Comment on “Dynamic Opinion Model and Invasion Percolation”

Arsalan Sattari, Maya Paczuski, and Peter Grassberger

1Complexity Science Group, University of Calgary, Calgary, Canada

(Dated: May 2, 2014)

In [1] Shao et al. claim, based on low statistics simulations, that a model with majority rule coarsening exhibits in $d = 2$ a percolation transition in the universality class of invasion percolation with trapping (IPT). They report also that the system reaches its final state rapidly with no diverging time scale. Since the original configurations are random and thus in the ordinary percolation (OP) universality class, it seems unlikely that long range correlations could develop in a finite time that would change this. Indeed it was proved rigorously [2] that similar 2-d models (called “dependent percolation” in [2]) belong to the OP universality class.

Here we present high statistics (up to $L = 2^{14}$, $> 10^4$ realizations) on $L \times L$ square lattices and confirm that the phase transition is in the OP universality class, thus refuting a central tenet of [1]. Initially each site $i$ is randomly assigned one of two opinions (or spins): $\sigma_i = +1$ with probability $f$, otherwise $\sigma_i = -1$. At each time step all sites are updated in parallel. If at least three of their four neighbors disagree with them, they change their opinion, otherwise they keep it. As noted in [1] this leads quickly (within $O(10)$ time steps) to a static state, except for sites that flip permanently with period 2. The critical probability $f_c$ where a “+1” cluster percolates depends slightly on how these flicker sites are treated (we treat them as “+”, if $\sigma_i = +1$ at even times), but the universal properties do not.

We first determine $f_c$ by measuring the chance that a cluster in the final state percolates through lattices with open boundary conditions. Using finite size scaling [3], we obtain $f_c = 0.506425(20)$, in agreement with the less precise estimate of [1]. After that, we measure the distribution of cluster sizes with $\sigma = +1$ in final states obtained with helical b.c. for $f \approx f_c$. Figure 1 confirms the above estimate of $f_c$ and shows that the data are excellently described by a power law $P(s) \sim s^{-\tau}$ with the OP critical exponent $\tau = 187/91 \approx 2.055$, ruling out the IPT exponent $\tau \approx 1.89$. We see also deviations from this power law at small masses $s$, as small clusters are eliminated by the coarsening. This, together with using open boundary conditions and neglecting finite size corrections, explains why $\tau$ was underestimated in [1]. A data collapse of the r.h.s peaks in Fig.1 gives $D_f = 1.895(15)$ as for OP, but in disagreement with IPT. Notice that the exponents obtained in [1] strongly violate the hyperscaling relation $\tau = d/D_f + 1$.

Shao et al. claim that IPT is relevant because local clusters get trapped. The difference between OP and IPT is that clusters can grow both outwards and inwards (into empty holes) in OP, while they can only grow outward in IPT. In this respect the model of [1] is exactly as OP.

We also simulated the process on random Erdös-Rényi networks. For small average degrees we confirm the claim of [1] that the percolation transition is in the OP class. But for large average degrees we find an unexpected first order transition [4].

As a model for opinion dynamics the model is of limited interest, since the dynamics leaves essentially unchanged all large clusters present in the initial state – except for clusters with hubs, in case of scale-free networks, who immediately adopt the majority opinion.

We thank a referee for pointing out the violation of hyperscaling in [1].

[1] J. Shao et al., Phys. Rev. Lett. 103, 18701 (2009)
[2] F. Camia et al., Commun. Math. Phys. 246, 311 (2004)
[3] R. Ziff and M. Newman, Phys. Rev. E 66, 016129 (2002)
[4] A. Sattari et al., to be published(2012)