Deterministic polarization chaos from a laser diode

Martin Virte1,2, Krassimir Panajotov2,3, Hugo Thienpont2 and Marc Sciamanna1*

Fifty years after the invention of the laser diode, and forty years after the butterfly effect signified the unpredictability of deterministic chaos, it is commonly believed that a laser diode behaves like a damped nonlinear oscillator and cannot be driven into chaotic operation without additional forcing or parameter modulation. Here, we counter that belief and report the first example of a free-running laser diode generating chaos. The underlying physics comprises a nonlinear coupling between two elliptically polarized modes in a vertical-cavity surface-emitting laser. We identify chaos in experimental time series and show, theoretically, the bifurcations leading to single- and double-scroll attractors with characteristics similar to Lorenz chaos. The reported polarization chaos resembles noise-driven mode hopping, but shows opposite statistical properties. Our findings open up new research areas for the creation of controllable and integrated sources of optical chaos.

The discovery of deterministic chaos—the aperiodic deterministic dynamics of a nonlinear system demonstrating sensitivity to initial conditions—has led to a major paradigm shift, overthrowing two centuries in which the Laplacian viewpoint of dynamical systems was dominant1–5. The analysis of system behaviour using chaos theory has helped to interpret and control many ordered and disordered behaviours prevalent today, such as the bifurcations leading to epilepsy and cancer6, the stabilization of cardiac arrhythmias7, and the improvement of complex behavioural patterns in robotics8.

Soon after the invention of the laser, the possibility of observing chaos in a light signal became a topic of interest. In 1975, Haken discovered an analogy between the Maxwell–Bloch equations for lasers and the Lorenz equations showing chaos9. The three Maxwell–Bloch equations describe the field E, polarization P and carrier inversion N, each of which has its own relaxation time. However, although the relaxation times of the dynamical variables in the Lorenz equations are of a similar order of magnitude, they may take very different values in lasers. If one variable relaxes much faster than the others, this variable is adiabatically eliminated, hence resulting in a reduced number of dynamical equations. Accordingly, so-called class A (He–Ne, Ar and dye), class B (Nd:YAG, CO2 and semiconductor) and class C (NH3) lasers have dynamics governed by a single equation (for the field), two equations (for the field and population inversion) or a full set of equations, respectively. In class A or B laser systems, chaos can be observed only if one or several independent control parameters are added10. Chaos has then been reported in, for example, free-running NH3 lasers11, He–Ne lasers with modulation of the external field12, CO2 lasers with loss modulation13, solid-state lasers with gain modulation14, injected field15 or global multimode coupling16, and diode lasers with optical feedback17, saturable absorption18 or optical injection19.

In this Article, we demonstrate deterministic chaos in a free-running laser diode that does not require external forcing or modulation. Chaos is unambiguously identified in the light polarization output of a vertical-cavity surface-emitting laser (VCSEL). Moreover, both the route to chaos and the mode dwell time (the time the laser remains emitting in one mode) characteristics qualitatively agree very well with the long-standing and so far unconfirmed prediction of the spin-flip model20, where a nonlinear coupling between two polarization modes induced by carrier spin relaxation leads to deterministic nonlinear dynamics.

Today, applications of chaotic laser diodes, such as secure communications21–23 and random number generation24,25, make use of high-dimensional chaos derived from time-delayed optical feedback. However, the chaos demonstrates correlation at the time delay value and multiples26,27, making it easy to reconstruct the attractor in a low-dimensional phase space, therefore reducing both security and randomness. The polarization chaos reported here is low-dimension, but (i) is obtained from a free-running device (that is, without the additional complexity of optical feedback) and (ii) uncovers a complex polarization dynamics that may improve randomness28 and security29 and allows for chaos multiplexing at high speed30.

**Route to polarization chaos**

Figure 1 presents typical measured dynamics (Fig. 1b,c) of the polarization-resolved laser output power in a single-mode 990 nm quantum dot VCSEL (Fig. 1a), where the quantum dot active region was grown using a submonolayer technique (that is, without wetting layer and with lateral compositional modulation due to strain distribution when depositing several InAs submonolayers in a GaAs matrix31). The device characteristics, in particular its polarization and spectral properties as a function of the injection current, have been described elsewhere32. On increasing the current, the VCSEL switches from lasing with linearly polarized emission to lasing in one of two elliptically polarized modes. The main axes of the ellipses make an angle of 40°, so the two polarization modes are not orthogonal. Figure 1b presents the polarization-resolved output measured at +45° according to the linear eigenaxes (x- and y-axes of Fig. 1a). The laser exhibits a polarization dynamic that resembles a random-like hopping between two polarization modes. At first sight it looks very similar to the so-called stochastic polarization mode hopping that has been extensively reported in VCSELS showing current- or temperature-driven polarization bistability33. However, this polarization dynamics has many features that are strikingly different from stochastic mode hopping. First, as schematically plotted in Fig. 1a, the competing polarization modes are elliptically polarized and not orthogonal. Second, a spectral analysis of the polarization-resolved output unveils an underlying self-pulsating dynamics.

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1Supélec OPTEL Research Group, Laboratoire Matériaux Optiques, Photonique et Systèmes (LMOPS) EA-4423, 2 rue Edouard Belin, 57070 Metz, France.
2Brussels Photonics Team, Department of Applied Physics and Photonics (B-PHOT TONA), Vrije Universiteit Brussels, Pleinlaan 2, 1050 Brussels, Belgium.
3Institute of Solid-State Physics, 72 Tzarigradsko Chaussee Boulevard, 1784 Sofia, Bulgaria. *e-mail: marc.sciamanna@supelec.fr
cesses for right and left circular polarization with two coupled
amplitude anisotropy using the following parameters (notations are taken from ref. 35):

\[ g = 100 \text{ ns}^{-1}, \quad \kappa = 600 \text{ ns}^{-1}, \quad \gamma = 1 \text{ ns}^{-1} \]

and normalized injection current \( \mu \) between 1 and 10.

At threshold, the laser emits linearly polarized light, but when the injection current is increased, it experiences a complex bifurcation sequence, leading to chaos. This route to polarization chaos is explained in Fig. 2, where we display the evolution of the system trajectory in the \((s_3; s_2)\) plane for different current values, where \(s_i\) is the Stokes parameter. At threshold, the laser operates in a linearly polarized steady state. As the current is increased, this state is destabilized by a pitchfork bifurcation, creating two symmetric elliptically polarized states (displayed as black and red crosses in Fig. 2a). When the current is increased, the polarization rotates and the ellipticity of the output beam increases. The two elliptically polarized states are never orthogonal for any given injection current. With a further increase in the current, both elliptically polarized states become unstable and two limit cycles are created, oscillating around these unstable steady states, as shown in Fig. 2b. A cascade of period-doubling bifurcations then leads to two symmetric single-scroll chaotic attractors (Fig. 2c). The trajectories of both attractors are centred on the unstable elliptically polarized states, and the size of the attractors grows as the current increases. Therefore, beyond a critical value of the injection current, they merge into a single double-scroll attractor that is akin to the ‘butterfly’ attractor of Lorenz chaos (Fig. 2d). This chaotic behaviour is displayed in Fig. 2e,f, where we plot the polarization-resolved output powers at an injection current of \( \mu = 1.425 \) for different projections. All time traces are averaged over 0.5 ns to simulate a slower photodetector response. In Fig. 2e we plot the projections at 0 and 90°, that is, the polarization at threshold and the orthogonal one. We clearly observe a ‘noisy’ but non-switching operation of the laser with an emission in both polarizations; however, for different projections we find a completely different behaviour. Indeed, in Fig. 2f the output power for projections at 45° and 45° displays irregular, random-like switching between two different states, as observed in the experiment. Using Wolf’s algorithm\(^{34}\) we find that the simulated dynamics of Fig. 2 is characterized by one positive ‘averaged’ Lyapunov exponent, like in the experimental time series analysis of Fig. 1, hence confirming the chaos-induced unpredictability of the free-running laser polarization dynamics. As in other systems showing low dimensions\(^{36}\), we find, however, that ‘finite time (or local)’ Lyapunov exponents have fluctuating signs and may all become negative during short, typically less than nanosecond, time intervals. Short time predictability can be improved by adding time-delayed feedback, which leads to hyperchaos with many positive Lyapunov exponents, but without the advantage of the free-running laser device and requiring an optimal adjustment to achieve time delay concealment\(^{26}\). Numerically, we were also able to demonstrate that (i) a similar chaotic polarization hopping is observed in a large range of laser parameters and (ii) the self-pulsation from which chaos originates has a frequency that is either close to the birefringence-induced frequency splitting or to the relaxation oscillation frequency. The laser’s dynamics therefore shows pulsations at a typical frequency of several gigahertz.

Statistics of deterministic mode hopping
We further analyse the statistics of the dwell time (the time the laser stays in one polarization state) and how the averaged dwell time scales with the laser injection current. Considering the similarity between the chaotic polarization mode hopping and the Lorenz chaotic attractor, we also simulate the case of the Lorenz chaos equations, where the \( r \) parameter plays the same role as the injection current in a laser\(^{27}\). The results are displayed in Fig. 3a,b for the laser system and in Fig. 3c,d for Lorenz chaos. In Fig. 3a and c we present the evolution of the dwell time for increasing injection current and \( r \), respectively. The dwell time distributions are given in Fig. 3b and d for an injection current of \( \mu = 1.425 \) and \( r = 27 \).

Figure 1 | Experimental observations showing chaotic polarization mode hopping in a quantum-dot VCSEL. a. Artist’s view of a quantum dot VCSEL with multiple quantum dots confined between two mirrors. At threshold, the laser emits a linearly polarized light. Driven by constant injection current, the device can exhibit chaotic mode hopping between two non-orthogonal elliptically polarized modes (red and blue ellipses) separated by \( 40° \). b,c. Polarization-resolved output power–time trace showing chaotic polarization mode hopping at constant injection currents of 2.0 and 2.6 mA, respectively. d. Estimation of largest Lyapunov exponent for the time series in c.
Figure 2 | Route to chaos in the SFM framework. Bifurcation sequence in Stokes parameter (s3; s2) plane for increasing injection current $\mu$. a, For $\mu = 1.34$, there are two symmetric non-orthogonal elliptically polarized states. b, For $\mu = 1.39$, two limit cycles oscillate around unstable elliptically polarized states. c, For $\mu = 1.41$, cascade of period doubling bifurcations leads to single-scroll chaotic attractors. d, For $\mu = 1.425$, the two attractors merge into a single double-scroll attractor. e, f, Polarization-resolved output power at $\mu = 1.425$: projections at 0° (black) and 90° (red) (e) and projections at 45° (black) and −45° (red) (f).

Figure 3a shows a significant decrease in the dwell time with current, from microseconds to nanoseconds. The dwell time statistical distribution follows an exponential decay law, as shown in Fig. 3b. These two statistical features agree qualitatively well with the observations in our experiment32. It is worth mentioning that the exponentially decaying statistical distribution of the dwell time is the result of polarization nonlinear deterministic dynamics and, in contrast to all previous studies, is not the result of a noise-driven Kramers hopping problem. In the latter case, polarization mode hopping between linearly polarized VCSEL modes displays the completely opposite behaviour, with the dwell time increasing exponentially with injection current33.

Kramer’s approach therefore fails to characterize the dwell time statistics of Fig. 3. Because the time series shows a random-like switching between two states, it is of interest to check whether the dynamics can be modelled, alternatively, as a hidden Markov process39,40—in other words, to analyse whether the jumps between states are influenced by hidden additional variables or any external perturbation. Such modelling would make it possible to identify the most probable noise-induced transitions. To achieve this, we applied the Baum and Welch algorithm to retrieve two matrices: the transition Markov matrix ($M$) and the hidden-related transition matrix ($N$). In the case of the experimental time series (Fig. 1c), we obtain off-diagonal elements of $N$ very close to zero (less than $1 \times 10^{-10}$), which means that the random-like jumps are not the result of noise, but of the internal system dynamics40. The same result that confirms deterministic chaos as a driving force for polarization mode hopping and random-like switching is also found from simulations on the SFM model, even in the presence of laser spontaneous emission noise, as well as for the Lorenz equations.

In summary, the similarities between the VCSEL polarization chaos experiments and simulations and the Lorenz chaos simulations, in addition to the fact that both Kramer’s and hidden Markov approaches fail to explain the mode hopping statistics, lead us to the conclusion that the physical mechanism underlying the mode hopping dynamics and statistics is a chaotic trajectory of the system in a double-scroll attractor. When increasing $r$ or the injection current, the two wings of the butterfly chaotic attractor grow, hence making the jumps between the two wings easier and therefore reducing the dwell time.

Experimental chaos identification

In this section, we unambiguously discriminate the deterministic (chaotic) polarization dynamics from stochastic dynamics, and then quantify chaos further. We use the Grassberger–Procaccia (GP) algorithm, which gives an estimate of the correlation dimension $D_2$ and of the $K_2$-entropy (Kolmogorov entropy)41, where $K_2$ is zero for periodic or quasiperiodic systems, positive for chaos and $K_2 = \infty$ for purely stochastic processes. Considering a sequence of $N$ points, we divide it into $N-D$ vectors of size $D$ and we compute

Figure 3 | Statistical properties of mode dwell time and comparison between polarization and Lorenz chaos. a–d. Dwell time statistics for simulated polarization chaos in a laser diode (a,b) and for Lorenz chaos (c,d). a. Semi-logarithmic plot of average dwell time for polarization chaos versus injection. b. Distribution of dwell time for polarization chaos at $\mu = 1.425$. c. Average dwell time evolution for Lorenz chaos versus $r$ parameter. d. Distribution of dwell time for Lorenz chaos with $r = 27$. 
the correlation integral $C_d(r)$ for this new sequence, that is, the average number of vectors that can be found in a sphere of radius $r$ around a single vector. If it converges, the slope of $\log(C_d(r))$ should give us an estimation of the correlation dimension $D_2$; it also allows us to estimate the $K_2$-entropy (see ref. 41). To improve the efficiency of the GP algorithm on noisy experimental time series we also use an extra re-embedding procedure based on singular-value analysis42.

The results of the GP algorithm are displayed in Fig. 4 for Lorenz chaos (Fig. 4a,b), theoretical polarization chaos (Fig. 4c,d) and experimental time series (Fig. 4e,f). Figure 4a,c,e shows logarithmic plots of the correlation integral versus sphere radius and Fig. 4b,d,f presents plots of the correlation integral slope versus sphere radius. In all three cases we obtain a clear convergence toward a correlation dimension of $D_2 \approx 2$; for each time series, a well-resolved plateau can be seen in the plot of the correlation integral slope. We obtain the following values for the $K_2$-entropy: $K_2 \approx 1.65$ for Lorenz chaos, $K_2 \approx 2.5$ for polarization chaos and $K_2 \approx 7.1$ for experimental data. On the other hand, we verified that no convergence appeared in a case of a stochastic polarization mode hopping, as can be simulated from the noise-driven two-mode equations (1) to (3) of ref. 43. As expected for a noise-induced process, $C_d(r)$ keeps increasing along with $D$ and never converges for any $r$, which would then lead to an infinite value of $K_2$.

Because specific coloured noise can also exhibit finite non-zero $K_2$-entropy, a few additional tests have been suggested to discriminate chaos from stochasticity44 (see Supplementary Information). One of these is described here.

For noise-driven processes two points will be correlated only if they are close in time. In contrast, for chaotic processes, a larger correlation exists between points as the system is driven by a low-dimensional attractor, and order can be found in the random-like dynamics. As the GP algorithm does not consider the time separation between points, specific noises can produce a finite $K_2$ value. In Fig. 5, we plot the separation in space versus the separation in time for a large number of pairs of points chosen randomly from our time series. Noise-induced processes will exhibit a stationary distribution after a small transient, because they are not correlated. However, we instead find frequent and sharp dropouts. These dropouts are manifolds, irregularly distributed and appear in the whole range of time shifts we consider, that is, several tens of nanoseconds. This result therefore confirms that our time series are not noise-driven.

**Discussion**

In summary, we report on the first observation of deterministic chaos in the dynamics of a free-running laser diode. The chaos results from the combination of (i) the light polarization degree of freedom in a VCSEL and (ii) nonlinear coupling mechanisms between two lasing modes with elliptical polarizations, as modelled within the so-called SFM approach for polarization switching. Increasing the injection current leads to polarization chaotic two-mode hopping similar to the trajectories of Lorenz chaos when jumping between the two wings of its double-scroll attractor. The chaotic property of the experimental and numerical time traces is demonstrated using appropriate chaos identification techniques.

**Figure 4 | Chaos identification.** a-f, Results of the GP algorithm for Lorenz chaos (a,b), theoretical polarization chaos (c,d) and experimental time series (e,f) showing logarithmic plots of correlation integral $C_d(r)$ versus sphere radius $r$ (a,c,e) and the slope of the correlation integral versus sphere radius $r$ (b,d,f). In all three cases the GP algorithm converges, creating a plateau on the slope of the correlation integral. The red dashed curves give estimates of the correlation dimension in all three cases: Lorenz chaos $D_2 \approx 2.05$, theoretical polarization chaos $D_2 \approx 1.89$ and experimental time series $D_2 \approx 1.82$.

**Figure 5 | Discrimination between chaos and coloured noise.** Plot of a space-time separation contour map for the experimental time series. Different curves correspond to different fractions of the distribution: above the blue line represents 10% of the points, then from blue to red the fractions are 30%, 50%, 70%, with 90% of points above the red curve. The dropouts indicate a particular proximity in space between pairs of point for a given time shift. Such dropouts can be found in the whole range of time shift we consider, that is, several tens of nanoseconds, demonstrating a strong dynamical correlation.
We anticipate our findings to have an impact in several ways. First, polarization chaotic hopping has dwell time properties opposite to those of noise-driven mode hopping. Which material and device properties lead to chaos polarization switching and which to stochastic polarization switching is a challenging question in the development of polarization-controlled microcavity lasers. Because the underlying physics comprises nonlinear carrier spin coupling, one could take advantage of recent works on spin control by carrier injection or variation of the quantum well composition. Second, the polarization chaos reported here has frequencies that are not limited by the laser relaxation oscillations, as suggested by the SFM model and recent works on spin-controlled VCSELs. Our findings therefore pave the way towards the development of integrated microcavity lasers generating multi-gigahertz chaos. Finally, polarization chaos encoding has only been used once, in an experiment showing message encoding and decoding at ~100 Mb s⁻¹ using synchronized polarization fluctuations in birefringence-modulated erbium-doped fibre ring lasers. Our work suggests the development of polarization chaos communication at higher modulation rates—beyond 10 GHz—and with compact, low-cost and low-threshold microcavity lasers. Today, state-of-the-art high-bit-rate optical communications indeed makes use of polarization multiplexing (in combination with phase encoding and coherent detection) and has overcome many of the limitations due to polarization mode dispersion. Integration of polarization encoding/decoding is therefore relevant. Polarization chaos also shows many advantages for chaos multiplexing, chaos synchronization and security, and ordering in globally coupled oscillators, which can now be explored and tackled practically.

Methods
Simulation details. For the SFM model, we used a classic four-step Runge–Kutta algorithm with a time step of 1 ps. For application of the GP algorithm we only considered the x-polarization output power. For the Lorenz equations, we again used a classic four-step Runge–Kutta algorithm with a time step of 0.004. For application of the GP algorithm we only consider the y variable.

Statistical analysis. The evolution of the dwell time for polarization chaos was measured over 2,000 mode-hopping events. For Lorenz chaos, we measured dwell time evolution over 10,000 events. Both distributions were calculated for 500,000 mode-hopping events.

Experimental data and re-embedding procedure. Our main contribution is the analysis of experimental data. Measurements of the data are already described elsewhere and we will therefore focus on the method to reduce the noise in our data analysis of experimental data. Measurements of the data are already described elsewhere and we will therefore focus on the method to reduce the noise in our data analysis of experimental data. Measurements of the data are already described elsewhere and we will therefore focus on the method to reduce the noise in our data analysis of experimental data. Measurements of the data are already described elsewhere and we will therefore focus on the method to reduce the noise in our data analysis of experimental data.

Chaos identification. To identify the chaotic dynamic and to find order in the experimental and theoretical time series, we used the GP algorithm. The goal of this algorithm is to provide an estimation of the correlation dimension D₂ (close to the fractal dimension of the attractor) and the K₂ entropy, which characterizes the randomness of a dynamic (close to the Kolmogorov entropy). We used the same parameters for the GP algorithm in all three cases. The N points sequence was divided into N-D vectors of size D, and for each value of D we computed the correlation integral C_D(r):

\[ C_D(r) = \frac{1}{N^2} \left( \sum_{i,j} \mathbb{1}(d(X_i, X_j) \leq r) \right) \]

This function is a numerical computation of the average number of vectors that can be found within a sphere of radius r around a given vector. Distance d is the Euclidian norm or norm 2.

According to Grassberger and Procaccia, we theoretically have \( C_D(r) \approx r^D \exp(-D \nu) \), where \( r \) is the sample rate of the time series, and \( \nu \) and \( K \) are the two parameters we want to approach. Thus we find

\[ D_2 = \lim_{r \to 0} \frac{\ln C_D(r)}{\ln r} \quad \text{and} \quad K_2 = \lim_{r \to 0} \frac{1}{r} \ln \left( \frac{C_D(r)}{C_D(r/r)} \right) \]

Obviously, infinite is out of range numerically, so we look for convergence in these functions as we increase the vector size D. In this contribution we present results for D between 12 and 15.

Discrimination between chaos and coloured noise. The goal of this method is to explicitly take into account the time correlation between points. For this test we directly used the experimental time trace without re-embedding. As in the GP algorithm, we divide the sequence into N-M vectors of size M = 0.6 ns. This discrimination is based on a space–time separation plot. For a pair of vectors \( \langle X_n, X_{n'} \rangle \), we consider the separation in space \( d(X_n, X_{n'}) \) and in time \( \Delta t = |n-m| \). To obtain a regular distribution in time, for each given value of \( \Delta t \) we select randomly, in our experimental data set, 500 pairs of points. We then plot the contour map, which gives us the evolution of this distribution for increasing time shift \( \Delta t \).

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Author contributions
M.S. and K.P. initiated the study. M.V. and M.S. performed the simulation of the laser dynamics. M.V., K.P. and M.S. carried out chaos identification from experimental and theoretical time traces. All authors discussed the results and contributed to writing the manuscript.

Additional information
Supplementary information is available in the online version of the paper. Reprints and permission information is available online at http://www.nature.com/reprints. Correspondence and requests for materials should be addressed to M.S.

Competing financial interests
The authors declare no competing financial interests.