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To cite this article: S Galovi et al 2010 J. Phys.: Conf. Ser. 214 012113

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Photothermal dynamic elastic bending in a semiconductor circular plate induced by a focused laser beam

S. Galović, M. Popović, D. M. Todorović

The "Vinča" Institute of Nuclear Sciences, P. O. Box 522, 11001 Belgrade, Serbia
Institute for Multidisciplinary Research, P.O.Box 33, 11030 Belgrade, Serbia

bobagal@vinca.rs

Abstract. The theoretical models of optically induced elastic bending for a semiconductor circular plate (clamped and simply supported) was derived including both plasmaselastic (PE) and thermoelastic (TE) wave influences. The PE and TE effects versus the modulation frequency of focused laser excitation were analyzed. Obtained results were compared with the optically excited elastic bending produced by homogeneous surface heat source and with experimental measurements.

1. Introduction

Photoacoustic (PA) and photothermal (PT) measurement techniques are being extensively developed and applied with increased success to the measurement of thermal, optical, electrical, and other related physical properties, as well as for the investigation of subsurface structure and macroscopic defects of various semiconductors and microelectronic and nanoelectronic devices [1-3]. During the last ten years, the PA and PT science and technology, together with micro- and nanosystem technologies, had an important role in the development and production of miniature sensors, actuators, resonators and electromechanical parts. The PA and PT effects that lay in the base of PA and PT methods can be important not only for inspection of various semiconductor systems but also as driving mechanisms for micromechanical structures [4]. In both employment of the PA and PT effect, the thermoelastic (TE) deformation and electronic deformation (ED), as a prominent deformation of semiconductor samples and the main drive mechanisms for micromechanical structures, should be investigated.

A few authors have been investigated optically induced thermoelastic bending in various three-dimensional (3D) and two-dimensional (2D) configuration [5-8], using tightly focused incident source and developing a new class of PT detecting techniques based on deflection of probe laser beam (for example, thermal mirror PT methods), but they didn't include ED phenomena. The TE and ED phenomena were experimentally and theoretically analyzed in some one-dimensional (1D) and 2D micromechanical structures [9-11], assuming non-focused optical source. In this work, the theoretical model including plasmaselastic (PE) and TE waves was developed to obtain the carrier density, the temperature distribution and the response of different deformation component (the dynamic TE and ED bending) in a semiconductor thin circular plate (2D geometry), generated by a tightly focused and intensity-modulated laser beam. This example is important as a model for semiconductor wafers in many PA and PT experimental configuration and as a model of some micromechanical devices.
2. Theory

The theoretical treatment of the PA and PT phenomena in semiconductors consists of modeling and solving a complex system of the coupled thermal, carrier and elastic transport equations. This permits quantitative account of the functional dependence of the amplitude and phase of the PA and PT response on modulation frequency, absorption coefficients, thermal, elastic and carrier transport properties of semiconductors, enabling the determination of mentioned properties of the sample from a measured PA and PT responses.

In cases which have the biggest practical value, the complex theoretical description of PA and PT effects may be simplified. The treatment of plasma wave is assumed to be completely decoupled from any temperature and elastic deformation effects. The heat caused by the elastic wave could also be neglected. This condition is almost always satisfied for solids and, consequently, this assumption does not significantly limit the application of the theory. On the other hand, thermal wave propagation includes a source term from the thermalization and nonradiative recombination decay of photoexcited plasma. So the plasma wave and thermal wave solutions can be obtained independently of elastic wave propagation, but elastic deformation depend on thermal and plasma wave.

We consider the PT elastic bending of a thin circular plate of thickness $l$ and radius $R >> l$. The excitation beam is assumed to be Gaussian with a $(1/e)$-radius equal to $a (a << R)$. The beam is modulated with an angular frequency $\omega$ ($\omega = 2 \pi f$) and focused onto the front side of laterally very thick (practically semi-infinite) semiconductor wafer. The configuration of the problem is shown in Figure 1. In accordance with the rotational symmetry of the problem it is possible to eliminate the dependence of excess carrier density, temperature variations and elastic displacements on the azimuth coordinate.

2.1. Model of PT induced dynamic elastic bending

Based on the elastic theory of thin plate with a rotational symmetry, the dynamic equation for the elastic bending in the case of periodical excitation is [3,10,11]

$$\left(\nabla^4 - \beta^4_3\right)v = - (1 + \nu) \nabla^2 g_m, \quad g_m = M(r, \omega) + \frac{6}{l} N(r, \omega), \quad \beta^4_3 = \frac{\rho l \omega^2}{R_f}, \quad \nabla^2_z v = \frac{\partial^2}{\partial r^2} + \frac{l}{r} \frac{\partial}{\partial r}$$

(1)

The $g_m$ is the source term from the thermal and plasma waves, i. e., the TE and ED moment generation term, given by [3,7]:

$$N(r, \omega) = \frac{1}{l} \int_0^l \left[ \alpha_r \vartheta(r, z, \omega) + d_n \Delta n(r, \omega, t) \right] dz,$$

$$M(r, \omega) = \frac{1}{l} \int_0^l \left[ z \alpha_r \vartheta(r, z, \omega) + d_n \Delta n(r, z, \omega) \right] dz.$$
The symbols \( \alpha \) and \( \delta \) stand for the coefficient of linear thermal expansion and the coefficient of electronic deformation, respectively. With \( \Delta n \) and \( \vartheta \) are designated plasma and thermal wave distribution. \( R_p = E_1 l^3 / (1 - \nu^2) \) is the flexural rigidity of the plate, \( E_1 \) is Young's modulus, \( \nu \) - Poisson's ratio and \( \rho \) - density of the sample.

The elastic circular plate can be mounted differently in the detection configuration. Generally, there are two characteristic boundary conditions:

a) circular plate clamped on the boundary
\[
w(r = R, \omega) = 0, \quad \frac{d}{dr}w(r = R, \omega) = 0 \quad (2)
\]
b) simply supported circular plate
\[
w(r = R', \omega) = 0, \quad \frac{d^2}{dr^2}w(r = R', \omega) + \frac{r}{R'} \frac{d}{dr}w(r = R', \omega) + (1 + \nu) \frac{g_m(r = R', \omega)}{r} = 0 \quad (3)
\]

The general solution of homogeneous biharmonic equation is
\[
w_m(r) = C_1 J_0(\beta_1 r) + C_2 K_0(\beta_2 r) \quad (4)
\]
where are: \( I_0, K_0 \) - the modified zero-order Bessel function of the first and second kind, respectively and \( J_0, Y_0 \) - the zero-order Bessel function of the first and second kind, respectively.

Particular solution can be obtained by applying Hankel transform if the \( \Delta n \) and \( \vartheta \) are known:
\[
w_{m}(r, \omega) = \int_{0}^{\infty} \mathcal{H}_{\lambda} J_0(\lambda r) \left[ \frac{1 + \nu}{\lambda^2} g_m(\lambda, \omega) \right] d\lambda \quad (5)
\]

The constants \( C_1 \) and \( C_2 \) are evaluated from boundary conditions, Eqs. (2), (3), and homogeneous and particular solutions, Eqs. (4), (5).

2.2. Plasma and thermal waves

The model of bipolar semiconductor is considered with a volume and a surface carrier recombination. For a low level of the optical excitation the linear volume recombination was taken into account. It is also assumed that the excitation energy \( E \) is greater than the energy gap \( E_G \) and then the excitation energy generates electrons from the valence energy band. Then, the density of excess electrons, \( \Delta n(r, z, \omega) \), is equal to the density of excess holes, \( \Delta \rho(r, z, \omega) \).

Under these assumptions, the optically injected carrier density was calculated from the carrier transport equation with the boundary conditions at the front \( (z = 0) \) and rear \( (z = -l) \) surfaces that include surface generation and recombination by applying the Hankel integral transform method:
\[
\Delta n(r, z, \omega) = \int_{0}^{\infty} \mathcal{H}_{\lambda} J_0(\lambda r) \left[ A_+ \exp(\beta_1 z) + A_- \exp(-\beta_1 z) + E_1 \exp(\alpha z) \right] d\lambda \quad (6)
\]

The periodic temperature distribution, \( \vartheta(r, z, \omega) \), in the sample can be given as a sum of three components
\[
\vartheta(r, z, \omega) = \vartheta^T(r, z, \omega) + \vartheta^{SR}(r, z, \omega) + \vartheta^{BR}(r, z, \omega) \quad (i = T, SR, BR)
\]
where \( \vartheta^T(r, z, \omega) \), \( \vartheta^{SR}(r, z, \omega) \), and \( \vartheta^{BR}(r, z, \omega) \), are thermalization, surface and bulk recombination components of temperature distribution. The different components of the solution of the heat transport equations with the boundary conditions at the front \( (z = 0) \) and rear \( (z = -l) \) surfaces that describe continuity temperature and heat flux, including surface generation and recombination, can be easily obtained by applying the Hankel integral transform method and are expressed as follows:
\[
\vartheta^i(r, z, \omega) = \int_{0}^{\infty} \mathcal{H}_{\lambda} J_0(\lambda r) \left[ F_i^T \exp(\beta_1 z) + F_i^T \exp(-\beta_1 z) + E_i \exp(\alpha z) \right] d\lambda \quad (i = T, SR, BR)
\]

The various constants are defined in our recently completed paper [12].
3. Analysis of the dynamic elastic bending

By using derived theoretical model, dynamic elastic bending in a Si wafer was calculated. The values of semiconductor parameters (for a typical Si wafer) used for calculating the theoretical curves are downloaded from [4].

Fig. 2 show the TE, ED and total dynamic elastic bending in the center of Si microplate (with thickness of 50 μm, and a radius of 500 μm) as a function of modulation frequency ω. For both types of bending, TE and ED bending, it is possible to see (Fig. 2) the similar behavior in all observed frequency range. In the low frequency range (ω < 10^4 Hz), the elastic bending is not in the function of modulation frequency. Also, TE mechanism of elastic bending generation is dominant in all observed frequency ranges.

![Figure 2. Elastic bending in the center of circular Si microplate (thickness 50 μm, radius 500 μm) induced by focused laser beam.](image)

As it could be observed from Fig. 2, the focused excitation produces larger elastic bending than non-focused (10^-7-10^-1 nm in compare to 10^-5-10^-1 nm [11]). It could be explained by important influence of lateral diffusion of thermal and plasma field to source of elastic deformation.

4. Conclusion

The dynamic TE and ED bending induced by focused laser beam in semiconductor thin circular plate for different boundary conditions was investigated. The elastic bending effect was analyzed as a function of modulation frequency including the generation and propagation processes of the plasma, thermal, and elastic waves.

It is demonstrated that TE and ED bending are alike in all observed frequency range and that the TE mechanism of elastic bending generation is dominant. Comparing to the elastic bending of thin circular plate induced by non-focused laser beam, our theoretically model predict the larger amplitude of elastic bending.

This investigation is important for different PA and PT techniques where various dynamic bending effects and mechanical resonant vibrations must be included.

Acknowledgement

This work is supported by Ministry of Science and Technological Development of the Republic of Serbia (Projects TR11027, 141013, and I.T.1.04.0062.B).

References

[1] A. Mandelis and P. Hess Eds, *Semiconductors and Electronic Materials*, SPIE Opt.Eng. Press, Bellingham, Washington, 2000
[2] A. Mandelis, *Solid-State Electronics*, **42**, pp.1-15, (1998).
[3] D.M. Todorovic, P.M. Nikolic, Ch. 9 in *Semiconductors and Electronic Materials* (A. Mandelis and P. Hess, Eds., SPIE Opt. Eng. Press, Bellingham, Washington, 2000), p. 273-318.
[4] Y. Song, B. Cretin, D. M. Todorovic, and P. Vairac, *J. Phys. D: Appl. Phys.*, **41**, 155106, (2008).
[5] T. Elperin and G. Rudin, *Int. J. Thermophys.*, **28**, 60, (2006).
[6] B. C. Li, *J. Appl. Phys.*, **68**, 482 (1990).
[7] N. G. Astrath, et. al., *Appl. Phys. Lett.*, **91**, 191908-1, (2007).
[8] F. Sato, et. al., *J. Appl. Phys.*, **104**, 053520-1, (2008).
[9] R.G. Stearns, G.S. Kino, *Appl. Phys. Lett.*, **47**, 1048-1050 (1985).
[10] D.M. Todorovic, *Rev. Sci. Instrum.*, **74**, 578-581 (2003).
[11] D. M. Todorovic, *Analytical Science*, **17**, s141-s144 (2001).
[12] S. Galovic, M. Popovic, A. Popovic, D. Todorovic, "Thermal wave in a semiconductor circular plate photogenerated by a focused laser beam", to be published in Thermal Science, **14**, 2010.