NEUTRINO OSCILLATIONS AND THE EARLY UNIVERSE

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Abstract. The observational and theoretical status of neutrino oscillations in connection with solar and atmospheric neutrino anomalies is presented in brief. The effect of neutrino oscillations on the early Universe evolution is discussed in detail. A short review is given of the standard Big Bang Nucleosynthesis and the influence of resonant and nonresonant neutrino oscillations on active neutrinos and on primordial synthesis of He-4. BBN cosmological constraints on neutrino oscillation parameters are discussed.

1. INTRODUCTION

Neutrino - a neutral weakly interacting particle, is of extreme interest for Physics and Astrophysics. It is a key to the investigation of the weak interactions and the physics beyond the standard electroweak model. On the other hand, being a very weakly interacting particle, and hence having a uniquely great penetrating capability, neutrinos carry precious information for the astrophysical processes in the most dense regions of the star cores and from the very early stages of the Universe evolution. Therefore, revealing neutrino characteristics is of great importance.

The contemporary particle physics theory neither requires nor forbids a nonzero neutrino mass. In the standard model of particle physics neutrinos are assumed massless. In the more general case of non-zero neutrino masses, the weak neutrino eigenstates may be a linear combination of the mass eigenstates, which means that transitions between neutrinos with different types (flavours), the so called neutrino oscillations are possible. Neutrino oscillations and their role in resolving the solar neutrino puzzle were first proposed by B. Pontecorvo (see Pontecorvo, 1958) and after 45 years they continue to be the theme of leading experimental and theoretical research.

In recent years positive indications for neutrino oscillations were obtained at the greatest neutrino experiments (evidence for solar neutrino oscillations: Homestake, Kamiokande, SuperKamioKa, Gallex, SAGE, SNO; evidence for atmospheric neutrino oscillations: Super-KamioKa, Macro, Soudan 2, IMB; evidence for neutrino oscillations at terrestrial experiments: LSND, KamLAND, K2K) (see refs. Gonzalez-Garcia & Nir, 2003; Smirnov, 2003; Giunti & Laveder, 2003). Each of these neutrino anomalies, namely the solar neutrino problem, atmospheric neutrino anomaly and the positive results of terrestrial LSND and KamLAND experiments may be resolved by

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the phenomenon of neutrino oscillations. These results have a great resonance as far as any experimental evidence for neutrino masses or mixing is a signal of new physics (NP) - physics beyond the standard model of electroweak interactions.

On the other hand, neutrino oscillations affect early Universe evolution by affecting expansion rate, neutrino densities and neutrino energy spectrum, neutrino - antineutrino asymmetry, thus influencing the neutrino involved processes, as for example cosmological nucleosynthesis, structure formation, etc. Cosmological nucleosynthesis, traditionally called Big Bang Nucleosynthesis (BBN) explains very successfully the data on the primordial abundances of the light elements D, He-3, He-4 and Li-7, and is traditionally used as a probe of the conditions of the early Universe, due to the high accuracy of the theoretically predicted abundances of light elements and to the good accuracy of their primordial values inferred from observations. Hence, BBN is a powerful probe for NP, like neutrino oscillations.

From BBN considerations most stringent constraints on neutrino oscillations parameters are obtained. In particular, LMA and LOW active-sterile solar oscillation solutions and atmospheric active-sterile solutions were excluded many years before the global analysis of experimental neutrino data pointed to the preference of flavour oscillations for solving these neutrino anomalies.

In the following we will present a brief introductory review of the solar and atmospheric neutrino anomalies and then discuss in more detail the role of neutrino oscillations in the early Universe and the cosmological constraints on oscillation parameters, following from BBN.

1. 1. NEUTRINO OSCILLATIONS

The basic idea of neutrino oscillations is that left-handed mass eigenstates $\nu_i$ are distinct from the left-handed flavour eigenstates $\nu_f$:

$$\nu_i = U_{if} \nu_f \ (f = e, \mu, \tau).$$

Then in the simple two-neutrino oscillation case in vacuum, the probability to find after a time interval $t$ a given neutrino type in an initially homogeneous neutrino beam of the same type is: $P_{ff} = 1 - \sin^2 \frac{2\vartheta}{2} \sin^2 \left(\frac{\delta m^2 t}{4E}\right)$, where $\delta m^2$ - the neutrino mass difference and $\vartheta$ - the oscillations mixing angle are the oscillation parameters, $E$ is the neutrino energy. I.e. the flavour composition changes with time.

The medium distinguishes between different neutrino types due to different interactions (Wolfenstein, 1978; Mikheyev & Smirnov, 1985). This leads to different average potentials $V_f$ for different neutrino types.

$$V_f = Q \pm L$$

where $f = e, \mu, \tau$, $Q = -bET^4/(\delta m^2 M_W^2)$, $L = -aET^3 L^\alpha/(\delta m^2)$, $L^\alpha$ is given through the fermion asymmetries of the plasma, $a$ and $b$ are positive constants different for the different neutrino types, $-L$ corresponds to the neutrino and $+L$ to the antineutrino case. The sterile neutrino does not feel the medium, hence $V_s = 0$.

The effects of the medium can be hidden in $\delta m^2$ and $\vartheta$. Namely, the matter mixing angle in the adiabatic case is expressed through the vacuum oscillation parameters and the characteristics of the medium, like its density and temperature. For the early Universe the following relation holds:

$$\sin^2 \vartheta_m = \sin^2 \vartheta / [\sin^2 \vartheta + (Q \mp L - \cos 2\vartheta)^2],$$
Neutrino Oscillations and the Early Universe

Although in general the medium suppresses oscillations by decreasing their amplitude, there also exists a possibility of enhanced oscillation transfer in case a resonant condition between the parameters of the medium and the oscillation parameters holds:

\[ Q \mp L = \cos 2\vartheta. \]

Then the mixing in matter becomes maximal, independently of the value of the vacuum mixing angle, i.e. resonant transfer takes place.

At high temperature of the early Universe for a lepton asymmetry of the order of the baryon one, \( Q > L \). So for \( \delta m^2 < 0 \) resonance is possible both for neutrino and antineutrino. At low \( T \), however, \( L > Q \), and as can be seen from the resonant condition, if \( \delta m^2 > 0 \) a resonance in the neutrino ensemble can take place, while for \( \delta m^2 < 0 \) the resonance is possible only for the antineutrinos.

Both the nonresonant and resonant oscillation cases are interesting from a cosmological point of view and from the viewpoint of the discussed neutrino anomalies.

2. NEUTRINO ANOMALIES AND NEUTRINO OSCILLATION EXPERIMENTS

2.1. SOLAR NEUTRINO DEFICIT

According to the contemporary astrophysical understanding the Sun is a Main Sequence star at the stage of hydrogen burning. It produces an intense flux of electron neutrinos as a result of its nuclear reactions generating the solar energy. Due to its weak interaction with matter, the solar neutrino reaching the Earth comes from the very deep solar core and carries valuable information about stellar structure and its evolution. Hence, the detection of the neutrino from the Sun has been recognized as a task of great importance as early as the 50ies - when Davis started a radiochemical experiment, aiming to detect neutrinos from the Sun, in the golden mine of Homestake. The solar neutrinos also present the unique possibility for investigation of the neutrino properties like neutrino mass and mixing, because the Sun is at a very large distance from the Earth, and also because the solar density varies strongly from the center to the surface and thus offers interesting conditions for the penetration of neutrino through layers with different density and thickness.

Since the first attempts to measure solar neutrinos, there has been different types of solar neutrino experiments, using Cl, Ga and H\(_2\)O and D\(_2\)O as targets for measuring electron neutrino from the Sun. The detected fluxes of solar neutrinos at these solar experiments (using different detection methods\(^2\) and sensitive to different energy ranges) are in qualitative agreement with the assumption that Sun burns due to nuclear reactions in its core. However, all the data of the solar neutrino experiments point to a considerably lower neutrino flux than the expected one in the standard Solar Model. Furthermore, the suppression is different in various experiments, sensitive to different energy range. This problem is called solar neutrino anomaly.

Despite the continuous improvements of the Solar Model and the predicted neutrino flux in the last 40 years, the discrepancies between the observations and the predictions of the model persist. Depending on the energy the measured fluxes consist 0.3 to 0.6 of the predicted values. The recent measurement of neutral currents

\(^2\)There exist radiochemical experiments like GALLEX, SAGE and Homestake and electron experiments like Kamiokande and SuperKamioka.
and charged currents fluxes at SNO experiment (Ahmad et al., 2002) provide $\sim 5\sigma$ signal for neutrino flavour transitions that is not strongly dependent on the Solar Model. Hence, there remain less doubts about the Solar Model prediction capability.

Thus, in case we exclude the possibility that most of the leading solar neutrino experiments are wrong, the experimental data points more and more convincingly to the necessity of new neutrino physics.

Neutrino oscillations are capable to explain the observational data and its discrepancies with the predictions of the Solar Model: The electron neutrino, produced in the solar core, undergoes transformations into other flavours while penetrating through the Sun and the cosmic space till the terrestrial detectors of electron neutrinos. Hence, the registered electron neutrino flux is reduced in comparison with the flux produced in the Sun core.\(^3\)

There existed different types of solar neutrino oscillations solutions - Small Mixing Angle (SMA) and Large Mixing Angle (LMA), depending on the mixing angles at around $\delta m^2 \sim 10^{-5}$ eV\(^2\) and LOW and vacuum oscillation solutions corresponding to very small $\delta m^2 \leq 10^{-7}$ eV\(^2\) mass differences and maximum mixing. The present solar neutrino data definitely prefers flavour oscillation solutions to active-sterile ones (as was pointed first from cosmology considerations - see section 4), and in the light of the recent results of the terrestrial experiment KamLAND, the LMA solution is split into two sub-regions, and is the chosen one. The best fit point values of $\delta m^2$ are $(7.3 \pm 0.8) \times 10^{-5}$ eV\(^2\) and $\sin^2 2\theta \sim 0.315 \pm 0.035$. For more details see (Bahcall et al., 2003, Balantekin & Yuksel, 2003, Holanda & Smirnov, 2003; Fogli et al., 2003; Maltoni et al., 2003).

2.2. ATMOSPHERIC NEUTRINO ANOMALY

A continuous isotropic flux of cosmic rays, consisting of protons and heavy nuclei, is bombarding the Earth's atmosphere. As a result of its interactions with the atmospheric particles pions and kaons are produced, which decay and produce muon and electron neutrinos with a wide energy range. The theoretical prediction for the ratio of the muon to the electron flux is $r \sim (\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e) = 2$ for energies less than 1 GeV. Besides, identical up-coming and down-coming fluxes are expected due to the isotropy of the cosmic rays flux and due to the spherical symmetry of the Earth's atmosphere. Any deviation from these predictions is an indication for new neutrino physics.

The underground neutrino experiments SuperKamioka, Soudan 2 and Macro, as well as the earlier experiments IMB and Kamiokande, have measured ratio $r$ considerably lower than the expected one (see for example Fukuda, 1998). This discrepancy, the so called atmospheric neutrino anomaly is known already for more than 10 years. Besides a dependence of the muon neutrino deficit on the zenith angle and distortion of the energy spectrum is observed.

The experimental data can be explained in terms of neutrino oscillations, namely by the transition of the muon neutrino into another type. The latest data analysis indicates the $\nu_\mu \leftrightarrow \nu_\tau$ channel as the dominant one. The oscillations into sterile neutrino are disfavoured, because of the absence of suppression of oscillations by the medium, expected in the sterile case at high energies. The best fit oscillation

\(^3\)Except the flux measured by SNO, which detects all flavour neutrinos, not only electron ones.
Neutrino Oscillations and the Early Universe

parameters for the available data are nearly maximal mixing and $\delta m^2 \sim (2.6 \pm 0.4) \times 10^{-3}$ eV$^2$. For more detail see (Guinti, 2003 and references therein).

2.3. LABORATORY EXPERIMENTS

Besides these two astrophysical indications for neutrino oscillations and non-zero neutrino mass, there exist also laboratory experiments, the so-called terrestrial experiments LSND (Aguilar et al., 2001), K2K (Ahn et al., 2003) and KamLAND (Eguchi et al., 2003), which data have given an indication for oscillations, too.

The short baseline Los Alamos Liquid Scintillation Neutrino Detector (LSND) experiment has registered appearance of electron antineutrino in a flux of muon antineutrino. This anomaly might be interpreted as $\nu_\mu \leftrightarrow \nu_e$ oscillations with $\delta m^2 = O(1 \text{ eV}^2)$ and $\sin^2 2\theta = O(0.003)$.

In case LSND result is confirmed\(^4\) an addition of a light singlet neutrino (sterile neutrino $\nu_s$) is required, because three different mass differences, needed for the explanation of the solar, atmospheric and LSND anomaly require 4 different neutrino masses. This simple extension already has difficulties, because oscillations into purely sterile neutrinos do not fit neither the atmospheric nor solar neutrino data. Both 2+2 and 3+1 oscillation schemes have problems. Active-sterile oscillations are strongly restricted by cosmological considerations as well (for review on 2 oscillation constraints see Kirilova & Chizhov, 2001; and for 4 neutrino oscillation schemes see (Bilenky et al., 1998; Di Bari, 2002; Dolgov & Villante, 2003).

$K2K$, a long baseline neutrino experiment, has probed the $\delta m^2$ region explored by atmospheric neutrinos. It has measured muon neutrino deficit in a beam coming from KEK to Kamiokande. The distance is 250 km and $E \sim 1.3$ GeV. 56 events were observed instead of the expected $80 \pm 6$ in case without oscillations. A hint of energy spectrum distortion is also indicated by the analysis.

The results are consistent with SuperKamioka atmospheric data and confirm atmospheric neutrino solution.

KamLAND (Kamioka Liquid Scintillator Anti-Neutrino Detector) experiment explored with reactor neutrinos the region of oscillation parameters relevant for the solar neutrinos. It has measured electron antineutrino deficit in the flux of antineutrinos coming from reactors at $\sim 180$ km distance. In the context of two-flavour neutrino oscillations KamLAND results single out LMA solution, as the oscillation solution to the solar neutrino problem. The allowed previously LMA region is further reduced by its results (see Eguchi et al., 2003). So, the KamLAND result appears to confirm in a totally independent and completely terrestrial way that solar neutrino deficit is indeed due to neutrino oscillations, which was suspected in many solar neutrino experiments over the last 40 years.

KamLAND reactor and KEK accelerator experiments strongly contributed to the reduction of the allowed range of mass differences for solving the solar and atmospheric neutrino anomalies. Their results mark the beggning of the precision epoch in determinations of neutrino characteristics.

The neutrino experiments results confirm non-zero neutrino mass and mixing. Non-zero neutrino mass is also cosmologically welcome, as it may play the role of the hot dark matter component essential for the successful structure formation (see the

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\(^4\)The LSND result was not confirmed by KARMEN. It will be tested in future by the ongoing MiniBoom experiment at Fermilab (Bazarko, 2002).
next section). The standard model of particle physics (SM) $SU_c(3) \times SU_W(2) \times U_Y(1)$ does not predict non-zero neutrino mass and mixing. To explain the smallness of the neutrino mass differences new physics beyond SM is required. Hence, neutrino data gathered at neutrino experiments and the cosmological considerations concerning neutrino mass and mixing, discussed below, point the way towards this NP - hopefully the true unified theory of elementary particles.

3. NEUTRINOS IN THE EARLY UNIVERSE

After the photons of the microwave background radiation, neutrinos are the most abundant particles in the Universe. Hence, in case they have non-zero mass they may contribute considerably to the total energy density of the Universe. From the requirement that the neutrino density should not exceed the matter density, and assuming that the Universe is older than the Earth, an upper bound on the neutrino mass was derived – "Gerstein-Zeldovich" limit (see Gerstein and Zeldovich, 1966). The contemporary version of it reads:

$$\Sigma m_{\nu} \leq 94eV\Omega_m h^2 = 15eV,$$

where $\Omega_m$ is the matter density in terms of the critical density and $h$ is the dimensionless Hubble parameter. In deriving this limit $\Omega_m < 0.3$ and $h = 0.7$ is assumed according to contemporary astronomical data. Hence, the neutrino mass of any neutrino flavour should be less than about 5 eV.

Much stronger limits on the neutrino mass may be obtained accounting for the considerable role of neutrinos in other important cosmological processes, like the primordial nucleosynthesis and the formation of large scale structure of the Universe, the formation of the cosmic microwave background radiation, etc.

3.1. LARGE SCALE STRUCTURE AND NEUTRINO

The mass of the visible (radiating) matter in the Universe is at most 0.01 of the total mass deduced from its gravitational effect. The remaining 0.99 consists of the so-called Dark Matter of the Universe (DM). Only a negligible portion of this DM may be in the form of invisible baryons, i.e. 0.04 of the total mass, i.e. DM is mainly non-baryonic. Massive neutrinos with a mass of a few eV could naturally be the candidates for DM component in clusters of galaxies (Cowsik & McCleland, 1972).

On the other hand, according to the accepted contemporary theory, structures in the Universe are a result of gravitational instabilities of overdensity perturbations. These perturbations result from the initial microscopical perturbations generated at the inflationary stage, which have been inflated during the exponential expansion. Neutrinos cluster more efficiently in larger potential wells. In case they play the role of DM mass-to-light ratio should increase with scale. That behavior was not confirmed by observations. Moreover, to be in agreement with the sizes of the structures observed today, it was found necessary to speed up the growth of the perturbations, which is naturally achieved by the presence of non-relativistic DM at the epoch of perturbations growth, which should be the main DM component. So, massive neutrinos are not the main DM component in galaxies and clusters.

- Their present day number density is $n_\nu = n_\nu \sim 56 \text{ cm}^{-3}$ for each neutrino flavor.
Still, the precise analysis of the recent microwave background anisotropy data and structures data at large red-shifts points to the necessity of some admixture of hot DM which can be naturally provided by light neutrinos with $\Sigma m_{\nu f} < 2.2$ eV or $m_{\nu} < 0.73$ eV (see for example Fukugita et al., 2000; Elgarøy et al., 2002). Recent WMAP measurements (see Spergel et al., 2003) together with the LSS analysis improved the limit: $\Sigma m_{\nu f} < 0.69$ eV and hence, $m_{\nu} < 0.23$ eV. See also recent analysis of the CMB, LSS and X-ray galaxy cluster data (Allen et al., 2003), which give for the preferred non-zero neutrino mass the bounds: $\Sigma m_{\nu f} \sim 0.64$ eV or $m_{\nu} \sim 0.21$ eV per neutrino.

It is remarkable that such mass value is in accordance with the picture of oscillation models, which predict oscillations between nearly degenerate neutrinos, with negligibly different masses (in case of solar neutrino anomaly with mass difference $\sim 10^{-5}$ eV and in case of the atmospheric anomaly $\sim 0.001$ eV). Each of these degenerate neutrino types, in case they have masses of the order of $\sim 0.2$ eV, can successfully play the role of the hot DM.

For comparison the laboratory bound on electron neutrino mass from Tritium $\beta$-decay experiments is $m_{\nu} < 2.2$ eV. Given the small mass differences pointed from solar and atmospheric neutrino data, this bound applies to each neutrino eigenstate, i.e. $\Sigma m_{\nu f} < 6.6$ eV (Barger, 1998). So, the cosmological bounds on neutrino masses at present are more restrictive than the laboratory constraints. For recent review on laboratory measurements of neutrino masses and also their cosmological and astrophysical constraints see (Bilenky et al., 2003; Sarkar, 2003; Dolgov, 2002; Raffelt, 2002).

3.2. BBN AND NEUTRINO

One of the most exciting events in the early Universe is the primordial nucleosynthesis of the light elements. The idea for the production of elements through nuclear reactions in the hot plasma during the early stage of the Universe evolution belongs to George Gamov and was proposed and developed in the 1930s and 1940s (Gamow, 1935, 1942, 1946). In the following 70 years this idea has grown to an elegant and famous theory - theory of the cosmological nucleosynthesis (Big Bang Nucleosynthesis), explaining successfully the data on the abundances of D, He-3, He-4 and Li-7 (see for example Esposito et al., 2000, and references therein).

Recently BBN theory was improved: nuclear data have been reanalysed, all nuclear reactions rates involving $\sim 100$ processes were updated, new processes were included, estimates of the weak rates were improved. Also statistical uncertainty of observational determination of the light elements was improved. And finally, thanks to the precise determination of the baryon density in CMB anisotropy measurements, it was used as an input in BBN. The present status of BBN after the recent measurements of CMB anisotropies by WMAP experiment is presented in detail in (Cuoco et al., 2003; Cyburt et al., 2003; Coc et al., 2003, Steigman, 2003).

Based on the excellent agreement between CMB, BBN predictions and the observational data, today we believe that we know to a great precision the physical processes typical for the BBN epoch. Hence, BBN is a most powerful probe for new physics, like the physics predicting neutrino oscillations and non-zero neutrino mass.
According to the Standard BBN model (SBBN) the cosmological nucleosynthesis proceeds when the temperature of the plasma falls down to 1 MeV, when the weak processes, governing the neutron-proton transitions become comparable with the expansion rate. As a result the neutron-to-proton ratio freezes out at temperature below 0.7 MeV. This ratio enters in the following rapid nuclear reactions leading to the synthesis of D and the rest light elements formed in the first hundred seconds from the Big Bang.

Only at $T < 80$ keV, a temperature well below the $D$ binding energy $\sim 2.2$ MeV, the building of complex nuclei becomes possible, the first step being: $n + p = D + \gamma$. At higher $T$ deuterons were quickly photo-disociated because of the large photon-to-baryon ratio $\eta^{-1}$.

So, actually in SBBN, $\eta$ is the only parameter. Most sensitive to $\eta$ among the light elements is D, therefore it was considered till recently the best baryometer. The observational $\eta$ values preferred today are namely, the one obtained on the basis of measurements of D in high redshift QSO Absorption Line Systems of (Kirkman et al., 2003), namely $\eta \sim (6.1 \pm 0.5) \times 10^{-10}$ and $\eta$ obtained from the CMB anisotropy measurements $\eta = (6.1 \pm 0.25) \times 10^{-10}$ (Spergel et al., 2003). Present day CMB measurements of $\eta$ are considered tighter than the BBN one, so $\eta_{CMB}$ is used as an input for BBN calculations.

According to the standard BBN during the early hot and dense epoch of the Universe only D, $^3$He, $^4$He, $^7$Li were synthesized in considerable amounts. $^4$He is with the highest binding energy among the light nuclides, hence $D$ and $^3$He were rapidly burned into it. $^4$He is the most abundantly produced. The production of heavier elements was hindered by the rapid decrease of the Universe density with the cooling of the Universe, growing Coulomb barriers and the absence of a stable mass 5 nuclide. The latter were formed much later in stars.

He-3 and Li-7 have a complex post BBN evolution. They are both created and destroyed in stars, hence are unreliable as a cosmological probes. Besides both their theoretically calculated and observational values suffer from large uncertainties. D although having a clear post BBN chemical evolution, namely it is believed to have been only destroyed after BBN, as far as it has the lowest binding energy of the light nuclides, still has large theoretical and observational uncertainties - up to 10%: $D/H = (2.6 \pm 0.4) \times 10^{-5}$ (Kirkman et al., 2003). Besides, there is a significant dispersion among the derived D abundances at low metallicity $Z$, a fact suggesting either the existence of systematic errors or a revision of our concepts about D post BBN evolution.

On the contrary, $Y_p$, predicted by BBN, is calculated with great precision (see Lopez and Turner, 1999; Esposito et al., 2000, Cyburt et al., 2003, Cuoco et al., 2003, Coc et al., 2003). The theoretical uncertainty is less than 0.1% ($|\delta Y_p| < 0.0002$) within a wide range of values of the baryon-to-photon ratio $\eta$. The predicted He-4 value is in in relatively good accordance with the observational data for He-4 and is consistent with other light elements abundances. The contemporary helium values, inferred from astrophysical observational data, are 0.238–0.245 (Olive et al., 1997; Izotov & Thuan, 1998). And although there exist some tension between the two different measurements giving different He values, and also between the observed He values and the predicted ones, using $\eta$ indicated either by D measurements or by CMB, taking the central Helium value of the 2 measurements 0.238 and assuming the
systematic error of 0.005 reestablishes the agreement both between different helium measurements and also between observed and the predicted He-4 values. I.e. there is an accordance at 2σ level and the uncertainty is only around 2%.

Measurements of primordial helium from CMB data are possible. Hopefully future Planck CMB measurements will be capable to determine the helium mass fraction within δY ∼ 0.01 in a completely independent way (see Trotta and Hansen, 2003). Primordial helium value is also in a good accordance with the initial helium content, necessary for the successful star evolution modelling (Bono et al., 2002; Cassisi et al., 2003).

In conclusion, 4He is the most abundantly produced (∼ 24% by mass), most precisely measured (∼ 2% uncertainty) and most precisely calculated element (∼ 0.1% uncertainty). This fact and its relatively simple chemical evolution make it the preferred element for probing non-standard physics. Particularly, for the analysis of the oscillations effect on BBN, He-4 is the traditionally used element, as well. Therefore, we will discuss it in more detail below.

According to the standard cosmological nucleosynthesis (SBBN) He-4 primordial yield essentially depends on the freezing of the reactions interconverting neutrons and protons:

\[ \nu_e + n \leftrightarrow p + e^-, \quad e^+ + n \leftrightarrow p + \bar{\nu}_e, \]

which maintain the equilibrium of nucleons at high temperature (\( T > 1 \text{MeV} \)) \( n/p \sim \exp(-\Delta m/T) \), where \( \Delta m = m_n - m_p \), \( T \) is the temperature \( T = T_e = T_\nu \) prior to electro-positron annihilation. For the radiation dominated epoch \( \rho \sim \rho_\gamma + \rho_\nu + \rho_e = g_{\text{eff}} T^4 \). The \( n/p \) freeze-out occurs when in the process of expansion the rates of these weak processes \( \Gamma_w \sim G_F^2 E^2 N_\nu \) become comparable and less than the expansion rate \( H(t) = 8\pi G \rho / 3 \sim \sqrt{g_{\text{eff}}} T^2 \). As far as the temperature of freezing is \( T_f \sim g_{\text{eff}}^{1/6} \) the neutron-to-proton frozen ratio \( (n/p)_f \) is sensitive to \( g_{\text{eff}} \).

Further evolution of the neutron-to-proton ratio is due to the neutron decays that proceed until the effective synthesis of D begins. Almost all available neutrons are sucked into He-4. So, the primordially produced mass fraction of He-4, to a good approximation, is

\[ Y_p(\text{He-4}) \sim 2(n/p)_f / (1 + n/p)_f \exp(-t/\tau_n). \]

Hence, the produced He-4 is a strong function of the effective number of relativistic degrees of freedom at BBN epoch, \( g_{\text{eff}} = 10.75 + 7/4 \delta N_\nu \), neutron mean lifetime \( \tau_n \), which parametrizes the weak interactions strength. He-4 depends weakly on the nucleon-to-photon ratio \( \eta \). It depends also on the electron neutrino spectrum and on the neutrino-antineutrino asymmetry, which enter through \( \Gamma_w \). In the standard BBN model three neutrino flavors (\( \delta N_\nu = 0 \)), zero lepton asymmetry and equilibrium neutrino number densities and spectrum distribution are postulated:

\[ n_\nu(E) = (1 + \exp(E/T))^{-1}. \]

Due to its strong dependence on \( g_{\text{eff}} \) He-4 abundance is used to constrain the number of the relativistic during BBN particles (Shvartsman, 1969), usually parametrized by \( \delta N_\nu \). For contemporary discussion of BBN constraints on \( \delta N_\nu \) see for example (Lisi et al., 1999). The present BBN upper bounds on \( \delta N_\nu \) depending on the concrete analysis vary in the range \( \delta N_\nu < 0.1 - 0.7 \) (Barger et al., 2003, 9).
Cyburt et al., 2003; Cuoco et al., 2003). The value we consider reliable enough for putting cosmological constraints on new physics parameters, corresponding to $\sim 3\%$ overproduction of He-4 is $\delta N_\nu < 0.64$.

BBN constraint is in agreement with the constraints based on LSS and WMAP data (Crotty et al., 2003).

The dependence of the primordial abundances on the density and on the nucleon kinetics was used also for constraining massive stable neutrinos and decaying massive neutrinos (Terasawa & Sato, 1987; Dolgov & Kirilova, 1988; Gyuk & Turner 1994). The contemporary status of these constraints is presented in (Dolgov et al., 1999).

For more details on neutrino role in cosmology see (Dolgov, 2002, 2003) and the references therein. In the following we will concentrate mainly on BBN and neutrino oscillations.

4. BBN WITH NEUTRINO OSCILLATIONS

The influence of neutrino oscillations depends on the type of oscillations: oscillation channels, resonant transitions, the degree of equilibrium of oscillating neutrinos (see the review of Kirilova & Chizhov, 2001 and the references therein). Flavour neutrino oscillations effect BBN negligibly in case different flavour neutrinos are in thermal equilibrium and with vanishing chemical potentials (Dolgov, 81). However, active-sterile oscillations may have considerable influence because they effect both expansion rate through exciting additional neutrino types, and the weak interactions rate due to shifting neutrino densities and energy spectrum from BBN equilibrium values.

On the other hand although solar and atmospheric neutrino anomalies can be explained without a sterile neutrino, and definitely do not allow active-sterile neutrino oscillations as a dominant channel, some subdominant admixture of steriles is not only allowed, but also desirable (Holanda & Smirnov, 2003). Hence, it is interesting to discuss the cosmological effects of active-sterile neutrino oscillations, and to provide cosmological limits to oscillation parameters, which can be helpful in constraining the possibilities for $\nu_s$ admixture.

The presence of neutrino oscillations invalidates BBN assumptions about three neutrino flavours, zero lepton asymmetry, equilibrium neutrino energy distribution, thus directly influencing the kinetics of nucleons during the weak freeze-out.

4.1. NEUTRINO OSCILLATIONS EFFECTS

Qualitatively, neutrino oscillations effects considerably influencing the neutrino involved processes in the Universe are

(a) Excitation of additional degrees of freedom: This leads to faster Universe expansion $H(t) \sim g_{eff}^{1/2}$, earlier $n/p$-freezing, $T_f \sim (g_{eff})^{1/6}$, at times when neutrons were more abundant (Dolgov, 1981)

$$n/p \sim \exp\left(-\frac{m_n - m_p}{T_f}\right)$$

This effect gives up to $5\%$ $^4$He overproduction (in case one additional neutrino type is brought into equilibrium by oscillations, $\delta N_s = 1$).

(b) Distortion of the neutrino spectrum: The effect of oscillations may be much stronger than $\delta N_s = 1$ in case of oscillations effective after $\nu$ decoupling, proceeding
Neutrino Oscillations and the Early Universe

between partially populated sterile neutrino state \(0 \leq \delta N_s < 1\) and electron neutrino (Kirilova 1988; Kirilova and Chizhov, 1996; Kirilova, 2002). The non-equilibrium initial condition, for most of the oscillations parameters of the model, leads to considerable and continuous deviations from the equilibrium \(\nu_e\) spectrum (spectrum distortion).

A study of the momentum dependent kinetic equations for oscillating neutrinos before decoupling, provided recently, showed that even in that case for some oscillation parameters kinetic equilibrium may be strongly broken, especially in the resonant case (Dolgov & Villante, 2003).

Since the oscillation rate is energy dependent \(\Gamma \sim \frac{\delta m^2}{E}\) the low energy neutrinos start to oscillate first, and later the oscillations concern more and more energetic neutrinos. Hence, the neutrino energy spectrum \(n_\nu(E)\) may strongly deviate from its equilibrium form (see Fig.1), in case oscillations proceed between nonequilibrium neutrino states.

The distortion leads both to a depletion of the active neutrino number densities \(N_\nu\):

\[
N_\nu \sim \int dE E^2 n_\nu(E)
\]

and a decrease of the \(\Gamma_w\). Thus spectrum distortion influences the nucleons kinetics, causing an earlier \(n/p\)-freezing and an overproduction of \(^4\)He yield.

The spectrum distortion may also cause underproduction of He-4, when due to oscillations the energy of the greater part of the neutrinos becomes smaller than the threshold for the reaction \(\bar{\nu}_e + p \rightarrow n + e^+\) and the \(n/p\)-ratio decreases leading to a decrease of He-4. However this effect is a minor one. Hence, the total effect is an overproduction of He-4.

The spectrum distortion is the greatest, in case the sterile state is empty at the start of oscillations, \(\delta N_s = 0\). It decreases with the increase of the degree of population of the sterile state at the onset of oscillations (see Kirilova, 2002) as illustrated

![Fig.1](image-url)
in the following figures (Fig.2).

Fig.2. The figures illustrate the spectrum distortion at different degrees of population of the steriles, namely $\delta N_s = 0$ (lower curve), $\delta N_s = 0.5$ and $\delta N_s = 0.8$ (upper curve). The dashed curve gives the equilibrium spectrum for comparison. It is obvious that the distortion of the spectrum is considerable and with time involves the whole neutrino ensemble.

Spectrum distortion effect may be considerable both for the vacuum oscillations (Kirilova, 1988) and oscillations in a medium (Kirilova and Chizhov, 1996), both for the nonresonant (Kirilova & Chizhov, 1998) and resonant oscillations case (Kirilova & Chizhov, 2000).

(c) Production of neutrino-antineutrino asymmetry: Neutrino-antineutrino asymmetry may be generated during the resonant transfer of neutrinos (Miheev & Smirnov, 1986; Langacker et al., 1987, Kirilova & Chizhov, 1996, Foot et al., 1996). Dynamically produced asymmetry exerts back effect to oscillating neutrino and may change its oscillation pattern. It influences $\nu$ and $\bar{\nu}$ number density evolution, their spectrum distortion and the oscillation pattern – all playing important role in $n-p$-kinetics.

Even when its value is not high enough to have a direct kinetic effect on the synthesis of light elements, i.e. even when $L << 0.01$ it effects indirectly BBN (Kirilova & Chizhov, 1996, 1999, 2000). This dynamically produced asymmetry suppresses oscillations at small mixing angles, leading to less overproduction of He-4 compared to the case without the account of asymmetry growth (see Fig.3), and hence alleviating BBN constraints on oscillation parameters.
In case, however, of initial asymmetry slightly higher than the baryon one \( (L \sim 10^{-6}) \), \( L \) can also enhance oscillations transfers, leading to an increased overproduction of He-4. However, for naturally small initial \( L \sim \) baryon asymmetry, the asymmetry effect is a subdominant one.

So, spectrum distortion effect is the dominant one and for a wide range of oscillation parameters it is considerable during the period of nucleons freezing and hence effects primordial nucleosynthesis. The rough calculations not accounting for spectrum distortion effects may underestimate oscillations effect on BBN even by several orders of magnitude of \( \delta m^2 \) (Chizhov & Kirilova, 1999).

4.2. PRODUCTION OF HE-4 IN THE PRESENCE OF NEUTRINO OSCILLATIONS

The case of oscillations effective before electron neutrino freezing, was considered both analytically (Barbieri & Dolgov, 1990; Barbieri & Dolgov, 1991) and numerically (Enqvist et al., 1992) accounting precisely for a) and partially for b) (namely, estimating the depletion of the neutrino number densities, assuming equilibrium neutrino energy spectrum).

Analytical description was found in the case of very small mixing angles and 'large' mass differences \( \delta m^2 > 10^{-6} \text{eV}^2 \), and for the case without spectrum distortion effects (Dolgov, 2002, 2003; Dolgov & Villante, 2003).

The nonequilibrium picture of neutrino oscillation effects (a)-(c) is hard to describe analytically. For nonequilibrium neutrino oscillations effective after active neutrino decoupling, i.e. for \( (\delta m^2/\text{eV}^2)\sin^22\theta < 10^{-7} \), the spectrum distortion effect was shown to play a considerable role. For that case a complete selfconsistent numerical
analysis of the kinetics of the oscillating neutrinos, the nucleons freeze-out and the asymmetry evolution was provided (Kirilova & Chizhov, 96, 98, 2000). Kinetic equations for neutrino density matrix and neutron number densities in momentum space (Kirilova & Chizhov, 96, 97) were used to make a proper precise account for spectrum distortion effect, neutrino depletion and neutrino asymmetry at each neutrino momentum.

The production of the primordial $^4$He, $Y_p$ in the presence of $\nu_e \leftrightarrow \nu_s$ oscillations, effective after $\nu_e$ decoupling, was calculated. The numerical analysis was provided for the temperature interval $[0.3 \text{ MeV}, 2 \text{ MeV}]$. He-4 production was calculated both in the nonresonant (Kirilova & Chizhov, 1998) and resonant (Kirilova and Chizhov, 2000) oscillation cases (see also Kirilova, 2003).

Primordial helium is considerably overproduced in the presence of active-sterile neutrino oscillations due to the effects (a)-(c). The kinetic effect of neutrino oscillations $\delta N_{\text{kin}}$ due to spectrum distortion usually comprises a major portion of the total effect, i.e. it plays the dominant role in the overproduction of $^4$He. It can be larger than the one corresponding to an additional degree of freedom. The overproduction is maximal for the case of initially empty $\nu_s$ state $\delta N_s = 0$ (Kirilova, 2003).

In the nonresonant case the effect of oscillations is proportional to the oscillation parameters. It becomes very small (less than 1%) for small mixings: as small as $\sin^2 2\theta = 0.1$ for $\delta m^2 = 10^{-7} \text{eV}^2$, and for small mass differences: $\delta m^2 < 10^{-10} \text{eV}^2$ at maximal mixing. The effect is maximal at maximal mixing for a given mass difference.

$Y_{p,\text{max}}$ increase with $\delta m^2$ till $\delta m^2 = 10^{-7} \text{eV}^2$ in our model. Further increase of the mass differences requires a decrease of the maximal mixing angle considered, such that oscillations remain effective after $\nu_e$ decoupling ($\sin^4 2\theta \leq 10^{-7} \delta m^{-2}$). Therefore, for higher $\delta m^2$ in the discussed oscillation model $\theta < \pi/4$, and $Y_{p,\text{max}}$ decreases with further increase of $\delta m^2$ beyond $\delta m^2 \sim 10^{-7} \text{eV}^2$ (see Fig.4).

In the resonant oscillation case, however, for a given $\delta m^2$ there exists some resonant mixing angle, at which the oscillations effects are enhanced by the medium due to the MSW effect (see Wolfenstein 1978; Mikheev & Smirnov, 1985), and hence, the overproduction of He-4 is greater than that corresponding to the vacuum maximal mixing angle. In case $N_s = 0$ $\delta N_{\text{kin}}^{\text{max}} > 1$ for $\delta m^2 > 10^{-9} \text{eV}^2$.

He-4 overproduction in the resonant case can be up to 31.8%, while in the nonresonant one - up to 13.8% (see Fig.4). So, the maximum overproduction of $^4$He corresponds to an increase of the neutrino effective degrees of freedom $\delta N_{\text{kin}}^{\text{max}} \sim 6$. 

14
Neutrino Oscillations and the Early Universe

Fig. 4. In the l.h.s. figure the maximal relative increase in the primordial $^4$He as a function of neutrino mass differences: $\delta Y_p^{\text{max}}/Y_p = \delta Y_p^{\text{max}}/Y_p (\delta m^2)$ is presented for $\vartheta = \pi/4$ in the nonresonant case, and for the resonant mixing angles in the resonant case (upper curve). In the r.h.s. figure maximum primordial $^4$He abundance for the resonant and the non-resonant oscillation case, as a function of the neutrino mixing angle at $\delta m^2 = 10^{-8}$ eV$^2$ (lower curve) and $\delta m^2 = 10^{-7}$ eV$^2$ is given.

4.3. COSMOLOGICAL CONSTRAINTS ON OSCILLATION PARAMETERS

*Cosmological Constraints — $\delta N_s = 0$ Case*

Observational data on primordial He-4 put stringent limits on the allowed active-sterile oscillation parameters. First BBN limits were derived in the pioneer works (Barbieri & Dolgov, 1990, 1991; Enqvist et al., 1990, 1992) under the assumption of kinetic equilibrium (neutrinos were described by a single momentum state with the thermal average energy). These constraints accounted for the effects a) and partially for b).

They were recently updated (Dolgov, 2002; Dolgov & Villante, 2003). In the nonresonant case they can be approximated:

$$ (\delta m_{\nu_e,\nu_s}^2/eV^2) \sin^4 2\theta_{\nu_e,\nu_s} = 3.16 \times 10^{-5} (\delta N_\nu)^2 $$

$$ (\delta m_{\nu_{\mu,\nu_s}}^2/eV^2) \sin^4 2\theta_{\nu_{\mu,\nu_s}} = 1.74 \times 10^{-5} (\delta N_\nu)^2 $$

assuming kinetic equilibrium and using stationary point approximation. The bounds are reasonably accurate for large mass differences in case of efficient repopulation of active neutrinos. For the exact constraints in the resonant case see (Dolgov & Villante, 2003).

However, as discussed in previous subsection, at lower mass differences, when sterile neutrino production takes place after active neutrino freezing, the re-population of active neutrino becomes slow and hence, kinetic equilibrium may be strongly broken.
due to active-sterile oscillations. The spectrum distortion of $\nu_e$ may be considerable, leading to strong influence on nucleons kinetics (effects (b) and (c)). The analysis of such oscillations with precise kinetic accounting for the effects (b) and (c) allowed to put stringent constraints to oscillations parameters at small mass differences. We will discuss below these BBN constraints (Kirilova & Chizhov, 1998, 2000, 2001; Kirilova, 2003).

We assume the uncertainty of observational helium-4 to be $\delta Y_p / Y_p < 3\%$ in accordance with observations of helium and also with the resent WMAP constraints on the additional relativistic degrees of freedom (Crotty et al., 2003, Cyburt et al., 2003, Cuoco et al., 2003). Then the range of cosmologically excluded electron-sterile oscillations parameters is situated above the 3\% contour at Fig.5:

![Fig.5. The combined iso-helium contours for the nonresonant and the resonant case, for $\delta Y_p = (Y_{osc} - Y_p) / Y_p = 3\%, 5\%, 7\%$ (Kirilova & Chizhov, 2001). The dashed curves present LOW sterile solution.](image)

The analytical fits to the exact constraints are:

$$\delta m^2 (\sin^2 2\theta)^4 \leq 1.5 \times 10^{-9} \text{eV}^2 \quad \delta m^2 > 0$$

$$|\delta m^2| < 8.2 \times 10^{-10} \text{eV}^2 \quad \delta m^2 < 0, \quad \text{large } \theta,$$

Due to the precise account of the kinetic effects of oscillations, these constraints are nearly an order of magnitude stronger at large mixings than other numerical calculations (Enqvist et al., 1992) of 2-neutrino mixing and much constraining the mass differences values than the constraints derived for oscillations effective before the electron neutrino freeze-out (Dolgov, 2002; Dolgov & Villante 2003). In the
Neutrino Oscillations and the Early Universe

resonant case, due to the proper account of the asymmetry generated in oscillations, they are less restrictive at small mixings \(^6\) than the constraints (Enqvist et al., 1992).

The cosmological constraints exclude almost completely sterile LOW solution to the solar neutrino problem, besides the sterile LMA solution and sterile atmospheric solution, excluded in previous works. This result is consistent with the global analysis (Holanda & Smirnov, 2003; Giunti & Laveder, 2003; Maltoni et al., 2003; Bahcall et al., 2003; see also Fogli et al., 2001, 2003) of the data from neutrino oscillations experiments KamLAND, SNO, SuperKamiokande, GALLEX+GNO, SAGE and Chlorine, which do not favour \(\nu_e \leftrightarrow \nu_s\) solutions.

These cosmological constraints should be generalized for the case of 4-neutrino mixing\(^7\), since the effect of mixing between active neutrinos on the BBN constraints of electron-sterile oscillations has been proved important in the resonant oscillation case (Dolgov & Villante 2003).

**Cosmological Constraints — \(\delta N_s \neq 0\) Case**

Sterile neutrinos \(\nu_s\) may be present at the onset of BBN epoch — they may be produced in GUT models, in models with large extra dimensions, Manyfold Universe models, mirror matter models, or in \(\nu_{\mu,\tau} \leftrightarrow \nu_s\) oscillations in 4-neutrino mixing schemes. Hence, the degree of population of \(\nu_s\) may be different depending on the \(\nu_s\) production model. Therefore, it is interesting to study the distortion of \(\nu_e\) energy spectrum due to oscillations \(\nu_e \leftrightarrow \nu_s\), and its influence on BBN for different degree of population of the initially present sterile neutrinos \(0 \leq \delta N_s \leq 1\). \(Y_p\), for different \(\delta N_s\) values and different sets of oscillation parameters \(Y_p(\delta N_s, \delta m^2, \sin^2 2\vartheta)\) was calculated (Kirilova, 2002).

\(\delta N_s \neq 0\) present before \(\nu_{\mu,\tau} \leftrightarrow \nu_s\) just leads to an increase of the total energy density of the Universe, and it is straightforward to re-scale the existing constraints. In the \(\nu_e \leftrightarrow \nu_s\) oscillations case, however, the presence of \(\nu_s\) at the onset of oscillations influences in addition the kinetic effects of \(\nu_e \leftrightarrow \nu_s\) on BBN. Larger \(\delta N_s\) decreases the kinetic effects, because the element of initial non-equilibrium between the active and the sterile states is less expressed (see the dashed curves in Fig.6).

\(^6\)Mind that the oscillations generated asymmetry suppresses oscillations at small mixing angles, reflecting in less overproduction of He-4 in comparison with the case where asymmetry generation was neglected

\(^7\)as was already done for the case of fast oscillations proceeding before neutrino decoupling in (Dolgov & Villante 2003)
Neutrino spectrum distortion effect is very strong even when there is a considerable population of the sterile neutrino state before the beginning of the electron–sterile oscillations. It always gives positive $\delta N_{\text{kin}}$, which for a large range of initial sterile population values, is bigger than 1. The kinetic effects are the strongest for $\delta N_s = 0$, they disappear for $\delta N_s = 1$, when $\nu_e$ and $\nu_s$ states are in equilibrium, and the total effect reduces to the SBBN with an additional neutrino (Fig.6).
Our numerical analysis has shown that up to $\delta N_s = 0.5$ the cosmological constraints are slightly changed and remain stringent, as before.

So, the presence of non-zero initial population of the sterile state does not change or remove the stringent cosmological constraints.

Cosmological constraints can be relaxed assuming even higher than the 0.007 systematic error. However, even for 0.25 isohelium contour the constraints on LOW sterile solution are not removed, but just relaxed.

Small relic fermion asymmetry $L < < 0.1$, however, is capable to suppress oscillations with small mass differences, thus removing BBN constraints for the corresponding oscillation parameters. For the oscillations effective after neutrino decoupling we have found that $L \sim 10^{-5}$ is large enough to suppress oscillations and remove BBN constraints (Kirilova & Chizhov, 2001). So, we expect that it is possible to remove the cosmological constraints only for rather small $\delta m^2$, as far as asymmetry larger than 0.1 is not allowed.

So, the presence of non-zero initial population of the sterile state does not change or remove the existing stringent cosmological constraints on active-sterile neutrino oscillations. They are not relaxed considerably also in case exaggerated He-4 abundance is assumed. Even assuming unnaturally large lepton asymmetry ($B << L < 0.1$), they are only relaxed, not removed. In case of naturally small $L$, BBN provides the most stringent constraints on active-sterile neutrino oscillations parameters, and restricts the fraction of the sterile neutrinos in the neutrino anomalies.

Besides, as discussed previously, cosmology provides still the most stringent upper limit for neutrino masses. So, it provides valuable precision probe of new neutrino physics.

5. CONCLUSIONS

During the last quarter of the 20th century strong evidence for non-zero neutrino masses and mixings has been provided from various neutrino oscillations experiments (astrophysical: solar neutrino experiments and atmospheric neutrino experiments and terrestrial: LSND and KamLAND).

The essential role of neutrino oscillations for processes in the early Universe was realized. Stringent cosmological constraints on the oscillation parameters were obtained on the basis of BBN considerations. Cosmological analysis of CMB, LSS and BBN data provide stringent upper limit for neutrino masses. Non-zero neutrino masses and mixings, required by the experimental data and cosmology, require new physics beyond the standard electroweak model. The scale of this NP is inversely proportional to the neutrino masses and appears to be of the order of the Grand Unification scale. Hence, the neutrino oscillation experiments and cosmology point the way towards the Grand Unified Theory of elementary particles.

8Such asymmetry in $\nu_e$ sector changes unacceptably the kinetics of nucleons and the He-4 yield. While in $\nu_{\mu,\tau}$ sectors it is constrained to the same level on the basis of redistribution of the asymmetries due to oscillations (Dolgov et al., 2002)
6. ACKNOWLEDGEMENTS

It is my pleasure to thank O. Atanacković-Vukmanović and the organizing committee of the NCYA Conference for the hospitality. I am grateful to I. Tkachev and the organizing committee of CAPP2003 for the financial support for my participation.

D.K. is a regular associate of ICTP, Trieste, Italy. This work is supported in part by Belgian Federal Government (office for scientific affairs grant and IAP 5/27)

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