Ziegler-Nichols optimization for quadrotor attitude control

Otimização de Ziegler-Nichols para o controle de atitude de quadrirrotores

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ABSTRACT
Quadrotor and multirotor aircraft, which do not need a pilot to fly, are used for recreational or work purposes, with the quadrotor being the type most used in practical implementations. The quadrotor represents a promising field of research due to its ability to fly and manoeuvre, gaining recognition as a technological solution in many areas. An aircraft quadrotor is a complex control system that allows great flexibility of flight. The identification and control of the variables in multirotor aerial systems such as quadrotors is challenging, however, because the quantities involved are not always available, known and accurate. Among the various control methods that have been investigated, proportional-integral-derivative (PID) control offers good results and simplicity for application in quadrotors, but achieving stability and high performance is challenging, with the fundamental task being tuning the controller gains. Improving the control system performance is also challenging, especially when the load varies. Here, Ziegler-Nichols (ZN) theory was used to tune the controller gains for pitch and roll attitude command, followed by evaluation of improvements in the system response. The optimized application of ZN theory to a quadrotor (termed optimized ZNAQ) is proposed and obtains a significant improvement in the control system response performance, demonstrating that optimized ZNAQ is valid for tuning the controller PID gains and more efficient than the original ZN theory approach.

Key words: Ziegler-Nichols, quadrotor control, PID tuning, multirotor.

RESUMO
As aeronaves multirrotores como do tipo quadrirrotores, que não precisam de um piloto para voar, são usadas para fins recreativos ou de trabalho, sendo o quadrirroto o tipo mais usado em implementações práticas. O quadrirrotores representam um campo promissor de pesquisa devido às suas capacidades de vôo e manobras, ganhando reconhecimento como solução tecnológica em muitas áreas. Um quadrirrotores é um sistema de controle complexo, mas que permite grande flexibilidade de vôo. A identificação e controle das variáveis em sistemas aéreos multirrotores como quadrirrotores é desafiadora, pois as grandezas envolvidas nem sempre estão disponíveis, são conhecidas e precisas. Entre os vários métodos de controle investigados, o controle proporcional-integral-derivado (PID)
oferece bons resultados e simplicidade para aplicação em quadirrotores, mas alcançar estabilidade e alto desempenho é desafiador, com a tarefa fundamental de ajustar os ganhos do controlador de voo. Melhorar o desempenho do sistema de controle também é desafiador, especialmente quando a carga varia. Neste trabalho, a teoria de Ziegler-Nichols (ZN) foi usada para ajustar os ganhos do controlador para o comando de atitude de rolagem e arfagem, seguido pela avaliação de melhorias na resposta do sistema. A aplicação otimizada da teoria de ZN em um quadirrotor (denominada ZNAQ otimizado) é proposta e obtém uma melhoria significativa no desempenho da resposta do sistema de controle, demonstrando que ZNAQ otimizado é válido para ajustar os ganhos do controlador PID e mais eficiente que a abordagem da teoria ZN original.

**Palavras-chave:** Ziegler-Nichols, controle de quadirrotor, sintonia PID, multirrotor.

**1 INTRODUCTION**

Multirotor aircraft are used for recreational and work purposes. Among the possible structural variations, the quadrotor contains a set of four propellers arranged at the ends of its wings. This aircraft does not need a pilot to fly and is the most commonly used variant in practical pilotless implementations. The quadrotor represents a promising field of research due to its applications from agricultural to military use (Federal Aviation Administration, 2015; Kumar & Loianno, 2016). To implement a control system in remotely piloted aircraft (RPA) of the quadrotor type, it is necessary to know the variables and parameters involved in the system and its dynamics and interaction, which can be represented by mathematical modelling.

A quadrotor has great complexity depending on the number of variables involved (Özbek, Önkol, & Efe, 2016; Zhang, Li, Wang, & Lu, 2014) but in return allows great flexibility of flight. The identification and control of the variables is challenging since the quantities involved are not always available, known and accurate. Proportional-integral-derivative (PID) control is sufficient for most performance requirements of these systems; however, it is challenging as well to design low-level flight controllers for quadrotors while ensuring stability and performance (Ireland, Vargas, & Anderson, 2015). Effective fields of investigation include the multirotor robustness against disturbances, transient response analysis, measurement errors of magnitudes and unmodulated dynamics (Ireland et al., 2015). The effectiveness of multirotor control in different situations can be investigated through control system research and experimental tests (Ireland et al., 2015). The system control aspects have been frequently investigated (Özbek et al., 2016).

Studies of high-level control systems for quadrotors have resulted in systems with high costs and hardware complexity, and it is preferable to deploy low-level controllers that offer ease of implementation and reduced cost. Low-level multirotor flight controllers depend on PID control and are sufficient for many requirements (Quan, 2017). However, the question remains of how to design controllers for multirotor aircraft to achieve stability, flight accuracy and manoeuvrability.
In (He & Zhao, 2016), the use of genetic algorithms in conjunction with the theory of Ziegler-Nichols (ZN) is presented, but the parameters of the controller PID are not provided. In (He & Zhao, 2014), a control system for a quadrotor is determined by ZN theory but again without evidence of the parameters used. It is verified that the studies of (He & Zhao, 2016) and (He & Zhao, 2014) consider only unloaded multirotor aircraft. The study carried out by (Khodja, Tadjine, Boucherit, & Benzaoui, 2017) applies ZN theory to an empty quadrotor, using an adaptation of the original theory proposed in the study of (Mallesham, Mishra, & Jha, 2011). This adaptation of (Mallesham et al., 2011) determines the parameters for the frequency control of loads and a microgrid to adjust the power generated to satisfy the load demand. The parameters used by (Khodja et al., 2017) were proposed by (Mallesham et al., 2011) in a system with different effects for a multirotor system. The ZN parameters developed in (Mallesham et al., 2011) and used for (Khodja et al., 2017) are different from the parameters originally developed in ZN theory (Ziegler, Nichols, & Rochester, 1942).

Three-dimensional stability spaces for PID controller design are proposed in (Bahavarnia & Tavazoei, 2013), enabling new results to be achieved through ZN theory. The so-called advanced adjustment for PID controller design with ZN theory is addressed in (Rodríguez, Han, Keel, & Bhattacharyya, 2017), relating the gain margin and phase margin specifications to the performance of curves projected through ZN theory to simultaneously achieve a controller design with the desired margin specifications. The review of ZN theory for PID controllers performed by (Astr, 2004) explores which processes are amenable to PID control and what information is necessary for a satisfactory design process of this control system. The set point is also considered in the study of problems with load disturbances.

Various methods have been investigated for quadrotor control, but it is challenging to achieve stability and performance, especially when the aircraft's load varies. Tuning the controller gains of the aircraft is critical, allowing for flight capability and manoeuvrability, as well as avoiding errors or system failures that could result in unsafe operating conditions leading to accidents. In this study, a PID controller for roll and yaw commands is selected for its simplicity and practical application in electronic systems; its effectiveness in stabilizing a quadrotor aircraft is verified, and optimization method are proposed. ZN theory (Ziegler et al., 1942) is implemented to determine the PID controller parameters in a quadrotor, and the optimization of ZN theory is applied to a quadrotor (ZNAQ). This study contributes to the area of quadrotors and control systems in addition to ZN theory optimization.

2 QUADROTOR ATTITUDE CONTROL

To carry out this study of the tuning gains for quadrotor attitude pitch and roll control, the characterization of the PID control system is presented first, and ZN theory is then applied for tuning
the controller parameters. Originally, ZN theory was implemented directly in the system, but due to the risks involved in aircraft experiments, a practical flight with a prototype of a quadrotor was performed, collecting response data regarding the replication of an impulsive manoeuvre on the rolling axis (which is equivalent to the pitch axis due to symmetry). The data collected were mathematically processed and used as the basis for the implementation of ZN theory to configure the PID controller. Based on ZN theory, optimized ZNAQ is proposed.

2.1 PID CONTROLLER

PID control systems are constructed with three elements, namely, proportional gain, integral gain and derived gain, which must be implemented to obtain a response to the control system in a closed loop. The selection of controller gains to satisfy desirable performance rules can be defined through controller tuning (Ogata, 2011). In the typical system of PID control in a closed loop, the process variables are compared with a set point so that the difference can then be compensated by the PID controller to enable the plant of the system or mathematical model to reach the desired response. Further information on PID controllers can be found in (National Instruments, 2019; Ogata, 2011).

The process variable of a control system is the quantity to be controlled, as in the case of the quadrotor, where the speed of the propellers is controlled to obtain the manoeuvring and flight capacities. The process variable is measured by sensors, providing closed loop feedback to the control system. The reference of the control system is called the set point, representing the value to be followed in the control system. The difference between the process variable and reference is used by the controller to obtain the desired output, thus compensating the system. This typical system is called a closed loop control system (National Instruments, 2019). Mathematically, PID control can be represented by equation 1 of $G_c(s)$, where $K_p$ is the proportional gain, $T_i$ is the integral time and $T_d$ is the derivative time.

$$G_c(s) = K_p (1 + \frac{1}{T_i s} + T_d s)$$

The design of a control system has the general objective of minimizing the disturbances on the process variable through the adaptation of the system parameters. The useful load projection of an aircraft changes the characteristics of the plant, necessitating new adaptation of the controller parameters. Determining the appropriate parameters to control the specific dynamics of a quadrotor system is challenging.
2.2 ZIEGLER-NICHOLS THEORY (ZN)

ZN theory is a method used to determine the parameters of PID controllers (Ziegler et al., 1942) based on the stability analysis of the system (Khodja et al., 2017). The determination of the PID gains by the ZN theory is employed through practical experiments. In this work, the PID gains are determined through the stability analysis of mathematical models as identified through data collected from a practical prototype in a controlled and safe manner, ensuring a low risk of loss or accident.

ZN theory for PID controller tuning can be applied through two methodologies. The first is based on the open loop response of the system, while the second is based on the closed loop response. In this work, the closed loop response is used because data are collected in a prototype that operates with a closed loop control system. To configure the closed loop PID controller, ZN theory allows definition of the final critical system gain (Kc) and the oscillation period (Pc) of critical gain. Then, the proportional gain (Kp) must be increased until the system initiates a shock, suffering oscillations equidistant in amplitude and sensitivity when critical values are encountered (Ziegler et al., 1942).

By performing a practical experiment with the quadrotor, the system response is collected in an adverse flight situation in a transitional period, providing data for the mathematical identification of the system. With the identified system response, the ZN theory is applied, and the critical values that determine the PID controller parameters are found. The determination of the parameters according to the theory by (Ziegler et al., 1942) for proportional (P), proportional-integral (PI) and PID controllers is presented in equations 2 to 7.

\[
\begin{align*}
    K_p(P) &= 0.5 \times K_c \quad (2) \\
    K_p(PI) &= 0.45 \times K_c \quad (3) \\
    K_i(PI) &= 1.2 / P_c \quad (4) \\
    K_p(PID) &= 0.6 \times K_c \quad (5) \\
    K_i(PID) &= 2 / P_c \quad (6) \\
    K_d(PID) &= P_c / 8 \quad (7)
\end{align*}
\]

2.3 CHARACTERIZATION OF THE QUADROTOR PROTOTYPE AND PARAMETER IDENTIFICATION

The prototype was built on a 500 mm frame, with four assemblies composed of 10×4.5” propellers and a 810 kV motor mounted at the ends of the arms in the X configuration, which is a configuration widely used in quadrotor prototypes. The prototype has a weight of 1800 g and moments of inertia \( J_{xx} = 0.0224 \text{ kg·m}^2 \), \( J_{yy} = 0.0224 \text{ kg·m}^2 \), and \( J_{zz} = 0.0414 \text{ kg·m}^2 \). When the load is
added, the assumed prototype moments of inertia are $J_{xx}=0.0254 \text{ kg}\cdot\text{m}^2$, $J_{yy}=0.0254 \text{ kg}\cdot\text{m}^2$, and $J_{zz}=0.0427 \text{ kg}\cdot\text{m}^2$.

The data collection for the determination of the parameters of the control system of the quadrotor are analysed with the system empty. Originally, ZN theory was implemented directly in the practical system, but because of the risk of crashing involved in aircraft experiments, a prototype flight was performed empty and at full load, collecting response data for an impulsive manoeuvre on the rolling axes in a controlled and safe experiment, allowing for ZN mathematical treatment. Due to system symmetry, the rolling axis data can also represent the pitch axis.

System identification was performed in MATLAB through the system identification toolbox, thus providing the empty model, represented by equation 8.

$$\frac{1106}{s^3+21.1s^2+490.9s+1729}$$ (8)

2.4 ZN THEORY APPLIED TO THE QUADROTOR (ZNAQ)

In conjunction with the identified plant of the empty quadrotor, the ZN methodology was applied in the mathematical model (equation 8) through simulation in MATLAB, resulting in a critical gain of $K_c=7.7865$ and a critical period of $P_c=0.286$ s. If the critical state of the system is found, it is possible to determine the gain parameters for a PID controller according to the theory originally proposed by Ziegler and Nichols (Ziegler et al., 1942) by implementing equations 2 to 7, with the results shown in Table 1.

Given the parameters for the controller, it is possible to simulate the response of the PID control in Simulink. The response to the step for the empty quadrotor model (equation 8) is analysed, with the controller parameters obtained from ZN theory shown in Table 1 and the results shown in Figure 1.

Figure 1: Response of the quadrotor with gains determined by ZN theory.
An analysis of Figure 1 indicates that in situations without a controller and with a proportional controller, the tracking of the unit reference is not achieved; the PI and PID controllers offer better performance. From these results for the unloaded plant, a refinement was made to the controller performance to obtain an optimized balance between reference tracking and disturbance rejection in MATLAB, obtaining the gains recorded in Table 1 and shown in Figure 2. The performance indicators are presented in Table 2.

### Table 1: Gains obtained through ZN theory versus the optimized parameters (no load).

| Controller | ZN parameters | Optimized parameters |
|------------|---------------|----------------------|
|            | Kp      | Ki     | Kd     | Kp     | Ki     | Kd     |
| P          | 3.89    | Zero   | Zero   | 4.31   | Zero   | Zero   |
| PI         | 3.50    | 4.2    | Zero   | 2.64   | 14.97  | Zero   |
| PID        | 4.67    | 6.99   | 0.03   | 2.90   | 17.36  | 0.09   |

Figure 2: Response of the quadrotor with optimized gains.

![Response of the quadrotor with optimized gains.](image)

### Table 2: Performance indicators for the quadrotor.

| Controller | ZN parameters | Optimized parameters |
|------------|---------------|----------------------|
|            | Rise time (ms) | Overshoot (%) | Rise time (ms) | Overshoot (%) |
| Without    | 496.75         | 0.50               | 496.75         | 0.50          |
| P          | 86.93          | 30.92              | 82.14          | 36.30         |
| PI         | 129.00         | -1.27              | 120.96         | 15.70         |
| PID        | 97.62          | 11.80              | 119.27         | 8.15          |
3 RESULTS

Considering as a reference the parameters of the present study, with the controller optimized for the quadrotor without a load, it is observed that the parameters obtained by ZN theory are proposed for the general application of dynamic systems but may produce better results through optimization for the specific quadrotor application. It is also verified that the best results were obtained with the PI and PID controllers, with the latter providing superior performance.

The PID controller obtained the best results in the previous simulations. The simulation of the quadrotor plant without a load was performed with controller parameters obtained through ZN theory and with parameters obtained through ZNAQ, with the results shown in Figure 5.

Figure 5: Response of the PID controller obtained from ZN theory versus the response of the PID controller obtained from ZNAQ.

Applying the optimized parameters presented in Table 1, which were obtained considering the specific characteristics collected in the identification of the empty quadrotor, in the structure originally treated with ZN theory, we propose the PID gain determination based on ZN theory (Ziegler et al., 1942) and the specific characteristics of the quadrotor, proposing the optimization of ZNAQ to determine the PID controller parameters, as shown in Table 5.

Table 5: Proposal for optimization of ZNAQ for tuning the PID controllers.

| Controller | Kp       | Ki      | Kd      |
|------------|----------|---------|---------|
| P (ZNAQ)   | 0.55×Kc  | Zero    | Zero    |
| PI (ZNAQ)  | 0.34×Kc  | 4.28/Pc | Zero    |
| PID (ZNAQ) | 0.37×Kc  | 4.96/Pc | Pc/3.18 |
In the case of the controllers determined for the systems without load, it was verified that the PID controller obtained through ZNAQ performs better than the controller obtained through ZN theory, demonstrating that the ZNAQ framework is valid and efficient, with an response improvement of up to 30%.

4 CONCLUSION

Ziegler-Nichols theory was implemented for quadrotor attitude pitch and roll control to determine PID controller gains, and the system response was verified. Optimized ZNAQ was proposed and found to obtain a significant improvement in the system response, demonstrating that the proposal is valid and efficient and contributing to the knowledge in this field.

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