GUT Scale Inflation, Non-Thermal Leptogenesis, and Atmospheric Neutrino Oscillations

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Abstract

Leptogenesis scenarios in supersymmetric hybrid inflation models are considered. Sufficient lepton asymmetry leading to successful baryogenesis can be obtained if the reheat temperature $T_r \gtrsim 10^6$ GeV and the superpotential coupling parameter $\kappa$ is in the range $10^{-6} \lesssim \kappa \lesssim 10^{-2}$. For this range of $\kappa$ the scalar spectral index $n_s \simeq 0.99 \pm 0.01$. Constraints from neutrino mixing further restrict the range of $\kappa$ that is allowed. We analyze in detail the case where the inflaton predominantly decays into the next-to-lightest right handed Majorana neutrino taking into account especially the constraints from atmospheric neutrino oscillations.

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1 Introduction

Supersymmetric hybrid inflation models [1, 2] provide a compelling framework for the understanding of the early universe. They account for the primordial density perturbations with a GUT scale symmetry breaking yet without any dimensionless parameters that are very small. As in any complete inflationary scenario, inflation in these models should be followed by a successful reheating accounting for the observed baryon asymmetry of the universe.

In SUSY hybrid inflation it is generally preferable (and in many models necessary) to generate the baryon asymmetry via leptogenesis, which is then partially converted into baryon asymmetry by sphaleron effects [3]. If the gauge symmetry $G = SO(10)$ or one of its subgroups (where inflation is associated with the breaking of a gauge symmetry $G \rightarrow H$), the inflaton decays into the right handed neutrinos, whose subsequent out of equilibrium decay leads to the lepton asymmetry [4]. The right handed neutrinos could also be produced thermally, although it is difficult to reconcile the high reheat temperature required by thermal leptogenesis with the gravitino constraint [5].

In thermal leptogenesis [6] the lightest right handed Majorana neutrino $N_1$ washes away the previous asymmetry created by the heavier neutrinos. If, on the other hand, $N_1$ as well as the heavier neutrinos are out of equilibrium ($T_r < M_1$), the lepton asymmetry could predominantly result from the inflaton $\chi$ decaying into the next-to-lightest neutrino $N_2$. ($\chi \rightarrow N_3 N_3$ is ruled out by the gravitino constraint.)

In this letter we focus on the latter scenario. It is easier to account for the observed baryon asymmetry in this case since the asymmetry per right handed neutrino decay is in general greater than the case where the inflaton decays into the lightest neutrino, and unlike thermal leptogenesis there is no washout factor.

The plan of the paper is as follows: In Section 2 we briefly review a class of supersymmetric hybrid inflation models. In Section 3 we qualitatively discuss leptogenesis scenarios for these models. In Section 4 we perform an analysis of the ‘next-to-lightest’ scenario, showing numerically that sufficient lepton asymmetry can be generated while satisfying, in particular, the constraints from atmospheric neutrino mixing.
2 Supersymmetric Hybrid Inflation

In a class of realistic supersymmetric models, inflation is associated with the breaking of either a grand unified symmetry or one of its subgroups. Here we will limit ourselves to supersymmetric hybrid inflation models \[2\]. The simplest such model \[1\] is realized by the renormalizable potential (consistent with a $U(1)$ $R$-symmetry) \[7\]

$$W_1 = \kappa S(\phi \bar{\phi} - M^2) \quad (1)$$

where $\phi(\bar{\phi})$ denote a conjugate pair of superfields transforming as nontrivial representations of some gauge group $G$, $S$ is a gauge singlet superfield, and $\kappa (> 0)$ is a dimensionless coupling. In the absence of supersymmetry breaking, the potential energy minimum corresponds to non-zero (and equal in magnitude) vevs ($= M$) for the scalar components in $\phi$ and $\bar{\phi}$, while the vev of $S$ is zero. (We use the same notation for superfields and their scalar components.) Thus, $G$ is broken to some subgroup $H$.

In order to realize inflation, the scalar fields $\phi, \bar{\phi}, S$ must be displayed from their present minima. For $|S| > M$, the $\phi, \bar{\phi}$ vevs both vanish so that the gauge symmetry is restored, and the tree level potential energy density $\kappa^2 M^4$ dominates the universe. With supersymmetry thus broken, there are radiative corrections from the $\phi - \bar{\phi}$ supermultiplets that provide logarithmic corrections to the potential which drives inflation.

The temperature fluctuations $\delta T/T$ turn out to be proportional to $(M/M_P)^2$, where $M$ denotes the symmetry breaking scale of $G$, and $M_P = 1.2 \times 10^{19}$ GeV is the Planck mass \[1\] \[2\]. Comparison with the $\delta T/T$ measurements by COBE \[8\] and WMAP \[9\] shows that the gauge symmetry breaking scale $M$ is naturally of order $10^{16}$ GeV.\[1\]

The inflationary scenario based on the superpotential $W_1$ in Eq. (1) has the characteristic feature that the end of inflation essentially coincides with the gauge symmetry breaking. Thus, modifications should be made to $W_1$ if the breaking of $G$ to $H$ leads to the appearance of topological defects such as monopoles, strings or domain walls. For instance, the breaking of $G_{PS} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$ \[10\] to the MSSM by fields belonging to $\phi(4, 1, 2), \bar{\phi}(4, 1, 2)$ produces magnetic monopoles that carry two quanta of Dirac magnetic charge \[11\]. As shown in \[12\], one simple resolution of the

\[1\text{We take } (\delta T/T)_{\text{h}} = 6.3 \times 10^{-6} \text{ where h denotes the horizon scale. This value corresponds to } A \simeq 0.76 \text{ for } n_s = 0.99, \text{ in agreement with } [9].\]
monopole problem is achieved by supplementing $W_1$ with a non-renormalizable term:

$$W_2 = \kappa S(\bar{\phi}\phi - \mu^2) - \beta \frac{S(\bar{\phi}\phi)^2}{M_S^2},$$

(2)

where $\mu$ is comparable to the GUT scale, $M_S \sim 5 \times 10^{17}$ GeV is a superheavy cutoff scale, and the dimensionless coefficient $\beta$ is of order unity. The presence of the non-renormalizable term enables an inflationary trajectory along which the gauge symmetry is broken. Thus, in this ‘shifted’ hybrid inflation model the magnetic monopoles are inflated away.

A variation on these inflationary scenarios is obtained by imposing a $Z_2$ symmetry on the superpotential, so that only even powers of the combination $\phi\bar{\phi}$ are allowed [13]:

$$W_3 = S \left( -\mu^2 + \frac{(\phi\bar{\phi})^2}{M_S^2} \right),$$

(3)

where the dimensionless parameters $\kappa$ and $\beta$ (see Eq. (2)) are absorbed in $\mu$ and $M_S$. The resulting scalar potential possesses two (symmetric) valleys of local minima which are suitable for inflation and along which the GUT symmetry is broken. The inclination of these valleys is already non-zero at the classical level and the end of inflation is smooth, in contrast to inflation based on the superpotential $W_1$ (Eq. (1)). An important consequence is that, as in the case of shifted hybrid inflation, potential problems associated with topological defects are avoided.

In all these models, for the symmetry breaking scale $M \sim 10^{16}$ GeV, one predicts an essentially scale invariant spectrum ($0.98 \lesssim n_s \lesssim 1$ depending on the value of $\kappa$ or of $M$ and $|dn_s/d\ln k| < 10^{-3}$ [14]) which is consistent with a variety of CMB measurements including the recent WMAP results [9, 15].

After the end of inflation, the system falls toward the SUSY vacuum and performs damped oscillations about it. The inflaton, which we collectively denote as $\chi$, consists of the two complex scalar fields $(\delta\bar{\phi} + \delta\phi)/\sqrt{2}$ ($\delta\bar{\phi} = \bar{\phi} - M$, $\delta\phi = \phi - M$) and $S$, with equal mass $m_{\chi}$. In the presence of $N = 1$ supergravity, SUSY breaking is induced by the soft SUSY violating terms in the tree level potential and $S$ acquires a vev comparable to the gravitino mass $m_3/2 \sim \text{TeV})$. This $(\text{mass})^2$ term provides an extra force driving $S$ to the minimum, but its effect is negligible for $\kappa \gtrsim 10^{-6}$.

More often than not, SUGRA corrections tend to derail an otherwise successful inflationary scenario by giving rise to scalar $(\text{mass})^2$ terms of order $H^2$, where $H$ denotes the Hubble constant. Remarkably, it turns out that for a canonical SUGRA
potential (with minimal Kähler potential $|S|^2 + |\phi|^2 + |\bar{\phi}|^2$), the problematic (mass)$^2$ term cancels out for the superpotential $W_1$ in Eq. (1) \[7\]. This property also persists when non-renormalizable terms that are permitted by the $U(1)_R$ symmetry are included in the superpotential.\(^2\)

As noted in \[19\] \[14\], for large values of $\kappa$ the presence of SUGRA corrections due to the minimal Kähler potential can give rise to $n_s$ values that exceed unity by an amount that is not favorably by the data on smaller scales. SUGRA corrections also become important for tiny values of $\kappa$. Nevertheless, they remain ineffective for a wide range of $\kappa$ ($10^{-6} \lesssim \kappa \lesssim 10^{-2}$). As we shall discuss below, leptogenesis consistent with the observed baryon asymmetry generally constrains $\kappa$ to a similar range.

3 Leptogenesis in SUSY Hybrid Inflation Models

The observed baryon asymmetry of the universe can be naturally explained via leptogenesis in SUSY hybrid inflation models. If inflation is associated with the breaking of the gauge symmetry $G = SO(10)$ \[20\] or one of its subgroups such as $G_{PS} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$ \[12\] and $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ \[21\], the inflaton decays into right handed neutrino superfields \[4\]. Their subsequent out of equilibrium decay to lepton and Higgs superfields leads to the observed baryon asymmetry via sphaleron effects \[3\].

Before discussing the constraints on $\kappa$ from leptogenesis, we note that an important constraint that is independent of the details of the seesaw parameters already arises from considering the reheat temperature $T_r$ after inflation, taking into account the gravitino problem which requires that $T_r \lesssim 10^{10}$ GeV \[5\]. We expect the heaviest right handed neutrino to have a mass of around $10^{14}$ GeV, which is in the right ballpark to provide via the seesaw a mass scale of about .05 eV to explain the atmospheric neutrino anomaly through oscillations. Comparing this with \[2\]

$$T_r = \left( \frac{45}{2\pi^2 g^*} \right)^{1/4} (\Gamma_\chi m_p)^{1/2} \lesssim \frac{1}{16} \frac{(m_p m_\chi)^{1/2}}{M} M_i \quad (4)$$

(where $m_p \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and the decay rate of the gravitino $\Gamma_\chi$ includes contributions from all charginos and neutralinos with masses up to $m_\chi$).

\(^2\)In general, $K$ is expanded as $K = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + \alpha |S|^4/M_p^2 + \ldots$, and only the $|S|^4$ term in $K$ generates a mass$^2$ for $S$, which would spoil inflation for $\alpha \sim 1$ \[16\] \[17\]. From the requirement $|S| < M_p$, one obtains an upper bound on $\alpha$ ($\lesssim 10^{-3}$) \[18\]. Since smaller values of $\alpha$ do not affect the dynamics of inflation significantly and other terms in $K$ are suppressed, we take the Kähler potential to be minimal for simplicity.
inflaton $\Gamma_x = (1/8\pi)(M_i^2/M^2)m_x), we see that for $m_x \gtrsim 10^5$ GeV, $M_i$ should not be identified with the heaviest right handed neutrino, otherwise $T_r$ would be too high $[21]$. Here we have assumed that the right handed neutrinos $N_i$ acquire mass from a non-renormalizable coupling $W \supset (1/m_P)\gamma_i\phi\phi N_i N_i$. Thus, we require that

$$\frac{m_x}{2} \leq M_3 \leq \frac{2M^2}{m_P}.$$  

(5)

The gravitino constraint expressed by Eq. (5) requires $\kappa \lesssim 10^{-3}$ independent of the details of seesaw parameters for the SUSY hybrid inflation model $[22, 2, 14]$. However, in shifted and smooth hybrid inflation the Majorana mass of the heaviest right handed neutrino $M_3 \leq 2M^2/M_S$ can remain an order of magnitude greater than the inflaton mass so that this constraint does not restrict $\kappa$ or $M$ (see Figs. 5, 7).

We now consider the case where the inflaton $\chi$ predominantly decays into a right handed neutrino that is heavy compared to the reheat temperature $T_r$. The ratio of the number density of the right-handed (s)neutrino $n_N$ to the entropy density $s$ is given by

$$\frac{n_N}{s} \simeq \frac{3}{2} \frac{T_r}{m_\chi} B_r,$$  

(6)

where $B_r$ denotes the branching ratio into the right handed neutrino channel. The resulting lepton asymmetry is

$$\frac{n_L}{s} = \frac{n_N}{s} \epsilon,$$  

(7)

where $\epsilon$ is the lepton asymmetry produced per right handed neutrino decay.

Note that unlike thermal leptogenesis, there is no washout factor in non-thermal leptogenesis since lepton number violating 2-body scatterings mediated by right hand ed neutrinos are out of equilibrium as long as the lightest right handed neutrino mass $M_1 \gg T_r$ $[23]$. More precisely, the washout factor is proportional to $e^{-z}$ where $z = M_1/T_r$ $[6]$, and can be neglected for $z \gtrsim 10$.

Suppose that the right handed Majorana masses are hierarchical, with $M_1 \ll M_2, M_3$ (but $M_1 > T_r$). If $M_2, M_3$ are heavier than $m_\chi/2$ the inflaton only decays into $2N_1$. With $B_r = 1$, the lepton asymmetry is then

$$\frac{n_L}{s} \simeq \frac{3}{2} \frac{T_r}{m_\chi} \epsilon_1,$$  

(8)

3In this paper we do not consider the possibility of a renormalizable Yukawa coupling $W \supset y_i\phi N_i N_i$. This would require $\phi$ to be a $SU(2)_R$ Higgs triplet (or a 126 of $SO(10)$), and the Yukawa couplings would have to be arranged to yield the intermediate scale Majorana masses consistent with the neutrino oscillation parameters.
with $\epsilon_1$ given by

$$\epsilon_1 = -\frac{1}{8\pi} \frac{1}{(hh^\dagger)_{11}} \sum_{i=2,3} \text{Im} \left[ \left\{ (hh^\dagger)_{i1} \right\}^2 \right] \left[ f^V \left( \frac{M_i^2}{M_i^2} \right) + f^S \left( \frac{M_i^2}{M_i^2} \right) \right],$$  

(9)

where

$$f^V(x) = \sqrt{x} \ln \left( 1 + \frac{1}{x} \right), \quad f^S(x) = \frac{2\sqrt{x}}{x - 1}. \tag{10}$$

Assuming $M_1 \ll M_2, M_3$, Eq. (9) simplifies to

$$\epsilon_1 \simeq -\frac{3}{8\pi} \frac{1}{(hh^\dagger)_{11}} \sum_{i=2,3} \text{Im} \left[ \left\{ (hh^\dagger)_{i1} \right\}^2 \right] \frac{M_1}{M_i}. \tag{11}$$

This formula leads to the upper bound \[24\]

$$\epsilon_1 \lesssim 2 \times 10^{-10} \left( \frac{M_1}{10^6 \text{ GeV}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right). \tag{12}$$

From the observed baryon to photon ratio $\eta \equiv n_B/n_\gamma \simeq 6.1 \times 10^{-10}$ \[9\], the lepton asymmetry is found to be $|n_L/s| \approx 2.4 \times 10^{-10}$, where we have used $n_B/s \simeq \eta/7.04$ \[25\] and $n_L/s = -(79/28)n_B/s$ \[26\]. Using Eqs. (11) \[8\] \[12\], together with the gravitino constraint $T_r \lesssim 10^{10}$ GeV, we find that sufficient lepton asymmetry requires

$$m_3^2 \leq 10^{12} \text{ GeV} \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right)^2 M^2, \tag{13}$$

which yields $m_\chi \lesssim 10^{15}$ GeV for $M \sim M_{\text{GUT}}$. From $M_1 < m_\chi/2$, we also obtain the lower bounds

$$T_r \geq 1.6 \times 10^6 \text{ GeV} \left( \frac{0.05 \text{ eV}}{m_{\nu_3}} \right), \tag{14}$$

$$m_\chi^2 \geq M^2 \times 10^{-3} \text{ GeV} \left( \frac{0.05 \text{ eV}}{m_{\nu_3}} \right), \tag{15}$$

yielding $T_r \gtrsim 10^6$ GeV and $m_\chi \gtrsim 10^{10}$ GeV or $\kappa \gtrsim 10^{-7}$. This remains so even with degenerate neutrinos, since the cosmological bound on the sum of neutrino masses leads to the limit $m_{\nu_i} < 0.23 \text{ eV}$ \[9\].

An alternative scenario \[27\] is the case where $M_1 \ll M_2 \ll M_3$ (but $M_1 > T_r$) and $M_2 < m_\chi/2$. Since the decay width of the inflaton is proportional to $M_i^2$, the branching ratios to $2N_1$ and $2N_2$ are $(M_1/M_2)^2$ and $1 - (M_1/M_2)^2$ respectively. Thus, provided $\epsilon_1 \lesssim \epsilon_2$, the contribution to the lepton asymmetry from $N_1$ is negligible. From Eq. (9) (with permuted indices) and Eq. (10)

$$\epsilon_2 \simeq -\frac{1}{8\pi} \frac{1}{(hh^\dagger)_{22}} \left[ \frac{2M_1}{M_2} \left( \ln \left[ \frac{M_1}{M_2} \right] - 1 \right) \text{Im} \left[ \left\{ (hh^\dagger)^{21} \right\}^2 \right] + 3 \frac{M_2}{M_3} \text{Im} \left[ \left\{ (hh^\dagger)^{23} \right\}^2 \right] \right] \tag{16}$$
or, since the first term is negligible for hierarchical Dirac neutrino masses

\[ \epsilon_2 \approx -\frac{3}{8\pi} \frac{M_2}{M_3} \text{Im} \left[ \left( hh^\dagger \right)_{33} \right] \]

\[ \left( hh^\dagger \right)_{22} \] 

(17)

We can also write this as Eq. (11) with permuted indices and recover Eq. (12) with $M_1$ replaced by $M_2$ (see [24], section 2.1). This indicates that $\epsilon_2$ can easily attain values $\gtrsim \epsilon_1$, so that the dominant contribution to the lepton asymmetry is from $N_2$. Qualitatively, Eq. (12) shows that lepton asymmetry sufficient to meet the observational constraint $|n_L/s| \approx 2.4 \times 10^{-10}$ can be generated with reasonable values for the phases.

We conclude this section by summarizing the various constraints on $\kappa$ and the symmetry breaking scale $M$. As noted in the previous section, for large $\kappa$ (or $M$) the SUGRA contribution gives rise to $n_s$ values that exceed unity by an amount that is not favored by the data on smaller scales ($n_s \leq 1$ at $k = 0.05 \text{ Mpc}^{-1}$ [9]). This provides an upper bound on $\kappa$ and $M$ for the shifted and smooth hybrid inflation models [14]. For SUSY hybrid inflation with the renormalizable potential Eq. (1), the gravitino constraint (Eq. (5)) provides a more stringent upper bound.

For small values of $\kappa$, the SUGRA correction and the soft SUSY breaking (mass)$^2$ term become important. We find by numerical calculation that the primordial density perturbations are too small for $\kappa \lesssim 10^{-6}$ for SUSY hybrid inflation and $\kappa \lesssim 10^{-7}$ for the shifted model. Sufficient leptogenesis requires Eqs. (13, 15), and these are satisfied for the range allowed by the constraints above (except for smooth hybrid inflation, for which Eq. (13) provides a lower bound for $M$). The allowed ranges of $\kappa$ and $M$ are shown in Fig. 1.  

4 Leptogenesis and Atmospheric Neutrino Oscillations

Two-family numerical calculations for SUSY hybrid inflation models discussed here have been carried out previously in refs. [27, 22, 12, 28]. Here we update and extend these calculations using recent measurements of neutrino oscillation parameters.

A comment is in order whether two-family calculations are physically relevant. We consider the case where the $\chi \rightarrow N_2 N_2$ branch is dominant, and Eq. (17) approx-

4To be specific we assumed that the SUSY breaking induces a mass of 1 TeV for $S$, a mass of 10 TeV would increase the relevant lower bounds by a factor of $\approx 1.5$. 

7
imates the lepton asymmetry in terms of two families only. Since the Dirac masses are assumed to be hierarchical, the $\mu\tau$ block is dominant. Furthermore, the gauge symmetries suggest a Dirac mixing matrix close to the CKM matrix $V_{\text{CKM}}$, which is close to the unit matrix especially in the $\mu\tau$ sector. Under these conditions the neutrino mixing matrix $U_{\text{MNS}}$ is approximately obtained by rotating the charged lepton and neutral Dirac sectors only in the $\mu\tau$ sector with respect to the weak basis and diagonalizing the resulting light neutrino mass matrix \cite{27}.

Note that the mixing angle obtained this way can only be identified with the atmospheric neutrino mixing angle if the mixing angles $\theta_{13}$ and $\theta_{12}$ are both small. While the solar mixing angle at weak scale is not small \cite{29}, its RG evolution can lead to a small angle at the reheat temperature \cite{30}. This occurs for a wide range of CP phases for large $\tan \beta$ ($\approx m_t/m_b \sim 50$ for $G \supset SU(4)_c$) and degenerate neutrino masses of $\approx 0.1$ eV \cite{31}. For hierarchical neutrino masses radiative effects on the mixing are in general small \cite{32,33}. The solar mixing in this case could be accounted for by non-diagonal Majorana masses of $\approx 10^{-3}$ eV that can arise from higher dimensional operators \cite{34}.

Thus, we can ignore the first family only if we consider the special case of a small solar mixing angle at large energy scales. For this special case the lepton asymmetry and the atmospheric neutrino mixing angle can be calculated without assuming any particular ansatz for the Dirac and Majorana mass matrices.

The lepton asymmetry in this case is given by \cite{27}

$$ n_L = \frac{9 T_R}{16\pi m_\chi} \frac{M_2}{M_3} \frac{c^2 s^2}{M_3} \sin 2\delta \left( m_D^2 - m_M^2 \right)^2 . \tag{18} $$

Here $\langle H_u \rangle = 174 \sin \beta$ GeV ($\approx 174$ GeV for large $\tan \beta$), where $\beta = \langle H_u \rangle / \langle H_d \rangle$. $m_{D,3}$ are the Dirac masses of the neutrinos (in a basis where they are diagonal and positive) and $c = \cos \theta$, $s = \sin \theta$, with $\theta$ and $\delta$ being the rotation angle and phase which diagonalize the right handed Majorana mass matrix.

The light neutrino mass matrix is given by the seesaw formula:

$$ m_\nu \approx -\tilde{m}_D^T \frac{1}{M_{\mu e}} m_D^T , \tag{19} $$

where $m_D$ is the Dirac neutrino mass matrix and $M_{\mu e}$ the right handed Majorana mass matrix. The atmospheric neutrino mixing angle $\theta_{23}$ lies \cite{27} in the range

$$ |\varphi - \theta^D| \leq \theta_{23} \leq \varphi + \theta^D , \text{ for } \varphi + \theta^D \leq \pi/2 , \tag{20} $$
where $\varphi$ is the rotation angle which diagonalizes the light neutrino mass matrix in the basis where the Dirac mass matrix is diagonal and $\theta^D$ is the Dirac mixing angle.

In our analysis we will assume $\theta^D \approx 0$ and so take $\varphi = \theta_{23}$. (From $SU(4)_c$ symmetry, $\theta^D \simeq |V_{cb}| \simeq 0.03$ [35].) We take $m^D_3$ compatible with $G$, i.e. $m^D_3 = m_\tau \times \tan \beta$ for $G_{LR}$, and $m^D_3 = m_t$ for $SO(10)$ or $G_{PS}$. These relations hold at $M_{GUT}$, while the relevant values of the parameters are those at the leptogenesis scale. We estimate the Dirac masses by using the above relations as approximations, with the values for the quark and lepton masses at $T_r = 10^9$ GeV given in [36].

The light neutrino masses are assumed to be either hierarchical with $m_\nu^2 = 8.5 \times 10^{-3}$ eV and $m_\nu^3 = 0.06$ eV [29, 37, 38], or degenerate with $m_\nu^2 = 0.104$ eV and $m_\nu^3 = 0.122$ eV. Note that the RG evolution of the masses is particularly important for the degenerate case. The latter values are calculated with $\tan \beta = 50$ [31].

Using these Dirac and light neutrino masses, we have numerically calculated the range of $\kappa$, the symmetry breaking scale $M$ and the reheat temperature $T_r$ consistent with the observed baryon asymmetry $n_B/s \simeq 8.7 \times 10^{-11}$ [2] and the near maximal atmospheric mixing $\sin^2 2\theta_{23} \gtrsim 0.95$ [39, 37, 38]. For the allowed range of $\kappa$ we also required that $M_2 \leq m_\chi/2 < M_3$ and $\gamma_3 \leq 1$ where $M_3 = 2\gamma_3 M^2/m_P$ (2$\gamma_3 M^2/M_S$ for shifted and smooth hybrid inflation). The results are summarized below.

1. SUSY hybrid inflation with $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
   a. Hierarchical neutrinos: The charged lepton masses at $10^9$ GeV are $m_\mu = 86$ MeV and $m_\tau = 1.47$ GeV. We set, as an approximation, $m^D_i = m_i \times \tan \beta$ with $\tan \beta = 10$. We obtain solutions for $\kappa \sim 10^{-3.5}$ with $T_r \simeq 10^9$ GeV, $M_2 \simeq 10^{10.5}$ GeV and $M_3 \simeq 10^{13}$ GeV (Fig. 2).
   b. Degenerate neutrinos: With $\tan \beta = 50$, solutions are obtained for $\kappa \sim 10^{-3}$ with $T_r \simeq 10^{10.5}$ GeV, $M_2 \simeq 10^{12}$ GeV and $M_3 \simeq 10^{13}$ GeV (Fig. 3).

2. SUSY hybrid inflation with $G = SO(10)$
   a. Hierarchical neutrinos: We set, as an approximation, $m^D_i = m^q_i$ where $m^q_i$ are the quark masses at $10^9$ GeV ($m_c = 0.427$ GeV and $m_t = 149$ GeV). We find that there are no solutions consistent with near maximal atmospheric mixing.

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5For details of the calculation, we refer the reader to refs. [27, 22]. Note that in ref. [27], the small angle MSW solution for $m_2$ was assumed and the atmospheric mixing angle was found to be small for a particular value of $\kappa$. Our results are different but not contradictory, since they hold for different values of $m_2$ and $\kappa$. Also, instead of fixing the heavy Majorana masses and calculating the Dirac masses, we have fixed the Dirac masses and calculated the heavy Majorana masses.
b. Degenerate neutrinos: Solutions are obtained for \( \kappa \sim 10^{-4} \) with \( T_r \sim 10^9 \) GeV, \( M_2 \sim 10^{11} \) GeV and \( M_3 \sim 10^{13} \) GeV (Fig. 4).

3. Shifted hybrid inflation with \( G_{PS} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R \)

a. Hierarchical neutrinos: As in case 2a, there are no solutions with \( m_i^P = m_i^q \), although solutions are obtained with higher values of \( m_2^D \). As a numerical example, \( m_2^D = 2 \) GeV allows solutions (with the coefficient of the non-renormalizable coupling \( \beta = 0.5 \)) for \( \kappa \sim 10^{-2.5} \), with the symmetry breaking scale \( M \sim M_{GUT} \) and \( T_r \sim 10^{9.5} \) GeV (Fig. 5).

b. Degenerate neutrinos: Solutions for \( \kappa \sim 10^{-3} \) and \( T_r \sim 10^9 \) GeV are obtained with \( m_2^D = 0.427 \) GeV and \( m_3^D = 149 \) GeV (Fig. 6). We have taken \( \beta = 0.5 \), for which \( \kappa \geq 3 \times 10^{-4} \) is required for the inflationary trajectory.

4. Smooth hybrid inflation with \( G_{PS} \)

a. Hierarchical neutrinos: As in case 3a, we take \( m_2^D = 2 \) GeV to allow solutions. The baryogenesis and neutrino mixing constraints can be satisfied with \( T_r \sim 10^{10} \) GeV, and the heavy Majorana masses are \( M_2 \sim 10^{11} \) GeV and \( M_3 \sim 10^{15} \) GeV (Fig. 7).

b. Degenerate neutrinos: Taking \( m_2^D = 2 \) GeV to allow solutions, the baryogenesis and neutrino mixing constraints can only be satisfied for a narrow range of masses, with the symmetry breaking scale \( M \lesssim 10^{16} \) GeV and \( T_r \gtrsim 10^{10} \) GeV.

Note that in our calculations we have assumed \( M_1 \gg T_r \) so that washout effects are negligible. Since \( M_2/T_r \) turns out to be in the range \( 10 - 100 \), this assumption conflicts with a strong hierarchy between \( M_1 \) and \( M_2 \). Eqs. (17) and (18) have to be suitably modified for \( M_2 \sim M_1 \). However, the resulting lepton asymmetry does not change significantly, unless the right handed neutrinos are quasi degenerate (\( (M_2 - M_1) \ll M_1 \)). On the other hand, if we drop the assumption \( M_1 \gg T_r \), the constraints to generate sufficient lepton asymmetry become more stringent [6, 40].

5 Conclusion

We have reviewed non-thermal leptogenesis in SUSY hybrid inflation models. For the simplest SUSY hybrid inflation model, sufficient lepton asymmetry can be generated provided that the dimensionless coupling constant appearing in the superpotential Eq. (11) satisfies \( 10^{-6} \lesssim \kappa \lesssim 10^{-2} \). SUGRA correction to the potential is negligible
for this range and the power spectrum is essentially scale invariant. For shifted and smooth hybrid inflation, leptogenesis with larger values of the coupling constant and the symmetry breaking scale is also possible.

Constraints from neutrino mixing could further restrict the range of $\kappa$ that is allowed. We have applied the constraint of maximal (or near maximal) atmospheric mixing, as observed by Super-Kamiokande and K2K, to the case where the inflaton predominantly decays into the next-to-lightest right handed Majorana neutrino. We have numerically shown, for this case, that sufficient lepton asymmetry can still be generated with hierarchical Dirac neutrino masses imposed by the gauge symmetries.

We conclude that SUSY hybrid inflation models can satisfactorily meet the gravitino and baryogenesis constraints, consistent with the observed neutrino (mass)$^2$ differences and near maximal atmospheric neutrino mixing.

Acknowledgments
This work was supported by DOE under contract number DE-FG02-91ER40626. Q. S. thanks George Lazarides for discussions.

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Figure 1: The allowed range of the dimensionless superpotential coupling $\kappa$ (left) and the symmetry breaking scale $M$ (right) for SUSY hybrid inflation with $G_{LR}$ (dash-dotted line), SUSY hybrid inflation with $SO(10)$ (dotted line), shifted hybrid inflation (solid line) and smooth hybrid inflation (dashed line).
Figure 2: From bottom to top, $T_r$, $M_2$ (dashed lines), $m_\chi/2$, $M_3$ (dotted lines) and $M$ as functions of $\kappa$, for SUSY hybrid inflation with $G_{LR}$ and hierarchical left handed Majorana neutrinos. The regions for $T_r$, $M_2$ and $M_3$ are bound by the baryon asymmetry and near maximal atmospheric mixing ($\sin^2 2\theta_{23} \geq 0.95$) constraints.

Figure 3: Same as Fig. 2, for degenerate left handed Majorana neutrinos. Note that the allowed regions for $M_2$ and $M_3$ are also constrained by $M_2 \leq M_\chi/2 < M_3$. 

14
Figure 4: Same as Fig. 2, for SUSY hybrid inflation with $G = SO(10)$ and degenerate left handed Majorana neutrinos.

Figure 5: Same as Fig. 2, for shifted hybrid inflation with $G = G_{PS}$ and hierarchical left handed Majorana neutrinos.
Figure 6: Same as Fig. 2, for shifted hybrid inflation with $G = G_{PS}$ and degenerate left handed Majorana neutrinos. Note that $M_3$ is bound below by $m_\chi/2$, and $\kappa > 3 \times 10^{-4}$ is required for the inflationary trajectory for $\beta = 0.5$.

Figure 7: From bottom to top, $T_r$, $M_2$ (dashed lines), $m_\chi/2$, and $M_3$ (dotted lines) as functions of the symmetry breaking scale $M = (\mu M_S)^{1/2}$, for smooth hybrid inflation with $G_{PS}$ and hierarchical left handed Majorana neutrinos.