A procedure to derive bounds on coupling strengths of exotic particles to nucleons from the neutrino signal of supernovae is outlined. The analysis is based on a model independent calculation for the emissivities for the exotic, detailed simulation for the evolution of the early proto-neutron star as well as a Likelihood analysis. As an example we derive confidence levels for the upper bound of the size of gravity only extra dimensions.

In the aftermath of a core collapse supernova we find a system described by extreme parameters: densities of several times nuclear matter density with simultaneous appearance of temperatures of several tens of MeV. Indeed, the matter is that dense and hot that even particles interacting as weakly as neutrinos—the by far most efficient conventional cooling mechanism—get trapped. Therefore, the cooling of the nascent proto–neutron star (PNS) has to happen on diffusion times scales of tens of seconds. It is this delayed cooling in the standard scenario that makes core collapse supernovae one of the most efficient elementary particle physics labs possible: any additional cooling mechanism will influence the time structure of the neutrino signal as long as its interaction with the medium is not too strong that it gets trapped as well.

To make this kind of an argument more quantitative several ingredients are in principle necessary: first one needs a reliable method to calculate the primary emission rates of the exotic under discussion in the inner core. The calculated emissivities are then to be fed into a simulation capable of describing the first several tens of seconds of a nascent PNS. The results of these simulations carried out for different coupling strengths of the additional cooling mechanism are to be compared with the data and analyzed with the appropriate statistical method. This leads then to bounds on the coupling strength of the exotic studied. The chain of arguments is illustrated in figure 1. Also indicated in the diagram there is a possible short–cut, here labeled as Raffelt criterium. This particular criterium is one of a group of criteria meant to allow to read a constraint off the emissivities directly. Although useful to get a very rough estimate of the allowed strength of an exotic one should keep in mind that the emission rates of particles in the nuclear medium are strongly temperature and energy dependent. The core temperature in the supernova, however, depends on whether or not there is an additional cooling mechanism present. Thus it is difficult if not impossible to fix a fiducial temperature and density to be used.

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in the criteria that is valid for all kinds of particles (note that the temperature dependence of a particular production process depends on the energy/momentum dependence of the fundamental coupling vertex of the exotic to the medium and thus depends of the properties of the particle studied). Thus, to get a reliable bound, a detailed simulation is required after all.

Let us now go through the structure of the analysis in more detail: the first step is the calculation of the emissivity of the exotic. In our work we follow the method developed in ref. 2. It is based on the observation that, if the radiation is soft, the production rate can be expressed in terms of the on–shell nucleon nucleon scattering data directly—the process is dominated by radiation off external legs. Obviously this kinematic requirement is best fulfilled in degenerate nuclear matter as it occurs in the late cooling phase of a PNS. It was shown in ref. 2 that our model independent approach leads to rates for neutrino–anti-neutrino bremsstrahlung that are significantly smaller than those used so far. This finding was confirmed recently, as reported in ref. 3. The separation of scales is not that clear when we move to the physical conditions present in the first 20 s after the collapse. With temperatures of some tens of MeV and densities of several times nuclear matter density, matter should be expected to be neither degenerate nor non–degenerate. However, the investigations of ref. 4 suggest, that the relevant expansion parameter, namely the radiation energy in units of the average nucleon–nucleon energy, still is smaller than one even here. Using this approach emissivities where already calculated for axions 2 as well as graviton only extra dimensions (GODs) 4. In addition the latter results were used as a basis for bounds derived on the ground of cosmological arguments 6, 7.

Independent of the production mechanism, it is always possible to cast the production rate in the following form

$$\frac{dE}{dt} = a_n \left( \frac{nB}{n_0} \right) \left( \frac{T}{10 \text{ MeV}} \right)^{p_n} \chi(X_n, X_p) \text{ MeV/baryon/s}. \quad (1)$$

To be concrete, let us concentrate on the production of gravitons in the presence of $n$ gravity only extra dimensions (GODs) 4. There we find $p_2 = 5.42$, $a_2 = 5.1 \times 10^4 R_2^2$, $p_3 = 6.5,
\[ a_3 = 1.4 \times 10^{16} R_3^3. \] Here, \( n_B \) is the baryon number density, \( T \) is the temperature. The size of the extra dimensions, \( R_n \), is given in mm. The emissivity depends weakly on the composition of matter through the function \( \chi(X_n, X_p) \), defined in ref. [4].

Once the emissivities are derived we have to quantify their influence on the neutrino signal of SN1987A. The by far cleanest way to do so is to implement the emission rate of eq. (1) in a simulation of the early PNS. For the present study we use the codes described in ref. [3]. The equilibrium diffusion approximation employed in the numerical code provides a fair description of the total neutrino luminosity and its time structure (c.f. figure [3]) and thus forms a sound basis for the present investigation. However, in order to deduce the anti–neutrino spectrum an additional assumption is needed, namely that the neutrino emission happens from a common neutrino sphere whose temperature is determined by the requirement for the optical depth to take some fixed value. In order to minimize the influence of this assumption on the bound derived we studied a variety of values for the neutrino sphere temperature around the central value. In addition, the mass of the PNS very strongly influences the neutrino luminosity. Since we only know, that the baryonic mass is bounded in the interval \( 1.4 M_\odot \leq M \leq 2.0 M_\odot \), we studied the whole mass range.

Having neutrino emission rates at hand we now have to compare to the data and develop a tool that quantifies the agreement. Thus we need to define a Likelihood function. Following ref. [3] we use

\[ \mathcal{L} (\{\text{data}\} | a_n, M, T_{\nu_e}, I) = \prod_D \left[ \prod_{t_i}^{N_{D,\text{tot}}} \frac{dN_{D}(t_{i}^{D})}{dt} \Delta t \right] e^{-N_{D}}, \] (2)

where \( N_{D} \) is the total number of observed neutrino arrivals, and \( t_{i}^{D}, t_{2}^{D}, \ldots \) are the times at which neutrinos actually arrived in the detector \( D \). \( N_{D}(t) \) is the total number of neutrinos that the model with assumptions \( I \), PNS mass \( M \), anti-neutrino temperature \( T_{\bar{\nu}_e} \), and exotic-coupling \( a_n \) predicts will have arrived in the detector \( D \) up until the time \( t \). \( \Delta t \) can be any interval small enough that the probability of detecting more than one count in any one bin can be taken to be negligible. Ultimately it will be absorbed into an overall normalization constant.

The likelihood function provides us a with quantitative tool to compare models of PNS evolution. Since we treat not only the coupling to extra dimensions, \( a_n \), but also \( M \) and \( T_{\nu_e} \), as parameters, the likelihood function is a function in a three-dimensional space. This function has a minimum at a baryonic mass of \( M = 1.5 M_\odot, a_2 = 0 \) and \( T_{\nu_e} = 1.1 T_\nu^o \). \( T_\nu^o \) denotes the “reference anti-neutrino temperature” defined above using the optical-depth prescription. We can now assess all other models by looking at the log of the ratio of their likelihood of a particular model to this “most likely” model:

\[ q(a_n, M, T_{\nu_e}) = -\log \left( \frac{\mathcal{L} (\{\text{data}\} | a_n, M, T_{\nu_e}, I)}{\mathcal{L} (\{\text{data}\} | 0, 1.5 M_\odot, 1.1 T_\nu^o, I)} \right). \] (3)

The function \( q \) is then a function in this same three-dimensional space.

For the case of two GODs we discuss first the situation where the extra dimensions have zero radius, and so \( a_2 = 0 \). Contours of \( q \) in the resulting two-dimensional \( x - M \) plane are shown in the left panel of the contour plots in Fig. [4]. Here the horizontal axis is the temperature normalized to the reference anti-neutrino temperature, \( x = T_{\nu_e}/T_\nu^o \), which is varied to explore the sensitivity of the results to deviations of the neutrino temperature from that obtained using the optical depth prescription. We see that although \( M = 1.5 M_\odot, T = 1.1 T_\nu^o \) is indeed a minimum of the function \( q \) it is a rather shallow minimum: varying the neutron-star mass and the anti-neutrino temperature over the range considered here does not reduce the likelihood greatly. Ultimately this reflects the weakness of the constraint that the SN 1987a data provides for these parameters.
The situation is rather different as we move away from $a_2 = 0$. In the middle panel we display similar likelihood contours in the $M - \log(a_2)$ plane for the case $T_{\bar{\nu}_e} = T^0_{\bar{\nu}_e}$, while the right panel shows contours in the $x - \log(a_2)$ plane for the baryonic mass $M = 1.6M_\odot$. These panels show that the likelihood function decreases rapidly for $a_2 \geq 10^{-2}$—regardless of the values of the poorly-known parameters $M$ and $T_{\bar{\nu}_e}$. Such large values of $a_2$ are essentially two orders of magnitude less likely than the “most likely” model. This is a contrast to smaller values of $a_2$, where the differences in likelihood are comparable to those seen in the $M - T_{\bar{\nu}_e}$ plane. Thus statements about the most likely value of $a_2$ in this smaller-$a_2$ regime will be sensitive to the ill-constrained information on these neutron-star parameters. Consequently we will not make any such statements here. However, the two rightmost panels give us confidence in our ability to derive a bound on $a_2$—as opposed to a most-likely value—since it is clear that certain values of this coupling can be well-excluded, completely independent of details of the PNS modeling. To actually derive a bound we now integrate (or “marginalize”) over all possible values of $M$ and $T_{\bar{\nu}_e}$, using appropriate weight functions (for details c.f. ref. 8). We find that the possibility that there are two compact extra dimensions with radii larger than 0.66 $\mu$m is excluded at the 95% confidence level—as is the possibility that there are three compact extra dimensions larger than 0.8 $\mu$m.

To summarize, in this talk a scheme to deduce bounds with a well defined statistical meaning for the coupling strength of exotic particles to nucleons was presented. The method is quite general and it was demonstrated that the bounds derived do not depend on parameters poorly constrained. However, improvements are possible, and should be studied. For instance, many-body effects are expected to modify the emissivity. We are working on this problem.

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1. G.G. Raffelt, in *Stars as Laboratories for Fundamental Physics*, University of Chicago Press, Chicago (1996).
2. C. Hanhart, D.R. Phillips, and S. Reddy, Phys. Lett. B 499 (2001) 9.
3. A. E. Dieperink, E. N. van Dalen, A. Korchin and R. Timmermans, *nucl-th/0012073*.
4. C. Hanhart, D. R. Phillips, S. Reddy and M. J. Savage, Nucl. Phys. B 595 (2001) 335.
5. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429, 263 (1998) and Phys. Rev. D 59, 086004 (1999).
6. M. Fairbairn, *hep-ph/0101131*.
7. S. Hannestad and G. Raffelt, *hep-ph/0103201*.
8. C. Hanhart, J. A. Pons, D. R. Phillips and S. Reddy, *astro-ph/0102063*.
9. J. A. Pons, J. A. Miralles, M. Prakash and J. M. Lattimer, Astrophys. J., in press (2001) *astro-ph/0008389*.
10. J. A. Pons, S. Reddy, M. Prakash, J. M. Lattimer and J. A. Miralles, Astrophys. J. 513, 780 (1999)
11. T.J. Loredo, D.Q. Lamb, in *Proceedings of the 14th Texas Symposium on Relativistic Astrophysics*, Ann. N.Y Acad. Sci. 571 (1989) 601