Curvaton Decay into Baryons, anti-Baryons and Radiation

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This paper calculates the amount of baryon/radiation isocurvature fluctuation produced through the decay of a curvaton field. It is shown in particular that if curvaton decay preserves baryon number and the curvaton dominates the energy density at the time of decay, the initial curvaton/radiation isocurvature mode is entirely transferred into a baryon/radiation isocurvature mode. This situation is opposite to that previously studied in three fluid models of curvaton decay; this difference is related to the conservation of the pre-existing baryon asymmetry and to the efficiency of the annihilation of all baryon/anti-baryon pairs produced in the decay. We study in detail the relevant cases in which the curvaton decay preserves or not baryon number and provide analytical and numerical calculations for each situation.

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I. INTRODUCTION

The curvaton scenario is a variant of the inflationary scenario in which the field driving the accelerated expansion (the inflaton field) is not necessarily that which produces all of the primordial fluctuations. Another field (the curvaton field), through its decay, can seed part of (or even all of) the cosmological perturbations. Therefore, in the most generic situation, these fluctuations originate from two different sources and the possibility of having isocurvature modes arises.

The existence of isocurvature fluctuations would lead to distortions of the multipole moments of cosmic microwave background anisotropies, as compared to pure adiabatic modes. It is thus possible to constrain the fraction of isocurvature modes using high accuracy measurements, and present-day constraints show that the contribution of isocurvature modes is sub-dominant, at least if the isocurvature components are considered separately. Therefore, the study of the production of isocurvature fluctuations in the curvaton scenario, although interesting per se, can also help us to constrain the free parameters describing the model.

The phenomenology of the curvaton scenario has been studied in the literature in a variety of cases, in particular in multi-fluid configurations (see for instance Refs. [5, 6, 7, 9, 23, 24, 25, 26]). The purpose of the present paper is to apply the formalism developed in Ref. [25] to the particular case of a net baryon/radiation isocurvature mode generated through curvaton decay. One peculiar feature that will emerge from the present study is the fact that the curvaton may induce a maximal isocurvature mode even if it dominates the energy density at the time of its decay, provided its decay preserves baryon number. This feature stands in sharp contrast with previous findings which showed that for curvaton decay into radiation and another fluid such as dark matter, the decay of a dominating curvaton would erase any pre-existing isocurvature mode. We study this case in detail and show that this particularity is related to the conservation of baryon number and to the efficient annihilation of all b ¯b pairs produced in curvaton decay (throughout this paper, “b” stands for a generic baryon, and ¯b for its antiparticle, not to be confused with bottom and anti-bottom quarks).

This paper is organized as follows. In Sec. III we describe the model and formulate the equations of motion at the background and perturbed levels. In Sec. III we numerically solve these equations in two cases, namely when the decay is symmetric in baryons and anti-baryons and when it is asymmetric meaning that the production of a net baryon number becomes possible. We show that these two cases correspond to very different phenomenologies. Finally, In Sec. IV we discuss and compare our main results and present our conclusions.

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II. DESCRIPTION OF THE MODEL

We consider a model where four fluids are present: baryons (denoted in what follows with the subscript “b”), anti-baryons (“\(\bar{b}\)”), radiation (“\(r\)”) and the curvaton field (“\(\sigma\)”). At the fundamental level, the curvaton is \textit{a priori} a massive scalar field but can effectively be treated as a pressureless fluid. One assumes that it can decay into radiation, baryons and anti-baryons. Each of these processes is controlled by a partial decay width denoted \(\Gamma_{\sigma}\), \(\Gamma_{b}\) and \(\Gamma_{\bar{b}}\) respectively. The curvaton decay occurs when the condition \(\Gamma_{\sigma} \sim H\) is met, where \(\Gamma_{\sigma}\) is the total decay width, namely \(\Gamma_{\sigma} = \Gamma_{\sigma\sigma} + \Gamma_{bb} + \Gamma_{\bar{b}\bar{b}}\) and \(H\) the Hubble parameter.

We do not discuss the phenomenology of curvaton to dark matter decay in the present paper. It is fair to assume that by curvaton decay, dark matter is effectively decoupled from radiation and baryons/anti-baryons. Even though the decay of curvaton may induce a dark matter - radiation isocurvature mode (see Ref. \[25\] for a detailed analysis), or a baryon - radiation isocurvature mode, both baryonic and dark matter sector will evolve independently. In this sense, the constraints obtained on dark matter or baryon isocurvature modes give complementary constraints on the physics of curvaton decay.

The freeze-out of baryon/anti-baryon annihilations is controlled by the velocity averaged cross-section

\[
\langle \sigma_{bb} v \rangle \simeq m_\pi^{-2},
\]

where \(m_\pi = 135 \text{ MeV}\). This relation originates from the fact that, in the present context, the pion can be viewed as the gauge boson mediating the strong force. Freeze-out of \(\bar{b}b\) annihilations occurs when \(\Gamma_{\bar{b}b} \equiv m_{\bar{b}b} \langle \sigma_{\bar{b}b} v \rangle\) and/or \(\Gamma_{b\bar{b}} \equiv m_{b\bar{b}} \langle \sigma_{b\bar{b}} v \rangle\) are of the order of the expansion rate \(H\) which corresponds to a temperature \(\sim 20\text{ MeV}\) in the absence of curvaton decay (that is to say, assuming that radiation always dominates the energy content of the Universe).

Big-Bang Nucleosynthesis (BBN) puts rather extreme upper bounds on the amount of energy density injected at temperatures \(T \lesssim 1\text{ MeV}\) (see Ref. \[27\] for a recent compilation). For all practical purposes, it suffices to impose that \(T_d \geq 1\text{ MeV}\) to satisfy these constraints. Furthermore, the late time decay of a scalar field at temperatures of order \(1 - 10\text{ MeV}\) is a fairly generic case in the framework of moduli cosmology. As is well known, such fields generically possess a very large energy density and a very small decay width \(\Gamma_{\sigma} \sim m_{\sigma}^3/m_{\sigma}^2\) hence they decay after big-bang nucleosynthesis if their mass is of the order of the weak scale. Therefore, in order to reconcile the existence of such fields with the success of big-bang nucleosynthesis, one has two choices: either the energy density of these fields at the time of big-bang nucleosynthesis is very small or their mass is large, leading to early enough decay. The mass also cannot be arbitrarily large, otherwise one has to face a hierarchy problem, hence the generic decay temperature is \(1 - 10\text{ MeV}\). Supersymmetric models with anomaly mediated supersymmetry breaking provide an explicit realization of particle physics model building in which the masses of moduli is of the order of \(m_{\sigma} \sim 10 - 100\text{ TeV}\), which leads to decay temperatures of the moduli/curvaton \(T_d \sim 1 - 10\text{ MeV}\) \[28\].

The above motivates the present study of the phenomenology of curvaton decay at temperatures of order \(1 - 10\text{ MeV}\). Out of simplicity, we keep this temperature fixed to a value \(T_d = 5.9\text{ MeV}\) in our numerical analysis, which corresponds to a total decay width \(\Gamma_{\sigma} = 1.6 \times 10^{-20}\text{ MeV}\). We will argue that the results obtained remain unchanged if the decay temperature is higher, in particular if \(T_d \gtrsim 20\text{ MeV}\).

At the background level, following the approach of Ref. \[25\], the above situation can be modelled by the following set of equations

\[
\frac{d\Omega_b}{dN} = \Omega_b \frac{H}{\Gamma_{bb}} \Omega_{\sigma} - \frac{3}{8\pi} \frac{\langle \sigma_{bb} v \rangle m_{\pi}^2}{m_b} \frac{H}{m_b} (\Omega_b \Omega_b - \Omega_{\sigma}^{eq} \Omega_{\sigma}^{eq}) ,
\]

\[
\frac{d\Omega_{\bar{b}}}{dN} = \Omega_{\bar{b}} \frac{H}{\Gamma_{\bar{b}\bar{b}}} \Omega_{\sigma} - \frac{3}{8\pi} \frac{\langle \sigma_{\bar{b}\bar{b}} v \rangle m_{\pi}^2}{m_{\bar{b}}} \frac{H}{m_{\bar{b}}} (\Omega_{\bar{b}} \Omega_{\bar{b}} - \Omega_{\sigma}^{eq} \Omega_{\sigma}^{eq}) ,
\]

\[
\frac{d\Omega_{\sigma}}{dN} = (\Omega_\sigma - 1) \Omega_\sigma + \frac{\Gamma_{\sigma}}{H} \Omega_\sigma + \frac{3}{8\pi} \frac{\langle \sigma_{bb} v \rangle m_{\pi}^2}{m_b} \frac{H}{m_b} (\Omega_b \Omega_b - \Omega_{\sigma}^{eq} \Omega_{\sigma}^{eq}) ,
\]

\[
\frac{d\Omega_{\bar{b}}}{dN} = \Omega_{\bar{b}} \frac{H}{\Gamma_{\bar{b}\bar{b}}} \Omega_{\sigma} - \frac{3}{8\pi} \frac{\langle \sigma_{\bar{b}\bar{b}} v \rangle m_{\pi}^2}{m_{\bar{b}}} \frac{H}{m_{\bar{b}}} (\Omega_{\bar{b}} \Omega_{\bar{b}} - \Omega_{\sigma}^{eq} \Omega_{\sigma}^{eq}) ,
\]

\[
\frac{dH}{dN} = \frac{3H}{2} \left( 1 + \frac{\Omega_\sigma}{3} \right) .
\]

Let us describe these equations in more detail. As usual, the parameters \(\Omega_{(\alpha)}\) are defined as the ratio of the energy density of the fluid \(\alpha\) to the critical energy density, \(\Omega_{(\alpha)} = \rho_{(\alpha)}/\rho_{cr}\). The time variable is the number of e-folds, \(N \equiv \ln a\), where \(a\) is the scale factor. The quantity \(\Omega_b^{eq}\) is defined by \(\Omega_b^{eq} = m_b n_b^{eq}/\rho_{cr}\), where \(n_b^{eq}\) is the particle density at thermal equilibrium, expressed as:

\[
n_b^{eq} = g_b \left( \frac{m_b T}{2\pi} \right)^{3/2} \exp \left( - \frac{m_b - m_b}{T} \right) ,
\]
with a similar expression for $n^\text{eq}_{\bar{b}b}$. The quantity $\mu_b$ is the chemical potential of the baryons and one has $\mu_b = - \mu_{\bar{b}}$. The temperature $T$ can be expressed in terms of the variables of the previous system of equations as:

$$T = \left( \frac{\pi^2 g_* 8\pi}{30 \cdot 3m_{Pl}^2} \right)^{-1/4} H^{1/2} \Omega_b^{1/4}.$$  \hfill (8)

Note that the above description implicitly assumes that the curvaton decay products thermalize instantaneously. This assumption will be discussed at the end of Section \textbf{III}. One should already underline that the above ratios $\Gamma_b/\Gamma_{\bar{b}}$, $\Gamma_{\bar{b}}/\Gamma_{\gamma}$ and $\Gamma_{\bar{b}}/\Gamma_{\sigma}$ should be understood as characterizing the number of curvaton energy that eventually goes into thermalized "b", "$\bar{b}$" and "e", rather than the branching ratios associated with curvaton decay channels.

For the sake of simplicity, we ignore any temperature dependence of the function $g_*$ and we take $g_* = 10.75$. If we compare with the equations of motion established in Ref. [25] in the case where the curvaton can decay into dark matter $\chi$ (rather than baryons and anti-baryons), the only difference is that terms like $\Omega_b^2$ or $\Omega_b^2 \Omega_{\chi,\text{eq}}^2$ are replaced by $\Omega_b \Omega_{\bar{b}}$ and $\Omega_{\bar{b}}^2 \Omega_{\bar{b}}^2$. Notice that, as a consequence, the evolution of the system does not depend on the chemical potential which cancels out, thanks to the fact that $\mu_b = - \mu_{\bar{b}}$. Finally, there is a factor $2$ in front of the last term in Eq. (8). This factor originates from the requirement that the total energy density be conserved.

Let us also discuss how the initial conditions are chosen. Initially, we start with thermal equilibrium and some baryons/anti-baryons asymmetry. This implies that

$$\Omega_b, \Omega_{\bar{b}} = \Omega_b^{(\text{eq})} \Omega_{\bar{b}}^{(\text{eq})}, \quad \Omega_b - \Omega_{\bar{b}} = \delta.$$  \hfill (9)

These two relations lead to

$$\Omega_b = \frac{\delta}{2} \left( 1 + \sqrt{1 + \frac{4\Gamma_{\bar{b}} \Omega_{\bar{b}} \Omega_{\bar{b}}^{(\text{eq})} \Omega_{\bar{b}}^{(\text{eq})}}{\delta^2} \right), \quad \Omega_{\bar{b}} = \frac{\delta}{2} \left( -1 + \sqrt{1 + \frac{4\Gamma_b \Omega_b \Omega_{\bar{b}}^{(\text{eq})} \Omega_{\bar{b}}^{(\text{eq})}}{\delta^2} \right).$$  \hfill (10)

Therefore, if the initial values of $\Omega_b^{(\text{eq})}$ and $\delta$ are known, one can deduce the initial values of $\Omega_b$ and $\Omega_{\bar{b}}$. The quantities $\Omega_b^{(\text{eq})}$ and $\delta$ can be expressed as

$$\Omega_b^{(\text{eq})} = \frac{g}{(2\pi)^{3/2}} \frac{8\pi}{3H^2 m_{Pl}^2} x^{-3/2} e^{-x}, \quad \delta = \frac{8\pi \zeta(3) g m_{\mu_b}^4 \epsilon_b}{3H^2 m_{Pl}^2 \pi^2 x^3},$$  \hfill (11)

where $x \equiv m_b/T$ and where the quantity $\epsilon_b$ is defined by

$$\epsilon_b \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma}.$$  \hfill (12)

The present-day value of the baryon asymmetry is $\epsilon_b \simeq 5.4 \times 10^{-10}$ [29]. The initial value of $\epsilon_b$ well before the freeze-out of $b\bar{b}$ annihilations must therefore be tuned in order to reproduce the final value after curvaton decay and entropy transfer from $e^+e^-$ to the photons. Curvaton decay may dilute any pre-existing asymmetry through entropy production or even produce net baryon number if the curvaton decay process violates baryon number. In all our calculations presented further below, we have tuned this initial asymmetry in order to match the observed present-day value.

In order to establish Eq. (12), we have used the fact that the number of photons is given by $n_\gamma = \zeta(3) g T^3 / \pi^2$. It is important to notice that the difference $n_b - n_{\bar{b}}$ is normalized with respect to the photon energy density (or number) and not to the total radiation energy density. In these formulas, $x = x_{\text{ini}} \sim 10$ (for instance) and $m_b \sim 0.9\text{GeV}$ are known (or chosen). Moreover, the Hubble parameter and $\Omega_{\gamma}$ are related through Eq. (8). Then, using the fact that the space-like sections are flat, i.e. $\Omega_{\sigma} + \Omega_{\chi} + \Omega_{\bar{b}} + \Omega_{\bar{b}} = 1$, and considering the (initial value of) $\Omega_{\sigma,\text{ini}}$ as a free quantity, one can derive the following expression

$$H^2 = \frac{1}{1 - \Omega_{\sigma}} \frac{8\pi^2 g_* m_{\mu_b}^4}{90 x^3 m_{Pl}^2} \left[ 1 \pm \frac{60 g_* \epsilon_b^2 e^{-x}}{\pi^3 (2\pi)^{3/2} g_* \sqrt{1 + \frac{2\zeta(3) \epsilon_b^2 e^{-2x}}{\pi^2 x^3}}} \right].$$  \hfill (13)

Therefore, for a given value of $\Omega_{\sigma,\text{ini}}$, $H_{\text{ini}}$ can be computed and the other quantities $\Omega_{\gamma,\text{ini}}$, $\Omega_{\bar{b},\text{ini}}$ and $\Omega_{\bar{b},\text{ini}}$, simply follow from the above equations.

Let us now consider the perturbations. In order to establish the gauge-invariant equations of motion, we follow the method of Ref. [25]. It consists in formulating the equations in a covariant way in order to be able to perturb them consistently. One can write

$$\nabla_\mu T^\mu_{\nu(\alpha)} = Q_{\nu(\alpha)} + Y_{\nu(\alpha)}$$  \hfill (14)
where \( Q^\mu = \Gamma T^{\mu\nu} u_\nu \) is the curvaton decay term, \( u_\nu \) being the four velocity of a fundamental observer, and the term \( Y^\mu \) is a phenomenological description of the interaction term. It reads
\[
Y^\mu = \frac{\langle \sigma b \rangle v}{m_b} \left[ T^{\mu\lambda(b)} - T^{\mu,eq\lambda(b)} \right] u_\beta. \tag{15}
\]

Of course, a rigorous treatment of the problem would rely on the full Boltzmann equation but this phenomenological description will be sufficient for our purpose. In particular, one can check that Eq. \((14)\) exactly reproduces the background equations \((2)-(6)\). Moreover, it is straightforward to perturb Eq. \((14)\). This leads to the following system
\[
\frac{d\Delta_b}{dN} = -\frac{\Gamma_{ab}}{H\Omega_b} (\Delta_b - \Delta_\sigma) - \frac{3}{2} \left( \Omega_\sigma \Delta_\sigma + \Omega_\pi \Delta_\pi + \Omega_b \Delta_b + \Omega_\delta \Delta_\delta \right) - \Phi \left( 3 - \frac{\Gamma_{ab}}{H\Omega_b} \right)
\]
\[
- \frac{3 \langle \sigma_b \rangle v m^2_{Pl}}{8\pi} \frac{H}{m_b\Omega_b} \left[ \Omega_b\Omega_b (\Delta_b - \Delta_\delta) - 2\Omega_{eq}^b\Omega_{eq}^b \Delta_{eq}^b \right] + \Phi \left( 3 - \frac{\Gamma_{ab}}{H\Omega_b} \right) \left( \Omega_b\Omega_b - \Omega_{eq}^b\Omega_{eq}^b \right), \tag{16}
\]
\[
\frac{d\Delta_\delta}{dN} = -\frac{\Gamma_{ab}}{H\Omega_b} (\Delta_\delta - \Delta_\sigma) - \frac{3}{2} \left( \Omega_\sigma \Delta_\sigma + \Omega_\pi \Delta_\pi + \Omega_b \Delta_b + \Omega_\delta \Delta_\delta \right) - \Phi \left( 3 - \frac{\Gamma_{ab}}{H\Omega_b} \right)
\]
\[
- \frac{3 \langle \sigma_b \rangle v m^2_{Pl}}{8\pi} \frac{H}{m_b\Omega_b} \left[ \Omega_b\Omega_b (\Delta_b - \Delta_\delta) - 2\Omega_{eq}^b\Omega_{eq}^b \Delta_{eq}^b \right] + \Phi \left( 3 - \frac{\Gamma_{ab}}{H\Omega_b} \right) \left( \Omega_b\Omega_b - \Omega_{eq}^b\Omega_{eq}^b \right), \tag{17}
\]
\[
\frac{d\Delta_r}{dN} = -\frac{\Gamma_{ab}}{H\Omega_r} (\Delta_r - \Delta_\sigma) - 2 \left( \Omega_\sigma \Delta_\sigma + \Omega_\pi \Delta_\pi + \Omega_b \Delta_b + \Omega_\delta \Delta_\delta \right) - \Phi \left( 4 - \frac{\Gamma_{ab}}{H\Omega_r} \right)
\]
\[
+ \frac{3 \langle \sigma_b \rangle v m^2_{Pl}}{8\pi} \frac{H}{m_b\Omega_r} \left[ \Omega_b\Omega_b (\Delta_b + \Delta_\delta) - 2\Omega_{eq}^b\Omega_{eq}^b \Delta_{eq}^b \right] + \Phi \left( 3 + \frac{\Gamma_{ab}}{H\Omega_r} \right) \left( \Omega_b\Omega_b - \Omega_{eq}^b\Omega_{eq}^b \right), \tag{18}
\]
\[
\frac{d\Delta_\pi}{dN} = -\frac{3}{2} \left( \Omega_\sigma \Delta_\sigma + \Omega_\pi \Delta_\pi + \Omega_b \Delta_b + \Omega_\delta \Delta_\delta \right) - \Phi \left( 3 + \frac{\Gamma_{ab}}{H} \right), \tag{19}
\]
\[
\frac{d\Phi}{dN} = -\Phi - \frac{1}{2} \left( \Omega_\sigma \Delta_\sigma + \Omega_\pi \Delta_\pi + \Omega_b \Delta_b + \Omega_\delta \Delta_\delta \right), \tag{20}
\]

where \( \Delta_\alpha \equiv \Delta \rho_\alpha / \rho_\alpha \) is the gauge-invariant density contrast for the fluid \( \alpha \). The quantity \( \Delta_{eq}^b \) is defined by the following expression
\[
\Delta_{eq}^b = \frac{1}{4} \left( \frac{3}{2} + x \right) \Delta_r. \tag{21}
\]

With this definition, it is easy to see that we deal with a "closed" system of equations since \( x \) must be viewed as a function of \( H \) and \( \Omega_r \), see Eq. \((3)\). Let us now turn to the discussion of the solutions of the two systems of equations presented in this section.

III. RESULTS

The main parameters that govern the cosmological consequences of curvaton decay into radiation and baryon/antibaryons are: \((i)\) the time of decay of the curvaton, which is encoded in the total decay width \( \Gamma_\sigma \), \((ii)\) the respective branching ratios \( \Gamma_{ab} / \Gamma_\sigma \) and \( \Gamma_{ab} / \Gamma_\sigma \); \((iii)\) the magnitude of the curvaton energy density at the time of decay, i.e. \( \Omega_{eq}^b \) when \( H = \Gamma_\sigma \). The main parameters are therefore the respective branching ratios and \( \Omega_{eq}^b \). Note that the branching ratios are constrained by the measured baryon asymmetry \( \epsilon_b \simeq 5.4 \times 10^{-10} \). In particular the baryon asymmetry \( \epsilon_b \) measured immediately after curvaton decay should be equal to \( 1.5 \times 10^{-9} \), in order to obtain the measured value after the reheating of the photon fluid by electron/positron annihilations. According to whether \( \Gamma_{ab} = \Gamma_{ab} \) or not, two possibilities may arise. In the case of symmetric decay, meaning \( \Gamma_{ab} = \Gamma_{ab} \), the baryon asymmetry is generated by some unspecified mechanism acting at a higher energy scale; it is simply diluted during curvaton decay by the extra entropy brought by the curvaton. In the case of asymmetric decay, the curvaton contributes to the net baryon asymmetry. We note that direct baryogenesis at a low temperature \( T \sim 10 \text{ MeV} \) is very contrived; we will nevertheless study this case for the sake of completeness and discuss the robustness of the results for higher decay temperatures. These two scenarios indeed exhibit different consequences, as discussed in turn in the following.

We will use the standard definition of the curvature perturbation in fluid \( \alpha \) \((30, 31, 32)\):
\[
\zeta_{(\alpha)} \equiv -\Phi - H \frac{\Delta \rho_{(\alpha)}}{\rho_{(\alpha)}} \simeq -\Phi + \frac{\Delta_{(\alpha)}}{3 \left[ 1 + \omega_{(\alpha)} \right]}, \tag{22}
\]
FIG. 1: Evolution of the background and perturbed quantities in the case in which the curvaton decays symmetrically into baryons and anti-baryons: $\Gamma_{\sigma b} = \Gamma_{\sigma \bar{b}} = 5 \times 10^{-4} \Gamma_{\sigma}$. The total decay width is $\Gamma_{\sigma} = 1.6 \times 10^{-20} \text{MeV}$, and $\Omega_{\sigma(0)} = 0.9$ at a temperature $T = 94 \text{MeV}$. The top panel shows the evolution of $\Omega_r$ (solid red line), $\Omega_{\sigma}$ (dotted green line), $\Omega_b$ (upper dashed dark blue line) and $\Omega_{\bar{b}}$ (lower dashed light blue line). The middle panel shows the evolution of $\zeta_r$ (solid red line) and $\zeta_b$ (dashed blue line). The bottom panel shows the evolution of the isocurvature transfer coefficient $S_{br}/S_{\sigma r}^{(i)}$. The transfer of isocurvature perturbation is maximal as the radiation but not the baryon fluid inherits the curvaton perturbation.

where, in order to express $\dot{\rho}_{(\alpha)}$, we have not considered the interaction term. The corresponding definitions for the isocurvature modes read

$$S_{br} \equiv 3 (\zeta_b - \zeta_r), \quad S_{br} \equiv 3 (\zeta_{\bar{b}} - \zeta_r), \quad S_{\sigma r} \equiv 3 (\zeta_{\sigma} - \zeta_r).$$

In particular, we will be interested in the transfer of the initial curvaton/radiation isocurvature perturbation into the final baryon/radiation isocurvature mode, as expressed by the ratio $S_{br}^{(i)}/S_{\sigma r}^{(i)}$. The quantity indexed with (f) [resp. (i)] is evaluated well after the decay (resp. well before). In the following, we also express quantities evaluated immediately before (resp. after) decay with the superscript $<d$ (resp. $>d$).

A. Symmetric decay

In this sub-section, we explore the phenomenology of models in which the curvaton decays symmetrically into baryons and anti-baryons, i.e. $\Gamma_{\sigma b} = \Gamma_{\sigma \bar{b}}$. We find that two situations may arise, according to whether the curvaton dominates the energy density at its decay, i.e. $\Omega_{\sigma d} \sim 1$, or not.

Consider first the case in which the curvaton dominates the energy density at decay, $\Omega_{\sigma d} \sim 1$. We find that the transfer coefficient of the isocurvature mode is maximal, as exemplified for instance in Fig. 1. The top panel of this figure shows the evolution of the background energy density in radiation (solid red line), curvaton (dotted green line), in baryons (top dashed dark blue line) and in anti-baryons (bottom dashed light blue line). The middle panel shows the evolution of the individual $\zeta_{(\alpha)}$ quantities, and the bottom panel the transfer of the isocurvature fluctuation. This latter shows clearly the emergence of a net baryon-radiation isocurvature fluctuation after the decay of the curvaton. The middle panel also reveals that in this case, the radiation fluid inherits the curvaton fluctuations (since $\zeta_{\gamma}/\zeta_{\gamma}^{(i)} \rightarrow 1$ at $T \ll T_d$), while the baryon fluid remains unaffected.

This result is quite different from a “standard” scenario of curvaton decay into radiation and dark matter, in which the domination of the curvaton at the time of decay ensures that only adiabatic modes subsist, as all fluids have inherited the same curvaton perturbations. This difference can be related to the annihilation of all $b\bar{b}$ pairs produced
by the curvaton, which effectively reduces to zero the net energy transfer of the curvaton to the baryon fluid. In order to put this statement on quantitative footing, it is useful to evaluate the ratio of the annihilation rates of baryons and anti-baryons to the expansion rate immediately after curvaton decay \( H = \Gamma_\sigma \):

\[
\begin{align*}
\Upsilon_b &= \frac{n_b^{\zeta_d} \langle \sigma v \rangle}{\Gamma_\sigma} \simeq 4.6 \times 10^{16} \left( \frac{T_d}{10 \text{ MeV}} \right)^2 \Omega_b^{\zeta_d}, \\
\Upsilon_{\bar{b}} &= \frac{n_{\bar{b}}^{\zeta_d} \langle \sigma v \rangle}{\Gamma_\sigma} \simeq 4.6 \times 10^{16} \left( \frac{T_d}{10 \text{ MeV}} \right)^2 \Omega_{\bar{b}}^{\zeta_d}.
\end{align*}
\]

The first equation gives the ratio \( \Upsilon_b \) of the annihilation rate of baryons to the expansion rate, while the second gives the corresponding ratio \( \Upsilon_{\bar{b}} \) of the annihilation rate of anti-baryons to the expansion rate. Considering \( \Upsilon_b \), the above formula shows that if \( \Omega_b \) exceeds \( \sim 10^{-16} \), annihilations are effective. In the absence of curvaton, the freeze-out of bb annihilations occurs as the abundance of anti-baryons is reduced to below this threshold. In the presence of a curvaton however, the decay of this field will regenerate the annihilations provided the amount of curvaton produced anti-baryons is sufficient, i.e. \( \Omega_{\bar{b}} \) has \( \Omega_{\bar{b}} > \Omega_b \). This composite fluid is isolated, as neither annihilation nor curvaton decay violate baryon number. Therefore, the energy density before decay, and transfers its energy to radiation, then:

\[\zeta_f = \zeta_i.\]

The theorem of Ref. [33] stipulates that the isocurvature mode between two fluids sharing thermal equilibrium are erased on a small timescale, unless there exists a conserved charge. In the present case, baryon number is conserved, or more precisely net baryon number does not couple to radiation, hence the above theorem does not apply. Consequently, once the isocurvature mode is produced, it remains conserved unless baryon number violating processes take place. In other words, one can extrapolate the above results to temperatures at least as high as the electroweak scale. Let us also remark that, if baryons are relativistic (at temperatures above the QCD scale), the above equations are slightly modified, but the above results remain unmodified.

One a more formal level, one can follow the evolution of the different variables as follows. Neglecting \( \Omega_{\bar{b}} \) in front of \( \Omega_b \) in Eq. \( (25) \) above indicates that \( \Omega_b \) scales as \( a \) when \( \Omega_b \simeq 1 \) (i.e. after curvaton decay), while \( \Omega_{\bar{b}} \) remains approximately constant when \( \Omega_{\bar{b}} \lesssim 1 \). These trends are observed in Fig. 1.

The behavior of \( \Omega_{\bar{b}} \) is less trivial to obtain (but its cosmological relevance is also much less). One can approximate Eq. \( (3) \) with the following, after curvaton decay:

\[
\frac{d\Omega_{\bar{b}}}{dN} \simeq -\frac{3m_b^2 \langle \sigma v \rangle H}{8\pi m_b} \Omega_b \Omega_{\bar{b}},
\]

where the term \( \Omega_b \Omega_{\bar{b}} \) has been neglected as the annihilations are dominant. Using the fact that \( H \propto a^{-2} \) and \( \Omega_b \propto a \) after curvaton decay, one derives the following late time value of \( \Omega_{\bar{b}} \):

\[
\Omega_{\bar{b}}^{(f)} \simeq \Omega_{\bar{b}}^{(i)} e^{-\Upsilon_{\bar{b}}^{\zeta_d}}.
\]

Since \( \Upsilon_{\bar{b}}^{\zeta_d} \) takes enormous values of order \( 10^9 \), the annihilations regenerated by curvaton decay essentially erase all trace of anti-baryons and the corresponding plateau cannot be observed in Fig. 1 because it is too small.

Let us now turn to the perturbations and assume that curvaton decay is instantaneous. If the curvaton dominates the energy density before decay, and transfers its energy to radiation, then:

\[\zeta_\tau^{(f)} \simeq \zeta_\tau^{(i)}.\]

The behaviors of the different quantities plotted in Fig. 1 can be understood in more detail along the following lines. Consider the variables associated to net baryon number, in particular \( \Omega_b - \Omega_{\bar{b}} \). Its equation of motion reduces to:

\[
\frac{d}{dN} (\Omega_b - \Omega_{\bar{b}}) = \Omega_\tau (\Omega_b - \Omega_{\bar{b}}).
\]
FIG. 2: Same as figure 1 except that $\Omega^{(i)}(i) = 0.01$, which corresponds to $\Omega^{< d}_{\sigma} = 0.09$. Essentially no isocurvature fluctuation is produced as neither the baryon nor the radiation fluctuations have been affected by curvaton decay.

This relation can be obtained through standard methods and corresponds to the conservation of the total curvature perturbation throughout curvaton decay. Similarly, one can build the variable associated to the perturbation of net baryon number, $\Omega_b \Delta_b - \Omega_{\bar{b}} \Delta_{\bar{b}}$, which for all practical purposes, can be approximated by $\Omega_b \Delta_b$. The equation of motion for this quantity reads:

$$\frac{d}{dN} (\Omega_b \Delta_b - \Omega_{\bar{b}} \Delta_{\bar{b}}) = 3 \frac{d\Phi}{dN} (\Omega_b - \Omega_{\bar{b}}) + \Omega_r (\Omega_b \Delta_b - \Omega_{\bar{b}} \Delta_{\bar{b}}) .$$

(29)

Since $\Phi$ is conserved both before and after curvaton decay, the first term on the r.h.s. can be neglected, and $\Omega_b \Delta_b - \Omega_{\bar{b}} \Delta_{\bar{b}}$ is approximately conserved when $\Omega_r \ll 1$. Approximating $\Omega_b \Delta_b - \Omega_{\bar{b}} \Delta_{\bar{b}}$ with $\Omega_b \Delta_b$, this implies that $\Delta_b$ is approximately conserved, since $\Omega_b$ is constant in this case (see before) and, hence, that $\zeta_b$ is also conserved. At late times, after curvaton decay, $\Omega_r \sim 1$ implies that $\Omega_b \Delta_b$ scales as $a$, hence that $\Delta_b$ (and therefore $\zeta_b$) is again approximately constant because $\Omega_b \propto a$. One thus finds that:

$$\zeta_b^{(f)} \sim \zeta_b^{(i)} .$$

(30)

As mentioned above, this property can be traced back to the fact that net baryon number behaves in the present case as an isolated fluid, hence its curvature perturbation is a conserved quantity. Finally, one derives from Eq. (28) and (30) above the transfer of isocurvature perturbation:

$$S_{br}^{(f)} \sim -S_{\sigma r}^{(i)} \quad (\Omega^{< d}_{\sigma} \simeq 1) .$$

(31)

These results match the numerical evolution observed in Fig. 1.

Obviously, the above discussion suggests that $S_{br}^{(f)} \to 0$ as $\Omega^{< d}_{\sigma} \to 0$ since the net baryon number must remain unaffected, while a decreasing curvaton energy density at the time of decay implies that a lesser amount of radiation is produced during the decay. In more detail, one should obtain

$$S_{br}^{(f)} \sim -\Omega^{< d}_{\sigma} S_{\sigma r}^{(i)} ,$$

(32)

since

$$\zeta_r^{(f)} \simeq (1 - \Omega^{< d}_{\sigma}) \zeta_r^{(i)} + \Omega^{< d}_{\sigma} S_{\sigma r}^{(i)} .$$

(33)

This trend is confirmed in Fig. 2 which provides an example with $\Omega^{(i)}_{\sigma} = 0.01$, corresponding to $\Omega^{< d}_{\sigma} \simeq 0.09$ at decay ($T_d \simeq 5.9 \text{ MeV}$). The final transfer coefficient is of order $-\Omega^{< d}_{\sigma}$ as expected.
B. Asymmetric decay

If the inflaton can decay asymmetrically, $\Gamma_{\sigma b} \neq \Gamma_{\sigma \bar{b}}$, the phenomenology is different, as all “$b$” and “$\bar{b}$” produced by the curvaton will not be able to annihilate with each other. In particular, the production of net baryon number during curvaton decay comes with the transfer of the curvaton perturbations to the baryon fluid. As already mentioned, known models of baryogenesis produce baryon number at a much higher scale than $1 - 10$ MeV. We nevertheless discuss this asymmetric case for the sake of completeness and because it provides useful insights into curvaton cosmology. Moreover, as we have argued in the previous section, the present results can be extrapolated to a higher decay temperature, possibly as high as the electroweak scale.

In the present case, one may expect cosmological consequences opposite to those found in the case of symmetric decay: if the curvaton dominates the energy density of the Universe shortly before decaying, and produces during its decay most of the baryon number, both baryon and radiation fluid will inherit its perturbations, hence there should be no final baryon/radiation isocurvature mode. On the contrary, if the curvaton energy density is small compared to the radiation density shortly before decay, but the curvaton still produces most of the baryon number, a maximal isocurvature mode between baryon and radiation should be produced.

These trends are confirmed by the numerical computations, as shown in Figs. 3 and 4. The first figure, Fig. 3 corresponds to the same value of $\Omega_\sigma^{(i)}$ as in Fig. 1 but with an asymmetric decay width $(\Gamma_{\sigma b} - \Gamma_{\sigma \bar{b}})/\Gamma_\sigma = 2 \times 10^{-8}$. In what follows, we will use the short-hand notation:

$$\Delta B_{bb} \equiv \frac{\Gamma_{\sigma b} - \Gamma_{\sigma \bar{b}}}{\Gamma_\sigma}.$$  

Assuming that the initial baryon asymmetry vanishes and that curvaton decay is instantaneous, one can obtain an order of magnitude of the decay asymmetry needed to reach the observed value of $\epsilon_b$ as follows:

$$\Omega_b^{\geq d} - \Omega_{\bar{b}}^{\geq d} \approx \Delta B_{bb} \Omega_\sigma^{\leq d},$$  

which implies:

$$\epsilon_b \approx 7.3 \times 10^{-2} \left(\frac{\Gamma_\sigma}{10^{20} \text{MeV}}\right)^{1/2} \Delta B_{bb} \Omega_\sigma^{\leq d}.$$  

Numerical calculations differ from this simple estimate by a factor of order unity.

In order to understand these results, it is instructive to express the time evolution of the baryon asymmetry using the system of Eqs. (2), (3), (4) and (5). The baryon asymmetry can indeed be written as:

$$\epsilon_b \equiv \frac{\pi^4}{60 \zeta(3)} \left(\frac{45}{4 \pi^4}\right)^{1/4} g^{3/4}_* \left(\frac{m_{\rho}}{m_b}\right)^{1/2} \left(\frac{H}{m_b}\right)^{1/2} \frac{\Omega_b - \Omega_{\bar{b}}}{\Omega_{\sigma}^{3/4}} \simeq 2.24 \times 10^{10} \left(\frac{H}{m_b}\right)^{1/2} \frac{\Omega_b - \Omega_{\bar{b}}}{\Omega_{\sigma}^{3/4}}.$$  

Hence the time evolution of the baryon asymmetry is governed by the following equation:

$$\frac{1}{\epsilon_b} \frac{d\epsilon_b}{dN} \approx \frac{\Omega_\sigma}{\Omega_b - \Omega_{\bar{b}}} \frac{\Gamma_{\sigma b} - \Gamma_{\sigma \bar{b}}}{H} - \frac{3 \Omega_\sigma \Gamma_\sigma}{4 \Omega_r} \frac{\Gamma_{\sigma r}}{H}.$$  

In order to obtain the above equation, we have neglected the baryon/anti-baryon annihilation term in the equation for $\Omega_\sigma$ [Eq. (4)], which is justified insofar as the amount of radiation produced in baryon/anti-baryon annihilations is negligible at or after freeze-out.

The above equation is interesting because it shows how the baryon number can be modified: either through baryon number violating curvaton decay (first term on the r.h.s), or through dilution due to entropy production (second term on the r.h.s). It also provides an estimate of the conditions under which the initial curvaton/radiation isocurvature mode is efficiently transfered to the baryon/radiation mixture. Such an efficient transfer can indeed be achieved if $|\Delta \epsilon_b/\epsilon_b| \approx 1$ at curvaton decay, without significant production of radiation by the curvaton. The latter condition amounts to negligible entropy production, or what is equivalent, to assuming that the second term on the r.h.s of Eq. (38) is negligible compared to unity. The former condition then implies that the first term on the r.h.s of Eq. (38) is larger than unity. All in all, efficient transfer of the isocurvature mode occurs if:

$$\frac{\Delta B_{bb} \Omega_\sigma^{\leq d}}{\Omega_b^{\geq d} - \Omega_{\bar{b}}^{\geq d}} \simeq \frac{\Gamma_{\sigma b} \Omega_\sigma^{\leq d}}{\Gamma_\sigma} \lesssim \Omega_r^{\leq d}.$$  

It is interesting to remark that this situation is very similar to that encountered for curvaton decay in a three-fluid model incorporating radiation and dark matter. Borrowing from the method of Refs. [23, 25], it is possible to express
FIG. 3: Same as figure 1 except that curvaton decay now violates baryon number, with $\Gamma_{\sigma b} - \Gamma_{\bar{\sigma} \bar{b}} = 1.7 \times 10^{-8} \Gamma_{\sigma}$. The final baryon number matches the observed value, for an initial asymmetry $\epsilon_b = 10^{-13}$. Other quantities remain unchanged, in particular $\Omega^{(i)}_{\sigma} = 0.9$ and $\Gamma_{\sigma} = 1.6 \times 10^{-20}$ MeV. Essentially no baryon/radiation isocurvature fluctuation results, since the baryon and the radiation fluctuations have been similarly affected by curvaton decay.

FIG. 4: Same as figure 3 for baryon violating curvaton decay, except that $\Omega^{(i)}_{\sigma} = 0.01$, which corresponds to $\Omega^{s, d}_{\sigma} = 0.09$. The baryon violating decay width is such that the final baryon number produced matches the observed value; this corresponds to $\Delta B_{b\bar{b}} = 1.3 \times 10^{-7}$ for an initial asymmetry $\epsilon_b = 10^{-13}$. A large isocurvature fluctuation is produced as the baryon (but not the radiation) fluctuations have been affected by curvaton decay.
the final baryon/radiation isocurvature fluctuation in terms of the initial curvaton and radiation curvature modes, as follows. One first constructs a composite fluid that has the property of being isolated, with energy density:

$$\rho_{\text{comp}} = \rho_b - \rho_b + \Delta B_{bb} \rho_\sigma .$$  \hspace{1cm} (40)

Notice that this construction is possible because each component of the composite fluid is pressureless. Its curvature perturbation, which is conserved by construction, is:

$$\zeta_{\text{comp}} = \frac{\Omega_b}{\Omega_b - \Omega_b + \Delta B_{bb} \Omega_\sigma} \zeta_b^b - \frac{\Omega_b}{\Omega_b - \Omega_b + \Delta B_{bb} \Omega_\sigma} \zeta_b^b + \frac{\Delta B_{bb} \Omega_\sigma}{\Omega_b - \Omega_b + \Delta B_{bb} \Omega_\sigma} \zeta_\sigma .$$  \hspace{1cm} (41)

Then, assuming that curvaton decay is instantaneous, one can match the value of $$\zeta_{\text{comp}}$$ after decay to that before decay, which gives:

$$\zeta_b^d \approx \zeta_{\text{comp}}^d = \zeta_{\text{comp}}^d .$$  \hspace{1cm} (42)

In order to obtain the first equality, we have used the fact that $$\Omega_b^d \ll \Omega_b^d$$ as a result of the efficient annihilation of $$bb$$ pairs after curvaton decay. Although the quantity $$\zeta_{\text{comp}}^d$$ is evaluated here immediately before decay, it can be evaluated at any initial time, since it is conserved.

The radiation perturbation is given by Eq. (33), hence the final baryon/radiation isocurvature perturbation can be written as:

$$S_{\text{br}}^{(f)} = \left[ \frac{\Delta B_{bb} \Omega_\sigma^{(f)}}{\Omega_b^{(i)} - \Omega_b^{(i)} + \Delta B_{bb} \Omega_\sigma^{(i)}} - \Omega_\sigma^{(i)} \right] S_{\text{br}}^{(i)} ,$$  \hspace{1cm} (43)

where we used the fact that $$S_{\text{br}}^{(i)} = S_{\text{br}}^{(i)} = 0$$. As expected, the isocurvature transfer vanishes as $$\Omega_\sigma^{(i)} \to 0$$ (since this also implies $$\Omega_b^{(i)} \to 0$$). When $$\Omega_\sigma^{(i)} \to 1$$, one can see that the first term in the bracket on the r.h.s. of the above equation also tends to one, and therefore the transfer coefficient of the isocurvature mode also vanishes. The initial isocurvature fraction is transferred efficiently only if the conditions expressed in Eq. (39) are fulfilled. Note also that in the limit $$\Delta B_{bb} \to 0$$, one recovers the result of Section III A presented in Eq. (32).

Finally, a last point is to be made concerning the assumption of instantaneous thermalization of the curvaton decay products. If the center of mass energy $$\sqrt{s} \sim (E_{\text{th}})^{1/2}$$ for an interaction between a high energy particle of energy $$E$$ and a thermalized particle of energy $$E_{\text{th}}$$ is well above the QCD scale, then the ratio of rates of thermalization processes to $$bb$$ producing ones is of the order of $$(\alpha_{\text{em}}/\alpha_s)^2 \ll 1$$. It is even less if $$\sqrt{s}$$ is smaller than the QCD scale. Therefore the above approximation is not strictly speaking justified. However, the neglect of these additional interactions would not modify our conclusions, for the following reason.

The only effect that could modify our conclusions is if one fluid (either radiation or baryon) were “contaminated” by the other fluid (respectively baryon or radiation) through the interaction of high energy particles produced through curvaton decay with thermalized particles. One typical example is given by the transfer of energy from the photon to the baryon fluid through $$\gamma + \gamma_{\text{th}} \rightarrow b + \bar{b}$$, where $$\gamma$$ stands for a high energy photon. However, net baryon number does not couple to radiation, hence transfers of energy between these two fluids cannot take place after curvaton decay (provided this latter occurs after any baryogenesis event).

Hence all the conclusions remain unaffected by these processes that occur between curvaton decay and thermalization. It is important to stress, however, that $$\Gamma_{\sigma_1}/\Gamma_\sigma$$, $$\Gamma_{\sigma_2}/\Gamma_\sigma$$ and $$\Gamma_{\sigma_3}/\Gamma_\sigma$$ should not be interpreted strictly speaking as the branching ratios of curvaton decay into radiation, baryons or anti-baryons, but rather as the fraction of curvaton energy eventually transferred into these fluids after all thermalization processes have occurred.

**IV. CONCLUSIONS**

In this section, we recap our main results. We have studied the production of isocurvature perturbations in the curvaton scenario where the curvaton field can decay into radiation, baryons and anti-baryons. Two different cases have been considered. The first one is the symmetric case in which the curvaton/baryon decay width equals the curvaton/anti-baryon one, i.e. curvaton decay preserves baryon number. We have found that if the curvaton dominates the energy density before decay, then a baryon/radiation isocurvature mode can be produced. In the opposite situation in which the curvaton contributes negligibly to the total energy density immediately before decaying, the isocurvature mode vanishes. This result is opposite to the standard prediction of the simplest curvaton scenario in which any pre-existing isocurvature mode is erased by curvaton decay if this latter dominates the energy density at the time of
decay. This difference can be traced back to the conservation of baryon number and to the annihilation of all $b\bar{b}$ pairs produced during curvaton decay.

One noteworthy consequence of the above is to forbid the liberation of a significant amount of entropy by a late decaying scalar field at temperatures below any baryon violating processes, such as is often invoked for the dilution of unwanted relics.

Another consequence of the above is that a baryon-radiation isocurvature mode $S_{br}$ cannot co-exist with a (WIMP) dark matter - radiation isocurvature mode $S_{xr}$, since the conditions to produce these modes are opposite to one another. Since $S_{xb} = S_{xr} - S_{br}$, the existence of a baryon-dark matter isocurvature mode appears generic in this case (unless $S_{x} > S_{br}$ is so small at the time of decay that the curvaton exerts essentially no influence on dark matter and baryon perturbations).

The asymmetric decay presents a different phenomenology. Since the curvaton decay does not produce the same number of baryons and anti-baryons, the annihilations cannot suppress all the baryonic decay product and, as a consequence, when the curvaton dominates at decay, the isocurvature perturbations are erased. In this case, most or all of the baryon and radiation fluctuations indeed originate from the curvaton. If the curvaton contribution to the energy density is smaller than unity at the time of decay, then radiation cannot be affected substantially, while the baryon fluid may be strongly affected; this situation results in a large baryon/radiation isocurvature fluctuation. In some sense, this case appears similar to the case of curvaton to dark matter decay studied in Ref. [25]. Contrary to the previous symmetric case, non vanishing $S_{br}$ and $S_{xr}$ can co-exist. We note however, that baryogenesis at low scales (below the electroweak phase transition) is rather contrived.

On more general grounds, the study presented in this article exemplifies how scenarios where scalar fields can decay at late times can be constrained not only at the background level, as it is usually done, but also by investigating the consequences at the perturbed level. It is clear that, if this type of information is taken into account, one can hope to improve our understanding of the feasibility of such theories. We hope to return to this question in future publications.

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