BHWIDE 1.00: $\mathcal{O}(\alpha)$ YFS Exponentiated Monte Carlo for Bhabha Scattering at Wide Angles for LEP1/SLC and LEP2†

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Abstract

We present $\mathcal{O}(\alpha)$ YFS exponentiated results for wide angle Bhabha scattering at LEP/SLC energies using a new Monte Carlo event generator BHWIDE 1.xx. Our calculations include two options for the pure weak corrections, as presented in Beenakker et al. and in Böhm et al. From comparison with the results of Beenakker et al., Montagna et al. and Cacciari et al., we conclude that the total precision of our BHWIDE results is 0.3%(0.5%) in the LEP1/SLC regime within $\pm100$ MeV ($\pm2.75/\pm2.5$ GeV) of the Z peak. For LEP2, the corresponding precision is currently estimated at 1.5%; the latter could be improved if the data in LEP2 so require. Both precision tags represent clear improvements over what is currently available in the literature.

Submitted to Physics Letters

† Work supported in part by the US DoE contract DE-FG05-91ER40627, Polish Government grants KBN 2P30225206, 2P03B17210 and Polish-French Collaboration within IN2P3.  
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UTHEP-95-1001  
Oct. 1995
1 Introduction

As the final LEP1 data analysis begins and the initial stages of LEP2 materialize while the SLD prepares for the beginning of what may be its final phase, the subject of precision calculations of wide angle Bhabha scattering becomes more and more interesting, particularly from the standpoint of realizing such calculations via a Monte Carlo event generator which would allow comparison between theory and experiment at the level of events in the presence of arbitrary detector cuts. Indeed, currently, the work of Beenakker et al. [1] and of Montagna et al. [2] represent the state of the art in the theoretical arena of the prospective calculations, and neither of these approaches has provided a genuine Monte Carlo event generator to allow comparison with arbitrarily cut experimental data. This has had the effect that the important Z physics parameter $\Gamma_{ee}$ has been extracted from the data using a subtraction of the contamination from the t-channel $\gamma$-exchange in the wide angle acceptance relevant to its measurement. A Monte Carlo event generator with sufficient precision would obviate the need for this procedure, for example. Motivated by this and other such Z and LEP2 applications, we have developed an $\mathcal{O}(\alpha)$ Yennie-Frautschi-Suura (YFS) [3] exponentiated Monte Carlo event generator for wide angle Bhabha scattering at LEP1/SLC and LEP2 energies. We present this development in what follows.

Specifically, our starting point will be the $\mathcal{O}(\alpha)$ YFS exponentiated Monte Carlo (MC) event generator BHLUMI 1.xx developed by two of us (S.J. and B.F.L.W.) in ref. [4] for low angle Bhabha scattering in the SLC/LEP luminosity regime $17 \text{ mrad} < \theta_{e,e} < 150 \text{ mrad}$, where $\theta_{e,e}$ are the respective $e, \bar{e}$ CMS scattering angles. In order to arrive at an event generator valid for wide angles at both LEP1/SLC and at LEP2 to sufficient accuracy, we have had to introduce the effects of the $Z$ exchange graphs into the calculations presented in ref. [4] and we have had to introduce the effects of the pure weak one-loop corrections into those calculations as well. We stress that we have made contact with the pioneering $\mathcal{O}(\alpha)$ MC program BABAMC of refs. [5, 6] as well as with the semi-analytical program ALIBABA of ref. [1] in that our pure weak corrections libraries are taken from these two cases: the user of our new wide angle Bhabha MC, BHWIDE 1.xx, may choose which library he uses and we shall discuss both of them in what follows. Our exact hard bremsstrahlung amplitude, in the presence of the full set of s-channel and t-channel $\gamma$ and $Z$ exchanges, we shall compute explicitly in what follows, using methods of the CALKUL-type [7], as formulated by Xu et al. [9]. It is in this way that we have arrived at our new $\mathcal{O}(\alpha)$ YFS exponentiated MC event generator BHWIDE 1.xx which simulates realistic multiple photon radiative effects for wide angle Bhabha scattering in the LEP1/SLC and LEP2 energy regimes.

We have compared our results with those of BABAMC, of the Monte Carlo integrator program SABSPV of ref. [10], of the semi-analytical program TOPAZ0 of refs. [11, 12], of ALIBABA, of the MC event generator BHAGENE3 of refs. [13, 14], of the MC event generator BHAGEN95 of refs. [15, 16, 17, 18], and of the MC event generator UNIBAB of ref. [19]. On this basis, we have arrived at a reliable estimate of the total precision of our

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1 We note that Kleiss and Stirling [8] have introduced analogous methods as well.
The applications of the type we pursue here, and where we note the usual connections representation is not readily available in the current literature and is essential for practical results. The basis of this estimate is presented in detail in ref. [21] and we will review only its main features and the main features of the aforementioned attendant comparisons in what follows.

More precisely, our BHLUMI 1.xx Monte Carlo event generator realizes the process

\[ e^+(p_1) + e^-(q_1) \rightarrow e^+(p_2) + e^-(q_2) + \gamma_1(k_1) + \ldots + \gamma_n(k_n) \]  

via the YFS exponentiated cross section formula

\[ d\sigma = e^{2\alpha Re B+2\alpha \bar{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j^0} \int \frac{d^4y}{(2\pi)^4} e^{ig(p_1+q_1-p_2-q_2-\sum_j k_j)+D} \frac{\tilde{\beta}_n(k_1,\ldots,k_n)d^3p_2d^3q_2}{p_2^0q_2^0}, \]  

where the real infrared function \( \tilde{B} \) and the virtual infrared function \( B \) are given in refs. [4, 21, 22, 23] (for definiteness, we record them in the form we shall use presently, as this representation is not readily available in the current literature and is essential for practical applications of the type we pursue here), and where we note the usual connections

\[ 2\alpha \tilde{B} = \int^{k_{max}} d^3k \frac{\tilde{S}(k)}{k_0}, \]

\[ D = \int d^3k \frac{\tilde{S}(k)}{k_0} \left( e^{-iyk} - \theta(K_{max} - k) \right) \]  

for the standard YFS infrared emission factor

\[ \tilde{S}(k) = \frac{\alpha}{4\pi^2} \left[ Q_f Q_{f'} \left( \frac{p_1}{p_1 \cdot k} - \frac{q_1}{q_1 \cdot k} \right)^2 + \ldots \right], \]  

if \( Q_f \) is the electric charge of \( f \) in units of the positron charge. Here, the “…” represent the remaining terms in \( S(k) \) obtained from the one given by respective substitutions of \( Q_f, p_1, Q_{f'}, q_1 \) with corresponding values for the other pairs of the respective external charged legs according to the YFS prescription in ref. [3] (wherein due attention is taken to obtain the correct relative sign of each of the terms in \( S(k) \) according to this latter prescription) and in refs. [24, 25], \( f \neq e, f' = \bar{f} \). We have explicitly the representations

\[ 2\alpha R\bar{R}(p_1,q_1,p_2,q_2) + 2\alpha \tilde{B}(p_1,q_1,p_2,q_2;k_m) = R_1(p_1,q_1;k_m) + R_1(p_2,q_2;k_m) + R_2(p_1,p_2;k_m) + R_2(q_1,q_2;k_m) - R_2(p_1,q_2;k_m) - R_2(q_1,p_2;k_m) \]  

with

\[ R_1(p,q;k_m) = R_2(p,q;k_m) + \left( \frac{\alpha}{\pi} \right)^2 \frac{\pi^2}{2}, \]  

\[ R_2(p,q;k_m) = \frac{\alpha}{\pi} \left\{ \ln \frac{2pq}{m_e^2} - 1 \right\} \ln \frac{k_m^2}{p^0q^0} + \frac{1}{2} \ln \frac{2pq}{m_e^2} - \frac{1}{2} \ln \frac{q^0}{p^0} - \frac{1}{4} \ln \frac{(\Delta + \delta)^2}{4p^0q^0} \]  

\[ \]
\[- \frac{1}{4} \ln^2 \frac{(\Delta - \delta)^2}{4p^0q^0} - \Re Li_2 \left( \frac{\Delta + \omega}{\Delta + \delta} \right) - \Re Li_2 \left( \frac{\Delta + \omega}{\Delta - \delta} \right) - \Re Li_2 \left( \frac{\Delta - \omega}{\Delta + \delta} \right) - \Re Li_2 \left( \frac{\Delta - \omega}{\Delta - \delta} \right) + \frac{\pi^2}{3} - 1 \right] , \tag{7} \]

where \( \Delta = \sqrt{2pq + (p^0 - q^0)^2} \), \( \omega = p^0 + q^0 \), \( \delta = p^0 - q^0 \), and \( k_m \) is a soft photon cut-off in the CMS \( (E^\text{soft}_\gamma < k_m < E^\text{beam}) \).

The YFS hard photon residuals \( \bar{\beta}_i \) in \( (2) \), \( i = 0, 1 \), are given in ref. \cite{4} for BHLUMI 1.xx so that this latter event generator calculates the YFS exponentiated exact \( \mathcal{O}(\alpha) \) cross section for \( e^+e^- \rightarrow e\bar{e} + n(\gamma) \) with multiple initial, initial-final, and final state radiation using a corresponding Monte Carlo realization of \( (2) \) in the low angle regime of the SLC/LEP luminosity monitor acceptance, where the \( e^\pm \) scattering angles \( \theta_{e^\pm} \) satisfy \( 17 \text{ mrad} < \theta_{e^\pm} < 150 \text{ mrad} \). In the next sections, we use explicit Feynman diagrammatic methods and the results in refs. \cite{1, 5, 6} to develop the corresponding Monte Carlo realization of the respective application of \( (2) \) to the wide angle scattering regime, \( \theta_{e^\pm} \geq 150 \text{ mrad} \), for the Bhabha process \( e^+e^- \rightarrow e^+e^- + n(\gamma) \).

We turn first to the implementation of the required wide angle physics effects in the YFS hard photon residual \( \bar{\beta}_0 \) through \( \mathcal{O}(\alpha) \). We follow this discussion with the corresponding analysis for the wide angle physics effects in the hard photon YFS residual \( \bar{\beta}_1 \). In this way, we arrive at an exact \( \mathcal{O}(\alpha) \) YFS exponentiated MC event generator valid for wide angle Bhabha scattering in the LEP1/SLC and LEP2 energy regimes.

2 Born and \( \mathcal{O}(\alpha) \) contributions to \( \bar{\beta}_0 \) for wide angle Bhabha scattering at high energies

In this section we develop the Born and \( \mathcal{O}(\alpha) \) contributions to the YFS hard photon residual \( \bar{\beta}_0 \) for the wide angle Bhabha scattering process. We use the low angle limit of the residual already presented in ref. \cite{4} as a starting point and as a limiting case cross-check.

Specifically, the hard photon residual \( \bar{\beta}_0 \) is defined as follows through \( \mathcal{O}(\alpha) \):

\[
\frac{1}{2} \bar{\beta}_0 = \frac{d\sigma^{1\text{-loop}}}{d\Omega} - 2\alpha \Re \frac{d\sigma^{\text{Born}}}{d\Omega} , \tag{8} \]

where \( d\sigma^{1\text{-loop}}/d\Omega \) is the differential cross section for wide angle Bhabha scattering computed through the 1-loop correction and \( d\sigma^{\text{Born}}/d\Omega \) is the respective Born differential cross section. In ref. \cite{4}, the right-hand side of \( (8) \) was computed in the low angle regime. Here, we need to compute \( (8) \) in the wide angle scattering regime. We do this as follows. First, we note that the exact expression for the wide angle scattering Born differential cross section in \( (8) \) is well-known

\[
\frac{d\sigma^{\text{Born}}}{d\Omega} = \frac{1}{64\pi^2s} |\mathcal{M}_0|^2 , \tag{9} \]

3
where $|\mathcal{M}_0|^2$ is a spin averaged lowest order matrix element squared, given e.g. in eq. (22).

Secondly, the complete 1-loop corrected wide angle Bhabha scattering differential cross section in (8) is also known and we use two different versions of it in our work, one taken from ref. [5] and one taken from ref. [1], where it is generally accepted that the latter version, which is the more recent of the two, is in fact the more up-to-date of the two [20] (and hence it is the option which we illustrate in our comparisons in Sect. 4). In our Monte Carlo event generator these two versions for $d\sigma^{1\text{-loop}}/d\Omega$ correspond to two choices for our electroweak library module, one of which the user specifies in his input file [26]. It is this way that we have realized the wide angle Bhabha scattering YFS hard photon residual $\tilde{\beta}_0$ through $\mathcal{O}(\alpha)$.

We turn next to the wide angle Bhabha scattering YFS hard photon residual $\tilde{\beta}_1$. This we do in the following section.

### 3 The YFS hard photon residual $\tilde{\beta}_1$ for wide angle Bhabha scattering at high energies

In this section we present our determination of the $\mathcal{O}(\alpha)$ YFS hard photon residual $\tilde{\beta}_1$ for wide angle Bhabha scattering at high energies. We start with the defining equation for $\tilde{\beta}_1$.

The hard photon residual $\tilde{\beta}_1$, to $\mathcal{O}(\alpha)$, is defined as follows

$$
\frac{1}{2} \tilde{\beta}_1 = \frac{d\sigma^{B_1}}{kdkd\Omega_\gamma d\Omega} - \tilde{S}(k) \frac{d\sigma_{\text{Born}}}{d\Omega},
$$

where $d\sigma^{B_1}/kdkd\Omega_\gamma d\Omega$ is the respective $\mathcal{O}(\alpha)$ bremsstrahlung differential cross section into the solid angles $d\Omega_\gamma$ and $d\Omega$ for the photon and positron respectively when the photon energy lies between $k$ and $k + dk$. Thus, to specify $\tilde{\beta}_1$, we need to specify the hard bremsstrahlung differential cross section on the RHS of (10). We now turn to this.

Specifically, the hard bremsstrahlung differential cross section required in (10) can be obtained via the standard methods from the corresponding helicity amplitudes $\mathcal{M}(\lambda_{e^+}, \lambda_{e^-}, \lambda'_{e^+}, \lambda'_{e^-}, \lambda\gamma)$, where $\lambda'_f$, $f = e^+, e^-$ is the incoming (outgoing) fermion helicity and $\lambda\gamma$ is the photon helicity. We have

$$
d\sigma^{B_1}/kdkd\Omega_\gamma d\Omega = \frac{1}{512\pi^5 s} \frac{(p_0^B)^2}{2E_b(E_b - k)} |\mathcal{M}|^2,
$$

with

$$
|\mathcal{M}|^2 = \frac{1}{4} \sum_{i=e^\pm, \gamma, j=e^\pm} |\mathcal{M}(\lambda_{e^+}, \lambda_{e^-}, \lambda'_{e^+}, \lambda'_{e^-}, \lambda\gamma)|^2
$$

being the spin averaged squared matrix element. The formula (11) is given in the CMS of the incoming beams, where $E_b$ is the beam energy. Thus, our determination of $\tilde{\beta}_1$ will
be completely specified when we give our results for $\mathcal{M}(\lambda_{\epsilon^+}, \lambda_{\epsilon^-}, \lambda'_{\epsilon^+}, \lambda'_{\epsilon^-}, \lambda_\gamma)$. We now turn to this.

More precisely, using the methods of ref. [9], we get the following representation for the non-vanishing helicity amplitudes:

\[
\begin{align*}
M_1 &\equiv M(++++) = -c s F_1 \left[ \frac{1}{t'_p} R(t'_p, a_L a_R) G_1 + \frac{1}{t'_q} R(t'_q, a_L a_R) G_2 \right] \\
M_2 &\equiv M(++++) = -c s' F'_2 \left[ \frac{1}{s'} R(s', a_L a_R) G'_1 + \frac{1}{s} R(s, a_L a_R) G'_2 \right] \\
M_3 &\equiv M(----) = -c s' F'_2 \left[ \frac{1}{t'_p} R(t'_p, a_L a_R) G_1 + \frac{1}{t'_q} R(t'_q, a_L a_R) G_2 \right] \\
M_4 &\equiv M(----) = -c s F_1 \left[ \frac{1}{s'} R(s', a_L a_R) G'_1 + \frac{1}{s} R(s, a_L a_R) G'_2 \right] \\
M_5 &\equiv M(+-+-) = -c t_q F_4 \left[ \frac{1}{s'} R(s', a_L a_R) G_3 + \frac{1}{s} R(s, a_L a_R) G_4 \right] \\
M_6 &\equiv M(+-+-) = -c t_p F_3 \left[ \frac{1}{s} R(s, a_L a_R) G_3 + \frac{1}{s'} R(s', a_L a_R) G'_4 \right] \\
M_7 &\equiv M(+-+-) = -c t_p F_3 \left[ \frac{1}{s} R(s, a_L a_R) G_3 + \frac{1}{s'} R(s', a_L a_R) G'_4 \right] \\
M_8 &\equiv M(+-+-) = -c t_q F_4 \left[ \frac{1}{s'} R(s', a_L a_R) G'_3 + \frac{1}{s} R(s, a_L a_R) G'_4 \right] \\
M_9 &\equiv M(+-+-) = +c u F_5 \left[ \frac{1}{t'_p} R(t'_p, a^2_L) G_1 + \frac{1}{t'_q} R(t'_q, a^2_L) G_2 \\
&\quad+ \frac{1}{s'} R(s', a^2_L) G_3 + \frac{1}{s} R(s, a^2_L) G_4 \right] \\
M_{10} &\equiv M(+-+-) = +c u' F'_6 \left[ \frac{1}{t'_p} R(t'_p, a^2_L) G'_1 + \frac{1}{t'_q} R(t'_q, a^2_L) G'_2 \\
&\quad+ \frac{1}{s'} R(s', a^2_L) G'_3 + \frac{1}{s} R(s, a^2_L) G'_4 \right] \\
M_{11} &\equiv M(+-+-) = +c u' F'_6 \left[ \frac{1}{t'_p} R(t'_p, a^2_R) G'_1 + \frac{1}{t'_q} R(t'_q, a^2_R) G'_2 \\
&\quad+ \frac{1}{s'} R(s', a^2_R) G'_3 + \frac{1}{s} R(s, a^2_R) G'_4 \right] \\
M_{12} &\equiv M(+-+-) = +c u F'_5 \left[ \frac{1}{s'} R(s', a^2_R) G'_3 + \frac{1}{s} R(s, a^2_R) G'_4 \right] \\
&\quad+ \frac{1}{t'_p} R(t'_p, a^2_R) G'_1 + \frac{1}{t'_q} R(t'_q, a^2_R) G'_2 \\
&\quad+ \frac{1}{s'} R(s', a^2_R) G'_3 + \frac{1}{s} R(s, a^2_R) G'_4 \right] \\
\end{align*}
\]

where our notation is as follows
• Invariants:

\[
s = 2p_1q_1, \quad s' = 2p_2q_2, \quad t_p = -2p_1p_2, \quad t_q = -2q_1q_2, \quad u = -2p_1q_2, \quad u' = -2q_1p_2. \quad (14)
\]

• Electroweak couplings:

\[
a_L = \frac{G}{e} (v_e + a_e), \quad a_R = \frac{G}{e} (v_e - a_e), \quad (15)
\]

where

\[
G = \frac{1}{\sin \theta_W \cos \theta_W}, \quad a_e = \frac{-1}{4 \sin \theta_W \cos \theta_W}, \quad v_e = a_e (1 - 4 \sin^2 \theta_W). \quad (16)
\]

• Spinor functions:

\[
F(p, q) = \frac{\langle pq \rangle}{\langle pq \rangle^*}, \quad G(p, q, r, s; k) = \frac{\langle pq \rangle^*}{\langle rk \rangle \langle ks \rangle}, \quad (17)
\]

where \( p, q, r, s \) denote fermion four-momenta in the massless limit, while \( k \) denotes the photon four-momentum, and the \( * \) stands for complex conjugation. Spinor products \( \langle .. \rangle \) are defined as

\[
\langle pq \rangle = \sqrt{p_+ q_+ e^{i\phi_p}} - \sqrt{p_- q_- e^{i\phi_q}}, \quad (18)
\]

where \( p\pm = p_0 \pm p_z, p_\perp = p_x + ip_y = \sqrt{p_+ p_-} e^{i\phi_p} \).

• Propagator factor:

\[
R(x, y) = 1 + \frac{xy}{x - M_Z^2 + i\theta(x)M_Z\Gamma_Z}. \quad (19)
\]

• Specific notation:

\[
F_1 = F(p_1, q_1), \quad F_2 = F(p_2, q_2), \quad F_3 = F(p_1, p_2), \quad F_4 = F(q_1, q_2), \quad F_5 = F(p_1, q_2), \quad F_6 = F(q_1, p_2),
\]
\[
G_1 = G(p_1, p_2, q_1, q_2; k), \quad G_2 = G(q_2, q_1, p_1, p_2; k), \quad G_3 = G(q_2, p_2, p_1, q_1; k), \quad G_4 = G(p_1, q_1, p_2, q_2; k),
\]
\[
c = i2\sqrt{2}e^3. \quad (20)
\]
The above helicity amplitudes have been obtained in the massless fermion approximation. However, to get a precise description of the photon radiation over the whole phase space (particularly for collinear configurations) the finite fermion masses have to be taken into account. This can be accomplished by adding to the matrix element $|M|^2$ of eq. (12) the mass correction term 

$$
\delta |M|^2_{mc} = -e^2 \left[ \frac{m_e^2}{(kp_1)^2} |M_0(s', t_q, u')|^2 + \frac{m_e^2}{(kq_1)^2} |M_0(s', t_p, u')|^2 
+ \frac{m_e^2}{(kp_2)^2} |M_0(s, t_q, u)|^2 + \frac{m_e^2}{(kq_2)^2} |M_0(s, t_p, u')|^2 \right],
$$

(21)

where

$$
|M_0(s, t, u)|^2 = e^4 \left\{ \frac{1}{s^2} \left[ |R(s, a_L^2)|^2 u^2 + |R(s, a_R^2)|^2 u^2 + 2 |R(s, a_L a_R)|^2 t^2 \right] 
+ \frac{1}{t^2} \left[ |R(t, a_L^2)|^2 u^2 + |R(t, a_R^2)|^2 u^2 + 2 |R(t, a_L a_R)|^2 s^2 \right] 
+ \frac{1}{s t} 2 \Re \left[ R^*(s, a_L^2) R(t, a_R^2) + R^*(s, a_R^2) R(t, a_L^2) \right] \right\}
$$

(22)

is the lowest order matrix element.

We have also used the results of ref. [7] for the matrix element $|M|^2$ as a cross check and we have found that the two sets of results are in very good agreement with one another, well below the desired technical precision of 0.01% of our analysis for example. We also note that an equivalent representation of the results in ref. [7] has been given by Kleiss in ref. [27]. On introducing these two sets of results into the formula (10) for $\beta_1$ and implementing the resulting expression into our YFS Monte Carlo program for Bhabha scattering, we arrive at the Monte Carlo event generator BHWIDE 1.00 for wide angle Bhabha scattering at high energies

We will now illustrate the application of BHWIDE 1.00 to LEP1/SLC and LEP2 physics scenarios in the next section.

## 4 Results and comparisons

In this section, we present sample Monte Carlo data which we use to compare our predictions from BHWIDE 1.00 to those of related calculations as reported in ref. [20]. We discuss both LEP1/SLC energies and LEP2 energies.

Specifically, for our comparisons we use the same event selection (ES) cuts as defined in ref. [20]:

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2We need to stress that, in implementing the results (11)–(22) and the corresponding results from ref. [7] into (10), care must be taken to compute the two terms on the RHS of (11) in the same manner with the same massive (massless) limits of the corresponding parts of each respective term so that the result for $\beta_1$ is numerically stable; we have done this.
Figure 1: Monte Carlo results for BARE trigger, for two values (10° and 25°) of acollinearity cut. Center of mass energies (in GeV) close to Z peak. In the plots cross section $\sigma_{\text{REF}}$ from TOPAZ0 is used as a reference cross section. Cross sections in nb. Two horizontal dotted lines indicate the 0.3% band, for reference.

- BARE ES – we require $40^\circ < \theta_{e^-} < 180^\circ$, $0^\circ < \theta_{e^+} < 180^\circ$, with acollinearity cuts of 10°, 25° and $E_{\text{min}} = 1$ GeV for both $e^-$ and $e^+$;
- CALO ES – we require the same cuts as in the BARE case but with $E_{\text{min}} = 20$ GeV
| No. | $E_{CM}$ | TOPAZ0   | BHWIDTH | BHAGENE3 | UNIBAB | BHAGEN95 |
|-----|----------|----------|---------|----------|--------|----------|
| 1.  | 88.45    | 0.4533 ± .0004 | 0.4523 ± .0004 | 0.4467 ± .0008 | 0.4490 ± .0010 | 0.4524 ± .0001 |
| 2.  | 89.45    | 0.6387 ± .0004 | 0.6377 ± .0006 | 0.6302 ± .0011 | 0.6358 ± .0012 | 0.6370 ± .0002 |
| 3.  | 90.20    | 0.9023 ± .0003 | 0.9016 ± .0008 | 0.8920 ± .0015 | 0.9021 ± .0014 | 0.8990 ± .0003 |
| 4.  | 91.19    | 1.1725 ± .0001 | 1.1707 ± .0010 | 1.1767 ± .0021 | 1.1772 ± .0016 | 1.1689 ± .0004 |
| 5.  | 91.30    | 1.1520 ± .0001 | 1.1505 ± .0009 | 1.1571 ± .0020 | 1.1559 ± .0016 | 1.1491 ± .0004 |
| 6.  | 91.95    | 0.8649 ± .0001 | 0.8646 ± .0007 | 0.8795 ± .0015 | 0.8689 ± .0012 | 0.8660 ± .0003 |
| 7.  | 93.00    | 0.4723 ± .0001 | 0.4725 ± .0005 | 0.4796 ± .0008 | 0.4733 ± .0008 | 0.4761 ± .0001 |
| 8.  | 93.70    | 0.3486 ± .0001 | 0.3486 ± .0004 | 0.3507 ± .0006 | 0.3486 ± .0007 | 0.3526 ± .0001 |

(b) CALO $\alpha_{\text{col}} = 25^\circ$

| No. | $E_{CM}$ | TOPAZ0   | BHWIDTH | BHAGENE3 | UNIBAB | BHAGEN95 |
|-----|----------|----------|---------|----------|--------|----------|
| 1.  | 88.45    | 0.4769 ± .0004 | 0.4742 ± .0004 | 0.4696 ± .0008 | 0.4733 ± .0010 | 0.4741 ± .0001 |
| 2.  | 89.45    | 0.6638 ± .0003 | 0.6615 ± .0006 | 0.6556 ± .0011 | 0.6619 ± .0012 | 0.6599 ± .0002 |
| 3.  | 90.20    | 0.9297 ± .0003 | 0.9278 ± .0008 | 0.9207 ± .0012 | 0.9302 ± .0014 | 0.9236 ± .0003 |
| 4.  | 91.19    | 1.2025 ± .0003 | 1.1994 ± .0010 | 1.2074 ± .0021 | 1.2073 ± .0016 | 1.1954 ± .0004 |
| 5.  | 91.30    | 1.1819 ± .0003 | 1.1790 ± .0010 | 1.1879 ± .0021 | 1.1860 ± .0016 | 1.1753 ± .0004 |
| 6.  | 91.95    | 0.8924 ± .0003 | 0.8909 ± .0007 | 0.9058 ± .0016 | 0.8965 ± .0012 | 0.8899 ± .0003 |
| 7.  | 93.00    | 0.4964 ± .0003 | 0.4954 ± .0005 | 0.5004 ± .0009 | 0.4976 ± .0008 | 0.4971 ± .0001 |
| 8.  | 93.70    | 0.3717 ± .0003 | 0.3704 ± .0004 | 0.3690 ± .0006 | 0.3720 ± .0007 | 0.3730 ± .0001 |

Figure 2: Monte Carlo results for CALO trigger, for two values (10° and 25°) of acollinearity cut. Center of mass energies (in GeV) close to Z peak. In the plots cross section $\sigma_{\text{REF}}$ from TOPAZ0 is used as a reference cross section. Cross sections in nb. Two horizontal dotted lines indicate the 0.3% band, for reference.

For the final ‘fermion’ energy which is the $e^-(e^+)$ energy if there are no photons nearby and which is the $e^-(e^+)$ plus the photon energy if the photon is within a cone of half-angle 1° from the $e^-(e^+)$, respectively.
Figure 3: Monte Carlo results for CALO trigger, for two values (10° and 25°) of acollinearity cut. Center of mass energies close to W-pair production threshold ($E_{CM}$: 1. 175 GeV, 2. 190 GeV, 3. 205 GeV). In the plots cross section $\sigma_{REF}$ from BHWIDE is used as a reference cross section. Cross sections in pb. Two horizontal dotted lines indicate the 1.5% band, for reference.

For the BARE ES acceptance cuts, we show in fig. 3 the comparison of the BHWIDE results with those of ALIBABA, BHAGEN95, BHAGENE3 and TOPAZ0, where for definiteness we plot the ratio $(\sigma_A - \sigma_{REF})/\sigma_{REF}$ for each calculation $A$, $A =$ ALIBABA, BHAGEN95, BHAGENE3, BHWIDE, and TOPAZ0, using the TOPAZ0 cross section as $\sigma_{REF}$. This we do for the 8 CMS energy values: 88.45, 89.45, 90.20, 91.19, 91.30, 91.95, 93.00, 93.70 GeV, which are denoted in the figure as energy points 1, 2, ..., 8, respectively. We see that BHWIDE agrees with the semi-analytical programs, ALIBABA and TOPAZ0, to within a few per mille at the $Z$ peak whereas off the peak BHWIDE remains generally within 1% of the semi-analytical programs. The difference between BHWIDE and the Monte Carlo program BHAGENE3 is also at the few per mille level at the $Z$ peak.

$^3$ Here we use recently updated results of BHAGEN95, obtained from the authors.
but it is as much as 2.27% off the peak. The agreement between BHWIDE and BHA-
GEN95 is also at the few per mille level at the Z peak; off the peak, two programs remain
within 1% of one another, with the better agreement holding for the looser acollinearity
cut of 25°. These results give us a handle on the total precision of BHWIDE in the Z
resonance regime, as we discuss presently.

Continuing in this way, we show in fig. 2 the similar type of comparison of the predic-
tions of the programs with the CALO ES. In fig. 2 ALIBABA is no longer applicable (it
does not handle the CALO ES) and UNIBAB appears — it was too slow to participate
in the BARE ES comparisons. Again, we use TOPAZ0 for the reference cross section and
we plot the same ratio \((\sigma_A - \sigma_{REF})/\sigma_{REF}\) for the same 8 energy points as in fig. 1. At the
peak, BHWIDE is within a few per mille of TOPAZ0 and BHAGEN95 and it is within 7
per mille of UNIBAB and BHAGENE3. Off peak, BHWIDE remains within 6 per mille
of TOPAZ0 whereas it remains within 0.8% of UNIBAB, within 1.2% of BHAGEN95 and
within 1.8% of BHAGENE3; the better agreement with the latter three programs holds
for the looser acollinearity cut whereas for TOPAZ0 the situation is reversed. Based on
these and related comparisons as described in ref. [20], including both the results and the
physics approximations in the various programs as presented in this last reference, we
conclude that, for the CALO ES, within \(\pm 100 \text{ MeV}\) of the Z peak, the total precision
of BHWIDE is 3 per mille and off peak, within \(+2.5/ -2.75 \text{ GeV}\) thereof, we set this
precision at 5 per mille in the LEP1 energy regime. For reference the three per mille
band is indicated by the two horizontal dotted lines in figs. 1 and 2. This precision tag
should be compared to that for the only other published wide angle Bhabha scattering
Monte Carlo event generator in wide use at LEP/SLC, namely BABAMC [6], whose total
precision on pure QED was set at 1% in the Z peak region in refs. [28, 29]. (In practice,
ALIBABA and/or TOPAZ0 would be used to determine the non-QED and higher order
QED part of the respective cross section.)

Turning next to LEP2 energies, we show in fig. 3 the comparison of the results of the
six programs BHAGEN95, BHAGENE3, BHWIDE, SABSPV, TOPAZ0, and UNIBAB
in the same format as in figs. 1, 2, using in this figure BHWIDE for the reference cross
section \(\sigma_{REF}\) for the three CMS energy points 175, 190, and 205 GeV. We use the CALO
ES only here. SABSPV and BHAGEN95 are within 2% of BHWIDE for both acollinearity
cuts, with SABSPV within 1.5% of BHWIDE and with BHAGEN95 maximally at 0.55%
above SABSPV. TOPAZ0 and UNIBAB deviate from BHWIDE by as much as 1.9% and
2.4% respectively, with the worse agreement holding for the looser (tighter) acollinearity
cut, respectively. These deviations are consistent with the leading logarithmic accuracy
one expects for these two Z peak optimized codes. Similarly, the deviations between
BHWIDE and BHAGENE3 by as much as 3.8% as shown in fig. 3 are also consistent with
what one expects from the leading logarithmic Z peak optimized nature of BHAGENE3.
On the basis of these and related comparisons, we estimate the total precision of BHWIDE
at the LEP2 energies as 1.5%, conservatively. For reference, the 1.5% band is indicated
by the two horizontal dotted lines in fig. 3.

We end this section by noting that, in addition to the comparisons just presented, we
have also checked that the pure QED \(O(\alpha)\) predictions of BHWIDE are within 0.05% of
those of the exact $\mathcal{O}(\alpha)$ MC OLDBIS of Ref. [22] — a modernized version of the MC of Ref. [30]. This gives us additional confidence in the technical component of our total precision estimate for BHWIDE 1.00.

5 Conclusions

In this paper, we have presented a new multiple photon Monte Carlo for wide angle Bhabha scattering at LEP1/SLC and LEP2 energies in which the respective multiple photon effects are realized on an event-by-event basis and in which the infrared singularities are cancelled to all orders in $\alpha$ via YFS exponentiation. This Monte Carlo calculation contains the exact $\mathcal{O}(\alpha)$ result and features two choices for the respective pure weak corrections at $\mathcal{O}(\alpha)$, that in ref. [1] and that in ref. [5]. It thus corresponds to the exact $\mathcal{O}(\alpha)$ YFS exponentiated treatment of wide angle Bhabha scattering.

We have illustrated our new calculation at both LEP1/SLC energies and at LEP2 energies, for two types of event selection, the BARE and CALO selections of ref. [20], which feature two choices of the acollinearity cut, 10° and 25°. In our illustrations, we compared our predictions with those of refs. [1, 10, 12, 18, 19]. We found in general a good agreement of the various calculations, an agreement consistent with the levels of approximations and realms of applicability of the respective codes. In this way, we arrived at the precision tags of 0.3% for BHWIDE at the $Z$ peak and of 1.5% at LEP2 energies. The program is available from the authors at the WWW URL [http://enigma.phys.utk.edu/pub/BHWIDE/]

In summary, we have developed a new Monte Carlo event generator for wide angle Bhabha scattering at LEP1/SLC and LEP2 energies in which the infrared singularities are cancelled to all orders in $\alpha$ via YFS exponentiation of the respective multiple photon effects. We look forward with excitement to its application to LEP1/SLC and LEP2 data analyses.

Acknowledgments

Two of us (S. J. and B. F. L. W.) thank Profs. G. Veneziano and G. Altarelli for the support and kind hospitality of the CERN Theory Division, where a part of this work was performed. One of us (B. F. L. W.) thank Prof. C. Prescott of SLAC for the kind hospitality of SLAC Group A while this work was completed.

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