On the measurement problem with entangled photons and the possibility of local hidden variables

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Abstract – It is shown that there is no remote action with polarization measurements of photons in singlet state. A model is presented introducing a hidden parameter which determines the polarizer output. This model is able to explain the polarization measurement results with entangled photons. It refutes Bell’s Theorem.

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Introduction ‘Bell’s Theorem is the collective name for a family of results, all showing the impossibility of a Local Realistic interpretation of quantum mechanics.’ See Shamony [1] who gives a good overview of the state of the discussion about Bell’s Theorem which was introduced in 1964 by the Irish physicist. In his paper “On the Einstein Podolsky Rosen Paradox” [2] he had developed the famous Bell inequality which any hidden variable theory describing entangled states has to obey in contrast to quantum mechanics (QM) which infringes that inequality. The inequality is a relation between expectation values of measurements taken at different settings of the instruments. A general approach to Logical Bell Inequalities and their relation to contextual models is given by [3]. Bell’s paper and many subsequent experiments [4] having proved the infringement of Bell’s inequality by QM established the belief of many physicists that nature was nonlocal [5]. The theory of relativity would not be violated because no information transport over the quantum channel of entangled photons [6] is possible.

Bell’s theorem can be refuted by presenting a counterexample which predicts correctly the expectation values of QM. Many authors have tried this. Some have developed models in which the influence of the measuring apparatus should cause the correlations [7]. Others blame various loopholes for measuring results that violate Bell’s inequality [1]. These are all ruled out after the results of Delft physicists [8] have proved that QM correctly describes the measured correlations without any reference to external conditions. Some models need to be discarded because of systematic errors for instance if the predicted readings at both stations are not independent [9]. One counterexample was presented by Muchowski [10]. But the derivation is not completely convincing as it contains ambiguous values for measurement results. A better model is presented in the current manuscript with more compelling model assumptions. Jung[11] derived the “polarization correlation based on the fact that circularly polarized wave packets associated with entangled photon pairs are phase shifted at the source.” He also states “the linear polarization assumption of a circularly polarized photon is not defined before the measurement has taken place” Therefore this derivation cannot be regarded as a deterministic hidden-variable model. De Zela [12] demonstrated the possibility of local contextual variables for the Bell theorem. He presented a hidden-variable model. But this “model includes a prescription that is absent in the quantum formulation. This prescription refers to the action of the measuring device upon the system being measured.”

Looking thoroughly what Bell has proved one sees that he only has ruled out a specific class of models namely those which are not contextual. Noncontextual models cannot describe QM measurement results as they infringe the Kochen Specker (KS) theorem. Noncontextuality is defined by KS as: ‘If a QM system possesses a property (value of an observable), then it does so independently of any measurement context.’ [13]. Contextual models do not infringe the KS theorem.

In this paper, a model is presented where the measurement results in the initial context are determined independent of the setting of the instruments but in an arbitrary context, the correlation depends on the settings. That is a contextual approach verified by the fact that the polarization of entangled photons is generally not defined. This view is supported by Amorim who suggests the consideration of information indistinguishability as an indicator for realistic theories [14]. The concept of indistinguishability of identical particles could be seen as one of the foundational principles of the quantum theory [15]. Quantum systems differ from classic particles, particularly by this effect. The current paper will help to understand the experimental results. Such an understanding can only be based on local effects. Although we refer to the singlet state as the basis of the investigation the model does not make use of the formalism of QM.

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A model describing the statistical behaviour of entangled photons

Model overview

The model should reproduce the QM predictions of polarization measurements with entangled photons.

Figure 1 shows how entangled photons are generated. Figure 2 shows the experimental arrangement with the coordinate system. With the system in singlet state the conditional probability $P_{B|A}=\sin^2(\alpha-\beta)$. (1)

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Figure 1: Entangled photons are generated, for example, by parametric fluorescence with a BBO crystal. The ordinary photon beam has the polarization 0° and the extraordinary photon beam comes with the polarization 90°. Both photons leave the source in a cone of light. Both cone shells intersect in beam 1 and beam 2. Their polarization is not defined and the photons are by superposition in singlet state. The SEPP (source of entangled photon pairs) emits photons $|H\rangle$ and $|V\rangle$ for system 1 at side 1 and $|H\rangle$ and $|V\rangle$ for system 2 at side 2. $|H\rangle$ and $|V\rangle$ correspond to the x-axis and $|V\rangle$ and $|H\rangle$ correspond to the y-axis.

Figure 2: The SEPP (source of entangled photon pairs) emits photons in singlet state propagating in opposite directions towards adjustable polarizers PA and PB and detectors DA-1 and DA-2 on wing A and DB-1 and DB-2 on wing B. A coincidence measuring device not seen in the picture encounters matching events. The polarization angles are defined in the x-y-plane which is perpendicular to the propagation direction of the photons. The coordinate system is left-handed and the same for both wings with the x-axis in horizontal and the y-axis in vertical direction. The z-axis is in propagation direction of photon 1 and opposite to the propagation direction of photon 2.

Model details

With polarization measurements, photons can choose one of two perpendicular polarizer exits. A hidden variable model has to describe which of these two possible exits a photon will take. Five model assumptions are introduced an overview of which is given below with a description afterwards in italic letters:

MA1 introduces the propensity state called p-state which represents the polarizer exit a photon will take.

MA2 introduces a statistical parameter $\lambda$ which controls the p-state.

MA3 describes how photons from the singlet state are coupled together.

MA4 describes the polarization of a selection of photons from an entangled pair.

MA5 says that photons don’t have a memory of their previous state after a measurement.

Model assumption MA1:

A propensity state (p-state) represents the polarizer exit $\alpha$ a photon with polarization $\phi$ will take. A photon with p-state $\alpha$ will pass the polarizer exit $\alpha$ with certainty.

Model assumption MA2:

The p-state is controlled by a parameter $\lambda$ which is equally distributed between 0 and +1. With $\delta = \alpha - \phi$ we define an indicator function $A(\delta,\lambda)$ which indicates the p-state of the photon before a subsequent measurement. $A(\delta,\lambda)$ can have the values +1 and -1.

For $0 \leq \delta < \pi/2$:

$A(\delta,\lambda) = +1 \text{ for } 0 \leq \lambda \leq \cos^2(\delta)$, (2)

meaning the photon is in p-state $\alpha$ given by the polarizer setting and

$A(\delta,\lambda) = -1 \text{ with } \cos^2(\delta) < \lambda \leq +1$, (3)

meaning the photon is in p-state $\alpha + \pi/2$ perpendicular to the polarizer setting.

Figure 3 shows the geometric relationships on which the model is based.
Predicting measurement results for single photons

Using equation (2) the photon with polarization $\varphi$ is found behind the output $\alpha$ of a polarizer with the probability

$$P_\alpha = \int_0^{\cos^2(\delta)} d\lambda = \cos^2(\delta).$$

(4)

With $\delta = \alpha - \varphi$ we obtain the same $P_\beta$ for a photon in state $\cos(\varphi)*|H> + \sin(\varphi)*|V>$ from a projection onto $\cos(\alpha)*|H> + \sin(\alpha)*|V>$ according to QM from Born’s rule.

Predicting measurement results for the initial context

Next, we see how entanglement affects the correlation of the photon states. Entangled photons are generated by a common source on wing A with the polarization $\varphi_1 = 0^\circ$ and on wing B with the polarization $\varphi_2 = 90^\circ$ or on wing A with the polarization $\varphi_1 = 90^\circ$ and on wing B with the polarization $\varphi_2 = 0^\circ$. This is the initial context. See figure 1.

First, we calculate measurement results for the pair of generated photon 1 with polarization $0^\circ$ and generated photon 2 with polarization $90^\circ$.

For instance, having a generated photon 1 with $\varphi_1 = 0^\circ$ and an assumed polarizer PA setting $\alpha$ we would get $\delta = \alpha - \varphi_1 = \alpha$ and from eq. (2)

$$A(\delta, \lambda) = +1 \quad \text{for} \quad 0 \leq \lambda \leq \cos^2(\delta),$$

(5)

meaning photon 1 is in p-state $\alpha$.

From eq. (3) we get

$$A(\delta, \lambda) = -1 \quad \text{for} \quad \cos^2(\delta) < \lambda \leq +1,$$

(6)

meaning photon 1 is in p-state $\alpha + \pi/2$ perpendicular to the polarizer PA setting.

Defining an indicator function $B(\delta, \lambda)$ for measurement results on wing B we can apply model assumption MA3 for the correlation between the entangled photons on both wings. Here $\delta$ is again the angle between the polarizer setting and the polarization of the generated photon. Equations (5) and (6) do also apply adding $90^\circ$ to all angles and exchanging $A(\delta, \lambda)$ with $B(\delta, \lambda)$.

Having thus a generated photon 2 with $\varphi_2 = 90^\circ$ and an assumed polarizer PB setting $\alpha + \pi/2$ we would get $\delta = \alpha + \pi/2 - \varphi_2 = \alpha$ and from eq. (2)

$$B(\delta, \lambda) = +1 \quad \text{for} \quad 0 \leq \lambda \leq \cos^2(\delta),$$

(7)

meaning photon 2 is in p-state $\alpha + \pi/2$ given by the polarizer PB setting.

From eq. (3) we get

$$B(\delta, \lambda) = -1 \quad \text{for} \quad \cos^2(\delta) < \lambda \leq +1,$$

(8)

meaning photon 2 is in p-state $\alpha$ perpendicular to the polarizer PB setting. Here p-state $\alpha$ and p-state $\alpha + \pi$ are equivalent.
As entanglement connects photons 1 on wing A with photons 2 on wing B by the same value of the parameter $\lambda$ we obtain from equations (5) and (7) and (6) and (8) respectively that the p-states of peer photons are perpendicular to each other meaning if photon 1 is detected by PA at $\alpha$ its peer photon 2 is detected with certainty by PB at $\alpha + \pi/2$.

In the same way, we calculate measurement results for the pair of generated photons 1 with polarization 90° and generated photons 2 with polarization 0°.

With the p-states perpendicular to each other, the model predicts correctly measurement results with perpendicular polarizers on both wings. The reason for this is a common parameter $\lambda$ and not a nonlocal action as we have seen.

So far we did not make use of the contextual assumption MA4 of the model. The derivation so far is local and noncontextual.

**Predicting measurement results for an arbitrary context**

We now calculate probabilities for arbitrary setting of the polarizers having polarizer PA set to $\alpha$ and polarizer PB set to $\beta$.

This means changing the selections of the photons. In the initial context 0°/90° the generated photons with 0° polarization and 90° polarization comprised the selection. Now the selection is changed. So is the polarization state of the photons which is defined by model assumption MA4.

If PA is set to $\alpha$, all selected photons 1 are in p-state $\alpha$ before selection. And the peer photons 2 belonging to the selected photons 1 are in p-state $\alpha + \pi/2$ as we have seen above. With MA4 the polarization state of the selected photons is equal to the p-state thus the polarization of the selected peer photon 2 is $\alpha + \pi/2$ where the polarization of the selected photon 1 is $\alpha$.

With the selected photons 1 in p-state $\alpha$ and peer photons 2 in polarization state $\alpha + \pi/2$ the selected peer photons 2 behave like a beam of single photons with polarization $\alpha + \pi/2$. The conditional probability $P_{\beta|\alpha}$ for those photons 2 to pass PB at $\beta$ where photon 1 passes PA at $\alpha$ can thus be obtained by equation 4 with $\delta = \beta - \alpha - \pi/2$ yielding

$$P_{\beta|\alpha} = P_{\delta} = \int_{\delta}^{\cos^{2}\beta} \cos^{2}\delta$$

where $\delta$ is the angle between the PB polarizer setting $\beta$ and the polarization $\alpha + \pi/2$ of the selected photon 2 which are peer to the selected photon 1. Thus we get

$$P_{\beta|\alpha} = \cos^{2}h(\delta/2) = \cos^{2}(\beta - \alpha - \pi/2) = \sin^{2}(\beta - \alpha)$$

in accordance with QM. The expectation value $E(\alpha, \beta)$ of a common measurement with polarizers PA and PB can be obtained from

$$E(\alpha, \beta) = 1*P_{\delta|\alpha} - 1*(1 - P_{\delta|\alpha}) = \cos^{2}h(\delta) - \sin^{2}h(\delta) = \sin^{2}(\beta - \alpha) - \cos^{2}(\beta - \alpha) = -\cos(2(\beta - \alpha))$$

in accordance with QM as well.

As the expectation value $E(\alpha, \beta)$ from equation (11) does exactly match the predictions of quantum physics it also violates Bell's inequality.

We have assumed that $\lambda$ is not changed with the change of the polarization. So we have left to prove that $\lambda$ is uniformly distributed in the interval $0 \leq \lambda \leq 1$ for the changed polarization. Equations (5) and (6) were derived assuming a generated photon 1 with $\phi = 0^\circ$ and polarizer PA setting $\alpha$. Now we assume a generated photon 1 with $\phi = 90^\circ$ and polarizer PA setting $\alpha + \pi/2$. Then we would get $\delta = \alpha + \pi/2 - 90^\circ$ and from eq. (2)

$$A(\delta, \lambda) = +1 \text{ for } 0 \leq \lambda \leq \cos^{2}(\delta)$$

meaning photon 1 is in p-state $\alpha + \pi/2$ given by the polarizer PA setting. From eq. (3) we get

$$A(\delta, \lambda) = -1 \text{ for } \cos^{2}(\delta) < \lambda \leq 1,$$

meaning photon 1 is in p-state $\alpha$ perpendicular to the polarizer PA setting. Note that p-state $\alpha$ and p-state $\alpha + \pi$ are equivalent.

Thus we see from equations (5) and (13) both generated photons with polarization 0° and 90° respectively contribute to the p-state $\alpha$ so that one half of photon 1 is in p-state $\alpha$ for $0 \leq \lambda \leq 1$ and from equations (6) and (12) we obtain that the other half of photon 1 is in p-state $\alpha + \pi/2$ also for $0 \leq \lambda \leq 1$. In both cases $\lambda$ is uniformly distributed in the interval $0 \leq \lambda \leq 1$ thus, the proof is complete.

**Extending the model to spin $1/2$ particles**

The model does also apply to spin $1/2$ particles (atoms) by simply replacing every angle with its half in equations (2) and (3) and figure 3 as well and in all subsequent derivations yielding figure 3a and

**Model assumption MA2a:**

With $\delta = \alpha - \phi$ we define an indicator function $A(\delta, \lambda)$ which indicates the p-state of the particle before a subsequent measurement. $A(\delta, \lambda)$ can have the values +1 and -1.

For $0 < \delta < \pi$

For a generated particle with spin $\sigma^*\phi$ and instrument PA setting $\alpha$ we get $\delta = \alpha - \phi$ and we define

$$A(\delta, \lambda) = +1 \text{ for } 0 \leq \lambda \leq \cos^{2}(\delta/2),$$

meaning the particle is in p-state $\alpha$ given by the instrument setting and

$$A(\delta, \lambda) = -1 \text{ for } \cos^{2}(\delta/2) < \lambda \leq 1,$$

meaning the particle is in p-state $\alpha + \pi$ opposite to the instrument setting.
The subsequent derivations have to be adapted appropriately with photon to be replaced by particle, polarization by spin and polarizer by instrument respectively. Perpendicular polarizer setting migrates to opposite instrument setting. Then $\delta = \beta - \alpha - \pi$ is the angle between the instrument PB setting $\beta$ and the spin direction $\alpha + \pi$ of the selected particle 2 which are peer to the selected particle 1 of spin direction $\alpha$. The expectation value of a common measurement is similar to equation (11)

$$E(\alpha, \beta) = \cos^2(\delta/2) - \sin^2(\delta/2) =$$

$$= \sin^2(\frac{1}{2} (\beta - \alpha)) - \cos^2(\frac{1}{2} (\beta - \alpha)) = -\cos(\beta - \alpha)$$

in accordance with QM.

### Results, Discussion, and Conclusion

A local contextual model was presented which correctly meets the predictions of QM for polarization measurements with photons in singlet state as well as for spin measurements with electrons in singlet state.

The rule determining which polarizer exit a photon will take is the same for both wings. Dependencies between the photons on either wing originate from the shared parameter $\lambda$ and not from a nonlocal influence of photon 1 upon photon 2. With the same rules acting upon the generated photons with polarization $0^\circ/90^\circ$ or $90^\circ/0^\circ$ on both wings, the measurement results are correlated without nonlocal effects. The model is also valid for single particles where the concept of a propensity state controlled by a parameter $\lambda$ allows to predict measurement results for incompatible states simultaneously.

Measurement values at one wing are determined independently of the setting of the polarizer at the other wing. Only the correlation between the measurements at both wings depends on the setting of the polarizers. The latter is a contextual effect that comes into play as the polarization of entangled photons is undefined. The model says that the polarization of a selection of photons is changed by entanglement. However, this effect is local as we have seen.

Photons and electrons in singlet state do thus not exhibit action at a distance. Nonlocality is therefore not a consequence of entanglement. Experimental results with spin measurements can be explained without assuming non-local effects. This means measured values are not generated upon the measurement, they already exist beforehand. Otherwise a strong correlation between the outcomes of measurements at different sides would demand nonlocal effects. This supports Einstein’s view of the meaning of the wave function as a description of an ensemble [16]. Thus, quantum mechanics does not violate the principle of causality, at least for spin measurements.

The purpose of the model was to show that a local contextual model is possible which correctly meets the predictions of QM thus refuting Bell’s theorem. The model is valid if it is free of contradictions.

After Bell’s theorem refuted we cannot conclude any more nature were nonlocal. This is no more a necessary consequence of QM infringing Bell’s theorem.

As it is now conceivable that quantum results are determined before measurement the concept of a quantum computer is in question as it relies upon the assumption that a quantum system bears simultaneously information about two possible outcomes [17]. If the model presented is a valid description of nature the propensity state of a photon is controlled by a single parameter defining the outcome of measurements for any chosen direction and thus considerably restricts the diversity of the solution of a quantum computer. The model presented is deterministic and as it exactly reproduces the predictions of QM it could be implemented on any ordinary computer in order to simulate a quantum computer.

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**Author Contribution**

E.M is the only author and solely responsible for the entire manuscript

**Competing interests**

None