Basic Simulations of Kinodynamic Control Using Local Environmental Information

Kimiko Motonaka  
Department of Electrical and Electronic Engineering  
Kansai University  
Osaka, Japan  
motonaka@kansai-u.ac.jp

Keigo Watanabe and Shoichi Maeyama  
Graduate School of Natural Science and Technology  
Okayama University  
Okayama, Japan  
{watanabe, maeyama}@sys.okayama-u.ac.jp

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Abstract—The kinodynamic control aims to design a control input so that we consider kinematic constraints, which is generated from an environment, and dynamic constraints of the airframe, simultaneously. In the previous research, we converted the environmental information to the harmonic potential field (HPF), and added its gradient information to the conventional dynamical control input. However, it was assumed in the previous research that the environmental information is known absolutely. In this research, we assume the actual situation that the quadrotor flies autonomously in an indoor environment. We propose the kinodynamic control that considers the measurable region of the sensor mounted on the quadrotor to obtain the environmental information. In this paper, we make a basic simulation experiment and confirm that the proposed control input works well.

Index Terms— Autonomous locomotion, kinodynamic control, quadrotor.

I. INTRODUCTION

Recently, there are many researches on autonomous flight for a quadrotor, which is the vertical take-off and landing (VTOL) aircraft with four rotors. It has received an attention in recent years, as surveillance or rescue robots, because of its highly maneuverability and a hovering ability, and the realization of the autonomous locomotion of a quadrotor has a possibility of leading to an improvement in these services. For the autonomous locomotion of a quadrotor, it needs to move and avoid obstacles while keeping its attitude. In the previous research, we designed a controller for guiding the quadrotor based on “kinodynamic motion planning” using the gradient of a harmonic potential field (HPF) [1]. The kinodynamic control aims to design a control input so that we consider kinematic constraints, which are generated from the obstacles in an environment, and dynamic constraints, which need to control the state of the airframe, simultaneously [2]-[6].

The control input designed in the previous research was able to guide the quadrotor to the arbitrary target point, and it was confirmed in some simulations and actual experiments. In these experiments, it was assumed that the environment was already known and the HPF of the environment was prepared in advance. However, autonomous flight only with information that can be gotten from the mounted sensor needs to work in various situations like a disaster site or living environment, and there exist several related researches [7]-[9]. Therefore, in this research, we consider the sensing region of the mounted sensor, and make some simulations of guiding the quadrotor using only the environmental information around the machine.

II. THE DYNAMICAL MODEL AND THE CONTROL INPUT

The dynamical model of a quadrotor, which has m [kg] was able to show as follows using the inputs for translational motion $U_1$, roll ($\phi$) angle motion $U_2$, pitch ($\theta$) angle motion $U_3$, and yaw ($\psi$) angle motion $U_4$:

$$\begin{align*}
\dot{x} &= -\left(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi\right) \frac{1}{m} U_1 \\
\dot{y} &= -\left(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi\right) \frac{1}{m} U_1 \\
\dot{z} &= -g + \left(\cos\phi\cos\theta\right) \frac{1}{m} U_1 \\
\dot{\phi} &= \dot{\phi}\left(I_y - I_z\right) - \frac{J_x}{I_x} \dot{\theta} \Omega + \frac{l}{I_x} U_2 \\
\dot{\theta} &= \dot{\theta}\left(I_z - I_x\right) - \frac{J_y}{I_y} \dot{\phi} \Omega + \frac{l}{I_y} U_3 \\
\dot{\psi} &= \dot{\psi}\left(I_x - I_y\right) + \frac{1}{I_z} U_4
\end{align*}$$

(1)
Note that, $l$ [m] is the length from the center of the airframe to the center of the rotor, $g$ [m/s$^2$] is the gravity acceleration, $I_x$, $I_y$, and $I_z$ [kg/m$^2$] are the moment of inertia around each axis respectively, and $J_r$ [kg/m$^2$] is the moment of inertia for a rotor.

In the proposed method, kinodynamic motion planning is achieved by combining nonholonomic control input and the

Gradient information which is calculated from the HPF. The system input $\mathbf{u}=[u_1,u_2,u_3,u_4]^T$, which is constructed by nonholonomic control input $\mathbf{u}$ and control input $\Delta \mathbf{u}$ based on the gradient of the HPF, is as follows:

$$U = u_i + \Delta u$$

(2)

Here, $U_1$ is a control input for acting on each translational motion, and $U_2$, $U_3$ and $U_4$ are control inputs for acting on roll angle $\phi$, pitch angle $\theta$ and yaw angle $\psi$, respectively. We describe the detail of the control input based on nonholonomic control $u_i$ and the control input $\Delta u$ based on the gradient of an HPF below.

The control input $u_i = [u_1,u_2,u_3,u_4]^T$ is added for Z-direction and three attitude angle and given as follows\[10\]:

$$u_{c1} = \frac{mg}{\cos \phi \theta} - \frac{m U_1}{\cos \phi \theta}$$

$$u_{c2} = -\frac{I_z}{I} (\phi - \phi_T) - k_1 \phi$$

$$u_{c3} = -\frac{I_z}{I} (\theta - \theta_T) - k_2 \theta$$

$$u_{c4} = -\frac{I_z}{I} (\psi - \psi_T) - k_3 \psi$$

(3)

where $U_1$ is

$$\dot{U}_1 = k_4 (z - z_T) + k_5 \dot{z}$$

(4)

and the $z_T$ is the target position in Z direction.

Next, added input $\Delta u$ for the translational motion is described. In this paper, it is assumed that the quadrotor moves on X-Y plane while hovering in constant altitude. For the control in the X-Y plane, the X- and Y-directional gradients of an HPF are added in the control input for $\theta$ and $\phi$ angles.

When the position coordinate of the quadrotor is $x=[x,y,z]^T$ and the gradient of the HPF is $\nabla V(x) = [f_x, f_y, f_z]^T$, then, using the gradient of the HPF, an added control input $\Delta \mathbf{u}$ is designed by

$$\Delta u = -K_v \nabla V(x)$$

(5)

Here, $K_v \in \mathbb{R}^{4 \times 3}$ is the gradient selection gain of the HPF.

The selection method of the selection gain depends on the movement characteristics and the form of the dynamic control law of the controlled object. The quadrotor can move its position by tilting the attitude. Example, in the X-Y plane, the quadrotor can move toward the X-axis by tilting the body to $-\theta$ angle, and move toward the Y-axis by tilting the body to $\phi$ angle. Therefore, it is assumed that the quadrotor is hovering in constant altitude using a nonholonomic controller. At that time, the control toward the X-Y direction can be achieved by adding the X- and Y-directional gradients of the HPF to the pitch ($\theta$ directional) controller $u_{c3}$ and the roll ($\phi$ directional) controller $u_{c2}$, which are based on nonholonomic control.

According to the above discussions, if the quadrotor only moves on the X-Y plane, then the selection gains $B_i$ and $K_i$ can be decided as below:

$$K_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_i & 0 \\ k_i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(6)

where, $k_i > 0$.

Using the extended gradient vector $\nabla V_e(x) \in \mathbb{R}^{4 \times 3}$, the control input based on the gradient of the HPF can be written by:

$$\Delta u = -k_i \nabla V_e(x)$$

(7)

Here, $\dot{x}_e = [0 \dot{y} \dot{z} 0]^T$ and $\nabla V_e(x) = [0 f_y f_z 0]^T$.

III. GENERATION OF HPF

In the previous research, we assumed that the environment is known, and we used the HPF of the environment prepared in advance. On the other hand, in this paper, we confirm under the unknown environment whether the controller can guide the quadrotor using only the information from the mounted sensor.

At first, the arbitrary final target point is given to the quadrotor. When the controller uses only the potential generated from the measurable region, the controller has to set a temporary target position because of the characteristic of the harmonic potential field method. In this research, if we consider the case where the mounted sensor can measure the
circular region around the airframe, then it can set the temporary target position by the following method.

First, the current position of the airframe \((x_0, y_0)\) and the final target position \((x_T, y_T)\) are connected by a straight line. Then, we set the temporary target position to a position shorter than an intersection, which is generated by crossing such a straight line and a circle with the radius \(R\), centered at the airframe. The parameter \(a\) can be set arbitrary, and we set it as 1 [m] in this research.

\[
L = \sqrt{(x_T - x_0)^2 + (y_T - y_0)^2} \\
x_n = x_0 + \frac{(R-a)(x_T - x_0)}{L} \\
y_n = y_0 + \frac{(R-a)(y_T - y_0)}{L}
\]  

(8)

Moving with updating the potential field in real time is not realistic considering the spec of the processor mounted on the quadrotor. Therefore, the controller updates the potential data when the quadrotor reaches the temporary target position. There is also the possibility that the quadrotor deviates from the field if the quadrotor cannot avoid the obstacles in the limited field. In that case, the quadrotor stops the movement in the X-Y directions, updates the sensor data, sets the temporary target position again, and restarts the movement. Moreover, if the generated temporary target position is placed where it cannot be reached, the temporary target position is reset in the reachable position considering the obstacle information.

![Fig. 2: Generation of the temporary target point](image)

**TABLE I. PHYSICAL PARAMETERS FOR THE QUADROTOR**

| Parameter | Description | Value | Unit |
|-----------|-------------|-------|------|
| \(g\)     | Gravity     | 0.80665 | m/s² |
| \(m\)     | Mass        | 0.4   | kg   |
| \(l\)     | Distance    | 0.248 | m    |
| \(I_x\)   | Roll inertia| 0.01467 | kg·m² |
| \(I_y\)   | Pitch inertia| 0.01467 | kg·m² |
| \(I_z\)   | Yaw inertia | 0.02331 | kg·m² |
| \(J_r\)   | Rotor inertia| 175.69 × 10⁻⁶ | kg·m² |
| \(b\)     | Thrust factor| 43.4 × 10⁻⁶ | |
| \(d\)     | Drag factor | 2.188 × 10⁻⁶ | |

The schematic diagram of this method is shown in Fig. 2. Then, each value can be calculated as follows:

\[
L = \sqrt{(x_T - x_0)^2 + (y_T - y_0)^2} \\
x_n = x_0 + \frac{(R-a)(x_T - x_0)}{L} \\
y_n = y_0 + \frac{(R-a)(y_T - y_0)}{L}
\]

Fig. 3 The assumed environment

![Fig. 4 An example of the harmonic potential field](image)

**IV. SIMULATION**

In this section, it is confirmed on the simulation using MATLAB that the controller can guide the quadrotor toward the target position by using the method described above.

**A. Conditions**

The assumed environment in this simulation is shown in Fig. 3. In the environment shown in Fig. 3, it is demonstrated that the quadrotor moves from the initial position \((x_0, y_0) = (16, 16) \text{ [m]}\) to the final target position \((x_T, y_T) = (4, 4)\), while keeping its altitude \(z = 1\).

As shown in the figure, a rectangle obstacle is placed in the environment. The measurable radius \(R\) of the mounted sensor is set to 5 \text{ [m]}, which is assumed to be the measureable range of a laser range finder (LRF). Each parameter like the mass or inertia moment of the airframe is set as shown in Table 1. These parameters were measured from the actual airframe. The gain \(k_v\) for the gradient of the HPF is set to 0.0001 by the empirical rule.
B. Results

Figure 4 shows an example of the potential field, which was generated at (13.4, 8.7) [m]. In addition, Fig. 5 shows the trajectory of the quadrotor on X-Y plane in the assumed environment. The black rectangle shows the obstacle, the pink line shows the trajectory of the quadrotor, and the circles show the region where the potential field was generated. In the assumed environment, the potential field was updated 6 times while the quadrotor moved from the initial position to the target position.

C. Discussions

From Fig 5, it was confirmed that the controller can guide the quadrotor to the arbitrary target position in an assumed environment using the proposed method. Then, it was also confirmed that the quadrotor avoided the obstacle while moving. In around the obstacle, the quadrotor sometimes deviated from the generated potential field because there was not enough space to avoid the obstacle. However, after deviation, the controller generated the potential field again, and finally the quadrotor was able to move to the target position.

V. Conclusion

We have aimed to the autonomous flight of the quadrotor in an indoor environment, and verified the proposed method by simulation. Especially, we proposed the method to generate the potential field using only information around the quadrotor and set the temporary target position when the arbitrary target position was given in an unknown environment. It was confirmed in simulation using MATLAB that the controller can guide the controlled object to the target position while avoiding the obstacle by using the proposed method. In future work, there are thought the operation verification in the various situations, considering the moving obstacles, the actual experiment, etc.

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