A Model for the Coexistence of $p$-wave Superconductivity and Ferroelectricity

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Abstract

A model for the coexistence of $p$-wave superconductivity (SC) and ferroelectricity (FE) is presented. The Hamiltonian of SC sector and FE sector can be diagonalized by using the $so(5)$ and $h(4)$ algebraic coherent states respectively. We assume a minimal symmetry-allow coupling and simplify the total Hamiltonian through a double mean-field approximation (DMFA). A variational coherent-state (VCS) trial wave-function is applied for the ground state. It is found that the ferroelectricity gives rise to the magnetic field effect of $p$-wave superconductivity.

Keywords: $p$-wave superconductivity, Ferroelectricity, Coexistence, A double mean-field approximation.

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I. INTRODUCTION

The coexistence of superconductivity (SC) and ferroelectricity (FE) and competition play a important role in motivating the work of Bednorz and Muller on the high temperature cuprates superconductors. Even earlier, work on the “old” superconductors of the $\beta$-W structure like $V_3Si$, $Nb_3Sn$ etc., where $T_c \sim 23K$ and a martensitic phase transition occurs in the same temperature range, gave rise to investigations on the possibility of a “ferroelectric metal” or a “polar metal” and thus to the study of SC-FE coexistence. Many theoretical papers have already studied microscopic models for the effect of lattice instability on superconductivity in the sodium tungsten bronze systems. These papers have illuminated many aspects of the interplay between the structure deformations, such as rotation of underlying octahedral units and coupling with electron pairs. Study of SC-FE coexistence problem can be relevant to recent work by Weger and collaborators, on the mechanism of high temperature superconductivity in the cuprates. In that work the presence of a nearby FE instability close to the SC transition is related to the anomalously large ionic dielectric coefficient in the cuprates, which reduces the electron-electron repulsion and then can lead to an enhanced net electron-electron attraction, producing higher $T_c$. Moreover the recent so(5) and su(4) models for multi-critical superconductor antiferromagnetic behavior in the high temperature superconductors have also been studied by Zhang et al. On the other hand, the recent experimental discovery of the coexistence ferromagnetism (FM) and SC in $VGe_2$, and subsequently in $ZrZn_2$ and $URhGe$, have shown clearly that the spin-triplet states of $p$-wave SC are realized in the nature. This naturally causes our interest in the relationship between FE and $p$-wave SC. This paper is organized as follows: In Sec. II we will construct a so(5)$\otimes$h(4) algebraic structure general model for the coexistence of $p$-wave SC and FE. In Sec. III and IV we will diagonalize the Hamiltonian of $p$-wave SC and FE according to so(5) and h(4) algebraic coherent state methods respectively, and get their energies, the eigenstates and the expection value of the order parameters in the ground coherent state. In Sec. V the total Hamiltonian including the biquadratic interaction is simplified to the bilinear Hamiltonian $H_{DMFA}$ by using a double-mean-field approximation. We can find the variational solution of $H_{DMFA}$ by forming a trial eigenstate analogous to the product coherent state. The energy spectrum and the eigenstates will be obtained based on the variational solution.
II. OUR MODEL

Here we will consider a general model for the coexistence between $p$-wave superconductivity and ferroelectricity. According to a known result there is so(5) structure in $p$-wave superconductivity [20, 21] that is formed by two $su(2)$ not commuting with each other, where one describes the attractive BCS interaction and the other the usual spin operators, as well as other four generators associated with the transitions. But ferroelectricity is formed by Heisenberg algebra $h(4)$ [22, 23, 24, 25]. Motivated by Ref. [26], we write the total Hamiltonian in three parts as follows:

$$H = H_{SC} + H_{FE} + H_{INT},$$

where the first term $H_{SC}$ is the BW type of $p$-wave superconductivity mean-field reduced Hamiltonian, namely

$$H_{SC} = \sum_{k} h_{k},$$

with the Hamiltonian at each given momentum $k$ given as

$$h_{k} = \epsilon_{k} E_{3}^{(k)} + [\Delta_{E}(k) E_{+}^{(k)} + \Delta_{U}(k) U_{+}^{(k)} + \Delta_{V}(k) V_{+}^{(k)} + H.c.] .$$

Here $\Delta_{E}(k) = \frac{1}{2} \sum_{k'} V_{kk'} < E_{-}^{(k')} >$, $\Delta_{U}(k) = \frac{1}{2} \sum_{k'} V_{kk'} < U_{-}^{(k')} >$ and $\Delta_{V}(k) = \frac{1}{2} \sum_{k'} V_{kk'} < V_{-}^{(k')} >$ are opposite and equal spin pairing (“gap”) energy respectively, and $p$-wave attraction pair interaction potential $V_{kk'} = -3V_{1}(k, k')k \cdot k'$.

The second term $H_{FE}$ is the displaced oscillator Hamiltonian of the displacive ferroelectric soft mode phonon [27]. It’s taken as

$$H_{FE} = \omega_{TO}(b^{\dagger}b + \frac{1}{2}) + \gamma_{1}\varepsilon(b^{\dagger} + b),$$

where $\omega_{TO}$ is the frequency of the soft TO mode, $\gamma_{1}$ is taken as a positive constant, and $\varepsilon$ is the magnitude of the electric field $\vec{E}$.

The third term $H_{INT}$ is the interaction term. According to Ginzburg-Landau (GL) theory [28], every term in the free energy shall be a scalar invariant under the relevant symmetry group, here the free energy density of system will be the form of $\delta F = a|\mathbf{P}|^{2} + \frac{b}{4}|\mathbf{P}|^{4} + \alpha|\Delta|^{2} + \beta|\Delta|^{4} + \kappa|\Delta|^{2}|\mathbf{P}|^{2}$. In such a GL theory, the lowest-order, generic coupling between superconducting $\Delta$ and the ferroelectric $\mathbf{P}$ will be a biquadratic term of the form proportional to $|\Delta|^{2}\mathbf{P}^{2}$, that is, the last term in $\delta F$. This lowest-order term satisfies gauge and parity symmetry requirements. From the correspondences $|\Delta|^{2} \sim (E_{+}E_{-} + U_{+}U_{-} + V_{+}V_{-} + H.c.)$.
and $P^2 \sim (b + b^\dagger)^2$ we can immediately translate this coupling term into the interaction term of our model as follows:

$$H_{INT} = \sum_k \gamma_{2k}(E_+^{(k)}E_-^{(k)} + U_+^{(k)}U_-^{(k)} + V_+^{(k)}V_-^{(k)} + H.c.)(b^\dagger + b)^2,$$

(5)

where $\gamma_{2k}$ is the initial pair-TO mode coupling coefficient.

In our model, the set \{\(E_+^{(k)}, E_3^{(k)}, F_+^{(k)}, F_3^{(k)}, U_+^{(k)}, V_-^{(k)}\)\} form so(5) algebra, its generators are expressed as

\[
\begin{align*}
E_+^{(k)} & = \frac{1}{\sqrt{2}}(a_{k\uparrow}^\dagger a_{k\downarrow}^\dagger + a_{k\downarrow}^\dagger a_{k\uparrow}^\dagger), & F_+^{(k)} & = \frac{1}{\sqrt{2}}(a_{k\uparrow}^\dagger a_{k\downarrow} + a_{k\downarrow}^\dagger a_{k\uparrow}), \\
E_-^{(k)} & = \frac{1}{\sqrt{2}}(a_{k\downarrow}^\dagger a_{-k\uparrow}^\dagger + a_{k\uparrow} a_{-k\downarrow}), & F_-^{(k)} & = \frac{1}{\sqrt{2}}(a_{k\uparrow}^\dagger a_{k\downarrow} + a_{-k\downarrow} a_{-k\uparrow}), \\
U_+^{(k)} & = a_{k\uparrow}^\dagger a_{k\uparrow}, & U_-^{(k)} & = a_{k\downarrow} a_{-k\downarrow}, \\
V_+^{(k)} & = a_{-k\uparrow}^\dagger a_{-k\downarrow}, & V_-^{(k)} & = a_{k\downarrow}^\dagger a_{-k\uparrow}.
\end{align*}
\]

(6)

and satisfy the following commutation relations:

\[
\begin{align*}
[E_+^{(k)}, V_-^{(k)}] & = \mp F_3^{(k)}, & [F_+^{(k)}, V_-^{(k)}] & = \mp E_3^{(k)}, & [E_-^{(k)}, U_+^{(k)}] & = \pm F_3^{(k)}, & [E_-^{(k)}, F_3^{(k)}] & = \mp E_3^{(k)}, \\
[F_+^{(k)}, U_+^{(k)}] & = -E_3^{(k)}, & [E_+^{(k)}, F_3^{(k)}] & = \mp E_3^{(k)}, & [E_+^{(k)}, U_-^{(k)}] & = \mp F_3^{(k)}, & [E_+^{(k)}, F_-^{(k)}] & = \mp E_3^{(k)}, \\
[F_3^{(k)}, U_-^{(k)}] & = \mp U_-^{(k)}, & [F_3^{(k)}, V_-^{(k)}] & = \pm V_-^{(k)}, & [F_3^{(k)}, V_-^{(k)}] & = \pm V_-^{(k)}, \\
[E_+^{(k)}, F_-^{(k)}] & = E_3^{(k)} - F_3^{(k)}.
\end{align*}
\]

(7)

Note that $H_{SC}$ is linearized with respect to so(5) algebraic generators for a given momentum $k$, $H_{FE}$ is written in terms of $h(4)$ generators, and $H_{INT}$ is expressed as a product of the bilinear form of so(5) and quadratic form of $h(4)$. Therefore the total Hamiltonian is a so(5) $\otimes$ h(4) direct product algebraic structure in each given momentum $k$.

**III. THE so(5) STRUCTURE OF p-WAVE SUPERCONDUCTIVITY MODEL**

From Hamiltonian $H_{SC}$ we can known that $h_k$ is written in terms of so(5) generators for a given $k$. The dynamical symmetry or spectrum generating algebra for each $k$ is so(5)$_k$, so the spectrum generating algebra of $H_{SC}$ is $\otimes_k$ so(5)$_k$. 
The eigenstates of $H_{SC}$ are expressed by a direct product of $so(5)_k$ coherent states $\otimes_k |\xi_k\rangle$.

Therefore the eigenstates $|\xi\rangle$ can be written as

$$|\xi\rangle = \otimes_k |\xi_k\rangle = \otimes_k W(\xi_k)|p, q >,$$

where

$$W(\xi_k) = \exp\{\xi_k(\sqrt{2}\cos\theta_k E_3^{(k)} - \sin\theta_k e^{i\phi_k}U_+^{(k)} + \sin\theta_k e^{-i\phi_k}V_+^{(k)}) - H.C.\},$$

with the real coherent parameters $\xi_k$, $(\theta_k, \phi_k)$ are angles in spin space for a given momentum $k$. $|p, q\rangle$ is the mutual eigenstates of the Cartan subalgebra $\{E_3^{(k)}, F_3^{(k)}\}$ of $so(5)$, and their eigenvalues are $p$ and $q$ respectively. By tedious calculations we can immediately diagonalize the Hamiltonian $h_k$ as

$$W^\dagger(\xi_k)h_kW(\xi_k) = \sqrt{\xi_k^2 + \Delta^2(k)E_3^{(k)}},$$

where $\Delta^2(k) = 2(|\Delta_E(k)|^2 + |\Delta_U(k)|^2 + |\Delta_V(k)|^2)$. Also we obtain the gap equation:

$$H = \begin{bmatrix} \Delta_E(k) \\ \Delta_U(k) \\ \Delta_V(k) \end{bmatrix} = \frac{1}{2} \sum_{k'} V_{kk'} \frac{\Delta(k')}{\sqrt{\xi_{k'}^2 + \Delta^2(k')}} \begin{bmatrix} -\frac{\sqrt{2}}{2} \cos\theta_{k'} \\ \frac{\sqrt{2}}{2} \sin\theta_{k'} e^{i\phi_{k'}} \\ -(p+q) \frac{\sqrt{2}}{2} \sin\theta_{k'} e^{-i\phi_{k'}} \end{bmatrix}.$$

(10)

From the above gap equation we can obtain the following results:

1. If both $p$ and $q$ are zero, the gap $\Delta = 0$ and the $p$-wave SC lies in disorder state. Its eigenstate is $|\xi_{dis}\rangle = \otimes_k W(\xi_k)|p = 0, q = 0 >.$

2. If either $p$ or $q$ are not zero, the gap $\Delta \neq 0$ and the $p$-wave SC lies in superconducting state, but when $p = -1$ and $q = 0$ the $p$-wave SC lies in the ground state. Its ground state is $|\xi_{BCS}\rangle = \otimes_k W(\xi_k)|p = -1, q = 0 >$ where $|p = -1, q = 0 >$ is the vacuum state.

IV. A h(4) STRUCTURE OF DISPLACIVE FERROELECTRIC SOFT MODE

The Hamiltonian $H_{FE}$ can be transformed by the displaces oscillator Bose operator

$$U(\zeta_0) = \exp[\zeta_0(b^\dagger - b)],$$

(11)

where the coherent parameter $\zeta_0$ is taken as real, so we obtain

$$U^\dagger H_{FE}U = \omega_{TO}(b^\dagger b + \frac{1}{2}) - (\gamma_1 \xi)^2 \omega_{TO}.$$
Note that $U^\dagger H_{FE}U$ is shifted to a new minimum, but it retains the same excitation frequency $\omega_{TO}$ as the original oscillator. The eigenstates and eigenvalues of $H_{FE}$ can be given as

$$|\zeta_0 > = U(\zeta_0)|n > = \exp[\zeta_0(b^\dagger - b)]|n >,$$  \hspace{1cm} (13)

and

$$W_n = \omega_{TO}(n + \frac{1}{2}) - \frac{(\gamma_1 \varepsilon)^2}{\omega_{TO}},$$ \hspace{1cm} (14)

where $|n >$ is a number eigenstate of the phonon number operator $N_b = b^\dagger b$ and the state $|\zeta_0 >$ is a Glauber coherent state for the FE oscillator [29, 30].

The order operator for the FE polarization is the coordinate operator $Q$ or $(b + b^\dagger)$, therefore the order parameter

$$\eta_{FE} = \begin{cases} < \zeta_0|b^\dagger + b|\zeta_0 > = 2\zeta_0 \\ < n|b^\dagger + b|n > = 0. \end{cases}$$

When $\eta_{FE} = 0$ it shows that the phonon is free, but $\eta_{FE} = 2\zeta_0$ it exhibits that the phonon is polarized spontaneously.

V. THE VARIATIONAL COHERENT STATE EIGENSTATES OF OUR HAMILTONIAN UNDER DOUBLE-MEAN-APPROXIMATION

The total Hamiltonian is turned into bilinear forms by double mean field approximation procedure which reduces biquadratic operators such as $A^2B^2$ to the following form

$$A^2B^2 \approx (2A < A > - < A >^2)(2B < B > - < B >^2),$$ \hspace{1cm} (15)

based on the assumption $(A - < A >)^2 \approx 0$ and $(B - < B >)^2 \approx 0$. After making the double-mean-approximation and isolating a single mode $k$, we have the effective Hamiltonian at each momentum mode $k$ in the DMFA:

$$H_{DMFA} = \epsilon E_3 + (\Delta'_E E_+ + \Delta'_U U_+ + \Delta'_V V_+ + H.c.) + \omega_{TO}(b^\dagger b + \frac{1}{2}) + \Gamma_1(b^\dagger + b) + (\Gamma_2^E E_+ + \Gamma_2^U U_+ + \Gamma_2^V V_+ + H.c.)(b^\dagger + b) + \Gamma_3.$$ \hspace{1cm} (16)
Therefore the double mean field approximation (DMFA) yields a bilinear effective interaction term, and it renormalizes the coefficients $\Delta'$ and $\Gamma_{1,2,3}$ in $H_{DMFA}$ as follows:

\[
\begin{align*}
\Delta_E' &= \Delta_E - \gamma_2 < b^+ b > < E_- >, \\
\Delta_U' &= \Delta_U - \gamma_2 < b^+ b > < U_- >, \\
\Delta_V' &= \Delta_V - \gamma_2 < b^+ b > < V_- >, \\
\Gamma_E^F &= 2\gamma_2 < b^+ b > < E_- >, \\
\Gamma_U^F &= 2\gamma_2 < b^+ b > < U_- >, \\
\Gamma_V^F &= 2\gamma_2 < b^+ b > < V_- >, \\
\Gamma_1 &= \gamma_1 \varepsilon - 2\gamma_2 < b^+ b > (< E_+ < E_- > + < U_+ < U_- > + < V_+ < V_- >), \\
\Gamma_3 &= \gamma_2 (< E_+ < E_- > + < U_+ < U_- > + < V_+ < V_- >) < b^+ b >^2.
\end{align*}
\]

This total Hamiltonian includes the pure $p$-wave SC and the pure FE prototype systems and their coupling via the soft-mode oscillator coupled to opposite and equal-spin pairing Hamiltonian. Noting that the initial Hamiltonian is the enveloping algebra of $so(5) \otimes h(4)$ because of the biquadratic interaction terms, while $H_{DMFA}$ is an element in the direct product algebra $so(5) \otimes h(4)$.

When $\gamma_2 \to 0$, we recover the sum of two separate sectors: for $p$-wave SC and FE. In order to obtain the ground state eigenstates and eigenvalue of our model $H_{DMFA}$, we will make use of the variational principle. Introducing an analogous trial variational coherent state (VCS) which is the product of two coherent-like states and is denoted $|\varphi_\nu >$:

\[
|\varphi_\nu >= |\xi > |\zeta >= \hat{V}_1 |p, q > \hat{V}_2 |n >,
\]

where $V_1(\xi) = \exp[\xi(\sqrt{2} \cos \theta E_+ - \sin \theta e^{i\phi} U_+ + \sin \theta e^{-i\phi} V_+) - H.c.]$ and $V_2(\zeta) = \exp[\zeta(b^+ b)]$. Here the real parameters $\xi$ and $\zeta$ are variational unknowns. The kets $|p, q >$ and $|n >$ are the same as before. Now we define the energy in state $|\varphi_\nu >$ as the diagonal value of $H_{DMFA}$ in the variational coherent state $|\varphi_\nu >$:

\[
\begin{align*}
E_{p,q,n}(\xi, \theta, \phi, \zeta) &= <\varphi_\nu |H_{DMFA}|\varphi_\nu > \\
&= p\varepsilon \cos 2\xi + \frac{p}{\sqrt{2}} \Delta_E' \cos \theta + \frac{p - q}{2} \Delta_U' e^{-i\phi} \sin \theta - \frac{p + q}{2} \Delta_V' e^{i\phi} \sin \theta + H.c.] \sin (2\xi) \\
&+ 2\zeta \left[ \frac{p}{\sqrt{2}} \Gamma_E^F \cos \theta + \frac{p - q}{2} \Gamma_U^F e^{-i\phi} \sin \theta - \frac{p + q}{2} \Gamma_V^F e^{i\phi} \sin \theta + H.c.] \sin (2\xi) \\
&+ \omega_{TO} (n + \zeta^2 + \frac{1}{2}) + 2\Gamma_1 \zeta + \Gamma_3 \\
&= p\varepsilon \cos (2\xi) + p\Delta \sin (2\xi) + \omega_{TO} (n + \zeta^2 + \frac{1}{2}) \\
&+ 6\gamma_2 \zeta^2 (p^2 + q^2 \sin^2 \theta) \sin^2 (2\xi) + 2\gamma_1 \varepsilon \zeta.
\end{align*}
\]
Here the coefficients $\Delta'$ and $\Gamma_{1,2,3}$ in the energy $E_{p,q,n}(\xi, \theta, \phi, \zeta)$ are given as

\[
\begin{align*}
\Delta'_E &= \Delta_E - 4\gamma_2\zeta^2 \sin(2\xi)(-\frac{p}{\sqrt{2}} \cos \theta), \\
\Delta'_U &= \Delta_U - 4\gamma_2\zeta^2 \sin(2\xi)(\frac{p-q}{2} \sin \theta e^{i\phi}) = \frac{1}{2} \sin \theta e^{i\phi}[\Delta - 4\gamma_2\zeta^2(p-q) \sin(2\xi)], \\
\Delta'_V &= \Delta_V - 4\gamma_2\zeta^2 \sin(2\xi)(-\frac{p-q}{2} \sin \theta e^{-i\phi}) = -\frac{1}{2} \sin \theta e^{-i\phi}[\Delta - 4\gamma_2\zeta^2(p+q) \sin(2\xi)], \\
\Gamma_2^E &= 4\gamma_2\zeta \sin(2\xi) \begin{pmatrix} -\frac{p}{\sqrt{2}} \cos \theta \\ \frac{p-q}{2} \sin \theta e^{i\phi} \\ -\frac{p+q}{2} \sin \theta e^{-i\phi} \end{pmatrix}, \\
\Gamma_2^V &= 4\gamma_2\zeta \sin(2\xi) \begin{pmatrix} -\frac{p}{\sqrt{2}} \cos \theta \\ \frac{p-q}{2} \sin \theta e^{i\phi} \\ -\frac{p+q}{2} \sin \theta e^{-i\phi} \end{pmatrix}, \\
\Gamma_1 &= \gamma_1 \epsilon - 2\gamma_2\zeta(p^2 + q^2 \sin^2 \theta) \sin^2(2\xi), \\
\Gamma_3 &= 2\gamma_2\zeta^2(p^2 + q^2 \sin^2 \theta) \sin^2(2\xi).
\end{align*}
\]  

(20)

We determine $\theta, \phi, \xi, \zeta$ from the following equations:

\[
\begin{align*}
\partial E/\partial \theta &= 0, \\
\partial E/\partial \phi &= 0, \\
\partial E/\partial \xi &= 0, \\
\partial E/\partial \zeta &= 0.
\end{align*}
\]  

(21)

Eqs. (21) have two solutions:

(i). $\sin \theta = 0$, 

\[
\begin{align*}
\tan 2\xi &= \frac{\Delta + 12\gamma_2\zeta^2 p \sin(2\xi)}{\epsilon}, \\
\xi &= -[\gamma_1 \epsilon + 6\gamma_2\zeta p^2 \sin^2(2\xi)]/\omega_{TO}.
\end{align*}
\]

This result exhibits that FE and SC can coexist in the opposite spin pairing state of $p$-wave SC but the equal spin pairing state of $p$-wave SC disappeared. Here the energy is

\[
E = p\epsilon \cos(2\xi) + p\Delta \sin(2\xi) + \omega_{TO}(n + \zeta^2 + \frac{1}{2}) + 6\gamma_2\zeta^2 p^2 \sin^2(2\xi) + 2\gamma_1 \epsilon \zeta.
\]

(ii). $\cos \theta = 0$, 

\[
\begin{align*}
\tan 2\xi &= \frac{\Delta + 12\gamma_2\zeta^2 (p + \frac{q^2}{p}) \sin(2\xi)}{\epsilon}, \\
\xi &= -[\gamma_1 \epsilon + 6\gamma_2\zeta(p^2 + q^2) \sin^2(2\xi)]/\omega_{TO}.
\end{align*}
\]

This result exhibits that FE and $p$-wave SC may coexist in the equal spin pairing state (ABM state) but the opposite spin pairing state of $p$-wave SC disappeared. Here the energy is

\[
E = p\epsilon \cos(2\xi) + p\Delta \sin(2\xi) + \omega_{TO}(n + \zeta^2 + \frac{1}{2}) + 6\gamma_2\zeta^2(p^2 + q^2) \sin^2(2\xi) + 2\gamma_1 \epsilon \zeta.
\]

Therefore from above two cases we get that the spin of $p$-wave SC is oriented because of the existence of FE. It indicates that the FE effect in the excite state of $p$-wave SC is equivalent to magnetic field.
VI. CONCLUSION

In this paper we have constructed a general model for the coexistence of $p$-wave superconductivity and ferroelectricity. The Hamiltonian of $p$-wave SC can been diagonalized based on $so(5)$ spectrum-generating algebra structure and has shown that the eigenstate is related to $so(5)$ coherent state. Also the Hamiltonian of FE is diagonalized by using $h(4)$ algebraic coherent state. The total Hamiltonian under the double mean-field approximation can been solved by making use of a variational coherent-state (VCS) procedure by virtue of the $so(5) \otimes h(4)$ algebraic structure in each given momentum $k$. This leads to the conclusion that the ferroelectricity gives rise to the magnetic field effect of $p$-wave superconductivity.

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[1] J. G. Bednorz and K. A. Muller, Rev. Mod. Phys. 60, 685 (1988).
[2] M. Weger and I. M. Goldberg, in Solid State Physics 28, (1973) edited by F. Seitz, D. Turnbull and H. Ehrenreich.
[3] P. W. Anderson and E. Blount, Phys. Rev. Lett. 14, 217 (1965).
[4] J. L. Birman, Phys. Rev. Lett. 17, 1216 (1966).
[5] C. S. Ting and J. L. Birman, Phys. Rev. B 12, 1093 (1975).
[6] R. N. Bhatt and W. Mc Millan, Phys. Rev. B 14, 1007 (1976).
[7] K. L. Ngai and R. Silberglitt, Phys. Rev. B 13, 1032 (1976).
[8] K. L. Ngai and T. Reinecke, Phys. Rev. Lett 38, 74 (1977); Phys. Rev. B 16, 1077 (1977); J. Phys. F: Metal Phys. 8, 151 (1978).
[9] A. H. Kahn and J. Ruvalds, Phys. Rev. B 19, 5652 (1979).
[10] M. Peter and M. Weger, “Influence of Ionic Dielectricity on the Dynamic Superconducting Order Parameter in High Temperature Superconductivity”, edited by S. E. Barnes et.al., Am. Inst. of Phys. (1999) p90-103.
[11] M. Peter, M. Weger and L. P. Pitaevskii, Ann. Physik 7, 174-200 (1998); Physica C 317-318, 252 (1999); M. Weger, J. Supercond. 10, 435 (1997).

[12] S. C. Zhang, Science 275, 1089 (1997).

[13] M. G. Zacher, et.al., Phys. Rev. Lett. 85, 824 (2000).

[14] S. C. Zhang and E. Demler, Phys. Rev. B 58, 5719 (1998).

[15] M. Guidry, L. A. Wu, W. Sun and C. L. Wu, Phys. Rev. B 63, 134516 (2001).

[16] S. Saxena, et.al., Nature 406, 587 (2000).

[17] E. Bauer, R. Dickey, V. Zapf and M. Maple, J. Phys. Cond. Matt. 13, L759 (2001).

[18] C. Pfleiderer, M. Uhlarz, et.al., Nature 412, 58 (2001).

[19] D. Aoki, A. Huxley, et.al., Nature 413, 613 (2001).

[20] S. Murakami, N. Nagaosa and M. Sigristn, Phys. Rev. Lett. 82, 2939 (1999).

[21] H. B. Zhang, M. L. Ge and K. Xue, J. Phys. A: Math. Gen. 35, L7-11 (2002).

[22] W. Cochran, Advances in Physics, 9, 387 (1960), ibid. 18, 157 (1969).

[23] A. D. Bruce and R. A. Cowley, J. Phys. C Solid State, 5, 595 (1972).

[24] R. Cowley, Phys. Rev. 134, A981 (1964).

[25] P. W. Anderson, Izd. AN. SSR., Moscow (1996).

[26] J. L. Birman and M. Weger, Phys. Rev. B 64, 174503 (2001).

[27] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, 1st ed. (Addison-Wesley, Reading, MA, 1960), p47 and footnote on p60.

[28] L. D. Landau, E. M. Lifshitz and L. P. Pitaevsky, Statistical Physics, 3rd ed. (Pergamon, Oxford, 1990).

[29] J. Klauder and B. S. Skagerstrom, “Coherent States”, World Science Press, Singapore (1996).

[30] R. J. Glauber, Phys. Rev. 231, 2778 (1960).