SUPERSYMMETRIC HIGGS BOSONS:
A THEORETICAL INTRODUCTION

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Abstract

After an introduction to the Higgs sector of supersymmetric extensions of the Standard Model, recent results on radiative corrections to Higgs boson masses and couplings are reviewed. The phenomenology of supersymmetric Higgs searches at large hadron colliders and at a possible linear $e^+e^-$ collider is also described.

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1 The Higgs sector of SUSY models

It has been known for a long time \[1\] that realistic supersymmetric extensions of the Standard Model (SM) require at least two Higgs doublets. Higgs fields must be embedded in chiral supermultiplets, whose physical degrees of freedom are a complex spin-0 boson and a two-component spin-1/2 fermion, and their Yukawa couplings to the matter fermions are encoded in the superpotential, an analytic function of the chiral superfields\[1\]. At least two Higgs doublets are then needed: 1) to give masses to all quarks and charged leptons via superpotential couplings; 2) to avoid higgsino-induced chiral anomalies (in the absence of other anomaly-cancellation mechanisms); 3) to avoid the existence of a massless charged spin-1/2 particle in the gaugino-higgsino sector.

1.1 The minimal model

In the Minimal Supersymmetric extension of the Standard Model (MSSM), the Higgs sector consists of just two doublets:

\[ H_1 \equiv \left( \begin{array}{l} H_1^0 \\ H_1^- \end{array} \right) \sim (1, 2, -1/2), \quad H_2 \equiv \left( \begin{array}{l} H_2^+ \\ H_2^0 \end{array} \right) \sim (1, 2, +1/2), \tag{1} \]

and, after imposing \(R\)-parity conservation, the superpotential reads

\[ w = h^U QU^c H_2 + h^D QD^c H_1 + h^E LE^c H_1 + \mu H_1 H_2. \tag{2} \]

The first three addends in (2) originate an acceptable set of Yukawa couplings, with automatic flavour conservation in tree-level neutral currents. The last one, containing the mass parameter \(\mu\), corresponds to a globally supersymmetric Higgs mass term, to be discussed in more detail later. After inclusion of general soft supersymmetry-breaking terms, which parametrize our ignorance about the mechanism for supersymmetry breaking in the underlying fundamental theory, the tree-level potential of the MSSM reads

\[ V_0 = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 H_2 + h.c.) + \frac{g^2}{8} \left( H_2^\dagger \bar{\sigma} H_2 + H_1^\dagger \bar{\sigma} H_1 \right)^2 + \frac{g'^2}{8} \left( |H_2|^2 - |H_1|^2 \right)^2, \tag{3} \]

where \(m_1^2, m_2^2, m_3^2\) are essentially arbitrary mass parameters, \(g\) and \(g'\) are the \(SU(2)_L\) and \(U(1)_Y\) coupling constants, respectively, and \(\bar{\sigma}\) are the Pauli matrices. A crucial difference between the Higgs potentials of the SM and of the MSSM is evident from (3): in the SM the quartic scalar coupling is determined by an arbitrary parameter \(\lambda\), proportional to the SM Higgs mass \(m_\phi\). In (3), despite the presence of two Higgs doublets, the quartic scalar couplings do not contain any arbitrary parameter, but are determined by the \(SU(2)_L \times U(1)_Y\) gauge couplings: supersymmetry strongly constrains the form of the scalar potential.

To discuss the mass spectrum, it is not restrictive to consider the following expansion

\[ H_1 = \left( v_1 + \frac{S_1 + iP_1}{\sqrt{2}} \right) \frac{1}{H_1^-}, \quad H_2 = \left( v_2 + \frac{H_2^+}{\sqrt{2}} \right) \frac{1}{S_2 + iP_2}, \tag{4} \]

\[ ^1\text{For a pedagogical review of the formalism of } N = 1 \text{ global supersymmetry, see e.g. ref. } [2]. \]
and to choose $m_A^2$ real and negative, so that the vacuum expectation values $v_1 \equiv \langle H_1^0 \rangle$ and $v_2 \equiv \langle H_2^0 \rangle$ can be taken to be real and positive.

One can then read from the MSSM Lagrangian the different mass terms in the $R$-even sector of the theory. The weak boson masses are given by

$$m_W^2 = \frac{g_2^2}{2} (v_1^2 + v_2^2), \quad m_Z^2 = \frac{g_2^2 + g'^2}{2} (v_1^2 + v_2^2),$$

and the fermion mass matrices by

$$m_U^2 = h_U v_2^2, \quad m_D^2 = h_D v_1, \quad m_E^2 = h_E v_1.$$  \hfill (5)

A physical constraint comes from the fact that the combination $(v_1^2 + v_2^2)$, which determines the $W$ and $Z$ boson masses, must reproduce their measured values. Once this constraint is imposed, at the classical level the MSSM Higgs sector contains only two independent parameters: they can be conveniently taken to be $\tan \beta \equiv v_2/v_1$ and one combination of the mass parameters (of the three original ones, two are removed by the minimization conditions).

The spin-0 boson mass matrices factorize into $2 \times 2$ blocks, corresponding to the charged, neutral CP-odd and neutral CP-even sectors. One can easily identify the Goldstone bosons $G^+ = -\cos \beta (H_1^-)^* + \sin \beta H_2^+$, $G^- = (G^+)^*$ and $G^0 = -\cos \beta P_1 + \sin \beta P_2$. The orthogonal combinations $H^+ = \sin \beta (H_1^-)^* + \cos \beta H_2^+$, $H^- = (H^+)^*$ and $A = \sin \beta P_1 + \cos \beta P_2$ correspond to physical particles, with masses

$$m_A^2 = -m_3^2 (\tan \beta + \cot \beta),$$

and

$$m_{H^\pm}^2 = m_W^2 + m_A^2.$$  \hfill (7)

A convenient choice, which will be adopted here, is to take as independent parameters $m_A$ and $\tan \beta$. The mass matrix of the neutral CP-even sector then reads

$$\left(\mathcal{M}_R^0\right)^2 = \begin{bmatrix} \cot \beta & -1 \\ -1 & \tan \beta \end{bmatrix} \frac{m_Z^2}{2} + \begin{bmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{bmatrix} \frac{m_A^2}{2} \sin 2\beta.$$  \hfill (9)

Defining the mass eigenstates as

$$h = -\sin \alpha S_1 + \cos \alpha S_2, \quad H = \cos \alpha S_1 + \sin \alpha S_2,$$

one obtains

$$m_{H,h}^2 = \frac{1}{2} \left[ m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta} \right],$$

and also celebrated inequalities such as $m_W, m_A < m_{H^\pm}$, $m_h < m_Z |\cos 2\beta| < m_Z < m_H$, $m_h < m_A < m_H$. Similarly, one can easily compute all the Higgs boson couplings by observing that the mixing angle $\alpha$, required to diagonalize the mass matrix (3), is given by

$$\cos 2\alpha = -\cos 2\beta \frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}, \quad -\frac{\pi}{2} < \alpha \leq 0.$$  \hfill (12)
Table 1: Correction factors for the couplings of the MSSM neutral Higgs bosons to fermion and vector boson pairs.

|        | $d\bar{d}, s\bar{s}, b\bar{b}$ | $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$ | $u\bar{u}, c\bar{c}, t\bar{t}$ | $W^+W^-, ZZ$ |
|--------|--------------------------------|---------------------------------|--------------------------------|--------------|
| $h$    | $-\sin \alpha/\cos \beta$    | $\cos \alpha/\sin \beta$     | $\sin (\beta - \alpha)$     |              |
| $H$    | $\cos \alpha/\cos \beta$    | $\sin \alpha/\sin \beta$     | $\cos (\beta - \alpha)$     |              |
| $A$    | $-i\gamma_5 \tan \beta$     | $-i\gamma_5 \cot \beta$      | $0$                           |              |

For example, the couplings of the three neutral Higgs bosons to vector boson and fermion pairs are easily obtained from the SM Higgs couplings: it is sufficient to multiply the latter by the $\alpha$- and $\beta$-dependent factors summarized in Table 1. The remaining tree-level Higgs boson couplings in the MSSM can be easily computed and are summarized, for example, in ref. [3].

An important consequence of the tree-level structure of the Higgs potential is the existence of at least one neutral CP-even Higgs boson with mass smaller than $m_Z$ ($h$) or very close to it ($H$), and significantly coupled with the $Z$ boson. In the past, this raised the hope that the crucial experiment on the MSSM Higgs sector could be entirely performed at LEP II (with sufficient centre-of-mass energy, luminosity and $b$-tagging efficiency), and took some interest away from higher energy colliders. However, it was recently pointed out [4] that the Higgs-boson masses are subject to large, finite radiative corrections, dominated by loops involving the top quark and its supersymmetric partners. We shall summarize the present theoretical status of these corrections in section 2, and their implications for future colliders in section 3 (the phenomenology of SUSY Higgs bosons at LEP I and LEP II is discussed in other talks at this Workshop [5,6]). We would now like to conclude this section with a few comments on non-minimal supersymmetric Higgs sectors, which might be useful to understand the relevance of the minimal case.

### 1.2 Non-minimal models

As in non-supersymmetric model building, also in the presence of low-energy supersymmetry one can abandon the economy principle, adding further chiral superfields to the MSSM Higgs sector. Some of these additions can create severe phenomenological problems, which can be solved only at the price of rather artificial constructions. Extra doublets, for example, are potentially dangerous sources of tree-level flavour-changing neutral currents and
charge-breaking minima. Extra Higgses in higher-dimensional representations of $SU(2)_L$ are difficult to reconcile with the measured value of $m_W/m_Z$; moreover, they do not seem to appear in the massless spectrum of realistic four-dimensional string constructions. A less dangerous option is to introduce one extra singlet (or more) under $SU(2)_L \times U(1)_Y$, as in the first models by Fayet [1]. A mildly attractive possibility along these lines is the introduction of just one extra singlet superfield, $N$, with a purely cubic superpotential [7]

$$\mu H_1 H_2 \rightarrow \lambda H_1 H_2 N + k N^3,$$  \hspace{1cm} (13)  

and a corresponding modification in the associated soft SUSY-breaking part of the scalar potential

$$m_3^2 (H_1 H_2 + \text{h.c.}) \rightarrow \left( \lambda A \lambda H_1 H_2 N + k A k N^3 + \text{h.c.} \right).$$ \hspace{1cm} (14)  

From the point of view of the low-energy effective theory, this ‘minimal-non-minimal’ model contains two more parameters and two more neutral states than the MSSM. The role of the mixing masses $\mu$ and $m_3^2$, necessary to obtain an acceptable breaking of the electroweak gauge symmetry, is effectively played by the quantities $\lambda x$ and $\lambda A \lambda x$, where $x \equiv \langle N \rangle$. The MSSM is recovered by taking the limit $x \rightarrow \infty$, while keeping $\lambda x$ and $k x$ fixed, in which case the two additional neutral spin-0 states become superheavy and decouple from the low-energy theory. In general, however, this model has a much more complicated phenomenology [8] than the MSSM. Before using it as an alternative paradigm for low-energy supersymmetry, it might be useful to carefully analyse its motivations.

One of the original motivations for the minimal-non-minimal model is the so-called $\mu$-problem [9] of the MSSM. From the point of view of the low-energy effective theory (the MSSM), the superpotential mass parameter $\mu$ is not related to the scale of supersymmetry breaking. On the other hand, $\mu = m_3 = 0$ would give an unacceptable axion, and in any case $\mu = 0$ is excluded by the present LEP data [10]. In the absence of a theoretical explanation for its existence, a supersymmetric mass term at the electroweak scale is clearly unsatisfactory. A second, more recent motivation is the observation that supergravity models describing the low-energy limit of four-dimensional string constructions have only cubic (or higher-dimensional) couplings among the light fields with non-trivial gauge quantum numbers. Two problems then have to be solved in the fundamental theory giving the MSSM in the low-energy limit: 1) Why is $\mu = 0$ (instead of $\mu = M$, where $M$ is some superheavy scale) in the limit of unbroken supersymmetry? 2) How can one generate a non-vanishing $\mu$, of the order of the electroweak scale, after the breaking of local supersymmetry?

As for the first problem, which is obviously present also in non-minimal models, one could simply argue that, if there is no supersymmetric mass term of this kind to begin with, non-renormalization theorems protect it from large radiative corrections. More ambitious suggestions to explain this fact resort to the so-called ‘missing-partner’ mechanism [11], or to the idea of seeing the Higgs doublets as pseudo-Goldstone bosons, associated with the quotient of some large global symmetry group over the grand-unification group [12]. As for the second problem, there are possible solutions that do without additional Higgs singlets at low energy. For example, one can start from a supergravity theory with a purely cubic superpotential in the observable sector and, if the geometrical structure of the theory is appropriate, generate [13] a non-vanishing $\mu$-term, proportional to the gravitino
mass $m_{3/2}$, in the low-energy effective theory with softly broken global supersymmetry, obtained by taking the flat limit $M_P \to \infty$. Alternatively, one can think of possible non-renormalizable superpotential terms of the form $\nu \phi^n H_1 H_2 / M^{n-1}$, where $M$ is some very large scale and the symbol $\phi$ stands for singlet fields getting a vacuum expectation value $\langle \phi \rangle \sim \tilde{M} < M$: a globally supersymmetric mass $\mu \sim \tilde{M}^n / M^{n-1}$ is then generated in the low-energy theory $\nu$.

From the previous considerations, it should be apparent that the solution of the $\mu$-problem does not necessarily imply an extension of the MSSM Higgs sector at the level of the low-energy effective theory. Moreover, the minimal-non-minimal model has also some potential drawbacks. First of all, models with singlets coupled with the superheavy sector of the theory might develop dangerous instabilities along the singlet direction $\nu$. Furthermore, in the minimal-non-minimal model it must be true that $k \neq 0$ to avoid a global axionic symmetry, spontaneously broken by the expectation values of the Higgs fields. On the other hand, $k \neq 0$ seems difficult to obtain in the low-energy limit of four-dimensional string models: singlets under the standard model gauge group are in general charged under some gauge group broken at high energy, and this is sufficient to forbid a purely cubic coupling in the superpotential.

In conclusion, non-minimal models are certainly not excluded by the present theoretical and experimental knowledge, but for the moment they do not appear to have stronger motivations than the MSSM.

2 Radiative corrections to SUSY Higgs masses and couplings

Radiative corrections to the parameters of the Higgs boson sector in the MSSM have recently received much attention. After the discovery $\nu$ that top and stop loops can cause large corrections to the masses of the neutral CP-even Higgs bosons, radiative corrections to Higgs boson masses and couplings have been computed by a variety of methods: the renormalization group approach $\nu$, the effective potential approach $\nu$, and the diagrammatic approach $\nu$.

The renormalization-group approach assumes that there are two (or more) widely separated mass scales, for example

$$M_{SUSY} \sim m_{t_1} \sim m_{t_2} \sim \ldots \sim m_H \sim m_{H^\pm} \sim m_A \gg m_Z \sim m_h \sim m_t,$$  \hspace{1cm} (15)

and considers the effective theory for the degrees of freedom lighter than $M_{SUSY}$. It then solves (non-supersymmetric) renormalization group equations to obtain running parameters down to the scale $Q = m_Z$, imposing the tree-level relations of the MSSM as boundary conditions at the scale $Q = M_{SUSY}$. This approach has the advantage of resumming the leading corrections, proportional to $\log(M_{SUSY} / m_Z)$, so that even the case of $M_{SUSY}$ orders of magnitude larger than $m_Z$ can be dealt with in perturbation theory. On the other hand, if supersymmetry is to solve the naturalness problem of the Standard Model, one expects the various mass parameters of the MSSM to be scattered around the electroweak scale, $G_F^{-1/2} \simeq 250$ GeV, so that assumption $\nu$ breaks down.
The effective-potential approach consists in identifying the Higgs boson masses and self-couplings with the corresponding derivatives of the one-loop effective potential, evaluated at the minimum. By definition, this approach evaluates all Higgs self-energies and vertices at vanishing external momentum. In the case of radiative corrections to Higgs boson masses, this was shown to be a rather accurate approximation\(^2\). Other possible drawbacks of the effective potential approach are the gauge- and scale-dependence of the associated quantities. These are not serious problems in the computation of the mass corrections: the dominant ones come from quark and squark loops, which introduce no spurious dependences on the gauge parameter into the results; also, wave-function renormalization effects, responsible for the scale dependence, are generally small with respect to the overall mass corrections.

The diagrammatic approach consists in performing the complete one-loop renormalization programme, specifying unambiguously the input parameters and the relations between renormalized parameters and physical quantities. This approach gives the most precise computational tool in the case of supersymmetric particle masses spread around the electroweak scale, and results that are formally gauge- and scale-independent. Since corrections can be numerically large, however, one has to pay attention and conveniently improve the naive one-loop calculations when necessary.

To simplify the discussion, in the following we shall take a universal soft supersymmetry-breaking squark mass, \(m_{sq}\), and assume negligible mixing in the stop mass matrix, \(A_t = \mu = 0\). More complete formulae for arbitrary values of the parameters are available, but the qualitative features corresponding to the above choices are representative of a very large region of parameter space. In the case under consideration, and working at leading order in the top-quark Yukawa coupling, the neutral CP-even mass matrix is modified at one loop as follows

\[
M^2_R = \left( M^0_R \right)^2 + \Delta M^2_R ,
\]

where

\[
\left( \Delta M^2_R \right)_{11,12,21} = 0 , \quad \left( \Delta M^2_R \right)_{22} = \frac{3}{8\pi^2} \frac{g^2 m_t^4}{m_W^2 \sin^2 \beta} \log \left( 1 + \frac{m_{sq}^2}{m_t^2} \right) .
\]

It is then a simple exercise to derive the one-loop-corrected eigenvalues \(m_h\) and \(m_H\), as well as the mixing angle \(\alpha\) associated with the one-loop-corrected mass matrix \(M^2_R\). The most striking fact in eq. (17) is that the correction \(\left( \Delta M^2_R \right)_{22}\) is proportional to \(\left( m_t^4 / m_W^2 \right)\) for fixed \((m_{sq} / m_t)\). This implies that, for \(m_t\) in the presently allowed range, the tree-level predictions for \(m_h\) and \(m_H\) can be badly violated, as for the related inequalities. The other free parameter in eq. (17) is \(m_{sq}\), but the dependence on it is much milder.

The above formulae have been generalized to arbitrary values of the parameters in the stop mass matrix, and the effects of other virtual particles in the loops have been included. Renormalization-group methods have been used to resum the large logarithms that arise when the typical scale of supersymmetric particle masses, \(M_{\text{SUSY}}\), is much larger than \(m_Z\). Two-loop corrections have been computed in the leading logarithmic approximation.

\(^2\) Actually, when the external momentum (i.e. the Higgs mass) approaches or exceeds the threshold of the internal particles, the full correction can be rather different from the zero-momentum one. However, in that case corrections themselves are small, either in the absolute sense or relatively to the (increased) tree-level mass.
and found to be small. After all these refinements, eq. (17) still gives the most important mass correction in the most plausible region of parameter space.

To illustrate the impact of eq. (17), we display in fig. 1 contours of $m_h^{\text{max}}$ (the maximum value of $m_h$, reached for $m_A \gg m_Z$ and $\tan \beta \gg 1$), in the $(m_t, m_{sq})$ plane. The calculation has been performed in the effective potential approach, including top, bottom, stop, sbottom loops, and neglecting mixing in the stop and sbottom mass matrices. For very large values of $m_t$ and $m_{sq}$, the renormalization-group improvement can lower the actual value of $m_h^{\text{max}}$ by a non-negligible amount. In the following, when making numerical examples we shall choose the numerical values $m_t = 140$ GeV, $m_{sq} = 1$ TeV: for this parameter choice, the effect of the renormalization-group improvement is of the order of a few GeV, comparable with other residual theoretical uncertainties.

The computation of radiative corrections can be extended to the other parameters of the MSSM Higgs sector. For example, one-loop corrections to the charged Higgs mass have been computed and found to be small, at most a few GeV, for generic values of the parameters.

Whilst radiative corrections to Higgs boson masses are by now well under control, the study of radiative corrections to Higgs boson couplings is still at a less refined stage. In the case of Higgs boson self-couplings, which control decays such as $H \rightarrow hh$, $H \rightarrow AA$ and $h \rightarrow AA$ when they are kinematically allowed, radiative corrections can be numerically large. Detailed computations of these corrections have been performed by a variety of methods. For example, the leading radiative correction to the cubic $Hhh$ coupling can be written as $\lambda_{Hhh} = \lambda^0_{Hhh} + \Delta \lambda_{Hhh}$, where

$$
\lambda^0_{Hhh} = -\frac{igt\bar{Z}}{2 \cos \theta_W} [2 \sin(\beta + \alpha) \sin 2\alpha - \cos(\beta + \alpha) \cos 2\alpha] \quad (2.9)
$$

is the tree-level coupling, and

$$
\Delta \lambda_{Hhh} = -\frac{igt\bar{Z}}{2 \cos \theta_W} \frac{3g^2 \cos^2 \theta_W}{8\pi^2} \frac{\cos^2 \alpha \sin \alpha}{m^4_W} \left(3 \log \frac{m^2_{sq} + m^2_{t}}{m^2_{t}} - 2 \frac{m^2_{sq}}{m^2_{sq} + m^2_{t}} \right). \quad (2.10)
$$

Notice the explicit dependence on the ratio $(m_t/m_W)^4$. Given the fact that, in addition to the masses of the virtual particles in the one-loop diagrams, two different mass scales are involved in the decays $H \rightarrow hh$, $H \rightarrow AA$ and $h \rightarrow AA$, the mass of the decaying particle and the mass of the decay products, one might suspect that momentum-dependent effects, which are neglected in the renormalization-group and in the effective-potential approaches, could play a role. This problem was recently studied in ref. [19], which performed a full diagrammatic computation of the decay rate $\Gamma(H \rightarrow hh)$, including top, bottom, stop and sbottom loops. A typical result is shown in fig. 2, which gives $\Gamma(H \rightarrow hh)$ as a function of $m_H$ for a representative parameter choice. One finds that, for $\tan \beta$ close to 1, there can be very large corrections to the ‘improved tree-level’ approximation, which amounts to plugging the one-loop corrected value of the mixing angle $\alpha$ into the tree-level formulae. Also, for $\tan \beta$ close to 1 and $m_H \gtrsim 2m_t$ the full diagrammatic result can significantly differ from the one obtained in the effective potential approach. The latter method, however, remains a good approximation in the region of parameter space which is most relevant for $H$ searches at future colliders.
One should also consider radiative corrections to Higgs couplings to vector boson and fermions. In most phenomenological studies, they have been taken into account only approximately, by improving the tree-level formulae with one-loop corrected values of the $H-h$ mixing angle, $\alpha$, and with running fermion masses, evaluated at the typical scale $Q$ of the process under consideration. Residual corrections are expected to be numerically small in the experimentally interesting regions of parameter space, as recently verified on a number of explicit examples.

Finally, it is interesting to ask what happens to the upper bound on the lightest Higgs mass if one goes from the MSSM to non-minimal models. From the point of view of the low-energy effective theory, the bound is no longer valid as long as supersymmetry and the particle content allow for some arbitrary quartic coupling in the tree-level potential. In the minimal-non-minimal model, for example, the tree-level bound is modified into

$$m_h^2 \leq m_Z^2 \left( \cos^2 2\beta + \frac{2\lambda^2 \cos^2 \theta_W}{g^2} \sin^2 2\beta \right).$$

(18)

Large values of the arbitrary coupling $\lambda$ can give large violations of the MSSM bound already at the tree level. On the other hand, a bound on the value of $\lambda$ at the electroweak scale, and thus on $m_h$, can be obtained by requiring that the running coupling constants of the model, including the Higgs self-coupling and the Yukawa couplings, remain perturbative up to the grand-unification scale \cite{21}. This argument can be put forward also in non-supersymmetric models, including the SM \cite{22}; but, in supersymmetric models it is particularly motivated by the success of supersymmetric grand unification. The existence of effective infrared fixed points for the top Yukawa coupling $h_t$ and the Higgs self-coupling $\lambda$ make this bound particularly stringent. Of course, one has to add the finite radiative corrections due to SUSY-breaking effects, as in the MSSM. In the case of the minimal-non-minimal model, the result is displayed in fig. 3, assuming a universal soft SUSY-breaking mass $M_{\text{SUSY}} = 1$ TeV. One can see that the absolute upper bound on the lightest neutral Higgs boson is of the order of 140 GeV, only slightly higher than the corresponding bound in the MSSM.

3 SUSY-Higgs searches at future colliders

The relevant processes for MSSM Higgs boson searches at LEP are $e^+e^- \to Z^* \to hZ^*$ and $e^+e^- \to Z^* \to hA$, which play a complementary role, since their rates are proportional to $\sin^2(\beta - \alpha)$ and $\cos^2(\beta - \alpha)$, respectively. Updated experimental limits that take radiative corrections into account have been presented at this Workshop by Grivaz \cite{5}. Neglecting mixing effects in the stop sector, and varying $m_t$ and $m_{sq}$ over plausible ranges, one gets $m_h > 43$ GeV, $m_A > 21$ GeV. More stringent limits can be obtained for specific choices of $m_t$ and $m_{sq}$.

The context in which the impact of radiative corrections is most dramatic is the search for MSSM Higgs bosons at LEP II. A detailed evaluation of the LEP II discovery potential can be made only if crucial theoretical parameters, such as the top-quark mass and the various soft supersymmetry-breaking masses, and experimental parameters, such as the centre-of-mass energy, the luminosity, and the $b$-tagging efficiency, are specified.
An updated assessment of the LEP II discovery potential has been presented at this Workshop by Treille [6], and further details can be found in a recent study by Janot [24]. The bottomline is that, with the planned machine parameters, LEP II will not be sensitive to some regions of the parameter space characterizing the SUSY Higgs sector, even in the most restrictive case of the MSSM. Of course, one should keep in mind that there is, at least in principle, the possibility of further extending the maximum LEP energy up to values as high as $\sqrt{s} \simeq 230–240$ GeV, at the price of introducing more (and more performing) superconducting cavities into the LEP tunnel. This could allow a probing of the most plausible region of the parameter space of the MSSM and of its extensions, up to Higgs mass values $m_h \lesssim 130–140$ GeV.

### 3.1 The LHC and the SSC

A natural question to ask is whether, assuming completion of the LEP II project with the foreseen parameters, the LHC and the SSC can explore the full parameter space of the MSSM Higgs bosons. A systematic study of this problem, including radiative corrections, has recently been started in refs. [20,25]. The analysis is complicated by the fact that the $R$-odd particles could play a role both in the production (via loop diagrams) and in the decay (via loop diagrams and as final states) of the MSSM Higgs bosons. For simplicity, one can concentrate on the most conservative case, in which all $R$-odd particles are heavy enough not to play any significant role. Still, one needs to perform a separate analysis for each $(m_A, \tan \beta)$ point, to include radiative corrections (depending on additional parameters such as $m_t$ and $m_{sq}$), and to consider Higgs boson decays involving other Higgs bosons. We make here only a few general remarks on the LHC case, for our representative parameter choice, sending the interested reader to refs. [20,25] for a more complete discussion and recent simulation work.

Beginning with the neutral states, when $h$ or $H$ are in the intermediate mass range (80–130 GeV) and have approximately SM couplings, the best prospects for detection are offered, as in the SM, by their $\gamma \gamma$ decay mode. In general, however, $\sigma \cdot BR(h, H \rightarrow \gamma \gamma)$ is smaller than for a SM Higgs boson of the same mass. As a rather optimistic estimate of the possible LHC sensitivity, we display in fig. 4 lines in the $(m_A, \tan \beta)$ plane corresponding to $\sigma \cdot BR(h, H \rightarrow \gamma \gamma) \sim 30$ fb. The contour line for $h$ is shown only for $m_h \gtrsim 80$ GeV. Only for $m_A \gtrsim 200$ GeV, $\tan \beta \gtrsim 3$ (in the case of $h$) and in the shaded area (in the case of $H$), does the $\gamma \gamma$ signal exceed the chosen reference value. Very similar considerations can be made for the production of $h$ or $H$, decaying into $\gamma \gamma$, in association with a $W$ boson or with a $t\bar{t}$ pair, giving an extra isolated lepton in the final state. When $H$ and $A$ are heavy, in general one cannot rely on the $ZZ \rightarrow 4l^\pm$ ($l=e, \mu$) decay mode, which gives the ‘gold-plated’ Higgs signature in the SM case, since $H$ and $A$ couplings to vector-boson pairs are strongly suppressed: only for small $\tan \beta$ and $150$ GeV $\lesssim m_H \lesssim 2m_t$ might the decay mode $H \rightarrow ZZ \rightarrow 4l^\pm$ still be viable despite the suppressed branching ratio. Taking into account that with sufficient experimental resolution one could exploit the small $H$ width, as an estimate of the possible LHC sensitivity we take $\sigma \cdot BR(H \rightarrow 4l^\pm) \sim 1$ fb ($l=e, \mu$). This contour defines the area in fig. 4 indicated by the label $H \rightarrow 4l$. For very large values of $\tan \beta$, and moderately large $m_A$, one could take advantage of the enhanced production cross-sections and of the unsuppressed decays into $\tau^+\tau^-$ to obtain a visible
signal for one or more of the MSSM neutral Higgs bosons, and in particular for $H$ and $A$, whose masses can be significantly larger than 100 GeV. The simulation work for this process is still at a rather early stage, so that no definite conclusion can be drawn yet. For reference, the dotted line in fig. 4 corresponds to a (somewhat arbitrary) interpolation of $\sigma \cdot BR(A, H \rightarrow \tau^+ \tau^-) \sim 10$ pb at $m_{H,A} \sim 100$ GeV and $\sigma \cdot BR(A, H \rightarrow \tau^+ \tau^-) \sim 1$ pb at $m_{H,A} \sim 400$ GeV.

Finally, in the region of parameter space corresponding to $m_A \lesssim m_Z$, the charged Higgs could be discovered via the decay chain $t \rightarrow bH^+ \rightarrow b\tau^+\nu$, which competes with the standard channel $t \rightarrow bW^+ \rightarrow bl^+\nu_l$ ($l = e, \mu, \tau$). A convenient parameter is the ratio $R \equiv BR(t \rightarrow \tau^+\nu,b)/BR(t \rightarrow \mu^+\nu,b)$, which measures the violation of lepton universality in top decays. As an estimate of the LHC sensitivity, we take $R > 1.15$. The corresponding region of the $(m_A, \tan \beta)$ plane is indicated by the label $H^+ \rightarrow \tau\nu$ in fig. 4.

For all processes considered above, similar remarks apply also to the SSC. For reference we also show, as dashed lines in fig. 4, contours associated with two benchmark values of the total cross-section $\sigma(e^+e^- \rightarrow hZ, HZ, hA, HA)$, which should give a rough measure of the LEP II sensitivity. The lower line corresponds to $\sigma = 0.2$ pb at $\sqrt{s} = 175$ GeV, which could be seen as a rather conservative estimate of the LEP II sensitivity. The upper line corresponds to $\sigma = 0.05$ pb at $\sqrt{s} = 190$ GeV, which could be seen as a rather optimistic estimate of the LEP II sensitivity.

In summary, a global look at fig. 4 shows that there is a high degree of complementarity between the regions of parameter space accessible to LEP II and to the LHC/SSC. However, for our representative choice of parameters, there is a non-negligible region of the $(m_A, \tan \beta)$ plane that is presumably beyond the reach of LEP II and of the LHC/SSC. This potential problem could be solved, as we said before, by a further increase of the LEP II energy beyond the reference value of $\sqrt{s} \lesssim 190$ GeV. Otherwise, one might need a higher-energy $e^+e^-$ collider, for a full exploration of the parameter space describing the MSSM Higgs sector. On the other hand, one should not forget that another important test of the MSSM will be provided by squark and gluino searches at the LHC/SSC, which should be sensitive to most of the theoretically motivated parameter space. One should also keep in mind that indirect information on the particle spectrum of the MSSM, including its extended Higgs sector, could come from lower-energy precision data. The possible effects of virtual supersymmetric particles on LEP observables have already been mentioned at this Workshop. Another interesting effect, recently re-emphasized in refs. [27], is the contribution of the charged-Higgs loop to the rare decay $b \rightarrow s\gamma$, which in the SM proceeds via a $W$-boson loop. The theoretical and experimental errors on the inclusive radiative $B$-decay could already be small enough to put non-trivial constraints on the particle spectrum of the MSSM. In particular, in the limit of very heavy $R$-odd particles one could identify an excluded region in the $(m_A, \tan \beta)$ plane, corresponding to low values of $m_{H^\pm}$: the precise form of such a region strongly depends on the assumed theoretical uncertainties. Furthermore, loops of (relatively light) virtual supersymmetric particles can give rise to partial cancellations with the $W$ and Higgs loops, thus allowing for values of the decay rate on both sides of the SM prediction [28].
3.2 High-energy linear $e^+e^-$ colliders

We now review, following ref. [29], the main production mechanisms of neutral SUSY Higgses in $e^+e^-$ collisions at very high energy, say $\sqrt{s} = 500$ GeV, namely:

\[
e^+e^- \rightarrow hZ, HA, h\nu\bar{\nu}, he^+e^- \quad [\sigma \propto \sin^2(\beta - \alpha)],
\]

\[
e^+e^- \rightarrow HZ, hA, H\nu\bar{\nu}, He^+e^- \quad [\sigma \propto \cos^2(\beta - \alpha)].
\]  

(19)

Other production mechanisms of interest are discussed in refs. [29,30], and details about experimental searches can be found in refs. [24]. It is useful to roughly estimate the cross-section for which we believe that any of the listed processes will be detectable. A cross-section of 0.01 pb will lead to 25 events for an integrated luminosity of 10 fb$^{-1}$ after multiplying by an efficiency of 25%: the latter is a crude estimate of the impact of detector efficiencies, cuts, and branching ratios to usable decay channels. One keep this benchmark cross-section value in mind as a rough criterion for where in parameter space a particular reaction can be useful.

Figure 5 shows contours of $\sigma(e^+e^- \rightarrow hZ)$ and $\sigma(e^+e^- \rightarrow HZ)$ in the $(m_A, \tan \beta)$ plane. Owing to the much higher energy with respect to the standard LEP II values, these two processes now become truly complementary, in the sense that everywhere in the $(m_A, \tan \beta)$ plane there is a substantial cross-section for at least one of them ($\sigma > 0.01$ pb). This should be an excellent starting point for experimental searches. Similar considerations hold for $hA$, $HA$ production, whose cross-sections are shown in fig. 6. As long as one of the two channels is kinematically accessible, the inclusive cross-section is large enough to provide a substantial event rate. Even in this case the two processes are complementary, and together should be able to probe the region of parameter space corresponding to $m_A < 200$ GeV. At a high-energy linear $e^+e^-$ collider one can also consider single Higgs production via vector-boson fusion. The cross-sections for $h$, $H$ production via $WW$ fusion can exceed 0.01 pb in large, complementary regions of the $(m_A, \tan \beta)$ plane. The $ZZ$ fusion processes are suppressed by an order of magnitude with respect to the $WW$ fusion ones, but could still be useful for experimental searches. Obviously, since the $AWW$ and $AZZ$ vertices are absent at tree level, one cannot have a substantial $A$ production with this mechanism for sensible values of the parameters.

The global picture that emerges from these results is the following. If no neutral Higgs boson is discovered until then, one must find, at a linear $e^+e^-$ collider with $\sqrt{s} = 500$ GeV (EE500) at least one neutral SUSY Higgs, otherwise the MSSM is ruled out (the same applies to its most plausible non-minimal extensions). If $m_A$ is not too large, at EE500 there is the possibility of discovering all of the Higgs states of the MSSM via a variety of processes, including charged-Higgs-boson production, which has not been discussed here. In the event that a neutral Higgs boson is already discovered at LEP or the LHC/SSC, with properties compatible with one of the MSSM Higgs states, EE500 would still be a very useful instrument to investigate in detail the spectroscopy of the Higgs sector, for example to distinguish between the SM, the MSSM and possibly other non-minimal supersymmetric extensions.
4 Conclusions

In conclusion, the search for Higgs bosons in the low and intermediate mass range is a crucial test of the MSSM, and more generally of the whole idea of low-energy supersymmetry. Compared with direct searches for $R$-odd supersymmetric particles, this test has a smaller dependence on subjective naturalness bounds on the amount of SUSY breaking. Under the generic assumption of $R$-odd particles not much above the TeV scale, the MSSM would be ruled out by the experimental exclusion of a Higgs boson below 130 GeV or so. Non-minimal models with extra gauge singlets could only survive up to Higgs masses of 140 GeV or so, under the only extra assumption that dimensionless couplings do not blow up below the grand-unification scale. More complicated non-minimal extensions could in principle escape the latter bound, but in that case one would need very artificial constructions to avoid the constraints coming from precision electroweak data and to preserve the successful predictions of grand unification. One could then say that ruling out the existence of a Higgs boson in the low or intermediate mass range would effectively rule out the idea of low-energy SUSY. A more optimistic scenario is the one in which a SM-like Higgs boson is indeed found in the low or intermediate mass range. This could not be taken as evidence for supersymmetry, but it would certainly give additional motivations to expect the existence of other Higgs states and of the $R$-odd SUSY particles at accessible mass scales. Finally, the gold-plated scenario is the one in which one finds from the beginning either a Higgs boson with non-standard properties or some $R$-odd supersymmetric particle: it is easy to imagine the theoretical and experimental excitement that such an event would generate.
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Figure 1: Contour curves of $m_h^{\text{max}}$ in the $(m_t, m_{sq})$ plane (from ref. [20]).

Figure 2: $\Gamma(H \to hh)$ as a function of $m_H$, for the indicated parameter choice. The solid line corresponds to the full diagrammatic calculation, the dashed line to the effective potential approach, the dash-dotted line to the ‘improved tree-level’ result (from ref. [19]).
Figure 3: Upper bound on the mass of lightest neutral Higgs boson, as a function of $m_t$, in the non-minimal supersymmetric extension of the SM with an extra singlet. The different solid lines correspond to the indicated values of $\tan \beta$, and all soft SUSY-breaking masses have been chosen equal to the common value $M_{SUSY} = 1$ TeV (from ref. [23]).
Figure 4: Pictorial summary of the discovery potential of the LHC in the $(m_A, \tan \beta)$ plane (from ref. [20]).
Figure 5: Contours of a) $\sigma(e^+e^- \rightarrow hZ)$ and b) $\sigma(e^+e^- \rightarrow HZ)$, in the $(m_Z, \tan \beta)$ plane (from ref. [29]).
Figure 6: Contours of a) $\sigma(e^+e^- \rightarrow hA)$ and b) $\sigma(e^+e^- \rightarrow HA)$, in the $(m_A, \tan \beta)$ plane (from ref. [29]).