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Compensation of Atmospheric Differential Color Refraction Bias in Ground-Based Optical Astrometric Observations of Satellites with Concurrent Spectroscopic Measurements

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Ground-based telescopes provide the majority of observations used to track geostationary satellites and orbital debris. Many sensors specifically designed for Space Situational Awareness (SSA) operate over almost the entire 300–1100 nm sensitivity band of silicon. Earth’s atmosphere is optically dispersive and observations made from the ground with wide passbands are subject to a systematic angle bias from differential color refraction (DCR). DCR bias affects both fiducial stars used to compute astrometric solutions for SSA images and the satellites being observed, and is on the order of 100 mas below 60 degrees elevation. With the release of the Gaia DR2 catalog, stellar DCR may be predicted to an accuracy of better than 20 mas above 20 degrees elevation but compensation of SSA target DCR requires per-target spectroscopic measurements. We have constructed a slitless spectrograph with COTS equipment and have used it to measure the silicon passband spectra of GPS and GLONASS satellites under a diversity of atmospheric and illumination conditions. The instrument multiplexes between two transmission gratings and suitable colored glass blocking filters to collect a full spectrum every 105 seconds on targets brighter than 12.5 magnitude. These spectra were used to infer a DCR bias for the target for that time period. Simultaneously, a separate telescope collected high frame-rate bare silicon images of the same targets. The images were processed into observations with color-aware astrometric reductions using the Gaia DR2 star catalog. The bare-silicon astrometry was compared with the final orbit solutions published by the International GNSS Service and the inferred DCR bias was compared against those observation residuals. 312 unique spectroscopic measurements on 14 distinct satellites were collected between 20 and 60 degrees elevation over seven nights from September 2018 to April 2019. Using these measurements, we demonstrate a 60% reduction in bias and 30% reduction in noise in the vertical component of astrometric residuals, relative to color-agnostic processing.

I. Introduction

In order to maximize the probability of detection of small debris fragments, sensors specifically designed for Space Situational Awareness (SSA) operate over almost the entire 300–1100 nm sensitivity band of silicon [1,2]. Earth’s atmosphere is a dispersive medium and because SSA targets are not uniform reflectors, observations made from the ground are subject to a systematic angle bias from differential color refraction (DCR) [3,4]. In observations of geosynchronous satellites, uncorrected DCR bias in ground-based optical observations precludes the accurate short-arc measurement of solar radiation pressure area-to-mass ratio. This, in turn, introduces a growing Cartesian error in the short-arc orbit solution derived from such data [5].

Prior to the advent of high-accuracy star catalogs such as UCAC4 [6], URAT1 [7], UCAC5 [8], and most recently, Gaia DR2 [9], the error imposed by DCR was on par with the fixed-pattern error in the fiducial catalogs used to reduce

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the raw SSA imagery. With the release of the sub-mas Gaia DR2 catalog, DCR has become a dominant systematic error source in ground-based optical SSA. The reduction in other sources of error motivates the search for a mitigation technique to enable ground-based optical SSA to realize the benefit of the recent improvements in star catalog accuracy.

In this paper, we report on our efforts to mitigate DCR bias in bare-CCD astrometry with concurrent low-resolution spectroscopy of active GPS and GLONASS satellites. Although there have been several efforts at spectroscopic characterization of on-orbit satellites [10–13], these have been focused on target classification, rather than on augmentation of astrometric accuracy, and we believe our approach to be novel.

A. Phenomenology

DCR is shown schematically in Figure 1. Short wavelengths refract more than long wavelengths and appear from the ground to be incident from a higher elevation. Target signals composed mostly of short wavelengths appear higher up than target signals composed mostly of longer wavelengths. The bias affects both fiducial stars and astrometry targets. The effect on the astrometric solution for a target is inverted for the DCR of fiducial stars: targets registered against redder stars (most stars are redder than Sun-like) appear to originate from a higher elevation and targets registered against bluer stars appears to originate from a lower elevation because the fiducial sources appear to be at a higher elevation and targets registered against redder stars (most stars are redder than Sun-like) appear to originate from a higher elevation because the stars defining the plate coordinate system appear systematically lower.

The calculation of DCR bias is as follows. Let \( \phi(\lambda, z) \) be the detectable photoelectron flux density at the imager and let \( \Delta z(\lambda, z) \) be the astronomical refraction at wavelength \( \lambda \) and zenith distance \( z \). Discretizing into wavelength bins of width \( \Delta \lambda \), the approximate vertical shift in the apparent incident direction of the target centroid \( \delta z_0 \) is given by:

\[
\delta z_0 \approx \frac{\sum_i \phi(\lambda_i, z) \Delta \lambda_i \Delta z(\lambda_i, z)}{\sum_i \phi(\lambda_i, z) \Delta \lambda_i}
\]  
(1)

The flux density \( \phi(\lambda, z) \) is the detectable photoelectron flux on the ground, which is a product of the solar illumination spectrum, the target spectral reflectance, the atmospheric transmittance at zenith distance \( z \), and the instrument overall quantum efficiency and optical transmittance. It is convenient to recenter the zero-point of \( \Delta z(\lambda, z) \) to the total DCR of a hypothetical fiducial star with a solar spectrum.

Satellite reflectance spectra may be approximated as combinations of common spacecraft materials such as solar panel and Kapton. To first order, satellite reflectance profiles may be parameterized by solar phase angle. As shown in Figure 2 at low phase angles, the front of the solar panels are seen face-on from the ground site and the satellites’ signature are likely to be dominated by solar panel reflectance, while at higher phase angles they may be dominated by spacecraft bus materials such as Kapton. When observing from the ground, low phase angles necessarily occur near eclipse. Not all satellite orbits go into eclipse, but if they do, shortly before entering eclipse, the satellite passes through a region of illumination by light that has been refracted and filtered by the Earth’s atmosphere. The short wavelength content of this light has been scattered away, leaving very red illumination. This region extends about one degree out from the limb of the Earth. At GPS/GLONASS altitude, this region spans about 450 km and a satellite at that altitude moving directly into the center of the umbra at 4 km/sec will take just under two minutes to traverse this region, while a geostationary satellite grazing eclipse may be subject to these illumination conditions for longer.

Using nominal laboratory measurements of common satellite materials [14][15], the DCR bias of satellites may be estimated over a variety of illumination conditions. Figure 3 plots the range of these possible DCR values as a function
of elevation angle and meteorological visibility. While there is slight sensitivity of DCR bias to temperature, barometric
pressure, and humidity, meteorological visibility is the most influential parameter other than the composition of the
target spectrum itself.

Stellar DCR may be estimated on a per-star basis using the multiband photometry information available in star
catalogs. The DCR bias of satellites depends on the spectral reflectance of the materials dominating the cross-section
presented by the satellite to the observing sensor, and this composition may change as the satellite undergoes normal
attitude control maneuvers and changes in illumination. Therefore, compensation for satellite DCR requires measurement
of the detectable photon spectrum at the observing sensor on a per-target basis.

B. DCR Compensation with Concurrent Spectroscopy

A 40 nm spectral resolution is necessary to enable DCR compensation to an accuracy of 20 mas, commensurate
with the accuracy of per-star DCR corrections [5, 10]. The accuracy of the target DCR estimate is a function of the
accuracy of the spectroscopic measurement. If the measurement is made in bins of width $\Delta \lambda_i$, and assuming that the
photoelectron counts in each measurement channel $\phi(\lambda_i, z) \Delta \lambda_i$ are large enough for Gaussian statistics to hold, the
uncertainty in DCR bias is given by:

$$\sigma_z \approx \sqrt{\frac{\sum_i \text{var}(\phi(\lambda_i, z) \Delta \lambda_i) (\Delta z(\lambda_i, z))^2 - \delta z_0^2)}{\sum_i \phi(\lambda_i, z) \Delta \lambda_i}}$$

(2)
Equation 2 is plotted in Figure 4 for satellites of all reasonable brightnesses and ratios of solar panel to Kapton ratios for a telescope with a one square meter aperture and a one second exposure. The calculation is made for both a slit spectrograph, where the contribution to the \( \text{var}(\phi(\lambda_i, z) \Delta \lambda_i) \) term comes solely from the shot noise in the target signal in each spectrographic bin, and for a slitless spectrograph where the noise is the combination target signal shot noise and the sky background shot noise.

The curves in Figure 4 may be used to compute the required time-aperture product to measure the spectrum of a target of known brightness to a desired level of DCR estimate accuracy. DCR reconstruction performance is plotted against both target brightness and target SNR for a variety of target material ratios and plausible sky background levels and point-spread function widths. The parameterization against SNR shows a clear distinction between a high-SNR regime where accuracy is a function only of directly measurable target SNR, and a low-SNR regime where the target signal’s spectral composition becomes important. In the case where the shot noise, and not camera read noise, is dominant, the \( \text{var}(\phi(\lambda_i, z) \Delta \lambda_i) \) term is proportional to the photoelectron count \( \phi(\lambda_i, z) \Delta \lambda_i \) and \( \sigma_z \) is inversely proportional to the square root of the total signal, and thus inversely proportional to the square root of the time-aperture product.

While there is a marked degradation in performance of the slitless design over the slit spectrograph in the low-SNR regime for faint targets where sky background noise becomes non-negligible, for desired accuracies around 100 mas, there is little difference in performance between slit and slitless configurations in the high-SNR regime. As such, a slitless design was chosen for a demonstration of the efficacy of DCR compensation with spectroscopic measurements. GPS and GLONASS targets were assumed to be no fainter than about 12.5 magnitude, and readily observable with a time-aperture product of under a few meter-squared-seconds in the worst case at elevations above 20 degrees.
II. Spectrograph Instrument

A. Design and Operation

A slitless spectrograph design was chosen for simplicity of implementation. The instrument consists of Richardson 606R and 906R transmission gratings and Schott colored glass BG40 and OG570 blocking filters placed in a Finger Lakes Instruments CL-1-10 dual filter wheel mated to a 16-inch Meade LX200 telescope. Figure 5 shows a cross-section of the spectrograph. An Andor iXon3 888 camera is used as the detector. The gratings are placed approximately 11 cm in front of the focal plane, giving a wavelength scale of approximately 4.2 nm / pixel with the 906R grating and 4.9 nm / pixel with the 606R grating.

A target’s full silicon passband spectrum is measured by multiplexing between the two gratings and their respective blocking filters. For the short end of the spectrum, the BG40 blocking filter is used along with the 606R grating. To measure the long end of the spectrum, the OG570 blocking filter is used with both the 606R and 906R grating. Sample grating images of a bright star are shown in Figure 6.

All gratings have some nonzero response from higher spectral orders overlapping with the \( m = 1 \) order. For measurements of short wavelengths, this is not an issue, but for measurements of wavelengths toward the long end of the silicon passband, contamination of the \( m = 1 \) order by energy from the \( m = 2 \) order of shorter wavelengths in the silicon passband is possible. The red-pass blocking filter was selected to minimize this contamination.

For measurements of the short end of the silicon passband, contamination by higher spectral orders is not an issue, but a blocking filter affords two benefits: clutter reduction and noise suppression. In slitless spectroscopy, the spectra of all sources in the field are present in grating images. Placing a short-pass filter in the optical path reduces the number of pixels which contain energy from sources other than the desired target. The second benefit comes from the fact that in slitless spectroscopy, the spectra are superimposed onto the full in-band sky background. Limiting the passband also limits the passband of the sky background, thereby reducing the contribution of sky background shot noise into the measurement of each spectroscopic bin. Figure 7 shows that the SNR boost is present in both combinations of gratings and blocking filters, but is most pronounced for the blue measurement.
Operationally, the sequence of gratings and filter changes is as follows:

1) 30 second exposure without filters or gratings. The purpose of this measurement is to collect bare CCD astrometry to identify the pixel location of the desired target in the frame in order to locate the origin of the spectrum in grating images when the $m = 0$ PSF is not detectable in grating images.

2) For observations of bright stars only, a measurement with the 606R grating and no filter is made for 30 seconds in order to calibrate the shapes of the blocking filter cutoffs in the middle of the silicon passband.

3) 30 second exposure with 606R/BG40 combination to measure the blue end of the target spectrum. Allowing for time to change filter positions, the interior 24 seconds of this measurement are useful.

4) 15 second exposure with 606R/OG570 combination. On bright stars, this measurement is used for grating tilt and plate scale calibration for the 606 grating, and for all targets it is used as a redundant measurement of the red portion of the spectrum. Allowing for time to change filter positions, the interior 9 seconds of this measurement are useful.

5) 30 second exposure with 906R/OG570 combination to measure the red end of the target spectrum. When observing bright stars, this measurement is used to calibrate the plate scale for the 906R grating, and used in concert with the 606R/OG570 measurement to calibrate the tilt of both gratings. Allowing for time to change filter positions, the interior 24 seconds of this measurement are useful.

Fig. 7  SNR Along Spectrum for 12.5 mag 80% Solar Panel Target
When observing bright stars, 120 msec exposures are used, and when observing fainter satellite targets, 500 msec exposures are used. Grating images are convolved with a kernel to capture the difference between the total energy in the spectrum pixels and the local mean background level measured between 20 and 40 pixels above and below the spectrum. Once spectra are extracted from individual grating images, a median in time is taken when observing moving satellite targets to mitigate the effect of passing star clutter contaminating the measurement. An arithmetic mean of median spectra over several measurement cycles may be combined to boost spectrum SNR, but in practice this was only done for a small number of measurements where the additional averaging made the difference between a marginal SNR and an SNR sufficient to achieve desired DCR reconstruction accuracy.

B. On-Sky Calibration

Spectrographic instruments are typically designed with kinematic stability in mind, and the use of cantilevered bracketry and a rotating carousel intended for use with filters rather than gratings presents challenges to the operation of the instrument. The relatively coarse 40 nm resolution requirement of this instrument permits some slop in the mechanical design. Nevertheless, it is still necessary to perform on-sky calibration to establish the kinematics of the instrument. During satellite measurements, calibration measurements on bright stars were performed approximately once or twice per hour at elevation angles comparable to those of the satellite tracks.

1. Wavelength Scale

As the optical tube undergoes changes in elevation angle, the telescope primary mirror shifts, necessitating a correction of the focal plane position to maintain proper focus of the instrument. This change in the distance between the gratings and the detector modulates the wavelength scale on the detector in grating images.

The wavelength scale is recovered using grating images of bright stars near tracks of the target satellites. The water vapor absorption band at 940 nm and the molecular oxygen absorption band at 765 nm are used to calibrate the plate scale for both the 606R and 906R gratings. Figure 8 shows sample calibration data on both a wet and a dry night. The water vapor band at 940 nm can, in practice, be washed out on dry nights. The molecular oxygen band at 765 nm is present at all times with constant strength and can be used as a fallback.

2. Grating Tilt

An optical element inserted into a converging beam of an imaging system gives an output at the focal plane that is a spatially scaled Fourier Transform of the optical element’s transmission function. For a purely transmissive element, the transmission function is an optical path difference, that is, a pure phase modulation. The spatial frequency of that phase modulation, and thus the spectral efficiency of the transmission grating, is dependent on its tilt relative to the optical axis.

In reference to Fig. 9(a), the optical path difference for a grating tilted by an angle $\theta$, with respect to the optical axis.
is given by

\[ T(x) = e^{j \tan(\theta_g) \left[ n_g(\lambda)(\tan(\theta_g + \theta_t) - \tan \theta_t) - n_a(\lambda)\tan(\theta_g + \theta_t) \right]} (x \mod (\Delta g \cos \theta_t))^2 \]  

where \( \Delta g \) is the nominal groove spacing, \( \theta_g \) is the groove angle, \( n_g(\lambda) \) is the grating material refractive index and \( n_a(\lambda) \) is the refractive index of air at wavelength \( \lambda \).

The Fourier transform of a periodic function is the Fourier transform of an individual periodic element multiplied by an array factor. The array factor is a periodic comb function, whose maxima correspond to the output angles of the individual spectral orders \( m \) for a given wavelength. The Fourier transform of the individual array element is a slowly-varying amplitude envelope that modulates the strength of the respective spectral orders at each wavelength [18]. The magnitude-squared of this envelope (evaluated at the array factor peak for a given wavelength corresponding to a given spectral order), is the grating efficiency at that wavelength in that order. For the tilted grating, this quantity is given by:

\[ I(\lambda) \propto \left[ \sin \left( \pi \frac{\Delta \cos \theta_t}{\lambda} \right) \left( \tan \left( \sin^{-1} \left( \frac{\lambda}{m\Delta} \right) - \frac{n_g - n_a}{\tan(\theta_g + \theta_t) - \tan \theta_t} \right) \right)^2 \]  

where the dependence on the grating tilt \( \theta_t \) is evident. The influence of a tilt perturbation to the ratio of grating responses can be significant. Figure 9(b) shows that a tilt of just 0.02 degrees, or about 17 microns over a 50 mm grating, can change the response ratio by several percent relative to an orthogonal assumption.

In a laboratory environment, a calibrated input spectrum may be used to compute the best-fit value of the grating tilt. However, in the absence of a calibrated source and under the assumption that grating tilt may vary over time, the ratio of responses in observations of a bright star made with both gratings is taken as the best-estimate of the grating tilts. To estimate the tilt of the gratings in images, a brute-force computation is made of grating responses for all reasonable tilts for both gratings. The computed ratio of responses that best matches (in a least-squares error sense) the observed ratio of responses in observations of a bright star made with both gratings is taken as the best-estimate of the grating tilts. Figure 10 shows the predicted zero-tilt grating response ratio, the measured grating response ratio for a bright star, and the predicted best-fit tilt response ratio, and the contours of the log squared error of the reconstructed-versus-measured response ratio for all reasonable grating tilts. The best-fit tilt angles give a cosmetically good reconstruction of measured data. The shape of the error function implies that the tilt of the 606 grating is more tightly constrained by the response ratio than the tilt of the 906 grating.
C. Atmosphere Model Validation

The DCR bias in a bare-CCD astrometric measurement of a satellite target comes both from target DCR and fiducial star DCR. In order for the target DCR estimate to be useful, the DCR estimate of the fiducial stars must also be correct to the same level of fidelity. The Gaia DR2 star catalog contains enough information to infer fiducial star DCR to an accuracy of better than 20 mas, and thus the fiducial stars’ nominal focal plane positions may be corrected during data reduction to positions of equivalent solar-analog stars. The target DCR, as inferred from spectrographic measurement, needs to be referenced to the same solar-analog zero-point to be meaningful.

The computation of a target’s DCR from the measurement of its spectrum does not explicitly require knowledge of atmospheric spectral transmittance, but it does require a model of spectral refraction. The computation of the solar-analog zero-point, against which the fiducial stars and target are referenced, requires a model of instrument overall quantum efficiency, atmospheric spectral transmittance, and atmospheric spectral refraction. Measurements of the spectra of bright stars are used to validate these models.

1. Atmospheric Transmittance and Overall Quantum Efficiency

Fig. 10  Estimation of Grating Tilt Using Response Ratio

Fig. 11  Quality of Model of Atmospheric Transmittance and Instrument Overall Quantum Efficiency

The model of atmospheric transmittance, telescope spectral throughput, and detector quantum efficiency were validated simultaneously by observing stars with known exo-atmospheric spectra and comparing the measured detectable photoelectron spectra with spectra predicted using models. MODTRAN [19] was used to generate the atmospheric spectral transmittance curves for the calculation, datasheet values were used for the camera quantum efficiency, and
handbook formulas were used to estimate the reflectivity of the telescope mirrors and transmissivity of the optical windows and corrector plates.

The Pickles spectral typing of Tycho2 stars \cite{20} were used for calibration sources. Stars brighter than about 7th magnitude had to be used in order to calibration measurements in a timely fashion. This requirement unfortunately meant sampling the brighter end of the Pickles catalog for which the accuracy of the modeled spectra is lower than for the bulk of the catalog. For many of these stars at the bright end of the catalog, the absolute luminosity predicted from the model does not match reality, and for some, the shape of the spectrum is also mismatched. Nevertheless, when both the measured and the modeled spectra are normalized to a common signal level, a sufficient number of stars’ spectra have approximately the correct shape, enabling the comparison of modeled-versus-measured spectra shapes to be meaningful.

Figure 11(a) shows a sample measured spectrum reconstructed from multiple grating images as well as both normalized and un-normalized predictions of that spectrum based on Pickles catalog information. Figure 11(b) shows the relative error between normalized measured and predicted spectra expressed as percentage points of error in the total atmospheric and instrument spectral throughput model for all observed photometric calibration stars. Outside of some mismatched stars and a disagreement centered around the area where the measurement from the 606R and 906R grating images overlap, agreement between model and measurement is mostly to within 5 percentage points.

2. Spectral Refraction

GPS and GLONASS satellites were observed with vertical gratings giving horizontally-aligned spectra. This was motivated by the fact that the majority of those satellites’ motion relative to the stars was in the vertical direction and by the fact that the wavelength scale on the focal plane of a horizontal spectrum is constant and is not perturbed by DCR. A side-effect of this decision is that the location of the spectrum’s vertical centroid versus wavelength is a direct measurement of DCR. This is illustrated in Figure 6 where in the blue portion of the spectrum, the presence of DCR is evident in the upward curl of the shortest wavelengths. This measurement was used to validate the spectral refraction model \cite{3, 21, 22} that was used to estimate DCR for satellite targets from measured spectra and to compute DCR corrections to fiducial stars.

The measurement of vertical centroid versus wavelength requires a horizon reference. The pixel grid of the camera is not perfectly aligned to the horizon and the gratings are not perfectly aligned to the pixel grid, thus regions of common wavelengths (subject to the same wavelength-dependent refraction) in the $m = -1$ and $m = +1$ spectral orders are used to establish the local horizon. In general, the grating efficiency in the $m = +1$ order is at most a few percent of the efficiency in the $m = +1$ order, and for the 906R/OG570 combination, efficiency peaks at under 1%. For this reason, the horizontal baseline can only be established on very bright stars, and the measurements for spectral refraction validation are fewer in number than the measurements for overall spectral throughput validation.

Figure 12 shows a sample of the measured vertical centroid versus wavelength from grating images, corrected to the horizon reference, subtracted from the expected spectral refraction for that wavelength, at that elevation (computed using the mean atmospheric conditions for that night). The reference wavelength for comparison is 550 nm, where the blue and red ends of the spectrum from different grating and blocking filter combinations are stitched together. The measurement is made separately for all three combinations of grating and blocking filter used to observe bright stars. Overplotted on top of the measured error in both $m = -1$ and $m = +1$ is a straight-line fit to the error computed from the high-SNR $m = 1$ centroid.

For most stars, the measurement of error with respect to the DCR model is of adequate quality, as indicated by agreement between the linear fit computed from the $m = +1$ data and the $m = -1$ data. Further, agreement between the slopes of the linear fits among the measurements with the three combinations of gratings and filters also indicates that the measured error is real, rather than an artifact of any one measurement. Thus, the model of atmospheric dispersion can differ from observation by as much as a few hundred milliarcseconds between the red and blue ends of the silicon passband. This is a relative error of a few percent with respect to the actual amount of dispersion.

The entire set of deviations from the nominal refraction model versus elevation angle is plotted in Figure 13(a). There does not appear to be any large-scale systematic trend to these deviations as a function of elevation or any systematic difference between different nights. There are two instances of large deviations from the overall trend: one at high elevation on 2019:094 and one at low elevation on 2018:272. The lack of agreement in the slope among the measurements with the three grating and filter combinations on 2019:094, combined with the large number of nearby measurements without any noticeable deviation from the model, implies that this is simply a bad measurement in a cluttered environment or on a low SNR star, which can be confirmed in Figure 12(d). The case of the spike at low elevation on 2018:272 is less clear-cut. Although it is possible this is a bad measurement, there is agreement among the
three measurements and inspection of Figure 12(b) indicates that this observation may indeed be real. The weather on 2018:272 was not photometric, with occasional pockets of clouds and water vapor going through the field of view. If an observation at low elevation happened to go through a patch of warm air, the overall dispersion may be lower than expected for that one observation, resulting in a larger positive slope.

Fig. 12 Example of DCR Verification Measurement on Several Stars

(a) V=3.8 A2V Star Observed 2018:272 Elv=21.0°
(b) V=5.0 A7III Star Observed 2018:272 Elv=26.7°
(c) V=6.5 B5II Star Observed 2018:342 Elv=47.4°
(d) V=5.7 A2V Star Observed 2019:094 Elv=56.7°

Fig. 13 Observed DCR Slope Error with Respect to Refraction Model vs. Elevation

The amount of deviation from the model is a small percentage of the absolute amount of dispersion. In Figure 13(b) the average error in the slope of a few hundred mas per micron is under 5% below 40 degrees elevation and under 30% at 60 degrees elevation. Although the relative error would seem to be large at high elevation, the implications for DCR prediction accuracy are minor. Also indiciated on Figure 13(b) are the elevation angle shifts equivalent to the relative
error. That is, below 50 degrees elevation, the amount of DCR error observed is equivalent to a shift of less than 3
degrees along the DCR slope vs. elevation curve, and above 50 degrees, the shift is under 7 degrees. From Figure[3], it is
clear that such a small shift at that elevation would introduce no more than a few tens of milliarcseconds of error into the
DCR estimate.

III. Satellite Measurements

Measurements on GPS and GLONASS satellites occurred on seven nights between September 2018 and April 2019. The
Firepond 48-inch telescope collected bare-CCD astrometry at 8 Hz using an Andor iXon 888 EMCCD camera
while the 16-inch spectrograph cycled through gratings and blocking filters as described in Section [I.A]. The planning
of satellite observations for each night attempted to capture the same satellite at the same elevation angle over a variety
of phase angles and to do so over a range of elevation angles with the goal of capturing a variety of both atmospheric
conditions and target illumination conditions. Special care was taken to observe satellites entering or exiting eclipse in
order to capture solar panel glints. One such glint was measured over the seven operational nights. The distribution of
GPS and GLONASS targets over elevation and phase angle is shown in Figure 14(a).

Fig. 14 Observation Geometry and Brightness of Satellites Observed with Spectrograph over Seven Operational Nights

Including calibration measurements and astrometric fiducial frames, a total of 920 GB of data was collected with the
16-inch spectrograph. A total of 1600 GB of bare-CCD astrometric images were collected with the 48-inch telescope. Bare-CCD images were reduced with Gaia DR2 and compared against final IGS truth ephemerides [23]. The observation
model used to make the comparison included the effects of light travel time [24], diurnal and annual aberration [22] and
parallactic refraction [25].

Of the approximately 800,000 bare-CCD images collected, 580,731 distinct images of GPS and GLONASS targets
were suitable for astrometric reduction. Further, an observation was considered to have quality suitable for inclusion in
the analysis if it was based on an astrometric solution containing at least five stars (preferably 20 or more stars), the
astrometric solution RMS residual was 300 mas or better, and the accuracy of the horizontal component of the residual
with respect to the IGS reference orbit was 300 mas or better. Approximately 75% of the observations produced were of
sufficient quality for meaningful analysis. Of those observations, approximately 60% coincided with a sufficiently high
target SNR to enable DCR compensation with measurements from the 16-inch spectrograph.

A. Target Brightness and DCR Estimation Accuracy

Figure 14(b) shows the average measured brightness versus phase angle of the satellites that were observed during
the measurement campaign. The observations are grouped by the type of satellite. Three types of GPS satellite were
observed: Block IIR, Block IIR-M, and Block IIF, while all GLONASS satellites observed were of the GLONASS-M
type. The design of the spectrograph assumed a worst-case brightness of 12.5 mag for all targets, and while this is
mostly true of GLONASS satellites, the GPS Block IIR and IIR-M satellites are fainter than that at phase angles above
60°, thereby reducing the diversity of target illumination conditions over which the DCR reconstruction technique was
exercised.

Also of note are the implications for the error budget on the astrometric measurement of these satellites. The GPS Block IIR and IIR-M satellites have a solar panel wingspan of 11.4 meters while the Block IIF satellites have a wingspan of 35.5 meters. The GLONASS satellites are about half a visual magnitude brighter than the GPS Block IIF satellites implying their wingspan is between 30% and 60% larger. At a typical range of 20,000 km, a one meter offset between the center of mass and center of illumination corresponds to an angular measurement bias of 10 mas. While it is very unlikely that the center of illumination of a satellite is at the very tip of its solar panels, if the center of illumination is somewhere in the middle of one solar panel, then the worst-case astrometric bias for a GPS Block IIR and IIR-M satellite would be 25 mas, but the worst case bias for a GPS Block IIF or a GLONASS-M would be between 87 mas and 140 mas at near zero phase angle and as high as 100 mas at 45 degrees phase angle. This is a paradox: the target with the brightest signal and therefore the best DCR reconstruction is also subject to the greatest astrometric bias from defect of illumination.

The original aspirational goal of the study was to achieve DCR reconstruction to an accuracy of 25 mas (commensurate with star catalog and stellar DCR correction accuracy), but the requirement of using a small aperture telescope, combined with the baseline error budget of an astrometric measurement of GPS and GLONASS satellites, and the fact that the targets were fainter than 12.5 magnitude at high phase angles, made that unrealistic.

Using Equation 2 and real-time sky background and target brightness measurements, the accuracy of the DCR reconstruction from the concurrent spectroscopy may be estimated on a per-measurement basis using the observed SNR of the satellite in the bare-CCD astrometric frames. Applying SNR thresholds to the dataset, Figure 15 shows that with a 100 mas desired accuracy, data is present up to a phase angle of 100°. Tightening the desired accuracy down

*GPS and GLONASS satellites have nadir-pointing antennas and solar panels aligned horizontally. If the solar panels track the Sun, the maximum extent is the satellite’s wingspan times the cosine of solar phase.
to 50 mas removes all instances of phase angles greater than 90° removes all observations of smaller GPS satellites above 60° phase. Lowering the desired accuracy further still down to the original design goal of 25 mas leaves data only at very low phase angles and thereby removes a large amount of target diversity over which the technique may be exercised. Therefore, the remaining analysis here will be restricted to data that meets the 50 mas and 100 mas accuracy requirement.

B. DCR Compensation Results

![Graphs showing predicted vs. observed vertical astrometric residual of satellites with measured spectra.](image)

Fig. 16 Predicted vs. Observed Vertical Astrometric Residual of Satellites with Measured Spectra.

Figure 16 shows the predicted vs. observed DCR bias for all GPS and GLONASS satellites observed over the seven operational nights. The astrometric observations are generated using the Gaia DR2 star catalog, processed with per-star DCR correction, and residuals are computed with parallactic refraction included in the observation model. For clarity, each point on the chart represents the mean of residuals over a 105-second averaging period for each spectroscopic measurement cycle. Astrometric observations are included in the average if they meet the relevant SNR requirement to achieve the desired level of DCR accuracy as well as the goodness criteria on the astrometric solution noted above. Only measurement periods with at least 30 distinct observations are included.

With no accuracy criterion applied, the 528 datapoints in Figure 16(a) show a weak correlation between observed and predicted DCR bias. When the accuracy criterion is tightened, but still left loose enough to allow at least one datapoint per observed satellite, the correlation becomes more evident in Figure 16(b). When the minimum SNR criterion is strictly applied at the expense of excluding observations of some satellites in Figures 16(c) and 16(d), the agreement between prediction and observation is quite good and deviations from predictions are within an order of magnitude of the error bounds used to generate the SNR threshold. It should be noted, however, that the distributions are not Gaussian, thus the relation between the 1-σ specification of the accuracy criterion and the fraction of residuals outside the 1-σ bounds is not expected to follow Gaussian statistics. The highlighted blue trace in Figures 16(c) and
16(d) represents the single instance of a satellite observed emerging from eclipse and glinting blue (see Section III.C).

Table 1 lists the statistics of the distributions of the vertical component of the astrometric residuals. Several kinds of corrections are applied to the residuals to generate the distributions to highlight the point that all atmospheric phenomena must be accounted for in order for per-target DCR corrections to be meaningful. The different distributions are defined by:

1) Raw astrometry: no DCR correction and no parallactic refraction in observation model;
2) No DCR correction but parallactic refraction in observation model;
3) Stellar DCR correction but no parallactic refraction;
4) Stellar DCR correction and parallactic refraction;
5) Stellar DCR correction, parallactic refraction, and per-target DCR correction based on concurrent spectroscopic measurements.
6) Stellar DCR correction, parallactic refraction, and per-target DCR correction based on an assumed target composition of 60% solar panel and 40% Kapton.

Where relevant, a distinction is made for data with no minimum SNR criterion and data meeting a DCR accuracy criterion.

| Criterion | Processing                                          | Mean [mas] | Std. Dev [mas] | N   |
|-----------|-----------------------------------------------------|------------|----------------|-----|
| All       | raw horizontal                                      | -15        | 35             | 528 |
| All       | raw vertical                                        | -73        | 116            | 528 |
| 100 mas   | raw vertical                                        | -61        | 100            | 312 |
| All       | parallax only                                       | -30        | 105            | 528 |
| 100 mas   | parallax only                                       | -29        | 94             | 312 |
| All       | no parallax, stellar DCR                            | -152       | 112            | 528 |
| 100 mas   | no parallax, stellar DCR                            | -131       | 102            | 312 |
| All       | parallax, stellar DCR                               | -106       | 87             | 528 |
| 100 mas   | parallax, stellar DCR                               | -94        | 77             | 312 |
| 100 mas   | parallax, stellar DCR, target correction            | 24         | 68             | 312 |
| 50 mas    | parallax, stellar DCR, target correction            | 27         | 61             | 223 |
| 100 mas   | parallax, stellar DCR, constant 60% solar panel/40% Kapton | 4         | 61             | 312 |
| 100 mas   | parallax only, target correction                     | 90         | 105            | 312 |

The horizontal residuals are essentially zero-mean given the error budget from defect of illumination, and their 35 mas standard error over the 105 second averaging period gives the baseline magnitude of the error floor for the rest of the other measurements. It is evident that the astrometry in the vertical direction is both noisier and more biased than in the horizontal direction. With the atmosphere-agnostic data reduction and observation model, the mean bias is 73 mas, or about 2σ with respect to the horizontal baseline, and the noise is more than three times worse. Including the effect of parallactic refraction reduces the bias but does not make an appreciable dent in the noise. Applying stellar DCR compensation but neglecting to include parallactic refraction in the observation model makes the bias considerably worse.

Only when the parallax correction is included in the observation model and the astrometric reductions are corrected for stellar DCR does the astrometric noise start to come down from about 100 mas to 77 mas. But again, if no per-target DCR correction is made, the bias is still large relative to the accuracy baseline. Once data reductions are made with stellar DCR corrections, parallactic refraction is included in the observation model, and per-target DCR corrections are applied, then the mean residual comes down to under 30 mas (comparable with the horizontal residual), and the noise level is reduced from near 100 mas to below 70 mas, a more than 30% performance improvement over atmosphere-agnostic processing.

The most important takeaway from these distributions is that everything has to be in place for per-target stellar DCR correction to make a quantifiable impact. If stellar DCR, target DCR, and parallactic refraction corrections are made in isolation, no performance improvement will be seen and in some cases performance will visibly degrade.

An important question to ask here is whether the variation in DCR bias spanning Figure 16 comes from a diversity of targets or a diversity of atmospheric conditions. This question is answered by plotting the observed vertical residual attributable to DCR bias against elevation. Figure 17 breaks down Figure 16(c) by elevation into 10-degree bands. Although there is some variation within the bands, most of the predicted and observed DCR within each elevation band
forms a single cluster about 100 mas across in both axes. That is, diversity of atmospheric conditions, and not diversity of target spectra, is responsible for much of the variation in DCR in the dataset, with the exception of the instance of 32260 emerging from eclipse.

![Graphs showing observed DCR vs. predicted DCR separated by elevation range.](image)

**Fig. 17** Observed DCR vs. Predicted (Data in Figure 16(c)) Separated by Elevation Range

![Graph showing observed DCR vs. elevation and DCR of solar panel/Kapton mixture.](image)

**Fig. 18** Observed DCR vs. Elevation and DCR of Solar Panel/Kapton Mixture
For most of the data for which there are good spectra, and for much of the data where there are not, the DCR bias is well-approximated by assuming that the target is a mixture of 60% solar panel / 40% Kapton (Figure 18). The correction made with the constant 60% solar panel / 40% Kapton mixture model is marginally better than the correction made with concurrent spectroscopy, with the mean error reduced from 24 mas to 4 mas and the standard deviation reduced from 68 mas to 61 mas. While this would seem to call into question the utility of spectroscopy when a blind correction seems better, it is important to recall that the spectra measured in this experiment are mostly confined to phase angles below 90°, with the majority of observations of GPS satellites occurring at phase angles below 60°. It is not surprising that in these restricted conditions, the satellites present what is essentially a constant mixture of materials to the observer. One would expect that at higher phase angles, the targets would look different than at low phase angle.

Further, the case of SCC 32260 exiting eclipse demonstrates why spectroscopy is necessary: while most of the time satellites’ spectra are steady, sometimes they are not. Occasionally the satellite will, either through chance geometric alignment or as part of its normal operations, present a radically different cross-section to the observer. Despite being observed over a small range of elevation angles with essentially a constant atmosphere, the variability in this target’s spectral signature was large enough to span a range of approximately 0.5 arcsec worth of DCR.

C. A Blue Glint

During the spectroscopic measurement campaign, instances where the satellites entered, exited, or grazed eclipse were prioritized for observation to try to capture a solar panel glint. Of the 14 distinct satellites observed over the seven operational nights from September 2018 to April 2019, only one instance of a blue glint was observed, that of SCC 32260 exiting eclipse on 2019:094. Interestingly, SCC 32384 was observed entering eclipse in the same part of the sky shortly afterward, but no glint was seen.

In observations of MEO navigation satellites, glints are rare, but in observations of geostationary satellites, they can be frequent around eclipse season. Furthermore, while a satellite going directly into eclipse may only present a glint to a terrestrial observer for a few minutes at most, there are periods of time around eclipse season where geostationary satellites graze eclipse. During this time, the glint duration may last longer and partial eclipse illumination conditions may last for many minutes. The rarity of these conditions merits further examination of the one instance that was observed.

Figure 19 shows the measured brightness of the satellite as it transitioned the partial eclipse region and entered full sunlight. Underlaid beneath the signature is the timing of the multiplexed gratings and blocking filters used to make the spectrographic measurement. The time between detection of the satellite signal and its peak brightness is on the order of two minutes and it is worth noting that during partial eclipse, the target brightness ranges over five visual magnitudes between the center of the blue end measurement with the 606/BG40 grating/filter combination and the measurement of the red end of the spectrum with the 906/OG570 grating/filter combination. The target saturated the 48-inch telescope at peak brightness, so it is likely that the blue glint peaked brighter than 5th magnitude, but the point is that during the glint, in the approximately 45 seconds between the center of the blue and the red measurement on the first filter wheel cycle to capture the glint, the target brightness dropped by at least 20%, and it dropped by about 40% during the second
cycle to capture the glint (Figure 19(b)).

The effect of the rapidly changing signature between measurement of the two ends of the target spectrum is visible in the reconstructed spectra in Figure 20. It is almost miraculous that the effect of the rapidly changing signature did not spoil the measured spectra to the point where DCR predictions derived from them were not usable.

Cycle 1 measures the target as it emerges from eclipse and undergoes a 7 visual magnitude change in brightness between the start of the blue measurement and the end of the red measurement. The spectra for each combination of grating and filter are derived from the median of the grating images, but that is not very satisfying as that still leaves about 5 visual magnitudes worth of signal growth between the red and the blue. What makes the reconstruction valid is that despite the nominal change between the blue and red ends of the measurement, while the satellite is in partial eclipse, almost all blue light in the illumination spectrum is filtered out by the atmosphere and there is no blue signal to measure during Cycle 1.

The effect of the changing signature is more apparent in the spectra reconstructed during Cycle 2 and Cycle 3. During both of these measurement cycles, the solar panels were glinting blue and the overall target brightness during the blue measurement was higher than during the red measurement. This is reflected in the straight line stitching together the red and blue ends of the reconstructed spectrum in Figure 20. Despite this artifact caused by the need to multiplex passbands and gratings during the measurement of the full spectrum, the cosmetic match between the reconstructed spectrum and a 100% solar panel model is quite good, validating the intuition that the signature of a satellite near zero phase is dominated by a solar panel blue glint. The rapid change in signature and DCR over the course of a few minutes highlights the necessity of realtime spectroscopic measurements, rather than models, to compensate for DCR in operational SSA measurements.

**IV. Conclusion**

**A. Summary**

We have demonstrated compensation of DCR bias in ground-based silicon passband observations of GPS and GLONASS satellites with concurrent spectroscopy. The compensation results in a 60% reduction in bias and 30% reduction in noise of the vertical component astrometric residuals relative to color-agnostic processing. The improvement is only meaningful when all atmospheric phenomena (target DCR, stellar DCR, and parallactic refraction) are accounted for. While GPS and GLONASS satellites’ spectral signatures at low-to-moderate phase angles are fairly steady and can be well-modeled by a constant 60% solar panel / 40% Kapton spectrum, rare illumination condition changes such as solar panel glints can cause large deviations in DCR from the nominal condition.

The compensation of stellar DCR on a per-star basis requires the use of models for both spectral transmittance and refractivity of the atmosphere. In addition to making measurements of satellites’ detectable photoelectron spectra, the
spectrographic instrument was also used to validate models of atmospheric transmittance and spectral refractivity by observing bright stars. Agreement between models and measurements was to within a few percentage points, and are accurate enough to enable measurable improvement in residuals statistics.

B. Future Work

The aggressive time-aperture requirement necessary to achieve DCR estimation accuracy on par with catalog accuracy is driven by the fact that a resolution of 40 nm is required. This resolution requirement stems from the assumption that the target spectrum is completely unknown. In practice, the spectra of satellites are not entirely free-form. Indeed, in terms of DCR accuracy, the spectra of the GPS and GLONASS satellites we measured for this study are well-approximated by mixtures of solar panel and Kapton, and most of the time this ratio is constant. This motivates a search for a lower-dimensional representation for satellite spectra that may be accurately estimated with a smaller time-aperture product, and yield comparable DCR accuracy.

While our observations of GPS and GLONASS satellite measurements seem to indicate that existing laboratory measurements of common spacecraft materials’ spectral reflectance may be sufficient, we must recall that our observations were confined to only four distinct satellite types and mostly at low phase angles. The spectrum of the one glint we observed with high SNR, while cosmetically a good match to a model based on laboratory measurements, is not a perfect match. For this reason, we believe that a long-term spectroscopic observing program against a wide variety of active and inactive geostationary satellites, over a variety of illumination conditions including partial eclipse, is warranted in order to determine if a suitable low-dimensional representation of on-orbit spectra exists.

We note that such a low-dimensional representation need not be universal. That is, if, for example, only half of all geostationary satellite’s spectra may be modeled by a two- or three-dimensional representation that permits a low time-aperture product spectroscopic measurement for accurate DCR compensation, then it would enable improved orbit solutions for half of the geostationary population.

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