Higgs and gauge boson phenomenology of the 3-3-1 model with CKS mechanism

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Abstract: The gauge boson sector of the renormalizable 3-3-1 model for the SM fermion masses and mixings is explored. The experimental data of the $\rho$ parameter shows that the vacuum expectation value (VEV) associated with the first spontaneous symmetry breaking (SSB) chain ranges from 3.6 TeV up to 6.1 TeV. Therefore the mass of the new heavy neutral gauge boson $Z'$ ranges from 1.42 TeV up to 2.42 TeV, which is consistent with estimations done in other 3-3-1 models. In that region of masses, we find that the total cross section for the production of the heavy neutral gauge boson $Z'$ at the LHC via Drell-Yan mechanism ranges from 46.2 pb up to 2.89 pb. On the other hand, in a future 100 TeV proton-proton collider the total cross section for the Drell-Yan production of a heavy $Z'$ neutral gauge boson gets significantly enhanced reaching values ranging from 1371 pb up to 235 pb. By the way, the masses of the new bilepton gauge bosons $Y^\pm$ and $X^0$ are around 800 GeV, which are quite good. The Higgs sector of the model is explored. The Higgs potential with lepton number conserving was considered in detail. The SM-like Higgs boson was identified and as expected, is mostly contained in the CP even part of $\eta^0_1$. It has couplings very close to SM expectation with small deviations of the order of $10^{-3}$. For the total scalar potential including lepton number violating interactions, excepting the CP-even sector, the situation is similar. The potential consists of enough number of Goldstone bosons corresponding to the longitudinal components of the massive gauge bosons. The scalar potential contains a complex scalar candidate for Dark Matter, namely $\phi^0_2$ and a Majoron but it is harmless because it is a gauge singlet. The constraints arising from the estimation of the Dark Matter (DM) relic density, set the mass of the scalar dark matter candidate in the range 300 GeV $\lesssim m_{\phi} \lesssim 600$ GeV, for a quartic scalar coupling $\lambda_{h^2,\phi^2}$ in the window 0.5 $\lesssim \lambda_{h^2,\phi^2} \lesssim 1$. 

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1 Introduction

Despite its great successes, the Standard Model (SM) still puzzles over the hierarchies and structure of the fermion sector, which remain without compelling explanation. It is known that in the SM, masses of the matter fields are determined through Yukawa interactions. In addition, the CKM matrix is also constructed from the same Yukawa couplings. To solve this puzzles, some mechanisms have been suggested. To the best of our knowledge, the first attempt to explain the huge differences in fermion masses in the SM is the Froggatt - Nielsen (FN) mechanism [1]. According to the FM mechanism, the mass differences between generations follow from suppression factors depending on FN charges of particles. It has been noticed that in order to implement the aforementioned mechanism, the effective Yukawa interactions were introduced, thus making this theory non-renormalizable. From this point of view, the recent mechanism proposed by Cárcamo, Kovalenko and Schmidt [2] (called by CKS mechanism) based on sequential loop suppression mechanism, is more natural since its suppression factor is arisen from loop factor \( l \approx (1/4\pi)^2 \).
One of the main purposes of the models based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$ (for short, 3-3-1 model) [3–9] is concerned with the search of an explanation for the number of generations of fermions. Combining with the QCD asymptotic freedom, the 3-3-1 models provide an explanation for the number of generations to be three. Some other advantages of the 3-3-1 models are: i) they solve the electric charge quantization [10, 11], ii) they contain several sources of CP violation [12, 13], and iii) they have a natural Peccei-Quinn symmetry, which solves the strong-CP problem [14–17].

In the framework of the 3-3-1 models, most of research are focused on radiative seesaw mechanisms, and some but involving nonrenormalizable interactions (see references in Ref.[18]). However, most researches on the 3-3-1 models are not concerned with vast different masses among the generations.

The FN mechanism was implemented in the 3-3-1 models in Ref.[19]. It is interesting that the FN mechanism does not produce new scale, i.e., the scale of the flavour breaking is the same as the breaking scale of the symmetry of the model.

The CKS mechanism has been included in the 3-3-1 model without exotic electric charges ($\beta = -1/\sqrt{3}$) in Ref. [18]. The implication of the CKS mechanism to the 3-3-1 model leads to the interesting 3-3-1 model in sense that the derived model is renormalizable, while it fits all current data on fermion masses and mixing [18]. It is worth mentioning that there exists a residual discrete $Z^2_{L_\nu}$ lepton number symmetry arising from the breaking of the global $U(1)_{L_\nu}$ symmetry. Under this residual symmetry, the leptons are charged and other particles are neutral [18].

However, in the mentioned work, the authors have just focused on the data concerning fermions (both quarks and leptons including neutrino mass and mixing), but some questions are open for the future study.

The aim in this work is to consider, in more details, the phenomenology of the model such as gauge and Higgs sectors from which we can get a bound on the model scale $v_\chi$ as well as on the mass of the new heavy $Z'$ boson. Due to the implemented symmetries, the Higgs sector is rather simple and can be completely solved. All Goldstone bosons and the SM like Higgs boson are defined.

The further content of this paper is as follows. In Sect. 2, we briefly present particle content and SSB of the model. Sect. 3 is devoted to gauge boson mass and mixing. Taking into account of data on the $\rho$ parameter, we get bounds on the VEV of the first step of the SSB and on the mass of the new heavy $Z'$ gauge boson. By the way, we also get a limit for masses of the bilepton charged/non-Hermitian bosons. The Higgs sector is considered in Sect. 4. The Higgs sector consists of two parts: the first part contains lepton number conserving terms and the second one is lepton number violating. We study in details the first part and show that the Higgs sector has all necessary ingredients. Sect. 5 is devoted to the production of the heavy $Z'$ and the heavy neutral scalar $H_4$. In Sect. 6, we deal with the DM relic density. We make conclusions in Sect. 7.
2 Review of the model

To implement the CKS mechanism, only the heaviest particles such as the exotic fermions and the top quark get masses at tree level. The next - medium ones: bottom, charm quarks, tau and muon get masses at one-loop level. Finally, the lightest particles: up, down, strange quarks and the electron acquire masses at two-loop level. To forbid the usual Yukawa interactions, the discrete symmetries should be implemented. Hence, the full symmetry of the model under consideration is

\[ SU(3)_C \times SU(3)_L \times U(1)_X \times Z_4 \times Z_2 \times U(1)_{L_9}, \]  

where \( L_9 \) is generalized lepton number defined in Refs. \([18, 20]\). It is interesting to note that, in this model, the light active neutrinos also get their masses by a combination of linear and inverse seesaw mechanisms at two-loop level.

As in the ordinary 3-3-1 model without exotic electric charges, the quark sector contains the following \( SU(3)_C \times SU(3)_L \times U(1)_X \) representations \([18]\)

\[
Q_{nL} = (D_n, -U_n, J_n)^T_L \sim (3, 3^*, 0), \quad Q_{3L} = (U_3, D_3, T)^T_L \sim (3, 3, 1/3), \quad n = 1, 2, \\
D_{iR} \sim \left(3, 1, -\frac{1}{3}\right), \quad U_{iR} \sim \left(3, 1, \frac{2}{3}\right), \quad i = 1, 2, 3, \\
J_{nR} \sim \left(3, 1, -\frac{1}{3}\right), \quad T_R \sim \left(3, 1, \frac{2}{3}\right), \\
\tilde{T}_{L,R} \sim \left(3, 1, \frac{2}{3}\right), \quad B_{L,R} \sim \left(3, 1, -\frac{1}{3}\right), \quad (2.2)
\]

where \( \sim \) denotes quantum numbers for the three above subgroups, respectively. Note that the \( SU(3)_L \) singlet exotic up type quarks \( \tilde{T}_{L,R} \), down type quarks \( B_{L,R} \) in the last line of Eq. (2.2) are newly introduced for implementation of the CKS mechanism.

In the leptonic sector, besides the usual \( SU(3)_L \) lepton triplets, the model contains more three charged leptons \( E_{jL(R)} \) \((j = 1, 2, 3)\) and four neutral leptons, i.e, \( N_{jR} \) and \( \Psi_R \) \((j = 1, 2, 3)\). The leptonic fields have the following \( SU(3)_C \times SU(3)_L \times U(1)_X \) assignments:

\[
L_{iL} = (\nu_i, e_i, \nu^c_i)^T_L \sim \left(1, 3, -\frac{1}{3}\right), \quad \nu_{iR} \sim (1, 1, -1), \quad i = 1, 2, 3, \quad (2.3) \\
E_{1L} \sim (1, 1, -1), \quad E_{2L} \sim (1, 1, -1), \quad E_{3L} \sim (1, 1, -1), \\
E_{1R} \sim (1, 1, -1), \quad E_{2R} \sim (1, 1, -1), \quad E_{3R} \sim (1, 1, -1), \\
N_{1R} \sim (1, 1, 0), \quad N_{2R} \sim (1, 1, 0), \quad N_{3R} \sim (1, 1, 0), \quad \Psi_R \sim (1, 1, 0). \quad (2.4)
\]

where \( \nu_{iL}, \nu^c \equiv \nu^c_{IR} \) and \( e_{iL} \) \((e_{L}, \mu_{L}, \tau_{L})\) are the neutral and charged lepton families, respectively.

The Higgs sector contains three scalar triplets: \( \chi, \eta \) and \( \rho \) and seven singlets \( \varphi^0_1, \varphi^0_2, \xi^0, \phi^+_1, \phi^+_2, \phi^+_3 \) and \( \phi^+_4 \). Hence, the content of the Higgs sector is

\[
\chi = \langle \chi \rangle + \chi' \sim \left(1, 3, -\frac{1}{3}\right), \quad (2.5)
\]
Table 1. Scalar assignments under $Z_4 \times Z_2$

|   | $\chi$ | $\eta$ | $\rho$ | $\varphi_1^0$ | $\varphi_2^0$ | $\varphi_3^0$ | $\phi_1^+$ | $\phi_2^+$ | $\phi_3^+$ | $\xi^0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $Z_4$ | 1 | 1 | $-1$ | $-1$ | $i$ | $i$ | $-1$ | $-1$ | 1 | 1 |
| $Z_2$ | $-1$ | $-1$ | 1 | 1 | 1 | 1 | $-1$ | $-1$ | 1 | 1 |

Table 2. Nonzero lepton number $L$ of fields

| $L$ | $T_{uR}$ | $J_{uR}$ | $J_{dR}$ | $\nu_{1L}$ | $\xi_{1R}$ | $E_{uL}$ | $N_{dR}$ | $\Psi_R$ | $\chi_1^0$ | $\chi_2^0$ | $\eta_1^0$ | $\eta_2^0$ | $\rho_1^0$ | $\phi_1^+$ | $\phi_2^+$ | $\phi_3^+$ | $\xi^0$ | $i = 1,2,3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $-2$ | 2 | 2 | $-1$ | 1 | 1 | $-1$ | 1 | 2 | 2 | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ |

\[
\langle \chi \rangle = \left(0,0,\frac{v_\chi}{\sqrt{2}}\right)^T, \quad \chi' = \left(\chi_1^0,\chi_2^0,\frac{1}{\sqrt{2}}(R_{\chi_3^0} - iI_{\chi_3^0})\right)^T,
\]

\[
\rho = \left(\rho_1^0,\frac{1}{\sqrt{2}}(R_{\rho} - iI_{\rho}),\rho_3^0\right)^T \sim \left(1,3,\frac{2}{3}\right),
\]

\[
\eta = (\eta) + \eta' \sim \left(1,3,-\frac{1}{3}\right),
\]

\[
\langle \eta \rangle = \left(\frac{v_\eta}{\sqrt{2}},0,0\right)^T, \quad \eta' = \left(\frac{1}{\sqrt{2}}(R_{\eta_1^0} - iI_{\eta_1^0}),\eta_2^0,\eta_3^0\right)^T,
\]

\[
\varphi_1^0 \sim (1,1,0), \quad \varphi_2^0 \sim (1,1,0),
\]

\[
\phi_1^+ \sim (1,1,1), \quad \phi_2^+ \sim (1,1,1), \quad \phi_3^+ \sim (1,1,1), \quad \phi_4^+ \sim (1,1,1),
\]

\[
\xi^0 = \langle \xi^0 \rangle + \xi'^0, \quad \langle \xi^0 \rangle = \frac{v_\xi}{\sqrt{2}}, \xi'^0 = \frac{1}{\sqrt{2}}(R_{\xi^0} - iI_{\xi^0}) \sim (1,1,0).
\]

The assignments under $Z_4 \times Z_2$ of scalar fields is presented in Table 1.

The fields with nonzero lepton number are presented in Table 2. Note that three singlets $N_{iR}$ as well as the elements in bottom of the lepton triplets $\nu_{iL}^c$ have lepton number equal $-1$.

In the model under consideration, the spontaneous symmetry breaking (SSB) occurs by two steps [18]. The first step is triggered by the vacuum expectation values (VEVs) of the $\chi_3^0$ and $\xi^0$ scalar fields. At this step, all new extra fermions, non-SM gauge bosons as well as the gauge singlet lepton $\Psi_R$ gain masses. In addition, the entries of the neutral lepton mass matrices with negative lepton number $(-1)$ also get values proportional to $v_\xi$. By this time, the initial group breaks down to that of the SM and $Z_4 \times Z_2^{(L_y)}$. The second step is triggered by $v_\eta$ providing masses for the top quark as well as for the $W$ and $Z$ gauge bosons and leaving the $SU(3)_C \times U(1)_Q \times Z_4 \times Z_2^{(L_y)}$ symmetry preserved. Here $Z_2^{(L_y)}$ is residual symmetry where only leptons are charged. Thus the interactions having an odd number of leptons are forbidden. This is crucial to guarantee the proton stability [18]. Thus

\[
SU(3)_C \times SU(3)_L \times U(1)_X \times Z_4 \times Z_2 \times U(1)_{L_y} \xrightarrow{v_\chi,v_\xi} SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_4 \times Z_2^{(L_y)} \xrightarrow{v_\eta} SU(3)_C \times U(1)_Q \times Z_4 \times Z_2^{(L_y)}.
\]
A consequence of the chain in (2.7) is
\[ v_\eta = v = 246 \text{GeV} \ll v_\chi \sim v_\xi \sim O(10) \text{TeV}. \] (2.8)

The corresponding Majoron is a gauge-singlet and, therefore, unobservable.

3 Gauge bosons

3.1 Gauge boson masses and mixing

After SSB, the gauge bosons get masses arising from the kinetic terms for the \( \eta \) and \( \chi \) \( SU(3)_L \) scalar triplets, as follows:
\[ L_{\text{mass}}^{\text{gauge}} = (D_\mu(\chi))'^\dagger D^\mu(\chi) + (D_\mu(\eta))'^\dagger D^\mu(\eta), \] (3.1)
with the covariant derivative for triplet defined as
\[ D_\mu = \partial_\mu - igA_\mu \frac{\lambda_a}{2} - igX \frac{\lambda_9}{2} B_\mu, \] (3.2)
where \( g \) and \( g_X \) are the gauge coupling constants of the \( SU(3)_L \) and \( U(1)_X \) groups, respectively. Here, \( \lambda_9 = \sqrt{2/3} \text{ diag}(1,1,1) \) is defined such that \( \text{Tr}(\lambda_9 \lambda_9) = 2 \), similarly as the usual Gell-Mann matrix \( \lambda_a, a = 1,2,3,\cdots,8 \). By matching gauge coupling constants at the \( SU(3)_L \times U(1)_X \) breaking scale, the following relation is obtained [8]
\[ t = \frac{g_X}{g} = \frac{3\sqrt{2} \sin \theta_W(M'_{Z'})}{\sqrt{3 - 4 \sin^2 \theta_W(M'_{Z'})}}. \] (3.3)

Let us provide the definition of the Weinberg angle \( \theta_W \). As in the SM, ones put \( g' = g \tan \theta_W \), where \( g' \) is gauge coupling of the \( U(1)_Y \) subgroup satisfying the relation [8]
\[ g' = \frac{\sqrt{3} gg_X}{\sqrt{18g^2 - g_X^2}}. \] (3.4)

Thus
\[ \tan \theta_W = \frac{\sqrt{3} gg_X}{\sqrt{18g^2 - g_X^2}}. \] (3.5)

Denoting
\[ W^\pm_\mu = \frac{1}{\sqrt{2}} (A_{\mu1} \mp iA_{\mu2}), \quad Y^\pm_\mu = \frac{1}{\sqrt{2}} (A_{\mu6} \pm iA_{\mu7}), \quad X^0_\mu = \frac{1}{\sqrt{2}} (A_{\mu4} - iA_{\mu5}), \] (3.6)
and substituting (3.2) and (3.6) into (3.1) ones get squared masses for charged/non-Hermitian gauge bosons as follows
\[ m_W^2 = \frac{g^2}{4} v_\eta^2, \quad M_{X^0}^2 = \frac{g^2}{4} (v_\chi^2 + v_\eta^2), \quad M_Y^2 = \frac{g^2}{4} v_\chi^2, \] (3.7)
and \( v_\eta = v = 246 \text{ GeV} \), as expected.
From (3.7) it follows a splitting of gauge boson masses
\[ M_{N_0}^2 - M_Y^2 = m_W^2. \]

For neutral gauge bosons, the squared mass mixing matrix has the form
\[ L_{mass}^{n gauge} = \frac{1}{2} V^T M_{n gauge}^2 V, \]

where \( V = (A_{\mu 3}, A_{\mu 8}, B_{\mu}) \) and
\[ M_{n gauge}^2 = g^2 \left( \begin{array}{cc} v_\eta^2 & v_\eta^2 \\ \frac{1}{3}(4v_\chi^2 + v_\eta^2) & \frac{2t\eta}{\sqrt{3}} (2v_\chi^2 - v_\eta^2) \\ \frac{2t\eta}{2\sqrt{2}} (v_\chi^2 + v_\eta^2) & \end{array} \right). \] (3.10)

The down-left entries in (3.10) are not written, due to the fact that the above matrix is symmetric.

The matrix in (3.10) has vanishing determinant, thus giving rise to massless gauge boson, which corresponds to the photon. Diagonalization of matrix in (3.10) separates into two steps. In the first step, the massive fields are identified as
\[ A_\mu = s_W A_{\mu 3} + c_W \left( -\frac{t_W}{\sqrt{3}} A_{\mu 8} + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \]
\[ Z_\mu = c_W A_{\mu 3} - s_W \left( -\frac{t_W}{\sqrt{3}} A_{\mu 8} + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \]
\[ Z'_\mu = \sqrt{1 - \frac{t_W^2}{3}} A_{\mu 8} + \frac{t_W}{\sqrt{3}} B_\mu, \]

where we have denoted \( s_W = \sin \theta_W, c_W = \cos \theta_W, t_W = \tan \theta_W \). After the first step, matrix \( M_{n gauge}^2 \) becomes block diagonal one, where the entry in the top is zero (due to the masslessness of the photon), while the 2 \( \times \) 2 matrix for \( (Z_\mu, Z'_\mu) \) in the bottom has the form
\[ M_{(2 \times 2)}^2 = \begin{pmatrix} M_{ZZ}^2 & M_{Z'Z'}^2 \\ M_{Z'Z}^2 & M_{ZZ'}^2 \end{pmatrix}. \] (3.12)

The matrix elements in (3.12) are given by
\[ M_{ZZ}^2 = g^2 \frac{v_\eta^2}{4v_W^2}, \]
\[ M_{Z'Z'}^2 = g^2 \frac{v_\eta^2(1 - 2s_W^2)}{4v_W^2 \sqrt{3 - 4s_W^2}}, \]
\[ M_{Z'Z}^2 = g^2 \frac{c_W^2}{4(3 - 4s_W^2)} \left[ 4v_\chi^2 + \frac{v_\eta^2(1 - 2s_W^2)^2}{c_W^4} \right]. \] (3.13)

Note that our formula of \( M_{Z'}^2 \) is consistent with that given in [21].
The last step of diagonalization is quite simple. The eigenstates are determined as

\[ Z_\mu^1 = Z_\mu \cos \phi - Z'_\mu \sin \phi, \]
\[ Z_\mu^2 = Z_\mu \sin \phi + Z'_\mu \cos \phi, \]  

(3.14)

where the mixing angle is given by

\[ \tan 2\phi = \frac{2M^2_{ZZ'}}{M^2_{ZZ'} - M^2_Z}. \]  

(3.15)

It is very easy to prove that our definition of \( \phi \) is consistent with that introduced in Ref. [22], which is needed to study the \( \rho \) parameter.

The masses of physical neutral gauge bosons are determined as

\[ M^2_{Z_\mu} = \frac{1}{2} \left\{ M^2_{Z'} + M^2_Z - \left[ (M^2_{Z'} - M^2_Z)^2 + 4(M^2_{ZZ'})^2 \right]^{\frac{1}{2}} \right\}, \]
\[ M^2_{Z_\mu'} = \frac{1}{2} \left\{ M^2_{Z'} + M^2_Z + \left[ (M^2_{Z'} - M^2_Z)^2 + 4(M^2_{ZZ'})^2 \right]^{\frac{1}{2}} \right\}. \]  

(3.16)

Ones approximate

\[ \Delta = M^4_{Z'} \left( 1 - \frac{M^4_Z}{M^4_{Z'}} + \frac{M^4_Z}{M^4_{Z'}} + 4(M^2_{ZZ'})^2 \right), \]
\[ \Rightarrow \sqrt{\Delta} \simeq M^2_{Z'} \left[ 1 - \frac{M^2_Z}{M^2_{Z'}} + \frac{2(M^2_{ZZ'})^2}{M^2_{Z'}} + O \left( \frac{M^4_{Z'}}{M^4_{Z'}} \right) \right]. \]  

(3.17)

Therefore

\[ M^2_{Z_1} \simeq M^2_Z - \frac{(M^2_{ZZ'})^2}{M^2_{Z'}} + M^2_Z \times O \left( \frac{v^4_\eta}{v^4_\chi} \right), \]  

(3.18)

\[ M^2_{Z_2} \simeq M^2_Z + \frac{(M^2_{ZZ'})^2}{M^2_{Z'}} + M^2_Z \times O \left( \frac{v^4_\eta}{v^4_\chi} \right) \simeq M^2_{Z'}. \]  

(3.19)

In the limit \( v_\chi \gg v_\eta \), the \( Z - Z' \) mixing angle is

\[ \tan \phi \simeq \frac{(1 - 2s^2_W)\sqrt{3 - 4s^2_W}}{4c^2_W} \left( \frac{v^2_\eta}{v^2_\chi} \right). \]  

(3.20)

Before turning to the next section, we remind the usual relation

\[ e = g s_W. \]  

(3.21)

### 3.2 Limit on \( Z' \) mass from the \( \rho \) parameter

The presence of the non SM particles modifies the oblique corrections of the SM, the values of which have been extracted from high precision experiments. Consequently, the validity of our model depends on the condition that the non SM particles do not contradict those
experimental results. Let us note that one of the most important observables in the SM is the \( \rho \) parameter defined as
\[
\rho = \frac{m_Z^2}{c_W^2 M_Z^2}.
\] (3.22)

For the model under consideration, the oblique correction leads to the following form of the \( \rho \) parameter \[22\]
\[
\rho - 1 \simeq \tan^2 \phi \left( \frac{M_Z^2}{m_Z^2} - 1 \right) + \frac{3\sqrt{2} G_F}{16\pi^2} \left[ M_+^2 + M_0^2 + \frac{2M_+^2 M_0^2}{M_+^2 - M_0^2} \ln \frac{M_0^2}{M_+^2} \right] \\
- \frac{\alpha(m_Z)}{4\pi s_W^2} \left[ t_W^2 \ln \frac{M_+^2}{M_+^2} + \frac{\varepsilon^2(M_+, M_0)}{2} + O(\varepsilon^3(M_+, M_0)) \right],
\] (3.23)

where \( M_0 = M_{X0}, \ M_+ = M_{Y+} \) and \( \varepsilon(M, m) = \frac{M^2 - m^2}{m^2} \).

Combining with Eq. (3.8), ones get
\[
\rho - 1 \simeq \tan^2 \phi \left( \frac{M_Z^2}{m_Z^2} - 1 \right) + \frac{3\sqrt{2} G_F}{16\pi^2} \left[ 2M_Y^2 + m_W^2 - \frac{2M_Y^2 (M_Y^2 + m_W^2)}{m_W^2} \ln \frac{M_Y^2 + m_W^2}{M_Y^2} \right] \\
- \frac{\alpha(m_Z)}{4\pi s_W^2} \left[ t_W^2 \ln \frac{M_Y^2 + m_W^2}{M_Y^2 + m_W^2} + \frac{m_W^4}{2(M_Y^2 + m_W^2)^2} \right],
\] (3.24)

where \( \alpha(m_Z) \approx \frac{1}{128} [23] \).

Taking into account \( s_W^2 = 0.23122 [23] \) and \( \rho = 1.00039 \pm 0.00019 \),
\] (3.25)

we have plotted \( \Delta \rho \) as a function of \( v_\chi \) in Fig. 1 (the left-panel). From figure 1 (the left-panel), it follows
\[
3.57 \, \text{TeV} \leq v_\chi \leq 6.09 \, \text{TeV}.
\] (3.26)

**Figure 1.** Left-panel: \( \rho \) parameter as a function of \( v_\chi \), upper and a lower horizontal lines are an upper a lower limits in (3.25). Right-panel: Relation between \( v_\chi \) and \( M_Z^2 \), upper horizontal lines are an upper and a lower limits of \( v_\chi \), respectively.
Substituting (3.26) into (3.19) and evaluating in figure 1 (the right-panel) we get a bound on the $Z'$ mass as follows

$$1.42 \text{ TeV} \leq M_{Z'} \leq 2.42 \text{ TeV}.$$ \hfill (3.27)

It is worth mentioning that the second term in (3.24) is much smaller the first one. Consequently, the limit deduced from the tree level is slightly different from the one with the oblique correction.

From LHC searches, it follows that the lower bound on the $Z'$ boson mass in 3-3-1 models is around 2.5 TeV [24]. Hence, the 3-3-1 scale $v_\chi$ is about 6.1 TeV, while from the decays $B_{s,d} \rightarrow \mu^+\mu^−$ and $B_d \rightarrow K^*(K)\mu^+\mu^−$ [21, 25–28], the lower limit on the $Z'$ boson mass ranges from 1 TeV to 3 TeV.

Then, the bilepton gauge boson mass is constrained to be in the range:

$$465 \text{ GeV} \leq M_Y \leq 960 \text{ GeV}.$$ \hfill (3.28)

Here we have used [23]

$$m_W = 80.379 \text{ GeV}.$$  

Note that the above limit is stronger than the one obtained from the wrong muon decay [29]

$$M_Y \geq 230 \text{ GeV}.$$  

For conventional notation, hereafter we will call $Z_1$ and $Z_2$ by $Z$ and $Z'$, respectively.

4 Higgs potential

The renormalizable potential contain three parts: the first one invariant under group $G$ in (2.1) is given by

$$V_{LNC} = \mu_\chi^2 \chi^\dagger \chi + \mu_\rho^2 \rho^\dagger \rho + \mu_\eta^2 \eta^\dagger \eta + \sum_{i=1}^{4} \mu_{\phi_i}^2 \phi_i^\dagger \phi_i^- + \sum_{i=1}^{2} \mu_{\varphi_i}^2 \varphi_i^0 \varphi_i^{0*} + \mu_{\xi}^2 \xi^0 \xi^0$$

$$+ \chi^\dagger (\lambda_{13} \chi^\dagger \chi + \lambda_{18} \rho^\dagger \rho + \lambda_{55} \eta^\dagger \eta) + \rho^\dagger \rho (\lambda_{14} \rho^\dagger \rho + \lambda_{66} \eta^\dagger \eta) + \lambda_{17} (\eta^\dagger \eta)^2$$

$$+ \lambda_7 (\rho^\dagger \rho) (\rho^\dagger \rho) + \lambda_8 (\eta^\dagger \eta) (\eta^\dagger \eta) + \lambda_9 (\rho^\dagger \rho) (\eta^\dagger \eta)$$

$$+ \chi^\dagger \chi \left( \sum_{i=1}^{4} \lambda_i^{\chi \phi} \phi_i^+ \phi_i^- + \sum_{i=1}^{2} \lambda_i^{\chi \varphi} \varphi_i^0 \varphi_i^{0*} + \lambda_{\chi \xi} \xi^0 \xi^0 \right)$$

$$+ \rho^\dagger \rho \left( \sum_{i=1}^{4} \lambda_i^{\rho \phi} \phi_i^+ \phi_i^- + \sum_{i=1}^{2} \lambda_i^{\rho \varphi} \varphi_i^0 \varphi_i^{0*} + \lambda_{\rho \xi} \xi^0 \xi^0 \right)$$

$$+ \eta^\dagger \eta \left( \sum_{i=1}^{4} \lambda_i^{\eta \phi} \phi_i^+ \phi_i^- + \sum_{i=1}^{2} \lambda_i^{\eta \varphi} \varphi_i^0 \varphi_i^{0*} + \lambda_{\eta \xi} \xi^0 \xi^0 \right)$$

$$+ \sum_{i=1}^{4} \phi_i^+ \phi_i^- \left( \sum_{j=1}^{4} \lambda_{ij}^{\phi \phi} \phi_j^+ \phi_j^- + \sum_{j=1}^{2} \lambda_{ij}^{\phi \varphi} \varphi_j^0 \varphi_j^{0*} + \lambda_{ij}^{\phi \xi} \xi^0 \xi^0 \right).$$
The scalar interactions needed for quark and charged lepton mass generation, read as

\[ L_{\text{Higgsqcl}} = \lambda_1 \rho \eta \varphi_1^0 + \lambda_2 \rho \varphi_1 \varphi_0^* + \lambda_3 \rho \varphi_1 \varphi_0^* + w_1 (\varphi_2^0)^2 \varphi_1 + w_2 \chi^\dagger \rho \varphi_3^0 + \text{h.c.} \]  

(4.5)
For the neutrino mass generation, beside the first term in (4.5), the additional part is given as

\[
L_{\text{Higgs neutrino}} = \lambda_{13}(\chi^\dagger\chi)^2 + \lambda_5(\chi^\dagger\chi)(\eta^\dagger\eta) + \left[ \lambda_{27}(\rho^\dagger\rho)(\chi^\dagger\eta + \eta^\dagger\chi) + \mu_2^2 \phi_4^+ \phi_3^- + h.c \right].
\] (4.6)

It is worth mentioning that for generation of masses for quark and charged lepton, only terms in conserving part \( V_{LNC} \) are enough, while for the generation of the light active neutrino masses, one needs the lepton number violating scalar interactions of \( V_{LNV} \) as well as the softly breaking part \( L_{\text{soft}} \) [the last term in (4.6)] of the scalar potential.

4.1 Potential with lepton number conservation

Below we present lepton number conserving part \( V_{LNC} \). Expanding the Higgs potential around VEVs, ones get the constraint conditions at the three levels as follows

\[
\frac{1}{\sqrt{2}} v_\eta v_\xi w_3 = 0,
\]
\[
\mu_\chi^2 v_\chi + v_\chi^2 \lambda_{13} + \frac{1}{2} v_\chi v_\eta^2 \lambda_5 + \frac{1}{2} \lambda_\chi v_\chi v_\xi^2 = 0,
\]
\[
\mu_\eta^2 v_\eta + v_\eta^2 \lambda_{17} + \frac{1}{2} v_\chi v_\eta v_\lambda + \frac{1}{2} \lambda_\eta v_\eta v_\xi^2 = 0,
\]
\[
\mu_\xi^2 v_\xi + \frac{1}{2} \lambda_\chi v_\xi^2 v_\chi + \frac{1}{2} \lambda_\eta v_\eta^2 v_\xi + \lambda_\xi v_\xi^3 = 0.
\] (4.7)

The simplified form is

\[
w_3 = 0, \quad (4.8)
\]
\[-\mu_\chi^2 = v_\chi^2 \lambda_{13} + \frac{1}{2} v_\chi v_\eta^2 \lambda_5 + \frac{1}{2} \lambda_\chi v_\chi v_\xi^2, \quad (4.9)
\]
\[-\mu_\eta^2 = v_\eta^2 \lambda_{17} + \frac{1}{2} v_\chi v_\eta v_\lambda + \frac{1}{2} \lambda_\eta v_\eta v_\xi^2, \quad (4.9)
\]
\[-\mu_\xi^2 = \frac{1}{2} \lambda_\chi v_\xi^2 v_\chi + \frac{1}{2} \lambda_\eta v_\eta^2 v_\xi + \lambda_\xi v_\xi^3.
\]

Applying the constraint conditions in (4.8), the charged scalar sector contains two massless fields: \( \eta^+ \) and \( \chi^\pm \) which are Goldstone bosons eaten by the \( W^+ \) and \( Y^+ \) gauge bosons, respectively. The other massive fields are \( \phi_1^+ \), \( \phi_2^+ \) and \( \phi_3^+ \) with respective masses

\[
m_\phi_1^2 = \mu_\phi_1^2 + \frac{1}{2} \left[ v_\chi^2 \lambda_1^\phi + v_\eta^2 \lambda_1^\phi \right],
\]
\[
m_\phi_2^2 = \mu_\phi_2^2 + \frac{1}{2} \left[ v_\chi^2 \lambda_2^\phi + v_\eta^2 \lambda_2^\phi + v_\xi^2 \lambda_2^\phi \right],
\]
\[
m_\phi_3^2 = \mu_\phi_3^2 + \frac{1}{2} \left[ v_\chi^2 \lambda_3^\phi + v_\eta^2 \lambda_3^\phi + v_\xi^2 \lambda_3^\phi \right].
\] (4.10)

In addition, in the basis \((\rho_1^+ , \rho_3^+ , \phi_3^+)\), there is the mass mixing matrix

\[
M_{\text{charged}}^2 = \begin{pmatrix}
A + \frac{1}{2} v_\eta^2 (\lambda_6 + \lambda_9) & v_\eta v_\xi \lambda_3 & 0 \\
0 & A + \frac{1}{2} \left( v_\eta^2 \lambda_7 + v_\eta^2 \lambda_6 \right) & \frac{1}{2} v_\eta v_\xi \eta_3 \\
\frac{1}{2} v_\eta v_\xi \lambda_3 & \frac{1}{2} v_\eta v_\xi \eta_3 & \mu_\phi_3^2 + B_3
\end{pmatrix},
\] (4.11)
where we have used the following notations

\[ A \equiv \mu_{\rho}^2 + \frac{1}{2} \left[ v_{\chi}^2 \lambda_{18} + \lambda_{28} v_{\xi}^2 \right], \]

\[ B_i \equiv \frac{1}{2} \left( v_{\chi}^2 \lambda_{1}^\phi + v_{\eta}^2 \lambda_{1}^\phi + v_{\xi}^2 \lambda_{1}^\phi \right), \quad i = 1, 2, 3, 4. \] (4.12)

From (4.11), it follows that in the limit \( v_\eta \ll v_\xi, \rho_1^+ \) is physical field with mass

\[ m_{\rho_1^+}^2 = A + \frac{1}{2} v_\eta^2 (\lambda_6 + \lambda_9), \] (4.13)

and two massive bilepton scalars \( \rho_3^+ \) and \( \phi_3^+ \) mixing each other.

Now we turn into CP-odd Higgs sector. There are three massless fields: \( I_{\chi_0^1}, I_{\eta_0^3} \) and \( I_{\xi_0}. \) The field \( I_{\varphi_2} \) has the following squared mass

\[ m_{I_{\varphi_2}}^2 = \mu_{\varphi_2}^2 + B'_2, \] (4.14)

where

\[ B'_n = \frac{1}{2} \left( v_{\chi}^2 \lambda_n^\varphi + v_{\eta}^2 \lambda_n^\eta + v_{\xi}^2 \lambda_n^\xi \right), \quad n = 1, 2. \] (4.15)

There are other two mass matrices, namely:

1. In the basis \( (I_{\chi_0^1}, I_{\eta_0^3}) \), the matrix is

\[ m_{CPodd1}^2 = \frac{\lambda_8}{2} \left( \begin{array}{cc} -v_\eta^2 -v_{\chi} v_\eta \\ -v_{\chi} v_\eta & v_\chi^2 \end{array} \right). \] (4.16)

The matrix in (4.16) provides two physical states

\[ G_1 = \cos \theta_a I_{\chi_0^1} + \sin \theta_a I_{\eta_0^3}, \]
\[ A_1 = -\sin \theta_a I_{\chi_0^1} + \cos \theta_a I_{\eta_0^3}, \] (4.17)

where

\[ \tan \theta_a = \frac{v_\eta}{v_\chi}. \] (4.18)

The field \( G_1 \) is massless while the field \( A_1 \) has mass as follows

\[ m_{A_1}^2 = \frac{\lambda_8 v_\chi^2}{2 \cos^2 \theta_a}, \] (4.19)

2. In the basis \( (I_{\varphi_1}, I_{\rho}) \), the matrix is

\[ m_{CPodd2}^2 = \left( \begin{array}{cc} \mu_{\varphi_1}^2 + B'_1 - C & \frac{1}{2} v_{\chi} v_\eta (\lambda_1 - \lambda_2) \\ \frac{1}{2} v_{\chi} v_\eta (\lambda_1 - \lambda_2) & A + \frac{\lambda_6}{2} v_\eta^2 \end{array} \right), \] (4.20)

where we have denoted

\[ C = v_{\chi}^2 \lambda_{22} + v_\eta^2 \lambda_{24} + v_{\xi}^2 \lambda_{25}. \] (4.21)
Generally, physical states of matrix (4.20) are

\[
\begin{pmatrix}
A_2 \\
A_3
\end{pmatrix} =
\begin{pmatrix}
\cos \theta \rho & \sin \theta \rho \\
-\sin \theta \rho & \cos \theta \rho
\end{pmatrix}
\begin{pmatrix}
I_{\phi_1} \\
I_{\rho}
\end{pmatrix},
\]
(4.22)

where the mixing angle is given by

\[
\tan 2 \theta \rho = \frac{v_\chi v_\eta (\lambda_1 - \lambda_2)}{\mu_{\phi_1}^2 - C + B_1' - A - \frac{1}{2} \eta v_\eta^2}.
\]
(4.23)

and their squared masses as follows

\[
m^2_{A_2} = \frac{1}{2} \left\{ A + D_1 - \sqrt{(A-D_1)^2 + v_\eta^2 [2(A-D_1)\lambda_6 + \mu_{\phi_2}^2 + v_\eta^2 (\lambda_{13} - \lambda_{14})^2]} \right\},
\]
\[
m^2_{A_3} = \frac{1}{2} \left\{ A + D_1 + \sqrt{(A-D_1)^2 + v_\eta^2 [2(A-D_1)\lambda_6 + \mu_{\phi_2}^2 + v_\eta^2 (\lambda_{13} - \lambda_{14})^2]} \right\},
\]
(4.24)

where

\[
D_1 = \mu_{\phi_2}^2 + B_1' - C + \frac{1}{2} \eta v_\eta^2 \lambda_6.
\]
(4.25)

Next, the CP-even scalar sector is our task. Ones have one massive field, namely \( R_{\phi_2} \) with mass

\[
m^2_{R_{\phi_2}} = m^2_{I_{\phi_2}} = \mu_{\phi_2}^2 + B_2'
= \mu_{\phi_2}^2 + \frac{1}{2} \left( v_\chi^2 \lambda_2 \phi + v_\eta^2 \lambda_{\phi\phi} + v_\xi^2 \lambda_{\phi\xi} \right).
\]
(4.26)

As mentioned in Ref. [18], the lightest scalar \( \phi_0 \) is possible DM candidate. Therefore from (4.26), the following condition is reasonable

\[
\mu_{\phi_2}^2 = -\frac{1}{2} \left( v_\chi^2 \lambda_2 \phi + v_\xi^2 \lambda_{\phi\xi} \right).
\]
(4.27)

In this case, the model contains the complex scalar DM \( \phi_0 \) with mass

\[
m^2_{R_{\phi_2}} = m^2_{I_{\phi_2}} = \frac{1}{2} \eta v_\eta^2 \lambda_{\phi\phi}.
\]
(4.28)

Other three mass matrices are

1. In the basis \( (R_{\chi_1}, R_{\eta_0}) \), the matrix is

\[
m^2_{CP\text{even}1} = \frac{\lambda_8}{2} \begin{pmatrix}
v_\eta^2 & v_\chi v_\eta \\
v_\chi v_\eta & v_\chi^2
\end{pmatrix}.
\]
(4.29)

The above matrix is similar to that in (4.16) except the mixing angle has the opposite sign. Thus, two physical states are

\[
R_{G_1} = \cos \theta \alpha R_{\chi_1} + \sin \theta \alpha R_{\eta_0},
\]

- 13 -
\[ H_1 = -\sin \theta R_{\chi_1} + \cos \theta R_{\eta_3}, \quad (4.30) \]

where \( R_{G1} \) is massless while the field \( H_2 \) has mass as follows

\[ m_{H_1}^2 = m_{A_1}^2 = \frac{\lambda_s v_\chi^2}{2 \cos^2 \theta_a}. \quad (4.31) \]

2. In the basis \((R_\rho, R_{\varphi_1})\), the matrix is

\[ m_{C_{\text{Peven}}}^2 = \begin{pmatrix} A + \frac{\lambda_6}{2} v_\eta^2 & - \frac{1}{2} v_\chi v_\eta (\lambda_1 + \lambda_2) \\ - \frac{1}{2} v_\chi v_\eta (\lambda_1 + \lambda_2) & \mu_{\varphi_1}^2 + C + B_1' \end{pmatrix}. \quad (4.32) \]

The physical states of matrix \((4.32)\) are

\[ \begin{pmatrix} H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} \cos \theta_r & \sin \theta_r \\ - \sin \theta_r & \cos \theta_r \end{pmatrix} \begin{pmatrix} R_\rho \\ R_{\varphi_1} \end{pmatrix}, \quad (4.33) \]

where the mixing angle is given by

\[ \tan 2\theta_r = \frac{v_\chi v_\eta (\lambda_1 + \lambda_2)}{\mu_{\varphi_1}^2 + C + B_1' - A - \frac{\lambda_6}{2} v_\eta^2}. \quad (4.34) \]

and their squared masses identified by

\[ m_{H_2}^2 = \frac{1}{2} \left\{ A + D_2 - \sqrt{(A - D_2)^2 + \frac{v_\eta^2}{4} [2(A - D_2)\lambda_6 + v_\chi^2 \lambda_5^2 + v_\xi^2 (\lambda_{13} + \lambda_{14})^2]} \right\}, \]
\[ m_{H_3}^2 = \frac{1}{2} \left\{ A + D_2 + \sqrt{(A - D_2)^2 + \frac{v_\eta^2}{4} [2(A - D_2)\lambda_6 + v_\chi^2 \lambda_5^2 + v_\xi^2 (\lambda_{13} + \lambda_{14})^2]} \right\}. \quad (4.35) \]

where

\[ D_2 = \mu_{\varphi_1}^2 + B_1' + C + \frac{1}{2} v_\eta^2 \lambda_6. \quad (4.36) \]

3. In the basis \((R_{\chi_3}, R_{\eta_1}, R_{\varphi})\), the matrix is

\[ m_{C_{\text{Peven}}}^2 = \begin{pmatrix} 2 v_\chi^2 \lambda_{13} & v_\chi v_\eta \lambda_5 & \lambda_\xi v_\chi v_\xi \\ v_\chi v_\eta \lambda_5 & 2 v_\eta^2 \lambda_{17} & \lambda_\xi v_\eta v_\xi \\ \lambda_\xi v_\chi v_\xi & \lambda_\xi v_\eta v_\xi & 2 \lambda_\xi v_\xi^2 \end{pmatrix}. \quad (4.37) \]

Let us summarize the Higgs content:

1. In the charged scalar sector: there are two Goldstone bosons \( \eta^- \) and \( \chi^- \) eaten by the gauge bosons \( W^- \) and \( Y^- \). Three massive charged Higgs bosons are \( \phi_1^+ \), \( \phi_2^+ \) and \( \phi_3^+ \). The remaining fields \( \rho_1^+ \), \( \phi_3^+ \) and \( \rho_3^+ \) are mixing.
2. In the CP-odd scalar sector: there is one massless Majoron scalar \( I_{\xi^0} \) which is denoted by \( G_M \). Fortunately, it is a gauge singlet, therefore, is phenomenologically harmless. Two massless scalars \( I_{\eta^0} \) and \( I_{\chi^0} \) which are Goldstone bosons for the gauge bosons \( Z \) and \( Z' \), respectively. There exists another massless state denoted by \( G_1 \), its role will be discussed below. Here we just mention that in the limit \( v_\eta \ll v_\chi \), this field is \( I_{\chi^0} \). The massive CP-odd field are \( I_{\varphi_2}, A_1 \) and other two \( I_{\varphi_1}, I_\rho \) are mixing.

3. In the CP-even scalar sector: There is one massless field: \( R_{G_2} \), and in the limit \( v_\eta \ll v_\chi \), it tends to \( R_{\chi^0} \). Combination of \( G_1 \) and \( R_{G_1} \) is Goldstone boson for neutral bilepton gauge boson \( X^0 \). Hence

\[
G_{X^0} = \frac{1}{\sqrt{2}} (R_{G_1} - i G_1). \tag{4.38}
\]

The massive fields are: \( R_{\varphi_2}, H_1, H_2 \) and three massive \( R_\chi, R_\eta, R_\xi \) and the SM-like Higgs boson \( h \). Note that there exists degeneracy when the contribution arising from \( Z_2 \times Z_4 \) soft breaking scalar interactions is not considered.

\[
m^2_{R_{\varphi_2}} = m^2_{I_{\varphi_2}} = \mu^2_{\varphi_2} + B'_2, \\
m^2_{H_1} = m^2_{A_1} = \frac{\lambda_8 v_\chi^2}{2 \cos^2 \theta_a}. \tag{4.39}
\]

Thus, the complex scalar \( \varphi_2 \) has mass given by

\[
m^2_{\varphi_2} = \mu^2_{\varphi_2} + B'_2. \tag{4.40}
\]

This result is consistent with the prediction in Ref. [18]. To be a DM candidate, the first term in (4.40) is suggested to be eliminated the terms with large VEVs such as \( v_\chi \) and \( v_\xi \). Then the above DM candidate \( \varphi_2 \) gets a mass given by

\[
m^2_{R_{\varphi_2}} = m^2_{I_{\varphi_2}} = \frac{1}{2} \lambda_8 v_\eta^2 \Rightarrow 0.04 \tag{4.42}
\]

According [23], the WIMP candidate has mass around 10 GeV, therefore

\[
\lambda_8^2 v_\eta^2 \approx 0.04 \tag{4.41}
\]

To get the second DM candidate, namely, \( \varphi_1^0 \), we have to carefully choose conditions. Looking at Eqs. (4.17), (4.30) and (4.39) we come to the fact that there is a new complex scalar

\[
\omega = \frac{1}{\sqrt{2}} (H_1 - i A_1), \tag{4.43}
\]

with mass determined by

\[
m^2_{\omega} = \frac{\lambda_8 v_\chi^2}{2 \cos^2 \theta_a}. \tag{4.44}
\]
Let us rewrite the Higgs content

\[
\chi \simeq \begin{pmatrix}
G_{X_0} \\
G_{Y^-} \\
\frac{1}{\sqrt{2}}(v_\chi + R_{X_0} - iG_{Z'})
\end{pmatrix},
\rho = \begin{pmatrix}
\rho_1^+ \\
\rho_3^+ \\
\frac{1}{\sqrt{2}}(R_\rho - iI_\rho)
\end{pmatrix},
\eta \simeq \begin{pmatrix}
1 \\
G_{W^-} \\
\frac{1}{\sqrt{2}}(v_\eta + h - iG_{Z'})
\end{pmatrix},
\varphi_2 = \frac{1}{\sqrt{2}}(R_{\varphi_2} - iI_{\varphi_2}) \sim (1, 1, 0, i, 1, 0) \sim \text{DM candidate},
\xi_0 = \frac{1}{\sqrt{2}}(v_\xi + R_{\xi_0} - iG_{M}) \sim (1, 1, 0).
\]

4.2 Simplified solutions

To find solutions in Higgs sector, we should make some simplification.

4.2.1 The CP-odd Higgs bosons

Looking at the potential in (4.1), the relations below are completely reasonable

\[
\lambda_1 = \lambda_2, \quad \lambda_{15} = \lambda_{16}, \quad \lambda_{19} = \lambda_{20}, \quad w_1 = w_4.
\]

The CP-odd Higgs sector contains three massless fields: \(I_{\chi_0^0}, I_{\eta_0^1}\) and \(I_{\xi_0^1}\). The field \(I_{\varphi_2}\) has the following squared mass

\[
m^2_{I_{\varphi_2}} = \frac{\mu^2}{2} + \frac{v_\chi^2}{2}(\lambda_{11}^\varphi + \lambda_{22}^\varphi) + v_\eta^2 + \frac{v_{\varphi_2}^2}{2}\lambda_{12}^\varphi \lambda_{21}^\varphi.
\]

Next, in the basis \((I_{\chi_0^0}, I_{\eta_0^1})\), we have one Goldstone boson \(G_1\) and a massive \(A_1\) with mass

\[
m^2_{A_1} = \frac{\lambda_8 v_\chi^2}{2 \cos^2 \theta_a},
\]

where

\[
\tan \theta_a = \frac{v_\eta}{v_\chi}.
\]

Taking into account (4.46), the mass matrix in Eq. (4.20) for \(I_{\varphi_1}\) and \(I_{\rho_1}\) becomes diagonal

\[
m^2_{CP_{odd}} = \begin{pmatrix}
m^2_{I_{\varphi_1}} & C + B'_1 & 0 \\
C + B'_1 & 0 & A + \frac{\lambda_6 v_\eta^2}{2}
\end{pmatrix}.
\]

Hence, according the above assumption, \(I_{\varphi_1}\) and \(I_{\rho_1}\) are physical states with respective masses as follows

\[
m^2_{I_{\varphi_1}} = \mu_{I_{\varphi_1}}^2 - C + B'_1 = \mu_{I_{\varphi_1}}^2 + \frac{v_\chi^2}{2} \left[ \lambda_{11}^\varphi + \lambda_{22}^\varphi - 2(\lambda_{22} + \lambda_{25}) \right] + \frac{v_\eta^2}{2}(\lambda_1^\varphi - \lambda_{21})
\]

\[
m^2_{I_{\rho_1}} = A + \frac{\lambda_6}{2} v_\eta^2 = \mu_{I_{\rho_1}}^2 + \frac{1}{2} \left[ (\lambda_{18} + \lambda_{26}) v_\varphi^2 + \lambda_6 v_\eta^2 \right].
\]

In this case, we have \(A_2 = I_{\varphi_1}\) and \(A_3 = I_{\rho_1}\).

In summary, under assumption of (4.46), the CP-odd scalar sector consists of four massless fields: \(I_{\chi_0^0}, I_{\eta_0}, G_M\) and \(G_1\). Four massive fields are \(A_1, A_2, A_3\) and \(I_{\varphi_2}\).

The content of the CP-odd scalar sector is summarized in Table 3.
In this case, the model contains the complex scalar DM. As mentioned in Ref. [18], the lightest scalar ϕ \( \phi \) is possible DM candidate. Therefore from (4.26), the following condition is reasonable

\[ m^2_{R_{\phi_2}} = m^2_{I_{\phi_2}} = \frac{1}{2} \left[ v^2_\chi (\lambda^2_2 + \lambda^2_1) + v^2_\eta \lambda_2^\phi \right] . \]  

(4.52)

As mentioned in Ref. [18], the lightest scalar ϕ \( \phi \) is possible DM candidate. Therefore from (4.26), the following condition is reasonable

\[ \mu^2_{R_{\phi_2}} = -\frac{1}{2} \left( v^2_\chi \lambda^2_2 + v^2_\eta \lambda^2_2 \right) . \]  

(4.53)

In this case, the model contains the complex scalar DM ϕ \( \phi \) with mass

\[ m^2_{R_{\phi_2}} = m^2_{I_{\phi_2}} = \frac{1}{2} v^2_\eta \lambda^2_2 . \]  

(4.54)

Some other components are:

In the basis \( (R_{\phi_2}, R_{\phi_1}) \), we have one massless \( R_G \) and one massive \( H_1 \) with mass equal to that of \( A_1 \) and the mixing angle is the same.

In the basis \( (R_\rho, R_\varphi) \), the matrix is

\[ m^2_{\text{CPeven2}} = \begin{pmatrix} \mu^2_\rho + \frac{1}{2} \left[ (\lambda_{18} + \lambda_{\rho\xi}) v^2_\rho + \lambda_{\rho\eta} v^2_\eta \right] & \frac{v^2_\rho}{2} \left[ \lambda_5^\rho + \lambda_6^\rho + 2(\lambda_{22} + \lambda_{25}) \right] + \frac{v^2_\eta}{2} (\lambda_{18}^\rho + \lambda_{24}^\rho) \\ -v_\chi v_\eta \lambda_1 & \mu^2_\varphi + \frac{v^2_\rho}{2} \left[ \lambda_5^\varphi + \lambda_6^\varphi + 2(\lambda_{22} + \lambda_{25}) \right] + \frac{v^2_\eta}{2} (\lambda_{18}^\varphi + \lambda_{24}^\varphi) \end{pmatrix} . \]  

(4.55)

The physical states of matrix (4.55) are \( H_2 \) and \( H_3 \) with mixing angle given by

\[ \tan 2\theta_\rho = \frac{2 v_\chi v_\eta \lambda_1}{(\mu^2_\rho + C + B_1^\rho - A - \frac{\lambda_{18}^\rho}{2} v^2_\eta)} . \]  

(4.56)

Now we turn to the sector where the SM Higgs boson exists, i.e., - the matrix in the basis \( (R_{\phi_2}, R_{\phi_1}, R_\rho) \) is given by

\[ m^2_{\text{CPeven3}} = \begin{pmatrix} 2 v^2_\rho \lambda_{13} & v_\chi v_\rho \lambda_5 & \lambda_\xi v_\chi v_\xi \\ v_\chi v_\rho \lambda_5 & 2 v^2_\rho \lambda_{17} & \lambda_\eta v_\rho v_\xi \\ \lambda_\xi v_\chi v_\xi & \lambda_\eta v_\rho v_\xi & 2 \lambda_\xi v^2_\xi \end{pmatrix} . \]  

(4.57)

Let us assume a simplified scenario worth to be considered is characterized by the following relations:

\[ \lambda_5 = \lambda_{13} = \lambda_{17} = \lambda_\xi = \lambda_\chi = \lambda_\eta = \lambda, \quad v_\xi = v_\chi. \]  

(4.58)

In this scenario, the squared matrix (4.37) for the electrically neutral CP even scalars in the basis \( (R_{\phi_1}, R_{\phi_2}, R_\rho) \) takes the simple form:

\[ m^2_{\text{CPeven3}} = \lambda \begin{pmatrix} 2 x^2 & x & x \\ x & 2 & 1 \\ x & 1 & 2 \end{pmatrix} v^2_\chi, \quad x = \frac{v_\eta}{v_\chi}. \]  

(4.59)
The charged scalar sector contains two massless fields: $G_{W^+}$ and $G_{Y^+}$ which are Goldstone bosons eaten by the $W^+$ and $Y^+$ gauge bosons, respectively. The other massive fields are $\phi_1^+, \phi_2^+$ and $\phi_3^+$ with respective masses

$$m_{\phi_1^+}^2 = \mu_{\phi_1^+}^2 + \frac{1}{2} \left[ v_h^2 (\lambda_1^\phi + \lambda_1^\rho) + v_\eta^2 \lambda_1^\eta \right],$$

$$m_{\phi_2^+}^2 = \mu_{\phi_2^+}^2 + \frac{1}{2} \left[ v_h^2 (\lambda_2^\phi + \lambda_2^\rho) + v_\eta^2 \lambda_2^\eta \right],$$

$$m_{\phi_3^+}^2 = \mu_{\phi_3^+}^2 + \frac{1}{2} \left[ v_h^2 \lambda_3^\phi + v_\eta^2 \lambda_3^\rho + v_\xi^2 \lambda_3^\xi \right].$$

\[ (4.65) \]

In this scenario, the squared mass matrix $m_{CPeven3}^2$ given above can be perturbatively diagonalized as follows:

$$R_{CPeven3}^2 m_{CPeven3}^2 R_{CPeven3} \simeq \begin{pmatrix} \frac{4}{3} \lambda v^2 & 0 & 0 \\ 0 & \lambda v^2 & 0 \\ 0 & 0 & 3 \lambda v^2 \end{pmatrix}, \quad R_{CPeven3} \simeq \begin{pmatrix} -1 + \frac{v^2}{\lambda^2} & 0 & \frac{\sqrt{2}v}{\lambda^2} \\ \frac{v}{\lambda} & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \frac{v}{\lambda} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix},$$

where we have taken into account that $v_\chi \gg v_\eta = 246$ GeV.

Thus, we find that the physical scalars included in the matrix $m_{CPeven3}^2$ are:

$$\begin{pmatrix} h \\ H_4 \\ H_5 \end{pmatrix} \simeq \begin{pmatrix} -1 + \frac{v^2}{\lambda^2} & \frac{v}{\lambda} & \frac{v}{\lambda} \\ \frac{v}{\lambda} & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \frac{v}{\lambda} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix},$$

where $h$ is the 126 GeV SM like Higgs boson, whereas $H_4$ and $H_5$ are physical heavy scalars acquiring masses at the scale of breaking of the $SU(3)_C \times U(1)_X \times Z_4 \times Z_2 \times U(1)_{L_h}$ symmetry. Thus, we find that the SM-like Higgs boson $h$ has couplings very close to SM expectation with small deviations of the order of $\frac{v_\chi}{\lambda^2} \sim O(10^{-3})$. In addition, the squared masses of the physical scalars included in the matrix $m_{CPeven3}^2$ take the form:

$$m_{H_4}^2 \simeq 3 \lambda v^2, \quad m_{H_5}^2 \simeq 3 \lambda v^2.$$  

Thus, from (4.62) we obtain

$$\lambda \approx 0.187.$$  

Combining with (3.26) yields

$$1.5 \text{ TeV} < m_{H_4} < 2.61 \text{ TeV},$$

$$2.6 \text{ TeV} < m_{H_5} < 4.5 \text{ TeV}.$$  

The content of the CP-even Higgs bosons is summarized in Table 4.

### 4.2.3 The charged Higgs bosons

The charged scalar sector contains two massless fields: $G_{W^+}$ and $G_{Y^+}$ which are Goldstone bosons eaten by the $W^+$ and $Y^+$ gauge bosons, respectively. The other massive fields are $\phi_1^+, \phi_2^+$ and $\phi_3^+$ with respective masses

### Table 4. Squared mass of CP-even scalars under condition in (4.58) and $v_\chi \gg v_\eta$.

| Fields | $R_{\chi^L} \not\subset G_{\phi^L}$ | $R_{\chi^Q} = H_4$ | $R_{\chi^b} = h$ | $R_{\chi^Q} = H_4$ | $R_{\phi} = H_4$ | $R_{\psi} = H_5$ | $R_{\psi} = H_5$ | $R_{\psi} = DM$ | $R_{\chi} \approx H_5$ |
|--------|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Squared mass | 0 | $\lambda v^2$ | $\frac{4}{3} \lambda v^2$ | $m_{H_4}^2$ | $m_{H_5}^2$ | $m_{\phi_2^+}^2$ | $m_{\phi_3^+}^2$ | $3 \lambda v^2$ | $3 \lambda v^2$ |
the masses of three charged scalars almost does not carry lepton number, while two others do. \( H^+ \) boson with mass around TeV (\( \propto \rho \)) in the basis (\( \rho^+ \), \( \rho^\pm \), \( \phi^\pm \)), the mass mixing matrix is given by

\[
M_{\text{charged}}^2 = \begin{pmatrix}
\mu^2 + \frac{1}{2}v^2_v(\lambda_1 + \lambda_\rho) + \frac{1}{2}v^2_\eta(\lambda_6 + \lambda_9) & 0 & \frac{\lambda_v^2 v_\eta v_\chi}{v^2_v v^2_\eta} \\
0 & \mu^2 + \frac{1}{2}v^2_v(\lambda_7 + \lambda_1 + \lambda_\rho) + \lambda_6 v^2_\eta & \frac{\lambda_v^2 v_\eta v_\chi}{v^2_v v^2_\eta} \\
\frac{\lambda_v^2 v_\eta v_\chi}{v^2_v v^2_\eta} & \frac{1}{\sqrt{2}}v_\chi w_2 & \mu_{\phi^+}^2 + \frac{1}{2}(v^2_\chi (\lambda_2^\phi + \lambda_2^\phi) + v^2_\eta^2) \end{pmatrix}.
\] (4.66)

Let us make effort to simplify the above matrix. Note that due to the conditions in (4.58) we have

\[
\mu^2 = -\frac{\lambda}{2}(3v^2_v + v^2_\eta) \approx -\frac{3}{2}v^2_v,
\]
\[
\mu_\eta^2 = -\lambda(v^2_\eta + v^2_\chi) \approx -\lambda v^2_\chi,
\]
\[
\mu_\xi^2 = -\frac{\lambda}{2}(v^2_\chi + v^2_\eta) \approx -\frac{1}{2}v^2_\chi.
\] (4.67)

Therefore, it is reasonable to assume

\[
\mu^2 = -\frac{v^2_\chi}{2}(\lambda_1 + \lambda_\rho) \approx \mu_\eta^2,
\]
\[
\mu_{\phi^+}^2 = -\frac{v^2_\chi}{2}(\lambda_2^\phi + \lambda_2^\phi).
\] (4.68)

Imposing these conditions, we get

\[
M_{\text{charged}}^2 = \begin{pmatrix}
\frac{1}{2}v^2_v(\lambda_6 + \lambda_9) & 0 & \frac{\lambda_v^2 v_\eta v_\chi}{v^2_v v^2_\eta} \\
0 & \frac{1}{2}(v^2_\chi \lambda_7 + \lambda_6 v^2_\eta) & \frac{\lambda_v^2 v_\eta v_\chi}{v^2_v v^2_\eta} \\
\frac{\lambda_v^2 v_\eta v_\chi}{v^2_v v^2_\eta} & \frac{1}{\sqrt{2}}v_\chi w_2 & \frac{1}{2}(v^2_\chi (\lambda_2^\phi + \lambda_2^\phi) + v^2_\eta^2) \end{pmatrix},
\] (4.69)

From (4.69), we get two charged Higgs bosons with masses at electroweak scale and one massive with mass around TeV (\( \propto v_\chi \)) as indicated in figures 2, 3 and 4. Figure 2 shows that the Higgs boson \( H^+_1 \) which is mainly composed of \( \rho^+_1 \) has mass around 100 GeV, while the \( H^+_2 \) gets mass in the range of 200 GeV and the mass of \( H^+_3 \) is around 3.5 TeV. In addition, the Higgs boson \( H^+_4 \) almost does not carry lepton number, while two others do.

The content of the charged scalar sector is summarized in Table 5. It is worth mentioning that the masses of three charged scalars \( \phi_i^+ \), \( i = 1, 2, 4 \) are still not fixed.

The potential including lepton number violations, i.e., \( V_{\text{full}} = V_{\text{LNC}} + V_{\text{LNV}} \) is quite similar to the previous one. There are some differences:

1. Masses of fields are added some new contributions.
2. The complex scalar \( \varphi_3^\pm \) with mass the same in mentioned two cases.
3. Majoron does not exist and its mass arises from only lepton number violating part.
4. The CP-even scalars mix more complicatedly.

| Fields | \( \eta^+ \) | \( \chi^+ \) | \( H^+_1 \) | \( H^+_2 \) | \( H^+_3 \) | \( \phi_1^+ \) | \( \phi_2^+ \) | \( \phi_4^+ \) |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Squared mass | 0 | 0 | \( m_{H^+_1}^2 \) | \( m_{H^+_2}^2 \) | \( m_{H^+_3}^2 \) | \( m_{\phi_1^+}^2 \) | \( m_{\phi_2^+}^2 \) | \( m_{\phi_4^+}^2 \) |
Figure 2. Correlation between the mass of the lightest charged scalar $H_1^+$ and the trilinear scalar coupling $w_2$.

5 Search for $Z'$ at LHC

In this section, we present two typical effects of the LHC, namely, production of single a particle in proton-proton collisions.

5.1 Phenomenology of $Z'$ gauge boson

In what follows we proceed to compute the total cross section for the production of a heavy $Z'$ gauge boson at the LHC via Drell-Yan mechanism. In our computation for the total cross section we consider the dominant contribution due to the parton distribution functions of the light up, down and strange quarks, so that the total cross section for the production of a $Z'$ via quark antiquark annihilation in proton proton collisions with center of mass energy $\sqrt{S}$ takes the form:

$$
\sigma_{pp \to Z'}^{(DrellYan)}(S) = \frac{g^2 \pi}{6c_4 W} \left\{ \left[ (g'_{uL})^2 + (g'_{uR})^2 \right] \int_{\ln \sqrt{m_{Z'}^2}}^{\ln \sqrt{m_{Z'}^2}} f_{p/u} \left( \sqrt{\frac{m_{Z'}^2}{S}}, e^{y}, \mu^2 \right) f_{p/u} \left( \sqrt{\frac{m_{Z'}^2}{S}}, e^{-y}, \mu^2 \right) dy \\
+ \left[ (g'_{dL})^2 + (g'_{dR})^2 \right] \int_{\ln \sqrt{m_{Z'}^2}}^{\ln \sqrt{m_{Z'}^2}} f_{p/d} \left( \sqrt{\frac{m_{Z'}^2}{S}}, e^{y}, \mu^2 \right) f_{p/d} \left( \sqrt{\frac{m_{Z'}^2}{S}}, e^{-y}, \mu^2 \right) dy \\
+ \left[ (g'_{sL})^2 + (g'_{sR})^2 \right] \int_{\ln \sqrt{m_{Z'}^2}}^{\ln \sqrt{m_{Z'}^2}} f_{p/s} \left( \sqrt{\frac{m_{Z'}^2}{S}}, e^{y}, \mu^2 \right) f_{p/s} \left( \sqrt{\frac{m_{Z'}^2}{S}}, e^{-y}, \mu^2 \right) dy \right\}
$$
Figure 3. Correlation between the mass of the charged scalar $H_2^+$ and the trilinear scalar coupling $w_2$.

Figure 5 displays the $Z'$ total production cross section at the LHC via Drell-Yan mechanism at the LHC for $\sqrt{S} = 13$ TeV and as a function of the $Z'$ mass, which is taken to range from 1.42 TeV up to 2.42 TeV to fulfill the constraints imposed by the $\rho$ parameter. In that region of masses for the heavy neutral gauge boson $Z'$, we find that the total cross section for its production at the LHC via Drell-Yan mechanism ranges from 46.2 pb up to 2.89 pb. On the other hand, in a future 100 TeV proton-proton collider the total cross section for the Drell-Yan production of a heavy $Z'$ neutral gauge boson gets significantly enhanced reaching values ranging from 1371 pb up to 235 pb, as indicated in figure 6.

It is worth noting that the produced $Z'$ boson will decay to $t$ and $\tilde{t}$, but in the model under consideration, there are no the decays of the top quark $t$ to the SM $h$ associated with $c$ or $u$ as well as $cZ$ [18].

5.2 Phenomenology of $H_4$ Heavy Higgs boson

In what follows we proceed to compute the LHC production cross section of the singly heavy scalar $H_4$. Let us note that the singly heavy scalar $H_4$ is mainly produced via gluon fusion mechanism mediated by a triangular loop of the heavy exotic quarks $T$, $J_1$ and $J_2$. Thus, the total cross section for the production of the heavy scalar $H_4$ through gluon fusion mechanism in proton proton collisions with center of mass energy $\sqrt{S}$ takes the form:

$$\sigma_{pp \rightarrow gg \rightarrow H_4}(S) = \frac{\alpha^2 m_{H_4}^2 |(R_{\text{CP even}})|^2}{64 \pi v^2 \sqrt{S}} \left[ I \left( \frac{m_{H_4}^2}{m_T^2} \right) + I \left( \frac{m_{H_4}^2}{m_{J_1}^2} \right) + I \left( \frac{m_{H_4}^2}{m_{J_2}^2} \right) \right]$$
Figure 4. Correlation between the mass of the heaviest charged scalar $H_3^+$ and the trilinear scalar coupling $w_2$.

$$\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{p/g}(x_1, \mu^2) f_{p/g}(x_2, \mu^2) \left( \sqrt{m_{H_4}^2 - \xi^2}, \mu^2 \right) dy$$

where $f_{p/g}(x_1, \mu^2)$ and $f_{p/g}(x_2, \mu^2)$ are the distributions of gluons in the proton which carry momentum fractions $x_1$ and $x_2$ of the proton, respectively. Furthermore $\mu = m_{H_4}$ is the factorization scale and $I(z)$ is given by:

$$I(z) = \int_0^1 dx \int_0^{1-x} dy \frac{1 - 4xy}{1 - zxy}$$  \hspace{1cm} (5.1)$$

Figure 7 displays the $H_4$ total production cross section at the LHC via gluon fusion mechanism for $\sqrt{S} = 13$ TeV, as a function of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$, which is taken to range from 3.57 TeV up to 6.1 TeV. The aforementioned range of values for the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ corresponds to a heavy scalar mass $m_{H_4}$ varying between 1.6 TeV and 2.7 TeV and was chosen to guarantee the consistency of our model with the constraints imposed by the $\rho$ parameter. Here, for the sake of simplicity we have restricted to the simplified scenario described by Eq. (4.58) and we have chosen the exotic quark Yukawa couplings equal to unity, i.e, $y^{(T)} = y^{(J_1)} = y^{(J_2)} = 1$. In addition, the top quark mass has been taken to be equal to $m_t = 173$ GeV. We find that the total cross section for the production of the $H_4$ scalar at the LHC takes a value close to about 0.5 fb for the lower bound of 3.57 TeV of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ arising from the $\rho$ parameter constraint and decreases when $v_\chi$ takes larger values. We see that the total cross section at the LHC for the $H_4$ production via gluon fusion mechanism is small to give rise to a signal for the allowed values of the $SU(3)_L \times U(1)_X$ symmetry breaking
Figure 5. Total cross section for the $Z'$ production via Drell-Yan mechanism at the LHC for $\sqrt{S} = 13$ TeV and as a function of the $Z'$ mass.

scale $v_{\chi}$, however in a future 100 TeV proton-proton collider, it can range from 134 fb up to 14 fb when $v_{\chi}$ is varied between 3.57 TeV up to 6.1 as shown in figure 8.

6 Dark matter relic density

In this section we provide a discussion of the implications of our model for DM, assuming that the DM candidate is a scalar. Let us recall that our goal in this section is to provide an estimate of the DM relic density in our model, under some simplifying assumptions motivated by the large number of scalar fields of the model. We do not intend to provide a sophisticated analysis of the DM constraints of the model under consideration, which is beyond the scope of the present paper. We just intend to show that our model can accommodate the observed value of the DM relic density, by having a scalar DM candidate with a mass in the TeV range and a quartic scalar coupling of the order unity, within the perturbative regime. We start by surveying the possible scalar DM candidates in the model. Considering that the $Z_4$ symmetry is preserved and taking into account the scalar assignments under this symmetry, given by Eq. (1), we can assign this role to either any of the $SU(3)_L$ scalar singlets, i.e., $Re\varphi^0_n$ and $Im\varphi^0_n$ ($n = 1, 2$). In this work we assume that the $\varphi_f = Im\varphi^0_1$ is the lightest among the $Re\varphi^0_n$ and $Im\varphi^0_n$ ($n = 1, 2$) scalar fields and also lighter than the exotic charged fermions, as well as lighter than $\Psi_R$, thus implying that its tree-level decays are kinematically forbidden. Consequently, in this mass range the $Im\varphi^0_1$ scalar field is stable.
Figure 6. Total cross section for the $Z'$ production via Drell-Yan mechanism at the LHC for a future $\sqrt{S} = 100$ TeV proton-proton collider and as a function of the $Z'$ mass.

The relic density is given by (c.f. Ref. [23, 30])

$$\Omega h^2 = \frac{0.1 pb}{\langle \sigma v \rangle}, \quad \langle \sigma v \rangle = \frac{A}{n_{eq}^2} = \frac{T}{32\pi} \int_{4m_\phi^2}^{\infty} \sum_{p=0}^{\infty} g_p^2 s \sqrt{s} \left( \sum_{p=0}^{\infty} \frac{\sigma v}{2} \right) v_{rel} (\varphi \varphi \rightarrow p\overline{p}) K_1 \left( \frac{s}{T} \right) ds,$$

where $\langle \sigma v \rangle$ is the thermally averaged annihilation cross-section, $A$ is the total annihilation rate per unit volume at temperature $T$ and $n_{eq}$ is the equilibrium value of the particle density. Furthermore, $K_1$ and $K_2$ are modified Bessel functions of the second kind and order 1 and 2, respectively [30] and $m_\phi = m_{10-\varphi}$. Let us note that we assume that our scalar DM candidate is a stable weakly interacting particle (WIMP) with annihilation cross sections mediated by electroweak interactions mainly through the Higgs field. In addition we assume that the decoupling of the non-relativistic WIMP of our model is supposed to happen at a very low temperature. Because of this reason, for the computation of the relic density, we take $T = m_\phi/20$ as in Ref. [30], corresponding to a typical freeze-out temperature. We assume that our DM candidate $\varphi$ annihilates mainly into $WW$, $ZZ$, $t\bar{t}$, $b\bar{b}$ and $hh$, with annihilation cross sections given by the following relations [31]:

$$v_{rel} (\varphi \varphi \rightarrow p\overline{p}) = \frac{\lambda_{h^2\varphi^2}^2}{8\pi} \frac{s \left( 1 + \frac{12m_W^4}{s^2} - \frac{4m_W^2}{s} \right)}{(s-m_h^2)^2 + m_h^2 \Gamma_h^2} \sqrt{1 - \frac{4m_W^2}{s}},$$
Figure 7. Total cross section for the $H_4$ production via gluon fusion mechanism at the LHC for $\sqrt{S} = 13$ TeV and as a function of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ for the simplified scenario described in Eq. (4.58).

\[ v_{rel} \sigma(\phi_I \phi_I \rightarrow ZZ) = \frac{\lambda_h^2 \phi^2}{16\pi} \frac{s}{(s - m_h^2)^2 + m_Z^2 \frac{s}{2}} \sqrt{1 - \frac{4m_Z^2}{s}}, \]

\[ v_{rel} \sigma(\phi_I \phi_I \rightarrow q\bar{q}) = \frac{N_c \lambda_h^2 \phi^2 m_q^2}{4\pi} \sqrt{1 - \frac{4m_q^2}{s}}, \]

\[ v_{rel} \sigma(\phi_I \phi_I \rightarrow hh) = \frac{\lambda_h^2 \phi^2}{16\pi s} \left( 1 + \frac{3m_h^2}{s - m_h^2} - \frac{4\lambda_h^2 \phi^2 v^2}{s - 2m_h^2} \right)^2 \sqrt{1 - \frac{4m_h^2}{s}}, \] (6.2)

where $\sqrt{s}$ is the centre-of-mass energy, $N_c = 3$ is the color factor, $m_h = 125.7$ GeV and $\Gamma_h = 4.1$ MeV are the SM Higgs boson $h$ mass and its total decay width, respectively. Note that we have worked on the decoupling limit where the couplings of the 126 GeV Higgs boson to SM particles and its self-couplings correspond to the SM expectation.

The vacuum stability and tree level unitarity constraints of the scalar potential are [32–34]:

\[ \lambda_{h^*} > 0, \quad \lambda_{\phi^*} > 0, \quad \lambda_{h^*}^2 \phi^2 < \frac{2}{3} \lambda_{h^*} \lambda_{\phi^*}. \] (6.3)

\[ \lambda_{\phi^*} < 8\pi, \quad \lambda_{h^*}^2 \phi^2 < 4\pi. \] (6.4)

The dark matter relic density as a function of the mass $m_\phi$ of the scalar field $\phi_I$ is shown in Fig. 9, for several values of the quartic scalar coupling $\lambda_h^2 \phi^2$, set to be equal to 0.7, 0.8 and 0.9 (from top
Figure 8. Total cross section for the $H_4$ production via gluon fusion mechanism for a future $\sqrt{S} = 100$ TeV proton-proton collider and as a function of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ for the simplified scenario described in Eq. (4.58).

to bottom). The horizontal line corresponds to the experimental value $\Omega h^2 = 0.1198$ for the relic density. We found that the DM relic density constraint gives rise to a linear correlation between the quartic scalar coupling $\lambda_{h^2,\varphi^2}$ and the mass $m_{\varphi}$ of the scalar DM candidate $\varphi_I$, as indicated in Fig. 10.

We find that we can reproduce the experimental value $\Omega h^2 = 0.1198 \pm 0.0026$ [35] of the DM relic density, when the mass $m_{\varphi}$ of the scalar field $\varphi_I$ is in the range $300 \text{ GeV} \lesssim m_{\varphi} \lesssim 600 \text{ GeV}$, for a quartic scalar coupling $\lambda_{h^2,\varphi^2}$ in the window $0.5 \lesssim \lambda_{h^2,\varphi^2} \lesssim 1$, which is consistent with the vacuum stability and unitarity constraints shown in Eqs. (6.3) and (6.4). Note that our range of values chosen for the quartic scalar coupling $\lambda_{h^2,\varphi^2}$ also allow the extrapolation of our model at high energy scales as well as the preservation of perturbativity at one loop level.

7 Conclusions

In this paper, we have studied the gauge and Higgs sectors of the renormalizable 3-3-1 model for the SM fermion masses and mixings. From the experimental data on the $\rho$ parameter, it follows that the mass of new heavy $Z'$ ranges from 1.42 TeV to 2.42 TeV. This bound is available for the LHC search and also implies that the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ ranges from 3.57 TeV up to 6.1 TeV. In the aforementioned region of masses for the heavy neutral gauge boson $Z'$, we find that the total cross section for the its production at the LHC via Drell-Yan mechanism ranges from 46.2 pb up to 2.89 pb. On the other hand, in a future 100 TeV proton-proton collider the total cross section for the Drell-Yan production of a heavy $Z'$ neutral gauge boson gets significantly
Figure 9. Relic density $\Omega h^2$, as a function of the mass $m_\varphi$ of the $\varphi$ scalar field, for several values of the quartic scalar coupling $\lambda_{h^2\varphi^2}$. The curves from top to bottom correspond to $\lambda_{h^2\varphi^2} = 0.7, 0.8, 0.9$, respectively. The horizontal line shows the observed value $\Omega h^2 = 0.1198$ [33] for the relic density.

Figure 10. Correlation between the quartic scalar coupling and the mass $m_\varphi$ of the scalar DM candidate $\varphi$, consistent with the experimental value $\Omega h^2 = 0.1198$ for the Relic density.

enhanced reaching values ranging from 1371 pb up to 235 pb. We find that the total cross section for the production of the $H_4$ scalar at the LHC with $\sqrt{S} = 13$ TeV takes a value close to about 0.5 fb for the lower bound of 3.57 TeV of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ required by the consistency of the $\rho$ with the experimental data. Despite that small value, the $H_4$ production total cross section can be as large as 134 fb in a future 100 TeV proton-proton collider that can be useful to probe the scalar and gauge sector of the 3-3-1 model and shed light in the understanding
of the underlying dynamics behind electroweak symmetry breaking.

By the way, the bilepton gauge bosons $Y^\pm$ and $X^0$ have masses around 800 GeV. For the further studies, the neutral currents have been also presented.

The general Higgs sector is separated into two parts. The first part consists of lepton number conserving terms and the second one contains lepton number violating couplings. The first part of potential was in details considered and the SM Higgs boson was derived and as expected, is most contained from $\eta^0_1$. We have showed that the total potential, except CP-even sector, has quite similar situation, i.e., the eigenmasses and eigenstates in the part with lepton number conservation and the total potential are similar. The potential consists of enough number of Goldstone bosons for massive gauge bosons. In the CP-odd scalar sector, there are four massive bosons and one of them is a DM candidate. The CP-even sector consists of seven massive fields including the SM Higgs boson and a DM candidate. The singly charged Higgs bosons sector contains six massive fields. Two of them have masses in the electroweak scale and the remaining has mass around 3.5 TeV. Masses of three charged bosons $\phi^\pm_i, i = 1, 2, 4$ are not fixed. The scalar potential contains a Majoron but it is harmless, because it is a scalar singlet. There is complex scalar DM candidate $\varphi^0_2$. To reproduce the Dark matter relic density, the mass of the scalar dark matter candidate has to be in the range $300 \, \text{GeV} \lesssim m_{\varphi} \lesssim 600 \, \text{GeV}$, for a quartic scalar coupling $\lambda_{\varphi^2 \varphi^2}$ in the window $0.5 \lesssim \lambda_{\varphi^2 \varphi^2} \lesssim 1$. In addition, it has been shown in Ref. [18] that requiring that the DM candidate $\varphi^0$ lifetime be greater than the universe lifetime $\tau_u \approx 13.8 \, \text{Gyr}$ and assuming $m_{\varphi} \sim 1 \, \text{TeV}$, we estimate the cutoff scale of our model $\Lambda > 3 \times 10^{10} \, \text{GeV}$. Thus we conclude that under the above specified conditions the model contains viable fermionic $\Psi^c_R$ and scalar $\varphi^0$ DM candidates. A sophisticated analysis of the DM constraints of our model is beyond the scope of the present paper and is left for future studies.

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