SCHRÖDINGER CATS, QUANTUM SLINGS AND AZIMUTHAL EFFECTS IN NUCLEUS-NUCLEUS COLLISIONS

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Abstract

Confinement of a chromodynamical string can result in specific effects in scattering processes and multiparticle production. In particular, due to its rotation secondary fragments of the splitting apart string can move like stones thrown from a sling. That would be detected as the azimuthal asymmetry of particle distributions in individual events. Thus we propose to explain the elliptic flow observed in AA-collisions as a sling-effect. It can provide information about confinement of quarks inside particles or binding forces in nuclei. Nonclassical states of the created particles like the Schrödinger cats are produced. Some classical and quantum-mechanical estimates of possible effects are given. Experimental facts which can be used for their verification are discussed.

The different types of nonclassical states of electromagnetic field quanta attract a lot of attention in quantum optics [1–4] and in quantum mechanics [5]. An essential property of the nonclassical states of the field quanta is a difference of the particles’ statistics from the Poissonian statistics. Mechanisms of generating different nonclassical states (squeezed states [6], correlated states [7, 8]) for photons were discussed recently [9–12]. The typical condition for nonclassical state generation is a sharp nonstationarity of the boundaries for photons or other quanta (e.g., in a cavity with moving walls). But an analogous situation can take place in the process of high energy particle interactions, as well. Thus, instant break-up of a particle or a nucleus and fast motion of some fragments can be considered as moving boundaries for other field quanta (nucleons, pions, gluons etc) involved in the process. In our opinion, such collective effects as particle (or fragment) flows observed in high-energy nucleus reactions (momentum flow, directed flow, squeeze-out, elliptic flow) are related to binding forces inside nuclei and to confinement of quarks in particles. We will shortly discuss below some aspects of the approaches used in high energy physics and possible effects, which can be explained as obviously following from the simple “mechanical” models.

In high energy physics, there coexist two seemingly incompatible approaches to particle interactions. Those are the parton model, which has been proposed by Feynman and treats each particle as a conglomerate of a large number of point-like free partons, and the string model with quarks strongly confined inside hadrons. At the very beginning, these models did not compete...
because regions of their applicability were separated. The parton model pretended to describe
the inelastic processes with high transferred momenta, while the string model was used for bound
states. In quantum chromodynamics (QCD), quarks and gluons play the role of partons. Its prop-
erty of the asymptotic freedom, i.e., of smallness of the coupling constant at small distances, has
been used for the processes with high transferred momenta. For strings, one should consider the
confinement phenomenon which can not be quantitatively described nowadays because it is purely
nonperturbative effect.

However, step-by-step, the convergence of these models became quite clear. From one side, the
higher perturbative approximations of QCD allowed us to describe many properties of compar-
atively soft inelastic processes as multiplicity distributions, inclusive rapidity distributions, some
correlations, etc. From another side, the same properties have been described by the string (Lund)
model with excitation and decay of strings considered as color dipoles. Probably, it implies that
above characteristics are not very sensitive to some specific effects caused by the binding forces of
quarks in such processes.

Our main goal is to search for such features which give some information on confinement. We
argue that the azimuthal collinearity in individual events of particle (nuclear) interactions at low
transferred momenta could give some hints to confinement effects in inelastic processes. Indeed,
any string defines an axis in the space which, in general, differs from the direction of the impinging
particle. Thus there should be the azimuthal asymmetry in a given event of a collision with
the string. The strength of this effect should depend on string properties. In particular, such
asymmetry can be observed if the colliding objects (e.g. nuclei) start rotating after the collision, in
some sense keeping memory about their initial binding forces during the process of their break-up.

Let us stress once more that only event-by-event analysis can provide necessary information.
From experimental point of view, it asks for $2\pi$-geometry in azimuthal angles with constant ac-
ceptance. Certainly, no azimuthal asymmetry exists when the trivial average over large ensemble
of events is considered because there is no common spatial axis of nucleus rotation or alignment
of a string in different events. Besides, one should separate the conservation of the transverse
momentum from dynamical effects. But the former effect must decrease at higher multiplicities.
Two-jet production would be a background of the effect sought for.

Here we give some examples borrowed from classical physics and nonrelativistic quantum me-
chanics, which show how the binding forces can give rise to the azimuthal collinearity in individual
events.

Let us consider the classical nonrelativistic problem where the ball with the radius $R$ and
mass $M$ gets a blow with the momentum $p$ at the impact parameter $b$. The kinetic energy of the ball as a whole is $T_{\text{kin}} = p^2/2M$ and its rotation energy is $T_r = 5p^2b^2/4MR^2$. The velocity at the surface of the ball at the blow side is $v_A = p(1 + 5b/2R)/M$, and at the opposite side $v_B = p(1 - 5b/2R)/M$. Thus $v_A$ is 3.5 times larger than the velocity of the center of mass $v = p/M$ for $b = R$. If a piece of mass $m_f$ of the surface flies off with the same velocity $v_A$, its momentum $p_f = p(1+5b/2R)m_f/M$ can be of the order of the initial momentum since the energy conservation asks just for $(1+5b/2R)m_f/M \leq 1$. It is clear that this fragment moves in an impact plane formed by the initial momentum vector and the center of the ball’s mass. It reminds of a stone from a sling or of sparks from a firewheel or a grindstone. Therefore, there should be azimuthal inhomogeneity in an individual event. The similar effects are observed with a rod or a hard string.

The same arguments can be applied to nucleus collisions studied in Dubna and Berkeley at moderate energies 2.5 GeV and 4.5 GeV per nucleon. For the sake of simplicity, we consider only those events where the emulsion target is hydrogen and work in the anti-laboratory system with the hydrogen nucleus (proton) impinging on the heavy nucleus at rest and fragmenting it. Even though the projectile is relativistic, the final system is not relativistic. The average rotation energy would be $\langle E_r \rangle = 70$ MeV at the primary energy 2.5 GeV, and 230 MeV at 4.5 GeV (with $\langle b^2/R^2 \rangle = 0.5$). For the isotropic decay of the nucleus in its rest system, the energy of each fragment along a definite axis is $E_0/N$ and its rotation energy is $\langle E_r \rangle$ in the azimuthal plane. The azimuthal asymmetry is large when they are of the same order, i.e., at $N \sim 20$. However, it can be observable at lower number of fragments, as well. The effective value of the azimuthal angle is estimated as $\tan \phi|_{\text{eff}} \approx (F_f, y/E_{f,x})^{1/2} \approx (1 + 3E_r N/2E_0)^{-1/2}$ that is equal (at $N=4$ and $E_0=4.5$ GeV) about 0.85 and differs from unity for isotropic angles. One could try to ascribe to this effect the azimuthal collinearity observed in recent experiments [13]. This alignment has been seen in positive values of the second Fourier coefficient of the series expansion of particle distributions in differences of the azimuthal angles.

Let us consider the effects of the nucleus rotation in quantum mechanics. This rotation is caused by bonds between different fragments inside a nucleus after one of the fragments is struck by a projectile moving along $z$-axis. The projectile feels these bonds as the fragment oscillations. We are interested in the angular distribution of scattered projectiles or of fragments of the impinging nucleus. Then the problem is similar to the old one treated by Bethe [14] for neutrons scattered by the paraffine molecule, but now at the other energy scale and other binding forces. The paraffine molecule reminds a string with very low binding energy. Now, for small polar angles, the angle between the transferred momentum and the direction of the strong bond ($x$ axis) is approximately
equal to the azimuthal angle so that the distribution is (see \[14\])

\[
\frac{d\sigma}{d\cos\theta d\phi} \propto \exp[-2(1 - \cos\theta)(\epsilon_1 \cos^2\phi + \epsilon_2 \sin^2\phi)],
\]

where \(\epsilon_i = E_0/h\omega_i\); \(\omega_i\) being the oscillation frequencies along the axes \(x\) and \(y\) with \(\omega_1 > \omega_2\).

Integrating the polar angles, one gets at small difference between the frequencies \(\Delta\omega = \omega_1 - \omega_2\):

\[
\frac{d\sigma}{d\phi} \propto (1 + \frac{\Delta\omega}{\omega} \sin^2\phi)^{-1},
\]

i.e. the scattering is stronger in the impact \(xz\)-plane (\(\phi = 0\) or \(\pi\)) of the strong bond and the projectile momentum. Note that the bond axis in each event has been chosen along the \(x\)-axis.

For the second Fourier coefficient describing the collinearity, one obtains

\[
\langle \cos 2\phi \rangle = \frac{\pi \Delta\omega}{4 \omega},
\]

which is small but different from zero and qualitatively agrees with tendencies observed in \[13\]. Namely, one can hope for smaller \(\Delta\omega/\omega\) for heavier (and more symmetrical) nuclei, what is found in \[13\] as the decrease of alignment with the atomic number increase.

One can calculate the azimuthal asymmetry in the scattering on a string in quantum mechanics in a following way. Consider the scattering of a particle on a system of two coupled scattering centers. If their recoil has been neglected, then according to \[15\] (see also the book \[16\], Eq. (311) in Chapter 11), the scattering amplitude is written as

\[
A^{(2)} = A_1 e^{-iqr_1} + A_2 e^{-iqr_2} + A_1 A_2 \frac{e^{ipR}}{R} \left( e^{i(pr_1 - kr_2)} + e^{i(pk_2 - kr_1)} \right)
\]

\[
\times \left( 1 - A_1 A_2 \frac{e^{2ipR}}{R^2} \right)^{-1}.
\]

Here, \(A_i\) are the scattering amplitudes on the centers \(i\). \(r_i\) denote the positions of the centers, and we place one of them at the origin \(r_1 = 0\) and the second at the distance \(R\) along the \(x\) axis, i.e., at \(r_2(R, 0, 0)\). Once again the direction of the strong bond is fixed. The primary momentum along the \(z\) axis is \(p(0, 0, p)\), and the final momentum \(k\) is equal to the initial one in absolute value \(|k| = |p|\). The transferred momentum is denoted by \(q = k - p\). For two identical centers with opposite (color) charges, one gets \(A_1 = -A_2 = A\). For small amplitudes \(A\) the multiple scattering effects can be neglected and only two linear in \(A\) terms of Eq. (4) survive. Then one obtains

\[
A^{(2)} \approx A(1 - e^{-ipR \sin\theta \cos\phi}).
\]
The scattering amplitude $A^{(2)}$ vanishes at $R \to 0$ due to the color screening (the so-called color transparency effect). The differential cross section is

$$\frac{d\sigma}{d\Omega} = |A^{(2)}|^2 = 2|A|^2[1 - \cos(pR \sin \theta \cos \phi)].$$

The cross section vanishes for $\cos \phi = 0 (\phi = \pm \pi/2)$ i.e. in plane perpendicular to the impact plane that corresponds to elliptic flow.

We have argued that different type azimuthal asymmetry can appear. The transition to ever higher energies, when the inelastic interactions of particles are studied, asks for the relativistic treatment of strings which is not yet developed. Here, one can hope that the analogy from the low energy region would work with some “natural” modifications, and one can propose such a picture where the string is treated as quark oscillations inside some confining potential and the external blows give rise to excitations and breaking of strings. The transverse momentum conservation would lead to azimuthal asymmetry of fragments.

Such breaks could be described as abrupt vanishing of the potential that would produce the quantum sling-effect suggested by Hacyan in [17]. He has solved the Schrödinger equation for the parametric oscillator with potential becoming zero at some time what describes a quantum sling. The specifics of the quantum sling is that the system of the fragments produced is a nonclassical state of even and odd coherent states introduced in [18] and modelling the Schrödinger cat states. They have been seen for the photons in cavities [19], and never discussed in particle or nuclear physics. The quantum-sling mechanism considered here could be in charge of some states like the Schrödinger cat in particle production. Its typical feature is the dominance of either even, or odd number of produced field quanta in a single process. Let us note that the isospin conservation could mimic such states as well.

There exists the special axis of a string and, therefore, the azimuthal asymmetry should be observable in individual events. It is preferred to work with the non-relativistic objects to treat the string rotation. Let us estimate what restrictions it would impose for scattering of leptons on hadrons (or nuclei). The simplest situation is where the target is almost unexcited but rotates as a whole. Then the nonrelativistic condition is provided by

$$E_r = W - M \approx M(1 + \frac{1 - x}{x} \frac{Q^2}{M^2})^{1/2} - M \approx \frac{1 - x Q^2}{2xM} \ll M$$

and leads to

$$Q^2 \ll \frac{2x}{1 - x} M^2.$$
Here \( W \) is the total target energy in its rest system, \( Q^2 \) is the squared transferred four-momentum (photon virtuality), \( x \) is Bjorken variable. One concludes that the nonrelativistic region is limited by very small values of \( Q^2 \), but heavier targets widen it.

There are many strings in typical inelastic processes at high energies, and the background due to their different orientations in space can be strong. However, one hopes that in some rare events a single string (or Pomeron?) can play a dominant role giving rise to strong alignment. It would ask for high orbital moments in such collisions. The events of that kind were, probably, seen in cosmic rays \[^{20}\text{20}\] if their alignment survives the secondary interactions in the atmosphere. To prove it one needs Monte-Carlo calculations.

At the same time, one can envisage that the ratio \( \Delta \omega/\omega \) in Eqs (2), and (3) becomes large due to strong disbalance of forces along the string and perpendicular to it. It would enlarge the azimuthal asymmetry.

Thus, we propose to experimentalists to look for the azimuthal collinearity in inelastic events by evaluating the alignment coefficients. They are introduced in \[^{13}\text{13}\] for an individual event with multiplicity \( n \) as

\[
\beta_2 = \frac{\sum_i \sum_{i>j} \cos 2(\phi_i - \phi_j)}{\sqrt{n(n-1)}},
\]

where \( \phi_i - \phi_j \) denotes the difference between the azimuthal angles of particles \( i \) and \( j \). The average value of \( \beta_2 \) over an ensemble of events must be calculated. An excess over the background of statistical fluctuations provided by traditional models, with jets accounted, would be a signature of confinement. Our preliminary analysis of some Pb-Pb collisions at 158 GeV supports the optimistic approach to the problem. For analysis of "Mercedes"-type events one should use \( \beta_3 \) instead \( \beta_2 \) with a factor in cosine being 3 instead 2. In general, \( \beta_m \) with any integer \( m \) can be used to study "\( m \)-leaves" topology of events. Another signature of creating the field quanta in nonclassical states is the difference of the multiplicity distributions from the Poissonian statistics and an excess of even or odd states. Beside Fourier coefficients, one can use the wavelet analysis \[^{21}\text{21}\] or other correlation methods \[^{22}\text{22}\] for event-by-event studies. We hope that these methods will help separate the nuclei rotations and the string tension to get some confinement parameters. We will study these problems in more detail later.

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