Non-Linear Finite Element Approach to Aerodynamic Effects on Horizontal Taut Cable

Nwabanne J. O., Chukwuneke J. L., Omenyi S. N.

Department of Mechanical Engineering, Nnamdi Azikiwe University, Awka, Nigeria
Corresponding E-mail: jl.chukwuneke@unizik.edu.ng

Abstract-
A horizontal taut cable which has application in a typical power transmission line was subjected to structural nonlinear static analysis. Finite Element Analysis (FEA) approach was employed in this study to ascertain the magnitude of tension on each node on the cable occasioned by induced aerodynamics forces. ANSYS 14.0 software helped in determining the behaviour of the system on the basis of finite element displacement method. Comparison was made between the two FEA cases with respect to the effectiveness of the analytical model which describes the response in form of a couple of degrees of freedom (DOF) while reflecting the complex features of the dynamics of the cable in reaction to the induced aerodynamic effect. In order to avert excessive displacement in the computations, it was necessary to incorporate the convergence procedure in the process. The optimum deflection of the parabolic response obtained from the analytical Finite Element Method (FEM) results in the absence of aerodynamic external effect was at the nodes 12 and 13 corresponding to cable weight of 1.002936N and cable tension of 189396.97kg/km. The cable also showed a maximum displacement of 4483.75mm for the X, Y and Z components at nodal point 2 while the minimum displacement value was 4218.75mm. The obtained result for this study revealed a complex modal behaviour as is quite expected.

Keywords–Aerodynamics, Analytical modal, Convergence procedure, Dynamics, Finite Element Method, Optimum deflection, Parabolic response, Transmission Line.

1. Introduction
Nonlinear responses of suspended cables under parametric excitation have attracted attention in recent years because of improved computational and modelling tools occasioned by the proliferation of different finite element analyses software. This has received much attention from the academia and industry globally [1]. Engineers and scientists are working hard to find answers to complex mathematical and structural problems involving cables using these abundant computational tools [2]. Modern engineering is replete with the use of structural cables for their lightweight, slender nature and toughness characteristics [2]. They have proved very effective in transporting and supporting static loads thus satisfying the question of stringent design requirements, aesthetic appeals and efficient material usage. It could be noted however, that recurrent high amplitude vibrations in the face of their rather extreme flexibility coupled with severe service conditions renders these cables susceptible to harmful environmental agents. Thus, aging structural cables tend to deteriorate as a result of dynamic fatigue, mechanical damage and galvanic corrosion [2–3].

It is noteworthy that quite a few researches explored the effects of potential damage on the mechanical behaviour of cables employing FEM [2, 4–6]. FEM in each of wires was discretized using geometry similar to independent helical springs in model formulation. Linear model was analyzed with the dynamics behaviour of transmission line cables [5]. Static cable segment model was analyzed [5] using symmetry properties and solid FEM but was restricted for static problems and with symmetrical loadings [6]. They focus only in beam structures and dynamic stiffness model and only technical problems are
addressed without analytical model [2, 7–8]. Thus, this study examines the aerodynamic effects on suspended cables using nonlinear finite element model approach.

2. Methodology

2.1. Basic Equations for a horizontal cable
The static form of a horizontal cable is approximated by a quadratic parabola shown in figure (1). The equations of static equilibrium of a differential cable segment of span \( l \) and mass \( m \) under tension \( T \) [9] are:

\[
\frac{d}{ds} \left( T \frac{dy}{ds} \right) = -mg \tag{1}
\]

\[
\frac{d}{ds} H = 0 \tag{2}
\]

\[
H = \frac{2dx}{ds} \tag{3}
\]

\[\text{Figure 1: Horizontal Cable Configuration}\]

Under an assumption that the cable sag-to-span ratio (\( D/L \)) is small. Ignoring the effect of \( mg \) on sag, the static equilibrium formation of the cable as described through the parabola [9]:

\[
y(x) = \frac{mg}{2H} \left[ \frac{x}{l} - \left( \frac{x}{l} \right)^2 \right] \tag{4}
\]

From figure 1; \( y(x) = d, \frac{x}{l} = \frac{1}{2} \), therefore equation (4) can be transformed as:

\[
d = \frac{mg}{2H} \left[ \frac{l}{2} - \left( \frac{1}{2} \right)^2 \right] = \frac{mgl^2}{8H} \tag{5}
\]

Therefore:

\[
H = \frac{mg}{8d} \tag{6}
\]

In the light of equation (6), equation (4) can be written as [9]:

\[
y(x) = 4d \left[ \frac{x}{l} - \left( \frac{x}{l} \right)^2 \right] \tag{7}
\]

Equation for the sag which is the same as equation of parabola as:

Taking moment about a point: \( + \sum M_D = 0 \)

\[
T \cdot y = \left( \frac{wx}{2} \right) \left( \frac{x}{2} \right) \tag{8}
\]

\[
y = \frac{wx^2}{2T} \tag{9}
\]

Substituting \( x = L/2 \)

\[
y = \frac{wL^2}{8T} \tag{10}
\]

Where \( w = mg \)

\[
y = \frac{mgL^2}{8T} \tag{11}
\]
Where; $T =$ static Tension, $y =$ Vertical distance between attachment point and lower point in parabola, $wx =$ weight that act at a horizontal distance $\frac{x}{2}$ from the attachment point, $g =$ acceleration due to gravity.

2.2. Aerodynamic Forces
Wind load may persuade instability and too much vibration in long span cable. Cable vibration and wind interaction results in motion dependent and motion independent forces. The motion dependent forces cause aerodynamic instability with stress on vibration of rigid bodies [10]. The motion dependent part for short span cable is irrelevant and as such no alarm about aerodynamic instability. Cable aerodynamic behaviour is controlled by structure parameter (i.e. cable layout, boundary condition, member stiffness, natural modes and frequencies) and aerodynamic parameter (i.e. wind climate, cable section shape). The aerodynamic equation of motion is expressed as [10–12]:

$$m \ddot{u} + c \dot{u} + ku = F_{umd} + F_{mi}$$  \hspace{1cm} (12)

Where: $F_{umd}$ is motion dependent aerodynamic force vector, $F_{mi}$ is motion independent wind force vector, $m$ is global mass, $c$ is damping, and $k$ is stiffness matrix, $\ddot{u}$ is acceleration, $\dot{u}$ is velocity and $u$ is nodal displacement.

A realistic model of a natural wind requires the knowledge of its three-dimensional velocity field, thus the assignment of the vectorial temporal law of the speed at a generic point. A quasi-static approach is adopted to describe the aerodynamic effects of the wind excitation on the oscillating body. The aerodynamic forces are considered to be the same as if the body is at rest in a uniform stream, or equivalently to develop instantaneously. Since cable cross-sections with polar symmetry are considered, the drag force does not depend on the angle of attack and remains aligned with the wind relative velocity [13–14].

According to quasi-steady theory, the wind flow exerts the following aerodynamic force on the cable section:

$$F_a = \frac{1}{2} \rho_a V r (C_d y) V + c_l(y) a_1 x V$$  \hspace{1cm} (13)

Where; $\rho_a$ is density of air, $Vr$ is relative velocity of the wind with respect to the section, $V = |V|$ its modulus, $y$ is the angle of attack, $C_d$ and $c_l$ are the two aerodynamic coefficients for the drag and lift respectively; $\gamma = -\arctan \left( \frac{V}{|V| a_2} \right)$  \hspace{1cm} (14)

2.3 Finite Element Equation Formulation for the Cable Problem

Approximation Functions: For a one-dimensional case, the best form of approximation functions is a first-order polynomial or straight line equation:

$$u_x = a_0 + a_1 x$$  \hspace{1cm} (15)

Where; $u_x =$ the dependent variable, $a_0$ and $a_1 =$ constraints, and $x =$ independent variable.

The above function will pass through the values of $u_x$ at the end points of the element at $x_1$ and $x_2$ to give the below equations:

$$u_1 = a_0 + a_1 x_1$$  \hspace{1cm} (16)

$$u_2 = a_0 + a_1 x_2$$  \hspace{1cm} (17)

Where; $u_1 = u_{x_1}$ and $u_2 = u_{x_2}$.

Equations (15), (16) and (17) can be solved by Cramer’s rule for

$$a_0 = \frac{u_1 x_2 - u_2 x_1}{x_2 - x_1}$$  \hspace{1cm} (18)

$$a_1 = \frac{u_2 - u_1}{x_2 - x_1}$$  \hspace{1cm} (19)
These results are substituted into the above equations and rearranged to yield:
\[ u = N_1 u_1 + N_2 u_2 \]  
(20)
Where:
\[ N_1 = \frac{x - x_3}{x_2 - x_3} \]  
(21)
And
\[ N_2 = \frac{x_2 - x}{x_2 - x_1} \]  
(22)
These are the shape functions; \( N_1 \) and \( N_2 \) are the interpolation functions.

2.4. Method of Weighted Residuals (MWR)

The differential equations [15]:
\[ \frac{d^2w}{dx^2} - \frac{a}{c^2} W = 0 \]  
(23)
And
\[ \frac{d^2T}{dx^2} - aT = 0 \]  
(24)
Will be solved by the Method of Weighted Residual and expressed as:
\[ \frac{d^2U}{dx^2} + f(x) = 0 \]  
(25)
Since this is not an exact solution, the equation is not equal to zero but to a residual \( R \).

Therefore;
\[ \frac{d^2V}{dx^2} + f(x) = R \]  
(26)
The method of weighted residuals consists of finding a minimum for the residual according to the general formula:
\[ \int_D RW_i \, dD = 0 \]  
(27)
Where: \( i = 1, 2, \ldots, m \), \( D \) is the solution domain and \( W_i \) = linearly independent weighting functions.

Now, we use the interpolating functions \( N_i \) as the weighting functions and this becomes:
\[ \int_D RN_i \, dD = 0 \]  
(28)
Where: \( i = 1, 2, \ldots, m \)

For one dimensional cable being acted upon by its weight:
\[ \int_{x_1}^{x_2} \left[ \frac{d^2 \omega}{dx^2} - f(x) \right] N_i \, dx \]  
(29)
Where \( i = 1, 2, \ldots, m \)

Expanding (29) we obtain:
\[ \int_{x_1}^{x_2} \frac{d^2 \omega}{dx^2} N_i(x) \, dx = \int_{x_1}^{x_2} f(x) N_i(x) \, dx \]  
(30)
Where \( i = 1, 2 \)

Now we use calculus to clear this level:
\[ \int_{x_1}^{x_2} \frac{d^2 \omega}{dx^2} N_i(x) \, dx = \int_{x_1}^{x_2} f(x) N_i(x) \, dx \]  
(31)

Choosing \( u \) and \( v \) to be \( N_i(x) \) and \( \left( \frac{d^2 \omega}{dx^2} \right) \) \( dx \) as \( dv \):
\[ \int_{x_1}^{x_2} N_i(x) \, \frac{d^2 \omega}{dx^2} \, dx = N_i(x) \frac{d^2 \omega}{dx^2} \, dx \mid_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{d^2 \omega}{dx^2} \, \frac{dN_i(x)}{dx} \, dx \]  
(32)
Where \( i = 1, 2 \)

The highest-order term has been reduced in the formula from a second to a first order derivative.

Therefore, for \( i = 1 \): \( N_1(x) \frac{d^2 \omega}{dx^2} \, dx \mid_{x_1}^{x_2} = N_1(x_2) \frac{d^2 \omega(x_2)}{dx^2} - N_1(x_1) \frac{d^2 \omega(x_1)}{dx^2} \)  
(33)
But \( N_1(x_2) = 0 \) and \( N_1(x_1) = 1 \),

Therefore;
\[ N_1(x) \frac{d^2 \omega}{dx^2} \mid_{x_1}^{x_2} = - \frac{d^2 \omega(x_1)}{dx^2} \]  
(34)

Similarly, for \( i = 2 \): \( N_2(x) \frac{d^2 \omega}{dx^2} \, dx \mid_{x_1}^{x_2} = \frac{d^2 \omega(x_2)}{dx^2} \)  
(35)

Regrouped for \( i = 1 \):
\[ \int_{x_1}^{x_2} \frac{d^2 \omega}{dx^2} \, dx \mid_{x_1}^{x_2} = - \frac{d^2 \omega(x_1)}{dx^2} - \int_{x_1}^{x_2} f(x) N_1(x) \, dx \]  
(36)

For \( i = 2 \):
\[ \int_{x_1}^{x_2} \frac{d^2 \omega}{dx^2} \, dx \mid_{x_1}^{x_2} = \frac{d^2 \omega(x_2)}{dx^2} - \int_{x_1}^{x_2} f(x) N_2(x) \, dx \]  
(37)

Putting the interpolating functions, \( 1 \) have for \( i = 1 \)
\begin{align}
\int_{x_1}^{x_2} \frac{u_1 - u_2}{(x_2 - x_1)^2} \, dx &= \frac{1}{x_2 - x_1} (T_1 - T_2) \\
\text{For } i = 2: \quad \int_{x_1}^{x_2} \frac{-u_1 + u_2}{(x_2 - x_1)^2} \, dx &= \frac{1}{x_2 - x_1} (-T_1 + T_2)
\end{align}

(38)

(39)

Putting all equations in matrix form:

\[
\frac{1}{x_2 - x_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} [T] = \begin{bmatrix} \frac{dT(x_1)}{dx} \\ \frac{dT(x_2)}{dx} \end{bmatrix} - \begin{bmatrix} \int_{x_1}^{x_2} f(x) N_1(x) \, dx \\ \int_{x_1}^{x_2} f(x) N_2(x) \, dx \end{bmatrix}
\]

(40)

3. Results and Analysis

The Power Holding Company of Nigeria standard parametric values for the erection of power transmission lines for 330KVA line is as follows: Weight (Mass) per unit length \( w_0 = 687.9\text{kg/km} \); Sag = 4.05m; Span = 460m. The power transmission lines are in the form of a catenary.

3.1. Solution to a Cable Problem

Discretization of the cable system: From our calculations, the cable length was approximated as 460 metres. If we divide this length by 20, we will have 23 elements with 24 nodes to solve.

Elemental equations of the cable system: An individual element is represented as [16]:

\[
\bar{W} = N_1 W_1 + N_2 W_2
\]

(41)

Where: \( N_1 \) and \( N_2 \) are the linear interpolation functions.

Assembly of the cable system: Before the assembling of these element equations, the numbering scheme is written to describe the system topology or spatial layout as shown in table 1. The table describes the mode of assembly of equations of the total cable system.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Element & Nodal Numbers & Element & Nodal Numbers & Element & Nodal Numbers \\
& Local & Global & & Local & Global & Local & Global \\
\hline
1 & 1, 2 & 1, 2 & 9 & 1, 2 & 9, 10 & 17 & 1, 2 & 17, 18 \\
2 & 1, 2 & 2, 3 & 10 & 1, 2 & 10, 11 & 18 & 1, 2 & 18, 19 \\
3 & 1, 2 & 3, 4 & 11 & 1, 2 & 11, 12 & 19 & 1, 2 & 19, 20 \\
4 & 1, 2 & 4, 5 & 12 & 1, 2 & 12, 13 & 20 & 1, 2 & 20, 21 \\
5 & 1, 2 & 5, 6 & 13 & 1, 2 & 13, 14 & 21 & 1, 2 & 21, 22 \\
6 & 1, 2 & 6, 7 & 14 & 1, 2 & 14, 15 & 22 & 1, 2 & 22, 23 \\
7 & 1, 2 & 7, 8 & 15 & 1, 2 & 15, 16 & 23 & 1, 2 & 23, 24 \\
8 & 1, 2 & 8, 9 & 16 & 1, 2 & 16, 17 & & & \\
\hline
\end{tabular}
\caption{Element topology}
\end{table}

The equation is assembled as showed in equation (42):
3.2. Solution of the Cable Weight without Considering External Effects

As the equations are assembled, the internal boundary conditions eliminated themselves therefore, solve for the first and last nodes. Assuming the nodes $W_1$ and $W_{24}$ are equal to zero i.e. $W(0) = 0$; $W(L) = 0$. The equations for evaluating $\frac{dw_{x_1}}{dx}$ and $\frac{dw_{x_24}}{dx}$ can be expressed as:

$$0.05W_1 - 0.05W_2 + \frac{dw_{x_1}}{dx} = -0.0003799$$

$$0.05W_1 - 0.05W_2 + \frac{dw_{x_24}}{dx} = -0.0003799$$

$$\frac{dw_{x_1}}{dx} = -0.0003799 + 0.05W_2$$

$$-0.05W_1 + 0.1W_2 - 0.05W_3 = -0.0007598$$

$$-0.05W(0) + 0.1W_2 - 0.05W_3 = -0.0007598$$

$$0.1W_2 - 0.05W_3 = -0.0007598$$

$$-0.05W_{22} + 0.1W_{23} - 0.05W_{24} = -0.0007598$$

$$-0.05W_{22} + 0.1W_{23} - 0.05(0) = -0.0007598$$

$$-0.05W_{22} + 0.1W_{23} = -0.0007598$$

$$0.05W_{24} - 0.05W_{23} - \frac{dw_{x_24}}{dx} = -0.0003799$$

**Boundary conditions:** As the equations are assembled, the internal boundary conditions eliminated themselves therefore, solve for the first and last nodes. Assuming the nodes $W_1$ and $W_{24}$ are equal to zero i.e. $W(0)=0$; $W(L)=0$. The equations for evaluating $\frac{dw_{x_1}}{dx}$ and $\frac{dw_{x_24}}{dx}$ can be expressed as:
From equation (44); \( \frac{dW_1}{dx} \) and \( \frac{dW_{23}}{dx} \) can be solved as follows:

\[
\frac{dW_1}{dx} = -0.0003799 + 0.05W_2 \tag{45}
\]

Substituting \( W_2 = -0.167156 \) into equation (45): \( \frac{dW_1}{dx} = -0.0087377 \)

Similarly; \( \frac{dW_{23}}{dx} = 0.0003799 - 0.05W_{23} \tag{46} \)

Substituting; \( W_{23} = -0.167156 \) into equation (46): \( \frac{dW_{23}}{dx} = 0.0087377 \)

Figure 2: Plot of cable weight against the nodal points
For the other property of the material given as \( \frac{d^2T}{dx^2} - aT = 0 \), comparing with earlier differential equation \( \frac{d^2w}{dx^2} - f(x) = 0 \). From literature, \( T = 4483.83 \) N and \( a = 0.0016 \).

Therefore; \( aT = 7.174128. f(x) = aT = 7.174128 \). Results obtained from this analytic FEM without the influence of aerodynamic show a parabolic response with the optimum deflection at nodal points 12 and 13 with cable weight at nodes 12 and 13 having value of -1.002936N.

### 3.3 Discretization of the cable system

From the calculations, cable length was approximated as 460 metres. If it is divided by 20, the cable reduces to 23 elements with 24 nodes.

#### Elemental equations of the cable system:

An individual element is represented as [16]:

\[
\bar{T} = N_1T_1 + N_2T_2
\]

(47)

Where: \( N_1 \) and \( N_2 \) are the linear interpolation functions. The elemental equations are the same as for those solved above. The assembly of the equations of the total system is performed accordance to the element topology given in table (1).

The cable tensions is solved and obtained as:

\[
\begin{bmatrix}
0.05 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.05 & 0.1 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6 \\
T_7 \\
T_8 \\
T_9 \\
T_{10} \\
T_{11} \\
T_{12} \\
T_{13} \\
T_{14} \\
T_{15} \\
T_{16} \\
T_{17} \\
T_{18} \\
T_{19} \\
T_{20} \\
T_{21} \\
T_{22} \\
T_{23} \\
T_{24} \\
\end{bmatrix}
\begin{bmatrix}
\frac{-dT_{s1}}{dx} = 71.74128 \\
\frac{-dT_{s24}}{dx} = 71.74128 \\
\end{bmatrix}
\]

(48)

#### Boundary conditions:

As the equations are assembled, the internal boundary conditions eliminated themselves therefore, solve for the first and last nodes. Recall we assumed these nodes \( T_1 \) and \( T_{24} \) are equal to zero [i.e. \( T(0) = 0 \); \( T(L) = 0 \)]. Thus, equations for evaluating \( \frac{dW_{s1}}{dx} \) and \( \frac{dW_{s24}}{dx} \) as:
0.05T_1 - 0.05T_2 + \frac{dw_{x_1}}{dx} = -71.74128
0.05(0) - 0.05T_2 + \frac{dw_{x_1}}{dx} = -71.74128
\frac{dw_{x_1}}{dx} = -71.74128 + 0.05T_2
-0.05T_1 + 0.1T_2 - 0.05T_3 = -143.48256
-0.05T(0) + 0.1T_2 - 0.05T_3 = -143.48256
0.1T_2 - 0.05T_3 = -143.48256
-0.05T_{22} + 0.1T_{23} - 0.05T_{24} = -143.48256
-0.05T_{22} + 0.1T_{23} - 0.05(0) = -143.48256
-0.05T_{22} + 0.1T_{23} = -143.48256
0.05T_{24} - 0.05T_{23} + \frac{dw_{x_24}}{dx} = -71.74128
\frac{dw_{x_24}}{dx} = 71.74128 - 0.05T_{23}

3.4. Solution of the Cable Tension Neglecting External Effects

\begin{bmatrix}
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6 \\
T_7 \\
T_8 \\
T_9 \\
T_{10} \\
T_{11} \\
T_{12} \\
T_{13} \\
T_{14} \\
T_{15} \\
T_{16} \\
T_{17} \\
T_{18} \\
T_{19} \\
T_{20} \\
T_{21} \\
T_{22} \\
T_{23}
\end{bmatrix}
= \begin{bmatrix}
-31566.1632 \\
-60262.6752 \\
-86089.536 \\
-109046.7456 \\
-129134.304 \\
-146352.2112 \\
-160700.4672 \\
-172179.072 \\
-180788.0256 \\
-186527.328 \\
-189396.9792 \\
-189396.9792 \\
-186527.328 \\
-180788.02556 \\
-172179.072 \\
-160700.4672 \\
-146352.2112 \\
-129134.304 \\
-109046.7456 \\
-86089.536 \\
-60262.6752 \\
-3566.1632
\end{bmatrix}

(50)

From equation (50); \frac{dr_{x_1}}{dx} and \frac{dr_{x_{24}}}{dx} is evaluated as follows:
\frac{dr_{x_1}}{dx} = -71.74128 + 0.05T_2

(51)

Substituting T_2 = -31566.1632 into equation (51); \frac{dr_{x_1}}{dx} = -1650.04944

Similarly;
\frac{dr_{x_{24}}}{dx} = 71.74128 - 0.05T_{23}

(52)

Substituting T_{23} = -31566.1632 into equation (52); \frac{dr_{x_{24}}}{dx} = 1650.04944
Results obtained from this analytic FEM without the influence of aerodynamic show a parabolic response with an optimum deflection with cable tension at nodal point 12 and 13 having value of $-9396.97\text{kg/km}$.

### 3.5. Considering Aerodynamic Influences on the Cable

The lift model can be expressed by [17–18]:

$$f_l = \frac{1}{2} \rho_a D U_l^2 C_l$$  \hspace{1cm} (53)

While the drag model by [17–18]:

$$f_d = \frac{1}{2} \rho_a D U_l^2 C_d$$  \hspace{1cm} (54)

Where; $\rho_a$ is density of air, $D$ is diameter of the cable, $C_l$ is a dimensionless lift coefficient, $C_d$ is a dimensionless drag coefficient, $U_l^2$ is relative velocity of wind: $R^2 = F_l^2 + F_d^2$  \hspace{1cm} (55)

Where; $R$ is the resultant force from the wind tunnel experiment, $F_l$ is the lift force, $F_d$ is the drag force. The dynamic pressure:

$$P_d = \frac{1}{2} \rho V^2$$  \hspace{1cm} (56)

The surface area ($A_s$) is function of chord and span; $A_s = c l$.

Bernoulli’s Equation is the function of velocity, density, pressure and specific weight:

$$P_1 + \frac{1}{2} \rho V_1^2 + \gamma Z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \gamma Z_2$$  \hspace{1cm} (57)

Assuming $Z_1 = Z_2$ meaning the heights are the same level then the equation (57) written as:

$$\Delta P = \frac{1}{2} \rho V^2$$  \hspace{1cm} (58)

$$V = \sqrt{2 \Delta P \rho}$$  \hspace{1cm} (59)

The net aerodynamic force on the model per unit length is expressed as [18]:

$$f_y = f_l \cos \beta - f_d \sin \beta = \frac{1}{2} \rho_a D U_l^2 (C_l \cos \beta - C_d \sin \beta)$$  \hspace{1cm} (60)

Applying the Fourier series equations generated from wind tunnel tests for the coefficients of drag and lift [17–18]:

$$C_d(\alpha) = 1.12 + 0.01 \cos \alpha - 0.30 \cos 2\alpha$$  \hspace{1cm} (61a)

$$C_l(\alpha) = 0.22 \sin \alpha + 0.32 \sin 2\alpha + 0.44 \sin 3\alpha + \sin 4\alpha + 0.02 \sin 5\alpha$$  \hspace{1cm} (61b)

Applying trigonometrically ratios rules to determine the coefficient lift and drag respectively: $\sin 2\alpha = 2 \sin \alpha \cos \alpha$; $\cos 2\alpha = 2 \cos^2 \alpha - 1$ and $\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$.

The calculated Drag coefficient was plotted against angle of attacks (see figure 4) which shows a sinusoidal harmonic behaviour. The data obtain from wind tunnel experiment conducted was used to calculate the lift coefficient which was plotted against Angle of attack; the graph behaves not exactly sinusoidal but non-linear behaviour.
Figure 4: Coefficients of Drag and Lift Coefficient against Angle of attack (Case two)

The cable properties used in this study are shown in table (2).

| S/No. | Property                          | Unit    | Value       |
|-------|----------------------------------|---------|-------------|
| 1.    | Conductor weight                 | kg/m    | 0.675       |
| 2.    | Length of the actual cable       | M       | 460         |
| 3.    | Pre-tension of cable, $T_0$      | N       | 4218.75     |
| 4.    | Elastic Modulus, $E$             | GPa     | 71.7        |
| 5.    | Shear Modulus, $G$               | GPa     | 26.9        |
| 6.    | Poisson’s ratio, $v$             |         | 0.333       |
| 7.    | Density                          | g/cm$^3$| 2.71        |
| 8.    | Cable Tension, $T$               | N       | 4483.83     |
| 9.    | Nominal overall diameter         | M       | 0.03        |
| 10.   | Wind speed                       | m/s     | 33          |
| 11.   | Net Aerodynamic force            | N       | 23.886      |
|       | Cross sectional Area             | m$^2$   | 13.8        |

3.6. ANSYS Parametric Design Language (APDL)

The flow chart of the analysis is shown as: Preferences ↔ Preprocessor ↔ General post processor → Solution. Figures (5 – 9) is the schematic diagram of the ANSYS parametric design language (APDL 14.0) shows Major Input Requirement: Geometry / Dimension; Type of forces; Boundary conditions; and Structural.
Figure 5: Cable Discretization

Figure 6: Undeformed shape of cable

Figure 7: Deformed shaped
Figure 8: Deformed shape of cable under loads and constraints

Figure 9: Displaced cable body under the influence of aerodynamic forces (Drag and Lift forces) with maximum displacement of 4483.83mm in X-direction

Figure (5) is shows a cable discretization. Figures (6 – 7) shows the analysis of undeformed shape of cable and deformed shape of cable respectively. Figure (8) present the deformed shape of cable under loads and constraints, while figure (9) shows a displaced cable body under the influence of aerodynamics forces (drag and lift) with maximum and minimum displacement occurring at 4483.83mm and 4218.75mm respectively in x-direction.

4. Conclusion
The mechanical behaviour of static suspended cable under aerodynamic influences has been presented in this study using ANSYS 14.0 FEA software. This study concludes as follows: computational efforts have been reduced using the ANSYS 14 software for the aerodynamic case; the analytic FEM results without the influence of aerodynamic show a parabolic response with optimum deflection at nodal points 12 and 13; cable weight with value -1.002936N at nodes 12 and 13 while the optimum value for cable tension is -189396.97kg/km at nodes 12 and 13 (the minus sign shows the direction of deflection of the cable); the formation of elements equations of FEM by method of weighted residuals is an appropriate method because the results of assemblage equations are distinctive and depicts general trend of analytical solution as shown in figures (2) and (3); deflection values are useful tools in analyzing average strength in structures because the two properties are inversely proportional; and the value of maximum
displacement for the cable in this study was obtained from ANSYS as 4483.83mm for X, Y and Z components of displacements at node 2 while the value of minimum displacement obtained is 4218.75mm for all the directional components.

References

[1] Lacarbonara W., Paolone A., Vestroni F. (2007). Nonlinear Modal Properties of non-shallow Cables. International Journal of Non-Linear Mechanics, Elsevier, 42(3): 542 - 576.

[2] Lepidi M., Gattulli F., Vestroni F. (2007). Static and Dynamic Response of Elastic Suspended Cables with Damage. International Journal of Solids and Structures, 44(25): 8194 - 8212.

[3] Rega G. (2004). Nonlinear Vibrations of Suspended Cables. Part I: Modeling and Analysis. Applied Mechanics Reviews, 57(1-6): 443 - 478.

[4] Nawrocki A. and Labrosse M. (2000). A finite element model for simple straight wire rope strands. Computers and Structures, 77: 345 - 359.

[5] Barbieri R, Barbieri N and Oswaldo Honorato de Souza Junior (2008). Dynamical analysis of transmission line cables. Part 3–Nonlinear theory. Mechanical Systems and Signal Processing, 22(4): 992-1007.

[6] Jiang, W. G., Henshall, J. L. and Walton, J. M., (2000). A concise finite element model for three-layered straight wire rope strand. International Journal of Mechanical Sciences, 42: 63 - 86.

[7] Vestrioni F., Capecci D. (2000). Damage Determination in Beam Structures based on Frequency Measurements. Journal of Engineering Mechanics, 126(70: 761 - 768.

[8] Khiem N. T. (2006). Crack determination for structure based on the dynamic stiffness model and the inverse problem of vibration. Inverse problems in Science and engineering (Abingdon: Taylor & Francis), 14(1): 85 - 96.

[9] Gattulli, V., Martinelli, L., Perrotti, F. and Vestrioni, F. (2004). Nonlinear Oscillations of Cables under harmonic loading using analytical and finite element models. Computational Methods Applied Mechanical Engineering, 193(1): 69-85.

[10] Cheng P. S. and Perkins N. C. (2010). Closed form Vibration analysis of sagged cable/mass suspensions. ASME journal of applied mechanics, 59(4): 923 - 928.

[11] Wenjing Wang and Yu Yueqing (2007). Analysis of Frequency Characteristics of Compliant Mechanisms. Frontiers of Mechanical Engineering in China, 2(3): 267-271.

[12] Kevin J Bentley (2000). Numerical analysis of kinematic response of single piles. Canadian Geotechnical Journal, 37(6): 1368-1382.

[13] Chaudhary M. T. A. (2014). Semi closed-form solution for static nonlinear analysis of extensible cables. Mechanics of solids, Springer, 49(4): 468 - 476.

[14] Gattulli, V., Martinelli, L., Perrotti, F. and Vestrioni, F. (2007). Dynamics of suspended cables under turbulence loading: Reduced models of wind field and mechanical system. Journal of Wind Engineering and Industrial Aerodynamics, 95(3): 183-207.

[15] Wood J. (2000). A Behavioral Approach to the Pole Structure of one-dimensional and Multidimensional Linear Systems. SIAM Journal on Control and Optimization, 33(2): 627-661.

[16] Yang X., Jinxial L., Wei L., and Peng G. (2011). Petri Net Model and Reliability Evaluation for Wind Turbine Hydraulic Variable Pitch Systems. Energies, 4(6): 978-997.

[17] Cao D. Q., Tucker R. W., Wang C. (2001). Aeroelastic stability of a Cosserat stay cable. Proceedings of the 4th International Symposium on Cable Dynamics, Montreal, Canada, 369 - 376.

[18] Cao D. Q., Tucker R. W., Wang C. (2003). A stochastic approach to cable dynamics with moving rivulets. Journal of Sound and Vibration, 268(2): 291-304.