Attitude control of self-balancing vehicle based on sliding mode variable structure control

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Abstract: The two wheeled self balancing vehicle belongs to a typical nonlinear system. Aiming at the problem of attitude control stability of two wheeled self-balancing vehicle. A new sliding mode reaching law of integral sliding mode variable structure control was proposed. The feasibility and effectiveness of the controller are proved by theory and experiment. The experiment results show that the designed controller has remarkable advantages of steady state characteristic, fast dynamic response and strong robustness to external disturbance compared with traditional PID controller.

1. Foreword

Fully enclosed self-balancing electric vehicle is a cross-domain conceptual product which should be classified to the field between motorcycle and traditional vehicle since it combines the safety of vehicle and the convenience of motorcycle, creating a new model of urban transportation featured with exquisiteness and unique. Control Moment Gyroscope which is widely used for keeping the motorcycle upright can help to realize the balance of the two wheeled balancing vehicles. The principle of Control Moment Gyroscopes (CMG) is that when a gyroscope is given a torque perpendicular to its rotor rotation axis, a precession torque which is perpendicular to the rotor rotation axis and to the torque axis will be produced to be the execution force for attitude control. When a vehicle is tilted due to external force or turning, the automatic control system will issue instructions to change the tilt angle of the control moment gyroscope according to the specific situation so as to ensure the balance of the vehicle and keep it upright under any circumstances.

Given the advantages, more and more researches on self-balancing electric vehicle are emerging at home and abroad. At present, among the various related reports and information, self-balancing vehicle is still in the development stage, and two of the more well-known ones are the LIT Motors C1 and the Lingyun automobile. Since the key technology and core difficulty of two wheeled balancing vehicle is its balance control, designing a controller with excellent performance and stability is still the main research direction at present. Through the tests of experiments, a synovial control algorithm used in Literature [2] is proved to be effective in suppressing overshoot. However, under the algorithm, the controller's output switch is too large and the change is too drastic, which makes demanding requirements on the hardware performance of the control moment gyroscope. While a state feedback control algorithm proposed in Literature [3] has to admit that it largely rely on the accuracy of the dynamic model of the vehicle and need a linearization of the dynamic model when it near the balance point. And the experiment only verifies the return to the balance position from the initial inclination angle of about 3°. Given the facts, this algorithm is suitable for the attitude control of the balancing vehicle with small angle changes since the linearization error cannot be eliminated.
and the roll range of the vehicle is limited. In Literature [4], a single control moment gyroscope stalled to the yaw axis is wielded for attitude control. But when the gyroscope is generating a moment in the roll direction, it also generates the disturbance moment in the yaw direction, which will affect the balance and stability of the self-balancing vehicle.

In this paper, self-balancing vehicle is equipped with dual-frame CMG structure and designed with a digital circuit control hardware scheme which combines DSP and FPGA. In terms of control algorithm, it adopts the integral sliding mode variable structure control and proposes a new sliding mode reaching law. Through the comparison with the traditional PID control algorithm, this algorithm is proved to have an edge in anti-interference ability and robustness, so that the self-balancing vehicle can restore its balance from a larger initial inclination angle.

2. Dynamic model of balancing vehicle

The attitude control system of a self-balancing vehicle is generally composed of an attitude sensor (IMU), an attitude controller and an actuator (control moment gyroscope). The actuator provides the control torque for the vehicle body in accordance to the instructions issued by the controller to realize the stability of the vehicle body or the posture maneuvering.

Simplified model of balance vehicle is shown in figure 1. In this model, the rotation direction of the two control moment gyroscope rotors is opposite to the precession angular velocity direction. According to the gyroscope effect, the righting moment in the same direction which works on the car and further controls its attitude can be produced. Given the constancy of the rotation speed of the control moment gyroscope rotor, the precession torque can be adjusted by changing the symmetrical rotation speed around the pitch axis, which further helps to control the stability of the vehicle’s body.

![Fig.1 Simplified model of balance vehicle](image)

When the vehicle is equivalent to a body with a certain mass that only has points to support on the ground and has only a structure with one degree of freedom that can be tilted left and right, a unstable balance point appears. Considering the control moment gyroscope is an angular momentum exchange type actuator, it’s tilt will produce precession torque on other axes. Therefore, two control moment gyroscope system is adopted in this paper to offset the interference torques. The system meets the following conditions:

a) The angular momentums of the two gyro rotors are the same, and their angular momentum directions are opposite at the initial zero position

b) The control frame corner α of the two frame motors are the same in size and opposite in direction;

The dynamic equation of the balancing vehicle is

\[ J\ddot{\theta} = mgh\sin{\theta} + \tau \]  

In equation (1), \( J \) is the moment of inertia of the balancing vehicle towards to the roll axis, \( \theta \) is the tilt angle of the vehicle towards to the vertical, \( \ddot{\theta} \) is the tilt angular acceleration of the vehicle towards to the vertical, \( \tau \) is the output torque of Control moment gyroscope. Thus, \( \tau \) is

\[ \tau = H \times \omega \]  

In equation (2), \( H \) is angular momentum of the Gyroscopes, \( \omega \) is precession angular velocity of the Gyroscopes. This paper adopts dual control moment gyroscope structure, and the precession angle dynamic equation is:
In equation (3), $\alpha$ is the precession angle of the control moment gyroscope, which can be adjusted to realize the balance of the two wheeled self-balancing vehicles.

3. Design of the controller

3.1. The dynamic model of the drive motor

The voltage balance equation of the motor is:

$$u_d = Ri_d + L \frac{di_d}{dt} + e$$  \hspace{1cm} (4)

In equation (4), $u_d$ is terminal voltage, $R$ is coil resistance, $L$ is the coil inductance, $e$ is back EMF.

And back EMF is:

$$E = C_en$$  \hspace{1cm} (5)

In equation (5), $C_e$ is back EMF coefficient, $n$ is motor speed.

$$i_d - i_i = \frac{T_m}{R} \frac{de}{dt}$$  \hspace{1cm} (6)

In equation (6), $i_i$ is disturbance load current, $T_m$ is electromechanical time constant of electric drive system, $T_m = \frac{GD^2R}{375C_eC_m}$.

The transfer function between voltage and current is:

$$\frac{I_d(s)}{U_d(s) - E(s)} = \frac{1/R}{Ls/R + 1} = \frac{1/R}{T_1s + 1}$$  \hspace{1cm} (7)

And in equation (7), $T_1 = L/R$.

The transfer function between current and electromotive force is:

$$\frac{E(s)}{I_d(s) - I_i(s)} = \frac{R}{T_ms}$$  \hspace{1cm} (8)

The motor model is shown in Figure 2:

![Fig.2 Dynamic model of frame motor](image)

3.2. Design of Discrete Integral Sliding Mode Variable Structure Controller

3.2.1. Design of synovial surface

Sliding mode variable structure control is a non-linear control. In the control process, the system structure can be constantly changed according to the current state, so the perturbation caused by factors such as external disturbances and inaccurate mathematical models will not influence the sliding mode.

The state variable taken from the attitude control system of the balancing vehicle is:

$$\begin{cases} x_1 = \theta_{ref} - \theta_{fb} \\ x_2 = \int_{-\infty}^{t} x_1 \, dt \end{cases}$$  \hspace{1cm} (9)
In equation (9), \( \theta_{mf} \) and \( \theta_{fb} \) are the expected inclination angle and the actual inclination angle.

In order to reduce the chattering in sliding mode, integration of state quantity is taken into consideration on the basis of conventional sliding mode surface, and the integration link is introduced. Thus, the switch function is:

\[
s(k) = Cx(k) = cx_i(k) + x_2(k) + k_1 \sum_{i=1}^{x_1} x(k)
\]

(10)

3.2.2 Design of new reaching law

A reaching law based on \( fal(x, a, \delta) \), the construction of nonlinear power combination functions is proposed in Literature (10):

\[
\dot{s} = -\varepsilon \cdot fal(s, a, \delta) - k \cdot \tanh(s)
\]

(11)

In equation (11), the function of \( fal(x, a, \delta) \) is:

\[
fal(x, a, \delta) = \begin{cases} 
| x |^{a} \cdot \text{sgn}(x) & | x | > \delta \\
\frac{x}{\delta^{(1-a)}} & | x | \leq \delta
\end{cases}
\]

(12)

and in which, \( 0 < a < 1 \), \( 0 < \delta < 1 \).

The reaching law in equation (11) can solve the high-frequency chattering problem caused by the discontinuity of the sign function \( \text{sgn}(x) \) at the origin. And the reduce of \( \varepsilon \) in the equation can reduce the chattering of the system and will reduce the approaching speed of the system to the switching surface. An ideal \( \varepsilon \) is expected to change with the change of the system state. Therefore, the new reaching law designed in this paper is:

\[
\dot{s} = -\frac{s}{2} \cdot fal(s, a, \delta) - k \cdot \tanh(s)
\]

(13)

3.2.3 Analysis of system stability

Defining the Lyapunov function:

\[
V = \frac{1}{2} s^2
\]

(14)

Using this formula to get the derivation:

\[
\dot{V} = s \cdot \dot{s} = -s^2 \cdot fal(s, a, \delta) - ks \tanh(s)
\]

(15)

if \( |s| > \delta \):

\[
\dot{V} = -|s|^{2(a)} - ks \tanh(s)
\]

(16)

if \( |s| \leq \delta \):

\[
\dot{V} = -\frac{|s|^3}{\delta^{(1-a)}} - ks \tanh(s)
\]

(17)

The analysis shows that the sliding mode tightening rate meets the conditions for the existence and reachability of the sliding mode, and the control system designed in this paper is stable.

4. Experiment and simulation results

Based on the above theoretical analysis, in order to verify the feasibility of the control system designed in this paper, Matlab/Simulink is used to simulate the system, and a physical system is built for experimental verification. The relevant parameters of the motor used in this paper are: moment of
inertia \( J_r = 0.0001 \text{Kg} \cdot \text{m}^2 \), stator inductance \( L = 0.54 \text{mH} \), stator resistance \( R = 0.13 \), rated torque 1.3Nm, rated voltage 76v, rated current 18A, torque constant \( K_m = 77.2 \text{mNm /A} \), no-load speed \( n = 6000 \text{ r/min} \).

The simulation results based on Matlab/Simulink is seen as figure 3. The initial inclination angle of the balance vehicle is 0.26rad, namely 15°, and 0° is the balance position of the balance car. The start-up adjustment time of the self-balancing vehicle is 2s, and it maintains a balanced state after 10s. The simulation results show that the controller designed in this paper is effective.

The self-balancing vehicle experimental platform built in this paper is shown in Figure 4, and the relevant parameters are:

1. Mass: 100kg;
2. The center of gravity of the whole vehicle: 0.5m;
3. Vehicle inclination angle: 10° ~ 15°;
4. Reduction ratio of harmonic reducer: 1:78;
5. Control moment gyro precession angle range: 0° ~ 60°;
6. Control torque gyro maximum output torque: 450Nm;
7. Rotation speed of control moment gyro rotor: 3000r/min.

The attitude angle curve of the self-balancing vehicle which uses PID control is shown in Figure 5. The adjustment time of the self-balancing vehicle is 2s. At the balance point, swings of about ±7° on the left and right sides may appear. And after 60 seconds of balance, the stability of the vehicle will be broken. While, the attitude angle curve of the vehicle that adopts the integral sliding mode variable structure control algorithm based on new reaching law is seen as Figure 6. Under this algorithm, the time for adjustment is shorten to 1s, the swing amplitude on both sides of the balance position is reduced to about ±4°, and the balance of the vehicle can be maintained. Compared with the traditional PID control, a new sliding mode reaching law of integral sliding mode variable structure control is not only performs better in robustness and dynamic response, but also contributes to improve the steady-state characteristics of the control system to a certain extent.
5. Conclusion
In order to optimize the control effect of balancing vehicles, reduce system chattering, and decrease the influence of external interference on the system, this paper adopts integral sliding mode variable structure control, and designs a new reaching law in accordance to the system characteristics. The effectiveness of the controller is verified by MATLAB simulation. Through experimental comparison, the controller in this paper performs better in robustness, anti-interference ability, keeps longer balance time and better steady-state characteristics than traditional PID controllers, which all show the improvements in vehicles’ balance effect.

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