Multi-objective optimization design of rocker arm on crown-mounted compensator

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Abstract. Aiming to solve the severe wear of wire of rocker arm on crown-mounted compensator, the working principle of rocker mechanism is analyzed and mechanical properties of wire is calculated by a model based on Simulink. To reduce the contact stress amplitude between the wire rope and three pulleys on the rocker mechanism, the multi-objective optimization design model of rocker mechanism is constructed. In addition, the length of rocker and linkage arm are considered as design parameters by using MATLAB software. The results showed that the contact stress of wire rope decreased by 21.6\%, when the length of rocker and linkage arm is respectively 3450mm and 5900mm, which significantly reduced the possibility of fatigue failure and the wear loss of wire rope.

1. Introduction

While floating drilling platform explore oil and gas in the deep sea, heave compensators must be equipped to ensure the safety and efficiency of drilling or other operations [1, 2]. Rocker mechanism is the key mechanisms of crown-mounted compensator (CMC) to ensure the tension of wire rope [3]. The movement of crown block caused the fluxently changes of the contact angle between wire rope and three pulleys, which results in the severe wear of wire rope [4, 5], threatening the safety of rocker mechanism and even the normal operation of CMC.

With high technical reliability, CMC is early and widely used as compensation system on drilling platforms or ships [6]. Burns [7] first proposed the use of rocker mechanism on the CMC to reduce the number of turns between wire rope and the winch; Rocker mechanism on passive heave compensator are designed by Yanting Zhang [8] but the optimization was not involved. S Takagawa [9] applied the rocker mechanism to ROV heave compensator, founding that the winding method of the rope would affect the wrap angle between wire and pulleys, which causes the contact stress changes but not taken into consideration. Through the combination of experiment and finite element analysis, the failure form of drilling wire rope is studied by S Moradi [10], who founded that the changes of alternating stress caused by wire rope overload is the main reason for its wear failure.
Aiming to solve those problems, the working principle of rocker mechanism is analyzed and mechanical properties of wire is calculated through a model established by Simulink. To reduce the contact stress amplitude between the wire rope and three pulleys on the rocker mechanism and to improve the stability and service life, the multi-objective optimization design model of rocker mechanism is constructed, in which the structural parameters of rocker mechanism is optimize.

2. Working principle of rocker mechanism

CMC is illustrated as Fig.1, which consists of floating crown block, rocker arm, supporting structure, compensator cylinder, accumulator and high-pressure vessels. Rocker mechanism is composed of two sets of rods and guide wheels, whose role is to tighten the wire rope and reduce the number of turns between wire rope and winch, thereby enhancing the service life of wire rope.

![Figure 1. Crown-mounted Compensator](image1)

![Figure 2. Simplified schematic diagram of rocker mechanism](image2)

When the platform (ship) rises, the crown block moves downward relative to the track, as the piston of compensator cylinder retracts and compress the gas of the main accumulator, compensating for the displacement of the platform (ship) and storing energy. When the platform (ship) sank, it moves in the opposite state.

The linkage arm connects with the crown block while the rocker arm with the water table. During the movement of the crown block along the rail, the linkage arm and crown block move like an offset rocker mechanism as illustrated in Fig.2.

3. Mechanical properties Analysis of rocker mechanism

3.1. Contact analysis of arms with pulleys

The winding method of wire rope is shown in Fig.3. Wire rope pass through the winch and guide pulley at the fast line then wrap on the crown block and travelling block, leading to the fixed point at another side.
As illustrated in Fig 4, there are three wrap angles between wire rope and three pulleys. Pulley 1 is installed at the hinge of rocker arm and the water table, while pulley 2 at the hinge of linkage arm and rocker arm, pulley 3 at the hinge of linkage arm and crown block.

According to the Fig. 4, angle equations between contact angle and internal angle are established.

\[ \angle 1 = \frac{\pi}{2} - (\angle 2 + \angle 3) \]  

\[ \angle 4 = \pi - \angle 5 \]  

\[ \angle 6 = \frac{\pi}{2} - \angle 7 \]  

\[ \angle 3 = \angle 8 \]

Where:

\[ \angle 1 \] is the wrap angle between pulley 1 and the wire rope;

\[ \angle 4 \] is the wrap angle between pulley 2 and the wire rope;

\[ \angle 6 \] is the wrap angle between pulley 3 and the wire rope.

According the law of cosines, equation between internal angle and length of rocker are established in \( \triangle ABC \) and \( \triangle ABD \).

\[ c = \sqrt{(h+s)^2 + e^2} \]  

\[ \angle 3 = \angle 8 = \arctan \left( \frac{h+s}{e} \right) \]  

\[ \angle 2 = \arccos \left( \frac{a^2 + c^2 - b^2}{2ac} \right) \]  

\[ \angle 5 = \arccos \left( \frac{a^2 + b^2 - e^2}{2ab} \right) \]
\[
\angle \gamma + \angle \beta = \arccos \left( \frac{c^2 + b^2 - a^2}{2cb} \right)
\]  

(9)

Where:

- \(a\) is the length of the rocker arm;
- \(b\) is the length of the linkage arm;
- \(e\) is the horizontal distance between the pulley I and the pulley III hinge;
- \(c\) is the distance between the pulley I and the pulley III hinge;
- \(h\) is the initial position of the crown block;
- \(s\) is displacement of crown block, which is similar to sinusoidal motion \(s = A \sin(2\pi ft + \varphi)\).

Contact angles between the wire rope and three pulleys on the rocker mechanism are as follow

\[
\begin{align*}
\theta_1 &= \frac{\pi}{2} - \arccos \left( \frac{a^2 + (h + s)^2 + e^2 - b^2}{2ab(h + s) + e^2} \right) - \arctan \left( \frac{h + s}{e} \right) \\
\theta_2 &= \pi - \arccos \left( \frac{a^2 + b^2 - (h + s)^2 - e^2}{2ab} \right) \\
\theta_3 &= \frac{\pi}{2} - \arccos \left( \frac{b^2 + (h + s)^2 + e^2 - a^2}{2bh(h + s) + e^2} \right) + \arctan \left( \frac{h + s}{e} \right)
\end{align*}
\]  

(10)

Where: \(\theta_1, \theta_2, \theta_3\) are the wrap angles between three pulleys and wire rope, respectively.

To determine the wrap angle between three pulleys and wire rope, a Simulink model is built as seen in Fig.5 to calculate those angles, in which \(a = 3050\) mm, \(b = 4900\) mm, \(h = 700\) mm, \(e = 3250\) mm, and the simulation time is set to 2 cycles (24 seconds). Output simulation data to the workspace by the oscilloscope and formed a curve described the wrap angle changes over time shown in Fig.6. As seen, while the crown block moves up and down, the maximum of wrap angle ups to 106.8°. Due to the changes of wrap angle and the heavy load the wire rope carries, contact stress between the pulley and wire has a significant change over time.

![Figure 5. Simulink calculation model of contact angle](image1)

![Figure 6. The curve of contact angle](image2)

3.2. Contact stress analysis of wire rope and pulleys

Due to the low rotate speed of rocker mechanism at the process of heave compensation, the bearing efficiency and friction efficiency is not taken into consideration. That is, the wire rope tension forces equal on both sides of the pulley. Take any pulley on the rocker mechanism, studying its relationship with the role of wire rope.
Take the pulley and the wire rope in contact as a separate body as illustrated in Fig 7 (a) (not consider the gravity) and build XOY Cartesian Coordinate System.

\[ F_n = 2T \sin(\theta/2) \]  

(11)

Where:
- \( \theta \) is the wrap angle between wire rope and pulley;
- \( T \) is the tension force of wire rope;
- \( F_n \) is the resistance of the hinge to the pulley.

The contact stress of wire rope and pulley can be calculated by Hertz formula [11] (Line contact situation as shown in Fig 7b), according to the contact state between wire rope and pulley.

\[ \sigma_H = \sqrt{\frac{P}{\pi L} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \left( 1 - \frac{\mu_1^2}{E_1} \right) \left( 1 - \frac{\mu_2^2}{E_2} \right)} \]  

(12)

Where:
- \( P \) is the load, \( P = F_n \);
- \( R_1 \) is the radius of wire rope;
- \( \mu_1 \) is the Poisson ratio of wire rope;
- \( E_1 \) is the elasticity modulus of wire rope;
- \( R_2 \) is the radius of wheel groove;
- \( D \) is the diameter of pulley;
- \( \mu_2 \) is the Poisson ratio of pulley;
- \( E_2 \) is the elasticity modulus of pulley;
- \( L \) is the length of contact, \( L = D \times \theta \);
- \( \sigma_H \) is the maximum contact stress.

Simulink model is built to calculate the contact stress between wire rope and pulley as shown in Fig 8. Take the drill string load 450t, Wire rope radius \( R_1 = 254 \text{mm} \), Wheel groove radius \( R_2 = 28 \text{mm} \), and the Pulley diameter \( D = 1600 \text{mm} \). Set two cycles to carry out the simulation and the curves of contact stress is shown in Fig 9.
As illustrated in Fig 9, contact stress change with time periodically. During one heave cycle, the pulley 1, which has the smallest contact stress amplitude in three pulleys of 7.24MPa. While the pulley 2 has the highest amplitude of 25.15MPa, which indicates that the pulley 2 is where the most serious parts of wear happens. Under the alternating contact stress, the place with biggest stress on the wire rope would form a crack first, then gradually expanded, leading to fatigue broken eventually [12].

4. Multi-objective Optimization Design Model of Rocker Mechanism

4.1. Design variable analysis
According to Eq. (10), we can see that the value of the intermediate variable \( \theta \) is related to \( a, b, s, h, e \), and substituting \( \theta \) into Eq. (12) shows that the maximum contact stress is related to the variables \( a, b, s, h, e \). \( T, R_1, R_2, D \) depends on the drill string load. \( h, e \) depends on the arrangement of the rocker mechanism. \( R_1, R_2, D \) are selected by configuration principle of wire rope and pulley. \( s \) depends on the heave displacement of the floating drilling platform, \( s = A \sin (2\pi ft + \phi) \), where, \( t \) is process control variables. Then determine the design variables as \( X = (x(1), x(2)) = (a, b) \).

4.2. Objective function construction
Reducing the contact stress amplitude of the wire rope can reduce the fatigue and wear on the surface of the wire rope. As can be seen from Fig. 9, the contact stress between the wire rope and the pulley 1 reaches the peak at \( t=0s \) and reaches the trough at 2.72s. Set the contact stress difference between \( t=0s, t=2.72s \) and \( t=6s, t=2.72s \) as the objective function \( f(1), f(2) \). The contact stress of the wire rope and pulley 2 reaches the maximum at \( t = 6s \) and achieves the minimum at \( t=1.18s \). The contact stress amplitude is the difference between the two extreme values, which is set as the objective function \( f(3) \). The contact stress of the wire rope and the pulley 3 reaches the maximum at \( t=0s \) and achieves the minimum at \( t=6s \). The contact stress amplitude is the difference between the two extreme values, which is set as the objective function \( f(4) \).

4.3. Constraint analysis and established
The rocker mechanism should satisfy the movement condition of offset rocker slider and the condition of the compensative travel, and should not exist the crank. That means when the rocker mechanism is working, there should not only can’t find the dead point position \( e < a+b \) and the \( \Delta ABC \) always exist, \( a+b > \sqrt{e^2 + (s+h)^2} \), but also satisfy the condition of \( a+e > b \).

Using MATLAB language to describe the mathematical model is


\[
\begin{align*}
  g(1) &= x(2) - x(1) - e \\
  g(2) &= e - x(1) - x(2) \\
  g(3) &= e - x(1) - x(2)
\end{align*}
\]

(13)

4.4. Multi-objective optimization model solving.

This optimization design of crown heave compensation rocker mechanism is a nonlinear multi-objective programming problem. Using the multi-objective programming problem function of fgoalattain of the MATLAB to solve the problem. Call the format for \([x,fval,attainfactor,exitflag]=fgoalattain(@myfun,x0,goal,w,A,b,Aeq,beq,Lb,Ub,@mycon)\). Which the \(x\) represent the optimal solution of multi-objective optimization model, the \(fval\) represent the value of the multi-objective function in the optimal solution \(x\), the \(attainfactor\) represent the planning factor in the optimal solution \(x\), and \(attainfactor > 0\) represent the target without overflow. The \(exitflag\) is the exit flag of the multi-objective programming solution, the \(@myfun\) and \(@mycon\) is the name of the invoking objective function and the function of constraint conditions.

\(x0\) is the initialization vector of the design variable. In this model \(x0= [3050; 4900]\).

Goal is the value vector of the designed objective function. The length of the vector is equal to the number of the objective function. In this model \(goal= [152; 150; 148; 146]\).

Weight is the weight value vector of the objective function. The length of the vector is equal to the number of the objective function. In this model \(weight=goal\).

\(A\) is the coefficient of the linear inequality constraints, while \(b\) is the corresponding item in the right of the inequality. And \(Aeq\) is the coefficient of the linear equation constraints, while \(beq\) is the corresponding item in the right of the equation. Because there don’t have the linear inequality constraints and the linear equation constraints. The \(A\), \(b\), \(Aeq\) and \(beq\) all is empty matrix.

\(lb\) is the lower limit of the design variable, while the \(ub\) is the upper limit. In this model \(lb= [2900; 4800]\), while \(ub= [5400; 6100]\).

Through iterative calculation, the optimal solution \(X= (3450, 5900)\). It means that through optimized the length of the rocker arm is 3450mm, and the length of the linkage arm is 5900mm. The final rocker mechanism optimization results is shown in Table 1.

| Parameter name                  | Original parameter | Optimize parameter |
|---------------------------------|--------------------|--------------------|
| The length of the rocker arm [mm] | 3050               | 3450               |
| The length of the linkage arm [mm] | 4900               | 5900               |
| The amplitude of contact stress on Pulley.1[Mpa] | 7.24               | 7.02               |
| The amplitude of contact stress on Pulley.2[Mpa] | 25.15              | 22.21              |
| The amplitude of contact stress on Pulley.3[Mpa] | 18.75              | 14.51              |

It can be seen from the table 1 that, the wire rope wear situation has been improved after the optimization. The amplitude of the contact stress that wire rope applying on the pulley .1 was reduced by 3.04%. The amplitude of the contact stress of the pulley.2 which is the most serious wear and tear was reduced by 11.69%. And the amplitude of the contact stress of the pulley.3 which is the second wear and tear was reduced by 22.61%.

5. Conclusion

The contact angle between wire rope and pulleys on the rocker mechanism is analysed, and contact stress between wire rope and pulleys is calculated through a model established by Simulink. The results showed that contact stress change with time periodically. During one heave cycle, the pulley 1, which has the smallest contact stress amplitude in three pulleys of 7.24MPa. While the pulley 2 has the highest amplitude of 25.15MPa, which indicates that the pulley 2 is where the most serious parts of wear happens.
The optimization goal is that reducing the contact stress amplitude between the wire rope and three pulleys on the rocker mechanism, and the design variable, length of rocker and linkage arm, is ascertained by analysing the effect of the structural parameters of rocker mechanism to the contact stress. The multi-objective optimization design model of rocker mechanism is constructed through MATLAB software and calculated by fgoalattain function which is one of the multi-objective nonlinear programming function.

The contact stress of wire rope decreased by 21.6% with optimization of structural parameters(rocker arm of 3450mm and linkage arm of 5900mm), which significantly reduced the possibility of fatigue failure and the wear loss of wire rope.

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