Kinetic coarsening in the random field Ising model : a different approach

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We report a study of nonequilibrium relaxation in a two-dimensional random field Ising model at a nonzero temperature. We attempt to observe the coarsening process from a different perspective with a particular focus on three dynamical quantities that characterize the kinetic coarsening. We give a simple generalised scaling theory of coarsening supported by numerical results. The excellent data collapse of the dynamical quantities justifies our proposition. The scaling relation turns out to be universal in characterizing the kinetic coarsening.

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Study of the effect of disorder and randomness on nondisordered magnetic systems has been a subject of intense interest for the last several years\textsuperscript{\textcopyright,\textcopyright}. When a system is quenched from a high temperature to a low temperature, it locally orders with the formation of domains separated by domain walls. The average linear size of the domains $R(t)$ grows with time. This linear size can also be understood as the non equilibrium correlation length of the system. The growth of the characteristic length scale $R(t)$ with time is known as the coarsening of the system. Although coarsening process in nondisordered systems is very well understood\textsuperscript{\textcopyright}, progress in understanding the same in disordered systems has been rather slow. In disordered systems, most aspect of non equilibrium relaxation remains under debate\textsuperscript{\textcopyright}. Unavailability of reliable theoretical tools makes it difficult to study the dynamics of disordered systems out of equilibrium. Moreover, the dynamics of disordered systems is typically so slow that we cannot access the truly asymptotic time regime in numerical simulations. Despite all these, last several years have witnessed appreciable effort in the study of disordered but unfrustrated systems. These include coarsening of disordered magnets\textsuperscript{\textcopyright,\textcopyright}, polymers in random media\textsuperscript{\textcopyright,\textcopyright,\textcopyright} or vortex lines in disordered type-II superconductors\textsuperscript{\textcopyright,\textcopyright,\textcopyright}.

The ideal model for studying coarsening in disordered systems is the random field Ising model (RFIM) in which the random fields or impurities couple directly to the local order parameter. The fundamental quantity of interest in the coarsening process is the growing length scale $R(t)$ and almost all studies of coarsening is primarily concerned with the determination of this characteristic length scale $R(t)$. But the growth law governing the coarsening process of disordered systems is at the center of some controversies. Some numerical simulations on disordered ferromagnets\textsuperscript{\textcopyright,\textcopyright} yielded an algebraic growth $R(t) \sim t^{1/z}$, with a nonuniversal dynamical exponent $z$ that depends on the temperature and on the nature of disorder. Huse and Henley\textsuperscript{\textcopyright} suggested a logarithmic increase of this characteristic length, $R(t) \sim (\ln t)^{1/\psi}$, with the barrier exponent $\psi > 0$. Later a series of papers on the dynamics of elastic lines in a random potential\textsuperscript{\textcopyright,\textcopyright,\textcopyright} claims a dynamic crossover from a pre-asymptotic algebraic regime to a asymptotic slow logarithmic regime.

In the present work, we attempt to study coarsening in a disordered spin system, namely the two-dimensional RFIM, from a different angle. To characterize the coarsening, we focus our attention on three dynamical quantities which are, in general, functions of the strength of the random fields ($\eta_0$) and the temperature ($T$). These are the total length of the interfaces ($\Pi(\eta_0, t)$), i.e., the total number of boundary spins of all the domains, the total number of domains ($\Lambda(\eta_0, t)$) in the system and the length of the interface of the domain with largest mass ($\Omega(\eta_0, t)$), i.e., the number of boundary spins of the domain containing maximum number of spins. In general, we denote by $\Psi(\eta_0, t)$ the three quantities which characterize the kinetic coarsening in the two-dimensional RFIM.

The Hamiltonian of the RFIM is given by

$$H = -J \sum_{\langle i,j \rangle} s_i s_j + \sum_i \eta_i s_i + H_{\text{ext}} \sum_i s_i$$  \hspace{1cm} (1)$$

where $s_i = \pm 1$ is the spin variable at site $i$, $J$ is the strength of the exchange interaction (conventionally set to unity) and $\eta_i$ is the quenched random fields taken from an uniform distribution with varying strength $\eta_0$. $\langle i,j \rangle$ indicates that the sum is over nearest neighbours. The external field $H_{\text{ext}}$ has been set to zero to observe the unbiased dynamics of the system. We consider a $L \times L$ square lattice (here $L = 256$) with periodic boundary conditions along both directions. We start our simulations from a completely random spin configurations, characteristic of a high temperature ($T = \infty$) phase.

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and then suddenly quench the system to a temperature $T = 0.50$, which is well below the critical temperature of a nondisordered system (Ising model) and then observe the time evolution of the system. The single spin flip Metropolis algorithm is used to simulate the system. Being a single spin flip dynamics, the Metropolis algorithm is believed to represent the natural way of the evolution of a system, since the acceptance ratio is given by the Boltzmann probability and the dynamics is local. The temperature is taken sufficiently low to reduce the thermal fluctuations. Coarsening proceeds through a compromise between the strength of the exchange interaction and the random fields with the thermal fluctuations serving only to renormalize the strengths of these couplings. The number of domains with their sizes are determined by the Hoshen-Kopelman algorithm. All the quantities are averaged over 50 independent simulations to get a precise estimate. The quantities are normalized with respect to the total number of spins ($L^2$) of the system.

We begin our analysis with the idea put forward by Imry and Ma. In their introductory article, they argued that if one reverses the spins within a domain of linear size $R$, the energy cost $E_{ex}$ is proportional to the domain wall area, i.e., $E_{ex} \propto JR^{d-1}$ where $d$ is the spatial dimension. This energy increase has to be compared with the energy gain from the interaction with the random field. The central limit theorem tells that the mean square random field energy $E_{RF}^2$ inside a region of volume $R^d$ is $\sim \eta_0^2 R^d$. The total energy involved in the creation of a domain of linear size $R$ is therefore

$$E(R) \approx JR^{d-1} - \eta_0 R^{d/2}$$  \hspace{1cm} (2)

The first term of Eq. (2) represents the contribution due to the boundary of the domain with linear size $R(t)$. The second term represents the contribution due to the fluctuations of the random fields in the bulk of the domain of linear size $R(t)$. We have extended this idea for determining the energy corresponding to the creation of all the domains at any instant of time and for a particular disorder strength ($\eta_0$). Taking a cue from the above relation, the surface energy of all the domains $E_{ex}^s \sim J\Pi(\eta_0, t)$ and the mean square bulk energy contained in all the domains of the system due to random fields $E_{RF}^2 \sim \frac{\eta_0^2 J^2}{\Lambda(\eta_0, t)}$, as the density of domains is inversely related to their characteristic volume. Thus the energy density ($\epsilon = E/l^d$) involved in the creation of all the domains in the system is given by

$$\epsilon(\eta_0, t) \approx \frac{J\Pi(\eta_0, t)}{L^d} - \frac{\eta_0}{\Lambda(\eta_0, t)^{1/2}}$$  \hspace{1cm} (3)

As $L \rightarrow \infty$, the surface energy contribution vanishes. This is true for any growing volume. For finite system size, the contribution from the surface energy term cannot be neglected. So the variation of both $\Pi(\eta_0, t)$ and $\Lambda(\eta_0, t)^{1/2}$ with time will govern the coarsening of the overall system. The log-log plot of $\Pi(\eta_0, t)$ and $\Lambda(\eta_0, t)$ against time are shown in Fig. 1 and Fig. 2 respectively. It is evident from Eq. (3) that the coarsening of the system energetically favours the minimization of the total length of the interfaces and also the decrease of the number of the domains. This is observed in Fig 1 and Fig 2 respectively. In view of the above discussions, we can redefine the coarsening as simply the minimization of $\epsilon(\eta_0, t)$ and the kinetic coarsening will be characterized by the scaling behavior of the two quantities $\Pi(\eta_0, t)$ and $\Lambda(\eta_0, t)^{1/2}$. With the flow of time, small domains coalesce to form relatively larger domains and from Eq. (2), it is clear that a domain with a typical size $R$ should grow in such a way that the length of the interface of that domain shrinks while the volume of the domain expands in order to minimize the energy of the domain. The domain

![Fig. 1](image1.png)

![Fig. 2](image2.png)
with largest mass should grow in the same fashion during its dynamical evolution and plays the most dominating role in governing the dynamic behavior of the system. Recently, we report in Ref. [22] that the size of a domain in two-dimension increases as a power law with time. Since the domain with largest mass predominates over all the other domains, the length of the interface of this domain \((\Omega(\eta_0, t))\) is expected to exhibit similar behaviour as that of \(\Pi(\eta_0, t)\). Log-log plot of \(\Omega(\eta_0, t)\) against time is shown in Fig. 3.

![FIG. 3. Plot of \(\Omega(\eta_0, t)\) against time along with the best fits according to (5).](image)

We now give a simple unified coarsening theory. A careful observation of the graphs suggests that the initial and asymptotic behaviour of the generalized function \(\Psi(\eta_0, t)\) is given by

\[
\Psi(\eta_0, t) \to \Psi_0 \text{ (constant), as } t \to 1
\]

and \(\Psi(\eta_0, t) \to \Psi_0 e^{-\mu(\eta_0)/\nu(\eta_0)}, \nu(\eta_0) > 0, \text{ as } t \to \infty\)

where \(\nu(\eta_0)\) is a disorder dependent scaling exponent. The decay rate of the function \(\Psi(\eta_0, t)\) at any time step for a particular value of \(\eta_0\) should depend on the following factors: first, on the value of the function itself at this time step. Secondly, from the nature of the variation of the functions \(\Pi(\eta_0, t), \Lambda(\eta_0, t)\) and \(\Omega(\eta_0, t)\), it is evident that the rate of decay of the function \(\Psi(\eta_0, t)\) also depends on the particular time step. As the time flows, the rate of decay of \(\Psi(\eta_0, t)\) slows down and this dependence is taken as a power law decay. In addition to these factors, another \(\eta_0\)-dependent parameter should come into the picture for controlling the decay rate of \(\Psi(\eta_0, t)\). This parameter considers the wandering of the interfaces in presence of the random field. Thus the decay rate of \(\Psi(\eta_0, t)\) is given by

\[
\frac{d\Psi}{dt} \sim -a(\eta_0)\Psi t^{-\mu(\eta_0)}, \mu(\eta_0) > 0
\]

\(a(\eta_0)\) is a disorder-dependent parameter. Integrating,

\[
\Psi(\eta_0, t) = \Psi_0 e^{\left(-\frac{\mu(\eta_0) - 1}{\nu(\eta_0)} + k(\eta_0)\right)t}
\]

where \(k(\eta_0)\) is a constant of integration and \(\nu(\eta_0) = \frac{\mu(\eta_0) - 1}{a(\eta_0)}\). Now from (3) as \(t \to 1\), \(\Psi(\eta_0, t) \to \Psi_0\) which gives \(k(\eta_0) = 0\) and as \(t \to \infty\), \(\Psi(\eta_0, t) \to \Psi_0 e^{-\mu(\eta_0)/\nu(\eta_0)}\) which gives \(\mu(\eta_0) > 1\). Thus the functional form of \(\Psi(\eta_0, t)\) at any time step is given by

\[
\Psi(\eta_0, t) = \Psi_0 e^{\left[-\frac{1 - t^{-\rho(\eta_0)}}{\nu(\eta_0)}\right]}
\]

where \(\rho(\eta_0) = \mu(\eta_0) - 1 > 0\). The scaling behaviour (7) of the functions characterizing the kinetic coarsening shows an universal nature with two disorder-dependent exponents \(\rho(\eta_0)\) and \(\nu(\eta_0)\). The plots of the data collapse for the three functions \(\Pi(\eta_0, t), \Lambda(\eta_0, t)\) and \(\Omega(\eta_0, t)\) characterizing the kinetic coarsening are shown in Fig. 4, Fig. 5 and Fig. 6 respectively.

![FIG. 4. Plot of the data collapse of the function \(\Pi(\eta_0, t)\). The inset shows the variation of \(\rho(\eta_0)\) and \(\nu(\eta_0)\) against \(\eta_0\).](image)

![FIG. 5. Plot of the data collapse of the function \(\Lambda(\eta_0, t)^{1/2}\). The inset shows the variation of \(\rho(\eta_0)\) and \(\nu(\eta_0)\) against \(\eta_0\).](image)

![FIG. 6. Plot of the data collapse of the function \(\Omega(\eta_0, t)^{1/2}\). The inset shows the variation of \(\rho(\eta_0)\) and \(\nu(\eta_0)\) against \(\eta_0\).](image)

It is evident from these figures that the scaling relation (7) yields an excellent data collapse, which confirms

\[
\Psi(\eta_0, t) = \Psi_0 e^{\left[-\frac{1 - t^{-\rho(\eta_0)}}{\nu(\eta_0)}\right]}
\]
FIG. 6. Plot of the data collapse of the function $\Omega(\eta_0, t)$. The inset shows the variation of $\rho(\eta_0)$ and $\nu(\eta_0)$ against $\eta_0$.

the validity of our proposition. We interpret $\nu(\eta_0)$ to be comprised of two parts. One part (say $\nu_1$) is fixed and it is a measure of the surface tension when the domains grow due to exchange interaction. The second part (say $\nu_2(\eta_0)$), that depends on $\eta_0$, prevents the domains to grow with time. This part may be thought to be a measure of the stiffness of the interfaces. Beyond a threshold value of $\eta_0$, $\nu_2(\eta_0)$ predominates after the time scale $t_x \sim 1/\nu(\eta_0)^{1/\nu(\eta_0)}$ and the domains are pinned, i.e., they are prevented from growing. For $t >> t_x$, the dynamic function $\Psi(\eta_0, t)$ becomes almost constant with time and the asymptotic value is $\sim \Psi_0 e^{-1/\nu(\eta_0)}$. With this interpretation, $\nu(\eta_0)$ should be a monotonically increasing function of $\eta_0$. It is shown in the inset of the figures. The variation of $\rho(\eta_0)$ against $\eta_0$ is also shown in the inset of the figures. As $t \to 1$, $\Psi(\eta_0, t)$ reaches a fixed value $\Psi_0$, independent of $\eta_0$.

In summary, we have studied the kinetic coarsening phenomena in a disordered but unfrustrated system, namely the RFIM in a different approach. The importance of this study lies in the fact that disorders are inherent in many physical systems and therefore the behaviour of these systems far from equilibrium is an active area of basic research in its own right. Most of the conventional studies of coarsening focuses considerable attention on the growth of the average linear domain size following a quench from a high temperature to a very low temperature. In the present work, instead we have devoted most attention on the time evolution of three dynamical quantities, introduced earlier in this paper. These three quantities characterize the kinetic coarsening of the system. We propose a dynamical scaling relation for the three quantities and have shown that the scaling is universal in characterizing the kinetic coarsening of the two-dimensional RFIM. We hope that the approach to study coarsening along the lines done here for the RFIM can also be followed in other disordered systems as well and the same dynamical scaling holds good.

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