Motion control of a space robot at launching and approaching a geostationary satellite

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Abstract. The problems of putting a space robot into a geostationary orbit using a combined scheme and its approach to an information geostationary satellite are considered. When approaching, the robot’s attitude and orbit control system uses an electric propulsion unit, a propulsion system based on eight electro-catalytic reaction engines with pulse-width modulation of their thrust, and a gyroscopic moment cluster based on four single gimbal control moment gyroscopes (gyrodines). Numerical results are presented that demonstrate the effectiveness of the developed discrete guidance and control algorithms.

1. Introduction

Information satellites (communications, meteorological Earth observation) in geostationary orbit (GSO) have a required life-time of up to 25 years if they are serviced by space robot-manipulators (SRMs), in particular refueling of their electric propulsion units (EPUs). In modern cosmonautics, there is a regular tendency to increase the payload mass of geostationary information satellites. Restrictions on the permissible mass of fuel consumption when launching a large-sized spacecraft (SC) on the GSO lead to the problem of ”add-launching” the spacecraft from a transition orbit to a geostationary one using own onboard EPU [1]. In this regard, the problematic tasks are to use electric propulsion both for add-launching the SRM and at the flybys of the geostationary satellite during its maintenance.

To launch geostationary spacecraft, the launch vehicles with an upper stage are used, which is capable of performing the necessary maneuvers to transfer the SC from initial elliptical geotransfer orbit (GTO) to the GSO. This scheme requires the availability on SC board own chemical propulsion unit (CPU) with large thrust, which is not an effective solution: mass of fuel for this additional launching can be up to 50 % of the SC starting mass. At the same time, low thrust electric reaction engines (EREs) as part of the EPU greatly increases the time of add-launching spacecraft, as well as the time spent in the area the most dangerous inner radiation belts at altitudes of from 2000 to 12,000 km, which imposes high requirements on radiation protection of both payload and service systems, including solar array panels (SAPs). Therefore, for the successful delivery of the SRM to the GSO with minimal fuel consumption in an acceptable time, it is rational to use a combined scheme based on the consequent operation of the CPU to form a transition orbit, when the SC quickly passes the earth’s internal radiation belts, and the EPU which is used for the subsequent add-launching the SRM on the GSO.
To implement such combined scheme, research and development has been intensively carried out over the past two decades. The first US satellites based on the Boeing 702sp platform with an electric propulsion were add-launched on GSO in 2015. JSC Reshetnev ISS also started to solve these problems in practice, the first Russian Express-AM5/AM6 communication satellites were delivered to the GSO using their own EPUs in 2013-2014 and 2015, respectively.

The paper deals with two problems: (i) choice the Russian drives for the SRM launching on GSO by combined scheme; (ii) synthesis of the guidance and control laws for the SRM when it approaching a target (geostationary satellite), and also a nonlinear dynamical analysis of the SRM attitude and orbit control system in this approaching.

2. Mathematical models and the problem statement
For the SRM and target motions in near-earth space it is very difficult to solve the above key problems. It is necessary to take into consideration the progressive and rotational motions of both the target and controlled SRM according to the laws of the space flight mechanics in the gravitational fields of the Earth, Moon and Sun. We use the inertial reference frame (IRF) $I$, the SRM body reference frame (BRF) $O_{xyz}$ and the target BRF $O_{x't'y't'z'}$, and also conventional symbols {·}≡col(·), [·]≡line(·), (·)$^t$, [·] and o, c for vectors, matrices and quaternions.

The thrust vector $P^e$ of the CPU is directed along the axis $O_{x'y'}$ of the robot’s BRF, as is the thrust vector $P^c$ of an electric propulsion unit.

In the EPU scheme (Fig. 1a) we present the unit vectors $e_p$, $p=1 \div 8$ of the ERE nozzle axes. Assume that $p_p$ define vectors of points $O_p$ for application of $p$-th thrust vector. Each electro-catalytic reaction engine has the pulse-width modulation (PWM) of its thrust $p_p(t)$, which is described by the nonlinear continuous-discrete relation

$$p_p(t) = P^m \text{PWM}(t - T^e_{zu}, t_r, \tau_m, v_{pr}) \forall t \in [t_r, t_{r+1})$$

with period $T^e_u$ and a time delay $T^e_{zu}$. Here $t_{r+1} = t_r + T^e_u$, $r \in \mathbb{N}_0 \equiv [0, 1, 2, \ldots)$ and functions

$$\text{PWM}(t, t_r, \tau_m, v_{pr}) \equiv \begin{cases} \text{sign} v_{pr} & t \in [t_r, t_r + \tau_{pr}), \\ 0 & t \in (t_r + \tau_{pr}, t_{r+1}); \end{cases} \quad \tau_{pr} \equiv \begin{cases} 0 & \text{sat} (T^e_u, |v_{pr}|) \\ |v_{pr}| & |v_{pr}| > \tau_m, \end{cases}$$

where $P^m$ is the nominal value of a thrust, similar for all catalytic EREs. In BRF the thrust vector of $p$-th ERE is calculated by relation $p_p(t) \equiv \{p_p\} = -v_p(t) e_p$, and vectors of the EPU force $P^e = \{p_p\}$ and torque $M^e = \{p_p\} \times \{p_p\}$.

Column $\mathbf{H}(\beta) = h_y \Sigma h_p(\beta_p)$ presents the angular momentum (AM) vector for the GMC scheme 2-SPE based on four gyrodines (GDs), Fig. 1b, where $|h_p| = 1$, $p = 1 \div 4$, and $h_y$ is a constant.

Figure 1. The schemes of EPU with 8 catalytic EREs (a) and GMC based on 4 gyrodines (b)
own AM of each gyroline. For the GD drive’s gear with a large transfer ratio, the command $u_p = \beta_p(t)$ and true $\dot{\beta}_p(t)$ angular rates are close to each other. Then vector $M^g$ of the GMC control torque is represented by the nonlinear relation

$$M^g = -\mathbf{H}(p) \dot{\beta}(t); \dot{\beta} = u^g(t) \equiv \{u^g_k(t)\}$$

with digital control $u^g_p = Z_h \text{sat}(\text{qnt}(u^g_{pk}, u^g_{pq}), u^g_{mu}), T_u$ for $k \in N_0$ and period $T_u$ where vector column $\beta = \{\beta_p\}$, matrix $A_h(\beta) = \partial \mathbf{H}(\beta)/\partial \beta$ and $(-)^*$ is the symbol of local time derivative.

In IRF the robot’s BRF orientation is determined by quaternion $\mathbf{A} = (\lambda_0, \lambda_1, \lambda_2, \lambda_3)$. We use a vector of modified Rodrigues parameters (MRP) $\mathbf{A} = e \text{tg} (\Phi/4)$ with Euler unit vector $e$ and angle $\Phi$ of own rotation which is uniquely connected with quaternion $\mathbf{A}$ by analytic relations. The kinematic equations for vector $\mathbf{r}$ of the robot’s location and quaternion $\mathbf{A}$ have the form $\dot{\mathbf{r}}_t = \mathbf{r}^*_t + \omega \times r$ and $\dot{\mathbf{A}} = \mathbf{A} \circ \omega/2$. If the SRM is considered to be a single solid with pole $O_r$ at its mass center, the model of the SRM spatial dynamics has the form

$$m(v^*_r + w \times v_r) = \mathbf{P}^r + \mathbf{R}^d; \quad \dot{\mathbf{K}} + w \times G = \mathbf{M}^k + \mathbf{M}^\iota + \mathbf{M}^d.$$

Here $v_r$ (index $r$, robot) is the velocity vector for SRM forward motion; $\mathbf{K} = Jw$ is vector of the SRM angular momentum, vector $\mathbf{G} = \mathbf{K} + \mathbf{H}(\beta)$, at last $\mathbf{R}^d$ and $\mathbf{M}^d$ are external disturbing force and torque vectors. The SRM location is defined by vector $r_r$ and equation $r^*_t + w \times r_r = v_r$. The vectors $r_r$ and $v_t$ (index $t$, target) represent the geostationary satellite’s location and velocity of its forward motion. The vectors of range to the target $\Delta \mathbf{r} = \{\Delta r_i\}$ and misalignment $\Delta \mathbf{v} = \{\Delta v_i\}$ between the SRM and target velocities are calculated by ratios $\Delta \mathbf{r} = r_r - r_r$ and $\Delta \mathbf{v} = v_r - v_r$.

For the SC guidance law $\mathbf{A}^\iota, \mathbf{w}^\iota, e^\iota = \dot{\mathbf{A}}^\iota \circ \mathbf{A}$ corresponds to Euler parameters’ vector $\mathbf{E} = \{e_0, e\}$ with $e = \{e_i\}$, the angular error matrix $C^\iota = C(\mathbf{E}) = I_3 - 2[e \times] Q^\iota, Q^\iota \equiv Q(\mathbf{E}) = I_3 e_0 + [e \times]$, the MRP vector $\mathbf{A}^\iota = e^\iota \text{tg} (\Phi^\iota/4)$ and the angular error vector $\delta \mathbf{E} = [\delta e_i] = 2[e, e_i]$. The error vector $\delta \omega$ in angular rate is calculated by the ratio $\delta \omega = \omega - C^\iota \omega^\iota$.

Measurement of kinematic parameters of the SRM spatial motion is carried out by the strapdown inertial navigation system (SINS) with correction by signals from the GPS/GLONASS satellites and star trackers. If the distance is less than 500 m, the parameters of the SRM motion regarding a moving geostationary satellite are determined using optoelectronic cameras and laser rangefinders.

We assume that the launch of a SRM with initial mass of 6300 kg to an elliptical GTO with perigee altitude $r_x = 200$ km, apogee altitude $r_a = 35786$ km and inclination $i = 51.6$ deg is performed by launching the Proton-M launch vehicle with Breeze upper stage from Baikonur cosmodrome. The applied strategy for the subsequent launch of the SRM to the GSO contains the following stages: 1) moving SRM to a transition orbit using CPU with thrust value $P^e = 200$ N when the steps (1a) of zeroing the orbit inclination and (1b) of raising its perigee altitude to 11,000 km are performed sequentially; 2) add-launching SRM from a transition orbit to GEO using EPU with thrust value $P^e = 0.58$ N to the target’s range of 500 m; 3) approaching SRM to the target at a distance of 25 m using the cluster of 8 catalytic EREs.

Estimates of fuel consumption (of 3012 kg) and duration (of 7 days) for orbital maneuvers at the first stage were obtained [2] by well-known methods. As a result, the main objectives of the paper are problems on synthesis of the guidance and control laws for the SRM when its add-launching on GSO and approaching a target, and also on nonlinear dynamical analysis of the SRM attitude and orbit control system in this approaching.

3. The add-launching a space robot on GSO by electric propulsion unit

At the first stage we have studied the problem of the minimum add-launching time using the simplest SRM model in the form of a point with variable mass and instantaneous change in
the EPU thrust direction [3]. The scheme for add-launching was selected with two consecutive sections: 1) increase the perigee radius \( r_\pi \) and 2) decrease the eccentricity \( e \). At the same time, the EPU worked non-stop and the duration of the sections was determined in such a way as to ensure that the required boundary conditions were met: the perigee radius \( r_\pi = 42786 \) km and the apogee radius \( r_\alpha = 42786 \) km. As a result, we got a duration of 45.1 days for first section and a duration of 67.0 days for second section with the total duration of 112.1 days and fuel consumption of 329.1 kg, see Figs. 2 and 3.

In the second stage, we studied this problem for the SRM model as a solid body (1), controlled by both the ERU and GMC for the SRM reversals when the ERU is not running [2]. As a result, we have obtained the total duration of add-launching of 122 days but the fuel consumption only of 270 kg. Here, the problematic challenges consist in providing electric power to the EPU when pointing large-sized SAPs at the Sun and in control the SRM orientation with minimizing the EPU fuel consumption during the SRM add-launching.

Figure 2. Changing the radius of robot’s orbit during its add-launching on GSO

Figure 3. Changes in the radii of perigee and apogee at add-launching the SRM on GSO

Figure 4. Scheme of the robot’s approach to a geostationary satellite
4. Guidance and control at the SRM approaching a geostationary satellite

At the initial moment of time $t_i$, the vectors of the location and speed of forward movement are known in the IRF both for the SRM $\mathbf{r}_r(t_i)$, $\mathbf{v}_r(t_i)$ and the target $\mathbf{r}_t(t_i)$, $\mathbf{v}_t(t_i)$. When introducing a reference circular orbit of radius $r(t_i) = \text{const}$ in the plane of the Earth’s equator, it is convenient to use a cylindrical reference frame (CRF) with standard coordinates $r, u$ and $z$ [4]. The SRM forward motion is determined by the relations

$$
\mathbf{r}_r = \{r C_u, r S_u, z\}; \quad \mathbf{v}_r = \{\dot{r} C_u - r S_u \dot{u}, \dot{r} S_u + r C_u \dot{u}, \dot{z}\}.
$$

Assume that $w^r$, $w^t$ and $w^z$ represent the radial, transversal and lateral components of the SRM control acceleration vector, and $\mu$ is the gravitational parameter of the Earth. The approach of the SRM to the target (Fig. 4) in the central gravitational field over a time interval $t \in [t_i, t_f]$ is described by the equations

$$
\ddot{r} - r \dot{u}^2 + \mu/r^2 = w^r; \quad r \ddot{u} + 2 \dot{r} \dot{u} = w^t; \quad \ddot{z} + \mu z/r^3 = w^z
$$

with the known boundary conditions on the orbital variables of the CRF. Here a forecast of the target’s location and speed, and also a calculation of the vectors $\mathbf{r}_t(t_f)$, $\mathbf{v}_t(t_f)$ are performed using the analytical relations [4].

The synthesis of the SRM guidance law in its forward motion is performed in the form of vector time spline with three sections of constant control acceleration, where there is no acceleration in the middle section. This guidance law determines the vectors $\mathbf{r}_p(t_f)$, $\mathbf{v}_p(t_f)$ and $\mathbf{w}_p(t_f)$ for the SRM design forward motion and further allows to calculate the differences between the target and the SRM locations $\Delta \mathbf{r}(t) = \mathbf{r}_t(t) - \mathbf{r}_r(t)$, their speeds $\Delta \mathbf{v}(t) = \mathbf{v}_t(t) - \mathbf{v}_r(t)$, as well as the differences $\Delta \mathbf{r}^p(t) = \mathbf{r}_p(t) - \mathbf{r}_r(t)$ and $\Delta \mathbf{v}^p(t) = \mathbf{v}_p(t) - \mathbf{v}_r(t)$. The SRM angular guidance law is determined by the design values of the quaternion $\Lambda_p(t)$, vectors of angular rate $\mathbf{\omega}^p(t) = \{\omega_i^p(t)\}$ and angular acceleration $\mathbf{\epsilon}^p(t) = \{\epsilon_i^p(t)\}$.

The catalytic EPU discrete control algorithm uses a vector $\delta \Delta \mathbf{r}_r = \Delta \mathbf{r}^p - \Delta \mathbf{r}_r$ of mismatch
between the design difference $\Delta r^p \equiv \Delta r^p(t_r)$ and the measured difference $\Delta r_r \equiv \Delta r(t_r)$ for the target and robot locations, moreover the values $\delta \Delta r_r$ are formed in the robot’s BRF within the design difference $\Delta r^p(t_r)$ and the measured difference $\Delta r(t_r)$ for the target and robot locations, moreover the values $\delta \Delta r_r$ are formed in the robot’s BRF within the period $T_e$ at the time moments $t_r, r \in N_0$. In this algorithm, in the first the command vector $I^e_r$ of the catalytic EPU thrust pulse is calculated over semi-interval $t_\in [t_{r+1}, t_{r+1}]$ using the relations

$$
g^e_{r+1} = k^e_b g^e_r - k^e_c \delta \Delta r_r; \quad \mathbf{\tilde{p}}_r = k^e_u (g^e_r - k^e_p \delta \Delta r_r); \quad I^e_r = T_u m (C^e_r w^p_r + \mathbf{\tilde{p}}_r),$$

and then for its implementation the durations of the PWM thrust activation for all eight EREs are computed by explicit relations [5]. In the algorithm of the SRM attitude digital control with a period $T_u$, the vectors of angular mismatch $\varepsilon_k = -\delta \phi_k$ and angular rate $\omega_k$ are determined to calculate the required control torque vector $M^g_k$ of the GMC in the form

$$
g^w_{k+1} = k^w_k g^w_k + k^w_c \varepsilon_k; \quad \mathbf{m}_k = k^m_k (g^w_k + k^w_p \varepsilon_k); \quad M^g_k = \omega_k \times G_k + J (C^w_k \varepsilon_k + [C^w_k \omega_k \times \omega_k + \mathbf{m}_k] (3)

with vector $G_k = J \omega_k + H_k$ of total angular momentum, and then vector $M^g_k$ is distributed between the GDs by explicit relations [6, 7]. As a result, the gyrodines’ digital control vector is formed as follows $u^g_k(t) = \dot{\beta}(t) \forall t \in [t_k, t_{k+1}]$.

5. Nonlinear dynamical analysis of the SRM attitude and orbit control system
Nonlinear analysis was performed for the model (1) with the specified guidance and control laws (2) and (3). There are two stages in the strategy for approaching the SRM with a target from 5000 m to a range of 50 m:

1) approaching the SRM at a distance of 500 m following the target along its orbit close to the geostationary one, using the EPU and GMC;

2) further approaching SRM the target up to a range of 50 m using the catalytic EPU and GMC, preparation of the SRM for visual inspection of the target with a nominal standing point $\Omega_g = 76$ deg East longitude on the GSO with the nominal radius $r_g = 42164172.93$ m.
We accepted the initial orbital parameters of the target $r_\pi = r_g$, $r_\alpha = r_g + 1000$ m, $i = 0$, $\Omega = \Omega_g + 24.46$ arc sec and the robot $r_\pi = r_g - 500$ m, $r_\alpha = r_g + 2000$ m, $i = -20$ arc sec, with zero values of the perigee arguments and the time moments of their passage.

In computer simulation the approaching of the robot with mass $m = 3000$ kg and the inertia tensor $J = \text{diag}(3248, 2348, 3640)$ kg m$^2$, we used the PWM period $T_e = 4$ s for the ERE thrust as part of the catalytic EPU with a time delay $T_{e_u} = 0.25$ s, the digital control period $T_u = 0.5$ s of the GDs as part of the GMC, and also the time counting from the value $t = t_0 = 0$.

At the beginning of stage 1, during the time semi-interval $t \in [0, 70]$ s, the SINS measurements are processed, the target movement forecast is made, and the robot guidance law is synthesized to achieve a range of 500 m to the target. Then follow:

(i) the first spatial rotational maneuver (RM-1) of the SRM $\forall t \in [70, 170]$ s at the angle 26 deg and the SRM angular stabilization $\forall t \in [170, 270]$ s;

(ii) accelerating pulse of the EPU thrust $\forall t \in [270, 4498]$ s, the RM-2 $\forall t \in [5660, 5860]$ s at the angle 156 deg, braking pulse of the EPU thrust $\forall t \in [5960, 10168]$ s and, finally, the RM-3 $\forall t \in [10168, 10368]$ s at the angle of 38.7 deg.
Stage 2 of the SRM approach with a target from a distance of 500 m to a range of 50 m begins at the time moment \( t^* = 10368 \) s, see Figs. 5 – 10.

The stage was simulated in the time interval \( t \in [10368, 14468] \) s, when the coordinates of the SRM spatial movement relative to a geostationary satellite are measured by the onboard optoelectronic devices and the digital control laws (2), (3) are applied.

The obtained results are presented in Figs. 5 – 11, where the changes are highlighted in color – blue, yaw axis \( x \); green, roll axis \( y \); red, pitch axis \( z \) and the distance module, black.

6. Conclusions
Methods of the guidance and control for a space robot-manipulator add-launching on GSO and approaching a geostationary satellite are briefly presented, as well as numerical results demonstrating the effectiveness of the developed algorithms.

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