Isospin analysis of $B \to D^* \bar{D} K$ and the absence of the $Z_c(3900)$ in $B$ decays

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**A B S T R A C T**

We study the isospin amplitudes in the exclusive $B \to D^* \bar{D} K$ decay process and fit the available $D^0 \bar{D}^0$ invariant mass distributions near threshold. The analysis demonstrates that the production of the isospin triplet $D^* \bar{D}$ state is highly suppressed compared to the isospin singlet one. That explains why the $Z_c(3900)$ has not been found in $B$ decays. In addition, the production of the negative charge-parity state might be further suppressed in the heavy quark limit. These two reasons which are based on the molecular assumption offer the first explanation why the $Z_c(3900)$ is absent in $B$ decays. Further studies of the absence from both the experimental and the theoretical side is extremely important for understanding the nature of the $X(3872)$ and the $Z_c(3900)$.

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In 2013, the BESIII and Belle Collaborations reported the charged charmonium-like state $Z_c(3900)\pm$ in the $\pi^{\pm} J/\psi$ invariant mass distribution of the $e^+e^- \to \pi^+\pi^- J/\psi$ process [1,2]. The observed channel, i.e. $\pi^{\pm} J/\psi$, reveals its minimal four-quark $c\bar{c}u\bar{d}$ constituent nature, making it more intriguing than other exotic candidates. The charged state was also confirmed by the reanalysis of the CLEOc data [3], which also discovered its neutral partner. In addition to the $\pi J/\psi$ invariant mass distribution in the $e^+e^- \to \pi^+\pi^- J/\psi$ process, the $Z_c(3900)$ was also observed in the $D^*\bar{D}$ channel\textsuperscript{1} by the BESIII Collaboration [4,5]. The angular analysis of Refs. [5,6] leads to its quantum numbers $I^G(J^P) = 1^+(1^-)$. The averaged mass is $3886.6 \pm 2.4$ MeV [7] which is slightly above the $D^*\bar{D}$ threshold, thus it is naturally to be regarded as a $D^*\bar{D}$ molecule state [8–13]. This proximity to the threshold also allows for an interpretation as a cusp effect [14,15]. However, it has been demonstrated that the treatment within the cusp scenario is not self-consistent [16] and the near threshold pronounced structure in elastic channels necessary requires a nearby pole. Besides these two interpretations, there are also others, such as tetraquark [17–21], hadro-charmonium [22], and hybrid [23].

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\textsuperscript{1} Here and in what follows, the charge-conjugated channels are considered implicitly.

In the $D^*\bar{D}$ hadronic molecular picture, the isospin singlet $Z_c(3900)$ and the isospin singlet $X(3872)$ share similar dynamics, such as the analogy of the processes $e^+e^- \to \gamma X(3872)$ and $e^+e^- \to \gamma Z_c(3900)$ [26,27]. As a conclusion, both the $Z_c(3900)$ and the $X(3872)$ have been observed in $e^+e^-$ annihilation. The most puzzling aspect of the $Z_c(3900)$ is its absence in $B$ decays which is different to the case of the $X(3872)$. The $Z_c(3900)$ is expected to be seen in the decay $B \to K J/\psi \pi$ due to the analogy of the $Z_c(3900)$ and the $X(3872)$, since the latter one has been observed in both $B \to K X(3872)$ [28] and $e^+e^- \to \gamma X(3872)$ [27] processes through its decay $X(3872) \to \pi^+\pi^- J/\psi$.

In the molecular picture, the $X(3872)$ and the $Z_c(3900)$ have the same constituents $D^*$ and $\bar{D}$, but different isospins and parities. Therefore, both of them can be formed from the interaction between $D^*$ and $\bar{D}$. The production of the $X(3872)$ in the process $B \to D^*\bar{D}K$ has already been studied in Ref. [29], where it occurs through the isospin conserved weak transition ($\Delta I = 0$) $b \to c\bar{s}s$ current. On the quark level, the $\Delta I = 0$ process is given in terms of two diagrams, i.e. the color suppressed internal W-emission and the external W-emission diagram, cf. Fig. 1. Besides those two, the $\Delta I = 1$ diagram, i.e. the second diagram of Fig. 2 in Ref. [31], could also contribute. However, it is CKM suppressed diagram and can safely be neglected. Further, the isospin decomposition of the penguin diagram, i.e. the first diagram of Fig. 2 in Ref. [31], is the same as diagram (B) in Fig. 1. Hence the contribution of the penguin diagram in the threshold region can be absorbed into the parameters of diagram (B) in Fig. 1. Based
on the above analysis, the $B \rightarrow D^* \tilde{D}K$ process conserves isospin $(\Delta I = 0)$ to the leading order of the expansion parameter of the Wolfenstein parametrization of the CKM matrix, although it is a weak decay process. It is the prerequisite that one can analyze the isospin amplitudes of the $B \rightarrow D^* \tilde{D}K$ process.

First, we derive the isospin relations for the $B \rightarrow D^* \tilde{D}K$ process through the quark-level Feynman diagrams, cf. Fig. 1. Since the light quark pair created from the vacuum is a flavor and isospin singlet, the decay amplitudes of the $B^0$ meson from the two diagrams of Fig. 1 are

$$
\mathcal{M}[B^0 \rightarrow D^{*0}D^{-}K^+] = -\frac{1}{\sqrt{2}} B_1.
$$

$$
\mathcal{M}[B^0 \rightarrow D^{*+}D^{-}K^0] = \frac{1}{\sqrt{2}} A_0 + \frac{1}{2} (B_0 + B_1) e^{i\theta},
$$

$$
\mathcal{M}[B^0 \rightarrow D^{*0}\bar{D}^0K^-] = -\frac{1}{\sqrt{2}} A_0,
$$

where $B_0$ ($B_1$) is the amplitude producing the $D^{*} \bar{D}$ with isospin 0 (1) through external $W$-emission, $A_0$ corresponds to the internal $W$-emission with isospin 0, and $\theta$ is the relative phase between diagram (A) and diagram (B) in Fig. 1. The decay amplitudes of the $B^+$ are

$$
\mathcal{M}[B^+ \rightarrow D^{*+}\bar{D}^0K^0] = \frac{1}{\sqrt{2}} B_1,
$$

$$
\mathcal{M}[B^+ \rightarrow D^{*0}\bar{D}^0K^+] = -\frac{1}{\sqrt{2}} A_0 + \frac{1}{2} (B_0 - B_1) e^{i\theta},
$$

$$
\mathcal{M}[B^+ \rightarrow D^{*+}D^-K^+] = \frac{1}{\sqrt{2}} A_0.
$$

Second, except for the short-distance direct production, final-state interactions also contribute as shown in Fig. 2. The amplitudes consist of two parts: one is the $D^* \bar{D}$ short-distance production amplitude and the other one is the long-distance $D^* \bar{D}$ scattering. The rescattering process proceeds in two pathways. Taking $D^{*0}\bar{D}^0$ production as an example, it includes the $D^{*0}\bar{D}^0 \rightarrow D^{*0}\bar{D}^0$ scattering following the $B \rightarrow D^{*0}\bar{D}^0K$ decay and $D^{*+}D^- \rightarrow D^{*0}\bar{D}^0$ scattering following the $B \rightarrow D^{*+}D^-K$ decay. The $D^* \bar{D}$ system we are concerned with is in the near-threshold energy region, as we consider the coalescence of the charm mesons into $X(3872)$ or $Z_c(3900)$. At the $D^* \bar{D}$ threshold, Lorentz invariance requires the short-distance decay amplitude to have the simple form [29]

$$
\mathcal{A}_{\text{short}}[B^{0(+)}] \rightarrow D^{*0}\bar{D}^0K^{0(+)}] = \mathcal{O}_{0(\pm)}^{0} P \cdot e^+, \\
\mathcal{A}_{\text{short}}[B^{0(+)}] \rightarrow D^{*+}D^-K^{0(+)}] = \mathcal{O}_{0(\pm)}^{0} P \cdot e^+,
$$

where $e$ is the polarization four-vector of the vector charmed meson, $P$ is the momentum of the bottom meson and $\mathcal{O}_{0(\pm)}^{0}$ are coefficients that need to be determined. For $B^{0(+)}$ are the combinations of $a_0, b_0$ and $b_1$, which are the constants corresponding to $A_0 = a_0 P \cdot e^+$ and $B_0 = b_0(1/P \cdot e^+)$.

After integrating over phase space, the differential decay width is written as

$$
\frac{d \Gamma}{d M} = \frac{\mu \lambda^{3/2}(m_B, M, m_K)}{256 \pi^3 m_B^2 M^2} \lambda^{1/2}(M, m_{D^0}, m_{\bar{D}^0}) |A(E)|^2,
$$

where $M$ is the invariant mass of $D^{*0}$ and $\bar{D}^0$, $E$ is the energy of the $D^{*0}\bar{D}^0$ system in its rest frame relative to the $D^{*0}\bar{D}^0$ threshold. $E = M - (m_{D^0} + m_{\bar{D}^0})$, $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(x^2y^2 + y^2z^2 + z^2x^2)$ is the triangle function and $A(E) = c_0 + c_1 T_{1,1} + c_2 T_{2,2}$ for $B^+ \rightarrow D^{*0}\bar{D}^0K^+$, while $A(E) = c_0 + c_2 T_{1,1} + c_1 T_{2,2}$ for $B^0 \rightarrow D^{*0}\bar{D}^0K^0$. The matrix $T(E)$ is the two-body scattering amplitude for the coupled channels $D^{*0}\bar{D}^0$ and $D^{*+}D^-$ [32,33]

$$
\frac{1}{T(E)} = \frac{1}{2\pi} \left( \frac{\mu_1(-1/a_{11} - \lambda p_0)}{\sqrt{\mu_1^0 \mu_2^0/a_{12}}} + \frac{\sqrt{\mu_1^0 \mu_2^0/a_{12}}}{\mu_2(-1/a_{22} - \lambda p_0)} \right),
$$

where $p_0 = \sqrt{\mu_1 E}$ and $p = \sqrt{\mu_2(E - \Delta)}$ are the binding momenta for the neutral $D^{*0}\bar{D}^0$ and charged $D^{*+}D^-$ channels, respectively, and $\Delta = m_{D^0} + m_{\bar{D}^0} - m_{D^{*0}} - m_{\bar{D}^{*0}}$ is the energy gap between the two channels. Thus, one can fit the $D^* \bar{D}$ invariant mass distributions through Eq. (8).

Next we turn to the production of the $D^* \bar{D}$ hadronic molecule $X(3872)$. Its production can be factorized as the short-distance production of the constituents and the long-distance formation of the $X(3872)$ state part. The factorization formulas for the prompt production of the $X(3872)$ at hadron colliders have been studied in Refs. [34,35]. For the production of $X(3872)$ through $B^0$ or $B^+$ decay, the factorization formula is written as

$$
\Gamma^{0(+) +} = \frac{\lambda^{3/2}(m_B, m_X, m_K)}{32 \pi m_B^2 m_K} |g_0 c_0 + c_0 g_0 + \bar{C} c_0|^2,
$$

where $g_0$ and $g_0$ are the coupling constants of the $D^{*0}\bar{D}^0$ and $D^{*+}D^-$ channel to the $X(3872)$ state, respectively, which are related to the residue of the scattering amplitudes at the pole position. The ratio of the $X(3872)$ production through $B^0$ and $B^+$ decays [25]

$$
\frac{B(B^0 \rightarrow XK^0)}{B(B^+ \rightarrow XK^+)} = 0.50 \pm 0.14 \pm 0.04
$$

is another constraint in the fit.
The dip structure at 3.88 GeV in the $B^0 \rightarrow D^{*0} \bar{D}^0 K^0$ process is due to the opening of the charged $D^{*+}D^-$ threshold. The fit parameters with 1σ error are shown in Table 1. The large uncertainties stem from those of the experimental data. The corresponding parameter correlation matrix is shown in Table 2, from which one can see that there are no particularly large correlations between the parameters. That also indicates there are no redundant parameters in our formulae. Further high statistics data will help to reduce the uncertainties. The most interesting parameters are $a_0$, $b_0$ and $b_1$ with the isospin of the $D^*\bar{D}$ system as subscripts, which reflect the production strengths of diagrams (A) and (B). From the fitted parameters, one can extract several interesting features which help us understand the nature of the $X(3872)$ and the absence of the $Z_c(3900)$ in $B$ decay.

- The $X(3872)$ behaves as a bound state with the binding energy $\epsilon = m_{D^{*0}} + m_{\bar{D}^0} - E_{\text{pole}} = 1.06^{+0.03}_{-0.05}$ MeV below the $D^{*0}\bar{D}^0$ threshold. It should not be the same as that, i.e. Eq. (53) of Ref. [37], extracted from the mass (or the peak position in another word) of the $X(3872)$. That is because that the peak positions are channel dependent, but the pole position is not. Especially, in its component channel, the peak position might be a little higher than the pole position due to the phase space limit.
- The contribution of diagram (B) to the isospin triplet channel is about three orders smaller than that to the isospin singlet channel, as $b_1/b_0 \sim 0(10^{-5})$.
- As shown by Eqs. (1) and (4), the small value of $b_1$ also indicates the small short-distance production amplitudes of the $B^0 \rightarrow D^{*0}\bar{D}^0 K^-$ and $B^+ \rightarrow D^{*+}\bar{D}^0 K^+$ processes at threshold. Thus the production of isotriplet state through these processes is quite small.
- Since diagram (A) also contributes to the isospin singlet channel, there is the coherent interference between diagrams (A) and (B) for the isospin singlet channel. As the result, for each individual channel, the ratios of the $I = 1$ and $I = 0$ components are
  
  
  $B^0 \rightarrow D^{*+}D^- K^0$;
  
  \[ \left| \frac{B_1}{B_0} \right|^2 = \frac{3.30 \times 10^{-6}}{-3.30 \times 10^{-6}}. \]

  $B^+ \rightarrow D^{*0}\bar{D}^0 K^+$:

  \[ \left| \frac{B_1}{B_0} \right|^2 = \frac{3.11 \times 10^{-5}}{-3.11 \times 10^{-5}}. \]

  From the above equations, one concludes that the production of $I = 1$ $D^{*0}\bar{D}^0$ and $D^{*+}D^-$ pairs in $B$ decay is highly suppressed. The large uncertainty of the upper limit in the second ratio stems from the destructive interference in the denominator, as the errors of the parameters are almost the same, thus generating this uncertainty.

Besides the isospin suppression, there might be another suppression coming from the C-parity as discussed in Ref. [29], where the productions of the states with $C = +$ and $C = −$ are constructive and destructive, respectively. In the heavy quark limit, the wave functions of $D$ and $D^*$ are the same, leaving the production of a state with $C = −$ in the transition from $B$ to $K$ equal to zero as shown by Eq. (8) in Ref. [29]. However, the branching ratio of $B^+ \rightarrow D^{*0}\bar{D}^0 K^+$ is about 3 times as that of $B^+ \rightarrow D^{*0}\bar{D}^0 K^+$. The deviation of the value 3 from the heavy quark limit value 1 indicates significant heavy quark symmetry breaking effect. Thus the suppression from the charge-parity is not sizable. In conclusion,
the production rate of the $Z_c(3900)$ in $B$ decays is dominantly suppressed by the small value of the isospin triplet production amplitude $B_1$.

On the contrary, in $e^+e^-$ annihilation, since the $Z_c(3900)$ and the $X(3872)$ are produced together with $\pi$ and $\gamma$ emissions, respectively, the $C$-parity will not suppress any of them. At the same time, since virtual photons do not have fixed isospin, the isospin factors for both isospin triplet and isospin singlet are the same. The only suppression happens for the production of the $X(3872)$ in $e^+e^-$ annihilation stemming from the additional factor of the fine-structure constant.

In summary, we have analyzed the isospin amplitudes of the $B \to D^*\bar{D}K$ process and fitted the presently available $D^0\bar{D}^0$ invariant mass distributions. The results indicate that the production of the isospin triplet $D^*\bar{D}$ state is highly suppressed due to the small value of $b_1$. In addition, the $Z_c(3900)$ will be further suppressed because of its negative charge-parity. These two reasons for the first time lead to a concise explanation why the $Z_c(3900)$ is absent in $B$ decays within the hadronic molecular structure. Stated differently, the absence of the $Z_c(3900)$ in $B$ decays clearly points towards its $D^*\bar{D}$ molecular nature. A detailed scan of the $D^*\bar{D}$ invariant mass distributions of the six processes $B^0 \to D^0\bar{D}^+K^+$, $B^0 \to D^{*+}\bar{D}^-K^0$, $B^0 \to D^0\bar{D}^-K_0$, $B^+ \to D^*\bar{D}^0K^0$, $B^+ \to D^\ast\bar{D}^0K^+$, $B^+ \to D^*\bar{D}^-K^+$ with high accuracy (near threshold), especially for the $B^0 \to D^0\bar{D}^-K^+$ and $B^+ \to D^*\bar{D}^0K^0$ reactions where only the isospin triplet production amplitudes contribute, will help to reduce the uncertainty of the conclusion, such as the uncertainty of the ratios between the isospin triplet amplitudes and the isospin singlet ones.

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