\( \pi-\eta \) mixing and charge symmetry violating NN potential in matter

Subhrajyoti Biswas, Pradip Roy, and Abhee K. Dutt-Mazumder
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata-700 064, INDIA
(Dated: February 26, 2010)

We construct density dependent Class III charge symmetry violating (CSV) potential due to mixing of \( \pi-\eta \) meson with off-shell corrections. The density dependence enters through the non-vanishing \( \pi-\eta \) mixing driven by both the neutron-proton mass difference and their asymmetric density distribution. The contribution of density dependent CSV potential is found to be appreciably larger than the contribution of vacuum CSV potential.

PACS numbers: 21.65.Cd, 13.75.Cs, 13.75.Gx, 21.30.Fe

I. INTRODUCTION

One of the interesting area of research in nuclear physics is the study of symmetries and their violation. The general goal of the research in this area is to find small but observable effects of charge symmetry violation (CSV) which might provide significant insight into the dynamics of CSV interaction.

CSV of the NN interaction refers to a difference between proton-proton and neutron-neutron interactions only. It is most clearly manifested in the \( {}^1S_0 \) scattering lengths i.e. the difference between \( pp \) and \( nn \) scattering lengths at \( {}^1S_0 \) state is non-zero \[1,3]. Other convincing evidence of CSV comes from the binding energy differences of mirror nuclei which is known as Okamoto-Nolen-Schifer (ONS) anomaly \[1,4]. The modern manifestation of CSV includes differences of neutron-proton form factors, hadronic correction to \( g-2 \) \[7], the observation of the decay of \( \Psi'(3686) \rightarrow (J/\Psi)\pi^0 \) etc \[8].

The current understanding of CSV is that at the fundamental level, caused by the finite mass difference between up (u) and down (d) quarks \[1,8-12\]. As a consequence, at the hadronic level, charge symmetry (CS) is violated due to non-degenerate mass of nucleons.

There are various mechanisms that can lead to CSV in NN interaction. For example, neutral mesons with same spin parity with different isospin can mix at the fundamental level due to quark mass difference (at the hadronic level due to neutron-proton mass splitting). The most important is the \( \rho-\omega \) mixing, which according to Ref. \[10,13-17\] is claimed to be successful to explain CSV observables. The other examples are \( \pi-\eta \) and \( \pi-\eta' \) mixing \[15,20\]. It is shown in Ref. \[21\] that \( \pi-\eta' \) is important as it is of opposite sign to \( \pi-\eta \) mixing where individual contribution is known to be small.

In all the previous works, the mixing amplitude is taken to be either constant or on-shell \[13,16\], which is not consistent for the construction of NN potential as the mixing amplitude has strong momentum dependence \[22,25\]. Even the \( \rho-\omega \) mixing amplitude changes sign as one moves away from \( \rho(\omega) \) pole to space-like region. It is important to note that the mixing amplitude in the space-like region is relevant for the construction of CSV potential.

Once the mixing amplitude is known, one can construct CSV potential by evaluating two body NN scattering diagram involving mixed intermediate states like \( \pi-\eta \) or \( \rho-\omega \). It is to be noted that external legs can also contribute to the CSV if one incorporates relativistic corrections as has recently been shown in Ref. \[29\].

In matter, there can be another source of symmetry breaking if the ground state contains unequal number of neutrons (n) and protons (p) giving rise to ground state induced mixing of various charged states like \( \rho-\omega \), \( \pi-\eta \), etc. meson even in the limit \( M_n = M_p \).

The matter induced mixing was studied in \[30-35\]. But none of these deal with the construction of two-body potential except \[36\] where density dependent CSV potential has been constructed considering only the effect of the scalar mean field on the nucleon mass excluding the possibility of matter driven mixing. Recently, the medium dependent CSV potential due to \( \rho-\omega \) mixing has been constructed in Ref. \[37\]. It is also to be noted that such mixing amplitudes, in asymmetric nuclear matter (ANM), have non-zero contribution even if the quark or nucleon masses are taken to be equal \[30,34\]. We consider both of these mechanisms to construct CSV NN potential due to \( \pi-\eta \) mixing.

Although the vacuum contribution of \( \pi-\eta \) mixing to CSV has been shown to be negligible as compared to \( \rho-\omega \) mixing, we, in this work, explore the role of medium dependent \( \pi-\eta \) mixing amplitude in constructing the CSV potential taking into account the contribution of external legs.

\[ \begin{array}{c|c|c} \pi_1 & \pi_2 & \pi_3 \\ \hline \Gamma_\pi & 1 & 2 \\ P_1 & \pi_1 & \pi_2 \\ P_2 & \pi_3 & P_4 & \pi_4 \\ \end{array} \]

\[ \begin{array}{c|c|c} \eta & \eta & \pi \\ \hline \Gamma_\eta & 1 & 2 \\ P_1 & \eta_1 & \pi_1 \\ P_2 & \eta_2 & \pi_2 \\ \end{array} \]

\[ \begin{array}{c|c|c} \eta' & \eta' & \pi \\ \hline \Gamma_\eta' & 1 & 2 \\ P_1 & \eta'_1 & \pi_1 \\ P_2 & \eta'_2 & \pi_2 \\ \end{array} \]

FIG. 1: Feynman Diagrams that contribute to the construction of CSV NN potential in matter. Solid lines represent nucleons and dashed lines stand for mesons. The crossed circles indicate the symmetry breaking piece.

Physically, in dense system, intermediate mesons
might be absorbed and re-emitted from the Fermi spheres. In symmetric nuclear matter (SNM) the emission and absorption involving different isospin states like \( \pi \) and \( \eta \) cancel when the contributions of both the proton and neutron Fermi spheres are added provided the nucleon masses are taken to be equal. In ANM, on the other hand, the unbalanced contributions coming from the scattering of neutron and proton Fermi spheres, lead to the mixing which depends both on the density \( (\rho_B) \) and the asymmetry parameter \( \alpha = (\rho_n - \rho_p)/\rho_B \). Inclusion of this process is depicted by the third diagram in Fig.1.

We present the formalism in Sec.II where the three momentum dependent \( \pi, \eta \) mixing amplitude is calculated to construct the CSV potential in matter. The numerical results are discussed in Sec.III. Finally, in Sec.IV, we summarise our results.

### II. FORMALISM

The following matrix element, which is required to construct the CSV \( NN \) potential is obtained from Fig.1

\[
\mathcal{M}_{\pi\eta}^{NN}(q) = [\bar{u}_N(p_3)\tau_3(1)\Gamma_\pi(q)u_N(p_1)]\Delta_\pi(q)\Pi_{\pi\eta}(q^2) \\
\times \Delta_\eta(q)[\bar{u}_N(p_4)\Gamma_\eta(-q)u_N(p_2)] \\
+ [\bar{u}_N(p_3)\Gamma_\pi(q)u_N(p_1)]\Delta_\eta(q)\Pi_{\pi\eta}(q^2) \\
\times \Delta_\pi(q)[\bar{u}_N(p_4)\tau_3(2)\Gamma_\pi(-q)u_N(p_2)].
\]

(1)

Here \( u_N \)'s represent Dirac spinors, \( \Pi_{\pi\eta}(q^2) \) is the \( \pi, \eta \) mixing amplitude, \( p_i, (i = 1 - 4) \) and \( q \) are the four momenta of nucleon and meson, respectively. \( \tau_3(1) \) and \( \tau_3(2) \) are isospin operators at vertices '1' and '2' (see Fig.1). The vertex factor is denoted by \( \Gamma_j(q), (j = \pi, \eta) \) and \( \Delta_j(q) \) stands for meson propagator given by

\[
\Delta_j^{-1}(q^2) = q^2 - m_j^2.
\]

In the limit \( q_0 \to 0 \), Eq.(1) gives the momentum space CSV \( NN \) potential, \( V_{CSV}^{NN}(q) \).

\[\begin{align*}
\pi & \quad \eta \\
p - \text{loop} & \quad \pi & \quad \eta \\
n - \text{loop}
\end{align*}\]

FIG. 2: The mixing amplitude is generated by the difference between proton and neutron loops.

In the present calculation mixing is assumed to be generated by the \( NN \) loops and the mixing amplitude \( \Pi_{\pi\eta}(q^2) \) is generated by the difference between proton and neutron loop contributions as shown in Fig.2

\[
\Pi_{\pi\eta}(q^2) = \Pi_{\pi\eta}^{\text{pro}}(q^2) - \Pi_{\pi\eta}^{\text{nn}}(q^2),
\]

(3)

where \( \Pi_{\pi\eta}^{\text{pro}}(q^2) \) or \( \Pi_{\pi\eta}^{\text{nn}}(q^2) \) is the \( \pi, \eta \) mixing self-energy. The origin of the relative sign between proton and neutron loops in Eq.(3) is due to the different signs involved in the coupling of \( \pi^0 \) and \( \eta \) to proton and neutron. The one loop contribution to the mixing self-energy is given by

\[
\Pi_{\pi\eta}^{\text{med}}(q^2) = \frac{d^4k}{(2\pi)^4}\text{Tr}\frac{\Gamma_\pi(q)G_N(k)\Gamma_\eta(-q)G_N(k+q)}{k^2 - M_N^2 + i\epsilon},
\]

(4)

where the subscript \( N \) stands for nucleon index (i.e. \( N = p \) or \( n \)), \( k = (k_0, \mathbf{k}) \) denotes the four momentum of the nucleon in the loops. The main ingredient of our calculation is the in-medium nucleon propagator \( G_N \) which consists of a free \( (G_N^0) \) and a density dependent \( (G_N^D) \) parts

\[
G_N^D(k) = \frac{iN}{E_N(k^2 + M_N^2)} \delta(k_0 - E_N(k^0 - |\mathbf{k}|)).
\]

(5b)

\( E_N = \sqrt{M_N^2 + k^2} \) is the nucleon energy where \( k_N \) and \( M_N \) denote Fermi momentum and mass of the nucleon, respectively. Note that delta function in Eq.(5b) indicates the nucleons are on-shell while \( \theta(k_0 - |\mathbf{k}|) \) ensures that propagating nucleons have momentum less than \( k_N \).

\[\begin{align*}
\pi & \quad \eta \\
p(n) - \text{loop} & \quad \pi & \quad \eta \\
\Pi_{\pi\eta}^{\text{med}} & \quad \Pi_{\pi\eta}^{\text{vac}}
\end{align*}\]

FIG. 3: The mixing self-energy contains a vacuum part and a medium part.

Likewise the in-medium nucleon propagator \( G_N \) the mixing self-energy \( \Pi_{\pi\eta}^{\text{med}}(q^2) \) contains a vacuum \( \Pi_{\pi\eta}^{\text{med}}(q^2) \) and a density dependent \( \Pi_{\pi\eta}^{\text{med}}(q^2) \) parts as shown in Fig.3. It is to be noted that the density dependent part is given by the combination of \( G_N^0G_N^0 + G_N^D\Pi_{\pi\eta}^{\text{med}} \) that we have discussed already, whereas the term proportional to \( G_N^D \) vanishes for low energy excitation. The vacuum part, \( \Pi_{\pi\eta}^{\text{vac}}(q^2) \) on the other hand involves \( G_N^0G_N^D \) which gives rise to usual CSV part of the potential due to the splitting of the neutron and proton mass.

The vacuum mixing contribution of CSV \( NN \) potential can be used to calculate the difference between \( nn \) and \( pp \) scattering lengths, \( \Delta a = a_{pp} - a_{nn} \), at \( ^1S_0 \) state \( \text{[29]} \).

#### A. Pseudoscalar coupling

First we consider pseudoscalar (PS) coupling of nucleons to the mesons to describe \( \pi NN \) and \( \eta NN \) inter-
actions which are represented by the following effective Lagrangians:

\[
\mathcal{L}^\text{PS}_{NN} = -i g_\rho \bar{\Psi} \gamma_5 \tau \cdot \Phi \Psi, \\
\mathcal{L}^\text{PS}_\eta NN = -i g_\eta \bar{\Psi} \gamma_5 \Phi \Psi, \\
\]

(6a) \hspace{2cm} (6b)

where \(\Psi\) and \(\Phi\) represent the nucleon and meson fields, respectively, and \(g_j\)'s stand for the meson-nucleon coupling constants. For PS coupling the vertex factor \(\Gamma_j = -i g_j \gamma_5\), \((j = \pi, \eta)\).

Now we proceed to calculate the CSV NN potential. For this purpose we need to calculate the \(\pi-\eta\) mixing self-energy using Eq. (4). First we consider mixing in vacuum. After performing trace calculation, the vacuum contribution of \(\pi-\eta\) mixing self-energy is found to be

\[
\Pi^{(N)}_\text{vac}(q^2) = 4 i g_\rho g_\eta \int \frac{d^4k}{(2\pi)^4} \left[ \frac{M_N^2 - k \cdot (k + q)}{(k^2 - M_N^2)(k + q)^2 - M_N^2(1 + \epsilon - \gamma_E + \ln(4\pi\mu^2))} \right]. \\
\]

(7)

From the dimensional counting it is found that the integral of Eq. (7) is divergent. We use dimensional \([40–42]\) regularization to isolate the singularities in Eq. (7) which reduces to \([20]\)

\[
\Pi^{(N)}_\text{vac}(q^2) = \frac{g_\rho g_\eta}{4\pi^2} \left[ \frac{q^2}{3} + \left( M_N^2 - \frac{q^2}{2} \right) \left( 1 + \frac{1}{\epsilon} - \gamma_E + \ln(4\pi\mu^2) \right) \right] \\
- \int_0^1 dx [M_N^2 - 3q^2x(1-x)] \ln(M_N^2 - q^2x(1-x)]. \\
\]

(8)

In Eq. (8) \(\mu\) is an arbitrary scale parameter, \(\gamma_E\) is the Euler-Mascheroni constant and \(\epsilon = 1 - D/2\), where \(D\) stands for the dimension of the integral. Notice, \(\epsilon\) in Eq. (8) contains the singularity and it diverges as \(D \to 4\). The divergences of Eq. (8) can be removed by adding appropriate counterterms \([43]\).

It is clear from Eq. (8) that unlike \(\rho-\omega\) mixing amplitude, the singularities cannot be removed by simply subtracting the neutron loop contribution from the proton loop contribution. This is because of the singular term proportional to the mass term. But one can eliminate this singular term by subtracting \(\Pi^{(N)}_\text{vac}(q^2 = 0)\) from \(\Pi^{(N)}_\text{vac}(q^2)\) which is

\[
\tilde{\Pi}^{(N)}_\text{vac}(q^2) = \Pi^{(N)}_\text{vac}(q^2) - \Pi^{(N)}_\text{vac}(q^2 = 0) \\
= \frac{g_\rho g_\eta}{4\pi^2} \left[ \frac{q^2}{3} + M_N^2 \ln M_N^2 \right] \\
- \frac{q^2}{2} \left( 1 + \frac{1}{\epsilon} - \gamma_E + \ln(4\pi\mu^2) \right) \\
- \int_0^1 dx [M_N^2 - 3q^2x(1-x)] \ln(M_N^2 - q^2x(1-x)]. \\
\]

(9)

Note that \(\tilde{\Pi}^{(N)}_\text{vac}(q^2)\) is not finite but the divergent part proportional to the mass term has been removed. Now one can easily obtain finite \(\pi-\eta\) mixing amplitude in vacuum by subtracting \(\tilde{\Pi}^{(N)}_\text{vac}(q^2)\) from \(\tilde{\Pi}^{(p)}_\text{vac}(q^2)\).

\[
\Pi^{PS}_\text{vac}(q^2) = \frac{g_\rho g_\eta}{4\pi^2} \left[ \frac{q^2}{2} \ln \left( \frac{M_p}{M_n} \right) \right] \\
+ q \sqrt{4M_p^2 - q^2} \tan^{-1} \left( \frac{q}{\sqrt{4M_p^2 - q^2}} \right) \\
- q \sqrt{4M_n^2 - q^2} \tan^{-1} \left( \frac{q}{\sqrt{4M_n^2 - q^2}} \right). \\
\]

(10)

Eq. (10) is \(q^2\) dependent \(\pi-\eta\) mixing amplitude. We obtain \(\Pi^{PS}_\text{vac}(q^2 = m_p^2) = -1197 \text{ MeV}^{-2}\) while experimentally it is found that \(\Pi^{PS}_\text{vac}(q^2 = m_n^2) = -4200 \text{ MeV}^{-2}\) \([20]\). In this equation we substitute \(q_0 = 0\) to obtain three momentum dependent mixing amplitude \(\Pi^{PS}_\text{vac}(q^2)\) which is required for the construction of CSV potential.

Now expanding the mixing amplitude \(\Pi^{PS}_\text{vac}(q^2)\) in terms of \(q^2/M_N^2\) and keeping the lowest order we obtain

\[
\Pi^{PS}_\text{vac}(q^2) = -a_1 q^2, \\
\]

(11)

where \(a_1 = \frac{g_\rho g_\eta}{4\pi^2} \ln \left( \frac{M_p}{M_n} \right)\) and if \(M_p = M_n\), mixing amplitude in vacuum i.e. \(\Pi^{PS}_\text{vac}(q^2)\) vanishes. This implies that CSV NN potential in vacuum does not exist for \(M_p = M_n\).

Now we calculate the density dependent part of the \(\pi-\eta\) mixing self-energy which is denoted by \(\Pi^{(N)}_\text{med}(q^2)\). After performing trace calculation and \(k_0\) integration it, reads

\[
\Pi^{(N)}_\text{med}(q^2) = -8g_\rho g_\eta \int_0^1 \frac{d^3k}{(2\pi)^3} E_N \left[ \frac{(k \cdot q)^2}{4q^4 - 4(k \cdot q)^2} \right] \theta(k_N - |k|). \\
\]

(12)

The above equation with the substitution \(E_N \simeq M_N\) and \(q_0 = 0\) yields

\[
\Pi^{(N)}_\text{med}(q^2) = \frac{g_\rho g_\eta}{\pi^2 M_N} \left[ \frac{k_N^3}{3} - \frac{q^2 k_N}{8} \right] \\
- \frac{q}{8} \left( k_N^2 - \frac{q^2}{4} \right) \ln \left( \frac{q^2 + 2k_N}{q^2 - 2k_N} \right). \\
\]

(13)

Eq. (13) represents three momentum dependent medium part of the \(\pi-\eta\) mixing self-energy. The mixing amplitude, as mentioned earlier, is again generated by the difference between contributions from the proton and neutron loops. This medium part of the \(\pi-\eta\) mixing amplitude, after suitable expansion in terms of \(\frac{1}{k_N^2}\) reads

\[
\Pi^{PS}_\text{med}(q^2) = a'_1 - b'_1 q^2, \\
\]

(14)
where the leading order contribution has been considered and

\[ a'_1 = \frac{g_\pi g_a}{3\pi^2} \left( \frac{k_p^3}{M_p^2} - \frac{k_n^3}{M_n^2} \right), \]  
(15a)

\[ b'_1 = \frac{g_\pi g_a}{4\pi^2} \left( \frac{k_p}{M_p} - \frac{k_n}{M_n} \right). \]  
(15b)

Following Eq. (11) and considering the contributions of external nucleon legs one obtains the momentum space potential given as

\[ V_{CSV}^{NN}(q^2) = T^+_{3}\frac{g_\pi g_\eta}{4M_N} (\sigma_1 \cdot q)(\sigma_2 \cdot q) \]
\[ \times \left\{ \frac{\Pi^{PS}(q^2)}{(q^2 + m_\pi^2)(q^2 + m_\eta^2)} \right\} \]
\[ \times \left[ 1 - \frac{q^2}{8M_N^2} - \frac{P^2}{2M_N^2} \right], \]
(16)

where \( T^+_{3} = \tau_3(1) + \tau_3(2) \) and \( P = (p_1 + p_3)/2 = (p_2 + p_4)/2 \). Eq. (16) presents CSV class II potential in momentum space. In contrast to \( \pi\eta \) mixing, \( \rho\omega \) mixing produces both class II and class IV NN interactions \[ \text{[20, 23]} \). Note that the terms within the square bracket in Eq. (16) are the contributions from the external legs.

The coordinate space CSV NN potential is obtained by Fourier transformation of Eq. (16).

\[ V_{CSV}^{NN}(r) = V_{vac}^{NN}(r) + V_{med}^{NN}(r), \]  
(17)

where \( V_{vac}^{NN}(r) \) represents CSV NN potential in vacuum and \( V_{med}^{NN}(r) \) is the CSV NN potential due to density driven mixing. Explicitly \( V_{vac}^{NN}(r) \) and \( V_{med}^{NN}(r) \) are given by

\[ V_{vac}^{NN}(r) = -T^+_{3}\frac{g_\pi g_\eta a_1}{48\pi M_N^2} \left[ m_\pi^5 U(x_{\pi}) - m_\eta^5 U(x_\eta) \right], \]  
(18a)

\[ V_{med}^{NN}(r) = -T^+_{3}\frac{g_\pi g_\eta}{48\pi M_N^2} \left[ a'_1 \left( m_\pi^5 U(x_{\pi}) - m_\eta^5 U(x_\eta) \right) \right] \]
\[ + \left( \frac{a'_1}{8M_N^2} + b'_1 \right) \left( m_\pi^5 U(x_{\pi}) - m_\eta^5 U(x_\eta) \right), \]  
(18b)

Here

\[ U(x_i) = Y_0(x_i)(\sigma_1 \cdot \sigma_2) + S_{12}(\hat{r})Y_2(x_i), \]  
(19a)

\[ Y_2(x_i) = \left( 1 + \frac{3}{x_i} + \frac{3}{x_i^2} \right) Y_0(x_i) \]  
(19b)

\[ S_{12}(\hat{r}) = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - (\sigma_1 \cdot \sigma_2) \]  
(19c)

where \( x_i = m_i r, i = \pi, \eta \) and \( Y_0(x_i) = \frac{e^{-x_i}}{x_i}. \)

Since mesons and nucleons are not point particles and they have internal structures one needs to incorporate vertex corrections which, in principle, can be calculated using renormalizable models based on hadronic degrees of freedom. In the present calculation following phenomenological form factors have been used to incorporate the vertex corrections,

\[ F_i(q^2) = \left( \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 + q^2} \right)^\eta, \]  
(20)

Here \( \Lambda_i \) is the cut-off parameter. With the inclusion of form factors Eqs. (18a) and (18b) reduce to

\[ V_{vac}^{NN}(r) = -T^+_{3}\frac{g_\pi g_\eta a_1}{48\pi M_N^2} \left[ a'_1 \left( \frac{a_\pi m_\pi^2 U(x_{\pi}) - a_\eta m_\eta^2 U(x_\eta)}{m_\pi^2 - m_\eta^2} \right) \right] \]
\[ + \left( \frac{a'_1}{8M_N^2} + b'_1 \right) \left( \frac{a_\pi m_\pi^2 U(x_{\pi}) - a_\eta m_\eta^2 U(x_\eta)}{m_\pi^2 - m_\eta^2} \right), \]  
(21)

and

\[ V_{med}^{NN}(r) = -T^+_{3}\frac{g_\pi g_\eta}{48\pi M_N^2} \left[ a'_1 \left( \frac{a_\pi m_\pi^2 U(x_{\pi}) - a_\eta m_\eta^2 U(x_\eta)}{m_\pi^2 - m_\eta^2} \right) \right] \]
\[ + \left( \frac{a'_1}{8M_N^2} + b'_1 \right) \left( \frac{a_\pi m_\pi^2 U(x_{\pi}) - a_\eta m_\eta^2 U(x_\eta)}{m_\pi^2 - m_\eta^2} \right), \]  
(22)

where \( X_i = \Lambda_i r \) and

\[ a_i = \left( \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - m_i^2} \right), \]  
(23a)

\[ b_i = \left( \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - m_i^2} \right), \]  
(23b)

\[ \lambda = \left( \frac{m_\pi^2 - m_\eta^2}{\Lambda_i^2 - \Lambda_j^2} \right). \]  
(23c)

B. Pseudovector coupling

Now we consider pseudovector (PV) coupling of nucleons to the mesons to describe \( \pi NN \) and \( \eta NN \) interactions which are represented by the following effective Lagrangians:

\[ \mathcal{L}_{\pi NN}^{{PV}} = -\frac{g_\pi}{2M_N} \bar{\Psi} \gamma_5 \gamma^\mu \partial_\mu \tau \cdot \Phi_\pi \Psi, \]  
(24a)

\[ \mathcal{L}_{\eta NN}^{{PV}} = -\frac{g_\eta}{2M_N} \bar{\Psi} \gamma_5 \gamma^\mu \partial_\mu \Phi_\eta \Psi, \]  
(24b)
where $\Psi$, $\Phi$, $\tau$ and $g$'s have already been defined in the previous subsection, and the vertex factors $\Gamma_j = ig_j\gamma^\mu q\nu$. The mixing self-energy in vacuum is given by

$$
\Pi_{\text{vac}}^{(N)}(q^2) = 4i \left( \frac{g_{\pi}}{2M_N} \right) \left( \frac{g_\eta}{2M_N} \right) \int \frac{d^4k}{(2\pi)^4} \times \frac{q^2 (M_N^2 - k \cdot (k+q) - 2q \cdot (k+q) (k \cdot q))}{(k^2 - M_N^2)((k+q)^2 - M_N^2)} \right).
$$

Note that the above integral is also divergent and we use dimensional regularization to isolate singularities which reduces to

$$
\Pi_{\text{vac}}^{(N)}(q^2) = \frac{g_{\pi} g_\eta}{8\pi^2} \left[ -\frac{1}{\epsilon} + \gamma_E - \ln(4\pi\mu^2) \right.
+ \left. \int_0^1 dx \ln(M_N^2 - q^2 x(1-x)) \right] q^2,
$$

where $\epsilon$, $\mu$ and $\gamma_E$ have been discussed previously. It is to be noted that unlike PS coupling, singularity in Eq. (26) is not proportional to the mass term. Therefore, simple subtraction of the neutron loop contribution from the proton loop contribution like $\rho$-$\omega$ mixing in vacuum will remove the divergent parts. Thus the finite $\pi$-$\eta$ mixing amplitude in vacuum is found to be

$$
\Pi_{\text{vac}}^{PV}(q^2) = \frac{g_{\pi} g_\eta}{4\pi^2} \left[ q^2 \ln \left( \frac{M_\pi}{M_N} \right) - q \sqrt{4M_p^2 - q^2} \tan^{-1} \left( \frac{q}{\sqrt{4M_p^2 - q^2}} \right)
- q \sqrt{4M_n^2 - q^2} \tan^{-1} \left( \frac{q}{\sqrt{4M_n^2 - q^2}} \right) \right].
$$

Similar to PS coupling, leading order contribution of the mixing amplitude in vacuum given by

$$
\Pi_{\text{vac}}^{PV}(q^2) = -a_2 q^2,
$$

where $a_2 = \frac{g_{\pi} g_\eta}{4\pi^2} \ln \left( \frac{M_\pi}{M_N} \right)$. Notice, the leading order contributions of $\pi$-$\eta$ mixing amplitudes in vacuum are same for both PS and PV coupling.

The three momentum dependent medium part of $\pi$-$\eta$ mixing self-energy reads

$$
\Pi_{\text{med}}^{(N)}(q^2) = \frac{g_{\pi} g_\eta}{8\pi^2 M_N} \left[ k_N^2 - \frac{q^2}{4} \right] \ln \left( \frac{q + 2k_N}{q - 2k_N} \right),
$$

where

$$
\Pi_{\text{med}}^{PV}(q^2) = -b_2 q^2,
$$

The leading order contribution of the medium part of the mixing amplitude, generated by the difference between proton and neutron loop contributions is

$$
\Pi_{\text{med}}^{PV}(q^2) = -b_2 q^2.
$$

Note, the density dependent mixing amplitude in PV coupling differs with that of PS coupling only by the term $a_1'$ and this will make difference between the medium part of the CSV potentials. The momentum space potential is given by

$$
V_{\text{CSV}}^{\pi\eta}(q^2) = T_3 + \frac{g_{\pi} g_\eta}{4(M_N^2 - M_\pi^2)} \left[ m_\pi^2 U(x_\pi) - m_\eta^2 U(x_\eta) \right]
\times \left[ 1 - \frac{q^2}{8M_N^2} - \frac{P^2}{2M_N^2} \right].
$$

In deriving Eq. (32) contributions from external leg have been considered which is given within the square braketch. These contributions are same as that of PS coupling. From this momentum space CSV $NN$ potential one can obtain the coordinate space potential.

$$
V_{\text{vac}}^{NN}(r) = -T_3 + \frac{g_{\pi} g_\eta a_2}{48\pi M_N^2} \left[ m_\pi^2 U(x_\pi) - m_\eta^2 U(x_\eta) \right],
$$

$$
V_{\text{med}}^{NN}(r) = -T_3 + \frac{g_{\pi} g_\eta b_2'}{48\pi M_N^2} \left[ m_\pi^2 U(x_\pi) - m_\eta^2 U(x_\eta) \right].
$$

The Eqs. (33a) and (33b) shows the coordinate space CSV $NN$ potential without form factors. It is to be noted that the CSV potentials in vacuum are same for both PS and PV couplings while density dependent parts are different. With form factors Eqs. (33a) and (33b) reduce to

$$
V_{\text{vac}}^{NN}(r) = -T_3 + \frac{g_{\pi} g_\eta a_2}{48\pi M_N^2} \left[ a_\pi m_\pi^2 U(x_\pi) - a_\eta m_\eta^2 U(x_\eta) \right]
- \lambda \left( b_\pi m_\pi^2 U(x_\pi) - b_\eta m_\eta^2 U(x_\eta) \right),
$$

$$
V_{\text{med}}^{NN}(r) = -T_3 + \frac{g_{\pi} g_\eta b_2'}{48\pi M_N^2} \left[ a_\pi m_\pi^2 U(x_\pi) - a_\eta m_\eta^2 U(x_\eta) \right]
- \lambda \left( b_\pi m_\pi^2 U(x_\pi) - b_\eta m_\eta^2 U(x_\eta) \right).
$$
III. RESULTS

In this section we present our numerical results. All the figures show the difference between CSV $nn$ and $pp$ potentials in $^3S_0$ state. To obtain density dependent CSV potential we consider nuclear matter density $\rho_B = 0.148 \text{ fm}^{-3}$ and asymmetry parameter $\alpha = 1/3$.

![Figure 4](image1.png)

**FIG. 4:** Difference between CSV $nn$ and $pp$ potentials in momentum space are shown.

![Figure 5](image2.png)

**FIG. 5:** Difference between CSV $nn$ and $pp$ potentials in coordinate space without form factors.

![Figure 6](image3.png)

**FIG. 6:** Difference between CSV $nn$ and $pp$ potentials in coordinate space with form factors

The CSV potential in coordinate space is presented in Fig. 5. In this figure we show the vacuum and medium contribution of the CSV potential without form factors. The same with the form factors are demonstrated in Fig. 6. Both the vacuum and medium parts contribute with the same sign. Note that CSV potentials change sign with the inclusion of form factors. Fig. 4 and Fig. 6 show that the medium contribution near the core region is much larger than the vacuum contribution.

IV. SUMMARY AND DISCUSSION

In the present work we have constructed CS violating density dependent two-body potential driven by the mixing of $\pi$-$\eta$ states. It is observed that density dependent contribution is larger than the vacuum contribution near the core region. This density dependent part might contribute significantly to the CSV observables. We estimate the contribution of $\pi$-$\eta$ mixing to the difference of $pp$ and $nn$ scattering lengths, $\Delta a$, where only the vacuum part contributes. Both for the density dependent and vacuum parts, we find that the role of $\pi$-$\eta$ mixing is smaller than that of $\rho$-$\omega$ mixing [29, 37].
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