Quantum optical response of a hybrid optomechanical device embedded with a qubit

Sabur A Barbhuiya and Aranya B Bhattacherjee

Department of Physics, Birla Institute of Technology and Science, Pilani, Hyderabad Campus, Telangana - 500078, India

E-mail: aranyabhuti@hyderabad.bits-pilani.ac.in

Received 23 June 2020, revised 25 August 2020
Accepted for publication 18 September 2020
Published 12 October 2020

Abstract
We theoretically investigate the optical response in a hybrid quantum optomechanical system consisting of two optically coupled micro-cavities in which a two-level system (qubit) is coupled with a mechanical membrane. The qubit can either be a defect which interacts with the mechanical oscillator via the linear Jaynes–Cummings interaction or a superconducting charge qubit coupled with the mechanical mode via non-linear interaction. We find that coherent perfect transmission (CPT), coherent perfect synthesis (CPS) and optomechanically induced absorption (OMIA) can be generated by suitably adjusting the system parameters. We find that the qubit and its interaction with the mechanical oscillator emerges as a new handle to control these quantum optical properties. The presence of the qubit results in four points where CPT and CPS can be realized compared to the pure optomechanical case (i.e. in the absence of a qubit) where only three points are attained. This shows that the presence of the qubit gives us more flexibility in choosing the appropriate parameter regime where CPT and CPS can be attained and controlled. We also find that OMIA shows three distinct peaks both in the linear and non-linear cases. In the absence of the qubit, OMIA is converted to optomechanically induced transmission (OMIT). An increase in the qubit decay rate also shows a transition from OMIA to OMIT. Our study reveals that the optical response of the non-linear case is relatively rapid and more sensitive compared to the linear case to changes in the system parameters. This demonstrates the potential use of this hybrid system in designing a tunable all-optical switch and photon router, both of which form important elements of a quantum information network.

Keywords: quantum optomechanics, qubit, coherent perfect transmission, coherent perfect synthesis, optomechanically induced transmission, optomechanically induced absorption

(Some figures may appear in colour only in the online journal)

1. Introduction

In the past few decades there has been tremendous advancement in our understanding of light–matter interaction in hybrid optomechanical systems [1]. In recent years, research in the area of micro- and nanoscale mechanical resonators has opened up the possibility of novel quantum devices [2, 3]. New and interesting physics have emerged by coupling mechanical resonators with other quantum objects such as superconducting charge qubits [4–15], transmission line resonators [16, 17], microwave cavities [18–20], optical cavities [21–24], quantum dots, nitrogen-vacancy centers [25–27], electron spin [28] and atoms [29–34]. As a result, research has advanced in the direction of demonstrating new quantum effects using hybrid optomechanics [29–34]. Experimental and theoretical results have demonstrated that mechanical resonators operating in quantum regime [36–38] can be used as quantum interfaces and quantum transducers [39–41]. In recent years, experiments with cavity optomechanics have successfully entered into the resolved sideband limit where
mechanical sidebands of the optical mode lie outside its linewidth [42]. It has been shown that the intracavity optical field can modify the effective loss factor of the mechanical mode [30] which leads to mechanical cooling [31] via the optomechanical interaction when the input field is red-detuned from the cavity resonance, where photons preferentially absorb a phonon from the mechanical oscillator and scatter upwards to the cavity resonance. This situation is quite similar to laser cooling of atomic and molecular motion in a cavity [48].

Hybrid electro-optomechanical systems demonstrate strong Kerr non-linearities even in the weak-coupling regime [49], and can be used for photon blockade and generation of nonclassical states of microwave radiation. There have been some interesting theoretical results based on single-photon strong coupling in optomechanical systems [50–55]. An interesting and useful development took place when a number of experiments demonstrated strong optomechanical coupling [32, 56–58]. Electromagnetically induced transparency (EIT) has played a crucial role in many subfields of quantum optics [59]. The quantum interference in the phonon excitation pathways leads to the optomechanical analog of EIT, the so-called optomechanically induced transparency (OMIT) [60, 61].

The OMIT phenomenon can be used to slow [62–64] and even stop light signals [62, 63] and can be used to store information in mechanical oscillators. Multimode optomechanical systems have also been studied to attain quantum entanglement [65]. OMIT [66] and single-photon non-linearity [67].

In this paper, we investigate a double-cavity optomechanical system with a movable mechanical membrane in the middle, in the presence of linear/non-linear interaction of a two-level system (TLS) and the mode of the mechanical oscillator. The linear interaction is achieved by coupling a quantum dot or a defect to the movable middle membrane while the non-linear interaction is generated by coupling a superconducting qubit with the mechanical oscillator. We have compared the two cases in terms of the optical response (coherent perfect transmission (CPT), coherent perfect synthesis (CPS), EIT) of the system.

2. Physical models

There are different ways in which the qubit optomechanics can be hybrid. In one possibility, the qubit can be coupled to the optical cavity mode and the optical cavity mode can be coupled to the mechanical mode. In another case of fully hybrid model, the qubit, optical mode and mechanical mode all are mutually coupled. In this work, we focus on a hybrid case where the qubit is coupled to the mechanical motion and the mechanical mode is interacting with the optical mode. Here, specifically, we consider a hybrid double-cavity optomechanical system composed of two fixed mirrors with partial transmissivity and one movable membrane located between the two fixed mirrors (membrane in the middle) as shown in figure 1 [68, 69]. The membrane in the middle oscillator has an eigen frequency $\omega_m$ and a decay rate $\gamma_m$. The movable membrane is perfectly reflective and is at its equilibrium position so that the system can be regarded as two identical Fabry–Perot cavities of length $L$ and frequency $\omega_0$. The left (right) cavity optical mode is described by the creation operator $c_1^\dagger (c_2^\dagger)$ and annihilation operator $c_1 (c_2)$ while the mechanical mode is described by the creation and annihilation operator $b^\dagger$ and $b$, respectively. These operators satisfy the bosonic commutation relation $[A_i, A_j^\dagger] = \delta_{ij} (A = c_1, c_2, b$ and $i, j = 1, 2$).

The system is driven from the left and right fixed mirrors by two control (probe) fields with amplitudes $\epsilon_{cl} = \sqrt{2 P_{cl}}$ and $\epsilon_{cr} = \sqrt{2 P_{cr}}$. Here the subscripts $L$ ($R$) denote the left (right) cavity. We assume both the cavities have the same decay rate $\kappa$. Both the left and right control (probe) modes have the same frequency $\omega_L (\omega_R)$. Here $P_{cl}, P_{cr}, P_L$ and $P_R$ are the relevant field powers. The subscripts $cL$ and $cR$ refer to the left and right control fields, respectively, while $L$ and $R$ refer to the left and right probe fields, respectively.

We now discuss the two cases that we will be analyzing in the paper. The first case is that of a TLS (qubit) linearly coupled to the mechanical oscillator. The qubit could be an intrinsic defect inside the mechanical resonator, a quantum dot or another TLS. The mechanical oscillator is coupled to the qubit via the linear Jaynes–Cummings interaction. Non-linear strain coupling of a single TLS defect to an optomechanical system (localized phonon mode) was recently proposed [70]. An example of such system is donor–acceptor impurity-doped silicon [71]. Another experimentally feasible scheme of linearly coupling a conducting cantilever to a Cooper box was analyzed [6]. The interaction between the cantilever and Cooper box considered was electrostatic in nature. Only the in-plane fundamental flexural mode of the cantilever was considered. The coupling constant between the Cooper box and the cantilever could be controlled by the charging energy of the box, gate voltage and cantilever electrode-island gap. Experimentally, linear coupling between a superconducting qubit and a mechanical resonator has also been demonstrated [29]. The high-frequency mechanical resonator fabricated was a piezoelectric material of aluminum nitride sandwiched between two aluminum metal electrodes. The qubit and the mechanical resonator were coupled using a capacitor. In yet another experiment, circuit cavity quantum electrodynamics was integrated with phonons [41]. A superconducting transmon qubit was simultaneously coupled to a microwave cavity and a micromechanical resonator. The coupling of the qubit to the mechanical resonator was linear in nature. The radiation pressure Hamiltonian describes the interaction between the cavity modes and the mechanical mode. However, there is no direct interaction between the qubit and the optical modes. Thus, the total Hamiltonian is the frame rotating with respect to the control field frequency $\omega_c$, and can be written as

$$H_{total} = H_{1,2} + H_{probe} + H_{qd-m}.$$ \hspace{1cm} (1)

$$H_{1,2} = \hbar \Delta c (c_1^\dagger c_1 + c_2^\dagger c_2) + \hbar g_0(c_2^\dagger c_2 - c_1^\dagger c_1)(b^\dagger + b) + \hbar \omega_m b^\dagger b + i \hbar \epsilon_{cl} (c_1^\dagger - c_1) + i \hbar \epsilon_{cr} (c_2^\dagger - c_2).$$ \hspace{1cm} (2)
Figure 1. Schematic diagram of the hybrid optomechanical system consisting of a double cavity with a movable membrane in the middle which corresponds to a mechanical oscillator (as shown in the top part). The oscillating membrane is coupled to a TLS which could be simply a defect (a) or a superconducting charge qubit (b). The TLS interacts with the mechanical mode via linear coupling in the case of a defect or non-linear coupling for the superconducting qubit. In addition, the mechanical oscillator couples to the two cavity modes via radiation pressure. Both the cavities are driven by two separate control fields as shown and two separate probe fields are also incident on the two cavities from either side which have a phase difference of $\theta$.

\[
H_{\text{probe}} = i\hbar(c_1^\dagger \epsilon_L e^{-i\delta t} - c_1^* \epsilon_L^* e^{i\delta t}) + i\hbar(c_2^\dagger \epsilon_R e^{i\theta} e^{-i\delta t} - c_2^* \epsilon_R^* e^{-i\theta} e^{i\delta t}),
\]

(3)

\[
H_{q\text{d} \rightarrow m} = \frac{1}{2} \omega_q \sigma_z + \hbar g(b\sigma^+ + b^\dagger \sigma^-).
\]

(4)

Here, $\Delta_c = \omega_0 - \omega_c$ denotes the detuning between the cavity mode and the control field, $g_0 = \frac{\Delta_c}{\sqrt{\hbar 2m \omega_m}}$ is the optomechanical coupling constant, $\delta = \omega_p - \omega_c$ is the detuning between the probe and the coupling field and $\theta$ is the relative phase between the left and right probe fields. Also, $\omega_q$ is the transition frequency of the TLS while $\sigma_+, \sigma_-,$ and $\sigma_z$ are the usual Pauli operators describing the TLS. The parameter $g$ describes the linear coupling strength between the mechanical resonator and the qubit. The linear system was studied earlier in the case of EIT [72].

We now describe an optomechanical system in which the qubit is interacting non-linearly with the mechanical mode [73]. The non-linear interaction can be achieved by coupling a superconducting charge qubit and a movable membrane in the middle of the cavity. This scheme can be achieved in an electromechanical system which operates in a strongly dispersive limit [74]. The strong dispersive limit gives rise to an effective non-linear coupling. The configuration of the superconducting charge qubit is based on a superconducting quantum interface device. The qubit can be coupled to a nano-mechanical oscillator via a variable capacitor. The capacitance depends on the position of the membrane inside the capacitor [74]. The coupling can be tuned by applying a gate voltage. Based on such proposals, strongly enhanced non-linear coupling was experimentally observed between a superconducting charge qubit and a mechanical resonator [75]. As a result, the qubit-mechanical mode term is written as

\[
H_{q\text{ubit} \rightarrow m} = \frac{1}{2} \omega_q \sigma_z + \hbar g_N (b^2 \sigma^+ + b^{2\dagger} \sigma^-).
\]

(5)

Here $g_N$ is the coupling strength between the superconducting qubit and the mechanical oscillator. The origin of $g_N$ is the Josephson coupling energy in a Cooper-pair box.

3. Heisenberg–Langevin equations, steady state and fluctuation dynamics

We now proceed to study the quantum dynamics of the linear and non-linear case systematically.

3.1. Linear case

Considering relevant dissipation and quantum or thermal noise, the quantum dynamics of the total system’s operators are given by the following quantum Langevin equations,

\[
\dot{b} = -i\omega_p b(t) - i\gamma_0 (c_2^\dagger c_2 - c_1^\dagger c_1) - i\gamma^- (t) - \frac{\gamma_0}{2} b(t) + \sqrt{\gamma_0} \eta(t),
\]

(6)
\[
\dot{\sigma} = -i\omega_\sigma \sigma(t) + igb\sigma_L(t) - \frac{k_d}{2} \sigma^-(t) + \sqrt{\Delta_2} \sigma^-_{in},
\]
\[
\dot{c}_1 = -[k + i\Delta_c - ig_0(b^\dagger + b)]c_1 + c_{eR} + c_{eL} e^{-ist} + \sqrt{2k_e} c_{in},
\]
\[
\dot{c}_2 = -[k + i\Delta_c + ig_0(b^\dagger + b)]c_2 + c_{eR} + c_{eL} e^{-ist} + \sqrt{2k_e} c_{in}.
\]

Here, \(b_{in}, \sigma^-_{in}\) are the zero-mean-value environmental noise operators of the mechanical oscillator and the two-level qubit, respectively. Also \(c_{in}^{\dagger}(c_{in})\) are the zero-mean-value quantum noise operators of the left (right) cavity. The probe fields are small compared to the control and hence can be considered as comparable to noise.

In the classical limit, we drop the fluctuations, the probe fields and replace the operators by their expectation values. We can generate the steady-state mean values by setting all the time derivatives to zero and with the factorization assumption \(<bc_1>=<b><c_1>\). By making this approximation, the coherence of the system is neglected while analyzing the mean-field dynamics.

\[
\langle b \rangle = b_1 = -i\frac{g_0(|c_2|^2 - |c_1|^2)}{\Delta_c + i\omega_q} - \frac{c_{eL}}{k + i\Delta_c},
\]
\[
\langle \sigma^- \rangle = \sigma^-_{e} = \frac{-igb\sigma_L}{\Delta_c + i\omega_q},
\]
\[
\langle c_1 \rangle = c_{1s} = \frac{c_{eR}}{k + i\Delta_c},
\]
\[
\langle c_2 \rangle = c_{2s} = \frac{c_{eR}}{k + i\Delta_c},
\]

Here, \(\Delta_{1,2} = \Delta_c + \pm g_0(b_c + b_c^*)\) is the effective detuning of the cavity modes. Note that the term \(g_0(b_c + b_c^*) \leq \Delta_c\), when \(g_0 \leq \omega_m\) and the number of photons in the two cavities is the same. This is evident from the expression for \(b_1\) since if \(|c_{1s}|^2 \approx |c_{2s}|^2\) then \(b_1 \ll 1\).

We now derive the quantum Langevin equations by substituting the ansatz \(b = b_1 + \delta b, c_1 = c_{1s} + \delta c_1, c_2 = c_{2s} + \delta c_2\) and \(\sigma^- = \sigma^-_{e} + \delta \sigma^-\) into equations (6)-(9) and retaining only the first-order terms in the fluctuations \(\delta b, \delta c_1, \delta c_2\) and \(\delta \sigma^-\). We are essentially obtaining the linearized quantum Langevin equations for the fluctuations. When the probe field is much weaker than the control field then we are justified in retaining only the linear terms of the fluctuation operators. So it is the relative strength of the control and the probe field that determines the choice of retaining the first order in fluctuations. The control field generates the mean-field values while the probe excites the fluctuations. We assume that each control drive corresponds to the corresponding cavity mode at the mechanical red sideband \((\Delta_1 \approx \Delta_2 \approx \omega_m \approx \omega_q)\) and simultaneously, the optomechanical system is operated in the resolved sideband regime \((\omega_m \gg k)\).

The quality factor \(Q\) of the mechanical oscillator is high \((\omega_m \gg \gamma_m)\). Introducing the slowly varying operators for the linear terms of the fluctuations as \(\delta b = \delta b e^{-i\omega t}, b_1 = b_{in} e^{-i\omega t}, \delta c_1 = \delta c_1 e^{-i\Delta t}, \delta c_2 = \delta c_2 e^{-i\Delta t}, \sigma^-_{in} = \sigma^-_{in} e^{-i\omega t}\), we thus obtain the linearized quantum Langevin equations for the fluctuations as

\[
\dot{\delta b} = -ig_0(c_{2s}^\dagger \delta c_2 - c_{1s}^\dagger \delta c_1) - igb\sigma_\sigma - \frac{\gamma_m}{2} \delta b + \sqrt{\gamma_m} b_{in},
\]
\[
\dot{\delta \sigma} = igb\delta b,\quad \dot{\delta c}_1 = -k\delta c_1 + ig_0 c_{1s} \delta b + \epsilon_{eR} e^{-ist} + \sqrt{2k_e} c_{in},
\]
\[
\dot{\delta c}_2 = -k\delta c_2 - ig_0 c_{2s} \delta b + \epsilon_{eL} e^{-ist} + \sqrt{2k_e} c_{in}.
\]

Note that we will be considering \(\omega_m > g_0|c_{1s}| \) and \(g_0|c_{2s}|\). Actually we are considering a high mechanical quality factor as we already mentioned before equation (14). At the same time, the optomechanical coupling is weak as mentioned just after equation (13). So overall \(\omega_m > g_0|c_{1s}| \) and \(g_0|c_{2s}|\) is valid if the cavity photon number in each cavity is not high. This considerably simplifies the analytical calculations. Here \(x = b - \omega_m\). We take the qubit to be in the ground state, i.e. \(\langle \sigma^- \rangle = -1\). We now use the ansatz \(\delta \sigma = \delta \sigma + \epsilon_{eL} e^{-ist} + \delta \sigma e^{-ist}\), with \(s = b, c_1, c_2\) and \(\sigma^-\). Under the steady-state condition \(\delta b = 0\), we obtain the following expressions,

\[
\delta b = \frac{-ig(-is + \frac{k_d}{2})(\epsilon_{eL} e^{-ist} - \epsilon_{eR})}{(-is + k)(-is + \frac{k_d}{2})(-is + \frac{k_d}{2}) - g^2 \sigma_{2s}^2} + \left[(\frac{-is + \frac{k_d}{2}}{2})G(\omega^2 + 1)\right],
\]

\[
\delta c_{1s} = \frac{G^2 \left[\epsilon_{eL} (-is + \frac{k_d}{2}) + \epsilon_{eR} e^{ist} (-is + \frac{k_d}{2}) + ig_0 c_{in}^\dagger \right]}{(-is + \frac{k_d}{2})(-is + \frac{k_d}{2}) - g^2 \sigma_{1s}^2} - \left[(\frac{-is + \frac{k_d}{2}}{2})G^2(\omega^2 + 1)(-is + k)\right],
\]

\[
\delta c_{2s} = \frac{G^2 \left[\epsilon_{eR} (-is + \frac{k_d}{2}) + \epsilon_{eL} e^{ist} (-is + \frac{k_d}{2}) + ig_0 c_{in}^\dagger \right]}{(-is + \frac{k_d}{2})(-is + \frac{k_d}{2}) - g^2 \sigma_{2s}^2} - \left[(\frac{-is + \frac{k_d}{2}}{2})G^2(\omega^2 + 1)(-is + k)\right].
\]
Here $G = g_0 c_{1x}$ is the effective optomechanical coupling related to the coupling power $P_c$. Without loss of generality, we assume $c_1$ and $c_2$, to be real-valued. In addition, $|c_{2x}/c_{1x}| = n$, as the photon number ratio of the two cavities.

### 3.2. Non-linear case

Following the procedure adopted in the linear case, we now write down the corresponding equations for the non-linear case. The quantum Langevin equations are derived as

\[
\dot{b} = -i\omega_mb(t) - ig_0(c_1^*c_2 - c_1^*c_1) - 2ig_0b\sigma^-(t) - \frac{\gamma_m}{2}b(t) + \sqrt{\gamma_m}b_m, \tag{21}
\]

\[
\dot{\sigma}^- = -i\omega_\sigma\sigma^-(t) + ig_0\sigma^2\sigma(t) - \frac{k_d}{2}\sigma^-(t) + \sqrt{k_d}\sigma^-_m, \tag{22}
\]

\[
\dot{c}_1 = [-k + i\Delta_e - ig_0(b_1 + b)]c_1 + \epsilon_{eL} + \epsilon_{eL}e^{-i\theta_1} + \sqrt{2\kappa}c_{1m}, \tag{23}
\]

\[
\dot{c}_2 = [-k + i\Delta_e + ig_0(b_1 + b)]c_2 + \epsilon_{eR} + \epsilon_{eR}e^{-i\theta_2} + \sqrt{2\kappa}c_{2m}. \tag{24}
\]

Note that we have now a non-linear term in the equation for $\sigma^-$. The corresponding steady-state values are found from the above equations as

\[
\langle b \rangle = b_s = \frac{-ig_0(|c_{2x}|^2 - |c_{1x}|^2)(k_e + i\omega_q)}{\left(\frac{2}{\pi} + i\omega_m\right)(k_e + i\omega_q) - 2G_N^2\langle \sigma \rangle_s}, \tag{25}
\]

where $G_N = g_0|b_1|$ is the effective qubit–mechanical coupling strength. The linearized quantum Langevin equations for fluctuations are now written as

\[
\dot{\sigma}^- = -i\omega_\sigma\sigma^-(t) + ig_0\sigma^2\sigma(t) - \frac{\gamma_m}{2}b + \sqrt{\gamma_m}b_m, \tag{29}
\]

\[
\dot{\sigma}^- = 2ig_0b\sigma^-\sigma - \frac{k_d}{2}\delta\sigma^-(t) + \sqrt{k_d}\delta\sigma^-_m, \tag{30}
\]

\[
\dot{c}_1 = -k\delta c_1 + ig_0c_{1x}\delta b + \epsilon_{eL}e^{-i\theta_1} + \sqrt{2\kappa}\delta c_{1m}, \tag{31}
\]

\[
\dot{c}_2 = -k\delta c_2 - ig_0c_{2x}\delta b + \epsilon_{eR}e^{-i\theta_2} + \sqrt{2\kappa}\delta c_{2m}. \tag{32}
\]

Analogous to equations (18)–(20) for the linear case, we now have the corresponding equations for the non-linear case as

\[
\delta b_s = \frac{-iG(-ix + \frac{k_e}{2})(\epsilon_{eR}e^{i\theta_2} - \epsilon_{eL})}{(-ix + k)(-ix + \frac{2}{\pi})(-ix + \frac{k_e}{2}) + 4G_N^2\sigma \langle \sigma \rangle_s + (-ix + \frac{k_e}{2})G^2(n^2 + 1)}. \tag{33}
\]

\[
\delta c_{1+} = \frac{G^2(\epsilon_{eR}e^{i\theta_2}(-ix + \frac{k_e}{2}) + n^2\epsilon_{eL}(-ix + \frac{k_e}{2})) + \epsilon_{eL}(-ix + k)(-ix + \frac{2}{\pi})(-ix + \frac{k_e}{2}) - 4G_N^2\sigma \langle \sigma \rangle_s}{(-ix + \frac{2}{\pi})(-ix + \frac{k_e}{2}) - 4G_N^2\sigma \langle \sigma \rangle_s + (-ix + \frac{k_e}{2})G^2(n^2 + 1)(-ix + k)}, \tag{34}
\]

\[
\delta c_{2+} = \frac{G^2(\epsilon_{eL}(-ix + \frac{k_e}{2}) + \epsilon_{eR}e^{i\theta_2}(-ix + \frac{k_e}{2})) + \epsilon_{eR}e^{i\theta_2}(-ix + k)(-ix + \frac{2}{\pi})(-ix + \frac{k_e}{2}) - 4G_N^2\sigma \langle \sigma \rangle_s}{(-ix + \frac{2}{\pi})(-ix + \frac{k_e}{2}) - 4G_N^2\sigma \langle \sigma \rangle_s + (-ix + \frac{k_e}{2})G^2(n^2 + 1)(-ix + k)}. \tag{35}
\]
In the next section we will investigate the optical response of the linear and non-linear system.

4. Optical response

To study the optical response of the system, we utilize the input–output theory [76] and the left-hand output field $\epsilon_{\text{out}L}$ and the right-hand output field $\epsilon_{\text{out}R}$ are written as

\[
\epsilon_{\text{out}L} + \epsilon_{L}e^{-ixt} = 2k(\delta c_{1}),
\]
\[
\epsilon_{\text{out}R} + \epsilon_{R}e^{i\theta}e^{-ixt} = 2k(\delta c_{2}).
\]

The oscillatory terms can be removed if we set $\epsilon_{\text{out}j} = \epsilon_{\text{out}j+}e^{-ixt} + \epsilon_{\text{out}j-}e^{ixt}$, $(j = L, R)$. Note that the components $\epsilon_{\text{out}L+}$ and $\epsilon_{\text{out}R+}$ have the same frequency $\omega_{p}$ as the probe fields $\epsilon_{L}$ and $\epsilon_{R}$ while the output components $\epsilon_{\text{out}L-}$ and $\epsilon_{\text{out}R-}$ have the frequency $2\omega_{p} - \omega_{p}$.

From equations (36) and (37), we obtain

\[
\epsilon_{\text{out}L+} = 2k\delta c_{1+} - \epsilon_{L},
\]
\[
\epsilon_{\text{out}R+} = 2k\delta c_{2+} - \epsilon_{R}e^{i\theta}.
\]

We will now discuss CPT and CPS for both the linear and non-linear system.

4.1. CPT

We consider here the possibility of achieving CPT in the parameter regimes where $|\epsilon_{\text{out}L+}| = 0$ and $|\epsilon_{\text{out}R+}| = 1$ with $\epsilon_{L} \neq 0$ and $\epsilon_{R} = 0$. These conditions essentially mean that we observe the left probe field from the right mirror after it passes through the double-cavity system and is perfectly transmitted through the right mirror. Note that the right probe field is taken to be absent.

Taking $\epsilon_{R} = 0$ and $n = 1$, we get from equations (29)–(32), (38) and (39) the four points where CPT will occur, in the limit $\gamma_{nl}, k_{d} \to 0$, as

\[
x_{1} \to -\sqrt{2G^{2} + g^{2} - k^{2} - \sqrt{4 g^{2}k^{2} + (-2G^{2} - g^{2} + k^{2})^{2}}} \sqrt{2},
\]
\[
x_{2} \to -\sqrt{2G^{2} + g^{2} - k^{2} - \sqrt{4 g^{2}k^{2} + (-2G^{2} - g^{2} + k^{2})^{2}}} \sqrt{2},
\]
\[
x_{3} \to -\sqrt{2G^{2} + g^{2} - k^{2} + \sqrt{4 g^{2}k^{2} + (-2G^{2} - g^{2} + k^{2})^{2}}} \sqrt{2},
\]
\[
x_{4} \to -\sqrt{2G^{2} + g^{2} - k^{2} + \sqrt{4 g^{2}k^{2} + (-2G^{2} - g^{2} + k^{2})^{2}}} \sqrt{2}.
\]

We first consider the linear case.

In figures 2 and 3, we plot the normalized output probe field energy $|\langle \omega_{L+}^{R} \rangle |^{2}$ and $|\langle \omega_{L+}^{R+k} \rangle |^{2}$, respectively, as a function of dimensionless input probe detuning $\frac{x}{k}$ for $G = 3k$; $g = 0$ (solid red line) and $G = 3k$; $g = k$ (dashed blue line). Figures 2(b) and 3(b) show the same plots near $x/k = 0$ for clarity. For $G = 3k$ and $g = k$, four distinct transmission points are noticed both from the plots as well as from equation (40). In the absence of two-level mechanical mode coupling ($g = 0$), CPT is observed at three points but on the other hand when $g$ is finite, i.e. $g = k$, the two points near $x/k$ do not demonstrate CPT. This perhaps indicates that some energy from the optical mode is taken away by the mechanical mode via optomechanical coupling and transferred to the TLS. The two points near $x/k = \pm 4.05k$ show perfect transmission (CPT).

Figure 4 shows a plot of $|\langle \omega_{L+}^{R+k} \rangle |^{2}$ for $g = 0.1k$ (solid red line) and $g = k$ (dashed blue line). Even at $g = 0.1k$, only three CPT points are visible. Actually, four transmission points start appearing when $g \geq 0.4k$.

We now consider the non-linear case corresponding to a superconducting charge qubit attached to the membrane in the middle. Proceeding in a manner similar to that for the linear case, the four transmission points appear at

\[
x_{1} \to -\sqrt{2G^{2} + 4G_{N}k^{2} - k^{2} - \sqrt{16G_{N}^{2}k^{2} + (-2G^{2} - 4G_{N}^{2} + k^{2})^{2}}} \sqrt{2},
\]
\[
x_{2} \to -\sqrt{2G^{2} + 4G_{N}k^{2} - k^{2} - \sqrt{16G_{N}^{2}k^{2} + (-2G^{2} - 4G_{N}^{2} + k^{2})^{2}}} \sqrt{2},
\]
\[
x_{3} \to -\sqrt{2G^{2} + 2G_{N}^{2} - k^{2} + \sqrt{16G_{N}^{2}k^{2} + (-2G^{2} - 4G_{N}^{2} + k^{2})^{2}}} \frac{2}{2},
\]
\[
x_{4} \to -\sqrt{2G^{2} + 2G_{N}^{2} - k^{2} + \sqrt{16G_{N}^{2}k^{2} + (-2G^{2} - 4G_{N}^{2} + k^{2})^{2}}} \frac{2}{2}.
\]

Figures 5 and 6 show the plots of $|\langle \omega_{L+}^{R+k} \rangle |^{2}$ and $|\langle \omega_{L+}^{R-k} \rangle |^{2}$, respectively, as a function of input probe detuning $\frac{x}{k}$ for $G = 3k$; $G_{N} = 0.1k$ (red line), and $G = 3k$; $G_{N} = 0.4k$ (blue dashed line). Now comparing figures 3 and 5, we note that CPT at four points is observed at $G_{N} = 0.1k$ while at $G_{N} = 0.4k$, complete transmission is only observed at two points near $x/k = \pm 4k$. The two points near $x = 0$ show near-perfect transmission ($|\langle \omega_{L+}^{R+k} \rangle |^{2} \approx 0.85$) for $G_{N} = 0.4k$, which is slightly higher than the $g = k$ case.

For all the above cases, exactly at $x = 0$, CPT is observed only when $g(G_{N}) = 0$. Thus in the presence of the TLS, we can design an ‘all-optical switch’ functioning around $x = 0$. From the above analysis, we can conclude that the non-linear system is comparatively more suitable to generate four CPT points. Around $x = 0$ points both for the linear as well as the non-linear case, we observe two transmission points. As one tunes $x$ across the $x = 0$ point, we can switch between zero transmission to large transmission (CPT in the case where $g$ or $G_{N}$ is very low). Thus, these systems have the potential to be
used as an all-optical switch. Moreover, the width of the transmission at $x = 0$ is very small for small $g(G_N)$ and it widens as we increase $g(G_N)$. This indicates that at small values of $g$ or $G_N$, the functioning of the optical switch is more sensitive, i.e., a small variation of $x$ around $x = 0$ causes a sharp change in the transmission.
4.2. CPS

In this sub-section, we consider the possibility of achieving CPS under the conditions $|\omega_{\text{out}L} + \omega_{\text{L}}| = 0$ and $|\omega_{\text{out}R} + \omega_{\text{L}}| = 2$ or $|\omega_{\text{out}L} + \omega_{\text{R}}| = 2$ and $|\omega_{\text{out}R} + \omega_{\text{R}}| = 0$ with $\epsilon_L = \epsilon_R \neq 0$. In order to avoid energy loss via fast mechanical decay, we consider a high-$Q$ quantum mechanical mode by taking $\gamma_m \rightarrow 0$. In addition we also assume that energy is not lost due to decay and decoherence of the TLS by taking $k_d \rightarrow 0$. A plot of $|\omega_{\text{out}L} + \omega_{\text{L}}|^2$ and $|\omega_{\text{out}R} + \omega_{\text{R}}|^2$ versus normalized detuning $x/k$ for the linear case is shown in figures 7(a) and (b), respectively.

Four perfect synthesis channels are produced for $g = k$, $G = 3k$ (solid red line) as well as for $G = 4k$ (dashed blue line). It is clear that at points where $|\omega_{\text{out}L} + \omega_{\text{L}}|^2 = 0$, we have $|\omega_{\text{out}R} + \omega_{\text{R}}|^2 = 2$ and vice-versa.

Plots for the non-linear case are depicted in figures 8(a) and (b), respectively. A similar observation is made as in the linear case (figure 7). Four perfect synthesis points are also visible here.

Figure 9 illustrates both the linear and non-linear case with the reduced range of $x/k$. This helps us to focus around the $x = 0$ point. Interestingly we notice that the variation of the output probe energy for the non-linear case is extremely rapid around $x = 0$ as compared to the linear case. This again points to the fact that the non-linear system can be used to design a comparatively more sensitive optical switch.

Thus we see that in CPS we can have coherent control over the perfect transmission and perfect reflection of the left and right probe fields. These observations are a result of constructive or destructive interference between $\epsilon_L$ and $\epsilon_R$ at the two cavity mirrors. This interference is found to be influenced by the coupling of the TLS with the movable membrane. We find that the transmission can be controlled by adjusting the two-level parameters, which emerges as a new handle.

5. Optomechanically induced absorption

In OMIT, probe excitations are transferred to mechanical oscillations and again converted back to the probe field. A perfect destructive interference can be set up between the intracavity probe field and the fluctuations that return to the cavity from the mechanical oscillator. As a result, the probe field cannot exist in the cavity, and the cavity then becomes...
Figure 7. The normalized output strength for linear case for (a) $|\epsilon_{\text{out}}^{L+}|^2$ and (b) $|\epsilon_{\text{out}}^{R+}|^2$ as a function of normalized probe detuning $\frac{x}{k}$. The parameters used are: $G = 3k$ and $g = k$ (solid red line), $G = 4k$ and $g = k$ (dashed blue line). The plots (c) and (d) are the focus around $\frac{x}{k} = 0$ corresponding to plots (a) and (b), respectively.

A system can also be designed such that a constructive interference take place that leads to optomechanically induced absorption (OMIA) [77,78]. In this section, we analyze the existence of OMIA in terms of the left-hand or right-hand output probe fields. The absorptive and dispersive behavior of the system is contained in the real and imaginary part of the transmission $\epsilon_T$. Defining $\epsilon_T = \frac{2k\delta c_1}{\epsilon_L}$, we obtain the following expressions for linear coupling,

$$\epsilon_T = \frac{G^22k(n\epsilon_{R}e^{\theta}(-ix + \frac{\delta k}{2}) + n^2\epsilon_L(-ix + \frac{\delta k}{2})) + 2k\epsilon_L(-ix + k)[(-ix + \frac{\delta k}{2})(-ix + \frac{\delta k}{2}) - g^2\sigma_z]}{\epsilon_L[(-ix + \frac{\delta k}{2})(-ix + \frac{\delta k}{2}) - g^2\sigma_z](-ix + k)^2 + (-ix + \frac{\delta k}{2})G^2(n^2 + 1)(-ix + k)\epsilon_L}$$

(42)

and for non-linear coupling,

$$\epsilon_T = \frac{G^22k(n\epsilon_{R}e^{\theta}(-ix + \frac{\delta k}{2}) + n^2\epsilon_L(-ix + \frac{\delta k}{2}) + 2k\epsilon_L(-ix + k)[(-ix + \frac{\delta k}{2})(-ix + \frac{\delta k}{2}) - 4G^2n^2\sigma_z]}{\epsilon_L[(-ix + \frac{\delta k}{2})(-ix + \frac{\delta k}{2}) - 4G^2n^2\sigma_z](-ix + k)^2 + (-ix + \frac{\delta k}{2})G^2(n^2 + 1)(-ix + k)\epsilon_L}$$

(43)
focus around the mechanical resonance frequency $\omega_x$ when the beat of the probe field and the control field with the probe field and hence leading to a constructive hand, presence of the qubit induces a sideband that is in-phase in a transparency window in the cavity output. On the other hand, leading to cancelation of the intracavity field, resulting in the absence of the qubit, destructive interference is generated due to the non-linearity in the system and the probe fields. In equations (42) and (43), if we put $g = 0$ and $G_N = 0$, respectively, and neglect $\gamma_m$ and $k_d$ compared to $k$, we obtain

$$\epsilon_T = \frac{G^2 2k (n^2 e^{\nu x} + n^2) - 2kx(x + ik)}{(k - ix)(G^2(n^2 + 1) - x(x + ik))}$$

Here in equation (44), we find that the numerator is quadratic in $x$ and the denominator is cubic in $x$. On the other hand, in equations (42) and (43), the numerator is cubic in $x$ and the denominator is quartic in $x$. These changes determine the physical behavior of the output field. Equation (44) is an expression for $\epsilon_T$ in the absence of qubit–mechanical resonator coupling (or extremely small values of $g$ and $G_N$). In such a case, the qubit plays no role in the optical response of the system and the system is reduced to a pure optomechanical device where the response is determined by the parameter $G$. Figure 10(b) shows the dispersion curves for the linear (red line) and non-linear case (blue dashed line). Clearly the non-linear curve is much steeper than the linear curve. The steep curve again indicates the possibility of using the non-linear hybrid system as an optical switch.

Figure 8. The normalized output strength for non-linear case for (a) $|\epsilon_{\text{out}}^{L+}|^2$ and (b) $|\epsilon_{\text{out}}^{R+}|^2$ as a function of normalized probe detuning $x/k$. The parameters used are: $G = 3k$ and $G_N = 0.1k$ (solid red line), $G = 4k$ and $G_N = 0.1k$ (dashed blue line). The plots (c) and (d) are the focus around $x/k = 0$ corresponding to plots (a) and (b), respectively.
Figure 9. The normalized output strength for both the linear and non-linear case for (a) $|\epsilon_{\text{out}} L|^2$ and (b) $|\epsilon_{\text{out}} R|^2$ as a function of normalized probe detuning $x/k$ near $x/k = 0$ for the following parameters: $(G = 3k, g = k)$ (orange dot-dashed line), $(G = 4k, g = k)$ (purple dotted line), $(G = 3k, G_N = 0.1k)$ (blue dashed line), $(G = 4k, G_N = 0.1k)$ (red solid line). The plots (c) and (d) are the focus around $x/k = 0$ corresponding to plots (a) and (b), respectively.

Figure 10. Real (a) and imaginary (b) part of the left-hand output probe field $\epsilon_T$ as a function of normalized probe detuning $x/k$. For linear case we have taken $g = k$ (solid red line) and for the non-linear, we choose $G_N = 0.1k$ (dashed blue line). For all curves the other parameters are $G = k, \sigma_z = -1, n = 1, \theta = 3\pi, k_d = 0$.

We now check the influence of $\sigma_z$ on the OMIA. If $\sigma_z = 0$ (both upper and lower level equally populated) then the influence of the qubit–mechanical oscillator vanishes and the OMIA is converted into OMIT as in [79]. Using $\sigma_z = 0.1$ (population of upper level is slightly more than the lower level), we generate the plots of Re[$\epsilon_T$] for the linear and non-linear case as shown in figures 11(a) and (b), respectively. For the linear case (figure 11(a)) the OMIA peak at $x = 0$ becomes
Figure 11. Real part of the left-hand output probe field $\varepsilon_T$ for the linear case (a) and non-linear case (b) as a function of normalized probe detuning $x/k$. For the linear case we have taken $g = k$ (solid red line) and for the non-linear case we have taken $G_N = 0.1k$ (blue dashed line). For both cases, the other parameters are $G = k$, $\sigma^2 = 0.1$, $n = 1$, $\theta = 3\pi$, $k_d = 0$.

Figure 12. Real part of the left-hand output probe field $\varepsilon_T$ for the linear case (a) and non-linear case (b) as a function of normalized probe detuning $x/k$. For the linear case we have taken $g = k$ , $k_d = 0.1$ (solid red line) and for the non-linear case we take $G_N = 0.1k$, $k_d = 0.01$ (blue dashed line). For both the plots, the other parameters are $G = k$, $\sigma^2 = 0.1$, $n = 1$, $\theta = 3\pi$. Plot (c) is the focus around $x/k = 0$ corresponding to (a).

narrow and the perfect transmission around $x = 0$ no longer exists. For the non-linear case (figure 11(b)) the OMIA peak at $x = 0$ becomes even more narrow compared to the linear case and partial transmission is observed. In figure 11(a), the dips around the OMIA peak at $x/k = 0$ tend to become more enhanced as the effective optomechanical coupling $G$
increases. On the other hand, the dips gradually vanish as $G$ decreases and we get a pure absorption peak as $G \to 0$. Interestingly, we notice the exact opposite impact on the dips when the linear qubit–mechanical resonator coupling strength $g$ is varied. Note that when $g \to 0$, the absorption peak is purely due to the optomechanical effect as mentioned earlier. This indicates that the interactions $G$ and $g$ play a competing role in the optical response of the system. In figure 11(b), the OMA narrow peak slowly vanishes as we increase $G$. Increasing the non-linear coupling $G_N$ broadens the OMA peak and makes it more prominent. This observation again demonstrates the competing role of $G$ and $G_N$. We also find that on increasing the qubit decay rate $k_d$, the constructive interference that leads to OMA starts to disappear and a transition towards OMIT occurs. This is illustrated in figures 12(a) and (b) for different values of $\sigma_z$ and $k_d$ for the linear and non-linear case. For the linear case a complete transition to OMIT occurs at $\sigma_z = 0.1$ and $k_d = 0.1 k$. On the other hand for the non-linear case, OMIT is seen to occur at $\sigma_z = 0.1$ and $k_d = 0.01 k$. Figure 12(c) illustrates the peculiar dip observed in figure 12(a) more clearly. We found this dip around $x/k = 0$ to be extremely sensitive to small variations in $G$ and $g$. The dip shifts downwards and eventually vanishes as we either increase $G$ or decrease $g$. On the other hand the dip shifts upwards (i.e. the minima of the dip moves up) on the $y$-axis and takes a finite value as we vary $G$ or $g$ on decreasing $G$ or increasing $g$. For the $\sigma_z = -1$ case, partial OMA is observed in figures 13(a) and (b), respectively.

All the parameters used in our calculations are accessible in earlier experiments as discussed in the following [24, 58, 80–83]. The length of the optical cavity may vary from $10^{-3} - 25 \times 10^{-3}$ m. The effective mass of the mechanical mirror can vary between $5$ and $145$ ng and its frequency varies between $1$ and $10$ MHz. The corresponding damping rate of the resonator is $\gamma_m = \omega_m/Q$, where $Q = 10^5$ is the quality factor of the optomechanical cavity. The external laser pump strength can vary from $0.2$ to $0.5 \omega_m$. Also, the damping rate of the intracavity optical field may vary from $2 \pi \times 0.1$ kHz–$2 \pi \times 1.0$ MHz. The damping rate of the TLS may vary from $2 \pi \times 0.1$ MHz–$2 \pi \times 0.66$ MHz and the linear and non-linear coupling can be around $2 \pi \times 1.0$ MHz–$2 \pi \times 2.0$ MHz with $g_N < g$ [72, 73]. The effective optomechanical coupling $G$ can be around $2 \pi \times 2.0 - 3.0$ MHz. This model can be realized experimentally by using known standard procedures. The two optically coupled cavities can be fabricated with the help of a set of distributed Bragg reflector (DBR) mirrors. Light in the $x$-direction can be confined by the DBRs and the confinement along the $y$–$z$ plane can be achieved by air guiding dielectric [84]. DBR mirror is fabricated using alternating quarter-wavelength-thick high and low refractive index layers. The reflectance of the DBR is dependent on the number of pairs and the difference between high and low index pairs [85]. The first and the last layers are made of AlGaAs, which increases the coupling of light in/out of the structure [85]. GaAs-based mechanical resonators are fabricated using the well-known method of micromachining with selective etching [86, 87].

6. Conclusion

In summary, we have studied the optical response properties of a hybrid double-cavity optomechanical system in the presence of a linear and non-linear qubit–mechanical oscillator interaction. Our results illustrate that CPT and synthesis can be achieved at four different points, which scan a wide parameter regime. From our studies it is clear that the qubit and its interaction with the mechanical mode appears as a new handle to control photon transport through the system. We further found the case of non-linear interaction to be more sensitive to variations in the system parameters compared to the linear case, thus making it a suitable candidate for all-optical switching. In addition, we have shown that the system exhibits OMA. The system can be made to switch between OMA and OMIT by tuning the parameters of the qubit and qubit–mechanical coupling. Overall we find that the optical properties of the proposed hybrid system are sensitive to the variations in $g$, $G$ and $G_N$ and...
this optical response can be used to our advantage for fabricating sensitive all-optical devices. Thus the hybrid system can be made to operate as a tunable photon router as well as an all-optical switch. Such a four-mode hybrid system with tunable and sensitive optical response properties provides a platform for novel photonic quantum devices which can form a part of a wider quantum network.

Acknowledgments

Sabar A Barbhuiya acknowledges BITS, Pilani Hyderabad campus for the doctorate institute fellowship.

References

[1] Treutlein P, Genes C, Hamerker K, Poggio M and Rabl P 2014 Cavity Optomechanics pp 327–51
[2] Blencowe M P 2004 Phys. Rep. 395 159
[3] Poet M and Vanderzant H S J 2012 Phys. Rep. 511 273
[4] Xiang Z-L, Ashhab S, You J Q and Nori F 2013 Rev. Mod. Phys. 85 623
[5] Irish E K and Kwon K 2003 Phys. Rev. B 68 155311
[6] Armour A D, Blencowe M P and Schwab K C 2002 Phys. Rev. Lett. 88 148301
[7] Sornberger A T, Cleland A N and Geller M R 2004 Phys. Rev. A 70 052315
[8] Zhang P, Wang Y D and Sun C P 2005 Phys. Rev. Lett. 95 097204
[9] Tian L 2009 Phys. Rev. B 79 193407
[10] Cleland A N and Geller M R 2004 Phys. Rev. Lett. 93 070501; Geller M R and Cleland A N 2005 Phys. Rev. A 71 032311
[11] Tian L 2005 Phys. Rev. B 71 195411
[12] Wei L F, Liu Y X, Sun C P and Nori F 2006 Phys. Rev. Lett. 97 237201
[13] Liu Y X, Minarownicz A, Gao Y B, Bajer J, Sun C P and Nori F 2010 Phys. Rev. A 82 032101
[14] LaHaye M D, Sub J, Echternach P M, Schwab K C and Roukes M L 2009 Nature 459 960
[15] Xue F, Liu Y X, Sun C P and Nori F 2007 Phys. Rev. B 76 064305
[16] Xue F, Wang Y D, Liu Y X and Nori F 2007 Phys. Rev. B 76 205302
[17] Li Y, Wang Y D, Xue F and Bruder C 2008 Phys. Rev. B 78 134301
[18] Rocheleau T, Ndukum T, Macklin C, Hertzberg J B, Clerk A A and Schwab K C 2010 Nature 463 72
[19] Hertzberg J B, Rocheleau T, Ndukum T, Savva M, Clerk A A and Schwab K C 2010 Nat. Phys. 6 213
[20] Massel F, Heikikila T T, Pirkkalainen J-M, Cho S U, Saloniemi H, Hakonen P J and Sillanpaa M A 2011 Nature 480 351
[21] Kimpenberg T J and Vahala K J 2008 Science 321 1172
[22] Aspelmeyer M, Groblacher S, Hammerer K and Kiesel N 2010 J. Opt. Soc. Am. B 27 A189
[23] Aspelmeyer M, Meystre P and Schwab K 2012 Phys. Today 65 29
[24] Aspelmeyer M, Kimpenberg T and Marquardt F 2014 Rev. Mod. Phys. 86 1391
[25] Arcizet O, Jacques V, Siria A, Poncharal P, Vincent P and Seidelin S 2011 Nat. Phys. 7 879
[26] Kolkowitz S, Jayich A C B, Unterreithmeier Q P, Rabl P, Harris J G E and Lukin M D 2012 Science 335 1603 SDBennett
[27] Bennett S D, Yao N Y, Otterbach J, Zoller P, Rabl P and Lukin M D 2013 Phys. Rev. Lett. 110 156402
[28] Rugar D, Budakian R, Mamin H J and Chui B W 2004 Nature 430 329
[29] Liu Z-X, Xiong H and Wu Y 2018 Phys. Rev. A 97 013801
[30] Mizra I M 2015 J. Opt. Soc. Am. B 32 1604
[31] Pflanzer A C, Isart O R and Cirac J I 2013 Phys. Rev. A 88 033804
[32] Mizra I M 2016 Opt. Lett. 41 2422
[33] Mizra I M and van Enk S J 2014 Phys. Rev. A 90 043831
[34] Restrepo J, Favero I and Ciuti C 2017 Phys. Rev. A 95 023832
[35] O’Connell A D et al 2010 Nature 464 697
[36] Teufel J D et al 2011 Nature 475 359
[37] Chan J, Alegre T P M, Safavi-Naeini A H, Hill J T, Krause A, Groblacher S, Aspelmeyer M and Painter O 2011 Nature 478 89
[38] Verhagen E, Deleglise S, Weis S, Schliesser A and Kippenberg T J 2012 Nature 482 63
[39] Wang Y D and Clerk A A 2012 Phys. Rev. Lett. 108 155603
[40] Dong C H, Fiore V, Kuzyk M C and Wang H 2012 Science 338 1609
[41] Marquardt F and Girvin S M 2009 Physics 2 40
[42] Verlof P, Tavernarakis A, Briant T, Cohadon P F and Heidmann A 2010 Phys. Rev. Lett. 104 133644
[43] Mahajan S, Kumar T, Bhattacherjee A and 2013 ManMohan Phys. Rev. A 87 013621
[44] Gao M, Liu Y X and Wang X B 2011 Phys. Rev. A 83 022309
[45] Sun C P, Wei L F, Liu Y X and Nori F 2006 Phys. Rev. A 73 022318
[46] PirkkalaJnen J-M, Cho S U, Hakonen P J and Sillanpaa M A 2013 Nature 494 211
[47] Arcizet O, Cohadon P-F, Briant T, Pinard M and Heidmann A 2006 Nature 444 71
[48] Phillips W D 1998 Rev. Mod. Phys. 70 721
[49] Lu X-Y, Zhang W-M, Ashhab S, Wu Y and Nori F 2013 Sci. Rep. 3 2943
[50] Nunnenkamp A, Borkje K and Girvin S M 2011 Phys. Rev. Lett. 107 063601
[51] He B 2012 Phys. Rev. A 85 063820
[52] Liao J Q, Cheung H K and Law C K 2012 Phys. Rev. A 85 023803
[53] Liao J Q and Nori F 2013 Phys. Rev. A 88 023853
[54] Xu X W, Li Y J and Liu Y X 2013 Phys. Rev. A 87 025803
[55] Kronwald A, Ludvig M and Marquardt F 2013 Phys. Rev. A 87 013847
[56] Enzian G et al 2019 Optica 6 77
[57] Teufel J D, Li D, Allman M S, Cicak K, Sirous A J, Whittaker J D and Simmons R W 2011 Nature 471 204
[58] Groblacher S, Hammerer K, Vanner M R and Aspelmeyer M 2009 Nature 460 724
[59] Veissier G L et al 2013 Phys. Rev. A 87 013823
[60] Agarwal G S and Huang S 2010 Phys. Rev. A 81 041803
[61] Weis S et al 2010 Science 330 1520
[62] Chang D E, Safavi-Naeini A H, Hafezi M and Painter O 2011 New J. Phys. 13 023003
[63] Fiore V, Yang Y, Kuzyk M C, Barbour R, Tian L and Wang H 2011 Phys. Rev. Lett. 107 133601
[64] Fiore V, Dong C H, Kuzyk M C and Wang H L 2013 Phys. Rev. A 87 023812
[65] Wang Y-D and Clerk A A 2013 Phys. Rev. Lett. 110 253601
[66] Karuza M, Biancofiore C, Bawaj M, Molinelli C, Galassi M, Natali R, Tombesi P, Di Giuseppe G and 2013 Phys. Rev. A 88 013804 DVitali
[67] Konar F, Bennett S D, Stannigel K, Habraken S J M, Rabl P, Zoller P and Lukin M D 2013 Phys. Rev. A 87 013839
[68] Ludwig M et al 2012 Phys. Rev. Lett. 109 063601
[69] Yan X-B et al 2014 Opt. Express 22 4886
[70] Ramos T et al 2013 Phys. Rev. Lett. 110 193602
[71] Soyeal O O, Rusakov R and Tahan C 2011 Phys. Rev. Lett. 107 235502
[72] Wang H et al 2014 Phys. Rev. A 90 023817
[73] Zhang Y et al 2018 Quant. Inf. Processing 17 209
[74] Sete E A and Eleuch H 2014 Phys. Rev. A, 89 013841
[75] Pirkkalainen J-M et al 2015 Nat. Commun. 6 6981
[76] Walls D F and Milburn G J 1994 Quantum Optics (Berlin: Springer)
[77] Prakash V N and Bhattacherjee A B 2019 J. Mod. Optics 66 1611
[78] Chen H J 2019 J. Russian Laser Res. 40 340
[79] Safavi-Naeini A H et al 2011 Nature 472 69
[80] Gigan S et al 2006 Nature 444 67
[81] Bariani F, Singh S, Buchmann L F, Vengalattore M and Meystre P 2014 Phys. Rev. A 90 033838
[82] Chakram S, Patil Y S, Chang L and Vengalattore M 2014 Phys. Rev. Lett. 112 127201
[83] Reithmaier J P 2009 Semicond. Sci. Technol. 23 123001
[84] Gudat J 2012 Cavity quantum electrodynamics with quantum dots in microcavities PhD Thesis University of Leiden
[85] Choy H K H 1996 Design and fabrication of distributed Bragg reflectors for vertical-cavity surface-emitting laser MSc. Thesis Mc Master University
[86] Yamaguchi H 2017 Semicond. Sci. Technol. 32 103003
[87] Bohm H R, Gigana S, Blaser F, Zeilinger A and Aspelmeyer M 2006 Appl. Phys. Lett. 89 223101