Quantum state tomography across the exceptional point in a single dissipative qubit

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Open physical systems can be described by effective non-Hermitian Hamiltonians that characterize the gain or loss of energy or particle numbers from the system. Experimental realization of optical1–7 and mechanical8–13 non-Hermitian systems has been reported, demonstrating functionalities such as lasing14–16, topological features17–19, optimal energy transfer20,21, and enhanced sensing22,23. Such realizations have been limited to classical (wave) systems in which only the amplitude information, not the phase, is measured. Thus, the effects of a system’s proximity to an exceptional point—a degeneracy of such non-Hermitian Hamiltonians where the eigenvalues and corresponding eigenmodes coalesce24–30—on its quantum evolution remain unexplored. Here, we use post-selection on a three-level superconducting transmon circuit to carry out quantum state tomography of a single dissipative qubit in the vicinity of its exceptional point. We observe the spacetime reflection symmetry-breaking transition30,31 at zero detuning, decoherence enhancement at finite detuning and a quantum signature of the exceptional point in the qubit relaxation state. Our experiments show phenomena associated with non-Hermitian physics such as non-orthogonality of eigenstates in a fully quantum regime, which could provide a route to the exploration and harnessing of exceptional point degeneracies for quantum information processing.

In introductory treatments of quantum mechanics, one typically assumes that a Hamiltonian describing a physical system is Hermitian, thus ensuring the reality of energy eigenvalues and a unitary time evolution. In recent years there has been growing interest in non-Hermitian systems, particularly those with spacetime reflection (parity–time; PT) symmetry that exhibit transitions from purely real to complex conjugate spectra. A canonical example of a PT-symmetric system is shown in Fig. 1a and consists of a bipartite system with balanced gain (part A) and loss (part B). Such systems have been experimentally studied in the quantum domain. The central feature of these systems is a transition from broken to unbroken PT symmetry. When the coupling, given by rate $J$, between the two parts is larger than the gain–loss rate $\gamma_A$, the system exhibits a real spectrum and simultaneous eigenmodes of both the Hamiltonian and the antilinear PT operator; yet when the coupling is small, this PT symmetry is broken by the emergence of complex conjugate eigenvalues. These two phases are joined by an exceptional point (EP). The EP degeneracy also occurs for a bipartite system with imbalanced losses. The schematic in Fig. 1b displays such a system in which parts A and B are coupled and part B exhibits loss. Here, we extend these studies to a fully quantum limit where these parts are realized as quantum energy levels—with no classical counterparts—in which the loss corresponds to transitions outside that manifold of states. This two-level system in the presence of coupling produced by a drive with detuning $\Delta$ can be described by an effective non-Hermitian Hamiltonian ($\hbar = 1$):

$$H_{\text{eff}} = J \langle f \rangle \langle e \rangle + \langle e \rangle \langle f \rangle + (\Delta - i \gamma_A / 2)\langle e \rangle \langle e \rangle$$

where $\langle e \rangle$ and $\langle f \rangle$ denote the first and second excited states of the quantum system and $\gamma_A$ is the occupation-number loss rate to the ground state $\langle g \rangle$ (Fig. 1b). At zero detuning, the complex eigenvalues of $H_{\text{eff}}$ have different real and imaginary parts. Here, the qubit dynamics is governed by eigenmode energy differences $\Re(\delta\lambda(\Delta, J))$ (Fig. 1c) and $\Im(\delta\lambda(\Delta, J))$ (Fig. 1d), where $\Re$ and $\Im$ indicate real and imaginary parts, respectively, and the eigenvalue difference $\delta\lambda$ is given by

$$\delta\lambda = (\lambda_+ - \lambda_-) = \sqrt{4J^2 - (\Delta - i \gamma_A / 2)^2}$$

Our experiment comprises a transmon circuit formed by a pair of Josephson junctions in a superconducting quantum interference device (SQUID) geometry shunted by a capacitor (Fig. 1e). The transmon circuit exhibits several quantum energy levels that can be individually addressed with narrow-bandwidth microwave pulses. By applying a magnetic flux through the SQUID loop, we can tune the spacing between energy levels. The coupling Hamiltonian $J_{\text{f\gamma}} = J \langle f \rangle \langle e \rangle + \langle e \rangle \langle f \rangle$ is realized by a coherent resonant drive of variable amplitude and detuning.

The transmon circuit is embedded in a three-dimensional waveguide cavity. The dispersive interaction between the transmon circuit and fundamental electromagnetic mode of the cavity results in a state-dependent shift in the cavity frequency. This frequency shift is detected by probing the cavity with a weak microwave tone; the resulting state-dependent phase shift is detected with homodyne measurement using a Josephson parametric amplifier. The lowest energy level $\langle g \rangle$ is the stable ground state, and we use it as an effective continuum—an environment that is ‘outside’ of the subspace of states $\langle e \rangle$ and $\langle f \rangle$ that form the qubit system under investigation. To implement $H_{\text{eff}}$, we require the respective energy decay rates $\gamma_A \gg \gamma_f$. The presence of a finite decay rate $\gamma_A$ shifts the EP to $H_{\text{eff}} = 4(\gamma_A - \gamma_f) / 4$. We achieve this hierarchy of decay rates by inserting an impedance mismatching element between the cavity and parametric amplifier. This causes an interference in the cavity field that alternately suppresses and enhances the density of states in the transmission line, resulting in a frequency dependence of the Purcell decay rate. Thus, by tuning the transition frequency between the $\langle g \rangle$ and $\langle e \rangle$ states to regions where the density of states is enhanced, we enhance the decay rate of the $\langle e \rangle$ state.
We first investigate the $PT$ symmetry-breaking transition, which occurs when $\Delta = 0$. We tune the transmon such that $\gamma_c = 6.7 \mu s^{-1}$ and $\gamma_r = 0.25 \mu s^{-1}$. We then initialize the system in the state $|0\rangle$ with $J = 0$, and at time $t = 0$, we switch $J$ to a finite value for a variable period of time. The experimental sequence is concluded with a projective measurement of the transmon energy. Evolution under
\( H_{\text{d}} \) leads to exponential decay of the norm of a given initial state. Experimentally, we focus on the evolution in the \(|e\), \(|f\) qubit manifold, which results in normalized populations \( P^0_f = P_f/(P_e + P_f) \) and \( P^0_e = P_e/(P_e + P_f) \) = \(1-P^0_f\). This is achieved through post-selection; experimental sequences conclude with a projective measurement of the transmon in the energy basis, and only experiments in which the transmon remains in the qubit manifold are included in the analysis. Thus, for longer experimental durations, the success rate decreases exponentially.

We now characterize the \( \mathcal{PT} \) symmetry-breaking transition using the observed experimental signatures in the populations and the coherences in the \(|e\), \(|f\) qubit manifold. In Fig. 2a we show \( P^0_f \) versus time for different coupling rates \( J \). For a large \( J \) we observe oscillatory dynamics in \( P^0_f \). These Rabi oscillations occur because the initial state \(|f\) can be expressed as a superposition of eigenstates of \( H_{\text{d}} \) with corresponding time evolution \( e^{-i\lambda t} \); the equal imaginary parts of \( \lambda_s \) for \( J > \gamma/4 \) result in the oscillatory evolution at angular frequency \( \Omega \) for the post-selected occupation probabilities. This region is referred to as the \( \mathcal{PT} \)-unbroken region. The time evolution of \( P^0_f \) shows a striking transition at the finite coupling rate, as detailed in Fig. 2b. Here, we observe that when \( J < \gamma/4 \), the oscillations cease due to the vanishing real parts of \( \lambda_s \). This is referred to as the \( \mathcal{PT} \)-broken region. Figure 2c displays time-trace cuts from Fig. 2b in the broken and unbroken regions with decaying and oscillating behaviour, respectively. Although Fig. 2 displays only data for which the transmon did not leave the \(|e\), \(|f\) qubit manifold, the post-selection on the qubit manifold leads to measurement back-action favouring the \(|f\) state, leaving a clear signature of the decay in the temporal evolution within this manifold.

The \( \mathcal{PT} \) symmetry-breaking transition can be quantified by looking at the Rabi frequency \( \Omega \) as a function of the coupling rate. This \( \Omega \) is obtained from a simple, exponentially damped sinusoidal fit to \( P^0_f(t) \) (Fig. 2c). In Fig. 2d we plot the observed \( \Omega \) versus \( J \), which displays a square-root singularity that is associated with increased sensitivity near the EP\(^{22,23}\). The solid curve displays a fit to \( \text{Re} \delta \lambda = 2 \text{Re} \sqrt{J^2 - J_0^2} \), where \( J_0 \) is the sole free parameter. From the fit we find \( J_0 \approx 1.71 \pm 0.07 \mu \text{s}^{-1} \), which is in agreement with the expected value based on the independently measured decay rates \( (\gamma_e - \gamma_f)/4 = \gamma/4 \approx 1.61 \mu \text{s}^{-1} \).

Next, we characterize the evolution of the qubit in the broken and unbroken regimes using quantum state tomography. Figure 2e shows a striking transition at the finite coupling \( \gamma \) for different values of the detuning. In the \( \mathcal{PT} \)-unbroken region \(|+\rangle\) and \(|-\rangle\) are represented by dashed lines. The non-orthogonality of the eigenstates across the \( \mathcal{PT} \) transition, including in the vicinity of the EP, is characterized in terms of the overlap \(|+\rangle\rangle = \text{min}(\chi, 1/\chi) \) of the two eigenstates, which were found from the zero-crossing of the \( \delta P^0_f \) plots, determined from \( \text{min}(\text{abs}(\delta P^0)) \). The error in the preparation angle was estimated as the distance to the next-nearest minima, and the error bars in c indicate this error propagated to the inner product. For some data points, the error bar is smaller than the marker.

With access to the quantum coherent dynamics in the vicinity of the EP, it is natural to investigate the role of decoherence in this regime. As shown in Fig. 1c,d, the eigenvalue difference \( \delta \lambda \) of \( H_{\text{d}} \) exhibits rich dependence on \( J \) and \( \Delta \), which in turn determine the time evolution of the dissipative qubit. Figure 4a depicts the time evolution of the qubit state given by Bloch coordinates \( x(t), y(t) \) and \( z(t) \), which were measured with quantum state tomography for different values of the detuning. In the \( \mathcal{PT} \)-symmetric phase, we fit the oscillations to determine both \( \Omega \) and the Rabi oscillation damping rate \( \Gamma_\Omega \) for different detunings, yielding, respectively, the real and imaginary parts of \( \delta \lambda \) (Fig. 4b). At \( \Delta = 0 \), the eigenmode decay rates are equal, and we observe only a residual, small coherence damping in the qubit manifold, characterized by \( \Gamma_\Omega = 0.6 \mu \text{s}^{-1} \); this damping is larger than expected from the small \( \gamma_e \) and is primarily due to charge and flux noise. As \( |\Delta| \) increases, the difference in the eigenmode decay rates leads to faster coherence damping.
The observed $\Omega$ and $\Gamma_y$ are in good agreement with the analytical predictions offset by $\Gamma_x$.

Quantum state tomography also allows us to study the steady states of the qubit system evolving under $H_{\text{eff}}$ in the vicinity of the EP. Figure 4c displays the steady-state results of quantum state tomography after 4μs of time evolution. Along the $PT$-symmetric phase line ($|\Delta| > 0$ and $J > j/4$), the qubit reaches a maximally mixed state. When $|\Delta| > 0$, the qubit reaches a mixed steady state in the $X$–$Z$ plane, that is, $y = 0$. In close proximity of the EP, when $\gamma$ approaches 0, the qubit reaches a steady state given by $(|\psi\rangle + |\phi\rangle)/\sqrt{2}$; that is, the single eigenmode of $H_{\text{eff}}$ at the EP (see Methods). In our experiment this appears as a peak in the $y$ component in the tomography in Fig. 4c along with a vanishing $x$ component and a $z$ component that is suppressed in magnitude. These results indicate that the dissipation of the system stabilizes the qubit to non-trivial steady states for different drive and detuning parameters.

While the dynamics of the three-level atom are described by a Lindblad equation with two dissipators that characterize spontaneous emission from levels $|\phi\rangle$ and $|\psi\rangle$, the non-Hermitian evolution and EP effects only manifest when quantum jumps to the $|\psi\rangle$ state are eliminated by post-selection. This approach, combined with the toolbox of circuit quantum electrodynamics, serves as a versatile platform to explore fundamental questions in the quantum mechanics of open systems. Recent work identifying enhanced sensitivities in the vicinity of the EP has spurred interest in the role of quantum noise in EP-based sensors. Our system forms an ideal platform for characterizing quantum sensing applications using non-Hermitian systems, including the role of noise introduced by dissipation (see Methods). Real-time control of parameters in $H_{\text{eff}}$ will enable studies of the topological features associated with adiabatic perturbations that encircle the EP and of higher-order exceptional surfaces that arise in time-periodic (Floquet) non-Hermitian dynamics. Finally, in concert with very recent work on superconducting circuits and nitrogen–vacancy centres, our study provides the opportunity for non-Hermitian physics to play a leading role in a range of quantum information-processing applications.

Online content

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**Author contributions**

K.W.M., M.N. and Y.N.J. conceived the project. K.W.M., M.A. and M.N. performed the experiments and analysed the data. Y.N.J. provided theory support. K.W.M., M.N. and Y.N.J. wrote the manuscript.

**Competing interests**

The authors declare no competing interests.

**Additional information**

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Methods

Here, we provide details of the experimental set-up and techniques used in this work. We also provide an analysis of the system as described by a Lindblad evolution in the three-state manifold, which is equivalent to the non-Hermitian Hamiltonian evolution in the two-state manifold. We provide further discussion regarding the interplay of Lindbladian dissipation and non-Hermitian dissipation as well as prospects for enhanced sensing near the EP.

Experimental set-up. The transmon circuit was fabricated by conventional double-angle evaporation and oxidation of aluminium on a silicon substrate. With zero flux threading the SQUID loop, the transition frequencies are \( \omega_{r}/2\pi = 6.1 \text{ GHz} \) and \( \omega_{f}/2\pi = 5.8 \text{ GHz} \). The transmon circuit is placed in a three-dimensional copper cavity with frequency \( \omega_{c}/2\pi = 6.86 \text{ GHz} \) and decay rate \( \kappa/2\pi = 5 \text{ MHz} \) with an embedded coil for adjusting the d.c. magnetic flux through the SQUID loop. The coupling rate between the transmon circuit and the cavity fields is \( g/2\pi = 65 \text{ MHz} \). Experiments are performed with a small flux threading the SQUID loop, resulting in \( \omega_{c}/2\pi \approx 5.7 \text{ GHz} \) and \( \omega_{f}/2\pi \approx 5.42 \text{ GHz} \). These are given by the charging energy \( E_{r}/h \approx 270 \text{ MHz} \) and the Josephson energy \( E_{J}/h = 16.6 \text{ GHz} \), where the dressed cavity resonance frequency is \( \omega_{c}/2\pi = 6.684 \text{ GHz} \). The dispersive coupling between the transmon and the cavity allows a high-fidelity, single-shot readout of the transmon states and quantum state tomography. The dispersive cavity resonance shifts are given by \( g/2\pi = -2 \text{ MHz} \) and \( g/2\pi = -11 \text{ MHz} \).

Data analysis and experimental error. In Fig. 3 we extract the locations of the eigenstates in the broken and unbroken regions. This is achieved through a two-point measurement technique. In the broken region, the eigenstates are simply found by comparing the change in \( P_{f} \) over 500 ns of evolution. States that are stationary exhibit no change, whereas non-stationary states exhibit oscillatory behaviour. In the broken regime, although the eigenvalues are strictly imaginary, the stationary states are still visible as regions where \( P_{f} \) is stationary. The data displayed in Fig. 3a have been scaled to account for the small decay over 500 ns. The preparation angles for the eigenstates were found from the zero crossing of the \( \langle \hat{J}_{z} \rangle \) plots, determined from \( \langle \hat{J}_{z} \rangle = \sum_{n} \sum_{m} \rho_{nm} \langle n | \hat{J}_{z} | m \rangle \rho_{mn} \).

Lindblad evolution of the three-state system. In the main text, we focused solely on the dynamics in a qubit subsystem that is governed by \( H_{q} \) (equation (1)). Instead, one can look at the dynamics for the entire three-level system, which can be described by a Lindblad master equation (\( \dot{\rho} \)) (refs. 15,66):

\[
\dot{\rho} = -i[H_{1}, \rho] + \sum_{k=1}^{8} \frac{1}{2} [L_{k} \rho L_{k}^{\dagger} - L_{k}^{\dagger} L_{k} \rho - \rho L_{k}^{\dagger} L_{k} \rho] \tag{2}
\]

where \( \rho(t) \) is a 3x3 density matrix, \( \rho \) is the time derivative, and \( H_{1} = \frac{1}{4} (|\psi_{r}^{\dagger} \rangle \langle \psi_{r}| + |\psi_{f}^{\dagger} \rangle \langle \psi_{f}|) - \Delta \). \((|\psi_{r}^{\dagger} \rangle \langle \psi_{r}| + |\psi_{f}^{\dagger} \rangle \langle \psi_{f}|) \) is a coupling Hamiltonian with \( \Delta \) in the rotating frame. The Lindblad dissipation operators \( L_{1} = \sigma_{r}^{\dagger} \rho_{r} \sigma_{f}^{\dagger} \) and \( L_{2} = \sigma_{f}^{\dagger} \rho_{f} \sigma_{r}^{\dagger} \) for the energy decay from \( |\psi_{r} \rangle \) to \( |\psi_{f} \rangle \) and from \( |\psi_{f} \rangle \) to \( |\psi_{r} \rangle \), respectively, and the dagger represents the Hermitian conjugate. Equation (2) leads to the following closed set of equations for the dynamics of the qubit levels:

\[
\begin{align*}
\dot{\rho}_{r} &= -i(\psi_{r}^{\dagger} \rho_{f} - \rho_{r} \psi_{f}^{\dagger}) - \gamma_{r} \rho_{r} \\
\dot{\rho}_{f} &= -i(\psi_{f}^{\dagger} \rho_{r} - \rho_{f} \psi_{r}^{\dagger}) - \gamma_{f} \rho_{f} \\
\dot{\rho}_{g} &= -i(\psi_{g}^{\dagger} \rho_{r} - \rho_{g} \psi_{r}^{\dagger}) - \gamma_{g} \rho_{g}
\end{align*} \tag{3}
\]

Since the drive only acts on the manifold of the excited states, the dynamics of the ground state are decoupled from the upper manifold. For a given initial condition, one can solve equation (3) to obtain the evolution of any observable. As shown in Supplementary Fig. 3a, the eigenvalues of the Liouvillian are strictly imaginary. In this case, the eigenstates of the three-state system are still visible as regions where \( P_{f} \) is stationary. The data displayed in Fig. 3a have been scaled to account for the small decay over 500 ns. The preparation angles for the eigenstates were found from the zero crossing of the \( \langle \hat{J}_{z} \rangle \) plots, determined from \( \langle \hat{J}_{z} \rangle = \sum_{n} \sum_{m} \rho_{nm} \langle n | \hat{J}_{z} | m \rangle \rho_{mn} \).

Quantum state tomography in the vicinity of the EP. Recent work with classical systems has indicated that EP degeneracies may yield measurement advantages\(^{23,24}\). These studies have motivated further investigation into whether these advantages persist in the fully quantum regime where quantum noise dominates the measurement process. Theoretical work on semiclassical optical systems\(^{33,47}\) has found that enhanced sensitivities near the EP are counteracted by enhanced fluctuations, curtailing measurement advantages. How these studies extend to the fully quantum regime explored here remains an open question. In this section, we briefly discuss how the Lindblad evolution of the three-state system can be used to characterize enhanced measurement sensitivities in terms of the quantum Fisher information (QFI) and how the post-selection process may hamper these advantages.

Quantum metrology, the Cramér–Rao bound gives a universal limit for the mean squared deviation in the estimate of a parameter:

\[
\left( \delta \theta \right)^{2} \geq \frac{1}{4} \frac{1}{\left( \partial^{2}/\partial \theta^{2} \right)} \tag{7}
\]

where \( \nu \) is a measure of the amount of data, \( \theta \) is an unbiased estimator of the parameter \( g \) formed from measurement data and \( 1/\left( \partial^{2}/\partial \theta^{2} \right) \) is the QFI, which can be expressed in terms of the Bures distance, \( d_{B}^{2} (\nu, \theta) = 2 \left( 1 - \langle \nu | D_{\theta} | \nu \rangle \right) \), where \( D_{\theta} \) is the state after evolution under parameter \( g \), and \( | \nu \rangle \) is the state after evolution under parameter \( g \). For larger \( \nu \) values, \( d_{B}^{2} \) increases, indicating a loss in information.

A simple approach to metrology near the EP is based on Rabi interferometry. For this, we consider preparing the qubit in \( | \nu \rangle \) and allowing evolution under \( H_{r} \) for certain durations of time. Supplementary Fig. 4a displays the evolution of \( P_{f} \) for parameter regimes that are near the EP calculated using equation (3) for the three-state system. The evolution near the EP is not purely sinusoidal; there are points where the state population varies rapidly with time. When \( J \) is changed by a small amount, we observe a large change in the state population compared with the case of a normal Hermitian qubit with no EP for the same evolution time. The fractional change in the state population with a fractional change in \( J \) is closely related to the QFI.

To determine the QFI, we simply vary \( J \) by a small amount to determine the slope \( dP_{f}^{2}/dJ \). For small changes near \( P_{f} = 0.5 \), we have
$P_{\theta}^{-1} = \cos(d\theta/2)\sin(d\theta/2) \approx d\theta/2$, where $d\theta$ is a small change in the polar angle near the equator of the Bloch sphere. Thus, near the equator of the Bloch sphere, the QFI about the coupling rate $J$ is simply given by $I_{J}^{(\theta)} = (dP_{\theta}^{2}/dJ)^{2}$.

Supplementary Fig. 4b displays the QFI for this measurement scheme near the EP using the parameters in Supplementary Fig. 4a. The QFI diverges near the EP, as has been observed for the classical Fisher information in classical systems. This improved QFI comes at a cost, however, due to the post-selection that is used to realize the effective non-Hermitian dynamics; near the EP, the post-selection efficiency is low, which ultimately decreases the amount of data available. The enhanced sensitivity near the EP bears similarities to weak value amplification, where low post-selection efficiency is at odds with amplified signals. Even in this case, there are advantages to post-selection when signals are dominated by technical noise\textsuperscript{41}.

**Data availability**
The data that support the plots within this paper and other findings of this study are available from K.W.M. on reasonable request.

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