Combined Nonlinear Control of Non-Affine MIMO System with Input and State Delays

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Abstract. The article studies algorithms synthesis of a multiply connected system for controlling non-affine plants, the state variables of which are not available for direct measurement. It is shown that the decentralized control law constructed on the basis of the hyperstability criterion eliminates the influence of non-linearities and compensates external and parametric perturbations in the tracking multiply connected system with the required accuracy.

1. Introduction
Multiply-connected control systems operating under conditions of a priori parametric uncertainty, in the presence of external uncontrolled disturbance and time delays in control channels occupy a special place among the variety of automatic control systems. A significant part of multiply connected systems is a class of decentralized multiply connected systems, in which a complex control plant is represented as a set of interconnected local subsystems with local control loops. Most of the known control schemes for multiply connected systems are affine plants \cite{1,2,3}, but from the point of view of applied significance, the most important is the problem of controlling non-affine multiply connected systems \cite{4,5,6,7,8,9}, including those containing control delays \cite{10}. It should be noted that the problems of controlling of the indefinite multiply-connected systems has a very high theoretical and practical importance. Examples of unstable plants can be found in various fields of knowledge. In mechanics, for example, it is multiply-connected inverted pendulum (which is multiply connected system itself), or the interconnected system of several inverted pendulums. In technology – operation of a gas turbine engine at low speeds, or control of statically unstable aircraft.

In this paper, the synthesis of the combined control law for one class of non-affine MIMO plants based on the hyperstability criterion \cite{11} is carried out; to compensate the negative impact of input delay on the functioning of a non-affine system, a predictor-compensator is used \cite{12,13,14}, to obtain estimates of the local subsystems inaccessible to direct measurements of variable state subsystems observers with strong feedback are used \cite{15,16,17}.

2. Mathematical Description of the Control System and the Problem Statement
We describe the dynamic processes in local subsystems of interconnected non-affine system by following equations:
\[\frac{dx_i(t)}{dt} = (A_i + B_0a_i^T(y_i(t)))x_i(t) + B_0d_i^Ty_i(t) - x_i(t) + B_i(y_i(t)) \times \]
\[\times \left( u_i(t - h_i)f_1(x_i(t), u_i(t - h_i)) + f_2(x_i(t), u_i(t - h_i)) + \sum_{j=1}^k \theta_{ij}(x_i(t)) \right) + \varphi_i(t), \]
\[y_i(t) = x_1(t); \quad x_i(0) = x_{0i}, x_i(\vartheta_i), \vartheta_i \in [-\tau; 0], u_i(\bar{\vartheta}_i) = \bar{v}_i(\bar{\vartheta}_i), \bar{\vartheta}_i \in [-h_i; 0],\]
where \(x_i(t) \in R^{n_i}\) are state variables of local subsystems; \(u_i(t) \in R\) are local control actions; \(y_i(t) \in R\) are subsystem outputs; \(\tau, h_i = const > 0\) are known permanent time delays; \(A_i, B_i\) are nilpotent matrices of \((n_i \times n_i)\) size; \(B_0 = [0, \ldots, 0, b_{0i}]^T\) are vectors of \((n_i \times 1)\) size, \(B_i(y_i(t)) = [0, \ldots, 0, b_{ni}(y_i(t))]^T, a_i(y_i(t)) = [a_{1i}(y_i(t)), a_{2i}(y_i(t)), \ldots, a_{ni}(y_i(t))]^T, d_i = [d_{1i}, d_{2i}, \ldots, d_{ni}]^T\) are nonlinear vector functions and constant vector respectively which elements satisfy the following constraints:

\[|a_{1i}(y_i(t))| \leq a_{1i}^+, \quad |a_{ni}(y_i(t))| \leq a_{ni}^+, \quad |a_{d1}| \leq d_{d1}^+, \ldots, |a_{dn_i}| \leq d_{dn_i}^+, b_{ni}^- \leq b_{ni}(y_i(t)) \leq b_{ni}^+, \]

where \(a_{ji}^+, d_{ji}^+, b_{ni}^-, b_{ni}^+ > 0\) are known numbers, \(i = \overline{1, k}, j = \overline{1, n}; b_{0i} = 1; \varphi_i(t) = [0, \ldots, 0, \varphi_{ni}(t)]^T \in R^{n_i}\) are vectors of external uncontrollable disturbance, which elements satisfy inequalities:

\[|\varphi_{ni}(t)| \leq \varepsilon_{ni}, \quad \forall t \geq 0,\]

where \(\varepsilon_{ni}\) are known numbers; \(f_1(x_i(t), u_i(t - h_i)), f_2(x_i(t), u_i(t - h_i))\) are unknown smooth nonlinear functions such that:

\[\varepsilon_{2i} < f_1(x_i(t), u_i(t - h_i)) \leq \varepsilon_{3i}, |f_2(x_i(t), u_i(t - h_i))| \leq \varepsilon_{4i}\]

where \(\varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i} = const > 0\) are known numbers; \(x_{0i}\) are initial conditions; \(\bar{v}_i(\bar{\vartheta}_i), \bar{v}_i(\bar{\vartheta}_i)\) are bounded continuous initial functions; \(\theta_{ij}(x_i(t))\) are functions describing cross links in a complex system; \(k\) is the number of local subsystems.

The cross links of the complex system (1)–(4) are described by the equations:

\[\frac{dx_{ij}}{dt} = P_{ij}x_{ij}(t) + W_{ij}y_j(t), \quad \theta_{ij}(x_i(t)) = L_{ij}^Tx_{ij}(t),\]

where \(x_{ij}(t) \in R^{n_{ij}}; W_{ij} = [0, \ldots, 0, 1]^T; P_{ij}\) is an unknown number matrix such that equation (5) describes a stable dynamic link.

The required dynamics in each subsystem (1) is determined by dynamic links – the local reference models. Moreover, two outputs of models are considered: the main one, which sets the desired dynamics of each subsystem, and the auxiliary one, which forms the dynamics of each local main control loop

\[\frac{dx_{m_i}(t)}{dt} = A_{m_i}x_{m_i}(t) + B_{m_i}r_i(t), y_{m_i}(t) = x_{m_1_i}(t), \quad z_{m_i}(t) = g_i^Tx_{m_i}(t),\]

where \(x_{m_i}(t) \in R^{n_i}\) are state vectors of the local reference models; \(r_i(t) \in R\) are control inputs, piecewise continuous, bounded functions; \(y_{m_i}(t)\) in \(R\) are main outputs of models; \(z_{m_i} \in R\)
are auxiliary outputs of reference models; \( A_{m_i} = (A_i - B_0 a_{m_i}^T) \) are Hurwitz matrices of \((n_i \times n_i)\) size; \( a_{m_i}^T = [a_{m1}, ..., a_{mni}]^T \), \( B_{m_i} = [0, ..., 0, b_{mi}]^T \) are constant vectors of \((n_i \times 1)\) size, \( b_{mi} = const > 0 \); \( g_i \) are given vectors which elements are chosen in a special way, described below.

Nonlinear multiply connected system (1) has a negative property that has a negative impact on the performance of the entire system. It is the control delay. To ease the lag effect on the operation of the control system as a whole, in each local subsystem the predictor-compensators, connected in parallel to each local control plant (1) are used [11, 12]. Similarly to the explicit reference models (6), we consider two predictor-compensators for each output:

\[
\frac{dx_{ki}(t)}{dt} = A_{m_i}x_{ki}(t) + B_{m_i}(u_i(t) - u_i(t - h_i)), y_{ki}(t) = x_{k1}(t), \quad z_{ki}(t) = g_i^T x_{ki}(t), \quad \text{(7)}
\]

where \( x_{ki}(t) \in R^{n_i} \) are the state vectors of local predictor-compensators; \( y_{ki}(t) \) and \( z_{ki}(t) \) are their main and auxiliary outputs respectively.

Let us formulate the problem statement: for a nonlinear multiply connected non-affine control system (1) — (7) in local subsystems it is required to synthesize combined control laws

\[
u_i(t) = u_i(x_i(t), x_i(t - \tau_i), x_{m_i}(t), x_{ki}(t), u_i(t - h_i), r_i(t)), \quad \text{(8)}
\]

so that under any initial conditions and at any level a priori uncertainty of the system (1) — (8) it would ensure the fulfillment of the relevant inequalities

\[
\lim_{t \to \infty} |y_{mi}(t) - y_i(t)| \leq \sigma_{0i}, \quad \sigma = const > 0, \quad i = 1, k.
\]

\[
\text{(9)}
\]

3. Synthesis of the Control Law

Following the methodology of the hyperstability criterion, the mismatch signal \( e_i = x_{mi}(t) - (x_i(t) - x_{ki}(t)) \) is considered and the equivalent mathematical description for the system under study (1) — (8) is written in the form of a linear stationary part

\[
\frac{de_i(t)}{dt} = A_{m_i}(t)e_i(t) + B_{m_i}\mu_i(t), \quad v_i(t) = z_{mi}(t) - g_i^T x_i(t) - z_{ki}(t), \quad \text{(10)}
\]

and non-linear non-stationary part

\[
\mu_i(t) = -\left[ u_i(t) - r_i(t) + b_{mi}^{-1} \times \left( (a_{m_i} + a_i(y_i(t))^T x_i(t) + d_i^T x(t - \tau_i) + b_i(y_i(t))(f_1(x_i(t), u_i(t - h_i) - 1)u_i(t - h_i) + f_2(x_i(t), u_i(t - h_i)) + \varphi_i(t)) \right) \right], \quad \text{(11)}
\]

where \( \mu_i(t) \) are modified controls; \( v_i(t) \) are errors on auxiliary outputs of local subsystems, \( i = 1, k \).

According to the hyperstability criterion, the explicit form of the control algorithms is determined using the integral inequality of V.M. Popov, ensuring its implementation:

\[
\eta_i(0, t) = -\int_0^t \mu_i(\xi)v_i(\xi)\,d\xi \geq \eta_{0i}^2, \quad \eta_{0i} = const, \forall t > 0, i = 1, k.
\]

\[
\text{(12)}
\]

When substituting (11) into the integral function of the inequality (12) and analyzing the obtained expressions, it was noted that when forming the structure of the control law (8), it is
reasonable for each local subsystem to equate three signals sum \((u_{1i}(t) + u_{2i}(t) + u_{3i}(t) + u_{4i}(t))\) to the difference of two signals \((u_i(t) - r_i(t))\), obtaining for each \(u_i(t)\) the equation

\[
u_i(t) = r_i(t) + u_{1i}(t) + u_{2i}(t) + u_{3i}(t) + u_{4i}(t), \tag{13}\]

where \(u_{1i}(t), u_{2i}(t), u_{3i}(t)\) and \(u_{4i}(t)\) are the control signal components, \(i = 1, \kappa\).

Integrals (12), taking into account (11), are transformed as follows:

\[
\eta_i(0, t) = \sum_{p=1}^{4} \eta_{ip}(0, t);
\eta_{1i}(0, t) = \int_{0}^{t} \left( u_{1i}(\varsigma) + b_{m_i}^{-1}(a_{m_i} + a_i(y_i(\varsigma)))^T x_i(\varsigma) \right) v_i(\varsigma) d\varsigma = \\
= \int_{0}^{t} \left( u_{1i}(t) + b_{m_i}^{-1} \sum_{j=1}^{n_i} (a_{mj_i} + a_j(y_i(\varsigma))) x_j(\varsigma) \right) v_i(\varsigma) d\varsigma;
\eta_{2i}(0, t) = \int_{0}^{t} \left( u_{2i}(\varsigma) + b_{m_i}^{-1} \sum_{j=1}^{n_i} d_j x_j(\varsigma - \tau_i) \right) v_i(\varsigma) d\varsigma;
\eta_{3i}(0, t) = \int_{0}^{t} \left( u_{3i}(\varsigma) + b_{m_i}^{-1} b_n(y_i(\varsigma))(f_{1i}(x_1(\varsigma), u_i(\varsigma - h_i)) - 1) u_i(\varsigma - h_i) \right) v_i(\varsigma) d\varsigma;
\eta_{4i}(0, t) = \int_{0}^{t} \left( u_{4i}(\varsigma) + b_{m_i}^{-1} b_n(y_i(\varsigma))(f_{2i}(x_1(\varsigma), u_i(\varsigma - h_i)) + \varphi_i(\varsigma)) \right) v_i(\varsigma) d\varsigma.
\]

The components \(u_{1i}(t), u_{2i}(t), u_{3i}(t)\) and \(u_{4i}(t)\), by analogy with [18], are synthesized as

\[
u_{1i}(t) = \sum_{j=1}^{n_i} \left( \gamma_{1ji} x_{j_i}(t) \int_{0}^{t} x_{j_i}(\varsigma) v_i(\varsigma) d\varsigma + \gamma_{2ji} x_{j_i}^2(t) v_i(t) \right), \tag{14}\]

\[
u_{2i}(t) = \sum_{j=1}^{n_i} \left( \gamma_{3ji} x_{j_i}(t - \tau_i) \int_{0}^{t} x_{j_i}(\varsigma - \tau_i) v_i(\varsigma) d\varsigma \right), \tag{15}\]

\[
u_{3i}(t) = \bar{\gamma}_4 u_i(t - h_i) \int_{0}^{t} u_i(\varsigma - h_i) v_i(\varsigma) d\varsigma, \tag{16}\]

\[
u_{4i}(t) = \bar{\gamma}_5 \int_{0}^{t} v_i(\varsigma) d\varsigma + \bar{\gamma}_6 v_i(t), \tag{17}\]

where \(\gamma_{1ji}, \gamma_{2ji}, \gamma_{3ji} = const > 0, \bar{\gamma}_4 = 2\tau_1 \bar{\gamma}_2, \bar{\gamma}_1i = \max \left| b_{m_i}^{-1} b_{nj_i} f_{1i}(x_i(t), u_i(t - h_i)) - 1 \right|, \bar{\gamma}_5 = 2\tau_3 (b_{nj_i})^2 (\varepsilon_1 + \varepsilon_4)^2, \tau_2, \tau_3, \bar{\gamma}_5i = const > 0, \forall t > 0, i = 1, \kappa, j = 1, n_i.

Thus, the combined control law, the general form of which was given by equation (8), taking into account relations (14) – (17), will be described by the equation

\[
u_i(t) = r_i(t) + \sum_{j=1}^{n_i} \gamma_{1ji} x_{j_i}(t) \int_{0}^{t} x_{j_i}(\varsigma) (z_{m_i}(\varsigma) - g_i^T x_i(\varsigma) - z_{ki}(\varsigma)) d\varsigma + \\
+ \sum_{j=1}^{n_i} \gamma_{2ji} x_{j_i}^2(t) (z_{m_i}(t) - g_i^T x_i(t) - z_{ki}(t)) + \\
+ \sum_{j=1}^{n_i} \gamma_{3ji} x_{j_i}(t - \tau_i) \int_{0}^{t} x_{j_i}(\varsigma - \tau_i) v_i(\varsigma) d\varsigma + \bar{\gamma}_5 \int_{0}^{t} v_i(\varsigma) d\varsigma + \bar{\gamma}_6 v_i(t). \tag{18}\]
+ \sum_{j=1}^{n_i} \gamma_{3j} x_j(t - \tau_i) \int_{0}^{t} x_j(\varsigma - \tau_i)(z_{m_i}(\varsigma) - g_i^T x_i(\varsigma) - z_{k_i}(\varsigma)) d\varsigma + \\
+ \tilde{\gamma}_4 u_i(t - h_i) \int_{0}^{t} u_i(\varsigma - h_i)(z_{m_i}(\varsigma) - g_i^T x_i(\varsigma) - z_{k_i}(\varsigma)) d\varsigma + \\
+ \tilde{\gamma}_5 \int_{0}^{t} (z_{m_i}(\varsigma) - g_i^T x_i(\varsigma) - z_{k_i}(\varsigma)) d\varsigma + \tilde{\gamma}_6_i (z_{m_i}(t) - g_i^T x_i(t) - z_{k_i}(t)) .

Next, it is required to ensure the fulfillment of the following inequality for the linear stationary part of the equivalent system (9):

\[ \text{Re}\left[ W_i(j\omega) \right] > 0, \forall \omega > 0, i = \overline{1,k}. \] (19)

Writing the generalized outputs of the equivalent system (10) as mismatch signals of the main circuits of the local subsystems of the studied multiply connected system we obtain following transfer functions for (10)

\[ W_i(s) = g_i^T \left(sE_{n_i} - A_{m_i}\right)^{-1} B_{m_i} = \frac{b_{m_i}g_i(s)}{a_{m_i}(s)}, \quad i = \overline{1,k}, \]

where \( a_{m_i}(s) \) are Hurwitz polynomials of \( n_i \) degrees; \( g_i(s) \) are polynomials of \((n_i - 1)\) degree, the coefficients of which are chosen to satisfy the inequalities (18). The easiest option of this choice is that the roots of the polynomials \( g_i(s) \) coincide with the corresponding to \((n_i - 1)\) roots of the polynomials \( a_{m_i}(s) \). Then the desired dynamics of the main contour of each local subsystem will correspond to the dynamics of the first-order inertial link for which the inequality (19) is obvious.

From fulfill the inequalities (12) and (19) on the basis of the hyperstability criterion, it can be affirmed that the control system (1), (5) – (7), (18) is hyper-stable in a given class of uncertainty (2) – (4) therefore, for each local subsystem, will be fulfilled the limit conditions

\[ \lim_{t \to \infty} |v_i(t)| = 0, i = \overline{1,k}, \] (20)

and, as a result, the control aim will be (9).

4. Technical Implementation of the Combined Regulator

The assumptions about the measurability of the state variables of system (1) made in the previous paragraph let us synthesize a combined control law for a multiply connected non-affine system on the basis of the hypersensitivity criterion. But when solving practical problems, only outputs of the local subsystems \( y(t) \) are available for measuring.

Therefore, for the technical implementability of the synthesized control algorithm, we need estimates of state variables that are not available for direct measurements, which can be obtained using, for example, strong observers in each subsystem

\[ \frac{dx_{N_i}(t)}{dt} = A_{m_i} x_{N_i}(t) + L_i (y_i(t) - C_i^T x_{N_i}(t)) + B_{m_i} u_i(t - h_i), \] (21)

\[ y_{N_i}(t) = C_i^T x_{N_i}(t), \quad v_{N_i}(t) = g_i^T x_{N_i}(t), i = \overline{1,k}, \]
where \( x_{Ni}(t) \in R^{m_i} \) are state variables of the observers; \( y_{Ni}(t) \in R \) are outputs of observers; \( v_{Ni}(t) \in R \) are generalized outputs of observers; \( L_i \) are constant vectors providing the specified dynamics of state estimates; \( C_i = [0, ..., 0, c_{0i}]^T \) are vectors of \((n_i \times 1)\) size , \( c_{0i} = 1 \).

To build up the parameters of the vector \( \vec{y}_i \) we set up following [13], [14], [15], where \( K_i \) are generalized outputs of the corresponding standards (6) and observers (21) coordination coefficients in steady mode: \( K_i = \lim_{s \rightarrow 0} g_i^T (sE_i - A_m)^{-1} L_i = -g_i^T A_m^{-1} L_i, \quad i = \overline{1,k}. \)

The components of the \( L_i \) vectors of observers (21) are chosen in such a way that the matrices \((A_{mi} - L_iC_i^T)\) will be Hurwitz. Since the corresponding pairs \((A_{mi}, C_i)\) are observable, such a choice is quite possible.

The choice of the values of the vectors \( L_i \) is made taking into consideration the desired distribution of the roots of the characteristic polynomials of local observers (21), which satisfy the inequality: \( \min Re(-\lambda_{ij}) \geq \theta_i, \max Re(-\lambda_{ij}), i = \overline{1,k}; \) where \( \lambda_{ij}, \lambda_{ij} \) are the characteristic numbers of the matrices \( A_{mi} \) and \((A_{mi} - L_iC_i^T)\) respectively; \( \theta_i \equiv const \) are scalars, determining the desired location of the poles on the complex plane with \( \theta_i \gg 1 \).

Thus, if in the combined control law (18) the state variables \( x_i(t) \) of the control plant (1) are replaced by their corresponding estimates \( x_{Ni}(t) \), then the system (1) – (7), (21), will be \( L\)-dissipative in a given class of uncertainty, and the regulator, synthesized with the help of hyperstability criterion, will be technically feasible.

As noted in [19], one of the drawbacks of observers with strong feedback (21) is the appearance of peaks in transient processes with increasing \( \theta_i \), which leads to system instability. Therefore, in order to weaken the influence of peaks on the behavior of the control plant, the control law uses saturation, which turns on when the observer’s state variables come out of a certain compact set:

\[
u_i(t) = r_i(t) + \sum_{j=1}^{n_i} \gamma_{1ji} \text{sat}(x_{Ni_j}(t)) \int_0^t \text{sat}(x_{Ni_j}(\varsigma))(z_{mi}(\varsigma) - g_i^T \text{sat}(x_{Ni}(\varsigma)) - \sum_{j=1}^{n_i} \gamma_{2ji} \text{sat}(x_{Ni_j}(t))^2(z_{mi}(t) - g_i^T \text{sat}(x_{Ni}(t))) - z_{ki}(t)) + \sum_{j=1}^{n_i} \gamma_{3ji} \text{sat}(x_{Ni_j}(t - \tau_i)) \int_0^t \text{sat}(x_{Ni_j}(\varsigma - \tau_i)) \times \left((z_{mi}(\varsigma) - g_i^T \text{sat}(x_{Ni}(\varsigma)) - z_{ki}(\varsigma))d\varsigma + \tilde{\gamma}_4 u_i(t - h_i) \int_0^t u_i(\varsigma - h_i)(z_{mi}(\varsigma) - g_i^T \text{sat}(x_{Ni}(\varsigma)) - z_{ki}(\varsigma))d\varsigma + \tilde{\gamma}_5 (z_{mi}(t) - g_i^T \text{sat}(x_{Ni}(t)) - z_{ki}(t)). \right]
\]

5. Illustrative Example of the Control System Functioning
Let us illustrate the results obtained using the example of a decentralized control of a multiply connected non-affine plant with a delay in control (1) consisting of two local subsystems:

\[
\dot{x}_{11}(t) = x_{21}(t), \quad \dot{x}_{21}(t) = x_{31}(t),
\]

\[
\dot{x}_{31}(t) = a_{11}(y_1(t))x_{11}(t) + a_{21}(y_1(t))x_{21}(t) + a_{31}(y_1(t))x_{31}(t) + d_{11}x_{11}(t - \tau_1) + d_{21}x_{21}(t - \tau_1) + d_{31}x_{31}(t - \tau_1) + b_1(y_1(t))u_1(t - h_1) f_{11}(x_1(t), u_1(t - h_1)) + b_1(y_1(t))f_{21}(x_1(t), u_1(t - h_1)) + \theta_{11}(x_1(t)) + \theta_{12}(x_2(t)),
\]

\[
\dot{x}_{21}(t) = x_{31}(t), \quad \dot{x}_{31}(t) = x_{11}(t),
\]

\[
\dot{x}_{11}(t) = a_{11}(y_1(t))x_{11}(t) + a_{21}(y_1(t))x_{21}(t) + a_{31}(y_1(t))x_{31}(t) + d_{11}x_{11}(t - \tau_1) + d_{21}x_{21}(t - \tau_1) + d_{31}x_{31}(t - \tau_1) + b_1(y_1(t))u_1(t - h_1) f_{11}(x_1(t), u_1(t - h_1)) + b_1(y_1(t))f_{21}(x_1(t), u_1(t - h_1)) + \theta_{11}(x_1(t)) + \theta_{12}(x_2(t)),
\]

\[
\dot{x}_{21}(t) = x_{31}(t), \quad \dot{x}_{31}(t) = x_{11}(t),
\]

\[
\dot{x}_{11}(t) = a_{11}(y_1(t))x_{11}(t) + a_{21}(y_1(t))x_{21}(t) + a_{31}(y_1(t))x_{31}(t) + d_{11}x_{11}(t - \tau_1) + d_{21}x_{21}(t - \tau_1) + d_{31}x_{31}(t - \tau_1) + b_1(y_1(t))u_1(t - h_1) f_{11}(x_1(t), u_1(t - h_1)) + b_1(y_1(t))f_{21}(x_1(t), u_1(t - h_1)) + \theta_{11}(x_1(t)) + \theta_{12}(x_2(t)),
\]

\[
\dot{x}_{21}(t) = x_{31}(t), \quad \dot{x}_{31}(t) = x_{11}(t),
\]

\[
\dot{x}_{11}(t) = a_{11}(y_1(t))x_{11}(t) + a_{21}(y_1(t))x_{21}(t) + a_{31}(y_1(t))x_{31}(t) + d_{11}x_{11}(t - \tau_1) + d_{21}x_{21}(t - \tau_1) + d_{31}x_{31}(t - \tau_1) + b_1(y_1(t))u_1(t - h_1) f_{11}(x_1(t), u_1(t - h_1)) + b_1(y_1(t))f_{21}(x_1(t), u_1(t - h_1)) + \theta_{11}(x_1(t)) + \theta_{12}(x_2(t)),
\]

\[
\dot{x}_{21}(t) = x_{31}(t), \quad \dot{x}_{31}(t) = x_{11}(t),
\]

\[
\dot{x}_{11}(t) = a_{11}(y_1(t))x_{11}(t) + a_{21}(y_1(t))x_{21}(t) + a_{31}(y_1(t))x_{31}(t) + d_{11}x_{11}(t - \tau_1) + d_{21}x_{21}(t - \tau_1) + d_{31}x_{31}(t - \tau_1) + b_1(y_1(t))u_1(t - h_1) f_{11}(x_1(t), u_1(t - h_1)) + b_1(y_1(t))f_{21}(x_1(t), u_1(t - h_1)) + \theta_{11}(x_1(t)) + \theta_{12}(x_2(t)),
\]

\[
\dot{x}_{21}(t) = x_{31}(t), \quad \dot{x}_{31}(t) = x_{11}(t),
\]

\[
\dot{x}_{11}(t) = a_{11}(y_1(t))x_{11}(t) + a_{21}(y_1(t))x_{21}(t) + a_{31}(y_1(t))x_{31}(t) + d_{11}x_{11}(t - \tau_1) + d_{21}x_{21}(t - \tau_1) + d_{31}x_{31}(t - \tau_1) + b_1(y_1(t))u_1(t - h_1) f_{11}(x_1(t), u_1(t - h_1)) + b_1(y_1(t))f_{21}(x_1(t), u_1(t - h_1)) + \theta_{11}(x_1(t)) + \theta_{12}(x_2(t)),
\]

\[
\dot{x}_{21}(t) = x_{31}(t), \quad \dot{x}_{31}(t) = x_{11}(t),
\]
where \( h_1 = 0.2, \tau_1 = 0.1, a_{11}(t) = a_{111} + a_{12} \sin(x_1(t)); a_{21}(t) = a_{211} + a_{22} \sin(x_2(t)); a_{31}(t) = a_{311} + a_{32} \sin(x_3(t)); b_1(y_1(t)) = b_{111} + b_{12} \sin(x_1(t)); f_1(x_1(t), u_1(t - h_1)) = 0.1 + \frac{d_0}{1 + |u_1(t-h_1)|}; f_2(x_1(t), u_1(t - h_1)) = 0.2 \tanh(0.15u_1(t-h_1)); y_2(0) = \dot{y}_2(0) = \ddot{y}_2(0) = 2; \varphi_1(t) = \varphi_0 \sin(0.01\pi t). \)

\[
\begin{align*}
\dot{x}_{12}(t) &= x_{22}(t), \quad \dot{x}_{22}(t) = x_{32}(t), \\
\dot{x}_{32}(t) &= a_{12}(y_2(t))x_{12}(t) + a_{22}(y_2(t))x_{22}(t) + a_{32}(y_2(t))x_{32}(t) + \\
&+ b_2(y_2(t))u_2(t - h_2)f_2(x_2(t), u_2(t - h_2)) + \\
&+ b_2(y_2(t))f_2(x_2(t), u_2(t - h_2)) + \theta_{21}(x_1(t)) + \theta_{22}(x_2(t)),
\end{align*}
\]

where \( h_2 = 0.7, a_{12}(t) = a_{112} + a_{12} \sin(x_1(t)); a_{22}(t) = a_{212} + a_{22} \cos(x_2(t)); a_{32}(t) = a_{312} + a_{32} \sin(x_3(t))^2; b_2(y_2(t)) = b_{212} + b_{222} \cos(x_3(t)); f_2(x_2(t), u_2(t - h_2)) = 0.1 + \frac{d_0}{1 + |u_2(t-h_2)|}; f_2(x_2(t), u_2(t - h_2)) = 0.2 \sin(0.15u_2(t-h_2)); y_2(0) = \dot{y}_2(0) = \ddot{y}_2(0) = 1; \varphi_2(t) = \varphi_0 \cos(0.1\pi t). \)

The level of a priori uncertainty of subsystems (23), (24) is determined as follows:

\[
|a_{111}| \leq 1, \quad |a_{112}| \leq 0.2, \quad |a_{211}| \leq 1, \quad |a_{221}| \leq 0.5, \quad 0.8 \leq b_{211} \leq 1.5, \quad -0.5 \leq b_{221} \leq 1, \quad 0 \leq d_{011} \leq 2, \quad 0 \leq \varphi_{01} \leq 0.5;
\]

\[
|a_{112}| \leq 1.1, \quad |a_{121}| \leq 0.2, \quad |a_{212}| \leq 0.2, \quad |a_{222}| \leq 2, \quad |a_{312}| \leq 4, \quad |a_{322}| \leq 0.6, \quad 1 \leq b_{212} \leq 2, \quad -1 \leq b_{222} \leq 1, \quad 0 \leq d_{022} \leq 2.2, \quad 0 \leq \varphi_{02} \leq 0.5;
\]

Cross-links (5) in the object under study were specified by following transfer functions: \( W_{12}(s) = \frac{1}{s+2}, \quad W_{21}(s) = \frac{1}{s+5} \).

Explicit reference models (6) that define the desired dynamics of the subsystems (23), (24) are defined as

\[
\begin{align*}
\dot{x}_{m1}(t) &= x_{m2}(t), \quad \dot{x}_{m2}(t) = x_{m3}(t), \\
\dot{z}_{m3}(t) &= -x_{m1}(t) - 3x_{m2}(t) - 3x_{m3}(t) + r_1(t), \quad \dot{y}_{m1}(t) = x_{m1}(t), \\
\dot{z}_{m1}(t) &= x_{m1}(t) + 2x_{m2}(t) + x_{m3}(t), \quad \dot{y}_{m1}(0) = \ddot{y}_{m1}(0) = 0, \quad i = 1, 2;
\end{align*}
\]

where \( r_1(t) = \sin(0.035t) \cos(0.7t), \quad r_2(t) = 0.2 + |0.5 - \cos(0.015\pi t)| \) are local command signals.

Local predictor-compensators (7) have parameters similar to (27).

Based on the specified values of the state matrices and control vectors of the reference models (27), we select values \( \theta_1 = 100, \quad \theta_2 = 100 \) and calculate the parameters of the observers (21) like:

\[
N_i = [117, 444, 5031]^T, \quad K_i = 1.0789, \quad g_i^T = [1.0789, 2.1578, 1.0789], \quad i = 1, 2.
\]

The Fig. 1–4 shows the dynamic characteristics of the control system (1), (6), (7), (21), (22) – (28) with the following parameters of the plant (1), (23), (24), satisfying inequalities (25), (26):

\[
\begin{align*}
a_{111} = 1, a_{121} = 0.1, a_{211} = -1, a_{221} = -0.2, a_{311} = -4a_{321} = -0.6, b_{211} = 1.8, \quad b_{221} = -0.8, d_{01} = 2, \quad \varphi_{01} = 0.5, \quad a_{112} = 1.1, a_{122} = -0.1, a_{212} = -2, a_{222} = -0.2, \quad a_{312} = 4, a_{322} = -0.5, b_{212} = 1.8, b_{222} = -0.1, d_{02} = 2, \quad \varphi_{02} = 0.5.
\end{align*}
\]
with the following selected values of the coefficients of the regulators (22):

\[ \gamma_{111} = 30000, \gamma_{211} = 300, \gamma_{121} = 800, \gamma_{221} = 100, \gamma_{131} = 200, \gamma_{231} = 2, \gamma_{311} = 30, \]
\[ \gamma_{321} = 100, \gamma_{331} = 3000, \tilde{\gamma}_{42} = 20, \tilde{\gamma}_{52} = 20000, \tilde{\gamma}_{62} = 600; \]  

\[ \gamma_{112} = 300, \gamma_{212} = 300, \gamma_{122} = 800, \gamma_{222} = 100, \gamma_{132} = 300, \gamma_{232} = 200, \]
\[ \tilde{\gamma}_{42} = 20, \tilde{\gamma}_{52} = 200, \tilde{\gamma}_{62} = 200; \]  

Diagram of the transient process in multiply connected system, allow us to conclude that the obtained algorithm of decentralized control ensures the required accuracy and magnitude of the mismatch between the plant and reference models output in local systems are from 5% to 6%.
6. Conclusion
In this paper, in view of using the hyperstability criterion and estimating non-measurable state variables with the help of observers with strong feedback, we consider an approach of design algorithms of decentralized control of multiply-connected non-affine system with scalar inputs and outputs.

The proposed control algorithm allows equalizing the presence of non-linearities and uncertainties in a multiply-connected plant as well as the effect of external interference, delayed control, and the availability of direct measurements only the output of the plant’s subsystems. A series of computational experiments demonstrated good quality tracking of the local subsystems outputs to the main outputs of explicit reference models and confirmed the theoretical results.

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References
[1] A.L. Fradkov and G.K. Grigoriev. Decentralized adaptive control of synchronization of dynamic system networks at bounded disturbances. Automation and Remote Control, 74(5):829–844, 2013.
[2] E.L. Eremin and E.A. Shelenok. Robust control for one class of multivariable dynamic plants. Automation and Remote Control, 78(6):1046–1058, 2017.
[3] E.L. Eremin, L.V. Chepak and E.A. Shelenok. Robust control of multi-connected nonlinear system. In Proc. 2015 Int. Siberian Conf. on Control and Communications (SIBCON), Omsk, 2015.
[4] C.H. Lee, J.C. Chien, H.H. Chang, et al. Direct Adaptive Backstepping Control for a Class of MIMO Non-affine Systems Using Recurrent Neural Networks. in Proc. Int. MultiConf. of Engineers and Computer Scientists 2009, V. I. IMECS, March 18–20, Hong Kong, 2009.
[5] R. Ghasemi, M.B. Menhaj and A. Afshar. A New Decentralized Fuzzy Model Reference Adaptive Controller for a Class of Large-scale Nonaffine Nonlinear Systems. Eur. J. Control, 5:534–544, 2009.
[6] R. Ghasemi. Adaptive State Tracking Controller for Multi-Input Multi-Output Nonaffine Non-linear Systems. Int. J. Comput. Electrical Engin., 3(3):426–431, 2010.
[7] M.A. Kolegov and V.D. Yurkevich. PI-controller design for nonaffine-in-control system. in Transaction of Scientific Papers of the Novosibirsk State Technical University, 2(64):13–18, 2011.
[8] E.L. Eremin. Robust Control for one Class Non-Affine Nonlinear SISO Systems. Information Science and Control Systems, 3(45):89–100, 2015.
[9] E.L. Eremin and E.A. Shelenok. Adaptive periodic servo-system for nonlinear control-affine objects. Optoelectronics, Instrumentation and Data Processing, 51(5):523–529, 2015.
[10] H. Wang, Q. Zhou, X. Yang, et al. Robust Decentralized Adaptive Neural Control for a Class of Nonaffine Nonlinear Large-Scale Systems with Unknown Dead Zones. Math. Probl. Engrin., Vol. 2014:640–660, 2014.
[11] E.L. Eremin. Hyperstability of Control System for Nonlinear Plant with Delay. Frunze: Frunzensk. polytech. in-t, 1987.
[12] E.L. Eremin and L.V. Ilyina. Adaptive Systems with Dynamic Surpass Compensator for Objects with Delay in Control Information Science and Control Systems, 1(3):97–102, 2002.
[13] E.L. Eremin and D.A. Telichenko. Algorithms of an Adaptive System with Delay on Control in the Scheme with an Augmented Error Signal and Reference Block of Forestalling. Mekhatronika, Avtomatizatsiya, Upravlenie, 6:9–16, 2006.
[14] E.L. Eremin and L.V. Chepak. Robust Control System with Filter-Corrector for Objects with Delay. Information Science and Control Systems, 2(40):138–146, 2014.
[15] A.E. Golubev, A.P. Krishchenko and S.B. Tkachev. Robust Stabilization of nonlinear dynamic systems using the system state estimates made by the asymptotic observer. Automation and Remote Control, 66(7):1021–1058, 2005.
[16] E.L. Eremin, N.V. Kvan and N.P. Semichevkaya. Robust Control of Nonlinear Object with High-Speed Obvious-Implicit Standard Model. Mekhatronika, Avtomatizatsiya, Upravlenie, 5:2–6, 2010.
[17] E.L. Eremin and L.V. Chepak. Adaptive System with Explicit-Implicit Standards and a Stationary Observer for an Object with a Delay in Control. Billetin of PNU, 2(21):13–22, 2010.
[18] E.L. Eremin and L.V. Chepak. Compound Controller for Non-Affine Multi-Connected Plants with Control Lag. Information Science and Control Systems, 1(59):118–130, 2019.
[19] H.K. Khalil. Nonlinear Systems. Prentice Hall, Upper Saddle River, New Jersey, 2002.