Simple refutation of the Eppley–Hannah argument

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Abstract

In an influential paper, Eppley and Hannah argued that gravity must necessarily be quantized, by proposing a thought experiment involving classical gravitational waves interacting with quantum matter. They argue the interaction must either violate the uncertainty principle or allow superluminal signalling. The feasibility of implementing their experiment in our universe has been challenged by Mattingly, and other limitations of the argument have been noted by Huggett and Callender and by Albers et al. However, these critiques do not directly refute the claim that coupling quantum theories with a Copenhagen collapse postulate to unentanglable classical gravitational degrees of freedom leads to contradiction. I note here that if the gravitational field interacts with matter via the local quantum state, the Eppley–Hannah argument evidently fails. This seems a plausibly natural feature of a hybrid theory, whereas the alternative considered by Eppley–Hannah is evidently inconsistent with relativity.

Keywords: classical gravity, entanglement, hybrid gravity theories

1. Introduction

In a paper [1] that has had significant influence in the quantum gravity community, Eppley and Hannah argued for the necessity of quantizing gravity, by claiming to show that a contradiction would arise in any dynamical theory in which a classical gravitational field interacts with quantum matter. Their argument invokes a thought experiment, in which the experimenter prepares gravitational waves of very short wavelength and low momentum (a consistent combination in classical theories) and uses these to probe the state of a localized quantum particle. They summarize their argument thus:
Briefly, we show that if a gravitational wave of arbitrarily small momentum can be used to make a position measurement on a quantum particle, i.e. to ‘collapse the wave function into an eigenstate of position’, then the uncertainty principle is violated. If the interaction does not result in collapse of the wave function, it is then possible to distinguish experimentally between superposition states and eigenstates. We show that this ability allows one to send observable signals faster than $c$ when applied to a state consisting of two spatially separated particles with correlated spins.

Huggett and Callender [2] noted that Eppley and Hannah’s arguments implicitly assume some version of quantum theory in which a collapse postulate applies, at least to standard measurements by standard matter-based devices\(^1\). It is not clear, for example, how to adapt their arguments to a theory in which matter is described by Everettian unitary quantum theory while the gravitational field is described classically. Similar comments apply to a de Broglie–Bohm quantum theory of matter coupled to classical gravity.

That said, versions of Copenhagen quantum theory with collapse postulates remain popular and may possibly be at least correct enough to justify Eppley–Hannah’s use of a projection postulate (though not, as we explain below, their inconsistent application of the projection postulate in relativistic quantum theory). The many versions [3] of Everettian quantum theory all have their own difficulties [3]. The most straightforward way of trying to couple Everettian quantum theory to classical gravity, via semi-classical gravity theories, is inconsistent with observation [4]. The same difficulty holds if a classical gravitational field is coupled to the unitarily evolving wave function in a de Broglie–Bohm theory of matter, while coupling a classical gravitational field to the de Broglie–Bohm particles seems likely to allow superluminal signalling. Huggett and Callender’s criticisms thus qualify the scope of the Eppley–Hannah argument, but (depending on one’s perspective on quantum foundations) may not necessarily remove much of its force.

Mattingly [5] challenged the argument on different grounds, arguing in impressive detail that the thought experiment is not merely impossible with any foreseeable technology but likely impossible in principle, given the known values of fundamental constants, the size of the observable universe and the amount of matter it contains. This makes the thought experiment’s status questionable: what logical force does it have if physics makes it impossible?

Here again, there is room for debate. Mattingly did not claim to show that no experiment along the lines of Eppley and Hannah’s proposal could ever be carried out, only that their specific proposal could not. The burden of proof may indeed be, as he noted, with Eppley and Hannah, but some may nonetheless have felt encouraged to think that some feasible experiment could be devised with more ingenuity, even if a concrete proposal presently eludes them.

A perhaps stronger counter-argument is that Eppley and Hannah’s thought experiment seems to be pointing to a conceptual problem preventing a unified theory of classical gravity and quantum matter. On one view of physics, if this problem is solvable, it ought to be solvable in a large class of logically possible universes, including universes that do not have the apparently contingent features that preclude carrying out the thought experiment in ours. On this view, if the thought experiment leads to contradiction in such universes, the problem is (most likely) unsolvable.

Albers, Kiefer and Reginatto (AKR) [6] noted a key error in Eppley–Hannah’s discussion of entangled systems and presented a model of a gravitational wave interacting with a quantized scalar field. AKR’s model is based on Nördstrom’s scalar theory of gravity in 1 + 1 dimensional Minkowski space, rather than a standard theory in 3 + 1 dimensions. This model

\(^1\)As we discuss below, Eppley and Hannah suppose that the gravitational field can also be used to carry out measurements on quantum systems, and that these measurements need not necessarily involve collapse.
was chosen to simplify the calculations. Their discussion is nonetheless still quite complex, and solutions for the relevant interacting case are presented only to first order in $g$, and are argued to be valid to this order only for specific quantum field states and gravitational waves satisfying particular conditions. Moreover, the formalism of interacting classical-quantum systems they consider leads them to conclude that ‘a consistent theory of interacting classical-quantum systems leads to final states that are entangled’. To be clear, this entanglement is meant to be between the classical and quantum sectors.

Clearly, there is no universal consensus on precisely what it means to say a theory is classical, or on how to delineate the class of classical-quantum hybrid theories. Nonetheless, from a modern information-theoretic perspective on physics, this conclusion seems misstated: a key difference between classical and quantum degrees of freedom is that classical degrees of freedom cannot be entangled with anything. The discussion of [6] thus does not seem to apply to classical-quantum hybrid theories as most theorists would currently understand the term.

2. Eppley–Hannah and collapse

In fact, Eppley–Hannah’s argument is flawed, even granting the assumption of a version of quantum theory with collapses, and regardless of the feasibility of their thought experiment.

To eliminate their contradiction, it is enough to show that one of the alternatives they consider leads to no contradiction. I will focus on their case 3, in which they consider ‘scattering of the gravitational wave from the wave function of the quantum particle with no collapse’. As they say, this could in principle be ‘an experimental method for observing the wave function without collapsing it’. It could also imply that ‘there exists a direct way of viewing [a] collapse event when it is produced by an ordinary measurement’. But it is not necessarily the case that this ‘would lead to the possibility of sending signals faster than $c$ in a hybrid theory, as they argue.

Eppley–Hannah present an experiment involving entangled photons split at each end by calcite crystals into polarization-dependent beams. They represent the individual photon states by position space wave functions. This may concern some readers, since the status of photon wave functions is controversial. However, it is not essential to their argument, which could equally well be carried out by considering entangled states of massive particles.

Eppley–Hannah proceed to argue as follows. Suppose we prepare an EPR-type state of the form

$$
\psi^L(x - x_0^L)\psi^R(x - x_0^R) + \psi^L(x - x_1^L)\psi^R(x - x_1^R),
$$

where the state $\psi^L(x - x_0^L)$ is localized around $x_0^L$ and so on. Suppose these factors of the wave function components are all localized within a scale $a$, so that

$$
\int_{|x| \leq a} |\psi^L(x)|^2 dx = 1,
$$

and similarly for $\psi^R$. Suppose also that $|x_0^L - x_0^R| \neq |x_0^R - x_0^L|$ where $b > 2a$, and that $|x_0^L - x_0^R| \gg b$.

Now Eppley–Hannah claim, oddly, that if no measurement has been carried out on $L$ then the $R$ system is described by the wave function $\psi^R(x - x_0^R) + \psi^R(x - x_1^R)$. There are two things wrong with this. First, one needs to fix a reference frame in order to relate two time-dependent
statements. A correct statement would be that, if we calculate and apply the collapse postulate in a given frame, and if system \( L \) has not been measured at time \( t \) in that frame, then in the standard relativistic quantum formalism we may say the wave function at time \( t \) in that frame has not collapsed. Second, as Albers et al note [6], even given such a frame choice, this does not give us a pure state wave function for system \( R \). The correct statement is that system \( R \) is in an improper mixed state, described by the density matrix giving a uniform mixture of \( \psi^R(x - x_0^L) \) and \( \psi^R(x - x_1^L) \). However, correcting this statement alone does not per se refute the next part of Eppley–Hannah’s argument, since this mixture is distinguishable from the individual states if one is given a system described by one of the three possible states and is able (by some hypothetical classical-quantum probe that does not follow standard quantum measurement rules) to identify the system’s quantum state.

Eppley–Hannah also claim that if a measurement has been carried out on \( L \), effectively localizing the position around either \( x_0^L \) or \( x_1^L \), then the \( R \) system is described either by \( \psi^R(x - x_0^L) \) or by \( \psi^R(x - x_1^L) \). Again, one needs to fix a reference frame to make such a statement, although it is true if a suitable frame is chosen. In standard relativistic quantum theory, of course, while the statement is mathematically true in a suitable frame, it is knowable only by an observer who knows that a localised position measurement has been carried out on \( L \). In order to know which description of the \( R \) system applies, the observer also needs to know the outcome of the measurement on the \( L \) system. Both of these are possible for an observer at \( L \) but not (until a time in the future light cone of the measurement at \( L \)) for an observer at \( R \). This frame-dependent description of the wave function does not lead to superluminal signalling or other observable consequences in standard relativistic quantum theory, because the wave function is not directly observable and the measurement postulates ensure that the observable information is frame-independent.

Finally, Eppley–Hannah claim that in a hypothetical hybrid theory in which interactions between gravitational waves and quantum matter do not collapse the wave function of the latter, an interaction between a classical gravitational wave and system \( R \) allows the first case (no measurement on \( L \)) to be distinguished from the second case (measurement on \( L \)).

The problem here is that, even if we restrict to Minkowski space rather than general space-times, it makes no sense to postulate a relativistic theory of quantum states, together with a collapse postulate, in which (i) collapse is a real physical phenomenon (since position space wave functions have objective and measurable meaning in the type of hybrid theory that Eppley–Hannah postulate), and (ii) collapse propagates instantaneously. In a relativistic theory instantaneity is frame-dependent, whereas a real physical effect cannot be. The fundamental problem with Eppley–Hannah’s proposal is thus not that it gives rise to superluminal signalling (which, although problematic and counter-intuitive in a relativistic theory, need not necessarily lead to contradiction (see e.g. [10])), but that it is incoherent.

There is, however, a consistent way of defining a relativistic theory of quantum states, together with a collapse postulate, in which collapse is a real physical phenomenon, local quantities defined by quantum states have objective and measurable meaning, and no superluminal signalling is possible [11, 12]. This is to suppose that the effects of collapse propagate causally, and that the objective and measurable quantity at a point \( x \) is the local quantum state \( \rho_{bc}(x) \).

We define \( \rho_{bc}(x) \) to be local density matrix of \( \psi_{\Lambda(x)} \) at \( x \), taken as the limit of the local density matrices for the wave functions of spacelike hypersurfaces \( S \) tending to the past light cone \( C = \Lambda(x) \) of \( x \). To understand the definition of \( \rho_{bc}(x) \), consider figure 1, adapted from [11].

Here \( C \) is the surface of the past light cone of \( x \). Take a family \( \{ H_n : n = 1, 2, \ldots \} \) of spacelike hypersurfaces which go through \( x \) and which asymptotically tend to \( C \). Write \( H_n \setminus \{ x \} \) for the hypersurface \( H_n \) with the point \( x \) removed.

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Let $|\psi^n\rangle$ be the wave function of the full quantum system on $H_n$, and define
\[ \rho_n(x) = \text{Tr}_{H_n}\{ |\psi^n\rangle\langle \psi^n | \}. \] (3)
Then the local state at $x$ is
\[ \rho_{\text{loc}}(x) = \lim_{n \to \infty} \rho_n(x). \] (4)
Thus the local state is given by taking the joint wave function of the complete system, defined by allowing for only collapses in the past light cone of the particle, and then tracing out the rest of the system.

To understand the essential point of this definition, consider a simplified example in which at time $t = 0$ there are two entangled pointlike particles at rest at spatial locations $L$ and $R$ in a state
\[ a|0\rangle_L|1\rangle_R + b|1\rangle_L|0\rangle_R, \] (5)
with zero Hamiltonian (in the absence of measurements). Let the separation between $L$ and $R$ in their mutual rest frame be $d$. At $t = 0$, the local state at $L$ is then
\[ |a|^2|0\rangle_L|0\rangle_R + |b|^2|1\rangle_L|1\rangle_R; \] (6)
the local state at $R$ is
\[ |b|^2|0\rangle_R|0\rangle_L + |a|^2|1\rangle_R|1\rangle_L. \] (7)
Here $|0\rangle, |1\rangle$ are orthonormal basis states of some physical qubit on each side. Now suppose that at $t = T$ a measurement is carried out in the $|0\rangle_L, |1\rangle_L$ basis on the $L$ particle, with outcome 0. At this point, the local state at $L$ becomes
\[ |0\rangle_L|0\rangle_R. \] (8)
However, for $T < t < T + \frac{d}{c}$, the local state at $R$ remains
\[ |b|^2|0\rangle_R|0\rangle_L + |a|^2|1\rangle_R|1\rangle_L. \] (9)
This changes only when $t \geq T + \frac{d}{c}$, when a light signal describing the outcome of the $L$ measurement could have reached $R$: the local state then becomes
\[ |a|^2|1\rangle_R|1\rangle_L. \] (10)
Note that this definition holds whether or not any communication actually occurs between observers at $L$ and $R$. Indeed, we do not necessarily need to assume there are any observers at $L$ and $R$, only that we are working with some version of quantum theory in which collapse or

Figure 1. Spacelike hypersurfaces tending to the past light cone.
measurement events are defined and localised, so that it makes sense to say that the $L$ particle is measured at $t = 0$.

In a more realistic model, the relevant systems are not pointlike particles but have wave functions with extended support, as in equation (1), or multi-particle wave functions, or field functionals. If classical gravitational fields interact with quantum states in such a theory, it is natural to postulate that they interact with $\rho^{loc}(x)$, which also generally has extended support at any given time. Such interactions allow descriptions of the quantum state of a localized system, in the sense that they could allow a complete classical readout of $\rho^{loc}(x)$ for $x$ in a localized region of space-time, without causing any collapse. A simulation argument shows that an effective readout device of this type would not allow superluminal signalling [11].

3. Conclusions

No-go theorems in physics are often presented as insuperable obstacles when they are actually only challenges to the imagination. This is the case with Eppley and Hannah’s argument, which is based on a limited and self-inconsistent model of the possible form of interactions between classical fields and quantum states.

It was shown some time ago [11] that perfect classical state readout devices can be consistently added to relativistic quantum theory without superluminal signalling, so long as the readout devices produce a classical readout of the local state $\rho(x)$. The implications of this result for non-standard theories and tests of gravity have been widely appreciated in the quantum foundations community; for example, it plays a significant part in the space QUEST mission proposal [13]. However, they seem to have been less widely appreciated in the quantum gravity community, and as far as I am aware the implications for the Eppley–Hannah argument have not previously been discussed. I hope this paper may help lay that argument to rest, and lead to a broader understanding of the possibilities for classical-quantum hybrid theories and a more balanced and nuanced view of the arguments for quantum gravity.

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References

[1] Eppley K and Hannah E 1977 The necessity of quantizing the gravitational field Found. Phys. 7 51–68
[2] Huggett N and Callender C 2001 Why quantize gravity (or any other field for that matter)? Phil. Sci. 68 S382–94
[3] Saunders S, Barrett J, Kent A and Wallace D 2010 Many Worlds? Everett, Quantum Theory and Reality (Oxford: Oxford University Press)
[4] Page D N and Geilker C 1981 Indirect evidence for quantum gravity Phys. Rev. Lett. 47 979
[5] Mattingly J 2006 Why Eppley and Hannah’s thought experiment fails Phys. Rev. D 73 064025
[6] Albers M, Kiefer C and Reginatto M 2008 Measurement analysis and quantum gravity Phys. Rev. D \textbf{78} 064051

[7] Kiefer C 2012 Quantum Gravity 3rd edn (Oxford: Oxford University Press)

[8] Hossenfelder S 2012 Eppley and Hannah’s thought experiment (http://backreaction.blogspot.com/2012/01/eppley-and-hannahs-thought-experiment.html)

[9] Weinstein S and Rickles D 2018 Quantum gravity The Stanford Encyclopedia of Philosophy ed E N Zalta (Stanford, CA: Stanford University) (summer 2018 edition) (https://plato.stanford.edu/archives/sum2018/entries/quantum-gravity/)

[10] Kent A 1998 Causality in time-neutral cosmologies Phys. Rev. D \textbf{59} 043505

[11] Kent A 2005 Nonlinearity without superluminality Phys. Rev. A \textbf{72} 012108

[12] Kent A 2005 Causal quantum theory and the collapse locality loophole Phys. Rev. A \textbf{72} 012107

[13] Joshi S K et al 2018 Space QUEST mission proposal: experimentally testing decoherence due to gravity New J. Phys. \textbf{20} 063016