Study on the anomalous quartic $W^+W^−\gamma\gamma$ couplings of electroweak bosons in $e^-p$ collisions at the LHeC and the FCC-he

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Abstract

In this paper, a study is carried out on the $e^- p \rightarrow e^- \gamma^* p \rightarrow pW^+ \gamma \nu_e$ production to probe quartic $W^+W^-\gamma\gamma$ couplings using 10, 100 fb$^{-1}$ of $e^- p$ collisions data at $\sqrt{s} = 1.30, 1.98$ GeV at the Large Hadron electron Collider (LHeC) and 100, 1000 fb$^{-1}$ with $\sqrt{s} = 3.46, 5.29$ GeV at the Future Circular Collider-hadron electron (FCC-he). Production cross-sections are determined for both at leptonic and hadronic decay channel of the $W$-boson. With the data from future $e^- p$ colliders, it is possible to obtain sensitivity measures at 95% C.L. on the anomalous $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ couplings which are competitive with the limits obtained by the LHC, as well as with others limits reported in the literature. The production mode $e^- p \rightarrow e^- \gamma^* p \rightarrow pW^+ \gamma \nu_e$ in $e^- p$ collisions offers a window for study the quartic $W^+W^-\gamma\gamma$ electroweak bosons couplings at the LHeC and the FCC-he, which provides a much cleaner collision environment than the LHC.

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I. INTRODUCTION

A property of the weak interaction is that its gauge bosons $W^\pm$ and $Z$ can couple to each other in certain combinations and also to $\gamma$. The gauge bosons $W^\pm$, $Z$, and $\gamma$ through mixing with each other represent some of the Standard Model (SM) particles most strongly coupled to Electroweak Symmetry Breaking (EWSB). Due to the non-Abelian nature of the SM electroweak theory, gauge bosons interact with each other and the SM predicts the existence of the anomalous Triple Gauge Couplings (aTGC) and the anomalous Quartic Gauge Couplings (aQGC). In particular, the aQGC $WW\gamma\gamma$ is the main topic in this article.

Studies for $WW\gamma\gamma$ aQGC have been theoretically carried out at lepton-lepton colliders with the processes $e^+e^- \rightarrow VVV$ [4–11], $e^+e^- \rightarrow VVFF$ [12, 13], $e\gamma \rightarrow VVF$ [14, 15], $\gamma\gamma \rightarrow VVV$ [16, 17], $\gamma\gamma \rightarrow VV$ [18], $e^+e^- \rightarrow e^+\gamma^*e^- \rightarrow VVFF$ [19] and at hadron-hadron colliders with the processes $pp \rightarrow VVV$ [20–26], $pp \rightarrow VVFF$ [27–29], $pp \rightarrow p\gamma^*p \rightarrow pVVF$ [29] and $pp \rightarrow p\gamma^*\gamma^*p \rightarrow pVVp$ [30–33], at lepton-hadron colliders with the process $ep \rightarrow VVFF$ where $V = W^\pm, Z, \gamma$ and $F = e, j, \nu$. Searches for processes containing aQGC have been performed by previous experiments, for instance $e^+e^- \rightarrow WW\gamma$ by the L3, DELPHI and OPAL Collaborations at the Large Electron Positron (LEP) collider [34–37], $p\bar{p} \rightarrow pW^+W^-\bar{p} \rightarrow pe^+\nu e^-\bar{\nu}\bar{p}$ by the D0 Collaboration at the Tevatron of Fermilab [38], $pp \rightarrow p\gamma^*\gamma^*p \rightarrow pWWp$ and $pp \rightarrow W\gamma jj$ by the CMS Collaboration [39, 40], $pp \rightarrow pW^+W^-p \rightarrow pe^\pm\nu\mu^\mp\nu p$ by the ATLAS Collaboration [41] at the LHC. In the post-LHC era the present and future colliders contemplate in their physics programs the study of the aQGC: the High-Luminosity Large Hadron Collider (HL-LHC), the High-Energy Large Hadron Collider (HE-LHC) [42], the Large Hadron electron Collider (LHeC) [43–47], the Future Circular Collider-hadron electron (FCC-he) [48, 49], the International Linear Collider (ILC) [50], the Compact Linear Collider (CLIC) [51], the Circular Electron Positron Collider (CEPC) [52] and the Future Circular Collider $e^+e^-$ (FCC-ee) [53].

The LHeC, is one of the proposed colliders in the new energy frontier at the LHC, is to inject an electron beam which will collide head-on with the available proton beam. For a first stage, this $ep$ collision option is for a center-of-mass energy $\sqrt{s} = 1.30$ TeV, where $e^-$ energy $E_e = 60$ GeV, and the proton beam energy $E_p = 7$ TeV. The second stage can be realised with $E_e = 140$ GeV and $E_p = 7$ TeV. The design for the new collider as well as the details can be found in Refs. [43–47]. Further upgrades to the HE-LHC would provide
proton beam energies up to 50 TeV. This is another available option, the FCC-he. This upgrade with \( E_p = 50 \) TeV, assumes potential reuse of the LHeC with \( E_e = 60 - 140 \) GeV.

It is possible that at a later stage the upgrade, with \( E_p = 50 \) TeV, assumes a maximum of \( E_e = 250 - 500 \) GeV.

In this paper, we present our results in a model-independent way for the total cross-section of the process \( e^−p \rightarrow e^−γ^*p \rightarrow pW^+γν_e \) at the \( e^−γ^* \) mode, as well as limits on \( WWγγ \) aQGC assuming \( \mathcal{L} = 10,100 \text{ fb}^{-1} \) of electron-proton collision data at 1.30 and 1.98 TeV at the LHeC and \( \mathcal{L} = 100,1000 \text{ fb}^{-1} \) of electron-proton collision data at 3.46 and 5.29 TeV at the FCC-he. For our study, we use an effective Lagrangian approach which provides a generic platform for introducing the effect of new physics Beyond the SM (BSM) by adding additional terms in the Lagrangian of the SM.

The paper is organized as follows: In Section II, we give the general expressions for the effective Lagrangian. In Section III, we evaluate the total cross-section of the reaction \( e^−p \rightarrow e^−γ^*p \rightarrow pW^+γν_e \) and derive the 95\% C.L. allowed sensitivity measures on the anomalous \( f_{M,i}/Λ^4 \) and \( f_{T,i}/Λ^4 \) couplings at the LHeC and the FCC-he. In Section IV, we summarize our conclusions.

II. DIMENSION-8 OPERATORS SET RELEVANT FOR \( e^−p \rightarrow e^−γ^*p \rightarrow pW^+γν_e \)

A suitable and relatively modern approach to observe the effects of new BSM physics in a model-independent formalism is to use an effective Lagrangian description of the SM.

Starting from our present theoretical, phenomenological and experimental understanding, treating the SM in an effective Lagrangian approach is an well-motivated starting point since we have no present evidence of BSM physics. In practice, this means defining a scale, \( Λ \), of new physics higher than the energy scale being probed in the experiment and using the fields of the SM to write higher dimension operators in addition to dimension-4 operators of the SM. Following the context of Refs. \[54\]–\[56\], the effective Lagrangian as well as the classes of genuine aQGC operators \[57\] of dimension-8 for \( WWγγ \) vertex are the following \[58\]:

\[
\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{i=1}^{2} \frac{f_{S,i}}{Λ^4} O_{S,i} + \sum_{i=0}^{9} \frac{f_{T,i}}{Λ^4} O_{T,i} + \sum_{i=0}^{7} \frac{f_{M,i}}{Λ^4} O_{M,i},
\] (1)
TABLE I: Set of genuine aQGC operators\(^5\) of dimension-8 for WW\(\gamma\gamma\) vertex. In these operators, each operator \(O_{S,i}, O_{M,i}\) and \(O_{T,i}\) is parametrized by Wilson coefficients.

| Operator name | Operator |
|---------------|----------|
| \(O_{S,0}\)   | \([(D_\mu \Phi)^\dagger(D_\nu \Phi)] \times [(D_\mu \Phi)^\dagger(D_\nu \Phi)]\) |
| \(O_{S,1}\)   | \([(D_\mu \Phi)^\dagger(D_\mu \Phi)] \times [(D_\nu \Phi)^\dagger(D_\nu \Phi)]\) |

| M-type operators |
|------------------|
| \(O_{M,0}\)   | \(\text{Tr}[W_{\mu\nu}W^{\mu\nu}] \times [(D_\beta \Phi)^\dagger(D_\beta \Phi)]\) |
| \(O_{M,1}\)   | \(\text{Tr}[W_{\mu\nu}W^{\nu\beta}] \times [(D_\beta \Phi)^\dagger(D_\mu \Phi)]\) |
| \(O_{M,2}\)   | \([B_{\mu\nu}B^{\mu\nu}] \times [(D_\beta \Phi)^\dagger(D_\beta \Phi)]\) |
| \(O_{M,3}\)   | \([B_{\mu\nu}B^{\nu\beta}] \times [(D_\beta \Phi)^\dagger(D_\mu \Phi)]\) |
| \(O_{M,4}\)   | \([(D_\mu \Phi)^\dagger W_{\beta\nu}(D_\mu \Phi)] \times B^{\beta\nu}\) |
| \(O_{M,5}\)   | \([(D_\mu \Phi)^\dagger W_{\beta\nu}(D_\nu \Phi)] \times B^{\beta\mu}\) |
| \(O_{M,6}\)   | \([(D_\mu \Phi)^\dagger W_{\beta\nu}W^{\beta\nu}(D_\mu \Phi)]\) |
| \(O_{M,7}\)   | \([(D_\mu \Phi)^\dagger W_{\beta\nu}W^{\beta\mu}(D_\nu \Phi)]\) |

| T-type operators |
|------------------|
| \(O_{T,0}\)   | \(\text{Tr}[W_{\mu\nu}W^{\mu\nu}] \times \text{Tr}[W_{\alpha\beta}W^{\alpha\beta}]\) |
| \(O_{T,1}\)   | \(\text{Tr}[W_{\alpha\nu}W^{\mu\beta}] \times \text{Tr}[W_{\mu\beta}W^{\alpha\nu}]\) |
| \(O_{T,2}\)   | \(\text{Tr}[W_{\alpha\nu}W^{\mu\beta}] \times \text{Tr}[W_{\beta\nu}W^{\mu\alpha}]\) |
| \(O_{T,3}\)   | \(\text{Tr}[W_{\mu\nu}W^{\mu\nu}] \times B_{\alpha\beta}B^{\alpha\beta}\) |
| \(O_{T,4}\)   | \(\text{Tr}[W_{\alpha\nu}W^{\mu\beta}] \times B_{\mu\beta}B^{\alpha\nu}\) |
| \(O_{T,5}\)   | \(\text{Tr}[W_{\alpha\nu}W^{\mu\beta}] \times B_{\beta\nu}B^{\mu\alpha}\) |
| \(O_{T,6}\)   | \(B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta}\) |
| \(O_{T,7}\)   | \(B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha}\) |

Here, 18 operators of dimension-8 which are classified in independent scalar operators, independent mixed operators and independent transverse operators are given in Table I. Table I contains the Wilson coefficient \(f_M^{i,j}/\Lambda^2\) which is relationship with \(a^W_{0,c}/\Lambda^2\) couplings as follows\(^5, 57, 59\):
TABLE II: Definition of the fiducial regions of the fully leptonic and hadronic $W^+\gamma\nu_e$ analyses.

| Fiducial Requirements |
|------------------------|
| Selected cuts for the $p_T$ |
| $p_T^\gamma > 10$ GeV, minimum $p_T$ for the photons |
| $p_T^l > 10$ GeV, minimum $p_T$ for the charged leptons |

| Selected cuts for the $\eta$ |
|-------------------------------|
| $|\eta_\gamma| < 2.5$, maximum rapidity for the photons |
| $|\eta_l| < 2.5$, maximum rapidity for the charged leptons |

| Selected cuts for the $\Delta R$ |
|----------------------------------|
| $\Delta R_{ll} = 0.4$, minimum distance between leptons |
| $\Delta R_{\gamma l} = 0.4$, minimum distance between $\gamma$ and lepton |

$$
\begin{align*}
\frac{f_{M,0}}{\Lambda^2} &= \frac{a_0}{\Lambda^2 g^2 v^2}, \\
\frac{f_{M,1}}{\Lambda^2} &= -\frac{a_c}{\Lambda^2 g^2 v^2}, \\
\frac{f_{M,2}}{2} &= \frac{f_{M,6}}{2}, \\
\frac{f_{M,1}}{2} &= \frac{f_{M,3}}{2} = -\frac{f_{M,5}}{2} = \frac{f_{M,7}}{2}.
\end{align*}
$$

III. CROSS-SECTION MEASUREMENTS AT THE LHEC AND THE FCC-HE

To investigate the effect of dimension-8 operators we focus on the process $W$-boson production in association with a neutrino plus a photon at the LHeC and the FCC-he.

Dimension-8 operators given in Table I directly enter into the process $e^- p \rightarrow e^- \gamma^* p \rightarrow pW^+\gamma\nu_e$ by modifying the $WW\gamma\gamma$ vertex. The part of $\sigma(e^- p \rightarrow e^- \gamma^* p \rightarrow pW^+\gamma\nu_e)$ that grows with the energy is controlled by dimension-8 operators alone. To quantitatively evaluate the effects we can expect at the electron-proton level, we turn to numerics. For which we consider the following cinematic cuts transverse momentum for the photon and charged leptons $p_T^\gamma$ and $p_T^l$, the rapidity for the photons and charged leptons $\eta_\gamma$ and $\eta_l$, distance
between leptons and distance between $\gamma$ and lepton $\Delta R_{\ell\ell}$ with the purpose of reducing the background and improve the sensitivity of the signal. To reconstruct the signal, we require at least one electron or muon with $p_T > 10$ GeV, least one photon with $p_T > 10$ GeV, rapidity for the photons and leptons with $|\eta_{\ell,l}| < 2.5$ and distance between leptons and distance between $\gamma$ and leptons with $\Delta R_{\gamma l,l\ell} = 0.4$. These selection cuts are summarized in Table II.

A. Photoproduction at the LHeC and the FCC-he

Photon interactions have been extensively studied at HERA [65], LEP [66], Tevatron [67] and LHC [68], in processes involving exchange of quasi-real photons collinear to the incoming lepton. In a similar manner, a significant fraction of a lepton-hadron collisions at the LHeC and the FCC-he will involve quasi-real photon interactions. The LHeC and the FCC-he can to some extend be considered as a high energy $e\gamma^*, \gamma^*p$ and $\gamma^*\gamma^*$ collisions. On this topic, the futures lepton-hadron colliders offer excellent new opportunities for the study of high energy particle collisions, thus significantly extending the physics capabilities of an lepton-hadron collider. With this options, a large number of new and exciting measurements become accessible with a $e\gamma^*, \gamma^*p$ and $\gamma^*\gamma^*$ collisions. Because the photons couple directly to all fundamental fields carrying the electromagnetic current leptons, quarks, $W$’s, etc.. High energy $e\gamma^*, \gamma^*p$ and $\gamma^*\gamma^*$ collisions will provide a comprehensive laboratory for exploring virtually every aspect of the SM and BSM physics. A review of the studies made on $e\gamma^*, \gamma^*p$ and $\gamma^*\gamma^*$ collisions physics on future colliders is made in Refs. [54–56, 69–82].

It is appropriate to mention that the studies of photon interactions at the LHC are possible due to experimental signatures of events involving photon exchanges such as the presence of very forward scattered protons and of large rapidity gaps in forward directions. However, to tag efficiently photon induced processes and to keep backgrounds under control, some processes require very forward proton detectors [83]. The photon induced processes have been measured in $p\bar{p}$ collisions at Tevatron-Fermilab using the large rapidity gap signature. The exclusive two-photon production of lepton pairs and the diffractive photoproduction of $J/\psi$ mesons were studied in Refs. [84–86], respectively. In both cases clear signals were obtained with low backgrounds.

As we mentioned above, scenarios like the LHeC and the FCC-he [69, 74] offer an unique
opportunity to build $ep$ collider, which can also be operated in $\gamma p$ collisions. These conversions are made by converting the incoming electrons or protons into an intense beam of high-energy photons. In addition, the $ep$ colliders also provide the opportunity to examine $\gamma^*\gamma^*$, $\gamma^*e$ and $\gamma^*p$ modes with quasi-real photons through the Equivalent Photon Approximation (EPA) \[83, 87, 88\], using the Weizsacker-Williams Approximation (WWA).

On the other hand, the phenomenological investigations at lepton-hadron colliders generally contain usual deep inelastic scattering reactions where the colliding hadron dissociates into partons. These reactions have been extensively studied in the literature, while the processes elastic and semi-elastic, such as $\gamma^*\gamma^*$ and $\gamma^*p$ have been much less studied. These processes have simpler final states with respect to lepton-hadron processes. In this case, these processes compensate for the advantages of lepton-hadron processes such as having high center-of-mass energy and high luminosity. In addition, $e\gamma^*$ have effective luminosity and much higher energy compared to the process $\gamma^*\gamma^*$ collisions. This may be significant because of the high energy dependencies of the cross-section containing the new physics parameters. For all the aforementioned, it is expected that the $\gamma^*p$ collisions to have a high sensitivity to the $WW\gamma\gamma$ aQGC.

Regarding $e\gamma^*$ collisions these can be discerned from usual deep inelastic scattering collisions by means of two experimental signatures. First signature is the forward large rapidity gap. Quasi-real photons have a low virtuality and scattered with small angles from the beam pipe. As the transverse momentum carried by a quasi-real photon is small, photon-emitting proton should also be scattered with small angles and exit the central detector without being detected. This causes a decrease in the energy deposit in the corresponding forward region. As a result of this, one of the forward regions of the central detector has a significant lack of energy. This defines the forward large-rapidity gap and usual $ep$ deep inelastic collisions can be rejected by applying a selection cut on this quantity. Second experimental signature is provided by the forward detectors. Forward detectors are capable to detect particles with a large pseudorapidity. When a photon emitting from proton is scattered with a large pseudorapidity, it exceeds the pseudorapidity coverage of the central detectors. The detection of this proton by the forward detectors provides a distinctive signal for $e\gamma^*$ collisions. In this regard, the LHeC Collaboration has a program of forward physics with extra detectors located in a region between a few tens up to several hundreds of metres from the interaction point \[89\].
B. The total cross-section for one exchanged quasi-real photon

\( \gamma^* \) photons emitted from proton beams collide with the incoming electron, and \( e\gamma^* \) collisions are generated. The process \( e^-\gamma^* \rightarrow W^+\gamma\nu_e \) participates as a subprocess in the process \( e^-p \rightarrow e^-\gamma^*p \rightarrow pW^+\gamma\nu_e \). In addition, the diagram of the process \( e^-p \rightarrow e^-\gamma^*p \rightarrow pW^+\gamma\nu_e \) is given in Fig. 1. The Feynman diagrams for the subprocess \( e^-\gamma^* \rightarrow W^+\gamma\nu_e \) are shown in Fig. 2. Therefore, we find the total cross section of the main process \( e^-p \rightarrow e^-\gamma^*p \rightarrow pW^+\gamma\nu_e \) by integrating the cross section for the subprocess \( e^-\gamma^* \rightarrow W^+\gamma\nu_e \). The total cross section of this process can be written as

\[
\sigma(e^-p \rightarrow pW^+\gamma\nu_e) = \int f_{\gamma^*}(x)\hat{\sigma}(e^-\gamma^* \rightarrow W^+\gamma\nu_e)dx. 
\]  

\( f_{\gamma^*}(x) \) is defined as follows \[87, 90\]:

\[
f_{\gamma^*}(x) = \frac{\alpha}{\pi E_p} \left\{ [1 - x] \left[ \varphi\left(\frac{Q_{\text{max}}^2}{Q_0^2}\right) - \varphi\left(\frac{Q_{\text{min}}^2}{Q_0^2}\right) \right] \right\}, \tag{7}
\]

with \( x = E_\gamma/E_p \), \( Q_{\text{max}}^2 = 2\,\text{GeV}^2 \) is the maximum virtuality of the photon and \( Q_{\text{min}}^2 \) is:

\[
Q_{\text{min}}^2 = \frac{m_p^2x^2}{1 - x}. \tag{8}
\]

In addition, the explicit form of function \( \varphi \) contained in Eq. (7) is:

\[
\varphi(\theta) = (1 + ay) \left[ -\ln(1 + \frac{1}{\theta}) + \sum_{k=1}^{3} \frac{1}{k(1 + \theta)^k} \right] + \frac{y(1 - b)}{4\theta(1 + \theta)^3} \\
+ c(1 + \frac{y}{4}) \left[ \ln \left( \frac{1 - b + \theta}{1 + \theta} \right) + \sum_{k=1}^{3} \frac{b^k}{k(1 + \theta)^k} \right], \tag{9}
\]

where explicitly \( a, b, c \) and \( y \) are:

\[
y = \frac{x^2}{(1 - x)}, \tag{10}
\]

\[
a = \frac{1 + \mu_p^2}{4} + \frac{4m_p^2}{Q_0^2} \approx 7.16, \tag{11}
\]
\[ b = 1 - \frac{4m_p^2}{Q_0^2} \approx -3.96, \]  

(12)

\[ c = \frac{\mu_p^2 - 1}{b^4} \approx 0.028. \]  

(13)

We calculate the total cross-section of the reaction \( e^- p \to e^- \gamma^* p \to pW^+ \gamma \nu_e \) through the expression given by Eq. (6) and in the presence of dimension-8 operators using the MadGraph5_aMC@NLO package. This requires events that pass the selection cuts to have \( p_T \) transverse moment, \( \eta \) rapidity and \( \Delta R \) distance between particles as specified in Table II. The effects of the kinematic selection cuts on the final-state particles, as well as of the dimension-8 \( f_{M,i}/\Lambda^4 \) and \( f_{T,i}/\Lambda^4 \) couplings on the total cross-section of the reaction \( e^- p \to e^- \gamma^* p \to pW^+ \gamma \nu_e \) for the center-of-mass energies at the LHeC and the FCC-he are shown in Tables III and IV.

For the aQGC \( f_{M,0-5,7}/\Lambda^4 \) and \( f_{T,0-2,5,6,7}/\Lambda^4 \) taking one at a time we get the results as shown in Figs. 3-10 and Tables III and IV at the LHeC for \( \sqrt{s} = 1.30, 1.98 \) TeV and the FCC-he for \( \sqrt{s} = 3.46, 5.29 \), respectively. The color lines in Figs. 3-10 show the deviation in \( e^- p \to e^- \gamma^* p \to pW^+ \gamma \nu_e \) from the SM value as a function of \( f_{M,i}/\Lambda^4 \) and \( f_{T,i}/\Lambda^4 \). The effects of the fiducial kinematics cuts given in Table II are required to rejects those particles misidentified as leptons and photons, as well as the effect of the dimension-8 operators \( O_{M,0-5,7} \) and \( O_{T,0-2,5,6,7} \) can be seen through Figs. 3-10. In these figures, we consider for \( e^- p \to e^- \gamma^* p \to pW^+ \gamma \nu_e \) signal, the leptonic and hadronic decays of the \( W \)-boson; \( W \to \nu_l l, W \to qq' \) with \( \nu_l = \nu_e, \nu_\mu \), \( l = e^-, \mu \) and \( q = u, c, \bar{d}, \bar{s}, q' = d, s, \bar{u}, \bar{c} \). Figs. 8 and 10 illustrate more clearly the effect of the dimension-8 operators on the total cross-section of the process \( e^- p \to e^- \gamma^* p \to pW^+ \gamma \nu_e \) with the leptonic and hadronic decay of the \( W \)-boson, and for the energy \( \sqrt{s} = 5.29 \) TeV for the FCC-he. The highest cross-section is obtained for \( \sigma(\sqrt{s}, f_{T,5}/\Lambda^4) = 2.29 \times 10^4 \) pb followed by \( \sigma(\sqrt{s}, f_{T,6}/\Lambda^4) = 1.11 \times 10^4 \) pb and \( \sigma(\sqrt{s}, f_{T,7}/\Lambda^4) = 2.73 \times 10^3 \) pb for the hadronic channel, respectively and as shown in Table IV as well as by Fig. 10.

To close this subsection, it is worth mentioning that our results shows that a nonzero aQGC enhances the production cross-section at large energies of the \( ep \) system with respect to the SM prediction, as can be seen in Figs. 3-10.
IV. PROJECTIONS ON THE AQGC $f_{M,i}/\Lambda^4$ AND $f_{T,i}/\Lambda^4$ AT THE LHEC AND THE FCC-HE

In this section, we present the bounds on the Wilson coefficients of 13 operators in question. We focus exclusively on the $\mathcal{O}_{M,i}$ and $\mathcal{O}_{T,i}$ operators with $i = 0, 1, 2, 3, 4, 5, 6, 7$.

To estimate the limits on anomalous couplings $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ through the process $e^- p \rightarrow e^- \gamma^* p \rightarrow pW^+ \gamma \nu_e$, we use the luminosities of $\mathcal{L} = 10, 100 \text{ fb}^{-1}$ of 1.30 and 1.98 TeV electron-proton collisions at the LHeC, and of $\mathcal{L} = 100, 1000 \text{ fb}^{-1}$ of 3.46 and 5.29 TeV electron-proton collisions at the FCC-he, respectively. In addition, we consider the leptonic and hadronic decay channel of the $W$-boson of the final state.

Furthermore, for the aQGC search, a restricted region of $p_T$, $\eta$ and $\Delta R$ is used, that is say, the fiducial region is defined in Table II. This is chosen to reduce the contribution of the background and to improve the sensitivity of the signal.

The presence of new physics characterized by the parameters $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ may be quantified by a simple $\chi^2$ method that varies the parameters $(f_{M,i}/\Lambda^4, f_{T,i}/\Lambda^4)$ and is based on:

$$\chi^2(f_{M,i}/\Lambda^4, f_{T,i}/\Lambda^4) = \left(\frac{\sigma_{SM}(\sqrt{s}) - \sigma_{BSM}(\sqrt{s}, f_{M,i}/\Lambda^4, f_{T,i}/\Lambda^4)}{\sigma_{SM}(\sqrt{s})\delta_{st}}\right)^2.$$  \hspace{1cm} (14)

In Eq. (14), $\sigma_{SM}(\sqrt{s})$ is the cross-section of the SM and $\sigma_{BSM}(\sqrt{s}, f_{M,i}/\Lambda^4, f_{T,i}/\Lambda^4)$ is the BSM cross-section, while $\delta_{st} = \frac{1}{\sqrt{N_{SM}}}$ is the statistical error and $N$ is the number of events:

$$N_{SM} = \mathcal{L}_{int} \times \sigma_{SM}.$$  \hspace{1cm} (15)

To get an idea of the LHeC and FCC-he constraining power, in Tables V-VI, we show the expected bounds on the aQGC $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ from the $e^- p \rightarrow e^- \gamma^* p \rightarrow pW^+ \gamma \nu_e$ production.

Our results for the anomalous $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ couplings are competitive with those reported in Ref. [92] through the $Z\gamma jj$ production in proton-proton collisions at $\sqrt{s} = 13$ TeV and integrated luminosity of 35.9 fb$^{-1}$ at the CMS Collaboration. A direct comparison of the anomalous $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ couplings given in Ref. [92] with our results reported in Tables V and VI, shows that in some cases our bounds for $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ are more stringent than those reported in Table 4 of Ref. [92] for the CMS Collaboration at the LHC.
Another paper presents a study of vector boson scattered in $WW$, $WZ$, and $ZZ$ channels using $pp$ collisions at $\sqrt{s} = 13$ TeV and integrated luminosity of $35.9 \pm 0.9 \text{ fb}^{-1}$ collected with the CMS detector at the LHC [93].

Other experimental results on the anomalous $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ couplings reported by the CMS and ATLAS Collaborations are the followings. With $\sqrt{s} = 8$ TeV and integrated luminosity of $19.7 \text{ fb}^{-1}$, the CMS experiment [39, 40] searching for exclusive or quasi-exclusive $WW$ production, via the signal topology $pp \rightarrow p\gamma^*\gamma^*p \rightarrow p^*W^+W^-p^*$ where the $p^*$ indicates that the final state protons either remain intact (exclusive or elastic production), or dissociate into an undetected system (quasi-exclusive or proton dissociation production). Their research are translated into limits on the aQGC $f_{M,0,1,2,3}/\Lambda^4$. In addition, the CMS experiment [39, 40] measure the electroweak-induced production of $W$ and two jets, where the $W$ boson decays leptonically, and experimental limits on the aQGC $f_{M,0-7}/\Lambda^4$ and $f_{T,0-2.5-7}/\Lambda^4$ are set at $95\%$ C.L.. In another investigation with $\sqrt{s} = 8$ TeV and $\mathcal{L}_{\text{int}} = 20.2 \text{ fb}^{-1}$ of proton-proton collisions the ATLAS experiment [94] studied the production of $WV\gamma$ events in $e\nu\mu\gamma$, $e\nu j j \gamma$ and $\mu\nu j j \gamma$ final states. The results reported in these studies are weaker than those reported in our present article.

Phenomenological results on the aQGC $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ at the LHeC with $\sqrt{s} = 1.30, 1.98$ TeV and the FCC-he with $\sqrt{s} = 3.46, 5.29$ TeV, respectively, are presented in Refs. [54–56]. In Ref. [56] through the $ep \rightarrow e^-\gamma^*p \rightarrow eW\gamma q'X \rightarrow e\nu q'X$ channel get sensitivity measures of the order of $10^{-1}$ for some anomalous $f_{T,i}/\Lambda^4$ couplings. Another, sensitivity measures on the aQGC of the order of $10^1$ are report by Refs. [54, 55] via the process $e^-p \rightarrow e^-\gamma^*\gamma^*p \rightarrow e^-W^+W^-p$ with the subprocess $\gamma^*\gamma^* \rightarrow W^+W^-$. In Tables V and VI, we summarize all of the sensitivity measures on the anomalous $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ couplings obtained at $\sqrt{s} = 1.30, 1.98$ TeV and $\sqrt{s} = 3.46, 5.29$ TeV with the production mode $e^-p \rightarrow e^-\gamma^*p \rightarrow pW^+\gamma\nu_e$. Our results on the aQGC $f_{M,0-5,7}/\Lambda^4$ and $f_{T,0-2.5-7}/\Lambda^4$ for the different energy stages above mentioned given sensitivity measures of the order of $10^{-1}$, which are similar to those sensitivity measures report by Refs. [54–56] at the LHeC and the FCC-he, with other channels. For other reviews experimental and phenomenological, the reader can check Refs. [39, 40, 54, 55, 94–97].

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TABLE III: Total cross-sections of the process $e^− p \rightarrow e^− γ^* p \rightarrow pW^+ γν_ε$ for $\sqrt{s} = 1.30, 1.98$ TeV at the LHeC and $\sqrt{s} = 3.46, 5.29$ TeV at the FCC-he depending on 13 anomalous couplings obtained by dimension-8 operators. Also, all anomalous couplings for the LHeC and the FCC-he are taken as equal to $1 \times 10^{-8}$ and $5 \times 10^{-9}$ GeV$^{-4}$, respectively. The total cross-sections for each coupling are calculated while fixing the other couplings to zero with the selections cuts defined in Table II.

| Couplings | $\sqrt{s} = 1.30$ TeV | $\sqrt{s} = 1.98$ TeV | $\sqrt{s} = 3.46$ TeV | $\sqrt{s} = 5.29$ TeV |
|-----------|------------------------|------------------------|------------------------|------------------------|
| $f_{M0}/Λ^4$ | $5.64 \times 10^{-3}$ | $1.07 \times 10^{-1}$ | $1.60 \times 10^{-2}$ | $8.12 \times 10^{-1}$ |
| $f_{M1}/Λ^4$ | $3.61 \times 10^{-3}$ | $6.22 \times 10^{-2}$ | $1.13 \times 10^{-2}$ | $6.26 \times 10^{-1}$ |
| $f_{M2}/Λ^4$ | $1.94 \times 10^{-1}$ | $4.51$ | $5.61 \times 10^{-1}$ | $3.46 \times 10^1$ |
| $f_{M3}/Λ^4$ | $1.08 \times 10^{-1}$ | $2.56$ | $3.57 \times 10^{-1}$ | $2.66 \times 10^1$ |
| $f_{M4}/Λ^4$ | $1.58 \times 10^{-2}$ | $3.46 \times 10^{-1}$ | $4.55 \times 10^{-2}$ | $2.65$ |
| $f_{M5}/Λ^4$ | $9.58 \times 10^{-3}$ | $2.00 \times 10^{-1}$ | $3.07 \times 10^{-2}$ | $2.03$ |
| $f_{M7}/Λ^4$ | $1.90 \times 10^{-3}$ | $1.83 \times 10^{-2}$ | $5.35 \times 10^{-3}$ | $1.62 \times 10^{-1}$ |
| $f_{T0}/Λ^4$ | $4.92 \times 10^{-1}$ | $12.71$ | $4.28$ | $2.41 \times 10^2$ |
| $f_{T1}/Λ^4$ | $8.89 \times 10^{-1}$ | $2.54 \times 10^1$ | $6.69$ | $6.33 \times 10^2$ |
| $f_{T2}/Λ^4$ | $1.24 \times 10^{-1}$ | $3.41$ | $9.49 \times 10^{-1}$ | $7.95 \times 10^1$ |
| $f_{T5}/Λ^4$ | $5.29$ | $1.36 \times 10^2$ | $4.60 \times 10^1$ | $2.60 \times 10^3$ |
| $f_{T6}/Λ^4$ | $9.55$ | $2.74 \times 10^2$ | $7.23 \times 10^1$ | $6.79 \times 10^3$ |
| $f_{T7}/Λ^4$ | $1.33$ | $3.67 \times 10^1$ | $10.20$ | $8.55 \times 10^2$ |

V. SUMMARY AND CONCLUSIONS

In this work, in the effective Lagrange approach, we study the $e^− p \rightarrow e^− γ^* p \rightarrow pW^+ γν_ε$ channel at the LHeC and the FCC-he as a way to perform sensitivity measures on the total cross-section and on the anomalous $f_{M,i}/Λ^4$ and $f_{T,i}/Λ^4$ couplings. We focus on new physics
TABLE IV: Total cross-sections of the process $e^- p \rightarrow e^- \gamma^* p \rightarrow p W^+ \gamma \nu_e$ for $\sqrt{s} = 1.30, 1.98$ TeV at the LHeC and $\sqrt{s} = 3.46, 5.29$ TeV at the FCC-he depending on 13 anomalous couplings obtained by dimension-8 operators. Also, all anomalous couplings for the LHeC and the FCC-he are taken as equal to $1 \times 10^{-8}$ and $5 \times 10^{-9}$ GeV$^{-4}$, respectively. The total cross-sections for each coupling are calculated while fixing the other couplings to zero with the selections cuts defined in Table II.

| Couplings | LHeC | FCC-he |
|-----------|------|--------|
| $\sqrt{s} = 1.30$ TeV | $3.93 \times 10^{-3}$ | $1.53 \times 10^{-2}$ |
| $\sqrt{s} = 1.98$ TeV | $8.70 \times 10^{-3}$ | $2.79 \times 10^{-2}$ |
| $\sqrt{s} = 3.46$ TeV | $1.31 \times 10^{-2}$ | $1.37 \times 10^{-1}$ |
| $\sqrt{s} = 5.29$ TeV | $1.75 \times 10^{-2}$ | $6.20 \times 10^{-1}$ |
| $f_{M0}/\Lambda^4$ | $4.46 \times 10^{-2}$ | $1.46$ |
| $f_{M1}/\Lambda^4$ | $1.31 \times 10^{-2}$ | $1.37 \times 10^{-1}$ |
| $f_{M2}/\Lambda^4$ | $6.51 \times 10^{-3}$ | $6.20 \times 10^{-1}$ |
| $f_{M3}/\Lambda^4$ | $0.41$ | $5.31$ |
| $f_{M4}/\Lambda^4$ | $1.38 \times 10^{-1}$ | $4.75$ |
| $f_{M5}/\Lambda^4$ | $3.58 \times 10^{-2}$ | $4.20 \times 10^{-1}$ |
| $f_{M7}/\Lambda^4$ | $6.51 \times 10^{-3}$ | $2.93 \times 10^{-1}$ |
| $f_{T0}/\Lambda^4$ | $2.69$ | $2.01 \times 10^{2}$ |
| $f_{T1}/\Lambda^4$ | $2.88$ | $9.14 \times 10^{1}$ |
| $f_{T2}/\Lambda^4$ | $4.74 \times 10^{-1}$ | $2.36 \times 10^{1}$ |
| $f_{T5}/\Lambda^4$ | $2.88 \times 10^{1}$ | $2.16 \times 10^{3}$ |
| $f_{T6}/\Lambda^4$ | $3.10 \times 10^{1}$ | $9.82 \times 10^{2}$ |
| $f_{T7}/\Lambda^4$ | $5.10$ | $2.53 \times 10^{2}$ |

effects that grow with energy, parameterized by dimension-8 effective operators within the effective Lagrange framework. In particular, we identify 13 operators that induce a growth with energy.

To get a quantitative idea of the sensitivity of our results, we give a summary of the projected for the total cross-section of the process $e^- p \rightarrow e^- \gamma^* p \rightarrow p W^+ \gamma \nu_e$ in Tables III-IV and Figs. 3-10, as well as 95% C.L. sensitivity measure on 13 operators listed in Tables
TABLE V: Sensitivity measures on aQGC at the 95% C. L. via $e^-p \rightarrow e^- \gamma^*p \rightarrow pW^+\gamma\nu_e$ for $\sqrt{s} = 1.30, 1.98$ TeV at the LHeC. The coupling are calculated while fixing the other couplings to zero.

| Couplings (TeV$^{-4}$) | LHeC, $\sqrt{s} = 1.30$ TeV | Hadronic channel |
|-------------------------|--------------------------------|-----------------|
|                         | 10 fb$^{-1}$                  | 100 fb$^{-1}$   | 10 fb$^{-1}$  | 100 fb$^{-1}$  |
| $f_{M0}/\Lambda^4$     | [-3.91;3.92] $\times 10^3$   | [-2.20;2.21] $\times 10^3$ | [-1.70;1.76] $\times 10^3$ | [-0.94;1.01] $\times 10^3$ |
| $f_{M1}/\Lambda^4$     | [-5.03;5.41] $\times 10^3$   | [-2.75;3.13] $\times 10^3$ | [-3.41;3.76] $\times 10^3$ | [-1.85;2.19] $\times 10^3$ |
| $f_{M2}/\Lambda^4$     | [-5.86;6.10] $\times 10^2$   | [-3.25;3.48] $\times 10^2$ | [-2.62;2.67] $\times 10^2$ | [-1.46;1.51] $\times 10^2$ |
| $f_{M3}/\Lambda^4$     | [-7.98;8.04] $\times 10^2$   | [-4.47;4.53] $\times 10^2$ | [-5.16;5.78] $\times 10^2$ | [-2.77;3.40] $\times 10^2$ |
| $f_{M4}/\Lambda^4$     | [-2.15;2.19] $\times 10^3$   | [-1.20;1.24] $\times 10^3$ | [-9.44;9.69] $\times 10^2$ | [-5.25;5.51] $\times 10^2$ |
| $f_{M5}/\Lambda^4$     | [-2.98;2.80] $\times 10^3$   | [-1.72;1.54] $\times 10^3$ | [-2.10;1.86] $\times 10^3$ | [-1.24;1.00] $\times 10^3$ |
| $f_{M6}/\Lambda^4$     | [-1.09;1.00] $\times 10^4$   | [-6.37;5.45] $\times 10^3$ | [-7.56;6.79] $\times 10^3$ | [-4.43;3.66] $\times 10^3$ |
| $f_{T0}/\Lambda^4$     | [-3.73;3.74] $\times 10^2$   | [-2.09;2.11] $\times 10^2$ | [-2.12;2.17] $\times 10^2$ | [-1.18;1.23] $\times 10^2$ |
| $f_{T1}/\Lambda^4$     | [-2.75;2.81] $\times 10^2$   | [-1.54;1.59] $\times 10^2$ | [-1.91;2.22] $\times 10^2$ | [-1.01;1.32] $\times 10^2$ |
| $f_{T2}/\Lambda^4$     | [-7.14;7.68] $\times 10^2$   | [-3.91;4.44] $\times 10^2$ | [-4.51;5.74] $\times 10^2$ | [-2.31;3.54] $\times 10^2$ |
| $f_{T5}/\Lambda^4$     | [-1.13;1.14] $\times 10^2$   | [-6.39;6.40] $\times 10^1$ | [-6.31;6.78] $\times 10^1$ | [-3.45;3.92] $\times 10^1$ |
| $f_{T6}/\Lambda^4$     | [-8.36;8.59] $\times 10^1$   | [-4.65;4.88] $\times 10^1$ | [-5.81;6.78] $\times 10^1$ | [-3.08;4.05] $\times 10^1$ |
| $f_{T7}/\Lambda^4$     | [-2.26;2.27] $\times 10^2$   | [-1.27;1.28] $\times 10^2$ | [-1.41;1.71] $\times 10^2$ | [-0.73;1.04] $\times 10^2$ |

| Couplings (TeV$^{-4}$) | LHeC, $\sqrt{s} = 1.98$ TeV | Hadronic channel |
|-------------------------|--------------------------------|-----------------|
|                         | 10 fb$^{-1}$                  | 100 fb$^{-1}$   | 10 fb$^{-1}$  | 100 fb$^{-1}$  |
| $f_{M0}/\Lambda^4$     | [-1.03;1.04] $\times 10^3$   | [-5.77;5.83] $\times 10^2$ | [-5.84;6.11] $\times 10^2$ | [-3.23;3.50] $\times 10^2$ |
| $f_{M1}/\Lambda^4$     | [-1.33;1.39] $\times 10^3$   | [-0.73;0.81] $\times 10^3$ | [-1.08;1.18] $\times 10^3$ | [-5.83;6.92] $\times 10^2$ |
| $f_{M2}/\Lambda^4$     | [-1.56;1.57] $\times 10^2$   | [-8.81;8.82] $\times 10^1$ | [-9.03;9.22] $\times 10^1$ | [-5.04;5.23] $\times 10^1$ |
| $f_{M3}/\Lambda^4$     | [-2.01;2.13] $\times 10^2$   | [-1.11;1.22] $\times 10^2$ | [-1.66;1.78] $\times 10^2$ | [-0.91;1.03] $\times 10^2$ |
| $f_{M4}/\Lambda^4$     | [-5.65;5.72] $\times 10^2$   | [-3.16;3.23] $\times 10^2$ | [-3.25;3.36] $\times 10^2$ | [-1.80;1.97] $\times 10^2$ |
| $f_{M5}/\Lambda^4$     | [-7.53;7.44] $\times 10^2$   | [-4.25;4.17] $\times 10^2$ | [-6.44;6.01] $\times 10^2$ | [-3.72;3.29] $\times 10^2$ |
| $f_{M6}/\Lambda^4$     | [-2.78;2.66] $\times 10^3$   | [-1.59;1.47] $\times 10^3$ | [-2.35;2.17] $\times 10^3$ | [-1.36;1.18] $\times 10^3$ |
| $f_{T0}/\Lambda^4$     | [-9.20;9.45] $\times 10^1$   | [-5.12;5.37] $\times 10^1$ | [-6.19;6.67] $\times 10^1$ | [-1.38;3.86] $\times 10^1$ |
| $f_{T1}/\Lambda^4$     | [-6.27;6.84] $\times 10^1$   | [-3.41;3.98] $\times 10^1$ | [-5.38;6.29] $\times 10^1$ | [-2.85;3.76] $\times 10^1$ |
| $f_{T2}/\Lambda^4$     | [-1.75;1.84] $\times 10^2$   | [-0.97;1.05] $\times 10^2$ | [-1.35;1.64] $\times 10^2$ | [-0.70;0.99] $\times 10^2$ |
| $f_{T5}/\Lambda^4$     | [-2.80;2.90] $\times 10^1$   | [-1.55;1.65] $\times 10^1$ | [-1.83;2.09] $\times 10^1$ | [-0.97;1.24] $\times 10^1$ |
| $f_{T6}/\Lambda^4$     | [-1.96;2.05] $\times 10^1$   | [-1.09;1.77] $\times 10^1$ | [-1.67;1.88] $\times 10^1$ | [-0.90;1.11] $\times 10^1$ |
| $f_{T7}/\Lambda^4$     | [-5.41;5.53] $\times 10^1$   | [-3.02;3.14] $\times 10^1$ | [-4.06;5.04] $\times 10^1$ | [-2.10;3.08] $\times 10^1$ |
TABLE VI: Sensitivity measures on aQGC at the 95\% C. L. via $e^-p \to e^-\gamma^*p \to pW^+\gamma\nu_e$ for $\sqrt{s} = 3.46, 5.29$ TeV at the FCC-he. The coupling are calculated while fixing the other couplings to zero.

| Couplings (TeV$^{-4}$) | FCC-he, $\sqrt{s} = 3.46$ TeV | FCC-he, $\sqrt{s} = 5.29$ TeV |
|------------------------|-------------------------------|-------------------------------|
|                        | Leptonic channel               | Hadronic channel              |
|                        | 100 fb$^{-1}$ | 1000 fb$^{-1}$ | 100 fb$^{-1}$ | 1000 fb$^{-1}$ |
| $f_{M0}/\Lambda^4$    | $[-8.18;8.26] \times 10^2$   | $[-4.59;4.66] \times 10^2$   | $[-1.15;1.16] \times 10^2$   | $[-0.64;0.65] \times 10^2$   |
| $f_{M1}/\Lambda^4$    | $[-0.97;1.09] \times 10^3$   | $[-0.52;0.64] \times 10^3$   | $[-3.80;4.07] \times 10^2$   | $[-2.08;2.35] \times 10^2$   |
| $f_{M2}/\Lambda^4$    | $[-1.25;1.26] \times 10^2$   | $[-7.04;7.07] \times 10^1$   | $[-1.74;1.78] \times 10^1$   | $[-0.97;1.01] \times 10^1$   |
| $f_{M3}/\Lambda^4$    | $[-1.51;1.62] \times 10^2$   | $[-0.83;0.93] \times 10^2$   | $[-5.87;6.13] \times 10^1$   | $[-3.24;3.51] \times 10^1$   |
| $f_{M4}/\Lambda^4$    | $[-4.43;4.65] \times 10^2$   | $[-2.45;2.67] \times 10^2$   | $[-6.28;6.45] \times 10^1$   | $[-3.50;3.67] \times 10^1$   |
| $f_{M5}/\Lambda^4$    | $[-5.67;5.62] \times 10^2$   | $[-3.20;3.15] \times 10^2$   | $[-2.35;2.00] \times 10^2$   | $[-1.41;1.06] \times 10^2$   |
| $f_{M7}/\Lambda^4$    | $[-2.12;1.98] \times 10^3$   | $[-1.23;1.08] \times 10^3$   | $[-8.26;7.51] \times 10^2$   | $[-4.82;4.07] \times 10^2$   |
| $f_{T0}/\Lambda^4$    | $[-4.46;4.60] \times 10^1$   | $[-2.47;2.62] \times 10^1$   | $[-0.86;1.11] \times 10^1$   | $[-0.44;0.69] \times 10^1$   |
| $f_{T1}/\Lambda^4$    | $[-3.52;3.68] \times 10^1$   | $[-1.95;2.11] \times 10^1$   | $[-1.27;1.65] \times 10^1$   | $[-0.65;1.02] \times 10^1$   |
| $f_{T2}/\Lambda^4$    | $[-0.90;1.02] \times 10^2$   | $[-0.48;0.60] \times 10^2$   | $[-2.32;3.51] \times 10^1$   | $[-1.11;2.31] \times 10^1$   |
| $f_{T5}/\Lambda^4$    | $[-1.38;1.39] \times 10^1$   | $[-7.76;7.81] \times 10^1$   | $[-2.66;3.32] \times 10^1$   | $[-1.37;2.04] \times 10^1$   |
| $f_{T6}/\Lambda^4$    | $[-1.07;1.13] \times 10^1$   | $[-5.88;6.48] \times 10^1$   | $[-3.75;5.15] \times 10^1$   | $[-1.87;3.27] \times 10^1$   |
| $f_{T7}/\Lambda^4$    | $[-2.87;2.99] \times 10^1$   | $[-1.59;1.71] \times 10^1$   | $[-6.98;10.80] \times 10^1$  | $[-3.34;7.15] \times 10^1$   |

FCC-he, $\sqrt{s} = 5.29$ TeV

| Couplings (TeV$^{-4}$) | FCC-he, $\sqrt{s} = 5.29$ TeV |
|------------------------|-------------------------------|
| $f_{M0}/\Lambda^4$    | $[-1.28;1.29] \times 10^2$   | $[-7.21;7.27] \times 10^1$   | $[-4.60;4.75] \times 10^1$   | $[-2.56;2.70] \times 10^1$   |
| $f_{M1}/\Lambda^4$    | $[-1.45;1.49] \times 10^2$   | $[-0.81;0.85] \times 10^2$   | $[-1.49;1.66] \times 10^1$   | $[-0.80;0.97] \times 10^1$   |
| $f_{M2}/\Lambda^4$    | $[-1.95;1.97] \times 10^1$   | $[-1.10;1.11] \times 10^1$   | $[-7.04;7.23] \times 10^1$   | $[-3.91;4.11] \times 10^1$   |
| $f_{M3}/\Lambda^4$    | $[-2.24;2.25] \times 10^1$   | $[-1.25;1.27] \times 10^1$   | $[-2.23;2.56] \times 10^1$   | $[-1.19;1.52] \times 10^1$   |
| $f_{M4}/\Lambda^4$    | $[-7.12;7.13] \times 10^1$   | $[-4.00;4.01] \times 10^1$   | $[-2.49;2.67] \times 10^1$   | $[-1.37;1.54] \times 10^1$   |
| $f_{M5}/\Lambda^4$    | $[-8.20;8.02] \times 10^2$   | $[-4.65;4.47] \times 10^1$   | $[-9.00;8.32] \times 10^1$   | $[-5.22;4.54] \times 10^2$   |
| $f_{M7}/\Lambda^4$    | $[-2.96;2.91] \times 10^2$   | $[-1.68;1.62] \times 10^2$   | $[-3.19;3.08] \times 10^2$   | $[-1.82;1.71] \times 10^2$   |
| $f_{T0}/\Lambda^4$    | $[-7.41;7.52] \times 10^2$   | $[-4.14;4.25] \times 10^2$   | $[-3.17;3.83] \times 10^2$   | $[-1.65;2.32] \times 10^2$   |
| $f_{T1}/\Lambda^4$    | $[-4.53;4.67] \times 10^2$   | $[-2.52;2.66] \times 10^2$   | $[-4.33;5.77] \times 10^2$   | $[-2.18;3.62] \times 10^2$   |
| $f_{T2}/\Lambda^4$    | $[-1.29;1.30] \times 10^1$   | $[-7.20;7.35] \times 10^1$   | $[-0.82;1.25] \times 10^1$   | $[-0.39;0.82] \times 10^1$   |
| $f_{T5}/\Lambda^4$    | $[-2.24;2.32] \times 10^1$   | $[-1.24;1.32] \times 10^1$   | $[-0.97;1.17] \times 10^1$   | $[-0.51;0.71] \times 10^1$   |
| $f_{T6}/\Lambda^4$    | $[-1.30;1.52] \times 10^2$   | $[-0.69;0.91] \times 10^2$   | $[-1.33;1.75] \times 10^1$   | $[-0.68;1.09] \times 10^1$   |
| $f_{T7}/\Lambda^4$    | $[-3.88;4.02] \times 10^2$   | $[-2.15;2.29] \times 10^2$   | $[-2.55;3.79] \times 10^1$   | $[-1.24;2.41] \times 10^1$   |
V-VI. We give two sets of results. The first results refer to the total cross-section of the $e^-p \rightarrow e^-\gamma^*p \rightarrow pW^+\gamma\nu_e$ signal for $\sqrt{s} = 1.30, 1.98$ TeV at the LHeC and $\sqrt{s} = 3.46, 5.29$ TeV at the FCC-he for each coupling $f_{M,i}/\Lambda^4$ or $f_{T,i}/\Lambda^4$ fixing one at a time. In our computation, we apply the selections cuts defined in Table II which is efficient in reducing the backgrounds while preserving most of the signal. An interesting feature of our results is the impact of the dimension-8 operators.

Since the aQGC $WW\gamma\gamma$ described through effective Lagrangian have dimension-8, they have very strong energy dependence. Therefore, the anomalous cross section containing the $WW\gamma\gamma$ vertex has a higher energy than the SM cross section. In addition, the future ep collider will possibly generate a final state with two or more massive gauge bosons. Hence, it will have a great potential to investigate aQGC. High-energy accelerated $e^-$ and $p$ beams at these colliders radiate quasi-real photons, and thus $e\gamma^*, \gamma^*p$ and $\gamma^*\gamma^*$ collisions are produced from the $e^-p$ process itself. Therefore, $ep$ colliders will provide an important opportunity to probe $e\gamma^*, \gamma^*p$ and $\gamma^*\gamma^*$ collisions at high energies. These collisions for the new physics searches at $ep$ colliders have a very clean experimental environment, since they have no interference with weak and strong interactions.

The second set of results, corresponds to the sensitivity measures on the aQGC $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$. For each of our sensitivity measures, we consider two benchmark scenarios characterized by different energies and luminosities, as well as by the leptonic and hadronic decay channels of the $W$-boson of the final state. Furthermore, the selection cuts used in our analysis are efficient in reducing the backgrounds while preserving most of the $e^-p \rightarrow e^-\gamma^*p \rightarrow pW^+\gamma\nu_e$ signal.

Regarding the comparison with present and future bounds from other collider experiments, we find that the $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ constraints are significantly competitive with the ones achievable at the CMS and ATLAS Collaboration at the LHC through the $Z\gamma jj$ production and of vector boson scattered in $WW$, $WZ$, and $ZZ$ channels at $\sqrt{s} = 13$ TeV. Also, via the observation of electroweak production of same-sign $W$-boson pairs in the two jet and two same-sign lepton final state in proton-proton collisions at 13 TeV. In Ref. [96, 99], is discuss the feature of the signals of aQGC and sensitivities to the aQGC in the $pp \rightarrow W\gamma jj$ channel at the LHC $\sqrt{s} = 13$ TeV. As well as with other limits reports with a projection at the LHeC and the FCC-he through the $ep \rightarrow \nu_e\gamma\gamma jj$ reaction, the process $ep \rightarrow e^-\gamma^*\gamma^*p \rightarrow e^-W^+W^-p$ and of the $ep \rightarrow e^-\gamma^*p \rightarrow eW\gamma'q'X \rightarrow e\nu_l l'q'X$
signal. In addition, of other limits reports in the literature \cite{39, 40, 54–56, 94, 95}.

We conclude by mentioning that our projections at the LHeC and the FCC-he are interpreted in the approach of dimension-8 effective field theory operators through the \( e^-p \rightarrow e^-\gamma^*p \rightarrow pW^+\gamma\nu_e \) channel. Confidence intervals are derived for all 13 parameters of aQGC this analysis is sensitive to. In this sense, our results indicate that the \( e^-p \rightarrow e^-\gamma^*p \rightarrow pW^+\gamma\nu_e \) production is convincing for searching for the dimension-8 operators \( O_{M,0-5,7} \) and \( O_{T,0-2,5,6,7} \), and as a consequence of the Wilson coefficients \( f_{M,0,2,3,4} \) and \( f_{T,0,1,2,5,6,7} \) with clean environments, as well as with good sensitivity.

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FIG. 1: Feynman diagram for the signal process $e^- p \rightarrow e^- \gamma^* p \rightarrow pW^+ \gamma \nu_e$. New physics (represented by a black circle) in the electroweak sector can modify the quartic gauge couplings.

FIG. 2: Representative Feynman diagrams contributing to the subprocess $e^- \gamma^* \rightarrow W^+ \gamma \nu_e$. 
FIG. 3: For leptonic channel, the total cross-sections of the process $e^- p \rightarrow e^- \gamma^* p \rightarrow p W^+ \gamma \nu_e$ as a function of the anomalous couplings for center-of-mass energy $\sqrt{s} = 1.30 \text{ TeV}$ at the LHeC.

FIG. 4: Same as in Fig. 3, but for $\sqrt{s} = 1.98 \text{ TeV}$ at the LHeC.
FIG. 5: Same as in Fig. 3, but for hadronic decay.

FIG. 6: Same as in Fig. 4, but for hadronic decay.
FIG. 7: For leptonic channel, the total cross-sections of the process $e^- p \rightarrow e^- \gamma^* p \rightarrow pW^+ \gamma \nu_e$ as a function of the anomalous couplings for center-of-mass energy $\sqrt{s} = 3.46$ TeV at the FCC-he.

FIG. 8: Same as in Fig. 7, but for $\sqrt{s} = 5.29$ TeV at the FCC-he.
FIG. 9: Same as in Fig. 7, but for hadronic decay.

FIG. 10: Same as in Fig. 8, but for hadronic decay.