The state of the art of the theoretical calculations for heavy quarks photoproduction is reviewed. The full next-to-leading order calculation and two possible resummations, the high energy one for total cross sections and the large $p_T$ one for differential cross sections, are described.

1 Introduction

Heavy quarks production processes provide a powerful insight into our understanding of Quantum Chromodynamics. The large mass of the heavy quark can make the perturbative calculations reliable, even for total cross sections, by setting a large scale at which, for instance, the strong coupling can be evaluated and found small enough. On the experimental side, the possibility to tag heavy flavoured mesons by means of microvertex detectors can provide quite accurate measurements.

All these potentialities must of course be matched by an accurate enough theoretical evaluation of the production cross section. In this talk I will describe the state of the art of such calculations for heavy quarks photoproduction. I will firstly review the next-to-leading order (NLO) QCD calculations recently presented by Frixione, Mangano, Nason and Ridolfi, for both direct and resolved photons. This calculation, available for total cross sections, one-particle and two-particle distributions, is now a consolidated result and provides a benchmark for future developments. Next I will describe the resummation of two kinds of large logarithms which appear in the NLO fixed order calculations and potentially make it less reliable in some regimes: $\log(S/m^2)$ is large when the center of mass energy $\sqrt{S}$ is much larger than the mass $m$ of the heavy quark, and $\log(p_T^2/m^2)$ becomes large when the transverse momentum $p_T$ of the observed quark is much larger than its mass.

2 Fixed Order Calculation

Heavy quarks photoproduction at leading order in the strong coupling $\alpha_s$ looks a very simple process: only the tree level diagram $\gamma g \to QQ$ con-
tributes at the partonic level, and the final answer for the total cross sec-
tion (to be convoluted with the gluon distribution function) reads
\[ \hat{\sigma}_{\gamma g}(\rho) = 2\pi\alpha\alpha_s\epsilon_Q^2\rho\beta \left( 1 + \rho - \frac{\epsilon_Q^2}{2} \right) \frac{1}{\beta^2} \ln \frac{1+\beta}{1-\beta} - 1 - \rho \] with \( \rho = 4m^2/S \) and \( \beta = \sqrt{1-\rho} \),\n\( m \) being the heavy quark mass and \( \sqrt{S} \) the center-of-mass energy of the inter-
action. This result is simple and well behaved, being finite everywhere.

At a deeper thinking, however, problems seem to arise. For instance, one
may ask himself why not to include initial state heavy quarks, coming from
the hadron and to be scattered by the photon, like \( \gamma Q \rightarrow Qg \). To include
consistently such a diagram is not an easy task, especially if one wants to keep
the quark heavy. Taking it massless, on the other hand, would not only be
a bad approximation but would also produce a divergent total cross section.
A way out of this was provided by Collins, Soper and Sterman\cite{1}, who argued
that for total cross section of heavy quarks in hadron collisions the following
factorization formula holds:
\[ \sigma = \sum \int f_{a/H_1} f_{b/H_2} \hat{\sigma}(ab \rightarrow Q\bar{Q}) \] (1)
The sum on the partons runs only on \( a \) and \( b \) being gluons or light quarks, and
the heavy quarks are only generated at the perturbative level by gluon splitting.
There is therefore no need to try to accomodate them in the colliding hadrons
and the relevant kinematics can be kept exact. Eq. (1) provides the basis for
an exact perturbative calculation of heavy quarks production to NLO. For
what concerns photoproduction, such a calculation has been first performed
by P. Nason and K. Ellis, and subsequently confirmed by J. Smith and W.L.
van Neerven\cite{2}. When going to order \( \alpha\alpha_s^2 \) in photon-hadron collision, however,
a new feature appears. The photon can now couple directly to massless quarks,
for instance in processes like \( \gamma q \rightarrow Q\bar{Q}q \), and in a given region of phase space
a collinear singularity will appear. It can be consistently factorized out, but
this requires the introduction of a photon structure function which, pretty
much like hadron structure functions, will describe the probability that before
the interaction the photon splits into hadronic components (light quarks or
 gluons, in this case). Such a behaviour is sometimes called resolved photon
(as opposed to direct). A full NLO calculation for heavy quark photoproduction
will therefore also require a NLO calculation for hadroproduction\cite{3}, where one
of the structure functions will be the photon’s one. A factorization scale \( \mu_r \),
related to the subtraction of the singularity at the photon vertex, will link the
two pieces and its dependence on the result will only cancel when both are
taken into account.

Frixione, Mangano, Nason and Ridolfi\cite{4} have recently presented Montecarlo
integrators (which are not, we stress, NLO Montecarlo generators) for these two
calculations, thereby allowing detailed comparisons with experimental data.
Rather than presenting here the plots with these comparisons I refer you to the original literature. The overall result can however be summarized as follows. Total cross sections seem to be reproduced by the calculation both at fixed target and HERA regimes, but the huge uncertainties present both on the experimental and the theoretical side do not allow the study of finer details like, for instance, the relevance of the resolved component at HERA. For what concerns transverse momentum distributions at fixed target, they can be reproduced after allowing for the heavy quark fragmentation to mesons and for a primordial transverse momentum of the incoming partons of the order of 1 GeV. These same non-perturbative corrections also allow for a description of two-particle correlations, thereby pointing towards a consistent picture. On the other hand, a word of caution is mandatory in the light of experimental results presented at this Workshop, which show a pseudorapidity distribution of the heavy quark at HERA not in agreement with the NLO calculation.

3 High Energy Resummation

Like any perturbative expansion, the NLO calculation for heavy quarks photoproduction is only reliable and accurate as long as the coefficients of the coupling constant remain small. Large terms of the kind \( \ln(S/m^2) \) (or \( \ln(1/x) \)), since \( xS = s \sim m^2 \) do however appear in the kernel total cross section, and for growing \( S \) they will eventually became large enough to spoil the convergence of the series order by order. Such terms need therefore to be resummed to all orders to allow for a sensible phenomenological prediction.

Theoretical frameworks for obtaining such a resummation have been provided in the papers of ref. The general procedure is that of replacing the usual “collinear pole” factorization for the kernel cross section

\[
4m^2 \sigma_{\gamma g}(\rho, m^2, Q_0^2) = \int_0^1 \frac{dz}{z} C \left( \frac{\rho}{z}, \alpha_s(m^2) \right) G(z, m^2, Q_0^2) \tag{2}
\]

with a new “high-energy” factorization, where the gluon which couples to the \( Q\bar{Q} \) system is kept off-shell by an amount \( k^2 \):

\[
4m^2 \sigma_{\gamma g}(\rho, m^2, Q_0^2) = \int d^2k \int_0^1 \frac{dz}{z} \hat{\sigma} \left( \frac{\rho}{z}, \frac{k^2}{m^2} \right) F(z, k, Q_0^2) \tag{3}
\]

The corresponding unintegrated gluon structure function \( F \), also depending on \( k \), will obey a BFKL evolution equation whose solution resums the dangerous small-\( x \) terms.

Not many phenomenological results are available within this framework. Bottom production at HERA has been estimated to grow by about 10% when these effects are taken into account.
4 Large Transverse Momentum Resummation

More potentially large terms, of the kind \(\ln(p_T^2/m^2)\), do appear when considering the one particle inclusive differential distribution at large transverse momentum. These terms should also be resummed to produce a reliable theoretical prediction in this region. Such a resummation has been performed in ref. 7 along the following lines. One observes that in the large-\(p_T\) limit the only important mass terms are those appearing in the logs, all the others being power suppressed. This means that an alternative description of heavy quark production can be achieved by using massless quarks and providing at the same time structure and fragmentation functions also for the heavy quark:

\[
\sigma = \sum \int f_{a/H_1} f_{b/H_2} \hat{\sigma}(ab \to c) dQ/c \tag{4}
\]

Indices \(a, b\) and \(c\) now also run on \(Q\), taken massless in \(\hat{\sigma}\). The key point is that the large mass of the heavy quark allows for the evaluation in perturbative QCD of its structure and fragmentation functions at a scale given by its mass. The logs will appear in these function, which can then be evolved with the Altarelli-Parisi equations up to the large scale set by \(p_T\). This evolution will resum the large logarithms previously mentioned. Phenomenological analyses show that the effect becomes sizeable only at very large \(p_T\), say greater than 20 GeV for charm photoproduction, and should therefore not be very important at HERA.

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References
1. J. Collins, D. Soper and G. Sterman, Nucl. Phys. B263 (1986) 37
2. R.K. Ellis and P. Nason, Nucl. Phys. B312 (1989) 551; J. Smith and W.L. van Neerven, Nucl. Phys. B374 (1992) 36
3. P. Nason, S. Dawson and R.K. Ellis, Nucl. Phys. B303 (1988) 607; Nucl. Phys. B327 (1989) 49; W. Beenakker et al., Phys. Rev. D40 (1989) 54; Nucl. Phys. B351 (1991) 507
4. S. Frixione, M.L. Mangano, P. Nason and G. Ridolfi, Nucl. Phys. B412 (1994) 225; Nucl. Phys. B431 (1994) 453; Phys. Lett. B348 (1995) 633; Nucl. Phys. B454 (1995) 3; hep-ph/9510253
5. K. Daum, these Proceedings; R. Graciani, ibidem
6. S. Catani, M. Ciafaloni and F. Hautmann, Nucl. Phys. B366 (1991) 135; J.Collins and R.K. Ellis, Nucl. Phys. B360 (1991) 3; E.M. Levin et al., Sov. J. Nucl. Phys. 53 (1991) 657
7. M. Cacciari and M. Greco, Z. Phys. C69 (1996) 459; Nucl. Phys. B421 (1994) 530; B. Mele and P. Nason, Nucl. Phys. B361 (1991) 626