Vortex matter in the charged Bose liquid at absolute zero

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The Gross-Pitaevskii-type equation is solved for the charge Bose liquid in an external magnetic field at zero temperature. There is a vortex lattice with locally broken charge neutrality. Remarkably, there is no upper critical field at zero temperature, so the density of single flux-quantum vortices monotonously increases with the magnetic field up to $B = \infty$ and no indication of a phase transition. The size of each vortex core decreases as about $B^{-1/2}$ keeping the system globally charge neutral. If bosons are composed of two fermions, a phase transition to a spin-polarized Fermi liquid at some magnetic field larger than the pair-breaking field is predicted.

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Charged Bose liquids (CBLs) have been solely of academic interest for a long time [1, 2, 3, 4, 5, 6, 7, 8]. Notwithstanding, experimental realization of the Bose-Einstein condensation (BEC) of trapped ultra-cold atoms [3, 10, 11, 12, 13, 14] made it possible to create ultra-cold plasmas [15] by using lasers to trap and cool neutral atoms to temperatures of 1 mK or lower. Another cold plasmas [15] by using lasers to trap and cool neutral atoms to temperatures of 1 mK or lower. Another cold plasmas [15] by using lasers to trap and cool neutral atoms to temperatures of 1 mK or lower. Another cold plasmas [15] by using lasers to trap and cool neutral atoms to temperatures of 1 mK or lower. Another cold plasmas [15] by using lasers to trap and cool neutral atoms to temperatures of 1 mK or lower. Another cold plasmas [15] by using lasers to trap and cool neutral atoms to temperatures of 1 mK or lower. 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The integro-differential equation (2) is quite different from original Ginzburg-Landau (GL) \[24\] and the Gross-Pitaevskii \[27\] equations, describing the order parameter in the BCS and neutral superfluids, respectively. As recognised by one of us \[22\] the coherence length in CBL is just the same as the screening radius, so the core of a single vortex is charged \[28\]. Indeed, introducing the dimensionless quantities: \( f = |\psi_0|/n^{1/2}, \rho = r/\lambda, \)
\( \mathbf{h} = \varepsilon_0 c \text{curl} \mathbf{A}/ \hbar c \) for the order parameter, length and magnetic field, respectively, one obtains the coherence length about the same as the screening radius at \( T = 0 \text{K} \). \[3\], \( \xi = (\hbar^2/2m^* \omega_p)^1/2, \) where \( \omega_p = (4\pi n e^2 / m)^{1/2} \) is the zero-temperature plasma frequency \[2\]. The London penetration depth is conventional, \( \lambda = (mc^2/4\pi ne^2)^{1/2}, \) but a new feature is an electric field potential, \( e\phi(r) = \int dr' V(r-r') |\psi_0(r')|^2 - n \). Moreover, the chemical potential \( \mu \) is zero, as it should be in the globally neutral CBL in the thermal equilibrium below the BEC critical temperature.

Any realistic CBL is an extreme type II with a very large Ginzburg-Landau parameter, \( \kappa = \lambda/\xi \gg 1 \). For example, the coherence length and the electric field inside the vortex core are about 1 nm or less and 10 mV, respectively, with the material parameters typical for cuprates \( (m = 10m_e, n = 10^{21} \text{cm}^{-3} \) and \( \epsilon_0 \gg 100), \) and \( \kappa \) is about \( 10^3 \) with these parameters. Hence, the magnetic field is practically homogeneous, and the ground state \( \psi_0(r) \) can be found by minimizing the energy functional \( E \) with respect to \( \psi_0(r) \),

\[
E(\psi_0) = \frac{1}{2m} \int dr (\hbar \nabla - ieA/c)\psi_0(r)^2 + \frac{1}{2} \int dr \int dr' V(r-r') |\psi_0(r')|^2 (|\psi_0(r')|^2 - 2n),
\]

where \( \mathbf{A} = \{0,Br,0\} \). In numerical simulations we consider a sample with the rectangular cross-section \( L \times L \) and the magnetic flux \( BL^2 = m\Phi_0 \), where \( m \) is an integer \( (\Phi_0 \) is the flux quantum). When the magnetic field \( \mathbf{B} \) is applied along z-direction, the order parameter \( \psi_0(x,y) \) does not depend on \( z \) obeying the following translation symmetry,

\[
\psi_0(x + L, y) = \exp(-ieBLy/\hbar c)\psi_0(x, y) \quad \psi_0(x, y + L) = \psi_0(x, y).
\]

These relations can be used as boundary conditions when \( m \) is an integer.

Because Eq.(3) does not contain the penetration depth, it is convenient to introduce new dimensionless coordinates \( \mathbf{x} = r/\xi \), the vector potential, \( \mathbf{a} = (0,2\pi Bx\xi^2/\Phi_0,0) \), and the Coulomb energy \( v(\mathbf{x}) = e\phi/\omega_p^2 m^* \xi^2 \). As a result, the problem is reduced to minimization of the functional

\[
E(f) = \frac{\hbar^2 n \xi}{2m} \int d\mathbf{x} \left[(\nabla - i\mathbf{a}) f(\mathbf{x}) \right]^2 + v(\mathbf{x})(|f(\mathbf{x})|^2 - 1),
\]

where the Coulomb field satisfies the Poisson equation,

\[
\Delta v(\mathbf{x}) = 1 - |f(\mathbf{x})|^2.
\]

To compare CBL vortex state with the Abrikosov vortex lattice we also minimize the conventional GL functional using the same dimensionless units,

\[
E_{\text{GL}}(f) = \frac{\hbar^2 n \xi}{2m} \int d\mathbf{x} \left[(\nabla - i\mathbf{a}) f(\mathbf{x}) \right]^2 - \frac{1}{2} |f(\mathbf{x})|^4,
\]

where \( \xi = \hbar^2/(2m|\alpha|)^{1/2}, \) \( n_s = |\alpha|/\beta \) and the order parameter \( f \) is normalised by \( \sqrt{n_s} \). Here \( \alpha \) and \( \beta \) are conventional GL coefficients \[26\]. We apply the standard discretization procedure described in Ref. \[24\], Eq.(6) for the electrostatic potential is solved by the Fourier transform in the discrete form, and the resulting energy is minimized with the conjugated gradient algorithm.

Since both functionals depend only on the dimensionless vector-potential \( \mathbf{a} \) which is proportional to the product \( B\xi^2 \), simulations can be performed at fixed \( L \) and \( \xi \) by changing \( B \) or at fixed \( L \) and \( B \) by changing \( \xi \). Our numerical results are shown in Figs. 1–4. At any value of the magnetic field we find the triangular vortex lattice.
the order parameter vanishes at and above the finite
they decrease in conventional superconductors, where
parameter increase with the magnetic field in CBL, while
result, the amplitude real-space modulations of the order
\( \xi \) "bare" coherence length
the absence of an equilibrium
breakdown of the local charge neutrality, Fig.2, is due to
charge redistribution caused by the magnetic field. The
itatively from the Abrikosov vortex \[23\] due to a local
Fig.2, in agreement with Ref. \[22\], which differs qual-
apart, their interaction yields only a small contribution
flux-quantum vortices, i.e.
only characteristic length is the distance between single
cores. The Coulomb energy is small compared with the
is the characteristic electrostatic potential inside vortex
cores. The Coulomb energy is small compared with the
spin (Pauli) contribution if
"trIPLETS are unstable, because the singlet binding en-
\( \xi \), then the left hand side of Eq.(6) takes the form \((1 - |f(x)|^2)d\delta(z)/\xi\). The
dimension analysis readily shows that the true co-
herence length, \( \xi_{2D} \) depends on the thickness as \( \xi_{2D} = (\xi^2/d)^{1/3} \) in that case. As a result the size of vortex cores
depends on the thickness of CBL films different from the
conventional films.

There is also an important consequence of the infi-
nite (orbital) upper critical field at absolute zero in such
CBLs, where singlet bosons are formed of two fermions
[16]. In this case sufficiently large magnetic field can
break bound pairs via a spin-flip of one of two fermions,
if triplets are unstable, because the singlet binding en-
ergy \( \Delta \) decreases with the field as \( \Delta(B) = \Delta - 2\mu_B B \)
\( (\mu_B = e\hbar/(2m_e) \) is the electron Bohr magneton \) \[30\].
A spin-polarised Fermi liquid appears at \( B > \mu_B \), where
\( \mu_B \) is the pair-breaking field. In this estimate we
neglect the orbital (Landau) diamagnetism of bosons
and fermions, and the Coulomb energy of the charged-
dominated vortex lattice. The latter is of the order of
\( e^2\phi_c n^2 B/\Phi_0 \) per unit volume, where \( \phi_c \sim \hbar^2/(em^2\xi^2) \) is the character-
istic electrostatic potential inside vortex cores. The Coulomb energy is small compared with the
spin (Pauli) contribution if \( m_e/m \ll 1 \), which we as-
sume to be the case, so diamagnetic contributions are
also small. However, bound pairs still survive up to a
higher field \( H^* = \mu_B n/(N\mu_B) > \mu_B \) due to the Pauli
exclusion principle, which prevents any further decay of
pairs, if the number of fermions \( \approx N(\Delta(B)) \) remains
smaller than \( 2n \) ( \( N \) is the fermion density of states).
There is a boson-fermion mixture, if \( \mu_B < B < H^* \), with
the fermion density modulated in real space because of
charged vortices. Normal fermions (as well as normal

While the field is small, there are only a few vortices
per sample cross-section, Fig.1a. When vortices are far
apart, their interaction yields only a small contribution
to the energy functional but even in that case a triangular
lattice of vortices is clearly seen in CBL, Fig.1.

Each vortex carries one flux quantum, as can be seen
from the phase profile in Fig.1b. It has an unusual core,
Fig.2, in agreement with Ref. \[22\], which differs qual-
itatively from the Abrikosov vortex \[23\] due to a local
charge redistribution caused by the magnetic field. The
breakdown of the local charge neutrality, Fig.2, is due to
the absence of an equilibrium normal state solution in
CBL at \( T = 0 \) with \( \psi_0 = 0 \), as explained in Ref. \[22\].

Increasing the field first increases the vortex density
with about constant size of the cores, as in conventional
superconductors, Fig.3 and Fig.4. However, quite differ-
ent from the Abrikosov lattice, increasing the field further
does not lead to a superfluid to normal phase transition,
but instead it increases the density of vortices by
decreasing the size of every individual core, Fig.3c,d. Re-
markably, each vortex carries one flux quantum at any
field. Keeping the global charge neutrality the charge
heterogeneity depends on the magnetic field, and the
core diameters decrease with the field, when the field is
large, \( \xi^2 > 2\pi \hbar c/(eB) \). Indeed, in this regime the
"bare" coherence length \( \xi \) becomes irrelevant, but the
only characteristic length is the distance between single
flux-quantum vortices, i.e. \( r \approx \sqrt{2\pi \hbar c/(eB)} \). As a re-
sult, the amplitude real-space modulations of the order
parameter increase with the magnetic field in CBL, while
they decrease in conventional superconductors, where
the order parameter vanishes at and above the finite
\( H_{c2} = \Phi_0/(2\pi \xi^2) \) (Fig.4c,d).

There is another difference between CBL and conven-
tional vortex matter in case of thin films. If we assume that
the film thickness \( d \) is small, \( d \ll \xi \), then the left
hand side of Eq.(6) takes the form \( (1 - |f(x)|^2)d\delta(z)/\xi \).
The dimension analysis readily shows that the true co-
herence length, \( \xi_{2D} \) depends on the thickness as \( \xi_{2D} = (\xi^2/d)^{1/3} \) in that case. As a result the size of vortex cores
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conventional films.

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FIG. 3: The vortex lattice in CBL for 30 flux quantum
per cross-section, (a) \( L/\xi = 33.67 \), (b) \( L/\xi = 25.25 \), (c)
\( L/\xi = 14.43 \), (d) \( L/\xi = 10.1 \) One can see from the scale near
each figure that the order parameter remains large outside
the cores, \( f > 1 \) at any \( \xi \) (or at any magnetic field).

FIG. 4: The Abrikosov vortex lattice for 30 flux quantum
per cross-section (a) \( L/\xi = 33.67 \), (b) \( L/\xi = 25.25 \), (c) \( L/\xi = 
14.43 \), and for (c) \( L/\xi = 13.87 \), which corresponds to \( B \) close
to \( H_{c2} = \Phi_0/(2\pi \xi^2) \). The order parameter decreases when \( B \)
approaches the conventional upper critical field.
bosons pushed up from the condensate by temperature) are distributed inhomogeneously across the sample with the maximum density in the vortex cores, where their potential energy is at minimum. The excess density of normal carriers inside the cores screens the electric field caused by the inhomogeneous condensate density. If the screening length due to normal fermions becomes smaller than the coherence length $\xi$, one can expect a nontrivial field dependence of the size of vortices, which disappear at $B = H^*$. 

In conclusion, we have found the triangular lattice of single-flux-quantum charged vortices in CBL which cannot be destroyed by any magnetic field at zero temperature. The vortex density monotonously increases and their core size decreases with the magnetic field up to $B = \infty$ with no indication of a phase transition. The core size depends on the thickness of CBL films. At finite temperatures $H_{c2}(T)$ is finite [20, 21]. Nevertheless, unusually large charge modulations with the scale depending on the magnetic field should persist at finite temperatures as well. The phase transition to the spin-polarized Fermi liquid at some magnetic field larger than the pair-breaking field has been predicted for preformed bosonic pairs. These results are relevant for real charged Bose-liquids in ultracold plasmas and in the superconducting cuprates, and for superconducting quantum dots and superconductor-insulator phase transitions described by a similar boson physics. There is also a close analogy between the vortex structure in CBL and the Josephson vortices. Since the normal phase is not defined below $T_c$, there is no "normal" vortex core in CBL, and there is no "normal" core in the Josephson vortex either. One can define the lower critical field $H_{c1}$ when a first vortex penetrates into CBL [22] and into the Josephson junction [31], but the upper critical field is infinite in both cases.

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