Variations of the problem-solving strategies in plane geometry of junior high school students

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Abstract. The purpose of this study was to explore the problem-solving strategies of junior high school students on plane geometry topics. Using a qualitative approach, as many as 40 students were given tests and one student was interviewed on each of the different problem-solving strategies. After checking the results of student answers, it turned out there were three strategies used by students. The first strategy was that students are able to answer questions by guessing the answer directly then testing them. The second strategy was that students use ratio in finding answers. The third strategy was that students used analogy strategies with variables. The results showed that 13 students used the first strategy, 20 students used the second strategy, one student used the third strategy, and six students did not use the strategy or did not answer. The most used problem-solving strategy by students is the ratio strategy, with the percentage of students using the strategy is 50% of the total students who participated in this study.

1. Introduction
Problem-solving skill is one of the important objectives of mathematics [1]. According to the National Council of Teachers of Mathematics (NCTM) problem-solving skill was one of the mathematical abilities in mathematics learning. Mathematical problem solving needs to be reflected so that students can apply their strategies to solve other problems and in other contexts [2]. Research in problem-solving has identified several variables that affect performance in doing problem solving, namely knowledge, cognitive processes and strategies, differences in individual abilities, and internal dispositions and factors [3].

Problem-solving is a process of how doing something so that it can make something desirable [4]. Problem-solving means being involved in a task with an unknown solution. In order to find the solutions, students must use their knowledge and skills in this process to develop an understanding of mathematics [2]. Problem-solving requires a variety of skills, including interpreting information, planning, checking results and trying various strategies [5]. Skills in using problem-solving strategies do not always end up in the solution level or results, but as a guide in solving the problem and describing the process carried out [6], [7].

Problem-solving starts when there is a representation of the problem. Prior knowledge will help in finding problem-solving strategies. Therefore, people will give different representations of the problem depend on their prior knowledge [8]. Prior knowledge is one of the important factors that make students able to solve mathematical problems. Hence, in order to find solutions, students must explore their prior knowledge, so they can develop new mathematical understandings [9].
Solving a problem can be pursued by using various strategies so that the resolution of the problem becomes simpler and more efficient [10]. Problem-solving strategies include (1) working backwards, (2) finding a pattern, (3) adopting a different point of view, (4) solving a simple or analogous problem, (5) considering extreme cases, (6) making a drawing (visual representation), (7) intelligent guessing and testing (approximation), (8) accounting for all possibilities, (9) organizing data, and (10) logical reasoning [11]. The four steps problem-solving strategy, namely examined the real world problem, form a suitable mathematical model, solve the mathematical model, and translate the solution in terms of the real world problem [6].

Geometry is a part of mathematics that is very close to students because almost all visual objects around them are geometric objects. By learning geometry, students will be helped in solving problems around them [12]. Students often misinterpret or misrepresent the intent and appearance of geometric problems [13]. Learning geometry emphasizes the importance of exploring different representations such as virtual manipulatives, written math formulas, and verbal explanations, which help students build math concepts and develop critical thinking [14].

The topic of geometry in this article is rectangular. The purpose of this study was to determine variations in student strategies in problem-solving, especially on rectangular topics. By describing the types of strategies students use in problem-solving, teachers may be able to develop instructions that are best suited for them, hence students can develop their creativity in problem-solving with a more precise and efficient strategy.

2. Research Methods

The type of research used is descriptive research with a qualitative approach. The subjects of this study were all 9th graders of private schools in Yogyakarta. The selection of 9th grade is because the students' have more prior knowledge than the 7th or 8th grade so that students are expected to have a more varied strategy. The ninth grade in the school under study has two classes. Class 9A consists of 20 students and 9B classes of 20 students. The characteristic of this school is that there is a rule that the teacher can only give questions to students in the form of descriptions so that students are accustomed to working on the questions accompanied by the steps to solve it.

The data in this study were obtained from the results of the completion of the test questions. The test in the form of a description of a problem that had previously been discussed with a junior high school mathematics teacher. Problem description form is chosen, so it looks like a problem-solving strategy. Students work on questions within 15 minutes independently and without opening a book. The following are test questions given to students.

![Figure 1. The problem about plane geometry.](image-url)

In the above problem (figure 1), it is known that the area sizes A, B, C are 80 cm\(^2\), 128 cm\(^2\), and 32 cm\(^2\), respectively. Students are asked to determine the area of D. The unique thing in this problem is
the lack of information on the length and width of each square so that students can be creative to use various strategies to solve the problem. From the results of tests that have known variations in problem-solving strategies, then an interview is conducted. Interviews were conducted to collect verbal data on the reasons students used the strategy. Interviews were conducted on one student in each of the different strategies, so that an analysis of problem-solving strategies could be carried out.

3. Results and discussion

The diagram in figure 2 explains the problem-solving process which is preceded by a problem representation and then uses various strategies for problem-solving. Prior knowledge of students has important roles and influences in the problem-solving process. The ability to represent problems and find problem-solving strategies can vary because it is influenced by students' prior knowledge. Problem-solving strategies describe the process that students do with problem-solving. The last step of the problem-solving process is to find a solution. The solution can be right also can be wrong, but in the most important problem-solving strategy is the process towards the solution.

![Figure 2. Diagram of the problem-solving process.](image)

The data in this study were obtained from students’ answers after working on the test questions. Of the 40 students found various kinds of problem-solving strategies used. The following is a table that shows the kinds of strategies used by students in solving mathematical problem-solving problems.

| No. | Students answer                                      | Many students | Correct answer | Wrong answer |
|-----|------------------------------------------------------|---------------|----------------|--------------|
| 1.  | The strategy of guessing the answers and testing them| 13 (32.5%)    | 10 (76.9%)     | 3 (23.1%)    |
| 2.  | Strategy with ratio                                  | 20 (50.0%)    | 20 (100%)      | 0 (0%)       |
| 3.  | Strategy of analogy with a variable                  | 1 (2.5%)      | 1 (100%)       | 0 (0%)       |
| 4.  | Not using a strategy/ No answer                      | 6 (15.0%)     | 0 (0%)         | 6 (100%)     |
|     | Total                                                | 40            | 31             | 9            |

In table 1 shows that as many as 13 students use the strategy of guessing answers and testing them, with details of 10 people correctly answering and 3 students the answer is wrong. A total of 20 students used a ratio strategy and all answered correctly. There is one student who uses the strategy of analogy with a variable and the answer is correct. Among the 40 students who worked on the problem,
there were 6 students who did not use the strategy and did not answer so the answer was wrong. In this article that will be discussed are some answers to students who use different strategies in answering questions. Some students were also interviewed regarding the strategies they applied.

The following is a strategy used by students for problem-solving, namely the strategy of guessing the answers and testing them, the ratio strategy, and the strategy of analogy with a variable.

3.1. The strategy of guessing the answers and testing them
This strategy is extremely powerful and quite sophisticated. The student makes a guess (and it must be an intelligent guess, not just an unformed stab at the problem) and then proceeds to test that guess within the conditions of the problem. The process continues until the child arrives at a guess that solves the problem [11]. The following in figure 3 is the answer of students with direct guessing strategies and testing them. A total of 13 students or 32.5% of all students used this strategy.

![Figure 3. Students answer with a guess and check strategy.](image)

Students try to guess each side length to four fields A, B, C, and D by giving information on the side length on the sides of the field. The student's guess is not immediately seen in the students' scribbles. The scribbles indicate that students tried several times to find the right side. On the upper side of field B, the student initially tries with number 16 but is changed to 8. On the top side of field A, the student initially tries number 10 but is crossed out and replaced by number 8. When students try to guess the sides of field A for example, it will result on the lengths of the sides of the fields B, C, and D. The student's answer is correct by writing the area of D in the image that is 20 even though it has not written down the unit area. Then students check the answer by looking at the suitability of the lengths of the sides of the fields in the picture. According to students, the problem is the level of difficulty being. The reason students consider the question to be because to determine the area of D, students must think hard first to determine the length of the sides.

3.2. Strategy with ratio
Interpretation of the right problem can often change problems that seem difficult to be problems that are easily solved. A problem can be changed in the form of an equivalent fraction or in the form of a fraction ratio [15]. One simple way to make a problem more manageable and usually produces good results is to turn the problem into an equivalent problem that might be easier to solve, namely by simplifying the numbers given in the problem. The use of ratio can simplify the numbers so that it is easy to work [11]. In figure 4 show students' answers using a ratio strategy. A total of 20 students or 50% of students uses this strategy.
According to students, this problem is moderate. First, the students write down the area of each known field, namely the area of fields A, B, and C. Then students pay attention to the area of B and field C and have an idea to divide 128 by 32 which produces a ratio of 4:1. Next, the students divide the area of field A by 4 to produce 20, the reason is that the width of field A and field B are the same, so that the width of the plane D and the plane C are the same. The uniqueness of students in answering this question is by writing the completion steps by describing them and also writing the mathematical steps.

3.3. The strategy of analogy with a variable
The letters used instead of numbers are called variables. The use of variable strategies, which is one of the most useful problems solving strategies, is widely used in algebra and mathematics involving algebra [15]. In figure 5 show the students' answers using the analogy strategies with the variables. There is only one student who uses this strategy from a total of 40 students studied.

**Figure 4.** Students’ answer with a ratio strategy.

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(a) **Figure 4**

(b) **Figure 4**
Students initially divide field B into two with the same estimated area of 64 cm$^2$. Then the students took the initiative to look for the roots of 64 so that the upper side of field B was 8 cm which was then able to find the lengths of the sides of field A. Student initiatives are carried out because they see field B which can be divided into two squares. This means that students also use a strategy to guess the answers. Then the students looked at the sides of field C by dividing the area by 16 so that the length of the upper side of field C was divided 2 cm. Here the student specifies the width of the C field with the variable $l$ (students write $l$ with a big "L" letter). Then the students calculate the total area of the field that is 260 cm$^2$. Then, with the addition and subtraction operation, the area of D is 20 cm$^2$. Students find this matter easy. The reason is that when you have obtained the total area of all fields, you only need to reduce it by the area of A, B, and C so that the area of D.

4. Conclusion
The most widely used problem-solving strategy for students is using ratio strategies. Students who use the ratio strategy are 20 students or 50% of the total subjects studied. Another strategy used by students is to guess directly and check it as many as 13 students or 32.5%. There is only one student who uses a variable strategy with variables. In addition to using these three strategies, there were 6 students who did not answer the reason they were still confused in understanding the problem and confused about how to do it. Although this study may be limited in terms of the number of participants, exploration reveals evidence that problem-solving strategies can vary according to
students’ prior knowledge. It is possible that the results of this study can be transferred to other mathematical topics, but further investigation is recommended.

5. References

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