Natural frequency analysis and simulation test of conical shell under support condition

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Abstract. This paper presents an analysis of the vibration characteristics of a conical shell with a fixed support. Firstly, a reasonable numerical calculation model has been established based on the Donnell shell theory, and the first 9 modes have been calculated by Numerical method with MATLAB. Then, the finite element model of conical shell has been built on the ANSYS Workbench simulation platform. With specific constraints set, modal analysis has been carried out on the model of conical shell to find and verify the corresponding numerical model is convergent.

1. Introduction

The conical shell model is used in a variety of mechanical applications, such as the large turbogenerator stator end winding and the subsea vehicle tail structure. It can be seen that it is necessary to study the vibration characteristics of conical shell. Crenwelge used the energy method to analyse the free vibration of the conical shell [1]. Hu.Yuda also obtained the vibration control equation of the laminated conical shell model through energy analysis [2]. Zhou.Yunze used the power series method of L.Tong to study the convergence and accuracy of the first two natural frequencies in the circumferential and axial modes of a conical shell [3][4]. In their research, the power series method simplifies the calculation process of the vibration equation and is more universal. In this paper, the power series method is integrated into the original Donnell shell theory to make the calculation results more accurate. At the same time, the reason why it is difficult to directly apply the theoretical model to the practical design is influencing factor between the theoretical model and the actual measurement value. This defect has been made up for by comparing the results of simulation analysis and numerical calculation.

This paper uses the method of power series for the free vibration analysis of conical shell. More detail of the derivation process of the vibration equation and the corresponding MATLAB calculation results is given in section 2. And the finite element model was established based on ANSYS Workbench in the section 3. The corresponding constraints of the model were designed and its intrinsic mode was calculated. Comparing the results with those of numerical calculation method can validate whether the established equation is reasonable. The section 4 is the conclusion of this paper.

2. The basic theory of conical shells

According to the Donnell shell theory, the coordinate system has been set up as shown in Figure 1. Take the orthogonal curvilinear coordinate system as \((s, \theta, z)\), where \(s\) and \(\theta\) are the directions of the bus and ring of the middle plane, and \(z\) is the normal direction. According to the assumption of
straight normal, the geometric equations of surface strain \((\varepsilon_s, \varepsilon_\theta, \gamma_{s\theta})\) and curvature \((k_s, k_\theta, k_{s\theta})\) in the shell are:

\[
\begin{align*}
\varepsilon_s &= \frac{\partial u}{\partial s} + \varepsilon_\theta = \frac{1}{s \sin \alpha} \frac{\partial v}{\partial \theta} + \frac{u + wetg \alpha}{s} \\
\gamma_{s\theta} &= \frac{\partial v}{\partial s} + \frac{1}{s \sin \alpha} \frac{\partial u}{\partial \theta} - \frac{v \cdot k_s}{s} = -\frac{\partial^2 w}{\partial s^2} \\
k_\theta &= -\frac{1}{s} \frac{\partial w}{\partial s} - \frac{1}{s^2 \sin^2 \alpha} \frac{\partial^2 w}{\partial \theta^2} \\
k_{s\theta} &= -\frac{2}{\sin \alpha} \frac{\partial}{\partial s} \left( \frac{1}{s} \frac{\partial w}{\partial \theta} \right) = 2 \tau
\end{align*}
\]

(1)

In the equation, \(u, v\) and \(w\) are the displacement components of the middle plane in \(s, \theta\) and \(z\) directions respectively. And the relationship between \(s\) and \(x\) is \(s = R_1 x \sin \alpha\).

The physical equations of conical shell model are:

\[
\begin{align*}
N_s &= \frac{E h}{1 - v^2} (\varepsilon_s + \varepsilon_\theta) \\
N_\theta &= \frac{E h}{1 - v^2} (\varepsilon_\theta + \varepsilon_s) \\
N_{s\theta} &= N_{\theta s} = \frac{E h}{2(1 + v)} \gamma_{s\theta} \\
M_s &= \frac{E h^3}{12(1 - v^2)} (k_s + v k_\theta) \\
M_\theta &= \frac{E h^3}{12(1 - v^2)} (k_\theta + v k_s) \\
M_{s\theta} &= M_{\theta s} = \frac{E h^3}{24(1 + v)} \tau
\end{align*}
\]

(2)

where \(N_s, N_\theta, N_{s\theta}\) and \(N_{\theta s}\) are the surface stress in the element and \(M_s, M_\theta, M_{s\theta}\) and \(M_{\theta s}\) are the bending moment in the shell element.

According to Donnell's shell theory, the following equilibrium equations can be obtained:

\[
\begin{align*}
\frac{1}{s} \frac{\partial N_s}{\partial s} + \frac{N_s - N_\theta}{s^2} + \frac{1}{s^2 \sin \alpha} \frac{\partial N_{s\theta}}{\partial \theta} &= \rho h \frac{\partial^2 u}{\partial t^2} \\
\frac{1}{s} \frac{\partial N_{s\theta}}{\partial s} + 2 \frac{N_{s\theta}}{s^2} + \frac{1}{s^2 \sin \alpha} \frac{\partial N_\theta}{\partial \theta} &= \rho h \frac{\partial^2 v}{\partial t^2} \\
\frac{1}{s^2 \sin^2 \alpha} \frac{\partial^2 (s M_s)}{\partial s^2} - \frac{1}{s^2} \frac{\partial M_\theta}{\partial s} + \frac{1}{s^2 \sin^2 \alpha} \frac{\partial^2 M_\theta}{\partial \theta^2} + \frac{2}{s^3 \sin \alpha} \frac{\partial^2 (s M_{s\theta})}{\partial s \partial \theta} + \frac{\cos \alpha}{s^2 \sin \alpha} N_\theta &= \rho h \frac{\partial^2 w}{\partial t^2}
\end{align*}
\]

(3)
In order to solve the equation, displacement expansion of power series is introduced as follow:

\[ u = \sum_{m=0}^{\infty} a_m x^m \cos(n\theta)e^{-j\omega t} \]

\[ v = \sum_{m=0}^{\infty} b_m x^m \sin(n\theta)e^{-j\omega t} \]

\[ w = \sum_{m=0}^{\infty} c_m x^m \cos(n\theta)e^{-j\omega t} \]  

(4)

\( n \) is the number of circumferential waves. Then multiply the first two equations of equation (3) by \( s^2 \sin^2 \alpha \), and multiply the last equation by \( s^2 \sin^2 \alpha \), and substitute in equation (6). After derivat and transformation, the factor of \( s \) with the same power is equal with each other, the recurrence relation can be obtained as follows:

\[ a_{m+2} = \sum_{i=1}^{2} A_{a,i} a_{m+i-1} + \sum_{i=1}^{2} B_{a,i} b_{m+i-1} + \sum_{i=1}^{2} C_{a,i} c_{m+i-1} \]

\[ b_{m+2} = \sum_{i=1}^{2} A_{b,i} a_{m+i-1} + \sum_{i=1}^{2} B_{b,i} b_{m+i-1} + \sum_{i=1}^{1} C_{b,i} c_{m+i-1} \]

\[ c_{m+4} = \sum_{i=1}^{2} A_{c,i} a_{m+i-1} + \sum_{i=1}^{1} B_{c,i} b_{m+i-1} + \sum_{i=1}^{4} C_{c,i} c_{m+i-1} \]  

(5)

Where \( m = 0, 1, 2, \ldots \). The value of \( m \) affects the convergence of the results. The larger \( m \) is, the stronger the convergence is. However, due to the limitation of operation, we can get the smaller \( m \) as much as possible on the premise of convergence.

We choose the most common fixed support as our boundary condition:

\[ u = v = w = \frac{\partial w}{\partial s} = 0 \]  

(6)

With the help of MATLAB, we get the calculation results in Table 1.
Table 1. Numerical simulation results

| Circumferential wave Numbers | Natural frequency (Hz) |
|-----------------------------|------------------------|
| 1                           | 35                     |
| 2                           | 24                     |
| 3                           | 22                     |
| 4                           | 15                     |
| 5                           | 13                     |
| 6                           | 12                     |
| 7                           | 15                     |
| 8                           | 16                     |
| 9                           | 21                     |

3. Modal analysis of conical shell

If the model was imported into ANSYS by other modelling software, there will be a certain probability of element loss. Since the model to be studied in this paper is not complex and ANSYS itself has a relatively powerful modelling module, this paper uses the Design Modeller inside ANSYS to build the conical shell we need. The model is shown in Figure 2.

![Figure 2.](image)

The material of this model is set as structural steel, and its main parameters are:

- Density $\rho = 7800 \text{ kg/m}^3$
- Young's modulus $E = 212 \text{ Gpa}$
- Poisson's ratio $\nu = 0.3$
- Radius of large end $R_m = 20 \text{ m}$
- Bus $L = 20 \text{ m}$
- Shell thickness $h = 0.2 \text{ m}$
- $\alpha = 30^\circ$

3.1. Finite element model of conical shell

The conical shell model we want to analyse is relatively simple. There is no special assembly process to be set up. Only the vibration characteristics of conical shell are studied. There are 103514 nodes and 14732 elements in the model (as shown in Figure 3). The model is discretized by hexahedral eight-node element, and the mesh quality is good. Since the modal was analysed under the fixed boundary condition in the numerical calculation, we also impose fixed constraints on the upper and lower ends of the conical shell in the finite element model.

3.2. Natural frequency solution of conical shell

There are not many modes we can excite in the actual structure. However, with ANSYS Workbench, we can find enough modes and corresponding natural frequencies to verify whether the results of our numerical analysis method have referenced value. The first nine modes of conical shell can be obtained by calculation. Figure 4 to 12 show the first nine modes of the conical shell. The axial wave number is 1, and the circumferential wave number is increasing according to the natural number. In this order, the natural frequency of the mode is decreased first and then increased. The specific data is shown in Table 2.
Figure 4. First mode

Figure 5. Second mode

Figure 6. Third mode

Figure 7. Fourth mode

Figure 8. Fifth mode

Figure 9. Sixth mode

Figure 10. Seventh mode

Figure 11. Eighth mode

Figure 12. Ninth mode
Table 2. Finite element modal analysis results

| Circumferential wave Numbers | Natural frequency (Hz) |
|-----------------------------|------------------------|
| 1                           | 38                     |
| 2                           | 27                     |
| 3                           | 20                     |
| 4                           | 16                     |
| 5                           | 14                     |
| 6                           | 13                     |
| 7                           | 14                     |
| 8                           | 16                     |
| 9                           | 19                     |

3.3. Comparison between numerical method and finite element method

The comparison of the results of the finite element method and the numerical calculation method is shown in Figure 13. It can be seen that the natural frequencies of corresponding modes are basically the same. So the results of the numerical calculation method are reliable.

4. Conclusion

In this paper, the numerical method and the finite element method are used to carry out the modal analysis of the conical shell. The conclusions are as follows:

(1) The numerical results in this paper are few affected by the order. In previous studies, comparing the results of numerical calculation method with finite element method, the low order coincidence was higher. The combination of power series and shell theory is proved to be correct;

(2) The conical shells studied in this paper are more flexible and have more natural frequencies. The numerical calculation model based on Donnell's shell theory is in good agreement with the results of the finite element model.

This model still needs further research:

(1) Theoretically, the result of numerical calculation should be close to that of finite element. However, since the convergence of the power series method and the calculation amount cannot be achieved simultaneously, how to optimize the numerical calculation method still need to be considered in the future;

(2) When analysing the vibration problem, the analysed object is often in a complex physical environment. The research on the complex boundary of conical shell is worth discussing.
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