CEMENT: Incomplete Multi-View Weak-Label Learning with Long-Tailed Labels

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Abstract
A variety of modern applications exhibit multi-view multi-label learning, where each sample has multi-view features, and multiple labels are correlated via common views. In recent years, several methods have been proposed to cope with it and achieved much success, but still suffer from two key problems: 1) lack the ability to deal with the incomplete multi-view weak-label data, in which only a subset of features and labels are provided for each sample; 2) ignore the presence of noisy views and tail labels usually occurring in real-world problems. In this paper, we propose a novel method, named CEMENT, to overcome the limitations. For 1), CEMENT jointly embeds incomplete views and weak labels into distinct low-dimensional subspaces, and then correlates them via Hilbert-Schmidt Independence Criterion (HSIC). For 2), CEMENT adaptively learns the weights of embeddings to capture noisy views, and explores an additional sparse component to model tail labels, making the low-rankness available in the multi-label setting. We develop an alternating algorithm to solve the proposed optimization problem. Experimental results on seven real-world datasets demonstrate the effectiveness of the proposed method.

1 Introduction
In many real-world applications, samples are often represented by several feature subsets, and meanwhile associated with multiple labels [Xu et al., 2013a]. For example, a natural scene image can be annotated with multiple tags \{house, sky, tree\}, and described by various visual features, such as histogram of oriented gradients, color features and scale-invariant feature transform. As an effective way to deal with such data, multi-view multi-label learning has attracted a lot of attention in various real-world applications [Wu et al., 2019; Zhang et al., 2020]. Though these approaches have achieved much success, there still exists two problems. The first one is that it is difficult to collect all the relevant labels of every sample. For example, in image annotations, an annotator may annotate an image with a partial label set from the large number of ground-truth labels. To address such problem, weak-label learning methods [Yu et al., 2014; Dong et al., 2018; Wu et al., 2018; Tan et al., 2018b] are proposed based on the assumption that similar instances have similar labels. Though these methods have shown promising results in real applications, they do not consider the second problem, i.e., samples may miss their representations on some views, which possibly leads to the performance degradation [Xu et al., 2015]. Many incomplete multi-view learning methods [Zhang et al., 2013; Liu et al., 2015; Xu et al., 2015; Yin et al., 2017] are then proposed to improve the performance by exploiting the complementary information from multiple incomplete views.

The co-existence of incomplete views and weak labels poses a severe challenge. To the best of our knowledge, only few studies [Tan et al., 2018a; Zhu et al., 2019; Li and Chen, 2021] take both two issues into consideration. However, iMVWL [Tan et al., 2018a] and IMVL-IV [Zhu et al., 2019] impose the low-rank constraint on the label matrix, which is usually violated in practice due to the presence of tail labels in multi-label learning [Li and Chen, 2021]. NAIM\textsuperscript{3}L [Li and Chen, 2021] assumes that the label matrix is high-rank, but treats all views equally just as iMVWL and IMVL-IV do, which probably suffers from the problem of noisy views.

To complete the label matrix under the aforementioned challenges, we propose a novel method for inCompletE Multi-view wEak-label leaRNing with long-Tailed labels (CEMENT) in this paper. Specifically, CEMENT first embeds both incomplete views and weak labels into low-dimensional subspaces with adaptive weights, which automatically detects noisy views by assigning relatively lower weights to them. It then adaptively correlates embedded views and labels via Hilbert-Schmidt Independence Criterion (HSIC) in Reproducing Kernel Hilbert Spaces (RKHSs). To capture tail labels, it separates an additional sparse component from weak labels, that makes the low-rankness valid in the multi-label setting. The framework of CEMENT is shown in Fig. 1. An alternating algorithm is developed to optimize the proposed problem, and its effectiveness is demonstrated on seven real-world datasets. The contributions of this work are summarized into three-folds:

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A novel method CEMENT is proposed to handle the incomplete multi-view weak-label issue. It jointly embeds incomplete views and weak labels into low-dimensional subspaces with adaptive weights, and adaptively correlates the embeddings via HSIC in RKHSs.

CEMENT enables to capture noisy views and tail labels in real-world datasets by learning adaptive embedding weights and exploring an additional sparse component from weak labels, respectively.

Experimental results on seven widely used real-world datasets show the effectiveness of CEMENT.

2 Related Work

In this section, we discuss the related works with this paper, and focus on the works from three research fields: Incomplete Multi-view Learning, Weak-label Learning and Incomplete Multi-view Weak-label Learning.

2.1 Incomplete Multi-View Learning

Multi-view learning handles the data represented by multiple views and aims to improve learning performance by discovering view correlations [Yin et al., 2017]. Under the incomplete multi-view setting, many algorithms have been proposed to handle the problem of missing views in recent years. Previous approaches have shown promising results in conjunction with semi-supervised learning [Xu et al., 2015; Yin et al., 2017], or with contrastive learning [Lin et al., 2021]. And some tried to seek shared information by projecting original multi-view data into a single low-dimensional subspace [Zhang et al., 2013; Liu et al., 2015].

2.2 Weak-Label Learning

Previous weak-label learning studies focus mainly on the single-view setting. MAXIDE [Xu et al., 2013b] uses the input feature data as side information to recover the label matrix, based on the assumption that the label matrix is low-rank. COCO [Xu et al., 2018] leverages a latent possibility matrix to generate the label matrix, and can recover the feature matrix and the label matrix simultaneously without the low-rank assumption. lrMMC [Liu et al., 2013] and McWL [Tan et al., 2018b] are multi-view weak-label learning methods, but they both need all the views to be complete.

2.3 Incomplete Multi-View Weak-Label Learning

As far as we know, there are only few studies focused on the incomplete multi-view weak-label learning. iMVWL [Tan et al., 2018a] learns a shared subspace from incomplete views with weak labels, and leverages both cross-view relationships and local label correlations. IMVL-IV [Zhu et al., 2019] designs a multi-view multi-label learning method with incomplete views and weak labels by learning label-specific features, label correlations, and complementary information of multiple views. These methods assume that the label matrix is low-rank, which is typically unsuitable in practice. NAIM3L [Li and Chen, 2021] explicitly exploits the high-rank structure of the multi-label matrix, and jointly takes incompleteness of views and missing of labels into account. However, the existing three methods treat all views equally, limiting the real applications in presence of noisy views.

Existing methods usually embed incomplete multi-view features into a common subspace with equal weights, but noisy views would degrade the generalization ability. Moreover, they typically assume that the partially observed label matrix is low-rank and ignore the existence of tail labels, which is probably contradictory to real-world applications. In contrast, the proposed CEMENT completes the unobserved labels by embedding both incomplete views and weak labels into multiple subspaces with adaptive weights, and captures tail labels by exploiting an additional sparse component, which makes the low-rank principle valid.

3 Methodology

3.1 Preliminaries

For the $i$-th instance, we denote its feature vector of $v$-th view by $x_{iv} \in \mathbb{R}^{d_v}$, and its corresponding label vector by
y_i \in \{0,1\}^t$, where $d_v$ is the feature dimension of the $v$-th view, and $t$ is the number of distinct labels. Let $\{X^v\}_{v=1}^m$, denote the input data with $n$ samples and $m$ views, where $X^v = [x_1^v, x_2^v, \ldots, x_n^v]^T \in \mathbb{R}^{n \times d_v}$ indicates the feature matrix in the $v$-th view. Let $Y = [y_1, y_2, \ldots, y_n]^T \in \{0,1\}^{n \times t}$ denote the label matrix, where $y_{ij} = 1$ means that the $j$-th label is assigned to the $i$-th instance, while $y_{ij} = 0$ otherwise. In the incomplete multi-view weak-label scenario, partial views and labels of some samples may be missing. Thus, we introduce $O_X^v \in \mathbb{R}^{n \times d_v}$ and $O_Y \in \mathbb{R}^{n \times t}$ to denote indexes of the missing entries in the feature matrix $X^v$ and the label matrix $Y$, respectively. $(O_X^v)_{ij} = 1$ or $(O_Y)_{ij} = 1$ if $(i,j)$ is an observed entry in $X^v$ or $Y$, and $(O_X^v)_{ij} = 0$ or $(O_Y)_{ij} = 0$ otherwise. Given incomplete multi-view data, the goal of CEMENT is to predict the unobserved labels, in presence of noisy views and long-tailed labels.

### 3.2 Formulation

Given a multi-view dataset, we can optimize the following problem to find a shared latent subspace $P \in \mathbb{R}^{t \times k}$ ($k < \min\{d_1, d_2, \ldots, d_m\}$), that integrates complementary information from different views [Gao et al., 2015]:

\[
\min_{P, \{K^v\}_{v=1}^m} \sum_{v=1}^m \|X^v - PU^v\|^2_F \\
\text{s.t. } P \geq 0, U^v \geq 0, v = 1, 2, \ldots, m,
\]

where $\| \cdot \|_F$ represents the Frobenius norm, and $U^v \in \mathbb{R}^{k \times d_v}$ is the coefficient matrix of the $v$-th view. Eq. (1) treats each view equally, whose objective actually equals to $\|X^v - PU^v\|^2_F$ with $X = [X^1, X^2, \ldots, X^m]$ and $U = [U^1, U^2, \ldots, U^m]$. Therefore, it might deviate from the true latent subspace, due to the existence of noisy views. Moreover, structured missing views in many applications also make Eq. (1) unreliable. A naive way to solve this problem is to fill the missing entries with average feature values, but it may introduce errors. To overcome the limitations, we propose the incomplete multi-view model as follows:

\[
\min_{\alpha, \{P^v\}, \{U^v\}} \sum_{v=1}^m \alpha_v \|O_X^v \odot (X^v - PU^v)\|^2_F \\
\text{s.t. } \alpha \geq 0, P^v \geq 0, U^v \geq 0, v = 1, 2, \ldots, m,
\]

where $\alpha_v$ weights the embedding importance of the $v$-th view, $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ and $\odot$ is the Hadamard product. According to Eq. (2), the $v$-th view data matrix $X^v$ is mapped to the view-specific latent representation $P^v \in \mathbb{R}^{d_v \times k_v}$ ($k_v < d_v$), with the view-specific adaptive weight $\alpha_v$. In addition, Eq. (2) minimizes the reconstruction error between $X^v$ and $P^vU^v$ based only on the observed entries, which is indexed by $O_X^v$. In this way, we successfully overcome the two limitations of Eq. (1).

Similarly, we map the label matrix $Y$ to its latent representation $P^* \in \mathbb{R}^{n \times k_s}$ ($k_s < l$) by $Y = P^*W$, where $W \in \mathbb{R}^{k_s \times l}$ is the coefficient matrix. However, the presence of long-tailed labels makes the low-rank assumption invalid in practice [Li and Chen, 2021]. Thus, it is desired to separate tail labels from the entire labels. To this end, we treat tail labels as outliers and decompose the label matrix $Y$ by

\[
Y = \hat{Y} + E.
\]

In Eq. (3), $\hat{Y} = P^*W$ models non-tail labels under the low-rank assumption, and $E$ captures tail labels with a sparse constraint. Besides, in the weak-label setting, the label matrix $Y$ is often incomplete and contains many missing entries. Thus, we propose to solve the following problem:

\[
\min_{P^*, W, E} \|O_Y \odot (Y - P^*W - E)\|^2_F + \lambda\|E\|_1 \\
\text{s.t. } P^* \geq 0, W \geq 0,
\]

where $\lambda > 0$ is a trade-off hyperparameter. Therefore, we successfully capture tail labels in the weak-label setting, and thus make the low-rankness valid.

Next, we adopt the Hilbert-Schmidt Independence Criterion (HSIC) [Gretton et al., 2005] to build the correlations among the embedded views $\{P^v\}_{v=1}^m$ and the embedded labels $P^*$ in an adaptive manner. HSIC computes the squared norm of the cross-covariance operator over $P^v$ and $P^*$, in Reproducing Kernel Hilbert Spaces (RKHSs) to estimate the dependency, which is empirically defined by:

\[
HSIC(P^v, P^*) = (n - 1)^{-2} tr(K_vHK_vH),
\]

where $K_v \in \mathbb{R}^{n \times n}$ and $K \in \mathbb{R}^{n \times n}$ are two Gram matrices, measuring the kernel induced similarity between row vectors of $P^v$ and $P^*$. $H = I - \frac{1}{n}11^T$ is the centering matrix, where $I \in \mathbb{R}^{n \times n}$ is an identity matrix, and $1 \in \mathbb{R}^n$ is an all-one vector. In theory, the larger the value of HSIC, the higher the dependence between $P^v$ and $P^*$. Thus, we promote the dependence between $P^v$ and $P^*$ by maximizing the value of HSIC:

\[
\max_{\alpha, \{P^v\}, P^*, \{U^v\}} \sum_{v=1}^m \beta_vHSIC(P^v, P^*) \\
\text{s.t. } ||\beta||_2 = 1, \beta \geq 0,
\]

where $\beta = [\beta_1, \beta_2, \ldots, \beta_m]$ and $\beta_v$ measures the importance of the correlation between the $v$-th view embedding $P^v$ and the label embedding $P^*$.

By incorporating Eq. (2), Eq. (4) and Eq. (6), we now have the optimization problem for the proposed CEMENT method:

\[
\min_{\alpha, \{P^v\}, P^*, \{U^v\}, W, E} - \sum_{v=1}^m \beta_vHSIC(P^v, P^*) \\
+ \sum_{v=1}^m \alpha_v \|O_X^v \odot (X^v - PU^v)\|^2_F \\
+ \|O_Y \odot (Y - P^*W - E)\|^2_F + \lambda\|E\|_1 \\
\text{s.t. } ||\beta||_2 = 1, \alpha, \beta, P^v, U^v, P^*, W \geq 0, \\
v = 1, 2, \ldots, m.
\]

In fact, Eq. (7) treats the label matrix as the $(m + 1)$-th view, and uses an additional non-negative parameter $\alpha_{m+1}$ to weight its embedding. It is worth noting that $\alpha_v$ weights the reconstruction between $X^v$ and $P^vU^v$, while $\beta_v$ weights the correlation between $P^v$ and $P^*$, $v = 1, 2, \ldots, m$. In other words, $\alpha_v$ will be assigned to a large value once $X^v$ is well recovered by $P^vU^v$, and $\beta_v$ will take a large value.
if $P^v$ is highly correlated to $P^*$. In this way, CEMENT adaptively embeds incomplete views and weak labels into low-dimensional subspaces, and correlates them with adaptive weights, enabling to complete the missing labels in presence of both incomplete noisy views and tail labels.

4 Optimization

The objective function $L$ in Eq. (7) is convex w.r.t $\alpha$, $\beta$, $P^v$, $P^*$, $U^v$, $W$ and $E$, respectively, that motivates us to develop an alternating optimization algorithm.

for simplicity, the linear kernel is used in HSIC, i.e., $K_v = P^v(T^v)^T$ and $K_s = P^*(P^*)^T$. It is easily extended to apply the other kernels. The algorithm repeats following steps until convergence.

Update $P^v$ with fixed others. When $\alpha$, $\beta$, $P^*$, $U^v$, $W$, $E$ are fixed, each $P^v$ can be updated individually, and the objective function becomes

$$
\min_{P^v} \beta_v \Vert (P^v)^T H P^v \Vert_F^2 + \alpha_v \Vert O_X \odot (X^v - P^v U^v) \Vert_F^2.
$$

(8)

We then optimize Eq. (8) with Project Gradient Descent algorithm (PGD) [Calamai and Moré, 1987]:

$$
P^v \leftarrow \text{Proj}(P^v - \eta \frac{\partial L}{\partial P^v}),
$$

where $\eta$ is a learning rate, and $\frac{\partial L}{\partial P^v}$ is the partial derivative of $L$ w.r.t. $P^v$. The projection function $	ext{Proj}(A_{ij}) = A_{ij}$ if $A_{ij} > 0$; $	ext{Proj}(A_{ij}) = 0$ otherwise.

Update $P^*$ with fixed others. When $\alpha$, $\beta$, $P^v$, $U^v$, $W$, $E$ are fixed, the objective function becomes:

$$
\min_{P^*} \sum_{v=1}^{m} \beta_v \Vert (P^v)^T H P^* \Vert_F^2 + \alpha_{m+1} \Vert O_Y \odot (Y - P^* W - E) \Vert_F^2.
$$

(10)

Similar with updating $P^v$, we use PGD to update $P^*$ by:

$$
P^* \leftarrow \text{Proj}(P^* - \eta \frac{\partial L}{\partial P^*}),
$$

(11)

where $\frac{\partial L}{\partial P^*}$ is the partial derivative of $L$ w.r.t. $P^*$.

Update $U^v$ with fixed others. With the others fixed, the computation of each $U^v$ is independent. The objective function w.r.t. $U^v$ is

$$
\min_{U^v} \Vert O_X \odot (X^v - P^* U^v) \Vert_F^2.
$$

(12)

Under the Karush-Kuhn-Tucker (KKT) condition [Boyd and Vandenberghe, 2004], we can derive the following updating rule:

$$
U^v_{ij} \leftarrow U^v_{ij} \frac{((P^v)^T(O_X \odot (X^v)))_{ij}}{(((P^v)^T(O_X \odot (P^v U^v)))_{ij}).
$$

(13)

Update $W$ with fixed others. With the others fixed, the objective function w.r.t. $W$ becomes

$$
\min_{W \geq 0} \Vert O_Y \odot (Y - P^* W - E) \Vert_F^2
$$

(14)

By using the KKT condition, we can derive the following updating rule:

$$
W_{ij} \leftarrow W_{ij} \frac{((P^*)^T(O_Y \odot (Y - E)))_{ij}}{(((P^*)^T(O_Y \odot (P^* W)))_{ij}.
$$

(15)

Update $E$ with fixed others. We solve the following problem to update the long-tailed label matrix $E$:

$$
\min_{E} \Vert O_Y \odot (Y - P^* W - E) \Vert_F^2 + \frac{\lambda}{2 \alpha_{m+1}} \Vert E \Vert_1.
$$

(16)

Eq. (16) can be easily optimized by soft-thresholding [Donoho, 1995], and the updating rule is

$$
E \leftarrow S_{\lambda/(2 \alpha_{m+1})}(O_Y \odot (Y - P^* W)),
$$

(17)

where $S_{\lambda/(2 \alpha_{m+1})}$ is the shrinkage operator, and it is defined as $S_{\lambda/(2 \alpha_{m+1})}(A_{ij}) = \max(A_{ij} - \frac{\lambda}{2 \alpha_{m+1}}, 0) - \max(-A_{ij} - \frac{\lambda}{2 \alpha_{m+1}}, 0)$.

Update $\alpha$ with fixed others. When $\beta$, $P^*$, $P^v$, $U^v$, $W$ and $E$ are fixed, updating $\alpha$ is to solve the following problem

$$
\min_{\alpha \geq 0} \sum_{v=1}^{m} \alpha_v \Vert O_X \odot (X^v - P^v U^v) \Vert_F^2 + \alpha_{m+1} \Vert O_Y \odot (Y - P^* W - E) \Vert_F^2.
$$

(18)

Based on [Li et al., 2021], each $\alpha_v$ is updated independently according to the following equation

$$
\alpha_v = \begin{cases} 
\frac{1}{\Vert O_X \odot (X^v - P^v U^v) \Vert_F^2}, & 1 \leq v \leq m; \\
\frac{1}{\Vert O_Y \odot (Y - P^* W - E) \Vert_F^2}, & v = m + 1.
\end{cases}
$$

(19)

Eq. (19) actually is an inverse distance weighting. Obviously, the larger the distance, the smaller the value of $\alpha_v$, $v = 1, 2, \ldots, m + 1$.

Update $\beta$ with fixed others. When $\alpha$, $P^v$, $P^*$, $U^v$, $W$, $E$ are fixed, the objective function w.r.t. $\beta$ becomes

$$
\max_{\beta \geq 0} \sum_{v=1}^{m} \beta_v \Vert (P^v)^T H P^* \Vert_F^2.
$$

(20)

Given $\beta_v \Vert (P^v)^T H P^* \Vert_F^2 \geq 0$, we have the following derivations according to Cauchy-Schwarz inequality [Steele, 2004]:

$$
\sum_{v=1}^{m} \beta_v \Vert (P^v)^T H P^* \Vert_F^2 \leq \left( \sum_{v=1}^{m} \Vert (P^v)^T H P^* \Vert_F \right)^2 = \sum_{v=1}^{m} \Vert (P^v)^T H P^* \Vert_F^2.
$$

(21)

The inequality in Eq. (21) holds when

$$
\beta_v \leftarrow \frac{\Vert (P^v)^T H P^* \Vert_F^2}{\sqrt{\sum_{v=1}^{m} \Vert (P^v)^T H P^* \Vert_F^2}}.
$$

(22)

which is the closed-form solution of Eq. (20).

4.1 Complexity Analysis

In terms of computational complexity, updating $\beta_v$ needs a cost of $O(mnk_{\text{max}}(n + k_{\text{max}}))$, updating $\alpha_v$ and $P^v$ cost $O(mnk_{\text{max}}(n + d_{\text{max}}))$, and updating $P^*$ costs $O(nk_{\text{max}}(mn + l))$, where $k_{\text{max}}$ and $d_{\text{max}}$ represent the largest dimensionality of the subspaces and feature matrices from all views, respectively. Thus, the total computational complexity of the algorithm at each iteration is $O(mnk_{\text{max}}(n + d_{\text{max}}))$.\footnote{We provide the pseudocode and the MATLAB code of CEMENT in the supplementary materials.}
5 Experiments

5.1 Experimental Settings

Datasets. We conduct a comprehensive experimental study to evaluate the performance of the proposed CEMENT on seven widely used multi-view multi-label datasets. The statistics of the used datasets are summarized in Table 1. The first five datasets (Corel5k, ESPGame, IAPRTC-12, Mirflickr, and Pascal07)\(^2\) are all image datasets, and obtained from [Guillaumin et al., 2010]. Each sample of these datasets is represented by six feature views. In the Yeast dataset\(^3\) [Bu et al., 2003], each gene is represented by a genetic expression and a phylogenetic profile. In the Emotions dataset\(^4\) [Tsoumakas et al., 2008], each music is represented by rhythmic and timbre feature views, and classified into emotions that it evokes.

Comparing Methods. We compare the proposed method CEMENT with four state-of-the-art methods: lrMMC [Liu et al., 2013], McWL [Tan et al., 2018b], iMVWL [Tan et al., 2018a], NAIM\(^3\)L [Li and Chen, 2021]. lrMMC and McWL are two multi-view weak-label learning methods, but they all assume that the views of features are complete. Thus, we adapt lrMMC and McWL by filling missing features with zero. iMVWL and NAIM\(^3\)L are two incomplete multi-view weak-label learning methods, which can be seen as the baselines. The implementations of the above algorithms are publicly available in corresponding papers.

Configurations. On the five image datasets, the hyperparameters of McWL, iMVWL and NAIM\(^3\)L are selected as recommended in the original papers. We tune the hyperparameters of lrMMC and CEMENT on all datasets, and the other three methods on the Yeast and Emotions datasets by grid search to produce the best possible results. We select the value of the hyperparameter \(\lambda\) from \({10^i | i = -5, \ldots, 1}\), and the ratio of \(\frac{\lambda}{d}\) and \(\frac{\lambda}{T}\) from \({0.2, 0.5, 0.8}\) for our method. We set the values of the hyperparameters of the other methods from the ranges recommended in the original paper. The prediction performance of all algorithms is evaluated by three widely used metrics: Hamming Score (HS), Ranking Score (RS) [Zhang and Zhou, 2013], and Area Under Roc-Curve (AUC) [Bucak et al., 2011]. We randomly sample 2000 samples of each image dataset, and use all samples from the Yeast and Emotions datasets in the experiment. Furthermore, we follow the protocol given in [Tan et al., 2018a] to create incomplete multi-view weak-label scenarios: we randomly remove \(r\)% sampled positive and negative samples for each label, and \(s\)% samples from each view by ensuring that each sample appears in at least one view. For all comparing algorithms, we repeat the experiment by ten times and report the average values and the standard deviations.

5.2 Experimental Results

Evaluations of Comparing Methods. Table 2 shows the experimental results of all comparing methods on seven real-world datasets with \(r\)% = 50%, and \(s\)% = 50%. From Table 2, we can see that CEMENT outperforms compared methods in most of the cases. The performance superiority probably comes from the ability of CEMENT on capturing noisy views and tail labels. The incompleteness of multi-view data causes the degradation of results on lrMMC and McWL. iMVWL and NAIM\(^3\)L are able to handle the incomplete multi-view and weak-label datasets, but perform worse than CEMENT. There are two possible reasons. One is that iMVWL assumes that the label matrix is low-rank, and the other is that both iMVWL and NAIM\(^3\)L treat every view equally. In contrast, CEMENT measures the importance of each view by adaptively choosing the appropriate values of \(\alpha\) and \(\beta\).

![Figure 2: Ablation study of CEMENT on the Corel5k dataset with \(r\%) = 50\%\) and different values of \(s\%).](image)

Ablation Study. We first introduce three variants of CEMENT, namely CEMENT-1, CEMENT-2 and CEMENT-3, to investigate the effects of the components of CEMENT\(^5\). CEMENT-1 only learns shared information from all feature views, and ignores individual information, i.e., \(P^v = P^{v'}\), \(\alpha_v = 1\) \((v, v' = 1, 2, \ldots, m, \text{ and } v \neq v')\). CEMENT-2 assumes that the label matrix \(Y\) is low-rank by ignoring the tail label matrix \(E\). CEMENT-3 only learns a single shared subspace \(P^*\) among all views and labels, i.e., \(P^v = P^*\) \((v = 1, 2, \ldots, m)\), which does not need HSIC. Fig. 2 shows the ablation study of CEMENT on the Corel5k dataset with \(r\%) = 50\%\) and different values of \(s\%). Among the variants, CEMENT-2 performs the worst due to its low-rank assumption on \(Y\). CEMENT-1 and CEMENT-3 perform worse than CEMENT, probably because of the common subspace constraint and their incapability for adaptive weighting. In contrast, CEMENT has the best performance in all metrics, indicating the effectiveness and necessity of its components.

![Figure 3: Case Study of CEMENT-2 and CEMENT for handling tail labels on the Corel5k, ESPGame and IAPRTC-12 datasets.](image)

Case Study on Handling Tail Labels. To demonstrate the significance of CEMENT on investigating long-tailed labels, we treat the labels satisfying \(\sum \frac{y_i}{n_i} \leq 0.1\) \((v)\) as tail labels, and the others as head labels on three image datasets.

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\(^2\)http://lear.inrialpes.fr/people/guillaumin/data.php
\(^3\)http://vlado.fmf.uni-lj.si/pub/networks/data/
\(^4\)http://www.uco.es/kdis/mlresources
\(^5\)The formulations of the three variants of CEMENT and more experimental results are provided in the supplementary materials.
Table 1: Statistics of seven multi-view multi-label datasets. #Samples is the number of samples; #Views is the number of views; #Features are the dimensions of all views; #Labels is the number of distinct labels; #Average is the average number of labels per sample.

| Dataset     | #Samples | #Views | #Features | #Labels | #Average | Domain         |
|-------------|----------|--------|-----------|---------|----------|----------------|
| Corel5k     | 4999     | 6      | 100/512/1000/4096/4096/4096 | 260     | 3.397    | image          |
| ESPGame     | 20770    | 6      | 100/512/1000/4096/4096/4096 | 268     | 4.686    | image          |
| IAPRTC-12   | 19627    | 6      | 100/512/1000/4096/4096/4096 | 291     | 5.719    | image          |
| Mirflickr   | 25000    | 6      | 100/512/1000/4096/4096/4096 | 38      | 4.716    | image          |
| Pascal07    | 9963     | 6      | 100/512/1000/4096/4096/4096 | 20      | 1.465    | image          |
| Yeast       | 2417     | 2      | 79/24     | 14      | 4.237    | biology        |
| Emotions    | 593      | 2      | 64/8      | 6       | 1.869    | music          |

Table 2: Experimental results on seven real-world datasets with $r\% = 50\%$ and $s\% = 50\%$. The best results are highlighted in boldface.

(Corel5k, ESPGame, and IAPRTC-12) with a relatively large number of labels. We evaluate the performances on the head labels and the tail labels separately, and show the results in Fig. 3. It can be seen that CEMENT outperforms CEMENT-2 in all the metrics on both head labels and tail labels. Moreover, the performance advantage of CEMENT over CEMENT-2 is higher on tail labels than on head labels, indicating that capturing tail labels is beneficial to recover the unobserved labels.

Figure 4: Hyperparameter sensitivity analysis of CEMENT under different combinations of $\lambda$ and $k$ on the Corel5k dataset.

**Parameter Analysis.** We analyze the sensitivity of CEMENT w.r.t. $\lambda$ and $k$. The hyperparameter $\lambda$ controls the sparsity of the tail label matrix $E$, and $k$ controls the dimension of the subspaces. The value of $\lambda$ is selected from $\{0.2, 0.5, 0.8\}$. The results in terms of HS and AUC on the Yeast dataset are reported in Fig. 4, and similar results are obtained on the other datasets. From Fig. 4, we can see that CEMENT achieves relatively stable and good performance when $\lambda \approx 10^{-3}$ and $k = 0.2$. And we can also observe that when $\lambda > 10^{-2}$, HS and AUC decrease sharply. The possible reason is that CEMENT may not successfully capture long-tailed labels, given a large penalty on $\|E\|_1$. It again confirms the contribution of capturing long-tailed labels in improving the performance of CEMENT.

6 Conclusion

In this paper, we propose a novel model named CEMENT to deal with incomplete multi-view weak-label data. CEMENT jointly embeds incomplete views and weak labels into low-dimensional subspaces with adaptive weights, and adaptively correlates them via HSIC. Moreover, CEMENT explores an additional sparse component to model tail labels, making the low-rankness available in the multi-label setting. An alternating algorithm is developed to solve the proposed optimization problem. Empirical evidence verified that CEMENT is flexible enough to handle the incomplete multi-view weak-label learning problems in presence of noisy views and tail labels, leading to improved performance.
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