On fuzzy priority weights of AHP for double inner dependence structure

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Abstract

The Analytic Hierarchy Process (AHP) is very popular method in decision making process, and its inner dependence extension is used for cases in which criteria or alternatives are not independent enough. Calculations and compositions of weights are very important steps in it, however using the AHP (inner dependence AHP) may cause results losing reliability because the comparison matrix is not necessarily sufficiently consistent. In such cases, fuzzy representation for weighting criteria or alternatives using results from sensitivity analysis is useful. In the previous papers, we defined local weights of criteria and alternatives for inner dependence AHP via fuzzy sets. In this paper we deal with overall weights of alternatives for double inner dependence structure AHP (among criteria and alternatives respectively). We propose fuzzy weights and two kinds of compositions. The compositions of weights depends if there are inconsistency in one level or not. Their results show the fuzziness of double inner dependence structure AHP in different way.

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1. Introduction

The Analytic Hierarchy Process (AHP) proposed by T.L. Saaty in 1977 [1][2] is widely used in decision making for selecting alternatives. It is useful for the system containing humans, because it can reflects humans feelings naturally. A normal AHP assumes independence among both criteria and alternatives, although it is difficult to choose enough independent elements. Inner dependence method AHP [3] is used to solve this problem even for criteria or alternatives having dependence.

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A comparison matrix may not, however, have enough consistency when AHP or inner dependence is used because, for instance, a problem may contain too many criteria or alternatives for decision making, meaning that answers from decision-makers, i.e., comparison matrix components, do not have enough reliability, they are too ambiguous or too fuzzy [4]. To avoid this problem, we usually have to revise again or abandon the data, but it takes a lot of time and costs [2][3]. Then, we consider that weights should also have ambiguity or fuzziness. Therefore, it is necessary to represent these weights using fuzzy sets.

Our research first applied sensitivity analysis [5] to inner dependence AHP to analyze how much the components of a pairwise comparison matrix influence the weights or consistency of a matrix [6]. This may enable us to show the magnitude of fuzziness in weights. We previously proposed new representation for criteria and alternatives weights in AHP, also representation for criteria weights for inner dependence, as L-R fuzzy numbers [7]. In the next step, we started to deal with double inner dependence structure [8]. In this paper, we consider compositions of weights to obtain overall alternative weights for double inner dependence structure AHP, using results from sensitivity analysis and fuzzy operations. We then consider fuzziness as a result of double inner dependence AHP when a comparison matrix among alternatives does not have enough consistency. The composition rules are different if consistency of a comparison matrix of criteria is good enough.

In section 2, we introduce AHP and its inner dependence method. The sensitivity analyses for AHP are described in section 3. Then the fuzzy weight representation is defined in section 4, we show examples in section 5, and section 6 is conclusions.

2. Inner dependence AHP

In this section, we introduce the process of normal AHP, inner dependence structure and consistency of pairwise comparison matrix proposed by Saaty [1][2].

2.1. Process of normal AHP

(Process 1) Representation of structure by a hierarchy. The problem under consideration can be represented in a hierarchical structure. The highest level of the hierarchy consists of a unique element that is the overall objective. At the lower levels, there are multiple criterion (i.e. elements within a single level) with relationships among elements of the adjacent higher level to be considered. The criterion are evaluated using subjective judgments of a decision maker. Elements that lie at the upper level are called parent elements while those that lie at lower level are called child elements. Alternative elements are put at the lowest level of the hierarchy

(Process 2) Paired comparison between elements at each level. A pairwise comparison matrix $A$ is created from a decision maker's answers. Let $n$ be the number of elements at a certain level. The upper triangular components of the comparison matrix $a_{ij}$ ($i < j = 1, \ldots, n$) are 9, 8, 7, 6, 5, 4, 3, 2, 1, 1/2, ..., or 1/9. These denote intensities of importance from activity $i$ to $j$. The lower triangular components $a_{ji}$ are described with reciprocal numbers as follows

$$a_{ij} = 1/a_{ji}$$

(1)

In addition, for diagonal elements, let $a_{ii} = 1$. The lower triangular components and diagonal elements are occasionally omitted from the written equation as they are evident if upper triangular components are shown. The decision maker should make $n(n-1)/2$ paired comparisons at a level with $n$ elements.

(Process 3) Calculations of weight at each level. The weights of the elements, which represent grade of importance among each element, are calculated from the pairwise comparison matrix. The eigenvector that corresponds to a positive eigenvalue of the matrix is used in calculations throughout in this paper.
**Process 4** Priority of an alternative by a composition of weights. The composite weight can be calculated from the weights of one level lower. With repetition, the weights of the alternative, which are the priorities of the alternatives with respect to the overall objective, are finally found.

2.2. Inner dependence structure and extension method

The normal AHP ordinarily assumes independence among criteria and alternatives, although it is difficult to choose enough independent elements. Inner dependence AHP [3] is used to solve this type of problem even for criteria or alternatives having dependence.

In the method, using a dependency matrix \( F = \{ f_{ij} \} \), we can calculate real weights \( w^{(n)} \) as follows,

\[
w^{(n)} = F w
\]

where \( w \) is weights from independent criteria or alternatives, i.e. normal weights of normal AHP, \( F \) consist of eigenvectors of influence matrices showing dependency among criteria or alternatives.

If there is dependence both lower levels, i.e., not only among criteria but also among alternatives, we call such kind of structure "double inner dependence". In the double inner dependence structure, we have to calculate modified weights of criteria and alternatives, \( w^{(n)} = (w_i^{(n)}) \) and \( u_i^{(n)} = (u_{ik}^{(n)}) \). Then we composite these two modified weights to obtain overall weights of alternative \( k \), \( v_k^{(n)} \) as follow:

\[
v_k^{(n)} = \sum_{i=1}^{m} w_i^{(n)} u_{ik}^{(n)}
\]

where \( m \) is number of criteria.

2.3. Consistency of pairwise comparison matrix

Since components of the comparison matrix are obtained by comparisons between two elements, coherent consistency is not guaranteed. In AHP, the consistency of the comparison matrix \( A \) is measured by the following consistency index (C.I.)

\[
C.I. = \frac{\lambda_A - n}{n-1},
\]

where \( n \) is the order of matrix \( A \), and \( \lambda_A \) is its maximum eigenvalue.

It should be noted that \( C.I. \geq 0 \) holds. And if the value of C.I. becomes smaller, then the degree of consistency becomes higher, and vice versa. The comparison matrix is consistent if \( C.I. < 0.1 \) holds. Also consistency ratio (C.R.) is defined as \( C.R. = C.I./M \) using random consistency value \( M \). However, we only employ C.I., since we mainly use 4 or 5-dimensional data whose random consistency value is not far from 1.

3. Sensitivity analysis of AHP

When AHP is used, the comparison matrix is often inconsistent or large differences among the overall weights of the alternatives do not appear. Thus, it is very important to investigate how the components of a
pairwise comparison matrix influence the consistency or weights. Sensitivity analysis is used to analyze how results are influenced when certain variables change. Therefore, it is necessary to establish a sensitivity analysis of AHP.

In our research, a previously proposed method [7] is used to evaluate the fluctuation of the consistency index and weights when a comparison matrix is perturbed. This method is useful as it does not change the structure of the data.

Evaluating the consistency index and the weights of a perturbed comparison matrix are performed as follows.

1. Perturbations \( \varepsilon a_{ij}d_{ij} \) are imparted to component \( a_{ij} \) of a comparison matrix, and the fluctuation of the consistency index and the weight are expressed by the power series of \( \varepsilon \).
2. Fluctuations of the consistency index and the weights are represented by the linear combination of \( d_{ij} \).
3. By the coefficient of \( d_{ij} \), it can be shown that how the component of the comparison matrix gives influence on the consistency index and the weight.

Since the pairwise comparison matrix \( A \) is a positive square matrix, the following Perron- Frobenius theorem [4] holds.

**Theorem 1 (Perron – Frobenius)** For a positive square matrix \( A \), the following holds true.

1. Matrix \( A \) has a positive eigenvalue. If \( \lambda_A \) is the largest eigenvalue then \( \lambda_A \) is a simple root. The positive eigenvector \( w \), corresponding to \( \lambda_A \), exists. \( \lambda_A \) is called the Frobenius root of \( A \).
2. Any positive eigenvectors of \( A \) are the constant multiples of \( w \).
3. The absolute value of the eigenvalues of \( A \), except for \( \lambda_A \), is smaller than \( \lambda_A \).
4. The Frobenius root of the transposed matrix \( A' \) is equivalent to the Frobenius root of \( A \).

This theorem ensures the existence of a weight vector in a pairwise comparison matrix.

From Theorem 1, the following theorem regarding a perturbed comparison matrix holds true [7].

**Theorem 2** Let \( A = (a_{ij}) \), \( i,j = 1,\ldots,n \) be a comparison matrix and let \( A(\varepsilon) = A + \varepsilon D \), \( D = (a_{ij}d_{ij}) \) be a matrix that has been perturbed. Moreover, let \( \lambda_A \) be the Frobenius root of \( A \) with \( w_1 \) being the corresponding eigenvector. Let \( w_2 \) be the eigenvector corresponding to the Frobenius root of transposed matrix \( A' \), then, the Frobenius root \( \lambda(\varepsilon) \) of \( A(\varepsilon) \) and the corresponding eigenvector \( w_1(\varepsilon) \) can be expressed as follows

\[
\lambda(\varepsilon) = \lambda_A + \varepsilon \lambda_A^{(1)} + o(\varepsilon),
\]

\[
w_1(\varepsilon) = w_1 + \varepsilon w_1^{(1)} + o(\varepsilon),
\]

where

\[
\lambda_A^{(1)} = \frac{w_2 D w_1}{w_2 w_1},
\]

\( w^{(1)} \) is an n-dimension vector that satisfies

\[
(A - \lambda_A I)w^{(1)} = -(D_A - \lambda_A^{(1)} I)w_1,
\]

where \( o(\varepsilon) \) denotes an n-dimension vector in which all components are \( o(\varepsilon) \).

Proof of this theorem can be found in Ohnishi’s paper [7].
3.1. Analysis for consistency

Regarding a fluctuation of the consistency index, the following corollary can be obtained from Theorem 2.

**Corollary 1** Using an appropriate $g_{ij}$, we can represent the consistency index $C.I.(ε)$ of the perturbed comparison matrix as follows

$$C.I.(ε) = C.I. + ε \sum_{i}^{n} \sum_{j}^{n} g_{ij} d_{ij} + o(ε).$$  \hspace{1cm} (9)

**(Proof)**

From the definition of the consistency index (3) and (5),

$$C.I.(ε) = C.I. + ε \frac{λ^{(1)}}{n-1} + o(ε).$$

Let $w_1=(w_{1i})$ and $w_2=(w_{2i})$ from (7). $λ^{(1)}$ can now be represented as

$$λ^{(1)} = \frac{1}{w_2} \sum_{i}^{n} \sum_{j}^{n} w_{2i} a_{ij} w_{1j} d_{ij},$$

therefore, the second part of the right side is expressed by a linear combination of $d_{ij}$. (Q.E.D)

$g_{ij}$ in equation (9) in Corollary 1 shows the influence of comparison matrix components on the consistency. On the other hand, since the comparison matrix $A(ε) = (a_{ij}(ε))$ is reciprocal, then

$$d_{ij} = -d_{ji}$$  \hspace{1cm} (10)

is obtained. The impact on the consistency can be easily shown by use of this property.

3.2. Analysis for weight vector

With regards to the fluctuation in weighs, the following corollary can also be obtained from Theorem 2.

**Corollary 2** Using an appropriate $h_{ij}^{(k)}$, we can represent the fluctuation $w^{(1)}=(w_k^{(1)})$ of the weight (i.e. the eigenvector corresponding to the Frobenius root) as follows

$$w_k^{(1)} = \sum_{i}^{n} \sum_{j}^{n} h_{ij}^{(k)} d_{ij}$$  \hspace{1cm} (11)

**(Proof)**

The $k$-th row component of the right side of (7) in Theorem 2 is represented as

$$\sum_{i}^{n} \sum_{j}^{n} \left\{ \frac{w_{1k}}{w_2} \frac{w_{2j} a_{ij} w_{1j}}{w_1} - δ(i,k) a_{ij} w_{1j} \right\} d_{ij},$$
and is expressed by a linear combination of $d_{ij}$. Here, $\delta(i,k)$ is Kronecker’s symbol

$$\delta(i,k) = \begin{cases} 1 & (i = k), \\ 0 & (i \neq k). \end{cases}$$

In contrast, since $\lambda_A$ is a simple root, $\text{Rank}(A-\lambda_A I) = n-1$. Accordingly, the weight vector is normalized as

$$\sum_k^n (w_k + \varepsilon w_k^{(1)}) = \sum_k^n w_k = 1,$$

then the condition is as follows.

$$\sum_k^n w_k^{(1)} = 0. \quad (12)$$

By using an elementary transformation to formula (8) in the condition above, we also can represent $w_k^{(1)}$ by linear combinations of $d_{ij}$. (Q.E.D)

As seen in equation (6) in Theorem 2, the component that has a great influence on weight $w_1(1)$ is the component which has the greatest influence on $w_k^{(1)}$. $q_{ij}^{(1)}$ in equation (11) from Corollary 2 shows how the influence by the components of a comparison matrix on the weights can be calculated.

The influence can also be shown easily by use of equation (10).

4. Fuzzy weight representation

The comparison matrix often has poor consistency (i.e. $0.1<C.I.<0.2$) because it encompasses too many criteria or alternatives. In these cases, the components of a comparison matrix are considered to have fuzziness since they result from the fuzzy judgment of humans. Therefore, weights should be treated as fuzzy numbers.

4.1. L-R fuzzy number

To represent fuzziness of weight $w_{1k}$, an L-R fuzzy number is used.

L-R fuzzy number

$$M = (m, \alpha, \beta)_{LR}$$

is defined as fuzzy sets whose membership function is as follows.

$$\mu_M(y) = \begin{cases} R \left( \frac{y - m}{\beta} \right) & (y > m), \\ L \left( \frac{m - y}{\alpha} \right) & (y \leq m). \end{cases}$$

where $L(y)$ and $R(y)$ are shape function which satisfies

1. $L(y) = L(-y)$,
(2) \( L(0) = 1 \),
(3) \( L(y) \) is a non-increasing function

4.2. Fuzzy weight of criteria or alternatives in normal AHP

A From the fluctuation of the consistency index, the multiple coefficient \( g_{ij}h_{ij}^{(k)} \) in Corollary 1 and 3 is considered as the influence on \( a_{ij} \).

Since \( g_{ij} \) is always positive, if the coefficient \( h_{ij}^{(k)} \) is positive, the real weight of criterion \( k \) is considered to be larger than \( w_{ik} \). Conversely, if \( h_{ij}^{(k)} \) is negative, the real weight of activity \( k \) is considered to be smaller.

Therefore, the sign of \( h_{ij}^{(k)} \) represents the direction of the fuzzy number spread. The absolute value \( g_{ij}|h_{ij}^{(k)}| \) represents the size of the influence. On the other hand, if C.I. becomes bigger, then the judgment becomes more fuzzy.

Consequently, multiple C.I. \( g_{ij}|h_{ij}^{(k)}| \) can be regarded as a spread of a fuzzy weight \( \tilde{W}_k \) concerned with \( a_{ij} \).

**Definition 1 (fuzzy weight)** Let \( w^{(n)}_k \) be a crisp weight of criterion \( k \) of inner dependence model, and \( g_{ij}|h_{ij}^{(k)}| \) denote the coefficients found in Corollary 1 and 3. If \( 0.1<\text{C.I.}<0.2 \), then a fuzzy weight \( \tilde{W}_k \) is defined by

\[
\tilde{W}_k = (w_k, \alpha_k, \beta_k)_{LR}
\]  

where

\[
\alpha_k = \text{C.I.} \sum_i^n \sum_j^n s(-, h_{ij}) g_{ij} |h_{ij}|,
\]

\[
\beta_k = \text{C.I.} \sum_i^n \sum_j^n s(+, h_{ij}) g_{ij} |h_{ij}|,
\]

\[
s(+, h) = \begin{cases} 1, (h \geq 0) \\ 0, (h < 0) \end{cases}, \quad s(-, h) = \begin{cases} 1, (h < 0) \\ 0, (h \geq 0) \end{cases}
\]

4.3. Fuzzy Weights for Double Inner Dependence Structure

For double inner dependence structure, we can define and calculate modified fuzzy local weights of a criteria \( \tilde{W}^{(n)} = (\tilde{W}_i^{(n)}) \), \( i = 1, \ldots, n \) and also weights of alternatives \( \tilde{U}^{(n)} = (\tilde{U}_k^{(n)}) \), \( k = 1, \ldots, m \) with only respect to criterion \( i \) using an dependence matrix \( F_C, F_A \), as follows

\[
\tilde{W}_i^{(n)} = (w_i^{(n)}, \alpha_i^{(n)}, \beta_i^{(n)})_{LR}
\]

\[
\tilde{U}_k^{(n)} = (u_k^{(n)}, \alpha_k^{(n)}, \beta_k^{(n)})_{LR}
\]

where
\[ w^{(n)} = (w_i^{(n)}) = F_c w \]  

\[ u_i^{(n)} = (u_{ik}^{(n)}) = F_a u_i \]  

\[ \alpha_i, \beta_i, \alpha_{ik}, \beta_{ik} \] are calculated by fuzzy multiple operations, equation (3) and definition 1.

Fuzzy overall weights of alternative \( k \) in double inner dependence AHP can be also calculated as follows using fuzzy multiple \( \otimes \) and fuzzy summation operations:

\[ \tilde{v}_k^{(n)} = \sum_i \tilde{w}_i^{(n)} \otimes \tilde{u}_{ik}^{(n)} \]  

Fuzzy weights \( \tilde{w}_i^{(n)} \) becomes crisp weights \( w_i^{(n)} \) if there is good consistency among criteria. Therefore

\[ v_k^{(n)} = \sum_i w_i^{(n)} \cdot u_{ik}^{(n)} \]  

In any cases we can evaluate fuzzy overall weights of alternatives with their centers and spreads.

5. Examples

In this section, we show an example of “Leisure in holiday” with 4 criteria and 4 alternatives. Criteria are \{popularity, good for rain (rain), fatigue, expense\} and alternatives are \{theme park (park), indoor theme park (indoor), cinema, zoo\}.

There are double dependency, i.e., dependencies in both 2 levels. Dependency matrices among criteria and alternatives (with only respect to criterion “expense”) are shown in Table 1 and 2 respectively. Table 3 shows normal weights (without considering dependency structure) of alternatives with respect to criteria and consistency index. There is inconsistency (bad consistency) among alternatives with only respect to criterion “rain”. Then using the results of sensitivity analyses (for consistency and weights), dependency matrix (Table 2) and equation (17), we can calculate fuzzy modified weights of alternatives shown in Table 4.

Table 1. Dependency matrix of criteria \( F_c \)

|        | popularity | rain | fatigue | expense |
|--------|------------|------|---------|---------|
| popularity | 1.000     | 0.000| 0.000   | 0.3072  |
| rain    | 0.000     | 1.000| 0.000   | 0.2056  |
| fatigue | 0.000     | 0.000| 1.000   | 0.3072  |
| expense | 0.000     | 0.000| 0.000   | 0.1799  |

Table 2. Dependency matrix of alternatives \( F_a \)

|        | park | indoor | cinema | zoo  |
|--------|------|--------|--------|------|
| park   | 0.277| 0.584  | 0.000  | 0.000|
| indoor | 0.171| 0.135  | 0.000  | 0.000|
| cinema | 0.477| 0.281  | 1.000  | 0.000|
| zoo    | 0.075| 0.000  | 0.000  | 1.000|
5.1. Inconsistency in only among alternatives

Table 5 shows a comparison matrix of criteria, weights, and consistency index, where its consistency is good (C.I. <0.1). Weights are normal weight (without considering dependency structure, weight) and modified weight (modified). Using a dependency matrix shown in Table 1, modified crisp weights of criteria are obtained. As described above, there are bad consistency and dependency among alternatives with only respect to criterion “rain”, then we can use fuzzy modified weights of alternatives shown in Table 4. Finally, using composition (21), we evaluate overall fuzzy weights of alternatives in Table 6.

Table 5. Comparison matrix of criteria and weight

| popularity | rain | fatigue | expense | weights | modified |
|------------|------|---------|---------|---------|----------|
| popularity | 1.000 | 0.500   | 0.333   | 0.333   | 0.214    |
| rain       | 5.000 | 1.000   | 5.000   | 3.000   | 0.598    |
| fatigue    | 2.000 | 0.200   | 1.000   | 0.333   | 0.115    |
| expense    | 3.000 | 0.333   | 3.000   | 1.000   | 0.249    |

C.I. = 0.0035

Table 6. Overall fuzzy weights of alternatives using crisp criteria weight.

| popularity | rain | fatigue | expense |
|------------|------|---------|---------|
| park       | 0.173| 0.0046  | 0.0036  |
| indoor     | 0.124| 0.0020  | 0.0013  |
| cinema     | 0.534| 0.0070  | 0.0101  |
| zoo        | 0.170| 0.0033  | 0.0084  |

Table 7. Comparison matrix of criteria and weights.

| popularity | rain | fatigue | expense | weights |
|------------|------|---------|---------|---------|
| popularity | 1.000| 0.500   | 0.333   | 0.214   |
| rain       | 2.000| 1.000   | 2.000   | 0.333   |
| fatigue    | 0.200| 0.500   | 1.000   | 0.333   |
| expense    | 3.000| 3.000   | 3.000   | 1.000   |

C.I. = 0.134

Table 8. Fuzzy weights of criteria.

| popularity | rain | fatigue | expense |
|------------|------|---------|---------|
| popularity | 0.358| 0.0079  | 0.0080  |
| rain       | 0.322| 0.0072  | 0.0071  |
| fatigue    | 0.236| 0.0052  | 0.0024  |
| expense    | 0.084| 0.0014  | 0.0043  |
5.2. Inconsistency in both among criteria and alternatives

Table 7 shows a comparison matrix of criteria and normal weights, where its consistency is not so good (C.I. >0.1). Then using results of sensitivity analyses of consistency and weights, we can calculate fuzzy weights. Next using a dependency matrix, modified fuzzy weights are obtained as shown in Table 8. Same as mentioned in subsection 5.1, we use fuzzy modified weights of alternatives shown in Table 4. Finally, using composition (20), we evaluate overall fuzzy weights of alternatives in Table 9.

6. Conclusions

As shown in the examples in the previous section, there are a lot of cases that data of AHP do not have enough consistency or reliability. For these cases, we propose fuzzy weight representation and two kinds of compositions. They depend on if there is inconsistency only among alternatives or in both levels (among criteria and among alternatives).

Our approach can also give how to represent weights and might be efficient to investigate how the result of AHP has fuzziness when element of hierarchy have dependency and data are not sufficiently consistent or reliable.

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