DISK MASSES AT THE END OF THE MAIN ACCRETION PHASE: CARMA OBSERVATIONS AND MULTI-WAVELENGTH MODELING OF CLASS I PROTOSTARS

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ABSTRACT

We present imaging observations at the 1.3 mm wavelength of Class I protostars in the Taurus star-forming region, obtained with the CARMA interferometer. Of an initial sample of 10 objects, we detected and imaged millimeter wavelength emission from 9. One of the nine is resolved into two sources and detailed analysis of this binary protostellar system is deferred to a future paper. For the remaining eight objects, we use the CARMA data to determine the basic morphology of the millimeter emission. Combining the millimeter data with 0.9 μm images of scattered light, Spitzer Infrared Spectrograph spectra, and broadband spectral energy distributions (all from the literature), we attempt to determine the structure of the circumstellar material. We consider models including both circumstellar disks and envelopes, and constrain the masses (and other structural parameters) of each of these components. We show that the disk masses in our sample span a range from \( \lesssim 0.01 \) to \( \gtrsim 0.1 M_\odot \). The disk masses for our sample are significantly higher than for samples of more evolved Class II objects. Thus, Class I disk masses probably provide a more accurate estimate of the initial mass budget for star and planet formation. However, the disk masses determined here are lower than required by theories of giant planet formation. The masses also appear too low for gravitational instability, which could lead to high mass accretion rates. Even in these Class I disks, substantial particle growth may have hidden much of the disk mass in hard-to-see larger bodies.

Key words: circumstellar matter – stars: formation – stars: individual (IRAS 04016+2610, IRAS 04166+2706, IRAS04169+2702, IRAS 04287+1801, IRAS 04295+2251, IRAS 04302+2247, IRAS 04361+2547, IRAS 04365+2535) – techniques: high angular resolution

1. INTRODUCTION

Stars form from clouds of dust and gas. As these clouds collapse and conserve angular momentum, matter that is not initially located near the polar axis falls onto a rotationally supported disk. Most of the material that eventually makes up the star falls initially onto the disk, with some matter subsequently losing angular momentum and accreting onto the protostar. The basic picture of star formation thus involves the free-fall collapse of material from an extended envelope (or cloud) onto a disk, and then viscous accretion through the disk and onto a protostar.

At the earliest stages of collapse, one would see a roughly spherical cloud with no embedded protostar. These so-called starless cores have now been identified in a number of star-forming regions (e.g., Schnee et al. 2007; Stutz et al. 2009). As collapse proceeds, a protostar and compact disk form, still surrounded by a substantial, roughly spherical envelope. Matter continues to fall onto the disk, while viscosity sources in the disk allow net outward angular momentum transport and inward mass accretion. In time the envelope completes its collapse, leaving a pre-main-sequence star surrounded by a massive disk. The disk continues to accrete onto the star, losing mass over time, eventually leaving the central pre-main-sequence object surrounded by only tenuous circumstellar matter.

Each of these stages of protostellar evolution produces observational diagnostics, and an empirical picture has been developed that links evolutionary phases with properties of the emergent spectral energy distributions (SEDs; Lada 1987; Andrè et al. 1993). Class 0 objects show highly obscured SEDs that peak at submillimeter wavelengths. These are thought to be protostars still deeply embedded in massive envelopes. Class I sources show SEDs that peak in the mid-IR. Class I objects are thought to be surrounded by massive disks embedded in non-spherically symmetric envelopes (e.g., Eisner et al. 2005). In Class II objects light from the central object is visible, although some short-wavelength emission is reprocessed and emitted at longer wavelengths. These are the T Tauri and Herbig Ae/Be stars, known to be pre-main-sequence stars surrounded by optically thick disks (e.g., Burrows et al. 1996; McCaughrean & O’Dell 1996; Corder et al. 2005). Finally, Class III sources are pre-main-sequence stars either devoid of circumstellar material or surrounded by only tenuous, optically thin circumstellar matter (e.g., Padgett et al. 2006).

The relative lengths of the Class 0, I, and II evolutionary stages can be estimated by counting the number of sources belonging to different classes in individual star-forming regions. Absolute timescales are determined by fitting the relatively unobscured Class II pre-main-sequence stellar photospheric properties to theoretical evolutionary tracks (e.g., White & Ghez 2001). The lifetime of the Class 0 phase has been estimated as \( \lesssim 0.1–0.2 \) Myr (Evans et al. 2009; Enoch et al. 2009), the Class I phase as \( \lesssim 0.2–0.5 \) Myr (Wilking et al. 1989; Greene et al. 1994; Kenyon & Hartmann 1995; Evans et al. 2009), and the Class II phase as \( \lesssim 2 \) Myr (e.g., Kenyon et al. 1990; Cieza et al. 2007).

The mass of circumstellar disks in each of the evolutionary stages outlined above has crucial implications for star and planet formation. Sufficiently massive disks may become gravitationally unstable, providing high effective viscosities and allowing rapid growth of protostellar mass. The disk mass at later stages is important to planet formation, since a certain mass is required to build giant planets (e.g., Weidenschilling 1977b).

During the Class 0 phase, disks are embedded in much more massive envelopes. High-presumed infall rates \( \gtrsim 10^{-5} M_\odot \) yr\(^{-1}\); e.g., Shu 1977; Hartmann et al. 1997) from envelopes to disks likely mean that these disks are gravitationally unstable. At the Class I stage, envelopes are less massive and the disks may dominate the total system mass (e.g., Jørgensen et al. 2009). Disks in Class II objects generally appear
gravitationally stable, accreting material slowly onto their central objects \((M \lesssim 10^{-7} M_\odot \text{yr}^{-1})\); e.g., Basri & Bertout 1989; Hartigan et al. 1995). Class I sources thus offer the prospect of observing disks at an intermediate state when they may transitioning from rolling, gravitationally unstable disks, to more stable disks that could provide a hospitable environment for the early stages of planet formation.

The masses of disks surrounding Class I sources have been constrained previously using millimeter wavelength observations (e.g., Hogerheijde 2001; Osorio et al. 2003; Wolf et al. 2003; Eisner et al. 2005; Jørgensen et al. 2007, 2009). Because much of the circumstellar dust around Class I objects is expected to be optically thin, the observed millimeter flux is correlated with mass. However, knowledge of the dust opacity and temperature (as a function of radius) is needed in order to convert flux to mass. Moreover, dense regions (e.g., close to the central protostars) may be optically thick. Radiative transfer modeling of multi-wavelength data is required to disentangle these effects and provide unambiguous circumstellar mass estimates.

Historically, constraints on the circumstellar matter in Class I sources came from modeling of SEDs (e.g., Adams et al. 1987; Kenyon et al. 1977a; Robitaille et al. 2007). These models include flattened, disk-like distributions of matter that are a consequence of angular momentum conservation (e.g., Ulrich 1976; Terebey et al. 1984). However, the SED data can often be explained with different models as well, including edge-on flared disk models (e.g., Chiang & Goldreich 1999). Imaging data, including scattered light images (e.g., Whitney et al. 2003) or millimeter images of continuum and line emission (e.g., Jørgensen et al. 2007, 2009), have also been used to constrain the circumstellar structure of Class I sources.

Modeling a single data set—e.g., an SED or scattered light image—is subject to ambiguities. Modeling multi-wavelength images and fluxes simultaneously provides tighter constraints on geometry, since emission at different wavelengths arises in different layers of circumstellar material. Scattered light at short wavelength traces low-density surface layers, while longer-wavelength (e.g., millimeter) emission arises in deeper, denser regions. Such modeling has been applied to a number of Class I sources, yielding constraints on circumstellar morphology better than afforded by modeling any single data set (Osorio et al. 2003; Wolf et al. 2003; Eisner et al. 2005).

Previous estimates of disk masses in Class I objects cover a broad range. Modeling millimeter images together with SEDs and scattered light images, disk masses of \(0.01–1 M_\odot\) were inferred from radiative transfer modeling of submillimeter continuum images of 10 Class I sources (Jørgensen et al. 2009). Some disk mass estimates are consistent with simulations of protostellar evolution, which find disk masses of \(\sim 0.2 M_\odot\) for Class I objects (Vorobyov 2011). For many objects, however, estimated disk masses are substantially lower.

For some objects, mass estimates place Class I disks near the limit of gravitational stability. This finding would fit naturally into the scenario of burst-mode accretion onto Class I protostars (Vorobyov & Basu 2005), and explain the discrepant infall rates from the envelope to the disk and from the disk onto the star (e.g., Kenyon & Hartmann 1987; White & Hillenbrand 2004). In a broader sense, the implied gravitational instability provides an easily explained viscosity source to enable angular momentum transport in protostellar disks.

Estimated disk masses in Class I sources appear higher than the disk masses determined for Class II objects. The median mass of disks in nearby star-forming regions is estimated as \(\sim 0.005 M_\odot\) (e.g., Andrews & Williams 2005, 2007; Eisner et al. 2008). These disks are about an order of magnitude less massive than the minimum mass solar nebula (Weidenschilling 1977b), suggesting that many stars in the Galaxy lack the circumstellar mass needed to form solar systems like ours. Higher inferred masses in Class I disks suggest that substantial particle growth (which depletes the observable population of small dust grains) has occurred between the Class I and II stages. Planet formation may already have advanced significantly during the first million years of disk evolution. Given the short lifetimes of particles with sizes of a few centimeters to a few meters (Weidenschilling 1977a), this would imply growth of large planetesimals in less than \(\sim 1\) Myr.

Substantial uncertainties remain in the estimated Class I disk masses, placing the conjectures above on somewhat shaky footing. Our aim in this paper is to better constrain the disk masses around a relatively large sample of Class I sources. We will improve upon previous constraints using high angular resolution millimeter imaging from the Combined Array for Research in Millimeter-wave Astronomy (CARMA). We model these data together with existing data at other wavelengths, including broadband SEDs, Spitzer Infrared Spectrograph (IRS) spectra, and J-band images of scattered light.

2. OBSERVATIONS

2.1. SAMPLE SELECTION

Our sample consists of Class I sources in Taurus (Table 1). We selected objects with existing \(0.9 \mu m\) scattered light imaging (Eisner et al. 2005), Spitzer/IRS spectra (Furlan et al. 2008,
Table 2

Properties of 1.3 mm Wavelength Images

| Source            | α (J2000) | δ (J2000) | Peak Flux (mJy) | rms (mJy) | θmaj (°) | θmin (°) | θPA (°) |
|-------------------|-----------|-----------|-----------------|-----------|----------|----------|---------|
| IRAS 04016+2610   | 04:04:43.07 | +26:18:56.3 | 17              | 1         | 1.13     | 0.85     | 54.     |
| IRAS 04166+2706   | 04:19:42.50 | +27:13:36.0 | 66              | 13        | 1.03     | 0.90     | 74.0    |
| IRAS 04169+2702   | 04:19:56.48 | +27:09:57.0 | 65              | 8         | 1.02     | 0.90     | 75.9    |
| IRAS 04181+2654   |           |           | 108             |           |          |          |         |
| IRAS 04263+2426N  | 04:29:23.74 | +24:33:01.0 | 37              | 4         | 1.02     | 0.85     | 72.5    |
| IRAS 04263+2426S  | 04:29:23.74 | +24:32:59.8 | 31              | 4         | 1.02     | 0.85     | 72.5    |
| IRAS 04287+1801   | 04:31:34.16 | +18:08:04.8 | 404             | 4         | 1.09     | 0.86     | 44.9    |
| IRAS 04295+2251   | 04:32:32.07 | +22:57:26.3 | 53              | 1         | 1.13     | 0.86     | 53.5    |
| IRAS 04302+2247   | 04:33:16.49 | +22:53:20.3 | 46              | 2         | 1.07     | 0.86     | 49.3    |
| IRAS 04361+2547   | 04:39:13.90 | +25:52:20.4 | 50              | 4         | 1.03     | 0.84     | 69.7    |
| IRAS 04365+2535   | 04:39:35.21 | +25:41:44.2 | 147             | 5         | 1.04     | 0.83     | 68.6    |

Notes. Coordinates indicate the positions of the peak fluxes. The major axis, minor axis, and position angle (east-of-north) of the beam FWHM are listed as θmaj, θmin, and θPA, respectively.

We used MIRIAD (Sault et al. 1995) to calibrate the CARMA data. Bandpass calibration utilized observations of bright, (nearly) spectrally flat sources (see Table 1). To calibrate time-dependent instrumental and atmospheric effects, we observed the blazar 3C 111 every 20 minutes. In between 3C 111 observations, we obtained data on two or three targets. Because our targets are bright enough to be detected in less than a full “track” (i.e., the length of time the source is above a reasonable elevation angle), we observed multiple targets in each track. Targets in a single track shared calibrators and comparison of calibrated data for these targets provides a consistency check on the calibration procedure. In a few of the tracks, Uranus was observed and used to set the instrumental flux scale. For the other tracks we took the flux of the gain calibrator, 3C 111, directly from the SMA submillimeter calibrators page,1 extrapolating to our observing date when necessary.

1 http://sma1.sma.hawaii.edu/callist/callist.html
of unweighted data. For our data, we use a scale factor of five for all objects except IRAS 04287+1801, where we use a scale factor of two.

Averaged visibilities for our targets are shown in Figure 2. The visibilities computed by averaging over a 2D grid of $u$ and $v$ are generally quite noisy. Azimuthally averaging these visibilities lowers the noise substantially and allows radial structure to be more easily discerned. We therefore plot the azimuthally averaged visibility amplitudes as well.

We fit inclined uniform disk models (e.g., Eisner et al. 2004) to the 2D CARMA visibilities, to provide rough constraints on the geometry of the millimeter emission. We did not fit a uniform disk model to IRAS 04263+2426 because it is a binary. The fitted fluxes, sizes, inclinations, and position angles (P.A.s) of these disk models are listed in Table 3. These fits show that the emission from most of our sample is resolved well in our observations. However, IRAS 04166+2706 appears to be unresolved, and the data for IRAS 04169+2702 and IRAS 04361+2547 lack the signal to noise and $uv$ coverage needed to constrain the geometry well.

2.3. Scattered Light Images

In our modeling we use Keck $I$-band imaging of our sample, published previously in Eisner et al. (2005). We calibrate the astrometry of these images by registering to the Two Micron All Sky Survey (2MASS) coordinate system. In each of our $I$-band images we searched for stars also seen in 2MASS images. We then computed a plate solution for our $I$-band images. The residuals in these fits are typically smaller than 0.3, which is comparable to the error in the 2MASS astrometry. For the angular resolution and signal to noise of our millimeter observations, we expect positional uncertainties of $\lesssim 0.1$ for the

![Image](image_url)

**Figure 1.** $I$-band scattered light images in gray scale with millimeter images overplotted as contours. Contours are plotted in intervals of 20% of the peak flux. The dashed ellipse at the bottom left is the FWHM of the synthesized beams of the CARMA observations.

**Table 3**

| Source             | Flux (mJy) | $\theta_{UD}$ (′′) | $i$ (°) | P.A. (°) |
|--------------------|------------|--------------------|--------|---------|
| IRAS 04016+2610    | 18 ± 2     | 2.49 ± 0.19        | 68 ± 7 | 66 ± 4  |
| IRAS 04166+2706    | 36 ± 8     | 0.75 ± 1.00        | 90 ± 90| 130 ± 130 |
| IRAS 04169+2702    | 71 ± 16    | 1.30 ± 0.64        | 50 ± 50| 0 ± 180 |
| IRAS 04287+1801    | 536 ± 35   | 1.61 ± 0.07        | 55 ± 9 | 167 ± 6 |
| IRAS 04295+2251    | 78 ± 8     | 2.05 ± 0.12        | 60 ± 7 | 73 ± 7  |
| IRAS 04302+2247    | 29 ± 15    | 2.76 ± 1.26        | 67 ± 22| 33 ± 20 |
| IRAS 04361+2547    | 62 ± 28    | 1.76 ± 0.84        | 60 ± 60| 160 ± 160 |
| IRAS 04365+2535    | 110 ± 48   | 2.14 ± 0.72        | 89 ± 44| 84 ± 25 |

**Notes.** For each source, we list the flux, angular diameter ($\theta_{UD}$), inclination, and position angle for best-fit inclined uniform disk models.
Figure 2. Averaged visibility amplitudes for our targets. Visibilities computed on a two-dimensional grid are shown with open circles. The error bars on these points are typically fairly large (often comparable to the plotted range) and are not shown here. We also plot the azimuthally averaged visibilities with solid points. The additional averaging leads to substantially smaller uncertainties, indicated here with plotted error bars. Note that the averaging is done coherently, and phase noise may lead to average amplitudes below the amplitudes measured for unaveraged data.

millimeter images. The registration of $I$-band and millimeter images is thus uncertain by $\lesssim 0.5\arcsec$.

For IRAS 04263+2426 we were unable to find enough reference stars to compute a plate solution. This dearth of reference stars is caused, in part, by the shorter integration time on this bright $I$-band object (60 s, compared to 120–300 s for the other objects in our sample). For this source we attempted to line up the scattered light structure seen in $J$-band 2MASS imaging with the structure seen in the $I$-band data. We estimate uncertainties of $\sim 1\arcsec$ in this procedure.

The $I$-band images, registered to the 2MASS coordinate system, are plotted in Figure 1. The scattered light morphologies for our sample vary widely, from bright, extended emission for IRAS 04016+2610, IRAS 04263+2426, IRAS 04287+1801, IRAS 04295+2251, and IRAS 04302+2247; to weak, more compact emission for IRAS 04361+2547 and IRAS 04365+2535; to a lack of any detectable scattered light toward IRAS 04166+2706 and IRAS 04169+2702.

2.4. Spectral Energy Distributions

We compiled broadband SEDs, including Spitzer/IRS spectra (Furlan et al. 2008, 2011), from the literature (Myers et al. 1987; Weaver & Jones 1992; Barsony & Kenyon 1992; Kenyon & Hartmann 1995; Padgett et al. 1999; Chandler & Richer 2000; Young et al. 2003; Moriarty-Schieven et al. 1994; Hogerheijde & Sandell 2000; Motte & André 2001; Andrews & Williams 2005; Kessler-Silacci et al. 2005; Eisner et al. 2005; Hartmann et al. 2005; Robitaille et al. 2007; Rebull et al. 2010). We ignore the error bars quoted for these photometric measurements (when available), and instead adopt a uniform uncertainty level. We adopt 10% errors for all photometric data. However, this value is somewhat arbitrary. As described in Section 3.3, we specify weights for various data sets in our modeling, and these weights effectively overrule the normalization of the internal uncertainties in a given data set.

3. MODELING

Following previous work (e.g., Eisner et al. 2005), we consider a model that includes a rotating, collapsing envelope with an embedded, hydrostatic, Keplerian disk. We describe the density distribution of this model in Section 3.1. We use the 2D Monte Carlo radiative transfer code RADMC (Dullemond & Dominik 2004) to compute the temperature distribution of our models. With the density and temperature distributions
computed, we use RADMC to produce 0.9 μm images, 1.3 mm images, and SEDs spanning wavelengths from 0.1 μm to 1 cm. We convert the 1.3 mm images to 2D visibilities over the same grid used to average the CARMA visibility data (Section 2.2). For ease of plotting, we also azimuthally average these 2D visibilities.

In all of our models we fix the dust opacities. We use dust opacities that were computed to fit extinction measurements in dense cores observed as part of the C2D project (K. M. Pontoppidan 2010, private communication; see also Shirley et al. 2011). These opacities are computed for spherical dust grains with a size distribution described by Weingartner & Draine (2001). The dust is a mixture of approximately 30% amorphous carbon, with grains between $6 \times 10^{-4}$ and 7 μm, and 70% silicate, with grains between $6 \times 10^{-4}$ and 0.6 μm. The grains are covered with ice mantles (70% H2O and 30% CO2), with an assumed ice abundance of $3 \times 10^{-4}$ per H2 molecule.

Changing dust opacities would affect the modeling, but we lack the computational power to vary these in concert with other parameters describing the stellar and circumstellar properties.

We consider three values of source luminosity in our analysis: 1, 3, and 10 $L_\odot$. Note that these are rough approximations of the true luminosities of our targets, which may lie between sampled values, but computational resources limit us to a few values. We take the stellar mass and temperature to be fixed in our modeling: $M_* = 0.5 M_\odot$ and $T_* = 4000$ K. These values are compatible with dynamical mass estimates and effective temperature measurements (e.g., Hogerheijde 2001; White & Hillenbrand 2004). Thus, we are assuming in our modeling that the only stellar parameter that varies is the radius.

A range of viewing geometries is considered. We allow models to be inclined between 0° and 90° with respect to the plane of the sky and to have P.A.s spanning the full range from 0° to 360°. The P.A. is defined east-of-north.

In Section 3.2, we present a parameter study that shows how synthetic data are affected by changes in various parameters. Varied parameters include properties of the circumstellar mass distribution (Section 3.1), protostellar luminosity, and viewing geometry. In Section 3.3, we describe the grid of models we computed, and discuss the procedure by which the best-fit model is found.

### 3.1. Input Density Distribution and Stellar Parameters

Our model includes both a collapsing envelope and an embedded disk. The density distributions of the two components are computed separately and then added together to produce the final model. We describe each of these components in turn and demonstrate the effects of changing input parameters on synthetic data.

#### 3.1.1. Collapsing Envelope

When a rotating, initially spherical cloud of material collapses, angular momentum conservation leads to a non-spherically symmetric distribution. Following Ulrich (1976), the density distribution of a rotating, infalling envelope is

$$\rho_{\text{env}}(r, \theta) = \frac{\dot{M}}{4\pi} (GM_\ast r^3)^{-1/2} \left(1 + \frac{\dot{\mu}}{\mu_0}\right)^{1/2} \left(\frac{\mu}{\mu_0} + 2\frac{\dot{\mu}}{\mu_0} \frac{R_c}{r}\right)^{-1}. \quad (1)$$

Here $r$ is the stellocentric radius, $\dot{M}$ is the mass infall rate, $M_\ast$ is the stellar mass, $\mu = \cos(\theta_0)$, and $\theta_0$ is the initial angle of infalling material. The centrifugal radius, $R_c$, is the location where material becomes rotationally supported against gravity and the density structure becomes less spherically distributed and more flattened:

$$R_c = \frac{R_0 \Omega^2}{G M_{\text{env}}}. \quad (2)$$

Here $R_0$ is the initial radius of the infalling matter, $\Omega$ is the angular velocity, and $M_{\text{env}}$ is the envelope mass. In our modeling, we will leave the centrifugal radius as a free parameter.

For each point in the envelope, matter can only have arrived from a specific initial location. We can therefore solve for $\mu_0$. The particle trajectories obey the relation (see, e.g., Ulrich 1976)

$$\frac{r}{R_c} = \frac{\cos \theta_0 \sin \theta_0^2}{\cos \theta_0 - \cos \theta} = \frac{1 - \mu_0^2}{1 - \mu/\mu_0}. \quad (3)$$

There are three solutions to this equation. We require that the solution be real and that $\sin \theta_0$ and $\sin \theta$ have the same sign. This means that a particle that starts out in one quadrant of the model is required to remain in that quadrant. The analytic solution is

$$\mu_0 = \xi = \frac{(r - R_c)}{3R_c \xi}, \quad (4)$$

where $\xi$ is given by

$$\xi = \frac{\left[27\mu_0 r^2 + \sqrt{729 \mu_0^2 r^4 R_c^4 + 108 R_c^2 (r - R_c)^2}\right]^{1/3}}{21^{1/3} 3 R_c}. \quad (5)$$

We modify this envelope model to include an outflow cavity along the rotation axis. We specify this cavity as

$$\rho_{\text{env}}(r, z > z_0 + r^\epsilon) = \rho_{\text{env}} \times f_{\text{env}}. \quad (6)$$

Here $z$ is the height above the midplane, $r$ is the stellocentric radius in the midplane, $z_0$ is the height above the midplane where the cavity begins, and $\epsilon$ describes the shape and opening angle of the cavity. For simplicity we fix $z_0 = 1$ AU in our models, but we explore the effects of $\epsilon$ and $f_{\text{env}}$ on synthetic data below.

The total envelope mass, $M_{\text{env}}$, is obtained from the integral of $\rho_{\text{env}}$. Since $M_{\text{env}} \propto \rho_{\text{env}} \propto M$ (Equation (1)), we may use the total envelope mass as a free parameter instead of $M$. The envelope mass is more relevant for our purposes, since it allows a more direct comparison with the disk mass, $M_{\text{disk}}$.

#### 3.1.2. Flared Disk

The flared disk model was first described by Shakura & Sunyaev (1973), where the density distribution is given by

$$\rho_{\text{disk}}(r, z = z_0 + r^\epsilon) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha} \exp\left(-\frac{1}{2} \left[\frac{z}{h(r)}\right]^2\right). \quad (7)$$

Here $r$ is the radial distance from the star, $z$ is the vertical distance above the midplane, and $r_0$ is assumed to be 1 AU. The scale height above the disk is given by

$$h(r) = h_{1\text{AU}} \left(\frac{r}{1\text{AU}}\right)^\beta. \quad (8)$$

We adopt $\beta = 58/45$, which is expected for a flared disk in hydrostatic equilibrium (e.g., Chiang & Goldreich 1997). This allows us to also fix the exponent of the radial volume density.
profile to $\alpha = 3(\beta - (1/2)) = 2.37$ based on viscous accretion theory (Shakura & Sunyaev 1973). Note that this implies that the surface density, $\Sigma(r) \propto r^{-1.37}$. The value of $\alpha$ is similar to assumed values in previous modeling of circumstellar disks (e.g., D'Alessio et al. 1999; Chiang & Goldreich 1997) and to the inferred value for the protosolar nebula (Weidenschilling 1977b). The free parameters for the flared disk are $M_{\text{disk}}$, $R_{\text{disk}}$, and $h_1\text{AU}$. We assume that $R_{\text{disk}} = R_c$.

3.1.3. Disk+Envelope

To produce a disk-envelope model, we take the envelope model from Section 3.1.1 and the disk model from Section 3.1.2 and add their density distributions. In our modeling we vary the disk and envelope masses so that our models encompass both disk-dominated and envelope-dominated morphologies.

3.2. Parameter Study

Here we vary the input parameters of the disk-envelope model described above, and illustrate the effect these parameters have on synthetic SEDs, 0.9 $\mu$m scattered light images, and millimeter wavelength visibilities. We vary parameters about a baseline model, which has the following parameters: $M_{\text{disk}} = 0.1 M_\odot$, $h_1\text{AU} = 0.05$ AU, $R_{\text{disk}} = R_c = 30$ AU, $M_{\text{env}} = 0.05 M_\odot$, $R_{\text{out}} = 1000$ AU, $f_{\text{cav}} = 0.2$, $\zeta = 1.0$, $L = 3 L_\odot$, $i = 45^\circ$, and P.A. = 90$^\circ$.

3.2.1. Disk Parameters

In our disk-envelope model, we have assumed a power-law radial surface density profile declining with radius with an exponent of $-1.37$. Since the disk surface area grows as the square of the radius, the radial mass distribution increases with radius: $M(r) \propto r^{0.63}$. However, since the disk outer radius is equal to the envelope centrifugal radius, material in the disk is still compact compared to the envelope mass distribution. Note, too, that the temperature is higher at smaller radii, and the flux from optically thin material can still be centrally concentrated even if the mass distribution is weighted to larger radii.

A higher mass corresponds to an increased millimeter-wavelength flux. As seen in Figure 3, the long-wavelength SED as well as the normalization of the millimeter-wavelength visibility is increased as the disk mass is increased. The disk emission is unresolved for our baseline disk model radius ($R_{\text{disk}} = 30$ AU). However for $R_{\text{disk}} \gtrsim 100$ AU, some disk emission may be resolved on the longest baselines.

The effect of disk mass on the scattered light image is small since most of the scattered light is generated on larger scales where the envelope component dominates. Figure 3 shows a slightly less extended scattered light morphology for the highest disk mass considered. However, this may reflect the fact that fewer photons escaped the high optical depth during the radiative transfer calculation.

Larger disk scale heights mean that the disk is better at intercepting short-wavelength light and reprocessing it to longer wavelengths. Thus, models with higher values of $h_1\text{AU}$ show depressed optical/near-IR fluxes and increased millimeter fluxes (Figure 4). Scattered light images become more compact for larger values of $h_1\text{AU}$, because photons can be scattered from vertically extended disk regions at smaller radii.

A larger disk outer radius means more resolved millimeter visibilities (Figure 5). Larger disks also re-distribute mass to larger radii, thus decreasing the optical depth at small radii and allowing more infrared emission to escape. Because the disk outer radius is also the envelope centrifugal radius in our model, its effects on the synthetic data are somewhat more complicated. We discuss how centrifugal radius impacts the synthetic data below.

The disk surface density profile may also influence the observed radial profile of 1 mm emission, since the mostly
optically thin dust emission correlates with mass. Shallower radial surface density profiles produce more millimeter emission at larger stellocentric radii, and hence more resolved visibilities. However, in the interest of minimizing the number of free parameters, we consider the surface density to be fixed in our modeling.

### 3.2.2. Envelope Parameters

Increasing the envelope mass increases the extinction toward the central object, thus decreasing the optical and infrared fluxes, and increases the mass of cooler material, thus increasing the millimeter fluxes (Figure 6). Unlike the disk mass, envelope
mass is distributed over larger spatial scales, and much of the additional millimeter emission is resolved by the long-baseline CARMA observations. Increasing the envelope mass thus tends to increase the visibility amplitudes more at shorter baselines than at longer baselines.

We see in Figure 6 that the effect on the synthetic scattered light images of changing the envelope mass is not monotonic. For lower envelope masses, the central object (protostar+inner disk) is less obscured and the extended scattered light is difficult to discern against the bright background. As envelope mass increases, the central light is blocked and it is easier to see the extended scattered component. For still higher envelope masses, scattered light simply cannot escape efficiently, and we see the obscured central object at a very low level.

As discussed above, since $R_c = R_{\text{disk}}$, a larger value of $R_c$ means that the disk mass is distributed over a larger range of radii. A larger centrifugal radius also implies that a larger fraction of the total envelope mass is distributed in a flattened distribution. The millimeter visibilities are thus more resolved for larger values of $R_c$.

Because a larger centrifugal radius means that the more spherical component of the envelope begins at larger stellocentric radii, it also means that $I$-band emission can escape more easily. As seen in Figure 5, the strength of the scattered light increases with $R_c$. We also see changes in the morphology of the scattered light images. It seems that when the disk size (defined by $R_c$) becomes large enough, scattered light from the disk may dominate the image. For smaller disks, and $R_c$ values, the scattering may come predominantly from the envelope component.

Figure 7 shows the effect on synthesized data of varying $R_{\text{out}}$. Because the envelope mass is held fixed, a larger value of $R_{\text{out}}$ means that the mass is more spread out and the mass on small scales is decreased. Models with larger outer radii are more resolved (i.e., display lower visibilities). They also show somewhat lower total millimeter fluxes, since more of the mass is shifted to lower temperature regions.

Envelopes with larger outer radii (and hence less mass on compact scales) let more short-wavelength flux escape to the observer. This is reflected clearly in the SED shown in Figure 7. For small $R_{\text{out}}$ values, little light escapes and the scattered light image shows only highly extincted flux from the central object. For high values of $R_{\text{out}}$, flux can escape from the less dense inner regions of the envelope. For still larger values of $R_{\text{out}}$, the inner regions of the envelope become sufficiently tenuous that the denser disk component produces most of the scattered light.

### 3.2.3. Outflow Cavity Properties

The outflow cavity in our models is described by three parameters: $\zeta$, $\zeta_v$, and $f_{\text{cav}}$ (Equation (6)). $\zeta$ is not particularly important to the synthetic data and so we fix its value. $\zeta$ and $f_{\text{cav}}$ influence both the shape of the cavity and its contrast with respect to the rest of the envelope (and whether there is a cavity at all).

A smaller value of $f_{\text{cav}}$, corresponding to a higher contrast between the densities within and outside the outflow cavity, facilitates the escape of short-wavelength emission. Smaller $f_{\text{cav}}$ values lead to more compact scattered light images, since more light from the central disk is visible. For very large values of $f_{\text{cav}}$, very little scattered light escapes, and we see the central source at a low level (Figure 8).

In Figure 9, we show the effects of varying $\zeta$ on the synthetic data. While Equation (6) describes a curved outflow surface, we can equate opening angles with various values of $\zeta$. The considered values of $\zeta = 0.8, 1.0, 1.2$ correspond roughly with half-opening angles of $70^\circ$, $45^\circ$, and $20^\circ$, respectively. Because mass is conserved, larger outflow opening angles (smaller $\zeta$) mean that more mass is distributed in a flattened density distribution. This facilitates the escape of short-wavelength radiation.
emission. Larger values of $\zeta$ produce scattered light images with larger offsets from the central object, since the observed line of sight intersects the cavity at larger radii than for smaller values of $\zeta$.

3.2.4. Source Luminosity

The central luminosity of the model strongly influences the synthetic fluxes. However, the fluxes do not scale simply with
luminosity. As seen in Figure 10, the infrared fluxes increase much more than millimeter fluxes as source luminosity rises. The morphology of the millimeter emission, as well as the scattered light emission, are relatively unaffected by the central luminosity.

3.2.5. Viewing Geometry

Viewing geometry dictates whether we see the disk+envelope model through the outflow cavity, through the dense disk midplane, or through some intermediate region. For more
edge-on inclinations, the central regions of the disk and envelope are more heavily obscured. Scattered light is thus more extended and offset from the center (Figure 11). While inclination does not change the size of the millimeter emission, it does affect the azimuthally averaged visibilities (since the minor axis becomes less resolved for higher inclinations). The P.A. simply rotates the scattered light images. P.A. does not alter the azimuthally averaged visibilities, but it does affect the 2D synthetic visibilities actually used in the modeling.

3.3. Model Fitting

We generate a grid of models and compute the \( \chi^2 \) residuals between each model and the combined data set. Each model takes \( \gg 1 \) hr to generate, and so we are limited to a fairly small grid. We cannot afford to vary all of the parameters discussed above simultaneously. We have chosen to vary \( M_{\text{disk}} \), \( h_{1\text{AU}} \), \( R_{\text{disk}} = R_{\text{c}}, M_{\text{env}}, R_{\text{out}}, f_{\text{cav}}, L, i \), and P.A. We fix \( \zeta = 1.0 \) and \( \alpha = -2.37 \).

We generated models for: \( M_{\text{disk}} = 0.005, 0.01, 0.1, \) and \( 0.5 M_{\odot}; h_{1\text{AU}} = 0.05 \) and 0.15 AU; \( R_{\text{disk}} = R_{\text{c}}, M_{\text{env}}, R_{\text{out}}, f_{\text{cav}}, L, i \), and P.A. We fix \( \zeta = 1.0 \) and \( \alpha = -2.37 \).

Because the weights of different data sets are somewhat arbitrary (for example, they depend on number of observed data points), we adjust the weights to ensure a roughly equal weighting of the different data sets. Following Eisner et al. (2005), we divide each data set by the minimum \( \chi^2 \) residual between all models and that data set

\[
\chi^2_{\text{TOT}} = \frac{\chi^2_{\text{mm}}}{\min(\chi^2_{\text{mm}})} + \frac{\chi^2_{\text{I band}}}{\min(\chi^2_{\text{I band}})} + \frac{\chi^2_{\text{SED}}}{\min(\chi^2_{\text{SED}})}. \tag{9}
\]

We include additional scaling factors here that we may adjust to ensure roughly equal weights given to each data set. Adjusting these scale factors also enables us to illustrate the constraints arising from individual data sets. The reduced \( \chi^2 \) we adopt is given by

\[
\chi^2_{r} = \left( w_{\text{mm}} \frac{\chi^2_{\text{mm}}}{\min(\chi^2_{\text{mm}})} + w_{\text{I band}} \frac{\chi^2_{\text{I band}}}{\min(\chi^2_{\text{I band}})} \right) \left( w_{\text{mm}} + w_{\text{I band}} + w_{\text{SED}} \right) \left( w_{\text{mm}} + w_{\text{I band}} + w_{\text{SED}} \right). \tag{10}
\]

where \( w \) indicates the weighting factor for an individual data set.

4. RESULTS

Table 4 lists the best-fit model parameters for each object in our sample, and Figures 12–19 show observed and synthetic data for these models. For each object, we varied the relative weights of different data sets, to highlight the constraints imposed by each set of observations. Changing the relative weights allows us to explore the region around the \( \chi^2 \) minimum, and hence to determine how shallow the \( \chi^2 \) surface is when projected onto different parameters. For example, if a parameter is very well constrained then it will remain constant as data weights are varied. In contrast, some parameters may explore large values without significantly affecting fit quality.
In general, several parameters are well constrained, staying roughly constant as relative weights are varied. These include $M_{\text{disk}}$, $L_*$, inclination, and P.A. This probably reflects the fact that these parameters tend to strongly influence one data set, and to produce an effect that does not strongly resemble the effects of other model parameters. The main effect of $M_{\text{disk}}$ is on the shape and strength of the millimeter visibilities (Figure 3). While $L_*$ does not directly translate into a flux normalization, its effects are mainly observed in the SED and the normalization of the millimeter visibilities. Since inclination and P.A. produce strong, easily discernible effects on the synthetic scattered light images, their values can be constrained well.

Our parameter grid is sampled only from 1, 3, and 10 AU. A dense sampling of the grid (with a step of 0.5 AU) could lead to a flux normalization that is affected by degeneracies in the model. However, similar conclusions are reached, regardless of the sparse sampling of the grid. The sparse sampling of our model grid occasionally leads to large apparent differences between models. However, the fitted values of these can be extracted. We compute the average disk mass over all the objects (using these averages) is $0.008 M_\odot$. For the overall sample, the envelope mass for each of the four best-fit models. The median disk mass for all the objects (using these averages) is $0.008 M_\odot$. The median envelope mass is $0.08 M_\odot$. For the overall sample, the envelope mass is $\sim 10$ times higher than the disk mass.

The main goals of our study are to constrain the disk mass, envelope mass, and their ratio. From Table 4, values for each of these can be extracted. We compute the average disk mass for each of the four best-fit models. The median disk mass for all the objects (using these averages) is $0.008 M_\odot$. The median envelope mass is $0.08 M_\odot$. For the overall sample, the envelope mass is $\sim 10$ times higher than the disk mass.

Another way to describe disk and envelope masses is to estimate the amount of mass on small scales relative to the more extended mass distribution. This circumvents uncertain interpretations of how much of the mass in our envelope model—which includes some material in a flattened, disk-like
Figure 12. Observed and synthetic data for IRAS 04016+2610. We plot: broadband SEDs, including Spitzer/IRS spectra (left panels), I-band scattered light images (middle panels), and azimuthally averaged λ1.3mm visibilities (right panels). Models are represented by solid curves in the left and right panels, and by contours in the middle panels. The different rows show best-fit models for different relative weightings of the data sets. The plotted models follow the same ordering (top-to-bottom) as the entries in Table 4, where data weights and parameters of best-fit models are listed.
Figure 13. Observed and synthetic data for IRAS 04166+2706. See Figure 12 for a description of the various panels.
Figure 14. Observed and synthetic data for IRAS 04169+2702. See Figure 12 for a description of the various panels.
Figure 15. Observed and synthetic data for IRAS 04287+1801. See Figure 12 for a description of the various panels.
Figure 16. Observed and synthetic data for IRAS 04295+2251. See Figure 12 for a description of the various panels.
Figure 17. Observed and synthetic data for IRAS 04302+2247. See Figure 12 for a description of the various panels.
Figure 18. Observed and synthetic data for IRAS 04361+2547. See Figure 12 for a description of the various panels.
Figure 19. Observed and synthetic data for IRAS 04365+2535. See Figure 12 for a description of the various panels.
distribution within $R_c$—should be counted as part of the disk mass.

In Figure 20, we plot the azimuthally averaged mass distributions for all of the models listed in Table 4. It is evident that for most objects in our sample, the majority of the mass is found at larger radii. This is true even for the disk component, since the assumed surface density profile ($\Sigma \propto r^{-1.37}$) implies a radially increasing mass distribution. Moreover, Table 4 shows that envelope masses typically exceed disk masses in best-fit models.

Table 5 lists the total (disk+envelope) masses within 100 AU for each object and model. This scale corresponds roughly with the resolution of our $\lambda 1.3$ mm observations. Constraints on the mass distribution on still more compact scales are uncertain. The mass found in the inner 100 AU has a median value of $0.007 M_\odot$ for our sample (with large variations). The fractional contribution of this “compact” mass distribution to the total mass varies from 0.03 to 0.44, with a median value of 0.18.

Even though the mass within 100 AU is often a small fraction of the total mass, it produces a relatively large fraction of the millimeter flux (Figure 21). The circumstellar material is considerably hotter at small stellocentric radii. At long wavelengths where emission strength is approximately proportional to temperature and circumstellar mass, even a modest temperature gradient can lead to higher millimeter fluxes at smaller radii. Thus, our modeling can constrain disk mass and geometry (as seen in Table 4) even if the disk contributes modestly to the total mass.

4.1. Comments on Individual Sources

4.1.1. IRAS 04016+2610

Models of this source generally fit all of the multi-wavelength data well. However, as seen in Figure 12, fitting the positional offset of the millimeter and $I$-band emission well leads to a
somewhat worse fit to the SED data (third row of Figure 12 and Table 4). If we allow our modeled scattered light image to have a smaller offset than seen in the data, we can easily reproduce other aspects of the data. Furthermore, the offset in the model image is affected strongly by the outflow cavity parameter $\xi$ (Figure 9), and so it seems that tweaking this parameter could improve the overall fits.

Our modeling of this source indicates a disk mass of $0.005 M_\odot$, a disk radius of 250–450 AU, an envelope mass of $0.05–0.10 M_\odot$, a luminosity of $\gtrsim 3 L_\odot$, an inclination of $\sim 35^\circ–65^\circ$, and a P.A. of $\sim 60^\circ$. The disk scale height, outer radius, and envelope outflow cavity properties are less well constrained.

The envelope appears to be relatively massive around this source, with $\sim 10$ times more mass in the envelope than in the disk. The ratio of compactly distributed (within 100 AU) to the total mass is $\sim 0.03$ (Table 5). The large ratio of envelope to disk mass suggests a younger, less evolved object. On the other hand, the values of $R_{\text{disk}} = R_e \geq 250$ AU, comparable to disk sizes in T Tauri stars (e.g., Dutrey et al. 1996). Thus, the disk may be fully formed in this object, despite continuing infall from an extended envelope.

Earlier modeling of millimeter wavelength continuum and line data suggested a higher disk mass ($\sim 0.02 M_\odot$) with a more highly inclined ($60^\circ–90^\circ$) viewing angle (Hogerheijde 2001). However, more recent millimeter data have been interpreted in the context of a model including disk and envelope masses close to those derived here (Jørgensen et al. 2009). Analysis of scattered light images also produces geometric parameters consistent with those derived here (e.g., Gramajo et al. 2007). This well-studied object has even been subjected to previous modeling of combined multi-wavelength data (Eisner et al. 2005; Brinch et al. 2007a, 2007b); again, results of this modeling are consistent with those presented here. We note, finally, that the luminosity inferred in our modeling is compatible with the effective temperature of this source compared to other objects in
Figure 21. Radial mass distribution (left) and azimuthally averaged λ 1.3 mm flux distribution (right) for the disk+envelope model listed in the first row (model “A” for IRAS 04016+2610) of Table 4. The disk component contributes most of the mass within 100 AU, although this represents only a small percentage of the total mass. However, the disk component dominates the observed flux at λ 1.3 mm, at least on the relatively compact scales (≲1000 AU) probed by our CARMA observations. Note that the component fluxes were determined from models where the disk or envelope mass was set to zero, while the flux of the disk+envelope model is determined from radiative transfer modeling for the combined density distribution.

| Source         | Mass Within 100 AU (M⊙) | Mass Within 100 AU/Total Mass |
|---------------|-------------------------|-----------------------------|
|                | A  | B  | C  | D  | Mean          |
| IRAS 04016+2610| 0.003 | 0.003 | 0.002 | 0.002 | 0.002±0.0004 |
| IRAS 04166+2706| 0.003 | 0.005 | 0.005 | 0.003 | 0.004±0.0009 |
| IRAS 04169+2702| 0.003 | 0.007 | 0.003 | 0.002 | 0.004±0.002  |
| IRAS 04257+1801| 0.05  | 0.18 | 0.09 | 0.18 | 0.12±0.06   |
| IRAS 04295+2251| 0.01  | 0.01 | 0.01 | 0.01 | 0.01±0.0008 |
| IRAS 04302+2247| 0.006 | 0.007 | 0.005 | 0.005 | 0.006±0.001 |
| IRAS 04361+2547| 0.005 | 0.003 | 0.005 | 0.005 | 0.005±0.0008|
| IRAS 04365+2535| 0.007 | 0.006 | 0.007 | 0.007 | 0.007±0.0007|

Notes. For each source, we list the mass within 100 AU for the four models listed in Table 4. We then give the mean value and the standard deviation. Finally, we list the fractional mass within 100 AU compared to the total mass.

For this object our models are able to reproduce well the observed SED and millimeter visibilities. Since no scattered light is detected from this source, the model scattered light image morphologies are not terribly important. However, models with low scattered light fluxes are more consistent with the data. The models shown in the second and third rows of Figures 13 (and seen in the second and third entries for IRAS 04166+2706 in Table 4) have smaller optical/near-IR fluxes. These models still reproduce well the SED and millimeter visibilities.

Inferred disk masses for this object are ∼0.01 M⊙. While a heavy weighting of the scattered light imaging data produces a disk mass of 0, we note that this model has a much smaller value of R⊙ than others, which means that it still has a significant amount of compactly distributed mass. Indeed, the mass within 100 AU is 0.003–0.005 M⊙ (Table 5) for all model fits to this object listed in Table 4.

The envelope mass is ∼0.1 M⊙, and the ratio of mass within 100 AU to the total is ∼0.05. The large ratio of extended mass to compact mass suggests a young age. While the large inferred disk radius (∼450 AU) suggests a possible older age, we note that a similar fit can be achieved with no disk at all, but with an envelope including a flattened density distribution within R⊙ = 30 AU.

The luminosity for this object is inferred to be ∼1–3 L⊙. The inclination and P.A. are 50°–65° and 120°–240°, respectively. The large range in allowed inclination and P.A. (compared with some other objects in our sample) is due to the lack of scattered light emission, which provides strong constraints on geometry. The disk scale height and envelope outer radius are not well constrained.

The results presented here are generally compatible with previous models of SEDs (e.g., Kenyon et al. 1993a; Furlan et al. 2008). However, our study is the first to model multi-wavelength images and the SED simultaneously.

4.1.3. IRAS 04169+2702

Our models for this source provide a reasonable fit to all of the data, although we deem the fits to be worse when we weight the scattered light imaging or SED data heavily. In these cases,
models tend to underpredict the observed millimeter visibilities. However, as seen in the second row of Figure 14, additional weighting of the millimeter data leads to a model that fits all data sets well.

The scattered light images show no detectable emission, and the weak $K$-band flux in our models is compatible with this observation. The scattered light morphology in our models is also consistent with weak scattered light emission seen in previous work (e.g., Kenyon et al. 1993b).

The best-fit models, listed in Table 4, yield a disk mass of $\sim 0.01 M_\odot\,$, a disk radius of $\sim 250-450\,AU$, and an envelope mass of $\sim 0.1 M_\odot\,$. The mass within 100 AU is $\sim 0.004 M_\odot\,$, which accounts for $\sim 0.04$ of the total mass (Table 5). The inferred luminosity is $\sim 1 L_\odot\,$, and the inclination and P.A. are $30^\circ-35^\circ$ and $90^\circ$, respectively. As for other objects in our sample, the disk scale height, envelope outer radius, and outflow cavity filling factor, are not well constrained.

4.1.4. IRAS 04287+1801 (L1551 IRS 5)

As seen in Figure 15, we were not able to provide a perfect fit to our entire multi-wavelength data set. We can fit the SED and scattered light image well together, but that model underpredicts the millimeter visibilities (top row of Figure 15). If we fit the scattered light image and millimeter visibilities well, the SED fit quality is degraded (second row of Figure 15). We can fit all of the data reasonably well if we allow the model to overpredict the optical/near-IR fluxes (bottom row of Figure 15). Some additional extinction, for example, from a different shape of the outflow cavity (Figure 9), may provide an even better fit.

The difficulty in achieving a high-quality fit to this object is due, in part, to the sparse sampling of parameter values in our modeling. Best-fit models often choose the highest sampled values of $M_{\text{disk}}$, $M_{\text{env}}$, and $L_*$ in particular, a value of $L_* > 10 L_\odot\,$ would produce a higher peak SED and larger overall millimeter visibilities (Figure 10). Thus, a luminosity larger than $10 L_\odot\,$ would probably result in a better fit to the data.

IRAS 04287+1801 is also known to be a multiple system, which may complicate fitting of our simple models to the data. This object is clearly resolved as a $\sim 0^\prime/3$ binary (Rodriguez et al. 1998), and even shows evidence for a third component $<0^\prime/1$ away from the northern component in high-resolution observations (Lim & Takakura 2006). All of these components would be within the resolution of our CARMA observations.

Previous authors have modeled SED data, scattered light imaging, and even millimeter imaging data for this source. Osorio et al. (2003) attempted to fit the SED and low-resolution submillimeter imaging with a binary protostar model. Their model specifies a circumbinary disk mass of $0.4 M_\odot\,$, a disk radius of $300\,AU$, an envelope mass of $\sim 0.01 M_\odot\,$, and $i = 50^\circ$. Subsequent modeling of the SED (Furlan et al. 2008) or scattered light imaging (Gramajo et al. 2007) finds similar results, but with smaller envelope masses and disk centrifugal radii.

Inferred parameter values from our modeling are: $M_{\text{disk}} = 0.1-0.5 M_\odot\,$, $R_{\text{disk}} = 100-450\,AU$, $M_{\text{env}} = 0.05-0.1 M_\odot\,$, $L_* \sim 10 L_\odot\,$, $i = 40^\circ-50^\circ$, and P.A. $= 160^\circ-180^\circ$ (Table 4). These are generally consistent with previous results, with the exception of a lower $L_*$. It seems that we recover most of the details of the circumstellar mass distribution, even for this complicated multiple system, using relatively simple disk+envelope models.

4.1.5. IRAS 04295+2251

Best-fit models provide a reasonable fit to the combined data set for IRAS 04295+2251, although better fits to individual data sets are possible at the expense of degrading the fits to other data (Figure 16). For example, the best-fit model to the millimeter visibility data does not fit the SED as well as other models. This may reflect the sparse sampling in our model grid to some extent. However, the fact that none of our models provide an excellent fit to the SED suggests that certain assumed parameters (e.g., dust opacities or outflow cavity shape) need to be adjusted to improve the overall fits.

Our modeling implies a disk mass of $0.01 M_\odot\,$ and a relatively compact disk radius of $30-100\,AU$. The envelope mass is inferred to be $0.005-0.05 M_\odot\,$, the envelope radius is $500-1000\,AU$, $L_*=1 L_\odot\,$, and the inclination and P.A. are $45^\circ-55^\circ$ and $300^\circ$, respectively (Table 4). This source has a high fraction of compact-to-extended mass, with $\sim 44\%$ of the matter distributed within $100\,AU$ (Table 5).

While most of the inferred parameter values are similar to those found in previous modeling of multi-wavelength data (Eisner et al. 2005), the disk mass presented here is substantially lower. We find $0.01 M_\odot\,$ while Eisner et al. (2005) found $1 M_\odot\,$. However, Eisner et al. (2005) assumed the disk mass was distributed over radii out to $500\,AU$, while the modeling presented here places the disk mass within $30-100\,AU$. Given the higher resolution of the data presented here, it is not surprising that the disk mass is now constrained to smaller radii. Since the matter at smaller radii produces most of the emission (Figure 21), a smaller disk requires less total mass to reproduce the observations.

4.1.6. IRAS 04302+2247

We are unable to find a single model that reproduces the combined multi-wavelength data set for this source. We can fit well the SED and millimeter data together, or the SED and scattered light data together, but not the millimeter and scattered light data together (Figure 17). However, the best-fit models for different data weightings produce fairly consistent values for some parameters. Disk mass is $0.005-0.01 M_\odot\,$, and the disk radius is $100-250\,AU$. The envelope mass is not well constrained, spanning values between 0.005 and $0.1 M_\odot\,$, while the envelope outer radius appears to be $\sim 500\,AU$. The inclination is $70^\circ-89^\circ$, and the P.A. is $10^\circ$. The mass within $100\,AU$ is found to be $\sim 0.006 M_\odot\,$, accounting for $\sim 20\%$ of the total mass (Table 5).

Previous modeling of millimeter and scattered light imaging data argued for different dust grain properties in the disk and envelope components (Wolf et al. 2003). We have not included such effects in our modeling, which may explain why we have difficulty fitting the millimeter visibilities and the scattered light images simultaneously. With a detailed model for this single object, Wolf et al. (2003) derived $M_{\text{disk}} = 0.07\,AU$, $R_{\text{disk}} = 300\,AU$, $h_{1\,AU} = 0.15\,AU$, $M_{\text{env}} = 0.05 M_\odot\,$, and $R_{\text{env}} = 450\,AU$. For the most part, our modeling (Table 4) is consistent with these results. The disk mass we infer is substantially lower, however, perhaps due to a somewhat smaller fitted disk radius, as well as different assumed opacities.

4.1.7. IRAS 04361+2547

We are able to fit the data for this source quite well, especially when the imaging data are given some additional weight compared to the SED data (Figure 18). The disk mass
is inferred to be $0.005 - 0.01 M_\odot$, and the envelope mass is comparable, $\sim 0.01 M_\odot$. The disk radius is not well constrained, while the outer radius of the envelope is inferred to be 500 AU. The total luminosity is $3 L_\odot$, the inclination is $\sim 45^\circ$, and the P.A. is $\sim 280^\circ$. This source exhibits a relatively high fraction of compact-to-extended mass content, with $\sim 40\%$ of the mass distributed within 100 AU (Table 5).

The disk mass we derive is consistent with Jørgensen et al. (2009, who found $M_{\text{disk}} = 0.009 M_\odot$), but the envelope mass we infer here is somewhat lower. This is probably a consequence of the fact that our millimeter observations are not sensitive to emission more extended than $\sim 1000$ AU, while Jørgensen et al. (2009) used single-dish observations to detect more extended emission. We predict a somewhat lower inclination than found in previous scattered light imaging (Kenyon et al. 1993b; Gramajo et al. 2007); however, yet lower inclination is inferred in previous SED modeling (Kenyon et al. 1993a). Thus, our inferred geometry is generally consistent with previous work.

4.1.8. IRAS 04365+2535

Our models for IRAS 04365+2535 fit the combined multiwavelength data set well, regardless of data weighting factors (Figure 19). Best-fit models imply $M_{\text{disk}} = 0.005 M_\odot$, $R_{\text{disk}} = 100$ AU, $M_{\text{env}} = 0.05 - 0.1 M_\odot$, $R_{\text{out}} = 500 - 1000$ AU, $L_\star = 1 L_\odot$, inclination $= 25^\circ$, and P.A. $= 310^\circ$. The mass enclosed within 100 AU is $\sim 0.007 M_\odot$, which comprises $\sim 11\%$ of the total mass.

Previous analysis of millimeter visibility data determined a disk mass of $0.03 M_\odot$, and an envelope mass of $0.12 M_\odot$ (Jørgensen et al. 2009). The disk mass in particular is higher than we infer in our analysis. This discrepancy is probably due to the lower angular resolution in the previous observations: $\sim 2\farcs5 = 350$ AU. In contrast, we infer a disk size of 100 AU. With a larger beam, the earlier observations could have easily inferred a larger mass. The results of our modeling also appear consistent with previous modeling of the SED or scattered light imaging (Kenyon et al. 1993a, 1993b; Gramajo et al. 2007), although our inferred inclination is somewhat lower than previous estimates.

5. DISCUSSION

The disk mass is a crucial parameter in understanding both star and planet formation. High disk masses, on the order of a tenth of the stellar mass, would imply that gravitational instability may be important. Such instability can lead to rapid accretion onto the central protostar and may explain why accretion rates onto the star appear low compared to estimated time-averaged accretion from the envelope onto the disk. Episodes of gravitationally enhanced accretion can lead to larger time-averaged rates but low typical instantaneous accretion rates (e.g., White & Hillenbrand 2004; Eisner et al. 2005).

Disk mass is also a critical initial condition for planet formation. To form gas giants like Jupiter and Saturn requires $> 0.01 M_\odot$ of matter (Weidenschilling 1977b). While measured disk masses around T Tauri stars are smaller (e.g., Andrews & Williams 2005; Eisner et al. 2008), this may reflect the agglomeration of small particles into larger bodies rather than a mass deficit. Data for the less evolved protostars in our sample can provide masses in systems where dust grain agglomeration is presumably less advanced.

We begin by examining the distribution of disk masses for our sample, drawing values from Table 4. Disk masses range from 0.005 to 0.5 $M_\odot$ across the sample, and the median disk mass is 0.008 $M_\odot$. The majority ($\sim 60\%$) of objects have disk masses $\geq 0.01 M_\odot$, and $\lesssim 10\%$ have $M_{\text{disk}} > 0.1 M_\odot$. For comparison, across $> 150$ (more evolved) Class II disks in Taurus, Andrews & Williams (2005) find a median mass of 0.005 $M_\odot$. In this sample of Class II objects, $\sim 20\%$ of disks are more massive than 0.01 $M_\odot$ and only $\sim 2\%$ are more massive than 0.1 $M_\odot$ (Andrews & Williams 2005).

Our sample of Taurus Class I objects has significantly more massive disks than the Class II population. Indeed, many of them may still possess sufficient mass in small ($\lesssim$mm-sized) dust grains for the eventual formation of planetary systems like our own (depending on how the mass is distributed on scales smaller than 100 AU). The lower masses in Class II disks likely reflect that some of the mass of small particles available at the Class I stage has grown into large ($>$meter-sized) bodies within $\sim 1$ Myr.

This argument can easily be extended to Class I sources themselves. While Class I disks appear more massive than Class II disks—at least as traced by small ($\lesssim$mm-sized) dust grains—particles in Class I disks may already have undergone substantial processing. That is, planet formation may already be underway even at these ages. Extending the analysis in this paper to younger, less evolved Class 0 objects provides a clear path to improving the estimates of the initial mass content of protoplanetary disks.

The range of disk masses observed for our sample indicates that few Class I disks are at the limit of gravitational instability. Even IRAS 04287+1801, which has a disk mass $\geq 0.1 M_\odot$, may remain stable against gravitational collapse because of the higher stellar mass contained in the multiple central system. If, however, substantial mass is hidden in larger bodies, gravitational instability may still be possible in these systems.

Measured envelope masses can be used to estimate the mass infall rate from the envelope onto the disk, $M$. Estimates of $M$ can then be compared to the accretion rate measured from the inner disk onto the star (e.g., White & Hillenbrand 2004). Such comparisons typically conclude that the infall rate from the envelope is higher than the accretion rate onto the star, suggesting that matter piles up in the disk and periodically accretes quickly in bursts (e.g., Kenyon & Hartmann 1987; White & Hillenbrand 2004; Eisner et al. 2005; Vorobyov & Basu 2005). The results presented here are generally compatible with this view.

However, the envelope masses estimated here are highly uncertain. For example, the range of $M_{\text{env}}$ allowed for IRAS 04016+2610 (Table 4) correspond to $M$ between $\sim 10^{-6}$ and $10^{-5} M_\odot$ yr$^{-1}$. Much of this uncertainty is due to degeneracies with other parameters (primarily $M_{\text{out}}$; see Section 3.2). Additional uncertainties arise because of the limited $uv$ coverage (i.e., range of telescope baseline separations) in our millimeter observations. The shortest baselines correspond to linear resolutions of $\sim 1000$ AU at the distance to our targets. More extended emission is largely resolved out. Thus, we are not sensitive to extended emission and, as a result, our total envelope masses are likely underestimated.

Despite uncertainties in inferred values of $M_{\text{env}}$, clear differences in the ratio of $M_{\text{disk}}/M_{\text{env}}$ are observed across our sample. The disk-to-envelope mass ratio ranges from $< 0.1$ for some objects (IRAS 04016+2610) to $> 1$ for others (IRAS 04287+1801). In most cases, the disk-to-envelope mass ratio is around 0.1. This suggests our sample is at an earlier evolutionary stage than Class II objects, which are essentially 100% disk. Furthermore,
one might be tempted to assign relative ages to sample objects based on $M_{\text{disk}}/M_{\text{env}}$ values.

Another method to constrain relative ages in our sample is to examine the disk/centrifugal radii. For collapsing, rotating envelopes, the centrifugal radius grows with time because of the inside-out nature of the collapse. For the envelope models used here, one expects the centrifugal, and hence disk, radius to grow with time as $R(t) \propto t^3$. If our sample objects had similar initial specific angular momenta (which is by no means guaranteed), then the inferred centrifugal radii can be used to estimate relative ages.

The disk radii inferred in our modeling have substantial uncertainties, but do reveal trends between objects (Table 4). For example, IRAS 04016+2610 seems to have a larger disk/centrifugal radius than IRAS 04295+2251. However, IRAS 04016+2610 has a lower disk-to-envelope mass ratio than IRAS 04295+2251. While a smaller disk/centrifugal radius suggests a younger source, a larger ratio of disk-to-envelope mass suggests an older object. We therefore do not place much confidence in our ability to distinguish ages among our sample.

To compare our inferred disk masses with expected minimum masses for planet formation theory, we turn now to the masses listed in Table 5. The table lists circumstellar masses enclosed within 100 AU, including both disk and inner envelope components. Since the inner regions of the envelope have a flattened density distribution (Equation (1)), they may appear disk-like, and may contribute to the mass budget for planet formation.

We choose 100 AU because this is approximately the smallest scale we can resolve with our millimeter imaging data. However, we would ideally like to probe smaller scales, since planet formation theories usually require a minimum mass within radii of 30–50 AU (e.g., Weidenschilling 1977b; Hayashi 1981). Thus, even our 100 AU masses (Table 5) may overestimate the mass available for planet formation. Even so, the masses listed in Table 5 are typically lower than the 0.01–0.1 $M_{\odot}$ usually required to form planetary systems like ours. As discussed above, this may reflect growth of small, observable dust grains into larger bodies. Probing younger sources may provide a clearer picture of how much mass is truly available to planet formation.

6. SUMMARY AND FUTURE DIRECTIONS

We described new millimeter imaging observations of a sample of Class I protostars, obtained with the CARMA interferometer. We presented basic properties of the millimeter emission, and a more detailed analysis of the millimeter data in conjunction with imaging and flux data at other wavelengths. We constructed models consisting of rotating, infalling envelopes and embedded, compact disks, and generated a grid of synthetic multi-wavelength data sets using the radiative transfer code RADMC. We fitted these models to the multi-wavelength data set for each target.

Our focus in this study is measuring disk and envelope masses. We found disk masses for our sample ranging from $\lesssim 0.01$ to $\gtrsim 0.1 M_{\odot}$. The compact disks typically only comprise a small percentage of the total mass ($\lesssim 10\%$), although this percentage is higher in a few objects. Envelope masses—and by extension, the disk-to-envelope mass ratios—are usually poorly constrained by our modeling. This is due, in large part, to the limited sensitivity of our observations to emission more extended than $\sim 1000$ AU. Better $uv$ coverage, for example, combining extended and compact configurations of CARMA or the Atacama Large Millimeter/submillimeter Array (ALMA), will alleviate this problem in the future.

The disk masses inferred for our sample are significantly higher than for samples of more evolved Class II objects. Thus, Class I disk masses probably provide a more accurate estimate of the initial mass budget for star and planet formation, before particle growth and planetesimal formation lock up mass in hard-to-see larger bodies. However, such particle agglomeration may already be underway by the Class I epoch, and so measured disk masses may still be underestimates. Observing Class I sources at multiple millimeter and submillimeter wavelengths with ALMA will help to constrain grain properties and how these depend on location in the disk, providing tighter constraints on disk masses and the extent of grain growth.

Another obvious extension of this project is to observe larger samples of protostars over a range of evolutionary states. In particular, observations and modeling of Class 0 protostars can reveal if their disk masses are larger than those inferred around Class I objects. Comparing disk masses for samples of objects at different ages will constrain how the mass in small dust grains evolves and give a better estimate of the true initial disk mass distribution.

The angular resolution of our observations is sufficient to constrain the disk mass distribution within $\sim 100$ AU. In contrast, planet formation models generally only consider the mass within 30–50 AU. Future work, using more extended array configurations of CARMA or ALMA, can reach these scales and provide constraints on disk masses that are directly comparable to critical initial conditions of planet formation theories.

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