The Blagonravov continuously variable transmission: computational model of oscillation generator loading

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Abstract. The kinematic diagram of the oscillation generator of the Blagonravov continuously variable transmission is considered in the work. It is shown that the proposed oscillation generator provides the same kinematics of each power flow. A mathematical model is presented in the form of a dependence of the vector load value acting on the crank of the generator on the internal gear ratio of the transmission and the design parameters of the power flows.

1. Introduction

Since the advent of mechanical gears, the search for continuous variable ratio drives has gone in two directions - friction transmission and gear transmission.

In the first direction, to ensure a continuous change in the gear ratio, only connections between link speeds can be used. These bonds cannot be integrated (nonholonomic) and cannot be represented as dependencies between coordinates (holonomic), as can be done with gears.

Designers and researchers have always been attracted by continuous variable gears with holonomic connections. Thus, from the 20s of the last century, exhibits with such continuous variable gears began to appear at international exhibitions. Among them were gears with internal automaticity [1,2], and IVTs [3] that did not have this property. However, a characteristic feature of all of them was presence of oscillatory motion of internal links. Much later A.A. Blagonravov [4] analytically proved that with holonomic connections, to ensure a continuous change in the gear ratio in the system, there must be a periodic (oscillatory) movement. Since there are vibrations inside, and there must be continuous rotation at the output, there must be rectifiers in the system. As mechanical rectifiers, free-wheeling mechanisms (FWM) can be used, for example, axial design with additional friction surfaces and intermediate rolling elements [5].

As a rule, such continuously variable transmissions consist of the following functionally related mechanisms: transforming - oscillation generator, regulating and free-wheeling mechanisms. The oscillation generator transforms the rotational motion of the drive shaft into mechanical vibrations of some intermediate links. As a generator, systems of linkage mechanisms are used consisting of crank-and-rocker four-link chains with a groove disk; geared linkage, consisting of two rocker four-link chains, six-link chains, clutch pins, etc. The greatest simplicity and high bearing capacity provide
mechanisms based on the jointed four-link chains. Regulatory mechanisms are designed to change the oscillations amplitude of the intermediate links [6].

In Blagonravov transmission (Figure 1), the free-wheeling mechanisms are supplemented with elastic shafts - torsions, because of which the well-known variator [3] turns into a new type of mechanical variable transmission with the ability to control the amplitude of oscillations of the internal links – the leading parts of rectifiers [7]. Let us recall the device and the principle of operation of such a continuously variable transmission.

Rotation of the drive shaft 1 through the general crank 2 is converted into a plane motion of the crank holder 3. This movement of the holder is provided by two jointed lever-type parallelograms connected by a rigid link 4, the mechanics of which are described in detail in [8]. This device allows to have the same kinematics of all five transforming mechanisms, consisting of a conditional crank 5, rotating about the O-O axis, which is equidistant to the general crank 2 and equal in magnitude to it; connecting rod 6; rocker arm 7 and conditional bearer 8; representing in this way five multilink mechanisms. The rocker arms are rigidly connected to the driving elements of mechanical rectifiers 9. When rocker arms 7 rotate counterclockwise, the rectifiers travel and the drive elements are locked with the driven parts 10, provided \( \omega_k \geq \omega_T \) where \( \omega_k \) is angular velocity of the rocker arm and \( \omega_T \) is angular velocity of gear 12. In this case, angular oscillations of the rocker arm are converted into unidirectional rotation of the driven elements of the rectifier. Using torsions 11 connecting the driven parts of rectifiers 10 with peripheral gears 12 through central gear 14 of the summing reducer, this rotation is transmitted to driven shaft 13. At the same time, torsions stretch the moment pulses in time, providing overlap and continuity of the torque on the driven shaft. Structurally, multithreading is provided by the phase shift of the rectifiers by \( \pi/5 \) rad and the presence of a summing reducer (gears 12 and 14). The drive shaft and the driven shaft rotate clockwise. This is ensured by the assembly of the jointed lever-type four-link chains according to the ‘antiparallelogram’ option. In this case, direct transmission can be obtained by connecting the drive shaft 1 with the driven shaft 13, for example, a friction clutch (not shown in the diagram).

Thus, general crank 2 through crank holder 3 and connecting rods 6 interacts with the rocker arms of five rectifiers at once. Some of the rectifiers are turned on and transmit to the input of the summing reducer a moment whose magnitude depends on the phase of the cycle. Others are idling. Such a discrete generator creates an uneven load on the drive shaft and the driven shaft. This was shown in work [9]. Changing the oscillations amplitude is carried out by adjusting the radius of general crank 2. Here, the principle of operation of the mechanism for changing the amplitude is not considered. We assume that the generator provides a fixed amplitude of oscillation. At the same time, the mechanism that regulates it through the control system [10] (not shown in Figure 1) experiences a complex type of loading in action and under load. Therefore, to perform engineering calculations during design, it is...
necessary to know the dependence of the load of the oscillation generator, in particular, general crank 2 on the transmission operation mode. This article is aimed at solving this problem.

2. Formation of loading of oscillation generator of the Blagonravov variable transmission

Figure 2 shows the kinematic diagram and the general view of the oscillation generator of the experimental Blagonravov transmission model. Here (Figure 2 a) the largest value of the radius r of the general crank is shown. Trailing connecting rods $l_2$ are pivotally connected to the crank holder at a distance of $l_1$ from the axis, connected pivotally to beam arms $l_3$ of the rectifiers, the axes of which are located at a distance $l_0$ from the axis of the drive shaft O.

![Figure 2. Blagonravov transmission oscillation generator](image)

The drive shaft rotates clockwise, and the rectifiers have a counterclockwise operation. Therefore, connecting rods $l_2$ during operation work in tension. The figure corresponds to the zero value of the crank angle. Unlike the slider-crank mechanism used in engines with a star-shaped arrangement of cylinders, where there is a main connecting rod and trailing connecting rods, here all the connecting rods are trailed. Otherwise, individual four-link chains have unfavorable kinematics. However, the lack of a main connecting rod gives an extra degree of freedom. The holder can be arbitrarily rotated on the crank axis. Four-link chains turn into five-link chains, and the kinematics of any of them may turn out to be unfavorable at random. Thus, when stretching the segments $l_1$ and $l_2$ in one line, the excess degree of freedom in the five-link chain $l_0l_1l_2l_3l_4$ ceases to exist and there is a sharp (shock) change in the relative speeds of the links. In order to exclude this, there is an additional connecting rod $l'_2$ equal in length to the connecting rod $l_2$, and an additional rocker arm $l'_3$, as well as a triangle $abc$ with hinges in the corners, made in such a way that $a$ is equal to the distance between the hinges of the connecting rods $l_2$ and $l'_2$ on the crank holder, and $b$ is equal to the distance between the rectifier axis and the hinge rocker arm $l'_3$ on the body. As a result, there are two jointed lever parallelograms connected by a rigid triangle, which, as in the drawing culmination, provides translational movement.
of the holder. Now, each connecting rod hinge of the holder moves in a circle with a radius \( r \) in the same way as its axis, but with an offset by value \( l \). Thus, we get five identical four-link chains, working with a phase shift of 72°. Figure 2a shows trajectories of the kinematic pairs for the studied mechanism.

On the side of the included rectifiers, the forces \( P_1 \ldots P_5 \) act on the crank holder. In the Figure they are shown in dotted lines. At any time during the cycle (cycle - one revolution of the drive shaft), some of the rectifiers are turned on, while others operate in idle mode. Moreover, the load on each of them is different. The magnitude of the force acting on the crank holder from the beam of the j-th rectifier is determined by the cycle phase and is equal to the torsion angle of rotation of the torsion bar \( \varphi_{stj} \), multiplied by its angular stiffness \( c_T \) and divided by the beam arm \( l_3 \). The direction of this force coincides with the direction of the connecting rods and is determined by the angle corresponding to the position of the j-th connecting rod of the rectifier relative to the selected reference point of the angle \( \alpha t \) of the drive shaft rotation. Therefore, the magnitude of the resultant force \( P \) acting on the general crank will be determined by the vector sum of forces \( P_1 \ldots P_5 \). In this case, the total moment acting on holder 3, relative to its axis of rotation on crank 2, is perceived by two jointed lever parallelograms connected by a rigid triangle 4 (see Fig. 1.). The direction of force \( P \) relative to the axis directed along radius \( r \) of the general crank is determined by angle \( \alpha \) (Figure 2.a.).

It is known [7] that in the stop mode, the torque on the drive shaft is zero, although force \( P \) can be quite large. This can only be if it is directed along the crank, i.e. angle \( \alpha \) between it and the crank is zero. In any other mode, angle \( \alpha \) is no longer zero. Since the generator is loaded by the type of antiparallel and the rods work in tension, the resultant force \( P \) applied to the general crank is always directed towards the axis of the drive shaft and deviates from the radius of the crank in the direction opposite to the direction of rotation as shown in Figure 2.a.

Therefore, when calculating, it is necessary to determine not only the magnitude of the load, which is important for calculating bearings on a general crank, but also its direction. The latter is important for determining the loading of other parts of the mechanism for controlling the amplitude of oscillations. This can be done by compiling a mathematical model of loading the oscillation generator.

3. The mathematical model of the oscillation generator load

The uneven loading of the drive shaft of the Blagonravov continuously variable transmission, caused by the discrete application of loads, was considered in work [9]. The coefficient of uneven load is not more than 1.25. In this work, we are interested in the average loads on the crank of the oscillation generator.

The dependence between the average moments at the input \( M_1 \) and output \( M_2 \) of the transmission without taking into account losses in kinematic pairs is determined by the known [6] dependence

\[ M_1 = iM_2 \]  

where \( i \) – total gear ratio.

Drive shaft torque

\[ M_2 = c_T \varphi_{stcp} n \cdot (i_p)^{-1} \]  

where \( \varphi_{stcp} \) – average per cycle torsion angle; \( n \) – number of torsion bars with rectifiers; \( i_p \) – reduction gear ratio.

The gear ratio is determined by the formula \( i = \varphi_0 i_p \)

where \( \varphi_0 \) – the amplitude of the rocker arm oscillations, for harmonic oscillations adopted, as in \( \varphi_0 = r / l_3 \) at \( \varphi_0 < 0.3 \); \( i_T \) – the internal gear ratio, which depends on the twist angle of the torsion shaft during the cycle and shows what proportion of the maximum per cycle angular speed of the end of the torsion shaft connected to the rectifier is the angular speed constant for the cycle of the end of the torsion shaft connected to the gear of the summing reducer.
Let the average moment per cycle on the drive shaft $M_1$ be created by a conditional constant per cycle force $P$, applied to the general crank at an angle $\alpha$ constant throughout the cycle. The force $P$ is the result of the interaction of the rocker arms $n$ of the rectifiers with the crank holder (see Figure 2a). So far, this force is unknown to us, as well as the angle $\alpha$. The design loading scheme of the general crank is shown in Figure 3a (on the left). The direction of forces $P_j$ is taken along the radius (equal to $l_1$ see Fig. 2a) of the circle with the center $O_2$. Such an assumption is acceptable for the previously adopted harmonic nature of the movement of the rocker arms $\varphi_3 = -\varphi_0 \cos(\omega t)$.

![Figure 3. Modeling the loading of the oscillation generator of the Blagonravov transmission.](image)

**Figure 3.** Modeling the loading of the oscillation generator of the Blagonravov transmission.  
*a* – calculation model of the oscillation generator; *b* - dependences of the resultant force $P$ (solid line) and angle $\alpha$ (dashed line) on the internal gear ratio $i_{iT}$

Then moment $M_1$ will be determined by the formula

$$M_1 = Pr\sin(\alpha)$$  \hspace{1cm} (4)

Substituting into equation (1) values $M_2$ from (2), $i$ from (3), and moment $M_1$ from (4) and taking into account that $\varphi_0 = r / l_1$ we obtain

$$P\sin(\alpha) = i_c c_r \varphi_{iT_{EP}} n \cdot (l_1)^{-1}$$  \hspace{1cm} (5)

Let us analyze the resulting expression. In this expression, we know: the angular rigidity of torsion $c_T$; rocker arm $l_1$; number of rectifiers $n$. The set value of the argument is the value of the internal gear ratio $i_{iT}$. The torsion angle averaged over a cycle is determined by the formula given in [6].

$$\varphi_{iT_{EP}} = \varphi_0 \cdot (2\pi)^{-1} \int_{\omega_T t_1}^{\omega_T t_2} [\cos(\omega t_1) - \cos(\omega t) - i_T (t - t_1)]d(\omega t),$$  \hspace{1cm} (6)

where $\omega_T t_1 = \arcsin(i_T)$ – rectifier switching phase angle; $\omega_1$ – constant angular speed of the drive shaft; $\omega_T t_2$ – rectifier turn-off phase angle determined from the solution of the transcendental equation

$$\cos(\omega_T t_1) - \cos(\omega_T t_2) - i_T \omega_1(t - t_1) = 0$$  \hspace{1cm} (7)

Determination of $\varphi_{iT_{EP}}$ using formulas (6) and (7) is carried out exclusively by a numerical method, for example, by iteration on a computer.

The expression $c_T \varphi_{iT_{EP}} n / l_1$ (see 5) can be interpreted as a conditional equivalent average force per cycle applied perpendicular to one beam, but which creates at the entrance to the summing reducer the same moment as $n$ of real rectifiers. The product $P\sin(\alpha)$ is the average force per cycle applied to the general crank perpendicular to the working radius of the crank and creates a moment on the arm $r$ loading the drive motor.
Let us return to expression (5). From this equation it is only possible to unambiguously determine the force applied perpendicularly to the working radius of the crank and creating the moment loading the engine. If we need to know the value of the force \( P \) and angle \( \alpha \), then this equation does not give an unambiguous answer, because we have only one equation with 2 unknowns.

In order to determine force \( P \) and angle \( \alpha \) separately, it is necessary to add one more equation to equation (5). Such an equation can be made up of considering loading a general crank with instantaneous forces acting on the crank holder from \( n \) rectifiers per cycle. We use the same loading scheme of the general crank, which is shown in Figure 3. a, but we supplement it by expanding the desired average force \( P \) into two projections: one - along the axis \( a-a \) of the crank \( P \cos(\alpha) \) and the other is perpendicular to the axis \( a-a \) \( P \sin(\alpha) \). These projections are the result of the interaction of the rocker arms \( n \) of rectifiers with the crank holder. Then you can write

\[
P \cos(\alpha) = \sum_{j=1}^{n} \left[ \frac{1}{2\pi} \int_{0}^{2\pi} P_j \cos \left( \omega t - \frac{2\pi(j-1)}{n} \right) d(\omega t) \right]
\]

\[
P \sin(\alpha) = \sum_{j=1}^{n} \left[ \frac{1}{2\pi} \int_{0}^{2\pi} P_j \sin \left( \omega t - \frac{2\pi(j-1)}{n} \right) d(\omega t) \right]
\]

Squaring and adding (8) and (9) we obtain the desired average force

\[
P = \left\{ \sum_{j=1}^{n} \left[ \frac{1}{2\pi} \int_{0}^{2\pi} P_j \cos(\omega t) d(\omega t) \right]^2 + \sum_{j=1}^{n} \left[ \frac{1}{2\pi} \int_{0}^{2\pi} P_j \sin(\omega t) d(\omega t) \right]^2 \right\}^{1/2}
\]

where \( P_j \) is the current value of the force on \( j \) rocker from twisting \( j \) torsion with a rectifier, determined by the formula

\[
P_j = \frac{c_{\gamma} \phi_0}{l_3} \cos(\omega t_{t_1}) \cos \left( \omega t - \frac{2\pi(j-1)}{n} \right) - i_{x} \omega(t_{3}-t_{1})
\]

considering that kinematics of all the rocker arms of the rectifiers are the same, but with a phase shift \( 2\pi(j-1)/(n) \), as well as the average projections of any of the forces per cycle, for example, \( P_1 \) on the axis \( a-a \) and perpendicular to it, will be the same for any of the \( n \) rectifiers, then expression (10) can be written as follows

\[
P = \frac{n}{2\pi} \left\{ \int_{0}^{2\pi} P_1 \cos(\omega t) d(\omega t) \right\}^2 + \left\{ \int_{0}^{2\pi} P_1 \sin(\omega t) d(\omega t) \right\}^2 \right\}^{1/2}
\]

Furthermore, taking into account that integration is carried out from the phase angle of jamming of the rectifier - \( \omega t_{t_1} \) to the phase angle of wedging out - , and the current value of the force \( P_1 \) is determined by formula (11), formula (12) takes the form of

\[
P = \frac{c_{\gamma} \phi_0 n}{2\pi l_3} \left\{ \left[ \int_{0}^{t_{f}} F \cos(\omega t) d(\omega t) \right]^2 + \left[ \int_{0}^{t_{f}} F \sin(\omega t) d(\omega t) \right]^2 \right\}^{1/2}
\]

where \( F(\omega t) = \left[ \cos(\omega t) - \cos(\omega t) - i_{x} \omega(t_{3}-t_{1}) \right] \) function depending on the internal gear ratio \( i_{x} \).

Thus, we obtained the desired value of the average force \( P \) acting on the crank spike continuously for one cycle. Comparing formulas (5), (6) and (13), we can determine the angle \( \alpha \) at which the force \( P \) is directed towards the crank axis.
\[
\sin(\alpha) = i_r \left[ \int_{-\alpha_i}^{\alpha_i} F(\omega t) d(\omega t) \right] \cdot \left[ \int_{-\alpha_i}^{\alpha_i} F(\omega t) \cos(\omega t) d(\omega t) \right]^2 + \left[ \int_{-\alpha_i}^{\alpha_i} F(\omega t) \sin(\omega t) d(\omega t) \right]^2 \right]^{1/2}
\]

Expression

\[ k(i_r) = \left[ \int_{-\alpha_i}^{\alpha_i} F(\omega t) d(\omega t) \right] \cdot \left[ \int_{-\alpha_i}^{\alpha_i} F(\omega t) \cos(\omega t) d(\omega t) \right]^2 + \left[ \int_{-\alpha_i}^{\alpha_i} F(\omega t) \sin(\omega t) d(\omega t) \right]^2 \right]^{1/2} \]

entering into equation (14) is not expressed in elementary functions, because the phase angle of the rectifier shutdown \( \omega t_i \) is a solution to the transcendental equation (7). Moreover, it does not depend on the amplitude of the rocker arm oscillations \( \varphi_0 \), does not depend on the number \( n \) of rectifiers with torsions, but depends only on \( i_r \). Therefore, using the numerical method, we find an approximating dependence for the expression in question \( k(i_r) \)

\[ k(i_r) = 2.138 - \left( 0.019 + 1.864 i_r - 0.597 i_r^2 \right)^{1/2} \]

This coefficient shows how many times the conditional equivalent average force per cycle applied perpendicularly to one rocker exceeds the conditional constant force \( P \) per cycle applied to the general crank at an angle \( \alpha \) constant throughout the cycle.

Then the force \( P \) acting on the crank is determined by the dependence

\[ P = \frac{c_r \varphi_{3g}}{l_1} \left[ 2.138 - \left( 0.019 + 1.864 i_r - 0.597 i_r^2 \right)^{1/2} \right] \]

and its direction is determined in accordance with the condition

\[ \alpha = \arcsin \left[ i_r \left[ 2.138 - \left( 0.019 + 1.864 i_r - 0.597 i_r^2 \right)^{1/2} \right] \right] \]

In figure 3.b. as an illustration, the calculations of \( P \) relative to the value \( 5c_r \varphi_{3g} / l_1 \) according to formula (15) and the angle \( \alpha \) according to formula (16) are given. The calculations were performed depending on the internal gear ratio \( i_r \) characterizing the self-regulating property of Blagonravov transmission. It can be seen from the figure that in stop mode, when the angles of torsion rotation can reach a maximum value, the force acting on the crank is two times less than the sum of the forces that create the moment at the entrance to the summing reducer. This is an important result.

4. Conclusions

The dependences of the average per cycle value of the force acting on the oscillator of the Blagonravov continuously variable transmission, as well as the average angle at which it is directed to the crank axis, are established. The value of this angle does not depend on the number of power flows of the transmission.

The maximum value of the force acting on the crank takes place in the stop mode. Moreover, it is two times less than the sum of the forces that create the moment at the entrance to the summing reducer by all transmission flows.

The obtained dependences are recommended for use in the calculations and selection of parameters of the mechanism controlling the amplitude of oscillations.
References

[1] Morales, F. and Benitez, F., A review of dynamic CVT-IVT transmissions, SAE Technical Paper 2014-01-1734, 2014, doi:10.4271/2014-01-1734

[2] Leonov A.I. 1978 Inertial automatic transformers of a torque (M.: Mashinostrojeniya [Mechanical Engineering]) 224 p.

[3] Maltsev V.F. 1978 Mechanical impulse transmissions (M.: Mashinostrojeniya [Mechanical Engineering]) 367 p.

[4] Blagonravov A.A. 1977 Mechanical stepless transmission of non-friction type. (M.: Mashinostrojeniya [Mechanical Engineering]) 143 p.

[5] Blagonravov A.A., Revnyakov E.N. Freewheel mechanisms of continuously variable gears (Avtomobil'naja promyshlennost' [Automotive industry]) 2008, No.6, pp. 16–18.

[6] Kropp A.E. Drive machines with pulse variators. (M. Mashinostrojeniya [Mechanical Engineering]) 144 p.

[7] Blagonravov A.A. 2011 Calculation of the external characteristics of a mechanical transformer with oscillatory movement of internal links (Vestnik mashinostroenija [Bulletin of mechanical engineering]) No. 10. - pp. 8-13.

[8] Blagonravov A.A., Tereshin A.V. Converter for mechanical continuously variable transmissions of transport and traction machines (Traktory i sel'hoz mashiny [Tractors and agricultural machines]). 2010, No. 2, pp. 12-15.

[9] Blagonravov A.A., Yurkevich A.V., Soldatkin V.A. Dynamic loading of external shafts of a stepless mechanical transformer with oscillatory movement of internal links in a transmission (Traktory i sel'hoz mashiny [Tractors and agricultural machines]). 2012, No. 5, pp. 39–42.

[10] Yurkevich A.V., Tereshin A.V., Soldatkin V.A. Electronic control system of continuously variable type of mechanical transmission named after Blagonravov, (IEEE Xplore 2019). DOI: 10.1109/FarEastCon.2019.8934163.