Theoretical basis of Higgs-spin analysis in $H \to \gamma\gamma$ and $Z\gamma$ decays

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A B S T R A C T

We chart the theoretical basis of radiative decays of the Higgs boson, $H \to \gamma\gamma$ and $Z\gamma$, for measuring the spin of the Higgs particle. These decay channels are complementary to other rare modes such as real/virtual $Z$-boson pair decays. In systematic helicity analyses the angular distribution for zero-spin is confronted with hypothetical spin-$2^+$ and higher assignments to quantify the sensitivity.

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1. After the discovery of the Higgs particle, the properties must be examined experimentally to identify the particle as the element proper of the Higgs mechanism for breaking the electroweak symmetries [1] (for recent general reviews see Ref. [2]). The dynamical steps include the confirmation of the scalar spin-zero character of the particle.

2. Zero-spin character of the Higgs boson reflects itself in the isotropic decay distribution. Since the $Z$-boson is produced in helicity states, the distribution would be $d J_{\gamma\gamma} / d \cos \theta = 1$ and $1/2$ in angular distributions.

3. The zero-spin character of the Higgs boson reflects itself in the isotropic decay distribution of the Higgs boson in the rest frame, i.e.

$$\frac{1}{\Gamma_{\gamma\gamma,Z\gamma}} \frac{d \Gamma_{\gamma\gamma,Z\gamma}}{d \cos \theta} = 1 \text{ and } 1/2$$

4. The flat distribution (1.2) is unique to zero-spin of the Higgs boson. For any other spin assignment $J$ the distribution would be given by the Wigner functions $|d^{I}_{m,J}\gamma_{\gamma}(\lambda)_{\gamma}(\lambda_{\gamma})(\theta)|^{2}$ and $|d^{I}_{m,J}\gamma_{\gamma}(\lambda)_{\gamma}(\lambda_{\gamma})(\theta)|^{2}$, where $m$ denotes the $S_{z}$ spin component being either 0 or ±2 for Higgs-boson production in gluon fusion, while $\lambda_{\gamma} - \lambda_{\gamma}(\theta) = 0, \pm 2$ and $\lambda_{\gamma} - \lambda_{\gamma}(\theta) = 0, \pm 1, \pm 2$. The explicit form of the Wigner $d$-functions $d^{I}_{m,J}(\theta)$ may be read off the tables in Ref. [23].
Fig. 1. (a) Kinematics of radiative Higgs-boson decays in gluon fusion; (b)–(d) Angular distributions of $\gamma\gamma$ and $Z\gamma$ axes in the rest frame of the subprocesses: the flat Higgs signal compared with the distributions of the background events with the invariant $q\bar{q}$ energy set to $M_H = 126$ GeV in (b) and all potential spin-2 distributions corresponding to the spin component along the collider axis in the rest frame of the Higgs boson, and the difference of the helicities of the vector bosons in the decay final states (all distributions normalized over the interval $|\cos \Theta| \leq 1/\sqrt{2}$). The upper indices refer to the allowed parity associated with the distributions in $\gamma\gamma$ decays. Allowed parities of $Z\gamma$ states are denoted in round brackets either if unique for $Z\gamma$ or if different from $\gamma\gamma$ final states. (Note that $Z\gamma$ states do not conform with the $gg$ initial states in contrast to $\gamma\gamma$ states.)

The general parity-invariant polar angular distribution can be cast into a compact form for any Higgs-spin $J$ decays to $\gamma\gamma$ and $Z\gamma$ in gluon fusion:

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \Theta} = (2J + 1) \left[ \lambda_0^J \lambda_0^J \mathcal{D}_0^0 + \lambda_0^J \lambda_2^J \mathcal{D}_0^2 + \lambda_2^J \lambda_0^J \mathcal{D}_2^0 + \lambda_2^J \lambda_2^J \mathcal{D}_2^2 \right]$$

and

$$\frac{1}{\sigma} \frac{d^2\sigma}{d \cos \Theta d \cos \theta_\ell} = \frac{3(2J + 1)}{16} \left[ \lambda_0^J \lambda_0^J \mathcal{D}_0^0 + \lambda_0^J \lambda_2^J \mathcal{D}_0^2 + \lambda_2^J \lambda_0^J \mathcal{D}_2^0 + \lambda_2^J \lambda_2^J \mathcal{D}_2^2 \right] (1 + \cos^2 \theta_\ell)$$

respectively, with the squared Wigner functions $\mathcal{D}_{\alpha \beta}^J = \frac{1}{4} \sum |m_\alpha(\theta)|^2 \sum |m_\beta(\theta)|^2$ symmetrized by the summing over all signs $\pm m, \pm \lambda$ of the spin/helicity components (index 0 counting twice). The functions read explicitly for $J = 0$ and $J = 2$:

$$\mathcal{D}_0^0 = 1$$
$$\mathcal{D}_0^2 = (3 \cos^2 \theta - 1)^2 / 4$$
$$\mathcal{D}_2^0 = T_{02} = 3 \sin^4 \theta / 8$$
$$\mathcal{D}_2^1 = D_{12}^2 = \sin^2 \theta (1 + \cos^2 \theta) / 4.$$

The functional forms displayed in Fig. 1(c), (d). The (non-negative) reduced production and decay helicity probabilities $\chi$ for gluon fusion, $\gamma$ for $\gamma\gamma$ and $\gamma'$ for $Z\gamma$ final states, are model-dependent parameters, obeying the sum rules

$$\mathcal{D}_0^0 = 1$$
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$$\mathcal{D}_2^0 = T_{02} = 3 \sin^4 \theta / 8$$
$$\mathcal{D}_2^1 = D_{12}^2 = \sin^2 \theta (1 + \cos^2 \theta) / 4.$$
The polar angle of the $Z$ decay distribution can easily be integrated out, equivalent to the substitutions $(1 + \cos^2 \theta_i) \rightarrow 8/3$ and $\sin^2 \theta_i \rightarrow 4/3$. Since the reduced helicity probabilities are non-negative, the coefficients of both the $(1 + \cos^2 \theta_i)$ term and the $\sin^2 \theta_i$ term inevitably generate non-vanishing maximum/minimum $\pm \cos^2 \theta_i$ terms. Thus, observing (in the experimentally ideal case) that the angular distribution is independent of $\cos \theta$, proves unambiguously the spin-zero character of the Higgs particle. At the same time the $\sin^2 \theta_i$ term in $\gamma \gamma$ final states is predicted to be absent, providing an independent check.

The observation of spin states in $gg \rightarrow H \rightarrow \gamma \gamma$ allows (partial) conclusions also on the parity of the states. From Bose symmetry and parity symmetry of the helicity amplitudes $T_{\lambda_{hv}}^{ij} = (-1)^{i-j} T_{\lambda_{hv}}^{ij} = \mathcal{P}(-1)^i T_{\lambda_{hv}}^{j-i}$, referring separately to initial and final states [3], the selection rules presented in Table 1 can easily be derived (see also Ref. [24]), complementing global rules noted earlier in the literature. Scalar and pseudoscalar Higgs bosons cannot be discriminated in the $\gamma \gamma$ decay mode, neither even/odd parity by observing $D_{00}$, and spin correlation effects [7,25,26] must be exploited for discrimination. But observing any state with helicity difference $\Delta \lambda = 2$ in initial or final state determines unambiguously the even-parity character of the Higgs boson. Analogous rules apply also to $Z\gamma$ final states in gluon fusion and $\gamma \gamma$ final states in $q\bar{q}$ annihilation. Either the first or the second index $\Delta \lambda$ in the $D$ functions coming with the production or decay amplitude, respectively, is restricted in parallel to the rules in Table 1 while the respective companion index is unrestricted apart from the standard spin constraints.

The $\gamma \gamma$ channel is described by two-by-two independent probabilities for production and decay, $\lambda_{0,2}^I$ and $\lambda_{0,2}^J$. Popular choices for experimental simulations are the complementary $\{J;0\}$ 'scalar-type' and the $\{J;2\}$ 'tensor-type assignments':

- **scalar-type assignment**: $\lambda_{0}^J = \lambda_{0}^J = 1$ and $\lambda_{2}^J = \lambda_{2}^J = 0$ ($J \geq 0$) (1.8)
- **tensor-type assignment**: $\lambda_{0}^J = \lambda_{0}^J = 0$ and $\lambda_{2}^J = \lambda_{2}^J = 1$ ($J \geq 2$), (1.9)

supplemented by the

Table 1

| $P$ | $J$ | $0$ | $1$ | $2,4,\ldots$ | $3,5,\ldots$ |
|-----|-----|-----|-----|---------------|---------------|
| even | $1$ | forbidden | $D_{00}^I$ | $D_{20}^I$ | $D_{10}^I$ |
| odd | $1$ | forbidden | $D_{00}^I$ | $D_{20}^I$ | $D_{10}^I$ |

$\lambda_{0}^I + \lambda_{2}^I = 1$\n
$\lambda_{0}^I + \lambda_{2}^I = 1$ and $\lambda_{0}^I + \lambda_{2}^I = 1$.\n
The formalism can easily be transferred to $q\bar{q}$ production with $S_{\pm} = 0$ or $\pm 1$ by substituting $\lambda_{0}^I$ for $\lambda_{0}^I$ and $D_{0k}^I$ for $D_{2k}^I$ ($k = 0,1,2$) in the cross sections and in the $\chi^2$ sum rule. Note however that the rate for signal Higgs production in $q\bar{q}$ collisions is negligibly small for light quark beams. The initial states $gg$ and $q\bar{q}$ mix incoherently in the most general configurations.

The polar angle of the $Z$ decay distribution contrasts with the distributions generated by hypothetical $\pi^{0}$, $\phi$ and $\eta$ decays [24,27], we will choose $J = 2$ for illustration. (Part of the distributions have also been noted in Refs. [8,28].) None of the possible helicity states would generate a flat distribution like spin-zero Higgs-boson decays. It is shown in Fig. 1 how the flat signal distribution contrasts with the distributions generated by hypothetical spin $J = 2$ assignments of both even and odd parity, and the angular distribution of the backgrounds as well (to be analyzed next). In the final section we will compare the spin-0 distribution, assigned $J = 0$ and $m = 3$, with the $J = 2$ assignment, specifically with the scalar-type ($2;0$) and the tensor-type ($2;2$) distributions, representing the state $J = 2$ with the two parities ±$\Delta$. The $2;0$ scalar-type distribution is pronounced in the center like spin-0 after angular cut, and rises in the forward/backward directions like the continuum backgrounds, thus providing a non-trivial analogue to be discriminated experimentally from the Higgs signal. The $2;2$ tensor-type distribution rises monotonically to the left and to the right of the center, providing also a valuable discriminant. Both types must be ruled out necessarily to reject experimentally the spin-2 assignment for even and odd parity.

A first global comparison between spin-0 and all the spin-2 distributions is offered by the moments of the polar-angle distributions of the $\gamma \gamma$ and $Z\gamma$ event axes\footnote{Angular asymmetries [9] can equivalently be used in global analyses.} noted in Table 2. The first moments of the spin-2 distributions are characteristically different\footnotemark[4] from the spin-0 distribution and may provide early information on the Higgs spin.

Table 2

| Polar moments $(\langle \cos \theta_i \rangle)/1$ | total cut | $Z$ decay |
|------------------------------------------|----------|-----------|
| $\gamma \gamma$ spin-0 $[0;0]$ | $1/2$ | $\sqrt{2}/4$ | $1$ |
| $\gamma \gamma$ spin-2 $[2;0]$ | $5/8$ | $5\sqrt{2}/36$ | $1 + \cos^2 \theta_i$ |
| $\gamma \gamma$ spin-2 $[2;2]$ | $5/16$ | $35\sqrt{2}/172$ | $1$ |
| $\gamma \gamma$ spin-2 $[2;21]$ | $65/96$ | $155\sqrt{2}/492$ | $1$ |
| $Z\gamma$ spin-2 $[2;01]$ | $5/8$ | $5\sqrt{2}/14$ | $\sin^2 \theta_i$ |
| $Z\gamma$ spin-2 $[2;21]$ | $5/12$ | $55\sqrt{2}/228$ | $1$ |

'mixed-type assignment': $\lambda_{0}^I = \lambda_{2}^I = 0$ and $\lambda_{0}^J = \lambda_{2}^J = 1$ ($J \geq 2$) and $'1,0'$ interchanged, (1.10)

for the signal $J = 0$ and the hypothetical alternatives $J \geq 2$. (The $\gamma \gamma$ coupling of the tensor-type assignment is equivalent to the KK graviton coupling in $d = 5$ scenarios [8].) The configurations can be exploited in two ways: (i) One of the two scalar- or tensor-type configurations for $J \geq 2$ for instance, is sufficient to prove that the spin-zero test of the signal is non-trivial; (ii) However, to prove experimentally that the spin-2 assignment is not realized, the three configurations, which are mutually independent, must necessarily be shown absent. Not observing the double index $[J;0]$, the state $[J;0]$ is ruled out, while neither observing distributions carrying at least one index 2, the state $[J;2]$ is ruled out, too. Thus any spin $J \geq 2$ can be rejected for both parities ±$\Delta$ by angular analyses. Due to potential longitudinal $Z$ polarization accounted for by $\lambda_{2}^I$, the $Z\gamma$ final state is described by three independent decay probabilities.

Disregarding $J = 1$, as forbidden by the Landau–Yang theorem in $\gamma \gamma$ decays [24,27], we will choose $J = 2$ for illustration. (Part of the distributions have also been noted in Refs. [8,28].) None of the possible helicity states would generate a flat distribution like spin-zero Higgs-boson decays. It is shown in Fig. 1 how the flat signal distribution contrasts with the distributions generated by hypothetical spin $J = 2$ assignments of both even and odd parity, and the angular distribution of the backgrounds as well (to be analyzed next). In the final section we will compare the spin-0 distribution, assigned $J = 0$ and $m = 3$, with the $J = 2$ assignment, specifically with the scalar-type ($2;0$) and the tensor-type ($2;2$) distributions, representing the state $J = 2$ with the two parities ±$\Delta$. The $[2;0]$ scalar-type distribution is pronounced in the center like spin-0 after angular cut, and rises in the forward/backward directions like the continuum backgrounds, thus providing a non-trivial analogue to be discriminated experimentally from the Higgs signal. The $[2;2]$ tensor-type distribution rises monotonically to the left and to the right of the center, providing also a valuable discriminant. Both types must be ruled out necessarily to reject experimentally the spin-2 assignment for even and odd parity.

The polar angle of the $Z$ decay distribution contrasts with the distributions generated by hypothetical spin $J = 2$ assignments of both even and odd parity, and the angular distribution of the backgrounds as well (to be analyzed next). In the final section we will compare the spin-0 distribution, assigned $J = 0$ and $m = 3$, with the $J = 2$ assignment, specifically with the scalar-type ($2;0$) and the tensor-type ($2;2$) distributions, representing the state $J = 2$ with the two parities ±$\Delta$. The $[2;0]$ scalar-type distribution is pronounced in the center like spin-0 after angular cut, and rises in the forward/backward directions like the continuum backgrounds, thus providing a non-trivial analogue to be discriminated experimentally from the Higgs signal. The $[2;2]$ tensor-type distribution rises monotonically to the left and to the right of the center, providing also a valuable discriminant. Both types must be ruled out necessarily to reject experimentally the spin-2 assignment for even and odd parity.

A first global comparison between spin-0 and all the spin-2 distributions is offered by the moments of the polar-angle distributions of the $\gamma \gamma$ and $Z\gamma$ event axes\footnote{The assignments $[2;01]$ and $[2;21]$ can also be discriminated from $[0;00]$ by identifying the $Z$-decay angular distributions $\sin^2 \theta_i$ versus $(1 + \cos^2 \theta_i)$.} noted in Table 2. The first moments of the spin-2 distributions are characteristically different\footnotemark[4] from the spin-0 distribution and may provide early information on the Higgs spin.
3. The large continuum background generated in $pp(qq) \to \gamma\gamma$ and $Z\gamma$ processes (and, to a lesser extent, loop-induced gluon fusion) requires stringent cuts in order not to dwarf the signals. The angular characteristics allow to reduce the continuum backgrounds considerably.

The angular distribution of the background events is strongly peaked in the forward and backward directions as a result of the $t$- and $u$-channel exchange mechanisms, in contrast to the flat distribution of the signal. In the rest frame of the parton system, cutting out the singular forward and backward directions (so long as the cross sections are not regularized properly):

$$\frac{d\sigma}{d\cos\theta}[q\bar{q} \to \gamma\gamma] = \frac{2\pi\alpha^2}{3s} Q_q^2 \left(1 + \cos^2\theta\right)$$

$$\frac{d\sigma}{d\cos\theta}[q\bar{q} \to Z\gamma] = \frac{2\pi\alpha^2}{3s} Q_q^2 \left[v_q^2 + a_q^2\right] \left[1 - M_Z^2/s\right]$$

$$\times \frac{1}{\sin^2\theta} \left[1 + \cos^2\theta + \frac{4M_Z^2s}{s - M_Z^2}\right]$$

for the invariant parton energy $\sqrt{s} = M_H$, electric and weak quark charges denoted by $Q$, $v$, and $a$ [29,30]; the helicity decomposition is familiar from electron–positron collisions, cf. Appendix in Ref. [31]. The parton subprocesses are peaked at small angles, regularized by the strong interaction scale $\Lambda_{QCD}$. For the given Higgs mass, the angular distribution of $gg \to \gamma\gamma$ [32] is surprisingly close to the $q\bar{q}$ process, cf. Fig. 1(b). Induced by radiative return [33] in $Z\gamma$, and self-evident in $\gamma\gamma$, the $\gamma$’s and the $Z$-bosons are traveling primarily along the LHC beam axis. By restricting $|\cos\theta|$ to less than $\cos\theta_{\text{cut}} \leq 1/\sqrt{2}$, the signal is reduced only modestly, but the background strongly.

4. For illustration of the $J^P$ sensitivity, we present a set of rough theoretical estimates, rescaled from available experimental data in $\gamma\gamma$ [11] or based on simulations in $Z\gamma$ [34]. One photon with transverse energy $E_T$ in excess of 25 GeV was required for $Z\gamma$ decays, and photons in excess of 40 GeV in $\gamma\gamma$ decays. Numerically, using $|\cos\theta| = |1 - 4E_{\text{perp}}^2/M_H^2|/\left(M_H^2 - M_Z^2\right)^2$, and $|1 - 4E_{\text{perp}}^2/M_H^2|^{1/3}$ for the corresponding polar angles in the Higgs rest frame, this is approximately equivalent to the restrictions $|\cos\theta| \leq 0.77$ and 0.55, close to the theoretical cut $1/\sqrt{2}$ used in the previous section. The $Z$-boson was assumed to decay leptonically to $e, \mu$ pairs.

The error estimates for the $\gamma\gamma$ final states were performed by rescaling the background event numbers of Ref. [11] in energy, using the theoretical energy dependence of $q\bar{q} \to \gamma\gamma$ (ignoring mis-identification from jets in this simplified theoretical estimate), and by raising the luminosity to 100 fb$^{-1}$, leaving us with 4.6 k (146 k) events after cuts for signal (background), in rough agreement, within a factor two with Ref. [35], after inserting the proper K-factor. The theoretical angular distribution of the photons in the center range left by the cuts was used as well according to Fig. 1. We have adopted the experimental efficiency of 40% and a resolution of ±2 GeV. Experimental refinements such as smearing effects etc. have not been considered in this coarse theoretical picture.

In the same way as Ref. [34] we determined the background event number from the cross section of the $q\bar{q} \to Z\gamma$ process which dominates the background compared with $gg$ collisions and the Drell–Yan process including final-state photon radiation. Similarly to the parameters in Ref. [34] we have adopted the values of 3 GeV for the mass resolution and 0.13 for the efficiency. The production cross section of the signal was recalculated theoretically by including the large $K$-value in Higgs-boson production $pp \to H$. Finally, 1.2 k (51 k) signal (background) events were predicted for a luminosity of 3 ab$^{-1}$.

These theoretical estimates of rates and errors, not including detailed experimental refinements, should serve only as a rough illustration of theoretical expectations for Higgs spin analyses in radiative decays.

In the framework defined above, the results for the signals and the size of the roughly expected backgrounds are shown in Fig. 2 for $H \to \gamma\gamma$ and $H \to Z\gamma$ in the theoretically cut $\cos\theta$ range. The solid lines are the (ideal) signals with very high statistics, the error bars, based on the event numbers defined above, represent

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5 These parameters were extracted by means of PYTHIA [36] and ACERDET [37], with support by D. Zerwas gratefully acknowledged. The transverse momentum of the photon was chosen in excess of 25 GeV. In $Z$ decays to leptons, electrons or muons were reconstructed with transverse energy/momentum of more than 25 GeV, separated from the photon by $\Delta R \geq 0.7$, where $\Delta R$ is the square root of the sum of the squares of the azimuthal angle and pseudo-rapidity differences. The invariant mass of the lepton pair was chosen compatible with the mass of the $Z$ boson within 5 GeV. The reconstructed mass of the Higgs boson was allowed in a window of 3 GeV centered on its nominal mass. The second window can be chosen more restrictive as the Higgs boson width is negligible compared to the experimental resolution, whereas for the $Z$ boson this is not the case. The resulting efficiency of 0.13 is close to the value reported in Ref. [34] when interpolating slightly different parameters.
the theoretical estimates of background fluctuations $\sqrt{B}$, containing the angular distributions of the signals $S \propto \langle q \rangle$. These distributions are compared with the expected decay characteristics of a hypothetical spin-2 particle, even/odd parity, derived for the representative angular tensor-type and scalar-type distributions in Eqs. (15). Note that the two distributions are mutually complementary to each other. The first $|\cos \theta|$ moments (normalized to zeroth moments) are in the ratio $(0.52 \pm 0.06)/(0.27 \pm 0.05)$ for the tensor/scalar assignment in $\gamma \gamma$ final states, and $(0.36 \pm 0.10)/(0.18 \pm 0.08)$ in $Z \gamma$ final states. Even if added up, the resulting flattish behavior is frustrated by the non-zero $D_{00} = D_{02}$ contributions.

The small leptonic $Z$ branching ratio together with the increased background cross section render experimental $Z \gamma$ analyses much more demanding than $\gamma \gamma$ analyses, and a large increase of luminosity is required.

Studying experimentally the $\gamma \gamma$ and $Z \gamma$ processes outside the Higgs mass window will yield a good understanding of the background shapes and normalizations. At the expense of a $\sqrt{2}$ increase of errors one could define a control region with a lower Higgs-type mass window to determine the shapes and use Monte Carlo to extrapolate from the control region to the signal region.

5. Summary. Dynamical characteristics of the Higgs boson in the Standard Model are particularly difficult to analyze experimentally in the mass region around 126 GeV since the overwhelming decays are $b$ decays. In this report we have analyzed the theoretical potential of two decay modes, $H \rightarrow \gamma \gamma$ decays and $H \rightarrow Z \gamma$ decays (the latter statistically more remote), to measure the spin of the Higgs boson. General helicity analyses prove the sensitivity of both decay modes to zero-spin of the Higgs boson, demonstrated by confronting $J = 0$ zero-spin for illustration to $J = 2^\pm$, i.e. spin = 2 and even/odd parity.

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References

[1] F. Englert, R. Brouz, Phys. Rev. Lett. 13 (1964) 321; P.W. Higgs, Phys. Lett. 12 (1964) 132; P.W. Higgs, Phys. Rev. Lett. 13 (1964) 508; P.W. Higgs, Phys. Rev. 145 (1966) 1156; G.S. Guralnik, C.R. Hagen, T.W.B. Kibble, Phys. Rev. Lett. 13 (1964) 585.

[2] W.D. Schlatter, P.M. Zerwas, Eur. Phys. J. H 36 (2012) 579, arXiv:1112.5127 [physics.hist-ph]; J. Ellis M, K. Gaillard, D.V. Nanopoulos, arXiv:1201.6045 [hep-ph].

[3] J. Ellis, D.S. Hwang, arXiv:1209.1037 [hep-ph];

[6] T. Plehn, M. Rauch, J. Zerwas, Phys. Rev. B 707 (2012) 512, arXiv:1112.3007 [hep-ph];

[9] A. Alves, arXiv:1209.1037 [hep-ph].

[10] C. Englert, D.S. Hwang, M. Takeuchi, JHEP 1206 (2010) 128, arXiv:1203.5788 [hep-ph];

[13] R. Boughezal T, J. LeCompte, F. Petriello, arXiv:1208.4311 [hep-ph];

[16] J.R. Ellis, M.K. Gaillard, D.V. Nanopoulos, Nucl. Phys. B 106 (1976) 292;

[19] A. Djoudi, M. Spira, J.J. van der Bij, P.M. Zerwas, Phys. Lett. B 257 (1991) 187;

[22] H.M. Georgi, S.L. Glashow, M.E. Machacek, D.V. Nanopoulos, Phys. Rev. Lett. 40 (1978) 149, arXiv:hep-ph/0105325.

[25] B. Coleppa, K. Kumar, H.E. Logan, arXiv:1208.2692 [hep-ph].