Searching for ultralight bosons within spin measurements of a population of binary black hole mergers

Ken K. Y. Ng,1,* Otto A. Hannuksela,2,1 Salvatore Vitale,1 and Tjonnie G. F. Li2

1LIGO Lab, Department of Physics, and Kavli Institute for Astrophysics and Space Research, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge MA 02139, USA
2Department of Physics, The Chinese University of Hong Kong, Shatin, NT, Hong Kong

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Ultralight bosons can form clouds around rotating black holes if their Compton wavelength is comparable to the black hole size. The boson cloud spins down the black hole through a process called superradiance, lowering the black hole spin to a characteristic value. It has thus been suggested that spin measurements of the black holes detected by ground-based gravitational-wave detectors can be used to constrain the mass of ultralight bosons. Unfortunately, a measurement of the individual black hole spins is often uncertain, resulting in inconclusive results. In this paper we use hierarchical Bayesian inference to combine information from multiple gravitational-wave sources and obtain stronger constraints. We show that 25+62 −57 (80+27 −12) high signal-to-noise ratio LIGO/Virgo detections are enough to exclude (confirm) the existence of bosons in the [10−11, 3 × 10−12] eV mass range. We then apply our method to the 10 binary black hole mergers detected by LIGO and Virgo in their first two observing runs, finding that we cannot draw statistically significant conclusion from the current sources, given their small number.

I. INTRODUCTION

Ultralight bosons have been proposed as a potential solution to various problems ranging from fundamental physics to cosmology [1–3]. These bosons encompass a broad range of particles, including dilatons and moduli [4], wave dark matter [5] and axion-like particles [6]. Indeed, bosons with masses ≲ 10−11 eV are usually termed ultralight bosons.

Intense efforts are underway to search for ultralight bosons using table-top experiments or astronomical observations based on electromagnetic probes [7–19]. The existence of ultralight bosons could also be revealed using astrophysical measurements of black hole (BH) spins. If ultralight bosons exist, BHs in the resonant mass range (defined below) will rapidly spin down through a process called “superradiance” [20–22], a classical wave-amplification process [23, 24]. Thus, ultralight bosons could leave measurable signatures in the spins of BHs detected in X-ray binaries or compact binaries [e.g. 22, 25–37].

Superradiance takes place when the Compton wavelength of the boson is comparable to the size of the BH, (i.e., when the “gravitational fine structure constant” αG satisfies: αG = GMμs/ℏc ≈ 1, where M is the BH mass and μs is the boson mass). If that condition is met, a bosonic cloud forms around the BH, which extracts rotational energy from the BH until it reaches a critical spin set by the Compton frequency of the boson [20–22].

The structure of the boson cloud is similar to the electronic structure of a hydrogen atom, and can be described by an analogous set of “quantum numbers” (n, l, m) corresponding to its eigenstates [38]. The cloud associated with the eigennumbers (n, l, m), grows over a timescale τgrow,mn (Supplementary Material), lowering the BH spin. As the cloud possesses a rotating quadrupole moment due to its non-axisymmetric nature [21, 22], it loses energy through gravitational-wave emission and eventually dissipates [22, 26]. We note that the timescale for the growth of the cloud is larger for higher modes (i.e. for larger l’s) [39], hence only modes for which the characteristic timescale is smaller than the Hubble time practically contribute to the process 1. Since this process depends on the BH mass, and leaves a clear imprint on the resulting BH spin, we can check for the existence of ultralight bosons by statistically measuring masses and spins of a population of BHs [2, 22, 25–29]. Thus, if ultralight bosons exist, the population of BHs with resonant mass should show a dearth of highly spinning BHs, as they will have been spun down due to superradiance, acquiring spins close to the critical values. The binary black holes mergers (BBHs) observed with gravitational-wave (GW) detectors such as LIGO [40] and Virgo [41] can theoretically be used to probe the existence of boson clouds. Intriguingly, all BHs detected by GW detectors to date are consistent with have small or no spin [42, 43]2.

The morphology of the GW signals emitted by BBHs encodes the properties of their sources, including the spins of the two compact objects. However, the spin measurements with ground-based detectors such as LIGO and Virgo are usually poor [46, 47], making it difficult to set stringent constraints on the boson masses with individual sources. Furthermore, the fraction of BHs with high spins at merger also depends on the distribution of spins at birth, which is currently unknown. One must thus disentangle the effect of potential ultralight bosons from the astrophysical spin distribution [48–50]. We tackle this problem by performing hierarchical Bayesian inference on a population of simulated BBHs, to simultaneously infer

1 In this work we assume non-interacting bosons, and note that if this were not the case, there can be other non-linear effect such as “bosenova” [2, 20].
2 As we finalizing this work, a group outside of the LIGO-Virgo collaboration has reported potential BBH sources with high spins [44, 45]. We do not consider these events in this study.
the mass of the boson and the initial spin distribution [51–54]. We show that the existence of ultralight bosons in the $[10^{-13}, 3 \times 10^{-12}]$ eV mass range can be ruled out (proven) with $25_{-20}^{+62} (80_{-57}^{+94})$ high signal-to-noise ratio (SNR) BBH detections, which corresponds to a total number of events equal to $1150_{-920}^{+2850} (3680_{-2620}^{+4320})$. LIGO and Virgo are currently discovering $\sim 1$ BBH per week, a number which is going to increase considerably as the detectors evolve to their design sensitivity [42, 55]. In a five year timescale, with the LIGO and Virgo detectors upgraded to use frequency-dependent squeezing [56], we can expect hundreds of BBH detections per year. That number will be in the hundreds of thousands per year when next-generation detectors are built [57].

We also apply our method to perform the first rigorous spin-based search for ultralight bosons with LIGO and Virgo detections from the first two observing runs [42]. We find no clear statistical evidence to support or reject the existence of ultralight bosons in current data.

II. METHOD

An astrophysical distribution of spins at birth which produces mainly small spins in absence of superradiance is partially degenerate with one that produces moderate (or high) spins, in presence of superradiance 3. Hence, we need to simultaneously infer the spin distribution at birth and the boson mass to properly account for this degeneracy.

We use hierarchical Bayesian inference [52–54], and consider two competing models: a) in the “boson model”, $\mathcal{H}_B$, we assume that bosons exist such that BHs can spin down to the corresponding critical spins, $\chi_{\text{crit}}$ (Supplementary Material) through superradiance; b) in the “astrophysical model”, $\mathcal{H}_A$, ultralight bosons do not exist, and the spin of BHs merging in binaries is entirely determined by their astrophysical evolution.

We distinguish the two hypotheses through the resulting distribution of the BH spins at merger. Specifically, for $\mathcal{H}_B$ we assume:

$$\mathcal{H}_B : \chi_M = \begin{cases} \chi_F, & \text{if } \chi_F < \chi_{\text{crit}}(M, \mu_s, \tau_s) \\ \chi_{\text{crit}}, & \text{otherwise} \end{cases}$$

(1)

where $\chi_F$ and $\chi_M$ are the values of the individual BH spins at formation and at merger, respectively, and $\tau_s$ is the timescale for the two BHs in the binary to merge. The fact that superradiance can only happen for up to a time $\tau_s$ (after which the two BHs merge) can be used to get an estimate of their critical spin, based on the instability time-scale of the first few dominant modes [2, 20, 26, 30] (Supplementary Material). For $\mathcal{H}_A$ there is no superradiance and one simply has:

$$\mathcal{H}_A : \chi_M = \chi_F.$$

For both models, we parametrize the distribution of BH spins at formation with a beta distribution, controlled by two unknown shape parameters $\alpha > 0$ and $\beta > 0$: $p(\chi_F | \alpha, \beta) \propto \chi_F^{\alpha-1} (1-\chi_F)^{\beta-1}$. This is a quite generic functional form that can capture multiple different formation pathways [43, 58]. The boson model thus depends on three hyper-parameters $\Lambda_{\mathcal{H}_B} = (\alpha, \beta, \mu_s)$, while the astrophysical model only has two hyper-parameters $\Lambda_{\mathcal{H}_A} = (\alpha, \beta)$. We aim at measuring the hyper-parameters $\Lambda$, given a set of $N$ GW observations $d = \{d_k\}$, whose morphology depends on a set of unknown parameters $\theta$ [59]. This can be written as [52–54]:

$$p(\Lambda | d) \propto \pi(\Lambda) \prod_k p(\theta | \Lambda) p(d_k | \theta) d\theta$$

where $p(\theta | \Lambda)$ is the expected distribution of the individual events parameters, given the hyper-population parameters; $\pi(\Lambda)$ are the priors of the hyper-parameters and $p(d_k | \theta)$ is the likelihood of the $k$-th GW source.

When working with the boson model, $\Lambda_{\mathcal{H}_B} = (\alpha, \beta, \mu_s)$ and $\theta_{\mathcal{H}_B} = (M_1, M_2, \chi_1^M, \chi_2^M, \tau_s)$, where $M_i$ and $\chi_i^M$ are the mass and spin (at merger) of the two compact objects in the binary. One thus has:

$$p(\chi_i^M | \alpha, \beta, \mu_s, \tau_s) = (1 - f_{\text{SR}}) p(\chi_i^F | \alpha, \beta) + f_{\text{SR}} \delta(\chi_i^M - \chi_{\text{crit}}(M_1, \mu_s, \tau_s))$$

(3)

where $f_{\text{SR}}$ is the fraction of BHs that undergo superradiance, i.e. for which $\chi_i^F > \chi_{\text{crit}}$: $f_{\text{SR}} \equiv \int_{\chi_{\text{crit}}}^{\infty} p(\chi^F | \alpha, \beta) d\chi^F$, and $\delta$ is the Dirac-delta function to describe the fraction of residue spins after superradiance.

In the astrophysical model, $\mathcal{H}_A$, one obtains a similar expression for $p(\Lambda_{\mathcal{H}_A} | d, \mathcal{H}_A)$ by replacing $\mathcal{H}_B \rightarrow \mathcal{H}_A$ every-

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3 Although in the latter case one would expect a characteristic peak at around the critical spin value.
where, and removing all references to \(\tau_s\), which is not a parameter of that model.

While calculating Eq. (2), we use a uniform prior for the masses, \(\pi(M_1, M_2)\), and we assume that \(\tau_s\) is known (for the \(\mathcal{H}_B\) model) and fixed at 10 Myr: \(\pi(\tau_s) = \delta(\tau_s - 10\text{Myr})\), which is toward the lower limit of typical merger times, according to numerical simulations (~10 Myr – ~10 Gyr) \([60–70]\). From one side, this choice is conservative as it allows for the least time for BHs to spin-down due to superspin- radiance, making it harder to find evidence for bosons. From the other, restricting the merger time prior overestimates the prior information thus overestimates the evidence for the boson hypothesis \(\mathcal{H}_B\) in Eq. (2). Nevertheless, the additional parameter space in \(\tau_s\) is expected to contribute modestly to the Bayes factor because the corrections to the available mass-spin parameter space are mostly smaller than the mass-spin measurement uncertainties. Therefore, while more realistic models for the merger time prior \(\pi(\tau_s)\) could be used, our choice is sufficient for advanced detectors, given their limited precision in the measurement of component masses and spins.

We do not need to account for selection effect on BH masses, as a fixed prior on the BH masses only contributes an overall normalization constant to the hierarchical posterior \([52–54]\). We also ignore selection effects of BH spins as the sensitive volume for different spins only varies by \(\lesssim 10\%\) \([51, 58, 71]\).

Integrating Eq. (2), and the equivalent expression for \(\mathcal{H}_A\), over the whole hyper-parameters space yields evidences \(Z_{\mathcal{H}_B}\) and \(Z_{\mathcal{H}_A}\) that can be used to calculate Bayes factor between the boson and astrophysical hypothesis: \(B_{\mathcal{A}}^{\mathcal{B}} = Z_{\mathcal{H}_B}/Z_{\mathcal{H}_A}\). We follow Ref. \([72]\) and strongly prefer the boson (astrophysical) hypothesis if \(B_{\mathcal{A}}^{\mathcal{B}} \geq 10\) (\(\lesssim 0.01\)).

### III. MOCK DATA ANALYSIS AND O1/O2 SEARCH

The method described above can be applied to both simulated and real detections. We first demonstrate its use to infer the properties of two sets of simulated BBHs: one for which we consider a boson scalar that can induce superspin-radiance (“boson population”), and one were such boson does not exist (“astrophysical population”). To create the mock-up populations, we generate BBHs with component masses \(M_{1,2}\) uniform in \([5, 50]\)\(\odot\), consistently with Ref \([43]\), luminosity distances uniform in source-frame comoving volume, and sky positions, orbital orientations and polarization angles uniform in the unit-sphere.

The astrophysical processes that set the initial spin magnitude and orientation are still to be fully understood \([48–50]\). We assume the formation spin magnitudes \(\chi_i^F\) to be uniform in \([0, 1]\), with an isotropic spin orientation.

When simulating signals for the boson population, we need to convert the spins at formation to the spins at merger using Eq. (1). For all the BHs in the boson population, we use a boson mass of \(\mu_b = 10^{-12}\text{eV}\), which is appropriate for stellar-mass BHs \([29]\), and assume all BBHs have a short merger time \(\tau_s = 10\) Myr. To maintain the computational cost of the analysis reasonable, of all the sources we generate, we only analyze those for which SNR > 30. These are the only sources that will contribute to the test anyways, since individual spins are hard to measure for low or medium SNR BBHs \([46, 47]\). The two populations of synthetic BBH sources are thus added into simulated noise of the LIGO and Virgo detectors at design sensitivity \([73, 74]\). We use the LALInference \([75, 76]\) algorithm with the IMRPhenomPv2 waveform family \([77]\) to obtain posterior and likelihood distributions for the compact binary parameters of the simulated sources, which can be used to infer the population hyper-parameters as described in the previous section. For all of the hyper-parameters, we use uniform-in-log priors, with ranges \(0.01 \leq \alpha, \beta \leq 10\) and \(10^{-13}\text{eV} \leq \mu_s \leq 3 \times 10^{-12}\text{eV}\) which is the range of boson masses that could be probed by stellar-mass BHs \([29]\). First, we show the evolution of the Bayes factor boson vs astrophysical model as more events are used for the test, Figure 1. The bottom x-axis show the number of loud events, while the top one shows the number of total events \(^4\).

![Figure 1](image-url)

**FIG. 1.** The \(\log_{10}\) Bayes factor between the boson and astrophysical hypothesis as a function of the number of sources \(N\) from the boson (blue) and astrophysical (orange) populations. The solid lines and colored bands are medians and 90\% credible intervals over 50 realizations of a population with \(N\) sources. The existence of bosons can be ruled out (confirmed) with \(25_{-20}^{+62}\) (80_{-57}^{+94}) high-SNR detections.

When analyzing the boson population, blue curves, the Bayes factor in favor of the boson model \(B_{\mathcal{A}}^{\mathcal{B}}\) increases steadily as more sources are used, as one would expect. Approximately \(80_{-57}^{+94}\) loud detections are required to reach \(B_{\mathcal{A}}^{\mathcal{B}} = 100\). Conversely, when the astrophysical population is analyzed, \(B_{\mathcal{A}}^{\mathcal{B}}\) decreases, and approximately \(25_{-20}^{+62}\) high SNR detections are required to rule out \(\mathcal{H}_B\). We notice that fewer sources are required to disprove the boson hypothesis than to confirm it. This is because even one highly spinning BH measurement contradicts the boson hypothesis, whereas multiple BHs that match the predicted critical spins are necessary to distinguish \(\mathcal{H}_B\) from an initially low-spin distributions.

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\(^4\) Since the distribution of SNRs for BBH detected by advanced detectors is known analytically and goes as \(P(\rho) \propto \rho^{-4}\) \([78]\), one can calculate that 1 in 46 events has SNR > 30 for a threshold SNR = 12 \([79]\).
Next, we look at the estimation of the individual hyper-parameters. In Fig. 2, top panel, we show the inference on the parameters of the boson population when \( H_B \) (blue) or \( H_A \) (orange) is used. The \( H_B \) model yields an accurate measurement of the boson mass of \( 1.02^{+0.81}_{-0.15} \times 10^{-12} \) eV. Conversely, the astrophysical model obtains biased posteriors for the two spin parameters.

While we only consider scalar field bosons in this study, the method we developed is applicable to vector or tensor boson fields, which have much shorter instability and GW emission timescales \([81, 82]\).

Our analysis on simulated BBHs has made a few simplifying assumptions which made it conservative. First, we have assumed that all BBH merge in 10 Myr, which is toward the lower limit of what is usually obtained in numerical simulations \([60–70]\). Longer merger times give more space for the boson clouds to form and reduce the BH spin, hence producing an easier-to-measure effect. Second, we have assumed that only sources with SNR > 30 will contribute to this test, as their spins are easier to measure. In reality, while the component spins of marginal events are indeed harder to measure, a large number of them will still be useful for the test.

The true distribution of spins at formation (which we assumed to be uniform in magnitude and isotropic in orientation) plays the most important role: the number of events needed to perform this test will be larger than what we found if the astrophysical distribution of spins at formation is such that smalls spins are preferred. Conversely, if many highly-spinning BHs are formed, potentially with significant misalignment between spin and angular momentum (both of which makes spins easier to measure), then fewer sources will be necessary.

Within the assumptions made in this study, it seems feasible to rule out the existence of ultralight bosons with a few years of Advanced LIGO/Virgo data. Statistically proving the existence of these bosons will take longer, as more sources are required: the planned upgrades of LIGO and Virgo to their “plus” configurations might yield thousands of BBH events per year, which will make it possible to gather evidence for the existence of ultralight bosons.

If we have shown that the BBHs detected by LIGO and Virgo can be used to either reveal or rule out the existence of ultralight bosons with masses in the range \( 3 \times 10^{-13} - 10^{-11} \) eV. First, we generated populations of simulated BBH with a uniform distribution of spins at formation, and shown that combining \( 25^{+62}_{-26} (80^{3}_{57}) \) high-SNR events we may rule out (confirm) the existence of an ultralight boson. Applying the same method to the 10 BBH published by LIGO and Virgo yields inconclusive results. This is not surprising, based on the results on simulated signals.

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Appendix A: Critical spin arising from superradiant instability

GW measurements only yield the mass and spins of black holes at merger. These are in general not equal to the values at formation, due to superradiance. To apply our method, we need to evolve the spin at formation to obtain the spin at merger. The initial and final mass and spin values can be related by solving the system of differential equations that governs the transfer of mass and angular momentum between the black hole and the axion cloud [20–22]. As the cloud grows, the mass and spin of the black hole decrease, until the cloud is massive enough to trigger the superradiance instability.

For each \((nlm)\) mode of the axion hydrogen-like cloud, the critical spin \(\chi_{nlm}\), i.e. the minimum spin for which superradiance can occur, can be analytically computed by using the eigenfrequency \(\omega_R(M, \mu_s, n, l, m)\) into the superradiance condition \(\omega_R = m\Omega_{BH}\), see Ref. [29] Eqs. (2-8). In this appendix, \(M\) is the black hole mass, \(\mu_s\) is the axion mass, \(\omega\) is the boson field, \(\chi\) is the dimensionless spin of the black hole, \(r_+ = M(1 + (1 - \chi^2)^{1/2})\) is the outer horizon, and \((j, n, l, m)\) is the set of total angular-momentum, radial, orbital azimuthal and magnetic quantum numbers.

The growth timescale of a given mode hence depends on the initial spin. For the \((011)\) mode it varies by roughly one order of magnitude for initial spins in the range \([1, \chi_{011}]\). Since \(\tau_{011} \ll 1\)Myr in most of the parameter space we cover, this difference can be neglected. Therefore, instead of calculating the growth timescale of each black hole individually, we calculate it for the average spin \((1 + \chi_{011})/2\), removing its dependence on the initial spin.

When we need to calculate the initial spin for higher order clouds, we set it to the critical spin of the preceding cloud, so for example the initial spin for the \((022)\) cloud is set to be \(\chi_{011}\).

Third, after the formation of the \((011)\) mode cloud, there begins a depletion phase during which the cloud evaporates through GW emission until it almost vanishes, starting the superradiance process for the \((022)\) mode [20]. Including the depletion phase would increase the time required for the host BH to develop the \((022)\) cloud and acquire a spin equal to \(\chi_{022}\). However, we verify that the growth timescales of higher order modes are much larger than the depletion timescales of previous modes (except for small \(\alpha C\)'s), for which the growth timescales are comparable to the depletion timescales [31,39]. We thus ignore the depletion time of the previous clouds when computing the final spin after superradiance, de facto assuming that the growth timescale of the last relevant mode dominates the total time interval.

With the approximations described above, we can ignore the dependence of the superradiance process on the initial spin \(\chi_F\) and calculate the expected final spin after superradiance for the \((nlm)\)-mode given the observed BH mass, the boson mass and merger timescale. We assume that BBHs merge \(t \sim 10\)Myr after formation, which is a conservative estimate [60–70]. Given this merger timescale \(\tau_s\), we calculate the boson model evidence and determine the expected fraction of BHs that has been spun down by superradiance \(f_{SR}\) by computing all possible \(\chi_{nlm}\) and the corresponding \(\tau_{nlm}^{\text{inst}}\) and \(\tau_{nlm}^{\text{grow}}\) as follows:

1. For each posterior sample (which gives the black hole mass, \(M_i\)) and a given set of population hyperparameters \((\mu_s, \alpha, \beta)k\), compute \(\chi_{011}\) from the superradiance condition in Ref. [29] by solving \(\Gamma_{011} = 0\);

\[
\Gamma_{nlm}^{\text{inst}} = \mu_s (\mu_s M)^{4l+4} (m\chi - 2\mu_s r_+^2) \\
\times \frac{2l+2}{(l+1+n)! \left[ (2l)! (2l+1)! \right]^2} \prod_{j=1}^l \left[ j^2 (1 - \chi^2) + (m\chi - 2\mu_s r_+^2)^2 \right],
\]

where \(\tau_{nlm}^{\text{inst}}\) is the inverse of the superradiant rate [20–22, 38]:

\[
\tau_{nlm}^{\text{inst}} = \frac{\mu_s}{\Gamma_{nlm}^{\text{inst}}},
\]

\[
= \left[ 190 + \log \left( \frac{M}{10M_\odot} \times 10^{-12}\text{eV} \right) \mu_s \right] \tau_{nlm}^{\text{inst}}, \tag{A1}
\]

\[
\tau_{nlm}^{\text{grow}} = \log \left( 10^{-4} M/\mu_s \right) \tau_{nlm}^{\text{inst}} \approx 10^{-4} M/\mu_s \tau_{nlm}^{\text{inst}}, \tag{A2}
\]

\[
\chi_{011} \approx \frac{1}{2} + \frac{1}{2} \log \left( \frac{M}{10M_\odot} \times 10^{-12}\text{eV} \right) \mu_s \]

\[
(022)\text{ superradiance cannot happen unless the initial spin is larger than } \chi_{011}.
\]

Remember that for any \((nlm)\), superradiance cannot happen unless the initial spin is larger than \(\chi_{nlm}\).
2. Compute $\tau_{\text{grow}}^{011}$ from Eq. (A1-A2) by setting the initial spin to be $\chi = (1 + \chi_{011})/2$ as discussed.

3. If $\tau_{\text{grow}}^{011} > \tau_s$, i.e., when the cloud does not have enough time to grow fully because the merger happens, then the merger spin is the same as the formation spin. If $\tau_{\text{grow}}^{011} < \tau_s$, then repeat steps 1-2 for the (022) mode using the same $M_i, (\mu_s, \alpha, \beta)_k$ and setting $\chi = \chi_{011}$ in Eq. (A1). There are three possible cases:

(a) The superradiance condition for the (022) mode is not satisfied, i.e., $\chi = \chi_{011} < \chi_{022}$.

(b) The superradiance condition for the (022) mode is satisfied and its growth timescale is larger than the merger timescale, i.e., $\chi = \chi_{011} > \chi_{022}$ but $\tau_{\text{grow}}^{022} > \tau_s$.

(c) The superradiance condition for the (022) mode is satisfied and its growth timescale is smaller than $\tau_s$, i.e., $\chi = \chi_{011} > \chi_{022}$ and $\tau_{\text{grow}}^{022} < \tau_s$.

If (a) or (b) happens, exit the calculation at $\chi_{\text{crit}} = \chi_{011}$ and set $f_{\text{SR}}$ as the portion of systems with $\chi > \chi_{011}$ in the hyper-population being considered, $p(\chi_{\mu_s, \alpha, \beta})$. If (c) happens, repeat steps 1-2 for the (033) mode, perform the same superradiance or timescale condition checks and evaluate $f_{\text{SR}}$ at $\chi_{\text{crit}} = \chi_{022}$, depending on whether (a) or (b) are satisfied for the (033) mode. If (c) is true for the (033) mode, stop the calculation at $\chi_{\text{crit}} = \chi_{033}$ and exit with $f_{\text{SR}}$ calculated as the fraction of systems with $\chi > \chi_{033}$ in the population. We stop at the (033) mode since the next order has instability timescales which are much larger than the inspiral time: $\tau_{\text{inst}}^{044} \gg 10$ Myr.

4. Repeat steps 1-3 with another posterior sample $M_{i+1}$ at the same population parameters $(\mu_s, \alpha, \beta)_k$ in order to evaluate the integral on the right-hand side of Eq. (2) with importance sampling.

5. Repeat steps 1-4 with another set of population parameters $(\mu_s, \alpha, \beta)_k$ in order to evaluate the model evidence with nested sampling.

Note that in principle, the merger could cause the cloud to fall back to its host through level mixing [37, 83]. By a naive angular momentum conservation, one could think that this results in a transfer of the cloud’s angular momentum back to the host black hole, which would be spun-up. However, most infalling modes have negative angular momentum (i.e. $m < 0$), which is why recent studies have suggested that the in-falling cloud instead spins down the host black hole and transfer angular momentum to the companion compact objects [37, 83–85]. This would further increase the size of the forbidden region on the mass-spin plane, making it easier to verify the existence of bosons with the method we developed.

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