Consequences of the H-Theorem from Nonlinear Fokker-Planck Equations

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It is well known that many real systems exhibit a dynamical behavior that falls out of the scope of the standard linear differential equations of physics. Although the linear Fokker-Planck equation is considered appropriate for the description of a wide variety of physical phenomena—typically those associated with normal diffusion—it is well accepted that this equation is not adequate for describing anomalous diffusion. In order to cope with such anomalous systems, modifications in the linear FPE have been carried out, and this subject has attracted the attention of many researchers recently. In the present work, a general type of nonlinear Fokker-Planck equation is derived directly from a master equation, by introducing generalized transition rates. The H-theorem is demonstrated for systems that follow those classes of nonlinear Fokker-Planck equations, in the presence of an external potential. For that, a relation involving terms of Fokker-Planck equations and general entropic forms is proposed. It is shown that, at equilibrium, this relation is equivalent to the maximum-entropy principle. Families of Fokker-Planck equations may be related to a single type of entropy, and so, the correspondence between well-known entropic forms and their associated Fokker-Planck equations is explored. It is shown that the Boltzmann-Gibbs entropy, apart from its connection with the standard—linear Fokker-Planck equation—may be also related to a family of nonlinear Fokker-Planck equations.