$e^+e^-$ pair production in relativistic ions collision and its correspondence to electron-ion scattering

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Abstract

It is shown that the amplitudes of electron-ion scattering and $e^+e^-$ pair production in the Coulomb field of two colliding ions are expressed in the terms of electron scattering amplitudes in the fields of the individual ions via the Watson expansion. We have obtained the compact expressions for these amplitudes valid in the high energy limit and discuss the crossing symmetry relations among the considered processes.

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1 Introduction

A lot of work has been done in past years [1-11] in the investigation of lepton pair production in the Coulomb fields of two colliding relativistic ions with charge numbers $Z_1, Z_2$

$$Z_1 + Z_2 \rightarrow e^+e^- + Z_1 + Z_2.$$  \(1\)

The main goal of this investigation is the attempt to obtain the compact expression for the amplitude of process (1) accounting for final state interaction of produced pair with ions in all orders of $Z_1\alpha, Z_2\alpha$, where $\alpha$ is fine structure constant. The solving of this issue can help to understand very important and unsolved problem of accounting the final state interaction of quarks and gluons in QCD. Unfortunately even in QED up to now this problem is not solved due to its complexity and so any progress in this direction is very useful.

The investigation of the process (1) becomes much more simple in the ultrarelativistic limit because of strong Lorentz contraction of electromagnetic fields of ions moving with the velocity close to the speed of light. An example of such simple solution which leads to an amplitude differing from the Born one only by phase factor was obtained in [1, 2, 3]. By virtue of procedure of the crossing symmetry the amplitude of the process (1) have been obtained from so-called “exact” result for amplitude of electron scattering in the Coulomb field of two colliding ions, which has been extracted from “exact” solution of Dirac equation.

1 On leave of absence from Yerevan Physics Institute
for electron in this field.
The further analysis of this problem in the framework of more familiar Feyn-
man diagrams leads to the conclusion that the result of [1, 2, 3] is incorrect.
This leads some authors [5, 11] to surprising conclusion that crossing symmetry
property is violated beyond the Born approximation.
Despite the essential progress achieved in this direction [8], the authors of which
succeed in the summing some class of main diagrams, the general structure of
amplitudes for electron scattering in the Coulomb field of two relativis-
tic nuclei and $e^+e^-$ production in this fields has not been established yet even in the limit
of ultrarelativistic energies. Taking into account the importance of this problem
and growing interest to this issue from the scientific community we would like
to do some remarks, which as we hope will be useful for understand-
ing of this problem.

2 Electron scattering and pair production in the
Coulomb fields of two colliding nucleus

The amplitudes of electron scattering and lepton pair production in the arbitrary
electromagnetic field $A_\mu(x)$ can be cast in the following form:

$$A^{(\text{scat})} = \bar{u}(p_f) \int d^4x_1 d^4x_2 e^{ip_f x_1 - ip_f x_2} T(x_2, x_1) u(p_i),$$

$$A^{(\text{prod})} = \bar{u}(p_2) \int d^4x_1 d^4x_2 e^{-ip_1 x_1 - ip_2 x_2} T(x_2, x_1) v(p_1),$$

(2)

where the function $T(x_2, x_1)$ is the same in both cases and obeys the following
equation

$$T(x_2, x_1) = V(x_2, x_1) - \int d^4x d^4x' V(x_2, x) G(x, x') T(x', x_1)$$

(3)

or in the short notation

$$T = V - V \otimes G \otimes T,$$
$$V(x_2, x_1) = e\gamma_\mu A_\mu(x_1) \delta^{(4)}(x_2 - x_1),$$
$$G(x, x') = \frac{1}{(2\pi)^4} \int d^4k \frac{\hat{k} + m}{k^2 - m^2 + i0} e^{-ik(x-x')},$$

(4)

where $\gamma_\mu$ and $u(p), v(p)$ are Dirac matrices and spinors.

From this relations it follow that for the two center problem ($A_\mu(x) = A_1^\mu(x) + A_2^\mu(x)$) the amplitude $T(x_2, x_1)$ can be represented in the form of infinitive
Watson series [12]

$$T = T_1 + T_2 - T_1 \otimes G \otimes T_2 - T_2 \otimes G \otimes T_1$$
$$+ T_1 \otimes G \otimes T_2 \otimes G \otimes T_1 + T_2 \otimes G \otimes T_1 \otimes G \otimes T_2...,$$

(5)
where $T_{1,2}$ obey the equations

$$
T_1 = V_1 - V_1 \otimes G \otimes T_1,
T_2 = V_2 - V_2 \otimes G \otimes T_2.
$$

(6)

In high energy limit when Lorentz factor of colliding ions $\gamma = \frac{E_M}{M} \to \infty$ this equations can be solved with the result:

$$
T_1(x_2, x_1) = \gamma_+ [U_1(x_1)\delta^4(x_2 - x_1)
+ \frac{i}{2\pi} \delta^2(\vec{x}_2 - \vec{x}_1)U_1(x_2)U_1(x_1) \exp \left( i \int_{x_1}^{x_2} U_1(x) dx_+ \right)
\times \int_{-\infty}^{\infty} dk_+ \left( \theta(k_+)\theta(x_2+ - x_1+) \right.
- \theta(-k_+)\theta(x_1+ - x_2+)) \exp \left( \frac{-ik_+(x_2- - x_1-)}{2} \right),
$$

(7)

$$
T_2(x_2, x_1) = \gamma_- [U_2(x_1)\delta^4(x_2 - x_1)
+ \frac{i}{2\pi} \delta^2(\vec{x}_2 - \vec{x}_1)U_2(x_2)U_2(x_1) \exp \left( i \int_{x_1}^{x_2} U_2(x) dx_- \right)
\times \int_{-\infty}^{\infty} dk_- \left( \theta(k_-)\theta(x_2- - x_1-) \right.
- \theta(-k_-)\theta(x_1- - x_2-)) \exp \left( \frac{-ik_-(x_2+ - x_1+)}{2} \right),
$$

(8)

where

$$
U_1(x) = e\gamma\Phi_1 \left( \sqrt{\left( \vec{b}_1 - \vec{x}_1 \right)^2 + \gamma^2 x_1^2} \right),
U_2(x) = e\gamma\Phi_2 \left( \sqrt{\left( \vec{b}_2 - \vec{x}_2 \right)^2 + \gamma^2 x_2^2} \right).
$$

(9)

Here $\Phi_{1,2}(r)$ are the Coulomb potentials of ions $Z_{1,2}$; $\gamma_\pm = \gamma_0 \pm \gamma_z$ is the Dirac matrices and we use the lightcone definition of momenta and coordinates $k_\pm = k_0 \pm k_z, x_{i\pm} = x_{i0} \pm x_{iz}; \vec{b}_1, \vec{b}_2$ are the the impact parameters of ions and $\vec{x}_1, \vec{x}_2$ the transverse coordinates of relevant four-vectors. There are no other simplifications in high energy limit. In particular there are no truncation of infinite Watson series in contrary to the statement done in [5].
3 The crossing symmetry relations

Let us discuss the property of crossing symmetry using as example the simplest crossed reactions - electron and positron scattering in the Coulomb field of ion. The amplitudes of these reactions read:

\[ A(e^{-}Z_{1,2} \rightarrow e^{-}Z_{1,2}) = \bar{u}(p_{f}^-)T_{1,2}(p_{f}^-, p_{i}^-)u(p_{i}^-) , \]
\[ A(e^{+}Z_{1,2} \rightarrow e^{+}Z_{1,2}) = -\bar{v}(p_{i}^+)T_{1,2}(-p_{i}^+, -p_{f}^+)v(p_{f}^+) , \]  

(10)

where 
\[ T_{1,2}(p_{2}, p_{1}) = \int d^{4}x_{1}d^{4}x_{2}e^{ip_{1}x_{1}-ip_{2}x_{2}}T_{1,2}(x_{2}, x_{1}) = (2\pi)^{2}\delta(p_{2\pm} - p_{1\pm}) \]
\[ \times \gamma_{\pm}[\theta(p_{1\pm})f_{1,2}^{\pm}(\tilde{p}_{2} - \tilde{p}_{1}) - \theta(-p_{1\pm})f_{1,2}^{-\pm}(\tilde{p}_{2} - \tilde{p}_{1})] , \]
\[ f_{1,2}^{\pm}(\tilde{q}) = \frac{i}{2\pi} \int d^{2}xe^{i\tilde{q}\tilde{x}}[1 - e^{\pm i\chi_{1,2}(\tilde{b} - \tilde{x})}] , \]
\[ \chi_{1,2}(\tilde{b} - \tilde{x}) = e \int_{-\infty}^{\infty} \Phi_{1,2} \left( \sqrt{(\tilde{b} - \tilde{x})^{2} + z^{2}} \right) dz . \]  

(11)

The crossing symmetry for reactions (10) means that the amplitudes for electron and positron scattering in the Coulomb field are expressed through universal function \( T(p_{2}, p_{1}) \) calculated at different values of its arguments. If this function would be analytical function of its arguments then it would be possible to express one amplitude through another using a simple substitution \( p_{i}^- \rightarrow -p_{f}^+ ; p_{f}^- \rightarrow -p_{i}^+ \) and trivial changing of the spinors \( u \rightarrow v \). But discontinuity of \( T(p_{2}, p_{1}) \) in the variable \( p_{+} \) doesn’t allow to do this in general case. Such substitution can be done only on the level of Born approximation as is easily seen from (11).

For the same reason the amplitude of pair production cannot be derived from the amplitude of electron scattering by procedure of “naive” crossing symmetry beyond the Born approximation.

4 Conclusions

There are two wrong points in the derivation of expressions for \( e^{\pm}e^{-} \) pair production amplitude in [1, 2, 3]. The first one is the oversimplified expression for electron scattering amplitude. Really the authors omit all higher terms of Watson expression (5) except the four first ones. The other mistake which leads the authors to wrong statement on absence of Coulomb corrections to the Born approximation is the wrong (naive) application of crossing symmetry property as we explained above.

Thus the problem of final state interaction to all orders in the fine structure constant even in abelian theory is a complex issue and demands the further and deeper investigation which will be done elsewhere.
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