High-efficiency wireless power transfer by optimal load and metamaterial slab

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Abstract: In this letter, a two-coil wireless power transfer system is analyzed mathematically to obtain optimal source and load impedances, which are the most important factors that affect system efficiency. In addition, a near-zero refractive index metamaterial slab with a negative permeability was inserted between two resonant coils close to the transmitter to improve the efficiency in a large range. Finally, the wireless energy transfer system with the metamaterial slab was designed, fabricated and measured to evaluate the analysis method. The power transfer efficiency of 73.1% with the optimal impedances was enhanced to 81.3% using a 5 × 5 array metamaterial slab at the frequency of 13.56 MHz and the distance of 20 cm between the transmitter and the receiver.

Keywords: metamaterial, mutual inductance, near-zero refractive index, optimal load, two-coil wireless power transfer

Classification: Power devices and circuits

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1 Introduction

Wireless power transfer (WPT) technology has been promoted for recharging portable device systems used in everyday life, such as consumer electronic products, robotics, electric vehicles and charging systems [1, 2, 3]. Maximum power transfer efficiency (PTE) is one of the biggest challenges of WPT design since the initial concept of WPT was pioneered by Nikola Tesla in 1914 [4]. Until now, there have been many techniques proposed to improve the efficiency. The wireless transfer of energy, based on the principle of non-radiative magnetic coupling, has gotten a great deal of attention. By using resonance, high PTE can be obtained even when the mutual coupling of the two-coil system is small. Besides, source and load impedances are the most crucial components that directly determine system performance. The optimal source and load impedances for maximum PTE of the magnetically coupled resonance WPT system were proposed in [5, 6], which represented a significant efficiency improvement. In this article, we propose another analysis approach with fully equivalent components of the two-coil WPT system to obtain the highest PTE with optimized load and source resistances.

In a WPT system, when the receiver (Rx) coil is placed further from the transmitter (Tx) coil, the PTE decreases significantly and rapidly because of a small mutual coupling and electromagnetic field leakage [7]. To overcome this restriction, another technique that can be used to enhance the PTE for a larger range is inserting a metamaterial (MTM) structure between the Tx and Rx coils. In this research, we proposed a negative permeability (mu negative - MNG) MTM slab with a near-zero refractive index (NZRI) property that can focus the magnetic field direction farther along the moving path of Rx coil. Typically, an MTM structure is placed in the middle of Tx and Rx coils, allowing the WPT system to achieve maximum PTE [7, 8]. However, such WPT systems are voluminous and difficult to implement in practice. In this article, the MTM slab was placed near the Tx coil to separate the transmitting module from the receiving module for realistic applications.
2 Two-coil WPT system analysis

A magnetically coupled resonator two-coil WPT system can be modeled in terms of equivalent lumped circuit elements. The schematic circuit that was used for hand analysis, as implemented below, is shown in Fig. 1. The schematic circuit is composed of two resonant circuits corresponding to the Tx and Rx coils, which are connected via a mutual inductance $M$. Traditionally, the mutual inductance is taken into account using the formula in elliptic integrals given by Maxwell for conventional coils such as circular or rectangular coils [9]. Referring to the equivalent circuit, the Tx is supplied by a sinusoidal voltage source $V_S$ with a source impedance of $R_S$. The Tx and Rx coils can be modeled as inductors $(L_1, L_2)$ with parasitic resistances $(R_1, R_2)$. Series capacitors $(C_1, C_2)$ are added to control the resonant frequency. Inductors $L_1$ and $L_2$ are magnetically coupled with the mutual inductance $M$. The Rx coil is connected with a load impedance $R_L$.

The magnetically coupled resonator system is analyzed by applying Kirchhoff’s Voltage Law to determine the currents in each circuit coil as demonstrated in Fig. 1:

\[
I_1 \left( R_S + R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \right) + j\omega M I_2 = V_S \tag{1}
\]
\[
I_2 \left( R_L + R_2 + j\omega L_2 + \frac{1}{j\omega C_2} \right) + j\omega M I_1 = 0. \tag{2}
\]

At resonance, the currents of each coil can be solved as:

\[
I_1 = \frac{(R_L + R_2)V_S}{\omega^2 M^2 + (R_L + R_2)(R_S + R_1)} - j\omega M V_S
\]
\[
I_2 = \frac{R_L + R_2}{\omega^2 M^2 + (R_L + R_2)(R_S + R_1)}.
\tag{3}
\]

Then, the transfer function is written by:

\[
\frac{V_L}{V_S} = \frac{-I_2 R_L}{V_S} = \frac{j\omega M R_L}{\omega^2 M^2 + (R_L + R_2)(R_S + R_1)}. \tag{4}
\]

For consistency, power transfer will be represented in terms of a linear magnitude scattering parameter ($|S_{21}|$), which is important experimentally since it can be measured with a vector network analyzer. The PTE is calculated according
to the $|S_{21}|$ by: $\text{PTE} = |S_{21}|^2 \times 100(\%)$. Hence, the maximum of $|S_{21}|$ leads to the maximum of PTE. The equivalent $S_{21}$ scattering parameter can be calculated in [10], which results in:

$$S_{21} = 2 \frac{V_L}{V_S} \sqrt{\frac{R_S}{R_L}}. \quad (5)$$

Substituting (4) into (5), the magnitude scattering parameter can be written as:

$$|S_{21}| = \frac{2\omega M \sqrt{R_S R_L}}{\omega^2 M^2 + (R_L + R_2)(R_S + R_1)} . \quad (6)$$

The first-order derivative of $|S_{21}|$ is taken to calculate the optimal $R_{L,\text{OPT}}$ for a maximum $|S_{21}|$, and then it gives:

$$R_{L,\text{OPT}} = \frac{\omega^2 M^2}{R_S + R_1} + R_1 . \quad (7)$$

The input impedance of the circuit model is:

$$Z_{\text{in}} = R_1 + \frac{j\omega M I_2}{I_1} = R_1 + \frac{\omega^2 M^2}{R_2 + R_L} . \quad (8)$$

Seen from the source, for no power reflection, it requires:

$$(Z_S = R_S) = Z_{\text{in}}^* . \quad (9)$$

Substituting the optimal load $R_{L,\text{OPT}}$ to (8), and from (9), we get the corresponding optimal source is:

$$R_{S,\text{OPT}} = \sqrt{\frac{\omega^2 M^2 R_1}{R_2} + R_1^2} . \quad (10)$$

In order to understand how magnetically coupled resonators can efficiently transfer energy, it is useful to characterize the interaction of the Tx and Rx by a coupling coefficient. The coupling coefficient is related to the mutual inductance through the given formula:

$$k = \frac{M}{\sqrt{L_1 L_2}}. \quad (11)$$

With the calculated example circuit parameters, the magnitude scattering parameter $|S_{21}|$ is a function of coupling coefficient $k$ and frequency as illustrated in Fig. 2(a). The $|S_{21}|$ can reach a peak at the resonant frequency and a critical coupling point $k_{\text{critical}}$ with the optimal source and load values. In Fig. 2(b), the efficiency is demonstrated by sweeping the source and load impedances with a fixed mutual coupling $M$ of 0.12 $\mu$H. At the optimal points, $|S_{21}|$ can obtain a high value of 0.97 with $R_{L,\text{OPT}} = 14$ $\Omega$ and $R_{S,\text{OPT}} = 8$ $\Omega$.

### 3 Two-coil WPT system with metamaterial slab

Metamaterial presents an interesting possibility for WPT applications based on recent research. An MTM with a NZRI can enhance the near-field evanescent waves by focusing the propagating electromagnetic waves. Macroscopic parameters, such as effective permittivity $\epsilon$ and effective permeability $\mu$, can be used to describe the electromagnetic properties of metamaterials [11]. In a WPT system, wireless energy is transferred via the coupling magnetic field leading to a negative
MTM that is efficient enough [12]. NZRI metamaterial could be applied in controlling wave directivity. According to Snell’s law, incident waves propagating from the NZRI slab are perpendicular to the outgoing surface. In this article, we designed a negative and near zero $\mu$ MTM to achieve NZRI property and enhance the magnetic field between the Tx and Rx coils.

### 3.1 MTM unit cell design

To obtain efficient performance, the imaginary value of the permeability $\mu_i$, which represents the magnetic loss tangent, should be minimized. In addition, its real value should be negative and near-zero at the operating frequency of the WPT system. From these requirements, a unit cell was designed using a Taconic CER-10 substrate with a relative permittivity of 10 and a thickness of 0.64 mm. The dimension of the MTM unit cell is $55 \times 55$ mm with a double-sided four-turn spiral structure, as shown in Fig. 3(a). This size is much smaller than a conventional wavelength, when the wavelength-to-unit-cell ratio is about 155. The permeability simulation of the proposed MTM unit cell is plotted in Fig. 3(b) with a real value $\mu_R$ of $-0.78$ and an imaginary value $\mu_I$ of 0.034 at 13.56 MHz, which satisfies the low loss and negative near-zero permeability MTM.

![Fig. 3.](image)

**Fig. 3.** a) A double-sided four-turn spiral MTM unit cell structure with the pattern width of 2 mm and space of 0.2 mm; b) Real and imaginary parts of permeability in simulation result.
3.2 Proposed optimal WPT system with MTM slab

The optimal Tx and Rx coils for the WPT system described in the section II were constructed using a copper wire with a radius of 2.5 mm. For both the Tx and Rx coils, the copper wire was folded into a square shape with a single turn loop and a dimension of 20 cm. The MTM slab with a size of $25.5 \times 25.5$ cm was fabricated by arraying a $5 \times 5$ MTM unit cell to match the size of the Tx and Rx coils. The slab position significantly affects the performance of the WPT system. Normally, the slab is placed near the middle, between Tx and Rx coils to obtain the highest PTE. However, the transmitting module, including Tx coil and MTM slab, of the WPT system is voluminous and unrealistic for applications. Therefore, the MTM slab in this research was optimized to place near the Tx coil to separate the transmitting module from the receiving module. Using the 3D EM simulation tool Ansoft HFSS, the power transfer, based on the magnetic resonance coupling, of a two-coil WPT system with an MTM slab was simulated to show the PTE as a function of the slab position. The maximum PTE was calculated in the frequency range of 13.3–13.8 MHz, as shown in Fig. 4. The highest PTE was found when the slab was located in the middle position between Tx and Rx coils. However, it was lower only by about 2% when the slab was at a 6 cm distance from the Tx coil. Therefore, the closed position was chosen to place the slab, as there was only an insignificant PTE decrease. A comparison of the H-field magnitude distribution between two WPT systems with and without the MTM slab also is shown in Fig. 4. It is obvious that the H-field intensity at Rx coil was increased strongly when the MTM slab was inserted next to Tx coil. The H-field around the MTM slab presents a strong field that is focused and driven perpendicularly to the outgoing surface of MTM slab.

Fig. 4. PTE as a function of the distance of the MTM slab from Tx, and the magnetic field distribution comparison between the Tx and Rx coils at a distance of 30 cm with and without the MTM slab.
In order to verify the analysis method, the two-coil WPT systems, with and without MTM slab, were fabricated and measured. The configuration of the resonant coupling system with a Protek A333 network analyzer is shown in Fig. 5. Two network analyzer ports were connected to the Tx and Rx coils to measure the scattering parameters. First, the Tx and Rx coils were measured to confirm the WPT efficiency with the optimal source and load impedances. The value of the inductance and resistance of the Tx \((L_1, R_1)\) and Rx \((L_2, R_2)\) coils were measured using the network analyzer, which were approximately \((0.76 \, \mu H, 0.23 \, \Omega)\) and \((0.73 \, \mu H, 0.36 \, \Omega)\), respectively. Series capacitances \((C_1, C_2)\) were added to control the resonant frequency at 13.56 MHz with the values of 182 pF and 189 pF. The optimal source \(R_{S,OPT}\) and load \(R_{L,OPT}\), calculated to obtain the highest performance, were 8.2 \(\Omega\) and 12.8 \(\Omega\). From these values, the scattering parameter \(S_{21}\) was measured and achieved approximately \(-1.36\, dB\) at a distance of 20 cm. Then, the MTM slab was inserted near the Tx coil at a 6 cm distance. The measurement result shows that the \(S_{21}\) enhances significantly at all distances, with the highest \(S_{21}\) of \(-0.9\, dB\). A comparison of PTE between the simulation and experiment results, with and without MTM slab, at all distances is represented in Fig. 6. With the optimal source and load method, the PTE of the two-coil WPT system reached up to 73.1\% in experiment at a 20 cm distance between the Tx and Rx coils. In addition, the MTM slab improved the PTE around 10\% to achieve the highest PTE of 81.3\%. There is a good agreement between the simulation and experiment results. A slight difference was caused by the tolerant fabrication and misalignment of the two-coil WPT and MTM slab.
4 Conclusion

In this letter, we proposed a novel analysis approach for optimized load and source impedances with a two-coil WPT system to obtain a high PTE. In addition, an MTM slab with negative and near-zero permeability was designed and inserted between the Tx and Rx coils of the WPT to enhance the efficiency in a larger range. In conclusion, we demonstrated an approximate 10% improvement in PTE, with the highest value of 81.3% in the experimental result when using a $5 \times 5$ array MTM slab at 13.56 MHz operating frequency placed at a 20 cm distance between the Tx and Rx coils.

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