ANNELING A FOLLOW-UP PROGRAM: IMPROVEMENT OF THE DARK ENERGY FIGURE OF MERIT FOR OPTICAL GALAXY CLUSTER SURVEYS

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ABSTRACT

The precision of cosmological parameters derived from galaxy cluster surveys is limited by the uncertainty in the cluster mass estimates. We demonstrate that a small mass-calibration follow-up program can significantly reduce this uncertainty and improve parameter constraints, particularly when the follow-up targets are judiciously chosen. To this end, we apply a simulated annealing algorithm to maximize the dark energy information at fixed observational cost, and find that optimal follow-up strategies can reduce the observational cost required to achieve a specified precision by up to an order of magnitude. Considering clusters selected from optical imaging in the Dark Energy Survey, we find that approximately 200 low-redshift X-ray clusters or massive SZ clusters can improve the dark energy figure of merit by 50%, provided that the follow-up mass measurements involve no systematic bias. In practice, the actual improvement depends on (1) the uncertainty in the systematic bias in follow-up mass measurements, which needs to be controlled at the 5% level to avoid severe degradation of the results; and (2) the scatter in the optical richness–mass distribution, which needs to be made as tight as possible to improve the efficacy of follow-up observations.

Subject headings: cosmology: theory — cosmological parameters — galaxies: clusters — galaxies: halos — methods: statistical

1. INTRODUCTION

The dynamical properties of dark energy can be constrained with two phenomena. The first is the expansion of the universe: dark energy has dominated the energy density of the universe for the past 4 billion years and has accelerated its expansion. The second is the growth of structure; since dark energy counteracts gravitational attraction, it slows the growth of structure. Galaxy cluster surveys explore both phenomena at the same time: the abundance and the correlation function of galaxy clusters depend on expansion history and structure growth, thus providing powerful probes of dark energy. Given the statistical power of ongoing and future surveys, galaxy clusters have become an indispensable probe of dark energy (e.g., Wang & Steinhardt 1998, Haiman et al. 2001, Holder et al. 2001, Levine et al. 2002, Hu 2003, Rozo et al. 2007a,b, 2009 and references therein).

Four cluster detection methods have been well established: the intracluster hot gas can be identified via X-ray (e.g., Ebeling et al. 1998, 2000, 2001, Vikhlinin et al. 1998, Böhringer et al. 2004) or Sunyaev-Zel’dovich (SZ) effects (e.g., Staniszewski et al. 2008, Hincks et al. 2009; see also Carlstrom et al. 2002); the mass concentrations can be identified using weak lensing shear (e.g., Wittman et al. 2001, 2006); or the galaxies in clusters can be identified in optical surveys (e.g., Postman et al. 1996, Koester et al. 2007, Eisenhardt et al. 2008). Large cluster surveys using each method are ongoing or forthcoming, and cosmological parameter constraints from cluster surveys have recently become competitive with other dark energy probes (e.g., Mantz et al. 2008, Henry et al. 2009, Rozo et al. 2009, Vikhlinin et al. 2009b).

The key issue for extracting cosmological information from clusters is the fidelity of the mass tracer. If a sample or sub-sample of clusters is observed with multiple methods, the cluster mass can be calibrated (e.g., Majumdar & Mohr 2003, 2004, Cunha 2009, Cunha et al. 2009). Determining the most effective approach to improve the constraining power of clusters using multiple mass tracers is particularly timely, as multi-wavelength observations will soon become available for large cluster samples.

In this work, we focus on follow-up observations for optical cluster surveys. We are particularly interested in how dark energy constraints from these surveys can be improved when a sub-sample of the clusters has better mass measurements from other methods, e.g., X-ray or SZ. Our goal is to characterize how follow-up observations should be designed and what precision is required in order for follow-up observations to maximize dark energy information.

Optical surveys identify massive clusters as agglomerations of galaxies. Since the physics of galaxy formation is much more complicated than the physics of hot intracluster gas, optical mass tracers are not as well understood as X-ray or SZ mass tracers. Nevertheless, the optical richness–mass distribution can be empirically determined, and precise cosmological parameters have been derived from optically-selected cluster samples (e.g., Gladders et al. 2007, Rozo et al. 2009). In the near future, optical surveys such as the Dark Energy Survey (DES3), the Panoramic Survey Telescope & Rapid Response System (PanSTARRS4), and the Large Synoptic Survey Telescope (LSST5) will be able to identify substantially larger and higher-redshift cluster samples, which will significantly improve our knowledge of dark energy equation of state \( w \).

In this work, we assume the statistical power and parameters relevant to DES, apply the self-calibration method proposed by Lima & Hu (2004) to calculate our fiducial cosmo-

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logical parameter constraints, and explore how follow-up observations will improve dark energy constraints. Throughout, we quantify dark energy constraints with the figure of merit (FoM) proposed in the Report of the Dark Energy Task Force (DETF) (Albrecht et al. 2006):

$$\text{FoM} = 1 / \sqrt{\det \text{Cov}(w_0, w_a)} = [\sigma(w_a) \sigma(w_p)]^{-1},$$

(1)

where $w = w_0 + (1 - a)w_a$, and $w_p$ is calculated at the pivot redshift for which $w$ is best constrained. The current value of the FoM from WMAP5, SNe, and BAO is 8.326 (Wang 2008). As a reference, the DETF report predicts that the FoM of the FoM from WMAP5, SNe, and BAO is 8.326 (Wang 2008). As a reference, the DETF report predicts that the FoM from WMAP5, SNe, and BAO is 8.326 (Wang 2008).

Assuming that the observable–mass distribution follows power laws in both mass and redshift, we find that the DES FoM from a self-calibrated analysis is 15.2. We apply a simulated annealing algorithm to design follow-up strategies that maximize the FoM, starting with the limiting case in which follow-up mass measurements have infinite precision. We then study how the FoM improvements depend on real-world complications, finding that the efficacy of follow-up observations is likely to be limited by the systematic bias in follow-up mass measurements. We finally consider observational issues and design different cost-effective follow-up strategies for X-ray and SZ.

Majumdar & Mohr (2003, 2004) have previously investigated how follow-up mass measurements of a fraction of the X-ray or SZ selected cluster sample can constrain the cluster evolution and improve dark energy constraints. Our major improvement upon these studies is the optimization of follow-up strategies. We also include counts-in-cells and mass binning, and explore how scatter and possible systematic bias of mass tracers can affect the efficacy of follow-up observations.

Cunha (2009) has recently studied the joint analysis of overlapping optical and SZ cluster surveys. By studying the cluster abundances in both surveys, the observable–mass distribution can be cross-calibrated. In contrast to this study, we concentrate on one survey and its follow-up observations. That is, our method does not require another complete survey but instead focuses on a small and optimized follow-up program.

This paper is organized as follows. We briefly review dark energy constraints from clusters and the self-calibration analysis in §2.1 and describe our survey and model assumptions in §2.2. We present the modeling for follow-up observations in §3.1 and the optimization procedure in §3.2. Real-world complications of follow-up observations are explored in §4. Optimizations considering observational issues specific to X-ray and SZ clusters are carried out in §5. We discuss other relevant studies in §6 and conclude in §7.

2. DARK ENERGY CONSTRAINTS FROM GALAXY CLUSTERS

2.1. Self-Calibration: A Review

In this section, we briefly review the self-calibration formalism proposed by Lima & Hu (2004). For detailed discussions, we refer the reader to Lima & Hu (2004, 2005, 2007), Hu & Coørdi (2006), and Wu et al. (2008).

In a galaxy cluster survey, we fit the mass function of dark matter halos to constrain dark energy. To infer the mass function from data, we need to relate the observable properties of clusters to the halo mass; thus, the uncertainty in this observable–mass distribution limits the constraining power of clusters. The observable–mass distribution can be self-calibrated using a counts-in-cells analysis, in which the survey volume is divided into small cells and the halo bias can be calculated from the sample variance among the cells. By measuring both the counts and the sample variance, the mass function and the halo bias are fit simultaneously, the observable–mass distribution can be self-calibrated, and the dark energy constraints can be improved.

Let us consider a cell of volume $V_c$ in a narrow redshift range in a survey. We denote the cluster mass proxy by $M_{\text{obs}}$ and further bin our cluster sample in $M_{\text{obs}}$ with a binning function $\phi(M_{\text{obs}})$, which equals unity within a $M_{\text{obs}}$ range and zero elsewhere. In this bin, the mean cluster counts ($\bar{m}$) and the mean cluster bias ($\bar{b}_h$) can be calculated from the mass function $dn/dM$, the halo bias $b_h(M)$, and the observable–mass distribution $P(M_{\text{obs}} / M)$:

$$\bar{m} = V_c \int d M \frac{dn}{dM} \langle \phi \rangle |ln M| ,$$

(2)

$$\bar{b}_h = \frac{V_c}{\bar{m}} \int d M \frac{dn}{dM} b_h(M) \langle \phi \rangle |ln M| ,$$

(3)

where

$$\langle \phi |ln M| = \int d M_{\text{obs}} P(M_{\text{obs}} / M) \phi(M_{\text{obs}}) .$$

(4)

We assume the redshift bin is thin enough and we simply use the function values at the middle redshift instead of averaging over the redshift bin.

The mean cluster bias $\bar{b}_h$ determines the number fluctuations among cells due to the large-scale clustering. If cell $i$ has cluster counts $m_i$, the corresponding sample variance among the cells is given by

$$S = (\langle m_i - \bar{m} \rangle^2) = \bar{m}^2 \sigma^2_{\bar{m}} / \sigma^2_{V_c} ,$$

(5)

where $\sigma^2_{V_c}$ is the variance of the dark matter density fluctuation in volume $V_c$. We assume that the cell volume is large enough for the covariance between cells to be negligible.

In a survey, one observes $\bar{m}$ and $S$, calculates $\bar{b}_h$, and self-calibrates $P(M_{\text{obs}} / M)$ based on Equations 2 and 5. To include multiple mass and redshift bins, both $\bar{m}$ and $S$ are generalized to matrices ($\bar{m}$ and $S$). We further define $C = diag(\bar{m}) + S$. The constraints on model parameters can be obtained from the Fisher matrix

$$F_{\alpha\beta} = \bar{m}_\alpha^T C^{-1} \bar{m}_\beta + \frac{1}{2} \text{Tr}[C^{-1} S_{\alpha\beta} C^{-1} S_{\beta\alpha}] ,$$

(6)

where the comma followed by a subscript indicates the partial derivative with respect to a model parameter, and both subscripts run through cosmological parameters and the parameters characterizing the observable–mass distribution. We invert this Fisher matrix to obtain the covariance matrix and the constraint on each parameter.

The second term in this Fisher matrix characterizes the information from the sample variance. Since the sample variance depends on both cosmology and observable–mass distribution, this “noise” in cluster counts actually provides “signal.” As we will see in §3.1, an analogous Fisher matrix is used to calculate the constraints from follow-up observations. In those observations, the variance in follow-up mass measurements will provide critical information for breaking the degeneracies between model parameters.

2.2. Survey and Model Assumptions
In this work, we consider a DES-like optical survey with a survey area of 5000 deg$^2$, and a survey depth such that clusters with redshift $z < 1$ are robustly identified. For the counts-in-cells analysis, the survey volume is divided into cells of area $10^4$ deg$^2$ and $\Delta z = 0.1$ (see, e.g., Lima & Hu 2004). We assume that the mass threshold for the survey is $M_{\text{obs}} = 10^{13.7} h^{-1} M_\odot$, and that the cluster sample is binned by $M_{\text{obs}}$ into 4 bins with logarithmic bin size $\Delta \log_{10} M_{\text{obs}} = 0.4$. Note that the mass threshold and binning are based on the mass proxy $M_{\text{obs}}$ rather than the true cluster mass $M$.

For the observable--mass distribution $P(\ln M_{\text{obs}} | M)$, we assume a Gaussian distribution with mean $(\ln M + \ln M_{\text{bias}})$ and variance $\sigma_{\text{obs}}^2$. Both $\ln M_{\text{bias}}$ and $\sigma_{\text{obs}}^2$ are assumed to vary linearly with $\ln M$ and $(1 + z)$ via

$$\ln M_{\text{bias}} = \ln M_0 + \alpha_M \ln (M/M_{\text{pivot}}) + \alpha_z \ln (1 + z),$$

$$\sigma_{\text{obs}}^2 = \sigma_0^2 + \beta_M \ln (M/M_{\text{pivot}}) + \beta_z \ln (1 + z),$$

Giving nuisance parameters: $(\ln M_0, \alpha_M, \alpha_z, \sigma_0^2, \beta_M, \beta_z)$. For our fiducial model, we assume an unbiased and non-evolving observable--mass distribution with $\sigma_0 = 0.5$ (all other parameters are set to zero), which is consistent with the results in Gladders et al. (2007) and Rozo et al. (2009). In addition, $M_{\text{pivot}}$ should be close to the mass scales of interest; we choose $M_{\text{pivot}} = 10^{13.7} h^{-1} M_\odot$, noting that the precise value of $M_{\text{pivot}}$ does not affect our results.

Throughout this work, we assume the WMAP5 cosmological parameters $^{\text{Komatsu et al.} 2008}$ $w_0 = -1$, $w_a = 0$, $\Omega_{\text{DE}} = 0.721$, $\Omega_k = 0$, $\Omega_m h^2 = 0.137$, $\Omega_b h^2 = 0.0227$, $n_s = 0.960$, $A_s = 4.66 	imes 10^{-5}$, and use the Planck-prior Fisher matrix provided by W. Hu and Z. Ma. We use the linear matter power spectrum calculated by CAMB ($^{\text{Lewis et al.} 2000}$), the updated mass function from Tinker et al. (2008), and the halo bias function from Sheth et al. (2001). Under these assumptions, we expect that DES will observe approximately $2 \times 10^5$ clusters in total.

3. IMPROVING DARK ENERGY CONSTRAINTS WITH OPTIMAL FOLLOW-UP STRATEGIES

3.1. Constraints from Follow-up Mass Measurements

In this section, we calculate the additional constraints from follow-up observations. Given the optical cluster sample from DES, we select some clusters from each mass and redshift bin to follow up — for example, to measure their properties in X-ray or SZ — estimating the cluster mass more precisely. These follow-up mass measurements provide further constraints on the observable--mass distribution, thus improving the dark energy constraints of the original survey. Throughout §3, we assume the follow-up observations provide mass measurements with infinite precision; the complications of follow-up mass tracers will be explored in §4.

Our goal is to constrain the scaling relation and the scatter of optical richness. We note that the term “scatter” usually has different meanings in theoretical and observational contexts. In the theoretical model, we use the scatter of $\ln M_{\text{obs}}$ at fixed $\ln M$ ($\sigma_{\text{obs}}$ in our notation) because the model is based on the mass function, which is a function of $M$. In contrast, we observationally constrain the scatter of $\ln M$ at fixed $\ln M_{\text{obs}}$ because we select follow-ups based on their $M_{\text{obs}}$.

An updated halo bias function is available in Tinker et al. (2009), the difference between the two functions do not significantly affect our results. We also note that the mass function and the halo bias function we use are not in a consistent framework; since we only use their dependence on cosmology, this inconsistency is not important.

In general, the theoretical model is based on $P(\ln M_{\text{obs}} | M)$, while observations put constraints on $P(M | \ln M_{\text{obs}})$. Our goal is therefore to convert constraints on the latter distribution to constraints on the former.

Let us return to the original survey, focusing on the follow-up observations in a bin specified by $\phi(\ln M_{\text{obs}})$ in a narrow redshift range. If the follow-up mass $M_f$ perfectly recovers the true mass, the mean and variance of $\ln M_f$ will read

$$\langle \ln M_f \rangle = E[\ln M_f | \phi(\ln M_{\text{obs}})]$$

$$\alpha_\phi \int d \ln M \frac{dn}{d \ln M} (\phi | \ln M) ,$$

and

$$V = \text{Var}[\ln M_f | \phi(\ln M_{\text{obs}})]$$

$$\alpha_\phi \int d \ln M (\ln M - \langle \ln M_f \rangle)^2 \frac{dn}{d \ln M} (\phi | \ln M) .$$

The information from a single follow-up mass measurement is given by an analog of Equation 6

$$\tilde{F}_{\alpha \beta} = \langle \ln M_f \rangle \alpha V^{-1} \langle \ln M_f \rangle \beta + \frac{1}{2} V^{-1} V_{\alpha \beta} V^{-1} ,$$

where the superscript $(i)$ indicates the mass and redshift bin from which we select follow-ups. If we follow up $N_i$ clusters in bin $i$, the Fisher matrix for the whole follow-up program reads

$$\tilde{F}_{\alpha \beta} = \sum_i N_i \tilde{F}_{\alpha \beta} (i) .$$

This Fisher matrix is added to the Fisher matrix of counts-in-cells (Equation 6) to improve the constraints on nuisance parameters.\footnote{Technical note: In principle, when we calculate the Fisher matrix $\tilde{F}_{\alpha \beta}$, the derivatives should include cosmological parameters. However, to correctly model the cosmological information in follow-up observations, we need to include the covariance between the follow-up observations and the original cluster survey. For simplicity, we ignore this covariance and only consider the information for nuisance parameters, noting that the cosmological information is negligible for up to 1000 follow-ups.} Note that the second term in Equation 11 plays the role of “noise as signal” as in the case of counts-in-cells, characterizing the information included in the mass variance of follow-ups.

Figure 11 shows how follow-up observations improve the dark energy FoM (defined in §1). We calculate the ratio between the improved FoM and the fiducial FoM from DES (without follow-ups) and demonstrate how this ratio improves as the number of follow-up mass measurements increases. For the two lower curves, we assume that follow-up targets are evenly selected across all bins (i.e., $N_i = N_i/40$, including 4 bins in mass and 10 bins in redshift). We also limit the number of follow-ups in each bin by the number of clusters that DES is expected to observe.

The red dashed curve shows the improvement of the FoM using the information from both the mean and the variance of follow-up mass measurements (i.e., both terms in Equation 11). As can be seen, the follow-ups can substantially improve the FoM; for example, for 100 follow-up mass measurements, the FoM can be improved by 47.0%. For comparison, the blue dotted curve shows the improvement in the FoM using the information only from the mean of follow-up mass measurements (i.e., only the first term in Equation 11). As can be seen, lacking the information from the variance can substantially reduce the effectiveness of follow-ups. This case may be relevant to the analysis of stacked cluster samples.
we design our algorithm as follows:

1. The system configuration is characterized by the number of follow-up targets in each mass and redshift bin, $N = (N_1, \ldots, N_{10})$. (We use 4 bins in mass and 10 bins in redshift.)

2. The rearrangement of clusters in bins, or the transition from one configuration to another, is designed as follows: given an initial configuration, for each “donor” bin $i$, we randomly pick a “receiver” bin $j$ and transfer $n_{\text{trans}}$ clusters to the receiver bin. Here $j$ is randomly chosen from all available bins, and $n_{\text{trans}}$ is a random integer between 0 and $n_{\text{limit}}$. We choose $n_{\text{limit}}$ to be 1% of the total number of follow-ups $N_i$; for example, if $N_i = 100$, we transfer 0 or 1 cluster at a time. We also require the number of follow-ups in each bin to be between zero and the number of clusters DES is expected to observe in that bin. After running $\theta$ through all bins, i.e., letting each bin play the role of donor once, we reach a new nearby configuration.

3. The objective function is the FoM, which we are trying to maximize by sampling different configurations. Applying the idea of the Metropolis algorithm, we start from the current configuration $(N_i, \text{FoM}_i)$ and sample a nearby configuration $(N_{\text{try}}, \text{FoM}_{\text{try}})$. If $\text{FoM}_{\text{try}} > \text{FoM}_i$, $N_{\text{try}}$ is accepted at this step; that is, we set $N_{i+1} = N_{\text{try}}$. If $\text{FoM}_{\text{try}} < \text{FoM}_i$, $N_{\text{try}}$ is accepted with probability $\exp[(\text{FoM}_{\text{try}} - \text{FoM}_i)/T]$. Here $T$ is the “temperature” parameter that determines the probability of moving to a smaller FoM value (analogous to thermal fluctuations).

4. To design our annealing schedule, we start with a $T$ value that roughly gives an acceptance rate of 0.2; this rate empirically indicates a fair sampling of the configuration space (see, e.g., Dunkley et al. 2005 for the case of Markov Chain Monte Carlo). After running $10^4$ iterations with this temperature, we lower the temperature by 10% and run 4000 iterations as one step of the annealing. We repeat this annealing procedure between 10 and 40 times (depending on the size of the configuration space) until the improvement in the FoM is negligible and the system “entropy” is low in the sense that clusters are concentrated in a few bins.

Since our configuration space is factorially large, we reduce its size to facilitate the sampling. We start with only 20 bins by doubling the size of the redshift bin and determine where the relevant bins are. We then return to our original binning and exclude irrelevant bins to reduce the size of the configuration space.

Regarding the convergence of our algorithm, we note that although the simulated annealing algorithm will eventually converge to the global optimum, pursuing this convergence is impractical due to the extremely large configuration space. We use a sufficiently high temperature at the beginning to ensure that the configuration space is fairly sampled, but we cannot guarantee that the global optimum is found at the end. However, after testing several different initial configurations, we find that various local optima are very close to each other. Each of these optima provides significant improvement when compared to the evenly-selected follow-ups. We thus expect...
that our method provides a good solution to the optimization problem.

Figure 2 illustrates two examples of our optimization; the upper panel corresponds to 10 follow-ups in total, $N_f = 10$, and the lower panel corresponds to $N_f = 100$. For $N_f = 10$, the optimal configuration focuses on only the highest and the lowest mass bins at the lowest redshift. This configuration can improve the FoM by 76\%. For $N_f = 100$, both strategies prefer the corners, because these bins provide the longest lever arms for constraining the mass and redshift dependence of the observable–mass distribution. Here we give each bin in mass and redshift equal weighting in choosing the follow-ups; the effects of observational cost will be explored in Figures 5 and 6.

Fig. 2.— Optimal follow-up strategies that maximize the FoM at a given number of follow-up observations $N_f$, calculated with a simulated annealing algorithm. Each pixel corresponds to one mass and redshift bin in the optical cluster survey, and the corresponding number of follow-ups in this bin is shown. Upper: Optimal strategy for 10 follow-ups. This configuration corresponds to following up the most and the least massive clusters in the lowest-redshift bin, and it can improve the FoM by 13.8\%. Lower: Optimal strategy for 100 follow-up observations. In addition to two low-redshift bins, this configuration also favors the highest-redshift and lowest-mass bin. This configuration can improve the FoM by 76.5\%, compared to 47.0% for evenly-selected follow-ups. Both strategies prefer the corners, because these bins provide the longest lever arms for constraining the mass and redshift dependence of the observable–mass distribution. Here we give each bin in mass and redshift equal weighting in choosing the follow-ups; the effects of observational cost will be explored in Figures 5 and 6.

4. REQUIREMENTS FOR THE FOLLOW-UP MASS PROXY

4.1. Scatter and Covariance of the Mass Proxies

So far we have assumed that follow-up observations can recover the true mass precisely; i.e., $M_f = M$. In reality, $M_f$ itself is a mass proxy and has a scatter $\sigma_f$ around the true mass $M$. Moreover, this scatter may correlate with $\sigma_{obs}$, the scatter of $M_{obs}$ around $M$. Therefore, a proper analysis of the effects of follow-up observations should include the observable–follow-up–mass distribution $P(ln M_{obs}, ln M_f | ln M)$. We assume that the follow-up mass tracer has a tighter relation with the true mass when compared with the original mass tracer in the survey; for example, X-ray and SZ mass proxies have lower scatter than any known optical mass proxy (e.g., Kravtsov et al. 2006). We have assumed $\sigma_{obs} = 0.5\%$ throughout our analysis; we further assume that $\sigma_f = 0.1$ and that the two mass proxies, $M_{obs}$ and $M_f$, have a correlation coefficient $\rho$. No prior knowledge is assumed about $\rho$.

Let us assume that the observable–follow-up–mass distribution $P(ln M_{obs}, ln M_f | ln M)$ is a bivariate Gaussian distribution in $(ln M_{obs} - ln M)$ and $(ln M_f - ln M)$ with the covariance matrix

$$
\Sigma = \begin{pmatrix}
\sigma_{obs}^2 & \rho \sigma_{obs} \sigma_f \\
\rho \sigma_{obs} \sigma_f & \sigma_f^2
\end{pmatrix}
$$

The mean of $ln M_f$ is given by

$$
E[ln M_f | \phi(\ln M_{obs})] \propto \int d ln M_f \frac{dn}{d ln M_f} \int d ln M_{obs} P(ln M_{obs}, ln M_f | ln M) .
$$

The variance of $ln M_f$ can be calculated similarly. Since this variance involves both $\sigma_{obs}$ and $\sigma_f$, in the limit $\sigma_f \ll \sigma_{obs}$, we expect the variance of $ln M_f$ to be dominated by $\sigma_{obs}^2$. On the other hand, if $\sigma_f$ is larger (i.e., the follow-up mass measurements are noisier), the resulting mass variance of follow-ups will be larger and the FoM improvement will be less significant.

We find that different values of $\rho$ barely affect the resulting FoM: Positive $\rho$ slightly improves the FoM, while negative
The top, labeled as (3), correspond to the case of a low scatter in $\ln M_{\text{obs}}$, $\sigma_{\text{obs}} = 0.2$. Comparing the red solid curve to the black solid curve, we can see that the FoM is significantly improved due to the smaller mass variance of the follow-ups. Since the follow-ups are selected by $M_{\text{obs}}$, lowering $\sigma_{\text{obs}}$ leads to follow-ups with a less spread in mass and provides better constraints. The red dashed curve shows that marginalizing over $\sigma_{\text{f}}$ and $\rho$ barely changes this result, since at this regime both mass tracers have very high fidelity.

We note that the FoM improvement due to a small $\sigma_{\text{obs}}$ comes from the follow-ups rather than self-calibration. When we lower $\sigma_{\text{obs}}$ from 0.5 to 0.2, the fiducial FoM from self-calibration (without follow-ups) barely changes. This result reflects the fact that simply reducing the value of the scatter is not as effective as improving the constraints on the scatter.

We also note that $\sigma_{\text{f}}$ and $\rho$ are assumed to be independent of mass and redshift; detailed properties of $\sigma_{\text{f}}$ and $\rho$ are beyond the scope of this work. However, possible dependence of these parameters on mass and redshift, if not well constrained, may severely degrade the FoM (see, e.g., Sahlen et al. 2009).

In summary, we have found that as long as $\sigma_{\text{f}}$ is sufficiently small, the effects of $\sigma_{\text{f}}$ and $\rho$ are negligible, and the uncertainty in $\rho$ has only a modest impact on the efficacy of the follow-up observations.

4.2. Systematic Bias in Follow-up Mass Measurements

In the previous section, we have learned that scatter and covariance do not affect our baseline result. Ignoring their impact, we now consider the possibility that follow-up mass measurements are systematically biased by a constant factor $b$. The mean of $M_{\text{f}}$ takes the form

$$E[\ln M_{\text{f}}|\phi(M_{\text{obs}})]=\ln b+ E[\ln M_{\text{f}}|\phi(M_{\text{obs}})].$$

The parameter $b$ characterizes the average systematic bias of the follow-up mass measurements and has no impact on the variance of $M_{\text{f}}$. We include $\ln b$ as an additional nuisance parameter in the Fisher matrix (Equation 12) and study its impact on the FoM. We are looking for the required constraints on $\ln b$ to avoid severe degradation of the FoM.

When comparing Equations 7 and 14, we note that $\ln M_{\text{bias}}$ and $\ln b$ are completely degenerate in determining $E[\ln M_{\text{f}}|\phi]$. On the other hand, since $\ln b$ does not affect the variance of $M_{\text{f}}$ in a bin (Equation 10), this variance can provide information for $\ln M_{\text{bias}}$ and break the degeneracy. Here we demonstrate again the importance of the mass variance in follow-up observations.

Figure 4 shows how the systematic bias degrades the efficacies of our optimal follow-up strategies. We set the fiducial value of $\ln b$ to be 0 and compare different prior constraints in $\ln b$. As can be seen, for approximately 200 follow-ups, a degradation of less than 10% in the FoM requires $\sigma_{\text{prior}}(\ln b) < 0.05$. The required prior constraint on $\ln b$ depends on the number of follow-up observations; larger follow-up programs require even higher precision. Consequently, it is very likely that the systematic bias in mass measurements will determine the efficacy of follow-up observations.

5. Optimizing X-ray and SZ Follow-up Programs: Observational Issues

In previous sections, we assumed that all optically-selected clusters have an equal chance to be followed up, regardless of their mass and redshift. In reality, observing optically-selected low-mass or high-redshift clusters may be very difficult or even impossible with some methods; the optimiza-
tion in §5.2 is thus impractical. For example, observing high-redshift clusters in X-ray requires substantially (if not prohibitively) more telescope time and sometimes has limited improvement in parameter constraints. Thus, we would like to optimize the follow-up strategy considering both the FoM and the cost of telescope time. In this section, instead of assuming a fixed number of follow-ups, we study how to optimize the follow-up strategy with limited observational cost. We first model the observational cost in §5.1 and then demonstrate the optimization in §5.2.

5.1. Observability and Cost Proxies

For X-ray, we expect that a precise mass measurement requires certain photon counts; therefore, we assume that the telescope time for observing a cluster is inversely proportional to its flux of X-ray photons. This flux is proportional to $L_X/D_2^2$, where $L_X$ is the X-ray luminosity and $D_2(z)$ is the luminosity distance. We assume a self-similar scaling relation from the fit of Vikhlinin et al. (2009b), $L_X \propto M_{500}^{1.6} E^{1.85}(z)$. This fit is based on the mass with overdensity 500 times the critical density ($M_{500}$), while we calculate the mass function using overdensity 200 times the mean matter density ($M_{200}$); therefore, we convert $M_{500}$ to $M_{200}$ using the fitting formula in Hu & Kravtsov (2003). We normalize the observational cost such that one unit corresponds to the telescope time for observing a cluster of mass $10^{15} h^{-1} M_\odot$ at redshift 0.05. Measuring the mass of such a cluster to 10% accuracy using $Y_X$ takes approximately 0.13 kilo-second with a single XMM MOS camera (A. Mantz 2009, private communication). We note that we use $\sigma_f = 0.1$ for X-ray clusters (e.g. Kravtsov et al. 2006).

For SZ, we expect that the observational time is proportional to the inverse square of signal-to-noise ratio, $\text{SNR}_X \propto Y/\sqrt{\Omega}$, where $Y$ is the integrated Compton-y parameter and $\Omega$ is the angular size of the cluster. In virial equilibrium, $Y \propto M_5^{1/3} \rho_m / D_2^2$, where $\rho_m(z)$ is the mean matter density and $D_2(z)$ is the angular diameter distance. The dependence on angular size comes from averaging the total cluster emission over some number of detectors. The SZ cost proxy is therefore proportional to $D_2^2 M_{850}^{2/3} (1+z)^{-4}$. In addition, we exclude clusters of redshift less than 0.1; these clusters have large angular sizes and are contaminated by the primary CMB anisotropy. We also exclude clusters of mass less than $10^{14} h^{-1} M_\odot$, because they are subject to significant background confusion (Holder et al. 2007). We normalize the observational cost such that one unit corresponds to the telescope time for observing a cluster of mass $10^{15} h^{-1} M_\odot$ at redshift 0.15. To observe such a cluster, it takes about 30 minutes to obtain SNR=10 with SPT (D. Marrone and B. Benson 2009, private communication). We use a slightly larger scatter for SZ clusters, $\sigma_f = 0.2$; simulations have suggested that this scatter may be intrinsic (e.g. Shaw et al. 2008), and projection effects can further increase the scatter.

The top panels in Figure 5 present the mass and redshift dependence of the cost proxies. As can be seen, X-ray and SZ cost proxies have different patterns. The cost of X-ray clusters increases rapidly with redshift, while the cost of SZ clusters is almost constant with redshift. The latter is primarily sensitive to mass rather than redshift; thus, high-redshift follow-ups are more available than low-mass ones for SZ. We will continue to factor in the total observational cost of a follow-up program, which is obtained by summing over the product of cluster number and cost in each bin.

5.2. Optimizing the Follow-up Strategy at a Given Observational Cost

Given limited telescope time for a follow-up program, we would like to find the strategy that maximizes the FoM. However, our optimization algorithm in §5.2 can not be applied directly; sampling a configuration at a given cost is not practical, since both the FoM and the cost depend on the configuration. We instead use a Monte Carlo approach: We sample a configuration and find its corresponding point on the cost–FoM plane. After sampling many configurations, we can find the boundary of these points and estimate the upper bound of the FoM at a given cost.

To generate these Monte Carlo points, we slightly modify the sampling algorithm in §5.2. At a given $N_f$, we sample $\sim 10^5$ configurations and compute their corresponding cost and FoM. We then plot all these points on the cost–FoM plane and find the maximum of FoM at a given cost. To make the sampling more efficient, we modify the algorithm to maximize (FoM/ln (cost)), which includes moderate dependence on cost. Since this objective function is not well justified, we only use it in the sampling. However, it turns out that the configuration maximizing (FoM/ln (cost)) at a given $N_f$ coincides with the boundary of the FoM at a given cost. We thus empirically propose that one can maximize (FoM/ln (cost)) at a given $N_f$ to design follow-ups.

The middle panels of Figure 5 show two examples of optimal configurations at a given cost, both of which improve the FoM by approximately 50%. We present both the number and the percentage of follow-ups in each bin. Comparing Figure 5 to Figure 2, we can see the impact of observational cost on designing follow-up strategies. For X-ray, as expected, the high-redshift and low-mass clusters are down-weighted because of their high cost; instead, clusters at low redshift are chosen. For SZ, due to its almost redshift-independent cost, massive clusters with a wide redshift range are preferred. We note that in this configuration, in addition to the lowest and the highest redshift bins, the middle-redshift bin ($z \approx 0.5$) is also

\[ Y \propto M_5^{1/3} \rho_m / D_2^2 \]
while the cost of SZ clusters is assumed to be proportional to $D_L^2/L_x$. The cost of X-ray is more sensitive to redshift, while the cost of SZ is more sensitive to mass. A wide redshift range.

50% improvement in the FoM. As can be seen, X-ray follow-ups include clusters only in the low-redshift bins, while SZ follow-ups include massive clusters in a wide mass range of mass and redshift is the most effective. We emphasize that the observable–mass distribution depends on mass and redshift dependence of the observable–mass distribution, thus further improving the FoM. The FoM and cost corresponding to 50% and 100% improvements in the FoM. The configurations are very different from the 50% cases in Figure 2. For X-ray, the follow-ups include a wider range of mass and redshift, while SZ follow-ups include massive clusters in a wide redshift range.

Two optimal cases that correspond to 100% improvement in the FoM. The configurations are very different from the 50% cases in Figure 2. For X-ray, the follow-ups include a wider range of mass and redshift, while SZ follow-ups include massive clusters in a wide redshift range.

For SZ, however, clusters of different redshift range are likely to be observed with different instruments with different normalizations in cost. These complications will be instrument specific and will change the optimization.

Figure 6 shows the FoM improvement due to optimization as a function of cost for both X-ray (left panel) and SZ (right panel) follow-ups. The lower x-axes correspond to the telescope time specific to XMM and SPT. We compare the optimal cases (solid curves) with the baseline cases (dashed curves), which correspond to equal number of follow-ups in all mass and redshift bins with cost less than 200. The import ance of optimizing follow-up strategies is abundantly clear: to achieve a specified FoM, our optimal strategies can completely included; this configuration reflects the fact that the two endpoints and the point in the middle are preferred for constraining the power-law dependence.

The bottom panels of Figure 5 present two examples of approximately 100% improvement in the FoM. As the allowed cost increases, follow-up strategies change. These optimal configurations now extend to bins on the corners, as in the cases in Figure 4. For X-ray, the follow-ups include a wider range of mass and redshift. For SZ, less-massive clusters are included, and the configuration favors the lowest and the highest redshift bins.

These follow-up strategies are related to our assumptions that the observable–mass distribution depends on mass and redshift via power laws. To constrain power laws, sampling a range of mass and redshift is the most effective. We emphasize that these follow-up strategies are targeting dark energy constraints alone and do not comprehensively consider cluster science. In reality, skipping follow-ups in some bins may be risky for cluster science. In addition, we will need some follow-ups in every mass and redshift bin to test our power-law assumptions. After the power-law assumptions are justified, we can more confidently use our follow-up strategies to improve dark energy constraints. We also emphasize that a single instrument is assumed for the follow-up observations. For SZ, however, clusters of different redshift range are likely to be observed with different instruments with different normalizations in cost.
reduce the required telescope time by about an order of magnitude for small follow-up programs.

In Figure 6, the optimal cases for X-ray and SZ show different features. The stars and crosses mark the 50% and 100% improvements, respectively; their corresponding configurations have been shown in Figure 5. As can be seen, for X-ray, as the allowed cost increases, the FoM increases less rapidly than SZ. Both curves show slope changes slightly below the 100% improvement. These slope changes are due to the changes in configurations, as we discuss below.

For X-ray, when the allowed cost is below $10^4$, the most effective strategy is to tighten the constraints of the observable–mass distribution in low-redshift bins (as shown in the middle left panel in Figure 5). When the allowed cost is high enough, the follow-ups can afford to constrain both low and high redshift bins (as shown in the bottom left panel). Since constraining two extreme redshift regimes gives much better constraints on the evolution, the slope of the FoM increases. Nevertheless, this slope increase is very close to the point where the optimal case approaches the baseline case. With such a large allocation of telescope time, optimization is no longer essential, and the uniform sampling can achieve the same FoM.

For SZ, two obvious slope changes can be seen. The first one occurs near the cost of 3000. Below this cost, only massive clusters are chosen (as shown in the middle right panel in Figure 5). Above this cost, less-massive clusters become affordable (as shown in the bottom right panel), and the mass dependence of the observable–mass distribution is better constrained, leading to the slope increase of the FoM. The second change occurs near the cost of $10^5$, where the slope suddenly drops and the optimal case approaches the baseline case. At this point, we exhaust the information from the two redshift ends, and sampling the middle regime cannot make significant improvement. The lack of further improvement leads to the decrease of slope, and optimization is no longer essential when so much telescope time is available.

Comparing X-ray and SZ, we can view the design of cost-effective follow-up strategies as a trade-off between constraining mass dependence and constraining redshift evolution of the observable–mass distribution. When the cost is more sensitive to redshift than mass, as in the case of X-ray, we should prioritize the constraints on mass dependence according to limited telescope time. On the other hand, when the cost is more sensitive to mass than redshift, as in the case of SZ, redshift evolution should be prioritized.

In the right panel of Figure 6, we add two points as references: the red triangle and the red square present the cases, respectively, of 1000 and 2000 deg$^2$ of SZ follow-ups for DES; these assumptions are relevant for SPT. We assume a mass threshold of $10^{14.5} h^{-1} M_\odot$, which roughly corresponds to the mass threshold of the ongoing SPT survey (L. Shaw and B. Benson, 2009, private communication). However, here we assume the follow-up mass measurements have a constant scatter and no systematic bias. In reality, the SPT survey may not have the same precision in mass measurements for all clusters, and degradation due to inaccurate mass estimates is possible.

6. DISCUSSION

In §4, we studied the impact of the correlation between different mass proxies. In principle, since the true mass is not observable, $\rho$ cannot be directly measured. However, this correlation can be studied using consistency of scaling relations for different mass proxies. For example, Rykoff et al. (2008) studied the scaling relation of the mean X-ray luminosity and mean weak lensing mass for optically-selected samples, which are binned by optical richness. They compared this scaling relation with the one derived from X-ray selected samples and found that the correlation between X-ray luminosity and optical richness is consistent with zero. Although these authors did not provide constraints on the correlation, they demonstrated an effective way to study it. In addition, Rozo et al. (2008) used a similar analysis to obtain the constraints on the correlation between X-ray luminosity and mass for a given optical richness, finding $\rho_{L_X M_{\text{opt}}} > 0.85$.

This correlation can also be studied with simulations. For example, the results of Wechsler et al. (2006) imply a slight
anti-correlation: at a given halo mass, halos with high concentration have lower richness on average. Cohn & White (2009) studied the joint SZ and optical cluster finding in simulations. They demonstrated that the cluster mass estimates from optical richness and SZ flux are positively correlated. Detailed comparisons for different mass proxies, however, will require further exploration.

In §4.2, we studied the impact of the systematic bias of follow-up mass measurements. Different follow-up methods have different sources of systematic bias; here we compare several different mass proxies studied in the literature. Nagai et al. (2007) simulated the X-ray mass measurements and found that the total cluster mass derived from hydrostatic equilibrium is biased low by about 5% to 20% (also see, e.g., Rasia et al. 2006; Mahdavi et al. 2008). They also found that the bias is less significant for relaxed systems and for the inner regions of the clusters. This underestimate can be attributed to the non-hydrostatic state of the intracluster medium that provides additional pressure support (Lau et al. 2009). On the other hand, they found that the estimate of the mass of the intracluster medium \( M_{\text{gas}} \) is robust.

Vikhlinin et al. (2009a) used multiple X-ray indicators, including \( M_{\text{gas}} \), the temperature \( T_X \), and estimated total thermal energy \( Y_X = M_{\text{gas}} \times T_X \) to calibrate the cluster mass. These mass indicators have been shown by simulations to have a tight scaling relation with respect to the total mass (e.g., Kravtsov et al. 2006). Vikhlinin et al. (2009a) also calibrated the total mass with low-redshift samples and cross-checked it with the weak lensing results. Therefore, X-ray clusters, when carefully calibrated, are likely to provide the most robust mass proxy and the most ideal method for follow-up observations.

On the other hand, SZ observations are still limited by statistics and have few observational studies on the scaling relation and the cluster profile (e.g., Mroczkowski et al. 2008; Bonamente et al. 2008). Their utility is thus yet to be fully demonstrated. In addition, Rudd & Nagai (2009) simulated the two-temperature model for clusters found that the non-equipartition of electrons and ions may lead to 5% underestimate of the mass derived from SZ. On the other hand, simulations have shown that the scatter of the SZ mass proxy is small. If the systematic bias can be well constrained, SZ follow-ups may become very influential given their statistical power in the near future.

Another possibility of follow-up observations is weak lensing mass measurements of individual clusters. Hoekstra (2007) and Zhang et al. (2008) compared the mass measurements from weak lensing and X-ray, finding good agreement. However, weak lensing mass measurements usually have 20% uncertainties due to projection along the line of sight, and the current statistics are still low. In addition, detailed understanding of the photometric redshift properties of source galaxies is required to avoid systematic bias in the recovered weak lensing mass (see e.g., Mandelbaum et al. 2008a).

Stacked weak lensing analysis has been used to measure the mean mass of the clusters for a given optical richness (e.g., Johnston et al. 2007; Mandelbaum et al. 2008a). This method does not suffer from projection effects due to uncorrelated structure and allows one to estimate the mass of low-mass clusters, for which individual weak lensing cannot be detected. However, the stacked analysis cannot provide the variance of mass, which, as we have shown, contains important information. If the variance of mass can be determined using this method in the future (for example, by resampling), the constraining power of this method will be improved.

Finally, we note that one caveat of our results is the assumption that the optical richness–mass distribution is well described by a log-normal distribution and the scaling and scatter follow power laws. The validity of these assumptions will need to be tested explicitly with both simulations and observations.

7. CONCLUSIONS

We studied the impacts of follow-up observations — more precise measurements of cluster mass — on the constraining power of large optical cluster surveys. Considering the self-calibrated cluster abundance data from the Dark Energy Survey, we demonstrated that the dark energy figure of merit can be significantly improved. Our primary findings are:

1. Optimal target-selection strategies are essential for maximizing the power of modestly-sized follow-up programs. For instance, 100 optimally-selected follow-ups can improve the FoM of DES by up to 76%, which is compared to a 47% improvement due to evenly-selected follow-ups. Random sub-sampling of the cluster catalog is even less effective. Generally speaking, one should always follow up low-redshift clusters first, and then extend to the higher-redshift and lower-mass regime (§3.1 and §3.2).

2. The scatter of the follow-up mass proxy and the correlation between the optical richness and the follow-up mass proxy have only modest effects on the FoM, provided that the follow-up mass proxy has sufficiently small scatter. On the other hand, although lowering the scatter of optical richness does not change the baseline self-calibration results, it will significantly enhance the efficacy of follow-ups (§4.1).

3. Systematic bias in follow-up mass measurements should be controlled at the 5% level to avoid severe degradation. In addition, if only the mean of cluster mass is measured, the systematic bias of follow-up mass proxy will be degenerate with the bias of optical richness; measuring the variance of cluster mass can break this degeneracy (§4.3).

4. We explored observational issues to propose more practical X-ray and SZ follow-up programs. The observational costs of X-ray and SZ are, respectively, sensitive to redshift and mass, which in turn leads to different follow-up strategies. To achieve 50% improvement in the FoM, the most cost-effective follow-up strategy involves approximately 140 low-redshift X-ray clusters or 200 massive SZ clusters. In general, our optimal strategies can reduce the observational cost required to achieve given dark energy constraints by up to an order of magnitude (§5).

A follow-up mass tracer that is unbiased at the 5% level will substantially benefit optical cluster surveys. On the other hand, reducing the scatter of optical mass tracer will significantly improve the efficacy of optically-selected follow-ups. Current observational resources allow a few hundred low-redshift X-ray clusters, and in the near future hundreds or thousands of SZ clusters will become available. Therefore, detailed follow-up studies of a small but optimally-selected cluster sub-sample have the potential to be a powerful complement to current and imminent cluster surveys.
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