Theory and Phenomenology of Spacetime Defects

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Abstract

Whether or not space-time is fundamentally discrete is of central importance for the development of the theory of quantum gravity. If the fundamental description of space-time is discrete, typically represented in terms of a graph or network, then the apparent smoothness of geometry on large scales should be imperfect – it should have defects. Here, we review a model for space-time defects and summarize the constraints on the prevalence of these defects that can be derived from observation.

1 Introduction

A theory of quantum gravity is necessary to describe the quantum behavior of space and time and to understand what happens in strong gravitational fields, when curvature reaches the Planckian regime. Finding this missing theory of quantum gravity is one of the big open problems in theoretical physics today, and it concerns the most fundamental ingredients of our existing theories: Space-time and its curvature, the arena in which physics happens.

But general relativity still stands apart from the quantum field theories of the standard model as a classical theory. There exists to date no known way to consistently couple a classical theory to a quantum theory, and neither do we know how to quantize gravity. While several theoretical approaches are being pursued with success, this success has so far been exclusively on the side of mathematical consistency, and the connection of these approaches to reality is still unclear.

The problem how to resolve the tension between quantum field theory and general relativity is more than an aesthetic unease. This tension signals that our understanding of nature is incomplete, but it also offers an opportunity to improve our theories. The missing theory of quantum gravity has the potential to revolutionize our understanding of space, time and matter.

Progress on the theory of quantum gravity however has been slow. The problem has been known since more than 80 years now. Since then we gained a great many insights about the

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nature of the problem but the big breakthrough has left us waiting. Next to the technical difficulties, the reason for the slow progress is lack of experimental guidance. The possibility that quantum gravitational phenomena might be observable has not been paid much attention to till the late 90s, and even now the awareness that this possibility exists is slow to sink into the minds of the community. However, without making contact to observation, no theory of quantum gravity can ever be accepted as a valid description of nature.

In the absence of a fully-fledged theory, this search for observable consequences proceeds by the development of phenomenological models. Such models parameterize properties that the theory of quantum gravity could have with the purpose of allowing to experimentally test or at least constrain the presence of these properties. This in turn guides the development of the theory. General reviews of phenomenological models for quantum gravity can be found in [1, 2]. In this contribution to the AHEP special issue on ‘Experimental Tests of Quantum Gravity and Exotic Quantum Field Theory Effects’ we will discuss a possible phenomenological consequence of quantum gravity that has so far received very little attention – the existence of space-time defects.

In many approaches to quantum gravity – such as causal sets, spinfoams, causal dynamical triangulation, loop quantum gravity, and emergent and induced gravity scenarios based on condensed matter analogies – space-time is fundamentally discrete and the smooth background geometry that we see only emerges as an approximation at low energies and large distances [3]. In this case, one expects that the apparently smooth background geometry is imperfect and has defects, just because perfection would require additional explanation.

In the following, we will review the recently proposed model for space-time defects [4, 5] and summarize the constraints on the prevalence of these defects that can be derived from observation. We will then discuss which steps can be taken to improve the model so that the constraints touch on the interesting parameter range.

2 Space-time Defects

Whether or not space-time is fundamentally discrete is a question of central importance for the development of the theory. But this discreteness typically makes itself noticeable at the Planck length, which is hard if not impossible to access experimentally. Thus, instead of searching for direct evidence for the Planck scale discreteness, we here propose to look for imperfections in this discreteness. Such imperfections in space-time will cause deviations from general relativity, since general relativity rests on the assumption that space-time is a manifold and locally smooth and differentiable. Because general relativity is an extremely well-tested theory even smallest deviations can become noticeable, making space-time defects a promising phenomenological consequence to search for. Looking for space-time defects as evidence for the presence of a discrete geometry is akin to looking for specks of dirt as evidence for the presence of a window.

That such defects should exist is a model-independent expectation for all approaches to quantum gravity in which geometry has a fundamentally discrete structure. But the prevalence, distribution, and properties of the defects will depend on the details of the underlying funda-
mental theory. This way, the phenomenological model can bridge the gap between theory and experiment. In the following we will aim to parameterize the consequences of space-time defects so that contact can be made to the underlying theory if the relevant parameters can be extracted.

In contrast to defects in condensed matter systems, space-time defects are not only localized in space but also in time. They do not have worldlines but are space-time events. Space-time defects can come in two different versions, local defects and nonlocal defects. The general case would be a hybrid of both, but treating these two types separately is helpful to develop the theory. We will first discuss the nonlocal defects.

2.1 Nonlocal Space-time Defects

One of the reasons it is expected that quantum gravity necessitates nonlocality is that the notion of the Planck length as a minimal length implies that it is meaningless to distinguish points below this distance – these points are not local (or not points, depending on your perspective). But besides this, during the last decades it has also become increasingly clear that a resolution of the black hole information loss problem requires some type of nonlocality \[6\]. Meanwhile, a completely different line of investigation has led to the conclusion that requiring the Planck energy to be an observer-independent component of a four-momentum, necessitates to give up absolute locality. Instead one might have to settle on a weaker locality requirement, which has been called ‘relative locality’ \[7\].

While Planck-scale nonlocality has received quite some attention, for example in the well-studied models for quantum field theory with a minimal length \[8\], here, we will focus on a feature whose phenomenology has so far received very little attention: Macroscopic nonlocality, where macroscopic means much larger than the Planck length.

Macroscopic nonlocality can be expected to arise in approaches towards quantum gravity in which space-time is only to good approximation a smooth manifold but fundamentally a network (graph) consisting of nodes and links which can carry additional charges or degrees of freedom. The reason is that the emergence of a manifold from the network will not be perfect, but it will have defects. And since our macroscopic notion of distance only emerges with the space-time and the metric that we define on it, there is no reason why these defects should respect the emergent macroscopic locality.

This has been demonstrated explicitly by Markopoulou and Smolin in \[9\] for the case of spin networks. Their argument can be briefly summarized as follows.

States of the spin-network describe spatial slices of space-time and they change in time by a set of allowed evolution moves, which are local according to the locality of the network. Certain spin network states, called ‘weave states’, match to good precision (up to Planck scale corrections), slowly varying classical spatial metrics. The structure of nodes and links of the network carries information about area and volumes and thus the geometry that the network fundamentally describes. It can then be shown that it is possible to act with a large number of local evolution moves on a state without nonlocal links and by this create a state that contains macroscopically nonlocal links. This is possible without changing the classical state that it
approximates, thus the existence of the classical approximation alone cannot be used to rule out these nonlocal links.

A nonlocal link is, intuitively, a link in the network that does not respect the emergent macroscopic locality. More strictly it can be identified by the number of nodes on the shortest closed loop that it is part of. For the nonlocal link this will be a large number of nodes, while all the local links have a small number of nodes (depending on the valence of the network). See Figure 1 for illustration.

![Figure 1](image)

Figure 1: Schematic picture for nonlocal links on a regular lattice. The lattice represents space-time. The link, which represents a defect in the regularity of the background, is long according to the distance measure of the background. Note that all nodes remain 4-valent. If space-time is fundamentally discrete, similar defects should be all around us.

In the example by Markopoulou and Smolin, the states with nonlocal links are still allowed solutions and thus valid semi-classical space-times, but do not respect the macroscopic locality. Because of simple combinatorics, one finds that there are in fact many more nonlocal states than local states. This means the locality of the state that we live in today is not perfect; nonlocal links should be all around us. This situation was aptly dubbed ‘disordered locality’ in [9].

The argument by Markopoulou and Smolin is an explicit example one can have in mind. But the expectation for the existence of nonlocal defects is more general than that, it arises because perfection requires additional explanations or selection criteria that we do not have.

Now such macroscopic locality might strike one as something to be avoided, since we do not seem to experience it, but this is a question of experimental constraints. To really understand the implications of such nonlocality one first needs to develop a phenomenological model that parameterizes the effects, and then contrast them with data. Such a model was developed in [4]. This model does not start with an underlying discrete structure, but instead deals with the defects in the local structure as deviations from the smooth background geometry of general relativity.

The central assumption for the model [4] is that Lorentz-invariance is preserved on the average since violations of Lorentz-invariance are strongly disfavored by the data. Lorentz-invariance, maybe not so surprisingly, proves to be very restrictive on the type of nonlocality
that is allowed. The distribution of nonlocal defects in this model is assumed to be given by
the only presently known Lorentz-invariant discrete distribution on Minkowski space, which is
defined by the Poisson-process described in [10, 11]. With this distribution, the probability of
finding N points in a space-time volume \( V \) is

\[
P_N(V) = \frac{(\beta V)^N \exp(-\beta V)}{N!},
\]

where \( \beta = L^4 \) is a constant space-time density and \( L \) a parameter of dimension length.

A particle that encounters a nonlocal defect will experience a translation in space-time. The translation vector is parameterized in a probability distribution which besides \( L \) introduces
a parameter of dimension mass, \( \Lambda \), and a parameter of dimension length \( \alpha \). \( \Lambda \) and \( \alpha \) (both
real-valued and positive) quantify the translation, \( y_\nu(\alpha, \Lambda) \) that the particle experiences at the
non-local defect

\[
p_\nu y_\nu' = \alpha \Lambda \quad , \quad y_\nu y_\nu'' = \pm \alpha^2,
\]

where the choice of sign determines whether the translation is timelike or spacelike. The interpretation of \( \Lambda \) is roughly speaking that a particle will be translated a distance of about \( \alpha \) in the
restframe in which its energy is about \( \Lambda \).

The translation that the particle experiences when it encounters the nonlocal defect is then
given by a probability distribution \( P_{NL}(\alpha, \Lambda) \) over the endpoints. By construction, this is all
entirely Lorentz-invariant. The density \( \beta \) together with the distribution \( P_{NL}(\alpha, \Lambda) \) determines
the phenomenology of the model. One can further simplify this situation by approximating the
probability distribution by a Gaussian with mean values \( \langle \alpha \rangle, \langle \Lambda \rangle \) and variances \( \Delta \alpha, \Delta \Lambda \). This
leaves one with 5 parameters. These can be further reduced by assuming that there is only one
new length scale \( L \sim \langle \alpha \rangle \), and that the width of the distributions is comparable to the mean
value \( \langle \alpha \rangle \sim \Delta \alpha \) and \( \langle \Lambda \rangle \sim \Delta \Lambda \). One is then left with two parameters, one length scale and
one mass scale, that can be constrained by experiment quite simply. While this might not be
the most general case, it allows one to get a first impression on what amount of nonlocality is
compatible with observation.

A massless particle that encounters a nonlocal defect will be deviated from the lightcone
and on the average travel either faster or slower than the normal speed of light (depending on
the choice of sign in (2)). It is possible to restrict translations to be timelike and velocities to
be subluminal to avoid causality problems because such a restriction does not violate Lorentz-
invariance. The repeated scattering on nonlocal defects creates a small effective mass for the
photon, that is the mass that a particle of the photon’s energy would have on the average trajec-
tory that contains nonlocal links. This is reflected in the average speed of the particle which,
in the presence of nonlocal defects, can deviate from the speed of light because the translations
that the particle experiences when it encounters a nonlocal defects may be spacelike or timelike
rather than lightlike.

It is important to note that the distribution of translation vectors in this model is not an
independent property of space-time but depends on the wave-function of the incident particle, in
the simplest case its momentum vector, in the general case the average momentum vector and the spatial width of the wave-function (at the moment it encounters the defect). It is this dependence on the incident particle that allows one to construct a normalizable and Lorentz-invariant distribution. While the full Lorentz-group is non-compact, using measurable properties of the incident particle as reference prevents the need to introduce a Lorentz-invariance violating cutoff while still maintaining observer-independence.

The central feature that distinguishes this model from other random-walk like models for propagation in a quantum space-time is that the probability of the particle being affected by the quantum properties, here the space-time defects, depends on the (Lorentz-invariant) worldvolume that is swept out by the particle’s wavefunction. Thus, the larger the position uncertainty of the particle and the longer its propagation time, the more likely the particle is to be affected by the space-time defects. This generically means that particles of long wavelength are better suited to find phenomenological consequences than highly energetic ones, in contrast to the phenomenology that arises for example within deformations or violations of Lorentz-invariance [2, 12].

With the use of this model for nonlocal space-time defects, constraints on the density of defects and the parameters of the model can then be derived from various observables.

For example, we have good evidence that protons of ultra-high energy which give rise to cosmic ray showers originate in active galactic nuclei. The protons have a finite mean-free path when traveling through the cosmic microwave background (CMB) because they can scatter on the CMB photons and produce pions. In the presence of nonlocal defects, the protons’ mean free path can increase because the particles effectively do not travel the full distance. If the mean free path increases substantially, this would be in conflict with observation and can thus be used to derive bounds on the density of defects. Other constraints come from the blurring of interference rings in images of distant quasars, and from the close monitoring of single photons in a cavity. For details and references please refer to [4]. The constraints on nonlocal defects can be visually summarized in Figure 2. Roughly speaking it can be concluded that the density of nonlocal defects has to be less than one in a space-time volume of fm$^4$.

In the cosmological context, a natural scale is given by the length scale associated to the measured value of the cosmological constant, which is about $1/10$ mm. The constraints on $L$ are presently about 10 orders of magnitude below this interesting parameter range. One can expect however that the existing data is actually sensitive to larger values of $L$. It’s just that the model in its present form cannot be used to reliably analyze much of the existing cosmological data because it does not take into background curvature. It is clearly desirable to generalize the model to at least a Friedmann-Robertson-Walker background to be able to analyze cosmological data for evidence of space-time defects.

2.2 Local Space-time Defects

A model for local defects can be developed based on a similar approach as that for the nonlocal defects [5]. Much like the nonlocal defects cause a statistically distributed translation in position space, local defects cause a statistically distributed translation in momentum space. In other
words, the local defect stochastically changes the momentum of the incident particle, thereby violating momentum conservation. Since the defect transfers a distribution of momenta, it consequently has a finite size. The finite size of the defect, together with the preservation of locality make the local defects much easier to treat and incorporating them into a quantum field theoretical framework is relatively straight-forward.

The distribution of local defects is again assumed to be given by the Poisson sprinkling \( \langle 1 \rangle \). The momentum non-conservation is then parameterized in the length scale \( L \) of the distribution and a mass scale \( M \). These could a priori be different from the parameters relevant for local defects.

The coupling of quantum fields to the local defects is incorporated by adding a term to the gauge covariant derivative \( \partial + eA \rightarrow \partial + eA + g\partial P \). Here \( A \) is some gauge field with coupling constant \( e \), \( g \) is a coupling constant for the space-time defects and \( P \) is essentially the Fourier transform of the distribution of the momentum that is stochastically transmitted by the defect. This then allows one to calculate the probability for different scattering processes in an \( S \)-matrix expansion as usual. Importantly, this particular coupling to the defects has the effect that an on-shell particle that scatters on a defect is necessarily off-shell after scattering.

For a massless particle with energy \( E \) in 1+1 dimensions, the assumption that the momentum has a Gaussian distribution over the model parameters leads to a space-time defect that also
has a Gaussian distribution in lightcone coordinates

\[ P(x^+, x^-) = \exp \left( \frac{(x^+)^2}{(2\sigma^+)^2} + \frac{(x^-)^2}{(2\sigma^-)^2} + \frac{i(k_+)x^+ + (k_-)x^-}{2\pi \sqrt{\sigma^+ \sigma^-}} \right), \quad \text{(3)} \]

with widths

\[ \sigma^+ = \sqrt{\frac{2E}{\Delta M^2}}, \quad \sigma^- = \sqrt{\frac{2E}{\Delta M^2}}, \quad \text{(4)} \]

where \( \Delta M^2 \) is the variance of the distribution of the parameter \( M^2 \). One sees that the typical space-time patch covered by the defect is

\[ \sigma^+ \sigma^- = 2 (\Delta M^2)^{-1} \quad \text{(5)} \]

The defect has a Lorentz-invariant volume independent of \( E \), though it will deform under boosts that red- or blueshift \( E \) as one sees from Eqs. (4). Care must be taken in higher dimensions to properly normalize the momentum distribution. As with the nonlocal defects, the normalization can be achieved using the same method that is commonly used in the evaluation of scattering amplitudes, by taking into account that in reality we strictly speaking never deal with plane waves. The finite spatial extension of the incident particle’s wavefunction serves to regularize the distribution.

Massive particles can be treated similar to the massless ones. Again, it is of central relevance that the momentum distribution of the defect is a function of the properties of the incident particle. For massive particles, the distribution can be assigned most easily in the restframe of the particle, and in that restframe it will have an especially simple form. Thus, while the distribution is not Lorentz-invariant in the sense that its expression changes under arbitrary Lorentz-transformations, observer-independence is maintained because all observers can use the incident particle’s momentum as a reference and obtain the same result.

With this setup, constraints can be derived from processes normally forbidden in the standard model, which are now allowed. The most important bounds come from long-lived particles that travel long distances and are the following:

1. **Photon decay**: After scattering on a defect, a photon is off-shell and subsequently decays into a fermion pair. This effect is similar to pair production in the presence of an atomic nucleus in standard quantum electrodynamics (QED). This process results in a finite photon lifetime, and leads to excess electron-positron pairs.

2. **Photon mass**: The presence of space-time defects makes a contribution to the photon propagator and creates a small photon mass. (Gauge invariance is violated.)

3. **Vacuum Cherenkov radiation**: An electron can emit a (real) photon after scattering on a defect. This is similar to QED Bremsstrahlung.
The constraints from these effects can be summarized in Figure 3. Again, note that the existing bounds on $L$ are several orders of magnitude, but not too far, below the interesting parameter range. (Making $1/M$ smaller than shown in the plot means it becomes comparable to the Planck length $\sim 10^{-35}$ m and then it doesn’t make sense any more to speak of defects.) It would thus be highly desirable to improve the model to tighten the bounds.

![Figure 3: Summary of constraints on local space-time defects, from [5]. The red (dark) shaded region is excluded. The peachy (light) shaded region indicates a stronger constraint from photon decay with the ad-hoc assumption that the typical distance between defects increases with the cosmological scale factor.](image)

Not much work has been done on local space-time defects prior to [5], except for the model proposed in [13]. The model in [13] differs from the one discussed here in three important ways.

First, in [13] the interaction with the defect is mediated by scalar field. Second, the treatment in [13] necessitates the introduction of a cut-off in the momentum-space integration which breaks Lorentz-invariance and defeats the point of using a Lorentz-invariant distribution of defects to begin with. Such a cut-off is unnecessary in the model discussed here where the regulator is essentially the spatial width of the wave-packet. Third, and most important, the coupling to the defect in [13] is different. The approach in [5] started from the assumption that the defects originate in a distortion of space-time regularity and make themselves noticeable as a modification in the covariant derivative. This leads to a specific structure of the coupling terms, which was not used in [13].

In summary it can be said that space-time defects are pretty much unexplored territory where not much previous work has been done. That makes the topic very exciting as it harbors a potential for breakthrough.
3 Discussion

The preliminary work \cite{4, 5} tested the potential of detecting space-time discreteness by the occurrence of defects in the background’s regularity. The models used in this preliminary work can deliver only rough estimates. They are suitable for these estimates, but are theoretically unsatisfactory. Since the estimates show that it seems possible to reach the interesting parameter ranges experimentally, a further investigation and improvement of the theory and phenomenology of space-time defects is desirable. In the following we will propose some steps into this direction.

First, the models proposed in \cite{4, 5} are for flat 3+1 dimensional Minkowski space. Since the best constraints on the presence of local and nonlocal defects come from particles that have traveled long times and distances, one could derive better constraints when background curvature in general, and an expanding Friedmann-Robertson-Walker (FRW) metric in particular, can be taken into account. This would then allow one to use cosmological data, eg from the cosmic microwave background, to constrain the density of defects.

First steps towards a cosmology with nonlocal defects have been taken in references \cite{14} and \cite{15} based on the quantum graphity model developed by Markopoulou et al \cite{16}. It was assumed in \cite{14} that the nonlocal connections lie within a timelike slice that is assumed to be identical to the cosmological time. This of course violates Lorentz-invariance, but in a time-dependent background this can be expected. However, this specific violation of Lorentz-invariance is very strong and artificial: There is really no reason why the time-evolution of the network should be identical to the cosmological time, which is only an approximation based on the assumption of a homogeneous matter distribution anyway. More realistically, one would expect both slicings to differ, so that the links would still have a spread in the time-like coordinate. This would alter phenomenological consequences. The model for defects discussed here provides a good basis to study this phenomenology.

Second, the model with nonlocal defects also so far only operates on a kinematical level and a full dynamical description in terms of a quantum field theoretical treatment is missing. It would be desirable to development a quantum field theoretical model for this case, and thus also be able to combine both local and nonlocal defects. One way to address this point would be to use the dual nature of the local and nonlocal defects and to make mathematically precise the idea that nonlocal defects act like local defects, just in position-space rather than in momentum space. This would allow one to express them as operators on the particles’ Hilbert space and facilitate their incorporation into quantum field theory.

Third, while the search for defects as a model-independent expectation for space-time discreteness is interesting in its own right, contact to theoretical approaches to quantum gravity would serve a better identification of the parameters. The density of defects might for example be possible to extract in approaches that display a phase-transition from a pre-geometrical phase to an approximately smooth geometry. In this case the density of remaining defects should depend on the properties of the phase transition.
4 Conclusion

If space-time is fundamentally of non-geometric origin, then the smooth background geometry of general relativity should have defects. The consequences of these defects can be parameterized and described in phenomenological models, which allow one to put constraints on the density of the defects and the strength of their effects. These models are in their infancy and much remains to be done, but they harbor the possibility that a targeted search for space-time defects will allow us to find evidence for quantum gravity.

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