The impact of filamentary accretion of subhaloes on the shape and orientation of haloes

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ABSTRACT

Dark matter haloes are formed through hierarchical mergers of smaller haloes in large-scale cosmic environments, and thus anisotropic subhalo accretion through cosmic filaments have some impacts on halo structures. Recent studies using cosmological simulations have shown that the orientations of haloes correlate with the direction of cosmic filaments, and these correlations significantly depend on the halo mass. Using high-resolution cosmological N-body simulations, we quantified the strength of filamentary subhalo accretion for galaxy- and group-sized host haloes (\(M_{\text{host}} = 5 \times 10^{11} - 13 \, M_\odot\)) by regarding the entry points of subhaloes as filaments and present statistical studies that how the shape and orientation of host haloes at redshift zero correlate with the strength of filamentary subhalo accretion. We confirm previous studies that found the host halo mass dependence of the alignment between orientations of haloes and filaments. We also show that, for the first time, the shape and orientation of haloes weakly correlate with the strength of filamentary subhalo accretion even if the halo masses are the same. Minor-to-major axis ratios of haloes tend to decrease as their filamentary accretion gets stronger. Haloes with highly anisotropic accretion become more spherical or oblate, while haloes with isotropic accretion become more prolate or triaxial. For haloes with strong filamentary accretion, their major axes are preferentially aligned with the filaments, while their angular momentum vectors tend to be slightly more misaligned.

Key words: methods: numerical – methods: statistical – galaxies: haloes – cosmology: dark matter

1 INTRODUCTION

According to the standard cosmological model, dark matter haloes are assembled via the hierarchical mergers of a number of smaller haloes (White & Rees 1978). Haloes reside in large-scale cosmic environments so-called "cosmic web" (Bond et al. 1996), which are classified as voids, sheets, filaments and clusters. We can recognize these environments by wide-field galaxy surveys, such as the Sloan Digital Sky Survey (York et al. 2000). Some properties of galaxies such as colour, age, size, and luminosity function of galaxies depend on cosmic environments (e.g., Murphy et al. 2011; Guo et al. 2015; Tempel et al. 2015; Chen et al. 2017), indicating that large-scale environments are responsible for formation histories of haloes and galaxies embedded in them.

Cosmological simulations have been suggesting that assembly histories of haloes depend on the environments around them (e.g. Hahn et al. 2007a,b; Maulbetsch et al. 2007), and such environmental effect would characterize some properties of haloes such as the shape, orientation, angular momentum and spin (e.g., Patiri et al. 2006; Hahn et al. 2007a,b; Zhang et al. 2009; Wang et al. 2011; Vera-Ciro et al. 2011; Libeskind et al. 2012, 2013; Trowland et al. 2013; Kang & Wang 2015; Lee et al. 2017; Xia et al. 2017; Wang & Kang 2017; Ganeshaiah Veena et al. 2018; Obuljen et al. 2019; Lee 2019). For example, major axes of host haloes tend to be preferentially aligned with the directions of filaments (Hahn et al. 2007b; Zhang et al. 2009; Libeskind et al. 2013; Ganeshaiah Veena et al. 2018) and entry points of subhaloes (Kang & Wang 2015; Shao et al. 2018). Hahn et al. (2007a) showed that less massive haloes in clusters tend to be less spherical and more prolate, and have higher spins than those in other cosmic environments (void, sheet and filament). Vera-Ciro et al. (2011) argued that the evolution of halo shape correlates well with the distribution

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of the infalling material and environments. They suggested that when haloes accrete mass through narrow filaments at early epochs, they tend to be prolate. On the other hand, when the accretion changes more isotropic at later epochs, the halo becomes oblate or triaxial.

These attempts imply that how haloes accrete mass through filaments is related to the shape and orientation of them. This also means how haloes accrete subhaloes through filaments is important because subhaloes contribute about 50 – 70% of the final host halo mass (Stewart et al. 2009). These values seem to be consistent with the picture that about 40% of halo mass comes from smooth accretion of dark matter particles, which were not bounded by any smaller haloes (Genel et al. 2010). The orbit of infalling subhaloes has a highly anisotropic distribution (e.g., Tormen 1997; Knebe et al. 2004; González & Padilla 2016), which would be related to that the spatial distribution of satellite galaxies around the Milky Way and the Andromeda galaxy are preferentially aligned in flattened planes (so-called "plane of satellites") (e.g. Lynden-Bell 1976; Kroupa et al. 2005; Ibata et al. 2013). Cosmological simulations have been suggesting that these anisotropic distributions of satellites may be influenced by infalling subhaloes along filaments (e.g., Libeskind et al. 2012, 2013).

González & Padilla (2016) showed that about 20% number of subhaloes that were accreted by their host haloes by $z = 1$ is coming from filaments, and this number fraction corresponds to 40% of the total subhalo mass, although there is a large halo-to-halo scatter. The strength of filamentary accretion of subhaloes would have some impacts on the shape of haloes as suggested by Vera-Ciro et al. (2011). However, Vera-Ciro et al. (2011) used only five haloes, which is not enough to fully capture the correlation of filamentary subhalo accretion with the shape and orientation of haloes.

To address these questions, we explore statistics of the correlation of filamentary subhalo accretion with the shapes (axis ratio) and orientations of abundant galaxy- and group-sized host haloes with mass range $5 \times 10^{11} - 12 M_\odot$, using two high-resolution and large cosmological N-body simulations (Ishiyama et al. 2015; Ishiyama & Ando 2019). We identify prime directions of filamentary subhalo accretion by a similar method proposed by Shao et al. (2018) and calculate the total numbers and masses of subhaloes accreted along the directions. The fractions of them to all subhaloes of each host halo represent the strength of the anisotropic assembly of the host halo. Then, we investigate the impact of anisotropic accretion of subhaloes on shapes and orientations of host haloes. This paper provides significant insight for understanding the galaxy formation histories in the large-scale universe.

This paper is organized as follows. In Section 2, we describe the details of our two cosmological N-body simulations and sample selection in our work. In Section 3, we explain how we calculate the accretion properties of subhaloes, and shapes and orientations of their host haloes. In Section 4, we present our statistical results of the impact of filamentary accretion of subhaloes on shapes and orientations of host haloes. Finally, we discuss and summarize our results in Section 5.

2 COSMOLOGICAL N-BODY SIMULATIONS

We use two large cosmological N-body simulations, the $\nu^2$GC-H2 (Ishiyama et al. 2015) and the Phi-1 (Ishiyama & Ando 2019) summarized in Table 1, and the cosmological parameters of them are $\Omega_0 = 0.31, \Omega_M = 0.048, A_0 = 0.69, h = 0.68, n_s = 0.96$, and $\sigma_8 = 0.83$, which are consistent with the observation of the cosmic microwave background obtained by the Planck satellite (Planck Collaboration et al. 2014, 2018). We identified haloes and subhaloes by ROCKSTAR (Behroozi et al. 2013a) and constructed their merger trees by CONSISTENT TREES code (Behroozi et al. 2013b). The halo/subhalo catalogues and merger trees of those simulations are publicly available at http://hpc.imit.chiba-u.jp/~ishiymtm/db.html.

We analyze galaxy-sized ($M_{\text{host}} = 5 \times 10^{11} - 12 M_\odot$) and group-sized host haloes ($M_{\text{host}} = 5 \times 10^{12} - 13 M_\odot$) at redshift $z = 0$, where $M_{\text{host}}$ is the halo virial mass. We exclude haloes that experience major mergers at $z < 1$ from analysis because recent major mergers are much more likely to affect structural properties of haloes than continuous accretion of subhaloes. We define mergers with the mass ratio greater than 0.3 as the major merger. The numbers of host haloes analyzed in the Phi-1 and the $\nu^2$GC-H2 simulations are 258 and 2405 for $M_{\text{host}} = 5 \times 10^{11} - 12 M_\odot$, respectively, and 26 and 2604 for $M_{\text{host}} = 5 \times 10^{12} - 13 M_\odot$, respectively. We select subhaloes with $M_{\text{halo}} > 5 \times 10^5$, where $M_{\text{halo}}$ is the mass ratio between their progenitor haloes at the accretion redshift $z_{\text{acc}}$ and host haloes at $z = 0$. The accretion redshift $z_{\text{acc}}$ is the time when progenitor haloes first pass through the virial radius of the most massive progenitors of the host haloes (so-called ‘main branch’).

3 METHODS

3.1 The shape and orientation of host haloes

To quantify the shape and orientation of haloes, we use axis ratios, $c/a$ and $b/a$ ($a \geq b \geq c$), and vectors of major axis $\vec{e}_{\text{major}}$, all of which Rockstar computed by the method in Allgood et al. (2006). The square-roots of the eigenvalues and eigenvectors of the inertia tensor of haloes correspond to the axis lengths, $a$, $b$ and $c$, and their vectors, respectively. We also use angular momentum vectors $\vec{J}$ provided by Rockstar to quantify the orientation of haloes.

3.2 Filamentary accretion of subhaloes

To investigate the impact of filamentary accretion of subhaloes on the shape and orientation of host haloes at $z = 0$, we detect directions from where subhaloes preferentially are accreted. First, we identify entry points of subhaloes with $M_{\text{halo}} > 5 \times 10^4$ on host haloes. We regard the direction of entry point of the most massive subhalo from the centre of host halo as a filament and assign the remaining subhaloes within the opening angle of 30$^\circ$ from the filament. Then we apply this procedure to the next most massive subhalo that is not assigned to any filaments, and repeat until all the subhaloes are assigned to any filaments. Finally, we regard the direction of filament $\vec{e}_{\text{filament}}$ that contains the largest number of assigned subhaloes as the main filament and use it to quantify the strength of filamentary accretion of subhaloes.

MNRAS 000, 1–8 (2018)
This method is based on Shao et al. (2018), which quantified filamentary accretion of subhaloes for their cosmological hydrodynamical simulations. The difference is that they applied the same iterative procedure for the top 11 to 80 massive satellites in the stellar mass. Although we use subhaloes with $M_{\text{ratio}} > 5 \times 10^{-4}$ to detect the main filament, we compared the results with $M_{\text{ratio}} > 5 \times 10^{-5}$ and $M_{\text{ratio}} > 5 \times 10^{-3}$. The choice of $M_{\text{ratio}}$ does not strongly affect the directions of main filaments, reinforcing the effectiveness of our method to detect filamentary accretion of subhaloes.

We quantify the strength of filamentary accretion of subhaloes for three groups of subhaloes with different sub-to-host halo mass ratio ranges of $M_{\text{ratio}} = (0.5 - 5) \times 10^{-4}$, $(0.5 - 5) \times 10^{-3}$, and $(0.5 - 5) \times 10^{-2}$. Then we calculate the number and mass fractions of subhaloes in the main filament relative to all subhaloes in each group, $F_{\text{number}}$ and $F_{\text{mass}}$, respectively. In the case of high-$F_{\text{number}}$ or $F_{\text{mass}}$, subhaloes are expected to be preferentially accreted into their host haloes from a specific direction, and its accretion is highly anisotropic. On the other hand, for haloes with low-$F_{\text{number}}$ or $F_{\text{mass}}$, accretion of their subhaloes is expected to be more isotropic than haloes with higher-$F_{\text{number}}$ or $F_{\text{mass}}$.

### 4 RESULTS

#### 4.1 Distributions of number fraction $F_{\text{number}}$ and mass fraction $F_{\text{mass}}$

Fig. 1 shows the probability distributions of the number fraction $F_{\text{number}}$ and the mass fraction $F_{\text{mass}}$ of filamentary subhalo accretion in host haloes with $M_{\text{host}} = 5 \times 10^{11-12}$ and $5 \times 10^{12-13} \, M_\odot$ for three different sub-to-host halo mass ratios, $M_{\text{ratio}} = (0.5 - 5) \times 10^{-4}$, $(0.5 - 5) \times 10^{-3}$, and $(0.5 - 5) \times 10^{-2}$. The results of two simulations agree well with each other regardless of the host halo mass and the sub-to-host mass ratio.

The distributions differ between the sub-to-host mass ratios. For the sub-to-host mass ratios with $M_{\text{ratio}} = (0.5 - 5) \times 10^{-3}$ and $(0.5 - 5) \times 10^{-4}$, both of the number and mass fractions peak at 0.13 and 0.18, respectively, and are in the specific ranges of 0.1 $\lesssim F_{\text{number}}$, $F_{\text{mass}}$ $\lesssim 0.2$ and 0.1 $\lesssim F_{\text{number}}$, $F_{\text{mass}}$ $\lesssim 0.3$, respectively. Therefore, host haloes typically accrete 13% $\sim 18\%$ of the total number or mass of subhaloes with these mass ratios through a specific direction, and these ratios are more than twice as large as expected fractions $\sim 6.7\%$ in case that the entry points of subhaloes are isotropically distributed. The peak shifts higher values with increasing mass ratio, consistent with previous studies that suggested more massive subhaloes come from filament directions (Libeskind et al. 2014).

Integrating the mass fraction with respect to $M_{\text{ratio}}$, we find that 25% on average of the total mass of subhaloes is accreted through filaments, which is comparable to the result founded by Kang & Wang (2015). Considering three filaments that consist of the three largest number of assigned subhaloes in a host halo, the average of the mass fraction increases by $\sim 40\%$. This value is consistent with González & Padilla (2016), which calculated the mass fractions of haloes that have an average of three filaments. These consistencies reinforce the effectiveness of our method to detect filamentary subhalo accretions, although several different methods have been proposed to date (e.g. Aragón-Calvo et al. 2007a; Forero-Romero et al. 2009; González & Padilla 2010; Sousbie et al. 2011; Hoffman et al. 2012; Tempel et al. 2014; Alpaslan et al. 2014; Libeskind et al. 2018, and references therein).

The fractions of massive subhaloes with $M_{\text{ratio}} = (0.5 - 5) \times 10^{-2}$ are widely distributed in the range of $0.0 \lesssim F_{\text{number}}, F_{\text{mass}} \lesssim 0.6$ and do not have specific peaks. One of the reason is that the average number of such massive subhaloes is only even per host halo, and thus the halo-to-halo scatter is large. For massive subhaloes, there are some differences between the distributions of $F_{\text{number}}$ and $F_{\text{mass}}$, but for less massive ones, as expected, there is no significant difference. Therefore, although we hereafter show the distributions of halo properties as a function of only $F_{\text{number}}$, we also confirm similar results as a function of $F_{\text{mass}}$. The curves show several peaks in the Phi-1 simulation for $M_{\text{host}} = 5 \times 10^{12-13} \, M_\odot$ because the number of host haloes is only 26, indicating that the statistics are not enough. Hereafter, we exclude the Phi-1 simulation for $M_{\text{host}} = 5 \times 10^{12-13} \, M_\odot$ from the analysis.

#### 4.2 Correlation between the host halo shape and filamentary subhalo accretion

Fig. 2 shows the axis ratios $c/a$ of host haloes with $M_{\text{host}} = 5 \times 10^{11-12}$ and $5 \times 10^{12-13} \, M_\odot$ at $z = 0$ as a function of the number fraction of filamentary accretion $F_{\text{number}}$ for three different sub-to-host mass ratios, $M_{\text{ratio}} = (0.5 - 5) \times 10^{-4}$, $(0.5 - 5) \times 10^{-3}$, and $(0.5 - 5) \times 10^{-2}$. We find that axis ratios depend on the number fraction regardless of the host halo mass and the sub-to-host mass ratio. The shape of haloes tends to be more elongated with increasing anisotropy of subhalo accretion, although the correlation is rather weak. For haloes with $M_{\text{host}} = 5 \times 10^{11-12}$ and $5 \times 10^{12-13} \, M_\odot$ in the $\nu^2$GC-H2 simulation, the median values of $c/a$ are $\sim 0.64$ and $\sim 0.58$, respectively, which exist between the first and third quantiles for most $M_{\text{ratio}}$ bins.

This weak correlation between $F_{\text{number}}$ and $c/a$ becomes slightly stronger with decreasing mass ratio $M_{\text{ratio}}$. We do not show the result of the Phi-1 simulation for $M_{\text{host}} = 5 \times 10^{12-13}$ because the statistics are not enough. However, the existence of the correlation is supported by the $\nu^2$GC-H2 simulation. The median axis ratios $c/a$ tend to be lower in more massive haloes than in less massive haloes, consistent with previous studies (e.g. Allgood et al. 2006).

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Table 1. Numerical parameters of the two simulations. $N$ is the number of simulated particles, $L$ is the comoving box size, $\epsilon$ is the gravitational softening length, and $m_p$ is the particle mass.

| Name     | $N$  | $L$ [Mpc] | $\epsilon$ [kpc] | $m_p$ [M$_\odot$] | reference        |
|----------|------|-----------|------------------|-------------------|-----------------|
| $\nu^2$GC-H2 | 2048 | 103.0     | 1.57             | $5.06 \times 10^6$ | Ishiyama et al. (2015) |
| Phi-1    | 2048 | 47.1      | 0.71             | $4.82 \times 10^5$ | Ishiyama & Ando (2019) |

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MNRAS 000, 1–8 (2018)
Figure 1. Probability distributions of the number fraction $F_{\text{number}}$ (left panels) and the mass fraction $F_{\text{mass}}$ (right panels) of filamentary accretion of subhaloes in host haloes with mass ranges of $M_{\text{host}} = 5 \times 10^{11-12} M_\odot$ (upper panels) and $M_{\text{host}} = 5 \times 10^{12-13} M_\odot$ (bottom panels). In each panel, dashed and solid curves show the different simulations $\nu$GC-H2 and Phi-1, respectively. Red, green and blue curves with different thickness show results with the sub-to-host mass ratio of $M_{\text{ratio}} = (0.5 - 5) \times 10^{-4}$, $(0.5 - 5) \times 10^{-3}$ and $(0.5 - 5) \times 10^{-2}$, respectively. Dotted vertical lines at $F_{\text{number}}$ and $F_{\text{mass}} \sim 0.067$ correspond to the expected fractions in case that the entry points of subhaloes are isotropically distributed.

4.3 Accretion alignment with the orientation of host halo major axis

Fig. 4 shows the cosine between the filamentary accretion directions $\epsilon_{\text{filament}}$ and the orientations of host halo major axis $\epsilon_{\text{major}}$ at $z = 0$ as a function of the number fraction of filamentary accretion $F_{\text{number}}$. Two simulations results agree well with each other. The median values of cosine are higher than 0.5 in any cases, $\sim 0.57 - 0.60$, signifying that the major axis tends to be preferentially aligned with the filamentary accretion direction. These results are consistent with previous studies that showed the orientations of major axes tend to be a little aligned with the directions of filaments (Hahn et al. 2007b; Zhang et al. 2009; Libeskind et al. 2013) and entry points of subhaloes (Kang & Wang 2015; Shao et al. 2018). We also find that the cosine increases with increasing the number fraction $F_{\text{number}}$, indicating that the major axis with higher anisotropic subhalo accretion tends to be more aligned with the filamentary accretion direction. We can see that these trends do not clearly depend on the sub-to-host halo mass ratio.

The correlation is slightly stronger in more massive host haloes than in less massive ones. Besides, major axes of more massive host haloes tend to be slightly more aligned with filamentary directions on average than those of less massive ones, consistent with Ganeshaiah Veena et al. (2018). Subhaloes are preferentially accreted along major axes of host haloes, and this trend is stronger with increasing host halo masses (Kang & Wang 2015). Therefore, halo mass dependence seen in Fig. 4 reflects these trends found by previous studies.
4.4 Accretion alignment with the orientation of host halo angular momentum vector

Fig. 5 shows the cosine between the filamentary accretion directions \( \mathbf{c} / \mathbf{a} \) of host haloes at \( z = 0 \) as a function of the number fraction of filamentary accretion \( F_{\text{number}} \) for subhaloes with sub-to-host halo mass ratios of \( M_{\text{host}} = (0.5 - 5) \times 10^{-4} \) (left panels), \( (0.5 - 5) \times 10^{-3} \) (middle panels) and \( (0.5 - 5) \times 10^{-2} \) (right panels). Upper and lower panels are for the host halo mass ranges of \( M_{\text{host}} = 5 \times 10^{11-12} M_\odot \) and \( M_{\text{host}} = 5 \times 10^{12-13} M_\odot \), respectively. Red solid and blue dashed curves are results from the Phi-1 and the \( \nu^* \)GC-H2 simulations, respectively. Squares and circles are the median values, and whiskers are the first and third quartiles in each \( F_{\text{number}} \) bin. The number of haloes in each bin for the Phi-1 and the \( \nu^* \)GC-H2 simulations are specified at the upper and bottom of whiskers, respectively. We only plot bins in which the number of haloes is greater than five. The results with \( M_{\text{host}} = 5 \times 10^{11-12} M_\odot \) and \( M_{\text{ratio}} = (0.5 - 5) \times 10^{-5} \) of the \( \nu^* \)GC-H2 simulation are not displayed because of its mass resolution limit.

5 DISCUSSION AND SUMMARY

Previous studies suggested that the shape and orientation of host haloes correlate with some properties of filaments, with significant dependence on the halo mass. For example, major axes of haloes are more aligned with filaments as the halo mass increases (Zhang et al. 2009; Kang & Wang 2015; Shao et al. 2016), and angular momentum vectors of less massive haloes \( \lesssim 10^{12} M_\odot \) tend to be slightly parallel to the filaments while those of more massive haloes \( \gtrsim 10^{12} M_\odot \) tend to be perpendicular (Aragón-Calvo et al. 2007b; Hahn et al. 2007b; Libeskind et al. 2013; Ganeshaiah Veena et al. 2018). In this paper, we have extended these studies in terms...
analyzed a large number of galaxy-sized haloes ($M_{10}$) in filamentary accretion directions, their alignment angles with the major axes and the angular momentum vectors tend to be slightly more misaligned with the directions. On the other hand, with decreasing the number fractions of subhaloes from filamentary accretion directions, their alignment angles with the major axes and the angular momentum vectors tend to be randomly distributed.

These correlations are seen in haloes with $M_{\text{host}} = 5 \times 10^{11-12}$ and $5 \times 10^{12-13} M_\odot$, and their strength is slightly different. This halo mass dependence is expected to result from different infall patterns of subhaloes with different host halo mass. On the other hand, there is no significant difference in the distributions of the shape and orientations as a function of number fraction for the different sub-to-host mass ratio of $M_{\text{ratio}} = (0.5 - 5) \times 10^{-4}$, $(0.5 - 5) \times 10^{-3}$ and $(0.5 - 5) \times 10^{-2}$.

Our studies have been highlighting that the shape and orientation of haloes correlate with not only large-scale cosmic environments but the strength of filamentary subhalo accretion. This implies that, as seen in intrinsic alignments of galaxies and haloes (Bailin & Steinmetz 2005; Tempel et al. 2013; Xia et al. 2017; Okumura et al. 2019), large-scale cosmic environments would leave some imprints on stellar streams, galaxies and their satellites. Their observational signatures are influenced by how haloes accrete subhaloes, such as the plane of satellites (Libeskind et al. 2012, 2013), and statistics of streams (Morinaga et al. 2019). In future studies, we will perform theoretical studies to connect these signatures and large-scale cosmic environments and compare with observations provided by new facilities, which help to understand galaxy formation histories over cosmic time.

ACKNOWLEDGEMENTS
We thank Takanobu Kirihara for valuable comments and discussions. Numerical computations were partially carried out on the K computer at the RIKEN Advanced Institute for Computational Science (Proposal numbers hp150226, hp160212, hp170231, hp180180), Aterui and Aterui II supercomputer at Center for Computational Astrophysics, CICA, of National Astronomical Observatory of Japan. This work has been supported by MEXT as “Priority Issue on Post-K computer” (Elucidation of the Fundamental Laws and Evolution of the Universe) and JICFuS. We thank the support by MEXT/JSPS KAKENHI Grant Number 17H04828, 17H01101 and 18H04337.

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Figure 3. Distribution of axis ratios $b/a$ and $c/a$ of host haloes with $M_{\text{host}} = 5 \times 10^{11-12} M_\odot$ in the $2\text{GC-H2}$ simulation as a function of number fraction of filamentary accretion $F_{\text{number}}$ for $M_{\text{ratio}} = (0.5 - 5) \times 10^{-3}$. Colour represents the median value of $F_{\text{number}}$ for haloes in each region, in which the number of haloes is greater than five.
Figure 4. Cosine of the angle between the filamentary accretion directions $e_{\text{filament}}$ and the orientations of host halo major axes $e_{\text{major}}$ at $z = 0$ as a function of the number fraction $F_{\text{number}}$, in the same format as Fig. 2. Dotted horizontal lines at $\cos \theta = 0.5$ correspond to the random distribution of angle between $e_{\text{filament}}$ and $e_{\text{major}}$.

Figure 5. Cosine of the angle between the filamentary accretion directions $e_{\text{filament}}$ and the vectors of host halo angular momentum $J$ at $z = 0$ as a function of the number fraction $F_{\text{number}}$, in the same format as Fig. 2. Dotted horizontal lines at $\cos \theta = 0.5$ correspond to the random distribution of angle between $e_{\text{filament}}$ and $J$. 
