Teleportation of two-mode squeezed states

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We consider two-mode squeezed states which are parametrized by the squeezing parameter and the phase. We present a scheme for teleporting such entangled states of continuous variables from Alice to Bob. Our protocol is operationalized through the creation of a four-mode entangled state shared by Alice and Bob using linear amplifiers and beam splitters. Teleportation of the entangled state proceeds with local operations and the classical communication of four bits. We compute the fidelity of teleportation and find that it exhibits a trade-off with the magnitude of entanglement of the resultant teleported state.

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I. INTRODUCTION

Quantum teleportation is an important and vital quantum information processing task where an arbitrary unknown quantum state can be replicated at a distant location using previously shared entanglement and classical communication between the sender and the receiver. A remarkable application of entangled states having many ramifications in information technology, quantum teleportation can also be combined with other operations to construct advanced quantum circuits useful for information processing[11]. The original teleportation protocol of Bennett et al.[2] for an unknown qubit using an EPR pair has been generalized to the case of non-maximally entangled or a noisy channel between the sender and the receiver[3]. The loss of fidelity for teleportation using non-maximally entangled channels could be compensated by schemes for probabilistic teleportation[4]. The first experimental demonstration of quantum teleportation was reported by Bouwmeester et al.[5].

Quantum teleportation is also possible for systems corresponding to infinite dimensional Hilbert spaces[6, 7, 8, 9, 10, 11, 12, 13, 14]. The teleportation process for continuous variables was originally formulated in terms of Wigner functions[7] and has also been extended in terms of characteristic functions[8] of the quantum systems involved. Schemes for obtaining optimal fidelity of teleportation using Gaussian[9] as well as non-Gaussian[10] resource states have been devised. The first experiment of continuous variable teleportation was performed by Furusawa et al.[11]. Since then there have been further improvements in the fidelity of teleportation obtained in experiments[12]. Recently, an experimental characterization of continuous variable quantum communication channels has been established by shared entanglement together with local operations and classical communications[13].

Since quantum entanglement is fragile and is easily destroyed in distribution, establishing entanglement between quantum systems at distant locations, and transporting entanglement from one location to another are rather challenging tasks. Various ingenious methods have been proposed to accomplish these, such as by using entanglement swapping protocols[14], quantum repeaters by combining operations of swapping with entanglement purification[15], and by continuous measurements[16]. For continuous variable systems, some protocols for entanglement swapping[13], establishing entanglement between distant stations through teleportation[15], testing the efficiency of teleportation with the aid of a third party[17], and combining teleportation with cloning[18] have been proposed. But no protocol for the explicit teleportation of an entangled continuous variable state exists in the literature, akin to a similar scheme for discrete variables for teleporting a two-qubit entangled state, that has been presented recently[19].

The aim of this work is to propose an explicit scheme for the teleportation of an unknown two-mode entangled state of continuous variables from one party (Alice) to the other distant party (Bob). For this purpose we first show how an entangled state of four modes can be generated and shared by Alice and Bob with the help of linear amplifiers and beam-splitters. Our protocol for teleportation can then proceed in the usual way with Alice making measurements on her side and communicating their results classically to Bob who in turns makes a local operation to obtain the teleported entangled two-mode state. The communication of four bits of information from Alice to Bob is required, similar to the case of the protocol for teleporting entangled states of two qubits[20]. We compute the entanglement of the teleported state with Bob, and also the fidelity of teleportation as functions of the squeezing parameters of the states generated by the source and the teleportation amplifiers. A trade-off between the entanglement of the teleported state and the fidelity of teleportation is observed with respect to the squeezing parameters.

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II. THE TELEPORTATION PROTOCOL

Our protocol is as follows. Let Alice hold the source parametric amplifier SA3 whose two output entangled modes are to be teleported. Bob possesses two teleportation amplifiers TA1 and TA2 which are required as compulsory accessories for our protocol of teleportation of two-mode entangled states. Alice’s task is to teleport the entangled state of the modes (x7, x8) to be teleported. Bob has two amplifiers TA1 and TA2 and two beam-splitters BS1 and BS2 using which he generates a four-mode state (x5, x6, x15, x16). He keeps two of these modes x5 and x16 with himself, and sends the remaining two modes x6 and x15 to Alice. Using the beam-splitters BS3 and BS4 Alice combines her modes x7 and x8 with those sent by Bob, and performs four measurements on the output modes x9, x10, x11, x12. She then communicates her results to Bob who uses these to apply a unitary transformation to displace the modes x5 and x16. The final teleported state is found in the modes x13 and x14.

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$$\sigma^{(1)(2)(3)(4)} = \sigma^{(1)(3)} + \sigma^{(2)(4)}$$

with

$$\alpha = \begin{pmatrix} c - hs & 0 \\ 0 & c + hs \end{pmatrix}$$

$$\beta = \begin{pmatrix} c + hs & 0 \\ 0 & c - hs \end{pmatrix}$$

$$\gamma = \begin{pmatrix} ks & 0 \\ 0 & -ks \end{pmatrix}$$

where c = Cosh(2r), s = Sinh(2r), k = Sin(2\phi), h = Cos(2\phi) with r being the squeezing parameter and \phi the amplifier phase. Bob uses two beam-splitters (BS1 and BS2) represented by the matrix B1 as

$$B_1 = \begin{pmatrix} I_2/\sqrt{2} & 0 & 0 & I_2/\sqrt{2} \\ 0 & I_2/\sqrt{2} & I_2/\sqrt{2} & 0 \\ 0 & I_2/\sqrt{2} & -I_2/\sqrt{2} & 0 \\ I_2/\sqrt{2} & 0 & 0 & -I_2/\sqrt{2} \end{pmatrix}$$

Bob then supplies the modes x15 and x6 to Alice and keeps the remaining modes x5 and x16 with himself. Hence, Alice and Bob share a four-mode entangled state to be used for the teleportation protocol.

Alice has the two entangled modes x7 and x8 originating from her source amplifier SA3 to teleport, in addition to the two modes x15 and x6 which Bob has sent her. The combined six-mode state (four modes with Alice and two with Bob) are represented by the CM

$$\sigma^{(5)(6)(15)(16)} = B_1 \sigma^{(1)(3)(2)(4)} B_1^T$$

Bob then uses the measurement results to displace the state of the modes x5 and x16 with him, by applying

$$\sigma^{(5)(6)(15)(16)(7)(8)} = \sigma^{(5)(6)(15)(16)} \oplus \sigma^{(7)(8)}$$

$$\sigma^{(7)(8)} = \begin{pmatrix} x - uy & 0 & vy & 0 \\ 0 & x + uy & 0 & -vy \\ vy & 0 & x + uy & 0 \\ 0 & -vy & 0 & x - uy \end{pmatrix}$$

where x = Cosh(2q), y = Sinh(2q), u = Cos(2\eta), v = Sin(2\eta), with q being the squeezing parameter of the two-mode state (x7 and x8) with Alice, and \eta being the phase of amplifier SA3.

To proceed further with the teleportation protocol, Alice uses two beam-splitters BS3 and BS4 represented by

$$B_2 = \begin{pmatrix} I_2/\sqrt{2} & 0 & 0 & I_2/\sqrt{2} \\ 0 & I_2 & 0 & 0 \\ 0 & 0 & I_2/\sqrt{2} & 0 \\ 0 & 0 & I_2/\sqrt{2} & -I_2/\sqrt{2} \end{pmatrix}$$

$$\sigma^{(5)(6)(15)(16)(7)(8)} = \sigma^{(5)(6)(15)(16)} \oplus \sigma^{(7)(8)}$$

Bob then makes measurements on these four modes. Without loss of generality, let us assume that her choice of measurements leads to the results (X9, P10, X11, P12), respectively, which she communicates to Bob.

Bob then uses these measurement results to displace the state of the modes x5 and x16 with him, by applying
the unitary transformation
\[
U = \begin{pmatrix}
-\sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sqrt{\frac{2}{3}} & 0 & 0 & 0 & \sqrt{\frac{1}{3}} & 0 \\
0 & 0 & 0 & \sqrt{\frac{2}{3}} & 0 & 0 & 0 & \sqrt{\frac{1}{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{2}{3}} & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{\frac{1}{3}} & 0 & -\sqrt{\frac{2}{3}} \\
\end{pmatrix}
\]

(7)
to obtain finally the modes \(x_{13}\) and \(x_{14}\). The gains \(\sqrt{2}\) on the classical measurements have been chosen such that the resultant matrix \(\sqrt{3}U KB_2\) (with \(K\) defined below) has all the elements either 0, 1 or −1. The combined physical processes (the beam splitters used by Alice, the measurements performed by Alice, and the unitary transformation performed by Bob) takes the CM \(\sigma^{(5)(6)(15)(16)(7)(8)}\) to

\[
\sigma^{(13)(14)} = (U KB_2)\sigma^{(5)(6)(15)(16)(7)(8)}(U KB_2)^\dagger
\]

(8)
where

\[
K = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

(9)
incorporates the measurements made by Alice. Note that the measurements on the four modes \((x_9, x_{10}, x_{11}, x_{12})\) with the choice of outputs \((X_9, P_{10}, X_{11}, P_{12})\) that we have made, corresponds to integrating out these variables, and hence in the CM [9] the corresponding rows are deleted.

Thus, the teleported signal and idler can be found as the modes \(x_{13}\) and \(x_{14}\) with Bob, represented by the CM

\[
\sigma^{(13)(14)} = \begin{pmatrix}
\sigma_{11} & 0 & \sigma_{13} & 0 \\
0 & \sigma_{22} & 0 & \sigma_{24} \\
\sigma_{31} & 0 & \sigma_{33} & 0 \\
0 & \sigma_{42} & 0 & \sigma_{44} \\
\end{pmatrix}
\]

(10)
where

\[
\begin{align*}
\sigma_{11} &= \frac{2c + 2ks + x - uy}{3} \\
\sigma_{13} &= \frac{-vy}{3} \\
\sigma_{22} &= \frac{2c + 2ks + x + uy}{3} \\
\sigma_{24} &= \frac{vy}{3} \\
\sigma_{33} &= \frac{2c + 2ks + x + uy}{3} \\
\sigma_{44} &= \frac{2c + 2ks + x - uy}{3}
\end{align*}
\]

(11)
The CM \(\sigma^{(13)(14)}\) can also be expressed as

\[
\sigma^{(13)(14)} = (\sigma')^{(7)(8)} + 2(c + ks)I
\]

(12)
where

\[
(\sigma')^{(7)(8)} = \begin{pmatrix}
x - uy & 0 & -vy & 0 \\
0 & x + uy & 0 & vy \\
-vy & 0 & x + uy & 0 \\
0 & vy & 0 & x - uy
\end{pmatrix}
\]

(13)
and \(I\) is the 4 × 4 identity matrix. For \(k = -1\), and in the limit \(r \rightarrow \infty\), one obtains \(\sigma^{(13)(14)} = (\sigma')^{(7)(8)}\). It can be shown that the Gaussian state with CM \((\sigma')^{(7)(8)}\) and the Gaussian state with CM \(\sigma^{(7)(8)}\) are equivalent under local linear unitary Bogoliubov operations (LLUBOs) [22]. Thus, in the limit of ideal input squeezing, the two-mode entangled state is teleported perfectly. Further, if the output states from the amplifiers TA1 and TA2 are coherent states, i.e., \(r = 0\), then the variances of the modes \(x_T\) and \(x_8\) are increased by twice the level of the vacuum noise. Thus our teleportation protocol in this special case reproduces the results obtained by Tam [18] for a scheme of teleporting a single-mode state.

III. ENTANGLEMENT AND FIDELITY OF THE OUTPUT MODES

FIG. 2: (Color online) The logarithmic negativity \(E_n\) for the final teleported state is plotted versus the squeezing \(q\) of the two-mode state generated from the source amplifier SA3 (x-axis), and the squeezing \(r\) of the two two-mode states from the teleportation amplifiers TA1 and TA2, respectively (y-axis).

In order to check that the teleported modes \(x_{13}\) and \(x_{14}\) indeed represent an entangled pair, we compute the symplectic eigenvalues of the partial transpose of the CM \(\sigma^{(13)(14)}\) given by Eq. (10). For the modes \(x_{13}\) and \(x_{14}\) to be entangled, the smallest symplectic eigenvalue of
\(\tilde{\nu}_{(13)(14)}\) has to be less than one \(^2\), i.e., \(\tilde{\nu}_- < 1\). For simplicity, we assume that the phase of the amplifier SA3 is chosen such that \(u = 0\) and \(v = 1\), and similarly, the phases of TA1 and TA2 are such that \(h = k = 1/\sqrt{2}\).

Then \(\tilde{\nu}_-\) is given by

\[
\tilde{\nu}_- = \frac{1}{3}\sqrt{(2c + x - y)^2 - 2s^2} \tag{14}
\]

The magnitude of entanglement in the teleported state is given by the logarithmic negativity defined as

\[
E_N = \max[0, -\log_2 \tilde{\nu}_-] \tag{15}
\]

\(E_N\) is plotted in Fig.2 versus the squeezing parameters \(q\) corresponding to the two-mode state \((x_7, x_8)\) to be teleported, and \(r\) corresponding to the states \((x_1, x_3)\) and \((x_2, x_4)\) originating from the amplifiers TA1 and TA2 respectively. One sees that the magnitude of entanglement of the teleported output two-mode state obtained by Bob goes up with increased squeezing of the states coming from both the source and the teleportation amplifiers.

We finally compute the fidelity of the teleported entangled state. The fidelity can be obtained from the expression \(^2\) given by

\[
F = \frac{1}{\sqrt{\det[\sigma_{in} + \sigma_{out}] + \delta - \sqrt{\delta}}} \tag{16}
\]

where

\[
\delta = 4(\det[\sigma_{in}] - \frac{1}{4})(\det[\sigma_{out}] - \frac{1}{4}) \tag{17}
\]

with \(\sigma_{in} = \sigma^{(7)(8)}\) and \(\sigma_{out} = \sigma^{(13)(14)}\) in the present case given by Eqs.(8) and (10) respectively. We plot the fidelity of teleportation versus the squeezing parameters in Fig.3. We notice that a maximum fidelity of 0.38 is possible in this scheme for an initial two-mode coherent state generated by the source amplifier. The fidelity stays nearly constant with variation of the squeezing of the states from the teleportation amplifiers. Thus coherent states generated by the teleportation amplifiers can also be used to implement this protocol. However, increased squeezing of the initial state from the source amplifier leads to the loss of fidelity. The latter feature is in sharp contrast to the magnitude of entanglement of the final two-mode state with Bob, which increases with the increase of squeezing of Alice’s two-mode state. This result seems to support the contention of Johnson et al.\(^9\) that the average fidelity may not be a good indicator of the quality of quantum teleportation. Though their results were obtained in the context of teleportation of single modes together with the physical transport of modes between three parties\(^19\), it might be more appropriate to use the measure of ‘entanglement fidelity’\(^7, 25\) to quantify, where possible, the ability of a process to preserve entanglement.

![Figure 3](image)

**FIG. 3:** (Coloronline) The fidelity of teleportation \(F\) is plotted versus the squeezing \(q\) of the state to be teleported (x-axis), and the squeezing \(r\) of the output modes of the amplifiers TA1 and TA2 (y-axis).

To summarize, we have presented the first explicit protocol for teleportation of two-mode entangled squeezed states. Our scheme is accomplished by the creation of a four-mode state shared initially by two distant parties through beams generated by two teleportation amplifiers and combined by two beam splitters. Teleportation takes place with the usual local measurements, unitary operations and the classical communication of four bits parallel to the case involving entangled states of discrete variables\(^21\). If coherent states from teleportation amplifiers are used to create the four-mode state shared by the two parties for enabling teleportation, the variance of the entangled two-mode teleported state increases by twice the level of the vacuum noise. In general, the entanglement obtained for the teleported state increases with the squeezing of the initial two-mode state which however leads to loss of fidelity of teleportation. It might be worthwhile to explore other schemes of teleportation to check whether fidelity of teleported entangled states could be substantially increased.

**IV. CONCLUSIONS**
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