Research Article

Research on Coordinated Passenger Inflow Control for the Urban Rail Transit Network Based on the Station-to-Line Spatial-Temporal Relationship

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This study proposes a coordinated inflow control organization for the urban rail transit network to improve train capacity utilization and reduce inbound delay rate, considering existing operational requirements and station-to-line spatial-temporal relationship of passenger flow. The coordinated passenger flow control model is proposed with the objective to maximize the inbound flow rate and the train full load rate. A station-to-line spatial-temporal correlation formula is constructed to characterize the relationship between station inbound passenger volume and section passenger volume. A two-stage approach is employed to solve this passenger flow control problem. The proposed model and solution strategy are evaluated on a well-known Beijing network with 10 operating lines. The refined inflow control scheme is displayed with the accurate inbound volume at each station during each time period. The comparison between the proposed control strategy and the current control strategy shows that the former can effectively improve the service level and capacity utilization.

1. Introduction

As more and more lines are brought into service, network operation has been a certain trend in urban rail transit. As a widely used means of urban transportation, some stations have suffered mass passenger flow or even super passenger flow, and this has gradually become normalization, which affects the safety and efficiency of passenger travel. When the inbound demand at stations exceeds the existing transport capacity, effective measures are taken to relieve the passenger flow pressure and ensure safety [1]. Those measures can be divided into two categories, the first is to adjust train operation organization to increase the transport capacity, and the second is to impose passenger flow restriction. Since the transport capacity is expandable and the headway can be shortened, the former measure has priority and the rail service can be improved to respond to the passenger demand. But if this approach does not work, passenger flow control becomes a general response taken by the operators, especially during rush hours. At present, this approach is used subjectively and qualitatively, leading to the lack of station-to-station coordination. The numbers of inbound passengers at some busy stations of Beijing metro during the morning rush hours for a certain day in May 2015 are shown in Figure 1. During this time, the carriages are crowded and many passengers gather together at platforms. A survey of the Beijing metro system shows that the number of passengers in the trains reaches or even exceeds 120% of the train capacity during rush hours [2]. According to relevant research, platform congestion has a direct impact on the boarding and alighting efficiency of passengers. Thus, the train may fail to depart on schedule and operational security cannot be guaranteed [2].

What’s worse, there were frequent safety accidents caused by passenger congestion. In 1999, a large number of concertgoers crowded into the nearby station, causing a
In 2014, a passenger got stuck between the train door and platform screen door due to the pushing of the crowd at one Beijing metro station. To avoid such accidents, all metro systems in China have been instructed and developed safety management plans [3].

In practice, passenger flow control refers to using guide railing, security inspection equipment, and other facilities to reduce the inbound flow rate. The objective of passenger flow control is to transport most passengers in the case of ensuring safety and comfort [4]. For the station, the basic executed unit, the passenger flow management can be divided into two levels according to the limiting strength, namely, control and sealing. The inflow control measure is an operative strategy taken at some stations in daily operation to mitigate the mismatch between inbound demand and the actual transport capacity. Sealing means that passengers are prohibited from entering and exiting the station with trains stop-skipping, and it is unreasonable in rush hours. So sealing is only implemented in special circumstances or the occurrence of important activities such as sports meets or concerts.

The inflow control strategy has been widely implemented in the population-intensive subway system around the world [5]. In terms of the scope of implementation, the inflow control includes station-level inflow control (hereafter referred to as station control), line-level inflow control (hereafter referred to as line control), and network-level inflow control (hereafter referred to as network control). The last one takes the dynamic characteristics of passenger flow inside the network into consideration. The control intensity is ultimately determined by decision-makers. For example, Beijing’s governing document [6] clearly states that flow restriction measures are required to ensure operational security when inbound demand reaches or exceeds 70% of capacity. However, this requirement cannot provide a quantitative inflow control strategy for stations. Therefore, it is necessary and urgent to develop an optimization framework to provide a reliable and effective passenger flow control scheme.

The purpose of this study is to provide a refined inflow control scheme for the whole urban rail transit network with limited transport capacity to maximize capacity utilization and inbound flow rate. Firstly, we develop a linear equation to characterize the relationship between the inbound passenger flow at stations and passenger flow through sections. Based on this, a coordinated inflow control model for the network is constructed to quantify the inbound flow rate during each time period. Thereby, the refined inflow control strategy is obtained.

In the following sections, the existing literature is reviewed and the contribution of this study is presented.

2. Literature Review

The inflow control in the traffic system was first proposed by Wattleworth [7] in 1965. A bottleneck flow theory was proposed to control the traffic volume in the expressway system during rush hours, and the linear programming method was used to solve the model aiming to maximize the entering traffic volume, which is a reference for normalized flow control on the rail transit network in rush hours. Later, some scholars researched integrated traffic control strategies for the expressway system. Kotsialos et al. [8] applied the optimal control method to the optimal coordinated and integrated control strategies to adjust traffic volume dynamically, which lacked a theoretically strict optimal solution. Gentile et al. [9] proposed a dynamic traffic assignment model on the road network, which introduced time-varying entry and exit capacities to limit the traffic inflow on downstream arcs. This approach can give some clues for passenger flow control at upstream and downstream stations on the rail transit network. Ibarra-Rojas et al. [10] summarized real-time control strategies in the bus transport system for transit network planning problems. Sanchez-Martinez et al. [11]
formulated a holding control optimization model with dynamic running time and demand, which outperformed the model with static inputs. The above methods have been successfully applied in bus transportation.

The rail transit network is different from the road traffic system in traffic control [12, 13]. It is necessary to construct an inflow control model and algorithm according to its characteristics. Station control refers to one or several stations control to reduce the number of inbound passengers. Some scholars [14–16] focus on the simulation of pedestrians at stations and the control of the number to guarantee service level. Bauer [17] established a macropassenger flow simulation model and calculated the bottleneck capacity of Vienna Metro Station. The model provides a basis for control parameters. Zhang et al. [18] established a passenger distribution model of the station, which indicated the contact of nodes passed by passengers inside the station. Based on previous research, Li et al. [19] optimized the passenger organization of the transfer station according to the number of entering, exiting, and transferring passengers. Wang et al. [16] proposed a multiple-position joint control model for the transfer station, aiming to meet safety requirements in key areas and improve the travel efficiency of passengers. According to the concepts of station limited-service capacity, station global service capacity, and station single service capacity, Xu et al. [20] proposed a passenger flow control strategy at the Beijing subway station. Bae et al. [21] established a microscopic model to simulate passenger behaviors and proposed different boarding/alighting strategies while reducing travel time to improve satisfaction.

However, the station control ignores the movement of passengers between sections. Many researchers have focused on developing inflow control strategies at multiple stations, namely, line control. By controlling inbound passenger flow at other stations, the load rate of trains arriving at transfer stations can be reduced, thereby improving the transfer service. Delgado et al. [22] established a new mathematical programming model for the vehicle bunching problem to achieve the minimum total delays. They introduced the boarding limits as an attractive control mechanism. From the ability constraints perspective, platform flow control and station inflow control are different. Some scholars combined passenger flow control and train operation organization to meet the demand. Jiang et al. [23] proposed a metro inflow model with boarding limiting and stop-skipping strategies to obtain the optimal control scheme considering its feedback to passenger original station choice. They optimized the boarding limiting and train operation organization simultaneously and then verified the model on a single line. Similar to Jiang et al. [23], Li et al. [2] also considered the simultaneous optimization of train regulation and inflow control. The dwell time of trains at each station is directly affected by the number of boarding and alighting passengers. Based on the evolution of train departure time and passenger load, a coupled state-space model was established, which solved the problem of joint optimal design of dynamic train regulation and inflow control. With passenger flow control strategy on a bi-directional metro line, Liu et al. [24] proposed a collaborative optimization model for the train timetable and train connections. The optimization results show that the model can significantly balance the congestion in stations as well as carriages. Shi et al. [25] formulated a collaborative method to optimize the train timetable and passenger flow control strategies with dynamic demand on an oversaturated metro line, which can effectively reduce the assembling numbers on the platform and the total waiting time. Yang et al. [26] focused on the commuting metro line during the peak period, which connects the suburb and urban areas. They proposed an integrated strategy of passenger flow control and bus-bridging service to relieve the congestion. Zhang et al. [27] constructed a coordinated passenger flow control model to maximize the number of boarding passengers with the consideration of fairness, and their experimental results show that the proposed method can reduce waiting time and line-level Gini coefficient. Yin et al. [28] constructed a single-line passenger flow control model to minimize the total passenger delay, in which the nonequilibrium and equilibrium flow control strategies can be reproduced by applying different forms of delay penalty function.

The network control will be enabled when the effect of line control does not meet expectations. With the increasing complexity of the passenger flow structure, most of the existing models and approaches focus on network control. Zeng et al. [29] and Kong et al. [30] proposed the passenger flow control strategy from the perspective of network controllability. Latora et al. [31] proved that the Boston underground transportation system was very efficient globally. The results showed the system was not fault-tolerant and the destroy at stations would greatly affect transportation efficiency. Another research direction was carried out by Dell’Olio et al. [32] who understood passenger behaviors on the trains in an abnormal situation and identified the most important variables that affected behaviors. The approach provides new ideas on how to deal with unexpected situations to minimize economic loss and passenger risk. According to Xu et al. [5], a new coordinated passenger flow control model for multiple stations, which adjusted the number of inbound and transfer passengers simultaneously, was put forward. The authors aimed at minimizing average travel time and got the number of inbound and transfer passengers every 15 minutes regardless of crowding. Considering the station capacity constraint, Guo et al. [33] established a network inflow control model to minimize the average delay time and the maximum delay time, which was solved by the heuristic algorithm. Huan et al. [34] developed a demand-responsive passenger flow control strategy considering service and fairness and passenger’s response to the control strategy but only applied it to local urban rail transit networks. Similarly, Yuan et al. [35] proposed a passenger flow control model to minimize the total waiting time of passengers and applied it to a local urban rail transit network, which is solved by Cplex.

It is acknowledged that few researchers have attempted to solve the problem of coordinated inflow control with dynamic demand on a real urban rail transit network and obtain a refined inflow control scheme. This study focuses on the coordinated passenger inflow control optimization for the network. The main contribution of this study includes the following:
3. Problem Description

The purpose of this study is to realize the maximum utilization of train capacity, reduce delays for passengers entering stations, and maximize the number of boarding passengers in the whole urban rail transit network. In this part, the network-level control for passenger flow is described, and the principle of coordinated inflow control is then illustrated.

3.1. Description of Passenger Flow Control for the Network

This study addresses the passenger flow control problem on urban rail systems during peak hours, with the shortage of transport capacity resources of some crowded lines. The network system includes the stations, the sections connecting adjacent stations, and the trains running on all lines. Trains run on fixed lines with fixed schedules. At each station, the passenger flow includes both inbound and outbound passenger flow, and it also includes transfer passenger flow for the transfer station. The inbound passengers will be limited to waiting at entrances, waiting to enter the station. The arrival of transfer passengers is strongly correlated with the train timetable. It is difficult to limit transfer passengers to queue in transfer channels or halls, especially during peak periods. Therefore, we only consider the control strategy for inbound passengers at each station on the network with limited capacity resources. The passenger flow control in this study can be stated as follows: given the topology and train service information of the network, determine the actual number of passengers entering each station during each time period based on passenger demand.

3.2. Principle of Coordinated Passenger Inflow Control for the Network

Different from the passenger flow control of the single station and the single line, the passenger flow control on the network needs to consider the transfer between lines and the route choice of passengers. Once passengers get on the trains, they will pass through several sections along their routes with train operation until they exit the stations. If the transfer occurs, they will occupy the capacity resources of more than one line. In this process, passengers at stations along lines successively board the train. In the saturation condition, there may not be enough capacity for all waiting passengers to board the train at some stations. Thus, those left-behind passengers have to wait for the next. The change of passenger flow in space over time is called the dynamic evolution process, which makes it necessary to comprehensively take the demand at all stations into account for network-level control. Figure 2 depicts this intricate but intrinsic process. The station \( n \) is a terminal, whereas stations \( k \) and \( h \) are intermediate ones. Additionally, \( t \) represents the time period divided. The number of inbound passengers at station \( n \) during time period \( t = 0 \) are denoted as \( Q_n^t = q_n^0 \), and they can board trains 1, 2, and 3. \( Q_a = q_1^{(k,h)} \) represents the number of passengers through the section \( (k, h) \) during time period \( t = 1 \). With trains operating, part of those passengers travel through the subsequent sections, such as \( (k, h) \), which directly affects the maximum number of passengers allowed to board at stations along the line, such as the station \( k \). Thus, the passenger flow control at each station will consider the influence on other stations and sections, to achieve the optimization of network-level control. In this study, the correlation between the number of inbound passengers at station \( n \) during time period \( i \), denoted by \( q_n^i \), and the number of passengers through section \( (k, h) \) during time period \( j \), denoted by \( q_1^{(k,h)} \), is defined as the station-to-line spatial-temporal relationship, which is the basis of the coordinated control.

There are two types of approaches in characterizing this relationship, namely, the based-sensitivity method [36] and the based-logit method [37]. The former one is expected to gain the derivatives of section passenger flow volume to inbound passenger flow volume by sensitivity analysis until it converges to the optimal solution. However, this approach has higher complexity because of the frequent passenger flow assignment. The latter gets the passenger flow assignment based on the logit model and then calculates the proportions of the inbound passenger flow at stations to passenger flow through sections. Based on this, the optimal control strategy is obtained by the optimization model. However, these fixed proportions ignore the time-varying characteristics of passenger flow, even though the complexity is reduced due to the time discretization. In this study, we introduce a linear method based on the station-section passenger flow proportion matrix, which can reduce the computational complexity while taking into account the dynamic characteristics of the passenger flow. This proportion matrix represents the spatial-temporal evolution of the inbound passenger flow at each station and passenger flow through each section, obtained according to the route choice model.
We set the study horizon as \([T_{\text{start}}, T_{\text{end}}]\), which is divided equally into \(M\) time periods named \(t\) of \(g\) in length. Let \(T\) be the set of time periods. Let \(G = (N, A, T)\) be a topology representing the urban rail transit network, where \(N\) is the set of stations and \(A\) is the set of sections connecting adjacent stations on the lines. Each section \((k, h) \in A\) represents that \(k\) and \(h\) are connected directly. The transfer arc is also considered a section object. Let \(q_n^i\) and \(q_{(k,h)}^i\) represent the number of inbound passengers at station \(n\) and the number of passengers through the section \((k, h)\) during time period \(t\), respectively. To quantify the relationship between \(q_n^i\), \(t \in T\) and \(q_{(k,h)}^i\), \(t \in T\), two types of parameters are proposed and introduced as follows.

\[ p_{(k,h)}^{i,j}, n, i : \text{the associated parameter, the coefficient of the number of inbound passengers at the station } n \in N \text{ during time period } i \in T \text{ to the number of passengers through the section } (k, h) \in A \text{ during time period } j \in T. \]

\[ \eta_{(k,h)}^i : \text{the passenger flow parameter, the number of passengers through the section } (k, h) \in A \text{ during time period } i \in T, \text{ who entered the network before the study horizon.} \]

Therefore, this relationship can be expressed as the constraint (1); these two kinds of parameters are fitted based on historical data, which will be introduced in detail in Section 5:

\[ q_{j}^{(k,h)} = \sum_{i \leq j} \sum_{n \in N} p_{(k,h)}^{i,j,n} \times q_n^i + \eta_{(k,h)}^j, 0 \leq i \leq j \leq M - 1, (k, h) \in A. \quad (1) \]

4. Mathematical Modeling

This section presents the coordinated passenger flow control model for the network, where inbound passenger flow is expressed for each station during each time period.

4.1. Basic Assumptions. Several assumptions are made within the model and are presented as follows:

**Assumption 1.** Trains run according to the predefined timetable, which provides the necessary operational parameters for the model. The headway of each line, running time in each section, and dwell time at each station are fixed during the study horizon [38], which will not be affected by the inflow control strategy.

**Assumption 2.** Passengers enter stations uniformly at a predetermined speed expressed by the inbound flow rate in the model. We assume that no inbound passengers will leave the rail transit system due to the inflow control.

**Assumption 3.** It is assumed that the outbound passengers can exit stations quickly, so the occupancy of the facility capacity by these passengers is not considered. Based on this, the exiting capacity of stations is not taken into consideration.

**Assumption 4.** The travel route choices for all different origin-destination (OD) passengers have been obtained by a logit route choice model based on historical data. This choice model provides the selection probability for each route by the utility of travel routes.

4.2. Definition of Symbols. Table 1 describes the input data and variables required by the model.

4.3. Objective Functions. The model takes the interests of multiple participants into account. From the perspective of operators, the coordinated inflow control scheme aims at improving the utilization of train capacity as much as
The set of stations, \( n \in N \)

The set of sections, \((k, h) \in A\), where \( k, h \in N \)

The set of time periods with passenger flow control, and \( T = \{0, 1, \ldots, M - 1\} \)

The set of lines, \( l \in L \)

The set of lines leaving the station \( n \in N \), and \( l \in L \)

The time serving the section \((k, h)\)

The time period \( t \in T \)

The length of each time period

\( g \)

\( r_n^m \)

\( H_l^{m|j} \)

\( C_{l|j} \)

\( C_{\text{transfer}, j} \)

\( \alpha \)

\( V_{n^{E}} \)

\( V_{n^{G}} \)

\( V_{n^{T}} \)

\( V_{n^{E}}^{\text{ES}} \)

\( P_{n^{A|j}, \alpha} \)

\( q_{n|j} \)

The capacity of the transfer section \((k, h) \in A\) during time period \( i \) at D-class service [6]

The maximum full load rate of the train

The actual passing capacity of the line \( l \in L \) at station \( n \in N \) during time period \( i \)

The actual passing capacity of entrance gates per minute at station \( n \in N \)

The actual passing capacity of channels per minute at station \( n \in N \)

The actual passing capacity of escalators per minute at station \( n \in N \)

The associated parameter, the number of inbound passengers at station \( n \in N \) during time period \( i \) to the number of passengers through the section \((k, h) \in A\) during time period \( j \in T \)

The passenger flow parameter, the number of passengers through the section \((k, h) \in A\) during time period \( i \in T \), who entered the network before the study horizon

The number of inbound passengers allowed to enter the station \( n \in N \) during time period \( i \in T \)

The number of passengers through the section \((k, h) \in A\) during time period \( i \in T \) under flow control strategy

The number of inbound passengers not allowed to enter the station \( n \in N \) during time period \( i \in T \)

 Equation (5):

\[
\begin{align*}
z_3 & = \sum_{0 \leq t \leq M-1} \sum_{(k, h) \in A} \left( \frac{q_{n|j}^{(k,h)}}{C_{n|j}^{(k,h)}} \right). \quad (5)
\end{align*}
\]

Finally, the objective function of the coordinated inflow control model can be expressed as the following equation:

\[
\max z = z_1 + z_2 + z_3. \quad (6)
\]

4.4. Constraints. The constraints of the proposed model include section capacity constraint, transfer capacity constraint, inbound capacity constraint, and inbound demand constraint.

Section capacity constraint (7) ensures that the number of passengers through the section during each time period is not more than the section capacity. According to the upper bound of the train full load rate and tracking interval between trains, section capacity \( C_{n|j}^{(k,h)} \) can be calculated by Equation (8):

\[
\begin{align*}
q_{n|j}^{(k,h)} & \leq C_{n|j}^{(k,h)}, \forall j \in T, \forall (k, h) \in A \setminus A_{\text{transfer}}, \quad (7)
\end{align*}
\]

\[
\begin{align*}
C_{n|j}^{(k,h)} & = \frac{C_{\alpha} \times \alpha \times g}{I_{l|j}}, \forall j \in T, \forall (k, h) \in \{A \setminus A_{\text{transfer}}\}. \quad (8)
\end{align*}
\]

To avoid transfer channel blocking, transfer capacity constraint (9) ensures that the number of transfer passengers is no more than the transfer capacity, which depends on the transfer facility capacity, as shown in Equation (10):
\[
q^{(k,h),j}_n \leq C^{(k,h),j}_n, \ j \in T, \ \forall (k,h) \in A_{\text{transfer}}, \quad (9)
\]
\[
C^{(k,h),j}_n = C^{(k,h),j}_{\text{transfer}}, \ \forall j \in T, \ \forall (k,h) \in A_{\text{transfer}}. \quad (10)
\]

Inbound capacity refers to the capacity of facilities and equipment passed by passengers entering the station. In this study, it is determined according to four types of facilities, namely, entrance corridors, entrance gates, inbound passages, and escalators. The following constraint refers to the inbound capacity constraint on inbound passenger flow:
\[
q^n_i \leq g \times \min \{V^n_{E}, V^n_{G}, V^n_{C}, V^n_{ES} \}, \ \forall i \in T, \ \forall n \in N. \quad (11)
\]

Inbound demand constraint ensures that the number of inbound passengers is not more than the number of inbound demand. Under the inflow control, the passengers who fail to enter during the previous time period will be counted as the inbound demand for the next time period. From the perspective of service requirements, it is unreasonable and impractical to have zero inbound passengers during a time period. Thus, the inbound demand constraint is described as the following constraints:
\[
0 < q^n_i, \ \forall i \in T, \ \forall n \in N, \quad (12)
\]
\[
q^n_i \leq \sum_{0 \leq t \leq j} x^n_{j,t} + r^n_i, \ \forall j \in T, \ \forall n \in N, \quad (13)
\]
\[
q^n_i \leq r^n_i, \ j = 0, \ \forall n \in N. \quad (14)
\]

### 5. Solution Strategy

Combined with the relational expression (constraint (1)), the constructed model above needs to be solved based on the parameters \( P_{n,i}^{(k,h),j} \) and \( q^{(k,h),j}_n \). According to Assumption 4, these two parameters were fitted as follows.

From constraint (1), it follows that for any station, the following constraint holds:
\[
P^{(k,h),j}_n = \frac{\partial q^{(k,h),j}_n}{\partial q^n_i}, \ 0 \leq i \leq j \leq M - 1, \ (k,h) \in A, \ \forall n \in N. \quad (15)
\]

According to the characteristics of the linear fitting method, it is assumed that among the section passenger flow \( q^{(k,h),j}_n \), the number of passengers who enter the station \( n \in N \) during time period \( i \in T \) is a linear function of the inbound volume of station \( n \in N \), as shown in the following constraint:
\[
q^{(k,h),j}_n = a^{(k,h),j}_n \times q^n_i + b^{(k,h),j}_n, \ 0 \leq i \leq j \leq M - 1, \\
\forall (k,h) \in A, \ \forall n \in N. \quad (16)
\]

According to the data obtained from the Automatic Fare Collection (AFC) system, we can get inbound passenger flow data, that is, \( q^n_i, i \in T, n \in N \). The number of passengers through each section, that is, \( a^{(k,h),j}_n \), can be calculated based on passenger flow assignment [39]. Thus, the values of parameters \( a^{(k,h),j}_n \) and \( b^{(k,h),j}_n \) in constraint (16) can be fitted by the least square method based on the passenger flow data for the previous days. Since it is assumed that the passenger flow entering from each station is independent of each other among the passengers through the section, the following constraint can be deduced:
\[
P^{(k,h),j}_n = \frac{\partial q^{(k,h),j}_n}{\partial q^n_i} = \frac{\partial q^{(k,h),j}_n}{\partial q^{(k,h),j}_n} = e^{(k,h),j}_n, 0 \leq i \leq j \leq M \quad (17)
\]

Then, a time-discrete linear relationship can be fitted for station-section passenger flows, and the values of the two types of parameters are given by the following constraints:
\[
P^{(k,h),j}_n = a^{(k,h),j}_n, 0 \leq i \leq j \leq M - 1, \ (k,h) \in A. \quad (18)
\]
\[
q^{(k,h),j}_n = \sum_{n \in N} b^{(k,h),j}_n, \forall j \in T, \ (k,h) \in A. \quad (19)
\]

After obtaining the values of parameters \( P_{n,i}^{(k,h),j} \) and \( q^{(k,h),j}_n \), the model constructed in Part 3 is an integer linear programming model, which is solved by the mathematical optimization tool Cplex. Thus, the solving framework of the coordinated inflow control model is summarized in Figure 3. The passenger flow data include the associated parameter \( P_{n,i}^{(k,h),j} \), the passenger flow parameter \( q^{(k,h),j}_n \), inbound demand \( r^n_i \), and path choice for passengers with different origins and destinations. The network data consist of network topology, train timetable, transport capacity, and facility capacities of all stations.

### 6. Numerical Experiments

The model and solution strategy are applied to the Beijing urban railway transit network to verify their rationality, as shown in Figure 4. There are 17 lines, 268 stations (including 49 transfer stations), and 621 sections in operation. This part aims at developing an inflow control strategy during rush hours for a day in May 2015, in which passenger flow data was obtained from the AFC system. According to the status of passenger volume during the whole day, the morning peak is from 7:00 AM to 9:00 AM, which is set as our study horizon. In actual operation without the coordinated inflow control strategy proposed above, the inflow restriction is imposed on 56 stations during this period. The maximum load rate of the train \( \alpha \) is set to 100%. The capacity of the facilities is obtained by reference to the relevant specification documents [6].

The length of the study horizon equals 120 minutes, which is divided into 8 time periods and with each time period equivalent to 15 minutes long. The inbound demand at 56 stations with inflow control is obtained through the AFC system and investigation. The statistics are shown in Table 2, in which the number columns display all station numbers. In the study horizon, the allocation of carrying capacities for all sections is displayed in Table 3. Column 1 in Table 3 indicates the station number. Column 5 gives the number of trains serving the sections during the time period in column 4. The last column refers to the capacity per train when \( \alpha \) is set to 100%. The data in Table 3 are used as the...
Determination of inflow control period and time discretization

Objectives
i Maximize the inbound flow rate
ii Maximize the number of boarding passengers
iii Maximize the full load rate of trains

Constraints
i Section capacity constraint
ii Transfer capacity constraint
iii Inbound capacity constraint
iv Inbound demand constraint

Refined inflow control strategy
i The number of inbound passengers not allowed to enter at each station
ii The number of inbound passengers allowed to enter at the station
iii The inbound flow rate at each station

Solved by Cplex

Figure 3: Solving process of the coordinated inflow control model.

Figure 4: Beijing urban rail transit network (May 2015).
input parameter $C_i^{(k,h)}$, $\forall (k, h) \in A_{\text{transfer}}, \forall i \in T$ of the model.

According to the solution strategy in Part 5, the AFC data for previous days are used for parameter fitting. Then, we obtained $268 \times 621 \times 8 \times 8$ associated parameters $p_{(k,h),j}$ and $621 \times 8$ passenger flow parameters $q_{(k,h)}$, part of which are presented in Table 4.

Next, this study will conduct three experiments to evaluate the performance of the coordinated inflow control model and approach on the network.

### 6.1. Comparative Experiments of Results

Firstly, the coordinated inflow control results are compared to the actual inflow control strategy. The number of stations under coordinated inflow control is similar to the number of stations with inflow control in actual operation, only 4 higher. However, the difference is significant in the inbound flow rate and the length of inflow-limit time at each station. The proposed control strategy has a shorter inflow-limit time in length and a lower inbound flow rate. The optimization result weakens the passenger flow peak, which reduces the impact of large-scale passenger flows on network operation in the morning rush hours. The comparison of the actual and coordinated control strategies is shown in Table 5. Columns 2 and 3 represent the number of stations with the inflow-limit strategy. It can be found in the case that Line 5 has the largest number of inflow-limit stations because it connects to 8 lines. The whole network has an inflow-limit time of 98.25 h, which is 8.6% lower than the actual one. From the perspective of the network, the inbound flow rate decreases by 12.84% per minute because the maximum train load rate is limited to 100%.

### 6.2. Analysis of Inflow Control Results for Line 1 and Line 5

In this subsection, the solution results of Line 1 and Line 5 are analyzed, which are shown in Tables 6 and 7, respectively. Line 1 passes through several urban and suburban areas of Beijing. Table 6 presents the optimization for Line 1, in which the time periods 1, 2, ... , 8 represent $7:00-7:15$, $7:15-7:30$, ... , $8:45-9:00$, respectively. Columns 2–9 list the number of inbound passengers at each station during each time period after optimization. The numbers in italics represent the difference between the optimization results and the actual results. For example, $-90$ of station PINGGUOYUAN during time period 1 means there are 1582 passengers entering the station in actual operation. Column 10 reports inbound demand $\sum r_i^n$ at each station on Line 1. The inbound delay rates before and after optimization are demonstrated in the last two columns, respectively. The significant differences are shown in bold. It can be seen that YUQUANLU station is newly added as the inflow-limit station for Line 5.
station and the inflow restriction is lifted at GONGZHUFEN station. Furthermore, at the end of morning rush hours, the limitation can be reduced or even lifted at some stations located in the suburbs, such as PINGGUOYUAN station, GUCHENG station, and BAJIAO Amusement Park.
than 100% in some sections during morning rush hours. The optimization results show that inflow-limit stations are located in both ends of line 5. Compared to the actual, inflow restriction can be lifted at DONGDAN station, CIQIKOU station, and TIANTANDONGMEN station. The time for passenger inflow control at stations HUIBEI and HUINAN can be shortened to one hour, that is, 7:30 AM to 8:30 AM. In the situation of tightening the limit on inbound passenger flow for stations at both ends, it can improve the inbound flow rate at stations located in the middle of the line.

### 6.3. Analysis of the Inflow Control Parameter

The strength of passenger flow control is restricted by the passing capacity of station facilities, and this restriction is stronger at stations with mass passenger flow. In the guidance document [6], it is stated that the number of inbound passengers cannot exceed 70 percent of the station capacity. Next, stations at which inbound demand is greater than 70% of station inbound capacity are taken for analysis. Figure 5 shows the ratio distribution of the optimized inbound passenger flow to the station inbound capacity, in which each bar indicates the frequency and cumulative percentage of the ratio of inbound passenger flow volume to maximum facilities capacity.

![Figure 5: Statistics on the utilization of the station inbound capacity under coordinated inflow control strategy.](image-url)
the number of stations at the corresponding capacity utilization rate, and the line shows the cumulative percentage of the number of stations. It is worth mentioning that the inflow restrictive measure is not adopted at some of these stations. After optimization, there are 16 stations with inbound passenger flow volume reaching the maximum inbound capacity of station facilities, and the coordinated inflow control strategy helps to limit inbound flow to less than 70% of capacity for about half of the stations. The average ratio of inbound demand to station inbound capacity for all stations in Figure 2 is 84.5%, and there are some inflow-limit stations whose demand does not exceed the limit specified in the document [6]. Therefore, the inbound capacity constraint is weak for the model and the stipulated 70% in document [6] should be recalibrated by further research according to the actual operation.

7. Conclusions and Further Research

The article proposes a coordinated inflow control model for the urban rail transit network with four types of constraints to improve the train capacity utilization and reduce the inbound delay rate. A combined method is presented to solve the model with a station-to-line spatial-temporal relationship, passenger flow OD matrix, and large-scale network topology. The proposed model and algorithm are tested on the Beijing urban rail transit network. The computational results display the inflow-limit stations and the number of inbound passengers at each station during each time period. In the case of limited transportation resources, the coordinated inflow control strategy is an effective method to weaken the influence of massive passenger flow on the network. The application of the quantitative method in this study can reduce the influence of subjective factors on the optimization results and provide a theoretical reference for developing a practical control strategy. Finally, the generality of a parameter related to passenger inflow control in the regulations [6] is analyzed and discussed. In this study, the station-to-line spatial-temporal relationship of inbound passenger flow is the key to realizing the numerical quantification for the network, which provides a new idea for network-level inflow control. Combining with the actual operation in Beijing, the values of two parameters are obtained by fitting based on historical passenger data, which can overcome the lag in formulating the inflow control scheme.

With the improvement of the network operation services, a simultaneous coordinated inflow control strategy with train plan and train timetable should also be explored. It is also expected that an inflow control method will be found to open with unexpected events, such as the train delay.

Data Availability

The data of passenger arrival rate, ratio of passengers in different pairs, and timetable and train capacity in numerical experiments used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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