On the impossibility of advection dominated accretion

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ABSTRACT
Using only the assumption that all interactions between particles in an accretion flow are electromagnetically mediated, it is shown that the time to establish equipartition between ions and electrons is shorter than the characteristic accretion time. Consequently, two-temperature fits to the spectra of accreting objects are unphysical, and models in which significant thermal energy is carried across the event horizon are effectively ruled out.

Key words: accretion, accretion disks – black hole physics – galaxies: active – X-ray: stars – binaries

1 INTRODUCTION

The supermassive black holes at the centres of galaxies are usually much less luminous than simple accretion theory would predict: the black hole is surrounded by thermal plasma whose properties may be known from X-ray observations, and the black hole should accrete this plasma at roughly the Bondi-Hoyle rate. When this accretion rate is combined with the radiative efficiency $\sim 0.1$ that follows from a comparison of the integrated luminosity of quasars with the observed space density of supermassive black holes (Yu & Tremaine, 2002), a luminosity is derived that generally exceeds that observed by 3–5 orders of magnitude (Loewenstein et al. 2001; Di Matteo et al. 2001, 2003; Pellegrini et al. 2003).

The low measured luminosities of nuclear black holes led to the development of the advection-dominated accretion flow – hereafter ADAF model – in which the energy released as plasma falls towards the black hole is retained as internal energy within the plasma and ultimately carried over the black hole’s event horizon rather than escaping to infinity (Ichimaru 1977; Rees et al. 1982; Narayan & Yi, 1994, 1995; Igumenshchev, Chen & Abramowicz, 1996; see also the lucid review in Chapter 11 of Frank, King & Raine, 2002). These ideas were subsequently applied to accreting stellar-mass black holes and neutron stars. The flows around these objects would be similar down to radii comparable to the neutron star surface. Hence the existence of an event horizon around a black hole would cause accreting stellar-mass black holes to be much less luminous than comparable neutron stars. Garcia et al. (2001) claim to observe this effect.

Since material reaching the event horizon has lost gravitational potential energy that is a substantial fraction of $c^2$ per unit mass, the ADAF model requires that the innermost plasma achieves temperatures of order $m_p c^2 \sim 1$ GeV. At such temperatures the electrons are highly relativistic ($\gamma \sim 1000$), the radiative efficiency of the plasma would be expected to be high because the electrons’ radiative losses increase with temperature faster than $T^7$ (Rees et al. 1982). In order to limit the radiative efficiency of the plasma, the ADAF model conjectures that the electrons decouple thermodynamically from the ions, which are the component of the plasma that receives the lion’s share of the released gravitational energy. Hence the thermal motions of the ions become mildly relativistic as the event horizon is approached, while the electrons, which dominate radiative processes, remain at temperatures that are orders of magnitude lower.

Proponents of the ADAF model motivate the decoupling of the electrons from the ions by pointing out that in a highly viscous low, the residence time of any given electron or ion in the flow is small, so the plasma density required to achieve a given accretion rate is low. At low densities and high temperatures, the time required for coulomb scattering to establish equipartition of energy between ions and electrons becomes long, and can easily exceed the residence time (Rees et al. 1982). In this paper I give a simple argument of considerable generality, which implies that the time $t_{\text{equal}}$ required to establish equipartition between ions and electrons is always shorter than the residence time $t_{\text{res}}$. Consequently, one may safely assume that the electrons are at the same temperature as the ions. This result invalidates most applications of the ADAF model, and effectively rules out all models in which significant thermal energy is carried across the event horizon.
2 THE EQUIPARTITION TIME

The plasma comprises particles that interact electromagnetically with each other and with a given gravitational field. I shall assume that the dynamics can be treated in a non-relativistic approximation – the generalization of the analysis to the relativistic case is not hard, but there is a loss of clarity and simplicity.

The Hamiltonian for motion of a particle of mass \( m \) and charge \( q \) in an electromagnetic field that is characterized by the potentials \( \psi \) and \( \mathbf{A} \) is

\[
H = \frac{(p - qA)^2}{2m} + q\psi + m\Phi.
\]

where \( \Phi \) is the gravitational potential. From standard Hamiltonian theory we know that

\[
\frac{dH}{dt} = \frac{\partial H}{\partial t} = -q\mathbf{v} \cdot \frac{\partial \mathbf{A}}{\partial t} - q\frac{\partial \psi}{\partial t} - \frac{q}{m} \mathbf{v} \cdot \frac{\partial \mathbf{A}}{\partial \mathbf{r}} + \frac{\partial q}{\partial t} \mathbf{v} = -q\mathbf{v} \cdot \frac{\partial \mathbf{A}}{\partial t} - q\frac{\partial \psi}{\partial t},
\]

where we have assumed that \( \Phi \) is time-independent and have identified the particle velocity \( \mathbf{v} = (p - q\mathbf{A})/m \). Since the right side of this equation is proportional to the charge \( q \), if at some location energy is lost by one species, it is gained by the oppositely charged species. Thus this equation describes the mechanism by which equipartition is established between ions and electrons; the net direction of the energy flow is determined by the potentials

\[
\text{In view of this distinction, the two terms will not cancel with each other, and we may obtain a lower limit on the rate of energy transfer between species by considering only one term. For reasons that should become clear later, we focus on the inductive term and obtain the following upper limit on the equipartition time}
\]

\[
l_{\text{eq}} \sim \frac{H}{|\mathbf{Q} \cdot \partial \mathbf{A}/\partial t|} < \frac{H}{|\mathbf{Q} \cdot \partial \mathbf{A}/\partial t|}.
\]

\[
\text{Inward drift through an accretion disk is controlled by loss of angular momentum } L_z. \text{ Again using standard Hamiltonian theory together with the result that } L_z = p_\phi \text{ is the momentum conjugate to the azimuthal angular coordinate } \phi, \text{ we have}
\]

\[
\frac{dL_z}{dt} = [p_\phi, H] = \frac{\partial H}{\partial \phi} = q\mathbf{v} \cdot \frac{\partial \mathbf{A}}{\partial \phi} - q\frac{\partial \psi}{\partial \phi},
\]

where the square bracket is a Poisson bracket.

The expressions above give the rates of energy and angular momentum change for a single particle. To obtain the corresponding rates for the plasma as a whole we have to sum over the particles in some volume.

\[
\frac{dL_z}{dt} = \sum q \left( (\mathbf{v}^+ - \mathbf{v}^-) \cdot \frac{\partial \mathbf{A}}{\partial \phi} - \frac{\partial \psi}{\partial \phi} \right)
\]

where \( \mathbf{j} \) is the current density and \( \rho \) is the charge density.

\[
\text{If we again neglect the term containing } \psi, \text{ we find that the residence time within the flow is}
\]

\[
l_{\text{res}} \sim \frac{L_z^2}{|\mathbf{v} \cdot \partial \mathbf{A}/\partial t|} \sim \frac{L_z^2}{|\mathbf{j} \cdot \partial \mathbf{A}/\partial \phi|}.
\]

\[
\text{We now return to our upper limit on the equipartition time. We average top and bottom over all particles in a given small region. This operation is bound to increase the ratio significantly since the Hamiltonian values on the top are all positive, while those on the bottom, being proportional to } q, \text{ have a tendency to cancel between species. Consequently, our upper limit remains an upper limit, and we may write}
\]

\[
l_{\text{eq}} \sim \frac{H}{|\mathbf{Q} \cdot \partial \mathbf{A}/\partial t|} < \frac{H}{|\mathbf{Q} \cdot \partial \mathbf{A}/\partial t|}.
\]

\[
\text{Dividing by equation } \text{we obtain}
\]

\[
l_{\text{eq}} < L_{\text{res}}^2 \left( \frac{\mathbf{j} \cdot \partial \mathbf{A}/\partial \phi}{|\mathbf{j} \cdot \partial \mathbf{A}/\partial \phi|} \right) \sim \frac{L_z^2}{|\mathbf{j} \cdot \partial \mathbf{A}/\partial \phi|}.
\]

\[
\text{Standard Hamiltonian theory tells us that for any particle the azimuthal frequency } \Omega_z \text{ is given by } \Omega_z = \partial H/\partial L_z \sim H/L_z, \text{ so the first ratio on the right side of } \text{can be approximated by the local circular frequency. The ratio of integrals we approximate by the ratio of the corresponding derivatives of } \mathbf{A}, \text{ so}
\]

\[
l_{\text{eq}} < \Omega_z \left( \frac{\partial \mathbf{A}/\partial \phi}{|\partial \mathbf{A}/\partial \phi|} \right) \sim \Omega_z \left( \frac{\partial \mathbf{A}/\partial \phi}{|\partial \mathbf{A}/\partial \phi|} \right).
\]

\[
\text{To get an idea of the relative sizes of the derivatives of } \mathbf{A}, \text{ consider the case in which } \mathbf{A}(t, \phi) \text{ can be approximated by a pattern that propagates at some phase frequency } \omega. \text{ Then } \mathbf{A} \text{ is a function of the single variable } \omega t - \phi \text{ and the ratio of derivatives becomes } 1/\omega. \text{ The important case is that in which the pattern propagates with some particles (the particles that are Landau-damping it). In this case } \omega \text{ is of order } \Omega_z \text{ and the right side of } \text{evaluates to unity. In principle the ratio } \Omega_z/\omega \text{ could be equal to some small integer because the Landau-damping resonance could be associated with a harmonic of } \omega. \text{ However, in view of the large amount by which the right side of } \text{exceeds that of } \text{we may state with confidence that}
\]

\[
l_{\text{eq}} < 1.
\]

3 DISCUSSION

In view of this simple and very general demonstration that the time to establish equipartition between electrons and ions is not greater (and realistically much smaller) than the time it takes plasma to move through the accretion disk, why has so much attention has been paid to the proposal that the electron and ion temperatures differ significantly? The answer lies in uncertainty as to what mechanism actually drives accretion through a disk. It was very early on
realised that the viscosity provided by standard kinetic theory falls short of that required by many orders of magnitude (e.g., Frank et al. 2002). In default of a proper physical theory, the field progressed by using the $\alpha$ parameter of Shakura & Sunyaev (1973) to connect viscosity to the pressure in a dimensional sense. The ADAF model assumed that a large value of $\alpha$ was appropriate, and thus that the viscosity was very much larger than kinetic theory would predict. Inconsistently, it assumed that the equipartition time was still correctly estimated by kinetic theory. All the derivation above does is to insist that whatever viscosity the disk experiences is mediated by electromagnetic fields, and to show that these fields inevitably establish equipartition on a timescale that is shorter than the accretion timescale.

Since the seminal paper of Balbus & Hawley (1991) it has become widely agreed that the magnetic-rotational instability (MRI) is the detailed mechanism that provides the high values of viscosity that observations imply. In this picture the electromagnetic field in the disk is dominated by time-varying magnetic fields with scale lengths comparable to the disk thickness. The derivation above implicitly focuses on this case in as much as it neglects the terms in (2) and (5) that contain the electrostatic potential $\psi$. If one were concerned about kinetic-theory viscosity, one would need to include these terms. It is easy to see that in the limit in which these terms dominate the terms in $A$, one would again arrive at our fundamental result (10).

4 SUMMARY

A time-dependent electromagnetic field transfers energy between oppositely charged particles very efficiently. Consequently, when one compares the rate of such energy transfer with the rate at which the field mediates the exchanges of angular momentum that drive accretion, one concludes that equipartition of energy between electrons and ions can be achieved well within the time it takes plasma to drift inwards. Hence we may state with confidence that $T_{\text{ion}} = T_{\text{electron}}$. Fits of the observed spectra of sources such as Sgr A* to the ADAF model (Narayan & Yi, 1995) assume that $T_{\text{ion}} \gg T_{\text{electron}}$. Such fits are unphysical.

In an $\alpha$ disk the density scales with the mass-flow rate as $\dot{m}^{11/20}$, while the residence time $t_{\text{res}}$ scales as $\dot{m}^{-3/10}$ (Frank et al. 2002). Since the time $t_{\text{rad}}$ required to radiate a given energy per particle scales inversely with density, the ratio $t_{\text{rad}}/t_{\text{res}}$ scales as $\dot{m}^{-1/4}$, and we conclude that at sufficiently small values of $\dot{m}$, even thermalized electrons will be unable to radiate the internal energy of a plasma that is at the virial temperature before the event horizon is reached. Hence the ADAF model is not completely excluded. However, for an ADAF to be possible, the ions must become relativistic, and, in view of the efficiency of equipartition, the electrons must become ultrarelativistic. Such electrons radiate extremely efficiently through a combination of bremsstrahlung, synchrotron radiation and the Compton scattering of photons. Consequently, the values of $\dot{m}$ at which radiative cooling of the electrons is unimportant are too low to be of astrophysical interest (Rees et al. 1982).

The nature of accretion onto black holes remains a puzzle. There is considerable observational evidence that energy released by accretion onto a black hole can be chan-