SDSS-IV MaNGA: Bayesian analysis of the star formation history of low-mass galaxies in the local Universe

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ABSTRACT
We measure the star formation histories (SFH) of a sample of low-mass galaxies with $M_\ast < 10^9M_\odot$ from the SDSS-IV MaNGA survey. The large number of IFU spectra for each galaxy are either combined to reach a high signal to noise ratio or used to investigate spatial variations. We use Bayesian inferences based on full spectrum fitting. Our analysis based on Bayesian evidence ratio indicates a strong preference for a model that allows the presence of an old stellar population, and that an improper model for the SFH can significantly underestimate the old population in these galaxies. The addition of NIR photometry to the constraining data can further distinguish between different SFH model families and significantly tighten the constraints on the mass fraction in the old population. On average more than half of the stellar mass in present-day low-mass galaxies formed at least 8 Gyrs ago, while about 30% within the past 4 Gyrs. Satellite galaxies on average have formed their stellar mass earlier than central galaxies. The radial dependence of the SFH is quite weak. Our results suggest that most of the low-mass galaxies have an early episode of active star formation that produces a large fraction of their present stellar mass.

Key words: galaxies: fundamental parameters – galaxies: stellar content –galaxies: formation – galaxies: evolution

1 INTRODUCTION
Star formation in galaxies is regulated by a wealth of complex physical mechanisms, such as the formation, growth and merger of dark matter halos, and the cooling and heating of baryon gas by radiative and feedback processes. Characterising the star formation history of observed galaxies represents, therefore, an important step in understanding how galaxies form and evolve. Despite being the most abundant type of galaxy in the universe, dwarf (low-mass) galaxies remain elusive as far as their formation and evolution is concerned. Their observed blue colour is usually taken as an indication that these galaxies are dominated by young stellar populations (e.g. Kauffmann et al. 2003). By studying stacked spectra from the Sloan Digital Sky Survey (SDSS), Heavens et al. (2004) found a very different formation history between low- and high-mass galaxies. While galaxies with stellar masses smaller than $10^{10}M_\odot$ could be well represented by a flat star formation rate over the past 3 Gyrs and a declining rate towards earlier epochs, more massive galaxies generally form most of their stellar masses earlier. Recent investigations about low-mass galaxies, however, challenge the stereotype that low-mass galaxies are all young. For local group dwarf galaxies in which individual stars can be resolved, the star formation history can be obtained through ‘archaeological’ age reconstruction (e.g. Mateo 1998; Dolphin et al. 2005; Aloisi et al. 2007; Tolstoy et al. 2009; Weisz et al. 2011; Annibali et al. 2013; Weisz et al. 2014; Sacchi et al. 2016; Albers et al. 2019). The analysis based on the colour-magnitude diagram of resolved stellar populations in general suggests that most stars in dwarf galaxies were formed more than 5 Gyr ago. For example, Weisz et al. (2011) analysed the star formation histories (SFHs) of 60 nearby (De4 Mpc) dwarf galaxies and found that these galaxies on average have formed half of their stars before $z \sim 2$, regardless of their morphological types. The existence of such an old stellar population is supported by other types of observations. Using the CANDELS survey, van der Wel et al. (2011) found a population of extreme emission line galaxies at redshift $z \sim 1.7$, with a number density so high that they can contribute a significant fraction of the total stellar mass contained in present-day dwarf galaxies with masses between $10^8$ and $10^9M_\odot$. These authors suggested that most of the stellar mass of these dwarf galaxies should have formed before $z \sim 1$. Kauffmann (2014) used the 4000 Å break and H$\delta_A$ indices in combination with SFR/M$_*$, derived from emission line measurements to constrain the SFHs of a sample of SDSS galaxies with stellar masses in the range $10^8 - 10^{10}M_\odot$, and concluded that galaxies with stellar masses.

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smaller than $10^9 M_\odot$ are not all young but with half-mass formation times ranging from 1 to 10 Gyr. Similar conclusions have also been reached by spectral energy distribution (SED) modelling of a sample of blue compact dwarf galaxies with masses between $10^7$ and $10^9 M_\odot$ (Janowiecki et al. 2017) and a sample of HII galaxies (Telles & Melnick 2018). Stars in most of these galaxies are best described by two or more stellar populations, with the oldest population often dominating the stellar mass.

In agreement with with those observations, the empirical model presented in Lu et al. (2014) and Lu et al. (2015) predicts that dwarf galaxies in small dark matter halos ($M_h < 10^{11} h^{-1} M_\odot$) had a strong episode of star formation at $z > 2$, producing significant amounts of old stars in them. However, such an old population is not predicted in many other models (Lim et al. 2017). SFHs of low-mass galaxies have also been investigated using hydrodynamical simulations. For example, Digby et al. (2019) analyzed the SFHs of a number of field and satellite dwarf galaxies in the APOSTLE and Auriga simulations, and found that the predicted star mass fractions of stars of different ages are quite different from those observed in real surveys. Garrison-Kimmel et al. (2019) analyzed about 500 dwarf galaxies in the FIRE-2 zoom-in simulations and found that the cumulative SFHs of the simulated galaxies do not match those observed.

Clearly, accurate measurements of SFHs of low-mass galaxies can provide important constraints on galaxy formation models. However, the investigations so far have their limitations. For example, the number of dwarf galaxies for which stars can be resolved is very limited (Weisz et al. 2011), and so it is difficult to draw reliable statistical conclusions. Methods based on SED fitting (e.g. Janowiecki et al. 2017; Telles & Melnick 2018) can make use of galaxy photometry over a wide wavelength coverage, but they may lose spectral features that contain information about the SFH. This shortcoming may be remedied by methods using galaxy spectra, but high SNR spectra are needed to probe the faint old stellar population. The stacking of spectra of individual galaxies has been used to achieve a high enough SNR for such analysis (e.g. Kauffmann 2014), but such stacking mixes the signals of individual galaxies.

Here we intend to make our contribution by analyzing a sample of low-mass galaxies selected from the Mapping Nearby Galaxies at Apache Point Observatory (MaNGA; Bundy et al. 2015). With its integral-field spectroscopy (IFS), MaNGA provides a large number of integral-field unit (IFU) spectra of individual galaxies. These not only allow us to obtain high signal-to-noise composite spectra for individual galaxies, which is essential for constraining the SFH in detail, but also to study spatial variations of the SFH within individual galaxies. The large sample of low-mass galaxies also makes it possible to study how the SFH depends on other galaxy properties. In addition, MaNGA is designed to overlap as much as possible with other observations, which allows us to make use of information from other observations, thereby to get more observational constraints. Our analysis is based on our newly developed stellar population synthesis (SPS) code, Bayesian Inference for Galaxy Spectra (BIFS), which has been successfully used to constrain the IMF of MaNGA early-type galaxies (Zhou et al. 2019). BIFS fits the full composite spectrum of a galaxy and constrains its SFH along with other properties of its stellar population. The Bayesian approach provides a statistically rigorous way to explore potential degeneracies in model parameters, and to distinguish between different models through Bayesian evidence. Moreover, the flexibility of BIFS also allows us to add new observational constraints in our inferences.

The paper is organised as follows. In §2 we present our data reduction process, including sample selection and spectral stacking procedure. We then introduce the SPS model and the Bayesian approach used to fit galaxy spectra in §3. Our main results are presented in §4, and we discuss some potential uncertainties in §5. Finally, we summarize and discuss our results in §6. Throughout this work we use a standard $\Lambda$CDM cosmology with $\Omega_M = 0.7$, $\Omega_M = 0.3$ and $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$.

2 DATA

2.1 The MaNGA survey

As one of the three core programmes in the fourth-generation Sloan Digital Sky Survey (SDSS-IV, Blanton et al. 2017), MaNGA aims to collect high resolution, spatially resolved spectra for about 10,000 nearby galaxies in the redshift range 0.01 < z < 0.15 (Yan et al. 2016b; Wake et al. 2017). MaNGA targets are selected from the NASA Sloan Atlas catalogue (NSA, Blanton et al. 2005), and are chosen to cover the stellar mass range $5 \times 10^8 M_\odot < M_* < 3 \times 10^{11} h^{-2} M_\odot$. For each target, the MaNGA IFU covers a radius up to either 1.5R$_e$ or 2.5R$_e$ (R$_e$ being the effective radius), to construct the â˘AIJPrimaryâ ˘A˙I and â˘AIJSecondaryâ ˘A˙I samples, respectively (Law et al. 2015). MaNGA observes the selected galaxies with the two dual-channel BOSS spectrographs (Smeee et al. 2013) on the Sloan 2.5 m telescope (Gunn et al. 2006), which provides simultaneous wavelength coverage over 3600 – 10,300 Å, with a spectral resolution $R \sim 2000$ (Drory et al. 2015). The spectrophotometry calibration of MaNGA is described in detail in Yan et al. (2016a), while the initial performance is given in Yan et al. (2016b). Raw data from MaNGA are calibrated and reduced by the Data Reduction Pipeline (DRP; Law et al. 2016) to produce spectra with a relative flux calibration better than 5% over more than 80% of the wavelength range (Yan et al. 2016b). In addition, MaNGA provides measurements of stellar kinematics (velocity and velocity dispersion), emission-line properties (kinematics, fluxes, and equivalent widths), and spectral indices for each spaxel through the MaNGA Data Analysis Pipeline (DAP; Westfall et al. 2019; Belfiore et al. 2019).

2.2 UKIDSS

Near-infrared (NIR) photometric data are commonly used to trace the stellar mass of galaxies, which can be compared with the mass estimated from the stellar population synthesis modeling of the optical spectra provided by MaNGA. As described in Yan et al. (2016a), the MaNGA targets are chosen to overlap as much as possible with the United Kingdom Infrared Telescope (UKIRT) Infrared Deep Sky Survey (UKIDSS) footprint. UKIDSS uses the Wide Field Camera (WFCAM) on the 3.8 m United Kingdom Infrared Telescope (UKIRT; Casali et al. 2007), providing ZYJHK images over a large sky coverage. The basic information of the survey can be found in Lawrence et al. (2007), the photometric system is described in Hewett et al. (2006), and the calibration is described in Hodgkin et al. (2009). The UKIDSS data is reduced by the official pipeline and the science products are released through the WFCAM Science Archive\textsuperscript{1} (hereafter WSA, Hambly et al. 2008).

\textsuperscript{1} http://www.nsatlas.org/

\textsuperscript{2} http://wsa.roe.ac.uk/
2.3 Sample selection

The sample used here is selected from the internal data release of MaNGA, the MaNGA Product Launch 7 (MPL-7), which includes a total of 4,621 unique galaxies and has been made public available together with the SDSS fifteen data release (SDSS DR15 Aguado et al. 2019). We select a set of the least massive galaxies ($M_\odot < 10^9$) according to their total stellar masses given by NSA. During the selection, we exclude galaxies with apparent problems in MaNGA DRP and DAP data processing. Galaxies that host AGN or with severe sky line contamination at the red end of the spectra are also excluded. After this selection process, we obtain a sample of 254 low-mass galaxies.

We then cross-match galaxies in this sample with data from WSA, selecting galaxies that have measurements of SDSS u, g, r, i and UKIDSS Y, J, H, K band magnitudes. This cross-match yields a total of 752 galaxies in MaNGA MPL7 and 22 of them are in the least massive sample. Considering potential differences between SDSS and MaNGA, such as flux calibrations, we convolve the spectrum of a galaxy obtained by stacking its spaxels within $1R_\text{e}$ (see below) with the SDSS filters to derive its MaNGA $(g-r)$ colour. We only select galaxies whose MaNGA $(g-r)$ colours are within $0.05$ mag of the $(g-r)$ colours listed in WSA. This yields a final sample of 19 low-mass galaxies with both optical and NIR photometry.

In what follows, we use the sample of 254 low-mass galaxies selected from MaNGA to investigate the statistical properties of the SFHs of these galaxies, and use the 19 low-mass galaxies with photometry from UKIDSS to study additional constraints on the SFHs from the NIR photometry.

2.4 Spectral stacking

The original spectra provided by MaNGA DRP have typical $r$-band SNRs of $4 - 8$ Å$^{-1}$ toward the outer radii of galaxies (Law et al. 2016). For the low-mass galaxies considered here, the SNR can be as low as $2$ Å$^{-1}$ due to their relatively low surface brightness. One thus needs to combine spectra in each IFU plate of each individual galaxy to obtain a stacked spectrum with a sufficiently high SNR.

We use two kinds of stacked spectra of every individual galaxy. First, to study the global SFHs of individual galaxies and the overall variations of SFH from galaxy to galaxy, we bin spaxels with elliptical annuli inside one $R_\text{e}$ of each galaxy to form a single spectrum. Second, to study the variations of the SFH within a galaxy, we divide spaxels of the galaxy into three radial bins, $(0.0 - 0.3)R_\text{e}$, $(0.3 - 0.7)R_\text{e}$ and $(0.7 - 1.2)R_\text{e}$, according to their normalised radii of elliptical annuli given in MaNGA DAP, and stack the spectra within individual radial bins. These radial bins are similar to those used in related MaNGA studies, such as Zheng et al. (2019).

The stacking procedure used here is similar to that in Zhou et al. (2019), and we refer the reader to that paper for details. As discussed in the MaNGA DAP paper (see Westfall et al. 2019, for details), the SNR of stack spectra deviates from the simple noise propagating formula, because the spaxels provided by DRP are not fully independent. Here we use the correction term given by Westfall et al. (2019) to account for the covariance between spaxels and to estimate the SNR. The correction term can be written as

$$n_{\text{real}}/n_{\text{covar}} = 1 + 1.62\log(N_{\text{bin}})$$

(1)

where $N_{\text{bin}}$ is the number of spectra used in the stacking, $n_{\text{real}}$ and $n_{\text{covar}}$ are the corrected noise vectors and noise vectors that assuming no covariance between pixels (namely, those generated from the simple noise propagating formula) respectively. With this correction, the typical SNR of the stacked spectra is around 40 pixel$^{-1}$.

3 ANALYSIS

The inferences of stellar population properties from galaxy spectra can be achieved by comparing stellar population models with the observed spectra. In practice, two complementary approaches, absorption-feature modeling and full-spectrum fitting, have been widely used. The full spectrum in principle contains more information, but the information can be fully used only when both the continuum shape and spectral features are all modelled accurately, and when the SNR of the spectra is sufficiently high. In contrast, absorption features can be selected to have the greatest sensitivity to the main parameters of interest, such as age, metallicity and $\alpha$ enhancement (Worthey 1994). Apparently because of the short wavelength window, absorption features may avoid influences from spectral regions that are not properly described by the model. However, the shortcoming is that some important information contained in the rest of the spectrum may be missed. In this paper, we adopt the full spectrum fitting method using Bayesian statistics to infer the SFH of individual galaxies and its co-variance with other properties.

3.1 The spectral synthesis model

To accurately model galaxy spectra, proper templates that meet the resolution and wavelength coverage of the data are crucial. Several popular codes are available to model simple stellar populations (SSPs) of given ages and metallicity, including BC03 (Bruzual & Charlot 2003), M05 (Maraston 2005), and E-MILES (Vazdekis et al. 2016). These SSP models can be combined with an assumption of the SFH of a galaxy to predict its spectrum. Different SSP models are based on different stellar templates and isochrones, and thus have their own merits and shortcomings. Given that the wavelength range of the original MaNGA data is $3600 - 10300$ Å, and the median redshift of MaNGA galaxies is $z \sim 0.03$, we select models that have uniform wavelength coverage to $\sim 9000$ Å. In addition, the UKIDSS data require an extension of the coverage to $\sim 2.5$ μm. With these considerations, we decide to adopt the E-MILES$^3$ model.

The MILES models first presented in Vazdekis et al. (2010) are constructed with the MILES (Sánchez-Blázquez et al. 2006) stellar library. Later on, EMILES models extend MILES both bluewards and redwards using CaT (Cenarro et al. 2001), Indo-U.S. (Valdes et al. 2004) and the IRTF stellar library (Cushing et al. 2005; Rayner et al. 2009). With all these stellar templates combined, the E-MILES SSP spectra cover the wavelength range from $1680.2$ Å to $5\mu m$ with a moderately high spectral resolution. In particular, the SSPs reach a resolution of $2.51$ Å (FWHM) over the range from $3540$ Å to $8950$ Å, which covers the main portion of the spectral range of MaNGA. The spectral resolution decreases towards longer wavelengths, but is sufficient for photometry calculations.

The E-MILES model is computed for several IMFs. As our
focus is on low-mass galaxies, which tend to have a bottom-light IMF (Li et al. 2018), we choose the model constructed with the Chabrier IMF. Moreover, E-MILES provides two sets of isochrones, the Padova+00 isochrones (Girardi et al. 2000) and the BaSTI isochrones. The impact of using different isochrones has not been investigated extensively in the literature. For MaNGA galaxies, Ge et al. (2019) found that using different isochrones does not lead to significant changes in the fitting quality of observed spectra. Using mock spectra, however, the authors found that the Padova+00 model works better at low metallicity ([Z/H]<-1.0), while the BaSTI model works better for galaxies of higher metallicity. Since low-mass galaxies in general are metal poor, we use E-MILES templates with Padova+00 isochrones.

3.1.1 Star formation history

In general, the details of the star formation history of a galaxy can be very complicated. However, limited by data quality, the observed galaxy spectra may be modeled in terms of a number of important stellar populations. For low-mass galaxies, observations based on resolved stars (e.g. Weisz et al. 2011) have shown that the SFHs of different galaxies are quite similar over most of the cosmic time, while differences are only seen in the recent few Gyrs. These observations also indicate that low-mass galaxies generally had enhanced star formation in the early universe (z > 1) where they formed more than half of their stars, and that some of them have gone through complex star formation histories during the recent 1 Gyr. Such two-phase star formation is also seen in empirical models such as that of Lu et al. (2015).

Traditionally, SFHs of galaxies have been modelled with two distinct approaches: a parametric approach that assumes a functional form specified by a small number of parameters, and a non-parametric approach that models the SFH as a histogram (a stepwise function) in a number of time bins. In our analysis, we use both approaches and make comparisons between them.

The parametric model adopted here is motivated by the empirical model of Lu et al. (2015), who found that the SFH of a dwarf galaxy can be well represented by a burst in the early universe followed by a continuous SFH. To describe such a SFH, we model a smooth component using a Ψ function:

$$\Psi(t) = \frac{1}{\tau \Gamma(\alpha, t_0 / \tau)} \left( \frac{t_0 - t}{\tau} \right)^{\alpha - 1} e^{-(t_0 - t)/\tau},$$

where $t_0 - t$ is the look-back time, and $\Gamma(\alpha, t_0 / \tau) \equiv \int_0^{t_0 / \tau} x^{\alpha - 1} e^{-x} \, dx$. The flexibility of the function allows cases where a galaxy is dominated by old stars (with both $\alpha$ and $\tau$ small) or dominated by younger populations (with both $\alpha$ and $\tau$ large). On top of this continuous SFH, we include an additional burst component to mimic the old stellar component in dwarf galaxies seen in resolved stars. The burst is specified by two free parameters: $t_b$ describing when the burst occurs, and $f_b$ specifying the relative fraction of stellar mass formed in the burst. Thus we consider the following two SFH model types:

- $\Gamma$ model: SFH given by equation (2);
- $\Gamma+B$ model: SFH given by equation (2) plus a burst.

Non-parametric models are more flexible. In real applications, however, the flexibility depends strongly on the number of time bins used, and the accuracy of the inference that can be achieved is still limited by data quality. In addition, using too many time bins will lead to degeneracy in the solution and increase the computational time for sampling the posterior distribution. Panter et al. (2007) used 11 time bins in their analysis of SDSS galaxies, but found the model to be too ambitious for most galaxies. To model the color-magnitude diagram (CMD) based on resolved stars, Weisz et al. (2011) used a 6-step model to describe the SFH of local dwarf galaxies. In our analysis, we adopt a stepwise SFH model similar to Weisz et al. (2011). The model is described by the average star formation rates in 7 time intervals: $0 \rightarrow 0.2, 0.2 \rightarrow 0.5, 0.5 \rightarrow 1.0, 1 \rightarrow 2, 2 \rightarrow 6, 6 \rightarrow 10$ and $10 \rightarrow 14$ Gyr. Since only the relative fraction of stars formed in each interval change the spectral shape, the model is specified by six free parameters.

We test the validity of these three forms of SFH using mock spectra generated from theoretical SFHs from the empirical model of Lu et al. (2015). To do this, we first convolve the theoretical SFHs with the E-MILES SSPs to obtain the corresponding noise-free composite spectra. Different levels of Gaussian noise are then added to the spectra to mimic real observations. An example of fitting such a mock spectrum with SNR=40 is shown in Fig. 1. Note that, in Fig. 1 and other figures, the burst component of the $\Gamma+B$ model is represented by a Gaussian peak with finite width, even though it is modelled as a single SSP in the fitting. As one can see, although both the $\Gamma$ and $\Gamma+B$ models can give a reasonable fit to the mock spectrum, the former fails to recover the input SFH. On the other hand, the $\Gamma+B$ model has the flexibility to recover the early burst component, although the exact burst time is not reproduced. Similar to $\Gamma+B$, the step-wise SFH can also reproduce the early burst component, although the specific shape of the SFH is not accurately reproduced due to the time resolution used.

For comparison, we also show the result obtained using both the optical spectrum and $(g-K)$ colour to constrain the model (see §3.2). In this case the recovered SFH (the yellow curve) closely matches the input SFH, with approximately the same burst strength and time. Fig. 2 shows another example in which the mock SFH does not contain any significant burst. Here the inclusion of a burst component in the model and the NIR photometry in the constraint does not lead to any significant changes in the inferred SFH, as the $\Gamma$ function already has the flexibility to approximate this kind of SFHs. From the cumulative plots shown in the right panels of Figs. 1 and 2, we can see that all models, except the $\Gamma$ model, predict similar half-mass formation times and old stellar fractions, although the predicted shapes of the SFH are quite different in early times. Because of this, we will use the old stellar fraction and half mass formation time to characterize the old population, without paying much attention to the exact shape of the derived SFH.

To compare the true to the estimated physical properties, we analyze 2,000 such mock spectra randomly chosen from the empirical models of Lu et al. (2015) which SNR ranging from 10 to 70. The differences in the mass fraction of the old stellar population (with stellar age >8 Gyrs) between the best-fit and input SFHs are shown in Fig. 3. It is seen that, the $\Gamma$ model systematically underestimates the mass fraction of the old population, regardless of the SNR. The $\Gamma+B$ model can well recover the input old fraction, with some exceptions of underestimation at low SNR. The stepwise model also recovers the old fraction well, but there is a systematical underestimate at low SNR. In addition, including the NIR photometry as an additional constraint can improve the accuracy of the derived mass fraction, especially when the SNR is low. Although these test results are for ideal cases, where the stellar populations are perfectly described by the the SSPs, they do indicate that the

4 http://www.oa-teramo.inaf.it/BASTI
Dust attenuation can also affect the inferred SFH. Since dust absorption is more significant at shorter wavelengths, a stellar population that contains more dust can mimic an older and/or more metal-rich population, producing the well known age-metallicity-dust degeneracy. Thus, dust attenuation has to be properly taken into account in order to make unbiased inferences from the observed spectra. In practice, dust attenuation is usually treated as an additional model parameter specifying the attenuation curve as a by-product.

3.1.2 Dust attenuation

Dust attenuation can also affect the inferred SFH. Since dust absorption is more significant at shorter wavelengths, a stellar population that contains more dust can mimic an older and/or more metal-rich population, producing the well known age-metallicity-dust degeneracy. Thus, dust attenuation has to be properly taken into account in order to make unbiased inferences from the observed spectra. In practice, dust attenuation is usually treated as an additional model parameter specifying the attenuation curve as a by-product.

Figure 1. An example of fitting mock spectra with different SFH models. The red line in the right panel shows the SFH used to generate the mock spectrum. White noise is added to the mock spectrum so that SNR=40. Blue solid and green dash lines are the best-fit results of the mock spectrum from 3800Å to 8900 Å using the Γ and Γ + B SFH models, respectively, while the orange solid lines show the best-fit results from the stepwise model. Result of the Γ + B model using the optical spectrum plus the (g − K) colour is shown as yellow dash-dotted lines. Residuals of the best-fit spectra are shown at the bottom of the left panel, with zero points of the blue, green and orange lines shifted by 0.2, 0.1, -0.1, respectively.

Figure 2. An example similar to Fig. 1, but for a mock SFH that does not contain a significant early burst.

Γ + B model and the step-wise model can both recover the the old stellar population predicted by the empirical model in some low-mass galaxies, although the details of the SFH may not be modeled accurately.

In what follows, we will apply these SFH models to MaNGA spectra and examine whether the data prefer a particular SFH model, and how model inferences are affected by the assumption of the SFH.

3.1.3 The implementation of BIGS

In complex problems, such as the spectrum fitting problem addressed here, the likelihood function can be complicated and may not be represented by simple analytical functions. One thus needs an efficient sampling method to sample the posterior distribution. In addition, as the Bayesian evidence ratio involves integration in high-dimensional space, an effective numerical method is also needed to evaluate it. BIGS adopts a Bayesian sampler, MULTINEST (Feroz et al. 2009, 2013), which uses the nest sampling algorithm to estimate the Bayesian evidence, and gives the posterior distribution as a by-product.

Briefly, BIGS works as follows. For each data spectrum, we pre-process it using pPXF (Cappellari 2017) and obtain the velocity distribution of the source. We then convolve our template spectra with a Gaussian that accounts for both the instrumental resolution and velocity dispersion of stars. The data spectrum and the
templates are then provided to BIGS. Using the prior distribution of model parameters, BIGS uses MULTINEST to generate a proposal parameter vector for the spectral synthesis model. These parameters, which specify the SFH, metallicity and dust attenuation of the stellar population, are used to generate a model spectrum to be compared with the data spectrum to calculate the corresponding likelihood. The MULTINEST sampler would either accept or reject the proposal according to the posterior probability and generates a new proposal for the model parameters, until a convergence criterion is reached. Once converged, the posterior distribution of model parameters, together with the Bayesian evidence, are stored for statistical analyses of the model.

3.2 The fitting procedure

We compare spectra predicted by the E-MILES SPS model to the observed MANGA spectra to infer the SFHs of the low-mass galaxies using the procedure described below. To begin with, we mask some of the spectral regions to ensure the validity of the fitting. For example, we mask the observed continua in the wavelength range 6800–8100 Å, as difficulties are commonly found in fitting the observed continua in this wavelength range, either due to issues in flux calibrations in the templates, residual telluric absorption, or even flux calibrations in the data (see Zhou et al. 2019 for more discussions). In addition, as the E-MILES templates do not contain the youngest stellar population (<0.06 Gyr), we also mask the very blue end of the spectra (<3800 Å) in the fitting.

To take into account effects of stellar kinematics and instrumental resolution, we first use the software pPXF to pre-fit the data spectra and get an effective velocity dispersion, σ_{\text{ppxf}}. This effective velocity dispersion is then used to convolve with the E-MILES template spectra to generate artificially broadened templates that are used to compute the synthesised spectra to be compared with the corresponding data spectra. In this step, apparent emission lines in the spectra are also identified, and masked out in subsequent analyses.

After this pre-processing, we first normalise the model and data spectra in the wavelength window 4500–5500 Å and then send them to BIGS. BIGS runs the fitting loop as described in §3.1.3, assuming a flat prior and a $\chi^2$-like likelihood function. In the fitting that uses only the MaNGA spectra, the likelihood function is defined as

$$\ln L(\theta) \propto -\frac{1}{2} \sum_{i,j=1}^{N} (f_{\theta,i} - f_{D,i}) \left( \mathcal{H}^{-1} \right)_{ij} (f_{\theta,j} - f_{D,j})$$

(3)

where $N$ is the total number of wavelength bins, $f_\theta$ and $f_D$ are the flux predicted from the parameter set $\theta$ and that of the data spectrum, respectively, and $\mathcal{H}_{ij} \equiv \left( \delta f_{D,i} \delta f_{D,j} \right)$ is the covariance matrix of the data. For spectra that have UKIDSS observations, we
\[ \ln \left( \frac{E}{\Delta E} \right) + \frac{B}{E} \]

**Table 1.** Priors of model parameters used to fit galaxy spectra

| Parameter                              | Description                        | Prior range  |
|----------------------------------------|------------------------------------|--------------|
| \( \log \left( \frac{Z}{Z_\odot} \right) \) | Metallicity                        | \([-2.3, 0.2]\) |
| \( \tau \)                             | SFH parameter in Eq. (2)           | \([0.0, 10.0]\) |
| \( \alpha \)                           | SFH parameter in Eq. (2)           | \([0.0, 20.0]\) |
| \( \tau_\text{v} \)                    | Dust optical depth at 5500 Å       | \([0.0, 2.0]\) |
| \( f_{\text{burst}} \)                  | Relative fraction of old populations | \([0.0, 1.0]\) |
| \( \log \left( \frac{Z_{\text{burst}}}{Z_\odot} \right) \) | Metallicity of the old population | \([-2.3, 0.2]\) |
| \( A_{\text{burst}} \) (Gyr)            | Age of the old population          | \([0.0, 14.0]\) |

use the following definition:

\[
\ln L(\theta) = -\frac{1}{2} \sum_{i,j=1}^{N} \left( f_{\theta,j} - f_{D,j} \right) \left( \sigma^{-1} \right)_{ij} \left( f_{\theta,j} - f_{D,j} \right) - \frac{(K_\theta - K_D)^2}{2\sigma_K^2}
\]

where \( K_\theta \) and \( K_D \) are the \((g-K)\) colour predicted from the parameter set \( \theta \) and that from the data, respectively. The uncertainty in the \((g-K)\) colour is denoted by \( \sigma_K \). In general, it is difficult to model \( \sigma_K \). The value of \( \sigma_K \) is related to the accuracy of UKIDSS photometry, which has an uncertainty of less than 2\% in the \( K \) band (Dye et al. 2006). In addition, \( \sigma_K \) should also include the relative flux variation between the MaNGA stacked spectra and the NIR photometry, which is hard to model. To minimize such influence, we only select galaxies with \( \Delta (g-r) < 0.05 \) between the MaNGA and UKIDSS archive data. In this case, our test shows that setting \( \sigma_K = 0.02 \) is appropriate to describe the constraints from the NIR observation. We list all the fitting parameters for the \( \Gamma+B \) model in Table 1, together with their prior distributions (assumed to be flat). For the priors of the step-wise SFH, we assume that the SFR at the last time interval, 10-14 Gyr, is a constant normalized to be 1 (0 in logarithmic scale), and that the prior distributions of the SFR in all other time bins are flat between –2 and 2 in logarithmic space. All other parameters are specified in the same way as for the \( \Gamma+B \) model.

4 RESULTS

4.1 Bayesian model selection

Our goal is to investigate the existence or absence of an old stellar population in low-mass galaxies. To this end, we fit the stacked spectra (each being a stack of pixels within the effective radius of a galaxy) of individual galaxies with three SFH models: \( \Gamma, \Gamma+B \) and stepwise, while keeping all other parts of the model intact. We first examine if the data shows a preference to one of the three models. As described above, the Bayesian evidence ratio provided by BIGS can serve as a discriminator between different model families. Fig. 4 shows the evidence ratio between the two continuous models, \( \ln \left( \frac{E_{\Gamma+B}}{E_{\Gamma}} \right) \), obtained from the stacked spectra, as a function of the signal-to-noise ratio (SNR) of the stacked spectra. The median values in five SNR bins are shown as blue stars to represent the global trend, with the error bars showing the \( 1\sigma \) scatter among galaxies in individual bins.

![Figure 4](image-url)  
**Figure 4.** The evidence ratio between the \( \Gamma+B \) and \( \Gamma \) models as a function of SNR of the stacked spectra. Each red dot stands for the result of a MaNGA galaxy obtained by fitting its stacked spectrum. Blue stars are the median values in five SNR bins and are linked by a blue line. Each of green triangles stands for the result obtained from fitting both the MaNGA stacked spectrum and the \((g-K)\) (see 4.3). Yellow stars are the median values in two SNR bins and are connected by a yellow line. Error bars are \( 1\sigma \) scatter among galaxies in individual bins.

Fig. 4 shows clearly that the median value of the evidence ratio increases with increasing SNR. This indicates that the SFHs of these low-mass galaxies are more likely a composite of two distinct stellar components than a single component. Although the young stellar component may dominate the luminosity and makes the galaxies blue, the faint old stellar component that formed early may contribute significantly to their total stellar masses, as we will quantify next.

As comparison, we plot in Fig. 5 the evidence ratio between \( \Gamma+B \) and the step-wise models. Although the number of free parameters in the step-wise model is much larger than that in the \( \Gamma+B \) model, the Bayesian evidences of the two models are comparable. The medians of the evidence ratio are close to 1, with large dispersion among different galaxies. This indicates that some of the
galaxies have a preference to the $\Gamma$+B model, while others have the opposite preference. The absent of a systematic trend suggests that both models have similar abilities to describe the overall SFH in the current data, while the preferences of different galaxies to different models indicate the intrinsic variations in the detailed shapes of their SFH.

It is, however, not straightforward to quantify the preference between different models. Bayesian model selection has been used for such purpose, but a quantitative criterion for model selection is still not established. The widely adopted Jeffreys’ scale uses a ratio of $E_1/E_2 > 150$, or $\ln(E_1/E_2) > 5$, where $E_1$ and $E_2$ are the evidences of the two competing models 1 and 2, respectively, to indicate a ‘strong’ preference to model 1 (e.g. Hobson 2009). But the validity of this scale has been questioned (e.g. Nesseris & García-Bellido 2013). In spectral fitting, the situation may be even worse. The number of data points is large (thousands of pixels for each spectra) and the current SSP models are not perfect, which may lead to very large evidence ratios between different models (e.g. Han & Han 2019). In what follows, we simply assume that a model is preferred when it has the largest evidence among all models considered.

4.2 The stellar populations in low-mass galaxies

4.2.1 The star formation history

We first obtain the best-fit SFH for each galaxy from the posterior distribution. Results for six representative galaxies are shown in Fig. 6. As can be seen, results from the $\Gamma$+B model and the stepwise model are broadly consistent with each other, both predicting the existence of an old stellar population for most (but not all) galaxies, while the $\Gamma$ model clearly misses such a population.

We plot the average cumulative SFH in Fig. 7. To estimate the statistical uncertainty, we divide the galaxy sample into 20 sub-samples of similar sizes, and make 20 different jackknife copies from these sub-samples by eliminating one of the sub-samples. The variances of the mean SFH inferred from the 20 jackknife copies are shown in Fig. 7 as the shaded regions around the corresponding lines. The results derived from the $\Gamma$+B model (red lines) indicate that low-mass galaxies on average formed about half of their stellar masses more than 8 Gyr ago, which may be associated with the starburst events observed in extreme emission line galaxies detected in the CANDELS survey (van der Wel et al. 2011). As a comparison, the black line shows the cumulative SFHs of local dwarf galaxies in the mass range $10^8 - 10^9 M_\odot$ obtained from resolved stars by Weisz et al. (2011). It is remarkable that the results obtained from the $\Gamma$+B model are in good agreement with that of Weisz et al. (2011). In contrast, the results obtained from the $\Gamma$ model (blue lines) indicate that most of the stellar mass in low-mass galaxies formed recently. This discrepancy is expected. Since an old stellar population is much fainter than the young population...
of the same mass, a model that is not sufficiently flexible to allow for the existence of both populations will miss the old population. As shown in §4.1, this limitation of the $\Gamma$ model weakens its ability to describe the true SFH, which leads to the smaller Bayesian evidence in comparison to the $\Gamma$+B model in fitting spectra of sufficiently high SNR. For comparison, we also show in Fig. 7 the result obtained from the stepwise model as a green line. In addition, we also obtain the best-fit SFH from the preferred model for each galaxy based on Bayesian model selection. The result, referred to as the preferred SFH, is plotted in Fig. 7 as a thick orange line. The two results are in qualitative agreement with those from the $\Gamma$+B model, indicating that the $\Gamma$+B model may be sufficiently broad for the current data, and that our conclusion does not depend on the exact form assumed for the SFH, as long as it is sufficiently flexible.

In what follows, we will thus focus on the results derived from the $\Gamma$+B model and use them to compare with those in the literature. In addition, we will also show results from the preferred SFH to demonstrate how our results may vary due to the use of a different SFH model.

In Fig. 8, we show results separately for central and satellite galaxies, using the central/satellite classification from Galaxy Environment for MaNGA Value Added Catalog (GEMA–VAC, Argudo–Fernández et al. 2015). This VAC uses the group catalogue of Yang et al. (2007) to separate MaNGA galaxies into centrals and satellites. Environmental effects can be seen from Fig. 8, in that satellite galaxies appear to form their stars slightly earlier than central galaxies. This is expected, as low-mass satellites may have their star formation quenched by environmental effects of their host halos (e.g. van den Bosch et al. 2008; Peng et al. 2012).

In addition to the cumulative SFH, we also derive the half-mass formation time, $t_{\text{half}}$, defined as the look-back time when a galaxy forms half of its final stellar mass, from the best-fit SFH models and plot the results in Fig. 9. Again, GEMA–VAC is used to divide our sample into centrals (red) and satellites (blue). For both the $\Gamma$+B model and the preferred SFH, $t_{\text{half}}$ varies from 2 Gyr to 12 Gyr, with a broad peak at about 9 Gyr. In contrast, $t_{\text{half}}$ inferred from the $\Gamma$ model is smaller on average, ranging from zero to 12 Gyr with a broad peak at $<4$ Gyr.

Comparing results obtained for centrals and satellites, one sees that the peak at 9 Gyr is weaker for centrals, and there is a weak excess of central galaxies at $t_{\text{half}} \sim 4$ Gyr. This excess may indicate a secondary star formation episode for some of the central galaxies, as is expected from the "gappy" star formation history found by Wright et al. (2019). These results are in rough agreement with those of Kauffmann (2014), who found that the distribution of $t_{\text{half}}$ for low-mass galaxies, derived from analysis of Dn4000 and H$_\alpha$ absorption features, is quite broad and shows double peaks.

We note that there is a significant peak in the distribution of $t_{\text{half}}$ at around 13 Gyr inferred from $\Gamma$+B. However, our extensive
test shows that this peak may not be real. We only use a single SSP to describe the burst in the early-Universe, and the age of the burst is confined to be within the age of Universe. As the spectra of old SSPs are insensitive to the age (e.g., Bruzual & Charlot 2003), a galaxy with an early starburst that contributes more than half of its stellar mass could have the best-fit $t_{\text{half}}$ at the edge of the prior. By setting the prior age range to be the maximum age of the SSP model, which is 18 Gyr for the EMILES model, we found that some best-fit ages would move out of the previous boundary, and the peak at 13 Gyr would disappear. These results indicate that the current model is not able to describe the ages of old stellar populations accurately. Because of this limitation, the exact ages of the old population predicted by the model should be treated with caution. We emphasise, however, that this limitation does not affect the conclusion that these galaxies contain large fractions of old stars. Indeed, the peak is much weaker in the distribution inferred from the preferred SFH, but the predicted fraction of galaxies with $t_{\text{half}} > 8$ Gyr is similar for both the $\Gamma+B$ and the preferred SFH.

4.2.2 The old stellar populations

Fig. 10 shows the mass fraction of stars in three populations, old (>8 Gyr), middle-age (4-8 Gyr), and young (<4 Gyr). We show the results for central and satellites galaxies separately, and separately for $\Gamma+B$ (upper panels), $\Gamma$ (middle panels) and the preferred SFH (bottom panels). For $\Gamma+B$ and the preferred SFH, the fractions are ~ 60%, 20% and 20% for the three populations, respectively, and are quite similar for both centrals and satellites. For the $\Gamma$ model, the corresponding fraction is about one third for each of the three populations.

Therefore, the $\Gamma$ model significantly underestimates the fraction of old stars while overestimating the young fraction in comparison with the $\Gamma+B$ model and the preferred SFH.
MaNGA spectra stacked in four radial bins, investigate the SFHs in different parts of a galaxy. Here we use and evolution of low-mass galaxies. This is not well reproduced in the predictions of \( \Gamma \). Note that the amount of stars in the middle age (4-8 Gyr) is relatively small both in the predictions of \( \Gamma \). In this subsection, we demonstrate this using the combination of optical spectra and NIR photometry is expected to provide better additional constraints on the stellar population of galaxies. Compared to spectroscopic observations, broad band measurements are much easier to make, although they may lose some spectral information. As old stars emit the majority of their light in NIR, a combination of optical spectra and NIR photometry is expected to provide better constraints on the SFH model which includes early star formation. In this subsection, we demonstrate this using the combination of UKIDSS K-band flux and MaNGA optical spectrum.

The fitting example of mock spectra in Fig. 1 illustrates the fitting results for radial bins [0.0-0.3]\( R_e \), [0.3-0.7]\( R_e \), and [0.7-1.2]\( R_e \), respectively. As a reference, results of stacking spaxels of the entire galaxy are shown by dash lines in all panels. Red and blue lines are for central and satellite galaxies, respectively. The shaded region around each line represents the variance of the mean SFH, estimated from the jackknife resampling method. The horizontal black dashed line marks the position of half of the total star formation.

### 4.2.3 Radial dependence

The spatially resolved spectra provided by MaNGA allow us to investigate the SFHs in different parts of a galaxy. Here we use MaNGA spectra stacked in four radial bins, [0.0 – 0.3]\( R_e \), [0.3 – 0.7]\( R_e \), and the entire galaxy with all spaxels, to investigate how the SFH varies with radius. We derive the average SFH from the radially stacked spectra using the \( \Gamma+B \) model and the step-wise model. Fig. 11 shows the cumulative distribution of the SFH obtained for the four radial intervals. Results are shown for the \( \Gamma+B \) model (top panels) and the preferred SFH (bottom panels), and separately for central and satellite galaxies. It is seen that the old stellar population exists not only in the central part, but spreads over the entire galaxy, making a significant contribution to the total stellar mass. Satellites form their stars slightly earlier than centrals and this is true for all radii. In addition, there is an indication that stars in the innermost part on average formed earlier, by \(~ 1\) Gyr, than in the outer part. However, the signal for the age gradient is too weak and the uncertainty in the result is too large to draw a definite conclusion.

### 4.3 Constraints from NIR

The results presented above are obtained using MANGA optical spectra as constraints. The SSP templates from the E-MILES model in fact have wavelength coverage from 1600 Å to 5 \( \mu \)m. Thus, these SSP templates allow us to predict the fluxes in both optical and NIR bands once a set of model parameters are given. By comparing the predicted colour with observations, we can get additional constraints on the stellar population of galaxies. Compared to spectroscopic observations, broad band measurements are much easier to make, although they may lose some spectral information. As old stars emit the majority of their light in NIR, a combination of optical spectra and NIR photometry is expected to provide better constraints on the SFH model which includes early star formation.

In this subsection, we demonstrate this using the combination of UKIDSS K-band flux and MaNGA optical spectrum.

The fitting example of mock spectra in Fig. 1 illustrates the improvement of the derived SFH when \(( g-K) \) colour is included in the fitting. As a real example, Fig. 12 shows the fitting results for MaNGA 9876-3703, one of the galaxies in our sample. From the optical image shown in the left panel, one can see that this galaxy...
is quite blue, indicative of ongoing star formation. Indeed, if we assume a simple $\Gamma$ model to fit its MaNGA spectra, the SFH obtained is dominated by recent star formation, with almost no population older than 8 Gyr, as shown by the blue lines. In contrast, the use of the $\Gamma^B$ model reveals the existence of a significant old stellar population. However, neither of the two constrained models can recover perfectly the observed $(g-K)$ colour for this galaxy. Using the posterior distribution of the model parameters obtained from the E-MILES templates, the best-fit of the $\Gamma$ and $\Gamma^B$ models predicts $(g-K) = 2.08$ and $(g-K) = 2.22$, respectively, while the measurement from WSA is $(g-K) = 2.43$. These results indicate that the lack of constraints from NIR may still lead to biased inferences of the stellar population, even if a proper SFH is assumed.

In order to check the effects of including the NIR photometry, we implement a new set of fitting, adopting the modified likelihood function described by equation (4). The results obtained for MaNGA 9876-3703 are plotted in Fig. 12 with solid lines. The inclusion of the $(g-K)$ colour changes the fitting result of the $\Gamma^B$ model significantly, in that the fraction of the old population increases significantly. The predicted $(g-K)$ colour is 2.38, very close to the observed value. In contrast, the result obtained from the $\Gamma$ model is not affected as much by the inclusion of the NIR data, with $(g-K) = 2.27$, which is still too blue.

We apply the same fitting to all the 19 galaxies that have UKIDSS NIR photometry. The green triangles in Fig. 4 show the evidence ratio between the $\Gamma^B$ and the $\Gamma$ models as a function of SNR of the stacked spectrum. Yellow stars are the median of the values in two SNR bins, divided at $SNR = 40$. Compared to the red dots that only use the MaNGA stacked spectra, there is a significant increase in the evidence ratio, indicating an increase in the ability of discriminating the two SFHs.

The red and blue dash lines in Fig. 7 show the cumulative SFHs for the NIR sample obtained from the best-fit $\Gamma^B$ and $\Gamma$ model, respectively. Compared to the solid lines that show results using MaNGA spectra alone, the result obtained with the $\Gamma^B$ model including the NIR data shows a significant increase in the stellar mass formed 8 Gyr ago, although the uncertainty is large owing to the limited sample size. In contrast, the result obtained by the $\Gamma$ model does not change much.

In Fig. 13, we compare the half mass formation times and the fractions of old (>8 Gyr) population derived from optical-only and optical plus NIR data. Results are shown for both the $\Gamma^B$ model (squares) and the stepwise model (stars). We also indicate the preferred SFH with a circle. It is seen that for most galaxies, the inclusion of the NIR constraint tends to increase $t_{\text{form}}$, or equivalently, to produce a higher fraction in the old population. This suggests that the old fraction inferred from the optical spectra alone may be an underestimate.

However, we should point out that those results also raise a concern. The tension between the predictions based on optical spectra and NIR photometry indicates that the model adopted may not be sufficiently general. To examine this problem in more detail, we apply the posterior predictive check method to the sample of 19 galaxies with NIR photometry. The detail is presented in the Appendix A. In general, we find that the model is often over-constrained by the optical spectra, so that the posterior predictive distribution (PPD) of the NIR photometry is very narrow and the observed NIR data is almost always rejected by the posterior predictive check (PPC). In Bayesian statistics, the inferences obtained from a data set apply only to the model (hypothesis) assumed. If the model is not general enough to accommodate all the information in the data, inferences may still be made for the assumed model. In this case, one might want to use all available data, hopefully to obtain a balance between different constraints.

In summary, the combination of the NIR photometry and optical spectra provide additional evidence that an early old stellar population exists in most of the low-mass galaxies. However, due to the limitation of the assumed model, the current analysis is unable to reach a quantitative consistency between the optical-only and optical+NIR results. This should be kept in mind when one is concerned with the details of the inferred SFH.
5 UNCERTAINTIES

The results obtained above are based on the analysis of MaNGA spectra and UKIDSS NIR photometry with the use of BIGS. We adopt the state-of-the-art SPS model based on the E-MILES SSP templates, and assume three models for the SFH. In this section we examine further potential uncertainties in different parts of our analysis.

5.1 SFH models

We first examine whether or not our SFH model is sufficient to characterise the SFH of real galaxies. To this end, we first extend our SFH model by including additional bursts in the fitting. Models that have two and three bursts are referred to as the $\Gamma+2B$ model and $\Gamma+3B$ model, respectively. Fig. 14 shows the evidence ratio between these models as a function of SNR. Compared with $\Gamma+B$ model, the $\Gamma+2B$ model is preferred by many galaxies, but the global trend for this preference is mild. The evidence ratio between $\Gamma+2B$ and $\Gamma+B$ is much smaller than that between $\Gamma+2B$ and $\Gamma$ as shown in Fig. 4, indicating a weaker preference for more complex models. Inspecting the evidence ratio between $\Gamma+2B$ and $\Gamma+3B$ model, one can see that, even for galaxies which prefer a second burst, a third burst is unnecessary. These results show again that, given the current data quality and the SPS model, the $\Gamma+B$ model seems to be sufficient.

We further test whether the additional burst changes our basic conclusion. To this end, we estimate the difference between $t_{\text{half}}$ derived from SFH models with one and two bursts, denoted by $\Delta t_{\text{half}}$, for individual galaxies. In addition, we compare $\Delta t_{\text{half}}$ with the inference uncertainty: $\text{err}_{t_{\text{half}}} \equiv \left( \sigma^2_{t_{\text{half}}, \Gamma+B} + \sigma^2_{t_{\text{half}}, \Gamma+2B} \right)^{0.5}$, where $\sigma_{t_{\text{half}}, \Gamma+B}$ and $\sigma_{t_{\text{half}}, \Gamma+2B}$ are the standard deviations of the posterior distribution of $t_{\text{half}}$, inferred from the $\Gamma+B$ and $\Gamma+2B$ models, respectively. We use the ratio between $\Delta t_{\text{half}}$ and $\text{err}_{t_{\text{half}}}$ to characterise how well the differences in the derived $t_{\text{half}}$ are accounted for by the inference uncertainty. The results plotted in Fig. 15 show that adding a new burst into the model have almost negligible effects on the derived $t_{\text{half}}$. The differences between the derived $t_{\text{half}}$ are only slightly larger than the inference uncertainty, indicating that the $\Gamma+B$ model is an acceptable choice for our purpose.

We also briefly discuss the differences between the parametric and non-parametric models. Non-parametric models avoid the limitation imposed by assuming a specific functional form, but the total number of time intervals (time resolution) is usually limited by the data. In practice, codes that focus on spectral fitting and stellar population parameters, such as STARLIGHT (Cid Fernandes et al. 2005) and pPXF (Cappellari & Emsellem 2004), usually adopt the non-parametric approach, while the ones that focus on constraining model parameters, such as CIGAL (Noll et al. 2009; Boquien et al. 2019) and BEAGLE (Chevallard & Charlot 2016), prefer the parametric approach. Our analysis uses a stepwise model with 7 time intervals. As shown by both the mock tests and the fitting to real data, the stepwise model in general give results similar to the $\Gamma+B$ model. This consistency indicate that, as long as the SFH model is sufficiently flexible to describe the major stellar populations, the results from our spectral fitting are robust. For brevity, we will use the $\Gamma+B$ model for the rest of our discussion.

5.2 SSP models

Spectral modelling is based on the linear combination of SSP templates. Thus, the accuracy and completeness of the SSP templates can influence fitting results. Unfortunately, the uncertainties of the SSP templates are not well-understood. As a test of this uncertainty, we use the BC03 model to perform a consistent check. In contrast to the E-MILES model, the BC03 SSPs are constructed with the STELIB (Le Borgne et al. 2003) empirical stellar templates. Models assuming Padova isochrones (Bertelli et al. 1994) and the Chabrier IMF are used in the comparison. The top panel of Fig. 16 shows the evidence ratio between the $\Gamma+B$ and $\Gamma$ models, derived from the fitting with BC03 templates, as a function of the SNR. The trend seen here is similar to that shown in Fig. 4 although is slightly weaker.

We have also made a test using a mixture of E-MILES and BC03. In this model, the model fluxes are calculated as $F = F_{\text{BC}} \times f_{\text{BC}} + F_{\text{EM}} \times (1 - f_{\text{BC}})$, where $F_{\text{BC}}$ and $F_{\text{EM}}$ are the fluxes obtained...
Figure 14. The evidence ratio between the $\Gamma + B$ and $\Gamma + 2B$ models (left panel) and between the $\Gamma + 2B$ and $\Gamma + 3B$ models (right panel) as a function of SNR of the stacked spectra. Each red dot stands for the result of a MaNGA galaxy. Blue stars connected by a blue line are the median values in five SNR bins. Error bars are $1\sigma$ scatter among galaxies in individual bins.

Figure 15. The distribution of the difference in the half mass formation time between the $\Gamma + B$ and $\Gamma + 2B$ models (top), and the distribution of the difference normalized by the inference uncertainty (bottom). The mean value and standard deviation are indicated for the histogram in each panel.

5.3 The NIR photometry

The accuracy of the predicted NIR photometry depends on the accuracy of the SSP templates in the NIR. For the E-MILES model, the SSPs at $\lambda > 8950$ Å are constructed using the empirical IRTF stellar template (Cushing et al. 2005; Rayner et al. 2009). This treatment provides a self-consistent E-MILES SSP spectra with moderately high resolution ($\sigma = 60$ km s$^{-1}$) at the NIR. In contrast, the BC03 model uses theoretical, low resolution NIR spectra of BaSeL (Westera et al. 2002). This difference may affect the predicted NIR photometry. To test this uncertainty, we apply the BC03 model to the sample that has both MaNGA and UKIDSS data. We
Figure 16. The evidence ratio between the $\Gamma+B$ SFH and the $\Gamma$ SFH as a function of SNR of the stacked spectra, derived from the BC03 model (top panel) and the mixed model (bottom panel). Each red dot stands for the result of a MaNGA galaxy. Blue stars are the median values in five SNR bins connected by a blue line. Error bars are $1\sigma$ scatter of galaxies in individual bins.

Figure 17. The cumulative SFH inferred from the best-fit SFH models using E-MILES templates (dash line), BC03 templates (dash-dotted line), and the mixed model (solid line). Lines show the mean SFHs of sample, while the shaded region shows the scatter among sample galaxies obtained from the mixed model. Red and blue colours are for the $\Gamma+B$ and $\Gamma$ models, respectively.

plot the half mass formation look-back time, $t_{\text{half}}$, and the fraction of old (>8 Gyr) stellar population derived from optical only data and optical plus NIR data in Fig. 18. Comparing with the results shown in Fig. 13, we see that the two SSP models reach the same conclusion that the inclusion of NIR data enhances the significance of the old stellar population.

In addition to the BC03 model, here we mention briefly another SSP model family, the SSP model of Maraston (Maraston 2005; Maraston & Strömbäck 2011). This model was first presented in Maraston (2005) using low-resolution theoretical BaSeL stellar libraries, and an updated version using a set of stellar templates was published in Maraston & Strömbäck (2011). The Maraston model contains treatments of TP-AGB and HB stars that are different from those in E-MILES and BC03. These treatments lead to redder colours for SSPs with ages around 1 Gyr. As the dwarf galaxies studied here have quite a significant population of such ages, the difference in the predicted $(g-K)$ colour can be as large as $\sim 0.15$. Unfortunately, the Maraston model cannot be compared with E-MILES. The high-resolution model, which is needed for our modeling, is based on theoretical MARCS templates that have very limited age and metallicity coverages. The models based on empirical templates have relatively wider coverage in age and metallicity, but they generally only cover the optical range. Due to these reasons, it is difficult to make a meaningful comparison with this model.

In summary, our current SPS modelling is sufficient to describe the stellar population in the galaxies studied here, and our method is in general robust against the variation in both the SSP model and the SFH model. Thus, the main conclusions we have reached are robust, while the details are still uncertain.

6 SUMMARY AND DISCUSSION

In this paper, we analyse the spectra of a sample of low-mass galaxies using the Bayesian inference code BIGS. Our sample galaxies are selected from the SDSS IV-MaNGA IFU survey, and some of the galaxies also have NIR photometry obtained from the WSA catalogue. We stack the spatially resolved spectra of each galaxy within a radius of $1.0R_e$ to obtain a representative high SNR spectrum for the galaxy. We also stacked spectra in three radial bins, $[0.0 - 0.3]R_e$, $[0.3 - 0.7]R_e$ and $[0.7 - 1.2]R_e$, to study possible radial variations. We analyse the stacked spectra using a full spectrum fitting approach, making use of the MaNGA spectra from 3400 Å to 8900 Å. In our analyses, we adopt the state-of-the-art E-MILES SSP templates, and assume different SFH models to derive the stellar population properties. We use Bayesian model selection to distinguish between different models, and use the posterior distribution to constrain model parameters. Our main results can be summarised as follows:
Old stellar population in a low-mass galaxy exists not only in the central part, but is spread over the entire galaxy. On average, the variation of the SFH with radius is rather weak.

A model of SFH needs to be sufficiently flexible to reproduce an observed optical spectrum and the \((g - K)\) colour simultaneously. A higher fraction of the old stellar population is obtained when the \((g - K)\) colour is included as an additional constraint. However, the results obtained from the optical and NIR data have some tension, indicating that the current spectral synthesis model may not be sufficiently flexible to accommodate the data.

We test potential uncertainties both in the SSP model and the SFH model, and find that our main results about the existence of an old stellar population in low-mass galaxies are robust. However, the details of the SFH are still poorly constrained.

Although SFHs in low-mass galaxies have been studied quite extensively, the underlying physical processes that regulate star formation are still poorly understood. For example, our results suggest that the SFH of low-mass galaxies may consist of an early star formation episode, where about half of the stellar mass was formed, followed by a secondary and more extended phase of star formation. This type of SFH is consistent with the empirical model of Lu et al. (2014), which predicts that many dwarf galaxies have experienced a phase of rapid star formation at \(z > 2\). This enhanced star formation at \(z > 2\) may be related to the fast accretion of dark matter halos (e.g. Zhao et al. 2003; Mo & Mao 2004) and seems to be required by the observed upturn in the low-mass end of the stellar mass function of galaxies (Lan et al. 2016), but is not predicted by many models of galaxy formation (Lim et al. 2017). In particular, the mass fractions in different stellar populations obtained from our analysis are not well reproduced by current hydrodynamic simulations (e.g. Digby et al. 2019). These suggest that the feedback effect assumed in the model to suppress star formation in low-mass halos may be too strong at high redshift. Clearly, the observational constraints on the SFH we obtain here can provide important information about the feedback processes operating in the population of low-mass galaxies.

Of all the approaches adopted to probe the SFH of low-mass galaxies, the most reliable methods are perhaps those based on resolved stars. However, such observations can be made only for a small number of nearby galaxies. In contrast, methods based on stellar population models can use a large sample of galaxies. Among the approaches based on spectral synthesis, SED fitting of broad-band photometry (e.g. Janowiecki et al. 2017; Telles & Melnick 2018) and absorption line analysis (e.g. Kauffmann 2014) have been used to infer the SFHs of low-mass galaxies. Compared to these two approaches, our method based on full spectral fitting can in principle extract more information from the spectra. However, SED fitting has the advantage that muti-band photometry is easier to obtain. As we have shown, analysis based on spectra with limited wavelength coverage can result in biases in the inferred stellar population. Our Bayesian analysis that combines MaNGA optical spectra and NIR photometry from UKIDSS is an attempt to overcome this difficulty. As shown in §4.3, this approach is promising in probing the stellar population in low-mass galaxies, in particular in revealing the old population that may be missed in earlier investigations. However, it should also be noted that this approach is still in its early stage, and more explorations are needed to take full advantage of it. Accurate and self-consistent SSP templates are crucial for this type of analysis. As seen in §5, although variations

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**Figure 18.** Comparison of the half mass formation time (top) and the fraction of old population (bottom) obtained from fitting only the MaNGA stacked spectra (horizontal axis) and fitting both optical spectra and NIR photometry (vertical axis), using the BC03 templates. Results are colour coded by the \((g - K)\) colour of the galaxy.

- Based on Bayesian model selection, we demonstrate that low-mass galaxies contain an old stellar population that may be missed in results obtained from low-resolution spectra and a too restrictive model for the SFH.
- The best fit SFHs for both parametric and non-parametric models all show that most of the low-mass galaxies have formed more than half of their stellar mass at \(z > 2\). The half mass formation time and the cumulative SFH from our spectra fitting of the unresolved stellar populations are in good agreement with those obtained from resolved stars (Weisz et al. 2011).
- The average mass fraction of the old stellar population derived from SFH models that are sufficiently flexible is as high as \(\sim 0.6\), while a simple \(\Gamma\) model significantly underestimates this fraction. This result is consistent with the resolved observation (Weisz et al. 2011), but inconsistent with some hydrodynamic simulations (e.g. Digby et al. 2019).
- Central galaxies on average have more recent star formations than satellite galaxies, indicating that star formation in satellite galaxies may be affected by their environment.
- Old stellar population in a low-mass galaxy exists not only in the central part, but is spread over the entire galaxy. On average, the variation of the SFH with radius is rather weak.

Although SFHs in low-mass galaxies have been studied quite extensively, the underlying physical processes that regulate star formation are still poorly understood. For example, our results suggest that the SFH of low-mass galaxies may consist of an early star formation episode, where about half of the stellar mass was formed, followed by a secondary and more extended phase of star formation. This type of SFH is consistent with the empirical model of Lu et al. (2014), which predicts that many dwarf galaxies have experienced a phase of rapid star formation at \(z > 2\). This enhanced star formation at \(z > 2\) may be related to the fast accretion of dark matter halos (e.g. Zhao et al. 2003; Mo & Mao 2004) and seems to be required by the observed upturn in the low-mass end of the stellar mass function of galaxies (Lan et al. 2016), but is not predicted by many models of galaxy formation (Lim et al. 2017). In particular, the mass fractions in different stellar populations obtained from our analysis are not well reproduced by current hydrodynamic simulations (e.g. Digby et al. 2019). These suggest that the feedback effect assumed in the model to suppress star formation in low-mass halos may be too strong at high redshift. Clearly, the observational constraints on the SFH we obtain here can provide important information about the feedback processes operating in the population of low-mass galaxies.
in the SSP model generally do not change our results qualitatively, they do affect the details of inferences. Care must be taken in calibrating different observations. In the future, with improvements of our understanding about stellar spectra and of stellar spectral templates, the method and analysis proposed in this paper are expected to provide an important avenue to explore the star formation in low-mass galaxies.

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DATA AVAILABILITY

The data underlying this article were accessed from: SDSS DR15 https://www.sdss.org/dr15/; UKIDSS http://www.roe.ac.uk/. The derived data generated in this research will be shared on reasonable request to the corresponding author.

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APPENDIX A: POSTERIOR PREDICTIVE DISTRIBUTION OF THE \((G – K)\) Colour

In Bayesian context, once the posterior distribution \(P(\theta | D, H)\) is obtained through a chosen posterior sampler, one can make predictions by marginalizing the desired likelihood function over the posterior distribution. For example, the posterior predictive distribution (PPD) of a set of observable quantities \(D’\), given the constraining data \(D\), can be written as

\[
P(D’ | H) = \int P(D’ | \theta, H) P(\theta | D, H) d\theta
\]  

(A1)

where \(P(D’ | \theta, H)\) is the desired likelihood function describing the probability distribution of \(D’\) expected from the model \(H\) specified by the parameter set \(\theta\). Note that the set of the predicted observable, \(D’\), is not necessarily related to the constraining data \(D\). \(D’\) can be anything that can be predicted by the model through \(P(D’ | \theta, H)\). For a complex model \(H\), such as the one we are concerned here, the PPD cannot be obtained analytically. In this case, one can select a large sample of models, \(\{\theta\}_l\) (\(l = 1, 2, \ldots, L\)) (with \(L\) the sample size), from the posterior distribution, make a prediction for \(D’\) using each of the \(\theta \in \{\theta\}_l\), and obtain a sample of the PPD, \(P(D’ | D, H)_l\), from the posterior distribution derived from the \(\{\theta\}_l\). The PPD so obtained can then be compared with the observational data of \(D’\) to check whether the original model can accommodate the observational data.

Once the PPD is obtained, one can check a specific model family using a procedure called the posterior predictive check (PPC). The basic idea is the following: if the model is correct then the data replicated from the model should have a distribution that can accommodate the observational data of \(D’\), which will be denoted as \(D’’\). Any significant discrepancy between the PPD and \(D’’\) will signify the inadequacy of the hypothesis \(H\). To compare the PPD with the data, one can define a test statistic \(T(D’\prime)\). The tail probability (the \(p\)-value) of the test statistic can then be used to assess the ‘goodness of fit’ of the model to the data. In the Bayesian context, the \(p\)-value can be defined as

\[
p = P(T(D’\prime) \geq T(D’)) = \frac{\int I(T(D’\prime) \geq T(D’)) P(D’ | \theta, H) P(\theta | D, H) d\theta dD’}{\int P(D’ | \theta, H) P(\theta | D, H) d\theta dD’},
\]

(A2)

where \(I\) is the indication function for the condition \(q = 1\) if \(q\) is true and 0 otherwise. If the observational data \(D’\prime\) is incompatible with the model, then the test statistic from the data, \(T(D’\prime)\), should be a significant outlier of the distribution of \(T(D’)\) predicted by the model. In practice, if the posterior predictive \(p\)-value is close to 0 or 1, then the model is most likely inadequate.

Here we use the models constrained by the MaNGA data to make predictions for the NIR photometry and check whether or not the models can also accommodate the NIR data. We use the fitting of galaxy 9490-6104 as an example. To get the PPD, we draw \(L = 1000\) samples from the posterior distribution obtained from the MaNGA spectra of this galaxy using the \(\Gamma\) model and \(\Gamma+B\) model, respectively. We then use the \((g – K)\) colour as the predicted observable \(D’\). The PPDs derived from the two SFH models are shown in Fig. A1. The PPD of the \(\Gamma\) model peaks at \((g – K) = 2.61\) while that of the \(\Gamma+B\) model at \((g – K) = 2.56\). The measurement from WSA for this galaxy is \((g – K) = 2.39\). These results indicate that even with the inclusion of an early burst population, the lack of constraints from NIR bands may still lead to some bias in the stellar population.

As we have chosen the \((g – K)\) colour (denoted as \(K’\) for convenience) as our test observable, we can use a \(z^2\)-like test statistic to perform the PPC on the fitting results. The corresponding test statistic can be written as

\[
T_z = \frac{(K’ – K)^2}{\sigma^2},
\]

(A3)

where \(K’\) and \(\sigma^2\) are, respectively, the mean and standard deviation of the PPD obtained from a posterior sample. For an observed \((g – K)\) colour, \(K_o\), the corresponding statistic is

\[
T_o = \frac{(K_o – K)^2}{\sigma^2}.
\]

(A4)

The \(p\)-value defined by equation (A2) can then be obtained. Fig. A2 shows the distribution of the test statistics \(T\) and \(T_o\) obtained from the observed \((g – K)\) colour, together with the \(p\)-value obtained from equation (A2). The \(p\)-value is 0.0 in both of the two fitting results, indicating that the models are over-constrained by the MaNGA spectra.

As a comparison, we can make inferences by including the observed \((g – K)\) colour as a constraint. Using the likelihood function in equation (4), we apply the posterior predictive check method to the fitting results. The results are shown in the bottom panels of Fig. A1 and Fig. A2, respectively. By including this additional constraint, the \(\Gamma+B\) model can now pass the PPC test, while the \(\Gamma\) model still fail to reproduce the observation.

To statistically characterize the results, we apply PPC to all the 19 galaxies that have UKIDSS NIR measurements. We derive the PPD from the posterior distributions of the fitting results and calculate the 3rd level of the distribution as an approximate boundary. Successful models should predict a PPD that covers the probable observational ranges, and so the observation data are expected to fall within the boundary. We plot the difference between the predicted and observational \((g – K)\) colours, denoted as \(K_o - K\), in Fig. A3, and show the corresponding 3σ boundary as error bars. As one can see, if only the MaNGA spectra are used in the fitting, both the SFH models fail to pass the PPC test in many case. In contrast, if the NIR data is included in the fitting, the \(\Gamma+B\) model can generally pass this test, while the \(\Gamma\) still fails to reproduce the observation in many cases (see blue points in Fig. A3). These results strengthen our conclusion that the \(\Gamma+B\) model is a better description of the true SFH. They also indicate the importance of using the NIR data in order to correctly model the stellar population.

However, those results also signify tensions between the predictions based on high-resolution spectra with limited wavelength coverage and those based on broadband photometry covering a larger wavelength range. Ideally, if the optical spectra do not provide tight constraints on the stellar populations that can affect the NIR fluxes significantly, the posterior distribution inferred from the optical data should be broad enough to accommodate the observed NIR fluxes. In reality, however, the situation may be
Figure A1. Histograms showing the posterior predictive distribution of the \((g-K)\) colour. Left panels are results obtained using the \(\Gamma+B\) model, while right panels are using the \(\Gamma\) model. Top panels are based on MaNGA optical spectra, while bottom panels are results that include the \((g-K)\) colour. Red dash line in each panel denotes the \((g-K)\) colour from UKIDSS.

more complicated. Spectral synthesis modeling of observed spectra is high dimensional, and the posterior distribution may be complex. If the model cannot describe the observed spectra perfectly, there is no guarantee that the 'best-fit' model derived from part of a spectrum is also the best-fit model for the entire spectrum. The posterior distribution inferred from part of the spectrum may then lead to biased predictions for the spectrum outside the observed window. In principle, any uncertainties in the model itself should be included in the likelihood function. Unfortunately, such uncertainties are difficult to quantify for the spectral synthesis model concerned here.

Uncertainties of the SSP templates are perhaps the most important in affecting the spectral synthesis model. Available Models based on different SSP templates, with their own merits and shortcomings, may work better in some cases but worse in others. To test this, we use our fitting results based on the BC03 model and the mixture model described in §5.2. The \(\Delta_{g-K}\) predicted by these two models are shown in Fig. A4 in comparison with those predicted by the E-MILES model. As we can see, the BC03 model can reach consistencies between the optical fitting and the NIR observations in some cases where E-MILES fails, and vice versa.

In summary, the conflict between optical predictions and the NIR constraints may indicate that the current model is not flexible enough. Allowing flexibility in, e.g. SSP templates, may alleviate such tension. As the factors that can contribute to the conflict are difficult to quantify, making full use of all available data to constrain model parameters may be a reasonable approach to reach a compromise among different constraints.
Figure A2. Posterior predictive check for the $\Gamma+B$ model (left) and the $\Gamma$ model (right). Histograms show the distribution of the test statistics $T_i$ for the 1000 samples drawn from the posterior distribution. Top panels are results based on fitting the MaNGA optical spectra, while bottom panels are results that include the $(g-K)$ colour. Red dash line in each panel denotes the test statistics $T_o$ calculated from the UKIDSS observation.

Figure A3. Differences between the posterior predicted $(g-K)$ colour from fitting MaNGA optical spectra and the WSA values. Results are derived from $\Gamma+B$ model (left) and $\Gamma$ model (right), red and blue dots are from MaNGA optical fitting and optical+NIR fitting, respectively. Error bars denote the 3$\sigma$ level of the PPD.
Figure A4. Differences between the posterior predicted \((g - K)\) colour from fitting MaNGA optical spectra and the WSA values. Results are derived from E-MILES (red), BC03 (blue), and mixed models (green). Error bars are the 3\(\sigma\) level of the PPD.