Scaling property and opinion model for interevent time of terrorism attack

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The interevent time of terrorism attack events is investigated by empirical data and model analysis. Empirical evidence shows it follows a scale-free property. In order to understand the dynamic mechanism of such statistic feature, an opinion dynamic model with memory effect is proposed on a two-dimension lattice network. The model mainly highlights the role of individual social conformity and self-affirmation psychology. An attack event occurs when the order parameter of the system reaches a critical value. Ultimately, the model reproduces the same statistical property as the empirical data and gives a good understanding of terrorism attack.

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I. INTRODUCTION

In recent years, terrorism attack events have occurred frequently. It has attracted interests of many scientists such as historians, politicians, physicists and so on. Many people might think terrorism attack is random and unpredictable. But, indeed there are some general, common statistic property, just like the result from other fields where the non-poison distribution is captured for the interevent time of the e-mail [1], surface mail[2], short message lending [3], web browsing [4,5], and rating of movies [6] etc.

As early as 1948, Richardson found the number of casualty follows a power-law distribution in interstate wars [7]. Recently the same statistic property is revealed for the casualty numbers in the global terrorism events [8]. Actually the terrorism events can be thought as wars in war. However, so far, the fundamental reason of terrorism events still remain unclear in view of its complexity and diversity. At present, there are mainly three kinds of viewpoints to explain the burst of terrorism events. The first kind is the self-organized critical notion. At first, Cederman [9] gave a possible interpretation for the finding of Richardson[7] by an agent-based model. After 9/11 event, The analysis for Iraq, Colombia, Afghanistan[10-12] displays the power law distribution with a scaling exponent $\alpha = 2.5$ which conforms with the non-G7 countries, the ones except the major industrialized nation: Canada, France, Germany, Italy, Japan, the United Kingdom and the United States, in old war [8]. Meanwhile the author proposed the self-organized critical model of interaction among terrorist groups who make decision of coalescence or fragmentation randomly or with some one probability. This model produces the results coinciding with the statistic data and give insight in term of the conception of complex system. Further, it is generalized and perfected by Clauset[13], and the solution of the steady-state behavior is obtained analytically under the conditions of constant number of terrorism-inclined individuals and the proportional relation between the severity and the size of the attacking cell. The second one proposed by Galam[14-17] is the terrorism model of percolation theory based on individual passive supporters. In this model one territory is under the terrorist threat if the density of the passive supporters exceeds percolation threshold in this territory. Further one clue is given to curb terrorism threat without harming the passive supporters. The specific scheme is increasing the value of terrorism percolation threshold by decreasing the space dimension but not the number of nearest neighbors. This interesting model describes the state of terrorism and gives an orient to fight against terrorism. The third one is the competition, selection viewpoint. Clauset et al. [18] found the scale-free property of frequency-severity ditribution has evident robustness on burst means and stability over time since 1968. they have developed a toy model to explain this kind of behavior by the mechanism of competition between states and the non-state actors successfully. In another reference [19], from the strategic selection, they illuminate the substitution and the competition in the Israel-Palestine conflict is the reason whether an organization resort to terrorism where the public standing is first thought a origin of attack occurrence.

From the aspect of fighting against terrorism, besides the model of percolation theory, the conditions of promoting a violence[20] are also referential. Lim et al.[20] think that a violence arises at boundaries between groups with culture differntiation when the group size achieves a critical scale and point out the violence might be prevented or minimized if appropriate boundaries is created for current geocultural regions. However that is not indisputable[27]. Recently, the variation of the interevent time over time have been concerned, and the research of Clauset et al.[21] shows the organizational growth leads to the decrease of the interevent time.
In this paper we focus on the interevent time of terrorism events in Iraq and Afghanistan from 2003 to 2007. It is found to obey Zipf’s law corresponding with the power-law distribution with scaling exponents $\alpha = 2.43$ and $\alpha = 2.35$ for Iraq and Afghanistan respectively. Considering the importance of memory [22-24], we propose an opinion dynamic model with memory effect to understand such a statistic property. Due to individual social conformity and self-affirmation psychology, the public opinion is formed and varies with time under the coaction of influence between individuals [25] and individual history memory [26]. Ultimately, under some certain social circumstance, an attack event occurs when the public consensus reaches some degree. We have found that this model can reproduce the same interevent time distribution of terrorism events.

II. THE EMPIRICAL DATA

Our data can be available from the database of MIPT (http://www.terrorisminfo.mipt.org/incidentcalendar.asp), which records the detailed information of terrorism attack events and includes the domestic event after 1998. Since there are huge differences of terrorism events for different countries, our statistics are distinguished as different countries.

As well known, terrorism attack events frequently occur in Iraq and Afghanistan due to the deep ethnic contradiction, intense religious struggle and increasing anti-Western emotion. The patterns of terrorism events occurrence in Iraq and Afghanistan are shown in Fig. 1. Apparently the succession of events takes on a pattern with long time inactivity. Then what statistic character does this kind of activity pattern has? In order to make it clear, we study the interevent time distribution of terrorism attack events in these two countries from 2003 to 2007.

The interevent time obtained from everyday event number. It is assumed that the events occur homogeneously and the interevent time is the reciprocal of event number in this day if the event number is more than one in a day. Otherwise the interevent time corresponds the number of days between two consecutive events. Because the diversity of interevent time is not enough, we draw the rank plots in Fig. 2. Obviously there exist Zipf’s laws. It suggests that the time between two consecutive attack events is usually very short, and the long interevent time is also can’t be ignored. Indeed, a Zipf’s law can be consider as a cumulative distribution with a power-law form. If the probability distribution of interevent time follows $p(\tau) = \tau^{-\alpha}$, the Zipf’s exponent $\alpha'$ should satisfy the relation [28,29]

$$R^{-1/\alpha'} = \int_{R}^{\infty} p(\tau) d\tau \propto R^{-(\alpha-1)}.$$ (1)

Here $R$ represents the rank. We find the Zipf’s exponent $\alpha' = 0.70$ for Iraq and $\alpha' = 0.73$ for Afghanistan. In term of the above relation, we have $\alpha = 1 + 1/\alpha'$. So the interevent time distribution has power exponent $\alpha = 2.43$ and $\alpha = 2.35$ for Iraq and Afghanistan respectively. For Afghanistan, the smaller exponent means the interevent time is more heterogenous than Iraq, namely, the occurrence of terrorism events in Afghanistan shows more burstiness property.
III. THE MODEL

To understand the underlying mechanism of the scaling property in terrorism events, we introduce an opinion dynamic model with memory effect to explain this statistic property. In the present agents model, every node is connected with its four adjacent neighbors inwardly and outwardly on a 2-dimension lattice network. Here nodes and links represent the individuals in a terrorism social system and the interaction between nodes respectively. Before an attack event occurs every individual has own viewpoint, support or opposition which are denoted by $\sigma_i = 1$ and $\sigma_i = -1$. Individual opinion is determined by two factors. One is the influence of its adjacent neighbors, it is described by

$$W_1(\sigma_{i,t}) = \sigma_{i,t} - \frac{1}{4} \sum_{j=1}^{4} \sigma_{j,t-1}$$ (2)

the other is the individual history memory effect measured by

$$W_2(\sigma_{i,t}) = \begin{cases} 1, & \sigma_{i,t-1}\sigma_{i,t-2} > 0 \\ 0, & \sigma_{i,t-1}\sigma_{i,t-2} < 0 \end{cases}$$ (3)

Individual opinion turnovers with time by the above factors in terms of next rule: $W_1 > 0$ means individual has consistent opinion with the majority of its adjacent neighbors and change his/her own opinion with the probability $\exp(-aW_1) + \exp(-bW_2)/T$. $W_1 < 0$ corresponds with the opposite case. In this case history memory effect dominates and individual changes his/her opinion with the probability $\exp(-bW_2)$. So the overturning probability is

$$P(\sigma_i) = \begin{cases} [\exp(-aW_1) + \exp(-bW_2)]/T, & W_1 > 0 \\ \exp(-bW_2), & W_1 < 0 \end{cases}$$ (4)

Here the parameters $a$ and $b$ are the main factors indicating the social conformity psychology and self-affirm psychology respectively. $T \geq 2.0$ is an index to describe the social chaotic degree. By this rule the system undergoes self-organization evolution in the non-equilibrium state.

Now we measure the order degree of the public opinion by an order parameter $m$ ($0 \leq m \leq 1$) on a lattice network with size $L \times L$ and periodic boundary,

$$m = \frac{1}{L^2} \sum_{j=1}^{L^2} \sigma_i, \sigma_i \in (+1, -1)$$ (5)

Generally the total population of a terrorism social system is approximately invariant. So let $L = 10$ and the initial states are given randomly. No matter how complicated that the practical reasons may be, we think an attack event is triggered when $m$ reaches a critical value $m_c$. Next we investigate the influence of different parameters on the interevent time statistic property by simulation.
the exponent mutual restraint to reach balance. The parameter opinion keeps consistent with the public opinion. Similar with Fig. 3 (a), the increase of $\alpha = 1$ exponent and reduce of interevent time of terrorism events from Fig. 3 (c). But, compared with the $T$ still has obvious influence on power exponent from the inset. In that the strengthening of self-affirmation reduces the time $\alpha$ increases as $a$ increases and converges to a steady value when $\alpha = 1.42$ and $\alpha = 2.85$ respectively. The top inset: $\alpha$ as a function of $T$. (d) The variation of distribution curves when the parameter $m_e$ is changed. Each data is obtained by averaging over 100 independent runs.

FIG. 3: (Color online) The normalized distribution of interevent time by simulation under the influence of one parameter among $a = 1.2$, $b = 0.5$, $T = 5.5$, $m_e = 0.7$. (a) The distribution curves when the parameter $a$ is changed. The two solid lines are the fitting of power-law with the exponent $\alpha = 1.75$ and $\alpha = 2.52$ respectively. The top inset: relationship between $a$ and $\alpha$. (b) The variation of distribution curves when the parameter $b$ is changed. (c) the distribution curves when the parameter $T$ is changed. Two curves fit by two solid lines have scaling exponent $\alpha = 1.42$ and $\alpha = 2.85$ respectively. The top inset: $\alpha$ as a function of $T$. (d) The variation of distribution curves when the parameter $m_e$ is changed. Each data is obtained by averaging over 100 independent runs.

IV. SIMULATION AND ANALYSIS

First, the social conformity will impact on the whole social order degree and smaller $a$ indicates individual has larger willingness to follow social public opinion. In Fig. 3 (a) the interevent time distribution is plotted for different values of $a$. It is shown that the distribution transits to power-law style from power-law-like style with a tail gradually when $a$ increases. For power-law-like style at $a = 0.5$ a natural cutoff of tail is executed, then the power exponent $\alpha$ is obtained by linear fitting. Obviously, the power exponent becomes larger with the increase of $a$, which means the distribution of terrorism events is more inhomogeneous when individual inclines to follow public opinion more easily. From the inset in Fig. 3 (a), one can see that $\alpha$ increases as $a$ increases and converges to a steady value when $a$ is large enough. This is because the effect of the public opinion is so weak for large $a$ that the dynamic evolution of terrorism attack is hardly affected.

In the evolution of terrorism events, self-affirmation psychology is also very important for social order degree. Together with the social conformity, they countermine wether an individual overturns his/her opinion when individual opinion agrees with the major opinion. However, in converse case, the self-affirmation play a decisive role in individual opinion selection. At the moment, individual will make a decision according to history selection in memory. Now let us pay attention to the effect of self-affirmation factor $b$ on time statistic property of terrorism event. As shown in Fig. 3 (b), the distribution of interevent time exceeds a power law when the self-affirmation is inadequate at large $b$ value. The distribution curve becomes a power-law and then tends to a stretched exponent style with the decrease of $b$ in that the strengthening of self-affirmation reduces the time interval between two consecutive terrorism events. Meanwhile it makes terrorism event with short interevent time occur with smaller probability. Indeed social conformity and self-affirmation are two factors who are complementary to each other and mutual restraint to reach balance.

The third tunable parameter is $T$, which implies the chaotic degree of a social circumstance. It will be effective as individual opinion keeps consistent with the public opinion. Similar with Fig. 3 (a), the increase of $T$ also leads to the increase of power exponent and reduce of interevent time of terrorism events from Fig. 3 (c). But, compared with the $a$, it is different that the large $T$ still has obvious influence on power exponent from the inset.

The above three factors determine the order degree of public opinion. We need to set a critical value of order parameter $m_e$ to
judge whether a terrorism event burst. Fig. 3 (d) displays the influence of different critical value \( m_c \) on the curve style. It is very easy to achieve consensus when \( m_c \) is small. So the terrorism event bursts frequently. Contrarily time interval of the terrorism event becomes longer in a way. The curve style transits to the stretched exponent because the proportion of the middling time interval is prominent relatively.

From Fig. 3, one can find that the style of distribution curve is decided by the self-affirmation psychology and the critical order parameter. The social conformity and the chaotic degree of a social circumstance decide the power exponent. So the specific power-law style with some exponent will be got if only we choose appropriate parameter values. According to the property above, we choose \( a = 1.31, b = 0.50, T = 5.50, m_c = 0.70 \) and \( a = 1.20, b = 0.50, T = 5.50, m_c = 0.70 \) to simulate the terrorism events in Iraq and Afghanistan respectively. The simulation results are shown in Fig. 4. The present model generates the power-law distribution with exponent \( \alpha = 2.43 \) and \( \alpha = 2.35 \) which accord with the empirical data. It is noted that the strong self-affirmation and weak social conformity are the significant character for Iraq. Nevertheless these two factors are almost equivalent for Afghanistan.

V. CONCLUSION

In conclusion, from real data we find the scale-free feature of interevent time distribution for terrorism event in Iraq and Afghanistan from 2003 to 2007. Here we consider the assumption that the burst of a terrorism event is closely relative to the formation of opinions. This formation process depends on not only the social influence but also the individual memory. Previous terrorism models have noted the former but ignored the later. So, to understand the observed statistic property from empirical data, we proposed an opinion dynamic model with memory effect in this paper. In the model, the order degree of public opinion determines the burst of a terrorism event. In certain social circumstance, the formation of public opinion depends individual psychology character of social conformity and self-affirmation. So individual psychology factor is the crucial reason whether a terrorism burst in a given social circumstance. This also alert us to poll is important in some wars. Winning morale to strengthen the social conformity is a possible means of reduce terrorism events. These results obtained by this model are coincide with the reality intuitively and it can reproduce the same power-law interevent time distribution of terrorism attack as the empirical data in Iraq and Afghanistan. It confirms the rationality of our assumption and provides a better understanding of the terrorism attack. In addition, terrorism events can be treated as a kind of collective behaviors of human. Our studies show that the memory and social effect could be an origin of the power-law properties in many collective behaviors of human.

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