Towards the understanding of Decoherence on Ion Traps

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Abstract

Two mechanisms of decoherence in ion traps are studied, specially related to the experiment [Kielpinski et al., Science 291 (2001) 1013]. Statistical hypothesis are made about unknown variables and the expected behaviour of the visibility of the best experimental pattern is calculated for each mechanism. Data from the experiment are analyzed and show to be insufficient to distinguish between them. We suggest improvements which can do this with slight modifications in the present facilities.

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Ions traps are physical systems over which experimentalists have a large amount of precise control of the relevant degrees of freedom. This naturally makes these systems good candidates for exploring quantum phenomena and even for small scale quantum computation. Decoherence is the major barrier to be overtaken. One way of understanding decoherence is as a kind of quantum noise[1]. Its consequence is the transformation of pure states, which can be described by a state vector $|\psi\rangle$, into mixed states to which only density operator description applies. Naturally, there is a huge set of decoherence sources, from spurious electromagnetic fields originated in the lab, passing through small oscillations in parameter controls, up to background radiation and gravitational waves or whatever other kind of perturbation which forbids the system to be

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completely isolated[2]. Usually they are all considered as an environment to which the system is coupled.

In this work we give special attention to a paper by the Nist group[3], in which they showed how to produce a decoherence “free” qubit from a pair of ions trapped together. The idea of decoherence free subspaces can be classically understood, despite the fact of being a quantum phenomenon: if two classical moving particles are subjected to exactly the same noise, their relative motion is unaffected. In fact, if one can determine the major source of decoherence, and find two state vectors which are affected in the same way by free Hamiltonian and this decoherence source, any quantum information encoded in the subspace generated by these two vectors will be preserved from this decoherence source. As a cat and rat dispute, the other sources of decoherence will prevent this subspace to be completely decoherence free, but decoherence times will be substantially raised. That is what was obtained in two different situations in the cited experiment.

We will start our analysis by considering that the major source of decoherence are oscillations in the magnetic fields used to split hyperfine structure of $^9\text{Be}^+$ ions. More generally speaking, in situations like this, one can just consider the system to be subjected to a Hamiltonian $H(\xi)$, which depends on a stochastic parameter $\xi$. Stochastic Hamiltonians generate stochastic evolutions, so pure states evolve to mixed ensembles in a quantum state diffusion picture[4]. If we consider a situation in which only the magnitude of the field is subjected to oscillations, then the eigenvectors of $H(\xi)$ do not depend on $\xi$, but their eigenvalues do. Let us consider a model like this for the ions: we are interested only in two internal states of each ion, so we consider the stochastic Hamiltonian to be ($\hbar = 1$)

$$H(\xi) = \frac{\omega_1(\xi)}{2} \sigma_{z1} + \frac{\omega_2(\xi)}{2} \sigma_{z2},$$

(1)

which can be rewritten as

$$H(\xi) = \frac{\omega_m(\xi)}{2} \{\sigma_{z1} + \sigma_{z2}\} + \frac{\omega_d(\xi)}{2} \{\sigma_{z1} - \sigma_{z2}\},$$

(2)

where $\omega_m = (\omega_1 + \omega_2)/2$ and $\omega_d = (\omega_1 - \omega_2)/2$. The form (2) makes evident the structure of the Hamiltonians $H(\xi)$, including their dependence on $\xi$. In the ideal case, $\omega_1 = \omega_2$, we get a doubly degenerate level with energy $E = 0$, and two other levels with energies $\pm \omega_m$. If the perturbations keep $\omega_1 = \omega_2$ we have a bidimensional decoherence free subspace generated by $\{|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle\}$. So the situation close to DFS is to have $|\omega_d| \ll |\omega_m|$, and to consider initial states in this subspace. In this subspace, the term proportional to $\omega_m$ is ineffective, and decoherence is only originated by the variations in $\omega_d$ (i.e.: only relative
variations are important, because they affect Born frequencies).

Now we address the question: how does a state in a decoherence “free” subspace decohere? Let us consider an initially pure state

$$|\psi(0)\rangle = \alpha |\uparrow\downarrow\rangle + \beta |\downarrow\uparrow\rangle.$$  \hfill (3)

For a fixed value $\xi$, the Hamiltonian (2) implies that at time $t$ one has

$$|\psi(t)\rangle_\xi = \alpha e^{-i\omega_d(\xi)t} |\uparrow\downarrow\rangle + \beta e^{i\omega_d(\xi)t} |\downarrow\uparrow\rangle.$$  \hfill (4)

It is interesting to stress that in this model the vector state (4) always belongs to the decoherence “free” subspace, but due to its dependence on $\xi$, decoherence emerges. Writing the state (4) as a density operator and taking the ensemble average over the stochastic parameter $\xi$, with weights $p(\xi)$, one gets

$$\rho(t) = \begin{bmatrix} |\alpha|^2 & k^* \alpha^* \beta \\ k \alpha \beta^* & |\beta|^2 \end{bmatrix},$$  \hfill (5)

where we defined the parameter

$$k = \left< e^{i\omega_d(\xi)t} \right> = \int e^{i\omega_d(\xi)t} p(\xi) d\xi.$$  \hfill (6)

Let us stop for a moment to discuss states like (5). A Bloch vector picture is simple, specially in cylindrical coordinates: $|\alpha|^2 - |\beta|^2$ determines $z$ coordinate, and $2 |k \alpha \beta^*|$ is the cylindrical radius (i.e.: the distance to the $z$ axis). The most interesting case is $|\alpha|^2 = |\beta|^2 = \frac{1}{2}$ (which corresponds to the experimental case), when $|k|$ directly gives the norm of Bloch vector, which can be interpreted as a direct measure of the purity of the state. In practice, the norm of a Bloch vector can be associated to the visibility of the best interferometer prepared with the state $\rho$. In interferometric experiments one naturally searches for the largest visibility[5], therefore we shall focus our attention on the factor $k$.

Now let us discuss the factor $k$ of the model here presented, eq. (6). As the only influence of $\xi$ we are considering is on the frequency $\omega_d(\xi)$, we can consider as stochastic parameter a frequency $\nu$ itself, with an ensemble weight $p(\nu)$. So we have

$$k = \int e^{i\nu t} p(\nu) d\nu,$$  \hfill (7)
where one can recognize the structure of Fourier transform of the stochastic weight \( p(\nu) \), so common in optics[6]. As \( k \) determines the visibility of the best interferometer prepared with state \( \rho \), the data of ref. [3] must fit this expression for \( |k| \). An interesting improvement on the experiment would be to independently monitor the magnetic field, and try to compare the fringe visibility and the Fourier transform of the statistical variations of the field. This would decide whether fluctuations of the magnetic field are the major decoherence source or not. As these fluctuations are not available, we can only speculate about them. One natural hypothesis is of Gaussian distribution. This would imply a Gaussian decay of \( |k| \) with time \( t \). Other distributions can also appear, but we have no a priori reasons for treating them. One last example is to suppose exponential decay for the visibility, what in this model would be consistent with Lorentzian stochastic weight

\[
p(\nu) \propto \frac{\Gamma}{(\nu - \nu_o)^2 + \Gamma^2}.
\]

(8)

In the cited experiment[3], one state in the decoherence “free” subspace (DFS state) and one out of it (test state) were submitted to two distinct situations each: autonomous evolution, just subjected to natural noise, and an engineered reservoir, consisting of a dissonant laser with random intensity. The first situation is supposed to be well modeled by the above discussion on variable magnetic field, but the second one deserves special attention. Experimental values allow one to consider only decoherence sources related to the presence of the dissonant laser. The most simple way to do it would be to consider the dispersive approximation to Jaynes-Cummings model for the interaction among one field mode and two two-level ions:

\[
H_{dJC} = H_o + H_{int},
\]

(9)

where

\[
H_o = \frac{\omega}{2} J_z + \omega f a^\dagger a,
\]

(10)

where \( J = \sigma_1 + \sigma_2 \), and

\[
H_{int} = g \left( J_+ a + J_- a^\dagger \right)
\approx \Omega \left\{ a a^\dagger \left( |\uparrow_1 \rangle \langle \uparrow_1| + |\uparrow_2 \rangle \langle \uparrow_2| \right) - a^\dagger a \left( |\downarrow_1 \rangle \langle \downarrow_1| + |\downarrow_2 \rangle \langle \downarrow_2| \right) \right\},
\]

(11)

where \( \Omega = \frac{g^2}{2\delta} \), \( g \) is a dipolar coupling constant, \( \delta = \frac{1}{2} (\omega - \omega_f) \) is the detuning, and dispersive approximation was applied, under the condition \( \delta^2 \gg \)
$g^2(n+1)$, $n$ the number of photons. However, this can not mimic the experimental situation, unless we include somehow the randomization of the laser field. This can be done by considering an environment coupled to this field. As usual, let us consider a huge set of harmonic oscillators and bilinear coupling as a model to the environment. In this sense, the full Hamiltonian we must consider is

$$H = H_{d JC} + \sum \omega_l \{b_l^\dagger b_l + \sum g_l (a_l^\dagger b_l + b_l^\dagger a_l)\}. \quad (12)$$

The laser-environment part of this Hamiltonian can be rewritten in terms of normal modes, and this formally shows the equivalence between two ions coupled to a laser coupled to an environment and the same two ions directly coupled to a reservoir. To go further one needs to make some hypothesis on the coupling of the laser to the environment. White noise is a natural choice, and a Wigner-Weisskopf like approximation[7] implies the form of the coupling between ions and environmental normal modes. This is once again consistent with exponential decay of fringe contrast in interferometry.

So, we considered two distinct sources of decoherence for a system like the one worked out in ref. [3]: one given by stochastic variations of the energy levels (probably due to variations of the applied magnetic field), and other given by the direct coupling to a “classical” field mode (“classical” in the sense of being strongly coupled to an environment). For both, we made statistically natural hypothesis, namely Gaussian fluctuations and white noise. The first model naturally gives Gaussian decay for the fringe contrast while the second implies exponential decay. We therefore strongly believe that when the laser is off, “natural” decoherence will fit a Gaussian, while with the laser on, “engineered” decoherence will be exponential. We take the data from the experiment, and used least square methods to obtain the best Gaussian and the best exponential to each set of data, and also used a kind of sieve to try to tell us if the points fit better a Gaussian or an exponential. We now discuss in detail the methods we used and the results we got.

In fact, the function which we statistically treated was $F(t) = \ln V(t)$, where $V$ is the visibility of the fringe pattern. For each set of data we search for the least square curves of the form

$$F = at + b,$$
$$F = At^2 + B, \quad (13)$$

for which is important to say that both have the same number of parameters, what make the comparison fair. For each set of data, and each fitting family, we accumulate the vertical distances between the measured points and the best fitting curve and compare: for each set of data, the one which accumulate
less distance is considered the best curve. For each initial state and laser situation the figure shows the experimental points, the best Gaussian (in fact, the best parabola for the logarithm) and the best exponential (resp. linear function). The accumulated distance is given in captions. The worked data are not enough to corroborate or deny our pre-conclusions about the form the coherence decays in each case, although a few more points could do the job. Also, as pointed out before, independently recording the field fluctuations and comparing its line shape to the visibility (with the laser off) could test the usually accepted viewpoint that the most important decoherence source are these field fluctuations. We expect that this work can stimulate more detailed experiments towards a better understanding of the mechanisms of decoherence on ion traps.

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Figure captions

Figure 1: For autonomous decoherence (laser off), the points and error bars are experimental[3], the curves are least square fits: (a) DFS state, $y = -.803 - .00224t$, with accumulated square distance (asd) .0095, and $y = -.997 - .393 \times 10^{-5}t^2$, with asd .062; (b) Test state, $y = -.109 - .00883t$, with asd .037, and $y = -.394 - .391 \times 10^{-4}t^2$, with asd .0040.

Figure 2: The same for engineered decoherence (laser on): (a) DFS state, $y = -.874 - .00330t$, with asd .0084, and $y = -.884 - .159 \times 10^{-3}t^2$, with asd .0083; (b) Test state, $y = -.174 - .175t$, with asd .084, and $y = -.0110 - .581t^2$, with asd .16.
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