Protecting a bosonic qubit with autonomous quantum error correction

To build a universal quantum computer from fragile physical qubits, effective implementation of quantum error correction (QEC) is an essential requirement and a central challenge. Existing demonstrations of QEC are based on an active schedule of error-syndrome measurements and adaptive recovery operations that are hardware intensive and prone to introducing and propagating errors. In principle, QEC can be realized autonomously and continuously by tailoring dissipation within the quantum system, but so far it has remained challenging to achieve the specific form of dissipation required to counter the most prominent errors in a physical platform. Here we encode a logical qubit in Schrödinger cat-like multiphoton states of a superconducting cavity, and demonstrate a corrective dissipation process that stabilizes an error-syndrome operator: the photon number parity. Implemented with continuous-wave control fields only, this passive protocol protects the quantum information by autonomously correcting single-photon-loss errors and boosts the coherence time of the bosonic qubit by over a factor of two. Notably, QEC is realized in a modest hardware setup with neither high-fidelity readout nor fast digital feedback, in contrast to the technological sophistication required for prior QEC demonstrations. Compatible with additional phase-stabilization and fault-tolerant techniques, our experiment suggests quantum dissipation engineering as a resource-efficient alternative or supplement to active QEC in future quantum computing architectures.

Error correction code and strategy

Following the accelerating progress in bosonic QEC research, we take advantage of the large Hilbert space and the long coherence time of microwave-photon states in a superconducting cavity to store a logical qubit. This logical qubit, \( |\psi_x y \rangle = |0\rangle + x |1\rangle \), is encoded in an odd-parity subspace of the cavity using the following Truncated 4-component Cat (T4C) code:

\[
|0\rangle = C_0 |B\rangle + C_1 |S\rangle, \quad |1\rangle = C_0 |B\rangle + C_1 |T\rangle,
\]

where \( C_0, C_1, C_2, C_3 = \sqrt{0.35}, \sqrt{0.9}, \sqrt{0.65}, \sqrt{0.1} \) are code word coefficients of the four Fock-state components, chosen to approximately balance the average photon numbers in \( |0\rangle \) and \( |1\rangle \). The \( x \) basis states (instead of \( z \) ) of the code, \( |\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2} \), resemble the Schrödinger cat superpositions of coherent states.

\( |\psi_x y \rangle \)

but they require hardware architectures yet to be developed. In this Article, we introduce and implement an AQEC scheme in a common transmon-based superconducting circuit QED device, which is solely enabled by the synthesis of a highly specific dissipation operator.

These simultaneous requirements dictate that the form of dissipation needed for AQEC must be exotic. Advances in quantum reservoir engineering (dissipation engineering) have paved the way to synthesize dissipation operators not naturally available, enabling stabilization of various non-classical states. More recently, dissipation has been tailored to confine a quantum harmonic oscillator to delocalized regions in phase space to suppress bit-flips. However, the stabilized manifold does not permit QEC of the dominant decoherence process in the system: single photon loss. Encouraging proposals for AQEC have emerged for a number of experimental platforms, but they require hardware architectures yet to be developed. In this Article, we introduce and implement an AQEC scheme in a common transmon-based superconducting circuit QED device, which is solely enabled by the synthesis of a highly specific dissipation operator.

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\( |\psi_x y \rangle \)
The coherent-state amplitude, $|a| = 1.87$, is a measure of the size of the cat-state encoding. Single-photon loss, the dominant intrinsic error in a superconducting cavity to be corrected, converts odd-numbered Fock components to even-numbered ones. The photon-number parity $P = a^\dagger a$ effectively plays the role of a QEC stabilizer operator, which can be repetitively measured in active QEC protocols.6,8

The main technical accomplishment of this work is the realization of parity recovery by selective photon addition (PReSPA) via a constantly applied dissipation operator

$$\hat{N} = |1|^\dagger \langle 0| + |2|^\dagger \langle 3| + |4|^\dagger \langle 5| + |6|^\dagger \langle 7|.$$  (2)

This operator stabilizes the photon-number parity, by converting even to odd states, while preserving coherence between the Fock components. Whenever a parity jump arises from photon loss, PReSPA performs AQEC by automatically adding a photon back to the cavity. PReSPA is not constructed from a true parity operator ($= e^{i\pi a^\dagger a}$). However, by explicitly constructing a superposition of four targeted dissipative processes, it achieves a workaround to the well recognized challenge of implementing continuous quantum non-demolition parity projection13,15, which has been a major obstacle for AQEC. It should be noted that $\hat{N}$ does not fully reverse the effect of a photon loss event ($\hat{N} a = a$); rather, it leads to a net gain in mean photon numbers for all logical states, which is approximately equivalent to an increase of $|a|$.

PReSPA also does not correct for the continuous decrease of $|a|$ in the absence of parity jumps1. However, the accumulated entropy in the size uncertainty of the cat-state encoding is not associated with substantial leakage of logical information (which is encoded in the phases of the cat states), and can be removed from the logical qubit effectively using a proper decoding transformation. Theoretically, instantaneous and exact $\hat{I}$ operations to this $T4C$ code can reduce its logical error rate from single-photon loss by approximately 50 times (or more if a larger $|a|$ is chosen for the encoding). See Methods for the theory of this approximate AQEC protocol.

**AQEC technique and device implementation**

Our experiment is carried out in a 3D-planar hybrid circuit QED architecture8 at a base temperature of about 10 mK. A high-coherence cylindrical post cavity $A$ (with $T_1A = 520 \mu s$, $T_2A = 380 \mu s$) is used to store the logical qubit10. A dispersively coupled transmon qubit $Q$ (with $T_1Q = 39 \mu s$, $T_2Q = 17 \mu s$) is used as an ancilla for encoding and decoding of the cavity state. A coaxial stripline resonator $R$ with fast decay rate ($\kappa/2\pi = 0.58 \text{ MHz}$) is used both for readout and as the source of dissipation.

The PReSPA operator is implemented with a fourfold degenerate two-stage pumping process, as illustrated in Fig. 1c. Two continuous-wave frequency combs, each consisting of four tones equally spaced in frequency by $2\chi_A$, are applied to drive transitions targeting the four even-number Fock states, $|2n\rangle_A (n = 0, 1, 2, 3)$. Under the rotating wave approximation, the drive Hamiltonian is

$$\hat{H}_\text{drive} = \frac{3}{h} \sum_{n=0} (|2n\rangle_A \langle 2n| + i\langle 2n| + 1, 0| \langle 2n + 1, 1| + 1\langle 2n + 1, 0|) + \text{h.c.}$$  (4)

where $h.c.$ is the Hermitian conjugate. The selectivity on individual levels relies on the photon-number-dependent transition frequencies of the dispersive Hamiltonian. Hence, it is critical that $\lambda < \chi_A$, so the odd-parity code space remains unperturbed. The other rates follow a hierarchy of $\lambda < \Omega < \chi_A$. When a photon emission occurs in $R$, the even-parity states are simultaneously projected to the odd-parity subspace by gaining a photon. By adiabatic elimination of the fast dynamics in $Q$ and $R$, we obtain $\hat{N}$ as the effective operator acting on cavity $A$. Phase coherence among the four converted states is expected to persist, since no which-path information leaks into the environment as long as (1) the transition rates along the four paths are identical, and (2) the frequency of the emitted reservoir photon is independent of the path choice. This path-degenerate pumping process is inspired by several theoretical proposals11,14,15,18, but our two-stage construction of the coherent drives is crucial for achieving path-independent reservoir emission frequencies while maintaining parity selectivity.

Each of the two frequency combs is generated by single-sideband modulation of a microwave carrier source with four intermediate-frequency control signals digitally combined. The amplitudes and phases of these intermediate-frequency signals can all be independently tuned. The ancilla state is measured using dispersive readout via mode $R$, which is boosted by a travelling wave parametric amplifier31 but not sufficiently optimized to enable single-shot state assignment. Measurement outcomes are converted to a nominal excited state probability by scaling the averaged demodulated signal relative to the reference ground and excited states. After a series of calibration experiments and compensating for multi-tone parametric
mixing effects (see Methods), we experimentally obtain $\lambda = 27 \text{ kHz}$, $\Omega = 88 \text{ kHz}$ for all transition paths.

**Characterization of dissipation operator**

We characterize the engineered PReSPA operator first by tracking the probability distribution of photon numbers in cavity A over time. This distribution can be measured using spectroscopy of the ancilla qubit (Fig. 2a), whose frequency shifts by $-\chi_n$ for every additional photon. As an example, Fig. 2b, c shows dissipative generation of a one-photon Fock state from vacuum under PReSPA as a result of the $|1\rangle\langle 0|$ element of the operator. To demonstrate the simultaneous action of the four parallel conversion paths, we apply PReSPA to an even-parity cat state $(|n\rangle + |n+1\rangle)/\sqrt{2}$, whose frequency shifts by $-\chi_n$ over time, showing the $|0\rangle$ and $|1\rangle$ conversion. For $0 < t < 20 \mu s$ an additional delay time of 20 $\mu s$ is inserted between PReSPA pumps and the transmon $\pi$-pulse to improve clarity of the spectroscopy data by allowing the partially excited transmon to relax.) Cuts of b at $t = 0$ and 25 $\mu s$ (grey dashed line). d, e, $P(\Delta \omega_q)$ for cavity A initialized in an even-parity cat state at: d, $t = 0$ and, e, $t = 25 \mu s$. All four spectroscopy peaks corresponding to even photon numbers (red) are shifted by $-\chi_n$ after $t = 25 \mu s$, indicating odd photon numbers (blue). f, Probability of achieving the target cavity state $|2n + 1\rangle_A$ as measured by $P(t)$ for fixed $\Delta \omega_q = -(2n + 1)\chi_n$ for cavity A initialized in $|2n\rangle_A$. Error bars reflect the standard error of the mean. These four time-domain curves are fitted using a numerical model of the cascaded pumping process (see Supplementary Fig. 1), resulting in $\Omega = 92 \text{ kHz}$, $88 \text{ kHz}$, $87 \text{ kHz}$ and $85 \text{ kHz}$; and $\lambda = 28 \text{ kHz}$, $27 \text{ kHz}$, $27 \text{ kHz}$ and $26 \text{ kHz}$, respectively. The inset shows a block of the cavity process $\chi$ matrix for 25 $\mu s$ of PReSPA. The matrix elements $\chi_{nm,\pm}$ are calculated from transmon spectroscopy measurements from all pairs of initial Fock states $|n\rangle$, and final Fock states $|m\rangle$.

![Fig. 2](image)

**Fig. 2** Characterization of the PReSPA operator: photon population conversion. a, Control pulse sequence of a transmon spectroscopy measurement to infer the cavity photon distribution after PReSPA. We initialize cavity A to a specific initial state using an optimal control theory (OCT) pulse$^{36}$, apply PReSPA for a variable time $t$, apply a spectrally selective $\pi$-pulse to transmon q at a variable detuning $\Delta \omega_q$ and measure the transmon excitation probability $P(\Delta \omega_q, t)$ by performing readout (RO). b, Transmon spectroscopy data $P(\Delta \omega_q, t)$ for cavity A initialized in vacuum. The bright feature is shifted from $\Delta \omega_q = 0$ to $-\chi_n$ over time, showing the $|0\rangle + |1\rangle$ conversion. For $0 < t < 20 \mu s$ an additional delay time of 20 $\mu s$ is inserted between PReSPA pumps and the transmon $\pi$-pulse to improve clarity of the spectroscopy data by allowing the partially excited transmon to relax.) Cuts of b at $t = 0$ and 25 $\mu s$ (grey dashed line). d, e, $P(\Delta \omega_q)$ for cavity A initialized in an even-parity cat state at: d, $t = 0$ and, e, $t = 25 \mu s$. All four spectroscopy peaks corresponding to even photon numbers (red) are shifted by $-\chi_n$ after $t = 25 \mu s$, indicating odd photon numbers (blue). f, Probability of achieving the target cavity state $|2n + 1\rangle_A$ as measured by $P(t)$ for fixed $\Delta \omega_q = -(2n + 1)\chi_n$ for cavity A initialized in $|2n\rangle_A$. Error bars reflect the standard error of the mean. These four time-domain curves are fitted using a numerical model of the cascaded pumping process (see Supplementary Fig. 1), resulting in $\Omega = 92 \text{ kHz}$, $88 \text{ kHz}$, $87 \text{ kHz}$ and $85 \text{ kHz}$; and $\lambda = 28 \text{ kHz}$, $27 \text{ kHz}$, $27 \text{ kHz}$ and $26 \text{ kHz}$, respectively. The inset shows a block of the cavity process $\chi$ matrix for 25 $\mu s$ of PReSPA. The matrix elements $\chi_{nm,\pm}$ are calculated from transmon spectroscopy measurements from all pairs of initial Fock states $|n\rangle$, and final Fock states $|m\rangle$.

![Fig. 3](image)

**Fig. 3** Characterization of PReSPA operator: preservation of coherence. a, Cavity Wigner tomography of six even-parity superposition states, $(|n\rangle + |n+1\rangle)/\sqrt{2}$, prepared by optimal control theory pulses, as input states for PReSPA. b, Wigner tomography of the six corresponding output states after 25 $\mu s$ of PReSPA. The two $\pm$ signs correspond approximately to odd-parity superpositions $(|n\rangle + |n+1\rangle)/\sqrt{2}$, with $n' = n + 1, m' = m + 1$. The Wigner function, $W(\alpha)$, a quasi-probability distribution in the oscillator phase space, is directly measured via photon number parity measurements after variable cavity displacements$^{37}$. From each Wigner function we reconstruct the density matrix, and the most important off-diagonal element is $\rho_{e,m}$ (or $\rho_{c,m}$), which reflects the coherence between $|n\rangle$ and $|m\rangle$ (or between $|n\rangle$ and $|m\rangle$). We also perform similar measurements with permutations of odd-parity superpositions as input states (not shown). The $\chi$ matrix block describing the coherence of the process can be computed by combining all the off-diagonal elements in these reconstructed density matrices (Extended Data Fig. 7). The result for the six key elements characterizing PReSPA coherence, $\chi_{nm,\pm}$, is shown next to the vertical arrows and to a good approximation is equal to $\rho_{e,m}/\rho_{c,m}$. The deviations of $\chi_{nm,\pm}$ from unity reflects the infidelity of the PReSPA process.
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can be greatly reduced with state-of-the-art filtering and shielding of the thermal environment.

The rest of the logical qubit decoherence can be mostly attributed to the finite success probabilities ($S_i$ and $S_f$) of PReSPA in correcting single-photon loss. We define $S_i = 1 - G_{ij}^{-1}$ (where subscript $i$ indicates longitudinal or transverse), where $G_i$ is the QEC gain factor between the natural photon-loss rate ($\Gamma_{\text{Na}} \approx 1/150 \, \mu s^{-1}$) and the part of the corrected logical qubit’s error rate ($\Gamma_f$ or $\Gamma_t$) that can still be attributed to photon loss. Various imperfections can each be analysed in terms of a correction failure rate that reduces $S_i$ and $S_f$. We estimate $S_i = 89\%$ and $S_f = 76\%$, with prominent failure modes including a second photon loss or ancilla decay during a PReSPA operation, virtually activated pumping processes and higher-order cavity nonlinearity (Extended Data Table 3). The QEC success probability should be understood as a hypothetical measure for a standalone PReSPA operation. Actual fidelity of a quantum state that experiences an error-and-correct cycle over a realistic time frame includes additional contributions from new errors at total rates of $\Gamma_f$ and $\Gamma_t$ after the initial error is corrected. For example, a logical quasiparticle state incurring a photon loss can be expected to recover after $t \cong 25 \, \mu s$ with fidelity $F = S_i S_f \cong 69\%$. This is consistent with the characterized $\chi$ matrix for PReSPA process, and is further confirmed in a separate experiment using initialized error states (Supplementary Information). The required QEC success probability to reach break-even in the absence of any undesirable transmon excitation is $\frac{1}{2} S_i S_f > 1 - \frac{2}{\pi^2} = 81\%$. Modest improvement in ancilla and cavity lifetimes can lift $S_i S_f$ sufficiently above this threshold to enable AQEC above break-even under realistic experimental conditions (see Methods for simulation results).

**Outlook**

The four-component cat code (and its truncated variants) has been pursued as a hardware-efficient paradigm for universal quantum computation, offering a full gate set and first-error-protection for logical qubits encoded in single cavities. However, there has been a caveat: the repetitive parity checks required to correct single-photon loss are not simultaneously compatible with the continuous driven dissipation needed for phase-space stabilization. A competing bosonic QEC approach based on two-component cat qubits with strongly biased noise channels defers the challenge of photon-loss correction to a next-level repetition code (for example, with a chain of cavities), but comes at the cost of increased hardware complexity.

Our introduction of PReSPA provides a non-invasive method for photon-loss correction, paving the road for concurrent logic operations or dissipative stabilization within the odd-parity subspace. For example, modest four-photon dissipation induced by a second reservoir can be applied together with PReSPA to evacuate entropy from cat-size changes and correct for phase drifts, hence completing fully passive first-order protection of a single-cavity logical qubit. Autonomously corrected quantum gates can be constructed from path-independent unitary operations acting on the code space and error space in parallel. To improve fault tolerance, forward propagation of ancilla relaxation errors can be suppressed by error-transparent processes through the second excited state of the transmon or by using a biased-noise ancilla. In addition, our cascaded pumping technique to realize path erasure can be generalized to construct a broad family of dissipation operators of the form $\hat{L} = \sum_j A_{ij} |0\rangle\langle j|$, serving as the basis for realizing various other AQEC schemes.

For reducing errors in quantum computing, the development of intrinsically protected physical qubits by Hamiltonian engineering and implementation of QEC codes based on redundancy have often been considered two distinct pursuits. The establishment of continuous AQEC of prominent errors, joined by recent developments of driven qubits with biased noise channels, bridges this divide. In our demonstration, the cavity qubit is protected by a QEC code executed through a driven-dissipative environment in quasi-equilibrium, as described by a time-independent rotating-frame Hamiltonian and dissipation operators. This feature of passive correction is similar to the scenario envisioned as self-corrected quantum memory, but is realized by adding corrective dissipation instead of increasing the energy barrier against error processes. Beyond specific implementations in circuit QED, our work suggests reservoir engineering as a unifying force that applies the flexibility of code-based error correction to improve the robustness of physical qubits with minimal resource overhead.

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at [https://doi.org/10.1038/s41586-021-03257-0](https://doi.org/10.1038/s41586-021-03257-0).
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Upon photon loss, such a cat code would enable evolution into the state $t_n N$ in equation (1), composed of odd-numbered Fock states $|n\rangle$ with $n \leq 7$.

Within T4C encoding, the joint effect of loss and instantaneous PRESPA, as described by a combined loss operator $\hat{N}_a$, generally induces an unwanted distortion of the original quantum state. Specifically, given a number of loss/PRESPA events within a time period $\tau$, the initial state $|\psi_{0\tau}\rangle = x|0\rangle + y|1\rangle$ evolves into the state

$$|\psi(t)\rangle = x n_{1/2}\langle t|\psi_{0}\rangle + y n_{1/2}\langle t|\psi_{1}\rangle + \text{yn}_{1/2}\langle t|\psi_{2}\rangle + \text{yn}_{1/2}\langle t|\psi_{3}\rangle.$$  (5)

Here, $|\psi_{i}\rangle = \Pi^q\exp(-nt/2T_{\text{BA})}|i\rangle/N(t)$ are orthonormal, and the coefficients recording the unwanted state distortion are $n_{1/2}(t) = \Pi^{|i\rangle}/\Pi^{|i\rangle}$, $n_{1/2}(t) = \Pi^{|i\rangle}/\Pi^{|i\rangle}$, and $n_{1/2}(t) = \Pi^{|i\rangle}/\Pi^{|i\rangle}$.

The decoding transformation seeks to transfer the amplitudes $x, y$ from the logical cavity qubit to the auxiliary transmon. Perfect transfer is impeded by two key factors. First, the coefficients $n_{1/2}(t)$ in equation (5) will generally differ and, hence, lead to the mentioned distortion of $x$ and $y$. Second, the code words undergo rotations within the $H_{15}$ and $H_{15}$ subspaces described by

$$|\psi(t)\rangle = x n_{1/2}\langle t|\psi_{0}\rangle + y n_{1/2}\langle t|\psi_{1}\rangle + \text{yn}_{1/2}\langle t|\psi_{2}\rangle + \text{yn}_{1/2}\langle t|\psi_{3}\rangle.$$  (6)

Here, the angles $\theta_j$ and $\phi_j$ depend both on time and the number of loss events. See Extended Data Fig. 1a for a visual description of the effect of jump and no-jump dynamics. The states $|u_0\rangle$, $|u_1\rangle$ and $|v_0\rangle$, $|v_1\rangle$ denote orthonormal bases of the $H_{15}$ and $H_{15}$ subspaces, respectively.

Constructing a decoding unitary $U_{\text{dec}}$ such that

$$|g\rangle \otimes |u_0\rangle \rightarrow |g\rangle 0.$$  (7)
$$|g\rangle \otimes |v_0\rangle \rightarrow |g\rangle 1.$$  (8)

one obtains the reduced transmon density matrix

$$\hat{A}_q = \sum_j \rho_j \begin{pmatrix} x n_{1/2}^2 & x^2 n_{1/2}^2 & x^2 n_{1/2}^2 & x^2 n_{1/2}^2 \\ x^2 n_{1/2}^2 & x^2 n_{1/2}^2 & x^2 n_{1/2}^2 & x^2 n_{1/2}^2 \\ x^2 n_{1/2}^2 & x^2 n_{1/2}^2 & x^2 n_{1/2}^2 & x^2 n_{1/2}^2 \\ x^2 n_{1/2}^2 & x^2 n_{1/2}^2 & x^2 n_{1/2}^2 & x^2 n_{1/2}^2 \end{pmatrix}.$$  (8)

with $\rho_j$ denoting the probability of observing $j$ loss events, and $\zeta = \theta_j - \phi_j$. In general, $\theta_j$ and $\phi_j$ will not be identical, thus causing a second contribution to amplitude distortions. Nevertheless, they vary in the same direction as $j$ varies, and this partial cancellation plays an important part in limiting the intrinsic loss of information in this approximate AQEC protocol. See Supplementary Information section I for analytic details.

We further note that the photon-number expectation value, when averaged over all trajectories, remains exactly constant under the effect of $\hat{N}_a$. In essence, despite its continuous decrease (under no-jump evolution) and stochastic increase (due to PRESPA after a photon loss), the effective cat size $\gamma$ of the encoding remains a constant on average. This is different from the cat states in parity-measurement-based bosonic QEC experiments. In these experiments, the cat states decrease deterministically in size over a timescale of $1/2\kappa$, hence quickly experiencing wavefunction overlap between $C_n$ and $C_{\gamma n}$. The T4C code thus has the advantage of being self-sustained in energy, and the loss of orthogonality at small $\kappa$ is only encountered through a diffusive process.

We quantify deviations of equation (8) from the intended target transmon states with the process fidelity $F$ obtained by averaging $\mathcal{F}_{\text{xy}}(t) = \langle \psi_{0\tau}(t)|\psi_{0\tau}(t)\rangle^2$ over the six cardinal Bloch sphere points and rescaling its range to $1/4 \leq F \leq 1$. Extended Data Fig. 1b compares the theoretical process fidelity for PRESPA-corrected T4C (using the code-word coefficients in our experiment as shown below equation (1)) to free evolution without PRESPA, for both T4C and Fock-state encoding.

Theoretically, the corrected logical qubit incurs no longitudinal relaxation, and the transverse relaxation rate $T_1$ is 40 times lower than the single-photon loss rate $R_k$, corresponding to an intrinsic QEC failure rate of $1 - S_4 = 2.5\%$.

The T4C code can also be viewed as an instance of the binomial code class, assuming the two logical codes contain equal photon numbers ($\Pi$), indeed, informational leakage associated with photon loss is minimized with equal $\Pi$ for $|0\rangle_L$ and $|1\rangle_L$, and the slight imbalance in our experimental code ($\Pi = 3.6, 3.4$ for $|0\rangle_L$, $|1\rangle_L$) was incidental and not optimal. However, we calculated that a revision to an equal $\Pi$ of 3.5 would only improve $S_n$ by 0.08%, which is extremely marginal compared with all the imperfections that lead to $1 - S_4 = 24\%$ in the experiment. Assuming balanced $\Pi$ in a binomial code, the value of $\Pi$ can be chosen to balance the trade-off between the intrinsic QEC failure rate (minimized at $\Pi = 4$) and photon-loss rate (minimized at $\Pi = 3$, the smallest possible in this code). Given the coherence parameters of the transmon and the cavity in this present study, the AQEC performance can be improved by choosing a smaller $\Pi$ close to 3.

Device and fabrication

Our device uses a 3D-planar-hybrid circuit quantum electrodynamics (QED) architecture, and the design specifics are similar to those in ref. 5. The cavity is machined from SNS Al (99.999% pure) and houses a large waveguide section containing two cylindrical re-entrant quarter-wave resonators and a small waveguide tunnel to fit a sapphire chip. The transmon and the low-$Q$ (quality factor) readout resonator are made from thin-film aluminium deposited on the sapphire chip. The transmon contains a single Al-AlO$_x$-Al Josephson junction and is fabricated using a Dolan bridge technique. Electron-beam lithography is carried out with a 30-keV JEOL JSM-7001F SEM, and the evaporation/oxidation processing is performed with a Plassy MEBS550 evaporator.

Of the two high-Q storage cavity modes, the lower-frequency mode A is used for this experiment while the higher-frequency (6.089 GHz) mode B, is idle in the vacuum state throughout the experiment. Cavity mode B has a dispersive coupling with the transmon of 6.5 MHz that we found too high for implementing PRESPA. It has a thermal population of 1–2%, which has a small contribution to the dephasing of mode A. All relevant device parameters are listed in Extended Data Table 1.

Driven Josephson circuit Hamiltonian under PRESPA

The Hamiltonian of the circuit QED system can be derived using the perturbation theory that adds weak anharmonicity to three harmonic oscillator modes corresponding to the cavity A, transmon ancilla q and the stripline resonator R.

$$\hbar \frac{\dot{E}}{\hbar} = \omega_d q^2 + \omega_n a^2 + \omega_n^* a^2 - E_1 \left( \cos \phi + \phi_n \right)$$  (9)

where $\omega_d, \omega_n, \omega_n^*$ are the frequencies of the eigenmodes of the linearized system, $\dot{q}, \dot{a}$ and $\dot{a}^*$ are their lowering operators, $E_1$ is the Josephson energy of the junction, and $\phi = \phi_q + \phi_a + \phi_n + h.c.$ is the phase operator of...
the Josephson junction. \( \phi_q, \phi_R, \phi_A \) are the mode zero-point phase fluctuations across the junction for modes \( q, A \) and \( R \).

Implementing PReSPA requires two frequency combs: one comb exciting the transmon, and one mixing comb converting that transmon excitation into excitations in \( A \) and \( R \). This mixing comb activates a four-wave mixing process using the nonlinearity of the Josephson junction. To account for the four mixing tones with frequencies \( \omega_m \) in our Hamiltonian, we make a unitary transformation by displacing the qubit annihilation operator \( \hat{q} \rightarrow \hat{q} + \sum_m \xi_m e^{i \omega_m t} \) and the phase across the junction is (see, for example, the Supplementary Information of ref. 1):

\[
\hat{\phi} = \phi_q \hat{q} + \phi_A \hat{a} + \phi_R \hat{a}^\dagger + \sum_{m=0}^{3} \phi_m \xi_m e^{i \omega_m t} + \text{h.c.} \tag{10}
\]

Expanding the cosine, going to a rotating frame, and employing a dispersive transformation that cancels 2nd-order terms, the relevant 4th- and 6th-order terms include non-driven terms (without \( \xi_m \)) and driven terms. The non-driven terms, generic to most circuit QED systems, are:

\[
\hat{H}_{\text{non-drive}} = -\frac{\hbar}{2} \chi^q q^\dagger q^\dagger a q a - \frac{\hbar}{2} \chi^q q^\dagger q^\dagger a a a q - \frac{\hbar}{2} \chi^q q^\dagger q^\dagger a q a q + \text{h.c.} \tag{11}
\]

where \( \hat{H}_{\text{non-drive}} \) are the non-driven terms, \( \chi^q, \chi^q, \chi^q \) are the dispersive couplings between the transmon and cavity, and \( A \), between the transmon and reservoir \( R \), and between \( A \) and \( R \), respectively. \( K \) is the self Kerr of cavity \( A \), and \( \chi^q \) is the 6th-order nonlinearity. We treat the transmon as a two-level system because the higher excited states are not accessed in this experiment. Likewise, for all operations except readout, \( R \) is either in the ground or first excited state and we ignore its higher order terms.

By setting \( \omega_m = \omega_A + \omega_R - \omega_q + m \eta \) we get stationary, or slowly rotating, four-wave mixing terms:

\[
\hat{H}_{\text{mix}} = -\frac{\hbar}{2} \chi^q q^\dagger q^\dagger a q a - \frac{\hbar}{2} \chi^q q^\dagger q^\dagger a a a q - \frac{\hbar}{2} \chi^q q^\dagger q^\dagger a q a q + \text{h.c.} \tag{12}
\]

where \( \eta \) is the difference in frequency between each nearest pair of mixing tones. The four mixing tones will each individually Stark shift the transmon but we can simply absorb those into the transmon frequency for the rotating frame transformation mentioned above. There will also be slowly rotating Stark shift terms that are a result of the cross terms of two different mixing tones:

\[
\hat{H}_{\text{Stark}} = -\frac{\hbar}{2} \chi^q q^\dagger q^\dagger a q a - \frac{\hbar}{2} \chi^q q^\dagger q^\dagger a a a q - \frac{\hbar}{2} \chi^q q^\dagger q^\dagger a q a q + \text{h.c.} \tag{13}
\]

The four mixing tones acting together drive the four \( (2n, c, 0, 0) \leftrightarrow (2n + 1, g, 1) \) \( (n = 0, 1, 2, 3) \) transitions (the black solid arrows in Fig. 1c). We can calculate the complex Rabi rate of these driven transitions under a weak-drive approximation of \( |\xi_m|^2 \ll \hbar \eta / (E \phi_q^2) \):

\[
\Omega_n = -\frac{\hbar}{2} \chi^q q^\dagger q^\dagger a q a - \frac{\hbar}{2} \chi^q q^\dagger q^\dagger a a a q - \frac{\hbar}{2} \chi^q q^\dagger q^\dagger a q a q + \text{h.c.} \tag{14}
\]

where \( \delta \) is the Kronecker delta function. This rate is caused by two different mechanisms: a direct drive by one of the four tones that is on resonance, and a multi-tone parametric effect. This parametric effect arises from the time-modulation of transmon frequency by the Stark shift induced by pairs of mixing tones. When the detuning of one of the off-resonant mixing tones exactly matches the modulation frequency, the transition can be parametrically driven by a combination of three tones. Depending on the relative phases of the three tones, these terms can contribute constructively or destructively to the transition rate.

This calculation has been done with the assumption that the spacing between tones \( \eta \) is constant, but we could also consider non-even frequency spacing. In this case, terms that rotate very slowly with time would appear in the mixing rates, which would cause our PReSPA operator to change undesirably with time. We make the choice of keeping \( \eta \) constant to avoid these complications.

A similar calculation can be done for the four selective transmon drives. To preserve path independence we use the same frequency-spacing magnitude \( \eta \) as the mixing drives but with the opposite sign. As the transmon drives are relatively weak, they do not cause a substantial single-tone or multi-tone Stark shift but the frequency modulation caused by the mixing drives will allow for similar three-tone parametric mixing effects (two mixing tones and one transmon tone). We can write the rates for the four transmon transitions as:

\[
\lambda_n = \lambda_n - \frac{2E \phi_q^4}{\eta \hbar} \sum_{m=m,K;k} \frac{A_m \xi_m \xi_{n-k} \xi_{n-m-k}}{m-n} \tag{15}
\]

where \( \lambda_n \) are the bare transmon Rabi rates without the mixing drives. For either comb, the effect of this parametric mixing is a renormalization of the Rabi rates \( \Omega_n \) and \( \lambda_n \) from the Rabi rates by individual resonant tones (that are proportional to \( \xi_m \) and \( \lambda_n \)).

PReSPA spectroscopy and rates

Implementing PReSPA requires initial parameter guesses supplemented with experimental corrections. We make the initial assumption that the comb frequency spacing \( \eta = 2\chi_q \), and that the \( 0|O\rangle \leftrightarrow |1\rangle \) (mixing) transition frequency is \( \omega_A + \omega_R - \omega_q \). Because of the strong off-resonant drives Stark shift the transmon, a detuning \( \Delta \) needs to be added to the mixing comb frequencies and subtracted from the transmon comb frequencies to best match the desired transitions.

In addition, the presence of non-zero \( \chi^q \) adds a quadratic shift to the transmon frequency based on the cavity photon occupancy. Therefore, there is no choice of evenly spaced tones that can drive all transitions on resonance. To best match the experimentally measured transitions we can fit \( \eta, \chi_q \), and \( \Delta \) through a set of spectroscopy measurements (Extended Data Fig. 2c). Experimentally, we choose \( \eta = 2.679 \) MHz and \( \Delta = 2.9 \) MHz (calibrated on a daily basis) to ensure no pair of tones are farther than 10 kHz (about \( 2 \chi_q \) off-resonant) to their corresponding transitions.

To match the dissipative processes across the four parallel paths in PReSPA, we measure the probability of photon addition, over time, to each of the four even states (see Supplementary Fig. 1 for further details). By fitting the curves of photon population and transmon excitation probability we can extract \( \Omega_n \) and \( \lambda_n \) for each of the four conversion paths.

From equations (14) and (15) we can also estimate \( \Omega_n \) and \( \lambda_n \) from the amplitudes and phases of the microwave tones. We find experimentally that achieving equal \( \Omega_n \) and \( \lambda_n \) for the four conversion paths requires different microwave amplitudes within each comb, in quantitative agreement with theoretical predictions (Extended Data Table 2). There is a discrepancy in a global pre-factor in the magnitude of \( \Omega_n \), possibly caused by the coarse estimates of zero point fluctuation parameters.
We therefore specified three additional con-straints for the numerical optimization. However aside from preserv-ing odd-parity initial state (for example, |2\rangle_L), it is required to act on the four relevant basis states
\begin{align*}
\begin{pmatrix}
|0\rangle_L + |2\rangle_L \\
|1\rangle_L + |3\rangle_L
\end{pmatrix},
\end{align*}
for nearly complete even-to-odd conversion. By fitting the Ramsey oscillations, we can extract the phase of the PReSPA-converted state (for example, \(\hat{U}_{\text{drive}}^{(0+10,0)}\)), which is calibrated against the phase extracted from another PReSPA Ramsey experiment on the corresponding odd-parity initial state (for example, \(\hat{U}_{\text{drive}}^{(0+10,1)}\)). Phase and amplitude parameters of the microwave combs are adjusted to minimize the imparted phases of the photon addition process.

PReSPA Ramsey is further used to measure the phase accumulation rate of our logical code words. When measuring the process fidelity of error correction we work in a frame where \(|0\rangle_L\) is stationary. By measuring the phase of \(|\psi_0\rangle\) as a function of time we can calculate the correct decoding angle.

We also perform a regular qubit Ramsey experiment for the transmon in the presence of the mixing combs of PReSPA (without exciting cavity A). This allows measurement of the Stark shift of the transmon rapidly and accurately, because to a very good approximation, only the mixing comb contributes to the Stark shift of the transmon. This experiment further shows that the coherence time of the transmon is not affected by the strong off-resonant drives of PReSPA (with \(T_{2q} = 17\) μs).

**GRAPE methods**

Pulses for state preparation and decoding are constructed using quantum optimal control (QOC) theory\(^{46,33,34,35}\), as adapted for arbitrary cavity-state controls in circuit QED in ref. \(^{44}\) (see Supplementary Information section 2 for additional details). QOC numerically optimizes the envelopes of tones acting on the transmon and storage cavity, such that deviations of the realized state or unitary from the target state or target unitary are minimized. Deviations are quantified as state or process infidelities and incorporated into a cost functional \(C\), which is subject to gradient-descent minimization via gradient ascent pulse engineering (GRAPE)\(^{36}\).

Two envelope-modulated drives with carrier frequencies \(\omega_A\) and \(\omega_k\) are applied to storage cavity and transmon for preparation of the cavity states shown in Figs. 3a and 4a. By adjusting the respective enve-lopes, GRAPE maximizes the state-transfer fidelity
\begin{align*}
F_{\text{prep}} = |\langle \psi_f | \psi_i \rangle|^2,
\end{align*}
where \(\psi_f\) and \(\psi_i\) are the cavity target state and the actual cavity state realized by the pulse. Closed-system time evolution is simulated based on the rotating-frame Hamiltonian (11).

GRAPE is further used in constructing a decoding unitary \(\hat{U}_{\text{drive}}\) seeking to map the T4C-encoded cavity state to the transmon. As minimal constraints on \(\hat{U}_{\text{drive}}\) it is required to act on the four relevant basis states as shown in equation (7).

\begin{align*}
\hat{H}_{\text{drive}}/\hbar &= \sum_{m=0}^{3} e^{-ik_\phi m - (\omega_m - 2\pi m/\tau)^2/\Omega_m^2} \sum_{n=0}^{7} \left( \lambda_n |m, e, 0\rangle \langle m, g, 0| \right) \\
&+ \Omega_m \left[ \frac{m+1}{2n+1} |m, e, 0\rangle \langle m+1, g, 1| + \text{h.c.} \right] \end{align*}

This drive Hamiltonian describes four transmon excitation tones and four four-wave mixing tones that are ±\(\Delta_N\) detuned from the transition frequencies, as shown in Extended Data Fig. 2B. \(\lambda_n\) and \(\Omega_m\) are the transmon drive rate and four-wave mixing rate for the intended transitions as listed in Extended Data Table 2, accounting for the small differences in rates within each comb. Crucially, equation (18) encapsulates the resonant and non-resonant dynamics of each photon-number state \((m = 0\) to \(8\)) under the effect of all four tones \((n = 0\) to \(3\)) in each PReSPA comb. We assume the constant Stark shift of all modes have been included in
the rotating frame. We also make the assumption that the effect of two-tone modulated Stark shift is fully captured by the renormalization of comb rates $\lambda_1$ and $\Omega_1$ in the spirit of equations (14) and (15).

In addition to single photon loss, we account for the spurious excitations, relaxation, dephasing of the ancilla and other causes of cavity dephasing. The QuTiP simulation thus solves the master equation,

$$\frac{\partial \rho(t)}{\partial t} = -\frac{i}{\hbar} \left( \hat{H}_{\text{non-drive}} + \hat{H}_{\text{drive}} \right) \rho(t)$$

$$+ \left[ \frac{1}{T_{1A}} D(\hat{a}) + \frac{1}{T_{1R}} D(\hat{f}) + \frac{1}{T_{1Q}} D(\hat{q}) \right]$$

$$+ \left[ \frac{1}{T_p} D(\hat{q}^\dagger \hat{q}) + \gamma_p D(\hat{q}^\dagger) + \gamma_p D(\hat{q}) \right] \rho(t)$$

(19)

where $D(\hat{O})\rho(t) = \hat{O}\rho\hat{O}^\dagger - \frac{1}{2} \hat{O}^\dagger \hat{O} \rho - \rho \hat{O}^\dagger \hat{O}$. The $T_{1A}$, $T_{1R}$, $T_{1Q}$ are the relaxation times for cavity, reservoir and ancilla, respectively. $y_p$ is the spurious ancilla excitation rate and $y_p$ is the storage cavity dephasing rate.

The density matrix of the system at any given time $t$ is transformed by the decoding unitary equation (7) so that the cavity logical information is decoded into the transmon state just like in the experiment. The cavity Kerr effect is taken into account by adjusting the phases of the decoding unitary as discussed earlier in Methods. The quantum state fidelity against the targeted ancilla state is then computed as a function of time $t$.

The simulation shows a corrected logical qubit lifetime of 290 ps, with $f_1^\dagger = 383 \mu s$ and $f_1 = 255 \mu s$, in excellent agreement with the experiment. We also performed sensitivity analysis to individual Hamiltonian or loss parameters in the simulation, and the results are consistent with our estimates of individual error channels presented in Extended Data Table 3 and the Supplementary Information section 3.

Since the simulation quantitatively captures all the error channels displayed in the experiment, we can evaluate the performance of our QResPA pumping scheme when the system parameter improves. With better cavity and transmon coherence, for the same static and driven Hamiltonian parameters, the logical error rates due to transmon decay and the second photon decay will be suppressed quadratically, as expected for an ideal first-order QEC protocol. However, other error channels due to higher-order nonlinearity and drive-tone selectivity will not scale down equally. Therefore, reduced $\chi_{1A}$, $\chi_1$, $\Omega_1$ and $\Omega_2$ should be chosen to partially trade away the gain from coherence improvement to suppress these error channels. This is analogous to the situation in active QEC experiments, where less frequent parity measurements may be desirable when coherence time improves.

Extended Data Fig. 5a demonstrates the simulated AQEC performance including all the imperfections intrinsic to a transmon-based QResPA scheme, which is computed using a specific set of Hamiltonian parameters (corresponding to 50% lower QResPA rates and a 80% lower dispersive shift than the experiment). The result shows logical lifetime 7–9 times longer than the physical photon loss time for a $f = 3.4$ T4C code, or 40–80% above the break-even point, using current state-of-the-art cavity and transmon of the same style as in our experiment. Extrinsic effects such as thermal excitation of the transmon is not included, but in Extended Data Fig. 5b we show for a specific attainable case of $T_{1A} = 1$ ms, $T_{1R} = 100$ $\mu$s, QEC break-even can be achieved with a few per cent of transmon thermal populations.

In Extended Data Fig. 5a, assuming constant $\lambda$ and $\chi_0$, the QEC gain factor decreases at the longest $T_{1A} (>2$ ms). This is because of the increased weight of error contribution from off-resonance excitation of the transmon due to imperfect frequency selectivity of the transmon comb. Further reduction of $\lambda$ (accompanied with smaller reduction of $\chi_0$) is needed to further improve the gain factor. A more promising alternative in this high-coherence regime is to use $|2n+1,f,0\rangle$ instead of $|2n,f,0\rangle$ as the first intermediate level of PResPA, which can immediately reduce all frequency-selectivity-related errors by a factor of 4 at the expense of a 2× faster transmon decay (a relatively small contribution in this regime).

Data availability

Data used in this work is available on reasonable request.

Code availability

Code used in this work is available on reasonable request.

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Author contributions

J.M.G. carried out the device design, microwave measurements, and data analysis of the experiment under the supervision of C.W. B.B. generated the numerical pulses for unitary control in the experiment under the supervision of J.K. J.L. fabricated the device and contributed to the cryogenic preparation of the apparatus. S.S. carried out numerical simulations for the experiment. B.B., J.K. and C.W. developed the approximate AQEC theory. C.W. conceived and oversaw this project. J.M.G., B.B., J.K. and C.W. wrote the manuscript with input from all authors.

Competing interests

The authors declare no competing interests.

Additional information

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Extended Data Fig. 1 | Dynamics and intrinsic performance of the approximate AQEC protocol. a, Diagram illustrating the quantum-trajectory state $|\psi(t)\rangle$ after a time period $t$ under the combined effect of photon loss and instantaneous PReSPA, given a certain number of jumps. The initial state is $|\psi(0)\rangle = |0\rangle_L$ in the subspace $\mathcal{H}_{11} = \text{span} \{ |\psi_0\rangle, |\psi_1\rangle \}$. Both jump and no-jump evolution lead to rotation of $|\psi(t)\rangle$ over time within the subspace, and the angle $\theta(t)$ parametrizes this rotation. For each quantum trajectory, $|\psi(t)\rangle$ slowly and continuously rotates clockwise in the absence of jumps and occasionally undergoes stochastic jumps counterclockwise. The diagram and

the dynamics for the states $|\psi_3(t)\rangle$ in the $\mathcal{H}_{37} = \text{span} \{ |\psi_0\rangle, |\psi_1\rangle \}$ subspace (not shown) follows an analogous pattern. b, Comparing the decay of process fidelities for three cases: T4C encoding using the ideal PReSPA scheme of this section (corrected T4C code, green), T4C encoding without using PReSPA (uncorrected T4C code, teal), and Fock state encoding (uncorrected Fock $|0\rangle$, $|1\rangle$, blue). Experimental values of $T_1$, and $K$ are used, and cavity dephasing is not considered. Exponential curves for the T4C fidelity use the equation $F(t) = 0.75e^{-t/\tau} + 0.25$ to extract decay rate $\tau$. 
Extended Data Fig. 2 | PReSPA spectroscopy. **a, b,** Control pulse sequence for two-dimensional spectroscopy to find the resonance conditions for the PReSPA mixing comb and transmon comb. We prepare an even-parity Fock state ($|0\rangle$, $|2\rangle$, $|4\rangle$, or $|6\rangle$), apply PReSPA for a fixed time (12 $\mu$s) with varying detunings of the transmon comb ($\Delta_q$) and the mixing comb ($\Delta_m$) in an attempt to activate dissipative photon addition. After a 1 $\mu$s wait time for the reservoir to relax, we either selectively $\pi$-pulse the transmon conditioned on cavity $A$ being in the targeted final state ($|1\rangle$, $|3\rangle$, $|5\rangle$, or $|7\rangle$) (**a**) or skip this pulse (for a background measurement, **b**), and proceed to read out the transmon state. The difference between the two measurements informs the likelihood of successful photon addition. **c,** Two-dimensional PReSPA spectroscopy data: probability of photon addition (colour scale) as a function of the comb detunings ($\Delta_q$ and $\Delta_m$) for the $|0\rangle$ to $|1\rangle$ transition. Note that the linewidth of the four-wave-mixing transition is an order of magnitude greater than that of the transmon excitation owing to the short reservoir $T_{1R}$. We can repeat this procedure to find all four sets of transition frequencies. **d,** Cartoon spectrum of PReSPA drive frequencies. Four transmon drives, left, and four mixing drives, right, compose PReSPA. The coloured ticks indicate the actual transition frequencies whereas the vertical black bars show the microwave drive frequencies in PReSPA. The transmon drive for the $|0\rangle$ to $|1\rangle$ conversion process is approximately at the Stark-shifted transmon frequency, $\omega_q - \Delta_{\text{Stark}}$, and the $|0\rangle$ to $|1\rangle$ mixing drive is near $\omega_m + \omega_q - \Delta_{\text{Stark}}$. Because of the equal frequency spacing $\eta$ in each comb and the unequal frequency spacing between the transitions with different photon numbers (owing to the 6th-order nonlinearity, $\chi''$), not all drives can be placed exactly on resonance. Experimentally, we settle for $\eta$ slightly greater than $2\chi''$ and $\Delta_q = \Delta_m$ slightly smaller than $\Delta_{\text{Stark}}$ to compensate for the effect of $\chi''$. 

\[ \text{Transmon comb detuning } \Delta_q (\text{MHz}) \]

\[ \text{Mixing comb detuning } \Delta_m (\text{MHz}) \]
Extended Data Fig. 3 | Cavity Wigner and PReSPA Ramsey measurements. 

a, Experimental Wigner function $W(\alpha)$ (colour scale, dimensionless) of $|0\rangle_L$, acquired by applying a cavity displacement operation $\hat{D}_\alpha = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a})$ with variable complex amplitude $\alpha$ followed by an ancilla-assisted photon-number-parity measurement (which is composed of two $\pi/2$ pulses of the ancilla and a delay time of $\pi/\chi$ and an ancilla readout $^{37,58}$). The Wigner function rotates around the origin over time at a rate proportional to the frequency difference between $|1\rangle$ and $|5\rangle$ in the rotating frame of the experiment.

b, Measured Wigner function values at a fixed phase-space position (as indicated by the cross in a, at $\alpha = 0.75$) as a function of time under PReSPA. Analogous to a qubit Ramsey measurement, this cavity PReSPA Ramsey experiment can be used to efficiently track the phase evolution of any two-component superposition states using the interference effect enabled by the coherent cavity displacement ($\hat{D}_\alpha$) before readout. The exponential envelope of the sinusoidal fit indicates the rate of decay for the coherence between $|1\rangle$ and $|5\rangle$ under the correction of PReSPA. Similar measurements are applied to various superposition states to provide direct calibration of the frequencies and phases of these states under PReSPA. PReSPA enhances the ability to use such Ramsey measurements at high photon numbers because it approximately preserves photon number distributions in the cavity.
Extended Data Fig. 4 | Tracking AQEC performance over time. a, We interleave measurements of the corrected logical qubits under two differently calibrated PReSPA parameter sets (PReSPA-1, red and PReSPA-2, blue) while also monitoring the cavity $T_2^A$. Each circle corresponds to the decay time of process fidelity extracted from measuring all six cardinal points of the logical Bloch sphere as described in Fig. 4. The decay rates for state fidelity are shown in triangles for the two logical pole states (upwards triangles) and the four equator states (downwards triangles). The state fidelity is measured by quantum state tomography of the ancilla after decoding the cavity state as described in the Methods. For ancilla state tomography, we measure all six Pauli operators ($\sigma_x$, $\sigma_y$, $\sigma_z$, $\sigma_x^-$, $\sigma_y^-$, $\sigma_z^-$) by performing ancilla rotations before readout. The over-complete measurement set is used for simultaneous calibration of the readout signal contrast, allowing for accurate determination of the transmon state. PReSPA-1 is calibrated by adjusting control parameters to achieve matched PReSPA rates and zero conversion phases as discussed in the Methods. For PReSPA-2, we employ further empirical parameter optimization to maximize equator state lifetime as described in the Supplementary Information section 5. Cavity A has a two-state switching behaviour of unknown origin (see notes in Extended Data Table 1). For distinct stretches of 2–8 h, cavity A shows fluctuating and abnormally low $T_2^A$, and all data recorded during such periods (with example data shown in the shaded region) are excluded in all other parts of the paper. b, Process fidelity averaged over the data, excluding the shaded region, for both PReSPA 1 and 2. Data reported in Fig. 4d for the corrected T4C encoding is a duplication of the blue points here. c, Equator and pole state fidelity for the same time period for both PReSPA 1 and 2. Oscillatory behaviour in the data are caused by the numerical differences between the two decoding pulses discussed in Methods section ‘GRAPE methods’.
Extended Data Fig. 5 | Predicted AQEC performance in numerical simulations. The results are based on master-equation simulations of a T4C qubit (with encoding \( \pi = 3.4 \) for both basis states) under the Hamiltonian equations (11) and (18), which captures the dynamics under the microwave combs of PReSPA. a, Gain factor (colour scale, dimensionless) of the corrected logical qubit lifetime over the physical photon lifetime \( (T_{1A}/T_{1q}) \) in the T4C encoding as a function of ancilla \( T_{1A}, T_{\phi} \) (which are made equal for convenience) and cavity \( T_{1q} \). To illustrate the intrinsic performance of our transmon-based PReSPA pumping scheme, we have assumed no ancilla thermal excitations and other cavity dephasing errors. However, ancilla excitations due to the imperfect frequency selectivity of PReSPA, which is unrelated to photon loss, are reflected in the simulation. Therefore, the gain factor shown is different from the \( G_i \) defined in the main text, and decreases slightly at long \( T_{1A} \). The QEC break-even ratio (colour scale, dimensionless), defined as the T4C qubit lifetime under PReSPA over the lifetime of the 0/1 Fock-state encoding. Here we use a specific set of achievable coherence times \( T_{1A} = 1 \) ms, \( T_{sw} = T_{\phi} = 100 \) \( \mu \)s (refs. 46,47) and show the degradation of AQEC performance in the presence of spontaneous transmon excitation \( (\gamma_{\uparrow}) \) errors caused by the stray thermal background (horizontal axis) or pump-tone-induced heating from PReSPA (vertical axis). QEC breakeven can be reached if the \( \gamma_{\uparrow} \) rate is kept reasonably low. The dashed lines in both a and b indicate where the QEC break-even ratio equals 1. Relevant system parameters: In a, we use \( \lambda/2\pi = 17.5 \) kHz, \( \Omega/2\pi = 45 \) kHz, \( \kappa/2\pi = 227 \) kHz, \( \chi_q/2\pi = 1.05 \) MHz, \( \chi_R/2\pi = 1.6 \) MHz, scaled from the experiment by 50%, 50%, 40%, 80%, 80% respectively. In b, we use \( \lambda/2\pi = 21.6 \) kHz, \( \Omega/2\pi = 72 \) kHz, \( \kappa/2\pi = 364 \) kHz, \( \chi_q/2\pi = 1.18 \) MHz, \( \chi_R/2\pi = 1.8 \) MHz, scaled from the experiment by 80%, 80%, 64%, 90%, 90% respectively. The choice of parameters here is guided by the scaling laws of various error channels (Extended Data Table 3) but did not go through optimization of individual parameters.
Extended Data Fig. 6 | Cavity heating effect caused by spurious transmon excitations. 

(a) Schematic mechanism of sequential two-photon gain triggered by spurious excitation of the transmon. Starting from $|\text{lg0}\rangle$, following the transitions labelled with blue-circled numbers, the system is excited to $|\text{le0}\rangle$ by a transmon $\gamma_\uparrow$ jump, and then driven unintentionally to $|\text{eg1}\rangle$ by mixing tones that are off-resonance by only $\chi/2\pi = \pm 1.3$ MHz (which does not provide strong enough frequency selectivity relative to the reservoir linewidth $\kappa/2\pi = 0.58$ MHz), and then relaxes to $|\text{eg0}\rangle$ following reservoir decay. Once a photon is added in this spurious odd-to-even conversion process, the PReSPA scheme by design will add a second photon, driving the system ultimately to $|\text{eg1}\rangle$. Similarly, a transmon $\gamma_\uparrow$ jump can add two photons to $|3\rangle$ and $|5\rangle$ states (but not $|7\rangle$). 

(b) Schematic illustration of the steady-state photon number distribution established between the ancilla $\gamma_\uparrow$-induced photon addition and the natural photon loss of the cavity. The figure corresponds to the configuration of the test experiment in (d) when only the mixing comb of PReSPA is applied, as in (b). Under this configuration, a transmon $\gamma_\uparrow$ jump may add just one spurious photon in the cavity, leading to an effective cavity heating rate out of its vacuum state $\gamma_01 \approx \gamma_\uparrow$ (when the transmon decay rate through the reservoir $\gamma_\uparrow \approx 1/T_m$). The relative probability of $|0\rangle$ versus $|1\rangle$ informs the balance between the cavity decay rate $1/T_1 = 1.9$ ms$^{-1}$ and the heating rate $\gamma_01$. 

(c) Steady-state cavity photon number distribution when only the mixing comb of PReSPA is applied, as in (b). Under this configuration, a transmon $\gamma_\uparrow$ jump may add just one spurious photon in the cavity, leading to an effective cavity heating rate out of its vacuum state $\gamma_01 \approx \gamma_\uparrow$ (when the transmon decay rate through the reservoir $\gamma_\uparrow \approx 1/T_m$). The relative probability of $|0\rangle$ versus $|1\rangle$ informs the balance between the cavity decay rate $1/T_1 = 1.9$ ms$^{-1}$ and the heating rate $\gamma_01$. 

(d) Steady-state cavity photon number distribution when only the mixing comb of PReSPA is applied, as in (b). Under this configuration, a transmon $\gamma_\uparrow$ jump may add just one spurious photon in the cavity, leading to an effective cavity heating rate out of its vacuum state $\gamma_01 \approx \gamma_\uparrow$ (when the transmon decay rate through the reservoir $\gamma_\uparrow \approx 1/T_m$). The relative probability of $|0\rangle$ versus $|1\rangle$ informs the balance between the cavity decay rate $1/T_1 = 1.9$ ms$^{-1}$ and the heating rate $\gamma_01$. 

(e) Blue triangles show the cavity heating rate $\gamma_01$, as measured with the technique described in (d), as a function of the Rabi amplitude of a single $|\text{le0}\rangle \leftrightarrow |\text{lg1}\rangle$ mixing tone. $\gamma_01$ initially follows the expected values corresponding to the transmon $\gamma_\uparrow = 1.4$ ms$^{-1}$ (green dashed curve), but additional heating effects due to the microwave pump is observed at higher four-wave mixing (FWM) rate $\Omega$. 

In this measurement, PReSPA is applied nearly at all times, only briefly interrupted by spectroscopy probes once every 2 ms. The peak amplitudes confirm that odd photon number parity is permanently stabilized, and also reveals the presence of spurious excitation processes in the cavity.
Extended Data Fig. 7 | Process χ matrix block for 25 μs of PReSPA. The matrix converts elements of input density matrices (top axis) to output density matrix elements (left axis) expressed in the Fock state basis. 

**a**, Amplitude values of the χ matrix elements. The upper left block (χ_{nn}^{mm}) describes the conversion of diagonal elements of the input and output density matrices, which is associated with transfer of photon occupation probabilities calculated from transmon spectroscopy experiments (as in Fig. 2d, e). The lower right block (χ_{nm}^{kl}) describes the conversion of relevant off-diagonal elements of density matrices, which is calculated from Wigner tomography and density matrix reconstruction59 (as in Fig. 3). The greyed blocks are assumed to be zero owing to the absence of interference between the four conversion paths in PReSPA.

**b**, Phase values of the χ matrix elements for the lower right block in **a**. For best illustration of the PReSPA process, the phases are reported in the full rotating frame where all Fock states have zero energy. Values in grey are measured but not statistically significant because the corresponding amplitude value is not large enough. In this frame, as prescribed by equation (2), PReSPA requires zero phase for the six χ_{nm}^{nm}(+1)(+1) elements representing the coherence of the even-to-odd conversion process, which is accomplished by our PReSPA calibration. The diagonal elements χ_{nm}^{nm}, representing the preservation of odd-state superpositions should have zero phase by definition. Their systematic deviation from zero was caused by parameter drift in the experiment as that block of data was acquired at a later time than the earlier rotating frame calibration.
| Symbol                          | Value       |
|--------------------------------|-------------|
| Transmon frequency $\omega_\text{q}/2\pi$ | 5489.59 MHz |
| Transmon anharmonicity $\alpha_\text{q}/2\pi$ | 201.22 MHz |
| Transmon $T_1$                  | $T_{1\text{q}}$ 39 $\mu$s |
| Transmon $T_2^*$ Ramsey         | $T_{2\text{q}}^*$ 17 $\mu$s$^1$ |
| Transmon $T_2$ Echo             | 36 $\mu$s |
| Transmon $|\psi\rangle$ population | 5%          |
| Reservoir frequency $\omega_\text{R}/2\pi$ | 7337.8 MHz |
| Reservoir-transmon coupling $\chi_\text{A}$/2$\pi$ | 2.8 MHz |
| Reservoir $T_1$                 | $\frac{1}{\kappa}$ 0.27 $\mu$s |
| Cavity A frequency $\omega_\text{A}/2\pi$ | 4067.445 MHz |
| Cavity A-transmon coupling $\chi_\text{AR}/2\pi$ | 1.313 MHz |
| Cavity A anharmonicity $K/2\pi$ | 1.7 kHz$^1$ |
| Cavity A 2nd order coupling $\chi_\text{A}/2\pi$ | 5.5 kHz |
| Cavity A $T_1$                  | $T_{1\text{A}}$ 520 $\mu$s |
| Cavity A $T_2$                  | $T_{2\text{A}}$ 380 $\mu$s$^#$ |
| Cavity A $|\phi\rangle$ population | $\sim$1% |

$^1$The transmon $T_1$ reflects any 1/e decay time of Ramsey oscillations. The transmon displays a random switching behaviour between two values of $\omega_\text{q}/2\pi$ that are 40 kHz apart, with a dwell-time split of approximately 85%:15%. The switching timescale is on the order of subseconds to seconds. All our experimental data reflects the averaged result from sampling the two ancilla frequencies.

$^#$The cross-Kerr $\chi_{\text{AR}}$ is derived from other measured parameters.

$^1$Cavity A has a distinctive switching behaviour between a regular state with stable $T_\text{coherence}$ (380 ± 25 $\mu$s) and occasional ‘bad periods’ lasting for 2–8 h where $T_\text{coherence}$ fluctuates wildly in the range of 200–340 $\mu$s. The reduced cavity coherence during these periods is not accompanied by any other changes of system parameters, and can be recovered by a Hahn echo pulse (with echo, cavity $T_{1\text{full}} = 390 \mu$s at all times). We exclude data during these unstable periods throughout the paper except for Extended Data Fig. 4 where its impact on AQEC performance is illustrated.
Using calibrated amplitudes and phases of the transmon comb and mixing comb in the experiment, the complex-valued PReSPA transition rates ($\lambda_n$ and $\Omega_n$) can be calculated based on equations (14) and (15). Here the transmon comb amplitude $\Lambda_n$ is calibrated from ancilla Rabi oscillations, and the amplitudes of mixing tones are approximately converted to the dimensionless displacement parameter $\xi_n$ by measuring the Stark shift $\Delta_{\text{Stark}}$ induced by that single tone: $\xi_n \approx \Delta_{\text{Stark}}/2\alpha_{\text{Stark}}$. The experimentally measured PReSPA transition rates result from fitting the time-domain dynamics of the photon addition processes (Fig. 2f, also see Supplementary Fig. 1 for further details), which do not contain phases. Note that $\Omega_n$ is intentionally set opposite to others in phase to suppress multi-tone mixing effects (see equations (14) and (15)) by destructive interference.

### Extended Data Table 2 | Comparison of calculated and measured PReSPA transition rates

| Index $n$ | Raw transmon amp. (a.u.) | $\xi_n$ (approx.) | $\Omega_n$ (kHz) (theory) | $|\Omega_n|$ (kHz) (fit) |
|-----------|--------------------------|------------------|---------------------------|--------------------------|
| 0 (|0⟩ to |1⟩) | -1.22 | -0.058 | -125 | 92 |
| 1 (|2⟩ to |3⟩) | 1.00 | 0.048 | 127 | 88 |
| 2 (|4⟩ to |5⟩) | 0.64 | 0.030 | 127 | 87 |
| 3 (|6⟩ to |7⟩) | 0.49 | 0.023 | 124 | 85 |

| Index $n$ | Raw mixing amp. (a.u.) | $\lambda_n$ (kHz) | $\lambda_n$ (kHz) (theory) | $|\lambda_n|$ (kHz) (fit) |
|-----------|--------------------------|-------------------|-----------------------------|--------------------------|
| 0 (|0⟩ to |1⟩) | -0.98 $e^{0.43i}$ | -21.4 $e^{-0.41i}$ | -27 $e^{-0.34i}$ | 28 |
| 1 (|2⟩ to |3⟩) | 1.52 $e^{0.06i}$ | 33.1 $e^{0.06i}$ | 28 $e^{0.06i}$ | 27 |
| 2 (|4⟩ to |5⟩) | 1.27 $e^{0.02i}$ | 26.7 $e^{0.02i}$ | 28 $e^{0.02i}$ | 27 |
| 3 (|6⟩ to |7⟩) | 1.14 $e^{0.35i}$ | 24.9 $e^{0.35i}$ | 27 $e^{0.34i}$ | 26 |
Extended Data Table 3 | Breakdown of the decoherence sources of the logical qubit under AQEC

| Source of Decoherence | Rate of occurrence | Longitudinal relaxation rate | Transverse relaxation rate | Rate scaling |
|-----------------------|--------------------|------------------------------|---------------------------|-------------|
| Unrelated to single-photon loss: |
| ancilla spuruous excitation $\gamma_T^\dagger$ | $1.8 \text{ ms}^{-1}$ | $1.9 \text{ ms}^{-1}$ | $1.8 \text{ ms}^{-1}$ | $< \gamma_m (\lambda/\chi_4)^2$ |
| off-resonant excitation by transmon comb | $0.2 \text{ ms}^{-1}$ | $0.3 \text{ ms}^{-1}$ | $0.2 \text{ ms}^{-1}$ | |
| other cavity dephasing | $0.3 \text{ ms}^{-1}$ | $0.3 \text{ ms}^{-1}$ | $0.3 \text{ ms}^{-1}$ | |
| Fail to correct single-photon loss: |
| ancilla relaxation $T_{1\alpha}$ | $7\% \cdot \langle \hat{n}/T_{1\alpha} \rangle$ | $0.5 \text{ ms}^{-1}$ | $0.5 \text{ ms}^{-1}$ | $1/(\gamma_0 T_{1\alpha}) \cdot \langle \hat{n}/T_{1\alpha} \rangle$ |
| 2nd photon decay | $6\% \cdot \langle \hat{n}/T_{1\alpha} \rangle$ | $0.7 \text{ ms}^{-1}$ | $0.4 \text{ ms}^{-1}$ | $\langle \hat{n}_{\text{even-odd}}/T_{1\alpha} \rangle \cdot \langle \hat{n}/T_{1\alpha} \rangle$ |
| incorrect pumping path by mixing comb | $3\% \cdot \langle \hat{n}/T_{1\alpha} \rangle$ | $0.2 \text{ ms}^{-1}$ | 
| $K$ & correction time uncertainty | $2\% \cdot \langle \hat{n}/T_{1\alpha} \rangle$ | $0.1 \text{ ms}^{-1}$ | 
| $\chi_A$ & correction time uncertainty | $1\% \cdot \langle \hat{n}/T_{1\alpha} \rangle$ | $0.1 \text{ ms}^{-1}$ | 
| $\chi_{AR}$ & correction time uncertainty | $< 0.5\% \cdot \langle \hat{n}/T_{1\alpha} \rangle$ | 
| intrinsic code word distortion | $2.5\% \cdot \langle \hat{n}/T_{1\alpha} \rangle$ | $0.2 \text{ ms}^{-1}$ | 
| other PReSPA phase errors | $\sim 2\% \cdot \langle \hat{n}/T_{1\alpha} \rangle$ | $\sim 0.1 \text{ ms}^{-1}$ | |
| Total | $2.8 \text{ ms}^{-1}$ | $3.8 \text{ ms}^{-1}$ | | |

See Supplementary Information section 3 for detailed discussions on all error channels. Some of the listed decoherence channels can be unambiguously associated with discrete quantum jumps, such as transmon $\gamma_T^\dagger$, 2nd photon loss and other cavity dephasing. Each event fully scrambles cavity photon phases and therefore translates to logical transverse relaxation at a 1:1 ratio. The bit-flip rate from the occurrence of double photon gain (about 50% of $\gamma_T^\dagger$ events, see Extended Data Fig. 6) and second-photon loss is converted to longitudinal relaxation rate via a factor of 2 by definition. Other decoherence channels are more continuous in nature, for which we define their effective rate of occurrence as the resultant logical transverse relaxation rate. They arise from various competing timescales in PReSPA such as dispersive shift against reservoir linewidth and dissipation rates against various higher-order nonlinearity. The scaling law for each contribution is also listed when applicable. $\gamma_m$ is the transmon decay rate via the reservoir in the presence of the mixing comb, approximately $0.35 \text{ ms}^{-1}$ in our experiment. Definition of other rates can be found in Extended Data Table 1 or in the main text. For intrinsic codeword distortion, see Methods section ‘Approximate AQEC and decoding unitary’. Other cavity dephasing effects are caused by thermal excitation in the reservoir mode and other cavity modes of the system. Other possible small PReSPA phase errors include imperfect matching of $\lambda$ and $\Omega$ for the four photon addition paths, and 2nd-order sensitivity to a low-frequency-noise problem present in our transmon ancilla (see the Extended Data Table 1 footnote).