Generalizing thawing dark energy models: the standard vis-à-vis model independent diagnostics

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ABSTRACT
We propose a two parameter generalization for the dark energy equation of state (EOS) \( w_X \) for thawing dark energy models which includes PNGB, CPL and Algebraic thawing models as limiting cases and confront our model with the latest observational data namely SNe Ia, OHD, CMB, BOSS data. Our analysis reveals that the phantom type of thawing dark energy is favoured upto 2σ confidence level. These results also show that thawing dark energy EOS is not unique from observational point of view. Though different thawing dark energy models are not distinguishable from each other from best-fit values (upto 2σ C.L.s) of matter density parameter (\( \Omega_m^0 \)) and hubble parameter (\( H_0 \)) at present epoch, best-fit plots of linear growth of matter perturbation (\( f \)) and average deceleration parameter (\( q_{av} \)); the difference indeed reflects in best-fit variations of thawing dark energy EOS, model-independent geometrical diagnostics like the statefinder pair \( \{r, s\} \) and \( Om3 \) parameter. We are thus led to the conclusion that unlike the standard observables (\( \Omega_m^0, H_0, f, q_{av} \)), the model-independent parameters \( \{r, s, Om3\} \) and the variations of EOS (in terms of \( w_X - w_X' \) plots) serve as model discriminators for thawing dark energy models.

Key words: cosmology: dark energy, thawing dark energy models, cosmological parameters, SNe Ia, OHD.

1 INTRODUCTION

Late time cosmic acceleration at the present epoch has almost been a de facto phenomenon since the late nineteenth. Advances in cosmological observations during the past two decades reveal strong evidences in favour of this accelerated expansion of the universe. These evidences have been brought forth à la independent astrophysical observations like Supernovae Type Ia (SNe Ia) luminosity distance modulus as a function of redshift [Riess et al. (1998); Perlmutter et al. (1999); Davis et al. (2007)] and [Riess et al. (2007); Wood-Vasey et al. (2007); Kowalski et al. (2008); Kessler et al. (2009); Riess et al. (2009); Amanullah et al. (2010); Suzuki et al. (2012)], Observational Hubble Data (OHD) [Jimenez et al. (2002); Abraham et al. (2004); Simon et al. (2005); Gaztanaga et al. (2009); Stern et al. (2010); Moreasco et al. (2012); Zhang et al. (2012)], Cosmic Microwave Background (CMB) Shift Parameter [Ratra et al. (1994); Podariu et al. (2001); Komatsu et al. (2003, 2011); Bennett et al. (2012)] and Baryon Oscillation Spectroscopic Survey (BOSS) Data [Sanchez et al. (2012)]. A good deal of attempts have been taken to explain this accelerated expansion assuming the presence of some exotic fluid, namely dark energy, in huge abundances in the universe. Though there exists a lot of dark energy models (see for example Bento et al. (2003); Bento et al. (2006); Chiba et al. (2003); Dutta et al. (2009); Ali et al. (2010); Dutta et al. (2010); Harko et al. (2010); Novosyadlyj et al. (2010); Chen et al. (2011a,b); Dutta et al. (2011); Hirano et al. (2011) and references therein) with standard as well as exotic ideas; the canonical and non-canonical scalar fields are the most promising candidates till date. Of late, Robert R. Caldwell and Eric V. Linder [Caldwell et al. (2005)] categorized these scalar field models in two broad classes namely “freezing” and “thawing” dark energy, based on the asymptotic behavior of the scalar field potential. In thawing models, dark energy equation of state \( w_X \) initially remains at \(-1\) and deviates from \(-1\) near present epoch whereas just the opposite behavior of \( w_X \) is witnessed in freezing models.

Thawing models, in which we are interested in the present article, are broadly classified into two categories: (i) quintessence (for which \( w_X \) moves to \( w_X^0 > -1 \)), and (ii) phantom (where \( w_X \) is less than \(-1\)). A third possibility has also been explored in [Clemson et al. (2009); Scherrer et al. (2008a,b); Dutta et al. (2009); Gupta et al. (2009); Sen et al. (2010)] which lead to both quintessence and phantom behavior of \( w_X \). In these slow-rolling scalar field models with nearly flat potential, initially the kinetic energy of the field is much smaller than the potential energy. This is because of the initial large Hubble damping which keeps the field nearly frozen at \( w_X = -1 \) at earlier era i.e., in radiation and matter dominated eras. Due to the expansion of the universe, energy density of the universe decreases. After the radiation and matter dominated eras, the field energy density becomes comparable to
the background energy density of the universe resulting in the deviation of the field from its frozen state, thereby leading to deviation of $w_X$ from $-1$.

Slow-roll scalar field thawing models can be characterized by different relations between $w_X$ and the scale factor $a$ of the universe. Some typical examples of CPL parametrization (Eq. (1)) Chevallier et al. (2001); Linder et al. (2003), PNGB models (Eq. (2)) and Algebraic thawing models (Eq. (3)) are included in the work by E. V. Linder (2008). The corresponding equation of state parametrizations are respectively given by,

$$\frac{dw_X}{d(\ln a)} = (1 + w_X)$$

(1)

$$\frac{dw_X}{d(\ln a)} = F(1 + w_X)$$

(2)

$$\frac{dw_X}{d(\ln a)} = (1 + w_X) \left( 3 - \frac{3 - p}{1 + \beta a^{-3}} \right),$$

(3)

where $F$ is a parameter which is inversely proportional to the symmetry breaking energy scale and $p$ and $b$ are two free parameters.

In the present work, we propose a two parameter generalization for this thawing dark energy models as

$$\frac{dw_X}{da} = (1 + w_X) f(a)$$

(4)

where $f(a)$ is an arbitrary function of scale factor $a$. In this article, we have chosen $f(a)$ as $f(a) = c/a^n$, where $c$ and $n$ are two arbitrary parameters. In this context we would like to mention that choice of $f(a)$ can be made otherwise and it would be interesting to see if there exists any observational constrain on the form of $f(a)$ which is beyond the scope of this article. With the chosen form of $f(a) = c/a^n$ for $n = 1$ and $c = 1$ our proposal exactly overlaps with CPL thawing dark energy model Linder et al. (2008). For $n = 1$ and $1 < c < 3$, our proposal leads to PNGB thawing dark energy model Linder et al. (2008) which have been studied exclusively for scalar fields dark energy with PNGB potential Frieman et al. (1995); Kaloper et al. (2006); Dutta et al. (2007); Rosenfeld et al. (2007). For suitable choice of the parameters $n$ and $c$, our model can approximately reflect Algebraic thawing Linder et al. (2008) as well.

As it turns out, all the existing (and probably, upcoming) thawing dark energy models fall in this broad minimal parametrization with different values of the parameters $n$ and $c$. So, rather than proposing individual models, it is quite reasonable to construct an algebraic thawing dark energy model Linder et al (2008) as well. Therefore it is difficult to provide a unique dark energy EOS $w_X$ for the thawing dark energy models as different values of $n$ with different values of $c$ lead to the same cosmological dynamics.

- Existing thawing dark energy models Linder et al. (2008) can be generalized in the form of Eq. (4) as we have presented in this article. Our minimal generalization of thawing dark energy models (Eq. (4)) with two parameters $n$ and $c$ leads to different existing thawing models namely CPL ($n = 1$, $c = 1$), PNGB ($n = 1$, $1 < c < 3$) and the Algebraic thawing (suitable choices of $n$, $c$).

- Results obtained for different $n$ values (with different values of $c$) barely differ from the observational point of view. Other way around, we can say that values of $n$ (with different values of $c$) can hardly affect the best fit values as well as the $1\sigma$ and $2\sigma$ C.L.s of matter density parameter $\Omega_m^0$ and Hubble parameter at the present epoch $H_0$. Also the best fit plots for redshift evolution of average deceleration parameter $q_{av}$ and the growth of matter perturbations in terms of evolution of growth factor $f$ with redshift $z$ (best fit plots) remain unaffected when the values of $n$ and $c$ are altered accordingly.

The major conclusions of the paper are as follows:

- The best fit values and the $1\sigma$ and $2\sigma$ C.L.s of EOS at the present epoch $w_X^0$ does leave little trace on model discrimination for thawing dark energy. Here we discuss the fact that the values of $n$ and $c$ can be constrained by $w_X - w_X^0$ plots and recent works on scalar field dark energy models point towards the non-linear $w_X - w_X^0$ plots.

- Most importantly, the model-independent parameters like statefinder pair $(r, s)$ Sahni et al. (2003) and the so called Om3 Sahni et al. (2008) parameter do play an important role in discriminating among different dark energy models. Study of these parameters in the context of our generalized thawing model, therefore, reveals the fact that unlike the standard parameters mentioned in 2nd major conclusion above, these parameters indeed serve as model discriminators. The non-linear $w_X - w_X^0$ plots can be realized for $n$ other than 1 with different values of $c$. This is an important issue as PNGB and CPL parameterizations can result only in linear $w_X - w_X^0$ plots and recent works on scalar field dark energy models point towards the non-linear $w_X - w_X^0$ plots.

The paper is organized as follows. In the next Sec. we propose the generalization for the thawing dark energy models and mention the standard as well as the model independent parameters. The Sec. 3 briefly describes the various observational data we used. In the Sec. 4 we present the results obtained by the analyses of the various observational data. In the Sec. 5 we discuss our results and put forward the conclusions of the present work.
2 THE SCHEME OF GENERALIZATION

2.1 Generalized thawing dark energy EOS

We propose a minimal two parameter generalization for thawing dark energy EOS $w_X$ as,

$$\frac{dw_X}{da} = (1 + w_X) f(a)$$

where $f(a)$ is an arbitrary function of scale factor $a$ of the universe. We study the dynamical universe with radiation, matter and thawing dark energy obeying the proposed EOS $w_X$ with $f(a) = \frac{a}{a_t}$. The proposed choice of $f(a)$ here, for the generalized thawing model is motivated by the following findings:

i) for $n = 1$ and $c = 1$, our model is exactly same as CPL parametrization (Eq 1).

ii) for $n = 1$ and $c = F$ (F being the parameter described in Sec. 1), our model is exactly same as PNGB model (Eq 2).

iii) Algebraic thawing case (Eq 3) can also approximated for certain choices of $c$ and $n$ in term of $b$ and $p$.

iv) for values of $n$ other than 1, generalized thawing dark energy EOS takes the form

$$w_X(a) = \frac{1}{1 - \Omega_m} \left( 1 + w_X(1 - a^{-1} - n) \right) \exp \left( \frac{c}{1 - 1} - a^{-1} - n \right),$$

where $w_X^0$ is the value of $w_X$ at the present epoch. Expansion of $w_X(a)$ about $a = 1$ gives,

$$w_X(a) = w_X^0 - c(1 + w_X)^2(1 - a) + \frac{1}{2} (1 + w_X)^2 c^2 - cn(1 - a)^2 + \text{higher order terms}.$$  

In order to test the validity of our generalized model we show in Fig. 1 the theoretically predicted $w_X = w_X^0$ plots for different thawing models that arise for different values of $n$ and $c$ (we will put constrains on this $w_X = w_X^0$ plane with direct observational data later in this paper). We find from Fig. 1 theoretically obtained $w_X = w_X^0$ plane for different combinations of $c$ and $n$ in our model satisfy the allowed regions for the same Caldwell et al (2005). In Fig. 1 the left plot is for quintessential thawing with $w_X^0 = -0.9$ and the right one is for the case of thawing originated in phantom scenarios with $w_X^0 = -1.1$. For $n = 1$ with $c = 1$ (dotted) we get CPL thawing (Eq. 1)) and for $n = 1$ with $c = 2, 3$ (solid and dot-dashed respectively) we get PNGB thawing (Eq. 2). The plots in black in Fig 1 indicate two models in the $w_X = w_X^0$ plane. The orange plots are for the Algebraic thawing model with $p = b = 1$ (dotted lines), $p = b = 2$ (solid lines) and $p = b = 6$ (dot-dashed lines). The results with higher values of $n$ are shown by the green ($n = 1.2$) and blue plots ($n = 1.5$). The dotted, solid and dot-dashed lines in these cases corresponds to $c = 1, 1.2, 1.5$ respectively.

### 2.2 Theoretical constraints on the models parameters $n$ and $c$

In this section we discuss the constraints on the model parameters of our generalized thawing dark energy EOS as proposed in the work by Caldwell et al Caldwell et al (2005). In Fig. 2 red region shows the allowed region of the parameter space $(n, c)$ which is allowed for thawing dark energy with our generalized EOS. It is also necessary to point out that our generalized EOS can represent dark energy models other than thawing. The region of $(n, c)$ parameter space except the red zone represents these models. This allowance of $n$ and $c$ values in Fig. 2 is also reflected in Fig. 1.

2.3 The standard and model independent parameters

As is well-known, any dark energy model must at least probe three parameters directly from observations:

i) the present value of equation of state (EOS) for dark energy ($w_X^0$)

ii) the present value of matter density ($\Omega_m^0$)

iii) the Hubble parameter today ($H_0$).

Nevertheless, dark energy model building today is tightly constrained by several observations, which, taken together, leave out a very narrow window through which the model should pass. So, from today’s perspectives, apart from the above three good old parameters, the supplementary parameters which one needs to address are the following:

The statefinder pair \{r, s\} Sahni et al (2003) serves as a geometrical diagnostic to probe the properties of dark energy in a model independent manner. This pair \{r, s\} has been studied extensively in the earlier works Panotopoulos et al (2008); Li et al (2009); Ali et al (2011); Tsujikawa et al (2010); Das et al (2011). For the late universe ($z < 10^5$), which is well approximated by the presence of matter and dark energy, the statefinder pair \{r, s\} can be expressed as,

\begin{align*}
    r &= 1 + \frac{9}{2} \Omega_X w_X (1 + w_X) - \frac{3}{2} \Omega_X \frac{dw_X}{da}, \quad (8) \\
    s &= 1 + w_X - \frac{1}{3} \frac{a dw_X}{w_X} \frac{da}{a}. \quad (9)
\end{align*}

where $a$ is the scale factor of the universe and $\Omega_X$ is the dark energy density parameter. In the late universe we have $\Omega_m + \Omega_X = 1$, $\Omega_m$ being the matter density parameter. For $\Lambda CDM$ model, it can be checked that the statefinder pair \{r, s\} takes the value $r = 1$ and $s = 0$. Any deviation in $r$ from 1 and $s$ from 0, indicates the existence of varying dark energy in the universe.

The $\Omega_m$ parameter proposed by Sahni et al Sahni et al (2003), is another tool to distinguish the dynamical dark energy from the cosmological constant. The uncertainty in matter density parameter allows significant errors in cosmological reconstruc-
Figure 1. Plots depicting generalized thawing EOS in terms of $w_X - w'_X$ plane for different parameter values as obtained from theoretical predictions. The left figure is for $w_X^0 = -0.9$ and the right figure corresponds to $w_X^0 = -1.1$. Black (orange) plots are for CPL, PNGB (Algebraic) thawing models for $w_X^0 = -0.9$ and $w_X^0 = -1.1$. The dotted, solid and dot-dashed black lines are for $c = 1$ (CPL) and $c = F = 2, 3$ for the PNGB thawing case and for Algebraic thawing case they are (in orange) for $p = b = 1, p = b = 2$ and $p = b = 6$. The blue and green curves are for our generalized thawing EOS. Green (dotted, solid, dot-dashed) lines are for $n = 1.2$ ($c = 1, 1.2, 1.5$). Similarly blue (dotted, solid, dot-dashed) lines are for $n = 1.5$ ($c = 1, 1.2, 1.5$). The area between solid red lines (overlapped with dotted and dot-dashed black lines) is the allowed thawing region [Caldwell et al. (2005)].

ditions of dark energy. $\Omega_m$ parameter can in practice differentiate between the models, independent of the matter density parameter. The $\Omega_m$ diagnostic has been studied well in the earlier works [Lu et al. (2009); Sahni et al. (2008); Nesseris et al. (2010); Shafieloo et al. (2010); Huang et al. (2011); Shafieloo et al. (2012)]. $\Omega_m$ parameter is defined in terms of Hubble parameter which can directly be measured in cosmological observations. The two-point $\Omega_m$ Sahni et al. (2008) diagnostic is given by,

$$\Omega_m(z_1, z_2) = \frac{h^2(z_2) - h^2(z_1)}{(1 + z_2)^3 - (1 + z_1)^3},$$  

(10)

where $h(z) = H(z)/H_0$.

It can be easily seen that for cosmological constant $\Omega_m(z_1, z_2) = 0$ and when $z_1 < z_2$, $\Omega_m(z_1, z_2) > 0$ ($\Omega_m(z_1, z_2) < 0$) represents the case of quintessence (phantom) Sahni et al. (2008). This is how $\Omega_m$ evaluated at two different redshifts ($z_1$ and $z_2$) can help in distinguishing the dark energy model. Needless to mention that this procedure is independent of $\Omega_{m0}$ and $H_0$. The three-point diagnostic $\Omega m 3$ Sahni et al. (2008) is defined by,

$$\Omega m 3 (z_1, z_2, z_3) = \frac{\Omega_m(z_2, z_1)}{\Omega_m(z_3, z_1)}.$$  

(11)

For $\Lambda CDM$ model $\Omega m 3 = 1$.

Another dimensionless parameter, which is useful for determining the beginning of cosmic acceleration in dark energy model, is the average deceleration parameter $q_{av}$, defined as [Sahni et al. (2008)].

$$q_{av} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} q(t)dt,$$  

(12)

where $q(t)$ is the deceleration parameter.

We use Eqs. (8, 9, 11, 12) for evaluating the statefinder pair \{r, s\}. $\Omega m 3$ and $q_{av}$ for the case of our generalization of thawing dark energy model.

Further more, we investigate the growth factor $f$ in the context of this proposed generalized thawing EOS. For this purpose we assume the generalized thawing dark energy models proposed here, are decoupled from the cold matter sector. This would lead to the effect that the galaxy cluster formation is not directly influenced by the existence of dark energy. But the presence of dark energy alters the Hubble expansion rate which affects the growth of inhomogeneities in the cold matter sector. In the linear regime of matter perturbations, the evolution of the inhomogeneities are governed by the relation [Wang et al. (1998)].

$$\frac{d^2 \ln \delta}{d\ln a^2} + \left(\frac{d\ln \delta}{d\ln a}\right)^2 + \frac{1}{2} \frac{d\ln \delta}{d\ln a} (1 - 3w_X(1 - \Omega_m)) = \frac{3}{2} \Omega_m,$$

(13)

where $\delta = \delta \rho_m/\rho_m$ is the matter density contrast with $\rho_m$ being the matter density. The growth factor $f$ is defined as [Wang et al. (1998)].

$$f = \frac{d\ln \delta}{d\ln a}.$$  

(14)

Eq. (13) can be written in terms of growth factor $f$ (defined in Eq. (14)) as,

$$\frac{df}{d\ln a} + f^2 + \frac{1}{2} f (1 - 3w_X(1 - \Omega_m)) = \frac{3}{2} \Omega_m.$$  

(15)

The growth equation can be expressed in terms of the redshift $z$ by the relation $\ln a = -\ln(1 + z)$. The growth factor is well approximated by the ansatz [Wang et al. (1998)].

$$f = \Omega_m(z)^{\gamma}.$$  

(16)

where $\gamma$ is termed as ”growth index”. The growth factor $f$ is affected by dark energy models via $\Omega_m(z)$.

3 COMPILATION OF COMBINED DATASETS

For the purpose of putting constraints on the generalized thawing dark energy EOS, we use the latest Supernova Type Ia (SNe Ia) Data from the Union 2.1 compilation [Suzuki et al. (2012)], Observational Hubble Data (OHD) [Jimenez et al. (2002); Abraham et al. (2004); Simon et al. (2005); Gaztanaga et al. (2009); Stern et al. (2010); Moresco et al. (2012); Zhang et al. (2012)]; Cosmic Microwave Background Data (CMB) from 9 year WMAP results [Bennett et al. (2012)] and BOSS data from SDSS-III [Sanchez et al. (2012)]. There are a total of 607 data points (580 data point from
SNe Ia, 25 from OHD, and 1 each from CMB and BOSS). We make a combined $\chi^2$ analyses of the data sets comprising of all 607 data points to constrain our model parameters $w,X, \Omega_m^0$, and $H_0$, as well as to confront with the model-independent parameters mentioned in Section 2. This makes our analysis robust.

3.1 Union 2.1 compilation of Supernova Type Ia Data

Luminosity distance ($d_L$) measurement of distant supernovae with redshifts $z$ is the first observational data to probe the current acceleration of the universe and the dark energy properties as well. The most recent compilation of the Supernova Type Ia Data is given by Union 2.1 dataset [Suzuki et al. (2012)]. The data is tabulated in terms of distance modulus $\mu(z)$ with redshift $z$. The distance modulus can be written as

$$\mu(z) = 5 \log_{10}(D_L(z)) + \mu_0,$$

where $D_L(z) = H_0 d_L(z)$ (speed of light in vacuum is normalized to unity) and $\mu_0 = 42.38 - 5 \log_{10} h$ with $h$ given by $H_0 = 100h$ Km.Sec$^{-1}$.Mpc$^{-1}$. $\chi^2$ of SNe Ia data is given by,

$$\chi^2_{SN}(w^0_X, \Omega_m^0, H_0) = \sum_i \left[ \frac{\mu_{obs}(z_i) - \mu(z_i; w^0_X, \Omega_m^0, H_0)}{\sigma_i} \right]^2,$$

Marginalizing over the nuisance parameter $\mu_0$, one gets the $\chi^2$ as,

$$\chi^2_{SN}(w^0_X, \Omega_m^0) = A - B^2/C,$$

where $A$, $B$ and $C$ are given by,

$$A = \sum_i \left[ \frac{\mu_{obs}(z_i) - \mu(z_i; w^0_X, \Omega_m^0, \mu_0 = 0)}{\sigma_i} \right]^2,$$

$$B = \sum_i \left[ \frac{\mu_{obs}(z_i) - \mu(z_i; w^0_X, \Omega_m^0, \mu_0 = 0)}{\sigma_i} \right],$$

$$C = \sum_i \frac{1}{\sigma_i^2}$$

3.2 Observational Hubble Data (OHD)

Measurements of Hubble parameters from differential ages of galaxies provide another way to probe the late time acceleration of the expanding universe. Jimenez et al. [Jimenez et al. (2002)] first utilized this idea of measuring Hubble parameter through the differential age method, Simon et al. [Simon et al. (2005)] and later Stern et al. [Stern et al. (2010)] provides the values of the Hubble parameter in the redshift range $0.1 \leq z \leq 1.8$ and $0.35 < z < 1$ respectively. A total of 21 OHD data points are recorded at present in the literature. Jimenez et al. (2002, 2008), Simon et al. (2005), Gaztanaga et al. (2008, 2009, 2010), Stern et al. (2010, 2012); Mozurkewich et al. (2012). With the data release 7 (DR7) from Sloan Digital Sky Survey (SDSS) Zhang et al. [Zhang et al. (2012)] provides 4 new values of Hubble parameters at different redshifts. All the 25 OHD data points, used in this work to constrain the model parameters, are listed in Table [II].

The $\chi^2$ function for the analysis of this observational Hubble data can be defined as

$$\chi^2_{OHD}(w^0_X, \Omega_m^0, H_0) = \sum_{i=1}^{15} \left[ \frac{H_{obs}(z_i) - H(z_i; w^0_X, \Omega_m^0, H_0)}{\sigma_i} \right]^2$$

| $z$ | $H(z)$ (km sec$^{-1}$ Mpc$^{-1}$) | $\sigma_H$ (km sec$^{-1}$ Mpc$^{-1}$) |
|-----|---------------------------------|----------------------------------|
| 0.900 | 69 | 12 |
| 0.170 | 83 | 8 |
| 0.270 | 77 | 14 |
| 0.400 | 95 | 17 |
| 0.900 | 117 | 23 |
| 1.300 | 168 | 17 |
| 1.430 | 177 | 18 |
| 1.530 | 140 | 14 |
| 1.750 | 202 | 40 |
| 0.480 | 97 | 62 |
| 0.880 | 90 | 40 |
| 0.179 | 75 | 4 |
| 0.199 | 75 | 5 |
| 0.352 | 83 | 14 |
| 0.593 | 104 | 13 |
| 0.680 | 92 | 8 |
| 0.781 | 105 | 12 |
| 0.875 | 125 | 17 |
| 1.037 | 154 | 20 |
| 0.24 | 79.6 | 3.32 |
| 0.43 | 86.45 | 3.27 |
| 0.07 | 69.0 | 19.6 |
| 0.12 | 68.6 | 26.2 |
| 0.20 | 72.9 | 29.6 |
| 0.28 | 88.8 | 36.6 |

Table 1. Hubble parameter ($H(z)$) versus redshift ($z$) data from Jimenez et al. (2002); Abraham et al. (2004); Simon et al. (2005); Gaztanaga et al. (2008, 2009, 2010); Mozurkewich et al. (2012)). Here $H(z)$ and $\sigma_H$ are in km sec$^{-1}$ Mpc$^{-1}$.

3.3 CMB Shift Parameter Data

CMB shift parameter $R$, to a great extent, is a model independent quantity extracted from CMB power spectrum. It is given by

$$R(z_s) = \frac{\Omega_m H_0^2}{(1+z_s)^3},$$

where $z_s$ is the redshift value at the time when photons decoupled from matter in the universe. $z_s$ can be calculated as (with $\Omega_b$ being the baryon density parameter)

$$z_s = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}[1 + g_1(\Omega_m h^2)^{g_2}],$$

where the functions $g_1$ and $g_2$ read as

$$g_1 = 0.0783(\Omega_b h^2)^{-0.238}(1 + 39.5(\Omega_b h^2)^{-0.763})^{-1},$$

$$g_2 = 0.560(1 + 21.1(\Omega_b h^2)^{1.81})^{-1}.$$ (25)

$\chi^2_{CMB}$ is defined as,

$$\chi^2_{CMB}(w^0_X, \Omega_m^0, H_0) = \frac{R(z_s, w^0_X, \Omega_m^0, H_0) - R}{\sigma_R}^2$$

From WMAP 9 year results [Bennett et al. (2012), we use $R = 1.728 \pm 0.016$ at the radiation-matter decoupling redshift $z_s = 1090.97$.

3.4 Baryon Oscillation Spectroscopic Survey (BOSS)

CMASS Data Release 9 (DR9) sample of Baryon Oscillation Spectroscopic Survey (BOSS) (a part of SDSS-III) provides constraint on the dimensionless combination $A(z) = D_H(z) \sqrt{\Omega_m H_0^2 / z}$ independent of $H_0$). We use the measured value of $A(z)$ at $z = 0.57$
(A_{obs}(0.57) = 0.444 \pm 0.014 \textbf{Sanchez et al} (2012)) to constrain our model parameters space.

The $\chi^2$ for the BOSS data is defined as

$$\chi_{\text{BOSS}}^2(w_X^0, \Omega_m^0) = \frac{[A_{obs}(0.57) - A(0.57, w_X^0, \Omega_m^0)]^2}{0.016^2}.$$  \hspace{1cm} (27)

### 3.5 Combined $\chi^2$ analyses

Combining all the datasets from Sections (3.1) - (3.4), comprising of altogether 607 data points, the combined $\chi^2$ can be evaluated as:

$$\chi^2(w_X^0, \Omega_m^0, H_0) = \chi^2_{\text{SN}}(w_X^0, \Omega_m^0) + \chi^2_{\text{HOD}}(w_X^0, \Omega_m^0, H_0) + \chi^2_{\text{CMB}}(w_X^0, \Omega_m^0, H_0) + \chi_{\text{BOSS}}^2(w_X^0, \Omega_m^0).$$  \hspace{1cm} (28)

In what follows, we minimize this combined $\chi^2$ with the observational data sets and search for possible consequences by confronting our generalized model directly with observations.

In the case we consider all the dark energy models i.e., thawing as well as non-thawing that can arise from our generalized EOS the total $\chi^2$ will be function of $n$, $c$, $\Omega_m^0$, $w_X^0$, $H_0$ when we consider combined data sets consisting of SNe Ia, BAO, OHD and CMB Shift parameter data. Marginalized $\chi^2$ in general is defined as [Nesseris et al. (2005); Perivolaropoulos et al. (2005)].

$$\chi^2(p_s, \theta) = -2 \ln \int_{\theta_2}^{\theta_1} \exp \left[ \frac{1}{2} \chi^2(p_s, \theta) \right] d\theta,$$

where $\chi^2(p_s, \theta)$ is marginalized over the parameter $\theta$ in the range $\theta_1 < \theta < \theta_2$.

### 4 DATA ANALYSIS AND RESULTS

In this section our primary objective is to make a combined $\chi^2$ analysis for our generalized model as proposed in Eq (4) with SNe Ia, OHD, CMB and BOSS data for the evaluation of the parameters space and their 1$\sigma$ and 2$\sigma$ confidence level (C.L.) limits. We further study these cases to compare between the results for $n = 1$ and other values of $n$ with the different values of $c$. Our results are tabulated in Table 2 and 3. There are five parameters in this generalized thawing model and they are $n$, $c$, $w_X^0$, $\Omega_m^0$, and $H_0$. We fix the values of $n$ at 1, 1.5, 2 with different values of $c$ so that we can compare different thawing models and find the best fit values of other three parameters by $\chi^2$ analyses. The results of $\chi^2$ analyses for PNGB and CPL models are furnished as Case I below and the $\chi^2$ analyses results for other thawing models with $n = 1.5$ and $n = 2$ are presented as Case II and Case III respectively.

#### 4.1 Standard parameters for different values of $n \neq c$

**Case I: $n = 1$ (CPL & PNGB)**

In what follows, we describe the results obtained for CPL and PNGB cases which can be obtained from the proposed generalization of $w_X$ (Eq. (4)) with $n = 1$. The $\chi^2$ analyses results for $n = 1$ with different values of $c$ are tabulated in Table 2. These are the cases of PNGB ($1 < c < 3$) and CPL ($c = 1$) thawing dark energy models. Here we choose the values of $c$ to be 1, 1.5, 2. It is seen from Table 2 that best-fit results ($w_X^0$) point towards the existence of phantom type thawing dark energy in the universe. As the parameter $c$ goes on taking higher values the phantom nature gets enriched i.e., the deviation of $w_X^0$ from $-1$ goes on increasing. During this change of EOS ($w_X^0$), the value of matter density parameter at present epoch and present epoch value of the Hubble parameter remain unaltered. Also needless to mention here that the values of total $\chi^2$ remain unchanged as is evident from Table 2.

In Fig. 3 the $1\sigma$ and $2\sigma$ contours of the different observables e.g., $w_X^0$, $\Omega_m^0$ and $H_0$ for $n = 1$ with different values of $c$ are shown by light blue and dark blue shaded regions respectively. The “o” in the plots represents the best fit values obtained by $\chi^2$ minimization (Table 2). Here one can see that the phantom kind of thawing dark energy is more favoured than the quintessence type upto 2$\sigma$ C.L.

| $n$ | $c$ | best-fit values of $(w_X^0, \Omega_m^0, H_0)$ | Minimum value of $\chi^2$ |
|-----|-----|---------------------------------------------|--------------------------|
| 1   | 1   | (-1.009, 0.28, 70.5)                       | 575.6                    |
| 1   | 1.5 | (-1.011, 0.28, 70.5)                       | 575.6                    |
| 1   | 2   | (-1.013, 0.28, 70.5)                       | 575.6                    |

**Table 2.** Best-fit values of parameters and minimum values of $\chi^2$ from combined $\chi^2$ analyses of SNe, CMB, OHD and BOSS data for Case I.

**Case II: $n = 1.5$**

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Here we investigate the other thawing model that can be originated for $n = 1.5$. The $\chi^2$ minimization results obtained for $n = 1.5$ with $c = 0.5, 1, 1.5$ are tabulated in Table 3. Here also the best-fit results suggest that the nature of thawing dark energy is of phantom kind and as $c$ increases the deviation of $w_X$ from $-1$ gets increased. One also sees from Table 3 that the best fit values of present epoch matter density parameter $\Omega_m$ remain unchanged as the values of $c$ changes. It is also observed that the best fit values of the Hubble parameters $H_0$ at the present epoch also have hardly undergone any changes in these cases. Like the previous case $\chi^2$ remains unchanged.

In Fig. 4 the best fit values (obtained from $\chi^2$ minimization) are shown with “*” symbol and the $1\sigma$ and $2\sigma$ contours for different observables e.g., $w_X^0$, $\Omega_m^0$ and $H_0$ are given by light blue and dark blue color shadings respectively. Here one can observe that the phantom type of thawing dark energy is more favoured over the quintessence upto $2\sigma$ confidence level.

**Case III: $n = 2$**

Moving onto the $n = 2$ thawing scenario, here the results for $n = 2$ with $c = 0.5, 1, 1.5$ are presented in Table 4. Like the previous two cases discussed above, it is also evident here that the best-fit $w_X$ points towards the phantom nature of thawing dark energy present in the universe. Also it is seen that $w_X$ decreases with the increasing value of $c$ leaving no significant signatures in the best-fit values of $\Omega_m$ and $H_0$. Also the $\chi^2$ in this case remains unchanged like the previous two cases.

### Table 3. Best-fit values of parameters and minimum values of $\chi^2$ from combined $\chi^2$ analyses of SNIa, CMB, OHD and BOSS data for Case II.

| $n$ | $c$ | best-fit values of $(w_X^0, \Omega_m^0, H_0)$ | Minimum value of $\chi^2$ |
|-----|-----|-----------------------------------------------|---------------------------|
| 1.5 | 0.5 | (-1.008, 0.28, 70.5)                           | 575.6                     |
| 1.5 | 1   | (-1.010, 0.28, 70.5)                           | 575.6                     |
| 1.5 | 1.5 | (-1.012, 0.28, 70.5)                           | 575.6                     |

### Table 4. Best-fit values of parameters and minimum values of $\chi^2$ from combined $\chi^2$ analyses of SNIa, CMB and OHD and BOSS data for Case III.

| $n$ | $c$ | best-fit values of $(w_X^0, \Omega_m^0, H_0)$ | Minimum value of $\chi^2$ |
|-----|-----|-----------------------------------------------|---------------------------|
| 2   | 0.5 | (-1.008, 0.28, 70.5)                           | 575.6                     |
| 2   | 1   | (-1.011, 0.28, 70.5)                           | 575.6                     |
| 2   | 1.5 | (-1.013, 0.28, 70.5)                           | 575.6                     |
As in the previous two occasions, best fit values (obtained from \( \chi^2 \) minimization) and 1\( \sigma \), 2\( \sigma \) contours are denoted by \( \sim \) and light blue, dark blue color shades respectively in Fig. 5. Here also it is easy to figure out that the thawing dark energy can be of both quintessence as well as phantom kind (more favoured).

Now we compare the results for different values of \( n \) with a particular value of \( c \). For \( c = 1 \), one can figure out from the Tables 2, 3, 4 that as \( n \) value increases from 1 to 2, \( w_X \) shifts from \(-1.009 \) to \(-1.011 \) indicating the enhancement of phantom nature of thawing. The present values of matter density parameter \( \Omega_{m}^{0} \) and Hubble parameter \( H_{0} \) remain unchanged in these cases. The same analogy goes for \( c = 1.5 \) case. From the above discussions this is apparent that all the three thawing models (that can be represented by a single form proposed in this work (Eq. (4))) produce identical \( \Omega_{m}^{0} \) and \( H_{0} \) values at least upto 2\( \sigma \) C.L.

In Fig. 5 the growth factor \( f \) is plotted against the number of \( e \)-foldings \( N = \log(a) \) for different best fit values of \( w_{X}^{0}, \Omega_{m}^{0} \), and \( H_{0} \) obtained in the Tables 2, 3, 4. The left (right) panel is with the initial condition \( f(N = -7) = 0.8 \) (\( f(N = -7) = 0.9 \)) for \( n = 1, 1.5, 2 \) with different values of \( c \) as described in Case I, Case II and Case III in this section. The evolution of the growth factor \( f \) is identical in all the cases suggesting the formation of the same large scale structure in all cases of thawing considered here (i.e., for CPL and PNGB \((n = 1)\), Algebraic thawing for \( n = 1.5 \) and \( n = 2 \)). Therefore the growth factor \( f \) does not serve as a model discriminator but acts as a supplementary probe to confirm correct estimation of cosmic structures formed.

Fig. 6 depicts the best-fit variation of \( w_{X} \) with \( w_{X} \) as obtained using the best fit values of \( w_{X}^{0} \) from the Tables 2, 3 and 4 for different combinations of \( n \) and \( c \). The plots show that we can indeed have non-linear behavior of \( w_{X} \) along with the linear behavior for the generalized thawing dark energy model from observations. Comparison of these plots with our theoretical predictions, as done in Fig. 1 will be interesting. Therefore Fig. 7 goes over Fig. 1 which was only a theoretical prediction. As it turns out from this figure, the \( w_{X} - w_{X} \) plane indeed serves as a model-discriminator for different thawing dark energy models.

### 4.2 Model-independent diagnostics

In Fig. 8 we show the best-fit variations of the statefinder parameters \( \{r, s\} \) with redshift \( z \) for \( n = 1 \) case (with the best-fit values of \( w_{X}^{0}, \Omega_{m}^{0} \), and \( H_{0} \) presented in the Tables 2) which is known as CPL for \( c = 1 \) or PNGB for other values of \( c \). The dashed, solid and dotted plots are for \( c = 1, 1.5, 2 \) respectively. These plots bear the clear signatures of thawing as one can see that for higher values of \( z \), the statefinder \( r \) tends to 1 and the statefinder \( s \) to 0. This is because \( w_{X} = -1 \) as \( z \) increases and since in present epoch \( w_{X} \) deviates from \(-1 \), \( r \) and \( s \) also deviates from 1 and 0 respectively. The same features are also observed in the cases of \( n = 1.5 \) and \( n = 2 \) in Fig. 9 and Fig. 10 respectively.

In Fig. 11 we show the best-fit variation of the Om3 parameters with the redshift \( z_{3} \) while \( z_{1} \) and \( z_{2} \) are kept at \( z_{1} = 0.2 \) and \( z_{2} = 0.57 \) for the best-fit values of \( w_{X}^{0}, \Omega_{m}^{0} \), and \( H_{0} \) presented in the Tables 2, 3, 4. The plot at the extreme left of Fig. 11 shows the variation of Om3 parameter for \( n = 1 \) with \( c = 1 \) (dashed), \( c = 1.5 \) (dotted) and \( c = 2 \) (dotted). The middle and the right plots of Fig. 11 show similar variations for \( n = 1.5 \) and \( n = 2 \) respectively with \( c = 0.5 \) (dashed), \( c = 1 \) (dotted) and \( c = 1.5 \) (dotted). As Om3 is a three point diagnostic, we need three redshift points to measure its value. We fix two redshift points \( z_{1} \) and \( z_{2} \) with \( z_{1} = 0.2 \) [Blake et al. (2011)] and \( z_{2} = 0.57 \) [Sanchez et al. (2012)] and allow \( z_{3} \) to be a variable. All the variation starts from a point where \( z_{3} = z_{2} \) that leaves Om3 = 1 and the immediate deviation of Om3 from 1 to less than 1 suggests the phantom nature of dark energy which is here the varying phantom thawing dark energy.

In Fig. 12 the best-fit variation of average deceleration parameter \( q_{av} \) has been plotted with the best fit values of the parameters \( w_{X}^{0}, \Omega_{m}^{0} \), and \( H_{0} \) obtained in the Tables 2, 3, 4. It is seen from the plots that all of them overlap with each other. It is thus evident that average deceleration parameter is not capable of being a model discriminator, but it does indicate the transition period from the deceleration to acceleration phase. In this case this transition occurs nearly at the redshift \( z \sim 7 \) as is evident from the best fit plots.
4.3 Observational constraints on the model parameters

In this section we present the result for marginalized contour of $n$ and $c$ (Fig. 13) with the other parameters $w_X^0$ and $\Omega^0_m$, marginalized over the ranges $-1.7 < w_X^0 < -0.2$ and $0.1 < \Omega^0_m < 0.9$. In Fig. 13 the light blue and the dark blue shades represent the 1$\sigma$ and 2$\sigma$ contours. We also present the marginalized contour of $w_X^0$ and $\Omega^0_m$ (Fig. 14) with the model parameters $n$ and $c$ marginalized over the ranges $0.1 < n < 3$ and $0.1 < c < 20$. In Fig. 14 the areas enclosed by the smaller inner contour and the bigger outer contour represents respectively, 1$\sigma$ and 2$\sigma$ allowed regions. In performing so the fact that we have included thawing dark energy models as well as dark energy models which are not thawing, is evident from the Fig. 2. We also would like to mention that in this process we have used the type Ia supernova data, baryon oscillations spectroscopic survey data and the cosmic microwave radiation shift parameter data.

The values of $n$ and $c$ leading to thawing dark energy models with our generalized dark energy EOS (Eq. (4)) is described in the subsection II A and II B. From Fig. 13 one can notice that present day data does not put any strong constraints on the dark energy models, i.e., claiming that dark energy is thawing is not perhaps completely justified from the observational point of view. In other words, data does not restricts us to thawing dark energy models only or present day data is insufficient to favour any particular class of dark energy models at present.
DISCUSSIONS AND CONCLUSIONS

In the present work, we proposed a two parameter generalized EOS, \( w_X \), for thawing dark energy models and studied the dynamics of spatially flat FRW universe containing radiation, matter and dynamical dark energy. This proposal of ours is a minimal generalization of thawing dark energy EOS and is given by,

\[
\frac{w'},{a^n} = \left(1 + w_X\right)^{\frac{c}{(n-1)}}(1 - a^{(1 - n)})
\]  

This leads to \( w_X(a) = -1 + (1 + w_X^0) a^c \) for \( n = 1 \) and for other values of \( n \), \( w_X(a) = -1 + (1 + w_X^0) \exp\left[\frac{c}{(n-1)}(1 - a^{(1 - n)})\right] \), where the scheme is that each value of the parameters \( n \) and \( c \) defines a specific thawing model, tuning them will lead to a second model, and so on. We have also demonstrated that this minimal generalization scheme is quite apt as it naturally goes over the well-known thawing dark energy models such as CPL, PNGB and Algebraic thawing, for suitable choice of the two parameters \( n \) and \( c \).

We have elaborately discussed the cases with \( n = 1 \), \( n = 1.5 \) and \( n = 2 \) for different values of \( c \) (\( c = 1, 1.5, 2 \) for \( n = 1 \) and \( c = 0.5, 1, 1.5 \) for both the cases of \( n = 1.5, 2 \)). We have shown
that though the parameter $c$ is very important for the slope of the $w_{\chi}(a)$ vs. $a$ plot, it barely changes the dynamics of the universe. This is quite evident from the average deceleration parameter ($q_{av}(z)$) vs redshift $z$ plot (Fig. 12), growth parameter plots (Fig. 6) etc and also from the present values of matter density parameter $\Omega_{m0}^0$ and the Hubble parameter $H_0$ as well. In this context it is therefore very important to mention that fine tuning of $c$ does not, at all, effectively change the observables like the values of density parameters and the Hubble parameter at the present epoch (vide Tables 2, 3 and 4 for best-fit values and Figs. 3, 4, 5 for plots arising from scalar field models. (e.g., PNGB and CPL cases), the present epoch value of radiation density parameter $\Omega_{r0}$ which will not change the total density parameter up to four decimal places and therefore not considering it as a parameter will not affect density parameters $\Omega_{m0}^0$ or $\Omega_{\lambda0}^0$ significantly.

Also we would like to conclude that different values of $n$ would lead to same cosmological dynamics for a particular value of $c$ which is evident from average deceleration parameter plot (Fig. 12) and growth factor plots (Fig. 6). These plots clear demonstrate that it is hardly possible to distinguish between the results for different thawing models (related to different values of $n$ and $c$). It is necessary here to mention that in calculating growth factor $f$, we have considered those thawing models that do not modify the Newton’s constant $G$. There exists a class of non-minimally coupled scalar field models that give rise to thawing and modify the Newton’s constant $G$, as well (see Ali et al (2012), Hossein et al (2012)). In those cases, no generic form for effective Newton’s constant $G_{\text{eff}}$ exists as the modification depends on the nature of non-minimal coupling. Therefore we exclude those thawing models in our proposal of generalized thawing EOS (Eq. (3)).

The analysis thus reveals a very crucial information about the general class of thawing dark energy models, namely, different thawing dark energy models can not be distinguished with the present-day values of matter density, Hubble parameter, the growth factor plots and the average deceleration parameter plots. The importance of our analysis further lies in the fact that this is a quite generic conclusion, since our proposition does take into account within itself almost all the thawing dark energy models. So, we claim that one indeed needs to go beyond these parameters in order to distinguish among thawing dark energy models.

These distinguishers come in the form of geometrical diagnostics like statefinder pair $r, s$ and the Om3 parameters. Even though observational data for these parameters are lacking till today, the analysis succeeds in giving some important predictions which, we believe, may show a direction of which way to proceed in near future. These statefinder pair $r, s$ and the Om3 parameter are shown in Figs. 8, 9, 10, 11 respectively.

We have also shown in Fig. 11 that the best-fit $w_{\chi} - w'_{\chi}$ plots can, as well, serve as another discriminator for those thawing models. Nevertheless, it is also shown in Fig. 7 that $w_{\chi} - w'_{\chi}$ plots are non-linear for $n = 1.5$ and $n = 2$. For the existing thawing models (e.g., PNGB and CPL cases), $w_{\chi} - w'_{\chi}$ plots are strictly linear. In a recent work, Ali et al (Ali et al (2009)) have found this kind of nonlinear $w_{\chi} - w'_{\chi}$ plots arising from scalar field models. So our generalization can also produce the values for $n$ other than 1 and they are also favoured well by the recent cosmological observations. Moreover from Fig. 7 it can be noted that as $n$ takes higher values, only lower values of $c$ are allowed for thawing dark energy models.

From the Fig. 7 one can finds the values of $n$ and $c$ that would lead to the thawing models with our generalized form of dark energy EOS (Eq. (4)). Therefore it easy to note that the values of $n$ studied in Tables 2, 3 i.e., $n = 1$ and some of $n = 1.5$ lead to thawing but others (Table 4) are not thawing which is also reflected in the Fig. 4. It leads us to also conclude that the values of $\Omega_{m0}$ and $H_0$ are same for the thawing as well as the non thawing dark energy models (Tables 2, 3 and 4 and the Figs. 3, 4, 5). Therefore the beauty of the parametrization lies in its form which generalizes the thawing models as well as includes other dark energy models which gives us the opportunity to study all the models together in the context of present day observational data.

Also from the Tables 2, 3 and 4 one can see that the $\chi^2$ values are a bit low. This is because the error bars in the data sets namely type Ia supernova data, baryon oscillation spectroscopy data, hubble parameter data and the cosmic microwave background shift parameter data are large with respect to this generalized model and in the definition of $\chi^2$ as the error bars appear in the denominator, we get the a bit low value of $\chi^2$. If the error bars are reduced in the data sets better results can be obtained and we hope to have well constraints on the model parameters in this generalized model in near future.

As mentioned, this is a minimal generalization with the two parameters $c$ and $n$ and one boundary condition given by $w_{\chi}(z = 0) = w'_{\chi0}$, $z$ being the redshift. There may exist other generalizations with more than two parameters. So selection can be made on the basis of Akaike Information Criteron (AIC) Akaike et al (1974) and the Bayesian Information Criteron (BIC) Schwarz et al (1978) that are defined as,

$$AIC = -2 \ln(\mathcal{L}) + 2p,$$

$$BIC = -2 \ln(\mathcal{L}) + p \ln N,$$

where $\mathcal{L}$ is the maximum likelihood value which is given by $\exp(-\chi^2_{\text{min}}/2)$, $p$ is the number of model parameters and $N$ is the number of data points used to find the minimum value of the $\chi^2$ denoted by $\chi^2_{\text{min}}$. We show the $\Delta AIC$ and $\Delta BIC$ values in Table 5.

| Model  | $\Delta AIC$ | $\Delta BIC$ |
|--------|--------------|--------------|
| CPL    | 0            | 0            |
| PNGB   | 2            | 6.39         |
| Our Model | 4           | 12.78        |

Table 5. A comparative analysis of the values of Information criteria using combined $\chi^2$ analyses of SNeIa, CMB, OHD and BAO data.
calculation, which gives a value after integrating over all probable states, and hence, does not suffer from any such limitations of AIC or BIC. Hence, nowadays, most of the cosmological models are relying more on Bayesian Evidence calculation, rather than \( \Delta AIC \) or \( \Delta BIC \) calculation. We hope to address this issue in near future.

We are in the era of precision cosmology. Observational data are improving day by day. But these are the error bars that the data come with makes the constraints on the models poor. Therefore it is very necessary to reduce the error bars which can improve the constraints on the model parameters further. We used Type Ia supernova data, Baryon Oscillation spectroscopic survey data, observational hubble data and the cosmic microwave shift parameter data to constrain the models parameters. Among all these data supernova data influences the analysis the most i.e., the constrained parameters space depend on supernova data to a great extent. Supernovae data are not that precise at the present moment as it comes with large error bars. Improving supernova data can probably give us the improved and satisfactory results in future.

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