Supersymmetric Gauged $U(1)_{L\mu-L\tau}$ Model for Neutrinos and Muon $(g-2)$ Anomaly

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Abstract: Gauged $U(1)_{L\mu-L\tau}$ model can provide for additional contributions to the muon anomalous magnetic moment by means of a loop involving the $Z'$ gauge boson. However, the parameter space of such models is severely constrained if one combines the latest muon $(g-2)$ data with various neutrino experiments such as, neutrino trident production, $\nu - e$ and $\nu - q$ elastic scattering, etc. In a supersymmetric $U(1)_{L\mu-L\tau}$ model, a larger region of parameter space opens up, thus enabling one to explore otherwise forbidden regions of parameter space in non-supersymmetric models. We show that the minimal model with the MSSM field content is strongly disfavored from $Z$-boson decay and neutrino data. We also show that the non-minimal model with two extra singlet superfields can lead to correct neutrino masses and mixing involving both tree level and one-loop contributions. We find that, in this model, both muon $(g-2)$ and neutrino data may be simultaneously explained in a parameter region consistent with experimental observations.

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Contents

1 Introduction 2

2 The minimal model 3
  2.1 Scalar sector 5
  2.2 Case of both $v_\mu$ and $v_\tau \neq 0$ 6
  2.3 Scalar mass matrices 6
  2.4 Mass mixing between $Z$ and $Z'$ bosons 8
  2.5 Case of either $v_\mu \neq 0$ or $v_\tau \neq 0$ 9
  2.6 Failure of the minimal model 10

3 The non-minimal model 10
  3.1 Free from gauge anomalies 11
  3.2 Vacua and scalar masses 12
  3.3 Minimization of the Potential 14

4 Neutralino and Neutrino Masses in the Non-Minimal Model 14
  4.1 Mass Models and Possible Mixing Patterns 17
  4.2 Numerical Analysis 18

5 Anomalous Magnetic Moment 20
  5.1 Outline of the Calculation 20
    5.1.1 Neutralino-Charged Scalar Loop 20
    5.1.2 Chargino-Neutral Scalar Loop 22
    5.1.3 $Z'$ Contribution 23
  5.2 Numerical Analysis 24

6 Conclusion 26

A Scalar Mass-Squared Matrices 28
  A.1 CP-even Mass Squared Matrix 28
  A.2 CP-Odd Mass Squared Matrix 29
  A.3 Charged Scalar Mass-Squared Matrix 29

B Chargino Mass Matrix 30
1 Introduction

The Standard Model (SM) of particle physics is a successful theory. However, it does not seem to be a complete one: it cannot explain the neutrino masses and mixing pattern. Neither can it explain the 3.6σ discrepancy between the SM prediction of anomalous magnetic moment and its experimental value [1–4]. In order to explain neutrino mass pattern and mixing [5] and muon anomalous magnetic moment, among other issues, one needs to look for physics beyond the SM.

One of the famous extensions of the SM is its minimal supersymmetric extension, popularly called MSSM [6, 7]. However, MSSM with R-parity conservation cannot explain the non-zero tiny masses of the neutrinos and their non-trivial mixing pattern as observed in experiments involving solar, atmospheric, accelerator and reactor neutrinos. A possible solution to explain the results coming from neutrino experiments is to extend the MSSM with additional singlet neutrino superfields giving rise to tiny neutrino masses through Type-I seesaw mechanism.

An intrinsically supersymmetric way of generating small neutrino mass pattern and mixing is to introduce R-parity violation (For a review, see, for example Ref[8]). Another way of extending MSSM is to enlarge the gauge group structure and a simplest possibility is to augment the SM gauge group with an additional $U(1)$ symmetry. Out of several models available in the literature, a very interesting $U(1)$ extension is gauged $U(1)_{\mu - \tau}$ extension of the SM. It was first studied in the three-generation minimal standard model of quarks and leptons in the absence of right-handed neutrinos[9, 10]. The contribution of the extra gauge boson $Z'$ of this model to the muon anomalous magnetic dipole moment was studied in Ref.[11]. Neutrino mass pattern and mixing angles in this class of models, with suitable field content, were discussed in Ref.[12]. The authors have also discussed signatures of this model in high energy colliders alongside an analysis of muon $(g - 2)$.

A detailed fit to electroweak data was performed in Ref.[13] in order to identify the allowed ranges of the mass of $Z'$ and its mixing with the SM $Z$ boson. The authors also studied this model in the context of neutrino mass model building. Constraints on the mass and the coupling of the new gauge boson have been derived from neutrino trident production in [14]. Dark matter candidates in this class of models and associated physics have been discussed in [15–18]. The possibility of detecting the gauge boson (assuming its mass in the range MeV-GeV) of $U(1)_{\mu - \tau}$ symmetry at the Belle-II experiment has been discussed in Ref.[20, 21]. In addition, constraints on such a light gauge boson have been imposed from neutrino beam experiments [21] and lepton flavor violating $\tau$ decays [22]. Constraints on gauged $L_\mu - L_\tau$ models have been derived from rare Kaon decays [23]. Higgs boson flavor violating decays have been studied in [24, 25]. Some recent anomalies involving B-meson decays have been addressed in [15, 24, 26, 27]. Neutrino masses and mixing have been studied very recently in a $U(1)_{\mu - \tau}$ symmetric model with additional scalars and vector-like leptons [28].

Supersymmetry is still one of the most attractive possibilities for physics beyond the standard model and the minimal supersymmetric standard model (MSSM) is a phenomenologically viable model, which has been studied extensively in the light of various experi-
mental observations.

In the gauged $U(1)_{\mu - \tau}$ model, non-zero neutrino masses can be obtained at the one-loop level with the introduction of additional scalar fields. Additional scalar fields are required for spontaneously breaking the $U(1)_{\mu - \tau}$ gauge symmetry. However, new scalar fields introduce new hierarchy problem and a possible solution is provided by embedding these models in a supersymmetric framework. As soon as one has a SUSY version of $U(1)_{\mu - \tau}$ model, new contributions to neutrino masses are available already at the tree level and it is tempting to see whether such a scenario can explain the results from neutrino oscillation experiments.

Supersymmetric version of $U(1)_{\mu - \tau}$ was studied earlier in [29] where the authors focused mainly on obtaining a leptophilic dark matter candidate in order to explain the PAMELA and AMS-02 results. However, our objective is to see how non-zero neutrino masses and non-trivial mixing pattern can be achieved in this kind of a set up along with a prediction for muon $(g - 2)$ consistent with experimental observations. The idea here is to first look at the possibility of a model with minimal field content, study it in detail and then point out the drawbacks, if any, of such a model to explain the experimental observations. Next we want to address all these issues in a model with non-minimal field content.

The plan of the paper is as follows. In section 2 we shall describe the minimal model and discuss its essential features. The constraints on the parameters of the minimal model will also be presented. The non-minimal model will be introduced in section 3 and the scalar sector of the model will be studied. Sections 4 will be devoted to the fermionic sector of this model and the neutralino mass matrix will be presented. We shall show how mixing of the neutrinos with the neutralinos along with the mixing involving the singlet fermions can generate tiny masses for the neutrinos. The mixing of the light neutrinos will be studied in detail in this section. Muon anomalous magnetic moment $(g - 2)_\mu$ will be studied in section 5. A detailed numerical analysis and allowed regions of the parameter space will be presented. Our conclusions and future directions will be presented in section 6. Analytical expressions for the scalar mass matrices and the chargino mass matrix for the non-minimal model have been included in the appendix.

## 2 The minimal model

The chiral superfield content of the minimal model is that of MSSM with the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{\mu - \tau}$. The $U(1)_{\mu - \tau}$ charge assignments of different chiral superfields are shown in Table 1. With the given charge assignments, we can write

| Superfields | $Q_1$ | $U^c_i$ | $D^c_i$ | $L_e$ | $E^c_e$ | $L_\mu$ | $E^c_\mu$ | $L_\tau$ | $E^c_\tau$ | $H_u$ | $H_d$ |
|-------------|-------|---------|---------|-------|---------|---------|---------|---------|---------|-------|-------|
| $U(1)_{\mu - \tau}$ | 0     | 0       | 0       | 0     | 0       | 1       | -1      | -1      | 1       | 0     | 0     |

**Table 1.** $U(1)_{\mu - \tau}$ charge ($Q_X$) assignments to the chiral superfields of the minimal model
the following superpotential.

\[ W^{\text{min}} = \epsilon_{ij} \left[ -y^u_{mn} H_u^i \tilde{Q}_n^i \tilde{U}_{m}^c + y^d_{mn} H_d^i \tilde{Q}_n^i \tilde{D}_{m}^c + y^e \tilde{H}_d^i \tilde{L}_j^i \tilde{E}_c^j + y^\mu \tilde{H}_d^i \tilde{L}_j^i \tilde{E}_c^j + y^\tau \tilde{H}_d^i \tilde{L}_j^i \tilde{E}_c^j \right. \\
\quad \left. - \mu_e \tilde{L}_e^i \tilde{H}_d^i - \mu_0 \tilde{H}_d^i \tilde{H}_d^i + \lambda_{122} \tilde{L}_e^i \tilde{L}_j^i \tilde{E}_c^j + \lambda_{133} \tilde{L}_e^i \tilde{L}_j^i \tilde{E}_c^j + \lambda_{1mn} \tilde{L}_e^i \tilde{Q}_n^i \tilde{D}_{m}^c \right]. \tag{2.1} \]

Here the lepton flavor indices are explicitly written for each individual flavor. We have considered baryon number parity so that \( \lambda''_{ijk} U_L^i D^j \) is not allowed. The presence of \( U(1)_{L_\mu - L_\tau} \) symmetry makes the Yukawa matrix for the lepton sector flavor diagonal. The gauge symmetries alone dictate the pattern or non-zero elements of the couplings for lepton number violating terms as follows:

\[ \lambda_{212}, \lambda_{122}, \lambda_{313} \quad \text{and} \quad \lambda_{133} \neq 0 \tag{2.2} \]

\[ \lambda'_{ijk} \neq 0 \quad \forall \ j & k \tag{2.3} \]

\[ \mu_c \neq 0. \tag{2.4} \]

The above superpotential has an accidental global symmetry: \( U(1)_{L_\mu + L_\tau} \). Soft SUSY breaking terms for this model are as follows:

\[ -\mathcal{L}^{\text{soft}}_{\text{min}} = \frac{1}{2} \left( M_2 (i \tilde{g})(i \tilde{g}) + M_2 (i \tilde{W})(i \tilde{W}) + M_1 (i \tilde{B})(i \tilde{B}) + M_0 (i \tilde{B}')(i \tilde{B}') + h.c. \right) - M_{10} (i \tilde{B})(i \tilde{B}') \]

\[ + \left( A_{ij}^u H_u \tilde{Q}_j \tilde{U}_i - A_{ij}^d H_d \tilde{Q}_j \tilde{D}_i - A_{ij}^e H_d \tilde{L}_{ij} \tilde{E}_c^j + A_{ij}^\mu \tilde{H}_d \tilde{E}_c^j - A_{ij}^\tau \tilde{H}_d \tilde{E}_c^j \right) - M_{122}^2 \tilde{Q}_j \tilde{Q}_j + M_{133}^2 \tilde{D}_{ij} \tilde{D}_{ij} + \sum_{a=\epsilon, \mu, \tau} \left( M_{L_a}^2 \tilde{L}_a \tilde{L}_a + M_{E_a}^2 \tilde{E}_c^j \tilde{E}_c^j \right) + M_{H_d}^2 \tilde{H}_d \tilde{H}_d + M_{H_u}^2 \tilde{H}_u \tilde{H}_u + h.c. \]

\[ \left. + (M_{L_d}^2 \tilde{H}_d \tilde{L}_c + h.c.) - \left( B_0 \tilde{H}_d \tilde{H}_d + B_e \tilde{L}_c \tilde{H}_u + h.c. \right) \right). \tag{2.5} \]

One can explicitly check that even after addition of the above soft SUSY breaking terms, the model still has the \( U(1)_{L_\mu + L_\tau} \) symmetry.

Without going into the details of calculations, we can make some comments based on symmetries. The electroweak symmetry is spontaneously broken by the vacuum expectation values (vevs) of the two Higgs fields \( H_u \) and \( H_d \). In addition, if the sneutrino fields \( \tilde{\nu}_\mu \) and \( \tilde{\nu}_\tau \) acquire non-zero vevs then both the \( U(1)_{L_\mu - L_\tau} \) and \( U(1)_{L_\mu + L_\tau} \) are broken down spontaneously to nothing. Thus we have two massless Goldstone bosons one of which makes the \( U(1)_{L_\mu - L_\tau} \) gauge boson massive and the other one, the Majoron, exists in the spectrum of particles. This Majoron is a CP-odd particle and the physical spectrum also has a very light CP-even scalar partner to the CP-odd massless Majoron. Hence, such a scenario is excluded as the Z-boson decay into the Majoron and its CP-even scalar partner has not been observed experimentally. We must study the scalar sector in some detail to see this explicitly.
2.1 Scalar sector

As the $U(1)_{\text{Le}}$ symmetry is explicitly broken, we cannot distinguish between $\hat{L}_e$ and $\hat{H}_d$ superfields because all of their quantum numbers are the same. In principle the scalar components of both $\hat{L}_e$ and $\hat{H}_d$ get non-zero VEVs. We use the above freedom of indistinguishability to choose a basis where only one of them gets non-zero VEV. In our subsequent discussion we shall work in a basis where the vev of the electron sneutrino $\tilde{\nu}_e$ is rotated away.

The total scalar potential is given by

$$V_{\text{scalar}} = V_F + V_D + V_{\text{soft}}. \quad (2.6)$$

where $V_F$ is calculated from 2.1 using,

$$V_F = \sum_i \left| \frac{\partial W}{\partial \Phi_i} \right|^2 \quad (2.7)$$

and

$$V_D = \frac{1}{2} D_a D_a + \frac{1}{2} D_Y^2 + \frac{1}{2} D_X^2 \quad (2.8)$$

where $D_a = \sqrt{2} g_a \phi^* T^a \phi$ and $V_{\text{soft}}$ is the scalar part of 2.5. In supersymmetric gauged $U(1)_{L_\mu-L_\tau}$ model the gauge kinetic term mixing affects the gauge fields, the gauginos as well as the auxiliary fields $D_Y$ and $D_X$, where $X = L_\mu - L_\tau$. The auxiliary fields can be written, using their equations of motion, as

$$D_Y = -\sum_i g Y^i \frac{1}{2} |\phi_i|^2, \quad D_X = -\sum_i \left( g_m Y^i \frac{1}{2} + g_X \frac{Q_i^X}{2} \right) |\phi_i|^2, \quad (2.9)$$

where $Y^i$ and $Q_X^i$ are the charges of the scalar fields $\phi_i$ corresponding to $U(1)_Y$ and $U(1)_{L_\mu-L_\tau}$ gauge symmetry, respectively. The couplings $g_X$ and $g_m$ are gauge couplings associated with $U(1)_{L_\mu-L_\tau}$ and that generated via kinetic mixing, respectively.

The contributions of the neutral scalar fields to the scalar potential is as follows:

$$V_{\text{neut}} = \left( m_{H_u}^2 + |\mu|^2 + |\mu_e|^2 \right) |h_u^0|^2 + \left( m_{H_d}^2 + |\mu|^2 \right) |h_d^0|^2 + \sum_{a=e,\mu,\tau} \left( M_{\tilde{L}_a}^2 + |\mu_e|^2 \delta_{e,a} \right) |\tilde{\nu}_a|^2$$

$$+ \left( \mu^* \mu h_u^0 \tilde{\nu}_e - B h_u^0 h_d^0 \tilde{\nu}_e + B e \tilde{\nu}_e h_u^0 + M_{\tilde{e}_L,a}^2 M_{\tilde{\nu}_a}^0 \tilde{\nu}_e + h.c. \right)$$

$$+ \frac{1}{8} \left( g_1^2 + g_2^2 \right) \left( |h_u^0|^2 - |h_d^0|^2 - \sum_{a=e,\mu,\tau} |\tilde{\nu}_a|^2 \right)^2$$

$$+ \frac{1}{8} \left( g_m \left( |h_u^0|^2 - |h_d^0|^2 - \sum_{a=e,\mu,\tau} |\tilde{\nu}_a|^2 \right) - g_X \left( |\tilde{\nu}_\tau|^2 - |\tilde{\nu}\mu|^2 \right) \right)^2. \quad (2.10)$$

We assume that only the neutral scalar fields $H_u$, $H_d$, $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$ acquire non-zero vacuum expectation values (vevs) while minimizing the scalar potential and the vevs are
defined as \( h_u^0 = v_u / \sqrt{2}, \ h_d^0 = v_d / \sqrt{2}, \ h_\mu = v_\mu / \sqrt{2} \) and \( h_\tau = v_\tau / \sqrt{2} \). The minimization equations are

\[
Bv_d + \frac{1}{8} g_x g_m (|v_\tau|^2 - |v_\mu|^2) v_u^* - \frac{1}{8} (g^2 + g'^2 + g_m^2) (|v_u|^2 - |v_0|^2) v_u^* \\
- (M^2_{H_u} + |\mu_0|^2) v_u^* = 0 \tag{2.11}
\]

\[
Bv_u - \frac{1}{8} g_x g_m (|v_\tau|^2 - |v_\mu|^2) v_d^* + \frac{1}{8} (g^2 + g'^2 + g_m^2) (|v_u|^2 - |v_0|^2) v_d^* \\
- (M^2_{H_d} + |\mu|^2) v_d^* = 0 \tag{2.12}
\]

\[
v_\mu^* \left[ M^2_{L_\mu} - \frac{g_x^2}{8} (|v_\tau|^2 - |v_\mu|^2) + \frac{1}{8} g_x g_m (|v_u|^2 - |v_d|^2 - 2|v_\mu|^2) \right] \\
- \frac{1}{8} (g^2 + g'^2 + g_m^2) (|v_u|^2 - |v_0|^2) \right] = 0 \tag{2.13}
\]

\[
v_\tau^* \left[ M^2_{L_\tau} + \frac{g_x^2}{8} (|v_\tau|^2 - |v_\mu|^2) - \frac{1}{8} g_x g_m (|v_u|^2 - |v_d|^2 - 2|v_\tau|^2) \right] \\
- \frac{1}{8} (g^2 + g'^2 + g_m^2) (|v_u|^2 - |v_0|^2) \right] = 0 \tag{2.14}
\]

\[
\left[ M^2_{\tilde{e}_L d} + \mu^* \mu_e \right] v_d^* = B_e v_u \tag{2.15}
\]

where \( |v_0|^2 \equiv |v_u|^2 + |v_\mu|^2 + |v_\tau|^2 \) and \( |\mu_0|^2 \equiv |\mu|^2 + |\mu_e|^2 \).

The vacuum expectation values are such that

\[
v \equiv (|v_u|^2 + |v_0|^2)^{1/2} = \frac{2 m_W}{g} \tag{2.16}
\]

### 2.2 Case of both \( v_\mu \) and \( v_\tau \neq 0 \)

If we demand that both \( v_\mu \) and \( v_\tau \) are non-zero, then we have two corresponding massless Goldstone bosons in the spectrum. There is always a Golstone boson arising because of non-zero vevs of the Higgs fields, \( H_u \) and \( H_d \). Two of these three Goldstone bosons can be eaten up by the neutral gauge bosons \( Z \) and \( Z' \). The remaining massless CP-odd Majoron is a physical particle and hence experimentally ruled out from the non-observation of such particles in the decay of the \( Z \) boson. We can understand this even better if we calculate the CP-even and CP-odd neutral scalar mass-squared matrices for these scenarios.

### 2.3 Scalar mass matrices

We can calculate the CP-even and CP-odd neutral scalar mass-squared matrices from the CP-even and CP-odd neutral scalar potential, using

\[
M^2_{ij} = \left. \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right|_{\text{min}} \tag{2.17}
\]
CP-even scalar mass matrix in the basis \((h_u, h_d, \tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)\), is given by

\[
M_{\text{even}}^2 = \begin{pmatrix}
B v_d/v_u & \frac{1}{3} \tilde{g}^2 v^2 u & -B - \frac{1}{3} \tilde{g}^2 v_u v_d & -B & -\frac{1}{4} (\tilde{g}^2 - g_m g_x) v_u v_\mu & -\frac{1}{4} (\tilde{g}^2 + g_m g_x) v_u v_\tau \\
-B & \frac{1}{3} \tilde{g}^2 v_u v_d & \tilde{m}^2_{h^0 h^0 u} + \frac{1}{8} \tilde{g}^2 v^2 d & \tilde{m}^2_{d e} + \mu \mu_e & \frac{1}{4} (\tilde{g}^2 - g_m g_x) v_d v_\mu & \frac{1}{4} (\tilde{g}^2 + g_m g_x) v_d v_\tau \\
-B & \tilde{m}^2_{d e} + \mu \mu_e & \tilde{m}^2_{\tilde{\nu}_e \tilde{\nu}_e'} & 0 & 0 \\
-\frac{1}{4} (\tilde{g}^2 - g_m g_x) v_u v_\mu & \frac{1}{4} (\tilde{g}^2 - g_m g_x) v_d v_\mu & 0 & \tilde{m}^2_{\tilde{\nu}_\mu \tilde{\nu}_\mu'} + \frac{1}{4} g^2 - \frac{g^2}{2} v^2_\mu & \frac{1}{4} (\tilde{g}^2 - g^2_\mu) v_\mu v_\tau \\
-\frac{1}{4} (\tilde{g}^2 + g_m g_x) v_u v_\tau & \frac{1}{4} (\tilde{g}^2 + g_m g_x) v_d v_\tau & 0 & \frac{1}{4} (\tilde{g}^2 - g^2_\tau) v_\mu v_\tau & \tilde{m}^2_{\tilde{\nu}_\tau \tilde{\nu}_\tau'} + \frac{1}{4} g^2_\tau v^2_\tau & \\
\end{pmatrix}
\]

(2.18)

Here \(\tilde{g}^2 = (g^2 + g'^2 + g_m^2)\) and \(g^2_x = \tilde{g}^2 \mp 2 g_m g_x + g^2_w\),

\[
\tilde{m}^2_{h^0 h^0 u} = m^2_{H_u} + \mu^2 - \frac{1}{8} g^2 (v^2_u - v^2_d - v^2_\mu - v^2_\tau) + \frac{1}{8} g_m g_x (v^2_\tau - v^2_\mu),
\]

(2.19)

\[
\tilde{m}^2_{\tilde{\nu}_e \tilde{\nu}_e'} = M^2_{\nu_e} + \mu^2 - \frac{1}{8} g^2 (v^2_u - v^2_d - v^2_\mu - v^2_\tau) + \frac{1}{8} g_m g_x (v^2_\tau - v^2_\mu),
\]

(2.20)

\[
\tilde{m}^2_{\tilde{\nu}_\mu \tilde{\nu}_\mu'} = M^2_{\nu_\mu} - \frac{1}{8} g^2 (v^2_u - v^2_d - v^2_\mu - v^2_\tau) + \frac{1}{8} g_m g_x (v^2_\mu - v^2_d - 2v^2_\mu) - \frac{1}{8} g^2_x (v^2_\mu - v^2_\tau),
\]

(2.21)

\[
\tilde{m}^2_{\tilde{\nu}_\tau \tilde{\nu}_\tau'} = M^2_{\nu_\tau} - \frac{1}{8} g^2 (v^2_u - v^2_d - v^2_\mu - v^2_\tau) - \frac{1}{8} g_m g_x (v^2_\mu - v^2_d - 2v^2_\tau) + \frac{1}{8} g^2_x (v^2_\tau - v^2_\mu).
\]

(2.22)

CP-odd scalar mass matrix in the basis \((h_u, h_d, \tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)\), is given by

\[
M_{\text{odd}}^2 = \begin{pmatrix}
B v_d/v_u & B & B_e & 0 & 0 \\
B & \tilde{m}^2_{h^0 h^0 u} & \tilde{m}^2_{d e} + \mu \mu_e & 0 & 0 \\
B_e & \tilde{m}^2_{d e} + \mu \mu_e & \tilde{m}^2_{\tilde{\nu}_e \tilde{\nu}_e'} & 0 & 0 \\
0 & 0 & 0 & \tilde{m}^2_{\tilde{\nu}_\mu \tilde{\nu}_\mu'} & 0 \\
0 & 0 & 0 & 0 & \tilde{m}^2_{\tilde{\nu}_\tau \tilde{\nu}_\tau'}
\end{pmatrix}
\]

(2.23)

When both \(v_\mu\) and \(v_\tau\) are non-zero then Eqs.(2.13) and (2.14) give us

\[\tilde{m}^2_{\tilde{\nu}_\mu \tilde{\nu}_\mu'} = 0 = \tilde{m}^2_{\tilde{\nu}_\tau \tilde{\nu}_\tau'}\]

(2.24)

This gives two massless Goldstone bosons from the CP-odd mass matrix as discussed earlier. In addition, the diagonalization of the upper 3 x 3 block gives another massless Goldstone boson which is absorbed by the Z-boson.

Let us now consider the CP-even scalar squared-masses by calculating the eigenvalues
of the matrix in Eq.(2.18). It is straightforward to check that the eigenvector
\[
\rho = \frac{1}{K} \begin{pmatrix}
2v_u v_\mu v_r \\
2v_l v_\mu v_r \\
0 \\
v_r (v_u^2 - v_d^2) \\
v_\mu (v_u^2 - v_d^2)
\end{pmatrix}, \quad K = \sqrt{v_\mu^2 (v_u^2 - v_d^2)^2 + v_r^2 (v_u^2 - v_d^2)^2 + 4v_u^2 v_r^2 (v_u^2 + v_d^2)}
\]
(2.25)
corresponds to a zero eigenvalue of \(M_{\text{even}}^2\). This means that at the tree level there exists a massless CP-even scalar, \(\rho\). However, \(\rho\) gains a small mass \(O(\sqrt{v_\mu^2 + v_\tau^2})\) when radiative corrections are incorporated since it is not a goldstone boson. The non-observation of the Z-boson decay \(Z \rightarrow \text{Majoron} + \rho\) in experiments rules out the minimal model described above.

### 2.4 Mass mixing between \(Z\) and \(Z'\) bosons

Although we have already seen that this scenario is ruled out, let us also check how the \(Z\) and \(Z'\) bosons mix in this model for the sake of completeness. Because of gauge kinetic term mixing in this model, the covariant derivative is written as
\[
D_\mu = \partial_\mu - ig\sigma^j 2 W^j_\mu - ig' B_\mu - i(g_m Y \frac{Q_X}{2} + g_x Q_X) B'_\mu
\]
(2.26)
where the relevant Lagrangian has been written in a canonically normalized basis [33]. Here \(Q_x\) is the \(U(1)_{L_\mu - L_\tau}\) charge and can be obtained from Table 1. A factor of \(\frac{1}{2}\) has been introduced as a convention.

Mass matrix of the neutral gauge bosons can be obtained from the following terms
\[
\mathcal{L} \supset \frac{i}{2} \begin{pmatrix}
g W^3_\mu + g' B_\mu + g_m B'_\mu & 0 & 0 \\
g W^3_\mu + g' B_\mu + g_m B'_\mu & -g W^3_\mu + g' B_\mu + g_m B'_\mu & \left( \frac{v_\mu}{\sqrt{2}} \right) \\
g W^3_\mu - g' B_\mu - g_m B'_\mu & 0 & \left( \frac{v_\mu}{\sqrt{2}} \right)
\end{pmatrix}^2
\]
(2.27)
where \(||\chi||^2 = \chi^\dagger \chi\) for any vector. Let us define
\[
\mathcal{Z}_\mu = \cos \theta_W W^3_\mu - \sin \theta_W B_\mu \\
A_\mu = \sin \theta_W W^3_\mu + \cos \theta_W B_\mu,
\]
(2.28)
where $\tan \theta_W = g'/g$. One can explicitly see that there is no mass term for the $A_\mu$ field; mass terms for the fields $Z_\mu$ and $B'_\mu$ are as follows

$$
\mathcal{L} \ni \frac{1}{2} \left( Z_\mu B'_\mu \right) \left( \begin{array}{l}
\frac{1}{4} \left( g'^2 + g^2 \right) v^2 \\
\sqrt{g'^2 + g^2 \left( \frac{1}{4} g_x (v^2_\mu - v^2_\tau) \right)} \\
\sqrt{g'^2 + g^2 \left( \frac{1}{4} g_x (v^2_\mu - v^2_\tau) \right)}^{\frac{1}{2}} \left( \frac{v^2_\mu + v^2_\tau}{4} \right) + \frac{g m g_x}{2} \left( v^2_\mu - v^2_\tau \right) \\
\frac{1}{4} g m v^2 \end{array} \right) \left( \begin{array}{c}
Z^\mu \\
B'^\mu 
\end{array} \right)
$$

(2.29)

where $v^2 = (v^2_u + v^2_d + v^2_\mu + v^2_\tau)$.

The real and symmetric matrix can be diagonalized by the following matrix

$$
\begin{pmatrix}
\cos \zeta & \sin \zeta \\
-\sin \zeta & \cos \zeta
\end{pmatrix},
$$

(2.30)

where $\tan 2\zeta = \frac{2 \sqrt{g'^2 + g^2 (g_x (v^2_\nu - v^2_\tau) - g_m v^2)}}{(g^2 + g'^2 - g_m^2) v^2 - g^2 (v^2_\nu + v^2_\tau) - 2 g_m g_x (v^2_\nu - v^2_\tau)$} and the physical massive neutral gauge bosons are

$$
Z_{1\mu} = \cos \zeta Z_\mu - \sin \zeta B'_\mu,
$$

(2.31)

$$
Z_{2\mu} = \sin \zeta Z_\mu + \cos \zeta B'_\mu.
$$

(2.32)

### 2.5 Case of either $v_\mu \neq 0$ or $v_\tau \neq 0$

On the other hand, the problem related to the massless Majoron discussed above can be ameliorated if only one of the two sneutrinos, namely, $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$ acquires vev. In this case we have two possibilities:

**Type – A**: $v_\mu \neq 0, v_\tau = 0$;

$$
U(1)_{L\mu - L\tau} \times U(1)_{L\mu + L\tau} \rightarrow U(1)_{L\tau}
$$

(2.33)

**Type – B**: $v_\mu = 0, v_\tau \neq 0$;

$$
U(1)_{L\mu - L\tau} \times U(1)_{L\mu + L\tau} \rightarrow U(1)_{L\mu}
$$

(2.34)

In both these cases there is no massless Majoron in the physical spectrum and either of these two scenarios are equally viable.

For $\tilde{\nu}_\kappa$ ($\kappa = \mu$ or $\tau$) the minimization equation is as follows (assuming all parameters to be real)

$$
v_\kappa \left[ M^2_{\tilde{\nu}_\kappa} + \frac{g^2}{8} v^2_\kappa + \frac{1}{8} Q_\kappa g_x g_m (v^2_u - v^2_d - 2 v^2_\nu) - \frac{1}{8} (g^2 + g'^2 + g^2_m) (v^2_u - v^2_\tau) \right] = 0
$$

(2.35)

where $Q_\kappa$ is the $U(1)_{L\mu - L\tau}$ charges corresponding to $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau.$
From Eq. 2.35 we get (for $\nu_\kappa \neq 0$)

$$
\tilde{m}_{\nu_\kappa}^2 = M_{\kappa}^2 + \frac{1}{8} (g_1^2 + g_2^2 + g_m^2) (v_\kappa^2 - v_u^2 + v_d^2) + \frac{1}{4} g_Z v_\kappa + \frac{1}{4} Q_\kappa g_m g_\kappa (v_u^2 - v_d^2 - 2v_\kappa^2)
$$

(2.36)

In the pseudoscalar mass matrix, all the off-diagonal entries of the column and row corresponding to the field $\tilde{\nu}_\kappa$ are zero and the diagonal entry is nothing but $\tilde{m}_{\nu_\kappa}^2$ (see, Eq.2.23). Thus if we demand $\nu_\kappa \neq 0$, which in turn implies that the condition of Eq.2.36 must be true, then there exists a corresponding massless pseudoscalar state as discussed in section 2.3. This massless pseudoscalar is eaten up by the neutral gauge field corresponding to $U(1)_{L_\mu - L_\tau}$ gauge symmetry. In addition, there is a Goldstone boson that gives mass to the Z boson. Thus there is no massless Majoron present in the physical spectrum of this model.

2.6 Failure of the minimal model

We have seen in the previous section that the models of Type-A [Eq.(2.33)] and Type-B [Eq.(2.34)] have residual global symmetries $U(1)_{L_\mu}$ and $U(1)_{L_\tau}$, respectively. Because of the presence of such global symmetries in each type of models after the electroweak symmetry breaking, textures of the Majorana neutrino mass matrix [in the basis $(\nu_e, \nu_\mu, \nu_\tau)$] and the charged lepton mass matrix [in the basis $(e, \mu, \tau)$] should have, in general, the following forms:

Type $- A$ : $m_\nu = \begin{pmatrix}
\checkmark & \checkmark & 0 \\
\checkmark & \checkmark & 0 \\
0 & 0 & 0 \\
\end{pmatrix}$, $m_\ell = \begin{pmatrix}
\checkmark & \checkmark & 0 \\
\checkmark & \checkmark & 0 \\
0 & 0 & \checkmark \\
\end{pmatrix}$

(2.37)

Type $- B$ : $m_\nu = \begin{pmatrix}
\checkmark & 0 & \checkmark \\
0 & 0 & 0 \\
\checkmark & 0 & \checkmark \\
\end{pmatrix}$, $m_\ell = \begin{pmatrix}
\checkmark & 0 & \checkmark \\
0 & 0 & \checkmark \\
\end{pmatrix}$

(2.38)

where $\checkmark$ means non-zero entries. Note that neutrino mass matrix has one less non-zero entry compared to the charged lepton mass matrix because of the Majorana nature of the neutrinos. With the above textures of these mass matrices, the resulting PMNS matrix will not be able to reproduce the correct pattern of neutrino mixing as observed in different neutrino experiments. Thus these two models with minimal field content are ruled out in the light of neutrino experimental data.

3 The non-minimal model

We have seen that the minimal model is not phenomenologically attractive. The source of this problem was essentially the fact that there is either an accidental $U(1)_{L_\mu + L_\tau}$ which
is broken along with $U(1)_{L_\mu-L_\tau}$, or that there is a residual $U(1)_{L_\mu}/U(1)_{L_\tau}$ that spoils the neutrino mass matrix texture. The solution is to have extra fields that are charged under $U(1)_{L_\mu-L_\tau}$ and couple to $\mu/\tau$ and thus make sure that none of the $U(1)_{L_\mu+L_\tau}$, $U(1)_{L_\mu}$ or $U(1)_{L_\tau}$ are symmetries of the theory. An additional benefit is the fact that now we have fields that are singlet under all SM gauged symmetries that can acquire vacuum expectation value to spontaneously break $U(1)_{L_\mu-L_\tau}$. While there is no problem even if the sneutrinos do acquire VEV, we consider the situation where they do not. This has more to do with simplifying the calculation than with any technical glitches, although one could argue that this minimizes tree-level $Z/Z'$ mixing and dissociates $U(1)_{L_\mu-L_\tau}$ breaking from EWSB. To this end we have also taken $g_m = 0$ in subsequent calculations. This ensures that there is no mixing between $Z$ and $Z'$ at tree level and the mass of the new gauge boson is given simply by,

$$M_{Z'}^2 = \frac{g_X^2}{4}(v_\eta^2 + v_{\bar{\eta}}^2). \quad (3.1)$$

Field content and $U(1)_{L_\mu-L_\tau}$ charges of the non-minimal model is as follows:

| Superfields | $Q_i$ | $U^c_i$ | $D^c_i$ | $L_\mu$ | $E^c_\mu$ | $L_\tau$ | $E^c_\tau$ | $H_u$ | $H_d$ | $\hat{\eta}$ | $\hat{\bar{\eta}}$ |
|-------------|------|--------|--------|--------|---------|--------|---------|------|------|---------|---------|
| $U(1)_{L_\mu-L_\tau}$ | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 1 | 0 | 0 | -1 | 1 |

Table 2. $U(1)_{L_\mu-L_\tau}$ charge assignments to the chiral superfields of the non-minimal model

The superpotential for the above choice of charges:

$$W = W^{\text{min}} + \epsilon_{ij} \left[ -y_\eta \hat{L}_\mu \hat{H}_d^i \hat{\eta} - y_{\bar{\eta}} \hat{L}_\tau \hat{H}_u^i \hat{\bar{\eta}} \right] + \mu_\eta \hat{\eta} \hat{\bar{\eta}}. \quad (3.2)$$

Here too we have considered baryon number parity as in the minimal model.

### 3.1 Free from gauge anomalies

Let us now discuss the conditions of anomaly cancellation[34] in this model.

1. It is not required to examine the anomaly condition involving all possible combinations of $SU(3), SU(2)$ and $U(1)_Y$ because MSSM is anomaly free.

2. The anomaly cancellation condition for \{SU(3), SU(3)\} $U(1)_{L_\mu-L_\tau}$ satisfies as none of the colored particles are charged under $U(1)_{L_\mu-L_\tau}$.

3. The $SU(2)$ fields which are charged under $U(1)_{L_\mu-L_\tau}$ are $L_\mu$ and $L_\tau$. As their charges are opposite to each other the \{SU(2), SU(2)\} $U(1)_{L_\mu-L_\tau}$ anomaly cancellation condition is also satisfied.

4. Similarly the \{U(1)$_Y$, U(1)$_Y$\} $U(1)_{L_\mu-L_\tau}$ anomaly cancellation condition is also satisfied because $U(1)_{L_\mu-L_\tau}$ charges of $L_\mu$ and $E^c_\mu$ are opposite to $L_\tau$ and $E^c_\tau$ respectively.
5. One can check that the \( \{ U(1)_{L_\mu-L_\tau}, U(1)_{L_\mu-L_\tau} \} U(1)_Y \) is also satisfied:

\[
2 \times 1^2 \times (-\frac{1}{2}) + 2 \times (-1)^2 \times (-\frac{1}{2})
\]
\[
+ (-1)^2 1 + 1^2 1 = 0
\]

(3.3)

6. The cubic anomaly for \( U(1)_{L_\mu-L_\tau} \):

\[
2 \times 1^3 + 2 \times (-1)^3 + (-1)^3 + 1^3 = 0
\]

(3.4)

7. And finally the mixed anomaly with gravity: It is also satisfied as trace of charges of fields for this new gauge group vanishes.

Thus all the gauge anomalies are cancelled out.

### 3.2 Vacua and scalar masses

We must consider the entire scalar potential of the model and minimize it to obtain the vacuum expectation values of the various fields. Just as in the case of minimal model, the total scalar potential is

\[
V = V_F + V_D + V_{soft},
\]

(3.5)

where \( V_F \) is calculated from 3.2 and \( V_{soft} \) comes from the soft SUSY breaking terms in the Lagrangian given by Equation 3.6.

\[
-\mathcal{L}_{soft} = -\mathcal{L}_{soft}^{min} - \left( A_\eta \eta \tilde{L}_\mu H_u + A_\eta \eta \tilde{L}_\tau H_u + h.c. \right) + M_\eta^2 \eta^{\dagger} \eta + M_\eta^{2 \frac{3}{2}} \eta^{\dagger} \bar{\eta}
\]
\[
+ (B_\eta \eta \bar{\eta} + h.c.).
\]

(3.6)

\( V_D \) is calculated in exactly the same way as for the minimal model (see Equation 2.9) apart from the fact that it includes contributions from two new scalar fields \( \eta \) and \( \bar{\eta} \).
The neutral scalar potential is,

\[
V_{\text{neut}} = \left( m_{H_u}^2 + |\mu|^2 + |\mu_e|^2 \right) |h_u^0|^2 + \left( m_{H_d}^2 + |\mu|^2 \right) |h_d^0|^2 + \sum_{a=e,\mu,\tau} \left( M_{H_a}^2 + |\mu_e|^2 \delta_{e,a} \right) |\tilde{\nu}_a|^2 \\
+ \left( M_\eta^2 + \mu_\eta^2 \right) |\eta|^2 \left( \frac{1}{2} y_\eta^2 |\bar{\nu}_\mu|^2 |h_u^0|^2 + \frac{1}{2} y_\eta^2 |\bar{\nu}_e|^2 |h_d^0|^2 \right) + \frac{1}{2} y_\eta^2 |\tilde{\nu}_e|^2 |h_d^0|^2 \\
+ \frac{1}{2} y_\eta^2 |\tilde{\nu}_\mu|^2 \left( -y_\eta \mu_\eta \tilde{\nu}_\mu^0 \tilde{\nu}_\mu - y_\eta \mu_\eta \tilde{\nu}_e \tilde{\nu}_\mu - \mu_\eta \mu_\eta \tilde{\nu}_\mu \tilde{\nu}_e \right) + y_\eta y_\eta^* \tilde{\nu}_\mu^* \tilde{\nu}_e + h.c. \right) \\
\left( 1 - y_\eta \mu_\eta \tilde{\nu}_\mu^0 \tilde{\nu}_\mu - y_\eta \mu_\eta \tilde{\nu}_e \tilde{\nu}_\mu - \mu_\eta \mu_\eta \tilde{\nu}_\mu \tilde{\nu}_e \right) + y_\eta y_\eta^* \tilde{\nu}_\mu^* \tilde{\nu}_e + y_\eta y_\eta^* \tilde{\nu}_\mu^* \tilde{\nu}_e + h.c. \right) \\
+ \frac{1}{8} \left( g_1^2 + g_2^2 \right) (|h_u^0|^2 - |\tilde{\nu}_e|^2 - |\tilde{\nu}_\mu|^2 - |\tilde{\nu}_\tau|^2)^2 \\
+ \frac{1}{8} \left( g_2 \left( |h_u^0|^2 - |\tilde{\nu}_e|^2 - |\tilde{\nu}_\mu|^2 - |\tilde{\nu}_\tau|^2 \right)^2 - g_X (|\eta|^2 - |\bar{\eta}|^2 - |\tilde{\nu}_e|^2 - |\tilde{\nu}_\mu|^2 \right)^2. \tag{3.7} \]

It is used to calculate the scalar and pseudoscalar mass-squared matrices. Replacing the fields by \((\phi_R + i\phi_I)/\sqrt{2}\) to separate out the CP-even and odd parts of the potential,

\[
V_{\text{even}} = \frac{1}{2} \left( M_{H_u}^2 + \mu^2 + \mu_e^2 \right) (h_u^0)^2 + \frac{1}{2} \left( M_{H_d}^2 + \mu^2 \right) (h_d^0)^2 + \frac{1}{2} \left( M_\eta^2 + \mu_\eta^2 \right) \tilde{\nu}_e^2 \\
+ \frac{1}{2} \left( M_{\eta_\mu}^2 + \frac{1}{2} y_\eta^2 \eta_R^2 \right) \nu_R^2 + \frac{1}{2} \left( M_{\eta_\mu}^2 + \frac{1}{2} y_\eta^2 \nu_R^2 \right) |\nu_R|^2 + \frac{1}{2} \left( M_{\eta_\mu}^2 + \frac{1}{2} y_\eta^2 \eta_R^2 \right) \tilde{\nu}_e^2 \\
+ \frac{1}{2} \left( M_{\eta_\mu}^2 + \frac{1}{2} y_\eta^2 \nu_R^2 + \mu_\eta^2 \eta_R^2 + \frac{1}{2} y_\eta^2 \tilde{\nu}_e^2 \right) \tilde{\nu}_e^2 + h.c. \right) \\
\left( \frac{1}{\sqrt{2}} y_\eta \mu_\eta \eta_R \tilde{\nu}_e - \frac{1}{\sqrt{2}} y_\eta \mu_\eta \nu_R \tilde{\nu}_e - \frac{1}{\sqrt{2}} y_\eta \mu_\eta \eta_R \tilde{\nu}_e - \frac{1}{\sqrt{2}} y_\eta \mu_\eta \nu_R \tilde{\nu}_e \right) \\
- \frac{1}{\sqrt{2}} A_{\eta} \eta_R \tilde{\nu}_e - \frac{1}{\sqrt{2}} \mu_\eta \nu_R \tilde{\nu}_e - \frac{1}{\sqrt{2}} y_\eta \mu_\eta \eta_R \tilde{\nu}_e - \frac{1}{\sqrt{2}} y_\eta \mu_\eta \nu_R \tilde{\nu}_e \\
+ \frac{1}{32} \left( g_1^2 + g_2^2 \right) \left( (h_u^0)^2 - (h_d^0)^2 \right)^2 - \tilde{\nu}_e^2 - \tilde{\nu}_e^2 - \frac{1}{16} g_1 g_2 \left( (h_u^0)^2 - (h_d^0)^2 \right)^2 - \tilde{\nu}_e^2 - \tilde{\nu}_e^2 - \frac{1}{16} g_1 g_2 \left( (h_u^0)^2 - (h_d^0)^2 \right)^2 \tag{3.8} \]

\[
V_{\text{odd}} = V_{\text{neut}} - V_{\text{even}}. \tag{3.9} \]

We can calculate the CP-even and CP-odd neutral scalar mass-squared matrices from 3.8 and 3.9 using 2.17.
3.3 Minimization of the Potential

At the minima of the potential all the first derivatives must vanish. The first derivatives thus give us a set of equations that we can plug in while calculating the second derivatives. The method is to first calculate the second derivatives of $V_{\text{even}}$ and $V_{\text{odd}}$ then replace the fields by their respective VEVs and the soft masses from the equations of minimization.

The minimization equations are,

\[
\begin{align*}
\left(\mu^2 + \mu_1^2 + \mu_2^2 + \mu_3^2 + M_{H_u}^2\right)v_u &+ \frac{(g_1^2 + g_2^2 + g_3^2)}{8}(v_u^2 - v_d^2)v_u - \frac{g_m g_x}{8}(v_u^2 - v_d^2)v_d = 0 \\
\left(\mu^2 + M_{H_d}^2\right)v_d - \frac{(g_1^2 + g_2^2 + g_3^2)}{8}(v_u^2 - v_d^2)v_d + \frac{g_m g_x}{8}(v_u^2 - v_d^2)v_d - B v_d = 0 \\
(M^2_{\xi L} + \mu_1) &v_d - B_v v_u = 0 \\
(\mu_2 - \mu_\eta \mu_2 t_\beta \cot \gamma) &v_d - A_\eta \frac{v_\eta}{\sqrt{2}} v_u = 0 \\
(\mu_3 - \mu_\eta \mu_3 t_\beta t_\gamma) &v_d - A_\eta \frac{v_\eta}{\sqrt{2}} v_u = 0 \\
(\mu_\eta^2 + \mu_2^2 \xi + M^2_\eta) &v_\eta + g_\eta^2 (v_u^2 - v_d^2)v_\eta - \frac{g_m g_x}{8}(v_u^2 - v_d^2)v_\eta + B_\eta v_\eta = 0 \\
(\mu_\eta^2 + \mu_3^2 \xi + M^2_\eta) &v_\eta - g_\eta^2 (v_u^2 - v_d^2)v_\eta + \frac{g_m g_x}{8}(v_u^2 - v_d^2)v_\eta + B_\eta v_\eta = 0
\end{align*}
\]

where,

\[
\begin{align*}
\mu_2 &= \frac{y_\eta v_\eta}{\sqrt{2}} = \sqrt{2} y_\eta \frac{M_Z^2 s_\beta}{g X}, & \mu_3 &= \frac{y_\eta v_\eta}{\sqrt{2}} = \sqrt{2} y_\eta \frac{M_Z^2 s_\beta}{g X}, \\
\tan \xi &= \frac{v_u}{v_\eta} = \frac{g_X M_W s_\beta}{\sqrt{2} g_2 M_Z^2 s_\gamma}, & \tan \gamma &= \frac{v_\eta}{v_\eta}.
\end{align*}
\]

We have used the notation where $t_\gamma$ or $t_\xi$ means $\tan \gamma$ and $\tan \xi$ respectively, $c_\beta$ or $s_\beta$ means $\cos \beta$ and $\sin \beta$ respectively. Henceforth this notation will be used in all expressions.

More about the scalar mass squared matrices is discussed in Appendix A. Full analytic expressions for the non-zero eigenvalues of the scalar mass-squared matrices are too complicated to write down under any approximations. We have, however checked for a consistent parameter space that there are no tachyonic modes in the spectra. To get a consistent non-tachyonic spectra, we were required to restrict both $\mu_\eta$ and $B_\eta$ in our formalism to negative sign. For almost the entire parameter space, the lightest CP-even Higgs has a tree level mass close to $M_Z$ and so at one-loop level it is possible to get a 125 GeV Higgs.

4 Neutralino and Neutrino Masses in the Non-Minimal Model

The Neutralino mass terms in the Lagrangian arise in this model in the basis

\[
\psi^0 = \left(\nu_e, \nu_\mu, \nu_\tau, i\tilde{B}', i\tilde{B}, i\tilde{W}, \tilde{h}_d^0, \tilde{h}_u^0, \tilde{\eta}, \tilde{\eta}\right)
\]

as,

\[
\mathcal{L} = -\frac{1}{2} \psi^{0T} M_N \psi^0 + \text{h.c.}
\]
where

\[ M_N = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \quad (4.3) \]

and,

\[ M_D = \begin{pmatrix} 0 & 0 & 0 & -\mu_e & 0 & 0 \\ 0 & 0 & 0 & -\mu_2 & -\mu_2 t_\xi & 0 \\ 0 & 0 & 0 & -\mu_3 & 0 & -\mu_3 t_\xi t_\gamma \end{pmatrix}, \quad (4.4) \]

\[ M_R = \begin{pmatrix} M_0 & 0 & 0 & 0 & 0 & -M_{Z'}_s \gamma & M_{Z'}_c \gamma \\ 0 & M_1 & 0 & -\frac{g_1}{g_2} M_W c_\beta & \frac{g_4}{g_2} M_W s_\beta & 0 & 0 \\ 0 & 0 & M_2 & M_W c_\beta & M_W s_\beta & 0 & 0 \\ 0 & -\frac{g_1}{g_2} M_W c_\beta & M_W c_\beta & 0 & -\mu & 0 & 0 \\ 0 & \frac{g_4}{g_2} M_W s_\beta & -M_W s_\beta & -\mu & 0 & 0 & 0 \\ -M_{Z'}_s \gamma & 0 & 0 & 0 & 0 & 0 & \mu_\eta \\ M_{Z'}_c \gamma & 0 & 0 & 0 & 0 & \mu_\eta & 0 \end{pmatrix}. \quad (4.5) \]

From this we can calculate the effective neutrino mass matrix \[ 36–39 \]

\[ m_{\nu}^{\text{eff}} = -M_D M_R^{-1} M_D^T. \quad (4.6) \]

Note that in this analysis we have taken both \( g_m \), the gauge coupling arising from kinetic mixing, and \( M_{10} \), the term corresponding to the \( \tilde{B}\tilde{B}' \) term in \( L_{\text{soft}} \), to be zero. Although the non-minimal model does not necessarily require these to be vanishing, under this approximation, not only is the neutrino mass matrix much more manageable, there is no \( Z-Z' \) mixing at the tree level.

Now we can write the effective neutrino mass matrix,

\[ m_{\nu}^{\text{eff}} = \frac{1}{\Lambda} \begin{pmatrix} \mu_1^2 & \mu_1 \mu_2 & \mu_1 \mu_3 \\ \mu_1 \mu_2 & \mu_2^2 \left( 1 - t_\xi \frac{M_{Z'}^2 c_\gamma + M_0 \mu_\eta}{M_W c_\beta} \frac{g_2^2 d_1}{M_d d_2} \right) & \mu_2 \mu_3 \left( 1 - t_\xi \frac{M_{Z'}^2 c_\gamma + M_0 \mu_\eta}{M_W c_\beta} \frac{g_2^2 d_1}{M_d d_2} \right) \\ \mu_1 \mu_3 & \mu_2 \mu_3 \left( 1 - t_\xi \frac{M_{Z'}^2 c_\gamma + M_0 \mu_\eta}{M_W c_\beta} \frac{g_2^2 d_1}{M_d d_2} \right) & \mu_3^2 \left( 1 - t_\xi \frac{M_{Z'}^2 c_\gamma + M_0 \mu_\eta}{M_W c_\beta} \frac{g_2^2 d_1}{M_d d_2} \right) \end{pmatrix}. \quad (4.7) \]

where,

\[ \begin{align*} 
\mu_2 &= \frac{y_{\eta} v_{\eta}}{g_X} = \sqrt{2} y_{\eta} M_{Z'} s_\gamma, & \mu_3 &= \frac{y_{\eta} v_{\eta}}{g_X} = \sqrt{2} y_{\eta} M_{Z'} s_\gamma, \\
d_1 &= 2 \mu(M_g v_u v_d - 2 \mu M_1 M_2), & d_2 &= 2 \mu_\eta (g_X v_u v_\eta + 2 M_0 \mu_\eta), \\
\tan \xi &= \frac{v_u}{v_\eta} = g_X M_W s_\beta, & \tan \gamma &= \frac{v_\eta}{v_\eta}, \\
M_g &= g_1^2 M_2 + g_2^2 M_1, & \Lambda &= \frac{g_2^2 d_1}{4 M_g M_W c_\beta}. \end{align*} \quad (4.8) \]
This matrix would resemble that obtained from bilinear R-Parity violation if the second terms inside the brackets of the lower \((2 \times 2)\) block were not there. That is, it would be a rank one matrix predicting two zero eigenvalues. This would mean that we would be unable to explain neutrino masses at the tree-level.

In addition to this effective light Majorana neutrino mass matrix that is generated by the see-saw effect, we have contributions to neutrino mass at one-loop level arising from the R-parity violating couplings through the diagram in Fig.1.

The contribution of this diagram is given by\(^{40}\),

\[
\left( m^{\nu(1)}_{\nu}\right)_{11} = \sum_{p=2}^{3} \frac{1}{32\pi^2} \lambda_{1pp}\lambda_{1pp} m_p \sin 2\phi_p \times \\
\left[ -\frac{M^2_{p1}}{m_p^2 - M^2_{p1}} \log \frac{m_p^2}{M^2_{p1}} + \frac{M^2_{p2}}{m_p^2 - M^2_{p2}} \log \frac{m_p^2}{M^2_{p2}} \right]
\]

where we assume a left-right slepton mixing matrix of the form,

\[
V = \begin{pmatrix} \cos \phi_p & \sin \phi_p \\ -\sin \phi_p & \cos \phi_p \end{pmatrix}
\]

and \(M^2_{p_i}\) are the slepton mass eigenvalues and \(m_p\) the lepton mass eigenvalues. The index \(p\) denotes \(\mu\) flavor when it takes the value \(2\) and \(\tau\) when it is \(3\) for both sleptons and leptons.

Similar contribution from the quark-squark loop through the \(\lambda'\) couplings are also present in our model along with those coming from the above lepton-slepton loop. The dominant contribution in this type of diagrams come from the bottom-sbottom pair. We can ignore this contribution to the one loop neutrino mass compared to the above contribution if we assume the soft SUSY breaking squark masses to be higher than a few TeV. For bounds on R-Parity violating couplings see for example Ref.\(^{41}\). The one-loop corrected neutrino mass matrix is,

\[
m_{\nu} = m_{\nu}^{eff} + m^{\nu(1)}_{\nu}. \tag{4.11}
\]

This matrix may be diagonalized by a unitary matrix \(U\) such that,

\[
Um_{\nu}U^T = \text{Diag}(m_1, m_2, m_3) \tag{4.12}
\]

\[\text{Figure 1. Charged lepton-slepton loop that contributes to neutrino mass at 1-loop}\]
which is called the PMNS matrix. The most general parametrization of the PMNS matrix,

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

(4.13)

contains three angles, $\theta_{13}$, $\theta_{12}$ and $\theta_{23}$ and the CP-violating phase $\delta_{CP}$.

### 4.1 Mass Models and Possible Mixing Patterns

Current neutrino data favors slightly non-maximal atmospheric mixing and a non-zero $\theta_{13}$ [4]. We find that, in our model, two very simple conditions,

$$y_\eta = y_\bar{\eta}$$
$$\tan \gamma = 1$$

(4.14)

leads to a mass matrix of the form,

$$M = \begin{pmatrix} a & b & b \\ b & c & d \\ b & d & c \end{pmatrix}$$

(4.15)

which is the most general $\mu - \tau$ exchange symmetric neutrino mass matrix. This matrix always predicts maximal atmospheric mixing and zero $U_{e3}$. It is with violation of the conditions 4.14 that we obtain mass matrices that satisfy neutrino oscillation data. We have not considered any CP-violation in our model, so $\delta_{CP} = 0$ for all subsequent calculations. Our modus operandi is to compare the mixing matrices obtained, with the matrix 4.13 and use,

$$\sin^2 \theta_{13} = |U_{e3}|^2$$
$$\sin^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2}$$
$$\sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}$$

(4.16)

to analyze how the mixing angles vary as we violate these conditions. We have quantified the deviation from the relations 4.14 by introducing two new parameters $\delta y_\eta$ and $\delta t_\gamma$,

$$\delta t_\gamma = (1 - t_\gamma) \times 100$$
$$\delta y_\eta = \frac{y_\eta - y_\bar{\eta}}{y_\eta} \times 100.$$

(4.17)

These parameters are just the percentage deviation from the conditions in Equation 4.14. In Figure 2 we have plotted the variation of the mixing angles with deviation in the conditions on the Yukawa couplings, $y_\eta$ and $y_\bar{\eta}$ (see Figures 2(a) and 2(b)) and $\tan \gamma$ (see Figures 2(c) and 2(d)). It is apparent from these figures that variation in either of the two parameters simultaneously shifts the mixing pattern towards non-maximal atmospheric mixing and real, non-zero $U_{e3}$.
4.2 Numerical Analysis

We have used Mathematica 11.1 for all the numerical analyses. For normal hierarchy of neutrino masses (NH) we found a large concentration of allowed parameter points in a region,

\[
\begin{align*}
800 \text{ GeV} &< M_{\tilde{L}} < 1 \text{ TeV}, & 400 \text{ GeV} &< M_1, M_2 < 800 \text{ GeV}, \\
200 \text{ GeV} &< \mu < 300 \text{ GeV}, & 50 \text{ GeV} &< M_0 < 100 \text{ GeV}, \\
-1 \text{ TeV} &< \mu_\eta < -1.5 \text{ TeV}, & 700 \text{ GeV} &< M_{Z'} < 800 \text{ GeV}, \\
0.4 &< g_X < 0.6, & 25 &< \tan \beta < 35, \\
10^{-6} &< y_\eta; y_{\bar{\eta}} < 2 \times 10^{-6}, & 7 \times 10^{-3} \text{ GeV} &< \mu_e < 10^{-2} \text{ GeV}, \\
10^{-4} &< \lambda_{122}, \lambda_{133} < 5 \times 10^{-4}. & 
\end{align*}
\]

Figure 2. Variation of $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ with deviation from $y_\eta = y_{\bar{\eta}}$ and $\tan \gamma = 1$ conditions on $m^\nu_{\tilde{E}E}$ with $\mu_\eta$ free. Red is for Normal hierarchy, blue is for inverted hierarchy.
while a similar concentration for inverted hierarchy (IH) was found in the region,

\[
\begin{align*}
550 \text{ GeV} &< M_{\tilde{L}} < 750 \text{ GeV}, & 300 \text{ GeV} &< M_1 < 500 \text{ GeV}, \\
1 \text{ TeV} &< M_2 < 1.2 \text{ TeV}, & 150 \text{ GeV} &< \mu < 250 \text{ GeV}, \\
0 \text{ GeV} &< M_0 < 20 \text{ GeV}, & -4 \text{ TeV} &< \mu_\eta < -2 \text{ TeV}, \\
1.2 \text{ TeV} &< M_{Z'} < 1.5 \text{ TeV}, & 0.3 &< g_X < 0.4, \\
30 &< \tan \beta < 40, & 3 \times 10^{-6} &< y_\eta, y_\bar{\eta} < 4 \times 10^{-6}, \\
2 \times 10^{-3} \text{ GeV} &< \mu_e < 1 \times 10^{-2} \text{ GeV}, & 1 \times 10^{-4} &< \lambda_{122}, \lambda_{133} < 2 \times 10^{-4}.
\end{align*}
\]

Here $M_{\tilde{L}}$ stands for all the slepton soft SUSY breaking masses. The scanned range of $M_{Z'}$ and $g_X$ is motivated from the restrictions laid down by the LHC data from the $Z \rightarrow 4\mu$ channel\cite{50, 51}, the observation of elastic neutrino nucleon scattering (CE$\nu$NS) by the COHERENT collaboration\cite{52–54} and the observation of elastic scattering of solar neutrinos off electrons by Borexino\cite{20, 55}. Apart from this the most stringent bounds on Sparticle masses\cite{56, 57} were also applied along with the kinematic bounds from the combined LEP data\cite{4}. Our neutrino data constitutes mostly of points where the lightest neutralino is at most 6 GeV lighter than the lightest chargino and hence evades much of the constrained parameter space. Both the conditions \ref{eq:4.14} were allowed to be violated upto 20% and we plot the points allowed by experimental data on a $(\delta t_\gamma - \delta y_\eta)$ plane in Figure 3. The points satisfying neutrino oscillation data are plotted in red while the blue background represents regions where Muon $g - 2$ is satisfied\footnote{A detailed analysis of muon anomalous magnetic moment in our model is presented later.}. Note that negative deviation in $\tan \gamma$ is preferred in both NH and IH from $(g - 2)_\mu$ in these cases, that is, a value of $t_\gamma$ greater than unity. However, this analysis is not exhaustive and there may be other regions where neutrino oscillation data may be fitted. We have only taken up two representative interesting regions where we found both neutrino and muon $(g - 2)$ data is satisfied simultaneously along with all the other experimental bounds as mentioned.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure3.png}
\caption{Points satisfying neutrino mixing angles and mass-squared difference in red and those satisfying Muon $g - 2$ constraint in blue on $(\delta t_\gamma - \delta y_\eta)$ plane. Figure 3(a) is for normal hierarchy and 3(b) for inverted hierarchy.}
\end{figure}
5 Anomalous Magnetic Moment

The magnetic moment of the muon is one of the most accurately measured physical quantities today with the final value \[4\],
\[
a_\mu^{exp} = (116592089 \pm 63) \times 10^{-11},
\]
which however does not agree with the theoretically predicted value from the Standard Model. The discrepancy,
\[
\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (28.8 \pm 8.0) \times 10^{-10},
\]
is a \(\sim 3.6\sigma\) deviation from the SM value. Given the accuracy of the \(g - 2\) measurement and the evaluation of its standard model prediction, it is an ideal testing ground for any new physics model, like Supersymmetry(SUSY). Supersymmetry, even in its minimal model (MSSM) has been shown to provide sizeable contributions to \(g - 2\) that are enough to explain its discrepancy from the SM prediction. The muon \(g - 2\) data is also ideal to constrain certain parameters of the model, for example, the sign of the “\(\mu\)-term” and the mass scale of the scalar and fermionic superpartners in the case of MSSM.

There are two main components of the MSSM contribution to the muon \(g - 2\): from the smuon-neutralino loop and from the chargino-sneutrino loop. When the mass scale of the superpartners are roughly of the scale \(M_{SUSY}\), this contribution is given by \([58–60]\),
\[
\Delta a_\mu^{MSSM} = 14 \text{Sign}(\mu) \tan \beta \left( \frac{100 \text{ GeV}}{M_{SUSY}} \right)^2 10^{-10}.
\]

Our model, which has a \(Z'\) boson coupling to the muon, can complement the SUSY contribution. This allows us to have a natural solution to the hierarchy problem and get a stable Higgs mass, while still explaining the anomalous magnetic moment of the muon.

5.1 Outline of the Calculation

In our model, we have non-trivial mixing between the smuons and other charged scalars, as well as the muons with other charged fermions. Otherwise the calculation is relatively straightforward and mimics that for the MSSM. Instead of the neutralino-smuon loop we consider the more general neutralino-charged scalar loops to allow for the mixing between smuons and other scalars. Similarly the chargino-sneutrino loop for the MSSM is expanded into a chargino-neutral scalar loop calculation. We have allowed the sign of the neutralino mass eigenvalues (\(\epsilon_i\)) and the chargino mass eigenvalues (\(\eta_i\)) to be either positive or negative. The real orthogonal diagonalizing matrices are suitably defined following the prescription in Appendix A of Ref.\([61]\).

5.1.1 Neutralino-Charged Scalar Loop

For this calculation we require the Neutralino mass matrix and the charged scalar mass matrix. In the basis \((\nu_e, \nu_\mu, \nu_\tau, i\tilde{B}, i\tilde{B}, i\tilde{W}, \tilde{h}_d, \tilde{h}_u, \tilde{\eta}, \tilde{\bar{\eta}})\), we can write the neutralino mass terms as,
\[
\mathcal{L} = -\frac{1}{2} \psi^{0T} M_N \psi^0 + h.c.
\]
which is diagonalized by the matrix $N$,

$$N^* M_N N^\dagger = m_{\tilde{\chi}_0}.$$  \hfill (5.5)

The charged scalar mass matrix ($M_{\tilde{S}^\pm}$) is written in the basis ($h^+_u, h^-_d, \tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{\mu}_R, \tilde{\tau}_R$) and diagonalized so that,

$$U_{\tilde{S}^\pm} M^2_{\tilde{S}^\pm} U_{\tilde{S}^\pm}^\dagger = m^2_{\tilde{S}^\pm}$$  \hfill (5.6)

which includes a Goldstone mode. More about the charged scalar mass-squared matrix is discussed in Appendix A.3.

Using these mixing matrices, the neutralino-charged scalar loop contribution to the muon ($g - 2$) was found to be[58–60],

$$a_{\mu}^\chi = -\frac{m_\mu}{16\pi^2} \sum_{i=1}^{10} \sum_{j=1}^{7} \left( |n^L_{ij}|^2 + |n^R_{ij}|^2 \right) \frac{m_\mu}{12m^2_{\tilde{S}^\pm_j}} F^N_1(x_{ij})$$

$$+ \frac{m_{\tilde{\chi}_0}}{3m^2_{\tilde{S}^\pm_j}} \text{Real}(n^L_{ij}n^R_{ij} F^N_2(x_{ij}))$$  \hfill (5.7)

where,

$$F^N_1(x) = \frac{2(1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x)}{(1 - x)^4}$$

$$F^N_2(x) = \frac{3(1 - x^2 + 2x \log x)}{(1 - x)^3}$$  \hfill (5.8)

with

$$x_{ij} = \frac{m_{\tilde{\chi}_0}^2}{m^2_{\tilde{S}^\pm_j}}$$

and

$$n^L_{ij} = \left( \frac{g_X}{\sqrt{2}} N^*_{i1} - \sqrt{2} g_1 N^*_{i5} - \sqrt{2} g_m N^*_{i4} \right) U^*_{\tilde{S}^\pm_j} \left( y_\mu N^*_{i7} + \lambda_{122} N^*_{i1} \right) U^*_{\tilde{S}^\pm_j}$$

$$+ \lambda_{122} N^*_{i2} U^*_{\tilde{S}^\pm j3} + y_\mu N^*_{i2} U^*_{\tilde{S}^\pm j2}$$  \hfill (5.9)

$$n^R_{ij} = \left( \frac{g_1}{\sqrt{2}} N^*_{i5} + \frac{g_2}{\sqrt{2}} N^*_{i6} + \frac{g_m}{\sqrt{2}} N^*_{i4} - \frac{g_X}{\sqrt{2}} N^*_{i4} \right) U^*_{\tilde{S}^\pm j4}$$

$$- \left( y_\mu N^*_{i7} + \lambda_{122} N^*_{i1} \right) U^*_{\tilde{S}^\pm j7} - y_\eta N^*_{i9} U^*_{\tilde{S}^\pm j1}.$$  \hfill (5.10)
In our case the external muons also mix with other charged fermions in the chargino mass matrix hence the expressions for the couplings \((n^L\text{ and } n^R)\) will include appropriate elements from the chargino mixing matrices \((V_{44}\text{ and } U_{44}^*)\) respectively. The most general formulae are given here taking non-zero \(g_m\) also. We took this to be zero in our numerical analysis.

### 5.1.2 Chargino-Neutral Scalar Loop

![Chargino-neutral scalar loop](image)

**Figure 5.** Chargino-neutral scalar loop that contributes to muon \((g - 2)\)

In this part of the calculation we will require the chargino mass matrix and the neutral scalar and pseudoscalar mass-squared matrices. Defining,

\[
\begin{align*}
\psi^- & = (i\tilde{W}^-, \tilde{h}^-, e^-_L, \mu^-_L, \tau^-_L) \\
\psi^+ & = (i\tilde{W}^+, \tilde{h}^+, e^+_R, \mu^+_R, \tau^+_R)
\end{align*}
\]

the chargino mass terms in the Lagrangian may be written as,

\[
\mathcal{L} \supset -\frac{1}{2}(\psi^-^T X \psi^+^T + \psi^+ T X^T \psi^-) + \text{h.c.}
\]

where \(X\) is the chargino mass matrix. It can be diagonalized by two matrices \(U\) and \(V\) so that,

\[
U^* X V^T = m_{\tilde{\chi}^\pm}.
\]

The chargino mass matrix is given and discussed in Appendix B. The neutral scalar mass-squared matrix \(M_{\tilde{S}_0}^2\), given in Appendix A.1, is written in the basis \((\tilde{\nu}_{eR}, \tilde{\nu}_{\mu R}, \tilde{\nu}_{\tau R}, h_{d}^0, h_{u}^0, \eta_R, \tilde{\eta}_R)\) and is diagonalized so that,

\[
U_{\tilde{S}_0}^T M_{\tilde{S}_0}^2 U_{\tilde{S}_0} = m_{\tilde{S}_0}^2.
\]

Similarly the pseudoscalar mass-squared matrix \(M_{\tilde{P}_0}^2\) from Appendix A.2 is written in the basis \((\tilde{\nu}_{eI}, \tilde{\nu}_{\mu I}, \tilde{\nu}_{\tau I}, h_{dI}^0, h_{uI}^0, \eta_I, \tilde{\eta}_I)\) and is diagonalized so that,

\[
U_{\tilde{P}_0}^T M_{\tilde{P}_0}^2 U_{\tilde{P}_0} = m_{\tilde{P}_0}^2.
\]
Using these mixing matrices we calculate the contribution of the chargino-neutral scalar loop to the muon \((g - 2)\)\cite{58–60},

\[
\begin{align*}
    a_{\mu}^{\tilde{\chi}^\pm} &= \frac{m_\mu}{16\pi^2} \sum_{i=1}^{5} \sum_{j=1}^{7} \left[ \frac{m_\mu}{12m_{\tilde{\chi}^\pm}^2} (|c_{i,j}^{Le}|^2 + |c_{i,j}^{Re}|^2) F_1^C(y_{ij}^e) + \frac{2m_{\tilde{\chi}^\pm}}{3m_{\tilde{\chi}^\pm}^2} \text{Re}(c_{i,j}^{Le} c_{i,j}^{Re}) F_2^C(y_{ij}^e) \right] \\
    &+ \frac{m_\mu}{16\pi^2} \sum_{i=1}^{5} \sum_{j=1}^{5} \left[ \frac{m_\mu}{12m_{\tilde{\chi}^\pm}^2} (|c_{i,j}^{Lo}|^2 + |c_{i,j}^{Ro}|^2) F_1^C(y_{ij}^o) + \frac{2m_{\tilde{\chi}^\pm}}{3m_{\tilde{\chi}^\pm}^2} \text{Re}(c_{i,j}^{Lo} c_{i,j}^{Ro}) F_2^C(y_{ij}^o) \right]
\end{align*}
\]

(5.15)

where,

\[
\begin{align*}
    F_1^C(x) &= \frac{2(2 + 3x - 6x^2 + x^3 + 6x \log x)}{(1 - x)^4} \\
    F_2^C(x) &= -\frac{3(3 - 4x + x^2 + 2 \log x)}{2(1 - x)^3}
\end{align*}
\]

(5.16)

with

\[
    y_{ij}^e = \frac{m_{\tilde{\chi}^\pm}^2}{m_{\tilde{S}^0}^2}, \quad y_{ij}^o = \frac{m_{\tilde{\chi}^\pm}^2}{m_{\tilde{P}^0}^2}
\]

and

\[
\begin{align*}
    c_{i,j}^{Le} &= \frac{y_{\mu}}{\sqrt{2}} V_{i4} U_{i4}^* S_{j2}^0 - \frac{\lambda_{122}}{\sqrt{2}} U_{i4} U_{i4}^* S_{j2}^0 + \frac{\lambda_{122}}{\sqrt{2}} U_{i4} U_{i4}^* S_{j1}^0 \\
    c_{i,j}^{Re} &= -\frac{g_2}{\sqrt{2}} V_{i1} U_{i4}^* S_{j2}^0 - \frac{y_{\eta}}{\sqrt{2}} V_{i2} U_{i4}^* S_{j6}^0 - \frac{\lambda_{122}}{\sqrt{2}} V_{i4} U_{i4}^* S_{j1}^0 \\
    c_{i,j}^{Lo} &= i \frac{y_{\mu}}{\sqrt{2}} V_{i2} U_{i4}^* S_{j2}^0 - i \frac{\lambda_{122}}{\sqrt{2}} U_{i4} U_{i4}^* S_{j6}^0 + i \frac{\lambda_{122}}{\sqrt{2}} U_{i4} U_{i4}^* S_{j1}^0 \\
    c_{i,j}^{Ro} &= -i \frac{g_2}{\sqrt{2}} V_{i1} U_{i4}^* S_{j2}^0 + i \frac{y_{\eta}}{\sqrt{2}} V_{i2} U_{i4}^* S_{j6}^0 + i \frac{\lambda_{122}}{\sqrt{2}} V_{i4} U_{i4}^* S_{j1}^0
\end{align*}
\]

(5.17)

Just as in the case of the neutralino-charged scalar loop, here too the external muons will mix with the other charged fermions and result in factors of \(V_{44}\) and \(U_{44}^*\) in \(c^L\) and \(c^R\) respectively.

### 5.1.3 \(Z'\) Contribution

![Figure 6. Z’ loop that contributes to Muon \((g - 2)\)](image)
In addition to the purely supersymmetric contribution to \( \Delta a_\mu \), \( Z' \)-boson also adds an important part to the total muon magnetic moment. The contribution of \( U(1)_{L_\mu - L_\tau} \) to muon \( g - 2 \) can be easily evaluated from the diagram in Figure 6. It is given by

\[
\Delta a_\mu^{Z'} = \frac{g_5^2 m_\mu^2}{4\pi^2} \int_0^1 dz \frac{z^2(1 - z)}{m_\mu^2 z + M_{Z'}^2(1 - z)}. \tag{5.18}
\]

Here too, the external muons and those inside the loop will mix with other leptons and charginos as in the previous sections. This calculation assumes no \( Z-Z' \) mixing at the tree level owing to the fact that \( g_m \) is zero and the sneutrinos do not acquire any VEV.

### 5.2 Numerical Analysis

Any gauged \( U(1)_{L_\mu - L_\tau} \) model is severely constrained by the process: neutrino trident production. That is, the production of \( \mu^+ \mu^- \) pair from the scattering of a muon neutrino off heavy nuclei. The CHARM-II[48] and CCFR[49] collaborations found reasonable agreement of observed cross section for this process to its SM prediction:

\[
\frac{\sigma_{\text{CHARM-II}}}{\sigma_{\text{SM}}} = 1.58 \pm 0.57, \quad \frac{\sigma_{\text{CCFR}}}{\sigma_{\text{SM}}} = 0.82 \pm 0.28. \tag{5.19}
\]

Thus it severely constrains the allowed parameter space for any new neutral gauge boson. In particular, when coupled with the restrictions laid down by the LHC data from the \( Z \to 4\mu \) channel[50, 51], the observation of elastic neutrino nucleon scattering (CE\( \nu \)NS) by the COHERENT collaboration[52–54] and the observation of elastic scattering of solar neutrinos by Borexino[20, 55], almost the entire parameter space relevant to muon \( g - 2 \) is ruled out. However, the situation of the SUSY version is not so bleak when it comes to resolution of muon \( g - 2 \) through an extra force. In our model the total contribution to muon \( (g - 2) \) from the two supersymmetric processes when added to that from the \( Z' \) loop allows for a much more liberal parameter space. We plot the region allowed by current \( (g - 2) \mu \) data in \((M_{Z'} - g_X)\) plane for two different scenarios in figure 7. The green shaded region shows the allowed parameter space in our model while the solid red shaded region shows that for a gauged \( U(1)_{L_\mu - L_\tau} \) model where SUSY plays no part. The dashed lines lay down the various exclusion limits from the different experiments. The red dashed line is for the Borexino experiment of elastic scattering of solar neutrinos, the purple dashed line is from the data for elastic scattering of solar neutrinos by the COHERENT collaboration[55]. The constraint from the neutrino trident observation by CCFR is shown in dashed black lines[14]. The exclusion limit from the LHC data of the process \( Z \to 4\mu \) is shown in blue dashed lines[14, 50, 51]. Figure 7(a) corresponds to \( M_{\tilde{\mu}_L} = M_{\tilde{\mu}_R} = 500 \text{ GeV}, M_0 = 70 \text{ GeV}, M_1 = 400 \text{ GeV}, M_2 = 800 \text{ GeV}, \mu = 400 \text{ GeV} \) and \( \tan \beta = 35 \). Figure 7(b) corresponds to \( M_{\tilde{\mu}_L} = M_{\tilde{\mu}_R} = 935 \text{ GeV}, M_0 = 100 \text{ GeV}, M_1 = 450 \text{ GeV}, M_2 = 650 \text{ GeV}, \mu = 400 \text{ GeV} \) and \( \tan \beta = 33.5 \). The rest of the SUSY parameters have been chosen judiciously for either plots: \( \tan \gamma = 1.1, \mu_e = 0.008, \mu_\eta = -3 \text{ TeV}, y_{\tau}/y_\eta = 3 \times 10^{-6} \) and the RPV \( \lambda \) couplings fixed at \( 10^{-4} \). The two plots were chosen to represent two different regions with differing magnitudes of the SUSY contribution to muon \( (g - 2) \). Figure 7(a) represents the scenario where there is a large SUSY contribution as opposed to Figure 7(b) where it is comparatively lower.
Figure 7. Parameter space for the $Z'$ gauge boson showing the regions relevant to $(g-2)_\mu$. Solid red shade is for contribution from gauged $U(1)_{L_\mu-L_\tau}$ without considering SUSY, solid green shaded region corresponds to our model. Overlap is shaded brown. The dashed lines denote respective exclusion limits: purple for COHERENT neutrino elastic scattering experiment, red for the data from Borexino, black for CCFR data for neutrino trident observation and blue for $Z \rightarrow 4\mu$ data from LHC. The CCFR and $Z \rightarrow 4\mu$ exclusion regions have been taken from Ref[14], the Borexino and COHERENT exclusion regions from Ref[55] and both the SUSY and $Z'$ contributions are by themselves insufficient to explain the anomalous magnetic moment of muon. It is very clear from these plots that large regions of the $(M_{Z'} - g_X)$ plane open up in terms of $(g-2)_\mu$ while the non-SUSY $U(1)_{L_\mu-L_\tau}$ models are already almost ruled out. More importantly, as we increase the SUSY contribution, the allowed region fills up the unconstrained parameter space. In a second analysis we have plotted the $(g-2)_\mu$ against the physical masses of the lightest neutralino and chargino and the slepton soft SUSY-breaking mass in figure 8. The SUSY parameters that affect our analysis were scanned randomly in the region,

$$100 \text{ GeV} < M_{\tilde{L}} < 2 \text{ TeV}, \quad 100 \text{ GeV} < M_1, M_2, \mu < 2 \text{ TeV},$$

$$1 \text{ GeV} < M_{Z'} < 1.5 \text{ TeV}, \quad 0.01 < g_X < 1,$$

$$10^{-6} < y_\eta < 5 \times 10^{-6}, \quad -10 \text{ TeV} < \mu_\eta < -1 \text{ TeV},$$

$$0 \text{ GeV} < M_0 < 2 \text{ TeV}, \quad 0 \text{ GeV} < A_t < 2 \text{ TeV},$$

$$100 \text{ GeV} < B, B_\epsilon < 2 \text{ TeV}, \quad 10 < \tan \beta < 50.$$
Figure 8. $\Delta a_{\mu}$ plotted against lightest neutralino and chargino masses, along with left-handed slepton soft mass. 2\(\sigma\) allowed region for $\Delta a_{\mu}$ is shown between dashed lines. The region shaded yellow is the allowed region from MSSM, it has been taken from Ref[62]

allowed in the MSSM and constrained only by LEP data is shown shaded yellow in the same plots[62]. We find that the $Z'$ contribution and the SUSY contribution complement each other so that we can have heavier sparticle masses than we could in the MSSM while still explaining $(g - 2)_{\mu}$. The approximate non-decoupling behavior that is observed is due to the extra contribution coming from $Z'$ loop. We have separately checked that the SUSY contribution alone shows the typical decoupling behavior as expected. However, it still allows for a heavier particle spectra than can be afforded in pure MSSM. We have shown the data considering the most stringent sparticle limits[4, 56, 57]. This comes from the 3\(l\) final state searches at LHC in chargino-neutralino pair production with slepton mediated decays. We have also obtained similar datasets considering more relaxed bounds, the 2\(l\) final state searches and just the LEP bounds. They show a similar feature where we can satisfy muon $(g - 2)$ for heavier sparticle masses compared to the MSSM.

6 Conclusion

We began with an attempt to explore whether gauged $U(1)_{L_{\mu} - L_{\tau}}$ extended SUSY could help improve the current situation when it comes to neutrino oscillation data and muon anomalous magnetic moment given the current experimental bounds on SUSY itself. The
minimal model with the MSSM field content required the sneutrinos to acquire non-zero vacuum expectation values so that the gauged $U(1)_{L_\mu-L_\tau}$ is spontaneously broken. This lead to two different problems: firstly, when both the sneutrinos charged under the new gauged symmetry acquired VEV the model suffered from the Majoron problem wherein we had a massless CP-odd scalar and its light CP-even partner which could couple to the $Z$-Boson. Secondly, even when only one of the sneutrinos acquired VEV we found the neutrino mass matrix had a texture that was impossible to fit to current oscillation data. Hence the minimal model was ruled out and the non-minimal model adopted.

Now, there are two extra fields that are singlets under all other gauge symmetries except $U(1)_{L_\mu-L_\tau}$ and acquire VEV to spontaneously break the symmetry instead of the sneutrinos. This allowed us to avoid the Majoron problem altogether. We found extremely intriguing results when it came to neutrino mixing. Under two very simple assumptions, the model resulted in the most $\mu-\tau$ symmetric mass matrix. These conditions were that the two new fields acquire VEVs of equal magnitude and sign, and that the two new Yukawa couplings be equal. Of course, this mass matrix yielded maximal atmospheric mixing and a zero $\theta_{13}$ which is ruled out by current data. So we parametrized deviations from this exact mixing pattern with two parameters for the two conditions and this allowed us to obtain multiple regions where the correct neutrino oscillation could be explained. Deviation in the Yukawa coupling equality complemented that in the equality of VEVs in the sense that the former alone is insufficient to fit neutrino data but is often necessary in conjunction with the latter.

In parallel, we conducted a numerical analysis of the muon anomalous magnetic moment in our model. A scan over the entire parameter space shows that we can explain the observed magnetic moment of muon for much larger values of the sparticle masses as compared to the MSSM. We also observe an intriguing non-decoupling behavior in the plots of $\Delta a_\mu$ vs sparticle masses owing to the presence of the $Z'$ boson which can make up for any fall in the SUSY contribution with heavier sparticle masses. Decoupling is still observed, as expected, if we do not consider the $Z'$ contribution. Finally we combine both of these analyses to show two representative regions of the parameter space in Figure 3 where neutrino oscillation data may be reconciled with muon magnetic moment measurements. It is possible to fit neutrino oscillation data with both normal and inverted hierarchy of the masses. On the plot showing these regions we superpose the regions explaining $(g-2)_\mu$, which further restricts the parameter space in both cases. Still, it is possible to obtain a parameter space where the model explains neutrino oscillation data along with muon anomalous magnetic moment.

Some interesting signatures for this model at the LHC would be the three or more leptons plus missing energy final state, involving supersymmetric particles in the intermediate states. For example, $pp\rightarrow \tilde{\mu}^+ \tilde{\mu}^- Z'/\tilde{\nu}_\mu \tilde{\nu}_\tau^* Z'/\tilde{\nu}_\mu \tilde{\nu}_\mu^* Z'$ processes can lead to multilepton final states along with $E_T$. In addition, contributions to multilepton final states (with or without $E_T$) involving SM particles and $Z'$ can also be present. A detailed analysis with all possible final states requires a dedicated study altogether which we plan to take up in a future work.
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A Scalar Mass-Squared Matrices

A.1 CP-even Mass Squared Matrix

The CP-even mass squared matrix was derived using 2.17 in the basis, \((\tilde{\nu}_R, \tilde{\nu}_R, \tilde{\nu}_R, h^0_R, h^0_{uR}, \eta_R, \bar{\eta}_R)\). We assume that \(g_m\) is zero. The mass matrix may be expressed as,

\[
M^2_{\tilde{S}0} = B_{even} + \tilde{M}^2_{even}
\]  

(A.1)

where

\[
B_{even} = \begin{pmatrix}
\mu_1^2 & \mu_1 \mu_2 & \mu_1 \mu_3 & B_e t_{\beta} & -B_e & 0 & 0 \\
\mu_1 \mu_2 & \mu_2^2(1 + t_{\xi}^2) & \mu_2 \mu_3 & \mu_2 \cot \beta & -\mu_2 \mu_2 t_{\xi} \cot \gamma & -\mu_2 \mu_2 t_{\xi} & 0 \\
\mu_1 \mu_3 & \mu_2 \mu_3 & \mu_3^2(1 + t_{\xi}^2 t_{\gamma}^2) & \mu_3 \mu_3 & -\mu_3 \mu_3 t_{\xi} t_{\gamma} & -\mu_3 \mu_3 t_{\xi} t_{\gamma} & 0 \\
B_e t_{\beta} & \mu_2 \mu_2 & \mu_3 \mu_3 & B \cot \beta & -B \cot \beta & 0 & B t_{\gamma} \\
-B_e & -\mu_2 \mu_2 t_{\xi} \cot \gamma & -\mu_2 \mu_3 t_{\xi} t_{\gamma} & 0 & 2 \mu_2 t_{\xi} & -B t_{\gamma} & \eta \\
0 & -\mu_2 \mu_2 t_{\xi} & \mu_3 \mu_3 t_{\xi} t_{\gamma} & 0 & 2 \mu_2 t_{\xi} & \eta & -B t_{\gamma}
\end{pmatrix}
\]

(A.2)

and

\[
\tilde{M}^2_{even} = \text{diag} \left( M^2_{\tilde{\nu}_L} + \tilde{m}^2_1, M^2_{\tilde{\mu}_L} + \tilde{m}^2_2, M^2_{\tilde{\tau}_L} + \tilde{m}^2_3, (M_Z)_{2\times 2}, (M_{Z'})_{2\times 2} \right)
\]  

(A.3)

and

\[
(M_Z)_{2\times 2} = \begin{pmatrix}
M^2_Z \cos^2 \beta & -M^2_Z c_{\beta} s_{\beta} \\
-M^2_Z c_{\beta} s_{\beta} & M^2_Z s_{\beta}^2
\end{pmatrix}
\]

\[
(M_{Z'})_{2\times 2} = \begin{pmatrix}
M^2_{Z'} s_{\gamma}^2 & -M^2_{Z'} s_{\gamma} c_{\gamma} \\
-M^2_{Z'} s_{\gamma} c_{\gamma} & M^2_{Z'} c_{\gamma}^2
\end{pmatrix}
\]

(A.4)

where all the parameters are as defined for the minimization equations and \(\tilde{m}_i^2 = \frac{M^2_Z}{2} c_{2\beta} + Q_X M^2_Z c_{2\gamma} \).
A.2 CP-Odd Mass Squared Matrix

The CP-odd mass square matrix is constructed by taking second derivatives according to 2.17 in the basis $(\tilde{\nu}_e I, \tilde{\nu}_\mu I, \tilde{\nu}_\tau I, h^0_d, h^0_u, \eta_I, \bar{\eta}_I)$. The symmetric matrix is given by,

$$M^2_{P^0} = B_{\text{odd}} + \tilde{M}^2_{\text{odd}} \quad (A.5)$$

where

$$B_{\text{odd}} = \begin{pmatrix}
\mu_1^2 & \mu_1 \mu_2 & \mu_1 \mu_3 & B_e t_\beta & B_e & 0 & 0 \\
\mu_1 \mu_2 & \mu_2^2 (1 + t_\xi^2) & \mu_2 \mu_3 & -\mu_2 \cot \beta & -\mu_\eta \mu_2 t_\xi \cot \gamma & -\mu_\eta \mu_2 t_\xi \\
\mu_1 \mu_3 & \mu_2 \mu_3 & \mu_3^2 (1 + t_\xi^2) & \mu_3 \cot \beta & -\mu_\eta \mu_3 t_\xi t_\gamma & -\mu_\eta \mu_3 t_\xi t_\gamma \\
B_e t_\beta & \mu_2 & \mu_3 & B t_\beta & B & 0 & 0 \\
0 & -\mu_\eta \mu_2 t_\xi \cot \gamma & -\mu_\eta \mu_3 t_\xi t_\gamma & 0 & 2 \mu_2^2 t_\xi & -B_\eta \cot \gamma & -B_\eta \\
0 & -\mu_\eta \mu_2 t_\xi & -\mu_\eta \mu_3 t_\xi t_\gamma & 0 & 2 \mu_3^2 t_\xi t_\gamma & -B_\eta & -B_\eta t_\gamma
\end{pmatrix} \quad (A.6)$$

and

$$\tilde{M}^2_{\text{odd}} = \text{diag} \left( M^2_{\tilde{\nu}_e}, M^2_{\tilde{\nu}_\mu}, M^2_{\tilde{\nu}_\tau}, M^2_{h^0_d}, M^2_{h^0_u}, M^2_{\eta}, M^2_{\bar{\eta}}, 0, 0, 0 \right). \quad (A.7)$$

Again, all the parameters are as defined for the minimization equations. There are two exactly zero eigenvalues of this mass matrix that correspond to the two goldstone modes arising from the spontaneous breaking of the gauge symmetries to $U(1)_{em}$. The corresponding matrix in the MSSM has just one zero eigenvalue. The extra goldstone mode corresponds to the breaking of $U(1)_{L_\mu - L_\tau}$ and gives mass to the $Z'$ Boson.

A.3 Charged Scalar Mass-Squared Matrix

The charged scalar mass square matrix is calculated from the total potential 3.5 using 2.17 in the basis $u^+ = (h^+_u, h^+_d, \tilde{\nu}_L, \tilde{\nu}_L, \tilde{\nu}_L, \tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{E}_e, \tilde{E}_\mu, \tilde{E}_\tau)^T$ and $u^- = (h^-_u, h^-_d, \tilde{\nu}_L, \tilde{\nu}_L, \tilde{\nu}_L, \tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{E}_e, \tilde{E}_\mu, \tilde{E}_\tau)^T$. The respective terms may be written down as,

$$u^+ T M^2_{S^\pm} u^- \quad (A.8)$$

where

$$M^2_{S^\pm} = B_\pm + \tilde{M}^2_\pm \quad (A.9)$$
\[ B_\pm = \begin{pmatrix}
B \cot \beta + & B^+ & B_e & \mu_2 \cot \beta & \mu_3 \cot \beta & m_e \mu_1 & m_\mu \mu_2 & m_\tau \mu_3 \\
M_{W}^2 c_\beta & M_{W}^2 c_\beta s_\beta & B^+ & B_t \beta & B_e t \beta & m_\mu & m_\mu \mu_2 t \beta & m_\mu \tau \mu_3 t \beta \\
M_{W}^2 s_\beta & M_{W}^2 s_\beta & B_e t \beta & \mu_2 & \mu_3 & m_e X_t & m_\mu \lambda_2 t \beta & m_\mu \lambda_3 t \beta \\
\mu_2 \cot \beta & \mu_2 & \mu_1 \mu_2 & \mu_2 + m_\mu^2 & m_\mu & 0 & m_\mu X_t & 0 \\
\mu_3 \cot \beta & \mu_3 & \mu_1 \mu_2 & \mu_2 \mu_3 & \mu_3 + m_\tau^2 & 0 & 0 & m_\tau X_t \\
m_e \mu_1 & m_e \mu_1 t \beta & m_e X_t & 0 & 0 & m_e^2 & 0 & 0 \\
m_\mu \mu_2 & m_\mu \mu_2 t \beta & \mu_2 \mu_3 t \beta & m_\mu X_t & 0 & 0 & m_\mu^2 & 0 \\
-m_\mu \mu_3 t \beta & -m_\mu \mu_3 t \beta & m_\mu X_t & 0 & 0 & m_\tau^2 & 0 & 0 \\
\end{pmatrix} \] (A.10)

and

\[ \tilde{M}_\pm^2 = \text{diag} (0, 0, M_{eL}^2 + \tilde{I}_{L1}, M_{\mu L}^2 + \tilde{I}_{L2}, M_{\tau L}^2 + \tilde{I}_{L3}, M_{eR}^2 + \tilde{I}_{R1}, M_{\mu R}^2 + \tilde{I}_{R2}, M_{\tau R}^2 + \tilde{I}_{R3}) \] (A.11)

with \(X_t = A - \mu \tan \beta, \tilde{I}_{Li} = Q_i^X M_{Z_\mu}^2 c_{27} + \left( \frac{1}{2} - c_{27}^2 \right) M_{Z_\mu}^2 c_{27} \) and \( \tilde{I}_{Ri} = -Q_i^X M_{Z_\rho}^2 c_{27} - (1 - c_{27}^2) M_{Z_\rho}^2 c_{27} \).

This matrix has one exact zero eigenvalue, as expected, that gives mass to the \( W^\pm \) gauge bosons.

### B Chargino Mass Matrix

The mass terms in the Lagrangian corresponding to charged fermions can be written as

\[ \mathcal{L}^\pm \supset -\frac{1}{2} (\psi^{-T} X \psi^+ + \psi^{+T} X^T \psi^-) + h.c. \] (B.1)

where

\[ \psi^- = (i \tilde{W}^-, \tilde{h}_d^-, \tilde{e}_L^- \mu_L^- \tau_L^-) \]

\[ \psi^+ = (i \tilde{W}^+, \tilde{h}_u^+, \tilde{e}_R^+ \mu_R^+ \tau_R^+) \]
and $X$ is the chargino mass matrix,

$$X = \begin{pmatrix} M_2 & \sqrt{2} M W s_{\beta} & 0 & 0 & 0 \\ \sqrt{2} M W c_{\beta} & \mu & 0 & 0 & 0 \\ 0 & \mu_e & m_e & 0 & 0 \\ 0 & \mu_2 & 0 & m_\mu & 0 \\ 0 & \mu_3 & 0 & 0 & m_\tau \end{pmatrix}$$

(B.2)

This may also be written in a more compact form by introducing the vector,

$$\psi_\pm = (\psi^-, \psi^+)$$

(B.3)

and

$$M_\pm = \begin{pmatrix} 0 & X \\ X^T & 0 \end{pmatrix}$$

(B.4)

such that,

$$\mathcal{L}_\pm = -\frac{1}{2} \psi_\pm^T M_\pm \psi_\pm + h.c.$$  

(B.5)

In order to diagonalize the mass matrix $X$ we need two matrices, one that transforms $\psi^-$ ($U$) and another that transforms $\psi^+$ ($V$) so that,

$$U^* X V^\dagger = m_{\tilde{\chi}^\pm}.$$  

(B.6)

The matrices $U$ and $V$ diagonalize the matrices $XX^T$ and $X^TX$ respectively. The charged leptons $e$, $\mu$ and $\tau$ also enter our chargino mass matrix and, in general, mix with the wino and higgsino. However, this mixing is extremely weak and hence the relevant mixing matrix elements in the calculations maybe taken to be unity. It is for this reason that they also do not enter into our neutrino mass matrix calculation.

References

[1] G. W. Bennett et al. [Muon g-2 Collaboration], “Measurement of the negative muon anomalous magnetic moment to 0.7 ppm,” Phys. Rev. Lett. 92, 161802 (2004) doi:10.1103/PhysRevLett.92.161802 [hep-ex/0401008].

[2] G. W. Bennett et al. [Muon g-2 Collaboration], “Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL,” Phys. Rev. D 73, 072003 (2006) doi:10.1103/PhysRevD.73.072003 [hep-ex/0602035].

[3] J. P. Miller, E. de Rafael and B. L. Roberts, “Muon (g-2): Experiment and theory,” Rept. Prog. Phys. 70, 795 (2007) doi:10.1088/0034-4885/70/5/R03 [hep-ph/0703049].

[4] C. Patrignani et al. [Particle Data Group], “Review of Particle Physics,” Chin. Phys. C 40, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001

[5] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler and T. Schwetz, “Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity,” JHEP 1701, 087 (2017) doi:10.1007/JHEP01(2017)087 [arXiv:1611.01514 [hep-ph]].

[6] P. Fayet, “Supersymmetry and Weak, Electromagnetic and Strong Interactions,” Phys. Lett. 64B, 159 (1976). doi:10.1016/0370-2693(76)90319-1
[7] S. P. Martin, “A Supersymmetry primer,” Adv. Ser. Direct. High Energy Phys. 21, 1 (2010) [Adv. Ser. Direct. High Energy Phys. 18, 1 (1998)] [hep-ph/9709356].

[8] R. Barbier et al., Phys. Rept. 420, 1 (2005) doi:10.1016/j.physrep.2005.08.006 [hep-ph/0406039].

[9] X. G. He, G. C. Joshi, H. Lew and R. R. Volkas, “NEW Z-prime PHENOMENOLOGY,” Phys. Rev. D 43, 22 (1991). doi:10.1103/PhysRevD.43.22

[10] X. G. He, G. C. Joshi, H. Lew and R. R. Volkas, “Simplest Z-prime model,” Phys. Rev. D 44, 2118 (1991). doi:10.1103/PhysRevD.44.2118

[11] S. Baek, N. G. Deshpande, X. G. He and P. Ko, “Muon anomalous g-2 and gauged L(muon) - L(tau) models,” Phys. Rev. D 64, 055006 (2001) doi:10.1103/PhysRevD.64.055006 [hep-ph/0104141].

[12] E. Ma, D. P. Roy and S. Roy, “Gauged L(mu) - L(tau) with large muon anomalous magnetic moment and the bimaximal mixing of neutrinos,” Phys. Lett. B 525 (2002) 101 doi:10.1016/S0370-2693(01)01428-9 [hep-ph/0110146].

[13] J. Heeck and W. Rodejohann, “Gauged L\(_\mu\) – L\(_\tau\) Symmetry at the Electroweak Scale,” Phys. Rev. D 84, 075007 (2011) doi:10.1103/PhysRevD.84.075007 [arXiv:1107.5238 [hep-ph]].

[14] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, “Neutrino Trident Production: A Powerful Probe of New Physics with Neutrino Beams,” Phys. Rev. Lett. 113, 091801 (2014) doi:10.1103/PhysRevLett.113.091801 [arXiv:1406.2332 [hep-ph]].

[15] W. Altmannshofer, S. Gori, S. Profumo and F. S. Queiroz, “Explaining dark matter and B decay anomalies with an L\(_\mu\) – L\(_\tau\) model,” JHEP 1612, 106 (2016) doi:10.1007/JHEP12(2016)106 [arXiv:1609.04026 [hep-ph]].

[16] A. Biswas, S. Choubey and S. Khan, “Neutrino Mass, Dark Matter and Anomalous Magnetic Moment of Muon in a U(1)\(_{L\mu-L\tau}\) Model,” JHEP 1609, 147 (2016) doi:10.1007/JHEP09(2016)147 [arXiv:1608.04194 [hep-ph]].

[17] A. Biswas, S. Choubey and S. Khan, “FIMP and Muon (g – 2) in a U(1)\(_{L\mu-L\tau}\) Model,” JHEP 1702, 123 (2017) doi:10.1007/JHEP02(2017)123 [arXiv:1612.03607 [hep-ph]].

[18] S. Patra, S. Rao, N. Sahoo and N. Sahu, “Gauged U(1)\(_{L\mu-L\tau}\) model in light of muon g – 2 anomaly, neutrino mass and dark matter phenomenology,” Nucl. Phys. B 917, 317 (2017) doi:10.1016/j.nuclphysb.2017.02.010 [arXiv:1607.04046 [hep-ph]].

[19] A. Biswas, S. Choubey, L. Covi and S. Khan, “Explaining the 3.5 keV X-ray Line in a L\(_\mu\) – L\(_\tau\) Extension of the Inert Doublet Model,” JCAP 1802, no. 02, 002 (2018) doi:10.1088/1475-7516/2018/02/002 [arXiv:1711.00553 [hep-ph]].

[20] T. Araki, S. Hoshino, T. Ota, J. Sato and T. Shimomura, “Detecting the L\(_\mu\) – L\(_\tau\) gauge boson at Belle II,” Phys. Rev. D 95, no. 5, 055006 (2017) doi:10.1103/PhysRevD.95.055006 [arXiv:1702.01497 [hep-ph]].

[21] Y. Kaneta and T. Shimomura, “On the possibility of search for L\(_\mu\) – L\(_\tau\) gauge boson at Belle-II and neutrino beam experiments,” PTEP 2017, no. 5, 053B04 (2017) doi:10.1093/ptep/ptx050 [arXiv:1701.00156 [hep-ph]].

[22] C. H. Chen and T. Nomura, “L\(_\mu\) – L\(_\tau\) gauge-boson production from lepton flavor violating \(\tau\) decays at Belle II,” Phys. Rev. D 96, no. 9, 095023 (2017) doi:10.1103/PhysRevD.96.095023 [arXiv:1704.04407 [hep-ph]].
[23] M. Ibe, W. Nakano and M. Suzuki, “Constraints on $L_\mu - L_\tau$ gauge interactions from rare kaon decay,” Phys. Rev. D 95, no. 5, 055022 (2017) doi:10.1103/PhysRevD.95.055022 [arXiv:1611.08460 [hep-ph]].

[24] A. Crivellin, G. D’Ambrosio and J. Heeck, “Explaining $h \rightarrow \mu^+\mu^-$, $B \rightarrow K^{*}\mu^+\mu^-$ and $B \rightarrow K\mu^+\mu^-/B \rightarrow Ke^+e^-$ in a two-Higgs-doublet model with gauged $L_\mu - L_\tau$,” Phys. Rev. Lett. 114, 151801 (2015) doi:10.1103/PhysRevLett.114.151801 [arXiv:1501.00993 [hep-ph]].

[25] W. Altmannshofer, M. Carena and A. Crivellin, “$L_\mu - L_\tau$ theory of Higgs flavor violation and $(g - 2)_\mu$,” Phys. Rev. D 94, no. 9, 095026 (2016) doi:10.1103/PhysRevD.94.095026 [arXiv:1604.08221 [hep-ph]].

[26] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, “Quark flavor transitions in $L_\mu - L_\tau$ models,” Phys. Rev. D 89, 095033 (2014) doi:10.1103/PhysRevD.89.095033 [arXiv:1403.1269 [hep-ph]].

[27] P. Ko, T. Nomura and H. Okada, “Explaining $B \rightarrow K^{(*)}\ell^+\ell^-$ anomaly by radiatively induced coupling in $U(1)_{\mu - \tau}$ gauge symmetry,” Phys. Rev. D 95, no. 11, 111701 (2017) doi:10.1103/PhysRevD.95.111701 [arXiv:1702.02699 [hep-ph]].

[28] C. H. Chen and T. Nomura, “Neutrino mass in a gauged $L_\mu - L_\tau$ model,” arXiv:1705.10620 [hep-ph].

[29] M. Das and S. Mohanty, “Leptophilic dark matter in gauged $L_\mu - L_\tau$ extension of MSSM,” Phys. Rev. D 89, no. 2, 025004 (2014) doi:10.1103/PhysRevD.89.025004 [arXiv:1306.4505 [hep-ph]].

[30] Y. Grossman and H. E. Haber, “The Would be Majoron in R parity violating supersymmetry,” Phys. Rev. D 67, 036002 (2003) doi:10.1103/PhysRevD.67.036002 [hep-ph/0210273].

[31] S. Choubey and W. Rodejohann, “A Flavor symmetry for quasi-degenerate neutrinos: $L(\mu)$ - $L(\tau)$,” Eur. Phys. J. C 40, 250 (2005) doi:10.1140/epjc/s2005-02133-1 [hep-ph/0411190].

[32] E. J. Chun and K. Turzynski, “Quasi-degenerate neutrinos and leptogenesis from $L(\mu)$ - $L(\tau)$,” Phys. Rev. D 76, 053008 (2007) doi:10.1103/PhysRevD.76.053008 [hep-ph/0703070].

[33] D. Suematsu, Phys. Rev. D 59, 055017 (1999) doi:10.1103/PhysRevD.59.055017 [hep-ph/9808409].

[34] A. Bilal, “Lectures on Anomalies,” arXiv:0802.0634 [hep-th].

[35] K. Huitu, J. Laamanen and S. Roy, “Non-universal gaugino masses and implications on the dark matter and Higgs searches,” In *Karlsruhe 2007, SUSY 2007* 914-917 [arXiv:0710.2058 [hep-ph]].

[36] R. N. Mohapatra and G. Senjanovic, “Neutrino Mass and Spontaneous Parity Violation,” Phys. Rev. Lett. 44, 912 (1980). doi:10.1103/PhysRevLett.44.912

[37] J. Schechter and J. W. F. Valle, “Neutrino Decay and Spontaneous Violation of Lepton Number,” Phys. Rev. D 25, 774 (1982). doi:10.1103/PhysRevD.25.774

[38] W. Grimus and L. Lavoura, “The Seesaw mechanism at arbitrary order: Disentangling the small scale from the large scale,” JHEP 0011, 042 (2000) doi:10.1088/1126-6708/2000/11/042 [hep-ph/0008179].
[39] G. Altarelli and F. Feruglio, “Models of neutrino masses and mixings,” New J. Phys. 6, 106 (2004) doi:10.1088/1367-2630/6/1/106 [hep-ph/0405048].

[40] Y. Grossman and H. E. Haber, “(S)neutrino properties in R-parity violating supersymmetry. 1. CP conserving phenomena,” Phys. Rev. D 59, 093008 (1999) doi:10.1103/PhysRevD.59.093008 [hep-ph/9810536].

[41] H. K. Dreiner, K. Nickel, F. Staub and A. Vicente, Phys. Rev. D 86, 015003 (2012) doi:10.1103/PhysRevD.86.015003 [arXiv:1204.5925 [hep-ph]].

[42] P. F. Harrison, D. H. Perkins and W. G. Scott, “Tri-bimaximal mixing and the neutrino oscillation data,” Phys. Lett. B 530, 167 (2002) doi:10.1016/S0370-2693(02)01336-9 [hep-ph/0202074].

[43] E. Ma and G. Rajasekaran, “Cobimaximal neutrino mixing from $A_4$ and its possible deviation,” EPL 119, no. 3, 31001 (2017) doi:10.1209/0295-5075/119/31001 [arXiv:1708.02208 [hep-ph]].

[44] E. Ma, “Neutrino mixing: $A_4$ variations,” Phys. Lett. B 752, 198 (2016) doi:10.1016/j.physletb.2015.11.049 [arXiv:1510.02501 [hep-ph]].

[45] G. Altarelli and D. Meloni, “A Simplest A4 Model for Tri-Bimaximal Neutrino Mixing,” J. Phys. G 36, 085005 (2009) doi:10.1088/0954-3899/36/8/085005 [arXiv:0905.0620 [hep-ph]].

[46] K. S. Babu, E. Ma and J. W. F. Valle, “Underlying $A(4)$ symmetry for the neutrino mass matrix and the quark mixing matrix,” Phys. Lett. B 552, 207 (2003) doi:10.1016/S0370-2693(02)03153-2 [hep-ph/0206292].

[47] W. Grimus and L. Lavoura, “A Nonstandard CP transformation leading to maximal atmospheric neutrino mixing,” Phys. Lett. B 579, 113 (2004) doi:10.1016/j.physletb.2003.10.075 [hep-ph/0305309].

[48] D. Geiregat et al. [CHARM-II Collaboration], “First observation of neutrino trident production,” Phys. Lett. B 245, 271 (1990). doi:10.1016/0370-2693(90)90146-W

[49] S. R. Mishra et al. [CCFR Collaboration], “Neutrino tridents and W Z interference,” Phys. Rev. Lett. 66, 3117 (1991). doi:10.1103/PhysRevLett.66.3117

[50] S. Chatrchyan et al. [CMS Collaboration], “Observation of Z decays to four leptons with the CMS detector at the LHC,” JHEP 1212, 034 (2012) doi:10.1007/JHEP12(2012)034 [arXiv:1210.3844 [hep-ex]].

[51] G. Aad et al. [ATLAS Collaboration], “Measurements of Four-Lepton Production at the Z Resonance in pp Collisions at $\sqrt{s} =$7 and 8 TeV with ATLAS,” Phys. Rev. Lett. 112, no. 23, 231806 (2014) doi:10.1103/PhysRevLett.112.231806 [arXiv:1403.5657 [hep-ex]].

[52] D. Akimov et al. [COHERENT Collaboration], “Observation of Coherent Elastic Neutrino-Nucleus Scattering,” Science 357, no. 6356, 1123 (2017) doi:10.1126/science.aao0990 [arXiv:1708.01294 [nucl-ex]].

[53] I. M. Shoemaker, “COHERENT search strategy for beyond standard model neutrino interactions,” Phys. Rev. D 95, no. 11, 115028 (2017) doi:10.1103/PhysRevD.95.115028 [arXiv:1703.05774 [hep-ph]].

[54] J. Liao and D. Marfatia, “COHERENT constraints on nonstandard neutrino interactions,” Phys. Lett. B 775, 54 (2017) doi:10.1016/j.physletb.2017.10.046 [arXiv:1708.04255 [hep-ph]].
[55] M. Abdullah, J. B. Dent, B. Dutta, G. L. Kane, S. Liao and L. E. Strigari, “Coherent Elastic Neutrino Nucleus Scattering (CEνNS) as a probe of Z’ through kinetic and mass mixing effects,” arXiv:1803.01224 [hep-ph].

[56] M. Aaboud et al. [ATLAS Collaboration], arXiv:1803.02762 [hep-ex].

[57] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001

[58] T. Moroi, “The Muon anomalous magnetic dipole moment in the minimal supersymmetric standard model,” Phys. Rev. D 53, 6565 (1996) Erratum: [Phys. Rev. D 56, 4424 (1997)] doi:10.1103/PhysRevD.53.6565, 10.1103/PhysRevD.56.4424 [hep-ph/9512396].

[59] S. P. Martin and J. D. Wells, “Muon anomalous magnetic dipole moment in supersymmetric theories,” Phys. Rev. D 64, 035003 (2001) doi:10.1103/PhysRevD.64.035003 [hep-ph/0103067].

[60] A. Chakraborty and S. Chakraborty, “Probing (g − 2)µ at the LHC in the paradigm of R-parity violating MSSM,” Phys. Rev. D 93, no. 7, 075035 (2016) doi:10.1103/PhysRevD.93.075035 [arXiv:1511.08874 [hep-ph]].

[61] J. F. Gunion and H. E. Haber, “Higgs Bosons in Supersymmetric Models. 1.,” Nucl. Phys. B 272, 1 (1986) Erratum: [Nucl. Phys. B 402, 567 (1993)]. doi:10.1016/0550-3213(86)90340-8, 10.1016/0550-3213(93)90653-7

[62] A. Kobakhidze, M. Talia and L. Wu, “Probing the MSSM explanation of the muon g-2 anomaly in dark matter experiments and at a 100 TeV pp collider,” Phys. Rev. D 95, no. 5, 055023 (2017) doi:10.1103/PhysRevD.95.055023 [arXiv:1608.03641 [hep-ph]].

[63] S. F. Ge, M. Lindner and W. Rodejohann, “Atmospheric Trident Production for Probing New Physics,” Phys. Lett. B 772, 164 (2017) doi:10.1016/j.physletb.2017.06.020 [arXiv:1702.02617 [hep-ph]].