The Optimal Timing of an Announcement for a Merger and Acquisition

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Abstract: In this paper, we propose a real options model for determining the optimal timing of a merger and acquisition (M&A). The estimated increased firm value of the acquirer after a merger may need to be reassessed when the economic situations or the two firms’ individual circumstances change dramatically during the period for preparing an M&A. Focusing on this point, we model the changes in increased firm value by using a discrete stochastic process. As the cost of an M&A is related to the market price of the target, we assume that the cost varies to follow a geometric Brownian motion. We derive explicit formulas for the optimal timing and expected waiting time to announce an M&A under the 2-dimensional stochastic process. Furthermore, we analyze the effects on the optimal timing and the expected waiting time by changing parameters’ value.

Keywords: Merger and acquisition; Optimal timing; Two-dimensional stochastic process; Real options model
1 Introduction

From the point of view of corporate finance, the gain of merger and acquisition (M&A) activity is measured by the difference between the acquirer’s increased firm value after the merger and the cost of purchasing the target firm. Therefore, to analyze the effect of an M&A, the acquirer has to estimate the potential increased cash flows after the merger, that is, to predict the differences between the cash flows which are consequent on taking over the target and those which are consequent on maintaining the present state, and then needs to evaluate their present value, which is referred to here as the increased firm value. To predict the cash flows after a merger, the acquirer has to analyze the synergies of the merger, for example, economies of scale, market share, effects in technologies, improvement in business risks. Thus operations of estimating the increased firm value are complicated and costly tasks. On the other hand, the purchasing cost is based on the stock price of the target at the time of an announcement to buy the target. If the target is a publicly traded company, the stock prices can be observed continuously. The acquirer has to determine the optimal timing of announcing to buy the target by observing the stock prices of the target, which maximize the expected present value of the gain of M&A.

The estimated increased cash flows may change under the influence of movements in economic situations, political environments, and the two firms’ individual circumstances in the period of preparing for M&A. So the acquirer may need to reassess the increased firm value. Because of the complexity and the costs of estimating the increased cash flows, reassessment may be difficult to carry out continuously. In this paper, we consider the situation that the events which cause the necessity of reassessing the increased firm value occur discretely. That is we consider the situation that the acquirer’s increased firm value changes discretely. In other words, the purpose of this paper is to analyze the optimal timing to announce an M&A under the assumption that the cost of M&A varies continuously, but the increased firm value changes discretely.

Recently, several papers discussed decisions of M&A activities in the real options framework. Lambrecht [5], Alvarez and Stenbacka [1], Lambrecht and Myers [6] assume that the two firms which consider to merge are facing one common uncertain factor which follows the geometric Brownian motion. Thus those papers use the one-dimensional real options model to analyze M&A activities. In Morellec and Zhdanov [7], Thijssen [9], two-dimensional models are studied. They use a correlated two-dimensional geometric Brownian motion to model the different risk factors that the two firms are facing respectively. In this paper, we assume that events that cause the increased firm value to shift up or down occur to follow a Poisson process respectively, and the sizes of up and down jumps of increased firm value have exponential distribution respectively. The cost of purchasing is relative to the stock prices of the target, and the movements in the stock price are assumed to follow a geometric Brownian motion. That is, we use a two-dimensional Lévy Process containing of a compound Poisson process and a geometric Brownian motion to analyze M&A activities. In the papers mentioned above, the thresholds of M&A are studied. In this paper, we also analyze the expected waiting time of the announcement of M&A.

Kou and Wang [4] obtains the optimal exercise boundary and the pricing formula for the perpetual American options when the stock prices follow a jump-diffusion process. In this paper, we convert the integral-differential equation related to the two-dimensional Lévy process into a one-dimensional
one by applying the homogeneity of the payoff function, and then, solve the problem. Eventually, the optimal solutions obtained here are very close to those in Kou and Wang [4].

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 shows the analytical solutions. Section 4 shows the results of comparative statics analysis of the threshold. Section 5 analyzes the effects of the variation patterns of the increased firm value by numerical examples, and discusses the relationship between the threshold, the expected present value of M&A, and the expected waiting time. Section 6 concludes.

2 The Model

Consider that an acquirer plans to take over a target firm. In order to measure the gain from M&A activities, the acquirer needs to estimate the differences between the future cash flows involved in acting and not acting, and evaluates their present value. We call the present value of the difference between the future cash flows of acting and not acting as the increased firm value of the acquirer. In the period for preparing an M&A, when an economic shock that influences the two firms’ business environment occurs, the acquirer may need to reassess the increased firm value. Economic shocks may have positive or negative effects on the future cash flows, this makes the increased firm value shift up or down uncertainly. The cost of acquisition, however, depends on the stock prices of the target at the time when an M&A is announced. In such an uncertain environment, therefore, the acquirer has the incentive to choose a proper timing to announce an M&A.

Although business environment changes frequently, for the acquirer, it may be difficult to reassess the increased firm value continuously, or it may be too costly to do so. We assume that reassessments only will be carried out when the economic situation changes dramatically, or when there occurs an event that has a serious impact on the two firms involved. Therefore, the estimated increased firm value shifts up or down discretely. Let $X_1(t)$ denote the increased firm value at time $t$, then $X_1(t)$ is a discrete stochastic variable. Assume that the events that cause upward jumps in the increased firm value occur following a Poisson process with parameter $\kappa$. When an upward jump occurs, $X_1(t)$ becomes $YX_1(t)$. The events that cause downward jumps in the increased firm value occur following a Poisson process with parameter $\lambda$. When a downward jump occurs, $X_1(t)$ becomes $X_1(t)/Z$. The multiples $Y > 1$ and $Z > 1$ are also stochastic variables, $y = \log Y$ and $z = \log Z$ to be assumed have exponential distributions with parameters $\zeta(>1)$ and $\eta > 0$. $Y$, $Z$, and the two Poisson processes are all mutually independent.\(^1\) Then the infinitesimal generator of $X_1(t)$ is given by

$$L(U(x_1)) = \kappa \{E[U(Yx_1)] - U(x_1)\} + \lambda \{E[U(x_1/Z)] - U(x_1)\}.$$ 

The purchase price is the stock price of the target at the time of an announcement of M&A by adding a premium which proportional to the stock price. For simplification, the acquisition cost $X_2(t)$ is assumed to be the purchase cost of issued stocks of the target. Assume that the stock price of the target varies to follow a geometric Brownian motion with parameters $\mu$ and $\sigma$. Therefore, $X_2(t)$ also follows a geometric Brownian motion,

$$dX_2(t) = \mu X_2(t)dt + \sigma X_2(t)dW(t).$$

\(^1\)Kou and Wang [4] uses a similar jump process adding to a geometric Brownian motion to model stock prices. As the expected value of $Y$ goes to infinity when $\zeta \leq 1$, here, we assume $\zeta > 1$. 

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Let $r$ denote the risk-adjusted discount rate, and $\tau$ denote the optimal timing of an announcement of the M&A. Then the expected present value of the gain of the M&A, hereinafter referred to as the expected present value of M&A, is given by

$$V(x_1, x_2) = \mathbb{E}[e^{-rt}(X_1(\tau) - X_2(\tau)) | \{X_1(0), X_2(0)\} = \{x_1, x_2\}],$$

(1)

The main purpose, below, is to derive the optimal timing of an announcement of M&A that maximizes the expected present value of M&A.

The infinitesimal generator of $\{X_1(t), X_2(t)\}$ is given by

$$\mathcal{L}V(x_1, x_2) = \lim_{t \to 0^+} \frac{\mathbb{E}[V(X_1(t), X_2(t)) | \{X_1(0), X_2(0)\} = \{x_1, x_2\}] - V(x_1, x_2)}{t},$$

$$= \frac{1}{2} \sigma^2 x_2^2 V_{22}(x_1, x_2) + \mu x_2 V_2(x_1, x_2)$$

$$+ \kappa \{\mathbb{E}[V(Y_1, x_2)] - V(x_1, x_2)\} + \lambda \{\mathbb{E}[V(x_1/Z, x_2)] - V(x_1, x_2)\},$$

(2)

where $V_2(x_1, x_2) = \partial V(x_1, x_2)/\partial x_2, V_{22}(x_1, x_2) = \partial^2 V(x_1, x_2)/\partial x_2^2$. Let $A$ denote the M&A announcement region in $\{x_1, x_2\}$, then the timing of an announcement is $\tau = \inf\{t | \{X_1(t), X_2(t)\} \in A\}$. Between time 0 and $s(\leq \tau)$, that is, before $\{X_1(t), X_2(t)\}$ reaches region $A$, $V(x_1, x_2)$ satisfies

$$V(x_1, x_2) = e^{-rt}\mathbb{E}[V(X_1(s), X_2(s)) | \{X_1(0), X_2(0)\} = \{x_1, x_2\}].$$

By the definition of the infinitesimal generator, we obtain

$$\mathcal{L}V(x_1, x_2) = rV(x_1x_2), \quad \{x_1, x_2\} \notin A.$$  

(3)

The boundary conditions of equation (3) are given by

$$V(0, x_2) = 0,$$

$$V(x_1, x_2) = x_1 - x_2, \quad \{x_1, x_2\} \in A.$$  

(4)

The first line of the boundary conditions means that the expected present value of M&A goes to zero as the increased firm value goes to zero. Under the above boundary conditions, we derive the solution of equation (3), and then derive the boundary of the region $A$ that maximize the expected present value of M&A.

In addition, the Lévy exponent $\varphi(s,t)$ of the 2-dimensional Lévy Process $\{X_1(t), X_2(t)\}$ satisfies

$$\mathbb{E}[X_1(\tau)^sX_2(\tau)^t | \{X_1(0), X_2(0)\} = \{x_1, x_2\}] = x_1^s x_2^t \exp[\varphi(s,t)\tau],$$

where the Lévy exponent is given by

$$\varphi(s,t) = \frac{1}{2} \sigma^2(t-1) + \mu t + \frac{\kappa s}{\zeta - s} - \frac{\lambda s}{\eta + s}.$$  

(5)

As the growth rate of expectation of $X_i(\tau), (i = 1, 2)$ is

$$m_i = \frac{d \mathbb{E}[X_i(\tau)]}{d\tau}/\mathbb{E}[X_i(\tau)], \quad i = 1, 2.$$
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\[
E[X_1(\tau) \mid \{X_1(0), X_2(0)\} = \{x_1, x_2\}] = x_1 \exp[\varphi(1,0)\tau],
\]
\[
E[X_2(\tau) \mid \{X_1(0), X_2(0)\} = \{x_1, x_2\}] = x_2 \exp[\varphi(0,1)\tau],
\]
it yields that
\[
m_1 = \varphi(1,0) = \frac{\kappa}{\zeta - 1} - \frac{\lambda}{\eta + 1}, \quad m_2 = \varphi(0,1) = \mu.
\]
Then we assume that \( r > m_i (i = 1, 2) \) below.\(^2\)

3 The Optimal Solution

As \( V(x_1, x_2) \) is a homogeneous function of degree one of \( \{x_1, x_2\} \), we can rewrite

\[
V(x_1, x_2) = x_2 V(x_1/x_2, 1).
\]

Let \( u = x_1/x_2 \), \( V(u, 1) = W(u) \), then equation (3) becomes\(^3\)

\[
\frac{1}{2} \sigma^2 u^2 W''(u) - \mu u W'(u) - (r-\mu)W(u) + \kappa \{E[W(Yu)] - W(u)\} + \lambda \{E[W(u/Z)] - W(u)\} = 0, \quad u < u^*,
\]
and the boundary conditions (4) becomes

\[
W(0) = 0; \quad W(u) = u - 1, \quad u \geq u^*.
\]

Where \( u^* \) denotes the threshold. Assume that

\[
W(u) = \sum_{j=1}^{2} A_j u^\alpha_j, \quad u < u^*
\]
to be the solution of equation (6), and combine with the boundary conditions (7), it yields

\[
W(u) = \begin{cases} 
\sum_{j=1}^{2} A_j u^\alpha_j, & u < u^*, \\
u - 1, & u \geq u^*.
\end{cases}
\]

The solutions of the problem are as follows.\(^4\) The M&A announcement region in \( \{x_1, x_2\} \) is

\[
\mathcal{A} = \{(x_1, x_2) \mid x_1/x_2 > u^*\},
\]
where the optimal boundary of the region \( \mathcal{A} \) is given by

\[
u^* = \frac{\zeta - 1}{\zeta} \frac{\alpha_1}{\alpha_1 - \frac{1}{\alpha_2}}.
\]

\(^2\)This condition is in order for the problem to have the optimal solution.

\(^3\)The transformation of a differential equation related with 2-dimensional geometric Brownian motion can be found in McDonald and Siegel [2], Dixit and Pindyck [3].

\(^4\)See Appendix A for details.
Where \( \alpha_1 \) and \( \alpha_2 \) are the two positive roots of the following equation

\[
F(x) = \frac{1}{2} \sigma^2 x(x-1) - \mu x - (r - \mu) + \frac{\kappa x}{\zeta - x} - \frac{\lambda x}{\eta + x} = 0.
\]  

(12)

The expected present value of M&A is

\[
V(x_1, x_2) = x_2 W(u),
\]

where \( W(u) \) is defined in equation (8), and

\[
A_1 = \frac{\zeta - \alpha_1}{\zeta} \frac{1}{\alpha_1 - 1} \frac{\alpha_2}{\alpha_2 - \alpha_1} u^{\alpha_1},
\]

(13)

\[
A_2 = \frac{\zeta - \alpha_2}{\zeta} \frac{1}{\alpha_2 - 1} \frac{\alpha_1}{\alpha_1 - \alpha_2} u^{\alpha_2}.
\]

(14)

The expected waiting time to announce an M&A is given by

\[
E[T(u)] = \begin{cases} 
\frac{1}{\bar{\mu}} \left[ \log \left( \frac{u^*}{u} \right) + \frac{\beta - \zeta}{\beta \zeta} \left( 1 - \left( \frac{u^*}{u} \right)^{-\beta} \right) \right], & \bar{\mu} > 0, \\
\infty, & \bar{\mu} \leq 0,
\end{cases}
\]

(15)

where

\[
\bar{\mu} = \frac{\kappa}{\zeta} - \frac{\lambda}{\eta} - \mu + \frac{\sigma^2}{2},
\]

(16)

and \( \beta \) is the positive root of the following equation

\[
G(x) = \frac{1}{2} \sigma^2 x(x-1) + (\sigma^2 - \mu) x + \frac{\kappa x}{\zeta - x} - \frac{\lambda x}{\eta + x} = 0.
\]

(17)

The first passage time \( T(u) \) is a time interval that form time 0 to the moment when \((X_1(t), X_2(t))\) reaches or exceeds the line \( x_2 = x_1/u^* \) for the first time, which starting from an initial point \((X_1(0), X_2(0))\) on a \((x_1, x_2)\) plane, where \( X_1(0)/X_2(0) = u < u^* \).

To show the meaning of \( \bar{\mu} \) in equation (16), let \( V(x_1, x_2) = \log(x_1/x_2) \), from equation (2), we obtain

\[
\mathcal{L}V(x_1, x_2) = \frac{\sigma^2}{2} - \mu + \frac{\kappa}{\zeta} - \frac{\lambda}{\eta} = \bar{\mu}.
\]

Thus, \( \bar{\mu} \) is the drift rate of \( \log[X_1(t)/X_2(t)] \).

4 Comparative Statics Analysis of the Threshold \( u^* \)

In this section, we analyze the influences on the threshold \( u^* \) of changing the parameters’ value. As equation (11) is a decreasing function of \( \alpha_1 \) and \( \alpha_2 \), it is clear that, except for \( \zeta \), \( u^* \) decreases or increases if \( \alpha_1 \) and \( \alpha_2 \) increase or decrease simultaneously when a parameter’s value changes. As \( \alpha_1 \) and \( \alpha_2 \) decrease simultaneously if equation (12) shifts upward by changing a parameter’s value, and \( \alpha_1 \) and \( \alpha_2 \) increase simultaneously if equation (12) shift downward by changing a parameter’s value, except for \( \kappa \) and \( \zeta \), the effects of the parameters are precise as shown in Table 1. The effects of
Table 1: Changes in the threshold $u^*$ as a parameter’s value increases.

| variables          | parameters            | changes in $u^*$ |
|--------------------|-----------------------|------------------|
| $X_2(t)$: cost of acquisition | $\mu$: drift rate    | decreasing       |
|                    | $\sigma$: volatility  | increasing       |
| $X_1(t)$: increased firm value | $\kappa$: expected frequency of upward jumps | increasing       |
|                    | $\lambda$: expected frequency of downward jumps | decreasing      |
|                    | $\zeta$: parameter of upward jump size | decreasing      |
|                    | $\eta$: parameter of downward jump size | increasing      |
|                    | $r$: discount rate    | decreasing       |
|                    | $E[Y]$: expected multiple of upward jumps | increasing      |
|                    | $E[1/Z]$: expected multiple of downward jumps | increasing     |

parameter $\kappa$ and $\zeta$ are confirmed by several numerical examples. As $\zeta$ and $\eta$ are the parameters of exponential distributions, it is hard to understand the substantive meaning of their effects, the effects of changes in the multiples of upward shifts $E[Y]$ and downward shifts $E[1/Z]$ are added to the last two rows of Table 1.

As shown in Table 1, an increase in the volatility ($\sigma$), or a strengthening in the upward jump (an increase in $\kappa$ or $E[Y]$) increases the threshold; an increase in the drift rate ($\mu$), or a strengthening in the downward jump (an increase in $\lambda$ or $E[1/Z]$) decreases the threshold.

5 Numerical Examples

Consider a situation that the drift rate and the volatility of the target firm’s stock price are $\mu = 0.02$ and $\sigma = 0.3$, and the risk-adjusted discount rate is $r = 0.2$. To show how the variation patterns of the acquirer’s increased firm value influence the optimal timing of the announcement, the expected present value of M&A and the expected waiting time, we consider the following seven cases. In case 1, the increased firm value is assumed to be constant. In case 2, there are 10% upward and downward jumps on average, which expected to occur once every 5 years. In case 3, there are bigger changes (20% upward and downward jumps on average) than case 2, but with lower frequency (once every 10 years on average). Case 4 and case 5 consider only the events which reduce the increased firm value may occur. In contrast, case 6 and case 7 consider only the events which augment the increased firm value may occur. The upper part of Table 2 shows parameters’ value of each case. The middle part of Table 2 shows $u^*$, $W(u)$, and the expectation of the first passage time $E[T(u)]$, where $u = x_1/x_2 = 1.4$ or 1.0 at time 0.6

In case 1, as the increased firm value is constant, the acquirer needs only to observe the variation in the stock prices of the target and waiting for the timing of the announcement. For example, if

5The derivation of the expectation of first passage time is described in Appendix B.

6When $\kappa = 0$, as equation (12) has only one positive root, the terms that contain $a_2$ and $\zeta$ are vanished in equations (11) and (13), and $A_2 = 0$ in equations (8) and (14). The expectation of the first passage time becomes

$$E[T(u)] = \begin{cases} \frac{1}{\bar{\mu}} \log \left( \frac{u^*}{u} \right), & \bar{\mu} > 0, \\ \infty, & \bar{\mu} \leq 0, \end{cases}$$

where $\bar{\mu}$ is as defined in equation (16) with $\kappa = 0$. 
Table 2: Variation patterns of the increased firm value and the results.

| Case | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|------|-----|-----|-----|-----|-----|-----|-----|
| \(\kappa\) | 0   | 0.1 | 0.05| 0   | 0   | 0.1 | 0.05|
| \(\lambda\) | 0   | 0.1 | 0.05| 0.1 | 0.05| 0   | 0   |
| \(E[Y]\) | 1.0 | 1.1 | 1.2 | 1.0 | 1.0 | 1.1 | 1.2 |
| \(E[1/Z]\) | 1.0 | 0.9 | 0.8 | 0.9 | 0.8 | 1.0 | 1.0 |
| \(u^*\) | 1.5409 | 1.5542 | 1.5635 | 1.5064 | 1.5110 | 1.5942 | 1.5987 |
| \(W(1.4)\) | 0.4116 | 0.4136 | 0.4151 | 0.4073 | 0.4078 | 0.4195 | 0.4205 |
| \(E[T(1.4)]\) | 3.837 | 4.593 | 5.409 | 5.274 | 6.104 | 3.844 | 4.056 |
| \(W(1.0)\) | 0.1574 | 0.1615 | 0.1646 | 0.1497 | 0.1508 | 0.1706 | 0.1726 |
| \(E[T(1.0)]\) | 17.296 | 19.256 | 21.656 | 29.500 | 33.022 | 13.724 | 14.202 |

The ascending order of \(u^*, W(u)\), and \(E[T(u)]\):

| Ascending order | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|-----------------|-----|-----|-----|-----|-----|-----|-----|
| \(u^*\)       | case 3 | case 4 | case 5 | case 1 | case 2 | case 6 | case 7 |
| \(W(1.4)\)     | case 3 | case 4 | case 5 | case 1 | case 2 | case 6 | case 7 |
| \(W(1.0)\)     | case 3 | case 4 | case 5 | case 1 | case 2 | case 6 | case 7 |
| \(E[T(1.4)]\)  | case 1 | case 4 | case 6 | case 5 | case 7 | case 2 | case 3 |
| \(E[T(1.0)]\)  | case 3 | case 4 | case 5 | case 6 | case 7 | case 1 | case 2 |

\((\mu = 0.02, \sigma = 0.3, r = 0.2)\)

the increased firm value \(X_1(t)\) is estimated to be 3.36 billion Yen, then the optimal timing of the announcement is that the acquisition cost \(X_2(t)\) goes below 2.18 billion Yen for the first time. If the number of the issued stocks of the target is 100 thousand, and the premium is 20%, then the timing of the announcement is when the stock price goes below 18,171 Yen per share for the first time. If the stock price now is 20,000 Yen, that is \(u = 1.4\), then the expected value of the M&A is \(V(x_1, x_2) = x_2 W(u) = 0.98785\) billion Yen. In addition, the expectation of the first passage time of \(u\) from 1.4 to \(u^*\) is 3.837 years.

As we assumed that the increased firm value changes discretely in this paper, \(X_1(t)\) remains unchanged until an event which causes necessity of reassessing the increased firm value occurs, thus, the timing of the announcement depends only on the stock price of the target as in case 1.

The lower part of Table 2 shows the ascending order of \(u^*, W(u), \text{ and } E[T(u)]\). We can see that the order of \(W(u)\) is the same as the order of \(u^*\), and the order of \(E[T(u)]\) does not depend on the order of \(u^*\). Furthermore, compared to the order of \(W(u)\) does not depend on the initial value \(u\), the order of \(E[T(u)]\) changes due to different values of \(u\). It could be considered naturally that an increase in threshold \(u^*\) delays the timing of an announcement of an M&A. Clearly, it is incorrect as shown in the above analysis. Although in one-dimensional real options model under geometric Brownian motion, an increase in threshold delays the timing of starting an investment, it is not the case under the situation that is considered here.\(^7\)

In order to elucidate the influences to \(W(u)\) and \(E[T(u)]\) from \(u\), values of \(W(u)\) and \(E[T(u)]\) which related to value of \(u\) are plotted in Figure 1 and Figure 2, where the scale of the horizontal axes is

\(^7\)Sarkar [8] shows some examples that a decrease in the threshold increases the probability of reaching the threshold in a certain period of time. This result is true only in the case that the expectation of the first passage time is infinite. In the case that the expectation of the first passage time is finite, the directions of change in the threshold and in the expected waiting time are the same.
set in $u/u^*$ which normalized by $u^*$. Curves on both figures can be divided into the “upward-jump group” (case 6 and 7), the “downward-jump group” (case 4 and 5), and the “middle group” (case 1, 2, and 3). In Figure 1, the upward-jump group, the middle group and the downward-jump group lie in decreasing order from top to bottom, the order of these groups in Figure 2 is the opposite to the order in Figure 1. (Although the order in a certain group is different in each figure, compared to the differences between the groups, the differences between curves in a certain group are smaller.)

The upward-jump group has the highest threshold, and also has the highest expected present value of M&A. Moreover, in spite of when the initial value ($u = x_1/x_2$ ) is far from the threshold, the expected waiting time does not becomes so long. In contrast, the downward-jump group has lowest expected present value of M&A, the expected waiting time becomes very long when the initial value is far from the threshold.

In order to ascertain that there is no direct relationship between the threshold and the expected waiting time, Table 3 shows changes in the expected waiting time (the “+” sign means increasing, and the “−” sign means decreasing) when the threshold changes that caused by changing a parameter’s value in case 2 of Table 2. Although the directions of change in the threshold and the expected waiting time are the same for some parameters ($\zeta$ and $r$), the directions of change in the threshold and the expected waiting time are not the same for most of the parameters.
6 Conclusion

In this paper, we develop a real options model for determining the optimal timing of an announcement of M&A. The problem is modeled by using a 2-dimensional stochastic process which contains a double exponential jump processes and a geometric Brownian motion. Closed form solutions for the optimal timing, expected value of the M&A, and the expectation of the first passage time are obtained.

We find that a strengthening in the upward jump of the increased firm value or an increase in the volatility increases the threshold; a strengthening in the downward jump of the increased firm value or an increase in the drift rate decreases the threshold. Although an increase (a decrease) in the threshold increases (decreases) the expected present value of M&A, it does not mean that a higher threshold leads to longer expected waiting time or a lower threshold leads to shorter expected waiting time. This result suggests that it is insufficient only depending on the threshold to make an investment decision, but also should take the expected waiting time into consideration when applying a two-dimensional real options model.

A Derivation of the Optimal Solution

Calculate the expectation of $W(uY)$ and $W(u/Z)$, substitute them into equation (6), and rearrange, it yields

$$
\sum_{j=1}^{2} A_j u^{\alpha_j} \left[ \frac{1}{2} \sigma^2 \alpha_j (\alpha_j - 1) - \mu \alpha_j - (r - \mu) + \frac{\kappa \alpha_j}{\xi - \alpha_j} - \frac{\lambda \alpha_j}{\eta + \alpha_j} \right] - \kappa \left( \frac{u}{u^*} \right)^{\zeta} \left[ \sum_{j=1}^{2} A_j u^{\alpha_j} \frac{\zeta}{\xi - \alpha_j} - u^* \frac{\zeta}{\xi - 1} + 1 \right] = 0.
$$

(A.1)
To ensure that equation (A.1) satisfies for arbitrary value of $u$, $\alpha_j$ must be the roots of equation (12). Combining the high-contact condition and the condition that the value in the square bracket of the second item in equation (A.1) must be zero, the optimal solution satisfies the following simultaneous equations:

\[
\begin{align*}
A_1 u^{*\alpha_1} + A_2 u^{*\alpha_2} - u^* + 1 &= 0, \\
\alpha_1 A_1 u^{*\alpha_1} + \alpha_2 A_2 u^{*\alpha_2} - u^* &= 0, \\
A_1 u^{*\alpha_1} \frac{\zeta}{\zeta - \alpha_1} + A_2 u^{*\alpha_2} \frac{\zeta}{\zeta - \alpha_2} - u^* \frac{\zeta}{\zeta - 1} + 1 &= 0.
\end{align*}
\]

(A.2)

Solving for $u^*$ and $A_1, A_2$ from the above simultaneous equations, we obtain the optimal solution.

B The Expectation of First Passage Times

Define $T(u)$ to be the first passage time of $U(t) = X_1(t)/X_2(t)$ starting from $U(0) = u$ to $u^*$, and define

\[ V(u) = E[e^{-rT(u)}]. \quad (B.1) \]

Then $V(u)$ satisfies the following equation.

\[ \frac{1}{2} \sigma^2 u^2 V''(u) + (\sigma^2 - \mu)uV'(u) + \kappa E[V(uY) - V(u)] + \lambda E[V(u/Z) - V(u)] = rV(u), \quad (B.2) \]

with boundary conditions

\[ V(0) = 0; \quad V(u) = 1, \quad u \geq u^*. \]

Assume that the solution of equation (B.2) is

\[ V(u) = \begin{cases} \displaystyle \sum_{j=1}^{2} A_j u^{\alpha_j}, & u < u^*, \\ 1, & u \geq u^*, \end{cases} \]

then equation (B.2) becomes

\[ \sum_{j=1}^{2} A_j u^{\alpha_j} \left[ \frac{1}{2} \sigma^2 \alpha_j (\alpha_j - 1) + (\sigma^2 - \mu)\alpha_j - r + \frac{\kappa \alpha_j}{\zeta - \alpha_j} - \frac{\lambda \alpha_j}{\eta + \alpha_j} \right] - \kappa \left( \frac{u}{u^*} \right) \zeta \left[ \sum_{j=1}^{2} A_j u^{*\alpha_j} \frac{\zeta}{\zeta - \alpha_j} - 1 \right] = 0. \quad (B.3) \]

This means that $\alpha_1, \alpha_2$ are the two positive roots of equation

\[ F(x) = \frac{1}{2} \sigma^2 x(x - 1) - (\sigma^2 - \mu)x - r + \frac{\kappa x}{\zeta - x} - \frac{\lambda x}{\eta + x} = 0, \]

and $A_1, A_2$ satisfy

\[
\begin{align*}
A_1 u^{*\alpha_1} + A_2 u^{*\alpha_2} &= 1, \\
A_1 u^{*\alpha_1} \frac{\zeta}{\zeta - \alpha_1} + A_2 u^{*\alpha_2} \frac{\zeta}{\zeta - \alpha_2} &= 1,
\end{align*}
\]
where

\[ A_1 = \frac{\zeta - \alpha_1}{\zeta} \frac{\alpha_2}{\alpha_2 - \alpha_1} \frac{1}{u^{\alpha_1}}, \]

\[ A_2 = \frac{\zeta - \alpha_2}{\zeta} \frac{\alpha_1}{\alpha_1 - \alpha_2} \frac{1}{u^{\alpha_2}}. \]

As the expectation of the first passage time is given by

\[ E[T(u)] = \lim_{r \to 0} \frac{\partial}{\partial r} V(u), \]

we obtain the result as in equation (14).

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