CP–Violation through Scalar Tau Oscillation

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Abstract

We point out that oscillation between the two scalar tau (\tilde{\tau}) mass eigenstates can give rise to CP–violation if some parameters appearing in the stau or chargino/neutralino mass matrices are complex. If \tilde{\tau}^+ and \tilde{\tau}^- decay into different charginos or neutralinos, rate asymmetries as large as 20\% are possible. If both staus decay directly into \tau^+LSP, CP–violation can in principle still be observed through an energy asymmetry of the \tau decay products, but this asymmetry never exceeds the percent level. Even the rate asymmetries become small if the mass splitting between the two stau mass eigenstates is larger than 1\%. 
1) Introduction

The currently most widely studied extensions of the Standard Model (SM) involve Supersymmetry (SUSY). Superparticles can stabilize the gauge hierarchy [1], and allow for the Grand Unification of all three gauge groups of the SM [2]. Fortunately both of these statements remain true in (softly) broken SUSY models, which satisfy experimental constraints from the unsuccessful searches for superparticles, most notably at LEP and the Tevatron [3]. Unfortunately neither of these arguments tells us anything about the dynamics, or even the scale, of supersymmetry breaking as long as (most) sparticles are not (much) heavier than 1 TeV. In phenomenological analyses it is therefore preferable to take as general an approach to SUSY breaking as possible, i.e. to simply parameterize it in terms of soft breaking operators. Many of these parameters can be complex, and therefore can give rise to many new CP–violating effects.

In this note we study CP–violation in the scalar tau (\(\tilde{\tau}\)) sector at a future (linear) \(e^+e^-\) collider. There are several reasons to assume that CP–violation might be much more prominent for \(\tilde{\tau}\) than for first or second generation sleptons. In the absence of generation mixing, all nontrivial phases in the lepton–slepton–chargino–neutralino part of Lagrangian of the Minimal Supersymmetric Standard Model (MSSM) can be shifted into terms involving lepton masses and Yukawa couplings. Moreover, present experimental bounds [3] on the CP–odd electric and weak dipole moments of the \(\tau\) lepton are too weak to significantly constrain CP–violation in the \(\tilde{\tau}\) sector, in sharp contrast to the case of (s)electrons and, to a lesser extent, (s)muons. In addition, the short lifetime of the \(\tau\) lepton allows one to determine its polarization, at least in a statistical sense, by measuring the energy distribution of its visible decay products; this opens the possibility to construct new CP–odd observables. Finally, unlike \(\tilde{\tau}\)’s, because of their small Yukawa couplings first and second generation sleptons could have masses in the tens of TeV range without destabilizing the gauge hierarchy [4]; indeed, this is one possible [4] (although not very attractive [5]) solution of the “SUSY flavor problem”.

In spite of these merits, CP–violation in \(\tilde{\tau}\) pair events can only be observable if the two \(\tilde{\tau}\) mass eigenstates are closely degenerate. The reason is that the two produced \(\tilde{\tau}\) sleptons, being scalar particles, decay independently of each other. All CP–odd variables are therefore proportional to the product of a CP–odd phase and a second, CP–even phase; this is analogous to the more familiar \(K^0 - \bar{K}^0\) and \(B^0 - \bar{B}^0\) systems. In the case at hand this second phase can only come from the Breit–Wigner propagators of the \(\tilde{\tau}\) sleptons [6] or, more specifically, from the interference of two different \(\tilde{\tau}\) propagators. These interference contributions, which can be understood in terms of oscillation between the two \(\tilde{\tau}\) mass eigenstates, will become very small if the mass difference is much larger than the (average) decay width of the two \(\tilde{\tau}\) states.

In this respect the situation is somewhat similar to the case of CP–violation through slepton flavor oscillation [4] [7] (see also refs. [8] for earlier, related work). There are two differences, however. First, the two \(\tilde{\tau}\) current eigenstates do not carry different quantum numbers that are conserved in the SM, while sleptons of different generations obviously do; even detecting the presence of mixing is therefore not entirely straightforward in our case [1]. Second, the two \(\tilde{\tau}\) current eigenstates have different \(SU(2) \times U(1)_Y\) quantum numbers. According to our current understanding of SUSY breaking there is therefore no good reason to assume that the soft breaking masses for these states should be close to each other. On the other hand, nothing forbids such an “accidental” near–degeneracy, either. It might therefore be useful to

\[\text{In general there can also be other dispersive phases, e.g. due to loop corrections to the } \tilde{\tau} \text{ decay vertices. However, they are too small to be of much use.}\]
point out that such a degeneracy can lead to interesting new phenomena.

The rest of this article is organized as follows. In the next Section we discuss rate asymmetries, which can occur if the positive and negative $\tilde{\tau}$ decay into final states that are not CP-conjugates of each other. We find that under favorable circumstances these asymmetries could be detectable at future $e^+e^-$ colliders. In Sec. 3 we discuss the pion energy asymmetry that can result from $\tilde{\tau} \rightarrow \tau \rightarrow \pi$ decays if both $\tilde{\tau}$ decay into the same neutralino. We find this asymmetry to be too small to be detectable with the currently foreseen luminosity of next-generation colliders, at least within the MSSM. Finally, Sec. 4 is devoted to a summary and conclusions.

2) Rate Asymmetries

We begin our discussion with a treatment of rate asymmetries in processes of the kind

$$ e^+e^- \rightarrow \tilde{\tau}^-\tilde{\tau}^+ \rightarrow (\tilde{\chi}_a^-)(\tilde{\chi}_b^+) \ . $$

Here $a, b$ labels the combination of a $\tau$ with a neutralino $\tilde{\chi}_n^0$ ($n = 1$ to 4 in the MSSM) or a $\nu_n$ with a chargino $\tilde{\chi}_m^\pm$ ($m = 1$ or 2). It is crucial to include all combinations $\tilde{\tau}_i^-\tilde{\tau}_j^+$ in the intermediate state, where $i, j \in \{1, 2\}$ labels the $\tilde{\tau}$ mass eigenstates. They are determined by the mass matrix in $\tau_L, \tau_R$ basis:

$$ \mathcal{M}_\tilde{\tau}^2 = \begin{pmatrix} m_{1\tau}^2 & -m_\tau (A_\tau + \mu \tan\beta) \\ -m_\tau (A_\tau + \mu \tan\beta) & m_{2\tau}^2 \end{pmatrix} \ . $$

We have absorbed the $D$-term contributions to the diagonal entries, as well as the supersymmetric contributions $+m_{\tilde{\tau}}^2$, into the definition of $m_{1\tau}^2$ and $m_{2\tau}^2$. Note that we allow the soft breaking parameter $A_\tau$ as well as the supersymmetric higgsino mass parameter $\mu$ to be complex. Defining the $\tilde{\tau}$ mass eigenstates through $(\tau_1, \tau_2) = (\tilde{\tau}_L, \tilde{\tau}_R) U_\tilde{\tau}^T$, with

$$ U_\tilde{\tau} = \begin{pmatrix} \cos \phi_\tilde{\tau} & e^{i\phi_\tilde{\tau}} \\ -e^{-i\phi_\tilde{\tau}} \sin \phi_\tilde{\tau} & \cos \phi_\tilde{\tau} \end{pmatrix} \ , $$

we have

$$ \phi_\tilde{\tau} = -\arg(A_\tau + \mu \tan\beta) \ ; $$

$$ \tan \theta_\tilde{\tau} = \frac{m_{2\tau}^2 - m_{1\tau}^2}{m_\tau |A_\tau + \mu \tan\beta|} \ . $$

The eigenvalues $m_{1,2\tau}^2$ are not sensitive to the phase $\phi_\tilde{\tau}$.

Since there are two $\tilde{\tau}$ mass eigenstates, the squared matrix element, or the cross section, for process (1) contains a total of 16 terms:

$$ \sigma(e^+e^- \rightarrow [\tilde{\chi}_a^-][\tilde{\chi}_b^+]) = \frac{e^4}{(2\pi)^3 s} \cdot \frac{1}{2^{10}} \cdot \sum_{i,j,k,l} \lambda^{3/2} \left( 1, \frac{m_{1\tau}^2}{m_\tau}, \frac{m_{2\tau}^2}{m_\tau} \right) \left( P_{ij}^L, c_{ij}^L c_{kl}^L, + P_{ij}^R, c_{ij}^R c_{kl}^R \right) 
\cdot \ A_{ik} A_{jl} \lambda^{1/2} \left( 1, \frac{m_{1\tau}^2}{m_{2\tau}^2}, \frac{m_{2\tau}^2}{m_{1\tau}^2} \right) \lambda^{1/2} \left( 1, \frac{m_{1\tau}^2}{m_{2\tau}^2}, \frac{m_{2\tau}^2}{m_{1\tau}^2} \right) D_{ik}^{a} D_{jl}^{b*} \ , $$

(5)
where $e$ is the QED gauge coupling, $s$ is the squared $e^+e^-$ c.m. energy, and $\lambda(a,b,c) = (a+b-c)^2 - 4ab$. The first two factors under the sum describe $e^+e^- \rightarrow \tilde{\tau}^- \tilde{\tau}^+$ production. We have allowed for non–degenerate $\tilde{\tau}$ states by introducing the average masses $m_{\tilde{\tau}_i} = (m_{\tilde{\tau}_i} + m_{\tilde{\tau}_h})/2$; nevertheless eq. (3) will not describe the (small) interference terms very well if $m_{\tilde{\tau}_2} - m_{\tilde{\tau}_1}$ is large. The coefficients $P_{\text{eff}}^L = (1 - P_{c^-}) (1 + P_{c^+})$ and $P_{\text{eff}}^R = (1 + P_{c^-}) (1 - P_{c^+})$ describe the longitudinal polarization of the $e^\pm$ beams. The effective couplings $c_{ij}^L$, $c_{ij}^R$ are given by:

$$
 c_{ij}^L = \delta_{ij} + \frac{1 - 2 \sin^2 \theta_W}{2 \sin^2 \theta_W \cos^2 \theta_W} \cdot \frac{s}{s - M_Z^2} c_{ij}^Z, \quad (6a)
$$

$$
 c_{ij}^R = \delta_{ij} - \frac{1}{\cos^2 \theta_W} \cdot \frac{s}{s - M_Z^2} c_{ij}^Z, \quad (6b)
$$

where the $Z\tilde{\tau}_i \tilde{\tau}_j$ couplings $c_{ij}^Z$ are defined as

$$
 c_{11}^Z = \frac{1}{2} \cos^2 \theta_\tau - \sin^2 \theta_W; \quad c_{22}^Z = \frac{1}{2} \sin^2 \theta_\tau - \sin^2 \theta_W; \quad c_{12}^Z = c_{21}^Z = - \frac{1}{4} \sin (2\theta_\tau) e^{i\phi_\tau}. \quad (7)
$$

The $A_{ik}$ factors in eq. (3) describe the convolution of two Breit–Wigner propagators. We follow ref. [7] in using the narrow width approximation, but allow for different decay widths of the two $\tilde{\tau}$ mass eigenstates:

$$
 \Re \{ A_{ik} \} = \frac{1}{m_{\tilde{\tau}_ik} \Gamma_{\tilde{\tau}_ik}} \cdot \frac{1}{1 + \frac{(m_2 - m_{\tilde{\tau}_k})^2}{\Gamma_{\tilde{\tau}_k}^4}} \left( 1 + \frac{2x_{ik}}{\pi} \log \frac{m_{\tilde{\tau}_h}}{m_{\tilde{\tau}_i}} \right); \quad (8a)
$$

$$
 \Im \{ A_{ik} \} = - \frac{1}{m_{\tilde{\tau}_ik} \Gamma_{\tilde{\tau}_ik}} \cdot \frac{x_{ik}}{1 + x_{ik}^2}, \quad (8b)
$$

where we have introduced the average widths $\Gamma_{\tilde{\tau}_ik} = (\Gamma_{\tilde{\tau}_i} + \Gamma_{\tilde{\tau}_h})/2$ as well as the “$\tilde{\tau}$ oscillation parameter”

$$
 x_{ik} = \frac{m_{\tilde{\tau}_2} - m_{\tilde{\tau}_1}}{\Gamma_{\tilde{\tau}_ik}}. \quad (9)
$$

The second (logarithmic) term in eq. (8a) has been introduced to improve agreement with a numerical convolution of two propagators for $m_{\tilde{\tau}_2} \geq 1.05m_{\tilde{\tau}_1}$; however, we will see that in this region of parameter space CP–violating effects are already very small. Notice that $x_{ki} = -x_{ik}$, i.e. $A_{ki} = A_{ik}$.

The last factors in eqs. (3) describe $\tilde{\tau} \rightarrow (\tilde{\chi} f)_a$ decays. In particular, $D_{ik}^a$ is given by:

$$
 D_{ik}^a = \left( m_{\tilde{\tau}_ik}^2 - m_{\tilde{\chi}_a}^2 - m_{f_a}^2 \right) (L_{ia} L_{ka}^* + R_{ia} R_{ka}^*) - 2m_{f_a} m_{\tilde{\chi}_a} (L_{ia} R_{ka}^* + R_{ia} L_{ka}^*). \quad (10)
$$

Here, $L_{ia}$ and $R_{ia}$ are the $\tilde{\tau}_i(\tilde{\chi} f)_a$ couplings for left– and right–handed $f_a$, respectively. They can be computed easily from the $\bar{f} f \tilde{\chi}$ interactions listed in ref. [11], together with eqs. (3) and (4) describing $\tilde{\tau}$ mixing; of course, care must be taken to allow for complex chargino and neutralino mixing matrices. Note that $m_{f_a} = R_{ia} = 0$ for $\tilde{\tau}^- \rightarrow \tilde{\chi}^- \nu_\tau$ decays.

CP–odd rate asymmetries can be defined as

$$
 A(\tilde{\chi}_{a,b}) = \frac{\sigma(a^{-} b^+) - \sigma(b^{-} a^+)}{\sigma(a^{-} b^+) + \sigma(b^{-} a^+)}, \quad (11)
$$


where we have used the short-hand notation \( a^\pm = (\tilde{\chi} f)_a^\pm \). From eq.(11) one finds

\[
A(\tilde{\chi}_a \tilde{\chi}_b) \propto \sum_{i,j,k,l} A_{ik} A_{jl} \left( c_{ij} c_{kl}^{*} D_{ik}^{a} D_{jl}^{b*} - c_{ij} c_{kl}^{*} D_{ik}^{b} D_{jl}^{a*} \right)
\]

\[
= \sum_{i,j,k,l} A_{ik} A_{jl} \left( c_{ij} c_{kl}^{*} D_{ik}^{a} D_{jl}^{b*} - c_{ji} c_{lk}^{*} D_{lk}^{b} D_{ij}^{a*} \right)
\]

\[
= 2i \sum_{i,j,k,l} A_{ik} A_{jl} \Im \left( c_{ij} c_{kl}^{*} D_{ik}^{a} D_{jl}^{b*} \right)
\]

\[
= -2 \sum_{i,j,k,l} \Im \left( A_{ik} A_{jl} \right) \Im \left( c_{ij} c_{kl}^{*} D_{ik}^{a} D_{jl}^{b*} \right),
\]

where we have used the abbreviation \( c_{ij} c_{kl}^{*} = P_{L}^{\text{eff}} c_{ij} c_{kl}^{*} + P_{R}^{\text{eff}} c_{ij} c_{kl}^{*} \). In the second term of the second line of eq.(12) we have simply relabelled the indices as \( i \leftrightarrow j, \ k \leftrightarrow l \), and in the third line we have used \( c_{ji} = c_{ij}^{*} \). In the last step we have symmetrized under \( i \leftrightarrow k, \ l \leftrightarrow j \) to show that the final result is real. (Recall that \( A_{ki} = A_{ik}^{*} \)).

Since there are four neutralinos and two charginos in the MSSM, one can in principle define 15 independent rate asymmetries. We found that for \( \tilde{\tau} \) masses within reach of a next-generation \( e^{+}e^{-} \) collider operating at \( \sqrt{s} \approx 500 \text{ GeV} \) the two most readily accessible asymmetries, \( A(\tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{+}) \) and \( A(\tilde{\chi}_{1}^{0} \tilde{\chi}_{2}^{0}) \), are also the most promising ones. We see from eqs.(12) and (14) that the rate asymmetries can only be sizable if the oscillation parameter \( x_{12} \) is not much larger (nor much smaller) than 1. Since the total width of \( \tilde{\tau} \) eigenstates with mass \( \leq 200 \text{ GeV} \) does not exceed 1 GeV, it is clear that a sizable effect can only be expected if the mass splitting is quite small. A numerical scan of the parameter space reveals that the rate asymmetries become maximal for \( m_{\tilde{\tau}_{L}} = m_{\tilde{\tau}_{R}} \), \( |A_{\tau}| = |\mu \tan \beta| \), and \( \arg(A_{\tau}) \approx \arg(\mu) - 3.2 \); the last two conditions ensure that the two contributions to the off–diagonal entries of the \( \tilde{\tau} \) mass matrix cancel approximately (but not exactly). The condition for \( |A_{\tau}| \) can usually only be satisfied for \( \tan \beta \leq 5 \), at least if we insist that the scalar potential should not have a deeper lying minimum where the \( \tilde{\tau} \) fields get non-vanishing vacuum expectation values.

In Fig. 1 we show a scatter plot in the plane spanned by the two “effective” asymmetries \( \tilde{A}(\tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{+}) \) and \( \tilde{A}(\tilde{\chi}_{1}^{0} \tilde{\chi}_{2}^{0}) \), where we have fixed \( m_{\tilde{\tau}_{L}} = m_{\tilde{\tau}_{R}} = 200 \text{ GeV} \), picked random combinations of \( M_{2} \), \( \mu \) and \( \tan \beta \), and fixed the absolute value and phase of \( A_{\tau} \) as described above. We have assumed that the \( SU(2) \) and \( U(1)_{Y} \) gaugino masses unify, which allows us to set both of their phases to zero without loss of generality.\footnote{In general, \( \arg(M_{1}) - \arg(M_{2}) \), \( \arg(\mu) + \arg(M_{2}) \) and \( \arg(A_{\tau}) + \arg(M_{2}) \) are the rephasing invariant CP–violating phases relevant to our problem. In our case the phase of \( A_{\tau} \) only enters through the \( \tilde{\tau} \) mixing phase \( \phi_{\tau} \) defined in eq.(14); however, the phase of \( \mu \) also appears independently in the chargino and neutralino mass matrices.} We have checked that introducing a relative phase between these two soft breaking parameters does not change the result qualitatively. The “effective” asymmetries are defined as

\[
\tilde{A}(\tilde{\chi}_{a} \tilde{\chi}_{b}) = A(\tilde{\chi}_{a} \tilde{\chi}_{b}) \cdot \sqrt{\sigma(a^{-}b^{+}) + \sigma(a^{+}b^{-})}.
\]

They directly determine the luminosity needed to experimentally detect an asymmetry with a significance of \( N_{\sigma} \) standard deviations:

\[
\epsilon_{\mathcal{L}}(N_{\sigma}) = \left( \frac{N_{\sigma}}{A(\tilde{\chi}_{a} \tilde{\chi}_{b})} \right)^{2}.
\]
where \( \epsilon \) is the overall efficiency. Fig. 1 shows that the anticipated luminosity of 500 GeV colliders, \( \mathcal{L} \approx 20 \) to 50 fb\(^{-1}\)/yr, should be sufficient to probe at least some regions of parameter space, if the more complicated \( \bar{\tau} \) decay modes can be reconstructed with an efficiency not far below the value of \( \sim 50\% \) found in refs.\(^{10} \) for \( \tau^+ \tau^- \tilde{\chi}_0^0 \tilde{\chi}_1^- \) final states where both \( \tau \) leptons decay hadronically.

We observe some correlation between the two effective asymmetries if they have the same sign.\(^1 \) The reason is that an enhancement of the \( \tilde{\chi}_1^0 \) mode in the negatively charged (\( \bar{\tau}^- \)) channel, or a suppression of this mode in the positively charged (\( \bar{\tau}^+ \)) channel, affects both asymmetries in the same way. On the other hand, the much weaker correlation between \( \hat{A}(\tilde{\chi}_1^0 \tilde{\chi}_1^+) \) and \( \hat{A}(\tilde{\chi}_1^0 \tilde{\chi}_2^0) \) for \( A(\tilde{\chi}_1^0 \tilde{\chi}_2^0) < 0 \) can be explained by the observation that both asymmetries would vanish in the limit of vanishing \( \tau \) Yukawa coupling, or vanishing gauge couplings. The final state chargino/neutralino therefore needs significant gaugino–higgsino mixing if the asymmetries are to be sizable. Since the parameters that describe the chargino mass matrix also appear in the neutralino mass matrix, large gaugino–higgsino mixing in one sector tends to lead to large mixing in the other sector as well. Finally, we note that the scenarios with the largest effective asymmetries tend to have a fairly small phase of the \( \mu \) parameter, of order 0.1 or less; this makes it a little easier to satisfy constraints from the electric dipole moments of the neutron and electron, either by choosing large values for first generation sfermion masses, or by tuning the phases of the relevant \( A \) parameters.

In Fig. 2 we show the reduction of the asymmetry \( A(\tilde{\chi}_1^0 \tilde{\chi}_1^+) \) as we deviate from near–degeneracy of the two \( \bar{\tau} \) mass eigenstates. As discussed above, three conditions need to be satisfied for this near–degeneracy to occur, which means that one can deviate from it in three different directions: One can introduce some splitting between the diagonal entries of the mass matrix (2), or one can vary the phase or absolute value of \( A_\tau \). These three directions are explored in the solid, short dashed and long dashed curves of Fig. 2, respectively, starting from a choice of parameters leading to a large asymmetry (\( \sim 20\% \)). Along the solid line, \( |m_{\tilde{\tau}_R} - m_{\tilde{\tau}_L}| \simeq m_{\tilde{\tau}_2} - m_{\tilde{\tau}_1} \), while \( |A_\tau| \) varies by more than 100 GeV along the long dashed curve; the impression that \( A(\tilde{\chi}_1^0 \tilde{\chi}_1^+) \) depends more sensitively on \( |A_\tau| \) than on \( m_{\tilde{\tau}_L} - m_{\tilde{\tau}_R} \) is therefore somewhat misleading. Finally, the phase of \( A_\tau \) varies by about \( \pi/10 \) along the short dashed curve. Fig. 2 clearly illustrates that rate asymmetries will be too small to be detectable at a next–generation \( e^+e^- \) collider if the two eigenvalues of the \( \bar{\tau} \) mass matrix (2) differ by more than 1%.

### 3) Energy Asymmetries

If \( \bar{\tau}^+ \) and \( \bar{\tau}^- \) decay into charge–conjugate final states no rate asymmetry can be measured. This will be true in particular if \( \tau \tilde{\chi}_1^0 \) is the only accessible \( \bar{\tau} \) decay mode. In this case one can still construct a CP–odd observable involving the spins of the two \( \tau \) leptons. While these spins are not directly measurable, they affect the energy distributions of the visible \( \tau \) decay products. Here we study this effect using the simplest \( \tau \) decay mode, \( \tau^+ \rightarrow \pi^+ \nu_\tau \), which also has the highest sensitivity to the \( \tau \) polarization.

Since the CP–odd observable we want to study here depends on kinematical quantities, we need an expression for the differential cross section, rather than the total one; this also allows us to implement acceptance cuts that are needed to isolate \( \bar{\tau} \) pair events from SM

\(^1\)Since all asymmetries change sign when the sign of all CP–odd phases in the Lagrangian is flipped, we only show results for \( \hat{A}(\tilde{\chi}_1^0 \tilde{\chi}_1^+) > 0 \).
to the couplings $\tau$ least one factor of the $\tau$ handed $\tilde{\tau}$ sufficient to treat

$$\begin{align*}
\text{Here, } \Theta_\tau \text{ is the angle of } \tau^- \text{ with respect to the } e^- \text{ beam direction in the lab frame, while} \\
\text{the starred variables } \Omega_{\pi\pm} \text{ and } E_{\pi\pm} \text{ refer to the rest frames of } \tau^{\pm}. \text{ Note that we have already} \\
\text{integrated analytically over the phase space of the invisible decay products (neutrinos and} \\
\text{neutralinos). This fixes the integration limits for } E_{\pi\pm} \text{ as} \\
\left. E_{\pi\pm} \right|_{\text{min,max}} = \frac{m_{\pi\pm}}{4} \left[ \left( 1 + \frac{m_\tau^2 - m_{\pi\pm}^2}{m_{\pi\pm}^2} \right) \left( 1 + \frac{m_{\pi\pm}^2}{m_\tau^2} \right) \pm \left( 1 - \frac{m_\tau^2}{m_{\pi\pm}^2} \right) \lambda^{3/2} \left( 1, \frac{m_\tau^2}{m_{\pi\pm}^2}, \frac{m_{\pi\pm}^2}{m_\tau^2} \right) \right], (16)
\end{align*}$$

with $m_{\pi^-} = m_{\pi_{ik}}$ and $m_{\pi^+} = m_{\pi_{jl}}$ in eq. (15).

The structure of eq. (15) is quite similar to that of eq. (5). However, the functions $D_{ik}^\pi$ now describe the entire $\tau^- \rightarrow \tau^- \tilde{\chi}_1^0 \rightarrow \pi^- \nu_\tau \tilde{\chi}_1^0$ decay chain, including all spin effects:

$$
D_{ik}^\pi = \frac{2m_\tau^2}{m_\tau^2 - m_{\pi}^2} \left[ \left( m_{\pi_{ik}}^2 - m_{\chi}^2 - m_{\tau}^2 \right) R_{i1} R_{k1}^* - m_{\tau} m_{\chi} \left( L_{i1} R_{k1}^* + R_{i1} L_{k1}^* \right) \right] + \frac{m_\tau^2}{m_\tau^2 - m_{\pi}^2} \left( R_{i1} R_{k1}^* - L_{i1} L_{k1}^* \right) \left( 2m_{\pi_{ik}}^2 E_{\pi}^+ - m_{\pi_{ik}}^2 + m_{\chi}^2 - m_{\tau}^2 \right), (17)
$$

where $L_{i1}, R_{i1}$ are the $\chi_1^0 \tau \tau$ couplings that already appeared in eq. (10). Note that it is not sufficient to treat $\tau^- \rightarrow \tau^- \tilde{\chi}_1^0$ decays as incoherent sum of $\tau^- \rightarrow \tau_L$ and $\tau^- \rightarrow \tau_R$ decays. The interference between left– and right–handed $\tau$ leptons gives a contribution of order $m_\tau$, shown in the second term of eq. (17); because this term involves the product of left– and right–handed $\tau$ couplings it allows to produce a CP–odd phase purely from gauge contributions to the couplings $L$ and $R$. In contrast, the other terms produce a CP–odd phase only if at least one factor of the $\tau$ Yukawa coupling appears in the relevant combination of $L$ and $R$ couplings. Of course, the expectation value of any CP–odd observable is again proportional to the imaginary part of $A_{12}$.

Unfortunately we found that the asymmetry in the $\pi^+$ and $\pi^-$ energy distributions is always quite small. The most easily measured quantity is the total energy asymmetry,

$$
\tilde{A}_{E_\pi} = \frac{\langle E_{\pi^+} \rangle - \langle E_{\pi^-} \rangle}{\langle E_{\pi^+} \rangle + \langle E_{\pi^-} \rangle}, (18)
$$

where the pion energies are now taken in the laboratory frame. In our scans of parameter space we did not find a scenario where this asymmetry exceeds 1%. The “differential asymmetry”

$$
A_{E_\pi}(E) = \frac{d\sigma}{dE_{\pi^+}}(E) - \frac{d\sigma}{dE_{\pi^-}}(E) \frac{d\sigma}{dE_{\pi^+}}(E) + \frac{d\sigma}{dE_{\pi^-}}(E) (19)
$$

$\tilde{A}_{E_\pi}$ is linear in the pion energies $E_{\pi\pm}$. It therefore involves the product of the $E_{\pi\pm}$–independent terms from one $D$–function with the term linear in $E_{\pi\pm}$ from the other $D$–function.
can reach the 10% level in some bins, but such a large asymmetry always coincides with a very small differential cross section in the same bin. This is illustrated in Fig. 3, which shows the differential asymmetry (solid) and the differential cross section (dashed, referring to the scale at the right) for one of the most optimistic scenarios we found. In particular, we have assumed a 100% right–handed $e^−$ beam here; for the given choice of parameters this increases the total cross section by about 30%, and increases the asymmetry by more than a factor of 2. We have applied the $τ$ pair acceptance cuts listed in the second ref. This reduces the total accepted cross section by about a factor of 2, and especially depletes the region of small $E_π$, but has no effect on the asymmetry. Note that the differential asymmetry changes sign near the value of $E_π$ where the differential cross section is maximal. This should allow one to construct an optimized CP–odd variable, which shows better sensitivity to CP–odd phases than the total energy asymmetry does, which amounts to only 0.77% in the example shown. Nevertheless the significance of the observed effect cannot exceed that obtained from simply adding the effective asymmetry observed in each bin in quadrature. This gives a combined effective asymmetry $\hat{A}_{E_\pi}$ in analogy to the effective rate asymmetries introduced in the previous section:

$$\hat{A}_{E_\pi}^2 = \int dE A_{E_\pi}^2(E) \left[ \frac{d\sigma}{dE_\pi^+} + \frac{d\sigma}{dE_\pi^-} \right].$$

(20)

Even for the optimistic case depicted in Fig. 3 $\hat{A}_{E_\pi}$ only amounts to 0.017 fb$^{1/2}$. This compares favorably to the product of the overall energy asymmetry and the square root of the total cross section, which only gives 0.0032 fb$^{1/2}$. However, even if we include events where only one of the two $τ$ leptons decays into the single pion mode, which increases the total available cross section by a factor of $1/B(τ^- → π^-ντ) ≃ 9$ if the efficiencies for the other decay modes are similar to that for the $π^+π^−$ mode, we would still need to accumulate about 400 fb$^{-1}$ of data to see an energy asymmetry at the level of one standard deviation. In contrast, the same choice of parameters yields an effective rate asymmetry $\hat{A}(\tilde{χ}_0^0\tilde{χ}_1^+) = 0.32$ fb$^{1/2}$, which would start to become visible after 20 fb$^{-1}$ have been collected (assuming an efficiency $ε ≃ 50%$).

4) Summary and Conclusions

In this note we have studied CP–violating phenomena that could arise from scalar $τ$ oscillations at future $e^+e^−$ colliders. In Sec. 2 we found that rate asymmetries, which can occur if $τ$ has several different decay modes, can be sizable. However, several conditions have to be satisfied in order to get asymmetries that might be detectable at a next–generation collider. The most critical requirement is that the mass difference between the two $τ$ eigenstates should not exceed 1%. In addition, one needs substantial higgsino/gaugino mixing in the chargino/neutralino sector of the theory and, of course, some significant CP–odd phases.

Note that the first of these conditions is almost impossible to fulfill for scalar $b$ or $t$ quarks, since here the off–diagonal entries in the relevant mass matrices are (much) larger. One might hope that in case of $b$ squarks at least their larger decay width could compensate for a larger mass splitting, if $\tilde{b} → \tilde{g} + b$ decays are allowed which proceed through strong interactions. However, in order to construct a rate asymmetry, at least one of the $\tilde{b}$ squarks must undergo a

*In order to reliably predict small asymmetries without generating hundreds of millions of events we have symmetrized our Monte Carlo phase space integration against exchange of $π^+$ and $π^−$ momenta, including an exchange $Θ_{τ} → −Θ_{τ}$. This ensures that the numerically computed asymmetry vanishes in the absence of CP–violation.
weak decay, with correspondingly reduced branching ratio. Moreover, determining the charges of the final state particles, which is crucial for constructing any CP-odd observable, is not easy for (s)quarks. We therefore do not expect third generation squark oscillations to be detectable at $e^+e^-$ colliders.

In Sec. 3 we studied asymmetries in the pion energy distribution in events where both $\tilde{\tau}$ decay as $\tilde{\tau} \rightarrow \tau \tilde{\chi}_1^0 \rightarrow \pi \nu_\tau \tilde{\chi}_1^0$. Unfortunately we found this asymmetry to be quite small even in the most optimistic case. The reason is that it vanishes in the limit where the tau mass and Yukawa coupling are set to zero. This is also true for the rate asymmetries discussed above; however, in that case substantial cancellations can and frequently do occur between different contributions to the relevant $\tilde{\tau}^{-}$ chargino/neutralino – $\nu_\tau/\tau$ couplings, which enhances the relative importance of the $\tau$ Yukawa coupling, as illustrated in Fig. 1. No such enhancement occurs in the pion energy asymmetry. The differential energy asymmetry has to change sign for some value of the pion energy, since the total decay width of the $\tau$ lepton is independent of its polarization. The structure of the decay amplitude implies that this change of sign occurs near the value of pion energy where the cross section is maximal; as a result, most events contribute only little to the total asymmetry. At least for the model with minimal particle content (the MSSM with general soft breaking terms) studied here, the energy asymmetry therefore remains too small to be detectable at next-generation $e^+e^-$ colliders. This remains true even if we allow for polarized beams, which can increase this asymmetry by more than a factor of two. On the other hand, at least in some regions of parameter space rate asymmetries should be detectable, and might provide us with the first evidence for CP-violation in the leptonic sector.

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Figure 1: Scatter plot of the effective rate asymmetries defined in eq. (13). We have fixed $m_{\tilde{\tau}_L} = m_{\tilde{\tau}_R} = 200$ GeV, $|A_\tau| = |\mu \tan \beta|$, and $\theta_A = \theta_\mu - 3.2$. The phases of the gaugino masses have been set to zero. The remaining parameters have been varied randomly, in the ranges $100 \text{ GeV} \leq |M_2|, |\mu| \leq 500$ GeV, $1 \leq \tan \beta \leq 5$ and $-\pi \leq \theta_\mu \leq \pi$, where $\theta_A$ and $\theta_\mu$ are the phases of $A_\tau$ and $\mu$, respectively. These results are for an $e^+e^-$ collider operating at $\sqrt{s} = 500$ GeV with unpolarized beams.
Figure 2: Dependence of the rate asymmetry \( A(\tilde{\chi}_1^0 \tilde{\chi}_1^+ ) \) defined in eq.(12) on the \( \tilde{\tau}_2 - \tilde{\tau}_1 \) mass difference. The point common to all three curves is defined by the parameters \( \sqrt{s} = 500 \) GeV, \( m_{\tilde{\tau}_L} = m_{\tilde{\tau}_R} = 200 \) GeV, \( |A_\tau| = 442.6 \) GeV, \( |M_2| = 240.2 \) GeV, \( |\mu| = 154.4 \) GeV, \( \tan\beta = 2.89 \), and \( \theta_\mu = \theta_A + 3.2 = 0.19 \). Gaugino masses are assumed to be real and to satisfy a unification condition. The solid, short dashed and long dashed curves have been obtained by varying \( m_{\tilde{\tau}_R} \), \( \theta_A \) and \( |A_\tau| \), respectively, as described in the text.
Figure 3: The differential pion energy asymmetry defined in eq. (19) (solid), and the differential cross section after acceptance cuts [9] (dashed); the latter refers to the scale at the right. Note that both $\tau$ leptons are assumed to decay into the single pion mode, which gives a combined branching ratio of only 1.2%; however, events where only one of the $\tau$ leptons decays into this mode can also be used for this measurement. The values of the relevant parameters are: $|A_{\tau}| = 454.6$ GeV, $|M_2| = 270.1$ GeV, $|\mu| = 152.3$ GeV, $\tan \beta = 2.98$, $\theta_A = -0.0358$, and $\theta_\mu = 3.1642$; the other parameters are as in Fig. 2, except that we have used an $e^-$ beam with 100% right–handed polarization.