### Abstract
The aim of this work was the investigation and improvement of a Structural Health Monitoring method. This method was based on guided elastic waves propagation. These waves propagate in thin-walled structures guided by their walls. This particular technique has been considered in the literature as very promising. This research concentrated on diagnostics using attached piezoelectric transducers that excite guided elastic waves. Non-contact method of measurement was used. The laser vibrometer was utilized to measure the velocities of out of plane vibrations related to propagating elastic waves. Excited waves propagate and reflect from the discontinuities encountered on their way, therefore registering them one can obtain information about the structural health of the structure. Several sensor arrangements (networks) were investigated. The focus of the research was on possible area of application and limitations of the investigated networks. Presented research was based on laboratory experiments on prepared specimens. Signals gathered in the discrete network nodes were processed in order to obtain diagnostics information for the whole surface monitored. The voltage signals in the time domain were transferred into spatial domain to indicate the damage position. The signal processing oriented on structural diagnostics was realized in the MATLAB environment.

### 1. Introduction
In this paper a research on the SHM topic is presented. A system that interrogate a structure on-line would increase the safety and reduce the cost connected with unexpected failure. One of the promising methods for such system is based on elastic wave propagation phenomena. The interaction of these waves with defects can bring the diagnostic information. In case of thin walled structures such as aircraft panels these wave are guided by the two parallel surfaces therefore they are called guided waves [1]. However if an isotropic material is considered they are called Lamb waves after the name of Horace Lamb who first described this kind of elastic waves [2]. One of the most important features of these waves is that they propagate as symmetric (S₀, S₁, S₂,...) and anti-symmetric (A₀, A₁, A₂,...) modes [3]. The number of both modes is infinite and they are highly dispersive. The dependence of phase velocity, \( c_p \), on frequency is given be the Rayleigh-Lamb equations [3]:

\[
c_p = \sqrt{\frac{\rho \omega^2}{\rho_k}}
\]

where \( \rho \) is the density of the material, \( \omega \) is the angular frequency, and \( \rho_k \) is the effective mass density.
\[
\tan(ad) = \left( \frac{4k^2ab}{(a^2-k^2)^2} \right)^{\pm 1},
\]

where \(d\) is the half of the plate thickness, \(k\) is the wave vector length and \(\omega\) is an angular frequency. Parameters \(a\) and \(b\) are defined by

\[
a^2 = \left( \frac{\omega}{c_T} \right)^2 - k^2, \quad b^2 = \left( \frac{\omega}{c_L} \right)^2 - k^2
\]

and

\[
k = \frac{\omega}{c_p}.
\]

The exponent +1 is for the symmetric modes, while -1 is for the antisymmetric modes. Parameters \(c_T\) and \(c_L\) denote the transverse and longitudinal wave velocities, respectively.

There are many ways to excite Lamb waves in structures. Methods worth mentioning are: angle beam transducers [3], EMATs [4], laser sources [5] or piezoelectric elements [6]. By the same means registration can be performed. However it seems that the most information about Lamb wave propagation phenomenon can be achieved using laser scanning vibrometry [7], [8], [9]. This method covers monitored structure with a grid of virtual points at which velocity or displacement is measured. In this way wave interaction with defects, notches, edges, stiffeners can be visualized.

In this paper damage detection, localisation and identification was performed using a single piezoelectric sensor as an elastic wave source and a 3D laser vibrometer for wavefield registration. A triangulation algorithm was used to extract information about damage from registered signals. The research showed that the method is very useful in diagnostics of structures.

In the second section of the paper the measurement set-up was described. In the third section the damage localisation algorithm was proposed. In the fourth section sensing networks are described and in the fifth section experimental results are presented. Finally the research work was summarised in the last section.

2. Experimental set-up
Measurement equipment consisted of: Polytec PSV-400-3D Laser Scanning Vibrometer, TTi TG1241 arbitrary waveform generator - Piezo Linear Amplifier EP-104 from Piezo Systems, Inc, and piezoelectric transducers. Measurements were conducted for thin panel with dimensions 1000 × 1000 × 1 mm³ made out of 5754 aluminum alloy. Excitation signal was generated using arbitrary waveform generator. This is a sine (with frequency \(f_w\)) modulated by a Hann window. In this particular experiment the number of modulated periods was \(n=5\). The duration of such signal is \(n/f_w\). What is important to reduce dispersion such signal is relatively narrow in the frequency domain. Generated signals were amplified by EP-104 amplifier and sent to the terminals of piezoelectric transducer. Elastic waves were registered using laser scanning vibrometer (Figure 1). Sampling frequency of PSV-400-3D was the highest as possible: 5.12 MHz.
3. Damage localisation using triangulation algorithm

The idea of proposed damage localisation algorithm is to search all registered signals for wave reflections from discontinuities. This can be done by introducing a damage index DI [10], [11] that will be defined later. The excitation signal has a finite length and thus can be surrounded by a time window. The time window can be arbitrarily placed in each of the registered signals resulting in a certain time instant, which is equivalent to a distance required for the propagating wave to travel from the excitation point to an arbitrary point \( P=(x, y) \) – possible damage location and then back to an appropriate sensing point (Figure 2). Next damage index for chosen point \( P \) is calculated. Mesh consisting of points for which damage index is calculated allow to create a damage map.

Time of propagation from actuator to arbitrary point \( P \) and from this point back to sensing point can be calculated using formulas (4) and (5), respectively:

\[
t_{AP} = \frac{d_{AP}}{c_g},
\]

(4)
where: \(d_{AP}\) – distance from actuator to arbitrary point \(P\), \(d_{PS}\) – distance from arbitrary point \(P\) to sensing point, \(c_g\) – group velocity of fundamental anti-symmetric Lamb wave mode \(A_0\) (this mode was used in this paper for damage localisation purposes).

The total time of wave propagation \(t_0\) can be determined using (6):

\[
t_0 = t_{AP} + t_{PS}
\]

(6)

This time is also used as a starting point of time window that is used in order to cut a portion of signal for feature extraction. Length of this time window is equal to the length of excitation signal \(an\) is denoted \(t_w\). Termination time of window can be calculated using (7):

\[
t_1 = t_0 + t_w
\]

(7)

Rectangular time window is described by formula (8).

\[
W(t) = \begin{cases} 
1 & t_0 \leq t \leq t_1 \\
0 & t < t_0 \lor t > t_1
\end{cases}
\]

(8)

Next, windowed part of signal must be prepared (9):

\[
S_w(t) = S(t) \cdot W(t)
\]

(9)

where: \(S(t)\) – time signal, \(S_w(t)\) – windowed part of time signal.

In next step squared amplitude of windowed part of signal is calculated (10):

\[
\hat{S}(t) = (S_w(t))^2.
\]

(10)

After this damage index is calculated as integral in the range from \(t_0\) to \(t_1\) using formula (11):

\[
DI(P) = \int_{t_{0}}^{t_{1}} \hat{S}(t)dt
\]

(11)

\(DI(P)\) – damage index calculated for arbitrary point \(P\). In practical application measured signal was discrete and the integral in (11) was replaced by sum.

This procedure leads to calculation of damage index for arbitrary point \(P\) and is repeated for all points from the mesh that should be created on monitored structure to cover its surface. Density of this mesh is connected with resolution of the damage map. That is clear if the part of the registered signal, within the considered time window, is free of the damage reflections the value of this damage index is very low and close to zero. On the other hand if the registered signal includes wave reflections from damage the value of the damage index is higher.

Based on developed damage index special damage influence map can be built by application of this procedure to all points of the panel and by summation of the obtained results for each sensing point.
4. Sensor networks

Sensor networks in Structural Health Monitoring (SHM) very often are consisted of some number of piezoelectric transducers. These transducers allow both to excite and sense of elastic waves. In proposed approach instead of sensing transducers non-contact measurement method is proposed based on laser vibrometer. This approach is very useful during experimental investigations and developing of SHM systems in laboratories.

As was mentioned in previous section damage map indicates places of monitored structure where wave reflections occur. Damage maps are created based on pairs of actuating and sensing points. In the case of damage for each pair of this points an ellipse consisting of higher value of damage index is created. Next, results in the form of calculated damage indexes for all combinations of sensing points are summed. In the case of large distance between neighborhood sensing points damage reflected wave allow to create ellipses and in the case of smaller sensing points spacing ellipses rather look like circles with almost the same radii. In consequence for smaller sensing points spacing one does not obtain directivity in the damage map. In the first case better directivity of damage map will be obtained (position of damage can be noticed on the damage map). On the other hand for the first case larger amount of signal will be interrupted by wave reflection between transducers in the real SHM system based on piezo sensors instead of scanning point for laser vibrometer. This part of signal is useless and must be rejected just as the part of signal containing reflections from the edges. These two parts of signal will be amplified on the damage map much stronger than reflections from damage and in consequence amplification of damage reflected wave cannot be seen due to linear scale of map of colors connected with damage index $DI$.

5. Experimental results

During experimental measurements elastic waves were excited using piezoelectric transducer placed in the middle of the aluminium panel. Excitation was in the form of tone burst signal with carrier frequency 16.5 kHz. Polytec 3D Laser scanning vibrometer were used for elastic waves sensing in chosen measurement points. In this paper few configurations of sensing point were proposed that are compared in respect of damage localisation results. It should be underlined that only one scanning
Figure 3 Damage maps and DI as a function of angle respectively for array of sensing point in the form of: a,b) square 3x3-64mm, c,d) square 3x3-96mm, e,f) square 5x5-3.2mm – simulated damage indicated by “X”
was used during measurements. In effect only transversal displacements of elastic wave were registered. Damage was simulated by additional mass.

In damage localisation procedure fundamental Lamb wave mode called $A_0$ was used. For used excitation frequency amplitudes of transversal displacements of $A_0$ mode are much larger than in-plane displacements. Experimental measurements were conducted for the configuration of sensing points in the form of square, star and cross arrays. In the case of square array of points three scenarios of point number and size are investigated. In the first case square array consisted of 9 point with spacing equal to 64 mm. In second case the same number of points were used but spacing was equal to 96 mm. In the third case 25 points with spacing 2.5 mm was used. In the case of star shape configuration 17 points with spacing 3.2 mm was used. The last configuration in the form of cross consisted of 5 points of 128 mm. Experimental results in the form of damage maps for described cases were presented in the Figure 3a,c,e) and Figure 4a,c) respectively. After thorough analysis it can be said that main influence on directivity of damage maps is related to the location of sensing points (configuration) and spacing of these points. For example Figure 4b shows strong response for all the angles (0-360°) while Figure 4d shows only response for narrow angular range (0-60° and 300-0°).
The increase the number of the sensing points does not necessarily improve the result of damage localisation. In order to assess the performance of configurations two types of relations were investigated. The first one DI as a function of $x, y$ coordinates and the second as maximum value of DI as a function of the angle around the configurations. These relations were presented in the form of plots in Figure 3a,c,e), Figure 4a,c) and Figure 3b,d,f), Figure 4b,d) respectively. For example the spacing for square (Figure 3e,f) and star (Figure 4a,b) arrays is the same but the star array gave better result despite lower number of sensing points in the array. Comparing square array 3x3 (Figure 3a,b) with 5x5 (Figure 3e,f) one can see that the reduction of sensing points and increase of the spacing improves the array performance. Taking larger 3x3 array (Figure 3c,d) gives even better results.

6. Summary

Proposed damage localisation algorithm do not need baseline data in order to create damage maps. It means that an SHM system based on this idea can be used on a structure without the need for reference (intact object) measurements.

Conducted research showed that directivity of damage map is strictly correlated with sensing points spacing. Better directivity will be obtained for larger spacing of sensing points. It was shown that by changing the distance between these points it is possible to control the damage map directivity. As was said in the previous section the reduction of sensing points and increase of the spacing improves the array directivity.

It should be kept in mind that in the case of SHM system that uses sensor network consisted of piezoelectric transducers additional wave reflections between transducers will occur.

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