INVESTIGATION OF InGaN/Si DOUBLE JUNCTION TANDEM SOLAR CELLS

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ABSTRACT
In this work, the solar power conversion efficiency of InGaN/Si double junction tandem solar cells was investigated under 1-sun AM1.5 illumination, using realistic material parameters. With this intention, the current-voltage curves are calculated for different front recombination velocities and the influence of the bottom cell thickness on efficiency has been studied. The results show that a front recombination velocity value of $10^3$ cm/s is most advantageous and the use of relatively thick bottom cell is necessary to obtain conversion efficiency greater than 27%, at 300°K cell temperature. This efficiency will decrease as the operating temperature increase.

Keywords: Photovoltaic, Efficiency, Carrier lifetimes, Recombination velocity, Temperature.

1. INTRODUCTION
Photovoltaic (PV) tandem cells have been widely demonstrated in recent years as an effective pathway to realize higher conversion efficiency, showing promising prospects in both terrestrial and space applications. Theoretical modelling of two-junction tandem solar cells shows that for optimal device performance, the bandgap of the top cell should be in the range of 1.6 to 1.8ev [1].
A tandem cell using Indium Gallium Nitride (In$_{x}$Ga$_{1-x}$N) for the top cell and Silicon (Si) for the bottom cell is advantageous in two respects: The direct bandgap of the In$_{x}$Ga$_{1-x}$N alloy system, extends continuously from Indium Nitride (InN) bandgap which is 0.7eV in the medium infrared, to that of the Gallium Nitride (GaN) which is 3.42eV [2] in the near ultraviolet, makes the In$_{x}$Ga$_{1-x}$N alloy a promising candidate for radiation and temperature resistant single or multi-junction solar cells [3]. In the other hand, Si is relatively cheap and plentiful and its processing techniques are well established, in addition to the fact that the Si bandgap of 1.1ev is ideally suited for the bottom junction of high efficiency two-junction solar cells [4].

In this paper, we have modelled the photovoltaic conversion efficiency of series-connected, two-junction, two-terminal In$_{x}$Ga$_{1-x}$N on Si solar cells in terms of their physical parameters, employing a simulation program developed for this reason, where the In$_{x}$Ga$_{1-x}$N has an alloy fraction close to In$_{0.53}$Ga$_{0.47}$N.

2. MODEL DESCRIPTION

Figure 1 show a simplified structure of the In$_{0.53}$Ga$_{0.47}$N/Si tandem, where $x_j$ is the junction depth, $w$ is the depletion region width and $d$ is the cell thickness.

![Fig.1. Simplified configuration of the In$_{0.53}$Ga$_{0.47}$N/Si tandem.](image)

In this work, calculations were all performed under 1-sun AM1.5 illumination and a temperature of 300°k using the one diode ideal model, and for convenience, several simplifying assumptions were made, including no series resistance losses, no reflection losses and contact shadowing. Currents calculation follows the general methodology described in ref. [5].

3. ANALYTICAL MODEL

The total output current drawn from single cells under illumination is given by [5] as:

$$I_{\text{Total}} = I_{\text{Light}} - I_{\text{Dark}}$$

\[1\]
\[ I_{\text{Light}} = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} [I_p(\lambda) + I_n(\lambda) + I_{\text{dr}}(\lambda)] d\lambda \]  

(2)

Where:

- \(I_p(\lambda)\) is the photocurrent due to holes collected at the depletion edge \(x_j\);
- \(I_n(\lambda)\) is the photocurrent due to electrons collected at the depletion edge \(x_{j+w}\);
- \(I_{\text{dr}}(\lambda)\) is the contribution of the depletion region to the photocurrent;
- \(\lambda_{\text{min}}\) is the wavelength corresponding for the bottom cell in case of double junction system to the top cell bandgap, and equals zero for the top cell;
- \(\lambda_{\text{max}}\) is the wavelength corresponding to the cell bandgap.

The dark current can be expressed as:

\[ I_{\text{Dark}} = I_0 \times \left[ \exp\left(\frac{qv}{kT}\right) - 1 \right] \]

(3)

Where:

- \(I_0\) is the saturation current which was calculated following the method described in the ref. [5], \(v\) is the applied voltage, \(k\) is the Boltzmann constant and \(T\) is the temperature.

The open circuit voltage is given by [5] as:

\[ V_{\text{oc}} = \frac{kT}{q} \ln\left(\frac{I_{\text{sc}}}{I_0} + 1\right) \]

(4)

Where \(I_{\text{sc}}\) is the short circuit current. The open circuit voltage of the tandem is taken to be the sum of the open circuit voltages of the tandem junctions:

\[ V_{\text{oc}} = \sum_{i=1}^{n} V_{\text{oc},i} \]

(5)

Where \(n\) is the number of junctions incorporated in the tandem.

The cell output power is given as:

\[ P = I_{\text{Total}} \times V \]

(6)

The cell conversion efficiency is usually taken to be:

\[ \eta = \frac{I_m \times V_m}{P_{\text{inc}}} \]

(7)

Where \(I_m\) and \(V_m\) are coordinates of the maximum power point, \(P_{\text{inc}}\) is the total incident solar power.

The fill factor is defined by:

\[ \text{FF} = \frac{I_m \times V_m}{I_{\text{sc}} \times V_{\text{oc}}} \]

(8)
**In$_x$Ga$_{1-x}$N parameter equations used in our program**

The equation relating the bandgap energy to the mole fraction $x$ is given by [6,7] as:

$$E_g(x) = x \times E_g(\text{InN}) + (1-x) \times E_g(\text{GaN}) - x \times (1-x) \times C$$  \(9\)

Where: $E_g(\text{InN}) = 0.7\text{eV}$, $E_g(\text{GaN}) = 3.42\text{eV}$ and $C$ is a bowing parameter which is taken to be equal to 1.43.

The electron and hole mobilities are calculated as a function of doping using [8]:

$$i_i(N) = \min_i + \frac{\max_i - \min_i}{1 + \left(\frac{N}{N_{g,i}}\right)^\gamma_i}$$  \(10\)

Where $i$ represent either electrons (e) or holes (h), $N$ is the doping concentration and the specific parameters $\mu_{\min}$, $\mu_{\max}$, $\gamma$ and $N_g$ are given in Table 1.

**Table 1.** Parameters used in the simulation of the InN and GaN carrier mobilities.

|        | $\mu_{\min,e}$ [cm$^2$/vs] | $\mu_{\max,e}$ [cm$^2$/vs] | $\mu_{\min,h}$ [cm$^2$/vs] | $\mu_{\max,h}$ [cm$^2$/vs] | $\gamma_e$ | $\gamma_h$ | $N_{g,e}$ [cm$^{-3}$] | $N_{g,h}$ [cm$^{-3}$] |
|--------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|------------|------------|----------------------|----------------------|
| GaN    | 55                          | 1000                        | 3                           | 170                         | 1          | 2          | 2e$^{17}$            | 3e$^{17}$            |
| InN    | 30                          | 1100                        | 3                           | 340                         | 1          | 2          | 2e$^{17}$            | 3e$^{17}$            |

In$_x$Ga$_{1-x}$N electron mobilities are taken as a linear interpolation between the InN and GaN values; however, hole mobilities of the In$_x$Ga$_{1-x}$N alloys are assumed to be similar to the GaN hole mobility.

The absorption coefficient of the In$_x$Ga$_{1-x}$N alloys is taken to be [9]:

$$\alpha(E) = 10^5 \times \sqrt{a \times (E - E_g) + b \times (E - E_g)^2} \text{ cm}^{-1}$$  \(11\)

Where $E$ is the incoming photon energy given in eV, $a$ and $b$ are dimensionless fitting parameters. The fitting parameters used in our program are shown in Table 2.

**Table 2.** Fitting parameters used to calculate the absorption coefficient of the In$_x$Ga$_{1-x}$N alloys.

| Indium composition | $a$    | $b$    |
|-------------------|--------|--------|
| 0.57              | 0.60946| 0.62182|
| 0.69              | 0.58108| 0.66902|
| 0.83              | 0.66796| 0.68886|
The relative dielectric constant for InN and GaN are 10.5 and 8.9 respectively [10,11], so a linear interpolation has been made to find the relative dielectric constant of the In\textsubscript{x}Ga\textsubscript{1-x}N alloys.

Effective masses for electrons and holes were determined using the following equations [12]:

\[
\begin{align*}
    m_e^* &= m_0 \times (0.2 - 0.105 \times x) \\
    m_h^* &= m_0 \times (1.25 + 0.8 \times x)
\end{align*}
\]

Where \(m_0\) is the electron rest mass.

**Si parameter equations used in our program**

The equation relating the bandgap energy to the temperature is given by [13] as:

\[
E_g (T) = 1.17 - \frac{4.73 \times 10^{-4} \times T^2}{636 + T}
\]

The electron and hole mobility components due to lattice scattering, ionized impurity scattering and carrier – carrier impurity scattering are calculated using the formula [14]:

\[
\frac{1}{\mu_{\text{e,h}}^{\text{tot}}} = \frac{1}{\mu_{\text{L,I}}^{\text{e,h}}} + \frac{1}{\mu_{\text{c}}}
\]

Where \(\mu_{\text{L,I}}\) is the combined lattice and ionized impurity scattering mobility and \(\mu_{\text{c}}\) is the carrier – carrier impurity scattering mobility component.

\[
\begin{align*}
    \mu_{\text{c}} &= 1.04 \times 10^{21} \times \left( \frac{T}{300} \right)^{1/2} \\
    \mu_{\text{L,I}}^{\text{e}} &= \frac{1252 \times \left( \frac{T}{300} \right)^{2.33}}{1 + \frac{1252 \times \left( \frac{T}{300} \right)^{2.33}}{1.432 \times 10^{17} \times \left( \frac{T}{300} \right)^{2.546} \times \text{CI}}} + 88 \times \left( \frac{T}{300} \right)^{-0.57} \\
    \mu_{\text{L,I}}^{\text{h}} &= \frac{407 \times \left( \frac{T}{300} \right)^{2.23}}{1 + \frac{407 \times \left( \frac{T}{300} \right)^{2.23}}{2.67 \times 10^{17} \times \left( \frac{T}{300} \right)^{2.546} \times \text{CI}}} + 54.3 \times \left( \frac{T}{300} \right)^{-0.57}
\end{align*}
\]

Where \(n\), \(p\) and \(\text{CI}\) are the electron, hole and total ionized impurity concentrations respectively.

For indirect transitions, light absorption is assisted by the absorption or emission of phonons. Theory predicts the absorption coefficient of crystalline Silicon to be [15]:

\[\text{Absorption Coefficient} = \text{constant} \times \left( \frac{T}{300} \right)^\gamma\]
\[ a(T) = \sum_{i,j=1,2} A_j C_{i,j} \times \left( \frac{(h\nu - E_{g,j} + E_{ph,i})^2}{\exp\left(\frac{E_{ph,i}}{kT}\right) - 1} + \frac{(h\nu - E_{g,j} - E_{ph,i})^2}{1 - \exp\left(-\frac{E_{ph,i}}{kT}\right)} \right) + A_d \times \frac{(h\nu - E_{g,d})^{3/2}}{h\nu} \]  

(19)

Where \(i\) and \(j\) are the phonons 1, 2 and band gaps 1, 2 respectively.

\(E_{g,1} = 1.16 \text{ [ev]}\) and \(E_{g,2} = 2.25 \text{ [ev]}\) are the indirect band gaps 1 and 2 respectively, and \(E_{g,d} = 3.2 \text{ [ev]}\) is the direct band gap 3 with all values for \(T\) being \((0^\circ\text{K})\).

\(E_{ph,1} = 212 \times k\) and \(E_{ph,2} = 670 \times k\) are the phonons with \(C_1 = 5.5, C_2 = 4, A_1 = 253, A_2 = 3312\) and \(A_d = 2.3 \times 10^7\) in \([\text{cm}^{-1} \text{ ev}^{-2}]\).

The first term describes the absorption of phonons while the second describes the emission of phonons during absorption of light. The third term gives the absorption for a direct transition.

4. RESULTS AND DISCUSSION

The analysis starts by assuming the physical and geometrical parameter values of the two-junctions presented in Table 3. Where \(N_a\) and \(N_d\) are the acceptor and donor concentrations, \(S_h\) and \(S_e\) are the recombination velocities in the N and P-type regions respectively.

| \(\text{In}_{0.53}\text{Ga}_{0.47}\text{N} - \text{Si}\) | \(E_g\) [ev] | \(N_a\) [cm\(^{-3}\)] | \(N_d\) [cm\(^{-3}\)] | \(\tau_e\) [s] | \(\tau_h\) [s] | \(S_e\) [cm/s] | \(S_h\) [cm/s] | \(x_j\) [\mu m] | \(d\) [\mu m] |
|-----------------|-----------|-------------|-------------|-------|-------|--------|--------|------|-------|
| \(1.622\) \(1.22\) | \(1 \times 10^{17}\) | \(1 \times 10^{17}\) | \(1 \times 10^{16}\) | \(4 \times 10^{-9}\) | \(4 \times 10^{-9}\) | \(1 \times 10^3\) | \(1 \times 10^3\) | \(2 \times 10^{-1}\) | \(3\) |

\(\text{In}_{0.53}\text{Ga}_{0.47}\text{N} \) minority carrier lifetimes effect on the conversion efficiency of the tandem

The minority carrier lifetime is one of the most critical parameters in determining the efficiency of a solar cell. We simulated the effect that lower lifetimes of the \(\text{In}_{0.53}\text{Ga}_{0.47}\text{N} \) top cell would have on device performance, and the results were shown in figure (2).
Fig. 2. Effect of minority carrier lifetimes of the In$_{0.53}$Ga$_{0.47}$N junction on the conversion efficiency of the tandem.

The large absorption coefficient in the direct bandgap In$_x$Ga$_{1-x}$N alloys causes the majority of the electron-hole pairs to be generated less than a diffusion length away from the junction. So for short carrier lifetimes, the carriers recombine quickly what implies a reduction in the conversion efficiency.

Hole lifetimes as high as 6.5ns and 5.4ns have been observed in GaN and InN respectively [16,17]. However, In$_x$Ga$_{1-x}$N alloys are likely to have lower lifetimes due to compositional fluctuations, and therefore a 4ns minority carrier lifetime was assumed in our calculations for both electrons and holes.

**Simulation of the current-voltage characteristics**

Figure (3) shows the current-voltage characteristic of the single junctions and that of the tandem, while Table 4 gives the computations of the open circuit voltage, short circuit current, fill factor and the conversion efficiency for the In$_{0.53}$Ga$_{0.47}$N/Si tandem.
Fig. 3. Simulated current-voltage characteristics of the In$_{0.53}$Ga$_{0.47}$N/Si tandem: (Left) for top and bottom single junctions, (Right) for top, bottom and the tandem.

Table 4. Photovoltaic parameters of the In$_{0.53}$Ga$_{0.47}$N/Si tandem.

|                        | Voc [V] | Isc [mA] | FF [%] | η [%] |
|------------------------|---------|----------|--------|-------|
| Top cell: In$_{0.53}$Ga$_{0.47}$N | 1.10    | 24.92    | 88.93  | 25.16 |
| Single bottom cell: Si | 0.47    | 38.94    | 79.45  | 15.17 |
| Bottom cell: Si        | 0.44    | 07.94    | 78.07  | 02.77 |
| Tandem: In$_{0.53}$Ga$_{0.47}$N / Si | 1.54    | 07.94    | 91.55  | 27.94 |

According to the results represented in Table 4 and the preceding curves, one note that, the current-voltage curve of the top cell stayed the same, while the current-voltage curve of the bottom cell had a current drop. This current decrease was expected because the incident spectrum was diminished while crossing the top cell, and since the junctions are mechanically stacked in series, the overall current-voltage characteristic is limited in its current level by the characteristic with the lowest current. In addition, we note a significant increase in efficiency from single junctions to the tandem.

Our calculations predict an efficiency of 27.94%, for the In$_{0.53}$Ga$_{0.47}$N/Si tandem by using realistic parameters. This result is very similar to that obtained by L. Hsu et al [4]. They calculated the conversion efficiency of InGaN/Si tandem cell, made from low quality
material, using a modified version of the standard equations for calculating the electrical characteristics of the tandem, using the air mass 1.5 global irradiance, and they obtained a conversion efficiency of about 27%.

The differences in efficiency results can be partly attributed to the difference in parameter formulas used. All formulas seek to duplicate results from actual measurements. Therefore, all formulas are approximations only and it is difficult to state which formula is more correct.

**In$_{0.53}$Ga$_{0.47}$N carrier front recombination velocities effect on the conversion efficiency of the tandem**

In this work, the back surface recombination velocity was fixed at $1 \times 10^3$ cm/s, and the influence of the front recombination velocity on the conversion efficiency has been simulated, where values of $1 \times 10^3$, $1 \times 10^4$, $1 \times 10^5$ and $1 \times 10^6$ cm/s was used.

Fig 4. (Left) Simulated current-voltage characteristics of the In$_{0.53}$Ga$_{0.47}$N/Si tandem for various front recombination velocities, (Right) Effect of In$_{0.53}$Ga$_{0.47}$N front recombination velocities on the conversion efficiency of the tandem.

From figure 4 above, it is apparent that the conversion efficiency drops notably with increasing the front recombination velocity, since most of the photo-generated carriers cannot be collected by the electrodes what implies a low electric current. Therefore, to reduce the power loss, the front surface recombination velocity should not exceed $1 \times 10^3$ cm/s.
Effect of the bottom cell thickness on the conversion efficiency of the tandem

It can be seen from Figure 5 below that the efficiency increases significantly with increasing the cell thickness.

![Graph showing the effect of the Si thickness on the conversion efficiency of the tandem.](image)

**Fig.5.** Effect of the Si thickness on the conversion efficiency of the tandem.

The curve show an improvement from 25.53% to 27.94% for 5µm and 200µm Si cell thickness respectively for the reason that, in case of thin thickness, most of the incident photons are not absorbed resulting in a lower efficiency. So a cell thickness of about 200µm is needed to lose not too much of the efficiency.

Effect of changing the operating temperature on the conversion efficiency of the tandem

According to the results represented in table 5 and figures (6) and (7) below, it is seen that the increase of the cells temperature causes an increase in the reverse saturation current because of the reduction in bandgaps. Thus, the open circuit voltage will increase and the mechanism of carrier’s production becomes increasingly significant what implies a weak increase in the short circuit current.
Table 5. Effect of temperature on the photovoltaic parameters.

| T (°k) | 300                | 340                | 380                |
|--------|--------------------|--------------------|--------------------|
| Cell   | Top | Bottom | Tandem | Top | Bottom | Tandem | Top | Bottom | Tandem |
| I_0 (A) | 7.62e-21 | 3.94e-10 | -         | 1.79e-17 | 1.39e-07 | -         | 8.55e-15 | 1.5e-05 | -         |
| V_oc (V) | 01.10 | 0.43 | 01.54 | 01.02 | 0.32 | 01.34 | 0.94 | 0.21 | 01.15 |
| I_sc (mA) | 24.92 | 07.94 | 07.94 | 24.97 | 08.03 | 08.03 | 25.02 | 08.11 | 08.11 |
| FF (%) | 88.93 | 78.07 | 91.55 | 87.19 | 70.88 | 89.64 | 85.16 | 59.73 | 87.27 |
| η (%) | 25.16 | 02.77 | 27.94 | 22.92 | 1.88 | 24.80 | 20.63 | 01.03 | 21.66 |

Fig.6. Current-voltage characteristics of the tandem for various Temperatures.

It may be seen also that the fill factor undergoes a reduction with the increase of the cells temperature following the increase in the dark saturation current, and owing to the fact that the reduction in the open circuit voltage is more significant with respect to the increase of the short circuit current, the conversion efficiency will also decrease.
5. CONCLUSION

In this study, we have improved a comprehensive model to predict the performance of double junction In$_{0.53}$Ga$_{0.47}$N/Si tandem cells under 1-sun AM1.5 illumination, using realistic parameters by a simulation program designed for this reason. It is found that higher values of the surface recombination velocity at the top In$_{0.53}$Ga$_{0.47}$N cell causes a considerable drop of the conversion efficiency. So, in order to achieve higher conversion efficiency, it is important to keep the front recombination velocity under $1 \times 10^3$ cm/s. Moreover, a cell thickness of about 200µm is privileged to not contribute significantly in recombination.

In addition, we have analyzed the effect of elevated operating temperature on the conversion efficiency and we found that the increase of the tandem’s temperature causes a degradation of their performance.

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