The stress raisers effect on the fatigue safety coefficients

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Abstract. The fatigue limits of metallic materials may be established using their limit cycle diagrams, for various values of stress ratio. On the other hand, the level of actual mechanical stresses in a certain component is taken into account, together with the possible existence of some stress raisers on it, in order to establish (using the schematic limit cycle diagrams) the component safety coefficients in operation. The limit cycle diagrams are plotted in a \( \sigma_a - \sigma_m \) coordinate axis system, i.e. the stress amplitude versus the mean stress graphical representation; any single point on this graph is corresponding to a specific fluctuating stress cycle. Its point of intersection with the abscissa axis (the \( \sigma_m \) value for \( \sigma_a = 0 \)) represents a stress limit of the component material, from its stress-strain curve in static tensile test, namely \( \sigma_{UTS} \) (the ultimate tensile strength) – for predominantly brittle materials, and respectively \( \sigma_y \) (the yield stress) – for ductile ones. When representing a limit cycle diagram, for a component with some stress raisers in its structure, the possible influence of stress raisers on the above mentioned static stress limits values should be considered. The present paper describes a practical method for determining the fatigue safety coefficients, for a certain component, using the schematic Sersensen limit cycle diagram, and considering the stress raisers influence on the static stress limits values.

1. The effective stress concentration coefficient in fatigue

The presence of stress raisers reduces the resistance of the structures, both in a static and in a dynamic loading regime [1]. For a static loading, the concentration effect of elastic stresses is taken into account using the concentration coefficient \( K_t \) that is defined by the relation \( K_t = \frac{\sigma_{max}}{\sigma_n} \); it is the ratio between the maximum stress value and the nominal stress in the respective cross section, obtained with some usual strength of materials calculation relationships. This coefficient depends on the stress raiser geometry, the mode of loading and the suitable nominal stress definition; it does not depend on the material or on the applied load values. The use of a static concentration coefficient is not allowed when any local plastic deformation occurs in the component area [2]. The presence of a stress concentrator will obviously reduce the fatigue strength of a structure; on the other hand, it is well known that similar specimens, of the same material, with the same \( K_t \), with dimensions and radii of curvature of the larger outer concentrator have a high fatigue limit [3, 4]. From this point of view, the effect of the concentrator size on the component fatigue strength is noticed. One possible method for taking into account such an effect may be described as follows: draw the durability curve \( \sigma_{a,N} \) for a specimen without stress concentrators, divide the ordinates of the curve by the specimen concentration coefficient \( K_t \), thus obtaining a theoretical curve of durability \( \sigma_{akN} \), for the specimen with stress raisers, under the same loading condition as it was presumed for the reference specimen. When the durability curve \( \sigma_{akN} \) is experimentally determined for a given value of the \( K_t \) coefficient, it is found that it lies between the theoretical curve and the curve obtained for the stress-raiser-free specimen. It follows that a stress...
concentrator reduces the fatigue strength value to a lesser extent than it was estimated using the theoretical static concentration coefficient $K_t$. As a result, one may consider that the use of that parameter in the fatigue limit calculation could lead to unacceptable errors.

Some fatigue durability curves, in rotating bending, are presented in figure 1 [5], for an aluminum alloy; the diameter of the stress-raiser-free specimen (having $K_t=1$) had the same value as the minimum cross-sectional dimension of the specimen with stress raiser, for which the $K_f$ value was calculated as being 2.2, by using the following relation:

Some examples:

$$K_f = \frac{\sigma_{lim\text{-}without\text{ Conc}}}{\sigma_{lim\text{-}with\text{ Conc}}}.$$

The ordinates for the points on the $\sigma_R/3.1$ curve, from the bottom of figure 1, were calculated by dividing the corresponding ordinates from the above $K_t=1$ curve, by the stress concentration coefficient $K_t=3.1$, that was found for the actual specimen stress raiser. One may observe that, for the specimen with stress raiser, the fatigue limit value does not decrease in proportion to $K_t$ stress concentration coefficient, that correspond to a specimen static loading: in fact, the experimental determined fatigue limit values – for the specimen with a stress raiser – were permanently greater than those of the bottom curve, obtained by dividing the reference ($K_t=1$) curve values by the actual $K_t$ value.

![Figure 1](image-url)

**Figure 1.** The influence of stress concentration on the fatigue limit values.

For the case of variable loading, the stress concentration effect is evaluated by using another characteristic value (instead of $K_t$), namely the fatigue actual stress concentration factor $K_f$ [6, 7]:

$$K_f = \frac{\sigma_R}{S_R}$$

where $\sigma_R$ is the fatigue limit for a stress-raiser-free specimen, and $S_R$ represents the fatigue resistance for the specimen containing the stress raiser.

Into the high durability domain, that coefficient may be used to evaluate the fatigue limit decrease for a given component, as an effect of the stress concentration phenomenon

**2. A method for establishing the actual values of stress concentration factors**

For a given linear elastic loading case, it is largely assumed that the value of stress concentration coefficient does not depend on the studied part material, but only on the shape and dimensions of the component stress raiser. On the other hand, it was experimentally observed the possible influence of some other practical issues – such as the material plastic deformation into the stress raiser vicinity, the working environment temperature, any heat treatments applied, a local strain hardening, the size of
crystalline grains, the speed and type of loading application. The actual stresses values in the stress raisers vicinity are usually situated above the yield stress of the component material, so the values of stress concentration factors are questionable, as being established for a linear elastic loading of the reference specimen.

A calculation rule for such cases was proposed by Neuber [9, 10, 11], on the connection between the theoretical value \( K_t \) and the actual values (\( K_{t\sigma} \) – for stresses and \( K_{te} \) – for strains) of the concentration factor:

\[
K_{t\sigma} \cdot K_{te} = K_t^2
\]  

(2)

Each of the three factors represent the ratio between the maximum and the nominal values, of the respective physical quantity, at the point chosen for analysis.

One may observe that the maximum local strain \( \varepsilon_{\text{max}} \) is corresponding to the maximum local stress \( \sigma_{\text{max}} \), but the connection between them is nonlinear, in principle, because it is assumed that the actual stress level has exceeded the yield stress of the component material.

Using the definition relationship of the actual concentration factors, the above equation may be written as:

\[
\sigma_{\text{max}} \cdot \varepsilon_{\text{max}} = K_t^2 \cdot \sigma_{\text{nom}} \cdot \varepsilon_{\text{nom}}
\]  

(3)

The theoretical factor \( K_t \), together with the nominal local stress \( \sigma_{\text{nom}} \) are usually established by suitable calculus relationships, in dependence with the stress raiser shape and dimensions, and respectively with the values of applied load and specimen cross-sectional area. On the other hand, the nominal local strain value \( \varepsilon_{\text{nom}} \) can be obtained, from the component material stress-strain curve, as corresponding to the about cited value \( \sigma_{\text{nom}} \). As a result, a constant value \( C \) can be calculated, for the right member of Eq. (3), as follows:

\[
\sigma_{\text{max}} \cdot \varepsilon_{\text{max}} = C
\]  

(4)

The values for the two physical quantities from the left could be, for example, graphically obtained, at the intersection of the hyperbolic curve \( x \cdot y = C \) with the stress-strain dependence curve (in uniaxial tension) for the component material; the coordinates of that intersection point can be assumed as being the maximum local values \( \sigma_{\text{max}} \) and \( \varepsilon_{\text{max}} \) in the chosen point of the studied component; those values may be used for determining the actual values of the concentration factors.

3. The calculus of actual concentration coefficient

The here described experimental analyzes were made using some flat samples, for tensile test, of Al 2014 aluminum alloy (AlCu4SiMg); each sample had 30x5 mm\(^2\) dimensions in the calibrated cross-sectional area. In order for a stress raiser to appear on the specimen, each sample was drill, in the center of its calibrated zone, at the middle of its width, so the geometric concentrator was a transverse hole with a diameter of 10mm (one third of the sample width \( H \)); as a consequence, the theoretical value of the concentration factor (in tensile stress) may be calculated as follows:

\[
K_t = 3.3.14 \left( \frac{d}{H} \right) + 3.667 \left( \frac{d}{H} \right)^2 - 1.527 \left( \frac{d}{H} \right)^3
\]  

(5)

The actual dimensional ratio is \( d/H=1/3 \), so the theoretical concentration factor (for the linear-elastic deformability domain) will be \( K_t = 2.3046 \), that can be rounded to the value \( K_t = 2.3 \).

As the nominal stress and strain local values – \( \sigma_{\text{nom}} \) and \( \varepsilon_{\text{nom}} \) – the coordinates of point A (100.68MPa and respectively 0.1449%) were used (figure 2), from the linear-elastic domain of the material stress-strain dependence curve, and consequently, from Eq. 3 – with \( K_t = 2.3 \) – the result \( \sigma_{\text{max}} \cdot \varepsilon_{\text{max}}=0.7716 \) was obtained.
On the other hand, by using the above described method of curves intersection, the coordinates values of point B (figure 2) indicated the maximum stress and strain local values for the studied situation, i.e. $\sigma_{\text{max}}=127\text{MPa}$ and $\varepsilon_{\text{max}}=0.6076\%$; one can observe the placement of point B into the plastic deformation domain of the sample material. It can be assumed that, for a stress raiser of the type described above, when the tensile loading of the specimen, in the elastic domain of its material, causes nominal stresses of about 100 MPa, the maximum stress in the stress raiser vicinity could reach the value of 127MPa.

Knowing the stress and strain values that were cited above, the actual concentration factors, for stresses and strains, can be calculated as follows:

$$K_{t\sigma}=\frac{127}{100.68}=1.26$$

and

$$K_{t\varepsilon}=\frac{60.76}{14.49}=4.19$$

It seems that the concentration effect is greater for strains, than for stresses; on the other hand, the obtained actual value of stress concentration factor $K_{t\sigma}$ is lower than the value $K_t$ – that was obtained from literature, for static tensile loading. It is important to note that the above cited quantities $\sigma_{\text{max}}$ and $\varepsilon_{\text{max}}$ represent the local maximum stress and strain values in close proximity to the stress raiser.

Regarding the graphical method from above, it is worth clarifying that from the stress-strain dependence curve of the sample material one can only obtain some conventional stress values (automatically calculated as $F/A_0$, so the loading force divided by the initial cross-sectional specimen area), and also some global strain values (obtained by averaging the elastic and plastic deformations occurring in the entire specimen volume); one can conclude that the here described method is a conventional one, which must be applied with caution, and only for ductile materials, that usually exhibit significant plastic deformations.

4. Drawing the modified Serensen diagram

A so-called Serensen schematization of the fatigue limit cycles diagram is presented in figure 3; the theoretical diagram is simplified, in its final part, with the $G'C'$ segment that is drawn with a slope of 45 degrees, and starting from the point of the material yield strength $\sigma'_y$ on the horizontal axis of the graph. Such a simplification is proposed to be used for materials with predominantly ductile behavior. The yield strength value is deduced from the yield limit $\sigma_y$ as follows: $\sigma'_y = \sigma_y/K_{t\sigma Y}$ where $K_{t\sigma Y}$ is the effective stress concentration factor at yield. For brittle materials, the schematic diagram, in its final part, follows the path $B'U'$; $U'$ is the point where $\sigma'_u = \sigma_u/K_{t\sigma U}$, whith $K_{t\sigma U}$ being the effective stress.
concentration factor of tensile stresses. In this case, the point marked as SnL₃ will be situated on the segment GU, while the point L₃ will be on the segment G'U'.

As a result of these changes, the fatigue safety factors can be calculated using the relationships from below, for the three distinct domains of Serensen diagram schematization.

For the domain OA'B'O, the safety factor that correspond to the point P₁ will be:

\[
c_{P₁} = \frac{\sigma_{-1}}{\varepsilon f} \left( \sigma_{amP₁} + \sigma_{apP₁} \frac{2\sigma_{-1} - \sigma_0}{\sigma_0} \right)
\]

The parameters that appear in this equation are:

- the fatigue limit of the sample material, for the reversed stress loading cycle (\(\sigma_{-1}\));
- the mean stress (\(\sigma_{mP₁}\)) and the stress amplitude (\(\sigma_{apP₁}\)) for the P₁ loading cycle;
- the fatigue limit of the sample material, for the pulsating stress loading cycle (\(\sigma_0\));
- the influencing factors for fatigue calculus, i.e. the actual stress concentration coefficient \(K_f\), the surface quality factor \(\gamma\), and the dimensional factor \(\varepsilon\).

It is worth noting that in the OA'B'O domain, the fatigue safety factor is not affected by the modification of \(\sigma_{UTS}\) and \(\sigma_r\) values, as a consequence of the stress raiser presence.

For the domain OB'G'O, the safety factor that correspond to the point P₂ will be:

\[
c_{P₂} = \left( \frac{\sigma_u}{K_{tot}} \right) \left( \frac{2\sigma_u K_f}{K_{tot} \varepsilon \gamma \sigma_0} \right) \left( \sigma_{amP₂} + \sigma_{apP₂} \frac{2\sigma_u - \sigma_0}{\sigma_0} \right)
\]

where \(K_{tot}\) is the actual value of the stress concentration factor corresponding to the moment of material fracture, when the actual stress value (that appear in the table from figure 3) is assumed to be \(\sigma_u = \sigma_u / K_{tot}\).
as previously obtained, from the curves intersection method, the value of \( K_{t\sigma U} (=1.26) \) - the actual stress concentration factor in the moment of material fracture (in a static tensile loading), is lower than the classical stress concentration factor \( K_t (=2.3) \) (that is valid only for the elastic domain of the material deformability).

Finally, for the domain OG’C’O, the safety factor that correspond to the point \( P_3 \) will be:

\[
c_{P_3} = \frac{\sigma_y}{\sigma_{\max P}} K_{t\sigma Y}
\]

where \( K_{t\sigma Y} \) is the actual stress concentration factor for the moment when the sample material reaches the yielding zone of its tensile stress-strain curve, and the actual stress value (that appear in figure 3) is assumed to be \( \sigma'_y = \sigma_y/K_{t\sigma Y} \).

5. Conclusions
The fatigue safety factors are established according to the nature of the component loading, the sample material characteristics, and also to some other influencing factors – the presence of stress raisers, the surface processing quality, the component shape and dimensions, etc. When considering a variable loading condition, some suitable values of influencing factors are used, in order to adjust the fatigue limits, when moving from the testing specimen to the studied component. One can assume that, on two of the third domains from the Serensen limit cycle schematic diagram, the fatigue safety factors are also influenced by some of the static characteristics of the sample material, namely \( \sigma_{UTS} \) and \( \sigma_y \). On the other hand, it was experimentally observed that those material characteristics are modified, to some extent, by the presence of stress raiser on the sample. This type of influence was quantified, in this paper, using a concentration factor \( K_{t\sigma} \), that was calculated from the Neuber relationship; it was assumed that the cited characteristics are both influenced, to the same extent, by the stress raiser presence, so it was considered that \( K_{t\sigma U} = K_{t\sigma Y} = K_{t\sigma} = 1.26 \); it was noted that applying this coefficient to the \( \sigma_{UTS} \) and \( \sigma_y \) values, some significant changes occur on the fatigue safety factors values.

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