Calculating the snow thermal diffusivity coefficient using snow temperature measurements

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Abstract. Measuring the density, thermal conductivity, and thermal diffusivity of snow is important for modeling the depth of soil freezing/thawing and water balance of the earth’s surface. A method of calculating the thermal diffusivity and snow depth refinement based on mathematical modeling is proposed.

1. Introduction
Since the mid-1960s, Russia has been predominantly experiencing climate warming [1]. The increase in the air temperature has a significant impact on the state of soils in the permafrost zone, which occupies more than 65% of the territory of the Russian Federation [2]. The most important climatic parameters that determine the temperature of the soil and its freezing/thawing are the air temperature and snow cover depth. Also of great importance are the modes of seasonal snow accumulation, its density, thermal characteristics, etc. [3, 4]. The thermal protective properties of the snow cover are determined by its thermal resistance Rs = hs/λs, where hs is the thickness of the snow cover, and λs is the coefficient of effective thermal conductivity of snow [5]. The thermal conductivity coefficient is used to calculate the depth of soil freezing and to assess the heat flow in the atmosphere-snow cover-underlying rocks in the modeling of climate change.

The dynamics of thermal resistance of the snow cover is determined by the change in its depth and density. In the first months of the cold period, the value of Rs increases. However, in the second half of winter for some areas these changes may be small, since the growth of Rs due to the increase in the height of the snow cover can be compensated by an increase in its density [5]. The density of snow varies over a wide range from 10 to 800 kg/m³ [10].

Thermal diffusivity (thermal diffusivity coefficient) is a physical quantity characterizing the rate of change (equalization) of the temperature of a substance in nonequilibrium thermal processes. It is numerically equal to the ratio of thermal conductivity to volumetric heat capacity at constant pressure (α = λs/(Cρ)). In the SI system it is measured in m²/s. The main method for measuring the thermal diffusivity of solids is the Parker method. In the classical paper of W. Parker et al. published in 1961 a heat impulse method for determining the thermal physical characteristics is proposed [6].

In this paper, we propose a method for calculating the coefficient of thermal diffusivity of snow cover based on the solution of the Fourier heat transfer equation by a numerical method based on measuring the temperature of snow in natural conditions.
2. **Materials and methods**

For studies of the temperature regime of soils and microclimate characteristics, a unique atmospheric soil measuring system called ASMS was developed at the Institute of Monitoring of Climatic and Ecological Systems SB RAS [7]. The ASMS is designed for mobile and stationary long-term automatic measurements and registration of the main parameters of the atmosphere, soils, and waters. Some modifications of the ASMS are equipped with sensors of depth and temperature of the snow cover, the level of bog water, the amount of precipitation, the speed and direction of the wind, the characteristics of solar radiation, the concentration of carbon dioxide, etc.

The measuring rail of the snow cover temperature sensor has the form of a three-wire printed circuit board with high-precision digital thermometers soldered to it, and is connected via a single-wire interface to the registrar. The rail is placed in a white shrink tube. The proposed design of the sensor allows one to reduce the parasitic heat transfer between the sensors on the rail, which improves the accuracy characteristics due to the lack of a large number of wires. Reducing the size of the rail and the use of digital thermometers has significantly reduced the impact on the natural snow cover, both due to the formation of "supercharges" and "weathering", and by reducing the absorption of heat penetrating solar radiation [8].

The proposed snow cover depth sensor has the ability to use a large number (up to 100) of digital thermometers, which allows one to increase the resolution and accuracy of measurement. The temperature sensors are placed above the soil surface in increments of 2.5 or 5 cm, and up to a height of 200 cm.

The method of calculating the coefficient of effective thermal diffusivity of snow is based on the numerical solution of the heat transfer equation in snow cover. The heat transfer equation has the form [9]

\[ \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial z^2}, \tag{1} \]

where \( \theta \) is the temperature of snow, °C; \( t \) is the time, s; \( z \) is the coordinate of the snow cover depth, m; and \( \alpha \) is the thermal diffusivity coefficient.

To solve equation (1), a finite difference method was used, which is based on the replacement of derivatives with difference schemes. Equation (1), written in finite differences, has the following form:

\[ \frac{(\theta_1(t_2) - \theta_1(t_1))}{(t_2 - t_1)} = \alpha \left( \frac{(\theta_2 - \theta_1)}{\Delta z} - \frac{(\theta_1 - \theta_0)}{\Delta z} \right), \tag{2} \]

where \( \Delta z \) is the depth step; \( \theta_0, \theta_1, \text{ and } \theta_2 \) are the snow temperatures at three depths with step \( \Delta z \) at time \( t_1 \); \( \theta_1(t_1) \) and \( \theta_1(t_2) \) are the snow temperatures at time points \( t_1 \) and \( t_2 \).

To solve equation (1) using the difference scheme (2), it is necessary to set initial and boundary conditions. The initial conditions will be the temperature values measured from the soil surface to the air-snow interface.

The boundary conditions for the solution of equation (1) will be the temperature values of the sensors at the lower and upper interfaces of the snow cover. The upper interface of the snow cover is determined by the snow cover height, for whose calculation an algorithm was developed based on analysis of the temperature gradient of snow and air.

The calculation algorithm is as follows: the snow-air interface is determined by exceeding the specified threshold value of the difference between the readings of two adjacent thermometers starting from the lower one. This algorithm allows one to determine the snow cover depth with an accuracy equal to the distance between the sensors. Under certain conditions (when the temperature in the air and snow is equalized), it is not possible to determine the snow depth.

The result of the calculation of the snow thickness by the algorithm will be the sensor number which is at the least distance from the air.
The solution of equation (1) will be the values of the snow temperature profile over time for an interval equal to 24 hours in steps of 12 minutes with an initial coefficient of thermal diffusivity $\alpha = 25 \times 10^{-7} \text{m}^2/\text{s}$, which is double the coefficient of thermal diffusivity of ice [11].

To determine the thermal diffusivity coefficient, it is necessary to choose a coefficient at which the calculated values of the snow temperature profile are closest to the measurement data through 24 hours. For comparison of the observed and modeled data, the standard deviation is calculated, with whose minimum value the thermal diffusivity value will be chosen which will be the initial value for the calculation on the next interval.

3. Results and discussion

The experimental calculations of the snow cover depth were carried out using the data of measurements of the ASMS with an ultrasonic sensor for measuring the snow cover depth installed on the Eastern shore of Lake Baikal. The measurement interval was 1 hour, and the number of temperature sensors in the measuring rail was 20 with a spacing of 25 mm. For the calculation we chose the time interval from 09.09.2017 to 31.03.2018. Readings of an ultrasonic snow depth sensor were used for verification of the snow depth calculation.

The results of the calculation of the snow cover depth and the measurement data of the ultrasonic sensor for the period from 09.09.2017 to 31.03.2018 are shown in Figure 1.

![Figure 1. Snow cover depth. 1 - Measurement data of the ultrasonic sensor ASMS-50000021. 2 - calculated data by the preliminary algorithm. 3 - Updated data on the simulation data.](image)

To estimate the accuracy of the snow depth calculation by the proposed method and the preliminary algorithm, the mean square deviation between the calculation results and the measurement data of the ultrasonic sensor was calculated. The ultrasonic sensor values were limited to a height of 500 mm, since the height of the measuring rail for this device is 500 mm; for the proposed calculation method the mean square deviation value is 69.3 mm, and for calculations using the preliminary algorithm it is 126.9 mm, which confirms higher accuracy of the proposed method.

To verify the calculation of the thermal diffusivity coefficient according to the proposed algorithm, an ASMS with a measuring rail height of 700 mm was installed on the measuring site, which has 15 sensors with a spacing of 50 mm. The period of measurement of the ASMS was 10 minutes; the depth and density of the snow were additionally measured using a standard snowmeter OS-1. These measurements are presented in Table 1.
Table 1. Results of the experiment. \( h \) is the measured snow depth, \( \rho \) is the snow density, \( H \) is the estimated snow depth, \( \alpha \) is the thermal diffusivity coefficient \( T_0, T_{14} \) are the temperatures at the bottom and top of the snowpack.

| Data            | Manual observations | Modelling  |
|-----------------|---------------------|------------|
|                | \( h \) (mm) \( \rho \) (g cm\(^{-3}\)) | \( H \) (mm) | \( \alpha \times 10^{-7} \) (m\(^2\) s\(^{-1}\)) | \( T_0 \) (°C) | \( T_{14} \) (°C) |
| 22.02.2018     | 650 0.244           | 650        | 13.61 | -1.84 | -11.28 |
| 27.02.2018     | 600 0.233           | 580        | 2.55  | -2.92 | -24.89 |
| 06.03.2018     | 580 0.271           | 370        | 4.64  | -2.79 | -8.5   |
| 14.03.2018     | 550 0.248           | 490        | 4.35  | -2.6  | -18.28 |
| 20.03.2018     | 530 0.262           | 530        | 5.95  | 0.05  | -7.8   |
| 27.03.2018     | 400 0.292           | 390        | 1.25  | 0.05  | -2.04  |
| 03.04.2018     | 380 0.312           | 380        | 5     | 0.05  | -9.9   |
| 11.04.2018     | 240 0.374           | 50         | 1.1   | 0.05  | 0.87   |

The snow cover depth and the thermal diffusivity coefficient were calculated for a time period from 22.02.2018 to 11.04.2018. The main criterion for the calculation is the presence of snow cover, which is calculated by the proposed algorithm.

The results of snow cover depth calculation, data of snow density and thickness measurement by OS-1 are presented in Figure 2.

![Figure 2](image)

Figure 2. Snow cover depth modeled (1), mm. Measured manually (2), mm. Snow density (3), kg/m\(^3\).

The calculation results of the thermal diffusivity coefficient are shown in Figure 3. The average thermal diffusivity value for the study period is \( 2.92 \times 10^{-7} \) m\(^2\) s\(^{-1}\). During the first 8 days of the experiment, there was a decrease in the diffusivity from \( 13.61 \times 10^{-7} \) to \( 2.55 \times 10^{-7} \) m\(^2\) s\(^{-1}\). The air temperature during this period varied from -12 to -25 °C.

Since March 15 there was an increase in the air temperature to +4 °C, which was accompanied by a short-term decrease in the thermal diffusivity values to \( 0.05 \times 10^{-7} \) m\(^2\) s\(^{-1}\).

Similar drops in the thermal diffusivity were observed on March 23, April 2 and 6. They are associated with the appearance of a significant amount of liquid moisture inside the snow thickness. At an air temperature close to 0 °C, the temperature of snow and air is equalized and when at the snow-air...
interface there is no layer of ice, the snow depth is calculated with an error. Since the change in the snow temperature near the soil surface for 24 hours is insignificant, the value of the coefficient reduced significantly. Since April 6th the air temperature has not fallen below 0°C, and the thermal diffusivity value was about $0.15 \times 10^{-7}$ m$^2$/s.

![Figure 3. Calculated thermal diffusivity coefficient ($\alpha$, m$^2$/s$^{-1}$) and air temperature (T, °C).](image)

According to the data of the manual snowmeter OS-1 measurement, the snow density during the experiment increased from 224 to 374 kg/m$^3$, and the snow cover depth decreased from 650 to 240 mm.

The dependence of the thermal diffusivity coefficient on the air temperature is inversely proportional in the absence of changes in the snow structure, i.e., when the temperature increases, the coefficient decreases.

According to the calculation data and conclusions [11], a sharp change in the thermal diffusivity is associated with the processes of water thawing or re-freezing, which characterizes changes in the snow structure.

4. Conclusions
The above-proposed method of calculation allows refining the values of snow cover depth calculated by a preliminary algorithm, as well as calculating the effective thermal diffusivity. The method requires a lot of calculations, which can be reduced by using the preliminary algorithm.

At a temperature close to 0°C and the absence of a layer of ice at the snow-air interface, the estimated snow cover depth has been highly understated. The calculation of the thermal diffusivity coefficient allows estimating the phase transitions occurring in the snow cover.

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