Refining the Experimental Extraction of the Number of Independent Samples in a Mode-Stirred Reverberation Chamber

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We investigate the number of independent samples in a chaotic reverberation chamber. Its evaluation as defined by the IEC standard can be made more precise when using not the index of the first value larger then the correlation length but using the value obtained by a linear interpolation instead. The results are validated by a juxtaposition with values from a measurement using a high stirrer-angle resolution. A comparison with estimates known from the literature validates our findings. An alternative approach using the local maxima of the parametric dependence of the transmission is presented in order to show the applicability of the extracted correlation length over a large range of frequencies.

I. INTRODUCTION

Mode-stirred reverberation chambers play an important role in electromagnetic compatibility. With their help it is possible to obtain statistically valuable results for the electromagnetic radiation emitted from the object under test. However, in order to get reliable statements about the fluctuations of the EM field it is necessary to use statistically independent experimental realizations. In other words, the statistical ensemble usually achieved by a so-called mode stirrer has to be mixing enough to make the intensity patterns of the chamber statistically independent from each other. As the solution to Maxwell’s equations under the given boundary conditions depend continuously on the latter, a sufficiently large change has to be performed. For mode stirrers, this usually means that the angle of rotation has to be sufficiently large, assuming the stirrer is not too small. Once this minimally necessary step width is determined the number of independent samples (NIS) from a full turn of the stirrer can be calculated [1]. This number is the main focus of this paper. Note that the results are obtained for an empty chamber. We do not expect that loading effects of typical devices under test will have a qualitative impact on our findings in a chaotic reverberation chamber (CRC).

Because of being such an important quantity there exist also rough estimates following geometrical arguments [2]. They are based on probabilistic estimates for rays of the EM field hitting the stirrer. Besides these predictions, one can use experimental data to extract the NIS. As the measured data is available only for a discrete set of stirrer positions the minimally necessary step width is determined from a full turn of the stirrer can be calculated [1]. This number is the main focus of this paper. Note that the results are obtained for an empty chamber. We do not expect that loading effects of typical devices under test will have a qualitative impact on our findings in a chaotic reverberation chamber (CRC).

The experiments were carried out in our chaotic reverberation chamber shown in Fig. 1. The Vector-Network Analyzer (VNA, Rohde&Schwarz ZVA67) was attached to two monopole antennas inside the chamber (see Fig. 1(right)). For a total range of frequencies $f$ from 0.5 GHz to 5.0 GHz we measured the complex transmission amplitude between the two antennas. The measurement was repeated for $N_\theta = 3600$ stirrer positions using a step of $\Delta \theta = 0.1^\circ$, Eq. (3). This range was chosen to cover frequencies from around the Lowest-Usable Frequency $f_{\text{LUF}}$ up to approximately $7 \cdot f_{\text{LUF}}$. Using the frequency of the 60th mode as a definition, the $f_{\text{LUF}}$ of our homemade chaotic chamber is approximately at 0.735 GHz.

Corresponding transmissions for three different frequency ranges with different values of the modal overlap

FIG. 1. Photograph of the chaotic reverberation chamber with length $L = 100 \text{ cm}$, width $W = 77 \text{ cm}$ and height $H = 62 \text{ cm}$. At the walls 54 spherical caps of radius $r_c = 10 \text{ cm}$ are used, 51 having a cap height of $h_c = 3 \text{ cm}$ and 3 having $h_c = 8 \text{ cm}$. The total internal volume is $V = 0.44 \text{ m}^3$ (left). At the bottom a stirrer with 5 paddles is placed which can be turned by a stepper motor and acts as mode stirrer. The two monopole antennas were mounted on polystyrene blocks having a perpendicular polarization direction (right).
for one fixed stirrer position versus frequency are shown in Fig. 2.

Besides the whole frequency range 0.5 – 5 GHz we focused on seven sub-intervals, where we measured with a higher frequency resolution to guarantee a proper extraction of the transmission maxima. The ranges have been chosen such that important parameters (see Tab. I) are sufficiently well defined. One important parameter is the mean frequency spacing of adjacent eigenmodes of the cavity, $\Delta f$, which we calculate using Weyl’s law, $\Delta f = c^3/(8\pi V f^2)$. The signal decay time $\tau$ is determined from the exponential decay of the square modulus of the Fourier transform of the transmission $|S_{21}|^2$ [5],

$$I(t) = |\text{FT}(S_{21})(t)|^2 = I_0 e^{-t/\tau},$$

where the transform is performed either over the frequency range of the seven sub-intervals or a frequency window of 100 MHz. The average quality factor $Q$ is thus given by $Q = 2\pi \tau(f)$, where $\langle f \rangle$ is the average frequency of the window used to calculate $\tau$. Finally, the modal overlap $d$ is obtained by

$$d = \frac{\text{mean decay rate}}{\text{mean eigenmode spacing}} = \frac{1/2\pi \tau}{\Delta f} = \frac{\langle f \rangle}{Q \Delta f}. \quad (2)$$

The extracted values can be found in Tab. I for the seven sub-intervals.

### TABLE I. Figures of merit for the reverberation chamber at different frequencies ranges ($f_{\text{min}} - f_{\text{max}}$). Shown are the values for the mean frequency spacing $\Delta f$, the signal decay time $\tau$, the mean distance between adjacent maxima $\delta f_{\text{max}}$, the quality factor $Q$, and the modal overlap $d$. The frequency ranges are also indicated in Fig. 4.

| $f_{\text{min}} - f_{\text{max}}$ | $\Delta f$ | $\tau$ | $\delta f_{\text{max}}$ | $Q$ | $d$ |
|------------------|------------|--------|-----------------|-----|-----|
| 0.75 – 0.85 GHz  | 734.4 kHz  | 0.85   | 122.05 GHz      | 8.0 | 0.3 |
| 1.1 – 1.25 GHz   | 1731.1 kHz | 1.25   | 1603.75 GHz     | 1.0 | 0.7 |
| 1.7 – 1.85 GHz   | 758.5 kHz  | 1.85   | 1422.05 GHz     | 2.0 | 1.2 |
| 2.2 – 2.40 GHz   | 451.8 kHz  | 2.4    | 1045.92 GHz     | 3.7 | 1.4 |
| 2.8 – 2.90 GHz   | 294.2 kHz  | 2.9    | 841.62 GHz      | 5.9 | 1.6 |
| 3.8 – 4.0 GHz    | 157.1 kHz  | 4.0    | 836.74 GHz      | 8.4 | 3.0 |
| 4.9 – 5.0 GHz    | 97.5 kHz   | 5.0    | 799.64 GHz      | 11.2| 4.5 |

### III. REFINING THE EXTRACTION OF THE NUMBER OF INDEPENDENT SAMPLES

The usual approach to extract the NIS is to use the autocorrelation function of the transmission data. Given $N_\theta$ equidistant angles at which the transmission ampli-
In case of small sample size the definition has to be redefined. We only use one stirrer although generalizations to multiple stirrers exist. A refined method can be obtained by linearly interpolating the two points before and after the critical value (5)

\[ \lambda_0 = \lambda_{\theta} - \frac{1}{2} g(\lambda_{\theta}) \]

is undercut,

\[ N^*(f) = \frac{N_{\theta}}{\lambda_0^2(f)} \]

This approach is justified by the fact that a larger \( N_{\theta} \) and therefore higher angle resolution (3) yields a finer resolution of the autocorrelation function. In the inset of Fig. 3 the values obtained by using this fit for two different \( N_{\theta} \) are shown. The resulting values are quite close, whereas using the index directly would lead to large differences in \( N \). In our experiment we checked this assumption by comparing the data with a reduced data set like in Ref. [7], see Sec. II. Note that due to the fact that \( \lambda_{\theta} - 1 < \lambda_0^* \leq \lambda_{\theta} \), the extracted NIS fulfills \( N^* \geq N \). Furthermore, using the decay of the autocorrelation function only estimates the number of uncorrelated samples. However, this approach has become normative for the determination of the NIS in the literature.

A. Number of Independent Samples

We calculated the number of independent samples (7) based on the integer-valued correlation length (5) and the interpolated value (9) based on Eq. (8), respectively. The analysis was done once for the whole frequency range 0.5 – 5 GHz. We determine the independent samples using either the whole set of measured angles \( (N_{\theta} = 3600, \Delta \theta = 0.1^\circ) \) or two reduced data sets \( (N_{\theta} = 450 \text{ (} \Delta \theta = 0.8^\circ) \text{ and } N_{\theta} = 72 \text{ (} \Delta \theta = 5^\circ) \text{).} \) The value of \( N_{\theta} = 450 \) is commonly used in the literature as suggested in the standard [1] but not without criticism [6, 7, 10].

In order to check whether the linear interpolation of formula (8) works for our experimental data, it is shown in Fig. 3 for \( f = 2.21 \text{ GHz} \) using the \( N_{\theta} = 450 \) (shown as blue dots) and for a strongly reduced number of \( N_{\theta} = 72 \) (shown as large crosses). On the one hand one can see that the values \( \lambda_{150} \) and \( \lambda_{72} \) obtained using the discrete numbers deviate, whereas the values of \( \lambda^*_{150} \) and \( \lambda^*_{72} \) are quite close (after an appropriate rescaling). In this example one can also see that the interpolation is justified as the decay of the correlation around \( 1/e \) is approximately linear. Have in mind that Fig. 3 is using a rescaled value of the integer \( j \cdot 450/N_{\theta} \) due to the reduction of angles, so that the axis corresponds to the index \( j \) in the case \( N_{\theta} = 450 \). Note that the interpolation also works close to \( f_{\text{LUF}} \) as our reverberation chamber is rendered fully chaotic due to the spherical caps at the walls [3].

The overall dependency of the NIS versus the full frequency range for different calculation methods is shown in Fig. 4. Because one observes large fluctuations with frequency, we applied a rectangular frequency filter. In general, such large fluctuations make the extraction of the NIS by using only a single frequency questionable. For increasing frequencies the averaged NIS rises as the
stirrer position is better resolved by the EM field in accordance with the estimates from Ref. [2]. This higher resolution of the stirred volume increases the sensitivity with respect to the stirrer position and thereby decreases the correlation length. The data using the 3600 (black curve) and 450 (orange curve) samples are following each other closely apart from small deviation at higher frequencies. In case of 72 samples the NIS is bounded at $N = 72$. All curves show frequency-smoothed values after applying a rectangular-filter of 100 frequency steps (10 MHz). (see Fig. 4, orange curve). This curve follows better the curves with larger $N_\theta$ and even gives values above 72. While technically the 72 sample cannot have more than 72 independent data sets, the linear interpolation outlines a possibility to estimate which angular resolution is a good choice for getting as many independent samples with a minimal amount of measurements.

Indicated in the figure are also the seven frequency subranges, for which parameters are detailed in Tab. I. Fig. 5 shows $N(f)$ and $N^*(f)$ for two of these frequency subsets as well as for different stirrer resolutions $N_\theta$. Here, no frequency average was applied. Note that the smaller sample size leads to larger step sizes in the frequency axis in case of the calculation via Eq. (7). On the one hand side we find that the larger $N_\theta = 3600$ is always above
the smaller $N_\theta = 450$ value but the value obtained by the linear interpolation for $N_\theta = 450$ follows nicely the $N_\theta = 3600$ case.

B. Prediction of the Number of Independent Samples

The above extracted values for the number of independent samples can be compared with an estimate based on the volume $V$ and quality factor $Q$ of the chamber [2]. The prediction is based on a probabilistic argument that the stirred volume $V_{\text{stirrer}}$ is hit by a beam in the chamber. It differs for large and small stirrers, $N_1$ and $N_s$, respectively. In our case the volume affected by the stirrer as defined in Ref. [2] is

$$V_{\text{stirrer}} = 0.0034 \text{ m}^3.$$  \hspace{1cm} (10)

For the low frequency range the estimate is given by [2]

$$N_s = C_s \frac{\lambda V_{\text{stirrer}}^{2/3}}{V}, \quad V_{\text{stirrer}} \ll \lambda^3$$  \hspace{1cm} (11)

whereas in the high frequency range it should be related to

$$N_1 = C_1 \frac{V_{\text{stirrer}}}{V}, \quad V_{\text{stirrer}} \gg \lambda^3.$$  \hspace{1cm} (12)

When both expressions are adjusted to the experimental data in the appropriate frequency range we obtain $C_s = 1.55$ and $C_1 = 1.56$, respectively. In Fig. 6 the averaged number of independent samples is compared to the two predictions. A good agreement is found to (11) (red circles) and Eq. (12) (light blue triangles). The interpolating formula (13) matches the whole frequency range (dark blue curve). The corresponding values are $C_s = 1.87, C_1 = 0.7$. The experimental curve show frequency-smoothed values after applying a rectangular-filter of 100 frequency steps (10 MHz).

![FIG. 6. Comparison of the number of independent samples with the estimate based on geometrical arguments. Based on the experimental data for $N_\theta = 3600$ we fit the prefactor in Eq. (11) (red circles) and Eq. (12) (light blue triangles). The interpolating formula (13) matches the whole frequency range (dark blue curve). The corresponding values are $C_s = 1.87, C_1 = 0.7$. The experimental curve show frequency-smoothed values after applying a rectangular-filter of 100 frequency steps (10 MHz).](image)

smoothed the $|S_{21}(f, \theta)|^2$ along the frequency axis using a Hann filter of window size 100 for all data sets. We then extracted the local maxima of the intensity for each value of the stirrer position. In the following we want to demonstrate that the statistical properties of the maxima are similar to one another across the frequency ranges shown, once the appropriate scaling has been applied. If the resonances are isolated (i.e., the modal overlap $d$ is small ($d \ll 1$)) the local maxima are defined by the frequencies of the eigenmodes $\nu_n$. The dependence of the eigenvalues on a parameter $p$ have been studied extensively in the framework of “Quantum Chaos” [11, 12] for scalar fields, but can be directly applied to the vectorial problem. Defining a level velocity $v_n = d\nu_n/d\nu$ one can distinguish global and local perturbation. In case of a global perturbation in a CRCthe distribution of the level velocities $\nu_n$ is Gaussian[13], whereas for local perturbations it shows a Bessel $K$ distribution [14]. This would be another possibility to characterize the quality of the stirring. In case of a chaotic system the levels show avoided crossing, whereas in case of regular, more precisely, integrable systems, the levels will cross[11, 12]. The frequency scale of importance is the mean frequency spacing $\Delta f$. In Fig. 7(top) the dynamics of the local maxima is presented for the low frequency range, which has $d = 0.3$, thus showing reasonably isolated resonance. The spectra should be uncorrelated when the local maxima go from one avoided crossing to another, which agrees visually with the calculated correlation length $\lambda_\theta$ indicated by the horizontal arrow. Studies also exist in the case of open systems [15] on eigenmode dynamics, but in the case of moderate or large modal overlap, the fre-
frequencies of the eigenmodes are not directly related to the local maxima we extracted here. To define the frequency scale in case of strong modal overlap we need to estimate the mean spacing between transmission maxima, which has been obtained by Schroeder and Kuttruff [16], \( \delta f_{\text{max}} = 1/(2\sqrt{3}\tau) \), an expression which is assumed to be valid for \( d > 3 \). If abscissa and ordinate are scaled appropriately the behavior of the ridges of maxima \( f_{\text{max}}(\theta) \) is similar with respect to the distances of close encounters and the steepness with respect to the stirrer angle, i.e., \( \lambda_{\text{max}}/\theta \). Using the correlation length \( \lambda_{\theta} \) for each of these frequency ranges as read from, e.g., Fig. 5, we can choose the scale of the abscissa of these plots to cover several correlation lengths. We chose the \( \theta \) range from \( \theta_{\text{min}} = 100^\circ \) to \( \theta_{\text{max}} = \theta_{\text{min}} + 3 \cdot \langle \lambda_{\theta}^2(f) \rangle_f \cdot \Delta \theta \) to cover 3 times the correlation length in each figure. The ordinate was scaled using the values from Tab. I in the following way: For frequency ranges with a modal overlap smaller than \( d \leq 1 \) we chose a plot range of width \( 8 \cdot \Delta f \). For the other ranges we used the estimate \( \delta f_{\text{max}} \) and chose \( 8 \cdot \delta f_{\text{max}} \). The corresponding plots are shown in Fig. 7 for three of the seven intervals from Tab. I. Each of the plots contains two arrows indicating the correlation length \( \lambda_{\theta} \) and the mean spacing between maxima, respectively. Due to the scaling of the abscissa the one for \( \lambda_{\theta} \) has the same length in every plot.

We can indeed see a qualitative agreement between the average distance between the extracted \( f_{\text{max}} \) ridges, thus confirming the results obtained in the previous chapter.

\[
\begin{array}{c}
\begin{align*}
\frac{\partial^2 f_{\text{max}}}{\partial \theta^2} & = 8 \cdot \lambda_{\theta}/(2\sqrt{3}\tau) \\
\frac{\partial f_{\text{max}}}{\partial \theta} & = 4 \cdot \sqrt{\frac{\lambda_{\theta}}{\tau}} \\
\end{align*}
\end{array}
\]

IV. CONCLUSION

This paper presents experimental data from a mode-stirred chaotic reverberation chamber and estimates the number of independent samples (NIS) for a 360° turn of the stirrer. We compare the reduced data set for \( N_\theta = 450 \) steps usually found in the literature with a finer \( \theta \) subdivision \( N_\theta = 3600 \) of the full angle. The corresponding values of the NIS extracted by the usual procedure (7) show a very coarse dependency if \( N_\theta \) is reduced. We compare this value with an improved estimate based on a linear interpolation (8) of the correlation length. The values obtained in this way for the coarser measurements resemble very much the full data set as shown, for example, in Fig. 5. We compare these values with predictions (11), (12) from Ref. [2] and find quantitative agreement if we use the interpolation formula (13), see Fig. 6. We check that the results are valid for conceptually different regimes, i.e. frequency ranges for which the modal overlap \( d \) of the chamber is smaller, around, or larger than 1, see Tab. I.

To emphasize the findings with an independent approach, we focus on the extraction of local maxima of the transmission [5]. The detailed resolution of \( N_\theta = 3600 \) allows to follow \( f_{\text{max}}(\theta) \) as the stirrer moves. The corresponding dynamics show fluctuations on a scale which is expected to be similar to the correlation length \( \lambda_{\theta} \).

Hence, we show that scaling a plot of \( f_{\text{max}} \) using \( \lambda_{\theta} \) yields qualitatively the same image.

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