Tunneling for Dirac Fermions in Constant Magnetic Field

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Abstract

The tunneling effect of two-dimensional Dirac fermions in a constant magnetic field is studied. This can be done by using the continuity equation at some points to determine the corresponding reflection and transmission coefficients. For this, we consider a system made of graphene as superposition of two different regions where the second is characterized by an energy gap $t'$. In fact, we treat concrete systems to practically give two illustrations: barrier and diode. For each case, we discuss the transmission in terms of the ratio of the energy conservation and $t'$. Moreover, we analyze the resonant tunneling by introducing a scalar Lorentz potential where it is shown that a total transmission is possible.

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1 Introduction

After the experimental realization of graphene in 2005, this kind of system became an attractive subject not only for experimentalists but also for theoretician physicists. This is because of the nature of its structures and the behavior of its relativistic particles. In addition of exhibiting the anomalous quantum Hall effect [1, 2], graphene gives an example of condensed matter physics where the quantum electrodynamics tools can be applied [3]. These new developments offered a laboratory for many investigations where interesting results are obtained by solving different problems. Because of the relativistic nature of their fermions, the system made of graphene renewed the interest of studying the Dirac fermions in two-dimensions.

One of characteristics of Dirac fermions in graphene is their ability to tunnel through a potential barrier with probability one [4, 5]. This so called Klein tunneling of chiral particles has long ago been proposed in the framework of quantum electrodynamics [6, 7, 8], but was never observed experimentally. As appealing as the Klein tunneling may sound from the point of view of fundamental research, its presence in graphene is unwanted when it comes to applications of graphene to nanoelectronics. This comes about because the pinch-off the field effect transistor may be very ineffective. The same may occur because of the minimum conductivity of graphene at the neutrality point. One way to overcome these difficulties is by generating a gap in the spectrum. From the point of view of Dirac fermions this is equivalent to the generation of a mass term.

A possibility of generating gaps in the graphene spectrum is to deposit graphene on top of hexagonal boron nitride (BN) [9]. This material is a band gap insulator with a boron to nitrogen distance of the order of 1.45 Å [12] (in graphene the carbon-carbon distance is 1.42 Å) and a gap of the order of 4 eV. It was shown that in the most stable configuration, where a carbon is on top of a boron and the other carbon in the unit cell is centered above a BN ring, the value of the induced gap is of the order of 53 meV. Depositing graphene on a metal surface with a BN buffer layer leads to $n-$doped graphene with an energy gap of 0.5 eV [10].

Theoretically, the tunneling effect of system type SiO$_2$-BN in zero magnetic field is discussed [11]. In fact, it is assumed that it is possible to manufacture slabs with SiO$_2$-BN interfaces, on top of which a graphene flake is deposit. This will induce spatial regions where graphene has a vanishing gap intercalated with regions where the BN will cause a finite gap. The graphene physics is considered in two different regions: the $k-$region, where the graphene sheet is standing on top of SiO$_2$, and a $q-$region, where a mass-like term is present, caused by BN, inducing an energy gap of value $2\ell'$. The effect of chiral electrons in graphene through a region is studied where the electronic spectrum changes from the usual linear dispersion to a hyperbolic dispersion, due to the presence of a gap. It is shown that contrary to the tunneling through a potential barrier, the transmission of electrons is, in this case, smaller than one for normal incidence.

Motivated by the reason discussed above and in particular the investigation made in [11], we would like to reply an interesting question. the fact that what happens to the tunneling effect of SiO$_2$-BN in the presence of an external magnetic field $B$. Otherwise, still we have the same conclusions reached for the case $B = 0$ in [11] or the will be affected. To answer these inquiries, we study such system in the same conditions as in [11] but taking into account the effect of the gauge field. This will allow us to deal with some issues and end up with different conclusions. Moreover, under some conditions we
show that there is possibility for a total transmission.

More precisely, we consider a system composed of two different regions, where the second is characterized by an energy gap $t'$, in a perpendicular magnetic field. The energy spectrum solutions are obtained for both regions in terms of two Landau levels and $t'$. From the energy conservation $E$, we obtain a set of the energy values that allows us to discuss the tunneling effect. By inspecting these values, one can end up with three limiting cases, which they have interesting consequences on the reflexion and transmission of the present system.

To be concrete, we give two examples of system made of graphene. In the beginning, we consider a barrier in magnetic field and study the tunneling effect. Indeed, from the continuity equation, we get different solutions, which allowed us to explicitly determine the reflexion and transmission coefficients for different regions. They are used to show that the probability condition is satisfied, namely the sum of these coefficients is one. To characterize the transmission behavior we give different figures, which underline its properties in terms of the energy ratio $E/t'$. Using the three limiting cases of energy we discuss the possibility to obtain a total transmission and give different interpretations.

As far as the diode in magnetic field is concerned, we discuss the tunneling effect by splitting the whole system in three regions where the second different from the first and third, which they are identical. After getting different coefficients, we use an appropriate definition to show that the sum of reflexion and transmission is one. Analyzing this under some conditions, we conclude that it is possible to obtain a total transmission. We also discuss a limiting situation where the barriers are described by a scalar Lorentz potential. Using the boundary conditions, we derive different coefficients, which lead again to verify the probability condition and give different discussions about such potential.

The present paper is organized as follows. In section 2, we formulate our problem by writing the Hamiltonian’s describing two regions of our system. The eigenvalue problems will be solved to obtain the energy spectrum and its eigenspinors. We inspect the eigenvalue solutions from the energy conservation point of view to underline their properties in section 3. After establishing all needed materials, we treat the first illustration of our system, i.e. the barrier, in section 4. In section 5, we consider the diode in magnetic field as a second illustration where an interesting limit will be investigated. We conclude and give some perspectives in last section.

## 2 Hamiltonian formalisms

As claimed before, our system is a superposition of two different regions (I) and (II), with (II) has an energy gap $t'$. For this, we start by setting the necessary tools needed to treat each region separately. These concerns to write the corresponding Hamiltonian’s and determine their eigenvalue solution as well as the eigenspinors.

Region (I) can be identified to a two-dimensional subsystem of Dirac fermions where the Hamiltonian for one massless relativistic fermion in the presence of a perpendicular magnetic field is

$$H^{(I)} = v_F \vec{\sigma} \cdot \vec{\pi}$$

(1)

where $v_F \approx 10^6 ms^{-1}$ is the Fermi velocity and $\vec{\sigma} = (\sigma_x, \sigma_y)$ are the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (2)$$
The conjugate momentum is \( \vec{\pi} = \vec{p} + e \vec{A} \), with \( \vec{A} \) is a gauge field. Recall that, region (II) is of finite mass due to \( t' \), then its appropriate Hamiltonian can be written as
\[
H^{(II)} = v_F \vec{\sigma} \cdot \vec{\pi} + t' \sigma_z. \tag{3}
\]
Its clear that, the mass term \( t' \sigma_z \) makes difference between both the above Hamiltonian’s. This will play a crucial role in the forthcoming analysis.

2.1 First region spectrum

To do our task, we start by determining the energy spectrum and its eigenspinors for each involved region. In doing so, let us start by writing the Hamiltonian (I) as
\[
H^{(I)} = v_F \begin{pmatrix} 0 & \pi_x - i\pi_y \\ \pi_x + i\pi_y & 0 \end{pmatrix}. \tag{4}
\]
Choosing the Landau gauge \( \vec{A} = B (0, x, 0) \), the momenta components take the form \( \pi_x = p_x \) and \( \pi_y = p_y + \frac{eB}{c} x \). These can be used to maps \( H^{(I)} \) into
\[
H^{(I)} = v_F \begin{pmatrix} 0 & p_x - ip_y - \frac{i eB}{c} x \\ p_x + ip_y + \frac{i eB}{c} x & 0 \end{pmatrix}. \tag{5}
\]

As usual to get the energy solutions of (5), one can use the eigenvalue equation for a given spinor \( \phi^{(I)} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \) of \( H^{(I)} \). This is
\[
H^{(I)} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = E^{(I)} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}. \tag{6}
\]
It implies the "kinetic balance" relation
\[
-iv_F \left( ip_x + p_y + \frac{eB}{c} x \right) \varphi_2 = E^{(I)} \varphi_1 \tag{7}
\]
\[
iv_F \left( -ip_x + p_y + \frac{eB}{c} x \right) \varphi_1 = E^{(I)} \varphi_2. \tag{8}
\]
To determine these spinor components, we can map for instance (7) into (8) to obtain a Schrödinger-like equation for \( \varphi_2 \). After calculation, we obtain
\[
v_F^2 \left[ p_x^2 + \left( p_y + \frac{eB}{c} x \right)^2 - \frac{eB\hbar}{c} \right] \varphi_2 = \left( E^{(I)} \right)^2 \varphi_2 \equiv \hbar \varphi_2. \tag{9}
\]
It is similar to that of the harmonic oscillator up to some constant term and thus \( \varphi_2 \) can be seen as its eigenfunction. To clarify this statement, let us consider the Landau Hamiltonian in the same gauge, such as
\[
H_0 = \frac{1}{2m} \left[ p_x^2 + \left( p_y + \frac{eB}{c} x \right)^2 \right]. \tag{10}
\]
One can easily show that its energy spectrum reads as
\[
E^{(I)'} = \hbar \omega_c \left( n + \frac{1}{2} \right), \quad n = 0, 1, 2, \cdots \tag{11}
\]
where the cyclotron frequency $\omega = \frac{eB}{mc}$. In terms of the cylindrical parabolic functions, the corresponding wavefunctions are given by

$$\phi(x, y) = D_n(Q) e^{ik_y y}$$

(12)

where $k_y$ is a wave vector along $y$-direction in region (I). We have set $Q = \frac{x + x_0}{l_B}$ and $x_0 = k_y l_B^2$, with the magnetic length $l_B = \sqrt{\frac{eh}{cB}}$. It is not hard to verify the property

$$D_n(-Q) = (-1)^n D_n(Q)$$

(13)

which will be used in the forthcoming analysis and more precisely when we start to investigate the tunneling effect. Explicitly, the states $\phi(x, y)$ are

$$\phi(x, y) = (l_B \sqrt{\pi n!}^2)^{-\frac{1}{2}} \exp \left( -\frac{Q^2}{2} \right) H_n(Q) e^{ik_y y}$$

(14)

where the Hermite polynomials $H_n(Q)$ are

$$H_n(Q) = (-1)^n \exp(Q^2) \frac{d^n}{dQ^n} \exp(-Q^2).$$

(15)

Now injecting (12) in (9), one can end up with

$$\left[ \frac{2v_F^2 \hbar^2}{l_B^2} (n + \frac{1}{2}) - \frac{v_F^2 \hbar^2}{l_B^2} \right] D_n(Q) = \left( E^{(\text{I})} \right)^2 D_n(Q)$$

(16)

which is leading to the form

$$\left[ \frac{2v_F^2 \hbar^2}{l_B^2} n \right] D_n(Q) = \left( E^{(\text{I})} \right)^2 D_n(Q).$$

(17)

It is clear that the eigenvalues read as

$$\left( E^{(\text{I})}_n \right)^2 = \alpha^2 n$$

(18)

where the constant $\alpha = \frac{2v_F^2 \hbar^2}{l_B^2}$. According to the interpretation of matter and antimatter, the bottom valley of the band structure takes the value $-n$ that correspond to negative energies (anti-matter). For this reason, we write the eigenvalues $H^{(\text{I})}$ as

$$E^{(\text{I})}_n = \text{sgn}(n) \alpha \sqrt{|n|}, \quad n \in \mathbb{Z}$$

(19)

where the eigenfunctions are given by

$$\varphi_2(x, y) = D_{|n|} \left( \frac{x + x_0}{l_B} \right) e^{ik_y y}. $$

(20)

To complete our analysis, we need to determine the second spinor component, which can be obtained from the "kinetic balance" relation. Doing so to obtain

$$\varphi_1 = \frac{1}{E^{(\text{I})}} \left[ -iv_F \left( ip_x + p_y + \frac{eB}{c} x \right) D_{|n|} \right] e^{ik_y y}.$$

(21)

It leads to the solution

$$\varphi_1 = -\text{sgn}(n) i D_{|n| - 1}(Q) e^{ik_y y}.$$  

(22)
Finally, combining all to get the normalized eigenspinors as
\[
\phi^{(I)}_{(n,k_y)}(x,y) = \frac{1}{\sqrt{2}} \begin{pmatrix} -siD_{|n|-1}(x+x_0) \\ D_{|n|}(x+x_0) \end{pmatrix} e^{ik_y y} \tag{23}
\]
where \( s = \text{sgn}(n) \) and the convention \( \text{sgn}(0) = 0 \) should be taken into account. The energy spectrum has zero-mode wavefunction, such as
\[
\phi^{(I)}_{(0,k_y)}(x,y) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ D_0(x+x_0) \end{pmatrix} e^{ik_y y}. \tag{24}
\]
These results are concerning the first region, which is assimilated to a subsystem of Dirac fermions in constant magnetic field. Next we will see how the above results will change when we move to the second region, which is also Dirac fermions but with a mass term.

### 2.2 Second region spectrum

From the nature of the system under consideration, we write the Hamiltonian corresponding to region (II) in terms of matrix as
\[
H^{(II)} = v_F \begin{pmatrix} 0 & \pi_x - i\pi_y \\ \pi_x + i\pi_y & 0 \end{pmatrix} + \begin{pmatrix} t' & 0 \\ 0 & -t' \end{pmatrix}. \tag{25}
\]
In the Landau gauge, we have the form
\[
H^{(II)} = v_F \begin{pmatrix} t' & p_x - ip_y - ieB/c x \\ p_x + ip_y + ieB/c x & -t' \end{pmatrix}. \tag{26}
\]
Note that, the energy gap behaves like a mass term. Certainly this will affect the above results and lead to interesting consequences in underlying the basics features of such system.

We need to derive the energy spectrum and its eigenspinors. In doing so, let us fix \( \phi^{(II)} = \begin{pmatrix} \varphi'_1 \\ \varphi'_2 \end{pmatrix} \) as a spinor of \( H^{(II)} \) to write
\[
H^{(II)} \begin{pmatrix} \varphi'_1 \\ \varphi'_2 \end{pmatrix} = E^{(II)} \begin{pmatrix} \varphi'_1 \\ \varphi'_2 \end{pmatrix} \tag{27}
\]
which implies two relations
\[
-iuv_F \left( ip_x + p_y + \frac{eB}{c} x \right) \varphi'_2 = \left( E^{(II)} - t' \right) \varphi'_1 \tag{28}
\]
\[
iv_F \left( -ip_x + p_y + \frac{eB}{c} x \right) \varphi'_1 = \left( E^{(II)} + t' \right) \varphi'_2. \tag{29}
\]
These can be treated as we have done before to get one equation for one component spinor. After injecting (28) in (29), we obtain a differential equation of second order for \( \varphi'_2 \). This is
\[
h\varphi'_2 = \left[ \left(E^{(II)}\right)^2 - t'^2 \right] \varphi'_2. \tag{30}
\]
It solution gives the second spinor component as
\[
\varphi'_2(x,y) = D_{|m|} \left( \frac{x+x'_0}{l_B} \right) e^{iq_y y}, \quad m \in \mathbb{Z} \tag{31}
\]
where \( q_y \) is a wave vector along \( y \)-direction in the second region and \( x'_0 = q_y l_B^2 \). From last equation, it is easy to obtain the energy spectrum

\[
E^{(II)} = \text{sgn}(m) \sqrt{\alpha^2 |m| + t'^2}.
\]  

We notice that the term \( t' \) makes difference with respect to spectrum of region (I). It is convenient for our task to write energy as

\[
E^{(II)} - t' = \frac{\alpha^2 |m|}{E^{(II)} + t'}.
\]  

This will be used to show how the tunneling effect behaves in terms of the energy conservation and the parameter \( t' \).

Returning to the "kinetic balance" relations (28) and (29), one can easily derive the first spinor component \( \phi_1' \). It is

\[
\phi_1'(x, y) = -\frac{i \alpha \sqrt{|m|}}{E^{(II)} - t'} D_{|m|-1} \left( \frac{x + x'_0}{l_B} \right) e^{i q_y y}.
\]  

After normalization the eigenspinors read as

\[
\phi^{(II)}_{(m, q_y)}(x, y) = \frac{1}{\sqrt{2}} \left( \begin{array}{c}
-a_m i D_{|m|-1} (x + x'_0) \\
-b_m D_{|m|} (x + x'_0)
\end{array} \right) e^{i q_y y}.
\]  

The normalization constants are given by

\[
a_m = s' \sqrt{\frac{E^{(II)} + s't'}{E^{(II)}},} \quad b_m = \sqrt{\frac{E^{(II)} - s't'}{E^{(II)}}}
\]  

where we have set \( s' = \text{sgn}(m) \). They verify the useful relation

\[
a_m^2 + b_m^2 = 2. \tag{37}
\]

We have also here a zero-mode wavefunction

\[
\phi^{(II)}_{(0, q_y)}(x, y) = \frac{1}{\sqrt{2}} \left( \begin{array}{c}
0 \\
D_0 (x + x'_0)
\end{array} \right) e^{i q_y y}.
\]  

Clearly, in the interface between two regions, there is conservation of the tangent components of the wave vector, i.e. \( k_y = q_y \), which is equivalent to \( x_0 = x'_0 \). This conclude the investigation of the energy spectrum for the present system in a constant magnetic field. Before using these results to solve some issues, it is interesting to start by underling properties of the eigenvalues.

### 3 Selection rules

Since we are considering two different regions characterized by two quantum numbers, one may investigate their behaviors in terms of the energy system and the parameter \( t' \). For this, we use the energy conservation to establish a relation between them, which will be employed in the next in dealing with different issues. In particular, it will used to discuss the tunneling effect for our system through the reflexion and transmission coefficients.
Recall that, from the above analysis, we ended up with two energy spectra: $E^{(I)}$ and $E^{(II)}$. On the other hand, due to the system nature one should have an energy conservation, such as

$$\left(E^{(I)}\right)^2 = \left(E^{(II)}\right)^2 = E^2. \tag{39}$$

After replacing the energies by their expressions, it is easy to observe that the allowed energy values should verify the relation

$$\frac{E^2}{t^2} = \frac{|n|}{|n| - |m|} \tag{40}$$

where the constraint $|m| \leq |n|$ must be fulfilled. According to (40), one can realize $n$ and $m$ in terms of a pair of quantum numbers $(p,q)$, such as

$$n = \pm kp, \quad m = \pm k(p-q) \tag{41}$$

where $\frac{p}{q}$ is an irreducible fraction with $p \geq q$ and $k$ is an integer value. This realization can be written in compact form as $(n,m) = \pm k(p,p-q)$, which is equivalent to say that this pair lies in the set of values

$$(n,m) = \{\pm(p,p-q), \pm2(p,p-q), \pm3(p,p-q), \pm4(p,p-q), \cdots\}. \tag{42}$$

This is very important because without such set one can not talk about tunneling effect in the present case. We will clarify this statement from next section and exactly when we start to calculate different quantities in order to check the probability condition.

The relation (40) is important in sense that its has some interesting consequences and can be used to give different interpretations of the present system. In fact, by inspecting some limiting cases, one can show

- $n = 0 \implies m = 0$.
- $\frac{E^2}{t^2} \to \infty \implies n = \pm m \implies a_m = s' = s, b_m = 1$.
- $\frac{E^2}{t^2} = 1 \implies m = 0$.

In the next, we will see how these cases affect the tunneling effect through the evaluation of the reflexion and transmission coefficients.

On the other hand, one can plot different quantum numbers to underline their behaviors. For this, we present three figures (1,2,3) illustrating some cases where the bold dots are exactly the allowed values for each numbers. They are...
Figure 1: Variation of $m$ in terms of $n$, it is showing that for $n = 5$ there are 6 possible values for $m$, i.e. $m = 0, 1, \cdots, 5$.

Figure 2: Variation of the ratio of energy conservation and $t', \frac{p}{q}$, in terms of $n$. For $n = 0$, we have $\frac{p}{q} = 1$, which is in agreement with the first item cited above.

Figure 3: Variation of the ratio of energy conservation and $t', \frac{p}{q}$, in terms of $m$. For $m = 0$, we have identical behavior as for $n = 0$.

4 Barrier in magnetic field

To treat a concrete example of the present system, we consider the barrier in magnetic field. This latter can be seen as superposition of two regions separated by interface. We study the tunneling effect by evaluating the reflexion and transmission coefficients at interface, which corresponds to the point zero. These will be used to show that the probability condition is exactly one and emphasis what makes difference with respect to the same system studied in zero field [11]. To do this task, we distinguish two cases: propagation with positive and negative incidences. In both cases we deal with propagation from region (I) to (II) and vice versa.

4.1 Propagation with positive incidence

We start by defining what does mean propagation with positive incidence in our case. In fact, with this we will be able to do our job and therefore use the continuity equation at interface to determine
different quantities. Indeed, we split the obtained eigenspinors for two regions in positive and negative directions of the variable \( x \), which is equivalent to write in region (I)

\[
\phi_+^{(I)} = \frac{1}{\sqrt{2}} \begin{pmatrix} -sD_{|n|-1}(x+x_0) \\ D_{|n|}(x+x_0) \end{pmatrix} e^{ik_y}, \quad \phi_-^{(I)} = \frac{1}{\sqrt{2}} \begin{pmatrix} -sD_{|n|-1}(-x-x_0) \\ D_{|n|}(-x-x_0) \end{pmatrix} e^{ik_y}
\]

and in (II) we can do the same, such as

\[
\phi_+^{(II)} = \frac{1}{\sqrt{2}} \begin{pmatrix} -a_m iD_{|m|-1}(x+x_0) \\ b_mD_{|m|}(x+x_0) \end{pmatrix} e^{ik_y}, \quad \phi_-^{(II)} = \frac{1}{\sqrt{2}} \begin{pmatrix} -a_m iD_{|m|-1}(-x-x_0) \\ b_mD_{|m|}(-x-x_0) \end{pmatrix} e^{ik_y}
\]

where we used the labels \((\pm)\) to show which direction is concerned. We will see how these materials will serve as a tool to deal with different issues. For this, one has to consider two different cases, which will be the next task.

**Propagation from region (I) to region (II):** This means that the waves are propagating from left to right and due to the continuity of the system, one can establish two equations. Indeed, at the interface \( x = 0 \) and for all \( y \), we can write

\[
\phi_+^{(I)} + r_{nm}^+ \phi_-^{(I)} = t_{nm}^+ \phi_+^{(II)}
\]

where \( r_{nm}^+ \) and \( t_{nm}^+ \) are reflexion and transmission coefficients, respectively, \((+\)) indicates sign of propagation. From (43) and (44), we end up with

\[
\begin{pmatrix} -sD_{|n|-1}(x_0) \\ D_{|n|}(x_0) \end{pmatrix} + r_{nm}^+ \begin{pmatrix} -sD_{|n|-1}(-x_0) \\ D_{|n|}(-x_0) \end{pmatrix} = t_{nm}^+ \begin{pmatrix} -a_m iD_{|m|-1}(x_0) \\ b_mD_{|m|}(x_0) \end{pmatrix}.
\]

Using the property (13), it is not hard to obtain

\[
\begin{align*}
-1)^{|n|} sD_{|n|-1}(x_0) + t_{nm}^+ a_m D_{|m|-1}(x_0) &= sD_{|n|-1}(x_0) \\
-(1)^{|n|} D_{|n|}(x_0) + t_{nm}^+ b_mD_{|m|}(x_0) &= D_{|n|}(x_0).
\end{align*}
\]

They can be solved to explicitly get the coefficients in terms of some constants, which are magnetic field dependent. They are

\[
\begin{align*}
|n| \left[ s_{mn} A_{nm}(x_0) - a_m B_{nm}(x_0) \right] \\
2sC_n(x_0)
\end{align*}
\]

where different involved quantities are given by

\[
\begin{align*}
A_{nm}(x_0) &= D_{|n|-1}(x_0)D_{|m|}(x_0), & B_{nm}(x_0) &= D_{|m|-1}(x_0)D_{|n|}(x_0), \\
C_n(x_0) &= D_{|n|-1}(x_0)D_{|n|}(x_0).
\end{align*}
\]

These coefficients are just part of a set of quantities that should be completely determined. In fact, we still need other quantities to achieve our task, which can be obtained from next consideration.

**Propagation from region (II) to region (I):** In similar way to the former case, we use the continuity at point zero to write

\[
\phi_+^{(II)} + r_{nm}^\prime \phi_-^{(II)} = t_{nm}^\prime \phi_+^{(I)}.
\]
Comparing this with \([15]\), we notice that there is an index interchange between different coefficients. By replacing, \([52]\) becomes
\[
\left( -a_m i D_{|m|-1}(x_0) \right)_{m} + r_{mn}^+ \left( -a_m i D_{|m|-1}(-x_0) \right)_{mn} = t_{mn}^+ \left( -isD_{|m|-1}(x_0) \right)_{mn} \ .
\]
These lead to the reflection and transmission coefficients
\[
r_{mn}^+ = \frac{(-1)^{|m|} \left[ a_m B_{mn}(x_0) - s_b A_{nm}(x_0) \right]}{s_b A_{nm}(x_0) + a_m B_{nm}(x_0)} \quad (54)
\]
\[
t_{mn}^+ = \frac{2a_m F_m(x_0)}{s_b A_{nm}(x_0) + a_m B_{nm}(x_0)} \quad (55)
\]
where \(F_m\) is given by
\[
F_m(x_0) = D_{|m|-1}(x_0) D_{|m|}(x_0) \quad (56)
\]
This summarizes our analysis for propagation with positive incidence, which together will be used to discuss different issues and before doing so, we need to analyze negative incidence.

4.2 Propagation with negative incidence

One may ask about propagation with negative incidence. To reply this inquiry, we use the same analysis as before but one should take into account the negative sign of variable. For the needness, we write the corresponding eigenspinors as
\[
\phi_+^{(I)} = \frac{1}{\sqrt{2}} \left( -siD_{|m|-1}(-x - x_0) \right)_{m} e^{-ik_y y}, \quad \phi_-^{(I)} = \frac{1}{\sqrt{2}} \left( -siD_{|m|-1}(x + x_0) \right)_{m} e^{-ik_y y} \quad (57)
\]
and for second region, we have
\[
\phi_+^{(II)} = \frac{1}{\sqrt{2}} \left( -a_m i D_{|m|-1}(-x - x_0) \right)_{mn} e^{-ik_y y}, \quad \phi_-^{(II)} = \frac{1}{\sqrt{2}} \left( -a_m i D_{|m|-1}(x + x_0) \right)_{mn} e^{-ik_y y} \quad (58)
\]
By analogy to the former case, we distinguish the sense of propagation and determine the corresponding coefficients in terms of the magnetic field.

**Propagation from region (I) to region (II):** Let us start from the continuity equation at point zero, such as
\[
\phi_+^{(I)} + r_{nm} \phi_-^{(I)} = t_{nm} \phi_+^{(II)} \quad (59)
\]
where label \((-\)
) carried by the involved coefficients describes sense of propagation. Using \([57]\) and \([58]\) to rewrite \([59]\) as
\[
\left( -siD_{|m|-1}(-x_0) \right)_{m} + r_{mn}^+ \left( -siD_{|m|-1}(x_0) \right)_{mn} = t_{mn}^+ \left( -a_m i D_{|m|-1}(-x_0) \right)_{mn} \quad (60)
\]
It can be solved to get
\[
r_{nm}^- = \frac{(-1)^{|m|} \left[ s_b A_{nm}(x_0) - a_m B_{mn}(x_0) \right]}{s_b A_{nm}(x_0) + a_m B_{mn}(x_0)} \quad (61)
\]
\[
t_{nm}^- = \frac{(-1)^{|m|} \left[ 2s C_n(x_0) \right]}{s_b A_{nm}(x_0) + a_m B_{mn}(x_0)} \quad (62)
\]
Propagation from (II) to region (I): In similar way, we have at zero point
\[
\phi_+^{(II)} + r_{mn}^+ \phi_+^{(II)} = t_{mn}^+ \phi_+^{(II)}
\]
(63)
showing the relation
\[
\begin{pmatrix}
-a_m i D_{|m|-1}(-x_0) \\
b_m D_{|m|}(-x_0)
\end{pmatrix}
+ r_{mn}^-
\begin{pmatrix}
-a_m i D_{|m|-1}(x_0) \\
b_m D_{|m|}(x_0)
\end{pmatrix}
= t_{mn}^-
\begin{pmatrix}
-si D_{|m|-1}(-x_0) \\
D_{|m|}(-x_0)
\end{pmatrix}.
\]
(64)
The solutions are given by
\[
r_{mn}^- = \frac{(-1)^{|m|}[a_m B_{nm}(x_0) - s b_m A_{nm}(x_0)]}{s b_m A_{nm}(x_0) + a_m B_{nm}(x_0)}
\]
(65)
\[
t_{mn}^- = \frac{(-1)^{|n|+|m|}2 a_m F_m(x_0)}{s b_m A_{nm}(x_0) + a_m B_{nm}(x_0)}
\]
(66)
Finally we derived all materials needed to do our job. In fact, we use the obtained results to give different discussions and interpretations.

4.3 Discussions
We show the relevance of different coefficients derived so far. In fact, we will combine all of them to underline the basic features of the barrier in constant magnetic field. Certainly, from the above analysis one can notice that there are some relations between coefficients, which can be interpreted as a kind of symmetry. This will be helpful in sense that one can check the probability condition and derive other results.

For the reason of do not repeating equations, let us choose a pair of index, such as \(i \neq j \in \{n, m\}\), to write them in compact forms. Indeed, from the above results, one can find
\[
r_{ij}^+(x_0) = r_{ij}^+(x_0), \quad t_{ij}^+(x_0) = t_{ij}^+(x_0) = -\frac{r_{ij}^+(x_0)}{r_{ij}^+(x_0)} = (-1)^{|n|+|m|}.
\]
(67)
To show the usefulness of these relations, let us define two coefficients in terms of the above quantities. The first one is
\[
\rho(x_0) = r_{ij}^+(x_0) r_{ji}^+(x_0) = \frac{[s b_m A_{nm}(x_0) - a_m B_{nm}(x_0)]^2}{[s b_m A_{nm}(x_0) + a_m B_{nm}(x_0)]^2}.
\]
(68)
It is not hard to obtain
\[
r_{ij}^+(x_0) r_{ji}^+(x_0) = \rho(x_0) \frac{t_{ij}^+(x_0)}{t_{ij}^+(x_0)} = \rho(x_0) \frac{r_{ij}^+(x_0)}{r_{ij}^+(x_0)} = (-1)^{|n|+|m|+1} \rho(x_0).
\]
(69)
In the same way we can write the second coefficient as
\[
\tau(x_0) = t_{ij}^+(x_0) t_{ji}^+(x_0) = \frac{4 s b_m a_m C_n(x_0) F_m(x_0)}{[s b_m A_{nm}(x_0) + a_m B_{nm}(x_0)]^2} = \frac{4 s b_m a_m A_{nm}(x_0) B_{nm}(x_0)}{[s b_m A_{nm}(x_0) + a_m B_{nm}(x_0)]^2}.
\]
(70)
This implies that all quantities defined here can be related as
\[
C_n(x_0) F_m(x_0) = A_{nm}(x_0) B_{nm}(x_0)
\]
(71)
which can easily check it from their expressions given before. Using (68) and (70), we show that they verify the condition
\[ \rho(x_0) + \tau(x_0) = 1. \]

(72)

This is among the interesting results derived so far. In fact, it tells us the transmission of barrier in magnetic field can not be greater than one, which is analogue to what obtained in zero field case [11].

At this level, one can inspect the relations (68) and (70) to see how they behave with respect to three items cited in section 3. As well see soon these will change the nature of the present system. Indeed, taking into account such items, we straightforwardly end up with the results

- \( n = 0 \Rightarrow \rho(x_0) = 0, \tau(x_0) = 1. \)
- \( \frac{E^2}{t'^2} \to \infty \Rightarrow \rho(x_0) = 0, \tau(x_0) = 1. \)
- \( \frac{E^2}{t'^2} = 1 \Rightarrow \rho(x_0) = 1, \tau(x_0) = 0. \)

These three cases have interesting interpretations in optics physics. More precisely, in the first and second points, the interface between two regions behaves like a non-reflective dioptr, which means that everything is transmitted, i.e. total transmission. However, in last point, it looks like a mirror where the reflexion is total.

To underline the basic properties of the transmission coefficient, we plot three figures in terms of the energy ratio \( E/t' \) where the conditions \( s = s' \) and \( E/t' \geq 1 \) are taking into account. These figures can be interpreted as follows.

- Figure 4: One can see that each curve rises to an asymptote, which decreases when \( B_{nm}/A_{nm} \) increases. Clearly, for \( B_{nm}/A_{nm} = 1 \) the asymptote goes to 1, for \( B_{nm}/A_{nm} = 4 \) to 0.6 and finally for \( B_{nm}/A_{nm} = 8 \) to 0.4.

- Figure 5: Each curve suddenly reached the value of 1 and then decreases to an asymptote, which decreases when \( B_{nm}/A_{nm} \) decreases. For \( B_{nm}/A_{nm} = 1 \) the asymptote goes to 1, for \( B_{nm}/A_{nm} = 1/4 \) to 0.6 and for \( B_{nm}/A_{nm} = 1/8 \) to 0.4.

- Figure 6: Both curves tend towards the same asymptote for \( B_{nm}/A_{nm} = 1/2 \). The curve decreases to the asymptote 0.7 of the upper, but for \( B_{nm}/A_{nm} = 2 \) the curve rises to the asymptote of the lower. This behavior can be generalized to any case where \( B_{nm}/A_{nm} = l \) and \( B_{nm}/A_{nm} = 1/l \), for all positive value \( l \), which is in agreement with the statement cited in item 2 that means that the transmission is total whenever \( \frac{E}{t} \to \infty \).
Figure 4: Variation of the transmission $\tau(x_0)$ in terms of the ratio $E/t'$ for three cases: $B_{nm} = A_{nm}$, $B_{nm} = 4A_{nm}$, $B_{nm} = 8A_{nm}$.

Figure 5: Variation of the transmission $\tau(x_0)$ in terms of the ratio $E/t'$ but this case for $B_{nm} = A_{nm}$, $B_{nm} = 1/4A_{nm}$, $B_{nm} = 1/8A_{nm}$.

Figure 6: Variation of the transmission $\tau(x_0)$ in terms of the ratio $E/t'$ for two cases: $B_{nm} = 2A_{nm}$, $B_{nm} = 1/2A_{nm}$.

5 Diode in magnetic field

In last section we focused on the study of two different regions, with this it is natural to ask about a generalization to three regions where the second is characterized also with the energy gap $t'$ and the first and third are identical. This is the case for instance of a diode in a constant magnetic field. To reply this inquiry, let us consider a system composed of a region indexed by the quantum number $m$ of length $2w$ separating two others indexed by the same $n$. In fact, we will apply the same machinery as before to analyze the system behavior in such consideration.

5.1 Reflexion and transmission coefficients

The present situation is different from the former one, i.e. the barrier. This means that actually we have two interfaces fixed at the points $w_1 = x_0 - w$ and $w_2 = w + x_0$, which of course will make difference with respect of the barrier analysis. We use the above tool to write the continuity equation at each point and therefore derive the needed quantities for discussing tunneling effect in such case. In doing so, we consider also propagation with positive and negative incidences.

In the first interface one can write

\[
\begin{pmatrix}
-siD_{n-1}(w_1) \\
D_{n}(w_1)
\end{pmatrix} + r^+ \begin{pmatrix}
-siD_{n-1}(-w_1) \\
D_{n}(-w_1)
\end{pmatrix} =
\]
using the former analysis one can show

\[ \alpha = \frac{sb_n A_{nm}(w_1) + a_m B_{nm}(w_1) + r^+(-1)^{|n|} [a_m B_{nm}(w_1) - sb_n A_{nm}(w_1)]}{2a_m b_m F_m(w_1)} \]  
\[ \beta = \frac{a_m B_{nm}(w_1) - sb_n A_{nm}(w_1) + r^+(-1)^{|n|} [a_m B_{nm}(w_1) + sb_n A_{nm}(w_1)]}{2(-1)^{|n|} a_m b_m F_m(w_1)} \]

These can be simplified to other equations by introducing two relevant relations in terms of \( w_1 \). Indeed, using the former analysis one can show

\[ \frac{sb_n A_{nm}(w_1) + a_m B_{nm}(w_1)}{2a_m b_m F_m(w_1)} = \frac{1}{t^+_{mn}(w_1)} \]  
\[ \frac{a_m B_{nm}(w_1) + sb_n A_{nm}(w_1)}{2a_m b_m F_m(w_1)} = (-1)^{|n|+|m|} t^+_{mn}(w_1) \]

With the help of these, we can rewrite \( \alpha \) and \( \beta \) as

\[ \alpha = \frac{1}{t^+_{mn}(w_1)} - r^+ \frac{r^+_{nm}(w_1)}{t^+_{mn}(w_1)} \]  
\[ \beta = (-1)^{|n|+|m|} \left[ \frac{r^+}{t^+_{mn}(w_1)} - \frac{r^+_{nm}(w_1)}{t^+_{mn}(w_1)} \right] \]

As we will see, they will be employed to set up another couple of equation and therefore explicitly determine the reflection and transmission coefficients for the diode case.

To analyze the second interface, we start by considering the corresponding continuity equation at the point \( w_2 = x_0 + w \). This is

\[ \alpha \left( \frac{a_m iD_{|m|-1}(w_2)}{b_m D_{|m|}(w_2)} \right) + \beta(-1)^{|m|} \left( \frac{-a_m iD_{|m|-1}(w_2)}{b_m D_{|m|}(w_2)} \right) = \]
\[ = t^+ \left( \frac{siD_{|n|-1}(w_2)}{D_{|n|}(w_2)} \right) \]

Note that, in obtaining (80) in such form we made use of the property (13). The coefficients solutions are given by

\[ \alpha = t^+ \frac{sb_m A_{nm}(w_2) + a_m B_{nm}(w_2)}{2a_m b_m F_m(w_2)} = \frac{t^+}{t^+_{mn}(w_2)} \]  
\[ \beta = t^+ (-1)^{|m|} \frac{a_m B_{nm}(w_2) - sb_m A_{nm}(w_2)}{2a_m b_m F_m(w_2)} = t^+ (-1)^{|n|+|m|} \frac{r^+_{nm}(w_2)}{t^+_{mn}(w_2)} \]

These lead to obtain

\[ r^+ = \frac{r^+_{nm}(w_1) - r^+_{nm}(w_2)}{1 - r^+_{nm}(w_1)r^+_{nm}(w_2)} \]

From the above relations, one can find

\[ t^+ = \frac{t^+_{mn}(w_2)}{t^+_{mn}(w_1)} \left[ \frac{1 - r^+_{nm}(w_1)r^+_{nm}(w_2)}{1 - r^+_{nm}(w_1)r^+_{nm}(w_2)} \right] \]
It convenient to write $t^+$ in terms of $\rho$ and $\tau$ as

$$
t^+ = \frac{t^+_{nm}(w_2)}{t^+_{nm}(w_1)} \left[ \frac{1 - \rho(w_1)}{1 - r^+_{nm}(w_1)r^+_{nm}(w_2)} \right] = \frac{t^+_{nm}(w_2)}{t^+_{nm}(w_1)} \left[ \frac{\tau(w_1)}{1 - r^+_{nm}(w_1)r^+_{nm}(w_2)} \right]. \tag{85}
$$

We close this part by nothing that the obtained results are only valid for the positive incidence. On the other hand, we will return to discuss the choice of fixing the points $w_1$ and $w_2$ in terms of $w$.

As far as the negative incidence is concerned, we can use a similar analysis to derive the corresponding coefficients. Indeed, from above we obtain

$$
r^- = \frac{r^-_{nm}(w_1) - r^-_{nm}(w_2)}{1 - r^-_{nm}(w_1)r^-_{nm}(w_2)} = \frac{r^+_{nm}(w_1) - r^+_{nm}(w_2)}{1 - r^+_{nm}(w_1)r^+_{nm}(w_2)}, \tag{86}
$$
as well as

$$
t^- = \frac{t^-_{nm}(w_2)}{t^-_{nm}(w_1)} \left[ \frac{\tau(w_1)}{1 - r^-_{nm}(w_1)r^-_{nm}(w_2)} \right] = \frac{t^-_{nm}(w_2)}{t^-_{nm}(w_1)} \left[ \frac{\tau(w_1)}{1 - r^+_{nm}(w_1)r^+_{nm}(w_2)} \right]. \tag{87}
$$

These results will used to deal with the same issues as we have done for the barrier in magnetic field and also discuss other points.

### 5.2 Collecting results

After deriving the needed tools, we start by discussing the usefulness of them. For example, we can check the probability to see how the present system behaves at some critical points. In doing so, let us make an appropriate definition of the reflexion and transmission coefficients. It is convenient for our task to write

$$
R = r^+r^- = r^2, \quad T = t^+t^-.
$$

Explicitly, they take the forms

$$
R = \frac{\rho(w_1) + \rho(w_2) - 2r^+_{nm}(w_1)r^+_{nm}(w_2)}{1 + \rho(w_1)\rho(w_2) - 2r^+_{nm}(w_1)r^+_{nm}(w_2)}, \tag{89}
$$

$$
T = \frac{1 - \rho(w_1) + \rho(w_2) + \rho(w_1)\rho(w_2)}{1 + \rho(w_1)\rho(w_2) - 2r^+_{nm}(w_1)r^+_{nm}(w_2)}. \tag{90}
$$

These show that how $R$ and $T$ for diode can be linked to the barrier coefficients at $w_1$ and $w_2$. Combining all to end up with probability one

$$
T + R = 1. \tag{91}
$$

This is among the interesting conclusion reached in this section, which analogue to that has been obtained for diode in zero field \[11\].

One can see how the obtained results for the diode case will change when we consider our system centered around the point $x_0$. In such case, its is not hard to show

$$
r^+_{nm}(w_1) = r^+_{nm}(w_2). \tag{92}
$$

After injecting this relation in the forms of different coefficient derived here, one can conclude that everything is transmitted, i.e. the transmission is total.
5.3 Limiting case

Another interesting case one should discuss is the resonant tunneling diode made of graphene in the presence of a magnetic field. This will allow us to characterize the system behavior and underline their properties. This limiting situation of the device is one where the barriers are described by a scalar Lorentz potential of the form [11]

$$V(x, y) = \lim_{\epsilon \to 0} \frac{1}{2\epsilon} [1 - \theta(|x| - \epsilon)] \sigma_z = g \sigma_z \delta(x). \quad (93)$$

The connection with the true barrier is made by identifying $g$ with $2\gamma'\omega a$ where $\gamma$ a numerical constant of dimensions inverse of length and $a$ is the carbon-carbon distance. This form of the potential is equivalent to a mass term and therefore to a gap in the spectrum. However, given the short range nature of the potential, its effect comes only in the boundary conditions imposed on the wavefunction at the potential position. Note in passing that, the problem of Dirac electrons in delta function potentials has been studied in the past [14] and for a recent review one can consult [11].

To study (93) in terms of our language, we adopt the same method as has been applied to the diode case for zero field [11]. It is based on the boundary condition around the point $x = 0$ of the eigenspinors. This is equivalent to write

$$\begin{pmatrix} \varphi_1(0^+) \\ \varphi_2(0^+) \end{pmatrix} = M \begin{pmatrix} \varphi_1(0^-) \\ \varphi_2(0^-) \end{pmatrix} \quad (94)$$

where the matrix $M$ is given by

$$M = \begin{pmatrix} \cosh(\tilde{g}) & i \sinh(\tilde{g}) \\ -i \sinh(\tilde{g}) & \cosh(\tilde{g}) \end{pmatrix}. \quad (95)$$

The points $0^\pm$ represent positive and negative infinitesimals, the constant is $\tilde{g} = \frac{2\gamma'\omega a}{\epsilon_F \hbar}$. For some reasons, we need the inverse of (94), which can be obtained by determining $M^{-1}$. It is easy to get

$$M^{-1} = \begin{pmatrix} \cosh(\tilde{g}) & -i \sinh(\tilde{g}) \\ i \sinh(\tilde{g}) & \cosh(\tilde{g}) \end{pmatrix}. \quad (96)$$

which together they verify the unitary condition $M^{-1}M = I_2$. Combining all to end up with the inverse of (94), such as

$$\begin{pmatrix} \varphi_1(0^-) \\ \varphi_2(0^-) \end{pmatrix} = M^{-1} \begin{pmatrix} \varphi_1(0^+) \\ \varphi_2(0^+) \end{pmatrix}. \quad (97)$$

After getting these two equations, one can use the former analysis to show that how the obtained results for diode in magnetic field will be affected. For this, we consider also two cases: propagation with positive and negative incidences.

**Propagation with positive incidence:** Returning to our tools, we can map (94) in terms of the cylindrical parabolic functions as

$$\begin{pmatrix} \varphi_1(0^-) \\ \varphi_2(0^-) \end{pmatrix} = \begin{pmatrix} -siD_{[n]-1}(x_0) & si(-1)^{|n|}D_{[n]-1}(x_0) \\ D_{[n]}(x_0) & (-1)^{|n|}D_{[n]}(x_0) \end{pmatrix} \begin{pmatrix} A^+ \\ B^+ \end{pmatrix}. \quad (98)$$
and in similar way, we have
\[
\begin{pmatrix}
\varphi_1(0^+) \\
\varphi_2(0^+)
\end{pmatrix}
= C^+ \begin{pmatrix}
-siD_{|n|-1}(x_0) \\
D_{|n|}(x_0)
\end{pmatrix}
\]  
(99)

where \( A^+, B^+ \) and \( C^+ \) are three parameters those should be determined. According to (93), the last equation takes the form
\[
\begin{pmatrix}
\varphi_1(0^+) \\
\varphi_2(0^+)
\end{pmatrix}
= M \begin{pmatrix}
-siD_{|n|-1}(x_0) & si(-1)^{|n|}D_{|n|-1}(x_0) \\
D_{|n|}(x_0) & (-1)^{|n|}D_{|n|}(x_0)
\end{pmatrix}
\begin{pmatrix}
A^+ \\
B^+
\end{pmatrix}
. 
\]  
(100)

This can be solved to obtain \( A^+ \) and \( B^+ \) as
\[
A^+ = \frac{2s \cosh(\tilde{g})D_{|n|}(x_0)D_{|n|-1}(x_0) + \sinh(\tilde{g}) \left[D_{|n|}^2(x_0) + D_{|n|-1}^2(x_0)\right]}{2D_{|n|}(x_0)D_{|n|-1}(x_0)}  
\]  
(101)

\[
B^+ = -\left(-1\right)^{|n|} \frac{\sinh(\tilde{g}) \left[D_{|n|-1}^2(x_0) - D_{|n|}^2(x_0)\right]}{2D_{|n|}(x_0)D_{|n|-1}(x_0)} .
\]  
(102)

The usefulness of these is to define the reflection and transmission coefficients for the present situation. Indeed, After calculation, we show
\[
r^+ = \frac{B^+}{A^+} = \frac{-\left(-1\right)^{|n|} \sinh(\tilde{g}) \left[D_{|n|-1}^2(x_0) - D_{|n|}^2(x_0)\right]}{2s \cosh(\tilde{g})D_{|n|}(x_0)D_{|n|-1}(x_0) + \sinh(\tilde{g}) \left[D_{|n|}^2(x_0) + D_{|n|-1}^2(x_0)\right]}  
\]  
(103)

\[
t^+ = \frac{C^+}{A^+} = \frac{2sD_{|n|}(x_0)D_{|n|-1}(x_0)}{2s \cosh(\tilde{g})D_{|n|}(x_0)D_{|n|-1}(x_0) + \sinh(\tilde{g}) \left[D_{|n|}^2(x_0) + D_{|n|-1}^2(x_0)\right]} .
\]  
(104)

**Propagation with negative incidence:** In this case, we can write for \( 0^+ \)
\[
\begin{pmatrix}
\varphi_1(0^+) \\
\varphi_2(0^+)
\end{pmatrix}
= \begin{pmatrix}
-siD_{|n|-1}(x_0) & si(-1)^{|n|}D_{|n|-1}(x_0) \\
D_{|n|}(x_0) & (-1)^{|n|}D_{|n|}(x_0)
\end{pmatrix}
\begin{pmatrix}
A^- \\
B^-
\end{pmatrix}
. 
\]  
(105)

as well as for \( 0^- \)
\[
\begin{pmatrix}
\varphi_1(0^-) \\
\varphi_2(0^-)
\end{pmatrix}
= C^- \begin{pmatrix}
-siD_{|n|-1}(x_0) \\
D_{|n|}(x_0)
\end{pmatrix}
. 
\]  
(106)

Using the same technique as above, one can obtain
\[
r^- = \frac{B^-}{A^-} = \frac{-\left(-1\right)^{|n|} \sinh(\tilde{g}) \left[D_{|n|}^2(x_0) - D_{|n|-1}^2(x_0)\right]}{2s \cosh(\tilde{g})D_{|n|}(x_0)D_{|n|-1}(x_0) - \sinh(\tilde{g}) \left[D_{|n|}^2(x_0) + D_{|n|-1}^2(x_0)\right]}  
\]  
(107)

\[
t^- = \frac{C^-}{A^-} = \frac{2sD_{|n|}(x_0)D_{|n|-1}(x_0)}{2s \cosh(\tilde{g})D_{|n|}(x_0)D_{|n|-1}(x_0) - \sinh(\tilde{g}) \left[D_{|n|}^2(x_0) + D_{|n|-1}^2(x_0)\right]} .
\]  
(108)

Having all ingredients, let us check the probability condition. This can be achieved by defining two quantities in terms of the above coefficients, such as
\[
R = r^+r^- = \frac{-\sinh^2(\tilde{g}) \left[D_{|n|}^2(x_0) - D_{|n|-1}^2(x_0)\right]^2}{4\cosh^2(\tilde{g})D_{|n|}^2(x_0)D_{|n|-1}^2(x_0) - \sinh^2(\tilde{g}) \left[D_{|n|}^2(x_0) + D_{|n|-1}^2(x_0)\right]^2}  
\]  
(109)

\[
T = t^+t^- = \frac{4D_{|n|}^2(x_0)D_{|n|-1}^2(x_0)}{4\cosh^2(\tilde{g})D_{|n|}^2(x_0)D_{|n|-1}^2(x_0) - \sinh^2(\tilde{g}) \left[D_{|n|}^2(x_0) + D_{|n|-1}^2(x_0)\right]^2} .
\]  
(110)
After a straightforward calculation, one can find a probability one

\[ R + T = \frac{4D_{|n|}^2(x_0)D_{|n|}^2(x_0) - \sinh^2(\tilde{g}) \left[ D_{|n|}^2(x_0) - D_{|n|}^2(x_0) \right]^2}{4 \cosh^2(\tilde{g})D_{|n|}^2(x_0)D_{|n|}^2(x_0) - \sinh^2(\tilde{g}) \left[ D_{|n|}^2(x_0) + D_{|n|}^2(x_0) \right]^2} = 1. \]  

(111)

This also has been obtained by studying the scalar Lorentz potential in zero field [11].

Let us analyze some limits of the transmission coefficient. This can be reached by inspecting (109) and as well as (110). Clearly, there is a trivial solution, i.e. \( T = 1 \), which can easily be obtained by requiring that the involved parameter \( \tilde{g} \) is nothing but zero. Furthermore, other interesting discussions can be reported by looking some cases involving the parameter \( g \). After calculation, we end up with the results

- \( g \ll v_F \hbar \implies T \to 1 \).
- \( g \gg v_F \hbar \implies T \to 0 \).

Of course, one can also try to plot different figures in order to give a full description of the transmission behavior with respect to the scalar Lorentz potential taken here.

6 Conclusion

The present paper is devoted to the study of the tunneling effect for Dirac fermions in the presence of a constant magnetic field \( B \). More precisely, we started by splitting our system in two different regions where the second one is characterized by an energy gap \( t' \). Moreover, we derived the energy spectrum and the corresponding eigenspinors for each region. They are used to deal with our task and in particular show how Dirac fermions behave in such system. From the energy conservation \( E \), we established an interesting relation between two quantum numbers \((n, m)\) involved in game and the system energy. This allowed us to control the behavior of the energy and therefore fix the allowed values of \((n, m)\) where their behaviors are plotted in the figures (1, 2, 3). In analyzing their properties, we noticed that there are three limiting cases those have significant influences on the study of the tunneling effect.

Subsequently, we concentrated on two examples of the present system. Indeed, by considering a barrier in magnetic field, we analyzed the propagation for positive and negative incidences. In fact, using the continuity equations we determined the corresponding reflexion and transmission coefficients. They are obtained as a function of the ratio \( \frac{E}{t'} \) and more importantly, they verified the probability condition, i.e. their sum is equal one. To characterize transmission of the system, we plotted the figures (4, 5, 6) showing its behaviors for different values of the involved parameters. Using the three limiting cases of \((n, m)\), we noticed that the system can be seen either as a diopter or mirror depending to which case is included. These actually are corresponding to a total reflexion or total transmission, respectively.

We focused on another interesting case that is a diode in magnetic field. This is equivalent to consider a system composed of three regions where the second regions is completely different from the first and third, which they are identical. After getting different coefficients for positive and negative
incidences, we defined two new quantities to end up with their sum is equal one, i.e. probability one. Requiring some constraint on the system, we showed that it is possible to have a total transmission. Moreover, we considered the case where the barriers are described by a scalar Lorentz potential of the form given in [93]. This allowed us to study the resonant tunneling of diode make in graphene where different coefficients are obtained and their probability condition is verified. Moreover, we showed that there is a possibility to have a total transmission in such case.

Finally, our findings so far will not remain at this stage. In fact, still some interesting questions one should answer to get a full descriptions of different issues. These concern for instance the Fabry–Pérot solid etalon in the usual optics [15] and the related finesse factor. Since we have transmission, one can also ask about the conductivity to discuss the anomalous quantum Hall effect for the present system. Hopefully to come back to these questions in a subsequent work.

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