Quantum computers are expected to offer phenomenal increases of computational power [1]. In spite of many proposals based on various physical systems, scalable quantum computation in a fault-tolerant manner is still beyond current technology. Optical models have some prominent advantages such as relatively quick operation time compared to decoherence time [2]. However, massive resource requirements and the gap between the fault tolerance limit and the realistic error rate should be significantly reduced [3]. Here, we develop a novel approach with all-optical hybrid qubits devised to combine advantages of well-known previous approaches. It enables one to efficiently perform universal gate operations in a simple and near-deterministic way using all-optical hybrid entanglement as off-line resources. Remarkably, our approach outperforms the previous ones when considering both the resource requirements and fault tolerance limits. Our work paves an efficient way for the optical realization of scalable quantum computation.

Certain properties of light can be useful to implement qubits, i.e., the basic units for quantum computation [4]. Typically, photons as "particles of light" are considered to encode information with a well-chosen degree of freedom such as horizontal and vertical polarization states, |H⟩ and |V⟩. A major difficulty in this approach is to realize two-qubit gates since photons seldom interact with each other, while single-qubit operations are straightforward [4]. In principle, scalable quantum computation can be achieved without inline non-linear interactions [5], which is often called linear optical quantum computing (LOQC). However, its practical implementation is difficult because fault-tolerant computation requires a very large number of resources [6–8] despite some progress in reducing the scheme-complexity and resource overhead [9,11].

In fact, any two distinct field states in phase space can be explored for a qubit basis [3]. Along this line, the coherent-state quantum computing (CSQC) has been developed with its own merit [12–15]. In CSQC, two coherent states, |α⟩ and |−α⟩ with amplitudes ±α, are used to form a qubit basis and equal superpositions of coherent states, e.g., |α⟩+|−α⟩ [16], are required as resources [14]. Using this encoding scheme, the Bell-state measurement (Bα) in Fig. 1 can be performed in a near-deterministic manner as α gets large [12,17]. On the other hand, single qubit rotations produce a cumbersome type of errors due to the non-orthogonality between |α⟩ and |−α⟩ [13,15], which makes it still difficult to implement deterministic gates [15]. The pros and cons of the two approaches are known: CSQC requires less resources than LOQC but suffers smaller fault-tolerance thresholds [15].

In our approach, the orthonormal basis to define optical hybrid qubits is

\[ \{ |0_L⟩ = |+⟩|α⟩, |1_L⟩ = |−⟩|−α⟩ \}, \]

where \( |±⟩ = (|H⟩ \pm |V⟩)/\sqrt{2} \). As we shall see, this approach enables us to overcome particular weak points of both LOQC and CSQC at the same time. The Z-basis measurement can be performed by a single measurement on either of the two physical modes. It can be done on the single-photon mode by a polarization measurement on the basis |+⟩ and |−⟩, or on the coherent-state mode using an ancillary coherent state [13].

In order to construct a universal set of gate operations, Pauli X, arbitrary Z (phase) rotation, Hadamard, and controlled-Z (CZ) gates suffice [1]. The Pauli X
Hybrid Entangler (HE)

φ
a

|Ψ
C
|

Operation
(even, 0) : j=0, k=0
(odd, 0) : j=0, k=1
(0, even) : j=1, k=0
(0, odd) : j=1, k=1

|Ψ
C
|

Table **

| Operation | Bα | BII |
|-----------|-----|-----|
| (H, H) or (V, V) | Flip k (0 ↔ 1) |
| or (H, V) or (V, H) | No flip |
| (H, V) or (V, H): | Flip k (0 ↔ 1) |
| j=0, k=1 |
| (0, 0) |

| Failure |
|---------|
| Otherwise |

FIG. 2: Generation of resource entanglement for quantum teleportation. a, Generation of a diagonal state \(|\chi φ γ⟩|α⟩ + |−⟩|α⟩\) (hybrid pair) using a hybrid entangler (HE) with an arbitrarily weak cross-Kerr nonlinearity. The cross-Kerr nonlinearity with strength \(χ\) and interaction time \(t\) can induce a transformation \((a|H⟩ + b|V⟩)|α⟩ → a|H⟩|α⟩ + b|V⟩|e^{iθ}|⟩\) where \(θ = χ t\) [20]. In a single-photon mode, \(±\) and \(H/V\) denote bases \(|±⟩\) and \(|H⟩, |V⟩\), respectively. The displacement operator is defined as \(D(γ) = \exp[γa^† − γ^∗ a]\), where \(a^†\) (\(a\)) is the photon creation (annihilation) operator and \(γ\) is assumed to be real.

b, Resource entanglement, \(|Ψ_C⟩\), is generated using a photon pair and a hybrid entangler with simple linear optics elements.

Operation (\(\hat{X}\)) can be performed by applying a bit flip operation on each of the two modes. The bit flip operation on the single-photon mode, \(|+⟩ \leftrightarrow |−⟩\), is implemented by a polarization rotator, and the operation on the coherent-state mode, \(|α⟩ \leftrightarrow |−α⟩\), by a π phase shifter. An arbitrary \(Z\) rotation (\(\hat{Z}_α\)) is performed by applying the phase shift operation only on the single-photon mode: \(|±⟩ \to \{|±\rangle e^{iθ}|−⟩\}\), and no operation is required on the coherent-state mode. This is a significant advantage over CSQC in which \(|Z\) states, \(|0_L⟩|0_L⟩ ± |1_L⟩|1_L⟩\) and \(|0_L⟩|1_L⟩ ± |1_L⟩|0_L⟩\). As described in Fig. 3, the Bell measurement for an optical hybrid qubit can be performed using two smaller Bell measurement units, \(B_α\) and \(B_{II}\) in Fig. 4. Appropriate Pauli operations in the table of Fig. 3 completes the teleportation process with failure probability \(P_f = e^{-2α^2}/2\), which outperforms the previous schemes that requires massive overheads with repetitive applications of teleporters [5, 15]. For example, 99% success rate of teleportation is achieved by encoding with \(α = 1.4\).

Entangled channels required for Hadamard and CZ gates, \(|Z⟩ \sim |0_L⟩|0_L⟩ + |1_L⟩|1_L⟩ + |L⟩|L⟩ - |L⟩|L⟩\) and \(|Z⟩ \sim |0000⟩ + |0011⟩ + |1100⟩ - |1111⟩\), where \(|0000⟩ = |0_L⟩|0_L⟩|0_L⟩|0_L⟩\) and so on, can be generated with two-photon pairs and hybrid entanglers as shown in Fig. 4. In order to generate \(|Z⟩\), either of two non-deterministic methods, \(G_0\) or \(G_α\), is employed using \(B_1\) or \(B_α\) (see Fig. 4), respectively. We emphasize that \(|Z⟩\) and
cluster state LOQC [6, 7], parity state LOQC (pLOQC) [8], and CSQC [15] were previously investigated. Their thresholds and resources are in a trade-off relation, and pLOQC and CSQC have shown the best compromise achievements [9]. In order to compare our approach with previous ones, we follow the same analysis using the 7-qubit STEANE code [25] based on the telecorrection protocol.

The results of our analysis are plotted in Fig. 5 for $G_1$ and $G_\alpha$. Remarkably, the threshold level is obviously higher than CSQC for all region of $\alpha$ and is comparable with pLOQC (about $2 \times 10^{-3}$ [8]), with a greatly reduced cost of resources compared to both CSQC and pLOQC. The threshold peak for each generation scheme appears around $\alpha \approx 1.08$ (Fig. 5a). However, further increase of $\alpha$ lowers the threshold level due to rapid increase of unlocatable errors which are more difficult to correct than locatable ones using the telecorrection protocol. Resource requirements are phenomenally reduced by $G_\alpha$ since the success rate of $B_\alpha$ grows rapidly as $\alpha$ increases. However, the success rate of $B_1$ is constant $(1/2)$ and so are the resource requirements with $G_1$. The noise thresholds of $G_1$ appear to be slightly larger than those of $G_\alpha$ because $G_\alpha$ requires preparation hybrid qubits of amplitudes $\sqrt{2\alpha}$ from the beginning as shown in Fig. 4.

We also note that one can construct a hybrid qubit with an arbitrary number of single-photon modes $n$. For $n > 1$, the same level of gate success rate is obtained with smaller $\alpha$ than the one required when $n = 1$. Then more errors become detectable using a detection scheme at single-photon modes [11], and the noise threshold raises (but this may consume more resources). Finally, as optical hybrid qubits are interconvertible with photon or coherent-state qubits by inline operations, one can construct hybrid architectures by integrating some devices developed in various approaches, such as quantum memories [26, 27] and photonic chips [28, 29]. Our approach provides an efficient way to implement scalable quantum computation considering resource requirements, fault-tolerance limits and compatibility. An efficient preparation of resource hybrid entanglement [20, 30] would be the next challenge of our scheme.

**APPENDIX**

**Error models.** We analyze locatable and unlocatable errors with loss rate $\eta$ using the master equation $d\rho/dt = \gamma (\hat{J} + \hat{L})\rho$, where $\hat{J} \rho = \sum_i \hat{a}_i \rho \hat{a}_i^\dagger$, $\hat{L} \rho = -\sum_i (\hat{a}_i^\dagger \hat{a}_i \rho + \hat{a}_i \rho \hat{a}_i^\dagger) / 2$, and $\hat{a}_i (\hat{a}_i^\dagger)$ is the annihilation (creation) operator for the $i$-th mode. Here, the loss rate is $\eta = 1 - e^{-\gamma t}$. When the master equation is applied to hybrid qubits, the failure probability $P_f$ for teleportation in Fig. 3 is modified to $P_f = (1 - \eta - \alpha')^2 \eta_{\alpha}^2 + \eta (1 + \alpha')^2 \eta_{\alpha}^2$ where $\alpha' = \sqrt{1 - \eta\alpha}$. If a gate operation fails, the tele-

![Figure 4: Generation methods for resource entanglement required for Hadamard and CZ gates. State $|Z\rangle$ is prepared as off-line resources for gate teleportation [19] while only linear optical elements with photon detections are used for in-line operations.](image)
in terms of $\eta$ and methods, we have analytically obtained probabilities of the output state of the channel [15]. Based on these models the teleportation scheme by changing the amplitude of $\alpha$ due to losses in the generation processes of Pauli $Z$ or coherent-state modes, the hybrid qubit experiences a Pauli operation to the qubit, i.e. $\text{I}$ becomes fully mixed. This is equivalent to applying a random Pauli operation to the qubit. Noise thresholds and resource requirements. A, Noise thresholds based on two generation schemes, $G_I$ and $G_a$, obtained using the 7-qubit STEANE code. B, Resource requirements estimated for one round of error correction based on the teleportation protocol. For $G_I$, a constant number (5260) of resources (two-photon pairs and hybrid entanglers) are required irrespective of amplitude $\alpha$, while for $G_a$ the required number of resource states tend to decrease rapidly down to 2173 as increasing $\alpha$. Two curves intersect at $\alpha \approx 0.675$. We can take the lower curve between them by choosing $G_a$ for $\alpha > 0.675$ and otherwise $G_I$. It shows a remarkable improvement compared to pLOQC (about $1.8 \times 10^5$ [8]) and CSQC (about $10^4$ [15]).

A ported qubit is assumed to experience depolarization and become fully mixed. This is equivalent to applying a random Pauli operation to the qubit, i.e. $Z$ and $X$ Pauli errors occurs independently with the equal probabilities. One can also assume that if a loss occurs in either photon or coherent-state modes, the hybrid qubit experiences a Pauli $Z$ error with probability $1/2$. We also model errors due to losses in the generation processes of $|Z\rangle$ and $|Z'\rangle$ as Pauli $X$ and $Z$ errors. We assume that the decrease in amplitude $\alpha$ by loss can be compensated whenever using the teleportation scheme by changing the amplitude of output state of the channel [13]. Based on these models and methods, we have analytically obtained probabilities of aforementioned errors in terms of $\eta$.

Error correction. We consider an error correction protocol with several levels of concatenation based on the circuit-based teleportation [6]. We assume our error model for the lowest level of concatenation. For higher levels, the noise model and error-correction protocol are identical to those of Ref. [8]. We perform a numerical simulation (Monte Carlo method using C++) for one round of the error correction for the first level concatenation. The modified telecorrection circuit is composed of $CZ$, Hadamard, $|+\rangle$ creation, and $X$-basis measurement [15].

We carried out a series of simulations for a range of loss rate $\eta$ and amplitude $\alpha$. The resulting rates of unlocatable and locatable errors are used for the next level of concatenation for the error correction. If the error rates tend to zero with certain values of $\eta$ and $\alpha$ in the limit of many levels of concatenation, fault-tolerant computing is possible with those values. In this way, the noise threshold curves are obtained.

Resource counting. We count the average total number of two-photon pairs and hybrid entanglers for one round of error correction in the lowest level of concatenation. It is assumed [3] [15] that the total number of operations in one round of teleportation is about 1,000. The fraction of each element used during the teleportation process can be estimated as [15]: 0.284 (memory), 0.098 (Hadamard), 0.343 (CZ), 0.164 (diagonal state), and 0.111 (X-basis measurement). We also assume parallel productions of resource states and no reuse of resources to avoid using complicated techniques.

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