Simulation Research on Trapped Oil Pressure of Involute Internal Gear Pump

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1. Introduction

In order to ensure the continuous and uniform oil supply of the gear pump, two pairs of gear teeth engaging in meshing are needed in the internal gear pump for a certain period of time. It results in a closed dead volume which is not connected with the inlet and outlet cavities [1]. The size of the closed dead volume changes periodically with the rotation of the gear, resulting in a dramatic change of the working pressure [2]. This phenomenon is usually called the oil trapping phenomenon of a gear pump, which is the main source of gear pump noise and cavitation [3]. It can cause hazards such as pressure swing, pressure shock, and flow pulsation in the hydraulic system [4]. Therefore, it is of great significance to avoid the so-called oil trapping phenomenon of the gear pump. However, because the trapped oil area is surrounded by complex curves such as the outer contour of gear, involute, or cycloid [5], it is very difficult to obtain the continuous and uniform oil supply. Up to date, there is much research on the oil trapping phenomenon of the external gear pump and cycloid gear [6], and great improvements have been made. In comparison with external gear pump and cycloid gear, internal gear pumps have advantages such as lighter weight, smaller volume, and higher pressure due to their geometric structure [7]. Because the internal gear has oil ports in the diameter direction and the oil trapping process is different from other gear pumps, the high-pressure internal gears are not easy to obtain by conventional fabrication methods of internal gears. The common internal meshing gears are arc tooth profile, conjugate tooth profile, involute tooth profile, and so forth. Most research is focused on arc tooth profile and conjugate tooth profile and there are a few studies on involute tooth profile [8]. In this study, in order to solve the oil trapping phenomenon of gear pump with involute tooth profile during the process of internal meshing, the contour of internal gear is obtained by means of an imaginary rack cutter generating internal gear, and the model of trapped oil...
volume is established. Then, the design parameters and the change of trapped oil volume in the process of gear rotation are described, and then the trapped oil pressure is simulated. The research conclusion can provide an effective reference for the design and manufacture of the high-performance internal gear pump.

2. Mathematical Model

2.1. Oil Trapped Model of Internal Gear Pump. Gear pairs are commonly manufactured by the generative method [9]. Figure 1(a) shows the processing of external gears by the rack tool generating method, and Figure 1(b) shows the processing of the internal gears by the virtual rack tool generating method. The outer contour of the rack cutter is composed of line segments AB, CD, DE, and arc BC. The parameters of \( h_f, c_r, h_f, \) and \( r_p \) are the addendum height, head clearance coefficient, total tooth height, and tool fillet, respectively. The parameters \( \alpha, \beta, \) and \( p \) are pressure angle, the profile shift coefficient, and pitch, respectively. A rectangular coordinate system \( s_i \) is established at the center of the rack tool, and the mathematical equation of the rack tool is described as [10]

\[
R_i^s(x_i) = \begin{bmatrix} R_i^{AB} & R_i^{BC} & R_i^{CD} & R_i^{DE} \end{bmatrix} = \begin{bmatrix} x_i^s & y_i^s & 1 \end{bmatrix}.
\]

The coordinate systems of gears \( O_s \) and \( O_r \) are \( s_i(O_s,x_i,y_i) \) and \( s_t(O_r,x_t,y_t) \), respectively. Their origins coincide with the gear axis and rotate with the gear axis. At the same time, the coordinate system \( s_i(O_s,x_s,y_s) \) fixedly connected with the earth is established. During the forming process, the rack tool translates in the \( x_t \) direction, and the displacement is \( s \). At this moment, the gear \( O_s \) rotates around \( O_s \) and the rotation angle is \( \varphi_i \), which satisfies the following motion relationship:

\[
s = r_s \varphi_i.
\]

In the above equation, \( r_s \) is the gear pitch radius [11]:

\[
r_s = 0.5mz,
\]

where \( m \) and \( z \) are the modulus of the gear and the number of teeth, respectively.

According to the generation principle, the relationship between the tooth profile of the gear and the tooth profile of the rack tool can be expressed by the following equation [12]:

\[
R_i^s(\varphi_i, x_i) = M_i(\varphi_i)R_i^s(x_i),
\]

where the vector \( R_i^s(\varphi_i, x_i) \) is the envelope of the tooth profile surface family of the rack tool and the matrix \( M_i(\varphi_i) \) is the transformation matrix from the coordinate system \( s_t \) to the coordinate system \( s_i \).

In the same way, the internal gear contour equation is obtained by the method of generating internal gears by imaginary rack tools:

\[
R_i^s(\varphi_i, x_i) = M_i(\varphi_i)R_i^s(x_i).
\]

In the formula, the vector \( R_i^s(\varphi_i, x_i) \) is the envelope of the rack tool tooth profile surface family, and the matrix \( M_i(\varphi_i) \) is the transformation matrix from the coordinate system \( s_t \) to the coordinate system \( s_i \).

2.2. Calculation of Trapped Oil Volume. The principle of the internal gear pump trapping oil is shown in Figure 2. The theoretical instantaneous flow \( Q_{sh} \) of the internal gear pump can be expressed as [13], which is derived [14] as

\[
Q_{sh} = B \omega (r_a^2 - r_j^2 \varphi^2).
\]

In equation (6), \( B \) is the gear face width, \( \omega \) is the angular velocity of the gear pump; \( r_a \) is the addendum circle radius; \( r_j \) is the gear reference radius; \( r_d \) is the gear base circle radius, and \( \varphi \) is the driving gear rotation angle.

The scanning volume \( V_{sh} \) changes with the change of the driving gear rotation angle \( \varphi \) and the rate of change is [15]

\[
\frac{dV_{sh}}{d\varphi} = Q_{sh} = B (r_a^2 - r_j^2 \varphi^2).
\]

Figure 3 shows the relationship between the meshing point and the angle of rotation. The rotation angles corresponding to \( F, G, G', \) and \( F' \) are \( \varphi_F, \varphi_G, \varphi_{G'} \), and \( \varphi_{F'} \), respectively. The length between \( G \) and \( G' \) is \( h_f \). The relationship between the rotation angles is \( \varphi_F = (\pi/z) - \varphi_F, \varphi_G = (1 + (2h_f/t_j))/\pi z - \varphi_F, \) \( \varphi_{G'} = (2\pi/z) - \varphi_{F'} \). The parameter of \( t_j \) is the base pitch.

The corresponding scan volume change rate is described as follows:

\[
\frac{dV_F}{d\varphi_F} = B (r_a^2 - r_j^2 \varphi_F^2),
\]

\[
\frac{dV_G}{d\varphi_G} = B (r_a^2 - r_j^2 \varphi_G^2),
\]

\[
\frac{dV_{G'}}{d\varphi_{G'}} = B (r_a^2 - r_j^2 \varphi_{G'}^2),
\]

\[
\frac{dV_{F'}}{d\varphi_{F'}} = B (r_a^2 - r_j^2 \varphi_{F'}^2).
\]

The change of trapped oil volume \( V, V_1, \) and \( V_2 \) with driving gear rotation angle \( \varphi \) is obtained by equations (8)–(11):

\[
\frac{dV}{d\varphi} = \frac{4\pi}{z} B r_a^2 \left( \varphi - \frac{\pi}{z} \right);
\]

\[
\frac{dV_1}{d\varphi} = \frac{2\pi}{z} \left( 1 - 2h_f/t_j \right) B \left( \varphi - \frac{\pi}{2z} \left( 1 - 2h_f/t_j \right) \right),
\]

\[
\frac{dV_2}{d\varphi} = \frac{2\pi}{z} B r_a^2 \left( \varphi - \frac{3\pi}{2z} \right).
\]

Integrating equations (12)–(14),
From equations (15)–(17), $V_0$, $V_1$, and $V_2$ are the quadratic functions of the rotation angle $\varphi$. When $\varphi = (\pi/z)$, $(\pi/2z)(1 - (2h_j/t_j))$, $(3\pi/z)$, the minimum values $V_0$, $V_{10}$, and $V_{20}$ are obtained, respectively.

2.3. Graphical Method to Solve the Minimum Trapped Oil Volumes $V_0$, $V_{10}$, $V_{20}$. According to the definition of the trapped oil [5] and Figure 3, let $V_{o_1F'}$, $V_{o_2F'}$, $V_{o_3G'}$, and $V_{o_4H'}$.
be the center of circle $O_1, O_2$ sector area, respectively. Let $V_{\Delta o_1o_2F}$ and $V_{\Delta o_1o_2F'}$ be the triangle area enclosed by $O_1, O_2, F, F'$, respectively. Let $\alpha'$ be the meshing angle of the gear pair. The trapped oil volume can be described as

$$V = V_{o_2G'F_3} - V_{o_1G'F_3} - (V_{\Delta o_1o_2F} + V_{\Delta o_1o_2F'}) = V_{o_2G'F_3} - V_{o_1G'F_3} - 0.5et_j \cos \alpha',$$

$$V_1 = V_{\Delta o_1G'F_3} - V_{o_1G'F_3} + (V_{\Delta o_2GF_3} - V_{o_2GF_3}),$$

$$V_2 = V - V_1.$$  \hspace{1cm} (18)

In the equation, $e$ and $\alpha'$ are the distance between the center of the gear and the meshing angle of the gear pair.

In Figures 4 and 5, as the driving gear angle $\varphi$ changes, $V_{o_2G'F_3}$ and $V_{o_1G'F_3}$ are composed of two parts: the unchanged area $V_A$ and the changing area $V_B$. Then, the trapped oil volume $V$ can also be expressed as

$$V = V_A + V_B - 0.5et_j \cos \alpha'.$$  \hspace{1cm} (19)

In the equation, $V_A$ consists of four sectors of area. The radius of the sector are the addendum circle $r_{a_1}, r_{a_2}$ and the tooth root circle $r_{f_1}, r_{f_2}$. The corresponding included angles of the sector are $\beta_{11}, \beta_{21}$ enclosed:

$$V_A = 0.5r_{a_1}^2 \beta_{21} + 0.5r_{f_2}^2 \beta_{22} - 0.5r_{a_2}^2 \beta_{12} - 0.5r_{f_1}^2 \beta_{11}.$$  \hspace{1cm} (20)

The derivation of area $V_B$ is shown in Figure 6. The volume $V_{B\theta}$ occupied by the spread angle $\theta$ can be obtained by the following equation [16]:

$$V_{B\theta} = 0.5B \int r_k^2 \mathrm{d}\theta + C.$$  \hspace{1cm} (21)

In the equation, the pressure angle corresponding to the $r_k$ radius on the tooth profile is $\theta$.

Solve equation (21) definite integral:
According to the principle of involute [17],
\[ r = \theta + \alpha = \tan \alpha. \] (23)

Therefore, when \( r = r_a \), equation (22) can be written as
\[ V_\theta = \frac{Br^3j}{6} \tan^3 r_a. \] (24)

It can be derived from this that
\[ V_\beta = \frac{Br^3j}{6} \left( \tan^3 \theta_{11} + \tan^3 \theta_{22} + \tan^3 \theta_2 - \tan^3 \theta_{11} - \tan^3 \theta_{12} - \tan^3 \theta_1 \right). \] (25)

\( \theta_1, \theta_{11}, \theta_{12}, \theta_2, \theta_{21}, \) and \( \theta_{22} \) are the spread angles corresponding to the meshing points.

2.4. Unloading Area. As shown in Figure 9, points \( P, F \) are node and meshing points, respectively. Let the length of \( P, F \) be \( f \), and the relationship between \( f \) and the gear rotation angle \( \phi \) is [18]
\[ f(\phi) = 0.5t_j - r_j \left( \phi - \frac{\pi}{z} \right). \] (26)
The unloading area is the area where the cross section of the trapped oil volume is located in the unloading groove [19]. In Figure 9, the calculation steps of $S_{u1v1}$ and $S_{u2v2}$ are as follows: (1) Calculate the expression of $f(\varphi)$ at points $F_1, F_2, F_3, F_4, F_5, F_6$ on the gear tooth profile. (2) Find the curve equations of $12, 23, 34, 4F_5, F_5, 56$. (3) Solve the integral to calculate the unloading area, the method, and steps referring to [20].

In the calculation process of $S_{u2v2}$, $f(\varphi)$ can be replaced with $[t_j - f(\varphi)]$, and the other steps can be calculated according to $S_{u1v1}$.

$V_a, V_b$ are the actual trapped oil volume:

\[ V_a = V_1 - S_{u1v1}, \]
\[ V_b = V_2 - S_{u2v2}. \]  

### 3. Oil Trapped Pressure Model of Internal Gear Pump

Figure 10 shows the trapped oil pressure model of the internal gear pump. Let the pressures of $V_1, V_2$, inlet chamber, and outlet chamber be $P_1, P_2, P_{in},$ and $P_{out}$, respectively. Let $q_1, q_2,$ and $q_h$ be the unloading flow rate from the trapped oil cavity $V_1$ to the oil outlet cavity, the unloading flow rate from the trapped oil cavity $V_2$ to the oil inlet cavity, and the unloading flow rate from the trapped oil cavity $V_2$ to trapped oil cavity $V_1$, respectively. According to fluid mechanics and dynamics, the following is derived:

\[
q_1 = CV_a \frac{e}{\rho} \sqrt{|P_1 - P_{out}|} \text{sign}(P_1 - P_{out}),
\]
\[
q_2 = CV_b \frac{e}{\rho} \sqrt{|P_2 - P_{in}|} \text{sign}(P_2 - P_{in}),
\]
\[
q_h = CV_h \frac{e}{\rho} \sqrt{|P_1 - P_2|} \text{sign}(P_1 - P_2),
\]

\[
q_1 + q_h = \frac{dV_a}{dt} - \frac{V_a}{\beta} \frac{dP_1}{dt},
\]
\[
q_2 - q_h = \frac{dV_b}{dt} - \frac{V_b}{\beta} \frac{dP_2}{dt},
\]
\[
\frac{dP_1}{dt} = \frac{\beta}{V_a} \left( (q_1 + q_h) + \frac{dV_a}{dt} \right),
\]
\[
\frac{dP_2}{dt} = \frac{\beta}{V_b} \left( (q_2 - q_h) + \frac{dV_b}{dt} \right).
\]

In above equations, $\rho, \beta$ are the density and bulk elastic modulus of the fluid, respectively. $C$ is the flow coefficient $0.60-0.65$. $V_h = h_j \times B$.

According to $d\varphi = \omega dt$, we have:

\[
\omega \frac{dt}{dt} = \frac{q_1 + q_h}{\rho V_a} \frac{dV_a}{dt} + \frac{q_2 - q_h}{\rho V_b} \frac{dV_b}{dt},
\]

\[
\frac{dP_1}{dt} = \frac{\beta}{V_a} \left( (q_1 + q_h) + \frac{dV_a}{dt} \right),
\]
\[
\frac{dP_2}{dt} = \frac{\beta}{V_b} \left( (q_2 - q_h) + \frac{dV_b}{dt} \right).
\]
Let, $k_1 = (\beta/\omega) C \sqrt{2\rho}$, $k_2 = (2\pi/\alpha) Br_j^2$, $k_3 = (2\pi/2z)(1 - (2h_j/t_j))$, $k_4 = (\pi/2z)(1 - (2h_j/t_j))$, $k_5 = (3\pi/2z)$, $k_6 = \beta k_2$, $k_7 = \beta k_3$, $k_8 = V_h k_1$. Then,

$$\begin{align*}
\frac{dP_1}{d\phi} &= -\frac{\beta}{\omega V_a} \left( q_1 + q_h \right) + \omega \frac{dV_a}{d\phi}, \\
\frac{dP_2}{d\phi} &= -\frac{\beta}{\omega V_b} \left( q_2 - q_h \right) + \omega \frac{dV_b}{d\phi}.
\end{align*}$$

Let $k_i$ be defined as above. Then,

$$\begin{align*}
\frac{dP_1}{d\phi} &= -\frac{k_1 \sqrt{P_1 - P_{out}}}{V_1 + 0.5k_3 (\varphi - k_4)} - \frac{k_8 \sqrt{P_1 - P_2}}{V_1 + 0.5k_3 (\varphi - k_4)^2} - \frac{k_7 (\varphi - k_4)}{V_1 + 0.5k_3 (\varphi - k_4)^2}, \\
\frac{dP_2}{d\phi} &= -\frac{k_1 \sqrt{P_2 - P_{in}}}{V_2 + 0.5k_2 (\varphi - k_5)} + \frac{k_8 \sqrt{P_1 - P_2}}{V_2 + 0.5k_2 (\varphi - k_5)^2} - \frac{k_6 (\varphi - k_5)}{V_2 + 0.5k_2 (\varphi - k_5)^2}.
\end{align*}$$
4. Simulation Research

4.1. Gear Parameters and Simulation. Input parameters, modulus \( m = 3 \), pressure angle \( \alpha = 20^\circ \), tooth number \( z_1/z_2 = 13/19 \), addendum height coefficient \( h_{a1}/h_{a2} = 1/1 \), radial clearance coefficient \( c_{n1}/c_{n2} = 0.25/0.25 \), total tooth height \( h_{f1}/h_{f2} = 6.734/6.776 \), tool fillet \( r_0 = 0.25 \), the profile shift coefficient \( x_1/x_2 = 0.432/0.504 \), and base pitch \( t_1 = 8.856 \). According to MATLAB simulation by equations (4) and (5), a pair of gears processed by the generative method is shown in Figure 11.

4.2. Simulation Results. When the inlet pressure \( p_i = 0 \text{ MPa} \) and the outlet pressure \( p_o = 10 \text{ MPa} \), the rotation process of the gear pair is simulated. During this process, the change trend of the trapped oil volumes \( V_1 \), \( V_2 \), \( V \) is shown in Figure 12. It can be seen that the change of \( V_1 \), \( V_2 \) is parabolic. When the rotation angle is 0.12 rad, the minimum value is 2.34 mm\(^3\), and when the rotation angle is 0.36 rad, the minimum value is 2.12 mm\(^3\). \( V \) is the sum of \( V_1 \) and \( V_2 \), and its changing trend is to first reach a low point and then present an increasing trend. Internal gear pumps and external gear pumps have similar changes [21, 22]. During a period of gear tooth meshing, simulate the pressure changes in \( V_1 \) and \( V_2 \). There is no backlash between the front gear and the rear gear. When \( h_j = 0 \), the changes of \( p_1 \) and \( p_2 \) are shown in Figure 13. \( p_1 \) and \( p_2 \) have a large variation range, and there are positive pressure and negative pressure in the interval. When the gear rotation angle reaches 0.12 rad, \( p_1 \) reaches the peak value of 35.5 MPa. When the corner reaches 0.36 rad, \( p_2 \) reaches the peak value of 40.2 MPa. If there is a backlash between the front gear and the rear gear, when \( h_j = 0.06 \text{ m} \), the changes of \( p_1 \) and \( p_2 \) are shown in Figure 14. It can be seen that the pressure change trend in the trapped oil zone is similar to that in Figure 13, but the change range is convergent. When the turning angle reaches 0.12 rad, \( p_1 \) reaches the peak value of 30.2 MPa. When the rotation angle reaches 0.36 rad, \( p_2 \) reaches the peak value of 20.24 MPa. If there is tooth side clearance \( h_j = 0.06 \text{ m} \), and a rectangular unloading groove is designed in the oil trapped area, the position of the boundary line of the unloading groove is \( B_v = B_u = 0.5 \text{ mm} \), and the pressure change trend in the oil trapped area is shown in Figure 15. The change trends of \( p_1 \) and \( p_2 \) are very similar to those in Figures 13 and 14, and the range of change is more convergent. When the rotation angle reaches 0.12 rad, \( p_1 \) reaches the peak value of 16.5 MPa, and when the rotation angle reaches 0.36 rad, \( p_2 \) reaches the peak value of 18.2 MPa. Internal gear pumps have lower trapped oil pressure than external gear pumps [23] and arc gear pumps [24]. The design of tooth side clearance and unloading groove can slow down trapped oil pressure and reduce peak pressure.
Figure 10: Trapped oil pressure model of the internal gear pump.

Figure 11: A pair of gears processed by the generative method.
The change trend of trapped oil volumes $V, V_1, V_2$

**Figure 12:** Changes in trapped oil volume.

The changes of $P_1$ and $P_1, h_j = 0$

**Figure 13:** $h_j = 0$, changes in trapped oil pressure.

The changes of $P_1$ and $P_1, h_j = 0.06 m$

**Figure 14:** $h_j = 0.06 m$, changes in trapped oil pressure.
5. Conclusions

(1) For a pair of internal gear pairs processed by the generative method, there are oil trapped areas between the gear teeth during meshing, and the trapped oil volumes $V_1$ and $V_2$ present a parabolic change law, and each has a minimum value, which changes periodically when the gear rotates.

(2) When the internal gear pump rotates, the trapped oil pressures $p_1$ and $p_2$ increase first and then decrease with the change of the rotation angle. There is a maximum peak value. When the volume of the trapped oil cavity is the smallest, the trapped oil pressure reaches the maximum.

(3) The tooth side clearance will improve the oil trapping characteristics of the internal gear pump and reduce the pressure peak in the trapped oil cavity.

(4) The design of the unloading groove will improve the fluidity of trapped oil, reduce the range of trapped oil pressure, and reduce the pressure peak in the trapped oil cavity.

Nomenclature

| Symbol | Description |
|--------|-------------|
| AB     | The outer contour of the rack cutter |
| BC     | The outer contour of the rack cutter |
| CD     | The outer contour of the rack cutter |
| DE     | The outer contour of the rack cutter |
| $h_a$  | Addendum height |
| $h_{sa}$ | Addendum height |
| $h_{sa2}$ | Addendum height |
| $c_o$ | Head clearance coefficient |
| $h_f$ | Total tooth height |
| $h_{f1}$ | Total tooth height |
| $h_{f2}$ | Total tooth height |
| $r_{o1}$ | Tool fillet |
| $\alpha$ | Pressure angle |
| $x$ | The profile shift coefficient |
| $x_i$ | The profile shift coefficient |
| $p$ | Pitch |
| $s_j(O_s,y_j)$ | A rectangular coordinate system |
| $O_z$ | Center of gear $O_z$ |
| $O_{1j}$ | Center of gear $O_{1j}$ |
| $s_j(O_s,y_j)$ | A rectangular coordinate system of gear $O_s$ |
| $s_j'(O_s,y_j')$ | A rectangular coordinate system of gear $O_s'$ |
| $x_i$ | Displacement direction |
| $s$ | Displacement |
| $O_c$ | Origin of $s_e$ |
| $\phi$ | The rotation angle |
| $r_{a}$ | The gear pitch radius |
| $m$ | The modulus of the gear |
| $z$ | The number of teeth |
| $z_{1}$ | The number of teeth |
| $z_{2}$ | The number of teeth |
| $R^p(\varphi_s, x_i)$ | The envelope of the tooth profile surface family of the rack tool |
| $M_{st}(\varphi_s)$ | The transformation matrix |
| $s_i$ | The coordinate system |
| $s_i'$ | The coordinate system |
| $R^p(\varphi_s, x_i)$ | The envelope of the rack tool tooth profile surface family |
| $Q_{sh}$ | The theoretical instantaneous flow |
| $B$ | The gear face width |
| $\omega$ | The angular velocity of the gear pump |
| $r_{a}$ | The addendum circle radius |
| $r_{c}$ | The gear reference radius |
| $r_{j}$ | The base circle radius |
| $\phi$ | The driving gear rotation angle |
| $V_{sh}$ | The scanning volume |
| $\phi_F$ | The rotation angle of $F$ |
| $\phi_G$ | The rotation angle of $G$ |
| $\phi_{G'}$ | The rotation angle of $G'$ |
| $\phi_{F'}$ | The rotation angle of $F'$ |
| $h_j$ | The length between $G$ and $G'$ |
| $t_j$ | The base pitch |
| $V$ | Trapped oil volume |
| $V_1$ | Trapped oil volume |
| $V_2$ | Trapped oil volume |
| $V_0$ | The minimum values of $V$ |
| $V_{10}$ | The minimum values of $V_{1}$ |
| $V_{20}$ | The minimum values of $V_{2}$ |
| $V_{o1F'}$ | Sector area $O_1, F', F'$ |
| $V_{o2F'}$ | Sector area $O_2, F', F'$ |
| $V_{o1G',F3}$ | Sector area $O_1, G', F_3$ |
| $V_{o1O2F'}$ | Triangle area $O_1, O_2, F', F'$ |
| $V_{Delta12F'}$ | Triangle area $O_1, O_2, F', F'$ |
| $\epsilon$ | The distance between the centers of the gear |
| $\alpha'$ | The meshing angle of the gear pair |
| $V_{sh}$ | The unchanged area |
| $V_{si}$ | The changing area |
| $r_{a1}$ | The addendum circle |
| $r_{a2}$ | The addendum circle |
| $\beta_{12}$ | Addendum angle |
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The authors declare that there are no conflicts of interest regarding the publication of this paper.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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\[ \beta_{21} \]: Addendum angle  
\[ r_{f,2} \]: The tooth root circle  
\[ r_{f,1} \]: The tooth root circle  
\[ \beta_{11} \]: Tooth root circle included angle  
\[ \beta_{21} \]: Tooth root circle included angle  
\[ \theta \]: The spread angle  
\[ r_{f} \]: The radius on the tooth profile  
\[ \theta_{1i} \]: The spread angles corresponding to the meshing points  
\[ \theta_{12} \]: The spread angles corresponding to the meshing points  
\[ \theta_{2i} \]: The spread angles corresponding to the meshing points  
\[ \theta_{21} \]: The spread angles corresponding to the meshing points  
\[ \theta_{22} \]: The spread angles corresponding to the meshing points  
\[ p \]: Node  
\[ F \]: Meshing point  
\[ f \]: The length of \( P \), \( F \)  
\[ S_{u1} \]: The unloading area  
\[ S_{u2} \]: The unloading area  
\[ f(\phi) \]: The relationship between \( f \) and the gear rotation angle \( \phi \)  
\[ P_{1} \]: The pressures of \( V_{1} \)  
\[ P_{2} \]: The pressures of \( V_{2} \)  
\[ P_{in} \]: The pressures of the inlet chamber  
\[ P_{out} \]: The pressures of the outlet chamber  
\[ q_{1} \]: The unloading flow rate from the trapped oil cavity \( V_{1} \) to the oil outlet cavity  
\[ q_{2} \]: The unloading flow rate from the trapped oil cavity \( V_{2} \) to the oil inlet cavity  
\[ q_{b} \]: The unloading flow rate from the trapped oil cavity \( V_{2} \) to trapped oil cavity \( V_{1} \)  
\[ \rho \]: The density  
\[ \beta \]: Bulk elastic modulus of the fluid  
\[ C \]: The flow coefficient  
\[ V_{h} \]: Tooth side clearance volume  
\[ c_{ni} \]: Radial clearance coefficient  
\[ c_{n2} \]: Radial clearance coefficient.

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Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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