Impact of dynamical charm quarks

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Motivation

- An enormous effort is made to generate $N_f = 2 + 1$ configurations
  - BMW, CLS, JLQCD, RBC/UKQCD, ...

Advantages

- Cheaper
  - odd numbers of flavors are expensive, e.g. inclusion of $s \sim$ doubling of costs
- Easier to define (and keep!) a chiral trajectory
- The charm quark has a very small impact on low energy observables (decoupling)
- No lattice artifacts $O([am_c]^{\alpha})$ which may be difficult to control
- With Wilson fermions: one may get away with $b_X$ from PT

Disadvantage

- Unknown systematical errors in high energy observables (where decoupling does not apply)

Goal

Estimate the difference between $N_f = 2 + 1 + 1$ and $N_f = 2 + 1$ for high energy observables
- e.g. charmonia masses or static force at small distances
Decoupling

The effect of heavy sea quarks at low energies

- Effective theory $\mathcal{L}_{\text{eff}} = \mathcal{L}^{N_l} + \frac{1}{M^2} \mathcal{L}_6 + \ldots$
  
  [S.Weinberg (1979)]
  
  - $N_q$ quarks in total
  - $N_l$ light quarks
  - Effective theory contains only the light quarks. Leading order describes full theory up to power-corrections $O((\Lambda_q/M)^2)$

- Detailed study
  
  [M.Bruno, J.Finkenrath, F.Knechtli, B.Leder, R.Sommer (2014)]
  
  [A.Athenodorou, M.Bruno, J.Finkenrath, F.Knechtli, B.Leder, M.Marinkovic, R.Sommer (2014)]
  
  - Factorization formula
    $$\frac{m_q^{\text{had}}(M)}{m_q^{\text{had}}(0)} = Q_{\text{had}}^{l,q} \times P_{l,q}(M/\Lambda_q) + O((\Lambda_q/M)^2)$$
  
  - Particularly simple: $\frac{m_q^{\text{had1}}(M)}{m_q^{\text{had2}}(M)} = r + O(M^{-2})$
  
  - Numerical study with $N_q = 2, N_l = 0$:
    $r_0/\sqrt{t_0}$ has a 0.1(6)% effect at $M \sim M_c/2$
We compare QCD with $N_f = 2$ heavy ($M \sim M_c$) quarks to quenched QCD

- We want to
  - Further confirm decoupling
    - Full theory $= \text{QCD}^{N_f=2}$
    - Effective theory $= \text{QCD}^{N_f=0}$
  - Investigate “high” energy observables for which decoupling does not apply

Simulate QCD$^{N_f=2}$ at $M = M_c$, several lattice spacings

Compute: $r_0/a$, $t_0/a^2$, $a m_P$, $a m_V$

Continuum extrapolate dimensionless ratios, e.g.

$r_0/\sqrt{t_0}$, $\sqrt{t_0} m_P$, $\sqrt{t_0} m_P$, $m_V/m_P$

Simulate QCD$^{N_f=0}$. Matching:

$$[t_0/a]^{N_f=0} \approx [t_0/a]^{N_f=2} \Rightarrow \beta \text{ for similar lattice spacings}$$

$$\left[\sqrt{t_0} m_P\right]^{N_f=0} \overset{!}{=} \left[\sqrt{t_0} m_P\right]^{N_f=2}_{\text{cont}} \Rightarrow \mu$$
Simulations

Dynamical quarks

- Gauge action: plaquette, $\beta \in \{5.7, 6.0, 6.2\}$
- Fermion action: doublet of twisted mass fermions $\psi = \begin{pmatrix} c \\ c' \end{pmatrix}$
  - $c_{SW}$ [K.Jansen, R.Sommer (1997)]
  - $\kappa_c$ interpolation of [P.Fritzsch et al (2012)], [P.Fritzsch, N.Garron, J.Heitger (2015)]
  - $a \mu = Z_P \times \frac{M_c}{\Lambda^{(2)}} \times \Lambda^{(2)} L_1 \times \frac{m}{M} \times \frac{a}{L_1}$
    [P.Fritzsch, F.Knechtli, B.Leder, M.Marinkovic, S.Schaefer, R.Sommer, F.Virotta (2012)]

Quenched quarks

- Gauge action: plaquette, $\beta \in \{6.34, 6.672, 6.9\}$
  Estimated from $\frac{r_0}{a}(\beta)$ [S.Necco, R.Sommer (2001)] and $t_0/r_0^2$ [M.Bruno]
- Valence quarks: doublet of twisted mass fermions
  - $c_{SW}$ [M.Lüscher, S.Sint, R.Sommer, P.Weisz, U.Wolff(1996)]
  - $\kappa_c$ interpolation of [M.Lüscher, S.Sint, R.Sommer, P.Weisz, U.Wolff (1996)]
  - $a \mu$: For each $\beta$, 3 values $\rightarrow$ interpolation to matching point
Ensembles

- Open boundaries in time, periodic in space
  - Milder critical slowing down than on torus
  - openQCD-1.2 [M. Lüscher, S. Schaefer (2013)]
  - $c_G = c_F = 1$

| $\beta$ | $\frac{L}{a} \times \frac{T}{a}$ | $a/\text{fm}$ | $a\mu$ | MDUs |
|---------|----------------------------------|--------------|--------|------|
| 5.700   | $32 \times 120$                  | 0.051        | 0.113200 | $17k$ (17k) |
| 6.000   | $48 \times 192$                  | 0.033        | 0.072557 | $22k$ (11k) |
| 6.340   | $32 \times 120$                  | 0.051        | -       | $20k$ (20k) |
| 6.672   | $48 \times 192$                  | 0.033        | -       | $74k$ (21k) |
| 6.900   | $64 \times 192$                  | 0.025        | -       | $100k$ (65k) |

critical slowing down compatible with $z \sim 2$
Measurements

- $r_0/a$, Sommer scale
  Computed as in [M.Donnellan, F.Knechtli, B.Leder, R.Sommer (2011)]
  - HYP-smeared Wilson loops, 3-4 levels
  - GEVP for static potential $aV(r)$
  - $r^2V'(r)\big|_{r=r_0} = 1.65$

- $t_0/a^2$, scale from gradient flow
  [M.Lüscher (2010)]
  - Wilson discretization of the gradient flow
  - Clover discretization of the action density $E = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$
  - $t^2\langle E(t) \rangle\big|_{t=t_0} = 0.3$

- Meson masses
  - From zero momentum correlation functions
    $f(x_0, y_0) = \sum_{x,y} \langle J(x)J^\dagger(y) \rangle$
  - Stochastic propagators
  - Local interpolating fields
    $J \in \{ \bar{c}\gamma_5 c, \bar{c}\gamma_k c \ldots \}$
    $\rightarrow m_P \quad \rightarrow m_V$
Effective masses

- Use T symmetry: \( g(x_0) = \frac{f(a, x_0) + f(T-a, T-x_0)}{2} \)
- Effective mass: \( a m_{\text{eff}}(x_0) = \log \left( \frac{g(x_0)}{g(x_0+a)} \right) \)
- Black = \( m_P \), blue = \( m_V \), finest lattice

![Graph showing effective masses](image_url)
Systematical errors

- \( a_\mu = Z_P \times \frac{M_c}{\Lambda^{(2)}} \times \Lambda^{(2)} L_1 \times \frac{\bar{m}}{M} \times \frac{a}{L_1} \)
  - \( \frac{M_c}{\Lambda^{(2)}} = 4.87 \)
  - Largest errors: \( \Lambda^{(2)} L_1, \frac{\bar{m}}{M} \)
    common to all points

- \( \kappa_c \) mistuning
  - Maximal twist: \( m_{PCAC} = 0 \)
  - \( \bar{m} = \frac{1}{Z_P} \sqrt{\mu^2 + Z_A^2 m_{PCAC}^2} \)
  - We have on all ensembles
    \( \frac{\bar{m} - \mu / Z_P}{\bar{m}} < 0.3\% (2\%) \)

- Finite volume effects:
  negligible \( \frac{L}{\sqrt{t_0}} > 10 \)

- Lattice artifacts:
  \( O(a^2) \)
Matching zero and two flavor QCD

\[ N_f = 2 \]

Linear & constant fits
\[ \rightarrow \] compatible continuum values

We work with value from linear fit
Matching zero and two flavor QCD

$N_f = 2$

$N_f = 0$

- Linear & constant fits → compatible continuum values
- We work with value from linear fit

- Black: $\sqrt{t_0} m_P$
- Blue: $\sqrt{t_0} m_V$
- $m_P$ linear in $\mu$ (like HQET)
Expected effect (decoupling): below 0.3%

No disagreement found at a precision of $\sim 2\%$
Results: masses

- Decoupling not applicable
- No effect resolvable at a precision of 0.7%
- Error dominated by $\Delta_{\sqrt{t_0} m_P}^{N_f=2}$

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Charm effects
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Errors in $\mu$ cancel to a large extent
No effect resolvable at a precision of 0.2%
\[
\bar{m} = \frac{1}{Z_P} \sqrt{\mu^2 + Z_A^2 m_{PCAC}^2}
\]

- \(Z_P^{N_f=2}, M/\bar{m}\)  
  [P.Fritzsch et al (2012)]
- \(Z_A^{N_f=2}\)  
  [M.Della Morte et al (2005)]
- \(Z_P^{N_f=0}, M/\bar{m}\)  
  [A.Jüttner (2004)]
- \(Z_A^{N_f=0}\)  
  [M.Lüscher et al (1997)]

\(\bar{m}\) values at different scales but \(M\) values comparable

\(~5\%\) effect, but large errors
Results: strong coupling

- Strong coupling from the static force: $\alpha_{qq}(\mu) = \frac{1}{C_F} r^2 V'(r)$
- Significant effect at $\mu = 1/r \sim 1.6$ GeV and above
- Not a lattice artifact
Conclusions

- Effects of dynamical charm quarks
  - tiny in charmonium masses
  - significant in $\alpha_{qq}$ at large energies
  - quite sizable in the RGI mass
- Lattice artifacts appear to be $O(a^2)$ below $a = 0.05$ fm

Outlook

- Higher statistics
- Third lattice spacing with $N_f = 2 \Rightarrow$ smaller errors everywhere
- More observables
  - Charmonium spectrum
  - Matrix elements
  - Quenched strange quark $\rightarrow f_{D_s}$