Unbiased Asymmetric Actor-Critic for Partially Observable Reinforcement Learning

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Abstract

In partially observable reinforcement learning, offline training gives access to latent information which is not available during online training and/or execution, such as the system state. Asymmetric actor-critic methods exploit such information by training a history-based policy via a state-based critic. However, many asymmetric methods lack theoretical foundation, and are only evaluated on limited domains. We examine the theory of asymmetric actor-critic methods which use state-based critics, and expose fundamental issues which undermine the validity of a common variant, and its ability to address high partial observability. We propose an unbiased asymmetric actor-critic variant which is able to exploit state information while remaining theoretically sound, maintaining the validity of the policy gradient theorem, and introducing no bias and relatively low variance into the training process. An empirical evaluation performed on domains which exhibit significant partial observability confirms our analysis, and shows the unbiased asymmetric actor-critic converges to better policies and/or faster than symmetric actor-critic and standard asymmetric actor-critic baselines.

1 INTRODUCTION

Partial observability is a key characteristic of many real-world reinforcement learning (RL) problems where the agent lacks access to the system state, and is restricted to operate based on the observable past, a.k.a. the history. Such control problems are commonly encoded as partially observable Markov decision processes (POMDPs) [Kaelbling et al.], which are the focus of a significant amount of research effort. Offline learning and online execution is an RL framework where an agent is trained in a simulated offline environment before operating in the real online environment, which offers the possibility of using latent information not generally available in online learning, such as the simulated system state or even the state belief from the agent’s perspective [Pinto et al., 2017] [Karkus et al., 2018] [Jonschkowski et al., 2018] [Nguyen et al., 2020] [Warrington et al., 2020] [Chen et al., 2020].

Offline learning methods are in principle able to exploit this additional information to achieve better online performance, so long as the resulting agent does not use the latent information during online execution. Specifically, actor-critic methods [Sutton et al., 2000] [Konda and Tsitsiklis, 2000] are able to adopt this approach via critic asymmetry, where the policy and critic models receive different types of information (e.g., the history and a latent state) [Pinto et al., 2017] [Foerster et al., 2017] [Lowe et al., 2017] [Li et al., 2019] [Wang et al., 2020] [Yang et al., 2018]; this is possible because the critic is purely a training construct, and is not required for the agent to operate online. By the very nature of actor-critic methods, critic models which are unable or slow to learn accurate values act as a performance bottleneck on the policy. Consequently, critic asymmetry is a powerful tool which, if carried out with rigor, may provide significant benefits and bootstrap the learning of the overall agent.

Unfortunately, existing asymmetric methods use asymmetric information heuristically, and rely extensively on empirical experimentation on selected environments to show their validity [Pinto et al., 2017] [Foerster et al., 2017] [Lowe et al., 2017] [Li et al., 2019] [Wang et al., 2020] [Yang et al., 2018] [Rashid et al., 2018] [Mahajan et al., 2019] [Rashid et al., 2020] [Nguyen et al., 2020]; however, the lack of a sound theoretical foundation leaves many doubts on whether these methods are able to generalize to other environments. Our main contributions are: (a) we analyze a standard variant of asymmetric actor-critic and expose analytical issues associated with the use of a state critic, namely that the state value function is, for most environments, either ill-defined, or is well-defined but causes learning bias; (b) we develop the asymmetric policy gradient theorem for partially observable
control, an extension of the policy gradient theorem which explicitly features latent state information; (c) we propose a novel unbiased asymmetric actor-critic method, which lacks the analytical issues of biased asymmetric actor-critic and is, to the best of our knowledge, the first of its kind to be theoretically motivated and sound; (d) we validate our theoretical findings through empirical evaluations on environments which feature significant amounts of partial observability, and demonstrate the performance gains of our unbiased variant.

This work opens the door for other principled asymmetric policy gradient methods that can learn with partial observability. In particular, although we focus on advantage actor-critic (A2C), our method can easily be extended to other critic-based learning methods such as off-policy actor-critic [Degris et al., 2012] [Wang et al., 2016], (deep) deterministic policy gradient [Silver et al., Lillicrap et al., 2015], and asynchronous actor-critic [Mnih et al., 2016]. Similarly, offline training is the dominant paradigm in multi-agent RL with many asymmetric actor-critic methods that could be similarly improved [Foerster et al., 2017] [Lowe et al., 2017] [Li et al., 2019] [Wang et al., 2020] [Yang et al., 2018] [Rashid et al., 2018] [Mahajan et al., 2019] [Rashid et al., 2020].

3 BACKGROUND

In this section, we review the topics relevant to our work, i.e., POMDPs, the reinforcement learning graphical model, standard actor-critic, and asymmetric actor-critic.

Notation We denote sets with calligraphy $\mathcal{X}$, set elements with lowercase $x \in \mathcal{X}$, random variables (RVs) with uppercase $X$, and the set of distributions over a set as $\Delta \mathcal{X}$. Occasionally, we will need absolute and/or relative time indices; We use subscript $t_i$ to indicate absolute time, and superscript $x^{(k)}$ to indicate the relative time of variables, e.g., $x^{(0)}$ marks the beginning of a sequence happening at an undetermined absolute time, and $x^{(k)}$ represents the variable $k$ steps later. We also use the bar notation to represent a sequence of superscripted variables $\bar{x} \equiv (x^{(0)}, x^{(1)}, x^{(2)}, \ldots)$.

3.1 POMDPs

A POMDP [Kaelbling et al.] is a discrete-time partially observable control problem described by a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, R, \gamma)$ consisting of: state, action and observation spaces $\mathcal{S}$, $\mathcal{A}$, and $\mathcal{O}$; transition function $T: \mathcal{S} \times \mathcal{A} \to \Delta \mathcal{S}$; observation function $O: \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \Delta \mathcal{O}$; reward function $R: \mathcal{S} \times \mathcal{A} \to \mathbb{R}$; and discount factor $\gamma \in [0, 1]$. The goal is that of maximizing the expected discounted sum of rewards $\mathbb{E} \left[ \sum \gamma^t R(s_t, a_t) \right]$, a.k.a. the expected return.

In the partial observable setting, the agent lacks access to the underlying system state, and action selection is based on the observable history $h$, i.e., the sequences of past actions and observations. We denote the space of all histories as $\mathcal{H} \equiv (\mathcal{A} \times \mathcal{O})^*$, and the space of histories of length $l$ as $\mathcal{H}_l \equiv (\mathcal{A} \times \mathcal{O})^l$. Generally, an agent operating under partial observability might have to consider the entire history to achieve optimal behavior [Singh et al., 1994], i.e., its policy should represent a mapping $\pi: \mathcal{H} \to \Delta \mathcal{A}$. The belief-state $b \equiv \mathcal{H} \to \Delta \mathcal{S}$ is the conditional distribution over states given the observable history, i.e., $b(h) = \Pr(\mathcal{S} \mid h)$, and a sufficient statistic of the history for optimal control [Kaelbling et al.]. We define the history reward function as $R(h, a) = \mathbb{E}_{\tilde{h} \mid h}[R(s, a)]$; from the agent’s perspective, this is the reward function of the decision process. We denote the last observation in a history as $\delta_h$, and say that an agent is reactive if its policy $\pi: \mathcal{O} \to \Delta \mathcal{A}$ uses the last observation rather than the entire history. A policy’s history value function $V^\pi: \mathcal{H} \to \mathbb{R}$ represents the expected future discounted returns when the agent finds itself in history $h$,

$$V^\pi(h^{(0)}) = \mathbb{E}_{s, a \mid h^{(0)}} \left[ \sum_{k=0}^{\infty} \gamma^k R(s^{(k)}, a^{(k)}) \right],$$

(1)
which supports an indirect recursive Bellman form,

\[ V^\pi(h) = \sum_{a \in A} \pi(a; h)Q^\pi(h, a), \]  
(2)

\[ Q^\pi(h, a) = R(h, a) + \gamma E_{o|h,a}[V^\pi(hao)]. \]  
(3)

3.2 THE RL GRAPHICAL MODEL

Some of the theory and results developed in this document concerns whether certain (RVs) of interest are well-defined; therefore, we review the RVs defined in the partially observable control case. Together, the environment and the agent induce a graphical model (see Figure 1) over timed RVs \( S_t, A_t, \) and \( O_t. \) Note that only timed RVs are defined directly, and there are no intrinsically time-less RVs. Any other RV must be defined in terms of the available ones, e.g., we can define a joint RV representing the timed history RVs \( H_t = (A_0, O_0, \ldots, A_{t-1}, O_{t-1}). \) Sometimes it is possible to define a limiting (stationary) state RV \( S = \lim_{t \to \infty} S_t; \) However, it is never possible to define a limiting (stationary) history RV \( H, \) since the sample space of each timed RV \( H_t \) is different and \( \lim_{t \to \infty} H_t \) does not exist.

A probability is a numeric value associated with the assignment of a value \( x \) from a sample space \( \mathcal{X} \) to an RV \( X, \) e.g., \( \Pr(X = x). \) Although it is common to use simplified notation and informally omit the RV assignment (e.g., \( \Pr(x) \)), it must always be implicitly clear which RV is involved in the assignment. In the reinforcement learning graphical model, a probability is well-defined if and only if (a) it is grounded (implicitly or explicitly) to timed RVs (or functions thereof); or (b) it is time-invariant (i.e., it can be grounded to any time index). For example, \( \Pr(s' \mid s, a) \) is implicitly grounded to the RVs of a state transition \( \Pr(S_{t+1} = s' \mid S_t = s, A_t = a), \) and although the time-index \( t \) is not clear from context, the probability is time-invariant and thus well defined. As another example, \( \Pr(s \mid h) \) is implicitly grounded to the RVs of a belief \( \Pr(S_t = s \mid H_t = h); \) in this case, the time-index \( t \) can be contextually grounded to the history length \( t = |h|, \) so the probability is well defined.

3.3 (SYMMETRIC) ACTOR-CRITIC FOR POMDPS

Policy gradient methods [Sutton et al. 2000] for fully observable control problems can be adapted to partially observable control problems by replacing occurrences of the system state \( s \) with the history \( h, \) which is the Markov-state of a history-MDP equivalent to the POMDP. In advantage actor-critic methods (A2C) [Konda and Tsitsiklis 2000], a policy model \( \pi: \mathcal{H} \to \Delta \mathcal{A} \) parameterized by \( \theta \) is trained using gradients estimated from sample data, while a critic model \( \hat{V}: \mathcal{H} \to \mathbb{R} \) parameterized by \( \vartheta \) is trained to predict history values \( V^\pi(h). \) Note that we annotate parametric critic models with a hat \( \hat{V}, \) to distinguish them from their analytical counterparts \( V^\pi. \) In A2C, the critic is used to bootstrap return estimates and as a baseline, both of which are techniques for the reduction of estimation variance [Greensmith et al. 2004]. The overall A2C objective is

\[ \mathcal{L}(\theta, \vartheta) \doteq \mathcal{L}_{\text{policy}}(\theta) + \mathcal{L}_{\text{critic}}(\vartheta) + \lambda \mathcal{L}_{\text{neg-entropy}}(\theta). \]  
(4)

**Policy Loss** The policy loss \( \mathcal{L}_{\text{policy}} \) is the agent’s performance, i.e., the episodic return

\[ \mathcal{L}_{\text{policy}}(\theta) \doteq -\mathbb{E}\left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \right]. \]  
(5)

The policy gradient theorem [Sutton et al. 2000, Konda and Tsitsiklis 2000] provides an analytical expression for the policy loss gradient w.r.t. the policy parameters,

\[ \nabla_{\theta} \mathcal{L}_{\text{policy}}(\theta) = -\mathbb{E}\left[ \sum_{t=0}^{\infty} \gamma^t Q^\pi(h_t, a_t) \nabla_{\theta} \log \pi(a_t; h_t) \right]. \]  
(6)

In A2C, the value function \( Q^\pi(h_t, a_t) \) is replaced with the temporal difference (TD) error \( \delta_t, \)

\[ \nabla_{\theta} \mathcal{L}_{\text{policy}}(\theta) = -\mathbb{E}\left[ \sum_{t=0}^{\infty} \gamma^t \delta_t \nabla_{\theta} \log \pi(a_t; h_t) \right], \]  
(7)

\[ \delta_t = R(s_t, a_t) + \gamma \hat{V}(h_{t+1}) - \hat{V}(h_t), \]  
(8)

to reduce variance at the cost of introducing modeling bias.

**Critic Loss** The critic is trained to minimize the TD error of the history-states; to make the policy and critic losses scale similarly to environments with different episode lengths, we adopt an unconventional time-discounted variant of the critic loss,

\[ \mathcal{L}_{\text{critic}}(\vartheta) \doteq \mathbb{E}\left[ \sum_{t=0}^{\infty} \gamma^t \delta_t^2 \right], \]  
(9)

the gradient of which should propagate through \( \hat{V}(h_t) \), but not through the bootstrapping component \( \hat{V}(h_{t+1}). \)
Negative-Entropy Loss Finally, the negative-entropy loss is commonly used, in combination with a decaying weight $\lambda$, to avoid premature convergence of the policy model and to promote exploration [Williams and Peng [1991]. As with the critic loss, we employ a time-discounted variant of the negative-entropy loss,

$$L_{\text{neg-entropy}}(\theta) = -E \sum_t \gamma^t \mathbb{H}[\pi(A_t; h_t)] .$$ (10)

3.4 ASYMMETRIC ACTOR-CRITIC FOR POMDPs

While asymmetric actor-critic can be understood to be an entire family of methods which use critic asymmetry, for the remainder of this document we will be specifically referring to a non-reactive and non-deterministic variant of the work by Pinto et al. [2017], which uses critic asymmetry to address image-based robot learning. Their work uses a reactive variant of deep deterministic policy gradient (DDPG) [Lillicrap et al., 2015] trained in simulation, and replaces the reactive observation critic $\hat{V}(o)$ with a state critic $\hat{V}(s)$; the variant we will be analyzing applies the same critic substitution to A2C. In practice, this state-based asymmetry is implemented by replacing the TD error of Equation (6) (used in both the policy and critic losses) with

$$\delta_t = R(s_t, a_t) + \gamma \hat{V}(s_{t+1}) - \hat{V}(s_t) ,$$ (11)

while every other aspect of A2C remains unchanged.

Although Pinto et al. [2017] claim that their work addresses partial observability, their evaluation is based on reactive environments which are virtually fully observable: while only an image is available to the agent, each image gives a virtually complete and collision-free view of the entire workspace. In practice, the images are low-level representations of the full state.

4 THEORY OF ASYMMETRIC ACTOR-CRITIC

In this section, we analyze the theoretical implications of using a state critic as described in Section 3.4, and expose critical issues. The primary result will be that the state value function $V^\pi(s)$ of a non-reactive agent under partial observability is generally ill-defined. Then, we show that the state value function $V^\pi(s)$ of a reactive agent under partial observability is well-defined, but introduces a bias into the training process which may undermine learning. Finally, we show that the state value function $V^\pi(s)$ of a reactive agent under an observation function which is equivalent to full observability is both well-defined and unbiased.

Note that replacing the history critic is intrinsically questionable: the policy gradient theorem for POMDPs (Equation (6)) specifically requires history values, and substituting them for another value which has a different expectation will result in the gradient estimates losing their theoretical guarantees of correctness. Therefore, we analyze state values $V^\pi(s)$ as estimators of history values $V^\pi(h)$, and consider the corresponding estimation bias, i.e., the difference between $\mathbb{E}_{s|h} [V^\pi(s)]$ and $V^\pi(h)$ for a given history $h$.

Informally, the fundamental issue with $V^\pi(s)$ is that the state does not contain sufficient information to determine the agent’s future behavior (which depends on the history) and is thus unable to meaningfully represent expected future returns. Ironically, state values suffer from a problem we call history aliasing, i.e., being unable to infer the agent’s history from the system’s state. Even when $V^\pi(s)$ is numerically well-defined, it usually introduces a bias caused by the imperfect correlation between histories and states; in essence, the average value of histories inferred from the current state is not an accurate estimate of the true current history’s value.

We begin with the definition of the state value function,

$$V^\pi(s^{(0)}) = \mathbb{E}_{s,a|s^{(0)}} \left[ \sum_{k=0}^{\infty} \gamma^k R(s^{(k)}, a^{(k)}) \right] ,$$ (12)

which, if well-defined, supports an indirect recursive Bellman form,

$$V^\pi(s) = \sum_{a \in A} \Pr(a | s) Q^\pi(s, a) ,$$ (13)

$$Q^\pi(s, a) = R(s, a) + \gamma \mathbb{E}_{s'|s,a} [V^\pi(s')] .$$ (14)

From Equation (13), we note the term $\Pr(a | s)$, which encodes the likelihood of an action being taken from a given state. Because the agent policy acts on histories, this term is not directly available, but must be derived indirectly by integrating over possible histories; and because there is no contextual information available to limit the integration to histories of a specific length, we can only integrate over the space of all possible histories,

$$\Pr(a | s) = \sum_{h \in H} \Pr(h | s) \pi(a; h) .$$ (15)

Equation (15) reveals the probability term $\Pr(h | s)$, which encodes the likelihood of an history having taken place given a state. While $\Pr(h | s)$ may look harmless, it is the underlying cause of serious analytical issues. As discussed in Section 3.2, a probability term is only well-defined if associated with well-defined RVs, and the fundamental issue with $\Pr(h | s)$ is that such RVs do not exist. On one hand, we cannot use timed RVs $\Pr(H_t = h | S_t = s)$, because Equation (15) integrates over the sample space of all histories, and not just those of a given length $t$. On the other hand, we cannot use time-less RVs $\Pr(H = h | S = s)$, because time-less RVs do not exist in the RL graphical model.
Ultimately, \( \Pr(h \mid s) \) is ill-defined, which causes \( \Pr(a \mid s) \) and \( V^\pi(s) \) itself to be ill-defined.

**Theorem 4.1.** For an arbitrary POMDP and policy, a time-invariant state value function \( V^\pi(s) \) is ill-defined. (proof in Appendix B)

In relation to the training procedure of a state critic model, the practical implications of an ill-defined value function are not obvious; even though the analytical value function is ill-defined mathematically, the critic’s training process performs valid calculations on sample data which results in valid updates of the critic parameters. However, convergence is unlikely, since empirical convergence to a meaningful value virtually requires the existence of a theoretical convergence value. Rather, it is likely that the critic’s training target will shift indefinitely, depending on the recent training data, inhibiting the convergence of the critic model even under ideal training circumstances, which itself will cause instabilities and divergence in the policy model; this is verified empirically in Section 6.

A natural solution to the underlying issue of state value functions \( V^\pi(s) \) is to define and employ timed value functions \( V^\pi_t(s) \); in Appendix A we show that timed value functions indeed address the primary issue, although learning a critic to model them is likely to pose a significantly harder learning challenge, due to the need to generalize well and accurately over different time-steps. Rather, in the next subsections, we show special cases of the general control problem which make non-timed \( V^\pi(s) \) well-defined; in some cases, this will lead to other theoretical issues involving the introduction of estimation bias.

### 4.1 REACTIVE POLICY UNDER PARTIAL OBSERVABILITY

We show that \( V^\pi(s) \) is well-defined if we make two assumptions about the agent and environment: (a) that the policy is reactive; and (b) that the POMDP observation function depends only on the current state, \( O: S \rightarrow \Delta O \), rather than the entire state transition. Under these assumptions, we can expand \( \Pr(a \mid s) \) by integrating over the space of all observations (rather than all histories),

\[
\Pr(a \mid s) = \sum_{o \in O} \Pr(o \mid s) \pi(a; o). \quad (16)
\]

In this case, the term \( \Pr(o \mid s) \) can be grounded to timed RVs \( \Pr(O_t = o \mid S_t = s) \); because that probability is time-invariant, it is well-defined, meaning that \( V^\pi(s) \) is well-defined in this case. However, we show that \( V^\pi(s) \) is biased compared to \( V^\pi(h) \).

**Theorem 4.2.** If the POMDP observation function depends only on the current state, \( O: S \rightarrow \Delta O \), and the policy is reactive, then \( V^\pi(s) \) is not necessarily an unbiased estimate of \( V^\pi(h) \), i.e., it is not guaranteed that \( V^\pi(h) = \mathbb{E}_{s \mid h}[V^\pi(s)] \). (proof in Appendix B)

Although we were able to show that the value function is well-defined in the case of reactive control, there are still two significant issues: (a) reactive policies are inadequate to solve many POMDPs; and (b) the bias of the value function \( V^\pi(s) \) may influence the agent learning capabilities catastrophically. Broadly speaking, this bias is caused by the fact that hidden in \( V^\pi(s) \) is an expectation over observations \( o \) which, while conditioned on the state \( s \), are not necessarily consistent with the true history \( h \). We discuss the cause of this bias more formally in the corresponding proof.

### 4.2 REACTIVE POLICY UNDER FULL OBSERVABILITY

We show that the state value function is not only well-defined but also unbiased if we make two assumptions about the agent and environment: (a) that the policy is reactive; and (b) that the there is a bijective abstraction \( \phi: O \rightarrow \Delta S \) between observations and states. The abstraction \( \phi \) encodes the fact that the environment is not truly partially observable, but rather that states and observations essentially contain the same information, albeit at different levels of abstraction, akin to the problems used by Pinto et al. [2017]. For example, an image displaying a workspace without occlusions could be a low-level abstraction (observation), while a concise vector representation of the object poses in the workspace could be a high-level abstraction (state).

In this case, the action probability term \( \Pr(a \mid s) \) does not need to be obtained indirectly by integrating other variables; rather, the state-observation bijection can be used to directly relate it to the policy model,

\[
\Pr(a \mid s) = \pi(a; \phi^{-1}(s)). \quad (17)
\]

Contrary to the previous cases, the overall state value function \( V^\pi(s) \) is not only well-defined, but also unbiased.

**Theorem 4.3.** If the POMDP states and observations are related by a bijection \( \phi: O \rightarrow \Delta S \), and the policy is reactive, then \( V^\pi(s) \) is an unbiased estimate of \( V^\pi(h) \), i.e., \( V^\pi(h) = \mathbb{E}_{s \mid h}[V^\pi(s)] \). (proof in Appendix B)

The benefit of using a state critic under this scenario is that the critic model can avoid learning a representation of the observations before learning the values [Pinto et al., 2017]. Naturally, the main disadvantage of this scenario is that most POMDPs do not satisfy the bijective abstraction assumption, which is virtually equivalent to full observability. Nonetheless, if a control problem only deviates mildly from full observability, it is very possible that a state critic might benefit the learning agent.
5 UNBIASED ASYMMETRIC ACTOR-CRITIC

In this section, we introduce unbiased asymmetric actor-critic, an actor-critic variant which is able to exploit asymmetric state information during offline training while avoiding the issues of state value functions exposed in Section 4.

Consider a history-state value function $V^\pi(h, s)$ (Bono et al. 2018), which represents the expected future discounted returns obtained when the history is $h$ and the state is $s$,

$$V^\pi(h^{(0)}, s^{(0)}) = \mathbb{E}_{s, a \mid h^{(0)}, s^{(0)}} \left[ \sum_{k=0}^{\infty} \gamma^k R(s^{(k)}, a^{(k)}) \right],$$

which supports an indirect recursive Bellman form,

$$V^\pi(h, s) = \sum_{a \in A} \pi(a; h) Q^\pi(h, s, a),$$

$$Q^\pi(h, s, a) = R(s, a) + \gamma \mathbb{E}_{s', a' \mid s, a} [V^\pi(bao', s')].$$

Providing the history information makes the history-state value function $V^\pi(h, s)$ not only well-defined even for non-reactive policies, but also an unbiased estimate of $V^\pi(h)$.

**Theorem 5.1.** For an arbitrary POMDP and policy, $V^\pi(h, s)$ is an unbiased estimate of $V^\pi(h)$, i.e., $V^\pi(h) = \mathbb{E}_{a \mid h} [V^\pi(h, s)]$. (proof in Appendix B).

As we have done for state values $V^\pi(s)$, we are interested in the properties of history-state values $V^\pi(h, s)$ in relation to history values $V^\pi(h)$. Theorem 5.1 shows that history and history-state values are related by $V^\pi(h) = \mathbb{E}_{a \mid h} [V^\pi(h, s)]$, i.e., history-state values are interpretable as Monte Carlo (MC) estimate of the respective history values. In expectation, history-state values provides the same information as the history values, therefore an asymmetric variant of the policy gradient theorem also holds.

**Theorem 5.2** (Asymmetric Policy Gradient). The policy gradient can be expressed using history-state values,

$$\nabla_\theta \mathcal{L}_{\text{policy}}(\theta) = - \mathbb{E} \left[ \sum_t \gamma^t Q^\pi(h_t, s_t, a_t) \nabla_\theta \log \pi(a_t; h_t) \right]$$

(proof in Appendix B).

As estimators, history-state values $V^\pi(h, s)$ can be described in terms of their bias and variance w.r.t. history values $V^\pi(h)$. Beyond providing the inspiration for the MC interpretation, Theorem 5.1 already proves that $V^\pi(h, s)$ is unbiased, while its variance is dynamic and depends on the history $h$ via the belief-state $P_T(S \mid h)$; in particular, low-uncertainty belief-states result in relatively low variance, and deterministic belief-states result in no variance. Given that operating optimally in a partially observable environment generally involves information-gathering strategies associated with low-uncertainty belief-states, the practical variance of the history-state value is likely to be relatively low once the agent has learned to solve the task to some degree of success.

Inspired by Theorem 5.2, we propose unbiased asymmetric A2C, which uses a history-state critic $\hat{V} : \mathcal{H} \times \mathcal{S} \rightarrow \mathbb{R}$ trained to model history-state values $V^\pi(h, s)$,

$$\nabla_\theta \mathcal{L}_{\text{policy}}(\theta) = - \mathbb{E} \left[ \sum_t \gamma^t \delta_t \nabla_\theta \log \pi(a_t; h_t) \right],$$

$$\delta_t = R(s_t, a_t) + \gamma \hat{V}(h_{t+1}, s_{t+1}) - \hat{V}(h_t, s_t).$$

Because $\hat{V}(h, s)$ receives the history $h$ as input, it can still predict reasonable estimates of the agent’s expected future discounted returns; and because it receives the state $s$ as input, it is still able to exploit state information while introducing no bias into the learning process, e.g., for the purposes of bootstrapping the learning of critic values and/or aiding the learning of history representations.

5.1 INTERPRETATIONS OF STATE

Although the history-state value is analytically well-defined, it is worthwhile to question why the inclusion of the state information should help the actor-critic agent at all. We attempt to address this open question, and consider two competing interpretations, which we call state-as-information and state-as-a-feature.

**State as Information** Under this interpretation, state information is valuable because it is latent information unavailable in the history, which results in more informative values. However, this interpretation is flawed for two reasons: (a) The policy gradient theorem specifically requires $V^\pi(h)$, which contains precisely the correct information required to accurately estimate policy gradients. In this context, there is no such thing as “more informative values” than history values. (b) In theory, the history-state value in Theorem 5.2 could use any other state sampled according to $s \sim b(h)$, rather than the true system state, which would also result in the same analytical bias and variance properties. In practice, we use the true system state primarily due to it being directly available during simulation; however, we believe that its identity as the true system state is analytically irrelevant, which leads to the next interpretation of state.

**State as a Feature** We conjecture an alternative interpretation according to which the state can be seen as a stochastic high-level feature of the history. Consider a history critic $\tilde{V}(h)$ to appropriately model the value function $V^\pi(h)$, the model must first learn an adequate history representation, which is in and of itself a significant learning challenge. The
We perform evaluations on seven gridworld environments which exhibit significant partial observability: Shopping-5, Shopping-6, Heavenhell-3, Heavenhell-4, and Rocksparse-5-6 are flat POMDPs where states and observations are represented by categorical indices, while Keydoor and Nine rooms are gridverse POMDPs where states and observations are represented by tensors of categorical indices which encode spatial relationships and other cell information. See Appendix C for a detailed description and graphical representations of all environments.

## 6 RESULTS AND DISCUSSION

Each method is evaluated in one of two ways: (a) we show empirical learning curve statistics for all environments, and (b) we show how critic values change for important history-state pairs over the course of training in Heavenhell-4.

### 6.1 Learning Curves

Figure 2 depicts the performance results in all the environments. First, we note that the symmetric baseline A2C(h) does not always learn to solve the task, but succeeds fully in Figures 2a, 2c and 2f partially in Figures 2d, 2e and 2g, and fails in Figure 2b. The asymmetric baseline A2C(s) also performs inconsistently across environments, with a mixture of successful and failing cases. We particularly note the strange learning curves of A2C(s) in Figures 2a and 2b, where performance improves quickly during the early training, but fails to improve further or even becomes unstable and collapses later on. While the exact dynamics of this collapse are not completely clear, we believe it is likely that this is a consequence of modeling the critic after an analytically unstable state value function, making convergence to stable values impossible. Using our proposed history-state critic, A2C(h,s) consistently exhibits either faster convergence or higher final performance in virtually all the flat environments, and competitive performance in the gridverse environments, taking a mild lead towards the end of Nine rooms. These results strongly demonstrate the importance of exploiting asymmetric information in ways which are theoretically justified and sound, as done in our work.

### 6.1.2 Critic Values

To further inspect the behavior of each critic, we show the evolution of critic values for important history-state pairs over the course of training. We perform this evaluation on Heavenhell-4, and use 4 deliberately chosen history-state pairs. In each case the agent is located at the fork, and the 4 cases differ according to heaven’s location (left or right) and whether the agent has previously visited the priest.

Figure 3 shows the resulting critic values, with one figure per environment. The figure shows the critic values for important history-state pairs over the course of training in Heavenhell-4.

---

**Algorithm 1** Each method follows the same algorithm structure, but uses different types of critics to compute the TD errors $\delta_t$ (see Equations (8), (11) and (23)). Values $N, B,$ and $E$ vary by environment.

Input: epochs $N$, episode batch $B$, evaluation period $E$ for epoch in 1 ... $N$ do

- training_episodes ← sample_episodes($\pi$, $B$)
- update $\theta, \vartheta$ via $\nabla L$(training_episodes) (see Equation (4))
- if epoch mod evaluation_period = 0 then
  evaluation_episodes ← sample_episodes($\pi$, $E$)
- report empirical_returns(evaluation_episodes)

End

critic would likely benefit from receiving auxiliary high-level black-box features of the history $\phi(h)$. The resulting critic $\hat{V}(h, \phi(h))$ remains fundamentally a history critic, the supplementary features being exclusively a modeling construct. Next, we consider what kind of high-level features $\phi(h)$ would be useful for control. While the specifics of what makes a good history representation depend strongly on the task, there is a natural choice which is arguably useful in many cases: the belief-state $b(h)$. Because the belief-state is a sufficient statistic of the history for control, providing it to the critic model $\hat{V}(h, b(h))$ is likely to greatly improve its ability to generalize across histories. Finally, we conjecture that any state sampled according to the belief-state $s \sim b(h)$—including the true system state—can be considered a stochastic realization of the belief-state feature, resulting in the history-state critic $\hat{V}(h, s)$. According to this interpretation, the importance of the state in the history-state critic is not in its identity as the true system state, but as a stochastic realization of hypothetical belief-state features, and presumably any other state sampled from the belief-state $s \sim b(h)$ could be equivalently used.
A2C(s)

120 k

60 k

75 k

60 k

120 k

180 k

(b)

30 k

(g)

120 k

60 k

(d) Heaven right, priest.

600 k

60 k

25 k

90 k

300 k

50 k

(d)

180 k

60 k

900 k

180 k

180 k

A2C(h,s)

Figure 2: Learning curve statistics over 20 independent training runs, where each run is periodically evaluated 20 times. Shaded areas are centered around the empirical mean performance, and show 2 standard errors (of the mean).

for each of the chosen history-state pairs. In each scenario, we note that all critic values exhibit convergence properties which resemble those of the agent’s performance in Figure 2d. This empirically confirms that agent performance and critic quality are strongly correlated factors; although it is not possible to make conclusions about the causality in this situation, we strongly believe that it is the critics which act as a learning bottleneck on policies.

Notably, the critics which focus on a single aspect of the joint history-state show the exact same values for different history-state; namely, A2C(s) is identical in the top and bottom plots, while A2C(h) is identical in the top-left and top-right plots. Although in practice left and right plots are similar for all critics, A2C(h,s) is the only critic capable of representing different values in each of the 4 scenarios, as none of its curves are identical. We also note that the state critic \( \hat{V}(s) \) is much less stable than the others, and shows no signs of convergence, which is consistent with our analysis in Section 4. In contrast, the history-state critic exhibits good convergence properties despite itself also using state information, which is consistent with our analysis in Section 5. Finally, we note again that the history-state critic \( \hat{V}(h, s) \) converges significantly faster than the history critic \( \hat{V}(h) \), confirming that state information is useful for training.

7 CONCLUSIONS

Asymmetric methods trained offline in simulated environments can use information which is normally unavailable in partially observable RL, such as the true system state. While the idea of exploiting such information has potential, current state-of-the-art methods are powered by empirical results rather than theoretical analysis. In this work, we exposed profound theoretical issues with a standard variant of asymmetric actor-critic, and proposed an unbiased asymmetric actor-critic variant which is analytically sound and theoretically justified. Empirical results confirm our analysis, the weaknesses of state critics and the strengths of history-state critics. Although we applied the history-state value function only to A2C, the same concepts are easily extensible to other critic-based RL methods [Silver et al., Lillicrap et al., 2015; Degris et al., 2012; Mnih et al., 2016].

In future work, we aim to extend the theory of history-state policy value functions \( Q^*(h, s, a) \) to optimal value functions \( Q^*(h, s, a) \), and develop theoretically sound asymmetric variants of other deep RL methods such as DQN [Mnih]...
soft Q-learning [Haarnoja et al., 2017], and soft actor-critic [Haarnoja et al., 2018]. We also plan to extend the theory and approach to multi-agent methods, potentially bringing theoretical rigor and improved performance [Foster et al., 2017, Lowe et al., 2017, Li et al., 2019, Wang et al., 2020, Yang et al., 2018, Rashid et al., 2018, Mahajan et al., 2019, Rashid et al., 2020].
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A TIMED VALUE FUNCTIONS

Section 4 shows that for a general POMDP and policy \( \pi : \mathcal{H} \rightarrow \Delta \mathcal{A} \), the state value function \( V^\pi(s) \) is not generally well-defined due to issues caused by the lack of time information. In this section, we consider addressing the primary issue by providing explicit time-index information via timed value functions, \( V^\pi_t(s) \) and \( Q^\pi_t(s, a) \), which represent the expected returns obtained when the agent finds itself in a state \( s \) at time \( t \),

\[
V^\pi_t(s) = \sum_{a \in \mathcal{A}} \Pr(A_t = a \mid S_t = s)Q^\pi_t(s, a),
\]

\[
Q^\pi_t(s, a) = R(s, a) + \gamma \mathbb{E}_{a' \mid s, a}[V^\pi_{t+1}(s')].
\]

Once again, we analyze the state-dependent action distribution term to verify correctness, and expand it by integrating over histories; this time, we can use the explicit time-index information to integrate over histories of a given length only,

\[
\Pr(A_t = a \mid S_t = s) = \sum_{h \in \mathcal{H}_t} \Pr(H_t = h \mid S_t = s)\pi(a; h).
\]

Because Equation 26 is now restricted to histories of a given length \( t \), the probability term \( \Pr(H_t = h \mid S_t = s) \) is well-defined, which means \( \Pr(A_t = a \mid S_t = s) \) is well-defined, and \( V^\pi_t(s) \) is well-defined.

Introducing the time-index makes the value function \( V^\pi_t(s) \) well-defined. However, its utility for the purpose of asymmetric reinforcement learning remains unclear because (a) it is still not formally proven whether the timed value function is unbiased, i.e., whether \( V^\pi_t(h) = \mathbb{E}_{s \mid h}[V^\pi_t(s)] \), and (b) it is harder for timed value critics to generalize appropriately across the additional discrete input \( t \).

B PROOFS

This section contains the proofs omitted from the main body of the document. For the sake of clarity, we repeat the main statements before showing the respective proof.

Theorem 4.1 For an arbitrary POMDP and policy, a time-invariant state value function \( V^\pi(s) \) is ill-defined.

Proof. The state value function, defined as

\[
V^\pi(s) = \sum_{a \in \mathcal{A}} \Pr(a \mid s)Q^\pi(s, a),
\]

requires the state-conditioned action probability term \( \Pr(a \mid s) \). Because a partially observable policy depends on the history and not the state, the state-conditioned action probability term must be expanded by integrating over the space of all histories,

\[
\Pr(a \mid s) = \sum_{h \in \mathcal{H}} \Pr(h \mid s)\pi(a; h),
\]

which requires the state-conditioned history probability term \( \Pr(h \mid s) \). However, it is impossible to associate that term to well-defined RVs: Clearly, the reinforcement learning graphical model (see Section 3) does not define a time-less history RV \( H \), so the term cannot be explicitly written as \( \Pr(H = h \mid S = s) \). Further, the probability associated with timed RVs \( \Pr(H_t = h \mid S_t = s) \) is clearly not time-invariant, hence identifying the time-index is crucial. Finally, it is not possible to restrict the integration in Equation 28 only to histories of a given time-index. Overall, the probability term \( \Pr(h \mid s) \) is necessarily time-variant, which means that \( \Pr(a \mid s) \) is itself time-variant, and therefore the state value function \( V^\pi(s) \) is time-variant, and a time-variant state value function \( V^\pi(s) \) is ill-defined.

Theorem 4.2 If the POMDP observation function depends only on the current state, \( O : S \rightarrow \Delta O \), and the policy is reactive, then \( V^\pi(s) \) is not necessarily an unbiased estimate of \( V^\pi(h) \), i.e., it is not guaranteed that \( V^\pi(h) = \mathbb{E}_{s \mid h}[V^\pi(s)] \).

As one of the salient theorems of our work, we will cover it from different angles and provide three proofs: first one which is simple but does not go into the detail about the causes of the bias, then a more detailed and analytical one, and finally one by example.
First proof. Consider two histories \( h, h' \in H \) which are different, \( h \neq h' \), but are associated with the same belief \( b(h) = b(h') \); a fairly common occurrence in many POMDPs. On one hand, because the two histories are different, the agent’s next action may differ, which may then lead to different immediate rewards and future trajectories, i.e., the respective values would also differ,

\[
V^\pi(h) \neq V^\pi(h').
\] (29)

On the other hand, because the two beliefs are the same, then the expected state values must be the same,

\[
E_s|h [V^\pi(s)] = E_s|h' [V^\pi(s)].
\] (30)

Therefore, it is not guaranteed that \( V^\pi(h) = E_s|h [V^\pi(s)] \).

Second proof, by contradiction. First, we assume that \( V^\pi(s) \) is unbiased and show that \( Q^\pi(s, a) \) (as defined by Equation (14)) is unbiased,

\[
E_s|h [Q^\pi(s, a)] = E_s|h [R(s, a) + \gamma E_{s'|s, a} [V^\pi(s')]]
\]

\[
= E_s|h [R(s, a) + \gamma E_{s'|s, a} [V^\pi(s)]]
\]

\[
= E_s|h [R(s, a) + \gamma E_{s'|h, a} [V^\pi(s)]]
\]

\[
= E_s|h [R(s, a) + \gamma E_{o|h, a} E_{s'|hao} [V^\pi(s)]]
\]

\[
= R(h, a) + \gamma E_{o|h, a} [V^\pi(hao)]
\]

\[
= Q^\pi(h, a).
\] (31)

Next, we show that even if \( Q^\pi(s, a) \) is unbiased, \( V^\pi(s) \) (as defined by Equation (13)) is biased, which contradicts the original assumption. To do that, we expand the expected state value function \( E_s|h [V^\pi(s)] \) and the history value function \( V^\pi(h) \), and show that there is a concrete difference between them:

\[
E_s|h [V^\pi(s)] = E_s|h \left[ \sum_{a \in A} \Pr(a \mid s)Q^\pi(s, a) \right]
\]

\[
= E_s|h \left[ \sum_{a \in A} \Pr(a \mid s) \pi(a; o_h)Q^\pi(h, a) \right]
\]

\[
= \sum_{a \in A} \pi(a; o_h)E_s|h [Q^\pi(s, a)]
\]

\[
= E_s|h \left[ \sum_{a \in A} \Pr(a \mid s) \pi(a; o_h)Q^\pi(s, a) \right].
\] (32)

\[
V^\pi(h) = \sum_{a \in A} \pi(a; o_h)Q^\pi(h, a)
\]

\[
= \sum_{a \in A} \pi(a; o_h)E_s|h [Q^\pi(s, a)]
\]

Equations (32) and (33) differ in terms of which observation is used by the policy; in Equation (32), an observation \( o \) inferred from a state \( s \) inferred from the history \( h \) is used, while in Equation (33) the final observation \( o_h \) of the history \( h \) is used. These two observations \( o \) and \( o_h \) are not generally the same, and the respective expectations are similarly not generally the same. The nested expectation in Equation (32) can be interpreted as a lossy round-trip inference from history to state and from state back to observation \( h \rightarrow s \rightarrow o \). Although histories and states tend to be somewhat correlated, both state aliasing and history aliasing make the roundtrip conversion imperfect, causing a mismatch between the expected state value function \( E_s|h [V^\pi(s)] \) and the history value function \( V^\pi(h) \) in the general control case of a general POMDP.

Third proof, by example. This is a proof by example (with a proof by contradiction element). We will define the good/bad POMDP and, for a specific policy and history, first calculate \( E_s|h [V^\pi(s)] \) exactly, and then \( V^\pi(h) \) using bootstrapping (while also assuming \( V^\pi(hao) = E_{s'|hao} [V^\pi(s')] \)). We show that the two values are numerically different.

In the good/bad POMDP, \( S = \{ \text{GOOD}, \text{BAD} \} \), \( A = \{ \text{GOOD}, \text{BAD} \} \), \( O = \{ \text{GOOD}, \text{BAD} \} \); At times, we will use the shorthands \( G \) and \( B \). The initial state distribution is uniform, and each state deterministically transitions into itself. The \( \text{GOOD} \) state always emits the \( \text{GOOD} \) observation, while the \( \text{BAD} \) state emits a random observation. Consider the reward function such that \( R(s, a) = 1[a = \text{GOOD}] \), i.e., the agent receives a reward whenever it choses the \( \text{GOOD} \) action. We will denote a history
as the concatenation of alternating observations and actions, starting with an observation. To keep the notation compact, we will occasionally use symbols $G$ and $B$ to represent GOOD and BAD states, observations and actions. Consider a deterministic policy $\pi(a; h) = I[a = o_h]$ which returns the action corresponding to the last observation. Note that this POMDP and this policy satisfy the requirements to guarantee that $V^\pi(s)$ is well defined.

Next, we calculate the state values $V^\pi(s)$. The GOOD state always emits the GOOD observation, so the agent will always choose the GOOD action and receive a reward of 1, then the state will always transition into itself,

$$V^\pi(s = \text{GOOD}) = 1 + \gamma V^\pi(s = \text{GOOD})$$

$$= \frac{1}{1 - \gamma}.$$  \hfill (34)

On the other hand, the BAD state will only emit the GOOD observation half of the times, so the agent will only choose the GOOD action and receive a reward of 1 half of the times, then the state will always transition into itself,

$$V^\pi(s = \text{BAD}) = \frac{1}{2} + \gamma V^\pi(s = \text{BAD})$$

$$= \frac{1}{2(1 - \gamma)}.$$  \hfill (35)

Next, we consider the history $h = G$ after a single initial GOOD observation, and calculate the history value $V^\pi(h)$. Before proceeding, we need to calculate a few intermediate quantities, such as the belief-distribution:

$$\Pr(s = \text{GOOD} \mid h = G) \propto \Pr(h = G \mid s = \text{GOOD}) \Pr(s = \text{GOOD})$$

$$= \frac{1}{2},$$ \hfill (36)

$$\Pr(s = \text{BAD} \mid h = G) \propto \Pr(h = G \mid s = \text{BAD}) \Pr(s = \text{BAD})$$

$$= \frac{1}{4}.$$ \hfill (37)

therefore

$$\Pr(s = \text{GOOD} \mid h = G) = \frac{2}{3},$$ \hfill (38)

$$\Pr(s = \text{BAD} \mid h = G) = \frac{1}{3}.$$ \hfill (39)

We also calculate the belief-state distribution after two other histories. First $h = GGG$,

$$\Pr(s = \text{GOOD} \mid h = GGG) \propto \Pr(h = GGG \mid s = \text{GOOD}) \Pr(s = \text{GOOD})$$

$$= \frac{1}{2},$$ \hfill (40)

$$\Pr(s = \text{BAD} \mid h = GGG) \propto \Pr(h = GGG \mid s = \text{BAD}) \Pr(s = \text{BAD})$$

$$= \frac{1}{8}.$$ \hfill (41)

therefore

$$\Pr(s = \text{GOOD} \mid h = GGG) = \frac{4}{5},$$ \hfill (42)

$$\Pr(s = \text{BAD} \mid h = GGG) = \frac{1}{5}. $$ \hfill (43)
Then \( h = GGB \),

\[
\Pr(s = G \mid h = GGB) \propto \Pr(h = GGB \mid s = G) \Pr(s = G) = 0, \quad (44)
\]

\[
\Pr(s = B \mid h = GGB) \propto \Pr(h = GGB \mid s = B) \Pr(s = B) = 1, \quad (45)
\]

therefore

\[
\Pr(s = G \mid h = GGB) = 0, \quad (46)
\]

\[
\Pr(s = B \mid h = GGB) = 1. \quad (47)
\]

We also need to calculate the observation emission probabilities,

\[
\Pr(o = G \mid h = G, a = G) = \Pr(s = G \mid h = G) \Pr(o = G \mid s = G) + \Pr(s = B \mid h = G) \Pr(o = G \mid s = B)
\]

\[
= \frac{2}{3} + \frac{1}{3} \frac{1}{2} = \frac{5}{6}, \quad (48)
\]

\[
\Pr(o = B \mid h = G, a = G) = \Pr(s = G \mid h = G) \Pr(o = B \mid s = G) + \Pr(s = B \mid h = G) \Pr(o = B \mid s = B)
\]

\[
= \frac{2}{3} + \frac{1}{3} \frac{1}{2} = \frac{1}{6} \quad (49)
\]

Next, we calculate \( V^\pi(h = G) \) under the assumption that the equality holds,

\[
V^\pi(h = G) = \mathbb{E}_{s \mid h = G} [V^\pi(s)]
\]

\[
= \Pr(s = G \mid h = G)V^\pi(s = G) + \Pr(s = B \mid h = G)V^\pi(s = B)
\]

\[
= \frac{2}{3} \frac{1}{1 - \gamma} + \frac{1}{3} \frac{1}{2(1 - \gamma)}
\]

\[
= \frac{5}{6(1 - \gamma)}. \quad (50)
\]

We can also apply the equality to other histories,

\[
V^\pi(h = GGG) = \mathbb{E}_{s \mid h = GGG} [V^\pi(s)]
\]

\[
= \Pr(s = G \mid h = GGG)V^\pi(s = G) + \Pr(s = B \mid h = GGG)V^\pi(s = B)
\]

\[
= \frac{4}{6} \frac{1}{1 - \gamma} + \frac{1}{6} \frac{1}{2(1 - \gamma)}
\]

\[
= \frac{9}{10(1 - \gamma)}. \quad (51)
\]

\[
V^\pi(h = GGB) = \mathbb{E}_{s \mid h = GGB} [V^\pi(s)]
\]

\[
= \Pr(s = G \mid h = GGB)V^\pi(s = G) + \Pr(s = B \mid h = GGB)V^\pi(s = B)
\]

\[
= 0 \frac{1}{1 - \gamma} + \frac{1}{2(1 - \gamma)}
\]

\[
= \frac{1}{2(1 - \gamma)}. \quad (52)
\]
Next, we calculate $V^\pi(h = G)$, this time by bootstrapping first, and then using the equality. Note that with the given history $h = G$, the agent will choose action $a = \text{GOOD}$. Then,

$$
V^\pi(h = G) = R(h = G, a = G) + \gamma E\left[\phi(h = G, o = G) | V^\pi(hao = GGo)\right] 
$$

$$
= 1 + \gamma \left( \frac{5}{6} \cdot \frac{9}{10(1 - \gamma)} + \frac{1}{6} \cdot \frac{1}{2(1 - \gamma)} \right) 
$$

$$
= \frac{60 - 60\gamma}{60(1 - \gamma)} + \frac{45\gamma}{60(1 - \gamma)} + \frac{5\gamma}{60(1 - \gamma)} 
$$

$$
= \frac{6 - \gamma}{6(1 - \gamma)}. 
$$

(53)

The values from Equations (50) and (53) contradict each other, therefore, for this POMDP, policy, and history, $V^\pi(h) \neq \mathbb{E}_s[h | V^\pi(s)]$.

**Theorem 4.3.** If the POMDP states and observations are related by a bijection $\phi: \mathcal{O} \rightarrow \mathcal{S}$, and the policy is reactive, then $V^\pi(s)$ is an unbiased estimate of $V^\pi(h)$, i.e., $V^\pi(h) = \mathbb{E}_s[h | V^\pi(s)]$.

**Proof.** Although there is no intrinsic state uncertainty, we continue to use probabilistic notation for notational consistency and simplicity. In the following derivation, we use the fact that states and observations contain the same system information to determine the first action and reward. This process can be repeated iteratively for all future actions and rewards (omitted, but represented by the ellipsis),

$$
\mathbb{E}_s[h | V^\pi(s)] = \mathbb{E}_s[h] \left[ \sum_{o \in A} \Pr(a | s) Q^\pi(s, a) \right] 
$$

$$
= \mathbb{E}_s[h] \left[ \sum_{o \in A} \pi(a; o_h) Q^\pi(s, a) \right] 
$$

$$
= \mathbb{E}_s[h] \left[ \sum_{o \in A} \sum_{a \in A} \pi(a; o_h) Q^\pi(s, a) \right] 
$$

$$
= \sum_{o \in A} \mathbb{E}_s[h] \left[ \mathbb{E}_s[h] \left[ R(s, a) + \gamma \mathbb{E}_s'[s, a] [V^\pi(s')] \right] \right] 
$$

$$
= \sum_{o \in A} \mathbb{E}_s[h] \left[ R(h, a) + \gamma \mathbb{E}_h[a, h] [V^\pi(s')] \right] 
$$

$$
= \sum_{o \in A} \mathbb{E}_s[h] \left[ R(h, a) + \gamma \mathbb{E}_h[a, h] [V^\pi(hao)] \right] 
$$

$$
= \sum_{o \in A} \mathbb{E}_s[h] \left[ R(h, a) + \gamma \mathbb{E}_h[a, h] [V^\pi(hao)] \right] 
$$

$$
= V^\pi(h). 
$$

(54)

In this case, the bijection between observations and states removes any error from the round-trip inference $h \rightarrow s \rightarrow o$ is perfect, which was the cause of the bias in Theorem 4.2.
**Theorem 5.1.** For an arbitrary POMDP and policy, \( V^\pi(h, s) \) is an unbiased estimate of \( V^\pi(h) \), i.e., \( V^\pi(h) = \mathbb{E}_{s|h} [V^\pi(h, s)] \)

Proof. Follows from Equations (1) and (18).

\[
V^\pi(h^{(0)}) = \mathbb{E}_{s,o|h^{(0)}} \left[ \sum_k \gamma^k R(h^{(k)}, a^{(k)}) \right] = \mathbb{E}_{s^{(0)}|h^{(0)}} \mathbb{E}_{s,a|h^{(0)},s^{(0)}} \left[ \sum_k \gamma^k R(h^{(k)}, a^{(k)}) \right] = \mathbb{E}_{s^{(0)}|h^{(0)}} \left[ V^\pi(h^{(0)}, s^{(0)}) \right]. \tag{55}
\]

**Theorem 5.2 (Asymmetric Policy Gradient).** The policy gradient can be expressed using history-state values,

\[
\nabla_\theta L_{\text{policy}}(\theta) = - \mathbb{E} \left[ \sum_t \gamma^t Q^\pi(h_t, s_t, a_t) \nabla_\theta \log \pi(a_t; h_t) \right]. \tag{56}
\]

Proof. Following Theorem 5.1, we have

\[
Q^\pi(h, a) = R(h, a) + \gamma \mathbb{E}_{o|h,a} [V^\pi(hao)] = R(h, a) + \gamma \mathbb{E}_{o|h,a} \left[ \mathbb{E}_{s'|h, o} [V^\pi(hao, s')] \right] = R(h, a) + \gamma \mathbb{E}_{s', o|h,a} [V^\pi(hao, s')] = \mathbb{E}_{s|h} \left[ R(s, a) + \gamma \mathbb{E}_{s', o|s,a} [V^\pi(hao, s')] \right] = \mathbb{E}_{s|h} [Q^\pi(h, s, a)]. \tag{57}
\]

Therefore,

\[
\nabla_\theta L_{\text{policy}}(\theta) = - \mathbb{E} \left[ \sum_t \gamma^t Q^\pi(h_t, a_t) \nabla_\theta \log \pi(a_t; h_t) \right] = - \sum_t \gamma^t \mathbb{E}_{h_t, a_t} [Q^\pi(h_t, a_t) \nabla_\theta \log \pi(a_t; h_t)] = - \sum_t \gamma^t \mathbb{E}_{h_t, a_t} \left[ \mathbb{E}_{s_t|h_t} [Q^\pi(h_t, s_t, a_t)] \nabla_\theta \log \pi(a_t; h_t) \right] = - \sum_t \gamma^t \mathbb{E}_{h_t, s_t, a_t} [Q^\pi(h_t, s_t, a_t) \nabla_\theta \log \pi(a_t; h_t)] = - \mathbb{E} \left[ \sum_t \gamma^t Q^\pi(h_t, s_t, a_t) \nabla_\theta \log \pi(a_t; h_t) \right]. \tag{58}
\]

\[
\square
\]

### C ENVIRONMENTS

The environments used in the evaluation can be split into two groups. **Shopping-5, Shopping-6, Heavenhell-3, Heavenhell-4 and Rocks-5-6** are flat POMDPs, while **Keydoor** and **Ninerooms** are gridverse POMDPs.
Table 1: Environment properties.

| Domain            | | | | | |
|-------------------|---|---|---|---|
|                   | $|S|$ | $|A|$ | $|O|$ | $\gamma$ |
| Shopping-5        | 625 | 6  | 50  | 0.99 |
| Shopping-6        | 1296 | 6  | 72  | 0.99 |
| Heavenhell-3      | 28  | 4   | 15  | 0.99 |
| Heavenhell-4      | 36  | 4   | 19  | 0.99 |
| Rocksmaple-5-6    | 1600 | 11 | 27  | 0.95 |

Figure 4: Layout of the Shopping environments.

Figure 5: Layout of the Heavenhell environments.

Figure 6: Layout of the Rocksmaple environment.
C.1 FLAT POMDPS

In the flat POMDPs, states, actions and observations are encoded by categorical indices with no inherent metric, which are intrinsically equally (dis)similar to each other. While it is not possible to generalize across states and observations via feature extraction, the primary challenge in these POMDPs is that of generalizing across different histories.

Because the flat POMDPs are finite, their state, action and observation spaces have well-defined sizes, shown in Table 1; note, however, that the size of the state space is not a significant measure of the complexity of partially observable tasks, while the time required to solve the task (i.e., history length) is a more relevant measure.

C.1.1 Shopping

This environment simulates an agent going to a shop to buy an item it forgot. The agent navigates a $5 \times 5$ or $6 \times 6$ gridworld trying to locate and select a randomly positioned item. The agent’s position is fully observable, while the item’s position is only observed when queried. Figure 4 depicts the gridworlds encoded by Shopping-5 and Shopping-6.

**States and Observations**  States encode the position of the agent and the position of the item in a single integer. Observations encode the position of the agent or the position of the item in a single integer.

**Actions**  Each time-step, the agent must choose an action from the set { LEFT, RIGHT, UP, DOWN, QUERY, BUY }. If the agent chooses the QUERY action, it observes the item’s position, otherwise it observes its own position. To solve the task optimally, the agents needs to query the item’s position and remember it, navigate to it, and then buy it.

**Rewards**  The agent receives the following reward signal:
- a reward of $-1.0$ for moving;
- a reward of $-2.0$ for performing a QUERY action;
- a reward of $-5.0$ for performing a BUY action in the wrong cell; and
- a reward of $10.0$ for performing a BUY action in the correct cell.

C.1.2 Heavenhell

The agent navigates a corridor-like gridworld composed of a fork and 3 dead-ends. Two dead-ends are exits which lead to heaven or hell, although the agent does not know which is which, while the third dead-end leads to a priest who can help the agent identify the heaven exit. Figure 5 depicts the gridworlds encoded by Heavenhell-3 and Heavenhell-4.

**States and Observations**  States encode the position of the agent and the position of the exit to heaven. Observations encode the position of the agent or the position of the exit to heaven.

**Actions**  Each time-step, the agent must choose an action from the set { NORTH, SOUTH, EAST, WEST }. If the agent is at the priest, it observes heaven’s location, otherwise it observes its own position. To solve the task, the agent needs to navigate to the priest, then back to the fork, and on to heaven.

**Rewards**  The agent receives a sparse reward signal composed of:
- a reward of $1.0$ for exiting to heaven; and
- a reward of $-1.0$ for exiting to hell.

C.1.3 Rocks Sample

This environment simulates an agent navigating a landscape to collect research material. The agent navigates a $5 \times 5$ gridworld which contains 6 rocks, each having either good or bad research value. The agent’s position is fully observable.

[https://github.com/abaisero/gym-pomdps](https://github.com/abaisero/gym-pomdps)
while each rock’s goodness is randomly sampled and unobserved unless the agent checks it. Each rock’s goodness can be queried via a stochastic check action, whose reliability decays with the agent-rock distance. Figure 6 depicts the gridworld encoded by Rocksample-5-6.

**States and Observations**  States encode the position of the agent and whether each rock is good or bad; the positions of the rocks are fixed so they do not need to be tracked by the state. Observations encode the position of the agent or whether the checked rock is good or bad.

**Actions**  Each time-step, the agent must choose an action from the set { NORTH, SOUTH, EAST, WEST, CHECK_0, CHECK_1, CHECK_2, CHECK_3, CHECK_4, CHECK_5, SAMPLE }. If the agent chose a CHECK_* action, it observes whether the corresponding rock is good or bad, otherwise it observes its own position. To solve the task, the agent needs to sample all the good rocks (and none of the bad rocks), dynamically adapting its path depending on the stochastic information gained by checking each rock.

**Rewards**  The agent receives a sparse reward signal composed of:

- a reward of $10.0$ for sampling a good rock; and
- a reward of $-10.0$ for sampling a bad rock.

## C.2 GRIDVERSE POMDPS

In the gridverse POMDPs, actions are still encoded by categorical indices, while states and observations are encoded as structures which do have an inherent similarity metric; they are split into two components:

https://github.com/abaisero/gym-gridverse
• A grid component: a $6 \times H \times W$ volume of categorical indices which encode cell types, colors, statuses, agent presence, and the spatial relationships between them. The observation grid component is a slice of the corresponding state grid component made to match the agent’s perspective: it is rotated to be a first-person view, and cells hidden behind walls are occluded, as shown in Figures 7b and 8b.

• An agent component: a 6-dimensional array of categorical indices representing the agent position and orientation, and the item held by the agent if any. The position and orientations in the state agent component are absolute, while those in the observation component are relative to the agent’s perspective—they are essentially constant, and not necessary for control.

C.2.1 Keydoor

The agent navigates a $5 \times 5$ gridworld split into two rooms split by a wall and a locked door; on one side is the agent and a key which can be used to open the door, while on the other side is the goal. The positions of the agent, the key, the door, and the goal are randomly sampled such that two instances of the same problem are unlikely to be the same. Figure 7 shows state and observation frames taken from an instance of the Keydoor environment.

States and Observations The state grid component is a $6 \times 5 \times 5$ volume. The observation grid component is a $6 \times 7 \times 7$ volume.

Actions Each time-step, the agent must choose an action from the set {MOVE_FORWARD, MOVE_BACKWARD, MOVE_LEFT, MOVE_RIGHT, TURN_LEFT, TURN_RIGHT, PICK_N_DROP, ACTUATE}. The MOVE_* actions result in a movement depending on the agent’s orientation, while the TURN_* allows the agent to change its orientation. With the PICK_N_DROP action, the agent can pick and/or drop the key from/to the cell in front, while with the ACTUATE action, the agent can open and/or close the door.

Rewards The agent receives a dense reward signal composed as the sum of the following terms:

• a living reward of $-0.05$ for every time-step;
• a reward of 0.2 for stepping closer to the goal, and $-0.2$ for stepping further away (ignoring walls);
• a reward of 1.0 for picking up the key, and $-1.0$ for dropping it;
• a reward of 1.0 for opening the door, and $-1.0$ for closing it; and
• a reward of 5.0 for reaching the goal.

C.2.2 Ninerooms

The agent navigates a $13 \times 13$ maze-like gridworld trying to locate and reach the goal. Figure 8 shows state and observation frames taken from an instance of the Ninerooms environment.

States and Observations The state grid component is a $6 \times 13 \times 13$ volume. The observation grid component is a $6 \times 7 \times 7$ volume.

Actions Each time-step, the agent must choose an action from the set {MOVE_FORWARD, MOVE_BACKWARD, MOVE_LEFT, MOVE_RIGHT, TURN_LEFT, TURN_RIGHT}. The MOVE_* actions result in a movement depending on the agent’s orientation, while the TURN_* allows the agent to change its orientation.

Rewards The agent receives a dense reward signal composed as the sum of the following terms:

• a living reward of $-0.05$ for every time-step;
• a reward of 0.2 for stepping closer to the goal, and $-0.2$ for stepping further away (ignoring walls); and
• a reward of 5.0 for reaching the goal.
(a) State, action, and observation representation models used for flat POMDPs.

(b) State, action, and observation representation models used for gridverse POMDPs.

(c) A2C architecture. The state, action, and observation representations $\phi(s)$, $\phi(a)$, and $\phi(o)$ are those shown in fig. 9a or fig. 9b depending on whether the type of environment. Dotted lines are present or omitted depending on whether a history critic $\hat{V}(h)$, state critic $\hat{V}(s)$, or history-state critic $\hat{V}(h,s)$ is being modeled.

Figure 9

D ARCHITECTURES AND HYPER-PARAMETERS

In this section, we describe the architectures used by the policy and critic models; a general overview is shown in Figure 9. The general architecture will be similar for all domains; however there will some differences to accommodate the fact that the flat POMDPs provide states and observations as categorical indices, while the gridverse POMDPs provide states and observations as volumes and arrays of categorical indices, whose structures include spatial relationships.

Features Extraction for Flat POMDPs  These components are shown in Figure 9a. Because flat POMDPs provide states, actions and observations as categorical indices, and we use 128-dimensional embedding models to represent each of them.

Features Extraction for Gridverse POMDPs  These components are shown in Figure 9b. Because gridverse POMDPs provide actions as categorical indices, and we use 128-dimensional embedding models to represent them. On the other hand, states and observations are provided in the format described in Appendix C.2, each composed of a grid and an agent component:

**grid** The grid component is a $6 \times H \times W$ volume of categorical data, which we pass through 4-dimensional embeddings, resulting in a $24 \times H \times W$ volume of numeric data. We further pass that data to two layers of CNN followed by ReLU non-linearities, resulting in a $16 \times H \times W$ volume of data, finally flattened into a $16HW$-dimensional array.

**agent** The agent component is a 6-dimensional array of categorical data, which we pass through 4-dimensional embeddings, resulting in a 24-dimensional array of numeric data.

The grid and agent representations are then concatenated to form the state or observation representation $\phi(s)$ or $\phi(o)$.

Remainder of the Architecture  These components are shown in Figure 9c. Concatenated action and observation embeddings form the to a 128-dimensional single-layer gated recurrent unit (GRU) [Cho et al., 2014], which acts as a history representation $\phi(h)$. The history representation is then fed into separate policy and critic models, each a 2-layer 128-dimensional feedforward model with ReLU non-linearities. A practical issue we found with A2C(h,s) is that the history and state representations $\phi(h)$ and $\phi(s)$ contain features with very different orders of magnitude; To address this, we use layer-normalization. To guarantee a fair comparison with the other methods, we do the same for other critics as well. Next, we describe architectural details specific to each method:

A2C(h): The history representation passes through layer-normalization first, and then is fed to the critic model;
A2C(s): The state embedding passes through layer-normalization first, and then is fed to the critic model;  
A2C(h,s): The history representation and state embedding individually pass through layer-normalization first, and then are concatenated to form a single feature vector, which is then fed to the critic model.