Real-Time Tuning Separate-Bias Extended Kalman Filter for Attitude Estimation

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This paper provides a new method for robust spacecraft attitude estimation in the presence of measurement biases. The proposed method is developed based on the separate-bias or two-state Kalman filter which was first introduced by Friedland. The separate-bias Kalman filter consists of two stages: the first stage, the “bias-free” filter, is based on the assumption that the bias is nonexistent; the second stage, the “bias” filter, is implemented to estimate bias vectors. The output of the first filter is then corrected with the output of the second filter. In this research, the authors propose a real-time tuning method for a parameter in the Kalman gain calculation process of the “bias” filter. The adaptive scale factor is optimized relying on the minimization of the cost function, which is calculated from the difference between the predicted and measurement values. The proposed filter has a faster convergence speed from large initial errors and an increased accuracy on unpredicted bias models than conventional methods. Moreover, to verify these advantages, the research also provides analyses and comparisons between the proposed method with conventional methods like the original separate-bias Kalman filter, unscented Kalman filter and extended Kalman filter in several numerical simulation scenarios for a microsatellite.

Key Words: Attitude Estimation, Kalman Filter, Adaptive Estimation, Bias Estimation

1. Introduction

Attitude estimation systems often need multiply sensors to increase the system reliability and estimated accuracy. Due to the complexity of the real working environment conditions, these sensors may be affected by unexpected biases or drift which reduce the stability and accuracy of estimators. In many practical applications, these biases are modeled as constant but unknown values. The standard technique is to augment the state vector of the original problem by adding additional components to represent estimated random biases. The filter then estimates the bias terms as well as those of the original problem. The increment of the state vector dimension may lead to the computations required by the filtering algorithm becoming excessive. In particular, when the unscented Kalman filter (UKF)1) is used, the computation cost becomes heavier because of the large amount of generated sigma points.

In an attempt to decouple the bias estimation from the state estimation, Friedland2) estimated the state as though the bias was not present and then added the contribution of the bias. Friedland showed that this approach is equivalent to augmenting the state vector. An alternate derivation of Friedland’s two-stage estimator has been given which provides some additional insights into the fundamental nature of the solution, and allows a number of extensions of the basic idea to be readily deduced.3) This technique, known as the two-stage Kalman filter or separate-bias Kalman filter (SKF), was then extended to incorporate a walk in the bias forced by white noise.4) To account for the bias walk, the process noise covariance was increased heuristically, and optimality conditions were derived.5,6) On the other hand, Zanetti and Bishop purposed algorithms for precise navigation are derived to include uncompensated bias terms in both the process and measurement noise.7) In their work, the effects of the noise and biases were considered as sources of uncertainty and not as elements of the state vector. This approach was applicable in situations when the biases are not observable or when there is not enough information to discern the biases from the measurements. One small drawback of this method is that the estimated values of biases, which may be needed for attitude control processes, are not provided.

In the case of nonlinear systems, such as the satellite attitude estimation problem, the Kalman filter (KF) and SKF were expanded to include nonlinear dynamics models, which led to the development of the extended Kalman filter (EKF), separate-bias extended Kalman filter (SEKF), or UKF. Due to the linearization effect and the approximation of the state distribution effect in EKF, SEKF and UKF, these filters have some limitations. They can achieve good results under prior assumptions of proper information of the noise distribution and proper initial conditions. But the prior information is not always available in practice so that assumptions have to be made. If the assumptions are not correct, it might lead the filters to diverge from the solution. One of the approaches to solving this problem is to introduce an adaptive mechanism into a normal filter; i.e., the adaptive law automatically tunes the filter parameters through the
identification of either the process and measurement noise covariance matrices or the system model parameters, or both simultaneously. There are some investigations in the area of adaptive filters, and most of them are constructed with respect to the EKF and UKF. Mohamed and Schwarz studied the performance of a multiple-model-based adaptive Kalman Filter for vehicle navigation using GPS.\textsuperscript{8} Loebis et al. proposed an adaptive EKF algorithm, which adjusted the measurement noise covariance matrix by fuzzy logic.\textsuperscript{9} An automated method forKF tuning has been introduced by Powell.\textsuperscript{10} The method formulated a stochastic cost function called a performance index based on the true estimation error, typically obtained from extensive Monte Carlo simulations. The proposed method solved the associated minimization problem using the downhill simplex algorithm. With the same technical, real-time tuning UKF, the reference attitude in the main estimator was used to calculate the optimal function for the optimal tuning process.\textsuperscript{11}

This paper introduces a new adaptive filter, real-time tuning separate-bias extended Kalman filter (RTSEK) for robust spacecraft attitude estimation in the presence of measurement biases. The proposed filter is especially suitable for microsatellites, which are limited in terms of computer performance and memory. The adaptive mechanism applied during calculation of Kalman gain in the “bias” filter could help the proposed filter provide a better response to the large initial estimated errors and unpredicted sensor bias models than conventional methods like EKF, SEKF and UKF.

The organization of this paper is as follows. In section 2, the state and bias estimations using the SEKF are reviewed and analyzed in detail. In section 3, the spacecraft rotation kinematics with quaternion representation and sensor models are briefly reviewed. In section 4, the details of the RTSEKF algorithm based on SEKF are presented and discussed. In section 5, some simulation results are given with the discussions of the computation cost, robustness, convergence speed and estimated accuracy. Finally, in section 6, the conclusions are given.

2. State and Bias Estimation by Kalman Filter

Consider the discrete-time stochastic dynamical system model

\[
\begin{align*}
    x_k &= \Phi_k x_{k-1} + u_k \\
    \hat{y}_k &= H_k x_k + C_k b_k + v_k \\
    b_k &= b_{k-1} + \xi_k
\end{align*}
\]

where \(x_k\) is a state vector, \(\hat{y}_k\) is an observation vector, \(b_k\) is an observation bias vector, \(\Phi_k\), \(H_k\) and \(C_k\) are time-variant coefficient matrices, \(u_k \sim N(0, Q_u)\) is a state process noise, \(v_k \sim N(0, R_k)\) is a measurement noise vector, and \(\xi_k \sim N(0, Q_x)\) is a measurement bias noise. The process, measurement and bias noises are assumed to be zero mean Gaussian noises with known covariance \(Q_u^k\), \(R_k\) and \(Q_x^k\).

The subscript \(k\) denotes a measurement that is taken at time \(t_k\).

2.1. KF for state estimation

The time update of mean and covariance are given by

\[
\begin{align*}
    \hat{x}_k &= \Phi_k \hat{x}_{k-1}^k \\
    P_k^\sim &= \Phi_k P_{k-1}^\sim \Phi_k^T + Q_k^u
\end{align*}
\]

and measurement update are calculated by

\[
\begin{align*}
    K_k &= P_k^\sim H_k^T (H_k P_k^\sim H_k^T + R_k)^{-1} \\
    \hat{x}_k &= \hat{x}_k^k + K_k (y_k - H_k \hat{x}_k^k) \\
    P_k &= (I - K_k H_k) P_k^\sim
\end{align*}
\]

where \(\hat{x}_k\) is the estimate of \(x_k\), \(P_k\) is the estimated error covariance matrix, \(K_k\) is Kalman gain matrix and the superscript \(\sim\) is used to denote a priori values of \(x_k\) and \(P_k\).

2.2. KF for state and bias estimation

If the bias states are appended directly to the variable states, the estimation of both sets of states can be achieved by utilizing the conventional Kalman estimator for the new state vector \(z = [x^T \ b^T]^T\) as

\[
\begin{align*}
    \hat{z}_k &= \Theta_k \hat{z}_{k-1} \\
    P_k^\sim &= \Theta_k P_{k-1}^\sim \Theta_k^T + Q_k \\
    K_k &= P_k^\sim L_k^T (L_k P_k^\sim L_k^T + R_k)^{-1} \\
    \hat{z}_k &= \hat{z}_k^k + K_k (y_k - L_k \hat{z}_k^k) \\
    P_k &= (I - K_k L_k) P_k^\sim
\end{align*}
\]

where \(\hat{z}_k\) is the estimate of \(z\), \(P\) is the estimation error covariance matrix, \(\Theta_k\) is the transition matrix from the \((n-1)\)th to \(n\)th update points, \(K\) is the Kalman gain matrix, \(L\) is the observation matrix, \(Q\) is the process noise covariance matrix and \(R\) is the measurement noise covariance matrix. In the application of interest here, the generalized estimator will be expressed in partitioned form utilizing the following partitions.

\[
L = [H \ C]
\]

\[
P = \begin{bmatrix} P_x & P_{xb} \\ P_{bx} & P_b \end{bmatrix}, \quad Q = \begin{bmatrix} Q_x & 0 \\ 0 & Q_b \end{bmatrix}
\]

\[
K = \begin{bmatrix} K_x \\ K_b \end{bmatrix}, \quad \Theta = \begin{bmatrix} \Phi & 0 \\ 0 & I \end{bmatrix}
\]

2.3. Optimal separate-bias Kalman filter

The idea of building a separate-bias Kalman filter is the separation of state and bias vectors which are obtained from each filter as follows. Firstly, the bias term is ignored and the state vector is obtained by a “bias-free” filter. Then, the bias term is considered and a “bias” filter is built whose input is the innovations of the bias-free filter and output is the bias estimate. Finally, based on the above two filters, the estimated result including state and bias vectors are corrected through an adjustment process. For the system defined by Eqs. (1)–(3), the optimal, linear, minimum variance separate-bias estimator to be derived is defined by the “bias-free” filter of \(x\) given by\textsuperscript{4,6}
\[ \bar{x}_k = \Phi_k \bar{x}_{k-1} \] (14)
\[ P_x^-(k) = \Phi_k P_x^-(k-1) \Phi_k^T + Q_x(k) \] (15)
\[ K_x(k) = P_x^-(k) H_k^T \left[ H_k P_x^-(k) H_k^T + R_x \right]^{-1} \] (16)
\[ x_k = \bar{x}_k + K_x(k) (y_k - H_k \bar{x}_k) \] (17)
\[ P_k = (I - K_x H_k) P_x^- \] (18)

where \( x_k \) is the bias-free estimate of \( x \), \( P \) is the bias-free estimation error covariance matrix and \( K_x \) is the bias-free Kalman gain matrix.

The “bias” filter of \( b \) is given by\(^{4-6}\)
\[ \bar{b}_k = b_{k-1} \] (19)
\[ P_b^-(k) = P_b(k-1) + Q_b(k) \] (20)
\[ K_b(k) = P_b^-(k) S_k^T \left[ S_k P_b^-(k) S_k^T + H_k P_x^-(k) H_k^T + R_b \right]^{-1} \] (21)
\[ b_k = \bar{b}_k + K_b(k) (y_k - H_k \bar{x}_k - S_k b_k) \] (22)
\[ b_k = \bar{b}_k + K_b(k) (y_k - H_k \bar{x}_k - C_k b_k) \] (23)

In which \( \bar{b} \) is the bias estimate, \( K_b \) is the bias gain, and \( P_b \) is the bias estimation error covariance matrix. The a priori and a posteriori sensitivity functions, \( U \) and \( V \), are given by\(^{4-6}\)
\[ U_k = \Phi_k V_{k-1} \] (24)
\[ V_k = U_k - K_x S_k \] (25)
\[ S_k = H_k U_k + C_k \] (26)

In the adjustment process, the a priori and a posteriori corrected estimates of \( x \) are expressed as
\[ \bar{x}_k = \bar{x}_k + U_k \bar{b}_k \] (27)
\[ \bar{x}_k = \Phi_k \bar{x}_{k-1} + U_k \bar{b}_k \] (28)
\[ \bar{x}_k = \Phi_k \bar{x}_{k-1} + U_k \bar{b}_k \] (29)
\[ \bar{x}_k = \Phi_k \bar{x}_{k-1} + U_k \bar{b}_k \] (30)

where \( \bar{b}_k \) is the bias estimate, \( K_b \) is the bias gain, and \( P_b \) is the bias estimation error covariance matrix. The a priori and a posteriori sensitivity functions, \( U \) and \( V \), are given by\(^{4-6}\)

In this filter, there are three covariance matrices, \( Q_x(k) \), \( Q_b(k) \) and \( R_x \), where the selection of them can affect the estimated qualities. Based on offline analysis when SEKF is applied for the attitude determination problem, the most effective factor is selection of the \( R_x \) at the computation of Kalman gain \( K_x \) in Eq. (21) inside the “bias” filter. In section 4, the new proposed method for attitude determination will tune this parameter by minimization of a cost function using a numerical optimal process. Through this real-time tuning process, the estimated qualities will be improved.

To do that, firstly, the mathematical models of attitude determination problem are briefly reviewed in the next section.

### 3. Mathematical Models

In this section, a brief review of the attitude kinematics equation of motion using quaternions representation is shown. Then, the models of gyro and attitude sensor are briefly reviewed.

#### 3.1. Attitude kinematics

The quaternion is defined by \( q = [q^T \ q_i]^T \), where \( q^T = [q_1 \ q_2 \ q_3]^T \) is the vector part and \( q_i \) is the rotation part. The quaternion representation is desirable because of its singularity free property. However, the norm constraint must be maintained. Since a four-dimension vector is used to represent three dimensions, the quaternion has a single constraint given by \( q^T q = 1 \). The attitude matrix is calculated as a quadratic function of \( q \); that is
\[ A(q) = \left( q^2 - ||q||^2 \right) I_{3 \times 3} + 2 \rho \rho^T - 2 q_4 [\rho \times] \] (31)

where \( I_{3 \times 3} \) is the \( 3 \times 3 \) identity matrix and \( [\rho \times] \) is the cross matrix defined as
\[ [\rho \times] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \] (32)

The quaternion kinematics differential equation is defined below.\(^{12}\)
\[ \frac{d}{dt} q = \frac{1}{2} \mathcal{S}(q) \omega = \frac{1}{2} \Omega(\omega) q \] (33)

where \( \omega \) is a \( 3 \times 1 \) vector of satellite angular velocity with respect to the initial frame and
\[ \Omega(\omega) = \begin{bmatrix} -[\omega \times] & \omega \\ -\omega^T & 0 \end{bmatrix} \] (34)
\[ \mathcal{S}(q) = \begin{bmatrix} q_4 I_{3 \times 3} + [\rho \times] \end{bmatrix} \] (35)

Equation (33) can be rewritten in the discrete form as
\[ q_{k+1} = \hat{\Omega}(\omega_k) q_k \] (36)

where
\[ \hat{\Omega}(\omega_k) = \begin{bmatrix} \gamma_k I_{3 \times 3} - [\Psi_k \times] & [\Psi_k \times] \\ -[\Psi_k \times] & \gamma_k \end{bmatrix} \] (37)

and
\[ \gamma_k = \frac{\sin \left( \frac{1}{2} \| \omega_k \| \Delta t \right)}{\| \omega_k \| \Delta t} \] (38)
\[ \Psi_k = \cos \left( \frac{1}{2} \| \omega_k \| \Delta t \right) \] (39)
3.2. Sensor models
The satellite angular rate is measured by the gyro and its widely used model is given by\(^{(13)}\)
\[
\begin{align*}
\omega(t) &= \dot{\omega}(t) - \beta(t) - \eta(t) \\
\beta(t) &= \eta_a(t)
\end{align*}
\]  
where \( \dot{\omega}(t) \) is the measured angular velocity, \( \omega(t) \) is the true angular velocity, \( \beta(t) \) is the gyro bias, and \( \eta_a(t) \) and \( \eta(t) \) are independent zero-mean Gaussian white-noise processes with standard deviations \( \sigma_\beta \) and \( \sigma_\eta \). Gyro bias \( \beta(t) = [\beta_1 \quad \beta_2 \quad \beta_3]^T \) is the unknown vector which is considered constant but can vary at each sampling step.

The measurement vector sensor can be modeled as\(^{(12)}\)
\[
y_k = \begin{bmatrix} A(q) r_1 \\ A(q) r_m \\ b_m \end{bmatrix} + v_k
\]  
where \( v_k \sim N_c(0, R_k) \) is a zero-mean Gaussian white noise with covariance \( R_k \), \( m \) is the number of vector measurements, \( b_m \) is a measurement bias vector in the spacecraft body frame of the \( i \)-th sensor, \( q_k \) is an attitude quaternion, \( A(q_k) \) is a corresponding attitude direction cosine matrix, \( r_m \) corresponds with the \( i \)-th reference vector model in the Earth-fixed frame and all values are represented at time \( t_k \).

4. Real-Time Tuning SEKF for Attitude and Bias Estimation
In this section, the proposed method, named the real-time tuning separate-bias extended Kalman filter (RTSEKF), for spacecraft attitude estimation in the presence of measurement biases is presented and discussed in detail. As mentioned above, this proposed filter includes a real-time tuning process, which will tune the matrix \( R_k \) inside the calculated function of Kalman gain \( K_k \) in the “bias” filter of the SEKF.

4.1. RTSEKF for attitude and bias estimation
To estimate the spacecraft attitude, gyro bias and measurement sensor bias, the state vector is chosen as
\[
\begin{bmatrix}
\delta \dot{\theta}_k \\
\delta \beta_k
\end{bmatrix}
\]
and the bias vector is chosen as
\[
\begin{bmatrix}
\tilde{b}_1 \\
\cdots \\
\tilde{b}_n
\end{bmatrix}
\]
where \( \delta \dot{\theta}_k \) is the estimated small angle approximation, \( \tilde{b}_i \) is the estimated gyro bias vector and \( n = 6 \) is the dimension of the state vector, \( \tilde{b}_i \) is the estimated measurement bias vector of the \( i \)-th sensor and \( m \) is the number of vector measurements.

The initial values of the estimated state, estimated bias vector and error covariance matrices are defined as
\[
\begin{align*}
\hat{x}(0) &= \hat{x}_0 \\
\hat{b}_0 &= b_0 \\
P_x(0) &= P_{x0} = E[\hat{x}(0)\hat{x}^T(0)] \\
P_b(0) &= P_{b0} = E[\hat{b}(0)\hat{b}^T(0)]
\end{align*}
\]  
The time update process:

The estimated angular velocities are calculated from the measured angular rate \( \omega_k \) by
\[
\dot{\hat{\omega}}_k = \omega_k - \hat{x}^\theta_k
\]  
where \( \hat{x}^\theta_k = \hat{x}^\theta_{k-1} \) because gyros bias is assumed to be the constant number. Next, the quadratures are updated using Eq. (36).

Then, the a priori covariance matrix can be calculated using Eq. (15) with the state transition matrix \( \Phi_k \) of
\[
\Phi_k = \begin{bmatrix} I_k \gamma_k \Delta t + \frac{1}{2} \gamma_k^2 \Delta t^2 + \frac{1}{3!} \gamma_k^3 \Delta t^3 + \cdots \end{bmatrix}
\]  
The bias-free Kalman gain \( K_k(k) \) is calculated by Eq. (16).

The measurement update process:

Firstly, the measurement sensitivity matrix is calculated as
\[
H_k = \begin{bmatrix} [A(\hat{\theta}_k)] r_k \times \end{bmatrix} 0_{3x3}
\]  
The residual vector is calculated by
\[
v_k = y_k - H_k \hat{x}_k - C_k b_k
\]  
The a priori and a posteriori sensitivity functions \( U \) and \( V \) are updated by Eqs. (24) to (26).

The “bias” estimator is realized using Eqs. (19) and (20). In the continuous part, a tuning process will tune the measurement noise covariance matrix, \( R_k^{\text{tuning}} \), which is modeled by
\[
R_k^{\text{tuning}} = \zeta_k R_k
\]  
where \( \zeta_k \) is a positive scaling factor and \( R_k \) is a static-optimal value of the measurement covariance matrix. The purpose of the tuning process is finding the optimal value of the positive scaling factor, \( \zeta_k^{\text{optimal}} \).

The tuning process calculates \( K_k^{\text{optimal}}(k), \tilde{b}_i^{\text{optimal}}(k) \) as a function of \( \zeta \) as below.
\[
K_k^{\text{optimal}}(k) = P_k^{\text{optimal}}(k) S_k^{\text{optimal}} \left[ S_k P_k^{\text{optimal}}(k) S_k^{\text{optimal}} + H_k P_k^{\text{optimal}}(k) H_k^T + \zeta_k R_k \right]^{-1}
\]  
\[
\tilde{b}_i^{\text{optimal}}(k) = \tilde{b}_i^{\text{optimal}} + K_k^{\text{optimal}}(k) v_i^{\text{optimal}}
\]  
\[
v_i^{\text{optimal}}(k) = y_k - H_i \hat{x}_k - C_i b_k
\]  
These calculations are looped until the cost function \( f_{\text{cost}} \) is minimized
\[
f_{\text{cost}} = \text{sqrt} \left[ (v_i^{\text{optimal}}(k))^T v_i^{\text{optimal}}(k) \right]
\]  
and the optimal value of the positive scaling factor, \( \zeta_k^{\text{optimal}} \), is achieved.
Then, based on this optimal scaling factor, the “bias” filter is updated using the optimal values of the bias Kalman gain $K_{bb}^+(k)$, optimal estimated bias vector $\hat{b}_k^+$ and optimal residual vector $v_k^+$, and optimal state vector $\hat{x}_k^+$:

$$P_b(k) = \left[I - K_{bb}^+(k)S_k\right]P_b(k)$$  \hspace{1cm} (58)

The adjusted estimates of $x$ and its error covariance matrix are given by

$$\hat{x}_k = \hat{x}_k^- + \left[K_x(k) + V_xK_{bb}^+(k)\right]v_k^+$$  \hspace{1cm} (59)

$$P_x(k) = \hat{P}_x(k) + V_xP_b(k)V_x^T$$  \hspace{1cm} (60)

Finally, the estimated attitude is updated and the state vector is reset by

$$q_t = \delta q_t \otimes q_k^-$$ with $\delta q_k = \left[\hat{x}_k^+/2 \quad 1\right]^T$$  \hspace{1cm} (61)

$$\hat{x}_k = \left[0 \quad 0 \quad 0\right]^T \hat{x}_k^+$$  \hspace{1cm} (62)

### 4.2. Numerical optimal algorithm for tuning process

In this part, we discuss the numerical optimal algorithm for the tuning process from Eqs. (54) to (57). The downhill simplex method\textsuperscript{13} is chosen as the optimal algorithm for the tuning process. This is a derivative free optimization (DFO) method which uses geometric relationships to aid in finding function minimums. One distinct advantage of this method is that it does not require the derivative of the function. Therefore it works well with a strongly nonlinear cost function.

In this approach, the RTSEKF needs to tune just one parameter, the positive scale factor $\xi$. This scale factor is modeled as

$$\xi_k = 10^{-1} \exp(\gamma)(v_k^+)^T v_k^+$$  \hspace{1cm} (63)

where $\gamma$ is tuning parameter, and it can be any arbitrary number in $\mathbb{R}$. Note that even though the tuning parameter $\gamma$ is a constant number, the scale factor $\xi_k$ is still a variable because of the residual vector part, $v_k^+$. There is only one parameter needed to tune. Therefore, the downhill simplex has two vertexes: $X_i \equiv [\gamma]$, and $i = 1, 2$. Each vertex has a function value that is exactly the same as the cost function defined in Eq. (57). The goal of this algorithm is then to move the simplex away from points with a larger value. The downhill simplex algorithm can employ several methods for moving the simplex downhill. These methods include reflection, expansion, contraction and reduction. The chosen one depends on both data known from the current simplex location as well as data from previous moves.

To implement the downhill simplex for RTSEKF, it is necessary to convert all equations from Eqs. (54) to (57) into the equivalence function, $F(X)$. This equivalence function also returns the same scalar value of the cost function as in Eq. (57). The downhill simplex loop is repeated until the difference between the maximum and minimum of cost functions at all vertexes are smaller than a predefined number $\epsilon$.

### 5. Numerical Simulation Results

In this section, the proposed method and three other conventional methods (including SEKF, EKF and UKF) are simulated with the same data for the attitude determination system (ADS) of a microsatellite named TSUBAME.\textsuperscript{14} TSUBAME is the fourth satellite developed in the Laboratory for Space Systems (LSS) at the Tokyo Institute of Technology and Institute of Space and Astronautical Science (ISAS/JAXA). It is a 50 kg demonstration microsatellite for Earth and astronomical observation technology. In this paper, we consider three low-cost types of attitude sensors in the TSUBAME including MEMS GYRO, three-axis magnetometer (TAM) and Sun Acquisition Sensor (SAS). The main specifications of all sensors are shown in Table 1.

All simulation scenarios are implemented as the Monte Carlo simulation with 100 runs in which the initial attitude error and gyro bias on each axis are randomly generated. The initial values of estimated state, estimated bias vectors and error covariance matrices for all filters are defined as

$$\hat{x}_0 = \hat{x}_0 = \left[0_{6 \times 1}\right]$$  \hspace{1cm} (64)

$$\hat{b}_0 = \hat{b}_0 = \left[0_{6 \times 1}\right]$$  \hspace{1cm} (65)

$$P_x(0) = P_{x0} = \begin{bmatrix} 10^{-1}I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 10^{-3}I_{3 \times 3} \end{bmatrix}$$  \hspace{1cm} (66)

$$P_b(0) = P_{b0} = \begin{bmatrix} I_{2 \times 2}b_{\text{ref}} \end{bmatrix}$$  \hspace{1cm} (67)

The process noise and measurement noise covariance matrices are selected as

$$Q_x = \frac{\Delta t}{2} \begin{bmatrix} \sigma_x^2 \Delta t + \frac{1}{3} \sigma_u^2 \Delta t^3 & \frac{1}{2} \sigma_u^2 \Delta t^2 \\ -\frac{1}{2} \sigma_u^2 \Delta t^2 & \sigma_u^2 \Delta t \end{bmatrix} I_{3 \times 3}$$  \hspace{1cm} (68)

$$Q_b = \frac{\Delta t}{2} \begin{bmatrix} 10^{-5} \sigma_{\text{TAM}} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 10^{-7} \sigma_{\text{SAS}} I_{3 \times 3} \end{bmatrix}$$  \hspace{1cm} (69)

$$R = \begin{bmatrix} \sigma_{\text{TAM}} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & \sigma_{\text{SAS}} I_{3 \times 3} \end{bmatrix}$$  \hspace{1cm} (70)

where $\sigma_{\text{TAM}}$ and $\sigma_{\text{SAS}}$ are the standard deviation of TAM and

| Sensor | Specification | Value |
|--------|---------------|-------|
| GYRO  | Accuracy ($3\sigma$) | 144 deg/$\sqrt{h}$ |
|       | Gyro bias error ($3\sigma$) | $144 \times 10^{-3}$ deg/$\sqrt{h}$ |
|       | Max. update rate (Hz) | $> 100$ |
| TAM   | Accuracy ($3\sigma_{\text{TAM}}$) | 40 nT |
|       | Max. update rate (Hz) | 100 |
| SAS   | Accuracy ($3\sigma_{\text{SAS}}$) | 0.025 (unit vector) |
|       | Max. update rate (Hz) | 100 |
SAS noises and $\sigma_u$ and $\sigma_v$ are introduced in the gyro model part.

The main simulation parameters are summarized in Table 2.

In the first simulation scenario, with small initial attitude error and the constant, randomly generated TAM and SAS bias vectors, there is not a great difference between estimated errors of the four methods. Figure 1 shows the simulated results of this scenario; from the figure we can also determine the calculation cost for one step (in ms), average and standard deviation (STD) of the estimated errors (in degree) in figure legends. Note for all simulation scenarios, the calculation cost is measured for the simulation in Matlab 2012a running on a computer. The average and STD value of estimated error are the average values of the last 100 s for all 100 simulation times.

Figure 2 shows the simulated results of the second simulation scenario in the first 1,500 s. In this scenario, the initial attitude error is generated within 180 degrees in each axis; other conditions remain as the first scenario. In this simulation, the RTSEKF clearly shows its advantage in convergence speed compared with the UKF, EKF or SEKF. The RTUKF converts to the stable error within 50 s while the UKF does so in 600 s, the EKF in 800 s and the SEKF in more than 1,000 s. At the end of the simulation, all filters’ errors are converted to the same accuracy level.

Regarding the second simulation scenario, Fig. 3 shows the values of the tuning parameter $\gamma$ and corresponding scale factor $\zeta$, which are optimized by the downhill simplex method.

Figure 4 shows the simulated results of the third simulation scenario. In this scenario, the initial attitude error is generated within 90 degrees in each axis. The sensor bias values are variable by time as

$$
\begin{align*}
    b_{\text{TAM}}(k) &= b_{\text{TAM}}(0) \left[ 1 + 0.6 \sin \left( \frac{2\pi \times k \times dT}{90 \times 30} \right) \right] \\
    &\quad + \text{randn} \left( \frac{\sigma_{\text{TAM}}}{5} \right) \\
    b_{\text{SAS}}(k) &= b_{\text{SAS}}(0) \left[ 1 + 0.7 \sin \left( \frac{2\pi \times k \times dT}{90 \times 30} + \frac{\pi}{3} \right) \right] \\
    &\quad + \text{randn} \left( \frac{\sigma_{\text{SAS}}}{5} \right)
\end{align*}
$$

(71)

Table 2. Simulation parameters.

| Specification | 1st | 2nd | 3rd | 4th |
|--------------|-----|-----|-----|-----|
| Angular rate (deg/s) | [0.1 – 0.1 – 5]^T | [0.1 – 0.1 – 5]^T | [0.1 – 0.1 – 5]^T | [0.1 – 0.1 – 0.1]^T |
| Max. initial attitude error (deg) | 20 | 180 | 90 | 180 |
| Max. gyro bias (deg/h) | 14,400 | 14,400 | 14,400 | 14,400 |
| Max. TAM bias (nT) | 500 | 500 | 500 | 500 |
| Max. SAS bias (unit vector) | 0.3 | 0.3 | 0.3 | 0.3 |
| Gyro sampling rate (Hz) | 5 | 5 | 5 | 5 |
| TAM&SAS samp. rate (Hz) | 5 | 5 | 5 | 5 |
| TAM&SAS bias model* | 1 | 1 | 2 | 3 |
| Small number $\epsilon$ (downhill) | $10^{-2}$ | $10^{-2}$ | $10^{-2}$ | $10^{-2}$ |
| Simulation duration (h) | 0.5 | 1 | 2 | 2 |

*TAM&SAS bias model type: 1: randomly generated and kept as constant, 2: randomly generated and variable by time, 3: randomly generated and variable as random walk.

Fig. 1. Estimated error of the first simulation scenario.

Fig. 2. Estimated error of the second simulation scenario.
where \( \text{randn}() \) is a zero-mean Gaussian white noise generating function. The other conditions remain as the first scenario. In this simulation, the RTSEKF also shows its advantage to adapt with the time-variable sensor bias model. At the end of simulation, where the biases values of TAM and SAS are maximized, the estimated error of the RTSEKF is only approximately 5% of the SEKF, 10% of the EKF and 12% of the UKF. The TAM bias values of the last simulation time (100th) are shown in Fig. 5.

Figure 6 shows the simulated results of the fourth simulation scenario. In this scenario, the initial attitude error is generated within 180 degrees in each axis. The sensor bias values are modeled as the random walk noises, which are available on the TSUBAME sensor’s models.

\[
\sigma_{\text{RW TAM}}(k) = \sqrt{10^{-3} \times k \times dT} \\
\sigma_{\text{TAM}}(k) = b_{\text{TAM}}(k - 1) + \text{randn}\left[\sigma_{\text{RW TAM}}(k)\right] \\
b_{\text{TAM}}(0) = \text{randn}(500/3) \\
\]

The other conditions remain as in the first scenario. In this simulation, the RTSEKF also shows its advantage to adapt with the random walk sensor bias model. At the end of simulation, where the biases values of TAM and SAS are maximized, the estimated error of the RTSEKF is only approximately 30% of the SEKF, 56% of the EKF, and 60% of the UKF. The TAM bias values of the last simulation time are shown in Fig. 7.

Table 3 shows the comparison of all methods in all four simulation scenarios. From this table we can draw some
conclusions. Firstly, in the case of constant-random sensor biases (1st, 2nd scenarios), estimated accuracy of the four methods are the same but the convergence speed of the RTSEKF is clearly the fastest one. Secondly, in the case of unpredicted sensor bias models (3rd, 4th scenarios), the RTSEKF shows advantages in both convergence speed and estimated accuracy. Thirdly, when the sampling frequency of measurement sensors is lower than the sampling frequency of the gyroscope (i.e., the filters perform the measurement update process less than the time update process, such as in the fourth scenario case where there are five time update processes for one measurement update process), the calculation cost of the UKF is still almost the same while those of the other methods are strongly reduced. This issue is caused by sigma points generating processes of the UKF. In addition, the computation cost of the RTSEKF can be controlled by choosing the accuracy of the optimal process through the small number in the looping condition inside downhill simplex method.

One more interesting point can be achieved during analysis of the value of the tuning parameter in Fig. 3. When the estimated result is converted into a stable value, the tuning parameter is almost "vibration" around to . Therefore, for several applications, in which the effect of bias models on estimated results is not so much or the behavior of bias models are known, the reduction form of the RTSEKF can be used. The reduction form of the RTSEKF is called the adaptive separate-bias extended Kalman filter (ASEKF). In this reduction method, the automated numerical optimization part is removed. Only several main points of the tuning parameter are checked and the optimal point is selected. This reduction helps the filter achieve a reasonable estimated accuracy with a controllable calculation cost. Figure 8 shows a simulation using this ASEKF method with three checking points of the tuning parameter. From the simulation results, the calculation cost of ASEKF is only approximately 1.5 times that of the EKF but the estimated result is much better. The simulation conditions here are the same as in the third simulation scenario.

6. Conclusion

This paper presented a new filter algorithm named the RTSEKF. The proposed filter used the downhill simplex method to tune a filter parameter by minimizing the cost function, which was calculated from the difference between the predicted and measurement values. The tuning parameter was inside the calculation process of the Kalman gain in the "bias" filter. This RTSEKF had better adaptive characteristics with large initial attitude error and unpredicted sensor bias models, which usually exist in practical applications. Therefore, it had higher accuracy and faster convergence speed than the conventional methods like SEKF, EKF and event UKF in those cases. The computation cost of RTSEKF could be controlled by choosing the accuracy of the optimal process or using the filter reduction form, ASEKF. Illustrated by the numerical simulation results, it was evident that there always exists the optimal value of the tuning parameter to minimize the cost function in each process of the filter. Moreover, with an adaptive mechanism applied in the general part of the filter, this proposed method may also be expanded for other applications in the field of optimal estimation of dynamic systems.

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