Considerations on combined-function optics for high-energy storage rings and colliders

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Abstract
In a recent paper, the proposal of using combined-function optics for the arcs of the Future Circular hadron Collider (FCC–hh) was presented and discussed in detail. In this paper, further considerations are presented on the same topic, reflecting the progress made since the previous publication. The studies presented here focus mainly on two aspects. Firstly, the layout of the combined-function periodic cell is optimised with the goal of fixing the number of main magnets. Secondly, possible layouts of the dispersion suppressor in the framework of this novel proposal for the optics of the arcs of the FCC hadron collider.

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1 Introduction
It is customary to use a periodic structure based on the FODO layout for the arcs of modern synchrotrons, and this is also the case for the lattice of the Future Circular Hadron Collider (FCC–hh) [1]. A FODO cell is made of a sequence of dipole magnets interspersed with focusing and defocusing quadrupoles. In such a cell, the bending and focusing of the charged particles are provided by two distinct types of magnet. In a recent paper [2], the proposal to consider combined function (CF) optics for the periodic cell of the FCC–hh ring has been made and discussed in detail, highlighting several advantages over a separate function (SF) layout. The key aspect of the proposed solution is that in a CF cell, the magnets are simultaneously providing bending and focusing to the charged particles. Such an approach has been in use for the design of the first synchrotrons, such as the CERN Proton Synchrotron (PS) [3–8], the BNL Alternating Gradient Synchrotron (AGS) [9–11], and the world’s first hadron collider, the CERN Intersecting Storage Ring (ISR) [12]. On the other hand, the combined-function optics concept was abandoned with the advent of the more modern and high-energy storage rings and colliders, based on superconducting magnets. The reason of this change in the design paradigm of energy frontier storage rings and colliders can only be speculated. From the standpoint of magnet design, an SF optics simplifies the task of conceiving the needed hardware, as each magnet has a separate function and its design can be more easily optimised. From the standpoint of accelerator operations, it is simpler to operate a ring in which the controlling parameters are fully decoupled between each other. Furthermore, the technological challenges
linked with the design and operation of superconducting magnets might have suggested the prudent approach to separate the main magnetic fields. However, the ever-increasing strength of the dipolar magnetic field, needed for building future colliders at the energy frontier, calls for a serious reconsideration of an accelerator design based on CF magnets, given the several advantages of this approach and the existence of CF function magnets already in operation, e.g. at the J-PARC transfer line [13–15], or proposed, e.g. at the High-Luminosity LHC D2 separation dipole [16]. First, let us consider the dipole filling factor defined as

$$\chi_m = \frac{nL_m}{L_c},$$

where $L_c$ and $L_m$ stand for the cell length and magnetic length of the dipoles (for a FODO cell) or main magnets (for a CF cell), respectively, and $n$ is the number of magnets in the periodic cell. $\chi_m$ represents the fraction of the cell length filled with magnets generating a dipole field, which provides the bending strength, and such a fraction is increased in the CF cell with respect to the standard FODO cell.

The impact on the filling factor is an important asset, as the planned 16 T-dipoles of the FCC–hh ring [1] are beyond the current state of the art of magnet technology. Therefore, increasing $\chi_m$ is an efficient way to alleviate some of the requirements in the challenging FCC–hh magnet design.

We remark that the increase of filling factor is achieved by removing the quadrupoles that are present in the FODO cell, with the additional beneficial side effect that the arcs will be made of a single type of magnet, thus simplifying the magnet production.

It is also worth highlighting that, in the design of the FODO cell of modern particle accelerators [1, 17], several auxiliary magnets are installed close to the quadrupoles, as well as diagnostic devices such as beam position monitors. These ancillary magnets are intended to provide appropriate control of the closed orbit, transverse tunes, chromaticity, linear coupling, and amplitude detuning to generate Landau damping. The removal of main quadrupoles when moving from a FODO to a CF cell leaves open the question of how best to deploy these ancillary magnets in the cell, and this will be the topic of future studies.

An additional advantage of the CF periodic cell is that it features smaller $\beta$- and dispersion-functions [2] with respect to a FODO cell of the same length, which represents an interesting aspect. Due to this intrinsic property of a CF cell, one can consider increasing the length of the proposed CF periodic cell, which would have two positive side effects: it would reduce the strength of the quadrupolar component to be superimposed on the dipole field and it would also reduce the strength of the chromatic sextupoles required.

In this paper, two fundamental aspects of possible CF optics for the FCC–hh ring are presented and discussed in detail, namely, the optimisation of the periodic cell length and a possible layout of the dispersion suppressor, i.e. the part of the ring lattice that is used to bring the dispersion function from the value it has in the arc to zero, as required in the straight sections.

2 Optimisation of the length of the combined-function periodic cell
The natural parameter to optimise in the layout of the CF cell is its length, or, equivalently, the number of main magnets. The LHC FODO cell is characterised by six dipoles
and two quadrupoles with a length of about 107 m [17], whereas the FCC–hh FODO cell features twelve dipoles and two quadrupoles for a total length of approximately 213 m [1]. When considering the difference in injection energy between the two machines, the transverse normalised emittance (the same for both machines), the linear scaling of the $\beta$-function with the cell length, one obtains $\sigma_{\text{LHC}} / \sigma_{\text{FCC–hh}} \approx 1.9$. This indicates clearly that there is a certain margin for increasing the FCC–hh cell length without impacting on the beam aperture requirements, as we assume the same mechanical aperture for both types of magnet.

The baseline layout of the proposed CF cell for FCC–hh is shown in Fig. 1 (bottom) together with a sketch of a standard FODO-cell layout (top).

Several variants of the CF cell layout have been considered, differing by the number of main magnets, which needs to be a multiple of four to respect the symmetry of the layout. Various optical parameters have been considered in the study, together with the maximum beam size and the beam aperture. The available beam aperture is a key figure of merit for assessing whether a given cell layout is acceptable. For the calculation of the beam aperture, the nominal shape of the proposed FCC–hh beam screen has been considered, which is shown in Fig. 2, and the standard approach has been used for the calculation of the aperture and to determine the target value set in Ref. [18] has been used.
We remark that the determination of the available beam aperture is not a plain conversion of the nominal mechanical dimensions of the beam screen in terms of beam sigma. Rather, the mechanical aperture is reduced by an amount that depends on the tolerances in the fabrication of the beam screen and on the overall alignment tolerances of the magnet in which the beam screen is installed. Tolerances on the absolute value of the beam orbit are also taken into account. Furthermore, the beam size is estimated by taking into account variations of the optical parameters due to beta- and dispersion-beating.

The computation of the optical parameters and the aperture estimates have been carried out using the MAD-X code [20], which is one of the standard codes for accelerator design. The results of these simulations are shown in Fig. 3 where various optical parameters are plotted as a function of the cell length for the different configurations considered. The crucial plot is the one showing the available beam aperture, as it provides the correct criterion to define the best cell length. In fact, for the case of a CF cell with 16 main magnets, corresponding to a cell length of about 260 m, the available aperture exactly matches the target value specified in Ref. [18], so that this seems to be the optimal layout. We remark that $\beta_{\text{max}}$ and $D_{\text{max}}$ are slightly above the corresponding values for the nominal FODO cell of the FCC–hh, but this is not a source of concern given that the beam aperture matches its target value.

![Figure 3](image-url)

**Figure 3** Top left: The maximum value of the $\beta$ and dispersion function over the periodic cell are shown as a function of the cell length. The horizontal dashed lines represent the corresponding value of the maximum $\beta$ and dispersion function for the nominal FODO cell of the FCC–hh ring. Top right: The maximum of the beam size over the periodic cell and available aperture are shown as a function of the cell length. The horizontal dashed lines represent the corresponding value of the maximum beam size and the available aperture for the nominal FODO cell of the FCC–hh ring. The target value for the aperture is also shown (dotted-dashed line). Bottom left: The quadrupolar and sextupolar gradients are shown as a function of the cell length. The horizontal dashed line represents the corresponding value of the sextupolar gradient for the nominal FODO cell of the FCC–hh ring. Bottom right: The longitudinal parameters ($\gamma_{\text{tr}}$, $\eta$) are shown as a function of the cell length. The horizontal dashed lines represent the corresponding value of $\gamma_{\text{tr}}$ and $\eta$ for the nominal FODO cell of the FCC–hh ring. In all plots, the number of CF magnets is also reported (the nominal FODO cell of the FCC–hh ring is based on twelve dipoles).
The increase of the CF cell length has a critical and very positive side effect that can be observed in Fig. 3 (bottom-left and bottom-right). First, an increase in $|\eta|$ has a beneficial impact on the threshold of some instabilities [21]. Secondly, the required strength of the chromatic sextupoles (used to set the chromaticity to $Q' = 10$, which is the target for the operational value in collision based on the LHC experience [22]) is about a factor of two smaller than in the case of the FCC–hh nominal lattice. This has additional positive side effects, since the longitudinal space allocated for the chromatic sextupoles can be reduced. Furthermore, the required strength of the quadrupolar component to be generated by the CF main magnets is considerably reduced with respect to the estimate presented in Ref. [2] and this has vital consequences on the performance reach of the optimised CF cell.

We recall that the superconducting CF magnet is limited by the combination of the current density and peak field in the coil. Hence, adding a quadrupolar component to a dipolar one increases the peak field $B_p$ in the coil by an amount that is proportional to the field gradient $G$ times the aperture radius $r$:

$$\Delta B_p = \lambda G r,$$

where the parameter $\lambda$ depends on the ratio between the coil width $w$ and the aperture radius, as discussed in Ref. [23]. A reasonable estimate for a CF magnet with the typical parameters needed for the FCC–hh lattice is $\lambda = 1.15$, which gives $\Delta B_p \approx 0.73$ T, and the available dipole field is only 15.27 T, instead of 16 T for the dipoles of an SF lattice. At this point, the CF option seems less advantageous, but the filling factor brings the key contribution. For the SF solution $\chi_m \approx 0.8$, while $\chi_m \approx 0.9$ for the CF layout. Finally, the effective dipole field $B_e = B \times \chi_m$ is higher for the CF solution, as $B_e^{\text{CF}} \approx 13.7$ T and $B_e^{\text{SF}} \approx 12.8$ T, with an increase of $\approx 7\%$. This margin, provided by the CF optics, can be used to increase the beam energy, keeping the tunnel length constant, or to reduce the tunnel length, keeping the beam energy constant.

### 3 Layout of a dispersion suppressor for CF optics

The dispersion suppressor is an essential component of the magnetic lattice of a modern collider or storage ring. It is used to control the dispersion function $D_x$ and its derivative $D'_x$ at the end of the arc so as to fulfil the condition $D_x = D'_x = 0$ at the beginning of the straight section. It is worth stressing that none of the existing particle accelerators based on the CF concept feature straight insertions with a corresponding dispersion suppressor. This is particularly surprising for the world’s first collider, the ISR, which did not feature any dispersion suppressor [24] due to the very short straight sections. Therefore, a detailed study of the feasibility of a dispersion suppressor for a regular CF cell is very pertinent.

One of the features that makes the CF concept appealing is that the arcs are made of a single type of CF magnet that comes with two possible values of the quadrupolar component. This is a great advantage in terms of industrial production of the main magnets. This advantage should not be lost with the need for a special version of the main magnet for the dispersion suppressor, either in terms of the dipole strength or of the quadrupolar component. Therefore, the main constraint in the design of the dispersion suppressor to be combined with a CF cell is to use the same main magnets as those of the arc cell. We remark, however, that the fact that individual quadrupole magnets may be required is not an issue given that stand-alone quadrupole magnets are in any case needed to define
the optics in the straight sections of the ring. Therefore, the only constraint is to ensure that the quadrupole magnets needed for the design of the dispersion suppressor are of the same type as those to be used in the straight sections in order to minimise the number of hardware variants. However, at the current level of study, one can deviate from this restriction.

There are different techniques for designing dispersion suppressors for circular accelerators. The standard approach consists of using the same FODO cell as that used in the arcs, which retains the same focusing properties, while the strength of the dipoles present in the dispersion suppressor cells is reduced. A comprehensive review of the available design options can be found in Ref. [25]. The various design alternatives depend on two parameters, namely, the cell phase advance and the dipole field level in the dispersion suppressor cell. We note that some very elegant solutions are based on the so-called missing dipole design, which is also the solution adopted for the LHC dispersion suppressor [17].

It is obvious that the standard design choices are intrinsically linked to the assumption that the optics is based on the SF paradigm, which implies the possibility of disentangling the focusing and bending functions of the magnets in each regular cell. Such a decoupling property is clearly lost in the CF paradigm; hence, the difficulties in the design of a dispersion suppressor.

The starting point of this study is the development of a dispersion suppressor based on half-field dipoles [25], which consists of two FODO cells with 90° phase advance and dipoles with a strength that is half that of the dipoles in the arc. This can be easily adapted to the optimised CF cell discussed in the previous section that features the appropriate phase advance. This layout is shown in Fig. 4(a) in which the main CF magnets have the same focusing strength as in the regular CF cell, with the bending angle halved relative to the regular CF cell. The starting point is a dispersion suppressor of about 550 m but is not compatible with the design criterion set above, as it does not use the same main magnets of the periodic CF cell in the arc, requiring dipoles with half the strength.

The first variant is based on the use of standalone quadrupoles to provide regular transverse focusing, keeping the phase advance constant at 90°. The number of CF main magnets is halved, so that the bending strength of each main magnet can be set to the nominal value used in the CF cell in the arc, while the integral field is halved. This provides a dispersion suppressor that is fully compatible with the constraints set above and is shorter than the starting design. The main parameters of this layout are shown in Fig. 4(b).

These two layouts are based on a concept that relies on two cells, but it can be further optimised to use only one and a half cells, thus shortening the layout even further. The optical parameters are shown in Fig. 4(c), where it is clearly seen that the beta-functions still retain the rather regular behaviour that they have in the arc, apart from in the last section of the dispersion suppressor.

The last three layouts of Fig. 4 are shorter variants of the one and a half cell design. For these three designs, the beta-functions have lost their regular character, at least in the second part of the dispersion suppressor. In Fig. 4(d) the last block of the eight main CF magnets is divided into two, and one block is located at the beginning of the dispersion suppressor, thus granting a length reduction relative to the previous design. Finally, in the two layouts shown in Figs. 4(e) and (f) the last magnet from the group of four main CF magnets is dropped at the end of the dispersion suppressor. The length of these dispersion suppressors is approximately 330 m, which represents a substantial improvement
Figure 4 Possible layouts of dispersion suppressor sections based on CF main magnets and standard quadrupoles to be used in combination with the optimised CF cell layout presented in the previous section. The evolution of the optical parameters and dispersion function as a function of $s$ is shown, together with a sketch of each layout on top of the plot: CF main magnets (rectangles centred around the middle line) and quadrupoles (rectangle above or below the centre line to represent their polarity) are shown. The layout (a) is the starting point, made of CF main magnets with half of the bending field of the magnets in the regular CF cell. All other cases refer to variants with CF main magnet with the same properties of those in the CF cell. The length of the dispersion suppressor, given by the condition $D_x = D'_x = 0$, decreases for the different variants over the starting design. Furthermore, one should consider that the length of the nominal dispersion suppressor of the FCC–hh lattice is about 301 m, which indicates that the studied solutions are indeed suitable for a realistic lattice design based on CF optics. The final choice between the last two layouts presented is a matter of finding which one is best adapted to the optics of the straight section downstream.

As a last point, we would like to stress that all the layouts of the dispersion suppressors presented here feature empty drift sections, which could be efficiently used to install collimators to improve the overall cleaning performance of the collimation system, particularly for the case of single diffractive events, which is an aspect considered in the upgrade of the collimation system for the HL–LHC [16] and is already part of the collimation system of the FCC–hh ring [1, 26].

4 Conclusions
In this paper, the optimisation of the layout of a combined-function periodic cell for the CERN FCC–hh lattice has been presented and discussed in detail. The number of main magnets, and hence, the overall cell length have been increased, which generates a beneficial reduction in the required strength of chromatic sextupoles and, much more importantly, an important reduction in the quadrupolar component needed to generate the transverse optics. All these positive consequences have been obtained fulfilling the standard beam aperture requirements. Thanks to this optimisation, the proposed combined-function cell provides 7% more integrated dipole field with respect to the nominal FODO cell of the FCC–hh lattice. Such a margin can be used to either change the value of the
nominal beam energy at constant ring length or to vary the ring circumference at constant beam energy, depending on the needs.

A second fundamental result presented comes from the study of possible layouts for the dispersion suppression of a circular accelerator based on combined function optics. Several designs have been found that are compatible with the constraint of being made-up of the same main magnets as the regular cell. Furthermore, it has been possible to optimise them to achieve a total length similar to the dispersion suppressor of the nominal FODO lattice of the FCC–hh. This provides another key element for the feasibility of a combined-function optics solution for a collider at the high-energy frontier.

The next step in the process of proposing a realistic design for a combined-function cell for the FCC–hh ring consists of studying the magnet systems needed to control the various aspects of the beam dynamics. The goal will be to define the layout of diagnostic devices such as beam position monitors as well as magnetic correctors to control the closed orbit, transverse tunes, linear coupling, chromaticities, non-linear magnetic imperfections, and to provide Landau damping. The definition of these magnetic systems will allow the final length of the optimised CF cell to be determined. Finally, the interaction between radiation effects, which are not negligible in the FCC–hh, and the proposed CF optics should be carefully assessed.

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Abbreviations
FCC–hh, Future Circular Hadron Collider; FODO, Focusing Off Defocusing Off; CF, combined-function; SF, Separate-function; PS, Proton Synchrotron; AGS, Alternating Gradient Synchrotron; ISR, Intersecting Storage Ring; LHC, Large Hadron Collider; HL-LHC, High-luminosity LHC.

Availability of data and materials
Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Declarations

Competing interests
The author declares that they have no competing interests.

Authors’ contributions
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