Chapter 5

Study of temperature-dependent fluid properties on MHD free stream flow and heat transfer over a non-linearly stretching sheet†

5.1 Introduction

The flow of an incompressible viscous fluid over a linearly stretching surface has been discussed respectively in Chapter 3 and 4 for steady-state and unsteady condition. In certain manufacturing processes, it is not always possible to have stretching surface which is linearly stretched. There are several applications which involve fluid flow over non-linearly stretching surfaces. The processes of extrusion of plastic sheets, wire drawing and aerodynamics shaping are generally non-linear in nature because a large force is applied initially which later reduces with time. The literature survey in the subsections 2.1 to 2.3 of Chapter 2 indicates that no work has been reported that examined the effect of temperature-dependent fluid properties with MHD free stream velocity on the flow and

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heat transfer over a non-linearly stretching sheet.

In this study, the non-linear stretching surface is assumed to have a power law stretching. Therefore, in this chapter we have addressed this problem and studied the effect of temperature-dependent fluid properties with MHD free stream velocity on flow and heat transfer rate over a non-linearly stretching sheet.

5.2 Mathematical Formulation

Here, we consider a steady, two-dimensional, electrically conducting flow of an incompressible fluid. The flow is considered over an impermeable stretching sheet coinciding with the $x$-axis. The continuous stretching surface is assumed to have a power law velocity $u = u_w = c x^m$, where $c$ is the stretching rate and $m$ is an exponent. We assume that the induced magnetic field is very small as compared to the applied magnetic field and hence, can be neglected. Due to small values, the viscous dissipation and the ohmic heating terms are not included in the energy equation. Under these assumptions, the governing equations of mass, momentum and energy for flow having temperature-dependent fluid viscosity and thermal conductivity are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \frac{\sigma B^2}{\rho} (u - U) \quad (5.2)$$

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( K \frac{\partial T}{\partial y} \right) \quad (5.3)$$

where $u$ and $v$ are the velocity components in the $x$ and $y$ directions respectively.

The viscosity $\mu$ in equation (5.2) can be expressed in terms of temperature as

$$\frac{1}{\mu} = \alpha(T - T_r), \quad \text{where} \quad \alpha = \frac{\delta}{\mu_{\infty}} \quad \text{and} \quad T_r = T_{\infty} - \frac{1}{\delta}.$$  

Here, $\alpha$, $\mu_{\infty}$ and $T_r$ are constants, and their values depend on the reference state. The parameter $\delta$ represents thermal property of the fluid. In general, $\alpha < 0$ corresponds to
gases whereas $\alpha > 0$ corresponds to liquids. This holds when the temperature at the sheet ($T_w$) is larger than that far away from the sheet ($T_\infty$).

In the above equations, $B^2$ represents strength of the magnetic field, $U$ is the free stream velocity and $\sigma$ is the electrical conductivity. The magnetic field is of the form

$$B = B_0 x \frac{m-1}{2}$$

and is chosen to get a similarity solution. Further, $c_p$ denotes the specific heat at constant pressure and $K$ denotes the temperature-dependent thermal conductivity which is considered to be of the form

$$K = K_\infty [1 + \frac{\epsilon}{\Delta T}(T - T_\infty)],$$

where $\Delta T = T_w - T_\infty$ and $\epsilon = \frac{K_w - K_\infty}{K_\infty}$ is small in magnitude.

Here, $K_w$ and $K_\infty$ are thermal conductivities of the fluid at the sheet and far away from the sheet respectively. Substituting these in equations (5.2) and (5.3), we obtain

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu_\infty \frac{\partial u}{\partial y} \right) - \frac{\sigma B^2}{\rho} (u - U)$$

(5.4)

$$\rho c_p u \frac{\partial T}{\partial x} + \left( \rho c_p v - \frac{K_\infty \epsilon}{\Delta T} \frac{\partial T}{\partial y} \right) \frac{\partial T}{\partial y} = \left( K_\infty (1 + \frac{\epsilon}{\Delta T}(T - T_\infty)) \right) \frac{\partial^2 T}{\partial y^2}$$

(5.5)

where $\epsilon$ represents the thermal conductivity parameter. The relevant boundary conditions are

$$u = u_w = c x^m, \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0$$

$$u \to U(x) = b x^m, \quad T \to T_\infty \quad \text{as} \quad y \to \infty$$

(5.6)

Here, we use the following dimensionless similarity variables

$$\eta = \frac{y}{x} \sqrt{\frac{m+1}{2} \sqrt{Re}}, \quad \psi = \left( \frac{2}{m+1} \right) u_w x (Re)^{-\frac{1}{2}} f(\eta) \quad \text{and} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

where $Re = \frac{u_w}{\nu_\infty}$ is the local Reynolds number and $\nu_\infty$ is the kinematic viscosity.
of the fluid.

The continuity equation (5.1) is identically satisfied with the relations

\[ u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \]

Using the above similarity variables, the equations (5.4) and (5.5) can be written as

\[ \beta (f'^2 - \lambda^2) - f f'' = \frac{1}{1 - \frac{\theta}{A}} \left[ f''' + \frac{f'' \theta'}{A - \theta} \right] - M (f' - \lambda) \quad (5.7) \]

\[ (1 + \epsilon \theta) \theta'' = -\epsilon \theta'^2 - Pr f \theta'. \quad (5.8) \]

The corresponding boundary conditions are

\[ f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \]
\[ f'(\infty) = \lambda, \quad \theta(\infty) = 0 \quad (5.9) \]

where primes denote differentiation with respect to \( \eta \).

Here, the expression for various parameters are given as:

\[ M = \frac{2 \sigma B^2}{\rho_\infty b(m + 1)} \quad \text{is the magnetic parameter,} \]
\[ \lambda = \frac{b}{c} \quad \text{is the free stream parameter,} \]
\[ Pr = \frac{\mu_\infty c_p}{K_\infty} \quad \text{is the Prandtl number,} \]
\[ \beta = \frac{2m}{m + 1} \quad \text{is the stretching parameter} \]
\[ \text{and} \quad A = -\frac{1}{\delta(T_w - T_\infty)} \left( \frac{2}{m + 1} \right) \quad \text{is the fluid viscosity parameter.} \]

The quantities of physical interest are the skin friction coefficient \( C_f \) and the local Nusselt number \( Nu \) which are defined as

\[ C_f = \frac{2 \tau_w}{\rho u w^2} = \frac{\sqrt{2(m + 1)} A}{A - 1} \frac{1}{\sqrt{Re}} f''(0, A) \]

and

\[ Nu = \frac{q_w}{K_\infty (T_w - T_\infty)} = -\frac{m + 1}{2} \sqrt{Re} \theta'(0, A). \]

Here,

\[ \tau_w = -\mu \frac{\partial u}{\partial y} \quad \text{is the local shear stress} \]
\[ \text{and} \quad q_w = -K_\infty \frac{\partial T}{\partial y} \quad \text{is the local heat flux.} \]
5.3 Results and Discussion

The coupled non-linear differential equations (5.7) and (5.8) along with the boundary conditions (5.9) are solved numerically using the Runge-Kutta Fehlberg scheme with the shooting technique. To validate the present study, we have compared our results (Table 5.1) with the results of a similar study published by Prasad et al. [65] as a limiting case of the present work. It is evident from Table 5.1 that the values of $f''(0)$ and $-\theta'(0)$ from the current analysis for various $\epsilon$ compare well with the results of [65].

Table 5.1: Computed values of $f''(0)$ and $-\theta'(0)$ for various values of thermal conductivity parameter $\epsilon$ when $M = 0$, $\beta = 0$, $Pr = 1$, $\lambda = 0$, $A = -5$: a comparison with similar study.

| $\epsilon$ | $f''(0)$ | $-\theta'(0)$ |
|-----------|----------|--------------|
| 0         | -0.70560 | 0.61693      |
| 0.1       | -0.70486 | 0.57429      |
| 0.2       | -0.70419 | 0.53809      |
| 0.3       | -0.70358 | 0.50693      |

From Table 5.2 which gives computed values of $f''(0)$ and $-\theta'(0)$ for various $M$ and $\lambda$, we notice that the skin friction ($C_f \propto f''(0)$) is negative for all values of $M$ when $\lambda < 1$. The negative values of $f''(0)$ show that fluid exerts a drag. In all the cases, we see that
Figure 5.1: The velocity profile $f'(\eta)$ for various values of magnetic parameter $M$ when $\lambda = 0$, $\beta = 0$, $Pr = 1$, $\epsilon = 0$ and $A^{-1} = 0$.

Figure 5.2: The velocity profile $f'(\eta)$ for various values of magnetic parameter $M$ when $\lambda = 0.5$, $\beta = 0$, $Pr = 1$, $\epsilon = 0$ and $A^{-1} = 0$. 
Figure 5.3: The velocity profile $f'(\eta)$ for various values of magnetic parameter $M$ when $\lambda = 2$, $\beta = 0$, $Pr = 1$, $\epsilon = 0$ and $A^{-1} = 0$.

the thickness of the boundary layer decreases with the increase in magnetic parameter $M$. Also, we observe that the velocity gradient is sharp when $\lambda > 1$ (Figure 5.3) as compared to $\lambda < 1$ (Figures 5.1 and 5.2) and it finally becomes zero at a certain distance from the surface.
Table 5.2: Computed values of $f''(0)$ and $-\theta'(0)$ for various values of magnetic parameter $M$ when $\beta = 0$, $Pr = 1$, $\epsilon = 0$ and $A^{-1} = 0$.

| $\lambda$ | $M$ | $f''(0)$ | $-\theta'(0)$ |
|-----------|-----|----------|---------------|
| 0         | 0   | -0.63001 | 0.63001       |
|           | 1   | -1.16337 | 0.54122       |
|           | 2   | -1.53149 | 0.49044       |
| 0.5       | 0   | -0.36031 | 0.72062       |
|           | 1   | -0.60439 | 0.69620       |
|           | 2   | -0.78173 | 0.68171       |
| 2.0       | 0   | 0.92980  | 0.92980       |
|           | 1   | 1.33840  | 0.95029       |
|           | 2   | 1.66016  | 0.96396       |
|           | 5   | 2.39037  | 0.98876       |

Figure 5.4: The temperature profile $\theta(\eta)$ for various values of magnetic parameter $M$ when $\lambda = 0$, $\beta = 0$, $Pr = 1$, $\epsilon = 0$ and $A^{-1} = 0$. 
Figure 5.5: The temperature profile $\theta(\eta)$ for various values of magnetic parameter $M$ when $\lambda = 0.5$, $\beta = 0$, $Pr = 1$, $\epsilon = 0$ and $A^{-1} = 0$.

Figures 5.4, 5.5 and 5.6 show the temperature profiles for various values of magnetic parameter $M$ for free stream parameter $\lambda = 0$, $\lambda = 0.5$ and $\lambda = 2$ respectively. We observe that the temperature increases as we increase the magnetic parameter $M$ when $\lambda < 1$ (Figures 5.4 and 5.5), whereas a reverse trend is seen when $\lambda > 1$ (Figure 5.6). The direction of the arrow in Figures 5.4-5.6 indicates change in temperature profile with increasing $M$. A comparison of corresponding curves in Figures 5.4 and 5.5, we notice that the temperature decreases with increasing free stream parameter $\lambda$, and the temperature gradient is sharper with higher $\lambda$. This implies that there is a reduction in thermal boundary layer thickness. The temperature profile curves in Figure 5.4 show greater separation as compared to the curves corresponding to $\lambda = 0.5$ and $\lambda = 2$ in Figures 5.5 and 5.6 respectively. When the free stream velocity exceeds the stretching sheet velocity ($\lambda > 1$), the temperature profile decreases with increasing $M$. This implies that the variation in magnetic parameter $M$ produces a more prominent impact of the temperature profile in the absence of free stream velocity ($\lambda = 0$). Further, we notice
Figure 5.6: The temperature profile $\theta(\eta)$ for various values of magnetic parameter $M$ when $\lambda = 2$, $\beta = 0$, $Pr = 1$, $\epsilon = 0$ and $A^{-1} = 0$.

from Table 5.2 that the local Nusselt number ($Nu \propto -\theta'(0)$) decreases as we increase the magnetic parameter $M$ for $\lambda < 1$, due to reduction in the heat transfer rate. This explains the increase in temperature with increasing $M$ in Figures 5.4 and 5.5. When the free stream velocity exceeds the stretching sheet velocity ($\lambda > 1$), the temperature decreases (Figure 5.6) with increasing $M$ due to higher heat transfer rate (Table 5.2).

The effects of stretching parameter $\beta$ on velocity profile curves are shown in Figures 5.7 and 5.8 for $\lambda = 0.5$ and $\lambda = 2$ respectively. As $\beta$ increases, the velocity decreases (Figure 5.7) for $\lambda < 1$ and increases (Figure 5.8) for $\lambda > 1$. In all the cases, the velocity gradient increases and momentum boundary layer thickness decreases with an increase in $\beta$. Hence, in order to reduce the momentum boundary layer thickness the stretching parameter $\beta$ should be increased.
Figure 5.7: The velocity profile $f'(\eta)$ for various values of stretching parameter $\beta$ when $\lambda = 0.5$, $M = 1$, $\epsilon = 0$, $Pr = 1$ and $A^{-1} = 0$.

Figure 5.8: The velocity profile $f'(\eta)$ for various values of stretching parameter $\beta$ when $\lambda = 2$, $M = 1$, $\epsilon = 0$, $Pr = 1$ and $A^{-1} = 0$. 
Figure 5.9: The temperature profile $\theta(\eta)$ for various values of stretching parameter $\beta$ when $\lambda = 0.5$, $M = 1$, $\epsilon = 0$, $Pr = 1$ and $A^{-1} = 0$.

Table 5.3: Computed values of $f''(0)$ and $-\theta'(0)$ for various values of stretching parameter $\beta$ when $M = 1$, $\epsilon = 0$, $Pr = 1$ and $A^{-1} = 0$.

| $\lambda$ | $\beta$ | $f''(0)$   | $-\theta'(0)$ |
|-----------|---------|------------|---------------|
| 0.5       | -1      | -0.24875   | 0.73009        |
|           | 0       | -0.60439   | 0.69620        |
|           | 1       | -0.83213   | 0.67935        |
| -0.5      | 0       | 0.55973    | 0.90487        |
| 2.0       | 0       | 1.33840    | 0.95029        |
|           | 1       | 2.24903    | 0.98595        |

Figures 5.9 and 5.10 show the temperature profiles for various values of stretching parameter $\beta$ for $\lambda = 0.5$ and $\lambda = 2$ respectively. It is clearly observed that, the temperature increases for $\lambda < 1$ and decreases for $\lambda > 1$ as we increase the stretching parameter $\beta$ (shown by the arrow). Table 5.3 gives the values of $f''(0)$ and $-\theta'(0)$ for various values of
The temperature profile \( \theta(\eta) \) for various values of stretching parameter \( \beta \) when \( \lambda = 2, M = 1, \epsilon = 0, Pr = 1 \) and \( A^{-1} = 0 \).

The effects of Prandtl number \( Pr \) on temperature profile curves are shown in Figures 5.11, 5.12 and 5.13 for free stream parameter \( \lambda = 0, \lambda = 0.5 \) and \( \lambda = 2 \) respectively. The temperature decreases as we increase the Prandtl number \( Pr \) in absence/presence of free stream parameter \( \lambda \). The temperature profile curves show greater separation in absence of \( \lambda \) (Figure 5.11) as compared to the curves corresponding to non-zero \( \lambda \) (Figures 5.12 and 5.13). Prandtl number \( Pr \) is the ratio of momentum diffusivity to thermal diffusivity. Large values of \( Pr \) correspond to relatively low thermal diffusivity. We know that with the increase in Prandtl number \( Pr \) the thermal boundary layer thickness de-
Table 5.4: Computed values of $f''(0)$ and $-\theta'(0)$ for various values of Prandtl number $Pr$ when $M = 0$, $\epsilon = 0$, $\beta = 1$ and $A^{-1} = 0$.

| $\lambda$ | $Pr$ | $f''(0)$ | $-\theta'(0)$ |
|-----------|------|----------|--------------|
| 0         | 0.7  | -1.00139 | 0.47157      |
|           | 1    | -1.00139 | 0.58718      |
|           | 2    | -1.00139 | 0.91049      |
| 0.5       | 0.7  | -0.66727 | 0.56975      |
|           | 1    | -0.66727 | 0.69250      |
|           | 2    | -0.66727 | 1.01157      |
| 2.0       | 0.7  | 2.01739  | 0.83106      |
|           | 1    | 2.01739  | 0.97869      |
|           | 2    | 2.01739  | 1.34343      |

creases. We observe that there is no change in skin friction with the change in $Pr$ and
hence velocity profiles remains unchanged Table 5.4. But, local Nusselt number increases
with increase in $Pr$ in all the cases of $\lambda$. 
Figure 5.11: The temperature profile $\theta(\eta)$ for various values of Prandtl number $Pr$ when $\lambda = 0, M = 0, \epsilon = 0, \beta = 1$ and $A^{-1} = 0$.

Figures 5.14, 5.15 and 5.16 show the effect of thermal conductivity parameter $\epsilon$ on temperature profiles for $\lambda = 0, \lambda = 0.5$ and $\lambda = 2$ respectively. We notice that there is a rise in temperature with the increase in thermal conductivity parameter $\epsilon$ (shown by the arrow). Table 5.5 gives computed values of $f''(0)$ and $-\theta'(0)$ for various values of thermal conductivity parameter $\epsilon$. From the table, we notice that there is a negligible change in skin friction with change in $\epsilon$, while Nusselt number decreases with increasing $\epsilon$. 
Figure 5.12: The temperature profile $\theta(\eta)$ for various values of Prandtl number $Pr$ when $\lambda = 0.5$, $M = 0$, $\epsilon = 0$, $\beta = 1$ and $A^{-1} = 0$.

Figure 5.13: The temperature profile $\theta(\eta)$ for various values of Prandtl number $Pr$ when $\lambda = 2$, $M = 0$, $\epsilon = 0$, $\beta = 1$ and $A^{-1} = 0$. 
Figure 5.14: The temperature profile $\theta(\eta)$ for various values of thermal conductivity parameter $\epsilon$ when $\lambda = 0$, $M = 0$, $\beta = 0$, $Pr = 1$ and $A = -5$.

Table 5.5: Computed values of $f''(0)$ and $-\theta'(0)$ for various values of thermal conductivity parameter $\epsilon$ when $M = 0$, $\beta = 0$, $Pr = 1$ and $A = -5$.

| $\lambda$ | $\epsilon$ | $f''(0)$  | $-\theta'(0)$ |
|-----------|-------------|-----------|---------------|
| 0         | 0           | -0.70560  | 0.61694       |
|           | 0.2         | -0.70420  | 0.53810       |
|           | 0.4         | -0.70302  | 0.47977       |
| 0.5       | 0           | -0.40560  | 0.71536       |
|           | 0.2         | -0.40479  | 0.62992       |
|           | 0.4         | -0.40411  | 0.56691       |
| 2.0       | 0           | 1.05340   | 0.93724       |
|           | 0.2         | 1.05124   | 0.83308       |
|           | 0.4         | 1.04937   | 0.75639       |
Figure 5.15: The temperature profile $\theta(\eta)$ for various values of thermal conductivity parameter $\epsilon$ when $\lambda = 0.5$, $M = 0$, $\beta = 0$, $Pr = 1$ and $A = -5$.

Figure 5.16: The temperature profile $\theta(\eta)$ for various values of thermal conductivity parameter $\epsilon$ when $\lambda = 2$, $M = 0$, $\beta = 0$, $Pr = 1$ and $A = -5$. 
The value of fluid viscosity parameter $A$ is determined by the viscosity of the fluid under consideration and the operating temperature difference. If $A$ is large, the effects of variable viscosity on the flow can be neglected. On other hand, for smaller values of $A$, either the fluid viscosity changes significantly with temperature or the temperature difference is high. In either case, the effect of the variable fluid viscosity is expected to be very important. In other words, the effect of viscosity is significant for smaller values of the fluid viscosity parameter $A$. The effect of fluid viscosity parameter $A$ on velocity profiles for $\lambda = 0$, $\lambda = 0.5$ and $\lambda = 2$ has been shown in Figures 5.17, 5.18 and 5.19 respectively. In the case of $\lambda < 1$, the velocity profile decreases with an increase in the fluid viscosity parameter $A$ whereas an opposite trend is observed for $\lambda > 1$. This shows up as an enhanced velocity profile (Figures 5.17 and 5.18). Also, Figure 5.20 shows the temperature profile for various values of fluid viscosity parameter $A$. There is an increase in temperature as we increase the fluid viscosity parameter $A$. Table 5.6 shows the variation in skin friction coefficient $C_f$ ($\propto f''(0)$) and local Nusselt number $Nu$ ($\propto -\theta'(0)$) for various values of the fluid viscosity parameter $A$. From the table, we notice that skin friction and Nusselt number decreases with the increase in fluid viscosity parameter $A$ when $\lambda < 1$. On the other hand, we observe an opposite trend for $\lambda > 1$ (Table 5.6).
Figure 5.17: The velocity profile $f'(\eta)$ for various values of fluid viscosity parameter $A$
when $\lambda = 0$, $M = 0$, $\beta = 0$, $Pr = 10$ and $\epsilon = 0$.

Figure 5.18: The velocity profile $f'(\eta)$ for various values of fluid viscosity parameter $A$
when $\lambda = 0.5$, $M = 0$, $\beta = 0$, $Pr = 10$ and $\epsilon = 0$. 
Figure 5.19: The velocity profile $f'(\eta)$ for various values of fluid viscosity parameter $A$ when $\lambda = 2$, $M = 0$, $\beta = 0$, $Pr = 10$ and $\epsilon = 0$.

Table 5.6: Computed values of $f''(0)$ and $-\theta'(0)$ for various values of fluid viscosity parameter $A$ when $M = 0$, $\beta = 0$, $Pr = 10$ and $\epsilon = 0$.

| $\lambda$ | $A$ | $f''(0)$ | $-\theta'(0)$ |
|-----------|-----|----------|---------------|
| 0         | -10 | -0.68056 | 2.36783       |
|           | -1  | -1.06660 | 2.30524       |
|           | -0.5| -1.39049 | 2.24792       |
| 0.5       | -10 | -0.38979 | 2.43923       |
|           | -1  | -0.61792 | 2.40655       |
|           | -0.5| -0.81370 | 2.37763       |
| 2.0       | -10 | 1.00825  | 2.70264       |
|           | -1  | 1.62577  | 2.76388       |
|           | -0.5| 2.16891  | 2.81399       |
Figures 5.20 and 5.22 show the temperature profiles for various values of free stream parameter $\lambda$ in presence and absence of fluid viscosity parameter $A$. In both the cases, the temperature decreases as we increase the free stream parameter $\lambda$ and hence, there is a reduction in the thermal boundary layer thickness. In other words, the boundary layer will be thinner for large $\lambda$. Thus, in order to reduce the thermal boundary layer thickness in industrial and engineering processes, the free stream parameter $\lambda$ should be increased. Figure 5.23 shows the velocity profile curves for various values of free stream parameter $\lambda$. It is clear from this figure that the velocity gradient increases with increase in $\lambda$. 

Figure 5.20: The temperature profile $\theta(\eta)$ for various values of fluid viscosity parameter $A$ when $\lambda = 0, M = 0, \beta = 0, Pr = 10$ and $\epsilon = 0$. 
Figure 5.21: The temperature profile $\theta(\eta)$ for various values of free stream parameter $\lambda$ when $A = -10$, $M = 0$, $\beta = 0$, $Pr = 10$ and $\epsilon = 0$.

Figure 5.22: The temperature profile $\theta(\eta)$ for various values of free stream parameter $\lambda$ when $M = 1$, $\epsilon = 0$, $Pr = 1$, $\beta = 0$ and $A^{-1} = 0$. 
Figure 5.23: The velocity profile $f'(\eta)$ for various values of free stream parameter $\lambda$ when $A = -10$, $M = 0$, $\beta = 0$, $Pr = 10$ and $\epsilon = 0$.

5.4 Summary

In the present chapter, numerical results have been reported for steady, two-dimensional, MHD free stream flow of a viscous and incompressible fluid along a continuously non-linearly stretching sheet. In this work, we have analyzed and discussed behavior of velocity and temperature with variations in Prandtl number $Pr$, magnetic parameter $M$, stretching parameter $\beta$, thermal conductivity parameter $\epsilon$ and fluid viscosity parameter $A$, by varying free stream velocity. The key findings of the present study in the presence of free stream are as follows:

1. For $\lambda < 1$: (a) the effect of increasing magnetic field produces a decrease in the velocity and an increase in temperature, (b) the effect of the variable viscosity parameter $A$ decreases the fluid velocity and increases the dimensionless temperature, (c) the velocity profile curves show a reduction whereas temperature profile curves
show a increment when we increase the stretching parameter $\beta$.

2. For $\lambda > 1$: (a) the velocity increases and the temperature decreases as we increase the magnetic parameter $M$, (b) the fluid velocity increases and temperature decreases with an increase in $A$ and $\beta$.

3. The effect of the Prandtl number $Pr$ is to decrease the thermal boundary layer thickness and the wall temperature gradient in the presence of effects of other physical parameters of the model.

4. We observed an reduction in momentum and thermal boundary layer thickness with $\lambda$. The velocity and the temperature gradient increases with $\lambda$. 