Radiation-mediated Shocks in Gamma-Ray Bursts: Pair Creation

Christoffer Lundman1,2,3, Andrei M. Beloborodov1, and Indrek Vurm1,4

1 Physics Department and Columbia Astrophysics Laboratory, Columbia University, 538 West 120th Street, New York, NY 10027, USA
2 Department of Physics, KTH Royal Institute of Technology, AlbaNova, SE-106 91 Stockholm, Sweden
3 The Oskar Klein Centre for Cosmoparticle Physics, AlbaNova, SE-106 91 Stockholm, Sweden
4 Tartu Observatory, Tõravere 61602, Tartumaa, Estonia

Received 2017 August 8; revised 2018 February 28; accepted 2018 March 24; published 2018 April 26

Abstract

Relativistic sub-photospheric shocks are a possible mechanism for producing prompt gamma-ray burst (GRB) emission. Such shocks are mediated by scattering of radiation. We introduce a time-dependent, special relativistic code which dynamically couples Monte Carlo radiative transfer to the flow hydrodynamics. The code also self-consistently follows electron–positron pair production in photon–photon collisions. We use the code to simulate shocks with properties relevant to GRBs. We focus on plane-parallel solutions, which are accurate deep below the photosphere. The shock generates a power-law photon spectrum through the first-order Fermi mechanism, extending upward from the typical upstream photon energy. Strong (high Mach number) shocks produce rising $\nu F_\nu$ spectra. We observe that in non-relativistic shocks the spectrum extends to $E_{\text{max}} \sim m_e v^2$, where $v$ is the speed difference between the upstream and downstream. In relativistic shocks the spectrum extends to energies $E > 0.1 m_e c^2$ where its slope softens due to Klein–Nishina effects. Shocks with Lorentz factors $\gamma > 1.5$ are prolific producers of electron–positron pairs, yielding hundreds of pairs per proton. The main effect of pairs is to reduce the shock width by a factor of $\sim Z^{-1}$. Most pairs annihilate far downstream of the shock, and the radiation spectrum relaxes to a Wien distribution, reaching equilibrium with the plasma at a temperature determined by the shock jump conditions and the photon number per proton. We discuss the implications of our results for observations of radiation generated by sub-photospheric shocks.

Key words: gamma-ray burst: general – plasmas – radiation mechanisms: non-thermal – radiative transfer – scattering

1. Introduction

Shocks in optically thin plasmas of low density are collisionless, i.e., mediated by collective plasma effects. Such shocks can produce radiation by passing a fraction of the dissipated energy to electrons, which may be capable of radiating the energy away. Shocks in optically thick media are different; they are typically mediated by photons, either pre-existing in the medium or generated by the shock itself. The most prominent examples of radiation-mediated shocks (RMSs) are found in supernovae and gamma-ray bursts (GRBs). They occur at the early stages of the explosion inside the progenitor star or inside optically thick ejecta.

Energy dissipation by sub-photospheric shocks has long been proposed as a mechanism for producing prompt GRB emission (see e.g., Eichler & Levinson 2000; Mészáros & Rees 2000; Pe’er et al. 2006; Ryde & Pe’er 2009; Giannios 2012). Levinson & Bromberg (2008), Bromberg et al. (2011), and Levinson (2012) emphasized that sub-photospheric shocks are in fact mediated by radiation (as opposed to collective plasma effects). Recently, Beloborodov (2017) (hereafter B17) performed time-dependent simulations demonstrating RMS formation in unmagnetized and magnetized flows.

RMSs differ from collisionless shocks in a few important ways. First, the dissipated kinetic energy is directly transferred to radiation through the first-order Fermi mechanism, as opposed to being transferred to photons via electron internal energy. Second, the width of an RMS is at least a few photon mean free paths, generally much larger than the Larmor radius of charged particles; therefore, RMSs are inefficient particle accelerators. Third, relativistic RMSs can heat photons to energies above the electron rest mass, leading to electron–positron pair production inside and around the shock. B17 estimated that $\sim 10^2$ pairs per ion should be created by internal shocks in GRB jets.

The photon spectrum within the RMS depends on the number of photons that share the dissipated kinetic energy. In general, the total number of photons downstream of the shock is the sum of photons advected from the upstream and the new photons generated by the shock itself. Two qualitatively different RMSs can then be identified, depending on the dominant photon source. The upstream is defined as photon-rich if the newly produced photons dominate the shock photon number (Bromberg et al. 2011). This is the case for shocks propagating in a cold stellar envelope. Relativistic photon-poor RMSs were studied by Katz et al. (2010), Budnik et al. (2010), and Nakar & Sari (2012). The upstream is defined as photon-rich if the advection of radiation from the upstream dominates the photon number downstream of the shock. This is typically the case for RMSs inside GRB jets (Levinson 2012), as the jet carries photon-rich plasma from the hot central engine.

Non-relativistic photon-rich RMSs were studied by Weaver (1976), Blandford & Payne (1981), Riffert (1988), and Becker (1988). Levinson & Bromberg (2008) studied the structure of mildly relativistic photon-rich RMSs (assuming that $e^\pm$ pair creation is negligible) by taking moments of the radiative transfer equation and accounting for the radiation anisotropy, which is strong in relativistic RMSs.

The recent work of B17 studied photon-rich RMSs using direct time-dependent simulations that couple the plasma hydrodynamics to Monte Carlo radiative transfer. B17 showed that, in the presence of sufficiently strong magnetic fields, a collisionless “subshock” forms inside the RMS. The subshock width is comparable to the ion Larmor radius, which is much
smaller than the wide RMS structure. A fraction of the total shock energy is dissipated in the subshock, heating the electron (or pair) and proton components. The hot $e^\pm$ quickly cool by inverse Compton scattering and emission of synchrotron photons, until they reach the Compton temperature of the downstream radiation. Additionally, the presence of neutrons can further complicate the shock structure and dissipation profile through nuclear collisions on a length scale longer than the Thomson mean free path (Beloborodov 2010, B17). In the present work, we consider RMSs in the limit of vanishing neutron component and magnetic fields. We will also assume that the photon number is conserved in the RMS. The validity of this approximation depends on the strength of the magnetic field and the shock speed. Magnetized RMSs generating synchrotron emission will be discussed elsewhere (C. Lundman & A. M. Beloborodov 2018, in preparation).

In the present paper we present a newly developed code for radiation hydrodynamics. Our code includes explicit treatment of $e^\pm$ pair production in photon–photon collisions, which for the first time allows for fully self-consistent, time-dependent simulations of $e^\pm$ loaded RMSs. Furthermore, the code utilizes an exact relativistic Riemann solver, which accurately captures any (collisionless) subshocks that might develop (depending on the upstream conditions). We focus on RMSs that occur deep below the photosphere, where the scattering time is much smaller than the jet expansion time. Such shocks quickly settle into a quasi-steady state after a few scattering times. Deep subphotospheric RMSs are also essentially plane parallel. In the simulations, we launch the shocks and let them propagate until they settle into a steady state, and then examine the shock structure.

The paper is organized as follows. The equations of special relativistic Lagrangian radiation hydrodynamics are given in Section 2 (and the Appendix). Our numerical implementations of hydrodynamics and Monte Carlo radiative transfer are presented and discussed in Sections 3 and 4, respectively. We qualitatively discuss GRB RMSs in Section 5, and numerically explore GRB shocks under four qualitatively different conditions in Section 6, confirming the main points of the previous section. Finally, the results are discussed in Section 7.

2. Equations of Radiation Hydrodynamics

Plasma and radiation will be treated using two distinct numerical methods. We first consider the plasma, which will be treated as a single fluid (this approximation is discussed in Section 7).

Conservation of energy and momentum is represented by the vanishing divergence of the stress–energy tensor; $T_{\alpha\beta}^{\alpha\beta} = 0$. Separating the stress–energy tensor into matter (including electron–positron pairs) and radiation parts, $T_{\alpha\beta}^{\alpha\beta} = M_{\alpha\beta}^{\alpha\beta} + R_{\alpha\beta}^{\alpha\beta}$, we can write

$$M_{\alpha\beta}^{\alpha\beta} = G^\alpha^\beta,$$

where $G^\alpha^\beta \equiv -R_{\alpha\beta}^{\alpha\beta}$ is considered as an energy and momentum source term for the fluid equations. The stress–energy tensor of the fluid is $M_{\alpha\beta}^{\alpha\beta} = u^\alpha u^\beta (p + e + p + n\eta^\alpha^\beta)$, where $u^\alpha = \Gamma(1, \beta^\alpha)$ is the four-velocity, $\beta^\alpha$ is the three-velocity, $\Gamma \equiv (1 - \beta^\alpha \beta^\alpha)^{-1/2}$ is the Lorentz factor, $\rho$ is the total rest mass density (ions, electrons, and positrons), $e$ is the internal energy density, $p$ is the pressure, $\eta^\alpha^\beta$ is the Minkowski metric (we consider a flat spacetime) of signature $(-, +, +, +)$, and we use units for which the speed of light equals unity.

Conservation of proton number and the equation for the pair number density are given by

$$\rho_p u^\alpha \partial_\alpha n_\pm = 0$$

and

$$n_\pm = \dot{n}_\pm$$

respectively, where $\rho_p$ is the proton rest mass density, $n_\pm$ is the pair (electron and positron) number density, and $\dot{n}_\pm$ is the net rate of pair production/annihilation, all measured in the local rest frame of the fluid.

In the Appendix, we rewrite Equations (1)–(3) in Lagrangian form and plane-parallel geometry. This gives

$$(V_p)_\dot{x} - \partial_m = 0,$$

$$(\dot{E}_p)_\dot{x} + (n_\beta)_m = V_p G^0,$$

$$(\dot{S}_p)_\dot{x} + \dot{p}_m = V_p G^1$$

and

$$(V_\pm)_\dot{x} = m_p V_p \dot{\pm}.$$  

Here $m$ is the Lagrangian mass coordinate ($\partial_m = (\Gamma \rho_p)^{-1} \partial_\lambda$) and $\partial_\lambda$ is the Lagrangian time derivative ($\partial_\lambda \rightarrow \partial_\lambda - \beta \partial_\lambda$). The new variables $V_p, E_p,$ and $S_p$ are the lab frame volume, energy, and momentum per proton rest mass respectively, defined as $V_p = (\Gamma \rho_p)^{-1}, E_p = V_p (\Gamma^2 (p + e + p) - p)$, and $S_p = V_p \beta \Gamma^2 (p + e + p)$. The pair loading factor is $Z_\pm = n_\pm / n_p$, and $m_p$ is the rest mass of a proton. The right-hand sides of Equations (5) and (6) are the rates of energy and momentum gain per proton, measured in units of the proton rest mass.

3. Hydrodynamics Implementation

The hydrodynamical Equations (4)–(6) are solved numerically using a standard Lagrangian scheme with an exact Riemann solver (e.g., Daigne & Mochkovitch 2000). In short, we discretize the equations (including Equation (7)) into fluid elements of a given proton mass, using finite differences on a mass grid. The time evolution of each fluid element is then integrated in the following way. The piecewise parabolic method (PPM; Colella & Woodward 1984), is used to find “left and right states” of the fluid at each grid interface (and time step). The left and right states are used to solve a Riemann problem in order to find the spatial (mass) derivative approximations.

A more detailed description of the hydrodynamics implementation is provided in the following subsections. The source terms ($G^\alpha$ and $\dot{n}_\pm$) are obtained from the Monte Carlo radiative transfer, as described in Section 4.

3.1. Discretization of the Hydrodynamical Equations

The Lagrangian grid is defined by the value of the mass coordinate at the interfaces between grid cells, and each grid cell represents a “fluid element.” The fluid element mass equals the difference between the mass coordinate values at the cell boundaries: $\Delta m_j = m_{j+1/2} - m_{j-1/2}$, where $j$ labels a cell, $j \pm 1/2$ labels the right and left cell boundaries, and $\Delta m_j$ is the proton mass contained in the cell.
Similarly, the pair-loading equation is discretized as
\[ \frac{d}{dt} \left( \rho \beta \right) = 0, \]
and the updated volume per proton is given by
\[ (V_p)_j^{n+1} = \frac{(x_j^{n+1/2} - x_j^{n-1/2})}{\Delta m_j}, \]
implicitly solving Equation (4). A straightforward discretization of the hydrodynamical equations for energy and momentum (Equations (5) and (6)) gives
\[
\begin{align*}
(E_p)_j^{n+1} &= (E_p)_j^n + (V_p)_j^n (G^0)^n_j \Delta t \\
&- \frac{\Delta t}{\Delta m_j} [\bar{p}^n_j - \bar{p}^{n-1}_j],
\end{align*}
\]
and
\[
\begin{align*}
(S_p)_j^{n+1} &= (S_p)_j^n + (V_p)_j^n (G^1)_j^n \Delta t \\
&- \frac{\Delta t}{\Delta m_j} [\bar{p}^{n-1}_j - \bar{p}^{n-2}_j].
\end{align*}
\]
Similarly, the pair-loading equation is discretized as
\[
(Z_\beta)_j^{n+1} = (Z_\beta)_j^n + m_p (V_p)_j^n (\hat{n}_\alpha)_\beta^n \Delta t.
\]

### 3.2. Variable reconstruction

The variables \( \beta, \rho, \) and \( p \) must be reconstructed numerically from \( V_p, E_p, \) and \( S_p \) in each grid cell and for each time step. Following Daigne & Mochkovitch (2000), we numerically solve the equation
\[
\frac{d}{dt} \left( \frac{\gamma_{\text{ad}}}{\gamma_{\text{ad}} - 1} \right) \hat{h} - \gamma_{\text{ad}} E_m (S_m^2 + \hat{h}^2)^{1/2} + \gamma_{\text{ad}} S_m^2 = 0 \tag{13}
\]
for the specific enthalpy, \( \hat{h} \equiv h / \rho = 1 + (e + p) / \rho, \) where \( \gamma_{\text{ad}} \) is the adiabatic index of the fluid (which is treated as an ideal gas) and \( E_m = E_p / (1 + Z \alpha m_e / m_p) \) is the energy per unit total mass (and similar definitions hold for \( S_m \) and \( V_m \)). We use the Newton–Raphson method to solve Equation (13). After \( \hat{h} \) is found, \( \beta, \rho, \) and \( p \) are computed as \( \beta = S_m (S_m^2 + \hat{h}^2)^{-1/2}, \)
\( p = (\Gamma V_m)^{-1}, \) and \( \rho = \rho (\hat{h} - 1) / \gamma_{\text{ad}} - 1 / \gamma_{\text{ad}}. \)

### 3.3. Finding Left and Right States Using the PPM Method

The PPM method is an extension of Godunov’s method, with the advantage of being second order accurate in time and third order in space. Parabolic (quadratic) polynomials are fit to each of the hydrodynamic grid quantities \( \beta, \rho, \) and \( p \) at a given time step. The parabolic fits provide continuous representations of the hydrodynamic quantities.

Only a fraction of the fluid in each grid cell can affect the conditions at the cell boundary during a time step (assuming that the time step is short enough for the computation to converge). The distance into each cell from which information can reach the cell boundary is found, and the time- and mass-averaged values of \( \beta, \rho, \) and \( p \) are computed using the continuous polynomials at each side of the boundary. The averaged values at each side of the boundary define the left and right fluid states which are needed for solving the Riemann problem at the boundary. We refer to Colella & Woodward (1984) for a detailed discussion of the parabolic fits and the averaging process.

### 3.4. Solving the Riemann Problem

A Riemann solver is designed to numerically compute the pressure and speed of the intermediate region which develops in the interaction between two initially separated fluid states. The intermediate region includes the contact discontinuity, which separates the fluids that were originally contained in the left and right states. Since we are solving Lagrangian equations, the contact discontinuity of the Riemann problem directly corresponds to the boundary between two grid cells. For a more detailed discussion on the Riemann problem see, e.g., Rezzolla & Zanotti (2013).

The Riemann problem admits three qualitatively different solution patterns: two shocks, one shock, and one rarefaction wave, or two rarefaction waves can be launched. We use the exact, special relativistic Riemann solver developed by Rezzolla & Zanotti (2001). It has the advantage of determining the solution pattern directly from the initial conditions. This makes the numerical implementation simpler, as the functional form of the solution is known before attempting numerical convergence. The Riemann solver gives \( \bar{p} \) and \( \bar{\beta} \) at each interface, which are then used to solve Equations (8)–(11), updating \( V_p, E_p, \) and \( S_p, \) and completing the hydrodynamical time step.

### 4. Radiation Implementation

Radiative transfer is performed using the Monte Carlo method. The radiation is described as a large collection of discrete Monte Carlo photons, or photon “packets.” A photon packet is defined by its spatial location \( x, \) angle \( \theta \) relative to the \( x \)-axis (or \( \mu = \cos \theta \)), energy \( E = \epsilon m_e c^2, \) and its statistical weight \( w. \) The packet weight gives the number of real photons (with assumed identical properties) represented by the packet (or, more precisely, photons per unit area for the plane-parallel problem). The weight is initially computed as
\[
\Delta N_w = \Gamma n_w \Delta x, \tag{14}
\]
where \( \Delta N_w \) is the chosen number of Monte Carlo photons within the spatial bin of width \( \Delta x, \) and \( n_w \) is the photon number density, as set by the initial conditions. An array of all photon packets is kept in memory. The photons propagate through the hydrodynamical Lagrangian grid, with different radiative processes contributing to the opacity. Photon packets are added to or removed from the array as soon as they are emitted or absorbed, with the corresponding energy and momentum differences subtracted or added to \( G^m \) at the relevant grid location so that total energy and momentum are conserved.
Below we will refer to photon packets simply as photons for brevity.

In this work we consider only the radiative processes of scattering and $\gamma\gamma$-absorption. However, additional interactions can be added if needed, and no specific number of radiative processes is assumed in the code description below.

### 4.1. Propagation Algorithm

The code picks a time step $\Delta t$ and then updates each photon in the array. The selected photon propagates and interacts with the plasma until it is either absorbed or it has propagated for a time $\Delta t$. The propagation algorithm is logically separated into "events." An event is here defined as either an interaction (scattering or absorption) or the crossing of a grid cell boundary into a neighboring mass bin. A typical propagation step consists of zero to several events, and most events are grid crossings.

Below is a more detailed description of the propagation algorithm. First, the mass bin where the photon is located is found (information regarding the photon location during the previous time step can be used here). The code computes local mean free paths in the photon propagation direction for all relevant radiative processes, using the plasma properties of the current spatial bin. (The computation of the mean free path for $\gamma\gamma$-annihilation is described in the next subsection.) The mean free path $\lambda$ is computed by adding the absorption coefficients for each process (e.g., $\lambda^{-1} = \sum_i \lambda_i^{-1}$, where $i$ labels each process). A lab frame propagation distance $l$ is drawn from the exponential distribution as $l = -\lambda \ln(u)$, where $u$ is a random number uniformly distributed between zero and one. If time $\Delta t$ has passed before an event occurs, the photon is simply propagated for the remaining time. Otherwise, the code moves the photon to the event location, updates the photon propagation time, and performs the event. The event type is determined by whatever happens first, either propagating distance $l$ or crossing a boundary. The crossing simply consists of moving the photon to the current boundary location and updating the current bin location. The boundaries are assumed to move during photon propagation, at a speed equal to the average speed of the neighboring bins. This gives better accuracy in flows with large bulk motion. As soon as an event has occurred, the algorithm computes new mean free paths $\lambda_i$ and draws a new $l$. The timestep is completed for the photon when its propagation time reaches $\Delta t$ or there is an absorption event.

The code must determine which radiative process has occurred at each event. Let $P_i$ be the probability that this was process $i$, $\Sigma P_i = 1$. The probabilities $P_i$ are simply proportional to the rates of the processes, and one can express them in terms of mean free paths $\lambda_i$ as

$$P_i = \frac{\lambda_i^{-1}}{\sum_j \lambda_j^{-1}},$$

which is independent of $l$. The code randomly chooses the process that has occurred in each event according to the probabilities $P_i$.

### 4.2. Scattering

The code uses the full Klein–Nishina cross-section for computing the scattering mean free path (as a function of photon energy and direction) and scattering angles. The gas temperature is determined by the hydrodynamic internal energy of the gas, and the electron (and positron) distribution is assumed to be thermal (the Maxwell–Jüttner distribution). As a test of the code, we have verified that initially non-thermal radiation interacting via scattering with thermal electrons relaxes to kinetic equilibrium: the spectrum assumes a Wien shape (as expected in the absence of photon production and stimulated scattering) with a temperature equal to the electron temperature.

### 4.3. Pair Production and Annihilation

The mean free path to $\gamma\gamma$-annihilation is not a function of the local plasma properties. Instead, it requires knowledge of the local radiation intensity. We define a grid in the photon energy ($E'$) and direction ($\mu'$), both measured in the fluid rest frame (the comoving frame). At each time step and grid cell, the comoving intensity is computed on the two-dimensional ($E'$, $\mu'$) grid by collecting the Monte Carlo photons. The absorption mean free path of a photon with energy $E_0'$ and direction $\mu_0'$ can be considered a function of $E_0'$, $\mu_0'$, and the photon location $x$, $\lambda\gamma_\gamma(x, E_0', \mu_0')$. Computation of $\lambda\gamma_\gamma$ (measured in the fluid comoving frame) is performed by integrating the target photon number intensity, $I_\gamma',\alpha'$, over the energies and directions of the target photons,

$$\lambda\gamma_\gamma^{-1} = \int (1 - \cos \alpha') \sigma\gamma\gamma I_\gamma'/d\Omega'dE',$$  \hspace{1cm} (16)

where $\sigma\gamma\gamma$ is the center-of-momentum frame cross-section and $\alpha'$ is the cosine of the angle between the primary and target photon directions. The mean free path $\lambda\gamma_\gamma(x, E_0', \mu_0')$ is tabulated on the grid of $x, E_0', \mu_0'$ at each time step, before the propagation of photons is initiated.

The comoving mean free path for a photon at any given $x, E_0', \mu_0'$ is found by interpolation in the grid of $x, E_0'$ and $\mu_0'$. The lab frame mean free path is then obtained by a Doppler boost, $\lambda\gamma_\gamma(x, E_0', \mu_0') = \lambda\gamma_\gamma(x, E_0', \mu_0')/\sqrt{1 + \beta(1 - \mu_0')}$.

The pair production source term can be written as $n\pm = n\pm_{\text{prod}} - n\pm_{\text{ann}}$. Each fluid element has a proton mass of $\Delta m$, which corresponds to $\Delta n_{\pm}/n_p$ protons (per unit area), and a Monte Carlo photon packet corresponds to $w$ photons (per unit area). After a photon–photon interaction the photon is absorbed, adding an equal number of pairs (per photon packet) to the plasma,

$$\frac{\delta n_{\pm,\text{prod}}}{n_p} = \frac{m_p w}{\Delta m}.$$

Requiring that a single annihilation event changes $Z_{\pm}$ only slightly, i.e., $\delta Z_{\pm} = \delta n_{\pm,\text{prod}}/n_p \ll 1$, we find a lower limit on the number of photon packets per bin, $\Delta N_{\gamma} \gg n_{\gamma}/n_p$, where Equation (14) was used.

At each time step the code computes the number of real photons emitted due to pair annihilation, equal to the number of annihilated $e^\pm$, $\delta n_{\pm} = n_{\pm,\text{ann}} \Delta t$, where $n_{\pm,\text{ann}} = (3/4)\sigma T n_{\pm}$, and $n_{-}$ ($n_{+}$) is the number density of electrons (positrons).
Using the relations \( n_{\pm} = n_+ + n_- \) and \( n_p = n_+ - n_- \), we have
\[
\frac{n_{\pm, \text{ann}}}{n_p} = \frac{3}{16} \sigma_T T_p (Z_\pm^2 - 1).
\] (18)

### 4.4. Photon Boundary Conditions

The hydrodynamic code is a Lagrangian code, which tracks the motion of individual fluid elements. The hydrodynamic boundary conditions used for the simulations presented in this paper consist of a stationary wall (i.e., a reflective boundary condition) to the left of the grid, and a piston to the right. The piston is moving at a constant speed toward the wall. The photons cannot propagate through the grid boundaries (i.e., the wall and the piston). As a photon reaches a boundary, its momentum is reflected in the comoving frame of the boundary.

### 4.5. Electron Cooling Time and Length Scales

Consider a plasma with a certain number of photons per proton, \( n_\gamma / n_p \), a pair-loading factor of \( Z_\pm \), and a typical photon energy \( \epsilon = E / m_e c^2 \). The scattering time for the photons is
\[
t_{sc} \approx \frac{1}{Z_\pm n_p \sigma_T \epsilon}.
\] (19)

The photon time step must resolve (at least) the scattering time, as this is the characteristic timescale for the RMS.

In this paper we report simulations of weakly magnetized RMSs, and we do not find any significant collisionless subshocks in the RMS. However, we need to make sure that subshocks are handled properly by the code should they appear. A key special feature of a collisionless subshock is the sudden heating of the plasma to a very high temperature. In particular, electrons (and positrons) can be heated to thermal Lorentz factors \( \gamma_e \gg 1 \). The corresponding electron cooling time is \( t_{\text{cool}}^{-1} \approx (4/3) \xi_{\text{KN}} \gamma_e n_\gamma \sigma_T \), where \( \xi_{\text{KN}} \approx (1 + 4 \gamma_e \epsilon)^{-3/2} \) describes the decrease of inverse Compton cooling efficiency due to Klein–Nishina effects (Moderski et al. 2005), so that
\[
t_{\text{cool}} \approx 3 Z_\pm n_\gamma (1 + 4 \gamma_e \epsilon)^{1/2} / \gamma_e \epsilon.
\] (20)

The number of photons per proton is in the range \( 10^4 - 10^6 \) for typical GRB conditions, and the electron cooling time is therefore typically several orders of magnitude shorter than the photon scattering timescale (depending on \( \epsilon \)). The photon time step must resolve the cooling timescale, as photons may otherwise “break the energy budget” by interacting with high-energy electrons for too long, consuming all internal fluid energy before the fluid can react by lowering its temperature. The corresponding length scale, \( l_{\text{cool}} \), must also be spatially resolved to capture accurate electron temperatures behind a subshock.

In the absence of a collisionless subshock, the RMS structure is smooth, with variations on scales \( \gtrsim \lambda \), where \( \lambda \) is the photon mean free path to scattering. The plasma passes through the RMS on a timescale \( \gtrsim t_{\text{sc}} = \lambda / \epsilon \), which is much longer than \( t_{\text{cool}} \). Therefore, the plasma everywhere is very close to the Compton temperature \( T_C \) at which it does not lose or gain energy from radiation. Furthermore, the fact that \( n_\gamma \gg Z_\pm \) implies that the photons everywhere dominate the internal energy budget. The electrons never carry any significant energy; they simply act as scattering targets for the photons, and serve to couple the photons to the protons, allowing the former to extract the bulk kinetic proton energy. Compton scattering also gradually redistributes the photon energies, “thermalizing” the photon spectrum. The relaxation to a Wien spectrum takes, however, a long time \( t \gg t_{\text{sc}} \).

Simply imposing the condition \( T_e = T_C \) is adequate for RMSs without subshocks. However, we wish to perform simulations that allow subshock formation and are capable of explicitly following the energy exchange between plasma and radiation, even when the plasma has a tiny heat capacity and its temperature closely tracks \( T_C \). Resolving energy exchange between components with vastly different heat capacities is computationally challenging, as the electron temperature tends to fluctuate wildly with affordable time-steps. However, a numerical trick can be used to increase the electron cooling time. A “fake heat” reservoir can be added to the plasma by simply multiplying the initial internal energy (of electrons, positrons, and protons) by a constant factor \( f \), and dividing the internal energy by the same factor when computing the fluid temperature as seen by the photons. This trick artificially increases the internal energy budget of the plasma and slows down its energy exchange with radiation (so that it can be resolved) while keeping the correct equilibrium temperature (equal to the Compton temperature). An upper bound on \( f \) is set by the fact that the electron cooling time must still be much shorter than the photon scattering time, so that electrons are still locked to the correct temperature inside the RMS.

The fake heat capacity also allows one to resolve the thermal plasma evolution behind a collisionless subshock should it form. The trick stretches the cooling length of the suddenly heated plasma \( l_{\text{cool}} \) by the factor \( f \) while still keeping \( l_{\text{cool}} \) much smaller than the RMS width. In this case, the fake heat capacity has one side effect: the plasma temperature immediately behind the subshock is artificially reduced by a factor \( f^{-1} \); however, the total energy carried by the plasma (which is quickly radiated) is correct.

### 4.6. Code Parallelization

Monte Carlo codes are easily parallelizable. Our implementation initiates a given number of photon packets on each CPU core. The hydrodynamics computations (which take much less CPU time compared with photon propagation) are performed only on the master core. At each time step, the master distributes the hydrodynamic grid to the worker cores. The workers propagate their photons in the grid and compute all hydrodynamical sources (\( G^{\text{em}} \) and \( n_\gamma \)). The master then collects the sources from all workers for the next hydrodynamics step. Similarly, the comoving radiation intensity (which is used for the pair production algorithm) is computed locally by each worker, and then collected by the master. The computation of the \( \gamma\gamma \)-annihilation mean free path is distributed over all workers, as this computation is fairly expensive. Each run presented below was executed on 12 nodes with 24 CPUs each, with a run time of at most a week (depending mostly on the amounts of pairs produced, which lowers the scattering time and therefore decreases the code time step).

### 5. RMS in GRB Jets

Shocks that occur deep below the GRB jet photosphere have a characteristic width which is much smaller than any
macroscopic flow length scale. Such shocks are therefore essentially plane-parallel and in a quasi-steady state. In this section we discuss the expected properties of steady, plane-parallel RMSs.

Given upstream values of \( w \equiv (1 + \rho, p, \rho) / \rho \) and \( v, \gamma \), where \( v \) is the upstream speed relative to the downstream (in units of the speed of light) and \( \gamma \) is the corresponding Lorentz factor, one can solve for the downstream \( w \) and shock compression ratio (B17). Besides these thermodynamic parameters, the photon spectrum in the shock transition region depends on the photon-to-proton ratio, \( n_\gamma / n_p \), of the upstream material (Levinson 2012).

### 5.1. The Shock Spectrum

The radiation spectrum inside an RMS is non-thermal. The RMS converts the incoming kinetic proton energy to radiation energy; each proton shares its energy with \( n_\gamma / n_p \) photons. The conversion is complete in the immediate downstream, so that the average photon energy must equal (in the limit of a cold upstream, \( w_u \ll 1 \))

\[
\tilde{\epsilon}_d = (\gamma - 1) \frac{m_p n_p}{m_e n_\gamma} 
\approx 1.8 \times 10^{-2} (\gamma - 1) \left( \frac{n_\gamma / n_p}{10^5} \right)^{-1}.
\]  
(21)

Photons gain energy inside the shock by scattering repeatedly within the converging fluid flow (i.e., the first-order Fermi mechanism). In order for photons to gain energy in the shock, so that they can mediate it, the RMS must structure itself so that the “shock \( \gamma \)-parameter” is of order unity,

\[
\left( \frac{\Delta \epsilon}{\epsilon} \right)_{\text{sc}} \sim 1,
\]  
(22)

where \( \Delta \epsilon / \epsilon \) is the fractional energy gain per scattering and \( N_{\text{sc}} \) is the typical number of scatterings for a photon that diffuses through the shock structure. The optical depth of an RMS is \( \tau_\text{sc} \sim \nu^{-1} \), and the number of scatterings is \( N_{\text{sc}} \sim \tau_\text{sc}^2 \). Thus one finds that the typical fractional energy gain per scattering in a non-relativistic RMS is small,

\[
\frac{\Delta \epsilon}{\epsilon} \sim \nu^2.
\]  
(23)

Since the photons greatly outnumber the electrons (and positrons) in a GRB RMS, the electrons carry essentially no heat capacity and are locally locked to the Compton temperature \( \theta_C \equiv kT_C / m_e c^2 \). The Compton temperature close to the immediate downstream is roughly \( \theta_C \sim \tilde{\epsilon}_d \) (within a factor of a few, depending on the spectral shape). Photons of energy \( \epsilon \ll 4\theta_C \) will not only gain energy by scattering in the bulk speed gradient, but also experience thermal Comptonization. The average thermal energy gain per scattering is \( \Delta \epsilon_{\text{th}} / \epsilon \approx 4\theta_C / (\nu^2/2) \left( m_p n_p / (m_e n_\gamma) \right) \), where Equation (21) was used. The thermal energy gain is much smaller than the energy gain due to bulk Comptonization as long as

\[
\frac{n_\gamma}{n_p} \gg \frac{m_p}{m_e}.
\]  
(24)

This condition is easily satisfied for GRBs which typically have \( n_\gamma / n_p \sim 10^5 \). Thus, thermal Comptonization can be neglected inside the shock and the non-thermal spectrum inside the RMS is shaped mainly by bulk Comptonization.

A fraction of photons scatter back toward the upstream, thereby spending longer inside the RMS and gaining more energy. As long as \( \epsilon \ll 1 \), both the relative energy gain \( \Delta \epsilon / \epsilon \), and the scattering cross-section are independent of photon energy. The problem therefore lacks an energy scale, and the photon spectrum at \( \epsilon \ll 1 \) will form a power law which extends upward from the typical upstream photon energy \( \epsilon_u \) (as shown by Blandford & Payne 1981 for non-relativistic shocks).

Mildly relativistic (and faster) RMSs are similar; the shock \( \gamma \)-parameter still has to be of order unity. Similar to non-relativistic shocks, there is a significant chance for a photon that just exited the shock in the downstream to scatter back toward the upstream and continue the energy gain. The resulting spectrum is again a power law.

The power law can be at most flat in \( \nu F_\nu \) if no upper photon energy exists (as found by Blandford & Payne 1981 in the limit \( v \ll 1 \)). A flat power law would imply a logarithmic divergence of radiation energy. However, an upper photon energy naturally exists due to electron recoil (and also pair production at energies above \( m_e c^2 \)). When the photon scatters through an angle \( \theta_{\text{rec}} \), the energy reduction factor due to electron recoil is \( \epsilon / \epsilon = 1/(1 + \epsilon (1 - \cos \theta_{\text{rec}})) \approx 1/(1 + \epsilon) \approx 1 - \epsilon \) (where we substituted the average \( \cos \theta_{\text{rec}} = 0 \) for scattering of photons with \( \epsilon \ll 1 \)). Therefore the typical energy loss due to recoil is

\[
\frac{\Delta \epsilon_{\text{rec}}}{\epsilon} \approx -\epsilon.
\]  
(25)

For non-relativistic shocks, the energy gains and losses (Equations (23) and (25)) balance when the photon reaches energy

\[
\epsilon_{\text{max}} \sim \nu^2.
\]  
(26)

We see that \( \epsilon_{\text{max}} \sim 1 \) for mildly relativistic shocks which have \( v \gamma \sim 1 \). Pair production is therefore expected to become relevant for shocks with \( v \gamma \gtrsim 1 \).

The RMS velocity profile must self-regulate into a shape that produces a photon spectrum with the average photon energy of \( \tilde{\epsilon}_d \) in the immediate downstream. The accurate shape of the non-thermal radiation spectrum inside an RMS is challenging to predict for mildly relativistic (and faster) shocks without resorting to numerical simulations, because of the coupled dynamics of the system and the non-trivial radiative transfer at energies close to and above the electron rest mass. The decrease of the Klein–Nishina cross-section with \( \epsilon \) results in a longer mean free path, so that photons can more easily propagate across the full width of the shock in a single free path. On the other hand, the longer mean free path can take the photons far downstream where they may become trapped after losing energy in scattering (which increases their scattering cross-section). Furthermore, high-energy photons scatter preferentially along their own forward direction, so that it becomes unlikely for the photon to turn around and scatter back toward the shock. If the photon manages to scatter through a large angle so that it may catch up with the shock, it will lose a significant fraction of its energy to electron recoil, and thus have its mean free path become shorter, decreasing the probability to reach the shock. The Klein–Nishina corrections are related to electron recoil in scattering and become noticeable already at \( \epsilon \gtrsim 0.1 \). Therefore, in a relativistic...
The non-thermal radiation exiting the RMS into the downstream tends to “thermalize” (or, rather, approach kinetic equilibrium with the electrons) by re-distributing their energy through scatterings. The thermal Compton y-parameter is

$$y_{th} = 4\theta_c N_{sc} \approx \bar{\varepsilon}_d N_{sc}$$

where $N_{sc}$ is the number of scatterings.

5.3. The Upstream Photon Precursor

A fraction of the non-thermal radiation generated inside the RMS will leak ahead of the shock into the upstream, and preheat the upstream plasma through scatterings. Scattered photons isotropize and propagate with the upstream plasma, so that even the isotropic component of the photon spectrum inside an upstream fluid element becomes increasingly non-thermal as it approaches the shock. The strength of the photon precursor naturally weakens with distance into the upstream as it is attenuated by scattering. Photons with energies $\epsilon \gtrsim 10^{-3}$ have longer mean free paths due to the energy dependence of the Klein–Nishina cross-section, and therefore propagate farther than low-energy photons, hardening the upstream spectrum with distance from the shock. Neglecting the fact that the scattering cross-section is energy dependent, the intensity of the photon precursor is proportional to $\exp(-\tau)$ where $\tau$ is the total Thomson optical depth (including pairs) as measured from the shock into the upstream.

If the shock contains photons of energies greater than the electron rest mass, then the precursor will also sprinkle pairs into the upstream, ahead of the shock. The high-energy photons close to the shock can easily collide and convert to pairs. At larger distances into the upstream the photon precursor quickly becomes collimated in the forward direction, so that photons of energy $\epsilon \gtrsim 1$ cannot efficiently pair produce on each other. On the other hand, a collimated hard photon can easily produce a pair when it changes its angle through scattering. Therefore, the rate of pair production in the far upstream is tied to the scattering rate.

We can estimate the pair-loading factor $Z_\perp$ as a function of distance into the upstream in the following way. Consider a steady-state shock. The pair-loading equation (Equation (7)) can be written as $(Z_\perp)_{\perp} = n_{\perp}/n_{\perp}$. The equation of motion for a fluid element that is advected from the upstream toward the shock is $x_{\perp} = -v_{\perp} t$, where the $x$ coordinate is measured from the shock toward the upstream (in the shock frame) and $v_{\perp} \gg 0$ is the upstream speed relative to the shock. The pair-loading equation for a fluid element then becomes

$$Z_{\perp} = -\frac{n_{\perp}}{\gamma n_{\perp} v_{\perp}}.$$

If we assume that all scattered high-energy photons are immediately converted to pairs, and that pair annihilation is negligible, then $\dot{n}_{\perp} \sim 2(\sigma_T / 5) Z_{\perp} n_{\perp} n_{\perp}$ (with two pairs created for each scattering, and the scattering mean free path approximately five times the Thomson mean free path). Here $n_{\perp} \propto \exp(-\tau)$ is the number density of high-energy photons in the precursor, and $\tau \approx \gamma_0 (1 + v_{\perp}) \int Z_{\perp} n_{\perp} \sigma_T dx$ is the optical depth into the upstream as measured from the shock. Changing the variable from $x$ to $\tau$, we find $(Z_{\perp})_{\perp} \sim \exp(-\tau)$, and integrating this equation from far in the upstream toward the shock, we find $Z_{\perp} \sim \exp(-\tau)$ (for $Z_{\perp} > 1$). This agrees well with the simulation results shown in the next section.

5.4. Downstream Spectrum “Thermalization”

The non-thermal radiation exiting the RMS into the downstream tends to “thermalize” (or, rather, approach kinetic equilibrium with the electrons) by re-distributing their energy through scatterings. The thermal Compton y-parameter is

$$y_{th} = 4\theta_c N_{sc} \sim \bar{\varepsilon}_d N_{sc}$$

where $N_{sc}$ is the number of scatterings.
Low-energy photons can significantly increase their energy when \( \gamma_{th} \gtrsim 1 \), or \( N_{sc} \gtrsim 1/\tilde{\epsilon}_d \). The number of scatterings \( \delta N_{sc} \) performed in time \( \delta t \) is \( \delta N_{sc} \approx \delta t/\tau_{sc} \), where \( \tau_{sc} = \lambda_{sc} \) is the scattering time, \( \delta t \approx \delta t/\nu_{th} \approx 3\delta t \) is the distance behind the shock, and \( \nu_{th} \approx 1/3 \) is the shock speed in the downstream frame of a relativistic shock. The number of scatterings is then related to the downstream optical depth, \( \delta \tau = \delta \lambda/\lambda_{sc} \), as measured from the shock into the downstream: \( \delta N_{sc} \approx 3\delta \tau \). Integrating the number of scatterings over the distance behind the shock, we find the characteristic optical depth that brings low-energy photons to the Wien peak at \( \epsilon \sim \tilde{\epsilon}_d \).

\[
\tau_{th} \sim \frac{1}{3\tilde{\epsilon}_d}.
\]

The process of thermal Comptonization shapes the low-energy slope of the Wien spectrum. If \( \tilde{\epsilon}_d \ll 1 \), this process is completed far downstream of the shock.

On the other hand, the high-energy spectrum at \( \epsilon > \tilde{\epsilon}_d \) is affected more quickly by downscattering, as \( N_{sc} \approx (\Delta \epsilon/\epsilon)^{-1} \sim 1/\epsilon \) scatterings are needed to reduce the photon energy. Thus, the spectrum at the highest energies is expected to progressively soften into the downstream. This high-energy spectrum evolution starts immediately behind the shock at \( \epsilon \ll 0.1 \) and is completed at an optical depth comparable to \( \tau_{th} \), given by Equation (28). This downscattering process shapes the exponential tail of the Wien spectrum.

The pairs created by high-energy photons inside the shock annihilate as they are advected into the downstream. In a steady-state the pair multiplicity satisfies \( \nu_{th}(Z_{\pm}) = \tilde{n}_\pm/\gamma n_p \) in the shock frame, where \( \nu \) is the distance as measured from the shock into the downstream. Pair production quickly ceases in the downstream, so that only pair annihilation is important; \( \tilde{n}_\pm \approx -n_{ann} \). From Equation (18) we find (for \( Z_{\pm} \gg 1 \))

\[
\delta Z_{\pm} \approx -\frac{Z_{\pm}^2 \sigma_T n_p}{5v_{th} \tilde{\epsilon}_d} \delta x.
\]

The number of scatterings experienced by a downstream photon in time \( \delta t \) is \( \delta N_{sc} \approx \delta t/\gamma \tilde{n}_{sc} = \delta x/\nu_{th} \gamma \lambda_{sc} \), where \( \lambda_{sc} = 1/Z_{\pm} \sigma_T n_p \). We then find \( \delta \ln Z_{\pm} \approx -\frac{1}{\gamma} \delta N_{sc} \), which gives (using \( N_{sc} \approx 3\tau \) and still assuming \( Z_{\pm} \gg 1 \)) \( Z_{\pm} \) as a function of optical depth behind the shock,

\[
Z_{\pm}(\tau) \approx Z_{\max} \exp(-0.6\tau).
\]

A typical photon which has passed through a shock with \( Z_{\max} \sim 200 \) will have scattered in the downstream \( N_{sc} \sim 25 \) times before the pairs are annihilated.

### 5.5. Parameter Space for RMSs in GRB Jets

The total lab frame energy per proton rest mass in a GRB jet is \( \Gamma(1+w) \), where \( \Gamma \gg 1 \) is the bulk Lorentz factor. The energy associated with radiation is \( \Gamma\mathcal{W} \), and the fraction of the total energy carried by radiation (i.e., the radiative efficiency) is

\[
\frac{L_{\gamma}}{L} = \frac{w}{1+w}.
\]

In the downstream, we have \( w_d = (e_d + p_d)/p_d = (4/3)\tilde{\epsilon}_d(m_\gamma n/m_p n_p) \). Using Equation (21) and assuming the upstream to be cold, \( w_u \ll \gamma - 1 \), one finds

\[
w_d = \frac{4}{3}(\gamma - 1).
\]

Only sufficiently relativistic shocks, \( \gamma_* \gtrsim 0.4 \), are capable of generating significant \( L_{\gamma}/L \gtrsim 0.1 \). We can then conclude that pair production is expected to occur in the RMSs that produce the most efficient GRB emission. The parameter space relevant to internal GRB shocks is shown in Figure 1. Shocks with \( \gamma_* \gtrsim 1 \) (or very large average photon energies) are expected to produce pairs. Shocks capable of producing the observed GRB emission are expected to populate the approximate region of \( 1/3 \lesssim \gamma_* \lesssim 3 \) and \( 10^4 \lesssim n_p/n_p \lesssim 10^5 \), which would result in reasonable observed average photon energies, \( E \sim \Gamma\tilde{\epsilon}_d m_\gamma c^2/(1+z) \), where \( z \) is the GRB redshift.

### 6. Numerical Simulations

In this section we present results from four simulations. We consider two faster shocks with \( \gamma_* = 3 \) and \( n_p/n_p = 2 \times 10^3 \), and two slower shocks with \( \gamma_* = 1 \) and \( n_p/n_p = 10^3 \) (these sets of parameters are marked with two red dots in Figure 1). The faster shocks have \( \gamma_* \gtrsim 1 \) and are therefore expected to produce large amounts of pairs, while the slower shocks should be close to the pair production boundary. The shocks run into an upstream which is either cold with \( w_u = 3 \times 10^{-2} \), or warm with \( w_u = 0.3 \). Warmer upstreams (\( w_u \lesssim 1 \)) are expected if the upstream material was recently heated, while fluid elements that were heated many expansion times ago will be colder (\( w_u \ll 1 \)).

We consider homogeneous initial conditions across the whole grid for all runs. The simulation starts with constant values of the hydrodynamical parameters \( \gamma_* \), \( \rho \), and \( p \). Photons are injected across the whole grid with a (comoving) Wien spectrum, so that they are initially in local kinetic equilibrium with the electrons.

Below we present the steady-state shock structure (as seen in the downstream frame) and photon spectra at different locations inside and around the shock. The structure is plotted versus the
original “proton” optical depth (before pair creation), defined as
\[ \tau_p(x) \equiv \int_0^x \gamma n_p \sigma_T dx', \]  
(33)

or the total optical depth that includes pairs,
\[ \tau_\pm(x) \equiv \int_0^x Z_\pm \gamma n_p \sigma_T dx'. \]  
(34)

These definitions correspond to the original electron or pair columns measured along the direction of the shock propagation. The actual Thomson optical depth as seen by a photon also depends on the photon direction and the speed of the fluid elements.

6.1. Initial Shock Evolution

All runs follow qualitatively similar dynamical evolutions before settling into a steady state. The initial conditions are set up so that the flow, which is initially moving to the left, immediately smashes into a lab frame wall (reflecting boundary) at the left end of the grid \((x = 0)\). A hydrodynamical shock is formed at the left boundary, propagating in the rightward direction, while the downstream fluid becomes stationary in the lab frame \((v = 0)\). The downstream region between the shock and the wall initially has a very small optical depth, so that photons are incapable of carrying the downstream pressure that is demanded by the shock jump conditions. The shock is therefore collisionless, and the downstream electron temperature is relativistic. Due to the large number of photons per electron, the electrons are quickly cooled in the downstream by a small fraction of the photons in the vicinity of the shock. The few photons that interact with the hot electrons quickly gain high energies. After a short time, the number of photons with energies \(\epsilon \gtrsim 1\) is large enough so that their free paths to \(\gamma\gamma\) collisions become smaller than the size of the downstream, triggering efficient pair production. The increase in the downstream optical depth causes more photons to scatter on the hot electrons, quickly cooling them and producing more pairs. Later the downstream plasma near the wall becomes sufficiently optically thick to reach the equilibrium state predicted by the shock jump conditions, with the Compton equilibrium between the electrons and photons.

A fraction of the photons with \(\epsilon \gtrsim 1\) leak ahead of the collisionless shock, sprinkling pairs into the upstream. As the upstream pair column becomes significant, photons can effectively “grip” the incoming upstream flow, and start gaining energy also by scattering back and forth across the collisionless shock. The rapid increase in photon pressure at the shock smears out the shock jump on a scale comparable to several photon mean free paths, smoothing out the collisionless shock and establishing proper radiation mediation. At this time, the shock has traversed a distance that corresponds to \(\tau_p\) significantly less than unity. The shock settles into a steady-state after propagating through \(\tau_p\) significantly larger than unity.

6.2. Faster Shock Into Cold Upstream

The fast shock simulation has parameters \(\nu \gamma = 3, n_e/n_p = 2 \times 10^5\) and a cold upstream with \(w_u = 3 \times 10^{-2}\). These values correspond to an average upstream photon energy of \(\bar{\epsilon}_\gamma \sim 6.5 \times 10^{-4}\), as measured in the downstream frame. (The corresponding \(\nu F_\nu\) peak of the Wien spectrum is \(\epsilon_{pk} \approx 4\bar{\epsilon}_\gamma\).) The speed difference between the upstream and downstream is large enough for pair production to become important, and the cold upstream ensures that essentially all downstream photon energy comes from the upstream proton kinetic energy.

The steady-state shock structure is shown in Figure 2 as a function of the original optical depth \(\tau_p\). A photon precursor is leaking into the upstream, pre-heating the electrons and sprinkling pairs ahead of the shock. The photon pressure gradient increases toward the shock, decelerating the incoming upstream flow. The pair multiplicity peaks immediately behind the shock due to pair production and annihilation balance, with \(Z_\pm \approx 225\) as its largest value. The large value of \(Z_\pm\) decreases the photon mean free path by about the same factor, causing the shock transition to occur on a very short length scale.

The detailed shock structure is more clearly visible in Figure 3, which shows the same shock profile as a function of the total optical depth \(\tau_\pm\). As expected, the shock transition region is smeared out over \(\tau_\pm \sim\) a few. The photon precursor and the pair multiplicity are decreasing roughly exponentially toward the upstream, ahead of the shock. The shock structure, when viewed as a function of the total optical depth, is similar to the structure of shocks with \(\nu \gamma \lesssim 1\), which do not produce pairs.

Photon number spectra at different locations within the shock structure are shown in Figure 4, as measured in the lab (i.e., downstream) frame. The spectra are collected at locations separated by an optical depth of unity, as indicated by the colored bars in Figure 3. The upstream spectra (red and dark red) have the Wien shape at energies of \(\epsilon \sim 2 \times 10^{-4}\), and a precursor of high-energy photons emitted by the shock. The photon number spectrum inside the shock is roughly flat at high energies (i.e., approximately constant photon number per logarithmic interval of energy), and the photon spectrum above \(1/\epsilon_{\max}\) is affected by \(\gamma\gamma\)-absorption. The spectrum at the base of the shock transition (yellow) is essentially a power law extending from \(\epsilon_{\nu\nu}\) which starts softening around \(\epsilon \gtrsim 10^{-1}\). A significant fraction \((\sim 10^{-2})\) of the photons inside the shock.
have energies above $\epsilon = 1$, giving rise to strong pair production. The downstream spectra (green to blue) show the gradual process of “thermalization” toward a Wien spectrum. The spectrum evolves more quickly at high energies, because of the fast energy transfer to electrons through recoil in scattering.

Figure 5 shows the same spectra as Figure 4, but zoomed in around the spectral peak. Downstream of the shocks the peak is shifting toward lower photon energies because of efficient recoil losses. At the same time, soft photons with energies well below the peak are gradually upscattered through thermal Comptonization.

6.3. Faster Shock into Warm Upstream

This simulation has the same parameters as the previous simulation ($v_\gamma = 3$, $n_\gamma/n_p = 2 \times 10^5$), but the upstream is warmer with $w_u = 0.3$, corresponding to $\tau_u \approx 6.5 \times 10^{-3}$ as measured in the downstream frame. This implies a smaller energy amplification factor of photons crossing the shock. The shock structure as a function of $\tau_\pm$ is very similar to the previous simulation. However, the higher $u_\parallel$ leads to a somewhat different radiation spectrum.

Figure 6 shows the same spectra as Figure 4, but now showing total energy density per logarithmic interval in energy (i.e., $\nu F_{\nu}$) and zoomed in around the spectral peak. The $\nu F_{\nu}$ shock spectra are shown in Figure 6, as a function of location within the shock. The colors correspond to the same locations as for the fast shock into the cold upstream. The smaller amplification factor of photon energies leads to the softer spectrum inside the shock (yellow curves). This reduces the number of photons with $\epsilon > 1$ and thus decreases pair loading to $Z_\pm \approx 100$.

6.4. Slower Shock into Cold Upstream

The slower shock has an upstream speed corresponding to $v_\gamma = 1$, which is right on the expected boundary for pair production. We consider $n_\gamma/n_p = 10^6$ and the cold upstream has $w_u = 3 \times 10^{-2}$, corresponding to $\tau_u \approx 3.0 \times 10^{-5}$. The shock structure is shown in Figure 7. Almost no pairs are created, $Z_\pm \approx 1$, although tiny “bumps” can be seen in the red
Z\(_{\pm}\) line, indicating that these shock parameters are just below the threshold for copious pair creation.

Figure 8 shows the \(\nu F_\nu\) spectrum at different locations within the shock. The upstream photon energy is very small, and the spectrum at the shock base (light blue) is a perfect power law for several decades in energy, extending up to \(\epsilon \sim 10^{-1}\). The precursor hardens slightly toward the upstream due to the increased mean free path for higher energy photons. The fraction of photons inside the shock with energy \(\epsilon \gtrsim 1\) is less than \(10^{-6}\), which is marginal for creating a noticeable number of pairs.

6.5. Slower Shock into Warm Upstream

Here we used the same parameters for the upstream speed and photon number as for the previous simulation (\(\nu \gamma = 1\) and \(n_\gamma/n_p = 10^6\)), but the upstream is warmer with \(w_u = 0.3\), corresponding to \(\tau_u \approx 3.0 \times 10^{-4}\). The hydrodynamic shock structure is the same as in the cold simulation. The \(\nu F_\nu\) spectrum is shown in Figure 9. Just as for the faster shocks, a warmer upstream leads to a softer power-law spectrum, since the shock must arrange itself to give the photons a smaller energy amplification factor. The fraction of photons with energy \(\epsilon \gtrsim 1\) in the shock is well below \(10^{-6}\), so that \(Z\pm = 1\) throughout the shock.

7. Discussion

7.1. Summary of the Main Results

In this paper we have presented a time-dependent, special relativistic radiation hydrodynamics code. The code is designed specifically for simulating RMSs, and incorporates full Klein–Nishina scattering and \(\gamma\gamma\)-pair production. We have used our code to calculate the fully self-consistent RMS structure in media where upstream photon advection is the main photon source; this is the case for RMSs in weakly magnetized jets. Photon production by sub-photospheric shocks in significantly magnetized outflows will be discussed elsewhere (C. Lundman & A. M. Beloborodov 2018, in preparation).

We simulated RMSs of various speeds and upstream conditions. The shocks were allowed to propagate until they settled into steady states, after which the shock structure was examined. In particular, the photon spectra and the pair-to-proton ratio \(Z\pm\) were analyzed as a function of location within the shock transition.

The main results may be summarized as follows.

1. Non-relativistic RMSs energize photons via the first-order Fermi process, which produces power-law photon spectra inside the shock transition region extending up to \(\epsilon_{\text{max}} = E_{\text{max}}/m_e c^2 \sim \nu^2/\gamma^2\), where \(\nu\) is the upstream speed relative to the downstream. At energy \(E_{\text{max}}\) the photon energy gain from the Fermi process is balanced by the energy loss due to electron scattering recoil.

2. GRB jets contain large numbers of photons, and the main RMS photon source is upstream advection. The average downstream photon energy \(\bar{\nu}_d\) is therefore known from the shock jump conditions and conservation of photon number. The power-law spectrum extends from the average upstream photon energy up to \(\epsilon_{\text{max}}\), and the power-law index has the value that gives the predicted \(\bar{\nu}_d\).
3. Photon energy gain by scattering on the thermal electrons (and positrons) inside the RMS can be neglected as long as \(n_e/n_p \gg m_p/m_e\), which is easily satisfied for GRBs.
4. Mildly relativistic shocks with \((\gamma/c) > 1\) have \(\epsilon_{\text{max}} > 1\) and produce pairs via photon–photon annihilation.
5. The power-law spectrum inside the RMS starts to curve downward at \(\epsilon \approx 10^{-1}\) due to Klein–Nishina effects. Photon–photon annihilation affects the spectrum strongly at \(\epsilon > 1\).
6. Pair production and annihilation is balanced inside the shock. This implies \(\lambda_{\gamma} \approx \lambda_{\text{sc}}\), where \(\lambda_{\gamma}\) is the photon scattering mean free path and \(\lambda_{\text{sc}}\) is the mean free path to photon–photon absorption at energies \(\epsilon > 1\).
7. GRB RMSs can generate huge amounts of pairs due to their large photon-to-proton ratios; we found \(Z\gamma > 200\) for a shock with \((\gamma/c) = 3\) and \(n_e/n_p = 2 \times 10^3\).
8. The spatial shock width is modified by a factor \(Z\gamma^{-1}\).
9. The pairs annihilate at an optical depth of \(\tau \sim \ln Z\gamma\) downstream of the RMS.
10. The photons reach kinetic equilibrium (i.e., a Wien spectrum) at an optical depth \(\tau \sim 1/3\tau_{\text{sc}}\) downstream of the RMS.

7.2. The “Single Plasma” Assumption

Our implementation of the hydrodynamics assumes that the plasma behaves as a single fluid, so that a single speed and temperature can be defined for each fluid element. In an RMS, the photons interact with electrons (or positrons), and the electrons are subsequently coupled to the protons. The coupling (i.e., isotropization of the electrons) is maintained on length scales of order the plasma skin depth, which is always much shorter than the proton mean free path. Coulomb collisions quickly establish an equilibrium Maxwellian distribution of electrons and positrons at a common temperature \(T_e\). In the case of \(\theta = kT_e/m_e c^2 \ll 1\), which is relevant for the shocks considered here, the e^\pm Coulomb relaxation timescale \(t_{\pm}\) may be written as (e.g., Stepney 1983),

\[
t_{\pm} \approx \frac{2\pi^{1/2}}{\ln \Lambda} \theta^{3/2} \approx 0.17 \theta^{3/2} \ll 1, \tag{35}
\]

where \(\ln \Lambda \approx 20\) is the Coulomb logarithm, and \(t_{\pm} = \lambda_{\text{sc}}/c = (Z\gamma n_p \sigma_{\text{TC}})^{-1}\) is the mean free path time of photons, which is comparable to the light crossing time of a relativistic RMS. The time for electron–electron relaxation is twice that for electron–positron relaxation. The times and length scales in an RMS (in the absence of a subshock) are set by \(t_{\text{sc}}\) and Equation (35) implies that electrons (and pairs) maintain a local Maxwellian distribution.

The timescale for electron–proton relaxation is longer,

\[
t_{\text{ep}} \approx \frac{Z\gamma m_p}{2\sqrt{\pi m_e \ln \Lambda}} \left( \frac{m_e}{m_p} \theta_e \right)^{3/2} \approx 120 Z\gamma \theta_e^{3/2}, \tag{36}
\]

so that electrons may not have time to exchange energy with the protons throughout the shock, depending on the shock parameters. On the other hand, the heat capacity of the protons is extremely small compared with that of radiation (due to the huge number of photons per proton), and the exact details of their internal energy are unimportant for the RMS problem. We therefore conclude that the “single plasma” assumption is valid for RMSs which propagate into unmagnetized, photon-rich upstreams.

7.3. Neutrons

GRB jets can have a significant neutron component (Derishev et al. 1999; Beloborodov 2003). Neutron-mediated shock waves were discussed by B17. The cross-section for nuclear collisions is smaller than the Thomson cross-section, \(\sigma_{\text{sc}}/\sigma_{\text{TC}} \approx 1/20\), and the neutron mean free path \(\lambda_n\) is related to the photon mean free path \(\lambda_{\text{sc}}\) by

\[
\lambda_n/\lambda_{\text{sc}} \approx 20 Z\gamma/(1 + Z_n), \tag{37}
\]

where \(Z_n \equiv n_n/n_p\) is the ratio of neutrons to protons in the flow. The neutron mean free path is larger than that of the photon (unless the flow is extremely neutron rich with \(Z_n > 20 Z\gamma\)). Therefore, the RMS can exist as a subshock inside a broader neutron-mediated shock. If the neutron component is small, \(Z_n \ll 1\), then the neutron transport creates a weak precursor to the RMS, and energy is dissipated mainly in the RMS.

In this work we considered a neutron-poor plasma (\(Z_n \ll 1\)). In principle, neutrons could be simulated as Monte Carlo particles along with the photons, although additional numerical challenges are introduced. Mildly relativistic neutron–proton collisions generate pions, which quickly decay into relativistic \(\gamma_{\ell} \approx m_e/m_p \sim 300\) electron–positron pairs (Derishev et al. 1999). The relativistic pairs subsequently launch a pair cascade (Beloborodov 2010; Vurm et al. 2011). The assumption of thermal electrons is not valid in this case.

7.4. Observations of RMS Spectra

In this work, we have presented radiation spectra in a steady RMS. The spectra are measured at different locations in the shock front, in a fixed frame where the downstream is at rest. Note, however, that RMSs can attain a steady state only as long as the local optical depth is large (or, equivalently, the scattering time is smaller than the jet expansion time). There is a qualitative difference between “deep” and “shallow” shocks. Deep shocks dissipate most of their energy at \(\tau > 1\), while shallow shocks dissipate most of their energy at about \(\tau \approx 10\).

Deep shocks are effectively planar while shallow shocks require simulations in the full spherical geometry. Note also that most of the e^\pm pairs produced by deep shocks have enough time to annihilate. Shocked fluid elements continue to expand (and perhaps will be shocked again) as they approach the photosphere. The shock-generated radiation continues to scatter until it reaches the photosphere and begins to stream freely. Scattering tends to “thermalize” the photon spectrum, and the combination of scattering and expansion leads to adiabatic energy losses. Thus the escaping spectrum from a deep shock is expected to appear like a partially thermalized RMS spectrum which has suffered adiabatic energy losses. A Wien spectrum will be formed if the shock occurred well inside the Wien zone (Beloborodov 2013), where the thermal Compton y-parameter is large, \(y \sim \tau_{\text{sc}} > 1\). Furthermore, the observed spectrum is necessarily integrated over the shock downstream due to the short time variability of the flow (Levinson 2012). The observed radiation can also be composed of several unresolved shock episodes (Keren & Levinson 2014).
Shallow shocks can be significantly different and will be studied in a separate paper (C. Lundman & A. M. Beloborodov 2018, in preparation). The planar approximation is expected to break down when the local scattering time becomes comparable to the expansion time (roughly at $\tau \lesssim 10$). A non-planar geometry causes the local comoving radiation intensity to become beamed along the local flow direction (Beloborodov 2011). The long scattering time makes photons less efficient in mediating the shock, and the flow is expected to try to develop a collisionless subshock as the shock “breaks out” of the photosphere. B17 pointed out that the shock will “dress” itself in pairs, maintaining a significant optical depth even far outside the nominal photosphere of the GRB jet. Non-planar, time-dependent numerical simulations are needed to fully assess the details of GRB shock breakout.

The authors would like to thank Hirotaka Ito for useful discussions. C.L. acknowledges the Swedish Research Council for financial support. A.M.B. is supported by NSF grant AST-1412485, NASA grant NNX15AE26G, and a grant from the Simons Foundation (#446228, Andrei Beloborodov). I.V. acknowledges support from the Estonian Research Council grant PUT1112.

Appendix

**Lagrangian Hydrodynamic Equations**

We here specialize Equations (1)–(3) to planar, one-dimensional flows. We denote the spatial coordinate as $x$. The equations for the conservation of proton number (Equation (2)), energy, and momentum (Equation (1)) then become

\[ (\Gamma_p \rho)_x + (\Gamma \beta \rho)_x = 0, \]
\[ (\Gamma^2h - p)_x + (\Gamma^2h \beta)_x = G^0, \]

and

\[ (\Gamma^2h \beta)_x + (\Gamma^2h \beta^2 + p)_x = G^1, \]

respectively, where $h \equiv \rho + e + p$.

We now define Lagrangian coordinates, for which the partial time derivative is taken for a given fluid element as opposed to at a fixed spatial coordinate: $\partial / \partial t \rightarrow \partial / \partial \tau - \beta \partial / \partial x$ and $\partial / \partial x \rightarrow \partial / \partial x$. The spatial coordinate is then replaced by the proton mass coordinate $m$, defined as

\[ m \equiv \int_{\alpha \tau}^{x} \Gamma_p \rho_p \, dx', \]

so that $\partial / \partial x \rightarrow \Gamma_p \partial / \partial m$. Rewriting Equations (38)–(40) in terms of the new coordinates gives

\[ (\Gamma_p \rho)_t + (\Gamma_p \beta)_t = 0, \]
\[ (\Gamma^2h - p)_t + \Gamma_p \beta \rho_m + \Gamma^2h \beta_m = G^0, \]

and

\[ (\Gamma^2h \beta)_t + \Gamma_p [\beta \rho_m + \Gamma^2h \beta_m] = G^1. \]

Finally, we introduce the lab frame volume, energy, and momentum per proton rest mass $V_p$, $E_p$, and $S_p$ as new variables,

\[ V_p \equiv \frac{1}{\Gamma \rho_p}, \]

\[ E_p \equiv \frac{\Gamma^2 h - p}{\Gamma \rho_p}, \]

\[ S_p \equiv \frac{\Gamma^2 \beta h}{\Gamma \rho_p}. \]

Rewriting Equations (42)–(44), we obtain the one-dimensional, planar equations of special relativistic Lagrangian hydrodynamics with energy and momentum source terms,

\[ (V_p)_t - \beta \rho_m = 0, \]
\[ (E_p)_t + (\beta \rho)_m = V_p G^0 \]

and

\[ (S_p)_t + p_m = V_p G^1. \]

The Lagrangian equation for the pair loading is found by noting that $Z_{\perp} \equiv n_{\perp} / n_p$ and $(n_{\perp} u^\perp)_\alpha = 0$, so that $(n_{\perp} u^\perp)_\alpha = n_p u^\alpha (Z_{\perp})_\alpha = n_{\perp}$. Changing to Lagrangian coordinates, $\partial_t \rightarrow \partial_t - \beta \partial_x$, we find the pair-loading equation,

\[ (Z_{\perp})_t = m_p V_p R_{\perp}. \]

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