Abstract

The Pisano-Pleitez-Frampton 3-3-1 model is revisited here within the framework of the general method for solving gauge models with high symmetries. This exact algebraical approach - proposed several years ago by one of us - was designed to include a minimal Higgs mechanism that spontaneously breaks the gauge symmetry up to the universal $U(1)_{em}$ electromagnetic one and, consequently, to supply the mass spectrum and the couplings of the currents for all the particles in the model. We prove in this paper that this powerful tool, when is applied to the PPF 3-3-1 model, naturally recovers the whole Standard Model phenomenology and, in addition, predicts - since a proper parametrization is employed - viable results such as: (i) the exact expressions for the boson and fermion masses, (ii) the couplings of the charged and neutral currents and (iii) a plausible neutrino mass pattern. A generalized Weinberg transformation is implemented, while the mixing between the neutral bosons $Z$ and $Z'$ is performed as a necessary step by the method itself. Some phenomenological consequences are also sketched, including the strange possibility that simultaneously $m(Z) = m(Z')$ and $m(W) = m(V)$ hold.

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1 Introduction

Among the various extensions of the Standard Model (SM) that emerged in the last decades in order to incorporate new phenomenology in the electro-weak sector (such as neutrino oscillations, extra-neutral bosons), or explain some features (such as mass hierarchy, fermion families replication, CP-phase question, etc), the well-known Pisano-Pleitez-Frampton (PPF) model earned a wide reputation. It is based on the gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_Y$ (hereafter 3-3-1) that undergoes a spontaneously symmetry breakdown (SSB) in two steps in order to provide with masses all the particles in the model. The model was first proposed [1, 2] at the beginning of the ’90s and
developed in the coming years with important results regarding topics related to the flavor changing neutral currents (FCNC) \[3\] - \[7\] - including a proper GIM mechanism for their suppression \[3\] - \[11\], the CP-phase issue \[6\] - \[11\], the mass generation in the fermion sector \[12\], and the quest for an appropriate scalar sector \[13\] - \[17\] for a realistic SSB. In order to make it a suitable gauge theory, the 3-3-1 class of models has to be anomaly-free. A systematic approach to the general case of the anomaly cancelation in 3-3-1 models can be found in Refs. \[18\] - \[19\].

These different ways of phenomenologically investigating the 3-3-1 models seemed to explain only particular issues. Therefore, different approximations were employed to solve certain troublesome aspects. Since a global view on the gauge models with high symmetries was still lacking. This state of affairs called for an elegant and systematic approach devoted to gauge theories with high symmetries in order to make them able to supply general predictions, once a parameter set is chosen from the very beginning of the calculus. Such an efficient tool was proposed by Cotăescu in Ref. \[20\] for the general case of a theory with the electro-weak sector’s symmetry given by the gauge group \(SU(n)_L \otimes U(1)_Y\) that undergoes a spontaneously breakdown up to the universal electromagnetic one \(U(1)_{em}\) in one step only. For this purpose, the main parameters of such a theory play the role of orthonormalization coefficients in the geometrized scalar sector of the model, so that only one physical Higgs real field finally survives the SSB. The method was successfully applied in a recent series of papers \[21\] - \[25\] by Palcu in the particular case of the 3-3-1 model with right-handed neutrinos, a particular version of 3-3-1 models initially proposed in \[3\], and ever since championed by Long and his collaborators \[26\] - \[33\]. The promising results given by the general method in that case encouraged us to revisit the PPF model in order to respectively reveal its rich phenomenology and embed the neutrino masses in it.

The general method is briefly reviewed in Sec. 2 and subsequently is applied to the PPF 3-3-1 model. We prove that it supplies viable results concerning both the boson mass spectrum and the neutral currents of the model (Sec. 3), and even if it is able to generate fermion masses (Sec. 4) - including a suitable neutrino mass pattern - in accordance with the available data \[34\]. In Sec. 5 we give some conclusions and phenomenological aspects of the results obtained, outlining particular values of the main free parameter that can supply simultaneously \(m(Z) = m(Z')\) and \(m(W) = m(V)\) which could, in turn, explain why those new bosons were not yet experimentally discovered.

2 The General Method of Solving Gauge Models

In this section we recall the main results of the method of exactly solving generalized \(SU(n)_L \otimes U(1)_Y\) electro-weak gauge models with a special type of Higgs mechanism proposed in Ref. \[20\].

2.1 \(SU(n)_L \otimes U(1)_Y\) electro-weak gauge models

In our general approach, the basic piece involved in the gauge symmetry is the group \(SU(n)\). Its two fundamental irreducible unitary representations (irreps) \(n\) and \(n^*\) play a
crucial role in constructing different classes of tensors of ranks \((r, s)\) as direct products like \((\otimes (\otimes n^r)^s)\). These tensors have \(r\) lower and \(s\) upper indices for which we reserve the notation, \(i, j, k, \cdots = 1, \cdots, n\). As usually, we denote the irrep \(\rho\) of \(SU(n)\) by indicating its dimension, \(n_\rho\). The \(su(n)\) algebra can be parameterized in different ways, but here it is convenient to use the hybrid basis of Ref. [20] consisting of diagonal generators of the Cartan subalgebra, \(D_i\), labeled by indices \(i, j, \cdots\) ranging from 1 to \(n - 1\), and the generators \(E^i_j = H^i_j/\sqrt{2}, i \neq j\), related to the off-diagonal real generators \(H^i_j\) [35 [36]. This way the elements \(\xi = D_i \xi^i + E^i_j \xi^j \in su(n)\) are now parameterized by \(n - 1\) real parameters, \(\xi^i\), and by \(n(n - 1)/2\) \(c\)-number ones, \(\xi^i_j = (\xi^i)^*\), for \(i \neq j\). The advantage of this choice is that the parameters \(\xi^i_j\) can be directly associated to the \(c\)-number gauge fields due to the factor \(1/\sqrt{2}\) which gives their correct normalization. In addition, this basis exhibit good trace orthogonality properties,

\[
Tr(D_i D_j) = \frac{1}{2} \delta_{ij}, \quad Tr(D_i E^i_j) = 0, \quad Tr(E^i_j E^k_l) = \frac{1}{2} \delta^i_j \delta^k_l. \tag{1}
\]

When we consider different irreps, \(\rho\) of the \(su(n)\) algebra we denote \(\xi^\rho = \rho(\xi)\) for each \(\xi \in su(n)\) such that the corresponding basis-generators of the irrep \(\rho\) are \(D_i^\rho = \rho(D_i)\) and \(E^i_j^\rho = \rho(E^i_j)\).

The \(U(1)_y\) transformations are nothing else but phase factor multiplications. Therefore - since the coupling constants \(g\) for \(SU(n)_L\) and \(g'\) for the \(U(1)_y\) are assigned the transformation of the fermion tensor \(L^\rho\) with respect to the gauge group of the theory reads

\[
L^\rho \rightarrow U(\xi_0^0, \xi^0) L^\rho = e^{-i(g\xi^0 + g'y_{ch}\xi^0)} L^\rho \tag{2}
\]

where \(\xi = su(n)\) and \(y_{ch}\) is the chiral hypercharge defining the irrep of the \(U(1)_y\) group parametrized by \(\xi^0\). For simplicity, the general method deals with the character \(y = y_{ch} g'/g\) instead of the chiral hypercharge \(y_{ch}\), but this mathematical artifice does not affect in any way the results. Therefore, the irreps of the whole gauge group \(SU(n)_L \otimes U(1)_y\) are uniquely determinate by indicating the dimension of the \(SU(n)\) tensor and its character \(y\) as \(\rho = (n_\rho, y_\rho)\).

In general, the spinor sector of our models has at least a part (usually the leptonic one) which is put in pure left form using the charge conjugation. Consequently this includes only left components, \(L = \sum_\rho \otimes L^\rho\), that transform according to an arbitrary reducible representation of the gauge group. The Lagrangian density of this part of the spinor sector may have the form

\[
\mathcal{L}_{\mathcal{S}_0} = \frac{i}{2} \sum_\rho \overline{\mathcal{T}^\rho} \partial \bar{\mathcal{T}}^\rho L^\rho - \frac{1}{2} \sum_{\rho \rho'} (\mathcal{T}^\rho \chi^{\rho \rho'} (L^{\rho'})^c + \text{h.c.}). \tag{3}
\]

Bearing in mind that each left-handed multiplet transforms as \(L^\rho \rightarrow U^\rho(\xi^0, \xi)L^\rho\) we understand that \(\mathcal{L}_{\mathcal{S}_0}\) remains invariant under the global \(SU(n)_L \otimes U(1)_y\) transformations if the blocks \(\chi^{\rho \rho'}\) transform like \(\chi^{\rho \rho'} \rightarrow U^\rho(\xi^0, \xi) \chi^{\rho \rho'} (U^{\rho'}(\xi^0, \xi))^T\), according to the representations \((n_\rho \otimes n_{\rho'}, y_\rho + y_{\rho'})\) which generally are reducible. These blocks will give rise to the Yukawa couplings of the fermions with the Higgs fields. The
spinor sector is coupled to the standard Yang-Mills sector constructed in usual manner by gauging the $SU(n)_L \otimes U(1)_Y$ symmetry \cite{20}. To this end we introduce the gauge fields $A_0^\mu = (A_0^\mu)^*$ and $A_\mu = A_\mu^+ \in su(n)$. Furthermore, the ordinary derivatives are replaced in Eq. (3) by the covariant ones, defined as $D_\mu L^\rho = \partial_\mu L^\rho - ig(A_\mu^\rho + y_\rho A_0^\mu) L^\rho$ thus arriving to the interaction terms of the spinor sector.

The Higgs sector, organized as the so called minimal Higgs mechanism \cite{20}, is able to produce maximal effects but with only one remaining Higgs neutral field, just as in SM. This sector consists of $n$ Higgs multiplets $\phi^{(1)}$, $\phi^{(2)}$, ... $\phi^{(n)}$ satisfying the orthogonality condition $\phi^{(i)} + \phi^{(j)} = y^{(i)} \delta_{ij}$ in order to eliminate the unwanted Goldstone bosons that could survive the SSB. $\phi$ is a gauge-invariant real scalar field while the Higgs multiplets $\phi^{(i)}$ transform according to the irreps $(n, y^{(i)})$ whose characters $y^{(i)}$ are arbitrary numbers that can be organized into the diagonal matrix

$$Y = \text{diag} \left( y^{(1)}, y^{(2)}, \ldots, y^{(n)} \right).$$

The Higgs sector is constructed by resorting to the parameter matrix

$$\eta = \text{diag} \left( \eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(n)} \right)$$

with the property $\text{Tr}(\eta^2) = 1 - \eta_0^2$. It will play the role of the metric in the kinetic part of the Higgs Lagrangian density which reads

$$L_H = \frac{1}{2} \eta_0^2 \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \sum_{i=1}^n \left( \partial_\mu \phi^{(i)} \partial^\mu \phi^{(i)} \right) - \frac{1}{2} \sum_{i=1}^n \eta^{(i)} \left( D_\mu \phi^{(i)} \right) - V(\phi)$$

(6)

where $D_\mu \phi^{(i)} = \partial_\mu \phi^{(i)} - ig(A_\mu + y^{(i)} A_0^\mu) \phi^{(i)}$ are the covariant derivatives of the model and $V(\phi)$ is the scalar potential generating the SSB of the gauge symmetry \cite{20}. This is assumed to have an absolute minimum for $\phi = \langle \phi \rangle \neq 0$ that is, $\phi = \langle \phi \rangle + \sigma$ where $\sigma$ is the unique surviving physical Higgs field. Therefore, one can always define the unitary gauge where the Higgs multiplets, $\tilde{\phi}^{(i)}$, have the components

$$\tilde{\phi}^{(i)}_k = \delta_{ik} \phi = \delta_{ik} (\langle \phi \rangle + \sigma).$$

(7)

This will be of great importance when the fermion masses will be computed, due to the fact that the fermion mass terms - provided by Eq. (3) via this minimal Higgs mechanism (mHm) - exhibit the Yukawa traditional form only when the theory is boosted towards the unitary gauge.

### 2.2 Neutral bosons

A crucial goal is now to find the physical neutral bosons with well-defined properties. This must start with the separation of the electromagnetic potential $A_\mu^{em}$ corresponding to the surviving $U(1)^{em}$ symmetry. We have shown that the one-dimensional subspace of the parameters $\xi^{em}$ associated to this symmetry assumes a particular direction in the parameter space $\{\xi^0, \xi^1\}$ of the whole Cartan subalgebra. This is uniquely determined by the $n - 1$ - dimensional unit vector $\nu$ and the angle $\theta$ giving the subspace equations
\( \xi^0 = \xi^{em} \cos \theta \) and \( \xi^i = \nu_i \xi^{em} \sin \theta \). On the other hand, since the Higgs multiplets in unitary gauge remain invariant under \( U(1)^{em} \) transformations, we must impose the obvious condition \( D_i \xi^i + Y \xi^0 = 0 \) which yields

\[
Y = -D_i \nu^i \tan \theta \equiv -(D \cdot \nu) \tan \theta. \tag{8}
\]

In other words, the new parameters \((\nu, \theta)\) determine all the characters \(y^{(i)}\) of the irreps of the Higgs multiplets. For this reason these will be considered the principal parameters of the model and therefore one deals with \( y^{(i)} \) which has \( n - 1 \) independent components instead of \( n - 1 \) parameters \( y^{(i)} \).

Under these circumstances, the generating mass term

\[
g^2 \langle \phi \rangle^2 Tr \left[ (A^\mu + Y A^0_\mu) \eta^2 (A^\mu + Y A^0\mu) \right], \tag{9}
\]

depends now on the parameters \( \theta \) and \( \nu_i \). The neutral bosons in Eq. \( \langle \phi \rangle \) being the electromagnetic field \( A^{em}_\mu \) and the \( n - 1 \) new ones, \( A^{i}_\mu \), which are the diagonal bosons remaining after the separation of the electromagnetic potential \([20]\).

This term straightforwardly gives rise to the masses of the non-diagonal gauge bosons

\[
M^i = \frac{g^2}{2} \langle \phi \rangle^2 Tr \left[ (A^\mu + Y A^0_\mu) \eta^2 (A^\mu + Y A^0\mu) \right], \tag{10}
\]

while the masses of the neutral bosons \( A^{i}_\mu \) have to be calculated by diagonalizing the matrix

\[
(M^2)_{ij} = \langle \phi \rangle^2 Tr(B_i B_j), \tag{11}
\]

where

\[
B_i = g \left( D_i + \nu_i (D \cdot \nu) \right) \eta, \tag{12}
\]

As it was expected, \( A^{em}_\mu \) does not appear in the mass term and, consequently, it remains massless. The other neutral gauge fields \( A^{i}_\mu \) have the non-diagonal mass matrix \([11]\). This can be brought in diagonal form with the help of a new \( SO(n-1) \) transformation, \( A^{i}_\mu = \omega^i_j Z^j_\mu \), which leads to the physical neutral bosons \( Z^i_\mu \) with well-defined masses. Performing this \( SO(n-1) \) transformation the physical neutral bosons are completely determined. The transformation

\[
A^0_\mu = A^{em}_\mu \cos \theta - \nu_i \omega^i_j Z^j_\mu \sin \theta, \quad A^k_\mu = \nu^k A^{em}_\mu \sin \theta + \left( \delta^k_i - \nu^k \nu_i (1 - \cos \theta) \right) \omega^i_j Z^j_\mu. \tag{13}
\]

which switches from the original diagonal gauge fields, \((A^0_\mu, A^i_\mu)\) to the physical ones, \((A^{em}_\mu, Z^i_\mu)\) is called the generalized Weinberg transformation (gWt).

The next step is to identify the charges of the particles with the coupling coefficients of the currents with respect to the above determined physical bosons. Thus, we find that the spinor multiplet \( L^\rho \) (of the irrep \( \rho \)) has the following electric charge matrix

\[
Q^\rho = g \left[ (D^\rho \cdot \nu) \sin \theta + y_\rho \cos \theta \right], \tag{14}
\]
and the $n-1$ neutral charge matrices

$$Q^\rho(Z^i) = g \left[D^\rho_k - \nu^\rho_k(D^\rho \cdot \nu)(1 - \cos \theta) - y^\rho \nu^\rho_k \sin \theta \right] \omega^k_i,$$  \hspace{1cm} (15)$$

corresponding to the $n-1$ neutral physical fields, $Z^i_\mu$. All the other gauge fields, namely the charged bosons $A^i_{\mu}$, have the same coupling, $g/\sqrt{2}$, to the fermion multiplets.

At this point one can change again the parametrization by using the electrical charges $q_i$ of the fundamental multiplet $(n,0)$ given by

$$Q \equiv \text{diag}(q_1, q_2, \cdots, q_n) = g(D \cdot \nu) \sin \theta,$$  \hspace{1cm} (16)$$

instead of the parameters $(g, \nu^i)$ but keeping the angle $\theta$ as the principal parameter of the model in order to remain in the spirit of the SM. This way $g$ and $\nu^i$ have to be expressed in terms of $q_i$ using the formulas $g \nu^i \sin \theta = 2 \text{Tr}(D_i Q)$ and $g^2 \sin^2 \theta = 2 \text{Tr}(Q^2)$. Moreover, the matrix $Q$ can be written now as $Y = -Q \tan \theta / \sqrt{2 \text{Tr}(Q^2)}$. Finally we have to replace $y^\rho$ with $y^\rho_{ch}(g'/g)$ in order to deal with the veritable chiral character of $\mathbb{U}(1)_Y$. The quantity $y_{ch}$ becomes the usual chiral hypercharge if we take

$$g' = e g \frac{\tan \theta}{\sqrt{2 \text{Tr}(Q^2)}},$$  \hspace{1cm} (17)$$

where $e$ is the elementary electric charge. This supplies at the same time the correct relation between the two couplings $g$ and $g'$ - once the $\theta$-angle is given as a function of the $\theta_W$ from SM - without resorting to any other supplemental condition. Particularly, with this assignment the chiral hypercharges of the Higgs multiplets take the simpler form $Y_{ch} = -Q/e$.

### 3 The Pisano-Pleitez-Frampton 3-3-1 Model Revisited

The general method - constructed in Ref. [20] and briefly presented in the above section - is based on the following assumptions in order to give viable results when it is applied to concrete models:

(I) the spinor sector must be put (at least partially) in pure left form using the charge conjugation (see for details Appendix B in Ref. [20])

(II) a minimal Higgs mechanism with arbitrary parameters ($\eta_0, \eta$) satisfying the condition $\text{Tr}(\eta^2) = 1 - \eta_0^2$ and giving rise to traditional Yukawa couplings in unitary gauge is employed

(III) the coupling constant, $g$, is the same with the first one of the SM

(IV) at least one $Z$-like boson should satisfy the mass condition $m_Z = m_W/\cos \theta_W$ established in the SM and experimentally confirmed.

Bearing in mind all these necessary ingredients, we proceed to solving the particular 3-3-1 model of PPF [1][2] by imposing from the very beginning the set of parameters we will work with.
3.1 The structure of the model

In what follows we denote the irreps of the electro-weak model under consideration here by \( \rho = (n_\rho, y_{ch}^\rho) \) indicating the veritable chiral hypercharge \( y_{ch} \) instead of \( y \). Therefore, the multiplets of the PPF 3-3-1 model will be denoted by \( (n_{\text{color}}, n_\rho, y_{ch}^\rho) \).

With this notation the irreps of the spinor sector are:

**Lepton families**

\[
\begin{align*}
  f_{\alpha L} & = \left( e^c_\alpha, e_\alpha, \nu_\alpha \right)_L \sim (1, 3, 0) \\
  (e_{\alpha L})^c & \sim (1, 1, -1)
\end{align*}
\]

**Quark families**

\[
\begin{align*}
  Q_{4L} & = \left( \begin{array}{c} J_i \\ u_i \\ d_i \end{array} \right)_L \sim (3, 3^*, -1/3) \\
  Q_{3L} & = \left( \begin{array}{c} J_i \\ -b \\ t \end{array} \right)_L \sim (3, 3, +2/3) \\
  (b_L)^c, (d_{iL})^c & \sim (3, 1, -1/3) \\
  (t_L)^c, (u_{iL})^c & \sim (3, 1, +2/3) \\
  (J_{3L})^c & \sim (3, 1, +5/3) \\
  (J_{iL})^c & \sim (3, 1, -4/3)
\end{align*}
\]

with \( \alpha = 1, 2, 3 \) and \( i = 1, 2 \). In the representations presented above we assumed, like in majority of the papers in the literature, that the third generation of quarks transforms differently from the other two ones. This could explain the unusual heavy masses of the third generation of quarks, and especially the uncommon properties of the top quark. The capital letters \( J \) denote the exotic quarks included in each family.

With this assignment the fermion families cancel all the axial anomalies by just an interplay between them, although each family remains anomalous by itself. Thus, the renormalization criteria are fulfilled and the method is validated once more.

Note that one can add at any time sterile neutrinos - i.e. right-handed neutrinos \( \nu_{\alpha R} \sim (1, 1, 0) \) that could pair in the neutrino sector of the Lagrangian density with left-handed ones in order to generate eventually tiny Dirac or Majorana masses by means of an adequate see-saw mechanism. These sterile neutrinos do not affect anyhow the anomaly cancelation, since all their charges are zero. Moreover, their number is not restricted by the number of flavors in the model.

Subsequently, we will use the standard generators \( T_a = \lambda_a/2 \) of the \( su(3) \) algebra connected to the usual Gell-Mann matrices which are differently displayed from those of Ref. [20]. So, the Hermitian diagonal generators of the Cartan subalgebra are

\[
D_1 = T_3 = \frac{1}{2} \text{diag}(1, -1, 0), \quad D_2 = T_8 = \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2).
\]

In this basis the gauge fields are \( A^0_\mu \) and \( A_\mu \in su(3) \) that is

\[
A_\mu = \frac{1}{2} \left( \begin{array}{ccc} A^3_\mu + A^8_\mu / \sqrt{3} & \sqrt{2} U_\mu & \sqrt{2} W_\mu \\
\sqrt{2} U_\mu^* & -A^3_\mu + A^8_\mu / \sqrt{3} & \sqrt{2} V_\mu^* \\
\sqrt{2} W_\mu^* & \sqrt{2} V_\mu & -2 A^8_\mu / \sqrt{3} \end{array} \right),
\]

\[
(23)
\]
Apart from the charged Weinberg bosons, $W$, there are new charged bosons, $V$ and $U$, among them $U$ is doubly charged - the so called "bilepton" - coupling different chiral states of the same charged lepton.

For our purpose it is convenient exploit the parametrization based on the $\theta$ angle and the electric charges of the lepton multiplet. The latter are supposed to be $Q = e \text{ diag}(1, -1, 0)$. On the other hand, Eq. (14) allows us to identify $\nu = (1, 0)$ and $g \sin \theta = 2e$. As long the SM condition $e = g \sin \theta_W$ holds, one obtains $\sin \theta = 2 \sin \theta_W$. It remains to observe that we have $Y_{ch} = -Q/e = \text{diag}(-1, 1, 0)$ which means that the irreps of the Higgs sector are $\phi^{(1)} \sim (3, -1)$, $\phi^{(2)} \sim (3, 1)$ and $\phi^{(3)} \sim (3, 0)$. Note that the Higgs components in unitary gauge satisfy Eq. (17) only if this numeration of the Higgs multiplets is kept.

### 3.2 Boson mass spectrum

The masses of both the neutral and charged bosons depend on the choice of the matrix $\eta$ whose components are free parameters. Here it is convenient to assume the following matrix

$$\eta^2 = (1 - \eta_0^2) \text{diag} \left(1 - a, a + b \frac{1}{2}, a - b \frac{1}{2}\right)$$

where, for the moment, $a$ and $b$ are arbitrary non-vanishing real parameters. Obviously, $\eta_0, a \in [0, 1)$. Note that with this parameter choice the condition (II) is accomplished. Under these circumstances, the mass matrix of the neutral bosons Eq. (11) reads

$$M^2 = m^2 \left(\begin{array}{c}
\frac{1}{\cos^2 \theta} \left(1 - \frac{1}{2}a + \frac{1}{2}b\right) \\
\frac{1}{\sqrt{3} \cos \theta} \left(1 - \frac{3}{2}a - \frac{1}{2}b\right) \\
\frac{1}{\sqrt{3} \cos \theta} \left(1 + \frac{3}{2}a - \frac{1}{2}b\right)
\end{array}\right)$$

with $m^2 = g^2 \langle \phi \rangle^2 (1 - \eta_0^2)/4$. Let us observe that the condition (IV) is fulfilled if and only if $b/a = -3 \tan^2 \theta_W$. That is, one remains with only one parameter - say $a$. In addition, there are terms which become singular for $\cos \theta = 0$ which corresponds to the value $\sin^2 \theta_W = 1/4$. On this reason the Weinberg angle is restricted in this particular model to values of $\sin^2 \theta_W$ less than $1/4$, which is in good accord to experimental measurements on it [34].

The mass spectrum of the gauge bosons (without insisting on the computing details) looks like

$$m^2(W) = m^2(Z) \cos^2 \theta_W = m^2a,$$

$$m^2(Z') = \frac{m^2}{1 - 4 \sin^2 \theta_W} \left\{ \frac{4}{3} \cos^2 \theta_W - a \left[1 - (1 - 4 \sin^2 \theta_W) \tan^2 \theta_W\right]\right\},$$

$$m^2(V) = m^2 \left[1 - \frac{a}{2}(1 - 3 \tan^2 \theta_W)\right],$$

$$m^2(U) = m^2 \left[1 - \frac{a}{2}(1 + 3 \tan^2 \theta_W)\right].$$

Obviously, $Z$ is the neutral boson of the SM, while $Z'$ is the new neutral boson of this model.
The mass scale is now just a matter of tuning the parameter $a$ in accordance with the possible values for $\langle \phi \rangle$. However, this mass spectrum exhibits a very strange feature. For the particular value
\begin{equation}
    a = a_c = \frac{2 \cos^2 \theta_W}{3(1 - 2 \sin^2 \theta_W)}
\end{equation}
a critical point arises. At that very value the following equalities $m(Z) = m(Z')$ and $m(W) = m(V)$ are simultaneously fulfilled, while the bilepton mass becomes
\begin{equation}
    m^2(U) = m^2(Z)(1 - 3 \sin^2 \theta_W). \tag{28}
\end{equation}
Numerically speaking if one inserts $\sin^2 \theta_W \sim 0.223$ in Eq. (27) then one gets that the critical point corresponds to $a_c \sim 0.934$ and $m(U) \sim 30 \text{ GeV}/c^2$. This phenomenon could give a plausible explanation for why the new bosons were not yet discovered and precisely weighted in the laboratory. However, although the data [34] suggest $m(Z) < m(Z')$ the possibility outlined above is not definitely ruled out unless an experimental argument is invoked. We are confident that this issue will be elucidated in the near future at LHC, when a precise experimental measurement of the masses of these new bosons predicted by the 3-3-1 theory will be available.

### 3.3 Electric and neutral charges

In the PPF 3-3-1 model under consideration here, assuming the versor choice $\nu = (1, 0)$, we obtain the generalized Wienberg transformation which was designed to reach the physical basis $(A_{em}, Z, Z')$ of the neutral bosons of the model. This reads
\begin{align*}
    A_0^\mu &= A_{em}^\mu \cos \theta - (\omega^1 Z'_\mu + \omega^2 Z_\mu) \sin \theta \\
    A_3^\mu &= A_{em}^\mu \sin \theta + (\omega^1 Z'_\mu + \omega^2 Z_\mu) \cos \theta \\
    A_8^\mu &= \omega^2 Z'_\mu + \omega^1 Z_\mu
\end{align*}
where $\omega$ acting as the required $SO(2)$ rotation. Its components
\begin{equation}
    \omega^1_1 = \omega^2_2 = -\frac{\sqrt{3}}{2 \cos \theta_W}, \quad \omega^1_2 = \omega^2_1 = \frac{1}{2} \sqrt{1 - 3 \tan^2 \theta_W}, \tag{30}
\end{equation}
ensure the diagonal form of the matrix (25). In order to recover all the results of SM and those of Ref. [20] (up to sign) the obvious identification has to be performed: $Z^2 = Z$ and $Z^1 = Z'$. It is worth observing that at the critical point, $a = a_c$, the matrix (25) becomes proportional with the unit matrix $I_{3 \times 3}$ so that the $\omega$-rotation can be arbitrarily chosen, offering thus a supplementary degree of freedom in defining $Z$-bosons. However, in order to avoid here a digression on this subject, we restrict ourselves to keep the rotation (30) at the critical point too, following to discuss about it elsewhere.

All the needed ingredients are now available in order to express the content of the gauge sector, $A_\mu^a + g Y^a_A A_\mu^a / g$, in terms of physical neutral bosons $(A_{em}, Z, Z')$ as well as the charged ones of Eq. (23), namely $(W^\pm, V^\pm, U^2 \pm)$. The latter charged
fields couple the currents of the spinor multiplets $L^\rho$ through the coupling constant $g = e / \sin \theta_W$, while from Eq. (17) straightforwardly results

$$g' = g \frac{\sin \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}. \quad (31)$$

Hereby we have to obtain the correct electric charges of the fermion irreps and, subsequently, the expected neutral charges for all the particles in the theory. In what follows all these coupling coefficients will be written in units of elementary electric charge, $e$. The electric charges of the components of a multiplet obeying the irrep $\rho$ read

$$Q^\rho(A^{cm}) = 2T^\rho_3 + y^\rho_{ch}, \quad (32)$$

while the neutral charges corresponding to the bosons $Z$ and $Z'$ are

$$Q^\rho(Z) = \frac{1}{\sin 2\theta_W} \left[ T^\rho_3 (1 - 4 \sin^2 \theta_W) - T^\rho_s \sqrt{3} - 2y^\rho_{ch} \sin^2 \theta_W \right], \quad (33)$$

$$Q^\rho(Z') = -\frac{\sqrt{1 - 4 \sin^2 \theta_W}}{\sin 2\theta_W} \left( T^\rho_3 \sqrt{3} + T^\rho_s - y^\rho_{ch} \frac{2 \sin^2 \theta_W}{1 - 4 \sin^2 \theta_W} \sqrt{3} \right). \quad (34)$$

It is remarkable that all the coupling coefficients of this model are independent of the parameter $a$ responsible for the boson mass spectrum. Computing the concrete values of these coefficients for all the fermion multiplets is presented in detail in Appendix and the results are displayed in Table.

4 Fermion Masses

Generating fermion masses is one of the most stringent issues in particle physics. This question is addressed in this section within the PPF 3-3-1 model, assuming that the technique of the “classical” Yukawa terms worked very well at SM level, although their couplings remained unrestricted parameters on theoretical ground. These values are exclusively determined by experimental reasons.

4.1 Quark masses

For all the quarks involved in the PPF 3-3-1 model, the traditional Yukawa couplings seem to be sufficient in order to supply their desired masses. That is - with the assignment of the Sec. 3.1. for the representations in the fermion and scalar sectors - one has the following terms in the quark mass sector:

$$G_u \bar{Q}_{1L} \phi^{(2)+} u_R + G_c \bar{Q}_{2L} \phi^{(2)+} c_R + H.c. \quad (35)$$

$$G_d \bar{Q}_{1L} \phi^{(3)+} d_R + G_s \bar{Q}_{2L} \phi^{(3)+} s_R + H.c. \quad (36)$$

$$G_t \bar{Q}_{3L} \phi^{(3)} t_R + G_b \bar{Q}_{3L} \phi^{(2)+} b_R + H.c. \quad (37)$$

$$G_1 \bar{Q}_{1L} \phi^{(1)+} J_R + G_2 \bar{Q}_{2L} \phi^{(1)+} J_R H.c \quad (38)$$

$$G_1 \bar{Q}_{3L} \phi^{(1)} J_{3R} + H.c \quad (39)$$
These terms are assumed to undergo necessary tuning of the complex coupling coefficients \( G_s \) in order to ensure the experimentally observed mass hierarchy \(^{[34]}\) in the quark sector. These coefficients remain - as in the SM - free parameters, once the vacuum expectation values of the scalar field \( \phi \) still has to be established.

At this point, one can identify the mass of each quark as

\[
m(q) = G_q \langle \phi \rangle
\]

where \( q \) in \(^{(40)}\) denotes any of the nine quarks in the model. Note that Eqs. \(^{(40)}\) introduce 9 parameters in the model.

### 4.2 Charged lepton masses

On the other hand, for charged leptons it was argued \(^{[12]}\) that a scalar sextet is a compulsory ingredient in the Yukawa lagrangian in order to have a realistic and consistent mechanism for generating masses.

We build this scalar sextet out of the scalar triplets - already existing in the Higgs sector of the model - as a tensor-like product in the following manner:

\[
S = \phi^{-1} \left( \phi^{(1)} \otimes \phi^{(2)} + \phi^{(2)} \otimes \phi^{(1)} \right)
\]

(41)

It plays the same role as the tensor blocks \( \chi^{\rho \rho'} \) in Eq. \(^{(3)}\). Evidently, \( S \sim (1, 6, 0) \) and thus the generating mass term in the charged leptons sector reads

\[
G_\alpha \bar{f}_{\alpha L} S f_{\alpha L}^c + H.c.
\]

(42)

Hence, consequently the SBB, only positions (12) and (21) in Eq. \(^{(41)}\) will remain non-zero. That is

\[
\langle S \rangle = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \langle \phi \rangle
\]

(43)

The lepton families in the model under consideration here acquire their masses through the above presented coupling terms \(^{(41)}\), since all couplings due to \( S \) get in the unitary gauge the traditional Yukawa form: \( G_\alpha \langle \phi \rangle \bar{e}_{\alpha L} e_{\alpha L}^c \) (according to a Dirac Lagrangian density put in the pure left form - see Appendix B in Ref. \(^{[20]}\)). Therefore, one can identify the mass of the charged lepton as

\[
m(e_\alpha) = G_\alpha \langle \phi \rangle
\]

(44)

Note that Eqs. \(^{(43)}\) introduce 3 more parameters in the model, in addition to those 9 necessary ones in the quark sector.

### 4.3 Neutrino Mass Matrix

Since the phenomenon of neutrino oscillations is an undisputable evidence, all the extensions of the SM must incorporate realistic mechanisms for generating tiny masses
in the neutrino sector of the theory. There are two main lines in the literature to obtain these tiny masses: see-saw mechanisms and radiative corrections. For a detailed overview on theoretical and phenomenological aspects in neutrino physics we refer the reader to several excellent papers published in the recent years [38] - [44].

We propose here a particular approach that naturally calls for the canonical see-saw mechanism. The neutrino mass matrix arises from certain mass terms - regardless they are of the Dirac or Majorana nature - at tree level in the Yukawa sector of the PPF 3-3-1 model. This model allows for both kinds of terms, since one can construct an additional tensor-like product of the form $\phi^{-1} (\phi^{(3)} \otimes \phi^{(3)})$ which leads to Majorana mass terms. For Dirac terms one can introduce terms like $f_{\alpha L}^{(3)} + \nu_{\beta R}$. A natural assumption here is to employ different couplings ($G'$s) in the Dirac sector, while the same parameters involved in the charged lepton sector ($G$s) are employed in the Majorana sector. This is quite a natural option, since both the charged lepton masses and the neutrino Majorana ones are supplied by some tensor-like products of scalar triplets.

**Majorana mass terms** The model allows for a pure Majorana mass matrix whose elements can be constructed as a tensor-like product in the manner

$$G_{\alpha\beta} f_{\alpha L} [\phi^{-1} (\phi^{(3)} \otimes \phi^{(3)})] f_{\beta L} + H.c. \quad (45)$$

Such terms develop the well-known Yukawa shape after the SSB only in unitary gauge. Therefore one has for the Majorana case - in which the matrix $M^M$ is a symmetric one - the following expression:

$$M^M (\nu) = \frac{1}{2} \left( \begin{array}{ccc} A & D & E \\ D & B & F \\ E & F & C \end{array} \right) \langle \phi \rangle \quad (46)$$

Obviously, the coupling constants are in our notation: $A = G^{ee}$, $B = G^{\mu\mu}$, $C = G^{\tau\tau}$, $D = G^{\mu e} = G^{\tau e}$, $E = G^{e\tau} = G^{e\mu}$, $F = G^{\tau\mu} = G^{\tau\tau}$. Moreover, $m(e) = A \langle \phi \rangle$, $m(\mu) = B \langle \phi \rangle$ and $m(\tau) = C \langle \phi \rangle$.

**Dirac mass terms** Assuming the existence of the right-handed neutrinos (see Sec.3.4.2) one can add to the Yukawa sector terms of the form

$$G'_{\alpha\beta} f_{\alpha L} \phi^{(3)} + \nu_{\beta R} + H.c \quad (47)$$

which develop pure Dirac masses. After SSB such a “classical” Yukawa term generates a Dirac neutrino mass matrix:

$$M^D (\nu) = \left( \begin{array}{ccc} A' & D' & E' \\ K' & B' & F' \\ L' & N' & C' \end{array} \right) \langle \phi \rangle \quad (48)$$

where primed couplings are self-explanatory.
See-saw mechanism With these distinct matrices - Eqs. (46) and (48) - one can construct a canonical see-saw mechanism in order to obtain the Majorana masses for both the left-handed neutrinos and right-handed ones. In the flavor basis, the neutrino mass matrix looks like

$$ M^{D+M} (\nu) = \begin{pmatrix} M^M & M^D \\ M^D & 0 \end{pmatrix} $$

(49)

After its diagonalization, one remains with the two following matrices assigning for the neutrino masses:

$$ M^M (\nu_L) \approx \begin{pmatrix} (A')^2 \\ (K')^2 \\ (L')^2 \end{pmatrix} \begin{pmatrix} D' \\ B' \\ E' \end{pmatrix} \begin{pmatrix} (E')^2 \\ (D')^2 \\ (L')^2 \end{pmatrix} \langle \phi \rangle $$

(50)

for the left-handed neutrinos, and

$$ M^M (\nu_R) \approx \frac{1}{2} \begin{pmatrix} A & D & E \\ D & B & F \\ E & F & C \end{pmatrix} \langle \phi \rangle $$

(51)

for the right-handed ones.

Neutrino mixing The physical neutrino basis can be determined by taking into consideration neutrino mixing performed by the unitary mixing matrix $U$ ($U^T U = 1$). It switches from the gauge-flavor basis to the physical basis of massive neutrinos in the manner

$$ \nu_{\alpha L}(x) = \sum_{i=1}^{3} U_{\alpha i} \nu_{iL}(x) $$

(52)

where $\alpha = e, \mu, \tau$ (corresponding to neutrino gauge eigenstates), and $i = 1, 2, 3$ (corresponding to massive physical neutrinos with masses $m_i$). In our case all neutrinos are Majorana fields $\nu_{\alpha L}(x) = \nu_{\alpha L}(x)$. Otherwise, one should consider in the case with neutrinos as Dirac fields $\nu_{\alpha L}(x) = \nu_{iR}(x)$. The mass term corresponding to neutrino mass yields:

$$ -L^{\text{mass}}_{\nu} = \frac{1}{2} \bar{\nu}_{\alpha L} M_{\alpha \beta} (\nu) \nu_{\beta L} + H.c $$

(53)

The mixing matrix $U$ that diagonalizes the neutrino mass matrix ensures the relation $U^T M (\nu) U = m_{ij} (\nu) \delta_{ij}$. It has in the standard parametrization the form:

$$ U = \begin{pmatrix} c_2 c_3 & s_2 c_3 & s_3 e^{-i \delta} \\ -s_2 c_1 - c_2 s_1 s_3 e^{i \delta} & c_1 c_2 - s_2 s_3 s_1 e^{i \delta} & c_3 s_1 \\ s_2 s_1 - c_2 c_1 s_3 e^{i \delta} & -s_1 c_2 - s_2 s_3 c_1 e^{i \delta} & c_3 c_1 \end{pmatrix} P $$

(54)

with $P = \text{diag} (e^{i \phi_1}, e^{i \phi_2}, 1)$ - the phase matrix. For, simplicity, we made the substitutions $\sin \theta_{23} = s_1, \sin \theta_{12} = s_2, \sin \theta_{13} = s_3, \cos \theta_{23} = c_1, \cos \theta_{12} = c_2, \cos \theta_{13} = c_3$ for the mixing angles, and $\delta$ is the CP Dirac phase and $\phi_1, \phi_2$ are Majorana phases. We note here that the later ones can not be removed by a simple redefinition of the phases, since they carry physical information for the Majorana neutrinos. They are not active if the Dirac case is considered.
**Mass squared differences**  For physical neutrinos, mass squared differences - which are experimentally accessible - are defined as $\Delta m^2_{ij} = m_j^2 - m_i^2$. Their right order of magnitude can be obtained for $\Delta m^2_{23} \leq 2 \cdot 10^{-3}$ eV$^2$ from Super Kamiokande atmospheric data [45, 46] and for $\Delta m^2_{12} \leq 8 \cdot 10^{-5}$ eV$^2$ from solar and KamLAND data [47, 48]. Considering that in Eq. (50), the coupling constants act as variables, the diagonalization of the matrix $M$ is equivalent to a system of 6 linear equations with 9 variables, as $M$ is symmetric, which leads to the following general solution for the physical neutrino masses:

$$m_i = F_i \left( \frac{(A')^2}{A} , \frac{(B')^2}{B} , \frac{(C')^2}{C} , \theta_{12}, \theta_{13}, \theta_{23} \right) \langle \phi \rangle \quad (55)$$

where $\theta_{12}, \theta_{13}, \theta_{23}$ stand for the mixing angles in the neutrino sector and $i = 1, 2, 3$.

The analytical functions $F_i$ could be determined in each particular case by solving the appropriate set of equations. For the case of Majorana neutrinos this task was accomplished in the general case of neutrino mixing without CP-phase violation in Ref. [49]. This case corresponds to the phenomenological situation $\sin \theta_{13} \simeq 0$.

The mass squared differences are now:

$$\Delta m^2_{ij} = (F_j^2 - F_i^2) \langle \phi \rangle^2 \quad (56)$$

With these expressions one can get the mass squared ratio $r_\Delta = \Delta m^2_{12} / \Delta m^2_{23}$ which is independent of the parameters of the scalar sector in the model - and thus is not affected by the SSB details - and depends only on the mixing angles and the couplings in the Yukawa sector.

**Phenomenological restrictions**  A great deal of experimental data (see [44] and Refs therein) confirm that phenomenological values of neutrino masses $m(\nu_i)$ are severely limited to a few eVs. Let us compute the sum of the neutrino masses. It is nothing but the trace of the neutrino mass matrix,

$$\sum_i m(\nu_i) = \text{Tr}[M^M (\nu_L)] = 2 \left( \frac{(A')^2}{A} + \frac{(B')^2}{B} + \frac{(C')^2}{C} \right) \langle \phi \rangle \quad (57)$$

In order to obtain the desired order of magnitude one has to tune these parameters or even enforce certain symmetries.

### 5 Concluding Remarks

In this paper we have proved that the well-known PPF 3-3-1 model can be investigated from an exact algebraical viewpoint, by simply using the method of solving gauge theories with high symmetries proposed in Ref. [20]. In this approach, all the phenomenological consequences regarding the boson mass spectrum in the model occur due to a natural tuning of a free parameter $a$. At the same time the correct couplings of the fermion currents with respect to the neutral and charged bosons are obtained. We
mention that the usual mixing (small $\phi$ angle) - worked out on the resulting couplings at the end of the calculus in other papers - is performed in our solution as an compulsory intermediate step by the method itself. Thus, the couplings in Table 1 being the exact ones for all the currents in the model. As one can easily observe, they do not depend on any parameter, except for the Weinberg angle $\theta_W$ well established in the SM.

A special Yukawa sector is constructed in the fermion sector of the model in order to generate the correct masses of the particles. Here a set of 9 free parameters (Yukawa couplings) are introduced in the quark sector and 3 more ones in the charged lepton sector. As long as the neutrino phenomenology is invoked, one can exploit it by just tuning 3 other parameters corresponding to the Yukawa couplings for the Dirac mass terms, while the same 3 couplings from charged lepton sector are employed to ensure the Majorana mass terms in a suitable see-saw mechanism. Since the unique breaking scale (with the vacuum expectation value $\langle \phi \rangle$) is responsible for the necessary SSB, one can establish that for a $\langle \phi \rangle$ at around TeV scale, the $A'$, $B'$, $C'$ have to be in the range $10^{-9}$ in order to give a viable order of magnitude for the neutrino mass spectrum $\sum_i m(\nu_i) \sim 1$eV.

Our solution presented above offers an exact algebraical framework for further investigations on interesting topics invoked in some papers already published on PPF 3-3-1 model. It is able to treat the case of adding an exotic charged lepton [50] which replaces the right-handed charged lepton in the third position of each lepton triplet. It can incorporate - if Majorana neutrinos are involved - phenomena regarding neutrino-less double decay [51] and thus lepton number violation. The particular behaviour of the extra neutral boson of the theory and its leptophobic character [52] - [54] is naturally obtained within our solution. It was argued that such models can well explain the electric charge quantization [55] - [58]. Regarding the neutrino masses, radiative mechanisms [59] - [62] could also be employed to generate tiny masses in contrast to the tree level attempts [63] [64], while the rich phenomenology of the see-saw mechanism could be further investigated [65] - [67]. Anomalous magnetic moment of the muon [68] and other static quantities [69] were calculated using this class of 3-3-1 models, and the perturbative border (including the Landau pole and the non-perturbative regime) [70] - [74] of such models can be also treated using our elegant parametrization. The search for doubly charged Higgs bosons [75] [76] can be naturally addressed within the framework of our solution. An attractive possibility stands in exploiting an additional $U(1)$ symmetry [77].

With such an efficient outcome, we consider that our method acts as an elegant and viable tool for solving the wide set of theoretical and phenomenological issues related to 3-3-1 models that, in addition, could suggest new ways and interpretations for the phenomena already experimentally confirmed.
Appendix: Calculating the coupling coefficients

Our model has three types of fermion triplets. The fundamental irrep \((3,0)\) of the lepton triplet defines the basic electric charges of the model, \(Q = Q^{(3,0)}(A^{em}) = \text{diag} (1, -1, 0)\). The electric charges in quark’s irreps, \((3^*, -\frac{1}{3})\) and \((3, +\frac{2}{3})\), are

\[
Q^{(3^*, -\frac{1}{3})}(A^{em}) = \text{diag}\left(\frac{-4}{3}, \frac{2}{3}, -\frac{1}{3}\right),
\]

\[
Q^{(3, +\frac{2}{3})}(A^{em}) = \text{diag}\left(\frac{5}{3}, -\frac{1}{3}, \frac{2}{3}\right),
\]

pointing out the presence of the exotic quarks with the electric charges \(\frac{5}{3}\) and \(-\frac{4}{3}\).

The neutral charges of the Weinberg neural boson \(Z\) result from Eq. (33) as

\[
Q^{(3,0)}(Z) = \frac{1}{\sin 2\theta_W} \text{diag} \left(-2 \sin^2 \theta_W, -1 + 2 \sin^2 \theta_W, 1\right),
\]

\[
Q^{(3^*, -\frac{1}{3})}(Z) = \frac{1}{\sin 2\theta_W} \text{diag} \left(\frac{8}{3} \sin^2 \theta_W, 1 - \frac{4}{3} \sin^2 \theta_W, -1 + \frac{2}{3} \sin^2 \theta_W\right),
\]

\[
Q^{(3, +\frac{2}{3})}(Z) = \frac{1}{\sin 2\theta_W} \text{diag} \left(-\frac{10}{3} \sin^2 \theta_W, -1 + \frac{2}{3} \sin^2 \theta_W, 1 - \frac{4}{3} \sin^2 \theta_W\right),
\]

recovering thus all the neutral charges of leptons and standard quarks predicted by the SM.

The neutral charges of our new neutral boson \(Z'\) calculated according to Eq. (34) read

\[
Q^{(3,0)}(Z') = \alpha (1 - 4 \sin^2 \theta_W) \text{diag} (-2, 1, 1),
\]

\[
Q^{(3^*, -\frac{1}{3})}(Z') = \alpha \text{ diag} \left(2 - 10 \sin^2 \theta_W, -1 + 2 \sin^2 \theta_W, -1 + 2 \sin^2 \theta_W\right),
\]

\[
Q^{(3, +\frac{2}{3})}(Z') = \alpha \text{ diag} \left(-2 + 12 \sin^2 \theta_W, 1, 1\right),
\]

where \(\alpha = \sqrt{3} \sin 2\theta_W \sqrt{1 - 4 \sin^2 \theta_W}^{-1}\).

For the singlets we obtain simpler formulas since in this case all the coupling coefficients are given by the chiral hypercharge. Thus for an arbitrary singlet \((1, y_{ch})\) we have

\[
Q^{(1, y_{ch})}(A^{em}) = y_{ch},
\]

\[
Q^{(1, y_{ch})}(Z) = -y_{ch} \tan \theta_W
\]

and

\[
Q^{(1, y_{ch})}(Z') = y_{ch} 6 \alpha \sin^2 \theta_W.
\]

Finally we remind the reader that all the charged bosons have the same coupling coefficient, \(g/\sqrt{2}\), which in units of \(e\) reads \(1/\sqrt{2} \sin \theta_W\).
Table: Coupling coefficients of the neutral currents in PPF 3-3-1 model

| Particle | Coupling | $Z \rightarrow ff$ | $Z' \rightarrow ff$ |
|----------|----------|---------------------|---------------------|
| $e_L, \mu_L, \tau_L$ | $2 \sin^2 \theta_W - 1$ | $\frac{\sqrt{1 - 4 \sin^2 \theta_W}}{\sqrt{3}}$ |
| $\nu_e L, \nu_\mu L, \nu_\tau L$ | $1$ | $\frac{\sqrt{1 - 4 \sin^2 \theta_W}}{\sqrt{3}}$ |
| $e_R, \mu_R, \tau_R$ | $2 \sin^2 \theta_W$ | $\frac{2 \sqrt{1 - 4 \sin^2 \theta_W}}{\sqrt{3}}$ |
| $\nu_e R, \nu_\mu R, \nu_\tau R$ | $0$ | $0$ |
| $u_L, c_L$ | $1 - \frac{4}{3} \sin^2 \theta_W$ | $\frac{-1 + 2 \sin^2 \theta_W}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $d_L, s_L$ | $-1 + \frac{2}{3} \sin^2 \theta_W$ | $\frac{-1 + 2 \sin^2 \theta_W}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $t_L$ | $1 - \frac{4}{3} \sin^2 \theta_W$ | $\frac{1}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $b_L$ | $-1 + \frac{2}{3} \sin^2 \theta_W$ | $\frac{1}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $u_R, c_R, t_R$ | $-\frac{4}{3} \sin^2 \theta_W$ | $\frac{4 \sin^2 \theta_W}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $d_R, s_R, b_R$ | $\frac{2}{3} \sin^2 \theta_W$ | $\frac{-2 \sin^2 \theta_W}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $J_{1L}, J_{2L}$ | $\frac{8}{3} \sin^2 \theta_W$ | $\frac{2(1 - 5 \sin^2 \theta_W)}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $J_{1R}, J_{2R}$ | $\frac{8}{3} \sin^2 \theta_W$ | $\frac{-8 \sin^2 \theta_W}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $J_{3L}$ | $-\frac{10}{3} \sin^2 \theta_W$ | $\frac{-2(1 - 6 \sin^2 \theta_W)}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
| $J_{3R}$ | $-\frac{10}{3} \sin^2 \theta_W$ | $\frac{10 \sin^2 \theta_W}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}$ |
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