Griffiths wings of electronic nematic phase transition

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Abstract

A phase transition is usually controlled by tuning some thermodynamic variables such as pressure, magnetic field, electron density, and temperature. A spatial $xy$ anisotropy generated by a uniaxial pressure or strain can also be used as a thermodynamic variable. In particular, the $xy$ anisotropy couples directly to electronic nematic order and thus is expected to play a crucial role in a system exhibiting nematicity. In this paper we clarify the phase diagram of electronic nematic instability in the presence of $xy$ anisotropy. While a second order transition cannot occur in this case, mean-field theory predicts that a first order transition occurs near van Hove filling and its phase boundary forms a wing structure, which we dub a Griffiths wing. When crossing the wing, the anisotropy of the electronic system exhibits a discontinuous change, leading to a meta-nematic transition, i.e., the analog to a meta-magnetic transition in a magnetic system. The upper edge of the wing corresponds to a critical end line, which shows a non-monotonic temperature dependence as a function of the external anisotropy and vanishes at a quantum critical end point for a strong anisotropy. The mean-field phase diagram is, however, sensitive even to weak fluctuations of the nematic order parameter, yielding a topologically different phase diagram. The Griffiths wing is broken into two pieces. A tiny wing appears close to zero anisotropy and the other is realized for a strong anisotropy. Consequently three quantum critical end points are realized.

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Nematic liquid crystals are well known. Rodlike molecules flow like a liquid, but are always oriented to a certain direction in the nematic phase. This state is characterized by breaking of the orientational symmetry, retaining the other symmetries of the system. Electrons are point particles, not molecules. Nevertheless electronic analogs of the nematic liquid crystals were observed in a number of interacting electron systems: Two-dimensional electron gases \[1,2\], high-temperature superconductors of cuprates \[3,4\] and pnictides \[5\], the bilayer strontium ruthenate \(\text{Sr}_3\text{Ru}_2\text{O}_7\) \[6\], and an actinide material \(\text{URu}_2\text{Si}_2\) \[7\]. The electronic nematicity is driven by electron-electron interactions. Depending on electronic degrees of freedom responsible for nematicity, we may distinguish between three kinds of nematicity: spin, orbital, and charge nematicity. The spin nematic state is driven by frustration between magnetic interactions \[8\]. The orbital nematic is due to an orbital order caused, for example, by a spontaneous occupation difference between \(d_{yz}\) and \(d_{zx}\) orbitals \[9,10\]. There are two routes toward a charge nematic state, via melting of spin-charge stripes \[11\] and a \(d\)-wave Pomeranchuk instability (\(d\)PI) of a normal phase \[12,13\].

The electronic nematic order couples directly to an external anisotropy, which is generated by applying a uniaxial pressure, strain, or sometimes by an intrinsic anisotropy of a crystal structure due to orthorhombicity. Hence the external anisotropy is expected to play a crucial role in a system exhibiting nematicity. On the other hand, it is generally not easy to quantify how much anisotropy is imposed on a sample. Quite recently, however, the anisotropy was calibrated by exploiting the piezoelectric effect \[14\]. In particular, a nematic susceptibility was extracted and its divergence was demonstrated near a nematic critical temperature.

Motivated by the experimental progress to control the external anisotropy, we clarify the phase diagram of the electronic nematic instability in the presence of anisotropy. In particular, we focus on the nematicity associated with a \(d\)PI \[12,13\], which exhibits interesting physics as we report in this paper.

In the mean-field theory of the \(d\)PI \[15,16\], the \(d\)PI occurs around van Hove filling with a dome-shaped transition line. The transition is of second order at high temperatures and changes to first order at low temperatures. The end points of the second order line are tricritical points.

The presence of a tricritical point (TCP) implies a wing structure when a conjugate field to the corresponding order parameter is applied to the system. This insight originates from the study of \(\text{He}^3-\text{He}^4\) mixtures by Griffiths \[17\]. However, the conjugate field to the
superfluid order parameter is not accessible in experiments. The wing structure predicted by Griffiths, which we dub the Griffiths wing, was not tested for He$^3$-He$^4$ mixtures.

It was found theoretically that itinerant ferromagnetism occurs generally via a first order transition at low temperatures and a second order one at high temperatures \cite{18}. The end point of the second order line is a TCP. The order parameter is magnetization and its conjugate field is a magnetic field in that case. Similar to Griffiths’s work \cite{17}, a wing structure emerges from the first order transition line and extends to the side of a finite magnetic field. Furthermore, it was found that the wing ultimately vanishes at a quantum critical end point (QCEP) in a strong magnetic field. When crossing the wing, the system exhibits a jump of the magnetization, leading to a metamagnetic transition. In contrast to the case of He$^3$-He$^4$ mixtures, a magnetic field is controlled in experiments. In fact the Griffiths wings were clearly observed in UGe$_2$ \cite{19} and UCoAl \cite{20}.

In the present paper, we report Griffiths wings of the electronic nematic phase transition associated with the $d$PI. A conjugate field to the nematic order parameter is $xy$ anisotropy, which is accessible in experiments. By applying the anisotropy, we obtain a wing structure. However, in contrast to the Griffiths wings obtained in previous studies \cite{17,18}, the critical end line (CEL) shows non-monotonic behavior as a function of the anisotropy. Furthermore the obtained wing structure is very sensitive to weak fluctuations of the order parameter, leading to a phase diagram topologically different from the mean-field result.

We study electronic nematicity associated with the $d$PI in the presence of $xy$ anisotropy. Our minimal model reads

$$H = \sum_{k,\sigma} (\epsilon_k^0 - \mu) c_{k\sigma}^\dagger c_{k\sigma} - \frac{1}{2N} \sum_q g(q) n_d(q)n_d(-q) - \mu d n_d(0),$$ \hspace{1cm} (1)

where $c_{k\sigma}^\dagger$ ($c_{k\sigma}$) is the creation (annihilation) operator of electrons with momentum $k$ and spin $\sigma$, $\mu$ is the chemical potential, and $N$ is the number of sites. The kinetic energy $\epsilon_k^0$ is given by a usual tight binding dispersion on a square lattice,

$$\epsilon_k^0 = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y.$$ \hspace{1cm} (2)

The interaction term describes a $d$-wave weighted density-density interaction \cite{21}; $n_d(q) = \sum_{k,\sigma} d_k c_{k+\frac{3}{2}\sigma}^\dagger c_{k+\frac{3}{2}\sigma}$ with a $d$-wave form factor $d_k = \cos k_x - \cos k_y$. The coupling strength $g(q)$ has a peak at $q = 0$, that is, forward scattering dominates. This interaction drives a $d$PI at low temperatures and is obtained in microscopic models such as the $t$-$J$ \cite{12}, Hubbard
The $d$-wave chemical potential $\mu_d$ gives an anisotropy of the nearest neighbor hopping integral $t$ between the $x$ and $y$ direction and is considered as a field applying $xy$ anisotropy to the system, for example, by a uniaxial pressure along the $x$ or $y$ direction. In mean-field theory (see Supplementary Materials), the order parameter of the $d$PI is proportional to $n_d(0)$ and thus $\mu_d$ is a conjugate field to that.

We consider a phase diagram in the three-dimensional space spanned by $\mu$, $\mu_d$, and temperature $T$. The phase diagram is symmetric with respect to the axis of $\mu_d = 0$ and is almost symmetric with respect to the axis of $\mu = 0$ as long as $t'$ in Eq. (2) is small. Hence we focus on the region of $\mu > 0$ and $\mu_d > 0$ by taking $t' = 0$. Considering previous studies [17, 18], we may expect a wing structure emerging from a first order transition of the $d$PI by applying the field $\mu_d$. By analogy with the case of ferromagnetism (an external field is a magnetic field [18], instead of $\mu_d$), the wing corresponds to a jump of the order parameter of $d$PI, namely a meta-nematic transition. The upper edge of the wing is a CEL, which is determined by the condition,

$$\frac{\partial \omega}{\partial \phi} = \frac{\partial^2 \omega}{\partial \phi^2} = \frac{\partial^3 \omega}{\partial \phi^3} = 0,$$

where $\omega$ is the free energy per lattice sites and $\phi$ is the order parameter of the $d$PI. All quantities of dimension of energy are units of $t$ in the present paper.

Figure 1(a) is a schematic phase diagram in the three-dimensional space of $\mu$, $\mu_d$, and $T$ obtained in a mean-field approximation to the Hamiltonian (1). At zero anisotropy ($\mu_d = 0$) a $d$PI occurs via a first order transition at low $T$. With increasing $\mu$, the band is eventually filled up and the band insulating (BI) state is realized in the striped region. A wing emerges from the first order line and extends to a region of a finite $\mu_d$. It stands almost vertically on the plane of $\mu$ and $\mu_d$ close to van Hove filling. We project the CEL on the plane of $\mu_d$ and $T$ in Fig. 1(b). The temperature of the CEL, $T_{CEL}$, is rapidly suppressed by applying the anisotropy $\mu_d$, but does not go to zero. It recovers to form a broad peak around $\mu_d = 2$ and eventually vanishes when it touches the BI phase, leading to a QCEP there. In fact, the electron density becomes two at the QCEP as seen in Fig. 1(b). When the system crosses the wing, the nematic order parameter exhibits a jump. Such a jump, $\Delta \phi$, is plotted in Fig. 1(c) along the bottom of the wing as a function of $\mu_d$. The magnitude of the jump exhibits behavior similar to $T_{CEL}$ shown in Fig. 1(b). It is interesting that $\Delta \phi$ around $\mu_d = 2$ becomes comparable to that at $\mu_d = 0$ in spite of the presence of a large external anisotropy.

The field $\mu_d$ gives an anisotropy to the hopping integral $t$ between the $x$ and $y$ direction.
FIG. 1: (Color online) Mean-field results. (a) Schematic phase diagram in the space spanned by \( \mu, \mu_d, \) and \( T \). The phase diagram is symmetric with respect to the axis of \( \mu_d = 0 \) and \( \mu = 0 \). At \( \mu_d = 0 \) a dPI occurs around van Hove filling, from which \( \mu \) is measured. The transition is of second order at high \( T \) (solid line) and of first order at low \( T \) (double line). The solid circle denotes the TCP. The BI state is realized in the striped region. The wing (colored in orange) stands almost vertically on the plane of \( \mu \) and \( \mu_d \) close to van Hove filling. The wing eventually vanishes at the QCEP; the index 1 implies that the system is almost one dimensional at the QCEP. The upper edge of the wing (solid line) is a CEL. (b) Temperature of the CEL (\( T_{CEL} \)) as a function of \( \mu_d \); The wing is projected on the plane of \( \mu_d \) and \( T \). The electron density at \( T_{CEL} \) is also plotted. (c) Jump of the nematic order parameter across the wing at \( T = 0.001t \).

[see Eq. (A3)],

\[
t_x = t(1 + \mu_d/2t), \quad t_y = t(1 - \mu_d/2t),
\]

which suggests that \( \mu_d \) may change the band structure. In fact, the saddle points of the non-interacting band dispersion are located at \((\pi, 0)\) and \((0, \pi)\) for \( \mu_d < 2 \), but they change to \((\pi, \pi)\) and \((0, 0)\) for \( \mu_d > 2 \). Around \( \mu_d = 2 \), therefore, the band becomes very flat, leading to an enhancement of the density of states. This is the reason why \( T_{CEL} \) as well as \( \Delta \phi \) is
FIG. 2: (Color online) Results in the presence of order-parameter fluctuations. (a) Schematic phase diagram. The phase of the $d$PI is slightly suppressed by fluctuations, compared with the mean-field result. The region of the BI is not computed, but is assumed to be the same as the mean-field result. The wing obtained in Fig. 1(a) is broken into two pieces: one tiny wing close to $\mu_d = 0$ and the other wing for a large $\mu_d$; the indices of QCEP$_2$ and QCEP$_1$ imply that the system can be regarded to be two and quasi-one dimensional, respectively, at the QCEP. The dashed line corresponds to a crossover. (b) and (c) The wings are projected on the plane of $\mu_d$ and $T$ for several choices of the cutoff $\Lambda_0$. (d) The critical anisotropy of the hopping integral, $\Delta t_c/t$, to obtain the QCEP$_2$ as a function of the strength of order-parameter fluctuations, which is quantified by the ratio of the tricritical temperature and its mean-field value.

enhanced around $\mu_d = 2$, yielding a peak there; the peak position is slightly deviated from $\mu_d = 2$ because of the presence of a finite order parameter $\phi$ [see Eq. (A3)]. Since $\mu_d$ is very large close to the QCEP$_1$, the system is almost one dimensional [24]. The wing, therefore, interpolates between a two- and (effectively) one-dimensional system by increasing $\mu_d$.

The mean-field results are, however, very sensitive to fluctuations of the nematic order
parameter. We apply a functional renormalization-group scheme \[25\] to the Hamiltonian (1) and compute the free energy \(\omega(\phi)\) in the presence of fluctuations [see Eq. (A9)]. Figure 2(a) is a schematic phase diagram in the presence of order-parameter fluctuations. The strength of fluctuations that we consider is weak in the sense that the mean-field phase diagram is still qualitatively correct at \(\mu_d = 0\) and the dPI exhibits a first order transition at low temperatures. Applying the anisotropy \(\mu_d\), the CEL rapidly drops to zero, leading to a tiny wing terminating at a QCEP\(_2\). We then have a crossover region depicted by the dashed line. The order parameter of the dPI shows a rapid change, but without a jump, by crossing the dashed line. With further increasing \(\mu_d\), another wing emerges with two QCEPs.

In Figs. 2(b) and (c) two Griffiths wings are projected on the plane of \(\mu_d\) and \(T\). The calculations are performed for several choices of the cutoff \(\Lambda_0\), which controls the strength of fluctuations (see Supplementary Materials). When \(\Lambda_0\) becomes larger \[26\], the CELs are suppressed more by order-parameter fluctuations, making the wings to be realized in smaller regions of the phase diagram. To quantify the suppression by fluctuations, we consider the critical external anisotropy to obtain a QCEP, which we express in terms of \(\Delta t_c/t = \frac{t_x - t_y}{t_x + t_y} = \mu_d/2t\). We plot \(\Delta t_c/t\) in Fig. 2(d) as a function of the ratio of the tricritical temperature and its mean-field value (\(T_{TCP}/T_{TCP}^{MF}\)) as the strength of fluctuations. We see that when \(T_{TCP}\) is suppressed by fluctuations, for example, by half, a very small anisotropy \((\Delta t_c/t \approx 0.01)\) is sufficient to yield a QCEP. Furthermore the strength of fluctuations to realize the QCEP is weak in the sense that the dPI phase diagram at \(\mu_d = 0\) is still well captured by mean-field theory. Therefore we conclude that the Griffiths wing is very sensitive to fluctuations and the QCEP\(_2\) can be reached with a weak anisotropy. This is sharply different from the mean-field result [Fig. 1(a)] where a QCEP can be reached only with a very strong anisotropy, i.e., \(\Delta t_c/t \approx 1.2\) for \(T_{TCP}/T_{TCP}^{MF} = 1\).

Our results can be related to cuprate superconductors. Neutron scattering experiments showed that the magnetic excitation spectrum becomes anisotropic in momentum space. The anisotropy observed in \(\text{YBa}_2\text{Cu}_3\text{O}_{6.85}\) and \(\text{YBa}_2\text{Cu}_3\text{O}_{6.6}\) \[27, 28\] was relatively weak. These experimental data are well captured in terms of competition of the tendency toward the dPI and pairing correlations \[29\]. For \(\text{YBa}_2\text{Cu}_3\text{O}_{6.45}\), however, Ref. 30 reported a very strong anisotropy in the magnetic excitation spectrum, which could not be interpreted in the same theory as Ref. 29. Instead two different theories were proposed: one invoking the presence of a quantum critical point of the nematic instability \[31, 32\] and the other invoking
a dominant nematic tendency over the pairing tendency \[33\]. The point is that the observed anisotropy seems to be suddenly changed by crossing the oxygen concentration around 6.45. Since superconducting samples of Y-based cuprates have an intrinsic \(xy\) anisotropy coming from the CuO chain structure, Y-based cuprates are located along the axis of a small \(\mu_d\) in Fig. 2(a). With decreasing \(\mu\) (hole picture), namely decreasing the oxygen concentration, the system can cross the tiny wing or pass close to the QCEP\(_2\) in Fig. 2(a), which may explain a sudden change of the anisotropy observed in the magnetic excitation spectrum \[30\].

The strontium ruthenate \(\text{Sr}_3\text{Ru}_2\text{O}_7\) exhibits an electronic nematic instability \[6, 34, 35\]. The observed phase diagram shows a first order transition at low \(T\) and a second order transition at high \(T\), similar to the mean-field phase diagram of the \(d\)PI except for a slope of the first order transition. The present theory predicts that the CEL rapidly drops by applying a strain along the \(x\) or \(y\) direction, possibly reaching a QCEP in experiments. While an \(xy\) anisotropy is also generated by introducing a magnetic field along the \(x\) or \(y\) direction \[6\], the coupling to electrons is different from our anisotropic field of \(\mu_d\).

A piece of the broken Griffiths wings is realized also for a strong anisotropy in Figs. 2(a) and (c). It is unrealistic to apply such a strong anisotropy to a two-dimensional electronic system in condensed matter. Instead we point out that quasi-one-dimensional metals such as organic conductors can be regarded as ones with a strong \(xy\) anisotropy. While those systems already have the strong anisotropy, our theory predicts that the anisotropy of the electronic system can change drastically by crossing the Griffiths wing, for example, by controlling carrier density, as implied from Fig. 1(c). Although we are not aware of papers discussing such a sudden change of anisotropy, it may be reasonable to wait for further experiments, because \(xy\) anisotropy of physical quantities in an already strongly anisotropic system was not likely recognized as something related to the physics associated with nematicity.

To test the whole structure of the broken Griffiths wings shown in Fig. 2 solid state systems are apparently not appropriate. Instead optical lattices in a cold atom system provide the best stage \[36\]. The \(\mu_d\) field can be easily simulated by controlling the anisotropy of the hopping integral \(t\) by changing the strength of laser beams between the \(x\) and \(y\) direction. Measurements should be done at temperatures much lower than the Fermi energy, especially to check the presence of a QCEP. This is not possible at present because of technical problems of cooling fermionic systems, which, however, will be overcome in the
near future.

The present theory provides many interesting insights into electronic nematicity and will likely promote further studies. A non-Fermi liquid ground state is stabilized at a quantum critical point of the $d\text{PI}$ [21, 37]. It is plausible to expect an anomalous ground state also at a QCEP of the $d\text{PI}$. In an intermediate region between the QCEP$_2$ and QCEP$_1'$, the dashed line in Fig. 2(a), quantum fluctuations associated with the $d\text{PI}$ completely wash out the wing even close to van Hove filling, suggesting a non-Fermi liquid there. If the system remained a Fermi liquid, the density of states would diverge at van Hove filling, which should lead to an instability because our Hamiltonian (1) has an attractive interaction of the $d\text{PI}$. However, we have obtained a crossover near the dashed line in Fig. 2(a). This consideration hints a possible non-Fermi liquid ground state at van Hove filling; the same conclusion was obtained also by Dzyaloshinskii in a different context [38]. Although the nematic instability cannot occur via a second order transition once $xy$ anisotropy is introduced to the system, critical nematic fluctuations exist not only around QCEPs but also near a CEL. The critical nematic fluctuations can be measured directly by electronic Raman scattering as an emergent central mode [39]. Raman experiments at very low energy, which are technically demanding, will thus play a crucial role to elucidate the physics associated with nematicity. Quite recently Raman scattering indeed detected a central mode for iron-based superconductors [40, 41], where nematicity associated with orbital and spin degrees of freedom is discussed [5].

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Appendix A: Supplementary Materials

Here we provide the mean-field theory of the Hamiltonian (1) and a formalism to analyze nematic order-parameter fluctuations.

1. Mean-field theory

We decouple the interaction term in the Hamiltonian (1) by introducing the order parameter

$$\phi = g n_d(0)/N,$$

where $g = g(0) > 0$. The mean-field Hamiltonian becomes

$$H_{MF} = \sum_{k,\sigma} \xi_k c_{k\sigma}^\dagger c_{k\sigma} + N \frac{g}{2} \phi^2,$$

and the renormalized band is

$$\xi_k = \xi_k^0 - \mu - (\phi + \mu_d) d_k .$$

Obviously the conjugate field $\mu_d$ breaks $xy$ symmetry and plays the same role as the order parameter $\phi$. It is straightforward to obtain the free energy per lattice site

$$\omega(\phi) = -\frac{2T}{N} \sum_k \log \left( 1 + e^{-\xi_k/T} \right) + \frac{1}{2g} \phi^2 .$$

We then solve Eq. (3) numerically. Since the schematic phase diagram Fig. 1(a) does not depend on the value of $g(> 0)$, we took $g = 1$ in Figs. 1(b) and (c).

2. Analysis of nematic fluctuations

As seen in the mean-field theory, the dPI occurs around the van Hove singularity. The proximity to the singularity hinders us from using a usual polynomial expansion of the potential. To circumvent such a problem, we employ a functional renormalization-group (fRG) scheme [25], which allows us to analyze fluctuations without any expansion of the potential [43].

First we derive the bare order-parameter action corresponding to the Hamiltonian (1). Using the path integral formalism, we introduce a Hubbard-Stratonovich field associated
with the fluctuating order parameter of the $d$PI, decouple the interaction of fermions, and integrate out the fermionic degrees of freedom. Retaining only the leading momentum and frequency dependencies of the two-point function and neglecting such dependencies in the higher-order vertex functions \[44\], we obtain the bare action,

$$S[\phi] = \frac{1}{2} \sum_q' \left[ \phi_q \left( A_0 \frac{|\omega_n|}{|q|} + Z_0 q^2 \right) \phi_{-q} \right] + U[\phi],$$

(A5)

where $\phi_q$ with $q = (q, \omega_n)$ is the momentum representation of the order-parameter field $\phi$ and $\omega_n = 2\pi n T$ with integer $n$ denotes the bosonic Matsubara frequencies. The momenta and frequencies contributing to the action $S[\phi]$ are restricted by the cutoff $\Lambda_0$ to the region $\frac{A_0 |\omega_n|}{Z_0 |q|} + q^2 \leq \Lambda_0^2$ as emphasized by adding the prime in the summation in Eq. (A5). In the fermionic representation, $\Lambda_0$ may be related to the maximal momentum transfer allowed by the interaction in the Hamiltonian $[\Pi]$. If $\Lambda_0$ is set to be zero, the action (A5) becomes equivalent to the mean-field theory. Hence the value of $\Lambda_0$ controls the strength of order-parameter fluctuations. The effective potential $U[\phi]$ is given by

$$U[\phi] = \int_0^{\frac{1}{2}} d\tau \int d^2 r U[\phi(r, \tau)],$$

(A6)

where $U(\phi)$ is equal to the mean-field potential Eq. (A4). In contrast to a perturbative analysis, we do not expand $U(\phi)$ in powers of $\phi$.

We carry out calculations within the one-particle irreducible scheme of the fRG by computing the flow of the effective action $\Gamma^\Lambda[\phi]$ in the presence of an infrared cutoff $\Lambda[23]$. The cutoff is implemented by adding a regulator $R^\Lambda$ to the inverse propagator. $\Gamma^\Lambda[\phi]$ interpolates between the bare action $S[\phi]$ [Eq. (A5)] at the ultraviolet cutoff $\Lambda^\text{UV}$ and the final effective action $\Gamma[\phi]$ in the limit of $\Lambda \rightarrow 0$. Its evolution is given by the exact functional flow equation \[43\]

$$\partial_\Lambda \Gamma^\Lambda[\phi] = \frac{1}{2} \text{tr} \frac{R^\Lambda}{\Gamma^\Lambda[\phi] + R^\Lambda},$$

(A7)

where $\dot{R}^\Lambda = \partial_\Lambda R^\Lambda$ and $\Gamma^\Lambda[\phi] = \delta^2 \Gamma^\Lambda[\phi] / \delta \phi^2$. The trace sums over momenta and frequencies.

The exact effective action is a complicated functional of $\phi$. We resort to an approximation of the flow based on the following ansatz:

$$\Gamma^\Lambda[\phi] = \frac{1}{2} \sum_q' \left[ \phi_q \left( A^\Lambda \frac{|\omega_n|}{|q|} + Z^\Lambda q^2 \right) \phi_{-q} \right] + U^\Lambda[\phi],$$

(A8)

with $\Lambda$-dependent parameters $A^\Lambda$ and $Z^\Lambda$ and a local potential $U^\Lambda[\phi]$ of the form of Eq. (A6) with a $\Lambda$-dependent function $U^\Lambda(\phi)$. The summation in Eq. (A8) is restricted to the region
\[ \frac{A^\Lambda(\phi)}{Z^\Lambda(q)} + q^2 \leq \Lambda_0^2; \] the cutoff \( \Lambda_0 \) is a quantity independent of the infrared cutoff \( \Lambda \). A classical version of the ansatz Eq. (A8), where Matsubara frequencies and the dynamical term are absent, has been used to analyze classical phase transitions \([45, 46]\). Inserting the ansatz Eq. (A8) into the exact flow equation Eq. (A7), and evaluating the resulting equation for uniform fields, one obtains the flow equation for \( U^\Lambda(\phi) \).

The evolution of the parameters \( A^\Lambda \) and \( Z^\Lambda \) is determined by the flow of \( \Gamma^\Lambda_2(q; \phi) \). We extract the \( Z \)-factor from the two-point vertex as
\[ Z^\Lambda = \frac{1}{\pi} \Delta_q \left[ \Gamma^\Lambda_2(q; \phi_0) \right]_{q=0}, \]
where \( \phi_0 \) is the position of the global minimum of \( U^\Lambda(\phi) \). Since the flow of the \( A \)-factor is of minor importance \([47]\), it is discarded here, that is, we set \( A^\Lambda = A_0 \). As a regulator we choose a Litim-type \([48]\) cutoff function used in Refs. 42 and 49.

The above formalism largely overlaps with previous works \([42, 49]\), which studied how nematic fluctuations change the mean-field phase diagram at \( \mu_d = 0 \). Compared with the previous calculations \([42, 49]\), we did the following extension in the present study. i) Introduction of two cutoffs, one is the physical cutoff \( \Lambda_0 \) which gives the upper cutoff to the summation in Eqs. (A5) and (A8), and the other is the ultraviolet cutoff \( \Lambda_{UV} \) which is in principle infinite. In the previous formalism \([42, 49]\) \( \Lambda_{UV} \) was assumed to be identical to \( \Lambda_0 \). ii) No additional approximations to compute the right hand side of flow equations, that is, we take account of quantum fluctuations in the anomalous dimension \( \eta = -\frac{\partial \log Z}{\partial \log \Lambda} \), the contribution from the term \( \frac{\partial Z}{\partial \Lambda} \), and an additional term coming from the momentum derivative of the regular for the flow of \( Z \).

In the present paper we calculate flows of the potential \( U^\Lambda(\phi) \) and \( Z^\Lambda \). Since we cannot set \( \Lambda_{UV} = \infty \) numerically, we first did calculations for various choices of large \( \Lambda_{UV} \) and confirmed that our conclusions do not depend on the value of \( \Lambda_{UV} (\geq \Lambda_0) \). In addition, our conclusions also do not depend on a precise choice of \( A_0 \) and \( Z_0 \). We take \( \Lambda_{UV} = \Lambda_0, A_0 = 1, \) and \( Z_0 = 10 \) in Fig. 2 and compute the free energy per lattice site as
\[ \omega(\phi) = \lim_{\Lambda \to 0} U^\Lambda(\phi). \quad (A9) \]

We then search for a solution of Eq. (3) in the three-dimensional space spanned by \( \mu, \mu_d, \) and \( T \). All these computational processes are performed numerically and require highly accurate numerics, otherwise higher order derivatives of the free energy would not become smooth enough to conclude a possible phase diagram. Technically, therefore, it is very challenging to solve Eq. (3) in the presence of fluctuations.