Proposed evolution in Marolf-Maxfield toy model obtained through correspondence to spontaneous collapse theory

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Abstract

We consider a general correspondence between wave function collapse and evolution through wormholes, which presumably occurs whenever a black hole evaporates after Page time. We use this correspondence in order to explore a possible evolution from the Hartle-Hawking state to one of the superselection sectors for a topological model. The model considered is the Marolf-Maxfield (MM) topological toy model for 2D gravity which gives the full spectrum of boundary theories. By equating the MM topological toy model to the Bonifacio model of spontaneous collapse, an equation for the evolution of a matrix element due to a generator of a parameter in MM model is obtained.

1 Introduction

Recently a unitary black hole evaporation was obtained. This was done by considering a quantum extremal surface (QES) which was created inside the black hole. It turns out that the QES mysteriously enables particles deep inside the black hole to no longer be part of the hole, but rather part of the radiation. One finds that in order to understand this formalism, one needs to include summing over wormhole geometries’ connecting boundaries and that these wormholes enable the entropy of Hawking radiation to follow the expected time dependence [1]. This evolution is unique and can not be obtained by using ordinary equations of motion. A topological toy model for 2D gravity, such as that of Marolf-Maxfield (MM) [2], can consider the presence of spacetime wormholes which connect boundaries. Although these models are purely topological and can not describe time evolution, they give the full spectrum of boundary theories. Assuming wormhole and other topological fluctuations lead to a unique evolution in gravity, we suggest to explore this evolution by considering the correspondence which exists between two processes: evaporation of black holes after Page time on the gravity side and collapse of the wave function on the quantum side.

Although black hole evaporation and wave function collapse occur at very different scales and are due to very different theories, a similar mathematical structure between the two scenarios could be expected not only in light of [3,4] which suggest that general relativity may already be a quantum theory, but also due to the existence of similarities between the two scenarios.

To start with they both have a kind of apparent loss of information. Black holes look as if they lose their information by Hawking radiation during their evaporation. According to Copenhagen interpretation,

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wave functions lose their information when collapsing to one of the eigenstates, since all information on the original function is lost during a measurement.

Moreover they both have a kind of action at a distance. Black holes look as if they act at a distance since they must connect Hawking radiation at infinity to the interior of the black hole in order for the black hole to be able to evaporate after Page time. Wave functions look as if they act at a distance whenever a collapse takes place since we expect a wave function to collapse all at once in all the points in the universe.

Finally, the most subtle and surprising resemblance is related to the suggestion that both scenarios involve a kind of decoherence. In general decoherence can be viewed as the loss of information from a system into an environment. When collapsing of a wave function is considered, decoherence provides a framework for an apparent collapse. In this case a quantum system begins to obey classical probability rules due to suppression of interference terms after interacting with its environment. This leads to a known outcome in quantum theory: observers can not consider the result of an experiment as a measurement of previously unknown value. On the other hand when evaporation of black hole through a wormhole is considered a different decoherence is obtained. As was first noted by Coleman [6]: "topological fluctuations cannot lead to an observable loss of quantum coherence." Considering wormholes as topological fluctuation one finds that a loss of quantum coherence cannot be obtained if black hole evaporates through wormholes and in this case, observers may consider the result of an experiment to be a measurement of a previously unknown value. To emphasize, whereas observers cannot consider the result of an experiment as a measurement of previously unknown value when a collapsing wave function is considered, they can do so when an evaporating black hole or any other evolution through wormholes is considered.

This important difference between the decoherence of a collapsing wave function and the decoherence due to topological fluctuations is crucial in our analogies, since it limits the relevant models of wave function collapse. In other words, in order to obtain mathematical correspondence between evolution due to topological fluctuations and wave functions collapse we must consider only those mathematical models of collapsing wave functions which cause non-observable decoherence.

The purpose of the present work is to use this apparent correspondence in order to obtain an equation for the evolution of the density matrix in MM topological toy model for 2D gravity. Marolf and Maxfield constructed a Hartle-Hawking state, after computing the full spectrum of the associated boundary theories and considering the presence of spacetime wormholes connecting these boundaries. However, there is no evolution in MM model, since this model is a topological model and there is no time. As a matter of fact, if one tries to consider evolution through wormholes, one may expect a significant change in the equations of motion, such as apparent loss of information and action at a distance.

In this short paper we suggest obtaining the differential equation for the evolution of the density matrix through wormholes, by equating it to a model of collapse that has similar mathematical and physical characteristics. We consider models of collapse which uses the Lindblad equation [7]. However, as expected, the observation that evolution through wormholes does cause non-observable decoherence restricts the Lindblad equation as well as the possible expected models which produce the collapse. This leads us to Bonifacio model [8] which does not cause observable decoherence. Moreover, when MM model is considered one finds that the two models have the same mathematical structure: the distribution

\[ \text{distribution} \]

\[ \text{distribution} \]

1Note that though we consider the Copenhagen interpretation, the approach that a quantum state represents only knowledge and “facts” can only exist relative to the observer as proposed by Brukner [5] is more relevant in our correspondence, since in this approach the loss of information is only an apparent property.
considered in both Bonifacio model and MM-model is the Poisson distribution. This enables us to obtain an equation of the "evolution" of an element of the density matrix in MM model.

This paper is organized as follows: Section 2 briefly summarizes recent relevant concepts on black hole evaporation and MM model. Section 3 briefly review relevant spontaneous collapse theories and the Bonifacio model. Section 4 obtains the density matrix for the MM model and its evolution equation. Then, by equating the two density matrix evolution equations, the evolution of a matrix element due to a generator of a parameter in MM model is obtained. Section 4 is a summary and conclusion.

2 Brief review of the BH evaporation and Marolf-Maxfield model

Recent years have seen significant progress towards the resolution the puzzle of black hole evaporation [1, 9, 10, 11, 12, 13, 14, 15]. This puzzle comes from semi-classical computations that imply that black hole evaporation is non-unitary, in stark contrast with the principles of quantum mechanics [16]. To see this, consider forming a black hole from a pure state with enough energy in a compact region of a quantum gravity system. Such a black hole seems to evolve from a pure state to a mixed thermal state through Hawking radiation. This amounts to a loss of information and is incompatible with unitary time evolution. Moreover, it turns out that if black hole evaporation is a unitary process, the entanglement entropy between the outgoing radiation and the quantum state associated to the remaining black hole is expected to follow the Page curve [17, 18]. According to the Page curve initially the black hole entropy grows, but at some point in time - the Page time - the entropy has to stop rising and start dropping. The reversal at the Page time would have to occur roughly halfway through the process, and not at the very end of the evaporation. Thus, although it was natural to expect this, it turns out that quantum gravity effects could not restore unitarity if they occur only towards the very end of the evaporation.

Recently an explicit description of evaporating black holes was obtained [1] and succeeded in obtaining the expected Page curve. The Page curve was obtained for a double sided black hole in two dimensional Jackiw-Teitelboim (JT) gravity with conformal matter, which is allowed to evaporate into a non-gravitational reservoir coupled to one side of the black hole [1, 9, 19]. This calculation applied the Engelhardt-Wall prescription for computing holographic entanglement entropy [20]. According to this prescription one can compute the entanglement entropy of a boundary subregion B by considering all possible co-dimension two surfaces $\sigma_B$ which are homologous to B. For each $\sigma_B$ one can define the generalized entropy

$$S_{gen}(\sigma_B) = \frac{A(\sigma_B)}{4G_N} + S_{bulk}(\Sigma_B)$$

(1)

where $\Sigma_B$ is a bulk codimension-one surface bounded by $\sigma_B$ and B, i.e., $\partial \Sigma_B = \sigma_B \cup B$. As we can see, the entanglement entropy of a boundary subregion B involves two terms: the Bekenstein-Hawking area term in the Hubeny-Rangamani-Ryu-Takayanagi prescription [21, 22] and the quantum corrections in the form of the von Neumann entropy of the quantum fields on $\Sigma_B$. According to the prescription, in order to compute the entanglement entropy of a boundary subregion B, one then extremizes the generalized entropy over all surfaces $\sigma_B$ that are homologous to B. Surfaces $\sigma_B$ that extremize the generalized entropy are referred to as quantum extremal surfaces (QES). It turn out that during the evaporation, a black
hole creates a QES. After the Page time the QES is located just inside the horizon of the black hole and mysteriously enables particles deep inside the black hole to no longer be part of the hole, but rather part of the radiation. This inner core of radiation is an “island”.

One of the ways to obtain the formalism that led to the Page curve is summing over wormholes. Wormholes can be viewed as an outcome of the Hilbert space interpretation that includes states of closed ‘baby’ universes that propagate between distinct asymptotic boundaries. The baby universe Hilbert space of any theory can be constructed by acting on a state with no-boundaries, typically called the Hartle-Hawking state, with boundary creation operators. Alpha-states are the special states in this space that are eigenfunctions of all such operators, leading to inner products in the baby universe Hilbert space.

To summarize, these recent works suggest that a unitary black hole evaporation can be obtained and that this evaporation uses wormholes geometries with connecting boundaries.

Marolf-Maxfield model for Hartle-Hawking no-boundary state

Marolf and Maxfield [2] considered a topological model with asymptotic boundaries and computed the full spectrum of the associated boundary theories by considering the presence of spacetime wormholes connecting these boundaries. They obtained a Hartle-Hawking state with Poisson distribution for the number of connected components.

Here we point out only the relevant features of their model.

The Marolf-Maxfield [2] topological model is a simple 2-dimensional model of the bulk with asymptotic AdS Hilbert space that is represented by a random variable $Z$ with nonnegative integer values. By using the Taylor series of the generating function $e^{uZ}$ and identifying the parameter $\lambda$ as the sum over connected compact surfaces, they obtain $Z$ as a Poisson random variable with mean $\lambda$. They characterize the state $|Z = d\rangle$ as a spacetime with $d$ connected components, and define an annihilation operator acting to shift functions of $Z$: $a |f(Z)\rangle = \sqrt{\lambda} |f(Z + 1)\rangle$, so that $\hat{Z} = N = a^\dagger a$, and $a |Z = 0\rangle = 0$. This enables Marolf and Maxfield to obtain the state $|Z = d\rangle$ as:

$$|Z = d\rangle = \frac{1}{\sqrt{d!}} (a^\dagger)^d |Z = 0\rangle \quad (2)$$

and the Hartle-Hawking no-boundary state as:

$$|HH\rangle = e^{\sqrt{\lambda} a^\dagger} |Z = 0\rangle . \quad (3)$$

This causes the number operator to follow a Poisson distribution in a coherent state. Marolf and Maxfield noted that the appearance of the Poisson distribution can be understood from the result that all connected components of spacetime contribute the same amplitude after summing over genus, independent of the number of boundaries.

3 Brief review of the spontaneous collapse theory and Bonifacio model

Models of spontaneous wave function collapse [23, 24] were formulated as a response to the measurement problem in quantum mechanics [25]. The fundamental idea is that the unitary evolution of the wave
function describing the state of a quantum system is approximate. In collapse theories, the Schrödinger equation is supplemented with additional nonlinear and stochastic terms (spontaneous collapses) which localize the wave function in space. These additional stochastic terms change the evolution equation of the density matrix and give the Lindblad equation [7]:

$$\frac{\partial}{\partial t} \rho(t) = -\frac{i}{\hbar} [H, \rho(t)] - \frac{\lambda}{2} [A, [A, \rho(t)]] \tag{4}$$

The second term on the RHS stochastically drives the state vector toward one of the eigenstates of the operator $\hat{A}$ with a probability equal to the Born rule [26, 27]. However, it is interesting to note that this does not always lead to an observable decoherence [28, 29]. This interesting and surprising property of a collapse theory is very important for our correspondence, since it is in agreement with the observation [6] that quantum decoherence due to information loss to baby universes is not experimentally observable.

Bonifacio model for spontaneous collapse theory

The Bonifacio model presents an interesting way to derive the structure of a collapse that does not lead to an observable decoherence [8]. This is done as follows. According to this model, instead of the continuous unitary evolution of the density matrix; i.e.:

$$\rho(t + \tau) = \exp(-\frac{i}{\hbar} H \tau) \rho(t) \exp(\frac{i}{\hbar} H \tau) \tag{5}$$

one assumes that a change of the system occurs with a certain probability: specifically, that there is a probability $p_n(\tau_0, \tau)$ such that $n$ random time shifts by $\tau_0$ lead to a total time shift of $\tau$. The density matrix then satisfies a probabilistic evolution equation:

$$\rho(t + \tau) = \sum_{n=0}^{\infty} p_n(\tau_0, \tau) \rho(t) \exp(-\frac{i}{\hbar} H n \tau_0) \exp(\frac{i}{\hbar} H n \tau_0). \tag{6}$$

In the limit $\tau_0 \to 0$ this yields the continuous equation [3]. Next assuming a Poisson distribution

$$p_n(\tau_0, \tau) = \frac{1}{n!} \left( \frac{\tau}{\tau_0} \right)^n e^{-\tau/\tau_0}, \tag{7}$$

one obtains an interesting result: the master equation takes the form

$$\frac{\partial}{\partial t} \rho(t) = \frac{1}{\tau_0} \left( \exp(-\frac{i}{\hbar} H \tau_0) \rho(t) \exp(\frac{i}{\hbar} H \tau_0) - \rho(t) \right). \tag{8}$$

We consider the parameter $\tau_0$ to be small, since this indicates the time scale where deviations from continuous evolution become apparent. Expanding in $\tau_0$ the master equation takes the form:

$$\frac{\partial}{\partial t} \rho(t) = -\frac{i}{\hbar} [H, \rho(t)] - \frac{\tau_0}{2\hbar^2} [H, [H, \rho(t)]] . \tag{9}$$

As pointed out by Anastopoulos and Hu [30], these considerations are in fact more general, and the semigroup equation [31] follows quite generally for any physical quantity and its corresponding generator. For

\[I want to thank Don Marolf for clarifying this important point for me.\]
example, the above results remain valid if time is replaced by position and the time translation generator
$H$ by the momentum $p$.

Since we construct a correspondence between Bonifacio collapse model and black hole evaporation, we
will need to write the Bonifacio model in a unitless form. For a unitless $\lambda$ and its unitless generator $B$,
the standard evolution for a density matrix $\rho(\lambda_0 + \lambda) = \exp(-iB\lambda)\rho(\lambda_0)\exp(iB\lambda)$ takes the form:

$$\rho(\lambda_0 + \lambda) = \sum_{d=0}^{\infty} p_d(\lambda)\exp(-iBd)\rho(\lambda_0)\exp(iBd). \quad (10)$$

For the choice of a Poisson distribution

$$p_d(\lambda) = \frac{1}{d!} (\lambda^d e^{-\lambda}) \quad (11)$$

the master equation takes the form

$$\frac{\partial}{\partial \lambda} \rho(\lambda) = \exp(-iB)\rho(\lambda)\exp(iB) - \rho(\lambda). \quad (12)$$

After expansion in the presumably small parameter $B$, the evolution becomes apparent:

$$\frac{\partial}{\partial \lambda} \rho(\lambda) \simeq -i \left[ B, \rho(\lambda) \right] - \frac{1}{2} \left[ B, \left[ B, \rho(\lambda) \right] \right]$$

and again the new term drives the state vector toward one of the eigenstates of the operator $B$.

## 4 Constructing the correspondence

In this section we equate Bonifacio model to Marolf-Maxfield’s topological model. The first subsection
gives the Hartle-Hawking no-boundary state density matrix for MM model and derives its "evolution"
equation. The second subsection equates the two density matrix equations: the master equation and
Hartle-Hawking evolution equation. This gives the condition for them to be the same.

"Evaporating" Hartle-Hawking state

In order to construct a correspondence between the MM model and the Bonifacio model, it is necessary to
identify the rate of change of the Hartle-Hawking density matrix before equating it to the unitless master
equation (12).

Using eq. (4), one finds that the Hartle-Hawking density matrix for the MM model can be written as

$$\hat{\rho} = \xi^{-1} \langle HH \rangle = e^{-\lambda} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\lambda^m}{\sqrt{m!}} \frac{\lambda^n}{\sqrt{n!}} \langle Z = n | Z = m \rangle. \quad (13)$$

This density matrix has a Poisson distribution, as does the Bonifacio model for spontaneous collapse theory.
Note that although in the original Bonifacio model the parameter $\lambda$ is a unitless time coordinate $\lambda = \tau/\tau_0$,
according to [26] this can be generalized to any physical quantity and its corresponding generator.

In order to construct an equation similar to (12), the rate of change of the Hartle-Hawking density
matrix should also be calculated with respect to the parameter $\lambda$. Using eq. (13) one finds (see Appendix)

\footnote{For example $B \equiv H\tau_0/h$, $\lambda = \tau/\tau_0$, and $\lambda_0 = t/\tau_0.$}
that:
\[
\frac{d}{d\lambda} \hat{\rho} = -\hat{\rho} + \frac{1}{2} \lambda^{-1/2} \left( \hat{a}^\dagger \hat{\rho} + \hat{\rho} \hat{a} \right).
\]  
(14)

This gives the equation for the rate of "evolution" of the Hartle-Hawking density matrix.

**Equating MM Hartle-Hawking evolution density matrix to the master equation.**

Next we require that the MM Hartle-Hawking evolution density matrix agree with the master equation obtained in the Bonifacio model. In other words, we equate the terms in the unit-less master equation for the Bonifacio model \([12]\) and the rate of change of Hartle-Hawking density matrix for the MM model \([14]\). This gives a connection between the two generating operators \(B\) and \(a\):

\[
\exp(-iB)\rho(\lambda)\exp(iB) = \frac{1}{2} \lambda^{-1/2} \left( \hat{a}^\dagger \rho(\lambda) + \rho(\lambda) \hat{a} \right).
\]  
(15)

This is a strange connection. Whereas the operator \(a\) promotes the number of the disconnected components of spacetime, the operator \(B\) promotes \(\lambda\), which is a free parameter in the theory. In order to try and understand this connection we use eq. \([2]\) for spacetimes with \(d\) connected components, and obtain that in order that the black hole density matrix stochastically drive the state vector toward one of the eigenstates, the operator \(B\) "evolves" the density matrix element \(|Z = n\rangle \langle Z = m|\) to:

\[
\exp(-iB) |Z = n\rangle \langle Z = m| \exp(iB) = \\
\frac{1}{2} (|Z = n + 1\rangle \langle Z = m| + |Z = n\rangle \langle Z = m + 1|).
\]  
(16)

Indeed, the operator \(B\) changes the density matrix element \(|Z = n\rangle \langle Z = m|\) to a "superposition of density matrix elements" with a different number of disconnected spacetimes.

**5 Summary and discussion**

This note obtains an "evolution" of the density metrics in MM model by considering a possible correspondence between collapsing wave functions and evolution through wormholes (as occur when black holes evaporate after Page time). By equating the Marolf-Maxfield topological toy model to the Bonifacio model for spontaneous collapse theory, a condition for their evolution equation to be the same is obtained. This gives the evolution that should be expected from the matrix element \(|Z = n\rangle \langle Z = m|\) in order for the Hartle-Hawking state to evolve from Hartle-Hawking state to one of the superselection sectors. This "evolution" is describe by \(\exp(-iB)\), where \(B\) is a generator of the parameter \(\lambda\) in MM topological toy model.\[4\]

By construction, we obtained that the evolution in MM-model should be related to changes in the parameter in the theory. This may look surprising at first glance, but looks reasonable if we consider that MM-model have the following property: different values of an apparently free parameter turn out to describe different states of the same theory. In this case, changes in the free parameter can be consider an evolution from one state to the another, as one may expects when evolution take place.

\[4\]So that the operator \(\exp(-\frac{\i}{\hbar}B)\) shifts the value of \(\lambda\) to \(\lambda + 1\)
This correspondence could shed some light on the nature of collapse. For example it may clarify the origin of the Bonifacio model. In order to see that, note that in the MM model the Hartle-Hawking state involves spacetimes with \( d \) connected components \(|Z = d\rangle\). If our correspondence is accurate, the Bonifacio model corresponds to the distribution of "\( d \) connected components" whose nature is not clear, but which somehow enables transmission of information without breaking causality in the same way that wormholes do. This would present an exciting avenue for further research on the nature of collapse which is still not properly understood.

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Appendix: Varying Hartle-Hawking density matrix with respect to \( \lambda \)

Since

\[
\hat{\rho} = \xi^{-1} |HH\rangle \langle HH| =
\]

\[
= \xi^{-1} e^{\sqrt{\lambda} a^\dagger} |Z = 0\rangle \langle Z = 0| e^{\sqrt{\lambda} a} =
\]

\[
= e^{-\lambda} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\lambda^{m/2} \lambda^{n/2}}{m! \sqrt{m!} \sqrt{n!}} (a^\dagger)^n |Z = 0\rangle \langle Z = 0| a^m
\]

\[
= e^{-\lambda} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\lambda^{m/2} \lambda^{n/2}}{\sqrt{m!} \sqrt{n!}} |Z = n\rangle \langle Z = m|,
\]

the derivation of eq. (19) with respect to \( \lambda \) is

\[
\frac{d}{d\lambda}\hat{\rho} = -e^{-\lambda} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\lambda^{m/2} \lambda^{n/2}}{m! \sqrt{m!} \sqrt{n!}} (a^\dagger)^n |Z = 0\rangle \langle Z = 0| a^m +
\]

\[
+ \frac{e^{-\lambda}}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\lambda^{m/2-1} \lambda^{n/2}}{m! \sqrt{m!} \sqrt{n!}} (a^\dagger)^n |Z = 0\rangle \langle Z = 0| a^m +
\]

\[
+ \frac{e^{-\lambda}}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\lambda^{m/2} \lambda^{n/2-1}}{m! \sqrt{m!} \sqrt{n!}} (a^\dagger)^n |Z = 0\rangle \langle Z = 0| a^m
\]

\[
\frac{d}{d\lambda}\hat{\rho} = -\hat{\rho} +
\]

\[
+ \frac{e^{-\lambda}}{2} \sum_{n=0}^{\infty} \frac{\lambda^{n/2-1}}{n!} (a^\dagger)^n |Z = 0\rangle \langle HH| +
\]
\[
+ \frac{e^{-\lambda}}{2} \sum_{m=0}^{\infty} m \frac{\lambda^{m/2-1}}{m!} |HH\rangle \langle Z = 0| a^m.
\]

Since \( m = 0 \) and \( n = 0 \) does not contribute, we can state the summation from \( m = 1 \) and \( n = 1 \)

\[
d\frac{d}{d\lambda} \hat{\rho} = -\hat{\rho} + \\
+ \lambda^{-1/2} e^{-\lambda} \frac{2}{2} a^\dagger \sum_{n=1}^{\infty} \frac{\lambda^{(n-1)/2}}{(n-1)!} \left( a^\dagger \right)^{n-1} |Z = 0\rangle \langle HH| + \\
+ \lambda^{-1/2} e^{-\lambda} \frac{2}{2} \sum_{m=1}^{\infty} \frac{\lambda^{(m-1)/2}}{(m-1)!} |HH\rangle \langle Z = 0| a^{m-1} a.
\]

We rename \( n - 1 \rightarrow n \) and \( m - 1 \rightarrow m \) thus the summation start again from 0:

\[
d\frac{d}{d\lambda} \hat{\rho} = -\hat{\rho} + \\
+ \lambda^{-1/2} e^{-\lambda} \frac{2}{2} a^\dagger \sum_{n=0}^{\infty} \frac{\lambda^{n/2}}{(n)!} \left( a^\dagger \right)^{n} |Z = 0\rangle \langle HH| + \\
+ \lambda^{-1/2} e^{-\lambda} \frac{2}{2} \sum_{m=0}^{\infty} \frac{\lambda^{(m)/2}}{(m)!} |HH\rangle \langle Z = 0| a^{m} a
\]

\[
\frac{d}{d\lambda} \hat{\rho} = -\hat{\rho} + \frac{1}{2} \lambda^{-1/2} \left( a^\dagger \hat{\rho} + \hat{\rho} a \right).
\]

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