DENSITY PROFILES AND CLUSTERING OF DARK MATTER HALOS

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1 N-body simulations with 17 million particles

Dark Matter (DM) halos carry important information about the formation of galaxies and of many other extragalactic objects. In particular, the density profile and the clustering of DM halos are two very important ingredients for understanding many observations of galaxies. In this talk, I present our new results for these two quantities from a large set of high quality N-body simulations.

The simulations were generated with a vectorized P^3M (i.e. Particle Particle Particle Mesh) code on the supercomputer VPP300/16R at the National Astronomical Observatory of Japan. Each simulation uses 256^3 simulation particles. The code adopts the standard P^3M algorithm. Twenty one simulations are generated for three representative cold dark matter (CDM) models of galaxy formation, and eighteen for six scale-free cosmologies. The CDM models are specified completely as regards the DM distribution by the density parameter \( \Omega_0 \), the cosmological constant \( \lambda_0 \), the shape \( \Gamma \) and the normalization \( \sigma_8 \) of the linear power spectrum. The scale-free models assume an Einstein-de Sitter universe (i.e. \( \Omega_0 = 1 \) and \( \lambda_0 = 0 \)) and a power-law \( P(k) \propto k^n \) for the linear density power spectrum. Table 1 summarizes the physical and simulation parameters used for these simulations.

This set of simulations has covered a large parameter space, which is often important in many studies. Each of our individual simulations has similar mass and force resolutions to that the Virgo Consortium (Gus August in these proceedings) has recently obtained (except for their Hubble-volume simulation which has many more particles but still poorer absolute mass resolution than the CDM simulations presented here), but our sample is much larger. The CDM simulations have been used to study the implications of the strong clustering discovered by Steidel et al. (1998) of the Lyman Break galaxies at high redshift (Jing & Suto 1998). Many works based on these high quality simulations are being done. As an incomplete list, we are 1) studying the DM distribution in the strongly clustering regime; 2) testing the Press-Schechter extension theories; 3) studying redshift distortion effect and investigating its potential to measure the cosmological parameters; 4) analyzing internal structures of DM halos; 5) investigating galaxy formation by hand-inputing gas physics; 6) applying the simulations to real observations.

2 The density profile of the dark matter halos

Recently Navarro, Frenk, & White (1996,1998; NFW) have found that the density profile \( \rho(r) \) of the DM halos follows a universal form:

\[
\rho(r) \propto \frac{1}{r(r+1/c)^2}
\]  

(1)
Table 1. List of simulations
1. All simulations use 256$^3$ particles, except the low density models with power-law $P(k)$ LSF1 to LSF4 use 200$^3$ particles;
2. Particle mass $m_p$ in CDM models is units of $h^{-1}M_\odot$;
3. Box size $L$ in CDM models is units of $h^{-1}$Mpc;
4. The rightmost column is the number of realizations (samples);

| Model | $\Omega_0$ | $\lambda_0$ | $\Gamma$ | $\sigma_8$ | $m_p$ | $L$ | Num. |
|-------|------------|-------------|---------|------------|------|-----|------|
| CDM1  | 1.0        | 0.0         | 0.5     | 0.6        | $1.7 \times 10^{10}$ | 100  | 3    |
| CDM2  | 0.3        | 0.0         | 0.25    | 1.0        | $5.0 \times 10^9$     | 100  | 3    |
| CDM3  | 0.3        | 0.7         | 0.20    | 1.0        | $5.0 \times 10^9$     | 100  | 3    |
| CDM4  | 1.0        | 0.0         | 0.5     | 0.6        | $4.6 \times 10^{11}$  | 300  | 4    |
| CDM5  | 0.3        | 0.0         | 0.25    | 1.0        | $1.3 \times 10^{11}$  | 300  | 4    |
| CDM6  | 0.3        | 0.7         | 0.20    | 1.0        | $1.3 \times 10^{11}$  | 300  | 4    |

| Model | $\Omega_0$ | $\lambda_0$ | $n$      | Num. |
|-------|------------|-------------|---------|------|
| SF1   | 1.0        | 0.0         | 1.0     | 3    |
| SF2   | 1.0        | 0.0         | 0.0     | 3    |
| SF3   | 1.0        | 0.0         | −0.5    | 3    |
| SF4   | 1.0        | 0.0         | −1.0    | 3    |
| SF5   | 1.0        | 0.0         | −1.5    | 3    |
| SF6   | 1.0        | 0.0         | −2.0    | 3    |
| LSF1  | 0.1        | 0.9         | −1.0    | 2    |
| LSF2  | 0.1        | 0.0         | −1.0    | 2    |
| LSF3  | 0.1        | 0.9         | −2.0    | 2    |
| LSF4  | 0.1        | 0.0         | −2.0    | 2    |

where $r$ is in units of the radius $R_{200}$ within which the overdensity around the halo center is 200, and the density profile depends only on the concentration parameter $c$. The parameter $c$ can be predicted for each cosmogonic model with a recipe given by NFW, and the predicted $c_{NFW}$ is a function of the halo mass only for a given cosmogonic model (i.e. the power spectrum and the background cosmology). These results are indeed very important and have many interesting applications. However, it is important to point out that NFW studied the density profile only for the halos which look in equilibrium, so it is unclear how much fraction of the DM halos really follows the universal form, since substructure is a common phenomenon for DM halos in all viable cosmological models (e.g. Jing et al. 1995). Even for halos which can be reasonably fit by the NFW profile, a dispersion in $c$ seems inevitable because of a different formation history. Quantifying this dispersion or more completely the Probability Distribution Function (PDF) of $c$ is of considerable importance when applying the NFW profile to interpret various observational facts. To an-
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Figure 1. The probability distribution function of the concentration parameter \( c \). The right solid histogram is for the halos with \( d v_{\text{max}} < 0.15 \), the middle dotted one for \( 0.15 < d v_{\text{max}} < 0.35 \), and the left solid one for \( d v_{\text{max}} > 0.35 \). For each subset of the halos, the distribution of \( c \) can be nicely fitted by a lognormal distribution.

swver these important questions, many halos with good resolution (certainly much more halos than that NFW had) are needed. Our simulations suit very well for this purpose since typically one model (either CDM or SF) has 300 to 500 massive halos with at least 10,000 particles within the virial radius. Both the force and the mass resolutions of our simulations are comparable to the NFW halos.

We have selected DM halos using the Friends-of-Friends method with the linking length 0.2 times of the mean particle separation. Then we choose the local potential minimum as the center of the halo, and compute the density \( \rho(r) \) in shells with logarithmic thickness \( \log_{10} \Delta r = 0.1 \) from \( R_{200} \) inward to \( \eta \), the force resolution limit. The density profiles are then fitted with Eq. (1) to get the parameter \( c \). Although we have computed the density profiles for the massive halos in all simulations, for the convenience of the discussion in this very limited space (and talk time), I present here the results only for one specific model, the scale-free model of \( n = -1 \). A detailed discussion of all the models will be given in a forthcoming paper (Jing & Suto 1998, in preparation).

Firstly we note that the goodness of the fit varies strongly from halo to halo. Since the Poisson noise in the simulation density profile is negligible, we define the goodness as the maximum relative deviation of the simulation \( \rho(r) \) from the fit \( \rho_{\text{NFW}}(r) \) in all the radial bins, i.e. \( d v_{\text{max}} = \max \{ \frac{|(\rho(r_i) - \rho_{\text{NFW}}(r_i))|}{\rho_{\text{NFW}}(r_i)} \} \). About 35, 50 and 15 percent of the halos have the maximum deviations \( d v_{\text{max}} > 35\% \), \( 15\% < d v_{\text{max}} < 35\% \), and \( d v_{\text{max}} < 15\% \) respectively. With this classification, it is fair to say that about 35% halos could not be fitted by the NFW profile, because of too significant substructures; another 50% halos with less substructures can be reasonably described by the NFW profile, and the rest about 15% halos with the least substructures can be fitted by the NFW profile very nicely.

The PDF of the fitted \( c \) is presented in Figure 1. We choose \( c/c_{\text{NFW}} \) as the abscissa in order
to correct for the mass dependence of the parameter $c$, though this correction for the mass range covered by the halos here is actually tiny. The PDF is shown separately for the halos with different amount of substructures. For each subset of the halos, the PDF can be fitted by a lognormal function

$$ p(c) dc = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\ln c - \ln \bar{c})^2}{2\sigma^2}\right) d\ln c. $$

(2)

As we discussed in the last paragraph, because of the poor fit for the halos with significant substructures, the PDF given by Fig. 1 for this subset is probably not much meaningful and we shall not discuss it. But for the other halos which can be reasonably fitted by the NFW profile, the PDF is indeed a very important quantity. Both for the most virialized halos ($d_{\text{vir}}_{\text{max}} < 15\%$) and for the halos with $15\% < d_{\text{vir}}_{\text{max}} < 35\%$, the dispersion $\sigma$ of the PDF is 0.27. The mean value $\bar{c}$ is $(0.93 \pm 0.03)_{\text{NFW}}$ for the halos with $d_{\text{vir}}_{\text{max}} < 15\%$ and $(0.84 \pm 0.02)_{\text{NFW}}$ for the halos with $15\% < d_{\text{vir}}_{\text{max}} < 35\%$. The PDF of the parameter $c$ depends weakly on the cosmological parameters, and the above results can be applied to other cosmological models with $\Omega_0 \geq 0.25$.

Our results are generally consistent with the results of NFW. Variation of the density profiles was also found by Kravtsov et al. (1998) but was discussed in a completely different way. What is new from our study is that we have quantified how much halos can really be fitted by the NFW profile and that we have derived the important quantity, the Probability Distribution Function of the concentration parameter $c$, for the halos which can be fitted by the NFW profile. These results would be important for properly interpreting many cosmological observations which are closely related to the density profile of halos.

Before concluding this section, it would be important to point out that because of the still limited resolution of our simulations, we could not address the important problem that density profile might be much steeper than $r^{-1}$ in the very inner region of the halos (Fukushige & Makino 1997; Moore et al. 1998). To definitely answer this problem, one needs to simulate many halos with much higher resolutions of both the mass and the force than the present work. Our work in this direction is also in progress.

3 The bias parameter of the dark matter halos

The clustering of the DM halos is the basic block for modeling and understanding the clustering of galaxies and of other extragalactic objects. We have recently measured the two-point correlation function of the DM halos for the scale free simulations with $n = -0.5$, $-1.0$, $-1.5$, and $-2.0$ (Jing 1998). The results confirmed with unprecedented accuracy that the bias of the halos $b$ is linear in the linearly clustering regime. For a scale-free model, the bias parameter $b$ is then expected to depend on the scaled mass $M/M_*$ only, where $M_*$ is a characteristic non-linear mass. In Fig. 2, I present the $b(M/M_*)$ for different simulation outputs of each model. The excellent scaling exhibited by the simulation data assures that any numerical artifacts indeed have negligible effect on the results of Fig. 2. The simulation results agree with the analytical formula of Mo & White (1996) only for massive halos with $M/M_* > 1$, but are significantly higher for much less massive halos. We also noted that there might be systematic differ-
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Figure 2. The $b^2(M/M_\star)$ for different evolution outputs (different symbols). The dotted lines are the analytical prediction of Mo & White (1996), and the solid lines are our fitting formula Eq. 3 which can accurately fit the simulation results.

ence between the analytical prediction and our simulation results even at $M/M_\star \gg 1$ (see Fig. 2), despite not with a high statistical confidence. Our simulation results could be fitted very well by

$$b(M) = \left(\frac{0.5}{\nu^4} + 1\right)^{(0.06-0.02n)} \times (1 + \frac{\nu^2 - 1}{\delta_c})$$

$$\nu = (M/M_\star)^{n+3/6}, \quad (3)$$

where $\delta_c = 1.68$. This fitting formula could be equally well applied to the CDM models with only a very simple modification (Jing 1998). This finding has profound implications for the clustering studies of galaxies. Further discussions are given in Jing (1998).

Acknowledgements

The author gratefully acknowledges Yasushi Suto for collaborations and the JSPS foundation for a postdoctoral fellowship. The simulations were carried out on VPP/16R and VX/4R at NAOJ, Japan.

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