ON THE ELECTRODYNAMICS WITH FASTER THAN LIGHT MOTION

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The version of electrodynamics is constructed in which faster-than-light motions of fields and particles with real masses are possible.

1. Introduction

For certainty, by faster-than-light motions we will understand the motions with velocities $v > 3 \cdot 10^{10}$ cm/sec. The existence of such motions is the question discussed in modern physics. In spite of the well-known conservatism, in the world science considerable attention is given to this question.

Blokhintsev (1946-1947) [1] paid attention to the possibility of formulating the field theory that permits the propagation of faster-than-light (superluminal) interactions outside the light cone. Later (1952) [2] he also noted the possibility of the existence of superluminal solutions in the nonlinear equations of electrodynamics.

Kirzhnits (1954) [3] showed that a particle possessing the tensor of mass $M^i_k = \text{diag}(m_0, m_1, m_1, m_1)$, $i, k = 0, 1, 2, 3$, $g_{ab} = \text{diag}(+,-,-,-)$ can move with the faster-than-light velocity $v = cm_0/m_1 > c$ if $m_0 > m_1$.

Terletsky (1960) [4] introduced into theoretical physics the particles with imaginary rest masses moving faster than light.

Feinberg (1967) [5] named these particles tachyons and described their main properties.

Research on the superluminal tachyon motions opened up additional opportunities which were studied by many authors, for example by Bilaniuk and Sudarshan [6], Recami [7], Mignani [8], Kirzhnits and Sazonov [9], Corben [10], Patty [11], Recami, Fontana and Caravaglia [12]. It has led to original scientific direction
(hundreds publications). The tachyon movements may formally be described by SR expanded to the area of motions where $s^2 < 0$. For comparison, the standard theory describes motions on the zero cone $s^2 = 0$ at $v = c$ and in the area $s^2 > 0$ when $v < c$.

The publications are also known in which the violation of invariance of the speed of light is considered [13] - [19]. We can note, for example, the Pauli’s monograph [13] where the elements of Ritz and Abraham theories are contained; the Logunov’s "Lections on Fundamentals of Relativity Theory" [14]; the Glashow’s paper [15] discussing the experimental consequences of violation of the Lorentz-invariance in astrophysics; publications [16] - [19] on the violation of invariance of the velocity of light in SR.

Below a version of the theory permitting faster-than-light motions of electromagnetic fields and charged particles with real masses is proposed as the continuation of such investigations.

2. Space-Time Metric. Transformation Law of Coordinates

Let us start from the condition of invariance of the infinitesimal space-time interval of 4-space $R^4$, the metric properties of which may depend on the velocity of a particle being investigated. Supposing that space is homogeneous and isotropic we take the metric in the form:

$$ds^2 = (c_0^2 + v^2)dt^2 - dx^2 - dy^2 - dz^2 = (c_0'^2 + v'^2)(dt')^2 - (dx')^2 - (dy')^2 - (dz')^2 = \text{invariant}.$$ (1)

Here $x, y, z$ are the spatial coordinates, $t$ is the time, $c_0, c_0'$ are the proper values of the speed of light, $v$ is the velocity of a particle with respect to the reference frame $K$. Let us connect the co-moving frame $K'$ with this particle. Let the proper speed of light be invariant

$$c_0 = c_0' = 3 \cdot 10^{10} \text{cm/sec} \quad \text{invariant.} \quad (2)$$

In this case, as follows from [1], the common time $t_0'$ similar to the Newton time may be introduced on the trajectory of the frame $K'$ movement with the velocity $v = dx/dt$:

$$dt = dt_0' \rightarrow t = t_0'.$$ (3)

The value $^a$

$$c = \pm c_0 \sqrt{1 + \frac{v^2}{c_0^2}} \quad (4)$$

$^a$In the form of $c' = c(1 - \beta^2)^{1/2}$ the expression $^b$ was obtained by Abraham in the model of ether [13].
is the velocity of light corresponding to the particle velocity \( v \). Hereafter let us agree to name the value \( 3 \cdot 10^{10} \) cm/sec the speed of light, the value \( c \neq c_0 \) the velocity of light. In accordance with the hypothesis of homogeneity and isotropy of the space-time, the velocity \( v \) of a free particle does not depend on \( t \) and \( x \). The velocity of light \( c \) is a constant value in this case. By substituting

\[
x^0 = \int_0^t c d\tau = \pm \int_0^t c_0 \sqrt{1 + \frac{v^2}{c_0^2}} d\tau,
\]

we may introduce the "time" \( x^0 \) and rewrite the expression (1) in the form

\[
ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2,
\]

where \( x^{1,2,3} = (x, y, z) \). As is known, the metric (6) describes the flat homogeneous Minkowski space-time \( M^4 \) with \( g_{ik} = \text{diag}(+1, -1, -1, -1) \), \( i, k = 0, 1, 2, 3 \). The infinitesimal space-time transformations, retaining the invariance of the form (6) take the form [20]

\[
dx^i = L^i_k dx^k, \quad i, k = 0, 1, 2, 3,
\]

where \( L^i_k \) is the matrix of the six-dimensional Lorentz group \( L_6 \). Let us, for example, write the matrix \( L^i_k \) for one-dimensional Lorentz group \( L_1 \) with the group parameter \( \beta = V/c = \text{const} \) [20]:

\[
L^i_k = \begin{pmatrix}
1 & 0 & 0 & 0 \\
\frac{\beta}{\sqrt{1 - \beta^2}} & \frac{1}{\sqrt{1 - \beta^2}} & 0 & 0 \\
\frac{\beta}{\sqrt{1 - \beta^2}} & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

Analogously to (1), we may introduce the formulae of transformation of the 4-velocity \( u^i = (dx^0/ds, dx^\alpha/ds) = (\sqrt{1 - u^2}, u^\alpha/\sqrt{1 - u^2}) = (c/c_0, cw^\alpha/c_0) \), where \( ds = \sqrt{1 - u^2} dx^0 = (c_0/c) dx^0 \), \( w^\alpha = v_{x,y,z}/c, \alpha = 1, 2, 3 \), \( u^2 = g_{ik} u^i u^k = 1 \):

\[
u^i = L^i_k u^k, \quad i, k = 0, 1, 2, 3.
\]

As a result, the one-dimensional infinitesimal transformations corresponding to the given matrix, are:

\[
dx^{i0} = \frac{dx^{i0} - \beta dx^{i1}}{\sqrt{1 - \beta^2}}; \quad dx^{i1} = \frac{dx^{i1} - \beta dx^{i0}}{\sqrt{1 - \beta^2}}; \quad dx^{i2} = dx^{i2}; \quad dx^{i3} = dx^{i3}; \quad c' = c \frac{1 - \beta u^1}{\sqrt{1 - \beta^2}}.
\]

Here the latest formula follows from the law of 4-velocity transformation \( u^{i0} = L^i_k u^k \) under the condition (2); \( c' = c_0/\sqrt{1 - u^2}, c = c_0/\sqrt{1 - u^2}, c' = \gamma c \). The
reciprocal transformations may be obtained by prime permutation. The group parameters are related by the ratios \( \beta' = -\beta \) and \( \gamma' = \gamma^{-1} \). The integral homogeneous transformations corresponding to \( (10) \) are

\[
x^0 = \frac{x^0 - \beta x^1}{\sqrt{1 - \beta^2}}; \quad x^1 = \frac{x^1 - \beta x^0}{\sqrt{1 - \beta^2}}; \quad x^2 = x^2; \quad x^3 = x^3; \quad c' = \frac{c - \beta u^1}{\sqrt{1 - \beta^2}}. \tag{11}
\]

They are induced by the operator \( X = x_1 \partial_0 - x_0 \partial_1 - u^1 c \partial_c \) which is the sum of Lorentz group \( L_1 \) generator \( J_{01} = x_1 \partial_0 - x_0 \partial_1 \) and the generator \( D = c \partial_c \) of scale transformations group \( \Delta_1 \). These generators act in 5-space \( M^4 X V^1 \) where \( V^1 \) is a subspace of the velocities of light. We may say that the transformations \( (11) \) belong to the group of direct product \( L_1 X \Delta_1 \). The generators \( J_{01} \) and \( D \) and transformations \( (11) \) are respectively the symmetry operators and symmetry transformations for the equation of the light cone \( s^2 \) in the 5-space \( V^5 = M^4 X V^1 \) where \( |c| < \infty \) includes the subset \( c_0 < |c| < \infty \). We have

\[
s^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = 0, \quad J_{01}(s^2) = 0, \quad D(s^2) = 0, \quad [J_{01}, D] = 0. \tag{12}
\]

Below we shall consider the case of positive values of the speed of light. In this case SR is realized on the hyperplane \( c = c_0 \).

The relationships between the variables for the space-time \( R^4 \) with metric \( (1) \) and for Minkowski space \( M^4 \) with metric \( (4) \) are as follows:

\[
\begin{align*}
\frac{\partial}{\partial t} &= \frac{\partial x^0}{\partial t} \frac{\partial}{\partial x^0} + \sum_\alpha \frac{\partial x^0}{\partial x^\alpha} \frac{\partial}{\partial x^\alpha} = c \frac{\partial}{\partial x^0}, \\
\frac{\partial}{\partial x} &= \frac{\partial x^0}{\partial x} \frac{\partial}{\partial x^0} + \sum_\alpha \frac{\partial x^\alpha}{\partial x} \frac{\partial}{\partial x^\alpha} = \left( \int \frac{\partial c}{\partial x} dx^0 \right) \frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^1}, \\
\frac{\partial}{\partial y} &= \frac{\partial x^0}{\partial y} \frac{\partial}{\partial x^0} + \sum_\alpha \frac{\partial x^\alpha}{\partial y} \frac{\partial}{\partial x^\alpha} = \left( \int \frac{\partial c}{\partial y} dx^0 \right) \frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^2}, \\
\frac{\partial}{\partial z} &= \frac{\partial x^0}{\partial z} \frac{\partial}{\partial x^0} + \sum_\alpha \frac{\partial x^\alpha}{\partial z} \frac{\partial}{\partial x^\alpha} = \left( \int \frac{\partial c}{\partial z} dx^0 \right) \frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^3}.
\end{align*} \tag{13}
\]

We restrict ourselves by studying a variant of the theory in which the velocity of light in the range of interactions may only depend on the time \( t \), i.e. \( c = c(t) \leftrightarrow c = c(x^0) \), where the relationship between \( x^0 \) and \( t \) may be deduced from the solution of Eq. \( (5) \). Then

\[
\nabla c(x^0) = 0, \quad c = c(x^0), \quad u = u(x^0); \quad \nabla c(t) = 0, \quad c = c(t), \quad v = v(t). \tag{14}
\]

Let us note some features of motions in the space \( M^4(x^0, x) \).

1. As in SR, the parameter \( \beta = V/c \) in the present work is in the range \( 0 \leq \beta < 1 \).
2. As in SR, the value \( dx^0 \) is the exact differential in view of the condition \( \nabla c = 0 \).

3. As distinct from SR, the "time" \( x^0 = ct \) in the present work is a function of the time \( t \) only for the case of a free particle. In the range of interaction the velocity of light depends on time \( t \). The value \( x^0 \) becomes the functional \( S \) of the function \( c(t) \) and takes into consideration the history of moving the particle.

4. The group parameter \( \beta = V/c \) of the matrix \( \mathbf{S} \) may be constant not only at the constant velocity of light, but also at \( c = c(t) \). Indeed we may accept that \( 0 \leq \beta = V(t)/c_0 (1 + v^2(t)/c_0^2)^{1/2} \) is constant \( \leq 1 \), which is not in contradiction with \( V = V(t), \ c = c(t) \). Supposing here \( V = 0 \) we find \( \beta = 0 \). Also, with \( V \to v, \ v \to \infty \), we find \( \beta \to 1 \). This property permits the using of the matrix \( \mathbf{S} \) for constructing Lorentz invariants in the range of interaction where \( c = c(t) \).

Keeping this in mind, let us construct in the Minkowski space \( M^4 \) a theory like SR, reflect it on the space \( R^4 \) by means of the formulas \( \mathbf{S} \) with \( \nabla c = 0 \) and consider the main properties of the constructed theory.

3. Action, Energy, Momentum

Following [20], analogously to SR we may construct the integral of action in the form:

\[
S = S_m + S_{mf} + S_f = -m_0 c_0 \int ds - \frac{e}{c_0} \int A_i dx^i - \frac{1}{16 \pi c_0} \int F_{ik} F^{ik} d^4 x = \\
\left[ -m_0 c_0 \sqrt{1 - u^2} + \frac{e}{c_0} (\mathbf{A} \cdot \mathbf{u} - \phi) \right] dx^0 - \frac{1}{8 \pi c_0} \int (E^2 - H^2) d^3 x dx^0 = \\
- m_0 c_0 \int ds - \frac{1}{c_0} \int A_i j^i d^4 x - \frac{1}{16 \pi c_0} \int F_{ik} F^{ik} d^4 x.
\]

(15)

Here in accordance with [20] \( S_m = -m_0 c_0 \int ds = -m_0 c_0 \int (c_0/c) dx^0 = -m_0 c_0 \int (1 - u^2) dx^0 \) is the action for a free particle; \( S_f = -1/16 \pi c_0 \int F_{ik} F^{ik} d^4 x \) is the action for free electromagnetic field, \( S_{mf} = -(e/c_0) \int A_i dx^i = -(1/c_0) \int A_i j^i d^4 x \) is the action corresponding to the interaction between the charge \( e \) of a particle and electromagnetic field; \( \mathbf{u} = \mathbf{v}/c \) is the "velocity" of a particle; \( A^i = (\phi, \mathbf{A}) \) is the 4-potential; \( A_i = g_{ik} A^k; \ j^i = (\rho, \mathbf{u}) \) is the 4-vector of the current density; \( \rho \) is the charge density; \( F_{ik} = (\partial A_k/\partial x^i - \partial A_i/\partial x^k) \) is the tensor of electromagnetic field; \( i, k = 0, 1, 2, 3; \ g_{ik} = \text{diag}(+, -, -, -) \); \( \mathbf{E} = -\partial \mathbf{A}/\partial x^0 - \nabla \phi \) is the electrical field; \( \mathbf{H} = \nabla \times \mathbf{A} \) is the magnetic field; \( F_{ik} F^{ik} = 2(\mathbf{H}^2 - \mathbf{E}^2) \); \( dx^4 = dx^0 dx^1 dx^2 dx^3 \) is the element of the invariant 4-volume. The proper value of the speed of light \( c_0 \), the rest mass \( m_0 \), the electric charge \( e \) are the invariant constants of the theory. The integral of action \( S \) we shall name the modified action.

In spite of the similarity, the action \( S \) differs from the action of SR [20]. The current density has taken in the form \( j^i = (\rho, \rho \mathbf{u}) = (\rho, \rho \mathbf{v}/c) \) instead of \( j^i = (\rho, \rho \mathbf{v}) \).
from [20]. The electromagnetic field has taken in the following form
\[ E = -\partial A / \partial x^0 - \nabla \phi = - (1/c) \partial A / \partial t - \nabla \phi \]
instead of the expression
\[ E = -(1/c_0) \partial A / \partial t - \nabla \phi \]
[20].

The current density [15] is similar to the current density from Pauli’s monograph [13] with the only difference that the 3-current density in [15] has taken in the form \( \rho v / c \) instead of \( \rho v / c_0 \) [13].

In addition to the Lorentz-invariance [20], the action [15] is also invariant with respect to any transformations of the velocity of light and, consequently, with respect to the transformations \( c' = \gamma c \), as the value \( c \) is not contained in the expression (15). As a result the action (15) is invariant with respect to the transformations (11) from the group of direct product \( L_1 \times \Delta_1 \subset L_6 \times \Delta_1 \), containing Lorentz group \( L_6 \) and the scale group \( \Delta_1 \) as subgroups.

The modified Lagrangian \( L \), generalized 4-momentum \( P \) and generalized energy \( H \) of a particle correspond to the modified action (15). We have:

\[
L = -m_0 c_0 \sqrt{1 - u^2} + \frac{e}{c_0} (A \cdot u - \phi); \tag{16}
\]

\[
P = \frac{\partial L}{\partial u} = \frac{m_0 c_0 u}{\sqrt{1 - u^2}} + \frac{e}{c_0} A = p + \frac{e}{c_0} A = m_0 v + \frac{e}{c_0} A; \tag{17}
\]

\[
H = P \cdot u - L = \frac{m_0 c_0 v + e\phi}{c_0} = \frac{E + e\phi}{c_0}. \tag{18}
\]

Here
\[ p = m_0 v \]

is the momentum of a particle.

\[
E = m_0 c_0 c = m_0 c_0^2 \sqrt{1 - \frac{v^2}{c_0^2}}, \quad T = m_0 c_0^2 \left( \sqrt{1 + \frac{v^2}{c_0^2}} - 1 \right). \tag{20}
\]

Let \( E \) be the relativistic energy, \( T \) be the kinetic energy of a particle. The momentum \( p \), energies \( E, T \) are the integrals of motion for a free particle. The energy \( E \) and the momentum \( p \) may be united into single 4-momentum (as in SR)

\[
p^i = m_0 c_0 u^i = \left( \frac{m_0 c_0}{\sqrt{1 - u^2}}, \frac{m_0 c_0 u^0}{\sqrt{1 - u^2}} \right) = \left( \frac{E}{c_0}, \frac{m_0 v}{c_0} \right). \tag{21}
\]

The components of \( p^i \) are related by the ratio:

\[
p_i p^i = \frac{E^2}{c_0^2} - p^2 = m_0^2 c_0^2; \]

\[
p = \frac{E}{c_0} v; \tag{22}
\]

\[
p = \frac{E}{c_0} c = \frac{E}{c_0} n, \quad n = \frac{c}{c}, \quad i f \quad m_0 = 0, \quad v = c.
\]

\[ b \]The choice of the action integral [15] is ambiguous. Instead of [15], we may introduce the action in the form \( cS \), where \( S \) is the action [20]. The “momentum” \( cp = m_0 c_0 v \) and the energy \( E = m_0 c_0 c \) are the integrals of motion in this case. The mass of movement \( M \) depends on the velocity \( v \) accordingly to the law \( M = m_0/(1 + v^2/c_0^2)^{1/2} \) [17].
4. Equations of Motion for Charged Particle and Electromagnetic Field

Let us start from the mechanical [20] and field Lagrange equations [21, 22]:

\[
\frac{d}{dx^0} \frac{\partial L}{\partial u} - \frac{\partial L}{\partial x} = 0; \quad \frac{\partial}{\partial x^k} \frac{\partial L}{\partial (\partial A_i/\partial x^k)} - \frac{\partial L}{\partial A_i} = 0. \tag{23}
\]

Here \( L \) is given by the expression (16); \( L \) is the density of the Lagrange function \( L = \frac{1}{c_0} A_{ij} - \frac{1}{16\pi c_0} F^{ik} F_{ik} \). Taking into account the vector equality \( \nabla(a \cdot b) = (a \cdot \nabla)b + (b \cdot \nabla)a + ax(\nabla xb) + bx(\nabla xa) \), the permutational ratios of the tensor of electromagnetic field, the expression \( \frac{\partial}{\partial x^k} \frac{\partial L}{\partial (\partial A_i/\partial x^k)} = -4F^{ik} \) and relations (13) with \( \nabla c = 0 \), we obtain the following equations of motions [17]:

\[
\frac{dp}{dt} = m_0 \frac{dv}{dt} = c_0 \frac{E}{c_0} + \frac{e}{c_0} v \times H; \tag{24}
\]

\[
\frac{dE}{dt} = e \frac{v \cdot E}{c_0} \rightarrow E(t) - E(0) = \frac{e}{c_0} \int_0^t v \cdot Ed\tau; \tag{25}
\]

\[
\nabla \times E + \frac{1}{c} \frac{\partial H}{\partial t} = 0; \quad \nabla \cdot E = 4\pi \rho; \tag{26}
\]

\[
\nabla \times H - \frac{1}{c} \frac{\partial E}{\partial t} = 4\pi \rho \frac{v}{c}; \quad \nabla \cdot H = 0.
\]

The equations (24)-(26), considered as the whole, form the set of nonlinear electrodynamics equations which describe the joint motion of an electrical charge and electromagnetic field. Taking into account the expression for the velocity of light

\[
c(t) = c_0 \sqrt{1 + \frac{v^2(t)}{c_0^2}} = c(0) \left[ 1 + \frac{e}{m_0 c_0 c(0)} \int_0^t v \cdot Ed\tau \right] = c(0) \left[ 1 + \frac{\mathcal{E}(t) - \mathcal{E}(0)}{m_0 c_0 c(0)} \right] = c(0) \frac{\mathcal{E}(t)}{\mathcal{E}(0)}, \tag{27}
\]

where \( c(0) \) is the velocity of light at \( t = 0 \), \( \mathcal{E} = m_0 c_0 c(0) \), we may rewrite the set (24)-(26) in the following form

\[
m_0 \frac{dv}{dt} = c(0) \left[ 1 + \frac{\mathcal{E}(t) - \mathcal{E}(0)}{\mathcal{E}(0)} \right] \frac{\mathcal{E}}{c_0} + \frac{e}{c_0} v \times H; \tag{28}
\]

\[
\frac{dE}{dt} = e \frac{v \cdot E}{c_0} \rightarrow E(t) - E(0) = \frac{e}{c_0} \int_0^t v \cdot Ed\tau; \]

\[
\left[ 1 + \frac{\mathcal{E}(t) - \mathcal{E}(0)}{\mathcal{E}(0)} \right] \nabla \times E + \frac{1}{c(0)} \frac{\partial H}{\partial t} = 0; \quad \nabla \cdot E = 4\pi \rho; \]

\[
\left[ 1 + \frac{\mathcal{E}(t) - \mathcal{E}(0)}{\mathcal{E}(0)} \right] \nabla \times H - \frac{1}{c(0)} \frac{\partial E}{\partial t} = 4\pi \rho \frac{v}{c(0)}; \quad \nabla \cdot H = 0.
\]
They coincide with the well-known SR equations [20] in the approximation \( |\mathcal{E}(t) - \mathcal{E}(0)|/\mathcal{E}(0) \ll 1 \) when \( c(0) = c_0 \).

5. Transformational Properties of 3-Velocity, Momentum, Energy and Electromagnetic Field

Let us start from the infinitesimal transformations (7). They induce the transformations of 4-velocity \( u_i' = L_{ik}u_k \), \( u_k = dx^k/ds \). As consequence, the formulas of transformations for the 3-velocity take the form:

\[
\begin{align*}
    v'_x &= \frac{v_x - V}{\sqrt{1 - \frac{v_x^2}{c^2}}}, \quad v'_y = v_y, \quad v'_z = v_z; \\
    v'_x &= \frac{v_x}{\sqrt{1 - \frac{v_x^2}{c^2}}}; \quad v'_y = v_y; \quad v'_z = v_z; \\
    c'\sqrt{1 - \frac{v'^2}{c'^2}} &= c\sqrt{1 - \frac{v^2}{c^2}}; \quad v'^2 = \frac{(v - V)^2 - (V\cdot v)^2}{1 - \frac{v^2}{c^2}}. 
\end{align*}
\] (29)

In accordance with these results, the velocity \( v \) does not exceed the velocity of light \( c \), i.e. \( v \leq c \) and analogously \( v' \leq c' \). The velocity of light \( c' \) in the reference frame \( K' \) corresponds to the velocity of light \( c \) in the frame \( K \) (as in SR). If \( c' = c \), all these formulas go into SR formulas [20]. Bearing in mind that \( c' = c(1 - Vv_x/c^2)/(1 - V^2/c^2)^{1/2} \), we find the transformational properties of 3-velocity in the present work:

\[
\begin{align*}
    v'_x &= \frac{v_x - V}{\sqrt{1 - \frac{v_x^2}{c^2}}}; \quad v'_y = v_y; \quad v'_z = v_z; \\
    c'\sqrt{1 - \frac{v'^2}{c'^2}} &= c\sqrt{1 - \frac{v^2}{c^2}}; \quad v'^2 = \frac{(v - V)^2 - (V\cdot v)^2}{1 - \frac{v^2}{c^2}}. 
\end{align*}
\] (31)

The first formula (32) also follows from the expression \((c'^2 - v'^2)dt'^2 = (c^2 - v^2)dt^2 = c_0^2dt_0^2\) for 4-interval \( ds^2 \), when \( dt_0 = dt = dt' \) in accordance with (4).

Analogously, from the expression \( p'^i = L_{ik}^ip^k \) one can be obtained the formulas, describing transformational properties of the 3-momentum \( p \) and energy \( \mathcal{E} \):

\[
\begin{align*}
    p'_x &= \frac{p_x - V\mathcal{E}/c_0c}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad p'_y = p_y, \quad p'_z = p_z; \quad \mathcal{E}' = \frac{\mathcal{E} - Vp_xc_0/c}{\sqrt{1 - \frac{v^2}{c^2}}}. 
\end{align*}
\] (33)

If here \( c' = c = c_0 \), we have the SR theory formulas [20].

The transformational properties of the density of electrical charge and electromagnetic field may be obtained from the expressions \( j'^i = L_{ik}^ij^k \) and \( F'_{ik} = \)
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$L^1 L^m_k F_{lm}$ in the case of the free motion of electrical charge when $c = \text{const}$:

$$\rho' = \rho - \frac{vV}{\sqrt{1 - \frac{V^2}{c^2}}}.$$  \hspace{1cm} (34)

$$E_x' = E_x; \quad E_y' = \frac{E_y - \frac{V H}{c}}{\sqrt{1 - \frac{V^2}{c^2}}}; \quad E_z' = \frac{E_z + \frac{V H}{c}}{\sqrt{1 - \frac{V^2}{c^2}}};$$

$$H_x' = H_x; \quad H_y' = \frac{H_y + \frac{V E_z}{c}}{\sqrt{1 - \frac{V^2}{c^2}}}; \quad H_z' = \frac{H_z - \frac{V E_y}{c}}{\sqrt{1 - \frac{V^2}{c^2}}}.$$ \hspace{1cm} (35)

Formally they are common both for SR theory [20] and for the present work. The expression (34) may be also written in the form $\rho'/c' = \rho/c$. Using the method of replacement of variables, the transformational properties of electromagnetic field can be applied to prove the invariance of Maxwell equations (26) with respect to space-time transformations [11].

6. Energy and Faster-than-Light Motion

Let us begin with the expression $v = \sqrt{E^2 - m_0^2 c_0^2}/m_0 c_0 > c_0$. It follows from here that in the framework of the present work a particle will move at the faster-than-light velocity, if the particle energy satisfies the condition:

$$\mathcal{E} > \sqrt{2E_0} = \sqrt{2m_0 c_0^2}.$$  \hspace{1cm} (36)

($\mathcal{E}_0 = m_0 c_0^2$), The energy $\mathcal{E} = \sqrt{2\mathcal{E}_0}$ is equal $\sim 723$ keV for the electron ($\mathcal{E}_0 \sim 510$ keV) and $\sim 1330$ Mev for the proton and neutron ($\mathcal{E}_0 \sim 938$ MeV). From here we may conclude that in the present work the neutron physics of nuclear reactors may be formulated in the approximation $v < c_0$ as in SR. The electrons with the energy $\mathcal{E} > 723$ keV would be faster-than-light particles (for example, the velocity of the 1 GeV electron would be $\sim 2000$ $c_0$); the particle physics on accelerators with the energy of protons more than 1.33 GeV would be the physics of faster-than-light motions if the present theory were realized in the field of validity of SR. The examples of using the space-time transformations [11] for the interpretation of Michelson, Fizeau and some other experiments, as well as for the interpretation of aberration of light and of Doppler effect, decay of unstable particles and creation new particles, faster-than-light motion of nuclear reaction products are given in [17].

7. Conclusion

The $L_6 \Delta_1 \triangle$ invariant theory has been constructed, where $L_6$ is the Lorentz group, $\Delta_1$ is the scale transformation group of the velocity of light $c' = \gamma c$. The field of application of the theory is yet unknown in the present time. Nevertheless in
according with the Blokhintsev papers [1] we may assume that the proposed theory will prove to be useful in the field of quantum physics of dimensional particles, where the property of elementary nature should not contradict to the existence of internal structure of the particle. Indeed, the elementary particles should be points in the $L_6$ invariant theory (SR) because of a finiteness of the speed of light $c_0$. In the $L_6 X \Delta_1$ invariant theory this requirement is not necessary because of the absence of the limit to the velocity of light $c$. The postulation $c' = c$ leads to SR.

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