Features of concrete creep numerical modeling in the calculation of building structures

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Abstract. The paper presents a study and justification of the concrete creep computational model for use in computer calculations of concrete and reinforced concrete structures of arbitrary complexity, with a possible change in the intensity of the current load over time. The computational model is based on the generalized Maxwell-Wiechert model of the viscoelastic material. The model was verified on the basis of data from experimental studies of concrete creep, was justified and calibrated in accordance with the requirements of regulatory documents for calculations of concrete and reinforced concrete structures, taking into account the material creep. The paper focuses on the situation where the creep of the material leads to a dangerous change in the structural scheme of a building, which in turn can cause its destruction. Verification of the creep computational model and calculation of structures using the selected computational model was carried out in the SIMULIA Abaqus software environment.

1. Introduction

The relevance of the calculation of concrete and reinforced concrete structures, taking into account the creep of the material, is considered in the paper in the aspect of a possible dangerous change in time of the structural scheme of a building due to material creep, which often leads to the destruction of the structure. The considered computational model of concrete creep was verified on the data of experimental studies of concrete rheological manifestations over time under load, and was also substantiated and calibrated in accordance with the requirements of regulatory documents for calculations of concrete and reinforced concrete structures, taking into account the material creep.

2. Materials and methods

2.1. Requirements for creep calculational models

The requirements for computational creep models are built on the experience of comprehensive experimental and theoretical studies of the processes of creep, relaxation and shrinkage of concrete over time [1 - 3] etc. These requirements are clearly stated in international and national regulatory documents. In this paper, they are based on the recommendations of Eurocode [4] (EC2), the Pre-norma fib Model Code [5] (PN), as well as the NIIZhB Recommendations [6]. In turn, the Recommendations [8] are based on the results of the relevant basic research [1, 2] and oth. and underlie the requirements of the SNiP [7].

The complete deformation of concrete $\varepsilon$ is usually represented by its three components:

$$\varepsilon = \varepsilon_0 + \varepsilon_{cr} + \varepsilon_{shr},$$

(1)
where $\varepsilon_0$ is the so-called “instantaneous” deformation which manifests itself at the moment of loading the structure, $\varepsilon_{cr}$ is the creep deformation that develops over time, and $\varepsilon_{shr}$ is the shrinkage deformation.

The key parameter characterizing the creep of concrete in EC2 and PN is the creep coefficient (creep factor) $\varphi(t,t_0)$ established between the moment of application of the load $t_0$ and the age of concrete $t$:

$$
\varphi(t,t_0) = \frac{\varepsilon_{cr}}{\varepsilon_0}.
$$

At the same time, the concept of creep measure $C(t,t_0)$ is also used in these regulatory documents:

$$
\varphi(t,t_0) = E_b(t_0) \cdot C(t,t_0),
$$

where $E_b(t_0)$ is the elasticity modulus of concrete at the time of loading $t_0$.

In turn, the creep measure is the initial key parameter in the NIIZhB Recommendations:

$$
C(t,t_0) = \frac{1}{E_b(t_0)} - \frac{1}{E_b(t)} + C(\infty,t_0),
$$

where $C(\infty,t_0 = 28)$ is the limit creep measure.

In this case the concept of creep characteristic (creep coefficient) $\varphi(t,t_0)$ is also used:

$$
C(t,t_0) = \frac{\varphi(t,t_0)}{E(t_0)}.
$$

An important requirement for the calculation models of concrete creep is the justified determination of the limits of the use of linear or non-linear dependencies of creep deformations on the stress values arising in the structure. EC2 permits the use of linear dependencies on $\sigma_c$ stresses arising in structures, based on (2), subject to

$$
\sigma_c \leq 0.45 \cdot f_{cm}(t_0),
$$

where $f_{cm}$ is the average compressive strength of concrete. And the PN condition in this case is more stringent:

$$
\sigma_c \leq 0.40 \cdot f_{cm}(t_0).
$$

The NIIZhB Recommendations establish the threshold for eligibility of a linear creep model by the condition

$$
\sigma_c \leq k_\sigma \cdot R_b,
$$

where the coefficient $k_\sigma$ varies in the range of $0.55 \div 1.00$ depending on the class of concrete (strength) and class of buildings (liability); $R_b$ is the prism strength of concrete.

EC2 recommends using the nonlinear exponential proportion of the change in the creep coefficient if the limit (5) is exceeded:

$$
\varphi_{nl}(t,t_0) = \varphi(t,t_0) \cdot e^{[1.5(\sigma_c - 0.45)]} \quad \text{for} \quad k_\sigma \geq 0.45,
$$

where $k_\sigma$ is the ratio of the compressive stress $\sigma_c$ and the average compressive strength of concrete $f_{cm}$ at the moment of loading, and the PN document here adds the condition:

$$
\varphi_{nl}(t,t_0) = \varphi(t,t_0) \cdot e^{[1.5(k_\sigma - 0.4)]} \quad \text{for} \quad 0.4 < k_\sigma \leq 0.6.
$$

The NIIZhB recommendations in case of non-observance of condition (6) establish the non-linear dependence of the change in the limiting creep measure:

$$
C(\infty,t_0) = C(\infty,t_0) \cdot \left[1 + v_c \cdot \eta(t_0) \right]^m,
$$

where $m$ is the exponent and $\eta(t_0)$ is the age factor.
wherein the values of $v_c$ and $m_c$ depending on the concrete class are taken from the table from [8], and the parameter $\eta(t_0)$ is calculated by the formula $\eta(t_0) = 0.78 \cdot k_e$, where $k_e = \frac{\sigma_b(t_0)}{R_{bn}(t_0)}$, $0.55 \leq k_e \leq 1.0$ - relative level of stress, $\sigma_b(t_0)$ is the compressive stress acting in concrete at the time of loading $t_0$, $R_{bn}(t_0)$ is the strength of concrete at the time of loading $t_0$.

3. Results

3.1. Calculative rheological model of concrete

The rheological model based on the generalized model of a viscoelastic material – the generalized Maxwell model (Wiechert model) [8, 9] was chosen to study and apply to the modeling of concrete creep. Elements of the mechanical interpretation of this model are $n$ springs ($i = 1 \div n$) with “temporary” elastic moduli $G_i$ and $n$ dampers with viscosity coefficients $\eta_i$ connected in series with each other in pairs, as well as an elastic element with “long-term” stiffness $G_\infty$ (figure 1a).

![Figure 1. Mechanical representation of the Maxwell model](image)

Fig. 1. Mechanical representation of the Maxwell model

\textit{a)} the generalized model; \textit{b)} the elementary model.

The basis of the generalized Maxwell model is the elementary model of the viscoelastic Maxwell material. According to this model, the material resistance is proportional to the speed of its strain $\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_v$, developing from the rate of elastic strain $\dot{\varepsilon}_e$ and the rate of viscous strain $\dot{\varepsilon}_v$. In turn, the elastic strain of the material is interpreted by the strain of the elastic spring with the stiffness $E$ (figure 1b)

$$\varepsilon_e = \frac{\sigma_v}{E},$$

from where $\dot{\varepsilon}_e = \frac{\sigma_e}{E}$,

$\varepsilon_e = \frac{\sigma_v}{E}$, and the rate of viscous strain in this model is represented by the properties of a damper with a coefficient of viscosity $\eta$:

$$\dot{\varepsilon}_v = \frac{\sigma_v}{\eta}.$$  \hspace{1cm} (12)

The serial connection of the spring and damper in the Maxwell model means that the stresses $\sigma$ in both its elements are the same, i.e. $\sigma = \sigma_e = \sigma_v$, but the strains in the elastic and viscous elements are different, i.e. $\varepsilon_e \neq \varepsilon_v$ ($\varepsilon = \varepsilon_e + \varepsilon_v$). Thus, according to (10) - (12), the strain rate of the material, represented by the Maxwell elementary model, will vary in proportion to time $t$

$$\dot{\varepsilon}(t) = \frac{\sigma(t)}{\eta} + \frac{\dot{\sigma}(t)}{E},$$ \hspace{1cm} (10a)

herewith at a constant stress $\sigma_0$ the strain will increase linearly (figure 2a).
The interdependencies of stresses and strains in the Maxwell elementary model

\begin{align*}
\sigma(t) &= \sigma_0 \cdot e^{-\frac{t}{\tau}} = E \cdot \varepsilon_0 \cdot e^{-\frac{t}{\tau}}, \\
\varepsilon(t) &= \varepsilon_0 + \sum_{i=1}^{n} E_i \cdot \varepsilon_0 \cdot e^{-\frac{t}{\tau_i}} = \varepsilon_0 \cdot \left[ E_x + \sum_{i=1}^{n} E_i \cdot e^{-\frac{t}{\tau_i}} \right],
\end{align*}

(13)

In turn, the solution (10a) with respect to \( \sigma(t) \):

\begin{align*}
\sigma(t) &= \sigma_0 \cdot e^{-\frac{t}{\tau}} = E \cdot \varepsilon_0 \cdot e^{-\frac{t}{\tau}}, \\
\varepsilon(t) &= \varepsilon_0 + \sum_{i=1}^{n} E_i \cdot \varepsilon_0 \cdot e^{-\frac{t}{\tau_i}} = \varepsilon_0 \cdot \left[ E_x + \sum_{i=1}^{n} E_i \cdot e^{-\frac{t}{\tau_i}} \right],
\end{align*}

(14)

where \( \tau_i = \frac{n_i}{E_i} \) is the relaxation time of the \( i \)-th damper (\( i = 1 \div n \)) in the generalized Maxwell model.

Thus, in the generalized Maxwell model, the viscoelasticity of a material is characterized by the generalized relaxation modulus depending on time (the so-called relaxation function):

\begin{align*}
E_R(t) &= \frac{\sigma(t)}{\varepsilon_0} = E_x + \sum_{i=1}^{n} E_i \cdot e^{-\frac{t}{\tau_i}},
\end{align*}

(15)

3.2. Computational implementation of the creep model

The generalized Maxwell model of viscoelastic properties of a material is implemented in the SIMULIA Abaqus software environment with the “viscoelastic” option [13]. Here it is represented by two relaxation functions, having the form of an exponential Prony series. This is the shear relaxation function:

\begin{align*}
G(t) &= G_x + \sum_{i=1}^{n} G_i \cdot e^{-\frac{t}{\tau_i}},
\end{align*}

(16)
where \( G = \frac{E}{2 \cdot (1 + \nu_0)} \) is the modulus of elasticity in shear, \( \nu_0 \) is the Poisson coefficient, and also the function of bulk relaxation:

\[
K(t) = K_\infty + \sum_{i=1}^{n} K_i \cdot e^{-\left(\frac{t}{\tau_i}\right)}, \quad (17)
\]

where \( K = \frac{E}{3 \cdot (1 - 2 \cdot \nu_0)} \) is the coefficient of volume stiffness.

In the SIMULIA Abaqus program, the parameters of the “viscoelastic” material model are set by the dimensionless shear relaxation modulus:

\[
g_R(t) = \frac{G(t)}{G_0} = 1 - \sum_{i=1}^{n} g_i \left(1 - e^{-\left(\frac{t}{\tau_i}\right)}\right), \quad (16a)
\]

where \( G_0 = E_0/(2 \cdot (1 + \nu_0)) \) is the instantaneous modulus of elasticity in shear, as well as the dimensionless modulus of bulk relaxation:

\[
k_R(t) = \frac{K(t)}{K_0} = 1 - \sum_{i=1}^{n} k_i \left(1 - e^{-\left(\frac{t}{\tau_i}\right)}\right), \quad (17a)
\]

where \( K_0 = E_0/(1 - 2 \cdot \nu_0) \) is the coefficient of instantaneous volume stiffness.

Obvious complexity here can be the assignment of values of the parameters of the material \( G_0, K_0, \tau_0 \), which can be obtained by static and dynamic tests of the material for creep and relaxation. However, under the condition of relatively small changes in stresses over time in the simulated structure, you can use the simple case of specifying the modules \( g_R(t) \) (16a) and \( k_R(t) \) (17a) with the introduction of the concept of effective modulus of elasticity [6], [8]:

\[
E_{\text{eff}} = \frac{E_0}{1 + \phi(t, t_0)} . \quad (18)
\]

Then the expression of the dimensionless shear relaxation modulus is considerably simplified:

\[
g_r(t) = \frac{g_R(t)}{G_0} = \frac{2 \cdot (1 + \nu_0) \cdot (1 + \left(t, t_0\right))}{E_0} \cdot \frac{E_0}{2(1 + \nu_0)} , \quad \text {or} \quad g_r(t) = \frac{1}{1 + \phi(t, t_0)}, \quad (16b)
\]

where \( \phi(t, t_0) \) is the creep coefficient of the material (2). In this case, the values of the time-varying coefficient \( \phi(t, t_0) \) in the computational model of creep can be set both on the basis of the material test results, and in accordance with the regulatory recommendations [6] - [8] and others. Similarly, the view of the dimensionless bulk relaxation modulus is simplified:

\[
k_r(t) = \frac{k_R(t)}{K_0} = \frac{1}{1 + \phi(t, t_0)} \quad (17b)
\]

4. Discussion

4.1. Verification of the computational creep model

To test the “viscoelastic” model, we used the results of experimental studies of two groups of concrete samples [9, 10] (figure 3).

Samples of the 1-st group were loaded (\( \sigma_1 = 10.67 \text{ [MPa]} \)) at the age of \( t_0 = 28 \text{ days} \), having on this day the modulus of elasticity \( E_{i(28)} = 36200 \text{ [MPa]} \). Samples of the 2-nd group were loaded (\( \sigma_0 = 2.47 \text{ [MPa]} \)) 1 day after their manufacture (\( t_0 = 1 \)), having the elastic modulus \( E_{i(1)} = 22201 \text{ [MPa]} \).
When specifying the dimensionless relaxation moduli of the material using formulas (16b) and (17b) in the “viscoelastic” model, the concrete creep factor $\varphi(t, t_0)$ was calculated based on the experimental results. The figures 4 and 5 show the results of a comparison of the dependences of the creep strain in the samples obtained in the experiment and in numerical simulation using the “viscoelastic” model.

Testing the performance of the “viscoelastic” model with setting the values of the dimensionless moduli of material relaxation using formulas (16b) and (17b) with an abrupt change in load value gave quite satisfactory results (figure 6).
Conclusion

Application of the computational creep model for calculating the shallow reinforced concrete structure

Justified consideration of the material creep when calculating characteristic reinforced concrete structures, such as, for example, flat arches and shells, can reveal a significant change in their stress-strain state over a long time. So when studying the destruction causes of reinforced concrete pavement of the Transvaal-Park sports and entertainment complex (SEC) in 2004 in Moscow at the age of 2 years [11, 12], the software package SIMULIA Abaqus was also used. The coating shell had the shape of a sector with the radius of 74.87 meters (figure 7).

![Figure 7. The deformed calculation model of the Transvaal-Park SEC coating structure.](image)

In the present work, when calculating this construction, the "viscoelastic" concrete creep model calibrated in accordance with the requirements of [6] was implemented. Calculation taking into account the creep of concrete showed a significant change in the stress-strain state of the structure over time, caused by a decrease in the curvature of the reinforced concrete shell of the coating. So Von Mises stresses

$$\sigma_{\text{Mises}} = \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2}{2} + 3 \cdot (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} }$$

(19)
in the middle steel support column of the Transvaal-Park SEC coating construction reached the elastic limit of the used steel $R_e = 245$[MPa] at the age of $t = 2$ years (figure 8).

Figure 8. Stresses in the middle support of the Transvaal-Park SEC at the age of $t = 2$ years.

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