Electroweak Symmetry Breaking via Bose-Einstein Mechanism

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(Dated: May 2003)

Recently the Bose-Einstein phenomenon has been proposed as possible physical mechanism underlyng the spontaneous symmetry breaking in cold gauge theories. The mechanism is natural and we use it to drive the electroweak symmetry breaking. It can be implemented in different ways while here we present a simple model in which the Bose-Einstein sector is felt only indirectly by all of the standard model fields. The structure of the corrections due to the new mechanism is general and independent on the model used here leading to experimental signatures which can be disentangled from other known extensions of the standard model.

INTRODUCTION

The Standard Model of particle interactions has passed numerous experimental tests [1] but despite the experimental successes it is commonly believed that it is still incomplete. The spontaneous breaking of the electroweak symmetry, for example, has been the subject of intensive studies and different models have been proposed to try to accommodate it in a more general and satisfactory framework. Technicolor theories [2] and supersymmetric extensions [3] of the Standard Model are two relevant examples.

In [4] we explored the possibility that the relativistic Bose-Einstein phenomenon [5, 6] is the cause of electroweak symmetry breaking. We introduced a new global symmetry of the Higgs field and associated a chemical potential $\mu$ with the generators of such a new symmetry. A relevant property of the theory was that the chemical potential induced a negative mass squared for the Higgs field at the tree level destabilizing the symmetric vacuum and triggering symmetry breaking. The local gauge symmetries were broken spontaneously and the associated gauge bosons acquired a standard mass term. We also showed that while the properties of the massive gauge bosons at the tree level are identical to the ones induced by the conventional Higgs mechanism, the Higgs field itself had specific Lorentz non covariant dispersion relations. The Bose-Einstein mechanism occurs, in fact, in a specific frame the effects of which are felt by the other particles in the theory via electroweak radiative corrections. Some of these corrections have been explicitly computed in [4] and the model presented in [4] is very predictive.

In this letter we present a new model in which a hidden Bose-Einstein sector drives the electroweak symmetry breaking while yielding small corrections to the standard Higgs dispersion relations. In this way the presence of a frame and hence Lorentz breaking corrections are suppressed with respect to the ones shown in [4]. Here the Bose-Einstein mechanism operates on a complex scalar field neutral under all of the Standard Model interactions. On general grounds this field interacts with the ordinary Higgs and we use these interactions to trigger the ordinary electroweak symmetry breaking. We investigate some of the general consequences of the present model. Due to the Bose-Einstein nature of the new mechanism the form of the corrections is insensitive to its specific realization while the mechanism is distinguishable from other sources of beyond standard model physics. Schematically the two classes of models we imagine are summarized in figure 1. In the first one the Higgs field feels directly the presence of a net background charge and communicates it to the gauge bosons and fermions via higher order corrections (left panel). In the second (the hidden case) a singlet field with respect to the standard model quantum numbers directly feels the effects of a frame via the interactions with the singlet field. Finally the rest of the standard model particles will be affected via higher order corrections (right panel). Since the chemical potential differentiates between time and space, and we have a scalar vacuum, all of the dispersion relations are isotropic. Theories with condensates of vectorial type have been studied in different realms of theoretical physics [5, 7, 10, 11, 12]. It is also worth mentioning that, although in a different framework, effects of a large lepton number on the spontaneous gauge symmetry breaking for the electroweak theory at high temperature relevant for the early universe have been studied [7, 8, 10, 11, 12, 13]. In [14] the concept of Bose-Einstein condensation in the gravitational systems was considered.
HIDDEN BOSE-EINSTEIN SECTOR

The Higgs sector of the Standard Model possesses, when the gauge couplings are switched off, an $SU_L(2) \times SU_R(2)$ symmetry. The full symmetry group is mostly easily recognized when the Higgs doublet field is represented as a two by two matrix in the following way:

$$M = \frac{1}{\sqrt{2}} (\sigma + i \vec{\tau} \cdot \vec{\pi}) .$$

(1)

The $SU_L(2) \times SU_R(2)$ group acts linearly on $M$ according to:

$$M \rightarrow g_L M g_R^\dagger \quad \text{and} \quad g_L, g_R \in SU_L/R(2) .$$

(2)

The $SU_L(2)$ symmetry is gauged by introducing the weak gauge bosons $W^a$ with $a = 1, 2, 3$. The hypercharge generator is taken to be the third generator of $SU_R(2)$. The ordinary covariant derivative acting on the Higgs, in the present notation, is:

$$D_\mu M = \partial_\mu M - ig W^a_\mu M + ig B^a_\mu B^a_\mu ,$$

$$W^a_\mu = W^a_\mu \frac{\tau^a}{2} , \quad B^a_\mu = B^a_\mu \frac{\tau^3}{2} .$$

(3)

We now extend the standard model to contain an extra complex bosonic field $\phi$ which is a singlet under all of the standard model charges. In this way we also introduced an extra $U(1)$ symmetry. The most general and renormalizable potential involving the standard model Higgs and the field $\phi$ respecting all of the symmetries at hand is:

$$V[\phi, M] = \left( M_R^2 - 8\hat{g} |\phi|^2 \right) \text{Tr}[M^\dagger M] + m^2 |\phi|^2 + \hat{\lambda} |\phi|^4 + \lambda \text{Tr}[M^\dagger M]^2 .$$

(4)

We have assumed the potential to be minimized for a zero vacuum expectation values of the fields and the couplings to be all positive. With this choice is clear that if $\phi$ acquires a non zero vacuum expectation value the ordinary Higgs also acquires a negative mass square contribution.

We introduce a non zero background charge for the field $\phi$. This is achieved modifying the $\phi$ kinetic term as follows:

$$\mathcal{L}_{\text{charge}} = D_\mu \phi^* D^\mu \phi ,$$

(5)

with

$$D_\nu \phi = \partial_\nu \phi - i A_\nu \phi , \quad A_\nu = \mu \left( 1, \vec{0} \right) ,$$

(6)

and $\mu$ is the associated chemical potential. Substituting (6) in (5) we have:

$$\mathcal{L}_{\text{charge}} = \partial_\mu \phi^* \partial^\mu \phi + i \mu (\phi^* \partial_\nu \phi - \partial_\nu \phi^* \phi) + \mu^2 |\phi|^2 .$$

(7)

The introduction of the chemical potential has broken Lorentz invariance $SO(1, 3)$ to $SO(3)$ while providing a negative mass squared contribution to the $\phi$ boson. When $\mu > m$ the spontaneous breaking of the $U(1)$ invariance is a necessity. Once the bosonic field has acquired a vacuum expectation value and for $8\hat{g} |\phi|^2 > M_R^2$ the Higgs field condenses as well.

In this model the Bose-Einstein mechanism, although indirectly, still triggers electroweak symmetry breaking while the effects of Lorentz breaking induced by the presence of the chemical potential are attenuated. The latter are controlled by the new coupling constant $\hat{g}$ as well as the ordinary higher order electroweak corrections. Our model potential is similar in spirit to the one used in hybrid models of inflation [17].

Without any loss of generality we set $M_R^2 = m^2 = 0$.

Defining

$$|\phi| = R \cos \alpha \quad \quad |\sigma| = R \sin \alpha ,$$

(8)

the potential has four extrema which are for $R = 0$ and any $\alpha$, for $R^2 = \mu^2 / 2\lambda$ and $\alpha = 0, \pi$ or:

$$\tan^2 \alpha = 2 \frac{\hat{g}}{\lambda} = \epsilon^2 ,$$

(9)

and

$$R^2 = \frac{\mu^2}{2} \frac{\cos^2 \alpha}{\lambda \sin^4 \alpha + \lambda \cos^4 \alpha - \hat{g} \sin^2(2\alpha)} .$$

(10)

The respective values of the potential evaluated on the extremum are $\langle V \rangle = 0$ for $R = 0$, $\langle V \rangle = -\mu^4 / (4\lambda)$ for $R \neq 0$ and $\alpha = 0, \pi$ while

$$\langle V \rangle = -\frac{\mu^4}{4\lambda} \left( 1 - \frac{\epsilon^2}{\lambda} \right) ,$$

(11)

for the last extremum, eq. (9) and eq. (10), proving that this is also the actual ground state of the theory. In the limit $\epsilon \ll 1$ we have

$$\langle |\phi|^2 \rangle = \frac{\mu^2}{2\lambda} + \mathcal{O}(\epsilon^4) , \quad \langle \sigma \rangle^2 = \frac{\mu^2}{2\lambda} \epsilon^2 + \mathcal{O}(\epsilon^6) ,$$

Assuming the new physics scale associated to $|\langle |\phi|^2 \rangle|$ to be within reach of LHC, i.e., of the order of $1 - 10$ TeV while the Higgs scale is $\langle \sigma \rangle \simeq 250$ GeV we determine:

$$\epsilon \simeq 0.25 - 0.025 .$$

(12)

All the corrections will be proportional to the fourth power of $\epsilon$. To explore further our model it is convenient to adopt the unitary gauge for the Higgs field and use the exponential map for the fields $\phi$. So we define:

$$\sigma = \langle \sigma \rangle + h , \quad \phi = \frac{\sqrt{2} (\sigma + h)}{\sqrt{2}} e^{i \frac{\pi}{2 \sqrt{2}} \sqrt{2} |\sigma|^2} .$$

(13)

In the broken phase $h$ and $\psi$ are not mass eigenstates. These are related to $h$ and $\psi$ via:

$$h = \cos \vartheta \tilde{h} - \sin \vartheta \tilde{\psi} , \quad \psi = \cos \vartheta \tilde{\psi} + \sin \vartheta \tilde{h} .$$

(14)
The mixing angle is:

\[
\tan 2\theta = 2\sqrt{2} \frac{\lambda}{\lambda} \frac{e^2}{1 - 2\lambda e^2} = 2\sqrt{2} \frac{\lambda}{\lambda} e^2 + \mathcal{O}(e^4). \tag{15}
\]

The eigenvalues which, in absence of the chemical potential, one calls the masses are:

\[
m_{h}^2 = m_{h}^2 \left[ 1 - 2 \left( \frac{\lambda}{\sqrt{\lambda}} \right)^2 + \mathcal{O}(e^6) \right],
\]

\[
m_{\psi}^2 = m_{\psi}^2 \left[ 1 + \frac{\lambda}{\sqrt{\lambda}} e^4 + \mathcal{O}(e^6) \right], \tag{16}
\]

with

\[
m_{\psi}^2 = 2\mu^2, \quad m_{h}^2 = 4\mu^2 \frac{\lambda}{\sqrt{\lambda}} e^2. \tag{17}
\]

The Lorentz breaking term induced by the Bose-Einstein condensation is the second term in (17). The new neutral Higgs field \( \tilde{h} \) due to the mixing with \( \psi \) feels feebly but directly the presence of the net background charge. The relevant terms in the Lagrangian are:

\[
\frac{1}{2} \partial_{\mu} \tilde{h} \partial^{\mu} \tilde{h} + \frac{1}{2} \partial_{\mu} \tilde{\psi} \partial^{\mu} \tilde{\psi} + \frac{1}{2} \partial_{\mu} \eta \partial^{\mu} \eta - \frac{m_{h}^2}{2} \tilde{h}^2 - \frac{m_{\psi}^2}{2} \tilde{\psi}^2 - \mu \cos \theta \left( \tilde{\psi} \partial_{\mu} \eta - \eta \partial_{\mu} \tilde{\psi} \right) - \mu \sin \theta \left( \tilde{h} \partial_{\mu} \eta - \eta \partial_{\mu} \tilde{h} \right). \tag{18}
\]

The propagator for \( \tilde{h} \) determined by inverting the three by three kinetic quadratic terms is:

\[
\frac{i}{p^{2} - m_{h}^2 - 4\mu^2 \mu^2 \sin^2 \theta \mathcal{F}[p, p_{0}]} \tag{19},
\]

with

\[
\mathcal{F}[p, p_{0}] = \frac{m_{\psi}^2 - p^2}{\left( m_{\psi}^2 - p^2 \right) p^2 + 4\mu^2 \mu^2 \cos^2 \theta}. \tag{20}
\]

We take \( \psi \) to be heavy so that the only corrections will be the ones induced by the small mixing between \( h \) and \( \psi \). The function (20) for large \( \mu \) is \( \mathcal{F} \approx 1/(3p_{0}^2 - p^2) \) and resembles the pole associated to the gapless state \( \eta \). If phenomenologically needed we can always give a small mass (with respect to the scale \( \mu \)) to \( \eta \) as shown in (14). The rest of the standard model particles are affected by the presence of a frame via weak radiative corrections. These corrections can be computed as in (14). The Higgs propagator here is more involved than the one in (14). However the relevant point is that the effects of the non standard dispersion relations in the present model are depleted with respect to the ones determined in (14) due to the presence of the suppression factor \( \sin^2 \theta \approx e^6 \) in the \( h \) propagator. Indeed since \( m_{h}^2 \approx \epsilon^2 \mu^2 \) the term in the propagator bearing information of the explicit breaking of Lorentz invariance due to the Bose-Einstein mechanism is down by \( \epsilon^4 \) with respect to the mass term. So we have a large suppression of Lorentz breaking effects induced by the presence of a frame needed for the Bose-Einstein mechanism to take place. Clearly also the dispersion relations of the ordinary Higgs are modified only slightly with respect to the standard scenario. The corrections due to the mixing with the \( \psi \) field should also be taken into account although in general they will be further suppressed due to the assumed hierarchy \( \mu \gg m_{h} \).

Let us roughly estimate, using the results of (14), the size of the corrections to some observables. We first consider the modification of the low energy effective theory for the electroweak theory due to the new sector. We focus on the charged currents since the neutral currents are affected in a similar way. The chemical potential leaves intact the rotational subgroup of the Lorentz transformations, so the effective Lagrangian modifies as follows (14):

\[
\mathcal{L}_{\text{Eff}}^{CC} = -2\sqrt{2} G J_{\mu}^{+} J_{\mu}^{-} \Rightarrow
\]

\[
- 2\sqrt{2} G \left( J_{\mu}^{+} J_{\mu}^{-} + \delta J_{\mu}^{+} J_{\mu}^{-} \right), \tag{21}
\]

where \( \delta \) is a coefficient effectively measuring the corrections due to modified dispersion relations of the gauge bosons. Using the previous Lagrangian the decay rate for the process \( \mu \rightarrow e\nu e\bar{\nu} \mu \) is (14):

\[
\Gamma[\mu \rightarrow e\nu e\bar{\nu} \mu] = \frac{G^2 M_{\mu}^5}{192\pi^3} \left( 1 + \frac{3}{2} \delta \right), \tag{22}
\]

where \( M_{\mu} \) is the muon mass and we neglected the electron mass. However the effects of a non zero electron mass are as in the Standard Model case (14). The parameter \( \delta \) in the model presented in (14) was explicitly computed yielding \( \delta \approx 0.007 \). For the hidden Bose-Einstein sector this result, on general grounds, is further suppressed by a multiplicative factor of the order \( \epsilon^4 \) yielding a new \( \delta \) of the order of \( 2.7 \times 10^{-5} - 2.7 \times 10^{-9} \) for \( \langle \phi \rangle \approx 1 - 10 \text{ TeV} \).

The fermion sector also bears information of the direct or indirect nature of the underlying Bose-Einstein phenomenon. We demonstrated for the direct case in (14) that the one loop corrections to the fermion velocities due to the exchange of the Higgs are tiny (\( \approx 10^{-15} \) for the electron). This is due to the smallness of the associated Yukawa’s couplings. However we have also argued that the higher order corrections due to the modified gauge boson dispersion relations may be relevant and estimated in this case a correction of the order of \( \delta g^2/4\pi \) with \( g \) the weak coupling constant. In the hidden Bose-Einstein case also the corrections to this sector are further suppressed by a factor of the order of \( \epsilon^4 \) with respect to the findings in (14).
DISCUSSION

The Bose-Einstein condensation phenomenon has been proposed as possible physical mechanism underlying the spontaneous symmetry breaking of cold gauge theories. In [4] the Higgs field was assumed to carry global and local symmetries and was identified with the Bose-Einstein field. The effects of a non zero background charge were, in this way, maximally felt in the Higgs sector and then communicated via weak interactions to the other standard model particles. Here we have presented a new model in which the Higgs mechanism is triggered by a hidden Bose-Einstein sector. The relevant feature of this model is that the effects of modified dispersion relations for the standard model fields are strongly suppressed with respect to the one in [4]. We have predicted the general form of the corrections and estimated their size. The fermion sector of the Standard Model, for example, is affected via higher order corrections leading to the appearance of modified dispersion relations of the type $E^2 = v_f p^2 + m_f^2$. The deviation with respect to the speed of light for the fermions is small when considering the direct Bose-Einstein mechanism [4] while it is further suppressed by a factor $\epsilon^2$ in the model presented here.

We emphasize that the form of the corrections, induced by the modified propagators in Eqs. (13) and (20), are general and dictated solely by the Bose-Einstein nature of our mechanism. The strength of the coupling between the standard model fields and the Bose-Einstein field is measured by the parameter $\epsilon$ which enters in some of the physical observables. Experiments can be used to constrain the possible values of $\epsilon$.

Spontaneous breaking of a gauge theory via Bose-Einstein condensation necessarily introduces Lorentz breaking since a frame must be specified differentiating time from space. We recall that the issue of Lorentz breaking has recently attracted much theoretical [14] and experimental attention [21]. In the Bose-Einstein case the underlying gravitational theory is not the cause of Lorentz breaking which is instead due to having immersed the theory in a background charge.

Modified dispersion relations for the standard model particles have also cosmological consequences since now one cannot simply assume the velocity of light as the common velocity for all of the elementary particles. In fact different particles will have, in general, different dispersion relations and hence speed. This is relevant, for example, when observing neutrino which propagated over long distances. Using the present model as guide for the structure of the corrections experiments can test the Bose-Einstein mechanism as possible source of electroweak symmetry breaking.

We thank P.H. Dangaard and J. Schechter for discussions and a critical reading of the manuscript. A.D. Jack-son and K. Kainulainen are thanked for discussions. The work of F.S. is supported by the Marie–Curie fellowship under contract MCFI-2001-00181.