Development of learning using Indonesian realistic mathematics education approach to build students’ relational understanding of derivative

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Abstract. This study used a design research method aiming to develop the local instructional theory on learning derivative to build relational understanding in class XI Cikarang Utara Senior High School using Indonesian Realistic Mathematics Education (PMRI) approach. Design research consists of three phases that are preparation and design, teaching experiment and retrospective analysis. Stages of building a relational understanding of students assisted by using the context of changes in body weight in infants. This study has 6 activities organized in Local Instructional Objectives. Subjects of this study consisted of 6 senior high school students XI MIPA Cikarang Utara. The data collection has been done by using paper and pencil method, interview method and the method ostensive. The validity of the data used internal and external validity. While the reliability of data using triangulation of data and cross interpretation. All data collected and analyzed in a retrospective analysis. A retrospective analysis showed that the context used can develop the students’relational abilities derivative. The teacher played a major role in directing the learning activities that the learning objectives achieved as instructional objectives that have been prepared.

1. Introduction
One of the lessons be taught in senior high school is derivative. We can find derivative applications in everyday life, one of them in the field of physics namely to calculate the rate of change in speed. But many students have difficulty in understanding concepts in the derivative, they argue that derivative has many formulas, in their applications require high understanding involving many concepts [2]. So far the target and orientation of mathematics learning is the mastery of mathematical material or concepts that tend to emphasize aspects of the problem-solving procedure with formulas so that the ability to memorize formulas and procedures is very important [7]. Many teachers give procedural questions, that is the problem with the completion step is in the form of a clear procedure, students complete the problem with a formula without knowing the meaning of the formula used [9]. Abba said that most teachers use conventional learning models and are dominated by teachers, resulting in low student activity [14]. The study conducted by Sahin that students’ understanding of derivative lesson is instrumental understanding, instrumental understanding is students knowing rules without reasons, students do not understand the relationship between limit and derivatives, the slope of tangent and derivatives, the rate of change with derivatives and the fact that the students have difficulties in solving problem that require using the relationship between the derivative and limit [12]. Skemp states understanding is not only remembering mathematical concepts or being able to follow procedures but
understanding is the ability to apply facts, concepts, and skills appropriately to new situations or problems [13]. Skemp defines two types of understanding, namely instrumental understanding and relational understanding [13]. Instrumental understanding is understanding which only memorizes knowledge without understanding its concepts and how those concepts are interrelated. Relational understanding is understanding using a mathematical procedure derived from the results of connecting various mathematical concepts that are relevant in solving a problem and knowing why the procedure can be used or relational understanding that is knowing what to do and why it is done [13]. Relational understanding will produce conceptual knowledge so students can understand the steps in solving problems.

Sahin argue when teaching derivatives, it is better if it starts with a level of change other than speed [12]. Sahin used distance- heigh graph of the roller coaster path in model exploration activity where students were asked to provide the graph of the first derivative [12].

Borgen & Manu stated that students can answer procedures that do not mean students understand concepts [1]. This is reinforced by Bosnan & Relly's research that students only consider derivatives as formulas and procedures that are remembered and forgotten, this concept which prevents students from better understanding conceptual understanding [1]. students must be active in learning to gain conceptual understanding. A study conducted by McCormic that students experience meaningful learning when they are actively involved in the learning process such as investigating and understanding important mathematical ideas [8], student activeness becomes a very important element in the success of learning, besides that learning is more concerned with the interaction between students and their groups [3]. Active student involvement in learning to attract interest and retention of learning by linking new knowledge with existing knowledge, according to Ausubel's theory, research is meaningful assimilation for students, lessons learned are assimilated and linked to possessed knowledge. So, students must relate derived learning with the knowledge they already have, one of which is a boundary. in addition to students actively involved in learning to gain meaningful learning, in the process of learning mathematics using context, which can develop students' understanding in more depth [8]. The use of context must be by the material that will be given to students. Related to the problems outlined, we need a learning approach that can develop students' relational understanding of the derivatives because the derivatives are interrelated with other sciences such as physics. good relational understanding allows students to have strong hereditary knowledge [1].

Based on observations made in the field by conducting interviews with students and tests on several students. The test consists of two questions namely conceptual questions and procedural questions from the test results obtained by the conclusion that students have difficulty when working on conceptual questions and the lack of student accuracy shows that students' relational understanding is still low and interviews with a number of students who have learned derivatives, obtained information that the method used by teachers in learning is a conventional method.

Based on research and theory from experts and conditions in the field that students' relational understanding is still low and the methods used by teachers in teaching are still conventional, learning design is needed that can help students in improving students' relational understanding, namely learning design with Pendidikan Matematika Realistik Indonesia (PMRI) approach. PMRI approach realistic mathematics education was adopted from Realistic Mathematics Education (RME).

According to Gravemeijer [5,6] RME always starts with contextual problems. The process of designing a sequence of learning activities in this study was inspired by five principles for RME defined by Treffers [16]. One of them uses a realistic context or problem that is used as a starting point for learning mathematics. The context must be known and can be imagined by students, so students use models and symbols for progressive mathematics. Students can develop the context to model of and then model for. One of the contexts that can be used to help develop students' relational understanding in children is the change in body weight in babies. The context is used to find the concepts \( \lim_{x \to 0} \) which are used as the basic for finding derivative concepts. This research answers the research question which is how to develop learning with PMRI approach to improve students' relational abilities.
2. Methodology

2.1 Research approach

The research method used for this study is Design Research which consists of three phases that are preparation and design, teaching experiment and retrospective analysis [6].

2.1.1 Preparing and Design

This phase begins with the classification of mathematics learning goals and thought experiment, that is image of how the teaching and learning process in the classroom. Result the thought is Hypothetical Learning Trajectory (HLT). HLT consists of learning objectives, plan of learning and conjecture of student’s strategies and thinking is developed [6].

2.1.2 Teaching Experiment

In this phase, the researchers tested the HLT that was created. HLT can changes when applied in the classroom after retrospective analysis because the expected students’ strategies do not appear or learning activities are too difficult for a student. Data collection in this phase is video, interview recording, photo, student’s results, and field notes.

2.1.3 Retrospective Analysis

The retrospective analysis phase consists of data analysis, reflection, interpretation of findings, and the formulation of recommendations for future research [15]. Data obtained from classroom learning activities were analyzed retrospectively [10].

2.2 Research subject

Subjects of this study consisted of 6 senior high school students XI MIPA Cikarang Utara academic year 2018/2019. Subjects of this study were selected based on general ability students in mathematics learning. Six students of the study were in one class with a total of thirty-six students. Addition subjects of this study can do it if data collected still less.

2.3 Data collection

In the preparation phase, there were some sorts of data that had been collected namely classroom observation, interviews with a teacher, and interviews with some students. Meanwhile, after teaching experiment there were some of the data that had been collected namely students’ written work, video recordings, photo, field notes, an evaluation test, and interview with some students who were analyzed retrospective.

3. Result and discussion

The retrospective analysis of data collected from teaching experiment activities, and the Hypothetical Learning Trajectory (HLT) served as a guideline in the analysis to investigated and explain students’ thinking in learning derivative.

3.1 Result

3.1.1 Activity 1. This activity is found derivative concepts \( f'(x) = nax^{n-1} \) using Context of changes in body weigh in infan. First activity students move the six months baby’s weight data on the cartesian diagram, a axis as month and ordinate as weight. Then connect each point formed into a line and formed five lines. Each point is given a symbol namely A, B, C, D, E, F. So, the five lines given symbol AB, BC, CD, DE, EF. Students look for slope every line. that can be seen on figure 1.
Next the students make an elbow triangle from one of the lines, namely line AB as a hypotenuse and other point given a symbol X. So, that an \( ABX \) triangle is formed. From this triangle students must search point A, \( \Delta x \), \( \Delta y \), point X, point B, that can be seen on figure 2.

To find the slope of the line in figure 1 used concept \( \frac{y_2 - y_1}{x_2 - x_1} \) or \( \frac{\Delta y}{\Delta x} \) because from figure 2 \( \Delta y = f(x + \Delta x) - f(x) \) so to find the slope of the line using \( \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \). From the \( ABX \) triangle if the point of X made close to point A so, the value of approaches zero. This is the same if the observation of changes in the baby’s weight is carried out from 1 month to 3 weeks, 2 weeks, 1 week, 1 day, 1 hour then changes in body weight get smaller or close to zero. To find the slope of the line can use the concept \( \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \).

Students search for information about definition derivative from the book and they are can define derivative is change if connected with context derivative is change body weight in infant or baby. The description of changes in body weight infants can be seen from the slope changes in the \( ABX \) triangle. So, to find a derivative concept can use the \( \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \).
Next activity student found derivative concept from the problem what is given by the teacher, that can be seen on figure 3. Mathematically students can find derivative concept use the $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ Finally students can conclude derivative $f(x) = ax^n$ is $f'(x) = na x^{n-1}$.

![Figure 3. Students’ work](image)

### Activity 2

This activity is to find derivative concepts for $f(x) = u(x)v(x)$ which is $f'(x) = u'(x)v(x) + u(x)v'(x)$ use the concept $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$, that can be seen on figure 4 and 5.

![Figure 4. Students’ work](image)

First step, students separated the two functions into two variable, namely $p(x)$ and $q(x)$ or $u(x)$ and $v(x)$, then searched the first derivative that is $v'(x)$ and $u'(x)$. Then students use the concept $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$, because the two functions, the concept used became $\lim_{\Delta x \to 0} \frac{u(x+\Delta x)v(x+\Delta x)-u(x)v(x)}{\Delta x}$. The multiplication process carried out in figure 4 dan 5 is unique. When the teacher asks why the multiplication like this figure 4 and 5. The student said to make it easier to reduce and find the formula. Not all the students can find concept in a way like figure 4 dan 5. So, the teacher explained using a concept where the final result must be there $u(x), v(x), u'(x)$, and $v'(x)$. Then Mathematically students look for derivative above so that it can be conclude derivative $f(x) = u(x)v(x)$ is $f'(x) = u'(x)v(x) + u(x)v'(x)$. 

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**Note:** The document is a transcription of content from a conference proceeding, focusing on educational content related to mathematics and derivative concepts. The text is structured to explain the process of finding derivatives and solving problems through examples and explanations.
3.1.3 Activity 3. This activity find derivative concepts $f(x) = (u(x))^n$ which is $f'(x) = n(u(x))^{n-1} u'(x)$ use the concept $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ that can be seen on figure 6, 7 and 8.
First step, students separating the functions into variable, namely \( p(x) \) or \( u(x) \), then search the first derivative functions that is \( p'(x) \) or \( u'(x) \). Then students use the concept \( \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \). All students experience difficulties when high rank multiplication, so the teacher reminds students to use multiplication of polynomials with pascal triangle. That can be seen on figure 6, 7 and 8 students used pascal triangle and concept multiplication same likes activity 2. Then Mathematically students look for derivative above so that it can be conclude derivative \( f(x) = (u(x))^n \) is \( f'(x) = n(u(x))^{n-1}u'(x) \).

3.1.4 Activity 4. This activity is find derivative concepts \( f(x) = \frac{u(x)}{v(x)} \) which is \( f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2} \) use the concept \( \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \), that can be seen on figure 9.
First step, students separating the two functions into two variable, namely \( p(x) \) and \( q(x) \) or \( u(x) \) and \( v(x) \), then search the first derivative functions that is \( v'(x) \) and \( u'(x) \). Then students use the concept 

\[
\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x},
\]

because the are two functions, the concept used became 

\[
\lim_{\Delta x \to 0} \frac{u(x+\Delta x)-u(x)}{\Delta x} \frac{v(x+\Delta x)-v(x)}{\Delta x}.
\]

The multiplication process carried out in figure 9. All students said this is difficult. So, the teacher explained the first process of working with the denominator is same, where the final result must be there \( u(x), v(x), u'(x), \) and \( v'(x) \). Then Mathematically students look for derivative functions above so that it can be conclude derivative \( f(x) = \frac{u(x)}{v(x)} \) is 

\[
\frac{d}{dx} \left( \frac{u(x)}{v(x)} \right) = \frac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2}.
\]

3.1.5 Activity 5. Activity 5 is the last activity before the evaluation test. This activity aims to find out that the concept of derivatives can be used in physics. First activity in figure 10, students presents data with a cartesian diagram based on speed and time data in the table. Then students look for speed with formula 

\[
v = \frac{\Delta s}{\Delta t}.
\]

**Figure 10. Students’ work**
Figure 11. Students’ work

In figure 11 student must look for the speed formula based on the graph provided. Based on the formula in figure 10, students can connect the formula is \( v = \frac{v_{akhir} - v_{awal}}{t_{akhir} - t_{awal}} = \frac{\Delta s}{\Delta t} \), so if connected with graphics \( v = \frac{\Delta s}{\Delta t} = \frac{s(t + \Delta t) - s(t)}{\Delta t} \). Then students asked to look for instantaneous velocities, namely the average speed for the time interval \((\Delta t)\) approaching zero, so the formula can be used with the relationship in first activity. By linking previously owned knowledge students can find the concept of looking for instantaneous speed, where the concept relates to the concept of the derivative. Then it can be concluded to look for in instantaneous velocities on moving object that can use a derivative concept. This proves that the concept of derivatives can be used in physics.

3.2 Discussion

The findings of the study by Sahin regarding the limit revealed that despite participant's knowledge of the existence of the limit formula in the algebraic definition of the derivative, they could not make sense of the role of limit in both the algebraic and geometric definitions of the derivative[12]. The reason for this might be the fact that students have difficulties in solving problems that require using the relationship between the derivative and limit [12]. From the perspective of Skemp regarding mathematical understanding, the results of Sahih's research show that participants' understanding of the concept of boundaries in relation to the derivative concepts is not relational[12]. But the findings in this study indicate students understand the relationship between limits and derivatives, it can be seen from the picture that students can find derivative concepts using concepts of \( \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \). In the study by Sahin can also be concluded that the participants’ understanding of derivative was not relational, they could not explain the role of the big ideas related to the concept of
derivative, despite the fact that they had been successful in the courses related to derivative [12].

Different from the findings in this study, that participants who numbered 6 students have a relational understanding, their relational understanding develops, these findings are proven by students being able to explain the concept to the audience during the interview and students are able to work on all evaluation questions. Where the problem is made based on indicators of relational understanding and has been validated by experts.

This research is not yet perfect, so researchers suggest that future research use a better context. one of the proposed contexts for future researchers is the length increase in infants.

4. Conclusion
The use of the context of changes in body weight in infants helps students find concepts $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ that are used as basic concepts to find derivative concepts. By using the RME approach, students' relational understanding develops. This can be seen from the ability of students to connect existing knowledge such as the concept of looking for slope lines and mathematical concepts to find derivatives.

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