Comparison of two-dimensional and quasi-one-dimensional scramjet models by the example of VAG experiment

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Abstract. In the paper two-dimensional and quasi-one dimensional models for scramjet combustion chamber are described. Comparison of the results of calculations for the two-dimensional and quasi-one dimensional code by the example of VAG experiment are presented.

1. Introduction
Recently, a study of combustion processes in the ramjet and scramjet is becoming more and more important in connection with attempts to create a hypersonic aircraft. Carrying out physical experiments in this field is connected with a lot of technical difficulties and is too expensive. Special requirements have to meet the fuel and combustion chamber geometry. Therefore, computational and theoretical studies of combustion processes in scramjet begin to play an increasing role. At the first stage of designing the scramjet it is very important to make an accurate prediction of the engine efficiency. In recent years, the quasi-one-dimensional scramjet model is widespread [1–14]. In these models to calculate the chemical concentrations, temperature and pressure fields, the set of ordinary differential equations should be solved, and the area of the combustion chamber is defined as a function of a longitudinal coordinate. However, the quasi-one-dimensional model does not allow to describe a structure of flow fields. There are two-dimensional [15] and three-dimensional [16] models for describing the structure of flows in scramjet chamber.

In [17] the comparison of the results of calculations for the two-dimensional and quasi-one dimensional code has been presented for HyShot-2 experiment. The purpose of this paper is to compares the quasi-one-dimensional and two-dimensional models by the example of numerical simulation of VAG experiment [18].

2. Description of VAG experimental setup
The VAG [18] experiment is similar to T4 at the University of Queensland.[19,20]. In [19] the comparison of data between this two experiments was carried out. The experiment was conducted for a rectangular channel expanding part, as shown in figure 1. The width of the camera was constant and equaled to 94.3 mm. Fuel injection is executed from the gap strut in the center of the chamber. On the top and bottom of the strut there were located 4 holes with a diameter of 2.5 mm, through which
hydrogen is carried out perpendicular to the flow. At the end of the strut there were located 4 holes with a diameter of 2.5 mm, through which hydrogen was injected parallel to the flow. The experiment studied the effect of the strut geometry on the results obtained. The length of the wedge spacers (33.5 mm, 37.6 mm, 39.6 mm, 43.8 mm and 44.8 mm) was varied. In the experiment the pressure distribution on the chamber wall was measured. In this paper only normal hydrogen injection and equivalence ratio $\varphi=0.61$ was considered. The following incoming air flow parameters were used for verification:

- Incoming flow pressure: $P = 0.58$ atm;
- Incoming flow temperature: $T = 1258$ K;
- Incoming flow Mach Number: $M = 2.44$;
- Incoming flow gas mixture: Air;
- Fuel Mach Number: $M = 1$.

![Figure 1. Scheme of the VAG[18] supersonic combustion experiment.](image)

### 3. Quasi-one-dimensional computational fluid dynamic model

In [17] the quasi-one-dimensional computational fluid dynamic model described in detail. To describe the fluid flow in scramjet the conservation laws for mass, momentum and energy should be specified. The governing equations are based on the following assumptions: 1) ideal gas law is fulfilled in the combustor, 2) quasi-one-dimensional flow (all flow variables are functions of the axial distance along the combustor), 3) continuous flow (changes in stream properties are continuous functions), 4) steady-state flow. Quasi-one-dimensional approach requires that we state these equations into the set of ordinary differential equations (ODE). The equations solved are:

$$
\frac{dU}{dx} = \frac{1}{\alpha} \left\{ - \frac{1}{A} \frac{dA}{dx} + \frac{1}{m} \frac{dM^2}{dx} \left( 1 - \varepsilon \right) - \frac{d\hat{h}}{dx} \frac{d\hat{m}}{dx} \right\} + \frac{1}{\hat{h}} \left( \sum_i h_i \frac{dY_i}{dx} + \frac{1}{m} \sum_i \left( h_i \frac{d\hat{m}_i}{dx} \right)_{\text{added}} \right) - \frac{1}{MW} \frac{dMW}{dx} + \left[ \gamma M^2 - \frac{c_p (T_{\text{sw}} - T_{\text{sw}})}{\hat{h} \cdot Pr^2 A} \right] \frac{2C_f}{D}.
$$

(1)
The skin friction is found using the Eckert

\[ \frac{dT}{dx} = \frac{1}{\hat{c}_p} \left[ -\sum_i h_i \frac{dY_i}{dx} + \frac{1}{m} \sum_i \left( h_i \frac{d\dot{m}_i}{dx} \right) \right] + \frac{2C_{J\gamma}(T_{AW} - T_w)}{Pr^2 D \cdot A} + h_0 \frac{\dot{m}}{m} - \frac{U}{dx} - \frac{U}{dx} \right], \]

where \( m \) is the ratio of injection velocity in the downstream direction to a flow (under perpendicular injection \( \varepsilon = 0 \)); \( A \) is the sectional area being a function of the coordinate \( x \). The skin friction is found using the Eckert Reference Temperature method. \( C_f = \frac{0.664\sqrt{C}}{\sqrt{Re}} \); \( C' = \left( \frac{T^*}{T} \right)^{1/2} \left( \frac{1 + R/T}{T^*/T + R/T} \right) \); \( R = 111 K \);

\( T' = 0.5T_u + 0.28T + 0.22T_{aw} \). The method is valid for estimating frictional drag at subsonic to hypersonic speed and is suitable for aircraft design. The method assumes that the vehicle surface is at the adiabatic wall temperature \( T_{aw} \).

We use the supersonic mixing model [2]. Hydrogen was considered as a fuel. Global kinetic mechanism suggested by Westbrook [21] was used. The set of ODE was solved numerically using the Runge-Kutta method and generalized Newton method [22].

4. Two-dimensional computational fluid dynamic model

For two-dimensional calculations we used the NERAT-2D computer code [15]. NERAT-2D realizes the time-relaxation method. At each time step the following groups of governing equations were integrated successively: the Navier–Stokes and continuity equations, the equations of mass conservation of chemical species, the equation of energy conservation. These equations are formulated in the following form:

\[ \frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{V}) = 0, \]

\[ \frac{\partial \rho u}{\partial t} + \text{div}(\rho u \mathbf{V}) = -\frac{\partial p}{\partial x} - \frac{2}{3} \frac{\partial}{\partial x} (\mu \text{div} \mathbf{V}) + \frac{1}{r \partial r} \left[ r \mu \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial r} \right) \right] + 2 \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} \right), \]

\[ \frac{\partial \rho v}{\partial t} + \text{div}(\rho v \mathbf{V}) = -\frac{\partial p}{\partial r} - \frac{2}{3} \frac{\partial}{\partial r} (\mu \text{div} \mathbf{V}) + \frac{\partial}{\partial x} \left[ \mu \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial r} \right) \right] + 2 \frac{\partial}{\partial r} \left( \frac{\partial v}{\partial r} \right) + 2 \frac{\partial}{\partial r} \left( \frac{v}{r} \right), \]

\[ \frac{\partial \rho}{\partial t} + \text{div} \rho \mathbf{V} = -\text{div} J_i + \dot{w}_i, \quad i = 1, 2, \ldots, N_i, \]
\[ \rho c_p \frac{\partial T}{\partial t} + \rho c_p \mathbf{V} \nabla T = \text{div}(\lambda \nabla T) + \frac{\partial p}{\partial t} + \mathbf{V} \nabla p + \Phi_{\mu} - \sum_{i=1}^{N_s} h_i \dot{w}_i + \sum_{i=1}^{N_s} \rho c_{p,i} D_i (\nabla Y_i \cdot \nabla T), \]

where \( \Phi_{\mu} = \mu \left[ 2 \left( \frac{\partial \mathbf{v}}{\partial r} \right)^2 + 2 \left( \frac{\partial \mathbf{u}}{\partial x} \right)^2 + \left( \frac{\partial \mathbf{v}}{\partial x} + \frac{\partial \mathbf{u}}{\partial r} \right)^2 - 2 \left( \frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial r} + \frac{\mathbf{v}}{r} \right)^2 \right] \) is the dissipative function; \( \mu, \lambda \) are the viscosity and heat conductivity coefficients, \( c_p \) is the specific heat capacity of gas mixture; \( c_p = \sum_{i=1}^{N_s} Y_i c_{p,i} \); \( Y_i \) is the mass fraction of species \( i \); \( c_{p,i}, h_i \) are the specific heat capacity at constant pressure and specific enthalpy of species \( i \); \( \dot{w}_i \) is the reaction rate for species \( i \); \( D_i \) is the effective diffusion coefficient of species \( i \); \( \rho, \mathbf{J}_i \) are the density and mass diffusion flux for species \( i \); \( \mathbf{J}_i = -\rho D_i \nabla Y_i \); \( N_s \) is the number of species. We used the kinetic mechanism proposed by Evans and Schexnayder [23].

5. Results

Comparison of the pressure distribution calculated by two-dimensional model (blue line) and quasi-one-dimensional model (red line) with experimental data (green triangles) are shown in figure 2. It should be noted that the pressure increase appears to be a more significant under two-dimensional modeling. This can be explained by the fact that the flow in the channel is actually three-dimensional, and therefore it is necessary and very important to consider the fuel injection features for a correct description of the flow and combustion in the channel. One can also see a good agreement between the quasi one-dimensional simulation results and experimental data.

![Figure 2](image-url)
6. Conclusion
Comparison of the pressure distribution calculated by two-dimensional model and quasi-one-dimensional model with experimental data for VAG experimental setup is presented. This work presents a continuation of our efforts on the verification and validation of numerical methods and computational codes for calculation of various hypersonic vehicles and energetic devices [22-31].

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