Ground State Energy of Massive Scalar Field in the Global Monopole Background

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We consider the ground state energy of scalar massive field in the spacetime of a pointlike global monopole. Using zeta function regularization method we obtain the heat kernel coefficients for this system. We show that the coefficient $B_1$ contains additional contribution due to the non-trivial topological structure of the spacetime. Taking into account the heat kernel coefficients we obtain that the ground state energy of the scalar field is zero. We also discuss our result using dimensional considerations.

I. INTRODUCTION

Different types of topological objects may have been formed during Universe expanding, such as domain walls, cosmic strings and monopoles [1]. These topological defects appeared due to breakdown of local or global gauge symmetries. Global monopoles are created due to phase transition when a global gauge symmetry is broken and may have been important for cosmology and astrophysics. The process of global monopole creation is accompanied by particles production [2]. Grand Unified Theory predicts great number of these objects in the Universe [3] but the problem may be avoided using inflationary models. From astrophysical point of view there is at most one global monopole in the local group of galaxies [4].

The spacetime of a global monopole in a $O(3)$ broken symmetry model has been investigated by Barriola and Vilenkin [5]. They have shown that far from the compact monopole’s core the spacetime is approximately described by spherical symmetry metric and with additional solid angle deficit (see also Ref. [6]). There is a global monopole with regular core and another one with pointlike singular core which we study in this paper and also call regular monopole. The spacetime is not locally flat, even for the case of pointlike global monopole spacetime.

The analysis of quantum fields on the global monopole background have been considered in Refs. [7,8]. It was shown, taking into account only dimensional and conformal consideration [7], that the vacuum expectation value of the energy momentum tensor of conformal massless scalar, spinor and vector fields on this background has the following general structure

$$T^i_k = S^i_k \frac{\hbar c}{4\pi},$$

where the quantities $S^i_k$ depend on solid angle deficit and spin of the fields. For scalar field this tensor was investigated in great details in the paper by Mazzitelli and Lousto [8].

The energy-momentum tensor has nonintegrable singularity at origin and therefore the ground state energy cannot be found by integrating the energy density. The same problem also appears for cosmic string spacetime [1] and Minkowsky spacetime with boundary condition on dihedral angle [3]. For the cosmic string spacetime this problem was considered in Refs. [1,3] by using another approaches for calculation of ground state energy. In spite of the energy density has singular form it has been found that the ground state energy of massive scalar field is zero. For infinitely thin cosmic string this may be explained taking into account only dimensional considerations (see below Sec. V).

Nontrivial topological structure of spacetime reveals itself via some contribution in the heat kernel coefficients. In the case of a cosmic string spacetime it appeared in the heat kernel coefficient $B_1$ as additional contribution along with usual volume and boundary terms [1,3,4,5]. But this question has not been yet investigated for global monopole spacetime.

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Nontrivial topological structure of spacetime leads to a number of interesting effects which are forbidden in flat space. For example, there appear additional self-interacting force which is non-zero even for particle in the rest. This force has been investigated in Refs. [15,16] for cosmic string and global monopole spacetimes respectively.

In this paper we would like to discuss the ground state energy of scalar massive field with arbitrary nonconformal coupling in the background of pointlike global monopole spacetime and calculate topological contribution to the heat kernel coefficients.

In the framework of zeta function regularization method [17] (see also [18]) the ground state energy of scalar massive field is given by

\[ E(s) = \frac{1}{2} M^2 \zeta_A(s - \frac{1}{2}), \]

which is expressed in terms of the zeta function \( \zeta_A \) of the Laplace operator \( \hat{A} = -\triangle + \xi R + m^2 \) on three dimensional spatial section of spacetime. Here, parameter \( M \) with dimension of mass keeps right dimension of energy. For calculation and renormalization of the ground state energy we use the approach which was suggested and developed in Refs. [19,20].

Zeta function of Laplace operator on the global monopole background has been considered in detail by Bordag, Kirsten and Dowker in Ref. [22] using the method given in Refs. [19,20]. There the general mathematical structure of zeta function and the heat kernel coefficients on the generalized cone were obtained. We shall rederive some formulas in our concrete case because the main emphasis of the present paper is ground state energy which was not considered in Ref. [22]. It is necessary also in order to compare our considerations with the general formulas of a nonsingular background and separate the additional topological contributions.

The organization of the paper is as follows. In Sec. II we give some geometrical data about global monopole spacetime which will be needed. In Sec. III, the zeta function of the Laplace operator on three dimensional section of a global monopole spacetime is calculated and the heat kernel coefficients are obtained including topological contributions. In Sec. IV the ground state energy of massive scalar field with arbitrary nonconformal coupling on global monopole background is considered. In last Sec. V we discuss our results. The signature of the spacetime, the sign of Riemann and Ricci tensors are the same as in Christensen paper [27]. We use units \( \hbar = c = G = 1 \).

II. THE GEOMETRY

Global monopoles are heavy objects probably formed in the early Universe by the phase transition which occurred in a system composed by a self-coupling scalar field triplet \( \phi^a \) whose original global symmetry \( O(3) \) is spontaneously broken to \( U(1) \).

The simplest model which gives rise a global monopole is described by the Lagrangian density below

\[ L = \frac{1}{2} \left( \partial_t \phi^a \right) \left( \partial_t \phi^a \right) - \frac{\lambda}{4} (\phi^a \phi^a - \eta^2)^2. \]

Coupling this matter field with the Einstein equation, Barriola and Vilenkin [5] have shown that the effect produced by this object in the geometry can be approximately represented by a solid angle deficit in the \((3 + 1)\) - dimensional spacetime, whose line element is given by

\[ ds^2 = -dt^2 + \alpha^{-2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

where the parameter \( \alpha^2 = 1 - 8\pi \eta^2 \) is smaller than unity and is connected with the symmetry breaking energy \( \eta \). The solid angle has the value \( 4\pi \alpha^2 < 4\pi \). Note that the spacetime given by (4) is not flat. Nonzero components of Riemann and Ricci tensors, and scalar curvature read

\[ R_{\theta\varphi} = R_{\varphi} = R_{\phi} = \frac{1-\alpha^2}{r^2}, \quad R = \frac{2(1-\alpha^2)}{r^2}. \]

For further application consider extrinsic curvature tensor on the sphere of radius \( R \) around the origin

\[ K_{ij} = \nabla_i N_j. \]

Here \( N_j \) is outward unit normal vector with coordinates \( N_j = (0, \alpha, 0, 0) \). This tensor has two nonzero components

\[ K^\theta_{\theta} = K^\varphi_{\varphi} = \frac{\alpha}{r}. \]
III. ZETA FUNCTION AND HEAT KERNEL COEFFICIENTS

In order to calculate the ground state energy given by [1] we have to obtain the zeta function of the operator \( \hat{A} = -\Delta + \xi \hat{R} + m^2 \) in the neighbourhood of point \(-1/2\). For calculation of zeta function we follow to Ref. [20].

The zeta function of operator \( \hat{A} \) is defined in terms of the sum over all eigenvalues of this operator by

\[
\zeta_A(s - \frac{1}{2}) = \sum_{l,j=0}^{\infty} (2l + 1)(\lambda_{l,j} + m^2)^{1/2-s} .
\]

Here \( \lambda_{l,j}^2 \) is the eigenvalue of operator \( \hat{B} = \hat{A} - m^2 \). The eigenfunctions which are regular at the origin have the form

\[
\Phi(r) = \sqrt{\frac{\lambda}{\alpha r}} Y_{lm}(\theta, \varphi) J_\mu\left(\frac{\lambda}{\alpha} r\right) ,
\]

where \( Y_{lm} \) is the spherical harmonics and \( J_\mu \) the Bessel function of the first kind with index

\[
\mu = \frac{1}{\alpha} \sqrt{(l + \frac{1}{2})^2 + 2(1 - \alpha^2)(\xi - \frac{1}{8})} .
\]

The eigenvalues \( \lambda_{l,j} \) can be found by some boundary condition imposed on this function. Let us consider the Dirichlet boundary condition at the surface of a sphere of radius \( R \)

\[
\sqrt{\lambda_{l,j}} J_\mu\left(\frac{\lambda_{l,j}}{\alpha} R\right) = 0 .
\]

So, the zeta function reads now as:

\[
\zeta_A(s - \frac{1}{2}) = \sum_{l=0}^{\infty} \sum_{j=0}^{\infty} (2l + 1)(\lambda_{l,j} + m^2)^{1/2-s} .
\]

The solutions \( \lambda_{l,j} \) of equation [4] can not be found in closed form. For this reason we use the method suggested in Refs. [19,20] which allows us to express the zeta function in terms of the eigenfunctions. According to this approach the sum over \( j \) may be converted into contour integral in complex \( \lambda \) plane using the princip of argument, namely

\[
\zeta_A(s - \frac{1}{2}) = \sum_{l=0}^{\infty}(2l + 1) \int d\lambda^2 \sum_{j=0}^{\infty} (\lambda_{l,j} + m^2)^{1/2-s} \frac{\partial}{\partial \lambda} \ln \lambda J_\mu\left(\frac{\lambda}{\alpha} R\right) ,
\]

where the contour \( \gamma \) runs counterclockwise and must enclose all solutions of eq. [4] on positive real axis. Next we use the uniform expansion for the zeta function (see [2] details)

\[
\zeta_A(s - \frac{1}{2}) = -\cos \frac{\pi s}{2} \sum_{l=0}^{\infty}(2l + 1) \int_m^{\infty} dk^2 k^{1/2-s} \frac{\partial}{\partial k} \ln k^{-\mu} I_\mu\left(\frac{k}{\alpha} R\right) .
\]

Here \( I_\mu \) is the modified Bessel function obtained from \( J_\mu \) in imaginary axis \((\lambda = ik)\). Next we use the uniform expansion for the Bessel function \( I_\mu(\mu z) \) as below

\[
I_\mu(\mu z) = \sqrt{\frac{t}{2\pi \mu}} e^{\mu \eta(z)} \left\{ 1 + \sum_{k=1}^{\infty} \frac{u_k(t)}{(\mu)^k} \right\} ,
\]

where \( t = 1/\sqrt{1 + z^2} \), \( \eta(z) = \sqrt{1 + z^2 + \ln(z/(1 + \sqrt{1 + z^2}))} \) and \( z = kR/\mu \alpha \). The first few coefficients \( u_k(t) \) are listed in [20]. This uniform expansion leads to power series over \( m \), and \( u_N \) gives the contribution \( \sim 1/m^{3-N} \). We shall make the calculations up to \( N = 3 \). Using the formula

\[
\sum_{l=0}^{\infty}(2l + 1)(\lambda_{l,j} + m^2)^{1/2-s} \frac{\partial}{\partial \lambda} \ln \lambda J_\mu\left(\frac{\lambda}{\alpha} R\right) ,
\]

for \( \lambda = ik \) one obtains

\[
\sum_{l=0}^{\infty}(2l + 1)(\lambda_{l,j} + m^2)^{1/2-s} \frac{\partial}{\partial \lambda} \ln \lambda J_\mu\left(\frac{\lambda}{\alpha} R\right) = \pi \gamma(\frac{1}{2} - s) \frac{\Gamma(s + \frac{n-3}{2})}{\Gamma(s + \frac{n+1}{2})} 2^{1-s} m^{-3-N} .
\]
we obtain the following expansion for the zeta function

\[
\zeta_A(s - 1/2) = \frac{m^{-2s}}{(4\pi)^{1/2}} \frac{4\pi^{3/2}m\beta/\alpha}{\Gamma(s - 1/2)} \left\{ \sum_{l=0}^{\infty} (2l + 1) \left[ \frac{\Gamma(s - 1)}{\sqrt{\pi}} \right] \right. \\
- \frac{\alpha}{2\beta} Z(0, s - 1/2) - \frac{\alpha^2}{4\beta^2\sqrt{\pi}} \left[ Z(0, s) - \frac{10}{3} Z(2, s + 1) \right] \\
- \frac{\alpha^3}{8\beta^3} \left[ Z(0, s + 1/2) - 6Z(2, s + 3/2) + 5Z(4, s + 5/2) \right] \\
- \frac{\alpha^4}{96\beta^4\sqrt{\pi}} \left[ 25Z(0, s + 1) - \frac{1062}{5} Z(2, s + 2) + \frac{884}{5} Z(4, s + 3) \right] \\
- \frac{1768}{63} Z(6, s + 4) + \ldots \}.
\]

Here \( {}_2F_1 = {}_2F_1(-1/2, s - 1/2; - (\mu/\beta)^2) \) is the hypergeometric function; \( \beta = mR \) and

\[
Z(p, q) = \Gamma(q) \sum_{l=0}^{\infty} \frac{2l + 1}{1 + \alpha^2\mu^2/\beta^2} \left( \frac{\alpha\mu}{\beta} \right)^p.
\]

The first expression in Eq. (7) given in terms of hypergeometric function

\[
T(s) = \sum_{l=0}^{\infty} (2l + 1) \left[ \frac{\Gamma(s - 1)}{\sqrt{\pi}} \right] {}_2F_1 - \frac{\alpha\mu}{\beta} \Gamma(s - 1/2),
\]

can be expressed in terms of the same function given in Eq. (8). Indeed, one can use analytical continuation of the hypergeometric function [123]

\[
{}_2F_1 \left( -1/2, s - 1/2; - \left( \frac{\alpha\mu}{\beta} \right)^2 \right) = \frac{\alpha\mu}{\beta} \frac{\Gamma(1/2)\Gamma(s - 1/2)}{\Gamma(s)}
\]

\[
+ \frac{\Gamma(1/2)\Gamma(1/2 - s)}{\Gamma(-1/2)\Gamma(3/2 - s)} \left( 1 + \left( \frac{\alpha\mu}{\beta} \right)^2 \right)^{1-s} {}_2F_1 \left( 1, s - 1; s + 1/2; \frac{1}{1 + \left( \frac{\alpha\mu}{\beta} \right)^2} \right). \]

So, the first term in the rhs. of the above equation cancels the second one, divergent term in the sum (8) which is due to term \( k^{-\mu} \) in (9) (see Ref. [12]). Next, one can use power series expansion for the hypergeometric function because its argument \( 1/(1 + (\alpha\mu/\beta)^2) \) is always smaller than unity, so we arrive at

\[
T(s) = \frac{1}{2\sqrt{\pi}} \Gamma(s - 1/2) \sum_{l=0}^{\infty} \frac{Z(0, n + s - 1)}{\Gamma(n + s + 1/2)}.
\]

Therefore for calculation of the zeta function we have to obtain an analytical continuation to the series \( Z(p, q) \) fact we may consider only

\[
Z(0, q) = \Gamma(q) \sum_{l=0}^{\infty} \frac{2l + 1}{1 + \alpha^2\mu^2/\beta^2} \left( \frac{\alpha\mu}{\beta} \right)^p.
\]

because the other functions with \( p = 2, 4, 6, \ldots \) can be expressed in terms of \( Z(0, q) \) only. Substituting the values for \( \mu \) given in (8) into (11) we obtain

\[
Z(0, q) = 2\Gamma(q)\beta^{2q} \sum_{l=0}^{\infty} \frac{l + 1/2}{((l + 1/2)^2 + b^2)^q},
\]

where \( b^2 = \beta^2 + 2(1 - \alpha^2)(\epsilon - 1/8) \). This series is convergent for \( Re \alpha > 1 \). For analytical continuation of this function.
we obtain the heat kernel coefficients:

\[ F(q, a, b^2) = \sum_{l=0}^{\infty} \frac{1}{((l + 1/2)^2 + b^2)^q} . \]

This series has been considered in great details by Elizalde [24]. He found analytical continuation of this series for great \( b \) reads

\[ F(q, a, b^2) \approx \frac{b^{-2q}}{\Gamma(q)} \sum_{l=0}^{\infty} \frac{(-1)^l \Gamma(l + q)}{l!} b^{-2l} \zeta_H(-2l, \alpha) + \frac{\sqrt{\pi} \Gamma(q - 1/2)}{2\Gamma(q)} b^{1-2q} \]

\[ - \frac{2\pi b^{-1/2-q}}{\Gamma(q)} \sum_{n=1}^{\infty} n^{q-1/2} \cos(2\pi na) K_{q-1/2}(2\pi nb) . \]

Here \( \zeta_H \) is the Hurwitz zeta function and \( K_n \) is the modified Bessel function. Differentiating this series with respect to \( a \) and putting \( a = 1/2 \) we obtain the analytical continuation we need which is the following

\[ \sum_{l=0}^{\infty} \frac{l + 1/2}{((l + 1/2)^2 + b^2)^q} \approx \frac{b^{-2q}}{2(q - 1)} + \sum_{l=0}^{\infty} \frac{(-1)^l \Gamma(l + q)}{l! \Gamma(q)} b^{-2l} \zeta_H(-1 - 2l, 1/2) . \]

Taking into account this expression we obtain analytical continuation for function \( Z(0, q) \):

\[ Z(0, q) \approx \left( \frac{b^2}{\beta^2} \right)^{-q} \left\{ b^2 \Gamma(q - 1) + 2 \sum_{l=0}^{\infty} \frac{(-1)^l \Gamma(l + q)}{l!} b^{-2l} \zeta_H(-1 - 2l, 1/2) \right\} , \]

where \( b^2/\beta^2 = 1 + 2(1 - \alpha^2)(\xi - 1/8)/\beta^2 \). This function has simple poles in integer numbers \( q = 1, 0, -1, -2, \ldots \). In order to calculate zeta function up to degree \( m^3 \) we need only two terms from series (12) in which \( \zeta_H(-1, 1/24), \zeta_H(-3, 1/2) = -7/960 \) and three terms of \( T(s) \) which is given by Eq. (11).

Putting this expression into (10) and (7) and expanding over \( 1/\beta = 1/mR \ll 1 \) and \( s \), and collecting terms similar degree on the mass \( m \) up to \( m^3 \) (we cannot here collect higher orders of \( m \) because we used uniform expansion up to this power) we get

\[ \zeta_A(s - 1/2) = \frac{m^{-2s}}{(4\pi R^3/3\alpha)^{3/2}} \left( \frac{4\pi R^3}{3\alpha} \right)^{m^4} \frac{\Gamma(s - 2)}{\Gamma(s - 1/2)} + \left[ 2\pi^{3/2} \right] - \frac{m}{\frac{1}{12} \left[ \frac{1}{12} \right] } \left[ \frac{2\pi^{3/2} \Gamma(s - 1/2)}{\Gamma(s - 1/2)} + \frac{\Gamma(s)}{\Gamma(s - 1/2)} + \ldots \right) . \]

Here \( \delta = 2(1 - \alpha^2)(\xi - 1/8) \). These are all pole contributions in zeta function, all next terms will be finite at \( s \to 0 \). Comparing this expression with that obtained by the Mellin transformation over trace of heat kernel (in 4 dimensions)

\[ \zeta_A(s - 1/2) = \frac{1}{\Gamma(s - 1/2)} \int_{0}^{\infty} dt t^{s-3/2} K(t) = \frac{m^{-2s}}{(4\pi R^3/3\alpha)^{3/2}} \left( \frac{4\pi R^3}{3\alpha} \right)^{m^4} \frac{\Gamma(s - 2)}{\Gamma(s - 1/2)} \]

\[ + B_{1/2} m^2 \frac{\Gamma(s - 3/2)}{\Gamma(s - 1/2)} + B_1 m^2 \frac{\Gamma(s - 1)}{\Gamma(s - 1/2)} + B_{3/2} m + B_2 \frac{\Gamma(s)}{\Gamma(s - 1/2)} + \ldots \]

we obtain the heat kernel coefficients:

\[ B_0 = \frac{4\pi R^3}{3\alpha}, \quad B_{1/2} = -2\pi^{3/2} R^2, \quad B_1 = \frac{7}{3} \pi \alpha R - \frac{4\pi R}{\alpha} (\delta - \frac{1}{12}) , \]

\[ B_{3/2} = 2\pi^{3/2} (\delta - \frac{1}{12}), \quad B_2 = \frac{\pi \alpha}{R} (\delta - \frac{1}{12} + \frac{2\pi \alpha^2}{24 + 105}) - \frac{2\pi \alpha^3}{\alpha R} (\delta^2 - \frac{\delta}{6} + \frac{7}{240}) . \]

Those terms which are proportional to inverse degree of \( \alpha \) come from exponential part of the uniform expansion and respectively for \( T(s) \) (10). Terms which are linear in \( \alpha^3 \) or \( \alpha^0 \) come from the series \( \sum u_k/\mu^k \) in (4).

Now we may compare our results with well-known formulas given in Refs. [25, 18, 26]. The coefficients \( B_0, B_{1/2}, B_1, B_{3/2}, \) and \( B_2 \) differ from those obtained by Elizalde [24] because of the difference in the series (12).

\[ F(q, a, b^2) = \sum_{l=0}^{\infty} \frac{1}{((l + 1/2)^2 + b^2)^q} . \]
\[ B_0 = \frac{4\pi R^3}{3\alpha} = \int_V dV , \]
\[ B_{1/2} = -2\pi^{3/2} R^2 = -\frac{\sqrt{\pi}}{2} \int_{\partial V} dS , \]
\[ B_{3/2} = -4\pi^{3/2} (1 - \alpha^2) \left( \frac{1}{6} - \xi \right) - \frac{\pi^{3/2} \alpha^2}{6} \]
\[ = -\frac{\pi^{1/2}}{192} \int_{\partial V} (-96\xi R + 16R + 8R_{ik} N^i N^k + 7(tr K)^2 - 10tr K^2) dS . \]

We divide the coefficient \( B_1 \) into volume and boundary contributions, according with general formulas and an additional topological contribution \( B^{\text{top}}_1 \) which has not obtained before:
\[ B_1 = \int_V b_1(x) dV + \int_{\partial V} c_1(x) dS + B^{\text{top}}_1 , \]
where
\[ b_1(x) = \left( \frac{1}{6} - \xi \right) R , \quad c_1(x) = \frac{1}{3} tr K , \quad B^{\text{top}}_1 = \frac{\pi R}{3} \left( \frac{1}{\alpha} - \alpha \right) . \]

Some problems are connected with the term \( B_2 \) because in this case the volume part of \( B_2 \) [27] is proportional to \( 1/r^4 \) and the integral over volume will divergent at origin. This problem has been already discussed by Cheeger [28], and Brüning and Seeley [29] using \( \text{partie finite} \) of the integral. Then we have good agreement with general formulas (for boundary contributions \( c_2 \) see Ref. [26])
\[ B_2 = \int_V Fb_2(x) dV + \int_{\partial V} Fc_2 dS \]
\[ = -\frac{4\pi}{\alpha R} \left( 2(1 - \alpha^2)^2 \left( \frac{1}{6} - \xi \right)^2 + 2\alpha^2 \left( \frac{1}{5} - \xi \right) \left( \frac{1}{6} - \xi \right) + \frac{1 - \alpha^2)^2}{90} \right) \frac{16\pi \alpha^3}{315R} . \]

IV. GROUND STATE ENERGY

Using the results of previous section and Eq.(1) we obtain the following formula for regularized ground state energy of massive scalar field in the global monopole background
\[ E(s) = \frac{1}{2} M^2 \zeta_A (s - 1/2) \]
\[ = \left( \frac{M}{m} \right)^{2s} \frac{1}{2(4\pi)^{3/2}} \left\{ B_0 m^4 \frac{\Gamma(s - 2)}{\Gamma(s - 1/2)} + B_{1/2} m^3 \frac{\Gamma(s - 3/2)}{\Gamma(s - 1/2)} + B_1 m^2 \frac{\Gamma(s - 1)}{\Gamma(s - 1/2)} + B_{3/2} m + B_2 \frac{\Gamma(s)}{\Gamma(s - 1/2)} + \ldots \right\} , \]
where the heat kernel coefficients \( B_k \) are given by expressions [13], [14] and [15].

This series has the form the Schwinger - DeWitt expansion with additional topological contribution term \( B^{\text{top}}_1 \) given by Eq.(14). All divergencies of the energy for \( s \to 0 \) are contained in these first five terms (three terms in the case without boundary [30]). The next terms are finite in the limit \( s \to 0 \) and they have the following structure:
\[ \sum_{k=1}^{\infty} \frac{D_k}{m^k R^{k+1}} . \]
\[ \lim_{m \to \infty} E_{ren} = 0. \]

For this reason for renormalization we subtract first five terms and because we are interested in the quantum effects which don’t depend on the boundary we must turn \( R \) to infinity. Obviously at the end of the calculation, the ground state energy will be zero.

### V. DISCUSSION AND CONCLUSION

In this paper we have investigated the zeta function of the Laplace operator and the ground state energy of massive scalar field on the global monopole background. In the calculations we have considered pointlike global monopole and took the metric in the form given by Eq.(2).

First of all we calculated zeta function of the Laplace operator \( \hat{A} = -\Delta + \xi R + m^2 \) on three dimensional section \( (t = \text{const}) \) of global monopole spacetime with Dirichlet boundary condition for the field on the surface of sphere of radius \( R \). We have rederived first five heat kernel coefficients (13), (14) and (15) which were firstly obtained in Ref.[22]. It is worth to mention that the term \( B_1 \) contains an additional contribution, \( B_{\text{top}}^1 \), which is due to the nontrivial topology of the spacetime itself, besides the usual volume and boundary parts. The topological contribution has the form below

\[ B_{\text{top}}^1 = \frac{\pi R}{3} \left( \frac{1}{\alpha} - \alpha \right). \]

As far as we know this is the first time where the purely topological effects could be separated from the geometrical one, in the calculation of the energy. As a matter of fact the parameter \( \alpha \) is related with the topology as well as with the curvature.

The above expression is in agreement with the case of a conical spacetime. Indeed, for \( t = \text{const} \) and \( \theta = \pi/2 \), Eq.(2) has the conical structure

\[ ds^2 = \alpha^{-2} dr^2 + r^2 d\varphi^2. \]

After trivial coordinate changing \( r = \rho \alpha \), \( \alpha = 1/\nu \) our topological term coincides with a similar one in conical spacetime (see for example [12]). We have to take into account only that in two dimensions coefficient \( B_1 \) is dimensionless and we must drop the radius \( R \) from \( B_{\text{top}}^1 \).

Next we obtained an expression for regularized ground state energy of massive scalar field with arbitrary nonconformal coupling \( \xi \). In the zeta regularization approach the ground state energy is proportional to the zeta function of the Laplace operator on three dimensional spatial section of global monopole manifold. Taking into account the renormalization prescription [30] we obtain that the ground state energy is zero. This result coincides with the result for cosmic string spacetime [12]. It is easy to understand this from dimensional consideration. In both cases there is no dimensional parameters. There are only two dimensionless parameters \( \nu = 1 - 4G\mu/c^2 \) and \( \alpha^2 = 1 - 8\pi G\eta^2/c^4 \) for cosmic string and global monopole spacetime, respectively. Parameters \( \mu \) and \( \eta^2 \) have dimensions of mass per unit length. Therefore, the ground state energy has the following structure

\[ E = \frac{\hbar c}{f(m,c)}, \]

where the function \( f \) with dimension \( L \) depends on mass \( m \) and velocity of light (gravitational constant \( G \) has already absorbed by dimensionless parameters \( \nu \) and \( \alpha \)). Obviously there is no way to construct quantity with dimension of length using only \( m \) and \( c \). In the case of energy density there is natural variable with dimension of length, the radial coordinate \( r \) and that is why one can construct some expression for energy density. In the case of global monopole with non zero core there will be a parameter with dimension of length, namely size of core and we expect that ground state energy will depend on this parameter. But this question should be investigated in a separate paper.

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