Compensation method of lever arm effect based on multi-accelerometers

Qiangjun Tan, Yongsheng Cheng, Bin Tang, Hao Zhou, Yinxin Li*
Institute of Electronic Engineering, China Academy of Engineering Physics, Mianyang, Sichuan, China

*Corresponding author e-mail: liyinxin4513@163.com

Abstract. In the wind tunnel model angle of attack test, the lever arm effect caused by the model vibration will greatly affect the attitude measurement accuracy. For the centrifugal acceleration term, there will be residual rectification errors using the low-pass filtering method; for the tangential acceleration, because of the same frequency as the attitude angle, the low-pass filtering method cannot filter it out. The mechanical compensation method will dramatically increase the compensation error of the tangential acceleration term due to the differential calculation. Aiming at the removal of the centrifugal acceleration term, this paper demonstrates that the error of the mechanical compensation method is far less than the low-pass filter method in the wind tunnel test environment. Aiming at the removal of the tangential acceleration term, this paper proposes a compensation method based on multi-accelerometers, which avoids the error amplification caused by differential calculation. Through theoretical derivation, analysis and simulation, the proposed method significantly improves the compensation accuracy of the lever arm effect.

1. Introduction

The lever arm effect is because the accelerometer's installation position does not coincide with the swing reference point in the angular motion environment, so the disturbance acceleration relative to the reference point is generated in the output of the accelerometer [1, 2]. The accelerometer has the characteristics of simple structure and convenient maintenance in wind tunnel model angle of attack attitude measurement. However, in a dynamic environment, the disturbance acceleration superimposed on the accelerometer's sensitive axis will greatly affect the attitude measurement accuracy.

According to the existing references, the analytical processing methods of lever arm effect can be divided into two categories: low pass filtering method and mechanical compensation method. (1) Low-pass filtering method. Since the lever arm effect is generated in the angular motion environment, it can be regarded as high-frequency disturbance and filtered out by low-pass filtering. Reference [3] used FIR and IIR low-pass filters to filter out the lever arm effect in the frequency domain. However, the low-pass filtering method cannot filter out the DC components and the components whose frequencies are consistent with the angular frequency of the attitude. (2) Mechanical compensation method. The angular velocity measured by the gyroscope and the angular acceleration obtained by the differential of the angular velocity are respectively compensated for the centrifugal acceleration term and tangential acceleration term in the lever arm effect. References [4, 5] used the mechanical compensation method to compensate the lever arm effect in the missile-borne environment. However, since the angular acceleration is obtained by differential calculation, the compensation error of...
tangential acceleration term is inevitably amplified. According to the characteristics of \( \mathbf{\dot{\omega}} \times \mathbf{r} \perp \mathbf{r} \), reference [6] arranged the sensitive axis of the accelerometer along the vector direction of the lever arm \( \mathbf{r} \) to avoid solving the tangential acceleration term. However, reference [6] only eliminated the tangential acceleration term on the accelerometer whose sensitive axis was parallel to the vector direction of the lever arm \( \mathbf{r} \), while the tangential acceleration terms on the other two axes accelerometers could not be ignored.

This paper analyzes the error mechanism of the lever arm effect, and proposes a compensation method of the lever arm effect based on multi-accelerometers. Compared with the existing methods, the proposed method significantly improves the compensation accuracy of lever arm effect.

2. Error mechanism of lever arm effect

Define the inertial coordinate system (\( i \) system) as \( o_i x_i y_i z_i \), the body coordinate system (\( b \) system) as \( o_b x_b y_b z_b \), the accelerometer is mounted on the carrier at point \( P \), \( o_b \) point is the reference point of the carrier, \( \mathbf{R} \) is the position vector of point \( P \) in the inertial system, \( \mathbf{R}_1 \) is the position vector of the carrier reference point \( o_b \) in the inertial system, and \( \mathbf{r} \) is the position vector from the carrier reference point \( o_b \) to the point \( P \), that is, the lever arm. The location diagram is shown in Figure 1.

![Figure 1. The sketch map of relationship between references systems](image)

In the angular motion environment, since the carrier reference point \( o_b \) does not coincide with the accelerometer installation point \( P \), the output of the accelerometer contains the lever arm effect relative to the reference point, as shown in equation (1)

\[
f^b_{ib} = f^b_{ib} + \mathbf{\omega}^b_{ib} \times \mathbf{\omega}^b_{ib} \times \mathbf{r}^b + \mathbf{\dot{\omega}}^b_{ib} \times \mathbf{r}^b
\]

In equation (1): \( f^b_{ib} \) is the projection of the accelerometer output specific force vector in the body system, \( f^b_{ib} \) is the projection of the specific force vector of the carrier reference point \( o_b \) in the body system, \( \mathbf{\omega}^b_{ib} \) is the projection of the angular velocity of the carrier relative to the inertial coordinate system in the body system, \( \mathbf{r}^b \) is the projection of the position vector from the carrier reference point \( o_b \) to the accelerometer installation point \( P \) in the body system.

The disturbance acceleration caused by the lever arm effect can be expressed as

\[
\delta f^b = \mathbf{\omega}^b_{ib} \times \mathbf{\omega}^b_{ib} \times \mathbf{r}^b + \mathbf{\dot{\omega}}^b_{ib} \times \mathbf{r}^b
\]

In equation (2), the first term is defined as the centrifugal acceleration term of the lever arm effect, and the second term is defined as the tangential acceleration term of the lever arm effect.

For the convenience of expression, \( \mathbf{a}_c \) is defined as the centrifugal acceleration term and \( \mathbf{a}_t \) as the tangential acceleration term, as shown in equation (3)

\[
\mathbf{a}_c = \mathbf{\omega}^b_{ib} \times \mathbf{\omega}^b_{ib} \times \mathbf{r}^b \quad \mathbf{a}_t = \mathbf{\dot{\omega}}^b_{ib} \times \mathbf{r}^b
\]

In the wind tunnel model angle of attack test, the model rotates around \( O \) point, as shown in figure 2. Under this working condition, the linear acceleration \( \mathbf{\dot{v}}^b \) and linear velocity \( \mathbf{v}^b \) of the model can be regarded as \( \mathbf{0} \), and the output specific force vector of the accelerometer under the carrier system can be expressed as

\[
f^b = -C_n^bg^n + \mathbf{\omega}^b_{ib} \times \mathbf{\omega}^b_{ib} \times \mathbf{r}^b + \mathbf{\dot{\omega}}^b_{ib} \times \mathbf{r}^b
\]
In equation (4), $C_{bn}$ is the vector cosine matrix between the body coordinate system ($b$ system) $o_bx_by_bz_b$ and the navigation coordinate system ($n$ system) $o_nx_ny_nz_n$. The lever arm vector $r^b$ is expressed in the body coordinate system as $r^b = [r \ 0 \ 0]^T$.

3. Analytical processing method of lever arm effect

Under the influence of wind tunnel airflow shock excitation, the model has angular vibration with a certain frequency and amplitude on the pitch axis, roll axis, and yaw axis, which can be expressed as

$$\theta(t) = \theta_m \sin \Omega t$$
$$\gamma(t) = \gamma_m \sin \Omega t$$
$$\psi(t) = \psi_m \sin \Omega t$$

In equation (6): $\Omega$ is the angular frequency of vibration, $\theta$ is the pitch angle, $\theta_m$ is the vibration amplitude of the pitch angle, $\gamma$ is the roll angle, $\gamma_m$ is the vibration amplitude of the roll angle, $\psi$ is the azimuth, and $\psi_m$ is Azimuth vibration amplitude.

According to the Euler angle differential equation, the angular velocity $\omega_{nb}$ of the carrier relative to the navigation coordinate system in a vibrating environment is shown in Equation (7)

$$\omega_{nb} = \begin{bmatrix}
\omega_{nrx}^b \\
\omega_{nry}^b \\
\omega_{nrz}^b
\end{bmatrix} = \begin{bmatrix}
0 \\
gamma_m^2 \sin \Omega t \\
\theta_m^2 \sin \Omega t
\end{bmatrix} + C_\gamma C_\theta \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}$$

In a small-amplitude vibration environment, $\theta, \gamma, \psi$ can be treated as small angles. After omitting the small quantities of higher order, the angular velocity $\omega_{nb}$ can be expressed as

$$\omega_{nb} = \Omega \cos \Omega t [\gamma_m \ \theta_m \ \psi_m]^T (8)$$

In the dynamic environment, earth rotation can be ignored, while the linear acceleration $v^b$ and linear velocity $v^b$ of the model in the wind tunnel model angle of attack test can be regarded as 0, so it can be considered as $\omega_{nb}^b \approx \omega_{nb}^b$. Substituting equation (8) into the centrifugal acceleration term of equation (5), $a_c$ can be expressed as

$$a_c = \frac{r^2 \Omega^2}{2} \begin{bmatrix}
-\psi_m^2 - \theta_m^2 \\
\theta_m \gamma_m \\
\gamma_m \psi_m
\end{bmatrix} (9)$$

By differential calculation of equation (8), the angular acceleration can be expressed as

$$\omega_{nb}^b = -[\gamma_m \Omega^2 \sin \Omega t \ \theta_m \Omega^2 \sin \Omega t \ \psi_m \Omega^2 \sin \Omega t]^T (10)$$

Substituting equation (10) into the tangential acceleration term of equation (5), $a_t$ can be expressed as

$$a_t = r \Omega^2 \sin \Omega t [\theta_m \ -\psi_m]^T (11)$$
At present, the main analytical methods for dealing with the lever arm effect are the low-pass filtering method and the mechanical compensation method, both of which are widely used but have their own limitations in practical applications.

3.1. Limitations of the low-pass filtering method

According to the existing references, the low-pass filtering method is mainly applied in the initial alignment scene. Under this environment, the frequency caused by the lever arm effect is much higher than that of Schuler and earth rotation, which can be regarded as high-frequency disturbance [3]. After low-pass filtering, the residual DC component of the centrifugal acceleration term can be expressed as

$$\Delta a_{cl} = \frac{r \Omega^2}{2} \left[ -\psi_m^2 - \theta_m^2 \right] \frac{\theta_m y_m}{\gamma_m \psi_m}$$

In equation (12), $\Delta a_{cl}$ is the residual error of the centrifugal acceleration term after low-pass filtering.

Because the frequency of the tangential acceleration of the lever arm effect is the same as the attitude angle, if it is directly filtered by low-pass filtering, useful attitude angle information will be filtered.

3.2. Limitations of the mechanical compensation method

The mechanical compensation method relies on the gyroscope to obtain the angular velocity $\omega_{ib}^b$ to compensate the centrifugal acceleration term, and to obtain the angular acceleration $\dot{\omega}_{ib}^b$ to compensate the tangential acceleration term by differentiating the angular velocity $\omega_{ib}^b$.

3.2.1. Limitations of mechanical compensation for centrifugal acceleration of the lever arm effect.

This paper defines $\Delta \omega_{ibx}^b, \Delta \omega_{iby}^b, \Delta \omega_{ibz}^b$ as gyroscope measurement errors. It can be considered that $\Delta \omega_{ibx}^b = \Delta \omega_{iby}^b = \Delta \omega_{ibz}^b$. The actual output value of the gyroscope can be expressed as

$$\dot{\omega}_{ib}^b = \omega_{ib}^b + \Delta \omega_{ib}^b$$

In equation (13): $\dot{\omega}_{ib}^b$ is the actual output angular velocity of the gyroscope; $\omega_{ib}^b$ is the true angular velocity; $\Delta \omega_{ib}^b$ is the angular velocity measurement error.

The error after the centrifugal acceleration is compensated by the measured value of the gyroscope is shown in equation (14)

$$\Delta a_{cm} = (\dot{\omega}_{ib}^b \times \omega_{ib}^b - \omega_{ib}^b \times \dot{\omega}_{ib}^b) \times r^b = r \Omega \cos \Omega t \left[ -2\psi_m \Delta \omega_{ibx}^b - 2\theta_m \Delta \omega_{iby}^b \right] \frac{\theta_m \Delta \omega_{ibx}^b + \gamma_m \Delta \omega_{iby}^b}{\psi_m \Delta \omega_{ibx}^b + \gamma_m \Delta \omega_{ibz}^b}$$

In equation (14), $\Delta a_{cm}$ is the residual error of the centrifugal acceleration term after mechanical compensation.

Due to the limited volume of the wind tunnel model and the large size of the high-precision gyroscope, a MEMS gyroscope is selected in this paper. Generally, the measurement accuracy of MEMS gyroscope can reach 0.2°/s. It is assumed that the measurement error of the gyro satisfies $\Delta \omega_{ibx}^b = \Delta \omega_{iby}^b = \Delta \omega_{ibz}^b = \Delta \omega$, and the vibration amplitudes of the pitch, roll, and yaw angle satisfy $\theta_m = \gamma_m = \psi_m$. The comparison of the residual error of centrifugal acceleration between the mechanical compensation method and the low-pass filtering method can be expressed as

$$\left\{ \begin{align*}
\frac{\Delta a_{clx}}{\Delta a_{cmx}} &= \left| \frac{\Delta \psi_m}{4 \cos \Omega t \Delta \omega} \right| > \left| \frac{\pi \psi_m}{2 \Delta \omega} \right| \\
\frac{\Delta a_{cly}}{\Delta a_{cmy}} &= \left| \frac{\Delta \gamma_m}{4 \cos \Omega t \Delta \omega} \right| > \left| \frac{\pi \gamma_m}{2 \Delta \omega} \right| \\
\frac{\Delta a_{clz}}{\Delta a_{cmz}} &= \left| \frac{\Delta \psi_m}{4 \cos \Omega t \Delta \omega} \right| > \left| \frac{\pi \psi_m}{2 \Delta \omega} \right|
\end{align*} \right.$$  

(15)

It can be seen from equation (15) that if $\frac{\psi_m}{\Delta \omega} \approx \frac{\gamma_m}{\Delta \omega} \approx 1s$, when the vibration frequency $f > 2/\pi$, there is $\Delta a_{cl} > \Delta a_{cm}$, that is, the mechanical compensation method is better than the low-pass
filtering method. According to the reference [7], the natural frequency range of the wind tunnel model support device is usually in the range of 25 to 40 Hz, which is much greater than $2/\pi$. Therefore, in the actual wind tunnel test, the mechanical compensation method is much better than the low-pass filtering method for the centrifugal acceleration term.

### 3.2.2 Limitations of mechanical compensation for tangential acceleration of the lever arm effect.

The mechanical compensation method obtains the angular acceleration by differential calculation of the angular velocity, which will cause the error of the tangential acceleration term to be amplified. This paper takes the CRG20 MEMS gyroscope produced by Silicon Sensing as an example, and samples 650 s under static conditions. The angular velocity error and its variance measured by the gyroscope are shown in figure 3, and the angular acceleration error and its variance obtained by differential calculation are shown in figure 4.

#### Figure 3. Angular velocity error and its variance

![Figure 3. Angular velocity error and its variance](image)

#### Figure 4. Angular acceleration error and its variance after differential calculation

Comparing figure 3 and figure 4, it can be seen that that the differential calculation leads to a dramatic amplification of the error.

### 4. Multi-accelerometers compensate for tangential acceleration of the lever arm effect

The tangential acceleration term in the lever arm effect is consistent with the frequency of the pitch angle and cannot be filtered out by low-pass filtering. The mechanical compensation method will cause a dramatic increase in the tangential acceleration term error. Therefore, this paper proposes an improved method to compensate the lever arm effect by using multi-accelerometers.

This paper designs a 4-accelerometer configuration, as shown in Figure 5. Four accelerometers are installed at the center of the four faces of the cube around the $y_b$ axis.
When the center of the 4-accelerometer configuration is at point $P$ and the side length of the cube is $2l$, the specific force vector $F^b$ of the position of the accelerometer in the 4-accelerometer configuration can be expressed as

$$F^b = f^b_{ip} + \omega^b_{ib} \times l^b + \dot{\omega}^b_{ib} \times l^b$$  \hspace{1cm} (16)

Assuming that $\theta^b$ is the sensitive direction vector of the accelerometer in the body system, the output of a single accelerometer can be expressed as

$$A = (\theta^b)^T (f^b_{ip} + \omega^b_{ib} \times l^b + \dot{\omega}^b_{ib} \times l^b)$$  \hspace{1cm} (17)

The installation parameters of 4-accelerometer configuration scheme can be expressed as

$$l^b = \begin{bmatrix} l_1^b & \cdots & l_4^b \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\theta^b = \begin{bmatrix} \theta_1^b & \cdots & \theta_4^b \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The angular acceleration obtained from the 4-accelerometer configuration can be expressed as

$$\begin{bmatrix} \dot{\omega}^b_{iby} \\ \dot{\omega}^b_{ibz} \end{bmatrix} = \frac{1}{2l} \begin{bmatrix} A_3 - A_4 \\ A_1 - A_2 \end{bmatrix} - \begin{bmatrix} \omega^b_{ibx} \omega^b_{iby} \\ \omega^b_{ibz} \omega^b_{iby} \end{bmatrix}$$  \hspace{1cm} (18)

Then the tangential acceleration term of the lever arm effect can be expressed as

$$a_{int} = r \begin{bmatrix} 0 \\ -\omega^b_{ibx} \omega^b_{iby} \\ \omega^b_{ibz} \omega^b_{ibx} \end{bmatrix} + \frac{r}{2l} \begin{bmatrix} A_1 - A_2 \\ A_1 - A_2 \end{bmatrix}$$  \hspace{1cm} (19)

In equation (19), $a_{int}$ is the tangential acceleration calculated by the 4-accelerometer configuration. Considering the measurement errors of the accelerometer and gyroscope, and omitting the small quantities of higher order, the error $\Delta a_{int}$ of the tangential acceleration of the lever arm effect is

$$\Delta a_{int} = \frac{r}{2l} \begin{bmatrix} \Delta A_1 - \Delta A_2 \\ \Delta A_1 - \Delta A_2 \\ \Delta A_4 - \Delta A_3 \end{bmatrix}$$  \hspace{1cm} (20)

According to equation (20), using the 4-accelerometer configuration to solve the angular acceleration avoids the differential calculation.

5. Simulation verification

It is assumed that the measurement error of each MEMS gyroscope is $\Delta \omega^b_{ibx} = \Delta \omega^b_{iby} = \Delta \omega^b_{ibz} = 0.2^\circ/s$; the measurement error of each accelerometer is $\Delta A_1 = \Delta A_2 = \Delta A_3 = \Delta A_4 = 10^{-4} \, g$; pitch, roll, and yaw vibration amplitudes are $\theta_m = \gamma_m = \psi_m = 0.2^\circ$; the length of the lever arm is $r = 1 \, m$; the side length of the 4 accelerometer configuration is $l = 0.05 \, m$.

Because the tangential acceleration of the lever arm effect is consistent with the attitude angular frequency, it cannot be filtered out by low-pass filtering. Therefore, figure 6 compares the mechanical compensation and the residual acceleration of the tangential acceleration of the lever arm effect with the 4-accelerometer configuration method.
As can be seen from figure 6, the 4-accelerometer configuration avoids differential operation, and the compensation precision is greatly improved compared with the mechanical compensation method.

6. Conclusion
This paper studied the compensation method of the lever arm effect. Aiming at the centrifugal acceleration term of the lever arm effect, this paper compared the low-pass filtering method with the mechanical compensation method, and verified that in actual wind tunnel tests, the mechanical compensation method was far better than the low-pass filtering method. For the tangential acceleration term of the lever arm effect, a 4-accelerometer configuration method was designed, which avoided the error amplification caused by differential calculation and provided a new idea for compensating the tangential acceleration of the lever arm effect.

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