Microwave photoconductivity of a periodically modulated two-dimensional electron system: Striped states and overlapping Landau levels

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We study the dc response of the striped state under microwave irradiation. The striped state can be modeled by a two-dimensional electron system under the influence of a unidirectional periodic modulation potential, which was examined in [J. Dietel et al., Phys. Rev. B 71, 045329 (2005)]. We further extend our study of the periodically modulated system to the case of strongly overlapping Landau levels and calculate the dark and photoconductivities. The strength of the modulation potential serves as an additional parameter which allows to vary the overlap of the Landau levels independently of the filling fraction.

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I. INTRODUCTION

Recent experiments [2–6] on a two-dimensional electron gas (2DEG) in weak magnetic fields show that these systems have an almost vanishing longitudinal resistivity when the microwave frequency \( \omega \) is (up to an additive constant) approximately a multiple of the cyclotron frequency. This triggered a number of theoretical and experimental papers [1–24]. As is shown in Ref. 7, zero resistance states can be due to a negative microscopic diagonal conductivity. There are two mechanisms producing negative microscopic longitudinal conductivity: The first mechanism, known as displacement mechanism, relies on disorder-assisted absorption and emission of microwaves [9–13]. Depending on the detuning \( \Delta \omega = \omega_c - \omega \), where \( \omega_c \) is the cyclotron frequency, the displacement of the electrons is preferentially in or against the direction of the applied dc field. An alternative mechanism, known as distribution function mechanism, relies on the fact that microwave absorption leads to a change in the electronic distribution function, which can result in a negative photoconductivity [5, 14, 15]. Detailed calculations within the self-consistent Born approximation suggest that the latter mechanism is larger by a factor \( \tau_{in}/\tau^*_{s} \) where \( \tau_{in} \) is the inelastic relaxation time and \( \tau^*_{s} \) the single particle scattering time in the presence of the magnetic field.

As is well known, the influence of a unidirectional periodic potential on a 2DEG in a homogeneous magnetic field leads to transport anisotropies [25] as well as commensurability oscillations in the resistivity known as Weiss oscillations [26]. In such systems the Landau level broadening is due to the periodic modulation induced by the additional potential. In a foregoing paper [1], we studied the dark conductivities as well as the microwave-induced photoconductivities for these systems using a Fermi’s golden rule approach which goes back to Titeica in Ref. 27. The dc response of the microwave-driven system in the presence of a modulation potential is similar to the magnetoresistivity response of a 2DEG to surface acoustic waves [19].

It is clear that one can calculate the current resulting from a perturbation of the electron system by calculating the transition rates. It is easy to establish a current formula from the transition rates between eigenstates when knowing their position expectation values [1]. Within the lowest order approximation, we can calculate these transition rates by the help of Fermi’s golden rule. Without microwave field, one can calculate the dark conductivity using the impurity potential as a perturbation. Under microwave irradiation, the microwave operator acts as an additional perturbation.

In Ref. 1, we carried out a calculation of the dark and the photoconductivity to the most obvious case where the periodic modulation wave length \( a \) of the unidirectional field is small compared to the cyclotron radius \( R_c \) and additional small to the inverse of the correlation length \( \xi \) times the square of the magnetic length, i.e. \( \xi^2/\xi \). Furthermore we restricted us to temperatures \( T \) large with respect to the Landau level width, \( 2VN \). \( N \) is the index of the topmost filled Landau level. Furthermore, we limited our calculation to the case that the amplitude of the unidirectional modulation potential \( V \) is smaller than \( \omega_c \) such that we were able to restrict ourselves to first order perturbation theory in \( V \). Finally, we discussed explicitly the conductivities for the case \( \omega \approx \omega_c \) which is known to be the dominant regime for zero resistance states. We obtained in Ref. 1 that the photocurrent parallel to the direction of the wavevector of the unidirectional field is governed by the distribution function mechanism, which is larger than the contribution of the displacement mechanism by a factor \( \tau_{in}/\tau^*_{s} \). In the perpendicular direction, we find a strong enhancement of the displacement mechanism photocurrent such that in this case both contributions are of the same order. In a recent paper [20], we calculated the various conductivities in the regime \( \omega/\omega_c \ll 1 \) which, experimentally, for the case without periodic modulation potential, has shown a strong suppression of the Shubnikov de-Haas oscillations [21, 22], in accordance with our calculation. Further theoretical work on this subject was done
in [23, 24].

There are other interesting applications of our formalism. The above system can be used to model different types of anisotropic quantum Hall systems, e.g. the striped state system [28, 29], which shows a more complex unidirectional modulation even without the application of a modulation potential. In the striped state, the modulation has a wavelength \( \alpha \approx R_c \). The available transport experiments in this regime were carried out at temperatures much smaller than the Landau level spacing [30, 31], but without microwave irradiation. We argue in this paper that interesting effects, among which zero resistance states, can arise when the striped system is irradiated by microwaves. Motivated by the physics of the striped state, we first calculate the various conductivities for the case \( T \ll V_N \) and \( \alpha \ll \ell_B^2/\xi, R_c \). We obtain a strong filling fraction dependence of these conductivities, except for the displacement mechanism contribution in the perpendicular direction. In addition, we discuss the dark conductivity and the photoconductivities in the striped state regime \( \alpha \approx R_c \) and obtain agreement of the dc (dark) conductivity in the striped state with prior calculations.

In our calculations in Ref. 1 we assumed that the impurity contribution to the Landau level bandwidth is negligible in comparison to the contribution from the modulation potential. This is in contrast to the experimentally realized system without unidirectional periodic modulation where the band width has its origin in impurity scattering. The band structure in this system depends mainly on the strength of the magnetic field. For weak magnetic fields, the system has highly overlapping Landau levels, while for strong magnetic fields, the Landau levels are well separated. By a variation of the modulation potential amplitude \( V \) – which is possible in the usual Weiss oscillation experiments by tuning the grid potential [26] – it is possible to reach the overlapping Landau level regime without changing the magnetic field and thus the effective filling fraction. It is physically plausible that the conductivity expressions parallel to the modulation potential should be in accordance to the system without unidirectional modulation given e.g. in Ref. 13, 14 up to numbers. This was shown explicitly in Ref. 1 for the non-overlapping Landau level regime, i.e. \( V_N \ll \omega_c \). In this paper we show the same for the highly overlapping Landau level regime, i.e. \( V_N \gg \omega_c \). We obtain also an oscillating photoconductivity but the explicit frequency dependence differ from the ones without modulation potential.

For calculating the various conductivities in the case \( V_N \gg \omega_c \) we have to discuss the single particle eigenvalue problem of an electron in a unidirectional periodic potential and a homogeneous magnetic field. This was done in Ref. 1 by solving the eigenvalue problem in first order perturbation theory in the oscillating potential strength, which is appropriate for \( V_N \ll \omega_c \). In this paper, we solve the eigenvalue problem without any restrictions to the relation between \( V_N \) and \( \omega_c \) by discussing the WKB approximation of this eigenvalue problem. These results are used in Section VI to calculate the conductivities for the case \( V_N \gg \omega_c \).

We chose to provide a self-contained summary of all main results and a discussion of their implications relevant to experiments in the next section of this paper.

This paper is organized as follows: Section II contains a summary of our results. In section III we calculate the various dark and photoconductivities for the distribution function and the displacement mechanism in the case where \( T \ll V_N \ll \omega_c \) and \( \alpha \ll \ell_B^2/\xi, R_c \). In section IV we generalize our discussion to arbitrary \( \alpha \) and \( R_c \), which is relevant for the discussion of the dark and microwave photoconductivity in the striped state. In section V, we discuss the WKB approximation of an electron in a homogeneous magnetic field and an additional periodic unidirectional potential. By using the results of section V, we discuss in section VI the dark as well as the photoconductivities for the case \( V_N \gg \omega_c \). The derivations of sections III-VI were carried out in the frequency regime \( \omega \approx \omega_c \) and for a linearly polarized microwave field in \( x \)-direction. In section VII, we generalize these result to an arbitrary microwave field of frequency \( \omega \gtrsim \omega_c \). We summarize our results in section VIII. Some technical details are deferred to a number of appendices. In the remainder of this paper we use the convention \( \hbar = 1 \).

## II. OVERVIEW OF THE RESULTS

For clarity we point out once more that the explicit conductivity formulas of sections III-VI are only valid for \( \omega \approx \omega_c \) and \( x \) polarization. We assume that the Landau level bandwidth due to impurity scattering is much smaller than the bandwidth due to the modulation potential. All conductivity formulas in this paper are expressed via the bandwidth \( 2V_n \) of the \( n \)th Landau band \( (2) \). The density of the states of the \( n \)th Landau level \( \nu_n^\pm(\epsilon) \) and of the whole system \( \nu^\pm(\epsilon) \) for \( V_N \ll \omega_c \) are given in \((3)-(5)\). The corresponding density of states \( \nu_n^\pm(\epsilon) \) in the large overlapping Landau level regime \( V_N \gg \omega_c \) is given in \((58)-(60)\). We have expressed the various conductivities by the help of the diffusion constants \( D_{xx}, D_{yy} \) (6) for \( V_N \ll \omega_c \) and \( D_{xx}^D, D_{yy}^D \) (63) for \( V_N \gg \omega_c \). The single particle \( \tau_n^\pm(\epsilon) \) and the transort time \( \tau_t^\pm(\epsilon) \) is defined below (6).

As mentioned above, in our paper Ref.1 we carried out the calculation of the various dark conductivities, i.e. \( \sigma_{xx}^D \) parallel to the modulation and \( \sigma_{yy} \) perpendicular to the modulation, as well as the photocconductivities \( \sigma_{xx}^{DF}, \sigma_{yy}^{DF} \) of the displacement mechanism and \( \sigma_{xx}^{DF}, \sigma_{yy}^{DF} \) of the distribution function mechanism in the parameter range
$V_N \ll T, \omega_c \ll N \omega_c$ and $a \ll \ell_B^2 / \xi, R_c$. Under this condition, an electron can jump over many periods of the modulation potential by microwave-assisted impurity scattering. On the way to calculate the various conductivities for the striped state we extend in section III the derivations to temperature $T \ll V_N \ll \omega_c$ but still $a \ll \ell_B^2 / \xi, R_c$. In this case the dark and photoconductivities are strongly filling fraction dependent except for the photoconductivity of the displacement mechanism in the direction perpendicular to the periodic potential, i.e. for $\sigma_{Dx}^{SP}$. In the case of integer filling fraction, we obtain the same photoconductivities as calculated earlier for $T \gg V_N$ in Ref. 1. The expressions for the dark conductivities are given in (7) and (8). The results for the photoconductivities are given in (9), (12), (15) and (19). The photoconductivity expressions depend via the functions $A_1, A_2, B_1, B_2$ on the frequency $\Delta \omega = \omega_c - \omega$ and the filling fraction $\Delta \mu = \mu - (N + 1/2) \omega_c$. These functions are shown in Fig. 1 in the case of half integer upper Landau level filling $\Delta \mu = 0$ most relevant for the striped state system. Corresponding functions for the integer filling system can be found in Ref. 1. In (21) we provide scaling relations for the photoconductivities of the displacement and distribution function mechanism in both directions.

In section IV we calculate the various conductivities without restrictions on $a$ and $R_c$. This is relevant for the striped state where $a \approx R_c$. For impurity correlation length $a \gg \ell_B^2 / \xi$ or $a \gg R_c$, respectively, which is the strong forward scattering case in the striped state system, we obtain (26) and (27) with (7) for the dark conductivities in both directions. The calculated dark conductivities fulfill the product rule (28) which is in accordance with the well known semicircle law for half filled systems derived before for striped states. In the case of the photoconductivities one has to distinguish between the regimes where the absolute value of $\Delta \omega$ is larger than $\Delta \omega_0 \sim V_N((\tau_s/\tau_\omega)(R_c/a))^{1/2}$ or smaller than $\Delta \omega_0' \sim V_N((\tau_s/\tau_\omega')(R_c/a))$. The exact definitions of $\Delta \omega_0$ and $\Delta \omega_0'$ are given in (34) and (35). When $|\Delta \omega| > \Delta \omega_0$, we obtain zero contribution to the photoconductivity for both directions (34). In the case $|\Delta \omega| < \omega_0'$, we obtain nonzero contributions to the photoconductivities for the two mechanisms. The exact expressions for the photoconductivities are given in (36), (37), (39) and (40). All formulas are also valid for $T \gg \omega_c$ which corresponds to the case of integer filling fraction, $\Delta \mu = \pm V_N$. We have summarized in (41) the scaling relations for the various photoconductivities in the striped state regime. We obtain that for both directions the photoconductivity of the distribution function mechanism is dominant over the displacement mechanism. Under the consideration of the conductivity expressions (39), (40) and Fig. 1 we obtain the remarkable fact that the sign of the photoconductivities of both directions at $\omega \approx \omega_c$ are reversed to each other. We did not provide explicit conductivity expressions in this paper for the case $a \lesssim R_c, \ell_B^2 / \xi$. The isotropic (delta-correlated) scattering regime for the striped state system is at the limit of this parameter range. When carrying out the calculation for the photoconductivities, we find a strong enhancement of $\sigma_{Dx}^{SP}$ and $\sigma_{Dx}^{SP}$ at $\Delta \omega$ for large Landau level filling $N$, fulfilling the conditions (42) and (43).

We also calculate the dark and photoconductivities for the case $\mu \gg V_N \gg \omega_c$ and $a \ll R_c, \ell_B^2 / \xi$. We obtain expression (59) for the density of states. In Fig. 2 we show the oscillatory part of the density of states. The dark conductivity in the direction parallel to the modulation is given in (61), in the perpendicular direction in (62). The various photoconductivities in the regime $V_N \gg \omega_c$ are given in (65), (67), (71) and (73). Their scaling relations are given in (74). We obtain that the photoconductivity parallel to the periodic modulation is governed by the distribution function mechanism and oscillates as a function of $\omega$. This should result in zero resistance states in certain frequency regions $\omega \approx \omega_c$. The photoconductivity perpendicular to the modulation direction is governed by the distribution function part when $\omega \gg V_N$. In the case $\omega \ll V_N$, the photoconductivity is dominated by the displacement mechanism with positive non-oscillating values. The oscillating photoconductivities all behave similarly as a function of $\omega, \omega_c$. In Fig. 3, we show the behavior of the various photoconductivities in the case $V_N \gg \omega_c$.

Finally, we give in (77) with (78)-(81) the various photoconductivities in the range $\omega \gtrsim \omega_c$ for an arbitrary form of the microwave field. We obtain a polarization dependence only for $\sigma_{Dx}^{SP}$, i.e. in the direction parallel to the modulation. This polarization dependent term is given by the last expression in (78).

Beside the results mentioned above, we also obtain results which are interesting from the theoretical point of view. For calculating the conductivities for $V_N \gg \omega_c$ we carried out a calculation of the eigenfunction and eigenvalues of an electron in a homogeneous magnetic field and an unidirectional periodic potential in the WKB approximation. As is well known, the physics in high Landau levels is described correctly within this approximation. We obtain no corrections to the eigenvalues in comparison to the first order result in the potential amplitude $\tilde{V}$ used in section III and IV and also in Ref. [1]. The WKB eigenfunction is a mixing of Landau level functions. The $x$-position operator matrix elements with respect to the WKB eigenfunctions are the same as in the first order in $\tilde{V}$ approximation. We furthermore calculated the expectation value of plane waves with respect to these WKB eigenfunctions. We obtain corrections to the first order result. When calculating the various conductivities we have to calculate the impurity averaged square of these matrix elements. We have shown in section V B that this quantity is the same as in first order perturbation theory in $\tilde{V}$ for large Landau level index $N$. Summarizing, we obtain only corrections in the conductivity expressions in comparison to the first order formulas of section III by the fact that the oscillating energy bands overlap for large $V_N/\omega_c$. These corrections were taken into account in section VI.
III. THE CALCULATION OF THE VARIOUS CONDUCTIVITIES FOR TEMPERATURE T \ll V_N AND ARBITRARY FILLING FRACTION

The system under consideration is a 2D electron system in the presence of a homogeneous magnetic field \( B \) and a periodic potential \( V(r) = \tilde{V} \cos(Qx) \) in \( x \)-direction with period \( a = 2\pi/Q \). The system is under influence of an impurity potential \( U \) of correlation \( \langle U(r)U(r') \rangle = W(r-r') \) and correlation length \( \xi \). In Ref. 1, we examined the two extreme cases of either a short range (delta-like) potential \( \xi \ll 1/k_F \) or a long range potential \( \xi \gg 1/k_F \) where all conductivity expressions were determined explicitly for these two limiting cases, where \( \xi \) is the correlation length of the impurity potential. By taking the full asymptotic expansions (C4) for the Laguerre polynomials, one can argue that all results of Ref. 1 are also valid for an arbitrary impurity potential.

In order to neglect quantum mechanical interference effects, we have to assume throughout this work that

\[ \xi \ll R_c. \]  

At first, we consider the case of a periodic potential of an amplitude which is small in comparison with the Landau level spacing, i.e. \( \tilde{V} \ll \omega_c \). In this limit, we can determine physical quantities within first order perturbation theory in \( \tilde{V} \). The energy dispersion is given by

\[ \epsilon_n^0 \simeq \omega_c(n + 1/2) + V_n \cos(QkF_B^2), \]

where \( \ell_B = (\hbar/eB)^{1/2} \) denotes the magnetic length. The eigenstates are the Landau level wavefunctions \( |nk\rangle \) in the Landau gauge. The amplitude \( V_n \) is given by

\[ V_n = \tilde{V}\exp[-Q^2\ell_B^2/4]L_n(Q^2\ell_B^2/2) \approx \tilde{V}J_0(2\pi R_c/a). \]  

It is useful to define the corresponding density of states (DOS) of the \( n \)-th Landau level by

\[ \nu_n^*(\epsilon) = \nu_n^0\tilde{\nu}_n^*(\epsilon). \]  

with

\[ \nu_n^0 = \nu_n^* = 1/2\tilde{V}\pi^2\ell_B^2, \quad \tilde{\nu}_n^*(\epsilon) = \frac{1}{\sqrt{1 - [(\epsilon - E_n)/\tilde{V}]^2}} \]  

and \( E_n = (n + 1/2)\omega_c \). We denote the DOS of the system by \( \nu^*(\epsilon) \). It can be expressed by

\[ \nu^*(\epsilon) = \nu^0\tilde{\nu}^*(\epsilon) = \nu^0\sum_n \tilde{\nu}_n^*(\epsilon). \]

In the following, we first calculate the \( dc \) conductivities in the \( x \)- and \( y \)-direction without microwave field and then the \( dc \) conductivities in the \( x \)- and \( y \)-direction when a microwave field is applied to the sample. These conductivities were calculated in Ref. 1 for arbitrary large filling fraction and \( T \gg V_N, a \ll R_c, \ell_B^2/\xi \). In this section we extend our analysis to the regime \( a \ll R_c, \ell_B^2/\xi \) by considering the various conductivities for \( T \ll V_N \).

As in Ref. 1, we formulate the results in terms of the diffusion constants in the \( x \)- and \( y \)-directions

\[ D_{xx}(\epsilon) = \frac{R_c^2}{2\tau_{xx}^*(\epsilon)} \quad \text{and} \quad D_{yy}(\epsilon) = v_y(\epsilon)^2\tau_{yy}^*(\epsilon) = \frac{1}{(\pi a \nu^*(\epsilon))^2} \tau_{yy}^*(\epsilon). \]  

Here \( \tau_{xx}^*(\epsilon) = \tau_s\nu_0/\nu^*(\epsilon) \) and \( \tau_{yy}^*(\epsilon) = \tau_t\nu_0/\nu^*(\epsilon) \) where \( \tau_s, \tau_t \) are the single particle and transport scattering times, respectively, and \( \nu_0 \) is the density of states without unidirectional potential and without external magnetic field. \( v_y(\epsilon) \) is the velocity of the electron with energy \( \epsilon \) in the \( y \)-direction. We define \( \tau^* \) by \( \tau^*(E_n) \) where \( \tau^* \) stands for the various transport times.

In the following, we calculate the dark conductivities in subsection A and the microwave induced photoconductivities in subsection B.

A. The dark conductivities for \( T \ll V_N, a \ll R_c, \ell_B^2/\xi \)

The dark conductivities in \( x \)- and \( y \)-direction are given by [1]

\[ \sigma_{xx,yy}(\epsilon) = \int d\epsilon \left( \frac{\partial f(\epsilon)}{\partial \epsilon} \right) \sigma_{xx,yy}(\epsilon) \]  

with the one particle distribution function \( f(\epsilon) = 1/(1 + \exp[-\beta(\epsilon - \mu)]) \). The conductivity per energy is given by

\[ \sigma_{xx,yy}(\epsilon) = e^2D_{xx,yy}(\epsilon)v_y(\epsilon). \]  

These expressions were derived explicitly without temperature restrictions in Ref. 1.
B. The photoconductivities for \( T \ll V_N, \ a \ll R_e, f_B^2/\xi \)

We now come to the discussion of the photoconductivities of both mechanisms

1. Photoconductivities of the displacement mechanism

For the conductivity in the \( x \)-direction \( \sigma_{xx}^{DP} \) we find a strong dependence of the \( T \ll V_N \) result on the special filling fraction. By carrying out a similar calculation as in Ref. 1 we obtain

\[
\sigma_{xx}^{DP} \approx \pi^2 \left[ e^2 D_{xx} \nu^* \right] \frac{\tau_T^2}{\tau_{tr}} \left( \frac{e E R_e}{4 \Delta \omega} \right)^2 A_1 \left( \frac{\Delta \omega}{2 V_N}, \frac{\Delta \mu}{V_N} \right)
\]  

with the function

\[
A_1 \left( \frac{\Delta \omega}{2 V_N}, \frac{\Delta \mu}{V_N} \right) = -\frac{6}{\pi^2} \frac{\partial}{\partial \Delta \omega} \sum_{\sigma \in \{\pm\}} \int_{E_{N+\Delta \mu}}^{E_{N}} d\epsilon \tilde{v}^*(\epsilon) \tilde{v}^*(\epsilon + \Delta \omega).
\]  

and \( \Delta \mu = \mu - (N + 1/2)\omega_c \), \( \Delta \omega = \omega_c - \omega \). The exact numerical prefactor in (9) depends on the details of disorder. For delta correlated impurities expression (9) is exact [32]. The integral in (10) is of incomplete elliptical type. It is easily seen that one gets \( A_1(x, \pm 1) = -(3/\pi^2)(\partial/\partial x)K(\sqrt{1-x^2}) \), which corresponds to a system of integer filling. We obtain for integer filling the same result for the conductivity as for \( T \gg V_N \) [1].

Next, we calculate the displacement current in the \( y \)-direction. This was done in Ref. 1 by calculating the eigenstates for electrons for a modulation potential in \( x \)-direction and a small additional \( dc \) field in \( y \)-direction, neglecting Landau level mixing. These eigenstates \( |nE_l\rangle \) are the dual to the basis states \( |nk\rangle \) in a similar manner as the basis of plane waves are the Fourier dual to the position eigenstates. By representing the ground state wavefunction as a linear combination of a Slater determinants of the functions \( |nE_l\rangle \), we do not get a single term, which means that one cannot apply Fermi’s golden rule formula of [1] to calculate the conductivities. By a generalization, we show in appendix A that one can nevertheless apply the simple Fermi’s golden rule formula given in (A5) when using for the distribution function in this formula

\[
f_{nl}^y = \frac{2\pi f_B^2}{(L_x L_y)} \sum_k f(\epsilon_{nk})
\]  

where \( f_{nl}^y \) is the value of the distribution function for the state \( |nE_l\rangle \). Thus, we obtain that \( f_{nl}^y \) depends only on the Landau level index \( n \). The reason for this is that the absolute value of the overlap of \( |nE_l\rangle \) and \( |nk\rangle \) does not depend on \( k \) and \( l \). From this, we find that the conductivity \( \sigma_{yy}^{DP} \) is independent of the filling fraction

\[
\sigma_{yy}^{DP} = \left[ e^2 D_{yy} \nu^* \right] \left( \frac{aB/\pi \sqrt{\tau_T^2/\tau_{tr}}}{E_{dc}} \right)^2 \left( \frac{e E R_e}{4 \Delta \omega} \right)^2 A_2 \left( \frac{\Delta \omega}{2 V_N}, \frac{\Delta \mu}{V_N} \right)
\]  

with

\[
A_2 \left( \frac{\Delta \omega}{2 V_N}, \frac{\Delta \mu}{V_N} \right) = \frac{\Delta \omega}{V_N} \int_{E_N}^{E_{N+\Delta \mu}} d\epsilon \tilde{v}^*(\epsilon) \tilde{v}^*(\epsilon + \Delta \omega).
\]  

This expression was also obtained for \( T \gg V_N \) in Ref. 1. By carrying out the integral in \( A_2 \), we obtain \( A_2(x, y) = 2\pi K(\sqrt{1-x^2}) \). Note the singular dependence of the displacement contribution to the transverse photoconductivity on the \( dc \) electric field \( E_{dc} \). This singularity is cut off for small \( dc \) electric fields by inelastic processes [1] when \( E_{dc} \sim \tilde{E}_{dc}^* \), where \( \tilde{E}_{dc}^* = Ba/2\pi \sqrt{\tau_{in}/\tau_s} \). For \( E_{dc} \ll \tilde{E}_{dc}^* \), the photoconductivity crosses over to Ohmic behavior, matching with Eq. (12) for \( E_{dc} \sim \tilde{E}_{dc}^* \).
2. Photoconductivities of the distribution function mechanism

First, we calculate the photoconductivity $\sigma_{xx}^{DF}$ in the $x$-direction. The calculation of Ref. 1 can be easily transferred to the case $T \ll V_N$. Due to the irradiation of the system by the microwave field, the distribution function is changed from its dark value. We obtain

$$\delta f(\epsilon) \approx \frac{2 \tau_m}{\tau_{tr}} \left( \frac{eER_c}{4\Delta \omega} \right)^2 \sum_{\sigma = \pm} \left[ f(\epsilon - \sigma \omega) - f(\epsilon) \right] \frac{\nu^* (\epsilon + \sigma \Delta \omega)}{\nu_0} .$$  \hspace{1cm} (14)$$

By using this distribution function we obtain

$$\sigma_{xx}^{DF} = 4 \left( \frac{\tau_m}{\tau_{tr}} \right) \left( \frac{eER_c}{4\Delta \omega} \right)^2 [\epsilon^2 D_{xx} \nu^*] B_1 \left( \frac{\Delta \omega}{2V_N}, \frac{\Delta \mu}{V_N} \right) \hspace{1cm} (15)$$

with

$$B_1 \left( \frac{\Delta \omega}{2V_N}, \frac{\Delta \mu}{V_N} \right) = -\frac{1}{2} \sum_{\sigma \in \{\pm\}} \left[ \frac{\partial}{\partial \Delta \omega} - \sigma \frac{\partial}{\partial \Delta \mu} \right] \left( \int_{E_N + \sigma \Delta \mu}^{E_N + \nu^* (\epsilon + \Delta \omega)} d\epsilon + \int_{E_N - \nu^* (\epsilon + \Delta \omega)}^{E_N - \Delta \omega + \sigma \Delta \mu} d\epsilon \right) .$$  \hspace{1cm} (16)$$

This results in

$$B_1(x, y) \approx Z[x, y] \frac{1}{16} \frac{1 - 2|x|}{|x|^2} \ln \left( \frac{V_N}{\Delta} \right) \sgn x .$$  \hspace{1cm} (17)$$

$\Delta$ is an effective energy cutoff at the band edges resulting from an additional energy broadening for example due to impurity scattering or a finite $dc$ field. The filling fraction dependent prefactor $Z[x, y]$ is given by $Z[\Delta \omega/2V, \Delta \mu/V_N] = 1$ for $T \gg V_N$. For $T \ll V_N$ and $\Delta \omega > 0$ we have

$$Z \left[ \frac{\Delta \omega}{2V_N}, \frac{\Delta \mu}{V_N} \right] = \begin{cases} 1 & \text{for } \Delta \omega \leq V_N - |\Delta \mu| \\ \frac{1}{2} & \text{for } V_N - |\Delta \mu| \leq \Delta \omega \leq V_N + |\Delta \mu| \\ 0 & \text{for } \Delta \omega \geq V_N + |\Delta \mu| . \end{cases}$$  \hspace{1cm} (18)$$

For $T \ll V_N$ and $\Delta \omega < 0$ we have $Z[\Delta \omega/2V_N, \Delta \mu/V_N] = 2 - Z[\Delta \omega/2V_N, \Delta \mu/V_N]$. The zero in the last line in (18) reflects the fact that the conductivity is not determined by the logarithm of the broadening energy cutoff parameter $\Delta$ which is assumed to be small compared to $V_N$.

Finally, we calculate the $dc$ current in the $y$-direction coming from the distribution function mechanism. One can immediately transfer the calculation of Ref. 1 for $\sigma_{yy}^{DF}$ to the case $T \ll V_N$ which yields

$$\sigma_{yy}^{DF} = 4 \left( \frac{\tau_m}{\tau_{tr}} \right) \left( \frac{eER_c}{4\Delta \omega} \right)^2 [\epsilon^2 D_{yy} \nu^*] B_2 \left( \frac{\Delta \omega}{2V_N}, \frac{\Delta \mu}{V_N} \right) \hspace{1cm} (19)$$

with

$$B_2 \left( \frac{\Delta \omega}{2V_N}, \frac{\Delta \mu}{V_N} \right) = \frac{1}{2} \sum_{\sigma \in \{\pm\}} \left| E_N + \nu^* (\epsilon + \Delta \omega) \right| \left( \int_{E_N + \sigma \Delta \mu}^{E_N + \nu^* (\epsilon + \Delta \omega)} d\epsilon + \int_{E_N - \nu^* (\epsilon + \Delta \omega)}^{E_N - \Delta \omega + \sigma \Delta \mu} d\epsilon \right) .$$  \hspace{1cm} (20)$$

This integral can be expressed as a sum of elementary functions \cite{1} which will not be stated due to its length. In the special case of integer filling (or $T \gg \omega_c$) \cite{1} we obtain $B_2(\Delta \omega/2V_N, \pm 1) = 4|x|/(\arcsin(1 - 2|x|) + \pi/2) - 4\sqrt{|x| - |x|^2} \sgn x \ [32]$.

By taking into account these calculations we obtain the following scaling relations for the various photoconductivities

$$\sigma_{xx}^{DF} \sim \left( \frac{\tau_m}{\tau_s} \right) \sigma_{xx}^{DP} , \quad \sigma_{yy}^{DF} \sim \sigma_{yy}^{DP} .$$  \hspace{1cm} (21)$$

Summarizing, we obtain a photoconductivity in the direction parallel to the modulation which is governed by the distribution function mechanism. In the perpendicular direction both mechanisms contribute equally. This was
derived before in [1] for \( T \gg V_N \). In the case of the photoconductivity experiments without the periodic potential the distribution function mechanism is the dominant contribution to the photoconductivity [15]. We point out that the photoconductivities of integer filling fraction \( \Delta \mu = \pm V_N \) for \( T < V_N \) are in accordance with the photoconductivities for \( T \gg V_N \) calculated in [1] which are odd functions of \( \Delta \omega \). This is except for \( \sigma_{\nu\nu}^{DP} \) not the case for systems of fractional filling and \( T \ll V_N \). We show in Fig. 1 the functions \( A_1(\Delta \omega/2V_N, 0) \) and \( B_1(\Delta \omega/2V_N, 0) \) corresponding to upper Landau level half filling fraction most relevant for the striped state. Note the singularity of \( A_1(\Delta \omega/2V_N, 0) \) at \( \Delta \omega = \pm V_N \) coming from the singularity of the density of states (4) at the band edge.

Summarizing, we obtain singularities of the photoconductivities at \( \Delta \omega = 0 \) but also at some other frequencies for \( \sigma_{xx}^{DP} \) and \( \sigma_{xx}^{DF} \). In realistic systems these photoconductivities are cut by impurity scattering. The origin of the singularities comes from the prefactors \((eER_c/4\Delta \omega)^2\) in the photoconductivity expressions (9), (12), (15), (19) resulting in the divergence of the photoconductivities at \( \Delta \omega = 0 \) and also from the singularities in the density of states at the band edge resulting in the singularities in \( A_1 \) and \( B_1 \). As was shown explicitly in Ref. [14] for the distribution function mechanism of the system without modulation potential, the \((eER_c/4\Delta \omega)^2\) term should be cut at \( |\Delta \omega| \sim 1/\tau_{1\nu}^s \). The cut off value due to corrections of the density of states at the band edge is equal to the Landau level broadening value of the system without modulation given by \(|\Delta \omega - \Delta \omega_s| \sim \sqrt{\omega_c/\tau_s}\) for a singularity at \( \Delta \omega = \Delta \omega_s \). We point out that a proper derivation of the form of the conductivity expressions at the various singularities requires a calculation of higher order corrections to the transition amplitude including vertex corrections which is out of the scope of this work.

IV. CALCULATION OF CONDUCTIVITIES WITHOUT RESTRICTIONS ON MODULATION WAVELENGTH AND CYCLOTRON RADIUS

There are different types of ground states for the various filling fractions of a 2DEG in a homogeneous magnetic field. One of the most prominent is relevant for filling fraction \( 2 \lesssim N \lesssim 8 \). This state consists of a charge density wave, which can be shown to be the ground state within Hartree-Fock approximation [28, 29]. The wavelength of the density wave is approximately given by \( a \approx 2.6 R_c \), i.e. out of the limits considered up to now. In this section, we therefore extend our previous considerations to the case without restrictions to \( a \) and \( R_c \). We believe that our model with the additional unidirectional periodic modulation can be used for the striped state system because when restricting the Hartree as well as the Fock potentials to the upper Landau level, both potentials have the same form [28]. Since the unidirectional periodic potential is of Hartree form, we believe that we can use this potential also for modelling the Fock potential by adjusting the potential amplitude \( V \) to the full amplitude of the Hartree-Fock potential of the striped state.

In the following two subsections, we discuss the dark and the photoconductivities for the case \( \min[1/\xi, 2k_F] \ll a/\ell_B^2 \), which is equivalent to \( \sqrt{\tau_{xx}/\tau_s}(a/R_c) \gg 1 \). This case is relevant for the striped state if scattering takes place only between nearest neighbor stripes. In the last subsection, we discuss the various conductivities in the transition region.
for smooth disorder, i.e. \( \min[1/\xi, 2k_F] \gtrsim a/\ell_B^2 \) or \( \sqrt{\tau_{tr}/\tau_s} (a/R_c) \lesssim 1 \), respectively. Throughout this section, we redefine \( 1/\xi \) to be the minimum of the inverse of the correlation length and \( 2k_F \).

### A. Calculation of the dark conductivities in the case \( \sqrt{\tau_{tr}/\tau_s} (a/R_c) \gg 1 \)

In general, the dark conductivities \( \sigma_{xx}, \sigma_{yy} \) can be calculated using Eq. (7) and Eq. (8) by replacing the scattering times \( \tau_{tr} \) and \( \tau_s \) in the diffusion constants, Eq. (6), by the energy dependent scattering times \( \tilde{\tau}_s(\epsilon) \) and \( \tilde{\tau}_{tr1}(\epsilon) \) which are defined by

\[
\frac{1}{\tilde{\tau}_s(\epsilon)} = a\epsilon_0 \sum_{j < z} dq_j e^{-q_j^2/\epsilon_0} \int [L_n(q_j^2/2)]^2 \tilde{W}(q_j) \frac{1}{\tilde{\tau}_s(\epsilon)} = \frac{a\epsilon_0}{4N} \sum_{j < z} dq_j e^{-q_j^2/\epsilon_0} \left( \frac{ja}{\ell_B} \right)^2 [L_n(q_j^2/2)]^2 \tilde{W}(q_j) + e^{-q_j^2/\epsilon_0} \left( \frac{2k_B^2 + ja}{\ell_B} \right)^2 [L_n(q_j^2/2)]^2 \tilde{W}(q_j). \tag{22}
\]

with \( q_j = q_x e_x + (ja/\ell_B^2)e_y \), \( q'_j = q_x e_x + (2k + ja/\ell_B^2)e_y \) and

\[
k = \arccos(\epsilon - (N + 1/2)\omega_c)/V_n/a/\ell_B^2. \tag{23}
\]

Here \( k\ell_B^2 \) and \( q_j\ell_B^2, q'_j\ell_B^2 \) in (22), (23) can be interpreted in the following way: The value \( k\ell_B^2 \) corresponds to the real space position of the electron for which we calculate the transition rate. \( e_x \times q_j\ell_B^2, e_x \times q'_j\ell_B^2 \) correspond to the displacement vectors of the transition. It is immediately seen that \( 1/\tilde{\tau}_s(\epsilon) = 1/\tau_s \) and \( 1/\tilde{\tau}_{tr1}(\epsilon) = 1/\tau_{tr} \) for \( a \ll \ell_B^2/\xi \) [1].

For \( \xi \gg \ell_B^2/a \), we can neglect the first term in (22) and restrict the \( j \)-sum in (22) and (23) only to the summand with \( |(2k + ja/\ell_B^2)| \leq a/2\ell_B^2 \). The conductivities can then no longer be expressed in terms of the single particle and transport time of a given impurity correlation function \( \tilde{W} \). We therefore carry out a scaling analysis of the conductivities. Nevertheless, we obtain from (22) the exact relation for \( \tilde{\tau}_s(\epsilon) \) and \( \tilde{\tau}_{tr1}(\epsilon) \)

\[
\frac{1}{\tilde{\tau}_{tr1}(\epsilon)} = \frac{\Delta_{ph}^2}{2R_c^2} \frac{1}{\tilde{\tau}_s(\epsilon)}. \tag{24}
\]

Here \( \Delta_{ph} = \min[2k\ell_B^2 + ja] \) is the smallest possible displacement of an electron jump, which, in the striped state, can be interpreted as the smallest distance between an electron and a hole stripe. By using the asymptotics of the Laguerre polynomials, given in App. C, we obtain (with \( \xi \sim \sqrt{\tau_{tr}/\tau_s} \ell_B^2/R_c \))

\[
\frac{1}{\tilde{\tau}_s(\epsilon)} \sim \begin{cases} \frac{\Delta_{ph}^2}{2R_c^2} \frac{1}{\tilde{\tau}_s(\epsilon)} & \text{for } \epsilon \text{ fulfilling } V_N - |\epsilon - (n + 1/2)\omega_c| < V_N \left( 1 - \cos \left( 2\pi \frac{\ell_B^2}{\xi} \right) \right) \sim V_N \frac{\ell_B^2}{\xi} \frac{1}{\tilde{\tau}_s(\epsilon)} \\
0 & \text{otherwise}. \end{cases} \tag{25}
\]

Summarizing, we have

\[
\sigma_{xx}(\epsilon) = e^2D_{xx}(\epsilon)\nu^x(\epsilon) \frac{\tau_{tr}}{\tilde{\tau}_{tr1}(\epsilon)}, \tag{26}
\]

\[
\sigma_{yy}(\epsilon) = e^2D_{yy}(\epsilon)\nu^x(\epsilon) \left[ \frac{\tilde{\tau}_s(\epsilon)}{\tau_s} \Theta \left( \frac{1}{\tilde{\tau}_s(\epsilon)} \right) + \frac{\tau_{in}}{\tilde{\tau}_s(\epsilon)} \left( 1 - \Theta \left( \frac{1}{\tilde{\tau}_s(\epsilon)} \right) \right) \right]. \tag{27}
\]

The values of \( \Theta(1/\tilde{\tau}_{tr1}(\epsilon)) \) are defined by unity in the case that \( 1/\tilde{\tau}_{tr1}(\epsilon) > 0 \) and zero otherwise. The dark conductivities are then calculated by the insertion of (26) and (27) in (7). Thus, the dark conductivity \( \sigma_{xx} \) can be calculated by the substitution of \( \tilde{\tau}_{tr1}(\epsilon) \) for \( \tau_{tr} \) in (8) [1]. This is not the case for \( \sigma_{yy} \). In this case \( \sigma_{yy} \) would get singular values for some energies \( \epsilon \) when we formally substitute \( \tilde{\tau}_s(\epsilon) \) for \( \tau_s \) in (8).

From Eq. (27), it is obvious that, due to the Theta function, there are two fundamentally different energy regions for the conductivity. For the energy region where \( 1/\tilde{\tau}_s(\epsilon) = 0 \), the conductivity is mainly due to inelastic scattering. In contrast, for the region where \( 1/\tilde{\tau}_s(\epsilon) \neq 0 \), the dominant process is the scattering by impurities. In order to derive the conductivity formula (27), one has to solve the Boltzmann equation containing also the inelastic scattering time as sketched in App. B of Ref. 1. For equation (27) to be valid, the condition \( E_{dc} \ll aB/\tau_{in} \) has to be satisfied. This can be shown by taking into account that the change in energy within the relaxation time \( \tau_{in} \) should be smaller than the effective modulation potential [1]. In the regime \( aB/\tau_{in} \ll E_{dc} \ll aB/\sqrt{\tau_{in}/\tilde{\tau}_s} \), where \( \tilde{\tau}_s \) is the
minimum of \( \tilde{\tau}(\epsilon) \) on the Landau band, we obtain an expression similar to Eq. (27) where the second term on the right hand side of (27) in the conductivity formula (7) gives the result only up to numbers with respect to the exact expression. For \( aB/\sqrt{\tau_n R^2} \ll E_{dc} \) inelastic scattering due to heating is dominant, which results in the behavior that \( \sigma_{yy} \sim (aB/E_{dc}\sqrt{\tau_n R^2})^2 e^{2} D_{yy} \nu^* \). This was shown explicitly in [1].

By use of Eqs. (24), (26), (27), we obtain the relation

\[
\sigma_{xx}(\epsilon) \sigma_{yy}(\epsilon) = e^4 \frac{\Delta^2_{ph}}{4a^2 \pi^2} \Theta \left( \frac{1}{\tilde{\tau}_{n, tr1}(\epsilon)} \right). \tag{28}
\]

We point out that Eq. (28) is an exact relation within our model although Eqs. (26) and (27) with Eqs. (24) and (25) are only scaling relations. This is due to the exact relation (24) which implies a universal value for \( \tilde{\tau}_n/\tilde{\tau}_{tr1} \) independent of the specific impurity configuration. In the case of half filling, i.e. \( \Delta_{ph} = a/2 \), and assuming that the correlation length allows jumps between nearby electron and hole stripes, i.e. \( \Theta(1/\tilde{\tau}_{n, tr1}(\epsilon)) = 1 \), we obtain in Eq. (28) the well known semicircle law for the conductivities of the striped state of half integer filling. This rule was derived before in Ref. 33 without taking into account the microscopic scattering process.

**B. Calculation of photoconductivities for \( \sqrt{\tau_{tr}/\tau_{n}} (a/R_c) \gg 1 \)**

As in the calculation of the dark conductivities above, we have to replace the scattering times in the integrals of section III by energy dependent scattering times in order to obtain the photoconductivities \( \sigma_{\text{photo}} \) in the range \( \xi \gg \ell_{B}/a \).

\[
\frac{1}{\tilde{\tau}_{tr2}(\epsilon)} = \frac{a v_{q_n}}{4} \sum_{j \in \mathbb{Z}} \int dq_y e^{-\frac{\tilde{\tau}_{tr2}^{2}}{2}} \left[ L_N(\frac{q_{j}^2 \ell_{B}^2}{2}) - L_{N-1}(\frac{q_{j}^2 \ell_{B}^2}{2}) \right]^2 \tilde{W}(q_j) + e^{-\frac{\tilde{\tau}_{tr2}^{2}}{2}} \left[ L_N(\frac{q_{j}^2 \ell_{B}^2}{2}) - L_{N-1}(\frac{q_{j}^2 \ell_{B}^2}{2}) \right]^2 \tilde{W}(q'_j) \tag{29}
\]

with \( q_j = q_x e_x + (ja + \Delta a)/\ell_{B} e_y, q'_j = q_x e_x + (2k + (ja - \Delta a)/\ell_{B}) e_y \) and \( k \ell_{B}^2 \) from (23).

The value \( \Delta a \) is defined by the condition for energy conservation

\[
\cos \left( \frac{2\pi k \ell_{B}^2 + \Delta a}{a} \right) = \cos \left( \frac{2\pi k \ell_{B}^2}{a} \right) + \frac{\Delta \omega}{V_N} = 0. \tag{30}
\]

This equation reflects the fact that electrons can only be scattered to positions of energy difference \( \omega \) under the combined influence of impurity scattering and microwave irradiation. The replacement of the scattering times by their energy-dependent values amounts to the following substitutions

\[
\tau_n/\tau_{tr}^2 \rightarrow 1/\tilde{\tau}_{tr2}(\epsilon) \quad \text{in (9), (10);} \quad 1/\tau_{tr} \rightarrow 1/\tilde{\tau}_{tr1}(\epsilon) \quad \text{in (12), (13);}
\]

\[
1/\tau_{tr}^2 \rightarrow 1/\tilde{\tau}_{n, tr1}(\epsilon) \tilde{\tau}_{tr2}(\epsilon) \quad \text{in (15), (16);} \quad \tau_{tr} \rightarrow \tilde{\tau}(\epsilon)/\tilde{\tau}_{tr2}(\epsilon) \quad \text{in (19), (20).} \tag{31}
\]

We point out that the derivative in (20) has to be applied to the energy dependent scattering time \( \tilde{\tau}_{n}(\epsilon) \) in \( D_{yy} \) (19) but not to \( \tilde{\tau}_{tr2}(\epsilon) \). Similarly as in the calculation of the dark conductivity (24), we have an exact relation between \( 1/\tilde{\tau}_{tr2} \) and \( 1/\tilde{\tau}_{tr2} \). This can be calculated from (29) resulting in

\[
\frac{1}{\tilde{\tau}_{tr1}(\epsilon)} = \frac{2}{3} \frac{(\Delta a)^2}{R_{\ell}^{2}} \frac{1}{\tilde{\tau}_{tr2}(\epsilon)} \sim \frac{a^2 (\Delta \omega \nu^*)(\epsilon)^2}{R_{\ell}^2 V_N^2} \frac{1}{\tilde{\tau}_{tr2}(\epsilon)} \tag{32}
\]

The scaling law at the right hand side of Eq. (32) can be derived by similar considerations as in Eq. (34) below. By using the asymptotics of the Laguerre polynomials from App. C we obtain the following scaling law for \( \tilde{\tau}_{tr2} \)

\[
\frac{1}{\tilde{\tau}_{tr2}(\epsilon)} \sim \begin{cases} \frac{a^2}{\ell_B^2 \tilde{\tau}^n} \sim \sqrt{\frac{2\pi}{\rho_n}} \frac{1}{\tau_{tr}} \quad \text{for some } n \text{ with } \left| \arccos \left( \frac{\Delta \omega \nu^*(\epsilon) - 1}{\sqrt{\frac{2\pi}{\rho_n}} \frac{1}{\tau_{tr}}} \right) \right| < \frac{\pi}{2} \frac{\ell_B^2}{a^2}, \tag{33} \\
0 \quad \text{otherwise.}
\end{cases}
\]
From Eq. (33), we obtain \( 1/\tilde{\tau}_{tr:2}(\epsilon) = 0 \) for \( \Delta \omega > 2V_N \sin[(2\pi)\tilde{\ell}_B^2/2a\xi] \). This results in the following expression for the photoconductivities

\[
\sigma^{\text{photo}} = 0 \quad \text{for} \quad |\Delta \omega| > \Delta \omega_0 := 2V_N \sin \left( \frac{(2\pi)\tilde{\ell}_B^2}{2a\xi} \right) \approx V_N \left( \frac{2\pi\tilde{\ell}_B^2}{a\xi} \right) \sim V_N \left( \frac{\tau_s}{\tau_{tr}} \right) \left( \frac{R_c}{a} \right)^2.
\]  

(34)

Here \( \sigma^{\text{photo}} \) stands for \( \sigma_{xx}^{\text{DP}}, \sigma_{yy}^{\text{DP}}, \sigma_{xx}^{\text{DF}} \) or \( \sigma_{yy}^{\text{DF}} \). The fact that we obtain zero photoconductivity for \( |\Delta \omega| > \Delta \omega_0 \) can be understood by taking into account that the microwave-mediated impurity jump of an electron has to fulfill an energy conservation condition which restricts the number of possible final states. The momentum transfer of the impurity to the electron is further restricted by the maximal value \( 1/\xi \). When taking into account that the momentum \( k \) of the electron and its \( X \) coordinate are related via \( X = klB^2 \) we obtain Eq. (34). By using Eq. (33) we are now able to calculate the various scaling laws for the photoconductivities of section III for the case \( |\Delta \omega| \ll \Delta \omega_0 \).

1. Photoconductivities for the displacement mechanism

By using Eqs. (9), (12) and (32) we obtain for the conductivity in the regime

\[
|\Delta \omega| < \Delta \omega'_0 = V_N \left( 1 - \cos \left( \frac{\pi\tilde{\ell}_B^2}{4a^2} \right) \right) \approx \frac{1}{2} V_N \left( \frac{\pi\tilde{\ell}_B^2}{a^2} \right)^2 \sim V_N \left( \frac{\tau_s}{\tau_{tr}} \right) \left( \frac{R_c}{a} \right)^2
\]  

(35)

the following expressions

\[
\sigma_{xx}^{\text{DP}} \sim -\text{sgn}(\Delta \omega) \pi^2 \left[ e^2D_{xx}v^* \right] \left( \frac{\tau_s}{\tau_{tr}} \right) \left( \frac{eER_c}{4\Delta \omega} \right) \left( \frac{\tau_{tr}}{\tau_s} \right) \left( \frac{a}{R_c} \right)^3,
\]

(36)

\[
\sigma_{yy}^{\text{DP}} \sim \left[ e^2D_{yy}v^* \right] \left( \frac{aB/\pi\sqrt{\tau_{tr}\tau_s}}{E_{dc}} \right) \left( \frac{eER_c}{4\Delta \omega} \right)^2 \left( \frac{\tau_{tr}}{\tau_s} \right)^2 A_2 \left( \frac{\Delta \omega}{2V_N} \right) \frac{\Delta \mu}{V_N} \left( \frac{\tau_{tr}}{\tau_s} \right) \left( \frac{a}{R_c} \right).
\]

(37)

As mentioned below Eq. (13), the singularity of \( \sigma_{yy}^{\text{DP}} \) is cut off for small \( dc \) fields by inelastic scattering processes. This cutoff was discussed extensively in Ref. 1. By transferring this discussion to the present case, we obtain a cutoff for small \( dc \) fields at \( E_{dc} \sim E_{dc}^* \) where

\[
E_{dc}^* = \frac{Ba}{2\pi \sqrt{\tau_{tr}\tau_s}} \sim \frac{Ba}{2\pi \sqrt{\tau_{tr}\tau_s}} \left( \frac{\tau_{tr}}{\tau_s} \right)^{1/4} \left( \frac{a}{R_c} \right)^{1/2}.
\]

(38)

For \( E_{dc} \ll E_{dc}^* \), the photoconductivity crosses over to Ohmic behaviour matching with (37) for \( E_{dc} \sim E_{dc}^* \).

In principle we can also get analytical formulas for the scaling laws in the transition region \( \Delta \omega'_0 < |\Delta \omega| < \Delta \omega_0 \). Due to their complexity we will not give explicit expressions in this range.

2. Photoconductivities for the distribution function mechanism

In the following, we calculate the photoconductivity in the regime \( |\Delta \omega| < \Delta \omega'_0 \). By using Eqs. (14), (15), (24) and (33) we obtain with \( (\Delta \rho_E/\alpha)^2 \approx (\tau_s/\tau_{tr})(R_c/\alpha)^2 \) that the \( \epsilon \) integral (16) has its main contribution from the band edges.

\[
\sigma_{xx}^{\text{DF}} \sim 4 \left( \frac{\tau_{in}}{\tau_{tr}} \right) \left[ e^2D_{xx}v^* \right] \left( \frac{eER_y}{4\Delta \omega} \right)^2 B_1 \left( \frac{\Delta \omega}{2V_N} \right) \frac{\Delta \mu}{V_N} \left( \frac{\tau_{tr}}{\tau_s} \right) \left( \frac{a}{R_c} \right)^2,
\]

(39)

\[
\sigma_{yy}^{\text{DF}} \sim 4 \left( \frac{\tau_{in}}{\tau_{tr}} \right) \left[ e^2D_{yy}v^* \right] \left( \frac{eER_y}{4\Delta \omega} \right)^2 B_2 \left( \frac{\Delta \omega}{2V_N} \right) \frac{\Delta \mu}{V_N} \left( \tau_{in}/\tau_s \right) \left( \tau_{tr}/\tau_s \right) \left( \frac{a}{R_c} \right).
\]

(40)

For calculating the photoconductivity in \( y \)-direction \( \sigma_{yy}^{\text{DF}} \) we have taken into account (19), (33) and the discussion below (27). This conductivity expression is valid for \( E_{dc} \ll a/B/\sqrt{\tau_{in}\tau_s} \) and \( |\Delta \omega| < \Delta \omega'_0 \). By using \( A_2 \sim (\Delta \omega'_0/V_N) \), \( B_1 \sim (\Delta \omega'_0/V_N)^{-3/2} \) and \( B_2 \sim (\Delta \omega'_0/V_N)^{1/2} \) we obtain for \( |\Delta \omega| < \Delta \omega'_0 \) with (35) the scaling relations

\[
\sigma_{xx}^{\text{DF}} \sim \frac{\tau_{in}}{\tau_s} \left( \frac{a}{R_c} \right)^2 \sigma_{xx}^{\text{DP}}, \quad \sigma_{yy}^{\text{DF}} \sim \frac{\tau_{in}}{\tau_s} \left( \frac{a}{R_c} \right)^2 \sigma_{yy}^{\text{DP}}.
\]

(41)
Summarizing, we obtain that the photoconductivities in the striped state regime are dominated by the distribution function mechanism where remarkably the scaling factors are the same for both directions. We do not expect a strong $\Delta \omega$ dependence of the exact proportionality factors to the photoconductivity expressions above for $|\Delta \omega| < \Delta \omega_0$ and for impurity correlation functions which do not vary strongly over $1/\xi$ in momentum space. In Fig. 1 we show $B_1$, $B_2$ for the half filled upper Landau level system. We expect from the figure zero resistance states for both directions at $\omega \approx \omega_c$. The region of possible zero resistance states near $\omega_c$ is given by $\Delta \omega < 0$ for $\sigma_{xx}^{DP}$ and $\Delta \omega > 0$ for $\sigma_{yy}^{DF}$. For the frequency range $|\Delta \omega| \gg \Delta \omega_0$ we obtain zero photoconductivities.

C. Calculation of the conductivities in the case $\sqrt{\tau_s/\tau_a (a/R_e)} \lesssim 1$

In this subsection we discuss the conductivities in the regime $1/\xi \geq a/\ell_B^2$. In this regime, one has to carry out explicitly the summations in the definition of the transport times (22) and (29). This can be done with the help of Poisson’s summation formula. We do not discuss the results of this calculation in detail. We obtain conductivity contributions beside the terms discussed in section III which have a damping term proportional to $a/\ell_B^2$ times an oscillating function of $2\ell_B^2/a\xi$ with period one. This can be understood by the functional dependence of the transport times in (22) and (29) because the additional conductivity expressions should be approximately periodic in the number of $j$-summands. These are given by $2\ell_B^2/a\xi$.

One of the main results of this calculation is that the derivatives with respect to $\Delta \omega$ in Eq. (10) and with respect to $\epsilon$ in Eq. (20) (we remind that the derivative has to be applied also on $\tilde{\tau}_s(\epsilon)$ in $D_{yy}$ (19)) have the effect that the photoconductivities $\sigma_{xx}^{DP}$ and $\sigma_{yy}^{DF}$ get singular for $N \to \infty$ due to the steps in the energy dependent transport times. This is because the support of $\tilde{W}(q)$ is zero outside the inverse of the correlation length such that the number of $j$-summands in (22) and (29) depends strongly on the energy $\epsilon$ of $\tilde{\tau}_s(\epsilon)$. $\sigma_{yy}^{DF}$ in Eq. (20) is singular when $\tilde{\nu}^*(\epsilon + \Delta \omega)$ and the step in $\tilde{\tau}_s(\epsilon)$ is in accordance. For $\sigma_{xx}^{DP}$ (12) we first transform the integral in Eq. (10) to displacement coordinates corresponding to $\Delta \omega$ in Eq. (30) (these were also used in Ref. 1 to calculate $A_1$). In these coordinates the impurity scattering time $\tilde{\tau}_2(\epsilon)$ does not depend explicitly on $\Delta \omega$. Thus we can change the order of the derivative with respect to $\Delta \omega$ and the integration in (10). Keeping this in mind, we find for the positions of the photoconductivity peaks

\[ 1 - \frac{1}{2} \left( \frac{\Delta \omega}{V_N} \right)^2 - \cos \left( 2\pi \frac{\ell_B^2}{a\xi} \right) = 0 \quad \text{for} \quad \sigma_{xx}^{DP}, \]  
\[ 1 - \left( \frac{\Delta \omega}{V_N} \pm \cos \left( \frac{\ell_B^2}{a\xi} \right) \right)^2 = 0 \quad \text{for} \quad \sigma_{yy}^{DF}. \]  

It is clear from the discussion above that these singularities should not be expected for general filling fraction and $T \ll V_N$. The reason is that the range of the integrals over the energy $\epsilon$ in the photoconductivity formulas, Eqs. (10) and (20), are strongly dependent on the filling fraction. Thus, the steps of the energy dependent transport times could lie outside this range. Nevertheless, for $T \gg V_N$ or integer filling fraction these singularities should be seen in general. We point out that in real systems the singularities of the photoconductivities will be smeared out for finite Landau level index $N$ due to the smoothness of the Laguerre wavefunctions in the monotonic region (see the discussion below Eq. (C4)).

V. ELECTRONS IN HIGH LANDAU LEVELS WITH UNIDIRECTIONAL PERIODIC MODULATION OF ARBITRARY STRENGTH

In the last two sections, we calculated the conductivities in $x$- and $y$-direction within a theory that uses the eigenvalues and eigenvectors calculated in first order perturbation theory in $V$. This is correct for $V_N \ll \omega_c$. In this section we generalize the calculation of the eigenvectors and the eigenfunctions to the case of large Landau level index $N$ without any restriction on the relation of $V_N$ and $\omega_c$. It is well known that one gets the exact eigenvalues and eigenfunctions for physical systems within the WKB approximation in the limit of large quantum numbers corresponding to high Landau levels in our system. This is the physical setting we are interested in.

In the following, we will calculate in subsection A the eigenvalues and eigenfunctions of the Hamiltonian of an electron in a homogeneous magnetic field and with a unidirectional periodic modulation in $x$-direction within the WKB approximation. We will use these wavefunctions to calculate simple matrix elements which are needed for the derivation of the microwave photoconductivities [1]. In subsection B we calculate the matrix elements of plane waves with respect to WKB eigenstates and discuss the implications on the calculation of impurity averaged absolute square of plane wave matrix elements, relevant for the calculation of conductivity values.
A. WKB-approximation for an electron in the background of a homogeneous magnetic field and a unidirectional periodic potential

By carrying out the standard separation ansatz \( \psi_{nk}(x) = e^{i k x}/\sqrt{T_x} \) for the eigenfunctions of the Hamiltonian under study, we obtain that \( \psi_{nk}(x) \) is an eigenfunction of the Hamiltonian

\[
H = \frac{1}{2m} \frac{\partial^2}{\partial x^2} + \frac{1}{2m} (x - X)^2 + \tilde{V} \cos \left( \frac{2\pi}{a} x \right)
\]

(44)

with \( X = k \ell_B^2 \) and \( m \) is the mass of the electron. The eigenfunction of \( H \) corresponding to the eigenvalue \( E_n \) is given in the WKB approximation by

\[
\psi_{nk}(x) = \frac{1}{N_{\psi}} \left( \frac{1}{2m(E_n - \tilde{V}_{\text{eff}}(x))} \right)^{1/4} \cos \left( \int_x^{x_B} \sqrt{2m(E_n - \tilde{V}_{\text{eff}}(x'))} \, dx' - \frac{\pi}{4} \right)
\]

(45)

with the effective one-dimensional potential

\[
\tilde{V}_{\text{eff}}(x) := \frac{1}{2m \ell_B^2} (x - X)^2 + \tilde{V} \cos \left( \frac{2\pi}{a} x \right).
\]

(46)

\( N_{\psi} \) is a normalization constant which will be determined below. \( x_L (x_R) \) is the left (right) classical reflection point of a particle in the potential \( \tilde{V}_{\text{eff}}(x) \) with energy \( E_n \), i.e.

\[
\frac{(x_{L,R} - X)^2}{2m \ell_B^2} + \tilde{V} \cos \left( \frac{2\pi}{a} x_{L,R} \right) = E_n.
\]

(47)

The WKB quantization rule is given by:

\[
\int_x^{x_B} \sqrt{2m(E_n - \tilde{V}_{\text{eff}}(x'))} \, dx' = \left( n + \frac{1}{2} \right) \pi.
\]

(48)

In the following, we will solve this equation for \( n \to \infty \). We will insert a \( \tilde{V} \) expansion of the solution for \( x_{L,R} \) in (47), at the left hand side of (48). Then it is easily seen that for \( n \to \infty \) it is sufficient to consider \( x_{L,R} \) up to the zeros order of \( \tilde{V} \). This is because the correction of \( x_{L,R} \) due to the oscillatory potential \( \tilde{V} \) is proportional to \( 1/\sqrt{n} \). Furthermore, one finds that the integrand of the left hand side of (48) has to be considered only up to first order in \( \tilde{V} \) for \( n \to \infty \). After some calculation, one gets from Eqs. (46) and (48) for \( n \to \infty \)

\[
E_n = \left( n + \frac{1}{2} \right) \omega_c + \tilde{V} \cos \left( \frac{2\pi}{a} x_{L} \right) J_0 \left( \frac{2\pi}{a} R_c \right).
\]

(49)

This energy corresponds to the first order in \( \tilde{V} \) result (2).

Next, we calculate explicitly the wavefunction \( \psi_{nk} \) (45). As in the calculation of the spectrum, we will calculate the integral in the cosine in Eq. (45) by the help of an expansion of the integrand with respect to \( \tilde{V} \). Then one can show that it is sufficient to take into account only the linear order to get all finite terms in the cosine for \( n \to \infty \). With the help of an elementary integration, we get

\[
\psi_{nk}(x) = \frac{1}{N_{\psi}} \left( \frac{1}{2m(E_n - \tilde{V}_{\text{eff}}(x))} \right)^{1/4} \sin \left( \Theta_n^V (x - X) + C_n^V (x - X) + \frac{\pi}{4} \right)
\]

(50)

with

\[
\Theta_n^V (x) = (n + 1/2) \arccos \left( \frac{x}{\sqrt{2(n + 1/2) \ell_B}} \right) - \frac{1}{2} (n + 1/2) \sin \left( 2 \arccos \left( \frac{x}{\sqrt{2(n + 1/2) \ell_B}} \right) \right) + \frac{\pi}{4}
\]

(51)

where \( \arccos(x) \in [0, \pi] \). \( C_n^V \) is a term proportional to \( \tilde{V} \). This term is given by

\[
C_n^V (x) = -m \ell_B^2 \int_{x/\sqrt{2(n+1/2) \ell_B}}^1 \frac{1}{\sqrt{1 - x'^2}} \tilde{V} \cos \left( \frac{2\pi}{a} \left( \sqrt{2(n + 1/2) \ell_B} x' + X \right) \right).
\]

(52)
By the help of a Fourier expansion we find

$$C_{nk}^V(x) = m\ell_B^2 \hat{V} \left\{ -J_0 \left( \frac{2\pi R_e}{a} \right) \cos \left( \frac{2\pi X}{a} \right) \left( \frac{\pi}{2} - \varphi_x \right) + \sum_{l>0} \frac{2}{2l-1} J_{2l-1} \left( \frac{2\pi R_e}{a} \right) \sin \left( \frac{2\pi X}{a} \right) \cos \((2l-1)\varphi_x) \right\}$$

(53)

with \( \varphi_x = \arcsin(x/\sqrt{2(n+1/2)\ell_B}) \). It should be emphasized that \( C_{nk}^V \) is a finite term for \( n \to \infty \).

Next, we calculate the normalization constant \( N_\psi \). This constant is determined by the requirement that \( \psi_{nk}(x) \) has norm one. Thus, we can calculate the one-dimensional integral of the function \( \psi_{nk}^2(x) \) for \( n \to \infty \). One can neglect in \( \psi_{nk}^2(x) \) the \( \hat{V} \) term in the prefactor \( 1/(E_n - V_{eff}(x))^{1/2} \) of \( \psi_{nk}^2(x) \) during this calculation. Furthermore, the integrand contains a factor \( \sin^2[...] \) whose argument is given by the argument of the sine in Eq. (50). For large Landau levels, \( (n \to \infty) \), we have \( \sin^2[...] = (1 - \cos[...])/2 \approx 1/2 \). This replacement is exact in leading order in \( n \) of \( N_\psi \) for \( n \to \infty \), since \( \cos[...] \) is a fast oscillating function for \( n \to \infty \). After carrying out the integral, we obtain

$$N_\psi = \sqrt{\frac{\pi}{2} \ell_B}$$

(54)

Now, we can compare \( <\hat{r}|nk> = \psi_{nk}(x)e^{iky}/\sqrt{\rho_y} \) for \( \hat{V} = 0 \), i.e. \( C_{nk}^V(x) = 0 \), with the exact eigenfunctions of the Hamiltonian in the Landau gauge. As is well known, these eigenfunctions consist of Hermite polynomials. By using the identities \( H_{2m}(x) = (-1)^m 2^m m! L_m^{1/2}(x^2) \), \( H_{2m+1}(x) = (-1)^m 2^m m! L_m^{1/2}(x^2) \) and further (C3) and (C4) for the asymptotics of the Laguerre polynomials we get accordance of the WKB wavefunctions and the exact eigenfunctions for Landau level \( n \to \infty \) and \( \hat{V} = 0 \).

Next we calculate the matrix element \( \langle nk|x|n'k' \rangle \). For calculating this quantity we have to determine \( \Theta_{nk}^V \) for a fixed \( \Delta n \) and \( n \to \infty \). With the help of a Taylor expansion we get

$$\Theta_{nk}^V \approx \Theta_{nk}^V + \Delta n \arccos \left( \frac{x}{\sqrt{2(n+1/2)\ell_B}} \right).$$

(55)

With the integration techniques stated below Eq. (53), we get for \( n \to \infty \)

$$\langle nk|x|n'k' \rangle = \frac{R_e}{2} \delta_{k,k'} \left( \delta_{n,n'+1} + \delta_{n,n'-1} \right).$$

(56)

This corresponds exactly to the first order perturbation result in \( \hat{V} \) [1]. We should mention that one can derive similarly the expected orthonormal basis property of the \( |nk \rangle \) eigenstates for \( n \to \infty \).

As was shown in Ref. 1 as well as in section III, when calculating the various dark and photoconductivities one has to know the eigenstates of the system with the unidirectional periodic potential (considered above) and an additional \( dc \) field in \( x \)- or \( y \)-direction. For calculating the linear response on this additional \( dc \) potential it is enough to consider the first order correction of the wavefunction and the energy due to this field. The generalization of the WKB approximation for the system with an additional \( dc \) field in \( x \)-direction is straightforward. It amounts to trivial change in wavefunctions and energies as in the well known transformations for the system without the applied periodic modulation potential. Next we calculate the eigenfunctions and eigenvalues in the case of the \( dc \) field in the \( y \)-direction. This was done in Ref. 1 for \( V_a \ll \omega_c \). One can carry out a similar calculation in the case of Landau level index \( n \to \infty \) without restrictions on \( V_a \) and \( \omega_c \) by using for the Landau level basis set named \( |nk \rangle \) in Ref. 1 the WKB basis set calculated above (see for example (A1) for the eigenfunctions).

Finally, we want to give a short application of our findings to the striped state physics. The striped state was experimentally identified by the measurement of an anisotropy in the resistivity of quantum Hall systems in high Landau levels \( N \geq 3 \). This anisotropy disappears above Landau levels \( N \approx 7 \). The disappearance of the striped state at very high Landau levels can be understood by considering the spectrum, Eq. (49). By taking into account that, at thermodynamic equilibrium, all states are occupied with energy values lower than the chemical potential and further that the period of the striped state is \( \alpha = 2.6 R_e \) we obtain that the striped state starts to diminish at \( V_N \approx \omega_c \). By using \( \omega_c = 2\pi e\rho/Nm \) where \( \rho \) is the density of the electron system and further due to the Poisson formula of electrodynamics \( V_N \approx \sqrt{\frac{ae^2\rho}{N}} \approx 2.6 \pi^2 R_e \rho /N \) where by approximation the electron density
in the highest occupied Landau level is given by \((\rho/N)\), we obtain a vanishing of the striped state at high Landau levels. In order to determine the correct Landau level index \(N\) of the transition, one should take into account the screening of the completely filled lower Landau levels as well as the total Hartree-Fock potential of the upper Landau level \([28]\). This trespasses the scope of this work.

B. Matrix elements of plane waves with respect to WKB wavefunctions and implications on the calculation of impurity averaged transition rates

In this subsection, we give a short outline of the calculation of matrix elements of plane waves with respect to the WKB states \(|nk\rangle\). These matrix elements are calculated explicitly in App. D. As is well known, the matrix elements of plane waves with respect to the Hermite polynomials which are the exact eigenstates for \(\tilde{V} = 0\) are given by Laguerre polynomials. As is outlined in App. D, we obtain the asymptotic forms given in App. C for these matrix elements when using the WKB approximated wavefunctions for \(\tilde{V} = 0\). These asymptotic forms of the Laguerre polynomials, for large Landau level index \(n\), consist in the relevant oscillatory region (see App. C) of a rapidly oscillating factor \(\sin(\Theta_n)\) (Eq. (C3)) times a non-oscillatory amplitude. Here, \(\Theta_n(x)\) is defined as in Eq. (C3) where the argument \(x\) is proportional to the absolute square of the wavevector of the plane wave. When switching on \(\tilde{V}\) we obtain for the WKB matrix elements a separation of the \(\sin(\Theta_n)\) asymptotics in two phase factors, in the following form

\[
\sin(\Theta_n(x)) \rightarrow \exp[i\Theta_n(x)] \exp[i\nu_n^\pm(x)] + \exp[-i\Theta_n(x)] \exp[-i\nu_n^\pm(x)]
\]

For large Landau level index \(n\), the functions \(\exp(\pm i\nu_n^\pm(x))\) are slowly oscillating functions in \(x\). The exact result is given in Eq. (D5). In contrast to the calculation of the matrix element of the \(x\)-operator (56) and the eigenenergies (49), we thus obtain a \(\tilde{V}\)-dependence of the plane wave matrix elements.

In the following, we will provide arguments that, despite this \(\tilde{V}\)-dependence of the plane wave matrix elements, we do not have a \(\tilde{V}\)-dependence of the impurity averaged square of these matrix elements relevant within the calculation of the conductivity \([1]\). The impurity averaged square of the plane wave matrix elements enters the conductivity expressions via the various scattering times \([1]\). We note that one gets the exact transport scattering times in high Landau levels when one uses the asymptotics (C3) and (C4) for the Laguerre polynomials. This was shown for smooth disorder in Ref. 1 but can be generalized to the case without restrictions on the correlation length \(\xi\). The calculation is similar to the calculation of the normalization constant of WKB states below (52) due to the resembling of Laguerre and Hermite polynomial asymptotics. From this we obtain with (57) that the impurity averaged square of the plane wave matrix elements remains the same for large Landau level index \(n\), irrespective of the value of \(\tilde{V}\) when using the WKB matrix elements.

In summary, we obtain that all inputs in the conductivity calculations, i.e. the eigenenergies (49), the \(x\)-operator matrix element (55) as well the various transport time integrals, do not depend on the oscillatory strength \(\tilde{V}\) for large Landau level index \(n\). Thus, we can easily generalize the calculations carried out in Ref. 1 and in the last sections for the various conductivities to general \(V_N\) and \(\omega_c\). It will become clear soon that the problem of calculating the various conductivities then is a mere summation problem. In the limit \(V_N \gg \omega_c\) we can carry out these summations easily.

VI. CONDUCTIVITIES FOR \(V_N \gg \omega_c\)

We calculated in section III and IV the conductivities in \(x\)- and \(y\)-direction within a theory that uses the eigenvalues and eigenvectors in first order perturbation theory in \(\tilde{V}\). This is correct in the limit \(V_N \ll \omega_c\). In this section, we will calculate the various conductivities in the opposite limit, \(\mu \gg V_N \gg \omega_c\), which excludes a perturbative treatment in \(V_N\). Due to the results of the last section we can still use the formalism derived in Ref. 1 for calculating the various conductivities but one has to take into account the overlapping of the Landau levels in the calculation of the transition probabilities.

It is clear, that the experimental situation where \(V_N \ll \omega_c\) considered up to now corresponds to the situation of well separated Landau levels in the microwave experiments without periodic modulation. This was also seen in the same scaling of the conductivity expressions of both systems \([1]\). As derived in the last section, the effective Landau level width in our system is proportional to \(V_N \sim VBa/\sqrt{\rho}\). In Weiss oscillation experiments, it is possible to vary \(\tilde{V}\) by changing the grid potential \([26]\). Thus, we obtain that by changing \(\tilde{V}\) we can reach the situation of large Landau level overlapping which is the low magnetic field regime in the microwave experiments without periodic modulation. The regime \(V_N \gg \omega_c\) is not relevant for the striped state, so that we can restrict ourselves to the case \(a \ll R_c, \ell_B/\xi\). Furthermore, we restrict us to the temperature regime \(\mu \gg T \gtrsim V_N \gg \omega_c\) which is the typical experimental regime for the system without periodic modulation. In the following, we calculate the dark conductivities in subsection A.
and the photoconductivities in subsection B. The density of states, Eq. (3), and the dimensionless density of states, Eq. (5), for $V_N \gg \omega_c$ are denoted by $\nu_\epsilon^*(\epsilon)$ and $\tilde{\nu}_\epsilon^*(\epsilon)$, respectively.

We obtain for the density of states

$$\nu_\epsilon^*(\epsilon) = \sum_{n \in \mathbb{Z}} \nu_\epsilon^*(\epsilon + n\omega_c) \approx \nu_0 + 2\nu_0 \sum_{n > 0} J_0 \left( \frac{2\pi n V_N}{\omega_c} \right) \cos \left( \frac{2\pi n \epsilon}{\omega_c} \right) + O \left( \frac{\omega_c}{V_N} \right).$$

(58)

By using the Bessel asymptotic expansion $J_0(x) \approx \sqrt{(2/\pi x)} \cos(x - \pi/4)$ for large $x$ we obtain

$$\nu_\epsilon^*(\epsilon) \approx \nu_0 + 2\nu_0 \sum_{n > 0} \left( \frac{\omega_c}{n\pi^2 V_N} \right)^{1/2} \cos \left( \frac{2\pi n V_N}{\omega_c} - \frac{\pi}{4} \right) \cos \left( \frac{2\pi n \epsilon}{\omega_c} \right) + O \left( \frac{\omega_c}{V_N} \right)$$

$$= \nu_0 + \nu_0 \left( \frac{\omega_c}{2\pi^2 V_N} \right)^{1/2} \sum_{\sigma \in \{\pm\}} \left( \frac{V_N}{\omega_c}, \frac{\epsilon - \omega_c/2}{\omega_c} \right) + O \left( \frac{\omega_c}{V_N} \right)$$

(59)

where the oscillatory part of the dimensionless density of states is given by

$$\Sigma(x, y) = \sum_{\sigma \in \{\pm\}} \zeta \left[ \frac{1}{2}, (x + \sigma y) \right] \text{ mod } 1$$

(60)

Here, $\zeta$ is the Riemann zeta function. The $\sqrt{\omega_c/V_N}$ correction to the density of states is oscillatory in $\epsilon$ with a wavelength $\omega_c$ and has singularities for $(V_N \pm (\epsilon - \omega_c/2))/\omega_c \text{ mod } 1 = 0$. These energy values correspond to the edges of the Landau band where the density of state contribution is singular (4). In realistic systems these singularities are cut at $\Delta \epsilon \sim \sqrt{\omega_c/\tau_s}$ due to impurity scattering when $\omega_c > \sqrt{\omega_c/\tau_s}$. The second term in (60) is then further changed by an additional damping factor $\sim \sqrt{\omega_c/\tau_s}$ in the case $V_N \gg \sqrt{\omega_c/\tau_s} > \omega_c$ by taking into account the Landau level broadening $\sqrt{\omega_c/\tau_s}$ for the system without modulation.

Fig. 2 depicts this oscillatory correction to the density of states leading to oscillatory contributions in the conductivities. We point out the similarity of the density of states formula (59) with the corresponding formula of the system without periodic modulation in the overlapping Landau level regime. In this system the density of states is given by $\nu = \nu_0 - 2\nu_0 \delta \cos(2\pi \epsilon/\omega_c)$ with a damping term $\delta = \exp[-\pi/\omega_c \tau_s]$. When comparing this expression with the density of states formula (59) it seems suggestive that we could get the conductivities in the direction of the periodic modulation up to numbers from the system without periodic modulation when carrying out the the replacement $\delta \to \sqrt{\omega_c/V_N}$ in the corresponding formulas. That this is in fact true will be shown below.
A. Calculation of dark conductivities

We start with a discussion of the dark conductivity. With Ref. 1, we obtain for the conductivities per energy $\sigma_{xx}(\epsilon)$ and $\sigma_{yy}(\epsilon)$, Eq. (8),

$$\sigma_{xx}(\epsilon) = e^2 D_{xx} \nu^* \left( \nu_{x}^*(\epsilon) \right)^2 \approx e^2 D_{xx}^\Sigma \nu_0 \left( 1 + \left( \frac{2\omega_c}{\pi^2 V_N} \right)^{1/2} \sqrt{\sum \frac{V_N}{\omega_c} \left( \frac{\epsilon - \omega_c/2}{\omega_c} \right)} \right),$$

$$\sigma_{yy}(\epsilon) = e^2 D_{yy} \nu^* \left( \nu_{y}^*(\epsilon) \right)^2 \approx e^2 D_{yy}^\Sigma \nu_0 \left( 1 - \left( \frac{2\omega_c}{\pi^2 V_N} \right)^{1/2} \sqrt{\sum \frac{V_N}{\omega_c} \left( \frac{\epsilon - \omega_c/2}{\omega_c} \right)} \right).$$

where we used $\sum_{n \in \mathbb{Z}} \frac{1}{\nu_{y}^*(\epsilon + n\omega_c)} = \pi V_N/2\omega_c + O(1)$. Note that $D_{xx}$, $D_{yy}$ are defined by Eq. (6) for $\epsilon = E_n$. The diffusion constants $D_{xx}^\Sigma$, $D_{yy}^\Sigma$ for $V_N \gg \omega_c$ are given by

$$D_{xx}^\Sigma = \frac{R_c^2}{\omega c} \text{ and } D_{yy}^\Sigma = \left( \frac{1}{\pi a\nu^*} \right)^2 \tau_s = \frac{1}{2} \left( \frac{1}{\pi a\nu^*} \right)^2 \tau_s .$$

That $D_{xx}^\Sigma$ and $D_{yy}^\Sigma$ are in fact the diffusion constants for $V_N \gg \omega_c$ can be shown by taking into account Eq. (6) with the replacements that the scatter time $\tau_s$ should be substituted by $\tau_{s}^\Sigma \nu^*/\nu_{x}^* \approx \tau_s$ in these expressions and furthermore the expression for $D_{yy}$ in (6) should be averaged over the various Landau levels intersecting one energy level weighted by the corresponding density of states contribution. We point out that the second term in $\sigma_{xx}(\epsilon)$ (61) corresponds to the Shubnikov de-Haas term of the conductivity without periodic modulation.

B. Calculation of photoconductivities

In this subsection, we calculate the various photoconductivities for $\mu \gg T \gg V_N \gg \omega_c$ by using the methods presented in Ref. 1.

1. Photoconductivities for the displacement mechanism

The photoconductivity $\sigma_{xx}^{DP}$ for $V_N \gg \omega_c$ is given by (9) with

$$A_1 = \frac{6}{\pi^2} \int_0^\infty d\epsilon \left[ f(\epsilon) - f(\epsilon + \omega) \right] \frac{\partial}{\partial \omega} \tilde{\nu}_{x}^*(\epsilon) \tilde{\nu}_{x}^*(\epsilon + \omega) \left( \epsilon \right).$$

Here we used that the impurity averaged matrix element $\frac{d^2 q}{d^2 q} \left[ L_{m+1} \left( \nu_{y}^* \right) \right] \left( \epsilon \right) \left( \nu_{y}^* \right) = \left( \epsilon \right)$ does not depend on $m$ which can be shown easily by using the asymptotic form (C4) for the Laguerre polynomials or by an explicit calculation of the integral in App. B for delta correlated impurities resulting in $12\pi N$. Now we insert the density of states expression (58) in (64). By taking into account that $\frac{d\epsilon \cos(2\pi n\epsilon/\omega_c)}{(\partial/\partial \epsilon) f(\epsilon)} \sim \exp[-2\pi^2 nT/\omega_c]$ is negligible for $T \gg \omega_c$ we obtain only non-exponentially vanishing corrections of the form $\int \frac{d\epsilon \cos(2\pi n\epsilon/\omega_c)}{(\partial/\partial \epsilon) f(\epsilon)} \approx -1/2$. Then by taking the cos asymptotics for the Bessel function in (58) one can carry out the resulting Bessel series giving

$$\sigma_{xx}^{DP} \approx -\frac{12}{\pi} \left[ e^2 D_{xx}^\Sigma \nu_0 \right] \frac{\tau_s}{\tau_tr} \left( \frac{eER_c}{4\Delta \omega} \right)^2 \left( \frac{\omega_c}{V_N} \right) \frac{\omega}{\omega_c} \frac{\sin \left( \frac{2\pi \omega_c}{\omega_c} \right)}{1 - \cos \left( \frac{2\pi \omega_c}{\omega_c} \right)}. \quad (65)$$

Next, we calculate the displacement photoconductivity $\sigma_{yy}^{DP}$ keeping in mind the remarks below Eq. (56) concerning the WKB wave functions with a $d$ field in the $y$-direction. With Eq. (12) we find

$$A_2 = \sum_{m,n \in \mathbb{Z}} \frac{m\omega_c - \omega}{V_N^2} \int_0^\infty d\epsilon \tilde{\nu}_{N}^*(\epsilon) \tilde{\nu}_{N}^*(\epsilon + m\omega_c - \omega) \left[ f_{mn}^y - f_{(n+m)}^y \right]. \quad (66)$$
By taking into account \( \sum_{n=0}^{\infty} [f_{nl}^y - f_{n+1l}^y] = m \) one can easily carry out the \( m \)-sum for \( V_N/\omega_c \gg 1 \) in Eq. (66) after having done the \( \epsilon \) integration. This results in

\[
\sigma_{yy}^{\text{DP}} \approx \frac{16 + 32G}{\pi^2} \left[ e^2 D_{yy}^{\Sigma}/\nu_0 \right] \left( \alpha B/\pi \sqrt{\tau_{ls} \tau_s} \right) \left( \frac{eER_c}{4\Delta E} \right)^2.
\]

(67)

where \( G \approx 0.915 \) is Catalan’s constant. As discussed in Ref. 1, Eq. (67) is only valid in the case that the electric \( dc \) field \( E_{dc} \) is not too small. The reason lies in inelastic scattering which we do not take into account when calculating Eq. (67). By carrying out a similar analysis as in Ref. 1, we find that Eq. (67) is valid only for

\[
E_{dc} \gg E_{dc}^* = \frac{Ba}{2\pi \sqrt{\tau_{ls} \tau_s}}.
\]

(68)

In the case \( E_{dc} \ll E_{dc}^* \) we have to substitute \( E_{dc} \) in the denominator in (67) by \( E_{dc}^* \). Finally, we remark that the results, Eqs. (67) and (68), are correct generally for \( T \ll \mu \), since \( f_{nl}^y \) is independent of \( l \) for fixed \( n \).

2. Photoconductivities for the distribution function mechanism

For calculating \( \sigma_{xx}^{\text{DF}} \) as well as \( \sigma_{yy}^{\text{DF}} \) one has to calculate the change in the distribution function due to absorption of microwaves. If one assumes that there is no heating, relaxation processes have to be taken into account. In the case \( V_N \ll \omega_c \), we calculated the distribution function (see Ref. 1) within the relaxation time approximation. In the following, we carry out a similar calculation for the distribution function in the case \( V_N \gg \omega_c \). By taking into account the overlap of Landau levels due to the large amplitude of the unidirectional modulation potential we get

\[
\delta f(\epsilon) \approx 2\frac{\tau_{ls}}{\tau_{tr}} \left( \frac{eER_c}{4\Delta E} \right)^2 \sum_{\sigma = \pm} |f(\epsilon - \sigma \omega) - f(\epsilon)| \nu_0^* \frac{\epsilon - \sigma \omega}{\nu_0}.
\]

(69)

By using this distribution function, we obtain for the photoconductivity \( \sigma_{xx}^{\text{DF}} \) (15) with

\[
B_1 = -\frac{1}{2} \sum_{\sigma \in \{\pm\}} \int_0^\infty \text{d} \epsilon \left( \nu_0^* \right)^2 \frac{\partial}{\partial \epsilon} \left( \nu_0^* (\epsilon \pm \sigma \omega) \right) \left( f(\epsilon - \sigma \omega) - f(\epsilon) \right).
\]

(70)

The calculation of this expression can be done similar to the calculation of \( \sigma_{xx}^{\text{DF}} \) above. This results in

\[
\sigma_{xx}^{\text{DF}} \approx -\frac{16}{\pi} \left( \frac{\tau_{in}}{\tau_{tr}} \right) \left( \frac{eER_c}{4\Delta E} \right)^2 \left[ e^2 D_{xx}^{\Sigma}/\nu_0 \right] \left( \frac{\omega_c}{V_N} \right) \frac{\omega_c}{\omega_c} \sin \left( \frac{2\omega \omega_c}{\omega_c} \right).
\]

(71)

Finally, we calculate the photoconductivity \( \sigma_{yy}^{\text{DF}} \) which is given by Eq. (19) with

\[
B_2 = \frac{1}{2} \sum_{\sigma \in \{\pm\}} \int_0^\infty \text{d} \epsilon \sum_{n \in \Lambda} \nu_0^* (\epsilon \pm \sigma \omega) \frac{\partial}{\partial \epsilon} \left( \nu_0^* (\epsilon \pm \sigma \omega) \right) \left( f(\epsilon - \sigma \omega) - f(\epsilon) \right)
\]

resulting in

\[
\sigma_{yy}^{\text{DF}} \approx -8 \left( \frac{\tau_{in}}{\tau_{tr}} \right) \left( \frac{eER_c}{4\Delta E} \right)^2 \left[ e^2 D_{yy}^{\Sigma}/\nu_0 \right] \left( \frac{\omega_c}{V_N} \right) \frac{\omega_c}{\omega_c} \sin \left( \frac{2\omega \omega_c}{\omega_c} \right).
\]

(73)

for large \( V_N/\omega_c \).

By taking into account the above expressions for the photoconductivities we obtain the result that the photoconductivities \( \sigma_{xx}^{\text{DF}}, \sigma_{xx}^{\text{DF}} \) and \( \sigma_{yy}^{\text{DF}} \) are oscillating in \( \omega/\omega_c \) whereas the photoconductivity \( \sigma_{yy}^{\text{DF}} \) is non-oscillating with positive values. We obtain the following scaling relations for the photoconductivities in the regime \( V_N \gg \omega_c \)

\[
\sigma_{xx}^{\text{DF}} \sim \frac{\tau_{in}}{\tau_{tr}} \sigma_{xx}^{\text{DF}}, \quad \sigma_{yy}^{\text{DF}} \sim \frac{\omega}{V_N} \sigma_{yy}^{\text{DF}}.
\]

(74)
Thus we find that the photoconductivity parallel to the modulation direction is dominated by the distribution function contribution. In the direction perpendicular to the modulation, the photoconductivity is dominated by the displacement contribution in the regime $V_N \gg \omega$ being always positive, or oscillating which should result also in zero resistance states in the regime $V_N \ll \omega$. The careful reader could have the objection that our derivations so far were done for $\omega \approx \omega_c$. We will show in the next section that the photoconductivities calculated in this section are also valid for all $\omega > 0$ up to a non-oscillating positive prefactor which has the same scaling for all photoconductivities $\sigma_{\text{photo}}$ depending on the polarization and $\omega, \omega_c$. The frequency dependent scaling of the photoconductivities is then given by

$$
\sigma_{xx,yy}^{\text{DP,DF}} \sim C = \omega_c \left( \frac{1 - \omega}{\omega} \right)^2 \sin \left( \frac{2\pi \omega_c}{\omega} \right), \quad \sigma_{yy}^{\text{DP}} \sim D = \left( \frac{\omega_c}{\omega} \right)^2 \left( 1 - \frac{\omega}{\omega_c} \right)^2
$$

Here we take into account the additional factor $(\omega_c/\omega)^2$ for the photoconductivities derived in section VII below such that the scaling functions are valid for all $\omega > 0$. In Fig. 3 we show the scaling functions $C(\omega/\omega_c)$ and $D(\omega/\omega_c)$.

Next, we compare $\sigma_{xx,yy}^{\text{DP,DF}}$ calculated above with the corresponding terms of Ref. 13 for $\sigma_{xx}^{\text{DP}}$ and Ref. 14 for $\sigma_{xx}^{\text{DF}}$ representing the photoconductivities for the system with no modulation potential by using the damping factor substitution $\delta \rightarrow \sqrt{\omega_c/V_N}$. We get immediately correspondence of the expressions up to numbers as was also the case in the non-overlapping Landau level regime shown in Ref. 1. Nevertheless the form of the photoconductivities differ. Especially the singularities of $\sigma_{xx,yy}^{\text{DP,DF}}$ at $\omega = \pm \omega_c$ (see Fig. 3) having their reason in the singularities of the density of states (59). They are not existent in the system without modulation. From the discussion below (60) and the calculation of the photoconductivities above we obtain that for $\omega > \sqrt{\omega_c/\tau_s}$ the photoconductities are cut at $|\omega - \pm \omega_c| \sim \sqrt{\omega_c/\tau_s}$. For $V_N \gg \sqrt{\omega_c/\tau_s} > \omega_c$ the singularities in photoconductivities are smeared out completely where the conductivity expressions obtain an additional damping factor $\sim (\omega_c/\tau_s)$ due to the damping of the oscillatory part of the density of states. As mentioned below (21) the $(eER_c/4\Delta\omega)^2$ prefactor in the photoconductivity expressions should be cut at $|\Delta\omega| \sim 1/\tau_{tr}$.

VII. PHOTOCONDUCTIVITIES FOR ARBITRARY POLARIZATION AND FREQUENCY

In this section, we generalize our results to the case of microwave irradiation of frequency $\omega > \omega_c$ and general polarization. An arbitrarily polarized microwave field is of the form

$$
\vec{E}(\alpha, \beta) = E \cos(\alpha) \cos(\omega t - \beta) \hat{e}_x + \sin(\alpha) \cos(\omega t + \beta) \hat{e}_y.
$$

(76)
We sketch the calculation of photoconductivities for this rather general microwave field for \( a \ll R_c, \ell_B^2/\xi \), in App. E. We obtain the result

\[
\sigma^{\text{photo}} = \sum_{m \geq 1} \Theta \left[ m\omega_c - \omega - \frac{\omega_c}{2} \right] (m\omega_c - \omega)^2 F \times (R_1 \sigma^{\text{photo}}(\Delta \omega \rightarrow m\omega_c - \omega, N) + R_2 \sigma^{\text{photo}}(\Delta \omega \rightarrow m\omega_c - \omega, \nu))
\]  

(77)

where \( \sigma^{\text{photo}}(\Delta \omega \rightarrow m\omega_c - \omega, \nu) \) is the placeholder for \( \sigma_{xx}^{\text{DP}}, \sigma_{yy}^{\text{DP}}, \sigma_{xx}^{\text{DF}}, \sigma_{yy}^{\text{DF}} \) calculated for the filling fraction \( \nu \) in the various regimes for \( \Delta \omega/\omega_c \ll 1 \). We carry out the replacement \( \Delta \omega \rightarrow m\omega_c - \omega \) in these conductivity expressions to get Eq. (77). We remark explicitly that this replacement should not be carried out in pure \( \omega \) terms in the photoconductivity expressions for \( V_N \gg \omega_c \) of section VI.

The frequency dependent prefactor \( F \) is given by

\[
F = \left( \frac{2(\omega^2 + \omega_c^2 + 2\omega_c \sin(2\alpha) \sin(2\beta))}{(\omega^2 - \omega_c^2)^2} \right) \cos(2\alpha) \right),
\]

(78)

The regime-dependent prefactors \( R_1 \) and \( R_2 \) are given by

\[
R_1 = \begin{cases} 
\frac{\omega_c - \omega}{\omega_c^2} \quad \text{for} \quad T \ll V_N \ll \omega_c \\
0 \quad \text{for} \quad V_N \ll \omega_c, T \ll \mu \\
\frac{\omega_c}{\omega} \quad \text{for} \quad \omega_c \ll V_N
\end{cases}
\]

\[(79)\]

and

\[
R_2 = \begin{cases} 
\frac{\omega_c}{\omega} \quad \text{for} \quad T \ll V_N \ll \omega_c \\
0 \quad \text{for} \quad V_N \ll \omega_c, T \ll \mu \\
\left( \frac{\omega_c}{\omega} \right)^2 \quad \text{for} \quad \omega_c \ll V_N
\end{cases}
\]

(80)

The polarization-dependent factor \( p_o \) is given by

\[
p_o = \begin{cases} 
1 \quad \text{for} \quad \sigma^{\text{photo}} = \sigma_{xx}^{\text{DP}} \quad \text{or} \quad \sigma_{yy}^{\text{DF}} \\
0 \quad \text{for} \quad \sigma^{\text{photo}} = \sigma_{xx}^{\text{DF}} \quad \text{or} \quad \sigma_{yy}^{\text{DP}}
\end{cases}
\]

(81)

These conductivity formulas are only correct for \( a \ll R_c, \ell_B^2/\xi \). The two different regime dependent prefactors \( R_1 \) and \( R_2 \) have their reason in the filling fraction dependence of the photoconductivity for \( T \ll V_N \). We point out that the above replacement for the photoconductivities for \( V_N \gg \omega_c \) is simply a multiplication of the conductivity expression of section VI by the factor \( (\Delta \omega)^2(\omega_c/\omega)^2 F \). The corresponding photoconductivity expressions are valid for all \( \omega > 0 \).

One can easily show that \( F \) is a positive function which is important for the determination of the range of zero resistance states, and that \( F \) is only zero at the values \( \alpha = -\beta = \pm \pi/4 \) and \( \omega = \omega_c \). This corresponds to a circularly polarized microwave field. From the calculation in App. E one further obtains that for \( a \gg R_c \) or \( a \gg \ell_B^2/\xi \) conductivity formulas similar to Eqs. (77)-(80) hold, with a polarization factor \( p_o \) depending on the specific impurity correlation function \( \tilde{W} \) which cannot be expressed in terms of the scattering times, as it was also the case for the photoconductivities in section IV.

In Ref. 1 we calculated the microwave photoconductivity in the case of \( \omega \approx \omega_c \) for the linearly as well as circularly polarized microwave field and found no dependence of the photoconductivities on the polarization direction in the case of a linearly polarized microwave field. These results are in agreement with Eq. (78), where the last term in \( F \) is approximately zero for \( \omega \approx \omega_c \). In contrast to this, for \( \omega \not= \omega_c \), we find a polarization dependence only for the displacement photoconductivity in \( x \)-direction. The displacement photoconductivity in \( y \)-direction does not exhibit a polarization dependence.

In the following, we compare the frequency terms \( F \) for \( \sigma_{xx}^{\text{DP}} \) (\( p_o = 1 \)) and \( \sigma_{xx}^{\text{DF}} \) (\( p_o = 0 \)) with the corresponding terms of Vavilov et al. [13] calculated for the case of conventional microwave experiments without unidirectional periodic field. We find agreement [34] for \( F \) (up to numbers) with the corresponding term of Ref. 13 for \( \sigma_{xx}^{\text{DF}} \). The same holds true for \( \sigma_{xx}^{\text{DF}} \) up to a sign difference in the second term in \( F \) (78). The prefactor of this term can be derived without explicit calculation as shown in App. E. Finally, we note that Ref. 13 restricts the calculation in the strong separated Landau level regime to circularly polarized microwave fields.
VIII. DISCUSSION

We refer to section II for an overview and a discussion of the results of this paper. Beside this, we want to emphasis further the connection of this work and our earlier work Ref. 1. One of the main theoretical consequences of Ref. 1 which was reconsidered in section III and summarized in the scaling relations (21) was that the photoconductivities for a $a < R_c, \ell_B^2/\xi$ and $V_N \ll \omega_c$ in the perpendicular direction to the modulation field are of the same order for the displacement mechanism as well as the distribution function mechanism. This is no longer the case for the striped state regime $\xi \gg \ell_B^2/a$ where the photoconductivities of the distribution function mechanism are for both directions a factor $(a/ R_c)^2 (\tau_{0\pi} \tau_{\pi}^x / (\tau_{\pi}^y)^2) (41)$ larger than the photoconductivities of the displacement mechanism. We expect zero resistance states for both directions at certain frequency regions in the range $\omega \approx \omega_c$. In the large overlapping Landau level regime the photoconductivity of the displacement mechanism in the perpendicular direction has no longer the oscillating property. Therefore we expect zero resistance states only parallel to the modulation direction.

Summarizing, we obtain in the non-overlapping regime [1] and in the overlapping Landau level regime parallel to the modulation direction accordance with the results of the photoconductivities without uniperiodic modulation potential. The concrete frequency dependence of the photoconductivities differ. For the system with modulation potential with weak disorder the photoconductivities can have large values at certain frequencies having its origin in the singularities of the density of states at the band edge for the system without disorder.

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APPENDIX A: THE CALCULATION OF THE PHOTOCONDUCTIVITY $\sigma_{yy}^{DP}$

In this section we generalize the displacement photoconductivity formula in the y-direction for $T \gg V_N$ [1] to a current formula without temperature restrictions. The main difference between the calculation of the displacement photocurrent in $x$ and in $y$-direction consists on the fact that the many particle ground state without field can be represented as a single Slater determinant of eigenstates in the x-direction where in the $y$-direction we have a linear combination of Slater determinants built from the eigenstates. For a single Slater determinant of eigenstates we can use the standard Fermi’s golden rule formula in order to calculate the current in $x$-direction where one works with single Fermi factors [1].

As is well known the one particle eigenstates with a $dc$ field in the x-direction can be represented in the Landau gauge by Hermite polynomials. As derived in [1] the corresponding one particle states with a $dc$ field in the y-direction are given by

$$|n_1\xi_1\rangle = \sqrt{\frac{2\pi \ell_B^2}{L_x L_y}} \sum_k \exp \left\{ i \int_0^k \frac{dk'}{eE_{dc}} (\xi_{l} - \xi_{l+1}) \right\} |nk\rangle. \quad (A1)$$

with the boundary condition $\xi_{l} = 2\pi l e E_{dc} \ell_B^2 / L_x$ and energies $\xi_{nl} = \hbar \omega_c (n + 1/2) + \xi_{l}$. The $y$ expectation value of these states are given by $\langle \overline{y}(\xi_l) := \langle n_1\xi_1\xi_1\rangle |n_1\xi_1\rangle = -\xi_{l} / eE_{dc}$. As mentioned above the ground state wavefunction $\Psi_0$ is a Slater determinant of the one particle states $|nk\rangle$. This can be represented as a linear combination of Slater determinants of $|n_1\xi_1\rangle$, i.e.

$$\Psi_0 = \frac{1}{N!} \left[ |n_1k_1\rangle, .., |n_Nk_N\rangle \right]$$

$$= \sum_{n_1l_1 < ... < n_Nl_N} a_{n_1l_1; ..; n_Nl_N} \left[ |n_1\xi_1\rangle, .., |n_N\xi_N\rangle \right]. \quad (A2)$$

Now we generalize the Fermi’s golden rule current formula of Ref. 1 to the case that the ground state wavefunction consists on linear combinations of Slater determinants. The current of such a state under microwave assisted impurity scattering is given by

$$j_{y}^{DP} = \sum_{n_1l_1 < ... < n_Nl_N} |a_{n_1l_1; ..; n_Nl_N}|^2 j_{y}^{DP}_{n_1l_1; ..; n_Nl_N}. \quad (A3)$$
with the current components

\[ j_{\text{DP}}^{y;n_1,l_1;\ldots;n_L L_y} = \frac{e}{L_x L_y} \sum_{n n'} \sum_{\ell_1, \ell_1'} |\gamma_{n \ell_1 \rightarrow n' \ell_1'}|^2 (\mathcal{F}(\mathcal{E}_\nu) - \mathcal{F}(\mathcal{E}_\ell)) [f^y(n_{1,l_1;\ldots;n_L L_y} - f^y(n'_{1,l_1';\ldots;n_L L_y})] \delta(\mathcal{E}_n - \mathcal{E}_{n'} - \omega). \] (A4)

Here \( f^y(n_{1,l_1;\ldots;n_L L_y}) = 1 \) if \((n,l) \in \{n_1,l_1;\ldots,n_L,L_y\}\) and zero otherwise. For the definition of the transition amplitude \(|\gamma_{n \ell_1 \rightarrow n' \ell_1'}|^2\) see Ref. 1. The current formula (A4) can be interpreted in the following way: The total transition rate of these electrons through a surface of constant \(r\) can be found in [36]. In the following we give expansions for large \(n\) of \(e^{-x^2/2}L_n^\alpha(x)\) for \(x\) near 0, in the oscillatory region, and in the monotonic region.

**APPENDIX B: SOME INTEGRALS OF LAGUERRE POLYNOMIALS**

In this paper we use a number of matrix elements of plane waves with respect to Laguerre polynomials in order to calculate the dark and photoconductivities. By the use of Ref. 35 the matrix elements are given by:

\[ \frac{d^2 q}{(2\pi)^2} \frac{\gamma_{n \ell_1 \rightarrow n' \ell_1'}}{2\omega} \left( L_0^m \left( \frac{q^2 B^2}{2} \right)^2 \right) (q_y B)^{2r} = \frac{1}{2 \pi q B} (\delta_{r,0} + \delta_{r,1} 2n), \]

\[ \frac{d^2 q}{(2\pi)^2} \frac{\gamma_{n \ell_1 \rightarrow n' \ell_1'}}{2\omega} \left( L_0^m \left( \frac{q^2 B^2}{2} \right)^2 \right) (q_y B)^{2r} = \frac{1}{2 \pi q B} (\delta_{r,0} + \delta_{r,1} 6n), \]

\[ \frac{d^2 q}{(2\pi)^2} \frac{\gamma_{n \ell_1 \rightarrow n' \ell_1'}}{2\omega} \left( L_0^m \left( \frac{q^2 B^2}{2} \right)^2 \right) (q_y B)^{2r} = \frac{1}{2 \pi q B} (\delta_{r,0} + \delta_{r,1} 16n), \]

\[ \frac{d^2 q}{(2\pi)^2} \frac{\gamma_{n \ell_1 \rightarrow n' \ell_1'}}{2\omega} \left( L_0^m \left( \frac{q^2 B^2}{2} \right)^2 \right) (q_y B)^{2r} = \frac{1}{2 \pi q B} (\delta_{r,0} + \delta_{r,1} 3n), \] (B1)

for \(r = 0\) or \(r = 1\), respectively. The results above are correct in the leading \(n\) order for \(n \rightarrow \infty\).

**APPENDIX C: THE ASYMPTOTIC EXPANSIONS OF THE LAGUERRE POLYNOMIALS**

In this section we review some of the known asymptotic expansions of the Laguerre polynomials. A full description can be found in [36]. In the following we give expansions for large \(n\) of \(e^{-x^2/2}L_n^\alpha(x)\) for \(x\) near 0, in the oscillatory region, and in the monotonic region.
The leading order term of an expansion which is valid for \( x \) near zero, i.e. \( 0 \leq x < n^{1/3} \), is given by

\[
e^{-x/2}L_n^0(x) = \frac{\Gamma(n + \alpha + 1)}{\Gamma(n + \alpha + 1/2)} \frac{1}{n^{\alpha+1/2}} J_\alpha[(\nu x)^{1/2}] + O \left( \frac{\nu x}{2} \right)
\] (C1)

where \( \nu = 4n + 2\alpha + 2 \). One can specialize this expansion in the range \( 0 < x < n^{1/3} \) with \( \epsilon > 0 \) to Fejér’s formula

\[
e^{-x/2}L_n^\epsilon(x) = \frac{n^{\epsilon+1/2}}{\pi^{\epsilon+1/2}} \cos \left( 2(nx)^{1/2} - \alpha - \frac{\pi}{4} \right) + O \left( \frac{\nu x}{2} \right).
\] (C2)

With the help of

\[
x = \nu \cos^2(\theta_\nu), \quad 0 < \theta_\nu < \pi/2, \quad 4\Theta_\nu(x) = \nu(2\theta_\mu - \sin(2\theta_\nu)) + \pi
\] (C3)

the Laguerre polynomials in the oscillatory region \( 0 < x < \nu \) are given by

\[
e^{-x/2}L_n^\nu(x) = 2(-1)^n \frac{1}{(2\cos(\theta_\nu))^n} \frac{1}{(\pi \nu \sin(2\theta_\nu))^{1/2}} \sin(\Theta_\nu) + O \left( \frac{1}{n} \right).
\] (C4)

One can find similar asymptotic expansions of \( e^{-x/2}L_n^\nu(x) \) at the transition point \( x \approx \nu \) and in the monotonic region \( x > \nu \) (see e.g. [36]). Because both regions give for \( n \to \infty \) vanishing contributions to the calculated physical quantities in this paper we shorten here the discussion of the asymptotics in these regions. In the monotonic region \( x > \nu \) the function \( e^{-x/2}L_n^\nu(x) \) behaves asymptotically as \( e^{-\nu x} \) with \( \Theta'_\nu = \nu(2\theta'_\mu - 2\theta'_\nu) \) and \( x = \nu \cosh^2(\theta'_\nu) \), \( \theta'_\nu > 0 \) (see [36]). Furthermore, one can show that both asymptotical expansions, i.e. in the oscillatory region as well as in the monotonic region, are valid at the transition point except in the range \( |x - \nu| < \nu^{1/3} \).

**APPENDIX D: THE CALCULATION OF PLANE WAVES MATRIX ELEMENTS WITH RESPECT TO WKB EIGENSTATES**

In this section, we calculate the matrix elements \( \langle nk|e^{i\Phi}\rangle |n'k'\rangle \). Before going on we will first quote for comparison the well known result of this matrix element for \( \tilde{V} = 0 \) by using the exact Hermite polynomial eigenfunctions. With

\[
A_{n,n'}(\tilde{r}) = \sqrt{\frac{n!}{n^1}} \frac{x + iy}{\sqrt{2\ell_B}} e^{-x^2/2} J_n(x) \frac{1}{(2\pi)^{1/2}}
\] (D1)

for \( n \geq n' \) and \( A_{n,n'}(\tilde{r}) = (-1)^{n-n'} A_{n,n'}^*(\tilde{r}) \) for \( n' \geq n \) we obtain

\[
\langle nk|e^{i\Phi}\rangle |n'k'\rangle =
\]

\[
\delta(k' - k + qy) e^{i\Phi(x + |q|k')/2} \frac{1}{(2\pi)^{1/2}} A_{n,n'}(\tilde{qy^2}).
\] (D2)

Now we come to the calculation of this matrix element in the WKB approximation:

For simplification, we denote \( \Theta_n^V(x), C_n^V(x) \); as a function of \( x \) by \( \Theta_n^V(x), C_n^V(x) \); as a function of \( x' = x/\sqrt{n + 1/2} \ell_B \) by \( \tilde{\Theta}_n^V(x'), \tilde{C}_n^V(x') \); and as functions of \( \varphi = \arcsin(x/\sqrt{2(n + 1/2)}\ell_B) \) by \( \tilde{\Theta}_n^V(\varphi), \tilde{C}_n^V(\varphi) \). Here \( \arcsin(x) \in [-\pi/2, \pi/2] \). Furthermore, we extend the range of definition of \( \tilde{\Theta}_n^V(\varphi) \) and \( \tilde{C}_n^V(\varphi) \) to \( [-\pi, \pi] \) by \( \tilde{\Theta}_n^V(\varphi) := \tilde{\Theta}_n^V(\arcsin(\sin(\varphi))) \) and \( \tilde{C}_n^V(\varphi) := \tilde{C}_n^V(\arcsin(\sin(\varphi))) \) for all \( \varphi \in [-\pi, \pi] \).

First, by using (50) we get for \( n \to \infty \)

\[
\langle nk|e^{i\Phi}\rangle |n'k'\rangle = \frac{\tilde{\Theta}_n^V}{N^q} \delta(k' - k + qy) e^{i\Phi(x + |q|k')/2}
\]

\[
\times \int_{-1+|\tilde{q}y|/2}^{1-|\tilde{q}y|/2} dx' e^{i(x + \tilde{q}y/2)} \frac{1}{(1 - (x' - |\tilde{q}y|/2)^2)^{1/4}} \frac{1}{(1 - (x' + |\tilde{q}y|/2)^2)^{1/4}}
\]

\[
\times \sin \left( \tilde{\Theta}_n^V(x' - \tilde{q}y/2 + \tilde{C}_n^V(x' - \tilde{q}y/2) + \frac{\pi}{4}) \sin \left( \tilde{\Theta}_n^V(x' + \tilde{q}y/2 + \tilde{C}_n^V(x' + \tilde{q}y/2 + \frac{\pi}{4}) \right) \right)
\] (D3)
with \( \tilde{q}_x = q_x \ell_B / \sqrt{2(n + 1/2)} \) and \( \tilde{q}_y = q_y \ell_B / \sqrt{2(n + 1/2)} \).

In the following, we calculate first the matrix element (D2) for \( q \ell_B \) near zero, i.e. \( q^2 \ell_B^2 \ll n^{1/3} \). Similar as in (55) we obtain for fixed \( \Delta n \), \( (\Delta x / \ell_B)^2 \ll n^{1/3} \) and \( n \to \infty \)

\[
\Theta^V_{n+\Delta n}(x - \Delta x) \approx \Theta^V_n(x) + 2(n + 1/2) \sqrt{1 - \frac{x^2}{2(n + 1/2) \ell_B^2}} \frac{\Delta x}{\sqrt{2(n + 1/2) \ell_B^2}} + \arccos \left( \frac{x}{\sqrt{2(n + 1/2) \ell_B^2}} \right) \Delta n. \tag{D4}
\]

Now we use this expansion in (D3). With the assumption \( q_x \ell_B / \sqrt{2(n + 1/2)} \ll 1 \) we obtain

\[
\langle nk | e^{i \bar{q} \bar{r}} | n' k' \rangle = \delta(k' - k + q_y) e^{i q_x (k + k')} \ell_B^2 / 2 \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\varphi_x e^{i(q_x \ell_B) \sin(\varphi_x) \sqrt{2(n + 1/2)}}
\]

\[
\times \cos \left( \frac{2(n + 1/2) \cos(\varphi_x)}{\sqrt{2(n + 1/2)}} + (n - n') \left( \frac{\pi}{2} - \varphi_x \right) + \tilde{C}^V_{nk}(\varphi_x) - \tilde{C}^V_{n'k'}(\varphi_x) \right). \tag{D5}
\]

When carrying out the integral in (D5) for \( C^V_{nk} = C^V_{n'k'} = 0 \) we obtain the exact result (D2) when inserting the asymptotics (C1) for the Laguerre polynomial.

Next, we calculate \( \langle nk | e^{i \bar{q} \bar{r}} | n' k' \rangle \) from (D5) for \( C^V_{nk} \neq 0 \). It is difficult to calculate the integral in this case without approximation. But for \( \sqrt{(k - k')^2 + q_x^2 \ell_B} > 0 \) one can get the leading term for \( n \to \infty \) with the help of a saddle point approximation. For doing this we first split the cosine in its two exponential terms. Then by carrying out a saddle point approximation we get by the help of the definitions \( \cos(\varphi_0) := (k - k') / \sqrt{q_x^2 + q_y^2} \) and \( \sin(\varphi_0) := -q_x / \sqrt{q_x^2 + q_y^2} \) for \( k - k' \geq 0 \) and no restrictions on \( n, n' \)

\[
\langle nk | e^{i \bar{q} \bar{r}} | n' k' \rangle = \delta(k' - k + q_y) e^{i q_x (k + k')} \ell_B^2 / 2 \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\varphi_x e^{i(q_x \ell_B) \sin(\varphi_x) \sqrt{2(n + 1/2)}}
\]

\[
\times e^{i(n-n') \varphi_0} \sum_{\sigma \in \{ \pm \}} \exp \left[ i \sigma \left( \frac{\sqrt{2(n + 1/2) \ell_B}}{2} + \left( - \frac{n}{n'} - \frac{1}{2} \right) + \tilde{C}^V_{nk}(-\sigma \varphi_0) - \tilde{C}^V_{n'k'}(-\sigma \varphi_0) \right) \right]. \tag{D6}
\]

By comparing (D6) for \( C^V_{nk} = C^V_{n'k'} = 0 \) with (D2) when inserting for the Laguerre polynomial the asymptotics (C2) we get correspondence of both formulas.

Next, we calculate \( \langle nk | e^{i \bar{q} \bar{r}} | n' k' \rangle \) in the case where \( q^2 \ell_B^2 / 8(n + 1/2) < 1 \) is larger than zero for \( n \to \infty \). This will be done, by carrying out a saddle point approximation for the integral in (D3) when the product of the two sinus terms are split into its four exponential parts. Then one gets after some algebraic manipulation by the use of (D4) that two of these exponents have stationary points only in the case when \( q^2 \ell_B^2 / 8(n + 1/2) < 1 \) \( (n \to \infty) \). According to (C4) this corresponds to the oscillatory region of the Laguerre polynomials with argument \( q^2 \ell_B^2 / 2 \). Which of the two exponential terms exhibit the stationary points depends on the relation of \( q^2 \) to \( 2q_y \). These two stationary points of the exponents in (D3) are given by \( x' = \pm (\tilde{q}_y / \tilde{q}_x) \sqrt{1 - q^2 / 4} \). Additionally to the angle \( \varphi_0 \) defined above (D6) it is useful to define the angle \( \varphi_q \) by \( \sin(\varphi_q) := \sqrt{1 - q^2 / 4} \) and \( \cos(\varphi_q) := \tilde{q}_y / 2 \). After some tedious calculation we get in the leading order for \( n \to \infty \) and \( k - k' > 0 \)

\[
\langle nk | e^{i \bar{q} \bar{r}} | n' k' \rangle = \delta(k' - k + q_y) e^{i q_x (k + k')} \ell_B^2 / 2 \frac{1}{\pi \nu} \sin \left( \frac{1}{2 \theta_\nu} \left( \frac{q^2 \ell_B^2}{2} \right) \right) \tag{D7}
\]

\[
\times \sum_{\sigma \in \{ \pm \}} \exp \left[ i \sigma \Theta_\nu \left( \frac{q^2 \ell_B^2}{2} \right) + i c_{\sigma,k} \tilde{C}^V_{nk} - \left( \varphi_0 + \frac{\pi}{2} \right) - \sigma \varphi_0 + i c_{\sigma,k'} \tilde{C}^V_{n'k'} + \left( \varphi_0 + \frac{\pi}{2} \right) - \sigma \varphi_0 \right] \right]
\]

with \( \nu = 2(n + 1/2) + 2 \) where \( \theta_\nu \) and \( \Theta_\nu \) are defined in (C3). The matrix \( c_{\sigma,k} \) is defined by \( c_{-k,k'} = c_{+,k} \) and \( c_{-,k} = c_{-,k'} \). Furthermore, we have \( c_{+,k} = c_{+,k'} = q_x / q_\sigma \) for \( |\varphi_0| \geq |\varphi_q| \), \( c_{-,k} = -1 \), \( c_{+,k'} = 1 \) for \( |\varphi_q| \geq |\varphi_0| \).

Finally, we compare the WKB formula (D7) for \( C^V_{nk} = C^V_{n'k'} = 0 \) with the exact formula (D2) when inserting in the Laguerre polynomial the asymptotics (C4). We get correspondence of both formulas.
APPENDIX E: THE CALCULATION OF THE PHOTOCONDUCTIVITIES IN THE CASE OF ARBITRARY FREQUENCY AND POLARIZATION OF THE MICROWAVE FIELD

Up to now, we restricted the determination of the photoconductivities to the case where the system is irradiated by a microwave field polarized in x-direction of frequency $\omega \approx \omega_c$. The knowledge of the polarization dependency under microwave irradiation could be useful when determining the physical mechanism of the microwave oscillations [37]. Therefore we calculate in this section the various photoconductivities in the case that the irradiation has the rather general form $\tilde{E} = \text{Re}\{Ee^{-i\omega t}\}$. Here $\tilde{E}$ is a two dimensional vector with complex values. This type of electric fields can be represented by the following potential

$$\phi(\alpha, \beta) = -eE (x \cos(\alpha) \cos(\omega t - \beta) + y \sin(\alpha) \cos(\omega t + \beta)) = \phi_+ (\alpha, \beta) e^{-i\omega t} + \phi_- (\alpha, \beta) e^{i\omega t}. \tag{E1}$$

Because we calculate dc response values we fixed the freedom of the $t = t_0$ starting point of the plane wave in (E1) such that the phase factor of the x-component of the electric field is minus the phase factor of the y-component. $\phi_+ (\alpha, \beta)$ is then given by

$$\phi_+ (\alpha, \beta) = -\frac{eE}{2} (x \cos(\alpha)e^{i\beta} + y \sin(\alpha)e^{-i\beta}) \tag{E2}$$

and $\phi_- (\alpha, \beta) = \phi_+^* (\alpha, \beta)$.

In the following, we give only a short outline of the calculation of the photoconductivities in the case $V_N \ll \omega_c$, $a \ll R_c$ and $T \gg V_N$ which was the regime discussed in [1]. By using the results for the transition matrix elements for the x-polarized and y-polarized microwave field calculated in [1] we obtain

$$|\langle n \pm m k'|T_{\pm}(\alpha, \beta)|n k\rangle|^2 = \left(\frac{e E R_c}{4}\right)^2 \int \frac{d^2 q}{(2\pi)^2} \delta_{\kappa, \kappa'} e^{-\frac{\omega_c^2}{2}} \left|U(q)|U(-q)\right|^2 \left[|M_1(q |q \ell_B)| \right|^2 \tag{E3}$$

with

$$M_1(q |q \ell_B) = \frac{\omega_c}{\omega(\omega_c - \omega)} \left(\frac{-q_y + i q_x |q \ell_B|}{\sqrt{2m}}\right)^{|m|^{-1}} \left[L_{n+1}^{-1} \left(\frac{q_y^2 |q \ell_B|^2}{2}\right) - L_n^{m-1} \left(\frac{q_y^2 |q \ell_B|^2}{2}\right)\right], \tag{E4}$$

$$M_2(q |q \ell_B) = \frac{\omega_c}{\omega(\omega_c + \omega)} \left(\frac{q_y + i q_x |q \ell_B|}{\sqrt{2m}}\right)^{|m|+1} \left[L_{n+1}^{m+1} \left(\frac{q_y^2 |q \ell_B|^2}{2}\right) - L_n^{-m+1} \left(\frac{q_y^2 |q \ell_B|^2}{2}\right)\right]. \tag{E5}$$

Here $T_{\pm}(\alpha, \beta)$ is the second order transfer matrix [1] containing the impurity potential and $\phi_{\pm}(\alpha, \beta)$. The calculation of the matrix elements was carried out by the insertion of intermediate states in between the microwave operator and the impurity operator in $T_{\pm}(\alpha, \beta)$ [1]. $M_1$ in (E4) was already considered before when we have calculated the photoconductivities for $\omega \approx \omega_c$. This term comes from intermediate states of Landau level index $n + 1$ and $n + m - 1$. $M_2$ in (E5) comes from matrix elements with intermediate states of Landau level index $n - 1$ and $n + m + 1$. It then immediately clear that the prefactor of $M_2$ can be deduced from the prefactor of $M_1$ by the substitution $\omega_c \rightarrow -\omega_c$.

By using the matrix elements in appendix B we obtain for delta correlated impurities

$$\sum_k |\langle n \pm m k'|T_{\pm}(\alpha, \beta)|n k\rangle|^2 = \left(\frac{e E R_c}{4}\right)^2 \frac{1}{2\pi \nu_0 \tau_{\perp}} \frac{1}{\pi \ell_B^2} \left(\frac{\omega_c}{\omega}\right)^2 \left(\frac{2(\omega_c^2 + \omega^2 + 2\omega \omega_c \sin(2\alpha) \sin(2\beta))}{(\omega^2 - \omega_c^2)^2}\right) \tag{E6}$$

and

$$\sum_k |\langle n \pm m k'|T_{\pm}(\alpha, \beta)|n k\rangle|^2 ((k - k') \ell_B)^2 \tag{E7}$$

$$= \left(\frac{e E R_c}{4}\right)^2 \frac{1}{2\pi \nu_0 \tau_{\perp}} \frac{6n}{\pi \ell_B^2} \left(\frac{\omega_c}{\omega}\right)^2 \left(\frac{2(\omega_c^2 + \omega^2 + 2\omega \omega_c \sin(2\alpha) \sin(2\beta))}{(\omega_c^2 - \omega^2)^2} + \frac{(\omega^2 - \omega_c^2) \cos(2\alpha)}{(\omega^2 - \omega_c^2)^2}\right).$$

By using the identity $x/n)e^{-x^2/2} L_n^m(x) = e^{-x^2/2} L_n^{m-1}(x)$ which is valid for $n \rightarrow \infty$ (C4) and further by taking into account the considerations in [1] we obtain that (E6) and (E7) are also valid for non delta correlated impurities.
The last term in (E7) stems from the term proportional to $M_1M_2$ in the integrand in (E3) resulting in the polarization dependency of $\sigma_{xx}^{DP}$. This interference term $M_1M_2$ does not exist in $\sum_k |\langle n \pm mk' |T_+(\alpha, \beta) |nk\rangle|^2$ (E6). By using the results above and further that

$$\sum_n (f_n - f_{n+m}) = m.$$  \hspace{1cm} (E8)

we obtain for the photoconductivities (77). The generalization of the calculation to the other regimes in (77) is straightforward.

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