Optimizing basket transplanting parameters using the mechanical properties of tomato plug seedlings

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Abstract. The collision between tomato plug seedlings and the planting device during basket transplanter function causes the seedlings’ root lump to lose some mass. To resolve this problem, we establish a contact mechanics model using Hertzian contact theoretical and Newtonian methods, enabling us to formally derive the maximum deformation of the root lump. The equation of motion describing the collision between the tomato plug seedling and planting device is constructed using the principles of impulse and momentum. Then, we identify the optimization function of contact angle − which consists of the maximum deformation of the root lump, the angular velocity of the tomato plug seedling, and the absolute velocity of the root lump — via multi-objective methods. When the angular velocity of the planting device is 1.05 rad/s or 1.15 rad/s, the optimal contact angle is 67°. Further, we examine the velocity of the tomato plug seedling after collision. A small plant height reduces the root lump’s mass loss; we found that 12-cm-tall tomato plug seedlings lose less root lump mass than taller seedlings. Finally, for angular velocities of 1.05 rad/s and 1.15 rad/s, we carry out tomato plug seedling crush tests with the optimized parameters, achieving minimum root lump mass losses of 8.76% and 7.43%, respectively.

1. Introduction

The basket transplanters can form holes and cast seedlings at one time and thus are applied into the transplanting of tomato plug seedlings [1]. To improve their efficiency, basket transplanters generally have auxiliary seedling transplanting machines [2]. Workers place tomato plug seedlings into the cast seedling cup of the feed device. When a tomato plug seedling falls freely from the cast seedling cup, its root lump collides with the inner wall of the planting device. This motion results in loss of the root lump’s mass and decreases the plug’s survival rate.

Under a theoretical framework, Song et al [3] revealed the relationship between rice seedlings’ movement and distance when the plugs are turned into a guide. Zhang et al [4] adopted a high-speed video camera to observe the collision between the rice plug seedling and the guide device; concluding that the rice plug seedling slides along the guide device. Wang et al [5] studied the breakage rate of broccoli seedlings during mechanized planting. Orthogonal experiments based on agronomic requirements have been carried out to obtain optimal parameters. Chen et al [6] developed a transplanting mechanism with planetary deformed elliptic gears for plug seedling transplant and built...
differential equations for the collision of broccoli plug seedlings and the planting derive. In all the studies discussed above, the collision between the root lump and planting device is plastic, the plug seedling slides along the wall of the planting device, and the velocity of the plug seedlings was neglected during the collision between the stem and planting device.

In this paper, we focus on the collision between tomato plug seedlings and the planting device of a basket transplanter. The contact model and collision model of tomato plug seedlings are established, and the relationships of the working parameters of transplanters with the deformation of root lumps and the movement speed of tomato plug seedlings are studied, and the working parameters are optimized to reduce the quality loss of root lumps.

2. Materials and method

2.1. Collision analysis
We assume uniform rotation of the planting device about a fixed axis in a plane perpendicular to this axis. When cast seedlings cup put tomato seedling plug, the attitude of the seedling (its verticality or slope) is a stochastic process. Two common sets of contacts are collisions between the root lump and planting device, and between the stem and the edge of the planting device.

![Figure 1. Structural diagram of a basket transplanting mechanism: 1) Cast seedling cup; 2) Feeding device; 3) Planting device.](image)

2.2. Mechanics model
We divide our analysis into two branches: a contact mechanics analysis and a collision analysis. Contact deals with the instant impact between a root lump and the planting device—that is, those that deform the surface—whereas collision addresses the velocity from just before to just after the collision.

2.3. Contact mechanics analysis
When the root lump collides with the planting device, deformation of the root lump occur. These deformations are classified as contact mechanics due to the instantaneous impact [7]. The colliding parts of the root lump can be compared to the face of sphere, and those of the planting device can be considered to be a surface. Thus, the instantaneous collision model can be simplified as describing contact between a flexible sphere and a rigid plane.

According to Hertzian contact theoretical analysis [7], the normal force $F$ can be described as
\[ F = \frac{4}{3} E^* r^2 \delta^2 \]  

(1)

Where

\[ \frac{1}{E^*} = \frac{1 - \gamma_1^2}{E_1} + \frac{1 - \gamma_2^2}{E_2} \]  

(2)

When the equation of motion is applied to each body of the system (the root lump and planting device), similar equations are derived by Newtonian methods.

\[ F = m \frac{du_1}{dt} = -M \frac{du_2}{dt} \]  

(3)

Then, equation (3) can be solved:

\[ \frac{M + m}{Mm} F = \frac{d}{dt} (u_2 - u_1) = \frac{d^2 \delta}{dt^2} \]  

(4)

Substituting equation (4) into equation (1) gives

\[ \left( \frac{Mm}{M + m} \right) \frac{d^2 \delta}{dt^2} = \frac{4}{3} E^* r^2 \delta^2 \]  

(5)

Thus, integrating the resulting equation (5) yields

\[ \frac{1}{2} \left( \frac{Mm}{M + m} \right) u_N^2 - \left( \frac{d \delta}{dt} \right)^2 = \frac{8}{15} E^* r^2 \delta^3 \]  

(6)

When the slope of deformation of the root lump is zero, the maximum compression of the root lump \( \delta_{\text{max}} \) is

\[ \delta_{\text{max}} = \left[ \frac{15 M m u_N^2}{16 (M + m) E^* r^2} \right]^\frac{2}{5} \]  

(7)

**Figure 2.** The configuration of the collision of the root lump and planting device: A, The collision between the root lump and the sloped wall of the planting device; B, The collision between the root...
lump and the vertical wall of the planting device.

In general, there are two types of impact. The motion in figure 2A is caused by the collision between the root lump and the sloped wall of the planting device. The motion in figure 2B occurs due to the collision between the root lump and the vertical wall of the planting device. Global inertial coordinates $XOY$ are considered. The symbols in figure 2 are listed in table 1.

**Table 1. Specification of analysis symbols.**

| Symbol | Implication | Symbol | Implication |
|--------|-------------|--------|-------------|
| $\omega$ | Angular velocity of the planting device | $u'_c$ | Velocity of the center of mass of the tomato plug seedling after collision between the root lump and planting device |
| $R$ | Radius of gyration of the planting device | $u'_{cx}$ | Velocity of the center of mass of the tomato plug seedling in the $X'$ direction after collision between the root lump and planting device |
| $H$ | Vertical distance between the cast seedling cup nozzle and the axis of rotation of the planting device | $u'_{cy}$ | Velocity of the center of mass of the tomato plug seedling in the $Y'$ direction after collision between the root lump and planting device |
| $\theta$ | Angle between the direction of the planting device and the horizontal plane after collision of the tomato plug seedling and planting device | $\omega_1$ | Angular velocity of the tomato plug seedling before collision between the root lump and planting device |
| $E_1$ | Modulus of elasticity of the planting device | $\omega_2$ | Angular velocity of the tomato plug seedling after collision between the root lump and planting device |
| $E_2$ | Modulus of elasticity of the root lump | $J_C$ | Moment of inertia of the collision model to the center of mass |
| $\gamma_1$ | Poisson’s ratio of the planting device | $m_1$ | Mass of ball A in the collision model |
| $\gamma_2$ | Poisson’s ratio of the root lump | $m_2$ | Mass of ball B in the collision model |
| $r$ | Radius of the root lump of in the contact mechanics model | $l$ | Distance between the center of mass of balls A and B in the collision model |
| $M$ | Mass of the planting device | $d$ | Distance between the center of mass of the tomato plug seedling and the center of mass of the root lump |
| $m$ | Mass of the tomato plug seedling | $r_1$ | Radius of ball A in the collision model |
| $u_1$ | Velocity of the tomato plug seedling in the collision direction | $r_2$ | Radius of ball B in the collision model |
| $u_2$ | Velocity of the planting device in the collision direction | $u'_{A}$ | Velocity of the center of mass of the root lump after collision between the root lump and planting device |
| $\delta$ | Compression of the root lump | $u'_{AX}$ | Velocity of the center of mass of the root lump in the $X'$ direction before collision between the root lump and planting device |
| $u_N$ | Relative velocity of the root lump to the planting device before collision | $u'_{AX}$ | Velocity of the center of mass of the root lump in the $X'$ direction after collision between the root lump and planting device |
| $\delta_{max}$ | Maximum compression of the root lump | $u'_{AX}$ | Velocity of the center of mass of the root lump
lump in the Y’ direction after collision between the root lump and planting device

Relative velocity of the center mass of the root lump to the center mass of the tomato plug seedling after collision

Coefficient of restitution after collision between the root lump and planting device

Coefficient of restitution after the collision between the stem and planting device

Angle between the stem and vertical plane before collision between the stem and planting device

Angular velocity of the tomato plug seedling after collision between the stem and planting device

Distance between the collision point and the center mass of the root lump

Velocity of the center mass of the root lump after collision between the stem and planting device

Velocity of center of mass of the root lump in the X’ direction before collision between the stem and planting device

Velocity of the center of mass of the root lump in the Y’ direction after collision between the stem and planting device

Velocity of the center of mass of the root lump in the X’ direction after collision between the stem and planting device

Velocity of the center of mass of the tomato plug seedling after collision between the stem and planting device

Velocity of the center mass of the tomato plug seedling in the Y’ direction before collision between the stem and planting device

Velocity of the center of mass of the tomato plug seedling in the Y’ direction after collision between the stem and planting device

Velocity of the center of mass of the tomato plug seedling in the X’ direction after collision between the stem and planting device

Velocity of the center of mass of the
direction after collision between the stem and planting device
tomato plug seedling in the Y' direction after collision between the stem and planting device

Just before the root lump strikes the planting device, its angular velocity $\omega_1$ and velocity $u_X$ in the $X$ direction is zero. Thus, the $Y$ direction with a velocity of $u_Y$ is

$$u_Y = \sqrt{2g(H - R \sin \theta)}$$ (8)

Neglecting air resistance, the planting device is subjected to a constant angular velocity about the axis of rotation. Then, the velocity $u_E$ of the planting device is

$$u_E = \omega R$$ (9)

Also neglecting the lump-device frictional force in the collision and applying the relative velocity equation to the collision points [8], the relative velocity $u_{N1}$ of the root lump to the planting device in the collision direction is

$$u_{N1} = \left(\omega R \sin \theta \cos \beta - \omega R \cos \theta \sin \beta\right) + \sqrt{2g(H - R \sin \theta)} \sin \beta$$ (10)

If the root lump crashes into the vertical wall of the planting device, angle $\beta$ between the inclined wall of planting device and the vertical plane is set to 0. Thus, equation (10) is reduced to

$$u_{N2} = \omega R \sin \theta$$ (11)

The relative velocity $u_{N1}$ and $u_{N2}$ are shown as functions of the planting device’s angular velocity $\omega$ and contact angle $\theta$. The structural parameters of the planting device and tomato plug seedling are given in table 2 [9]. Then, three-dimensional representations of the solutions given by both equations (10) and (11) can be obtained. As shown in figure 3, velocity $u_{N1}$ is higher than velocity $u_{N2}$. Thus, attention is focused on the collision between the root lump and the sloped wall of the planting device.

**Table 2.** The relative parameters of the contact mechanics.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $H$ (mm)   | 490    | $\gamma_1$| 0.31   |
| $R$ (mm)   | 240    | $\gamma_2$| 0.44   |
| $\beta$ (°) | 20     | $M$ (g)   | 1000   |
| $E_1$ (Pa) | $2.1 \times 10^{11}$ | $m$ (g) | 14.7 |
| $E_2$ (Pa) | $3 \times 10^6$ | $r$ (mm) | 16.7 |

From equation (7), the maximum deformation $\delta_{\text{max}}$ of the root lump depends on the relative velocity before collision. The influence of the interaction between angular velocity $\omega$ of the planting device and contact angle $\theta$ on the maximum deformation $\delta_{\text{max}}$ of the root lump is plotted in figure 4. From the curved surface diagram, when angular velocity $\omega$ is fixed at a low speed, maximum deformation $\delta_{\text{max}}$ gradually decreases with increasing contact angle $\theta$. When angular velocity $\omega$ is fixed at a high velocity, the trend in maximum deformation $\delta_{\text{max}}$ has the opposite effect on contact angle $\theta$. When contact angle $\theta$ is fixed at a low value, the change in maximum deformation $\delta_{\text{max}}$ with angular velocity $\omega$ is at approximately one level. However, when the contact angle $\theta$ was fixed at a large angle, the maximum deformation $\delta_{\text{max}}$ gradually increased with the angular velocity $\omega$. Therefore, contact angle $\theta$ is the major factor influencing the deformation of the root lump.
Figure 3. The effect of angular velocity \( \omega \) and contact angle \( \theta \) on relative velocity \( u_N \).

Figure 4. The effect of angular velocity \( \omega \) and contact angle \( \theta \) on the maximum deformation \( \delta_{\text{max}} \) of the root lump.

Manual planting frequencies of 50 plants/minute and 55 plants/minute were set as a design parameter, based on the worker productivity [9]. There are five planting devices in the transplanting mechanism in figure 1, resulting in rotation velocities of \( \omega = 1.05 \) rad/s and \( \omega = 1.15 \) rad/s, respectively. According to figure 4, when angular velocity \( \omega = 1.05 \) rad/s, the maximum deformation of the root lump \( \delta_{\text{max}} \) gradually increases from 0.94 mm to 0.96 mm when the contact angle \( \theta \) increases from 0\(^\circ\) to 90\(^\circ\). When angular velocity \( \omega = 1.15 \), the maximum compression \( \delta_{\text{max}} \) changed from 0.92 mm to 0.97 mm when increasing \( \theta \).

2.4. Collision motion analysis

2.4.1. Collision between the root lump and planting device. To describe the characteristics of this collision, we assume a dumbbell shape for the distribution of the tomato seedling plug’s structure [10]. The link mass between balls A and B is negligible. Figure 5 shows an example of such a body. The origin of the X and Y coordinates is located at the ground O. The origin of the relative X’ and Y’ coordinates has been placed at collision point O’. The symbols in figure 5 are listed in table 1.

Using impulse and momentum [11], it is possible to write three equations that define the particle’s motion:

\[
m u_X' - m u_{CX'} = \sum I_{X'}
\]  
(12)
\[ m u'_{Cy} - m u_{Cy'} = \sum I_y' \]  
\[ J_c \omega_2 - J_c \omega_1 = \sum M_c (I'^e_c) \]  
(13)
(14)

The first two of these equations represent the linear impulse in the \( X' \) and \( Y' \) directions, and the third equation represents the impulse momentum about point \( C \) of the center of mass. Neglecting fractional terms, impulse \( I_x' \) in the \( X' \) directions is zero. The \( X' \) and \( Y' \) components of the velocity are indicated in figure 5; then,

\[ u_{Cx'} = \sqrt{2g(H - R\sin \theta)} \cos \beta, \quad u'_{Cx'} = \sqrt{2g(H - R\sin \theta)} \cos \beta \]  
\[ u_{Cy'} = \sqrt{2g(H - R\sin \theta)} \sin \beta \]  
(15)
(16)

Substituting equation (16) into equation (13) leads to

\[ m u'_{Cy'} - m \sqrt{2g(H - R\sin \theta)} \sin \beta = I_y' \]  
(17)

Since the tomato plug seedling is in translation just before collision, angular velocity \( \omega_1 = 0 \). equation (14) can be written as

\[ J_c \omega_2 = I_y' d \cos \beta \]  
(18)

Where

\[ J_c = \frac{2(m_1 r_1^2 + m_2 r_2^2)}{5} + \frac{2m_1 m_2 l^2}{m_1 + m_2} \]  
(19)

\[ d = \frac{m_2 l}{m_1 + m_2} \]  
(20)

To determine the relation between the velocities of points \( A \) and \( C \), the absolute velocity can be determined:

\[ u'_A = u'_C + u'_{AC} \]  
(21)

Then, the \( X' \) and \( Y' \) components of absolute velocity \( u'_{Ax'} \) and \( u'_{Ay'} \) can be written as

\[ u'_{Ax'} = u'_{Cx'} + \omega_2 d \sin \beta \]  
\[ u'_{Ay'} = u'_{Cy'} + \omega_2 d \cos \beta \]  
(22)
(23)

The ratio of the restitution impulse to the deformation impulse is called the coefficient of restitution \( k_1 \) [11]. From the above equations, this value for ball \( A \) can be expressed in terms of the particles’ initial and final velocities as

\[ k_1 = \frac{u'_{Ay'} - (oR \sin \theta \cos \beta - oR \cos \theta \sin \beta)}{(oR \sin \theta \cos \beta - oR \cos \theta \sin \beta) + \sqrt{2g(H - R \sin \theta) \sin \beta}} \]  
(24)

Equations (17), (18), (22), (23), and (24) can be solved simultaneously to obtain the \( X' \) and \( Y' \) components of velocities \( u'_{Ax'} \) and \( u'_{Ay'} \) of ball \( A \) and the angular velocity \( \omega_2 \) of the tomato plug seedling just after collision.
\[ u'_{AX} = \frac{\cos \beta (d^2(1+k_1)mR\omega \cos(2\beta - \theta) - d^2(1+k_1)mR\omega \cos \theta)}{2(J_c + d^2m \cos^2 \beta)} + \frac{(2J_c + d^2k_1m - d^2(2 + k_1)m\cos 2\beta)(1)}{2(J_c + d^2m \cos^2 \beta)} \sqrt{2g(H - R\sin \theta)} \] (25)

\[ u'_{AY} = \frac{\sqrt{2g(H - R\sin \theta)}k_1 \sin \beta - (1 + k_1)R\omega \sin (\beta - \theta)}{J_c + md \cos \beta} \] (26)

\[ \omega_2 = \frac{md(-1 + k_1)\sin \beta \sqrt{2g(H - R\sin \theta)} - md(1 + k_1)\omega R \sin (\beta - \theta)}{J_c + md \cos \beta} \] (27)

The absolute velocity \( u_A' \) of ball \( A \) has a magnitude that is found from

\[ u_A' = \sqrt{u'_{AX}^2 + u'_{AY}^2} \] (28)

In the studies discussed above, the planar movement equation of the tomato plug seedling was obtained. The relative parameters in the above equations are given in tables 2 and 3 [6].

Table 3. The relative parameters of collision motion.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| \( m_1 \) (g) | 11.2   | \( r_2 \) (mm) | 5.6    |
| \( m_2 \) (g) | 3.5    | \( k_1 \) | 0.42   |
| \( r_1 \) (mm) | 16.7   | \( k_2 \) | 0.75   |

Given equation (27), the effects of the angular velocity \( \omega \) of the planting device and contact angle \( \theta \) on the angular velocity \( \omega_2 \) of the tomato plug seedling can be examined in figure 6. When the angular velocity \( \omega \) of the planting device is fixed, the angular velocity \( \omega_2 \) of the tomato plug seedling decreases as contact angle \( \theta \) increases. When contact angle \( \theta \) is fixed at a low value, angular velocity \( \omega_2 \) increases with the rotation velocity \( \omega \); however, when the contact angle \( \theta \) is fixed at a high value, there is a single decreasing curve. Thus, the angular velocity \( \omega_2 \) of the tomato seedling decreases from -5.78 rad/s to -2.04 rad/s or from -5.89 rad/s to -1.93 rad/s when the angular velocity \( \omega \) of the planting device is 1.05 rad/s and 1.15 rad/s, respectively.

Figure 6. The effect of angular velocity \( \omega \) and contact angle \( \theta \) on angular velocity \( \omega_2 \) of the tomato seedling plug.

Figure 7. The effect of angular velocity \( \omega \) and contact angle \( \theta \) on the absolute velocity \( u_A' \) of the root lump.

From equation (28), the influence of the interaction between angular velocity \( \omega \) of the planting device and contact angle \( \theta \) on the absolute velocity \( u_A' \) of the root lump is graphed in figure 7. When
the angular velocity $\omega$ of the planting device is fixed, the absolute velocity $u'_A$ decreases as contact angle $\theta$ increases. If the contact angle $\theta$ is a small constant, absolute velocity $u'_A$ is on the order of angular velocity $\omega$. However, when contact angle $\theta$ is fixed at a high value, absolute velocity $u'_A$ increases asymptotically with angular velocity $\omega$. When the angular velocity $\omega$ is 1.05 rad/s, the absolute velocity $u'_A$ decreases from 2.87 m/s to 2.19 m/s, and when the angular velocity $\omega$ is 1.15 rad/s, the absolute velocity $u'_A$ decreases from 2.88 m/s to 2.18 m/s.

2.4.2. Collision between the stem and planting device. The free-body diagram of the inclined tomato plug seedling shown in figure 8 indicates that during the brief time from just before to just after the collision, the weights of the stem and resistance force have been ignored. The symbols in figure 8 are listed in table 1.

![Figure 8. Collision diagram between the stem and planting device.](image)

According to both impulse and momentum theories, impulse $I_X'$ in the $X'$ direction is zero because fractional terms are ignored. Then

$$v_{CX'} = \sqrt{2g(H - R \sin \theta) \cos \alpha}, \quad v_{CY'} = \sqrt{2g(H - R \sin \theta) \cos \alpha}$$

$$v_{CY'} = \sqrt{2g(H - R \sin \theta) \sin \alpha}$$

Substitution of equation (30) into equation (13) leads to

$$mv_{CY'} - m\sqrt{2g(H - R \sin \theta) \sin \alpha} = I_{Y'}$$

Because the tomato plug seedling is not rotating just before collision, angular velocity $\omega_h = 0$. Equation (14) can be rewritten as

$$J_c \omega_h = I_{Y'} (b - d)$$

To study the relation between the velocities of points $D$ and $C$, the absolute velocity after the collision can be determined:

$$v'_D = v'_C + v'_{DC}$$

From absolute motion analysis, the $Y'$ projections of absolute velocity $v'_{DY'}$ can be written as
\[ v'_{DY} = v'_{CY} + (b - d)\omega_3 \] (34)

From the definition of the coefficient of restitution, \( k_2 \) for particle \( D \) can be expressed in terms of the particles’ initial and final velocities as

\[ k_2 = \frac{v'_{DY} - (\omega R \sin \theta \cos \alpha - \omega R \cos \theta \sin \alpha)}{\omega R \sin \theta \cos \alpha - \omega R \cos \theta \sin \alpha + \sqrt{2g(H - R \sin \theta)} \sin \alpha} \] (35)

Equations (31), (32), (34), and (35) can be solved simultaneously to obtain the relative velocity \( v'_{CY} \) of the center of mass and angular velocity \( \omega_3 \) of the tomato plug seedling just after collision:

\[ v'_{CY} = \frac{\left( J_c k_2 + m(b - d)^2 \right) \sin \alpha \sqrt{2g(H - R \sin \theta)} - J_c (l + k_2) \omega R \sin (\alpha - \theta)}{J_c + m(b - d)^2} \] (36)

\[ \omega_3 = -\frac{m(b - d) \left( (l + k_2) \omega R \sin (\alpha - \theta) + (l - k_2) \sin \alpha \sqrt{2g(H - R \sin \theta)} \right)}{J_c + m(b - d)^2} \] (37)

According to planar kinematics analysis, the \( X' \) and \( Y' \) components of the velocity of ball \( A \) are

\[ v'_{AY} = \sqrt{2g(H - R \sin \theta)} \cos \alpha \] (38)

\[ v'_{AX} = \frac{\left( J_c k_2 + m(b - d)^2 \right) \sin \alpha \sqrt{2g(H - R \sin \theta)}}{J_c + m(b - d)^2} \] (39)

The absolute velocity \( v'_A \) of ball \( A \) has a magnitude that is found from

\[ v'_A = \sqrt{v'_{AX}^2 + v'_{AY}^2} \] (40)

We now discuss the collision of the stem, whose physical structure is defined in tables 2 and 3. From equation (37), we can plot the data as shown in figure 9, allowing us to examine the effects of the angular velocity \( \omega \) of the planting device and contact angle \( \theta \) on the rotation velocity \( \omega_3 \) of the tomato seedling plug. When angular velocity \( \omega \) is fixed, angular velocity \( \omega_3 \) decreases as contact angle \( \theta \) increases. When contact angle \( \theta \) is fixed at a low value, rotation velocity \( \omega_3 \) increases with angular velocity \( \omega \); however, when contact angle \( \theta \) is fixed at a high value, the reverse occurs. Thus, the angular velocity of a tomato seedling \( \omega_3 \) decreases from -8.27 rad/s to -1.96 rad/s or from -8.56 rad/s to -1.89 rad/s when the angular velocity \( \omega \) of the planting device is 1.05 rad/s or 1.15 rad/s, respectively.

Using equation (40), we can show the influence of the interaction between angular velocity \( \omega \) of the planting device and contact angle \( \theta \) on the absolute velocity \( v'_A \) of the root lump in figure 10. When angular velocity \( \omega \) of the planting device is fixed, absolute velocity \( v'_A \) decreases as contact angle \( \theta \) increases; when contact angle \( \theta \) is fixed, absolute velocity \( v'_A \) decreases in roughly inverse proportion with angular velocity \( \omega \) of the planting device. These results reveal that when angular velocity \( \omega \) is 1.05 rad/s, absolute velocity \( v'_A \) decreases from 2.67 m/s to 2.17 m/s, and when angular velocity \( \omega \) is 1.15 rad/s, absolute velocity \( v'_A \) decreases from 2.65 m/s to 2.19 m/s.
Using equation (37), we examine the effects of distance $b$ from collision point $D$ to the center $A$ of the root lump mass and incline angle $\alpha$ on the angular velocity $\omega_3$ of the tomato plug seedling in figure 11. When distance $b$ is equal to $d$, collision point $D$ coincides with point $C$—the center of mass. Angular velocity $\omega_3$ equals zero and the tomato plug seedling is in translation. When collision point $D$ is far from point $C$, rotation velocity $\omega_3$ increases with incline angle $\alpha$. When the distribution of collision point $D$ is subjected to either the upward or downward sides of point $C$, note that the directions of rotation of the tomato plug seedling are reversed.

We next discuss the effects of distance $b$ from collision point $D$ to the center $A$ of the root lump mass and incline angle $\alpha$ on the absolute velocity $v'_A$ of ball $A$ in figure 12. When $b$ is fixed, absolute velocity $v'_A$ decreases as incline angle $\alpha$ increases. When incline angle $\alpha$ is fixed at a low value, absolute velocity $v'_A$ remains the same. However, if incline angle $\alpha$ is fixed at a large angle, absolute velocity $v'_A$ is concave. Thus, when distance $b$ is equal to $d$, absolute velocity $v'_A$ is effectively at its minimum.

3. Results and discussion

3.1. Optimization of working parameters
Decreasing the velocity of the tomato plug seedling during the collision and the deformation of the root lump can reduce the root lump mass loss [12]. Thus, the maximum deformation $\delta_{\text{max}}$, angular velocities $\omega_2$ and $\omega_3$ of the tomato seedling plug, and absolute velocities $u'_A$ and $v'_A$ of the root lump should be decreased. However, when the angular velocity $\omega$ of the planting device equal 1.05 rad/s or 1.15 rad/s in previous research of contact and collision analysis, the contact angle $\theta$ is decreased to
minimize the maximum deformation $\delta_{\text{max}}$, and increased for other parameters. Therefore, a multi-objective method is utilized to obtain the optimizing equations for contact angle $\theta$ [13]. A series of sub-objective functions—equations (7), (27), (28), (37), and (40)—are combined to construct objective function $f(\theta)$. Thus, the optimization function is:

$$f(\theta) = W_1 \left( \frac{\delta_{\text{max}} - \delta^*_\text{max}}{F_{\text{max}}} \right)^2 + W_2 \left( \frac{V'_{\text{A}} - V'_{\text{A}}^*}{V_{\text{A}}^*} \right)^2 + W_3 \left( \frac{\omega_{\text{B}} - \omega_{\text{B}}^*}{\omega_{\text{B}}^*} \right)^2$$

$$+ W_4 \left( \frac{u'_{\text{A}} - u_{\text{A}}^*}{u_{\text{A}}^*} \right) + W_5 \left( \frac{\omega_{\text{3}} - \omega_{\text{3}}^*}{\omega_{\text{3}}^*} \right)^2$$

(41)

where $W_1$, $W_2$, and $W_3$ are weighting factors, and $\delta^*_\text{max}$, $v'_{\text{A}}$, $\omega_{\text{3}}$, $u_{\text{A}}$, and $\omega_{\text{3}}^*$ represent the most reasonable values arising from the minimum sub-objective function. These factors are listed in table 4.

Table 4. The relative parameters of the optimization function.

| Parameters         | Values               | $\omega = 1.05 \text{ rad/s}$ | $\omega = 1.15 \text{ rad/s}$ |
|--------------------|----------------------|-------------------------------|-------------------------------|
| $W_1$              | 0.6                  |                               |                               |
| $W_2$              | 0.2                  |                               |                               |
| $W_3$              | 0.2                  |                               |                               |
| $\delta_{\text{max}}$ (mm) | 0.94               | 0.92                          |                               |
| $\omega_{\text{3}}$ (rad/s) | -2.04          | -1.93                         |                               |
| $u_{\text{A}}$ (m/s)   | 2.19                | 2.18                          |                               |
| $\omega_{\text{3}}$ (rad/s) | -1.96           | -1.89                         |                               |
| $v'_{\text{A}}$ (m/s)   | 2.17                | 2.19                          |                               |

![Figure 13. The optimizing function curves of contact angle $\theta$.](image)

The curve in figure 13 is established by solving equation (41), specific parameters of which are listed in tables 2 and 3. Figure 13 reveals the different optimizing function curves of contact angle $\theta$. When the angular velocity $\omega$ of the planting device is $1.05 \text{ rad/s}$ or $1.15 \text{ rad/s}$, $f(\theta)$ is minimized and the contact angles are $68.1^\circ$ and $66.5^\circ$, respectively. Hence, the optimal contact angle is $67^\circ$ because the manual planting frequency was changed from 50 to 55 plants/minute.

According to figures 11 and 12, as the distance between collision point $D$ and the center of mass $C$ decrease, the angular velocity $\omega_{\text{3}}$ of the tomato plug seedling and the velocity $v'_{\text{A}}$ of the root lump also decreases. If collision point $D$ is located below center of mass $C$, angular velocity $\omega_{\text{3}}$ is positive and is identical to the direction of $\omega_{\text{3}}$ in figure 8. The results indicated that the root lump failed to enter the planting device due to its rotation. Reducing the plant height of tomato plug seedlings decreases the distance between the center of mass and the root lump, which can reduce the occurrence of collision phenomena between the planting device and the bottom of the center of mass of the tomato plug.
According to agronomic requirement, the general plant height range is 12–19 cm [14]. Then, based on the studies discussed above, 12-cm-tall tomato plug seedlings have an advantage.

### 3.2. Experimental verification test

The experiments described here were designed to confirm the optimized working parameters in crush resistance experiments. In this experiment, the plug seedling tray has 128 cells, the tomato seedlings were 45 days old and 12–19 cm in height. The transplant mechanism experimental bed is shown in figure 14, and includes the planting device, feeding device, cast seedling cup.

**Figure 14.** The transplanting mechanism test bench. 1) Planting device; 2) Cast seedling cup; 3) Feeding device.

Table 5 provides a set of direct test parameters. Conditions 1 and 5 is the optimum work parameters through optimizing analysis. The conditions were chosen by varying the contact angle and plant height. In this experiment, 200 tomato plug seedlings were fed in succession, and each condition was repeated three times.

**Table 5.** Test parameters and results.

| Test No. | Planting frequency (plants min⁻¹) | Contact angle (°) | Plant height (cm) | Percent of weight loss (%) |
|----------|-----------------------------------|-------------------|-------------------|----------------------------|
| 1        | 55                                | 67                | 12                | 8.76                       |
| 2        | 55                                | 67                | 19                | 9.52                       |
| 3        | 55                                | 80                | 12                | 11.93                      |
| 4        | 55                                | 60                | 12                | 13.84                      |
| 5        | 50                                | 67                | 12                | 7.43                       |
| 6        | 50                                | 67                | 19                | 9.17                       |
| 7        | 50                                | 80                | 12                | 10.75                      |
| 8        | 50                                | 60                | 12                | 12.41                      |

The weight of the tomato plug seedling was measured before and after testing. The percent of weight loss during collision is

\[
P = \frac{M_B - M_A}{M_B} \times 100\% \tag{42}
\]

Here, \(P\) is the percent of weight loss, \(M_B\) is the weight of the tomato plug seedling before collision, and \(M_A\) is the weight of the tomato plug seedling after collision.

The crush test results are shown in table 5. The percent of weight loss under Conditions 1 and 5 were 8.76% and 7.43%, respectively. Those experimental values are generally smaller than other test values. We therefore have experimentally validated the optimal working parameters. Wang et al [5] transplanted broccoli plug seedlings with a planetary elliptic gear transplanting machine. Their results...
showed that the percentage of damage to the root lump is 12.2%. Although the types of vegetables to be transplanted were different, it could still prove that the optimized working parameters in this paper have good working performance.

4. Conclusions
   - We employed Hertzian contact theory and the principles of impulse and momentum to derive equations to maximize root lump compression and formalize the motion of tomato seedling plugs, respectively. The trends in maximum compression and velocity were discussed in detail. Under angular velocity conditions $\omega = 1.05$ rad/s and 1.15 rad/s of the planting device, reducing the contact angle decreases the maximum deformation, whereas the contact angle is increased for other parameters (the angular velocity of the tomato plug seedling and the absolute velocity of the root lump).
   - We derived optimizing equations for contact angle $\theta$ by a multi-objective method. Using angular velocities of $\omega = 1.05$ rad/s or 1.15 rad/s for the planting device, the optimum contact angle was 67°. We then studied the effects of the location of the collision point and the incline angle on the velocity of the tomato seedling plug, revealing that 12-cm-tall tomato plug seedlings experience minimal root lump mass loss.

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