Filamentation of Fast Radio Bursts in magnetar winds

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**ABSTRACT**

Magnetars are the most promising progenitors of Fast Radio Bursts (FRBs). Strong radio waves propagating through the magnetar wind are subject to non-linear effects, including modulation/filamentation instabilities. We derive the dispersion relation for modulations of strong waves propagating in magnetically-dominated pair plasmas focusing on dimensionless strength parameters \(a_0 \lesssim 1\), and discuss implications for FRBs. As an effect of the instability, the FRB radiation intensity develops sheets perpendicular to the direction of the wind magnetic field. When the FRB front expands outside the radius where the instability ends, the radiation sheets are scattered due to diffraction. The FRB scattering timescale depends on the properties of the magnetar wind. In a cold wind, the typical scattering timescale is \(\tau_{sc} \sim \mu s - ms\) at the frequency \(\nu \sim 1\) GHz. The scattering timescale increases at low frequencies, with the scaling \(\tau_{sc} \propto \nu^{-2}\). The frequency-dependent broadening of the brightest pulse of FRB 181112 is consistent with this scaling. From the scattering timescale of the pulse, one can estimate that the wind Lorentz factor is larger than a few tens. In a warm wind, the scattering timescale can approach \(\tau_{sc} \sim ns\). Then scattering produces a frequency modulation of the observed intensity with a large bandwidth, \(\Delta \nu \sim 1/\tau_{sc} \gtrsim 100\) MHz. Broadband frequency modulations observed in FRBs could be due to scattering in a warm magnetar wind.

**Key words:** fast radio bursts – radio continuum: transients – plasmas – instabilities – relativistic processes

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1 INTRODUCTION

Fast Radio Bursts (FRBs) are bright flashes of millisecond duration (e.g. Lorimer et al. 2007; Thornton et al. 2013; Spitler et al. 2014, 2016; Petroff et al. 2016; Shannon et al. 2018; CHIME/FRB Collaboration et al. 2019a,b,c). Magnetars are promising progenitors of FRBs. The FRB-magnetar connection, which was initially proposed on statistical grounds (e.g. Popov & Postnov 2010, 2013), is strongly supported by the discovery of weak FRBs from the Galactic magnetar SGR 1935+2154 (e.g. CHIME/FRB Collaboration et al. 2020, Bochenek et al. 2020).

Due to the huge luminosity of FRBs, the electromagnetic field of the radio wave accelerates the electrons in the magnetar wind up to a significant fraction of the speed of light (e.g. Luan & Goldreich 2014). Strong FRB waves can experience non-linear propagation effects, including modulation/filamentation instabilities (Sobacchi et al. 2021). The instability produces a spatial modulation of the intensity of the FRB wave. As the FRB front expands, the structures generated by the instability are scattered due to diffraction, and may interfere with each other. This process leaves an imprint on the time-frequency structure of FRBs.

In our previous work (Sobacchi et al. 2021), we studied the modulation/filamentation instabilities of FRBs propagating in a weakly magnetised electron-ion plasma. Such environment may be found at large distances from the FRB progenitor. The instability develops at nearly constant electron density since the effect of the ponderomotive force is suppressed due to the large inertia of the ions. The frequency-dependent broadening of the plasma frequency due to relativistic corrections to the effective electron mass. In regions of enhanced radiation intensity, the plasma frequency decreases because the electrons oscillate with a larger velocity in the field of the wave, and therefore have a larger effective mass. Modulations of the radiation intensity perpendicular to the direction of propagation of the wave grow because the refractive index of the plasma increases, which creates a converging lens that further enhances the radiation intensity. Modulations along the direction of propagation grow because the group velocity of the wave depends on the local radiation intensity. It turns out that
the spatial scale of the modulations is shorter along the direction of propagation than in the perpendicular direction.

In this paper, we study the modulation/filamentation of FRBs propagating in a magnetar wind, which is modelled as a magnetically-dominated pair plasma. We find that the instability develops because the ponderomotive force pushes particles out of regions of enhanced radiation intensity. The refractive index of the plasma increases in the regions where the particle number density is smaller, thus creating a converging lens that further enhances the radiation intensity. Since the ponderomotive force preferentially pushes particles along the magnetic field lines, the instability produces sheets of radiation intensity perpendicular to the direction of the wind magnetic field. Consistent with previous studies focusing on unmagnetised plasmas (e.g. Kates & Kaup 1989), we do not find significant modulations of the radiation intensity along the direction of the wave propagation.

As the FRB front expands outside the radius where the instability ends, the radiation sheets are diffracted, effectively scattering the arrival time of the FRB wave. In a cold magnetar wind, we find significant modulations of the radiation intensity along the direction of propagation. In a warm magnetar wind, the scattering timescale is much shorter, \( \tau_{sc} \sim \mu s \). Then scattering produces frequency modulations with a large bandwidth, \( \Delta \nu \sim 1/\tau_{sc} \gtrsim 100 \text{ MHz} \). Such broadband frequency modulations are often observed in FRBs (e.g. Shannon et al. 2018; Hessels et al. 2019; Nimmo et al. 2021).

The paper is organised as follows. In Section 2, we briefly review some relevant properties of magnetar winds and FRBs. In Section 3, we study the filamentation instability of FRBs. We refer the reader not interested in the technical details to the calculation of Tables 1 and 2 where we summarise our results. In Section 4, we discuss the scattering of FRBs. In Section 5, we conclude.

2 FAST RADIO BURSTS IN MAGNETAR WINDS
The magnetar wind forms outside the light cylinder, at radii \( R \gtrsim R_{LC} = cP/2\pi \), where \( P \) is the magnetar rotational period and \( c \) is the speed of light. The magnetic field strength in the wind proper frame is \( B_{bg} = \mu/\gamma_{w} R_{LC}^{2} R \), where \( \mu \) is the magnetar magnetic dipole moment, and \( \gamma_{w} \) is the wind Lorentz factor. The wind magnetic field is nearly azimuthal.

The ratio of the Larmor frequency, \( \omega_{L} = eB_{bg}/mc \), where \( e \) is the electron charge \( m \) is the electron mass, and the angular frequency of the FRB wave in the wind frame, \( \omega_{0} = \pi v/\gamma_{w} \), where \( v \) is the observed frequency, is

\[
\frac{\omega_{L}}{\omega_{0}} = 2.5 \times 10^{-4} \mu_{33} P_{0}^{-2} v_{0}^{-1} R_{15}^{-1},
\]

where we have defined \( \mu_{33} = \mu/10^{33} \text{ G cm}^{2} \), \( P_{0} \equiv P/1 \text{ s} \), \( v_{0} \equiv v/1 \text{ GHz} \), and \( R_{15} \equiv R/10^{15} \text{ cm} \). At the radii where \( \omega_{L} \ll \omega_{0} \), the electron motion is weakly affected by \( B_{bg} \). The electrons reach a maximum velocity of \( a_{0}c \), where \( a_{0} = eE/\omega_{0}mc \). We consider radii where \( a_{0} \lesssim 1 \), so that the electrons are sub-relativistic. The peak electric field of the wave in the wind frame, \( E \), can be calculated from the isotropic equivalent of the observed FRB luminosity, \( L = 2c^{2} \mu_{33} E^{2} R^{2} \). One finds

\[
\omega_{0} = 2.3 \times 10^{-2} L_{42}^{1/2} v_{0}^{-1} R_{15}^{-1},
\]

where \( L_{42} \equiv L/10^{42} \text{ erg s}^{-1} \). The condition \( a_{0} \lesssim 1 \) is satisfied at radii \( R \gtrsim 2.3 \times 10^{13} L_{42}^{1/2} v_{0}^{-1} \text{ cm} \). Since \( \omega_{L}/\omega_{0} \ll a_{0} \), the electric field of the FRB wave is larger than the wind magnetic field.

It is useful to define the wind magnetisation, \( \sigma_{w} \), as twice the ratio of the magnetic and rest mass energy densities of the plasma. One finds

\[
\sigma_{w} = \frac{\omega_{0}^{2}}{\omega_{p}^{2}},
\]

where \( \omega_{p} = \sqrt{8\pi N_{0} e^{2}/m} \) is the plasma frequency (the particle number density is \( 2N_{0} \)). We consider a magnetically-dominated wind with \( \sigma_{w} \gg 1 \).

3 FILAMENTATION INSTABILITY

3.1 Fundamental equations
We consider an electromagnetic wave propagating through a magnetised pair plasma, with mean particle number density \( 2N_{0} \), and background magnetic field \( B_{bg} \). We study the stability of slow, long-wavelength modulation of the initial wave.

The electromagnetic field of the wave can be expressed using the vector potential \( \mathbf{A} \). We are interested in the regime where the angular frequency of the wave, \( \omega_{0} \), is much larger than both the Larmor frequency, \( \omega_{L} = eB_{bg}/mc \), and the plasma frequency, \( \omega_{p} = \sqrt{8\pi N_{0} e^{2}/m} \). When \( \omega_{0} \gg \omega_{L}, \omega_{p} \), the non-linear wave equation is (e.g. Montgomery & Tidman 1964; Sluijter & Montgomery 1965; Ghosh et al. 2021)

\[
\frac{\partial^{2} \mathbf{A}}{\partial t^{2}} - c^{2} \nabla^{2} \mathbf{A} + \omega_{0}^{2} \langle N \rangle_{0} \left( 1 - \frac{1}{2} \frac{e^{2} (A^2)}{m c^{2}} \right) \mathbf{A} = 0,
\]

where \( \langle \ldots \rangle \) denotes the average on the spatial scale of many wavelengths of the initial wave. We have defined \( \langle N \rangle \) as half the total particle number density, namely \( 2(N_{p} + N_{e}) \) where \( \langle N_{p} \rangle \) and \( \langle N_{e} \rangle \) are the positron and electron densities. To avoid a lengthy notation, below we write \( N_{c} \) instead of \( \langle N_{c} \rangle \).

Eq. (4) contains two non-linear terms. The term proportional to \( \langle A^2 \rangle \) originates from the relativistic corrections to the effective electron mass, and from the heating by the density oscillations at the frequency \( 2\omega_{0} \) and the velocity oscillations at the frequency \( \omega_{0} \) (for a detailed discussion, see Appendix A of Ghosh et al. 2021). The term \( \langle N \rangle \) describes plasma density modulations produced by the ponderomotive force.

We use the same approach that is customarily adopted to study non-linear propagation effects in unmagnetised electron-ion plasmas (e.g. Krueer 2019). The plasma is described using a two-fluid model. The evolution of the positron and electron number densities is described by the continuity equation

\[
\frac{\partial N_{c}}{\partial t} + \nabla \cdot (N_{c} V_{c}) = 0,
\]

where \( V_{p} \) and \( V_{e} \) are the positron and electron coordinate velocities. The evolution of the velocities is described by the Euler’s...
where we have neglected quadratic terms in the perturbed quantities. The average is made on a spatial scale much longer than \(k_0^{-1}\), and much shorter than \(k^{-1}\).

Below we use Eqs. (9)-(10) to calculate the density perturbation \(\delta N = \delta N_e + \delta N_z\) as a function of \(\delta A_+\) and \(\delta A_-.\) Then we substitute \(\delta N\) into Eq. (4) and derive two homogeneous equations for \(\delta A_+\) and \(\delta A_-\). The condition that the determinant of the coefficients vanishes gives the dispersion relation.

### 3.3.1 Two-fluid equations

Substituting \(N_e = N_0 + \delta N_e\), \(V_{e+} = \delta V_{e+}\), into Eqs. (5)-(10), and neglecting quadratic terms in the perturbed quantities, one finds

\[
\frac{\partial \delta n_e}{\partial t} + N_0 V \cdot \delta V_e = 0
\]

\[
\frac{\partial \delta V_e}{\partial t} = -c_s^2 \frac{\partial^2 \delta n_e}{\partial x^2} - \frac{e}{m} \left[ \delta E + \frac{\delta V_e}{c} \times B_{bg} \right] - \frac{e^2}{2 m} \nabla \delta (A^2)
\]

It is convenient to introduce new variables defined as \(\delta \Omega = \delta \Omega_{e+} + \delta \Omega_{e-}\), \(\delta \Omega_n = \delta \Omega_{e+} - \delta \Omega_{e-}\), \(\delta \Omega = \delta \Omega_{e+} + \delta \Omega_{e-}\), and \(\delta \Omega_a = \delta \Omega_{e+} - \delta \Omega_{e-}\). From Eq. (16), one finds

\[
\frac{\partial \delta \Omega}{\partial t} + N_0 V \cdot \delta \Omega = 0
\]

\[
\frac{\partial \delta \Omega_n}{\partial t} + N_0 V \cdot \delta \Omega_a = 0
\]

From Eq. (17), one finds

\[
\frac{\partial \delta V}{\partial t} = -c_s^2 \frac{\partial \delta \Omega}{\partial x} - \frac{e}{m} \frac{\delta V_e}{c} \times B_{bg} - \frac{e^2}{2 m} \nabla \delta (A^2)
\]

\[
\frac{\partial \delta \Omega_a}{\partial t} = -c_s^2 \frac{\partial \delta \Omega_n}{\partial x} - \frac{e}{m} \left[ \delta E + \frac{\delta \Omega_a}{c} \times B_{bg} \right]
\]

Substituting \(E = \delta \Omega, B = \delta B\) into Eqs. (7)-(10), one finds

\[
\nabla \cdot \delta \Omega = 8 \pi e \delta \Omega_n
\]

\[
\nabla \cdot \delta \Omega_a = 0
\]

\[
\nabla \times \delta \Omega = \frac{1}{c} \frac{\partial \delta B}{\partial t}
\]

\[
\nabla \times \delta B = \frac{8 \pi}{c} e N_0 \delta \Omega_a + \frac{1}{c} \frac{\partial \delta E}{\partial t}
\]

It is convenient to introduce a system of coordinates \((x',y',z')\) so that \(k = k_e x'\) and \(B_{bg} = B_{bg} \sin \theta e_x + B_{bg} \cos \theta e_y\). Since \(\nabla (A^2)\) is directed along \(k\), the solution has the form \(\delta \Omega = \delta \Omega_{e+} e_{x'} + \delta \Omega_{e-} e_{x'} + c.c., \delta \Omega_a = \delta \Omega_{a+} e_{x'} + c.c., \delta \Omega_n = 0, \delta \Omega = \delta \Omega_{e+} e_{x'} + c.c., \delta \Omega_a = \delta \Omega_{a+} e_{x'} + c.c.\). Since \(\nabla (A^2)\) is proportional to \(\exp(i \omega t - i k \cdot x)\), all the variables depend on the coordinates as \(|\exp(i \omega t - i k \cdot x)|\).

Equation (24) gives \(\delta B_{e'} = (ck/\omega) \delta E_{e'}\). Substituting \(\delta B_{e'} = (ck/\omega) \delta E_{e'}\) into Eq. (25), one finds

\[
\frac{e}{m} \frac{\partial \delta E_{e'}}{\partial t} = i \frac{\omega_0}{\omega_{e'}} \frac{\delta T}{\omega_{e'}^2} \delta \Omega_{e'}
\]

The \(y'\) component of Eq. (20) gives

\[
\delta \Omega_{a'} = \frac{\omega_0}{\omega_{e'}} \cos \theta \delta \Omega_{a,e'}
\]

Using Eqs. (26)-(27), from Eq. (21) one finds

\[
\sin \theta \delta \Omega_{e'} + \frac{i \omega_{e'}}{\omega_0} \left(1 - \frac{\omega_0^2}{\omega_{e'}^2} - \frac{\omega_0^2}{\omega_{e'}^2} \cos^2 \theta \right) \delta \Omega_{a,e'} = 0
\]
Since $\delta N/N_0 = (k/\omega) \delta V_z$, which follows from Eq. (18), the $z'$ component of Eq. (20) gives

$$\dot{\delta V}_z = \frac{i e^2}{2 m^2} k_0 \omega (\delta A_+ + \delta A_-) \exp(\text{i} \omega - \text{i} k \cdot x),$$

where we have used Eq. (15) to calculate $V(A^2)$. Obtaining $\delta V_z$ from Eqs. (23)-(29), and using the fact that $\langle N \rangle / N_0 = 1 = (\langle N \rangle / N_0) \delta V_z + \text{c.c.}$, one eventually finds

$$\frac{\langle N \rangle}{N_0} = 1 = \frac{Q}{2 m^2 c^2} A_0 (\delta A_+ + \delta A_-) \exp(\text{i} \omega - \text{i} k \cdot x) + \text{c.c.},$$

where

$$Q = \frac{c k^2}{\omega^2} \left[ \left( 1 - \frac{\omega_0^2}{\omega^2 - c^2 k^2} - \frac{\omega_0^2}{\omega^2 + c^2 k^2} \cos^2 \theta \right) \delta A_+ + \frac{1}{4} (1 - Q) \frac{\omega_0^2}{\omega^2 - c^2 k^2} \delta A_- \right].$$

Note that the density perturbation is independent of $\omega_0$ when the ponderomotive force is aligned with the magnetic field. Indeed, for $\theta = 0$ one finds $Q = c^2 k^2 / (\omega^2 - c^2 k^2)$.

### 3.3.2 Dispersion relation

Substituting Eqs. (14), (15), and (30) into Eq. (4), and neglecting quadratic terms in the perturbed quantities, one finds

$$S_+ \exp(\text{i} \omega t - \text{i} k_+ \cdot x) + S_- \exp(\text{i} \omega t - \text{i} k_- \cdot x) + \text{c.c.} = 0,$$

where

$$S_\pm = \left[ \omega_0^2 - c^2 k_\pm^2 - \omega_0^2 \left( 1 - \frac{1}{4} a_0^2 \right) \right] \delta A_\pm + \frac{1}{4} (1 - Q) a_0^2 \omega_0^2 \left( \omega_0^2 - c^2 k^2 \right) \delta A_-.$$

Eq. (32) requires $S_+ = S_- = 0$, which is a homogeneous system of two equations for $\delta A_+$ and $\delta A_-$. The condition that the determinant of the coefficients vanishes gives the dispersion relation. Since $\omega_0^2 - c^2 k_\pm^2 - \omega_0^2 \left( 1 - \frac{1}{4} a_0^2 \right)$ is $\pm 2(\omega_0^2 - c^2 k_0^2 - \text{i} k)$, which follows from Eq. (12), the dispersion relation can be presented as

$$4 \left( \omega_0^2 - c^2 k_0^2 \right) = \left( \omega^2 - c^2 k^2 \right)^2 + \frac{1}{2} (1 - Q) a_0^2 \omega_0^2 \left( \omega^2 - c^2 k^2 \right),$$

where $a_0$ and $Q$ are given by Eqs. (12) and (31).

### 3.4 Evolution of the wavebands

Below we solve the dispersion relation, Eq. (34), and show that the wavebands grow exponentially. We are interested in a magnetically-dominated magnetar wind with $\sigma_w = \omega_0^2 / \omega_0^2 \gg 1$. We focus on a pump wave that propagates in the direction perpendicular to the background magnetic field, as expected since the wind magnetic field is nearly azimuthal. We introduce a system of coordinates $(x, y, z)$ so that $k_0 = k_y e_y + k_z e_z$ and $B_{bg} = B_{bg} e_y + B_{bg} e_z$. The cosine of the angle between the ponderomotive force (which is directed along $k_0$) and the background magnetic field is $\cos^2 \theta = k_y^2 r_{bg}^2 / (k_y^2 + k_z^2) (B_{bg}^2 + B_{bg}^2)$.

#### 3.4.1 Case $\omega^2 \gg \omega_0^2$

It is convenient to start with the case $\omega^2 \gg \omega_0^2$, since the back-ground magnetic field does not affect the development of the instability. Indeed, one can approximate $Q \approx c^2 k^2 / (\omega^2 - c^2 k^2)$, which is independent of $\omega_0$. Since the dispersion relation depends only on $k_y$ and $k_z$, when $\omega^2 \gg \omega_0^2$ there is a rotational symmetry about the direction of propagation of the pump wave.

One can find approximate analytical solutions of Eq. (34) as follows. Far from the resonances the right hand side of Eq. (33) is small. Then the solution can be presented as $\omega = c^2 k_0^2 / \omega_0 + \Delta \omega$, with a small $\Delta \omega$. Substituting $\omega = c^2 k_0^2 / \omega_0 + \Delta \omega$, the left hand side of Eq. (34) becomes $(\omega_0^2 - c^2 k_0^2) \Delta \omega = \omega_0^2 (\Delta \omega)^2$.

Now we discuss the approximation of $\omega^2 = c^2 k^2_0$ on the right hand side of Eq. (33). Substituting $\omega = c^2 k_0^2 / \omega_0 + \Delta \omega$, one finds $\omega^2 - c^2 k^2 = c^2 k^2_0 (1 - c^2 k_0^2 / \omega_0^2)$. We have neglected the terms $(\Delta \omega)^2$ and $ck_0 (\Delta \omega)$, which are much smaller than $c^2 k^2_0$ (this can be verified a posteriori from Eqs. (37)-(41)). Since $1 - c^2 k_0^2 / \omega_0^2 \approx \omega_0^2 / \omega_0^2$, one finds $\omega^2 - c^2 k^2 = c^2 k^2_0 - c^2 k^2 - c^2 k^2_0\omega_0^2 / \omega_0^2$.

Finally, we need to approximate $Q$. We discuss the two cases $\omega^2 \gg c^2 k^2_0$ and $\omega^2 \ll c^2 k^2_0$. When $\omega^2 \gg c^2 k^2_0$, one finds $Q = c^2 k^2_0 / \omega^2$. When $c^2 k^2_0 \ll (\Delta \omega)^2$, one can approximate $Q \approx c^2 k^2 / (\Delta \omega)^2$ and $\omega^2 - c^2 k^2 = -c^2 k^2 / (\Delta \omega)^2$. Then the dispersion relation can be approximated as

$$4 a_0^2 \omega_0^2 (\Delta \omega / c k_0)^4 = -c^2 k^2_0 / (\Delta \omega)^2 - 2 c^2 k^2 / (c s)^2 \
$$

The wave number of the most unstable modes is $ck_0 \gg \sqrt{\omega_0 a_0 \omega_0}$, and the corresponding growth rate is determined by $\omega_0 / (\Delta \omega)^2 = a_0^2 \omega_0 \omega_0^2 / 2$. Since $\Delta \omega$ is purely imaginary, the perturbation moves along $z$ with the group velocity of the pump wave, $c^2 k_0 / \omega_0$. The conditions $c^2 k^2_0 / \omega^2$ and $c^2 k^2_0 \ll (\Delta \omega)^2$ give $c k_0 \ll a_0 \omega_0$ and $ck_0 \ll a_0 \omega_0$. Following the procedure that we used to derive Eq. (35), one sees that the instability does not develop for $c^2 k^2_0 \gg (\Delta \omega)^2$, which gives $Q \approx c^2 k^2_0$.

When $\omega^2 \ll c^2 k^2_0$, one finds $Q \approx -c^2 k^2_0 / (cs)^2$. Then the dispersion relation can be approximated as

$$4 a_0^2 \omega_0^2 (\Delta \omega / c k_0)^4 = c^2 k^2_0 (c s / 2)^2 - \frac{1}{2} a_0^2 \omega_0^2 = 0.$$

The maximum growth rate of the instability is found when $c^2 k^2_0 = a_0^2 \omega_0^2 / 4 c^2 s^2$, which gives $(\Delta \omega)^2 = -a_0^2 \omega_0^2 / 64 a_0^2 c^2 s^2$. The condition $\omega^2 < c^2 k^2_0$ requires $(\Delta \omega)^2 < c^2 k^2_0$ and $c^2 k^2_0 < c^2 k^2_0$, which give $a_0^2 < c^2 k^2 / 4 \omega_{\text{pc}}$ and $ck_0 \ll a_0 \omega_0$ respectively.

The instability is robust since it can be excited over a broad range of wave numbers. As we show in Appendix A the instability develops for all wave numbers $k > 0$ in a cold plasma where the dispersion relation is given by Eq. (35). The instability develops for $0 < k_0 / a_0 \omega_0 / \sqrt{2} c_0$, in a warm plasma where the dispersion relation is given by Eq. (36).

We summarise our results in Eqs. (37)-(41) below. The growth

4 Since $Q > 1$, in Eq. (41) one has $\delta N / N_0 \gg (\epsilon^2 / m^2 c^2) \delta A^2$. Then the density modulations produced by the ponderomotive force are the dominant non-linear effect leading to the exponential growth of the instability.

5 As discussed by Chosh et al. (2021), when $\omega^2 > c^2 k^2$ the ponderomotive force is balanced by the electron inertia. The reason is that one may neglect $c^2 k^2 V_N / N_0$ with respect to $\delta A^2 / \epsilon t$ in Eq. (29). When $\omega^2 \ll c^2 k^2$, the thermal pressure and the ponderomotive force balance each other since one may neglect $\delta A^2 / \epsilon t$ with respect to $c^2 k^2 V_N / N_0$ in Eq. (20).
The growth rate remains same order of the maximal one for $k_z \gg \Omega_0/c$. When $a_0 \gg \beta_0^2 \Omega_0/\Omega_p$, the growth rate remains same order of the maximal one for $\sqrt{\Omega_0 \Omega_p / \Omega_0} / c \ll k_z < a_0 \beta_0^4 / \Omega_p$. Since $k_z \gg k_x$, the modulations are elongated in the direction of propagation of the electromagnetic pump wave.

The modes described by Eqs. (37)-(41) also exist in unmagnetised electron-ion plasmas (e.g. Drake et al. 1974; Sobacchi et al. 2021), with the only difference that the ion plasma frequency replaces the electron plasma frequency. Our results are consistent with those of Ghosh et al. (2021), who studied the filamentation of electromagnetic waves in unmagnetised pair plasmas.

### 3.4.2 Case $\omega^2 \ll \Omega_0^2$, with $B_{bg,y} = 0$

Since $\omega^2 \gg \Omega_0^2$, the condition $\sigma_w^2 \ll \Omega_0^2$ could be satisfied only for a weak magnetisation $\sigma_w \ll a_0^2 \lesssim 1$, one should study the case $\omega^2 \ll \Omega_0^2$.

When $B_{bg,y} = 0$, the ponderomotive force is nearly parallel to the background magnetic field. Indeed, one finds $\cos^2 \theta = k_x^2 / (k_x^2 + k_z^2)$, which gives $\theta \sim 0$ for $k_x \ll k_z$. Then one can approximate $Q \approx c^2 k_x^2 / (\omega^2 - c^2 k_x^2)$, which is the same as in the weakly magnetised case discussed in the previous section. The most unstable wave number and the growth rate are given by Eqs. (37)-(41). These results are summarised in Table 1. Since $\omega^2 \sim c^2 k_x^2 + 1^2 \sim a_0^2 \omega_0^2$, the condition $\sigma_w^2 \ll \Omega_0^2$ is satisfied in a magnetically-dominated plasma.

We remark that the wave number and the growth rate of the most unstable modes are the same as in the weakly magnetised case. The reason is that the particles can move freely along the background magnetic field under the effect of the ponderomotive force when $\theta \sim 0$.

### 3.4.3 Case $\omega^2 \ll \Omega_0^2$, with $B_{bg,y} = 0$

When $B_{bg,y} = 0$, the ponderomotive force is perpendicular to the background magnetic field. For $\cos \theta = 0$, one finds $Q \ll 1$ because $\sigma_w^2$ appears only in the denominator of $Q$, and $\omega^2, c^2 k_x^2, \Omega_0^2$ are much smaller than $\Omega_0^2$ in magnetically-dominated plasmas. For $Q \ll 1$ the dispersion relation can be approximated as

$$4 \Omega_0^2 (\Delta \omega)^2 = \left( c^2 k_x^2 + \frac{\Omega_0^2}{\omega_0^2} - c^2 k_z^2 - \frac{1}{2} \frac{\omega_0^2}{\omega_p^2} \right).$$

The condition $\omega^2 < \Omega_0^2$ gives $c^2 k_x^2 < \Omega_0^2$. Since $(\omega_p^2 / \Omega_0^2) c^2 k_z^2 \ll \omega_0^2 / \omega_p^2$, the terms proportional to $c^2 k_z^2$ can be neglected in Eq. (42). The wave number of the most unstable modes is $c^2 k_x^2 = a_0^2 \Omega_0^2 / 4$, and the corresponding growth rate is determined by $(\Delta \omega)^2 = -a_0^2 \omega_0^2 / 64 \Omega_0^2$. We conclude that the growth rate can be estimated as

$$\Gamma \approx \frac{1}{8} \frac{\omega_p^2}{\omega_0^2}$$

and the most unstable wave number can be estimated as

$$k_z \approx \frac{\omega_p}{c}$$

These results are summarised in Table 2. Comparing Eqs. (37)-(41) and (43)-(45), one sees that the growth rate is faster when the ponderomotive force is nearly parallel to the direction of the background magnetic field. This may explain the formation of density sheets nearly perpendicular to the pre-shock magnetic field in three-dimensional simulations of the relativistic magnetised shocks (Sironi et al. 2021).

In the modes described by Eqs. (43)-(45), the dominant non-linear effect is the relativistic correction to the electron motion. The effect of the ponderomotive force is suppressed since the particles cannot move in the direction perpendicular to the magnetic field, and the instability develops at nearly constant electron density. The same modes also exist in unmagnetised electron-ion plasmas (e.g. Max et al. 1974; Sobacchi et al. 2021), where the ponderomotive force can be suppressed due to the inertia of the ions.

In magnetically-dominated plasmas the particle distribution could be anisotropic. Then the thermal velocity $c_s$ may be different along the magnetic field lines and in the perpendicular direction.

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**Table 1.** Wave number in the transverse direction ($k_x$) and in the longitudinal direction ($k_z$), and growth rate ($\Gamma$) of the unstable modes (the electromagnetic pump wave is propagating along the $z$ direction). Results are the same for a weakly magnetised wind ($\sigma_w \ll a_0^2$), and for a magnetically-dominated wind with the background magnetic field along $k_z$. The growth rate remains same order of the maximal one for $k_z \ll \Omega_0/c$, while there is no instability for $k_z \gg \Omega_0/c$. When $a_0 \gg \beta_0^2 \Omega_0/\Omega_p$, the growth rate remains same order of the maximal one for $\sqrt{\Omega_0 \Omega_p / \Omega_0} / c \ll k_z < a_0 \beta_0^4 / \Omega_p$. Since $k_z \gg k_x$, the modulations are elongated in the direction of propagation of the electromagnetic pump wave.

| $a_0$ | $k_x$ | $k_z$ | $\Gamma$ |
|------|------|------|--------|
| $\gg \beta_0^2 \Omega_0 / \Omega_p$ | $k_x \ll a_0 \beta_0^{-1} \Omega_p / c$ | $a_0 \beta_0^{-2} \Omega_0 / \Omega_p$ | $\ll a_0 \Omega_0 / \Omega_p$ |
| $< \beta_0^2 \Omega_0 / \Omega_p$ | $k_x \approx a_0 \beta_0^{-2} \Omega_0 / \Omega_p$ | $\ll a_0 \beta_0^{-2} \Omega_0 / \Omega_p$ | $\ll a_0 \Omega_0 / \Omega_p$ |

**Table 2.** Same as Table 1 but for a magnetically-dominated wind with the background magnetic field perpendicular to $k_x$ and $k_z$. The growth rate remains same order of the maximal one for $k_z \ll \Omega_0 / c$, while there is no instability for $k_z \gg \Omega_0 / c$. Since $k_z \gg k_x$, the modulations are elongated in the longitudinal filament (direction). Results are the same for a weakly magnetised wind ($\sigma_w \ll a_0^2$), and for a magnetically-dominated wind with the background magnetic field along $k_z$. The growth rate remains same order of the maximal one for $k_z \ll \Omega_0 / c$, while there is no instability for $k_z \gg \Omega_0 / c$. When $a_0 \gg \beta_0^2 \Omega_0 / \Omega_p$, the growth rate remains same order of the maximal one for $\sqrt{\Omega_0 \Omega_p / \Omega_0} / c \ll k_z < a_0 \beta_0^4 / \Omega_p$. Since $k_z \gg k_x$, the modulations are elongated in the direction of propagation of the electromagnetic pump wave.

| $a_0$ | $k_x$ | $k_z$ | $\Gamma$ |
|------|------|------|--------|
| $\gg \beta_0^2 \Omega_0 / \Omega_p$ | $k_x \ll a_0 \beta_0^{-1} \Omega_p / c$ | $a_0 \beta_0^{-2} \Omega_0 / \Omega_p$ | $\ll a_0 \Omega_0 / \Omega_p$ |
| $< \beta_0^2 \Omega_0 / \Omega_p$ | $k_x \approx a_0 \beta_0^{-2} \Omega_0 / \Omega_p$ | $\ll a_0 \beta_0^{-2} \Omega_0 / \Omega_p$ | $\ll a_0 \Omega_0 / \Omega_p$ |
Since the unstable wave numbers and the growth rate are independent of $c_s$, when the ponderomotive force is perpendicular to the background magnetic field, our results depend only on the value of the thermal velocity along the field lines.

4 SCATTERING OF FAST RADIO BURSTS

We apply these results to the propagation of a FRB through the magnetically-dominated magnetar wind. The observed duration of the FRB is $T \sim 1$ ms. Since the FRB light curve is typically variable, we consider the possibility that the burst is made of pulses with duration $\tau < T$, during which the radiation intensity remains constant. We do not assume any specific emission mechanism.

The wind may be thought of as a sequence of plasma slabs of thickness $\Delta R \sim R$ with a decreasing plasma density $n(R)$. The instability develops when:

(i) The longitudinal wavelength of the unstable modes, $\lambda_c \approx 2\pi/k_c$, is shorter than the length of the pulse in the wind frame, $\ell \approx 2\gamma_w c\tau$.

(ii) The timescale on which the instability grows, $\tau_{gr} \approx 10/\Gamma$, is shorter than the expansion time of the wave front in the wind frame, $\tau_{exp} \approx R/\gamma_w c$.

The conditions (i) and (ii) are satisfied for $R \lesssim R_1$ and $R \lesssim R_2$ respectively. The values of $R_1$ and $R_2$ depend on the thermal velocity, and we calculate them below in the relevant cases.

The instability breaks the wave packet into sheets of radiation perpendicular to the direction of the wind magnetic field. We estimate the transverse size of the radiation sheets as $\lambda_c \approx 2\pi/k_c$, where $k_c(R)$ is the wave number of the most unstable modes. At radii $R \ll \min[R_1, R_2]$, the most unstable transverse wavelength slowly increases with the radius, namely $d\lambda_c/dR \ll \theta_{sc}(R)$, where $\theta_{sc}(R) \sim \lambda_c/\lambda_y$ is the scattering angle at the radius $R$ ($\theta_{sc}$ is measured in the observer’s frame), and $\lambda_y = \pi c/\gamma_w 0\beta_0$ is the FRB wave-length in the observer’s frame. Then the transverse scale of the sheets is gradually adjusted to $\lambda_y$. Scattering occurs at a large radius

$$R_{sc} = f \min[R_1, R_2]$$

where $f \sim 1$ is a numerical factor, since the instability no longer develops for $R \gtrsim R_{sc}$. The corresponding scattering time is $\tau_{sc} \approx R_{sc}^2 \theta_{sc}^2 / 2c$. The outlined scenario is sketched in Figure 1.

Different frequency components of the same burst have different scattering times. Since filamentation is a non-linear process, the transverse scale of the sheets, $\lambda_y$, depends on the power-weighted frequency of the burst. On the other hand, low frequency components are more diffracted. The scattering angle is $\theta_{sc} \sim \lambda_c/\lambda_y \propto \nu^{-1}$, and the corresponding scattering time is $\tau_{sc} \sim R_{sc} \theta_{sc}^2 / 2c \propto \nu^{-2}$.

We consider the case when the transverse component of the perturbation wave vector is parallel to the wind magnetic field (see Section 3.4.2 and Table 1), which gives the largest growth rate of the instability. First we discuss the case of a warm plasma, and then the case of a cold plasma.

4.1 Warm magnetar wind

We start considering the case of a warm plasma with $00 \beta_0^2 \theta_0/\omega_0$, which is satisfied for

$$\beta_0 \gg 2 \times 10^{-3} L_{12}^{1/4} P_{33}^{1/2} P_{10}^{-1} \nu_{15}^{-1} \sigma_{m}^{-1} \sim$$

$$\sim 8 \times 10^{-4} L_{12}^{1/4} P_{42}^{1/2} P_{10}^{-1} \nu_{15}^{-1} \sigma_{m}^{-1/4} .$$

We have expressed $\beta_0$ as a function of the rate of particle outflow in the wind, $\dot N \sim L_{12}^{1/4} \sigma_{m} m c^2$, where $L_{12} \sim \mu^2 (2\pi P) / c^3$ is the luminosity of the wind ($\mu$ and $P$ are the magnetic dipole moment and...
the rotational period of the magnetar, $\gamma_w$ and $\sigma_w$ are the Lorentz factor and the magnetisation of the wind). For our fiducial parameters, we find $N \sim 7 \times 10^3 \mu^2 \, P^3 \, \mu_{20} \, B_{12} s^{-1}$. We have defined $N_{\mu^2} \equiv N/10^4 s^{-1}$, which is the appropriate normalisation for values of $\gamma_w \sigma_w$ of the order of a few tens, as we find below.

The wave numbers and the growth rate of the most unstable mode are $c_k \approx a_0 \beta_{\mu^2}/\sqrt{2}$, $\delta \kappa \approx a_0 \sigma_{\mu^2}$, and $\Gamma \approx a_0^2 \beta_{\mu^2}^2 \sigma_{\mu^2}/80 \omega_0$. Then one finds

$$R_1 \sim 2 \times 10^{14} L_{\mu^2}^{1/3} P_{33}^{-1/2} \nu_9^{-1/2} \gamma_w^{-1/2} \sigma_w^{-1/4} \text{ cm} \sim 8 \times 10^{14} L_{\mu^2}^{1/3} P_{33}^{-1/2} \nu_9^{-1/2} \gamma_w^{-1/2} \sigma_w^{-1/4} \text{ cm} \quad (48)$$

$$R_2 \sim 4 \times 10^{14} L_{\mu^2}^{1/3} P_{33}^{-1/2} \nu_9^{-1/2} \gamma_w^{-1/2} \sigma_w^{-1/4} \text{ cm} \sim 9 \times 10^{14} L_{\mu^2}^{1/3} P_{33}^{-1/2} \nu_9^{-1/2} \gamma_w^{-1/2} \sigma_w^{-1/4} \text{ cm} \quad (49)$$

where we have defined $\tau_{-3} \equiv \tau/1 \text{ ms}$. The conditions $\kappa \lesssim \ell$ and $\tau_{\text{GR}} \lesssim \tau_{\text{exp}}$ are satisfied at radii $R \lesssim R_1$ and $R \lesssim R_2$ respectively.

The value of the scattering time depends on the Lorentz factor of the wind. The critical wind Lorentz factor that gives $R_1 = R_2$ is

$$\gamma_{\text{cr}} \sim 2 L_{\mu^2}^{1/3} P_{33}^{-1/2} \nu_9^{-1/2} \gamma_w^{-3/4} \tau_{-3}^{-3/4} \beta_{\mu^2}^{-1} \sigma_w^{-1/8} \sim 1.7 L_{\mu^2}^{1/3} \nu_9^{-6/7} \gamma_w^{-6/7} \tau_{-3}^{-3/7} \beta_{\mu^2} \sigma_w^{-1/7} . \quad (50)$$

Note that $\gamma_{\text{cr}} \gg 1$ for $\beta_{\mu^2} \ll 1$. When $\gamma_w \lesssim \gamma_{\text{cr}}$, one finds $R_{\text{sc}} = f R_1$, and the scattering time is

$$\tau_{\text{sc}} \sim 3 L_{\mu^2}^{1/4} P_{33}^{1/2} \nu_9^{-5/2} \gamma_w^{-2} \tau_{-3}^{-2} \beta_{\mu^2}^{-2} \sigma_w^{-1/4} \text{ ns} \sim 0.9 L_{\mu^2}^{1/4} P_{33}^{1/2} \nu_9^{-5/2} \gamma_w^{-2} \tau_{-3}^{-2} \beta_{\mu^2}^{-2} \sigma_w^{-1/4} \text{ ns} . \quad (51)$$

When $\gamma_w \gtrsim \gamma_{\text{cr}}$, one finds $R_{\text{sc}} = f R_2$, and the scattering time is

$$\tau_{\text{sc}} \approx 0.8 \nu_9^{-1} \text{ ns} . \quad (52)$$

Scattering in a warm wind produces a frequency modulation with a large bandwidth, $\Delta \nu \sim 1/\tau_{\text{sc}} \gtrsim 100 \text{ MHz}$ (see Eqs. [51] and [52]). Such broadband frequency modulations are observed in FRBs (e.g., Shannon et al. 2018; Hessels et al. 2019; Nimmo et al. 2021). The bandwidth increases with the burst frequency (Eqs. [51] and [52] give $\Delta \nu \sim \nu^2$ with $\beta_{\mu^2} \sim 1 - 2.5$), consistent with observations of FRB 121102 (Hessels et al. 2019).

When the transverse component of the perturbation wave vector is perpendicular to the wind magnetic field (see Section 3.4.2 and Table 3), the unstable modes have $c_k \approx a_0 \omega_{\mu^2}/\sqrt{2}$, $\delta \kappa \approx a_0 \sigma_{\mu^2}$, and $\Gamma \approx a_0^2 \beta_{\mu^2}^2 \sigma_{\mu^2}/80 \omega_0$. Since the condition $\kappa \lesssim \ell$ is easily satisfied, one finds $R_{\text{sc}} = f R_1$. The scattering time is the same as in Eq. [52], regardless of $\beta_{\mu^2}$.

### 4.2 Cold magnetar wind

In a cold plasma with $a_0 \gg \beta_{\mu^2}^2 \omega_{\mu^2}/\sqrt{2}$, we have $c_k \approx a_0 \omega_{\mu^2}/\sqrt{2}$ and $\delta \kappa \approx a_0 \sigma_{\mu^2}$, and $\Gamma \approx a_0^2 \beta_{\mu^2}^2 \sigma_{\mu^2}/80 \omega_0$. To determine the dominant transverse scale of the radiation sheets, $\lambda_{\nu}$, one should study how the instability saturates for various wave numbers, which is out of the scope of the paper. Nevertheless, our results can be used to place a lower limit on the Lorentz factor of the magnetar wind. Since $\lambda_{\nu} \lesssim 2\mu / \sqrt{\gamma_w \sigma_w}$, one finds a lower limit for the scattering time that is independent of $\beta_{\mu^2}$,

$$\tau_{\text{sc}} \gtrsim 4 \nu_9^{1/8} L_{\mu^2}^{1/4} P_{33}^{1/2} \nu_9^{-1/2} \gamma_w^{-1/4} \sigma_w^{-1/8} \text{ ms} \sim 4 \nu_9^{1/8} L_{\mu^2}^{1/4} P_{33}^{1/2} \nu_9^{-1/2} \gamma_w^{-1/4} \sigma_w^{-1/8} \text{ ms} . \quad (53)$$

We remark that $\nu_0$ corresponds to the power-weighted frequency of the burst. As discussed above, the frequency components of one burst have different scattering times, with $\tau_{\text{sc}} \sim \nu^{-2}$.

Interestingly, the frequency-dependent broadening of the brightest pulse from FRB 181112 is consistent with $\tau_{\text{sc}} \sim \nu^{-2}$ (Cho et al. 2020). The rise time of the pulse is $\tau \sim 15 \mu s$, and the scattering time is $\tau_{\text{sc}} \sim 25 \mu s$. The observed scattering can be an effect of propagation through a cold magnetar wind. Substituting $\tau \sim 15 \mu s$ and $\tau_{\text{sc}} \sim 25 \mu s$ into Eq. [53], one can estimate the wind Lorentz factor,

$$\gamma_w \gtrsim 60 L_{\mu^2}^{1/8} P_{33}^{1/2} \nu_9^{-1/4} \sigma_w^{-1/8} \sim 50 L_{\mu^2}^{1/8} P_{33}^{1/2} \nu_9^{-1/4} \sigma_w^{-1/8} \text{ ms} . \quad (54)$$

This Lorentz factor is not far from $\gamma_w \sim 10 – 30$ estimated for magnetar winds (Beloborodov 2020).

### 5 CONCLUSIONS

We have studied the modulation/filamentation instabilities of FRBs propagating in a magnetar wind. We have modelled the wind as a magnetically-dominated pair plasma. We have focused on the regime of sub-relativistic electron motion, i.e. dimensionless wave strength parameter $a_0 \lesssim 1$.

The instability modulates the intensity of the radio wave, producing radiation sheets perpendicular to the direction of the wind magnetic field. As the FRB front expands outside the radius where the instability ends, the radiation sheets are diffraeted, effectively spreading the arrival time of the FRB wave. The imprint of the scattering on the time-frequency structure of FRBs depends on the properties of the wind.

In a cold wind with $\beta_{\mu^2} \ll 10^{-3}$ ($\beta_{\mu^2} = c_{\mu^2}/c$ is the ratio of the thermal velocity along the magnetic field lines and the speed of light), the typical FRB scattering time is $\tau_{\text{sc}} \sim \mu s - \text{ms}$ at the frequency $\nu \sim 1 \text{ GHz}$. Low frequencies have longer scattering times, with $\tau_{\text{sc}} \sim \nu^{-2}$. Such frequency-dependent broadening has been observed in the brightest pulse of FRB 181112 (Cho et al. 2020). From the rise and scattering timescales of the pulse, we estimate the wind Lorentz factor, $\gamma_w \gtrsim 50$. Within the accuracy of this estimate (a factor of a few), $\gamma_w$ is consistent with theoretical expectations for magnetar winds (Beloborodov 2020).

In a warm wind with $\beta_{\mu^2} \gtrsim 10^{-3}$, the FRB scattering time can approach $\tau_{\text{sc}} \sim \text{ns}$. Then scattering produces a frequency modulation of the observed intensity with a large bandwidth, $\Delta \nu \sim 1/\tau_{\text{sc}} \gtrsim 100 \text{ MHz}$. The modulation bandwidth increases with the burst frequency. Broadband frequency modulations observed in FRBs (e.g., Shannon et al. 2018; Hessels et al. 2019; Nimmo et al. 2021) could be due to scattering in a warm magnetar wind.

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DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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APPENDIX A: SOLUTION OF EQS. (35), (36), (42)

We start with the case when the ponderomotive force is nearly parallel to the background magnetic field. In a cold plasma, the dispersion relation is given by Eq. (35). The growth rate of the exponen-

![Figure A1](Image)

Figure A1. Imaginary part of $\Delta \omega$ as a function of $k_y$ in a cold plasma (top panel) and in a warm plasma (bottom panel). The ponderomotive force is nearly parallel to the background magnetic field.

The solution can be presented as

$$\langle \Delta \omega \rangle^2 = -\frac{2\Gamma_{\text{cold}}^2}{1 + \sqrt{1 + \frac{k_y^2}{a_s^2}}}$$

(A1)

where $\Gamma_{\text{cold}} = \frac{a_0 \omega_p}{2}$ and $c k_{\text{cold}} = \sqrt{8 \Gamma_{\text{cold}} a_0 \omega_p}$. The maximal growth rate $\Gamma_{\text{cold}}$ is achieved for $k_y \gg k_{\text{cold}}$. In the top panel of Figure A1 we plot $\text{Im}(\Delta \omega)/\Gamma_{\text{cold}}$ as a function of $k_y/k_{\text{cold}}$. The instability develops for all $k_y$.

In a warm plasma, the dispersion relation is given by Eq. (36). The solution can be presented as

$$\langle \Delta \omega \rangle^2 = -\Gamma_{\text{warm}}^2 \left[ 1 - \left( 1 - \frac{k_y^2}{k_{\text{warm}}^2} \right)^2 \right]$$

(A2)

where $\Gamma_{\text{warm}} = a_0^2 \beta^2 \omega_p^2 / 8 \omega_0$ and $c k_{\text{warm}} = a_0 \beta^{-1} \omega_p / 2$. The maximal growth rate $\Gamma_{\text{warm}}$ is achieved for $k_y = k_{\text{warm}}$. In the bottom panel of Figure A1 we plot $\text{Im}(\Delta \omega)/\Gamma_{\text{warm}}$ as a function of $k_y/k_{\text{warm}}$. The instability develops for $k_y < \sqrt{2} k_{\text{warm}}$.

When the ponderomotive force is perpendicular to the background magnetic field, the dispersion relation is given by Eq. (42). The terms proportional to $k_x^2$ are negligibly small. Then Eq. (42) is identical to Eq. (36) after the formal substitution $c_x \to c$ in Eq. (36). The dependence of $\text{Im}(\Delta \omega)$ on $k_y$ is analogous to the bottom panel of Figure A1.