Kinematics modeling and workspace analysis of large offshore trestle based on wave compensation

Xingyao Wang¹, Yifan Xue¹, Yanjun Liu², *

¹School of Institute of Marine Science and Technology, Shandong University, Qingdao, China
²School of Mechanical Engineering, Shandong University, Jinan, China

*Corresponding author: lyj111@sdu.edu.cn

Abstract. Referring to the modeling method of the series manipulator, the D-H parameter method is used to model the kinematics of the large-scale offshore trestle, and the forward kinematics equation of the offshore trestle is calculated, and the algebraic method is used to solve the inverse kinematics. At the same time, using the robotics toolbox in Matlab to verify the feasibility of the D-H parameter method modeling and the correctness of the forward and inverse solutions of the kinematics equation, and using the Monte Carlo method to analyze the working space of the offshore trestle in Matlab, finally, the motion of the offshore trestle is wave compensated. It provides a theoretical basis for the kinematics modeling, workspace analysis and control system design of large-scale offshore trestle.

1. Introduction

China has put forward the strategic goal of speeding up the construction of a maritime power, vigorously supports the development of the maritime industry, and has made policy guidance and overall requirements for the Marine engineering equipment in the form of documents. The research on the Marine engineering equipment will also become a key strategic technology for the national development. With the continuous increase of maritime operations, people's demand for maritime transfer is also increasing [1]. The traditional sea transfer technology has strict requirements on sea conditions and can only be transferred in a relatively stable environment, so its application scope is limited. Due to the ship will be affected by waves and sea breeze, when working in the sea, there will be horizontal movement of swing, swell and vertical swing, as well as roll, pitch and bow rotation movement. Traditional transfer device may collide with the ship and other dangerous actions, resulting in poor safety and stability. Therefore put forward the new sea transfer technique (large Marine pier), large Marine engineering trestle is set floating body ocean splicing, wave compensation function in the integration of large Marine engineering equipment, which can realize complex sea condition of personnel and goods supply operation, widely used in large offshore platforms, sea fan ship maintenance, supply vessels, etc. In complex sea conditions, it is necessary to overcome the disturbance of waves and sea breeze, and successively complete the floating body bridging, wave compensation and floating body separation operations.

Large-scale Marine trestle is a non-fixed terminal ship transfer device, and the swing, swell and heave movement caused by the Marine trestle following the ship are compensated by the active wave
compensation device. The active wave compensation system of large Marine trestle can control the hydraulic system to produce a motion opposite to the ship's motion in real time after detecting the ship's motion by the detection device, so as to ensure that the terminal of the trestle is stationary relative to the transfer point. It is mainly reflected in the rotation around the rotary device, the rotation around the pitching device and the translation movement of the telescopic device, so that the terminal position of the trestle is static relative to the offshore platform, so as to realize the smooth and safe transfer between the trestle and the offshore building.

Figure 1. Large offshore trestle bridge abroad.

The Marine trestle in this paper is mainly composed of four parts: base, rotary device, pitching device and expansion device. The slewing is attached to a base which is attached to the deck of the ship and is driven by gears.

Figure 2. Base and rotary device.

The pitching device is composed of two identical hydraulic cylinders fixedly connected between the base and the bridge, and the pitching movement of the Marine trestle is realized through the telescopic movement of the hydraulic rods.

Figure 3. Pitching device.
The telescopic device realizes the telescopic movement of the Marine trestle bridge through the rack movement.

![Figure 4. Telescopic device.](image)

The following picture is a general view of the pier:

![Figure 5. Overall view.](image)

2. Kinematics analysis of offshore trestle

2.1. Forward kinematics

Kinematics analysis mainly includes two aspects, forward kinematics analysis and inverse kinematics analysis [2]. Forward kinematics is the process of calculating the end-state variables of the manipulator based on the known arm parameters and the state information of each joint. There is a unique solution to the forward kinematics. The inverse kinematics is the state information of the end of the manipulator, and the motion state variables of each joint are obtained when the end of the manipulator reaches the state.

In 1995, Denavit and Hartenberg published a paper in "ASME Journal of Applied Mechanics". Later, they used this paper to represent and model robots, and derived their kinematics equations. Now, it has become a standard method of representing robots and modeling robot motion, D-H representation [3].

The manipulator is composed of a series of rods connected by rotating joints and sliding joints. Each pair of joints and members constitute a degree of freedom. The n-joint robot needs to establish n + 1 coordinate system, so the 3-DOF offshore trestle needs to establish 4 coordinate systems. Where, the coordinate system of the end of the manipulator is $O_{n+1}x_{n+1}y_{n+1}z_{n+1}$. In addition, the reference (base) coordinate system is $O_{0}x_{0}y_{0}z_{0}$, The coordinate system at the i joint is $O_{i}x_{i}y_{i}z_{i}$. The determination and establishment of each coordinate system shall be based on the following three principles:
Figure 6. Definition method of connecting rod coordinate system and D-H parameters.

(a) Axis $z_{i-1}$ of coordinate system $\{i-1\}$ is collinear with joint axis $\{i-1\}$, pointing to arbitrary;
(b) The $x_{i-1}$ axis of coordinate system $\{i-1\}$ coincides with the common perpendicular of connecting rod $\{i-1\}$, pointing from joint $\{i-1\}$ to joint $\{i\}$. When $a_{i-1}=0$, take $x_{i-1} = \pm z_i \times z_{i-1}$ [4];
(c) The $y_{i-1}$ axis of coordinate system $\{i-1\}$ is established according to the requirements of right-handed coordinate system.

D-H method represents the four parameters of the relationship between adjacent connecting rods:
(1) Angle $\theta$ between two connecting rods: joint Angle pointing from $x_{i-1}$ axis to $x_i$ axis by rotating around $z_i$ axis;
(2) Distance $d_i$ between two connecting rods: the distance from $x_{i-1}$ to $x_i$ along the $z_i$ axis;
(3) Connecting rod length $a_{i-1}$: the distance from $z_{i-1}$ to $z_i$ along axis $x_{i-1}$;
(4) Torsion Angle of connecting rod $\alpha_{i-1}$: the Angle from $z_{i-1}$ axis to $z_i$ axis around $x_{i-1}$ axis.

Figure 7. 3-DOF offshore trestle coordinate system.
Set up the coordinate system as shown in Figure 8, with the axis of rotary mechanism as Axis $z_0$, the direction is vertical and upward, set up the base coordinate system $O_{x_0y_0z_0}$, the origin is located at the intersection point of the rotary axis and the horizontal plane of the base. Also, the axis of the rotary mechanism is $z_1$, vertically upward, set up the coordinate system $O_{x_1y_1z_1}$, $x_1$ as the common perpendicular of $z_1$ and $z_2$, with the axis of the pitching mechanism as $z_2$, the direction is perpendicular to the rotation axis and vertically outward, set up the coordinate system $O_{x_2y_2z_2}$, $x_2$ as the common perpendicular of $z_2$ and $z_3$, take the direction of the telescopic mechanism as axis $z_3$, establish the coordinate system $O_{x_3y_3z_3}$, $x_3$ as the common perpendicular of $z_3$ and $z_4$, an end-effector coordinate system $O_{x_4y_4z_4}$ is established at the end of the telescopic mechanism, which is parallel to $O_{3x_3y_3z_3}$ along the telescopic direction.

### Table 1. Variable range of each joint.

| joint | variable | range         |
|-------|----------|---------------|
| 1     | $\theta_1$ | $[-10°~+195°]$ |
| 2     | $\theta_2$ | $[-113°~+63°]$ |
| 3     | $d_4$     | $[8.2m~23.2m]$ |

### Table 2. D-H parameter table of mechanical arm of offshore trestle.

| $i$ | $a_{i-1}$ | $\alpha_{i-1}$ | $\theta_i$ | $d_i$ |
|-----|-----------|----------------|------------|-------|
| 1   | 0         | 0              | $\theta_1$ | 5.062m|
| 2   | 0.6m      | $-90°$         | $\theta_2$ | 0     |
| 3   | 0         | $90°$          | 0          | 23.9m |
| 4   | 0         | 0              | 0          | $d_4$ |

Let the transformation matrix from coordinate system $\{i-1\}$ to coordinate system $\{i\}$ be $^{i-1}_iT$, that is, the coordinate representation of the origin pose of coordinate system $\{i\}$ in $\{i-1\}$. According to the "D-H" method, it can be further written as:

$$^{i-1}_iT = \begin{bmatrix}
C\theta_i & -S\theta_i & 0 & a_{i-1} \\
S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -d_i S\alpha_{i-1} \\
S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & d_i C\alpha_{i-1} \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(1)

In the formula, $S\theta_i$ is the abbreviation of $\sin(\theta_i)$; $C\theta_i$ is short for $\cos(\theta_i)$.

By substituting the parameters in the D-H parameter table of the offshore trestle manipulator into the above equation, the transformation matrix of the coordinate system $\{1\}$ relative to the base coordinate system $\{0\}$ is obtained as follows:

$$^0_1T = \begin{bmatrix}
C\theta_1 & -S\theta_1 & 0 & 0 \\
S\theta_1 & C\theta_1 & 0 & 0 \\
0 & 0 & 1 & d_1 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(2)

Similarly, the transformation matrices of coordinate system $\{2\}$ relative to the coordinate system $\{1\}$, coordinate system $\{3\}$ relative to the coordinate system $\{2\}$, and coordinate system $\{4\}$ relative to the coordinate system $\{3\}$ are respectively:
\[ T = \begin{bmatrix} C\theta & -S\theta & 0 & a_1 \\ 0 & 0 & 1 & 0 \\ -S\theta & -C\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (3)

\[ T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (4)

\[ T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (5)

Then, the pose matrix \( R_T \) of the end-gripper coordinate system \( \{4\} \) in frame \( \{0\} \) can be ultimately expressed as:

\[
R_T = T_1^0 T_2^0 T_3^0 T_4^0 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \end{bmatrix} \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}
\] (6)

Where, \( \vec{n} \), \( \vec{o} \), and \( \vec{a} \) are the position and pose vectors of end-effector coordinate system \( \{3\} \) in the reference coordinate system, and are respectively represented by three components of the reference coordinate system. \( R \) is the end-effector pose rotation matrix, and \( P \) is the end-effector coordinate position matrix, which can be expressed as follows:

\[
R = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}, \quad P = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}
\] (7)

Joint coordinate system \( \{1\} \) is represented in coordinate system \( \{0\} \):

\[
T_1^0 = \begin{bmatrix} C\theta & -S\theta & 0 & 0 \\ S\theta & C\theta & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\] (8)

Joint coordinate system \( \{2\} \) is represented in coordinate system \( \{0\} \):

\[
T_2^0 = \begin{bmatrix} C\theta_1C\theta_2 & -C\theta_1S\theta_2 & -S\theta_1 & a_1C\theta_1 \\ S\theta_1C\theta_2 & -S\theta_1S\theta_2 & C\theta_1 & a_1S\theta_1 \\ -S\theta_2 & -C\theta_2 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\] (9)

Joint coordinate system \( \{3\} \) is represented in coordinate system \( \{0\} \):

\[
T_3^0 = \begin{bmatrix} C\theta_1C\theta_2 & -S\theta_1 & C\theta_1S\theta_2 & a_1C\theta_1 + d_3C\theta_1S\theta_2 \\ S\theta_1C\theta_2 & C\theta_1 & S\theta_1S\theta_2 & a_1S\theta_1 + d_3S\theta_1S\theta_2 \\ -S\theta_2 & 0 & C\theta_2 & d_1 + d_3C\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\] (10)
End-effector coordinate system \( \{4\} \) is represented in coordinate system \( \{0\} \):

\[
\begin{bmatrix}
C\theta_1 C\theta_2 & -S\theta_1 & C\theta_1 S\theta_2 & a_1 C\theta_1 + (d_3 + d_4) C\theta_2 S\theta_2 \\
S\theta_1 C\theta_2 & C\theta_1 & S\theta_1 S\theta_2 & a_1 S\theta_1 + (d_3 + d_4) S\theta_1 S\theta_2 \\
-S\theta_2 & 0 & C\theta_2 & d_1 + (d_3 + d_4) C\theta_2 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

Finally, the forward kinematics equation of the Marine trestle is obtained as follows:

\[
\begin{bmatrix}
r_{11} & r_{12} & r_{13} & p_x \\
r_{21} & r_{22} & r_{23} & p_y \\
r_{31} & r_{32} & r_{33} & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(12)

Among them:

\[
\begin{align*}
& r_{11} = C\theta_1 C\theta_2, \quad r_{12} = -S\theta_1, \quad r_{13} = C\theta_1 S\theta_2, \\
& r_{21} = S\theta_1 C\theta_2, \quad r_{22} = C\theta_1, \quad r_{23} = S\theta_1 S\theta_2, \\
& r_{31} = -S\theta_2, \quad r_{32} = 0, \quad r_{33} = C\theta_2
\end{align*}
\]  

(13)

\[
\begin{align*}
p_x &= a_1 C\theta_1 + (d_3 + d_4) C\theta_2 S\theta_2, \\
p_y &= a_1 S\theta_1 + (d_3 + d_4) S\theta_1 S\theta_2, \\
p_z &= d_1 + (d_3 + d_4) C\theta_2
\end{align*}
\]  

(14)

2.2. The inverse kinematics

The inverse kinematics is the state information of the end of the manipulator, and the motion state variables of each joint are obtained when the end of the manipulator reaches the state. Solving methods can be divided into two categories: closed method and numerical method. The algebraic method is suitable for solving the inverse kinematics of the manipulator with fewer degrees of freedom, with less computation and fast solving speed [5]. For the practical solution of inverse kinematics, there may be unique solution or multiple solution set. Marine trestle is a three-degree-of-freedom manipulator, and there are three unknown quantities to be solved, and there is a unique solution, so the algebraic method is used to solve the inverse kinematics. Assume that the expected pose of the end-effector is given as:

\[
\begin{bmatrix}
n_x & o_x & a_x & p_x \\
n_y & o_y & a_y & p_y \\
n_z & o_z & a_z & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(15)

The forward kinematics equation of the Marine trestle is finally obtained as follows:

\[
\begin{bmatrix}
C\theta_1 C\theta_2 & -S\theta_1 & C\theta_1 S\theta_2 & a_1 C\theta_1 + (d_3 + d_4) C\theta_2 S\theta_2 \\
S\theta_1 C\theta_2 & C\theta_1 & S\theta_1 S\theta_2 & a_1 S\theta_1 + (d_3 + d_4) S\theta_1 S\theta_2 \\
-S\theta_2 & 0 & C\theta_2 & d_1 + (d_3 + d_4) C\theta_2 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(16)

Equations can be obtained simultaneously:

\[
\begin{align*}
p_x &= 0.6 C\theta_1 + (23.3 + d_4) C\theta_1 S\theta_2, \\
p_y &= 0.6 S\theta_1 + (23.3 + d_4) S\theta_1 S\theta_2, \\
p_z &= 5.062 + (23.3 + d_4) C\theta_2
\end{align*}
\]  

(17)
Four-quadrant arc tangent method is used to express:

\[
\begin{align*}
\theta_1 &= \text{ATAN2}(-p_y, -p_x) \\
\theta_2 &= \text{ATAN2}\left(\frac{p_x}{\sin \theta_1} - 0.6, (p_x - 5.062)\right) \\
d_4 &= \frac{p_x - 5.062}{\cos \theta_2} - 23.3
\end{align*}
\] (18)

2.3. Methods validation

Robotics Toolbox in Matlab can be used to verify the forward and inverse solutions of kinematics by D-H parameter method [6]. Input the D-H method parameters of offshore trestle in the Robotics Toolbox to get its simulation model, as shown in Fig. 12. By moving the buttons \( q_1, q_2, q_3 \) and \( q_4 \) in the teaching panel, the values of \( \theta_1, \theta_2 \) and \( d_4 \) can be changed to obtain the position and posture of the end of the trestle in the base coordinate system.

![Figure 8. Kinematics model of Robotics Toolbox.](image)

Given the values of variables \( \theta_1, \theta_2 \) and \( d_4 \) are \((10^\circ, -80^\circ, 8.2\text{m})\), and put it into the Matlab simulation program, the kinematics solution of the offshore trestle can be obtained as follows:

\[
T = \begin{bmatrix}
0.1710 & -0.1736 & -0.9698 & -30.541 \\
0.0301 & 0.9848 & -0.1710 & -5.385 \\
0.9848 & 0 & 0.1736 & 10.638 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (19)

The results are the same as those obtained by substituting the parameters into the formula for the forward kinematics. Similarly, the given homogeneous transformation matrix is put into the Matlab simulation program to obtain the corresponding values of \( \theta_1, \theta_2 \) and \( d_4 \), which verifies the correctness of the D-H parameter method for solving the forward and inverse kinematics.
3. Workspace analysis

The working space of a trestle is its effective range of action under the limit of joint Angle, and it is also an important kinematics index to measure the working ability of a trestle. At the same time, it can provide spatial data for reference and ensure that the trajectory of the task execution is located in the interior of the joint space when the user is planning the transfer task, otherwise the transfer task cannot be performed. At present, there are three main methods to solve the workspace: analytical method, graphical method and numerical method [7, 8].

Geometric method is to show the working space of the manipulator intuitively in the form of graphics. However, it is only applicable to the manipulator with degrees of freedom, because too many degrees of freedom lead to difficulties in the drawing process. Analytical rules are solved by mathematical equations, which are seldom applied to engineering practice because of the complexity and difficulty of solving equations. Numerical law is the application of computer technology to simulate, can get all the manipulator can reach the point, and the workspace is the set of the points. This method usually uses the computer simulation software MATLAB, which has the advantages of simple process and fast solving speed [9, 10]. The method adopted in this paper is based on the forward kinematics equation, Monte Carlo method in Matlab simulation, to solve the work space of the sea laborer trestle.

The calculation steps for solving the working space of Marine trestle based on Monte Carlo method are as follows:

1. According to the forward kinematics equation of the offshore trestle, the position vector of the end-effector in the base coordinate system can be expressed as follows:

\[ P = [P_x \ P_y \ P_z]^T \] (20)

2. Set a cycle number N, within the limit range of each joint Angle, call Rand (N) function in Matlab, N random numbers between 0 and 1 can be obtained as random step size variables (N =1,2..., N), is \((\theta_{imax} - \theta_{imin})\text{rand}(n)\), and then the pseudo random number of the Angle variables of each joint of the Marine trestle is obtained:

\[ \theta_i = \theta_{imin} + (\theta_{imax} - \theta_{imin})\text{rand}(n) \] (21)

Where, \(i\) is the joint ordinal number, \(i=1, 2, 3\), \(\theta_{imin}\) represents the minimum Angle of the \(i\) joint, and \(\theta_{imax}\) represents the maximum Angle of the \(i\) joint.

3. The pseudo-random number is substituted into the position vector of the end-effector in the base coordinate system to obtain the mapping point set of all joint angles of the Marine trestle to the spatial position of the end-effector, which is equivalent to the effective working space of the Marine trestle. And the larger the number of cycles, the more mapping points will be, the more close to the actual working space.

4. Carry out 3D drawing of all the mapping points obtained, and get the point cloud map of the working space of Marine engineering trestle. Taking the number of cycles \(N=20000\), draw the three-dimensional map of the workspace and the projection map of XOY, XOZ and YOZ planes respectively.
**Figure 9.** Three-dimensional working space of Marine trestle.

**Figure 10.** XOY plane projection.

**Figure 11.** XOZ plane projection.
4. Analysis of wave compensation for offshore trestle

4.1. Ship motion

The ship motion can be divided into three translational motions and three rotational motions. Translational motion includes: transverse, longitudinal and heave. Rotational movements include roll, pitch and bow. The actual movement of the ship's six degrees of freedom can be measured through MRU, and the following reference coordinate system is established to describe the ship's movement.

Earth coordinate system O_xeYeZe: the coordinate system is fixedly connected to the earth, the origin of coordinate O_e is parallel to the static horizontal plane, due north is the positive direction of axis X_e, Z_e is vertically upward, and the direction of axis Y_e is determined by the right hand rule. Use this coordinate system to define the direction of the ship and the wave.

Ship coordinate system O_xvYvZv: the coordinate system is fixedly connected to the ship, the origin of coordinate O_v is located on the static horizontal plane, the midpoint of the ship's length, the positive direction of axis X_v points to the bow, the positive direction of axis Y_v points to the port side, and the
direction of axis $Z_v$ is determined by the right hand rule. The coordinate system is used to define the position and pose of the trestle.

The purpose of active wave compensation for Marine trestle is to make the trestle end fixed over the offshore platform and keep motionless, which requires the response motion of the ship relative to the earth coordinate system to be transformed into the motion of the end effector of the trestle relative to the earth.

Then there is a rotation relationship between coordinate systems $O_{vX_vY_vZ_v}$ and $O_{eX_eY_eZ_e}$, namely, bow rotation $\psi$ about the Z axis, pitch rotation $\zeta$ about the Y axis, and roll rotation $\phi$ about the X axis.

Thus, the forward rotation transformation matrix can be obtained as follows:

$$
\text{Rot}(Z, \psi) = \begin{bmatrix}
c \psi & -s \psi & 0 \\
s \psi & c \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

(22)

Pitch rotation transformation matrix:

$$
\text{Rot}(Y, \zeta) = \begin{bmatrix}
c \zeta & 0 & s \zeta \\
0 & 1 & 0 \\
-s \zeta & 0 & c \zeta
\end{bmatrix}
$$

(23)

Roll rotation transformation matrix:

$$
\text{Rot}(X, \phi) = \begin{bmatrix}
1 & 0 & 0 \\
0 & c \phi & -s \phi \\
0 & s \phi & c \phi
\end{bmatrix}
$$

(24)

The rotation matrix of frame $O_{vX_vY_vZ_v}$ relative to frame $O_{eX_eY_eZ_e}$ can be obtained:

$$
0_v^eR = \text{Rot}(Z, \psi) \times \text{Rot}(Y, \zeta) \times \text{Rot}(X, \phi) = \begin{bmatrix}
c \psi c \zeta & c \psi s \zeta s \phi - s \psi c \phi & c \psi s \zeta c \phi + s \psi s \phi \\
- s \psi c \phi & s \psi s \zeta s \phi + c \psi c \phi & c \psi s \zeta c \phi - s \psi s \phi \\
- s \zeta & c \zeta s \phi & c \phi
\end{bmatrix}
$$

(25)

There is a translation relation between the coordinate system $O_{vX_vY_vZ_v}$ and the coordinate system $O_{eX_eY_eZ_e}$. The homogeneous transformation matrix is expressed as:

$$
0_v^eT_{cart} = [P_x \ P_y \ P_z]^T
$$

(26)

Finally, the homogeneous transformation matrix of frame $O_{vX_vY_vZ_v}$ relative to frame $O_{eX_eY_eZ_e}$ is obtained:

$$
0_v^eT = \begin{bmatrix}
c \psi c \zeta & c \psi s \zeta s \phi - s \psi c \phi & c \psi s \zeta c \phi + s \psi s \phi & P_x \\
- s \psi c \phi & s \psi s \zeta s \phi + c \psi c \phi & c \psi s \zeta c \phi - s \psi s \phi & P_y \\
- s \zeta & c \zeta s \phi & c \phi & P_z \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(27)

4.2. Compensatory motion of Marine trestle

When the Marine trestle is used, it usually needs the replenishment ship to berth at a certain point in the sea, so the dynamic positioning system of the ship is needed. This system uses thrusters in different directions to reduce the yaw, swell and roll motion of the ship at rest [11]. The dynamic positioning
system module is provided in the MSS (Marine Systems Simulator) Toolbox. The ship response motion when using the dynamic positioning system can be obtained by entering the ship parameters and wave modeling parameters. The overall modeling of Simulink is shown in the figure below:

![Simulink simulation model](image)

**Figure 14.** Simulink simulation model.

Ship parameters and wave modeling parameters are shown in the following table:

| name       | displacement/t | ship length /m | molded breadth /m | moulded depth /m | depth of immersion /m |
|------------|----------------|----------------|-------------------|------------------|-----------------------|
| Supply ship| 6360           | 82.8           | 19.2              | 7.3              | 6                     |

| Wave level | Spectrum model | significant wave height | peak frequency | Mean wave direction | Peak factor |
|------------|----------------|-------------------------|----------------|---------------------|-------------|
| three      | JONSWAP        | 0.9m                    | 1.3rad/s       | 30°                 | 3.3         |

The Marine trestle is fixedly connected to the supply ship. It is affected by the waves together with the ship and makes the same motion response. Therefore, the motion required by the Marine trestle can be compensated as shown in the figure:

![Displacement in X direction](image)

**Figure 15.** Displacement in X direction
Figure 16. Displacement in Y direction.

Figure 17. Displacement in Z direction.

Figure 18. Rotation angle around X axis $\phi$.

Figure 19. Rotation angle around Y axis $\zeta$.

Figure 20. Rotation angle around Z axis $\psi$. 
A fixed position vector $\mathbf{p}_0 = [P_{x0} \ P_{y0} \ P_{z0}]^T$ is selected from the working space at the end of the Marine trestle to keep it fixed and stationary, and the selected fixed and stationary points are substituted into the position vector equation to obtain:

$$
\begin{align*}
\mathbf{p}_{x0} &= (a_1C\theta_1 + d_3C\theta_1S\theta_2 + d_4C\theta_1S\theta_2)c\psi c\zeta + \\
&\quad (a_1S\theta_1 + d_3S\theta_1S\theta_2 + d_4S\theta_1S\theta_2)(c\psi s\zeta s\phi - s\psi c\phi) \\
&\quad + (d_5C\theta_2 + d_1 + d_4C\theta_2)(c\psi s\zeta c\phi + s\psi s\phi) + p_x
\end{align*}
$$

(28)

$$
\begin{align*}
\mathbf{p}_{y0} &= (a_1C\theta_1 + d_3C\theta_1S\theta_2 + d_4C\theta_1S\theta_2)s\psi c\zeta + \\
&\quad (a_1S\theta_1 + d_3S\theta_1S\theta_2 + d_4S\theta_1S\theta_2)(c\psi c\phi + s\psi s\zeta s\phi) + \\
&\quad (d_3C\theta_2 + d_1 + d_4C\theta_2)(s\psi s\zeta s\phi - c\psi s\phi) + p_y
\end{align*}
$$

(29)

$$
\begin{align*}
\mathbf{p}_{z0} &= -(a_1C\theta_1 + d_3C\theta_1S\theta_2 + d_4C\theta_1S\theta_2)s\zeta + \\
&\quad (a_1S\theta_1 + d_3S\theta_1S\theta_2 + d_4S\theta_1S\theta_2)c\zeta s\phi + \\
&\quad (d_3C\theta_2 + d_1 + d_4C\theta_2)c\zeta c\phi + p_z
\end{align*}
$$

(30)

If the fixed point $\mathbf{p}_0 = [16 \ \ 5 \ \ 20]^T$ is selected and substituted into the above formula, the rotation Angle $\theta_1$, elevation Angle $\theta_2$ and expansion amount $d$ can be obtained in Matlab simulation.

**Figure 21.** rotation angle $\theta_1$.

**Figure 22.** pitch angle $\theta_2$.

**Figure 23.** Expansion and contraction quantity $d$.

5. Conclusion
Based on D-H parameter method, the kinematics model of large offshore trestle based on wave compensation is established, the space transformation matrix of offshore trestle is deduced, the feasibility and correctness of forward and inverse kinematics are solved and verified, and the working space of offshore trestle is analyzed. In addition, the wave compensation response under three-level sea
condition is obtained through Matlab/ Simulink simulation, which provides a foundation and reference for the analysis and research of the hydraulic control system of Marine trestle in the future.

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