On analogues of black brane solutions in the model with multicomponent anisotropic fluid

V. D. Ivashchuk

Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya ul., Moscow 119361, Russia
Institute of Gravitation and Cosmology, Peoples’ Friendship University of Russia, 6 Miklukho-Maklaya ul., Moscow 117198, Russia

Abstract

A family of spherically symmetric solutions with horizon in the model with $m$-component anisotropic fluid is presented. The metrics are defined on a manifold that contains a product of $n-1$ Ricci-flat “internal” spaces. The equation of state for any $s$-th component is defined by a vector $U^s$ belonging to $\mathbb{R}^{n+1}$. The solutions are governed by moduli functions $H_s$ obeying non-linear differential equations with certain boundary conditions imposed. A simulation of black brane solutions in the model with antisymmetric forms is considered. An example of solution imitating $M_2 - M_5$ configuration (in $D=11$ supergravity) corresponding to Lie algebra $A_2$ is presented.

1 Introduction

In this paper we continue our investigations of spherically-symmetric solutions with horizon (e.g., black brain ones) defined on product manifolds containing several Ricci-flat factor-spaces (with diverse signatures and dimensions). These solutions appear either in models with antisymmetric forms and scalar fields [1]-[11] or in models with (multi-component) anisotropic fluid [12]-[15]. For black brane solutions with 1-dimensional factor-spaces (of Euclidean signatures) see [16, 17, 18] and references therein.

These and more general brane cosmological and spherically symmetric solutions were obtained by reduction of the field equations to the Lagrange equations corresponding to Toda-like systems [2, 19]. An analogous reduction for models with multicomponent anisotropic fluids was performed earlier in [20, 21]. For cosmological-type models with antisymmetric forms without scalar fields any brane is equivalent to an anisotropic fluid with the equations of state:

$$\hat{p}_i = -\hat{\rho} \quad \text{or} \quad \hat{p}_i = \hat{\rho},$$

(1.1)

when the manifold $M_i$ belongs or does not belong to the brane world volume, respectively (here $\hat{p}_i$ is the effective pressure in $M_i$ and $\hat{\rho}$ is the effective density).

In this paper we present spherically-symmetric solutions with horizon (e.g the analogues of intersecting black brane solutions) in a model with multi-component anisotropic fluid (MC AF), when certain relations on fluid parameters are imposed. The solutions are governed by a set of moduli functions $H_s$ obeying non-linear differential master equations with certain boundary conditions imposed. These master equations are equivalent to Toda-like equations and depend upon the non-degenerate $(m \times m)$ matrix $A$. It was conjectured earlier that the functions $H_s$ should be polynomials when $A$ is a Cartan matrix for some semi-simple finite-dimensional Lie algebra (of rank $m$) [9]. This conjecture was verified for Lie algebras: $A_m, C_{m+1}, m \geq 1$ [7, 8]. A special case of black hole solutions with MCAF corresponding to semisimple Lie algebra $A_1 \oplus ... \oplus A_1$ was considered earlier in [13] (for $m = 1$ see [12]).

The paper is organized as follows. In Section 2 the model is formulated. In Section 3 spherically-symmetric MCAF solutions with horizon corresponding to black-brane-type solutions, are presented. In Section 4 a polynomial structure of moduli functions $H_s$ for semi-simple finite-dimensional Lie algebras is discussed. In Section 5 a simulation of intersecting black brane solutions is considered and an analogue of $M2 - M5$ dyonic solution is presented.

2 The model

In this paper we deal with a family of spherically symmetric solutions to Einstein equations with an anisotropic matter source

$$R^M_N - \frac{1}{2} \delta^M_N R = k^2 T^M_N,$$

(2.1)
defined on the manifold
\[ M = \mathbb{R} \times (M_0 = S^{d_0}) \times (M_1 = \mathbb{R}) \times \ldots \times M_n, \] (2.2)
with the block-diagonal metrics
\[ ds^2 = e^{2\gamma(u)} du^2 + \sum_{i=0}^{n} e^{2\beta_i(u)} h_{m_i n_i} dy^m dy^n. \] (2.3)

Here \( \mathbb{R} \subseteq \mathbb{R} \) is an open interval. The manifold \( M_i \) with the metric \( h^{[i]} \), \( i = 1, 2, \ldots, n \), is a Ricci-flat space of dimension \( d_i \):
\[ R_{m_i n_i}[h^{[i]}] = 0, \] (2.4)
and \( h^{[0]} \) is the standard metric on the unit sphere \( S^{d_0} \), so that
\[ R_{m_0 n_0}[h^{[0]}] = (d_0 - 1) h_{m_0 n_0}; \] (2.5)
\( u \) is a radial variable, \( \kappa^2 \) is the gravitational constant, \( d_1 = 1 \) and \( h^{[1]} = -dt \otimes dt \).

The energy-momentum tensor is adopted in the following form for each component of the fluid:
\[ (T^M_N) = \text{diag}(-\hat{\rho}, \hat{p}^{0}, \hat{p}^{1}, \ldots, \hat{p}^{n}), \] (2.6)
where \( \hat{\rho} \) and \( \hat{p}_i \) are the effective density and pressures respectively, depending on the radial variable \( u \).
We assume that the following "conservation laws"
\[ \nabla_M T^{(s)}_N^M = 0 \] (2.7)
are valid for all components.
We also impose the following equations of state
\[ \hat{p}_i^s = \left( 1 - \frac{2U^s_i}{d_i} \right) \hat{\rho}, \] (2.8)
where \( U^s_i \) are constants, \( i = 0, 1, \ldots, n \).
The physical density and pressures are related to the effective ones (with "hats") by the formulae
\[ \rho = -\hat{\rho}, \quad p = -\hat{p}, \quad p_i = \hat{p}_i \quad (i \neq 1). \] (2.9)
In what follows we put \( \kappa = 1 \) for simplicity.

### 3 Spherically symmetric solutions with horizon

We will make the following assumptions:
1°. \( U^0_0 = 0 \Leftrightarrow \hat{p}_0 = \hat{\rho}, \)
2°. \( U^1_1 = 1 \Leftrightarrow \hat{p}_1 = -\hat{\rho}, \)
3°. \( (U^s, U^s) = U^s_j G^{ij} U^j_s > 0, \)
4°. \( 2(U^s, U^l)/(U^l, U^l) = A_{st}, \)
where \( A = (A_{st}) \) is non-degenerate matrix,
\[ G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D}, \] (3.1)
are components of the matrix inverse to the matrix of the minisuperspace metric [22]
\[ (G_{ij}) = (d_i \delta_{ij} - d_i d_j), \] (3.2)
i, j = 0, 1, ..., n and \( D = 1 + \sum_{i=0}^{n} d_i \) is the total dimension.
The conditions 1° and 2° in brane terms mean that brane "lives" in the time manifold $M_1$ and does not "live" in $M_0$. Due to assumptions 1° and 2° and the equations of state (2.8), the energy-momentum tensor (2.6) reads as follows:

$$(T^{(c)}_{MN}) = \text{diag}(-\rho^s, \rho^s \delta_{k_0}, \rho^s, p^0 \delta_{k_2}, \ldots, p^0 \delta_{k_n}).$$

(3.4)

Under the conditions (2.8) and (3.1) we have obtained the following black-hole solutions to the Einstein equations (2.1):

$$ds^2 = J_0\left(\frac{dr^2}{1 - 2\mu/R^d} + R^2 d\Omega_{d_0}^2\right) - J_1 \left(1 - \frac{2\mu}{R^d}\right) dt^2 + \sum_{i=2}^{n} J_i h_{m,n_i}^{[i]} dy^{m_i} dy^{n_i},$$

(3.5)

$$\rho(s) = -\frac{A_s}{\int_0^{R_0} \prod_{l=1}^{m} H_l^{-A_{s,l}},}$$

(3.6)

which may be derived by analogy with the black brane solutions [4, 8]. Here $d = d_0 - 1$,

$$d\Omega_{d_0}^2 = h_{m,n_0}^{[0]} dy^{m_0} dy^{n_0}$$

(3.7)

is the $d_0$-dimensional spherical element (corresponding to the metric on $S^{d_0}$),

$$J_i = \prod_{s=1}^{m} H_s^{-2h_{s,U_{si}}},$$

(3.8)

$i = 0, 1, \ldots, n$, $\mu > 0$ is integration constant and

$$U^{si} = G^{ij} U_j^s = \frac{U_j^s}{d_i} + \frac{1}{2 - D} \sum_{j=0}^{n} U_j^s,$$

(3.9)

$$h_s = K_s^{-1}, \quad K_s = (U^s, U^s).$$

(3.10)

It follows from 1° and 3.9 that

$$U^{s0} = \frac{1}{2 - D} \sum_{j=1}^{n} U_j^s.$$  

(3.11)

Functions $H_s > 0$ obey the equations

$$R^{d_0} \frac{d}{dR} \left[\left(1 - \frac{2\mu}{R^d}\right) \frac{R^{d_0} dH_s}{H_s} \frac{dH_s}{dR}\right] = B_s \prod_{l=1}^{m} H_l^{-A_{s,l}},$$

(3.12)

with $B_s = 2K_s A_s$ and the boundary conditions imposed:

$$H_s \rightarrow H_{s0} \neq 0, \quad \text{for} \quad R^d \rightarrow 2\mu,$$

(3.13)

and

$$H_s(R = +\infty) = 1,$$

(3.14)

$s = 1, \ldots, m$, i.e. the metric (3.5) has a regular horizon at $R^d = 2\mu$ and has an asymptotically flat $(2 + d_0)$-dimensional section.

Due to to (3.1) and (3.9) the metric reads

$$ds^2 = J_0\left[\frac{dr^2}{1 - 2\mu/R^d} + R^2 d\Omega_{d_0}^2 - \left(\prod_{s=1}^{m} H_s^{-2h_s}\right) \left(1 - \frac{2\mu}{R^d}\right) dt^2 + \sum_{i=2}^{n} Y_i h_{m,n_i}^{[i]} dy^{m_i} dy^{n_i}\right],$$

(3.15)

where

$$Y_i = \prod_{s=1}^{m} H_s^{-2h_{s,U_{si}}/d_i}. $$

(3.16)

The solution (3.5), (3.16) may be verified just by a straightforward substitution into equations of motion. A detailed derivation of this solution will be given in a separate paper [26]. A special orthogonal case when $(U^s, U^s) = 0$, for $s \neq 1$, was considered earlier in [13] (for $m = 1$ see [12]) More general solutions in orthogonal case (with more general condition instead of 2°) were obtained in [15] (for $m = 1$ see [14]).
4 Polynomial structure of $H_s$ for Lie algebras

Now we deal with solutions to second order non-linear differential equations (3.12) that may be rewritten as follows

$$\frac{d}{dz} \left( \frac{1-2\mu z}{H_s} \frac{d}{dz} H_s \right) = \bar{B}_s \prod_{i=1}^{n_2} H_{l}^{-A_{s_i}},$$  \quad (4.1)

where $H_s(z) > 0$, $z = R^{-d} \in (0, (2\mu)^{-1})$ ($\mu > 0$) and $\bar{B}_s = B_s/d^2 \neq 0$. Eqs. (3.13) and (3.14) read

$$H_s((2\mu)^{-1} - 0) = H_s(0) \in (0, +\infty),$$  \quad (4.2)

$$H_s(+0) = 1,$$  \quad (4.3)

$s = 1, \ldots, m$.

It was conjectured in [6] that equations (4.1)-(4.3) have polynomial solutions when $(A_{s_i})$ is a Cartan matrix for some semisimple finite-dimensional Lie algebra $G$ of rank $m$. In this case we get

$$H_s(z) = 1 + \sum_{k=1}^{n_s} P_s^{(k)} z^k,$$  \quad (4.4)

where $P_s^{(k)}$ are constants, $k = 1, \ldots, n_s$; $P_s^{(n_s)} \neq 0$, and

$$n_s = b_s \equiv 2 \sum_{l=1}^{m} A_{s_l}$$  \quad (4.5)

$s = 1, \ldots, m$, are the components of twice the dual Weyl vector in the basis of simple co-roots [24]. Here $(A_{s_l}) = (A_{s})^{-1}$.

This conjecture was verified for $A_m$ and $C_{m+1}$ series of Lie algebras in [7, 8]. In extremal case ($\mu = +0$) an analogue of this conjecture was suggested (implicitly) in [25].

A$_1 \oplus \ldots \oplus$ A$_1$ - case.

The simplest example occurs in orthogonal case: $(U^s, U^l) = 0$, for $s \neq l$ [12] (see also [16, 17, 18] and refs. therein). In this case $(A_{s_l}) = \text{diag}(2, \ldots, 2)$ is a Cartan matrix for semisimple Lie algebra $A_1 \oplus \ldots \oplus A_1$ and

$$H_s(z) = 1 + P_s z,$$  \quad (4.6)

with $P_s \neq 0$, satisfying

$$P_s(P_s + 2\mu) = -\bar{B}_s = -2K_s A_s/d^2,$$  \quad (4.7)

$s = 1, \ldots, m$. When all $A_s < 0$ (or, equivalently, $\rho^s > 0$) there exists a unique set of numbers $P_s > 0$ obeying (1.4).

A$_2$ - case.

For the Lie algebra $G$ coinciding with $A_2 = sl(3)$ we get $n_1 = n_2 = 2$ and

$$H_s = 1 + P_s z + P_s^{(2)} z^2,$$  \quad (4.8)

where $P_2 = P_s^{(2)}$ and $P_1^{(2)} \neq 0$ are constants, $s = 1, 2$.

It was found in [13] that for $P_1 + P_2 + 4\mu \neq 0$ (e.g. when all $P_s > 0$) the following relations take place

$$P_s^{(2)} = \frac{P_s P_{s+1} (P_s + 2\mu)}{2(P_1 + P_2 + 4\mu)}, \quad \bar{B}_s = -\frac{P_s (P_s + 2\mu) (P_s + 4\mu)}{P_1 + P_2 + 4\mu},$$  \quad (4.9)

$s = 1, 2$.

Here we denote $s + 1 = 2, 1$ for $s = 1, 2$, respectively.

Other solutions.

At the moment the "master" equations were integrated (using the Maple) in [9, 10] for Lie algebras $C_2$ and $A_3$, respectively.

Special solutions $H_s(z) = (1 + P_s z)^{b_s}$ with $b_s$ from (4.5) appeared earlier in [3, 4, 5] in a context of so-called block-orthogonal configurations.
5 Examples

5.1 Simulation of intersecting black branes

The solution from the previous section for MCAF allows to simulate the intersecting black brane solutions \[2\] in the model with antisymmetric forms without scalar fields. In this case the parameters \(U_i^a\) and pressures have the following form:

\[
U_i^a = d_i, \quad p_{i}^{(s)} = -\rho^s, \quad i \in I_s; \\
0, \quad \rho^s, \quad i \notin I_s.
\]

(5.1)

Here \(I_s = \{i_1^s, \ldots, i_n^s\} \in \{1, \ldots n\}\) is the index set \[1\] corresponding to brane submanifold \(M_{i_1^s} \times \ldots \times M_{i_n^s}\). The relation \(4^o \ (3.11)\) leads us to the following dimensions of intersections of brane submanifolds ("worldvolumes") \[2\] \[11\]:

\[
d(I_s \cap I_l) = \frac{d(I_s)d(I_l)}{D - 2} + \frac{1}{2} K_l A_{st},
\]

(5.2)

\(s \neq l; \ s, l = 1, \ldots, m\). Here \(d(I_s)\) and \(d(I_l)\) are dimensions of brane world-volumes.

5.2 \(M_2 - M_5\)-analogue for Lie algebra \(A_2\)

In \[13\] examples of simulation by MCAF of \(M2 \cap M5, \ M2 \cap M2, \ M5 \cap M5\) black brane solutions in \(D = 11\) supergravity, with the standard (orthogonal) intersection rules were considered.

Now we consider a solution with 2-component anisotropic fluid that simulates \(M_2 - M_5\) dyonic configuration in \(D = 11\) supergravity \[6\], corresponding to Lie algebra \(A_2\).

The solution is defined on the manifold

\[
M = (2\mu, +\infty) \times (M_0 = S^2) \times (M_1 = \mathbb{R}) \times M_2 \times M_3
\]

(5.3)

where \(\dim M_2 = 2\) and \(\dim M_3 = 5\). The \(U^s\)-vectors corresponding to fluid components obey \(5.1\) with \(I_1 = \{1, 2\}\) and \(I_2 = \{1, 3\}\).

The solution reads as following

\[
g = H_1^{1/3} H_2^{2/3} \left\{ \frac{dR \otimes dR}{R^2} + R^2 d\Omega_2^2 \left( 1 - \frac{2\mu}{R} \right) dt \otimes dt + H_1^{-1} h[^2] + H_2^{-1} h[^3] \right\}.
\]

(5.4)

\[
- H_1^{-1} H_2^{-1} \left( 1 - \frac{2\mu}{R} \right) dt \otimes dt + H_1^{-1} h[^2] + H_2^{-1} h[^3],
\]

(5.5)

\[
\rho^{(1)} = -A_1 J_0 H_1^3 H_2^{-2} H_2, \quad \rho^{(2)} = -A_2 J_0 H_1 H_2^{-2},
\]

(5.6)

where \(J_0 = H_1^{1/3} H_2^{2/3}; \ h[^2]\) and \(h[^3]\) are Ricci-flat metrics of Euclidean signatures, \(\mu > 0\) and \(H_s\) are defined by \[13\], where \(z = R^{-1}\) and parameters \(P_s, P_s^{(2)}, B_s = B_s = 4A_s (s = 1, 2)\) obey \[19\].

This solution simulates \(A_2\)-dyon from \[6\] consisting of electric \(M2\)-brane with worldvolume isomorphic to \((M_1 = \mathbb{R}) \times M_2\) and magnetic \(M5\)-brane with worldvolume isomorphic to \((M_1 = \mathbb{R}) \times M_3\). The branes are intersecting on the time manifold \(M_1 = \mathbb{R}\). Here \(K_s = (U^s, U^s) = 2\), for all \(s = 1, 2\).

For \(A_2\)-dyon from \[6\] we had \(B_s = B_s = -2Q_s^2\), where \(Q_s\) is the charge density parameter of \(s\)-th brane. Thus, for fixed \(Q_s\) the fluid parameters should obey the relations \(A_s = -\frac{1}{2} Q_s^2\) and hence \(A_s\) are negative.

5.3 The Hawking temperature

The Hawking temperature of the black hole \(5.5\) (see also \(3.15\)) may be calculated using the relation from \[23\]. It has the following form:

\[
T_H = \frac{d}{4\pi(2\mu)^{1/d}} \prod_{s=1}^{m} H_{s0}^{-h_s},
\]

(5.7)

where \(H_{s0}, \ s = 1, 2\), are defined in \[13\].

For the dyonic solution from the previous subsection we get

\[
T_H = \frac{1}{8\pi\mu}(H_{10} H_{20})^{-1/2},
\]

(5.8)

where \(T_H\) is a function of fluid parameters \(A_s < 0, \ s = 1, 2\).
6 Conclusions

Here we have presented a family of spherically symmetric solutions with horizon in the model with multi-component anisotropic fluid with the equations of state \( \frac{2}{3} \) and the conditions 3.1 imposed. The metric of any solution contains  \((n-1)\) Ricci-flat “internal” space metrics.

As in [6, 7, 8] the solutions are defined up to solutions of non-linear differential equations (equivalent to Toda-like ones) with certain boundary conditions imposed. These solutions may have a polynomial structure when the matrix  \( A \) from 3.1 is coinciding with the Cartan matrix of some semi-simple finite-dimensional Lie algebra.

For certain equations of state (with  \( p_i = \pm \rho \)) the metric of the solution may coincide with the metric of intersecting black branes (in a model with antisymmetric forms without dilatons). Here we have considered an example of simulating of  \( M^2 - M^5 \) black brane (dyonic) solution in  \( D = 11 \) supergravity with intersection rules corresponding to Lie algebra  \( A_2 \).

An open problem is to generalize this formalism to the case when scalar fields are added into consideration. In a separate paper we also plan to calculate the post-Newtonian parameters  \( \beta \) and  \( \gamma \) corresponding to the 4-dimensional section of the metric (for  \( d_0 = 2 \) ) and analyze the thermodynamic properties of the black-brane-like solutions in the model with MCAF.

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V. D. Ivashchuk

Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya ul., Moscow 119361, Russia
Institute of Gravitation and Cosmology, Peoples’ Friendship University of Russia, 6 Miklukho-Maklaya ul., Moscow 117198, Russia

Abstract

A family of spherically symmetric solutions with horizon in the model with \( m \)-component anisotropic fluid is presented. The metrics are defined on a manifold that contains a product of \( n - 1 \) Ricci-flat “internal” spaces. The equation of state for any \( s \)-th component is defined by a vector \( U^s \) belonging to \( \mathbb{R}^{n+1} \). The solutions are governed by moduli functions \( H_s \) obeying non-linear differential equations with certain boundary conditions imposed. A simulation of black brane solutions in the model with antisymmetric forms is considered. An example of solution imitating \( M_2 - M_5 \) configuration (in \( D = 11 \) supergravity) corresponding to Lie algebra \( A_2 \) is presented.

1 Introduction

In this paper we continue our investigations of spherically-symmetric solutions with horizon (e.g., black brane ones) defined on product manifolds containing several Ricci-flat factor-spaces (with diverse signatures and dimensions). These solutions appear either in models with antisymmetric forms and scalar fields \([1]-[11]\) or in models with (multi-component) anisotropic fluid \([12]-[15]\). For black brane solutions with 1-dimensional factor-spaces (of Euclidean signatures) see \([16, 17, 18]\) and references therein.

These and more general brane cosmological and spherically symmetric solutions were obtained by reduction of the field equations to the Lagrange equations corresponding to Toda-like systems \([2, 19]\). An analogous reduction for models with multicomponent anisotropic fluids was performed earlier in \([20, 21]\). For cosmological-type models with antisymmetric forms without scalar fields any brane is equivalent to an anisotropic fluid with the equations of state:

\[
\hat{p}_i = -\hat{\rho} \quad \text{or} \quad \hat{p}_i = \hat{\rho},
\]

when the manifold \( M_i \) belongs or does not belong to the brane worldvolume, respectively (here \( \hat{p}_i \) is the effective pressure in \( M_i \) and \( \hat{\rho} \) is the effective density).

In this paper we present spherically-symmetric solutions with horizon (e.g the analogues of intersecting black brane solutions) in a model with multi-component anisotropic fluid (MCAF), when certain relations on fluid parameters are imposed. The solutions are governed by a set of moduli functions \( H_s \) obeying non-linear differential master equations with certain boundary conditions imposed. These master equations are equivalent to Toda-like equations and depend upon the non-degenerate \( (m \times m) \) matrix \( A \). It was conjectured earlier that the functions \( H_s \) should be polynomials when \( A \) is a Cartan matrix for some semisimple finite-dimensional Lie algebra (of rank \( m \)) \([6]\). This conjecture was verified for Lie algebras: \( A_m, C_{m+1}, m \geq 1 \) \([7, 8]\). A special case of black hole solutions with MCAF corresponding to semisimple Lie algebra \( A_1 \oplus \ldots \oplus A_1 \) was considered earlier in \([13]\) (for \( m = 1 \) see \([12]\)).

The paper is organized as follows. In Section 2 the model is formulated. In Section 3 spherically-symmetric MCAF solutions with horizon corresponding to black brane type solutions, are presented. In Section 4 a polynomial structure of moduli functions \( H_s \) for semisimple finite-dimensional Lie algebras is discussed. In Section 5 a simulation of intersecting black brane solutions is considered and an analogue of \( M2 - M5 \) dyonic solution is presented.

2 The model

In this paper we deal with a family of spherically symmetric solutions to Einstein equations with an anisotropic matter source

\[
R^M_N - \frac{1}{2} s^M_N R = k^2 T^M_N,
\]

\(^1\)e-mail: ivashchuk@mail.ru
defined on the manifold

\[ M = \mathbb{R} \times (M_0 = S^{d_0}) \times (M_1 = \mathbb{R}) \times \ldots \times M_n, \]

with the block-diagonal metrics

\[ ds^2 = e^{2\gamma(u)} du^2 + \sum_{i=0}^{n} e^{2\beta^i(u)} h^{[i]}_{m_i,n_i} dy^m_i dy^n_i. \]  

(2.2)

Here \( \mathbb{R}^* \subseteq \mathbb{R} \) is an open interval. The manifold \( M_i \) with the metric \( h^{[i]} \), \( i = 1, 2, \ldots, n \), is a Ricci-flat space of dimension \( d_i \):

\[ R_{m_i,n_i}[h^{[i]}] = 0, \]

(2.4)

and \( h^{[0]} \) is the standard metric on the unit sphere \( S^{d_0} \), so that

\[ R_{m_0,n_0}[h^{[0]}] = (d_0 - 1) h^{[0]}_{m_0,n_0}. \]

(2.5)

\( u \) is a radial variable, \( \kappa^2 \) is the gravitational constant, \( d_1 = 1 \) and \( h^{[1]} = -dt \otimes dt \).

The energy-momentum tensor is adopted in the following form for each component of the fluid:

\[ (T^{sM})_{MN} = \text{diag}(-\hat{\rho}^s, \hat{p}^s_0 \delta^{m_0}_{k_0}, \hat{p}^s_1 \delta^{m_1}_{k_1}, \ldots, \hat{p}^s_n \delta^{m_n}_{k_n}), \]

(2.6)

where \( \hat{\rho}^s \) and \( \hat{p}^s_i \) are the effective density and pressures respectively, depending on the radial variable \( u \).

We assume that the following “conservation laws”

\[ \nabla_M T^{(s)M} = 0 \]

are valid for all components.

We also impose the following equations of state

\[ \hat{p}^s_i = \left(1 - \frac{2U^s_i}{d_i}\right) \hat{\rho}^s, \]

(2.8)

where \( U^s_i \) are constants, \( i = 0, 1, \ldots, n \).

The physical density and pressures are related to the effective ones (with “hats”) by the formulae

\[ \rho^s = -\hat{\rho}^s, \quad p^s_u = -\hat{\rho}^s, \quad p^s_i = \hat{p}^s_i (i \neq 1). \]

(2.9)

In what follows we put \( \kappa = 1 \) for simplicity.

3 Spherically symmetric solutions with horizon

We will make the following assumptions:

1°. \( U^s_0 = 0 \iff \hat{p}^s_0 = \hat{\rho}^s, \)

2°. \( U^s_1 = 1 \iff \hat{p}^s_1 = -\hat{\rho}^s, \)

3°. \( (U^s, U^s) = U^s_i G^{ij} U^s_j > 0, \)

4°. \( 2(U^s, U^s) / (U^s, U^s) = A_{st}, \)

where \( A = (A_{st}) \) is non-degenerate matrix,

\[ G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D}, \]

(3.1)

are components of the matrix inverse to the matrix of the minisuperspace metric [22]

\[ (G_{ij}) = (d_i \delta_{ij} - d_i d_j), \]

(3.3)

\( i, j = 0, 1, \ldots, n \) and \( D = 1 + \sum_{i=0}^{n} d_i \) is the total dimension.
The conditions 1° and 2° in brane terms mean that brane “lives” in the time manifold $M_1$ and does not “live” in $M_0$. Due to assumptions 1° and 2° and the equations of state (2.8), the energy-momentum tensor (2.6) reads as follows:

$$ \langle T^{\mu \nu} \rangle = \text{diag}(-\rho^s, \rho^s \delta^m_{k_0}, -\rho^s, \rho^s \delta^m_{k_1}, \ldots, \rho^s \delta^m_{k_n}). $$

(3.4)

Under the conditions (2.8) and (3.1) we have obtained the following black-brane-like solutions to the Hilbert-Einstein equations (2.1):

$$ ds^2 = J_0 \left( \frac{dR^2}{1 - 2\mu/R^2} + R^2 dY_{d0}^2 \right) - J_1 \left( 1 - \frac{2\mu}{R^2} \right) dt^2 + \sum_{i=2}^n J_i h^{[i]}_{m_1n_1} dy^{m_1} dy^{n_1}, $$

(3.5)

$$ \rho^s = \frac{A_s}{J_0 R^{2d_0} \prod_{i=1}^m H_i^{-A_i}}, $$

(3.6)

which may be derived by analogy with the black brane solutions [7, 8]. Here $d = d_0 - 1$,

$$ dY_{d0}^2 = h_{0n_0}^{[0]} dy^{m_0} dy^{n_0} $$

(3.7)

is the $d_0$-dimensional spherical element (corresponding to the metric on $S^{d_0}$),

$$ J_i = \prod_{s=1}^m H_s^{-2h_s U^{s_i}}, $$

(3.8)

$$ i = 0, 1, \ldots, n, \mu > 0 $$

is integration constant and

$$ U^{s_i} = G^{ij} U^s_j = \frac{U^s_j}{d_j} + \frac{1}{2 - D} \sum_{j=0}^n U^s_j, $$

(3.9)

$$ h_s = K_s^{-1}, \quad K_s = (U^s, U^s). $$

(3.10)

It follows from 1° and (3.9) that

$$ U^{s0} = \frac{1}{2 - D} \sum_{j=0}^n U^s_j. $$

(3.11)

Functions $H_s > 0$ obey the equations

$$ R^{d_0} \frac{d}{dR} \left[ \left( 1 - \frac{2\mu}{R^2} \right) R^{d_0} \frac{dH_s}{H_s} \frac{dR}{dR} \right] = B_s \prod_{i=1}^m H_i^{-A_i}, $$

(3.12)

with $B_s = 2K_s A_s$ and the boundary conditions imposed:

$$ H_s \to H_{s0} \neq 0, \quad \text{for} \quad R^d \to 2\mu, $$

(3.13)

and

$$ H_s(R = +\infty) = 1, $$

(3.14)

$s = 1, \ldots, m$.

Here we also impose the following (additional) condition on the solutions

$$ H_s(R) > 0 \quad \text{is smooth in} \quad (R, +\infty), $$

(3.15)

$s = 1, \ldots, m$, where $R_c = (2\mu)^{1/\epsilon} e^{-\epsilon}$, $\epsilon > 0$. Then the metric (3.5) has a regular horizon at $R^d = 2\mu$ and has an asymptotically flat $(2 + d_0)$-dimensional section.

Due to (3.1) and (3.9) the metric reads

$$ ds^2 = J_0 \left[ \frac{dR^2}{1 - 2\mu/R^2} + R^2 dY_{d0}^2 - \left( \prod_{s=1}^m H_s^{-2h_s} \right) \left( 1 - \frac{2\mu}{R^2} \right) dt^2 + \sum_{i=2}^n Y_i h^{[i]}_{m_1n_1} dy^{m_1} dy^{n_1} \right], $$

(3.16)

where

$$ Y_i = \prod_{s=1}^m H_s^{-2h_s U_{s_i}/d_i}. $$

(3.17)

The solution (3.10), (3.16) may be verified just by a straightforward substitution into equations of motion. A detailed derivation of this solution will be given in a separate paper [27]. A special orthogonal case when $(U^s, U^l) = 0$, for $s \neq l$, was considered earlier in [13] (for $m = 1$ see [12]). More general solutions in orthogonal case (with more general condition instead of 2°) were obtained in [15] (for $m = 1$ see [14]).
4 Polynomial structure of \( H_s \) for Lie algebras

Now we deal with solutions to second order non-linear differential equations \( 3.12 \) that may be rewritten as follows

\[
\frac{d}{dz} \left( \frac{(1-2\mu z)}{H_s} \frac{d}{dz} H_s \right) = \tilde{B}_s \prod_{l=1}^{m} H_l^{-A_{sl}},
\]

where \( H_s(z) > 0, \ z = R^{-d} \in (0, (2\mu)^{-1}) \ (\mu > 0) \) and \( \tilde{B}_s = B_s/d^2 \neq 0 \). Eqs. \( 3.13 \) and \( 3.14 \) read

\[
H_s((2\mu)^{-1} - 0) = H_{s0} \in (0, +\infty),
\]

\[
H_s(+0) = 1,
\]

\( s = 1, ..., m. \)

The condition \( 3.15 \) reads as follows

\[
H_s(z) > 0 \text{ is smooth in } (0, z_c),
\]

\( s = 1, ..., m, \) where \( z_c = (2\mu)^{-1} e^{s^d}, \ \epsilon > 0. \)

It was conjectured in \( 6 \) that equations \( 4.1-4.3 \) have polynomial solutions when \( (A_{s'}) \) is a Cartan matrix for some semisimple finite-dimensional Lie algebra \( G \) of rank \( m. \) In this case we get

\[
H_s(z) = 1 + \sum_{k=1}^{n_s} p_s^{(k)} z^k,
\]

where \( p_s^{(k)} \) are constants, \( k = 1, \ldots, n_s; \ p_s^{(n_s)} \neq 0, \) and

\[
n_s = b_s \equiv 2 \sum_{l=1}^{m} A^{sl}
\]

\( s = 1, ..., m, \) are the components of twice the dual Weyl vector in the basis of simple co-roots \( 24. \) Here \( (A^{sl}) = (A^{-sl}_{sl}). \)

This conjecture was verified for \( A_m \) and \( C_{m+1} \) series of Lie algebras in \( 7, 8. \) In the extremal case \( (\mu = +0) \) an analogue of this conjecture was suggested (implicitly) in \( 25. \)

**Remark.** We note that the substitution of \( 2.6, 2.8, 3.5, 3.6 \) into Hilbert-Einstein equations \( 2.1 \) gives an extra equation

\[
E_T = \frac{d^2}{dz} - \frac{1}{4} \sum_{s,l=1}^{m} h_s A_{sl} \left[ F^2 \frac{d}{dz} \ln H_s + \mu b_s \right] \left[ F^2 \frac{d}{dz} \ln H_l + \mu b_l \right] + \sum_{s=1}^{m} A_s F \prod_{l=1}^{m} H_l^{-A_{sl}} = \frac{1}{2} \sum_{s=1}^{m} h_s b_s (\mu d)^2,
\]

where \( F = 1 - 2\mu z. \) \( E_T \) is an integral of motion for the set of equations \( 4.1. \) The constraint \( 4.7 \) is satisfied identically due to \( 4.2, 4.3 \) (one can check this by putting \( 2\mu z = 1 \)).

**A_1 \oplus \ldots \oplus A_1 - case.**

The simplest example occurs in the orthogonal case : \( (U^s, U^l) = 0, \) for \( s \neq l \) \( 12 \) (see also \( 16, 17, 18 \) and refs. therein). In this case \( (A_{sl}) = \text{diag}(2, \ldots, 2) \) is a Cartan matrix for the semisimple Lie algebra \( A_1 \oplus \ldots \oplus A_1 \) and

\[
H_s(z) = 1 + P_s z,
\]

with \( P_s \neq 0, \) satisfying

\[
P_s(P_s + 2\mu) = -\tilde{B}_s = -2K_s A_s/d^2,
\]

\( s = 1, ..., m. \) When all \( A_s < 0 \) (or, equivalently, \( \rho^s > 0 \) ) there exists a unique set of numbers \( P_s > 0 \) obeying \( 4.9. \)

**A_2 - case.**

For the Lie algebra \( G \) coinciding with \( A_2 = sl(3) \) we get \( n_1 = n_2 = 2 \) and

\[
H_s = 1 + P_s z + P_s^{(2)} z^2,
\]

where \( P_s = P_s^{(1)} \) and \( P_s^{(2)} \neq 0 \) are constants, \( s = 1, 2. \)
It was found in [6] that for $P_1 + P_2 + 4\mu \neq 0$ (e.g. when all $P_2 > 0$) the following relations take place

$$P_s^{(2)} = \frac{P_s P_{s+1} (P_s + 2\mu)}{2(P_1 + P_2 + 4\mu)}, \quad \dot{B}_s = -\frac{P_s (P_s + 2\mu) (P_s + 4\mu)}{P_1 + P_2 + 4\mu},$$

(4.11)

$s = 1, 2$.

Here we denote $s + 1 = 2, 1$ for $s = 1, 2$, respectively.

**Other solutions.**

At the moment the “master” equations were integrated (using Maple) in [9, 10] for Lie algebras $C_2$ and $A_3$, respectively. (For $D_4$-polynomials in the extremal case $\mu \to +0$ see [25].)

Special solutions $H_s(z) = (1 + P_2 z^{b_s}$) with $b_s$ from (4.6) appeared earlier in [3, 4, 5] in a context of so-called block-orthogonal configurations.

**Extremal case.** For $\mu \to +0$ the conditions (4.2), (4.4) should be omitted but we should impose the relation

$$E_T = \frac{d^2}{dz^2} \sum_{s, \bar{s}} b_s A_{\bar{s}l} \frac{d}{dz} \ln H_s \left( \frac{d}{dz} \ln H_l + \sum_{s = 1}^{m} A_{s} F \prod_{l = 1}^{m} \frac{H_{s}^{-A_{\bar{s}l}}}{P_{s}} = 0, \right.$$

(4.12)

following from (4.14). The functions $H_s(z) > 0$ obeying (4.13) (with $\mu \to +0$) are smooth on $(0, +\infty)$. For certain relations on $U^s$-vectors imposed the solution under consideration has a horizon for $R \to +0 (z \to +\infty)$, e.g. it may describe analogues of extremal black brane solutions [11].

When the boundary condition (4.3) is omitted we get a special solution with

$$H_s(z) = C_s z^{b_s},$$

(4.13)

and

$$C_s = \prod_{l = 1}^{m} (-b_l/\dot{B}_l)^{-A_{\bar{s}l}},$$

(4.14)

where $b_s/\dot{B}_s < 0$, $s = 1, \ldots, m$. The metric (3.5) with $H_s$ from (4.13) has no an asymptotically flat $(2 + d_0)$-dimensional section.

It should be noted that the solutions obeying $H_s(+0) = 1$ and $b_s/\dot{B}_s < 0$ have an asymptotical behaviour $H_s(z) \sim C_s z^{b_s}$ for $z \to +\infty$ (e.g. in the near-horizon limit $R \to +0$ of extremal black-brane-type solutions).

## 5 Examples

### 5.1 Analogues of intersecting black brane solutions

The solution from the previous section for MCAF allows one to simulate the intersecting black brane solutions [11] in the model with antisymmetric forms without scalar fields. In this case the parameters $U^s_i$ and pressures have the following form:

$$U^s_i = d_i, \quad p^s_i = -\rho^s, \quad i \in I_s;$$

$$0, \quad \rho^s, \quad i \notin I_s.$$

(5.1)

Here $I_s = \{i^s_1, \ldots, i^s_k\} \subset \{1, \ldots, n\}$ is the index set [11] corresponding to brane submanifold $M_{i^s_1} \times \ldots \times M_{i^s_k}$.

The relation 4° (3.4) leads us to the following dimensions of intersections of brane submanifolds (“worldvolumes”) [2, 11]:

$$d(I_s \cap I_l) = \frac{d(I_s) d(I_l)}{D - 2} + \frac{1}{2} K_{1} A_{st},$$

(5.2)

$s \neq l; \; s, l = 1, \ldots, m$. Here $d(I_s)$ and $d(I_l)$ are dimensions of brane worldvolumes.
5.2 $M_2 - M_5$-analogue for Lie algebra $A_2$

In examples of MCAF-analogues of $M2 \cap M5$, $M2 \cap M2$, $M5 \cap M5$ black brane solutions in $D = 11$ supergravity, with the standard (orthogonal) intersection rules were considered.

Now we consider a solution with 2-component anisotropic fluid that simulates $M_2 - M_5$ dyonic configuration in $D = 11$ supergravity [4], corresponding to Lie algebra $A_2$.

The solution is defined on the manifold

$$M = (2\mu, +\infty) \times (M_0 = S^2) \times (M_1 = \mathbb{R}) \times M_2 \times M_3,$$

(5.3)

where $\text{dim} M_2 = 2$ and $\text{dim} M_3 = 5$. The $U^s$-vectors corresponding to fluid components obey (5.1) with $I_1 = \{1, 2\}$ and $I_2 = \{1, 3\}$.

The solution reads as following

$$g = H^{1/3}H_2^{2/3} \left\{ \frac{dR \otimes dR}{1 - 2\mu/R} + R^2 h[S^2] - H_1^{-1}H_2^{-1} \left( 1 - \frac{2\mu}{R} \right) dt \otimes dt + H_1^{-1}h^{[2]} + H_2^{-1}h^{[3]} \right\},$$

$$\rho^1 = -\frac{A_1}{J_0R^1}H_1^{-2}H_2, \quad \rho^2 = -\frac{A_2}{J_0R^1}H_1H_2^{-2},$$

(5.4) (5.5)

where $J_0 = H_1^{1/3}H_2^{2/3}$; $h[S^2]$ is the canonical metric on 2-dimensional sphere $S^2$, $h^{[2]}$ and $h^{[3]}$ are Ricci-flat metrics of Euclidean signatures defined on the manifolds $M_2$ and $M_3$, respectively; $\mu > 0$ and $H_s$ are defined by (4.10), where $z = R^{-1}$ and parameters $P_s, P_2$, $B_s = B_t = 4A_s$ ($s = 1, 2$) obey (4.11).

This solution simulates $A_2$-dyon from [6] consisting of an electric $M_2$-brane with a worldvolume isomorphic to $(M_1 = \mathbb{R}) \times M_2$ and a magnetic $M_5$-brane with a worldvolume isomorphic to $(M_1 = \mathbb{R}) \times M_3$. The branes are intersecting on the time manifold $M_1 = \mathbb{R}$. Here $K_s = (U^s, U^s) = 2$, for all $s = 1, 2$.

For the $A_2$-dyon from [6] we had $B_s = B_t = -2Q_s^2$, where $Q_s$ is the charge density parameter of $s$-th brane. Thus, for fixed $Q_s$ the fluid parameters should obey the relations $A_s = -\frac{1}{2}Q_s^2$ and hence $A_s$ are negative.

Let us consider the extremal case $\mu \to +0$ of the solution (5.2) with $A_1 < 0$ and $A_2 < 0$. The near-horizon limit ($R \to +0$) gives us an exact solution (see (4.13))

$$g = C_1^{1/3}C_2^{2/3} \left\{ h[AdS^2] + h[S^2] + C_1^{-1}h^{[2]} + C_2^{-1}h^{[3]} \right\},$$

$$\rho^1 = -A_1C_1^{-7/3}C_2^{2/3}, \quad \rho^2 = -A_2C_2^{5/3}C_1^{-2/3},$$

(5.6) (5.7)

where $C_1 = 2[A_1]^{2/3}[A_2]^{1/3}$ and $C_2 = 2[A_2]^{2/3}[A_1]^{1/3}$, $h[AdS^2] = R^{-2}(dR \otimes dR - R^4d\tilde{t} \otimes d\tilde{t})$ is the metric of (the half of) the anti-deSitter space AdS$^2$ (here $\tilde{t} = C_1^{-1/2}C_2^{-1/2} t$). Thus, we have obtained a static configuration defined on (the half of) the product space $AdS^2 \times S^2 \times M_2 \times M_3$. (For the solution with two branes and $C_1 = C_2 = 1$ see [20].)

**Analogy of $M_2$, $M_5$ and D3 Solutions.** Here we outline for completeness the analogues of non-marginal $M2$, $M5$ ($D = 11$) and $D3$ ($D = 10$) black brane solutions. The solutions are defined on the product manifolds $M = (2\mu, +\infty) \times (M_0 = S^{d_0}) \times (M_1 = \mathbb{R}) \times M_2$, where $\text{dim} M_2 = d_2 = 2, 5, 3$ and $d_0 = 7, 4, 5$ for $M_2$, $M_5$, $D3$ branes, respectively. The vector $U = U^1$ has the components: $U_0 = 0$ and $U_i = 1$ for $i > 0$. The solutions read as follows

$$g = H^r \left\{ \frac{dR \otimes dR}{1 - 2\mu R^{-d}} + R^2 h[S^{d_0}] - H_1^{-1} \left( 1 - 2\mu R^{-d} \right) dt \otimes dt + H_1^{-1}h^{[2]} \right\},$$

$$\rho = p^1 = -AH^{-2-r}R^{-2d_0},$$

(5.8) (5.9)

where $h^{[2]}$ is a Ricci-flat metric on $M_2$, $h[S^{d_0}]$ is the canonical metric on $d_0$-dimensional sphere $S^{d_0}$, $H = 1 + PR^{-d}$, $P(P + 2\mu) = -4A/d^2$ ($A < 0$, $P > 0$, $\mu > 0$), $d = d_0 - 1$ and $r = 1/3, 2/3, 1/2$ for $M_2$, $M_5$, $D3$ branes, respectively. In the extremal case $\mu \to +0$ the near-horizon limit gives an exact solution for the flat space ($M_2 = \mathbb{R}^{d_2}$, $h^{[2]}$)

$$g = P^r \left\{ h[S^{d_0}] + \frac{4}{(d - 2)^2} h[AdS^{d_2 + 2}] \right\},$$

$$\rho = -AP^{-2-r}, \quad P^2 = -4A/d^2,$$

(5.10) (5.11)

where $h[AdS^{d_2 + 2}] = du \otimes du + e^{2u}(-dy^0 \otimes dy^0 + \sum_{i=1}^k dy_i \otimes dy_i)$ is the metric of (the part of) the anti-deSitter space $AdS^{d_2 + 2}$. Thus, we are led to a static configurations defined on (the parts of) the product spaces $S^{d_0} \times AdS^{d_2 + 2}$, i.e. $S^7 \times AdS^4$, $S^4 \times AdS^7$ and $S^5 \times AdS^5$ for $M_2$, $M_5$ and $D3$ branes, respectively. It should be pointed out that the solutions with $AdS^k$ factor spaces may be of interest due to possible application in a context of AdS/CFT approach [28] [29] [30] (or its modifications).
5.3 The Hawking temperature

The Hawking temperature of the black hole (3.5) (see also (3.16)) may be calculated using the relation from [23]. It has the following form:

\[ T_H = \frac{d}{4\pi(2\mu)^{1/d}} \prod_{s=1}^{m} H_{s0}^{-h_s}, \]  

(5.12)

where \( H_{s0} \), \( s = 1, 2 \), are defined in (3.13).

For the dyonic solution from the previous subsection we get

\[ T_H = \frac{1}{8\pi\mu} (H_{10}H_{20})^{-1/2}, \]  

(5.13)

where \( T_H \) is a function of fluid parameters \( A_s < 0, s = 1, 2 \).

6 Conclusions

Here we have presented a family of spherically symmetric solutions with horizon in the model with multi-component anisotropic fluid with the equations of state (2.8) and the conditions (3.1) imposed. The metric of any solution contains \((n-1)\) Ricci-flat "internal" space metrics.

As in [6, 7, 8] the solutions are defined up to solutions of non-linear differential equations (equivalent to Toda-like ones) with certain boundary conditions imposed. These solutions may have a polynomial structure when the matrix \( A \) from (3.1) is coinciding with the Cartan matrix of some semi-simple finite-dimensional Lie algebra.

For certain equations of state (with \( p_i = \pm \rho \)) the metric of the solution may coincide with the metric of intersecting black branes (in a model with antisymmetric forms without dilatons). Here we have considered an example of simulating of \( M2 - M5 \) black brane (dyonic) solution in \( D = 11 \) supergravity with intersection rules corresponding to the Lie algebra \( A_2 \). We have also outlined the analogues of non-marginal \( M2, M5 \) and \( D3 \) black brane solutions. In the extremal case \( \mu \rightarrow +0 \) the near-horizon limits of all these solutions were found.

An open problem is to generalize this formalism to the case when scalar fields are added into consideration. In a separate paper we also plan to calculate the post-Newtonian parameters \( \beta \) and \( \gamma \) corresponding to the 4-dimensional section of the metric (for \( d_0 = 2 \)) and analyze the thermodynamic properties of the black-brane-like solutions in the model with MCAF.

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