Unitary symmetry of sum rules for hadron photoproduction on octet baryons
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Dubnicka-Dubnickova-Kuraev (DDK) sum rules are considered. It is shown that integrals over differences of the total photoproduction cross-sections on octet baryons could be understood in terms of unitary symmetry approach. All the DDK sum rules for these quantities are expressed in terms of only three parameters.

High Energy Physics - Phenomenology

1 Introduction

Many years ago Gottfried [1] wrote dispersion sum rule interlacing proton magnetic moment and proton charge radius with integral over total photoproduction cross-section on protons:

$$\int_0^{\infty} \frac{d\nu}{\nu} \left[ \sigma_{\gamma p \to X}^{p} (\nu) \right] = 4\pi^2 \alpha \frac{1}{3} \langle r_{Ep}^2 \rangle + \frac{1 - \mu_p^2}{4m_p^2}$$

where $\langle r_{Ep}^2 \rangle$ is the proton mean square charge radius and $\mu_p = 1 + \kappa_p$ and $\kappa_p$ are total and anomalous magnetic moments of proton in terms of nuclear magnetons.

As years ago it was proved experimentally that total photoproduction cross-section on protons is arising with energy [2] the Gottfried sum rule results to contain diverging integral and therefore cannot be valid.

But recently an interesting sum rules for photoproduction of baryons were proposed which overcome this difficulty considering instead differences between integrals over total cross sections on various baryons [3]. In this way convergency of the integral was achieved and series of sum rules were evaluated [4, 5]. The main assumption for the convergency lies in equality of the Pomeron exchange for all the baryons of the octet. It is seems to be valid for the baryons of the same isomultiplet and plausible for the whole unitary octet.

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2 DDK universal sum rules

The DDK universal sum rule [4] can be written as the equality with left-hand side (LHS) in terms of baryon anomalous magnetic moments and Dirac baryon mean square radii and right-hand side (RHS) as integral over difference of total photoproduction cross-sections on octet baryons

\[
\frac{1}{3} \left[ F_{1B}(0) \langle r_{1B}^2 \rangle - F_{1B'}(0) \langle r_{1B'}^2 \rangle \right] = \left[ \frac{\kappa_B^2}{4m_B^2} - \frac{\kappa_{B'}^2}{4m_{B'}^2} \right] = \frac{2}{\pi^2\alpha} \int_{\omega_B}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma B \to X}^\text{tot}(\omega) - \sigma_{\gamma B' \to X}^\text{tot}(\omega) \right],
\]

which relates Dirac baryon mean-square radii \( \langle r_{1B}^2 \rangle, \langle r_{1B'}^2 \rangle \) and anomalous magnetic moments \( \kappa_B, \kappa_{B'} \) to the convergent integral due to presumed cancellation of the otherwise arising high energy total cross-sections at \( \omega \to \infty \).

Electric form factors \( F_{1B}(q^2), F_{1B'}(q^2) \) reduce to electric charges \( e_B, e_{B'} \) at zero momentum transfer squared \( q^2 = 0 \). Instead the Dirac baryon mean-square radii \( \langle r_{1B}^2 \rangle \) can be reliably taken from the relation

\[
\langle r_{EB}^2 \rangle = \langle r_{1B}^2 \rangle + 3 \frac{\kappa_B}{4m_B^2}
\]

either from experimental data (for \( p, n \) and \( \Sigma^- \))[2] or from theory [6]. We put them into the Table 1. ( Note that sum of the 4th and 5th columns just give the 3rd one. )

We remind the first sum rule of [3]

\[
\frac{1}{3} \left( \langle r_{1p}^2 \rangle - \frac{\kappa_p^2}{4m_p^2} + \frac{\kappa_n^2}{4m_n^2} \right) = \frac{2}{\pi^2\alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma p \to X}^\text{tot}(\omega) - \sigma_{\gamma n \to X}^\text{tot}(\omega) \right]
\]

with \( \omega_N = m_\pi + m_N^2/2M_N \).

Here \( \langle r_{1p}^2 \rangle \) is electric square radius of the proton and \( \kappa_{p,n} \) mean anomalous magnetic moments of proton and neutron. It agrees well with experiment as LHS=1.93±0.18 mb and RHS=1.92±0.32 mb [3].

It was generalized in [4] to all the octet baryons writing 28 relations. But as it is easy to see only 7 of them are linearly independent, and we choose them as Eq.(1) plus other 6 relations below. The important issue is that while treating photoproduction on \( \Lambda \) ( or \( \Sigma^0 \) ) we should also add the \( \Sigma^0 \Lambda \) contribution. For the differences \( \Sigma^0 - \Lambda \) these contributions cancel each
other. But in other cases with single Σ⁰ or single Λ in pair with any other baryon we have found noticeable effects.

\[
\frac{1}{3} \langle r_{1Σ^+}^2 \rangle - \frac{\kappa_{Σ^+}^2}{4m_{Σ^+}^2} + \frac{\kappa_{Σ⁰}^2}{4m_{Σ⁰}^2} + \frac{\kappa_{ΣΛ}^2}{4m_{ΣΛ}^2} = \frac{2}{\pi^2\alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{Σ^+\rightarrow X}(\omega) - \sigma_{tot}^{Σ⁰\rightarrow X}(\omega)];
\]

\[
\frac{1}{3} \langle r_{1Σ^-}^2 \rangle - \frac{\kappa_{Σ^-}^2}{4m_{Σ^-}^2} + \frac{\kappa_{Σ⁰}^2}{4m_{Σ⁰}^2} + \frac{\kappa_{ΣΛ}^2}{4m_{ΣΛ}^2} = \frac{2}{\pi^2\alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{Σ^-\rightarrow X}(\omega) - \sigma_{tot}^{Σ⁰\rightarrow X}(\omega)];
\]

\[
\frac{1}{3} \langle r_{1π^+}^2 \rangle - \frac{\kappa_{π^+}^2}{4m_{π^+}^2} + \frac{\kappa_{Σ^-}^2}{4m_{Σ^-}^2} + \frac{\kappa_{ΣΛ}^2}{4m_{ΣΛ}^2} = \frac{2}{\pi^2\alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{π^+\rightarrow X}(\omega) - \sigma_{tot}^{Σ^-\rightarrow X}(\omega)];
\]

\[
\frac{1}{3} \langle r_{1π^-}^2 \rangle - \frac{\kappa_{π^-}^2}{4m_{π^-}^2} + \frac{\kappa_{Σ⁻}^2}{4m_{Σ⁻}^2} = \frac{2}{\pi^2\alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{π^-\rightarrow X}(\omega) - \sigma_{tot}^{Σ⁻\rightarrow X}(\omega)];
\]

We control relations by putting also two corollaries:

\[
\frac{2}{\pi^2\alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{Σ^+\rightarrow X}(\omega) - \sigma_{tot}^{Σ⁻\rightarrow X}(\omega)] = 4.0131 \text{ mb} (4.2654 \text{ mb} [4])
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{π^-\rightarrow X}(\omega) - \sigma_{tot}^{Σ⁻\rightarrow X}(\omega)] = -0.4075 + 0.0875 = -0.3200 \text{ mb} (-0.3156 \text{ mb} [4])
\]

Fig.1. Inclusive photoproduction on octet baryon B 1/2⁺
3 Unitary symmetry relations

We study now unitary symmetry of these sum rules for the differences of the integrals of total photoproduction cross-sections on baryons of the octet $1/2^+$. As we will see all of them could be described in terms of only 3 parameters.

We remind universal formula for magnetic moments which transfers either in the NRQM formula (with $F=2/3$ and $D=1$ and $e_q \rightarrow \mu_q$) or into the unitary symmetry result (by putting quark electric charge explicitly) [9]:

$$\mu_{\Sigma^0} = (e_u + e_d)F + e_s(F - D).$$  \hspace{1cm} (6)

All the other baryon magnetic moments but that of $\Lambda$ are obtained just by changing properly quark indices. Instead that of the $\Lambda$ baryon could be written as

$$\mu_{\Lambda} = \frac{1}{3}[2\mu_{\Sigma^0} + 2\mu_{\Sigma^*} - \mu_{\Sigma^0}] = \frac{1}{3}[(e_u + e_d + 4e_s)F + (2e_u + 2e_d - e_s)](F - D)].$$  \hspace{1cm} (7)

For the corresponding transition moment one get

$$\sqrt{3}\mu_{\Sigma^0\Lambda} = \mu_{\Sigma^0 s} - \mu_{\Sigma^* s} = (e_u - e_d)D,$$  \hspace{1cm} (8)

where subscribes $ds$ and $us$ mean just that we interchanged quarks $d \leftrightarrow s$ and $u \leftrightarrow s$ in the $\Sigma^0$ wave function.

We have seen that similar reasoning can be applied to the QCD sum rules not only for magnetic moments [10] but also for other vertex quantities [8]. But in the QCD sum rules the quantities analogues to $F, D$ would contain the dependence on the quark parameters absent in the simple unitary symmetry model [10].

We now proceed with differences of the total cross-sections on octet baryons by using as operator not the electric charge of quarks but their electric charge squared. That is, we describe finite parts of the integrals in Eqs.(4,5) which we denote just by symbol of the target, in terms of $F, E$’s depending on the quark parameters at this stage only phenomenologically putting subindices $s$ and $ss$ to indicate number of strange quarks in baryon

$$\mathcal{P} = 2e_u^2F + e_d^2E$$  \hspace{1cm} (9)

$$\Sigma^0 = (e_u^2 + e_d^2)F^s + e_s^2E^s$$  \hspace{1cm} (10)

$$\Xi^0 = 2e_s^2F^{ss} + e_s^2E^{ss}$$  \hspace{1cm} (11)
Table 1: Contributions of magnetic moments and charge radii of octet baryons $1/2^+$ in mb

| B    | $\kappa_B [\mu_N]$ | $\langle r_{EB}^2 \rangle$ [mb] | $3\kappa_B / 2m_B^2$ [mb] | $\langle r_{1B}^2 \rangle$ [mb] | $\kappa_B^2 / 4m_B^4$ [mb] |
|------|---------------------|----------------------------------|---------------------------|-------------------------------|----------------------------|
| p    | 1.7928              | 7.17                             | 1.19                      | 5.98                          | 0.3560                     |
| n    | -1.9130             | -1.13                            | -1.27                     | 0.14                          | 0.4075                     |
| $\Lambda$ | -0.6130          | 1.10                             | -0.29                     | 1.39                          | 0.0295                     |
| $\Sigma^+$ | 1.4580             | 6.00                             | 0.60                      | 5.40                          | 0.1458                     |
| $\Sigma^0$ | 0.6490             | -0.30                            | 0.27                      | -0.57                         | 0.0293                     |
| $\Sigma^-$ | -0.1600            | 6.70                             | -0.07                     | 6.77                          | 0.0019                     |
| $\Xi^0$ | -1.250              | 1.30                             | -0.42                     | 1.72                          | 0.0875                     |
| $\Xi^-$ | 0.3493              | 4.90                             | 0.12                      | 4.78                          | 0.0070                     |

Upon using our relations [9, 10] we write for the finite $\Lambda$ contribution

$$\Lambda = \frac{1}{3}[(e_u^2 + e_d^2 + 4e_s^2)F^s + [2e_u^2 + 2e_d^2 - e_s^2]E^s]$$

(12)

with $E^s = E - D$ in the unitary limit.

The structure $F$ corresponds to the contributions of two (quasi)similar quarks ($uu, dd, ud, ss$), while the structure $E$ corresponds to single-quark contribution [9, 10]. Subindices $s$ and $ss$ indicate number of strange quarks in baryon.

Thus we can write the righthand sides of the DDK sum rules, that is, the integral over differences of the total cross-sections on two different baryons $B$ and $B'$. We choose $p$ and $\Sigma^+$ putting the rest into the Table 2 and put them into the form

$$[2e_u^2F + e_d^2E] - [2e_u^2F^s + e_d^2E^s] = \int_{\omega_{\text{thresh}}}^{\infty} \frac{d\omega}{\omega}(\sigma_{\gamma p \rightarrow X}(\omega) - \sigma_{\gamma \Sigma^+ \rightarrow X}(\omega)).$$

(13)

We repeat that while treating photoproduction on $\Lambda$ (or $\Sigma^0$) we should also add the $\Sigma^0\Lambda$ contribution. For the differences $\Sigma^0 - \Lambda$ these contributions cancel each other. But in other cases with single $\Sigma^0$ or single $\Lambda$ in pair with any other baryon we have obtained noticeable effects.

We have succeeded to fit the RHS’s of Eqs(13) with $F = E = F^s = E^s = 6$ and $F^{ss} = 3.3, E^{ss} = 4.5$ (see Table 2). Only cascade hyperon contributions differ strongly from unitary symmetry parameters which can
be explained by presence of two heavy quarks and only one light quark in these hyperons. Discovery of doubly charmed baryons $\Xi_{cc}$ [2] can help us to solve this discord.

### 4 Conclusion

We have shown that unitary symmetry of the BDDK sum rules for the differences of the integrals of total photoproduction cross-sections on baryons of the octet $1/2^+$ holds within some reasonable lines. In fact we succeeded in describing DDK sum rules in terms of only 3 parameters. It is also shown that one should take into account not only $\Sigma^0$ and $\Lambda$ baryons while analysing total cross-sections $\sigma^{\gamma \Sigma^0 \to X}$ and $\sigma^{\gamma \Lambda \to X}$ but also take into account the $\Sigma^0\Lambda$ transition mode.

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