538}{1+e^{-(2.7938t+8.5594)}} + 4.0139 \\
\frac{0.2102t+0.3699}{1+e^{-(2.7938t+8.5594)}} - 3.0240 \\
\frac{1.01746}{1+e^{-(2.3578t-3.3338)}} \\
\frac{1.6509}{1+e^{-(2.6845t+0.0899)}} - 4.7675
\end{cases}, \tag{33}$$

$$\hat{x}_c = \begin{cases} 
2.1160 \\
\frac{1+e^{-(2.0180t+2.7664)}}{1+e^{-(3.5157t-4.0161)}} - 1.2446 \\
\frac{1+e^{-(1.0225t-0.1344)}}{1+e^{-(3.0748t+3.1021)}} - 6.6985 \\
\frac{1+e^{-(2.3287t-7.9767)}}{1+e^{-(3.8073t-4.2497)}} + 0.7325 \\
\frac{1+e^{-(1.559t-3.4066)}}{1+e^{-(1.3713t+0.7520)}} + 3.4242
\end{cases}, \tag{34}$$
A. Khan et al.: Analysis of Oscillatory Behavior of Heart by Using a Novel Neuroevolutionary Approach

FIGURE 14. The graphs of fitness values for every case of both problems for fifty independent runs of heartbeat model. (a) and (c) shows scenario-1 of problem-1 and problem-2 for unlike values of $\alpha$ respectively, while (b) and (d) shows scenario-2 of problem-1 and problem-2 for unlike values of $(v_1, v_2)$.

Best solution of $\hat{x}_{c_1}$ in the present scenario and $\hat{x}_{c_1}$ in previous scenario are similar. We got our results based on the inputs in the interval $[0, 2]$ with $h = 0.1$ taken as step size and the AE in our solutions and reference numerical solutions are given in figure (13). It is evident that the accuracy of the order between $10^{-11} - 10^{-07}$ is achieved by our designed technique. Fifty independent runs are simulated based on our hybrid scheme HHO-IPA. Our experimental outcomes are presented in terms of the best error in solution, worst error in solution, STD and Mean of error which is listed in the table (4). It is observed that the mean values for these three cases i.e. $C_2$, $C_3$ and $C_4$, respectively are of order $10^{-05} - 10^{-06}$, $10^{-04} - 10^{-06}$ and $10^{-04} - 10^{-05}$. Additionally, lower values of standard deviation revealed the exactness of our proposed scheme.

V. DISCUSSION ON RESULTS

In this research, we have considered a VdP heartbeat model see figure (4). It is a second-order, non-linear ordinary differential equation with initial conditions. Our analysis is divided into two main problems and sixteen subcases, see figure (4).
Below we present details of our experimental outcome in this paper:

**A. PROBLEM-I, SCENARIO-I**

In this scenario, we have analyzed the effects of variations in pulse shape modification factor $\alpha$ in heartbeat model.

We have subdivided this scenario into four cases by choosing different values of $\alpha$. It is worth noting that the exact solution doesn’t exist for equation (12). Results obtained by RKM are used as reference solutions. It is observed that results obtained by our hybrid scheme of ANNs and HHO-IPA are better than RKM solutions. Our results for $C_1$, $C_2$, $C_3$, and $C_4$...
A. Khan et al.: Analysis of Oscillatory Behavior of Heart by Using a Novel Neuroevolutionary Approach

FIGURE 16. The graph of RMSE values considering sixteen cases of HBM for fifty independent runs of heartbeat model. (a) and (c) shows scenario-1 of problem-1 and problem-2 for unlike values of $\alpha$ respectively, while (b) and (d) shows scenario-2 of problem-1 and problem-2 for unlike values of $(v_1, v_2)$.

are $6.6486E-12$, $1.0447E-11$, $4.7537E-13$, and $2.0005E-12$ respectively, see figure (6b). These solutions are obtained by using the best weights obtained by our optimizer the HHO-IPA, see figure (7). As in figure (6b), solutions for $\alpha = 1$ are best in terms of residual errors. On the other hand, solutions at $\alpha = 2, 3$ are comparatively worse. For $\alpha = 0.001$, we got average solutions. This means that higher values of pulse shape modification factor results in exponentially growing solutions, see figure (6a).

B. PROBLEM-I, SCENARIO-II

In this scenario, we have analyzed the effects of variations in asymmetric damping terms $(v_1, v_2)$ on the dynamic heartbeat model in the absence of forcing term. Four subcases are considered by varying values of $(v_1, v_2)$. The exact solution to this problem is not known. We have considered the solutions obtained by Adam’s numerical technique as reference solutions. The same experimental settings are used as in scenario-I. After training weights of ANNs, our optimizer HHO-IPA, successfully got better solutions with lower errors. Fitness values of our solutions for $C_1$, $C_3$, and $C_4$ are $7.9429E-12$, $2.3429E-13$, and $3.0588E-13$ respectively, see figure (8). In all four cases, errors are in the range of E-15 to E-11. Moreover, the graphs of our solutions are similar with slight variations. This points to the better conditionality of the mathematical heartbeat model. Best, mean, worst, and standard deviations in our errors are listed in the table (3). The best weights for this scenario are given in figure (9).

C. PROBLEM-II, SCENARIO-I

In this problem, the forcing term is considered. It is given in equation (23). In scenario-I, we study the effects of pulse shape modification factor $\alpha$. We have considered four subcases for different values of $\alpha$. The mathematical model
for this scenario is given in equation (24). As there is no exact solution for this problem, so we have considered solutions obtained by Adam’s numerical technique as reference solutions. The same experimental setup is used as in previous scenarios. We have obtained better fitness values for $C_1$, $C_2$, $C_3$, and $C_4$ as 2.8836E-10, 2.7479E-10, 8.0300E-11, and 2.7565E-11. These errors are graphically illustrated in figure (10) and corresponding weights are shown in figure (11). It is obvious from our experiments that errors in our solutions are lower and are ranging between E-13 and E-10. It is interesting to note that with forcing term solutions are almost similar for all four cases, see figure (10a).

**D. PROBLEM-II, SCENARIO-II**

In this scenario, we have kept values of $\alpha$ constant, and analyzed the effects of variations in $(v_1, v_2)$ on the heartbeat model in the presence of forcing term. For this purpose, we have considered four cases. Reference solutions by Adam’s technique are used for calculating errors in our results. The set of trained weights optimized through HHO-IPA, respectively for cases $C_2$, $C_3$, and $C_4$ are shown in figure (13). We have plotted our solutions in figure (12a). It is observed that there are variations in solution graphs for different values of $(v_1, v_2)$. This points to the significance of terms $(v_1, v_2)$ in the heartbeat model in presence of forcing term. Absolute errors are given in figure (12b), where for all cases errors in our solutions are ranging between E-13 to E-09. Detailed statistical analysis of absolute errors in our solutions are given in table (4).

**VI. COMPARATIVE PERFORMANCE-INDEX TESTS**

In this section, we have analysed our experimental outcome base on performance indicators as given in equations (36-39), and (40-43). These indicators have further revealed the better
TABLE 6. Convergence complexity for the proposed algorithm based on global performance indicators for every change in heartbeat model.

| Problem | Scenario | Cases | Mean | STD | Mean | STD | Mean | STD |
|---------|----------|-------|------|-----|------|-----|------|-----|
| P-1 | S-1 | C_1 | 3.4684E-10 | 1.8603E-09 | 3.4689E-07 | 6.5535E-07 | 4.5802E-07 | 8.3307E-07 |
|       | C_2 | 5.7078E-09 | 2.6016E-08 | 1.1774E-06 | 5.2350E-06 | 1.4989E-06 | 6.9023E-06 |
|       | C_3 | 1.0163E-07 | 6.3987E-07 | 4.0972E-06 | 2.1303E-05 | 4.3935E-06 | 2.2124E-05 |
|       | C_4 | 2.0360E-11 | 3.5027E-11 | 3.5677E-08 | 4.0765E-08 | 5.0289E-08 | 5.0289E-08 |
| S-2 | C_1 | 2.7992E-07 | 1.3005E-07 | 5.9548E-05 | 2.6248E-04 | 8.2296E-05 | 3.6333E-04 |
|       | C_2 | 1.0554E-11 | 1.2088E-11 | 4.6992E-08 | 3.7813E-08 | 5.3285E-08 | 4.4522E-08 |
|       | C_3 | 2.9152E-11 | 6.9976E-11 | 5.0076E-08 | 7.0578E-08 | 5.7294E-08 | 8.0962E-08 |
|       | C_4 | 5.7226E-11 | 1.0351E-10 | 1.1424E-07 | 6.0168E-08 | 1.4306E-07 | 6.5825E-08 |
| P-2 | S-1 | C_1 | 2.5815E-10 | 2.7569E-10 | 1.5904E-07 | 7.8847E-08 | 2.0894E-07 | 9.5420E-08 |
|       | C_2 | 1.1790E-10 | 1.2296E-10 | 1.3077E-07 | 6.2412E-08 | 1.7440E-07 | 7.2350E-08 |
|       | C_3 | 4.5783E-11 | 5.4976E-11 | 1.1538E-07 | 5.4058E-08 | 1.4923E-07 | 6.0747E-08 |
|       | C_4 | 4.4185E-10 | 4.6161E-10 | 3.2257E-07 | 1.0926E-07 | 4.3560E-07 | 1.3646E-07 |

TABLE 7. Computational complexity analysis results for various changes in heartbeat model.

| Problem | Scenario | Cases | Mean | STD | Mean | STD | Mean | STD |
|---------|----------|-------|------|-----|------|-----|------|-----|
| P-1 | S-1 | C_1 | 38.3 | 2.1 | 1578.0 | 36.7 | 72,490.4 | 1671.7 |
|       | C_2 | 38.1 | 2.9 | 1547.9 | 67.0 | 73,456.9 | 3201.1 |
|       | C_3 | 38.2 | 2.1 | 1588.8 | 17.3 | 73,204.9 | 2967.3 |
|       | C_4 | 38.8 | 5.8 | 1538.7 | 57.4 | 71,937.7 | 4572.1 |
| S-2 | C_1 | 38.1 | 2.1 | 1573.6 | 30.7 | 78,673.8 | 921.4 |
|       | C_2 | 38.2 | 1.9 | 1589.5 | 24.1 | 74,765.9 | 2908.5 |
|       | C_3 | 39.3 | 6.9 | 1529.0 | 114.7 | 73,724.0 | 2512.3 |
|       | C_4 | 38.7 | 2.0 | 1550.0 | 26.2 | 74,536.2 | 213.0 |
| P-2 | S-1 | C_1 | 39.2 | 2.2 | 1590.8 | 94.8 | 72,951.4 | 211.2 |
|       | C_2 | 39.2 | 2.1 | 1584.3 | 99.0 | 73,657.8 | 301.8 |
|       | C_3 | 39.3 | 2.4 | 1591.4 | 0.0 | 76,847.8 | 721.8 |
|       | C_4 | 39.9 | 2.1 | 1573.7 | 0.0 | 75,939.3 | 1029.2 |
| S-2 | C_1 | 38.4 | 2.7 | 1576.9 | 38.1 | 74,573.9 | 2192.7 |
|       | C_2 | 38.5 | 2.8 | 1580.0 | 23.7 | 73,948.0 | 2900.0 |

The values of performance indicators containing fitness, ENSE, MAD, and RMSE for fifty independent runs are calculated, and the consistency and effectiveness of the designed computing approach are inspected.

Mathematical expressions for mean absolute derivation (MAD), root-mean-square error (RMSE) and error in Nash–Sutcliffe efficiency (ENSE) are given as follows:

\[
MAD = \frac{1}{N} \sum_{m=1}^{N} \left( \hat{x}(t_i) - x(t_i) \right),
\]

\[
RMSE = \sqrt{\frac{1}{N} \sum_{m=1}^{N} \left( \hat{x}(t_i) - x(t_i) \right)^2},
\]

\[
NSE = 1 - \left( \frac{\sum_{i=1}^{N} \left( \hat{x}(t_i) - x(t_i) \right)^2}{\sum_{i=1}^{N} \left( x(t_i) - \frac{1}{N} \sum_{j=1}^{N} x(t_j) \right)^2} \right).
\]

\[
ENSE = |1 - NSE|.
\]

The values of performance indicators containing fitness, ENSE, MAD, and RMSE for fifty independent runs are calculated, and the consistency and effectiveness of the designed computing approach are inspected.

Graphical illustrations of MAD, fitness, ENSE and RMSE respectively are presented on semi-log scale for 50 runs, see figures (14), (15), (16) and (17).

It is observed that different values of MAD, RMSE and ENSE performance indicators varied directly with fluctuations in fitness values between low and high. It is noted that for problem-1 scenario-1, and C_1 these variations are negligible as seen from values of MAD, ENSE and RMSE. Additionally, for mentioned case the value of these indicators are comparatively decreased. Reliability of our designed technique is further inspected through by percentage of converged runs on the basis of pre-described criterion of MAD, fitness, ENSE and RMSE values. The successfully converged runs (Cr) for fifty separate simulations is calculated for each case and our results are tabulated in table (5) for both problems. These calculations are done based on the following criteria, i.e. \((Cr_{FIT}) \leq E - 0.07, (Cr_{MAD}) \leq E - 0.05, (Cr_{RMSE}) \leq E - 0.05 \text{ and } (Cr_{ENSE}) \leq E - 0.07\). It is interesting to note that the average rate of convergence of our scheme is almost 100%.

Further estimation of the performance of our designed technique is carried out by describing its efficiency through global indicators, i.e.global MAD, global RMSE, global ENSE and global fitness. Formulations for these indicators are following:

\[
G_{MAD} = \frac{1}{R_{p}} \sum_{r=1}^{R_{p}} \left( \frac{1}{G_{p}} \sum_{i=1}^{G_{p}} \left( \hat{x}(t_i) - x(t_i) \right)^2 \right),
\]

\[
G_{RMSE} = \frac{1}{R_{p}} \sum_{r=1}^{R_{p}} \left( \frac{1}{G_{p}} \sum_{i=1}^{G_{p}} \left( \hat{x}(t_i) - x(t_i) \right)^2 \right).\]
TABLE 8. Notations and abbreviations used in this paper.

| Abbreviation | Description |
|--------------|-------------|
| ANNs         | Artificial neural networks |
| HHO          | Harris Hawks Optimizer |
| IPA          | Interior Point Algorithm |
| VdP          | Van der Pol |
| MAD          | Mean absolute deviation |
| RMSE         | Root-mean-square error |
| AE           | Absolute error |
| AM           | Adams method |
| HBM          | Heart beat model |
| Gb           | Global indicators |
| GbFIT        | Fitness function |
| STD          | Standard deviation |
| Cr           | Converged runs |
| GbENSE       | Error in Nash–Sutcliffe efficiency |
| ENSE         | Nash–Sutcliffe efficiency |
| RKM          | Runge Kutta Method |
| PP           | Computational complexity analysis |
| Cr           | Converged runs |
| Rn           | Number of total runs |
| Cr           | Converged runs |
| Cr           | Converged runs |
| Cr           | Converged runs |

\[
G_{bENSE} = \frac{1}{R_n} \sum_{r=1}^{R_n} \left( \frac{1}{N} \sum_{i=1}^{N} \left( \hat{x}(t_i) - x(t_i) \right)^2 \right),
\]

\[
G_{bFIT} = \frac{1}{R_n} \sum_{r=1}^{R_n} \varepsilon_r,
\]

where \( G_p \) in above equations is a number of total input values, \( R_n \) is number of total executed runs, \( \varepsilon_r \) is objective value of \( r^{th} \) experiment, \( \hat{x}(t) \) and \( x(t) \) are the standard solutions for same number run. The inputs \( t \in [0, 2] \) with the step size 0.1 are taken in this study, i.e., \( G_p = 21 \) and \( R_n = 50 \). Results of \( G_{bFIT}, G_{bMAD}, G_{bRMSE} \) and \( G_{bENSE} \) are listed in Table (6) for all problems. Furthermore, the lower values of global performance indicators for most of the cases shows the coherent accuracy and consistency of HHO-IPA.

Computational complexity analysis (CCA) is performed for the designed algorithm based on average time taken for the calculation of unknown parameters of ANNs by HHO-IPA, the average number of function evaluations and population creation. Values of CCA operators are given in Table (7) together with mean and standard deviation considered for 50 independent runs of designed technique for all case studies of both problems. It is evident that the mean values of population creation, number of function evaluations and time for problem-1 are about 1570, 74, 536 and 38s respectively. While these values for problem-2 are about 1577, 75, 259, and 39s respectively. All calculation and evaluation for this research are done on HP Laptop AMD A4 – 4300 APU with Radeon(TM) HD Graphics CPU @2.50 GHz 2.50 GHz, 8.00 GB RAM, 64 bit operating system, ×64 based Processor, in Microsoft Windows 10 Education edition running R2015a version of MATLAB.

VII. CONCLUSIONS

We conclude this research by stating the following key findings and contributions which are revealed from our experiments:

- A soft computing procedure is designed to analyse the mathematical model of Van der Pol type equations. These equations represent the heartbeat dynamics. Series solutions are constructed with the help of artificial neural networks. Unknown weights are finely tuned by a combination of a global search technique the Harris Hawks Optimizer (HHO) and a local search technique the Interior Point Algorithm (IPA) named as HHO-IPA.
- Approximate series solutions of the VdP heartbeat model are proposed and graphically plotted. Our outcome is in strong agreement with the reference solutions. We have considered two scenarios and sixteen different cases to analyse the mathematical model.
- To check the consistency and accuracy of HHO-IPA, we analysed our results by calculating values of performance indicators, like, absolute errors in solutions, mean and standard deviations in errors. Lower values of these indicators suggested that HHO-IPA can tune unknown weights consistently and accurately for all problems considered in this study.
- Values of MAD, ENSE, RMSE, GbMAD, GbENSE, and GbRMSE are calculated based on our outcome. Optimal values of these indicators dictate that we have attained better results for HBM.
- Computational complexity analysis is carried out by considering function evaluations, mean execution time, number of iterations taken to calculate optimal design weights for our problems. In all cases, negligible fluctuations in these values is observed. It indicated that our algorithm is stable and can handle difficult operations by consuming less time.
Additionally, interested readers can replace different activation functions. Using orthogonal polynomials to construct the weighted series solution is still worth to investigate.

By our results we have shown that VdP oscillatory model for hear dynamics is a well-conditioned model.

Our methodology can be implemented to solve problems in biomathematics, and physics.

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VOLUME 8, 2020
86695

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