Microwave properties of Yttrium Vanadate at cryogenic temperatures

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Yttrium Vanadate (YVO₄) is a birefringent crystal material used in optical isolators and circulators with potentials for application in cryogenic microwave devices. As microwave properties of the YVO₄ are not known, we measured the complex permittivity at the frequency of 25 GHz, using the Hakki-Coleman dielectric resonator technique in the temperature range from 13 K to 80 K. The real part of relative permittivity ε, turned out to be similar to that of Sapphire - one of popular dielectric materials, used at microwave frequencies. The measured loss tangent tan δ of the YVO₄ was of the order of 10⁻⁶ at cryogenic temperatures. As Yttrium Vanadate (YVO₄) is easy to synthesize and machine, it may replace the expensive Sapphire in some microwave applications.

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Introduction

Yttrium Vanadate (YVO₄) is a birefringent crystal grown by the Czochralski method and is characterized by good mechanical and physical properties [1-3]. YVO₄ is considered very suitable for optical polarizing components due to its wide transparency range and large birefringence. The YVO₄ crystals are mainly used in optical components such as fiber optical isolators and circulators, beam displaces and other polarizing optical systems. The Neodymium-doped Yttrium Vanadate (Nd:YVO₄) is one of the most promising commercially available diode-pumped solid state laser materials for optical communication. The YVO₄ has better thermal stability and physical and mechanical properties than CaCO₃, more than three times larger birefringence than LiNbO₃, and lower hardness than rutile TiO₂, what greatly reduces the cost of fabrication. Also, the YVO₄ crystal growth and fabrication are easy than similar birefringent crystals.

Even though the YVO₄ is well characterized at optical frequencies, there is hardly any data on microwave properties of this material. In this paper, we present, for the first time, the results of precise measurements of the permittivity and loss tangent of the YVO₄ crystals fabricated by [3] at the frequency of 25 GHz and the cryogenic temperatures from 13 K to 80 K. We have used a superconducting dielectric technique combined with the multi-frequency Transmission Mode Q-Factor (TMFQ) method [4, 5] for data processing to ensure the high accuracy of calculated values of ε and tan δ.

Dielectric resonator measurement method

The dielectric resonator technique can be used for the microwave characterization of dielectric materials [6-10]. The permittivity and loss tangent of the dielectric material under test can be calculated from the unloaded Q-factor and the resonance frequency of the resonator. In our work, the Hakki-Coleman dielectric resonator, containing the Yttrium Vanadate (YVO₄) under test, schematically shown in Fig. 1, was used as the resonating structure. The resonator consisted of the copper cavity with diameter of 9.5 mm and height 3 mm with High Temperature Superconducting (HTS) endplates to increase the sensitivity and reduce the uncertainty in the loss tangent measurements. The YVO₄ sample was machined into the cylindrical shape sample with the aspect ratio (the diameter of height) equal to 1.61, with the height of 3.09 mm and the diameter of 4.99 mm. The sample was oriented and machined in such a way that its z-axis was parallel to the crystal optical axis with accuracy better than 0.5°.

Fig. 1. Hakki-Coleman dielectric resonator.
Measurements were carried out, using the TE_{011} mode of the resonator for which the electric field is perpendicular to z-axis, and hence the component of the complex permittivity of YVO_4 perpendicular to the optical axis was determined. The real part of relative permittivity \( \varepsilon_r \) was determined from measurements of the resonance frequency as the first root of the following transcendental equation [11], using the software SUP12 [12]

\[
k_{\rho_1} J_0(k_{\rho_1}b) F_1(b) + k_{\rho_2} J_1(k_{\rho_2}b) F_1(b) = 0 \tag{1}
\]

The loss tangent (\( \tan \delta \)) of the YVO_4 was computed from the measured \( Q_0 \)-factor of the resonator on the basis of the well known loss equation [11], namely

\[
\tan \delta = \frac{1}{\rho_e} \left[ \frac{1}{Q_0} - \frac{R_{SS}}{A_S} - \frac{R_{SM}}{A_M} \right] \tag{2}
\]

where \( Q_0 \) is the unloaded \( Q \)-factor of the resonant structure with the sample, \( R_{SS} \) and \( R_{SM} \) are the surface resistances of the superconducting and the metallic parts of the cavity respectively, \( A_S \) and \( A_M \) are the geometric factors of the superconducting and the metallic parts of the cavity respectively, \( \rho_e \) is the electric energy filling factor.

The geometric factors \( A_S, A_M \) and \( \rho_e \) to be used in eq. (2), were computed, using the incremental frequency rules [11] and calculated values for the YVO_4 and Sapphire (used for the measurements of \( R_{SS} \) and \( R_{SM} \)) are given in Tab. 1.

| Dielectric | Yttrium Vanadate (YVO_4) | Sapphire |
|------------|--------------------------|----------|
| \( f_{res} \) | 24.4 GHz | 24.65 GHz |
| \( A_S \) | 19593 | 22319 |
| \( A_M \) | 291.6 | 280.6 |
| \( \rho_e \) | 0.97 | 0.97 |

Tab. 1. Calculated geometric factor of the dielectric resonator with Yttrium Vanadate (YVO_4) and Sapphire rods.

**Measurements of microwave properties of Yttrium Vanadate (YVO_4) crystals**

The measurement system, we used for the microwave characterization of the YVO_4, is shown in Fig. 2. The system consisted of the Network Analyzer (HP 8722C), closed cycle refrigerator (APD DE-204), temperature controller (LTC-10) vacuum Dewar, a PC, and the Hakki-Coleman dielectric resonator in the transmission mode.

**Measurements of surface resistances of HTS thin film (R_{SS}) and copper walls (R_{SM}) of Hakki-Coleman cavity**

The surface resistances of the HTS thin films \( R_{SS} \) and the Copper walls \( R_{SM} \), necessary for calculations of \( \tan \delta \) of YVO_4 sample, were measured in the same measurement system with the same dielectric resonator, but with the Sapphire rod instead of YVO_4. To obtain the precise values of surface resistances, we have measured the \( S \)-parameters (\( S_{21}, S_{11} \) and \( S_{22} \)) around the resonance. The measured data sets were processed with the Transmission Mode Q-Factor (TMFQ) technique [4, 5] to obtain the loaded \( Q \)-factor (\( Q_L \)), coupling coefficients and unloaded \( Q \)-factor as mentioned in the Introduction. The TMFQ method accounts for the noise, delay due to the un-calibrated transmission lines and its frequency dependence, and crosstalk in measurement data, and hence provides the accurate values of the loaded \( Q \)-factor (\( Q_L \)) and the coupling coefficients \( \beta_1 \) and \( \beta_2 \). The unloaded \( Q \)-factor was subsequently calculated, using the exact equation

\[
Q_0 = Q_L \left( 1 + \beta_1 + \beta_2 \right) \tag{3}
\]

Assuming the loss tangent of the Sapphire rod as \( 10^{-7} \), the surface resistance of Copper \( R_{SM} \) and superconductor \( R_{SS} \) were calculated, using the eq. (2) [11]. The comprehensive information on the surface resistance \( R_S \) measurements, using the TMFQ technique, is given in reference [5].

**Measurements of real part of relative permittivity \( \varepsilon_r \) and loss tangent \( \tan \delta \) of Yttrium Vanadate (YVO_4) crystal**

The Hakki-Coleman dielectric resonator with the HTS endplates, containing the YVO_4 sample, was cooled from the room temperature down to the temperature of 13 K approximately, and the resonant frequency of 24.4GHz was obtained. The \( S_{21}, S_{11} \) and \( S_{22} \) parameters data sets around the resonance were measured as a function of increasing temperature from 13 K to 81 K, and the \( Q_0 \)-factor and \( f_{res} \) were calculated, using the TMFQ technique and the eq. (3). The real part of
relative permittivity $\varepsilon_r$ of the Yttrium Vanadate ($\text{YVO}_4$) sample was calculated from the measured resonant frequency, using the eq. (1) and taking the thermal expansion phenomenon into the consideration. The values of expansion coefficients ($\alpha$) of the $\text{YVO}_4$ for the cryogenic temperatures were not available, and hence we assumed the thermal expansion coefficients as for another optical material $\text{CaF}_2$ [14]. At the room temperature, the expansion coefficient ($\alpha$) for the $\text{CaF}_2$ is around $17\times10^{-6}/\text{K}$, while it is $8.5\times10^{-6}/\text{K}$ for the $\text{YVO}_4$. Therefore, our assumption of the expansion coefficient ($\alpha$) is most probably higher than the real values.

The measured real part of relative permittivity of $\text{YVO}_4$ as a function of temperature $\varepsilon_r(T)$ at frequency of 24.4 GHz in the temperatures range from 13 K to 80 K is shown in Fig. 3. The “circles” represent the values of the $\varepsilon_r$, computed assuming the thermal expansion coefficient of the $\text{CaF}_2$, and the “squares” represent the permittivity with the thermal expansion neglected. The difference between the values of permittivity with and without thermal expansion considered, is approximately 1 %. The $\varepsilon_r$ exhibited the magnitude of approximately 9.3 and increased with the temperature by approximately 0.1 %, from 9.318 to 9.326.

The Fig. 4 shows the measured temperature dependence of loss tangent of $\text{YVO}_4$ at frequency of 24.4 GHz, calculated using the eq. (2) from the measured unloaded $Q$-factor. The loss tangent exhibits an increase of $125 \%$ in the temperatures range from 13 K to 80 K; the measured $\tan \delta$ of the $\text{YVO}_4$ were $2.265\times10^{-6}$ and $5.1\times10^{-6}$ respectively. Using a linear scaling, the calculated loss tangent of the $\text{YVO}_4$ at the temperature of 13 K and the frequency of 10 GHz is $9.2\times10^{-7}$ only.

Fig. 3. Real part of relative permittivity of $\text{YVO}_4$ as a function of temperature $\varepsilon_r(T)$ at frequency of 24.4 GHz.

Fig. 4. Loss tangent of $\text{YVO}_4$ as a function of temperature $\tan \delta(T)$ at the frequency of 24.4 GHz.

**Conclusion**

The complex permittivity of the Yttrium Vanadate ($\text{YVO}_4$) birefringent crystal has been precisely measured at the frequency of 24.4 GHz at the cryogenic temperatures, using the Hakki-Coleman dielectric resonator with superconducting endplates. Our measurements accounted for the noise, crosstalk and uncompensated cables and adaptors, and were based on the accurate equations for the unloaded $Q$-factor. The $\text{YVO}_4$ was found to exhibit the $\varepsilon_r$ varying from 9.31 to 9.32, comparable to the permittivity of the Sapphire, and the $\tan \delta$ from $2.2\times10^{-6}$ to $5.1\times10^{-6}$ in the temperatures range from 13 K to 80 K. Our measurements have shown that the $\text{YVO}_4$ is a very low loss material at the cryogenic temperatures. Therefore, apart from the optical applications, the Yttrium Vanadate ($\text{YVO}_4$) can be useful in the cryogenic microwave circuits as a replacement for the Sapphire.

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