Assessment of Speckle Noise Reduction in Digital Images using Nonlinear Anisotropic Diffusion: An Experimental Investigation

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Abstract

Objectives: To investigate on choice of suitable image quality assessment techniques and effect of parameters such as gradient threshold, time step, number of iterations etc. on anisotropic diffusion filtering. Methods/Statistical Analysis: Various diffusivity functions and the individual contribution of associated parameters are compiled and evaluated. Edge preservation in smoothing is an important issue while filtering digital images. A recently proposed image quality metrics is used for evaluating the edge preserving ability of filters. A comparative study is carried out on the basis of experiments and their performance is tested on standard test images using 16 different image quality metrics. Findings: The experimental findings show that most of the true edges got lost for higher number of iterations along with more smoothening of images in techniques namely, Perona and Malik diffusion, Speckle Reducing Anisotropic Diffusion (SRAD) and Weickert anisotropic diffusion. Further, in robust anisotropic diffusion it is found that conduction coefficient plays significant role in filtering process. Higher value of conduction coefficient results in more smoothening with blurring of sharp details and edges. It is concluded that image quality measures such as universal Quality index (Q), Edge Retrieval Index (ERI) and Structural Content (SC) can give significant information about image quality even in cases where conventional image quality metrics such as Picture Signal To Noise Ratio (PSNR), Mean Square Error (MSE), Root Mean Square Error (RMSE), geometric average error (GAE), normalized absolute error (NAE) etc. fail to do so. Proposed approach can give significant information when conventional metrics fail to assess the filtered image quality. Application/Improvements: The behavioural characteristics of techniques studied will be suggestive when applied on real time noisy images such as ultrasound images which inherently contain speckle.

Keywords: Diffusion, Diffusivity Function, Edge Preservation, Edge Retrieval Index, Noise Filtering, Speckle

1. Introduction

Noise is a major problem in almost all imaging modalities. Noise may be introduced in images due to intrinsic artifacts, atmospheric turbulence and noisy sensors. Image filtering is an important pre-processing step in digital image processing. This paper focuses specifically on removal of speckle noise from digital images. If \( \eta_s \) be the speckle noise introduced by acquisition device then the generalized model for speckle imaging neglecting additive noise is given by:

\[
I_n = \eta_s I_o \tag{1}
\]
where, $I_o$ is the original noise free image, $I_n$ is the noisy image and $\eta$ is the multiplicative noise. Effective filtering of speckle noise while preserving object boundaries, fine details, sharp discontinuities and high resolution are major research areas in computer vision and image processing.

Linear filtering techniques like Gaussian filtering for removal of speckle noise results in blurring of edges and boundaries. Linear filters sometimes also dislocate some important image features such as edges resulting in poor resolution. To deal with the drawbacks of linear filtering techniques Perona and Malik introduced the concept and formulation of anisotropic diffusion filtering. Anisotropic diffusion filtering has found wide number of applications in removing speckle noise from digital images including medical images such as ultrasound and MRI (Magnetic Resonance imaging) images. Diffusion algorithms use partial differential equation to denoise image. In its simplest form diffusion equation is represented as:

$$\partial I_t = \text{div}(D \nabla I)$$  \hspace{2cm} (2)

$$I(t = 0) = I_o$$

where ‘$\nabla$’ is the gradient operator, ‘$\text{div}$’ is the divergence operator, ‘$D$’ is diffusion coefficient, ‘$I_o$’ is the initial image and $I$ is the smoothed image at time $t$. In linear diffusion filtering, $D$ is constant while in nonlinear diffusion filtering $D$ is function of some differential characteristics of image. Representing $D$ as function of edge gradient in Perona and Malik diffusion, the initial image $I_o$ is modified through anisotropic diffusion as given by:

$$\partial I_t = \text{div}(g\|\nabla I\|\nabla I)$$  \hspace{2cm} (3)

$$I(t = 0) = I_o$$

where $g\|\nabla I\|$ is called diffusivity function. For simplicity $g\|\nabla I\|$ is represented as $g(x)$. The function $g(x)$ is usually a non-negative monotonically decreasing function approaching zero at infinity. Perona and Malik diffusion proposed two diffusivity functions of form:

$$g(x) = \frac{1}{1 + \left(\frac{x}{\delta}\right)^2}$$ \hspace{2cm} (4)

and,

$$g(x) = e^{-\left(\frac{x}{\delta}\right)^2}$$ \hspace{2cm} (5)

where $\delta$ is noise threshold also called contrast parameter or conduction coefficient. It is usually fixed or determined by some noise parameter. The plot of diffusivity function given by equation 4 and 5 are shown in Figure 1(a) and Figure 1(b) respectively for $\delta = 8$. It can be observed that diffusivity function is positive and monotonically decreasing.

The fall time of diffusivity function is mainly responsible for retaining sharp boundaries and edges in anisotropic diffusion filtering. Perona and Malik diffusion however suffers from stability issues and may produce false step edges. Over the last years, a considerable amount of research has been carried out for understanding of mathematical properties anisotropic diffusion and its variants in both continuous and discrete form. A number of researchers have proved the ill-posedness of continuous anisotropic diffusion. In semi implicit scheme was presented based on discrete nonlinear diffusion. Later a spatial filtering process based on discrete implementation of anisotropic diffusion was presented.
Table 1. Anisotropic diffusion filters and their important parameters

| Name of filter                        | Important parameters                                                                 |
|---------------------------------------|--------------------------------------------------------------------------------------|
| Perona and Malik anisotropic diffusion| Number of iterations, Conduction coefficient and lambda (\(\lambda\)) for stability |
| Speckle reducing anisotropic diffusion | Number of iterations and time step                                                  |
| Weickert diffusivity function         | Conduction coefficient (\(\delta\)) or contrast parameter, \(\delta\), \(C_m\), no of iterations, space regularization parameter, sigma and time step (stability factor) |
| Robust anisotropic diffusion          | Conduction coefficient \(\delta\) to determine contrast parameter, stability factor, no of iterations |

Figure 1. Plot of diffusivity functions given by equation 4 and 5 for \(\delta = 8\).
in\(^{14}\). New diffusivity functions were proposed and experimented in\(^{14}\). In\(^{4}\) anisotropic diffusion was combined with Lee\(^{12}\) and frost filter\(^{13}\). Relation between local directional variance and local geometry of image was reported in\(^{1}\) which matrix anisotropic diffusion for filtering across the image contours and in principal curvature directions was proposed. In\(^{14}\) relation between anisotropic diffusion and adaptive smoothening was studied.

Table 1 shows most commonly used anisotropic diffusion filters which are investigated in this work along with important parameters which affect the performance of filter. In anisotropic diffusion appropriate choice of diffusivity function and other filter specifications is extremely important for preserving edges, boundaries and detailed structures. Further they play critical role in performance of anisotropic diffusion filtering. In this work choice of suitable image quality assessment techniques and filter specifications such as gradient threshold, time step, number of iterations etc., for appropriate diffusion is investigated. Effect of each associated parameter on performance of filter is studied. Further a new image quality metrics is proposed for evaluating and comparing various techniques.

The rest of the paper is organized as follows. In Section 2 data used and methodology adopted are explained along with various image quality evaluation metrics used for evaluating the filters performance. Section 3 presents results and discussion followed by conclusion and future directions in Section 4.

### 2. Materials and Methods

Experiments were conducted on 25 noise free digital images available in MATLAB\(^{*}\) image processing toolbox. Speckle noise was later added to these images using MATLAB\(^{*}\) software. Figure 2 shows three images from database and Figure 3 shows corresponding images after adding speckle noise to them. The noisy images were subjected to filters discussed in section two and contribution of various filter parameters on filters performance was studied and evaluated by computing thirteen image

![Figure 2. Original noise free images.](image-url)
quality metrics as discussed below. Finally to evaluate the average performance mean of each image quality metrics was calculated.

The performance of various filtering techniques can be evaluated by computing the difference between original image and filtered image. One of the major objectives of any filtering technique is to minimize blurring of edges and boundaries and retain important image features such as edges resulting in good resolution. To evaluate the edge retrieval capability of various techniques, a new quality metric proposed recently in called 'Edge Retrieval Index' (ERI) is used. It is defined as the ability of the filter to retain sharp edges and boundaries in filtered image as that in original noise free image. Its value approaches maximum to 1 for best transformation and minimum to 0 for worst transformation.

Assuming that the original noise free image is represented by $I(i, j)$ and filtered image is represented by $F(i, j)$ then ERI is calculated using following algorithm.

**Step.1:** Number of edges in $I(i, j)$ is calculated using any standard edge detector like Canny, Prewitt, Sobel etc. In our work we have used Canny edge detector with standard deviation of 0.4. Number of edges of $I(i, j)$ is denoted by $n_i$.

**Step.2:** Step one is repeated for filtered image $F(i, j)$ using same edge detector. Number of edges of $I(i, j)$ is denoted by $n_o$.

**Step.3:** ERI is then computed using:

$$\text{ERI} = 1 - \frac{n_i - n_s}{n_i} \quad (6)$$

where $n_s$ is number of edges which are present both in $I(i, j)$ and $F(i, j)$ at similar pixel locations. It is calculated by comparing edges of $I(i,j)$ and $F(i, j)$.

The algorithm is illustrated in Figure 4. Figure 4(a) shows the original image $I(i, j)$ and figure 4(b) shows its edges. Figure 4(b) is used to calculate $n_i$. Figure 4(c)
Figure 4. (a) Original image (top left). (b) Edges of original image obtained using Canny edge detector with $\sigma = 0.4$ (top middle). (c) Image after adding speckle noise (top right). (d) Image obtained after filtering (bottom left). (e) Edges of filtered image obtained using Canny edge detector with $\sigma = 0.4$ (bottom right).
shows the noisy image after adding speckle noise to I(i, j). Figure 4(d) shows the filtered image F(i, j) using Peronal and Malik anisotropic diffusion and Figure 4(e) shows its edges. Now ERI is computed using step 3.

Following additional image quality metric were used for evaluating the performance of filters:

1. **Geometric Average Error (GAE)** is defined as-
   \[ \text{GAE} = \left( \prod_{i=1}^{M} \prod_{j=1}^{N} \left| I(i,j) - F(i,j) \right| \right)^{\frac{1}{MN}} \]  
   (7)

   Value of GAE approaches zero for very good transformation.

2. **Mean Square Error (MSE) and Root Mean Square Error (RMSE):** MSE is given by-
   \[ \text{MSE} = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (I(i,j) - F(i,j))^2 \]  
   (8)

   and RMSE = \( \sqrt{\text{MSE}} \). Low value of RMSE and MSE is desired for good transformation.

3. **Signal to Noise Ratio (SNR) is computed using the following formula-**
   \[ \text{SNR} = 10 \log_{10} \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (I_{i,j}^2 + F_{i,j}^2)}{\sum_{i=1}^{M} \sum_{j=1}^{N} (I_{i,j}^2 - F_{i,j}^2)} \]  
   (9)

4. **Peak signal to noise ratio (PSNR) is calculated using-**
   \[ \text{PSNR} = -10 \log \frac{\text{MSE}}{\left( I_{i,j}^2 \right)_{\text{max}}} \]  
   (10)

where \( (I_{i,j})_{\text{max}} \) is the maximum intensity of original image I(i, j). Both SNR and PSNR are higher for better transformed image. They measure the resemblance between original and filtered image.

5. The Minkowski metric is measure of dissimilarity between the original and filtered image and is defined as-
   \[ M_n = \left( \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} |I(i,j) - F(i,j)|^n \right)^{\frac{1}{n}} \]  
   (11)

   which is computed for \( n = 1 \) (\( M_1 \) also called as average difference (AD)), \( n = 3 \) (\( M_3 \)) and \( n = 4 \) (\( M_4 \)). Low value of \( M_n \) is desirable for good transformation.

6. **Universal quality index (Q) is defined as-**
   \[ Q = \frac{\sigma_I \sigma_F}{\sigma_I^2 + (\bar{I})^2} - \frac{2\bar{I}\bar{F}}{\sigma_I^2 + \sigma_F^2} \]  
   (12)

where \( \sigma_I^2 \) is the covariance between original and filtered image windows.

\( \bar{I} \) and \( \bar{F} \) mean of intensities of original and filtered image.

\( \sigma_I \) and \( \sigma_F \) are standard deviation of original and filtered image.

The value of Q lies between -1 and 1 for bad and good transformation respectively.

7. **Mean structural similarity index (MSSIN) is given by-**
MSSIN = \frac{(2\overline{TF} + k_1)}{(\overline{I}^2 + \overline{F}^2 + k_1)} \cdot \frac{(2\sigma_{MF} + k_2)}{(\sigma_I^2 + \sigma_F^2 + k_2)} \quad (13)

where \( k_1 = 0.01d_I, k_2 = 0.01d_F \) and \( d_I \) is the dynamic range of original images. MSSIN ranges between -1 and +1 for bad and good transformation respectively.

8. Structural Content (SC) is the measure of image similarity based on similar small patterns in an image. Large value of SC indicates that filtered image is of poor quality. It is calculated using following equation -

\[
SC = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (I(i, j))^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} (F(i, j))^2} 
\]

9. Normalized cross correlation (NC) is the measure of similarity between two patterns. It is given by-

\[
NC = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (I(i, j) \times F(i, j))}{\sum_{i=1}^{M} \sum_{j=1}^{N} (I(i, j))^2} 
\]

Its value is high (maximum = 1) for good transformation.

10. Maximum difference (MD) is defined as –

\[
MD = \max \left[ |I(i, j) - F(i, j)| \right] 
\]

Its value is low for good transformation.

11. Laplacian Mean Squared Error (LMSE) is given by

\[
LMSE = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (\nabla^2 I(i, j) - \nabla^2 F(i, j))^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} (\nabla^2 I(i, j))^2} 
\]

where \( \nabla^2 \) is Laplacian operator given by-

\[
\nabla^2 I = \frac{I_{i,j+1} + I_{i,j-1} + I_{i+1,j} + I_{i-1,j} - I(i, j)}{4} 
\]

For perfect transformation LMSE = 0 otherwise it increases up to 1.

12. Normalized Absolute Error (NAE) is numerical difference between original image and filtered image. It is given by-

\[
NAE = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} |I(i, j) - F(i, j)|}{\sum_{i=1}^{M} \sum_{j=1}^{N} (I(i, j))} 
\]

3. Results and Discussions

All the steps of filtering and associated parameters were carefully studied so that best choice among various options can be selected. Various types of anisotropic diffusion filters experimented and evaluated are-

3.1 Perona and Malik Anisotropic Diffusion

Table 2 shows values of various image quality metrics obtained after applying Perona and Malik anisotropic diffusion3 with diffusivity function given by equation 4. Number of iterations is varied while keeping conduction coefficient and \( \lambda \) constant. It is observed that along with PSNR all other metric approximately remain constant.
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Figure 5. (a) Original noise free image (top left). (b) Image corrupted by speckle noise (Top right). (c) Filtered image after first iteration (middle left). (d) Second iteration (middle right). (e) Fifth iteration (bottom left) and tenth iteration (bottom right).

Figure 6. Effect of varying number of iterations on image quality metric.
Table 2. Effect of varying no. of iterations on performance of Perona and Malik anisotropic diffusion with diffusivity function given by Equation 3

| Image quality metric | No. of iterations (conduction coefficient = 20, \( \lambda = 0.2 \)) |
|----------------------|---------------------------------------------------------------|
|                      | 1     | 2     | 5     | 10    | 20    |
| GAE                  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| MSE                  | 15489.253 | 15489.824 | 15490.876 | 15491.903 | 15493.203 |
| SNR                  | 0.034 | 0.033 | 0.033 | 0.033 | 0.032 |
| RMSE                 | 113.219 | 113.221 | 113.225 | 113.229 | 113.234 |
| PSNR                 | 9.492 | 9.491 | 9.491 | 9.491 | 9.490 |
| M3                   | 121.164 | 121.167 | 121.173 | 121.179 | 121.186 |
| M4                   | 127.299 | 127.303 | 127.311 | 127.318 | 127.326 |
| Q                    | 0.103 | 0.099 | 0.085 | 0.067 | 0.042 |
| MSSIN                | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 |
| ERI                  | 0.685 | 0.670 | 0.614 | 0.517 | 0.361 |
| AD                   | 101.271 | 101.271 | 101.272 | 101.274 | 101.276 |
| SC                   | 65952.620 | 66936.588 | 68246.509 | 69320.366 | 70611.567 |
| NC                   | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| MD                   | 214.886 | 214.900 | 214.957 | 215.004 | 215.051 |
| LMSE                 | 0.995 | 0.997 | 0.998 | 0.999 | 1.000 |
| NAE                  | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 |
### Table 3. Effect of varying conduction coefficient on performance of Perona and Malik anisotropic diffusion with diffusivity function given by Equation 3

| Image quality metric | Conduction coefficient (Number of iterations = 1, $\lambda = 0.2$) |
|----------------------|---------------------------------------------------------------|
|                      | 1  | 20  | 40  | 60  | 80  | 100 |
| GAE                  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| MSE                  | 15489.367 | 15489.254 | 15489.253 | 15489.253 | 15489.253 | 15489.253 |
| SNR                  | 0.034 | 0.034 | 0.034 | 0.034 | 0.034 | 0.034 |
| RMSE                 | 113.219 | 113.219 | 113.219 | 113.219 | 113.219 | 113.219 |
| PSNR                 | 9.492 | 9.492 | 9.492 | 9.492 | 9.492 | 9.492 |
| M3                   | 121.164 | 121.164 | 121.164 | 121.164 | 121.164 | 121.164 |
| M4                   | 127.299 | 127.299 | 127.299 | 127.299 | 127.299 | 127.299 |
| Q                    | 0.103 | 0.103 | 0.103 | 0.103 | 0.103 | 0.103 |
| MSSIN                | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 |
| ERI                  | 0.685 | 0.685 | 0.685 | 0.685 | 0.685 | 0.685 |
| AD                   | 101.271 | 101.271 | 101.271 | 101.271 | 101.271 | 101.271 |
| SC                   | 66061.121 | 65953.480 | 65952.620 | 65952.517 | 65952.461 | 65952.365 |
| NC                   | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| MD                   | 214.890 | 214.886 | 214.886 | 214.886 | 214.886 | 214.886 |
| LMSE                 | 0.995 | 0.995 | 0.995 | 0.995 | 0.995 | 0.995 |
| NAE                  | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 |
except Q, ERI and SC. Figure 5 shows the original image, image corrupted by speckle noise and result of filtering using Perona and Malik (Equation (4)). It is observed that with increase in number of iterations the image becomes more smoothened there by reducing speckle noise but at the same time blurring edges and sharp details of the image. The effect of varying number of iterations on PSNR, Q, ERI and SC is shown in Figure 6. It is observed that with increase in number of iterations though PSNR remain constant, Q and ERI decreases while SC increases thereby dictating poor transformation for higher number of iterations. After studying the effect of number of iterations, the effect of conduction coefficient was analyzed. In this case λ (stability factor) and number of iterations were kept fixed. Table 3 observed that on an average, the performance of the filter remains same throughout for all the values of conduction coefficient. Now to study the effect of variations in λ, conduction coefficient and number of iterations are fixed at 20 and 1 respectively.

Figure 7 shows the filtered images for various values of λ. It is observed that with increase in value of λ the resulting image becomes smooth and edges get blurred. The corresponding image quality evaluation metrics is shown in Table 4 and illustrated in Figure 8. Change in value of λ does not affect most of the image quality metrics including PSNR. However with increase in λ value of ERI decreases and SC increases thereby indicating blurring and loss of details in image. The results are even worst for λ greater than 0.3. Hence for satisfactory transformation using diffusivity function given by equation 3, number of iterations should be small and λ must be less than 0.3. It is also observed that when filter parameters are varied, most of the image quality metrics such as GAE, MSE, RMSE, PSNR, M3, M4, MSSIN, AD, NC, MD, LMSE, and NAE
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Figure 7. (a) Filtered image for $\lambda=0.1$ (top left). (b) $\lambda = 0.2$ (top right). (c) $\lambda=.3$ (bottom left). (d) $\lambda=0.5$ (bottom right).

Figure 8. Effect of varying stability factor on image quality metric.
Table 4. Effect of varying $\lambda$ on performance of Perona and Malik anisotropic diffusion with diffusivity function given by Equation 3

| Image quality metric | $\lambda$ (Number of iterations = 1, Conduction coefficient = 20) | $\lambda$ | $\lambda$ | $\lambda$ | $\lambda$ | $\lambda$ | $\lambda$ | $\lambda$ |
|----------------------|-----------------------------------------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
|                      | 0.01                                                             | 0.05      | 0.1       | 0.15      | 0.20      | 0.25      | 0.30      | 0.50      |
| GAE                  | 0.000                                                           | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     |
| MSE                  | 15488.438                                                       | 15488.609 | 15488.823 | 15489.038 | 15489.253 | 15489.468 | 15489.684 | 15490.549 |
| SNR                  | 0.034                                                           | 0.034     | 0.034     | 0.034     | 0.034     | 0.034     | 0.034     | 0.033     |
| RMSE                 | 113.216                                                         | 113.216   | 113.217   | 113.218   | 113.219   | 113.219   | 113.220   | 113.223   |
| PSNR                 | 9.492                                                           | 9.492     | 9.492     | 9.492     | 9.492     | 9.491     | 9.491     | 9.491     |
| M3                   | 121.159                                                         | 121.160   | 121.161   | 121.162   | 121.164   | 121.165   | 121.166   | 121.172   |
| M4                   | 127.292                                                         | 127.293   | 127.295   | 127.297   | 127.299   | 127.300   | 127.302   | 127.309   |
Table 4 Continued

|     | Q    | 0.103 | 0.103 | 0.103 | 0.103 | 0.103 | 0.103 | 0.103 |
|-----|------|-------|-------|-------|-------|-------|-------|-------|
| MSSIN | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 |
| ERI  | 0.684 | 0.685 | 0.686 | 0.687 | 0.685 | 0.681 | 0.679 | 0.655 |
| AD   | 101.270 | 101.270 | 101.271 | 101.271 | 101.271 | 101.271 | 101.271 | 101.272 |
| SC   | 63776.662 | 64315.287 | 64930.857 | 65478.043 | 65952.620 | 66350.824 | 66669.428 | 67105.747 |
| NC   | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| MD   | 214.898 | 214.893 | 214.888 | 214.885 | 214.886 | 214.889 | 214.894 | 214.959 |
| LMSE | 0.994 | 0.994 | 0.995 | 0.995 | 0.995 | 0.996 | 0.996 | 0.999 |
| NAE  | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 |
etc. remain constant. Hence ERI can be used as useful metric for evaluating filters performance.

Similar methodology is adopted for analyzing the performance of Perona and Malik anisotropic diffusion with diffusivity function given by equation 5. It was observed that with variations in filter specifications, most of the image quality metrics remain constant as in previous case. Further following observations were made:

- When numbers of iterations were increased from 1 to 20, Q and ERI decreases while SC increases thereby resulting in poor transformation.
- When conduction coefficient was increased from 1 to 100, no variations in image quality metrics were found.
- When stability factor $\lambda$ was varied from 0.001 to 0.5 it was observed that most of the edges were lost for $\lambda$ greater than 0.3 as depicted by ERI.

### 3.2 Speckle Reducing Anisotropic Diffusion

After analyzing Perona and Malik anisotropic diffusion, we analyzed another anisotropic diffusion filter called as "Speckle Reducing Anisotropic Diffusion (SRAD) filter". SRAD is the edge-sensitive anisotropic diffusion filter for speckled images. This technique enhances edges by inhibiting diffusion across the edges and allowing diffusion on either side of edges. It is based on Minimum Mean Square Error (MMSE) approach to filtering. It is useful for reducing noise from images particularly corrupted by multiplicative noise such as speckle noise. The details of this technique can be found in[8]. Effect of two parameters on SRAD is considered in this study i.e. number of iterations and time step as mentioned in Table 1. Initially time step was fixed at 0.15 and number of iterations was varied from 1 to 100. It is found that all quality metrics except Q, ERI and SC showed no variations. With increase in number of iterations there is no significant improvement in PSNR as illustrated in Figure 9. Its value remains constant and approximately same as that of Perona and Malik diffusion. Further Q and ERI decreases while SC increases indicating loss of visual details and resolution in filtered image. Now to analyze the influence of time step on performance of SRAD, number of iterations was fixed at 1 and time step was varied from 0 to 0.5. It is observed that

![Figure 9](image.jpg) **Figure 9.** Effect of varying number of iterations on image quality metric in SRAD.
with increase in time step ERI decreases and SC increases. This is illustrated in Figure 10. For time step greater than 0.35, image gets highly blurred and most of the edges get lost.

### 3.1 Weickert Anisotropic Diffusion

The next anisotropic diffusion filter considered in this study is called Weickert diffusivity function\textsuperscript{10} given by-

\[
g(x) = 1 - \exp\left[-C_m \left(\frac{x}{\delta}\right)^m\right] \quad \text{for } x > 0 \\
    = 1, \quad \text{otherwise.} 
\]

(20)

where constant $C_m$ is determined such that flux reaches its maximum at noise threshold $\delta$ (Conduction coefficient) and ‘m’ affects the rise time and fall time of the function in the neighborhood of the noise threshold. In this study $C_m$ is fixed at 3.5 and $m = 4^{10}$.

To study the influence of other parameters on performance of Weickert’s anisotropic diffusion filter, initially number of iterations and time step were fixed at 1 and 0.15 respectively while conduction coefficient was increased from 1 to 100. It is observed that there is no significant change in various image quality metrics as in case of Perona and Malik diffusion and SRAD filter. However, value of SC is low (as desired) for conduction coefficient greater than 10. Hence for satisfactory transformation conduction coefficient should be greater than 10.
The effect of varying number of iterations on performance of Weickert’s anisotropic diffusion filter. Conduction coefficient and time step were fixed at 30 and 0.15 respectively. It is observed that with increase in number of iterations Q, ERI and SC showed significant variations while PSNR and other quality metrics.
remain approximately constant as illustrated in Figure 11. With increase in number of iterations the image gets more smoothed, PSNR remains constant but Q and ERI decreases while SC increases thereby indicating poor transformation. Finally to study the effect of variations in time step on filters performance, the conduction coefficient and number of iterations are fixed at 30 and 1 respectively. It is found that except ERI and SC, no significant variations were observed in other image quality metrics. However with increase in time step ERI decreases and SC increases as illustrated in Figure 12. At time step = 0.25, SC reaches its highest value and ERI reduces drastically which is undesired. Hence for satisfactory transformation time step should be less than 0.25.

3.4 Robust Anisotropic Diffusion

We consider the robust anisotropic diffusion in which a new edge stopping function based on Tukey’s bi weight robust estimator was proposed. Again three parameters namely number of iterations, conduction coefficient and stability factor influence the performance of filter. First the effect of varying conduction coefficient was analyzed while number of iterations and stability factor are fixed at 20 and 0.15 respectively. With increase in conduction coefficient it is observed that ERI and SC showed significant variations unlike that in other anisotropic diffusion filters. All other image quality metrics except ERI and SC were approximately constant while ERI decreases with increase in conduction coefficient as shown in Figure 13. Thus to retain edges and sharp details conduction coefficient should be less than 10 in case of robust anisotropic diffusion. Figure 14 shows the filtered images for various values of conduction coefficient. It is observed that with increase in value of conduction coefficient image gets more and more smooth and edges get blurred. Next we study the effect of varying number of iterations and time step. With increase in number of iterations and time step no significant variations are observed on various image quality metrics considered in this study. It is found that unlike other types of anisotropic diffusion discussed in this work, in robust anisotropic diffusion conduction coefficient plays critical role in filtering process.

Figure 13. Effect of varying conduction coefficient on image quality metric in robust anisotropic diffusion.
Figure 8. (a) Filtered image for conduction coefficient = 5 (top left). (b) Conduction coefficient =15 (top right. (c) Conduction coefficient = 20 (bottom left). (d) Conduction coefficient = 40 (bottom right).
4. Conclusions and Future Scopes

Speckle reduction is an important preprocessing step to improve image quality. Appropriate selection of despeckle filter and associated parameters are very important in enhancement, segmentation and feature extraction of digital images corrupted by speckle noise. Anisotropic diffusion has been successfully used by many researchers for removal of speckle noise. In this paper four most commonly used anisotropic diffusion filters and their associated parameters were analyzed and their individual contribution on performance of filter was studied. Further traditional methods to assess image quality were studied. A new image quality evaluation metric called Edge Retrieval Index (ERI) was used to evaluate edge retaining capability of filter. Experiments were conducted on 25 digital images using MATLAB® software. In Perona and Malik, SRAD and Weickert anisotropic diffusion, the analysis shows that most of the true edges got lost for higher number of iterations along with more smoothing of images. In robust anisotropic diffusion conduction coefficient plays important role in filtering process. Higher value of conduction coefficient results in more smoothing with blurring of sharp details and edges. It is concluded that Q, ERI and SC can give significant information about image quality even in cases where conventional image quality metrics such as PSNR, MSE, RMSE, GAE, LMSE, NAE etc. fail to do so. Combination of these measures may prove more useful in describing image quality. In future this analysis can be carried out on real time images such as medical images. Further visual evaluation by experts may be combined with proposed work method based on quality evaluation metrics to improve the results.

5. References

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