Formation Control of Multiple Dubins Airplane System with Geometric Approach

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Abstract. Formation Control is very important for multi agent systems (Swarm). This paper analyses control formation on the Dubins airplane that consists of three airplanes, where an airplane as the leader and two other airplanes move to follow leader airplane’s track. First, Doing design of leader airplane’s controls by using the method of tracking error dynamics. Controls on the leader follow the trajectory tracking that desired. Desired path is in the form of parametric functions. Second, geometric approaches method is used to design two other airplanes, so the move follows path of the airplane as the leader with a certain distance. Simulation result shows that airplane leader can be controlled to the desired path crosses with good results. Therefore, error has been defined as the distance between leader airplane’s paths with the desired path while tracking also depends on the shape of path. The more difficult trajectory tracking of airplane also serves as a leader in passing through it. The result of the simulations also indicates that the value of the mean percentage error (MPE) of 26.93 percent. This suggests that the methods used in this study quite effective either used to control design leader or follower.

1. Introduction

Natural phenomenons are very interesting to analyse mathematically. One of the examples is the phenomenon called swarming which occurs in various groups of organism. Swarming’s behaviour or aggregation of organism in groups is abundant in nature. For example, the behaviour can be seen in animal aggregation, such as flocks of birds (geese), schools of fish, herds of mammals etc. Flocks of geese often fly along in the inverted "V" formation. By flying in the inverted "V" formation they obtain some advantages. The motion of swarm can be used in engineering for cooperative control (multi robots) and formation control (aircraft and ship). The formation preservation of the swarm has received considerable attentions. Gazi, et al. [1],[2] and Miswanto, et.al[3] study the coordination and tracking control problem of the motion of the swarm. In the literature, some researchers have discussed the formation control of mobile robots. In [4], the authors study a stable and decentralized control strategy for multi agent system to capture a moving target in a specific formation. In [5] the authors study control and coordination for many robots moving in formation using decentralized controllers. They investigate feedback law used to
control multiple robots moving in formation. They proposed a method for controlling formations that uses only local sensor based information, in a leader – follower motion. They use methods of feedback linearization to exponentially stabilize the relative distance and orientation of follower and show that the zero dynamics of the system are also stable. Another researchers have discussed the control design of a mobile robot to track a desired path. In [6], the authors discuss the tracking control of mobile robot using integrator back stepping. Many mechanical systems with non-holonomic constrains can be locally or globally converted to the chained form under coordinate change. In [7], the authors study the tracking control problem of non-holonomic system in chained form. They derive semi global tracking controllers for general chained form systems by means of back stepping and they achieve global tracking result for some special cases. In [8] an adaptive tracking control problem is studied for a four wheel mobile robot. The authors propose a formulation for the adaptive tracking problem that meets the natural prerequisite such that it reduces to the state feedback tracking problem if the parameters are known. They derive a general methodology for solving their problem. In [9] the authors study the formation control of swarm whose agents are Dubins cars. The agents of swarm are moving to track a desired path. They consider the swarm model with presence of a leader. First, they design the control of the leader with tracking error dynamics. The control of the leader is designed for tracking the desired path. Furthermore, this paper also carried out computation the tracking error of the path of the leader tracking. A desired path is sufficiently small and the distance between the leader path and the desired path is preserved. In the next section, the formal problem statement is described. In section 3, Doing design control of the leader using tracking error dynamics. In section 4, Design the control of each agent follower by using geometric approach and the last section will show the results of numerical simulation trajectory of the leader airplane and the desired path also error between the trajectory of the leader airplane and desired path.

2. Problem Formulation

In this paper, the dynamics system of airplane is taken from Fozen (2011) and three airplanes which described as:

\[
\begin{align*}
\dot{x}_i &= u_i \cos \theta_i + v_i \sin \theta_i \\
\dot{y}_i &= -u_i \sin \theta_i + v_i \cos \theta_i \\
\dot{\theta}_i &= w_i \\
u_i &= a_i \\
v_i &= b_i \\
w_i &= c_i
\end{align*}
\]  

(1)

Where \((x_i, y_i) \in \mathbb{R}^2\) represents the position of the \(i\)-th airplane, and \(\theta_i \in [0,2\pi]\) represents the orientation of the \(i\)-th airplane. The symbol \(u_i\) is the linear velocities for moving forward and back, \(v_i\) is the linear velocities to move to the side and \(w_i\) represents airplane maneuvering motion. In this paper, the desired path \(\gamma\) that would be tracked by the leader airplane is obtained by using calculus variation method. The path is denoted by \(\gamma(t) = [x(t), y(t)]^T\). There are two problems that will be completed. First, Design the control of the leader airplane for tracking the desired path by tracking error dynamic. Second, Design the control of the other agents by geometric approach to follow the leader’s path with a certain distance.
3. The Control Design of The Leader Airplane

Design control of the leader by tracking error dynamics for minimizing the tracking error in order to keep the position of the leader close to the desired path. Tracking error $e(t)$ as the difference between the actual leader path and desired path.

$$e(t) = [x_2(t) - y_2(t), y_1(t) - y_1(t)]^T$$

(2)

Differentiating the error equation (2) with respect to time yields.

$$\dot{e}(t) = [\dot{x}_2(t) - \dot{y}_2(t), \dot{y}_1(t) - \dot{y}_1(t)]^T = [u_1 \cos \theta_1 + v_1 \sin \theta_1 - \dot{y}_1(t), -u_1 \sin \theta_1 + v_1 \cos \theta_1 - \dot{y}_1(t)]^T.$$  (3)

And,  

$$\ddot{e}(t) = [\ddot{x}_2(t) - \ddot{y}_2(t), \ddot{y}_1(t) - \ddot{y}_1(t)]^T = [\ddot{u}_1 \cos \theta_1 - u_1 \omega_2 \sin \theta_1 + (v_1 \sin \theta_1 + v_1 \omega_2 \cos \theta_1) - \dot{y}_1(t), -\ddot{u}_1 \sin \theta_1 - u_1 \omega_2 \cos \theta_1 + (v_1 \cos \theta_1 + v_1 \omega_2 \sin \theta_1) - \dot{y}_1(t)]^T.$$  (4)

Now, we define the tracking error dynamics $\dot{F}$ where $F = [f_1, f_2]^T$ and $\dot{f}_i = (e_i, \dot{e}_i) = 0, i = 1, 2$

$$f_1(t) = \dot{e}_1(t) + k_1 e_1(t)$$

$$f_2(t) = \dot{e}_2(t) + k_2 \dot{e}_2(t)$$

(5)

Where $k_1$ and $k_2$ are positive contants. Differenting the system (5) with respect to time, one obtains

$$\dot{f}_1(t) = \dot{e}_1(t) + k_1 \dot{e}_1(t)$$

$$\dot{f}_2(t) = \dot{e}_2(t) + k_2 \dot{e}_2(t)$$

(6)

The control of the leader can be determined from equation (1), (2), (3), (4), (5) and (6).

To get of the $u_2, v_2$ and $w_2$ done by way of elimination and multiplying with $\cos \theta_2$ and $\sin \theta_2$ so obtained;

$$w_1 = \frac{(u_1 \sin \theta_1 + v_1 \cos \theta_1)(\dot{u}_1 \cos \theta_1 - v_1 \sin \theta_1 + \ddot{y}_1(t) + k_1 (u_1 \cos \theta_1 + v_1 \sin \theta_1 - \dot{y}_1(t))) + (u_1 \cos \theta_1 + v_1 \sin \theta_1)(-u_1 \sin \theta_1 + v_1 \cos \theta_1 - \dot{y}_1(t) + k_1 (-u_1 \sin \theta_1 + v_1 \cos \theta_1 - \dot{y}_1(t)))}{u_1^2 - v_1^2}$$

Where,

$$u_1 = \ddot{x}_1(t) \cos \theta_1 - \ddot{y}_1(t) \sin \theta_1 - k_1 x_1(t) \cos \theta_1 - k_1 y_1(t) \sin \theta_1 - k_1 x_1(t) \cos \theta_1 - k_1 y_1(t) \sin \theta_1$$

$$v_1 = \ddot{y}_x(t) \sin \theta_1 + \ddot{y}_y(t) \cos \theta_1 - k_1 x_1(t) \sin \theta_1 - k_1 y_1(t) \cos \theta_1 + k_1 y_1(t) \cos \theta_1 + k_1 y_1(t) \cos \theta_1$$

(7)

Then this control $(u_1, v_1, \text{and } w_1)$ is substituted to the system (1). Thus, if one uses (7), one obtains differential equations:

$$\dot{x}_1 = (\ddot{x}_1(t) \cos \theta_1 - \ddot{y}_1(t) \sin \theta_1 - k_1 x_1(t) \cos \theta_1 - k_1 y_1(t) \sin \theta_1 - k_1 x_1(t) \cos \theta_1 - k_1 y_1(t) \sin \theta_1 - k_1 y_1(t) \cos \theta_1 + k_1 y_1(t) \cos \theta_1 + k_1 y_1(t) \cos \theta_1) \sin \theta_1$$

$$\dot{y}_1 = -(\ddot{y}_1(t) \cos \theta_1 - \ddot{y}_1(t) \sin \theta_1 - k_1 x_1(t) \cos \theta_1 + k_1 y_1(t) \sin \theta_1 + k_1 y_1(t) \cos \theta_1 + k_1 y_1(t) \cos \theta_1 + k_1 y_1(t) \cos \theta_1 + k_1 y_1(t) \cos \theta_1) \sin \theta_1$$

$$\dot{\theta}_1 = \frac{(u_1 \sin \theta_1 + v_1 \cos \theta_1)(\dot{u}_1 \cos \theta_1 - v_1 \sin \theta_1 + \ddot{y}_1(t) + k_1 (u_1 \cos \theta_1 + v_1 \sin \theta_1 - \dot{y}_1(t))) + (u_1 \cos \theta_1 + v_1 \sin \theta_1)(-u_1 \sin \theta_1 + v_1 \cos \theta_1 - \dot{y}_1(t) + k_1 (-u_1 \sin \theta_1 + v_1 \cos \theta_1 - \dot{y}_1(t)))}{u_1^2 - v_1^2}$$

(8)

This system is a system of dynamical airplane that is a leader and been given a control so that it can pass the desired trajectory. The initial and final condition of the state variables $(x_1, y_1, \theta_1)$ are known. The solution of this system of differential equations use numerical approximation by substituting the desired path in system (8).
4. The Control Design of the Following Agent

In this section, Design control of the follower using geometry approach. Figure (1) shows three dubins cars. Where $d_1$ and $d_2$ are the distance of agents to the leader. $\phi_1$ and $\phi_2$ are the orientation of agents to position of the leader.

![Figure 1. Three Dubins Airplane.](image)

4.1 The Control Design of The First Follower

From the figure (1), one obtain

$$x_1 - x_2 = d_1 \cos(\theta_2 + \phi_1) \text{ and,}$$

$$y_1 - y_2 = d_1 \sin(\theta_2 + \phi_1)$$

Differentiating the equation above with respect to time, one obtains,

$$\dot{x}_1 - \dot{x}_2 = -d_1 \sin(\theta_2 + \phi_1) (\dot{\theta}_2 + \dot{\phi}_1)$$

$$\dot{y}_1 - \dot{y}_2 = d_1 \cos(\theta_2 + \phi_1) (\dot{\theta}_2 + \dot{\phi}_1) \tag{9}$$

Then from the above equation is obtained:

$$\dot{x}_2 = \dot{x}_1 + d_1 \sin(\theta_2 + \phi_1) (\dot{\theta}_2 + \dot{\phi}_1)$$

$$\dot{y}_2 = \dot{y}_1 - d_1 \cos(\theta_2 + \phi_1) (\dot{\theta}_2 + \dot{\phi}_1) \tag{10}$$

After that, by substitution equation $\dot{x}_2 = u_2 \cos \theta_2 + v_2 \sin \theta_2$ dan $\dot{y}_2 = -u_2 \sin \theta_2 + v_2 \cos \theta_2$

One obtain;

$$u_2 \cos \theta_2 + v_2 \sin \theta_2 = \dot{x}_1 + d_1 \sin(\theta_2 + \phi_1) (\dot{\theta}_2 + \dot{\phi}_1)$$

$$-u_2 \sin \theta_2 + v_2 \cos \theta_2 = \dot{y}_1 - d_1 \cos(\theta_2 + \phi_1) (\dot{\theta}_2 + \dot{\phi}_1)$$

To get control of the $u_2$ and $v_2$ elimination done so obtained,

$$u_2 = \dot{x}_1 \cos \theta_2 - \dot{y}_1 \sin \theta_2 + d_1 (\dot{\theta}_2 + \phi_1) \sin((\theta_2 + \phi_1) + \theta_2)$$

$$v_2 = \dot{x}_1 \sin \theta_2 + \dot{y}_1 \cos \theta_2 - d_1 (\dot{\theta}_2 + \phi_1) \cos((\theta_2 + \phi_1) + \theta_2)$$

And from $u_2$ and $v_2$ obtained,

$$w_2 = \frac{(u_2 - \dot{x}_1 \cos \theta_2 + \dot{y}_1 \sin \theta_2) d_1 (\cos((\theta_2 + \phi_1) + \theta_2)) + (v_2 - \dot{x}_1 \sin \theta_2 - \dot{y}_1 \cos \theta_2) d_1 \sin((\theta_2 + \phi_1) + \theta_2)}{d_1 (\sin((\theta_2 + \phi_1) - \theta_2)) d_1 (\cos((\theta_2 + \phi_1) + \theta_2))}$$

Then, this control ($u_2$, $v_2$, and $w_2$) is substituted to the system (1) with $i = 2$. Thus obtained a system of the differential equations of the first follower:

$$\dot{x}_2 = (\dot{x}_1 \cos \theta_2 - \dot{y}_1 \sin \theta_2 + d_1 (\dot{\theta}_2 + \phi_1) \sin((\theta_2 + \phi_1) + \theta_2)) \cos \theta_2 + (\dot{x}_1 \sin \theta_2 + \dot{y}_1 \cos \theta_2 - d_1 (\dot{\theta}_2 + \phi_1) \cos((\theta_2 + \phi_1) + \theta_2)) \sin \theta_2$$
\[
\dot{y}_2 = - (\dot{x}_1 \cos \theta_2 - y_1 \sin \theta_2 + d_1 (\dot{\theta}_2 + \dot{\phi}_1) \sin ((\theta_2 + \phi_1) + \theta_2)) \sin \theta_2 + (\dot{x}_1 \sin \theta_2 + \dot{y}_1 \cos \theta_2 - d_1 (\dot{\theta}_2 + \dot{\phi}_1) \cos ((\theta_2 + \phi_1) + \theta_2)) \cos \theta_2
\]

\[
\dot{\theta}_2 = \frac{(u_2 - x_1 \cos \theta_2 + y_1 \sin \theta_2) d_1 (\cos ((\theta_2 + \phi_1) + \theta_2)) + (v_2 - x_1 \sin \theta_2 - y_1 \cos \theta_2) d_1 (\sin ((\theta_2 + \phi_1) + \theta_2))}{d_1 (\sin ((\theta_2 + \phi_1) - \theta_2)) d_1 (\cos ((\theta_2 + \phi_1) + \theta_2))}
\]

The system in equation (11) is a system of dynamical airplane which is first follower and been given a control so that it can be controlled dependent on the leader.

4.2 The Control Design of the Second Follower
From the figure (1) is obtained,
\[x_1 - x_3 = d_2 \cos (\theta_3 - \phi_2)\] and,
\[y_1 - y_3 = d_2 \sin (\theta_3 - \phi_2)\]
Using similar steps such as in 4.1 one may design the control of the second follower. Thus obtain a system of differential equations of the first follower.
\[
u_3 = x_1 \sin \theta_3 + y_1 \cos \theta_3 - d_2 (\theta_3 + \phi_2) \cos (\theta_3 + \phi_2) + \theta_3)
\]

And from \(u_3\) and \(v_3\) is obtained,
\[
w_3 = \frac{(u_3 - x_1 \cos \theta_3 + y_1 \sin \theta_3) d_2 \cos (\theta_3 + \phi_2) + \theta_3) + (v_3 - x_1 \sin \theta_3 - y_1 \cos \theta_3) d_2 \sin (\theta_3 + \phi_2) + \theta_3)}{d_2 \sin ((\theta_3 + \phi_2) + \theta_3) d_2 \cos ((\theta_3 + \phi_2) + \theta_3)}
\]

Then, this control \((u_3, v_3, \text{and } w_3)\) is substituted to the system (1) with \(i = 3\). System of the differential equations of the second follower:
\[
\dot{x}_3 = (\dot{x}_1 \cos \theta_3 - \dot{y}_1 \sin \theta_3 + d_2 (\theta_3 + \phi_2) \sin (\theta_3 + \phi_2) + \theta_3) \sin \theta_3 + (\dot{x}_1 \sin \theta_3 + \dot{y}_1 \cos \theta_3 - d_2 (\theta_3 + \phi_2) \cos (\theta_3 + \phi_2) + \theta_3) \cos \theta_3
\]

\[
\dot{y}_3 = - (\dot{x}_1 \cos \theta_3 - \dot{y}_1 \sin \theta_3 + d_2 (\theta_3 + \phi_2) \sin (\theta_3 + \phi_2) + \theta_3) \cos \theta_3 + (\dot{x}_1 \sin \theta_3 + \dot{y}_1 \cos \theta_3 - d_2 (\theta_3 + \phi_2) \cos (\theta_3 + \phi_2) + \theta_3) \sin \theta_3
\]

\[
\dot{\theta}_3 = \frac{(u_3 - x_1 \cos \theta_3 + y_1 \sin \theta_3) d_2 \cos (\theta_3 + \phi_2) + \theta_3) + (v_3 - x_1 \sin \theta_3 - y_1 \cos \theta_3) d_2 \sin (\theta_3 + \phi_2) + \theta_3)}{d_2 \sin ((\theta_3 + \phi_2) + \theta_3) d_2 \cos ((\theta_3 + \phi_2) + \theta_3)}
\]

Such as is the case in the previous system(11), the system in equation (12) is a system of dynamical airplane that role as second follower and been given a control so that it can be controlled dependent on the leader.

5. Numerical Simulation
In this section, some numerical simulations which is to illustrate the system(8),(11) and (12) are reported. For illustration, a desired path is the following parametric curve:
\[
y_x(t) = \frac{3}{49} t^3 - \frac{25}{29} t^2 + 5t + 3
\]
\[
y_y(t) = \frac{2}{99} t^3 - \frac{15}{500} t^2 + 5t + 4
\]

The problem in this section is to design the control for three airplanes, so the three airplane move to follow the desired path from the initial position \((t = 0\) seconds) to the end \((t = 25\) seconds). First of all is designing motion control of the leader airplane for track a desired path. Design control of the leader airplane used the tracking error dynamics. Figure 2 below shows the trajectory of the leader airplane tracing the desired path by using the method. From figure 3 above shows that the trajectory of the leader airplane can track the desired path from the position 0 seconds to the end position 25 seconds with a small
enough distance. This means that the leader airplane can move from one area (the starting position) to another area (final position), hope of results numerical simulations are the first follower airplane and the second follower airplane also move to follow the trajectory of the leader airplane from the start position \((t = 0 \text{ seconds})\) to the end \((t = 25 \text{ seconds})\). Trajectory error between the trajectories of the leader airplane with desired path can be seen in Figure 3.

![Figure 2. Trajectory of the leader airplane and the desired path.](image)

From the results of simulations can be seen that the tracking aircraft that serves as a leader originally indeed outside the path, but in the 5th second tracking planes can already be on the desired trajectory.

![Figure 3. Trajectory error between the trajectory of the leader airplane and desired path.](image)

Based on the results of the simulations the above error is getting smaller over time. This indicates that the method used is quite effective because the airplane was able to do tracking on the path until the end.

6. Conclusion

From the numerical simulation result above, it can be seen that the tracking error of the path of the leader airplane tracing a desired path is sufficiently small and the distance between the path of leader airplane and the desired path is preserved. A geometry approach for formation control of a group of airplane is investigated in this paper. A small error that indicates that the method used is a fairly effective method. In the future works, we will discuss the movement control of model swarm consisting of more three airplanes with a specific geometry formation.

References

[1] Miswanto, Pranoto, H, Muhammad, D Mahayana 2015 The Control Design of Ship Formation with the presence of a Leader International Journal of Robotics and Automation (IJRA) vol. 4 No.1 March pp 53-62

[2] V Gazi, B Fidan, Y S Hanay, and M Ilter Koksal 2007 Aggregation, Foraging and formation control
of swarm with non-Holonomic Agents Using potential Functions and Sliding Mode Technique

 Turk, J. Elec. Engin. Vol.15 No.2

[3] Miswanto, I Pranoto, and H Muhammad 2006 A model of swarm movement with the presence of a leader Proceeding of the International Conference on mathematics and natural Sciences pp 740-2 (Bandung: ITB)

[4] Gazi and R Ordonez 2007 Target Tracking Using Artificial Potentials and Sliding Mode Control International Journal of Control Vol.80 No.10 pp 1626-35

[5] D Wang and G Xu 2000 Full state tracking and internal Dynamics of Nonholonomic Wheeled Mobile Robots Proceedings of the American control Conference Chicago Illinois pp 3274-8

[6] C Y Tzeng and J F Chen 1999 Fundamental Properties of Linear Ship Steering dynamic models, Journal of Marine Science and Technology Vol.7 No.2 pp 79-88

[7] A P Aguir and J P Hespanha 2003 Position Tracking of underactuated Vehicles Proceedings of the American control Conference Denver Colorado pp 1988-93

[8] A Behal, D M. Dawson, B Xian and P Setlur 2001 Adaptive tracking control of underactuated Surface Vessels Proceedings of the IEEE International Conference on Control Application Mexico City

[9] Miswanto, I Pranoto, J Naiborhu, S Achmadi 2012 Formation Control of Multiple Dubin's Car System with Geometric Approach IOSR Journal of Mathematics (IOSRJM) 1 pp 16-20