Black hole spin-down by truncated disc emission

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ABSTRACT

The influence of disc radiation capture upon black hole rotational evolution is negligible for radiatively inefficient discs. For the standard thin disc model it is a slight but potentially important effect leading to the equilibrium spin parameter value of $a_{eq} \simeq 0.998$. For optically thin discs, the fraction of disc radiation captured by the black hole is however about two times larger. In some disc radiation models, inner parts of the accretion flow are optically thin, advection-dominated flows, and the thin disc ends at some transition radius $R_{tr}$. The thermal energy of the disc stored in trapped radiation is released at this radius. Angular distribution of the radiation released at this radial photosphere facilitates its capture by the black hole. For accretion rates close to critical and disc truncation radius $R_{tr} \simeq (2 \div 4)GM/c^2$, radiation capture is most efficient in spinning the black hole down that may lead to $a_{eq} \sim 0.996 \div 0.997$ or less depending on the mass accretion rate. For an accretion flow radiating some constant fraction $\epsilon$ of dissipated energy, the equilibrium Kerr parameter is shown to obey the relation $1 - a_{eq} \propto \epsilon^{3/2}$ as long as $1 - a_{eq} \ll 1$. Deviations from Keplerian law near the last stable orbit dominate over the radiation capture effect if they exceed $1 \div 2\%$.

Key words: accretion, accretion discs – relativity – black hole physics

1 INTRODUCTION

Black hole is probably the simplest astrophysical system described, apart from its position and velocity, by only one scalar (mass) and one vector parameter (angular momentum). Angular momentum $J$ of a black hole is convenient to normalize by its highest possible value:

$$J = \frac{GM^2}{c} a$$

Here, $-1 < a < 1$ is dimensionless Kerr parameter that we will consider as a positive scalar assuming that the accretion disc lies within the black hole equatorial plane and all the matter being accreted has angular momentum collinear with that of the black hole. The primary reason for this is Bardeen–Petterson effect (Bardeen & Petterson 1975) that aligns the inner parts of an accretion flow with the black hole spin. A tilted disc exchanges angular momentum with the black hole through Lense–Thirring precession but it does not affect the absolute value of the spin.

In the case of equatorial disc, evolution of a black hole is described by the two first-order equations for mass and for Kerr parameter (see Bardeen (1970)). As long as the mass and angular momentum accreted by the black hole depend linearly on the mass accretion rate, the evolution may be expressed in terms of spin parameter change with mass:

$$\frac{da}{d\ln M} = \frac{c^3}{c^3} \frac{L^\dagger}{GM E^\dagger} - 2a$$

Here, $L^\dagger$ and $E^\dagger$ are the net (per unit mass) angular momentum and energy of the matter absorbed by the hole. For the
case of thin disc accretion of ideal matter with no additional stress terms, black hole spin evolution proceeds towards the maximal possible value of $a = 1$ and formally even further. For black holes close to the extreme Kerr case, unexpected effects such as radiation capture may play the main role in stopping the spin-up. In particular, black hole should absorb stellar light and cosmic microwave background. Net angular momentum of distant photons is around zero (see section 2.3) hence they simply dilute the rotational energy of the accreting black hole by irreducible mass. If accretion is present, its impact and impact of its radiation upon rotation of the black hole are evidently much stronger. Since Kerr parameter is likely to differ from zero by a small but still significant amount, it is reasonable to operate with $\delta a = 1 - a$.

A relatively strong limit upon the rotational parameter is set by selective capture of the radiation of the disc pointed out by Thorne (1974). For the thin radiatively efficient disc model (Shakura & Sunyaev 1973; Novikov & Thorne 1973), black hole spin-up proceeds towards the equilibrium value of $a \simeq 0.998$ (or $\delta a \simeq 2 \times 10^{-3}$).

While for radiatively efficient thin disc accretion is relatively well understood, there is still lack in understanding of geometrically thick discs and accretion flows that are generally radiatively inefficient and hence may spin up black holes to higher values of $a$ (Abramowicz & Lasota 1980; Sadowski et al. 2011). The non-Keplerian nature of thick radiatively inefficient flows may revert the effect by lowering the specific angular momentum. Equilibrium spin values found by Popham & Gammie (1998) are considerably lower ($a_{eq} \sim 0.8 \div 0.9$). Inclusion of different effects such as magnetic stresses and minor mergers (in spin evolution of massive black holes) also leads to the relatively small values of $a_{eq} \sim 0.9$ (Gammie et al. 2004). Intermediate Kerr parameters are also generally found in fitting the observational data (Li et al. 2005). Here, I will consider primarily the effect of radiation capture paying little attention to deviations from Keplerian law (they are considered approximately in section 2.3) and not considering the effect of additional momentum transfer (for instance, by magnetic fields).

There are accretion flow models where radiatively efficient and inefficient parts coexist, primarily in the form of a nearly standard outer disc with optically thin advection-dominated inner parts (see for example Meyer et al. 2000). In the inner parts of thin accretion discs in X-ray binaries, radiation pressure dominates over gas pressure, and the transition from the standard disc to an optically thin flow should be accompanied by more or less abrupt emission of the internal energy stored in radiation trapped inside the disc. More gradual transition to an optically-thin flow should be still accompanied by radiation of all the trapped emission, but the shape of the photosphere will be more complex. Below, I will consider the inner disc face consisting either of one cylindrical surface of a constant radius or of two conical surfaces inclined by some angle $\eta$ toward the surface of the thin disc. The effect of the transition photosphere becomes more pronounced with growing mass accretion rate and is enhanced by radial advection of trapped radiation. The radial photosphere position is determined either by disc evaporation (Honma 1996) or by the sonic surface situated close to $r_{ISC}$ (Penna et al. 2012). In the latter case, the disc becomes transparent because of the density drop after transition to free-fall regime. Since I restrict myself to the standard disc model with Keplerian rotation law, I will not consider solutions with the transition radius situated inside the last stable orbit.

Honma (1996) estimates the transition radius due to evaporation as:

$$R_{tr} \simeq 2.1 \times 10^3 \alpha^4 \frac{GM}{m^2 c^2}$$

where $\dot{m} = \dot{M} c^2 / L_{Edd}$, therefore the normalization in this expression is different from that used in the original work. Strong dependence on the poorly known viscosity parameter $\alpha$ makes a broad range of truncation radii possible. Transition radius will be treated as a free parameter spanning a broad range of values between the last stable orbit and several tens of $GM/c^2$.

The primary goal of this work is to estimate how does the maximal possible spin depend on the properties of the inner disc such as disc truncation and existence of a radial photosphere. In the next section I describe the technique used to calculate the radiation braking term based on the method used by Thorne (1974). In section 3 I report the results obtained for the general case of an accretion disc truncated from inside and the case of a geometrically and optically thick disc where only photons emitted from its inner rim may reach the black hole. In section 4 I discuss the implications and limitations of my results.

2 CALCULATION TECHNIQUE

2.1 Local disc radiation field

Let us consider that in the co-moving frame, a unit surface element of disc surface radiates some known energy flux $F = F(R, a)$. For instance, for the standard thin accretion disc this flux equals:

$$F_{SD} = \frac{3}{8\pi} \frac{G M \dot{M}}{R^3} \frac{Q}{B \sqrt{C}} = \frac{3}{2} \frac{c^5}{\pi G M} \frac{\dot{m}}{r^3} \frac{Q}{B \sqrt{C}}$$

(1)
Below I will use dimensionless variables \( r = R c^2 / G M, \dot{m} = \dot{M} c^2 / L_{\text{Edd}} \), where \( L_{\text{Edd}} \) is Eddington luminosity, \( \chi \) is (Thomson) opacity. The calligraphic letters denote the coefficients used in the relativistic thin disc model as given by Penna et al. (2012). This flux may be converted to the fluxes of energy-at-infinity and angular momentum only if the angular dependence of the intensity of the outgoing radiation is known. This distribution is different, for instance, for optically-thin and optically thick discs and for discs with contributions of different opacity sources in the atmosphere. An interesting possibility is the possible inclination of the disc photosphere caused by disc thickness dependence on radius.

Locally measured flux leaving the disc in a unit solid angle may be expressed as \( F \times i(\Theta, \Phi) \), where \( \int i d\Omega = 1 \) and \( \Theta \) and \( \Phi \) characterize the direction in the frame co-rotating with the disc, \( d\Omega = \sin \Theta \sin \Phi \, d\Theta \, d\Phi \) (see details in Appendix A). This normalization is different from the intensity normalization used by Thorne (1974) by a factor of \( \cos \Theta \). For an isotropic source, \( i = 1/4\pi \). For the more general case of radiation field symmetric with respect to some axis \( i = i(\mu) \), where \( \mu = \cos \Theta \) where \( \sin \Theta \sin \Phi \) for the axis lying in the plane perpendicular to the direction of disc rotation and inclined by some angle \( \eta \) with respect to the vertical. Minus sign means that we consider \( \eta \) positive if the face of the inclined photosphere is oriented toward the black hole. In particular, \( \mu = \cos \Theta \) if the disc surface is horizontal and \( \mu = -\sin \Theta \cos \Phi \) for the inner disc face case. For a thin static plane-parallel photosphere:

\[
i(\Theta, \Phi) = \begin{cases} 
\frac{\mu}{2\pi} & \text{Lambert’s cosine law} \\
\frac{3}{4\pi} \mu \times (1 + 2\mu) & \text{atmosphere affected by electron-scattering opacity}
\end{cases}
\]

The only place where the \( 1 \times \frac{1 + 2\mu}{2\pi} \) law for outgoing intensity is derived seems to be the Chandrasekhar’s monograph on radiation transfer (Chandrasekhar 1960). For \( \mu < 0 \), I assume \( i = 0 \) in both cases.

Outer parts of accretion discs may have considerable thickness due to disc thickness dependence on radius and due to irradiation effects (flaring discs, see Shakura & Sunyaev (1973)), but their impact on the black hole is smaller by a factor of \( 1/r \). In the inner parts of the disc, strong but poorly known dependence of disc thickness on radius creates inclined portions of disc photosphere. Here, I assume that disc truncation is abrupt enough to make cylindrical photosphere with \( \eta = 0 \).

### 2.2 Radiation capture braking term

To calculate the effect of captured radiation upon black hole evolution, it is convenient to perform integration over solid angles in the co-moving frame and then over the disc surface. The integrand is \( \sqrt{-g} \tilde{F} n_t \) in the case of energy and \( \sqrt{-g} \tilde{F} n_\phi \) in the case of angular momentum, where \( n_\phi \) and \( n_t \) are the components of the momentum of a photon having unit energy in the orbiting frame (see Appendix A). The multiplier \( \sqrt{-g} = \alpha \times \sqrt{\eta^2 \eta_{\phi}^2} = r \) takes into account the difference in the proper and coordinate time (\( \alpha \)) and for the elementary disc surface area. First integration should be performed only over the photon trajectories that finally encounter the black hole. Condition for hitting the black hole was considered by Thorne (1974) in terms of effective potential. Since some parts of the disc may be geometrically thick, photons will be at average emitted at some distance from the equatorial plane of the disc, equal to the disc height for the standard disc and smaller for the optically thick case and for a radially-oriented photosphere. In the last case the gain is even higher because the radiation is channelled toward the black hole. In figure I the dotted and dot-dashed curves correspond to the optically thick disc case for \( H/R = 0 \) and \( H/R = 0.5 \). The amplitude of disc thickness effects remains of the order of \( (H/2R)^2 \approx 6\% \) for the considered range of radial distances.

Radiation contributions to black hole mass growth and angular momentum evolution are:

\[
\left( \frac{d\dot{M}}{dt} \right)_{\text{rad}} = \frac{1}{c^2} \int_{\text{disc surface}} F(R) \left( \int_{\text{co-moving}} \frac{i(\Theta, \Phi, R) n_t C_{BH}(\Theta, \Phi, R) d\Omega}{\Omega} \right) R dR d\phi
\]

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Figure 1. Co-moving flux fraction absorbed by the black hole as a function of distance. Solid lines correspond to the thin disc case, dotted for isotropic emission (optically thin case), dashed for radial photosphere (transition region). Hereafter grey lines correspond to the case of electron-scattering atmosphere solution. The dot-dashed curve corresponds to the optically thin case without the effects of disc thickness. Kerr parameter is set to $a = 0.998$.

\[
\frac{dJ}{dt}_{rad} = \frac{GM}{c} \int \text{disc surface} F(R) \left( \int \text{co-moving} \ i(\Theta, \Phi, R)n_{\varphi}C_{BH}(\Theta, \Phi, R)d\Omega \right) RdRd\varphi
\]

Here, $dA = RdRd\varphi$ is surface area element, $C_{BH} = 1$ if the photon encounters the black hole and 0 otherwise. The form of $i(\Theta, \Phi, R)$ is set explicitly in accordance with the considered emission regime. Apart from the case of optically-thin disc and the standard disc case considered by Thorne (1974), the most expected picture is a standard disc truncated from inside with its inner parts replaced by an optically-thin, geometrically thick flow. In this case, the two above integrals may be expressed as sums of three terms corresponding to the inner transparent part, the inner face of the disc and the outer standard thin disc. I treat the radial coordinate of the inner face of the disc is a free parameter and assume that it coincides with the transition radius $R_{tr}$, dividing the optically thin and optically thick parts of the disc.

The two quantities conserved along the path of the photon, $-n_t$ and $n_{\varphi}$, are calculated as functions of its comoving-frame properties (see Appendix A):

\[
-n_t = \frac{1}{\sqrt{c}} \times \left( \sqrt{r} + \sqrt{\frac{D}{r}} \sin \Theta \sin \Phi \right)
\]

\[
n_{\varphi} = \frac{1}{\sqrt{c}} \times \left( \sqrt{rF} + rB \sqrt{c} \sin \Theta \sin \Phi \right)
\]

For the optically thin part, I will assume the accretion disc flux to be equal to the standard disc flux $F_{SD}$ multiplied by a constant factor of $\epsilon < 1$. This is presumably a very much simplified picture but sufficient to estimate the principal effect of disc truncation in its inner parts.

Kerr parameter evolution is governed by a first-order equation of the form:
\[ \frac{\text{d}a}{\text{d}M} = \frac{c}{GM^2} \frac{\text{d}J}{\text{d}t} - \frac{2a}{\dot{M}} \frac{\text{d}M}{\text{d}t} \]

To exclude the black hole mass from the right-hand side it is convenient to divide the expression by \( \frac{1}{3} \frac{\text{d}M}{\text{d}t} \). This leads to the following expression:

\[ \frac{\text{d}a}{\text{d}M} = \frac{c^3}{GM} L^1 + \frac{(\text{d}J/\text{d}t)_{\text{rad}}}{\dot{M}} - 2a \]

(4)

\( L^1 \) and \( E^1 \) are the net angular momentum and energy-at-infinity at the last stable orbit. Note that \( \dot{M} \) is rest-mass accretion rate and is not equal to \( dM/dt \). The two terms produced by the absorbed radiation may be written as follows:

\[ \left( \frac{\text{d}E}{\text{d}t} \right)_{\text{rad}} = \int_{R_{\text{in}}}^{R_{\text{tr}}} RF(R) dR \int_{\Omega} n_v C_{BH}(\Theta, \Phi, R) d\Omega + \]

(5)

\[ + L_{\text{tr}} \times \int_{\Omega} i_s(\Theta, \Phi) n_v C_{BH}(\Theta, \Phi, R_{\text{tr}}) d\Omega - L_{\text{diff}} \times \int_{\Omega} n_v C_{BH}(\Theta, \Phi, R_{\text{tr}} + \Delta R) i_s(\Theta, \Phi) d\Omega + \]

\[ + \int_{R_{\text{diff}}}^{R_{\text{in}}} RF(R) dR \int_{\Omega} n_v C_{BH}(\Theta, \Phi, R_{\text{tr}} + \Delta R) i_s(\Theta, \Phi) d\Omega \]

and

\[ \left( \frac{\text{d}J}{\text{d}t} \right)_{\text{rad}} = \frac{c}{GM} \int_{R_{\text{in}}}^{R_{\text{tr}}} RF(R) dR \int_{\Omega} n_v C_{BH}(\Theta, \Phi, R) d\Omega + \]

(6)

\[ + L_{\text{tr}} \times \int_{\Omega} i_s(\Theta, \Phi) n_v C_{BH}(\Theta, \Phi, R_{\text{tr}}) d\Omega - L_{\text{diff}} \times \int_{\Omega} n_v C_{BH}(\Theta, \Phi, R_{\text{tr}} + \Delta R) i_s(\Theta, \Phi) d\Omega + \]

\[ + 4\pi \int_{R_{\text{in}}}^{R_{\text{diff}}} RF(R) dR \int_{\Omega} n_v C_{BH}(\Theta, \Phi, R_{\text{tr}} + \Delta R) i_s(\Theta, \Phi) d\Omega \]

Here, \( i_s \) and \( i_r \) are the normalized intensities for the outer thin disc (\( \eta = 0 \)) and for the radial photosphere (\( \eta = \pi/2 \)), respectively. Their dependence on the angular variables for different cases was considered in section 2.1.

Co-rotating luminosity \( L_{\text{tr}} \) of the inner face of the disc may be expressed as:

\[ L_{\text{tr}} = L_{\text{diff}} + L_{\text{adv}} \cong 4\pi RH \hat{F}_{SD} \times 2\pi \times U_{\text{rad}} v^* = \]

(7)

\[ = 2\pi RH \hat{F}_{SD} \times \left( 1 + 4K^2 \left( \frac{\hat{C}}{v^*} \right)^4 v^* \right) = 2\pi RH \hat{F}_{SD} \times \left( 1 + 4K^2 v^* \times \left( 1 + \frac{3}{5} \hat{C} \right) \right) , \]

where \( H \) is disc half-thickness, \( U_{\text{rad}} \) is vertically integrated energy density in the disc, \( v^* \) is the locally-measured radial velocity in the disc. The first term describes the radiation diffusing out of the inner face of the disc (hence its effective temperature is close to the local effective temperature of the disc). Since the radiation diffusing out of the transition photosphere can not contribute to the radiation of the outer standard disc, its contribution is subtracted from the radiation of the disk (the negative term proportional to \( L_{\text{diff}} \) in the formulae \( \text{A4} \) and \( \text{A6} \) above) setting \( \Delta R = H(R_{\text{tr}}) \). The lacking disc radiation has smaller effect upon black hole rotation than the inner disc face since the orientation of the photosphere is different. The second term corresponds to the radiation energy advected out of the optically thick region. Temperature ratio \( T_c/T_{\text{tr}} \) was obtained in Shakura & Sunyaev (1973) by considering vertical radiation diffusion. Second term becomes important if the accretion rate is high and radiation trapping works efficiently outside the transition radius. In this case, it is reasonable to connect \( R_{\text{tr}} \) with the sound surface near the last stable orbit rather than with disc evaporation. \( K \) multiplier in the above equation takes into account the vertical structure of the disk. Vertical structure of a radiation-supported disc is fairly approximated by a polytropic model with polytropic index \( n \simeq 1 \) (see for example Shakura et al. (1978)). For \( n = 1 \), \( p_{\text{rad}} \propto \left( 1 - (z/H)^2 \right) \) and \( K = 2/3 \). This value was used in all the calculations. The particular value of \( K \) varies slightly with the polytropic index reaching 16/35 \( \approx 0.457 \) for the extreme value of \( n = 3 \) that makes the relevant spin-down term about 10\% smaller. Mean value of \( (z/H)^2 \) for \( n = 1 \) is 2/5 that justifies the choice of \( \mu_0 = H/R = 2/5 \) for the transition region case (see above this section).

In all the simulations, we use the viscosity \( \alpha \) parameter value of \( \alpha = 0.1 \).

2.3 Asymptotic behaviour for \( \delta a \ll 1 \)

Let \( J'_{\text{rad}} \) and \( E'_{\text{rad}} \) be the angular momentum and energy emitted and absorbed by the hole for a unit accreted rest mass. To my knowledge, there are no convenient expressions for \( L^1 \) and \( E^1 \) as functions of \( a \), but since the considered Kerr parameter values are very close to unity it is reasonable to apply series expansion in \( \delta a^{1/3} \). To the second order in \( \delta a^{1/3} \), using expressions \( \text{A3} \), \( \text{A1} \) and \( \text{A2} \), one obtains:

\[ L^1 \times c \left( \frac{a}{GM} \right) = \frac{2}{\sqrt{3}} \times \left( 1 + 2^{2/3} \delta a^{1/3} + 2^{-5/6} \delta a^{2/3} \right) + O(\delta a) \]

(8)

\[ E^1 \times c^{-2} = \frac{1}{\sqrt{3}} \times \left( 1 + 2^{2/3} \delta a^{1/3} - \frac{5}{3} 2^{-8/3} \delta a^{2/3} \right) + O(\delta a) \]

(9)

For Kerr parameter evolution, one obtains through direct substitution of the above expansions into the spin-up law \( \text{B1} \):
\[
\frac{d\delta a}{d\ln M} \simeq 2 \left[ \left( 2^{-5/6} + \frac{5}{3} 2^{-8/3} \right) \delta a^{2/3} + \sqrt{3} \left( \frac{c}{2GM} J_{\text{rad}}' - \frac{1}{c^2} E_{\text{rad}}' \right) \right],
\] (10)

This expression provides a general scaling for the equilibrium rotation parameter provided that \(\delta a_{eq}\) is small and the impact of radiation is much smaller than that of accreted matter:

\[
\delta a_{eq} \propto \left( \frac{c}{2GM} J_{\text{rad}}' - \frac{1}{c^2} E_{\text{rad}}' \right)^{3/2}
\]

Here, \(J_{\text{rad}}'\) and \(E_{\text{rad}}'\) scale with the local radiative efficiency of accretion \(\epsilon\), that implies \(\delta a \propto \epsilon^{3/2}\). For the standard disc case, they do not depend on the mass accretion rate, but dependence on accretion rate may arise for the inner disc face. While in the thin-disc limit, \(E_{\text{rad}}'\) and \(J_{\text{rad}}'\) do not depend on \(\dot{m}\), existence of an inner face with \(H \propto \dot{m}\) leads to \(J_{\text{rad}}' \propto E_{\text{rad}}' \propto \dot{m}\) and hence \(\delta a_{eq} \propto \dot{m}^{3/2}\). Broader applicability of this scaling is supported by the more comprehensive numerical results given in section 3.1.

If the disc is non-Keplerian but its inner rim is fixed to the ISCO radius, deviations from the Keplerian may play the main role. Spin evolution is then determined (in the \(\delta a \ll 1\) limit) by the following expression:

\[
\frac{d\delta a}{d\ln M} \simeq 2C_1 \left[ 3 \times 2^{-2/3} \delta a^{2/3} - \left( \frac{1}{C_1} - 1 \right) \right],
\]

where \(C_1\) is the angular momentum at the ISCO in the units of Keplerian angular momentum. Equilibrium spin value may be supported without radiation capture in this case:

\[
\delta a_{eq} \simeq 2 \times 3^{-3/2} \left( \frac{1}{C_1} - 1 \right)^{3/2}
\]

3 RESULTS

3.1 Inner edge of a thick disc

Most of the internal energy in the inner parts of X-ray binary discs is stored in the pressure of radiation diffusing upward towards the disc surface. If the disc abruptly becomes optically thick (that is expected in the case of disc evaporation or near the ISCO), this radiation escapes due to diffusion and advection. The luminosity created by this “transition” radiation source is estimated by the expression (7) above. This radiation is much more efficient in spinning down the black hole than standard disc radiation.

Since disc thickness is proportional to mass accretion rate, the equilibrium Kerr parameter becomes dependent on the mass accretion rate. Maximal spin-down occurs for \(r \approx 2\sqrt{5}\) and results in \(\delta a_{eq} \sim 4 \times 10^{-4} \dot{m}^{-3/2}\) (see figure 2). The dependence on mass accretion rate is easily explained in the large-radius limit but holds to \(\sim 20\%\) accuracy even if the inner disc rim is close to the ISCO. One should expect near-critical and mildly super-critical accretion to be efficient in spinning down the BH to the probable \(a_{eq} \sim 0.995\).

Since the disc has some non-trivial vertical structure, its inner face is expected to be not exactly cylindrical. To account for this, I considered a photosphere inclined by different angles \(\eta\) (see figure 3). \(\eta\) is the angle between the normal to the emitting surface and vertical direction. The photosphere is assumed symmetric with respect to the disc plane that implies bi-conical shape of its surface.

It should be noted that for large spin parameter values, the shape of the inner parts of the thin relativistic disc deviates strongly from plane-parallel approximation. It may be shown that for \(\delta a \lesssim 10^{-2}\), the inner parts of the thin disc are inclined by \(\gtrsim 20^\circ\) to the equatorial plane.

Results of this subsection may be used to make a rough estimate for the maximal possible \(\delta a\) in the optically and geometrically thick supercritical disc if it is due to some reason (such as high viscosity or magnetic pressure) truncated outside the ISCO. The maximal possible disc thickness is \(H \sim R\) that corresponds to \(\dot{m} \simeq 2/3\eta(a) \simeq 3\), where \(\eta(a) \sim 0.32\) is accretion disc efficiency. For \(r_r = 2\), this implies \(\delta a \simeq 4 \times 10^{-4} \times (3)^{3/2} \sim 2 \times 10^{-3}\). Hence, equilibrium Kerr parameter is unlikely to become smaller than \(\sim 0.998\) through radiation capture from an optically and geometrically thick disc if the outer disc parts are invisible.

3.2 Truncated disc case

In figure 4 I show the dependence of equilibrium \(a\) on the inner truncation radius \(r_r\) in two extreme cases: if the optically thin disc part emits nothing (in this case, there is no Thorne spin-down term in the limit \(r_r \to \infty\)) and if it produces...
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Figure 2. Equilibrium Kerr parameter values (for the “radial photosphere” case considered in section 3.1) normalized by dimensionless mass accretion rate $\dot{m}^{3/2}$ for $\dot{m} = 0.1$ (solid lines), 1 (dotted) and 10 (dashed).

Figure 3. Equilibrium Kerr parameter values for the case of thick disc photosphere inclined by some angle $\eta$. Solid, dotted, dot-dashed and multiple dot-dashed lines correspond to $\eta = 0, 22, 44, 67$ and 90 degrees. Mass accretion rate is fixed to $\dot{m} = 1$.

exactly the same amount of radiation as standard disc, but the radiation is emitted isotropically. An optically thin disc has a potential for somewhat stronger radiative spin-down than the standard disc, leading (if it happens to be radiatively efficient and geometrically thin) to the equilibrium Kerr parameter value of $\delta a_{\text{eq}} \simeq (4.02 \pm 0.05) \times 10^{-3} \epsilon^{3/2}$.

There is a range of $r_{tr} \sim 1.5 \div 5$ where the efficiency of truncated disc radiation in spinning the black hole down may be higher than for the standard disc case. The prominent bump at these $r_{tr}$ is produced by the radial photosphere (see previous subsection). Its contribution grows rapidly with mass accretion rate and is responsible for the difference between the individual curves in figure 4. For mass accretion rates $\dot{m} \lesssim 0.01$, the variations in spin-down term with $\dot{m}$ are $\lesssim 20\%$.
Figure 4. Equilibrium Kerr parameter values as functions of truncation radius $r_{tr}$ for different mass accretion rate values ($\dot{m} = 10^{-2}, 4 \times 10^{-2}, 0.17, 0.72$ and $3$ are shown by solid, dotted, dashed, dot-dashed and double dot-dashed lines, respectively). In the left panel, the inner ADAF disk part emits nothing, in the right panel its locally emitted flux equals that for the standard disc. Horizontal lines mark the Thorne limits of $\delta a \simeq 0.0018$ and $0.0022$ and the maximal possible $\delta a \simeq 0.004$ for the optically thin case (not taking into account disc thickness, hence this value is higher than the limiting $\delta a$ for a hybrid disc).

The maximal possible $\delta a$ and the position of the maximum depend on the efficiency of the optically thin part of the flow and change from $r_{tr} \simeq 3$, $\delta a_{\text{max}} \simeq 3 \times 10^{-3}$ for $\epsilon = 0$ to $r_{tr} \simeq 4$, $\delta a_{\text{max}} \simeq 5 \times 10^{-3}$ for $\epsilon = 1$. One additional assumption needed for this estimates to work is direct visibility of the emitting surface of the disc from the black hole. If the disc launches wind or its inner parts are geometrically thicker the efficiency of radiation-capture mechanism is always smaller than of in the standard disc case (see previous subsection).

4 DISCUSSION

The effect of radiation spin-down is probably of little importance because magnetic fields are expected to spin black holes down much more efficiently (Blandford & Znajek 1977; Uzdensky 2005; Gammie et al. 2004). The role of mechanisms involving magnetic fields depends on the geometry of the field and on the existence of some extended load (such as jet). Hence the situation when magnetic fields are insufficient to stop the spin-up should not be excluded. Unlike the effect of radiation capture, the impact of Blandford–Znajek and similar process changes smoothly when $\delta a$ approaches zero and may be safely ignored unless it already provides an equilibrium $a < 1$ by itself.

Radiation of optically thin ADAF discs is of little importance for black hole evolution since the radiation is emitted inefficiently and hence the amplitude of the radiation contribution is several orders smaller than for the radiatively-efficient standard disc case. Potentially interesting case is hyperaccretion through a neutrino-emitting accretion disc (Chen & Beloborodov 2007). For stellar-mass black holes and $M \sim 10^{-3} \div 10^{-1} M_\odot$ yr$^{-1}$, accretion disc emits neutrinos in an optically-thin but radiatively efficient regime.

Another complication that should be taken into account in more comprehensive models is deviation from the Keplerian law possibly important for radiatively inefficient discs. Substantially sub-Keplerian flows are unable to spin the black hole up to Kerr parameters where radiation capture effect becomes important (Popham & Gammie 1998). It can be checked that if the net angular momentum at the last stable orbit differs from Keplerian by a factor of $c < 1$, spin-up proceeds up to some equilibrium value of $a < 1$. Estimates made in section 2.3 suggest scaling $\delta a_{eq} \simeq 2 \times 3^{-3/2} \times (1/c - 1)^{3/2}$ in this case. Deviations from Keplerian rotation become important if deviations from Keplerian law are $\gtrsim (1 / 2)^{\%}$. For thick discs with $H/R \gtrsim 0.1$, sub-Keplerian rotation may be a more important factor than radiative spin-down. Disc rotation faster than Keplerian by several percent makes it impossible to balance black hole rotation by radiation capture. Super-Keplerian slim discs considered by Sadowski et al. (2011) provide the black hole with matter having not only higher net angular momentum but also exceedingly high net energy hence the overall spin-up may be still stopped by radiation. Inner structure for these accretion disc models is profoundly different from the thin-disc approximation and also shows strong dependence on viscosity.
5 CONCLUSIONS

I come to the conclusion that radiative spin-down is sensitive to the geometry and optical depth of the emitting material. Existence of a radially oriented disc photosphere at several gravitational radii may increase the spin-down term by about a factor of 1.5 for large (near-critical) accretion rates in the disc if the inner edge of the standard disc part lies in the range \((2 \div 4)GM/c^2\). In other cases the effect of disc radiation is much smaller due to lower radiative efficiency. Non-Keplerian rotation becomes more important than radiation capture if deviations from Keplerian law exceed \(1\div 2\%\).

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APPENDIX A: SOME ESSENTIAL PROPERTIES OF THE THIN RELATIVISTIC DISC MODEL AND TRANSITION FROM THE ORBITING FRAME

This Appendix was introduced only for reference and does not contain any original research results. Since all the lengths scale either with the black hole mass or with the Kerr parameter, I assume here \(GM = c = 1\). Kerr metric in the Boyer-Lindquist coordinates may be expressed near the equatorial plane as:

\[
\frac{ds^2}{c^2} = -\alpha^2 dt^2 + \frac{\Sigma^2}{r^2} (d\varphi - \omega_{LT} dt)^2 + \frac{r^2}{\Delta} dr^2 + dz^2
\]

Here:

\[
\alpha^2 = \frac{r^2 \Delta}{\Sigma^2}
\]

\[
\Sigma^2 = r^4 + r^2 a^2 + 2ra^2 = r^4 A
\]

\[
\Delta = r^2 - 2r + a^2
\]

Lense-Thirring precession frequency:

\[
\omega_{LT} = \frac{1}{r^{3/2} + a} = B^{-1} r^{-3/2}
\]

The theory of relativistic thin accretion disk as introduced by Novikov & Thorne (1973); Page & Thorne (1974) operates a series of auxiliary factors depending on the radial coordinate and rotation parameters \(a\) that I here denote with calligraphic letters following the notation given by Penna et al. (2012).

Net angular momentum and energy on equatorial Keplerian orbits are expressed as:

\[
L^\dagger = \sqrt{\frac{\mathcal{F}}{\mathcal{C}}}, \quad (A1)
\]

\[
E^\dagger = \sqrt{\frac{G}{\mathcal{C}}}, \quad (A2)
\]

The inner rim of the disc is set by the innermost stable orbit radius that may be expressed as follows:

\[
\tau_{ISCO} = 3 + Z_2 - \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}; \quad (A3)
\]

where:

\[
Z_1 = 1 + (1 - a^2)^{1/3} \left((1 + a)^{1/3} + (1 - a)^{1/3}\right),
\]

\[
Z_2 = \sqrt{3a^2 + Z_1^2}
\]

If \(\delta a \ll 1\), the last stable orbit radius is a smooth function of \(\delta a^{1/3}\) that justifies the approximation I use in section...
Let us consider the orbiting frame moving with the matter with the four-velocity of \( u^t, u^\theta = 0, \ u^\varphi = \Omega u^t \). Normalization yields

\[
u^t = \frac{1}{\sqrt{\alpha^2 - g_{\varphi\varphi} (\Omega - \omega_{LT})^2}}
\]

I use the orbiting frame tetrad (see for example Novikov & Thorne (1973)) that is more convenient to express in terms of covariant (1-form) basis:

\[
\omega^t = \frac{1}{u^t} \times \left( dt - \frac{\alpha^2}{g_{\varphi\varphi}} (\Omega - \omega_{LT}) \times (d\varphi - \Omega dt) \right) = C^{-1/2} \times \left( G dt - \sqrt{r} F d\varphi \right)
\]

\[
\omega^\varphi = \frac{1}{u^t} \times \frac{\sqrt{g_{\varphi\varphi}}}{\alpha} (-\Omega dt + d\varphi) = C^{-1/2} \times \left( -\sqrt{\frac{D}{r}} dt + r B \sqrt{D} d\varphi \right)
\]

\[
\omega^r = \sqrt{g_{rr}} dr = D^{-1/2} dr
\]

\[
\omega^z = dz
\]

In the orbiting frame, a photon is characterized by the unit vector \( n^a \):

\[
n_t = -1; \quad n_\varphi = \sin \Theta \sin \Phi; \quad n_r = \sin \Theta \cos \Phi; \quad n_\varphi = \cos \Theta;
\]

These quantities may be connected to the coordinate-frame vector components as \( n^a = n_i e^a_i \). This allows to express the energy-at-infinity \(-u_t\) and angular momentum \( u_\varphi\) of a given photon as:

\[
-n_t = -\omega^a n_\varphi = \frac{1}{\sqrt{c}} \times \left( G + \sqrt{\frac{D}{r}} \sin \Theta \sin \Phi \right)
\]

\[
n_\varphi = \omega^a n_\varphi = \frac{1}{\sqrt{c}} \times \left( \sqrt{r} F + r B \sqrt{D} \sin \Theta \sin \Phi \right)
\]

These quantities multiplied by the local intensity and integrated over the solid angle give the energy-at-infinity and angular momentum fluxes (see section 2.2).

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