Controlling the interferometers of zero-line modes in graphene by pseudomagnetic field

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Abstract

Networks of graphene-based topological domain walls function as nano-scale interferometers of zero-line modes, with magnetic field and (or) scalar potential as the controlling parameters. In the absence of externally applied magnetic or electrical field, strain induces pseudomagnetic field and scalar potential in graphene, which could control the interferometers more efficiently. Two types of strains are considered: (i) horizontally bending the graphene nanoribbon into circular arc induces nearly uniform pseudomagnetic field; (ii) helicoidal graphene nanoribbon exhibit nonuniform pseudomagnetic field. Both types of strain induce small scalar potential due to dilatation. The interferometers are studied by transport calculation of the tight binding model. The transmission rates through the interferometer depend on the strain parameters. An interferometer with three loops is designed, which could completely switch the transmitting current from one export to the other.

Keywords: graphene, zero-line mode, interferometer, pseudomagnetic field

(Some figures may appear in colour only in the online journal)
of the transmission rate. Two types of strain are considered: (i) bending a graphene nanoribbon horizontally into circle (as shown in figures 1(c) and (d)) induces pseudomagnetic field that is nearly uniform [20]. (ii) Twisting the graphene around the open boundary (or the axis) of the nanoribbon into helicoid (as shown in figures 2(a) and (b)) induces nonuniform pseudomagnetic field and scalar potential. The transmission rates through the interferometer were numerically calculated and discussed.

The design of triple interferometer that consists of two interferometers in parallel enable the complete switching of the transmitting current from one export to the other. Because the strain as small as 1% could generate pseudomagnetic field as large as 10 T, the complete switching of incident current requires small strain.

The article is organized as following: section 2 described the system and theoretical model. Section 3 presents and discusses the numerical results of strain dependent transmission rate. Section 4 describe the design of the triple interferometer. Section 5 is the conclusion.

2. System and model

Zigzag nanoribbons of monolayer graphene with staggered sublattice potential are studied. The Hamiltonian of the tight binding model is given as

$$H = \sum_{(i,j)} t_{ij} c_i^\dagger c_j + \sum_i [\eta_i \Delta_i + V(r_i)] c_i^\dagger c_i$$  \hspace{1cm} (1)$$

where $t_{ij}$ is the hopping parameter between the nearest neighbor sites $(i,j)$, $\Delta_i = \pm \Delta_0$ if the $i$th site belongs to A(B) sublattice, $\eta_i = \pm 1$ if the $i$th site belongs to the region with positive(negative) staggered sublattice potential, $V(r_i)$ is the scalar potential due to dilatation and external gated pads. For graphene without strain, the hopping parameter is $t_0 = -2.7$ eV. We considered the systems with $\Delta_0 = 0.1t_0$ as example. The spatial structures of the ZLM splitter and AB interferometer in the nanoribbons are shown in figure 1. At the intersections of the splitters, the crossing angle between two importing domain walls is $2\pi/3$. Connecting two ZLM splitters in series form an AB interferometer. In our numerical calculation, we used the parameters that $L_x = 23.61$ nm in (a), and $L_x = 47.22$ nm, $L_x' = 15.74$ nm in (b); $L_y = 13.63$ nm for both of the ZLM splitter and AB interferometer. The four leads that connect to the four external domain walls are plotted as yellow pads. The transmission rate $T_{ij}$ with import from the $i$th lead and export to the $j$th lead is numerically calculated by non-equilibrium Green’s function method [25–29].
In the presence of strain, the coordinate of each lattice site \( r \) is changed to \( r + u_i \), so that the bond length between nearest neighbor sites is changed. The hopping parameter is modified as [22, 24]

\[
t_{ij} = t_0 e^{-\beta \left( \frac{\Phi_{ij}}{\hbar} - 1 \right)}
\]  

(2)

where \( \beta = 3.37 \), \( a_c = 0.142 \text{nm} \) is the bond length of unstrained graphene, \( d_{ij} \) is the distance between the nearest neighbor sites \((i, j)\) of the graphene with strain. For the low energy excitation near to the \( K \) point of the Brillouin zone, the effective vector potential of the pseudomagnetic field at the \( i \)th lattice site could be estimated as

\[
A_i(r_i) - iA_i(r_i) = \frac{2\hbar}{3}\alpha a_c e \sum_{j \neq (i,j)} (t_{ij} - t_0) e^{iK(r_i - r_j)}. 
\]  

(3)

The dilatation of the graphene is given as \( \nabla \cdot u \), and the scalar potential is given as \( V_0 \nabla \cdot u \) with \( V_0 \approx 3 \text{ eV} \) [20].

For the systems with the first type of strain, the spatial structures are shown in figures 1(c) and (d). The scattering regions that contain the ZLM splitter or the AB interferometer (the regions within the range of \( L_e \) in figures 1(a) and (b)) are bent into circle; the domain walls that connect to the leads remain straight. Assuming that the axial length of the nanoribbon is not changed by the bending, the bending angle is \( \alpha = L_e/R \), with \( R \) being the bending radius. With small bending angle, the pseudomagnetic field is nearly uniform with strength \( B_0 \approx \hbar / a_c R \), where \( \Phi_0 = h/2e \) is the magnetic flux quantum with \( h \) being the Planck constant and \( e \) being the charge of an electron. The scalar potential is given as \( V(r) \approx V_0 y/R \), with \( y \) being the coordinate of the lattice site in the corresponding unstrained graphene. If the bending angle is large, the pseudomagnetic field becomes nonuniform, and the scalar potential has more complicated form. The exact formula of the pseudomagnetic field and scalar potential can be found in [20].

For the systems with the second type of strain, the spatial structures are shown in figures 2(a) and (b). The helicoidal graphene nanoribbon is characterized by the twisting angle \( \alpha \). Only the scattering regions (the regions within the range of \( L_e \) in figures 1(a) and (b)) are twisted, and the remaining regions are unstrained. The pseudomagnetic field could be numerically calculated by equation (3). The dilatation is calculated by measuring the expansion of the surface area of the helicoidal surface, which is given as

\[
V(r) = V_0 \left( \frac{1}{1 + \frac{y^2\alpha^2}{4\pi^2L_e^2}} - 1 \right). 
\]  

4. Numerical results

The interfere pattern of the AB interferometer with pseudomagnetic field is different from that with real magnetic field. For the AB interferometer with real magnetic field, the partition ratio of each ZLM splitter, \( \alpha \), is independent of the magnetic field; the magnetic field is assumed to be uniform, so the AB phases of the two arms, \( \phi \), are the same; the scalar potential is absent, so the dynamical phases of the two arms, \( k_j l_0 \), are the same. Adopting the transfer matrix method, the transmission rates are given as [19]

\[
T_{1,2} = \frac{(1 - \alpha)^2}{(1 - \alpha)^2 + 4\alpha \cos^2 \phi + k_j l_0}, 
\]  

(5)

\[
T_{1,3} = 1 - T_{1,2} \quad \text{and} \quad T_{1,4} = 0, \text{where } k_j \text{ is the wave number of the ZLM along the domain wall and } l_0 \text{ is the length of each arm. By contrast, for the AB interferometer with pseudomagnetic field, } \alpha \text{ are dependent on the bending angle; } \phi \text{ of the two arms are different because the pseudomagnetic field is in general nonuniform; } k_j l_0 \text{ of the two arms are different due to the scalar potential. In subsection (A), the AB interferometers consisted of horizontally bent nanoribbons are studied in detail to compare with the AB interferometers with real magnetic field. We firstly studied the dependence of the transmission rate through the ZLM splitter on the pseudomagnetic field, and secondly studied the transmission rate through the AB interferometer. In subsection (B), the numerical result of the AB interferometers consisted of helicoidal nanoribbon is presented.}

3.1. Horizontally bent interferometer

The transmission rates through a single ZLM splitter versus the pseudomagnetic field are plotted in figures 3(a) and (b), where the Fermi energy equate to zero and 0.1 eV, respectively. Because of the conservation of the valley index, the transmission rates from the \( P_1 \) port to the other three ports satisfy the conditions \( T_{1,4} = 0 \) and \( T_{1,3} = 1 - T_{1,2} \). Thus, only \( T_{1,2} \) are plotted. The calculation results that neglect and consider the scalar potential are plotted together for comparison. The scalar potential is proportional to the bending angle, which in turn is proportional to the pseudomagnetic field. If the scalar potential is smaller than the energy difference between the Fermi energy and the conduction(valence) bulk band edge, only the dynamical phases of the ZLMs are changed. In contrast, if the scalar potential is large enough, the strong coupling between the ZLMs and the bulk states induces reflection, which interferes with the ZLM splitter and changes the transmission rate. As a result, if the pseudomagnetic field is larger than 60 T, the transmission rates are significantly impacted by the scalar potential. As the pseudomagnetic field varies within the range of 0–60 T, the transmission rates smoothly change. The change is more dramatic for the systems with larger Fermi energy. These phenomena can be explained by inspecting the mechanism of the current partition at the intersection [16]. The current partition originates from the coupling of the incoming ZLM to the two adjacent outgoing ZLMs. With a given distance from the intersection, the coupling strength is determined by the lateral separations between the adjacent ZLMs. The two outgoing ZLMs have different angle from the incoming ZLM, thus have different lateral separations, so the transmission rates to the two outgoing ZLMs are different. In the presence of strain, the lateral separations are changed, so the coupling strength is modified. Thus, the transmission rates to the two outgoing ZLM are changed. For the ZLM with
larger Fermi energy, the wave number along the ZLM, $k_{||}$, is larger. Thus, the coupling between adjacent ZLMS occur in a shorter distance, $L_c \approx 1/k_{||}$. As a result, the transmission rate is more sensitive to the change of lateral separations as the Fermi energy increases.

The transmission rates through the AB interferometer are plotted in figures 3(c) and (d), where the Fermi energy equate to zero and 0.1 eV, respectively. If the scalar potential is neglected, the interference pattern have negligible change. The oscillation frequency become larger as $\theta$ increases. For the system in figure 2(b), the pseudomagnetic field is odd function of $y$, so that the magnetic flux through the loop is zero; the scalar potential at the two arms are the same. Thus, no interference pattern would be exhibited.

3.2. Helicoidal interferometer

For the AB interferometers consisted of helicoidal nanoribbon in figure 2(a), the numerical results of the transmission rate are plotted in figures 3(e) and (f). If the scalar potential is neglected, the interference pattern have negligible change. The oscillation frequency become larger as $\theta$ increases. For the system in figure 2(b), the pseudomagnetic field is odd function of $y$, so that the magnetic flux through the loop is zero; the scalar potential at the two arms are the same. Thus, no interference pattern would be exhibited.

4. Triple interferometer as current switch

Although the AB interferometer induces interfere pattern of the transmission rates, the nano-structure does not completely switch the imported current from one export to the other. More precisely, $T_{1,3}$ could equate to zero as the dynamical phase satisfy $(\phi + k_{||}l_0)/2 = \pi (1/2 + N)$ with $N$ being integer, but $T_{1,2}$ could never equate to zero as shown in equation (5). We designed the triple interferometer in figure 4(a), which consists of two AB interferometers in parallel. The structure parameters are shown in figure 4(a), with $L_s = 7.87$ nm and $L_c = 13.63$ nm as example.

In the presence of uniform real magnetic field, the transmission rates to each export can be deduced by applying the scattering matrix method [19]. In the network of ZLMSs in figure 4(a), the length of each domain wall are marked as $L_s$. Path integral of the vector potential along each domain wall is marked as $\phi_i$. It is convenient to denote the round-trip phase of the upper and lower loops as $\Phi_1 = k_{||}t_1 + k_{||}l_2 + \phi_1 - \phi_2$, and the round-trip phase of the middle loop as $\Phi_2 = 2k_{||}l_2 + 2k_{||}l_3 - 2\phi_2 - \Phi_1$. The transmission amplitudes to the exports are given as

$$t_{1,2} = -\frac{2(\alpha - 1)(\cos(\Phi_1) + \alpha \cos(\Phi_2)) \cos(\frac{\Phi_2}{2}) e^{i\phi_1} e^{i(2k_{||}l_2 + \phi_1 + \phi_2)} + e^{2i\phi_2}}{(\alpha - 1)^2 e^{2ik_{||}l_2} + e^{2i\phi_2} (\alpha e^{i(3k_{||}l_2 + \phi_1 + \phi_2)} + e^{i\phi_2})^2}$$

$$t_{1,3} = \frac{i\alpha e^{i(k_{||}l_2 + \phi_1 + 2\phi_2)} (\alpha e^{\phi_1} + 1)^2}{(\alpha - 1)^2 e^{2ik_{||}l_2} + e^{2i\phi_2} (\alpha e^{i(3k_{||}l_2 + \phi_1 + \phi_2)} + e^{i\phi_2})^2}.$$

The transmission rates are $T_{1,2} = |t_{1,2}|^2$ and $T_{1,3} = |t_{1,3}|^2$. The transmission rates are plotted as black(solid) lines in figures 4(c) and (d). The resonant peak of $T_{1,2}$ consists of a wide peak and a sharp dip, as shown in figures 4(c) and (d). The inserts in figures 4(c) and (d) show that the sharp dip has asymmetric line shape, which implies a Fano type of resonance. The peak with $T_{1,2} = 1$ is determined by the condition $\Phi_1 = \pi (1 + 2N)$ with $N$ being integer. The sharp dip with $T_{1,2} = 0$ is determined by the condition $\cos(\Phi_2) + \alpha \cos(\Phi_1) = 0$. The sharp dip is due to the interference between the two tunneling
processes, i.e. the near-resonant tunneling through the upper and lower loops with round-trip phases being $\Phi_1$ and the off-resonant tunneling through the middle loop with round-trip phase being $\Phi_2$. The transmission valley in figures 4(c) and (d) with $T_{1,2} = 0$ is determined by the condition $\Phi_2 = \pi (1 + 2N)$. In summary, the triple interferometer can completely switch the current from one export to the other by changing the real magnetic field.

If the nanoribbon is horizontally bent, as shown in figure 4(b), the pseudomagnetic field control the transmission rates, as shown by the numerical result in figures 4(c) and (d). The transmission rates are different from equations (6) and (7) due to the same reasons as those for the AB interferometers. With energy $\varepsilon = 0$, the resonant peak with $T_{1,2} = 1$ is slightly different from the peak given by equation (6), as shown in figure 4(c). The sharp dip due to Fano resonance vanishes. With energy $\varepsilon \neq 0$, the resonant peak with $T_{1,2} = 1$ is significantly different from the peak given by equation (6), as shown in figure 4(d), because the dynamical phase is more relevant. The transmission valleys with $T_{1,2} = 0$ also appear within the range of the pseudomagnetic field in figures 4(c) and (d). Thus, the triple interferometer can completely switch the transmitting current from one export to the other. The largest pseudomagnetic field being considered in figure 4 is 6 T, which corresponds to bending angle as small as 4.80° and maximum stretch of bond length as small as 2%. As a result, the bending of the nanoribbon is not as exaggerated as being shown in figure 4(b).

If the nanoribbon is twisted into helicoid, the transmission rate has interference patterns that are significantly different from equations (6) and (7), because the pseudomagnetic field is nonuniform. If the twisting is around the open boundary of the nanoribbon (similar to that in figure 2(a)), the transmission rate is plotted in figures 5(a) and (b). Because the
pseudomagnetic field along all of the four arms are different, the interference patterns include many resonant peaks. As energy increase, more resonant peaks appear. If the twisting is around the axis of the nanoribbon (similar to that in figure 2(b)), the transmission rates are shown in figures 5(c) and (d). Because the pseudomagnetic field at the upper two arms has the same magnitude as that at the lower two arms, the interference patterns have much fewer resonant peaks.

5. Conclusion

In conclusion, quantum interferometers based on network of domain walls and the presence of pseudomagnetic field are theoretically studied. The strain in the horizontally bent graphene nanoribbon or helicoidal graphene nanoribbon induces pseudomagnetic field and scalar potential. In addition to inducing the AB phases, the strain changes the dynamical interaction ratio of each interaction. As a result, comparing to the interferometer with real uniform magnetic field, the interferometer with pseudomagnetic field has more complicated interference pattern. The benefit of designing the interferometer with pseudomagnetic field is that large pseudomagnetic field can be obtained by small strain. Triple interferometers are designed to completely switch the transmission of current between two exports. These designs could be implemented experimentally and be developed into practical highly integrated nano-electronic devices.

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References

[1] Semenoff G W, Semenoff V and Zhou F 2008 Phys. Rev. Lett. 101 087204
[2] Ivar Martin, Blanter Ya M and Morpurgo A F 2008 Phys. Rev. Lett. 100 036804
[3] Zarenia M, Pereira J M Jr, Farias G A and Peeters F M 2011 Phys. Rev. B 84 125419
[4] Klinovaja J, Ferreira G J and Loss D 2012 Phys. Rev. B 86 235416
[5] Zarenia M, Leenaerts O, Partoens B and Peeters F M 2012 Phys. Rev. B 86 085451
[6] Vaezi A, Liang Y, Ngai D H, Yang L and Kim E-A 2013 Phys. Rev. X 3 021018
[7] Zhang F, MacDonald A H and Mele E J 2013 Proc. Natl Acad. Sci. USA 110 10546–51
[8] Bi X, Jung J and Qiao Z 2015 Phys. Rev. B 92 235421
[9] Lee C, Kim G, Jung J and Min H 2016 Phys. Rev. B 94 125438
[10] Ren Y, Zeng J, Wang K, Xu F and Qiao Z 2017 Phys. Rev. B 96 155445
[11] Hou T, Cheng G, Tse W-K, Zeng C and Qiao Z 2018 Phys. Rev. B 98 245417
[12] Giovannetti G, Khomyakov P A, Brocks G, Kelly P J and van den Brink J 2007 Phys. Rev. B 76 073103
[13] Dean C R et al 2010 Nat. Nanotechnol. 5 722–6
[14] Zhou S Y, Gweon G H, Fedorov A V, First P N, de Heer W A, Lee D H, Guinea F, Castro Neto A H and Lanzara A 2007 Nat. Mater. 6 770–5
[15] Qiao Z, Jung J, Niu Q and MacDonald A H 2011 Nano Lett. 11 3453–9
[16] Qiao Z, Jung J, Lin C, Ren Y, MacDonald A H and Niu Q 2014 Phys. Rev. Lett. 112 206601
[17] Anglin J R and Schulz A 2017 Phys. Rev. B 95 045430
[18] Wang K, Ren Y, Deng X, Yang S A, Jung J and Qiao Z 2017 Phys. Rev. B 95 245420
[19] Cheng S-G, Liu H, Jiang H, Sun Q-F and Xie X C 2018 Phys. Rev. Lett. 121 156801
[20] Guinea F, Geim A K, Katsnelson M I and Novoselov K S 2010 Phys. Rev. B 81 035408
[21] Wang Z, Fu Z-G, Zheng F and Zhang P 2013 Phys. Rev. B 87 125418
[22] Qi Z, Kitt A L, Park H S, Pereira V M, Campbell D K and Castro Neto A H 2014 Phys. Rev. B 90 125419
[23] Zhu S, Stroscio J A and Li T 2015 Phys. Rev. Lett. 115 245501
[24] Settnes M, Power S R, Brandbyge M and Jauho A-P 2016 Phys. Rev. Lett. 117 276801
[25] Lopez Sancho M P, Lopez Sancho J M and Rubio J 1984 J. Phys. F: Met. Phys. 14 1205–15
[26] Nardelli M B 1999 Phys. Rev. B 60 7828
[27] Xu F, Li B, Pan H and Zhu J-L 2007 Phys. Rev. B 75 085431
[28] Diniz G S, Latg A and Ulloa S E 2012 Phys. Rev. Lett. 108 126601
[29] Lewenkopf C H and Mucciolo E R 2013 J. Comput. Electron. 12 203C231
[30] Yokoyama T 2008 Phys. Rev. B 77 073413
[31] Haugen H, Huertas-Hernando D and Brataas A 2008 Phys. Rev. B 77 115406
[32] Inoue T, Bauer G E W and Nomura K 2016 Phys. Rev. B 94 205428