Hard Leptoproduction of Charged Vector Mesons$^*$

L. Mankiewicz$^†$, G. Piller and T. Weigl

Physik Department, Technische Universität München, D-85747 Garching, Germany

Abstract

We present an analysis of twist-2, leading order QCD amplitudes for hard exclusive leptoproduction of charged vector mesons. These processes are determined by nonforward parton distribution functions which are nondiagonal in quark flavor. We derive relations between flavor diagonal and nondiagonal distribution functions based on isospin symmetry. Furthermore, we discuss general features of $\rho^+$ and $\rho^−$ production cross sections, and present estimates based on a simple model for nonforward distribution functions.

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$^†$On leave of absence from N. Copernicus Astronomical Center, Polish Academy of Science, ul. Bartykowa 18, PL-00-716 Warsaw (Poland)
1 Introduction

Recently much effort has been devoted to study properties of nonforward parton distribution functions. They can be viewed as generalizations of ordinary parton distributions. In addition they are closely related to nucleon form factors. Thus, they combine different aspects of the nucleon structure and offer new insights. Although being discussed already some time ago [1, 2, 3], new detailed investigations of their properties were initiated recently in the context of the nucleon spin structure [4, 5].

To summarize the relation of nonforward to ordinary parton distributions, recall that the latter can be represented at the twist-2 level as normalized Fourier transforms of forward nucleon matrix elements of nonlocal QCD operators which are constructed as gauge-invariant overlap of two quark or gluon fields separated by a light-like distance [6, 7]. Nonforward parton distributions are defined by the same nonlocal operators – just sandwiched between nucleon states with different momenta and eventually spin. Equally close is the relation to nucleon form factors which are defined by the nonforward matrix elements taken in the local limit.

Nonforward parton distributions are probed in processes where the nucleon target recoils elastically. To select twist-2 correlations a large scale has to be involved. One possible process is deeply-virtual Compton scattering [4, 5]. Another promising class of reactions sensitive to nonforward distribution functions is exclusive hard meson production as discussed initially in Ref. [3]. In a recent publication [8] we have studied the production of neutral mesons. In this work we focus on charged vector meson production. Here new distribution functions are probed which are nondiagonal in flavor. They describe a situation where, for example, an up-quark from a proton is removed and a down-quark is returned to form a neutron. In this letter we derive relations between flavor diagonal and nondiagonal distribution functions based on isospin symmetry. Subsequently, we use these relations to estimate $\rho^+$ and $\rho^-$ production cross sections within a simple model for the involved nonforward distribution functions.

Data on the exclusive production of neutral vector mesons have been taken lately at high center of mass energies at HERA (for a review and references see Ref. [9]). In the measured kinematic domain the corresponding production cross sections are controlled by the nonforward gluon distribution of the target. This will be quite different in charged vector meson production where gluon contributions are absent in leading order.

This letter is organized as follows: in Sec. 2 we present the production amplitude for charged vector meson production. Relations between flavor diagonal and nondiagonal distribution functions are derived in Sec. 3. In Sec. 4 we discuss results for $\rho^+$ and $\rho^-$ production. Finally we give a short summary.

2 Production amplitude

Solid QCD descriptions of hard exclusive meson production processes are based on the factorization of long- and short-distance dynamics which has been proven recently in
Ref. [10]. In this work it has been shown that at large photon virtualities $Q^2 \gg \Lambda_{QCD}^2$ and moderate momentum transfers $|t| \sim \Lambda_{QCD}$ the amplitudes for the production of mesons from longitudinally polarized photons can be split into three parts: the perturbatively calculable hard photon-parton interaction arises from short distances, while the long-distance dynamics can be absorbed in nonperturbative meson distribution amplitudes and nucleon nonforward parton distributions. In [8] we have outlined the derivation of the amplitudes for neutral meson production. Amplitudes for charged vector meson production can be obtained in a similar way. In leading order in the strong coupling constant $\alpha_s$ we obtain for the $\rho^+$-production amplitude:

$$A_{\rho^+} = \pi \alpha_s \frac{C_F}{N_c} \frac{1}{Q} \hat{N}(P', S') \hat{n} N(P, S) f^L_{\rho} \int_0^1 d\tau \frac{\Phi^L_{\rho}(\tau)}{\tau \bar{x}} \int_0^1 dx \int_0^\omega dy$$

$$\times \left[ (e_d F^{ud} + e_u \bar{F}^{ud}) \frac{\bar{\omega}}{x + 2y + x\bar{\omega} - i\epsilon} - (e_u F^{ud} + e_d \bar{F}^{ud}) \frac{\bar{\omega}}{x + 2y - x\bar{\omega} - i\epsilon} \right] + K - \text{terms.}$$

(1)

$N(P, S)$ ($N(P', S')$) stands for the Dirac spinor of the initial (scattered) nucleon with four momentum $P$ ($P'$) and spin $S$ ($S'$). Here and in the following we do not explicitly specify the contribution arising from the so-called “K-terms” which are proportional to the momentum transfer $r = P - P'$ (see e.g. [11]). The average nucleon momentum is denoted by $\bar{P} = (P + P')/2$. The four momentum of the incident virtual photon is $q$ and $Q^2 = -q^2$. The produced vector meson carries the four momentum $q'$ and $\bar{q} = (q + q')/2$. In addition we have introduced the variable $\bar{\omega} = 2\bar{q} \cdot \bar{P}/(-q^2)$. Furthermore, $n$ denotes a light-like vector with $n \cdot a = a^+ = a^0 + a^3$ for any vector $a$, and $\hat{n} = \gamma_\mu n^\mu$. In Eq.(1) the perturbative photon-parton interaction has been calculated to leading twist accuracy. As compared to the virtuality of the photon $Q^2$ we have therefore neglected the momentum transfer $t = (P - P')^2$, the invariant mass of the nucleon target $P^2 = P'^2$, and the mass of the produced vector meson $q^2$.

All information about the long-distance dynamics of the produced longitudinally polarized $\rho$ meson is contained in the decay constant $f^L_{\rho}$ and the distribution amplitude $\Phi^L_{\rho}$ [11]:

$$\langle \rho^+(q') | \bar{\psi}(x) \hat{n} \psi(y) | 0 \rangle = \pm q' \cdot n f^L_{\rho} \int_0^1 d\tau \Phi^L_{\rho}(\tau) e^{iq' \cdot (\tau x + \tau y)}.$$ 

(2)

Here the quark fields $\psi$ are understood to carry proper flavor quantum numbers.

Contrary to neutral meson production [8] the nucleon part of the amplitude in Eq.(1) is determined by nucleon double distribution functions $F^{ud}$ and $\bar{F}^{ud}$ which are nondiagonal in flavor. They correspond to a situation where e.g., an up-quark with electromagnetic charge $e_u$ is removed from the target but a down-quark with charge $e_d$ is returned. The operator definition of these genuine new object reads:

$$\langle p(P', S') | \bar{\psi}_u(0) \hat{n} [0, z] \psi_d(z) | n(P, S) \rangle_{z^2 = 0} = \hat{N}(P', S') \hat{n} N(P, S)$$

$$\times \int_0^1 dx \int_0^\omega dy \left[ e^{-i(z(P, z) - i\hat{n} (r, z)) F^{ud}(x, y, t)} - e^{i(z(P, z) - i\hat{n} (r, z)) \bar{F}^{ud}(x, y, t)} \right] + K - \text{terms.}$$

(3)
Here ψ_u and ψ_d represent up- and down-quark fields which are separated by a light-like distance \( z \sim n \). The proton and neutron states are denoted by \(|p(P', S')\rangle\) and \(|n(P, S)\rangle\). Gauge invariance is guaranteed by the path-ordered exponential \([0, z] = P\exp[-igz\mu - \int_0^1 d\lambda A^\mu(z\lambda)]\) which reduces to one in axial gauge \( n \cdot A = 0 \) (\( g \) stands for the strong coupling constant and \( A^\mu \) denotes the gluon field).

The amplitude for \( \rho^- \)-production can be obtained from Eq.(1) by exchanging quark flavors, \( u \leftrightarrow d \), and applying an overall minus sign due to the definition of the corresponding distribution amplitude (2).

Note that due to the necessary charge transfer in the t-channel, gluons do not contribute in leading order to hard exclusive leptoproduction of charged mesons.

### 3 Isospin relations

In the following we use isospin symmetry to relate flavor nondiagonal double distribution functions to flavor diagonal ones. Because the argument of the nonlocal twist-2 string operator in Eq.(3) is a light-like vector, it is convenient to introduce conserved isospin charges defined on a light-like hyper-surface:

\[
\hat{\tau}^i = \frac{1}{2} \int d^2 x_\perp dx^- \tilde{\Psi}(x^+ = 0, \vec{\tau}) \gamma^+ \tau^i \Psi(x^+ = 0, \vec{\tau}),
\]

where \( \vec{\tau} = (x^-, x_\perp) \), \( \Psi = (\psi_u, \psi_d)^T \), and \( \tau^i \) are common Pauli matrices. Since the definition of \( \hat{\tau}^i \) involves only "good" components of the spinor fields, canonical anticommutation relations on the light-cone can be used [12]. They lead to:

\[
[\hat{\tau}^i, \hat{\tau}^j] = i\epsilon^{ijk} \hat{\tau}^k.
\]

Hence the charges in Eq.(4) are indeed generators of the isospin symmetry. As a consequence in an isospin symmetric world proton and neutron states can be related to each other by the usual ladder operators \( \hat{\tau}^\pm = \hat{\tau}^x \pm i\hat{\tau}^y \):

\[
\hat{\tau}^+ |n\rangle = |p\rangle, \quad \hat{\tau}^- |p\rangle = |n\rangle, \\
\hat{\tau}^+ |p\rangle = 0, \quad \hat{\tau}^- |n\rangle = 0.
\]

As a next step we consider the nonforward matrix element (3) which defines the flavor nondiagonal double distribution function \( F^{ud} \) and \( \bar{F}^{ud} \). Using the notation

\[
\hat{O}^{qq'}(z) = \tilde{\psi}_q(0)\hat{n}[0, z] \psi_{q'}(z)|_{z^2 = 0},
\]

with quark flavors \( q \) and \( q' \) one obtains from Eq.(3):

\[
\langle p| \hat{O}^{ud}(z) |n\rangle = \langle p| \hat{O}^{ud}(z) \tau^- |p\rangle = \langle p| [\hat{O}^{ud}(z), \hat{\tau}^-] |p\rangle = \langle p| \hat{O}^{uu}(z) |p\rangle - \langle p| \hat{O}^{dd}(z) |p\rangle.
\]
and, similarly:

\[
\langle p | \hat{O}^{ud}(z) | n \rangle = \langle n | \hat{O}^{du}(z) | p \rangle - \langle n | \hat{O}^{uu}(z) | n \rangle,
\]

\[
\langle n | \hat{O}^{du}(z) | p \rangle = \langle p | \hat{O}^{uu}(z) | p \rangle - \langle p | \hat{O}^{dd}(z) | p \rangle,
\]

\[
\langle n | \hat{O}^{du}(z) | p \rangle = \langle n | \hat{O}^{dd}(z) | n \rangle - \langle n | \hat{O}^{uu}(z) | n \rangle.
\] (9)

These relations enable us to express the charged meson production amplitudes in Eq. (1) in terms of flavor diagonal double distribution functions $F_u \equiv F^{uu}$ etc.:

\[
\mathcal{A}^{\rho^\pm} \sim \int_0^1 dx \int_0^x dy \left[ (F^u - \bar{F}^u) - (F^d - \bar{F}^d) \right] \left( \frac{\bar{\omega}}{x + 2y + x\bar{\omega} - i\epsilon} + \frac{\bar{\omega}}{x + 2y - x\bar{\omega} - i\epsilon} \right)
\]

\[
\pm \frac{1}{3} \int_0^1 dx \int_0^x dy \left[ (F^u + \bar{F}^u) - (F^d + \bar{F}^d) \right] \left( \frac{\bar{\omega}}{x + 2y + x\bar{\omega} - i\epsilon} - \frac{\bar{\omega}}{x + 2y - x\bar{\omega} - i\epsilon} \right) + \chi - \text{terms}.
\] (10)

Here we have inserted the electromagnetic charges $e_u + e_d = \frac{1}{3}$, and $e_u - e_d = 1$. Note that the second term in Eq. (10) can be related to the isovector part of the $\rho^0$ production amplitude from Ref. [8], $\mathcal{A}_{(i=1)}^{\rho^0} = \frac{1}{2\sqrt{2}}(\mathcal{A}^{\rho^+} - \mathcal{A}^{\rho^-})$.

4 Model

In the following we discuss the production cross sections for $\rho^+$ and $\rho^-$ production using model distribution functions introduced recently in Ref. [8]. For flavor diagonal distributions we choose [13]:

\[
F(x, y, t; \mu_0^2) = h(x, y) f(x, \mu_0^2) f(t),
\] (11)

where $f(x, \mu_0^2)$ stands for the corresponding ordinary quark distribution which we take from Ref. [14]. As shown in Ref. [8], double distribution functions have to be symmetric with respect to an exchange of variables, $y \leftrightarrow 1 - y - x$. In accordance with this constraint we use:

\[
h(x, y) = 6 \frac{y(1 - x - y)}{(1 - x)^3}.
\] (12)

The form factor $f(t)$ is responsible for the $t$-dependence of double distributions. Motivated by the relationship between double distributions and nucleon form factors we assume $f(t) = e^{bt/2}$ with $b = 5 \text{GeV}^{-2}$ (see e.g. discussion in [8]):

For the $\rho$ meson distribution amplitude we use the asymptotic form [13, 16]:

\[
\Phi_\rho(\tau) = 6\tau(1 - \tau).
\] (13)

Finally we fix the $\rho$ meson decay constant $f_\rho^L = 195 \text{ MeV}$ [1].

In our numerical analysis we neglect the practically unconstrained "K-terms". As their contribution enters proportional to the momentum transfer they are bound to be small at small $r$. 
In Fig.1 we present the $\rho^+$ and $\rho^-$ total production cross sections from a proton and neutron target, respectively, for $Q^2 = 10\, \text{GeV}^2$. At small values of $x_{Bj}$ the cross sections are approximately proportional to $x_{Bj}$. This results from the small-$x_{Bj}$ behavior of the flavor nonsinglet combination of ordinary quark distributions which enter in Eqs.(10,11). At large $x_{Bj}$ the production cross sections drop with rising $x_{Bj}$.

It is important to note that the shape of the production cross sections can be motivated from quite general grounds, applicable beyond our leading order, leading twist treatment. The rise at small $x_{Bj}$ with increasing $x_{Bj}$ is expected from the exchange of the leading reggeon poles $\rho$ and $a_2$ [18]. On the other hand at large values of $x_{Bj}$ the minimal momentum transfer to the nucleon target which is required for the production of a vector meson, $t_{\text{min}} \approx -x_{Bj}^2 M^2/(1 - x_{Bj})$, becomes of the order of typical nucleon scales. For example at $x_{Bj} = 0.5$ one has $-t_{\text{min}} \approx 0.5\, \text{GeV}^2$. For such values of $t$ elastic production processes from nucleons are suppressed. A decrease of $\rho^+$ and $\rho^-$ production at large $x_{Bj}$ is the consequence.

Since electromagnetic processes are not isospin invariant the production cross sections for $\rho^+$ and $\rho^-$ may differ. In Fig.2 we show the ratio of both cross sections. In the kinematic region $x_{Bj} \sim 0.3$, where both cross sections are relatively large, $\rho^+$ production from a proton is around 30% larger than $\rho^-$ production from a neutron. At small-$x_{Bj}$ both cross sections are of equal size. Also this feature is quite general and is due to the approximate flavor symmetric sea $\bar{F}_u(x,y) \approx \bar{F}_d(x,y)$, and the small-$x$ behavior $F_u(x,y) - \bar{F}_u(x,y) \sim x^{-0.5}$ which is expected from Regge theory [18].

Finally we should mention that a QCD study of neutral vector meson leptoproduction processes emphasizes the role of higher twist effects [17,19]. A systematic analysis of their importance, also in charged vector meson production, is needed.

5 Summary

We have discussed hard exclusive leptoproduction of charged vector mesons at twist-2 leading order $\alpha_s$ accuracy. In this framework the considered reactions are determined by nonforward parton distribution functions which are nondiagonal in quark flavor. Through isospin symmetry these are related to flavor diagonal distribution functions. Within a simple model for flavor diagonal double distribution functions estimates for $\rho^+$ and $\rho^-$ production have been given. Independent of specific model assumptions we find that the corresponding production cross sections are peaked at moderate values of $x_{Bj}$. Furthermore, at small $x_{Bj}$ both cross sections are of similar size. Differences are expected to show up at moderate $x_{Bj}$ where the $\rho^+$ and $\rho^-$ cross sections assume their maximum.

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Figure 1: Total cross sections for $\rho^+$ (full) and $\rho^-$ (dashed) production from a nucleon through the interaction of a longitudinally polarized photon.

Figure 2: Ratio of cross sections for $\rho^+$ and $\rho^-$ production from a nucleon through the interaction of a longitudinally polarized photon.