Casimir stress on parallel plates in de Sitter space

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March 27, 2022

Abstract

The Casimir stress on two parallel plates in de Sitter background for massless scalar field satisfying Robin boundary conditions on the plates is calculated. The metric is written in conformally flat form to make maximum use of the Minkowski space calculations. Different cosmological constants are assumed for the space between and outside of the plates to have general results applicable to the case of domain wall formations in the early universe.

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1 Introduction

The Casimir effect is one of the most interesting manifestations of nontrivial properties of the vacuum state in quantum field theory [1,2]. Since its first prediction by Casimir in 1948 [3] this effect has been investigated for different fields having different boundary geometries [4-7]. The Casimir effect can be viewed as the polarization of vacuum by boundary conditions or geometry. Therefore, vacuum polarization induced by a gravitational field is also considered as Casimir effect.

In the context of hot big bang cosmology, the unified theories of the fundamental interactions predict that the universe passes through a sequence of phase transitions. These phase transitions can give rise to domain structures determined by the topology of the manifold $M$ of degenerate vacuua [8, 9, 10]. If $M$ is disconnected, i.e. if $\pi(M)$ is nontrivial, then one can pass from one ordered phase to the other only by going through a domain wall. If $M$ has two connected components, e.g. if there is only a discrete reflection symmetry with $\pi_0(M) = \mathbb{Z}_2$, then there will be just two ordered phase separated by a domain wall. In the domain wall formation models, in the early universe, the space-time changes from de Sitter to the geometry induced by the presence of a domain wall. In [11] the effects of particle production and vacuum polarization attendant to the domain wall formation have been studied. Casimir stress for parallel plates in the background of static domain wall in four and two dimensions is calculated in [12, 13]. Spherical bubbles immersed in different de Sitter spaces in- and out-side is calculated in [14].

Our aim is to calculate the Casimir stress on two parallel plates with constant comoving distance having different vacuums between and outside, i.e. with false/true vacuum between/outside. Our model may be used to study the effect of the Casimir force on the dynamics of the domain wall formation appearing in the simplest Goldston model. In this model potential of the scalar field has two equal minima corresponding to degenerate vacuua. Therefore, scalar field maps points at spatial infinity in physical space nontrivially into the vacuum manifold [15]. Domain walls occur at the boundary between these regions of space. One may assume that the outer regions of parallel plates are in $\Lambda_{\text{out}}$ vacuum corresponding to degenerate vacuua in domain wall configuration. In section two we calculate the stress on two parallel plates with Robin boundary conditions. The case of different de Sitter vacua between and outside of the plates, is considered in section three. The last section conclude and summarize the results.

2 Parallel Plates with Robin boundary conditions in de Sitter Space

Consider a massless scalar field coupled conformally to a de Sitter background space. The scalar field satisfies the following Robin boundary condition on two parallel plates within an arbitrary space-time is defined as [16]:

\begin{equation}
(1 + \beta_m (-1)^{m-1} \partial_x) \Phi(x^\nu)|_{x=a_m} = 0, \quad m = 1, 2,
\end{equation}

Here we have assumed that the two plates are normal to the cartesian $x$-axis at $x = a_{1,2}$. The Robin boundary condition may be interpreted as the boundary condition on a thick
plate \[\text{II}\]. Rewriting (1) in the following form

\[
\partial_x \Phi(x^\nu) = (-1)^m \frac{1}{\beta_m} \Phi(x^\nu),
\]

(2)

where \(|\beta_m|\), having the dimension of a length, may be called skin-depth parameter. This is similar to the case of penetration of an electromagnetic field into a real metal, where the tangential component of the electric field is proportional to the skin-depth parameter.

It is known that in the Minkowski space-time for the conformally coupled scalar field the perpendicular pressure, \(P\), is uniform in the region between the plates and is given by \[\text{III}\]

\[
P = 3\varepsilon_c,
\]

(3)

where \(\varepsilon_c\) is the Casimir energy density. This Casimir energy has been calculated to be

\[
\varepsilon = \varepsilon_c = \frac{-A}{8\Gamma(5/2)\pi^{3/2}a^4},
\]

(4)

where \(A\) depends on \(\beta_{1,2}\) and may be inferred from the Eq.(4.15) from the reference \[\text{II}\]. It has also been shown that for \(\beta_1 = -\beta_2\)

\[
\varepsilon = \varepsilon_c = \frac{-\zeta_R(4)\Gamma(2)}{(4\pi)^2a^4} = \frac{-\pi^2}{1440a^4},
\]

(5)

which is the same as for the Dirichlet and Neumann boundary conditions.

Consider now two parallel plates in the de Sitter space-time. To make the maximum use of the flat space calculation we use the de Sitter metric in the conformally flat form:

\[
ds^2 = \frac{\alpha^2}{\eta^2} [d\eta^2 - 3 \sum_{i=1}^{3} (dx^i)^2],
\]

(6)

where \(\eta\) is the conformal time:

\[
\infty < \eta < 0.
\]

(7)

The relation between parameter \(\alpha\) and cosmological constant \(\Lambda\) is given by

\[
\alpha^2 = \frac{3}{\Lambda}.
\]

(8)

Using the standard relation between the energy-momentum tensor for conformally coupled situations \[\text{I}\]

\[
<T_{\nu}^\mu[g_{\alpha\beta}] > = \left[\left(\frac{g}{\bar{g}}\right)\right]^\frac{1}{2} < T_{\nu}^\mu[g_{\alpha\beta}] > - \frac{1}{2880} \left[\frac{1}{6} \bar{H}_{\nu}^{(1)\mu} - \bar{H}_{\nu}^{(3)\mu}\right],
\]

(9)

where \(g_{\mu\nu}\) and \(\bar{g}_{\mu\nu}\) are conformal to each other, with their respective determinants \(g\) and \(\bar{g}\). We are going to assume now that \(g_{\mu\nu}\) is the Minkowski metric. Now, \(< T_{\nu}^\mu[g_{\alpha\beta}] >\), the regulatized energy momentum tensor for a conformally coupled scalar field for the case of parallel plate configuration in flat space-time is given by

\[
<T_{\nu}^\mu[g_{\alpha\beta}] > = \text{diag}(\varepsilon, -P, -P_\perp, -P_\perp) = \text{diag}(\varepsilon, 3\varepsilon, -\varepsilon, -\varepsilon).
\]

(10)
The second term in (9) is the vacuum polarization due to the gravitational field, without any boundary conditions. The functions $H^{(1,3)}_{\nu}^\mu$ are some combinations of curvature tensor components (see [18]). For massless scalar field in de Sitter space, the term is given by [18, 19]

$$-\frac{1}{2880} [\frac{1}{6} \tilde{H}^{(1)}_{\nu}^\mu - \tilde{H}^{(3)}_{\nu}^\mu] = \frac{1}{960\pi^2\alpha^4} \delta^\mu_\nu. \quad (11)$$

From (3,5,8,9,10) one can obtain vacuum pressure due to the boundary acting on the plates:

$$P_{b}^{(1,2)} = P_{b}(x_{1,2}) = \left(\frac{g}{\tilde{g}}\right)^{\frac{1}{2}} \left(\frac{-3\pi^2}{1440a^4}\right) \left(\frac{\eta^4}{\alpha^4}\right) (\frac{-3\pi^2}{1440a^4}) = -\frac{\eta^4\Lambda^2}{3} \frac{\pi^2}{1440a^4}. \quad (12)$$

which is attractive. It has been shown that this pressure is zero for $x < a_1$ and $x > a_2$ [12, 13]. The gravitational part of the pressure according to (11) is equal to

$$P_g = -< T^1_1 > = \frac{-1}{960\pi^2\alpha^4}. \quad (13)$$

This is the same from both sides of the plates, and hence leads to zero effective force. Therefore the effective force acting on the plates are given only by the boundary part.

3 Parallel plates with different cosmological constants between and out-side

Now, assume there are different vacuua between and out-side of the plates, corresponding to $\alpha_{betw}$ and $\alpha_{out}$ in the metric(6). As we have seen in the last section, the vacuum pressure due to the boundary is only non-vanishing between the plates. Therefore, we have for the pressure due to the boundary

$$P_{b}^{(1,2)} = \frac{\eta^4}{\alpha^4_{betw}} \frac{-3\pi^2}{1440a^4} = -\frac{\eta^4\Lambda^2_{betw}}{3} \frac{\pi^2}{1440a^4}. \quad (14)$$

Now, the effective pressure created by gravitational part(11), is different for different part of the space-time:

$$P_{g}^{betw} = -< T^1_1 >_{betw} = \frac{-1}{960\pi^2\alpha^4_{betw}} \frac{-\Lambda^2_{betw}}{9} \frac{1}{960\pi^2}, \quad (15)$$

$$P_{g}^{out} = -< T^1_1 >_{out} = \frac{-1}{960\pi^2\alpha^4_{out}} \frac{-\Lambda^2_{out}}{9} \frac{1}{960\pi^2}. \quad (16)$$

Therefore, the gravitational pressure acting on the plates is given by

$$P_g = P_{g}^{betw} - P_{g}^{out} = \frac{-1}{9 \times 960\pi^2} (\Lambda^2_{betw} - \Lambda^2_{out}). \quad (17)$$

The total pressure acting on the plates, $P$, is then given by

$$P = P_g + P_b = \frac{-1}{9 \times 960\pi^2} (\Lambda^2_{betw} - \Lambda^2_{out}) - \frac{\eta^4\Lambda^2_{betw}}{3} \frac{\pi^2}{1440a^4}. \quad (18)$$

The term $P_b$ is always negative corresponding to an attractive force on the plates. The term $P_g$, however, may be negative or positive, depending on the difference between the
cosmological constants in the two parts of space-time. Given a false vacuum between the plates, and true vacuum out-side, i.e. $\Lambda_{\text{betw}} > \Lambda_{\text{out}}$, then the gravitational part is negative. Therefore, the total pressure $P$ is always negative leading to a attraction of the plates. For the case of true vacuum between the plates and false vacuum out-side, i.e. $\Lambda_{\text{betw}} < \Lambda_{\text{out}}$, the gravitational pressure is positive. Therefore, the total pressure may be either negative or positive. For $P > 0$, the initial repulsion of the parallel plates may be stopped or not depending on the detail of the dynamics. Given $P < 0$ initially, it remains negative and there is an attraction between the plates.

4 Conclusion

We have considered two parallel plates in de Sitter background with a massless scalar field, coupled conformally to it, satisfying the Robin boundary conditions with constant comoving distance. Our calculation shows that for the parallel plates with false vacuum between and true vacuum outside, the total Casimir force leads to an attraction of the plates. The boundary term is proportional to the forth power of the inverse distance between the plates, and is always negative, which means a huge attractive force for small distances. Therefore, parallel plates with false vacuum in between always attract each other. In contrast, plates with true vacuum between them may repel each other to a maximum distance and attract again. The result may be of interest in the case of formation of the cosmic domain walls in early universe, where the wall orthogonal to the $x-$axis is described by the function $\Phi_i(x)$ interpolating between two different minima at $x \to \pm \infty$ [15].

Acknowledgement

We would like to thank Prof. A. A. Saharian for his valuabel hints and comments.

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