Determination of the nuclear incompressibility from the rapidity-dependent elliptic flow in heavy-ion collisions at beam energies 0.4A - 1.0A GeV

Yongjia Wang\textsuperscript{a,}\textsuperscript{*}, Chenchen Guo\textsuperscript{b}, Qingfeng Li\textsuperscript{a,c,\textsuperscript{*}}, Arnaud Le Fèvre\textsuperscript{d}, Yvonnes Leifels\textsuperscript{e}, Wolfgang Trautmann\textsuperscript{d}

\textsuperscript{a}School of Science, Huzhou University, Huzhou 313000, China
\textsuperscript{b}Sino-French Institute of Nuclear Engineering and Technology, Sun Yat-sen University, Zhuhai 519082, China
\textsuperscript{c}Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
\textsuperscript{d}GSI Helmholtzzentrum für Schwerionenforschung GmbH, D-64291 Darmstadt, Germany

Abstract

Background: The nuclear incompressibility ($K_0$) plays a crucial role in understanding diverse phenomena in nuclear structure and reactions, as well as in astrophysics. Heavy-ion-collision measurements in combination with transport model simulations serve as important tools for extracting the nuclear incompressibility. However, uncertainties in transport models (or model dependence) partly affect the reliability of the extracted result. Purpose: In the present work, by using the recently measured data of rapidity-dependent flows, we constrain the incompressibility of nuclear matter and analyse the impact of model uncertainties on the obtained value. Method: The method is based on the newly updated version of the ultrarelativistic quantum molecular dynamics (UrQMD) model in which the Skyrme potential energy-density functional is introduced. Three different Skyrme interactions which give different incompressibilities varying from $K_0=201$ to $271$ MeV are adopted. The incompressibility is deduced from the comparison of the UrQMD model simulations and the FOPI data for rapidity-dependent elliptic flow in Au+Au collisions at beam energies 0.4A - 1.0A GeV. Results: The elliptic flow $v_2$ as a function of rapidity $y_0$ can be well described by a quadratic fit $v_2 = v_{20} + v_{22} y_0$... It is found that the quantity $v_2$ defined by $v_2 = [v_{20} + v_{22}]$ is quite sensitive to the incompressibility $K_0$ and the in-medium nucleon-nucleon cross section, but not sensitive to the slope parameter $L$ of the nuclear symmetry energy. Conclusions: With the FU3FP4 parametrization of the in-medium nucleon-nucleon cross section, an averaged $K_0 = 220\pm40$ MeV is extracted from the $v_{20}$ of free protons and deuterons. However, remaining systematic uncertainties, partly related to the choice of in-medium nucleon-nucleon cross sections, are of the same magnitude ($\pm40$ MeV). Overall, the rapidity dependent elliptic flow supports a soft symmetric-matter equation-of-state.

Keywords: Nuclear equation of state, transport model, nuclear symmetry energy, heavy ion collision, collective flow

PACS: 21.65.-f, 21.65.Mn, 25.70.-z

The nuclear incompressibility ($K_0$) is defined as the derivative of pressure $P$ with respect to the density $\rho$, $K_0 = 9 \left( \frac{\rho}{\rho_0} \right) \left( \frac{E}{A} \right)_{\rho_0 = \rho_0}$, or the curvature of the energy per nucleon $E/A$ in nuclear matter at the saturation density ($\rho_0$), $K_0 = 9 \rho_0^2 \left( \frac{dE}{d\rho} \right)_{\rho_0 = \rho_0}$. The saturation density $\rho_0 \sim 0.16$ fm$^{-3}$ and the saturation energy $E_0 \sim -16$ MeV have been widely accepted and appear in textbooks. The equation of state (EOS) of symmetric nuclear matter can be expanded as $\frac{1}{K_0} (\rho) = E_0 + \frac{1}{K_0} (\frac{\rho}{\rho_0})^2 + ..., therefore, a more accurate value of $K_0$ means a better understanding of the EOS in the vicinity of the saturation density. As the knowledge of the EOS is essential for studying nuclear structure and reactions, as well as astrophysics, many attempts have been made to infer $K_0$ by using experimental data on the properties of nuclei (such as the giant monopole/dipole resonance and the nuclear masses and radii) or heavy-ion collisions (HIC) since 1960s.

Constraints on $K_0$ through comparing experimental data on nuclear properties (such as, the giant monopole resonance (GMR) energies in some nuclei, nuclear masses and charge radii) and theoretical model (as, e.g., the Skyrme-Hartree-Fock or relativistic mean-field extended by quasiparticle random-phase approximation) calculations have been summarized recently in Ref.\cite{1}. In Ref.\cite{2}, the authors studied the GMR energies of $^{208}$Pb and $^{120}$Sn, based on the constrained Hartree-Fock-Bogoliubov (CHFB) approach, and pointed out that $K_0$ varies in the region of $190 < K_0 < 270$ MeV. However, different models offer a wide range of results for $K_0$ (see, e.g., \cite{1, 2, 3} and references therein). Heavy-ion collisions provide the unique way to compress nuclear matter to high densities in the laboratory. They can serve as a powerful tool for studying the EOS of dense nuclear matter. However, the compressed nuclear matter exists only for a very short time (typically from several to several tens of fm/c, $10^{-23}$ to $10^{-22}$ s), therefore, $K_0$ cannot be measured directly but only inferred from the comparison of experimental measurements with transport model simulations.

Since 1960s, several heavy-ion collision facilities became available, such as the BEVALAC at Berkeley and the NSCL at Michigan State University in US, the SIS of GSI at Darmstadt in Germany, the GANIL cyclotron at Caen in France, and the
Table 1: Saturation properties of nuclear matter as obtained with selected Skyrme parameterizations used in this work.

| Parameterization | $K_0$ (MeV) | $S_0$ (MeV) | $L$ (MeV) |
|------------------|-------------|-------------|------------|
| Skxs15           | 201         | 31.88       | 34.79      |
| MSK1             | 234         | 30.00       | 33.92      |
| SKX              | 271         | 31.10       | 33.18      |
| SV-sym34         | 234         | 34.00       | 80.95      |

To permit a better description of the recent experimental data in HICs at intermediate energies, the Skyrme potential energy density functional has been introduced into the mean-field potential part of the UrQMD code. It is found that with an appropriate choice of the in-medium nucleon-nucleon cross section, the recent published experimental data can be reproduced fairly well. In this work, the Skxs15, MSK1, and SKX interactions are chosen which give quite similar values of nuclear symmetry energy (the symmetry energy coefficient $S_0$ and the slope parameter $L$) but the incompressibilities $K_0$ varies from 201 MeV to 271 MeV (e.g., see Table I). It should be pointed out that, the slope parameter $L$ given by the three selected interactions is approximately 34 MeV, i.e., smaller than the average value of 59 MeV. Because the main purpose of this work is to study $K_0$ from elliptic flow, with the introduction of the Skxs15, MSK1, and SKX interactions are illustrated in Fig.1. For comparison, constraints obtained by Danielewicz et al. and by Le Fèvre et al. are also shown with shaded bands. The pressure predicted by SKX lies close to the upper limit of the result of Danielewicz et al., and the pressure given by Skxs15 lies roughly in the center of the two bands.

Besides the mean field potential part, the in-medium nucleon-nucleon cross section in the collision term, which is still not well-established, also noticeably affects the collective flow. Based on our previously studies, it is found that, by considering a density- and momentum- dependent reduction factor on the free nucleon-nucleon elastic cross section, i.e., the so-called FU3FP4 parametrization, the collective flow and the stopping power data in Au+Au collision at intermediate energies can be reproduced quite well and better. The FU3FP4 set is, therefore, also here adopted as the preferred parametrization. To
show how the in-medium nucleon-nucleon cross section affects the incompressibility; the FU3FP5 parametrization, which represents a stronger reduction of the in-medium elastic cross section, is also considered. Details about the FU3FP4 and FU3FP5 parametrizations of the in-medium elastic nucleon-nucleon cross section can be found in our previous publication [32].

Both the slope of \( v_1 \) and the absolute value of \( v_2 \) at mid-rapidity calculated with SKX, i.e., with a stronger repulsive potential, are larger than that with Skxs15 but the differences are small. At mid-rapidity, the elliptic flow \( v_2 \) calculated with SKX is more negative than that calculated with Skxs15; it means protons are more preferentially emitted perpendicular to the reaction plane in the SKX case. Towards the target or projectile rapidity (\( y_0 = 1 \) or \( -1 \)), the \( v_2 \) calculated with SKX is more positive and crosses the zero line at smaller absolute values of \( y_0 \) than with Skxs15, indicating that the sideways deflection of the spectator matter for SKX is stronger than that for Skxs15. The reason is that, at the mid-rapidity (the overlapping region of the target and projectile), the expanding participant matter will be blocked by the spectator matter and will preferentially be squeezed out in directions perpendicular to the reaction plane. A higher pressure generated by the higher incompressibility of SKX leads to a stronger expansion and, consequently, a more negative \( v_2 \).

Around the target or projectile rapidity (spectator nucleons), a higher pressure with SKX causes a stronger deflection of spectator nucleons in the reaction plane resulting in a larger transverse momentum \( p_t \) and thus to larger coefficients \( v_1 \) and a more positive \( v_2 \) [44]. It has been found that both the directed and elliptic flow at intermediate energies exhibit approximately the scaling behavior, see, e.g., Refs. [41, 42, 43]. The newly measured high-precision collective flow data provides a new opportunity to investigate the scaling behavior. Further studies are certainly required to understand the physics behind the scaling deviations.

In addition, it has been found that the \( v_2 \) as a function of rapidity \( y_0 \) can be well described by a quadratic fit \( v_2 = v_{20} + v_{22} y_0^2 \). According to the calculated results shown in Fig.2, it can be inferred that a stiff EOS leads to larger values of both \( v_{20} \) and \( v_{22} \). Thus the sensitivity to the EOS can be enhanced by using the observable \( v_{2n} = |v_{20}| + |v_{22}| \), as discussed in Ref. [28].

The \( v_{2n} \) of free protons produced in \( {}^{197}\text{Au}+{}^{197}\text{Au} \) collisions at \( E_{\text{lab}} = 0.4 \text{A GeV} \) are shown in Fig.3. Calculations with Skxs15, MSK1, and SKX interactions are compared to the FOPI experimental data, as well as to the IQMD calculations, taken from Ref. [28]. First, it can be seen that, the \( v_{2n} \) increases strongly with increasing \( K_0 \) in both the IQMD and UrQMD model calculations, implying that the \( v_{2n} \) is indeed sensitive to the incompressibility \( K_0 \), though this slope dependence is not exactly the same for the two models. At 0.4A GeV, the values of \( v_{2n} \) calculated with the IQMD model are significantly larger than that of the UrQMD model, and the difference steadily decreases with increasing beam energy. Reasons for this will be discussed later.

Second, the results of the UrQMD model exhibit an approximate linearity between the \( v_{2n} \) and the incompressibility \( K_0 \). The intersections between the lines and the shaded bands provide the range of expectation for the incompressibility \( K_0 \). The \( v_{2n} \) calculated with the FU3FP4 and FU3FPS parametrizations are well separated at 0.4A GeV, but overlap quite well at 1.0A GeV. The main difference between the FU3FP4 and FU3FPS parametrizations consists in a larger reduction of the nucleon-nucleon elastic cross section at lower relative momenta in FU3FPS than in FU3FP4. The reduction factor for both

Figure 2: (Color online) The directed flow \( v_1 \) (upper panels) and elliptic flow \( v_2 \) (lower panels) of free protons produced in \( {}^{197}\text{Au}+{}^{197}\text{Au} \) collisions at \( E_{\text{lab}} = 0.4 \text{A GeV} \) (a,e), 0.6A GeV (b,f), 0.8A GeV (c,g), and 1.0A GeV (d,h) with centrality \( 0.25 < b_0 < 0.45 \) and the scaled transverse velocity \( u_0 > 0.4 \). Results calculated with Skxs15 (solid line) and SKX (dashed line) together with the FU3FP4 parametrization of the in-medium nucleon-nucleon cross section are compared with the FOPI experimental data (stars).
FU3FP4 and FU3FP5 approaches unity at higher relative momentum \[12\] hence at higher incident energies.

On average, the central value of the incompressibility \(K_0\) is obtained to be 240 MeV for calculations with the FU3FP4 parametrization, while it reaches 275 MeV for FU3FP5. Those results are larger than that from the IQMD model simulations using the same observable, which is about 222 MeV (evident also from Table 2 in Ref.\[28\]). The main difference comes from the collision term in the two models, i.e., the free nucleon-nucleon cross section is used in the IQMD model, while a density- and momentum-dependent in-medium nucleon-nucleon cross section is used in the UrQMD model. The difference between the two model calculations becomes smaller at higher beam energies. This can be understood from the near equivalence of the in-medium and free cross sections at the higher relative momenta prevailing at higher beam energies. The remaining difference between the two models may stem from (I) different treatments in the Pauli blocking which also determine the collision rate. It can be seen from the transport model comparison paper \[12\] that the Pauli blocking rate in the IQMD model is higher than that in the UrQMD model. Therefore, a larger sensitivity of \(v_{2n}\) to the incompressibility \(K_0\) is seen in the IQMD model. They may further stem from (II) different values of the width of the Gaussian wave packet, as well as different parameters used in the cluster recognition criteria. Influences of those treatments on the \(v_{2n}\) deserve further studies. With a weaker reduction of the in-medium nucleon-nucleon cross section, the extracted \(K_0\) will be smaller. It may explain the reason why the \(K_0\) obtained from the IQMD model is smaller than that from the UrQMD model. \(K_0 = 240 \pm 20\) MeV (\(K_0 = 275 \pm 25\) MeV) for the FU3FP4 (FU3FP5) parametrization of the in-medium nucleon-nucleon cross section, which best describes the experimental data, can be extracted within a 2-\(\sigma\) confidence limit from the chi-square test.

Third, we note that with both models, the value of extracted \(K_0\) increases with increasing beam energy, e.g., \(K_0 = 180 \pm 20\) MeV is favored at the beam energy of 0.4A GeV while \(K_0 = 280 \pm 17\) MeV fits best to the data taken at 1.0A GeV in the UrQMD model description with FU3FP4. It is known that the emitted free protons are sensitive on both the maximum densities reached in the collision and to the contributions of inelastic channels (mainly the Delta degree of freedom), which become larger with an increasing incident energy. If these dependencies are not fully reproduced by the transport model and the employed Skyrme forces, e.g., the high orders of the EOS dependence on density like the skewness, they may result in an energy dependence of the deduced \(K_0\) parameter as it is observed here.

Further, one sees clearly that the results for \(v_{2n}\) calculated with SV-sym34 and MSK1, i.e., with the same value of \(K_0\), are very close to each other even though the difference in \(L\) is as large as 47 MeV. It illustrates the \(v_{2n}\) is much more sensitive on the nuclear incompressibility than on the nuclear symmetry energy. Because the nuclear symmetry potential is relatively weak compared to the isoscalar part of the nuclear potential, its weak effect on observables is not easy to see, usually, the difference or ratio between isospin partners can provide some
hints for the isovector part of the nuclear potential.

In summary, by comparing the UrQMD model calculations with the recent FOPI data for the elliptic flow in Au+Au collisions in the beam energy range 0.4A - 1.0A GeV, it is found that the nuclear incompressibility $K_0$ is quite sensitive to the $v_{2n}$, a quantity obtained from a quadratic fit ($v_2 = v_{20} + v_{22} \cdot v_{20}^2$) of the elliptic flow as a function of rapidity by adding the coefficients as $v_{2n} = |v_{20}| + |v_{22}|$, and the $v_{2n}$ increases almost linearly with increasing the incompressibility $K_0$. The influences of the in-medium nucleon-nucleon cross section and the nuclear symmetry energy on the $v_{2n}$ are also analyzed. It is found that the $v_{2n}$ can be affected by the in-medium nucleon-nucleon cross section but hardly influenced by the nuclear symmetry energy. With the FUSFP4 parameterization (i.e., the preferred choice in the present version of the UrQMD model) of the in-medium nucleon-nucleon cross section, $K_0 = 240 \pm 20$ MeV and $K_0 = 190 \pm 20$ MeV are extracted from the $v_{2n}$ of free protons and deuterons, respectively. By combining the error intervals of the proton and deuteron results, an averaged $K_0 = 220 \pm 40$ MeV is obtained. The extracted $K_0$ will be smaller (larger) if a weaker (stronger) reduction on the in-medium nucleon-nucleon cross section is used. Additional calculations with other model assumptions and/or transport models will be certainly required to

Figure 4: (Color online) The same as Fig but for the $v_{2n}$ of deuterons.
confirm the sensitivity of the $v_{2n}$ to the EOS and assess the obtained result.

Acknowledgements

Fruitful discussions with Ch. Hartnack are greatly appreciated. The authors acknowledge support by the computing server C3S2 in Huzhou University. The work is supported in part by the National Natural Science Foundation of China (Nos. 11505057, 11375062, 11647306, and 11747312), and the Zhejiang Provincial Natural Science Foundation of China (No. LY18A050002).

References

[1] J. R. Stone, N. J. Stone and S. A. Moszkowski, Phys. Rev. C 89, no. 4, 044316 (2014).
[2] E. Khan and J. Margueron, Phys. Rev. C 88, no. 3, 034319 (2013); E. Khan and J. Margueron, Phys. Rev. Lett. 109, 092501 (2012).
[3] G. Giuliani, H. Zheng and A. Bonasera, Prog. Part. Nucl. Phys. 76, 116 (2014).
[4] L. W. Chen, Sci. China Phys. Mech. Astron. 54, 124 (2011).
[5] G. F. Bertsch and S. Das Gupta, Phys. Rept. 160, 189 (1988).
[6] J. Aichelin, Phys. Rept. 202, 233 (1991).
[7] H. H. Gutbrod, A. M. Poskanzer, and H. G. Ritter, Rep. Prog. Phys. 52, 1267 (1989).
[8] Reisdorf W, Ritter H G. Ann Rev Nucl Part Sci, 1997, 47, 663-709.
[9] J. J. Molitoris and H. Stöcker, Nucl. Phys. A 447, 13c (1986).
[10] J. J. Molitoris and H. Stöcker, Phys. Rev. C 32, 346 (1985).
[11] H. Kruse, B. V. Jacak and H. Stöcker, Phys. Rev. Lett. 54, 289 (1985).
[12] J. Aichelin and C. M. Ko, Phys. Rev. Lett. 55, 2661 (1985).
[13] H. Stöcker and W. Greiner, Phys. Rept. 137, 277 (1986).
[14] W. Cassing, V. Metag, U. Mosel and K. Nitta, Phys. Rept. 188, 363 (1990).
[15] Y. M. Zheng, C. M. Ko, B. A. Li and B. Zhang, Phys. Rev. Lett. 83, 2534 (1999).
[16] D. Persram and C. Gale, Phys. Rev. C 65 (2002) 064611.
[17] A. Andronic et al. [FOPI Collaboration], Phys. Lett. B 612, 173 (2005).
[18] T. Gaifanos, C. Fuchs and H. H. Wolter, Phys. Lett. B 609, 241 (2005).
[19] B. A. Li and L. W. Chen, Phys. Rev. C 72, 064611 (2005).
[20] Y. Zhang and Z. Li, Phys. Rev. C 74, 014602 (2006); Y. Zhang, Z. Li and P. Danielewicz, Phys. Rev. C 75, 034615 (2007).
[21] B. Li, L. W. Chen and C. M. Ko, Phys. Rep. 464, 113 (2008).
[22] Q. Li, C. Shen, C. Guo, Y. Wang, Z. Li, J. Lukaski, W. Trautmann, Phys. Rev. C83, 044617 (2011).
[23] M. Kaur and S. Gautam, J. Phys. G 43, no. 2, 025103 (2016).
[24] M. D. Partlan et al. [EOS Collaboration], Phys. Rev. Lett. 75, 2100 (1995).
[25] P. Danielewicz, R. Lacey, and W. G. Lynch, Science 298,1592 (2002).
[26] C. Sturm et al., Phys. Rev. Lett. 86, 39 (2001); C. Fuchs, A. Faessler, E. Zabrodin and Y. -M. Zheng, Phys. Rev. Lett. 86, 1974 (2001); C. Hart- nack, H. Oeschler and J. Aichelin, Phys. Rev. Lett. 96, 012302 (2006).
[27] Z. Q. Feng, Phys. Rev. C 83, 067604 (2011).
[28] A. Le Fevre, Y. Leifels, W. Reisdorf, J. Aichelin and C. Hartnack, Nucl. Phys. A 945, 112 (2016).
[29] Jun Xu, Lai-Wen Chen, ManYee Betty Tsang et al., Phys. Rev. C 93, no. 4, 044609 (2016).
[30] W. M. Guo, G. C. Yong, Y. Wang, Q. Li, H. Zhang and W. Zuo, Phys. Lett. B 726, 211 (2013).
[31] W. M. Guo, G. C. Yong, Y. Wang, Q. Li, H. Zhang and W. Zuo, Phys. Lett. B 738, 397 (2014).
[32] Y. Wang, C. Guo, Q. Li, H. Zhang, Z. Li, and W. Trautmann, Phys. Rev. C 89, 034606 (2014).
[33] Y. Wang, C. Guo, Q. Li, H. Zhang, Y. Leifels, and W. Trautmann, Phys. Rev. C 89, 044603 (2014).
[34] M. Dutra, O. Lourenco, J. S. Sa Martins, A. Delfino, J. R. Stone and P. D. Stevenson, Phys. Rev. C 85, 035201 (2012).