Interference quantum correction to conductivity of Al$_x$Ga$_{1-x}$As/GaAs double quantum well heterostructures near the balance

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Abstract. We present the results of experimental investigations of the interference quantum correction to the conductivity of the gated double quantum well Al$_x$Ga$_{1-x}$As/GaAs/Al$_x$Ga$_{1-x}$As heterostructures. Analyzing the positive magnetoconductivity we obtain the interwell transition rate and the phase relaxation rate under the conditions when one and two quantum wells are occupied. It has been found that the interwell transition rate resonantly depends on the difference between the electron densities in the wells in accordance with the theoretical estimate. The central point, however, is that the dephasing rate in the lower quantum well is independent of whether the upper quantum well contributes to the conductivity or not. The results obtained are interpreted within framework of the recent theory for the dephasing and electron-electron interaction in the double well structures [Burmistrov I S, Gornyi I V and Tikhonov K S 2011 Phys. Rev. B 84 075338].

1. Introduction

The quantum corrections to the conductivity of the two-dimensional (2D) electron gas with the single-valley parabolic energy spectrum are extensively investigated last three decades. A reasonably good agreement between experimental and theoretical results evident at relatively high conductivity, $\sigma \gtrsim (10-15)G_0$, where $G_0 = e^2/\pi \hbar$, attests adequate understanding of the role of the quantum interference and electron-electron (e-e) interaction in the transport properties of 2D systems. The quantum corrections in the double quantum well (DQW) structures are studied significantly less. Specific features of the corrections in this case are governed by relationship between the following parameters: the transport time $\tau$, the phase relaxation time $\tau_\phi$, and the interwell transition time $\tau_{12}$, the temperature length $L_T = \sqrt{D/T}$ (where $D$ is the diffusion coefficient), the interwell distance $d$, and the screening length $r_0$. In the limiting cases such structures can look like a single quantum well structure (at $\tau_{12} \ll \tau_\phi$, $1/T$) or a structure with two uncoupled wells (at $\tau_{12} \gg \tau_\phi$, $1/T$ and $d \gg r_0$). The intermediate cases are more interesting and diversified. Concerning the interaction correction, the gain of the interaction in a multiplet channel can be so significant that it can lead to the change of the sign of the temperature dependence of the conductivity from dielectric to metallic like as it takes place in the 2D systems with double valley energy spectrum. The interference correction, even without interwell transitions ($\tau_{12} \gg \tau_\phi$), can differ from the case of non-interacting wells due to
inelasticity of the $e-e$ interaction of carriers in the different wells. This interaction can change the dephasing rate significantly.

2. Experimental details
Two structures grown by the molecular beam epitaxy on a semiinsulating GaAs substrate were investigated. The first structure 3243 consists of 200 nm-thick undoped GaAs buffer layer, 50 nm undoped Al$_{0.3}$Ga$_{0.7}$As layer, two 8 nm GaAs quantum wells separated by 10 nm Al$_{0.3}$Ga$_{0.7}$As barrier, 70 nm Al$_{0.3}$Ga$_{0.7}$As undoped layer. The thickness of the GaAs cap layer is 130 nm. The main doping $\delta$ layer of Si is situated in the barrier center. Because the gate voltage in such a type of structures can effectively decrease the electron density only, an additional Si $\delta$ layer is located above the upper quantum well at distance of 18 nm from the well interface. The second structure 3154 differs by the doping level and has lower electron density and mobility. Such structure design allows us to have the close mobilities in the wells at equal electron density that is very important for reliable determination of the phase relaxation time as will be discussed below. The samples were mesa etched into standard Hall bars and then an Al gate electrode was deposited by thermal evaporation onto the cap layer through a mask.

3. Interference quantum correction
It is well known that the suppression of the weak localization (WL) by in the transverse magnetic field $B \parallel n$, where $n$ is the normal to the structure plane results in negative magnetoresistivity. In single quantum well (SQW) systems the shape of magnetoconductivity $\Delta \sigma(B) = 1/\rho_{xx}(B) - 1/\rho_{xx}(0)$ is described by the following expression [1, 2, 3]:

$$\frac{\Delta \sigma(B)}{G_0} = \alpha \mathcal{H} \left( \frac{\tau \phi}{\tau} \frac{B}{B_{tr}} \right), \quad \mathcal{H}(x) = \psi \left( \frac{1}{2} + \frac{1}{x} \right) + \ln x,$$

where $G_0 = e^2/\pi h$, $B_{tr} = h/4eD\tau$ is a transport magnetic field, $\psi(x)$ is a digamma function, $\alpha$ is prefactor, which appears due to not rigorous fulfilment of the diffusion approximation $\tau/\tau_{\phi} \ll 1$ [4], due to the magnetic field dependence of the dephasing time [5] and the contributions of the second order corrections [6]. As for the DQW systems, the shape of the magnetoconductivity curve is widely varied. For instance, even in the case of low interwell transition rate, when $\Delta \sigma(B)$ is barely the sum of the contributions from each well

$$\frac{\Delta \sigma(B)}{G_0} = \alpha_1 \mathcal{H} \left( \frac{\tau_{\phi1}}{\tau} \frac{B}{B_{tr1}} \right) + \alpha_2 \mathcal{H} \left( \frac{\tau_{\phi2}}{\tau} \frac{B}{B_{tr2}} \right),$$

it depends, as seen, on the large number of parameters: the two phase relaxation times, $\tau_{\phi1}$ and $\tau_{\phi2}$, the two transport times, $\tau_1$ and $\tau_2$, and two transport magnetic fields, $B_{tr1}$ and $B_{tr2}$. Obviously it is impossible to find six parameters while fitting the single smooth curve. Reliable obtaining of the phase relaxation time is possible only when the corresponding parameters $\tau_i$, $\tau_{\phi i}$, and $B_{tr i}$ ($i = 1, 2$) in the wells are close to each other.

In order to find experimentally the gate voltage, at which the DQW structure is in the balance (i.e., electron densities in the wells $n_1$ and $n_2$ are identical) and to ascertain that the mobility of electrons in the wells are close to each other, we have analyzed the magnetic field dependences of the Hall coefficient, the classical magnetoresistivity and the Shubnikov-de Haas effect (for more details, see [7]). The parameters obtained are listed in table 1. From here on the regime when the upper quantum well is depleted and only the lower well is occupied is referred to as SQW regime.

The magnetic field dependences of $\Delta \sigma(B)$ at $T = 1.35$ K for different gate voltages $V_g$ are presented in Fig. 1(a). At first sight all the curves are the same in the shape and no peculiarity
Table 1. Parameters of the structures investigated.

| Structure | #3243 | #3154 |
|-----------|-------|-------|
| Regime    | SQW balance | SQW balance |
| $V_g$ (V) | 4.1   | -1.5  | -3.6  | -2.0  |
| $n$ ($10^{11}$ cm$^{-2}$) | 7.0   | 7.5   | 4.0   | 4.5   |
| $\mu$ ($10^3$ cm$^2$/Vs) | 14.5  | 15.0  | 4.8   | 6.5   |
| $\tilde{\gamma}_{t,0}$ | 0.30  | —     | 0.35  | —     |
| $\tilde{\gamma}_s$ | —     | -0.77 | —     | -0.77 |
| $\tilde{\gamma}_t$ | —     | 0.30  | —     | 0.35  |
| $\tilde{\gamma}_t$ | —     | 0.007 | —     | 0.009 |

Figure 1. (a) The interference-induced positive magnetoconductivity for different gate voltages near the balance. For clarity, the curves are shifted in the vertical direction. (b) The magnetoconductivity measured at $V_g = -1.5$ V for two orientations of the magnetic field. Solid lines are the data, the dashed line is the best fit by (2) with the parameters $\alpha = 1.9$, $\tau_\phi = 85$. $T = 1.35$ K for all the cases, $B_{tr} = 8.4$ mT.

is observed in the balance. This, indeed, should be happening when the interwell transition rate is low, $1/\tau_{12} \ll 1/\tau_\phi$, and $\Delta \sigma(B)$ is the sum of the contributions from each well.

Closeness of the mobilities in the balance means closeness of transport fields, $B_{tr1} = B_{tr2} = B_{tr}$ so that $\Delta \sigma(B)$ should be described by (1), which in this case reduces to expression (1) with $\alpha = 2$. Therefore, let us start analysis from $V_g = -1.5$ V. The result of the best fit within the magnetic field range $(0 - 0.3)B_{tr}$ for $T = 1.35$ K is shown in Fig. 1(b). It is evident that (1) perfectly fits the data with the prefactor which value is really about two. The temperature dependences of the fitting parameters $\tau_\phi$ and $\alpha$ are presented in Fig. 2. It is seen that the $T$ dependence of $\tau_\phi$ is close to $1/T$ (some deviation will be discussed below). Slight decrease of the prefactor with the growing temperature is a consequence of the lack of the diffusion regime due to the decrease of the $\tau_\phi$ to $\tau$ ratio [4] from 85 at $T = 1.35$ K to 30 at $T = 4.2$ K.

Let us now analyze the data in the vicinity of $V_g = -1.5$ V. The gate voltage dependence of the fitting parameter $\tau_\phi^*$ presented in Fig. 3(a) by the diamonds exhibits the sharp minimum at $V_g = -1.5$ V (the superscript * implies that the $\tau_\phi$ value found from the fit can differ from the true dephasing time). It stands to reason that this minimum results from the interwell transitions neglected in Eq. (2). Theoretically, their role in the positive magnetoconductivity was considered in Refs. [8] and [9]. However, our attempts to use the expressions from these papers for the data fit have failed. The values of $\tau_\phi$ and $\tau_{12}$ are obtained with very large uncertainty. This is
because the shape of the magnetoconductivity curve in actual case is mainly controlled by some combination of $\tau_\phi$ and $\tau_{12}$ but not by $\tau_\phi$ and $\tau_{12}$ individually. It becomes clear when considering the parabolic (at $B \to 0$) and logarithmic (at $B \gg B_\text{tr}/\tau_\phi$) asymptotics of Eq. (24) in Ref. [9] at $\tau_{12} > 2 \tau_\phi$. They coincide with corresponding asymptotics, which one can extract from Eq. (1) by using $\alpha = 2$ and $\tau_\phi^* = (1/\tau_\phi + 1/\tau_{12})^{-1}$ instead of $\tau_\phi$. So, the sum $1/\tau_\phi + 1/\tau_{12}$, but not the rates $1/\tau_\phi$ and $1/\tau_{12}$ separately are experimentally obtained when the interwell transitions are relatively rare.

According to Ref. [9], the interwell transition rate resonantly depends on the difference between the Fermi energies in the wells. When the electron density in one well (in lower one in our case) depends on the gate voltage weakly, the $V_g$ dependence of the transition rate is given by the following expression

$$\frac{1}{\tau_{12}(V_g)} = \frac{1}{\tau_{12}^h} \left[ 1 + \left( \frac{\pi h}{m} \frac{dn}{dV_g}(V_g - V_g^h) \tau_\phi \right)^2 \right]^{-1},$$

which can be directly compared with the data. Here, $1/\tau_{12}^h$ is the rate in the balance and $m$ is the electron effective mass. Experimentally, the rate $1/\tau_{12}$ near the balance can be found as the difference between $1/\tau_\phi^*$ and $1/\tau_\phi$, if one supposes that the interwell transitions do not contribute to $\tau_\phi^*$ at $V_g = -1.8 \text{ V}$ and $V_g = -1.2 \text{ V}$, and $1/\tau_\phi$ linearly depends on $V_g$ between

Figure 2. The temperature dependences of $\tau_\phi$ (a) and $\alpha$ (b) found from the fit of the magnetoconductivity curves by (1). Solid symbols are obtained at $V_g = -1.5 \text{ V}$ when the structure is in the balance. Open symbols correspond to $V_g = -3.7 \text{ V}$ when the conductivity over the upper well is negligible. The solid and dashed lines are the functions $6 \times 10^{-11}/T$ and $(T/6.4 \times 10^{-11} + 1/2 \times 10^{-10})^{-1}$, respectively.

Figure 3. (a) The values of the fitting parameter $\tau_\phi^*$ plotted against the gate voltage for structures 3243 and 3154. Solid symbols are obtained near the balance. The open symbols correspond to SQW regime. The lines are provided as a guide to the eye. (b) The $1/\tau_\phi^*$ vs $V_g$ plot for structure 3243 near the balance. (c) The $V_g$ dependence of the transition rate for structure 3243. Symbols are the experimental data, the line is equation (3).
these gate voltages as shown in Fig. 3(b). The results are presented in Fig. 3(c) by the circles. As seen the experimental $V_g$ dependence of $1/\tau_{12}$ is really close to the theoretical one calculated from Eq. (3) with $\tau_{12}^b = 1.7 \times 10^{-10}$ s and $\tau = 5.5 \times 10^{-13}$ s found from the mobility value at $V = V_g^b$ given in table 1.

Thus, we obtain from the above analysis that the dephasing time $\tau_\phi$ in the balance is about $5 \times 10^{-11}$ s at $T = 1.35$ K, whereas the interwell transition time is approximately three times larger, $\tau_{12} \simeq 1.7 \times 10^{-10}$ s. The fact that $\tau_\phi^* \sim 10^{-11}$ s, which value is close to $1/[2 \pi c (\hbar)]$, determines the tendency to saturation of the $\tau_\phi^*$ vs $T$ dependence at low temperature [see the solid symbols and the dashed curve in Fig. 2(a)].

It is impossible to find the values of $\tau_{\phi 1}$ and $\tau_{\phi 2}$ within the gate voltages range from $-3.5$ V to $-2.0$ V because six independent parameters govern the magnetoconductivity [see discussion below Eq. (2)]. However, it can be easily done at lower gate voltage, $V_g < -3.7$ V, when the conductivity and, thus, the magnetoconductivity are determined by the lower well only. The $\Delta \sigma$ vs $B$ curve for this case is well fitted by Eq. (1) with the prefactor $\alpha$, which value is close to 1 [see Fig. 2(b)]. Therewith the temperature dependence of $\tau_\phi$, as seen from Fig. 2(a), is close to $1/T$.

**Figure 4.** (a) The conductivity dependence of the phase relaxation time for the SQW regime. The solid line is calculated according [10]. The dashed line is obtained by multiplying the solid curve by the factor 0.7. (b) and (c) The phase relaxation time plotted against the conductivity per the well for the SQW regime (open symbols, dashed lines) and for the DQW regime near balance (solid symbols, solid lines). The dashed lines are calculated in the same way as in the panel (a). The solid lines are the theory for the DQW regime.

As known, the dephasing time $\tau_\phi$ should mainly depend on the conductivity, at least for the SQW structure [10]. In Fig. 4(a), we show the data obtained for both structures in the SQW regime. In the same figure, the theoretical dependence [10] is depicted by the solid line. As seen, being multiplied by the factor 0.7 it perfectly describes the data within the entire conductivity range. We believe that the appearance of the additional factor can result from the fact that the expression used for the data treatment does not take into account the magnetic field dependence of $\tau_\phi$, the contributions of the second order corrections, and the finiteness of the $\tau_\phi$ to $\tau$ ratio.

Let us again direct the reader’s attention to the data obtained near the balance. These data are presented in Figs. 4(b) and 4(c) by the solid symbols. As seen the experimental $\tau_\phi$ values in SQW and DQW regimes fall on the common curve. This means that the inelastic interaction of an electron in the one well with electrons in the other one does not contribute noticeably to the dephasing rate. At first glance the last seems very strange because the distance between the wells is about screening length so that the interaction between the carriers located
in the different wells should not be significantly less than that for carriers in the one well. On the other hand, the inelastic interaction of an electron with the electrons in the other well is not sole effect influencing the dephasing rate. Another effect is an additional screening of the interaction between the electrons in the one well by the carriers located in the other one. The former effect should lead to the \( \tau_\phi \) decrease, while the second one to its increase. In our previous paper [7] we have compared the data obtained with the results of simple theory which takes into account both these effects in singlet channel only. Remarkable discrepancy has been mentioned. The theory predicts the increase in the dephasing time in double-layer structures as compared with the single-layer case [shown by solid lines in figures 4(b) and 4(c)], which is not observed experimentally.

In recent paper [11], the theory, which takes into account contributions to the dephasing rate from the interaction both in singlet and multiplet channels has been developed. According to [11] the dephasing rate at the balance can be found from equation

\[
\frac{\tau_\phi^0}{\tau_\phi} = \frac{A}{A_0} \frac{\ln T}{\ln T_0}, \quad A_0 = 1 + \frac{3\tilde{\gamma}_{t,0}^2}{2 + \tilde{\gamma}_{t,0}}, \quad A = \frac{1}{2} \left( 1 + \frac{\tilde{\gamma}_s^2}{2 + \tilde{\gamma}_s} + 6\tilde{\gamma}_t^2 + 8\tilde{\gamma}_v^2 \right),
\]

(4)

where \( \tau_\phi^0 \) denotes the dephasing time of a single layer, and \( \tilde{\gamma}_{t,0}, \tilde{\gamma}_s, \) and \( \tilde{\gamma}_v \) are the interaction constants. Using the values of interaction constants listed in table 1 for our samples, we obtain the value 0.77 for \( A_0 \) to \( A \) ratio instead of 0.75 in [7] for both samples. The difference between these values is very small and does not practically change the run of sold curves in Fig. 4(c). Thus, the taking into account of the interaction contribution in the multiplet channel does not explain the experimental results concerning the dephasing rate in the DQW heterostructures.

4. Conclusion
We have studied the interference quantum correction in the gated double quantum well \( \text{Al}_x\text{Ga}_{1-x}\text{As/GaAs/Al}_x\text{Ga}_{1-x}\text{As} \) heterostructures. Analyzing the positive magnetococonductivity we have obtained the interwell transition rate and the phase relaxation rate under the conditions when one and two quantum wells are occupied. It has been found that the dephasing rate in the lower quantum well is independent of whether the upper quantum well contributes to the conductivity or not. This observation is inconsistent with theory, which predicts the increase of the dephasing time in double layer structures as compared with the single layer case.

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