Simulation of the strain diagram of a slag-concrete element subject to bending

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Abstract. In the modern world of information technology personal computers considerably dominate our everyday life. All real, field experiments and tryouts replace computer-aided simulations as they often save a lot of time. It is much easier to make the multiple calculations related inter alia to reinforced concrete structures using strain diagrams for the concrete and re-bars. This is the method ensuring the results similar to the field tests and the only challenge is that many equation parameters have to be reduced. Therefore, the objective of the work is to propose a simplified mathematical model for describing the strain diagram of a bended slag-concrete element based on air-cooled slag (ACS) and the aggregate made of sieves (0...5 mm fraction) from crashed ACS originating from NLMK’s Blast Furnace Operations, while implementation of the model helps to avoid complicated equilibrium experiments. As a result of the research and mathematical simulation of the bended element diagram a model capable of reflecting the sample behavior at any loading stage using maximum load and initial modulus of elasticity was proposed.

1. Introduction

Due to intensive development of IT-equipment we face a problem of scheming an automated method for computation and design of building structures and creating such mechanical models of concrete and reinforced concrete which are general enough to approximate the state-of-the art strength models and theories of deformable solid body mechanics and closely related thereto [1].

No development of modern enhanced computation methods for reinforced concrete structures in terms of strength and far less deformability is feasible without considering nonlinear properties of concretes: heavy-weight high-strength concretes [2], light-weight concretes with artificial porous aggregates [3] as well as concretes using by-products of different industries and local aggregates [4,5].

It is noted that structural damage and deformability is strongly impacted by the features of the aggregate. For dense (heavy-weight) concretes four compression failure stages are specified [2]: 1) initial loading stage having intensity up to about (0.1-0.15) Rₘ with characteristic development of the concrete permanent deformation, low modulus of elasticity; 2) the stage of the sample volume compaction and reduction when loading with intensity up to about (0.15-0.3)Rₘ; 3) the stage of the
material decompaction and disintegration (the first reference point) - characterizes the threshold of the concrete elastic behavior at the moment when first micro cracks are generated in the concrete structure when loading with intensity up to about \((0.3-0.6) R_b\); 4) extreme stage of the concrete equilibrium condition effecting the cracks (beyond the boundaries of which the proportion of stresses and deformation is upset - characterizes the threshold (the second reference point) when loading with intensity up to about \((0.5-0.85) R_b\).

Experiments show [3] that for light-weight concretes not four but three areas of stressed conditions are revealed. Such statement is supported by higher deformability of porous vs. compact aggregate by its ideal bond with mortar fraction and shrinkage-induced 3D reduction. Hence, for light-weight concretes based on artificial porous aggregates [3], as well as concretes based on air-cooled slag (ACS) and the aggregate made of sieves (0...5 mm fraction) from crashed ACS originated from NLMK’s Blast Furnace Operations [5] the level fluctuation for the first parametrical point is possible within \((0.3-0.7) R_b\) (i.e. higher than for heavy-weight concretes), and for the second parametrical point this level comes up to \((0.1-1.0)R_b\) with concrete strength approximately identical.

When developing a model of the structure real behavior it is necessary to take into account an important feature of the experimental diagrams of the concrete compression [2], characterized by the fact that at the time of initial load application to the sample, which has never been loaded before, significant residual strain is revealed characterized by low values of the modulus of deformation. Normally, when testing concrete samples, these deformations are not noted while they reveal themselves during the sample centering for the next test. Nevertheless, these deformations impact the real behavior of the structure especially the one non identified statically.

For analytical description of the entire concrete deformation model the third reference point should be taken into account (the threshold condition of the concrete) beyond which significant crack formation and concrete failure take place. The second and third reference points in the ultimate design behavior are supposed to be determined simultaneously as the meeting point of the respectively rising and down-coming curves of the real diagram with horizontal section [6].

The stress-deformation method for computation of reinforced concrete structures using strain diagrams of concrete and re-bars has recently attained high priority as ensuring high level of reliability in assessing the strength and deformational properties [7-14].

Along with deformational models the engineering practice has a long experience of using conventional computation method based on the ultimate behavior approach.

Research shows that for reinforced concrete elements with regular shapes (rectangular, T-beams, I-beams) the strength computations can be made using simplified diagrams of the material behavior [16-18].

Based on comparative analysis [19] the conclusion has been made that the strength calculations obtained using simplified stress-deformation model and those according to SP 63.13330.2012 «Concrete and reinforced concrete structures», differ greatly, though, the curve and respectively the bends in the middle of the span calculated using the stress-deformation model is a sequence higher than according to code of practice (SP), therefore, it is necessary to limit the norms for limit and threshold values of stress deformation in the diagrams of the concrete deformation induced by axial compression and the re-bar deformation induced by axial tension.

2. Materials and methods

Samples of slag-concrete, bending test.

In order to fully reflect the real physical processes of damage accumulation in the material during mechanical tests it is recommended to use fully stable strain diagram (FSSD) obtained during bending test [20].

It is known that during bending test using the samples of elastoplastic material (e.g. a slag-concrete element) inelastic bending can sometimes take place, i.e. the material fails to follow Hooke Law. The simplest case of inelastic bending can be plastic hinge taking place with elastic-ideally-plastic material. Such material follows Hooke Law until the tension value reaches the yield limit and then
plastic deformations under constant tension [21] start developing.

As an example, we consider the behavior of a rectangular-shape freely supported beam made of elastic-ideally-plastic material under concentrated force $P$ applied right in the middle (Figure 1.a).

![Figure 1. Plastic zone and plastic hinge: a) appearance of plastic zone and generation of plastic hinge; b) bending-moment diagram; c) curve diagram; d) general view of three-point bending test of the sample; e) pre-fracture of the sample during three-point bending](image)

Bending-moment diagram represents a triangle and max. momentum $M_{\text{max}}$ equals to $PL/4$ (Figure 1,b). When max. momentum value is higher than $M_t = PL/6 = 0.67M_{\text{max}}$, but less than ultimate momentum $M_{\text{ult}} (M_{\text{ult}} = M_{\text{max}} = P_{\text{max}}L/4)$, then limited plastic yield will take place in the central part of the beam, while the plastic zone length for a rectangular shape will be $L_r = L/3$ (Figure 1,a).

Figure 1,c shows the curve distribution $(1/\rho)$, depending on the bending moment $M$, modulus of elasticity $E$ and second area moment $I$, and equals to $M/(EI)$. Bends in the middle of the span were measured using dial test indicator accurate to 0.001 mm (Figure 1,d). The curve grows following the linear law in the section from the beam end to the beginning of the plastic zone, where the curve equals to $(1/\rho)_t$. Then the curve growth rate increases and in the middle of the beam the maximum value $(1/\rho)_{\text{max}}$ is obtained. The maximum curve value stays final until elastic zone is maintained in the middle of the beam. When the load increases even more and the value of maximum bending moment approaches the ultimate moment value $M_{\text{ult}}$, the plastic zones in the middle of the beam start quickly developing inside towards the neutral axis. Finally, when $M_{\text{max}}$ reaches $M_{\text{ult}}$ value, the cross section in the middle of the beam completely turns into plastic (Figure 1). The curve in the middle of the beam becomes extremely high, the bend in the middle of the span quickly increases which is measured during the experiments using a dial test indicator accurate to 0.001 mm (Figure 1,d) and unlimited plastic yield appears. The maximum momentum value cannot grow any more while the load reaches the peak value. The beam is weakened by extreme turns appearing in the middle cross section while the two parts of the beam remain relatively rigid. Thus, the beam behaves like two rigid rods, connected by a plastic hinge allowing these two rods rotate against each other influenced by constant momentum $M_{\text{ult}}$ (Figure 1,e).

Figure 1.e shows the curve distribution $(1/\rho)$, depending on the bending moment $M$, modulus of elasticity $E$ and second area moment $I$, and equals to $M/(EI)$. Bends in the middle of the span were measured using dial test indicator accurate to 0.001 mm (Figure 1,d). The curve grows following the linear law in the section from the beam end to the beginning of the plastic zone, where the curve equals to $(1/\rho)_t$. Then the curve growth rate increases and in the middle of the beam the maximum value $(1/\rho)_{\text{max}}$ is obtained. The maximum curve value stays final until elastic zone is maintained in the middle of the beam. When the load increases even more and the value of maximum bending moment approaches the ultimate moment value $M_{\text{ult}}$, the plastic zones in the middle of the beam start quickly developing inside towards the neutral axis. Finally, when $M_{\text{max}}$ reaches $M_{\text{ult}}$ value, the cross section in the middle of the beam completely turns into plastic (Figure 1). The curve in the middle of the beam becomes extremely high, the bend in the middle of the span quickly increases which is measured during the experiments using a dial test indicator accurate to 0.001 mm (Figure 1,d) and unlimited plastic yield appears. The maximum momentum value cannot grow any more while the load reaches the peak value. The beam is weakened by extreme turns appearing in the middle cross section while the two parts of the beam remain relatively rigid. Thus, the beam behaves like two rigid rods, connected by a plastic hinge allowing these two rods rotate against each other influenced by constant momentum $M_{\text{ult}}$ (Figure 1,e).

Hence, in case of inelastic bend, it can be assumed that the material has followed Hooke Law at the momentum value of $M = 0.9 M_t = 0.6P_{\text{max}}L/4 = 0.6M_{\text{max}}$.

The same assumption can be made for the trial beams used for four-point-bending test (Figure 2).
Figure 2. Four-point-bending test of a beam: a) test general view; b) load application chart

The value of the initial modulus of elasticity can be obtained from the compression test or by measuring US speed in the material, or directly measured during bending [18,22,23].

Determine the value of the initial modulus of elasticity during bending at $M = 0.6M_{\text{max}}$ and $f_y = 0.6f_{\text{max}}$. It is known that the bend in the middle of the span at the elastic stage of the beam behavior equals to [21]:

- for three-point bending
- for four-point bending

Then from formulas (1) and (2) determine the value of the initial modulus of elasticity:

- for three-point bending
- for four-point bending

In formulas (3) and (4) $k_M = L^2/I_{\text{single yield during bending, mm}^2}$. During computation of reinforced concrete structures by stress deformations based on non-linear deformation model (NLDM) we should use the modulus of deformation (transversal or tangential) which, as opposed to the initial modulus of elasticity ($E_b$ is a constant value), changes with the increase of the load value and exposure time, and tends to zero when they decrease while deformations achieve their extreme values [1,6,7,13,18].

With multiple dependencies proposed the problem here is to minimize the equation parameters.

Analyzing behavior of the real structures and models made of heavy-weight and light-weight concrete [2,3], as well as structures based on slag-pumice-concrete [5,18,23], in order to make mathematical description of the dependency between the applied load ($M$) and the bend ($f$) during bending we assume the following parameters as characteristic parameters of the diagram “$M – f$” (Figure 3): angle $\alpha$ of the rising curve of the diagram to abscissa axis ($\tan \alpha = 0.6M/f$), point 1 at the end of the straight-line portion of the rising curve, point 2 the peak of the deformation curve, point 3 bends of the down-coming curve of the diagram and point 4 the sample defragmentation.

Analysis of the common strain diagrams for concrete and re-bars [1,6-14,18] provides for describing the diagram using the following expression:

where $A$, $B$ and $C$ – parameters of the deformation curve

Parameters $A$, $B$ and $C$ of the equation (5) shall be found based on physical and geometrical assumptions [18], as well as on the obtained test data.
Figure 3. Parametrical points of the “bending moment - bend” diagram
a) assumption diagram;  b) real diagram (first application of the load)
Mstr – bending moment (structure); Msmp – bending moment (sample)

To validate the simulation of the strain diagram of a slag-concrete element subject to bending experimental strain diagrams [18,24] were used.

Compound №1 of B20 class concrete, based on ACS (sample BSh-1) from «Zhelezobeton Plant» (Lipetsk) for production of self-supporting structures (in particular, bar-type lintels of 1.038.1-1 series) and experimental compound №2 of fine-grain concrete matrix (sample OA-0-0-6) are shown in Table 1.

Table 1. Compounds of concrete based on ACS and sieves from crashing

| No. Item | Component | Component consumption, kg/m$^3$ |
|---------|-----------|----------------------------------|
|         |           | Compound №1 | Compound №2 |
| 1       | Portland-blastfurnace cement M400 | 328 | 531 |
| 2       | Air-cooled slag (5-20 mm fraction) | 1030 | - |
| 3       | Sieves from crashed ACS (0-5 mm fraction) | - | 1423 |
| 4       | Sand (pit-run) | 700 | - |
| 5       | “Universal P-2” additive | 19 | 30.6 |
| 6       | Water | 170 | 224 |

Table 2 shows geometrical characteristics and main physical and mechanical properties of concrete of the beam samples at the time of testing.

Table 2. Sample characteristics

| Sample grade (age of concrete) | Loading chart | Geometrical characteristics | Initial modulus of elasticity $E_b$, MPa$^{10^3}$ |
|-------------------------------|---------------|-----------------------------|-----------------------------------------------|
| OA-0-0-6 (714 days)           | Figure 1      | $b$, mm; $h$, mm; $L$, mm; $I_b$, mm$^4$; $k_{M_r}$, mm$^2$ | During compression; During bending acc. to formulas (3) and (4) |
| BSh-1 (912 days)              | Figure 2      | 40; 40; 150; 213333.33; 0.105 | 33.6; 34.54 |

Figure 4 shows experimental strain diagrams of the samples. Parameters $A$, $B$ and $C$ of the equation (5), found based on physical and geometrical assumptions [18], are shown in Table 3. Parametrical points of the diagram (Figure 2) based on the obtained test data are given in Table 4.

Table 3. Stress deformation values

| Sample | $A$, N/mm$^{-1}$ | $B$ | $C$, l/mm$^{-1}$ |
|--------|------------------|-----|-----------------|
| OA-0-0-6 | 1455422.2 | 0.7066 | -7.8508 |
| BSh-1     | 71506-90.9    | 0.3742 | -0.6803 |
Table 4. Values of the strain diagram parametrical points

| Parametrical points   | Parametrical points function | OA-0-0-6 (experimental) | OA-0-0-6 acc.to formula (5) | BSh-1 (experimental) | BSh-1 acc.to formula (5) |
|-----------------------|-----------------------------|-------------------------|-----------------------------|----------------------|--------------------------|
|                       |                             | 0.6 M₀, Nmm             | 0.6f₁, mm                  | 0.6f₂, mm            | 1.5f₂, mm                |
| Point «1»             |                             | 120300                  | 130988                      | 108863               | 83400                    |
| Point «2»             |                             | 0.05                    | 0.09                        | 0.14                 | 0.18                     |
| Point «3»             |                             | 118376                  | 130998                      | 120888               | 1055471                  |
| Point «4»             |                             |                        |                             |                      |                          |

Figure 4. Experimental strain diagram

3. Results

As is seen from Table 4 and Figure 4 the down-coming area shows deviation between the experimental and theoretical (5) values of the applied load, therefore, the formula (5) should be adjusted taking into account the dynamic movement of the main crack. It is also noted that for beam BSh-1 the formula (5) does not take into account the real behavior of the structure at the initial loading stage (M=0.1Mₘₙₙ).

In order to eliminate the revealed deviations and to simplify the mathematical model for describing the material strain diagram during bending via single parameter M₀ (max. load value) providing that the modulus of elasticity is known, we can express the diagram description as follows:

The method to identify A and B parameters is given in [18].

Below the expression, please, find «M-f» dependencies:
Using formulas (7) - (12) and taking into account the experimental values in Figure 5 a fully stable diagram “М – f” can be plotted.

![Figure 5](image)

**Figure 5.** Strain diagram “M – f”, plotted acc. to formulas (7)-(12), vs. experimental values

4. Conclusion

The proposed mathematical model for describing the fully stable strain diagram of a slag-concrete element subject to bending using a single experimental parameter (max. load $M_c$) and known initial modulus of elasticity of the material allows to avoid complicated equilibrium experiments related to constant recording of the bends at each loading stage and is capable of simulating the material behavior at each loading stage up to the material failure.

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