K-Inflation with a Dark Energy Coupling

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It is usually thought that the quintessence as a fundamental scalar field was already present during the inflationary epoch. While there are various models in which the quintessence couples to other species, it is attractive to anticipate a coupling between the quintessence and the inflaton in the very early universe as well. We consider such a coupling in the context of k-inflation. The coupling function and the potential of the quintessence are chosen to be of inverse power law forms. We show such a coupling affects the speed of sound for the inflaton field as well as the power spectra of perturbations.

I. INTRODUCTION

In the context of the inflationary universe scenario [1, 2, 3, 4], the universe experienced a quasi-exponential expansion at the very early stages of its evolution, leading to the generation of density perturbations via quantum fluctuations. These small density perturbations seed the formation of large scale structure of our universe. Various kinds of interesting inflationary models could successfully solve the flatness and homogeneity problems in the hot Big Bang theory. K-inflation model is one of such candidates. It was first proposed in [5, 6]. Unlike traditional inflation models with a canonical kinetic term, in k-inflation paradigm the expansion is driven by a noncanonical kinetic term. In k-inflation, the potential of the scalar field is not necessary any more.

In the past decade, several independent but complementary cosmological observations including supernovae(SN) IA [7], large-scale structure(LSS) [8], and cosmic microwave background (CMB) anisotropy [10], have provided strong evidences to indicate that our universe has recently again embarked upon an epoch of accelerating expansion, driven by so-called dark energy. The nature of dark energy remains a mystery. The most well-known candidate of dark energy is the cosmological constant. Even though such a candidate is still consistent with the current data of observation, it suffers from fine-tuning and coincidence problem. Another class of candidates of dark energy is to take the dark energy as a dynamical scalar field. Among various dynamical dark energy models [11], quintessence could be the most famous one [12]. Quintessence is described by an ordinary scalar field minimally coupled to gravity but with particular potential that leads to the late time acceleration.

The quintessence field is usually thought to have already existed during the inflationary stage. Therefore, it is natural to ask whether there is a coupling between the quintessence field and the inflaton field, and what is the physical implications of such a coupling. In [13] the authors investigated the initial conditions for the quintessence field under the assumption that the quintessence field is sufficiently weakly coupled and is not affected by the inflaton decay at the end of inflation, they found that the quintessence field was typically driven to large values. In [14], the authors investigated the effects of perturbations in a dark energy component with a constant equation of state on large scale CMB anisotropy, and found that the inclusion of the perturbations would increase the power spectrum. In these studies, there is no direct coupling between dark energy and the inflation field. Very recently, in [15] the authors considered a quintessence field coupled directly to the inflaton field with a square potential. In their model, both the inflation field and the dark energy field were chosen to be canonical scalar fields, and the coupling function was chosen to be exponential form. They found the power spectra of perturbations were slightly modified.

In this paper, we construct a phenomenological model: a k-inflation model with a quintessence field coupled to it. The coupling function are chosen to be inverse power law form. In our model, the quintessence field has a heavy effective mass, and settles in its effective potential. In next section, we set up our model and discuss the dynamics of the quintessence field and the evolution of the scale factor. In section III, we study the perturbations of our model and calculate the power spectra. We find that the influence of the quintessence could be encoded in a modified speed of sound. As an effect, the power spectra turn to be a little larger. At the end of section III, we use the WMAP data [16] to restrict the parameters in our model. In section IV, we justify our assumption via numerical analysis. We end with conclusions.

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II. K-INFLATION WITH A DARK ENERGY COUPLING

The action of the theory we consider is
\[
S = \int \! d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R + \mathcal{L}_Q + \mathcal{L}_\phi \right]
\]
with
\[
\mathcal{L}_Q = -\frac{1}{2} g^{\mu\nu} (\partial_\mu Q)(\partial_\nu Q) - V(Q)
\]
\[
\mathcal{L}_\phi = A(Q)F(\phi, X).
\]

Where \( R \) is the Ricci scalar, and \( \phi \) is the inflaton field. \( Q \) is another scalar field which possibly serves the role of dark energy and \( g \) is the determinant of the metric tensor. \( X \) is defined by
\[
X = -\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi).
\]

In k-inflation models, the pressure of the inflaton field is usually written as \( F(\phi, X) \). In this model, we investigate the possibility that the dark energy field contributes an extra factor \( A(Q) \) to the pressure \( F(\phi, X) \). In this case, the pressure of the inflaton field is written as \( A(Q)F(\phi, X) \), where \( Q \) denotes the dark energy field. The equations of motion for the field \( \phi \) and \( Q \) in a homogeneous and isotropic universe are
\[
(F_X + 2XF_{XX})\ddot{\phi} + 3HF_X \dot{\phi} + (2X \frac{\partial F_X}{\partial \phi} - \frac{\partial F}{\partial \phi}) = -\frac{F_X}{A(Q)} \frac{\partial A(Q)}{\partial Q} \dot{Q}\dot{\phi},
\]
(1)
\[
\ddot{Q} + 3H \dot{Q} + \frac{\partial V(Q)}{\partial Q} - \frac{\partial A(Q)}{\partial Q} F(\phi, X) = 0.
\]
(2)

Here \( F_X \) and \( F_{XX} \) refer to \( \frac{\partial F(\phi, X)}{\partial X} \) and \( \frac{\partial^2 F(\phi, X)}{\partial X^2} \) respectively. \( H \) is the Hubble parameter, given by \( \frac{\dot{a}}{a} \).

In this paper, we choose the factor \( A(Q) \) to be a dimensionless coupling function
\[
A(Q) = A_0 \left( \frac{Q}{M_{Pl}} \right)^\beta,
\]
where \( A_0 \) is a constant. We choose the potential of the quintessence field to be an inverse power-law potential,
\[
V(Q) = V_0 M_{Pl}^4 \left( \frac{Q}{M_{Pl}} \right)^{-\lambda},
\]
where \( \lambda \) is positive, and \( V_0 \) is a constant. We use the Planck units \( M_{Pl} = 1 \) throughout this paper.

The energy density and the pressure of the inflaton field \( \phi \) are
\[
\rho = A(Q)(2XF_X - F), \quad p = A(Q)F.
\]
(5)

The energy density and the pressure of the \( Q \) field are
\[
\rho_Q = \frac{1}{2} \dot{Q}^2 + V(Q), \quad p_Q = \frac{1}{2} \dot{Q}^2 - V(Q).
\]
(6)

The Friedmann equations read
\[
H^2 = \frac{1}{3M_{Pl}^2} \left( \frac{1}{2} \dot{Q}^2 + V(Q) + A(Q)(2XF_X - F) \right),
\]
(7)
\[
\dot{H} = -\frac{1}{2M_{Pl}^2} (\dot{Q}^2 + A(Q)2XF_X).
\]
(8)

During inflation, the inflaton field dominates the evolution, and the \( Q \) field moves in an effective potential. In this paper we consider the case in which the effective potential for the \( Q \) field has a minimum, and it exists for a positive \( \beta \). The existence of such a minimum was already explored in [15], and the method used here is similar to that in [15].
From eqn. (2), we require

$$\frac{\partial V_{eff}(Q)}{\partial Q} = \frac{\partial V(Q)}{\partial Q} - \frac{\partial A(Q)}{\partial Q} F(\phi, X) = \frac{1}{Q}(\beta A(Q)F(\phi, X) - \lambda V(Q)) = 0,$$

(9)

where $F(\phi, X) = -F(\phi, X)$ (during inflation $F(\phi, X) < 0$). We have used expressions for the potential $A(Q)$ and $V(Q)$. This gives a relation between the potential of the Q field and the pressure of the inflaton field,

$$V(Q_{\text{min}}) = \frac{\beta}{\lambda} A(Q_{\text{min}}) F(\phi, X) = -\frac{\beta}{\lambda} p.$$

(10)

From the above equation we can see the value of $\frac{\beta}{\lambda}$ determines the ratio of the energy density of the dark energy field to the inflaton field (during inflation $p + \rho \approx 0$). If we require the inflaton field dominates the evolution, the value of $\frac{\beta}{\lambda}$ must be small. The value of the Q field near the minimum of the effective potential is

$$Q_{\text{min}} = \left(\frac{\lambda V_0}{\beta A_0 F(\phi, X)}\right)^{\frac{1}{3 + \epsilon}}.$$

(11)

The expression for $A(Q_{\text{min}})$ is

$$A(Q_{\text{min}}) = A_0\left(\frac{\lambda V_0}{\beta A_0 F(\phi, X)}\right)^{\frac{2}{3 + \epsilon}}.$$

(12)

Eqn. (7) simplifies to be

$$H^2 \approx \frac{1}{3}(\beta A(Q)F + A(Q)(2XF_X - F)) \approx \frac{1}{3}(1 + \frac{\beta}{\lambda})A(Q)(2XF_X - F) \approx \frac{1}{3}(1 + \frac{\beta}{\lambda})\rho,$$

(13)

in the first step we use the eqn. (10) to eliminate $V(Q)$, and in the second step we use the slow roll condition

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{3}{2}(1 + \frac{\beta}{\lambda})^{-1} \frac{2XF_X}{2XF_X - F},$$

(14)

to add a term $\frac{2}{3}A(Q)2XF_X$ to the expression.

Using eq. (13), the mass square of the Q field near the minimum of the effective potential is

$$m_{\text{eff}}^2 = \frac{\partial}{\partial Q}\left(\frac{1}{Q}(\beta A(Q)F(\phi, X) - \lambda V(Q))\right)$$

$$= -\frac{1}{Q^2}(\beta A(Q)F(\phi, X) - \lambda V(Q)) + \frac{1}{Q^2}(\beta^2 A(Q)F(\phi, X) + \lambda^2 V(Q))$$

$$\approx \frac{1}{Q^2}(\beta^2 A(Q)(2XF_X - F) + \lambda^2 V(Q))$$

$$\approx \frac{\partial^2 V(Q)}{\partial Q^2} + \frac{3H^2}{Q^2} \frac{\beta^2}{1 + \frac{\beta}{\lambda}}.$$

In the second line, the first term vanishes because of eq. (9). In the third line, we replace $-A(Q)F$ with $A(Q)(2XF_X - F)$, because of $p + \rho = 0$ in the slow roll limit. In the last line we have used eqn. (13). For $\beta \ll 1$, $\lambda \sim 1$ and $\beta^2/Q^2 > 1$, $m_{\text{eff}}^2/3H^2$ exceeds order unit. The range of $Q$ is restricted by experimental data, and is very small in our model. In [21] the authors give a lower limit of the energy density:

$$\rho_Q \geq 10^{-109} M_{\text{Pl}}^4.$$

(16)

This corresponds to $Q \leq 1.8 \times 10^{-6} M_{\text{Pl}}$ with the $\lambda$ and $V_0$ used in section (IV). So even for a $\beta$ as small as $10^{-3}$, the effective mass is rather large, and the Q field would quickly settle in the minimum of its effective potential. This was also justified in section (V) via numerical analysis.

From eqn. (11) we get

$$\dot{Q} = -\frac{Q}{\beta + \lambda} \frac{\dot{F}}{F}.$$

(17)
III. PERTURBATIONS

It is useful to calculate perturbations to see how the coupling with a dark energy field influences the power spectra of curvature and tensor perturbations of the inflaton field. Below we follow the standard procedures in \[6, 17\] and restrict ourselves in a flat background. First we rewrite eqn. \((7)\) and eqn. \((8)\)

\[
H^2 = \frac{1}{3}(-\frac{\beta}{\lambda}p + \rho) \quad (18)
\]

\[
\dot{H} = -\frac{1}{2}(p + \rho) \quad (19)
\]

in here, together with a redundant equation which can be derived using the above two

\[
\dot{\rho} = -3H(p + \rho) + \frac{\beta}{\lambda} \dot{\phi}. \quad (20)
\]

In deriving the above expressions we have neglected the term \(\dot{Q}^2\), and used eqn.\((5)\) and eqn.\((10)\). The last expression differs from usual form by an extra term \(\frac{\beta}{\lambda} \dot{\phi}\). In this section we adopt the signs and conventions as in \[6\]. The energy momentum tensor is

\[
T_{\mu\nu} = (\rho + p)u_\mu u_\nu - (1 + \frac{\beta}{\lambda})pg_{\mu\nu}, \quad (21)
\]

where

\[
u_\mu = \frac{\phi,\mu}{(2X)^{\frac{1}{2}}}. \quad (25)
\]

The perturbations of the components of the energy momentum tensor can then be expressed as

\[
\delta T_{00} = \delta \rho - \frac{\beta}{\lambda} \delta p = \frac{\rho + p}{c_s^2}((\frac{\delta \phi}{\phi}) \cdot - \Phi) - 3H(\rho + p)\frac{\delta \phi}{\phi} + \frac{\beta}{\lambda} \delta \phi \frac{\delta \phi}{\phi}, \quad (26)
\]

\[
\delta \rho = \rho,\delta X + \rho,\phi \delta \phi = \frac{\rho + p}{c_s^2}((\frac{\delta \phi}{\phi}) \cdot - \Phi) - 3H(\rho + p)\frac{\delta \phi}{\phi} + \frac{\beta}{\lambda} \delta \phi \frac{\delta \phi}{\phi}, \quad (24)
\]

\[
\delta p = p,\phi \delta \phi = (\rho + p)((\frac{\delta \phi}{\phi}) \cdot - \Phi) + \rho \frac{\delta \phi}{\phi}. \quad (22)
\]
\[ \delta T^0_i = (\rho + p)(\delta \phi)_{\phi}i. \]

The above two expressions should be substituted into the 00 and 0i linearized Einstein equations. It is convenient to change from the independent variables \( \Phi \) and \( \delta \phi / \phi \) to the new variables \( \xi \) and \( \zeta \) (defined in eqn.(19) and eqn.(20) in [6]).

The action can be inferred from the equations for the variables \( \xi \) and \( \zeta \).

We find if we make the substitution
\[
\frac{c_s^2}{1 - c_s^2 \beta} \rightarrow \tilde{c}_s^2
\]
in eqn.(26), the following derivation can be repeated as in the standard case. The symbol \( \tilde{c}_s \) denotes a modified speed of sound. Further introducing the Mukhanov variable \( v = z \zeta \), where \( z \) is defined as
\[
z \equiv \frac{a(\rho + p)^{1/2}}{\tilde{c}_s H}
\]
in a flat universe, we obtain an equation for the variable \( v \)
\[
v'' - \tilde{c}_s^2 \Delta v - \frac{z''}{z} v = 0.
\]
In the above equation, the prime represents derivative with respect to the conformal time. The speed of sound is modified in this model, and it recovers the normal form in the limit of \( \beta \rightarrow 0 \). We consider parameters to satisfy \( \frac{\beta}{\lambda} \ll 1 \), therefore, in this model we stay in the scope of positive speed of sound. Before we write down the expressions for the power spectra, let’s first introduce another two slow roll parameters (see [19]),
\[
\eta \equiv \frac{\dot{\epsilon}}{H \epsilon}, \quad \kappa \equiv -\frac{\tilde{c}_s^{-1}}{H \tilde{c}_s^{-1}}.
\]

The power spectrum of curvature perturbations is
\[
P_R \approx \frac{1}{8\pi^2 M_{Pl}^2} \frac{H^2}{\tilde{c}_s \epsilon} \big|_{aH=\tilde{c}_s k}.
\]
The scalar spectral index is
\[
n_S - 1 = \frac{d \ln P_R}{d \ln k} \approx -(2\epsilon + \eta + \kappa).
\]

Tensor power spectrum
\[
P_g \approx \frac{2H^2}{\pi^2 M_{Pl}^2} \big|_{aH=k}.
\]
The tensor to scalar ratio is
\[
r \equiv \frac{P_g}{P_R} = 16\tilde{c}_s \epsilon.
\]

The tensor spectral index is
\[
n_T = \frac{d \ln P_g}{d \ln k} \approx -2\epsilon.
\]

These expressions are similar to the standard case, but both \( \tilde{c}_s \) and \( \epsilon \) in these expressions are slightly modified, also the Hubble parameter \( H \) contains an extra factor \( A(Q) \).
IV. CONSTRAINING THE PARAMETERS

In this section we set up to estimate the parameters in our model. First we need to specify an expression for $F(\phi, X)$. Several forms of $F$ have been already used in the literature. The form $F(\phi, X) = f(\phi)(-X + X^2)$ was first discussed in [3], where an inverse square pole-like $f(\phi)$ led to a power law $k$-inflation. A discussion about such form can also be found in [7], where the authors argued that for a general form of $f(\phi) = \phi^s$ with $s > 0$, inflation could be successfully produced. Here we choose $F(\phi, X) = f(\phi)(-X + X^2)$, $f(\phi) = \phi^2$.

In slow roll limit, $X$ is fixed in the point $X_0 = \frac{1}{2}$ to satisfy $F_X = 0$. In post slow roll stage, we can expand $X$ to $X_0 + \delta X$.

The model has four parameters, namely $\lambda$, $\beta$, $A_0$, and $V_0$. First one can use eqn. (11) to eliminate $Q$ in eqn. (13) to get

$$H^2 \approx \frac{1}{3}(1 + \frac{\beta}{\lambda})A_0(\frac{\lambda V_0}{\beta A_0 F})^{\frac{3}{4}}(2X F_X - F) \approx \frac{1}{3}(1 + \frac{\beta}{\lambda})A_0(\frac{\lambda V_0}{\beta A_0})^{\frac{3}{4}}\frac{1}{4}f(\phi). \quad (35)$$

For convenience, we set

$$C = \frac{1}{3}(1 + \frac{\beta}{\lambda})A_0(\frac{\lambda V_0}{\beta A_0})^{\frac{3}{4}}. \quad (36)$$

And $H^2$ writes as

$$H^2 = \frac{C}{4}f(\phi). \quad (37)$$

The requirement of 60 e-foldings of expansion is

$$N = \int H dt = \int \frac{H}{\phi} d\phi = \frac{\sqrt{C}}{4}(f(\phi_i) - f(\phi_e)) = 60. \quad (38)$$

Here and below the subscripts $i$ and $e$ denote the value at the beginning and the end of inflation. Using eqn. (20), one can get the derivative of $X$ in term of $\phi$,

$$(\dot{X})_i = \frac{1}{4\phi_i}. \quad (39)$$

Expanding eqn. (20) to the first order in $\delta X$, and using

$$F_X = F_{XX} \delta X = 2f(\phi)\delta X, \quad (40)$$

one gets

$$\delta X = \frac{1}{6\sqrt{C}f(\phi)}. \quad (41)$$

The inflation ends when $\frac{\delta X}{X_0} = 1$, and one gets

$$f(\phi_e) = \frac{1}{3\sqrt{C}}. \quad (42)$$

Using eqn. (38), the value of $f(\phi_i)$ is

$$f(\phi_i) = \frac{1}{\sqrt{C}}(4N + \frac{1}{3}). \quad (43)$$

By substituting it back into eqn. (11), one has

$$\delta X_i = \frac{1}{24N + 2}. \quad (44)$$
We can express the value of speed of sound and the slow roll parameters at the beginning of inflation in term of $\delta X_i$ and $\dot{X}_i$,

$$
(c_s^2)_i = \frac{F_X}{F_X + 2XF_{XX}} = \delta X_i, \quad \epsilon = 12(1 + \frac{\beta}{\lambda})^{-1}\delta X_i, \quad \eta = \frac{\dot{X}_i}{H_0\delta X_i}, \quad \kappa = \frac{\dot{X}_i}{2H_0\delta X_i}.
$$

(45)

All the above parameters are evaluated at the beginning of inflation. The WMAP data of the amplitude of curvature perturbations $\Delta_R^2$ is $2.41 \times 10^{-9}$, and we have

$$
8\pi^2 M_{Pl}^2 c_s \epsilon \Delta_R^2 = H^2.
$$

(46)

One gets

$$
H_i \approx 6.46 \times 10^{-6}.
$$

(47)

By substituting eqn.(37) into eqn.(43), one gets

$$
C = \frac{H_i^4}{(N + \frac{1}{12})^2} = 4.82 \times 10^{-25}, \quad \phi_i = (\frac{4H_i^2}{C})^{\frac{1}{2}} = 7.32 \times 10^9.
$$

(48)

One can also get $\phi_e = 6.93 \times 10^5$ and $H_e = 2.40 \times 10^{-7}$. In [20] and references there, flat inverse power law model of dark energy is favored by the current data, for example $0.5 \leq \lambda \leq 2$. As is discussed below eqn.(10), $\frac{\beta}{\lambda}$ must be small if one requires the inflaton to dominate the inflation. The parameter $V_0$ should equate to $\Lambda^4 \approx 3 \times 10^{-121} M_{Pl}^4$, where $\Lambda$ is the cosmological constant. In this paper we consider the values of $\lambda$ and $\beta$ in the below ranges,

$$
0.5 \leq \lambda \leq 2, \quad 0 \leq \beta \leq 0.1.
$$

(49)

Now the value of $C$ is known, and then the value of $A_0$ can be determined in terms of $\lambda$ and $\beta$. The result is in fig.(1).

![log(A0)](image)

FIG. 1: $log(A_0)$

Now we can calculate the below parameters,

$$
\epsilon \approx 8.32 \times 10^{-3}, \quad c_s \approx 2.63 \times 10^{-2}, \quad \eta \approx 7.62 \times 10^{-3}, \quad \kappa \approx 3.81 \times 10^{-3}.
$$

(50)

Using the above results, we can evaluate the power spectrum and spectral index.

$$
n_S \approx 0.972, \quad P_g \approx 8.45 \times 10^{-12}, \quad r \approx 0.004, \quad n_T \approx -0.016.
$$

(51)

These results are consistent with the current observed data. At the end of inflation, the inflaton oscillates around its minimum and decays. After that, the quintessence field recovers its freedom and behaves like a standard canonical field.
V. NUMERICAL SIMULATION

In section (II), we considered the theory of k-inflation coupled with a dark energy field. In the analysis we assumed slow roll for both fields and neglected the kinetic term of the Q field in Friedmann equations. We argued the Q field, despite of different initial values, would quickly settle into the minimum of the effective potential within a few e-foldings. In this section, we reserve all of the terms in the equations of motion for the Q field and the inflaton field as well as in Friedmann equations, and do a numerical analysis.

Fig. (2) shows the Q field with different initial values (marked by blue and green dashed lines) typically evolves toward the minimum of the effective potential (marked by solid yellow line as calculated by eqn.(11)). As inflation continues, the denominator of eqn.(11) gradually declines and the value of Q field in the minimum of effective potential slowly grows. The numerical simulation is consistent with this simple analysis-the Q field is attracted to the solid yellow line and subsequently increases as a function of $N = \ln a$.

VI. CONCLUSIONS

In this paper we described a model of k-inflation coupled with a quintessence field, which played the role of dark energy. We chose an inverse power law potential: $V(Q) = V_0 M_{Pl}^4 (\frac{Q}{M_{Pl}})^{-\lambda}$ for the dark energy field, and a particular coupling function $A(Q) = A_0 (\frac{Q}{M_{Pl}})^\beta$. We assumed for proper parameters the dark energy field, despite of varied initial values, would quickly settle in the minimum of its effective potential. Further we did a strict numerical simulation to justify our assumption about the behavior of the Q field. The effective mass of the dark energy field is usually heavy compared with the Hubble parameter during inflation. We studied the perturbations of the inflaton field and found that the speed of sound was modified. We also found the modification to the tensor to scalar ratio only came from the modification of speed of sound and the slow roll parameter, and was independent of the coupling function $A(Q)$.

Due to the particular choice of the quintessence potential and the coupling function, we find that in a sense our model suffers from a fine-tuning problem. Technically we note that in this type of directly-coupled model, in order to treat the models analytically, we have to choose both of the quintessence potential and the coupling function to be of a power law or exponential form at the same time. The latter case has been investigated in [13], where both of the quintessence and inflaton are canonical field. In this model, the quintessence potential and the coupling function are of the forms $exp(-\lambda \frac{Q}{M_{Pl}})$ and $exp(-\beta \frac{Q}{M_{Pl}})$ respectively. Similarly, the amplitude of curvature perturbation is modified but the situation is better since the modification is an exponential factor which is always greater than unit, which in our case, due to the smallness of $V_0$, one has to fine tune $A_0$ such that the amplitude of the perturbation would not be suppressed too much. Nevertheless, since the inverse power law potential is one of the most popular models for quintessence, it makes our model an interesting complement to the works in [13]. Obviously, it would be interesting to study the K-inflation with the quintessence potential and the coupling function being of exponential forms.

In our case, we chose the K-inflation model among many other kinds of inflation models. We note that even with other kinds of inflation models, the fine-tuning problem could not be bypassed in a simple way. This is because that in this class of directly coupled models between the inflaton and the quintessence field, the modification of the cosmological perturbation is almost certain to be proportional to $A_0$ and a power of $V_0$. One lesson from our study is that the coupling function may affect the amplitude of the perturbation significantly and therefore is restricted very much by the experimental data.
Acknowledgments

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