Finite-element modelling of superconductors in over-critical regime with temperature dependent resistivity

J Duron¹, F Grilli², L Antognazza³, M Decroux³, S Stavrev¹, B Dutoit¹, and Ø Fischer³
¹ Swiss Federal Institute of Technology - Lausanne, Station 14, 1015 Lausanne, CH.
² Los Alamos National Laboratory - MST/STC, Los Alamos, NM 87545, USA
³ University of Geneva - DPMC, 24 quai E. Ansermet, 1211 Geneva, CH.
E-mail: joseph.duron@epfl.ch

Abstract.
In this paper, we present a new numerical model, in which both the thermal and the electromagnetic aspects of the over-critical current regime of HTS materials are taken into account. The electromagnetic and thermal equations have been implemented in finite-element-method (FEM) software in order to obtain a novel, closer to reality model for investigating the behaviour of the superconductor when the current exceeds \( I_c \). This model has been applied for studying the behaviour of strip lines of an YBCO/Au FCL with a sapphire substrate. Simulations with currents largely exceeding \( I_c \) have been performed, showing that the total current limitation occurs only when the temperature dependence of the electrical parameters is taken into consideration. Such modelling can replace experiments with currents far exceeding \( I_c \), which may damage or destroy the studied sample or HTS device.

1. Introduction
The temperature dependence of the physical properties of HTS materials, such as resistivity and critical current density, is an important factor when the current is higher than the critical current of the device, or in cases where the current density exceeds locally the critical current density of the material. This is for example the case in superconducting fault current limiters (FCLs) where the peak current during a short circuit can easily reach 3-4 times the critical current. In this case, the temperature of the HTS increases above the critical one and the material goes from superconducting to normal state. Even when the critical temperature is not reached, the power losses in the HTS augment as a result of the increase of its resistivity with the temperature.

The aim of this article is to provide a simulation method for studying both the electromagnetic and the thermal behaviour of superconducting materials, particularly in over-critical-current regime.

2. Interaction between electromagnetic and thermal model
For studying the thermal effects inside the device, new FEM simulations have been made with the general electromagnetic software package Flux 3D [1]. As the software does not allow
Table 1. Value of the material properties at $T = 77$ K.

| Material | Thickness | $J_c$ | $\rho$ | $\mu_r$ | $C_{Th}$ | $\sigma_{Th}$ |
|----------|-----------|-------|--------|---------|----------|-----------|
| YBCO     | Real values | $0.3$ | $3 \cdot 10^6$ | $-$ | $1$ | $-$ | $-$ |
|          | Scaled values | $300$ | $3 \cdot 10^3$ | $-$ | $1000$ | $-$ | $-$ |
| Gold     | Real values | $0.120$ | $7.5 \cdot 10^{-4}$ | $1$ | $-$ | $-$ | $-$ |
|          | Scaled values | $120$ | $7.5 \cdot 10^{-1}$ | $1000$ | $-$ | $-$ | $-$ |
| Sapphire | Real values | $500$ | $-$ | $-$ | $2.44 \cdot 10^5$ | $1131$ |
|          | Scaled values | $500$ | $-$ | $-$ | $2.44 \cdot 10^5$ | $1131$ |

Simultaneous coupling between electromagnetic and thermal equations within the same model, we have separated the computation into electromagnetic and thermal parts, using at each time step the loss results of the former as input for the latter [2].

In this paper, the electromagnetic modelling of FCL previously presented in [3] has been used. In particular, the YBCO and gold resistivity and the YBCO critical current density have been scaled as explained in [3] in order to avoid a too large geometric aspect ratio for the meshing process. In this work, several improvements have been included in order to reduce the complexity of the problem. Firstly, since the modified YBCO critical current density and gold resistivity lead to a change on a dimensional effect similar to the skin effect in massive conductors (as explained in more details in [3]), the $\mu_r$ value has been scaled in order to have the same skin depth. Secondly, in addition to the periodicity along, the symmetry with respect to the Y axis is taken into account. Thirdly, only one quarter of the total geometry of the FCL cell has been simulated. Figure 1 shows the final implemented geometry. Numbers represent the volumes where the power losses are calculated. The thin gold and YBCO layers are deposited onto a thicker sapphire substrate.

We have included the temperature dependence of the electrical parameters, which is a new feature in the FEM software Flux 3D for modelling FCLs. The set of equations (1) describes the thermal dependence of the critical current density and resistivities of the superconducting material in normal state and gold with the reference temperature $T_{ref} = 77$ K [4].

Figure 1. Implemented geometry. Numbers represent volumes where the losses are computed. YBCO/Au is not drawn to scale compared with the sapphire thickness (420 nm vs 0.5 mm).
The resistivity of the superconducting material, represented in figure 2, is modelled by the following equation, as explained in [4].

\[
\begin{align*}
\rho(T) &= \rho(T_{ref}) \left(1 + \beta(T - T_{ref})\right) \\
J_c(T) &= J_c(T_{ref}) \left(\frac{T^\alpha - T_{ref}^\alpha}{T_c^\alpha - T_{ref}^\alpha}\right)
\end{align*}
\] (1)

The resistivity of the superconducting material, represented in figure 2, is modelled by the following equation, as explained in [4].

\[
\rho = \frac{(\rho_{SC} + \rho_0) \cdot \rho_{sat}}{(\rho_{SC} + \rho_0 + \rho_{sat})}
\] (2)

\(\rho_{SC}\) is the resistivity of the superconducting material when \(T < T_c\). Its expression can be derived from the power law \(|E| = E_0(|J|/J_c - 1)^n\) (see [7]). \(\rho_{sat}(T_{ref}) = 9 \cdot 10^{-5} \Omega\text{cm}\) is the resistivity of the superconducting material in normal state at 77 K and \(\rho_0 = 10^{-15} \Omega\text{cm}\) is the residual resistivity, calculated from the criteria presented in [3]. All these values are scaled by 1000 inside the FEM model, due to the multiplication of the thicknesses (see table 1).

For calculating the power losses, we have divided the geometry in five different volumes (see figure 1), where the integral of \(E \cdot J\) is computed at the end of each time step in the electromagnetic model. The results of these computations are injected into the second series of simulations, where a thermal time step is solved.

In the thermal modelling, since the sapphire wafer is much thicker than the YBCO/Au thin film (0.5 mm compared to 420 nm), the thicknesses of the YBCO/Au thin film is supposed to be negligible and the power losses are assumed to be localized on the sapphire surface only. This means that in the thermal model only the sapphire material is taken into account. As a first approximation of the problem, all other faces which are not concerned by symmetries or periodicities are supposed to be at the nitrogen bath temperature of \(T = 77\) K. The average temperature of the heating surfaces is therefore computed and used as input for the next electromagnetic time step, where the resistivity and the critical current density have new values according to the temperature dependence of equation (1).

3. Results and discussion
The electrical circuit used to feed the FCL is composed by a voltage source \(V = V_{max} \sin(2\pi ft)\) with \(f = 50\) Hz and a resistance \(R_{load}\), in series with the superconducting device. The value of \(R_{load}\) is not constant and passes from 2.2 Ω to 70 mΩ at \(t = 10\) ms in order to simulate a short
circuit. The values of the total current $I$ and of the average temperature in region number 5 (see figure 1) are given in figure 3. When the short circuit occurs, the current increases rapidly and exceeds 60 A ($4 \times I_c$). Below $2 \times I_c$, the electric field $|E| = E_0(|J|/J_c - 1)^n$ and the temperature do not increase significantly due to the specific power-law used. After 1 ms, the temperature also begins to augment rapidly because of a fast increase of the power losses caused by the increase of $E$. The current limitation starts approximately at 12 ms (2 ms after the onset of the short circuit), when the critical current density of the YBCO material begins to decrease due to the thermal effect expressed by equation (1). As shown in figure 4, the current in the FCL is limited to approximately 50 A (more than $3 \times I_c$).

Figure 4 shows the repartition of the current in the YBCO and gold layers in the centre of the constriction, with temperature dependent parameters and fixed at 77 K. One can clearly see the current limitation inside the superconducting material when the temperature variation is taken into account. Even if the current passing through the gold layer increases in this case, the total current, sum of the YBCO and gold current, is limited by the FCL, as expected.

Finally, figure 5 shows an example of the current density in the gold and YBCO layers, and the temperature map in the sapphire substrate for $t = 12.9$ ms. In particular, the current distribution between YBCO and gold material along the constriction is clearly visible.

4. Conclusions and further work
We have used a new modelling technique in the FEM software Flux 3D for novel simulations of YBCO/Au thin film fault current limiters in order to simulate both the electromagnetic and thermal behaviour of the materials composing the device. We have demonstrated the importance of the temperature dependence of the electrical properties (resistivity and critical current density) in situations where the total current is higher than the critical current of the FCL (e.g. in fault current events).

This 3D FEM model can reproduce both the local and global behaviour of FCLs, and allows studying the effects occurring during a fault. The results may be used to replace experiments, where repeated tests lead to damages or destruction of the superconducting device.

However, the approximations made in the thermal part of the model do not allow computing precise local values, even if the average estimation of the temperature is good. The next stage of this 3D modelling technique is to improve the thermal part by modelling the thermal exchange with the nitrogen bath, and by increasing the number of volumes where power losses and average
temperature are computed.

Acknowledgments
This work has been supported by the Swiss National Science Foundation through the National Center of Competence in Research ”Materials with Novel Electronic Properties - MaNEP” and by the US Department of Energy Office of Electricity Delivery and Energy Reliability.

References
[1] CEDRAT S.A., Meylan, France, http://www.cedrat.com
   Flux 2D-3D software is distributed in Switzerland by Applied Magnetics, Morges, Switzerland
[2] Rettelbach T, Schmitz G J, 2003 Supercond. Sci. Technol. 16 645
[3] Duron J, et al 2005 IEEE Trans. Appl. Supercond. 15 1998
[4] Duron J, Grilli F, Dutoit B, Stavrev S 2004 Physica C 401 231-35
[5] Chen N Y, Jonker R, Matijasevic V C, Jaeger H M, Mooij J E Appl. Phys. Lett. 67 1995
[6] Sambles J R, Elsom K C, Jarvis D J, Solid State Communications 32 1979
[7] Decroux M, et al 2001 IEEE Trans. Appl. Supercond. 11 2046
[8] Decroux M, et al 2003 IEEE Trans. Appl. Supercond. 13 1988