The Effect of Dust Size Distribution on the Propagation of Electromagnetic Waves in a Weakly Ionized Dusty Plasma

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Abstract. The propagation characteristics of electromagnetic (EM) waves in dusty plasma of rocket exhaust are studied. Usually, the dust sizes are different in a general dusty plasma. However, most previous studies have focused on the mono-sized dusty plasma. The influence of dust size distribution on the EM wave attenuation of dusty plasmas has been investigated. The dependence of the EM wave attenuation on the electron density of dusty plasmas, the neutral gas density and the EM wave frequency are obtained and discussed. The results illustrate that the EM wave attenuation constant has a maximum value when the electron-neutral particles collision frequency is close to the EM wave frequency. The EM wave attenuation constant increase with the increase of electron density and dust particles density. Moreover, the EM wave attenuation constant decreases as the dust size decrease for mono-sized dust particles. It indicates that the attenuation of the EM wave is mainly determined by the largest dust particles. Finally, we conclude that the larger the dust particle, the stronger the attenuation to the EM wave.

1. Introduction
The electromagnetic (EM) wave propagating in plasmas is carried out extensive research. This interest can be attributed to their widespread applications in the aircraft stealth, space communication, microwaves for the plasma diagnostics, and electromagnetic protection[1-4]. Studies show that there exists power attenuation of EM waves propagating in plasmas on account of the interactions between them[5]. Moreover, EM waves are reflected and absorbed by the plasma[6]. Therefore, it is very important to study the interaction mechanism between EM waves and plasmas, and further research is needed on the effects of their various parameters.

When the EM waves propagate in a plasma, reflecting, absorbing and transmitting of the EM waves will take place[7-10]. Xu et al. researched the attenuation properties of EM wave in partially ionized plasma found that the EM wave frequency, plasma density and other factors affect strongly on the energy attenuation of the EM wave[7]. Recently, researchers have carried out many experiments works on the EM waves and plasmas[10-13]. Zheng et al[13], focusing on the EM wave propagation in non-magnetized plasma. The variations of the EM wave attenuation with plasma density, collision frequency and EM wave frequency are obtained. However, the EM waves transmit very differently in dusty plasma and general plasma (with no dust grains)[8,13]. It is found that many studies have gnored the influence of dust grains on the EM wave transmission that led to an inaccurate attenuation rate of EM waves[8,14]. So it is very necessary to research the influence of dust grains on the EM wave transmission.
2. Basic Equations

To get more insight into the EM wave attenuation in a plasma and how it is affected by the plasma parameters, let us consider the EM wave propagation in a weak ionization plasma. In this case, the microwave attenuation constant and the phase constant are

\[ \alpha = \frac{\omega}{2c} \left( -\gamma_1 + \left( \gamma_1^2 + \gamma_2^2 \right)^{\frac{1}{2}} \right), \]

\[ \beta = \frac{\omega}{2c} \left( \gamma_1 + \left( \gamma_1^2 + \gamma_2^2 \right)^{\frac{1}{2}} \right), \]

then \( \gamma_1 = \left( 1 - \frac{\omega^2}{\omega^2 + \omega^2_e} + \epsilon_{dr} \right), \gamma_2 = \frac{\omega^2}{\omega^2 + \omega^2_e} \frac{\nu_{en}}{\omega} + \frac{\sigma_{d}}{\epsilon_{r} \omega}, \) where \( \omega \) is the EM wave frequency, \( c = 3 \times 10^8 \text{ m/s} \) is the light speed in a vacuum, \( \omega_{pe} = \left( \frac{n_e e^2}{m_e} \right)^{\frac{1}{2}} \) is the electron frequency of the plasma, \( n_e \) is the electron density, \( \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \) is the permittivity of vacuum, \( m_e = 9.107 \times 10^{-31} \text{ kg} \) is the mass of the electron, \( e = 1.6021862 \times 10^{-19} \text{ C} \) is the electric charge of an electron, \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \) is the permeability of vacuum, \( k_0 = 1.38 \times 10^{-23} \text{ J/K} \) is the Boltzmann constant. The collision angular frequency between the electrons and the neutrals \( \omega = \frac{n_e e^2}{m_e} \) is electron thermal velocity, \( \sigma_{en} \) is effective collision cross section between the molecular and electron, \( N_n \) is the molecular density, dust charging frequency is \( \omega_{ch} = \frac{n_e e^2}{m_i} \left( \frac{1}{2} + \frac{Z_e e^2}{4\pi n_0 Y_d k_i T_e} \right), \)

where \( \omega_{pi} = \left( \frac{n_e e^2}{m_i} \right)^{\frac{1}{2}} \) is the ion frequency of the plasma, \( n_i \) is the ion density, \( m_i \) is the mass of the ion, \( V_{Ti} = \left( \frac{kT_i}{m_i} \right)^{\frac{1}{2}} \) is ion thermal velocity, \( T_i \) is the ion temperature, the charge of dust particles is \( Z_d = k_z r_d \), where \( r_d \) is the radius of the dust grains, \( k_z = 4\pi n_0 e^2 V_0 / \epsilon_0 \) is approximately constants, \( V_0 \) is the electric surface potential at equilibrium, and \( e_0 \) is the vacuum permittivity. The re-lative complex permittivity is

\[ \epsilon_{dr} = \frac{\eta_{ed} c (v_{ch} + v_{en})}{\epsilon_0 (\omega^2 + v_{ch}^2 + \nu_{en})}, \]

the complex conductivity is

\[ \sigma_{d} = \frac{\eta_{ed} c (\omega^2 - v_{ch} v_{en})}{(\omega^2 + v_{ch}^2 + \nu_{en})}, \]

where \( v_{ch} \) is the charging frequency, \( \eta_{ed} \) is the response factor of charging. For a simple case in which all the dust particles are the same with the same radius \( r_d \) and the same mass \( m_d \), \( \eta_{ed} = \frac{\pi e^2 n_d a_d r_d^2}{m_e} \), where \( n_d \) is the density of the dust particles.

3. The Dust Size Distribution Effect

As well known, the size of dust particles is different in dusty plasmas in a real situation. Gaussian distribution is used to describe the dust size distribution in laboratory dust plasma, and a power-law distribution is used to describe the dust size distribution in space plasma. The distribution function relies on the plasma environment and other conditions. If we consider the dust size distribution effect, the response factor of charging is.
We consider a power-law distribution of a dusty plasma describing a space plasma. The dust size distribution function in this case, \( n(r) = kr^{-\beta} \), where \( k \) is the normalization constant, \( \beta \) is power-law index, and in the region \( r_1 < r < r_2 \). \( n(r) = 0 \), in the other region, where \( r_1 \) is the minimum size of the dust grains, and \( r_2 \) is the maximum size of the dust grains. In this case, the average dust size is \( r = \frac{r_1 + r_2}{2} \), the number density of the dust grains is

\[
N_{\text{tot}} = \frac{k}{1-\beta} (r_2^{-\beta} - r_1^{-\beta}),
\]

where \( k = \frac{N_{\text{tot}}(1-\beta)}{r_2^{-\beta} - r_1^{-\beta}} \), then the charging response influence factor is

\[
\eta_{ed} = \frac{\pi e^2 n_e}{m_e} \cdot \frac{k}{(3-\beta)} (r_2^{-\beta} - r_1^{-\beta}).
\]

4. The Analysis for the Results Introduction

The parameters of rocket exhaust plume measured by experiment indicate that the dusty density is about \( 10^{13} \) to \( 10^{15} \) m\(^{-3}\) and the size of the dust particle is typically 1 \( \mu \)m to 10 \( \mu \)m\(^{16}\). We take electron density \( n_e = 1 \times 10^{17} \sim 1 \times 10^{19} \text{m}^{-3} \) in the region of \( T_e = 3000 \text{k} \) of rocket exhaust plume, dust particles density is \( n_d = 1 \sim 10 \mu \text{m}, N_n = 1 \times 10^{24} \sim 5 \times 10^{25} \text{m}^{-3} \). In the weak collision dusty plasma, \( Z_d = k_n Z_d^\gamma, 1 \leq \gamma \leq 2, 10^8 \leq k_n \leq 10^9 \) if \( \gamma = 1.1 \) and \( k_n = 10^9 \). \( Z_d \) will obtain different values due to different radius of dust particles. Here, the collision frequency between the electron and neutral is calculated by \( f_{en} = \frac{v_{en}}{2\pi}, \) which is taken \( \sigma_{en} = 1 \times 10^{-20} \text{m}^2, N_n = 1 \times 10^{24} \text{m}^{-3} \).

The dependence of the microwave attenuation constant on the frequency of the microwave is shown in figure 1. There are five curves which represent the different dust size. Three of them stand for that all the dust particles are the same as the different radius of 1 \( \mu \)m, 5.5 \( \mu \)m, 10 \( \mu \)m, respectively. One of them represents a power-law distribution, while the other one is for the case in which there are no dust particles.

![Figure 1](image_url)

**Figure 1.** Dependence of the microwave attenuation constant on the frequency of the microwave, where \( n_i = 1.1 \times 10^{17} \text{m}^{-3}, n_d = 1 \times 10^{13} \text{m}^{-3}, n_e = 1 \times 10^{16} \text{m}^{-3}, Z = 1 \times 10^4, \omega_{pe} = 5.64 \times 10^9 \text{rad}/\text{s}, f_{en} = 5 \text{GHz} \) in figure 1(a) and \( f_{en} = 0.1 \text{THz} \) in figure 1(b).

It is noted from figure 1 that the microwave attenuation constant decreases as the dust size decreases for mono-sized dust particles. The smaller the dust size, the less the microwave attenuation constant. Furthermore, the microwave attenuation constant increases until the frequency of the microwave reach a critical point \( f_c \), and then decreases. The critical value of \( f_c \) is approximately equal to the collision frequency between the electrons and the neutrals \( f_{en} \). It seems that it is closer between \( f_c \) and \( f_{en} \) for...
larger dust particle dusty plasma than that of with the smaller dust particles. The larger the dust particle, the closer between two.

The interesting phenomena are found that the microwave attenuation constant considering the dust size distribution is close to that of the mono-sized dust particles with the minimum dust size of $r_d = 1\mu m$. It indicates that the attenuation of the microwave is mainly determined by the largest dust particles. It is also noted that the microwave attenuation constant considering the dust size distribution is close to that with no dust particles. Finally, we conclude that the larger the dust particle, the stronger the attenuation to the microwave.

\[ \text{Figure 2. Dependence of the microwave attenuation constant on the collision frequency between the electrons and the neutrals, where } n_e = 1.1 \times 10^{17} \text{m}^{-3}, n_d = 1 \times 10^{13} \text{m}^{-3}, n_e = 1 \times 10^{16} \text{m}^{-3}, Z = 1 \times 10^4, \omega_{pe} = 5.64 \times 10^9 \text{rad/s}, \text{the regime of the collision frequency is } 1 \times 10^9 Hz < f_{en} < 1 \times 10^{12} \text{Hz}. \text{The frequency of the microwave is 5 GHz in figure 2(a) and 0.1 THz in figure 2(b).} \]

Figure 2(a) shows the dependence of the microwave attenuation constant on the collision frequency between the electrons and the neutrals in the region $1 \times 10^9 Hz < f_{en} < 1 \times 10^{12} Hz$ where the frequency of the microwave is the 5GHz. It shows that the attenuation constant decreases as the collision frequency between the electrons and the neutrals increases for the larger dust particles such as either the $r_d = 10 \mu m$ or the $r_d = 5.5 \mu m$, while for smaller dust particles such as $r_d = 1 \mu m$ the attenuation constant nearly a constant, i.e. it is independent on the collision frequency. This phenomenon can also be observed for a plasma when there are no dust particles. By considering the dust size distribution, similar results are observed.

The dependence of the microwave attenuation constant on the collision frequency between the electrons and the neutrals are shown in figure 2(b) in the THz regime of the microwave. It is noted that the microwave attenuation constant increases as the collision frequency between the electrons and the neutrals increases until it reaches a critical point $f_{en}$, and then it decreases suddenly as the collision frequency further increases. It indicates that the microwave attenuation constant of the THz microwave is about two order magnitude smaller than that of the GHz microwave.

Figure 3 shows the dependence of the microwave attenuation constant on the electron density. Both the microwave frequency and the collision frequency between the electrons and the neutrals are 5 GHz in figure 3(a), while they are 0.1 THz in figure 3(b). It indicates that the attenuation constant increases as the electron density increase if the electron density is smaller than a critical point of $n_e^c$, then it decreases as the electron density increases further when $n_e > n_e^c$. In the curve of $r_d = 10 \mu m$, when the electron density is $n_e = 5.8 \times 10^{16} \text{m}^{-3}$, then the density of the dust particles is $n_d = 5.2 \times 10^{12} \text{m}^{-3}$ and the electron frequency of the plasma $\omega_{pe} = 1.36 \times 10^{10} \text{rad/s}$, the maximum of the attenuation constant is 50.46 in figure 3(a). It is also noted that the microwave attenuation constant of the THz microwave is about two order magnitude smaller than that of the GHz microwave.
Figure 3. Dependence of the microwave attenuation constant on the density of the electron, where \( n_e = 1.1 \times 10^{17} \text{m}^{-3} \), \( Z = 1 \times 10^4, 1 \times 10^{10} \text{m}^{-3} < n_e < 1.1 \times 10^{17} \text{m}^{-3} \), 1.78 \times 10^9 \text{rad/s} < \omega_{pe} < 1.87 \times 10^{10} \text{rad/s} \), the micro frequency and the collision frequency are 5GHz in figure 3(a), while they are 0.1THz in figure 3(b).

Figure 4. Dependence of the microwave attenuation constant on the density of the dust particle, where \( n_i = 1.1 \times 10^{17} \text{m}^{-3} \), \( Z = 1 \times 10^4, 1 \times 10^{10} \text{m}^{-3} < n_d < 1.1 \times 10^{13} \text{m}^{-3} \). The micro frequency and the collision frequency are 5GHz in figure 4(a), while they are 0.1THz in figure 4(b).

Figure 4 show the dependence of the microwave attenuation constant on the density of the dust particles. It is found that the attenuation constant increases as the density of the dust particles increases if the density of the dust particles is smaller than a critical point \( n_d^c \) for larger dust size such as \( r_d = 10 \mu m \) or \( r_d = 5.5 \mu m \), then it decreases if the density of the dust particles increases further, i.e.\( n_d > n_d^c \), when the frequency of the microwave is \( f = 5 \text{GHz} \), and the collision frequency between the electrons and the neutrals is also \( f_{en} = 5 \text{GHz} \), see figure 4(a). In the curve of \( r_d = 10 \mu m \), when the density of the dust particles is \( n_d = 5.2 \times 10^{12} \text{m}^{-3} \), then the electron density is \( n_e = 5.8 \times 10^{16} \text{m}^{-3} \) and the electron frequency of the plasma \( \omega_{pe} = 1.36 \times 10^{10} \text{rad/s} \), the maximum of the attenuation constant is 50.46 in figure 4(a). It is observed that the attenuation constant increases slowly as the density of the dust particles increases for a smaller dust size such as \( r_d = 1 \mu m \) or if the dust size distribution is considered.

Different phenomena are observed from figure 4(b) for the larger frequencies of the microwave and the larger collision frequency between the electrons and the neutrals where \( f = 0.1 \text{THz} \), \( f_{en} = 0.1 \text{THz} \). It is noted for this case that the microwave attenuation constant is first independent on the density of the dust particles, then it decreases as the density of the dust particles increases for either the smaller dust size \( (r_d = 1 \mu m, 5.5 \mu m) \) or considering the dust size distribution effect except for the larger dust size of \( r_d = 10 \mu m \). It is also concluded from figure 4 that the microwave attenuation constant of the THz microwave is about two order magnitude smaller than that of the GHz microwave.
5. Conclusion

In summary, the effects of dust size distribution on the EM wave attenuation of dusty plasmas have been investigated. The results show that the EM wave attenuation is much smaller for the THz wave than that of the GHz wave, and it decreases monotonously as the frequency of the EM wave increases. The results illustrate that the EM wave attenuation constant has a maximum value when the electron-neutral particles collision frequency is close to the EM wave frequency. The EM wave attenuation constant increases with the increase of electron density and dust particles density. Moreover, the EM wave attenuation constant decreases as the dust size decrease for mono-sized dust particles. The smaller the dust size, the smaller the EM wave attenuation constant. Therefore, the attenuation of the EM wave is mainly determined by the largest dust particles. Finally, we conclude that the larger the dust particle, the stronger the attenuation to the EM wave.

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