Dirac-fermions and conductance-oscillations in $(s, d)$-wave superconductor/normal graphene junctions

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We investigate quantum transport in a normal/superconductor graphene heterostructure, including the possibility of an anisotropic pairing potential in the superconducting region. We find that under certain circumstances, the conductance displays an undamped, oscillatory behaviour as a function of applied bias voltage. Also, we investigate how the conductance spectra are affected by a $d$-wave pairing symmetry. These results combine unusual features of the electronic structure of graphene with the unconventional pairing symmetry found for instance in high-$T_c$ superconductors.

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Graphene is a monoatomic layer of graphite with a honeycomb lattice structure [1]. The electronic properties of graphene display several intriguing features, such as a six-point Fermi surface and Dirac-like low-energy energy dispersion around the Fermi-points. Condensed matter systems with such ‘relativistic’ electronic structure properties constitute fascinating examples of low-energy emergent symmetries (in this case Lorentz-invariance). Another example where precisely this occurs is in one-dimensional interacting fermion systems, where phenomena like breakdown of Fermi-liquid theory and spin-charge separation take place. Graphene features certain similarities to, but also important differences from, the nodal Dirac fermions emerging in the low-energy sector of the pseudogap phase of $d$-wave superconductors such as the high-$T_c$ cuprates. When Lorentz-invariance emerges in the low-energy sector of higher-dimensional condensed matter systems, it is bound to attract much interest from a fundamental physics point of view.

Various aspects of resonant tunneling phenomena in N/N and N/I/N graphene structures have recently been investigated [2]. Although superconductivity does not appear intrinsically in graphene, it may nonetheless be induced by means of the proximity effect [3]. Motivated by this, the authors of Refs. [4, 5] considered quantum transport in N/S and N/I/S graphene junctions for the case where the pairing potential is isotropic, leading to $s$-wave superconductivity. However, the hexagonal symmetry of the graphene lattice also admits unconventional order parameters such as $p$-wave or $d$-wave. The possible pairing symmetries on a hexagonal lattice up to $f$-wave pairing ($l = 3$) was given in Ref. [6]. Interestingly, among the allowed order parameters, one finds the $d_{x^2-y^2}$ symmetry, which is believed to be the dominant pairing symmetry in high-$T_c$ superconductors. Consequently, it should be possible to induce superconductivity with nodes in the gap in graphene by manufacturing heterostructures of graphene and unconventional superconductors. It is of interest to investigate how this would affect coherent quantum transport in junctions with normal and superconducting graphene. In particular, it is essential to study possible zero-energy states (ZES) at the interface of such a junction. Such states are known to give rise to zero-bias conductance peaks (ZBCPs) in metallic N/S junctions [7], and will influence the conductance spectra of N/I/S junctions.

In this Letter, we take into account the possibility of an anisotropic pairing potential induced in graphene, and study coherent quantum transport in both N/S and N/I/S junctions. In addition, we show that in the latter structure, novel conductance-oscillations as a function of bias voltage are present both for $s$-wave and $d$-wave symmetry of the superconducting condensate due to the presence of low-energy ‘relativistic’ nodal fermions on the N-side. The period of the oscillations decreases with increasing width $w$ of the insulating region, and persists even if the Fermi energy in $I$ is strongly shifted. This contrasts sharply to metallic N/I/S junctions, where the presence of a potential barrier causes the transmission of the junction to go to zero with increasing $w$. The feature of conductance-oscillations is thus unique to N/I/S junctions with low-energy Dirac-fermion excitations. Moreover, we contrast the N/S or N/I/S conductance spectra for the cases where $s$-wave and $d_{x^2-y^2}$-wave superconductor constitutes the S-side. The former has no nodes in the gap and lacks Andreev bound states. The latter has line-nodes that always cross the Fermi surface in the gap, and thus features in addition to Andreev bound states, also nodal relativistic low-energy Dirac fermions. The quantum transport properties in a heterostructure of two such widely disparate systems, both featuring a particular intriguing emergent low-energy symmetry, is of considerable importance.

The Brillouin zone of graphene is hexagonal and the energy bands touch the Fermi level at the edges of this zone, amounting to six discrete points. Out of these only two are inequivalent, denoted $K$ and $K'$ and referred to as Dirac points. The energy dispersion in the Brillouin zone was calculated within a tight-binding model [8], revealing a conical structure of the conduction and valence bands close to the six Fermi points, giving rise to an essentially linear dispersion. Graphene N/S interfaces contain a new phenomenology compared to their metallic counterpart, namely the possibility of specular Andreev-reflection (AR) [4]. In the process of normal AR, an incident electron from the N side is reflected as...
a hole which retraces the trajectory of the electron. In specular AR, the reflected hole follows the trajectory which a normally reflected electron would have. Depending on whether the graphene is doped or not, specular and normal AR will differ. In specular AR, the reflected hole follows the trajectory which a normal hole retraces the trajectory of the electron. In specular reflection, (\(\theta_A = \pm \pi/2\)), and consequently no subgap conductance, for angles of incidence above the critical angle \(\theta_c = \text{asin}(|E - E_F|/(E + E_F))\).

The conductance of the N/I/S junction is given by \[ G(e V) = \frac{G_N}{4} \left[ 1 + \sin(\theta)p e V \right] \left[ 1 + \sin(\theta)\theta_p e V \right] \left[ 1 + \sin(\theta)\theta_p e V \right], \] where \(G_N\) is the conductance of the normal metal, \(\theta_p\) is the phase difference between the normal and superconducting sides, and \(\theta_F = \frac{4 \cos(\theta)}{2 - 4 \cos(\theta)}\) is the Fermi vector mismatch (FVM) between the normal and superconducting sides. The case \(E' \neq E_F\) corresponds to a heavily doped superconducting region, while \(E' = E_F\) describes undoped graphene. Since we are using a mean-field approach to describe the superconducting part of the Hamiltonian, it is implicitly understood that phase-fluctuations of the order parameter must be small. This amounts to imposing the restriction \(|\xi|/\lambda \gg 1\), or equivalently, \(E' \gg \Delta\).

The conductance of the N/I/S junction is given by \[ G(e V) = G_N \int_0^{2\pi} \frac{d\theta}{\pi} \cos 2\theta \left[ 1 - |r| e V(\theta) \right]^2 + P r_A(-e V, \theta), \] where \(r\) and \(r_A\) are the reflection coefficients for normal and Andreev reflection, respectively, \(P = |p| e V \cos \theta_A/(|p| e V \cos \theta),\) and \(G_N = \int_{-\pi/2}^{\pi/2} \cos 2\theta \left( 4 \cos^2 \theta \right) / (4 \cos^2 \theta + Z^2) \) is a renormalization constant corresponding to the N/N metallic conductance [12]. In this case, we have zero intrinsic barrier such that \(Z = 0\). We will apply the usual approximation \(|r_A(-e V, \theta)| \simeq |r_A(e V, \theta)|\), which holds for subgap energies. Although it is not valid for energies above the gap, this is of little consequence for the final result, since Andreev reflection is suppressed for \(e V > \Delta\). The reflection and transmission coefficients constitute a unitary scattering matrix, a property that essentially expresses a conservation of probability. In deriving the conductance, we have ensured that the scattering coefficients have been normalized by the incoming current through the factor \(P\). In order to obtain these coefficients, we make use of the boundary conditions \(\psi_{l,x=0} = \psi_{l,x=\pi}, \psi_{l,x=d} = \psi_{l,x=-d}\), where we have defined the wavefunction in the insulating region \(\psi = \hat{t}_1 \psi_{L} + \hat{t}_2 \psi_{R} + \hat{t}_3 \psi_{L}^{\ast} + \hat{t}_4 \psi_{R}^{\ast}\). The wavefunctions \(\psi\) differ from \(\psi\) in that the Fermi energy is greatly shifted by means of \(e V\). An external potential, such that \(E_F = E_V + V_0\) where \(V_0\) is the barrier (equivalent to the Fermi level).
Consider first a N/I/S graphene junction. In the thin-barrier limit defined as \( d \to 0 \) and \( V_0 \to \infty \) with \( s \)-wave pairing, Ref. \([5]\) reported a \( \pi \)-periodicity of the conductance with respect to the parameter \( \chi = V_0d/\sqrt{4eV} \). In the present study, we do not restrict ourselves to isotropic pairing, nor to the thin-barrier limit, and show that new physics emerges from the presence of a finite-width barrier. We measure the width \( w \) of region I in units of \( d/\lambda_F \) and the potential barrier \( V_0 \) in units of \( E_F \). The linear dispersion approximation is valid up to \( \sim 1 \text{ eV} \) \([8]\), and we will consider typical Fermi energies in graphene of \( E_F = 100 \text{ meV} \) in the undoped case and a gap \( \Delta = 1 \text{ meV} \) \([1]\). In the doped case, we set \( E_F' = 10E_F \), and we also fix \( V_0 = 10E_F \) in order to operate within the regime of validity of the linear dispersion approximation. The undoped situation originally refers to the case where the Fermi level is located at the Dirac point, although real experimental graphene samples may have free carriers, such that \( E_F \) is pushed upwards. The doped case denotes a large FVM between the N and S region which may be induced by chemical doping or by a gate voltage.

![Diagram](image)

**FIG. 1:** Tunneling conductance of N/I/S graphene junction for both \( s \)-wave and \( d \)-wave pairing in the undoped and doped case (see main text for parameter values). It is seen that for increasing \( w \), a novel oscillatory behaviour of the conductance as a function of voltage is present in all cases.

Consider Fig. 1 where we plot the normalized tunneling conductance in the two cases of \( s \)-wave and \( d \)-wave pairing, for both doped and undoped graphene. The most striking new feature compared to the thin-barrier limit is the strong oscillations in the conductance as a function of \( eV \). We also include the thin-barrier limit with \( \chi = 0 \) and \( \chi = \pi \) to illustrate the \( \pi \)-periodicity in this limit. For subgap energies, we regain the N/S conductance for undoped graphene when \( \chi = 0 \), with nearly perfect Andreev reflection. To model the \( d \)-wave pairing, we have used the \( d_{x^2-y^2} \) model \( \Delta(\theta) = \Delta \cos(2\theta - 2\alpha) \) with \( \alpha = \pi/4 \). The parameter \( \alpha \) effectively models different orientations of the gap in \( \mathbf{k} \)-space with regard to the interface, and \( \alpha = \pi/4 \) corresponds to perfect formation of ZES in N/S metallic junctions. For \( \alpha = 0 \), the \( d \)-wave spectra are essentially identical to the \( s \)-wave case, since the condition for formation of ZES is not fulfilled in this case \([7]\). It is seen that in all cases shown in Fig. 1 the conductance exhibits a novel oscillatory behavior as a function of applied bias voltage \( eV \) as the width \( w \) of the insulating region becomes much larger than the Fermi wavelength, i.e. \( w \gg \lambda_F \).

The oscillatory behavior of the conductance may be understood as follows. Non-relativistic free electrons with energy \( E \) impinging upon a potential barrier \( V_0 \) are described by an exponentially decreasing non-oscillatory wavefunction \( e^{i\chi x} \) inside the barrier region if \( E < V_0 \), since the dispersion essentially is \( k \sim \sqrt{E - V_0} \). Relativistic free electrons, on the other hand, have a dispersion \( k \sim (E - V_0)^{1/2} \alpha \) inside the tunneling region, and hence damped oscillatory behavior of the wave function. Relativistic massless fermions are unique in the sense that only in this case (\( \alpha = 1 \)) the momentum is purely real. Hence, the undamped oscillatory behavior at subgap energies appears as a direct manifestation of the relativistic low-energy Dirac fermions in the problem. This observation is also linked to the so-called Klein paradox which occurs for electrons with such a relativistic dispersion relation, which has been theoretically studied in normal graphene \([2]\).

We next discuss why the illustrated conductance spectra are different for \( s \)-wave and \( d \)-wave symmetry, in addition to comparing the doped and undoped case. The doping level may be considered as an effective FVM, acting as a source of normal reflection in the scattering processes. This is why the subgap conductance at thin barrier limit is reduced in the doped case. Moving away from the thin barrier limit, it is seen that oscillations emerge in the conductance spectra. For \( s \)-wave pairing, the amplitude of the oscillations is larger in the doped case than in the undoped case, and the period of oscillations remains the same. This period depends on \( w \), while the amplitude of the oscillations is governed by the wavevectors in the regions I and S. The maximum value of the oscillations occurs when \( 2w \) equals an integer number of wavelengths, corresponding to a constructive interference between the scattered waves. Physically, the amplitude-dependence of the oscillations on doping originates with the fact that doping effectively acts as an increase in barrier strength. By making \( V_0 \) larger, one introduces a stronger source of normal reflection. When the resonance condition for the oscillations is not met, the barrier reflects the incoming particles more efficiently. This is also the reason why increasing \( V_0 \) directly and increasing \( E_F' \) has the same effect on the spectra.

We now turn to the difference between the \( s \)-wave and \( d \)-wave for the undoped case. It is seen that the conductance is reduced in the \( d \)-wave case compared to the \( s \)-wave case. One may understand the reduction in subgap conductance in the undoped case as a consequence of tunneling into the nodes of
the gap, which is not present in the \( s \)-wave case. Hence, Andreev reflection which significantly contributes to the conductance, is reduced in the \( d \)-wave case compared to the \( s \)-wave case. Moreover, we see that a ZBCP is formed in the doped case, equivalent to a stronger barrier, and this is interpreted as the usual formation of ZES leading to a transmission at zero bias with a sharp drop for increasing voltage.

From Fig. 2, we see that the peak of the conductance is quite distinct from that encountered in a N/S metallic junction in the superconducting part of the system. Consider Fig. 2 for the case of doped graphene, where we plot the conductance to see how it evolves upon a rotation of the gap. The behaviour for the case of doped graphene, where we plot the conductance spectra actually mimicks a lower value of the gap than what is the case, if one were to infer the gap magnitude from the position of the singularity in the spectra. This should be an easily observable feature in experiments, and provides a direct way of testing our theory. For undoped graphene, we found very little difference in the conductance spectra upon varying \( \alpha \). The inset of Fig. 2 illustrates the undoped case for \( \alpha = \pi/4 \).

Finally, we briefly investigate how the conductance spectra of a N/S graphene junction (without the insulating region) change when going from a \( s \)-wave to a \( d \)-wave order parameter in the superconducting part of the system. Consider Fig. 2 for the case of doped graphene, where we plot the conductance, is reduced in the \( d \)-wave case compared to the \( s \)-wave case. Moreover, we see that a ZBCP is formed in the doped case. This is different from what is observed in metallic N/S junctions, where the formation of a ZBCP starts immediately as one moves away from \( \alpha = 0 \) in the presence of a FVM, corresponding to the doped case here. In Fig. 2, the conductance spectra actually mimicks a lower value of the gap than what is the case, if one were to infer the gap magnitude from the position of the singularity in the spectra. This should be an easily observable feature in experiments, and provides a direct way of testing our theory. For undoped graphene, we found very little difference in the conductance spectra upon varying \( \alpha \). The inset of Fig. 2 illustrates the undoped case for \( \alpha = \pi/4 \), where the deviation from perfect Andreev reflection for \( eV < \Delta \) is due to tunneling into the nodes of the gap.

In summary, we have studied coherent quantum transport in N/S and N/I/S graphene junctions, investigating also the role of \( d \)-wave pairing symmetry on the tunneling conductance. We report a new oscillatory behaviour of the conductance as a function of bias voltage for insulating regions that satisfy \( d > \lambda \), which is present both for \( s \)- and \( d \)-wave pairing. In the latter case, we have also studied the conductance of an N/S junction and find very distinct behaviour from metallic N/S junctions: a rotation of \( \alpha \) is accompanied by a progressive shift of the peak in the conductance. All of our predictions should be easily experimentally observable.

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