Inconsistency of Naive Dimensional Regularizations and Quantum Correction to Non-Abelian Chern-Simons-Matter Theory Revisited

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Abstract

We find the inconsistency of dimensional reduction and naive dimensional regularization in their applications to Chern-Simons type gauge theories. Further we adopt a consistent dimensional regularization to investigate the quantum correction to non-Abelian Chern-Simons term coupled with fermionic matter. Contrary to previous results, we find that not only the Chern-Simons coefficient receives quantum correction from spinor fields, but the spinor field also gets a finite quantum correction.

I. INTRODUCTION

There has been a considerable amount of popularity in perturbative Chern-Simons-type theory due to the particular feature of the finite renormalization of its coupling constant. Almost all the old regularization schemes [1–4] and even some newly developed ones [5, 6] have been applied. The existence of the antisymmetric tensor in Chern-Simons term makes the implement of the regularization method much more non-trivial. Specially the one-loop quantum correction to a general three-dimensional field theory is very delicate [7]. Different regularization methods can easily produce ambiguity in the quantum corrections. In particular, it appears that some old regularization schemes can bring non-physical quantum corrections [8, 9]. This makes one be cautious about the use of some regularization methods. In this paper we show that dimensional reduction and naive dimensional regularization are inconsistent when they are applied to Chern-Simons type theories. Indeed, when we use the consistent dimensional regularization to re-calculate the one-loop quantum correction for one typical example, Chern-Simons term coupled with spinor field, we obtain a result different from the previous ones [10].

Regulating Chern-Simons theory by dimensional reduction means evaluating all the antisymmetric tensor algebra in three dimensions but performing the loop momentum integration in $n$-dimension [11, 12]. The concrete definition in three dimensions is as follows:
The inconsistency of this regularization method in four-dimensional supersymmetric field theories had already been found by its inventor [11]. For three-dimensional case, it can also be easily shown that this regularization method is not consistent. From Eq. (1), we have

\[ \delta^\mu_\nu = \tilde{\delta}^\mu_\nu + \delta^\mu_\nu, \delta^\mu_\nu \tilde{\delta}^\nu_\rho = \delta^\mu_\rho, \tilde{\delta}^\nu_\rho = \delta^\mu_\rho, \tilde{\delta}^\mu_\nu = 0, \]

\[ k_\mu \tilde{\delta}^\mu_\nu = k_\mu, k_\mu \delta^\mu_\nu = 0; \tilde{\delta}^\mu_\nu = n, \delta^\mu_\nu = 3 - n. \]  

(1)

So we can obtain

\[ 0 = (\tilde{\epsilon}^{\mu \nu \rho} \epsilon_{\alpha \beta \gamma}) (\tilde{\epsilon}_{\mu \nu \rho} \epsilon^{\alpha \beta \gamma}) = (\tilde{\epsilon}^{\mu \nu \rho} \epsilon^{\alpha \beta \gamma}) (\epsilon_{\alpha \beta \gamma} \tilde{\epsilon}^{\mu \nu \rho}) 
= n(3 - n)(n - 1)^2(2n - 2)^2. \]

(3)

Therefore, it is only valid for \( n = 0, 1, 2, 3 \), and thus it is not the analytic dimensional continuation as that required by dimensional regularization.

As for the naive dimensional regularization, it defines the antisymmetric tensor algebra to satisfy [10]

\[ \epsilon_{\mu \sigma \eta} \epsilon^{\mu \lambda \tau} = (\delta^\lambda_\sigma \delta^\tau_\eta - \delta^\tau_\sigma \delta^\lambda_\eta) \Gamma(n - 1), \delta^\sigma_\sigma = n. \]  

(4)

We can show that this definition in essence makes the theory defined in three dimensions, but not in \( n \)-dimension as it should be. This can be seen from the following simple algebraic manipulations. Consider the quantity \( \epsilon_{\mu \sigma \eta} \epsilon^{\mu \lambda \tau} \epsilon^{\alpha \lambda \tau} \). On one hand, it equals to

\[ (\epsilon_{\mu \sigma \eta} \epsilon^{\mu \lambda \tau} \epsilon^{\alpha \lambda \tau}) \epsilon^{\alpha \lambda \tau} = \Gamma(n - 1)(\delta^\lambda_\sigma \delta^\tau_\eta - \delta^\tau_\sigma \delta^\lambda_\eta) \epsilon_{\alpha \lambda \tau} 
= \Gamma(n - 1)(\epsilon_{\alpha \sigma \eta} - \epsilon_{\alpha \eta \sigma}) = 2\Gamma(n - 1)\epsilon_{\alpha \sigma \eta}; \]

(5)

on the other hand, we have

\[ \epsilon_{\mu \sigma \eta} (\epsilon^{\mu \lambda \tau} \epsilon^{\alpha \lambda \tau}) = \epsilon_{\mu \sigma \eta} \Gamma(n - 1)(\delta^\alpha_\mu \delta^\lambda_\sigma \delta^\lambda_\alpha - \delta^\mu_\lambda \delta^\sigma_\alpha \delta^\lambda_\alpha) 
= \Gamma(n - 1)(n - 1)\epsilon_{\alpha \sigma \eta}. \]

(6)

Comparing Eq. (5) with Eq. (3), we can see that only \( n = 3 \), otherwise \( \Gamma(n - 1) = 0 \). Thus the naive dimensional regularization also does not make the theory well defined.

This motivates us to re-consider Chern-Simons type theory in the dimensional regularization schemes proposed by ’t Hooft and Veltman [13]. To our knowledge, up to now this is the only dimensional continuation scheme compatible with gauge symmetry to deal with \( \gamma_5 \) and the similar problems. In the following we use this consistent dimensional regularization to investigate the one-loop quantum corrections of non-Abelian Chern-Simons term coupled to spinor field, the classical action of which in Minkowski space is

\[ S = \int d^3x \left[ \epsilon^{\mu \nu \rho} \left( \frac{1}{2} A^{\alpha}_\mu \partial_\nu A^\alpha_\rho + \frac{1}{3!} g f^{abc} A^a_\mu A^b_\nu A^c_\rho \right) + \bar{\psi}(i\partial - m + g A^a T^a) \psi \right], \]

(7)
where \( \psi \) belongs to the fundamental representation of gauge group, and for simplicity we only consider the one-flavour case; \( \gamma_\mu (\mu = 0, 1, 2) \) are usually chosen as follows [14],

\[
\gamma^0 = \sigma_2, \quad \gamma^1 = i\sigma_3, \quad \gamma^3 = i\sigma_1; \\
\gamma_\mu \gamma_\nu = g_{\mu\nu} - i\epsilon_{\mu\nu\rho}\gamma^\rho, \quad g_{\mu\nu} = \text{diag}(1, -1, -1).
\] (8)

This model has become revived in recent years owing to its possible physical application to condensed matter theory. The coefficient of Chern-Simons term (called statistical parameter) plays a crucial role in transmuting the spin and the statistics of the anyon particles. The quantum corrections up to two-loop for this model was investigated in dimensional reduction and naive dimensional regularization [10]. It was found that there exist no quantum corrections and all the renormalization constant are identically equal to one.

For the case of Chern-Simons typical theory, the dimensional continuation proposed by 't Hooft and Veltman, which had been been explicitly written down in Refs. [5, 16], is as follows,

\[
e^{\mu_1\mu_2\mu_3}e_{\nu_1\nu_2\nu_3} = \sum_{\pi \in \mathbb{P}_3} \text{sgn}(\pi)\Pi_{i=1}^3 \delta^{\mu_i}_{\nu_{\pi(i)}}, \quad g_{\mu\nu} = \tilde{g}_{\mu\nu} \oplus \hat{g}_{\mu\nu}, \quad p_\mu = \tilde{p}_\mu \oplus \hat{p}_\mu,
\]

\[
e^{\mu\nu\rho}\tilde{g}_{\rho\alpha} = 0, \quad e^{\mu\nu\rho}\hat{g}_{\rho} = 0, \quad \tilde{\delta}^\mu = 3, \quad \hat{\delta}^\mu = n - 3. \quad (9)
\]

Then \( n \) is continued to a complex variable to regulate the theory. However, the price that must be paid for this consistent definition, as that pointed out in Ref. [4], is first performing higher covariant derivative regularization. Otherwise this dimensional continuation will lead to linear dependence of Chern-Simons kinetic operator even after gauge-fixing. As usual, we choose the simplest higher covariant derivative term, the Yang-Mills Lagrangian,

\[
-\frac{1}{4M}F^a_{\mu\nu}F_{a\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu.
\] (10)

The ghost and gauge-fixing terms have the well known form in the covariant gauge,

\[
S_{\text{ghost}} + S_{g.f.} = \int d^3x \left[ -\partial^\mu e^a_\mu \left( \partial_\mu e^a_\mu + gf^{abc}A^b_\mu e^c_\mu \right) - \frac{1}{2\alpha} (\partial^\mu A^a_\mu)^2 \right]. \quad (11)
\]

There still exists another difficulty, that is, this dimensional prescription in fact defines the \( n \)-dimensional \( \epsilon^{\mu\nu\rho} \) effective only in three dimensions. This makes the regulated theory possess the \( SO(3) \otimes SO(n-3) \) covariance rather than \( SO(n) \), the regulated propagator will not only take very complicated form, but it is also not \( SO(n) \) covariant. This will make the loop integration very difficult to carry out. However, thanks to [4], one can prove that the propagator of gauge field can be decomposed into two parts: one part is composed of evanescent quantity, which has no contribution to the loop integration in the limit \( n \to 3 \); then one can make use of the second part as an effective propagator

\[
C^{ab}_{\mu\nu}(p) = -\frac{iM}{p^2 (p^2 - M^2)} \left( iM\epsilon^{\mu\nu\rho}p^\rho + p^2 g_{\mu\nu} - p_\mu p_\nu \right)
\] (12)

in order to perform calculation. Note that we have chosen the Landau gauge \((\alpha = 0)\). The other Feynman rules are listed as below:
• Fermion propagator

\[ S(p) = i \left( \frac{p^2 + m}{p^2 - m^2} \right) \delta_{ij}; \]  

(13)

• Quark-Gluon vertex

\[ ig\gamma^\mu T^a_{ij} (2\pi)^3 \delta^{(3)}(p + q + r). \]  

(14)

In Sect.II, using the consistent dimensional continuation, we re-consider some of the new one-loop two-point Green functions such as the fermionic self-energy, the ghost self-energy and the contribution to vacuum polarization tensor from the fermionic loop. Sect.III is about the one-loop three-point functions such as the fermion-gluon vertex, the ghost-gluon vertex and so on. Since it is quite complicated to straightforwardly calculate the fermion-gluon vertex in a non-Abelian gauge theory, we make use of the Slavnov-Taylor identity between fermion-gluon vertex and the composite ghost-fermion vertex to facilitate the calculation. In Sect.IV, we define the finite renormalization of the coupling constant with mass-shell renormalization convention and we show that the result is different from that presented in the literature previously [3]. Finally, in Sect.V we emphasize our conclusions and discuss the justification for our results. For clarity and completeness, a derivation of the needed Slavnov-Taylor identities from BRST symmetry is presented in Appendix.

II. ONE-LOOP TWO-POINT FUNCTION

A. Contribution to Vacuum Polarization Tensor from Fermionic Loop

The contributions to vacuum polarization tensor from the self-interaction of gauge fields and the ghost loop have been shown in many works [3, 4, 17]. Here we only consider the extra contribution from the spinor field.

The relevant Feynman diagram is shown in Fig.1 and the amplitude is

\[ i\Pi_{\mu\nu}^{(f)}_{ab} = -g^2 \text{Tr}(T^a T^b) \int \frac{d^4k}{(2\pi)^n} \frac{\text{Tr} \left\{ \gamma_\nu \left[ k + \beta + m \right] \gamma_\mu \left( k + m \right) \right\}}{(k^2 - m^2)(p^2 - m^2)} \]

\[ = \frac{1}{2} g^2 \delta^{ab} \int \frac{d^4k}{(2\pi)^n} \left\{ -im\epsilon_{\mu\nu\rho}p^\rho + 2k_\mu k_\nu + k_\mu p_\nu + k_\nu p_\mu - g_{\mu\nu} [k \cdot (k + p) - m^2] \right\}, \]  

(15)

where we choose the normalization of group factor as,

\[ \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}. \]  

(16)

Calculation (after taking the limit \( n \rightarrow 3 \)) gives

\[ i\Pi_{\mu\nu}^{(f)}_{ab} = \frac{ig^2}{16\pi} \delta^{ab} \left\{ i\epsilon_{\mu\nu\rho} \frac{m}{p} \ln \left[ \frac{1 + p/(2m)}{1 - p/(2m)} \right] \right\} \]

\[ - \left( p^2 g_{\mu\nu} - p_\mu p_\nu \right) \frac{1}{m} \left[ -\frac{m^2}{p^2} + \left( \frac{1}{4} \frac{m}{p^2} + \frac{m^3}{p^3} \right) \ln \left( \frac{1 + p/(2m)}{1 - p/(2m)} \right) \right], \]  

(17)
where \( p \equiv |p| \). Using the expansion near \( p = 0 \),

\[
\ln \left[ \frac{1 + p/(2m)}{1 - p/(2m)} \right] = \frac{p}{m} + \frac{1}{12} \frac{p^3}{m^3} + \frac{1}{80} \frac{p^5}{m^5} + \cdots ,
\]

we have

\[
\Pi^{(f)ab}(0) = \frac{g^2}{8\pi} \delta^{ab} \left[ i \epsilon_{\mu\nu\rho} p^\rho - \frac{1}{3m} (p^2 g_{\mu\nu} - p_\mu p_\nu) \right] .
\]

Combining Eq.(17) with the contributions to polarization tensor from gluon and ghost loops [3, 4, 17],

\[
\Pi^{(g)}_{\mu\nu} (p) + \Pi^{(gh)}_{\mu\nu} (p) = -\frac{7}{3} \frac{g^2}{4\pi} C_V \delta^{ab} \epsilon_{\mu\nu\rho} p^\rho ,
\]

and choosing the renormalization condition on gluon mass-shell \( p = 0 \),

\[
\Pi^{abR}_{\mu\nu}(0) = 0,
\]

we can define the gluon wave function renormalization constant,

\[
Z_A = 1 - \frac{g^2}{4\pi} \left( \frac{7}{3} C_V + \frac{1}{2} \right) .
\]

### B. Self-energy for Spinor Field

Let us consider fermionic self-energy. Its Feynman diagram is shown in Fig.2 and the amplitude is read as

\[
-i \Sigma(p, M) = -g^2 (T^a T^a) M \int \frac{d^n k}{(2\pi)^n} \frac{\gamma_\nu [\gamma_\mu (p + k + m) + m]}{[(k + p)^2 - m^2] k^2 (k^2 - M^2)} [\gamma_\nu (k - m) + m (k^2 - \gamma_\alpha k^\alpha M)]
\]

\[
= -2M g^2 C_2(R) 1 \int \frac{d^n k}{(2\pi)^n} \frac{(M - k) \cdot (k + p) + m (k^2 - \gamma_\alpha k^\alpha M)}{[(k + p)^2 - m^2] k^2 (k^2 - M^2)} ,
\]

where 1 is the unit matrix in colour space, and we have used Eq.(8) and the identity

\[
\int \frac{d^n k}{(2\pi)^n} \frac{\epsilon_{\mu\nu\rho} k^\nu k^\rho}{[(k + p)^2 - m^2] k^2 (k^2 - M^2)} = 0 .
\]

Using the decomposition

\[
\frac{1}{k^2 (k^2 - M^2)} = \frac{1}{M^2} \left( \frac{1}{k^2 - M^2} - \frac{1}{k^2} \right) ,
\]

\[1\]For pure Chern-Simons theory, this renormalization scheme is equivalent to taking the large mass limit \( M \to \infty \). However here, owing to the mass parameter \( m \), they are not equivalent.
and
\[ k \cdot p = \frac{1}{2} \left\{ \left[ (k + p)^2 - m^2 \right] - k^2 - (p^2 - m^2) \right\} \]
\[ = \frac{1}{2} \left\{ \left[(k + p)^2 - m^2\right] - (k^2 - M^2) - (p^2 - m^2 - M^2) \right\}, \tag{26} \]

we can write Eq.\((22)\) as follows:
\[
- i \Sigma(p, M) = -2M g^2 C_2(R) \text{i} \int \frac{d^n k}{(2\pi)^n} \left\{ \left( \frac{p^2 - m^2}{2M^2} - \frac{1}{2} - \frac{m}{M} \right) \frac{k - M}{(k^2 - M^2)[(k + p)^2 - m^2]} \right. \\
+ \frac{1}{2M} \frac{1}{k^2 - M^2} + \frac{p^2 - m^2}{2M} \frac{1}{k^2[(k + p)^2 - m^2]} \\
- \left( \frac{p^2 - m^2}{2M^2} - \frac{m}{M} \right) \frac{k}{k^2[(k + p)^2 - m^2]} \right\}. \tag{27} \]

The standard integration gives
\[
- i \Sigma(p, M) = -\frac{i}{4\pi} g^2 C_2(R) \text{i} M \left\{ 1 + \frac{p^2 - m^2}{M p} \ln \left( \frac{1 + p/m}{1 - p/m} \right) \right. \\
- \gamma_\mu p^\mu \left( \frac{p^2 - m^2}{2M^2} - \frac{m}{M} \right) \left[ \frac{m}{p^2} + \left( \frac{1}{p} - \frac{m^2}{p^3} \right) \ln \left( \frac{1 + p/m}{1 - p/m} \right) \right] \\
\left. + \frac{\not{p}}{p^2} \left( \frac{p^2 - m^2}{2M^2} - \frac{1}{2} - \frac{m}{M} \right) \left[ M - m - \frac{p^2 - m^2 + M^2}{2p} \ln \left( \frac{1 + (p + m)/M}{1 - (p + m)/M} \right) \right] \\
\right. + \left. \left( \frac{m}{p} + \frac{M}{2p} - \frac{p^2 - m^2}{2Mp} \right) \ln \left( \frac{1 + (p + m)/M}{1 - (p + m)/M} \right) \right\}. \tag{28} \]

After taking the large-\(M\) limit, we obtain the quark self-energy
\[
- i \Sigma(p) = -\frac{i}{4\pi} g^2 C_2(R) \text{i} \left( 2M + m + \frac{p^2 - m^2}{p} \ln \left( \frac{1 + p/m}{1 - p/m} \right) \right) \\
- \frac{\not{p}}{p^2} \left[ \frac{m^2}{p^2} + \left( \frac{m}{p} - \frac{m^3}{p^3} \right) \ln \left( \frac{1 + p/m}{1 - p/m} \right) - \frac{2}{3} \right] \right\}. \tag{29} \]

As usual, this quark self-energy can be written in the form of quark mass expansion,
\[
\Sigma(p) = \frac{1}{2\pi} g^2 C_2(R) \text{i} \left( M + \frac{m}{3} \right) + \frac{1}{4\pi} g^2 C_2(R) \text{i} \frac{5}{3} (\not{p} - m) \\
+ \frac{1}{4\pi} g^2 C_2(R) \text{i} \left( 2m + \frac{p^2 - m^2}{p} \ln \left( \frac{1 + p/m}{1 - p/m} \right) \right) \\
- \frac{\not{p}}{p^2} \left[ \frac{m^2}{p^2} + \left( \frac{m}{p} - \frac{m^3}{p^3} \right) \ln \left( \frac{1 + p/m}{1 - p/m} \right) \right] \right\} \\
= \delta m \text{i} - (Z_\psi^{-1} - 1)(\not{p} - m) \text{i} + Z_\psi^{-1} \Sigma_R(p). \tag{30} \]

Thus in the quark mass-shell renormalization scheme, we have the renormalization constants and the radiative correction of quark self-energy as:
\[ m_{ph} = m - \delta m = m - \frac{g^2}{2\pi} C_2(R)(M + m); \]

\[ Z_\psi = 1 + \frac{5}{3} \frac{g^2}{4\pi} C_2(R); \]

\[ \Sigma_R(p) = \frac{1}{4\pi} g^2 C_2(R) \left\{ 2m_{ph} + \frac{p^2 - m_{ph}^2}{p} \ln \left( \frac{1 + p/m_{ph}}{1 - p/m_{ph}} \right) \right. \]
\[ \left. - \hat{p} \left[ 1 + \frac{m_{ph}^2}{p^2} + \left( \frac{m_{ph}}{p} - \frac{m_{ph}^3}{p^3} \right) \ln \left( \frac{1 + p/m_{ph}}{1 - p/m_{ph}} \right) \right] \right\}. \]

(31)

C. Self-energy for Ghost Field

The self-energy for ghost field had been explicitly shown in [17] in a different method. Here for completeness and later use, we re-calculate it in terms of consistent dimensional regularization (Fig.3),

\[ i\Sigma^{(1)ab}_g(p)p^2 = \lim_{M \to \infty} g^2 C_V \delta^{ab} \int \frac{d^Dk}{(2\pi)^D} \frac{M}{k^2(k^2 - M^2)(k + p)^2} \left[ k^2p^2 - (k.p)^2 \right] \]
\[ = \lim_{M \to \infty} g^2 C_V \delta^{ab} \int \frac{d^Dk}{(2\pi)^D} \left[ \frac{Mp^2}{(k^2 - M^2)(k + p)^2} - \frac{1}{M} \frac{(k.p)^2}{(k^2 - M^2)(k + p)^2} \right] \]
\[ = \lim_{M \to \infty} g^2 C_V \delta^{ab} \left[ \frac{1}{2} \frac{1}{p^2} + \frac{1}{4} \frac{M^2}{p^3} - \frac{1}{4} \frac{M^3}{p^3} \right] \ln \left( \frac{1 + p/M}{1 - p/M} \right) \]
\[ = g^2 C_V \delta^{ab} \frac{i}{4\pi} p^2 \frac{2}{3}. \]

(32)

Consequently, one can define the wave function renormalization constant for ghost field,

\[ Z_c = 1 + \frac{g^2}{4\pi} C_V. \]

(33)

III. ONE-LOOP THREE-POINT FUNCTION

A. One-loop On-shell Quantum Correction to Fermion-Gluon Vertex

Let us see the one-loop quantum correction to quark-gluon vertex, which receives contributions from two Feynman diagrams (Fig.4). The first diagram is quite simple and can be calculated analytically. However the calculation for the second diagram is quite complicated since it contains one three-gluon vertex and two gauge field propagators. Thus we shall make use of the Slavnov-Taylor identity to convert the calculation of fermion-gluon vertex into that of composite fermion-ghost vertices, whose amplitude can be easily calculated. The detailed derivation of this identity and its one-loop form are listed in Appendix.

From Eq.(A13), we can see that to calculate the quark-gluon vertex, three parts need to be considered. The first part is associated with the ghost field self-energy, which can be easily obtained from Eq.(32),

\[ i\Sigma^{(1)ab}_g(p)p^2 = \lim_{M \to \infty} g^2 C_V \delta^{ab} \int \frac{d^Dk}{(2\pi)^D} \frac{M}{k^2(k^2 - M^2)(k + p)^2} \left[ k^2p^2 - (k.p)^2 \right] \]
\[ = \lim_{M \to \infty} g^2 C_V \delta^{ab} \int \frac{d^Dk}{(2\pi)^D} \left[ \frac{Mp^2}{(k^2 - M^2)(k + p)^2} - \frac{1}{M} \frac{(k.p)^2}{(k^2 - M^2)(k + p)^2} \right] \]
\[ = \lim_{M \to \infty} g^2 C_V \delta^{ab} \left[ \frac{1}{2} \frac{1}{p^2} + \frac{1}{4} \frac{M^2}{p^3} - \frac{1}{4} \frac{M^3}{p^3} \right] \ln \left( \frac{1 + p/M}{1 - p/M} \right) \]
\[ = g^2 C_V \delta^{ab} \frac{i}{4\pi} p^2 \frac{2}{3}. \]
\[ \Sigma_g^{(1)}(p) = \frac{g^2}{4\pi^3} \frac{2}{3} C_V. \quad (34) \]

Now we turn to the second part, which is connected with the quark self-energy. To calculate its contribution to the on-shell quark-gluon vertex, we should first pull out the factor \((q - p)_\mu\) and then put it on mass-shell. From Eq.\((29)\), we have

\[
-i \left[ \Sigma^{(1)}(q) - \Sigma^{(1)}(p) \right] = -2g^2C_2(R)M \left\{ \left( m + \frac{M}{2} \right) \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 - M^2} \left[ \frac{1}{(k + q)^2 - m^2} - \frac{1}{(k + p)^2 - m^2} \right] \right. \\
\left. - \left( \frac{m}{M} + \frac{1}{2} \right) \int \frac{d^n k}{(2\pi)^n} \left[ \frac{1}{(k + q)^2 - m^2} - \frac{1}{(k + p)^2 - m^2} \right] + \frac{m}{M} \int \frac{d^n k}{(2\pi)^n} \left[ \frac{1}{(k + q)^2 - m^2} - \frac{1}{(k + p)^2 - m^2} \right] \right. \\
\left. + \frac{m}{M} \int \frac{d^3k}{(2\pi)^3} \frac{k}{(k^2 - M^2) [(k + q)^2 - m^2]} - \frac{p^2 - m^2}{2M} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 [(k + p)^2 - m^2]} \right\} \\
= 2(q - p)^\mu g^2C_2(R)M \left\{ \left( m + \frac{M}{2} \right) \int \frac{d^3k}{(2\pi)^3} \frac{2k_\mu}{(k^2 - M^2) [(k + q)^2 - m^2] [(k + p)^2 - m^2]} \right. \\
\left. - \left( \frac{m}{M} + \frac{1}{2} \right) \int \frac{d^3k}{(2\pi)^3} \frac{2k_\mu}{k^2 [(k + q)^2 - m^2] [(k + p)^2 - m^2]} \right. \\
\left. + \frac{m}{M} \int \frac{d^3k}{(2\pi)^3} \frac{2k_\mu}{k^2 [(k + q)^2 - m^2] [(k + p)^2 - m^2]} \right. \\
\left. - \left( \frac{m}{M} + \frac{1}{2} \right) \int \frac{d^3k}{(2\pi)^3} \frac{2k_\mu}{k^2 [(k + q)^2 - m^2]} \right. \\
\left. + \frac{m}{M} \int \frac{d^3k}{(2\pi)^3} \frac{2k_\mu}{k^2 [(k + q)^2 - m^2]} \right\}, \quad (35) \]

where we have thrown away the vanishing terms in the large-\(M\) limit.

As above, to compute the terms in Eq.\((35)\), we cannot take the large-\(M\) limit directly. So we still make use of above decomposition, and then we have

\[
-i \left[ \Sigma^{(1)}(q) - \Sigma^{(1)}(p) \right] = 2(q - p)^\mu g^2C_2(R) \left\{ - \int \frac{d^n k}{(2\pi)^n} \frac{k_\mu}{(k^2 + 2k\cdot p)(k^2 + 2k\cdot q)} \right. \\
\left. - \left( m + \frac{M}{2} \right) \int \frac{d^n k}{(2\pi)^n} \frac{kk_\mu}{(k^2 - M^2)(k^2 - m^2)^2} + m \int \frac{d^n k}{(2\pi)^n} \frac{kk_\mu}{k^2 [(k + q)^2 - m^2]} \right\}, \quad (36) \]
\[ \gamma^{(1)}_{\mu} = -\frac{g^2 C_V T^a}{2} \int \frac{d^3 k}{(2\pi)^3} \left[ \frac{2(q_{\mu} - m \gamma_{\mu})}{(k^2 + 2k \cdot p)(k - r)^2} + \frac{2(p_{\mu} - m \gamma_{\mu})}{(k^2 + 2k \cdot q)(k + r)^2} \right] + \left( m \gamma_{\mu} - p_{\mu} \right) \frac{1}{p} \ln \left( \frac{1 + p/m}{1 - p/m} \right) \bigg|_{p^2 = m^2} \]

\[ \gamma^{(2)}_{\mu} = \frac{g^2 C_V T^a}{2} \int \frac{d^3 k}{(2\pi)^3} \left[ \frac{2q \cdot k \gamma_{\mu}}{k^2(k^2 + 2p \cdot k)(k - r)^2} + \frac{2p \cdot k \gamma_{\mu}}{k^2(k^2 + 2q \cdot k)(k + r)^2} \right] \]
and the mass-shell condition above procedure, we have used the identities there is an IR pole term, which is induced purely by the mass-shell condition. During the

\[
\gamma^\mu - \frac{1}{2} \frac{g^2 C V T^\alpha}{2} \int \frac{d^n k}{(2\pi)^n} \left[ \frac{k^\mu}{k^2(k^2 + 2k \cdot p)(k - r)^2} + \frac{k^\mu}{k^2(k^2 + 2k \cdot q)(k + r)^2} \right] \left[ C V T^\alpha g^2 p_\mu + q_\mu \right],
\]

where \( \epsilon_{IR} = 3 - n \) and \( \mu \) is the artificial parameter with mass dimension. One notices that there is an IR pole term, which is induced purely by the mass-shell condition. During the above procedure, we have used the identities

\[
2p \cdot k = (k^2 + 2p \cdot k) - k^2,
\]

\[
2k \cdot r = (k + r)^2 - k^2 - r^2 = k^2 + r^2 - (k - r)^2,
\]

and the mass-shell condition

\[
2p \cdot q - 2m^2 = p^2 + q^2 - (q - p)^2 - 2m^2 = -r^2.
\]

As for \( \gamma^{(4)}_\mu \), it is a little more complicated, namely

\[
\gamma^{(4)}_\mu = \frac{1}{2} \frac{g^2 C V T^\alpha}{2} \int \frac{d^n k}{(2\pi)^n} \left[ \frac{4p \cdot k q_\mu - 4q \cdot k p_\mu + 2(p \cdot q - m^2)k^\mu}{k^2(k^2 + 2k \cdot p)(l - r)^2} \left( \epsilon_{IR} \right) + \frac{4mq_\mu k - 4mk \cdot q_\mu}{k^2(k^2 + 2k \cdot q)(l + r)^2} \right].
\]

However, from Eq.(13) one can see that we only need two Feynman integrals (on-shell),
\[
\int \frac{d^nk}{(2\pi)^n} \frac{k_\mu}{k^2 + 2k \cdot p} |_{p^2 = m^2} = A p_\mu + B r_\mu;
\]
\[
\int \frac{d^nk}{(2\pi)^n} \frac{k_\mu}{k^2 + 2k \cdot q} |_{q^2 = m^2} = A q_\mu - B r_\mu,
\]
where \(A\) and \(B\) are the form factors needed to be determined. It is easy to obtain that

\[
A = \frac{1}{2(p^2 - q^2)} \left\{ \int \frac{d^nk}{(2\pi)^n} \left[ \frac{1}{((k + p)^2 - m^2) (k + r)^2} - \frac{1}{((k + q)^2 - m^2) (k + r)^2} \right] |_{p^2 = q^2 = m^2} + \int \frac{d^nk}{(2\pi)^n} \left[ \frac{q^2 - m_0^2}{k^2 (k + q)^2 - m^2} (k + r)^2 - k^2 (k + p)^2 - m^2 (k + r)^2 \right] |_{p^2 = q^2 = m^2} \right\}
\]
\[
= \frac{i}{16\pi m(p^2 - q^2)} \left[ \left. \frac{1}{2} \ln \left( \frac{1 + q/m}{1 - q/m} \right) - \frac{1}{2} \ln \left( \frac{1 + p/m}{1 - p/m} \right) \right|_{p^2 = q^2 = m^2} - \frac{1}{2} \right.
\]
\[
= \frac{i}{16\pi} \frac{1}{2} \frac{1}{m^{3+\epsilon}} \int_0^1 dx \left[ \frac{1}{(x - q^2/m^2x(1 - x))^{(1+\epsilon)/2}} \right] |_{p^2 = q^2 = m^2} - \frac{1}{2}
\]
\[
= \frac{i}{16\pi} \frac{1}{2} \frac{1}{m^{3+\epsilon}} \int_0^1 dx x(1 - x)
\]
\[
\times \int_0^1 dy \left\{ \left[ \frac{1}{(x - q^2/m^2x(1 - x))^{(3+\epsilon)/2}} \right] |_{p^2 = q^2 = m^2} - \frac{1}{2} \right.
\]
\[
= \frac{i}{16\pi} \frac{1}{2} \frac{1}{m^{3+\epsilon}} \int_0^1 dx (1 - x) x^{-2-\epsilon} - \frac{1}{2} = \frac{i}{32\pi m^2} \left( \frac{1}{\epsilon_{IR}} + \ln \left( \frac{\mu}{m} \right) \right) - \frac{1}{2},
\]
where we have used the relation

\[
\frac{1}{a^{(1+\epsilon)/2}} - \frac{1}{b^{(1+\epsilon)/2}} = \frac{1 + \epsilon}{2} \int_0^1 dy \frac{b - a}{[a + (b - a)y]^{(3+\epsilon)/2}},
\]
and

\[
I = \int \frac{d^nk}{(2\pi)^n} \frac{1}{k^2 [(k + q)^2 - m^2]} |_{q^2 = m^2}
\]
\[
= \int \frac{d^nk}{(2\pi)^n} \frac{1}{k^2 [(k + p)^2 - m^2]} |_{p^2 = m^2}
\]
\[
= \frac{i}{4\pi^2} \Gamma \left( \frac{3 - \epsilon}{2} \right) \Gamma \left( \frac{3 + \epsilon}{2} \right) \frac{1}{m^{3+\epsilon}} \left( \frac{\mu}{m} \right)^\epsilon
\]
\[
\times \int_0^1 dx \int_0^1 dy \left[ (1 - x)^2 y^2 - k^2/m^2 x y(1 - y) \right] |_{p^2 = q^2 = m^2}^{(3+\epsilon)/2}.
\]
Thus we obtain

\[
\gamma^{(4)}_{\mu} = -\frac{1}{2} g^2 C_V T^a A \left[ (4m^2 - 2r^2)(p_\mu + q_\mu) + (2mr^2 - 8m^3) \gamma_\mu \right]
\]
\[
= -\frac{i}{32\pi} g^2 C_V T^a \left( \frac{1}{\epsilon_{IR}} + \ln \left( \frac{\mu}{m} \right) - 16m^3 I \right) \left[ \left( 2 - \frac{r^2}{m^2} \right) \frac{p_\mu + q_\mu}{m} + \left( \frac{r^2}{m^2} - 4 \right) \gamma_\mu \right] .
\]
We define the vertex at the renormalization point then the quark-gluon vertex renormalization constant is
\[ \Gamma_{\mu}^{(1)}(r) = \left( p_\mu + q_\mu \right) \frac{g^2 C_V T^a}{8\pi} \left\{ \frac{1}{2} \left( \frac{1}{\epsilon_{IR}} + \ln \frac{\mu}{m} \right) \left( 1 + \frac{r^2}{2m^2} \right) - \frac{1}{2} + 4m^3 I \left( 2 - \frac{r^2}{m^2} \right) \right\} \].

From Eqs. (A13), (34), (36) and (49), we finally obtain the on-shell one-loop fermion-gluon vertex,
\[ \Gamma_{\mu}^{(1)a}(r) = -\frac{g^2}{4\pi} T^a \gamma_\mu \left\{ C_V \left( \frac{5}{12} + \frac{1}{4} \left( \frac{1}{\epsilon_{IR}} + \ln \frac{\mu}{m} \right) \left( 1 + \frac{r^2}{2m^2} \right) - 2m^3 I \left( \frac{2}{m^2} - 4 \right) \right) + C_2(R) \left( \frac{2}{3} + \frac{m}{r} \ln \frac{1 + r/(2m)}{1 - r/(2m)} \right) \right\} \]
\[ +\frac{p_\mu + q_\mu}{m} \left\{ C_2(R) \left( \frac{m}{2r} \ln \frac{1 + r/(2m)}{1 - r/(2m)} \right) - \frac{2}{4 - r^2/m^2} - \frac{1}{2} \left( \frac{1}{\epsilon_{IR}} + \ln \frac{\mu}{m} \right) \right\} \]
\[ + C_V \left[ \frac{1}{8} \left( \frac{1}{\epsilon_{IR}} + \ln \frac{\mu}{m} \right) \left( 1 + \frac{r^2}{2m^2} \right) - \frac{1}{8} + 2m^3 I \left( 2 - \frac{r^2}{m^2} \right) \right] \}
\[ = -\frac{g^2}{4\pi} T^a \gamma_\mu \left\{ C_V \left[ \frac{3}{2} + \frac{3}{2} \frac{m}{r} \ln \frac{1 + r/(2m)}{1 - r/(2m)} \right] - \frac{2}{4 - r^2/m^2} \right\} \]
\[ + C_2(R) \left[ \frac{2}{3} + \frac{3}{2} \frac{m}{r} \ln \frac{1 + r/(2m)}{1 - r/(2m)} \right] \]
\[ +\frac{g^2}{4\pi} T^a \frac{i\epsilon_{\mu\nu\lambda\rho}}{m} \left\{ C_2(R) \left[ \frac{m}{2r} \ln \frac{1 + r/(2m)}{1 - r/(2m)} \right] - \frac{2}{4 - r^2/m^2} \right\} \]
\[ + C_V \left[ \frac{1}{8} \left( \frac{1}{\epsilon_{IR}} + \ln \frac{\mu}{m} \right) \left( 1 + \frac{r^2}{2m^2} \right) - \frac{1}{8} + 2m^3 I \left( 2 - \frac{r^2}{m^2} \right) \right] \}
\[ = \gamma_\mu T^a f_1(r) + iT^a \epsilon_{\mu\nu\lambda\rho} r^\nu \gamma^\lambda f_2(r), \] (50)

where we have used the three-dimensional analogue of the Gordon identity:
\[ \gamma_\mu = \frac{1}{2m} \left[ (p_\mu + q_\mu) + i\epsilon_{\mu\nu\lambda\rho} r^\nu \gamma^\lambda \right]. \] (51)

We define the vertex at the renormalization point \( r = 0 \) as that done in [18],
\[ \Gamma_{\mu}^{(R)}(0) = 0; \]
\[ \Gamma_{\mu}^{(R)}(r) = T^a \gamma_\mu (Z_3^{-1} - 1) + Z_3^{-1} \Gamma_{\mu}^{(R)}(r), \] (52)

then the quark-gluon vertex renormalization constant is
\[ Z_3^{-1} = 1 + f_1(0), \]
\[ Z_3 = 1 + \frac{g^2}{4\pi} \left[ \frac{2}{3} C_V + C_2(R) \frac{5}{3} \right]. \] (53)

One can notice that actually \( f_2(0) \) does not vanish. This will induce a non-minimal (colour) magnetic moment interaction between two three-dimensional quarks, which may play a certain role in the application of this model to condensed matter physics. The similar result had also been obtained in Abelian case [19].
B. One-loop Ghost-Gluon Vertex

For discussing the renormalization of coupling constant, we shall have a brief look at the one-loop ghost-gluon vertex (Fig.6), whose value had been predicted in Ref. \[16\] from the general result in Landau gauge \[2\] and was explicitly calculated in Ref. \[16\]. It is obvious that after taking the large-$M$ limit, the amplitude indeed vanishes. Therefore we can always define the gluon-ghost vertex renormalization constant as,

$$Z_2 = 1.$$  (54)

IV. FINITE RENORMALIZATION

Now let us consider the renormalization of the coupling constant. There are two ways to implement this: one way is using the Slavnov-Taylor identities and the known one-loop results to determine the local part of the one-loop quantum effective action \[4, 6\]; another way, which we shall adopt in the following, is to use the relations among various coupling constants imposed by the Slavnov-Taylor identities to determine the finite renormalization of the coupling constant \[3, 17\]. Since the on-shell renormalization is compatible with the Slavnov-Taylor identity and the renormalized coupling constant is unique, we can write the local effective action in terms of the renormalized fields in the following two forms,

$$S = \int d^3x \left[ \frac{1}{2} Z_A \epsilon^{\mu\nu\rho} A_\mu^a \partial_\nu A_\rho^a + Z_c \bar{c}^a \partial_\mu c^a + Z_\psi \bar{\psi} (i\partial - Z_m m_{ph}) \psi + \frac{1}{3!} Z_1 g f^{abc} A_\mu^a A_\nu^b A_\rho^c - Z_2 g f^{abc} A_\mu^a \partial_\mu c^b \bar{c}^c + Z_3 g \bar{\psi} A \psi \right]$$

$$= \int d^3x \left[ \frac{1}{2} Z_A \epsilon^{\mu\nu\rho} A_\mu^a \partial_\nu A_\rho^a + Z_c \bar{c}^a \partial_\mu c^a + Z_\psi \bar{\psi} (i\partial - Z_m m_{ph}) \psi + \frac{1}{3!} g_B Z_A^{3/2} \epsilon^{\mu\nu\rho} A_\mu^a A_\nu^b A_\rho^c - g_B Z_c Z_A^{1/2} f^{abc} \partial_\mu c^b \bar{c}^c + g_B Z_\psi Z_A^{1/2} \bar{\psi} A \psi \right],$$  (55)

where the correspondence between the renormalized quantities and the bare ones is defined as usual,

$$A_B^\mu = Z_A^{1/2} A_\mu^a, \quad c_B^a = Z_c^{1/2} c^a, \quad \bar{c}_B = Z_c^{1/2} \bar{c}^a, \quad A_B^a = Z_c^{1/2} A^c, \quad \psi_B = Z_\psi^{1/2} \psi,$$

$$\bar{\psi}_B = Z_\psi^{1/2} \bar{\psi}, \quad m = m_{ph} + \delta m = Z_m m_{ph}. \tag{56}$$

Eq. (55) gives

$$g = g_B Z_1^{-1} Z_A^{3/2} = g_B Z_2^{-1} Z_c Z_A^{1/2} = g_B Z_3^{-1} Z_\psi Z_A^{1/2};$$

$$\frac{Z_A}{Z_1} = \frac{Z_c}{Z_2} = \frac{Z_\psi}{Z_3}. \tag{57}$$

From Eqs. (51), (53), (54) and (57), we can see that the relation $Z_c/Z_2 = Z_\psi/Z_3$ is indeed satisfied. Using Eq. (54), we obtain

$$Z_1 = 1 - \frac{g^2}{4\pi} \left( 3C_V + \frac{1}{2} \right).$$  (58)
V. SUMMARY AND DISCUSSION

We have found the inconsistency of dimensional reduction and naive dimensional regularization when they are applied to Chern-Simons type theories. Further we use the consistent dimensional continuation to re-investigate the one-loop quantum correction of Chern-Simons term coupled with spinor fields. As it is pointed out in Ref. [4], the practice of consistent dimensional regularization requires the introduction of the higher covariant derivative term like Yang-Mills term, since this special prescription of dimensional continuation results in the linear dependence of the $n$-dimensional kinetic operator, even though the gauge fixing has been performed. Therefore the regularization we adopt in essence consists of higher covariant derivative regularization combining with consistent dimensional continuation.

With this regularization prescription, we have calculated all the one-loop two-point amplitudes and have given the analytical result of one-loop on-shell quark-gluon vertex with aid of the Slavnov-Taylor identity. In the mass-shell renormalization convention, we have found that not only the coupling constant receives an extra finite renormalization from the fermionic loop, but the fermionic matter also has a finite renormalization. This is different from the result given in Ref. [4], where it was shown that all the renormalization constants are defined as $Z_i = 1$.

Of course, purely from the viewpoint of renormalization, our results do not contradict the ones of Ref. [3] since a difference in a finite renormalization can always be explained as a different choice of renormalization convention. However, since Chern-Simons type theory is finite at one-loop level, the $\beta$-function and the anomalous dimensions of all the fields vanish identically, and we have no objects like renormalization group equation to show the renormalization convention independence. As pointed out in Ref. [21], the only criterion for the equivalence among different renormalization conditions is that all the regularization schemes preserving the fundamental symmetry such as gauge invariance should give the same gauge invariant radiative corrections. Therefore, we interpret these differences as the inconsistency of naive dimensional regularization.

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APPENDIX A: SLAVNOV-TAYLOR IDENTITY FROM BRST SYMMETRY

The BRST transformation reads as follows:

$$\delta A^a_\mu = -D^a_\mu \bar{c} \cdot b, \quad \delta \bar{c} = \frac{g}{2} f^{abc} c^b \bar{c}^c, \quad \delta c^a = -\frac{1}{\alpha} \partial^\mu A^a_\mu,$$

$$\delta \psi = -i g T^a c^a \psi, \quad \delta \bar{\psi} = i g \bar{\psi} T^a c^a,$$

(A1)

which is nilpotent,

$$\delta^2 = 0.$$

(A2)
The generating functional with all the external source terms is

\[ Z \equiv Z[J^\mu_\alpha, \eta, \bar{\eta}, \bar{K}, K, \bar{L}, L, u_\mu, v^a] = \int \mathcal{D}X \exp \left\{ i \left[ S + \int d^3 x \left( J^{\mu a} A^\mu_a + \bar{\psi} \eta + \bar{\eta} \psi + \bar{K} a + e^a K^a \right) \right. \right. \]

\[ + \int d^3 x \left( -Lg T^a c^a \psi + \bar{\psi} g T^a c^a L - u_\mu D^{\mu ab} c^b + v^a q f^{abc} c^b c^c \right] \right\}. \tag{A3} \]

The BRST invariance of the generating functional lead to the following general Ward identity:

\[ \delta Z = \int \mathcal{D}X \left\{ \int d^3 u \left[ J^\mu_a \delta A^\mu_a + \bar{\psi} \delta \eta - \bar{\eta} \delta \psi + \delta e^a K^a - \bar{K}^a \delta e^a \right] \right. \]

\[ \times \exp \left\{ iS + i \int \text{(the source term)} \right\} = 0, \]

\[ \int d^3 u \left[ J^\mu_a \delta u^a_\mu - i\bar{\eta} \delta L - i \delta \bar{\eta} \eta - \bar{K}^a \delta v^a - \frac{1}{\alpha} \left( \frac{\partial_{\mu} \delta}{\delta J^\mu_a} \right) K^a \right] Z = 0. \tag{A4} \]

It can be directly written out the Ward identities for the generating functional of the connected Green functions due to the linearity of the functional differential operator in Eq.(A4)

\[ \int d^3 u \left[ J^\mu_a \delta u^a_\mu - i\bar{\eta} \delta L - i \delta \bar{\eta} \eta - \bar{K}^a \delta v^a - \frac{1}{\alpha} \left( \frac{\partial_{\mu} \delta}{\delta J^\mu_a} \right) K^a \right] W = 0, \tag{A5} \]

where \( Z = \exp[iW] \). Acting \( \delta/\delta \eta(x), \delta/\delta \eta(y) \) and \( \delta/\delta K^a(z) \) on above identity and then the external sources to zero, we obtain the Ward identity containing the quark-gluon vertex,

\[ \left[ \frac{1}{\alpha} \frac{\delta}{\delta \eta(x)} \frac{\delta}{\delta \eta(y)} \frac{\partial_{\mu} \delta}{\delta J^\mu_a(z)} + i \frac{\delta}{\delta \eta(y)} \frac{\delta}{\delta \bar{K}^a(z)} \right] W | \text{all the external sources} = 0 = 0, \]

\[ \frac{1}{\alpha} \frac{\partial}{\partial z_\mu} (\bar{\psi}(x) \psi(y) A^a_\mu(z)) + igT^b(\bar{\psi}(y) c^a(z) c^b(x) \psi(x))_C \]

\[ -igT^b(\bar{\psi}(x) \bar{c}^a(z) c^b(y) \psi(y))_C = 0, \tag{A6} \]

where the subscript " C " means the connected part of the Green functions. Decomposing the above Green functions into 1PI part, we get

\[ \frac{1}{\alpha} \frac{\partial}{\partial z_\mu} \int d^3 u d^3 v d^3 w iD^{a\alpha}_\mu(z-w) iS(x-u) g \Gamma^a(u, v, w) iS(v-y) \]

\[ + ig \int d^3 u d^3 v \left[ \gamma^a(x, u, v) iS(u-y) - iS(x-u) \gamma^a(u, y, v) \right] iD^{a\alpha}(v-z) = 0, \tag{A7} \]

where \( \Gamma^a(u, v, w) \) is the 1PI part of the fermion-gluon vertex function, \( \gamma^a(x, u, v) \) and \( \gamma^a(u, y, v) \) are the composite ghost-gluon vertex functions. After Fourier transformation, we obtain
\[
\frac{1}{\alpha} r^\mu D^{\alpha \prime \nu}(r) S(p) \Gamma^{\alpha \prime}(p, q, r) S(q) + \gamma^{\alpha \prime}(p, q, r) D^{\alpha \prime \nu}(r) S(q) \\
-S(p) \gamma^{\alpha \prime}(p, q, r) D^{\alpha \prime \mu}(r) = 0,
\]

(A8)

where \( r_\mu = q_\mu - p_\mu \). Considering the fact that the longitudinal part of gauge field receives no quantum correction, i.e.

\[
r^\mu D^{\mu \nu}(r) = r^\mu D^{\mu \nu}(0) = -\alpha \frac{r_\nu}{r^2} \delta^{\mu \nu},
\]

(A9)

and using the general form of the full ghost propagator

\[
D^{\mu \nu}(r) = -\frac{i \delta^{\mu \nu}}{r^2 [1 + \Sigma_g(r^2)]},
\]

(A10)

we get the required Ward identity

\[
r^\mu \Gamma^a_\mu(p, q, r) \left[1 + \Sigma_g(r^2)\right] = \gamma^a(p, q, r) S^{-1}(q) - S^{-1}(p) \gamma^a(p, q, r).
\]

(A11)

Expanding the above identity up to one-loop order and using

\[
\Gamma^a_\mu(p, q, r) = \gamma^a_\mu T^a + g^2 \Gamma^{(1)a}_\mu + \mathcal{O}(g^4), \quad \Sigma_g(r^2) = g^2 \Sigma^{(1)}_g(r^2) + \mathcal{O}(g^4), \\
S^{-1}(p) = \hat{p} - m - g^2 \Sigma^{(1)}(p) + \mathcal{O}(g^4), \quad \gamma^a(p, q, r) = T^a + g^2 \gamma^{(1)a}(p, q, r) + \mathcal{O}(g^4),
\]

(A12)

we obtain the desired one-loop Slavnov-Taylor identity:

\[
(q^\mu - p^\mu) \Gamma^{(1)a}_\mu(p, q, r) = -(\hat{q} - \hat{p}) T^a \Sigma^{(1)}_g(r^2) - T^a \left[\Sigma^{(1)}(q) - \Sigma^{(1)}(p)\right] \\
+ g^2 \left[\gamma^{(1)a}(p, q, r)(\hat{q} - m) - (\hat{p} - m) \gamma^{(1)a}(p, q, r)\right].
\]

(A13)
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FIGURES

FIG. 1. Contribution to vacuum polarization from fermionic loop

FIG. 2. Self-energy for fermionic field

FIG. 3. Self-energy for ghost field

FIG. 4. One-loop quark-gluon vertex
\[ \gamma^a(p, q, r) \]

FIG. 5. One-loop ghost-fermion composite vertex

\[ r = q - p \]

\[ \gamma^a(p, q, r) \]

FIG. 6. One-loop ghost-gluon vertex