Four Generations, Higgs Physics and the MSSM

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Abstract

We consider the effects of a fourth generation of chiral fermions within the MSSM. Such a model offers the possibility of having the lightest neutral Higgs boson significantly heavier than in the three generation MSSM. The model is highly constrained by precision electroweak data, along with Higgs searches at the Tevatron. In addition, the requirements of perturbative unitarity and direct searches for heavy quarks imply that the four generation MSSM is only consistent for \(\tan \beta \sim 1\) and highly tuned 4\textsuperscript{th} generation fermion masses.
I. INTRODUCTION

The Standard Model offers no clue as to why only three generations of chiral fermions are observed. It is thus natural to consider the consequences of a fourth family of heavy fermions\[1, 2\]. The allowed parameter space for a fourth generation is severely restricted by experimental searches, by precision electroweak measurements, and by theoretical constraints from the requirements implied by the perturbative unitaritv of heavy fermion scattering amplitudes and the perturbativity of the Yukawa coupling constants at high energy.

A model with a fourth generation contains charge $2/3$ and $-1/3$ quarks, $t'$ and $b'$, and a charged lepton, $e'$, with its associated neutrino, $\nu'$. Tevatron searches for direct production of a $b'$ $[3]$ imply $m_{\nu'} > 338$ GeV, assuming $b' \rightarrow Wt$, and $m_{\nu} > 335$ GeV, assuming $t' \rightarrow Wq$, with $q = d, s, b [4]$. Relaxing the mixing assumptions changes the limits somewhat, but the $b'$ limits vary by less than 20%, while the $t'$ limits increase in some mixing scenarios to $m_{\nu'} > \mathcal{O}(400 \text{ GeV}) [5]$. In all cases, a fourth generation quark is excluded up to a mass of $\mathcal{O}(300 \text{ GeV})$. We consider 4th generation neutrinos heavier than $M_Z/2$, so there is no constraint from the invisible $Z$ width. From direct production searches for $e'$ and $\nu'$ at LEPII, there is a limit of $\mathcal{O}(100 \text{ GeV})$ on the masses of 4th generation charged leptons and unstable neutrinos. Current bounds on 4th generation Standard Model like fermions are reviewed in Ref. [6–9]. We will typically consider 4th generation lepton masses greater than $\sim 200$ GeV and quark masses greater than $\sim 300-400$ GeV, which are safely above direct detection bounds. Furthermore, we will neglect CKM mixing between the 4th generation and the lighter 3 generations [10].

Precision electroweak measurements place strong constraints on the the allowed fermion masses of a 4th generation, but it is possible to arrange the masses such that cancellations occur between the contributions of the heavy leptons and quarks. By carefully tuning the fourth generation fermion masses, the Higgs boson can be as heavy as $M_h \sim 600$ GeV [6, 11–13]. In a four generation model, therefore, Higgs physics can be significantly altered from that of the Standard Model: Higgs production from gluon fusion is enhanced by a factor of roughly 9 [14], and the Higgs branching ratio to 2 gluons is similarly enhanced [6]. The D0 experiment has recently excluded a SM-like Higgs mass between 131 GeV and 204 GeV produced from gluon fusion in a four generation scenario [15].

It is interesting to consider scenarios with heavy fermions and a neutral Higgs boson heavier than expected from Standard Model electroweak fits. A model of this type is the MSSM with a fourth generation of chiral fermions (4GMSSM). This model has a number of interesting features. Since the mass-squared of the lightest Higgs boson in the MSSM receives corrections proportional to the (mass)$^4$ of the heavy fermions, it is potentially possible to significantly increase the lightest Higgs boson mass in the four generation version of the MSSM [16]. In general, a 4th generation of heavy quarks can contribute to electroweak baryogenesis [17, 18] and Ref. [19] argues that the 4GMSSM with $\tan \beta \sim 1$ can yield a first order electroweak phase transition for 4th generation quark and squark masses just beyond the current Tevatron search bounds.

We discuss the features of the model in Section II and derive unitarity constraints on the fermion masses in Section III. In the 4GMSSM, these constraints can be quite different from those of the four generation version of the Standard Model [20]. Section IV contains limits on the four generation MSSM from precision electroweak measurements. Section V contains some conclusions.
II. THE MODEL

We consider an $N = 1$ supersymmetric model which is an exact replica of the 3 generation MSSM except that it contains a 4th generation of chiral superfields described by the superpotential\cite{21–25}

$$W_4 = \lambda_t \hat{t}' \hat{\psi}_4 (\hat{t}')^c \hat{H}_2 + \lambda_b \hat{b}' \hat{\psi}_4 (\hat{b}')^c \hat{H}_1 + \lambda_e \hat{e}' \hat{\psi}_4 (\hat{e}')^c \hat{H}_1 + \lambda_{\nu} \hat{\nu}_4 (\hat{\nu}')^c \hat{H}_2,$$

(1)

where $\hat{\psi}_4$ is the 4th generation $SU(2)_L$ quark and squark doublet superfield, $\hat{l}_4$ is the 4th generation $SU(2)_L$ lepton and slepton doublet superfield, and $\hat{H}_i$ are the $SU(2)_L$ Higgs superfields. Similarly, $\hat{t}'$, $\hat{b}'$, $\hat{e}'$ and $\hat{\nu}'$ are the 4th generation superfields corresponding to the right-handed fermions. We assume no mixing between $W_4$ and the superpotential of the 3 generation MSSM\cite{1}. The new particles in the 4GMSSM are the 4th generation quarks and leptons (including a right-handed heavy neutrino), along with their associated scalar partners. We assume that the 4th generation neutrino receives a Dirac mass, although our conclusions are relatively insensitive to these assumptions.

The Higgs sector is identical to the 3 generation MSSM and consists of 2 neutral scalars, $h$ and $H$, a pseudo-scalar, $A$, and a charged scalar, $H^\pm$. The Higgs Yukawa couplings of $t'$, $b'$, $e'$ and $\nu'$ are,

$$\lambda_t = \frac{m_t}{v \sin \beta}, \quad \lambda_b = \frac{m_b}{v \cos \beta}, \quad \lambda_e = \frac{m_e}{v \cos \beta}, \quad \lambda_{\nu} = \frac{m_{\nu}}{v \sin \beta},$$

(2)

where $\tan \beta$ is the usual ratio of Higgs vacuum expectation values\cite{26}. Because of the large masses of the 4th generation fermions which are required in order to satisfy restrictions from the experimental searches, the Yukawa couplings quickly become non-perturbative. Requiring perturbativity at the weak scale, a strong bound comes from the restriction $\lambda_{b'}^2 < 4\pi$ which implies\cite{27},

$$\tan \beta < \sqrt{2\pi \left(\frac{v}{m_{b'}}\right)^2 - 1} \sim 1.8,$$

(3)

for $m_{b'} \sim 300 \text{ GeV}$. The evolution of the Yukawa couplings above the weak scale has been studied in Refs. \cite{21, 22, 24} with the conclusion that it is not possible for the 4GMSSM to be perturbative above scales on the order of $10 \sim 1000 \text{ TeV}$. The 4GMSSM thus leads to a picture with an intermediate scale of physics such as that present in gauge mediated SUSY models.

In the 4GMSSM, the lightest Higgs boson mass has an upper bound which receives large corrections proportional to the 4th generation fermion masses. The masses of the neutral Higgs bosons can therefore be significantly heavier than in the case of the 3 generation MSSM and are shown in Fig. \ref{fig:masses} for $\tan \beta = 1$ and representative 4th generation masses\cite{16}. The dominant contributions to the neutral Higgs masses in the 4GMSSM are given in Appendix A\cite{28, 31}.

\footnote{Limits on the 4 generation Standard Model suggest that the mixing between the 3rd and 4th generation is restricted to be small, $\theta_{34} < 0.1$\cite{6, 10}.}
FIG. 1: Predictions for the neutral Higgs boson masses in the four generation MSSM. The squarks and sleptons are assumed to have degenerate masses of 1 TeV. The mass of the lighter Higgs boson, $M_h$, is insensitive to the value of $M_A$. (Not all masses shown here are allowed by the restrictions of perturbative unitarity and electroweak precision measurements, as discussed in Sects. III and IV.)

FIG. 2: Feynman diagrams contributing to $f_i \bar{f}_i \rightarrow f_j \bar{f}_j$ in the high energy limit. $\phi_\alpha$ is a scalar, pseudo-scalar, or Goldstone boson.

III. TREE LEVEL UNITARITY

Chiral fermions have an upper bound on their masses from the requirement of perturbative unitarity of fermion anti-fermion scattering at high energy, originally derived in Ref. [20]. In the MSSM, the unitarity bounds on heavy fermions can be quite different from those of the Standard Model, due to the effects of the additional scalars present in the MSSM, and also to the different fermion Yukawa couplings in the MSSM relative to those of the Standard Model.

Consider an $SU(2)_L$ doublet of heavy left-handed fermions, along with their corresponding right-handed fermion partners,

$$\psi_L = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_L, \ f_1R, f_2R,$$  \hspace{1cm} (4)
with masses $m_1$ and $m_2$. At high energy, $\sqrt{s} >> m_i$, the scattering amplitudes can be most conveniently written in terms of helicity amplitudes. The positive and negative helicity spinors are $u_\pm(p) = P_{L,R}u(p)$ and $v_\pm = P_{L,R}v(p)$, where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. The fermions interact with the scalars of the MSSM and the Goldstone bosons of electroweak symmetry breaking via the interactions,

$$L = \overline{f}_i \left( a_{L}^{i\alpha} P_L + a_{R}^{i\alpha} P_R \right) {f}_j \phi_0^i + \left\{ \overline{f}_i \left( a_{L}^{i2\alpha} P_L + a_{R}^{i2\alpha} P_R \right) {f}_j \phi_1^i + h.c. \right\},$$

(5)

where $\phi_0^i$ and $\phi_1^i$ are generic neutral and charged scalars.

The scattering of $\overline{f}_i f_j \rightarrow \overline{f}_i f_j$ can be found using the Goldstone Boson equivalence theorem to obtain the high energy limits (where $\lambda$ are the helicity indices). The Feynman diagrams are shown in Fig. 2. In the $s-$ channel, the contribution from neutral scalar or pseudo-scalar exchange, $\phi_0^\prime$, to the generic amplitude for $f_i \overline{f}_j \rightarrow f_j \overline{f}_j$ in the high energy limit is,

$$M_s = \overline{u}\phi_0(p_3)(a_{L}^{i\alpha} P_L + a_{R}^{i\alpha} P_R)\overline{v}\phi_0(p_4) \overline{u}\phi_0(p_2)(a_{L}^{i\alpha} P_L + a_{R}^{i\alpha} P_R)u_\lambda(p_1).$$

(6)

The high energy limits of the helicity amplitudes from the $s-$channel contributions are thus,

$$M_s(\lambda^\prime \rightarrow \lambda) = +a_{L}^{i\alpha}\lambda_\lambda$$

$$M_s(\lambda \rightarrow \lambda^\prime) = -a_{R}^{i\alpha}\lambda_\lambda$$

where $s = (p_1 + p_2)^2$, $t = (p_1 - p_2)^2$, and we have assumed $s >> m_1^2, M_\beta, M_W^2$, and $M_Z^2$.

Similarly, the high energy limit of the amplitude resulting from the exchange of a scalar or pseudo-scalar in the $t-$ channel is,

$$M_t = \overline{u}\phi_0(p_3)(a_{L}^{i\alpha} P_L + a_{R}^{i\alpha} P_R)\overline{v}\phi_0(p_4) \overline{u}\phi_0(p_2)(a_{L}^{i\alpha} P_L + a_{R}^{i\alpha} P_R)u_\lambda(p_1),$$

(8)

which yields the helicity amplitudes,

$$M_t(\lambda^\prime \rightarrow \lambda) = -(a_{L}^{i\alpha})^2t$$

$$M_t(\lambda \rightarrow \lambda^\prime) = -(a_{R}^{i\alpha})^2t$$

where $s = (p_1 - p_2)^2$, $t = (p_1 + p_2)^2$, and we have assumed $s >> m_1^2, M_\beta, M_W^2$, and $M_Z^2$.

(We have assumed that all couplings are real).

From the results in Eqs. 7 and 8 it is straightforward to read off the contributions to the partial wave amplitudes for a specific model. The MSSM couplings of the fermions to the scalars can be found in Ref. [26], for example. First consider the scattering of $\overline{f}_i f_1 \rightarrow \overline{f}_i f_i$ in the 4GMSSM. In the $s-$ channel, $h, H, A$, and $G^0$ contribute and their contributions are listed in Table II while the $t-$ channel contributions are shown in Table III. It is apparent that there are many cancellations between the various contributions that are not present in

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2 We have defined $s_\beta = \sin \beta$, $c_\beta = \cos \beta$, $s_\alpha = \sin \alpha$ and $c_\alpha = \cos \alpha$. The mixing in the neutral Higgs sector is described by the angle $\alpha$ which is defined in Appendix A and in Ref. [26].
\[
\begin{array}{|c|c|c|c|}
\hline
\lambda \lambda \rightarrow \lambda' \lambda' & M_h & M_H & M_A \\
\hline
++ \rightarrow ++ & -\frac{c^2}{s^2} & -\frac{s^2}{c^2} & -\cot^2 \beta & -1 \\
++ \rightarrow -- & +\frac{c^2}{s^2} & +\frac{s^2}{c^2} & -\cot^2 \beta & -1 \\
-- \rightarrow ++ & +\frac{c^2}{s^2} & +\frac{s^2}{c^2} & -\cot^2 \beta & -1 \\
-- \rightarrow -- & -\frac{c^2}{s^2} & -\frac{s^2}{c^2} & -\cot^2 \beta & -1 \\
\hline
\end{array}
\]

TABLE I: Contributions from \( s \)– channel exchange of \( h, H, A \), and \( G^0 \) to helicity scattering amplitudes for \( \overline{f}_1 f_1 \rightarrow \overline{f}_1 f_1 \) in the high energy limit of the 4GMSSM. The contributions given in the table must be multiplied by \( \sqrt{2} G_F m_1^2 \).

\[
\begin{array}{|c|c|c|c|}
\hline
\lambda \lambda \rightarrow \lambda' \lambda' & M_h & M_H & M_A \\
\hline
++ \rightarrow -- & +\frac{c^2}{s^2} & +\frac{s^2}{c^2} & -\cot^2 \beta & -1 \\
-- \rightarrow ++ & +\frac{c^2}{s^2} & +\frac{s^2}{c^2} & -\cot^2 \beta & -1 \\
++ \rightarrow -- & -\frac{c^2}{s^2} & -\frac{s^2}{c^2} & -\cot^2 \beta & -1 \\
-- \rightarrow ++ & -\frac{c^2}{s^2} & -\frac{s^2}{c^2} & -\cot^2 \beta & -1 \\
\hline
\end{array}
\]

TABLE II: Contributions from \( t \)– channel exchange of \( h, H, A \), and \( G^0 \) to helicity scattering amplitudes for \( \overline{f}_1 f_1 \rightarrow \overline{f}_1 f_1 \) in the high energy limit of the 4GMSSM. The contributions given in the table must be multiplied by \( \sqrt{2} G_F m_1^2 \).

the Standard Model. The amplitudes for \( \overline{f}_2 f_2 \rightarrow \overline{f}_2 f_2 \) are found by making the replacements \( m_1 \rightarrow m_2, \beta \rightarrow \beta + \frac{\pi}{2}, \alpha \rightarrow \alpha - \frac{\pi}{2} \).

Flavor changing fermion anti-fermion scattering, \( \overline{f}_1 f_1 \rightarrow \overline{f}_2 f_2 \), also yields interesting limits on heavy fermion masses in the 4GMSSM. The \( s \)–channel contributions to the high energy limits of the helicity scattering amplitudes are shown in Table III and the \( t \)– channel contributions from \( H^+ \) and \( G^+ \) exchange in Table IV.

Bounds on the fermion masses come from the coupled channel \( J = 0 \) partial wave amplitudes for \( f_i^\lambda \overline{f}_1^\lambda \rightarrow f_j^\lambda \overline{f}_2^\lambda \) \([20, 32]\),

\[
a_0 = \left. \frac{1}{16 \pi s} \int_{-s}^{0} | M | \right|, \tag{10}
\]

where \( | M | \) is the sum of the \( s \)– and \( t \)– channel helicity amplitudes given in the tables. Perturbative unitarity requires that the eigenvectors of the scattering matrix satisfy \( | a_0 | < 1 \)\([33]\). In the scattering basis, \( f_1^+, \overline{f}_1^+, f_2^+, \overline{f}_2^+, f_1^-, \overline{f}_1^-, f_2^-, \overline{f}_2^- \), the high energy limit of the \( J = 0 \) coupled partial wave scattering matrix is,

\[
| a_0 | \equiv B = \frac{G_F}{4\sqrt{2}\pi} \begin{pmatrix} m_1^2 & 0 & 0 \\ \frac{c^2}{s^2} & 0 & 0 \\ 0 & 0 & \frac{m_1^2}{s^2} \end{pmatrix}.
\tag{11}
\]
TABLE III: Contributions from $s-$ channel exchange of $h, H, A$ and $G^0$ to helicity scattering amplitudes for $f_1 f_1 \rightarrow f_2 f_2$ in the high energy limit. An overall factor of $\sqrt{2} G_F m_1 m_2$ is omitted.

| $\lambda \bar{\lambda} \rightarrow \lambda \bar{\lambda}$ | $M_h$ | $M_H$ | $M_A$ | $M_{G^0}$ |
|-----|-----|-----|-----|-----|
| $++ \rightarrow ++$ | $\sin 2\alpha$ | $-\sin 2\alpha$ | $-1$ | $+1$ |
| $++ \rightarrow --$ | $\sin 2\beta$ | $\sin 2\beta$ | $-1$ | $+1$ |
| $-- \rightarrow ++$ | $\sin 2\alpha$ | $\sin 2\alpha$ | $-1$ | $+1$ |
| $-- \rightarrow --$ | $-\sin 2\alpha$ | $-\sin 2\alpha$ | $-1$ | $+1$ |

TABLE IV: Contributions from $t-$ channel exchange of $H^+$ and $G^+$ to helicity scattering amplitudes for $f_1 f_1 \rightarrow f_2 f_2$ in the high energy limit. An overall factor of $2\sqrt{2} G_F$ is omitted.

| $\lambda \bar{\lambda} \rightarrow \lambda \bar{\lambda}$ | $M_{H^+}$ | $M_{G^+}$ |
|-----|-----|-----|
| $++ \rightarrow --$ | $+m_1 m_2$ | $-m_1 m_2$ |
| $-- \rightarrow ++$ | $+m_1 m_2$ | $-m_1 m_2$ |
| $++ \rightarrow --$ | $-m_2^2$ | $-m_2^2$ |
| $-- \rightarrow ++$ | $-m_1^2$ | $-m_1^2$ |

Enforcing the unitarity condition, $|a_0| < 1$, on the eigenvalues of Eq. (11) gives the restrictions,

$$m_1^2 < s_\beta^2 4\sqrt{2} \pi \frac{G_F}{m_1^4}$$

$$m_2^2 < c_\beta^2 4\sqrt{2} \pi \frac{G_F}{m_2^4}.$$  

(12)

A further interesting limit is found from the coupled channel scattering of the helicity amplitudes $f_1^+ f_1^-, f_1^+ f_2^-, f_2^+ f_2^-$, with

$$|a_0| = \frac{G_F}{4\sqrt{2} \pi} \begin{pmatrix} 0 & m_2^2 & 0 & m_2^2 \\ m_2^2 & 0 & m_2^2 & 0 \\ 0 & m_2^2 & 0 & m_2^2 \\ m_2^2 & 0 & m_2^2 & 0 \end{pmatrix}.$$  

(13)

Requiring the largest eigenvalue of Eq. (13) to be $< 1$,

$$\lambda_{\text{max}} = \frac{G_F}{4\pi} \sqrt{\frac{m_1^4}{c_\beta^4} + \frac{m_2^4}{c_\beta^4}} < 1.$$  

(14)

The bounds of Eqs (12) and (14) are relevant for a heavy lepton doublet in the 4GMSSM and the allowed regions are shown in Fig. 3. These bounds can be compared with the Standard Model bounds, $m_{\text{lepton}}^2 < 4\sqrt{2} \pi \frac{G_F}{m_1^4} = (1.2 \text{ TeV})^2$. For $\tan \beta = 1$, the bound is reduced from the Standard Model value to $m_{\text{lepton}} < 750 \text{ GeV}$. For $\tan \beta = 10$, the value of $m_2$ ($m_\nu$) allowed by unitarity is $m_\nu < 100 \text{ GeV}$, which is excluded by experimental searches.
FIG. 3: Unitarity restriction on a 4th generation lepton doublet in the 4GMSSM. The allowed region with the vertical (diagonal) cross-hatches corresponds to \( \tan \beta = 10(1) \).

The bounds on a heavy quark doublet in the 4GMSSM can be found by considering the color neutral scattering amplitudes\(^3\). In the basis, \( f_1^+ f_1^+ , f_2^+ f_2^+ , f_1^- f_1^- , f_2^- f_2^- \), the coupled channel scattering matrix for the \( J = 0 \) partial wave amplitude is a 12 \( \times \) 12 matrix of the form,

\[
| a_0 | \sim \begin{pmatrix} B & B & B \\ B & B & B \\ B & B & B \end{pmatrix},
\]

where the 3 \( \times \) 3 color neutral matrix \( B \) is given in Eq. [11]. Restricting the eigenvectors to be less than 1 gives the restrictions on 4th generation quark masses shown in Fig. [4].

\[
m_1^2 < s_\beta^4 \frac{4\sqrt{2}\pi}{3G_F},
\]

\[
m_2^2 < c_\beta^4 \frac{4\sqrt{2}\pi}{3G_F}.
\]

It is apparent that the experimental bounds of \( m_\nu > 335 \text{ GeV} \) and \( m_\nu > 338 \text{ GeV} \) are close to violating perturbative unitarity in the 4GMSSM with \( \tan \beta = 1 \). For larger \( \tan \beta \), the parameters are even more restricted.

IV. LIMITS FROM PRECISION ELECTROWEAK MEASUREMENTS

The limits on the 4GMSSM from precision electroweak measurements can be studied assuming that the dominant contributions are to the gauge boson 2-point functions\(^{[34, 35]}\).

\(^3\) The logic is identical to Ref. [20].
FIG. 4: Unitarity restriction on a 4th generation quark doublet in the 4GMSSM. The allowed region with the vertical (diagonal) cross-hatches corresponds to $\tan \beta = 10(1)$.

\[
\Pi_{XY}(p^2) = \Pi_{XY}(p^2)g^{\mu\nu} + B_{XY}(p^2)p^\mu p^\nu, \text{ with } XY = \gamma\gamma, \gamma Z, ZZ \text{ and } W^+ W^-,
\]

\[
\alpha S = \left( \frac{4s_W^2 c_W^2}{M_Z^2} \right) \left\{ \Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) - \Pi_{\gamma\gamma}(M_Z^2) \right\}
- \frac{c_W^2 - s_W^2}{c_W s_W} \left( \Pi_{\gamma Z}(M_Z^2) - \Pi_{\gamma Z}(0) \right)
\]

\[
\alpha T = \left( \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{2s_W \Pi_{\gamma Z}(0)}{c_W M_Z^2} \right)
\]

\[
\alpha U = 4s_W^2 \left\{ \frac{\Pi_{WW}(M_W^2)}{M_W^2} - \frac{\Pi_{WW}(0)}{M_W^2} \right\}
- \frac{c_W^2}{M_Z^2} \left( \Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) \right)
- 2s_W c_W \left( \Pi_{\gamma Z}(M_Z^2) - \Pi_{\gamma Z}(0) \right)
- s_W^2 \frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2} \right\}, \tag{17}
\]

where $s_W \equiv \sin \theta_W$ and $c_W \equiv \cos \theta_W$ and any definition of $s_W$ can be used in Eq. 17 since the scheme dependence is higher order. The contributions to $S, T, \text{ and } U$ from fourth generation fermions, squarks, and the scalars of the MSSM are given in Appendix B[34–44]. Our definition of $U$ differs from that of Ref. [40] and so should not be compared with those results. The potential contributions from other MSSM particles such as charginos and neutralinos decouple for heavy masses[41] and we omit them here.

Considerable insight can be gained from various limits of $S, T \text{ and } U$. We begin by considering the contributions from a heavy fermion generation as defined in Eq. [34, 40, 44]. The potentially large isospin violating contributions to $\Delta T_f$ imply that fermions in an $SU(2)_L$ doublet must have nearly degenerate masses. For a fermion doublet with $m_f^2 = \ldots$
\[m^2_f + \delta m^2_f \ll m^2_{1,2}, M^2_W, M^2_Z, \text{ and } m^2_{1,2} \gg M^2_W, M^2_Z,\]

\[
\Delta S_f \rightarrow \frac{N_c}{6\pi} \left\{ 1 - 2Y_f \left( \frac{\delta m^2}{m^2_f} \right) \right\},
\]

\[
\Delta T_f \rightarrow \frac{N_c}{48\pi^2 s_W^2 M^2_W} \left( \frac{\delta m^2}{m^2_f} \right)^2,
\]

\[
\Delta U_f \rightarrow \frac{N_c}{30\pi} \left( \frac{\delta m^2}{m^2_f} \right)^2,
\]

(18)

where \(N_c = 3(1)\) and \(Y_f = \frac{1}{6}(-\frac{1}{2})\) for a quark or lepton doublet. Both \(\Delta U_f\) and \(\Delta T_f\) are isospin violating, but \(\Delta U_f\) is suppressed by a factor of \(M^2_f/m^2_f\) relative to \(\Delta T_f\) and is numerically small. \(\Delta S_f\) does not decouple for large fermion masses and so poses a potential problem for consistency with the experimental limits from precision electroweak measurements\[12, 13\]. By carefully arranging the 4th generation quark and lepton masses, however, it is possible to find values of the fermion masses where the contribution to \(\Delta S_f\) is reduced from its value for degenerate fermion partners of \(\frac{N}{6\pi}\), while still respecting the limits on \(\Delta T_f\[6\]. This possibility is due to the strong correlation between the experimental limits on \(\Delta S\) and \(\Delta T\[6, 11\].

The 4th generation squarks and sleptons are denoted by,

\[
\left( \tilde{\nu}^L, \tilde{b}^L \right), \left( \tilde{\nu}'^L, \tilde{c}'^L \right), \tilde{\nu}^R, \tilde{b}^R, \tilde{c}^R, \tilde{\nu}^R
\]

(19)

Consider the limit of no mixing between the left- and right- handed sfermion partners, and also no mixing between the sfermion generations. (The mixing between left- and right-handed sfermions is included in the formulae in Appendix B). In this limit, the the contribution from sfermions with small mixing between the isospin partners, \(\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_R} \ll \tilde{m}^2_{\nu_L}, \tilde{m}^2_{\nu_R}\), is \[36, 39\],

\[
\Delta S_{sf} \rightarrow -\frac{1}{12\pi} \left[ 3Y_q \left( \frac{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_R}}{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}} \right) + Y_f \left( \frac{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}}{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}} \right) \right] + O\left( \frac{1}{m^4} \right).
\]

(20)

(Note that only the scalar partners of the left-handed sfermions contribute in this limit). For intermediate values of the sfermion masses, it is possible to arrange cancellations between the slepton and squark contributions. In the same limit, the contributions from squarks and sleptons to the \(\Delta T_{sf}\[36, 41, 43\], and \(\Delta U_{sf}[36\) parameters are \[36, 41, 43\],

\[
\Delta T_{sf} \rightarrow \frac{1}{48\pi s_W^2 M^2_W} \left[ \frac{3}{2} \left( \frac{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_R}}{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}} \right) + \frac{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}}{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}} \right] + O\left( \frac{1}{m^4} \right).
\]

\[
\Delta U_{sf} \rightarrow -\frac{1}{30\pi} \left[ \frac{3}{2} \left( \frac{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_R}}{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}} \right) + \frac{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}}{\tilde{m}^2_{\nu_L} - \tilde{m}^2_{\nu_L}} \right] + O\left( \frac{1}{m^4} \right).
\]

(21)

The sfermion contributions decouple for heavy masses and for sfermions with \(TeV\) scale masses, the effects on precision electroweak constraints are small. In our numerical results, we use the complete amplitudes given in Appendix B. The major effect of heavy sfermions
in the 4GMSSM is to increase the predictions for the neutral Higgs masses, as shown in Fig. 11.

We study the restriction on the 4GMSSM using the fits to $\Delta S$, $\Delta T$, and $\Delta U$ given by the GFITTER collaboration[13].

\[
\begin{align*}
\Delta S &= S - S_{SM} = 0.02 \pm 0.11 \\
\Delta T &= T - T_{SM} = 0.05 \pm 0.12 \\
\Delta U &= U - U_{SM} = +0.07 \pm 0.12
\end{align*}
\]  

(22)

with the Standard Model values defined by $M_{h,ref} = 120 \text{ GeV}$ and $M_t = 173.2 \text{ GeV}$. The associated correlation matrix is,

\[
\rho_{ij} = \begin{pmatrix}
1.0 & 0.879 & -0.469 \\
0.879 & 1.0 & -0.716 \\
-0.469 & -0.716 & 1.0
\end{pmatrix}.
\]

$\Delta \chi^2$ is defined as

\[
\Delta \chi^2 = \sum_{ij} (\Delta X_i - \Delta \hat{X}_i)(\sigma^2)_{ij}^{-1}(\Delta X_i - \Delta \hat{X}_i),
\]

where $\Delta \hat{X}_i = \Delta S, \Delta T, \text{ and } \Delta U$ are the central values of the fit in Eq. 22, $\Delta X_i = \Delta S, \Delta T, \text{ and } \Delta U$ include the 4th generation fermions, sfermions, and MSSM scalars, $\sigma_i$ are the errors given in Eq. 22 and $\sigma^2_{ij} = \sigma_i \rho_{ij} \sigma_j$. The 95\% confidence level limit corresponds to $\Delta \chi^2 = 7.815$.

In Fig. 5 we show the 95\% confidence level allowed region for $m_{t'} = 400 \text{ GeV}$, $M_A = 300 \text{ GeV}$, and $m_{s} = 1 \text{ TeV}$ including 4 generations of sfermions with degenerate masses, $m_{sl} = 1 \text{ TeV}$. The difference between the masses of the quark and lepton isospin $+\frac{1}{2}$ and $-\frac{1}{2}$ doublet partners is scanned over (while imposing the requirement of perturbative unitarity as discussed in the previous section) to find the allowed regions. We note that the point with all 4th generation masses degenerate is not allowed. As pointed out in Refs. [6, 11, 45] for the Standard Model case, the fermion masses must be carefully tuned to find agreement with precision electroweak measurements. As $\tan \beta$ is increased, the allowed region shrinks and for the parameters of Figs. 5 and 6 there is no allowed region with $\tan \beta > 2.5$. The Higgs boson masses vary within these plots according to Eq. 28. Fig. 7 demonstrates the effects of increasing the charged lepton mass. The effect of increasing the $t'$ mass is shown in Fig. 7 and we see that the allowed parameter space is significantly shrunk from Figs. 5 and 6. In Fig. 8 we show the allowed range of Higgs masses corresponding to the scan of Fig. 5 and imposing the experimental constraints on 4th generation masses. It is apparent that the 4GMSSM requires highly tuned fermion masses in order to be viable.

The effect of increasing $m_A$ (and hence $M_h$) is shown in Fig. 9 and we see only a very small region of allowed parameters. In Fig. 10, we show the effects of lowering the sfermion masses to 500 GeV and see that the allowed region shrinks considerably. This is due not to the effects of sfermion contributions to the electroweak limits, but rather to the change in Higgs mass corresponding to the heavier squark masses.

V. CONCLUSIONS

We have studied an extension of the MSSM with 4 generations of chiral fermions. The existence of a 4th generation allows the lightest neutral Higgs boson mass to be considerably
heavier than in the Standard Model. However, imposing the restrictions of perturbative unitarity and constraints from precision electroweak measurements requires \( \tan \beta \sim 1 \) and extremely fine-tuned values of the fermion masses.

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Appendix A: Neutral Higgs Mass

The neutral Higgs masses are found from the eigenvectors of the matrix[26]:

\[
M^2 = \begin{pmatrix}
M_{11} & M_{21} \\
M_{12} & M_{22}
\end{pmatrix}
\] (24)

where \( M_{ij} \equiv M_{ij,\text{tree}} + \Delta_{ij} \) The tree level values are (where \( c_\beta = \cos \beta \) and \( s_\beta = \sin \beta \)),

\[
\begin{align*}
M_{11,\text{tree}} &= M_A^2 s_\beta^2 + M_Z^2 c_\beta^2 \\
M_{12,\text{tree}} &= -(M_A^2 + M_Z^2) s_\beta c_\beta \\
M_{22,\text{tree}} &= M_A^2 c_\beta^2 + M_Z^2 s_\beta^2
\end{align*}
\] (25)
Constraints from oblique parameters and unitarity

$m_t = 400 \text{ GeV}, \ m_e = 400 \text{ GeV}, \ m_A = 1 \text{ TeV}, \ m_A' = 300 \text{ GeV}$

FIG. 6: 95 % confidence level allowed regions from fits to S, T, and U in the 4GMSSM. The requirement of perturbative unitarity is also imposed. From top to bottom, the curves correspond to $\tan \beta = 2$ and $1$. The only difference from Fig. 5 is that $m_{e'} = 400 \text{ GeV}$ here.

At one-loop the effects of a heavy 4th generation on the neutral Higgs masses, including only the leading logarithms and assuming no mixing in the sfermion sector, are

$$
\Delta_{11} = \hat{e}_b = \sum_{i=b',e'} \frac{N_i G_F}{2 \sqrt{2} \pi^2} \frac{m_i^4}{c_\beta} \ln \left( \frac{\tilde{m}_{11}^2 \tilde{m}_{12}^2}{m_i^4} \right)
$$

$$
\Delta_{22} = \hat{e}_t = \sum_{i=t',\nu'} \frac{N_i G_F}{2 \sqrt{2} \pi^2} \frac{m_i^4}{s_\beta} \ln \left( \frac{\tilde{m}_{11}^2 \tilde{m}_{12}^2}{m_i^4} \right)
$$

$$
\Delta_{12} = 0,
$$

where $\tilde{m}_{11}$ and $\tilde{m}_{12}$ are the physical sfermion masses associated with $f_i$.

The neutral Higgs boson masses are then,

$$
m_{H,h}^2 = \frac{1}{2} \left\{ M_A^2 + M_Z^2 + \hat{e}_b + \hat{e}_t \pm \left[ (M_A^2 + M_Z^2)^2 - 4 c_{2\beta} M_A^2 M_Z^2 \right]
+ \left( \hat{e}_b - \hat{e}_t \right) \left[ 2 c_{2\beta} (M_Z^2 - M_A^2) + \hat{e}_b - \hat{e}_t \right] \right\}^{1/2}.
$$

Formulae including non-logarithmic terms and 2-loop contributions to the neutral Higgs boson masses can be found in Ref. [42]. However, since we assume very heavy 4th generation quarks, we expect Eq. 28 to be a good approximation.

The mixing angle (which we use to define the fermion and sfermion couplings) is

$$
sin 2\alpha = \frac{2 M_{11}^2}{\sqrt{(M_{11}^2 - M_{22}^2)^2 + 4 M_{12}^4}}
$$
Constraints from oblique parameters and unitarity
\[ m_t = 500 \text{ GeV}, \quad m_{\nu} = 300 \text{ GeV}, \quad m_{\chi} = 1 \text{ TeV}, \quad m_{\chi} = 300 \text{ GeV} \]

**FIG. 7:** 95 % confidence level allowed regions from fits to S, T, and U in the 4GMSSM. The requirement of perturbative unitarity is also imposed. From top to bottom, the curves correspond to \( \tan \beta = 2 \) and 1. The only difference from Fig. 5 is that \( m_t = 500 \text{ GeV} \) here.

**Appendix B: Summary of S,T,U formula**

From a heavy \( SU(2) \) Standard Model like doublet of fermions with masses \((m_1, m_2)\) and isospin \( Y_f^{[40, 44, 46]} \),

\[
\Delta S_f = \frac{N_c}{6\pi M_Z^2} \left\{ 2Y_f \left[ -M_Z^2 \ln \left( \frac{m_1^2}{m_2^2} \right) + (2m_1^2 + M_Z^2) f_0(m_Z^2, m_1^2) \right] \right.
\]
\[
- \left. (2m_2^2 + M_Z^2) f_0(m_Z^2, m_2^2) \right] + 3m_1^2 f_0(m_Z^2, m_1^2) + 3m_2^2 f_0(m_Z^2, m_2^2) \right\}
\]

\[
\Delta T_f = \frac{N_c}{16\pi s_W^2 M_W^2} F(m_1^2, m_2^2)
\]

\[
\Delta U_f = -\frac{N_c}{\pi M_W^2} \left\{ \left[ M_W^2 \left[ \frac{1}{3} - \frac{m_1^2 + m_2^2}{6} - \frac{(m_1^2 - m_2^2)^2}{6M_W^2} \right] f(M_W^2, m_1^2, m_2^2) + \frac{m_1^2 + m_2^2}{4} \right] \right.
\]
\[
+ \frac{(m_1^2 - m_2^2)^2}{6M_W^2} \left[ \frac{m_1^4 - m_2^4}{12M_W^4} + \frac{m_1^2 m_2^2}{2(m_1^2 - m_2^2)} \right] \ln \left( \frac{m_1^2}{m_2^2} \right)
\]
\[
\left. + c_W^2 \left[ \frac{m_1^2 - M_Z^2}{6} f_0(M_Z^2, m_1^2) + \frac{m_2^2 - M_Z^2}{6} f_0(M_Z^2, m_2^2) \right] \right\} \]

where

\[
F(x, y) = x + y - \frac{2xy}{x - y} \log \left( \frac{x}{y} \right)
\]

\[
f(p^2, m_1^2, m_2^2) = -\int_0^1 dx \log \left[ \frac{x m_2}{m_1} + (1 - x) \frac{m_1}{m_2} - x(1 - x) \frac{p^2}{m_1 m_2} \right]
\]
FIG. 8: Neutral Higgs boson masses allowed by precision electroweak measurements and by unitarity in the 4GMSSM. The mass difference between $b'$ and $t'$ is scanned over. The experimental constraints on the 4th generation masses are also imposed.

$$f_0(p^2, m^2) = f(p^2, m^2, m^2) = 2 - 2\sqrt{\frac{4m^2}{p^2}} - 1 \tan^{-1}\left(\frac{\frac{1}{2}}{\sqrt{\frac{4m^2}{p^2}} - 1}\right).$$

(31)

$N_c = 3$ for quarks and 1 for a lepton doublet. The hypercharge is $Y_q = \frac{1}{6}$ for a quark doublet and $Y_l = -\frac{1}{2}$ for a lepton doublet.

We define $\tilde{m}_{t_1}, \tilde{m}_{t_2}, \tilde{m}_{b_1}$, and $\tilde{m}_{b_2}$ to be the physical masses of the 4th generation squarks with the mixing angles, $\theta_t$ and $\theta_b$, $(s_b = \sin\theta_b$, etc). From the 4th generation squarks [36, 41]:

$$\Delta S_{sf} = -\frac{N_c}{2\pi M_Z^2} \left\{ Q_t \left[ c_5^2 F_5(M_Z^2, \tilde{m}_{t_1}^2, \tilde{m}_{t_1}^2) + s_5^2 F_5(M_Z^2, \tilde{m}_{t_2}^2, \tilde{m}_{t_2}^2) \right] 
- Q_b \left[ c_b^2 F_5(M_Z^2, \tilde{m}_{b_1}^2, \tilde{m}_{b_1}^2) + s_b^2 F_5(M_Z^2, \tilde{m}_{b_2}^2, \tilde{m}_{b_2}^2) \right] 
- \frac{1}{2} \left[ c_t^2 F_5(M_Z^2, \tilde{m}_{t_1}^2, \tilde{m}_{t_1}^2) + 2c_t s_t F_5(M_Z^2, \tilde{m}_{t_1}^2, \tilde{m}_{t_2}^2) + s_t^2 F_5(M_Z^2, \tilde{m}_{t_2}^2, \tilde{m}_{t_2}^2) \right] 
+ s_t^2 F_5(M_Z^2, \tilde{m}_{b_1}^2, \tilde{m}_{b_1}^2) + c_t^2 F_5(M_Z^2, \tilde{m}_{b_1}^2, \tilde{m}_{b_1}^2) 
+ 2c_t s_t F_5(M_Z^2, \tilde{m}_{b_2}^2, \tilde{m}_{b_2}^2) + s_t^2 F_5(M_Z^2, \tilde{m}_{b_2}^2, \tilde{m}_{b_2}^2) \right\}$$

$$\Delta T_{sf} = -\frac{N_c}{16\pi M_W^2 s_W^2} \left\{ -s_t^2 c_t^2 F(\tilde{m}_{t_1}^2, \tilde{m}_{t_2}^2) - s_t^2 c_b^2 F(\tilde{m}_{b_1}^2, \tilde{m}_{b_2}^2) 
+ c_t^2 c_b^2 F(\tilde{m}_{t_1}^2, \tilde{m}_{b_1}^2) + c_t^2 s_b^2 F(\tilde{m}_{t_1}^2, \tilde{m}_{b_2}^2) 
+ s_t^2 c_b^2 F(\tilde{m}_{t_2}^2, \tilde{m}_{b_1}^2) + s_t^2 s_b^2 F(\tilde{m}_{t_2}^2, \tilde{m}_{b_2}^2) \right\}.$$
where, \( \beta \) to \( \tan \) requirement of perturbative unitarity is also imposed. From top to bottom, the curves correspond to \( \tan \beta = 2 \) and 1. The only difference from Fig. 5 is that \( m_A = 1 \ TeV \) here.

\[
\Delta U_{sf} = -\frac{N_c}{4\pi M_W^2} \left\{ c_W^2 \left( c_t^4 F_5(M_Z^2, \tilde{m}_{t1}^2, \tilde{m}_{t2}^2) + s_t^4 F_5(M_Z^2, \tilde{m}_{t1}^2, \tilde{m}_{t2}^2) \right)
+ 2s_t^2 c_t F_5(M_Z^2, \tilde{m}_{t1}^2, \tilde{m}_{t2}^2) + c_b^4 F_5(M_Z^2, \tilde{m}_{b1}^2, \tilde{m}_{b2}^2)
+ s_b^4 F_5(M_Z^2, \tilde{m}_{b1}^2, \tilde{m}_{b2}^2) + 2s_b^2 c_b^2 F_5(M_Z^2, \tilde{m}_{b1}^2, \tilde{m}_{b2}^2) \right\} - 2 \left\{ c_t^2 s_b c_b F_5(M_W^2, \tilde{m}_{t1}^2, \tilde{m}_{b2}^2) + s_t^2 c_b^2 F_5(M_W^2, \tilde{m}_{t2}^2, \tilde{m}_{b2}^2) + s_b^2 c_t^2 F_5(M_W^2, \tilde{m}_{t1}^2, \tilde{m}_{b2}^2) + s_t^2 s_b^2 F_5(M_Z^2, \tilde{m}_{t2}^2, \tilde{m}_{b2}^2) \right\}
\]

where,

\[
F_5(p^2, m_1^2, m_2^2) = \int_0^1 dx \left[ (1 - 2x)(m_1^2 - m_2^2) + (1 - 2x)^2 p^2 \right] \Lambda
\]
\[
\overline{F}_5(p^2, m_1^2, m_2^2) = F_5(p^2, m_1^2, m_2^2) - F_5(0, m_1^2, m_2^2)
\]
\[
\Lambda = \log \left( (1 - x)m_1^2 + xm_2^2 - x(1 - x)p^2 \right)
\]

The contributions from the 4th generation sleptons are found in an analogous manner.

From Higgs scalars, (where \( M_{h, ref} \) is the value of the Standard Model Higgs boson mass for which the fits are performed.), the contribution of the MSSM scalars is \([41, 47]\),

\[
\Delta S_H = S_H(M_h, M_H, M_A, M_{H\pm}) - S_{SM}(M_{h, ref})
= \frac{1}{4\pi} \left\{ \sin^2(\beta - \alpha) \left[ \frac{1}{M_Z^2} \overline{F}_5(M_Z^2, M_H^2, M_A^2) + F_2(M_Z^2, M_H^2, M_A^2) \right] \right\}
\]

FIG. 9: 95 % confidence level allowed regions from fits to S, T, and U in the 4GMSSM. The requirement of perturbative unitarity is also imposed. From top to bottom, the curves correspond to \( \tan \beta = 2 \) and 1. The only difference from Fig. 5 is that \( m_A = 1 \ TeV \) here.
FIG. 10: 95% confidence level allowed regions from fits to S, T, and U in the 4GMSSM. The requirement of perturbative unitarity is also imposed. The only difference from Fig. 5 is that $m_u = 500$ GeV here.

\[ + \cos^2(\beta - \alpha) \left[ \frac{1}{M_Z^2} T_5(M_Z^2, M_h^2, M_A^2) + F_2(M_Z^2, M_h^2, M_A^2) \right] \]

\[ - \frac{1}{M_Z^2} T_5(M_Z^2, M_h^2, M_A^2) - F_2(M_Z^2, M_h^2, M_A^2) \}

\[ \Delta T_H = T_H(M_h, M_H, M_A, M_{H\pm}) - T_{sm}(M_{h,ref}) \]

\[ = \frac{1}{32\pi M_W^2 s_W} \left\{ \cos^2(\beta - \alpha) \left[ G_3(M_W^2, M_Z^2, M_A^2) + G_3(M_Z^2, M_{H\pm}^2, M_A^2) \right] \right. \]

\[ + \sin^2(\beta - \alpha) \left[ G_3(M_W^2, M_Z^2, M_A^2) + G_3(M_Z^2, M_{H\pm}^2, M_A^2) \right] \]

\[ - 8M_Z^2 G_4(M_Z^2, M_{H\pm}^2, M_h^2) + 8M_W^2 G_4(M_W^2, M_{H\pm}^2, M_h^2) \]

\[ + F(M_{H\pm}^2, M_A^2) - G_3(M_W^2, M_Z^2, M_{H\pm}^2) \]

\[ + 8M_Z^2 G_4(M_Z^2, M_{H\pm}^2, M_{h,ref}^2) - 8M_W^2 G_4(M_W^2, M_{H\pm}^2, M_{h,ref}^2) \} \]

\[ \Delta U_H = U_H(M_h, M_H, M_A, M_{H\pm}) - U_{sm}(M_{h,ref}) \]

\[ = \frac{1}{4\pi} \left\{ -\frac{1}{M_Z^2} T_5(M_Z^2, M_{H\pm}^2, M_{H\pm}^2) + \frac{1}{M_W^2} T_5(M_W^2, M_A^2, M_{H\pm}^2) \right. \]

\[ + \sin^2(\beta - \alpha) \left[ \frac{1}{M_W^2} T_5(M_W^2, M_{H\pm}^2, M_{H\pm}^2) - \frac{1}{M_Z^2} T_5(M_Z^2, M_{H\pm}^2, M_A^2) \right] \]

\[ + F_2(M_W^2, M_{H\pm}^2, M_h^2) - F_2(M_Z^2, M_{H\pm}^2, M_h^2) \] + \cos^2(\beta - \alpha) \left[ -\frac{1}{M_Z^2} T_5(M_Z^2, M_h^2, M_A^2) \right] \]
\[
\begin{align*}
&\left. + \frac{1}{M_W} \mathcal{F}_5(M_W^2, M_{H^\pm}^2, M_H^2) + F_2(M_W^2, M_W^2, M_H^2) - F_2(M_Z^2, M_Z^2, M_H^2) \right] \\
&\left. - F_2(M_W^2, M_W^2, M_{h,ref}^2) + F_2(M_Z^2, M_Z^2, M_{h,ref}^2) \right] \\
&= (34)
\end{align*}
\]

where

\[
\begin{align*}
\hat{B}_0(p^2, m_1^2, m_2^2) &= B_0(p^2, m_1^2, m_2^2) - B_0(0, m_1^2, m_2^2) \\
B_0(p^2, m_1^2, m_2^2) &= \frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{m_1 m_2} \right)^\epsilon \Gamma(1+\epsilon) + f(p^2, m_1^2, m_2^2) \\
F_2(p^2, m_1^2, m_2^2) &= \frac{1}{p^2} \mathcal{F}_5(p^2, m_1^2, m_2^2) - 4\hat{B}_0(p^2, m_1^2, m_2^2) \\
G_3(m_1^2, m_2^2, m_3^2) &= F(m_1^2, m_3^2) - F(m_2^2, m_3^2) \\
G_4(m_1^2, m_2^2, m_3^2) &= m_3^2 \log \left( \frac{m_1^2}{m_3^2} \right) - m_2^2 \log \left( \frac{m_1^2}{m_2^2} \right)
\end{align*}
\]

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