Modeling of fractional-order COVID-19 epidemic model with quarantine and social distancing

Muhammad Farman | Muhammad Aslam | Ali Akgül | Aqeel Ahmad

Different countries of the world are facing a serious pandemic of coronavirus disease (COVID-19). One of the most typical treatments for COVID-19 is social distancing, which includes lockdown; it will help to decrease the number of contacts for undiagnosed individuals. The main aim of this article is to construct and evaluate a fractional-order COVID-19 epidemic model with quarantine and social distancing. Laplace homotopy analysis method is used for a system of fractional differential equation (FDEs) with Caputo and Atangana–Baleanu–Caputo (ABC) fractional derivative. By applying the ABC and Caputo derivative, the numerical solution for fractional-order COVID-19 epidemic model is achieved. The uniqueness and existence of the solution is checked by Picard–Lindelof’s method. The proposed fractional model is demonstrated by numerical simulation which is useful for the government to control the spread of disease in a practical way.

KEYWORDS
ABC fractional-order derivative, Caputo fractional derivative, COVID-19 model, quarantine, social distancing

MSC CLASSIFICATION
26A33

1 INTRODUCTION

Epidemiological study plays an essential role to understand the impact of infectious disease in a community. In mathematical modeling, we check models, estimate parameters, measure sensitivity through different parameters, and compute their numerical simulations through building models. The control parameters and ratio spread of disease can be understood through this type of research. These types of disease models are often called infectious diseases (i.e., diseases which are transferred from one person to another person). Measles, rubella, chicken pox, mumps, AIDS, gonorrhea, and syphilis are examples of infectious diseases.

Coronavirus causes severe respiratory syndrome (SARS) and plays an important role in its investigation. According to the group of investigators that has been working for 30 years on the coronavirus family, SARS and coronavirus have many similar features like biology and pathogenesis. RNA-enveloped viruses known as coronavirus are spread in populations of humans, mammals, and birds. Many respiratory, enteric, hepatic, and neurological diseases are caused by coronavirus. Human disease is caused by six types of coronavirus. In 2019, A major coronavirus 2019 (COVID-19) outbreak was faced by China, and after some time this outbreak became pandemic to the world. Interventions and real-time data are necessary for the control of this outbreak of coronavirus. In previous studies, the transfer of the virus from one person...
to another person, its severity, and the history of the pathogen in the first week of the outbreak has been explained with the help of real-time analyses. In December 2019, a group of people in Wuhan were admitted to the hospital; all were suffering from pneumonia, and the cause of pneumonia was idiopathic. Most people linked the cause of pneumonia with eating of wet market meat and seafood. Investigation on the etiology and epidemiology of the disease was conducted on December 31, 2019, by the Chinese Center for Disease Control and Prevention (China CDC) with the help of the Wuhan city health authorities. Changing in epidemiology was measured by time-delay distributions including date of admission to hospital and death. According to the clinical study on the COVID-19, symptoms of coronavirus appear after 7 days of onset of illness, and fatality risk cases are underestimated, which is critical.8,9

The fractional order that involves integration and transects differentiation with the help of fractional calculus can also help to better understand the explanation of real-world problems than ordinary integer order, as well as in the modeling of real phenomena due to a characterization of the memory and hereditary properties.10,11 The idea of fractional derivative is introduced by Riemann Liouville which is based on power law. The new fractional derivative which is applying the exponential-decay kernel is proposed by Caputo and Fabrizio12 after Caputo–Frabrizio13–16 represents non-singular kernel fractional derivative which includes the trigonometric and exponential function, and the literature17–25 shows some related approaches for models of epidemic. The proposed outbreak of this virus effectively catches the timeline for the COVID-19 disease conceptual model.26–28 Analysis of a mathematical model for the spread of the COVID-19 disease by different mathematical techniques is discussed.29–34 For more details see other works.35–38

We analyse the mathematical model of COVID-19 with quarantine and social distancing in Section 2. In Section 3, Laplace homotopy analysis method for system of FDEs with Caputo sense is used for simulation including numerical solution by Atangana–Baleanu–Caputo (ABC) sense using Picard–Lindelof which presents the existence of the solution. The proposed scheme which provides better results is discussed in Section 4. We give the conclusion of this study in Section 5.

2 | PRELIMINARIES

The Liouville–Caputo fractional derivative (C) is presented as

\[
\begin{align*}
\frac{\mathcal{C}^\xi}{\mathcal{I}^\eta} g(t) & = \frac{1}{\Gamma(1-\xi)} \int_{t_0}^{t} \frac{d}{dt} g(\tau)(t-\tau)^{-\xi}, \quad n-1 < \xi \leq n, \\
\end{align*}
\]

where gamma function is represented by \(\Gamma(.)\).11,12,39,40 Laplace transform for CF-order derivative is as follows

\[
\mathcal{L}\{\frac{\mathcal{C}^\xi}{\mathcal{I}^\eta} g(t)\}(s) = S^\xi G(s) - \sum_{k=0}^{n-1} S^{\xi-k-1} g^{(k)}(0)
\]

Recently, a fractional derivative proposed by Atangana and Baleanu with the combination of Mittag-Leffler function as the kernel of differentiation. This kernel is non-singular and nonlocal in character and has benefits of the above mentioned Liouville–Caputo derivative as preservation.39,40

Let \(f \in H^1(a, b), \ a > b, \xi \in [0,1]\). The definition of the ABC derivative in Caputo sense which is the latest work presented in the literature is as follows:

\[
\mathcal{A}^{\mathcal{B}C}_t \frac{\mathcal{D}^\xi}{\mathcal{I}^\eta} g(t) = \frac{B(\xi)}{1-\xi} \int_{t_0}^{t} g'(\tau) E_\xi \left[ -\frac{(t-\tau)^\xi}{1-\xi} \right] \, d\tau, \quad n-1 < \xi(t \leq n)
\]

where normalization function is represented by \(\xi \in \mathbb{R}\), \(B(\xi)\) and Mittag-Leffler function is represented by \(B(0) = B(1) = 1\) and \(E_\xi(.)\). We can write the above equation in Laplace transform

\[
\mathcal{L}\{\frac{\mathcal{A}^{\mathcal{B}C}_t}{\mathcal{I}^\eta} g(t)\}(s) = \frac{B(\xi)}{1-\xi} \mathcal{L} \left[ \int_{t_0}^{t} \frac{d}{dt} g(\tau) E_\xi \left[ -\frac{(t-\tau)^\xi}{1-\xi} \right] \, d\tau \right](s) = \frac{B(\xi) s^\xi \mathcal{L}[g(t)](s) - s^{\xi-1} g(0)}{s^\xi + \frac{\xi}{1-\xi}}
\]
3 | FRACTIONAL-ORDER EPIDEMIC MODEL WITH LAPLACE HOMOTOPY METHOD

3.1 | Caputo derivative for epidemic model

The combination of Laplace transform and homotopy analysis technique makes an analytical method which is known as Modified Homotop Analysis Transform Method (MHATM) in the literature.\textsuperscript{39,41,42} The main steps of the modified technique are written as follows:

\textbf{Step 1.} Suppose that, we have the following equation

\[ D_\rho^\alpha [g(a, t)] + \beta \{a\}g(a, t) + \Lambda [a]g(a, t) = \Psi(a, t), \quad t > 0, \quad a \in \mathbb{R}, \quad 0 < \rho \leq 1, \]  

(5)

where fixed linear operation in $a$ is $\beta [a]$.  

\textbf{Step 2.} In applying the methods suggested, we get the equation of $m$th order deformation which is as below:

\[ g_m(a, t) = (X_m + h)g_{m-1} - h(1 - X_m) \sum_{i=0}^{j-1} t^i g^{(i-1)}(0) \]

\[ + h \mathcal{L}^{-1} \left( \frac{1}{s^\rho} \mathcal{L} \left( \Xi_{m-1}[a]g_{m-1}(a) + \sum_{k=0}^{m-1} P_k(g_0, g_1, \ldots, g_m) - \Psi(a, t) \right) \right) \]

(6)

\textbf{Step 3.} $\Lambda [a]g(a, t)$ is the nonlinear definition, and it has polynomial homotopes.

\[ \Lambda [g(a, t)] = \Lambda \left( \sum_{k=0}^{m-1} g_m(a, t) \right) = \sum_{m=0}^{\infty} P_m g^m \]

\textbf{Step 4.} Extend a nonlinear term to (6), a sequence of homotopical polynomials, for $g_m(a, t)$ to $m > 1$, and for an explanation of the solutions for Equation (5), which normally converge rapidly to exact solutions

\[ g(a, t) = \sum_{m=0}^{\infty} g_m(a, t). \]

Kuniya and Inaba\textsuperscript{43} studied a classical form of model for the first time. Time-fractional funding model which is solved by the derivative of the fractional-order Liouville–Caputo is as below\textsuperscript{31} on the basis of this methodology.

\[ \begin{align*}
\frac{\partial^\alpha}{\partial \tau^\alpha} S(\tau) &= -\beta_1 E(\tau) S(\tau) - \beta_2 I(\tau) S(\tau) - k S(\tau) + k q S(\tau) \\
\frac{\partial^\alpha}{\partial \tau^\alpha} E(\tau) &= \beta_1 E(\tau) S(\tau) + \beta_2 I(\tau) S(\tau) - \epsilon E(\tau) - k p E(\tau) \\
\frac{\partial^\alpha}{\partial \tau^\alpha} I(\tau) &= \epsilon E(\tau) - \gamma I(\tau) - k p I(\tau) \\
\frac{\partial^\alpha}{\partial \tau^\alpha} R(\tau) &= \gamma I(\tau) + \eta Q(\tau) \\
\frac{\partial^\alpha}{\partial \tau^\alpha} Q(\tau) &= k S(\tau) - k q S(\tau) + k p E(\tau) + k p I(\tau) - \eta Q(\tau) 
\end{align*} \]

(7)

with initial conditions $S(0) = 1 - \frac{1}{1.25 \times 10^9}, I(0) = \frac{1}{1.25 \times 10^9}, E(0) = R(0) = Q(0) = 0$

Let $S$ be the susceptible, $E$ the asymptomatic infected, $I$ the symptomatic infected, $R$ the recovered, and $Q$ be the quarantined population, where $\beta_1$ denotes the asymptomatic transmission rate, $\beta_2$ the symptomatic transmission rate, $\epsilon$ the onset rate, $\gamma$ the recovery rate for infected, and $\eta$ denotes the recovery rate for the quarantined population. Next, here we consider a situation that the susceptible and infected are all assumed to be exposed to massive testing (PCR test) with testing rate $k$ followed by case isolation. Let $p \epsilon (0, 1)$ be the sensitivity of the test, and $q \epsilon (0, 1)$ be the specificity of the test.

\textbf{Solution.} We utilize the transform Laplace (1) for the first system Equation (7).

\[ s^\rho \mathcal{L}[S(s)] - s^{\rho-1} S(0) = \mathcal{L} \{ -\beta_1 E(\tau) S(\tau) - \beta_2 I(\tau) S(\tau) - k S(\tau) + k q S(\tau) \} \]
By using the initial condition, we get

\[ \bar{S}(s) = \frac{S(0)}{s} + \mathcal{L}\{-\beta_1 E(r)S(r) - \beta_2 I(r)S(r) - kS(r) + kqS(r)\} \tag{8} \]

through applying Laplace inverse at (8), we obtain the following results

\[ S(r) = S_0 + \mathcal{L}^{-1}\left(\frac{1}{s^p}\mathcal{L}\{-\beta_1 E(r)S(r) - \beta_2 I(r)S(r) - kS(r) + kqS(r)\}\right) \]

We have for the other Equations (7) shown

\[
\begin{align*}
E(r) &= E_0 + \mathcal{L}^{-1}\left(\frac{1}{s^p}\mathcal{L}\{\beta_1 E(r)S(r) + \beta_2 I(r)S(r) - eE(r) - kpE(r)\}\right) \\
I(r) &= I_0 + \mathcal{L}^{-1}\left(\frac{1}{s^p}\mathcal{L}\{eE(r) - \gamma I(r) - kpI(r)\}\right) \\
R(r) &= R_0 + \mathcal{L}^{-1}\left(\frac{1}{s^p}\mathcal{L}\{\gamma I(r) + \eta Q(r)\}\right) \\
Q(r) &= Q_0 + \mathcal{L}^{-1}\left(\frac{1}{s^p}\mathcal{L}\{kS(r) - kqS(r) + kpE(r) + kpI(r) - \eta Q(r)\}\right)
\end{align*}
\]

By using linear operator, we get

\[ [\phi_j(r; q)] = \mathcal{L}[\phi_j(r; q)], \quad j = 1, 2, 3 \]

c = 0 with constants, where constant is represented by c. Then the device is established

\[
\begin{align*}
N[\phi_1(r; q)] &= \mathcal{L}[\phi_1(r; q)] - S_0 - \frac{1}{s^p}\mathcal{L}\{-\beta_1 \phi_2 \phi_1 - \beta_2 \phi_3 \phi_1 - k\phi_1 + kq\phi_1\} \\
N[\phi_2(r; q)] &= \mathcal{L}[\phi_2(r; q)] - E_0 - \frac{1}{s^p}\mathcal{L}\{\beta_1 \phi_2 \phi_1 + \beta_2 \phi_3 \phi_1 + e\phi_2 - kp\phi_2\} \\
N[\phi_3(r; q)] &= \mathcal{L}[\phi_3(r; q)] - I_0 - \frac{1}{s^p}\mathcal{L}\{e\phi_2 - \gamma \phi_3 - kp\phi_3\} \\
N[\phi_4(r; q)] &= \mathcal{L}[\phi_4(r; q)] - R_0 - \frac{1}{s^p}\mathcal{L}\{\gamma \phi_3 + \eta \phi_5\} \\
N[\phi_5(r; q)] &= \mathcal{L}[\phi_5(r; q)] - Q_0 - \frac{1}{s^p}\mathcal{L}\{k\phi_1 - kq\phi_1 + kp\phi_2 + kp\phi_3 - \eta Q\phi_5\}
\end{align*}
\]

The ostensible zero order deformation equation is stated as follows

\[(1 - q)[\phi_j(r; q) - u_0(r)] = qh[\phi_j(r; q)], \quad j = 1, 2, 3, 4, 5\]

We have \(q = 0\) and \(q = 1\),

\[ \phi_j(r; 0) = u_0(r), \quad \phi_j(r; 1) = u(r), \quad j = 1, 2, 3 \]

A system in \(m\)th-order deformation is given as

\[
\begin{align*}
\mathcal{L}\{S_m(r) - P_mS_{m-1}(r)\} &= hA_m(S_{m-1}^-, r) \\
\mathcal{L}\{E_m(r) - P_mE_{m-1}(r)\} &= hA_m(E_{m-1}^-, r) \\
\mathcal{L}\{I_m(r) - P_mI_{m-1}(r)\} &= hA_m(I_{m-1}^-, r) \\
\mathcal{L}\{R_m(r) - P_mR_{m-1}(r)\} &= hA_m(R_{m-1}^-, r) \\
\mathcal{L}\{Q_m(r) - P_mQ_{m-1}(r)\} &= hA_m(Q_{m-1}^-, r)
\end{align*}
\]  

(9)  

(10)

by applying the inverse Laplace transformation, we get Equation (9)
\[ S_m(\tau) = P_m S_{m-1}(\tau) + h A_m(S_{m-1}^{-}, \tau) \]
\[ E_m(\tau) = P_m E_{m-1}(\tau) + h A_m(E_{m-1}^{-}, \tau) \]
\[ I_m(\tau) = P_m I_{m-1}(\tau) + h A_m(I_{m-1}^{-}, \tau) \]
\[ R_m(\tau) = P_m R_{m-1}(\tau) + h A_m(R_{m-1}^{-}, \tau) \]
\[ Q_m(\tau) = P_m Q_{m-1}(\tau) + h A_m(Q_{m-1}^{-}, \tau) \]

where

\[ A_m(S_{m-1}^{-}, \tau) = \mathcal{L}[S_{m-1}(\tau)] - (1 - P_m) \left( S_0 + \frac{1}{S_0} \mathcal{L}\{ - \beta_1 X_m - \beta_2 Y_m - k S_{m-1} + k q S_{m-1} \} \right) \]
\[ A_m(E_{m-1}^{-}, \tau) = \mathcal{L}[E_{m-1}(\tau)] - (1 - P_m) \left( E_0 + \frac{1}{S_0} \mathcal{L}\{ \beta_1 X_m + \beta_2 Y_m - \epsilon E_{m-1} - k p E_{m-1} \} \right) \]
\[ A_m(I_{m-1}^{-}, \tau) = \mathcal{L}[I_{m-1}(\tau)] - (1 - P_m) \left( I_0 + \frac{1}{S_0} \mathcal{L}\{ \epsilon E_{m-1} - \gamma I_{m-1} - k p I_{m-1} \} \right) \]
\[ A_m(R_{m-1}^{-}, \tau) = \mathcal{L}[R_{m-1}(\tau)] - (1 - P_m) \left( R_0 + \frac{1}{S_0} \mathcal{L}\{ \gamma I_{m-1} + \eta Q_{m-1} \} \right) \]
\[ A_m(Q_{m-1}^{-}, \tau) = \mathcal{L}[Q_{m-1}(\tau)] - (1 - P_m) \left( Q_0 + \frac{1}{S_0} \mathcal{L}\{ k S_{m-1} - k q S_{m-1} + k p E_{m-1} + k p I_{m-1} - \eta Q_{m-1} \} \right) \]

The solution of Equation (9) is written as

\[ S_m(\tau) = (P_m + h) S_{m-1} - h(1 - P_m)(S_0) - h \mathcal{L}^{-1} \left\{ \frac{1}{S_0} \mathcal{L}\{ - \beta_1 X_m - \beta_2 Y_m - k S_{m-1} + k q S_{m-1} \} \right\} \]
\[ E_m(\tau) = (P_m + h) E_{m-1} - h(1 - P_m)(E_0) - h \mathcal{L}^{-1} \left\{ \frac{1}{S_0} \mathcal{L}\{ \beta_1 X_m + \beta_2 Y_m - \epsilon E_{m-1} - k p E_{m-1} \} \right\} \]  \hspace{1cm} (11)
\[ I_m(\tau) = (P_m + h) I_{m-1} - h(1 - P_m)(I_0) - h \mathcal{L}^{-1} \left\{ \frac{1}{S_0} \mathcal{L}\{ \epsilon E_{m-1} - \gamma I_{m-1} - k p I_{m-1} \} \right\} \]
\[ R_m(\tau) = (P_m + h) R_{m-1} - h(1 - P_m)(R_0) - h \mathcal{L}^{-1} \left\{ \frac{1}{S_0} \mathcal{L}\{ \gamma I_{m-1} + \eta Q_{m-1} \} \right\} \]
\[ Q_m(\tau) = (P_m + h) Q_{m-1} - h(1 - P_m)(Q_0) - h \mathcal{L}^{-1} \left\{ \frac{1}{S_0} \mathcal{L}\{ k S_{m-1} - k q S_{m-1} + k p E_{m-1} + k p I_{m-1} - \eta Q_{m-1} \} \right\} \]

where

\[ X_m = \frac{1}{\Gamma m + 1} \left[ \frac{d^m}{dq^m} N[(q \phi_1(\tau; q))(q \phi_2(\tau; q))] \right]_{q=0'} \] \hspace{1cm} (12)
\[ Y_m = \frac{1}{\Gamma m + 1} \left[ \frac{d^m}{dq^m} N[(q \phi_1(\tau; q))(q \phi_3(\tau; q))] \right]_{q=0'} \] \hspace{1cm} (13)

Eventually, the solutions of Equation (7)

\[ S(\tau) = \sum_{m=0}^{\infty} S_m(\tau) \]
\[ E(\tau) = \sum_{m=0}^{\infty} E_m(\tau) \]
\[ I(\tau) = \sum_{m=0}^{\infty} I_m(\tau) \]
\[ R(\tau) = \sum_{m=0}^{\infty} R_m(\tau) \]
\[ Q(\tau) = \sum_{m=0}^{\infty} Q_m(\tau) \] \hspace{1cm} (14)
The solution of Equation (7) with iterative method is given as

\[
S_n(\tau) = S_0 + \mathcal{L}^{-1} \left\{ \frac{1}{s^\rho} \mathcal{L} \left\{ -\beta_1 E_{n-1}(\tau) S_{n-1}(\tau) - \beta_2 I_{n-1}(\tau) S_{n-1}(\tau) - k S_{n-1}(\tau) + k q S_{n-1}(\tau) \right\}(s) \right\}(\tau)
\]

\[
E_n(\tau) = E_0 + \mathcal{L}^{-1} \left\{ \frac{1}{s^\rho} \mathcal{L} \left\{ \beta_1 E_{n-1}(\tau) S_{n-1}(\tau) + \beta_2 I_{n-1}(\tau) S_{n-1}(\tau) - \epsilon E_{n-1}(\tau) - k p E_{n-1}(\tau) \right\}(s) \right\}(\tau)
\]

\[
I_n(\tau) = I_0 + \mathcal{L}^{-1} \left\{ \frac{1}{s^\rho} \mathcal{L} \left\{ \epsilon E_{n-1}(\tau) - \gamma I_{n-1}(\tau) - k p I_{n-1}(\tau) \right\}(s) \right\}(\tau)
\]

\[
R_n(\tau) = R_0 + \mathcal{L}^{-1} \left\{ \frac{1}{s^\rho} \mathcal{L} \left\{ \gamma I_{n-1}(\tau) + \eta Q_{n-1}(\tau) \right\}(s) \right\}(\tau)
\]

\[
Q_n(\tau) = Q_0 + \mathcal{L}^{-1} \left\{ \frac{1}{s^\rho} \mathcal{L} \left\{ k S_{n-1}(\tau) - k q S_{n-1}(\tau) + k p E_{n-1}(\tau) + k p I_{n-1}(\tau) - \eta Q_{n-1}(\tau) \right\}(s) \right\}(\tau)
\]

In the initial conditions of \(S_0, E_0, I_0, R_0\), and \(Q_0\), if \(n\) tends to infinity, it is supposed that the solution is a limit

\[
S(\tau) = \lim_{n \to \infty} S_n(\tau), E(\tau) = \lim_{n \to \infty} E_n(\tau), I(\tau) = \lim_{n \to \infty} I_n(\tau), R(\tau) = \lim_{n \to \infty} R_n(\tau), Q(\tau) = \lim_{n \to \infty} Q_n(\tau)
\]

**Theorem 3.1.** A firm recursive system is provided through Equation (12).

**Proof.** We assume it as a given. Five positives \(K, L, M, N, O\) can be written as \(0 \leq \tau \leq \infty\)

\[
\|S(\tau)\| < K; \|E(\tau)\| < L; \|I(\tau)\| < M; \|R(\tau)\| < N; \|Q(\tau)\| < O;
\]

Now we take a \(L^2(e, f)(0, W)\) subset as defined by

\[
\Xi = \left\{ \rho : (e, f)(0, W) \rightarrow \Xi, \frac{1}{\Gamma(\rho)} \int (\tau - \beta)^{(\rho-1)} \psi(\beta) u(\beta) d\beta < \infty \right\}
\]

by supposing \(\Theta\) is known as the following operator

\[
\Theta(S, E, I, R, Q) = -\beta_1 E(\tau) S(\tau) - \beta_2 I(\tau) S(\tau) - k S(\tau) + k q S(\tau)
\]

\[
= \beta_1 E(\tau) S(\tau) + \beta_2 I(\tau) S(\tau) - \epsilon E(\tau) - k p E(\tau)
\]

\[
= \epsilon E(\tau) - \gamma I(\tau) - k p I(\tau)
\]

\[
= \gamma I(\tau) + \eta Q(\tau)
\]

\[
= k S(\tau) - k q S(\tau) + k p E(\tau) + k p I(\tau) - \eta Q(\tau)
\]

Then

\[
< \Theta(S, E, I, R, Q) - \Theta(S_1, E_1, I_1, R_1, Q_1), (S - S_1, E - E_1, I - I_1, R - R_1, Q - Q_1) >,
\]

\[
< -\beta_1 (E(\tau) - E_1(\tau)) (S(\tau) - S_1(\tau)) - \beta_2 (I(\tau) - I_1(\tau)) (S(\tau) - S_1(\tau)) - k (S(\tau) - S_1(\tau)) + k q (S(\tau) - S_1(\tau)) >
\]

\[
< \beta_1 (E(\tau) - E_1(\tau)) (S(\tau) - S_1(\tau)) + \beta_2 (I(\tau) - I_1(\tau)) (S(\tau) - S_1(\tau)) - \epsilon (E(\tau) - E_1(\tau)) - k p (E(\tau) - E_1(\tau)) >
\]

\[
< \epsilon (E(\tau) - E_1(\tau)) - \gamma (I(\tau) - I_1(\tau)) - k p (I(\tau) - I_1(\tau)) >,
\]

\[
< \gamma (I(\tau) - I_1(\tau)) + \eta (Q(\tau) - Q_1(\tau)) >
\]

\[
< k (S(\tau) - S_1(\tau)) - k q (S(\tau) - S_1(\tau)) + k p (E(\tau) - E_1(\tau)) + k p (I(\tau) - I_1(\tau)) - \eta (Q(\tau) - Q_1(\tau)) >
\]

where

\[
S(\tau) \neq S_1(\tau); E(\tau) \neq E_1(\tau); I(\tau) \neq I_1(\tau); R(\tau) \neq R_1(\tau); Q(\tau) \neq Q_1(\tau)
\]
Thus, we get the norm and the absolute value on both sides

\[
< \Theta(S, E, I, R, Q) - \Theta(S_1, E_1, I_1, R_1, Q_1), (S - S_1, E - E_1, I - I_1, R - R_1, Q - Q_1) >
\]

\[
< \{ -\beta_1\|E(\tau) - E_1(\tau)\| - \beta_2\|I(\tau) - I_1(\tau)\| - \kappa + kq \} \|S(\tau) - S_1(\tau)\|^2
\]

\[
\left\{ \beta_1\|S(\tau) - S_1(\tau)\| + \beta_2\frac{\|I(\tau) - I_1(\tau)\|\|S(\tau) - S_1(\tau)\|}{\|E(\tau) - E_1(\tau)\|} - \epsilon - kp \right\} \|E(\tau) - E_1(\tau)\|^2
\]

\[
< c\frac{\|E(\tau) - E_1(\tau)\|}{\|I(\tau) - I_1(\tau)\|} - \gamma - kp \right\} \|I(\tau) - I_1(\tau)\|^2
\]

\[
< \left\{ \gamma \frac{\|I(\tau) - I_1(\tau)\|}{\|R(\tau) - R_1(\tau)\|} + \eta \right\} \|Q(\tau) - Q_1(\tau)\|^2
\]

\[
< \left\{ k \frac{\|S(\tau) - S_1(\tau)\|}{\|Q(\tau) - Q_1(\tau)\|} - \kappa \right\} \|S(\tau) - S_1(\tau)\|^2 + kp \frac{\|E(\tau) - E_1(\tau)\|}{\|Q(\tau) - Q_1(\tau)\|}
\]

\[
< \left\{ k \frac{\|I(\tau) - I_1(\tau)\|}{\|Q(\tau) - Q_1(\tau)\|} - \eta \right\} \|Q(\tau) - Q_1(\tau)\|^2
\]

where

\[
< \Theta(S, E, I, R, Q) - \Theta(S_1, E_1, I_1, R_1, Q_1), (S - S_1, E - E_1, I - I_1, R - R_1, Q - Q_1) >
\]

\[
< K\|S(\tau) - S_1(\tau)\|^2,
\]

\[
< L\|E(\tau) - E_1(\tau)\|^2,
\]

\[
< M\|I(\tau) - I_1(\tau)\|^2,
\]

\[
< N\|R(\tau) - R_1(\tau)\|^2,
\]

\[
< O\|Q(\tau) - Q_1(\tau)\|^2,
\]

with

\[
K = \frac{\|z(\tau) - z_1(\tau)\|}{\|x(\tau) - x_1(\tau)\|} + \|y(\tau) - y_1(\tau)\| - \alpha
\]

\[
L = \left\{ \frac{1}{\|y(\tau) - y_1(\tau)\|} - b - \frac{\|x(\tau) - x_1(\tau)\|^2}{\|y(\tau) - y_1(\tau)\|} \right\}
\]

\[
M = \left\{ -\frac{\|x(\tau) - x_1(\tau)\|}{\|z(\tau) - z_1(\tau)\|} - c \right\}
\]

\[
N = \left\{ \frac{1}{\|y(\tau) - y_1(\tau)\|} - b - \frac{\|x(\tau) - x_1(\tau)\|^2}{\|y(\tau) - y_1(\tau)\|} \right\}
\]

\[
O = \left\{ \frac{\|x(\tau) - x_1(\tau)\|}{\|z(\tau) - z_1(\tau)\|} - c \right\}
\]

If a non-null vector \((S_1, E_1, I_1, R_1, Q_1)\) is also considered, by applying a certain routine like which is mentioned above, we obtain

\[
< \Theta(S, E, I, R, Q) - \Theta(S_1, E_1, I_1, R_1, Q_1), (S - S_1, E - E_1, I - I_1, R - R_1, Q - Q_1) >,
\]

\[
< K\|S(\tau) - S_1(\tau)\|\|S(\tau)\|
\]

\[
< L\|E(\tau) - E_1(\tau)\|\|E(\tau)\|
\]

\[
< M\|I(\tau) - I_1(\tau)\|\|I(\tau)\|
\]

\[
< N\|R(\tau) - R_1(\tau)\|\|R(\tau)\|
\]

\[
< O\|Q(\tau) - Q_1(\tau)\|\|Q(\tau)\|
\]

The conclusion which we obtain from our results of Equations (13) and (14) is that the iterative process is stable.
3.2 | ABC derivative for epidemic model

The methodology is explained in Morales-Delgado et al.\textsuperscript{31} and Atangana and Baleanu\textsuperscript{32}; we solve the time-fractional COVID-19 system which is as below, through ABC fractional-order derivative

\[
\begin{align*}
\frac{ABC D^\alpha S(\tau)}{0} &= -\beta_1 ES - \beta_2 IS - kS + kqS \\
\frac{ABC D^\alpha E(\tau)}{0} &= \beta_1 ES + \beta_2 IS - cE - kpE \\
\frac{ABC D^\alpha I(\tau)}{0} &= \epsilon E - \gamma I - kpI \\
\frac{ABC D^\alpha R(\tau)}{0} &= \gamma I + \eta Q \\
\frac{ABC D^\alpha Q(\tau)}{0} &= kS - kqS + kpE + kpI - \eta Q
\end{align*}
\] (20)

with given initial conditions. \(S(0) = 1 - \frac{1}{1.26 + 10^9}\), \(I(0) = \frac{1}{1.26 + 10^9}\), \(E(0) = R(0) = Q(0) = 0\)

**Solution.** By using The Laplace transform given in Equation (3) for system (16), we have

\[
\frac{B(\rho)}{1 - \rho} \frac{s^\alpha \hat{S}(s)}{s^\alpha + \frac{\rho}{1 - \rho}} = \mathcal{L}\{ -\beta_1 E(\tau)S(\tau) - \beta_2 I(\tau)S(\tau) - kS(\tau) + kqS(\tau) \}
\]

We get

\[
\hat{S}(s) = \frac{S(0)}{s} + \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ -\beta_1 E(\tau)S(\tau) - \beta_2 I(\tau)S(\tau) - kS(\tau) + kqS(\tau) \}, \quad (21)
\]

\[
S(\tau) = S_0 + \mathcal{L}^{-1}\left\{ \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ -\beta_1 E(\tau)S(\tau) - \beta_2 I(\tau)S(\tau) - kS(\tau) + kqS(\tau) \} \right\}
\]

We have

\[
E(\tau) = E_0 + \frac{(1 - \rho)}{B(\rho)s^\alpha} + \frac{\rho}{B(\rho)s^\alpha \rho} + \mathcal{L}^{-1}\left\{ \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ \beta_1 E(\tau)S(\tau) + \beta_2 I(\tau)S(\tau) - cE(\tau) - kpE(\tau) \} \right\},
\]

\[
I(\tau) = I_0 + \mathcal{L}^{-1}\left\{ \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ \epsilon E(\tau) - \gamma I(\tau) - kpI(\tau) \} \right\}, \quad (22)
\]

\[
R(\tau) = R_0 + \mathcal{L}^{-1}\left\{ \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ \gamma I(\tau) + \eta Q(\tau) \} \right\},
\]

\[
Q(\tau) = Q_0 + \mathcal{L}^{-1}\left\{ \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ kS(\tau) - kqS(\tau) + kpE(\tau) + kpI(\tau) - \eta Q(\tau) \} \right\},
\]

by using linear operator, we have

\[
[\phi_n(\tau; q)] = \mathcal{L}[\phi_n(\tau; q)], \quad n = 1, 2, 3.
\]

Then we have

\[
N[\phi_1(\tau; q)] = \mathcal{L}[\phi_1(\tau; q)] - S_0 - \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ -\beta_1 \phi_2 \phi_1 - \beta_2 \phi_3 \phi_1 - k\phi_1 + kq\phi_1 \}
\]

\[
N[\phi_2(\tau; q)] = \mathcal{L}[\phi_2(\tau; q)] - E_0 - \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ \beta_1 \phi_2 \phi_1 + \beta_2 \phi_3 \phi_1 - \epsilon \phi_2 - kp\phi_2 \}
\]

\[
N[\phi_3(\tau; q)] = \mathcal{L}[\phi_3(\tau; q)] - I_0 - \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ \phi_2 - \gamma \phi_3 - kp\phi_3 \}
\]

\[
N[\phi_4(\tau; q)] = \mathcal{L}[\phi_4(\tau; q)] - R_0 - \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ \gamma \phi_3 + \eta \phi_3 \}
\]

\[
N[\phi_5(\tau; q)] = \mathcal{L}[\phi_5(\tau; q)] - Q_0 - \frac{(1 - \rho)s^\alpha + \rho}{B(\rho)s^\alpha} \mathcal{L}\{ k\phi_1 - kq\phi_1 + kp\phi_2 + kp\phi_3 - \eta Q\phi_3 \}
\]
Zero-order deformation is given as

\[(1 - q)[\phi_j(\tau; q) - u_0(\tau)] = qhN[\phi_j(\tau; q)], j = 1, 2, 3\]

We have, when \(q = 0\) and \(q = 1\),

\[\phi_j(\tau; 0) = u_0(\tau), \phi_j(\tau; 1) = u(\tau), j = 1, 2, 3\]

Hence, \(m\)th-order deformation is given as

\[
\begin{align*}
\mathcal{L}\{S_m(\tau) - P_mS_{m-1}(\tau)\} &= hA_m(S_{m-1}^-\tau) \\
\mathcal{L}\{E_m(\tau) - P_mE_{m-1}(\tau)\} &= hA_m(E_{m-1}^-\tau) \\
\mathcal{L}\{I_m(\tau) - P_mI_{m-1}(\tau)\} &= hA_m(I_{m-1}^-\tau) \\
\mathcal{L}\{R_m(\tau) - P_mR_{m-1}(\tau)\} &= hA_m(R_{m-1}^-\tau) \\
\mathcal{L}\{Q_m(\tau) - P_mQ_{m-1}(\tau)\} &= hA_m(Q_{m-1}^-\tau)
\end{align*}
\]

Using Laplace inverse transform, we have

\[
\begin{align*}
S_m(\tau) &= P_mS_{m-1}(\tau) + hA_m(S_{m-1}^-\tau) \\
E_m(\tau) &= P_mE_{m-1}(\tau) + hA_m(E_{m-1}^-\tau) \\
I_m(\tau) &= P_mI_{m-1}(\tau) + hA_m(I_{m-1}^-\tau) \\
R_m(\tau) &= P_mR_{m-1}(\tau) + hA_m(R_{m-1}^-\tau) \\
Q_m(\tau) &= P_mQ_{m-1}(\tau) + hA_m(Q_{m-1}^-\tau)
\end{align*}
\]

where

\[
\begin{align*}
A_m(S_{m-1}^-\tau) &= \mathcal{L}\{S_{m-1}(\tau)\} - (1 - P_m)S_0 + \frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{-\beta_1X_0 - \beta_2Y_0 - kS_{m-1} + kqS_{m-1}\} \\
A_m(E_{m-1}^-\tau) &= \mathcal{L}\{E_{m-1}(\tau)\} - (1 - P_m)E_0 + \frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{\beta_1X_0 + \beta_2Y_0 - \epsilon E_{m-1} - kpE_{m-1}\} \\
A_m(I_{m-1}^-\tau) &= \mathcal{L}\{I_{m-1}(\tau)\} - (1 - P_m)I_0 + \frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{\epsilon E_{m-1} - \gamma I_{m-1} - kpI_{m-1}\} \\
A_m(R_{m-1}^-\tau) &= \mathcal{L}\{R_{m-1}(\tau)\} - (1 - P_m)R_0 + \frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{\gamma I_{m-1} + \eta Q_{m-1}\} \\
A_m(Q_{m-1}^-\tau) &= \mathcal{L}\{Q_{m-1}(\tau)\} - (1 - P_m)Q_0 + \frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{kS_{m-1} - kqS_{m-1} + kpE_{m-1} + kpI_{m-1} - \eta Q_{m-1}\}
\end{align*}
\]

Hence, we have

\[
\begin{align*}
S_m(\tau) &= (P_m + h)S_{m-1} - h(1 - P_m)S_0 + h\mathcal{L}^{-1}\left\{\frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{-\beta_1X_0 - \beta_2Y_0 - kS_{m-1} + kqS_{m-1}\}\right\} \\
E_m(\tau) &= (P_m + h)E_{m-1} - h(1 - P_m)E_0 + h\mathcal{L}^{-1}\left\{\frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{\beta_1X_0 + \beta_2Y_0 - \epsilon E_{m-1} - kpE_{m-1}\}\right\} \\
I_m(\tau) &= (P_m + h)I_{m-1} - h(1 - P_m)I_0 + h\mathcal{L}^{-1}\left\{\frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{\epsilon E_{m-1} - \gamma I_{m-1} - kpI_{m-1}\}\right\} \\
R_m(\tau) &= (P_m + h)R_{m-1} - h(1 - P_m)R_0 + h\mathcal{L}^{-1}\left\{\frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{\gamma I_{m-1} + \eta Q_{m-1}\}\right\} \\
Q_m(\tau) &= (P_m + h)Q_{m-1} - h(1 - P_m)Q_0 + h\mathcal{L}^{-1}\left\{\frac{(1 - \rho)s^\rho + \rho}{B(\rho)s^\rho} \mathcal{L}\{kS_{m-1} - kqS_{m-1} + kpE_{m-1} + kpI_{m-1} - \eta Q_{m-1}\}\right\}
\end{align*}
\]
where

\[ X_m = \frac{1}{\Gamma(m+1)} \left[ \frac{d^m}{dq^m} N[(q\phi_1(t; q))(q\phi_2(t; q))] \right]_{q=0}^{q=\tau} \]  
\[ Y_m = \frac{1}{\Gamma(m+1)} \left[ \frac{d^m}{dq^m} N[(q\phi_1(t; q))(q\phi_3(t; q))] \right]_{q=0}^{q=\tau} \]  

Finally, the solutions of Equation (7)

\[ S(\tau) = \sum_{m=0}^{\infty} S_m(\tau), E(\tau) = \sum_{m=0}^{\infty} E_m(\tau), I(\tau) = \sum_{m=0}^{\infty} I_m(\tau), R(\tau) = \sum_{m=0}^{\infty} R_m(\tau), Q(\tau) = \sum_{m=0}^{\infty} Q_m(\tau) \]

Another model solution (16) with Equation (4) can be acquired. In the sense of Atangana–Baleanu, the systems (16) are equal to the Volterra type. The exact solution is obtained by following scheme; a large value of \( n \) is also measured through this scheme.

\[ S_{n+1}(\tau) = \frac{1-\rho}{B(\rho)} \left\{ -\beta_1 E_n(\tau) S_n(\tau) - \beta_2 I_n(\tau) S_n(\tau) - kS_n(\tau) + kqS_n(\tau) \right\} + \frac{\rho}{B(\rho)\Gamma(\rho)} \int_0^\tau (\tau - w)^{\rho-1} \left\{ -\beta_1 E_n(w) S_n(w) - \beta_2 I_n(w) S_n(w) - kS_n(w) + kqS_n(w) \right\} dw, \]

\[ E_{n+1}(\tau) = \frac{1-\rho}{B(\rho)} \left\{ \beta_1 E_n(\tau) S_n(\tau) + \beta_2 I_n(\tau) S_n(\tau) - cE_n(\tau) - kpE_n(\tau) \right\} + \frac{\rho}{B(\rho)\Gamma(\rho)} \int_0^\tau (\tau - w)^{\rho-1} \left\{ \beta_1 E_n(w) S_n(w) + \beta_2 I_n(w) S_n(w) - cE_n(w) - kpE_n(w) \right\} dw, \]

\[ I_{n+1}(\tau) = \frac{1-\rho}{B(\rho)} \left\{ cE_n(\tau) - \gamma I_n(\tau) - kpI_n(\tau) \right\} + \frac{\rho}{B(\rho)\Gamma(\rho)} \int_0^\tau (\tau - w)^{\rho-1} \left\{ cE_n(w) - \gamma I_n(w) - kpI_n(w) \right\} dw, \]

\[ R_{n+1}(\tau) = \frac{1-\rho}{B(\rho)} \left\{ \gamma I_n(\tau) + \eta Q_n(\tau) \right\} + \frac{\rho}{B(\rho)\Gamma(\rho)} \int_0^\tau (\tau - w)^{\rho-1} \left\{ \gamma I_n(w) + \eta Q_n(w) \right\} dw, \]

\[ Q_{n+1}(\tau) = \frac{1-\rho}{B(\rho)} \left\{ kS_n(\tau) - kqS_n(\tau) + kpE_n(\tau) + kpI_n(\tau) - \eta Q_n(\tau) \right\} + \frac{\rho}{B(\rho)\Gamma(\rho)} \int_0^\tau (\tau - w)^{\rho-1} \left\{ kS_n(w) - kqS_n(w) + kpE_n(w) + kpI_n(w) - \eta Q_n(w) \right\} dw, \]

**Theorem 3.2.** The condition for the Picard–Lindelof method in Morales Delgadillo et al.\(^{39}\) is considered to show the existence of the system.

**Proof.** Considering the following operator, we have

\[ \Xi_1(t, \zeta) = -\beta_1 E(t) S(t) - \beta_2 I(t) S(t) - kS(t) + kqS(t) \]
\[ \Xi_2(t, \zeta) = \beta_1 E(t) S(t) + \beta_2 I(t) S(t) - cE(t) - kpE(t) \]
\[ \Xi_3(t, \zeta) = cE(t) - \gamma I(t) - kpI(t) \]
\[ \Xi_4(t, \zeta) = \gamma I(t) + \eta Q(t) \]
\[ \Xi_5(t, \zeta) = kS(t) - kqS(t) + kpE(t) + kpI(t) - \eta Q(t) \]
In which the contractions of \( \Xi_1(\tau, \varsigma), \Xi_2(\tau, \varsigma), \Xi_3(\tau, \varsigma), \Xi_4(\tau, \varsigma), \) and \( \Xi_5(\tau, \varsigma) \) for the first, second, third, fourth, and fifth functions, respectively, are \( \theta, \alpha, \delta, \zeta, \) and \( \nu. \) Let

\[
\begin{align*}
\Omega_1 &= \sup \|\gamma_{\tau,k_1}\Xi_1(\tau, \varsigma)\|; \\
\Omega_2 &= \sup \|\gamma_{\tau,k_2}\Xi_2(\tau, \varsigma)\|; \\
\Omega_3 &= \sup \|\gamma_{\tau,k_3}\Xi_3(\tau, \varsigma)\|; \\
\Omega_4 &= \sup \|\gamma_{\tau,k_4}\Xi_4(\tau, \varsigma)\|; \\
\Omega_5 &= \sup \|\gamma_{\tau,k_5}\Xi_5(\tau, \varsigma)\|;
\end{align*}
\]

where

\[
\begin{align*}
\gamma_{\tau,k_1} &= |\tau - a, \tau + a| \times [\theta - k_1, \theta + k_1] = e_1 \times k_1 \\
\gamma_{\tau,k_2} &= |\tau - a, \tau + a| \times [\theta - k_2, \theta + k_2] = e_1 \times k_2 \\
\gamma_{\tau,k_3} &= |\tau - a, \tau + a| \times [\theta - k_3, \theta + k_3] = e_1 \times k_3 \\
\gamma_{\tau,k_4} &= |\tau - a, \tau + a| \times [\theta - k_4, \theta + k_4] = e_1 \times k_4 \\
\gamma_{\tau,k_5} &= |\tau - a, \tau + a| \times [\theta - k_5, \theta + k_5] = e_1 \times k_5
\end{align*}
\]

Picard’s operator can be used; then it follows

\[\Theta : \gamma(e_1, k_1, k_2, k_3, k_4, k_5) \rightarrow \gamma(e_1, k_1, k_2, k_3, k_4, k_5)\]

(29)

given as

\[\Theta \Omega(\tau) = \Omega_0(\tau) \Delta(\tau, \Omega(\tau)) + \frac{1 - \rho}{B(\rho)} \int_0^\tau (\tau - w)^{-1} \Delta(w, \Omega(w))dw,\]

where

\[\Omega(\tau) = \{G(\tau), X(\tau), I(\tau)\} = \{g_1, g_2, g_3, g_4, g_5\},\]

and

\[\Delta(\tau, \Omega(\tau)) = \{\Xi_1(\tau, \theta(\tau)), \Xi_2(\tau, \theta(\tau)), \Xi_3(\tau, \theta(\tau)), \Xi_4(\tau, \theta(\tau)), \Xi_5(\tau, \theta(\tau))\}.\]

The bounded solutions are to be assumed

\[\|\Omega(\tau)\|_{\infty} \leq \max\{k_1, k_2, k_3, k_4, k_5\},\]

\[\|\Omega(\tau) - \Omega_0(\tau)\| = \|\Delta(\tau, \Omega(\tau)) + \frac{1 - \rho}{B(\rho)} \int_0^\tau (\tau - w)^{-1} \Delta(w, \Omega(w))dw\|\]

\[\leq \frac{1 - \rho}{B(\rho)} \|\Delta(\tau, \Omega(\tau))\| + \frac{\rho}{B(\rho) \Gamma(\rho)} \int_0^\tau (\tau - w)^{\rho - 1} \|\Delta(\tau, \Omega(\tau))\|dw\]

\[\leq \frac{1 - \rho}{B(\rho)} X = \max\{k_1, k_2, k_3, k_4, k_5\} + \frac{\rho}{B(\rho)} \xi \theta^\rho \leq \theta \xi \leq k = \max\{k_1, k_2, k_3, k_4, k_5\}\]

Here consider

\[\theta < \frac{k}{\xi}\]
The fixed point theorem for the Banach space is as follows

\[ \|\Theta \Omega_1 - \Theta \Omega_2\|_\infty = \sup_{\tau \in \varepsilon} |\Omega_1 - \Omega_2|, \]

\[ \|\Theta \Omega_1 - \Theta \Omega_2\| = \|\{\Delta(\tau, \Omega_1(\tau)) - \Delta(\tau, \Omega_2(\tau))\}\frac{1 - \rho}{B(\rho)} + \frac{\rho}{B(\rho)\Gamma(\rho)} \int_0^\tau (\tau - w)^{\rho - 1} \{\Delta(w, \Omega_1(\tau)) - \Delta(w, \Omega_2(\tau))\} dw, \]

\[ \leq \frac{1 - \rho}{B(\rho)} \|\Delta(w, \Omega_1(\tau)) - \Delta(w, \Omega_2(\tau))\| + \frac{\rho \omega}{B(\rho)\Gamma(\rho)} \int_0^\tau (\tau - w)^{\rho - 1} \|\Omega_1(\tau) - \Omega_2(\tau)\| dw, \]

\[ \leq \left\{ \frac{1 - \rho}{B(\rho)} \omega + \frac{\rho \omega \rho}{B(\rho)\Gamma(\rho)} \right\} \|\Omega_1(\tau) - \Omega_2(\tau)\| dw, \]

\[ \leq \omega \|\Omega_1(\tau) - \Omega_2(\tau)\|, \]

\( \Omega \) is considered a contraction that’s why \( \omega < 1 \) cents. The defined operator \( \Theta \) is also a contraction and hence the solution exits.

\[ \boxdot \]

**FIGURE 1** Modeling of \( S(\tau) \) with Caputo derivative [Color figure can be viewed at wileyonlinelibrary.com]

**FIGURE 2** Modeling of \( E(\tau) \) with Caputo derivative [Color figure can be viewed at wileyonlinelibrary.com]
4 | NUMERICAL RESULTS AND DISCUSSION

The ABC derivative is used to present a nonlinear system as the analytical solution for the fractional differential equation. The numerical consequences of $S$ the receptive, $E$ the asymptomatic infected, $I$ the symptomatic infected, $R$ the recovered, and $Q$ the quarantined populations at fractional order are acquired according to the parameter values given in Kuniya and Inaba; also, details of the related reproductivity number of the model are in Kuniya and Inaba. It should be observed that the behavior of the COVID-19 epidemic model is almost same, but the ABC technique has better results which shows comfortable behavior in fractional system for control and for improving the strategy to overcome the risk of spread.

**FIGURE 3**  Modeling of $I(t)$ with Caputo derivative [Color figure can be viewed at wileyonlinelibrary.com]

**FIGURE 4**  Modeling of $R(t)$ with Caputo derivative [Color figure can be viewed at wileyonlinelibrary.com]

**FIGURE 5**  Modeling of $Q(t)$ with Caputo derivative [Color figure can be viewed at wileyonlinelibrary.com]
diseases. Figures 1–10 demonstrate the simulations, and the step size was $h = 0.001$, which is used to evaluate approximate solutions. In terms of the values of the travel restriction, the decreased contact rate and high level of quarantine is only possible when the number of cases from the epicenter is low. Furthermore, the numerical solution of the model represents different non-integer values. In Figure 1, the susceptible population starts increasing by decreasing the fractional values, whereas in Figures 2–5 exposed, infected, recovered, and quarantined populations start decreasing by decreasing the fractional values. A similar behavior can be seen from Figures 6–10 but more appropriate as it gives better and fast results according to steady state. The main approach to slowing down the transmission of COVID-19 is to remain home, and the sick individuals must be at a distant location or protected place as far as possible. The following values of param-

**FIGURE 6**  Modeling of $S(t)$ with Atangana–Baleanu–Caputo derivative [Color figure can be viewed at wileyonlinelibrary.com]

**FIGURE 7**  Modeling of $E(t)$ with Atangana–Baleanu–Caputo derivative [Color figure can be viewed at wileyonlinelibrary.com]

**FIGURE 8**  Modeling of $I(t)$ with Atangana–Baleanu–Caputo derivative [Color figure can be viewed at wileyonlinelibrary.com]
eters are used for numerical simulation: $\beta_1 = 0.23, \beta_2 = 0.15, \epsilon = 0.2, \gamma = 0.1, \text{ and } \eta = 0.07$. Ultimately, efficient and appropriate care of sick patients must be given, and medicines, tones, and nutrients must be distributed to non-infected individuals to prevent them from getting sick.

5 | CONCLUSIONS

In this article, the COVID-19 is analyzed through combination of ABC derivative with the dynamical fractional-order model. In the derivation of the ABC, exponentially decreasing non-singular kernels are present, which is used as the basis for this fractional-order model. In order to control the spread of COVID-19, the representation of fractional-order COVID-19 model with theoretical and numerical investigations is used. The numerical simulation has been developed by numerical solver using Caputo and ABC fractional-order derivative, which provides more absorbing characteristics as compared to the classical derivative. The ABC derivative gives more appropriate and fast results according to steady state. The main approach to slowing down the transmission of COVID-19 is to remain home, and the sick individuals must be at a distant location or protected place as far as possible. This method is easy for understanding the complex COVID-19 system and provides appropriate results which are helpful to understand the actual behavior of COVID-19. The solution covers the overall situation occurring during COVID-19 using fractional values, whereas the classical model represents the behavior at a single integer value.

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CONFLICT OF INTEREST

This work does not have any conflicts of interest.
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