The TeV Gamma-Ray Luminosity of the Milky Way and the Contribution of H.E.S.S. Unresolved Sources to Very High Energy Diffuse Emission

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Abstract

H.E.S.S. has recently completed the first systematic survey of the Galactic plane in the TeV energy domain. We analyze the flux, latitude, and longitude distributions of γ-ray sources observed by H.E.S.S. in order to infer the properties of the Galactic TeV source population. We show that the total Milky Way luminosity in the 1–100 TeV energy range is $L_{\text{MW}} = 1.7^{+0.5}_{-0.4} \times 10^{37}$ erg s$^{-1}$. Evaluating the cumulative flux expected at Earth by the considered population, we show that H.E.S.S. unresolved sources provide a relevant contribution to the diffuse Galactic emission. Finally, in the hypothesis that the majority of bright sources detected by H.E.S.S. are powered by pulsar activity, like, e.g., pulsar wind nebulae or TeV halos, we estimate the main properties of the pulsar population.

Unified Astronomy Thesaurus concepts: High energy astrophysics (739); Gamma-ray sources (633); Pulsars (1306)

1. Introduction

The field of TeV astronomy is rapidly evolving thanks to the data obtained by recent experiments. Imaging Atmospheric Cherenkov Telescopes (IACT), like H.E.S.S. (Aharonian et al. 2006), MAGIC (Alekšić et al. 2016), and VERITAS (Weekes et al. 2002), and air shower arrays, such as Argo-YBJ (Bartoli et al. 2013), Milagro (Atkins et al. 2004), and HAWC (Abeysekara et al. 2016), provided a detailed description of Galactic γ-ray emission in the energy range of 0.1–100 TeV. Large-scale diffusion emission from different regions of the Galactic plane has been measured by H.E.S.S. (Abramowski et al. 2014), Argo-YBJ (Bartoli et al. 2015), HAWC (Zhou et al. 2018), and Milagro (Abdo et al. 2008) while catalogs of point-like and extended sources have been recently produced by H.E.S.S. (Abdalla et al. 2018a) and HAWC (Abeysekara et al. 2017a). Moreover, the Cherenkov Telescope Array (CTA; Acharya et al. 2018) will perform a systematic survey of the entire Galaxy with unprecedented sensitivity covering a huge range in photon energy from 20 GeV to 300 TeV. At energies ~100 TeV or more, the IceCube neutrino telescope has reported the existence of an astrophysical population of neutrinos (Aartsen et al. 2013, 2014). This signal is believed to be mainly due to extragalactic sources but a subdominant Galactic component, produced by cosmic-ray (CR) interactions with interstellar gas and/or Galactic TeV sources, should also exist. The search for this Galactic contribution is in progress and potentially within the reach of the IceCube experiment (Aartsen et al. 2017, 2020).

Even if the knowledge of our Galaxy in the TeV domain has greatly progressed, several problems remain unsolved. In most cases, we are not able to determine whether the observed gamma-ray signals are produced at TeV energies by leptonic or hadronic mechanisms. This limits the possibility to use the gamma–neutrino connection, implied by hadronic production, to estimate the neutrino signal from gamma-ray observed sources. In addition, we still miss a robust determination of the diffuse γ-ray flux produced at TeV energies by CR interactions with the gas contained in the Galactic disk. At these energies, the situation is substantially different from the one observed in the 1–100 GeV energy range by the Fermi-LAT experiment (Ackermann et al. 2013; Ajello et al. 2017) where the CR diffuse emission overshines the contribution of individual sources. The relatively large diffuse flux measured at TeV by Milagro (Abdo et al. 2008), H.E.S.S. (Abramowski et al. 2014), and HAWC (Zhou et al. 2018) could be explained either as the cumulative contribution of unresolved sources, see, e.g., Linden & Buckman (2018) or by considering nonconventional CR propagation models characterized by position-dependent transport properties, e.g., Pothast et al. (2018).

Although different astrophysical objects, such as supernova remnants (SNRs) and pulsar wind nebulae (PWNe), can generate TeV γ-rays, we still do not know which (if any) class of sources dominate Galactic emission. Recent observations of Geminga and PSR B0656+14 by Milagro (Abdo et al. 2009) and HAWC (Abeysekara et al. 2017b), provided evidence for a new class of objects powered by pulsar activity, the so-called TeV halos, that could potentially explain a large fraction of bright TeV sources observed in the Sky (Sudoh et al. 2019).

In this work, we perform a population study of the H.E.S.S. Galactic Plane Survey (HGPS) catalog with the goal of addressing some of the above open issues. The HGPS catalog is particularly useful for our purposes because it provides the optimal sky coverage, encompassing about ~80% of the Galactic plane within its observation region. We analyze the flux, latitude, and longitude distributions of sources detected by H.E.S.S. in order to infer the properties of the TeV source population. To avoid selection effects, we include in our analysis the brightest sources with a flux above 1 TeV larger than 10% of the CRAB flux. By performing a general analysis based on suitable assumptions for the source space and luminosity distributions, we show that the HGPS data permit us to estimate with relatively good accuracy the total Milky Way luminosity produced by TeV sources and the total Galactic flux due to both resolved and unresolved sources in the H.E.S.S. observational window (OW) covering in longitude the range $-110^\circ \leq l \leq 60^\circ$ and in latitude $|b| < 3^\circ$. This allows us to quantify the contribution of unresolved sources to the total flux, showing that the unresolved contribution is possibly the dominant component of the large-scale diffuse
signal observed at TeV by H.E.S.S. (Abramowski et al. 2014) and Milagro (Atkins et al. 2005). We then consider the regime where all bright sources observed by H.E.S.S. (which are not firmly identified as SNRs) are powered by pulsar activity, e.g., PWNe and/or TeV halos, as suggested by Abdalla et al. (2018a), and we discuss the constraints on the pulsar properties, namely the initial spin period and magnetic field, that are obtained by HGPS data. Our analysis of the TeV source population improves and complements previous discussions on the subject, such as, e.g., the one provided by Casanova & Dingus (2008), by considering different aspects and an original approach and by taking advantage of more recent observational data.

The plan of the paper is as follows. In Section 2 we discuss the HGPS catalog. In Section 3 we present our method to describe the TeV source population. In Section 4 we show our results and we discuss their robustness. In Section 5 we draw our conclusions.

2. H.E.S.S. HGPS

The HGPS catalog (Abdalla et al. 2018a) includes 78 VHE sources observed in the longitude range $-110^\circ < l \leq 60^\circ$ and for latitudes $|b| < 3^\circ$, measured with an angular resolution of $0.08^\circ$ and a sensitivity $\approx 1.5\%$ Crab flux for point-like objects. The integral flux above 1 TeV of each source is obtained from the morphology fit of flux maps, assuming a power-law spectrum with index $\beta = 2.3$. In order to be consistent with this procedure, we adopt the same assumption to describe the spectrum of Galactic sources in the TeV domain. The value $\beta = 2.3$ is compatible with the average spectral index obtained by fitting HGPS sources by using a power-law or a power-law plus exponential cutoff in the energy range 0.2 TeV $\leq E_{\gamma} \leq 100$ TeV.

In the following, we focus on the bright sources that produce a photon flux above 1 TeV larger than 10% of that produced by the Crab nebula. Above this threshold, the HGPS catalog can be considered complete (Abdalla et al. 2018a) and consists of 32 sources: 19 are unidentified, 3 are firmly associated with SNRs (Vela Junior, RCW 86, and RX J1713.7-3946), 2 are objects showing evidence of both shell and nebular emission, which we refer to as composite objects, and 8 are associated with PWNe.

The HGPS survey provides optimal sky coverage to perform Galactic population studies. Indeed, the observation window $-110^\circ \leq l \leq 60^\circ$ and $|b| < 3^\circ$ includes about 80% of potential sources located in the Galactic plane, according to PWN and SNR distributions parameterized by Lorimer et al. (2006) and Green (2015), respectively. As a comparison with H.E.S.S., we report in the following the sky coverage of other TeV gamma-ray detectors. The HAWC experiment provides the longitudinal gamma-ray profile in the angular region $0^\circ < l < 180^\circ$ and $|b| < 2^\circ$, for a photon median energy $E_{\gamma} = 7$ TeV (Zhou et al. 2018). The Argo-YBJ experiment measures the total gamma-ray emission in the longitudinal region $40^\circ < l < 100^\circ$ and latitudes $|b| < 5^\circ$ for $E_{\gamma} = 600$ GeV (Bartoli et al. 2015). At higher energy, $E_{\gamma} = 15$ TeV, the Milagro experiment reports the total gamma-ray emission for longitudes $30^\circ < l < 110^\circ$ and $136^\circ < l < 216^\circ$ and for latitudes $|b| < 10^\circ$ (Abdo et al. 2008). The sky regions probed by Milagro, Argo-YBJ, and HAWC contain a smaller fraction of the potential sources in the Galactic plane, equal to $\approx 20\%$, and $\approx 40\%$, respectively. Therefore, H.E.S.S. appears to be the best choice for our purposes.

3. Method

In order to predict the signal observed by H.E.S.S., we need to consider the spatial and intrinsic luminosity distribution of the TeV sources. We assume that this can be factorized as the product:

$$\frac{dN}{d\ell dL} = \rho(r) Y(L), \quad (1)$$

where $r$ indicates the position in the Galaxy and $L$ is the $\gamma$-ray luminosity integrated in the energy range $1–100$ TeV probed by H.E.S.S.. The function $\rho(r)$, which is conventionally normalized to one when integrated in the entire Galaxy, is assumed to be proportional to the pulsar distribution in the Galactic plane parameterized by Lorimer et al. (2006). The source density along the direction perpendicular to the Galactic plane is assumed to scale as $\exp(-|z|/H)$, where $H = 0.2$ kpc represents the thickness of the Galactic disk.

We assume that the intrinsic luminosity distribution $Y(L)$ can be parameterized as a power law:

$$Y(L) = \frac{N}{L_{\text{max}}} \left( \frac{L}{L_{\text{max}}} \right)^{-\alpha}, \quad (2)$$

that extends in the luminosity range $L_{\text{min}} \leq L \leq L_{\text{max}}$ (Strong 2007). We take $\alpha = 1.5$ as a working hypothesis, since this value can be motivated in the context of sources connected with pulsar activity, such as PWNe and/or TeV halos. Other options for the power-law index $\alpha$ (and other assumptions in the analysis) will also be considered (see Table 1) in order to test the stability of our results.

The parameter $N$ defined in Equation (2) determines the high-luminosity normalization of the function $Y(L)$; it represents the number of sources per logarithmic luminosity interval at the maximal luminosity (i.e., $dN/d\ln L = N$ for $L = L_{\text{max}}$); its physical meaning in the context of a fading source population is discussed in the next section.

The last necessary ingredient to predict the expected signal in H.E.S.S. is the relationship between the intrinsic luminosity $L$ of sources and the flux produced at Earth, which can be generally written as:

$$\Phi = \frac{L}{4\pi r^2 \langle E \rangle}, \quad (3)$$

where $r$ is the source distance and $\langle E \rangle$ is the average energy of photons emitted in the range $1–100$ TeV. In our calculations, we consider the average spectrum observed by HESS as a reference (Abdalla et al. 2018a), i.e., we assume that all sources can be described by a power law in energy with spectral index $\beta = -2.3$ that corresponds to $\langle E \rangle = 3.25$ TeV.

In our analysis, we determine the maximal luminosity $L_{\text{max}}$ and the normalization $N$ of the luminosity function by fitting H.E.S.S. observational results. This approach is original and different from previous studies on the subject (Casanova & Dingus 2008) where the value of the maximal luminosity is instead assumed a priori. The determination of $L_{\text{max}}$ and $N$ allows us to estimate the total TeV luminosity produced by the
The different cases are described in the text. The source spatial distribution above values are specified as the average value of their inverse square distance. While the quantity represents the fraction of sources of the considered population that did not have enough time to lose their initial luminosity and that are expected to be more easily detected by HESS. Note that the observational determination of $N^*$ can be converted into a bound on the fading timescale $\tau$, if the source formation rate is known.

The above description can be applied to potential TeV sources in the Galaxy, such as PWNe (Gaensler & Slane 2006) or TeV Halos (Linden & Buckman 2018), which are connected with the explosion of core-collapse SN and the formation of a pulsar. The birth rate of these objects can be assumed proportional to that of SN explosions in our Galaxy, i.e., $R_{SN} = 0.019$ yr$^{-1}$ as recently measured by Diehl et al. (2006). We thus write $R = \varepsilon R_{SN}$ assuming $\varepsilon = 1$ for simplicity, unless otherwise specified. If the TeV emission is powered by pulsar activity it is reasonable to assume that TeV luminosity is proportional to the pulsar spin-down power, i.e.: 

$$L = \lambda E,$$

(10)

where $\lambda \leq 1$ and:

$$E = E_0 \left(1 + \frac{t}{\tau_{sd}}\right)^{-2}$$

(11)

for energy loss dominated by magnetic dipole radiation (braking index $n = 3$), with:

$$E_0 = \frac{8\pi^4 B_0^2 R^6}{3c^3 P_0^n}$$

$$\tau_{sd} = \frac{3c^3 P_0^2}{4\pi^2 B_0^2 R^6},$$

(12)

where $t \leq \tau_{sd}$ indicates the time passed since source formation, $\tau_{sd}$ is the total duration of TeV emission and $L_{max}$ is the initial luminosity. Assuming that the birth rate $R$ of these sources in the Galaxy is constant in time, we can calculate the luminosity function $Y(L)$ that is given by:

$$Y(L) = \frac{R}{L_{max}} \tau \left(\frac{L}{L_{max}}\right)^{\alpha},$$

(9)

where $\alpha = 1/\gamma + 1$ and $L_{min} \equiv R(\tau_{sd})$. Under this assumption, the normalization factor $N^* = R(\tau_{sd})$ of the luminosity distribution has a precise physical meaning; it basically represents the total number of young sources in the Galaxy that did not have enough time to lose their initial luminosity and that are expected to be more easily detected by HESS.

Note that the different cases are described in the text. The $\Delta \chi^2$ is calculated with respect to our reference case (first row in the table).

### Table 1

| Ref. | $\log_{10}(L_{max}/\text{erg s}^{-1})$ | $N^*$ | $\log_{10}(\rho_{MW}/\text{cm}^{-3})$ | $\rho_{tot}$ | $\tau$ | $\Delta \chi^2$ |
|------|--------------------------------------|------|------------------------------------|--------------|------|--------------|
| SNR  | 35.69 $^{+0.21}_{-0.28}$            | 18 $^{+6}_{-7}$ | 17 $^{+14}_{-6}$ | $3.6^{+0.1}_{-0.1}$ | $3.8^{+1.0}_{-1.0}$ | 1.4 |
| $H = 0.1$ kpc | 35.69 $^{+0.22}_{-0.25}$ | 18 $^{+5}_{-15}$ | 17 $^{+16}_{-6}$ | $3.8^{+1.0}_{-1.0}$ | $3.8^{+1.0}_{-1.0}$ | 1.4 |
| $H = 0.05$ kpc | 35.65 $^{+0.27}_{-0.26}$ | 15 $^{+34.5}_{-6}$ | 20 $^{+20}_{-4}$ | $3.7^{+0.3}_{-0.1}$ | $5.0^{+0.4}_{-0.2}$ | 1.6 |
| $d = 20$ pc | 35.47 $^{+0.19}_{-0.20}$ | 28 $^{+13}_{-19}$ | 20 $^{+20}_{-4}$ | $3.7^{+0.4}_{-0.3}$ | $4.4^{+0.9}_{-1.0}$ | 2.9 |
| $d = 40$ pc | 35.72 $^{+0.03}_{-0.03}$ | 26 $^{+21}_{-34}$ | 25 $^{+24}_{-3}$ | $3.7^{+0.1}_{-0.0}$ | $3.5^{+1.1}_{-1.0}$ | 4.3 |
| $\alpha = 1.3$ | 35.69 $^{+0.57}_{-0.27}$ | 28 $^{+13}_{-19}$ | 17 $^{+16}_{-6}$ | $3.7^{+0.3}_{-0.1}$ | $4.4^{+0.9}_{-1.0}$ | 2.9 |
| $\alpha = 1.8$ | 35.83 $^{+0.24}_{-0.24}$ | 25 $^{+24}_{-3}$ | 7 $^{+4}_{-6}$ | $3.7^{+0.1}_{-0.0}$ | $5.0^{+1.8}_{-0.2}$ | 0.5 |

$N_{obs} = 32$

$37.26^{+0.12}_{-0.12}$

$4.2^{+1.3}_{-1.0}$

...
where \( P_0 \) is the initial spin period and \( B_0 \) is the inertial magnetic field (Shapiro & Teukolsky 1983) while the inertial momentum is \( I = 1.4 \times 10^{38} \text{g cm}^2 \) and the pulsar radius \( R = 12 \text{km} \) (Lattimer & Prakash 2007). This implies that the fading timescale is determined by the pulse spin-down timescale, i.e., \( \tau = \tau_{sd} \). Moreover, if the efficiency of TeV emission does not depend on time (\( \lambda \sim \text{const} \)), the exponent in Equation (8) is \( \gamma = 2 \), motivating our working hypothesis that the luminosity distribution scales as \( Y(L) \propto L^{-1.5} \). Finally, \( P_0 \) and \( B_0 \) can be determined from \( L_{\text{max}} \) and \( \tau \) by using:

\[
\frac{P_0}{1 \text{ ms}} = 94 \left( \frac{\lambda}{10^{-3}} \right)^{1/2} \left( \frac{\tau}{10^{4} \text{yr}} \right)^{-1/2} \left( \frac{L_{\text{max}}}{10^{34} \text{erg s}^{-1}} \right)^{-1/2}
\]

\[
\frac{B_0}{10^{12} \text{G}} = 5.2 \left( \frac{\lambda}{10^{-3}} \right)^{1/2} \left( \frac{\tau}{10^{4} \text{yr}} \right)^{-1} \left( \frac{L_{\text{max}}}{10^{34} \text{erg s}^{-1}} \right)^{-1/2}
\]

(13)

provided that the fraction \( \lambda \) of the spin-down power that is converted into TeV \( \gamma \)-ray emission is known.

The parameter \( \lambda \) is highly uncertain; it is determined by the conversion of the spin-down energy into \( e^\pm \) pairs (that can be very efficient, see, e.g., Sudoh et al. 2019; Manconi et al. 2020) and by the subsequent production of TeV photons. The values obtained for firmly identified PWNe in the HGPS catalog fall between \( 5 \times 10^{-5} \) and \( 6 \times 10^{-2} \), see Table 1 of Abdalla et al. (2018b). For comparison, the value \( \lambda \sim 3 \times 10^{-3} \) is obtained in Lindén & Buckman (2018) by studying the TeV \( \gamma \)-ray emission of Geminga. In this work, we consider \( \lambda \) as a free parameter, taking the value \( \lambda = 10^{-3} \) as a reference in numerical calculations.

The possibility of \( \lambda \) being correlated to the spin-down power, i.e., \( \lambda = \lambda_0 (E/E_0)^{\delta} \), is suggested by the results of Abdalla et al. (2018b) that found \( L = \lambda \dot{E} \propto E_0^{1.5 \pm 0.5} \) with \( 1+\delta = 0.59 \pm 0.21 \) by studying a sample of PWNe in the HGPS catalog. In this case, one obtains \( \gamma \simeq 1.2 \) in Equation (8) that corresponds to a source luminosity function \( Y(L) \propto L^{-1.8} \). This scenario is also discussed in our analysis and does not introduce relevant changes in our conclusions. The initial spin period \( P_0 \) and magnetic field \( B_0 \) can still be derived from Equations (13) by using the value \( \lambda_0 \) referred to initial efficiency of TeV emission.

Finally, we consider the effects of dispersion of the initial period and magnetic field around reference values indicated as \( P_0 \) and \( B_0 \). This in turn implies a dispersion in \( L_{\text{max}} \) and \( \tau \). The source luminosity function can be obtained by integrating Equation (9) calculated by assuming \( \tau = \tau_{sd}(B_0, P_0) \) and \( L_{\text{max}} = \lambda \dot{E}_0(B_0, P_0) \), over \( B_0 \) and \( P_0 \) probability distributions. We obtain:

\[
Y(L) = \frac{R}{L} \left( \frac{\alpha}{L} - 1 \right) \left( \frac{L}{L_{\text{CRAB}}} \right)^{-\alpha} G \left( \frac{L}{L_{\text{CRAB}}} \right),
\]

(14)

where \( \bar{\tau} \equiv \tau_{sd}(B_0, \bar{P}_0) \) and \( \bar{L} \equiv L_{\text{max}}(B_0, \bar{P}_0) \) are the spin-down timescale and maximal luminosity for the reference values \( \bar{P}_0 \) and \( B_0 \). The obtained luminosity function differs from Equation (9) for the presence of the function \( G(L/L_{\text{CRAB}}) \) that is defined according to:

\[
G(x) \equiv \int dp h(p) p^{\delta-4\alpha} \int db g(b) b^{2\alpha-4} \theta(p^{-4} b^2 - x),
\]

(15)

where \( p \equiv P_0/\bar{P}_0, b \equiv B_0/\bar{B}_0 \), while \( h(p) \) and \( g(b) \) describe the probability distributions of initial period and magnetic field. We assume that these functions can be modeled as Gaussian distributions in \( \log_{10}(p) \) and \( \log_{10}(b) \), centered at zero and having widths given by \( \sigma_{\log p} = \log_{10}(f_p) \) and \( \sigma_{\log B} = \log_{10}(f_b) \) with the parameters \( f_p \) and \( f_b \) described in the next section. Under this assumption, the parameters \( \tau \) and \( L \) represent the central values of the log-normal (correlated) distributions of \( \tau \) and \( L_{\text{max}} \) that are obtained as a result of the introduction of \( P_0 \) and \( B_0 \) dispersion.

4. Results

Flux, latitude, and longitude distributions of the sources observed in HGPS are fitted by using an unbinned likelihood (see Appendix A for details) with the goal of constraining the source luminosity distribution. In order to avoid selection effects, we restrict our analysis to the brightest sources that produce a photon flux above 1 TeV larger than 10% of that produced by the CRAB nebula \( \phi_{\text{CRAB}} = 2.26 \times 10^{-11} \text{cm}^{-2} \text{s}^{-1} \). Above this threshold, the catalog consists of 32 sources (3 of which are identified as SNRs) and is considered complete (Abdalla et al. 2018a). This allows us to perform our analysis in full generality without being forced to hypothesize a prescribed physical dimension for the sources because the angular extension does not discriminate the possible identification. A possible exception is provided by very close and very extended sources that cover angular regions larger than \( \sim 1^\circ \) and could escape detection due to background subtraction procedure employed by H.E.S.S. We checked, however, that this situation is unlikely and, thus, does not affect our constraints unless one assumes that the majority of the observed sources have physical extensions much larger than a few \( \sim 10 \text{ pc} \). In conclusion, the obtained results may be applied to PWNe as well as to TeV halos, provided that they have dimension that do not exceed \( \sim 40 \text{ pc} \).

The best-fit values and the allowed regions for the maximal luminosity \( L_{\text{max}} \) and the normalization \( \mathcal{N} \) of the source luminosity distribution are shown in Figure 1. We obtain:

\[
L_{\text{max}} = 4.9_{-2.1}^{+10} \times 10^{35} \text{erg s}^{-1}, \quad \mathcal{N} = 17_{-4}^{+14},
\]

(16)

where the quoted uncertainties correspond to 1\( \sigma \) confidence level (CL). The constraint on the maximal luminosity can be also expressed as \( L_{\text{max}} = 13_{-8}^{+8} L_{\text{CRAB}} \) by considering that the CRAB luminosity (above 1 TeV) is \( L_{\text{CRAB}} = 3.8 \times 10^{34} \text{erg s}^{-1} \). The above results are obtained for our reference case where we assume that the source distribution is proportional to that of pulsars given by Lorimer et al. (2006), the disk thickness is \( H = 0.2 \text{ kpc} \) and the power-law index of the luminosity distribution is \( \alpha = 1.5 \). Moreover, we include 29 HGPS sources neglecting the three sources that are firmly identified as SNRs. This is motivated by the fact that we discuss, in the next section, the possible interpretation of our results in terms of a population of fading sources powered by pulsar activity. The dependence and/or stability of the obtained results with respect to this and other assumptions in our analysis are discussed in detail in Table 1 and further commented on at the end of this section.
The obtained bounds are connected with specific features of the H.E.S.S. data. The constraint on the maximal luminosity essentially originates from the flux distribution of HGPS sources, as can be understood by looking at Figure 2, where we compare the cumulative number \( N(\Phi) \) of observed sources with a flux larger than \( \Phi \) with the predictions obtained for different \( L_{\text{max}} \) values. The theoretical calculations are normalized in such a way that the expected number of sources with \( \Phi \geq 0.1 \Phi_{\text{CRAB}} \) is equal to the observational value \( N_{\text{obs}} = 29 \). This corresponds to moving along the cyan dashed line in Figure 1 that maximizes the likelihood for each assumed \( L_{\text{max}} \). The black line in Figure 2 corresponds to the best-fit value \( L_{\text{max}} = 13 L_{\text{CRAB}} \) and well reproduces the flux distribution in the range \( \Phi \geq 0.1 \Phi_{\text{CRAB}} \) considered in our analysis. For comparison, we also show with a red dashed line the expected behavior of \( N(\Phi) \) for \( L_{\text{max}} = 30 L_{\text{CRAB}} \). This value is disfavored at the \( \sim 2\sigma \) level by HGPS data because bright sources are overproduced with respect to observational results.

A more complete understanding of the above points can be obtained by considering the magenta dotted–dashed line and the blue dotted line in Figure 2 that correspond to the limiting cases \( L_{\text{max}} \to 0 \) and \( L_{\text{max}} \to \infty \), respectively. The limit \( L_{\text{max}} \to 0 \) represents the case in which all sources have a very low luminosity, therefore, the detector is able to resolve only a small surrounding region, with the sources that are far away being too faint to produce a detectable flux. The limit \( L_{\text{max}} \to \infty \), instead, corresponds to the possibility for the detector to investigate the whole Milky Way. For both these assumptions, the flux distribution can be derived analytically, as is discussed in Appendix B. Namely, for \( L_{\text{max}} \to \infty \), the source flux distribution \( dN/d\Phi \) is described by a power law with the same index of the luminosity function, so that the cumulative distribution scales as \( N(\Phi) \propto \Phi^{1-\alpha} \). When \( L_{\text{max}} \to 0 \), one instead obtains \( dN/d\Phi \propto \Phi^{-5/2} \), predicting \( N(\Phi) \propto \Phi^{-3/2} \) independently from the assumed source luminosity function. The cumulative distribution of sources observed by H.E.S.S. has a different behavior with respect to both cases and thus it requires a specific \( L_{\text{max}} \) value in order to be reproduced. The possibility to determine \( L_{\text{max}} \) from the flux distribution automatically implies the ability to fit the normalization \( N \) of the source luminosity function by considering the additional constraint provided by the total number of observed sources, as is understood by looking at the cyan dashed line in Figure 1.

By using Equations (4) and (5), we obtain a determination of the total luminosity of the Galaxy in the energy range \( 1–100 \text{ TeV} \) and of the total flux (in the same energy range) produced by sources in the H.E.S.S. OW. We get:

\[
L_{\text{MW}} = 1.7^{+0.5}_{-0.4} \times 10^{37} \text{ erg s}^{-1} \\
\Phi_{\text{tot}} = 3.8^{+1.0}_{-1.0} \times 10^{-10} \text{ cm}^{-2} \text{ s}^{-1}
\]

that correspond to \( L_{\text{MW}} = 445^{+128}_{-128} L_{\text{CRAB}} \) and \( \Phi_{\text{tot}} = 16.8^{+6.4}_{-5.8} \Phi_{\text{CRAB}} \) in CRAB units. We note that the uncertainties on these quantities are relatively small because they are proportional to the product \( N L_{\text{max}} \) that is well constrained by observational data, as is also understood by considering the green dotted–dashed line in Figure 1.

The total TeV luminosity is only a factor of \( \sim 4 \) smaller than that obtained in the energy range \( 1–100 \text{ GeV} \) by fitting the Fermi-LAT 3FGL (Ajello et al. 2017) and 1FHL (Ackermann et al. 2013) catalogs. The total flux at the Earth \( \Phi_{\text{tot}} \) should be compared with the cumulative emission produced by all 78 resolved sources in the HGPS catalog, i.e., \( \Phi_{\text{HGPS}} = 10.4 \Phi_{\text{CRAB}} \). We obtain by subtraction the unresolved flux \( \Phi_{\text{NR}} = 6.4^{+4.4}_{-3.5} \Phi_{\text{CRAB}} \), which is due to sources in the considered population that are too faint to be identified by H.E.S.S. We see that unresolved emission \( \Phi_{\text{NR}} \) is relatively large, comparable to the resolved source contribution. This is naturally expected because the observational horizon for H.E.S.S. is limited, while sources are expected to be distributed everywhere in the Galaxy.\(^4\) In agreement with our previous estimate of this quantity (Cataldo et al. 2019), we obtain \( \Phi_{\text{NR}} \approx 60\% \Phi_{\text{HGPS}} \).

\(^4\) As an example, a source with intrinsic luminosity \( L \approx L_{\text{CRAB}} \) produces a flux larger than \( 0.1 \Phi_{\text{CRAB}} \) only at a distance smaller than \( \approx 6 \text{ kpc} \).
In conclusion, our results show that unresolved sources are likely to provide a relevant contribution to the diffuse large-scale γ-ray signal observed by H.E.S.S. and other experiments, with profound implications for the interpretation of observational results in the TeV domain. The unresolved flux Φ_{NR} is comparable to or larger than expectations for the truly diffuse contribution produced by the interaction of high-energy CRs with the gas contained in the Galactic disk. This diffuse component can be estimated to be in the range Φ_{diff} = (5–15)Φ_{CRAB} following the approach of Cataldo et al. (2019) and Pagliaroli et al. (2016), depending on the assumed CR space and energy distribution. The estimate Φ_{diff} ≈ 15 Φ_{CRAB} is, e.g., obtained by assuming CR spectral hardening toward the Galactic center, as recently emerged from analysis of Fermi-LAT data at lower energies (Acero et al. 2016). It was noted in Cataldo et al. (2019) that, if an unresolved contribution is large (namely, Φ_{NR} ≳ 0.5Φ_{HPGS}), this possibility is disfavored by H.E.S.S. (Abramowski et al. 2014) because the total flux (resolved + unresolved + truly diffuse signal) obtained in this hypothesis exceeds the total observed emission from the Galactic plane. Here, we strengthen this conclusion by noting that the total flux measured by Milagro at 15 TeV (dΦ/hE ∼ 2.9 × 10^{-12} cm^{-2} s^{-1} sr^{-1} TeV^{-1} for 30 < l < 65 and |b| < 2) is consistent (within uncertainties, see the next section) with the total flux produced by the HGPS source population in the same observation window (dΦ_{HPGS}/dE ∼ 3.4 × 10^{-12} cm^{-2} s^{-1} sr^{-1} TeV^{-1}). This suggests that the anomalous diffuse emission reported by Milagro is due to unresolved sources and provides an additional constraint to the possibility of a large truly diffuse contribution produced by CR interactions in the Galactic disk.

4.1. Robustness of the Results

In the following we briefly discuss the stability of our results with respect to the assumptions adopted in our analysis. In Table 1 we consider different scenarios identified by the ingredient that has been modified with respect to the reference case (e.g., the space distribution, the disk thickness, the source physical dimension, the power-law index of the luminosity distribution, etc.). For each case, we give the best-fit results and the 1σ allowed regions for the source luminosity function parameters (N and L_{max}), the total TeV luminosity of the Galaxy L_{MW}, the total flux produced at Earth Φ_{tot}, the fading timescale τ and the level of agreement with data, expressed in terms of the Δχ² with respect to our reference case.

We thereby illustrate that the inclusion of the three sources firmly identified as SNRs in the HGPS catalog (case labeled as N_{obs} = 32 in Table 1) does not alter our conclusions, marginally affecting the maximal luminosity L_{max} and increasing by less than 10% the normalization N of the source luminosity distribution. No significant effects are produced by assuming that sources follow the SNR distribution parameterized by Green (2015; case labeled as SNR) instead of the pulsar distribution of Lorimer et al. (2006). The results of our analysis are only slightly modified when we reduce the thickness of the Galactic disk from our reference choice H = 0.2 kpc to H = 0.1 kpc or H = 0.05 kpc. In this last case, the total Milky Way luminosity has a variation of ∼27% with respect to our reference case while the variation of the total flux is ∼17%. However, the quality of the fit substantially improves with respect to our reference choice (Δχ² ≈ -7 and Δχ² ≈ -10.5) the more the thickness of the disk is reduced (H = 0.1 kpc and H = 0.05 kpc). This is due to the fact that the latitudinal distribution of HGPS sources is quite narrow, having an rms latitude of 0.017, as is expected for a population of young sources connected with the site of past core-collapse supernova explosions. In particular, this information can be used in favor of a fading sources population, as young PWNs, not old enough to drift off the Galactic plane (Abdalla et al. 2018b). This specific hypothesis and its implications will be further discussed in the next section. The cases labeled as d = 20 pc and d = 40 pc are obtained by assuming that all sources in the Galaxy have a prescribed physical dimension and that objects with angular extension larger than ∼1° are not observed by H.E.S.S. We see that our results are not modified in this assumption. Although not listed in the table, we also check possible variation of the source spectral index in the range 2.2 < β < 2.4. We notice that our results are not modified. In particular, the total Milky Way luminosity has a maximum variation of ∼10% with respect to the reference case while the total flux remains almost the same.\(^5\) We do not consider larger variations on this parameter because different values are less compatible with the observed data sample (Abdalla et al. 2018a).

Finally, we consider the effects produced by a variation of the power index α of the luminosity distribution by considering two cases: α = 1.3 and α = 1.8. We obtain an ∼10% decrease (∼50% increase) of the TeV Milky Way luminosity and of the total flux at Earth for α = 1.3 (α = 1.8), with a slight preference for the case with power index 1.3. In conclusion, the cumulative source contribution to the Milky Way luminosity in the 1–100 TeV range and to the total γ-ray flux in the H.E.S.S. OW are included in the ranges: L_{MW} = (1.2 – 2.5) × 10^{37} erg s^{-1}, Φ_{tot} = (3.5 – 5.9) × 10^{-10} cm^{-2} s^{-1} sr^{-1}, showing that the Milky Way luminosity and the total γ-ray flux can be constrained within a factor of 2.1 and 1.7, respectively, by present observational data.

4.2. Interpretation in Terms of a Fading Source Population

If we consider a fading source population connected with the explosion of core-collapse SNe, we can convert the limits on the normalization parameter N of the source luminosity function into a determination of the fading timescale τ through the relationship N = R τ(α – 1). By assuming that the source formation rate R is approximately equal to the SN rate R_{SN} = 0.019 yr^{-1}, we get:

\[
τ = 1.8^{+1.5}_{-0.6} \times 10^{3} \text{ yr}
\]

for our reference case, that corresponds to the orange solid line in the left panel of Figure 3. Similar values are obtained in the other cases, as reported in Table 1.

In the assumption that the observed objects are PWNe and/ or TeV halos that are powered by the formation and the subsequent spin-down of a pulsar, the above value can be used to determine through Equations (13) the initial period P_{0} and magnetic field B_{0} of the considered population. We get the

\(^5\) In general, the total flux is slightly more stable than the luminosity with respect to variations of β and 1/α as a consequence of the fact that HGPS data constrain the fluxes (and not the luminosities) of observed sources.
The small uncertainty for the period that correspond to the orange solid line in the right panel of Figure 3. Left panel: the best fit and the 1σ and 2σ allowed regions in the plane $(L_{\text{max}}, \tau)$. The red shaded area is excluded by the data because it corresponds to $N(0.1\Phi_{\text{CRAB}}) \leq 10$ with the assumption of $\lambda = 5 \times 10^{-2}$, which is a large value for the fraction of pulsar spin-down energy converted to TeV emission. Right panel: the best fit and the 1σ and 2σ allowed regions in the plane $(P_0, B_0)$, calculated with the assumption that $\lambda = 10^{-3}$. The red shaded area corresponds to $N(0.1\Phi_{\text{CRAB}}) \leq 10$ with the assumption of $\lambda = 5 \times 10^{-2}$.

Figure 3. Left panel: the best fit and the 1σ and 2σ allowed regions in the plane $(L_{\text{max}}, \tau)$. The red shaded area is excluded by the data because it corresponds to $N(0.1\Phi_{\text{CRAB}}) \leq 10$ with the assumption of $\lambda = 5 \times 10^{-2}$, which is a large value for the fraction of pulsar spin-down energy converted to TeV emission. Right panel: the best fit and the 1σ and 2σ allowed regions in the plane $(P_0, B_0)$, calculated with the assumption that $\lambda = 10^{-3}$. The red shaded area corresponds to $N(0.1\Phi_{\text{CRAB}}) \leq 10$ with the assumption of $\lambda = 5 \times 10^{-2}$.

Constraints:

$$P_0 = 33.5^{+5.4}_{-4.3} \times \left(\frac{\lambda}{10^{-3}}\right)^{1/2}$$

$$B_0 = 4.3(1 \pm 0.45) \times 10^{12} \times \left(\frac{\lambda}{10^{-3}}\right)^{1/2}$$

(19)

that correspond to the orange solid line in the right panel of Figure 3. The small uncertainty for the period $P_0$ is connected with the fact that this quantity is determined by the product $L_{\text{max}}$, which is relatively well determined by observational data, with the possible variations of $L_{\text{max}}$ and $\tau$ anticorrelated.

We note that inferred magnetic field agrees with the value $\log_{10}(B_0/\text{G}) \simeq 12.65$ obtained by pulsar population studies (Faucher-Giguere & Kaspi 2006). The inferred period is consistent with the value $P_0 \sim 50$ ms obtained in Watters & Romani (2011) by studying a $\gamma$-ray pulsar population. The value $P_0 \sim 300$ ms that is obtained from pulsar radio observation (Faucher-Giguere & Kaspi 2006) is instead excluded by our analysis, unless one assumes that a very large fraction $\lambda \sim 10^{-1}$ of the spin-down power is converted to TeV $\gamma$-ray emission.

The above results are obtained under the assumption that all the sources in the HGPS catalog with flux $\Phi \geq 0.1\Phi_{\text{CRAB}}$ (except those firmly identified as SNRs) are powered by pulsar activity. A conservative upper bound for the period $P_0$ can be obtained by considering that no less than 10 of these sources have to be necessarily included in this population, being firmly identified as PWNe or composite sources. The lines $N(0.1\Phi_{\text{CRAB}}) = \text{const}$ corresponding to a fixed number of sources above the adopted flux threshold $0.1\Phi_{\text{CRAB}}$ are shown by the gray dashed lines in planes $(L_{\text{max}}, \tau)$ and $(P_0, B_0)$ in Figure 3. It can be shown analytically (see Appendix B) that $N(\Phi)$ scales as:

$$N(\Phi) \propto \tau L_{\text{max}}^{3/2} \propto B_0 P_0^{-4} \lambda^{3/2}$$

(20)

for the limiting case $L_{\text{max}} \to 0$, while it scales as:

$$N(\Phi) \propto \tau L_{\text{max}}^{\alpha-1} \propto B_0^{2\alpha-4} P_0^{6-4\alpha} \lambda^{\alpha-1}$$

(21)

for $L_{\text{max}} \to \infty$. If $1 < \alpha < 2$, the condition $N(\Phi) = \text{const}$ always individuates a maximum allowed period $P_0$ (at the transition between the above regimes) whose specific value depends on the fraction $\lambda$ of the pulsar spin-down energy that is converted to TeV $\gamma$-ray emission. In particular, the red shaded area in Figure 3 can be excluded because it corresponds to $N(0.1\Phi_{\text{CRAB}}) \leq 10$ and to the relatively large value $\lambda = 5 \times 10^{-2}$. This allows us to obtain the bound $P_0 \leq 500$ ms that can be strengthened if an upper limit for the magnetic field $B_0 \leq 10^{14}$ G is introduced.

In order to test stability of the constraints given in Equation (19), we repeat our calculation for the case $\alpha = 1.8$ obtained by assuming that $\lambda$ is correlated with the spin-down power as suggested by Abdalla et al. (2018b). In this case, the fading timescale is $\tau = 0.5^{+0.4}_{-0.2} \times 10^3$ yr, while the initial period and magnetic field are given by:

$$P_0 = 51.0^{+8.1}_{-6.4} \times \left(\frac{\lambda_0}{10^{-3}}\right)^{1/2}$$

$$B_0 = 12.7^{+9.6}_{-5.8} \times 10^{12} \times \left(\frac{\lambda_0}{10^{-3}}\right)^{1/2}.$$
inferred value for $\bar{R}_0$ is basically insensitive to assumed dispersion while the preferred magnetic field $B_0$ is slightly reduced with respect to the reference case, as a consequence of the high-luminosity tail of the source luminosity function that is obtained by assuming $f_p = 0$ and $f_b = 0$.

In summary, the results displayed in Figure 4 show that the bounds on the initial period and magnetic field do not critically depend on the adopted assumptions, with $P_0$ being constrained to the narrow range 25–60 ms for $\lambda = 10^{-3}$. The fact that the inferred values for $B_0$ and $P_0$ are consistent with expectations justifies the working assumption that a large fraction of bright sources observed by H.E.S.S. belongs to a population of young pulsars, and supports the hypothesis, formulated, e.g., by Linden & Buckman (2018) and Sodoh et al. (2019), that PWNe and/or TeV halos could produce the majority of TeV bright sources in the sky. On the contrary, the large values for the initial period $P_0 \sim 300$ ms can explain the HGPS results, only if we assume that a limited fraction of observed sources belong to the considered population and/or a consistent fraction of the spin-down energy is converted into TeV $\gamma$-ray emission.

As a further check of this point, we calculate the expected number of sources in the H.E.S.S. OW by using the $P_0$ and $B_0$ distributions obtained by Faucher-Giguere & Kaspi (2006) from pulsar radio observations, i.e., a Gaussian centered in $P_0 = 300$ ms with standard deviation $\sigma_P = 150$ ms, and a log-normal centered in $\log B_0 = 12.65$ with standard deviation $\sigma_{\log B} = 0.55$. By using the reference value $\lambda = 10^{-3}$, we obtain only $\sim 1$ source above the adopted flux threshold $0.1 \Phi_{CRAB}$. In order to reproduce the 10 sources firmly identified as pulsars, we have to assume $\lambda = 1.6 \times 10^{-2}$, while to predict all the 29 sources observed by H.E.S.S. the value of the efficiency $\lambda$ has to be as large as $\sim 5 \times 10^{-2}$.

5. Conclusions

Recently the H.E.S.S. observatory completed the first systematic survey of the Galactic plane in the very high-energy domain. Remarkably, the astrophysical nature of the majority of detected sources is still unknown. In this work, we present a novel analysis of the flux, longitude, and latitude distributions of the brightest sources ($\Phi \geq 10\% \Phi_{CRAB}$) of the HGPS catalog showing that the luminosity distribution of Galactic TeV sources can be effectively constrained.

More precisely, by assuming that the luminosity function is described by a power law (see Equation (2)) we extract the source maximal luminosity $L_{\text{max}} = 4.9_{-2.1}^{+3.0} \times 10^{35}$ erg s$^{-1}$ and the high-luminosity normalization of the source distribution $N = 17_{-5}^{+14}$ by fitting HGPS data. This allows us to determine the total Milky Way luminosity $L_{\text{MW}} = 1.7_{-0.4}^{+0.5} \times 10^{37}$ erg s$^{-1}$ in the energy range $1–100$ TeV and the total Galactic flux in the H.E.S.S. OW given by $\Phi_{\text{tot}} = 3.8_{-1.0}^{+1.0} \times 10^{-10}$ cm$^{-2}$ s$^{-1}$. The luminosity $L_{\text{MW}}$ is only a factor of $\sim 4$ smaller than that obtained in the energy range $1–100$ GeV by fitting Fermi-LAT 3FGL and 1FHL catalog. In addition, the total source flux is relatively large, implying that unresolved source contribution is not negligible (about 60% of the resolved signal measured by H.E.S.S.) and potentially responsible for a large fraction of the diffuse large-scale gamma-ray signal observed by H.E.S.S.

This could have important implications for the interpretation of current observations of other experiments in the TeV domain. The unresolved contribution can e.g., explain the excess reported by Milagro at 15 TeV (Atkins et al. 2005). Our results can also be used to investigate the capability of future experiments, like, e.g., CTA, to probe the Galactic TeV source population.

Moreover, we consider the possibility that the bright sources observed by H.E.S.S., which are not firmly identified as SNRs, are powered by pulsar activity, like, e.g., PWNe and TeV halos. We evaluate the constraints on the physical properties of the pulsar population that follow from this hypothesis. For our reference case, assuming that the fraction of the pulsar spin-down energy converted into TeV photons is $\lambda = 10^{-3}$, we obtain the best-fit values $P_0 = 33.5_{-4.3}^{+5.4}$ ms and $B_0 = 4.32(1 \pm 0.45) \times 10^{12}$ G, the initial spin period and magnetic field, respectively. The above constraints are consistent with the $B_0$ values obtained in Faucher-Giguere & Kaspi (2006) and $P_0$ constrains described in Watters & Romani (2011) by studying the gamma-ray pulsar population.

Finally, by considering that 10 sources in the HPGS catalog have been firmly identified as PWNe and considering $\lambda \leq 5 \times 10^{-2}$ as an upper bound for efficiency of TeV emission, we obtain that the initial spin-down period of the considered pulsar population is constrained to be $P_0 \leq 500$ ms.

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Appendix A

Likelihood Definition

In order to determine the maximal luminosity $L_{\text{max}}$ and the normalization $N$ of the luminosity function, see Equation (2), we use the maximum likelihood technique. The H.E.S.S. catalog contains 78 sources with their Galactic coordinates $(b_i, l_i)$, the observed fluxes $\Phi_i$ in the energy range $1\text{–}100\text{ TeV}$, and the respective uncertainty $\delta \Phi_i$. In our work we considered only the 32 brightest sources with a flux above $1\text{ TeV}$ larger than $0.1\Phi_{\text{CRAB}}$ for which the H.E.S.S. catalog can be considered complete.

Given this data set we define an unbinned likelihood function $L$, according to:

$$\log L = -\mu_{\text{tot}} + \sum_i \log(\mu_i), \quad (A1)$$

where $\mu_{\text{tot}}$ represents the number of expected sources, while $\mu_i$ is the probability to observe an object with coordinates $(b_i, l_i)$ and measured flux $\Phi_i$. These quantities are calculated by considering that the source distribution per unit of flux $\Phi$ and solid angle $d\Omega$ is given by:

$$\mu(b, l, \Phi) = \int \! d\Omega \, 4\pi r^2 \langle E \rangle \, Y(4\pi r^2 \langle E \rangle \Phi) \rho(r, b, l) \quad (A2)$$

with the functions $Y(L)$ and $\rho(r)$ defined in Section 3. The parameter $\mu_{\text{tot}}$ is obtained by integrating the function $\mu(b, l, \Phi)$ in the HESS OW and in the flux range $\Phi \geq 0.1\Phi_{\text{CRAB}}$. The coefficients $\mu_i$ are obtained as:

$$\mu_i = \int \! d\Phi \, \mu(b_i, l_i, \Phi)P(\Phi_i, \Phi, \delta \Phi_i), \quad (A3)$$

where the function $P(\Phi, \Phi, \sigma)$ represents the probability that the measured flux $\Phi$ is obtained for a source emitting the real flux $\Phi$. We assume that this can be described by a Gaussian with a dispersion $\sigma$ equal to the uncertainty of the measured flux, i.e.,

$$P(\Phi_i, \Phi, \delta \Phi_i) = \frac{1}{\sqrt{2\pi \delta \Phi_i^2}} \exp \left[ -\frac{(\Phi - \Phi_i)^2}{2 \delta \Phi_i^2} \right] \quad (A4)$$

Finally, the best-fit values and the allowed regions for the parameters in our analysis are obtained by studying the $\chi^2$ behavior, defined according to:

$$\chi^2 = -2 \log L. \quad (A5)$$

Appendix B

The Flux Distribution

The flux distribution can be calculated as:

$$\frac{dN}{d\Phi} = \int \! d\Omega \, 4\pi r^4 \langle E \rangle \, Y(4\pi r^2 \langle E \rangle \Phi) \, \bar{\rho}(r), \quad (B6)$$

where $\bar{\rho}(r) \equiv \int_{\text{OW}} \! d\Omega \, \rho(r, n)$ is the source spatial distribution integrated over the longitude and latitude intervals probed by H.E.S.S. Note that the integration in Equation (B6) is limited to the distance range $r \leq d(L_{\text{max}}, \Phi)$, where $D(L, \Phi) \equiv \sqrt{L/4\pi \langle E \rangle \Phi}$ represents the distance below which a source with intrinsic luminosity $L$ produces a flux larger than $\Phi$. Moreover, in the assumption that sources have a physical dimension $d$, one also has a lower integration limit $r \geq d/L_{\text{max}}$, where $\theta_{\text{max}}$ is the maximal angular dimension that can be probed by H.E.S.S. The function $dN/d\Phi$ can be calculated analytically in the two limit cases $L_{\text{max}} \to \infty$ and $L_{\text{max}} \to 0$.

For $L_{\text{max}} \to \infty$, hence $D(L_{\text{max}}, \Phi) \to \infty$, the function is:

$$\frac{dN}{d\Phi} = R(\alpha - 1) \, L_{\text{max}}^{-1} \Phi^{-\alpha} \times \int_{0}^{\infty} \! dr \, (4\pi \langle E \rangle)^{1-\alpha} r^{-2\alpha} \, \bar{\rho}(r); \quad (B7)$$

Here the integral is only dependent on the coordinate $r$ and is therefore a constant. The dependence on $\Phi$ is only given by the term $\Phi^{-\alpha}$; the total number of sources $N(\Phi)$ above a flux $\Phi$, which is shown in Figure 2, is therefore proportional to $\Phi^{-\alpha+1}$. For the limit case $L_{\text{max}} \to 0$, hence $D(L_{\text{max}}, \Phi) \to 0$, the integral over $r$ is extended to a small region where the distribution function $\bar{\rho}(r)$ can be considered constant and equal to its value at $r = 0$, i.e., $\bar{\rho}(r) \approx \bar{\rho}(0)$. We thus obtain:

$$\frac{dN}{d\Phi} \approx (4\pi \langle E \rangle)^{1-\alpha} \bar{\rho}(0) R(\alpha - 1) \, L_{\text{max}}^{-1} \Phi^{-\alpha} \times \int_{0}^{D_{\text{max}}(\Phi, \Phi)} \! dr \, r^{-2\alpha} = \bar{\rho}(0) R(1-\alpha) \left( \frac{L_{\text{max}}}{4\pi \langle E \rangle} \right)^{\frac{1}{2}} \Phi^{-\frac{\alpha}{2}}. \quad (B8)$$

The cumulative function $N(\Phi)$ is therefore independent from the index $\alpha$ considered and is proportional to $\Phi^{-\frac{\alpha}{2}}$ as is shown in Figure 2.

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