Fuzzy cross-model cross-mode method and its application to update the finite element model of structures

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Abstract. As a novel updating technique, cross-model cross-mode (CMCM) method possesses a high efficiency and capability of flexible selecting updating parameters. However, the success of this method depends on the accuracy of measured modal shapes. Usually, the measured modal shapes are inaccurate since many kinds of measured noises are inevitable. Furthermore, the complete testing modal shapes are required by CMCM method so that the calculating errors may be introduced into the measured modal shapes by conducting the modal expansion or model reduction technique. Therefore, this algorithm is faced with the challenge of updating the finite element (FE) model of practical complex structures. In this study, the fuzzy CMCM method is proposed in order to weaken the effect of errors of the measured modal shapes on the updated results. Then two simulated examples are applied to compare the performance of the fuzzy CMCM method with the CMCM method. The test results show that proposed method is more promising to update the FE model of practical structures than CMCM method.

1. Introduction
FE model updating of structures using vibration test data has received considerable attentions in recent years due to its crucial role in fields ranging from establishing a reality-consistent structural model for dynamic analysis and control, to providing baseline model for damage identification in structural health monitoring. Structural model updating is to correct the analytical finite element model using test data to produce a refined one that better predict the dynamic behavior of structure. FE model updating of structures usually ends up with a nonlinear optimization problem. Many techniques have been developed to address the model updating problem, as discussed by Mottershead and Friswell [1, 2].

Present FE model updating methods can be broadly divided into two categories based on whether it needs amounts of iterative calculation. One is called direct updating method and the other is iterative updating method. Usually, the former can directly obtain the global stiffness and mass matrix by solving constrained optimization problems. Such a method does not require amounts of iterative search, which has high efficiency, but the updated model cannot be interpreted in a physical way. The later is easier to select the updating parameters, and the updated structure has explicit physical meaning. However, the disadvantage of iterative method is that the process of model updating usually requires a lot of iteration, and sometimes it is easy to fall into the local optimal results.
CMCM method [3] is a novel model updating method. From the perspective of solution method, this method is similar to the direct updating method since CMCM method does not require iterative search. At the same time, this method is deemed as the iterative updating method by reason of flexible choosing a variety of updating parameters. Therefore, CMCM method is a direct and efficient updating method for updating the physical parameters of structures. However, the success of this method depends on the accuracy of measured modal shapes. The measured modal shapes are usually inaccurate since, on one side, many kinds of measured noises are inevitable to identify the modal shapes; on the other side, of the calculating errors are generated by conducting the modal expansion or model reduction technique in order to obtain the complete modal shapes. Therefore, this algorithm is faced with the big challenge of updating the FE model of practical structures. In order to improve the problem of CMCM method, this paper proposes an improved method called the fuzzy CMCM method.

2. CMCM updating method

2.1. Introduction

Assuming that the mass and stiffness matrices of the structure are denoted by \( M \) and \( K \) respectively, and the mass and stiffness matrices of the actual model are denoted by \( M^\ast \) and \( K^\ast \). And then the stiffness matrix of the actual (experimental) model is a modification to be formulated as,

\[
K^\ast = K + \sum_{n=1}^{Ne} A_n K_n
\]  

where \( K_n \) is the stiffness matrix corresponding to the \( n \)th element, \( A_n \) are correction factors to be determined, and \( Ne \) is the number of updating elements. Here, for simplicity in presentation, it is assumed that each element involves a parameter to be updated, such as the Young’s modulus of each element.

Likewise, one writes the corresponding expression for a updated mass matrix \( M^\ast \) as,

\[
M^\ast = M + \sum_{n=1}^{Ne} B_n M_n
\]

where \( M_n \) is the mass matrix corresponding to the \( n \)th element, and \( B_n \) are correction parameters to be determined. Again, it is assumed that each element involves a mass quantity to be corrected, such as the mass density of each element.

Using the FE model of the actual structure, the characteristic equation may be defined as,

\[
K^\ast \phi_j = \lambda_j^\ast M^\ast \phi_j
\]

where \( \lambda_j^\ast \) is the \( j \)th measured eigenvalue obtained from modal tests. \( \phi_j^\ast \) is the \( j \)th measured mode shape from modal tests.

\[
K \phi_i = \lambda_i M \phi_i
\]

where \( \lambda_i \) is the \( i \)th eigenvalue of the initial analytical FE model. \( \phi_i \) is the \( i \)th modal shape of the initial analytical FE model.

If \( N_a \) analytical modes and \( N_m \) measured modes are applied to update the FE model of structures, the following equation is obtained (the detail can be found in reference [3]),

\[
CA + EB = f
\]
where $C$ and $E$ are $N_m \times N_e$ matrix ($N_m = N_j \times N_j$), and $A$ and $B$ are column vectors of size $N_e$, and $f$ is a column vector of size $N_m$. And $E_{n,m} = -b_n D_{n,m}$ and $f_m = b_m - \frac{\lambda_j^*}{\lambda_i}$, and

$$C_{n,ij} = \frac{(\varphi_i)^T K \varphi_{j}^*}{(\varphi_i)^T K \varphi_{j}^*} \quad (6)$$

$$D_{n,ij} = \frac{(\varphi_i)^T M \varphi_{j}^*}{(\varphi_i)^T M \varphi_{j}^*} \quad (7)$$

Furthermore, one can rewrite Eq. (15) as

$$G\gamma = f$$

where

$$G = [C \ E] \text{ and } \gamma = \begin{bmatrix} A \\ B \end{bmatrix} \quad (9)$$

If $N_m > 2N_e$, the least-squares solution can be taken as,

$$\hat{\gamma} = (G^T G)^{-1} G^T f$$

Using Eq. (17), the updated results may be obtained. In addition, if assuming the mass matrix of the finite element model is more accurate ($M^* = M$), the following equation can be obtained,

$$CA = f$$

Similar, if assuming the stiffness matrix of FE model is more accurate($K^* = K$), the following equation can be taken as

$$EB = f$$

In addition, assuming that $M$ represents the main freedom and $S$ represents the subsidiary freedom, the structure of the test modes can be transformed into complete modals by the following equation,

$$\varphi_j = \begin{bmatrix} (\varphi_j)_M \\ (\varphi_j)_S \end{bmatrix} = T(\varphi_j)_M \quad (13)$$

where $T$ is transformation matrix and $\varphi_j$ is the $j$th test mode of the structure. Therefore Eq. (12) and (13) can be replaced by

$$C_{n,ij} = \frac{\left( (\varphi_i)_M^T T^T K_n T (\varphi_{j}^*)_M \right)}{\left( (\varphi_i)_M^T T^T K (\varphi_{j}^*)_M \right)} \quad (14)$$

$$D_{n,ij} = \frac{\left( (\varphi_i)_M^T T^T M_n T (\varphi_{j}^*)_M \right)}{\left( (\varphi_i)_M^T T^T M (\varphi_{j}^*)_M \right)} \quad (15)$$
2.2. The challenges of CMCM algorithm
As the CMCM method is direct updating method, the successes of this method depends on the accuracy of the Eq. (10), i.e., the accuracy of $C_{n,ij}$ and $D_{n,ij}$. As shown in Eqs (6) and (7), the accuracy of $C_{n,ij}$ and $D_{n,ij}$ depends upon the accuracy of measured mode shape $\phi_j^*$, which are influenced by the following two factors. One, the inevitable measure noises decrease the accuracy of measured modal shapes. Two, some errors are introduced into modal shapes by using the method of mode expansion and model reduction. Therefore, this study adopts the idea of fuzzy method, and the fuzzy CMCM method is proposed.

3. Fuzzy CMCM method

3.1. Fuzzy sets and membership functions
The fuzzy set is an extension of classical set theory. While a classical set clearly distinguish its members from the non-members, a fuzzy set introduces the degree of membership, described by the membership function, to weigh the possibility of members belonging to the set. For a fuzzy set $\tilde{x}$, the membership function is defined as for all $x$ in domain $\mathbb{X}$,

$$\tilde{x} = \{(x, \mu_x(x)) | x \in \mathbb{X}, \mu_x(x) \in [0,1]\} \quad (16)$$

where $\mu_x(x)$ indicates that element $x$ is a member of the set $\tilde{x}$. $\mu_x(x) = 0$ means that $x$ is not a member of the set $\tilde{x}$. For $0 < \mu_x(x) < 1$, element $x$ is not surely a member or not a member of set $\tilde{x}$, but belongs to set $\tilde{x}$ with a certain degree of membership.

3.2. Procedure of fuzzy CMCM method
The measured modal shapes can be seen as fuzzy variables owing to the existence of measured noises. If the mode shapes are fuzzy variables, $C_{n,ij}$ and $D_{n,ij}$ shown in Eqs. (14) and (15) are deemed as fuzzy variables in order to describe the uncertainty of modal $C_{n,ij}$ and $D_{n,ij}$. Therefore, the basic idea of fuzzy CMCM is described as follows. Firstly, parameters $\tilde{C}_{n,ij}$ and $\tilde{D}_{n,ij}$ are transformed into fuzzy variables. Secondly, a set of updated model is obtained by updating the FE model considering different fuzzy degrees of $\tilde{C}_{n,ij}$ and. Finally, a updated FE model may be selected from the set of updated FE models, which meets the clear physical meaning of the practical structures. The specific description of fuzzy CMCM is shown as follows.

Using the fuzzification of parameters $C_{n,ij}$ and $D_{n,ij}$ shown in Eqs. (14) and (15), the following equations are defined as,

$$\tilde{C}_{n,ij} \equiv \left(\phi_i^\top\right)^T K \phi_j^* \quad (17)$$

$$\tilde{D}_{n,ij} \equiv \left(\phi_i^\top\right)^T M \phi_j^* \quad (18)$$

where symbol $\equiv$ means fuzzy equality.

In this study, the middle-form Cauchy distribution function (Fig. 1) is applied to be the membership function of fuzzy variable $\tilde{C}_{n,ij}$, which is defined as,
Here, there are two reasons that the middle-form Cauchy distribution function is used to describe the fuzzy degree of parameter $\tilde{C}_{n,j}$. On one side, if the mode shapes have no error, and then $\tilde{C}_{n,j}$ is equal to $C_{n,j}$. On the other side, with the increase of errors contained in modal shapes, the fuzzy degree of $\tilde{C}_{n,j}$ would increase. At a certain fuzzy degree, the value of $\tilde{C}_{n,j}$ would both increase and decrease. Similarly, the membership function of $\tilde{D}_{n,j}$ can be shown as follows

$$
\mu_{\tilde{C}_{n,j}}(x) = \begin{cases} 
\frac{1}{1 + (x - C_{n,j})^2} & (x \neq C_{n,j}) \\
1 & (x = C_{n,j})
\end{cases} \quad (19)
$$

As shown in Fig.1, at a certain fuzzy level $\alpha$, determined parameters $\tilde{C}_{n,j}$ and $\tilde{D}_{n,j}$ of fuzzy variable $\tilde{C}_{n,j}$ can be obtained (The same as $\tilde{D}_{n,j}$ and $\tilde{C}_{n,j}$). Therefore, at different fuzzy level $\alpha$, different updated models may be obtained by updating the FE model of structures with parameters $\tilde{C}_{n,j}$ and $\tilde{D}_{n,j}$.

It should be noted that all the $\tilde{C}_{n,j}$ can take the same fuzzy level since the fuzzy degree of $\tilde{C}_{n,j}$ is determined by the uncertainty of $j$th modal shape of structures. Therefore, if choosing $J$ ($j=1,2,\cdots,J$) modes of modal shapes during FE model updating, then the number of combinations of $C_{n,j}^a$ is $2^J$ at a given fuzzy level. For example, if $J$ takes 2, the number of combinations goes to 4, which are $\{\tilde{C}_{n,j1}^a, \tilde{C}_{n,j2}^a\}$, $\{\tilde{C}_{n,j1}^a, \tilde{C}_{n,j2}^a\}$, $\{\tilde{C}_{n,j1}^a, \tilde{C}_{n,j2}^a\}$ and $\{\tilde{C}_{n,j1}^a, \tilde{C}_{n,j2}^a\}$ respectively. In addition, because $\tilde{C}_{n,j}$ and $\tilde{D}_{n,j}$ can be determined by the same the mode shapes, $\tilde{C}_{n,j}$ and $\tilde{D}_{n,j}$ may take the same fuzzy level.
The procedure of fuzzy CMCM algorithm is described in detail as follows.

Step 1: Give the membership function of fuzzy parameters $\tilde{C}_{n,j}$ and $\tilde{D}_{n,j}$.

Step 2: Determine the modes of analytical modal shapes $I_i (i = 1, 2, \cdots, I)$ and measured modal shapes $J_j (j = 1, 2, \cdots, J)$.

Step 3: Give the fuzzy level of $j$th measured modal shapes $\alpha_{j}$, and the initial fuzzy level $\alpha_{j0}$ takes 1.

Step 4: At the $t$th iteration, the fuzzy level $\alpha_{jt} = \alpha_{jt-1} - \Delta \alpha_{j}$, and then determine the corresponding parameter values of $\tilde{C}_{n,j}$, $\tilde{C}_{n,j}$, $\tilde{D}_{n,j}$ and $\tilde{D}_{n,j}$.

Step 5: According to Step 4, $2^J$ combinations of parameters the $J$th measured modal shapes can be obtained by fuzzification. And different updated models are obtained by taking different combination into Eq. (10).

Step 6: If the updated results satisfy the convergence conditions, the optimal search is stop. Else, repeat steps (4) to (6) until the convergence conditions are satisfied.

4. Numerical examples

In this section, both CMCM correction method and fuzzy CMCM correction method are applied to update the FE model of two stimulation examples in order to compare the performance of two methods. Among them, the modified example 1 uses comprehensive test modes. Modified example 2 uses incomplete test modes. Also, each example is used with different modes of error correction tests to compare the performance of two methods.

4.1. A 5- story shear structure

The first example is a 5-story shear structure that is the same as the one shown in paper [1]. As shown in Fig.2, the structure consists of five elements. Element stiffness matrix is defined as:

$$
\mathbf{k}_n = \begin{bmatrix}
  k_n & -k_n \\
  -k_n & k_n
\end{bmatrix}, \quad (n = 1, 2, 3, 4, 5)
$$

According to the finite element theory, the global stiffness matrix of the structure is defined as,
where \( K \) is the element stiffness matrix in global coordinate. \( k_1 = k_2 = \cdots = k_5 = 2.9 \times 10^7 \text{N/m} \).

Assuming that the mass of each story is \( m_n \), the global mass matrix of this structure is defined as,

\[
M = \sum_{n=1}^{5} M_n = \begin{bmatrix}
n_1 & 0 & 0 & 0 & 0 \\
0 & n_2 & 0 & 0 & 0 \\
0 & 0 & n_3 & 0 & 0 \\
0 & 0 & 0 & n_4 & 0 \\
0 & 0 & 0 & 0 & n_5
\end{bmatrix}
\]

where \( M_n \) is the element mass matrix in global coordinate. \( n_1 = n_2 = \cdots = n_5 = 260 \text{ kg} \), \( m_5 = 220 \text{ kg} \).

Using the stiffness matrix and mass matrix shown in Eqs. (22) and (23), the frequencies of this structure are obtained, which are listed in Table 1. For simplicity, this study assumes that modeling error comes mainly from the elements of stiffness matrix, while the mass matrix is accurate. According to the results of paper [3], stiffness correction factors of the real structure are \( A_1 = -0.0433 \), \( A_2 = -0.1666 \), \( A_3 = 0.0125 \), \( A_4 = 0.0288 \) and \( A_5 = -0.1146 \) respectively. Using the stiffness correction factors, the simulated measured frequencies of this structure are shown in Table 1.

| Mode | 1   | 2   | 3   | 4   | 5   |
|------|-----|-----|-----|-----|-----|
| Analytical frequency (Hz) | 15.56 | 45.26 | 70.89 | 90.34 | 102.30 |
| Measured frequency (Hz)    | 15.00 | 44.61 | 68.38 | 85.77 | 100.79 |

Assuming that the measured modal shapes are complete, but there exist different levels of test noise. Three different level of noise (10%, 20% and 30%) are considered during FE mode updating. The simulated modal shapes with noise are defined by the following equation [4],

\[
\vec{\phi}_{hj} = \phi_{hj}^* \left(1 + \mu R \phi_{max,j}^*\right)
\]

where \( \vec{\phi}_{hj} \) represents the \( h \)th element of the \( j \)th measured modal shape with noise. \( \phi_{hj}^* \) represents the \( h \)th element of the \( j \)th measured modal shape without noise. \( \mu \) is a random number whose mean value and variance are 0 and 1 respectively. \( \mu \) is the noise level. \( \phi_{max,j}^* \) represents the maximum element of the \( j \)th measured modal shape.

| Updating parameters | \( A_1 \) | \( A_2 \) | \( A_3 \) | \( A_4 \) | \( A_5 \) |
|---------------------|-----------|-----------|-----------|-----------|-----------|
| Real values         | -0.0433   | -0.1666   | 0.0125    | 0.0288    | -0.1146   |
| Updated values (10% noise level) | -0.0507 | -0.1734 | 0.0047 | 0.0166 | -0.1239 |
| Updated values (20% noise level) | -0.0565 | -0.1861 | -0.0006 | 0.0113 | -0.1288 |
| Updated values (30% noise level) | -0.1264 | -0.2071 | -0.0942 | -0.0439 | -0.1859 |
Here, the CMCM and fuzzy CMCM method both are applied to update the FE model of this shear structures, and each method used the modal shapes (the first two measured modal shapes). The updated results obtained by CMCM under different noise level are listed in Table 2, and the updated results acquired by fuzzy CMCM under different noise level are listed from Table 3 to Table 5.

### Table 3 Updated results obtained by fuzzy CMCM (10% noise level)

| Updating parameters | Real values | Fuzzy level $\alpha$ |
|---------------------|-------------|----------------------|
|                     | $\alpha_1=0.9900$ | $\alpha_2=0.9898$ | $\alpha_1=0.9694$ | $\alpha_2=0.4457$ | $\alpha_1=0.4102$ | $\alpha_2=0.9665$ | $\alpha_1=0.6625$ | $\alpha_2=0.4824$ | $\alpha_1=0.4536$ | $\alpha_2=0.4792$ |
| $A_1$               | $-0.0433$   | $-0.0488$            | $-0.1041$            | $-0.0311$            | $-0.2029$            | $-0.1949$            |
| $A_2$               | $-0.1666$   | $-0.1616$            | $-0.0196$            | $0.0943$             | $-0.1284$            | $-0.1472$            |
| $A_3$               | $0.0125$    | $0.01658$            | $-0.0572$            | $-0.0860$            | $-0.1502$            | $-0.1409$            |
| $A_4$               | $0.0288$    | $0.0285$             | $0.0241$             | $-0.1001$            | $-0.0997$            | $-0.1010$            |
| $A_5$               | $-0.1146$   | $-0.1020$            | $-0.0581$            | $0.0521$             | $-0.0885$            | $-0.0905$            |

Note: $\alpha_1$ and $\alpha_2$ represent the fuzzy degree of the first modal shapes respectively.

### Table 4 Updated results obtained by fuzzy CMCM (20% noise level)

| Updating parameters | Real values | Fuzzy level $\alpha$ |
|---------------------|-------------|----------------------|
|                     | $\alpha_1=0.9700$ | $\alpha_2=0.9798$ | $\alpha_1=0.7943$ | $\alpha_2=0.8554$ | $\alpha_1=0.8603$ | $\alpha_2=0.4125$ | $\alpha_1=0.9887$ | $\alpha_2=0.6742$ |
| $A_1$               | $-0.0433$   | $-0.0424$            | $-0.0645$            | $0.0002$             | $0.0089$             | $-0.2535$            |
| $A_2$               | $-0.1666$   | $-0.1619$            | $0.0658$             | $-0.1309$            | $-0.1241$            | $-0.2002$            |
| $A_3$               | $0.0125$    | $0.0135$             | $-0.1202$            | $0.0564$             | $0.0644$             | $-0.1961$            |
| $A_4$               | $0.0288$    | $0.0254$             | $-0.1329$            | $0.0661$             | $0.0780$             | $-0.1569$            |
| $A_5$               | $-0.1146$   | $-0.1046$            | $0.0110$             | $-0.0680$            | $-0.0727$            | $-0.1418$            |

### Table 5 Updated results obtained by fuzzy CMCM (30% noise level)

| Updating parameters | Real values | Fuzzy level $\alpha$ |
|---------------------|-------------|----------------------|
|                     | $\alpha_1=0.9763$ | $\alpha_2=0.9695$ | $\alpha_1=0.9073$ | $\alpha_2=0.7245$ | $\alpha_1=0.8851$ | $\alpha_2=0.7160$ | $\alpha_1=0.5498$ | $\alpha_2=0.7058$ | $\alpha_1=0.5950$ | $\alpha_2=0.6742$ |
| $A_1$               | $-0.0433$   | $-0.0656$            | $-0.0264$            | $-0.0465$            | $-0.2240$            | $-0.2247$            |
| $A_2$               | $-0.1666$   | $-0.1466$            | $-0.1101$            | $-0.1698$            | $-0.1917$            | $-0.1854$            |
| $A_3$               | $0.0125$    | $-0.0334$            | $0.0054$             | $0.0154$             | $-0.1778$            | $-0.1775$            |
| $A_4$               | $0.0288$    | $0.0168$             | $0.0555$             | $0.0254$             | $-0.1421$            | $-0.1427$            |
| $A_5$               | $-0.1146$   | $-0.1250$            | $-0.0871$            | $-0.0867$            | $-0.1506$            | $-0.1439$            |

From the results shown from Table 2 to Table 5, with the increasing of noise degree, the results in CMCM method deviates from the true results gradually. Comparing with the CMC method, a set of different updated results is obtained. As shown from Table 3 to Table 5, at the fuzzy level $\alpha_1=0.9900$, $\alpha_2=0.9898$ (Table 3), the fuzzy level $\alpha_1=0.9700$, $\alpha_2=0.9798$ (Table 4) and the fuzzy level $\alpha_1=0.8851$, $\alpha_2=0.7160$ (Table 5), the updated results are close to the real results. Therefore, comparing with the CMCM method, the fuzzy CMCM method has stronger ability to resist the measured noise exiting in modal shapes.
4.2. Two-dimension truss structure

The other simulated example is a two-dimension truss structure. The structure consists of 12 nodes and 21 rods. The outer diameter of each pipe rod is $1.71 \times 10^{-2}$ m and the thickness is $3.14 \times 10^{-3}$ m. Detailed dimension and the number of rods are shown in Fig. 3.

| Mode | 1  | 2  | 3  | 4  | 5  |
|------|----|----|----|----|----|
| Analytical frequency (Hz) | 141.87 | 343.50 | 449.93 | 651.87 | 887.18 |
| Measured frequency (Hz)   | 135.60 | 328.00 | 430.00 | 627.66 | 862.12 |

When building up the FE model of this truss structure, each node of the element is considered as 3 degrees of freedom (DOF). The first five frequencies obtained by FE analysis are listed in Table 6. Assuming that the modeling error comes mainly from the stiffness of each rod, the mass of each rod is accurate. Here, the stiffness correction factors of the real truss structure are shown in Fig. 4, and the first five measured frequencies are shown in Table 6.

Assuming that only the DOF along Y direction of each node (Fig. 3) is measured and the rotational DOF and DOF along Y direction of each node are not measured. Therefore, the measured modal shapes are not complete. In this section, the GUYAN method [5] is applied to extend the modal shapes firstly, and then the FE model of this structure is updated by two updating methods. During the procedure of model updating, the first two modes of extended modal shapes and all the analytical modal shapes are used in two methods, and the noise level of measured modal shapes is 20%.

From the results in Fig. 4, when the measured modal shapes contain both measurement noise and calculation error of modal expansion, the fuzzy CMCM method is more prone to obtain the ideal updated results than the CMCM method.

5. Conclusion

CMCM method is a novel updating method, however, the success of this method depends on the accuracy of the measured modal shapes. Usually, modal shapes inevitably contain varying errors such as measured noise and calculation error of modal expansion. Therefore, the CMCM faces the big challenge to update the FE model of the practical structures. To overcome the disadvantages of CMCM method, the fuzzy CMCM is proposed in this study.

In the fuzzy CMCM method, parameters $C_{n,j}$ and $D_{n,j}$ shown in Eqs. (14) and (15) are deemed as fuzzy variables i.e. $\tilde{C}_{n,j}$ and $\tilde{D}_{n,j}$. By updating the FE model considering different fuzzy degrees of
and \( \tilde{\mathbf{D}}_{n,i,j} \), a set of updated models are obtained, which represent different error level of modal shapes. The most reasonable updated model may be chosen from the set of updated results. Finally, two simulated examples are applied to test the performance of two methods, and the testing results shown that the fuzzy CMCM method is more promising to update the FE model of practical structures than CMCM method.

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