Complete twist-decomposition for non-local QCD vector operators in $x$-space

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A general procedure is introduced allowing for the complete, infinite twist decomposition of non-local vector operators in QCD off the light-cone. The method is applied to the operators $\bar{\psi}(x)\gamma_\mu\psi(-x)$ and $\bar{\psi}(x)\sigma_{\mu\nu}x^\nu\psi(-x)$ as well as to their matrix elements thereby determining all power (resp. target mass) corrections being relevant for the related distribution amplitudes.

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I. INTRODUCTION

The dependence of the non-perturbative distribution amplitudes on the momentum transfer $Q^2$ is very important in phenomenological considerations in QCD. In the past it became clear that the non-local light-cone expansion together with the renormalization group equation is a suitable tool to determine this dependence. Experimental applications are, for example, the parton distributions in deep inelastic scattering (DIS), the non-forward double distribution amplitudes in deeply virtual Compton scattering (DVCS) and the (vector) meson wave functions. Furthermore, it became obvious that all the phenomenological quantities appearing in the above experimental settings are related to the Compton amplitude for non-forward scattering of a virtual photon off a hadron given by

$$T_{\mu\nu}(P, Q; S_i) = i \int d^4x \ e^{i Q x} \hat{S} \left[ J_{\mu}(x/2) J_{\nu}(x/2) \right] S \left| P_1, S_1 \right> \left< P_2, S_2 \right|, \quad (1)$$

where $P_1(P_2)$ and $S_1(S_2)$ are the momenta and spins of the incoming (outgoing) hadrons, $Q^2 = -q^2$, $q = q_2 - q_1 = P_1 - P_2$ denotes the momentum transfer and $S$ is the (renormalized) $S$–matrix.

To find an adequate representation of the Compton amplitude in terms of non-local operators on the light-cone one applies the non-local operator product expansion \cite{1,2,3} to the renormalized time-ordered product occurring in the matrix element (1). This leads immediately to compact expressions for the coefficient functions and the corresponding bilocal light-cone operators. If the quark propagator near the light-cone is approximated by its most singular parts one finds the well-known expression \cite{23}

$$T \left[ J_{\mu}(x) \right] J_{\nu}(x) \approx \left[ \frac{1}{2\pi^2 (x^2 - i\epsilon)^2} + \frac{m^2}{8\pi^2 (x^2 - i\epsilon)} \right] \left( g_{\mu\nu} O_{(5)}(x, x) - 2 x_{(\mu} O_{\nu)}(x, x) \right)$$

$$- \frac{i m}{4\pi^2 (x^2 - i\epsilon)} \left( g_{\mu\nu} N(x, x) + M_{\mu\nu}(x, x) \right). \quad (2)$$

Thereby, the ‘centered’ non-local chiral-even (axial) vector operators $O_{(5)}^{(\mu)}(x, x)$ and the corresponding scalar operators $O^{(5)}(x, x) := x^\mu O_{(5)}^{(\mu)}(x, x)$ are given by the following (anti)symmetrized operators (here, and in

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the following, corresponding phase factors are suppressed)

\[
O_\mu (\kappa x, -\kappa x) = \bar{\psi} (\kappa x) \gamma_\mu \psi (-\kappa x) - \bar{\psi} (-\kappa x) \gamma_\mu \psi (\kappa x),
\]

(3)

\[
O_5^\mu (\kappa x, -\kappa x) = \bar{\psi} (\kappa x) \gamma^5 \gamma_\mu \psi (-\kappa x) + \bar{\psi} (-\kappa x) \gamma^5 \gamma_\mu \psi (\kappa x),
\]

(4)

\[
O (\kappa x, -\kappa x) = \bar{\psi} (\kappa x) \gamma^\nu \not{\psi} (-\kappa x) - \bar{\psi} (-\kappa x) \gamma^\nu \not{\psi} (\kappa x),
\]

(5)

\[
O_5 (\kappa x, -\kappa x) = \bar{\psi} (\kappa x) \gamma^5 \gamma^\nu \not{\psi} (-\kappa x) + \bar{\psi} (-\kappa x) \gamma^5 \gamma^\nu \not{\psi} (\kappa x),
\]

(6)

whereas the ‘centered’ chiral-odd skew tensor operator \(M^\mu_ \nu| (\kappa x, -\kappa x)\) and the scalar operator \(N (\kappa x, -\kappa x)\) are given by the following (anti)symmetrized operators

\[
M^\mu_ \nu| (\kappa x, -\kappa x) = \bar{\psi} (\kappa x) \sigma_\mu_ \nu \psi (-\kappa x) - \bar{\psi} (-\kappa x) \sigma_\mu_ \nu \psi (\kappa x),
\]

(7)

\[
N (\kappa x, -\kappa x) = \bar{\psi} (\kappa x) \psi (-\kappa x) + \bar{\psi} (-\kappa x) \psi (\kappa x).
\]

(8)

For scalar operators, like \(O_5^\mu (\kappa x, -\kappa x)\) and \(N (\kappa x, -\kappa x)\), as well as operators which can be derived from them, e.g. by applying derivatives w.r.t. \(x\), the complete twist decomposition has been studied previously (see Ref. [4]).

Now, we extend these considerations to vector operators, like \(O_\mu^5 (\kappa x, -\kappa x)\) and \(M^\mu (\kappa x, -\kappa x)\), the latter being obtained from the skew tensor operator \(M^\mu_ \nu| (\kappa x, -\kappa x)\) by ‘external’ contraction with \(x^\nu\),

\[
M^\mu (\kappa x, -\kappa x) := x^\nu \cdot M^\mu_ \nu| (\kappa x, -\kappa x).
\]

(9)

Obviously, the operators \(O_\mu^5 (\kappa x, -\kappa x)\) contribute to the leading light-cone singularity being independent of the quark-mass \(m\) and also to the lower light-cone singularities being proportional to \(m^2\). The operator \(M^\mu_ \nu| (\kappa x, -\kappa x)\) only contributes to quark-mass terms and is therefore not present if only leading contributions are considered. In addition, its complete twist decomposition is much more involved and will be studied separately.

All the non-perturbative distribution amplitudes which appear in the above mentioned physical processes are Fourier transforms of matrix elements whose unique input are the light-cone operators (3) – (9). Accordingly, all interesting evolution equations of the distribution amplitudes result from the renormalization group equation of these operators. This is the reason why we consider the operators themselves and not their phenomenologically relevant matrix elements.

With growing accuracy of experimental data for the various parton distribution amplitudes more and more terms of the expansion of the Compton amplitude (11) w.r.t. the variable \(M^2/Q^2\) will become accessible, where \(M\) is the nucleon (or, eventually, the meson) mass. These terms are of quite different origin. The first source are radiative corrections and a second source are target mass corrections resulting from higher twist contributions.

According to Gross and Treiman (2) the various contributions of definite (geometric) twist \(\tau\) (= canonical dimension \(d\) minus spin \(j\)) are obtained by a decomposition of the local components \(O_{\alpha_1, \ldots, \alpha_n}\) of the bilocal operators \(O_\tau (\kappa x, -\kappa x)\) into irreducible tensor representations of the Lorentz group. As is well-known, these representations are given by traceless tensors which, in addition, are characterized by their symmetry type under index permutations. In the case of the Lorentz group \(SO(3,1)\) there exist four generic symmetry types which we call I, II, III and IV, and which are characterized by corresponding (normalized) Young operators \(Y^\mu_{\{m\}}, i = 1, \ldots, 4\), with patterns \(\{m\}\) of \(m\) boxes, \(|m| = (m), (m - 1, 1), (m - 2, 1, 1)\) and \((m - 2, 2)\), respectively (for a more detailed group theoretical characterization see, e.g., Refs. [3, 7]). In case of scalar operators only the first pattern corresponding to totally symmetric tensors is relevant because \(\Gamma = 1\) and all tensor indices are to be contracted by the symmetric tensor \(x^\alpha x^\beta x^\gamma x^\delta\). In case of vector operators only the first two patterns are relevant with the ‘free’ index \(\Gamma = \mu\) being symmetrized (pattern of type I) or antisymmetrized (pattern of type II) with the remaining indices \(\alpha_i, i = 1, \ldots, m - 1\). In the case of antisymmetric and symmetric tensor operators of rank 2 also patterns III and IV, respectively, have to be taken into account.

Using this purely group theoretical procedure the (finite) twist decomposition of non-local light-cone operators in configuration space has been performed in Refs. (13, 14). However, if one wants to calculate target mass corrections stemming from higher twist operators one is forced to consider the (infinite) twist decomposition off the light-cone. This infinite decomposition includes all trace terms which, after Fourier transformation to momentum space, lead to contributions suppressed by powers of \(M^2/Q^2\).

The target mass corrections resulting from leading and, eventually, next-to-leading twist contributions have first been discussed by Nachtmann (8) in unpolarized deep inelastic scattering. Later on, this method has been applied to polarized deep inelastic scattering in Refs. (10, 11, 12, 13). Another method for the determination of target mass effects was first given by Georgi and Politzer (14) for unpolarized deep inelastic scattering and then extended to polarized scattering and to general electro-weak couplings by Refs. (13, 16).

However, all the above mentioned articles treated only the leading twist contributions since, up to now, a complete off-cone decomposition for non-local operators carrying free tensor indices in \(x\)-space has been possible only when
their \( n \)th moments are totally symmetric allowing for the application of the group theoretical results of Bargmann and Todorov \(^{17}\). Especially, this holds for all scalar operators, see Ref. \(^{4}\) for the infinite twist decomposition of such objects. But once free indices are involved the non-trivial Young patterns occur. Here, we solve that problem when one free tensor index appears.

The paper is organized as follows. Section 2 is devoted to the deduction of the complete off-cone twist decomposition of generic vector operators thereby determining all power corrections in \( x \)-space contained in these operators. In section 3 we apply these results to the operators \( O_{\mu}(\kappa x, -\kappa x) \) and \( M_{\mu \nu}(\kappa x, -\kappa x) \). In section 4 the forward and non-forward matrix elements of these operators are taken thereby representing our results in a manner which can be used for further phenomenological considerations.

II. COMPLETE TWIST-DECOMPOSITION FOR GENERIC VECTOR OPERATORS IN \( x \)-SPACE

In this Chapter we solve the problem of the twist decomposition for a generic vector operator. Thereby, we make use of the polynomial technique which already has been applied in order to complete the twist-decomposition on the light-cone \(^{3, 7}\). By this method the totally symmetric part of any (local) operator is (sometimes only partly) truncated by the coordinates \( x^{\alpha_i} \) (see, Eqs. \(^{14}\) and \(^{15}\) below). In the following, we first list the necessary ingredients, then we briefly illustrate the method in case of scalar operators, and then we generalize to generic vector operators.

To begin with let us first perform a formal Fourier transformation of the non-local operators \( O_{\mu}(\kappa x, -\kappa x) \) and \( M_{\mu \nu}(\kappa x, -\kappa x) \) followed by an expansion into local operators (here, and in the following, without changing the notation we suppress w.r.t. \( \kappa \) being part of the definition of the operators \(^{10}\) and \(^{11}\)).

\[
O_{\mu}(\kappa x, -\kappa x) = \int \text{d}^4 u \; O_{\mu}(u) \; e^{i \kappa (ux)} = \sum_{n=0}^{\infty} \frac{(i \kappa)^n}{n!} \; O_{\mu|n}(x) , 
\]

\[
M_{\mu \nu}(\kappa x, -\kappa x) = \int \text{d}^4 u \; M_{\mu \nu}(u) \; e^{i \kappa (ux)} = \sum_{n=0}^{\infty} \frac{(i \kappa)^n}{n!} \; M_{\mu \nu|n}(x) . 
\]

(For notational simplicity we understand the factor \( 1/(2\pi)^4 \) to be included into the measure of the Fourier transformation.) Obviously, the operators \(^{10}\) and \(^{11}\) are homogeneous in \( \kappa, u \) and \( x \) whereas those which are obtained after ‘external’ multiplication with \( x^\mu \) are not. Therefore, the explicit expressions for the moments of the vector operators \(^{3}\) and \(^{10}\) read

\[
O_{\mu|n}(x) = \int \text{d}^4 u \; O_{\mu}(u) \; (ux)^n = \int \text{d}^4 u \; (\bar{\psi} \gamma_\mu \psi)(u) \; (ux)^n ,
\]

\[
M_{\mu|n+1}(x) = \int \text{d}^4 u \; M_{\mu \nu}(u) \; (ux)^n = \frac{1}{n+1} \int \text{d}^4 u \; (\bar{\psi} \sigma_{\mu \nu} \psi)(u) \; \partial_\nu (ux)^{n+1} .
\]

If we perform a local expansion of the operators without a preceding Fourier transformation, the moments \( O_{\mu|n}(x) \) and \( M_{\mu|n+1}(x) \) receive the following (equivalent) representation

\[
O_{\mu|n}(x) = (-i)^n \; x^{\alpha_1} \ldots x^{\alpha_n} \; O_{\mu \alpha_1 \ldots \alpha_n} ,
\]

\[
M_{\mu|n+1}(x) = (-i)^n \; x^\nu x^{\alpha_1} \ldots x^{\alpha_n} \; M_{\mu \nu \alpha_1 \ldots \alpha_n} ,
\]

with

\[
O_{\mu \alpha_1 \ldots \alpha_n} = (\bar{\psi}(0) \gamma_\mu \overset{\leftrightarrow}{\partial}_{\alpha_1} \ldots \overset{\leftrightarrow}{\partial}_{\alpha_n} \psi(0) ,
\]

\[
M_{\mu \nu \alpha_1 \ldots \alpha_n} = (\bar{\psi}(0) \sigma_{\mu \nu} \overset{\leftrightarrow}{\partial}_{\alpha_1} \ldots \overset{\leftrightarrow}{\partial}_{\alpha_n} \psi(0) .
\]

\( \overset{\leftrightarrow}{\partial}_{\alpha} \) is the covariant derivative,

\[
\overset{\leftrightarrow}{\partial}_{\alpha} = \partial_{\alpha} - \partial_{\alpha} + 2i \; g \; A_{\alpha}(x) ,
\]

with \( g \) being the strong coupling constant and \( A_{\alpha}(x) \) the gluon field in the fundamental representation.

Let us begin with the construction of the infinite twist decomposition for generic scalar operators \( O(\kappa x, -\kappa x) \) with local components \( O_n(x) = (-i)^n \; x^{\alpha_1} \ldots x^{\alpha_n} \; O_{\alpha_1 \ldots \alpha_n} \). Using the group theoretic results of Bargmann and Todorov.
twist and, therefore, no further symmetrization is necessary. This key observation can be generalized also to operators with vector operators or operators being antisymmetric in \( \mu \) from the trace terms, \(|\alpha|^2 \rho \) of all possible traces of the operators the trace terms from the traceless operators we have to specify which kind of trace terms could appear. Obviously, generically will be denoted by \( O \) operator (20).

\[
\sum_{j=0}^{[\frac{n}{2}]} c(j,n) (x^2)^j H_{n-2j} (x^2, \square) \square^j O_n (x),
\]

with the projection operator \( H_n (x^2, \square) \) onto traceless homogeneous polynomials of degree \( n \),

\[
H_n (x^2, \square) = \sum_{k=0}^{[\frac{n}{2}]} d(k,n) (x^2)^k \square^k \equiv \sum_{k=0}^{[\frac{n}{2}]} (-1)^k (n-k)! \frac{1}{4^k k! n!} (x^2)^k \square^k;
\]

thereby, the corresponding coefficients are given by

\[
c(j,n) = \frac{(n+1-2j)!}{4^j j! (n+1-j)!}, \quad (21)
\]

\[
d(k,n) = \frac{(-1)^k (n-k)!}{4^k k! n!}. \quad (22)
\]

The (dimensionless) projector \( H_n (x^2, \square) \) is obtained as the unique solution of the requirement of tracelessness of a (scalar) homogeneous polynomial of degree \( n \) according to

\[
\square H_n (x^2, \square) O_n (x) = 0. \quad (23)
\]

Formula (19) provides us with a separation of the traceless operators of definite twist, \( H_{n-2j} (x^2, \square) \square^j O_n (x) \), from the trace terms, \( (x^2)^j \), which are obtained by contracting the indices of the various products of the metrics by \( x^\alpha \)'s. Since scalar operators always obey symmetry type I, all traceless scalar operators are automatically of definite twist and, therefore, no further symmetrization is necessary. This key observation can be generalized also to operators with free tensor indices (which however require subsequent application of Young symmetrizers, see below).

With the notations (20) – (22), after exchanging the summations and changing summation indices from \((j,k)\) to \((j,r = k + j)\), the result (19) can be rewritten in the form

\[
O_n (x) = \sum_{r=0}^{[\frac{n}{2}]} (x^2)^r \square^r \sum_{j=0}^{r} c(j,n) \ d(r-j,n-2j) \ O_n (x), \quad (24)
\]

Since the undecomposed operator \( O_n (x) \) occurs on both sides of the equation the differential operator on the right hand side must be equal to the identity \( I \). This is only possible if the coefficients \( c \) and \( d \) fulfill the equations

\[
\sum_{j=0}^{r} c(j,n) \ d(r-j,n-2j) = \delta_{0r}, \quad \text{for} \quad r = 0, \ldots, \left[ \frac{n}{2} \right], \quad (25)
\]

which can be checked by explicit calculation. In this sense, \( c \) and \( d \) are inverse to each other. However, if \( c \) is regarded as an unknown coefficient, equation (25) can be used to determine \( c \) since \( d \) is already defined by the projection operator (20).

Now, we generalize these observations to the case of the vector operators under consideration, \( O_\mu \) and \( M_\mu \), which generically will be denoted by \( O_\mu \). First, in order to get an ansatz which provides us with an analogous separation of the trace terms from the traceless operators we have to specify which kind of trace terms could appear. Obviously, all possible traces of the operators \( O_{\mu_1 \cdots \mu_n} \) and \( M_{\mu \nu} \alpha_1 \cdots \alpha_n \) are multiplied by products of metrics \( g_{\beta_1 \beta_2} \), with \( \{\beta_i\} = \{\mu, \nu, \alpha_1, \ldots, \alpha_n\} \). After contraction with the symmetric tensor \( x^{\alpha_1} \cdots x^{\alpha_n} \) these metrics obtain the form of products of \( x^\alpha \), \( x_\mu \), \( x_\nu \) and \( g_{\mu \nu} \) depending on the number of \( x^\alpha \)'s being contracted with the different \( g_{\beta_1 \beta_2} \). Since we are dealing with vector operators or operators being antisymmetric in \( \mu \) and \( \nu \) only two types of trace terms appear, namely, \( x^2 \)-traces and \( x_\alpha \)-traces.

Traces of type \( x^2 \) must be multiplied by a traceless vector operator (carrying the free index \( \alpha \)) and traces of type \( x_\alpha \) must be multiplied by a traceless scalar operator. Therefore, a preliminary ansatz reads

\[
O_{\alpha | n} (x) = \sum_{j=0}^{[\frac{n+1}{2}]} (x^2)^j H_{\alpha | n-2j}^\rho (x, \partial) + x_\alpha (x^2)^{j-1} H_{n+1-2j}^\rho (x, \partial) O_{\rho | n} (x), \quad (26)
\]
where $\mathbf{H}^\rho_{\alpha|n-2j}(x, \partial)$ is a projection operator on traceless vector operators of order $n-2j$ and $\mathbf{H}^\rho_{\alpha|n+1-2j}(x, \partial)$ is a projection operator on scalar traceless operators of order $n+1-2j$. The most general ansatz for these projectors is

$$
\mathbf{H}^\rho_{\alpha|n-2j}(x, \partial) = c_1(j, n) \cdot H^\rho_{\alpha|n-2j}(x, \partial) \square^j + c_2(j, n) \cdot \partial_\alpha H_{n+1-2j}(x^2, \square) \square^{j-1} \partial^\rho + c_3(j, n) \cdot \partial_\alpha H_{n+1-2j}(x^2, \square) \square^{j} x^\rho,
$$

$$
\mathbf{H}^\rho_{\alpha|n+1-2j}(x, \partial) = c_4(j, n) \cdot H_{n+1-2j}(x^2, \square) \square^{j-1} \partial^\rho + c_5(j, n) \cdot H_{n+1-2j}(x^2, \square) \square^{j} x^\rho,
$$

where $\mathbf{H}^\rho_{\alpha|n}(x, \partial)$ is the (primary) projector onto traceless homogeneous polynomials of degree $n$ bearing a free Lorentz index and obeying the defining equations

$$
\square \mathbf{H}^\rho_{\alpha|n} \mathcal{O}_\rho (n) (x) = 0, \quad \partial^\alpha \mathbf{H}^\rho_{\alpha|n} \mathcal{O}_\rho (n) (x) = 0,
$$

which has been determined already in Refs. [6, 7].

$$
H^\rho_{\alpha|n} (x, \partial) = \left[ \delta^\rho_\alpha - \frac{1}{(n+1)^2} \left( n x_\alpha - \frac{1}{2} x^2 \partial_\alpha \right) \partial^\rho \right] H_n(x^2, \square) = \delta^\rho_\alpha H_n(x^2, \square) - \frac{1}{n(n+1)^2} x_\alpha H_{n-1}(x^2, \square) \mathbf{d}^\rho.
$$

Here, the set of traceless vector operators is enlarged by differentiated scalar traceless operators, which is allowed since a differentiation does not destroy the condition of tracelessness. Namely, by construction, $\mathbf{H}^\rho_{\alpha|n-2j} \mathcal{O}_\rho (n) (x)$ and $\mathbf{H}^\rho_{\alpha|n+1-2j} \mathcal{O}_\rho (n) (x)$ fulfill the equations (in the following, the arguments of the projectors $\mathbf{H}$ and $H$ will be omitted)

$$
\square \mathbf{H}^\rho_{\alpha|n-2j} \mathcal{O}_\rho (n) (x) = 0, \quad \partial^\alpha \mathbf{H}^\rho_{\alpha|n-2j} \mathcal{O}_\rho (n) (x) = 0, \quad \square \mathbf{H}^\rho_{\alpha|n+1-2j} \mathcal{O}_\rho (n) (x) = 0.
$$

Above, for later convenience we introduced the operators

$$
x_\alpha = x_\alpha (x \partial + 1) - \frac{1}{2} x^2 \partial_\alpha,
$$

$$
\mathbf{d}_\alpha = (x \partial + 1) \partial_\alpha - \frac{1}{2} x_\alpha \square,
$$

with the properties

$$
x_\alpha H_n(x^2, \square) = H_{n+1}(x^2, \square) x_\alpha (x \partial + 1),
$$

$$
(x \partial + 1) \partial_\alpha H_n(x^2, \square) = H_{n-1}(x^2, \square) \mathbf{d}_\alpha.
$$

Using these relations it is easily seen that also the following equation holds,

$$
x^\beta H^\rho_{\beta|n} = H_{n+1} x^\rho,$$

reflecting the fact that a traceless vector operator remains traceless once it is contracted with $x$. Let us also introduce the generators of angular-momentum, $X_{[\mu \nu]} = -x_{[\mu} \partial_{\nu]}$, and of dilations, $X = x \partial + 1$, which commute with $H_n$,

$$
[X, H_n] = 0, \quad [X_{[\mu \nu]}, H_n] = 0.
$$

$\mathbf{d}_\alpha$ is a generalization of the interior derivative on the light-cone which has been introduced by Bargman and Todorov [17]. It is nilpotent on the light-cone but off the light-cone the square is proportional to $x^2$:

$$
\mathbf{d}^\sigma \mathbf{d}_\sigma = \frac{1}{4} x^2 \square^2.
$$

Let us remark, that the operators $X, x_\mu, \mathbf{d}_\mu$ and $X_{[\mu \nu]}$ obey the conformal algebra also off the light-cone.

The set of unknown objects is now reduced to the coefficients $c_1$ to $c_5$ which can be determined by the same argument as has been used in the scalar case to fix the coefficient $c$. We therefore expect five equations similar to (38), which have to be solved iteratively in $r$. As a preparatory step we insert the explicit expressions for $H^\rho_{\alpha|n}$ and
$H_\alpha$ into the equations $^{27}$ and $^{28}$ and perform the necessary differentiations followed by a change of summation indices from $(j, k)$ to $(j, r)$. The result for the five coefficients $c_1(j, n)$ to $c_5(j, n)$ is:

$$
\sum_{j=0}^{\frac{n}{2}} (x^2)^j c_1(j, n) \ H^\rho_{\alpha|n-2j} \ O^j_{\rho|n} (x)
$$

(39)

$$
= \sum_{r=0}^{\frac{n}{2}} \sum_{j=0}^{r} c_1(j, n) \cdot d(r-j, n-2j) \frac{(x^2)^{r-1}}{(n+1-2j)^2} \\
\times \left( (r-j + (n+1-2j)^2) x^2 \delta^\rho_\alpha \Box - 2 (r-j) (n+1-r-j) x_\alpha x^\rho \Box \\
- (n-r-j) x^2 x_\alpha \partial^\rho \Box + (r-j) x^2 x^\rho \partial_\alpha \Box + \frac{1}{2} (x^2)^2 \partial_\alpha \partial^\rho \Box \right)^{r-1} O_{\rho|n} (x),
$$

$$
\sum_{j=0}^{\frac{n+1}{2}} (x^2)^j c_2(j, n) \ H_{n+1-2j} \ \partial^\rho \Box^{j-1} O_{\rho|n} (x)
$$

(40)

$$
= \sum_{r=0}^{\frac{n+1}{2}} \sum_{j=0}^{r} c_2(j, n) \cdot d(r-j, n+1-2j) \ (x^2)^{r-1} \left[ 2 (r-j) \cdot x_\alpha \partial^\rho + x^2 x^\rho \partial_\alpha \Box \right]^{r-1} O_{\rho|n} (x),
$$

$$
\sum_{j=0}^{\frac{n+1}{2}} (x^2)^j c_3(j, n) \ H_{n+1-2j} \ \partial^\rho O_{\rho|n} (x)
$$

(41)

$$
= \sum_{r=0}^{\frac{n+1}{2}} \sum_{j=0}^{r} c_3(j, n) \cdot d(r-j, n+1-2j) \ (x^2)^{r-1} \\
\times \left[ x^2 \delta^\rho_\alpha \Box + 2 (r-j) \cdot x_\alpha x^\rho \Box + 4 r (r-j) \cdot x_\alpha \partial^\rho + x^2 x^\rho \partial_\alpha \Box + 2 r \cdot x^2 \partial_\alpha \partial^\rho \right]^{r-1} O_{\rho|n} (x),
$$

$$
\sum_{j=0}^{\frac{n+1}{2}} (x^2)^j c_4(j, n) \ x_\alpha H_{n+1-2j} \ \partial^\rho \Box^{j-1} O_{\rho|n} (x)
$$

(42)

$$
= \sum_{r=0}^{\frac{n+1}{2}} \sum_{j=0}^{r} c_4(j, n) \cdot d(r-j, n+1-2j) \ (x^2)^{r-1} \left[ x_\alpha \partial^\rho \Box \right]^{r-1} O_{\rho|n} (x),
$$

$$
\sum_{j=0}^{\frac{n+1}{2}} (x^2)^j c_5(j, n) \ x_\alpha H_{n+1-2j} \ \partial^\rho \Box^{j-1} O_{\rho|n} (x)
$$

(43)

$$
= \sum_{r=0}^{\frac{n+1}{2}} \sum_{j=0}^{r} c_5(j, n) \cdot d(r-j, n+1-2j) \ (x^2)^{r-1} \left[ x_\alpha x^\rho \Box + 2 r \cdot x_\alpha \partial^\rho \Box \right]^{r-1} O_{\rho|n} (x).$$

Looking onto these equations we observe five independent tensor structures of dimension zero,

$$(x^2)^{r-1} \left\{ x^2 \delta^\rho_\alpha \Box, \ x_\alpha x^\rho \Box, \ x_\alpha \partial^\rho, \ x^2 x^\rho \partial_\alpha \Box, \ x^2 \partial_\alpha \partial^\rho \right\} \Box^{r-1}, \quad (44)$$

which should be taken as basis of reference operators. Now we collect all terms belonging to the structures $^{14}$ by imposing the constraint

$$
\sum_{j=0}^{\frac{n+1}{2}} \left( (x^2)^j H^\rho_{\alpha|n-2j} + x_\alpha (x^2)^{j-1} H^\rho_{n+1-2j} \right) = \delta^\rho_\alpha,
$$

(45)

which again reflects the fact that the trace decomposition $^{29}$ must be a formal representation of $\delta^\rho_\alpha$. As expected
we find a set of five equations which, in the order of the reference operators, read:

\[
\sum_{j=0}^{r} c_1(j,n) d(r-j,n-2j) \left( \frac{n+1-2j}{n+1-2j} \right)^2 + c_3(j,n) d(r-j,n+1-2j) = \delta_{0r}, \tag{46}
\]

\[
\sum_{j=0}^{r} c_1(j,n) d(r-j,n-2j) \left( \frac{n+1-2j}{n+1-2j} \right)^2 \left( \frac{2(r-j)}{n+1-2j} \right) - d(r-j,n+1-2j) \left[ 2(r-j) c_3(j,n) + c_5(j,n) \right] = 0, \tag{47}
\]

\[
\sum_{j=0}^{r-1} c_1(j,n) d(r-1-j,n-2j) \left( \frac{n-r-1-j}{n+1-2j} \right)^2 \left( \frac{r-j}{n+1-2j} \right) - \sum_{j=0}^{r} d(r-j,n+1-2j) \left[ 2(r-j) c_2(j,n) + 4r(r-j) c_3(j,n) + c_4(j,n) + 2rc_5(j,n) \right] = 0, \tag{48}
\]

\[
\sum_{j=0}^{r} c_1(j,n) d(r-j,n-2j) \left( \frac{r-j}{n+1-2j} \right)^2 + c_3(j,n) d(r-j,n+1-2j) = 0, \tag{49}
\]

\[
\sum_{j=0}^{r-1} c_1(j,n) d(r-1-j,n-2j) \left( \frac{1}{2(n+1-2j)} \right)^2 + \sum_{j=0}^{r} d(r-j,n+1-2j) \left[ c_2(j,n) + 2rc_3(j,n) \right] = 0. \tag{50}
\]

The shift \( r \to r-1 \) in the first term of equations \( r \) and \( r \) is due to the fact that the third and the fifth of the structures \( r \) are shifted by \( r \to r+1 \) in equation \( r \). For \( r = 0 \) these two sums are empty. This shift must be compensated otherwise \( c_5 \) would not contribute to the correct power of \( r \).

The set of equations \( r \) can be regarded as a generalization of equation \( r \). It can be solved iteratively starting with \( r = j = 0 \). We find \( c_1(0,n) = 1 \) and \( c_1(0,n) = 0, i = 2, ..., 5 \). Then, we insert this solution into the set of linear equations for \( r = 1 \). The resulting system is easily solved and the solution determines \( c_1(2,n) \) to \( c_5(2,n) \), and so on. This iterative procedure leads to the following general solution for \( c_1 \) to \( c_5 \):

\[
c_1(j,n) = \frac{(n+1-2j)!}{4^j j! (n+1-j)!} = c(j,n), \tag{51}
\]

\[
c_2(j,n) = -\frac{2j (n+1-2j)!}{4^j j! (n+3-2j) (n+1-j)!} = -\frac{2j}{n+3-2j} \cdot c(j,n), \tag{52}
\]

\[
c_3(j,n) = \frac{2j (n+1-2j)!}{4^j j! (n+3-2j) (n+1-j)!} = \frac{2j}{2(n+2-j)} \cdot \frac{1}{2n+3-2j} \cdot c(j,n), \tag{53}
\]

\[
c_4(j,n) = \frac{4j (n+2-2j)!}{4^j j! (n+3-2j) (n+1-j)!} = \frac{2j}{n+3-2j} \cdot c(j,n), \tag{54}
\]

\[
c_5(j,n) = -\frac{2j (n+2-2j)!}{4^j j! (n+3-2j) (n+2-j)!} = -\frac{2j}{(n+2-j)} \cdot \frac{(n+2-2j)!}{(n+2-j)(n+3-2j)} \cdot c(j,n). \tag{55}
\]

For a direct proof of the above solution we should have a closer look at \( r \) and \( r \). A subtraction of both equations yields a relation for \( c_1(j,n) \) which coincides with Eq. \( r \). For \( c_1(j,n) \) is therefore identical to \( c(j,n) \) given by Eq. \( r \). The proof of the remaining solutions \( r \) to \( r \) for \( c_2(j,n) \) to \( c_5(j,n) \) is obtained by induction. Let us show this for \( c_3 \). Namely, inserting \( c_1 \) into Eq. \( r \) it is easily seen that this equation is valid for \( r = 0 \) and for \( r = 1 \). Now, assume that \( r \) is valid for \( c_3(r,n) \), then Eq. \( r \) is chosen for \( r + 1 \) and resolved w.r.t. \( c_3(r+1,n) \),

\[
c_3(r+1,n) = -\sum_{j=0}^{r} c(j,n) d(r+1-j,n-2j) \left( \frac{r+1-j}{n+1-2j} \right) + \frac{j (n-r-j)}{n+1-2j} \left( \frac{j (n-r-j)}{(n+2-j)(n+3-2j)} \right). \tag{56}
\]

Performing this sum we find the desired solution for \( c_3(r+1,n) \),

\[
c_3(r+1,n) = \frac{(r+1) (n+1-2(r+1))!}{4^{r+1} (n+3-2(r+1)) (r+1)! (n+2-(r+1))}. \tag{57}
\]

One can now continue to prove by induction also the remaining three results for \( c_2 \), \( c_4 \) and \( c_5 \). We omit the explicit calculations since they yield no additional information.
Now, inserting the solution for $c_1$ to $c_5$ into the projectors (27) and (28) of the decomposition (26) we get the complete trace decomposition as follows

$$O_{\alpha|n}(x) = \sum_{j=0}^{\lfloor n/2 \rfloor} \frac{(n+1-2j)!}{4^j j! (n+1-j)!} \left(x^2\right)^{j-1} \left[ x^2 H^\rho_{\alpha|n-2j} \square \right.$$  
$$+ \frac{4j}{(n+3-2j)(n+2-j)} x_\alpha H_{n+1-2j} \rho \right] \square^{j-1} O_{\rho|n}(x). \quad (56)$$

This is the generalization of the scalar trace decomposition (19) to vector operators carrying a free index $\alpha$.

In the scalar case the trace decomposition already is equal to the infinite twist decomposition because traceless scalar operators always have well-defined twist. However, once free indices are involved we need to apply suitable projection operators onto the desired symmetry type. As has been mentioned before there exist two different symmetry types I and II when only one free index occurs. The corresponding Young projection operators have to be adjusted to the polynomial method. They have been determined in Refs. [4, 6] and are given by

$$\gamma_{\mu|n}^1 = \frac{1}{n+1} \partial_\mu x^\alpha, \quad (57)$$
$$\gamma_{\mu|n}^2 = \frac{2}{n+1} x^\alpha \delta_{\mu}^\sigma \partial_\sigma. \quad (58)$$

As is easily proven, $\gamma_{\mu|n}^1$ and $\gamma_{\mu|n}^2$ fulfill the correct projection properties,

$$\sum_{k=1}^{2} \gamma_{\mu|n}^k \alpha \ O_{\alpha|n}(x) = \delta_{\mu}^\alpha \ O_{\alpha|n}(x), \quad (59)$$
$$\gamma_{\nu|n}^k \gamma_{\mu|n}^l \alpha \ O_{\alpha|n}(x) = \delta_{\nu}^\alpha \gamma_{\mu|n}^l \alpha \ O_{\alpha|n}(x). \quad (60)$$

Now, we must have a closer look at the trace decomposition (56) in order to see where the representation (59) of $\delta_{\mu}^\alpha$ must be inserted:

(a) In the second term of the expansion, behind the terms $(x^2)^{j-1}x_\alpha$, only traceless scalar operators occur belonging to symmetry type I. Therefore, they have already well-defined twist which is not destroyed by applying $\partial_\alpha$.

(b) In the first term, however, the operator $H^\rho_{\alpha|n-2j} \square \ O_{\rho|n}(x)$, despite being traceless, has no well-defined twist because it has no definite symmetry behavior and, therefore, here we must insert the decomposition (59).

(c) Furthermore, the expansion (56) must have a definite overall-symmetry. So we have to multiply the whole expression with the decomposition of $\delta_{\mu}^\alpha$.

Following these arguments we find

$$O_{\mu|n}(x) = \sum_{k=1}^{2} \gamma_{\mu|n}^k \sum_{j=0}^{\lfloor n/2 \rfloor} \frac{(n+1-2j)!}{4^j j! (n+1-j)!} \left(x^2\right)^{j-1} \left[ x^2 \sum_{l=1}^{2} \gamma_{\mu|n}^l \beta \ H^\rho_{\beta|n-2j} \square \right.$$  
$$+ \frac{4j}{(n+3-2j)(n+2-j)} x_\alpha H_{n+1-2j} \rho \right] \square^{j-1} O_{\rho|n}(x). \quad (61)$$

Let us remark that the symmetry projectors $\gamma_{\mu|n}^k$ and $\gamma_{\mu|n}^l$ in (56), although being of the same form (57) and (58), respectively, depending on their symmetry type $k$ and $l$ have a different interpretation. If a symmetry projector acts on a traceless operator it generates an operator of well-defined twist whereas the global projectors $\gamma_{\mu|n}^k$ fix the symmetry of the undecomposed operator. According to this, the symmetry properties of the operators of well-defined twist are nested within the decomposition of the different global Young patterns being determined by appropriate symmetry projectors.

To find the infinite twist-decomposition in $x$-space we must now apply all symmetry projectors $\gamma_{\mu|n}^k$. This is done by calculating the following expressions,

$$\gamma_{\mu|n}^k \left(x^2\right)^{j-1} x_\alpha H_{n+1-2j} \quad \text{for} \quad k = 1, 2, \quad (62)$$
$$\gamma_{\mu|n}^k \left(x^2\right)^{j} \gamma_{\alpha|n-2j}^l \beta H_{\beta|n-2j}^\rho \quad \text{for} \quad k, l = 1, 2,$$
in terms of operators of well-defined twist. We begin with $k = 1$ which corresponds to symmetry type I. The calculation is almost trivial and we find

$$\mathcal{Y}^{\alpha}_{\mu|n} (x^2)^{j-1} x_{\alpha} H_{n+1-2j} = \frac{n+3-2j}{2(n+1)} \partial_{\mu} (x^2)^j H_{n+1-2j}, \quad (63)$$

$$\mathcal{Y}^{\alpha}_{\mu|n} (x^2)^j \mathcal{Y}^{\beta}_{\alpha|n-2j} H_{\beta|n-2j}^\rho = \frac{1}{n+1} \partial_{\mu} (x^2)^j H_{n+1-2j} x^\rho, \quad (64)$$

$$\mathcal{Y}^{\alpha}_{\mu|n} (x^2)^j \mathcal{Y}^{\beta}_{\alpha|n-2j} H_{\beta|n-2j}^\rho = 0, \quad (65)$$

where we have used the relation (64). Inserting these results into the expansion (61) we find the infinite twist-decomposition for vector operators of symmetry type I as it already has been given in Ref. [4], Chapter III.C.

$$\mathcal{Y}^{\rho}_{\mu|n} O_{\rho|n} (x) = \left[ \sum_{j=0}^{[\frac{n+1}{2}]} \frac{(n+2-2j)!}{4^j j! (n+1-2j)!} \partial_{\mu} (x^2)^j H_{n+1-2j} \quad (66)$$

This result also shows, that the trace decomposition (60) fulfills the consistency condition $x^\alpha O_{\alpha|n} (x) = O_{n+1} (x)$ which must hold since an infinite twist-decomposition of symmetry type I always reduces to the infinite twist decomposition of a scalar operator for any number of free indices. All indices are then freed from their contraction with the coordinates by partial differentiations, see Ref. [4], page 121, for the details.

The calculation for $k = 2$ is a little bit more involved but leads straightforwardly to the following result:

$$\mathcal{Y}^{\alpha}_{\mu|n} (x^2)^{j-1} x_{\alpha} H_{n+1-2j} = -\frac{2(n+2-j)}{n+1} (x^2)^{j-1} x^\sigma X_{[\mu|\sigma]} H_{n+1-2j}, \quad (67)$$

$$\mathcal{Y}^{\alpha}_{\mu|n} (x^2)^j \mathcal{Y}^{\beta}_{\alpha|n-2j} H_{\beta|n-2j}^\rho = \frac{4 j}{n+1} (n+1-2j) (x^2)^{j-1} x^\sigma X_{[\mu|\sigma]} H_{n+1-2j} x^n, \quad (68)$$

$$\mathcal{Y}^{\alpha}_{\mu|n} (x^2)^j \mathcal{Y}^{\beta}_{\alpha|n-2j} H_{\beta|n-2j}^\rho = \frac{2 n+1-2j}{n+1} (x^2)^j x^\sigma \left( \delta_{[\mu|\sigma]} + \frac{1}{n+1-2j} X_{[\mu|\sigma]} \partial^\rho \right) H_{n+1-2j}. \quad (69)$$

Again, we have used the relation (60). Inserting these results into the expansion (61) we find the following decomposition:

$$\mathcal{Y}^{\rho}_{\mu|n} O_{\rho|n} (x) = \sum_{j=0}^{[\frac{n+1}{2}]} \frac{(n+2-2j)!}{4^j j! (n+1-2j)!} \left[ (x^2)^j x^\sigma \frac{2}{n+1-2j} \left( \delta_{[\mu|\sigma]} + \frac{1}{n+1-2j} X_{[\mu|\sigma]} \partial^\rho \right) H_{n+1-2j} \quad (70) \right.$$

At this point we introduce the **complete set of local operators of well-defined twist** which appear in the resulting infinite twist-decomposition:

First we define **two scalar operators** which appear behind $x_{\mu}$-traces,

$$O_{n+1-2j}^{tw(\tau+2j)} A (x) := H_{n+1-2j} \quad (71)$$

$$O_{n+1-2j}^{tw(\tau+2j)} B (x) := H_{n+1-2j} \quad (72)$$

Secondly, we define **two vector operators** which are deduced from the previous two scalar operators by a differentiation $\partial_{\mu}$ and multiplication with the pre-factor $1/(n+1-2j)$, which is included for later convenience,

$$O_{\mu|n-2j}^{tw(\tau+2j)} A (x) := \frac{1}{n+1-2j} \partial_{\mu} O_{\mu|n-2j}^{tw(\tau+2j)} A (x) \quad (73)$$

$$O_{\mu|n-2j}^{tw(\tau+2j)} B (x) := \frac{1}{(n+1-2j)^2} H_{n+1-2j} \quad (75)$$
Finally, we define an **antisymmetric tensor operator** and a related **vector operator**, 

\[
\mathcal{O}_{\mu n}^{\text{tw}(\tau_0+1+2j)} (x) := \frac{2}{n+1-2j} \left( \delta_{\mu j} \partial_\sigma + \frac{1}{n+1-2j} X_{[\mu\sigma]} \partial^\rho \right) H_{n-2j} \boxtimes^j \mathcal{O}_{\rho n} (x)
\] (77)

\[
= \frac{2}{(n-2j)(n+1-2j)} H_{n-1-2j} \left( \delta_{\mu j} d_\sigma + \frac{1}{n+1-2j} X_{[\mu\sigma]} d^\rho \right) \boxtimes^j \mathcal{O}_{\rho n} (x)
\] (78)

\[
\mathcal{O}_{\mu n}^{\text{tw}(\tau_0+1+2j)} (x) := x^\sigma \mathcal{O}_{\mu [\sigma n 1-2j]}^{\text{tw}(\tau_0+1+2j)} (x)
\] (79)

Obviously, these scalar, vector and tensor operators vanish for \( j > \left[ \frac{n+1}{2} \right], \left[ \frac{n}{2} \right] \) and \( \left[ \frac{n-1}{2} \right] \), respectively, by construction. Here, \( \tau_0 \) denotes the **minimal twist** of the operator \( \mathcal{O}_\mu (\kappa x, -\kappa x) \), given by its canonical dimension \( d \) minus its spin \( j \), thereby also observing external contractions with \( x^\mu \) and/or \( \partial^\mu \):

(i) For the vector operator \( \mathcal{O}_\mu (\kappa x, -\kappa x) \) we find \( \tau_0 = 2 \). Then, the first four operators \( (71) - (75) \) are of even twist beginning with twist-2 and the last two operators of type C are of odd twist beginning with twist-3.

(ii) For the operator \( M_\mu (\kappa x, -\kappa x) \) we find \( \tau_0 = 1 \). In that case operators of type \( (71) \) and \( (73) \) do not exist due to the ‘internal’ antisymmetry of \( \mu \) not allowing symmetry type I which is related to operators of type A.

It is important to remark that \( \tau_0 \) only enumerates the twist content of various operators but has no major impact on the twist decomposition itself.

The first two scalar operators, \( \mathcal{O}_{n+1-2j}^{\text{tw}(\tau_0+2j)} A (x) \) and \( \mathcal{O}_{n+1-2j}^{\text{tw}(\tau_0+2j)} B (x) \), lie in the tensor space \( T \left( \frac{n+1-2j}{2} \right) \) while the two related vector operators, \( \mathcal{O}_{\mu n}^{\text{tw}(\tau_0+2j)} A (x) \) and \( \mathcal{O}_{\mu n}^{\text{tw}(\tau_0+2j)} B (x) \), lie in the space \( T \left( \frac{n-2j}{2}, \frac{n-2j}{2} \right) \). The forth operator, \( \mathcal{O}_{\mu n}^{\text{tw}(\tau_0+1+2j)} C (x) \), lies in the space \( T \left( \frac{n-2j}{2}, \frac{n-2j}{2} \right) \). The vector operators obey the relations

\[
x^\mu \mathcal{O}_{\mu n}^{\text{tw}(\tau_0+2j)} A/B (x) = \mathcal{O}_{n+1-2j}^{\text{tw}(\tau_0+2j)} A/B (x)
\] (80)

\[
x^\mu \mathcal{O}_{\mu n}^{\text{tw}(\tau_0+1+2j)} C (x) = 0
\] (81)

The first of these relations is obtained due to the factor \( 1/(n+1-2j) \), the second one due to the ‘internal’ antisymmetry of the vector operator \( (71) \).

We have written the operators \( \mathcal{O}_{\mu n}^{\text{tw}(\tau_0+2j)} A (x) \), \( \mathcal{O}_{\mu n}^{\text{tw}(\tau_0+2j)} B (x) \) and \( \mathcal{O}_{\mu [\sigma n 1-2j]}^{\text{tw}(\tau_0+1+2j)} C (x) \) also in the form \( (71) \), \( (73) \) and \( (78) \) because in the on-cone limit, \( x^2 \to 0 \), the projector \( H_n \) reduces to the identity.

With these definitions the expansion \( (66) \) related to symmetry type I, i.e., totally symmetric local operators, reads

\[
\mathcal{Y}_{\mu n}^1 \mathcal{O}_{\rho n} (x) = \frac{1}{n+1} \sum_{j=0}^{\left[ \frac{n+1}{2} \right]} \frac{(n+2-2j)!}{4^j j! (n+2-2j)!} \partial_\mu \left( x^2 \right)^j \mathcal{O}_{n+1-2j}^{\text{tw}(\tau_0+2j)} A (x)
\] (82)

\[
= \frac{1}{n+1} \sum_{j=0}^{\left[ \frac{n+1}{2} \right]} \frac{(n+2-2j)!}{4^j j! (n+2-2j)!} \left( x^2 \right)^{j-1} \left[ x^2 (n+1-2j) \mathcal{O}_{\mu n}^{\text{tw}(\tau_0+2j)} A (x) + 2j x^\mu \mathcal{O}_{n+1-2j}^{\text{tw}(\tau_0+2j)} A (x) \right]
\]

and the expansion \( (70) \), globally related to symmetry type II, reads

\[
\mathcal{Y}_{\mu n}^2 \mathcal{O}_{\rho n} (x) = \sum_{j=0}^{\left[ \frac{n+1}{2} \right]} \frac{(n+1-2j)!}{4^j j! (n+1-j)!} (x^2)^{j-1} x^\sigma
\]

\[
\times \left[ x^2 \mathcal{O}_{[\mu n 1-2j]}^{\text{tw}(\tau_0+1+2j)} C - \frac{8j (n+2-2j)}{n+3-2j} x_{[\mu} \left( \frac{1}{n+1} \mathcal{O}_{\sigma n}^{\text{tw}(\tau_0+2j)} A - \mathcal{O}_{\sigma n}^{\text{tw}(\tau_0+2j)} B \right) \right]
\]
separately. However, for the operator \( O_{-\kappa} \) present in this case. One can therefore not expect to find a useful combination of these operators in terms of need the full off-cone twist decomposition of a general 2nd rank tensor operator which is not available by now.

Furthermore, vector operators of type B do not appear in the on-cone limit while vector operators of type A are related to symmetry type I have to be omitted. This chapter is devoted to the infinite twist decomposition of the nonlocal operators \( O_{\mu} \). Eqs. (3) and (9), by summing up their corresponding local decomposition. Also here we suppress anti-symmetrization w.r.t. \( \kappa \) which can be done at the end. Obviously, for the operator \( O_{\mu} \) the above expansion is already the complete twist-decomposition of \( O_{\mu} \) and will not be repeated for that reason. However, for the operator \( M_{\mu} \) the expansion reduces to operators of type B and C only, i.e., operators of type A which are related to symmetry type I have to be omitted.

When the explicit expressions for the moments \( O_{\mu} \) and \( M_{\mu} \) given by Eqs. (1) and (2), respectively, are put into the local operators \( \tau_{\mu} \) these operators of well-defined twist may be expressed in terms of Gegenbauer polynomials. For this reason let us introduce some useful definitions and relations. First, a set of polynomials \( h_n(u|x) \), being symmetric and homogeneous in the variables \( u \) and \( x \), is defined as follows:

\[
\begin{align*}
h_n^\nu(u|x) & := \left( \frac{1}{2} \sqrt{u^2 x^2} \right)^n G_n^\nu \left( \frac{u x}{\sqrt{u^2 x^2}} \right) \quad \text{for } n \geq 0, \\
h_n^\nu(u|x) & := 0 \quad \text{for } n < 0,
\end{align*}
\]

III. COMPLETE TWIST-DECOMPOSITION OF THE NONLOCAL OPERATORS

Now, putting together both contributions, Eqs. (3) and (4), we finally obtain the infinite twist-decomposition of the complete local operator as follows:

\[
O_{\mu;n}(x) = \sum_{j=0}^{[\frac{n+1}{2}]} \frac{(n + 1 - 2j)!}{4^j j! (n + 1 - j)!} (x^2)^{j-1} x^2 O_{\mu;n-2j}^{(n+1+2j)} C (x)
+ n + 2 - 2j \left\{ \frac{1}{n + 2 - j} \left( (n + 3) x^2 O_{\mu;n-2j}^{(n+1+2j)} A (x) - 2j x_{\mu} O_{\mu;n+1-2j}^{(n+1+2j)} A (x) \right) - 4j \left( x^2 O_{\mu;n-2j}^{(n+1+2j)} B (x) - x_{\mu} O_{\mu;n+1-2j}^{(n+1+2j)} B (x) \right) \right\}.
\]

At this stage we observe that one can add up both twist contributions appearing behind the \( x_{\mu} \) trace terms to a single operator. Namely, we find an expression containing the generalized interior derivative \( d^\rho \):

\[
- \frac{1}{2} O_{n+1-2j}^{(n+1+2j)} A (x) + (n + 2 - j) O_{n+1-2j}^{(n+1+2j)} B (x) = H_{n+1-2j} d^\rho \Box^{j-1} O_{\mu;n}(x).
\]

However, since the vector operators of type A and B cannot be combined in similarly this may just be a coincidence. Furthermore, vector operators of type B do not appear in the on-cone limit while vector operators of type A are present in this case. One can therefore not expect to find a useful combination of these operators in terms of \( d^\rho \). Of course, this justifies the distinction of operators into type A and type B. The open question therefore is whether relation is genuine for general trace terms surviving the on-cone limit or not. To judge this question we would need the full off-cone twist decomposition of a general 2nd rank tensor operator which is not available by now.

The result is valid for a generic local vector operator \( O_{\mu;n}(x) \) and we are now going to apply it to the local operators \( O_{\mu;n}(x) \) and \( M_{\mu;n}(x) \) which, then, will be summed up to the non-local operators. Since the ‘internal’ structure of these operators is different (compare the moments and ) we will now treat these operators separately.
where $C_n^\nu (z)$ are the Gegenbauer polynomials (see Ref. [18], Appendix II.11) given by

\[
C_n^\nu (z) = \frac{1}{(\nu - 1)!} \sum_{k=0}^{\lfloor \nu/2 \rfloor} \frac{(-1)^k (n-k+\nu-1)!}{k! (n-2k)!} (2z)^{n-2k} \quad \text{for} \quad \nu > 0 ,
\]

(87)

\[
C_n^0 (z) = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{(-1)^k (n-k-1)!}{k! (n-2k)!} (2z)^{n-2k} \quad \text{for} \quad n > 0 , \quad C_n^0 (1) = 1 .
\]

(88)

For $\nu > 0$, the polynomials $h_n^\nu (u|x)$ obey the following relations,

\[
H_n (ux)^\nu = h_n^1 (u|x) ,
\]

(89)

\[
\partial_\nu^\alpha h_n^\nu (u|x) = \nu \left( u_\alpha h_n^{\nu+1} (u|x) - \frac{1}{2} u^2 x_\alpha h_n^{\nu+1} (u|x) \right) ,
\]

(90)

\[
\partial_\nu^\alpha h_n^\nu (u|x) = \nu \left( x_\alpha h_n^{\nu+1} (u|x) - \frac{1}{2} x^2 u_\alpha h_n^{\nu+1} (u|x) \right) ,
\]

(91)

\[
\Box h_n^\nu (u|x) = \nu (\nu - 1) u^2 h_n^{\nu+1} (u|x) ,
\]

(92)

\[
\Box u h_n^\nu (u|x) = \nu (\nu - 1) x^2 h_n^{\nu+1} (u|x) ,
\]

(93)

\[
(n + \nu) h_n^\nu (u|x) = \nu \left( h_n^{\nu+1} (u|x) - \frac{1}{2} u^2 x^2 h_n^{\nu+1} (u|x) \right) ,
\]

(94)

which can be traced back to the definition and corresponding relations of the Gegenbauer polynomials. They will be used to derive the representations $\text{(95)} - \text{(99)}$ below. Relation $\text{(88)}$ is the origin for all Gegenbauer polynomials appearing in these expressions.

### A. Infinite twist-decomposition of the operator $O_\mu (\kappa x, -\kappa x)$

Now, as announced, let us rewrite the operators of definite twist related to $O_\mu (\kappa x, -\kappa x)$ according to $\text{(71)} - \text{(77)}$ in terms of the polynomials $\text{(88)}$.

For the two scalar operators we obtain:

\[
O_{n+1-2j}^{\text{tw}(2+2j)} A (x) = \frac{n!}{2 (n+1-2j)!} \int d^4u \ O_\rho (u) \ (u^2)^{j-1} \times \left[ 4 j u^\rho h_{n+1-2j}^1 (u|x) + u^2 \left( 2 x^\rho h_{n-2j}^2 (u|x) - x^2 u^\rho h_{n-1-2j}^2 (u|x) \right) \right] ,
\]

(95)

\[
O_{n+1-2j}^{\text{tw}(2+2j)} B (x) = \frac{n!}{(n+1-2j)!} \int d^4u \ O_\rho (u) \ (u^2)^{j-1} u^\rho h_{n+1-2j}^1 (u|x) .
\]

(96)

The two vector operators $\text{(73)}$ and $\text{(75)}$ are obtained from the previous expressions as follows:

\[
O_{\mu|n+1-2j}^{\text{tw}(2+2j)} A (x) = \frac{n!}{2 (n+1-2j) (n+1-2j)!} \int d^4u \ O_\rho (u) \ (u^2)^{j-1} \times \partial_\mu \left[ 4 j u^\rho h_{n+1-2j}^1 (u|x) + u^2 \left( 2 x^\rho h_{n-2j}^2 (u|x) - x^2 u^\rho h_{n-1-2j}^2 (u|x) \right) \right] ,
\]

(97)

\[
O_{\mu|n+1-2j}^{\text{tw}(2+2j)} B (x) = \frac{n!}{(n+1-2j) (n+1-2j)!} \int d^4u \ O_\rho (u) \ (u^2)^{j-1} u^\rho \partial_\mu h_{n+1-2j}^1 (u|x) .
\]

(98)

The vector operator of type C is given by

\[
O_{\mu|n+2j}^{\text{tw}(3+2j)} C (x) = \frac{2 n! x^\sigma}{(n+1-2j) (n+1-2j)!} \left( \delta_\mu^\rho \partial_\sigma |n+1-2j| + X_{[\mu|\sigma]} \partial^\rho \right) \int d^4u \ O_\rho (u) \ (u^2)^{j} h_{n-2j}^1 (u|x) .
\]

(99)

In order to have a cross-check on our result $\text{(81)}$ we perform the on-cone limit $x \rightarrow \tilde{x}$, $\tilde{x}^2 = 0$, for $O_{\mu|n} (x)$ and find

\[
O_{\mu|n} (\tilde{x}) = O_{\mu|n}^{\text{tw}2} A (\tilde{x}) + O_{\mu|n}^{\text{tw}3} C (\tilde{x}) - \frac{\tilde{x}_\mu}{2 (n+1)^2} \left( O_{n-1}^{\text{tw}4} A (\tilde{x}) - 2(n+1)O_{n-1}^{\text{tw}4} B (\tilde{x}) \right)
\]

(100)
with
\[
O^{\text{tw}2}_{n/n} (\tilde{x}) = \frac{1}{n+1} \left( \partial_{\mu} - \frac{1}{2(n+1)} x_{\mu} \partial \right) x^{\mu} O_{\rho|\mu|n} (x) \bigg|_{x=\tilde{x}},
\]
\[
O^{\text{tw}3}_{n/n} (\tilde{x}) = \frac{1}{n+1} \left( n \delta^{\mu}_{\rho} - x^{\mu} \partial_{\mu} - \frac{x_{\mu}}{n+1} ((n-1) \partial^{\mu} - x^{\mu} \partial) \right) O_{\rho|\mu|n} (x) \bigg|_{x=\tilde{x}},
\]
\[
O^{\text{tw}4}_{n-1} (\tilde{x}) = \Box x^{\mu} O_{\rho|\mu|n} (\tilde{x}),
\]
\[
O^{\text{tw}4}_{n-1} (\tilde{x}) = \partial^{\rho} O_{\rho|\mu|n} (\tilde{x}).
\]

Thus, only the twist-2, the twist-3 and two twist-4 parts survive. The twist-4 parts are combined according to
\[
- \frac{1}{2} O^{\text{tw}4}_{n-1} (\tilde{x}) + (n+1) O^{\text{tw}4}_{n-1} (\tilde{x}) = \left( n+1 \right) \partial^{\rho} - \frac{1}{2} \Box x^{\rho} \bigg|_{x=\tilde{x}} = \partial^{\rho} O_{\rho|\mu|n} (x) \bigg|_{x=\tilde{x}},
\]
which can be directly deduced from \[85\]. Comparing these results with those of Ref. \[19\] we find that the twist-2, the twist-3 and the twist-4 parts obtain the form \[22.22\], \[22.23\] and \[22.24\], respectively. If we perform the summation to a non-local operator we can compare with Ref. \[6\]. The twist-2, twist-3 and twist-4 parts are then given by \[32.6\], \[32.42\] and \[32.45\], respectively. The expansion \[84\] therefore provides us with the correct on-cone limit.

In order to get the infinite twist decomposition of the non-local operator \(O_{\mu} (\kappa x, -\kappa x)\) we perform the summation over \(n\) also off-cone. To this purpose let us introduce an integral representation for the coefficient \(c(j, n)\) by using the integral representation of Euler’s beta function,
\[
\frac{(n+1-2j)!}{(n+1-j)!} = \frac{1}{(j-1)!} \frac{\Gamma (n+2-2j) \Gamma (j)}{\Gamma (n+2-j)} = \frac{1}{(j-1)!} \int_{0}^{1} dt \ (1-t)^{j-1} t^{n+2-j},
\]
which is valid for \(j \geq 1\). In addition, we introduce also integral representations for the different fractions in the second term of the expansion \[84\],
\[
\frac{1}{(n+2-j)(n+3-2j)} = \int_{0}^{1} d\lambda \ (1-\lambda^{j-1}) \lambda^{n+2-2j},
\]
\[
\frac{1}{n+3-2j} = \int_{0}^{1} d\lambda \ \lambda^{n+2-2j}.
\]
These representations hold for \(j \geq 0\), since the first representation is correct also in the limit \(j = 1\) with the result \(\int_{0}^{1} d\lambda \ln \lambda \lambda^{j}\).

In the same manner as in Ref. \[3\], the polynomials \(h^{\nu}_{n} (u|z)\) which are defined in terms of Gegenbauer polynomials may be summed up to the functions \(\mathcal{H}^{\nu} (u|x)\) being defined in terms of Bessel functions,
\[
\mathcal{H}^{\nu} (u|x) := \sqrt{\pi} \left( \frac{2}{(u^{2})^{2} - u^{2}x^{2}} \right)^{1/2 - \nu} J_{\nu - 1/2} \left( \frac{1}{2} \sqrt{(u^{2})^{2} - u^{2}x^{2}} \right) e^{i (ux)/2}.
\]
Namely, with that definition taking into account Eq. II.5.13.1.3 of Ref. \[18\] (or formula (2.29) from Ref. \[4\]) we can deduce the formula
\[
\sum_{n=0}^{\infty} \frac{i^{n}}{(n+l)!} K^{n-m}_{n-m} h^{\nu}_{n-m} (u|x) = \frac{i^{m}}{(\nu-1)!} \left[ \prod_{k=1+l+m}^{2 \nu - 1} (k + x \partial) \right] \mathcal{H}^{\nu} (u|x)
\]
for \(m \in \{0, \ldots, 2 \nu - 1\}\) and \(l \in \{-m, \ldots, 2 \nu - m - 1\}\) for \(k = 2 \nu\) the product in \[110\] is empty. The ranges for \(l\) and \(m\) have been chosen such that integral representations for factors like \(1/(n+i)\) are avoided. In principle these factors could be included in a more general formula which, however, will not be needed.

Now we define non-local operators of well-defined twist by
\[
O^{\text{tw}(2+2j)}_{\mu A/B} (\kappa x, -\kappa x) := \sum_{n=0}^{\infty} \frac{i^{n}}{n!} K^{n+2j}_{n+2j} O^{\text{tw}(2+2j)}_{n+2j} A/B (x),
\]
\[
O^{\text{tw}(2+2j)}_{\mu A/B/C} (\kappa x, -\kappa x) := \sum_{n=0}^{\infty} \frac{i^{n}}{n!} K^{n-2j}_{n-2j} O^{\text{tw}(2+2j)}_{n-2j} A/B/C (x),
\]
which are introduced as functions of $\kappa x$ because they will be used in the twist decomposition of a non-local vector operator which obeys that property! This will be very important for the corresponding summations.

Performing the above sums, using equation (110), and the integral representation $1/(n + 1 - 2j) = \int_0^1 d\lambda \lambda^{n-2j}$ we find by straightforward calculations,

$$O_{\mu}^{\text{tw}(2+2j)} (\kappa x, -\kappa x) = \frac{1}{2} \int d^4 u \frac{O_\mu (u)}{(-\kappa^2)^{j-1}} \left[ 4j \ln^\mu (1 + x\partial) \mathcal{H}^1 (u|\kappa x) \right. $$

$$\left. - u^2 \left( 2\kappa x^\rho (2 + x\partial) - iu^\rho \kappa^2 x^2 \right) (3 + x\partial) \mathcal{H}^2 (u|\kappa x) \right],$$

(113)

$$O_{\mu}^{\text{tw}(2+2j)} (\kappa x, -\kappa x) = \int d^4 u \frac{O_\rho (u)}{(-\kappa^2)^{j-1}} \mathcal{H}^1 (u|\kappa x), \quad j \geq 1,$$

(114)

$$O_{\mu}^{\text{tw}(2+2j)} (\kappa x, -\kappa x) = \frac{1}{2\kappa} \int d^4 u \frac{O_\rho (u)}{(-\kappa^2)^{j-1} \mathcal{H}^1 (u|\kappa x)}, \quad j \geq 1,$$

(115)

$$O_{\mu}^{\text{tw}(2+2j)} (\kappa x, -\kappa x) = \frac{1}{\kappa} \int d^4 u \frac{O_\rho (u)}{(-\kappa^2)^{j-1}} \mathcal{H}^1 (u|\kappa x), \quad j \geq 1,$$

(116)

$$O_{\mu}^{\text{tw}(3+2j)} (\kappa x, -\kappa x) = 2 \int d^4 u \frac{O_\rho (u)}{(-\kappa^2)^{j-1} \mathcal{H}^1 (u|\kappa x)} .$$

(117)

In the case $j = 0$ we can again compare with Ref. [2]: The twist-2 operator $O_{\mu}^{\text{tw}2 \ A} (\kappa x, -\kappa x)$ is then equal to (3.15) and the twist-3 operator $x^\rho O_{[\mu\nu\rho]}^{\text{tw}3 \ C} (\kappa x, -\kappa x)$ is equal to (4.27). Operators of type B, [114] and [116], have not been considered in [2] because the full decomposition of operators related to symmetry type II was not available there.

Now we are in a position to sum up to a non-local operator using equations (106) and (107) which are valid for

$$\sum_{j=1}^{\infty} \frac{(\kappa^2 x^2)^{j-1}}{4j! (j-1)!} (x\partial) \left[ 4j \ln^\mu (1 + x\partial) \mathcal{H}^1 (\lambda \kappa x, -\lambda \kappa x) \right. $$

$$\left. - 2j (\lambda \kappa x) x_\mu O_{\mu}^{\text{tw}(2+2j)} (\lambda \kappa x, -\lambda \kappa x) \right]$$

$$- 4j \left( (\lambda \kappa x)^2 x^2 O_{\mu}^{\text{tw}(2+2j)} (\lambda \kappa x, -\lambda \kappa x) \right)$$

(118)

$$+ (t \kappa x)^2 O_{\mu}^{\text{tw}(3+2j)} (t \kappa x, -t \kappa x)$$

Therefore, $\lim_{j \to 1} \left( (1 - \lambda^j)/(j-1) \right) = -\ln \lambda$ has to be observed.

The above formula shows explicitly the origin of the different twist contributions. The twist-2 and twist-3 contributions are given by single terms which are traceless but twist-4 and also all higher contributions of even twist consist of two terms where the traceless operators $O_{\mu}^{\text{tw4} \ A/B} (\lambda \kappa x, -\lambda \kappa x)$ and $O_{\mu}^{\text{tw4} \ A/B} (\lambda \kappa x, -\lambda \kappa x)$ appear behind trace terms of type $x^2$ and $x_\mu$. Therefore, the sum of these two twist-4 contributions can never be traceless. Contracting the expression (118) with $x^\mu$ we obtain after a straightforward computation the well-known infinite twist decomposition of $O (\kappa x, -\kappa x)$, due to Eqs. [30] and [31]:

$$O (\kappa x, -\kappa x) = O_{\mu}^{\text{tw}2 \ A} (\kappa x, -\kappa x) + \sum_{j=1}^{\infty} \frac{(\kappa^2 x^2)^{j-1}}{4j! (j-1)!} (x\partial) \left[ 4j \ln^\mu (1 + x\partial) \right. $$

$$\left. - 2j (\lambda \kappa x) x_\mu O_{\mu}^{\text{tw}(2+2j)} (\lambda \kappa x, -\lambda \kappa x) \right]$$

(119)

$$\times \int_0^1 d\lambda \frac{1 - \lambda^j}{j-1} (\lambda \kappa x)^2 x^2 O_{\mu}^{\text{tw}(2+2j)} (\lambda \kappa x, -\lambda \kappa x) .$$


which already has been determined in Ref. [4], Eqs. (3.14,15).

It is obvious that the contributions of even twist, beginning with twist-4, look rather complicated. In principle, it is possible to express the double integral \( \int_0^1 dt \int_0^1 d\lambda \) in Eqs. (118 and 119) by a single integral representation but then it yields combinations of some generalized hypergeometric functions \( F_{[p,q]}([a_1,\ldots, a_p], [b_1,\ldots, b_q], z) \). We did not make use of them.

### B. Infinite twist-decomposition of \( M_\mu (\kappa x, -\kappa x) \)

In this subsection we apply the result [8] to the operator \( M_\mu (\kappa x, -\kappa x) \). Since operators of type A, (14) and (18), are not present in this case the infinite twist-decomposition for the moments \( M_{\mu|\nu+1} (x) \) obtains a simpler form:

\[
M_{\mu|\nu+1} (x) = \sum_{j=0}^{[\frac{\nu+2}{2}]} \frac{(n + 2 - 2j)!}{4^j j! (n + 2 - j)!} \left( x^2 \right)^{j-1} x^\sigma \]

\[
\times \left[ x^2 M_{[\mu|\sigma+2j]}^{tw(2\nu+2)} \left( x \right) + \frac{8 j(n + 3 - 2j)}{n + 4 - 2j} x^{[\mu|\sigma]n+1} \right] \]

\[
\times \left[ x^2 M_{[\mu|\sigma+2j]}^{tw(1\nu+2\nu)} \left( x \right) - x^{[\mu|\sigma]n+1} \right] ,
\]

globally being of symmetry type II. Of course, the operators \( M_{[\mu|\nu+1]} (x) \) which already has been determined in Ref. [4], Eqs. (3.14,15).

In principle this result follows from the complete off-cone decomposition of the skew tensor operator \( M_{[\mu|\nu]} (x) \) (see the definition [9]) which contains contributions of symmetry type II and III. Type III contributions cancel completely once they are contracted with \( x^\nu \) while the type II contributions yield additional trace terms which are contained in (120) after the contraction. The decomposition (120) is therefore not equivalent to the type II part of the antisymmetric tensor case.

If we insert the explicit form of the moments \( M_{\mu|\nu+1} (x) \) given by (13) into (121), (122) and (123) and use (89) – (91) we find the following three operators of well-defined twist, again expressed in terms of Gegenbabara polynomials,

\[
M_{\mu|\nu+1}^{tw(1\nu+2\nu)} (x) = \frac{n!}{(n + 2 - 2j)!} \int d^4 u \ M_{[\mu|\sigma]}^{\nu+1} \left( u \right) \left( u^2 \right)^{j-1} u^{[\nu|\sigma]} \ h_{n+1-2j}^2 \left( u|x \right) , \quad j \geq 1 ,
\]

\[
M_{\mu|\nu+1}^{tw(1\nu+2\nu)} (x) = \frac{n!}{(n + 2 - 2j)!} \int d^4 u \ M_{[\mu|\sigma]}^{\nu+1} \left( u \right) \left( u^2 \right)^{j-1} \partial u^{[\nu|\sigma]} \ h_{n+1-2j}^2 \left( u|x \right) , \quad j \geq 1 ,
\]

\[
M_{\mu|\nu+1}^{tw(2\nu+2\nu)} (x) = \frac{n!}{(n + 2 - 2j)!} \int d^4 u \ M_{[\mu|\sigma]}^{\nu+1} \left( u \right) \left( u^2 \right)^{j-1} \partial u^{[\nu|\sigma]} \ h_{n+1-2j}^2 \left( u|x \right) ,
\]

In the on-cone limit, \( x^2 \to 0 \), using \( x^\sigma \ M_{[\mu|\sigma]}^{tw(3\nu+3\nu)} (x) = n/2 \ M_{[\mu|\sigma]}^{tw(3\nu+3\nu)} (x) \), we find two contributions, one of twist-2 and another one of twist-3,

\[
M_{\mu|\nu+1} (x) = M_{\mu|\nu+1}^{tw(2\nu+2\nu)} (x) + \frac{x^{[\mu|\nu+1]} (x)}{n + 2} M_{\mu|\nu+1}^{tw(3\nu+3\nu)} (x)
\]

with

\[
M_{\mu|\nu+1}^{tw(2\nu+2\nu)} (x) = \frac{2}{(n + 1)! (n + 2)} x^{[\nu+1]} \left( \delta_{[\mu|\nu]} + \frac{1}{n + 2} \ X_{[\mu|\nu]} \ d^4 \phi \right) M_{\mu|\nu+1} (x)
\]

\[
= \left( \delta_{[\mu|\nu]} + \frac{1}{n + 2} \ M_{[\mu|\nu+1]} (x) \right)
\]

\[
M_{\mu|\nu+1}^{tw(3\nu+3\nu)} (x) = \partial^\mu M_{[\mu|\nu+1]} (x) .
\]
Here we have used the relation $\tilde{x}^\rho M_{\rho \mu n + 1} (\tilde{x}) = 0$. After the summation we again compare with Ref. [8]. The twist-2 part obtains the form (3.74) and the twist-3 part is equal to (3.76). In Ref. [19] the twist-2 part is given by (2.28) and the twist-3 part by (2.29). Again, we confirm the correct on-cone limit.

To perform the summation to a non-local infinite twist decomposition we first define non-local operators of well-defined twist by

$$M^{tw(1+2j)}_\mu (\kappa x, -\kappa x) := \sum_{n=0}^{\infty} \frac{\lambda_n}{n!} \kappa^{n+2-2j} M^{tw(1+2j)}_{\mu n+2-2j} (x) ,$$

$$M^{tw(1+2j)}_\mu (\kappa x, -\kappa x) := \sum_{n=0}^{\infty} \frac{\lambda_n}{n!} \kappa^{n+1-2j} M^{tw(1+2j)}_{\mu n+1-2j} (x) ,$$

$$M^{tw(2+2j)}_\mu (\kappa x, -\kappa x) := \sum_{n=0}^{\infty} \frac{\lambda_n}{n!} \kappa^{n+1-2j} M^{tw(2+2j)}_{\mu n+1-2j} (x) .$$

Again, we have chosen them homogeneous in $\kappa$ and $x$ because they are related to the (infinite) twist decomposition of the skew tensor operator $M_{[\mu \rho \mu]} (\kappa x, -\kappa x)$. After a straightforward calculation, using the summation properties of $h_n^\nu(u|x)$, Eq. (106), and the integral representation for $1/(n + 2 - 2j)$, we find

$$M^{tw(1+2j)}_\mu (\kappa x, -\kappa x) = \int d^4 u M_{[\mu \rho \mu]} (u) \left( -u^2 \right)^{j-1} i u^\nu \kappa x^\rho (2 + x \partial) (3 + x \partial) \mathcal{H}^2 (u|\kappa x) , \quad j \geq 1 ,$$

$$M^{tw(1+2j)}_\mu (\kappa x, -\kappa x) = \int d^4 u M_{[\mu \rho \mu]} (u) \left( -u^2 \right)^{j-1} \partial_\mu i u^\nu \kappa x^\rho (2 + x \partial) (3 + x \partial) \int_0^1 d\lambda \mathcal{H}^2 (u|\kappa x) , \quad j \geq 1 ,$$

$$M^{tw(2+2j)}_\mu (\kappa x, -\kappa x) = \int d^4 u M_{[\mu \rho \mu]} (u) \left( -u^2 \right)^{j-1} x^\rho \left( \delta_{[\mu} \partial_{\sigma]} (1 + x \partial) + X_{[\mu \sigma]} \partial^\rho \right)$$

$$\times \int_0^1 d\lambda \left[ 4j i u^\beta \mathcal{H}^1 (u|\kappa x) - u^2 \left( 2 \kappa x^\beta (3 + x \partial) - i u^\beta (\kappa x)^2 \right) \mathcal{H}^2 (u|\kappa x) \right].$$

Now we get the off-cone representation for the complete twist-decomposition of $M_\mu (\kappa x, -\kappa x)$ as follows:

$$\kappa M_\mu (\kappa x, -\kappa x) = M^{tw}_{\mu 2} C (\kappa x, -\kappa x)$$

$$\quad + \sum_{j=1}^{\infty} \frac{(\kappa^2 x^2)^{j-1}}{4^j j! (j-1)!} \int_0^1 \frac{dt}{t} (1 - t)^{j-1} \left[ (\kappa t)^2 x^2 M^{tw(2+2j)}_\mu (\kappa t x, -\kappa t x) \right.$$

$$\quad - 4j (x \partial) \int_0^1 d\lambda \left( (\lambda t \kappa)^2 x^2 M^{tw(1+2j)}_\mu (\lambda t \kappa x, -\lambda t \kappa x) - (\lambda t \kappa) x_\mu M^{tw(1+2j)}_\mu (\lambda t \kappa x, -\lambda t \kappa x) \right) \right].$$

This result will be much simpler than the decomposition because we only need the integral representations and which are both valid starting from $j = 1$.

### IV. POWER CORRECTIONS OF (NON)FORWARD MATRIX ELEMENTS IN $x$–SPACE

In this sections we are going to consider the implication of our results to deep inelastic scattering and deeply virtual Compton scattering. To this purpose we have to take the (non)diagonal matrix elements of the infinite off-cone decompositions and for the non-local operators $O_\mu (\kappa x, -\kappa x)$ and $F_\mu (\kappa x, -\kappa x)$, respectively, with the incoming and outgoing hadron states $(P_1, S_1)$ and $(P_2, S_2)$. Before doing this, again separately for the various operators of well-defined twist $\langle P_2, S_2 \mid O^{tw(\tau)}_\Gamma (\xi, -\xi) \mid P_1, S_1 \rangle$, let us shortly review the parametrization of such matrix elements as it has been introduced in Ref. [4]. Here, $\Gamma$ denotes the Dirac structure of the operator and $\xi$ denotes any of the different products of $\lambda$, $t$ and $\kappa$ appearing in the arguments of the non-local operators.

According to that approach the matrix elements of operators of well-defined twist are parametrized by Lorentz invariant double distributions $f_{\alpha}^{(\tau)} (Z, \mu^2)$, not suffering from any power corrections, and suitable kinematical factors $K^{\mu}_\Gamma (P_1, P_2; S_1, S_2)$ referring to the $\Gamma$-structure of the considered operator on the hadronic level:

$$\langle P_2, S_2 \mid O^{tw(\tau)}_\Gamma (\xi, -\xi) \mid P_1, S_1 \rangle \equiv \langle P_2, S_2 \mid P^{(\tau)}_\Gamma \Gamma' O^{tw(\tau)}_\Gamma (\xi, -\xi) \mid P_1, S_1 \rangle$$

$$= P^{(\tau)}_\Gamma \Gamma' (x, \partial) K^{\mu}_\Gamma (P, S) \int DZ e^{i \alpha (\partial x) Z} f_{\alpha}^{(\tau)} (Z, \mu^2) .$$

(134)
Here, $\mathcal{P}_\tau^{(\tau)T'}(x, \partial)$ are the corresponding projections onto operators of well-defined twist $\tau$,

$$
\mathcal{O}_\tau^{tw(\tau)}(\zeta x, -\zeta x) = \mathcal{P}_\tau^{(\tau)T'}(x, \partial) \mathcal{O}_{T'}(\zeta x, -\zeta x),
$$

(135)

which follow from their local versions in Eqs. (71) – (79) in the form (113) – (117) and (130) – (132), respectively, eventually including integrations over $\lambda$. In the representation (134) these projection operators act only on the $\Gamma$–structure and the exponential $e^{i\omega(x^p)^2}$. In addition, we have used the notation $\mathbb{F} = \{P_+, P_-\}$ and $\mathbb{Z} = \{z_+, z_-\}$ with $P_{\pm} = P_2 \pm P_1$ and $z_{\pm} = 1/2(z_2 \pm z_1)$. The integration measure is given by $DZ = dz_1 dz_2 \theta(1 - z_1) \theta(z_1 + 1) \theta(1 - z_2) \theta(z_2 + 1)$ and reflects the fact that the double distributions $f_\alpha^{(\tau)}(x\mathbb{F}, \mu^2)$ are entire analytic functions in $x\mathbb{F}$-space. We assumed summation convention with respect to $\alpha$. For deep inelastic scattering we have only one momentum $P$ and one distribution variable $z$.

Applying this parametrization, after the formal Fourier transformations (10) and (11), the replacements

$$
\langle P, S | \mathcal{O}_\Gamma(u) | P, S \rangle = K^\kappa_\tau(P, S) \int dz \delta^{(4)}(u - 2Pz) \hat{f}_\alpha^{(\tau)}(z, \mu^2),
$$

(136)

$$
\langle P_2, S_2 | \mathcal{O}_\Gamma(u) | P_1, S_1 \rangle = K^\kappa_\tau(P, S) \int DZ \delta^{(4)}(u - \mathbb{F}Z) \hat{f}_\alpha^{(\tau)}(Z, \mu^2),
$$

(137)

are deduced, which hold under the application of the twist projection $\mathcal{P}_\tau^{(\tau)T'}$. For a detailed discussion of (136) and (137), see Ref. [1].

A. Power corrections of forward matrix elements

In the case of deep inelastic scattering we have $P_1 = P_2$ and $S_1 = S_2$. For the operator $O_\mu(\kappa x, -\kappa x)$ there remains only the Dirac structure $K^\kappa_\mu = 2P_\mu$, see also Ref. [19]. Then, after inserting the replacement (136), for the two non-local scalar operators (113) and (114) we obtain the following forward matrix elements:

$$
\langle P | O^{tw(2+2j)}_\mu A (\zeta x, -\zeta x) | P \rangle = 2P_\rho \int dz \hat{F}_A^{(2+2j)}(z) \left( - (2Pz)^2 \right)^{j-1} \frac{4j}{\lambda} iz P^\rho (1 + x\partial) \mathcal{H}^1(2Pz|\zeta x) \left[ 4j iz P^\rho (1 + x\partial) \mathcal{H}^1(2Pz|\zeta x) \right],
$$

(138)

$$
\langle P | O^{tw(2+2j)}_\mu B (\zeta x, -\zeta x) | P \rangle = 4P_\rho \int dz \hat{F}_B^{(2+2j)}(z) \left( - (2Pz)^2 \right)^{j-1} iz P^\rho (1 + x\partial) \mathcal{H}^1(2Pz|\zeta x).
$$

(139)

The matrix elements of the two related vector operators (115) and (116) obtain a quite similar form:

$$
\langle P | O^{tw(2+2j)}_\mu A (\zeta x, -\zeta x) | P \rangle = \frac{2}{\zeta} P_\rho \int dz \hat{F}_A^{(2+2j)}(z) \left( - (2Pz)^2 \right)^{j-1} \frac{1}{\lambda} \frac{d\lambda}{P^\rho} (1 + x\partial) \mathcal{H}^1(2Pz|\zeta x) \left[ 4j iz P^\rho (1 + x\partial) \mathcal{H}^1(2Pz|\zeta x) \right],
$$

(140)

$$
\langle P | O^{tw(2+2j)}_\mu B (\zeta x, -\zeta x) | P \rangle = \frac{4}{\zeta} P_\rho \int dz \hat{F}_B^{(2+2j)}(z) \left( - (2Pz)^2 \right)^{j-1} iz P^\rho (1 + x\partial) \mathcal{H}^1(2Pz|\zeta x).
$$

(141)
The last matrix element, giving all contributions of odd twist being contained in the operator \( O_\mu (\kappa x, -\kappa x) \), reads

\[
\left\langle P \left| O_\mu^{t_w(3+2j)} C (\zeta x, -\zeta x) \right| P \right\rangle = 4 P_\rho \int \, dz \left\langle \hat{F}_C^{(3+2j)} (z) \left( - (2 P z)^2 \right)^j x^\sigma \left( \delta_\mu^\rho \partial_\sigma \right) (1 + x \partial) + X_{[\mu\sigma]} \partial^\rho \right\rangle \int_0^1 \, d\lambda \, \mathcal{H}^j (2 P z | \lambda \zeta x) .
\]

All these matrix elements are written down for unpolarized scattering, where only the momentum \( P_\mu \) can be used for the construction of kinematical factors.

The operator \( M_\mu^5 (\kappa x, -\kappa x) \) only contributes to polarized deep inelastic scattering. Here, we have to use the kinematical factor \( K_1^{\mu} (P, S) = 2 \left( S_\mu P_\nu - S_\nu P_\mu \right) / M \). The three related matrix elements read

\[
\left\langle PS \left| M_\mu^5 t_w(1+2j) B (\zeta x, -\zeta x) \right| PS \right\rangle = \frac{4}{M} \left( S_\mu P_\beta - S_\beta P_\mu \right) \int \, dz \, \hat{H}_B^{(1+2j)} (z) \left( - (2 P z)^2 \right)^j \partial_\mu \, i z P^{[\mu \varsigma} (2 + x \partial) (3 + x \partial) \mathcal{H}^j (2 P z | \zeta x) ,
\]

\[
\left\langle PS \left| M_\mu^5 t_w(1+2j) C (\zeta x, -\zeta x) \right| PS \right\rangle = \frac{4}{M} \left( S_\mu P_\beta - S_\beta P_\mu \right) \int \, dz \, \hat{H}_C^{(1+2j)} (z) \left( - (2 P z)^2 \right)^j (2 + x \partial) (3 + x \partial) \int_0^1 \, d\lambda \, \mathcal{H}^j (2 P z | \lambda \zeta x) ,
\]

where we have used \( S P = 0 \) and \( P^2 = M^2 \).

If we want to take polarized matrix elements of the operator \( O_\mu^5 (\kappa x, -\kappa x) \) we have to use the replacement

\[
\left\langle PS \left| O_\mu^5 (u) \right| PS \right\rangle = 2 S_\rho \int \, dz \, \delta^{(4)} (u - 2 P z) \hat{G}_a^\mu (z, \mu^2) (146)
\]

under the different twist projectors. The resulting matrix elements will be of the form \( 133 \) to \( 142 \) with \( P_\rho \) replaced by \( S_\rho \) and \( \hat{F} \) replaced by \( \hat{G} \).

Forward matrix elements on the light-cone of non-local operators of well-defined twist have been constructed in Ref. \( 19 \) for a large number of operators. If we perform the on-cone limit of our results \( 133 - 145 \) we again find the correct on-cone limits.

### B. Power corrections of non-forward matrix elements

To apply the parametrization \( 134 \) to the operators \( O_\mu (\kappa x, -\kappa x) \) and \( O_\mu^5 (\kappa x, -\kappa x) \) we have to fix the kinematical factors \( K_1^{\mu(5)} (P, S) \) which are given by the Dirac and Pauli structures,

\[
K_1^{\mu(5)} 1 (P, S) = \bar{u} (P_2, S_2) (\gamma^5) \gamma_\mu \, u (P_1, S_1) , \quad (147)
\]

\[
K_1^{\mu(5)} 2 (P, S) = \frac{1}{M} \bar{u} (P_2, S_2) (\gamma^5) \sigma_{\mu\nu} P_\nu^\nu \, u (P_1, S_1) , \quad (148)
\]

(see also Ref. \( 20 \)) and then insert the replacement \( 134 \) into the different matrix elements.
The non-forward matrix elements of the two non-local scalar operators \( \bar{M}_\rho \) and \( \bar{M}_{\rho a} \) obtain the form

\[
\left\langle P_2, S_2 \left| O^{\text{tw}(2+2j)}_\Lambda (\zeta x, -\zeta x) \right| P_1, S_1 \right\rangle = \frac{1}{2} K^a_\rho (P, S) \int DZ F_{Aa}^{(2+2j)} (Z) \left( - (PZ)^2 \right)^{j-1} \left[ 4j \, i\rho^\mu Z (1 + x\partial) \, \mathcal{H}^1 (PZ|\zeta x) - (PZ)^2 \left( 2\zeta x^\rho (2 + x\partial) - i\rho^\mu Z \zeta^2 x^2 \right) (3 + x\partial) \, \mathcal{H}^2 (PZ|\zeta x) \right],
\]

\[
\left\langle P_2, S_2 \left| O^{\text{tw}(2+2j)}_\mu (\zeta x, -\zeta x) \right| P_1, S_1 \right\rangle = K^a_\rho (P, S) \int DZ F_{Ba}^{(2+2j)} (Z) \left( - (PZ)^2 \right)^{j-1} i\rho^\mu Z (1 + x\partial) \, \mathcal{H}^1 (PZ|\zeta x) \right). \tag{150}
\]

The matrix elements of the two related vector operators \( \bar{M}_\rho \) and \( \bar{M}_{\rho a} \) again obtain the similar form

\[
\left\langle P_2, S_2 \left| O^{\text{tw}(3+2j)}_\Lambda (\zeta x, -\zeta x) \right| P_1, S_1 \right\rangle = \frac{1}{2} \zeta K^a_\rho (P, S) \int DZ F_{Aa}^{(3+2j)} (Z) \left( - (PZ)^2 \right)^{j-1} \partial_\mu \int_0^1 \frac{d\lambda}{\lambda} \left[ 4j \, i\rho^\mu Z (1 + x\partial) \, \mathcal{H}^1 (PZ|\zeta x) - (PZ)^2 \left( 2\zeta x^\rho (2 + x\partial) - i\rho^\mu Z \zeta^2 x^2 \right) (3 + x\partial) \, \mathcal{H}^2 (PZ|\zeta x) \right],
\]

\[
\left\langle P_2, S_2 \left| O^{\text{tw}(3+2j)}_\mu (\zeta x, -\zeta x) \right| P_1, S_1 \right\rangle = \frac{1}{\zeta} K^a_\rho (P, S) \int DZ F_{Ba}^{(3+2j)} (Z) \left( - (PZ)^2 \right)^{j-1} i\rho^\mu Z \partial_\mu (1 + x\partial) \int_0^1 \frac{d\lambda}{\lambda} \mathcal{H}^1 (PZ|\zeta x) \right). \tag{152}
\]

The last matrix element of the type C vector operator \( \bar{M}_\rho \) gives all contribution of odd twist for unpolarized deeply virtual Compton scattering

\[
\left\langle P_2, S_2 \left| O^{\text{tw}(3+2j)}_\mu (\zeta x, -\zeta x) \right| P_1, S_1 \right\rangle = 2 K^a_\rho (P, S) \int DZ F_{C\rho}^{(3+2j)} (Z) \left( - (PZ)^2 \right)^{j-1} x^\sigma \left( \delta^\mu_\sigma \partial_\sigma (1 + x\partial) + X_{\mu \sigma} \partial^\sigma \right) \int_0^1 \frac{d\lambda}{\lambda} \mathcal{H}^1 (PZ|\zeta x) \right). \tag{153}
\]

A parametrization for polarized scattering is found by the replacement of \( K^a_\rho (P, S) \) by \( K^5_\rho a (P, S) \). For a parametrization of the operator \( M_\mu (\kappa x, -\kappa x) \) for polarized and unpolarized deeply virtual Compton scattering we use the kinematical factors

\[
K^{(5), 1}_{\mu \nu} (P, S) = \frac{1}{M} \bar{u} (P_2, S_2) (\gamma^5) \gamma_\mu P^+_{\nu} u (P_1, S_1), \tag{154}
\]

\[
K^{(5), 2}_{\mu \nu} (P, S) = \frac{1}{M} \bar{u} (P_2, S_2) (\gamma^5) \gamma_\mu P^-_{\nu} u (P_1, S_1), \tag{155}
\]
and find the matrix elements

\[ \langle P_2, S_2 | M^{(5) \text{tw}(1+2)}_\mu (\zeta, -\zeta) | P_1, S_1 \rangle \]

\[ = \mathcal{K}_{[\rho\delta]}^{(5) \mu} (P, S) \int DZ H_{Ba}^{(1+2 \mu)} (Z) \left( - (PZ)^2 \right)^{j-1} i\not{\partial} Z \zeta x^\delta (2 + x\partial) (3 + x\partial) \mathcal{H}^2 (PZ|\zeta x) , \]

\[ \langle P_2, S_2 | M^{(5) \text{tw}(1+2 \mu)}_\mu (\zeta, -\zeta) | P_1, S_1 \rangle \]

\[ = \mathcal{K}_{[\rho\delta]}^{(5) \mu} (P, S) \int DZ H_{Ba}^{(1+2 \mu)} (Z) \left( - (PZ)^2 \right)^{j-1} \partial_\mu i\not{\partial} Z x^\delta (2 + x\partial) (3 + x\partial) \int_0^1 d\lambda \mathcal{H}^2 (PZ|\lambda \zeta x) , \]

\[ \langle P_2, S_2 | M^{(5) \text{tw}(2+2 \mu)}_\mu (\zeta, -\zeta) | P_1, S_1 \rangle \]

\[ = \mathcal{K}_{[\rho\delta]}^{(5) \mu} (P, S) \int DZ H_{Ca}^{(2+2 \mu)} (Z) \left( - (PZ)^2 \right)^{j-1} x^\sigma \left( \delta^{\mu \sigma} \partial_\sigma (1 + x\partial) + X_{[\mu\sigma]} \partial^\rho \right)
\times \int_0^1 d\lambda \left[ 4 j i\not{\partial} Z \mathcal{H}^1 (PZ|\lambda \zeta x) - (PZ)^2 \left( 2 \lambda \zeta x^\delta (3 + x\partial) - i\not{\partial} Z (\lambda \zeta^2 x^2) \mathcal{H}^2 (PZ|\lambda \zeta x) \right) \right]. \]

Notice that the kinematical factor \( \mathcal{K}_{[\mu\rho]}^{5 \mu} (P, S) \) vanishes for forward scattering since \( P_- \) vanishes in this case. For unpolarized deep inelastic scattering \( \mathcal{K}_{[\mu\rho]}^{3 \mu} (P) \) also gives zero since \( \bar{u} (P, S) \gamma_{[\rho\sigma]} P_+ \gamma_{\lambda} u (P, S) = 4 P_{\rho\sigma} P_\mu = 0 \). For polarized deep inelastic scattering we find \( \mathcal{K}_{[\mu\rho]}^{5 \mu} (P, S) = 2 (S_\mu P_\nu - S_\nu P_\mu) / M. \)

V. CONCLUSION

In this paper we introduced a procedure which allows to determine the decomposition of arbitrary non-local vector operators into an infinite sum of non-local vector operators of arbitrary twist. The procedure generalizes the much simpler decomposition of non-local operators introduced in Ref. [1]. As examples we considered the bilocal operators \( O_5^{(5)} (\kappa x, -\kappa x) \) and \( M_\mu (\kappa x, -\kappa x) = x^\sigma M_{[\mu\sigma]} (\kappa x, -\kappa x) \) together with their forward and non-forward matrix elements. The latter operator has been chosen because it shows some peculiarities which are related to its ‘internal’ antisymmetry and the ‘external’ contraction with \( x^\sigma \). However, the procedure is much more general applying to any kind of non-local vector fields. Other examples could be purely gluonic bilocal operators or trilocal operators like \( x^\sigma \psi (\kappa_1 x) F_{[\mu\rho]} (\kappa_2 x) \psi (\kappa_3 x) \) where \( F_{[\mu\rho]} \) is the gluon field strength. (In the case of tri- and multi-local operators one has to apply a prescription of local operators which was introduced in [7], Chapter 4.) Furthermore, let us remark that the trace decomposition as a prerequisite of that approach - which, in fact, is more complicated than the application of the symmetry projections - is also separately of interest.

The procedure introduced here can be extended straightforwardly to antisymmetric and symmetric tensor operators of rank 2. Thereby, a hard part is the determination of the construction of traceless local operators \( T_{\mu\nu}(x) = T_{\mu\nu;\alpha_1\cdots\alpha_n} x^{\alpha_1} \cdots x^{\alpha_n} \) as well as the implementation of the various symmetry types I to IV through the differential operators corresponding to \( J_{[\alpha_i]}, i = 1, \ldots, 4. \)

This work is motivated by the aim to study the target mass contributions to the various QCD-distribution amplitudes which occur in the phenomenological considerations of hard hadronic scattering processes, like deep inelastic scattering and deep virtual Compton scattering, as well as in hadronic form factors. In the case of hadronic wave functions, e.g. for the pion or the \( \rho \)-meson, the obtained twist decompositions, after corresponding (anti-) symmetrization w.r.t. \( \kappa \), could be used for their power (resp. mass) corrections. However, to give a comprehensive description the complete twist decomposition of \( M_{[\mu\rho]} (\kappa x, -\kappa x) \) had to be known. On the other hand, in order to determine the target mass corrections of deep virtual Compton scattering in terms of \( M^2 / Q^2 \), as has been done in Ref. [21] up to twist-3, the Fourier transform of the (anti-) symmetrized operators of definite twist with the corresponding coefficient functions according to Eqs. (11) and (21) has to be performed. In the case of the scalar operators this has been done in [22], and in the case of vector operators this is under consideration.
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