Inflatonless Inflation

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Abstract

We consider a $4 + N$ dimensional Einstein gravity coupled to a non-linear sigma model. This theory admits a solution in which the $N$ extra dimensions contract exponentially while the ordinary space expand exponentially. Physically, the non-linear sigma fields induce the dynamical compactification of the extra dimensions, which in turn drives inflation. No inflatons are required.

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I. INTRODUCTION

Inflation has been the most convincing scheme to solve the horizon, flatness and monopole problems \([1, 2]\). Usually, theoretical models of inflation involve some kind of scalar fields called inflatons, whose potential energy dominates over the kinetic energy during the phase of accelerated expansion \([3]\). The stage right after inflation is called reheating \([4, 5]\) at which these inflatons decay into all other observed particles. As pointed out in \([6]\), inflationary models involving inflatons generally suffer from issues of fine-tuning and initial singularity. This appears to call for the exploration of possible alternatives to the conventional inflation schemes \([7–10]\). However, it is fair enough to say that the standard scalar field inflation is the best scenario proposed so far. Despite this fact, we think that it is still important to look for other alternatives. For instance, a direct question could be: Can inflation happen without inflatons? This question was also addressed in \([11]\) by assuming that the universe has undergone cascading energy transitions, but these authors did not provide a specific model that realized the assumption.

On the other hand, superstring theory \([12]\) proposes that we are actually living in a ten-dimensional spacetime, six of which are compactified. The natural size of these compactified extra dimensions is expected to be of the same order as the Planck length. This explains why we have never observed them at the energy scale that we have been probing. But at the same time, this immediately poses a fundamental question: Does the compactification of these extra dimensions have a dynamical origin? Some interesting works concerning this issue include \([13–15]\).

The purpose of this article is to search for a simultaneous solution to both of the above questions. Similar attempts by invoking a \(4 + N\) dimensional Einstein gravity with or without matter sources, where \(N\) is the total number of extra spatial dimensions, have been conducted in \([16–19]\). However, a common problem associated with these schemes is that they cannot lead to an exponential expansion of the ordinary space.

In this article, we will consider a \(4 + N\) dimensional Einstein gravity coupled to a non-linear sigma model. A similar model was first proposed in \([20–22]\) to induce a spontaneous compactification of the extra dimensions. It was then utilized to induce both compactification and “inflation” in \([23]\). But again the expansion of the ordinary space is not exponential. In order to resolve this problem, we will perform a simple extension to the original model.
As a consequence, we are able to find a cosmological solution where exponential inflation and dynamical compactification of the extra dimensions occur simultaneously.

In fact, since the work of [19-22], the idea of the connection between inflation and dynamical compactification has been continually explored, particularly in string inflation models as reviewed in [23]. However, what we have set out to achieve is not to improve any of these string inflation models which contain inflatons in our 4D spacetime in their low-energy effective Lagrangian. The main achievement of our article is to provide a model in which inflation happens without inflatons, but instead is solely driven by the dynamical compactification of extra dimensions. This was precisely what [19-22] had hoped to achieve.

The article is organized as follows. In Section II, we will construct our model and obtain the equations of motion. In Section III, we obtain a simple solution in which the extra dimensions contract exponentially while the ordinary space expand exponentially. Finally, we will draw our conclusions and include some discussions in Section IV.

II. THE MODEL

Before getting into the dynamics by specifying an action for our model, we will first lay out the geometric part of the Einstein field equations. We start with a generalized Friedmann-Robertson-Walker (FRW) metric in a $4 + N$ spacetime:

$$ds^2 = dt^2 - a^2(t) g_{ij} dx^i dx^j - b^2(t) g_{mn} dy^m dy^n,$$

where $i, j = 1, 2, 3$ and $m, n = 5, 6, ..., 4 + N$, $a(t)$ and $b(t)$ are the scale factors associated with the ordinary and extra (spatial) dimensions respectively. For instance, we have $N = 6$ in superstring theory. However, to retain the generality of our study, we will keep $N$ unspecified.

The various components of the Ricci tensor $R_{AB}$ resulting from the above metric are

$$R_{00} = -3 \frac{\dddot{a}(t)}{a(t)} - N \frac{\dddot{b}(t)}{b(t)},$$

$$R_{ij} = a^2(t) g_{ij} \left\{ \dot{H}_a(t) + [3 H_a(t) + N H_b(t)] H_a(t) \right\},$$

$$R_{mn} = b^2(t) g_{mn} \left\{ \dot{H}_b(t) + [3 H_a(t) + N H_b(t)] H_b(t) + \frac{2K}{b^2(t)} \right\},$$

where $K$ is the possible constant curvature of the compact extra dimensions and

$$H_a(t) = \frac{\dot{a}(t)}{a(t)} ; \quad H_b(t) = \frac{\dot{b}(t)}{b(t)}.$$
As anticipated, our theory is an extension of the model proposed by [20–23] in which the \(4+N\) dimensional Einstein gravity is coupled to a non-linear sigma model. Our \(4+N\) dimensional action is given by

\[
S = \int d^{4+N}Z \sqrt{G} \left[ -\frac{1}{2} M^{2+N} R + \frac{1}{\lambda^2} G^{AB} h_{CD}(\phi) \partial_A \phi^C \partial_B \phi^D + g G^{AB} h_{AB}(\phi) (\partial_C \phi^C)^2 \right],
\]

(2.6)

where \(Z = \{t, x^i, y^m\}\), \(G_{AB}\) and \(R\) are the metric and the Ricci scalar of the \(4+N\) dimensional spacetime respectively, \(1/\lambda^2\) and \(g\) are coupling constants, \(M\) is the \((4+N)\) dimensional Planck mass, \(\phi^C\) are the non-linear sigma fields, and the target space manifold metric \(h_{CD}\) determines the dynamics of the non-linear sigma fields. We choose the number of non-linear sigma fields to be exactly the same as the dimension of the spacetime. Thus, throughout the entire article, both of the spacetime and target-space indices run from 1 to \(4+N\). We note in passing that this choice for the number of non-linear sigma fields is crucial for the feasibility of our model.

The last term in the action, \(g G^{AB} h_{AB}(\phi) (\partial_C \phi^C)^2\), is a new term that we have added to the original model in [20–23]. As we will see, it plays a crucial role in giving the simultaneous solution to inflation and dynamical compactification. However, this new term in the action is not generally covariant. It is because the spacetime and target-space indices have been contracted in a mixed way. In order to make the action covariant, we begin by imposing the following ansatz:

\[
\begin{align*}
\phi^0 &= 0, \\
\phi^i &= 0, \\
\phi^m &= \alpha y^m + \text{constant,}
\end{align*}
\]

(2.7) \hspace{1cm} (2.8) \hspace{1cm} (2.9)

where \(\alpha\) is a characteristic constant carrying the dimension of inverse length squared. This is precisely the same ansatz that was imposed by [20–23] to induce dynamical compactification and hence inflation, although they failed to generate an exponential inflation. Physically, it means that we have identified the non-linear sigma fields with the coordinates of the extra dimensions. This is intuitively conceivable because the non-linear sigma fields themselves are coordinates of the target space manifold. Mathematically, this ansatz is justified as the non-linear sigma fields are functions of the spacetime coordinates, and it simply specifies an
explicit dependence of the non-linear sigma fields on the spacetime coordinates. Interestingly, 't Hooft made a similar ansatz in [25] for other reasons.

The above ansatz also implies that the non-linear sigma fields are totally independent of the ordinary 4D spacetime coordinates. Thus, they only live in the extra dimensions. Except for the metric associated the ordinary space, everything in the theory is independent of the ordinary 4D spacetime coordinates. Note that the "constant" appearing in (2.9) has been set to zero in [20–23]. While we keep it for generality, none of the results in our work or in [20–23] depend on them.

In the light of the above ansatz, the spacetime indices coincide exactly with the target-space indices. Now, the action becomes

\[
S' = \int d^{4+N} Z \sqrt{G} \left[ -\frac{1}{2} M^{2+N} R + \frac{\alpha^2}{\lambda^2} G^{AB} h_{CD}(z) \partial_A z^C \partial_B z^D + g \alpha^2 G^{AB} h_{AB}(z) (\partial_C z^C)^2 \right],
\]

(2.10)

where \( z^C = 0 \) for \( C = 0, 1, 2, 3 \) and \( z^C = y^C \) for \( C = 5, 6, ..., 4 + N \). Originally, \( h_{AB}(\phi) \) was a tensor in the target space and depended on the non-linear sigma fields. Once the ansatz is imposed, \( h_{AB}(z) \) is solely dependent on the spacetime coordinates. However, the operation of the ansatz, which is mathematically consistent, should not affect the tensor identity of \( h_{AB} \). Thus, we expect the components of \( h_{AB}(z) \) to be proportional to components of \( G_{AB} \). This will be confirmed later on when we solve for the actual expressions for \( h_{AB}(z) \). Also, there is now no more uncontracted spacetime indices or target-space indices in \( S' \). As a result, the action \( S' \) is generally covariant. In this article, we will simply assume that there exists a more fundamental theory that leads to \( S' \) in the low energy limit.

III. INFLATION FROM DYNAMICAL COMPACTIFICATION

The equations of motion following from the variation of the action \( S' \) with respect to the metric \( G^{AB} \) is given by

\[
R_{AB} = \frac{2 \alpha^2}{M^{2+N}} \left( \frac{1}{\lambda^2} h_{CD}(z) \partial_A z^C \partial_B z^D + g h_{AB}(z) (\partial_C z^C)^2 \right),
\]

(3.1)
which leads to following components

\[ R_{00} = \frac{2 \alpha^2}{M^{2+N}} N^2 g h_{00}, \quad (3.2) \]
\[ R_{ij} = \frac{2 \alpha^2}{M^{2+N}} N^2 g h_{ij}, \quad (3.3) \]
\[ R_{mn} = \frac{2 \alpha^2}{M^{2+N}} \left( \frac{1}{\lambda^2 + N^2 g} \right) h_{mn}. \quad (3.4) \]

It is clear that if \( g = 0 \), then \( R_{00} = R_{ij} = 0 \), which is in agreement with \cite{20, 23}. In that case, we will have a non-exponential expansion of the ordinary space which is unacceptable. Hence, the new term we introduced to the original model will be a hope for a successful inflation.

To have a simultaneous solution to both inflation and dynamical compactification, we expect a cosmological solution with the extra dimensions contract exponentially while the ordinary space expands exponentially, namely

\[ a(t) = a_0 e^{H(t-t_0)}; \quad b(t) = b_0 e^{-h(t-t_0)}. \quad (3.5) \]

By Einstein’s field equation, the various components derived from (3.1) represent the matter source that leads to the geometry realized by the metric (2.1). Thus, we proceed to solve the equations of motion (3.2), (3.3) and (3.4) by using the geometry of the metric (2.1). This requires matching (3.2) with (2.2), (3.3) with (2.3), (3.4) with (2.4), while having the solution (3.5) admitted at the same time. A consistent solution requires the metric of the target space manifold to satisfy:

\[ h_{00} = -\frac{M^{2+N}}{2 N^2 g \alpha^2} \left( 3 H^2 + N h^2 \right), \quad (3.6) \]
\[ h_{ij} = \frac{M^{2+N}}{2 N^2 g \alpha^2} (3 H - N h) H a_0^2 e^{2H(t-t_0)} g_{ij}, \quad (3.7) \]
\[ h_{mn} = \frac{M^{2+N}}{2 \left( 1/\lambda^2 + N^2 g \right) \alpha^2} \left[ -(3 H - N h) h b_0^2 e^{-2h(t-t_0)} + 2 K \right] g_{mn}. \quad (3.8) \]

The above expressions for \( h_{AB}(z) \) verify what we have stated earlier, namely blocks of \( h_{AB}(z) \) are proportional to blocks of \( G_{AB} \).

In fact, a simple inspection of the above equations reveals that we can actually further simplify our solution in a physically motivated way. The strategy goes as follows. Since the extra dimensions are contracting exponentially while the three ordinary dimensions are expanding exponentially, the total spatial volume \( V \) of the \( 3 + N \) dimensional space will be
given by

\[ V \propto e^{(3H-Nh)t}. \] (3.9)

Imposing \(3H - Nh = 0\) keeps the total spatial volume \(V\) of the \(3 + N\) dimensional space constant. This implies that the exponential contraction of the extra dimensions is exactly compensated by the exponential expansion of the ordinary space. In a sense, inflation appears to be driven by dynamical compactification, and vice versa. As a bonus, the condition \(3H - Nh = 0\) gets rid of all the time dependence in \(h_{ij}\) and \(h_{mn}\).

By imposing the volume-preserving condition \((3H - Nh = 0)\), the form of the target space manifold metric required to realize the solution (3.5) is greatly simplified, namely

\[
h_{00} = -\frac{M^{2+N}(N+3)}{6Ng\alpha^2}h^2, \tag{3.10}\\
h_{ij} = 0, \tag{3.11}\\
h_{mn} = \frac{M^{2+N}K}{(1/\lambda^2 + N^2g)\alpha^2}g_{mn}. \tag{3.12}
\]

Apart from simplicity, the time independence of \(h_{AB}\) also ensures that the target space is static, despite the exponential expansion and contraction of the ordinary and extra spaces respectively.

Let us give a physical interpretation of our results. First of all, since the non-linear sigma fields are only dependent on the coordinates of the extra dimensions as dictated by (2.9), they only exist in the extra dimensions. However, if the volume-preserving condition is not imposed, then the non-trivial time dependence in \(h_{ij}\) and \(h_{mn}\) will back-react on the time-coordinate. This means that while the ordinary 4D spacetime appears to be completely empty, the non-linear sigma fields are affecting it indirectly. As a result, the dynamics of the non-linear sigma fields simultaneously drive both of the exponential expansion and dynamical compactification in a non-trivial way.

On the contrary, suppose that the volume-preserving condition is imposed. As we can observe from (3.10), (3.11) and (3.12), all of the non-trivial time dependence in \(h_{ij}\) and \(h_{mn}\) disappears. In this case, the non-linear sigma fields can no longer back-react on the time-coordinate, so the ordinary 4D space-time is completely empty and free of any indirect effects. Since the non-linear sigma fields only exist in the extra dimensions, they will preferentially trigger the dynamical compactification. The extra dimensions are rolling up to form a manifold exhibiting the same shape as that of the target space manifold, which
can be understood from (3.12). At the same time, the exponential contraction of the extra dimensions is being exactly compensated by the exponential expansion of the ordinary space. Therefore, the non-linear sigma fields induce the dynamical compactification of the extra dimensions, which in turn drives inflation. No inflatons are required in the ordinary 4D spacetime.

The volume-preserving condition is physically motivated and has led to some simplifications to our solution. When it is imposed, we can interpret inflation as driven by dynamical compactification. This means that the two physically important processes, inflation and dynamical compactification, are both explained, and connected in a remarkable way.

IV. DISCUSSIONS AND CONCLUSIONS

By considering a 4+N dimensional Einstein gravity coupled to a non-linear sigma model, we have been able to provide a simultaneous solution to inflation in 4D and dynamical compactification of the extra dimensions. The non-linear sigma fields induce the dynamical compactification, which in turn drives the inflation without inflatons. Our solution is valid only if the number of non-linear sigma fields is exactly the same as the dimension of the spacetime. To elaborate, we have provided a model of inflation without inflatons, where the inflation is solely driven by the dynamical compactification of extra dimensions. The reason that we have no inflatons in our 4D spacetime is because none of the non-linear sigma fields depends on any of our 4D spacetime coordinates. In other words, the non-linear sigma fields only live in the extra dimensions. Our 4D spacetime is completely empty until we add matter fields.

One may contend that any true and physical solution to inflation must be able to stop it at some point. How this happens in our model has yet to be determined, but we can still offer a heuristic argument of how this may be achieved. The extra dimensions contract exponentially and will eventually reach the Planck length scale. Since the Planck length is generally believed to be the minimum fundamental length scale in Nature, we expect that the contraction will be forced to stop when this scale is reached. In this case, \( h \) will be identically zero. Due to the volume-preserving condition \( (3H - Nh = 0) \), the ordinary space is forced to stop expanding accordingly.

Another issue is particle production after inflation. The question is: How do we produce
particles given that the 4D spacetime is completely empty? When the extra dimensions are forced to stop contracting at the Planck length, they will undergo a period of abrupt deceleration. This will produce a huge amount of entropy which is manifest as the production of superheavy Kaluza-Klein (KK) particles. The natural mass scale of these KK particles will be the Planck mass, and they will readily decay into all of the observed particles in our 4D spacetime.

Of course, what we have just discussed are only the possible physical pictures. A concrete understanding of how inflation ends and particle production occurs is beyond the scope of the current work.

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