Parity doubling in particle physics

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Abstract

Parity doubling in excited hadrons is reviewed. Parity degeneracy in hadrons was first experimentally observed 40 years ago. Recently new experimental data on light mesons caused much excitement and renewed interest to the phenomenon, which still remains to be enigmatic. The present retrospective review is an attempt to trace the history of parity doubling phenomenon, thus providing a kind of introduction to the subject. We begin with early approaches of 1960s (Regge theory and dynamical symmetries) and end up with the latest trends (manifestations of broader degeneracies and AdS/QCD). We show the evolution of various ideas about parity doubling. The experimental evidence for this phenomenon is scrutinized in the non-strange sector. Some experiments of 1960s devoted to the search for missing non-strange bosons are re-examined and it is argued that results of these experiments are encouraging from the modern perspective.

PACS: 12.90.+b, 12.38.-t, 12.39.Mk

Keywords: Parity doubling, Hadron symmetries

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1 Introductory remarks

Parity doubling in particle physics is the occurrence of opposite-parity states of equal spin value. The problem of parity doubling is that experimentally these states are often approximately mass degenerate. In particle physics the phenomenon is mainly a feature of unflavored (light non-strange) hadron spectrum.

One has always been inspired by a hope that parity doubling is able to shed light on various knotty problems of the strong interaction dynamics. The subject has already more than 40 years history, nevertheless only recently two reviews appeared [1, 2], which cover some related ideas in a more or less systematic manner. We would like to make a reservation from the very outset that the present review differs from them mainly by two aspects. First, the issue of parity doubling in hadrons is addressed broader, in particular, we place emphasis on the historical development of the subject. Second, the theoretical presentation is simplified as much as we could with the aim of making it more readable for experimentalists, the given review is designed in many respects for experimentalists seeing that presently only experiment is able to make a major contribution to clarification of modern situation with parity doubling in particle physics.

In view of renewed interest to the parity doubling phenomenon, a comprehensive review is certainly called for. This is, however, a pretty formidable task which we do not pretend to do. To a greater extend the difficulty is caused by the fact that the ideas and approaches invoked for explanation of the phenomenon come from quite different branches of physics, and it is hardly possible to be a specialist in all these fields. On the other hand, in a situation when the final truth is far from being established in a subject, it is not easy to propose an unprejudiced view on the subject for non-specialists. As a result, a choice of material and references, authors’ comments, etc. can be somewhat questionable, let alone a tendency to over-concentration on authors’ personal work. In trying to escape this in our subject, we will provide mostly a guide on the relevant literature (in the first part of the review) with brief explanations of proposed ideas and without giving any preferences or criticism, an interested reader is further referred to the original literature. In the second part of the review (Sections 6 and 7) we scrutinize experimental evidences for the parity doubling phenomenon, discuss clustering of states near certain values of masses and further perspectives.

The phenomenon of parity doubling in hadron spectrum has experienced two waves of interest — in late 1960s and in late 1990s. The first wave was caused by the discovery of many baryon states in 1960s. The origin of the second wave (growing up to now) is more intricate, partly it was inspired by
the appearance of many experimental data on light mesons. We will try to
describe the related ideas in a more or less chronological order.

Our discussions will concern many forgotten papers, the choice of such
a retrospective style has a motivated ground — a known wisdom says that
the new is a well-forgotten old. We would be happy if reading of this review
stimulated someone to put forward new ideas...

2 1960s: Search for hidden order

2.1 Beginning of 1960s: The first precursors of problem

Historically the first discovered hadron resonances gradually formed the \( J^P \)
octets \( 0^-, 1^-, 1^+, \) and decuplet \( \frac{3}{2}^+ \). The minimal group containing such
representations is \( SU(3) \). After experimental establishing of these multiplets
the \( SU(3) \) symmetry was finally accepted as a group of internal symmetry
for strong interactions [3]. Nearly at the same time it turned out that many
approaches used in that epoch for description of strong interactions were
requiring the existence of multiplets with the opposite parity. This need was
in Regge theory (for a short review see [4]), in some bootstrap models [5,
6], a bit later in the dynamical symmetry approaches [7]. The proposed
extensions of \( SU(3) \) also often demanded the opposite parity multiplets (see,
\textit{e.g.}, a review [8]). The competition won the Gell-Mann’s [9] \( SU(3) \times SU(3) \)
chiral symmetry\(^2\), which gave rise to current algebra and later became an
approximate classical symmetry of Quantum Chromodynamics. Despite the
success of current algebra, at the beginning the chiral symmetry was not
widely accepted because it predicted the opposite parity multiplets which
had to be mass degenerate with the known multiplets. This situation was far
from the experimental one, to say the least. The attitude was considerably
changed when Weinberg derived his famous formula [12], \( m_{a_1} = \sqrt{2} m_\rho \),
assuming the chiral symmetry at large four-momentum. It became clear
that the chiral invariance can be regarded as an asymptotic symmetry of
strong interactions. This somewhat solved the problem of unwanted mass
degeneracy for parity partners.

\(^2\) To be precise, this is the minimal three-flavor chiral symmetry. \textit{Say}, Freund and
Nambu proposed \( SU(3) \times SU(3) \times SU(3) \times SU(3) \) chiral symmetry [10]. The word “chiral”
stems from the greek word “\( \chi e \rho \)” – “hand”. In various branches of science an object is
called chiral if it differs from its mirror image, like left and right hands. The first systematic
study of chiral symmetries in particle physics was performed by Coleman and Glashow [11].
In what follows we will often prefer to discuss the baryons and mesons separately.

2.2 Baryons in 1960s

The first theoretical hints on a possible existence of parity doublets appeared before the corresponding experimental observations. The first one was the MacDowell symmetry [13]: The slopes of baryon Regge trajectories of equal isospin and signature but with opposite parity must coincide. The second one was due to Gribov [14, 15]: The Regge trajectories of opposite parity fermions are complex conjugated. Both results indicated that baryons must form parity doublets if the corresponding Regge trajectories are linear.

Thus, the first explanations for parity doubling were tightly related to the linearity of Regge trajectories motivated by the linear dependence of hadron spin \( J \) from the hadron mass squared,

\[
J = \alpha(0) + \alpha' m^2, \tag{1}
\]

the Chew-Frautschi conjecture [16, 17]. It is important to stress that Regge theory itself did not provide convincing arguments in favor of relation (1) as this theory establishes the fact of certain dependence of spin from the mass squared and some restrictions on this dependence, but it does not yield an explicit form for this dependence. Typically the linear trajectories appear in the relativistic descriptions while the non-linear trajectories emerge in the non-relativistic approaches. Why do not the straight trajectories become curved at some higher energy scale like in the non-relativistic scattering theory based on the notion of scattering potential? The linearity was an experimental fact, in addition, the linear trajectories were inherent in the Veneziano model [18], which was extremely popular at that time. On the other hand, this model had problems with the incorporation of the MacDowell symmetry. The universal slope of Regge trajectories \( \alpha' \) (of the order of 1 GeV\(^{-2}\)) is naturally related to the universal range of strong interactions (of the order of \( 10^{-13} \) cm). If the trajectories are curved at much higher energy scale, this means then that strong interactions have an additional characteristic scale. In this case one observes the linear trajectories simply because every curve looks as a straight line at sufficiently small interval. If it were the case, the self-consistency of the analytical \( S \)-matrix approach would be questionable (say, one of postulates of the \( S \)-matrix theory is decomposability of the \( S \)-matrix due to finite range of strong interactions). There were proposals that this scale at high energies (high compared to the known resonance region) could be provided by quark masses as long as quarks were very heavy in the...
old quark models, of the order of 5 GeV or more. Later a more convincing justification for the linearity of trajectories was proposed — the relativistic hadron strings. But this is out of the scope of our topic.

In several years parity doubling among some nucleon resonances was indeed observed experimentally. An ”explosion” of these observations happened in 1967 (see, e.g., [19–23]). Barger and Cline [24, 25] immediately attributed the phenomenon to a manifestation of the MacDowell symmetry. However, along with the parity doublets one observed some notable parity singlets, e.g., the ground states. This fact seemed to contradict the MacDowell symmetry and caused much discussions. Different ways out were proposed, for instance, vanishing residues for the corresponding parity partners [26], but such ad hoc solutions did not seem to be satisfactory [4]. Different authors tried to adjust the situation in the framework of representations of the Lorentz group or its extensions (see, e.g., [27–32]). The proposed schemes indeed required the parity duplication of some baryons since they were (partly) based on the Toller analysis [29, 30]. In Toller’s scheme one assigns hadrons (in the rest frame) to irreducible representations \((j_1, j_2)\) of the Lorentz group, then one considers the ”Toller” quantum number

\[
M = |j_1 - j_2|.
\]  

The states with \(M = 0\) are parity singlets while the states having \(M \neq 0\) are parity doublets. Inasmuch as baryon spin \(J\) is half-integer and \(|j_1 - j_2| \leq J \leq j_1 + j_2\), the pair of indices \((j_1, j_2)\) has to consists of one integer and one half-integer numbers, hence, all baryons transforming under the representation \((j_1, j_2)\) are parity doubled.

At the same time it was realized that parity doubling in the Regge theory is a particular solution for the so-called ”conspiracy” among different Regge trajectories (see, e.g., [33] for references): In order to avoid kinematic singularities of invariant amplitudes at vanishing momentum transfer, some linear combinations of certain partial wave amplitudes have to be equal to zero [34]. This problem emerges when one takes into account the spin of particles and differences in masses. Generally speaking, a solution of the conspiracy problem is not unique. Consequently, a natural question emerged, why parity doubling is preferred? Various proposals appeared that this is a consequence of \(SO(4)\) space-time symmetry of scattering amplitude at vanishing momentum transfer (see, e.g., [29, 30, 35–37] and references therein), for some dynamical reasons one also observes an imprint of this symmetry at non-vanishing momentum transfer. The Lorentz invariance (or \(SO(4)\) after the Wick rotation) of scattering amplitude was argued to result in the existence of ”daughter trajectories” for any Regge trajectory (earlier this result
was deduced from the analyticity properties of scattering amplitude) and in
the appearance of parity doubled type of conspiracies. Extending $SO(4)$ by
parity, one thus can conclude that parity doubling is a consequence of $O(4)$
symmetry of the spectrum. However, a certain care must be exercised there-
upon. The invariance group of a scattering amplitude need not coincide with
the classification group for its spectrum of the bound states. The coincidence
takes place for a scattering amplitude with all the external particle masses
equal [33].

We shortly remind the origin of ideas related to the $O(4)$ symmetry for
Regge theory. In 1954 Wick [38] introduced his famous "rotation" from
Minkowski space to Euclidean one. It was proposed for mathematical sim-
plification of the Bethe-Salpeter equation. Cutkosky [39] immediately made
use of Wick’s trick to find a complete set of bound state solutions in the case
of the Bethe-Salpeter equation for two scalar particles. The degeneracy of
solutions turned out to be identical to that of the nonrelativistic hydrogen
atom. The method itself happened to be, in a sense, dual to Fock’s treat-
ment of hydrogen atom [40] where the $O(4)$ symmetry is manifest. In ten
years Domokos and Surányi [41] noted that such a higher symmetry implies
interesting consequences for Regge trajectories. They found that every singu-
larituy in the angular momentum plane induces a series of other singularities
of the same nature following the original one at unit steps. This situation is
a natural consequence of $O(4)$ symmetry: There is, in fact, one singularity in
the complex $O(4)$ angular momentum variable, which generates the series of
singularities above when one decomposes according to the usual $O(3)$ angu-
lar momentum. Stated differently, one four-dimensional pole is equivalent to
a superposition of poles in the usual three-dimensional angular momentum
plane. In that way the daughter trajectories emerge. The $O(4)$ theory of
Regge trajectories was further elaborated by Freedman and Wang [42–45].
In particular, they examined the reason of Coulomb degeneracy in Bethe-
Salpeter models. The group $O(3)$ of three-dimensional rotations is the in-
variance group of Bethe-Salpeter equations for nonzero total energy as the
total energy-momentum four-vector is fixed under $O(3)$ rotations. For zero
total energy, however, this four-vector vanishes, and the equation becomes
invariant under $O(4)$ transformations of its integration variables. This very
extra degree of invariance ensures the existence of daughter trajectories in
much the same way that the extra degree of invariance for some infinite range
potentials ensures the Coulomb degeneracy of bound states. As a byproduct,
the higher symmetry (with the ensuing decomposition of amplitudes in $O(4)$
harmonics) automatically resolved a long-standing problem with the ambi-
guity of the asymptotic behavior of the unequal-mass scattering. Although
in general case (unequal mass scattering) $O(4)$ is not an exact symmetry of
the scattering amplitude, this higher symmetry can be a good symmetry for the spectrum of the amplitude, at least in the first approximation. For this reason the spectrum of $\pi N$ resonances should follow the underlying higher symmetry. This point was scrutinized by Domokos [46].

Let us present the key features of $O(4)$ partial-wave analysis. One decomposes an amplitude in the four-dimensional spherical harmonics,

$$Z_{nl}^m(\beta, \theta, \phi) = p_{nl}(\beta)Y_{lm}(\theta, \phi),$$

where $Y_{lm}(\theta, \phi)$ is a usual three-dimensional spherical harmonic, $n = 0, 1, 2, \ldots$ is analogous to the principal quantum number in the hydrogen atom, and $p_{nl}(\beta)$ can be expressed through Legendre or Gegenbauer functions of $\cos \beta$, which gives the restriction $l \leq n$. The spectrum (both poles and branch cuts) appear as simple singularities in the $n$ plane, in the $l$ plane it shows the pattern required by the higher symmetry $O(4)$. Then one introduces the integer quantum number $\kappa$,

$$n = l + \kappa.$$ 

It is called ”relative-time parity” and bears a close analogy with the radial quantum number in the hydrogen atom. The even values of $\kappa$ give rise to the daughter Regge trajectories. The odd values do not correspond to observable particles (the odd-$\kappa$ poles in the physical region violate unitarity). Thus, starting, e.g., from a parent trajectory with the states at $l = 0, 2, 4, \ldots$, one obtains the daughter states corresponding to the even valued $O(4)$ spherical harmonics.

The $O(4)$ partial-wave analysis may be regarded as a particular realization of generalized partial-wave analysis concept for the $S$-matrix, which was put forward by Salam and Strathdee [47]. According to this concept, the partial-wave analysis can be probed by almost any complete set of orthogonal functions and if a certain choice turns out to be successful phenomenologically and the corresponding set realizes a representation of some higher symmetry group, then the corresponding higher symmetry is a good candidate for the underlying dynamical symmetry generating the observed spectral recurrences. The concept was illustrated in [47] by decomposition in the $O(6)$ spherical harmonics. Being isomorphic to Wigner’s higher symmetry $SU(4)$, the group $O(6)$ was assumed to include internal symmetries.

The previous note makes a bridge to another approach to the description of parity doubling — the dynamical symmetry formalism. By a dynamical symmetry group one means here a group which gives the actual quantum numbers and degeneracy of a quantum-mechanical system (sometimes it is called ”hidden”, ”accidental” or ”spectrum-generating” symmetry). In this
approach symmetries of Hamiltonian do not play an important role. Physically the dynamical group reflects the internal structure of the system.

Let us explain the idea by a classical example — the hydrogen atom (H). It has the $O(3)$ rotational invariance, hence, each state of discrete spectrum can be labelled by $|lm\rangle$, where $l$ and $m$ are the usual angular momentum quantum numbers — the angular momentum and its projection. However, as was first discovered by Fock [40], the actual symmetry of discrete spectrum for the H-atom is $O(4)$. It is manifested by the existence of the principal quantum number $n$ numerating the energy levels,

$$E_n \sim \frac{1}{n^2}, \quad n = l + n_r + 1,$$

(5)

where $n_r$ is the radial quantum number. As a consequence, the discrete states of H-atom are labelled by three numbers, $|nlm\rangle$. All wave functions corresponding to states with the same energy, i.e. labelled by the same $n$, fall into one irreducible representation$^3$ of $O(4)$ [48]. In thirty years Malkin and Man’ko made the next breakthrough in the group theory of H-atom [49, 50]: the full dynamical symmetry group is the conformal group $O(4,2)$ which includes $O(4)$ as a maximal compact subgroup. Soon alternative derivations of this result were proposed (see references in [51, 52]). It turned out (see references in [53]) that all states of discrete spectrum as well as the continuum spectrum and all radiative transitions can be compactly described within the $O(4,2)$ dynamical group, i.e. the whole relativistic theory of H-atom (without account of electron spin) can be formulated in terms of this group, with the $O(4,1)$ subgroup being the dynamical group of the bound states and the $O(3,2)$ that of the scattering states. This is tightly related with the fact that the Kepler problem can be formulated as $O(4,2)$ dynamical group theory [54].

What is the physical meaning of $O(4)$ and $O(4,2)$ dynamical symmetries for the H-atom? The $O(4)$ symmetry tells us that if we know a wave function of state with a given energy then acting by generators of $O(4)$ on this wave function we are able to obtain the wave functions of all states with the same energy without solving the Schrödinger equation. The larger $O(4,2)$ symmetry$^4$ tells us that by applying the same procedure we will get the whole set of wave functions for discrete spectrum.

$^3$Although $O(4)$ has two Casimir operators, i.e. irreducible representations are labelled by two indices $(j_1,j_2)$, one index is enough for labelling of irreducible representations in the Coulomb (Kepler) problem. The reason is that the Casimir operators happen to be equal in the case of the Coulomb potential, hence, only the representations with $j_1 = j_2$ are realized in nature.

$^4$More exactly, its $O(4,1)$ subgroup when discussing the discrete spectrum of H-atom. The dynamical group $O(4,1)$ connects states with different principal quantum numbers $n$
The success of dynamical symmetry approach in the H-atom inspired to apply similar ideas to hadron physics. It was assumed that the quantum theory of hadrons can be formulated in terms of irreducible representations of some dynamical groups (both compact and noncompact) with no Hamiltonian or space-time coordinates at all. The problem was to identify an appropriate dynamical group and find its relevant irreducible representations. Indeed, in the usual dynamical approach one finds a discrete spectrum by solving an eigenvalue equation. On the other hand, if one knows all solutions of an eigenvalue equation one can always assign the corresponding eigenfunctions to one irreducible representation of some group (at least for the differential eigenvalue equations). In this sense a search for the solutions of dynamical equations might be equivalent to a group-theoretical search for higher symmetry.

The experimental spectrum of baryons happened to be qualitatively similar to that of the H-atom. This observation inspired Barut et al. \cite{28,53,55–57} to apply the dynamical \( O(4,2) \) group to description of baryons. The unitary irreducible representations of \( O(4,2) \) contain the states which for given quantum numbers are characterized in the rest frame by \( |njm,\pm\rangle \). Here \( \pm \) refers to the parity determined from the parity of the ground state. There are two possible ways of parity doubling in the \( O(4,2) \) representations. In the first case all states have their opposite parity counter-part. In the second case all states for a given \( n \) are parity doublets, except one parity singlet state emerging at \( j = n - 1 \) (see Fig. 1). The latter case is realized in the H-atom, it seemed to be preferable also for nucleons. The obtained accordance with the experimental data (both on mass spectrum and on formfactors) was rather encouraging.

Originally the dynamical symmetry approach was introduced to hadron physics independently of the group theory for the H-atom. The corresponding ideology was formulated by Dashen and Gell-Mann \cite{58}. A general scheme for accommodation of states with different parities was discussed in \cite{59}. In short, one deals with a finite number of energy levels (hadron masses) in hadron physics. Before those papers, the situation was usually accommodated by a finite-dimensional irreducible representations of compact groups, like \( O(4) \) in the H-atom. However, if there are many states, an infinite sequence of discrete energy levels can be a permissible idealization. In this case the use of an infinite-dimensional representation can turn out to be a more effective approximation than the use of a finite-dimensional one. A and contains \( O(4) \) as a subgroup. The totality of all the bound-state wave functions carry a representation of \( O(4,1) \).
group possessing such a unitary irreducible representation has to be non-compact. After this justification, the use of noncompact dynamical groups became quite popular, the conformal group $O(4,2)$ is an example.

The program for determining the whole hadron mass spectrum and form-factors with the help of some underlying dynamical group was very ambitious, the peak of activity occurred in 1967-1968. Finally the program failed, the number of papers on the spectrum-generating approach decreased exponentially, although this method was not forgotten completely (see, e.g., a classification of meson Regge trajectories based on the $SO(4)$ dynamic symmetry in [60]).

At that time the success of current algebra and partially conserved axial-vector current hypothesis made apparent the fact that strong interactions are approximately symmetrical under the $SU(3)_L \times SU(3)_R$ chiral group [9] and, hence, all hadrons should fall into multiplets of chiral group (see, e.g., the related discussions in [61, 62]) containing degenerate states of positive and negative parity. This symmetry (more precisely, its Wigner-Weyl realization) is broken to the vector $SU(3)_V$ subgroup and the broken part of the chiral symmetry manifests itself through the appearance of eight nearly massless Goldstone bosons. In other words, the chiral symmetry is realized in the Nambu-Goldstone mode. In 1969 Dashen noticed [63], however, that

Figure 1: The weight diagram of the hydrogen-like $O(4,2)$-representation for the nucleon $J^P$ states (a simplified figure from [55]).
the residual symmetry of hadron spectrum could be $SU(3)_V \times Z$, where $Z$ is a discrete symmetry, which leaves the vacuum invariant and leads to parity doublets. Namely, the discrete group $Z$ consists of six elements \( \{1, P, Z, Z^\dagger, PZ, PZ^\dagger\} \), where $P$ is the parity operator and the discrete operation $Z$ is related to the axial hypercharge $Y_5$: $Z \equiv e^{i2\pi Y_5}$. The group $Z$ has two one-dimensional representations that are parity singlets and one two-dimensional representation which contains the states with opposite parities, the latter representation exists only if $n_f > 2$. The particles will then fall into multiplets corresponding to one of these irreducible representations. In the second case one must observe parity doubling in the mass spectrum. Within this picture all states on a given Regge trajectory must be either parity singlets or doublets. The related phenomenology was occasionally appearing in the literature. In thirty years, however, this possibility was excluded by the rigorous QCD inequalities [64].

A few years later, in 1973, the fundamental theory of strong interactions, QCD, was introduced [65] after the discovery of its asymptotic freedom [66–69] and many theoreticians switched over QCD. Nevertheless, QCD was not shedding light on the problem of parity doubling for a long time. Meanwhile, experimentalists were discovering and confirming more and more new parity doubles in the baryon sector...

### 2.3 Mesons in 1960s

Because of a shortage of experimental data the story in the meson sector is not so rich. The same as in the baryons the first arguments were based on Regge theory and on the dynamical group approach.

Barger and Cline [70] associated the absence of backward peaks in $\pi^+\pi^-$, $\pi^+K^-$, $K^+K^-$, and $\bar{N}N$ elastic scattering with the occurrence of meson resonances in highly correlated sequences of angular momentum states with alternating parities called ”towers”. The first $JP$ tower is $(0^+, 1^-)$ (of both isospin), the second one is $(0^+, 1^-, 2^+)$, the third one is $(0^+, 1^-, 2^+, 3^-)$, etc. (see Fig. 2). According to modern knowledge, Regge trajectories of different isospin, the $(\omega, \rho)$ and $(f_2, a_2)$ trajectories in our case, are practically degenerate due to a negligible admixture of strange quark. Experimentally the four trajectories $(\omega, \rho, f_2, a_2)$ coalesce into one master trajectory, in Regge theory this fact is known as exchange degeneracy\(^5\). The tower hypothesis

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\(^5\)Exchange degeneracy is the approximate dynamical degeneracy of two sets of trajectories with opposite signature and $G$ parity, e.g., the $\rho$ and $a_2$ trajectories. Using the Mandelstam variables $(s, t, u)$, exchange degeneracy originates from the absence of contribution of $u$ channel resonances to an amplitude $A(s, t)$. Like linearity of trajectories, exchange degeneracy does not rigorously follow from Regge theory, it was a feature of the
predicted for linear rising meson trajectories the existence of large number
of meson states in the mass regions called $R(\sim 1700\text{ MeV})$, $S(\sim 1930\text{ MeV})$, $T(\sim 2100\text{ MeV})$, and $U(\sim 2300\text{ MeV})$. In addition, in order to build up full nucleon-antinucleon elastic scattering amplitude one required a strong local parity degeneracy of the meson states of the kind that towers could provide. Making use of the fact that $\bar{N}N$ inelastic cross section is bigger than the elastic one, it was concluded that mesons should be strongly coupled to the $\bar{N}N$ annihilation inelastic channels, hence, the discovery of many meson resonances in $\bar{N}N$ annihilation would provide a crucial test for the tower hypothesis. In about thirty years all these conclusions were qualitatively confirmed by the Crystal Barrel experiment on $\bar{p}p$ annihilation in flight [72–78]. It is quite remarkable that a recently obtained preliminary picture of non-strange meson spectrum (see Fig. 8) had been qualitatively anticipated in the preQCD time.

A bit earlier Barut [79,80] applied to the non-strange mesons the hydrogen-like description based on the $O(4,2)$ dynamical group. As a result, a similar picture of meson ”towers” emerged. Say, the states in the pion towers of $O(4,2)$ are $(0^-)^{n=1}; (0^-, 1^+, 2^-)^{n=2}; (0^-, 1^+, 2^-, 3^+)^{n=3}; \ldots$. The states belonging to the same tower are naturally degenerate because they have equal ”principal” quantum number $n$. The parity conjugated towers ”grow out” of the $\rho$-meson. The same as in the baryon case, there are two ways of parity doubling within the rest-frame dynamical group $O(4,2)$ — either with parity doubled ground state or with parity singlet one.

An interesting proposal made Alessandri [81]. He tried to apply to mesons the Gribov’s mechanism of parity doubling for the fermion Regge

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Veneziano model [18] and was explained by the old quark hypothesis: The exchange forces stem from direct interaction between heavy quark and antiquark, the exchange mesons cannot be lighter than quarks, hence, exchange forces are very short-range, i.e. negligible. Exchange degeneracy was first proposed by Arnold by analogy with potential theory [71].
trajectories [14, 15] (the parity-doubled conspiracy at zero momentum transfer). It was shown that this can be achieved if the wave functions of spin $J$ bosons belong not to the $(\frac{1}{2}, J, \frac{1}{2}, J)$ irreducible representation of the Lorentz group (because $M = 0$ in this case, see Eq. (2)) but to the reducible representation

$$[(1, 0) \oplus (0, 1)] \times \left[ \frac{J - 1}{2}, \frac{J - 1}{2} \right].$$

This representation is analogous to the Rarita-Schwinger representation for half-integer spin.

Domokos et al. [32] introduced a certain complex extension of Lorentz group (isomorphic to the "chiral" Lorentz group $SL(2, \mathbb{C}) \times SL(2, \mathbb{C})$). Covariance under this group requires parity doubling not only for baryons, but also for mesons with $M \neq 0$ if the corresponding Regge trajectories are linear.

The exercises above looked rather academical as experimentally there was no example of degenerate parity partners in the mesonic sector. In addition, making use of the fact that residues of states with $M = 0$ and $M \neq 0$ behave differently at vanishing momentum transfer, the analysis of various helicity amplitudes showed that all known mesons had $M = 0$, i.e. they are parity singlets if the Toller theory is correct.

Just as in baryons, after advent of QCD the problem of parity doubling was forgotten for a long time.

3 End of 1980s: Revival of interest

In the late 1980s the problem was risen again by Iachello [82]. The purpose of his work was "... to bring attention to a major feature of baryon spectra that, although extensively investigated in the late 1960s within the context of both chiral symmetry [83] and Regge-pole theory [84], has, in recent years, been somewhat overlooked. This is the occurrence in the spectra of parity doublets..." It was argued that the occurrence of parity doubling in some states and nonoccurrence in others are a consequence of the geometric structure of hadrons, i.e. the underlying dynamics is similar to that of some molecules.

To reveal the underlying physics Iachello proposed some baglike and string-like models. We will describe the idea confining ourselves to the string case only.

Consider the following model for baryon: Three identical quarks are on the tops of Y-type string (the Mercedes-Benz type of string junction, the

\[ ^{6} \text{In fact, at that time the well established mesons were either the Goldstone bosons or belonged mostly to the principal Regge trajectories. According to the modern experimental data, all meson states on these trajectories are indeed parity singlets, this will be concerned in Sections 6 and 7 (see Fig. 8).} \]
picture resembles the ammonia molecules \( \text{NH}_3 \) where parity doubling of energy levels is known to occur. Aside from the rotational invariance, there is a symmetry with respect to the permutations of quarks. This symmetry is isomorphic to the geometric point group \( C_{3v} \). In the theory of point groups, the symbol \( C_n \) denotes the symmetry with respect to rotation on the minimal angle \( 2\pi/n \). This angle is equal to 120° in our case, hence, \( n = 3 \). The symbol \( v \) means the reflectional symmetry with respect to the vertical plane. Consider the mesons. They are made of quark and antiquark attached by a string. Since quark and antiquark are not identical particles, this system has the geometric symmetry \( C_{\infty v} \) (the same as the symmetry of linear molecule). The transformations of this group consist in rotations and reflections on a plane, i.e. it is isomorphic to \( O(2) \). At enlarging angular momentum \( l \) the \( Y \)-string produces an elongated shape, i.e. it becomes reminiscent of the quark-diquark structure. Thus, at large \( l \) the geometric group of baryons \( C_{3v} \) converts into \( C_{\infty v} \). The discrete group \( C_{3v} \) has two one-dimensional representations, called \( A_1 \) and \( A_2 \), and one two-dimensional representation called \( E \). In this respect it is similar to Dashen’s \( Z \)-invariance discussed above. Hadrons possess also internal symmetries, for baryons the internal symmetry is usually believed to be \( SU(3)_c \times SU_{sf}(6) \). Hence, the geometric wave functions (w.f.) must be combined with the internal w.f. in such a way that the total w.f. are antisymmetric for baryons. The spin-flavor group \( SU_{sf}(6) \) has the representations referred to as \( 56, 70, \) and \( 20 \). All baryons are commonly assumed to fall down to the corresponding multiplets. Following the w.f. antisymmetry principle, it was argued that \( A_1 \) must be combined with \( 56, A_2 \) with \( 20 \), and \( E \) with \( 70 \). Thus, the states belonging to the representation \( 70 \) are expected to be parity doubled, while the states in \( 56 \) (they are known to include the ground states) should be parity singlets. The geometrical considerations based on a baglike analysis resulted in the claim that parity doubling in mesons does not occur. Since at large \( l \) the baryons and mesons become similar, parity doubling in baryons should gradually disappear as \( l \) increases. The overall picture was in accord with the available experimental data at that time.

Iachello’s paper [82] was followed by Robson’s comment [85] and Iachello’s reply [86]. The discussion concerned a possibility for inclusion of the center of \( Y \)-type string junction to the geometrical symmetries.

We would like to mention two instructive comments of current importance which appeared in [82, 85]. First, Robson [85] noted that the relativistic motion of the quarks and strings does not allow a simple separation of total spin into orbital and intrinsic spin components. The impact of relativity on such type of models is difficult to assess. Second, Iachello [82] anticipated the failure of the nonrelativistic quark models in the description of parity...
doubling. For instance, within quark models with harmonic-oscillator potentials the states of opposite parity have different numbers of oscillator quanta, hence, parity doubling can be only accidental.

Another baryon string model explaining parity doubling was proposed by Khokhlachev [87]. The effect was attributed to a large centrifugal potential for quarks in the rotating gluon string. In this model two quarks are frozen at the ends of linear gluon string and the third one moves along the string. There are two levels with nearly equal energy corresponding to "left" and "right" diquark states. These two states can evolve into each other by quark tunneling under the centrifugal barrier. The transition amplitude is small for large $l$, hence, the mass difference of parity partners is small too. An interesting prediction of the considered model is that the mass difference dies off exponentially with increasing $l$,

$$
\Delta m_\pm \sim \sqrt{\frac{\mu}{L}} \exp(-\mu L),
$$

where $L$ is the length of the string ($L^2 = 4(2l + 1/2)/\pi\sigma$, $\sigma$ is the string tension) and $\mu$ is the effective mass of travelling quark when it moves at a large distance from the ends.

Independently, the available experimental information for meson and baryon Regge trajectories of hadrons built of light quarks was summarized and discussed by Kaidalov [88]. The data seemed to favor the idea of approximate dynamical supersymmetry between mesons and baryons (the related discussions have been occasionally appearing in the literature, see [89] for a review). It was emphasized that the existing quark models are unable to reproduce the observed regularities in hadronic mass spectra, in particular, parity doubling among baryons. The latter phenomenon was conjectured to happen due to the Chiral Symmetry Restoration (CSR) for large masses. It was noted also that CSR does not occur for the principal boson Regge trajectories and that the behavior of boson Regge trajectories can be explained by a smallness of spin-orbital interaction between quark and antiquark. All these observations anticipated qualitatively the main lines of later development of the subject under consideration.

Nearly at the same time DeTar and Kunihiro proposed [90] a generalization of the linear sigma model where two parity partner nucleons form a multiplet of the chiral group and they can be degenerate with a non-vanishing mass. This model, however, was intended to describe CSR at high temperatures with entailing parity doubling of the baryon spectrum known from the lattice simulations. But the idea itself was exploited later for description of CSR in highly excited baryons.
4 1990s: New ideas

4.1 Baryons in 1990s

In 1990s the following idea independently came to mind of different people: The systematic parity doubling in excited baryons is nothing but a manifestation of effective chiral symmetry restoration in the upper part of baryon spectrum. We have just mentioned the idea of CSR in relation with Kaidalov’s work [88]. Kirchbach arrived at this idea in 1992 (see [91]) in a rather philosophical way, inspired by an analogy with chirality in chemistry and biology. A manifest realization of chiral symmetry above 1.3 GeV in non-strange baryons was explicitly stated in [92]. However, the systematic occurrence of parity unpaired states and spin-parity clusters forced her to abandon the idea of CSR in such a straightforward interpretation and to propose an alternative scheme (to be discussed below). Nevertheless the idea itself was not forgotten [93, 94] (we refer to [95] for relevant discussions). Nearly at the same time CSR in excited baryons was independently observed by J. Dey and M. Dey within a dynamical symmetry model (inspired by 1960s Barut’s work on dynamical conformal $O(4,2)$ group) based on $U(15|30)$ graded Lie group reduced to $SU(3)$ subgroup [96] (see also [97]). In framework of this approach the baryons are supersymmetric partners of mesons.

After some years of recess the idea about different realization of chiral symmetry of QCD in the low-energy and high-energy sectors (the Nambu-Goldstone mode and the Wigner-Weyl one correspondingly) was again repeated in the beginning of review [98], although the review itself was devoted to the description of baryons within a constituent quark model with the harmonic confinement potential. In several years the potential models were criticized by Glozman [99]: They cannot explain the appearance of systematic parity doublets, this is especially evident for the harmonic confinement. In essence, the 10-years old Iachello’s and Kaydalov’s conclusion [82, 88] was rediscovered. The paper [99] seems to be the first attempt to reveal the dynamics underlying CSR. The effect was ascribed to the strong residual interactions between valence constituent quarks due to Goldstone boson exchange. A parallel with the chiral phase transition at high temperature was drawn. The proposed explanation, however, did not work for mesons (the meson spectra indeed did not exhibit parity doubling at that time).

To proceed further we should present the experimental spectrum for non-strange baryons, see Fig. 3 for nucleons and Fig. 4 for $\Delta$-baryons. One can immediately notice the main features of displayed spectrum — parity doubling of many states and clustering of masses near certain values of energy. The Particle Data Group [100] averages the data over different experiments, this
Figure 3: The experimental spectrum of nucleons [100] in units of the proton mass squared. Experimental errors are indicated. The most reliable values reported in [100] are denoted by circles. The filled and open strips (circles) stay for the positive and negative parity states correspondingly. The approximate positions of clustering are shown by dashed lines.

obscures clustering because of accumulation of experimental errors. For this reason it is instructive to demonstrate the results of a separate comprehensive analysis. In Fig. 5 we show the data provided by Höhler (this data is cited by the Particle Data Group [100] under the name ”Hoehler”) for Δ-baryons (for nucleons the picture is very similar). Clustering in Fig. 5 becomes much sharper. Höhler seems to be the first who emphasized that baryons appear as spin-parity clusters rather than as separate states [101–103]. Now these clusters often carry his name. We draw attention to the (quasi)systematic parity singlets in Fig. 5 (or Fig. 4), especially the lowest $\frac{3}{2}^+$, $\frac{7}{2}^+$, and $\frac{11}{2}^+$ states. One can expect that all states inside a cluster are parity doubled except, in some cases, the state with the highest spin. The existence of such parity unpaired states represents a stumbling-block in interpretation of the parity doubling phenomenon. Are they regular or we are simply dealing with a lack of experimental data? At present this is not known, this very point generates various models and speculations.
Figure 4: The experimental spectrum of $\Delta$-baryons [100] in units of the $\Delta(1232)$ mass squared. The notations are as in Fig. 3.

Figure 5: The spectrum of $\Delta$-baryons from Höhler analysis [100]. The notations are as in Fig. 4.
The first theoretical explanation for H"ohler’s clusters was proposed by Kirchbach [104]. The symmetry of all reported nonstrange baryon excitations was advocated to be governed by $O(4) \times SU_1(2)$ rather than by $O(3) \times SU(6)_{sf}$ which is the usual textbook symmetry for classification of the baryon states. The clusters appear due to the $O(4)$ partial wave decomposition of the $\pi N$ amplitude, where only even valued four-dimensional harmonics should be taken into account. In a sense, it was a revival of old ideas of 1960’s (see discussions before and after Eqs. (3) and (4)). These ideas, however, were developed towards accommodation of many new experimental data. The clusters of non-strange baryons in Fig. 3 and Fig. 4 are assigned to $n = 2, 4, 6$ poles of $O(4)$ partial wave decomposition for the $\pi N$ amplitude. The states inside each cluster fall into the Rarita-Schwinger-like Lorentz multiplet (compare to Eq. (6)),

$$\left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right] \times \left[ \frac{n-1}{2}, \frac{n-1}{2} \right].$$

(8)

According to the proposed theory, the states belonging to $n = 2$ possess a natural parity $(-1)^l$, the states in $n = 4, 6$ carry an unnatural parity $(-1)^{l+1}$. The different assignment of parity is interpreted as appearance of low-lying states on the top of the scalar vacuum and that of high-lying states on the top of the pseudoscalar vacuum (as the parity was defined as $\eta(-1)^l$, with $\eta$ being the parity of underlying vacuum). The change of underlying vacuum when passing to high excitations is suggested to signal the chiral symmetry restoration in highly excited states. The corresponding ideas and phenomenology were developed in [105–110]. In particular, a solution of the Velo-Zwanziger problem (the violation of causality at propagation of the Rarita-Schwinger fields when minimally coupled to an external electromagnetic field) was proposed: The low-spin states entering the Rarita-Schwinger spinors should not be eliminated as redundant components by some auxiliary conditions, instead they should be treated as physically observable resonances reflecting the composite character of baryons. Stated differently, a pathology-free Rarita-Schwinger field describes a H"ohler cluster as a whole rather than a separate state (see [111–114] for the latest results). Kirchbach’s classification allowed to describe successfully the H"ohler clusters and to reduce significantly the number of ”missing” states.

With regard to clustering in baryons we would like to make the following remark. The first who predicted this phenomenon seems to be Feynman. Basing on unpublished 1969 Caltech Lecture Notes, he suggested certain approximate regularities among the square masses of the baryons. His scheme was elaborated and published in [115]. Now it appears to be timely to remind the results. The proposed classification of baryons was guided by the
SU_{sf}(6) quark model and the principle of Regge recurrence, in other words, it was guided by certain "clustering" principle. In the non-strange sector, a crucial test for the suggested mass degeneracies had to be the discovery of six "missing" states. In addition, the confirmation of these states was claimed to be equivalent to "... the statement that the spin-orbit contribution to the mass splitting in the quark model is small". This guess-work, likely, was not taken seriously by specialists (at least, Feynman et al.'s paper [115] has an extremely low citation by non-authors, which is quite unusual for such a physicist as Richard Feynman). Curiously enough, later all these six "missing" states were gradually discovered with the masses close to Feynman’s predictions! At present [100] they are (we display the star rating): \(N_{3/2}^+(1720)^{****}, N_{3/2}^-(1750)^{***}, \Delta_{3/2}^+(1920)^{***}, N_{1/2}^-(2080)^{**}, N_{1/2}^-(2200)^{**}, \) and \(N_{7/2}^-(2250)^{****}.\) Unlike Höhler's spin-parity clusters, Feynman’s clusters are only "spin" ones, they do not predict parity doubling.

Another approach to the problem of baryon parity doublets was suggested by Balachandran and Vaidya [116, 117]. They noticed that parity doublets occur typically in systems with two differing time scales. There are numerous examples of this phenomenon in molecular and nuclear physics. The possible parity doubles in particle physics were supposed to have the same origin. The idea was then realized in [118], where the baryon was modeled by slow Skyrmion and fast light quarks whizzing around.

### 4.2 Mesons in 1990s

In 1990s there was increasing evidence that meson states of different spin fall into degenerate towers at a given mass, this interesting tendency attracted some attention within the framework of relativistic quark models [119], although the related problem of parity doubling was not directly addressed. The experimental data did not unambiguously show a systematic parity doubling among mesons. As a consequence, physicists were not enthusiastic to work in the given direction. We are aware of one attempt to address the problem directly, within a combined analysis of effective quark models and asymptotic sum rules from QCD. Before the relevant discussions we remind some prehistory of asymptotic sum rules.

In early 1960s the idea of asymptotic chiral symmetry appeared\(^7\). This symmetry was supposed to become rigorous at sufficiently high energy region

\(^7\)The roots of this idea go back to 1950s when different authors were attempting to uncover a "higher symmetry" of strong interactions, which is broken at low energies but perhaps becomes exact in some high energy limit (see, e.g., [11] for references).
where the symmetry breaking effects are negligible. For instance, the axial nucleon current $j_\mu^A$ is not conserved by itself, the Partially Conserved Axial Current (PCAC) hypothesis states that

$$j_\mu^A \sim \bar{\psi}_N (i \gamma_\mu \gamma_5 + 2m_N \gamma_5 p_\mu / p^2) \psi_N,$$

where the second term is associated with the pion and $m_N$ is the nucleon mass. However, if the momentum is so large that $p^2 \gg m_N^2$ then one does not need PCAC, the axial current is conserved by itself. This is a reflection of the fact that the kinetic term $\bar{\psi}_N \gamma_\mu \partial_\mu \psi_N$ in effective strong-interaction Lagrangians becomes dominant in the high-energy region. Such a point of view was often stressed by Nambu [121] (see also [122]). Consider as an example the $\pi N$ system. The corresponding amplitude possesses a broken chiral invariance, the chirality is conserved due to pions. However, if the momentum is so large that the nucleon mass may be neglected, one does not need the pions to conserve chirality. The $\pi N$ scattering amplitude becomes chirally invariant by itself, hence, the soft pion emission process will vanish. This observation results in interesting predictions [121].

The Weinberg’s sum rules [12] are, perhaps, the most famous application of the asymptotic chiral symmetry concept. Consider a two-point correlation function for hadron currents (for the sake of convenience we consider the momentum representation in Euclidean space),

$$\Pi_k(Q^2) \sim \int d^4 x e^{iQx} \langle j_k(0) j_k(x) \rangle,$$

where $k$ denotes a set of indices characterizing the hadron current $j_k(x)$. Let $j_+(x)$ and $j_-(x)$ be parity (chiral) partner currents. Consider the difference of their two-point correlators and impose the condition

$$\Delta(Q^2) = \Pi_+(Q^2) - \Pi_-(Q^2) \xrightarrow{Q^2 \rightarrow \infty} 0,$$

This condition can be regarded as a mathematical expression for the asymptotic chiral symmetry [123]. It gives relations for the hadron masses when one makes use of the pole approximation. Weinberg considered the vector and axial-vector isovector currents, and assumed the dominance of the ground state in the pole approximation and the convergence condition

$$Q^4 \Delta^4(Q^2) \xrightarrow{Q^2 \rightarrow \infty} 0,$$

In the case of badly broken symmetries, it is necessary to indicate the limit, where the symmetry is present in pure form. By the beginning of 1960s it became clear that the relevant limit is the limit of high frequencies, i.e. of high energies [120].
where $\Delta^t$ means that the transverse part $(-\delta_{\mu\nu}Q^2 + Q_\mu Q_\nu)$ is factorized out. Taking into account PCAC and the so-called KSFR relation ($Z_\rho = 2f_\pi^2$, here $Z_\rho$ is the $\rho$-meson residue and $f_\pi \approx 93$ MeV is the weak pion decay constant), he derived the relation $m_{a_1}^2 = 2m_\rho^2$, which was in impressive agreement with the experimental data at that time. The idea turned out to be very fruitful. For instance, very soon Das et al. [124] calculated the pion electromagnetic mass difference due to the Weinberg sum rules. Schechter and Venturi [125] showed that under some assumptions the Weinberg relation can be reproduced numerically from the values of neutron and proton magnetic moments and the axial constant. One of their assumptions was that the baryons can be assigned to a definite chiral representation at very large momentum transfer (compared to the baryon masses), i.e. again one used the asymptotic chiral symmetry. The concept of asymptotic symmetries became a standard topic in the textbooks on elementary particles of that time (see, e.g., [123]). Later Weinberg’s assumptions were somewhat justified. The pole approximation, i.e. the approximation of infinitely narrow resonances, is equivalent to the large-$N_c$ limit in QCD [126,127]. The convergence condition was derived within the Operator Product Expansion (OPE) method [128,129].

Consider the exact planar limit of QCD (infinite number of colours). As a rule, this limit is known to work well within 10% accuracy. The meson correlators then have to be saturated completely by the infinitely narrow meson resonances [126,127]. The number of resonances with identical quantum numbers should be infinite in order to reproduce the perturbative logarithmic asymptotics of correlators. Thus, one has

$$\Delta_{\text{planar}}(Q^2) = \sum_{n=0}^{\infty} \frac{Z_+(n)}{Q^2 + m_+^2(n)} - \sum_{n=0}^{\infty} \frac{Z_-(n)}{Q^2 + m_-^2(n)} \xrightarrow{Q^2 \to \infty} 0. \quad (13)$$

Here $n$ is analogous to the radial quantum number. The OPE [128,129] predicts a quite rapid convergence at large Euclidean momentum in the r.h.s. of Eq. (13) (say, as $O(Q^{-4})$ for the scalar case and as $O(Q^{-6})$ for the vector one). On the other hand, the dominance of ground state ($n = 0$) is typically a good approximation. In order to reconcile these facts one can deduce that the masses and residues of opposite-parity states should be rapidly converging with $n$. A similar reasoning forced A. A. Andrianov and V. A. Andrianov [130] to conclude that a rapid restoration of chiral symmetry in Eq. (13) suggests a rapid CSR in the spectrum of radially excited mesons. Consequently, any effective quark model describing the strong dynamics above the chiral symmetry breaking scale ($\approx 1$ GeV) has to reproduce the asymptotic restriction (13) dictated by OPE, i.e. it has to reproduce the CSR at high energies. This is a powerful test for QCD-motivated effective quark models.
even if they do not describe the radial excitations. The corresponding concept was formulated earlier [131]. Later, matching of some effective models to the short distance behavior of two-point correlators was performed [132–136]. In addition, since chiral symmetry is restored quite rapidly, already the first radial excitation might reveal this phenomenon, i.e. one should have then

\[ m_+ (1) - m_- (1) \ll m_+ (0) - m_- (0). \]

This property was demonstrated for the case of so-called quasilocal quark model in the scalar channel [130]. If one assigns the first scalar and pseudoscalar “radial” excitations to the states \( f_0 (1370) \) and \( \pi (1300) \) then this prediction is fulfilled indeed. Moreover, a fast CSR in the spectrum was argued [130] to entail the decoupling of heavy parity doublets from the low-energy pion physics. In practice, this statement means that contribution of radial excitations to the constants of low-energy effective chiral Lagrangians [137] is negligible, these constants are mostly saturated by the ground states.

5 2000s: Golden age

5.1 General discussions

The beginning of this decade is marked by an experimental breakthrough in the unflavored meson sector. The analysis of Crystal Barrel Collaboration experimental data on proton-antiproton annihilation in flight in the energy range 1.9-2.4 GeV revealed more than thirty new resonances (see, e.g., [72–75]). Subsequently, all known light mesons were systematized by Anisovich with collaborators in [76, 77], which resulted in the experimental discovery of approximately linear trajectories on the \((n, M^2)\) and \((J, M^2)\) planes (\(n\) is the “radial” quantum number and \(J\) is the meson spin). In particular, on the \((n, M^2)\) plane the light mesons can be fitted with a good accuracy by the linear parametrization:

\[ M^2 (n) = m_0^2 + an, \quad n = 0, 1, 2, \ldots, \quad (14) \]

where \(m_0\) is the mass of basic meson and \(a\) is the trajectory slope parameter. The latter turned out to be approximately the same for all trajectories, \(a = 1.25 \pm 0.15 \text{ GeV}^2\). It is exactly a string-like spectrum predicted by many dual models and effective boson string models starting since 1960s. However, since these experimental results were extracted by a single group, many of them are still listed by Particle Data [100] as not well confirmed states. The latest review on the Crystal Barrel results is contained in Bugg’s work [78], the
averaged slope of meson trajectories was reported there to be \( a = 1.14 \pm 0.013 \text{ GeV}^2 \).

The analysis of Crystal Barrel data was criticized for adopting resonances from the outset, this interpretation of data, however, is in accord with the general principles of quantum field theory, such as analyticity. This point seems to be underestimated by other groups who have the data. It should be mentioned that the analysis was performed without any intentions to obtain something like linearity. It is quite remarkable that the final systematization of the best fits yielded (unexpectedly!) the linear trajectories and numerous cases of parity doubling.

We display some typical examples of meson trajectories from [77] in Fig. 6. A prominent feature of presented plots is duplication of some trajectories. This effect is trivial for the scalar mesons: The lower trajectory corresponds to the states in which the strange component dominates. In other cases the explanation can be given in terms of nonrelativistic quantum mechanics [76]. Let \( \vec{l} \) and \( \vec{s} \) be relative angular momentum of quark-antiquark pair and its spin. The \( P \) and \( C \) parities are defined for quark-antiquark pair as \( P = (-1)^{l+1} \) and \( C = (-1)^{l+s} \). Following the rule for the total spin \( \vec{J} = \vec{l} + \vec{s} \), the vector \( IJ^{PC} \) state \( 11^{--} \) can be constructed by two ways, \( (l, s) = (0, 1), (2, 1) \) (the S- and D-wave vector mesons in usual spectroscopic language), while its parity partner \( 11^{++} \) is made by one way only, \( (l, s) = (1, 1) \). The same can be repeated for all other cases. Thus, at \( J > 0 \) one of parity conjugated radial trajectories is always duplicated.

At the beginning only a little attention was paid to the Crystal Barrel results. Many specialists somehow overlooked them. The baryon sector was remaining richer, hence, more interesting.

Jido et al. [140] basing on ideas of DeTar and Kunihiro [90] proposed to organize low-lying baryon fields into the representations \( (1, \frac{1}{2}) \oplus (\frac{3}{2}, 1) \) of \( SU(2)_L \times SU(2)_R \) chiral group (the so-called quartet scheme or mirror assignment). Soon Cohen and Glozman [141], basing on the quark-hadron duality, advocated the point of view that the quartet scheme should be applied to physical states of highly-lying baryons (in [140] it was applied to low-lying baryon fields, the physical states are obtained after acting by fields on the vacuum, the difference would not be critical were the chiral symmetry unbroken in the physical vacuum), the CSR then ensues. In addition, CSR was substantiated by an OPE-based reasoning applied to the case of baryon currents. This reasoning is somewhat similar to that of described at

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\(^9\)This value coincides with the fit obtained independently in [149] and is very close to an earlier estimation [139] which did not make use of the Crystal Barrel data. A minor discrepancy between Anisovich’s and Bugg’s fits originates from a different treatment of the scalar sector.
Figure 6: Some plots for the $IJ^{PC}$ spectrum of unflavored mesons from [77]. Open circles stay for the predicted states. Chiral partners are marked by "⇔".
the end of previous subsection for mesons. In this respect the baryon case, however, is more involved as the baryonic excitations are not well separated even in the planar limit, where they emerge as solitons forming rather a continuum [126,127].

We explain briefly the $SU(2)_L \times SU(2)_R$ classification of baryons performed in [141]. A general irreducible representation is marked by two indices $(I_1, I_2)$. It transforms under parity into $(I_2, I_1)$. As long as QCD is invariant under parity reversal, one cannot ascribe any definite parity to the states in $(I_1, I_2)$ unless $I_1 = I_2$. Hence, if chiral symmetry is restored the multiplets must either be $(I, I)$ or $(I_1, I_2) \oplus (I_2, I_1)$. The latter is an irreducible representation of the parity-chiral group $^{[10]}$ which is isomorphic to $O(4)$. As baryons in two flavor QCD have half integral isospin, they cannot be in $(I, I)$ (since the isospin $I$ of states in $(I_1, I_2)$ is $|I_1 - I_2| \leq I \leq I_1 + I_2$). Thus, if chiral symmetry is effectively restored, the baryons fall into $(I_1, I_2) \oplus (I_2, I_1)$ parity-chiral multiplets with $I_1$ integral and $I_2$ half integral. The simplest representations are: i) $(1/2, 0) \oplus (0, 1/2)$ (nucleons), ii) $(3/2, 0) \oplus (0, 3/2)$ (deltas), iii) $(1, 1/2) \oplus (1/2, 1)$ (the nucleons and deltas in one multiplet — the mirror assignment). Consider baryons with a fixed spin. If the cases i) and ii) are realized then the mass of parity doublets in nucleon spectrum $a$ priori do not coincide with that of in delta one because these doublets belong to different representations. In case iii) the masses of nucleon and delta parity doublets coincide. The phenomenological analysis favored the quartet realization iii) in the experimental spectrum.

The upshot is that the effective CSR is not simply equivalent to parity doubling (such an equivalence, however, takes place for the effective axial $U(1)_A$ symmetry restoration), it manifests itself through the existence of complicated multiplet structure. Originally this point was accentuated by Jaffe [142].

Subsequently Glozman et al. elaborated the ideology of effective restora-

\footnote{Such a group-theoretical machinery is familiar from the representation theory for the groups where representations are formed in terms of the sum of two objects, and a parity reversal changes the sign of one, but not the other. A classical example is the Lorentz group, whose representations are formed by the sum of rotation generators (pseudovector) and boost generators (vector). An irreducible representation $(j_1,j_2)$ of the restricted Lorentz group (i.e. without time and parity reversal) is carried into the $(j_2,j_1)$ representation by a parity reversal. Thus it is not a representation any more for the full Lorentz group, unless $j_1 = j_2$. The irreducible representations of the latter are given by $(j_1,j_2) \oplus (j_2,j_1)$. As a consequence, the relativistic field Lagrangians respecting parity should not contain the left- and right-handed Weyl spinors because they are Lorentz $(1/2,0)$ and $(0,1/2)$, one must use Dirac spinors $(1/2,0) \oplus (0,1/2)$, the vector potentials carry $(1/2,1/2)$, while the field tensors transform under $(1,0) \oplus (0,1)$, the $(1,1/2) \oplus (1/2,1)$ is the Rarita-Schwinger field representation, etc.}
tion of chiral and axial symmetries, having already written about two tens papers on the subject. A comprehensive review of this line of research is contained in [2], we refer to this review for a detailed reading.

We will discuss only some aspects described in [2]. The parity-chiral classification was applied to mesons, the idea was partly borrowed from Cohen and Ji work [143], where possible chiral representations for hadron interpolating currents were systematically classified. This allowed to explain the duplication of some trajectories in Fig. 6 without use of nonrelativistic notions. For instance, the reason for duplication of $\rho$-meson trajectory is that there are two kinds of $\rho$-meson falling into different representations of parity-chiral group $^{12}$. The $\rho$-mesons of the first kind transform as $(1,0) \oplus (0,1)$, their chiral partners are the $a_1$-mesons. The interpolating currents are $\bar{q}\gamma_\mu q$ and $\bar{q}\gamma_\mu \gamma_5 q$ respectively. The $\rho$-mesons of the second kind carry $(1/2, 1/2)$ representation, their chiral partners are the $h_1$-mesons which are isosinglets. The interpolating currents are $\bar{q}\sigma_{\alpha\beta} q$ (or $\bar{q}\partial_\mu q$) and $\bar{q}\varepsilon_{\alpha\beta\gamma\delta} \sigma_{\gamma\delta} q$ (or $\bar{q}\gamma_5 \partial_\mu q$) respectively. Thus, the underlying reason for duplication of vector mesons is that they have two different interpolating currents belonging to different representations of parity-chiral group. The naive interpretation of CSR as simply parity doubling would lead to inconsistency: Experimentally there exist about two times more excited vector states than the axial-vector ones. At low energies the multiplets are mixed and $\rho(770)$ meson is roughly an equal mixture of both representations. If the chiral symmetry is restored at high energies this mixing disappears. Then the combined amount of highly excited $a_1$ and $h_1$ mesons must coincide with the amount of $\rho$-meson excitations. This situation recurs for higher spins. The overall scheme enjoined a phenomenological success in description of the Crystal Barrel data on unflavored mesons $^{[77, 78]}$.

The justification for the effective restoration of chiral and axial symmetries of two flavor QCD Lagrangian in the spectrum of highly excited states relies heavily on quasiclassical considerations. It was advocated that quantum fluctuations are suppressed at energies large enough, hence, the highly excited systems necessarily become quasiclassical. As a consequence, the classical symmetries of QCD should be effectively restored. Various exam-

$^{11}$The first who suggested the parity-chiral assignment to known and predicted mesons (within more general $SU(3)_L \times SU(3)_R$ chiral group) seems to be Arnold $^{[71]}$. This proposal was essentially based on exchange degeneracy introduced in the same paper. The problem was posed to relate dynamics to the suggested symmetry and in this way to derive the pattern of mass splitting inside multiplets. The general solution of this problem remains to be a dream up to now.

$^{12}$The existence of vector mesons differing by chiral properties has been proposed earlier, we refer to $^{[144]}$ for relevant discussions.
ples are cited in [2] showing that in high excitations the low-energy effects become inessential and the broken symmetries are restored high in the spectrum. As a matter of fact, in the quantum theory this property is well known — properties of any quantum system approach to its classical ones while the quantum numbers defining the stationary states of this system are large enough (see, e.g., a standard textbook [145]). For instance, the wave function corresponding to the \( n \)-th radial excitation has approximately \( n \) nodes in the coordinate space, the distance between two neighboring nodes is of order of de Broglie wavelength, hence, at large \( n \) one inevitably obtains a quasiclassics. The assumption is that one has something similar in QCD in the resonance region.

The semiclassical expansion suggests that the higher Fock components in the quark wave functions are suppressed for high radial and orbital excitations. In this situation, the string picture with massless relativistic quark and antiquark at the ends seems to be an admissible approximation for non-strange mesons. An idea was put forward that CSR then follows if relativistic quark and antiquark at the ends have definite chiralities. In this picture any degenerate chiral pair belongs to the same intrinsic quantum state of the string, the opposite parity of states in the pair results from different chiral configurations of the quarks at the ends. It was noted also that the string picture should lead to a much higher degeneracy than just parity doubling. This issue will be discussed in Section 7.

In the recent review by Jaffe et al. [1], three candidates for possible origin of parity doubling among non-strange baryons have been finally selected. The first possibility is a dynamical suppression of the violation of flavor singlet axial symmetry of QCD. This scenario is substantially different from \( U(1)_A \) restoration (i.e. an effective Wigner-Weyl realization of axial symmetry in highly excited baryons) and can be realized if the matrix elements \( \tilde{G}^{\mu\nu}G_{\mu\nu} \) \((G_{\mu\nu} \text{ is the gluon field tensor})\) taken between baryons of opposite parity are very small for some reasons.

The second possibility is that parity doubling in the baryon spectrum might be related to the internal geometrical structure of baryons (”deformed shapes”) by analogy with similar phenomena in nuclear and molecular physics. We have already discussed such possibilities in relation with Iachello’s [82] and Balachandran and Vaidya’s [116–118] models. The underlying observation is that if an intrinsic state (whose collective quantization describes a system in question) spontaneously violates parity and the deformation of this state is relatively rigid, then the low-lying excitations of the system will display parity doubling. The term ”rigid” means that the Hamiltonian matrix element between the intrinsic state and its parity image is small. In case of baryons this intrinsic state might be an elongated quark-diquark structure.
which violates reflection symmetry. If tunneling of a "mobile" quark from one end to the other were effectively suppressed, then parity doubling would result. Khokhlachev’s model [87] discussed above is nothing but a particular realization of this idea.

The third possibility consists in accommodation of parity doubling within models which do not naturally lead to this phenomenon. Weinberg’s ”mended symmetry approach” [62, 146] was considered as an example. The underlying hypothesis of this picture is that chiral symmetry may be realized on the matrix of the couplings of the Goldstone bosons rather than on the mass eigenstates. The mass matrix $\hat{m}^2$ at any given helicity may be then written as the sum of a chiral scalar $\hat{m}_0^2$ and the fourth component of a chiral four-vector $\hat{m}_4^2$ with respect to $SU(2)_L \times SU(2)_R$ formed by the isospin and the axial vector coupling matrix. The term $\hat{m}_4^2$ appears due to existence of vacuum Regge trajectories and destroys the algebraic chiral symmetry for the eigenstates. To obtain a model of hadron one should choose a reducible representation of this $SU(2)_L \times SU(2)_R$ and mixing angles. Parity doubling emerges for some particular choices of representation contents and corresponding mixing angles.

Klempt [147] proposed that the appearance of parity doublets in light baryons does not reflect the chiral symmetry but rather the vanishing of spin-orbit forces ($\vec{l} \cdot \vec{s}$) in quark interactions; the chiral symmetry itself might lead only to a weak attraction between chiral partners. The idea was somewhat inspired by Feynman et al’s. analysis [115] of baryon mass regularities mentioned above. In comparison with the hypothesis of CSR, this interpretation gives different predictions for the spectrum of highly excited baryon resonances, grouping them into $(l, s)$ multiplets. Baryons emerge as approximately mass degenerate clusters where both parity doublets and parity singlets can live. In addition, baryons can be assigned to $(l, s)$ multiplets so that the linear mass formula for baryons holds [148]

$$m^2 = m_{0,k}^2 + a(l + n_r),$$

where the intercept $m_{0,k}^2$ depends on the type of baryons (labelled by $k$), the slope $a$ is the slope of principal meson Regge trajectories (compare to Eq. (14)), and $n_r$ is the "radial" quantum number. Taking into account the nonrelativistic definition of parity for baryons, $P = (-1)^l$, Eq. (15) yields a pattern of parity doubling among baryons.

Finally, we mention a recent new scheme for parity doubling among light mesons based on the MacDowell symmetry [149]. The idea is that if a kind of dynamical meson-baryon supersymmetry exists indeed, then the MacDowell symmetry among the baryons should have an imprint on the meson spectrum.
The phenomenological analysis carried out in [149] using the Crystal Barrel data [78] showed that this proposal looks really plausible, at least formally.

5.2 Parity doubling in effective quark models

Under ”effective quark models” we mean the following. Let us imagine that we have ”integrated” over all gluons and over other degrees of freedom above some energy scale in the QCD Lagrangian. The obtained Lagrangian should then describe the strong interactions below the chosen scale, with the result of ”integration” being encoded in types of residual interactions and in values of coupling constants (in some models the ”integration” over all fermion degrees of freedom is also assumed). As long as nobody knows how to perform this analytically, one often resorts to models in studying the low-energy dynamics of strong interactions. Any such model puts forward an effective Lagrangian (or Hamiltonian) as an interpolating Lagrangian for the ”genuine” effective Lagrangian of QCD. The better the relevant effective degrees of freedom are guessed, the better is the effective model.

In most cases parity doubling and CSR within effective quark models have been studied within various extensions of the Nambu–Jona-Lasinio (NJL) model [150, 151]. The NJL model approximates the low-energy strong interaction dynamics by a local four-fermion interaction, the corresponding Lagrangian density can be written as

\[ L_{\text{NJL}}(x) = \bar{q}(x)(i\gamma - m)q(x) + g_1 j_1(x)j_1(x), \quad j_i(x) = \bar{q}(x)\Gamma_i q(x). \] (16)

where \( \Gamma_i \) is a Lorentz and isospin structure fixing the quantum numbers of interpolating current \( j_i(x) \) (the scalar one in the original NJL) and the summation over \( i \) is assumed. To specify a model completely one needs to fix a calculation method and momentum cutoff (of the order of the chiral symmetry breaking scale, 1 GeV, in conventional NJL). Originally the idea came from solid state physics where such Lagrangians are used for phenomenological description of superconductivity\(^\text{13}\). In particle physics it was first applied to nucleons and later, after advent of QCD, to quarks. The Lagrangian (16) can be easily made chiral-invariant, e.g. for \( j_1(x) = i\bar{q}(x)q(x) \), \( j_2(x) = \bar{q}(x)\vec{\tau}\gamma_5 q(x) \), and \( g_1 = g_2 \) one has a chiral-invariant scalar-pseudoscalar effective model. The scalar four-fermion interaction dynamically breaks the chiral invariance if the value of coupling constant \( g_1 \) exceeds some critical value — the corresponding mass-gap equation reveals a non-zero solution for the vacuum average \( \langle \bar{q}q \rangle \). The overall effective theory has a substantially right low-energy

\(^{13}\)The BCS theory is meant. The situation with superconductivity is similar — nobody knows how to derive the BCS Lagrangian from the underlying Quantum Electrodynamics.
physics as practically all important relations of current algebra can be naturally reproduced within the NJL model. This model describes commonly the ground boson states, although sometimes it is used for baryons as well.

The number of proposed extensions for the NJL model is so large that even all extensive reviews on this model taken together reflect only a small fraction of related researches. Loosely speaking, we would classify the proposed extensions by means of (or mixing of) four directions. The first way is to incorporate the quark currents with new quantum numbers with the aim to describe the hadrons possessing these new quantum numbers. The second direction is to include higher-dimensional vertices. For instance, to take into account the $U(1)_A$ symmetry breaking one needs the six-fermion 't Hooft term. These two extensions, however, describe generically the ground states only. As long as parity doubling is expected in high excitations, one needs something qualitatively different. The third and fourth types of extensions have been developed for accommodation of higher excitations.

The third possibility is to consider nonlocal interactions,

$$L_{\text{int}}(x) \sim g_i \int V_i(x - y) j_i(x) j_i(y) d^4y.$$ \hspace{1cm} (17)

The functions $V_i(x - y)$ are called formfactors or potentials (the latter name is inherent in the Hamiltonian formulation when one solves the Bethe-Salpeter equation for bound states, the term "kernel" is also used). The choice of these functions specifies a model (see, e.g., \cite{152, 153}). To our knowledge, the first successful application of such an extension to the parity doubling problem was proposed by Le Yaouanc et al. \cite{154} in the mid 1980s, i.e. long before any experimental evidence. Solving the corresponding Bethe-Salpeter equation with a confining Lorentz-vector potential, it was observed that the splitting between the parity doublets decreases as one goes to high masses, large compared to the scale of spontaneous chiral symmetry breaking. After appearance of the Crystal Barrel data \cite{78} we can say that the meson spectrum obtained in \cite{154} is in a bad agreement with the experiment quantitatively, but in a good agreement qualitatively: Fixing a mass scale large enough, the number of states in the $J^{PC}$ channel $J^--$ is a sum of $J^{++}$ and $J^{-+}$ states ($J > 0$, $J$ is odd; for $J$ even the situation is converse), i.e. the right duplication of trajectories was predicted. As one often says now, an effective CSR was observed. It is worth noting that only one kind of interpolating currents was exploited for each channel. Later various modifications of this model were proposed and a better agreement with the experimental data was achieved. We refer to the review \cite{2} and to \cite{155} for references and relevant discussions.
The fourth possibility consists in inclusion of derivatives into interaction vertices preserving the locality of interactions,

\[ L_{\text{int}}(x) \sim g_i j_i(x) F(\partial_x^2) j_i(x). \]  

(18)

where the formfactor \( F \) is some polynomial function. The function \( F \) can be chosen so that this extension also describes the "radially" excited states. This property was demonstrated by Andrianov et al. in [156–158]. The corresponding models are usually called quasilocal quark models. The issue of parity doubling and CSR was later elaborated in [130, 159–162]. Within these models, a clear signal of CSR can be easily demonstrated analytically for the first meson radial excitations.

Another lines of research have been undertaken for baryons. Löring et al. [163–167] developed a relativistic quark model for baryons on the basis of the three-particle Bethe-Salpeter equation. Parity doubling within this picture naturally arose as an instanton-induced effect.

Finally we mention an algebraic rather than effective model for baryons. Following some ideas from the spectroscopy of diatomic molecules, Kirchbach et al. [168] constructed a group-theoretical "rovibron" quark-diquark model describing the Rarita-Schwinger type of baryon clusters, which we have concerned in Section 4.1.

5.3 Parity doubling from QCD sum rules

We have already discussed the underlying idea of emergence of parity doubling within asymptotic sum rules in Section 4.2. On the one hand, the OPE dictates a rapid convergence of difference of two-point correlators for chiral partners at large space-like momenta. On the other hand, in the large-\( N_c \) limit, the meson excitations become narrow and asymptotic chiral symmetry imposes certain relations among masses and residues of chiral partners, they are often called sum rules. In 2000s this issue attracted much attention (see, e.g., [144, 169–179]). An explanation of parity doubling due to a strong suppression of direct instanton contributions to the two-point correlators at large space-like momenta was proposed in [180]. In this case the effect is interpreted as a partial restoration of \( U(1)_A \) symmetry.

Beane deduced in [169] that the joint constraints of quark-meson duality in the large-\( N_c \) approximation and chiral symmetry imply degeneracy of excited vector and axial-vector mesons. According to [169], asymptotically one has for the masses of parity partners of linearly rising spectrum

\[ m^2_+(n) - m^2_-(n) \xrightarrow{n \to \infty} 0. \]  

(19)
A natural question appears, what is the rate of asymptotic CSR? Later it became clear that the OPE by itself is hardly able to answer this question. It is an asymptotic expansion at large Euclidean momenta (it has zero radius of convergence), which does not contain enough information to provide an answer. One needs to invoke some additional assumptions besides the behavior of spectrum at large \( n \). Different assumptions resulted in different patterns of asymptotic rate for CSR. In [170–173] the exponential decreasing was inferred,

\[
m^2_+(n) - m^2_-(n) \sim e^{-bn}, \quad b > 0.
\]

The polynomial decreasing was obtained in [144],

\[
m^2_+(n) - m^2_-(n) \sim \frac{1}{n}.
\]

The constant behavior is also possible [176–179],

\[
m^2_+(n) - m^2_-(n) \sim \text{const}.
\]

It must be emphasized that even in the latter case one would see parity doubling as long as masses are growing, because

\[
m_+(n) - m_-(n) \sim \frac{\text{const}}{m_+(n) + m_-(n)} \quad \underset{n \to \infty}{\longrightarrow} 0.
\]

We will discuss this case in Section 6.

### 5.4 Parity doubling in heavy-light mesons

Parity doubling in heavy-light quark systems was expected from simple considerations. Following Bardeen and Hill [181], let us perform a gedanken experiment: What happens to the heavy-light meson spectrum if we could somehow restore the chiral symmetry, maintaining the other features of confining QCD? One can naturally expect that the heavy meson masses must remain unaffected in the first approximation as they arise primarily from the mass of the heavy constituent quark and the chiral mass gap is rather a perturbation. In this respect they are qualitatively different from the ground nucleons. This gedanken experiment suggests that the explicit chiral \( SU(2)_L \times SU(2)_R \) symmetry should somehow be realized in the heavy meson mass spectrum as long as the heavy constituent acts as a spectator in the chiral dynamics. Consequently, even the ground states should appear as approximate parity doublets. In [181] this situation was described by means of a generalization of the NJL model, where the chiral symmetric and chiral broken phases can be fine-tuned by an appropriate choice of coupling.
constant. As a result, the nearly degenerate chiral partners for the known $(0^-,1^-)$ heavy-light $D$-mesons (the corresponding $(0^+,1^+)$ multiplet) were theoretically predicted.

A bit earlier Nowak et al. [182] described the same phenomenon within a version of constituent chiral quark model of Manohar and Georgi [183]. The latter can be viewed as a bosonized version (i.e. when one formally introduces collective boson variables constructed from fermion ones) of the NJL model, which was shown in [181] and in some other papers. The mass difference between parity doublets has a particularly simple form in the given model,
\begin{equation}
m_\pm \simeq m_h \pm m_{\text{con}}, \quad m_{\text{con}} \ll m_h, \tag{24}
\end{equation}
where $m_h$ is the bare heavy quark mass and $m_{\text{con}}$ is the constituent (dynamical) quark mass. Thus, $m_+ - m_- \simeq 2m_{\text{con}}$ gives a simple pattern of mass splitting inside a chiral multiplet due to the chiral mass gap\cite{14}.

In ten years the predicted $(0^+,1^+)$ multiplet of heavy-light mesons was discovered experimentally. The two groups above wrote the reminder papers [185, 186]. The experimental mass splitting turned out to be even less, $m_+ - m_- \simeq m_{\text{con}}$ (for a generally accepted value $m_{\text{con}} \approx 300 \pm 50$ MeV), but the qualitative agreement is undoubtful.

Parity doubling among heavy-light mesons was also studied in [187] within a version of extended nonlocal NJL model with a confining instantaneous potential. The effective CSR was demonstrated analytically for the spectrum of orbitally excited states. In fact, a similar result was obtained in framework of a more early version of extended nonlocal NJL model by Bicudo et al. [188]. It was observed that the spectrum of heavy-light quarkonia becomes almost parity independent for high spin excitations. This result, however, was not attached a particular significance.

\footnote{In a formal language we would interpret this result as follows. Let $|L\rangle$ and $|R\rangle$ be degenerate left-handed and right-handed eigenstates of parity-invariant Hamiltonian operator $\hat{H}$. They are related by parity operator, $\hat{P}|L, R\rangle = |R, L\rangle$, which commutes with $\hat{H}$. Construct parity-even and parity-odd eigenstates of $\hat{H}$, $|\pm\rangle = 1/\sqrt{2}(|L\rangle \pm |R\rangle)$, which diagonalize the parity operator, $\hat{P}|\pm\rangle = \pm|\pm\rangle$. Parity invariance, however, does not imply equality of masses. A relevant example is the parity-invariant Hamiltonian (with $\hat{P}$ Hermitian) $\hat{H} = \hat{H} + \varepsilon \hat{P}$, where the second term is a perturbation to the Hamiltonian above [184]. This perturbation removes the degeneracy, $m_+ - m_- = 2\varepsilon$. Parity can be replaced by chirality, and the perturbation $\varepsilon$ mimics the term with $m_{\text{con}}$ in the Lagrangian of [182] differing by sign for chiral partners. Note in passing that the same example can serve for a formal demonstration of effective CSR. Let $\varepsilon$ be not small in comparison with energies of ground $|\pm\rangle$ states. Consider the $n$-th excitations, $\hat{H}|\pm\rangle^{(n)} = E_\pm^{(n)}|\pm\rangle^{(n)}$. If $E_\pm^{(n)} \gg \varepsilon$ the same logic works.}
5.5 Parity doubling among glueballs?

In the recent literature a couple of proposals has appeared that some observed parity doubled states are actually glueballs...

Faddeev et al. [184] put forward a geometrical mechanism for parity doubling of glueballs. It is widely accepted that glueballs are likely related to closed QCD strings, i.e. to closed toroidal fluxtubes of chromoelectric field. These objects can be emitted by a usual long linear string. Energy of string is proportional to its length, hence, a closed string should be unstable against shrinkage away by minimizing its length. On the other hand, within the purely gluonic part of QCD, the mass gap and color confinement should prevent such a shrinkage. This implies that there must be additional contributions to the energy of closed gluonic string, which is absent for an open string. It is natural to assume that the source of this stabilizing force is in the three-dimensional geometry of toroidal configuration. This configuration is prepared when one bends a finite length open string and joins its ends. But before joining the ends, the string can be twisted once around its core, the resulting topology may prevent from shrinking. The twist can be either a left-handed or a right-handed rotation around the core. Thus, degenerate left and right twisted configurations appear which are related by parity. Experimentally such states could be prepared from left-handed and right-handed polarized gluons. It was argued also that in pure Yang-Mills theory these states could emerge as solitons. The approximate mass degenerate $\eta_L(1410)$ and $f_0(1500)$ states were advocated to be the first $(0^+, 0^{++})$ glueball parity pair.

Kochelev and Min [180] applied the instanton mechanism of partial $U(1)_A$ symmetry restoration to the problem of low mass glueballs. Analysing the direct instanton contribution to the difference of two correlators of glueball currents with opposite parities, they proposed that the recently observed $X(1835)$ resonance is the lowest mass pseudoscalar glueball, which is parity doubled with the presumably lowest mass scalar glueball $f_0(1710)$.

Finally, we would mention that another kind of gluonic excitations has attracted attention recently, the gluelumps. At present, they do not reveal the parity doubling, nevertheless, these idealized gluonic constructions are not free from puzzles with the parity as well, namely they show an unusual ordering of the spin-parity energy levels (see, e.g., [189]).

5.6 Parity doubling and AdS/QCD

Nowadays a new fashionable approach to quantum field theory has flourished, the so-called AdS/CFT correspondence (Anti de Sitter/Conformal
Field Theory), which establishes a duality between string theories defined on the 5-dimensional AdS space-time and conformal field theories in physical space-time. The fact that QCD becomes nearly conformal field theory in the regime, where its effective coupling is approximately constant and the quark masses can be neglected, i.e., at high momentum transfer, inspired to apply the AdS/CFT correspondence to QCD, assuming that QCD can be approximated as a conformal theory even at relatively small momentum transfer, this conjecture is often referred to as AdS/QCD approach. Loosely speaking, one tries to derive the hadron spectrum and strong dynamics from a holographic dual string theory defined on five-dimensional AdS space, whose metrics is a function to be guessed. Such a ”bottom-up” approach is often regarded as a useful (not yet proven) semi-classical approximation to QCD, which incorporates both color confinement and conformal short-distance behaviour, see a recent review [190] for references.

The AdS/QCD models are believed to provide insights into non-perturbative aspects of strongly coupled QCD such as hadron spectra. It is natural thus to wonder if this approach may be useful for the subject in question. To the best of our knowledge, the only paper which directly addressed the parity doubling within AdS/QCD is [191]. The proposed model gives a certain pattern for the parity doubling among the unflavored baryons with different angular momenta, which implies a larger symmetry than the effective chiral symmetry restoration. In a sense, the holographic models put forward in [192–194] predict the parity doubling of hadrons as a particular case of the clustering of resonances expressed by relation \( (33) \), the subject of clustering will be considered in Section 7.

A problem of existing holographic models of QCD is that typically they lead to the spectrum (see, e.g., [144] for references)

\[
m^2_n \sim n^2, \quad (25)
\]

rather than to the linear one, \( m^2_n \sim n \) (the papers [192–194] are among the exceptions).

We would make a funny observation that exactly the pattern \( (25) \) was obtained in 1960s (as a particular case) by Barut et al. [57, 79, 80] within the dynamical group approach. This could be regarded as a mere coincidence if it were not a curious fact that both approaches are essentially based on the conformal group \( O(4,2) \). In Barut et al’s. analysis, this group is underlying dynamical symmetry, while in holographic duals, the conformal symmetry is fundamental — it is taken as a first approximation to the real-life QCD (where it is broken down by the conformal anomaly) as long as the gravity/gauge correspondence was originally established for conformal field theories. Is this a fortuitous coincidence?
Independently of answer, one might observe some symptomatic similarities between the impetuous group-theoretical activity of 1960s, which was eager to find a “genuine” spectrum-generating group for the hadron world, and the present AdS/QCD (more generally, AdS/CFT) one. In both cases one looks for a theoretical control over the strong interactions with the help of some “other” theory, and tries to find the fittest one (the search for underlying dynamical symmetry group and its physical representation in one case and the search for metrics of underlying AdS space and physical boundary conditions in its holographic variable in the other). In both cases the activity was inspired by a successful model example demonstrating a complete realization of proposed ideology (the hydrogen atom in one case and Maldacena’s example [195] in the other). In both cases in order to take into account new real-life features one usually needs more and more contrived descriptions. The dynamical group approach finally did not justify ambitious hopes, it bootstrapped itself into complexity (following the fate of bootstrap models in 1960s) and a large interest to this approach faded away...

We hope that such historical parallels are premature. Revelation of parity doubling mechanism is certainly a challenge for the AdS/QCD models.

6 Forms of parity doubling

Up to now, discussing the parity doubling we have skipped a delicate question: In what situation do we deal with a real parity doubling, or put it differently, at what mass splitting between states with opposite parity but equal spin may we say that they are parity partners? In this respect the situation with parity doubling in hadron spectrum needs a further specification as long as in the literature there is no unique criterium. The resonances have a width which usually grows for highly excited states and eventually the excitations become practically indistinguishable from the perturbative continuum. But, at least for mesons, in the limit of infinite number of colors [126,127], where the meson states are infinitely narrow, the problem looks as well defined. We consider the radially excited states, the case of orbitally excited ones is similar.

Denote the masses of the \( n \)-th radial excitations of parity partners as \( m_+(n) \) and \( m_-(n) \). Parity doubling, in fact, can be understood in different ways. We would propose the following classification of forms for parity
doubling in the hadron spectrum,

\[
\begin{align*}
\frac{m^2_+(n) - m^2_-(n)}{n \to \infty} &\to 0 & \text{superstrong;} \\
\frac{m^2_+(n) - m^2_-(n)}{n \to \infty} &\to \text{const} & \text{strong;} \\
\frac{m_+(n) - m_-(n)}{n \to \infty} &\to 0 & \text{moderate;} \\
\frac{m_+(n) - m_-(n)}{n \to \infty} &\to \text{const} & \text{weak;} \\
m_+(n) + m_-(n) &\to 0 & \text{superweak.}
\end{align*}
\] (26)

Each subsequent definition is less strong than the previous one. For instance, consider the mass spectrum where the main asymptotic in \(n\) is linear like in Eq. (14). The difference of masses squared can behave as:

\[
m^2_+(n) - m^2_-(n) \to \text{const} \times n^{1-\epsilon}, \quad \epsilon > 0.
\] (27)

Then one has according to classification (26): \(\epsilon > 1\) — superstrong form of parity doubling; \(\epsilon = 1\) — strong form; \(1/2 < \epsilon < 1\) — moderate form; \(\epsilon = 1/2\) — weak form; \(0 < \epsilon < 1/2\) — superweak form. In papers [144, 169, 170, 173] the authors arrived at the superstrong form of parity doubling (interpreted as CSR at high energies). The difference of results in these approaches was in the estimation of the rate of CSR. The criterion chosen in [179] coincides with the superstrong requirement if the spectrum for large \(n\) is linear. The spectrum of Ademollo-Veneziano-Weinberg dual amplitude [196] (a generalized Lovelace-Shapiro amplitude [197, 198]) reveals the strong form of parity doubling: The chiral partner trajectories have an equal slope but different intercepts. Similar results were obtained in [148, 176–179, 192] (see also a toy model [199] in the chiral broken phase). Another example of strong form is given by the two-dimensional QCD in a specific sequence of \(N_c \to \infty\) limits, \(m_q \to 0\) while \(m_q \gg g \sim 1/\sqrt{N_c}\) (\(m_q\) denotes current quark mass and \(g\) is coupling constant), the so-called ’t Hooft model [200–202] where the linear spectrum \(m^2_n \sim n\) alternates in parity as one increases \(n\) by one unit (this model, however, has a little to do with the dim4 QCD, see, e.g., [2, 203, 204]).

The same situation takes place in the models where \(n\) is replaced by the ”principal quantum number” of the kind \(n = l + 2n_r\) (see, e.g., [119, 205–210]) as long as the meson parity is \((-1)^{l+1}\). The choice of moderate form for parity doubling seems to be natural for the potential and other nonrelativistic models. Throughout the review on CSR [2] the criterion for effective CSR was adopted in the superweak form. A similar assumption was used also in [211].

Various nonlocal extensions of NJL model (see references in [2,155]) typically reveal the superweak form of parity doubling at low spins which gradually
converts into the superstrong one at high spins. If the slopes of chiral partner trajectories turn out to be different (as in the models [212–214] where \( \epsilon = 0 \)) one has no parity doubling in any sense.

Different ways of understanding of parity doubling can sometimes lead to confusing situations. For instance, the 't Hooft model was treated as an example of parity doubling in [215] and as counterexample in [179,216]. The reason is that the superweak form was meant in the former case and the superstrong one in the latter.

For the case of the weak and superweak forms the difference \( m_+(n) - m_-(n) \) does not converge at all, just it becomes negligible in comparison with values of masses. This kind of parity doubling can be called "effective". This is opposed to the "genuine" one which should be defined. In relativistic theories the latter could be the superstrong form of parity doubling for bosons. Indeed, if, say, a "genuine" restoration of chiral and axial symmetries occurs in a part of spectrum, the corresponding states forget completely about violations of these symmetries. But parity doubling in the strong form means that all states are equally influenced by chiral and axial symmetry breakings at low energies. In this case we observe parity doubling high in spectrum because chirally non-invariant contributions to masses remain constant while the masses are growing, i.e. low-energy effects equally persist at any energies but become relatively unimportant high in energy. In this sense the strong form is also an "effective" parity doubling. This automatically excludes the moderate form as a candidate for "genuine" parity doubling in relativistic theories.

Thus, if reply to the question "There is or there is no parity doubling in the hadron spectrum?" is positive, the next question is "What form of parity doubling is realized in nature?" Are we dealing with a "genuine" parity doubling or with simply "effective" one?

The Crystal Barrel data [78] allows to offer a preliminary answer for the unflavored mesons. Consider the principal \( \rho \)-meson Regge trajectory. It consists of \( IJ^{PC} \) states \( 1J^{--} \), \( J = 1,3,5... \). Consider the principal \( f_2^{-} \)-meson Regge trajectory. It consists of \( IJ^{PC} \) states \( 0J^{++} \), \( J = 2,4,6... \). Both trajectories are known to coalesce into one master trajectory (isospin + exchange degeneracy, see Section 2.3). Consider the daughter trajectories and the corresponding chiral partners (the \( a_J \) and \( \pi_J \) states respectively). The introduced states are better known experimentally in comparison with their isospin counterparts, so we confine ourselves by the states above. The known averaged masses squared are depicted in Fig. 7 in units of \( \rho(770) \)-meson mass squared. Master trajectory is known to be populated by the parity singlet states only. As to the differences of \( (\text{masses})^2 \) between chiral partners, there
Figure 7: The averaged $(m^2)_{\rho}$ of states on master trajectory and on daughter trajectories (filled circles) in units of $\rho$($770$)-meson (mass)$^2$ (see text). The chiral partners are denoted by open circles. The solid line is the master trajectory, the dashed lines mark the clustering positions. The following experimental $J^P$ states have been used [78,100] (the states discovered in the Crystal Barrel experiment are marked by star; if a duplication of states happens (like for some $\rho$- and $\rho_3$-mesons in Fig. 6) the most degenerate chiral partner is chosen). $0^+ (f_0$-mesons): 1350±150, 1770 ±12, 2020 ±38(*), 2337 ±14(*); $0^-(\pi$-mesons): 140, 1300 ±100, 1812 ±14, 2070 ±35(*), 2360 ±25(*); $1^-(\rho$-mesons): 776, 1459 ±11, 1720 ±20, 1900±?, 2265 ±40(*); $1^+(a_1$-mesons): 1230 ±40, 1647 ±22, 1930_{+30}^{−70}(*)}, 2270_{+55}^{−40}(*)}; $2^+(f_2$-mesons): 1275 ±1, 1638 ±6, 2001 ±10(*), 2240 ±15(*); $2^−(\pi_2$-mesons): 1672 ±3, 2005 ±15(*), 2245 ±60(*); $3^−(\rho_3$-mesons): 1689 ±2, 1982 ±14(*), 2260 ±20(*); $3^+(a_3$-mesons): 2031 ±12(*), 2275 ±35(*); $4^+(f_4$-mesons): 2018 ±6, 2283 ±17; $4^−(\pi_4$-mesons): 2250 ±15(*); $5^−(\rho_5$-mesons): 2300 ±45.
is a certain tendency to converge systematically high in the spectrum. This convergence means that nature seems to prefer the superstrong form of parity doubling among light mesons. In addition, the resonances apparently cluster near approximately equidistant values of masses squared (approximately near \(1.3^2, 1.7^2, 2^2, \) and \(2.3^2 \text{ GeV}^2\)), this issue will be discussed in the next Section.

Let us believe that the superstrong form is indeed realized. The next qualifying question is then "What is the actual rate of parity doubling?" We have already concerned this issue in Section 5.3 from the theoretical side. But what about experiment? As a rough estimate, we can propose an averaged splitting between (masses)\(^2\) of chiral partners within each cluster. Define the averaged splitting as

\[
\Delta_i = \frac{1}{N_i} \sum_{k=1}^{N_i} \left| m_{ik,+}^2 - m_{ik,-}^2 \right|, \tag{28}
\]

where \(N_i\) is the number of chiral pairs inside the \(i\)-th cluster (in fact, they are related by \(N_i = i\) for \(i > 1\), see Fig. 7), \(i\) enumerates the clusters in increasing energy, \(i = 1, 2, 3, 4, 5\) (\(i = 1\) corresponds to the lowest cluster where there is no chiral pair), and \(m_{ik,\pm}\) stays for the mass of \(\pm\) state in the \(k\)-th chiral pair of the \(i\)-th cluster. The mass of \(\rho(770)\)-meson can be regarded as QCD mass gap. Thus, the quantities \(\Delta_i\) "measure" averaged chiral symmetry breaking effects at different energy scales in the units of QCD mass gap. We will regard also the lowest \(\pi\) and \(\rho\) mesons as "would be nonlinear" chiral pair. The deviation of quantity \(\Delta_1\) from unity "measures" then an explicit chiral symmetry breaking in the QCD Lagrangian (of the order of \(m_{\text{current}}/m_{\text{constituent}}\)).

The results of our "measurements" for the states in Fig. 7 are

\[
\Delta_1 \approx 0.97, \quad \Delta_2 \approx 0.62, \quad \Delta_3 \approx 0.28, \quad \Delta_4 \approx 0.22, \quad \Delta_5 \approx 0.12. \tag{29}
\]

The series of numbers \(\{0.97, 0.62, 0.28, 0.22, 0.12\}\) seems to be convergent. Keeping in mind the discussions in Section 5.3, one could ask a question of the kind "What continuous function does interpolate this series in the best possible way?" We think that any answer to such a question is hardly able to help to the parity doubling problem from the theoretical point of view.

A more constructive question may be the following. If the superstrong form is indeed realized, the mass splitting within chiral pairs can rapidly become less than the typical experimental errors in determination of masses. Suppose that parity doubling occurs due to internal symmetries (chiral and axial). The experimental situation is then indistinguishable from the explicit Wigner-Weyl realization of chiral symmetry (provided that pions are decoupled). This means a complete restoration of chiral (and axial) symmetries.
of the classical QCD Lagrangian in the hadron spectrum. Thus, what is a scale of this restoration? We are aware of three such estimations in the literature: from a kind of nonlocal extension of the NJL model [187], from a phenomenological analysis [217] (the position of "would be" the next cluster in Fig. 7), and from analysis of decreasing of the constituent quark mass in response to increasing of momentum [218]. These three very different approaches remarkably converged to one number: 2.5 GeV. Unfortunately, we do not know whether the unflavored meson resonances persist systematically at such an energy scale. It may be that the perturbative QCD continuum sets in, and CSR becomes trivial. The Crystal Barrel experiment measured the relevant resonances up to 2.4 GeV only. One certainly needs new experiments devoted to the search for new meson resonances, which cover the range at least up to 2.6 GeV. These extra 200 MeV would be decisive in checking various proposals about the global features of meson spectrum.

Let us summarize the distinctive global features of unflavored meson spectrum which attracted attention recently:
1) The systematic appearance of parity doublets (presumably everywhere except pion and the states belonging to the master trajectory).
2) The systematic appearance of parity singlets — they occupy completely the master trajectory.
3) The states with different spins cluster near certain values of energy.

We propose to compare Fig. 7 with Fig. 1 (a hydrogen-like assignment of hadron levels within the dynamical $O(4,2)$ conformal group). The same pattern of spectral degeneracies was suggested for baryons in the preQCD time!

Let us pass on to baryons. The mass of ground nucleon is known to be mostly induced by the chiral symmetry breaking [219, 220]. Consider the quantities $\Delta_i$ defined as in Eq. (28) with the replacement

$$m_{\rho(770)} \to m_{N(939)}. \quad (30)$$

Thus, we will "measure" parity doubling in the units of chiral mass gap in QCD. The first cluster will be simply the ground nucleon state. As fermion masses enter relativistic equations linearly, it may be useful to consider also an analogous to $\Delta_i$ quantities for linear masses,

$$\delta_i = \frac{1}{N_i} \sum_{k=1}^{N_i} \frac{|m_{ik,+} - m_{ik,-}|}{m_{N(939)}}. \quad (31)$$

A convergence of $\delta_i$ signals that at least the weak form of parity doubling is realized (or the moderate one for the convergence to zero). We will also
Table 1: The values of quantities $\Delta_i$, $\delta_i$, and $\chi_i$ (see text) for the experimental spectrum of nucleons and deltas [100].

| $\Delta_i$ | $\delta_i$ | $\chi_i$ |
|------------|------------|----------|
| $i$ | $N$ | $\Delta$ | $N$ | $\Delta$ | $N$ | $\Delta$ |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0.32 | 0.41 | 0.10 | 0.11 | 0.032 | 0.032 |
| 3 | 0.10 | 0.15 | 0.03 | 0.04 | 0.008 | 0.009 |
| 4 | 0.58 | 0.76 | 0.13 | 0.15 | 0.030 | 0.031 |

check the superweak form of parity doubling in the non-strange baryons with the help of the quantities

$$\chi_i = \frac{1}{N_i} \sum_{k=1}^{N_i} \frac{|m_{ik,+} - m_{ik,-}|}{m_{ik,+} + m_{ik,-}}.$$  \hspace{1cm} (32)

The mass values for parity partners are contained in the Particle Data [100] (see also [2]). The results of our "measurements" are summarized in Table 1.

It can be readily seen that without account for the highest cluster one likely has the superstrong form of parity doubling. However, the last cluster spoils this nice picture — there is a vague hope for the superweak form, at best. Qualitatively the deterioration of parity doubling in the highest known non-strange baryons can be foreseen from a glance at Fig. 3 and Fig. 4. On the other hand, these figures show visually how poor is the experimental determination of masses in the highest clusters. The spectral regularities cease to be clear-cut. Thus, one is in great need of high precision experiments both for determination of masses of unflavored baryons and for search for new states.

### 7 The latest idea: Broader degeneracy?

We have already noted several times that the non-strange hadrons tend to cluster into fairly narrow mass range (see discussions on meson "towers" in Section 2.3, on H"ohler’s clusters in Section 4.1, and on qualitative features of meson spectrum in relation to Fig. 7 in Section 6). Clustering of unflavored mesons was clearly observed in Crystal Barrel experiment (see Fig. 4 in [78]). A preliminary picture for non-strange meson spectrum, as we know it now, is displayed in Fig. 8.
Figure 8: The spectrum of non-strange mesons in units of $\rho(770)$-meson mass squared (plot from [221]). The data for the first three clusters is contained in Particle Data [100], while the data for the last two ones is mostly due to Crystal Barrel [78]. Experimental errors are indicated. Circles stay when errors are negligible. The dashed lines mark the mean (mass)$^2$ in each cluster. The open strips and circles denote either the states with the lowest star rating according to [78] or the states which are somewhat doubtful as candidates for the unflavored mesons. The arrows indicate the $J > 0$ mesons which have no chiral partners (the candidates for chiral singlets).
The clustering is a striking phenomenon which theoretically is even more enigmatic than parity doubling. Why do many states with different spins and other quantum numbers have close masses? In the last year this subject has been attracting much attention [2,138,155,217,221–224]. As a matter of fact, in some approaches parity doubling is a mere consequence of some kind of clustering (see, e.g., Kirchbach’s and Klempt’s models discussed above). If some force drives hadron masses to cluster into well separated narrow mass regions then parity doubling ensues automatically provided that opposite parity states exist systematically in the resonance region. In this case the mass splitting between parity (chiral) partners is of the order of “width” of narrow mass region occupied by given cluster. A surprising observation is that exactly this picture seems to be the case experimentally.

The present experimental data shows three qualitative distinctions between the baryon clusters in Fig. 3, Fig. 4 and the meson clusters in Fig. 8 (or its abridged form, Fig. 7). First, the meson clusters are approximately equidistant, while this is not the case for the baryon ones. Second, the baryon clustering becomes much worse high in energy, while the meson one is progressively improving up to the highest available resonance region. Third, all meson states belonging to the lowest principal Regge trajectories are parity singlets (they are indicated by arrows in Fig. 8), the baryon sector has several exceptions from this rule.

If the linear parametrization of meson spectrum (14) is approximately valid with the universal slope $a$, then clustering in the meson sector is essentially equivalent to an approximate universality of the intercept $m_0^2$. What is the averaged value of $m_0^2$ in the units of $a$? This question was addressed in [221]. Averaging over all states in Fig. 8, the value is $m_0^2/a \approx 1/2$, like in the spectrum of Lovelace-Shapiro dual amplitude [197,198]. More precisely, the following averaged spectrum was obtained for the states in Fig. 8

$$m^2(l, n_r) = a(l + n_r + c).$$

with $a \approx 1.1$ GeV$^2$ and $c \approx 0.6$. Compare the classical hydrogen spectrum, Eq. (5), with the approximate meson spectrum, Eq. (33) (see also Klempt’s formula, Eq. (15)). The outcome is that a broad degeneracy emerges due to the existence of single ”principal” quantum number

$$n = l + n_r + \text{const}. \quad (34)$$

The validity of such nonrelativistic relations presupposes a smallness of spin-orbital interactions. Recalling the nonrelativistic definition of parity for quark-antiquark pair, $P = (-1)^{l+1}$, relation (33) immediately reproduces the absence of parity doubling for the leading meson trajectories: The corresponding states have $n_r = 0$ and the minimal value of $l$ at given spin, while
the closest parity partners have $l$ larger by one unit (hence, according to Eq. (33), they are degenerate with the first "radial" excitations of the states lying on the principal trajectories). This reasoning does not work for baryons if Eq. (15) is valid — the intercept is not universal, thus, parity doubling on principal trajectories is not forbidden.

The clustering constitutes, perhaps, a problem of paramount importance for modern spectroscopy of non-strange hadrons. What is the underlying physics? The restoration of chiral and axial symmetries cannot lead to the multipin clustering since the corresponding transformations relate states with equal spin only. Many other approaches put forward for explanation of parity doubling have also problems with a natural accommodation of clustering. The hadron strings are somewhat encouraging (see, e.g., [225] where Eq. (33) was qualitatively derived), but a consistent relativistic theory of hadron strings predicts plenty of unobserved "hybrid" states, not to mention the unresolved problem with tachyons in four dimensions. It would be interesting to address the problems of parity doubling and clustering within the framework of presently fashionable AdS/QCD approach, for the time being only a few of holographic models reproduced a spectral pattern in light mesons [192–194], which is similar to the observed one, Eq. (33). The description of large degeneracy is a challenge for quark models, recently this problem was emphasized by Bicudo [155]. The most ambitious approach in this field is to find a quark model with a principal quantum number, at least in some approximate sense. The existence of effective principal quantum number in the spectrum is a strong argument in favor of the hydrogen like classification of unflavored mesons, which was put forward recently [149]. The underlying qualitative motivation for such a classification is that both the hydrogen atom and the mesons represent quantum two-body systems interacting via central forces, so it looks plausible that they could have some general dynamical symmetries.

The dependence of spectrum on one quantum number only, Eq. (34), may be regarded as a compact form to express the combined effect of suppression of spin-orbital and exchange forces. To explain the statement, consider a $J^-$ state on leading Regge trajectory $R_-$. If the spin-orbital interaction is small, the state is approximately degenerate with $(J-1)^-$, $(J-2)^-$, and so on, in the baryon case we referred to such a tower as Feynman’s cluster (see Section 4.1). The same can be repeated for a $J^{+}$ state on leading Regge trajectory $R_+$. However, due to exchange degeneracy (see Section 2.3), the trajectories $R_-$ and $R_+$ coincide. Consequently, the $(J-1)^-$ state will be degenerate with the $(J'-1)^+$ lying on the first daughter of $R_+$ (the first "radial" excitation). This chain can be continued, and as a result one obtains parity doublets for all daughter trajectories. The described mechanism produces the observed
There is a hope to obtain this mechanism from confinement in QCD [226]. If the confinement dynamics follows from the area law for large Wilson loops, then linearity and other properties of Regge trajectories can be derived under some assumptions (see, e.g., [227,228]). Along this line, Eq. (33) was derived in [229] in a model-dependent way for large angular momentum, in this limiting case the interquark separation is large and, hence, only the large area asymptotics for Wilson loops seems to be essential. Similar results were obtained also for baryons [230] assuming a large quark-diquark separation. The existence of single quantum number, Eq. (34), is expected due to hidden conformal invariance resulting from reparametrization invariance for Wilson loops [226].

Thus, at present there is no understanding whether the observed large degeneracy (and parity doubling as a particular case) is related with fundamental symmetries of QCD or with some dynamical symmetry. It may be that the separation itself between ”fundamental” and ”dynamical” symmetries is somewhat artificial. For instance, the chiral $U(n_f) \times U(n_f)$ invariance of QCD Lagrangian (in the chiral limit) is broken down to the diagonal $U(n_f)$ subgroup, which is a symmetry of spectrum. The ”remainder” is realized dynamically through massless Goldstone bosons, thus, giving rise to all low energy dynamics. In this sense the fundamental chiral symmetry is converted into the dynamical one [62]. Initially, QCD possesses a mass gap, but the dynamical realization of chiral symmetry removes a ban, and generates gapless Goldstone excitations. Higher in energy the fundamental chiral symmetry, likely, gradually restores and not far from the onset of complete restoration (the perturbative continuum) the hadron spectrum reveals the fundamental chiral symmetry through parity doubling. One could imagine the following rather unorthodox situation. The conformal $O(4,2)$ invariance of QCD Lagrangian (in the chiral limit) is broken down to the maximal compact subgroup $O(4)$, which is a symmetry of spectrum. The ”remainder” is somehow realized dynamically. Initially, the fundamental conformal symmetry prohibits any massive excitations — spectrum of conformal theories is massless or continuum — but a dynamical realization of conformal symmetry removes a ban, and massive states are allowed. Higher in energy the fundamental conformal symmetry, likely, gradually restores and not far from the onset of complete restoration (the perturbative continuum) the hadron spectrum reveals the fundamental conformal symmetry through clustering — the gradual restoration simply means a gradual cleaning from ”singled out” regions in the energy distribution, a liberation from enhancements of any kind, the resonances represent these very regions, and hadrons are thereby ”expelled” in the resonance region forming narrow clusters, like Abrikosov
vortices are formed in a type-II superconductor when the strength of magnetic field lies between the first and second critical values. These values would correspond to the scales of breaking (mass gap) and restoration of conformal symmetry in this analogy as at the first critical value the magnetic vortices appear inside a type-II superconductor while at the second one the superconductivity is destroyed. Unfortunately, all such suggestive analogies are highly speculative at present stage, so it is worth finishing them here.

Descending to down-to-earth discussions, it should be mentioned that the results of the Crystal Barrel experiment have not yet been confirmed convincingly, this uncertain situation gives rise to a rather widespread opinion that all related discussions, like parity doubling and clustering in mesons, do not have a solid ground — it is not excluded that the systematic character of the observed effects is absent at all. Such a point of view may be correct when discussing some separate unconfirmed states, but it is hardly correct when discussing clustering of states. It seems to be timely to remind some forgotten experiments.

The clusters are observed as peaks in differential cross sections. A high-precision experiment is able to distinguish the saw-tooth structure of these peaks, which depends on a concrete reaction. The determination of separate resonances then follows. The point is that peaks like in Fig. 8 with close positions were observed many times in 1960s, but the instrumental resolution usually did not allow to distinguish separate resonances. Perhaps, the most prominent old experiment devoted to the search for missing resonances is the one performed with the use of the CERN missing-mass spectrometer in the mid 1960s [231]. The charged non-strange bosons $X^-$ were produced in the reaction $\pi^- + p \rightarrow p + X^-$. Mass spectra were obtained by measuring the missing-mass of the recoil protons in the range 0.5 - 2.5 GeV. Apart from the known at that time peaks (in MeV) $\rho(768 \pm 5)$ and $a_2(1286 \pm 8)$, the following peaks were observed: $R(1691 \pm 30)$ (consisting of three separate peaks), $S(1929 \pm 14)$, $T(2195 \pm 15)$, and $U(2382 \pm 24)$. We have already mentioned the mass regions $R$, $S$, $T$, and $U$ in Section 2.3 in relation to another experiments (see references in [70]). A remarkable observation was that the masses square $M^2_X$ of the $\rho$, $a_2$, $R$, $S$, $T$, and $U$ regions lie on a straight line! The slope turned out to be 1.05 GeV$^2$, this value was very close to the baryon slope 1.04 GeV$^2$ known at that time (such observations resulted in the idea of dynamical supersymmetry between baryons and mesons [89]). Even more forgotten is the fact that this experiment was continued with the CERN boson spectrometer exploiting the same reaction. At the first stage, the mass region 2.5 - 3.0 GeV was measured. Two peaks were observed, at 2.62 and 2.80 (with a close peak at 2.88) GeV [232]. The corresponding masses square happened to lie on the extrapolated $\rho - a_2 - R - S - T - U$
linear trajectory! At the second stage, the mass region 3.0 - 3.8 GeV was investigated. The spectrum showed four prominent peaks at 3.025 (with a close peak at 3.075), 3.145 (with a close peak at 3.180), 3.475, and 3.535 GeV [233]. A less significant peak at 3.605 GeV was detected as well. The compiled spectrum is displayed in Fig. 9. Some of the discovered peaks were observed also by other experiments. For instance, near 3.01 GeV in an inelastic \( \pi p \) reaction [234], near 3.03 and 3.4 GeV in a \( \bar{p}p \) annihilation [235], Particle Data [100] cites (in section "Further States") a resonance structure of unknown quantum numbers near mass regions 2.38, 2.62, and 3.02 GeV, which was produced in various inelastic \( \pi p \) and \( \bar{p}p \) reactions.

The plot displayed in Fig. 9 suggests that the resonance region is not limited by the \( U \)-region, it extends at least up to 3.6 GeV. In addition, the linear behavior of objects in Fig. 9 hints at some Regge-like recurrence, possibly a recurrence of meson clusters. It would be highly desirable to recommence such experiments using the progress in resolution technics achieved for the last 40 years.

## 8 Conclusions

We have tried to trace the development of parity doubling ideas in particle physics since 1960s and up to the latest publications. The history of parity doubling is tightly interrelated with the history of strong interactions. The development of many approaches invented for description of strong interactions may be looked at from the point of view of parity doubling. For instance, the evolution of effective quark models has passed two stages. At
the first stage the quark models were not able to reproduce the systematic parity doubling. At the second stage, more refined models naturally accommodated the phenomenon. It seems that soon we will be witnesses of the third stage — the creation of models which describe the clustering of hadrons at certain energies, i.e., a broader degeneracy than parity doubling.

There is still no agreement whether parity doubling is a reflection of approximate classical symmetries of QCD or it is a dynamical effect emerging due to certain internal space structure of hadrons. Interpreting parity doubling as a manifestation of some fundamental symmetries, one encounters a bifurcation point — are they space-time or internal symmetries? A delicate problem of two-flavor sector is that these alternatives turn out to be somewhat dual from the group-theoretical point of view: Extending both the Lorentz and the chiral group by parity, one obtains a group isomorphic to $O(4)$, where the irreducible representations for parity eigenstates are given by $(j_1, j_2) \oplus (j_2, j_1)$ if $j_1 \neq j_2$. This circumstance hampers to reveal the genuine character of underlying symmetry.

We have demonstrated that experimentally parity doubling in non-strange baryons and mesons is not of the same type. May be this is an artefact of insufficient experimental data. The deficiency of reliable experimental information is a serious problem in modern spectroscopy of non-strange hadrons. It would be desirable if parity doubling in baryon and meson sector had the same origin. In this case one obtains a powerful selective principle: Any model for parity doubling which treats baryons and mesons differently is missing essential physics, hence, it has to be ruled out. The idea of approximate meson-baryon dynamical supersymmetry is suggestive in this respect. Unfortunately, one does not have enough arguments to postulate that principle.

To summarize, at present the issue of parity doubling has much more questions than ready answers and definite conclusions. In the review we have tried to convince in the extreme importance of experimental searches for missing states in the non-strange sector and confirmation of preliminarily known states. In this respect it should be added that presently the lattice simulations are not able to shed light on the spectrum of high excitations, hence, on parity doubling. It should be added also that a more precise specification of light hadron spectrum requires the rather modest amounts of money and resources. In particular, a rich experimental information is accumulated at Jefferson Lab (TJNAF), a careful analysis of this data could be invaluable for the spectroscopy of excited nucleons and deltas. The same can be said about the meson spectrum near 1.7 GeV, which could be finally established using the VES and E852 data. The future experiment of the PANDA Collaboration at GSI could refine and extend significantly the spectroscopic results

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of the Crystal Barrel one on unflavored mesons. The same task would be accomplished, at least partly, in a closer perspective if a new polarised target were used inside the old Crystal Barrel detector (or any other detector, e.g., the Babar detector after its present experiment ends). All this is quite realizable, a good will is needed only. We hope that future experiments will give the long-awaited whole and precise picture of light hadron spectrum, thus providing a key for ultimate explanation of the parity doubling phenomenon as well as other spectroscopic puzzles.

Acknowledgments

I would like to thank the participants of XLI PNPI Winter School on Nuclear and Particle Physics for stimulating discussions, especially A. B. Kaidalov for reading the manuscript and enlightening comments. The correspondence with D. V. Bugg and M. Kirchbach is gratefully acknowledged. The work was supported by RFBR, grant 05-02-17477, by the Ministry of Education of Russian Federation, grant RNP.2.1.1.1112, and by grant LSS-5538.2006.2.

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