The developed hypothetical learning trajectory for integral topic based on realistic mathematics education

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Abstract. This article discussed the development of hypothetical learning trajectory for integral topic which based on realistic mathematics education for class XI Senior High School (SMA). The study focused on the phase of one to one evaluation by using observation. In this phase, we try it out to three students which divided into low, medium and high mathematics ability. Based on the observation, it can be concluded that the designed Hypothetical Learning Trajectory (HLT) can guide the students to achieve the expected purpose, that is the students can construct and discover the concept of integral by themselves based on their experience. Students use their reasoning ability to solve the given real problem so that their mathematical reasoning ability is improved.

1. Introduction
Integral is one of the most important topics to learn when students studied mathematics. This topic can be used to solve any problem in our life such as finding the position of an object toward a certain time with any velocity, designing sturdy building, calculating the volume of object that used in any industry, calculating the area under the curve, determining the center of mass, predicting the flow of water or gas and much more. Other than that, our era development can’t be happen without the benefit of integral application. Since integral is so important for student to learn Permendikbud No 21 of 2016 stated that Integral must be learned by student in Mathematics Learning.

Despite its importance, integral is not really popular topic. Students tend to think this topic is so difficult to learn [1]. The research by Orton showed that worst result of student evaluation in senior high school was integral topic which score on 1,895. The problem of student on understanding integral was the inability to draw the relevant graph and the lack of understanding of some symbols. The teacher should not be too fast when teaching integral procedural without any good understanding [2]. For this case, it is so important to design a suitable learning method for student to learn integral [3].

In other hand, the research of Tasman showed that student understanding of derivative topic played an important role in understanding integral. Tasman suggested to create a discussion to build the comprehension of integral topic. The suggestion based on though that student would able to get any idea to determine the anti-derivative of a function [4]. An instrument is needed as a whole learning process and also a suitable teaching material which can be used to improve reasoning ability on integral concept [5].

The description of the problem above is caused by the teachers are too focused on text book when teaching integral so far. Even though the available text book tend to have teacher teach mathematics topic mechanically and algorithmic [6]. The learning is still teacher centered. These learning method
bring such a bad impact to student ability to construct their own concept without knowing the meaning in the concept.

This condition was also found when conducting a preliminary study in several schools especially in XI grade student in SMA Negeri 10 Padang and SMA Pembangunan Laboratorium which hold on 10 and 15 January 2019. Based on observations and interviews with some mathematics teacher that the teacher was only guided by the text book when teaching integral so far. The text book was focused only on remembering the formula and doing any related exercise. The learning trajectory in the book was not contributive enough to improve student learning especially in the mathematical reasoning ability.

Based on the problem description, it is necessary to design an integral learning trajectory which can develop student ability in mathematics reasoning to solve a problem. The mathematics learning expect student to be active in learning, which is not just using certain formulas and procedure to solve problem. Mathematics is not only a lesson to memorize facts, but also a lesson that require thinking skills, problem solving and making conclusions [7]. So it can be said that learning mathematics used reasoning to construct a concept. Knowledge cannot be transferred from teacher to student but it must be constructed by the student [8]. It means the student have to be active to construct their on knowledge. The students who learn mathematics must be given a chance to find the concept and idea of mathematics [9]. The process of concept discovery was done on two mathematical process that is horizontal and vertical mathematics [9]. In other word, the students have to solve the problem informally then they are guided to use formal language to create an algorithm.

These designs of learning trajectory is known as Hypothetical Learning Trajectory (HLT). HLT is a supposed learning trajectory which used as guide on learning process during this research. HLT can predict how the ability of thinking and student’s comprehension will be improving during learning activities and also its anticipations. This HLT contains the phase of activities to solve the problem in integral topic.

During designing the learning trajectory which focused on giving the students a chance to discover their own concept, it is necessary to have a suitable learning approach. In the process of developing concept and mathematical thinking it must be associated with the real life [10]. Therefore, one of learning approach which suitable with learning trajectory is the Realistic Mathematic Education (RME) approach. RME departs from real things for student, emphasizes the skills of doings mathematics, discussing and collaborating, arguing with classmates so that they can discover the concept by themselves and use mathematics concept to solve any related problem both individually or in groups. Through classroom discussion, learning becomes more effective [11]. In using the RME approach, student will be guided in finding the concept of mathematics through horizontal and vertical mathematical processes.

By using RME approach, students can improve their mathematical reasoning ability. This is supported by the result of [12] and [13] which can be concluded that the design of RME-based learning can improve students reasoning ability. In this study, an integral learning trajectory was designed which consisted of several activities to improve student’s mathematical reasoning ability.

2. Methods
The development model which used in this development research by using design research model is a combination of design model research that are Plomp design model and Gravemeijer & Cobb. Development of learning trajectory is designed using design research Gravemeijer and Cobb. Then in implementing this learning trajectory, a product is designed in the form of teacher books and students books which using Plomp’s design research to produce the products that are valid, practical and effective.

Gravemeijer & Cobb which consist of three phases namely preparing for the experiment, conducting the experiment, and retrospective analysis [14]. The activity begins with the thought experiment which thinking the learning trajectory that will be traversed by students then doing a reflection on the result of the experimental followed by the next though experiment.
The phases combined with Plomp stated that there are three phases in design research, namely Preliminary Research, Development or Prototyping Phase, and Assessment Phase [15]. In the first phase, there is needs analysis, student analysis, curriculum analysis and concept analysis. Then in the second phase, development design activities are carried out and also doing product formative evaluation. Formative evaluation is carried out in order to ensure that the products developed are valid, practical and effective. Formative evaluation including self-evaluation, expert reviews, one to one evaluation, small group evaluation and field test. One to one evaluation is done to one mathematics teacher and three students. This evaluation will be conducted face-to-face between the researchers and students sequentially from those who have, medium and low level by using observation and interview. In third phase a semi-summative evaluation was conducted to see the effectiveness of the products that had been developed.

This article focuses on the one-to-one phase of this learning design research. This problem that will be answered is whether the HLT design can directing student to achieve the expected goals.

3. Result and discussion

The one to one stage was done after the product had been declared valid by the experts. At this stage information was obtained whether the HLT could direct the student to achieve the expected goals. At one to one stage the products was applied to three student of class XI SMA Negeri 10 Padang which were conducted separately with 5 meeting each. Students were given products in the form of student books which contained activities and problems that had be solved by students. Students were given the opportunity to try to solve these problems themselves by using their reasoning abilities they have and try to find their own concept. Then the researcher conducted a probing question in accordance with the anticipation answer given by students which had been designed in HLT so that the students were able to solve the problem correctly. At the end of the meeting the student was interviewed about the provided product.

The learning trajectories of integral topic were to find the concept of integral as the area under the curve, to find the concept of an indefinite integral as anti derivative, to find the general form of indefinite integral algebraic function, to find the formula of indefinite integral algebraic function, and to find the properties of indefinite integral algebraic function. The following was an example of given activities to students.

![Figure 1. The example of given activity to students.](image)

The purpose of solving this problem was the students were able to find and recognize the concept of integral as the limit of the sum of rectangular area under the curve.

For the high-ability students, after he read the given problem, he immediately drew a number of square shapes in the area to be counted for as in picture 2(a). Because the purpose of solving the
problem was to be able to introduce the concept of integral as the limit of the sum of rectangular areas under curve, the researcher gave probing questions. For example “Why do you use a square? What if the curve is big? Are you able to draw as many plot lines? Can we use another shape to shorten the time to calculate the area?”. Then the student responded by using another shape which was rectangle as shown in picture 2(b). Then the student calculated the area as predictions.

But there were still many empty areas that had not been counted. It could causes the counted area was less accurate. Therefore the researcher conducted a probing question to direct the student to use smaller rectangular for example “what about the countless empty area? Is it accurate if there still many countless area?”. The smaller the size, the more accurate the calculation would be. However it would be more difficult to calculate the area manually because the numbers of rectangle were very close to infinity. In the end, the researcher was able to introduce the concept of integral as the limit of the sum of rectangular areas under the curve. It could be seen in picture 2(c).

For the medium-level student, after she read the given problem, she drew a number of rectangular shapes on the curve area as shown in picture 3(a). In order to achieve the expected result, the researcher gave a probing question. For example “In addition of horizontal one, is there any other way to draw the rectangle so that more rectangles can be calculated?”. Then the student responded by drawing the shape vertically or upright rectangles with sizes as shown in picture 3(b). Because there were still many empty areas, the researcher performed another probing question that was the same as previous high-level student so that she could be able to construct the concept of integral as the limit of the sum of rectangular areas under the curve. It could be seen in picture 3(c).
As for the low-level student, he had no idea how to find the area of the curve. Therefore the researcher gave other shape to help on calculating the area. It was a rectangle which contained unit squares. Then the researcher conducted probing question such as “What is the area of the shapes? How many square units are there under the curve? What is the relationship between the shape of rectangle and square unit in the area?” Then the student got the idea to calculate the area by drawing the unit square. It could be seen in picture 4(a). As the purpose of this problem solving was to introduce the concept of integral as the limit of the sum of rectangular areas under the curve then the student was given the same probing question like previous student. Then the student responded by using rectangles as in picture 4(b). Because of still empty space which are not counted then the researcher asked the probing question like previous student. It could be seen in picture 4(c).

(a) Before Probing Question  (b) After First Probing Question  (c) After Second Probing Question

Figure 4. Low level student’s answer sheet.

Based on observation in one to one phase, high, medium and low ability students could discover and construct their own concept of integral through their experience and anticipation that have been designed in HLT. It could cause the learning to be meaningful. Then based on the results of interviews to them after the one to one activities were done, in general they like the appearance of student books because the design of the book was colorful and there were several animations that made it more attractive. Then they were also happy learning integral topic with the RME approach because they knew the application of integrals in real life. They were also easier to understand the concept because they discovered the concept by themselves.

4. Conclusion
Based on the analysis in one to one phase, it can be concluded that the designed HLT can guide the students to achieve the expected purpose, that the student can construct and discover the concept of integral by themselves based on their experience. Students use their reasoning ability to solve the given real problem so that their mathematical reasoning ability is improved.

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