Quantum oscillations without magnetic field

Tianyu Liu, D. I. Pikulin, and M. Franz

Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada V6T 1Z1 and Quantum Matter Institute, University of British Columbia, Vancouver BC, Canada V6T 1Z4

(Dated: August 17, 2016)

When magnetic field $B$ is applied to a metal, nearly all observable quantities exhibit oscillations periodic in $1/B$. Such quantum oscillations reflect the fundamental reorganization of electron states into Landau levels as a canonical response of the metal to the applied magnetic field. We predict here that, remarkably, in the recently discovered Dirac and Weyl semimetals quantum oscillations can occur in the complete absence of magnetic field. These zero-field quantum oscillations are driven by elastic strain which, in the space of the low-energy Dirac fermions, acts as a chiral gauge potential. We propose an experimental setup in which the strain in a thin film (or nanowire) can generate pseudomagnetic field as large as 15T and demonstrate the resulting de Haas-van Alphen and Shubnikov-de Haas oscillations periodic in $1/b$.

Dirac and Weyl semimetals [1-3] are known to exhibit a variety of exotic behaviors owing to their unusual electronic structure comprised of linearly dispersing electron bands at low energies. This includes the pronounced negative magnetoresistance [4,11] attributed to the phenomenon of the chiral anomaly [12-14], theoretically predicted nonlocal transport [15-16], Majorana flat bands [17], as well as an unusual type of quantum oscillations (QO) that involve both bulk and topologically protected surface states [18-19]. In this theoretical study we establish a completely new mechanism for QO in Dirac and Weyl semimetals that requires no magnetic field. These zero-field oscillations occur as a function of the applied elastic strain and, similar to the canonical de Haas-van Alphen and Shubnikov-de Haas oscillations [20], manifest themselves as oscillations periodic in $1/b$, where $b$ is the strain-induced pseudomagnetic field, in all measurable thermodynamic and transport properties. To the best of our knowledge this is the first instance of such zero-field quantum oscillations in any known substance.

Materials with linearly dispersing electrons respond in peculiar ways to the externally imposed elastic strain. In graphene, for instance, the effect of curvature is famously analogous to a pseudomagnetic field [21] that can occur in the complete absence of magnetic field. These zero-field quantum oscillations are analogous, in terms of its low-energy properties, to an unstrained film subject to magnetic field $B$. Such quantum oscillations reflect the fundamental reorganization of electron states into Landau levels as a canonical response of the metal to the applied magnetic field. We predict here that, remarkably, in the recently discovered Dirac and Weyl semimetals quantum oscillations can occur in the complete absence of magnetic field. These zero-field quantum oscillations are driven by elastic strain which, in the space of the low-energy Dirac fermions, acts as a chiral gauge potential. We propose an experimental setup in which the strain in a thin film (or nanowire) can generate pseudomagnetic field as large as 15T and demonstrate the resulting de Haas-van Alphen and Shubnikov-de Haas oscillations periodic in $1/b$.

![FIG. 1: Proposed setup for strain-induced quantum oscillation observation in Dirac and Weyl semimetals. a) Bent film and Na$_3$Bi [30,31] which is the best characterized representative of this class of materials. Our results are directly applicable also to Na$_3$Bi whose low-energy description is identical, and are easily extended to other Dirac and Weyl semimetals [30,31]. We start from the tight-binding model formulated in Refs. [30,31] which describes the low-energy physics of Cd$_3$As$_2$ by including the band inversion of its atomic Cd-5s and As-4p levels near the $\Gamma$ point. In the basis of the spin-orbit coupled states $|P_+, \frac{3}{2}\rangle, |S_-, \frac{1}{2}\rangle, |S_x, \frac{1}{2}\rangle$ and $|P_-, \frac{3}{2}\rangle$ the model is defined by a 4 x 4 matrix Hamiltonian

$$H_{\text{latt}} = \epsilon_k + \begin{pmatrix} h_{\text{latt}} & 0 \\ 0 & -h_{\text{latt}} \end{pmatrix},$$

(1)

on a simple rectangular lattice with spacings $a_{x,y,z}$, where

$$h_{\text{latt}}(k) = m_k \tau^z + \lambda (\tau^x \sin a_x k_x + \tau^y \sin a_y k_y),$$

(2)

$\tau$ are Pauli matrices in the orbital space and $m_k = t_0 + t_1 \cos a_z k_z + t_2 (\cos a_x k_x + \cos a_y k_y)$. For the analytic calculations below we will assume $a_i = a$, while
The gauge potential is given by $\eta$ of the atoms. To see how this leads to an emergent vector by modifying the electron tunneling amplitude along the $\hat{z}$ direction according to $t_1 \tau^z \rightarrow t_1 (1 - u_{33}) \tau^z + i \Lambda \sum_{j \neq 3} u_{3j} \tau^j$, (3)

where $u_{ij} = \frac{1}{2} (\partial_i u_j + \partial_j u_i)$ is the symmetrized strain tensor and $\mathbf{u} = (u_1, u_2, u_3)$ represents the displacement of the atoms. To see how this leads to an emergent vector potential we study the low-energy effective theory. We expand $h^{\text{lat}}(\mathbf{k})$ in the vicinity of the Weyl points $\mathbf{K}_\pm$ by writing $\mathbf{k} = \mathbf{K}_\pm + \mathbf{q}$ and assuming small $|\mathbf{q}|$. To leading order we obtain the linearized Hamiltonian of the distorted crystal [20]

$$h_\eta(\mathbf{q}) = v_\eta \tau^j \left( h q_j - \eta e A_j \right),$$

with the velocity vector

$$v_\eta = h^{-1} a (\Lambda, -\eta t_1 \sin aQ).$$

For Cd$_3$As$_2$ parameters and lattice constant $a = 4\AA$ this gives $h v_{\eta} = (0.89, 0.89, -1.24\eta)eV\AA$. The strain-induced gauge potential is given by

$$\tilde{A} = -\frac{hc}{ea} (u_{13} \sin aQ, u_{23} \sin aQ, u_{33} \cot aQ).$$

We see that elements $u_{ij}$ of the strain tensor act on the low-energy Weyl fermions as components of a chiral gauge field because according to Eq. [1] $\tilde{A}$ couples with the opposite sign to the Weyl fermions with opposite chirality $\eta$. Ordinary electromagnetic gauge potential couples through the replacement $\hbar \mathbf{q} \rightarrow \hbar \mathbf{q} - \frac{e}{c} \mathbf{A}$, independent of $\eta$. Ref. [20] noted that application of a torsional strain to a nanowire made of Cd$_3$As$_2$ (grown along the 001 crystallographic direction) results in a uniform pseudomagnetic field $\mathbf{b} = \nabla \times \tilde{A}$ pointed along the axis of the wire. The strength of this pseudomagnetic field was estimated as $b \lesssim 0.3T$ which would be insufficient to observe QO. Our key observation here is that a different type of distortion, illustrated in Fig. [1f], can produce a much larger field $b$.

One reason why the torsion-induced $b$-field is relatively small lies in the fact that it originates from the $A_x$ and $A_y$ components of the vector potential. According to Eq. [6] these are suppressed relative to the strain components by a factor of $\sin aQ$. This is a small number in most Dirac and Weyl semimetals because the distance $2Q$ between the Weyl points is typically a small fraction of the Brillouin zone size $2\pi/a$. Specifically, we have $aQ \approx 0.132$ in Cd$_3$As$_2$ [31]. Note on the other hand that the $A_z$ component of the chiral gauge potential comes with a factor $\cot aQ \approx 1/aQ$ and is therefore enhanced. A lattice distortion that produces nonzero strain tensor element $u_{33}$ will therefore be much more efficient in generating large $b$ than $u_{13}$ or $u_{23}$. Specifically, for the same amount of strain the field strength is enhanced by a factor of $\cot aQ/\sin aQ \approx 1/(aQ)^2 \approx 57$ for Cd$_3$As$_2$.

To implement this type of strain we consider a thin film (or a nanowire) grown such that vector $K_\eta$ lies along the $z$ direction as defined in Fig. [1h]. More generally we require that $K_\eta$ has a nonzero projection onto the surface of the film or on the long direction for the nanowire. Cd$_3$As$_2$ films [29], microribbons [44] and nanowires [45, 46] satisfy this requirement. Bending the film as shown in Fig. [1b] creates a displacement field $\mathbf{u} = (0, 0, 2\alpha xz/d)$, where $d$ is the film thickness and $\alpha$ controls the magnitude of the bend. (If $R$ is the radius of the circular section formed by the bent film then $\alpha = 2d/R$. $\alpha$ can also be interpreted as the maximum fractional displacement $\alpha = u_{33}/a$ that occurs at the surface of the film.) This distortion gives $u_{33} = 2\alpha x/d$ and through Eq. [6] yields a pseudomagnetic field

$$\mathbf{b} = \nabla \times \tilde{A} = \hat{y} \left( \frac{2\alpha}{a} \right) \frac{hc}{ea} \cot aQ. \quad (7)$$

Noting that $\Phi_0 = hc/e = 4.12 \times 10^6 \text{T}\AA$ we may estimate the resulting field strength for a $d = 100\text{nm}$ film as

$$b \simeq \alpha \times 246\text{T}. \quad (8)$$

The maximum pseudomagnetic field that can be achieved will depend on the maximum strain that the material can support.
sustain. Ref. [45] characterized the Cd$_3$As$_2$ nanowires as “greatly flexible” and their Figure 1a shows some wires bent with a radius $R$ as small as several microns. This implies that $\alpha$ of several percent can likely be achieved. From Eq. (8) we thus estimate that field strength $b \approx 10-15T$ can be reached, providing a substantial window for the observation of the strain-induced QO.

To substantiate these claims we now present the results of our numerical simulations based on the lattice Hamiltonian $H$. Magnetic field $B$ is implemented via the standard Peierls substitution while the strain-induced field $b$ through Eq. (4). Geometry outlined in Fig. 1 is used with periodic boundary conditions along $y$ and $z$, open along $x$. Fig. 3 provides the summary of our results. The unstrained crystal at zero field (panel a) shows the expected band structure with bulk Weyl nodes close to $k_x a = \pm 0.2$ and a pair of linearly dispersing surface states corresponding to Fermi arcs. The density of states (DOS) exhibits the expected quadratic behavior $D(E) \sim E^2$ at low energies with some deviations apparent for $|E| \gtrsim 12\text{meV}$ due to the departure of the lattice model from the perfectly linear Weyl dispersion. At $E_{\text{Lif}} \approx 20\text{meV}$ Lifshitz transition occurs where two small Fermi surfaces associated with each Weyl point merge into a single large Fermi surface as illustrated in Fig. 2a.

In Fig. 3, magnetic field $B = \hat{y}B$ is seen to reorganize the linearly dispersing bulk bands into flat Landau levels. In the continuum approximation given by Eq. (4) the bulk spectrum of such Dirac-Landau levels is well known and reads

$$E_n(k_y) = \pm h \sqrt{v_y k_y^2 + 2nu_z v_z |B|/\hbar}, \quad n = 1, 2, \ldots , \tag{9}$$

The corresponding DOS shows a series of spikes at the onset of each new Landau level and is in a good agreement with the DOS calculated from the lattice model. Deviations occur above $\sim 12\text{meV}$ because the energy dispersion of the lattice model is no longer perfectly linear at higher energies. The peak positions $E_n$ agree perfectly with the Lifshitz-Onsager quantization condition [20], which takes into account these deviations. It requires that $S(E_n) = 2\pi n(eB/\hbar c)$, where $S(E)$ is the extremal cross-sectional area of a surface of constant energy $E$ in the plane perpendicular to $B$ (see Fig. 2b), and $n = 1, 2, \ldots , \ldots$.

Pseudomagnetic field $b = \hat{y}b$, induced by strain using Eq. (3) with $\nu_{33} = 2\alpha x/d$, also generates flat bands (panel c), as expected on the basis of arguments presented above. The corresponding DOS is in agreement with that obtained from Eq. (9) upon replacing $B \to b$. Remarkably the agreement is nearly perfect for all energies up to $E_{\text{Lif}}$. We attribute this interesting result to the fact that strain couples as the chiral vector potential only to the Weyl fermions. If we write the full Hamiltonian as $h(p) = h_W(p) + \delta h(p)$ where $h_W$ is strictly linear
in momentum $p$ and $\delta h$ is the correction resulting from the lattice effects, then strain causes $p \rightarrow p - \frac{\delta}{2} \hat{A}$ only in $h_W$ but does not to leading order affect $\delta h$. The real vector potential $A$ affects $h_W$ and $\delta h$ in the same way.

These results imply that QO will occur when either $B$ or $b$ is present. If we vary $B$ then $D(E_F)$, together with most measurable quantities, will exhibit oscillations periodic in $1/B$. The same is true for the strain-induced pseudomagnetic field $b$. This is illustrated in Fig.3 which shows oscillations in DOS and longitudinal conductivity $\sigma_{yy}$ at energy $10$meV as a function of $1/b$ and $1/B$. Conductivity is calculated using the standard relaxation time approximation as described in SM. Strain-induced QO show robust periodicity in $1/b$. Their period $0.329T^{-1}$ is in a good agreement with the period $0.324T^{-1}$ expected on the basis of the Lifshitz-Onsager theory and $0.336T^{-1}$ obtained from Eq. [9]. Small irregularities that appear at low fields can be attributed to the finite size effects as the Landau level spacing becomes comparable to the subband spacing apparent e.g. in Fig. 3. We verified that similar oscillations occur at other energies below the Lifshitz transition. Remarkably, we find strain-induced oscillations periodic in $1/b$ also above $E_{Lif}$. In addition, we expect that in the presence of both $b$ and $B$ fields the peaks split as two Weyl cones feel different effective magnetic fields. These effects are further discussed in SM.

Results presented above extend trivially to the full Cd$_3$As$_2$ Hamiltonian Eq. [1] where the spin-down block makes an identical contribution and the p-h symmetry breaking terms contained in $\epsilon_k$ bring only quantitative changes (see SM for discussion). Experimental studies [92, 93] indicate that the linear dispersion in Cd$_3$As$_2$ extends over a much wider range of energies than theoretically anticipated [31] with the Lifshitz transition occurring $\sim 200$meV. We therefore expect the zero-field strain-induced QO predicted in this work to be easily observable in suitably fabricated Cd$_3$As$_2$ films and nanowires and potentially also in other Dirac and Weyl semimetals. Our results show that conditions for their observability are identical to those required to detect ordinary QO. The continuous tunability of the pseudomagnetic field in large parameter range provides a new experimental basis for the study of emergent gauge fields in three-dimensional crystalline solids.

The authors are indebted to D.A. Bonn, D.M. Broun, A. Chen, I. Elfimov and W. N. Hardy for illuminating discussions, and thank NSERC, CIFAR and Max Planck - UBC Centre for Quantum Materials for support.

FIG. 4: Strain-induced QO. Top panel shows oscillations in DOS at energy $10$meV as a function of inverse strain strength expressed as $1/b$. For comparison ordinary magnetic oscillations are displayed, as well as the result of the bulk continuum theory Eq. [9]. Crosses indicate peak positions expected based on the Lifshitz-Onsager theory. Bottom panel shows oscillations in conductivity $\sigma_{yy}$ assuming Fermi energy $E_F = 10$meV. To simulate the effect of disorder all data are broadened by convolving in energy with a Lorentzian with width $\delta = 0.25$meV. The same geometry and parameters are used as in Fig. 3.

[1] X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov. Phys. Rev. B 83, 205101 (2011).
[2] A. A. Burkov, M. D. Hook, and L. Balents, Phys. Rev. B 84, 235126 (2011).
[3] O. Vafek and A. Vishwanath, Annual Review of Condensed Matter Physics 5, 83 (2014).
[4] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008).
[5] D. T. Son and B. Z. Spivak, Phys. Rev. B 88, 104412 (2013).
[6] H.-J. Kim, K.-S. Kim, J.-F. Wang, M. Sasaki, N. Satoh, A. Ohnishi, M. Kitaura, M. Yang, and L. Li, Phys. Rev. Lett. 111, 246603 (2013).
[7] X. Huang, L. Zhao, Y. Long, P. Wang, D. Chen, Z. Yang, H. Liang, M. Xue, H. Weng, Z. Fang, et al., Phys. Rev. X 5, 031023 (2015).
[8] J. Xiong, S. K. Kushwaha, T. Liang, J. W. Krizan, M. Hirschberger, W. Wang, R. J. Cava, and N. P. Ong, Science 350, 413 (2015).
[9] A. A. Burkov, Journal of Physics: Condensed Matter 27, 113201 (2015).
[10] Q. Li, D. E. Kharzeev, C. Zhang, Y. Huang, I. Pletikosic, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu, and T. Valla, Nat Phys 12, 550 (2016).
[11] C.-L. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, Z. Lin, B. Tong, G. Bian, N. Alidoust, C.-C. Lee, S.-M. Huang, et al., Nat Commun 7, 10735 (2016).
[12] S. L. Adler, Phys. Rev. 177, 2426 (1969).
[13] J. S. Bell and R. Jackiw, Il Nuovo Cimento A (1971-1996) 60, 47 (1969).
[14] H. Nielsen and M. Ninomiya, Physics Letters B 130, 389 (1983).
[15] S. A. Parameswaran, T. Grover, D. A. Abanin, D. A. Pesin, and A. Vishwanath, Phys. Rev. X 4, 031035 (2014).
[16] Y. Baun, E. Berg, S. A. Parameswaran, and A. Stern, Phys. Rev. X 5, 041046 (2015).
Additionally, we model the particle-hole asymmetry of the real CdAs using the familiar formula for the DC conductivity

\[ \sigma_{k} = r_{0} + r_{1} \cos a_{z} k_{z} + r_{2}(\cos a_{x} k_{x} + \cos a_{y} k_{y}), \]

with \( r_{0} = 5.9439eV \), \( r_{1} = -0.8472eV \), and \( r_{2} = -2.5556eV \).

The results for the dispersion and DOS for the realistic particle-hole asymmetric case are shown in Fig. 5. We note the similarity of the results with those displayed in the main text and in Fig. 5. Specifically, both the real magnetic field and the spin-induced pseudomagnetic field give rise to pronounced Landau levels. We thus conclude that all our predictions remain valid for this realistic Dirac semimetal.

**Conductivity calculation**

First we obtain the analytical expression for the conductivity of Dirac-Landau levels in the bulk and in the continuum limit. From the dispersion relation \( \epsilon(k) \) we obtain the velocity in \( y \) direction,

\[ v^{\star}(k_{y}) = \frac{1}{\hbar} \frac{\partial E_{n}^{\star}}{\partial k_{y}} = s v_{y} \frac{k_{y}}{\sqrt{k^{2} + n^{2} \Omega}}, \]

where \( s \) is the sign of the energy and \( \Omega = \frac{2eB v_{f}^{\star} \omega_{A}}{\hbar} \). Then we use the familiar formula for the DC conductivity \( \sigma_{yy} \) due to the \( n^{\star} \)th Landau level in the relaxation time approximation

\[ \sigma_{n}(\mu) = e^{2} \int \frac{dk_{y}}{2\pi} \tau_{n}(E_{n}^{2}(k_{y}))(\nu_{n}(k_{y}))^{2} \left( -\frac{\partial f(E - \mu)}{\partial E} \right) E_{n}^{*}(k_{y}). \]

**Model parameters**

We model CdAs using the Hamiltonian \( \mathbf{H} \) with parameters taken from the first principles band structure calculations \([17, 18]\) and with the lattice constants corresponding to the actual material, \( a_{x,y} = 3\AA, a_{z} = 5\AA \). This implies that the constants used in \([2]\) are: \( \Lambda = 0.296eV, t_{0} = -7.4811eV, t_{1} = 1.5016eV, \) and \( t_{2} = 3eV \). Additionally, we model the particle-hole asymmetry of the real CdAs using the familiar formula for the DC conductivity

\[ \sigma_{k} = r_{0} + r_{1} \cos a_{z} k_{z} + r_{2}(\cos a_{x} k_{x} + \cos a_{y} k_{y}), \]

with \( r_{0} = 5.9439eV \), \( r_{1} = -0.8472eV \), and \( r_{2} = -2.5556eV \).

The results for the dispersion and DOS for the realistic particle-hole asymmetric case are shown in Fig. 5. We note the similarity of the results with those displayed in the main text and in Fig. 5. Specifically, both the real magnetic field and the spin-induced pseudomagnetic field give rise to pronounced Landau levels. We thus conclude that all our predictions remain valid for this realistic Dirac semimetal.

**Conductivity calculation**

First we obtain the analytical expression for the conductivity of Dirac-Landau levels in the bulk and in the continuum limit. From the dispersion relation \( \epsilon(k) \) we obtain the velocity in \( y \) direction,

\[ v^{\star}(k_{y}) = \frac{1}{\hbar} \frac{\partial E_{n}^{\star}}{\partial k_{y}} = s v_{y} \frac{k_{y}}{\sqrt{k^{2} + n^{2} \Omega}}, \]

where \( s \) is the sign of the energy and \( \Omega = \frac{2eB v_{f}^{\star} \omega_{A}}{\hbar} \). Then we use the familiar formula for the DC conductivity \( \sigma_{yy} \) due to the \( n^{\star} \)th Landau level in the relaxation time approximation

\[ \sigma_{n}(\mu) = e^{2} \int \frac{dk_{y}}{2\pi} \tau_{n}(E_{n}^{2}(k_{y}))(\nu_{n}(k_{y}))^{2} \left( -\frac{\partial f(E - \mu)}{\partial E} \right) E_{n}^{*}(k_{y}). \]
After the change of the integration variable from \( B \) to \( s \), the angle-independent relaxation time, and substitute \( f \) for \( \mu \), we find

\[
\sigma_n^*(\mu) = e^2 \tau_n^* \int \frac{dk_y}{2\pi} \frac{v_n^2 k_y}{k_y^2 + n\Omega} \delta(E_n^*(k_y) - \mu). \tag{13}
\]

After the change of the integration variable from \( k_y \) to \( E_n^*(k_y) \) and integration we find

\[
\sigma_n^*(\mu) = \frac{e^2 \tau_n^* v_n}{\hbar \pi} \left. \frac{\text{Re} \sqrt{E_n^2 - n\hbar^2 v_n^2 \Omega}}{E_n^*} \right|_{E_n^* = \mu}, \tag{14}
\]

and the total conductivity is

\[
\sigma = \frac{2e^2 v_y}{\hbar} \sum_n \tau_n(\mu) \text{Re} \sqrt{\frac{\mu^2 - n\hbar^2 v_y^2 \Omega}{\mu^2}}. \tag{15}
\]

Finally, we estimate the relaxation time in the lowest order Born approximation

\[
\frac{1}{\tau} = 2\pi D(\mu) n_{\text{imp}} C, \tag{16}
\]

where \( D(\mu) \) is the density of states at the Fermi level and \( n_{\text{imp}} \) is the impurity concentration. Constant \( C \) depends on the details of scattering from impurities. Thus the final formula we use for the conductivity computation in the main text is

\[
\sigma^{yy} = \frac{e^2 v_y}{\pi \hbar D(\mu) n_{\text{imp}} C} \sum_n \text{Re} \sqrt{\frac{\mu^2 - n\hbar^2 v_y^2 \Omega}{\mu^2}}. \tag{17}
\]

Numerically we use the same formula \((12)\), but input the actual velocities and energies into it.

**Quantum oscillations above Lifshitz transition**

In this section we present the results for QO at energy 28meV, above the Lifshitz transition (at approximately 20meV). In Fig. 6 we see that the area of the Fermi surface causing the oscillations in \( B \) and \( b \) fields is different by slightly larger than a factor of 2. For the external magnetic field case the effective area of the Fermi surface is approximately doubled as compared to the gauge field. Strain couples only to the linear part of the Hamiltonian as a gauge field, therefore only the

FIG. 5: Bandstructure and density of states for the model of Cd₃As₃ with the particle-hole asymmetric part \((10)\) included. Top row is for the pseudomagnetic field \( b = 4.25T \) (corresponding to \( \alpha = 0.04 \), stronger strain than in the main text), and bottom row is for real magnetic field \( B = 4.25T \). From left to right – bandstructure for spin up band, bandstructure for spin down band, and normalized total DOS.
oscillations around each of the Weyl points are possible. Notice also that the electron in the pseudomagnetic field travels clockwise around one of the Weyl points and counterclockwise around the another. The precise nature of the corresponding quasiclassical trajectories above the Lifshitz transition is therefore an interesting open question which we leave for further study. We speculate that they include tunneling between the opposite points of the Fermi surface as depicted in Fig. 6b. Such trajectories would define an extremal area consistent with our numerical results.

**Equivalence of external and gauge fields**

In this section we additionally substantiate the proposed equivalence of $b$ and $B$ fields and suggest an additional experimental test. We propose to apply external magnetic field of fixed strength and then slowly turn on strain (or vice versa, whichever is more convenient in a particular experimental design). This will result in splitting of the first peak in DOS as seen in Fig. 7. This happens because the two Weyl cones will feel different effective magnetic fields, $B + b$ and $B - b$, which result in two independent sequences of peaks in DOS. Observation of the splitting would prove the identical nature of the gauge and external magnetic fields in each of the Weyl cones, and establish that the two cones feel opposite effective field due to $b$. 

**FIG. 6:** a) QO above the Lifshitz transition due to ordinary magnetic field and due to the gauge field. Period difference by more than a factor of 2 is seen. The low-energy analytics does not apply anymore, as expected. b) Corresponding hypothesized quasiclassical trajectories of electrons in the Brillouin zone. Green – for $B_y$ field, and red – for $b_y$ field.
FIG. 7: Normalized density of states for both fields present, $B = 1$T and $b = 0.0184$T. Each of the DOS peaks due to ordinary magnetic field splits due to torsion thus proving the equivalence of the external and gauge fields. Inset gives closer view of the first two peaks.