The Relative $J/\Psi$ to $\Psi'$ Suppression in Proton-Nucleus and Nucleus-Nucleus Collisions

Jörg Hufner

Instut für Theoretische Physik der Universität ,
Philosophenweg 19, 69120 Heidelberg, Germany
E-mail: huefner@zooey.mpi-hd.mpg.de

Boris Kopeliovich

Max-Planck Institut für Kernphysik
Postfach 103980, 69029 Heidelberg, Germany
and
Joint Institute for Nuclear Research, Laboratory of Nuclear Problems,
Dubna, 141980 Moscow Region, Russia
E-mail: bzk@dxnhd1.mpi-hd.mpg.de

Abstract

We calculate the nuclear suppression for $J/\Psi$ and $\Psi'$ production within a coupled channel approach in the subspace of the $J/\Psi$ and $\Psi'$ states. We are able to explain, why (i) the $J/\Psi$ and $\Psi'$ show the same suppression from 200 GeV to 800 GeV in proton-nucleus collisions and why (ii) the $\Psi'$ is absorbed more strongly than the $J/\Psi$ in nucleus-nucleus collisions at 200 GeV. The numerical result which includes only interactions with nucleons accounts for half of the observed suppression in sulphur-uranium collisions.
The E772 collaboration [1] was the first who claimed that $\Psi$ (we use hereafter this abbreviation for $J/\Psi$) and $\Psi'$ produced in proton-nucleus ($p-A$) collisions at 800 GeV and $x_F \geq 0.1$ experience the same nuclear suppression. This result was confirmed by the NA38 collaboration [2, 3, 4] at an energy of 200 GeV and for $x_F \approx 0.1$, though with larger error bars. The two observations contradict the simple-minded expectation, namely that the $\Psi'$ should be more strongly suppressed, because absorption cross section scales with the mean square radius $\langle r^2 \rangle$ of the meson [5, 6] and the $\Psi'$ is much larger than the $\Psi$. To our opinion, no satisfactory explanation of the data has yet been proposed.

Recently the situation became even more mysterious by the observation [2, 4] that in Sulfur-Uranium (S-U) collisions at 200 GeV/A the $\Psi'$ is significantly more strongly absorbed than the $\Psi$. Does this result signal dense hadronic gas or quark-gluon plasma formation?

In this letter we treat the propagation of a $c\bar{c}$ pair through nuclear matter as a coupled system of the $\Psi$ and $\Psi'$ states. In addition to elastic collisions $\Psi N \bar{\Psi} N$ and $\Psi' N \bar{\Psi}' N$ with amplitudes $f(\Psi, \Psi)$ and $f(\Psi', \Psi')$ respectively, we consider the conversion amplitudes $f(\Psi, \Psi')$ and $f(\Psi', \Psi)$ for the processes $\Psi N \leftrightarrow \Psi' N$ during propagation through nuclear matter. The inelastic amplitudes turn out to be nearly as big as the elastic ones.

We define as nuclear suppression factor of a $\Psi$ meson in $pA$ collisions the ratio

$$S_{pA}^{\Psi}(x_F, E) = \frac{1}{A} \frac{d\sigma(pA \bar{\Psi} X; x_F, E)/dx_F}{d\sigma(pN \bar{\Psi} X; x_F, E)/dx_F},$$

where $E$ is the lab. energy. We also introduce the relative $\Psi'$ to $\Psi$ nuclear suppression function by

$$S_{\Psi'/\Psi}^{pA} = \frac{S_{pA}^{\Psi'}}{S_{pA}^{\Psi}},$$

Then the results of the E772 and NA38 experiments can be summarized by $S_{\Psi'/\Psi}^{pA} \approx 1$ for the values of $x_F$ and $E$ considered.

In order to exhibit the physics of our coupled channel approach, we first calculate eq. (2) perturbatively, i.e. restricting ourselves to only one $\Psi N$ ($\Psi'N$) interaction after the creation of the $c\bar{c}$ pair. This interaction can be an elastic one as well as a conversion event. Then
\[
S_{\Psi'/\Psi}^{pA} = \frac{1 - \frac{1}{2}\sigma_{\Psi N}^{\Psi}(r + \epsilon/R)\langle T \rangle_A}{1 - \frac{1}{2}\sigma_{\Psi N}^{\Psi}(1 + \epsilon R)\langle T \rangle_A},
\]

where \( r = f(\Psi'\Psi')/f(\Psi\Psi) \), \( \epsilon = f(\Psi\Psi')f(\Psi\Psi) \) and \( \sigma_{\Psi N}^{\Psi} = 2Imf(\Psi, \Psi) \). Furthermore, \( R \) denotes the relative amplitude of \( \Psi' \) to \( \Psi \) production in the initial \( pN \) collision, and \( \langle T \rangle_A = 1/A \int d^2b T^2(b) \), where \( T(b) = \int_{-\infty}^{\infty} dz \rho_A(b, z) \) is the nuclear thickness.

If one neglects the conversion rate \( (\epsilon = 0) \), eq. (3) reduces to the conventional result, namely that the \( \Psi \) suppression depends only on \( \sigma_{\Psi N}^{\Psi} \), while \( r\sigma_{\Psi N}^{\Psi} = \sigma_{\Psi' N}^{\Psi'} \) determines the suppression of the \( \Psi' \).

The ratios \( r \) and \( \epsilon \) in eq. (3) can be fairly reliably calculated since at high energies the scattering amplitudes \( f(\alpha, \beta) \), where \( \alpha \) and \( \beta \) stand for \( \Psi \) and \( \Psi' \), are proportional to \( \langle \alpha | r^2_T | \beta \rangle \), as long as the meson radius in the transverse direction, \( \langle r^2_T \rangle \), is small. With \( 1S \) and \( 2S \) harmonic oscillation functions for \( \Psi \) and \( \Psi' \), respectively, one has \( r = 7/3 \) and \( \epsilon = -\sqrt{2/3} \). The ratio \( R \) of initial \( \Psi \) to \( \Psi' \) production cannot be calculated in this way. We turn the argument around and calculate from eq. (3) that value of \( R_{cal} \) which leads to the observed relative suppression \( S_{\Psi'/\Psi}^{pA} = 1 \) and compare it to \( R_{exp} \) deduced from \( pN \) collisions. A quadratic equation for \( R \) leads to \( R_{cal} = \sqrt{5/3} \pm \sqrt{2/3} \), the smaller one being \( R_{cal} = 0.47 \).

If one uses the experimental intensities of \( \Psi' \) and \( \Psi \) produced in \( pp \) collisions and corrects them for the feedings \( \Psi' \Sigma \Psi \) and \( \chi \Sigma \Psi \), one arrives at an amplitude ratio \( |R_{exp}| = 0.48 \pm 0.06 \) \( [7, 4] \), which agrees well with the calculated one, indicating that the initially produced state \( |\Phi_{\psi}^{\psi} \rangle = (|\Psi \rangle + R|\Psi' \rangle)/\sqrt{1 + R^2} \) is such as to lead to the same final state attenuation for \( \Psi \) and \( \Psi' \). Mathematically spoken, \( |\Phi_{\psi}^{\psi} \rangle \) is an eigenstate of the final state interaction matrix,

\[
\hat{f} = \begin{pmatrix}
1 & \epsilon \\
\epsilon & r
\end{pmatrix} f(\Psi, \Psi).
\]

The property of \( |\Phi_{\psi}^{\psi} \rangle \) being eigenstate of \( \hat{f} \) is equivalent to the statement that \( |\Phi_{\psi}^{\psi} \rangle \) has an extreme value for its transverse size:

\[
\frac{d}{dR} \langle \Phi_{\psi}^{\psi} | r^2_T | \Phi_{\psi}^{\psi} \rangle = 0.
\]

The experimental value \( |R_{exp}| \) selects the physical state as that with minimal transverse
Strictly speaking the result eq. (3) is only valid for energies $E_{\text{lab}} \infty$. For finite lab. energies, especially for $pA$ and $AA$ collisions at 200 GeV one has to include the effect of the longitudinal momentum transfer $q$ associated with the conversion reaction $\Psi \leftrightarrow \Psi'$:

$$q = \frac{M_{\Psi}^2 - M_{\Psi'}^2}{2E_F x_1},$$

where $x_1 = (x_F + \sqrt{x_F^2 + 4M_{\Psi}^2 / s}) / 2$ with $s$ the c.m. energy. One arrives at an expression like eq. (3) where $\epsilon$ is replaced by $\epsilon_F A(q)$ with

$$F_A(q) = \frac{2}{A(T)} \int_{-\infty}^{\infty} dz \rho_A(b, z) \int_{z}^{\infty} dz' \rho_A(b, z') \exp(iqz')$$

being a kind of nuclear formfactor. For the E772 experiment $q \leq 0.06 \text{ fm}^{-1}$ for $x_F \geq 0.2$ and $F_A = 1$ is a very good approximation. Thus, the observed result $S_{\Psi'/\Psi} = 1$ is reproduced.

For the NA38 experiment at 200 GeV/A especially for the nucleus–nucleus collisions, the effect of the formfactor becomes crucial: In the $SU$ collision the produced $\Psi$ (or $\Psi'$) moves with fractional momentum $x_F$ (in the c.m. system) with respect to the target nucleus $U$, but moves with $-x_F$ relative to the projectile nucleus $S$ (inverse kinematics). Therefore the $\Psi'/\Psi$ suppression arises from two sources,

$$S_{\Psi'/\Psi}^{SU} = S_{\Psi'/\Psi}^{pS}(-x_F, E)S_{\Psi'/\Psi}^{pU}(x_F, E).$$

For 200 GeV and $x_F = 0.2$ we have $q = 0.16 \text{ fm}^{-1}$ to be used in the second factor of eq. (8) and $q = 0.56 \text{ fm}^{-1}$ in the first one. This is the inverse kinematics which leads to a much stronger $\Psi'/\Psi$ suppression.

For the detailed comparison with experiment we have to go beyond the perturbative expression (3) and use numerical methods. We describe the propagation of $c\bar{c}$ pair through nuclear matter by a differential equation

$$i \frac{d}{dz} |\Phi^{c\bar{c}}(z, \vec{b})\rangle = \hat{U}(z, \vec{b}) |\Phi^{c\bar{c}}(z, \vec{b})\rangle,$$

where the limitation to the $\Psi, \Psi'$ subspace invites the use of matrix notations: $|\Phi^{c\bar{c}}\rangle = \binom{\alpha}{\beta}$ and
\[
\hat{U} = \begin{pmatrix}
0 & 0 \\
0 & q
\end{pmatrix} - \frac{i}{2} \frac{\sigma_{\text{tot}} \rho_A(b, z)}{\Psi_{\text{Ntot}}} \begin{pmatrix}
1 & \epsilon \\
\epsilon & r
\end{pmatrix}
\]

(10)

with initial condition \( |\Phi_{cc}^e\rangle = \left( \frac{1}{R} \right) / \sqrt{1 + R^2} \) at the point \((\vec{b}, z_0)\) of \(c\bar{c}\) creation. At \(z = +\infty\) the wave function \( |\Phi_{cc}^e\rangle \) is projected on the \(\Psi\) and \(\Psi'\) states. The result is squared and averaged over the coordinates \((\vec{b}, z_0)\) of the production point. Note that this is only true if the longitudinal momentum transfer at the production point, \(q_0 = M_{\Psi}^2/2Ex_1 \gg q\), is much larger than the reversed mean internucleon distance in a nucleus. Otherwise one should take into account coherence between different production points as well (see discussion in terms of production and formation times in [10, 11]). This would be equivalent to an effective increase of the length of path of the \(q\bar{q}\) pair in nuclear matter. However it does not affect the relative \(\Psi'/\Psi\) production rate if \( |\Phi_{cc}^e\rangle \) is an eigenstate of interaction.

The relative \(\Psi'/\Psi\) suppressions are calculated for \(p-A\) collisions and with the help of eq. (8) also for \(A-A\) collisions. The numerical results shown in Figs. 1 and 2 are calculated with realistic nuclear densities and with the values of \(\epsilon\) and \(r\) as given by the harmonic oscillator model and for the absolute value of the \(\Psi N\) total cross section \(\sigma_{\text{tot}}^{\Psi N} \approx C \langle r_T^2 \rangle_{\Psi} = 5.7 \text{ mb}\), where we use the perturbative QCD estimate of [11] or systematics of [8] for a value of \(C\).

While at 800 GeV the values for \(S_{\Psi'/\Psi}^{pA}\) are measured directly, the values given at 200 GeV for \(B_{\Psi'}\sigma_{\Psi'}/B_{\Psi}\sigma_{\Psi}\) in nuclear collisions were renormalized by us using the value \(1.80 \pm 0.10\%\) for this quantity from \(pp\) and \(pd\) collisions [4]. Fig. 1 shows the suppression functions \(S_{\Psi'/\Psi}\) for \(p-W\) at 800 GeV and for \(p-U\) and \(p-W\) at 200 GeV. Although we predict for 200 GeV some reduction of the \(\Psi'/\Psi\) relative suppression it is still in agreement with the data of the NA38 experiment within rather large error bars. Fig. 2 shows the predicted and observed [2, 4] suppression \(S_{\Psi'/\Psi}^{SU}\) for the nucleus-nucleus case. The solid curve is the product of the suppression curves for the \(p-U\) and the \(S-p\) collisions (each dashed). Experiment and calculation agree in that the \(\Psi'\) should be significantly more strongly suppressed than the \(\Psi\). However, the measured suppression seems to be stronger than our expectation, indicating that there may be room for other effects. The NA38 group has published [2, 4] also four points for the relative \(\Psi'\) to \(\Psi\) suppression in \(S-U\) collisions as a function of the transverse energy \(E_T\). The suppression is larger for larger values of \(E_T\), corresponding to
more central collisions. On the basis of our model we expect such a behaviour, but we lack precise quantitative information for the association of a value of $E_T$ to a definite geometric configuration.

In Fig. 3 we show the $\Psi'/\Psi$ suppression as a function of $x_F$ for $Au - Au$ collisions calculated for RHIC and LHC energies. The result is predicted to be energy independent at high energies and the two curves coincide.

The model of coupled channels presented in this letter ”naturally” explains, why at high energies of the charmonia $\Psi$ and $\Psi'$ should be similarly suppressed as observed in proton-nucleus collisions at 800 GeV, and why in nucleus-nucleus collisions one should see significant differences in $\Psi'$ and $\Psi$ suppressions as was reported at 200 GeV/A.

While the model is free of adjustable parameters, the precision of the two-channel approximation is questionable. In order to evaluate the corrections to $S_{\Psi'/\Psi}$ from inclusion of higher charmonium excitations we switch from the hadronic basis to the quark one in coordinate representation. In this case the nuclear suppression of $\Psi$ production is calculated as

$$S_{\Psi}^{pA} = \frac{|\langle \Phi_{c\bar{c}}^{\Psi}\rangle \hat{V}(b,z,r_T)\Phi_{c\bar{c}}^{\Psi} \rangle^2}{|\langle \Phi_{c\bar{c}}^{\Psi} \rangle|^2} \quad (11)$$

and the same for $\Psi'$. Here the evolution operator $\hat{V}$ at high energy (when the fluctuations in $r_T$ are frozen by Lorentz time dilation) reads, $\hat{V}(b,z,r_T) = \exp[-Cr_T^2/2 \int_z^\infty dz' \rho_A(b,z')]$. The averaging $\langle \ldots \rangle_A$ in eq. (11) denotes the integration over the coordinates of the production point weighted with nuclear density.

Eq. (11) (valid only for $q = 0$) generalizes the two-channel approach in that $|\Phi_{c\bar{c}}^{\Psi}\rangle$ may have other components in addition to the $\Psi$ and $\Psi'$ states (see an alternative interpretation of enhancement of the $\Psi'$ production rate on nuclei in [11, 12]). We have evaluated the suppression eq. (11) for $\Psi$ and $\Psi'$ and the relative suppression $S_{\Psi'/\Psi}^{pA}$ for various trial functions for $\langle \vec{r} | \Phi_{c\bar{c}}^{\Psi} \rangle$ like $\exp(-\alpha r^2)$, $r_T^2 \exp(-\beta r^2)$, $r_T^2 K_0(\lambda r_T)$ ($K_0$ is a modified Bessel function), where the constants $\alpha$, $\beta$, $\lambda$ have been chosen to give the same amplitude ratio $R$ for the $\Psi'$ content of $|\Phi_{c\bar{c}}^{\Psi}\rangle$ relative to the $\Psi$ one. The resulting relative $\Psi'/\Psi$ nuclear suppression exceeds the prediction of the two-channel model $S_{\Psi'/\Psi}^{pA} = 1$ only by $5 - 10\%$.
for heavy nuclei, a deviation which is still compatible with the data. Another correction of the same order of magnitude is expected arising from the $\chi$-component of the initial $c\bar{c}$ state. A more complete analysis of these effects as well as recalculation of the $\Psi'/\Psi$ nuclear suppression using path-integral methods \[11\] will be presented in a forthcoming paper.

Note that the initially produced $c\bar{c}$ wave packet, which is a combination of $\Psi$ and $\Psi'$, attenuates in the nucleus less than each of two charmonia. This fact is important for the nuclear suppression of $\Psi$, taken separately. This should be checked with available experimental data.

In the case of nucleus-nucleus collisions, we have assumed that charmonia attenuate only due to interaction with the projectile nucleons, having $x_F = \pm 1$. However, particle production (and possibly a dense hadronic gas or a quark-qluon plasma) should cause an additional suppression of $S_{\Psi'/\Psi}^{A-A}$ down from our prediction and may explain the deviation of our calculations from the results of the NA38 experiment depicted in Fig. 2. Recently, Satz \[13\] has proposed to use the value of $S_{\Psi'/\Psi}$ as a probe for dense matter formation.

Acknowledgement: We are grateful to P. Giubellino and H. Satz, who stimulated our interest to the problem under discussion. We thank Dr. C. Gerschel for several discussions on the experimental situation. B.K. thanks E. Predazzi for useful discussion and MPI für Kernphysik, Heidelberg, for financial support. The work was partially supported by a grant from the BMFT, grant number 06HD742(0).

References

[1] D.M. Adle et al., Phys. Rev. Lett. 66 (1991) 133

[2] The NA38 Collaboration, B. Ronceux et al., Nucl. Phys. A566 (1994) 371c

[3] The NA38 Collaboration, C. Lourenco et al., Nucl. Phys. A566 (1994) 77c

[4] The NA38 Collaboration, C. Baglin et al., Phys. Lett. B345 (1995) 617 and M.C. Abreu et al., presented at QUARK MATTER’95

[5] J.F. Gunion and D. Soper, Phys. Rev. D15 (1977) 2617
[6] J. Hüfner and B. Povh, Phys. Lett. B245 (1990) 653

[7] The E705 Collaboration, L. Antoniazzi et al. Phys. Rev. Lett. 70 (1993) 383

[8] B.Z. Kopeliovich and L.I. Lapidus, Sov. Phys. JETP Lett. 32 (1980) 592

[9] B.K. Jennings and B.Z. Kopeliovich, Phys. Rev. Lett. 70 (1993) 3384

[10] S.J. Brodsky and A. Mueller, Phys. Lett. B206 (1988) 685

[11] B.Z. Kopeliovich and B.G. Zakharov, Phys. Rev. D44 (1991) 3466

[12] O. Benhar, B.Z. Kopeliovich, Ch. Mariotti, N.N. Nikolaev and B.G. Zakharov, Phys. Rev. Lett. 69 (1992) 1156

[13] H. Satz, In ”Aachen 1992, Proceedings, QCD: 20 Years Later”, v. 2, p. 748

**Figure capture**

**Fig. 1** The relative $\Psi'/\Psi$ nuclear suppression in $p - W$ collisions. The full circles and the solid curve are the data of the E772 experiment at 800 GeV and our calculation, respectively. The open circle and the dashed curve are the result of the NA38 experiment at 200 GeV and our prediction, respectively.

**Fig. 2** The relative $\Psi'/\Psi$ nuclear suppression in $S - U$ collisions at 200/GeV. The dashed curves are the relative $\Psi'/\Psi$ suppression in $p - U$ and $S - p$ (inverse kinematics) collisions at 200 GeV. The solid curve, which is the two dashed lines describes our prediction for $S - U$ collisions. The data-point is the result of NA38 experiment.

**Fig. 3** Prediction for the relative $\Psi'/\Psi$ suppression in $Au - Au$ collisions for the expected energies of the RHIC and LHC accelerators.
