Brane world corrections to Newton’s law

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Abstract

We discuss possible variations of the effective gravitational constant with length scale, predicted by most of alternative theories of gravity and unified models of physical interactions. After giving a brief general exposition, we review in more detail the predicted corrections to Newton’s law of gravity in diverse brane world models. We consider various configurations in 5 dimensions (flat, de Sitter and AdS branes in Einstein and Einstein-Gauss-Bonnet theories, with and without induced gravity and possible incomplete graviton localization), 5D multi-brane systems and some models in higher dimensions. A common feature of all models considered is the existence of corrections to Newton’s law at small radii comparable with the bulk characteristic length: at such radii, gravity on the brane becomes effectively multidimensional. Many models contain superlight perturbation modes, which modify gravity at large scale and may be important for astrophysics and cosmology.

1. Introduction

In this paper, we are going to discuss one of the most important predictions of a large and actively discussed class of modern theories of gravity, related to different brane world scenarios, namely, possible violations of Newton’s law of gravity for macroscopic bodies. It seems useful to begin with giving a brief but more general exposition of the to-date experimental and theoretical situation with possible Newton’s law violations.

Most of the existing generalized theories of gravity as well as unified theories of basic physical interactions predict time and also spatial variations of the gravitational constant \cite{1–4} which may manifest themselves as new interactions in addition to Newton’s law

\[ F = G m_1 m_2 / r^2. \] (1)

Such interactions can violate the equivalence principle (EP) if they depend on the bodies’ composition, modify the inverse square law and are often described in terms of new particles — new interaction carriers \cite{5–9}.

The laboratory experimental data and observations of planet and satellite motion in the Solar system exclude, with great accuracy, any deflections from Newton’s law and, accordingly, the existence of new interaction carriers in almost all length ranges, except the range much shorter than a millimeter and the one from meters to hundreds of meters. This possible deflection may be described by an additional Yukawa-type contribution to the gravitational potential,

\[ V = -(G m_1 m_2 / r)(1 + \alpha \exp(-r/\lambda)), \] (2)

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characterized by the interaction strength $\alpha$ and the scale $\lambda$ related to the mass of the interaction carrier. Among laboratory experiments, there is only one result on a possible inverse square law violation [10] in the range of 20 to 500 m with the interaction strength between 0.13 to 0.25. It was obtained by using a set of gravimeters and an energy storage station, where water was pumped to a lake during the night and then was released at daytime giving additional electric power. Though, an independent verification of this result using other schemes is absent.

More serious evidence on a possible violation of Newton’s law has come to us from space, namely, from data processing on the motion of the spacecrafts Pioneer 10 and 11, referring to length ranges of the order of or exceeding the size of the Solar system. The discovered anomalous (additional) acceleration is [12]

$$(8.60 \pm 1.34) \cdot 10^{-8} \text{ cm/s}^2,$$

it acts on the spacecrafts and is directed towards the Sun. This acceleration is not explained by any known effects, bodies or influences related to the design of the spacecrafts themselves (leakage etc.), as was confirmed by independent calculations.

Many different approaches have been analyzed both in the framework of standard theories and invoking new physics (Chongming Xu’s talk at ICGA-7, Taiwan, November 2005), but none of them now seems to be sufficiently convincing and generally accepted. There are the following approaches using standard physics:

— an unknown mass distribution in the Solar system (Kuiper’s belt), interplanetary or interstellar dust, local effects due to the Universe expansion [11];

— employing the Schwarzschild solution with an expanding boundary [12, 13] etc.

Among the approaches using new physics one can mention:

— a variable cosmological constant [14];

— a variable gravitational constant [17];

— a new PPN theory connecting local scales with the cosmological expansion [15];

— the five-dimensional Kaluza-Klein (KK) theory with a time-variable fifth dimension and varying fundamental physical constants [16];

— Moffat’s [18] non-symmetric gravitational theory;

— Milgrom’s [19, 20] modified Newtonian dynamics (MOND);

— special scalar-tensor theories of gravity [21];

— approaches using some ideas of multidimensional theories [22];

— modified general relativity with a generalized stress-energy tensor [23] etc.

This Pioneer anomaly has caused new proposals of space missions with more precise experiments and a wide spectrum of research at the Solar system length range and beyond:

— Cosmic Vision 2015-2025, suggested by the European Space Agency, and

— Pioneer Anomaly Explorer, suggested by NASA [24].
As to other theoretical schemes leading to Newton’s law modifications, from the general relativistic viewpoint, if we admit that the physical constants may vary with time, it is quite natural to consider their spatial variations, as was done in Ref. [6].

As is well known, in Einstein’s general relativity
  — the gravitational interaction is carried by massless gravitons,
  — they are described by second-order differential equations, and
  — they interact with matter with an invariable strength related to the constant $G$.

If at least one of these features is violated, we shall in general arrive at some distance-dependent Newton’s law violation and consequently to a modification of Einstein’s theory. One can single out a number of classes of generalized theories known in the scientific literature:

1. Theories with massive gravitons, e.g., bimetric theories.
2. Scalar-tensor theories with a variable effective gravitational constant [4].
3. Theories containing, in addition to the metric, space-time torsion and nonmetricity [6].
4. Theories whose equations contain higher-order derivatives (e.g., due to quantum effects).
5. Theories with interaction carriers other than gravitons (the so-called partners): supergravity, superstrings, M-theory which promises to unify the latter but so far unfinished, etc.
6. Other nonlinear theories induced by any known interaction type, also creating effective non-zero masses [6].
7. Various phenomenological theories which do not specify the cause of Newton’s law violation (the fifth force etc.)
8. Multidimensional theories: Kaluza-Klein and brane world approaches.

All such theories lead to real or effective masses leading to additional Yukawa or power-law corrections to Newton’s law.

There are some model-dependent estimates of such forces. Thus, according to Sherk [25], in supergravity where the graviton is accompanied by a partner (graviphoton) of spin 1, there appears a Yukawa-type repulsion. Moody and Wylcek [26] introduced a pseudoscalar particle leading to an additional attraction in the range between $2 \cdot 10^{-4}$ and $20$ cm with a strength between $1$ and $10^{-10}$. In Fayer’s [27] supersymmetric model a spin-1 graviphoton leads to an additional repulsion in a $10$ km range with a strength about $10^{-13}$. S. Weinberg’s model [28] with a scalar field, aimed at generating a cosmological constant, predicted an extra interaction at lengths smaller than $0.1$ mm. Most of the multidimensional theories [1–3], including those developed and analyzed in Ref. [6], also predict Newton’s law violations and the values of post-Newtonian parameters other than in general relativity.

Most of the brane world models, to be discussed below in more detail, also predict deflections from Newton’s law at lengths smaller than $1$ mm. At present, very active experimental and theoretical studies are being performed in this range, see, e.g., [9]. No Newton’s law violations have been detected in the submillimeter range by now [40]. It is expected that a few coming years will be marked with an improvement of such estimates in the range from a nanometer to a centimeter by several orders of magnitude, and hopefully such experiments will be able to probe gravity in extra dimensions.
2. The brane world concept

The brane world concept, broadly discussed in the recent years, treats our Universe as a distinguished three-dimensional (or four-dimensional if time is included) surface or layer, called the brane, in a multidimensional space-time where the extra dimensions are large or infinite. The Standard Model fields are supposed to be concentrated on the brane while gravity (and, as a rule, only gravity) propagates in the ambient space called the bulk. The history of such models traces back to the early 80s, beginning with Akama’s [29], Rubakov and Shaposhnikov’s [30] and some other works. The recent outburst of interest in such models is mostly related to the progress in string and M-theory, in particular, with Hořava and Witten’s well-known 11D model [31], in which one of the extra dimensions is of much larger size than the others. This approach has suggested, in particular, a natural mechanism for solving the hierarchy problem in particle physics (Randall and Sundrum’s model [32] and others), while retaining the Newtonian behaviour of weak gravitational fields on the brane in agreement with modern experiments. The brane world models have led to an appreciable progress in both particle physics and cosmology.

By now, there are thousands of publications in this area, with a great number of particular models and scenarios suggested, see, e.g., the reviews [33]– [39]. The models differ in the following essential properties:

- Total dimension $D$ (most of the studies consider $D = 5$).
- Models with single or multiple branes.
- Flat or curved branes.
- Flat or curved bulk.
- Compact or non-compact bulk.
- Possible fields other than gravity (spin 0, 1/2, 1) in the bulk.
- Thin or thick branes. (An infinitely thin brane may only be considered as an approximation since any underlying theory contains a fundamental length, such that at smaller scales a classical description is meaningless.)
- Various symmetries of the brane(s) and the bulk, etc.

Even this rough classification shows how diverse should be the observational predictions: each model gives, in principle, effects of its own.

Here we will consider the predictions of various brane world models for deviations from Newton’s law of gravity for material bodies situated (as well as the observer) on the brane. We will be restricted, by necessity, to a certain set of most elaborated models.

Laboratory measurements [40] have not discovered any deviations from Newton’s law at distances larger than 0.1 mm, which seriously constrains the parameters of any generalized theories of gravity.

On the other hand, the astronomical observations show that Newton’s law holds with high accuracy at least up to stellar cluster scales. At the galactic scale, however, there exists the well-known dark matter (DM) problem, and modification of Newton’s law [41] is one of the ways of its solution. Some brane world models also predict changes in Newton’s law at kiloparsec and higher scales. Some of these changes are able to remove (or at least ease) the DM problem in galaxies and galaxy clusters. This set of problems is discussed in Refs. [33], [42]– [50].
3. Modified Newton’s law in RS2 model

Apparently, the simplest brane world model (at least, the one containing a minimum of details) is Randall and Sundrum’s second model (RS2) [51], in which a single brane, endowed with the Minkowski metric and a positive tension, is embedded in a 5D anti-de Sitter (AdS) space-time with a constant negative curvature. According to [51], the gravitational perturbations of this model (gravitons) contain a zero (i.e., zero-mass) mode, concentrated on the brane and responsible for the validity of Newton’s law on the brane, and the so-called Kaluza-Klein (KK) modes corresponding to all possible finite mass values.

The RS2 model realizes a solution to the 5D vacuum Einstein equations with the cosmological constant $\Lambda_5 < 0$ and a source in the form of a $\delta$-like matter distribution in a certain spatial direction $y$. The total action has the form

$$ S = S_{\text{grav}} + S_{\text{brane}}, $$

$$ S_{\text{grav}} = \int d^4x \int dy \sqrt{|g_5|}(2m_5^3 R_5 - \Lambda_5), $$

$$ S_{\text{brane}} = \int d^4x \sqrt{|g_4|} V_{\text{brane}}, $$

where $R_5$ is the 5D scalar curvature, $g_5$ and $g_4$ are the determinants of the 5D metric $g_{MN}$ and the 4D metric $g_{\mu\nu}$ at the hypersurface $y = 0$, respectively; $m_5$ is the 5D Planck mass, which, in general, does not coincide with the conventional one; the quantity $V_{\text{brane}}$ describes the brane tension. The solution possesses a mirror ($\mathbb{Z}_2$) symmetry with respect to $y = 0$ and is described by the metric

$$ ds_5^2 = e^{-2|y|/l} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, $$

(4)

(where $\eta_{\mu\nu}$ is the Minkowski metric, $l$ is the curvature radius of the AdS bulk) and exists under the conditions

$$ V_{\text{brane}} = 24m_5^3/l, \quad \Lambda_5 = -24m_5^3/l^2. $$

(5)

Thus a “fine tuning” is required, which connects the input constants $m_5$, $V_{\text{brane}}$ and $\Lambda_5$ and providing a zero 4D cosmological constant $\Lambda_4$. The 4D Planck mass value $m_4 = 1/\sqrt{G}$ (where $G$ is the Newton constant of gravity) is related to the 5D constants: $m_4^2 = m_5^2 l$.

Beginning with Ref. [51], a number of authors have calculated the Newtonian potential on the brane with the result (for sufficiently large $r$)

$$ V(r) \approx \frac{GM}{r} [1 + \Delta(r)], $$

(6)

and in all cases a correction of the form $\Delta \sim 1/r^2$ was obtained, but with different numerical coefficients. A calculation of the scalar part of the graviton propagator gave $\Delta(r) = l^2/(2r^2)$ at radii $r \gg l$ [52,53], whereas a calculation of the $h_{00}$ component of the metric perturbation led to $\Delta(r) = 2l^2/(3r^2)$ [54–57]. The difference between these results in the form of the factor 4/3 was ascribed to “brane bending” by the point source.

Ref. [49] explains the advent of the factor 4/3 by the effect of the full tensor structure of the graviton propagator, without need for effects like “brane bending”. The following expressions for the correction $\Delta$ have been obtained ($\mu = 1/l$):

$$ \mu r \ll 1 : \quad \Delta = \frac{4}{3\pi \mu r} - \frac{1}{3} - \frac{1}{2\pi} \mu r \ln \mu r + 0.089237810 \mu r + \mathcal{O}(\mu^2 r^2), $$

$$ \mu r \gg 1 : \quad \Delta = \frac{2}{3\mu^2 r^2} \ln \frac{\mu r}{\mu^4 r^4} + 16 - 12 \ln 2 + \mathcal{O}\left(\frac{(\ln \mu r)^2}{\mu^6 r^6}\right). $$

(7)
For intermediate values of \( r \), the correction was obtained numerically and unifies these two asymptotic expressions. An approximate analytic expression, valid for all \( r \), was obtained in Ref. [44].

Let us note that at small \( r \) the main term of the expansion has the form \( \Delta \simeq 4/(3\pi \mu r) \). In this case, \( \Delta \gg 1 \), and the whole potential behaves as \( V \sim 1/r^2 \): gravity becomes effectively 5-dimensional at radii smaller than the bulk curvature radius. The model space in this approximation has almost the same properties as 5D Minkowski space.

In this model, the curvature radius is \( l \lesssim 0.1 \) mm according to the laboratory constraint [40].

4. Other brane models in 5D space

4.1. Branes in 5D Einstein-Gauss-Bonnet theory

One of the generalizations of the RS2 models is a similar model in which 5D gravity is described, instead of general relativity, in the framework of the 5D Einstein-Gauss-Bonnet theory (see [44] and references therein) with the Lagrangian

\[
L'_{\text{grav}} = R_5 - 2\Lambda_5 + \alpha(R_5^2 - 4R^{LM}R_{LM} + R^{LMNP}R_{LMNP}),
\]

(8)

where \( \alpha \) is a constant of dimension \( l^2 \), \( R_5 \), \( R_{LM} \) and \( R_{LMNP} \) are the 5D scalar curvature, Ricci tensor and Riemann tensor, respectively. At \( \alpha = 0 \) this Lagrangian reduces to that of general relativity (in notations slightly different from (3)). The 5D metric in this model also has the form (4), i.e., describes an AdS space-time with the curvature radius \( l \), but the perturbations, which determine the form of corrections to the Newton’s law, behave, in general, differently.

The analytical expression obtained by Deruelle and Sasaki [44] for all values of \( r \) and \( \alpha \), are very cumbersome and will not be presented here, while their main conclusion is as follows: for values of the parameter \( \tilde{\alpha} = 4\alpha/l^2 \) close to unity, the gravitational potential of an attracting centre reproduces the Newtonian potential much more precisely than it is the case for an “Einstein brane” corresponding to \( \alpha = 0 \) (in other words, the RS2 model). Namely, at small radii the potential retains the form \( \sim 1/r \), and consequently the modern experiments do not constrain the curvature radius \( l \) to values smaller than 1 mm: the authors even assert that even values of \( l \) of the order of 1–100 km are compatible with the experiment. At distances \( r \gg l \), the \( 1/r \) dependence is also preserved, but with another value of the gravitational constant. Since, according to the astrophysical data, the constant \( G \) at astronomical distances should not differ from the one measured in laboratory by more than 10 per cent, this theory is subject to the constraint \( 0.85 < \tilde{\alpha} < 1.15 \) if \( l \) is of the order of 1–100 km.

4.2. Inclusion of induced 4D gravity

Some authors [46, 58, 59] consider RS2-like models, adding into the 4D Lagrangian \( L_{\text{brane}} \) a term proportional to the 4D scalar curvature \( R_4 \). It is explained as a contribution from one-loop quantum effects of the matter fields existing on the brane. Outside the brane, gravity is described by the 5D Einstein Lagrangian.

In the Dvali-Gabadadze-Porrati (DGP) model [59], belonging to this class and assuming a flat metric in the bulk, there appears a modification of Newton’s law at large distances, \( r \gg r_c = m_3^2/m_5^3 \), which is of potential interest for explaining the observed acceleration of the Universe expansion. To obtain such an effect, the radius \( r_c \) should be comparable with the Hubble radius \( \sim 10^{25} \) m, and then the 5D Planck mass \( m_5 \) is comparable with the proton mass. However, as shown by Rubakov [46], at such values of the parameters, the gravitational interaction turns out to be anomalously strong at distances of the order of meters. This effect is caused by the behaviour of
the scalar (in the 4D sense) part of perturbations of the 5D Minkowski metric. So the model [59] loses its appeal as a possible explanation of the cosmological acceleration.

More general models with induced 4D gravity and an AdS bulk have been analyzed in Ref. [58, 60]. The action of the form [60]

\[
S = m_5^3 \left[ \int_{\text{bulk}} (R_5 - 2\Lambda_5) - 2 \int_{\text{brane}} K \right] + \int_{\text{brane}} \left( m_4^2 R - 2\sigma \right) + \int_{\text{brane}} L(h_{ab}, \phi)
\]

(9)

contains the constants \(m_5\) and \(m_4\) (the five-dimensional and four-dimensional Planck masses, respectively), the bulk cosmological constant \(\Lambda\) and the brane tension \(\sigma\). The quantity \(K\) is the trace of the symmetric tensor of extrinsic curvature of the brane, while \(L(h_{ab}, \phi)\) denotes the Lagrangian density of the four-dimensional matter fields \(\phi\) confined to the brane and interacting only with the its induced metric \(h_{ab}\).

The corrections to Newton’s law are of the same nature depend on the interplay of scales related to the constants involved in (9) and generalize those of the RS2 and DGP models. It is shown, in particular, that, for some values of the constants, the \(\sim 1/r\) dependence of the potential is preserved at small radii [58], similarly to the results [44] for branes in Einstein-Gauss-Bonnet gravity.

4.3. Curved branes in 5D space-time

So far we have been describing Newton’s law modifications on Minkowski branes, i.e., 4D gravity was considered as a small perturbation in a flat background. This approach is justified by that the brane curvature (in other words, the observed space-time curvature) is small as compared to the curvature of multidimensional space which leads to the formation and existence of the brane itself. Many authors nevertheless consider curved branes, most frequently de Sitter (dS) branes. This maximally symmetric space-time is attractive observationally since it describes a Universe in accelerated expansion and are of interest from a theoretical viewpoint, in particular, due to the conjectural dS/CFT correspondence [61, 62].

Refs. [53, 63] (see also references therein) calculate the gravitational potential on a dS brane by the perturbation method, assuming three forms of bulk space-times: dS, Minkowski and AdS. In the first case [63], for distances \(r \gg l\) (where, as before, \(l\) is the curvature radius of the AdS bulk), the result is the same as in the RS2 model: \(\Delta = 2l^2/(3r^2) + o(l^2/r^2)\) — cf. Eqs. (9) and (7).

At small \(r\) the potential also behaves “five-dimensionally”, i.e., \(\sim 1/r^2\). It is of interest that the brane curvature, whose radius is assumed to be much larger than \(l\), does not affect the effective Newtonian potential.

A different behaviour of the gravitational perturbations is found for dS branes in dS bulk [63]. First, at \(r \gg l'\) (where \(l'\) is the curvature radius of the 5D de Sitter space) the correction is negative: \(\Delta = -2l^2/(3r^2) + o(l^2/r^2)\). Second, tachyonic perturbation modes are revealed, which are probably related to instabilities of the initial solution to the gravitational field equations.

Ref. [53] describes another result for 5D Minkowski and dS spaces: it is claimed that it is only possible to calculate the corrections in the near zone, i.e., for small radii, and the corrections exceed the basic Newtonian potential related to the zero graviton mode. It is concluded that only AdS space-time is suitable for describing the bulk. A similar conclusion is made in Ref. [64] in an analysis of gravitational perturbations outside a dS brane.

It should be noted that the cited papers have only discussed thin branes for which the corresponding solution of 5D gravity equations are easily found. Refs. [65, 66] have shown that a Minkowski brane treated as a thick domain wall, appearing in 5D space due to the violated \(Z_2\)
symmetry of a scalar field, can only exist in an asymptotically AdS bulk; in this case, the RS2 model with a thin brane is obtained in a universal way (independently of the shape of the symmetry breaking potential) as a properly defined limit. These results correlate with that of [53]: it is, e.g., natural to suppose that a weakly curved dS brane with the curvature radius \( R \) cannot exist in a 5D dS bulk with the curvature radius \( l \ll R \). This apparently means that those solutions of the 5D gravity equations for thin branes which are, according to Refs. [53, 63, 64], unstable or unrealistic, simply cannot be obtained as well-defined limits from thick brane solutions.

4.4. Brane world in dilaton gravity

Among the multidimensional gravity theories assuming other fields in the bulk in addition to the metric, the simplest one is apparently the theory with a single scalar field which is sometimes called dilaton gravity. The corresponding 5D Lagrangian is

\[
L_{DG} = R_5 + \frac{1}{2} g^{AB} \partial_A \phi \partial_B \phi - V(\phi),
\]

where \( V(\phi) \) is the dilaton potential. Ref. [67] has shown that there are brane world solutions with the bulk metric

\[
ds_5^2 = e^{2A(y)} \gamma_{\mu\nu} dx^\mu dx^\nu - dy^2,
\]

where the 4-metric \( \gamma_{\mu\nu} \) may be, in particular, Minkowski, dS or AdS while the functions \( A(y) \) and \( \phi(y) \) are determined from the 5D dilaton gravity equations. The inclusion of a scalar is motivated by that the graviton is accompanied by scalar partners in all models of string and M-theory and by considerations related to the AdS/CFT correspondence. As in the RS2 model, the brane is situated at a fixed hypersurface \( y = \text{const} \), at which the function \( A(y) \) suffers a fracture. It has been proved that gravity can be localized on the brane in the limit of a small 4D cosmological constant, and the corrections to Newton’s law due to KK models have been calculated. The modified Newtonian potential on the brane has the form \( \Theta \) for sufficiently large radii [67], but, unlike the previously discussed models, the correction has the form \( \Delta \sim 1/r^3 \), with a coefficient depending on the choice of the specific solution. The additional potential is thus proportional to \( r^{-4} \) instead of \( r^{-3} \) in other model, making gravity stronger at small radii.

For a similar class of models with an exponential potential \( V(\phi) \), Ref. [68] has studied all possible tensor and scalar perturbations. Apart from the corrections to the Newtonian potential (which, in a certain special case, coincides with the RS2 result), an effective scalar-tensor interaction on the brane has been discovered. This interaction contains both short-range and long-range components and has been estimated as a potentially dangerous one for the brane world scenario.

4.5. Incomplete localization of gravitons

Some authors consider the possibility of obtaining phenomenologically acceptable models with infinite extra dimensions under the assumption that the 5D gravitons form, instead of a usual zero mode, a metastable bound state with a small but finite probability of leaving the brane.

One such model was studied in the papers by Gregory, Rubakov and Sibiryakov [69, 70] (see also references therein). In 5D space-time, one assumes the existence of three parallel branes: the central one, with the positive tension \( \sigma \), and two identical lateral ones with the negative tension \(-\sigma/2\), at equal spacings from the central brane. The whole system possesses \( \mathbb{Z}_2 \) symmetry. The usual matter is located on the central brane. The cosmological constant \( \Lambda_5 \) is negative between the branes and is zero outside them. At proper values of \( \Lambda_5 \), there are solutions to the 5D Einstein equations with
the AdS metric between the branes and the flat metric outside. A linear perturbation study reveals the following: 1) There is an intermediate range of distances where the 4D Einstein equations hold (and consequently the usual Newton law). 2) Phenomenologically acceptable bounds of this range may be obtained by a proper choice of the model parameters and, as is claimed, “without strong fine tuning”. 3) At both small and large distances, there are contributions to the gravitational potential $\sim 1/r^2$. For small distances, as in other models, it means a transition to an effectively 5D nature of gravity. 4) At very large (cosmological) distances, there appears a contribution of the form $G_4/(3r)$, where $G_4$ is Newton’s constant from the intermediate range. Thus at large $r$ the gravitational field turns out to be repulsive and represents “scalar antigravity”.

Ref. [50] has studied tensor perturbations of sufficiently general 5D models of flat branes, with the metric

$$ds^2 = e^{-A(z)}(\eta_{\alpha\beta}dx^\alpha dx^\beta - dz^2),$$  \hspace{1cm} (12)

where $A(z)$ is an even function, non-decreasing at $z > 0$. The graviton zero mode turns out to be quasi-localized if $A(z)$ grows sufficiently slowly at large $z$, in particular, if it tends to a constant, which happens if the metric (12) has a flat asymptotic. It has been shown that the conclusion [70] on the existence of an intermediate range of radii with Newtonian gravity is general for all such models. As to the behaviour of the effective gravitational potential at large distances, its asymptotic has the form $1/r^{1+\alpha}$, $0 < \alpha \leq 1$, where the constant $\alpha$ depends on the model parameters and is equal to 1 for an asymptotically flat 5-geometry. However, the contribution of the scalar sector of the metric perturbations was not analyzed; meanwhile, it was this sector that led in Ref. [70] to an antigravitational potential at large radii. The authors of [50] stress that this sector does not follow such universal laws as the tensor perturbations and requires consideration in a more specific problem setting.

4.6. 5D models with multiple branes

In this class of models, the most well-known is Randall and Sundrum’s first model (RS1) [32], containing two branes and providing a solution to the interaction hierarchy problem due to the exponential factor in the AdS metric in the bulk. The extra dimension is compact and has the form of a ring, and the branes are located at its antipodal points. In the RS1 “scenario”, the Standard model fields of particle physics are assumed to be localized on the “Planckian” brane with negative tension while the other (TeV) brane, possessing a positive tension, may contain “shadow” matter which is invisible from the observable Planckian brane. A characteristic feature of such models is the existence of perturbations in a mode related to a variable distance between the branes, called the radion. The latter behaves in an effective 4D theory as a scalar field which does not directly interact with matter but makes an appreciable contribution to the predicted law of gravity.

The effect of the radion and a possible shadow matter on the observable gravitational field has been analyzed, in particular, in Refs. [54, 71–73] (including RS1 and other two-brane models). It has been shown [54] that, subject to the radion, a weak field on each brane is described by the linearized Brans-Dicke theory of gravity with different parameters $\omega$: $\omega > 0$ on the positive-tension brane, and $\omega > 3000$ if the brane separation exceeds the AdS curvature radius by a factor of four. Newton’s law holds on each brane but with different effective gravitational constants and with complicated corrections at both small and large radii. The gravity of shadow matter from the other brane is also felt, and, in some variants of the theory, even stronger than that on the observed brane. Similar results have been obtained in Ref. [71], but with other numerical values of some coefficients. It has been shown [72] that gravity of point particles on the TeV brane is
much stronger than on the Planckian brane. Thus the results of different authors slightly vary, and, probably, some further analysis will follow in order to remove the discrepancies. Of particular interest is, however, the possible existence of shadow matter and its gravitational interaction with the observed matter. Though, some arguments have been put forward for the non-existence of any matter on the second brane [71, 74]. On the other hand, it has been shown [74, 75] that in some RS1 type models (including 4D gravity of each brane, i.e., with induced 4D gravity, as in Sec. [4.2]), the radion may be absent as a result of a certain additional symmetry between the branes.

Rather general two-brane models with AdS bulk have been studied in Ref. [74]: the action is written in a form like (9) but the brane integrals encompass two branes with different values of \( m_4 \) and \( \sigma \). Negative values of \( m_5 \) and the brane tensions are allowed. The gravitational fields are calculated for sources placed on any of the two branes. As in other models, there is a vast range of sufficiently large radii where Newton’s law holds (with the effective gravitational constant depending on the model parameters), and there appear corrections \( \sim r^{-2} \) and \( \sim \ln r \) at small radii; their nature and ranges again depend on the interplay of the parameters.

As is noted in [74], exploring both signs of \( m_5 \), related to the bulk gravitational constant, and negative brane tensions may be of interest in cosmological applications, such as models with disappearing dark energy.

Configurations with a larger number of branes are described in detail in the PhD thesis by Mouslopoulos [42], written on the basis of studies conducted in the Oxford group, see, e.g., [76]–[79].

A new phenomenon of interest that appears in multiple-brane systems is the so-called multi-localization. It occurs if the potential \( V(z) \) in the corresponding Schrödinger equation (for any physical fields and, in particular, for gravitational perturbations of the background model) contains at least two wells. In a quantum-mechanical description, a brane with a positive tension \( \sigma > 0 \) creates a \( \delta \)-function shaped potential well while a brane with \( \sigma < 0 \) creates a similar potential barrier. The possible tunnelling in such models leads to nontrivial spectra of KK states including superlight localized KK states. Tunnelling removes the degeneracy of zero modes and creates an exponentially small splitting in the mass spectrum. Other levels, which do not correspond to bound states, form a usual KK spectrum.

Multi-localization is of particular interest because it creates, in a system with a single mass scale existing from the outset, another, exponentially smaller mass scale.

In the simplest case, we obtain a picture in which the observed gravitational interaction is a summed effect of a massless graviton and a massive super-light KK state. A large energy gap between the first KK state and the remaining tower leads to the existence of Newtonian gravity at intermediate scales. A radical prediction of the model, apart from the short-distance modification of gravity (owing to “heavy” KK modes), is its modification at superlarge distances due to a superlight KK mode with nonzero mass [79].

In more complex multi-brane systems there can be multiple superlight modes making the predictions more complex and diverse. However, as noted by the authors themselves [42], this class of models either contains a “ghost” radion scalar field (with a wrong sign of energy) or it is necessary to introduce a negative cosmological constant \( \Lambda_4 \) on the observed brane that leads to the AdS geometry, whereas the cosmological observations are only compatible with \( \Lambda_4 > 0 \). The authors hoped to get rid of these difficulties by considering higher-dimensional models [77].
5. Models with multiple extra dimensions

Models with more than five dimensions are also various and also predict essential changes in Newton’s law at small distances. There is a common physical reason for such a behaviour: the higher dimension manifests itself at scales comparable with the bulk curvature radius.

In Roessl’s PhD thesis [80], which completes a number of works with the author’s participation (see, e.g., [81, 83]), multidimensional models with a single brane are classified as follows:

1) “Strictly local” branes, i.e., branes in which the stress-energy tensor related to brane matter is either zero in the bulk or decays at least exponentially.

2) Global topological defects (strings, monopoles) with sets of scalar fields \( \phi^a \) of sigma-model type with “hedgehog” configurations \( \phi^a = \phi(x) n^a, n^a n_a = 1 \) and a spontaneously broken global symmetry, e.g., \( O(N) \) [84–86].

3) Models similar to the ’t Hooft-Polyakov magnetic monopole, with a gauge field in extra dimensions [83].

The cited papers considered \( D = 4 + n \)-dimensional metrics of the form

\[
ds_D^2 = A(l)\eta_{\mu\nu}dx^\mu dx^\nu + dl^2 + R^2(l)d\Omega^2_{n-1},
\]

where \( \eta_{\mu\nu} \) is the 4D Minkowski metric while \( d\Omega^2_{n-1} \) is the linear element on a unit \( (n-1) \)-dimensional sphere. In the extra-dimensional spherical coordinates, the brane is situated at the centre, \( l = 0 \). The following expression for the Planck mass was used:

\[
M_P^2 = A_{n-1}M_D^{n+2}\int_0^\infty A(l)R^{n-1}(l)dl,
\]

where \( A = 2\pi^{n/2}/\Gamma(n/2) \) is the \( (n-1) \)-dimensional area of a unit sphere. The finiteness of the expression served as a criterion for localization of gravity.

According to [80], under the “strictly local” assumptions, no acceptable multidimensional solutions have been found, and it is asserted that this class does not contain any (at least simple) models with localized gravity.

In models with global topological defects [81, 82] with a Mexican-hat potential for the scalar fields, \( V = \lambda(\phi^a\phi^a - \eta^2)^2 \) where \( \lambda \) and \( \eta \) are constants, the authors sought models with the function \( A(l) \), decaying far from the brane by the law \( e^{-cl} \). Such a solution was found, such that the spherical radius \( R \to \text{const} \) at large \( l \), i.e., the extra dimensions form an \( n \)-dimensional cylinder \( \mathbb{R}_+ \times S^{n-1} \). It is found that such a scalar structure is only necessary for obtaining solution with a decreasing function \( A(l) \) for \( n \geq 3 \). It leads, just as the 5D models, to corrections to Newton’s law of the form \( \sim 1/r^2 \), which is physically explained by the fact that only one extra dimension remains non-compact.

At other value of the parameters, one obtains singular solutions with \( R \to 0 \) as \( l \to \infty \). Assuming that the singularity may be smoothed away by higher string corrections, a modified Newtonian potential was also calculated for this model, with the result

\[
V(r) \approx \frac{GM}{r} \left[ 1 + \frac{\Gamma(n+2)}{2\Gamma^2[(n+3)/2]} \frac{1}{(cr)^{n+1}} \right],
\]

where the correction rapidly decays at large \( r \) and with growing number of dimensions \( n \).

The following remark is in order here. In the above model, the “cylinder end”, which is infinitely remote in the static reference frame, is accessible for geodesics at finite proper time and represents a killing horizon at which the function \( A \) turns to zero. Consequently, the spatial part of the 4D metric \( \eta_{\mu\nu} \) becomes degenerate, and an arbitrarily small matter density reaching this region should
turn to infinity at such a horizon, leading to a space-time singularity [90]. This actual instability leads, in our view, to serious doubts that this and similar models can be viable.

The third type of multidimensional models [83] includes gauge p-forms with the strength tensors $F_{A_1...A_{p+1}}$ in the bulk, where $p$, the rank of the form, may be different, up to $D - 1$ (in the latter case, however, they only create an addition to the $D$-dimensional cosmological constant). The necessity of considering such models is motivated by the inevitable emergence, in the case of global symmetry violation, of a massless Nambu-Goldstone boson, leading to difficulties with stability [80]. Ref. [83] describes a model with $p = n - 2$ and a regular cylinder-like geometry in the extra dimensions, with a constant radius $R$ as $l \to \infty$. It is a direct generalization of the 't Hooft-Polyakov monopole. Corrections to Newton’s law are proportional to $1/r^2$, as in similar models with global monopoles and for the same reason.

6. Concluding remarks

This review briefly describes the corrections to Newton’s law for only some most well-known brane world models, and the authors bring their apologies to the colleagues whose work, being maybe not less interesting and important, is not mentioned here.

With all the diversity of models, the nature of the corrections is more or less common: at small distances between the gravitating bodies, close to the bulk curvature radius, the world’s multidimensional geometry comes to the scene. For the number of dimensions higher than five, one could expect a universal dependence of the correction on the total dimension $D$. The analysis has shown, however, that it is the number of non-compact extra dimensions that is essential rather than the total dimensionality.

Of special interest are apparently the models with superlight metric perturbation modes which predict a modification of the gravitational interaction for matter on the brane at astronomical scales. Such corrections to Newton’s gravity may be promising for resolving the problem of dark matter (DM) in galaxies and galaxy clusters and even the cosmological dark energy problem.

This is, however, only one of many approaches to these problems. Thus, apart from the well-known attempts to attribute the DM to dark celestial bodies and weakly interacting massive particles (WIMPs) or to the modified laws of gravity in four dimensions [41, 87], the DM effect may be explained in a different way in the brane world framework. Thus, it has been shown [88] that the astrophysical observations which are usually ascribed to DM can be explained by extra terms in the effective Einstein equations for gravity on a curved brane of RS2 type [89]. It is also claimed [88] that future gravitational lensing observations will be able to discriminate between different explanations of the DM effects.

It should be noted that the enumerated models in which the coefficients by $\eta_{\mu\nu}$ (frequently called warp factors) decay as one moves away from the brane, so successfully used for the description of localized gravity, are not free of problems with non-gravitational matter localization [66,90–92]. It has been shown, in particular, that in all $\mathbb{Z}_2$-symmetric 5D models with a single brane (i.e., all RS2 type models), any scalar field mode has an infinite energy per unit volume on the brane since the corresponding integral over the fifth dimension diverges [66]. Such problems emerge owing to the repulsive (from the brane) nature of the bulk gravity: the warp factor is actually a multidimensional generalization of the conventional gravitational potential, and particles tend to rolling down to its minimum, the AdS horizon in the present case. In our view, it makes sense to analyze, more thoroughly than it has been done so far, the brane world models that are free of these problems, namely, those with attractive gravity in the bulk. Some such models are described in Refs. [90–92].
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