A design of mathematical modelling for the mudharabah scheme in shariah insurance

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Abstract. Indonesian Shariah Insurance Association (AASI) believes that 2014 is the year of Indonesian Shariah insurance, since its growth was above the conventional insurance. In December 2013, 43% growth was recorded for shariah insurance, while the conventional insurance was only hit 20%. This means that shariah insurance has tremendous potential to remain growing in the future. In addition, the growth can be predicted from the number of conventional insurance companies who open sharia division, along with the development of Islamic banking development which automatically demand the role of shariah insurance to protect assets and banking transactions. The development of shariah insurance should be accompanied by the development of premium fund management mechanism, in order to create innovation on shariah insurance products which beneficial for the society. The development of premium fund management model shows a positive progress through the emergence of Mudharabah, Wakala, Hybrid (Mudharabah-Wakala), and Wakala-Waqf. However, ‘model’ term that referred in this paper is regarded as an operational model in form of a scheme of management mechanism. Therefore, this paper will describe a mathematical modeling for premium fund management scheme, especially for Mudharabah concept. Mathematical modeling is required for an analysis process that can be used to predict risks that could be faced by a company in the future, so that the company could take a precautionary policy to minimize those risks.

1. Introduction
The development of shariah insurance started in 1994 with the establishment of PT Syarikat Takaful Indonesia (STI), initiated by the Indonesian Muslim Intellectuals Association (ICMI) through Abdi Bangsa Foundation, Bank Muamalat Indonesia, PT Asuransi Jiwa Tugu Mandiri, the Ministry of Finance, as well as some Indonesian muslim businessmen. [12]

Shariah Insurance as an alternative protection for Muslims needs to be managed in accordance to the Islamic law. Shariah insurance can also be an option for other faiths who consider that the shariah concept is fair for them, wherein shariah is a universal concept and can be applied by everyone.

National Shariah Board (MUI) determined the meaning of shariah insurance (ta'imin, takaful, tadamun) as an effort to protect and mutual assistance among persons or parties through investment funds, in form of assets or tabarru, which has a refund arrangement to face particular
risks, based on a contract which corresponds to shariah (DSN MUI Fatwa No.21 / DSN-MUI / X / 2001 regarding General Guidelines for Shariah Insurance). [12]

Indonesian Sharia Insurance Association (AASI) believes that 2014 is the year of Indonesian Sharia Insurance, since its growth was above the conventional insurance. In December 2013, 43% growth was recorded for sharia insurance, while the conventional insurance was only hit 20%. This means that sharia insurance has tremendous potential to remain growing in the future. In addition, that growth can be predicted from the number of conventional insurance companies who open sharia division, along with the development of Islamic banking development which automatically demand the role of sharia insurance to protect assets and banking transactions.

The development of sharia insurance should be accompanied by the development of premium fund management mechanism, in order to create innovation on shariah insurance products which beneficial for the society. The mechanism of fund management on sharia insurance premium is divided into two, namely with saving portion and without saving portion [13]. The mechanism with saving portion divides the premium fund into two accounts, which are saving account (saving premium) and special account (tabarru premium). Both premiums are managed and invested by the company in accordance with the concept of sharia, in places that are lawful and do not contain usury. Special account (tabarru) is paid by participants with the aim to help (taáwun).[11]

The development of fund management mechanism model on sharia insurance shows an immense progress through the emergence of Mudharabah, Wakala, Hybrid (Mudharabah-Wakala), and Wakala-Waqf concepts [1, 6, 7, 14]. However, ‘model’ term that referred in this paper is regarded as an operational model in form of a scheme of premium fund management mechanism.

This paper will discuss about a mathematical modelling design for the scheme of premium fund management mechanism, particularly Mudharabah. Mathematical modelling is required for analysing process to predict what will happen in the future if certain changes occur in the system. This model can also be utilized to identify the most influential variable in the system. Firstly, the authors will discuss principles in shariah insurance which is applied as a base of the premium fund management mechanism. Afterward, the design of mathematical modelling will be discussed to translate operational model in form of schemes of premium fund management mechanism into mathematical language as a mathematical model.

The applied theory in sharia insurance calculation, among other, are:

a. Probability theory

Probability is defined as the extent to which an event is likely to occur, measured by the ratio of the favourable cases to the whole number of cases possible. For an illustration, an event occurred in m ways and failed to occur in n ways, then the probability of the event occurrence is: [11]

\[ p = \frac{m}{m + n} \]

and the probability of the failed occurrence is:

\[ q = \frac{n}{m + n} \]

b. Mortality table

This table contains of probability of a person to die based on their age, in particular population, in this case the insured group. The number of people in age of x is stated in \( l_x \) symbol. While the number of people from the \( l_x \) group who died before they reach \( x+1 \) is stated in \( d_x \) symbol,

\[ d_x = l_x - l_{x+1}. \]

A probability of a person with age of x died before they reach \( x+1 \) of age, or a probability of a person with age of x died between age of x and \( x+1 \) are stated in \( q_x \) symbol,
The mortality table is utilized to determine the amount of compensation from the probability of losses caused by the decease, and to predict a person life expectancy. Calculating the relation between age and time is useful to determine life-death probability.

1) Life probability
A person’s life expectancy with \( x \) of age is \( n \) years.

\[
\frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x} = n \]

2) Death probability
A person death probability with \( x \) of age is \( n \) years, or before they reach \( x+n \) of age

\[
n \hat{q}_x = 1 - nP_x = 1 - \frac{l_{x+n}}{l_x}
\]

c. Gross Contribution
Gross Contribution is gross contribution that have to collect by the shariah insurance company in the form of insurance premium participants. In other words, this fund will be used as based of shariah insurance in determining the amount of the premium per participant.

General formula that used to count GC, defined as:

\[
GC = RC + \lambda + E
\]

where:
RC = Risk Contribution ; \( \lambda \) = Reserve Fund ; E = Dismissal of company
d. Risk Contribution
Risk Contribution is amount of fees that will be used in shariah insurance company to fulfill the needs of the insured claims.
e. The reserve fund
This reserve fund is collected by shariah insurance company as the fund that will be used in deficit situation underwriting.
f. Dismissal of company
Dismissal of Shariah insurance company consists of various cost as Fees from the company wakala, operational cost, employee salaries, company profit, etc.

2. Methodology
Two steps were taken in this research, namely literature study and drafting mathematical modeling
a. Literature Study
In this step the researchers reviewed principles that related the shariah insurance as a foundation for premium fund management. The sources of references are books, papers, proceedings, and articles on the internet.

b. Drafting Mathematical Model
This step was aimed to construct a plan to translate the Mudharabah scheme into mathematical language to produce a mathematical model for premium fund management mechanism.
3. Results and Discussion

3.1. The principles of Shariah Insurance [12]

1. Tawhid Principle
Tawhid is a basic principle in sharia insurance. Fundamentally, every moslem should base their life and do every activity by tawhid (monotheism), including in muamalat. This means that the basic intention in performing shariah insurance is to obtain blessing form Allah SWT.

2. Justice Principle
This principle means that the shariah insurance should be fair, impartial, especially in making the relationship pattern whether between participants, or between the shariah insurance companies and their customer, concerning their respective rights and obligations. Shariah insurance should not inflict disadvantage or detriment to customers.

3. Mutual Assistance Principle
The spirit of mutual assistance is a very important aspect in the shariah insurance business. According to this principle, the concept of shariah insurance is fellow participants give charity for other participants who suffer from misfortune. Participants do not donate to the shariah insurance company, but they give it to fellow participants, while the companies merely act as fund managers. As a result, the companies are not entitled to claim or take participants’ tabarru fund. The companies obtain ujrah (fee) upon the tabarru fund management, which is paid by the participants along with the contribution payment (premiums). The insurance companies manage the tabarru funds to be invested in shariah manner (syar’iyyah), and then allocate its result for other participants who suffer from misfortune.

4. Cooperation Principle
Good cooperation should be established between participants and insurance companies. If both participants and the companies favorably fulfill their rights and responsibilities, it will establish a good and cooperative relationship pattern; which, in God’s will, will bring blessing for both sides.

5. Amanah Principle
Amanah (trustworthiness/reliability) is also a very important principle, because essentially this life is a mandate and everyone should be responsible to Allah SWT. The companies are required to be reliable in managing the premium fund. Likewise, the participants should be true and fair about any occurred risk that happen to them. Do not let the participants lie or exaggerate and claim something that does not exist which will give disadvantages to other participants. Equally, the companies should not arbitrarily taking advantages, and detriment participants.

6. Mutual Willingness Principle
Mutual willingness should accompany every transaction. The participants are willing their fund is managed by the insurance companies professionally and trustworthy, and the companies should solemnly perform the entrusted responsibility. In addition, the participants should agree and willing that their funds being allocated to other participants who suffered, to ease their burden. With this principle, shariah insurance has made mutual assistance broader and deeper meaning, since every party helping, cooperating, and transacting with each other in sincere manners.

7. Non-Riba Principle
Riba (usury) is a form of transaction that should be avoided as far as possible, especially in insurance. Contributions (premiums) fund from the participants must be invested in shariah investment and has a clear halal status. Likewise, its operating system must applying risk sharing concept, which is based on tabarru contract, in order to eliminate the element of riba in participants’ insurance benefits (claims).

8. Non-Maisir principle
A conventional managed insurance has maisir (gambling) element, since participants may pay premium up to dozen times but never claim. On the other hand there is a new participant
who only pays premium once but claims immediately. This happens because the basic concept in conventional insurance is the transfer of risk. According to this concept, whenever the company receives premiums, it automatically becomes the company’s possession. Any claim will be paid directly from the company’s account. The company will gain a huge profit if they receive high premiums fund but low claims. Vis a versa, they will lose whenever the amount of claims is high while the collected premiums fund is low.

9. Non-Gharar Principle

Gharar is obscurity, because the risk could happen or not. In conventional insurance, the participants do not know whether they get a claim or not, since it depends on risk occurrence. A participant can claim whenever a particular risk happens, but otherwise the participant will not be able to claim. This condition becomes gharar because the applied concept is the transfer of risk. Meanwhile, if the company applies the sharing of risk concept, the obscurity will not become gharar, because any claim will be carried together with other participant based on tabarru concept.

10. Non-Risywah Principle

In its operational, both shariah insurance companies and its participants must avoid risywah (bribery) aspect. For example, a participant should not bribe an insurance official in order to get benefits (claims), likewise the company should not bribe the participant in order to obtain insurance premiums (contributions).

3.2. Design Modeling

The stages of mathematical modeling of a system in general can be illustrated through the following figure:

![Figure 1. The Stages of Mathematical Modelling of a System](image.png)

It shows that the first step that needs to be performed is to collect data in form of information about the existing system. The next step is to model the system or translate the existing system into mathematical language through mathematical equations. Afterward, the mathematical equations are transformed into programming language such as syntax. A computer simulation is performed to test, verify and validate the obtained mathematical models.

The Mudharabah scheme in the mechanism of premium-fund management in shariah insurance can be described as follows:
The design of mathematical modeling for Mudharabah scheme in Figure 2 is started with determining research variables, as a basis to create mathematical equations, namely:

| Research Variable | Mudharabah Scheme |
|-------------------|--------------------|
| Participant’s Contribution | Participant’s Contribution |
| Participant’s | Participant’s |
| Participant’s Special (Tabarru) | Participant’s Special (Tabarru) |
| Investement of Takaful Fund | Investement of Takaful Fund |
| Claims payment | Claims payment |
| Contingency Reserve | Contingency Reserve |
| Underwriting costs | Underwriting costs |
| Contingency Reserve | Contingency Reserve |
| Underwriting Surplus | Underwriting Surplus |
| Participant | Participant |
| Shareholders | Shareholders |

Afterward, defining the research variables into following symbols:

- **a.** Participant’s Contribution = \( Z = X + Y \)
- **b.** Participant’s = \( X = (100-p) \% \times Z \)
- **c.** Participant’s Special (Tabarru) = \( Y = p \% \times Z \)
- **d.** (investment level) Participant’s = \( U \)
  Participant’s Special (Tabarru) = \( V \)
- **e.** Claims payment = \( W \)
- **f.** Underwriting costs = \( T \)
- **g.** Contingency Reserve = \( R \)
- **h.** Underwriting Surplus = \( S = a \% \times T \) (if available)
i. Profit Sharing Investment of Takaful (example)
   Participant = 60%
   Shareholders = 40%

j. Profit Sharing of Underwriting Surplus (if available)
   Participant = 60%
   Shareholders = 40%

The next step is to build a mathematical modeling formulation by arranging the said symbols into mathematical equations and a system of mathematical equations.

| Table 1. Mathematical Modeling Formulation |
|-------------------------------------------|
| t = 1 | t = 2 | t = 3 | … | t = n |
| Number of collected participants’ contribution = Z |
| t = 1 | t = 2 | t = 3 | … | t = n |
| Z(1) = 1 * Z | Z(2) = 2 * Z | Z(3) = 3 * Z | … | Z(n) = n * Z |
| Number of collected participant's special (Tabarru) = Y |
| t = 1 | t = 2 | t = 3 | … | t = n |
| Y(1) = p% * Z(1) | Y(2) = p% * Z(2) | Y(3) = p% * Z(3) | … | Y(n) = p% * Z(n) |
| Number of collected participants = X |
| t = 1 | t = 2 | t = 3 | … | t = n |
| X(1) = (1 - p%) * Z(1) | X(2) = X(1) + (1 - p%) * Z | X(3) = X(2) + (1 - p%) * Z | … | X(n) = X(n - 1) + (1 - p%) * Z |
| Collected profit sharing from Y |
| t = 1 | t = 2 | t = 3 | … | t = n |
| M(1) = q% * V * Y(1) | M(2) = q% * V * (Y(2) + M(1)) | M(3) = q% * V * (Y(3) + M(2)) | … | M(n) = q% * V * (Y(n) + M(n - 1)) + M(n - 1) |
| Collected profit sharing from X |
| t = 1 | t = 2 | t = 3 | … | t = n |
| N(1) = r% * U * X(1) | N(2) = r% * U * (X(2) + N(1)) | N(3) = r% * U * (X(3) + N(2)) | … | N(n) = r% * U * (X(n) + N(n - 1)) + N(n - 1) |
| Cash value |
| t = 1 | t = 2 | t = 3 | … | t = n |
| L(1) = X(1) + N(1) | L(2) = X(2) + N(2) | L(3) = X(3) + N(3) | … | L(n) = X(n) + N(n) |

Afterward, is to translate the mathematical model into syntax, a programming language, which will be utilized in the final stage, which is performing a computer simulation to verify and validate the obtained model.

4. Conclusion
Mudharabah scheme in the perspective of mathematics in shariah insurance is translating the scheme into the language of mathematics to produce mathematical models. Generally, mathematical modeling stage consists of: 1) learn the real system 2) translate the real system into the language of mathematics through the model 3) translate the mathematical models into computer programming language 4) do simulations for testing, verification and validation of the model by comparing the result of modeling with reality at the system. In this paper, research is still in the second stage and will be continued until the final stage.

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