Quantum State Readout, Collapses, Probes and Signals

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Theories involving localized collapse allow the possibility that classical information could be obtained about quantum states by mechanisms other than the standard quantum dynamical maps defined by positive operator valued measurements, but nonetheless without allowing superluminal signalling. We can model this by extending quantum theory to include hypothetical devices that read out information about the local quantum state at a given point, which is defined by considering unitary evolution modified only by the effects of collapses in its past light cone. Like Popescu-Rohrlich boxes, these hypothetical devices would have practical and scientific implications if realisable. These include signalling through opaque media, probing the physics of distant or opaque systems without needing a reflected signal and giving detailed information about collapse dynamics without requiring direct observation of the collapsing system. These potential applications add to the motivation for systematic searches for possible signatures of these nonstandard extensions of quantum theory, and in particular for relevant gravitational effects, such as the validity of semi-classical gravity on small scales.

INTRODUCTION

Nature extracts classical information from quantum states, as this classical sentence, apparently generated by the matter in the brain of one organism purportedly described by a quantum state, and now represented in the mind of another, shows. The gravitational fields around us may be an independent example, if (for instance) gravity is not quantized but fundamentally described by a (quasi-)classical theory.

According to standard textbook quantum theory (see e.g. [1]), the most general way of obtaining classical information from quantum states is via positive operator valued measurements (POVMs). These define a quantum operation via the Kraus representation

$$E(\rho) = \sum_k A_k \rho A_k^\dagger,$$

where $k$ labels the different classical outcomes and

$$\sum_k A_k^\dagger A_k = I.$$  

Quantum operations respect the no-signalling principle. For example, if $\rho_{LR}$ is a density matrix for a bipartite system with Hilbert space $H_L \otimes H_R$ and $E_L$ is a quantum operation on the $L$ system, then

$$\text{Tr}_L(E_L \otimes I)(\rho_{LR}) = \text{Tr}_L(\rho_{LR}),$$

so that the fact that a measurement is carried out on the $L$ subsystem has no measurable effect on the $R$ subsystem. Predictions for measurements on the $R$ subsystem do, of course, generally change if one conditions on the measurement outcome at $L$, but for an observer at $R$ to do this requires information to be sent from $L$ after the measurement. Hence measurements cannot be used to signal superluminally. Stinespring’s theorem shows that quantum operations [1] can be derived from unitary maps on a larger Hilbert space, on which the POVMs are represented as projective measurements. Hence this treatment of measurement leaves the boundary between quantum system and classical information ambiguous. One line of thought on this, following Everett, is that quantum evolution is fundamentally unitary, defined on a Hilbert space for the universe. On this view, the appearance of classical information must be explained by the connection between consciousness and the universal wave function: whether this explanation follows naturally or requires ad hoc postulates is disputed [2]. Another view is that quantum wave function collapse is an objective phenomenon governed by mathematically well-defined rules whose form we can try to conjecture (e.g. [3–7]).

Both lines of thought have problems (see e.g. [2, 7, 8] for some discussions). It is also worth stressing that even if there were a consensus on a consistent and confirmable version of many-worlds quantum field theory, we would still not have strong reasons to believe it without a consistent and confirmed quantum gravity theory.
Fig. 1: Action of a readout device on an entangled subsystem

Here, we work with the alternative hypothesis: that POVMs are applied at definite points in time defined by some objective collapse model. These collapses may be discrete or continuous in time; we assume they are localized in space.

This still leaves the question: might nature possibly extract classical information in other ways than POVMs? Could the right theory of gravity (or possibly some future theory characterising the contents of consciousness from quantum states, in the way that integrated information theory [9] attempts for classical networks) involve different rules? The honest scientific answer is that, while there is no evidence for this, we do not know. As noted above, POVMs are the most general class of measurements that can alternatively be represented by unitary operations in a larger Hilbert space. But whether quantum theory is universally valid is very much an open question in the context of gravity (and indeed of consciousness). There is, in any case, a consistent way of extending quantum mechanics, at least in the idealised semi-relativistic setting (see e.g. [10]) often used in discussing relativistic quantum information, to allow rules that go beyond POVMs. This is [11] to postulate hypothetical devices that give information about quantum states, following rules that are defined to be compatible with standard quantum theory and to ensure no superluminal signalling.

A simple illustration, adequate for our discussion here, is to take the devices to give information about internal degrees of freedom such as spin, for systems of particles with relatively negligible spread in position space, which we treat as effectively pointlike. More precisely, when applied at a space-time point $x$, the devices produce classical descriptions of (or some information about) the local quantum state $\rho_{\text{loc}}(x)$, which is defined as the local reduced density matrix at $x$ for the relevant degrees of freedom obtained from the quantum state on (spacelike hypersurfaces tending to) the past light cone $\Lambda(x)$. We consider the devices in the context some version of quantum theory with objective localised collapses. A device at $x$ is then sensitive to the effects of collapses within $\Lambda(x)$, but not outside. Any collapses arising from a measurement on a subsystem at a point $y$ space-like separated to $x$ take place at $y$ or in its causal future, and so do not affect the value of $\rho_{\text{loc}}(x)$. A measurement at $y$ thus has no observable effect at $x$, either via the standard quantum state or via the devices, and so there is no direct way of signalling superluminally via measurements. More generally, it can be shown that the devices do not allow any way of superluminal signalling. [11] For example, if we have two particles $L$ and $R$ at spacelike separated points $x$ and $y$, in an entangled spin state, a spin measurement on $R$ that induces collapse at $y$ does not affect the output of the device at $x$, but does affect the output at points in the causal future of $y$. (See Figure 1.)

We assume here that all the readout devices we discuss leave the quantum state uncollapsed and unaltered. Other possibilities are also interesting: for example, consistent nonlinear versions of quantum theory can be defined by allowing the local unitary evolution to depend on the readout [11]. However, we focus here on the simplest case of readout devices that have no direct effect on the quantum state. This is approximately the case for the simplest semi-classical gravity models, when the gravitational self-interaction is relatively negligible. In these models, the semi-classical gravitational field (1 below) is effectively a readout. However, we need to consider its precision.

If some physical theory allowed us to actually build readout devices, any device actually built presumably would work only to finite precision, though it might perhaps be theoretically possible to attain arbitrarily high precision at the cost of greater technological resources. For example, as we discuss further below, if semi-classical gravity were valid in some regime, we would be able to infer information about quantum systems in that regime from their classical
gravitational fields. A superposition state of the form \( \sum_{i=0}^{1} a_i |x_i\rangle \), where \( |x_i\rangle \) is a state in which a small object of mass \( m \) has wave-function peaked around \( x_i \), with radius \( r \) and spreads \( \delta_i \) such that \( r, \delta_0, \delta_1 \ll |x_0 - x_1| \), would generate a gravitational field

\[
\Phi(y) = Gm \left( \frac{|a_0|^2}{|x_0 - y|} + \frac{|a_1|^2}{|x_1 - y|} \right),
\]

at points \( y \) with \( \min_i (|y - x_i|) \gg \max_i (\delta_i) \). In models where measuring the gravitational field has no effect on the quantum state, this allows estimates of the Born probabilities \( |a_i|^2 \). In principle, by applying suitable unitaries to a qubit and converting the computational degrees of freedom to position space, the full form of the qubit \( \sum_{i=0}^{1} a_i |i\rangle \) could thus be estimated.

In practice, though, gravitational fields cannot be measured to infinite precision. It might perhaps be possible to increase the precision arbitrarily by improving the measuring devices, isolating the system as far as possible from unknown gravitational fields and other forces, and so on. Even if so, we should expect it to become greatly, perhaps exponentially, more difficult to add to the precision beyond a certain point.

We will not generally make any detailed assumptions about this tradeoff in our discussion, simply supposing that the relevant readout devices can be built to sufficiently high precision. To simplify the notation, when the applications we consider involve distinguishing significantly different states, we treat the precision as effectively infinite.

One can imagine various potentially interesting types of readout device. A state readout device \( RD(x) \) applied at \( x \) would print out a classical description of \( \rho^{loc}(x) \) to given (maybe infinite) precision, expressed in some given basis. An alternative idealisation, which is perhaps more elegant (particularly for an infinite precision readout device for qubit states), is for it to represent \( \rho^{loc}(x) \) physically with a classical pointer. An expectation value readout device \( RD(A, x) \) prints out the expectation value \( \langle A \rangle \) of some hermitian observable \( A \) in the local state. A stochastic eigenvalue readout device \( SRD(A, x) \) prints out an eigenvalue \( \lambda_i \) of \( A \), randomly chosen using the Born probabilities

\[
\langle A \rangle \text{ or } |\langle A \rangle| \leq \Delta_{\text{max}},
\]

where the problem \( \Delta_{\text{max}} \) is non-degenerate, then in principle multiple applications of either of the last two devices, combined with appropriate unitaries, can be used tomographically to produce finite precision versions of the state readout device.

One way of viewing these readout devices is as analogous to Popescu-Rohrlich (PR) nonlocal boxes. PR boxes extend quantum theory without violating special relativity, and bring computational advantages. In particular, both the infinite and finite precision versions of \( RD \) would allow computational speedups. It follows from results of Abrams and Lloyd \[17\] that a single infinite precision readout device that can be applied to a single qubit would allow quantum computers to solve NP and \#P problems in polynomial time. Abrams and Lloyd’s algorithm is framed in terms of an oracle that calculates a function \( f : \{0, 1\}^n \rightarrow \{0, 1\} \), where the problem is to determine whether there exists an input string \( x \) such that \( f(x) = 1 \). They create a uniform superposition of all possible inputs, and apply the oracle once, obtaining

\[
|\psi\rangle = 2^{-n+1} \sum_{i=0}^{2^n-1} |i, f(i)\rangle.
\]

With probability at least \( \frac{1}{2} \), applying a Hadamard transformation to the first \( n \) qubits, followed by a measurement in the computational basis, produces the state

\[
\psi_{n,s} = C |00\ldots0\rangle \otimes \left( \frac{2^n - s}{2^n} |0\rangle + \frac{s}{2^n} |1\rangle \right),
\]

where \( s \) is the number of values of \( x \) such that \( f(x) = 1 \) and \( C \) is the normalisation factor. Applying an infinite precision \( RD \) to the last qubit thus gives the value of \( s \) and in particular distinguishes the cases \( s = 0 \) and \( s > 0 \) as required.

A sensible rough definition of a finite precision \( RD \) is that, with some given high probability \( p \leq 1 \), it is able to distinguish \( \psi_{n,0} \) from \( \psi_{n,1} \) (and more generally \( \psi_{n,s} \) from \( \psi_{n,s'} \) for \( s \neq s' \)) for \( n \leq m \), with the maximal such value of \( m \) effectively defining the precision. This would efficiently solve the search problem for sets of size up to \( 2^n \), a potentially valuable speed-up if \( m \) is large. It would be interesting to know if stronger results are possible. An obstacle
to straightforwardly using $RD$ within Abrams-Lloyd’s algorithms for general $n$ is that $RD$ is defined to read out the local density matrix, which for a single qubit readout device is the reduced state of the relevant qubit, so that applying it on the penultimate qubit in

$$\sum_{i=0,1} a_i \psi_i \otimes \psi_{m,i} \otimes |0\rangle$$

would generally produce a readout of a mixed state: for example if $\langle \psi_0 | \psi_1 \rangle = 0$ it would produce the readout

$$\sum_{i=0,1} |a_i|^2 |\psi_{m,i}\rangle \langle \psi_{m,i}|.$$  

The readout thus cannot be directly used to implement transformations of the form

$$\sum_{i=0,1} a_i \psi_i \otimes \psi_{m,i} \otimes |0\rangle \rightarrow \sum_{i=0,1} a_i \psi_i \otimes \psi_{m,i} \otimes |i\rangle,$$

which would be desirable, since it would leave the final qubit registering the search result on $2^n$ strings and available for further quantum processing. We leave open here the question whether there may be other ways of using finite precision single qubit $RD$ devices to obtain faster speedups, and the more general question of the power of multi-qubit finite precision readout devices, and turn to other potential applications of readout devices.

**PROBES AND SIGNALS**

Consider an entangled pair of subsystems at rest in state

$$|\Phi_+\rangle_{LR} = \frac{1}{\sqrt{2}}(|0\rangle_L |0\rangle_R + |1\rangle_L |1\rangle_R),$$

where the $L$ and $R$ subsystems are separated by distance $d$ in some inertial frame and $|0\rangle, |1\rangle$ are orthogonal states of each subsystem. We take $d$ to be large compared to the wave function spread of either subsystem and fix units with $c = 1$. Suppose that a state readout device is applied on the $L$ subsystem. Suppose now that at time $t = 0$ (in the same frame) a projective measurement in the $|0\rangle_R, |1\rangle_R$ basis is applied to system $R$, which, according to the relevant collapse hypothesis, ensures a rapid collapse (taking time negligible compared to $d$) onto the measured outcome state. The readout device at $L$ produces readout

$$\frac{1}{2} I_L$$

up to time $t = d$, where $I_L$ is the uniform mixed state in the relevant two dimensional Hilbert space. After time $t = d$ it produces a readout

$$|i\rangle_L \langle i|_L,$$

where $i = 0$ or 1 is the outcome obtained at $R$. (See again Figure 1).

An observer at $L$ reading the readout, who knows the initial state $|\Phi_+\rangle_{LR}$ and the locations of the subsystems, thus learns at $t = d$ that a measurement was carried out at $R$ at time $t = 0$. That is, the readout device gives observers at $L$ and $R$, who have previously shared the state and preagreed their locations, a means of signalling at light speed. The signal involves no carrier subsystem, and so this mechanism works regardless of how opaque any intervening material is to ordinary signals. For example, this would allow a simple means of signalling at light speed between antipodal points on Earth, something which with current technology requires a strong neutrino source and neutrino detector.

An expectation value readout device for the observable $A = a_0 |0\rangle \langle 0| + a_1 |1\rangle \langle 1|$, where $a_0 \neq a_1$ and both are real, would work similarly: the observer at $L$ would obtain readout $\frac{1}{2} (a_0 + a_1)$ up to time $t = d$ and then either $a_0$ or $a_1$ thereafter. A stochastic eigenvalue readout device for $A$, if applied repeatedly, would produce a random sequence of $a_0$ and $a_1$ up to time $t = d$ and then either a sequence of $a_0$’s or a sequence of $a_1$’s.

**Probes of distant systems**

Assume now that some specific collapse model has been empirically confirmed, and that the readout devices function as specified with respect to collapses in this model. Turning the previous observation around, if the $R$ system is sent
into an unknown distant environment, then an observer monitoring the L system can infer, from the transition between (10) and (11), that it has collapsed at some point on the past light cone of the observed transition point.

If the R system’s trajectory is known (for instance, if it is known to travel at fixed velocity) then the location of the collapse can be inferred (at least to within a small region). In other words, properties of distant environments – specifically, their propensity to cause collapse within the given model – can be probed, even though no particle or field perturbation is reflected back to the observer.

No existing technology allows this form of probing. Although “interaction free” measurement [18] or imaging (e.g. [19]) may seem somewhat analogous, they still require well-defined trajectories with non-zero amplitudes to and from the region of the imaged object, to distinguish between reflection and absorption (or scattering).

Tests of collapse hypotheses

Suppose now that readout devices have been found to work in combination with standard measurements carried out by human observers using macroscopic apparatuses. This would be compelling evidence for some form of objective collapse model, but the details of this model might not immediately be fully clear. Measurements on the L system can then be used in order to obtain empirical evidence about precisely when and under what conditions collapses occur on the R system, by exposing the R system to a variety of potentially collapse-inducing measurement-like interactions.

For example, the R system’s superposition state could be amplified towards the macroscopic by correlating it with a variety of mass distribution states, in order to test and refine hypotheses about state reduction associated with superposed mass distributions or gravitational fields. Alternatively, the R system could be “observed” by (and so correlated with the information processing states of) a variety of candidate observers – humans, small animals, photosynthesis mechanisms in plants, small quantum computers, and so on – to test and refine speculative hypotheses about observer- or consciousness-induced state reduction (e.g. [7, 20–22]).

Again, nothing comparable is possible with existing technology using standard quantum theory. Tests of collapse models currently require either very challenging interferometry, or indirect evidence from small violations of conservation laws or small anomalous wave function spreads. These require near complete isolation of the relevant system from environmental decoherence, which has very similar effects. It is also not completely clear whether violations of conservation laws are essential corollaries of any plausible collapse model, although they are features of all models considered to date.

THE EXAMPLE OF SEMI-CLASSICAL GRAVITY

It may seem far-fetched to imagine that any of our readout devices could be found anywhere in nature. However, the continuing interest in semi-classical gravity models (e.g. [23–29]) gives one reason not to dismiss the possibility. The literature on semi-classical gravity is inspired by the equation

\[ G_{\mu \nu} = \langle T_{\mu \nu} \rangle, \]  

which is easy to write but hard to interpret given (inter alia) that the quantum matter whose stress-energy tensor appears on the right hand side is propagating in the space-time whose metric determines the left hand side.

In the non-relativistic limit with N fixed particles, with mass density operator

\[ \hat{M}(x) = \sum_i m_i \delta(x - \hat{x}_i), \]  

we can define (see e.g. [27]) the classical Newtonian potential \( \Phi \) obeying

\[ \nabla^2 \Phi(x) = 4\pi G \langle \hat{M}(x) \rangle. \]  

Semi-classical gravity is then defined by a modified Schrödinger equation

\[ i \frac{\partial}{\partial t} |\psi\rangle = (\hat{H}_{\text{matter}} + \hat{H}_{\text{gravity}})|\psi\rangle = (\hat{H}_{\text{matter}} + \int \hat{M}(x)\Phi(x)dx)|\psi\rangle. \]

Although there are many unresolved problems with semi-classical gravity theories [25, 27], there are at least ways of interpreting the non-relativistic equations [11, 27, 29, 30] that avoid the pathological superluminal signalling [31].
arising from applying standard measurement postulates directly to (15). While we do not see averaged gravitational fields from superposed position states of large masses [32], this leaves open the possibility that semi-classical gravity may hold within an objective collapse model [28] with collapses preventing macroscopically distinct mass distributions from remaining in superposition, and with their effects on the gravitational field propagating at light speed [11].

We can replicate our earlier discussion in the context of semi-classical gravity by considering an entangled pair of subsystems at rest, of the form

$$\frac{1}{\sqrt{2}}(|x⟩_L|0⟩_R + |x'⟩_L|1⟩_R),$$

where $|x⟩, |x'⟩$ correspond to a mesoscopic object of mass $m$ with centre-of-mass well localized around the points $x, x'$ respectively, and $|0⟩, |1⟩$ are orthogonal states of the other subsystem which can be distinguished by some practical measurement method that implies a rapid collapse onto the measured outcome state, according to the relevant collapse hypothesis. We take the wave functions of the states $|x⟩, |x'⟩$ to have spreads $δ, δ' \ll |x - x'|$, and the L and R subsystems to be separated by $d \gg \Delta = |x - x'|$.

Semi-classical gravity [12] implies that in this state the L subsystem generates a Newtonian gravitational potential

$$\Phi(y) \approx \frac{Gm^2}{2} \left( -\frac{1}{|x - y|} + \frac{1}{|x' - y|} \right)$$

at points $y$ with $|y - x|, |y - x'| \gg δ, δ'$.

A signal can then be sent from R to L by measuring the R subsystem, in a way that we know theoretically or empirically must cause a rapid collapse, at time 0 in the rest frame of the two subsystems. By hypothesis [11], for a consistent combination of semi-classical gravity and collapse, the Newtonian potential contributed by L in its vicinity remains of the form (10) until approximately time $d$, when either

$$\Phi(y) \approx -\frac{Gm}{|x - y|}$$

or

$$\Phi(y) \approx -\frac{Gm}{|x' - y|},$$

depending on the R measurement outcome.

For suitable values of $m$ and $\Delta$, the cases (17), (18) and (19) can be distinguished by gravitational phase interferometry [29], quickly compared to $d$ (for sufficiently large $d$). This allows signalling, remote probing and investigation of collapse dynamics to be carried out as above.

SEARCHES FOR GRAVITY-BASED READOUT DEVICES

Models involving semi-classical gravity suggest one way that readout devices might possibly be found in nature. More generally, any measurable gravitational effect different from that predicted by perturbatively quantized general relativity implies some extension or breakdown of quantum theory, and hence the possibility of readout devices. Another example in this direction, which if confirmed would have enabled the signalling and probe mechanisms discussed above, is the event formalism investigated by Ralph and collaborators [33–36], which predicts anomalous gravitationally induced decoherence. While this prediction appears to have been falsified [37], it illustrates that there are other (albeit also speculative and incomplete) theoretical motivations for nonlinear quantum effects connected to gravity.

The various applications of readout devices add further motivation to proposed tests of gravitationally-induced entanglement [38–40] or other tests distinguishing perturbatively quantized general relativity from possible classical models of gravity (e.g. [29]) that might confirm or exclude such effects. They also motivate more systematic theoretical and experimental investigations.

A particular feature of readout device based models is that a local system in the neighbourhood of a space-time point $x$ behaves differently depending on whether or not it was entangled with a distant system up to the boundary of its past light cone $\Lambda_x$. The differences could, as in the case of semi-classical gravity, involve non-quantum effects whose signature might anyway be detected in experimental tests not involving distant entanglement. But they could also be subtle. It would be worth systematically searching for such effects by extending experiments such as those
Particle/photon entangled with the path state of $S_2$

$S_1$ mass $m_1$, radius $r_1$

$S_2$ mass $m_2$, radius $r_2$

$P$ in the state

$$\frac{1}{\sqrt{2}}(|L| + |R|).$$

(Equivalently, $P$ could be a photon, with its polarization degrees of freedom entangled.) A suitably macroscopically amplified measurement (which, by hypothesis, in the relevant collapse model, is swiftly collapse-inducing) on $P$ in the basis $\frac{1}{\sqrt{2}}(|\uparrow| + |\downarrow|)$ would effectively place $S_2$ in one of the states $\frac{1}{\sqrt{2}}(|L| \pm |R|)$. By hypothesis, this would occur with a time lapse (as measured in the lab frame) due to light speed transmission of the effect of the collapse. So, by appropriate choices of the measurement time, it could be made to occur before, during, or after the interferometry phase of the experiment. Similarly, a measurement on $P$ in the basis $\{|\uparrow|, |\downarrow|\}$ would effectively place $S_2$ in one of the states $\{|L|, |R|\}$; again, by hypothesis, this could be made to occur before, during or after the interferometry. Each basis choice and each outcome defines an ensemble of experiments, allowing a test of whether the results within that ensemble depend in any way on the time of the distant measurement and collapse. (See Figure 2)

**DISCUSSION**

Nonlinear quantum effects and objective collapses are both speculative hypotheses, and it is unclear whether they can be extended to fully consistent relativistic theories. Nonetheless, there are theoretical motivations to consider both, and they have been extensively studied (separately and together) in work on unifications of quantum theory and gravity. The potential information-theoretic, technological and scientific applications of combining these hypotheses has however not been fully appreciated. For example, even the fact that semi-classical gravity would transform our understanding of the physical basis of computing seems not to have been noted, although it could be argued that this application alone should motivate far more theoretical and experimental attention on this topic.

The possibilities we have discussed here – signalling, remote probing, and alternative ways of uncovering the details of collapse mechanisms – add to the potential technological and scientific payoffs of finding any measurable nonlinear effect of the type we consider. From a more foundational perspective, readout device models add to the taxonomy of possible extensions of quantum theory and of possible principles that might identify the true theory of nature. We hope that these observations may stimulate further theoretical and experimental work.

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