Determination of stress concentration factors from photometric analysis data

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Abstract. The determination of stress concentration factors in local areas of metal structures plays an important role in the evaluation of structural strength. Existing computational methods for their evaluation take into account only the geometric characteristics of the stress concentrators, and the experimental methodologies are notable for their laboriousness and are unsuitable for use under operational conditions. The paper proposes a new method for evaluating the stress concentration factors and discusses the results of its use on the example of aluminium alloy with notches sample under fatigue test conditions.

1. Introduction
Modern metal constructions often have features in the form of holes, the place of pairings of sheets with different thicknesses, cuts, and welded joints. When they are loaded, local perturbations of the stress field occur at the locations of these features, which are taken into account in strength calculations using stress concentration factors (SCF). In the framework of the theory of elasticity, the SCS takes into account mainly the geometric characteristics [1, 2]. In reality, the role of stress concentrators can be played by local structural inhomogeneities and macroscopic defects in technological processing of a material, whose effect on strength can be theoretically difficult to estimate. For this reason, experimental methods of various types have become widespread for evaluating SCF [3, 4]. Their diversity, complexity, and in some cases, partial local structural damage are vital in the process of determining the SCF. Of particular value are non-contact methods of obtaining information necessary to determine the SCF. The purpose of this work is to present the results of the development and experience of applying the method of determining SCF according to the analysis of the reflection brightness spectra from the surfaces of objects of visible light under study. To study such spectra, photometric analysis of structural images (PASI) was developed.

2. Materials and Methods
PASI is based on a differential scheme comparing the fragments of the surfaces studied and the reflection brightness spectra before and after the effects on them of factors of different physical or chemical nature. The physical bases of PASI and some experience of its application are described earlier [5, 6]. Below are the results of its use for evaluating SCF. This study was conducted on samples made of D16 aluminum alloy sheet with a thickness of 1.5 mm. The sample form and the location of the SCF in the form of electric-spark cuts with a length of 4.7 ± 0.1 are shown in figure 1. The cuts are applied in increments along the length of the sample of 40 mm. Sample width 20 mm, length 230 mm. The tests were carried out the vibrating table. Before and after testing, the surface of the images was scanned on
a scanner with a resolution of 6x10^6 pixels. To assess the SCF in the image of the surface of the sample in the locations of the cuts, fragments were isolated, each of which was divided into two approximately equal parts. Figure 2 shows an example of such a separation of the selected fragments. Analysis of the development of deformation at the tip of the notch under cyclic loading was carried out using photometric analysis of structural images, which was developed in the team of authors. Its basis is the software and analytical complex operating on the basis of a personal computer. The analysis begins with the construction of the brightness spectra of the reflection of visible light from the surface of the studied fragments in the coordinates “The spectral density of the intensity of reflection of visible light p (I) is the intensity of the reflected light I”.

Figure 1. Appearance of sample for fatigue tests on alternating bending with a frequency of 32 Hz.

With the help of PASI, the total estimate of the energy of the reflection spectra of the fragments of the selected subpictures in arbitrary units was determined. The results of these measurements were calculated values of the coefficients of stress concentration. The sample was tested prior to the occurrence of fatigue cracks near the sample head. The tests continued for 243.75 minutes.

Figure 2. Scheme of separation of a sample fragment into parts.

Figure 3. The results of photometric analysis of the fragment No. 1 sample alloy D16.

The sample was loaded with inertial forces, the magnitude of which was determined from the solution of the problem of the deflection at the end of the sample and the experimentally measured values of this deflection. As a result, a formula was obtained describing the distribution of effective stresses along the length of the sample, excluding notches [7, 8]:

\[ \sigma(x) = \frac{P}{A} \left( 1 - \frac{x}{L} \right) \]

where \( P \) is the applied load, \( A \) is the cross-sectional area, \( x \) is the distance from the end of the sample, and \( L \) is the length of the sample.
\[ \sigma(z) = \frac{M(z)}{W} = \frac{q \cdot (l^2 - z^2)}{2W} \]  

(1)

where: \( q \) is the intensity of the distributed load, \( E \) is the normal modulus of the sample material, \( l \) is the sample length, \( J_s \) is the moment of inertia of the sample section relative to the horizontal axis of symmetry. The sample axis coincides with the direction of the 0z coordinate axis. From the formula (1) follows the expression to determine the intensity of the distributed inertial load \( q \) in the form:

\[ q = \frac{y(x)EJ_x}{(l^3 - x^3)} \]  

(2)

where \( y(x) \) is the value of the sample deflections at the point with \( x \) coordinate when the sample axis is oriented along the X axis; \( E \) is the normal modulus of the sample material; \( J_y \) is the moment of inertia of the sample section; \( W \) is the moment of resistance of the sample cross section; \( l \) is the length of the working part of the sample. Figure 4 shows the distribution of effective stresses on the sample surface. According to the PASI, estimates of the fatigue damage of the sample surface were obtained, which were calculated by the formula:

\[ D_s = \frac{S_i(t_r) - S_i(0)}{S_{max}(t_r) - S_{max}(0)} \]  

(3)

where: \( S_i \) is the area under the spectral curve of the \( i \)-th sample fragment before the test, \( S_i(t_r) \) is the area under the spectral curve of the \( i \)-th sample fragment after the sample was destroyed, \( S_{max}(0) \) is also, but for the fragment along which the destruction occurred for the moment before the start of the tests, \( S_{max}(t_r) \), too, but after the destruction of the sample. Structural damage expressed by the formula (3) satisfies the requirements imposed on the damage function:

when \( t = 0 \), \( D_s = 0 \)
when \( t = t_r \), \( D_s = 1 \)  

(4)

Figure 5 shows the dependence on stress of damage to the surface of the sample tested for fatigue. It can be seen from the figure 5 that the maximum of the material damageability (\( D_s \)) is reached at \( \sigma = 34.5 \) MPa, which is close to the endurance limit of the alloy D16. Using the dependence \( D_s = f(x) \), one can estimate the length of the sample, beyond which the effect of acting stresses on the damageability of the sample is imperceptible. This dependence is given in figure 6.

3. Results and Discussions

Applying a notch reduces the local width of the specimen, which led to a decrease in the cross section resistance moment and a local increase in stresses. The stress concentration ratio due to the notch reaches a value of 1.31. According to [9], the energy emitted by the body (\( \phi \)) with internal energy \( E \) can be expressed by the formula:

\[ \phi = A \cdot E \]  

(5)

where: \( A \) is the coefficient reflecting the probability of spontaneous radiation by a body with internal energy \( E \). The ratio of the energies in parts of fragments 1 and 2 can be written, as \( \frac{(Ka)^2E}{\sigma^2E} \), on the other hand with Taking into account formula (5), this ratio is written as \( \frac{A\phi_1}{A\phi_2} \). Equating these expressions after reducing these and extracting the root, and resolving the result relative to \( K \), we obtain a formula for estimating the component of the stress concentration factor, which takes into account the effect of a notch in the form:

\[ K_n = \sqrt{\frac{\phi_1}{\phi_2}} \]  

(6)
4. Distribution of effective stresses along the length of the sample when tested for fatigue.

Figure 5. Dependence \( D_s = f(\sigma) \).

Figure 6. The dependence of \( D_s = f(x) \) for the sample alloy D16, tested for fatigue.

Figure 7. The dependence of the integral SCF on the magnitude of the current stresses for a sample of alloy D16 after fatigue.

For the integral stress concentration factor, we finally have:

\[
K_i = 1.31 \cdot \frac{\varphi_1}{\varphi_2}
\]  

The results of such calculations are shown in figure 7.

4. Conclusions

1. A method has been developed for determining the stress concentration coefficient from measurements of the areas under the spectral curves of the brightness of reflection of visible light from a fragment with a stress concentrator according to photometric analysis.

2. The method was experimentally tested during fatigue tests of samples of aluminum alloy D16 under cyclic loads.

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References

[1] Neuber G 1947 *Concentration of stresses* (Moscow OGIZ)

[2] Peterson R 1977 *Stress concentration factors. Graphs and formulas for the calculation of structural elements of strength* (Moscow Mir)
[3] Kasatkin BS et al 1981 Experimental methods for the study of deformations and stresses (Kiev Naukova Dumka)
[4] Bell F 1984 Experimental mechanics of solids (Moscow Nauka)
[5] Ermishkin V A, Murat D P and Podbelsky V V 2007 Information technologies 2007(11) 65–70.
[6] Ermishkin V A, Murat D P, Podbelsky V V 2008 Instruments and systems. Management, monitoring, diagnostics 10 38-44.
[7] Tumanov A T 1975 Handbook of aviation materials. Part 1 (Moscow Mashinostroenie)
[8] Feodosyev V I 1999 Strength of materials (Moscow Publishing House of the Bauman MSTU)
[9] Astapenko V A 2010 The interaction of radiation with atoms and nano particle (Dolgoprudny Intellect)