Search for lepton flavor violation in supersymmetric models via meson decays

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Abstract

Considering the constraints from the experimental data on $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu - e$ conversion etc., we analyze the Lepton Flavor Violating decays $\phi(J/\Psi, \Upsilon(1S)) \rightarrow e^+\mu^- (\mu^+\tau^-)$ in the scenarios of the minimal supersymmetric extensions of Standard Model with seesaw Mechanism. Numerically, there is parameter space that the LFV processes of $J/\Psi(\Upsilon) \rightarrow \mu^+\tau^-$ can reach the upper experimental bounds, meanwhile the theoretical predictions on $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu - e$ conversion satisfy the present experimental bounds. For searching of new physics, Lepton Flavor Violating processes $J/\Psi(\Upsilon) \rightarrow \mu^+\tau^-$ may be more promising and effective channels.

Keywords: Lepton flavor violating, supersymmetry, see-saw.

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1. Introduction

As an evidence to discover new physics beyond the Standard Model (SM), searching for Lepton Flavor Violating (LFV) processes in charged lepton sector have attracted a great deal of attention. The theoretical predictions on those lepton flavor violating processes are suppressed by small masses of neutrinos in SM, and exceed the detecting extent of experiment in near future. Nevertheless, the corrections to the branching ratios of LFV decays $\phi \rightarrow e^+\mu^-$, $J/\Psi \rightarrow \mu^+\tau^-$ and $\Upsilon \rightarrow \mu^+\tau^-$ are enhanced by the new sources of LFV in various extensions of the SM, such as grand unified models [1].
supersymmetric models with and without R-parity \[2\], left-right symmetry models \[3\] etc. Although nonzero neutrino masses supported by the neutrino oscillation experiments \[4\] imply the non-conservation of lepton flavor, it is very important to directly search the LFV processes of charged lepton sector in colliders running now.

Using a sample of \(5.8 \times 10^7\) \(J/\Psi\) events collected with the BESII detector, Ref.\[5\] obtains the upper limits on \(\text{BR}(J/\Psi \to \mu\tau) < 2.0 \times 10^{-6}\) and \(\text{BR}(\Upsilon \to \mu\tau) < 8.3 \times 10^{-6}\) at the 90% confidence level (C.L.). Adopting the data collected with the CLEO III detector, the authors of Ref.\[6\] estimate the upper limits on \(\text{BR}(\Upsilon(1S) \to \mu\tau) < 6.0 \times 10^{-6}\), \(\text{BR}(\Upsilon(2S) \to \mu\tau) < 1.4 \times 10^{-5}\) and \(\text{BR}(\Upsilon(3S) \to \mu\tau) < 2.0 \times 10^{-5}\) respectively at the 95% C.L. Additionally, the study of LFV processes involving light unflavored meson is an effective way maybe to search for new physics beyond the SM, and the SND Collaboration at the BINP (Novosibirk) presents an upper limit on the \(\phi \to e^+\mu^-\) branching fraction of \(\text{BR}(\phi \to e^+\mu^-) \leq 2 \times 10^{-6}\) \[7\].

In literature, several stringent limits on LFV decays of both light and heavy unflavored mesons are derived already. Assuming that a vector boson \(M_i\) (\(M_i\) could be either a fundamental state, like the \(Z_0\), or a quark-antiquark bound state like the \(\phi, J/\Psi, \Upsilon\)) couples to \(\mu^\pm e^\mp\) and \(e^\pm e^\mp\) as:

\[
\mathcal{L}_{\text{eff}} = g_{M_i\mu\mu}\bar{\mu}\gamma_\mu M_i^\mu + g_{M_i\mu e\mu}\bar{\mu}\gamma_\mu e M_i^\mu + h.c. ,
\]

(1)

where \(g_{M_i\mu\mu}\) and \(g_{M_i\mu e\mu}\) denote the corresponding couplings of a meson to lepton flavor conservation and violation currents, and by unitarity its exchange contributes to \(\mu \to 3e\), the authors of Ref.\[8\] deduce upper bounds on the LFV decay of mesons from the LFV process \(\mu \to 3e\). Under a similar assumption that a vector meson \(M_i\) couples to \(\mu^\pm e^\mp\) and \(NN\) as:

\[
\mathcal{L}_{\text{eff}} = (\xi_{V,M}^\mu \bar{\mu}\gamma_\mu \mu + \xi_{A,M}^\mu \bar{\mu}\gamma_\mu \gamma_5 \mu) M_i^\mu + g_{MNN}\bar{N}\gamma_\mu N M_i^\mu + h.c. ,
\]

(2)

where \(N\) is a nucleon, \(\xi_{V,A}^M\) are effective vector and axial couplings of a meson to the LFV lepton currents, authors of Ref.\[9\] studies the LFV decays of vector mesons by taking account of the experimental constraint on \(\mu - e\) conversion. It shows the constraint from \(\mu - e\) conversion on LFV decays of vector mesons is more stronger. Likewise, authors of Ref.\[8\] also deduce upper bounds on other LFV decay of mesons from the LFV processes \(\tau \to 3e\) and \(\tau \to 3\mu\). Making the assumption that fermion mixing and mass hierarchy originate from mass matrix rotation, authors of Ref.\[10\] also get some upper limits on the LFV decays of heavy unflavored mesons and Z boson. Searching
for new physics beyond the SM is also a goal of LHC. In LHC, vector mesons can be produced by photo fusion \([11]\).

In SM, the LFV decays mainly originate from the charged current with the mixing among three lepton generations. The fields of the flavor neutrinos in charged current weak interaction Lagrangian are combinations of three massive neutrinos:

\[
\mathcal{L} = -\frac{g_2}{\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma_{\mu} \nu_{lL}(x) W^\mu(x) + h.c.,
\]

\[
\nu_{lL}(x) = \sum_{i=1}^{3} (U_{MNS})_{li} \nu_{iL}(x), \tag{3}
\]

where \(g_2\) denotes the coupling constant of gauge group SU(2), \(\nu_{lL}\) are fields of the flavor neutrinos, \(\nu_{iL}\) are fields of massive neutrinos, and \(U_{MNS}\) corresponds to the MNS neutrino mixing matrix \([12, 13]\). In the standard parametrization \([14]\), the leptonic mixing matrix is given by:

\[
U_{MNS} = \begin{pmatrix}
  c_1 c_3 & c_3 s_1 & s_3 e^{-i\delta} \\
  -c_1 s_3 s_2 e^{i\delta} - c_2 s_1 & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & c_3 s_2 \\
  s_1 s_2 - c_1 s_3 c_2 e^{i\delta} & c_1 s_2 - s_1 c_2 s_1 e^{i\delta} & c_3 c_2
\end{pmatrix}
\times \text{diag}(e^{i\Phi_1/2}, 1, e^{i\Phi_2/2}), \tag{4}
\]

where \(s(c)_1 = \sin(\cos)\theta_{12}, s(c)_2 = \sin(\cos)\theta_{23}, s(c)_3 = \sin(\cos)\theta_{13}\). The phase \(\delta\) is the Dirac CP phase, and \(\Phi_1, \Phi_2\) are the Majorana phases. A global fit of the neutrino oscillation data points out: \(\theta_{12} \sim 34^\circ\) and \(\theta_{23} \sim 45^\circ\). Recently, the observing \(\bar{\nu}_e\) disappearance in reactor experiments Daya Bay \([15]\) and RENO \([16]\) have definitely established that \(\theta_{13} > 0\) at \(\sim 5\sigma\) level. The Daya Bay and RENO have measured \(\sin \theta_{13} \simeq 0.024\) and \(\sin \theta_{13} \simeq 0.029\), respectively. However, the theoretical predictions on branching ratios of any LFV decays are suppressed strongly by the tiny neutrino masses in SM and fall out the reach of experiment in near future. In this work, we analyze the LFV decays: \(\phi \rightarrow e^+\mu^-\), \(J/\Psi \rightarrow \mu^+\tau^-\) and \(\Upsilon(1S) \rightarrow \mu^+\tau^-\) in the framework of accommodating supersymmetry with type I seesaw mechanism simultaneously. With the accumulation of events on BEPC \([5]\) and SuperKEKB \([17]\), the updated experimental data on those LFV decays maybe constrain the concerned models more stringent. To shorten the length of text, we just present the upper bounds on those branching ratios of \(\rho(\omega, J/\Psi, \Upsilon) \rightarrow e^+\mu^-\) under our assumptions on parameter space.
The paper is organized as follows. In Section 2, we firstly provide a simple overview for the origin of lepton flavor changing and corresponding interaction lagrangian in the framework of MSSM with type I seesaw mechanism. Then, as an example, we derive the analytic results of amplitude for one diagram in detail. The numerical results are presented in Section 3, and the conclusion is drawn in Section 4. All the simplified amplitudes corresponding to the Feynman diagrams in Fig. 1 and Fig. 2 are given in Appendix A.

2. Formalism

In the minimal supersymmetric extension of SM with R-parity conservation, the general form of the superpotential involving the lepton and Higgs superfields is given by [18]:

$$W_{MSSM} = \epsilon^{ij} \left( \mu \hat{H}^1_i \hat{H}^2_j + Y_{IJ}^l \hat{H}^1_i \hat{L}^j \hat{R}^j \right), \quad (5)$$

where $\mu$ is the mu-parameter, the $3 \times 3$ matrix $Y_l$ is the charged lepton Yukawa couplings. For convenience, we assume $Y_{IJ}^l = Y^l \delta_{IJ}$ $(1, J = 1, 2, 3)$ in this work. Then, the relevant soft supersymmetry breaking terms involving the slepton sector and sneutrino sector are:

$$V_{soft}^{MSSM} = \left( m^2_L \right)^{IJ} \tilde{L}_i^* \tilde{L}_i^j + \left( m^2_R \right)^{IJ} \tilde{R}_i^* \tilde{R}_i^j - A_{IJ}^l e^{ij} \hat{H}^1_i \hat{L}^j \hat{R}^j - A_{IJ}^l \hat{H}^1_i \hat{L}^j \hat{R}^j - h.c. , \quad (6)$$

where $m^2_L$ is left $3 \times 3$ soft slepton mass matrix, $m^2_R$ is right $3 \times 3$ soft slepton mass matrix, the $3 \times 3$ matrix $A_l$ is the trilinear scalar couplings, the $3 \times 3$ matrix $A'_l$ is the non-standard trilinear scalar couplings, respectively. The LFV interactions mainly originate from the potential misalignment between the leptons and sleptons mass matrices in the MSSM. In other words, the sources of LFV are the off-diagonal entries of the $3 \times 3$ soft supersymmetry breaking matrices $m^2_L, m^2_R, A_l$ and $A'_l$ in $6 \times 6$ slepton mass matrix $M^2_L$, which are listed below:

$$\left( M^2_L \right)^{IJ}_{LL} = \frac{e^2 (v^1_i - v^2_i) (1 - c^2_w)}{8 s^2_w e^2_w} \delta^{IJ} + \frac{v^2_i (Y_i^l)^2}{2} \delta^{IJ} + (m^2_L)^{IJ} , \quad (7)$$

$$\left( M^2_L \right)^{IJ}_{RR} = - \frac{e^2 (v^1_i - v^2_i)}{4 c^2_w} \delta^{IJ} + \frac{v^2_i (Y_i^l)^2}{2} \delta^{IJ} + (m^2_R)^{IJ} , \quad (8)$$

$$\left( M^2_L \right)^{IJ}_{LR} = \frac{1}{\sqrt{2}} (v_2 \mu^* Y_i^l \delta^{IJ} - A'^{IJ}_i) + v_1 A^{IJ}_i , \quad (9)$$

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where $s_w = \sin \theta_w, c_w = \cos \theta_w$ with $\theta_w$ denoting the Weinberg angle, and $\nu_{1,2}$ are the non zero vacuum expectation values (VEVs) of two Higgs doublets.

In the minimal supersymmetric extension of the seesaw extended SM \[19, 20, 21, 22, 23, 24\], there are three generation right handed neutrino superfields $\tilde{N}^I$ ($I = 1, 2, 3$) with zero hypercharge. The most general form of the superpotential involving the lepton and Higgs superfields in the R-parity conserving scenario is given by:

$$\mathcal{W} = \mathcal{W}_{\text{MSSM}} + \epsilon_{ij} Y_{\nu}^{IJ} \tilde{H}_I^2 \tilde{L}_J \tilde{N}_J + \frac{1}{2} M_{IJ}^1 \tilde{N}_I^1 \tilde{N}_J^1,$$

(10)

where $Y_\nu$ is the $3 \times 3$ neutral lepton Yukawa coupling, $M$ is the $3 \times 3$ Majorana mass matrix. Here, we adopt the parameterization in \[19\] to reproduce the PMNS mixing matrix:

$$ (Y_{\nu})^{ij} = i \sum_{k=1}^{3} \sqrt{2} (m_{\nu_L} M_{\nu_R})^{1/2} R_{jk} (U_{\text{MNS}}^*)_{ik} / \nu_2,$$

(11)

where $U_{\text{MNS}}$ is the MNS mixing matrix in Eq.(4), $m_{\nu_L} (i = e, \mu, \tau)$ are the masses of left handed neutrinos, and $M_{\nu_R} (i = e, \mu, \tau)$ are right handed neutrino masses. Furthermore, $R$ is an arbitrary orthogonal matrix \[25\] determined by three angles $\alpha_1, \alpha_2, \alpha_3$:

$$ R = \begin{pmatrix} c_2 c_3 & -c_1 s_3 - s_1 s_2 c_3 & s_1 s_3 - c_1 s_2 c_3 \\ c_2 s_3 & c_1 c_3 - s_1 s_2 s_3 & -s_1 c_3 - c_1 s_2 s_3 \\ s_2 & s_1 c_2 & c_1 c_2 \end{pmatrix},$$

(12)

in which $c_i = \cos \alpha_i$ and $s_i = \sin \alpha_i, i = 1, 2, 3$. In the scenarios of MSSM with Seesaw mechanism, the corrections from right handed Majorana neutrinos to the branching ratios of vector meson LFV decays can be ignored since they are suppressed by the huge masses of right handed neutrinos. In addition, the mass term for the light sneutrinos is given by:

$$ -\mathcal{L}_{\tilde{\nu}}^{\text{mass}} = \frac{1}{2} \left( \tilde{\nu}_L^I, \tilde{\nu}_L^{I*} \right) \mathcal{M}_{\tilde{\nu}}^2 \left( \tilde{\nu}_L^I, \tilde{\nu}_L^{I*} \right),$$

(13)

with $I, J = 1, 2, 3$ are the indices of generation, and the $6 \times 6$ mass matrix is

$$ \mathcal{M}_{\tilde{\nu}}^2 = \begin{pmatrix} (M_{LC}^2)^{IJ} & (M_{LV}^{2*})^{IJ} \\ (M_{LC}^{2*})^{IJ} & (M_{LV}^2)^{IJ} \end{pmatrix}. $$

(14)
Here $M_{LC}^2$ and $M_{LV}^2$ are $3 \times 3$ matrices. If $M_{LV}^2 = 0$, the six light sneutrinos are comprised of three sneutrino-antisneutrino pairs. If $M_{LV}^2 \neq 0$, the lepton number is violated and the sneutrinos and antisneutrinos can mix and yield six non-degenerate sneutrinos. The elements of $M_{LC}^2$ and $M_{LV}^2$ are given by, in a simple form at GUT scale:

\begin{align}
(M_{LC}^2)^{IJ} &= (m_L^2)^{IJ} + \frac{1}{2} M_Z^2 \cos 2\beta \delta^{IJ}, \\
(M_{LV}^2)^{IJ} &= -\frac{(\nu_2)^2 \mu^* \cot \beta}{2} (Y_{\nu} M_{-1}^{-1} Y_{\nu}^T)^{IJ},
\end{align}

where $M$ is the right handed neutrino mass matrix in Eq.(10) and $\tan \beta = \nu_2/\nu_1$. $M_{LC}^2$ reproduces the well known $3 \times 3$ light sneutrino matrix in MSSM.

In the CP-base

\begin{align}
\tilde{\nu}_L^{(+)} &= \frac{1}{\sqrt{2}} \left( \tilde{\nu}_L^I + \tilde{\nu}_L^I \right), \\
\tilde{\nu}_L^{(-)} &= -\frac{i}{\sqrt{2}} \left( \tilde{\nu}_L^I - \tilde{\nu}_L^I \right),
\end{align}

the mass term for the light sneutrinos is rewritten as:

\begin{align}
-\mathcal{L}_{\tilde{\nu}}^{mass} &= \frac{1}{2} \left( \begin{array}{c}
\tilde{\nu}_L^{(+)} \\
\tilde{\nu}_L^{(-)}
\end{array} \right) \tilde{\mathcal{M}}_{\tilde{\nu}}^2 \left( \begin{array}{c}
\tilde{\nu}_L^{(+)} \\
\tilde{\nu}_L^{(-)}
\end{array} \right).
\end{align}

Here, the $6 \times 6$ mass-squared matrix $\tilde{\mathcal{M}}_{\tilde{\nu}}^2$ is

\begin{align}
\tilde{\mathcal{M}}_{\tilde{\nu}}^2 &= \left( \begin{array}{cc}
\Re \left((M_{LC}^2)^{IJ} + (M_{LV}^2)^{IJ}\right) & \Im \left((M_{LC}^2)^{IJ} + (M_{LV}^2)^{IJ}\right) \\
\Im \left((M_{LC}^2)^{IJ} - (M_{LV}^2)^{IJ}\right) & \Re \left((M_{LC}^2)^{IJ} - (M_{LV}^2)^{IJ}\right)
\end{array} \right).
\end{align}

The effective squared-mass matrix can be diagonalized by $6 \times 6$ orthogonal matrix, $Z_{\tilde{\nu}}$ via:

\begin{align}
Z_{\tilde{\nu}}^T \tilde{\mathcal{M}}_{\tilde{\nu}}^2 Z_{\tilde{\nu}} &= (m_{S_1}^2, m_{S_2}^2, ..., m_{S_6}^2),
\end{align}

where $S_i (i = 1, ..., 6)$ correspond to the physical sneutrino mass eigenstates. The sneutrino interaction eigenstates, $\tilde{\nu}^I$, can be expressed in terms of the physical sneutrino mass eigenstates $S_k$ by:

\begin{align}
\tilde{\nu}_L^I &= \frac{1}{\sqrt{2}} \sum_{k=1}^{6} (Z_{\tilde{\nu}}^{I+k} + i Z_{\tilde{\nu}}^{(I+3)k}) S_k,
\end{align}

Here, the 6×6 mass-squared matrix $\tilde{\mathcal{M}}_{\tilde{\nu}}^2$ is given by:

\begin{align}
\tilde{\mathcal{M}}_{\tilde{\nu}}^2 &= \left( \begin{array}{cc}
\Re \left((M_{LC}^2)^{IJ} + (M_{LV}^2)^{IJ}\right) & \Im \left((M_{LC}^2)^{IJ} + (M_{LV}^2)^{IJ}\right) \\
\Im \left((M_{LC}^2)^{IJ} - (M_{LV}^2)^{IJ}\right) & \Re \left((M_{LC}^2)^{IJ} - (M_{LV}^2)^{IJ}\right)
\end{array} \right).
\end{align}
Correspondingly, the relevant Lagrangian is given as:

\[
\mathcal{L} = \chi_j^0 \left[ \left( \frac{e}{\sqrt{2}s_w c_w} Z_{1}^{l} (Z_{N}^{l} s_w + Z_{N}^{2j} c_w) + Y_{l}^{l} Z_{L}^{(J+3)i} Z_{N}^{3j} \right) P_L \\
+ \left( -\frac{e}{\sqrt{2} c_w} Z_{L}^{(l+3)i} Z_{N}^{j} + Y_{l}^{l} Z_{L}^{l} Z_{N}^{2l} \right) P_R \right] e^I \tilde{L}_i^+ \\
- \chi_i^C \left( \frac{e}{s_w} Z_{+}^{l} P_L + Y_{l}^{l} Z_{-}^{2l} P_R \right) \left( Z_{\tilde{\nu}}^{l} - i Z_{\tilde{\nu}}^{(l+3)i} \right) e^I S_j + h.c., \quad (23)
\]

where \( Z_\pm \) are the mixing matrices of chargino sector, \( Z_N, Z_L \) and \( Z_{\tilde{\nu}} \) are the mixing matrices of neutralino sector, slepton sector and sneutrino sector, respectively. \( e^I \) denote the SM charged leptons. \( \tilde{L}_i^+ \) and \( S_j \) denote the sleptons and sneutrinos. \( \chi_i^C \) and \( \chi_j^0 \) stand for the charginos and neutralinos. \( P_{L/R} = \frac{1}{2}(1 \mp \gamma_5) \). The relevant Feynman diagrams contributing to the LFV decays are presented in Fig.1 and Fig.2.

In the quark picture, mesons are composed of a quark and an anti-quark. As we analyze those LFV processes mentioned above, we do not want to
calculate the complicated loop integrations at quark-gluon level since the lack of a completely reliable way to calculate the non-perturbative QCD effects. We adopt a phenomenological model where the amplitude of hard process involving a s-wave meson can be described by the matrix elements of gauge invariant nonlocal operators, which are sandwiched between the vacuum and the meson states. For our case, the matrix is given by [26]:

$$\langle 0 | \bar{q}(y) \Gamma [y, x] q(x) | \phi \rangle ,$$  

(24)

where the momentum of \( \phi \) is on-shell, i.e. \( p^2 = m^2_\phi \), \( \varepsilon_\phi \) is the polarization vector, \( f_\phi \) and \( f^T_\phi \) are the decay constants of \( \phi \) meson, \( \phi_\parallel \) and \( \phi_\perp \) are the leading-twist distribution functions corresponding to the longitudinally and transversely polarized meson, respectively. For the cases of \( J/\Psi \) and \( \Upsilon \), there are similar distribution amplitudes. The integration variable \( u \) corresponds to the momentum fraction carried by the quark, \( \bar{u} = 1 - u \) stands for the momentum fraction carried by the anti-quark, \( \alpha \) and \( \beta \) are the indices of matrix elements, and \( N_c \) is the number of colors, separately. Since the leading-twist light-cone distribution amplitudes of meson are close to their asymptotic form [27], so we set \( \phi_\parallel = \phi_\perp = \phi(u) = 6u(1 - u) \).
Taking the diagram in Fig 1 as an example, we show how to write the corresponding correction to the LFV decay \( \phi \rightarrow e^+ \mu^- \) in MSSM with seesaw mechanism. At quark level, the relevant amplitude is written as:

\[
\mathcal{A}_Q = -\frac{e^2 g_{\mu\nu}}{4 s_w^2 c_w^2} \int \frac{d^D k}{(2\pi)^D} \frac{(Z_L^{2i} Z_L^{1j*} - 2 s_w^2 \delta^{ij})(p_3 + p_4)\nu}{[(p_1 + p_2)^2 - m_Z^2][k^2 - m_{\chi_0}^2]} \times \bar{u}_s(p_2)\gamma^\mu(P_L - \frac{2}{3} s_w^2 u_s(p_1)) [\nu(p_3) + \frac{2}{3} s_w^2 u_s(p_1)] \times \{ \left[ Z_L^{2i} (Z_N^{1k} s_W + Z_N^{2k} c_W) + Y_i Z_L^{2i} Z_N^{3k} \right] P_R + \left( Z_L^{1j} (Z_N^{1k} s_W + Z_N^{2k} c_W) + Y_i Z_L^{1j} Z_N^{3k} \right) P_L \} \frac{\nu(p_3)}{k + m_{\chi_0}}.
\]

In the frame of center of mass, one can write down the amplitude at hadron level using Eq. (25):

\[
\mathcal{A}_H = \frac{-e^2}{24 N_c s_w^2 c_w^2} \int \frac{d^D k}{(2\pi)^D} \frac{(Z_L^{2i} Z_L^{1j*} - 2 s_w^2 \delta^{ij})}{[(p_1 + p_2)^2 - m_Z^2][k^2 - m_{\chi_0}^2]} \times \bar{f}_\phi m_\phi (4 s_w^2 - 3) \varepsilon(p) \cdot (p_3 + p_4) \frac{\nu(p_4)}{[(p_3 - k)^2 - m_{\chi_0}^2][k^2 - m_{\chi_0}^2]} \times \{ \left[ Z_L^{2i} (Z_N^{1k} s_W + Z_N^{2k} c_W) + Y_i Z_L^{2i} Z_N^{3k} \right] P_R + \left( Z_L^{1j} Z_N^{1k} + Y_i Z_L^{1j} Z_N^{3k} \right) P_L \} \frac{\nu(p_3)}{k + m_{\chi_0}}.
\]
\[ \begin{align*}
&\times \{ \left[ Z_L^{ij}(Z_N^{ik} s_W + Z_N^{jk} c_W) + Y_L^i Z_L^{kj} Z_N^{lk} \right] P_L \\
+ \left[ Z_L^{kj} Z_N^{ik} + Y_L^i Z_L^{lj} Z_N^{3k} \right] P_R \} \nu_e(p_3). \\
\end{align*} \] (27)

Applying the high energy physics package FeynCalc \[28\], one can simplify the amplitude in terms of invariant Passarino-Veltman integrals \[29\]:

\[ A_H = \frac{ie^2 \pi^2 f_\phi m_\phi (4 s_\phi^2 - 3)}{24 N_c s_\omega^2 c_\omega (m_\phi^2 - m_\mu^2)^6} \sum_{i,j,k=1}^{6,6,4} (p_3 + p_4) \cdot \varepsilon(p) \]

\[ \times A_i^{ij} \bar{u}_\mu(p_4) \left\{ C_1 m_\mu(A_3^{kj} A_1^{ik} P_L + A_4^{kj} A_2^{ik} P_R) \\
+ C_2 \left[ (m_e A_3^{kj} A_1^{ik} - m_\mu A_4^{kj} A_2^{ik}) P_L \right] \\
+ (m_e A_4^{kj} A_2^{ik} - m_\mu A_3^{kj} A_1^{ik}) P_R \right\} \nu_e(p_3) \] (28)

All of integrals can be calculated through another high energy physics package LoopTools \[30\]. In a similar way, we can write down the corrections from other diagrams in Fig.1 and Fig.2 at hadron level, and list the simplified amplitudes in Appendix A.

Using the summation formula

\[ \sum_{\lambda=\pm 1,0} \varepsilon_\lambda^\mu(p) \varepsilon_\lambda^{\mu'}(p) \equiv -g^{\mu\nu} + \frac{p_\mu p_\nu}{m_\phi^2}, \] (29)

we express the branching ratio of \( \phi \rightarrow e^+ \mu^- \) as

\[ Br(\phi \rightarrow e^+ \mu^-) = \frac{\sqrt{[m_\phi^2 - (m_e + m_\mu)^2][m_\phi^2 - (m_e - m_\mu)^2]}}{16\pi m_\phi^3 \Gamma_\phi} \times \sum_i A_i A_i^*, \] (30)

in which \( \Gamma_\phi \) is the total decay width, \( A_i \) are the amplitudes in Appendix A. The branching ratios for \( J/\Psi(\Upsilon) \rightarrow \mu^+ \tau^- \) can be formulated in a similar way.
3. Numerical Analysis

In the numerical analysis, we adopt the following value for mass of mesons $M_\phi = 1.019\text{GeV}$, $M_{J/\Psi} = 3.096\text{GeV}$, $M_\Upsilon = 9.460\text{GeV}$. For the decay constants, we take $f_\phi = 0.231\text{GeV}$, $f_{J/\Psi} = 0.405\text{GeV}$, $f_\Upsilon = 0.715\text{GeV}$ \[31\]. Furthermore, the electromagnetic coupling is determined by $\alpha(m_Z) = 1/127$.

Coinciding with the neutrino oscillation data and not losing generality, we always assume the lightest neutrino mass as: $m_{\nu_e} = 1.0 \times 10^{-14}\text{GeV}$, and the masses of three neutrinos satisfy following relations from experiment: $\Delta m^2_{\text{sol}} = 8.0 \times 10^{-5}\text{eV}^2$, $\Delta m^2_{\text{atm}} = 3.0 \times 10^{-3}\text{eV}^2$. Here, we also assume three right handed neutrinos are degenerate, i.e., $M_{\nu_R} \sim M_{\nu_R} \sim M_0$, $M_0$ is the mass scale of three right handed neutrinos. The recent results of the LHC experiments indicate that the lower limit of the squark mass is roughly given as $800\text{ GeV}$\[32\]. Not losing generality, we assume the degenerate spectrum in scalar quark sector ($m^2_{QIJ} = m^2_{UIJ} = m^2_{DJJ} = \tilde{m}^2_{Q}_{\delta_{IJ}} = 1\text{TeV}$), $A_{lIJ} = 0$ ($I, J = 1, 2, 3$) at GUT scale to satisfy the constraint. Through the calculation of mass spectrum and mixing matrices, a publicly available fortran77 program SUSY-FLAVOR is used \[33\].

In our numerical analysis, we assume that the gaugino masses are GUT-related, that is,

$$M_1 = \frac{5s_w^2}{3c_w^2}M_2, \quad M_2 = \frac{\alpha_2}{\alpha_s}M_3 \approx \frac{1}{3}M_3. \quad (31)$$

In order to decrease the number of free parameters involved in our calculation, we suppose that the diagonal entries of two $3 \times 3$ matrices $m^2_L$, $m^2_R$ in Eq.\[9\] are equal $(m^2_L)_{II} = (m^2_L)_{IJ} = (m^2_D)_{JJ} = m^2_{\tilde{E}}\delta_{IJ} = 1\text{TeV}^2$, $A^I_{IJ} = 0$ ($I, J = 1, 2, 3$) at GUT scale to satisfy the constraint. Therefore, the calculation of mass spectrum and mixing matrices, a publicly available fortran77 program SUSY-FLAVOR is used \[33\].

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At first, we discuss the LFV decays of vector mesons $\phi \rightarrow e^+\mu^-$. The corrections from Higgs to the LFV branching ratios of vector mesons $\phi \rightarrow e^+\mu^-$ can be neglected safely since they are suppressed by the tiny masses of leptons.

In the MSSM with type I seesaw, the LFV processes originate from the mass insertions $\delta_{ij}^L$, $\delta_{ij}^R$. The most challenging experimental prospects arise for the CR($\mu-e$) in heavy nuclei such as titanium ($^{48}_{22}$Ti). The experimental upper bounds on the conversion rate reach CR($\mu-e$, $^{48}_{22}$Ti) $\leq 4.3 \times 10^{-12}$ [14]. In the MSSM with type I seesaw, the conversion rate in nuclei can be calculated by [20]:

\[
CR(\mu - e, X) = \frac{\Gamma(\mu + X \rightarrow e + X)}{\Gamma(\mu + X \rightarrow \text{capture})} = 4\alpha^5 \frac{Z_{eff}^4}{Z} |F(q)|^2 m_\mu^5 \left[ Z(A_1^L - A_2^R) - (2Z + N)\bar{D}_u^L - (Z + 2N)\bar{D}_d^L \right]^2 \\
+ \left[ Z(A_1^R - A_2^L) - (2Z + N)\bar{D}_u^R - (Z + 2N)\bar{D}_d^R \right]^2,
\]

(36)

where $Z$ and $N$ denote the proton and neutron numbers in a nucleus, $F(q)$ is the nuclear form factor and $Z_{eff}$ is an effective atomic charge. $A_{1,2}^{L,R}$ stand for the contributions from penguin-type diagram, $\bar{D}_{u,d}^{L,R}$ stand for the contributions from box-type diagrams. In $^{48}_{22}$Ti, $F(q^2) \sim 0.54$ and $Z_{eff} = 17.6$ [34]. After a scan over the parameter space, we will discuss $\phi \rightarrow e^+\mu^-$ and $\mu-e$ conversion for two cases: (I) $\delta_{12}^L$ dominance, $\delta_{23}^L = \delta_{13}^L = 0$; (II) $\delta_{23}^L\delta_{13}^L$ dominance, $\delta_{12}^L = 0$. We assume $\tan\beta = 10$, $\mu = 200$GeV, $M_2 = 200$GeV, $a_1^L = 1$ and $\delta_{LR}^L = 0$.

**Case (I) Taking** $m_\phi = 1$TeV, $M_0 = 10^{10}$GeV, we plot the theoretical prediction of BR ($\phi \rightarrow e^+\mu^-$) (solid line) and CR($\mu + ^{48}_{22}$Ti $\rightarrow e + ^{48}_{22}$Ti) (dash line) versus $\delta_{12}^L$ in Fig.4(a), where the gray shadow is the excluded region for CR($\mu + ^{48}_{22}$Ti $\rightarrow e + ^{48}_{22}$Ti). The CR($\mu + ^{48}_{22}$Ti $\rightarrow e + ^{48}_{22}$Ti) exceeds the current experiment limit at $\delta_{12}^L \sim 2.0 \times 10^{-6}$. The parameter space of $\delta_{12}^L$ has been highly suppressed with respect to the prediction of CR($\mu + ^{48}_{22}$Ti $\rightarrow e + ^{48}_{22}$Ti). Both BR ($\phi \rightarrow e^+\mu^-$) and CR($\mu + ^{48}_{22}$Ti $\rightarrow e + ^{48}_{22}$Ti) tend to be not sensitive to $\delta_{12}^L$ when its value is below $10^{-7}$. In [20, the authors investigate the LFV processes $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and deduce a constraint with $\delta_{12}^L \leq 3 \times 10^{-4}$. 12
Figure 4: Case (I): BR ($\phi \to e^+ \mu^-$) (solid line) and CR($\mu + ^{48}_{22}$Ti $\to e + ^{48}_{22}$Ti) (dash line) vs mass insertion $\delta_{12}^L$, slepton mass sector $m_{\tilde{E}}$ and right handed neutrinos mass scale $M_0$. The shadow is the excluded region for CR($\mu + ^{48}_{22}$Ti $\to e + ^{48}_{22}$Ti).
Figure 5: Case (II): BR ($\phi \rightarrow e^+\mu^-$) (solid line) and CR($\mu + ^{48}_{22}$Ti $\rightarrow e + ^{48}_{22}$Ti) (dash line) vs mass insertion $\delta^2_{\tilde{E}}, \delta_{\tilde{L}}^{13}$, slepton mass sector $m_{\tilde{E}}$ and right handed neutrinos mass scale $M_0$. The shadow is the excluded region for CR($\mu + ^{48}_{22}$Ti $\rightarrow e + ^{48}_{22}$Ti).
Comparing with the constraint on $\delta^{12}_L$ from $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, It displays the constraint from $\mu - e$ conversion is more stronger.

Taking $M_0 = 10^{10}$ GeV, $\delta^{12}_L = 10^{-6}$, we plot the theoretical prediction of BR ($\phi \rightarrow e^+\mu^-$) (solid line) and CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$) (dash line) versus slepton mass sector $m_{\tilde{E}}$ in Fig 4(b), where the gray shadow is the excluded region for CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$). Different to Fig 4(a) and Fig 4(c), Both BR ($\phi \rightarrow e^+\mu^-$) and CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$) decrease as $m_{\tilde{E}}$ varies from 0.5 TeV to 3 TeV. For lower slepton mass, the prediction on CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$) is also out of the current experiment limit.

Taking $m_{\tilde{E}} = 1$ TeV, $\delta^{12}_L = 10^{-6}$, we plot the theoretical prediction of BR ($\phi \rightarrow e^+\mu^-$) (solid line) and CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$) (dash line) versus the right handed neutrino mass scale $M_0$ in Fig 4(c), where the gray shadow is the excluded region for CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$). There is a sharp decrease around $\delta^{23}_L \delta^{13}_L \sim 1.17 \times 10^{-6}$ with a minimum BR ($\phi \rightarrow e^+\mu^-$) of order about $10^{-23}$, which is about two orders smaller than the most stringent prediction in [9]. In [20], the authors also give a expected value for $\delta^{23}_L \delta^{13}_L$ deduced from the processes $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$, which is ($\sim 10^{-6}$) and compatible with ours.

Taking $M_0 = 10^{10}$ GeV, $\delta^{23}_L \delta^{13}_L = 1.2 \times 10^{-6}$, we plot the theoretical prediction of BR ($\phi \rightarrow e^+\mu^-$) (solid line) and CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$) (dash line) versus $m_{\tilde{E}}$ in Fig 4(b), where the gray shadow is the excluded region for CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$). Here and followed, we will assume $\delta^{23}_L = 4 \times 10^{-3}$, $\delta^{13}_L = 3 \times 10^{-4}$ and these are also expected values for $\delta^{23}_L$ and $\delta^{13}_L$ reported in [20] evaluated from LFV decays $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$. We also find a resonating absorption around $m_{\tilde{E}} = 1$ TeV that originates from the interference between the corrections from sneutrino sector and that from charged slepton sector. Fig. 5(b) displays that no constraint on $m_{\tilde{E}}$ has arisen with respect to $\mu - e$ conversion and LFV decay $\phi \rightarrow e^+\mu^-$.

Taking $m_{\tilde{E}} = 1$ TeV, $\delta^{23}_L \delta^{13}_L = 1.2 \times 10^{-6}$, we plot the theoretical prediction of BR ($\phi \rightarrow e^+\mu^-$) (solid line) and CR($\mu +_{22}^{48} Ti \rightarrow e +_{22}^{48} Ti$) (dash line) versus $M_0$ in Fig 4(c), where the gray shadow is the excluded region for
CR(μ +32 Ti → e +32 Ti). There is a resonating absorption around $M_0 = 10^{10}$GeV that originates from the interference between the corrections from sneutrino sector.

Comparing Case (I) with Case (II), we find: (i) In Case (II), the prediction for $\text{BR}(\phi \to e^+\mu^-)$ are more compatible with $\phi$. In [8] and [9], the constraints are $\text{BR}(\phi \to e^+\mu^-) \leq 4.0 \times 10^{-17}$ and $\text{BR}(\phi \to e^+\mu^-) \leq 1.3 \times 10^{-21}$. It is noted worthwhile that the prediction in [10] also satisfies the constraint from $\mu - e$ conversion, even if it is derived by the constraint from $\mu \to 3e$. (ii) Compared with $\delta_{L}^{23} (\sim 10^{-3})$ and $\delta_{L}^{13} (\sim 10^{-4})$, the value for $\delta_{L}^{12} (10^{-6} \text{ or little})$ is so small that it can be neglected.

Then, we will investigate meson decays $J/\Psi(\Upsilon) \to \mu^+\tau^-$ in Case (II) not only for reasons above, but also for the aim to generate a large enough BR $(J/\Psi(\Upsilon) \to \mu^+\tau^-)$ to be observed in experiment. As it displays in Fig[6] and Fig[7], the mass insertion $\delta_{L}^{23}$ affects the theoretical evaluation of BR $(J/\Psi(\Upsilon) \to \mu^+\tau^-)$ strongly. In formula, there is a simple relation $[20]$:

$$\frac{\text{BR}(\tau \to 3\mu)}{\text{BR}(\tau \to \mu\gamma)} \approx \frac{\alpha}{8\pi} \left(\frac{16}{3} \ln \frac{m_\tau}{2m_\mu} - \frac{14}{9}\right) \simeq 0.003. \quad (37)$$

So, we just consider the constraint from $\tau \to \mu\gamma$.

(I) $J/\Psi \to \mu^+\tau^-$

Taking $m_\tilde{E} = 1$TeV, $M_0 = 10^{10}$GeV, we plot the theoretical prediction of BR $(J/\Psi \to \mu^+\tau^-)$ (solid line) and BR$(\tau \to \mu\gamma)$ (dash line) versus $\delta_{L}^{23}$ in Fig[8](a). Taking $M_0 = 10^{10}$GeV, $\delta_{L}^{23} = 4 \times 10^{-3}$, $\delta_{L}^{13} = 3 \times 10^{-4}$, we plot the theoretical prediction of BR $(J/\Psi \to \mu^+\tau^-)$ (solid line) and BR$(\tau \to \mu\gamma)$ (dash line) versus $m_\tilde{E}$ in Fig[8](b). The gray shadow is the excluded region for $\text{BR}(\tau \to \mu\gamma)$ from the experiment. A linear relationship is displayed between BR $(J/\Psi \to \mu^+\tau^-)$ and $\delta_{L}^{23}$ in logarithmic scale, which shows the great dependence of BR $(J/\Psi \to \mu^+\tau^-)$ on $\delta_{L}^{23}$. We also investigate the dependence of BR $(J/\Psi \to \mu^+\tau^-)$ on $M_0$ and it shows BR $(J/\Psi \to \mu^+\tau^-)$ is not sensitive to $M_0$.

In [8] and [10], the authors calculate the BR $(J/\Psi \to \mu^+\tau^-)$ with BR $(J/\Psi \to \mu^+\tau^-) \leq 4.1 \times 10^{-9}$ and BR $(J/\Psi \to \mu^+\tau^-) \leq 6.3 \times 10^{-10}$, that is three or four orders below the experiment limit $2.0 \times 10^{-6}$. Under our assumption, $m_\tilde{E} = 1$TeV, $M_0 = 10^{10}$GeV, $\delta_{L}^{23} = 4 \times 10^{-3}$, $\delta_{L}^{13} = 3 \times 10^{-4}$, we get the BR $(J/\Psi \to \mu^+\tau^-)$ can be enhanced as large as $1.6 \times 10^{-7}$, that is more promising to detect directly in experiment in near future.

(II) $\Upsilon \to \mu^+\tau^-$
Figure 6: BR ($J/\Psi \rightarrow \mu^+ \tau^-$) (solid line) and BR($\tau \rightarrow \mu \gamma$) (dash line) vs mass insertion $\delta_{23}^L$, slepton sector $m_{\tilde{E}}$. The gray shadow is the excluded region for BR($\tau \rightarrow \mu \gamma$).
Figure 7: BR(Υ $\rightarrow \mu^+\tau^-$) (solid line) and BR(τ $\rightarrow \mu\gamma$) (dash line) vs mass insertion $\delta^{23}_L$, slepton sector $m_{\tilde{E}}$. The gray shadow is the excluded region for BR(τ $\rightarrow \mu\gamma$).
Similar to the process $J/\Psi \to \mu^+\tau^-$, the LFV decay $\Upsilon \to \mu^+\tau^-$ has the same behavior as a function of $\delta^2_{L,3}$, $m_{\tilde{E}}$ and $M_0$. Taking $m_{\tilde{E}} = 1\text{TeV}$, $M_0 = 10^{10}\text{GeV}$, we plot the theoretical prediction of $\text{BR}(\Upsilon \to \mu^+\tau^-)$ (solid line) and $\text{BR}(\tau \to \mu\gamma)$ (dash line) versus $\delta^2_{L,3}$ in Fig.7(a). Taking $M_0 = 10^{10}\text{GeV}$, $\delta^2_{L,3} = 4 \times 10^{-3}$, $\delta^2_{L,3} = 3 \times 10^{-4}$, we plot the theoretical prediction of $\text{BR}(\Upsilon \to \mu^+\tau^-)$ (solid line) and $\text{BR}(\tau \to \mu\gamma)$ (dash line) versus $m_{\tilde{E}}$ in Fig.7(b). The gray shadow is the excluded region for $\text{BR}(\tau \to \mu\gamma)$ from the experiment. Similarly, the mass insertion $\delta^2_{L,3}$ affects the theoretical evaluation of $\text{BR}(\Upsilon \to \mu^+\tau^-)$ strongly, and $\text{BR}(\Upsilon \to \mu^+\tau^-)$ depends on $M_0$ mildly.

The most stringent prediction of $\text{BR}(\Upsilon \to \mu^+\tau^-)$ in theory is given in [10] with $\text{BR}(\Upsilon \to \mu^+\tau^-) \leq 2.9 \times 10^{-6}$, and that is very close to the experiment limit. In [8], the author calculates the $\text{BR}(\Upsilon \to \mu^+\tau^-) \leq 7.2 \times 10^{-5}$, and it shows the updated data for $\text{BR}(\tau \to 3\mu)$ from experiment is necessary. Under our assumption, $m_{\tilde{E}} = 1\text{TeV}$, $M_0 = 10^{10}\text{GeV}$, $\delta^2_{L,3} = 4 \times 10^{-3}$, $\delta^2_{L,3} = 3 \times 10^{-4}$, we get $\text{BR}(\Upsilon \to \mu^+\tau^-) \leq 5.3 \times 10^{-7}$. That is also promising to detect directly in experiment in near future.

We can evaluate the branching ratios of LFV decays $\rho(\omega, J/\Psi, \Upsilon) \to e^+\mu^-$ using above method. To shorten the length of text, we just present the upper bounds on those branching ratios under the same assumptions as $\phi \to e^+\mu^-$. After considering the constraints from $\mu - e$ conversion, $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to 3e$ etc, we give a summary of upper bounds of experiment data and corresponding theoretical predictions in Tab.1.

### Table 1: The upper bounds on the branching ratios of vector bosons

| Decay            | Experiment | Ref. [8] | Ref. [9] | Our prediction |
|------------------|------------|----------|----------|----------------|
| $\phi \to e^+\mu^-$ | $\leq 2.0 \times 10^{-6}$ | $\leq 4.0 \times 10^{-17}$ | $\leq 1.3 \times 10^{-21}$ | $\leq 5.0 \times 10^{-20}$ |
| $\rho \to e^+\mu^-$ | $-$ | $\leq 3.8 \times 10^{-20}$ | $\leq 3.5 \times 10^{-24}$ | $\leq 1.0 \times 10^{-20}$ |
| $\omega \to e^+\mu^-$ | $-$ | $\leq 8.1 \times 10^{-16}$ | $\leq 6.2 \times 10^{-27}$ | $\leq 1.8 \times 10^{-20}$ |
| $J/\Psi \to e^+\mu^-$ | $< 1.1 \times 10^{-6}$ | $\leq 4.0 \times 10^{-13}$ | $\leq 3.5 \times 10^{-13}$ | $\leq 1.9 \times 10^{-18}$ |
| $\Upsilon \to e^+\mu^-$ | $-$ | $\leq 2.0 \times 10^{-9}$ | $\leq 3.9 \times 10^{-6}$ | $\leq 3.6 \times 10^{-18}$ |
| $J/\Psi \to \mu^+\tau^-$ | $< 2.0 \times 10^{-6}$ | $\leq 4.1 \times 10^{-9}$ | $-$ | $\leq 1.6 \times 10^{-7}$ |
| $\Upsilon \to \mu^+\tau^-$ | $< 6.0 \times 10^{-6}$ | $\leq 7.2 \times 10^{-5}$ | $-$ | $\leq 5.3 \times 10^{-7}$ |

4. Conclusions

Considering the constraints from $\mu - e$ conversion, $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to 3e$ etc, we analyze the LFV decays of $\phi \to e^+\mu^-$, $J/\Psi(\Upsilon) \to \mu^+\tau^-$ in
the framework of MSSM with type I seesaw extended.

In the MSSM with type I seesaw extended, the theoretical evaluation on BR ($\phi \rightarrow e^+\mu^-$) is affected by the mass insertion $\delta_{13}^2\delta_{13}^1 L$. After considering the constraints from $\mu - e$ conversion, $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$, the prediction on BR ($\phi \rightarrow e^+\mu^-$) can reach $5.0 \times 10^{-20}$, which is far below the present experimental upper bound. In a similar way, the mass insertion $\delta_{13}^2 L$ affects the theoretical evaluations on BR ($J/\Psi \rightarrow \mu^+\tau^-$) and BR ($\Upsilon \rightarrow \mu^+\tau^-$) sensitively. Considering the constraints from $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow 3\mu$, the predictions on BR ($J/\Psi \rightarrow \mu^+\tau^-$) and BR ($\Upsilon \rightarrow \mu^+\tau^-$) can reach $10^{-7}$, which are very promising to be observed in near future experiment.

In the future, the expected sensitivities for BR ($\mu \rightarrow e\gamma$) would be of order $10^{-13}$ [35]. For BR ($\tau \rightarrow e\gamma$) and BR ($\tau \rightarrow \mu\gamma$), it would be $10^{-9}$ [36]. For CR ($\mu - e, Ti$) in nuclei, it would be as low as $10^{-16} \sim 10^{-17}$ [37]. Thus, the $\mu - e$ conversion experiments would represent the most promising tool to probe new physics. The study of LFV decays via vector mesons is also waiting for the new data from the experiment.

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Appendix A. The simplified amplitude in seesaw extended MSSM

In this appendix we present the simplified amplitudes in Fig.1

$$A_a = \frac{ie^2 \pi f_\phi}{3m_\phi N_c} (p_3 + p_4) \cdot \varepsilon(p) \sum_{i,j=1}^{6} \sum_{k=1}^{4} \bar{u}_\mu(p_4) \left\{ C_1 m_e (A_{3k}^{kj} A_{1}^{iks} P_L + A_{4k}^{kj} A_{2}^{iks} P_R) \right. \nonumber$$
$$+ C_2 \left[ (m_e A_{3k}^{kj} A_{1}^{iks} - m_\mu A_{2}^{kj} A_{2}^{iks} P_R) P_L \right. \nonumber$$
$$+ (m_e A_{2}^{kj} A_{4}^{ik} - m_\mu A_{3k}^{kj} A_{1}^{iks} P_R) ] \nonumber$$
$$+ C_0 \left[ (m_e A_{2}^{iks} A_{1}^{ik} - m_\chi A_{2}^{iks} A_{1}^{ik} P_R) \right. \nonumber$$
$$+ A_{3k}^{kj} (m_e A_{4}^{iks} - m_\chi A_{2}^{iks}) P_L \right\} u_e(p_3)$$
where
\[
A_{ik}^1 = \frac{e(s_w Z_N^{1k} + c_w Z_N^{2k}) Z_N^{2i}}{\sqrt{2} c_w s_w} + Y_l^2 Z_N^{3k} Z_l^{5i}
\]
\[
A_{ik}^2 = Y_l^2 Z_N^{2i} (Z_N^{3k})^* - \frac{\sqrt{2} e Z_N^{5i} (Z_N^{1k})^*}{c_w}
\]
\[
A_{kj}^3 = \frac{e(s_w Z_N^{1k} + c_w Z_N^{2k}) Z_l^{1j}}{\sqrt{2} c_w s_w} + Y_l^1 Z_N^{3k} Z_l^{4j}
\]
\[
A_{kj}^4 = Y_l^1 Z_N^{3k} (Z_l^{1j})^* - \frac{\sqrt{2} e Z_N^{1k} (Z_l^{1j})^*}{c_w}
\]
and
\[
C_0 = C_0[m^2, m_e^2, m_{\mu}^2, m_{L_i}^2, m_{L_j}^2, m_{\chi_0}^2]
\]
\[
C_i = C_i[m_{\mu}^2, m_e^2, m_{\phi}^2, m_{L_i}^2, m_{\chi_0}^2, m_{L_j}^2]
\]
with \(i = 1, 2\).

\[
A_b = \frac{i e^2 \pi^2 f_o m_\phi (4 s_w^2 - 3)}{24 N_c s_w^2 c_w^2 (m_\phi^2 - m_\omega^2)} (p_3 + p_4) \cdot \varepsilon(p) \sum_{i,j=1}^6 \sum_{k=1}^4
\]
\[
\times A_{ij}^{\tilde{\mu}}(p_4) \bigg\{ C_1 m_e (A_3^{kj} A_1^{ik*} P_L + A_4^{kj} A_2^{ik*} P_R)
\]
\[
+ C_2 \left[ (m_e A_3^{kj} A_1^{ik*} - m_\mu A_4^{kj} A_2^{ik*}) P_L + (m_e A_4^{kj} A_2^{ik*} - m_\mu A_3^{kj} A_1^{ik*}) P_R \right]
\]
\[
+ C_0 \left[ (m_e A_4^{kj} A_2^{ik*} - m_{\chi_0} A_4^{kj} A_1^{ik*}) P_R + (m_e A_3^{kj} A_1^{ik*} - m_{\chi_0} A_3^{kj} A_2^{ik*}) P_L \right] \bigg\} \varepsilon(p_3)
\]

where
\[
A_{ij}^{\tilde{\mu}} = Z_N^{2i} Z_l^{1j*} - 2 \delta_{ij} s_w^2
\]
and
\[
C_0 = C_0[m^2, m_e^2, m_{\mu}^2, m_{L_i}^2, m_{L_j}^2, m_{\chi_0}^2]
\]
\[
C_i = C_i[m_{\mu}^2, m_e^2, m_{\phi}^2, m_{L_i}^2, m_{\chi_0}^2, m_{L_j}^2]
\]
with $i = 1, 2.$

$$A_c = \frac{ie^3 \pi^2 f_\phi m_\phi (4s_w^2 - 3)}{48\sqrt{2}c_w^4 (m_\phi^2 - m_\phi^2) N_c s_w^4} \sum_{i,j=1}^{4} \sum_{k=1}^{6} \left[ \bar{\mu}_\mu (p_4) \left\{ 2\bar{\alpha}(p) \left[ \left( m_\mu m_\mu A_6^{ij} A_7^{jk} C_0 + C_1 + C_2 \right) \right] + 2s_w A_8^{ik} \left\{ m_\mu m_\chi_\phi^0 A_6^{ij} C_1 \right\} A_9^{ik*} + 2s_w A_8^{ik*} \left( A_6^{ij} m_\chi_\phi^0 C_0 + A_6^{ij} m_\chi_\phi^0 C_2 \right) \left[ 1 + 2C_2 m_\phi^2 \right] + 2p_3 \cdot \varepsilon(p) \left[ \left( A_6^{ij} A_7^{jk} \{ (C_1 + C_{11}) m_\epsilon P_L + C_{12} m_\mu P_R \} \right) + 2s_w \left( A_6^{ij} A_7^{jk} m_\chi_\phi^0 C_1 P_L \right) \right] + 2p_3 \cdot \varepsilon(p) \left[ \left( A_6^{ij} s_w m_\chi_\phi^0 C_2 P_L + 2s_w \left( A_6^{ij} A_7^{jk} m_\chi_\phi^0 C_2 \right) \right) P_R \right] \right] P_L \right]$$

where

$$A_6^{ij} = Z_N^{4i} Z_N^{4j} - Z_N^{3i} Z_N^{3j}$$

$$A_7^{jk} = Z_L^{1k} \left( Z_N^{1j} s_w + Z_N^{3j} c_w \right) + Y_1 Z_L^{1k} Z_N^{3j}$$

$$A_8^{ik} = Z_L^{ik} Z_N^{ij*} + Y_1 Z_L^{1k} Z_N^{2j*}$$
and

\[ A_{i0}^{ik} = Z_L^{2k} (Z_N^{2i} s_w + Z_N^{2i} c_w) + Y_1^i Z_L^{5k} Z_N^{3i}, \]
\[ A_{10}^{ik} = Z_L^{5k} Z_N^{1i} + Y_1^2 Z_L^{2k} Z_N^{3i}. \]

with \( i = 1, 2. \)

\[
A_d = \frac{ie^2 \pi^2 f_\phi}{6m_\phi N_c s_w^2} \sum_{i,j=1}^{N_c} \sum_{k=1}^{6}
\times \bar{u}_\mu(p_1) \left\{ \frac{\xi(p)}{2} \left[ A_{11}^{ik*} \left( \left\{ 2(C_2 + C_{22})m_e^2 + 2(C_1 + C_{11})m_\mu^2 + 2C_{12}(m_e^2 + m_\mu^2 - m_\phi^2) + 2C_0(m_{\chi_i^+} m_{\chi_j^+} + 2) - 1 \right\} e^2 A_{13}^{ik} + 2es_w A_{14}^{ik} m_{\mu} \left\{ (C_0 + C_2)m_{\chi_i^+} + C_2 m_{\chi_j^+} \right\} \right) \right] P_L + \xi(p) \left[ A_{12}^{ik*} \left( 2es_w A_{14}^{ik} m_{\mu} \left\{ (C_0 + C_2)m_{\chi_i^+} + C_2 m_{\chi_j^+} \right\} + s_w^2 A_{14}^{ik} (2(C_2 + C_{22})m_e^2 + 2(C_1 + C_{11})m_\mu^2 + 2C_{12}(m_e^2 + m_\mu^2 - m_\phi^2) + 2C_0(m_{\chi_i^+} m_{\chi_j^+} + 2) - 1) \right) - 2m_\epsilon A_{11}^{ik*} \left( e^2 A_{13}^{ik} (C_0 + C_2)m_\mu - es_w A_{14}^{ik} \{ C_{11} m_{\chi_i^+} + (C_0 + C_1)m_{\chi_j^+} \} \right) \right] P_R - 4(p_4 \cdot \varepsilon(p)) \left[ A_{11}^{ik*} \left( e^2 A_{13}^{ik} \{ C_1 + C_{11} \} m_\mu P_L + C_{12} m_\mu P_R \right) + es_w A_{14} C_{11} m_{\chi_i^+} + s_w A_{12}^{ik*} \times \left( eA_{13}^{ik} C_{11} m_{\chi_i^+} P_L + s_w A_{14}^{ik} \{ C_{12} m_\mu P_L + C_{11} m_\mu P_R \} \right) \right] - (p_3 \cdot \varepsilon(p)) \left[ A_{11}^{ik*} \left( e^2 A_{13}^{ik} \{ C_{12} m_\mu P_L + (C_2 + C_{22})m_\mu P_R \} - es_w A_{14}^{ik} \{ (C_2 + C_{22})m_\mu P_L + C_{12} m_\mu P_R \} \right) + A_{12}^{ik*} \left( s_w^2 A_{14}^{ik} \{ (C_2 + C_{22})m_\mu P_L + C_{12} m_\mu P_R \} - es_w A_{14}^{ik} \{ C_{12} m_\mu P_L \} \right) \right] \varepsilon(p_3) \]

where

\[
A_{11}^{ik} = Z_+^{1i} (Z_\nu^{2k} - IZ_\nu^{5k})
\]
\[ A_{12}^{jk} = Y_l^2 Z_{2^k}^{2^k} (Z_\nu^{2^k} - IZ_\nu^{5^k}) \]
\[ A_{13}^{jk} = Z_{1^k}^{1^k} (Z_\nu^{1^k} - IZ_\nu^{4^k}) \]
\[ A_{14}^{jk} = Y_l^1 Z_{2^j}^{2^j} (Z_\nu^{1^j} - IZ_\nu^{4^j}) \]

and

\[ C_0 = C_0 [m_{\phi}^2, m_e^2, m_\mu^2, m_{\chi_1^\pm}^2, m_{\chi_2^\pm}^2, m_{\nu_\mu}^2] \]
\[ C_{i,ij} = C_{i,ij} [m_{\mu}^2, m_{\phi}^2, m_e^2, m_\mu^2, m_{\chi_1^\pm}^2, m_{\chi_2^\pm}^2] \]

with \( i = 1, 2. \)

\[
A_e = \frac{i\pi^2 e^2 f_\phi m_\phi (4s_w^2 - 3)}{48c_w^2 (m_\phi^2 - m_e^2)^4 N_{c_w}} \sum_{i,j=1}^{2} \sum_{k=1}^{6} \times \delta(p) \left[ s_{w} A_{12}^{ik}\left( 2 e A_{13}^{jk}(A_{10}^{ij} m_{\chi_1^\pm} + A_{15}^{ij} m_{\chi_2^\pm}) C_2 m_\mu \right) + s_{w} A_{14}^{ik}(A_{10}^{ij} 2C_2 m_e^2 + 2C_2 m_e^2 + 2(C_1 + C_{11}) m_\mu^2 - 1 + 2C_{12} \times (m_e^2 + m_\mu^2 - m_\phi^2) ) - 2 em_e A_{11}^{ik}\left( e A_{13}^{jk} A_{16}^{ij} (C_1 + C_2) m_\mu \right) - s_{w} A_{14}(A_{15}^{ij} m_{\chi_1^\pm} + A_{16}^{ij} m_{\chi_2^\pm}) C_1 + 2 \left( s_{w} A_{12}^{ik} \{ A_{13}^{jk} A_{16}^{ij} C_1 \} - s_{w} A_{14}(2A_{15}^{ij} + A_{16}^{ij} m_{\chi_1^\pm} m_{\chi_2^\pm}) \} - 2 em_e A_{11}^{ik}\left( e A_{13}^{jk} A_{16}^{ij} \{ (C_1 + C_{11}) m_e P_L + C_{12} m_\mu P_R \} + s_{w} A_{14}^{ik} C_{15} m_{\chi_1^\pm} P_R \right) + s_{w} A_{12}^{ik}\left( e A_{13}^{jk} A_{16}^{ij} \{ (C_1 + C_{11}) m_e P_R \} + C_{12} m_\mu P_L \} \right) + \delta(p) \left[ e A_{11}^{ik}\left( e A_{13}^{jk} A_{16}^{ij} \{ 2(C_2 + C_{22}) m_e^2 + 2(C_1 + C_{11}) m_\mu^2 + 2C_1 (m_e^2 + m_\mu^2 - m_\phi^2) - 1 \} + 2 s_{w} A_{14}^{ik} \times (A_{15}^{ij} m_{\chi_1^\pm} + A_{16}^{ij} m_{\chi_2^\pm}) C_2 m_\mu + 2C_0 \{ 2 e A_{13}^{jk} A_{16}^{ij} + A_{15}^{ij} m_{\chi_1^\pm} \times (e A_{13}^{jk} m_{\chi_2^\pm} + s_{w} A_{14}^{ik} m_\mu) \} + 2 m_e s_{w} A_{12}^{ik}\left( e A_{13}^{jk} A_{16}^{ij} C_{15} m_{\chi_1^\pm} + A_{15}^{ij} (C_0 + C_1) m_{\chi_1^\pm} \} - s_{w} A_{14}^{ik} A_{15}(C_0 + C_1 + C_2) m_\mu \right) P_L \right] + 4(p_3 \cdot \varepsilon(p)) \left[ e A_{11}^{ik} A_{16}^{ij}\left( e A_{13}^{jk} C_{12} m_e P_L + \{ e A_{13}^{jk} (C_2 + C_{22}) m_\mu \right) \right]
\]
\[-s_w A_{14}^{ik} C_2 m_{\chi_i^+} P_R \right) + s_w A_{12}^{ik} A_{15}^{ij} \left( s_w A_{14}^{ik} \{(C_2 + C_{22}) m_\mu P_L + C_{12} m_e P_R \} - e A_{13}^{jk} C_2 m_{\chi_j^+} P_L \} \}\} u_e(p_3)

where

\[ A_{15}^{ij} = Z_+ Z_+^{ij} + \delta^{ij} (c_w^2 - s_w^2) \]
\[ A_{16}^{ij} = Z_- Z_-^{ij} + \delta^{ij} (c_w^2 - s_w^2) \]

and

\[ C_0 = C_0 [m_{\chi_1^0}, m_{\chi_2^0}, m_{\mu_1}, m_{\mu_2}, m_{\chi_1^\pm}, m_{\chi_2^\pm}] \]
\[ C_i^{i,j} = C_i^{i,j} [m_{\mu_1}, m_{\phi}, m_{\mu_1}, m_{\phi}, m_{\mu_2}, m_{\phi}, m_{\chi_1^\pm}, m_{\chi_2^\pm}] \]

with \( i=1,2 \).

\[ A_f = \frac{i \pi^2 e^2 f_\phi m_\phi (3 - 4 s_w^2)}{24 N_c s_w^2 c_w^2 (m_\phi^2 - m_\chi^2)} (p_3 + p_4) \cdot \varepsilon(p) \sum_{i,j=1}^6 \sum_{k=1}^2 \]
\[ \times \bar{u}_\mu(p_4) \left\{ C_1 m_\mu \left[ e^2 A_{11}^{kj} A_{13}^{kl} P_R + s_w A_{12}^{ki} A_{14}^{kj} P_L \right] P_R \right) \]

\[ + C_2 \left[ e^2 A_{11}^{kj} A_{13}^{kl} (m_\mu P_R - m_e P_L) + s_w A_{12}^{ki} A_{14}^{kj} (m_\mu P_L \right) \]

\[-m_e P_R \] + \( C_0 \left[ e A_{11}^{kj} (A_{13}^{kl} m_\mu - s_w A_{14}^{kj} P_R \right) \]

\[ + s_w A_{12}^{ki} (s_w m_\mu A_{14}^{kj} - e m_\mu A_{13}^{kj} P_R \right) \}\} v_e(p_3)

where

\[ C_0 = C_0 [m_{\chi_1^0}, m_{\chi_2^0}, m_{\mu_1}, m_{\mu_2}, m_{\chi_1^\pm}, m_{\chi_2^\pm}] \]
\[ C_i = C_i [m_{\mu_1}, m_{\phi}, m_{\mu_2}, m_{\phi}, m_{\chi_1^\pm}, m_{\chi_2^\pm}], i = 1,2 \]

the simplified amplitudes in Fig 2

\[ A_g = \frac{i \pi^2}{48 N_c s_w^2} \sum_{i,m=1}^2 \sum_{j=1}^6 \sum_{l=1}^6 \]
\[ \times \bar{u}_\mu(p_4) \left\{ 2 f_\phi m_\phi \left[ A_{13}^{mj} e D_0 A_{17}^{ml} A_{17}^{kl} (e m_\mu A_{14}^{ij} \right) \]

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+s_w m_e m_{\chi_i}^\pm A_{(12)}^{ijs}) - A_{14}^{mj} s_w D_0 A_{17}^{mls} A_{17}^{il}(em_{\chi_i}^\pm m_{\mu} A_{11}^{ij})
+ s_w m_{\chi_i}^\pm m_{\chi_i}^\pm A_{(12)}^{ijs}) - A_{14}^{mj} s_w A_{12}^{ij} A_{18}^{il}(2C_0
+ 2D_1 m_{\nu}^2)) \right] \frac{1}{(p - \bar{p})} P_R + D_0 f_{\phi}^T A_{17}^{mls} A_{17}^{il}
\times (2C_0 + 2D_1 m_{\nu}^2) + A_{14}^{mj} D_0 A_{18}^{mls} A_{18}^{il}(s_w(em_{\chi_i}^\pm
\times A_{11}^{ij} + m_e m_{\mu} s_w A_{12}^{ij}) - em_{\chi_i}^\pm
\times m_{\chi_i}^\pm A_{11}^{ij})) \right] \frac{1}{(p - \bar{p})} P_L
+ D_0 f_{\phi}^T A_{17}^{mls} A_{17}^{il}
\times \left[ s_w A_{14}^{mj}(em_{\chi_i}^\pm m_{\chi_i}^\pm A_{11}^{ij} + m_e m_{\mu} s_w A_{12}^{ij}) - em_{\chi_i}^\pm
\times (em_{\chi_i}^\pm m_{\chi_i}^\pm A_{11}^{ij} + m_e m_{\mu} s_w A_{12}^{ij}) \right] \frac{1}{(p - \bar{p})} P_L \right] v_{e}(p_3)

where

A_{17}^{(i,m)l} = \left[ - \frac{e}{s_w} Z_{U}^{Jls} Z_{+}^{1(i,m)} + Y_{u}^{l} Z_{U}^{(J+3)l} Z_{+}^{2(i,m)} \right] K_{J2}

A_{18}^{(i,m)l} = \left[ - Y_{d}^{2} Z_{U}^{Jls} Z_{-}^{2(i,m)l} \right] K_{J2}

and

C_0 = C_0[m_{\phi}^2, m_{s}^2, m_{s}^2, m_{\chi_i}^\pm, m_{\chi_i}^\pm, m_{U_i}^2]

D_0 = D_0[m_{\mathcal{E}}^2, m_{\mathcal{E}}^2, m_{\mathcal{E}}^2, m_{\mu}^2, m_{\phi}^2, m_{\nu}^2, m_{\chi_i}^\pm, m_{\chi_i}^\pm, m_{U_i}^2, m_{\phi}^2, m_{\nu}^2, m_{\chi_i}^\pm, m_{\chi_i}^\pm]

D_1 = D_0[m_{\mathcal{D}}^2, m_{\mathcal{D}}^2, m_{\mu}^2, m_{\phi}^2, m_{\nu}^2, m_{\chi_i}^\pm, m_{\chi_i}^\pm, m_{U_i}^2, m_{\phi}^2, m_{\nu}^2, m_{\chi_i}^\pm, m_{\chi_i}^\pm]

where m_{\mathcal{D}}^2 = m_{\mu}^2 + m_{\phi}^2 - 0.5m_{\phi}^2

A_h = \frac{i\pi^2}{48N_c} \sum_{i,m=1}^{4} \sum_{j,l=1}^{6}
\times \bar{u}_h(p_4) \left\{ 2f_{\phi} m_{\phi} \left[ A_{12}^{ijs} (A_{19}^{mls} D_0 A_{19}^{il} m_{\phi}^0 (A_{13}^{mj} m_{\mathcal{E}})
-A_{14}^{mj} m_{\chi_i}^0) - A_{20}^{mls} A_{20}^{il}(2C_0 A_{20}^{mj} + 2D_1 A_{20}^{mj} m_{L_j}) \right) \right\}
\[ + A_{19}^{j,i} D_0 A_{19}^{m,l} A_i^j (A_3^{m,j} m_e m_\mu - A_4^{m,j} m_\chi^0 m_\mu) \hat{\phi}(p) \bar{P}_R \\
+ 2 f_\phi m_\phi \left[ A_{19}^{j,i} D_0 A_{19}^{m,l} A_i^j (A_3^{m,j} m_e m_\mu - A_4^{m,j} m_\chi^0 m_\mu) \hat{\phi}(p) \bar{P}_R \\
+ A_1^{j,i} (A_2^{m,l} D_0 A_i^j m_\chi^0 (A_3^{m,j} m_e - A_4^{m,j} m_\chi^0 m_\mu) - 2 A_1^{m,l} \\
\times A_{19}^{j,i} (C_0 A_3^{m,j} + D_1 A_3^{m,j} m_{L_j})) \hat{\phi}(p) P_L + D_0 f_\phi^T A_{19}^{m,l} \\
\times A_{20}^{j,i} (A_1^{j,i} m_\chi^0 + A_2^{j,i} m_\mu) (A_3^{m,j} m_e - A_4^{m,j} m_\chi^0 m_\mu) \hat{\phi}(p) \phi \\
- \hat{\phi}(p) \bar{P}_R + D_0 f_\phi^T A_{20}^{m,l} A_{19}^{j,i} (A_1^{j,i} m_\mu + A_2^{j,i} m_\chi^0) \\
\times (A_4^{m,j} m_e - A_3^{m,j} m_\chi^0 m_\mu) (\hat{\phi}(p) \phi - \hat{\phi}(p) P_L) \right] v_\epsilon (p_3) \]

where
\[ A_{19}^{j,i,m} = - \frac{e}{s_w} Z_{19}^2 \left( \frac{Z_{19}^2}{3} - \frac{Z_N^2}{3} e_{\phi} \right) + Y_D^2 Z_{19}^2 Z_N^2 \]
\[ A_{20}^{j,i,m} = - \frac{\sqrt{2} e}{3 c_w} Z_{20}^2 Z_N^2 + Y_D^2 Z_{20}^2 Z_N^2 \]

and
\[ C_0 = C_0 [m_\phi^2, m_s^2, m_{s_\chi}^2, m_{s_\phi}^2, m_{s_{\chi_\phi}}^2, m_{s_{\phi_{\chi_\phi}}}^2, m_\mu^2] \\
D_0 = D_0 [m_s^2, m_{s_\chi}^2, m_{s_\phi}^2, m_{s_{\chi_{\mu}}}^2, m_{s_{\phi_{\mu}}}^2, m_{s_{\phi_{\chi_{\mu}}}}^2, m_\mu^2] \\
D_1 = D_0 [m_{s_\phi}^2, m_{s_{\phi_{\mu}}}^2, m_{s_{\phi_{\chi_{\mu}}}}^2, m_{s_\chi}^2, m_{s_{\chi_{\mu}}}^2, m_{s_{\phi_{\chi_{\mu}}}}^2, m_\mu^2] \]

where \( m_D^2 = m_\mu^2 + m_s^2 - 0.5 m_\phi^2 \)

\[ A_i = \frac{i \pi^2}{48 N_c} \sum_{i,m=1}^{4} \sum_{j=1}^{6} \]
\[ \times \bar{u}_e (p_3) \left\{ 2 f_\phi m_\phi \left[ A_{19}^{j,i} (A_1^{m,l} D_0 A_i^j m_\chi^0 (A_3^{m,j} m_e \\
- A_2^{m,j} m_{\chi^0}) - A_1^{m,l} A_{19}^{j,i} (A_3^{m,j} m_e m_\mu - A_4^{m,j} m_{\chi^0} m_\mu) \hat{\phi}(p) \bar{P}_R \\
+ 2 f_\phi m_\phi \left[ A_{19}^{j,i} D_0 A_{19}^{m,l} A_i^j (A_3^{m,j} m_e m_\mu - A_4^{m,j} m_{\chi^0} m_\mu) \hat{\phi}(p) \bar{P}_R \\
+ A_3^{j,i} (A_2^{m,l} D_0 A_i^j m_\chi^0 (A_3^{m,j} m_e - A_4^{m,j} m_{\chi^0}) - 2 A_1^{m,l} \\
\times A_{19}^{j,i} (C_0 A_3^{m,j} + D_1 A_3^{m,j} m_{L_j})) \hat{\phi}(p) P_L + D_0 f_\phi^T A_{19}^{m,l} \\
\times A_{20}^{j,i} (A_1^{j,i} m_\chi^0 + A_2^{j,i} m_\mu) (A_3^{m,j} m_e - A_4^{m,j} m_{\chi^0} m_\mu) \hat{\phi}(p) \phi \\
- \hat{\phi}(p) \bar{P}_R + D_0 f_\phi^T A_{20}^{m,l} A_{19}^{j,i} (A_1^{j,i} m_\mu + A_2^{j,i} m_{\chi^0}) \\
\times (A_4^{m,j} m_e - A_3^{m,j} m_{\chi^0} m_\mu) (\hat{\phi}(p) \phi - \hat{\phi}(p) P_L) \right] v_\epsilon (p_3) \right\} \]
\[ \times A_{20}^{il}(A_{3}^{ij*} m_{\chi_0} + A_{4}^{ij*} m_{\mu})(A_{1}^{im} m_e - A_{2}^{im} m_{\chi_0}) \\
(\bar{\psi}(p)\bar{f}_R + D_0 f_T A_{20}^{il*} A_{19}^{ij*} (A_{3}^{ij*} m_{\mu} + A_{4}^{ij*} m_{\chi_0}) \times m_{\chi_0} (A_{2}^{im} m_e - A_{1}^{im} m_{\chi_0}))(\bar{\psi}(p)\bar{f}_R + D_0 f_T) \} v_\mu(p_4) \]

where

\[
C_0 = C_0[m_{\phi}, m_s, m_s, m_{\phi}, m_{\chi_0}, m_{\chi_0}, m_{D}] \\
D_0 = D_0[m_{\mu}, m_s, m_{\phi}, m_s, m_{D}, m_{\phi}, m_{L}, m_{\chi_0}, m_{D}, m_{\chi_0}, m_{L}, m_{\chi_0}] \\
D_1 = D_0[m_{D}, m_s, m_{\phi}, m_s, m_{D}, m_{\phi}, m_{L}, m_{\chi_0}, m_{D}, m_{\chi_0}, m_{L}, m_{\chi_0}] \\
\]

where \( m_{D}^2 = m_e^2 + m_s^2 - 0.5 m_{\phi}^2 \)

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