On the Effective Description of Dynamically Broken SUSY-Theories

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Abstract
We introduce a new class of effective actions describing dynamically broken supersymmetric theories in an essentially non-perturbative region. Our approach is a generalization of the known supersymmetric non-linear sigma models, but allows in contrast to the latter the description of dynamical supersymmetry breaking by non-perturbative non-semiclassical effects. This non-perturbative breaking mechanism takes place in confined theories, where the effective fields are composite operators. It is necessary within the context of quantum effective actions and the associated concept of symmetry breaking as a hysteresis effect. In this paper we provide a mathematical definition and description of the actions, its application to specific supersymmetric gauge theories is presented elsewhere.

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1 Introduction and Outline of the Problem

Effective Lagrangian techniques for about twenty years have been a successful tool to explore the low-energy dynamics of supersymmetric gauge-theories. While most of the models investigated are in the Higgs or Coulomb phase at low energies, mainly due to the work by Dijkgraaf and Vafa, confined models are of particular interest in these days. There exists a simple but far-reaching difference between confined theories and theories in the Higgs or Coulomb phase: The effective fields of a confined theory are always composite operators. Although this is not solely a technical difficulty, but a main characteristic of the phenomenology of confined theories, a detailed discussion of its consequences for the construction of effective actions within supersymmetry does not seem to exist in the literature. In this paper we propose a consistent treatment of composite effective fields within supersymmetry and we show that confinement indeed leads to fundamental changes in the understanding of the effective action.

Before going into the technical details of our construction we shortly review the problems arising in the context of composite effective superfields. As a standard example of a confined supersymmetric theory we consider \( N = 1 \) SYM. We want to construct an effective action for \( N = 1 \) SYM theories based on classical fields from composite operators that represent (by assumption) the relevant low energy degrees of freedom. It has been shown that we obtain such an effective action by extending the complex coupling constant \( \tau \) of SYM to a chiral superfield \( J(x) = \tau(x) + \theta \eta - 2 \theta^2 m \). The effective action is obtained by Legendre transformation and is formulated in terms of three classical fields \( \varphi, \psi \) and \( F \) that represent the gluino condensate, a spinor (the goldstino in case of dynamical supersymmetry breaking) and the classical Lagrangian, respectively. The source extension is unique in the sense that there exists no other extension that preserves gauge invariance and supersymmetry covariance \( \mathcal{R} \). By combining the three effective fields to a chiral superfield \( \Phi = \varphi + \theta \psi + \theta^2 F \), the effective action can be written as an integral over superspace.

This system had been studied in refs. \( [5, 9, 6] \), the ansatz for the effective action used therein can be written as

\[
\mathcal{L} = \int d^4x \left( \int d^4 \theta \ K(\Phi, \bar{\Phi}) - \int d^2 \theta \ W(\Phi) + h.c. \right) ,
\]

where the superpotential is determined by the anomaly-structure

\[
W(\Phi) = \Phi (\log \frac{\Phi}{\Lambda^3} - 1)
\]

and the Kähler potential is a polynomial function in \( \Phi \). The most important consequence from the ansatz \( [1, 4] \) is: The effective composite operator \( F \) must be an auxiliary field. As this effective field is build up from composite operators, this constraint is neither motivated nor consistent. Indeed, in this situation there exist two types of “auxiliary” fields:
1. The auxiliary field of the fundamental theory, in $N = 1$ SYM typically denoted by $D$. The theory must be ultra-local in this field and it can be eliminated consistently even in the full quantum theory. This field will be denoted as 1st generation auxiliary field in the following.

2. The effective composite field appearing in the effective superfield at a place, where one usually expects an auxiliary field ($F$ in $N = 1$ SYM, 2nd generation “auxiliary” field in the following). As the field $F$ is itself a composite operator of fundamental fields, it is not directly related to the 1st generation auxiliary field $D$. It even exists, if the 1st generation field has been eliminated by its algebraic equations of motion!

The imperative to distinguish strictly between 1st and 2nd generation “auxiliary” fields must lead to the conclusion that the ansatz (1.1) is inconsistent. This has been discussed extensively in [10], in the following the main conclusions are summarized:

1. Within the ansatz (1.1) the field $F$ has the typical potential of an auxiliary field: It is not bounded from below but falls off to $-\infty$ as $F$ goes to $+\infty$. In fact it does not even have a local minimum, but instead an absolute maximum, cf. figure 1. All supersymmetric Lagrangians with a polynomial Kähler potential have auxiliary fields with a potential of this type. This implies that one is forced to interpret the physical ground-state of the theory by the absolute maximum of the auxiliary field potential.

2. Point 1 is equivalent to the statement that $F$ must be an auxiliary field that can be eliminated by its algebraic equations of motion. Indeed, the spectrum of the theory obtained in [5] can be found after this elimination, only.

3. As a consequence of 1 and 2 the action must be ultra-local in $F$ exactly, i.e. derivatives acting on $F$ do not exist. This point is important as the action (1.1) is certainly not the complete effective action of $N = 1$ SYM, but at most an approximation for small momenta ($p^2 \ll \Lambda^2$). The fact that $F$ must be auxiliary then means that derivative terms on this field do not vanish solely in the approximation (1.1), but within the complete effective action. Indeed, when introducing derivative terms on $F$ as higher order effects the auxiliary fields become dynamical (the action is no longer ultra-local in this field). Consequently the absolute maximum must become an unstable point of the theory and $F$ moves towards the correct ground-state of this situation: $\langle \Omega | F | \Omega \rangle = \infty$! Of course such effects are suppressed at low energies compared to the dynamics of $\varphi$. It is of main importance to notice that this is irrelevant: Simple classical stability considerations show that any finite contribution to the dynamics of $F$ is sufficient to generate the instability (this point is discussed more in detail in [11]).

1We emphasize that we are using a quantum effective action, whose description is a purely classical object. Indeed the question of dynamical auxiliary fields is quite different, if the non-linear model is not seen as a classical but a quantum object. We will comment on this below.
Thus either the higher order derivatives vanish exactly or \( \langle \Omega | F | \Omega \rangle = \infty \) is the “correct” ground-state. In the latter case we would have to conclude, that we did not identify our low-energy degrees of freedom correctly.

4. As \( F \) is not related to the fundamental auxiliary field, an interpretation as sketched in point 3 simply fails to capture the physics of this field. \( F \) contains the operators \( F_{\mu\nu}F^{\mu\nu} \) and \( F_{\mu\nu} \bar{F}^{\mu\nu} \), which are dynamical variables of pure YM theory and certainly the same applies to SYM. The misinterpretation of the composite field \( F \) within the ansatz (1.1) (points 1-3) led to the mystery of the missing glue-balls: As \( F \) has not been interpreted as an independent degree of freedom, the resulting theory is (by assumption) confined and has a mass-gap, but nevertheless there appears no glue-ball in its spectrum.

5. To escape the conclusion of the last point one could try to include the glue-ball without identifying it with the operator \( F \) (this has been suggested in [12], from our point of view the result found therein is not consistent with \( N = 1 \) SYM [10]). But this would not solve the problem, as such an effective action would still be strictly local (one derivative for the spinor, two derivatives for the gluino condensate). If we identified the low-energy degrees of freedom correctly higher order derivatives are indeed suppressed but they are not allowed to vanish exactly. An effective action of this type is not acceptable as long as the underlying theory is not free.

Besides this general objection a careful analysis within the supersymmetric framework shows that the locality of supersymmetric non-linear sigma models is not just a harmless peculiarity but has drastic consequences: By writing down the effective action as superspace integral we assumed that the invariance under extrinsic supersymmetry (transforming the quantum fields as well as the sources) is realized thereby and the generic form of (1.1) is correct for any value of the sources (up to possible spurion fields, which are irrelevant for the following arguments). By decoupling the gluino, (1.1) becomes completely non-dynamical. From physical arguments we would expect that this effective action breaks down at some value of the gluino mass: As we increase \( m \) the mass of the lightest gluino state increases as well and for some critical value \( m_c \) reaches the scale of the lightest glue-ball. At this point the description (1.1) should break down as it does not include all relevant low-energy degrees of freedom. This should be seen by the breakdown of the expansion in the derivatives. But this breakdown can never take place in (1.1) as higher order derivatives are always strictly zero. Notice that the alternative indication of a breakdown of the description –instabilities in the potential– is excluded as well. For details we refer the reader to [10].

An acceptable ansatz for the quantum effective action could be found by dropping the assumption that the latter can be written as an integral over superspace. Without further specifications such a model has been discussed in [10]. There indeed exist possibilities for descriptions of this type for both, broken and unbroken supersymmetry. However in its most
general form such an ansatz seems to be excluded by symmetry-arguments: As the three classical fields do transform under supersymmetry they build a representation thereof and thus supersymmetry would be realized non-linearly. But this contradicts the assumption that we can expand our effective action in the momenta, as non-linear representations mix different orders in $p^2$. This would then lead to the conclusion that we did not correctly identify the low-energy degrees of freedom.

In this paper we show how to construct a model obeying all requirements outlined above and leading to an effective action for SYM with

- linear realization of supersymmetry,
- dynamical glue-ball,
- infinite orders of derivatives, where higher orders are suppressed but present.

As most important consequence of a consistent implementation of these three points we will find that supersymmetry breaks dynamically. The breaking mechanism is of essentially non-perturbative character and is not comparable to any other breaking mechanism known in literature.

In the present paper we explain the mathematical definition the model and its basic physical properties. The application of these ideas to SYM is worked out in [11].

2 Definition of the Model

2.1 The Basic Idea

In principle a non-local effective Lagrangian and dynamical auxiliary fields do not stand in contradiction to (linearly realized) supersymmetry. When considering a single chiral superfield $\Phi = \varphi + \theta \psi + \theta^2 F$, then the expression

$$L_{\text{kin}} = \int d^4 \theta \left( c_0 \bar{\Phi} \Phi - c_1 \bar{\Phi} \Box \Phi - c_2 \bar{\Phi} \Box^2 \Phi - \ldots \right)$$

(2.1)

is invariant under supersymmetry. As outlined in the introduction the simple extension of the ansatz (1.1) with such higher order derivative terms is excluded by stability arguments as they introduce kinetic terms for the auxiliary field $F$:

$$L_{\text{kin}} = c_0 |F|^2 - c_1 \bar{F} \Box F - c_2 \bar{F} \Box^2 F + \ldots$$

(2.2)

To construct a physically meaningful Lagrangian containing terms of the form (2.1) it is indispensable to construct a potential for the 2nd generation "auxiliary" field $F$, which is
Figure 1: The potential of the highest component $F$ in a theory with standard Kähler potential as non-holomorphic part (left hand side) and a possible shape of the potential within the extension proposed in this work (right hand side).

bounded from below\(^2\) (cf. figure 1). At the same time of course, the potential in $\varphi$ and $\psi$ must be bounded from below as well and the dynamics in all fields must be stable at least for small momenta.

When representing the non-holomorphic part of the action by a standard non-linear sigma-model (cf. eq. (1.1)) a stable potential for both, the auxiliary as well as the physical fields cannot be obtained. As by its construction the non-linear sigma model is the most general Lagrangian obeying all symmetries, we have to weaken some conditions compared to this approach. This concerns the understanding of a stable potential. We insist on the potential being bounded from below and having an unique absolute minimum, identified with the physical minimum (we do not consider models with a quantum moduli space, as the classical moduli space must getting lifted when exploring the hysteresis line \([10]\)). In contrast to (1.1) we however accept potentials that become flat above some value of the fields (cf. figure 2). This is motivated by the following observation: Our description is valid below some energy-scale $\Lambda$ (or equivalently within restricted local excitations of the sources) as well as within a certain range of the global sources, only. A breakdown of the description outside of this range is rather a necessity than just a possibility. This breakdown can either be seen in the momentum-expansion or in the potential. However we have to insist on a potential bounded from below, as the physical minimum is defined as the absolute minimum of the effective potential (for a detailed discussion of this point see \([10]\)). Thus a potential becoming flat above some scale of the fields is indeed the most general situation. Of course

\(^2\)In a situation with more than one effective superfield, an alternative route to include higher derivative terms has been discussed in the literature \([13, 14, 15, 16]\). But within that approach, the 2\textsuperscript{nd} generation auxiliary fields keep their usual behavior and thus these models deal with a physically different situation than the one discussed here.
Figure 2: Acceptable potentials within effective descriptions. Left hand side: The physical potential from standard non-linear sigma models as non-holomorphic part. Right hand side: Possible potential in our class of models.

this scale has now direct physical implications. As we discuss in this paper the general model without any reference to a concrete application, we do not attend to this point within this work.

This general choice of acceptable potentials as well as other steps of our construction will lead to an ansatz for the effective action, which is not an acceptable (classical) field theory for its own. It is of particular importance in the discussion on hand that this need not be the case – in fact we will see that any acceptable description of the effective action must disobey important features of classical supersymmetric field theories. From the point of view of the underlying quantum field theory these non-supersymmetric aspects of the effective description will turn out to be in perfect agreement with all symmetries. We emphasize that supersymmetry (or any other symmetry realized in the system) can be understood from this point of view, only. Many problems in the description of dynamically broken supersymmetry and its hysteresis line can be resolved by dropping the unfounded assumption that such a model must be described by the classical supersymmetric non-linear sigma model of equation (1.1). In this context it is important to note that our model should be understood in a complete non-perturbative study of quantum field theories, only. Clearly a perturbative analysis of the same models must be compatible with standard superspace geometry even when formulated in terms of the same operators as used in the non-perturbative region.

To avoid misunderstandings we should shortly comment on the notion of an effective action used in this work.

- We consider as effective action a quantum effective action obtained by Legendre transformation. This action is a purely classical object, where all quantum effects have been summed up. We emphasize that we cannot escape the difficulties discussed so far by switching to an alternative low-energy description, especially to a Wilsonian
low-energy effective action. Indeed, some of the mentioned problems may be absent in the Wilsonian action, but instead of solving them it simply gets rid of an important part of the dynamics by introducing an arbitrary infrared regulator. In consequence the quantities appearing in the Wilsonian action (e.g. the coupling constant or the low-energy fields) are not physical, but the physical quantities are found after a (perturbative and non-perturbative) renormalization step, only. It has been discussed in detail in [10] why the Wilsonian action alone cannot serve as an alternative to the quantum effective action.

- Dynamical auxiliary fields appear in a quite different context as well: If the non-linear model is not seen as a purely classical object (as in this paper) but still has quantum degrees of freedom the auxiliary fields become dynamical without introducing any kinetic terms by hand. This is the direct consequence of supersymmetry Ward identities, which read for a chiral superfield

\[ \langle \Omega | T \bar{F}(x) F(x') | \Omega \rangle = \Box \langle \Omega | T \bar{\psi}(x) \psi(x') | \Omega \rangle. \]  

(2.3)

In a linear theory this relation defines \( F \) to be an auxiliary field, in a non-linear theory it makes \( F \) dynamical. Though this is a quite different mechanism than the one proposed in this paper, our discussion is not irrelevant for this case. The important observation is the following: If \( F \) is a dynamical field, the potential must be bounded from below without elimination of the auxiliary fields. Whether the origin of the kinetic terms in \( F \) are quantum dynamics or just some terms written into the Lagrangian by hand is completely irrelevant. From this point of view mainly the discussion of the new interpretation of the superpotential given in this paper is valid for a non-linear quantum model as well. As an example, the spectrum of the superpotential by Veneziano and Yankielowicz [5] is getting changed compared to that work in our models as well as in a quantum treatment of this Lagrangian. In both cases the elimination of the \( F \) field performed in [5] is forbidden and e.g. the second derivative of the superpotential is no longer proportional to the mass of \( \varphi \) (nevertheless it does still define a mass term of \( \psi \)). Although our discussion takes place on a completely classical level this aspect of the problem is of fundamental importance in a non-linear quantum model as well.

### 2.2 Constraint Kähler Geometry with Dynamical Auxiliary Fields

A physical potential of the 2nd generation auxiliary field within the ansatz (1.1) would be possible with \( g_{\varphi \bar{\varphi}} < 0 \) (equivalent to \( c_0 < 0 \) in (2.1)), only. It is easy to check, that the instabilities caused by this “wrong” sign cannot be removed. On the other hand, a physical potential is possible, if and only if the highest power in \( F \) comes together with a positive sign in the effective potential. The only way out is thus an effective Lagrangian containing higher powers in \( F \) (at least \( |F|^2 \)). Therefore we have to define a new superfield, where \( \bar{F} \)
appears as lowest component. Starting from the effective superfield \( \Phi \) we construct further dependent effective fields according to

\[
\Phi_0 \equiv \Phi, \quad \Phi_n = D^2 \Phi_{n-1}, \quad \Phi_{2n} = (-1)^n \Box^n \Phi_0, \quad \Phi_{2n+1} = (-1)^n \Box^n \Phi_1. \tag{2.4}
\]

The most general effective Lagrangian of the chiral field \( \Phi \) is now given by

\[
\mathcal{L} = \int d^4 x \left( \int d^4 \theta A(\Phi_0, \Phi_1, \bar{\Phi}_0, \bar{\Phi}_1) + \left( \int d^2 \theta H(\Phi_0) + h.c. \right) \right). \tag{2.5}
\]

As this effective Lagrangian allows the construction of a potential bounded from below in all fields (as motivated in the previous section), we can—in contrast to (1.1)—relax the constraint of \( A \) and \( H \) being polynomial functions in \( \Phi_0 \) and \( \Phi_1 \). Instead they are allowed to include explicit space-time derivatives. To make contact to the standard formulation of effective Lagrangians in terms of a Kähler- and a superpotential we may define a related action as

\[
\mathcal{L} = \int d^4 x \left( \int d^4 \theta K(\Phi_n, \bar{\Phi}_n) - \left( \int d^2 \theta W(\Phi_0) + h.c. \right) \right). \tag{2.6}
\]

Here the index \( n \) runs from zero to infinity according to equation (2.4). \( K \) and \( W \) are polynomial functions, i.e. before using the constraint (2.4) they describe the standard Kähler- and superpotential. Both actions (2.5) and (2.6) have the same effective potential. However, (2.6) is a restricted version of (2.5) as it does not include all derivative terms of the latter. To obey fundamental stability conditions certain constraints among the different geometrical objects in (2.5) and (2.6) have to be fulfilled.

Before going into a detailed discussion of (2.5)/(2.6) the two important characteristics of these actions are set out again:

1. The 2nd generation “auxiliary” field \( F \), being the highest component of \( \Phi = \Phi_0 \), appears as lowest component of \( \Phi_1 \). Thus both scalar fields, \( F \) and \( \varphi \), can appear with any power in the effective potential.

2. The equation of motion of \( F \) is not algebraic. Thus this field cannot be eliminated. Instead it must be treated as an independent physical degree of freedom.

To discuss the effective potential of (2.5)/(2.6) we can suppress all \( \Phi_n (n > 2) \), or equivalently, restrict the functions \( A \) and \( H \) to be polynomials in the fields. The corresponding action is given by

\[
\mathcal{L} = \int d^4 x \left( \int d^4 \theta K(\Phi_0, \Phi_1; \bar{\Phi}_0, \bar{\Phi}_1) - \left( \int d^2 \theta W(\Phi_0) + h.c. \right) \right). \tag{2.7}
\]

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3 If the mass dimension of \( \Phi \) is not 1 the modified sequence \( \Psi_0 = \Phi^{1/d}, \Psi_n = D^2 \Psi_{n-1} \) should be considered. An example is \( N = 1 \) SYM discussed in ref. [11].

4 It had been realized in [17] that the symmetries of \( N = 1 \) SYM allow terms similar to the actions (2.6) or (2.6). However, a detailed analysis of the system had not been given therein, which led the author to conclusions different from the ones presented in this work.
and in a first step we analyze its effective potential, the dynamics around the minimum of $V$ are discussed in a second step. Integrating out superspace the potential becomes (all relevant superspace integrals used in the following are listed in the appendix):

$$V = -g_{\varphi\bar{\varphi}}F\bar{F} + \frac{1}{2}g_{\varphi\varphi,F}F\bar{F}(\bar{\psi}\psi) + \frac{1}{2}g_{\varphi\varphi,\varphi}\bar{F}(\psi\bar{\psi}) - \frac{1}{4}g_{\varphi\varphi,\bar{\varphi}}(\psi\bar{\psi})(\bar{\psi}\psi) + FW_{,\varphi} + \bar{F}\bar{W}_{,\bar{\varphi}} - \frac{1}{2}(\psi\bar{\psi})W_{,\varphi\bar{\varphi}} - \frac{1}{2}(\bar{\psi}\psi)\bar{W}_{,\varphi\bar{\varphi}} \tag{2.8}$$

This potential is not equivalent to the one of the model (1.1), as the Kähler metric $g_{\varphi\bar{\varphi}}$ is a function of both scalar fields, $F$ and $\varphi$

$$g_{\varphi\bar{\varphi}} = g_{\varphi\bar{\varphi}}(\varphi, F; \bar{\varphi}, \bar{F}) \tag{2.9}$$

This dependence can include arbitrary powers in $\varphi$ as well as in $F$. Analyzing the potential we insist on minima in all three fields $\varphi$, $\psi$ and $F$ as condition of the ground-state, a maximum for $F$ is not acceptable due to the dynamics of this field. The minima for $F$ are found at

$$\delta_F V(\varphi, F; \bar{\varphi}, \bar{F})|_{F = F_0} = -g_{\varphi\bar{\varphi}}F - g_{\varphi\varphi,F}\bar{F}F + \bar{W}_{,\varphi}\bigg|_{F = F_0} = 0 , \quad \delta_F \delta_F V(\varphi, F; \bar{\varphi}, \bar{F})|_{F = F_0} = -g_{\varphi\bar{\varphi}} - g_{\varphi\varphi,F}\bar{F}F - (g_{\varphi\varphi,F}F + g_{\varphi\bar{\varphi},\bar{F}}\bar{F})|_{F = F_0} > 0 . \tag{2.10}$$

If supersymmetry is unbroken the solution $F = 0$ must be the minimum of the potential and thus the minimum condition $\delta_F \delta_F V|_{F = F_0} > 0$ becomes $g_{\varphi\bar{\varphi}} < 0$. This leads to an unstable kinetic term for $\varphi$. We could try to fix this by a mixing of $\varphi$ with $\bar{F}$ through $g_{\varphi\varphi,F} \neq 0$. From the point of view of fundamental stability properties there may exist acceptable models of this type, but they are certainly irrelevant in any physical application: The above equations tell us that the potential for $\varphi$ must be completely flat for $F_0 = 0$. Thus there exists neither the possibility of a vacuum expectation value for this field (chiral symmetry breaking) nor for a mass term. Thus a model of this type could never generate a mass gap.

In the light of our discussion of the last section this means that unbroken supersymmetry does not allow to deform the flat potential of $\varphi$ as sketched in figure 2. Together with broken supersymmetry this deformation is possible, but obviously any potential of this type becomes flat in the field $\varphi$ for a (non-minimum) solution $F = 0$. At this point we use of the “semi-stable” potentials.

A relevant simplification in the discussion of broken supersymmetry is the constraint on $F_0$, the latter must be real and positive, as it is the order parameter of supersymmetry breaking. The Goldstino is represented by $\psi$, as this is the field transforming into the order parameter under supersymmetry transformations. For simplicity we set the superpotential to zero and the minimum of the potential in F is found to be at

$$F_0 = \frac{-g_{\varphi\bar{\varphi}}}{g_{\varphi\varphi,F}} \tag{2.11}.$$
The metric $g_{\phi\bar{\phi}}$ itself is real and thus the same applies to $g_{\phi\bar{\phi},FF}$ and we find that either $g_{\phi\bar{\phi}}$ or $g_{\phi\bar{\phi},FF}$ are smaller than zero\(^5\). If the Kähler potential depends on the real combination $\bar{F}F$ only, the vacuum expectation value reduces to

$$ (\bar{F}F)_0 = -\frac{g_{\phi\bar{\phi},FF}}{g_{\phi\bar{\phi}}} .$$

(2.12)

A positive mass term for $\bar{F}F$ moreover tells us that

$$ g_{\phi\bar{\phi}} - g_{\phi\bar{\phi},FF} \left( \frac{g_{\phi\bar{\phi}}}{g_{\phi\bar{\phi},FF}} \right)^2 > 0 .$$

(2.13)

Analogously we find the conditional equation for the vacuum expectation value of $\phi$ and the corresponding mass term

$$ g_{\phi\bar{\phi}}|_{\phi=\phi_0} = 0 , \quad \bar{F}Fg_{\phi\bar{\phi},\phi\phi} = g_{\phi\bar{\phi},\phi\phi} \left( \frac{g_{\phi\bar{\phi}}}{g_{\phi\bar{\phi},FF}} \right)^2 |_{\phi=\phi_0} < 0$$

(2.14)

and thus $g_{\phi\bar{\phi},\phi\phi} < 0$. This last constraint follows from the stability of the $\psi$ potential as well. In addition we see that $\psi$ is indeed a massless Goldstone particle as

$$ m_\psi \propto g_{\phi\bar{\phi},\phi\phi}\bar{F} \neq 0 .$$

(2.15)

More involved than the straightforward discussion of the potential is the derivation of consistent dynamics around the minimum described above. Evaluating the momentum expansion (A.4)-(A.7) at the minimum we find the following bilinear terms

$$ \mathcal{L}^{(1)} = -\frac{i}{2} g_{\phi\bar{\phi}} \psi \sigma^\mu \leftrightarrow \partial_\mu \bar{\psi} ,$$

(2.16)

$$ \mathcal{L}^{(2)}_{sc} = -g_{\phi\bar{\phi}} \partial_\mu \bar{\phi} \partial^\mu \phi + g_{\phi\bar{\phi},FF} \bar{F} \partial_\mu \bar{F} \partial^\mu \phi - \left( g_{\phi\bar{\phi}} \frac{g_{\phi\bar{\phi},FF}}{g_{\phi\bar{\phi}}} \partial_\mu \bar{F} \partial^\mu \phi + \text{h.c.} \right)$$

(2.17)

$$ + \left( (2g_{\phi\bar{\phi}} - g_{\phi\bar{\phi}} \frac{g_{\phi\bar{\phi},FF}}{g_{\phi\bar{\phi}}} ) \partial_\mu \bar{F} \partial^\mu \phi + \text{h.c.} \right) - \left( g_{\phi\bar{\phi}} \frac{g_{\phi\bar{\phi},FF}}{g_{\phi\bar{\phi}}} \partial_\mu \phi \partial^\mu \phi + \text{h.c.} \right) ,$$

$$ \mathcal{L}^{(2)}_{fer} = \left( g_{\phi\bar{\phi}} - \frac{1}{2} g_{\phi\bar{\phi}} \frac{g_{\phi\bar{\phi},FF}}{g_{\phi\bar{\phi}}} \right) \psi \square \bar{\psi} + \left( g_{\phi\bar{\phi}} - \frac{1}{2} g_{\phi\bar{\phi}} \frac{g_{\phi\bar{\phi},FF}}{g_{\phi\bar{\phi}}} \right) \bar{\psi} \square \bar{\psi} ,$$

(2.18)

$$ \mathcal{L}^{(3)} = -\frac{i}{2} g_{\phi\bar{\phi}} \psi \sigma^\mu \leftrightarrow \partial_\mu \bar{\psi} ,$$

(2.19)

$$ \mathcal{L}^{(4)} = g_{\phi\bar{\phi}} \square \bar{\phi} \square \phi .$$

(2.20)

If $g_{\phi\bar{\phi}} > 0 \mathcal{L}^{(2)}_{sc}$ shows potential instabilities in the kinetic term of $\phi$. The existence of positive eigenvalues depends on the details of $g_{\phi\bar{\phi}}$, nevertheless we can distinguish the following two alternatives:

\(^5\)of course all quantities have to be evaluated in the minimum $F = F_0$, $\phi = \phi_0$, $\psi = 0$, but we suppress the index 0 wherever the meaning is obvious.
\( \mathbf{g}_{\varphi\bar{\varphi}} > 0 \) In this case we find the minimum of the potential to be negative: \( V_0 < 0 \). In the classical and perturbative region such a potential would not be compatible with supersymmetry, but it has been pointed out in [10] that this may happen in the non-perturbative region. The application of this type of models could be important in a theory with non-vanishing Witten index, if the corresponding state shall be part of the Hilbert space.

\( \mathbf{g}_{\varphi\bar{\varphi}} < 0 \) This is the somehow more natural solution, as the positivity of \( F \approx \langle \Omega | T^\mu_\mu | \Omega \rangle \) comes together with \( V_0 > 0 \).

In the following we demonstrate that stable dynamics can be introduced among both types of minima at least for some finite range of \( p^2 \). The two types have relevantly different characteristics and are strictly separated from each other, as \( g_{\varphi\bar{\varphi}} = 0 \) is not a consistent solution. To arrive at stable dynamics we have to add different terms including explicit space-time derivatives to the Lagrangian. Its most general form would thus be the action (2.5)/(2.6). But these additional terms do not contribute to the effective potential and thus play a special role. The main reason to consider a quantum effective action is to find the minimum of the effective potential. For this task it is sufficient to show that for a certain model stable dynamics can be introduced, its most general form is not of main interest. The following simpler considerations are thus sufficient in the present context.

**Dynamics with \( g_{\varphi\bar{\varphi}} > 0 \)**

To ensure stable \( p^2 \) fluctuations we add \( L_c \) of (A.8) to the Lagrangian. To get the correct sign for \( \varphi \Box \bar{\varphi} \) we impose

\[
c_1 > \frac{g_{\varphi\bar{\varphi}}}{(FF)_0} = \frac{(g_{\varphi\bar{\varphi}F})^2}{g_{\varphi\bar{\varphi}}} . \tag{2.21}
\]

If \( g_{FF} \) has the wrong sign, \( e \) in (A.13) should be chosen appropriately. Considering the off-diagonal terms, all of them can be canceled by choosing \( d, f \) and \( g \) correctly in (A.12)-(A.15), as this allows independent coefficients for all combinations appearing in \( L^{(2)} \).

Among the even powers in the momenta we see that \( L^{(4)} \) is now unstable. We can correct this by choosing \( c_2 > 0 \) with

\[
c_2 > \frac{1}{(FF)_0} (g_{FF} + c_1(\varphi\bar{\varphi})_0) . \tag{2.22}
\]

Again additional contributions may cancel off-diagonal terms. At this point the term \( p^6 \) is unstable, which could be changed by an appropriate choice of \( c_3 \). This way we arrive at a Lagrangian with stable \( p \)-expansion up to any given order. Denoting by \( \Lambda \) the typical scale of the theory (scale of supersymmetry breaking) and assuming \( g_{\varphi\bar{\varphi}} = O(1) \) we get the following list

\[
\begin{align*}
\varphi_0 &= O(\Lambda) \\
F_0 &= O(\Lambda^2) \\
c_k &= O(\Lambda^{-k+2})
\end{align*}
\]
Although we need a cancellation of the terms in $c_i$ by terms in $c_{i+1}$ it is not surprising that higher order terms are indeed suppressed for $p^2 \ll \Lambda^2$ as all tunings are of order one.

Two important points of the above construction should be clarified here:

- The reference point of any scaling argument is now the minimum found in eqs. (2.10)-(2.15). Thus dominance or suppression of any term cannot be compared with the situation of a static auxiliary field: Indeed, the vacuum of a situation with static auxiliary field ($F_0 = 0$) is at a distance of $O(\Lambda)$ from the correct minimum. At this point, the momentum-expansion discussed here breaks down. This again illustrates that models with dynamical 2nd generation “auxiliary” fields must be discussed without any reference to models with static 2nd generation auxiliary fields.

- Most terms including explicit space-time derivatives added above exist in the formulation (2.5), only. The instabilities may be lifted in the formulation (2.6) using more complicated structures. But apart from its nice mathematical construction there exists no reason to prefer (2.6) compared to (2.5). Thus we do not go into the details of this problem.

Before we go over to the discussion of the case $g_{\phi \bar{\phi}} < 0$ we want to make some comments. As mentioned in the previous section our Lagrangian can be seen as an effective description of some more fundamental theory, only. This can be illustrated by means of several properties of the model:

- It is well known that the order parameter of spontaneous supersymmetry breaking is directly related to the coupling of the goldstino and is restricted to positive values [18]. We could try to construct the supercurrent of our model and read off the above quantities. At first sight these relations seem to be broken by the models discussed here: The typical representative of the order parameter is the vacuum expectation value of the energy-momentum tensor. From $(\mathcal{L})_0 = g_{\phi \bar{\phi}}(F \bar{F})_0$ we find $(T_{\mu \nu})_0 = -g_{\mu \nu}g_{\phi \bar{\phi}}(F \bar{F})_0 < 0$, but obviously this quantity has nothing to do with a goldstino coupling.

To understand this behavior one should notice that the positivity of supersymmetric potentials is actually a constraint on the maximum of the auxiliary field potential, the latter must be positive semi-definite. By eliminating the auxiliary field the system is put on top of the auxiliary field potential, which becomes the minimum of the physical potential. In our model we minimize the potential in all fields and the positivity property is lost.

Nevertheless, the correct goldstino coupling is still realized: From the point of view of an effective theory it is determined from the order parameter of the underlying quantum field theory, which will typically be equivalent to the 2nd generation auxiliary field $F$. The goldstino coupling must be in agreement with the restrictions from the current algebra of this underlying quantum field theory, only. We cannot expect that
similar relations from the effective theory have a direct (physical or mathematical) interpretation.

- In contrast to standard supersymmetry-breaking models we did not break supersymmetry by a splitting of the masses of the physical fermion and boson states, \( \psi \) and \( \varphi \). In a physical application \( \varphi \) will typically be a massive state, but one should notice that we can break supersymmetry even with massless \( \varphi \) (an example is given in section 3). Instead we have arrived at a non-zero vacuum expectation value of the auxiliary field just by manipulating the potential thereof. This still generates the typical transformation-rule of a goldstino for \( \psi \): 
  \[ \delta \alpha \psi_\beta = i \epsilon_{\alpha\beta} F_0 + \text{local} \]
and the effective theory is in agreement with all current algebra relations of the fundamental theory.

- Closely related to the above observations is the atypical form of the potential with \( V_{\text{eff}}(\Phi_0) < 0 \). It has been discussed in [10] that such a potential need not contradict supersymmetry, but its realization needs the presence of non-perturbative non-semiclassical effects. Our model leads to an effective description of this type of supersymmetry breaking by means of a Lagrangian written in superspace. Thus by its definition the current-algebra relations of this model (taken for its own) cannot be consistent with standard results from classical supersymmetric field theories.

- The price we paid to arrive at the model is a wrong sign in the \( p \)-fluctuations of the goldstino. It is easy to check that all terms of odd order in the momentum have the same “wrong” sign. This may look unesthetic but one should rate any effective model according to stability conditions and correct realization of symmetries, only. From this point of view the wrong sign of the kinetic term is acceptable, it does not introduce instabilities but can be removed by interchanging positive and negative frequencies of the goldstino. Nevertheless this sign could be problematic if we try to couple the model to additional matter fields.

- Without reference to an underlying theory, the model violates the equality of bosonic and fermionic degrees of freedom. Again the underlying theory may resolve this: The equality obviously holds on the level of the (quantum-)field content of the classical fields in \( \Phi \). Whether the equality is realized on the level of \( \Phi \) by the suggestive solution with \( \varphi, \psi \) physical and \( F \) auxiliary or not is not at all obvious, physical \( \varphi \) and \( F \) need not stand in contradiction to the equality as they are usually subject to constraints from the fundamental theory.

To prevent misunderstandings we notice again that the treatment of the 2nd generation “auxiliary” fields is completely independent from the 1st generation, i.e. it does not change when using Wess-Zumino gauge on the level of the fundamental theory. This may on the contrary motivate the above statement: By eliminating the fundamental auxiliary field, all fields of the effective superfield are built up from solely physical fields. Inspecting e.g. the effective superfield of SYM, the constraint on its highest
component being auxiliary looks completely arbitrarily. Our construction shows how to avoid this at least for a certain class of models.

**Dynamics with \( g_{\phi \bar{\phi}} < 0 \)**

If \( g_{FF} > 0 \) the fluctuations to order \( p^2 \) can be stable even when choosing \( c_1 = 0 \). As before all off-diagonal terms can be canceled in \( \mathcal{L}^{(2)} \). Again \( \mathcal{L}^{(4)} \) is unstable and we choose appropriate values for \( c_2 \) or similar higher derivative terms. But in contrast to the above discussion it is impossible to write down a Lagrangian with a consistent (though non-convergent) \( p \)-expansion for any value of the momentum. While \( \mathcal{L}^{(1)} \) has now the standard sign, this does not apply to the other terms of odd order in the momentum. These wrong signs are closely related to fundamental characteristics of the geometry of chiral fields and cannot be changed with the techniques presented in this paper. Thus the expansion breaks down at some \( p = p_0 \) of the order of \( \Lambda \). As a peculiarity this is not a tachionic instability (we are still able to remove all of them), but the goldstino becomes static at this point and interchanges positive and negative frequencies for momenta larger than \( p_0 \).

**Including the Superpotential**

Finally models with a superpotential are shortly discussed. Using the potential (2.23) its minima are found at

\[
F_0 = -\frac{1}{g_{\phi \bar{\phi},F}} \left( g_{\phi \bar{\phi}} \pm \sqrt{(g_{\phi \bar{\phi}})^2 + 4g_{\phi \bar{\phi},F} \mathcal{W}_{,\bar{\phi}}} \right).
\]

(2.23)

Independent of the sign of \( g_{\phi \bar{\phi}} \) the + solution is the absolute minimum of the potential. For \( g_{\phi \bar{\phi}} > 0 \) one finds the conditions

\[
g_{\phi \bar{\phi},F} < 0, \quad \mathcal{W}_{,\phi} < -\frac{(g_{\phi \bar{\phi}})^2}{4g_{\phi \bar{\phi},F}},
\]

(2.24)

if \( g_{\phi \bar{\phi},F} \) and \( \mathcal{W}_{,\phi} \) are both real in the minimum (there exist of course solutions where these quantities are complex, but \( F_0 \) still real). For \( g_{\phi \bar{\phi}} < 0 \) the sign of \( g_{\phi \bar{\phi},F} \) is not fixed as long as \( \mathcal{W}_{,\phi} \) is negative and

\[
|\mathcal{W}_{,\phi}| < \frac{(g_{\phi \bar{\phi}})^2}{4|g_{\phi \bar{\phi},F}|}.
\]

(2.25)

The generalization of the \( \psi \) and \( \varphi \) potentials is straightforward. The goldstino is again massless and stable if \( g_{\phi \bar{\phi}, \varphi \varphi} < 0 \). The vacuum expectation value of \( \varphi \) is determined by

\[
-g_{\phi \bar{\phi},\varphi}(\bar{F}F)_0 + \mathcal{W}_{,\phi \phi} F\big|_{\varphi = \varphi_0} = 0.
\]

(2.26)

In the \( p \)-expansion some constraints look more complicated when introducing a superpotential, but nothing changes fundamentally. We will illustrate its effect at hand of an example, a more realistic application is discussed in [11].
3 Examples

As illustration we provide some examples of the model developed in the previous section. They should give some feeling about the possibilities within this approach and do not have a specific application in a real model. For simplicity we introduce the notation \( \Phi_0 = \Phi \), \( \Phi_1 = \Psi \).

The simplest consistent constraint Kähler potential is given by
\[
K(\Phi, \Psi; \bar{\Phi}, \bar{\Psi}) = \bar{\Psi}\Psi + \bar{\Phi}\Phi I(\Psi, \bar{\Psi}) ,
\]
its minimum found at
\[
F_0 = -\frac{I(F, \bar{F})}{\partial_F I(F, \bar{F})}, \quad \varphi_0 = 0 .
\]

As \( \varphi_0 = 0 \), the kinetic terms of the scalars are automatically diagonal to all orders. Moreover the ones of the goldstino appear with odd powers of the momentum, only. For \( g_{\varphi \varphi} > 0 \) we can choose e.g. \( I = g_2 + g_4 \bar{F}F \) with \( g_2 > 0 \) and \( g_4 < 0 \), the standard Mexican-hat potential. The minimum of the potential is given by \( (\bar{F}F)_0 = \frac{g_2}{2|g_4|} \) and \( (g_{\varphi \varphi})_0 = \frac{g_2}{g_4} \) and we derive the stability constraints
\[
c_1 > |g_4|, \quad c_2 > 2\frac{|g_4|}{g_2} .
\]

With this choice the Lagrangian has a stable \( p \)-expansion stopping at order \( p^4 \). Higher order derivatives can be introduced by choosing \( c_2, c_3 \ldots \) and the related series from \( L^{(1)} \) non-zero. A similar phenomenology has a model with \( g_{\varphi \varphi} < 0 \), e.g. \( I = -\frac{\alpha}{(\bar{F}F)^2} + \beta \), \( \alpha > 0 \) and \( \beta > 0 \), with
\[
V = \frac{\alpha}{\bar{F}F} + \beta \bar{F}F , \quad (\bar{F}F)_0 = \sqrt{\frac{\alpha}{\beta}} , \quad g_{\varphi \varphi} = -2\beta .
\]

The momentum expansion again stops at order \( p^4 \) and we have to choose \( c_2 > \sqrt{\frac{\beta}{\alpha}} \). Notice the difference in the signs of \( L^{(1)} \) and of \( L^{(3)} \). These two examples illustrate the fact that we can break supersymmetry without a split of the masses of \( \varphi \) and \( \psi \), but they are both massless.

We can immediately generalize the above example to models of the type
\[
K(\Phi, \Psi; \bar{\Phi}, \bar{\Psi}) = f_2 \bar{\Psi}\Psi + \bar{\Phi}\Phi I(\Psi, \bar{\Psi}) + J(\Phi, \bar{\Phi}) .
\]

These models have a minimal coupling between \( \Phi \) and \( \Psi \) with the potential
\[
V = -\bar{F}F \left( I(F, \bar{F}) + \partial_{\varphi} \partial_{\bar{\varphi}} J(\varphi, \bar{\varphi}) \right) .
\]
If in addition the vacuum expectation value of \( \varphi \) vanishes, all constraints on the dynamics are equivalent to the free model above. For \( \varphi_0 \not= 0 \) however, the dynamics become more complicated. Even for the simple minimal coupling of \( \Psi \) to \( \Phi \) we get off-diagonal terms in the kinetic Lagrangian. An example of this type with \( g_{\varphi \bar{\varphi}} > 0 \) is the double-Mexican-hat with the potentials

\[
I(F, \bar{F}) = g_2 + g_4 \bar{F} F, \quad J(\varphi, \bar{\varphi}) = \frac{h_2}{4}(\bar{\varphi} \varphi)^2 + \frac{h_4}{9}(\bar{\varphi} \varphi)^3 .
\]  

(3.7)

The potential has the minimum at

\[
(\bar{\varphi} \varphi)_0 = \frac{h_2}{2|h_4|} , \quad (\bar{F} F)_0 = -\frac{1}{2g_4}(g_2 + \frac{h_2^2}{4|h_4|}) .
\]

(3.8)

The complete \( \mathcal{L}^{(2)} \) of this example reads:

\[
\mathcal{L}^{(2)} = -\left(\frac{g_2}{2} + \frac{1}{8}\frac{h_2^2}{|h_4|}\right)\partial_\mu \varphi \partial^\mu \bar{\varphi} + \left(f_2 + g_4 \frac{h_2}{2|h_4|}\right)\partial_\mu F \partial^\mu \bar{F}
\]

\[
- \left(\frac{g_4}{2} + \frac{1}{8}\frac{h_2^2}{|h_4|}\right)\left(\frac{\bar{\varphi}}{F}ight)_0 \partial_\mu \varphi \partial^\mu \bar{\varphi} + \text{h.c.}
\]

\[
+ \left(g_4(\bar{\varphi} \bar{F})_0(2\partial_\mu F \partial^\mu \varphi + \psi \Box \psi) + \text{h.c.}\right) .
\]

(3.9)

In this equation \( F_0 \) must be real and positive, while \( \varphi_0 \) has a free phase. As \( f_2 \) does not contribute to the potential we can assume without loss of generality that \( g_{F \bar{F}} > 0 \). The diagonal sector then has positive eigenvalues if \( c_1 > g_4 \). This will lead to new contributions \( \propto \partial_\mu \varphi \partial^\mu \bar{F} \), all of them can be canceled by an appropriate choice of \( d \) in (A.12). Finally we choose in (A.15)

\[
g = \frac{|g_4|}{2(F \bar{F})_0} ,
\]

(3.10)

which cancels the last expression of (3.9). This way we arrive at a completely diagonal and bosonic \( \mathcal{L}^{(2)} \).

In a similar way the higher order derivatives can be arranged. Adding a superpotential leads to cubic equations in \( F_0 \), whose explicit form is not very illuminating. Notice however that the cancellation of off-diagonal terms and the resulting structure of the kinetic term matrix does not depend on the absence of a superpotential. Especially one can again use \( f_2 > 0 \) to get two positive eigenvalues. An interesting but simple superpotential is \( W = \frac{w_3}{6} \Phi^3 \). The minimum \( \varphi_0 \) is then found at

\[
(\bar{\varphi} \varphi)_0 = \frac{1}{2|h_4|}(h_2 - \frac{1}{F_0} \frac{\varphi_0}{\varphi_0} w_3)
\]

(3.11)

As \( h_2, h_4 \) and \( F_0 \) are all real the phase of \( w_3 \) directly determines the phase of \( \varphi \):

\[
\arg w_3 = -\frac{1}{2} \arg \varphi_0
\]

(3.12)
Finally we want to give an example with singular potential at the origin. For simplicity we choose again the same shape of the $F$ and $\varphi$ potential, namely

$$I = -(\frac{\alpha}{(FF)^2} + \beta) \quad J = -(\tilde{\alpha} \ln \frac{\varphi}{\Lambda} \ln \frac{\bar{\varphi}}{\Lambda} + \frac{\tilde{\beta}}{4}(\bar{\varphi} \varphi)^2),$$

where all coupling constants are positive and $\Lambda$ denotes the scale of the theory. The minima are found at

$$\langle \bar{\varphi} \varphi \rangle_0 = \sqrt{\frac{\tilde{\alpha}}{\beta}} \quad \langle \tilde{F}F \rangle_0 = \sqrt{\frac{\alpha}{\beta + 2\sqrt{\tilde{\alpha} \tilde{\beta}}}}$$

As in the same model with vanishing $\varphi_0$ (eq. (3.4)), the potential is positive definite and thus $g_{\varphi \bar{\varphi}} < 0$. Without loss of generality we can choose $g_{FF} > 0$ and thus the diagonal part of $\mathcal{L}^{(2)}$ is stable even for $c_1 = 0$. The term $\propto \partial_\mu \varphi \partial^\mu \varphi$ vanishes again while the coefficients of the $\varphi$-$F$ and $\varphi$-$\bar{F}$ kinetic terms are found to be:

$$2g_{\varphi F} - g_{\varphi \varphi} g_{\varphi F,F} = -2\left(\beta + \sqrt{\tilde{\alpha} \tilde{\beta}}\right) \frac{\bar{\varphi}_0}{F_0} \quad -g_{\varphi \varphi} g_{\varphi F,\bar{F}} = -4\left(\beta + \sqrt{\tilde{\alpha} \tilde{\beta}}\right) \frac{\bar{\varphi}_0}{F_0}$$

These off-diagonal terms can be canceled by choosing

$$d = 2\frac{(\beta + \sqrt{\tilde{\alpha} \tilde{\beta}})^3}{\alpha^2} \quad g = \frac{1}{2} \frac{(\beta + \sqrt{\tilde{\alpha} \tilde{\beta}})^3}{\alpha^2}$$

Again we have to choose the parameters of the higher order derivatives according to the discussion in the previous section.

We have illustrated in this section at hand of a few simple examples, how an effective description of a non-perturbatively broken SUSY theory could be realized. Of course the examples have been too simple for any realistic application and from this point of view we conclude this section with a few remarks:

- In a theory with dynamical “auxiliary” fields, the superpotential looses most of its power. This has a simple reason: As long as we can eliminate the auxiliary fields the superpotential produces automatically super-stable potentials (i.e. they are automatically positive-semidefinite). In a theory with dynamical auxiliary fields, the contrary is true: The superpotential alone is necessarily unstable, as its holomorphic character in $\varphi$ cannot be changed through the equations of motion of $F$. We emphasize again that this fact does not depend on the origin of the dynamics. Here they came from explicit new terms written into the (classical) Lagrangian, but the same conclusion applies if they are an effect of quantum dynamics
Nevertheless in a concrete application the superpotential is important. Veneziano and Yankielowicz [5] have shown how to realize the anomalies of SYM in the superpotential. Of course a similar term must be present in a generalized construction of this effective action based on the Lagrangian (2.5)/(2.6) [11].

- Closely related to the above observations is the question of the spectrum of our models. In the above examples the Lagrangian respects the two global $U(1)$ symmetries from the complex fields $F$ and $\varphi$ (except for the example including a superpotential). Consequently one ($\varphi_0 = 0$) or two Goldstone bosons are present in addition to the goldstino. At least one of them (associated with $F$) must be absent: $F$ is the order parameter of supersymmetry breaking, its value must be real and positive and thus the direction of the vacuum is fixed. If $\varphi_0 \neq 0$ is related to chiral symmetry breaking, the corresponding Goldstone boson is absent as well. Such a potential can be realized together with holomorphic terms from a superpotential, as demonstrated in one example.

- Finally it is important to note that a concrete application of our ideas relies on a quantum effective action, defined via a source extension of the classical system. Such a source extension breaks intrinsic supersymmetry but should preserve the latter extrinsically [6][10]. By taking the limit of constant sources we thus arrive at a system with softly broken supersymmetry and the (pseudo-)goldstino receives a mass. Therefore it will be important to include a spurion field into the description. Some comments on this problem for $N = 1$ SYM are given in [11], for a recent discussion of related problems in perturbation theory we refer to [19, 20, 21, 22].

4 Summary and Conclusions

We have introduced in this paper a class of low-energy descriptions of supersymmetric gauge theories and demonstrated its features at hand of some simple examples. Our description is suitable for models where supersymmetry is broken dynamically by non-perturbative non-semiclassical effects. The specific problem in the construction of the low-energy approach lies in the realization of supersymmetry: In a sense the model must be both, supersymmetric and non-supersymmetric. The supersymmetric and non-supersymmetric aspects are:

- Supersymmetry is realized linearly in the sense of transformation rules between the different low-energy degrees of freedom. This behavior is included in the assumption of a correct identification of the low-energy degrees of freedom. In practice it simply means that our ansatz must be expressible as integrals over superspace.

- Taken for its own the effective model must disobey standard characteristics of (classical and perturbative) supersymmetric models. This can be motivated as follows: Our
model shall describe by means of a classical Lagrangian the behavior of a theory, in which important characteristics of classical and perturbative supersymmetry have been changed by non-perturbative effects. This classical (effective) Lagrangian must thus in certain aspects be non-supersymmetric. This especially includes:

1. The usual splitting of the fields into physical ones (with an equal number of bosonic and fermionic degrees of freedom) and into auxiliary fields does not hold. Instead the auxiliary fields turn into physical fields.

2. The minimum of the potential can be negative. In classical and perturbative supersymmetry this is excluded by the analogy of the minimum of the potential and the vacuum expectation value of the energy-momentum tensor due to the non-renormalization theorem. The latter must be positive semi-definite from current algebra relations. In the non-perturbative region this relation can be broken and thus there exists no constraint on the value of the potential in its minimum.

3. Supersymmetry is broken dynamically, but the Goldstino coupling on the level of the effective model is not realized as in classical and perturbative supersymmetry. We should emphasize that all these non-supersymmetric characteristics can be seen as non-supersymmetric from the point of view of the effective model (taken for its own and not referring to the underlying theory), only. From the point of view of the underlying theory all seeming discrepancies have its definite interpretation. Especially it has been worked out in [10] that the possibility of dynamical auxiliary fields, of infinite orders of derivatives and of a classically non-supersymmetric shape of the potential are of fundamental importance.

The ansatz used in this work is consistent together with dynamically broken supersymmetry, only. Typically we assume in this type of theories the existence of a mass-gap where solely the goldstino lives below the latter. One then might ask what our formulation gains compared to a low-energy description of the goldstino, which can be found within non-linear realizations of supersymmetry [23,24]. We think that there exists an important difference between the two situations: Indeed our specific model has to assume dynamically broken supersymmetry, but the motivation stems from a quantity, whose existence does not depend on this question, the quantum effective action. Our analysis together with the discussion of [10] shows that a single ansatz for the description of the quantum effective action is probably insufficient, instead we need two, one for broken and one for unbroken supersymmetry. Stability and consistency conditions evaluated in both cases then should show, whether supersymmetry actually breaks or not. But this program does not allow to replace the description of the broken case by the above mentioned non-linear effective goldstino-Lagrangian: The field-content is determined by the source-extension of the system and supersymmetry must be realized on this set of fields. Moreover the description must hold even for softly broken supersymmetry, which can easily be included in our approach. From this point of view we think that our description is more fundamental: Although the specific realization had to assume broken supersymmetry
it can be used within a program that does not presume this behavior. Once we have found therefrom that supersymmetry is actually broken, the above goldstino-Lagrangian may be a simpler and more convenient way to describe the theory for vanishing sources. For a more detailed discussion of this relation between fundamental models describing the quantum effective action and effective approaches describing the dynamics once we have found the ground-state, we refer the reader to [10].

Many questions are still open. A concrete application to a physical system has not been given within this work. Its application to $N = 1$ SYM is discussed in [11]. In addition the model is restricted to one classical chiral field (the goldstino field). It would be interesting to see, whether one can couple additional fields to the system, either from higher supersymmetry or as Goldstone bosons from the breakdown of bosonic symmetries. Finally the existence of the non-perturbative non-semiclassical effects has been derived in [10] from physical arguments, only. Classical field-configuration that could lead to such an effect are unknown. In other words: It would be interesting to turn the non-perturbative non-semiclassical effect into a non-perturbative semiclassical one.

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**A Appendix**

In this appendix we list the complete momentum expansion of the constraint double-Kähler model and of some other superspace integrals used in the paper. The fundamental Kähler potential is given by:

$$
\int \! d^4 \theta \ K(\Phi_1, \Phi_2; \bar{\Phi}_1, \bar{\Phi}_2) = g_{ij} \left( \partial_{\mu} \varphi^i \partial^\mu \varphi^j + \frac{i}{2} \psi^i \sigma^\mu \bar{D}_\mu \bar{\psi}^j + F^i \bar{F}^j \right) - \frac{1}{2} g_{ij} \Gamma^k_{kl} F^i \bar{\psi}^k \bar{\psi}^l - \frac{1}{2} g_{ij} \Gamma^i_{kl} \bar{F}^j \psi^k \psi^l + \frac{1}{4} g_{ij} \bar{F}^i \psi^k \bar{\psi}^j \bar{\psi}^l
$$

(A.1)

$\Phi_2 = \bar{D}^2 \Phi_1$ and thus

$$
\Phi_1 = \varphi + \theta \psi + \theta^2 F \\
\Phi_2 = \bar{F} - i \theta \sigma^\mu \partial_\mu \bar{\psi} - \theta^2 \Box \bar{\varphi}
$$

(A.2)
Denoting the components of the Kähler potential by $1 = \varphi$, $\bar{1} = \bar{\varphi}$, $2 = \bar{F}$ and $\bar{2} = F$ we get the momentum expansion

\begin{align}
\mathcal{L}^{(0)} &= -V = g_{\varphi\bar{\varphi}} \bar{F} F - \frac{1}{2} g_{\varphi\varphi} \bar{F} \bar{F} + \frac{1}{2} g_{\varphi\bar{\varphi}} \bar{F} \bar{F} + \frac{1}{4} g_{\varphi\varphi} \bar{F} \bar{F} \psi \bar{\psi}
\end{align}

\begin{align}
\mathcal{L}^{(1)} &= i \left( \frac{1}{2} g_{\varphi\bar{\varphi}} + g_{\varphi\varphi} \bar{F} - \frac{1}{2} g_{\varphi\bar{\varphi}} \bar{F} \right) \psi \sigma^\mu \partial_\mu \bar{\psi} - i \left( \frac{1}{2} g_{\varphi\bar{\varphi}} + g_{\varphi\varphi} F - \frac{1}{2} g_{\varphi\bar{\varphi}} F \right) \partial_\mu \psi \sigma^\mu \bar{\psi}
+ \frac{i}{2} \left( g_{\varphi\varphi} \bar{\partial}_\mu \varphi + g_{\varphi\varphi} \partial_\mu F - g_{\varphi\varphi} \bar{\partial}_\mu \varphi - g_{\varphi\varphi} \partial_\mu \bar{F} \right) \psi \sigma^\mu \bar{\psi}
\end{align}

\begin{align}
\mathcal{L}^{(2)} &= g_{\varphi\bar{\varphi}} \partial_\mu \varphi \partial^\mu \varphi + g_{F\bar{F}} \partial_\mu \bar{F} \partial^\mu F + g_{F\varphi} \partial_\mu F \partial^\mu \varphi + g_{F\bar{\varphi}} \partial_\mu \bar{F} \partial^\mu \bar{\varphi}
+ \frac{1}{2} g_{\varphi\psi} \sigma^\mu \bar{\sigma}^\nu \partial_\mu \partial_\nu \psi - \frac{1}{2} g_{\varphi\psi} \partial_\nu \bar{\psi} \bar{\sigma}^\nu \sigma^\mu \partial_\mu \bar{\psi}
+ \frac{1}{2} \left( g_{\varphi\varphi} \partial_\mu \bar{\varphi} + g_{\varphi\varphi} \partial_\mu F - g_{\varphi\varphi} \partial_\mu \varphi - g_{\varphi\varphi} \partial_\mu \bar{F} \right) \psi \sigma^\mu \bar{\psi}
\end{align}

\begin{align}
\mathcal{L}^{(3)} &= \frac{i}{2} g_{F\bar{F}} \left( \partial_\mu \psi \sigma^\mu \partial_\nu \bar{\psi} - \square \psi \sigma^\mu \partial_\nu \bar{\psi} \right)
+ \frac{i}{2} \left( g_{F\varphi} \partial_\mu \bar{\varphi} + g_{F\varphi} \partial_\mu F - g_{F\varphi} \partial_\mu \varphi + g_{F\varphi} \partial_\mu \bar{F} \right) \partial_\nu \bar{\psi} \bar{\sigma}^\nu \sigma^\mu \partial_\rho \psi
+ i \left( g_{F\bar{\varphi}} \square \bar{\varphi} + \frac{1}{2} g_{F\bar{\varphi}} \partial_\mu \bar{\psi} \bar{\sigma}^\nu \partial_\nu \bar{\psi} \right) \rho \psi \bar{\sigma}^\rho \bar{\psi}
\end{align}

\begin{align}
\mathcal{L}^{(4)} &= g_{F\bar{F}} \square \bar{\varphi} \bar{\varphi} + \frac{1}{4} g_{F\bar{F}} \partial_\mu \psi \sigma^\mu \sigma^\nu \partial_\nu \psi \partial_\lambda \bar{\psi} \bar{\psi} \sigma^\lambda \partial_\rho \bar{\psi}
+ \frac{1}{2} g_{F\bar{F}} \square \bar{\varphi} \partial_\nu \psi \sigma^\nu \bar{\sigma}^\rho \partial_\rho \psi
\end{align}
To get a stable p-expansion up to some given order we have to add terms including explicit space-time derivatives. The following integral is of main importance:

\[ \mathcal{L}_c = \int d^4 \theta \sum_k c_k \bar{\Phi} \Phi \partial_\mu \Phi \partial^\mu \Box^{-1} \Phi \] (A.8)

Evaluated with respect to a minimum \( \varphi_0 \), \( F_0 \) and \( \psi_0 \equiv 0 \) we get the following bilinear terms

\[ \mathcal{L}_c^{(2)} = c_1 \left( \bar{F} F \partial_\mu \varphi \partial^\mu \bar{\varphi} \right) + F \bar{\varphi} \partial_\mu \varphi \partial^\mu \bar{F} + \text{h.c.} \] (A.9)

\[ \mathcal{L}_c^{(2k)} = \left( c_k \bar{F} F - c_{k-1} \bar{\varphi} \varphi \right) \partial_\mu \varphi \partial^\mu \Box^{-1} \bar{\varphi} \]
\[ + c_k \left( \bar{\varphi} \varphi \partial_\mu \varphi \partial^\mu \Box^{-1} \bar{F} \right) + \left( F \bar{\varphi} \varphi \partial^\mu \Box^{-1} \bar{F} \right) + \text{h.c.} \] (A.10)

\[ \mathcal{L}_c^{(2k+1)} = -i c_k \bar{\varphi} \varphi \psi \sigma^\mu \partial_\mu \Box^{k+1} \psi \] (A.11)

To cancel off-diagonal terms additional contributions with explicit space-time derivatives can be introduced. A possible choice, which straightforwardly generalizes to specific restrictions on the dimensions of the coupling constants, reads:

\[ \mathcal{L}_d = \int d^4 \theta \left( \partial_\mu \Phi \partial^\mu \bar{\Phi} \right) \] (A.12)
\[ = d(2 \bar{F} F \bar{\varphi} \varphi \partial_\mu \bar{F} \varphi \partial^\mu F \bar{\varphi} + \text{O}(p^3)) \]

\[ \mathcal{L}_e = \int d^4 \theta \left( \partial_\mu \bar{F} \partial^\mu \Phi \right) \] (A.13)
\[ = e \bar{F} F \partial_\mu \bar{F} \partial^\mu F + \text{O}(p^3) \]

\[ \mathcal{L}_f = \int d^4 \theta \left( \partial_\mu \bar{\Phi} \partial^\mu \Phi \right) \] (A.14)
\[ = f \bar{F} \left( \frac{F}{\varphi^2} \varphi \partial^\mu \varphi + \frac{1}{\varphi^2} \left( 2 \partial^\mu \varphi \varphi + \psi \Box \psi \right) \right) + \text{O}(p^3) \]

\[ \mathcal{L}_g = \int d^4 \theta \left( \partial_\mu \bar{\Phi} \partial^\mu \Phi \right) \] (A.15)
\[ = 2g \bar{F} F \bar{\Phi} \left( \varphi \partial^\mu \varphi + \psi \Box \psi \right) + \text{O}(p^3) \]

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