Parameter study on a phase-field model for fatigue fracture

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Recently, the problem of modelling fatigue fracture has been approached by the phase-field method. The approach of Seiler et al. \cite{1} focuses on a reduction of computational cost by combining the phase-field method for fracture with a classic fatigue concept. For this model, we perform a parameter study analysing the influence of the fatigue degradation function and the crack propagation behaviour under cyclic loads with different mean loads. Eventually, we compare the results to a cyclic compact tension test.

1 Model framework

Modelling fatigue phenomena is still a challenging task. Recently, the phase-field method has come into focus for this, being able to model arbitrary crack paths in a straightforward way. Among others who, e.g., focus on fatigue fracture for ductile \cite{2} or viscous \cite{3} materials, we developed a model \cite{1} which especially aims at reducing the computational cost - a crucial point when dealing with at least \(10^4\) load cycles in high cycle fatigue.

Within the model, the crack topology is described using an additional phase-field variable \(d \in [0, 1]\) smoothly bridging the fully intact \((d = 0)\) and fully damaged \((d = 1)\) state. Regularizing the phase-field with the length scale parameter \(\ell\), the energy functional can be written as

\[
\Pi_\ell = \int_\Omega g(d) \psi^\ell(\varepsilon) \, dV + \int_\Omega \alpha(D) G_c \frac{1}{2e} (d^2 + \ell^2 \nabla d^2) \, dV.
\]

The standard phase-field formulation is thereby extended to cyclic loading by introducing a scalar fatigue degradation function

\[
\alpha(D) = (1 - \alpha_0)(1 - D)^\xi + \alpha_0 \quad \text{with} \quad \alpha(D) \in [\alpha_0, 1]
\]

with the parameters \(\alpha_0\) and \(\xi\). It reduces the fracture toughness \(G_c\) locally in order to model the progressive material weakening. This reduction function depends on a lifetime variable \(D \in [0, 1]\). The lifetime variable \(D\) is determined with the notch strain concept \cite{4}. According to this concept, the damaging effect of a load cycle only depends on its stress and strain amplitudes as well as the mean stress. Those quantities can be computed in a single elastic simulation for several load cycles in a row, saving immense computational cost compared to an explicit simulation of the full load cycle. The damage \(D\) is computed using material data in the form of cyclic stress-strain curves (CSSC) and strain Wöhler curves (SWC) which are approximated with the Ramberg-Osgood and the Manson-Coffin-Morrow approach, respectively. For an extensive description of the model as well as numerical studies, see \cite{1}.

2 Parameter study

The following studies were performed with a compact tension (CT) specimen. The characteristic dimension according to testing guidelines ASTM E647-05 and ASTM E1820-01 is \(W = 100\) mm, the thickness is 4.8 mm. The material parameters were chosen for aluminium AA2024-T3, see Tab. 1. The characteristic length \(\ell\) is set to 1 mm. The specimen were loaded with a cyclic load characterised by the maximum and minimum load \(\bar{F}_{\max}\) and \(\bar{F}_{\min}\) (see Fig. 1) with the corresponding load ratio \(R = \bar{F}_{\max}/\bar{F}_{\min}\). Fig. 2a) shows an exemplary crack path after 403500 load cycles with \(\bar{F}_{\max} = 4\) kN and \(R = 0.1\).

Fatigue degradation function: The influence of the exponent \(\xi\) of the fatigue degradation function in studied in Fig. 2b). A larger exponent \(\xi\) shifts the Paris curve upwards, i.e. increases the crack propagation rate.

Load ratio: Consistent with literature data, e.g. \cite{7}, increasing the load ratio \(R\) shifts the specimen Wöhler curve to the right, see Fig. 3a). That means, given a fixed load range \(\Delta \bar{F}\) increasing the mean load \(\bar{F}_m = 0.5(\bar{F}_{\max} + \bar{F}_{\min})\) into the tension range will lead to a shorter lifetime of the specimen.

Parameterisation with experiment: The simulations are compared to an experiment performed by Keller et al. \cite{8}, as shown, in Fig. 3b). With the fatigue degradation parameters \(\alpha_0 = 0.002\) and \(\xi = 1500\), the intermediate range of the crack propagation can be described fairly well. However, at the beginning of the crack the model predicts too low crack propagation rates. This could be due to the fact that the creation of the precrack (of 5 mm length according to ASTM norm) is tracked in the experiment whereas it is applied as an initial condition in the simulation.

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Table 1: Material parameters for aluminium AA2024-T3 taken from given references [5, 6].

| Parameter          | Value          |
|--------------------|----------------|
| Elastic constants  | $E = 74.6$ GPa, $\nu = 0.33$ |
| CSSC               | $K' = 0.453$ GPa, $n' = 0.201$ |
| SWC                | $\sigma_f = 0.314$ GPa, $\epsilon_f = 0.162$, $b = -0.091$, $c = -0.452$ |
| Fracture toughness | $G_c = 0.165$ MPa m |

Fig. 1: Cyclic loading.

Fig. 2: (a) Crack path for loading with 403500 cycles with $\tilde{F}_{\text{max}} = 4$ kN, $R = 0.1$. Fatigue degradation function: $\alpha_0 = 0.002$, $\xi = 1000$. (b) Paris plot for varying fatigue degradation exponent $\xi$, $\alpha_0 = 10^{-5}$, $F_{\text{max}} = 4$ kN, $R = 0.1$.

Fig. 3: (a) Wöhler curve for CT specimen for different load ratios $R$. Load cycles until crack initiation $N$ given a cyclic load range $\Delta \tilde{F}$. Fatigue degradation function: $\alpha_0 = 0.002$, $\xi = 1$. (b) Comparison of experiment [8] and simulation: Load $\tilde{F}_{\text{max}} = 4$ kN, $R = 0.1$; fatigue degradation function: $\alpha_0 = 0.002$, $\xi = 1500$.

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