Magnetically Activated Thermal Vacuum Torque

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We theoretically demonstrate the existence of a torque acting on an isotropic particle activated by a static magnetic field when the particle temperature differs from the surrounding vacuum. This phenomenon originates in time-reversal symmetry breaking of the particle interaction with the vacuum electromagnetic field. A rigorous quantum treatment of photons and particle excitations predicts a nonzero torque in a motionless particle, as well as exotic rotational dynamics characterized by stability points in which the particle rotates without friction at a constant temperature that differs from that of vacuum. Magnetically activated thermal vacuum torques and forces offer a unique way of exploiting time-reversal symmetry-breaking for nanoscale mechanics.

I. INTRODUCTION

Coupling between the bosonic excitations of moving objects (e.g., plasmons or phonons) and the vacuum electromagnetic field can produce net transfers of momentum and emission of real photons at the expense of mechanical motion [1][32]. These phenomena have been explored in accelerated mirrors [10,14], sliding surfaces [15,19], rotating objects [20,26], optical cavities [27,29], and moving particles [30,32]. For example, two planar homogeneous surfaces in relative parallel motion undergo contactless friction due to exchanges of surface excitations that interact through the vacuum electromagnetic field [14,17]. Friction can additionally occur by emitting photon pairs if the two media are transparent and their relative velocity exceeds the Cherenkov condition [16,18]. The continuous change in the dielectric boundaries associated with the rotation of a nonspherical object made of a nonabsorbing material also leads to stopping assisted by the emission of photon pairs [20,21]. More intriguing is the case of a spinning lossy sphere: despite the apparent preservation of dielectric boundaries, it undergoes a frictional torque even when the entire system is at zero temperature [22], while the torque can be enlarged by the presence of a planar surface [24], giving rise to a lateral force [26].

Vacuum friction is closely related to time-reversal symmetry (T-symmetry) of the electromagnetic field in the vicinity of the involved materials. Considering again two moving parallel surfaces [17], T-symmetry implies that the excitations of one of them have equal local density of states (LDOS) in its rest frame regardless of the orientations of their wave vectors. However, T-symmetry is broken for the surrounding electromagnetic field due to the Fresnel drag associated with the moving surface, a result that has been recently exploited to design optomechanically-induced nonreciprocal optical devices [33,37]. T-symmetry breaking is a direct consequence of the different Doppler shifts experienced by excitations propagating along opposite directions in the moving surface, which understandably exhibit a LDOS asymmetry. This produces an imbalance in the momentum exchanged during transfers of excitations between the two surfaces, giving rise to a net stopping force. In a similar fashion, the rotational Doppler effect in a rotating spherical particle induces T-symmetry breaking between excitations circulating in clockwise and anticlockwise directions, which also results in a vacuum frictional torque. From these general considerations, one would expect the emergence of vacuum forces in geometrically symmetric structures composed of nonreciprocal materials, in which T-symmetry is broken for example by applying a static magnetic field.

In this paper, we show that a spherical particle experiences a counterintuitive torque due to T-symmetry breaking induced by a static magnetic field. We formulate a rigorous quantum-electrodynamic model to describe the system and show that a finite torque is exerted parallel to the magnetic field even on a motionless particle, provided its temperature differs from that of the surrounding vacuum. The torque originates in the asymmetric thermal population of particle internal boson excitation modes with opposite angular momentum (AM). We find the particle temperature and rotation frequency to follow an exotic dynamics characterized by spontaneous changes in the direction of rotation and stability points in which the particle rotates indefinitely at a temperature different from the vacuum. We anticipate that similar vacuum forces should generally appear in nonmagnetic nanostructures when optical T-symmetry is broken by means of static magnetic fields.
These states experience a Zeeman splitting \(2\Delta = (2\propto \gamma)\). These states experience a Zeeman splitting \(2\Delta = (2e/mc)B\) in the presence of the magnetic field. These states experience a Zeeman splitting \(2\Delta = (2e/mc)B\) in the presence of the magnetic field.

II. INTUITIVE INTERPRETATION

Thermal vacuum torques activated by a magnetic field can be understood by analyzing energy exchanges between the vacuum field and the bosonic excitations of a spherical nanoparticle, as illustrated in Fig. 1. For simplicity, we consider a particle with two degenerate bosonic states \(|x\rangle\) and \(|y\rangle\) of energy \(\hbar \omega_0\) and polarization in the \(x\)-\(y\) plane. A magnetic field \(B\) along \(z\) induces a Zeeman splitting \(2\hbar \Delta = (2e/mc)B\) of the excited states \(|\pm\rangle = (|x\rangle \pm i|y\rangle)/\sqrt{2}\), whose frequencies become \(\omega_{0\pm} = \omega_0 \pm \Delta\) [Fig. 1(b)]. Their populations are therefore different when the particle is at finite temperature \(T\), as determined by the Bose-Einstein distribution \(n_1(\omega_0^+)\) [Fig. 1(c)]

- Conclusively, their rates of exchange with photons in the surrounding vacuum is also different. For example, when the particle is motionless (\(\Omega = 0\)) and the vacuum rests at zero temperature \((T_0 = 0)\), the photon emission rates are given by the blackbody spectrum \(\gamma_e^+ \propto (\omega_0^+)^3 n_1(\omega_0^+)\) [Fig. 1(c)], whereas the absorption rates are \(\gamma_e^- = 0\).
- Additionally, the excited states possess opposite AM \(\pm \hbar\) [red and blue arrows in Fig. 1(a)], which is gained (lost) by the particle rotational motion during photon absorption (emission) because the ground state \(|g\rangle\) has zero AM. We conclude that there is a net transfer of AM, which results in a nonzero torque

\[
M = h(\gamma_e^- - \gamma_e^+) - (\gamma_\alpha^- - \gamma_\alpha^+).
\] (1)

In the absence of rotation (\(\Omega = 0\)) and magnetic fields (\(\Delta = 0\)), both states \(|\pm\rangle\) have the same rates and energies, so we find \(M = 0\). However, when a magnetic field is applied, we have \(\gamma^+ \neq \gamma^-\) [Fig. 1(c)], thus leading to \(M \neq 0\).

III. THEORETICAL MODEL

We assume the particle to be small compared to the light wavelength associated with the excitation, rotation, and Zeeman frequencies \(\omega_0, \Omega,\) and \(\Delta\), respectively, so that photon emission and absorption are adequately described through the particle transition dipoles. It is important to realize that the Zeeman-split boson states \(|\pm\rangle\) consist of electronic or phononic modes that are rigidly rotating with the particle. These states have transition dipoles \(\propto \hat{x}' \pm i\hat{y}'\) and frequencies \(\omega_0^\pm\) in the rotating frame, defined by the axes \(\hat{x}'\) and \(\hat{y}'\). The radiation field sees however transition dipoles in the lab frame of axes \(\hat{x}\) and \(\hat{y}\). Using the relations \(\hat{x}' = \hat{x} \cos(\Omega t) + \hat{y} \sin(\Omega t)\) and \(\hat{y}' = -\hat{x} \sin(\Omega t) + \hat{y} \cos(\Omega t)\), we find that the dipole becomes \(\propto \hat{x} \pm i\hat{y}\) in the lab frame and picks up an additional phase \(e^{\pm i\Omega t}\), which shifts the transition frequencies to \(\omega_0^\pm \Omega\).

The populations of vacuum photons and particle bosons are then determined by the Bose-Einstein distributions \(n_j(\omega) = (e^{h\omega/kT} - 1)^{-1}\) at the vacuum \((j = 0)\) and particle \((j = 1)\) temperatures [Fig. 1(a)], evaluated at the lab-frame \((\omega = \omega_0^\pm \Omega)\) and rotating-frame \((\omega = \omega_0^\pm)\) transition frequencies, respectively. From these considerations, we can directly write the emission and absorption rates as

\[
\gamma_\alpha^\pm = \frac{\gamma_0}{\omega_0^\pm (\omega_0^\pm \Omega)^3} [n_1(\omega_0^+) + 1] n_0(\omega_0^+ \pm \Omega),
\] (2)

\[
\gamma_e^\pm = \frac{\gamma_0}{\omega_0^\pm (\omega_0^\pm \Omega^3) n_1(\omega_0^+) [n_0(\omega_0^+ \pm \Omega) + 1],
\] (3)

where \(\gamma_0 = 4\omega_0^3(p_0/3\hbar c)^3\) is the radiative decay rate of the intrinsic particle dipole \(p_0 = \langle x|x|g\rangle = \langle y|y|g\rangle\) in the absence of rotation and magnetic field [35]. We then calculate the torque from Eq. 1 using, from Eqs. 2 and 3,

\[
\gamma_e^- - \gamma_\alpha^+ = \frac{\gamma_0}{\omega_0^+ (\omega_0^+ \Omega)^3} [n_1(\omega_0^+) - n_0(\omega_0^+ \pm \Omega)].
\] (4)
Equation 1 clearly indicates the existence of a torque acting on a nonrotating particle \((\Omega = 0)\) when its temperature differs from the vacuum \((n_1 \neq n_0)\). Similarly, the absorption power, associated with changes in the total energy of the particle boson modes, reduces to

\[
P_{\text{abs}} = -\hbar \left[ \omega_0^2 (\gamma_0^- - \gamma_0^+) + \omega_0 (\gamma_+^+ - \gamma_+^-) \right] + P_z^\text{abs}, \tag{5}
\]

where \(P_z^\text{abs} = -\hbar \omega_0 |\langle n_1(\omega_0) - n_0(\omega_0) \rangle|\) is the contribution of polarization along the rotation axis \(z\).

This formalism can be readily extended to particles that sustain many excitation modes. A rigorous quantum-electrodynamics formulation is provided in the Appendix, where we show that the torque and absorption power can be expressed in terms of the sphere polarizability at rest \(\alpha(\omega)\)

\[
M = 4\pi \hbar \sum_{\nu = \pm 1} \nu \int_0^\infty \omega \rho^0(\omega) d\omega \, N_\nu(\omega) A_\nu(\omega), \tag{6}
\]

\[
P_{\text{abs}} = -4\pi \hbar \sum_{\nu = 0, \pm 1} \int_0^\infty \omega \rho^0(\omega) (\omega + \nu \Omega) d\omega \, N_\nu(\omega) A_\nu(\omega),
\]

where \(A_\nu(\omega) = \text{Im} \left\{ \alpha(\omega + \nu (\Omega + \Delta)) \right\}\) gives the response of the rotating particle in the magnetic field, \(N_\nu(\omega) = n_1(\omega + \nu \Omega) - n_0(\omega)\) is the imbalance of particle and vacuum mode populations, and \(\rho^0(\omega) = \omega^2 / 3 \pi^2 e^3\) is the projected vacuum LDOS \cite{39}. These expressions readily reduce to Eqs. (1) and 5 for a two-level particle \((\text{i.e., with } \text{Im} \{ \alpha(\omega) \} = \pi p_0^2 / h)\) \([\text{for } \delta(\omega_0 - \omega) - \delta(\omega_0 + \omega)]\) \cite{13}.

**IV. TORQUE ON A STATIC TWO-LEVEL NANOPARTICLE**

Figure 2 shows the torque calculated from Eqs. (1) and 4 for various nanoparticle temperatures \(T_1\) as a function of magnetic field strength \(\Delta\) when the vacuum is at temperature \(T_0 = 0\). The rotation symmetry of the particle implies that the torque changes sign when the direction of the magnetic field \(B\) is reversed. The direction of the torque also depends on particle temperature: it is roughly parallel (anti-parallel) to \(B\) at low (high) \(T_1\). This behavior is clearly illustrated by the expression

\[
M = (\hbar \gamma_0 / \omega_0^2) \left[ (\omega_0^2)^3 n_1(\omega_0^-) - (\omega_0^2)^3 n_1(\omega_0^+) \right],
\]

which is valid for \(T_0 = 0\), \(\Omega = 0\), and \(|\Delta| < \omega_0\); under the conditions of Fig. 2(c) \((\text{low } T_1)\), the high-energy state \(|+\rangle\) decays more slowly than \(|-\rangle\), and hence \(M > 0\), whereas the opposite is true when the state energies lie to the left of the emission maximum \((\text{high } T_1)\).

**V. EXOTIC DYNAMICS**

The existence of a nonzero torque for a static nanoparticle suggests that \(\Omega = 0\) is not necessarily an equilibrium configuration. We explore this possibility for disk-like particles in which polarization along \(z\) can be disemissed \((\text{i.e., } P_z^\text{abs} = 0)\). Figure 3(a) shows the rotation frequencies for which there is either no friction \((M = 0\), solid curves) or no absorption \((P_{\text{abs}} = 0\), broken curves), calculated from Eqs. (1), (4), and (5) and plotted as a function of magnetic splitting \(\Delta\). Besides the trivial conditions \(T_1 = T_0 = 0\) and \(\Omega = 0\), the plot reveals other stationary situations \((\text{arrows})\) where both \(M\) and \(P\) vanish. Indeed, from Eqs. (1), (4), and (5), two nontrivial equilibrium configurations are found at disk temperatures \(T_1 = (T_0/2)(1 \pm \Delta / \omega_0)\) and rotation frequencies \(\Omega = \pm \omega_0 - \Delta\). In these points, one of the particle bosons has zero frequency in the lab frame \((\omega_0^\pm \pm \Omega = 0)\) and therefore does not couple to radiation, whereas the other one has the same rate of emission and absorption \((\text{i.e., } n_0(n_1 + 1) = (n_0 + 1)n_1)\). Interestingly, stable points also exist without magnetic field \((T_1 = T_0/2, \Omega = \pm \omega_0)\).

The dynamical evolution of the particle \((\Omega\) and \(T_1\) as a function of time) is ruled by the equations \(\dot{\Omega} = M/I\) and \(\dot{T}_1 = P_{\text{abs}}/C\), where \(I\) is the moment of inertia and \(C\) is the heat capacity. A set of universal landscapes are found for the evolution of \(k_B T_1 / h\) and \(\Omega / \omega_0\) as a function of the fixed parameters \(\Delta, \omega_0, k_B T_0 / h\), and \(C/I\). A particular numerical solution is plotted in Fig. 3(b), where we observe evolution lines that are strongly influenced by the trivial (red solid circle) and one of the nontrivial (open circle) equilibrium points. The system is shown to evolve toward the nontrivial point over a significant range of initial solutions, although it is a metastable position because small perturbations from it lead, after a long evolution time, to the trivial point (see Appendix). Interestingly, evolution toward the trivial point is often involving stopping of the particle out of equilibrium and
FIG. 3: (a) Rotation frequencies for which the torque (solid curves) or the absorption power (broken curves) vanish at different normalized vacuum temperatures $\tilde{T}_0 = k_B T_0 / h \omega_0$ as a function of magnetic splitting $\Delta$ for a two-level particle. The condition for $P = 0$ is plotted both for spheres (dashed curves) and disks [dotted curves, obtained by assuming $P_{abs} = 0$ in Eq. [5]]. The normalized particle temperature is $\tilde{T}_1 = 0.2$. (b) Temporal evolution in the space of rotation frequency $\Omega$ and particle temperature $T_1$ for a nanodisk with $\Delta = 0.6 \omega_0$ and $T_0 = 0.6$. The dashed curves and corresponding numerical labels indicate evolution times in units of $\tau = I \omega_0 / h \gamma_0$ for $C/I = k_B \omega_0 / h$, where $I$ is the moment of inertia and $C$ is the heat capacity of the particle. A red circle indicates a nontrivial stability point ($M = 0$ and $P = 0$).

VI. NONTRIVIAL STATIONARY EQUILIBRIUM OF DRUDE PARTICLES

For low enough temperatures and rotation velocities, the response frequencies involved in lossy particles are sufficiently small as to consider that they are well represented by the Drude limit $\text{Im}\{\alpha(\omega)\} = g \omega$, where $g$ is a shape- and material-dependent constant (e.g., $g = 3 a^3 / 4 \pi \sigma$ for a metal sphere of radius $a$ and conductivity $\sigma$). This simple frequency dependence leads to analytical expressions for the torque and absorption power (see Appendix for detailed expressions), which simplify the analysis of the dynamical evolution. In particular, we find a rich family of equilibrium positions, in which both $M = 0$ and $P_{abs} = 0$, corresponding to the rotation frequencies and particle temperatures that we plot in Fig. 4(a) as a function of magnetic splitting. Interestingly, this is a universal plot valid for any Drude sphere, as it is independent of particle size and material conductivity. Apart from the trivial stationary point, nontrivial stable equilibrium is taking place for magnetic splitting

FIG. 4: (a) Universal conditions for stable equilibrium of Drude spheres. We represent the equilibrium particle temperature $T_1$ and rotation velocity $\Omega$ as a function of magnetic splitting $\Delta$. All quantities are normalized to the scaled vacuum temperature $\theta_0 = 2 \pi k_B T_0 / h$. The trivial configuration ($T_1 = T_0$ and $\Omega = 0$) is only stable for $|\Delta| \lesssim 1.03 \theta_0$. (b) Temporal dynamics in the space of rotation frequency $\Omega$ and particle temperature $T_1$ for $\Delta = 0.8 \theta_0$. The dashed curves and corresponding numerical labels indicate evolution times in units of $\tau = I \theta_0 / M_0$, for $C/I = k_B \theta_0 / h$, where $I$ is the moment of inertia and $C$ is the heat capacity of the particle, and $M_0 = h g \theta_0^2 / 90 \pi c^3$ (see main text). Red circles indicate stability points.
The Zeeman effect could reveal additional physics in connected neutron stars. In this respect, the resulting nonlinear magnetic fields are generated near stars, and in particular to a large range of vacuum temperatures and magnetic cosmic dust could be a potentially suitable testbed for exposed to an in-plane magnetic field. We also note that refraction of neutral particles incident on a planar surface an effect that could be observed through the lateral deflection 

\[ |\Delta| \gtrsim 3k_B T_0 / \hbar, \]
which even becomes degenerate at larger \( \Delta \). Surprisingly, \( \Omega = 0 \) is no longer an stable equilibrium condition for \( |\Delta| \gtrsim 6.5k_B T_0 / \hbar \). A simple analysis of the effect of perturbations around the equilibrium positions (see Appendix) reveals that points along the curves of Fig. 3(a) are stable. This is illustrated by the temporal evolution chart depicted in Fig. 3(b), where these points are shown to act as attractors.

**VII. CONCLUDING REMARKS**

The emergence of thermal vacuum torques in nonmagnetic particles subject to static magnetic fields suggests a radically new way of mechanically controlling nanoscale objects. Remarkably, these torques exist even when the particle is nonrotating. Importantly, there are nontrivial equilibrium points in which the particle rotates at a temperature that differs from the vacuum. These points dominate the dynamics of the system, and remarkably, for Drude particles there are several stable conditions that co-exist if the Zeeman frequency is comparable to the vacuum temperature frequency, in which case \( \Omega = 0 \) is no longer an equilibrium condition. These findings could be explored by observing the dynamical evolution of small-particle gases (e.g., through rotational frequency shifts \( |\Delta| \sim \bar{\Delta} \) held in vacuum inside a container that is subject to an external magnetic field. The sum of torques of an ensemble of particles contained inside a dielectric matrix could be also measured macroscopically. Additionally, one could use a low-frequency electric field polarized along the rotation axis to heat the particle and control its temperature, so that dynamical equilibrium is then established at a rotation frequency that depends on both the applied heating and the external magnetic field.

For these proposed experiments, the torque can reach a multiple value of the magnetic splitting (see Appendix), which for \( B \sim 1 \)T leads to \( \sim \text{pN} \cdot \text{nm} \) torques (i.e., experimentally accessible values \( |\Delta| \)). In the presence of a planar surface parallel to the magnetic field, the torque is increased and a lateral force emerges due to AM conservation, even in the absence of rotation (see Appendix), an effect that could be observed through the lateral deflection of neutral particles incident on a planar surface exposed to an in-plane magnetic field. We also note that cosmic dust could be a potentially suitable testbed for these ideas, as it contains submicron particles exposed to a large range of vacuum temperatures and magnetic fields for very long periods of time. For example, gigantic magnetic fields are generated near stars, and in particular neutron stars. In this respect, the resulting nonlinear Zeeman effect could reveal additional physics in connection with vacuum friction.

**Appendix A: Quantum-mechanical description of the particle-vacuum system**

In what follows, we present a self-contained quantum-mechanical description of the frictional torque and radiative emission for a spherical particle rotating with angular frequency \( \Omega \) at temperature \( T_1 \) in a vacuum at temperature \( T_0 \), exposed to a static magnetic field. The axis of rotation \( z \) is aligned with the direction of this field. Following previous work \( |\Delta| \), we describe the spherical particle and the vacuum electromagnetic field using a basis of \( |m, \{n_l\}, \{n_i\} \) states encompassing rotational, internal, and photonic degrees of freedom, respectively. More precisely, \( m \) is the azimuthal number corresponding to a rotational wave function \( e^{i m \varphi} \), where \( \varphi \) is the rotation angle, \( l \) labels internal bosonic states of the particle of energies \( h \epsilon_l \) with occupation numbers \( n_l \), and \( \{n_i\} \) is a complete set of photon occupation numbers in the surrounding vacuum. Photons and particle excitations are coupled through the interaction Hamiltonian \( |\Delta| \) (we use Gaussian units in what follows)

\[
H_1 = i \sum_{i,l} \sqrt{\frac{2 \pi \hbar \omega_i}{V}} \mathbf{e}_i \cdot (a_i^+ - \epsilon_i) (p_l^+ b_l^+ + p_l b_l), \quad (A1)
\]

where \( V \) is the quantization volume, \( a_i \) and \( b_l \) (\( a_i^+ \) and \( b_l^+ \)) are the annihilation (creation) operators of a photon in mode \( i \) and a particle excitation \( l \), respectively, \( \omega_i \) and \( \mathbf{e}_i \) are the frequency and unit polarization vector of the photon, and \( p_l \) is the transition dipole moment associated with the internal excitation \( l \) expressed in the nonrotating lab frame. In Eq. (A1), we contemplate the possibility that \( p_l \) is complex in order to deal with Zeeman excited states (see below). Additionally, by describing photon-particle interactions through the excitation dipoles we are assuming the the particle is small compared with the wavelengths of the involved photons.

**Appendix B: Frictional torque**

The rotational state of the particle is generally a combination of different angular momentum numbers that are piled up near a value \( m = I \Omega / \hbar \gg 1 \), where \( I \) is the moment of inertia. For large \( I \), it is a good approximation to consider the particle to be prepared in a pure state \( m \), as determined by \( I \Omega \). We then calculate the torque acting on the particle by summing all the rates of transition to final states \( m' \), multiplied by the transferred angular momentum \( (m' - m) \hbar \). Using Fermi’s golden rule, we find
\[
M = \frac{2\pi}{\hbar} \sum_{m'} \sum_{m}(m' - m) \sum_{\{n_i\}} \sum_{\{n_i'\}} \frac{1}{Z_0} e^{-\sum_i n_i \hbar \omega_i/k_BT_0} \\
\sum_{\{n_i\}} \sum_{\{n_i'\}} \frac{1}{Z_1} e^{-\sum_i n_i' \hbar \varepsilon_i/k_BT_1} \mid \{m', \{n_i'\}, \{n_i\}, \{H_1\}|m, \{n_i\}, \{n_i\}\} \mid^2 \times \delta[(m' - m)\Omega + \sum_i (n_i' - n_i) \varepsilon_i + \sum_i (n_i' - n_i) \omega_i].
\]

where we sum over all possible final populations of vacuum photons and particle bosons, \(\{n'_i\}\) and \(\{n_i\}\), and perform the thermal average over initial populations \(\{n_i\}\) and \(\{n_i\}\) at their respective temperatures \(T_0\) and \(T_1\) using the partition functions

\[
Z_0 = \sum_{\{n_i\}} \exp \left(-\sum_i n_i \hbar \omega_i/k_BT_0\right) = \prod_{i} \sum_{n_i=0}^{\infty} e^{-n_i \hbar \omega_i/k_BT_0},
\]

\[
Z_1 = \sum_{\{n_i\}} \exp \left(-\sum_i n_i' \hbar \varepsilon_i/k_BT_1\right) = \prod_{i} \sum_{n_i=0}^{\infty} e^{-n_i' \hbar \varepsilon_i/k_BT_1}.
\]

Energy conservation is ensured by the \(\delta\) function, in which \(\hbar (m' - m) \Omega\) accounts for the change in mechanical rotation energy.

From Eq. (A1), it is clear that the matrix elements only involve changes in at most one of the occupation numbers of the internal states \((l)\) and the photon states \((i)\). Additionally, it is important to realize that the particle bosons consist of electronic or phononic modes (i.e., charge disturbances or atomic displacements in the material) that are rigidly rotating with the particle, and consequently, we need to rewrite the lab-frame moments \(p_i\) in terms of their corresponding rotating-frame moments \(p'_i\) in order to relate our results to the particle polarizability at rest (see below). More precisely,

\[
p_{ix} = p'_{ix} \cos \varphi - p'_{iy} \sin \varphi,
\]

\[
p_{iy} = p'_{ix} \sin \varphi + p'_{iy} \cos \varphi,
\]

\[
p_{iz} = p'_{iz},
\]

for rotations along the \(z\) axis. The only nonzero matrix elements are then

\[
\langle m', n_l + 1, n_l + 1 | H_1 | m, n_l, n_i \rangle = \sqrt{(n_l + 1)(n_l + 1)} \Delta_{m,-m}^{m'\ast -m},
\]

\[
\langle m', n_l + 1, n_l - 1 | H_1 | m, n_l, n_i \rangle = -\sqrt{(n_l + 1)n_l} \Delta_{m,-m}^{m'\ast -m},
\]

\[
\langle m', n_l - 1, n_l + 1 | H_1 | m, n_l, n_i \rangle = \sqrt{n_l(n_l + 1)} \Delta_{m,-m}^{m'\ast -m},
\]

\[
\langle m', n_l - 1, n_l - 1 | H_1 | m, n_l, n_i \rangle = -\sqrt{n_l n_l} \Delta_{m,-m}^{m'\ast -m},
\]

where

\[
\Delta_{m,-m}^{m'\ast -m} = i \sqrt{\frac{\hbar \omega_i}{2V}} \left[(p'_{l,ix} + ip'_{l,iy}) (e_{ix} - ie_{iy}) \delta_{m',m+1} + (p'_{l,ix} - ip'_{l,iy}) (e_{ix} + ie_{iy}) \delta_{m',m-1} + 2p_{lizen} e_{iz} \delta_{m',m}\right],
\]

\[
\Delta_{m,-m}^{m'\ast -m} = i \sqrt{\frac{\hbar \omega_i}{2V}} \left[(p'^{\ast}_{l,ix} + ip'^{\ast}_{l,iy}) (e_{ix} - ie_{iy}) \delta_{m',m+1} + (p'^{\ast}_{l,ix} - ip'^{\ast}_{l,iy}) (e_{ix} + ie_{iy}) \delta_{m',m-1} + 2p'^{\ast}_{lizen} e_{iz} \delta_{m',m}\right].
\]

Because the particle has rotational symmetry around the \(z\) axis, its bosons in the absence of a magnetic field consist of degenerate modes \(|l_z\rangle\) and \(|l_y\rangle\) with polarizations along \(x\) and \(y\), as well as modes \(|l_z\rangle\) with polarization along \(z\). However, this degeneracy is broken due to the Zeeman effect when a magnetic field \(B \parallel \mathbf{z}\) is applied, which leads to new eigenstates with \(\pm \mathbf{y}\) polarizations and energies shifted by \(\pm \hbar \Delta\) with \(\Delta = (e/m)cB\) [45]. Using the notation \(|l_{\pm}\rangle = (|l_x\rangle \pm i|l_y\rangle)/\sqrt{2}\) for these Zeeman states, we can write their transition dipoles as

\[
p'_{l_z} = -e(g|r|l_{\pm}\rangle = (p_l/\sqrt{2})(\hat{x} \pm i\hat{y})
\]

in the rest frame of the particle, where \(|g\rangle\) is the ground state and \(p_l = -e(g|x|l_x\rangle = -e(g)|l_y\rangle\). From here we find that the only nonzero matrix elements are

\[
\Delta_{l_z,i}^{\pm 1} = i \sqrt{\frac{\hbar \omega_i}{V}} p_{l_z} (e_{ix} \pm ie_{iy}),
\]

\[
\Delta_{l_z,i}^{\pm 1} = i \sqrt{\frac{\hbar \omega_i}{V}} p_{l_z} (e_{ix} \pm ie_{iy}),
\]

\[
\Delta_{l_z,i}^{\pm 0} = i \sqrt{\frac{2\pi \hbar \omega_i}{V}} p_{l_z} e_{iz},
\]

where \(p_{l_z} = -e(g|z|l_z\rangle\) is the transition dipole of modes with polarization along \(z\), the (de-)excitation of which
does not change the quantum number $m$. We conclude that the boson angular momentum $\pm \hbar$ of the $|l_{\pm}\rangle$ states is directly transferred to the particle orbital angular momentum (i.e., they couple to $m \rightarrow m \pm 1$ transitions).

Before inserting the above matrix elements into Eq. (B1), it is useful to realize that the occupation numbers only appear as multiplicative factors in the squared matrix elements, and therefore, we can perform the thermal average sums by factoring out all modes except the ones that are changing during the transition. The remaining sums become

\[
\sum_{n_i} n_i e^{-n_i \hbar \omega_i / k_B T_0} = n_0(\omega_i),
\]

\[
\sum_{n_i} n_i e^{-n_i \hbar \epsilon_i / k_B T_0} = n_1(\epsilon_i),
\]

where

\[
n_j(\omega) = \frac{1}{\exp(h\omega/k_B T_j) - 1}
\]

is a Bose-Einstein distribution at temperature $T_j$.

Now, inserting the above matrix elements into Eq. (B1) and performing the thermal sums, the torque becomes

\[
M = \frac{2\pi^2}{V} \sum_{i,j} \omega_i p_i^2 \sum_{\pm} (1) |e_{i,x} \mp i e_{i,y}|^2 \times \{ [n_0(\omega_i) + 1] [n_1(\epsilon_i^+ \mp \omega_i) + n_1(\epsilon_i^- \mp -\omega_i)] + n_0(\omega_i) [n_1(\epsilon_i^+ \mp \omega_i) + n_1(\epsilon_i^- \mp -\omega_i)] + [n_0(\omega_i) + 1] n_1(\epsilon_i^+) \delta(\pm \Omega - \epsilon_i^+ + \omega_i) + n_0(\omega_i) n_1(\epsilon_i^+) \delta(\pm \Omega - \epsilon_i^+ - \omega_i) \},
\]

where we have already introduced the Zeeman states, so that

\[
\epsilon_i^\pm = \epsilon_i \pm \Delta
\]

are the frequencies of bosons $l_{\pm}$, which are involved in transitions with $m' - m = \pm 1$. A plane-wave representation of the photon states allows us to make the following substitution for the sum over $i$:

\[
\sum_{i} \rightarrow \frac{V}{(2\pi)^3} \sum_{\sigma} \int d^3 k,
\]

where $k$ and $\sigma$ run over photon wave vectors and polarizations, respectively. The integral over photon directions and the sum over polarizations can be readily performed to yield

\[
\sum_{\sigma} \int d^3 k \left( e_{i,x}^2 \mp i e_{i,y}^2 \right) \rightarrow \frac{16\pi}{3\epsilon^3} \int_0^\infty \omega^2 d\omega,
\]

which allows us to recast Eq. (B2) as

\[
M = \frac{4}{3\epsilon^3} \int_0^\infty \omega^3 d\omega \sum_{l} p_l^2 \sum_{\pm} (1) \times \{ [-n_0(\omega) + 1] n_1(\omega \mp \Omega) \delta(\pm \Omega + \epsilon_i^\pm + \omega) + n_0(\omega) [n_1(\omega \mp \Omega) + 1] \delta(\pm \Omega + \epsilon_i^\pm - \omega) + [n_0(\omega) + 1] n_1(\omega \mp \Omega) \delta(\pm \Omega - \epsilon_i^\mp + \omega) + n_0(\omega) n_1(\epsilon_i^+) \delta(\pm \Omega - \epsilon_i^+ - \omega) \}.
\]

Here, we have taken advantage of the $\delta$ functions to express the boson frequencies $\epsilon_i^\pm$ in terms of $\omega$ and $\Omega$ inside $n_1$, and further used the relation $n_j(-\omega) = -n_j(\omega) - 1$. Finally, after some straightforward algebra, Eq. (B3) can be
written as
\[ M = \frac{4\hbar}{3\pi c^3} \int_0^\infty \omega^3 d\omega \left( [n_1(\omega + \Omega) - n_0(\omega)] \Im \{\alpha(\omega + \Omega + \Delta)\} - [n_1(\omega - \Omega) - n_0(\omega)] \Im \{\alpha(\omega - \Omega - \Delta)\} \right), \]
or equivalently
\[ M = \frac{4\hbar}{3\pi c^3} \sum_{\nu=\pm 1} \nu \int_0^\infty \omega^3 d\omega \left( [n_1(\omega + \nu\Omega) - n_0(\omega)] \Im \{\alpha(\omega + \nu(\Omega + \Delta))\} \right), \quad (B4) \]
where
\[ \alpha(\omega) = \frac{1}{\hbar} \sum_l p_l^2 \left( \frac{1}{\varepsilon_l - \omega - i0^+} + \frac{1}{\varepsilon_l + \omega + i0^+} \right) \quad (B5) \]
is the polarizability of the static sphere without magnetic fields (see Sec. E below). Equation (B4) is reproduced in the main text using a more compact notation and relating \( \omega^2/3\pi^2 c^3 \) to the projected local density of states (LDOS) of photons in vacuum.

**Appendix C: Absorption power**

During a transition \(|m, \{n_i\}, \{n_i\}| \rightarrow |m', \{n'_i\}, \{n'_i\}|\), the internal energy of the particle varies in accordance to the change in boson occupation numbers by \( \sum_l (n'_l - n_l)\hbar \varepsilon_l \). Consequently, the absorption power can be obtained from an expression similar to Eq. (B1), with the factor \((m' - m)\hbar\) (change in mechanical angular momentum) replaced by \( \sum_l (n'_l - n_l)\hbar \varepsilon_l \) (change in internal energy). Proceeding in a similar way as for the calculation of the torque, we obtain the equivalent of Eq. (B2) for the absorption power due to \(x\) and \(y\) polarization,
\[ P_{xy}^{\text{abs}} = \frac{2\pi^2}{V} \sum_{i,l} \omega_i p_l^2 \sum_{\pm} |e_{i,x} \mp ie_{i,y}|^2 \]
\[ \times \left\{ [n_0(\omega + 1) + n_1(\omega)] \delta(\pm \Omega + \varepsilon_{l,x}^\pm + \omega) \right. \]
\[ +\varepsilon_{l,x}^\dagger n_0(\omega) [n_1(\varepsilon_{l,x}^\pm) + 1] \delta(\pm \Omega + \varepsilon_{l,x}^\pm - \omega) \]
\[ -\varepsilon_{l,x}^\dagger n_0(\omega) n_1(\varepsilon_{l,x}^\pm) \delta(\pm \Omega - \varepsilon_{l,x}^\pm + \omega) \]
\[ \left. -\varepsilon_{l,x}^\dagger n_0(\omega) n_1(\varepsilon_{l,x}^\pm) \delta(\pm \Omega - \varepsilon_{l,x}^\pm - \omega) \right\}, \]
and from here the equivalent of Eq. (B3),
\[ P_{xy}^{\text{abs}} = \frac{4\hbar}{3\pi c^3} \int_0^\infty \omega^3 d\omega \sum_l p_l^2 \]
\[ \times \sum_{\pm} \left\{ [n_0(\omega + 1) + n_1(\omega)] \delta(\pm \Omega + \varepsilon_{l,x}^\pm + \omega) + (\omega + \Omega) n_0(\omega) [n_1(\omega + \Omega) + 1] \delta(\pm \Omega + \varepsilon_{l,x}^\pm - \omega) \right. \]
\[ - (\omega + \Omega) [n_0(\omega) + 1] n_1(\omega) \delta(\pm \Omega - \varepsilon_{l,x}^\pm + \omega) - (\omega + \Omega) n_0(\omega) [n_1(\omega + \Omega) + 1] \delta(\pm \Omega - \varepsilon_{l,x}^\pm - \omega) \right\}. \]
Again, after some algebra, we can rewrite this expression in terms of the static polarizability as
\[ P_{xy}^{\text{abs}} = -\frac{4\hbar}{3\pi c^3} \int_0^\infty \omega^3 d\omega \left( (\omega + \Omega) [n_1(\omega + \Omega) - n_0(\omega)] \Im \{\alpha(\omega + \Omega + \Delta)\} \right) \]
\[ + (\omega - \Omega) [n_1(\omega - \Omega) - n_0(\omega)] \Im \{\alpha(\omega - \Omega - \Delta)\} \right\}. \quad (C1) \]
This result must be supplemented by the contribution of polarization along \(z\), which is unaffected by the rotation and the magnetic field, and therefore, it must coincide with half of the value of \( P_{xy}^{\text{abs}} \) evaluated at \( \Omega = 0 \) and \( \Delta = 0 \). We find
\[ P_z^{\text{abs}} = -\frac{4\hbar}{3\pi c^3} \int_0^\infty \omega^4 d\omega \left[ n_1(\omega) - n_0(\omega) \right] \Im \{\alpha(\omega)\}. \]
The total absorption power is then
\[ P_{\text{abs}} = P_{xy}^{\text{abs}} + P_z^{\text{abs}} = -\frac{4\hbar}{3\pi c^3} \sum_{\nu=0,\pm 1} \int_0^\infty \omega^3 d\omega (\omega + \nu \Omega) [n_1(\omega + \nu \Omega) - n_0(\omega)] \text{Im}\{\alpha(\omega + \nu \Omega + \Delta)\}, \]
which is the expression reproduced in the main text.

**Appendix D: Energy balance**

The analysis presented above can be straightforwardly extended to calculate the mechanical power acting on the rotation of the particle \( P_{\text{mech}} \) simply by substituting the \((m' - m)\hbar\) factor in Eq. (B1) by the change in mechanical energy \((m' - m)\hbar\Omega\). This leads to the expected relation \( P_{\text{mech}} = \Omega M \). Additionally, the radiated energy \( P_{\text{rad}} \) is found by weighting the transition probabilities by the emitted/absorbed photon energy. This also leads to the expected relation \( P_{\text{abs}} + P_{\text{mech}} + P_{\text{rad}} = 0 \), which is trivially satisfied because each transition includes a \( \delta \) function for energy conservation.

**Appendix E: Particle polarizability under a magnetic field**

We now calculate the polarizability of the spherical particle using linear response theory. Because dipole components along \( z \) are unaffected by the particle rotation, it is clear that the polarizability remains unchanged for polarization along that direction, and additionally, there are not diagonal terms that mix \( z \) with \( x \) or \( y \). Using the Zeeman basis set, the \( x\)-\( y \)-subspace polarizability tensor in the particle rest frame becomes \( \tilde{\alpha}(\omega) = \frac{1}{\hbar} \sum_{i=1}^4 \sum_{l=\pm} \left( \frac{\hat{p}_{l+}^i \otimes \hat{p}_{l+}^i}{\varepsilon_{l+}^i - \omega + i0^+} + \frac{\hat{p}_{l+}^i \otimes \hat{p}_{l-}^i}{\varepsilon_{l-}^i + \omega + i0^+} \right) \),

where the state frequencies \( \varepsilon_{l+}^i = \varepsilon_i \pm \Delta \) are shifted by \( \pm \Delta \) due to the magnetic field along \( z \) (see above). This can be readily rewritten as \( \tilde{\alpha}(\omega) = \frac{1}{\hbar} \sum_{i=1}^4 \sum_{l=\pm} \left( \frac{1}{\varepsilon_{l+}^i - \omega + i0^+} + \frac{1}{\varepsilon_{l-}^i + \omega + i0^+} \right) \).

\[ \alpha(\omega) = \frac{1}{\hbar} \sum_{i=1}^4 \sum_{l=\pm} \left( \frac{1}{\varepsilon_{l+}^i - \omega + i0^+} + \frac{1}{\varepsilon_{l-}^i + \omega + i0^+} \right) \]

is the polarizability for a field \( \propto \hat{x} \pm i\hat{y} \), yielding an induced dipole \( \propto \hat{x} \pm i\hat{y} \). Incidentally, we can rewrite \( \alpha(\omega) = \alpha(\omega \mp \Delta) \) in terms of the unperturbed polarizability given by Eq. (B5).

**Appendix F: Drude particle**

Lossy materials such as graphite, SiC, and metals exhibit a low-frequency response that is well represented by the Drude limit. This leads to an imaginary part of the polarizability given by

\[ \text{Im}\{\alpha(\omega)\} \approx g \omega, \quad (F1) \]

where the coefficient \( g \) depends on the size, material, and morphology of the particle. For example, for a metallic sphere of radius \( a \) described by the Drude permittivity \( \epsilon(\omega) = 1 + 4\pi\sigma/\omega \), where \( \sigma \) is the conductivity, we have \( g = 3\pi\sigma/4\pi\sigma \). However, the linear scaling of \( \text{Im}\{\alpha(\omega)\} \) with \( \omega \) is general property of lossy particles at low frequency, so the analysis presented below should be applicable for any lossy particle with axially symmetry, provided we are in the limit of small rotation velocities, thermal photon frequencies, and Zeeman splittings compared with the excitonic or plasmonic resonances of the particle, a situation that should be common at (or below) room temperature.

The simple scaling of \( \text{Im}\{\alpha(\omega)\} \) with \( \omega \) leads to closed-form expressions for the torque and the absorption power. We start by inserting Eq. (F1) into Eq. (B4), which permits writing the torque as

\[ M = M_0 + M_1 \Delta, \quad (F2) \]

where

\[ M_0 = \frac{4\hbar g}{3\pi c^3} \int_0^\infty \omega^3 d\omega \left\{ [n_1(\omega + \Omega) - n_0(\omega)] (\omega + \Omega) - [n_1(\omega - \Omega) - n_0(\omega)] (\omega - \Omega) \right\}, \quad (F3) \]

\[ M_1 = \frac{4\hbar g}{3\pi c^3} \int_0^\infty \omega^3 d\omega \left[ n_1(\omega + \Omega) + n_1(\omega - \Omega) - 2n_0(\omega) \right]. \]
These integrals can be carried out analytically by transforming them into the well-known expressions

\[
\int_0^\infty \omega^n \, d\omega \, n_j(\omega) = (\theta_j/2\pi)^{n+1} \int_0^\infty \frac{x^n \, dx}{e^x - 1} = n! \zeta(n+1) (\theta_j/2\pi)^{n+1} = \begin{cases} 
\theta_j^2/24, & (n = 1) \\
\theta_j^4/240, & (n = 3) \\
\theta_j^6/504, & (n = 5)
\end{cases} (F4)
\]

where \(\theta_j = 2\pi k_B T_j / \hbar\)

is a thermal frequency corresponding to the temperature \(T_j\) and \(\zeta(s) = \sum_{j=1}^\infty j^{-s}\) is the Riemann zeta function [e.g., the values \(\zeta(2) = \pi^2/6\), \(\zeta(4) = \pi^4/90\), and \(\zeta(6) = \pi^6/945\) have been used to write the examples given in the rightmost part of \(F4\)]. First, we change variables in the \(\omega\) integrals, so that \(\omega \pm \Omega\) becomes \(\omega\) in the argument of \(n_1\). We find

\[
M_0 = -\frac{4h g}{3\pi c^3} \left[ 2 \int_0^\infty d\omega \left[ (3\omega^3 + \omega\Omega^3)n_1(\omega) + \omega^3 n_0(\omega) \right] + \int_0^\Omega d\omega \omega(\omega - \Omega)^3 n_1(\omega) + \int_{-\Omega}^0 d\omega (\omega + \Omega)^3 n_1(\omega) \right],
\]

\[
M_1 = \frac{4h g}{3\pi c^3} \left[ 2 \int_0^\infty d\omega \left[ (\omega^3 + 3\omega\Omega^2)n_1(\omega) - \omega^3 n_0(\omega) \right] - \int_0^\Omega d\omega (\omega - \Omega)^3 n_1(\omega) + \int_{-\Omega}^0 d\omega (\omega + \Omega)^3 n_1(\omega) \right],
\]

where two finite-interval integrals are introduced in each of these equations to set the lower limit of integration to 0 in the infinite-interval integrals after the change of variables. We then use Eq. (F4) to work out the infinite-interval integrals, leading us to

\[M_0 = -\frac{4h g}{360\pi c^3} \Omega (6\Omega^4 + 10\Omega^2 \theta_1^2 + 3\theta_1^4 + \theta_0^4), \quad (F5)\]

\[M_1 = -\frac{4h g}{360\pi c^3} (30\Omega^4 - 30\Omega^2 \theta_1^2 - \theta_1^4 + \theta_0^4), \quad (F6)\]

where we have also worked out the finite-interval integrals by changing \(\omega\) to \(\omega\), and using the relation \(n_1(-\omega) = -1 - n_1(\omega)\) in the rightmost term of \(M_0\) and \(M_1\), resulting in the cancellation of \(n_1\) terms.

We proceed in the similar fashion with the absorption power. Inserting Eq. (F1) into Eq. (C1), we readily find

\[P_{xy}^{\text{abs}} = P_{xy,0}^{\text{abs}} + P_{xy,1}^{\text{abs}} \Delta, \quad (F7)\]

where

\[P_{xy,0}^{\text{abs}} = -\frac{4h g}{3\pi c^3} \int_0^\infty \omega^3 d\omega \left\{ [n_1(\omega + \Omega) - n_0(\omega)](\omega + \Omega)^2 + [n_1(\omega - \Omega) - n_0(\omega)](\omega - \Omega)^2 \right\},\]

\[P_{xy,1}^{\text{abs}} = -\frac{4h g}{3\pi c^3} \int_0^\infty \omega^3 d\omega \left\{ [n_1(\omega + \Omega) - n_0(\omega)](\omega + \Omega) - [n_1(\omega - \Omega) - n_0(\omega)](\omega - \Omega) \right\} = -M_0.\]

The rightmost equality in the last equation is directly obtained by comparison with Eq. (F3). The \(P_{xy,0}^{\text{abs}}\) contribution can be calculated using the same methods as for the torque, so we first change \(\omega \pm \Omega\) to \(\omega\) in the terms of the \(\omega\) integral that involve \(n_1\). We obtain

\[P_{xy,0}^{\text{abs}} = -\frac{4h g}{3\pi c^3} \left[ 2 \int_0^\infty \omega^3 d\omega \left[ (\omega^2 + 3\Omega^2)n_1(\omega) - (\omega^2 + \Omega^2)n_0(\omega) \right] - \int_0^\Omega d\omega \omega^2(\omega - \Omega)^3 n_1(\omega) + \int_{-\Omega}^0 d\omega \omega^2(\omega + \Omega)^3 n_1(\omega) \right], \quad (F8)\]

and from here, using Eq. (F4) and changing \(\omega\) to \(-\omega\) in the rightmost integral, we find

\[P_{xy,0}^{\text{abs}} = \frac{4h g}{360\pi c^3} \left[ 2\Omega^6 + \Omega^2 (\theta_0^4 - 3\theta_1^4) + (10/21)(\theta_0^6 - \theta_1^6) \right]. \quad (F8)\]
Finally, for a disk-like particle we can neglect $P_{z\text{abs}}$, whereas for a sphere we have $P_{z\text{abs}} = (1/2)P_{xy\text{abs}}$ for $\Delta = \Omega = 0$, and therefore,

\[ P_{z\text{abs}} = \frac{4\hbar g}{360\pi e^2} (5/21)(\theta_0^6 - \theta_1^6). \quad \text{(F9)} \]

In summary, the torque and absorption power of a Drude particle are given by Eqs. (F2) and (F7), with the coefficients of those equations given by Eqs. (F5), (F6), and (F8), as well as the identity $P_{z\text{abs}} = -M_0$. Additionally, the absorption associated with $z$ polarization can be neglected in a disk-like particle, and it is given by Eq. (F9) for a sphere. Incidentally, these expressions agree with previous results in the $\Delta = 0$ limit [22].

**Appendix G: Dynamical equilibrium**

The thermal and rotational dynamics of the particle are controlled by the equations

\[ \dot{\Omega} = M/I, \]
\[ \dot{T}_1 = P_{\text{abs}}/C, \]

where $I$ is the moment of inertia and $C$ is the heat capacity. For fixed vacuum temperature $T_0$ and magnetic field $B$ (i.e., fixed splitting $\Delta = (e/mc)B$), the condition of equilibrium becomes $M = 0$ and $P_{\text{abs}} = 0$, which can be fulfilled for specific values of $\Omega = \Omega^{\text{eq}}$ and $T_1 = T_1^{\text{eq}}$. Incidentally, we are assuming that the variation of $C$ with temperature is negligible within a small region around that position. For simplicity, we ignore the $T_1$ dependence of $C$, so that the positions of equilibrium derived from our analysis involve a value of $C$ at the corresponding particle temperature.

We discuss several points of equilibrium in the main text for two-level and Drude particles. However, a study of the stability of these points requires a more careful analysis. We proceed by considering small perturbations in the values of $\Omega$ and $T_1$ around the equilibrium position $\Omega^{\text{eq}}$ and $T_1^{\text{eq}}$. Then, we follow the temporal evolution of the system from that position. Because we have two first-order equations of motion, the evolution must result from the combination of two homogeneous solutions, with temporal dependences $\propto e^{\lambda t}$, where $\lambda$ refers to the two eigenvalues of the problem (see below). The condition for stability then becomes $\text{Re}\{\lambda\} < 0$.

We thus use the ansatz

\[ \Omega = \Omega^{\text{eq}} + e^{\lambda t} A, \]
\[ T_1 = T_1^{\text{eq}} + e^{\lambda t} B, \]

where $A$ and $B$ are eigenmode constants. Additionally, we linearize the system by expanding $M$ and $P_{\text{abs}}$ to first order around the equilibrium position as

\[ M \approx \partial_\Omega M \left( \Omega - \Omega^{\text{eq}} \right) + \partial_{T_1} M \left( T_1 - T_1^{\text{eq}} \right), \quad \text{(G1)} \]
\[ P_{\text{abs}} \approx \partial_\Omega P_{\text{abs}} \left( \Omega - \Omega^{\text{eq}} \right) + \partial_{T_1} P_{\text{abs}} \left( T_1 - T_1^{\text{eq}} \right), \quad \text{(G2)} \]

where the derivatives are evaluated at $\Omega = \Omega^{\text{eq}}$ and $T_1 = T_1^{\text{eq}}$. Combining these elements, we find

\[ \lambda \begin{bmatrix} A \\ B \end{bmatrix} = \left[ \begin{array}{cc} \partial_\Omega M/I & \partial_{T_1} M/I \\ \partial_\Omega P_{\text{abs}}/C & \partial_{T_1} P_{\text{abs}}/C \end{array} \right] \cdot \begin{bmatrix} A \\ B \end{bmatrix}, \]

and by solving the secular determinant, we obtain the eigenvalues

\[ \lambda = \frac{1}{2} \left[ \partial_\Omega M/I + \partial_{T_1} P_{\text{abs}}/C \pm \sqrt{(\partial_\Omega M/I + \partial_{T_1} P_{\text{abs}}/C)^2 + 2/IC} \right]. \]

From this expression, the condition $\text{Re}\{\lambda\} < 0$ implies

\[ (\partial_\Omega M/I + \partial_{T_1} P_{\text{abs}}/C) - \sqrt{(\partial_\Omega M/I + \partial_{T_1} P_{\text{abs}}/C)^2 + 2/IC} < 0, \quad \text{(G3)} \]
\[ (\partial_{T_1} M)(\partial_{T_1} P_{\text{abs}}) - (\partial_\Omega M)(\partial_{T_1} P_{\text{abs}}) < 0. \quad \text{(G4)} \]

In all cases that we have examined, the trivial equilibrium position $\Omega^{\text{eq}} = 0$ and $T_1^{\text{eq}} = T_0$ is found to be stable according to the above analysis.

1. Two-level particles

For a two-level particle (see main text), the condition $\text{Re}\{\lambda\} = 0$ easily seen to be satisfied at the nontrivial equilibrium points. However, the left-hand side of the inequality

\[ \text{Re}\{\lambda\} = 0 \]

...
and thus, the condition (G3) is also universally satisfied, with independence of the values of I and C. We conclude that Fig. 4(a) constitutes a universal plot valid for any spherical particle, regardless of its size and material conductivity. We supplement these results in Fig. 4 by including unstable equilibrium points, showing that the trivial configuration $T_1 = T_0$ and $\Omega = 0$ stops being stable when it crosses another equilibrium branch.

Incidentally, similar results are obtained for disk-like particles, because the only difference between them and spheres is the term $P_{abs}^z$ [Eq. (F9)], which is small compared with other contributions to $P_{abs}$ within the range of parameters explored in Fig. 4 of the main text.

### Appendix H: Spontaneous lateral force near a planar surface

When the particle is spinning near a planar surface, the AM transfer rates must be corrected by the change in LDOS relative to vacuum, in the same way as the decay rates of optical point emitters are modulated by the LDOS due to the Purcell effect [46]. More precisely, $\tilde{\rho}$ needs to be replaced by the average of the projected LDOS $\bar{\rho}$ within the $x$-$y$ plane in Eq. (6) of the main text. When $\hat{z}$ (rotation axis and direction of $\mathbf{B}$) is parallel to the surface, considering for simplicity a homogeneous surface material of permittivity $\epsilon_s$ at a small distance $d \ll \omega/c$ from the particle (quasistatic limit), we find $\bar{\rho} = (3/16\pi^2\omega d^3)\text{Im}\{-1/(\epsilon_s + 1)\}$, which leads to a torque $M \propto d^{-3}$. Additionally, bosonic states with AM produce directional emission when they decay to evanescent surface modes [47], thus giving rise to a lateral force acting on the particle [48–51]. A direct generalization of the formalism presented in Ref. [26] in order to account for Zeeman splitting reveals that this lateral force $F$ is simply related to the torque through $M = Fd$, as expected from conservation of total AM in the system.

### Appendix I: Order of magnitude of the magnetically activated thermal torque

1. Two-level particles

The two-level particle model applies to a large range of particle sizes and physical systems. It describes for example molecules in which $\omega_0$ is a dipole-active vibrational state, but also plasmonic spheres in which $\omega_0 = \omega_p/\sqrt{3}$ is determined by the bulk plasma frequency $\omega_p$ of the metal permittivity $\epsilon(\omega) = 1 - \omega_p^2/\omega(\omega + i\gamma)$. Under the reasonable assumption of small magnetic splitting $\Delta \ll \omega_0$, the static ($\Omega = 0$) torque in a vacuum at temperature $T_0 = 0$
the torque as $\lambda$ electric matrix of radius equal to $a$

We consider particles with low plasmon frequencies, which could be obtained from metalodielectric multishell spheres with a small metal filling fraction $f$, so that $\omega_0 \propto \sqrt{f}$. Opaque homogenous metal particles also display plasmons at low frequencies, with a reduction relative to $\omega_p \propto 1/(\text{aspect-ratio})$. As another possibility, plasmon-supporting doped graphene islands exhibit their plasmons at midinfrared frequencies [52]. We then conclude that there are several realistic realizations of two-level plasmonic particles that permit obtaining static torques that are a multiple of $\hbar \Delta$.

In SI units, for a magnetic field of 10 T, we have $\hbar \Delta = 1.2 \text{meV}$, or equivalently, $M = 5\hbar \Delta = 25 \text{pN-nm}$, which is an experimentally measurable torque [44].

2. Drude particles

According to the results of Sec. 7, the magnetically activated torque acting on a static Drude sphere reduces to

$$M = \hbar \Delta \frac{V}{160 \pi^3 e^2 \sigma} (\theta_1^4 - \theta_0^4).$$

In contrast to the two-level particle, the absence of an absorption gap leads to a polynomial temperature dependence, instead of an exponential cutoff at low $T$. A good conductivity prevents radiative absorption and is thus detrimental to obtain a large torque, so we consider a poorly conducting material (e.g., a doped semiconductor) with a conductivity of $10^3 \Omega^{-1}\text{m}^{-1}$ (i.e., $\sigma \approx 6 \text{meV}$). Then, for $T_0 = 0$ and taking a sphere of $2 \mu\text{m}$ radius at temperature $T_1 = 1000 \text{K}$, we find $M \approx 12 \hbar \Delta$. Drude particles are therefore good candidates to obtain magnetic torques that are a multiple of the magnetic splitting as well.

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[1] M. Castagnino and R. Ferraro, Ann. Phys. (N.Y.) 154, 1 (1984).
[2] G. Calucci, J. Phys. A: Math. Gen. 25, 3873 (1992).
[3] E. Sassaroli, Y. N. Srivastava, and A. Widom, Phys. Rev. A 50, 1027 (1994).
[4] M. Kardar and R. Golestanian, Rev. Mod. Phys. 71, 1233 (1999).
[5] S. K. Lamoreaux, Am. J. Phys. 67, 850 (1999).
[6] M. Bordag, U. Mohideen, and V. M. Mostepanenko, Phys. Rep. 353, 1 (2001).
[7] V. V. Dodonov, Phys. Scr. 82, 038105 (2010).
[8] D. A. R. Dalvit, P. A. M. Neto, and F. D. Mazzitelli, Fluctuations, Dissipation and the Dynamical Casimir Effect (Springer-Verlag, Berlin, 2011), pp. 419–457.
[9] K. A. Milton, Am. J. Phys. 79, 697 (2011).
[10] G. T. Moore, J. Math. Phys. 11, 2679 (1970).
[11] C. K. Law, Phys. Rev. Lett. 73, 1931 (1994).
[12] H. Saito and H. Hyuga, J. Phys. Soc. Jpn. 65, 3513 (1996).
[13] C. Yuce and Z. Ozccakmakli, J. Phys. A: Math. Theor. 41, 265401 (2008).
[14] C. M. Wilson, G. Johansson, A. Pourkabirian, M. Simoen, J. R. Johansson, T. Duty, F. Nori, and P. Delsing, Nature 479, 376 (2011).
[15] J. B. Pendry, J. Phys. Condens. Matter 9, 10301 (1997).
[16] J. B. Pendry, J. Mod. Opt. 45, 2389 (1998).
[17] J. B. Pendry, New J. Phys. 12, 033028 (2010).
[18] M. F. Maghrebi, R. Golestanian, and M. Kardar, Phys. Rev. A 88, 042509 (2013).
[19] T.-B. Wang, N.-H. Liu, J.-T. Liu, and T.-B. Yu, Eur.
[20] Y. Pomeau, J. Stat. Phys. 121, 1083 (2005).
[21] Y. Pomeau, Europhys. Lett. 74, 951 (2006).
[22] A. Manjavacas and F. J. García de Abajo, Phys. Rev. Lett. 105, 113601 (2010).
[23] A. Manjavacas and F. J. García de Abajo, Phys. Rev. A 82, 063827 (2010).
[24] R. Zhao, A. Manjavacas, F. J. García de Abajo, and J. B. Pendry, Phys. Rev. Lett. 109, 123604 (2012).
[25] M. F. Maghrebi, R. L. Jaffe, and M. Kardar, Phys. Rev. Lett. 108, 230403 (2012).
[26] A. Manjavacas, F. J. Rodríguez-Fortuño, F. J. García de Abajo, and A. V. Zayats, Phys. Rev. Lett. 118, 133605 (2017).
[27] A. Lambrecht, M.-T. Jaekel, , and S. Reynaud, Phys. Rev. Lett. 77, 615 (1996).
[28] A. Lambrecht, J. Opt. B: Quantum Semiclass. Opt. 7, S3S10 (2005).
[29] A. V. Dodonov and V. V. Dodonov, Phys. Rev. A 85, 055805 (2012).
[30] G. Barton, New J. Phys. 12, 113044 (2010).
[31] G. Barton, New J. Phys. 12, 113045 (2010).
[32] H. Bercegol and R. Lehoucq, Phys. Rev. Lett. 115, 090402 (2015).
[33] S. Manipatruni, J. T. Robinson, and M. Lipson, Phys. Rev. Lett. 102, 213903 (2009).
[34] M. Hafezi and P. Rabl, Opt. Express 20, 7672 (2012).
[35] D. W. Wang, H. T. Zhou, M. J. Guo, J. X. Zhang, J. Ev- ers, and S. Y. Zhu, Phys. Rev. Lett. 110, 093901 (2013).
[36] F. Ruesink, M. A. Miri, A. Alù, and E. Verhagen, Nat. Commun. 7, 13662 (2016).
[37] Z. Shen, Y. L. Zhang, Y. Chen, C. L. Zou, Y. F. Xiao, X. B. Zou, F. W. Sun, G. C. Guo, and C. H. Dong, Nat. Photon. 10, 657 (2016).