MAGNETOSPHERIC ECLIPSES IN THE DOUBLE PULSAR SYSTEM PSR J0737−3039

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ABSTRACT

In the binary radio pulsar system PSR J0737−3039, the faster pulsar, A, is eclipsed once per orbit. A clear modulation of these eclipses at the 2.77 s period of pulsar B has recently been discovered. We construct a simple geometric model that successfully reproduces the eclipse light curves, based on the idea that the radio pulses are attenuated by synchrotron absorption on the closed magnetic field lines of pulsar B. The model explains most of the properties of the eclipse: its asymmetric form, the nearly frequency-independent duration, and the modulation of the brightness of pulsar A at both once and twice the rotation frequency of pulsar B in different parts of the eclipse. This detailed agreement confirms the dipolar structure of the star’s poloidal magnetic field. The inferred parameters are the physical width of the region that causes this periodic modulation and the angle between the rotation axis of pulsar B and its magnetic moment, which retains enough plasma to effect an eclipse. An alternative possibility, which we disfavor, is that the eclipse is caused by charged particle bunches generated near the magnetic axis of B. In addition, there are narrow, transparent windows in which the radio pulses are generated near the magnetic axis of B. In these spikes in the radio flux are tied to the rotation of pulsar B and provide key constraints on the geometry of the absorbing plasma.

Subject heading: pulsars: individual (PSR J0737−3039)

1. INTRODUCTION

The double-pulsar system PSR J0737−3039A/B contains a recycled 22.7 ms pulsar (A) in a 2.4 hr orbit around a 2.77 s pulsar (B) (Burgay et al. 2003; Lyne et al. 2004). Pulsar A is eclipsed once per orbit, for a duration of ~30 s, centered around superior conjunction. The width of the eclipse is a weak function of the observing frequency (Kaspi et al. 2004). Recently, the pulsar A radio flux has been found to be modulated by the rotation of pulsar B during the eclipse (McLaughlin et al. 2004). The eclipse is longer when the magnetic axis of pulsar B is approximately aligned with the line of sight (assuming that radio pulses are generated near the magnetic axis of B). In addition, there are narrow, transparent windows in which the flux from pulsar A rises nearly to the unabsorbed level. These spikes in the radio flux are tied to the rotation of pulsar B and provide key constraints on the geometry of the absorbing plasma.

The physical width of the region that causes this periodic modulation is comparable to or smaller than the estimated diameter of the magnetosphere of pulsar B. Combined with the rotational modulation, this provides a strong hint that the absorption is occurring within the magnetosphere of pulsar B, which retains enough plasma to effect an eclipse. An alternative possibility, which we disfavor, is that the eclipse is caused by absorption in the relativistic wind of pulsar A after it is shocked and decelerated by its interaction with the magnetosphere of B (Lyutikov 2004; Arons et al. 2005).

Distinguishing between these two models is greatly aided by a quantitative model of the eclipse light curve. In this paper, we construct such a model and show that the light curve is consistent, in considerable detail, with synchrotron absorption in the pulsar B magnetosphere. The agreement is detailed enough to constitute direct evidence for the presence of a dipolar magnetic field around pulsar B. We assume that the absorbing plasma is confined within a set of poloidal field lines that is rotationally symmetric about the magnetic dipole axis. Considerations of the sourcing and heating of this plasma suggest that its density exceeds the Goldreich-Julian density by a few orders of magnitude and that the absorbing particles are relativistic. Similar conclusions have been reached independently by Rafikov & Goldreich (2005) under different assumptions about the sourcing and heating mechanisms. Note that if the plasma has a finite transverse temperature, then particles will be reflected from regions of high magnetic field and trapped on the closed field lines. Since the trapped particles cool slowly, a large equilibrium density can be established. From this perspective, it is clear that the eclipse model of Lyutikov (2004) and Arons et al. (2005) had to make three fine-tuning assumptions. First, the wind of pulsar A must be extremely dense: about ~10^6 times the Goldreich-Julian density at the pulsar speed of the light cylinder, in contradiction to those in models of pair creation, which suggest values closer to ~10^7 (e.g., Harding et al. 2002; Hibschman & Arons 2001). Second, the pulsar A wind must be slow, with a typical Lorentz factor of γ_A ~ 10, while at the distance of ~1000 light cylinder radii of pulsar A it is expected...
that $\gamma_A \sim 1000$ (Michel 1969). Third, the orbital inclination angle is $\leq 86^\circ$, and new data indicate that the inclination angle is very close to $90^\circ$ (Coles et al. 2005; Ransom et al. 2004). The model also does not offer a simple explanation for the observed transparent windows in the eclipse light curves: while the position of the magnetosheath is expected to vary by $\sim 25\%$ between different phases, it is difficult to see how the sheath can become fully transparent. In addition, the sheath is expected to be wider when the magnetic axis is perpendicular to the line of sight, while, in contrast to this, the eclipse is narrow at the corresponding rotational phases of pulsar B (McLaughlin et al. 2004).

The presence of such a dense plasma within the magnetosphere of pulsar B is contrary to what is usually claimed for isolated radio pulsars. Thus, a source of charged particles—of both signs—is required on the most extended closed magnetic field lines. We describe two related mechanisms by which the absorbing particles can be supplied by the star itself or by pair field lines. We describe two related mechanisms by which the absorption of high-energy emission from pulsar A can occur near the magnetic poles, and the Goldreich-Julian current. A second, more subtle, effect involves the pumping of magnetic helicity from the outer to the inner magnetosphere. We outline a simple energetic reason why this occurs when the outgoing and return currents are not precisely balanced in each magnetic hemisphere. The dipolar magnetic field then supports a static twist, and a persistent current flows along magnetic field lines that close within the magnetosphere.

The plasma on closed field lines is quickly heated to relativistic energies. The outer magnetosphere of pulsar B is heated both by the damping of relativistic turbulence and by the resonant absorption of the radio waves from pulsar A. (The resonant absorption of high-energy emission from pulsar A can occur only very close to the neutron star, where the geometric cross section is small.) Since the energy flux in the wind of pulsar A exceeds the energy flux in its radio emission by several orders of magnitude, we argue that turbulent heating is likely to dominate, especially in regions of high synchrotron optical depth.

Turbulence in the surrounding magnetosheath generates Alfven waves on closed field lines. The nonlinear interaction between the waves creates higher wavenumber modes and, eventually, the dissipation of turbulent energy on small scales. Inside the magnetopause, where the pressure of the background magnetic field exceeds the particle pressure, the inner scale of the turbulent spectrum occurs when the fluctuations become too large to be supported by the available free charges: the plasma becomes charge starved at small scales (Thompson & Blaes 1998). As a result, charges are accelerated electrostatically along the background magnetic field. Bundles of such relativistic charges generate high-brightness radio emission (wiggler radiation) when they scatter off the highest frequency Alfvén modes. We show that this radiation can self-consistently excite the transverse motion of the particles. The conclusion is that self-absorbed radio photons that are generated within the optically thick plasma are the dominant heating mechanism.

The plan of the paper is as follows. In § 2 we review the basic parameters of the PSR J0737–3039 system. In § 3, we describe how electric currents may be excited in the magnetosphere of a neutron star and how the particles that support the current are supplied. In § 4 we describe particle heating at large synchrotron optical depth and calculate the distribution of synchrotron-absorbing particles that is used to model the eclipse. Section 5 presents our calculations of the eclipse light curves. In § 6 we discuss the modification of the model due to the presence of ions in the magnetosphere. The polarization of the radio waves transmitted through the absorbing magnetosphere is calculated in § 7. In § 8 we present predictions and compare with alternative eclipse models. In the final section, § 9, we summarize the broader implications of our work.

2. BASIC PARAMETERS

The semimajor axis of the PSR J0737–3039 system is $a \sim 8.8 \times 10^{10}$ cm, the orbital period is 2.45 hr, and the orbital eccentricity is small, $e \sim 0.088$. At the time of eclipse, the separation of the two pulsars is $D_{AB} \approx 8 \times 10^{10}$ cm, their relative orbital velocity is $V_{\perp} = 680 \text{ km s}^{-1}$ transverse to the line of sight, and 1 orbital degree near eclipse corresponds to a time interval of 20.5 s. The observed eclipses cover $\Delta \phi \sim 0.035$ radians (2°) of orbital phase. The corresponding half-width of the eclipsing material, transverse to the line of sight, is $\frac{\pi}{4} \Delta \phi D_{AB} = 1.4 \times 10^6$ cm. This is much smaller than the pulsar B light cylinder, which has the radius $c/\Omega_B = 1.3 \times 10^{10}$ cm. The orbital inclination is close to $90^\circ$ (Coles et al. 2005; Ransom et al. 2004).

The spin periods and period derivatives, $P_{A,B}$ and $\dot{P}_{A,B}$, have been measured for the two pulsars. Normalizing the moments of inertia $I_{A,B}$ to $10^{45}$ g cm$^2$, the spin-down luminosities are $I_A = I_A \Omega_A \dot{\Omega}_A = 5.8 \times 10^{33} I_{A,45} \text{ ergs s}^{-1}$ and $I_B = I_B \Omega_B \dot{\Omega}_B = 1.6 \times 10^{33} I_{B,45} \text{ ergs s}^{-1}$ (Burgay et al. 2003).

The magnetosphere of pulsar B is truncated, compared with the magnetosphere of an isolated pulsar of the same spin, by the relativistic wind flowing out from pulsar A. The spin-down torque of pulsar B is therefore modified. The surface magnetic field can be estimated self-consistently only by modeling the interaction between wind and magnetosphere. Different effects can contribute to the actual torque (Lyutikov 2004; Arons et al. 2005). For example, the open-field current is increased compared with that of an isolated dipole, which implies

$$L_B = N_B B_{\text{NS}}^2 R_{\text{NS}}^2 c \left( \frac{\Omega_B R_{\text{NS}}}{c} \right)^2 \left( \frac{R_{\text{NS}}}{R_{\text{mag}}} \right)^2,$$

where $B_{\text{NS}}$ is the average surface dipole field and $N_B$ is a parameter that rescales the torque acting on pulsar B.

The magnetospheric radius $R_{\text{mag}}$ is determined by the pressure balance between the wind of pulsar A and the magnetic pressure of pulsar B, evaluated at

$$\frac{B_{\text{mag}}^2}{8\pi} = \frac{I_A \Omega_A \dot{\Omega}_A}{4\pi c D_{AB}^2}.$$

Thus, the magnetic field at $R_{\text{mag}}$ depends only on the parameters of pulsar A’s wind. Using our fiducial numbers, we find

$$B_{\text{mag}} = \sqrt{\frac{2I_A \Omega_A \ddot{\Omega}_A}{D_{AB}^2}} = 7 I_{A,45}^{1/2} \text{ G}.$$
On the other hand, the magnetospheric radius $R_{\text{mag}}$ and surface magnetic field $B_{\text{NS}}$ depend on details of the wind-magnetosphere interaction. By using eqs. (1)–(2), we find

$$\begin{align*}
B_{\text{NS}} &\approx \frac{D^{1/2}(I_B \dot{\Omega}_B / \Omega_B)^{3/4} c}{N_B^{3/4} (I_B \Omega_A \Omega_3)^{1/4} R_{\text{NS}}} = 4 \times 10^{11} N_B^{-3/4} I_B^{3/4} \dot{\Omega}_B^{1/4} \Omega_A^{1/4}, \\
I_{\text{mag}} &\approx \left( \frac{(cD_{\text{AB}} I_B \dot{\Omega}_B)}{N_B I_A \Omega_3 \Omega_4} \right)^{1/4} = 4.0 \times 10^{9} N_B^{-1/4} I_B^{1/4} \Omega_A^{1/4} \text{ cm.}
\end{align*}$$

Below, we use these parameters as our fiducial numbers.

3. INJECTION OF PLASMA ON CLOSED FIELD LINES

To calculate the eclipse light curve, one needs the basic plasma properties in the outer magnetosphere of pulsar B, most importantly, the equilibrium density, particle energy distribution, and composition. In this section and § 4, we motivate our basic assumption that this confined plasma has a large synchrotron optical depth and is relativistically hot. The implications of a significant ion component of the plasma are addressed in § 6.

One advantage of positioning the eclipsing plasma on the closed magnetic field lines of pulsar B is that a large particle density can build up slowly, over many rotation periods. Heating of the particles in the outer magnetosphere, where cooling is long, allows them to be trapped through the effect of magnetic bottling. Qualitatively, if particles are leaking out of the outer parts of magnetosphere on a timescale $\lambda$ times the pulsar period, then a source of particles that generates the Goldreich-Julian density each period would result in an equilibrium multiplicity $\lambda$.

The typical lifetimes of particles in the outer magnetosphere are indeed long, $\sim 10^5$ seconds; see eq. (9). Since the required density at the edge of the magnetosphere is $\sim 10^5$ times the Goldreich-Julian density (see § 5.3), plasma should be sourced at a rate that is comparable to the Goldreich-Julian density per period or higher. This sourcing can be driven either by torsional Alfvén waves (AC sourcing) or by pumping magnetic helicity from the outer to the inner closed field lines (DC sourcing).

3.1. Electrodynamics of a Perturbed Magnetosphere

The magnetopause of pulsar B is expected to be strongly turbulent, as particle-in-cell simulations demonstrate (J. Arons et al. 2005, in preparation). Turbulence in the sheath launches (torsional) Alfvén waves on the closed magnetic field lines, which carry charge and current. The sign of the current alternates on a timescale comparable to the spin period $P$, which is a few times longer than $R_{\text{mag}}/c$. It also alternates on a timescale comparable to the flow time behind the wind shock (which is comparable to or somewhat larger than $P$). The depth to which turbulence may be excited in the magnetosphere of pulsar B depends on more subtle details, such as the resonant interaction between compressive and torsional modes. We do not examine them in this paper.

Torsional Alfvén waves, launched by turbulence in the magnetosheath, impose a twist on closed field lines. If the typical fluctuating component of the magnetic field in the wave at radius $R_{\text{mag}}$ is $B_\phi = \Delta \phi B_\psi$, then there is an associated current $J \sim cB_\phi / 4 \pi R_{\text{mag}}^2 = \Delta \phi c B_\psi / 4 \pi R_{\text{mag}}$, where $B_\psi$ is the toroidal magnetic field and $\Delta \phi$ is the angle through which the poloidal magnetic field $B_\psi$ is twisted. As the twist in the magnetic field (the Alfvén wave) propagates along the closed field line toward the star, the associated current density increases. The ratio of the current density to the local corotation current density remains approximately constant,

$$\frac{J}{\rho_{\text{GJ}}} \approx \frac{\Delta \phi}{\cos \theta_3} \left( \frac{\Omega_3 B_{\text{mag}}}{c} \right)^{-1} = 1.1 N_B^{1/4} \left( \frac{\Delta \phi}{0.1} \right) \left( \frac{\cos \theta_3}{0.3} \right)^{-1},$$

where

$$\rho_{\text{GJ}} = - \frac{\Omega_3 B}{2 \pi c} = - \cos \theta_3 \frac{\Omega_3 B}{2 \pi c}$$

is the corotation charge density (Goldreich & Julian 1969) and $\theta_3$ is angle between the local magnetic field and the spin vector of pulsar B, $\Omega_3 B$. The net current is

$$I \approx \frac{\Delta \phi}{2} c R_{\text{NS}}^2 / R_{\text{mag}}^3.$$

In order to supply this current, a minimum particle density is required. If at some point the actual particle density is below the minimum value $\sqrt{\nabla \times B / 4 \pi e}$, then charges will be supplied from the star or created locally through the reaction $^{\gamma} - e^+ + e^-$. A minimum charge density is always present on closed field lines, the corotation charge density (eq. [6]). When pulsar B is near an orthogonal rotator (as is implied by our eclipse model; see § 5), $\theta_3$ is close to $90^\circ$, and the corotation charge density can fail to supply the fluctuating current even for a small twist angle $\Delta \phi \sim 0.1$. In this case, the current cannot be supplied by an upward or downward drift of the corotation charge; instead, charges of both signs will be supplied to the outer magnetosphere. The sign of the instantaneous charge flow will fluctuate in tandem with the sign of the current.

Electrons and positrons cool slowly by cyclo/synchrotron emission in the outer parts of the magnetosphere. Beyond the cooling radius,

$$R_{\text{cool}} = 1.1 \times 10^8 \left( \frac{(B_{\text{NS}} R_{\text{NS}}^3 B_3)}{5 \times 10^{29} \text{ G cm}^3} \right)^{2/5} \text{ cm,}$$

the cyclotron timescale $t_{\text{cy}} \approx 3m_e^2 c^4/2e^4 B_3^2$ becomes shorter than $\sim c/R$. Only those particles reaching the cooling radius can precipitate to the surface. If the particles in the outer magnetosphere are isotropized, then the cooling fraction is small (§ 4.3),

$$f_{\text{cool}} \approx \frac{R_{\text{cool}}}{R_{\text{mag}}}^{-3} \approx 2.6 \times 10^{-6} \left( \frac{(B_{\text{NS}} R_{\text{NS}}^3 B_3)}{5 \times 10^{29} \text{ G cm}^3} \right)^{6/5} \left( \frac{R_{\text{mag}}}{4 \times 10^9 \text{ cm}} \right)^{-3}.$$
obtain an estimate of the equilibrium multiplicity of electrons (or positrons)

\[ \lambda_{eq}(r) = \lambda_j \frac{B(R_{cool})}{B(r)} \]

\[ = 9.6 \times 10^4 \lambda_j \left( \frac{B_{\text{NS}}R_{\text{NS}}^3}{5 \times 10^{29} \text{ G cm}^3} \right)^{-6/5} \left( \frac{r}{4 \times 10^9 \text{ cm}} \right)^3, \]

in the slow-cooling region, where \( r \) is the distance from pulsar B at the magnetic equator. Note that this expression is independent of the electron temperature in the outer magnetosphere. It also implies a characteristic number density,

\[ n_e = \lambda_{eq} \cos \theta_B \frac{\Omega_B B}{2 \pi e c} \]

\[ = 1.9 \times 10^4 \lambda_j \cos \theta_B \left( \frac{B_{\text{NS}}R_{\text{NS}}^3}{5 \times 10^{29} \text{ G cm}^3} \right)^{-1/5} \text{ cm}^{-3}, \]

that is approximately independent of \( r > R_{cool} \).

An independent upper bound on the multiplicity comes from the requirement that the particle pressure not exceed the magnetic dipole pressure:

\[ \lambda_{\max} = \left( \frac{eB(r)c}{4 \cos \theta_B} \right) \frac{k_B T_e}{m_e c^2} \]

\[ = \frac{1.5 \times 10^7}{\cos \theta_B} \left( \frac{B_{\text{NS}}R_{\text{NS}}^3}{5 \times 10^{29} \text{ G cm}^3} \right) \left( \frac{k_B T_e}{m_e c^2} \right)^{-1} \left( \frac{r}{4 \times 10^9 \text{ cm}} \right)^3. \]

Electrons or positrons with a density of \( \lambda_{eq} \) have, equivalently, a maximum temperature

\[ k_B T_{e,\max} = \frac{300}{\lambda_j \cos \theta_B} \left( \frac{B_{\text{NS}}R_{\text{NS}}^3}{5 \times 10^{29} \text{ G cm}^3} \right)^{11/5} \left( \frac{r}{4 \times 10^9 \text{ cm}} \right)^6. \]

It will be noted that the equilibrium multiplicity \( \lambda_{eq} \) (and the equilibrium particle pressure) is an increasingly small fraction of the maximum possible value as one moves inward through the magnetosphere of pulsar B.

The supply of charges to the magnetosphere can be greatly augmented by multiple stages of pair creation (e.g., Hilschman & Arons 2001). This requires the formation of large gaps (regions of uncompensated parallel electric field) in the inner magnetosphere. From the perspective of the near-field electrodynamics of the neutron star, the twist in the magnetic field is modulated only very slowly (over \( \sim 10^7 \) times the light crossing time of the star). This part of the circuit therefore bears some resemblance to the polar cap of an ordinary isolated radio pulsar.

Electric gaps can appear higher in the magnetosphere if the particles at a given point have a fixed sign and a charge density equal to \( \rho_G \). These gaps form along the surfaces where \( \Omega \cdot \mathbf{B} = 0 \) (e.g., Holloway 1977). The magnitude of the electrostatic potential drop that builds up across this gap during one torsional oscillation would approach \( \Delta \phi \sim eBR\Delta \phi \). However, the presence of a dense, polarizable plasma would largely prevent these gaps from forming on the closed magnetic field lines, outside the cyclotron cooling radius (eq. [8]). The gaps would also be suppressed if the magnetosphere supported a persistent twist with a magnitude greater than \( \Delta \phi \sim \Omega r / c \), a possibility that we entertain in the Appendix.

3.2. Comparison: Absorption of Particles from the Wind of Pulsar A

It is also worth considering the wind of pulsar A as a competing source of absorbing particles in the magnetosphere of pulsar B. The transfer of particles from wind to magnetosphere occurs rapidly through reconnection between the oscillating magnetic field that is advected by the wind and the forward surface of the magnetosphere (e.g., Thompson et al. 1994). The particle density in the pulsar A wind at the position of pulsar B is

\[ n_B(R_{mag}) = \frac{\lambda_A}{2 \pi D_{AB}^2} \frac{\Omega_A}{\Omega_B} \frac{I_A}{I_B} \frac{\Omega_A}{\Omega_B} = 4 \times 10^{-2} \lambda_A \text{ cm}^{-3}, \]

where \( \lambda_A \) is the multiplicity of pairs created by a cascade on the open field lines of pulsar A. Comparing with the characteristic charge density \( n_B = \lambda_{eq} \Omega_B B = \lambda_{eq} \Omega_B B(R_{mag}) / 2 \pi e c \) in the magnetosphere of pulsar B, at a distance \( R_{mag} \) from the star one has

\[ \frac{n_B(R_{mag})}{n_A} \sim \frac{\lambda_A}{\lambda_B} \left( \frac{R_{mag}}{4 \times 10^9 \text{ cm}} \right)^3 \left( \frac{B_{\text{NS}}R_{\text{NS}}^3}{5 \times 10^{29} \text{ G cm}^3} \right). \]

The density of particles trapped from the wind of pulsar A does not, generally, exceed the density in the wind itself: otherwise, the return of particles from the magnetosphere back to the wind would balance the gain. Models of pair cascades (e.g., Hilschman & Arons 2001) suggest that the multiplicity \( \lambda_A \) is much smaller than \( \sim 10^5 \) in weak-field radio pulsars with modest spin-down voltages. So there is a theoretical motivation for expecting that \( \lambda_{eq} > 10^5 > \lambda_A \) and that the outer magnetosphere of pulsar B is populated primarily by particles that have been pulled from its inner magnetosphere. Indeed, the eclipse model detailed in § 5 indicates that the absorbing particles are concentrated well inside the magnetopause of pulsar B.

4. PARTICLE HEATING IN THE OUTER MAGNETOSPHERE

There are two principal sources of free energy in the outer magnetosphere of pulsar B: the relativistic wind emitted by pulsar A and the radio emission of pulsar A. Both can be effective at heating trapped particles to relativistic energies. The wind energy flux is, nonetheless, several orders of magnitude larger (in spite of the uncertain effects of wind collimation and beaming). In addition, the equilibrium plasma density estimated in § 3 is too large to allow effective heating through radio absorption throughout the bulk of the pulsar B magnetosphere.

The kinetic power incident on the magnetosphere (radius \( R_{mag} \), eq. [4]) at a separation of \( D_{AB} = 8 \times 10^{10} \) cm is

\[ P_{\text{wind}} = \left( \frac{R_{mag}}{2 D_{AB}} \right)^2 L_A = 4 \times 10^{38} N_B^{-1/2} (I_{A,43} I_{B,45})^{1/2} \text{ ergs s}^{-1}. \]

The radio-energy flux incident on pulsar B is \( \nu F_\nu = (D_{9073}) / (D_{AB})^2 (\nu F_\nu)_{\text{obs}} \sim 1.2 \times 10^{4} (D_{9073}/0.6 \text{ kpc})^2 \text{ ergs cm}^{-2} \text{ s}^{-1} \) at 1.4 GHz, where \( (F_\nu)_{\text{obs}} \sim 1.6 \text{ mJy} \) is the observed flux at this frequency and \( D_{9073} \sim 0.6 \text{ kpc} \) is the estimated distance of
the PSR J0737–3039 system (Lyne et al. 2004). The incident radio power is smaller by

\[
P_{1.4\text{ GHz}} \approx 1.5 \times 10^{-7} \left( \frac{D_{\text{J0737}}}{0.6 \text{ kpc}} \right)^2.
\]

(17)

4.1. Heating by External Radio Photons

In spite of the small total radio power output, the absorption of the radio waves can have an important influence on the motion of trapped electrons. (See Rafikov & Goldreich [2005] for an independent and more detailed analysis of this heating mechanism.) For our present purposes, we assume the existence of a flux of electrons (and possibly positrons) moving trans-relativistically along the closed poloidal magnetic field line, away from the star. We now show that these particles are heated sufficiently to mirror and become trapped in the outer magnetosphere. The trapped particles are further heated by the radio beam.

A nonrelativistic electron absorbs (unpolarized) radio photons with a cross section \( \sigma_{\text{cycle}}(\nu) = (\nu e^2 / 2 m_e) (1 + \cos^2 \kappa) b(\nu - \nu_{\text{B,c}}) \), where \( \nu_{\text{B,c}} = eB / 2m_{e}c \) and \( \kappa \) is the angle between the direction of magnetic field and the photon wavevector. The energy absorbed by one particle from the radio beam at a radius \( r \) (outside the cooling radius; eq. [8]) is

\[
\frac{r}{v} \frac{dE}{dt} \approx \frac{r}{v} \int \sigma_{\text{cycle}}(\nu) F_{\nu} \, d\nu,
\]

(18)

when the particle motion is transrelativistic. This works out to

\[
\frac{r}{v} \frac{dE}{dt} = \frac{\pi}{4} \left( 1 + \cos^2 \kappa \right) \frac{e\nu}{B(r)D_{\text{AB}}} \nu_{\text{B,c}}^3 \approx 400 \left( \frac{1 + \cos^2 \kappa}{v/c} \right) \left( \frac{\nu_{\text{B,c}}}{100 \text{ MHz}} \right)^{4/3} \left( \frac{\nu F_{\nu}}{10^3 \text{ erg s}^{-1} \text{ cm}^{-2}} \right) \text{keV},
\]

(19)

when \( 1 - (v^2/c^2)^{1/2} = O(1) \). This energy is absorbed far outside the cooling radius, and so the particle mirrors soon after it begins to return to the star.

Particles trapped in the magnetosphere continue to be heated. The rate of heating at any one position in the magnetosphere depends on the column of intervening particles and the pump spectrum. Using the synchrotron absorption coefficient for a thermal distribution of particles at low frequencies (eq. [39]), we find that the time to heat the particles up to a temperature \( T \gg m_e c^2 / k_B \) is short:

\[
\tau_{\text{heat}} = \frac{3 k_B T_e n_e}{\alpha_{\nu}(\nu F_{\nu})},
\]

\[
= 74 \nu_{\phi} \left( \frac{k_B T_e}{m_e c^2} \right)^{8/3} \left( \frac{\nu F_{\nu}}{10^3 \text{ ergs s}^{-1} \text{ cm}^{-2}} \right)^{-1} \left( \frac{\nu_p}{\nu_{\text{B,c}}} \right)^{2/3} \text{s},
\]

(20)

where \( \nu_{\phi} = \nu_p / 10^9 \text{ Hz} \). Balancing optically thin synchrotron cooling at the rate \( 16(k_B T / m_e c^2)^2 (n_e \sigma_T c) (B^2 / 8\pi) \) per unit volume, with synchrotron heating at the rate \( \alpha_{\nu}(\nu F_{\nu}) \nu_p \) per unit volume, one obtains the equilibrium Lorentz factor

\[
k_B T_{\text{eq}} \approx 9.6 \left( \frac{B_{\text{NS}}}{5 \times 10^{11} \text{G}} \right)^{-9/11} \left( \frac{\nu F_{\nu}}{10^3 \text{ ergs s}^{-1} \text{ cm}^{-2}} \right)^{3/11} \left( \frac{\nu_p}{\nu_{\text{B,c}}} \right)^{-2/11}.
\]

(21)

This is fairly close to the limiting temperature (eq. [13]) obtained by balancing the particle pressure in the outer magnetosphere with the magnetic pressure.

4.2. Electrostatic Heating and Thermalization of an Optically Thick Plasma

The wind energy of pulsar A that is incident on the magnetosphere of pulsar B is converted, with some efficiency, to internally generated radio waves. Even if the radio output of the magnetosphere \( L_{\text{mag}} \) is much weaker than that of pulsar A, its heating effect can dominate by the factor \( 4(D_{\text{AB}}/R_{\text{mag}})^2 L_{\text{mag}} / L_A \sim 2000(R_{\text{mag}}/4 \times 10^9 \text{ cm})^{-2} L_{\text{mag}} / L_A \). We consider the possibility that the magnetospheric plasma is, itself, a source of low-frequency photons (\( \nu < 100 \text{ MHz} \)), which are created and absorbed in situ. The unpulsed emission from the PSR J0737–3039A/B system is several times brighter at 1400 MHz than is the combined pulsed emission of the two neutron stars (Burgay et al. 2003). It is possible that some of this unpulsed emission is generated in the magnetosphere of pulsar B.

Internal heating has another advantage over the absorption of radio photons from an external pulsar: it is more effective when the seed particles move relativistically along the magnetic field. The freshly injected particles will only resonantly absorb photons of a very low frequency, \( \nu \sim \gamma_{\text{eq}} \nu_{\text{B,c}} \). At this frequency, the external radiation sees a very large optical depth (from the previously injected particles, which have already acquired large perpendicular energies).

When the density \( n_e \) of electrons (and positrons) greatly exceeds the corotation charge density, torsional waves generated with a wavelength of \( k \parallel r \sim 1 \) couple nonlinearly to higher frequency waves. If the energy density in the light charges is still small compared with that of the background magnetic field, the inner scale of the resulting turbulent spectrum is determined by balancing the fluctuating current density with the maximum conduction current that can be supported by the plasma (Thompson & Blaes 1998). At high frequencies, the cascade is expected on general grounds to be strongly anisotropic. The wavenumber \( k \perp \) of a wave packet perpendicular to the background magnetic field is much greater than the parallel component \( k \parallel \) and related to it by \( k \perp B \sim k \parallel B \) (Goldreich & Sridhar 1997). Balancing \( k \perp B \sim 4\pi n_e e \) and relating \( n_e \) to the corotation charge density through \( n_e = \lambda_{\text{eq}}(\Omega B / 2\pi e) \) gives

\[
k_{\parallel} r = 2\lambda_{\text{eq}} \left( \frac{\Omega_e}{c} \right),
\]

(22)

Relativistic electrons moving antiparallel to such an Alfvén wave packet with a Lorentz factor of \( \gamma_e \) emit photons of a low frequency (wiggler radiation),

\[
\omega_{\text{wig}} \sim \frac{\gamma^2 k_{\parallel}}{c}.
\]

(23)
Large parallel Lorentz factors are most easily achieved by particles that are freshly injected with small perpendicular energies into the outer magnetosphere and then electrostatically accelerated in regions of high turbulent intensity. These same charges absorb the wiggler photons at their rest-frame cyclotron resonance (lab frame frequency $eB/2\pi m_e c^2 \gamma$) if

$$\gamma_\parallel \sim \frac{1}{2 \lambda_{eq}} \frac{eB}{m_e c \Omega_B} = \left( \frac{3 \times 10^7}{\lambda_{eq}} \right) B_{mag}.$$  \hspace{1cm} (24)

This condition is easily satisfied for the multiplicities ($\lambda_{eq} \sim 10^5$) that we encountered in §3. It is straightforward to check that the implied wiggler frequency (eq. [23]) lies comfortably above the plasma frequency at density $\rho_{eq}$. It works out to

$$\omega_{wig} = 3.0 \left( \frac{\lambda_{eq}}{10^5} \right)^{1/3} \left( \frac{B}{B_{mag}} \right)^{2/3} \text{MHz}. \hspace{1cm} (25)$$

The radio power needed to heat the freshly injected electrons is a small fraction of the net power dissipated in the magnetosphere. For example, if the particles carry a current $J$ that supports a twist $A \phi \sim 1$ in the background magnetic field, then their kinetic power (before electrostatic acceleration) is

$$L_{kin} \sim \frac{4 \times 10^{23}}{e} \left( \frac{R_{mag}}{4 \times 10^9 \text{ cm}} \right) \left( \frac{r}{R_{mag}} \right)^{-3} \text{ergs s}^{-1}. \hspace{1cm} (26)$$

The minimum radio power needed to heat these particles is no larger than $L_{kin}$. (It should, nonetheless, be emphasized that some bunching of the radiating particles within the turbulent plasma is required to exceed this requirement.)

Relativistic motion of the injected particles reduces the radio power even further. The particles start in their lowest Landau states close to the star. As they move out beyond the cooling radius (eq. [8]), they begin to decelerate as they absorb photons. The Lorentz factor parallel to the magnetic field is halved when the energy of the absorbed photons in the particle rest frame is

$$\Delta E \sim \gamma_\parallel^2 m_e c^2,$$  \hspace{1cm} where $\gamma_\parallel = [(p_\perp/m_e c)^2 + 1]^{1/2}$. The energy of the absorbed photons in the star’s frame is smaller by a factor of $\sim 1/\gamma_\parallel$. The net density of the absorbed photons is therefore smaller than the beam energy density,

$$U_\gamma \sim \frac{\gamma_\parallel}{\gamma_\parallel} n_{beam} m_e c^2 = \frac{1}{\gamma_\parallel} U_{beam} \propto \gamma^{-1}. \hspace{1cm} (27)$$

The equilibrium electron temperature can be much higher in this situation than the value we found by assuming the external radio photons from pulsar A to be the sole radiative pump. Balancing the rate of turbulent heating with the incoherent synchrotron output of the thermalized particles and assuming that a fraction $\varepsilon_{\text{turb}}$ of the wind energy density $B_{mag}^2/8 \pi$ is damped on the timescale $r/c$, one finds

$$\lambda_{eq} \left( \frac{k_B T_e}{m_e c^2} \right) = 2 \times 10^{15} \varepsilon_{\text{turb}} \left( \frac{r}{R_{mag}} \right)^8 N_B^{1/4}. \hspace{1cm} (28)$$

So, for example, if the energy density in torsional Alfvén waves is a fraction $\varepsilon_A$ of the wind energy density at the magnetopause of pulsar B, then the three-wave-damping timescale is $\sim (\delta B/B)^2 (r/c) \sim \varepsilon_A (B/B_{mag})^2 (r/c).$ One finds

$$\varepsilon_{\text{turb}} = \varepsilon_A \left( \frac{B}{B_{mag}} \right)^{-2} \left( \frac{r}{R_{mag}} \right)^6. \hspace{1cm} (29)$$

The implied equilibrium temperature is

$$k_B T_e = 230 \varepsilon_A \left( \frac{\lambda_{eq}}{10^5} \right)^{-1/2} N_B^{15/8} \left( \frac{r}{1.5 \times 10^9 \text{ cm}} \right)^7. \hspace{1cm} (30)$$

For $\varepsilon_A \sim 0.1$ this estimate is consistent with equation (13).

4.3. Density Distribution of the Heated Particles

Suppose that at the edge of the magnetosphere, located at radius $R_{mag}$, relativistic particles with a typical Lorentz factor $\gamma_0$ are isotropized. Particles captured from the wind of pulsar A and particles injected into an optically thick plasma from below (§ 4.2) are both expected to satisfy this criterion approximately. If we neglect for the moment the effects of cyclo- or synchrotron emission, the pitch angle $\psi$ can be related to the value $\psi_{mag}$ at the radius $r \sim R_{mag}$ from the conservation of the first adiabatic invariant $p_\perp^2/B$ and the constancy of the Lorentz factor $\gamma_0$ where $p_\perp = \sin \psi (\gamma_0 - 1)^{1/2} m_e c$ is the is particle momentum perpendicular to the magnetic field. One finds

$$\sin \psi = \left( \frac{B}{B_{mag}} \right)^{1/2} \sin \psi_{mag}, \hspace{1cm} (31)$$

Reflection occurs when $\psi = \pi/2$, where the field strength is

$$\frac{B_{refl}}{B_{mag}} = \frac{1}{\sin^2 \psi_{mag}}. \hspace{1cm} (32)$$

Near the axis of the dipolar field, the reflection radius is $R_{refl}/R_{mag} \simeq \sin^{-2/3} \psi_{mag}$. The particle spends only a short time $\sim R_{refl}/c$ near the reflection point, as may be seen from the solution for the speed of the particle parallel to the magnetic field,

$$v_\parallel = v_0 \cos \psi = v_0 \left( 1 - \frac{B}{B_{mag}} \sin^2 \psi_{mag} \right)^{1/2}. \hspace{1cm} (33)$$

Here $v_0/c = (1 - \gamma_0^{-2})^{1/2}$. The density of particles with initial pitch angle $\psi_{mag}$ is obtained from conservation of the particle flux,

$$\frac{dn}{d\psi_{mag} B} = \frac{dn}{d\psi_{mag}} \left| \frac{v_0 \cos \psi_{mag}}{B_{mag}} \right|. \hspace{1cm} (34)$$

Integrating over initial pitch angles up to the maximum value $\psi_{\text{max}} = \arcsin (B_{mag}/B)^{1/2}$ gives the local density of particles,

$$n = 2 \frac{dn}{d\psi_{mag} B} \int_{\psi_{\text{max}}}^{\varepsilon_{\text{max}}} \left( \frac{B_{mag}}{B_{mag} \cos \psi_{mag}} \sin \psi_{mag} d\psi_{mag} \right) \left[ 1 - \frac{(B/B_{mag}) \sin^2 \psi_{mag}}{1 - (B/B_{mag}) \sin^2 \psi_{mag}} \right]^{1/2} \frac{1}{2} n_{mag}. \hspace{1cm} (35)$$

(The factor of 2 in front of the expression on the right accounts for the reflected flux of particles.) Thus, the density of particles is approximately constant, in spite of the strong convergence of the magnetic field lines toward smaller radius.
The adiabatic invariant is no longer conserved if a particle reaches deep enough into magnetosphere that it cools significantly. This occurs for electrons (and positrons) inside the radius (eq. [8]). The consequences for the equilibrium density of particles are discussed in § 3.1.

5. GEOMETRIC MODEL OF ECLIPSES

In this section we assume that closed field lines within the pulsar B magnetosphere are populated by relativistically hot plasma. We calculate the synchrotron optical depth over a large number of lines of sight, taking into account the three-dimensional structure of the magnetosphere. We also assume, for simplicity, that the magnetic field is described by a vacuum dipole and that the absorbing plasma is truncated outside some radius. This simple model can reproduce all the salient features of the eclipse light curve, including an essential part of the eclipse phenomenology, the weak frequency dependence of the eclipse duration and the dependence of phases of B. The optical depth to synchrotron absorption at all frequencies (especially at the highest) has a strong gradient near the eclipse boundary and quickly becomes large ($\tau_p \geq 1$) over a small range of orbital phase. This can be achieved if the absorbing particles are relativistic and can absorb in a wide frequency range.

5.1. Synchrotron Absorption

We assume that the closed field lines are populated by relativistic electrons with a thermal distribution at a temperature of $k_B T_e/m_e c^2 \approx 10$ (see eqs. [21] and [30], with $\varepsilon_A \approx 0.1$). In this case, the peak of the synchrotron emission is at the frequency $\nu_p \approx 4(k_B T_e/m_e c^2)^2 \nu_{\nu,e}$. The cyclotron frequency is $\nu_{\nu,e}(B_{\text{mag}}) \sim 2 \times 10^{12}$ Hz at the edge of magnetosphere and increases inward. Therefore, radio waves propagate in the low-frequency regime, $\nu \ll \nu_p$, when the observing frequency is in the gigahertz range.

The eigenmodes of the electromagnetic wave are linearly polarized when ions are absent from the plasma. The synchrotron absorption coefficients of the two eigenmodes are then (Rybicki & Lightman 1979)

$$\alpha^{(1,2)}_\nu = -\frac{e^2}{8\pi m_e c^2} \int d\varepsilon \frac{d}{d\varepsilon} \frac{n(\varepsilon)}{\varepsilon} p^{(1,2)}_\nu d\varepsilon. \quad (36)$$

The polarization-averaged absorption coefficient is

$$\alpha_\nu = \frac{1}{2} (\alpha^{(1)}_\nu + \alpha^{(2)}_\nu). \quad (37)$$

In these expressions, the spectral power density emitted by a single particle is denoted by

$$p^{(1,2)}_\nu = \frac{\sqrt{3} e^3 B \sin \kappa}{4\pi m_e c^2} \left[ \tilde{p} \int_0^\infty K_{5/3}(\eta) d\eta \pm \tilde{p} K_{2/3}(\tilde{p}) \right],$$

$$\tilde{p} \equiv \left( \frac{e}{m_e c^2} \right)^2 \frac{2\nu}{3(\sin \kappa)\nu_{\nu,e}}, \quad \nu_{\nu,e} \equiv \frac{eB}{2\pi m_e c}, \quad (38)$$

and we assume an isotropic distribution of pitch angles. The electromagnetic wave is absorbed on particles with pitch angle equal to the angle $\kappa$ between $\mathbf{B}$ and the direction $\mathbf{k}$ of wave propagation. In polarization state (1) the electric vector is orthogonal to the $\mathbf{k}$-$\mathbf{B}$ plane (the plus sign in eq. [38]), whereas in polarization state (2) it lies in the $\mathbf{k}$-$\mathbf{B}$ plane (the minus sign in eq. [38]). Also $\kappa$ is the energy of the emitting particle, $n(\varepsilon)$ is particle distribution function, and $K_{5/3}$ and $K_{2/3}$ are modified Bessel functions.

For a relativistic thermal distribution $n(\varepsilon) \propto e^{-\varepsilon/k_BT}$ characterized by a temperature $T$ and total density $n_e$, the absorption coefficient below the peak frequency is

$$\alpha_\nu = \frac{4\pi^2 e_n}{3\sqrt{3} B \sin \kappa} \left( \frac{m_e c^2}{k_B T} \right) \nu_{\nu,e} \sin \kappa \nu_{\nu,e}^{5/3} \nu_{\nu,e}, \quad (39)$$

5.2. Eclipse Light Curve

We introduce a Cartesian system of coordinates $x$, $y$, and $z$ centered on pulsar B (see Fig. 1). The plane of the orbit is taken to coincide with the $x$-$y$ plane, from which the line of sight is offset by a vertical distance $z_0$ [eq. (67)]. The observer is located at $x \to \infty$. The spin axis of pulsar B is inclined at an angle $\theta_{B_0}$ to the orbital normal and at angle $\phi_{B_0}$ with respect to the $x$-$z$ plane. The magnetic moment of pulsar B has the magnitude $\mu_B = (B_{NS} R_{NS})^2$ and is inclined at an angle $\chi$ with respect to $\Omega_B$. To calculate the intensity of the transmitted radiation, we need to work out the magnetic polar angle $\theta_{B_0}$ at each position $x = (x, y, z_0)$ along the line of sight,$^4$ where

$$\gamma(t) = \frac{\pi a}{2} \left[ 1 - \left( \frac{\phi}{90} \right)^2 \right] \quad (40)$$

is expressed in terms of the semimajor axis, $a$, in cm, and the orbital phase, $\phi$. The distance from a given point to pulsar B is

$$r(x, t) = \sqrt{x^2 + y^2(t) + z_0^2} \quad (41)$$

The magnetic polar angle of coordinate $r$ is determined from the unit vector $\hat{\mu}(t)$ parallel to $\mu_B(t)$,

$$\cos \theta_{B_0} = \frac{\hat{\mu} \cdot r}{r} \quad (42)$$

$^4$ For the purposes of an initial calculation, we neglect the possible presence of a toroidal magnetic field (see the Appendix).
The components of \( \tilde{\mu} \) are easy to write down in a coordinate system aligned with \( \Omega_B \):

\[
\tilde{\mu}_x = \sin \chi \cos (\Omega_B t), \quad \tilde{\mu}_y = \sin \chi \sin (\Omega_B t), \quad \tilde{\mu}_z = \cos \chi. \tag{43}
\]

Transforming to the observer’s coordinate system gives

\[
\begin{align*}
\hat{\mu}_x &= \left( \tilde{\mu}_x \cos \theta_{t1} + \tilde{\mu}_y \sin \theta_{t1} \right) \cos \phi_{t1} - \tilde{\mu}_z \sin \phi_{t1}, \\
\hat{\mu}_y &= \left( \tilde{\mu}_y \cos \phi_{t1} + \tilde{\mu}_z \sin \phi_{t1} \right) \sin \phi_{t1}, \\
\hat{\mu}_z &= \tilde{\mu}_z \cos \theta_{t1} - \tilde{\mu}_y \sin \theta_{t1}. \tag{44}
\end{align*}
\]

The strength of the magnetic field at position \( x \) is then given by

\[
B = \frac{\sqrt{1 + 3 \cos^2 \theta_{t1}}}{r} \mu_B. \tag{46}
\]

The condition that a given point is located on closed field lines is that the maximum extension of a field line is less than \( R_{\text{mag}} \):

\[
R_{\text{max}} = \frac{r}{\sin \theta_{t1}} < R_{\text{mag}}. \tag{47}
\]

We also need to know the angle between the line of sight and the local direction of the magnetic field:

\[
\cos \kappa = \frac{B_x}{B} = \frac{3 \cos \theta_{t1} (x/r) - \hat{\mu}_y}{(1 + 3 \cos^2 \theta_{t1})^{1/2}}. \tag{48}
\]

For calculation of polarization properties, the electric vectors of the two eigenmodes are parallel to the unit normals,

\[
E_1 = \frac{B_z \hat{z} - B_y \hat{y}}{(B_y^2 + B_z^2)^{1/2}} \quad \text{and} \quad E_2 = \frac{B_z \hat{z} + B_y \hat{y}}{(B_y^2 + B_z^2)^{1/2}}, \tag{49}
\]

which lie in the plane of the sky.

We first calculate the total intensity of the transmitted radiation. Our examination of polarization effects is deferred to \( \S \ 7 \).

The optical depth at time \( t \), frequency \( \nu \), and impact parameter \( z_0 \) is obtained by integrating the polarization-averaged absorption coefficient (eq. [39]) along the line of sight,

\[
\tau_r(t, z_0) = \int_{-\infty}^{\infty} \alpha_r(x, y(t), z_0) dx. \tag{50}
\]

We normalize the density, \( n_e \), of absorbing particles to a characteristic Goldreich-Julian density at the magnetospheric boundary (\( B = B_{\text{mag}} \)):

\[
n_e = \frac{\lambda_{\text{mag}} \nu G (R_{\text{mag}})}{2 \pi \varepsilon c} = 0.2 \lambda_{\text{mag}} \text{ cm}^{-3}. \tag{51}
\]

(This is only a convenient normalization: in fact, \( \mu_B \) and \( \Omega_B \) are nearly orthogonal in our best eclipse model.) We also normalize the line-of-sight coordinate and field strength to \( R_{\text{mag}} \) and \( B_{\text{mag}} \) (eq. [4]). For our fiducial parameters, the optical depth is

\[
\tau_r(t, z_0) = \frac{4.5 \times 10^{-6}}{\nu G \lambda_{\text{mag}} (R_{\text{mag}})} \int \left( \frac{x}{R_{\text{mag}}} \right)^{1/4} \lambda_{\text{mag}} \left( \frac{k_B T_e}{10 \text{m} \cdot \text{e}^2} \right)^{-5/3} \left( \frac{B \sin \kappa}{B_{\text{mag}}} \right)^{2/3}, \tag{52}
\]

where \( \nu_{\text{GHz}} \) is the wave frequency normalized to 1 GHz and we have set \( I_\lambda = I_0 = 10^{45} \text{ g cm}^2 \). In this expression, the parameter \( N_B \) encapsulates the uncertainty in the magnetospheric radius through the normalization of the spin-down torque acting on pulsar B (eq. [4]). The code also tests whether the local cyclotron frequency satisfies the condition \( \nu_{\text{cycl}} > \langle \gamma_B \rangle = 3 k_B T_e / m_e c^2 \) and whether a given point is located outside the cooling radius (eq. [8]). Both of these effects are important only for extremely small impact parameters \( z_0 \).

As a simple prescription for density distribution, we assume that only those field lines are loaded with absorbing particles for which the maximum extension (calculated at the magnetic equator) lies within a prescribed range,

\[
R_{\text{abs} -} < R_{\text{max}} < R_{\text{abs} +}. \tag{53}
\]

Motivated by \( \S \ 4.3 \), the plasma density \( n_e \) and temperature \( T_e \) are assumed to be constant along each field line that lies in this range. We further assume that \( n_e \) does not vary between field lines.

The onset and termination of the eclipse are then determined by the physical boundary of the absorbing plasma. We find that \( R_{\text{abs}} \) must be fixed at a value \( \sim 1.5 \times 10^9 \text{ cm} \) in order to reproduce the width of the observed eclipses. The optical depth must quickly become large on lines of sight passing just inside the plasma boundary (at a radius \( \sim R_{\text{abs}} \)). The implied multiplicity is large, \( \lambda_{\text{mag}} \sim 10^3 \), but still consistent with our estimates in equations (10) and (12).

One expects a dipole to be only a rough approximation near the plasma boundary when \( R_{\text{abs} -} \sim R_{\text{mag}} \). However, the dipole pressure rises rapidly inward, so the dipole approximation becomes increasingly accurate as \( R_{\text{abs} +} / R_{\text{mag}} \) becomes smaller. Modest deviations between the model and the data near the edges of the eclipse could be used to probe the distortion of magnetic field lines from a true dipole.

5.3. Successful Eclipse Parameters

The calculation of the eclipse light curve requires a choice of several parameters: the angles \( \theta_{t1}, \phi_{t1}, \) and \( \chi \); the electron temperature \( T_e \) and density multiplicity \( \lambda_{\text{mag}} \); the inner and outer radii, \( R_{\text{abs} -} \) and \( R_{\text{abs} +} \); and the impact parameter \( z_0 \).

Our choice of the parameters was guided by a qualitative sense of how they would influence the shape of the light curve, in the following key respects:

1. the eclipse is asymmetric, with the ingress shallower and longer than the egress;
2. near ingress, transparent windows appear twice per rotation of pulsar B;
3. near the center of the eclipse, the transparent windows appear once per rotation;
4. near egress, the transparent windows are not well defined;
5. the duration of eclipse at a given rotational phase of pulsar B depends on the phase of B and is larger when the magnetic moment is along the line of sight; and
6. the duration of eclipse at half-maximum flux is \( \sim 30 \text{ s} \), while the full duration is \( \sim 40 \text{ s} \).

The modulation of the radio flux during the eclipse is due to the fact that at some rotational phases of pulsar B the line of sight passes only through open magnetic field lines, where the absorption is assumed to be negligible. One of the main successes of the model is its ability to reproduce both the single and double periodicities of these transparent windows at appropriate places.
in the eclipse. This requires that \( \mu_B \) be approximately—but not quite—orthogonal to \( \Omega_B \). To understand this result geometrically, consider Figures 2 and 3. The rotation of \( \mu_B \) out of the plane of the sky brings the observer’s line of sight to within a small angle of the magnetic pole (in the favored geometry). The cross-sectional area of the absorbing plasma is maximized at this moment, and it projects an ellipsoidal shape on the sky. On the other hand, when \( \mu_B \) lies in the plane of the sky, the absorbing plasma fills a region bounded by a set of closed dipole field lines (Fig. 3). The shifting inclination of \( \mu_B \) through one-half a rotation of pulsar B allows the line of sight to intersect this dipolar region once or twice. The proportions of the eclipse in which either periodicity is observed depend on the impact parameter \( z_0 \) and the extent to which \( \mu_B \) is nearly—but not quite—orthogonal to \( \Omega_B \).

Our simulated light curve is displayed in Figures 4 and 5. Variations in the parameters of the model modify the light curve in the following ways. A large asymmetry between ingress and egress results from a combination of finite \( \theta_1 \) (the inclination between the spin axis of pulsar B and the orbital plane normal) and finite \( z_0 \) (the impact parameter). The asymmetry is largely erased if \( \phi_1 \) is in the range \( \pm 60^\circ \) (the rotation axis is out of the plane of the sky). There can be one or two transparent windows per rotation of pulsar B, depending on \( z_0 \) and \( \chi \) (the angle between rotational angle and magnetic moment). If \( \chi \) differs considerably from \( \pi/2 \), then both eclipse center and egress show

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**Fig. 2.**—View of the magnetosphere at different rotational phases separated by \( \pi/4 \). For a full movie of the eclipse, see [http://www.physics.mcgill.ca/~lyutikov/movie.gif](http://www.physics.mcgill.ca/~lyutikov/movie.gif).
The absorbing plasma is located between the two drawn magnetic surfaces with the pulsar when the line of sight crosses both the solid and the dashed field lines. A full eclipse occurs when the line of sight crosses both the solid and the dashed field lines. The model readily reproduces many fine details of the eclipses. It explains the modulation that is observed at the first and second harmonics of the spin frequency of pulsar B and the deepening of the eclipse after superior conjunction. The average eclipse duration is almost independent of frequency when the

The orbital inclination is therefore predicted to be close to 90°55. This is somewhat larger than that inferred by Ransom et al. (2005; 90°26 ± 0°13) but consistent with the estimate of Coles et al. (2004; 88°7 ± 0°9). Note that there is a symmetry in these parameters, \( z_0 \rightarrow -z_0 \) and \( \theta_\Omega \rightarrow -\theta_\Omega \), which preserves the shape of the light curve. In view of the results of Coles et al. (2005), we have chosen \( z_0 < 0 \).

The dependence of the eclipse profile on the parameters \( z_0 \), \( \theta_\Omega \), and \( \chi \) is illustrated in Figure 6. Overall, the best-constrained parameter is \( z_0 \sim -7.5 \times 10^8 \) cm, which must lie within a range of \( \sim \pm 10\% \) of this value. There is, however, a degeneracy between \( \theta_\Omega \) and \( \chi \) that produces nearly identical light curves if both angles are varied up or down by the same factor. The results are not sensitive to \( |\theta_\Omega + 90°| \leq 30° \).

The required minimum multiplicity is \( \lambda_{\text{mag}} \sim 3 \times 10^5 \text{G}_{\text{mag}} \left( k_B T_{\text{B}}/10^4 m_e^2 \right)^{\frac{1}{2}} \) to reduce the transmitted radio flux by \( \sim 90\% \) at an orbital phase of \( \sim 90°55 \). This electron density is, in fact, close to the estimates in equations (10) and (12). The full duration of eclipse is about 40 s. The size of the eclipsing region is \( R_{\text{abs}} = 1.5 \times 10^8 \) cm, so the absorbing plasma must be truncated well inside the expected radius of the magnetopause. We comment on the significance of this result in § 5.4.
multiplicity is larger than $\lambda_{\text{mag}} \sim 3 \times 10^5$ (Fig. 7). The eclipse is broader when the magnetic moment of pulsar B is pointing closest to the observer, just as is observed (McLaughlin et al. 2004). Figure 8 shows the eclipse profile averaged over different orientations of the magnetic moment. When $\mu_B$ is pointing toward the observer, the “doughnut” of the closed field lines is seen nearly face-on, producing broad eclipses; when $\mu_B$ is in the plane of the sky, the doughnut is seen edge-on, producing narrower eclipses.

It should also be noted that the details of eclipse ingress and egress are not fully reproduced by the model: in particular the rise in the radio flux at egress is sharper than that observed (especially if $\lambda_{\text{mag}}$ is high enough to give the eclipses a weak frequency dependence). This could be due to deviations of the actual magnetic field from our assumption of a pure dipole, or it could be an artifact of our assumption of a sharp plasma boundary. We examine the effects of a smoother plasma profile and a variable electron temperature profile in §6.

Fig. 6.—Change in the eclipse light curve due to a variation in the impact parameter $z_0$ and the inclination $\chi$ between the magnetic and spin axes of pulsar B. The parameters are the same as in Fig. 4.

Fig. 7.—Frequency dependence of the eclipse profile, smoothed on the timescale $\Delta t = 2.77\,\text{s}$. The parameters are the same as in Fig. 4. At high frequencies the eclipse duration should become strongly frequency dependent. This can be used to place tighter constraints on the plasma density.

Fig. 8.—Average eclipse profiles for different orientations of the magnetic moment of B. The solid line shows the eclipse profile averaged over points $315^\circ < \phi_B < 45^\circ$ and $135^\circ < \phi_B < 225^\circ$, where $\phi_B$ is a rotational phase of B ($\phi_B \sim 0^\circ$ and $180^\circ$ correspond to the orientations in which the magnetic moment points nearly toward an observer, while $\phi_B \sim 90^\circ$ and $270^\circ$ correspond to the orientations in which the magnetic moment is in the plane of the sky). The dotted line shows the eclipse profile averaged over $45^\circ < \phi_B < 135^\circ$; the dashed line shows the eclipse profile averaged over $225^\circ < \phi_B < 315^\circ$. The eclipse is broader when the pulsar is face-on. The parameters are the same as in Fig. 4.
5.4. Eclipse Duration

Our eclipse calculations show that the optical depth of the absorbing plasma undergoes a sharp drop at a distance $R_{abs} \approx 1.5 \times 10^{10}$ cm from pulsar B. This is about 2.5 times smaller than the expected radius of the magnetopause, $R_{mag} \approx 4 \times 10^{10}$ cm (assuming that the torque parameter $N_B$ is unity in eq. [4]). The disagreement with the maximum lateral extension of the closed field lines is closer to a factor of 4 (e.g., Lyutikov 2004).

The synchrotron optical depth (eq. [52]) reflects both the geometric distribution of absorbing particles and their temperature profile. One possibility is that $R_{mag}$ has been overestimated and that $R_{abs}$ reflects the actual boundary of the pulsar B magnetosphere. Alternatively, the suppression in $\tau_e$ could be due to particle loss and overheating in the outer magnetosphere. Let us consider these possibilities in turn (we clearly favor the latter).

Recall that the torque parameter $N_B$ is presently unknown. The surface magnetic field of pulsar B could be smaller than is implied by the simplest estimate of the spin-down torque ($N_B \sim 1$; eq.[1]). The torque formula (1) assumes an increase in the opening of the dipole field lines that is expected from simple geometry. But other effects are at work. The pressure is distributed asymmetrically about pulsar B, so even if its spin were in co-rotation with the orbit, it would be still be torqued by the pulsar A wind (the Magnus torque; e.g., Thompson et al. 1994, Arons et al. 2004). In addition, a strong surface resistivity at the interface between the pulsar B magnetosphere and the shocked wind of pulsar A would result in an enhanced spin-down torque, due to the resistive dragging of the poloidal field lines of pulsar B.

There is a clear upper bound on the torque and spin-down luminosity of pulsar B:

$$L_B \leq B_{SS}^2 R_{SS}^2 c \left( \frac{R_{NS}}{R_{mag}} \right)^3 \frac{\Omega_{NS}}{\Omega_{mag}} \frac{\Delta n_{NS} B_{SS}}{c}, \quad (54)$$

which comes from the limit of comparable toroidal and poloidal fields at the magnetospheric boundary. Equation (54) implies that $N_B < c/(\Omega_{NS} R_{mag}) \sim 10$. If $N_B$ saturated this bound, the magnetospheric radius $R_{mag}$ would be

$$R_{mag} = \left( \frac{B_{SS} \Omega_{NS}}{I_B \Omega_{NS} \Omega_{A}} \right)^{1/3} \left( \frac{c d_{AB}^2}{2} \right)^{1/3} = 2.4 \times 10^9 \text{ cm}. \quad (55)$$

The lateral radius of the closed field lines is 50% larger, or 3.6 $\times 10^{10}$ cm, which is still a factor of 2 too large. Thus, one cannot explain the small eclipse duration as entirely due to a large torque or a small magnetic field of pulsar B.

The short eclipse duration may also reflect the loss of plasma from the outermost closed field lines, resulting from a change in the topology of the field lines. Experience with planetary magnetospheres suggests that the boundary between open and closed field lines can vary by a factor of a few orders of magnitude (see a factor of 2 or so; Lyutikov 2004, Arons et al. 2005). This disfavors synchrotron absorption in the shocked wind of pulsar A as the explanation for the eclipse.

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5 Note that the shock that deaccelerates the wind of pulsar A has a yet larger transverse dimension (by a factor of 2 or so; Lyutikov 2004, Arons et al. 2005). This disfavors synchrotron absorption in the shocked wind of pulsar A as the explanation for the eclipse.
The corresponding lower bound on the multiplicity of the neutralizing electrons (mass $m_e$) is

$$\lambda_{\text{eq}}^{-1} = \left( \frac{180}{\cos \theta_B} \right) \left( \frac{k_B T_e}{m_e c^2} \right)^{-1} \left( \frac{B_{\text{NS}} R_{\text{NS}}^3}{5 \times 10^{29} \text{ G cm}^3} \right) \left( \frac{r}{4 \times 10^9 \text{ cm}} \right)^{-3}. \quad (57)$$

This can be competitive with expression (eq. 10) somewhat inside the magnetopause, e.g., at $r \lesssim 1 \times 10^9$ cm.

From the requirement that the electron pressure not exceed the magnetic dipole pressure and by using the electron multiplicity (eq. [57]), the maximum temperature is

$$k_B T_{e,\text{max}} = \frac{m_p}{2m_e} \left( 1 + \frac{T_p}{T_e} \right)^{-1/2} = 9 \times 10^2 \left( 1 + \frac{T_p}{T_e} \right)^{-1/2}, \quad (58)$$

which applies as long as $k_B T_p \approx m_p c^2$.

We therefore set the electron density to the threshold value for significant ion heating. If we keep in mind the results of § 4.3, $n_e$ is fixed at a constant value (eq. [56])

$$n_e(R_{\text{mag}}) = \frac{k_B T_e}{m_p c^2} \frac{B_{\text{mag}}^2}{2 \pi m_p c^2} \left( \frac{R_{\text{max}}}{R_{\text{mag}}} \right)^{-6} \quad (59)$$

along each field line (labeled by the maximum radius $R_{\text{mag}}$).

The corresponding multiplicity parameter $\lambda_{\text{eq}}^{-1}$ is given by equation (57) or, equivalently, by

$$\lambda_{\text{mag}} = \lambda_{\text{eq}}^{-1}(R_{\text{max}}) \left( \frac{R_{\text{max}}}{R_{\text{mag}}} \right)^{-3}. \quad (60)$$

The factor of $(R_{\text{mag}}/R_{\text{mag}})^{-3}$ in this expression accounts for the normalization of the multiplicity $\lambda_{\text{mag}}$ at the magnetospheric boundary $R_{\text{mag}}$.

One also expects a very strong dependence of $T_e$ on $r$ when the electrons absorb the energy of large-scale torsional motions in the magnetosphere and cool by incoherent synchrotron emission (§ 4.2). The equilibrium temperature of the absorbing electrons is expected to increase rapidly with radius (eq. [30]). The synchrotron optical depth given by equations (39) and (50), with electron multiplicity (eq. [57]), drops precipitously with distance from pulsar B:

$$\tau_{\text{e}}(r) \propto \frac{m_e B_{\text{mag}}^2}{T_e} \frac{R_{\text{mag}}^{5/3}}{T_e^{5/3}} \propto \varepsilon_A^{-4/9} \nu^{-5/3} r^{-9/9}. \quad (61)$$

Here $\varepsilon_A$ is the energy density in torsional motions at radius $r$ in the pulsar B magnetosphere, relative to the magnetic pressure at the magnetopause boundary. One finds, generically, a sharp transition from large to small optical depth, because of the much higher heating rates in the outer magnetosphere. Taking $\varepsilon_A$ to be constant gives the frequency scaling

$$\Delta \phi(\nu) \sim \nu^{-15/9}. \quad (62)$$

We therefore choose a temperature profile $T_e(r) \propto r^7$. After setting the magnetospheric radius to $R_{\text{mag}} = 4 \times 10^9$ cm ($N_0 = 1$ in eq. [4]), the only free parameter in this model is the normalization of $T_e$, which is set to $k_B T_e = 10 m_e c^2$ at $r = 1.4 \times 10^9$ cm. The resulting eclipse light curves are displayed in Figure 9. One obtains an equally good fit to the central parts of the eclipse and a smoother fall and rise in the radio flux at ingress and egress. The eclipse duration has a weak frequency dependence (Fig. 10), and the depth of the eclipse is less dependent on frequency than in the constant-$n_e$ model (compare Fig. 7).

A rapid increase in the temperature toward the edge of the magnetosphere (combined with a drop in $n_e$ to maintain pressure...
balance) can therefore provide an explanation for the duration of the observed eclipses. Indeed, the dependence of \( T_e \) on \( r \) could be even stronger than we assume if the amplitude of the turbulence increases outward in the magnetosphere of pulsar B and if there is enhanced plasma loss from the outer magnetosphere.

It should also be noted that our model of electron heating involves the absorption of low-frequency (1–10 MHz) radio waves (§ 4.2). This means that \( \tau_\nu \), maintains the scaling (eq. [61]) and continues to drop off rapidly with radius, even when the plasma is optically thin at GHz frequencies.

7. POLARIZATION

We now consider polarization effects associated with the propagation and absorption of the two electromagnetic modes in the eclipse region. In a magnetized, relativistic plasma these modes are linearly polarized below the synchrotron peak frequency, \( \nu \lesssim \nu_B = \nu_B(k_B T_e/m_e c^2)^{1/2} \), even if the plasma is composed of electrons and ions (e.g., Sagiv et al. 2004). The linear absorption coefficients of the two modes are given by equation (39). We present a sample calculation of the polarization of the transmitted radio pulses in our favored eclipse model, under the assumption that the incident radiation is unpolarized. In this case, the radiation can become polarized as it propagates through an absorbing medium.

In an inhomogeneous plasma (where the density and/or the magnetic field are functions of position), it is more convenient to describe radiation transfer in terms of modes polarized along fixed directions in space. We choose these reference polarization states to lie along the normal to the orbital plane (a), and within the orbital plane (b). The radiation field is then characterized by four Stokes parameters, \( I, Q, U, \) and \( V \). The first two parameters can be reexpressed in terms of the intensities in the two reference polarization states, \( I = I^a + I^b \) and \( Q = I^a - I^b \). The polarization fraction is \( \Pi = [(U^2 + Q^2)^{1/2}] / I \).

Our calculation of the transmitted polarization neglects the effects of synchrotron reemission and Faraday rotation. The first effect can be safely neglected if one is interested in the pulsed component. Faraday rotation is absent from a pair plasma, while for electron-ion plasma at low frequencies it is suppressed by a factor of \( \sim \nu/\nu_B \) compared with the effects of synchrotron absorption.

Some circular polarization can be generated as the result of a relative phase delay between the two linear eigenmodes, combined with a rotation of the background magnetic field along the line of sight. In a relativistic thermal plasma, the difference \( n_2 - n_1 \) between the mode indices of refraction depends on the angle between \( \mathbf{k} \) and \( \mathbf{B} \): one has approximately

\[
(n_2 - n_1) \frac{\omega}{c} \sim \frac{\Gamma(1/3)}{2^{2/3} \pi} \alpha_\nu = 0.54 \alpha_\nu
\]

\[
\left[ \nu \ll \nu_B, (k_B T_e/m_e c^2)^{1/2} \right],
\]

where \( \alpha_\nu \) is the polarization-averaged synchrotron absorption coefficient (e.g., Sagiv et al. 2004). In this situation, the phase shift \( n_2 - n_1 \omega L_0 / c \) is not large in regions where the absorption optical depth is modest, \( \tau_\nu \lesssim 1 \). This means that the instantaneous polarization angle, \( \chi = (1/2) \tan^{-1} (U/Q) \), cannot adjust adiabatically to the changing direction of the magnetic field (e.g., Thompson et al. 1994).

Within the above approximations, the equations of polarization transfer read

\[
\frac{dI^a}{dx} = -I^a \left[ \alpha_\nu (1) \sin^2 \chi_B + \alpha_\nu (2) \cos^2 \chi_B + \frac{1}{2} \alpha_\nu \sin^2(2\chi_B) \right] + \frac{1}{4} U (\alpha_\nu (1) - \alpha_\nu (2)) \sin(2\chi_B) - \frac{1}{2} V \left[ \frac{\omega}{c} (n_2 - n_1) \right] \sin(2\chi_B),
\]

\[
\frac{dI^b}{dx} = -I^b \left[ \alpha_\nu (1) \cos^2 \chi_B + \alpha_\nu (2) \sin^2 \chi_B + \frac{1}{2} \alpha_\nu \cos^2(2\chi_B) \right] + \frac{1}{4} U (\alpha_\nu (1) - \alpha_\nu (2)) \sin(2\chi_B) + \frac{1}{2} V \left[ \frac{\omega}{c} (n_2 - n_1) \right] \sin(2\chi_B),
\]

\[
\frac{dU}{dx} = -\alpha_\nu U + \frac{1}{2} (I^a + I^b) \left[ \alpha_\nu (1) - \alpha_\nu (2) \right] \sin(2\chi_B) + V \left[ \frac{\omega}{c} (n_2 - n_1) \right] \cos(2\chi_B),
\]

\[
\frac{dV}{dx} = -\alpha_\nu V + \left[ (I^a - I^b) \sin(2\chi_B) - U \cos(2\chi_B) \right] \frac{\omega}{c} (n_2 - n_1)
\]

(e.g., Pacholczyk & Swihart 1970), where \( \chi_B \) is the angle between the reference direction \( a \) and the projected magnetic field (at a certain point along the line of sight).

The Stokes parameters are plotted in Figure 11 for our best-fit eclipse model. The transmitted radiation is predicted to be strongly polarized in the deepest part of the eclipse, with a polarization fraction reaching 25%. (This is, unfortunately, the time when the signal-to-noise ratio is smallest.) The polarization angle evolves smoothly throughout eclipse. One finds \( U \lesssim Q \) when \( \Pi \) is largest, which means that the transmitted polarization tends to be aligned with one of the two reference polarization directions. The main contributions to \( U \) and \( Q \) come from regions of the magnetosphere that have finite transparency.
When the background radio source is unpolarized, we find that the circular polarization \( V \) is much smaller than \( U \) and \( Q \), by 1 or 2 orders of magnitude. This is partly the result of a cancelation between the ingoing and outgoing parts of the ray trajectory.

Polarization provides an independent test of the eclipse mechanism and the geometry of the system. The radio pulses of pulsar A are, in fact, strongly polarized (up to 50%; Ransom et al. 2004). In order to predict correctly the transmitted polarization, one needs to know the direction of polarization with respect to the orbital plane. This can be achieved by calibrating the polarization angle, i.e., by referring to a source with known position angle and by determining the orientation of the orbit on the sky (by combining the scintillation measurements with the anticipated proper-motion measurements).

8. PREDICTIONS AND COMPARISON WITH OTHER MODELS

8.1. Predictions of the Model

We now summarize the predictions of our model and how further observations can be used to probe the geometry of the system and the properties of the eclipsing material.

1. The impact parameter \( z_0 \) between the line of sight and the orbital plane is predicted to be \( |z_0| = 7.5 \pm 0.7 \times 10^8 \) cm. The corresponding orbital inclination is \( 90.55 \pm 0.05 \) (or possibly \( 89.45 \pm 0.05 \)). This prediction is intermediate between the measurements of Coles et al. (2005) and Ransom et al. (2004).

2. The spin \( \Omega_B \) of pulsar A is expected to undergo geodetic precession on a \( \sim 75 \) yr timescale (Burgay et al. 2003). We have provided predictions for how the eclipse light curve varies as a result. In particular, the orbital phase at which the radio flux reaches a minimum shifts back toward superior conjunction (Fig. 12). Since \( \phi_0 \) is not well constrained, the time for the eclipse to become symmetric is between \( \sim 12 \) yr (if \( |\phi_0| + 90^\circ| \approx 30^\circ \) and \( \Omega_B \) is drifting away from \( \phi_0 = -90^\circ \)) and \( \sim 25 \) yr (if \( |\phi_0| \) is drifting toward \(-90^\circ\)).

3. High time-resolution observations (see Fig. 13) are a sensitive probe of the distribution of plasma properties (density and temperature) on the closed field lines. If plasma is depleted from the outermost field lines, at high temporal resolution the flux should return to unity. On the other hand, if the absorbing plasma does not have a sharp truncation in radius, then the flux does not return to unity in all the transparent windows.

4. The eclipses must regain a strong frequency dependence at sufficiently high frequencies. The critical frequency above which significant transmission occurs can be used to place tight constraints on the plasma density. The electron cyclotron frequency...
at the distance \( r \sim z_0 \sim 7.5 \times 10^8 \) cm from pulsar B is estimated to be \( v_{\text{B, e}} \sim 3 \) GHz. One therefore expects the eclipses to develop a significant frequency dependence at higher frequencies.

5. There are a number of definite predictions for the polarization of the transmitted radiation, as discussed in § 7. The propagation of the radio waves through the intervening magnetosphere is in the quasi-transverse regime, where the polarization eigen-modes are linear and Faraday rotation can be neglected. The rotation measure should therefore not change significantly during the eclipse, and the variation in the dispersion measure is predicted to be small, on the order of \( 10^{13} \) cm\(^{-2}\).

Finally, we note that the model predicts some radio emission from the magnetosphere of pulsar B, but its precise amplitude and spectrum cannot be easily determined from first principles. A straightforward upper bound to the radio output is obtained with respect to the line of sight is (Appendix). If the closed dipolar field lines of pulsar B are twisted, the cyclotron resonance (of either type of charge) is in the quasi-transverse regime, where the polarization eigen-modes are linear and Faraday rotation can be neglected. The rotation measure should therefore not change significantly during the eclipse, and the variation in the dispersion measure is predicted to be small, on the order of \( 10^{13} \) cm\(^{-2}\).

The corresponding expression is

\[
\nu_{\text{B, e}} = 400N_B^{1/4} \left( \frac{2^2}{R_z^2} + 1 \right)^{-3/2} \text{MHz} \tag{68}
\]

when the magnetic moment is oriented perpendicular to the orbital plane. Inverting these expressions, one obtains \( R_z \) as a function of \( \nu_{\text{B, e}} \) for a fixed impact parameter. In both cases, it is a stronger function of frequency than \( R_z \sim \nu_{\text{B, e}}^{1/3} \) when \( b \neq 0 \). In addition, one observes that absorption must occur at a harmonic >1 unless the torque parameter \( N_B > 1 \) (see eq. [4]).

Cyclotron absorption inside the magnetosphere of pulsar B has some advantages: the cross section is larger than it is for synchrotron absorption by relativistic particles, and the required particle density is thereby reduced. To have an appreciable optical depth at a radius \( r \) [and corresponding frequency \( \nu_{\text{B, e}}(r) \)] the particle density should exceed the local Goldreich-Julian density by a factor of \( (\Omega_B r/c)^{-1} \sim 10 \). In addition, the eclipse is observed to begin roughly where the line of sight passes deep enough into the magnetosphere of pulsar B that \( \nu_{\text{B, e}} \sim \nu \). Cyclotron absorption is, nonetheless, strongly disfavored for at least two reasons: first, the expected frequency dependence of the eclipse duration is \( \Delta \phi \propto \nu^{1/3} \) (or a stronger function of frequency), in contradiction to the much weaker observed scaling \( \Delta \phi \propto \nu^{-0.1} \), and, second, the absorbing particles are inevitably heated to relativistic temperatures. Note also that a fluctuating component of the current would cause a heavy fossil disk of cold particles, which could be centrifugally suspended in the outer magnetosphere, to be drained on a very short timescale (see Thompson et al. 2002).

Our favored plasma parameters differ substantially from those advocated by Rafikov & Goldreich (2005), basically because we invoke a much stronger heating mechanism and a different source of relativistic particles. At high optical depth, particles are isotropized much more easily, and the equilibrium particle density is regulated to a much larger value (due to the competition between injection from below and precipitation through the cyclotron cooling radius). If the external radio photons are the primary heat source, as advocated by Rafikov & Goldreich (2005), then the optical depth at the cyclotron fundamental frequency is regulated to value of the order of unity. One infers that the eclipse duration should vary as \( \Delta \phi \sim \nu^{-1/3} \) (or as a stronger function of \( \nu \) if the radio pump radiation is brighter at \( \sim 100 \) MHz than it is at \( \sim 1 \) GHz).

We have also considered the effects of induced Compton scattering of pulsar A radio emission by plasma in pulsar B magnetosphere and by the pulsar B radio beam (e.g., Thompson et al. 1994). Both effects are negligible due to a small optical depth.

9. CONCLUSION

We have developed a model of the radio eclipses of pulsar PSR J0737–3039A, in which synchrotron absorption on relativistic particles occurs inside the intervening magnetosphere of the companion pulsar B. We believe that the value of such modeling is threefold. First, it provides a strong test of the long-standing assumption that isolated neutron stars are surrounded by co-rotating, dipolar magnetic fields. Second, one is probing how a pulsar magnetosphere interacts with an external wind, in particular the mechanism by which turbulence is damped in a relativistic and magnetically dominated plasma. Third, an understanding of how charges are supplied to the magnetosphere may have broader implications for the electrodynamics of radio pulsars.
Given the simplicity of the model, it is in excellent agreement with observations, especially in the middle of the eclipse, where the interaction with the external wind has only a modest effect on the shape of the poloidal field lines. The model can explain the asymmetry of the eclipse between ingress and egress; the weak frequency dependence of its duration; the modulation of the pulsar A emission at both the spin period of pulsar B and half its spin period, in different parts of the eclipse; the detailed shape of the luminosity spikes in the middle of the eclipse; and the dependence of the eclipse duration on the rotational phase of pulsar B. The model implies that pulsar B is nearly, but not exactly, an orthogonal rotator. It also requires the spin axis of pulsar B to be tilted from the normal to the orbital plane.

We have demonstrated that at intermediate distances the poloidal magnetic field of a neutron star is well approximated by a dipole. This is a valuable confirmation of a fundamental assumption made in models of pulsar electrodynamics. In addition, our results are consistent with models that place the source of the radio emission close to the magnetic axis. Note that McLaughlin et al. (2004) define phases with respect to the arrival of radio pulses from pulsar B, whereas we define them with respect to the magnetic axis of B. On the other hand, the dependence of eclipse duration on pulsar B phase in our model is the same as that inferred by McLaughlin et al. (2004), so the two definitions of phases are close to each other.

In the present model we have neglected the fact that the radio pulse profile of pulsar B has a single peak. This can be used to put additional constraints on the geometry of the system, assuming that the radio emission mechanism is the same at both magnetic poles. In particular, in order for the radio beam from one of the magnetic poles to miss the observer, the angle \( \phi_B \) should not be equal to \( \pm 90^\circ \). This comes from the fact that for \( \phi_B = \pm 90^\circ \) the angle between the direction of the magnetic moment and the line of sight is the same every half-period. Unfortunately, in the absence of a pulsar radio emission model with predictive power, this does not provide any meaningful constraint at the moment.

Our eclipse model has implications for the formation and evolution of the PSR J0737−3039A system. The tilt of the spin axis of pulsar B with respect to the normal to the orbital plane is consistent with a large natal kick (as has been inferred from the spatial velocity of the system; Ransom et al. 2004). The kick would change the orientation of the orbital plane and disrupt any preexisting alignment between the orbit and the spin of the progenitor star. Our measurement also does not support the suggestion of Demorest et al. (2004) that the spin axis should become aligned with the normal to the orbital plane due to torque from pulsar A.

We have also described how an optically thick, relativistic, thermal plasma may be maintained in the outer magnetosphere of pulsar B, where synchrotron cooling is slow. We have argued that the interaction between the wind of pulsar A and the magnetosphere is much more effective at heating the particles than is the absorption of the radio pulses of pulsar A. This interaction also generates two sources of particles on the closed magnetic field lines. First, bunches of relativistic charges are created by a pulsar-type mechanism close to the neutron star surface as the dipolar field lines are twisted back and forth close to the magnetic poles. Second, magnetic helicity builds up on the dipolar field lines, as the result of the asymmetry between the outgoing and return currents in each magnetic hemisphere. The resulting steady current supplies a constant flow of electrons and ions from the star to the outer magnetosphere.

The relativistic charges flowing in the magnetosphere of pulsar B are generally a more potent source of radio photons than is the companion pulsar. The requirement for this to be true is that the efficiency of conversion of turbulent energy to radio photons is larger than \( \sim 10^{-7} \). (For example, if the unpulsed radio emission originates in the magnetosphere of pulsar B, then this minimal efficiency is exceeded.) As a result, heating of the magnetospheric particles occurs mainly through self-absorption of these internally generated photons. Heating of ions can have an important regulatory effect on the equilibrium electron density: if \( n_e \) exceeds a critical value, then radio photons that resonate with the ion gyromotion are absorbed by the plasma. A plausible source of low-frequency pump photons has also been identified. This involves a two-step emission process: a cascade of torsional Alfvén waves to high frequencies at which the waves become charge starved, followed by wiggler emission of radio waves by the electrostatically accelerated charges in the fluctuating magnetic field.

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**APPENDIX**

**HELICITY PUMPING ON THE CLOSED MAGNETIC FIELD LINES**

The excitation of large-scale torsional motions provides a robust mechanism for drawing charges into the outer magnetosphere of pulsar B. Damping of this turbulence also heats the suspended particles (\( \xi \leq 2 \)), and if the heating rate is too high, then the plasma may not absorb radio waves effectively. This leads us to examine a subtler effect, one involving a static component of the magnetospheric twist, which operates deeper in the magnetosphere of pulsar B, where the fluctuating current is small, \( |\mathbf{j}| \ll |\mathbf{\rho}_\phi|/c \).

The exterior magnetic field of a neutron star can store a considerable amount of magnetic helicity. When it does, the current flowing nearly parallel to the magnetic field has a zero-frequency component. In the magnetically active soft gamma repeaters and anomalous X-Ray pulsars, the unwinding of a strong internal magnetic field is a repeating source of magnetic helicity for the exterior of the star (Thompson et al. 2002). One does not expect such a mechanism to be active in the much older (and weakly magnetic) PSR J0737−3039.

Nonetheless, we offer a simple argument for why magnetic helicity should be pumped from the outer to the inner magnetosphere of pulsar B. There are two simple components to this argument. First, if a fixed amount of helicity is injected into a dipole magnetic field, then the energy in the toroidal magnetic field is minimized if the helicity

\[
H \sim (\pi r^2)^2 B_r B_\phi \sim (\pi B_\phi r^2)^2 \Delta \phi
\]  

(A1)
is concentrated close to the star (Thompson et al. 2002), where \( B_p \) is the poloidal magnetic field at radius \( r \) and \( B_\phi \sim B_p \Delta \phi \). The energy in the toroidal magnetic field is

\[ E_\phi \sim \frac{1}{12} B_\phi^2 r^3 \sim \frac{H^2}{12 \pi^2 r^3 B_p^2} \sim H^2 r. \]  
\( \text{(A2)} \)

If a magnetosphere carrying the helicity \( H \) is not in the minimum-energy state, the winding of the magnetic field can be rapidly transferred by reconnection between different flux surfaces (e.g., Taylor 1974).

The second observation is that the current flowing out along an open bundle of magnetic field lines is not precisely compensated for by the return current in the same magnetic hemisphere. There is, as a result, a residual current connecting the two magnetic poles, which flows along the outermost closed magnetic field lines. This closed-field current imparts a small amount of helicity to the magnetosphere. The net rate of transfer of helicity is then

\[ \frac{dH}{dt} \bigg|_+ \sim \varepsilon_H \Phi^2 \Omega, \]  
\( \text{(A3)} \)

where \( \Phi \) is the magnetic flux \( \Phi(r) = \pi B_p(r)r^2 \) evaluated at the boundary radius of the magnetosphere.

In the case of an isolated neutron star, an imbalance between the outgoing and return current can result from an offset, \( \Delta R_{\text{pol}} \), of the magnetic dipole from the center of the star. One then deduces

\[ \varepsilon_H \sim \frac{\Omega \Delta R_{\text{pol}}}{c} < \frac{\Omega R_{\text{NS}}}{c}. \]  
\( \text{(A4)} \)

Making use of \( \Phi_{\text{open}} = \pi B_{\text{pol}} R_{\text{NS}}^2/ c \), one can relate the helicity increase (eq. [A3]) to the magnetic dipole luminosity \( \dot{L}_{\text{MDR}} = (1/6)(\Omega R_{\text{NS}}/c)^3 B_{\text{pol}}^2 R_{\text{NS}}^2 c \),

\[ \frac{dH}{dt} \bigg|_+ \sim 6 \pi^2 \dot{L}_{\text{MDR}} \Delta R_{\text{pol}}. \]  
\( \text{(A5)} \)

where \( B_{\text{pol}} \) is the magnetic field at the magnetic pole of the star.

The effect is much stronger in the case of PSR J0737–3039B. The strong asymmetry in the external stress of the pulsar A wind enforces a large asymmetry between the outgoing and return currents in each magnetic hemisphere. We therefore estimate

\[ \varepsilon_H \sim \Delta \phi(R_{\text{mag}}) \sim \frac{\Omega B_{\text{mag}}}{c} \sim 0.1-0.2. \]  
\( \text{(A6)} \)

The equilibrium twist angle established at smaller radii depends on how rapidly the helicity is damped in the inner magnetosphere. We do not examine this problem here but simply note that the inward flux of helicity allows one to establish a minimal static twist angle \( \Delta \phi \) in the outer parts of the magnetosphere. Given that the twist is transferred between flux surfaces with a speed \( \sim c \), one has

\[ \frac{dH}{dt} \bigg|_+ \sim \Phi^2(r) \Delta \phi(r) \frac{c}{r}. \]  
\( \text{(A7)} \)

Combining this with equations (A3) and (A6) gives

\[ \Delta \phi(r) \sim \frac{\Omega B_{\text{mag}}}{c} \left( \frac{r}{R_{\text{mag}}} \right)^3. \]  
\( \text{(A8)} \)

The corresponding current is

\[ \frac{J}{|J|} \sim 3 \left( \cos \theta_{\text{B}} \right)^{-1} \left( \frac{r}{R_{\text{mag}}} \right)^2. \]  
\( \text{(A9)} \)

It will be noted that the presence of a zero-frequency component of the current influences the nonlinear couplings between torsional waves in the magnetosphere (Goldreich & Sridhar 1997; Thompson & Blaes 1998).

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