Higher derivative terms in the $\pi\Delta N$ interaction: some phenomenological consequences

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Abstract.
In this paper we implement for the first time the use of a $\pi N\Delta(1232)$ vertex interaction containing both first and second order derivative terms, as required by renormalization considerations. As was previously shown both interactions present formal problems, but can be implemented pertubatively. We put an end point to the discussion of which type of interaction is the appropriate one: both the usual pi-derivative and the ”spin 3/2” gauge invariant that include also a derivative in the Delta field should be included in amplitude calculations. We show that when working within a tree level approach the description of total $\pi N$ scattering in the different channels is improved.

keywords: DELTA ISOBAR; PION-NUCLEON-ISOBAR INTERACTION; PION-NUCLEON ELASTIC SCATTERING

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1. Introduction

For decades, the phenomenology of the $\Delta(1232\text{ MeV})$ resonance, was modelled as a Rarita-Schwinger (RS) vector spinor field $\Psi_\mu$ with a free Lagrangian

$$L_{\text{free}} = \overline{\psi}_\mu(x)K(\partial, A)^{\mu\nu}\Psi_\nu(x)$$  \hspace{1cm} (1)

where

$$K(\partial, A) = R \left(-\frac{1}{2}(1+A)\right)^{\mu\nu} \left[ \epsilon_{\mu'\nu'\alpha\beta} \partial^{\alpha} \gamma^{\beta} \gamma_5 + im\sigma_{\mu'\nu'} \right] R \left(-\frac{1}{2}(1+A)\right)^{\nu'\mu}$$  \hspace{1cm} (2)

being $\sigma_{\mu,\nu} = i\frac{1}{2}[\gamma_\mu, \gamma_\nu] = i\gamma_\mu\gamma_\nu$, $\epsilon_{0123} = 1$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and $R_{\mu\nu}(a) = g_{\mu\nu} + a\gamma_\mu\gamma_\nu$.  

Note that the matrices $R \left(-\frac{1}{2}(1+A)\right)$ appear because by construction the field $\Psi_\mu$ has a spurious spin 1/2 component and as a consequence the Lagrangians $L_{\text{free}}$ are connected by the contact transformation $\Psi_\mu \rightarrow R^{\mu\nu}\Psi_\nu$, $A \rightarrow \frac{A}{1+4a}(A \neq -\frac{1}{2})$, which change the proportion of the 1/2 states while leaving the equations of motion invariant.

Interaction with nucleon $\psi$ and pion $\phi$ fields have been studied using the chiral invariant term which dominates at small energies [1]:

$$L_{I_1} = g_1\overline{\psi}_\mu R \left(\frac{1}{2}(1 + 4Z_1)A + Z_1\right)^{\mu\nu} \Psi_\nu + \text{c.c.},$$  \hspace{1cm} (3)

where $I_1$ indicates that we have an interaction with a first derivative, i.e at order $q_\pi$, and it is also invariant (at the level of the equations of motion and the amplitudes) under contact transformations. It is clear that the $A$-dependence cancels up in any physical amplitude, whose dependence on $Z_1$ persists. $Z_1$ is thus a free parameter of the interaction [2].

But this interaction has been shown to be problematic from the formal relativistic field theory point of view since for certain background pion fields the Fock space becomes non-positive definite, [3]. However, this term can be used to get low energy amplitudes since it is the most general first derivative Lagrangian respecting covariance and chiral symmetry, and admits a nonproblematic perturbative order by order approach for the amplitude [4]. Other concern about Delta interaction amplitudes, the existence of the so-called “spin 1/2 background”, has been proved baseless since lowest spin representation contributions are present in other cases [6].

In order to try solving these shortcomings of the interaction $I_1$, and based on the supposition that the key for doing so was the decoupling of the spin 1/2 sector from amplitudes, during the last decade a new interaction of second derivative order, i.e $\sim q_\pi q_\Delta$, [7] has been proposed and extensively used. It has been conjectured to solves both the signature problem and the concern about the spin 1/2 background. It reads

$$L_{I_2} = -g_2\partial_\alpha \Psi_\mu R \left(\frac{1}{2}(1 + 4Z_2)A + Z_2\right)^{\mu\nu} \epsilon^{\sigma\nu\alpha\beta} \gamma_\beta \gamma_5 \partial_\nu \Phi + \text{c.c.}$$  \hspace{1cm} (4)

‡ These are Bjorken and Drell conventions.
It is important to note that this interaction term is the most general second order interaction derivative in the pion (which is necessary for chiral invariance), provided all free parameters are set such that Lagrange multiplier fields of the free theory do not acquire dynamics due to the interaction, as explained in the appendix. This criterion, subject to some controversy \[6\], is nonetheless the one used to fix \(Z_1 = 1/2\) for \(I_1\) in \[1\], and for \(I_2\) it leads to \(Z_2 = -1/2\), which corresponds to the interaction originally proposed by \[7\].

Nevertheless we have shown recently that this new interaction presents the same signature problems than the conventional \(L_{I_1}\) \[10\], it couples to a spin 1/2 background in radiative amplitudes and renormalization considerations force the reintroduction of conventional terms \[9\].

In addition as expected from general considerations from Effective Field Theory in the resonance region, (EFT) that consider a contribution of the pion momentum to the power counting of \(\delta = (m_N - m_\pi)/\Lambda_{\chi PT}\) or \(m_\pi/\Lambda_{\chi PT}\) \(\delta^2\) depending of its value, both interactions are of the same order since momentum coming from \(\partial_\mu \Psi_\nu\) behaves as order 1 at threshold \[9\]. Furthermore, from the phenomenological point of view, the fit to data from the new interaction is by no means superior to the conventional \[2\] in the resonance region. All these comments suggest that we should consider \(I_1\) and \(I_2\) together. We consider \(I_2\) of higher order in consideration of the dimension of the coupling constant and the number of derivatives in the term, in line with \[5\].

The newer interaction \(I_2\) could be easily seen to be simply the next order in derivatives from the conventional coupling \[9\] \[10\]. The addition of each term \(I_k\) implies the incorporation of a new parameter to fit (the corresponding coupling constant). In this paper we will work with both the \(I_1\) and the next derivative order \(I_2\), and will explore the possible phenomenological consequences of considering its coexistence in the case pion-nucleon scattering. Then we consider the Lagrangian

\[
\mathcal{L}_\Delta = \mathcal{L}_{\text{free}} + \mathcal{L}_{I_1} + \mathcal{L}_{I_2}.
\]

The structure of the paper is as follows. In Section (2) we build the Feynman amplitude for the elastic \(\pi N\) cross section individualizing the cases \(g_1 = 0, g_2 = 0, g_1, g_2 \neq 0\) in the Lagrangian \[5\]. In section (3) we show the numerical results for the s and u-channel \(\Delta\)-exchange contributions to the total cross section and compare the results obtained in both cases for the different \(\pi^+ p, \pi^- p, \pi^- p \rightarrow \pi^0 n\) channels. In section(4) we summarize our conclusions and in the appendix we show the fixing of the off-shell parameters \(Z_1\) and \(Z_2\) and the spin projectors in the RS space.

2. Elastic pion-nucleon amplitude

We first calculate the tree level elastic nucleon-pion amplitude from the Lagrangian \[5\]. Let us first cast the propagator of the RS field in terms of projectors on the different spin

\[\text{In spite we use the terminology “elastic” we also will try together the charge exchange } \pi^- p \rightarrow \pi^0 n \text{ channel with the same approach.}\]
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\[ R_{\pi N \Delta} = + \]

\[ \text{Figure 1. Elastic amplitude of pion ($\pi$) nucleon (N) scattering split in s(left) and u or cross (right) contributions for $\Delta$ resonance.} \]

sectors. This will allow us to identify the contributions from each one in the amplitude. First, recall that $A$ in eq. (1) is an arbitrary parameter while $Z_1$ and $Z_2$ are parameters regulating the coupling to the off-shell sector $\gamma_\mu \gamma_\nu \Psi^\nu$, as can be seen from the definition of $R(a)$, and from the fact that on shell we have the constraint $\gamma^\nu \Psi_\nu = 0$. The fixing for $Z_k$ from theoretical arguments, as we mention above, remains controversial (see [6]) but we stick to the usual criterion used in [1, 10] and the appendix in order to compare our results with the literature.

Using the properties of $R$ matrices in $\mathcal{L}_{\text{free}}$ at (5) we can write the general propagator in terms of the propagator for $A = -1$ (which renders the calculations simpler) as

\[ G(p, A)^{\mu\nu} = R^{-1} \left( -\frac{1}{2}(1 + A) \right)_\mu^\alpha G(p, -1)^{\alpha\beta} R^{-1} \left( -\frac{1}{2}(1 + A) \right)_\beta^\nu \]  

where $G(p, -1)^{\mu\nu}$ can be put in terms of the well known projectors $P^{3/2}$, $P_{11,22}^{1/2}$, $P_{21}^{1/2}$ and $P_{12}^{1/2}$ (see the appendix) as

\[ G(p, -1)^{\mu\nu} = - \left[ \begin{array}{c} \frac{\not{p} + m}{p^2 - m^2} P^{3/2}_{\mu\nu} - \frac{2}{3m^2} (\not{p} + m)(P_{22}^{1/2})_{\mu\nu} + \frac{1}{\sqrt{3m}} (P_{12}^{1/2} + P_{21}^{1/2})_{\mu\nu} \end{array} \right]. \]  

Consider the Lagrangian [5] with $\mathcal{L}_{I_1}$ and $\mathcal{L}_{I_2}$ given by eqs. (3) and (4) respectively, with $A = -1$, $Z_1 = 1/2$, $Z_2 = -1/2$; those values for $Z$ were already adopted in refs. [1] and [7]

\[ \mathcal{L}_\Delta = \bar{\psi}_\mu \left[ \epsilon^{\mu\nu\alpha\beta} \partial_\alpha \gamma_\beta \gamma_5 + (im\sigma^{\mu\nu} \equiv mR(-1)^{\mu\nu}) \right] \Psi_\nu + g_1 \bar{\psi}_\mu \phi^\dagger \cdot T R(-1)^{\mu\nu} \Psi_\nu + g_1 \bar{\Psi}_\mu R(-1)^{\mu\nu} \partial_\nu \phi \cdot T^\dagger \psi \]

\[ - g_2 \bar{\psi} \partial_\mu \phi^\dagger \cdot T \epsilon^{\mu\nu\alpha\beta} \partial_\alpha \gamma_\beta \gamma_5 \Psi_\nu + g_2 \bar{\alpha} \bar{\psi}_\mu \epsilon^{\mu\nu\alpha\beta} \gamma_\beta \gamma_5 \partial_\nu \phi \cdot T^\dagger \psi, \]  

(8)
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where $T$ are the $N \to \Delta$ isospin excitation operators. Now from it and the propagator
we can calculate the resonance elastic amplitude contribution($R_{\pi N}$) shown in the
fig. (2). Omitting the nucleon spinors and isospin factors, letting the incoming and
outgoing pions momentum be noted $k$ and $k'$ respectively and the $\Delta$ momentum by $p$
The $s$-channel amplitude reads

$$ R_{\pi N} = g_1^2 k'_\mu R^{\mu \alpha} G_{\alpha \beta} (p) R^{3\nu} k_\nu + g_2^2 k'_\mu (-i) \Gamma^{\mu \alpha} (p) G_{\alpha \beta} (p) (-i) \Gamma^{3\nu} (p) k_\nu $$

$$ -g_1 g_2 k'_\mu R^{\mu \alpha} G_{\alpha \beta} (p) (-i) \Gamma^{3\nu} (p) k_\nu - g_1 g_2 k'_\mu (-i) \Gamma^{\mu \alpha} G_{\alpha \beta} (p) R^{3\nu} k_\nu $$

where $\Gamma^{\mu \nu} (p) = \epsilon^{\mu \nu \alpha \beta} \gamma_5 p_\alpha$, $R \equiv R(-1)$ and where we have used $\partial_\mu \psi, \phi, \Psi_\nu \sim -i q_\mu \psi, \phi, \Psi_\nu$. The $u$(cross) channel contribution is obtained by simply replacing $p$ by
$p - k - k'$ and $k$ by $k'$ in the former expression. Observe that the first two terms
correspond to the first and second order (in derivatives) contributions to the Lagrangian
respectively while the last two can be construed as interference terms between them.
Let us analyze first the amplitude for the first order derivative interactions. If $g_2 = 0$
we get the $s$-chanel amplitude

$$ R_{\pi N}^{g_2=0} = -(g_1)^2 \frac{\not{p} + m}{p^2 - m^2} P^{3/2}_{\mu \nu} k_\mu k'_\nu $$

$$ -\frac{(g_1)^2}{m^2} \left[ 2(\not{p} + m) P_{11}^{1/2} + m \sqrt{3} (P_{12}^{1/2} + P_{21}^{1/2})_{\mu \nu} \right] k_\mu k'_\nu, $$

(10)

where the relations

$$ R^{\mu \nu} = P^{3/2}_{\mu \nu} - \sqrt{3} (P_{12}^{1/2} + P_{21}^{1/2})_{\mu \nu} - 2 (P_{11}^{1/2})_{\mu \nu}, $$

$$ (p^\mu \gamma_\nu - \gamma^\mu p_\nu) = \sqrt{3} \not{p} (P_{12}^{1/2} + P_{21}^{1/2})_{\mu \nu} = -\sqrt{3} (P_{12}^{1/2} + P_{21}^{1/2})_{\mu \nu} \not{p}, $$

$$ \left[ P^{3/2}_{\mu \nu}, \not{p} \right] = \gamma^\mu P^{3/2}_{\mu \nu} = P^{3/2}_{\mu \nu} \gamma_\nu = 0, $$

(11)

were used. The second term in (10) is the so-called “spin 1/2 background”, but what
is relevant for the asymptotic behavior of the amplitude is that it represents a non-pole
(without a pole) contribution which grows with $p$, since the projectors go as $p^0$ (see
Appendix). On the other hand, if $g_1 = 0$ the second order derivative amplitude can be
expressed also as a pole and non-pole term:

$$ R_{\pi N}^{g_1=0} = -(g_2)^2 m^2 \frac{\not{p} + m}{p^2 - m^2} P^{3/2}_{\mu \nu} k_\mu k'_\nu $$

$$ -\frac{(g_2)^2}{m^2} (\not{p} + m) P^{3/2}_{\mu \nu} k_\mu k'_\nu $$

(12)

where we have used eqs. [11] and that

$$ -i \Gamma (p)^{\mu \nu} = - \not{p} R^{\mu \nu} - (p^\mu \gamma_\nu - \gamma^\mu p_\nu) = -R^{\mu \nu} \not{p} + (p^\mu \gamma_\nu - \gamma^\mu p_\nu). $$

(13)

Observe that, except for the dimensions of the coupling constants, the pole terms for
both amplitudes are identical in form. Additionally there is also a non-pole background
term as badly behaved asymptotically as the non-pole term in (10). We will might call
it a “spin 3/2 background”. Putting now both interactions together the amplitude reads
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\[ R_{\pi N} = \left( \frac{g_1 + mg_2}{m} \right)^2 k'_\mu \left\{ -p^2 \frac{\not{p} + m}{p^2 - m^2} P_{3/2}^{\mu\nu} \right\} k_\nu 
+ \left[ \left( \frac{g_1 + mg_2}{m} \right)^2 - g_2^2 \right] k'_\mu \left\{ (\not{p} + m) P_{3/2}^{\mu\nu} + (p^\mu \gamma^\nu - \gamma^\mu p^\nu) \right\} k_\nu 
- 2g_1g_2k'_\mu R^{\mu\nu} k_\nu, \]  
\[ = - \left( \frac{g_1 + mg_2}{m} \right)^2 k'_\mu \left\{ \frac{\not{p} + m}{p^2 - m^2} P_{3/2}^{\mu\nu} \right\} k_\nu 
- \left( \frac{g_1 + mg_2}{m} \right)^2 k'_\mu \left\{ 2(\not{p} + m)(P_{11/2}^{1/2})^{\mu\nu} + m\sqrt{3}(P_{12}^{1/2} + P_{21}^{1/2})^{\mu\nu} \right\} k_\nu 
+ g_2^2k'_\mu \left\{ 2(\not{p} + m)(P_{11/2}^{1/2})^{\mu\nu} + m\sqrt{3}(P_{12}^{1/2} + P_{21}^{1/2})^{\mu\nu} \right\} k_\nu 
- g_2^2k'_\mu (\not{p} + m)P_{3/2}^{\mu\nu} k_\nu 
- 2g_1g_2k'_\mu \left( P_{3/2}^{\mu\nu} - 2(P_{11/2}^{1/2})^{\mu\nu} - \sqrt{3}(P_{12}^{1/2} + P_{21}^{1/2})^{\mu\nu} \right) k_\nu. \]  
\[ = - \left( \frac{g_1 + mg_2}{m} \right)^2 k'_\mu \left\{ \frac{\not{p} + m}{p^2 - m^2} P_{3/2}^{\mu\nu} \right\} k_\nu 
- \frac{g_2^2}{m^2} k'_\mu \left\{ 2(\not{p} + m)(P_{11/2}^{1/2})^{\mu\nu} + m\sqrt{3}(P_{12}^{1/2} + P_{21}^{1/2})^{\mu\nu} \right\} k_\nu 
- g_2^2k'_\mu (\not{p} + m)P_{3/2}^{\mu\nu} k_\nu 
- \frac{2g_1g_2}{m} k'_\mu \left( 2\not{p}(P_{11/2}^{1/2})^{\mu\nu} + mP_{3/2}^{\mu\nu} \right) k_\nu, \]  
where we have assumed that the projectors are defined for each $p^2$, that is the case for the $s$-channel contribution for which $p^2 > 0$ always, while for the $u$-channel one, since $p^2$ could be arbitrary small, it is preferable to express the amplitude without separating pole from non-pole terms, as in eq. (14). As can be seen in eq. (18), the first term corresponds to the pole contribution both in (10) with $g_2 = 0$ and (12) with $g_1 = 0$, but now with a coupling constant $g = (g_1 + mg_2)$. The third and fourth terms are the corresponding backgrounds from (10) and (12), the last term being a background contribution coming from the interference of both vertices. If we use these interactions in the region of the resonance, of course, it is unimportant the individual values of $g_1$ and $g_2$, only the combination $g = (g_1 + mg_2)$ would be observable, but for higher values the backgrounds become relevant, and we might ask if by a judiciously adjustment of both coupling constants the bad high energy behavior (generated by the non-pole background) of the amplitude can be moderated. Let us write the complete amplitude for the $s$-channel in terms of $g$ and a parameter $\kappa$ assuming that $g_1 + mg_2 = g$, in order to keep the old peak adjustment, and $g_2 = \kappa g/m$. In this way

\[ g_1 = (1 - \kappa)g \]  
\[ g_2 = \frac{\kappa g}{m} \]  
\[ (16) \]  
\[ (17) \]
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Observe that for $\kappa = 0$ we obtain the amplitude for pure leading first derivative interaction, while in the limit $\kappa = 1$ we get pure second derivative interaction. The amplitude reads

$$R_{\pi N} = -g^2 k'_\mu \left\{ \frac{\not{p} + m}{p^2 - m^2} P_{3/2}^{\mu\nu} \right\} k_\nu$$

$$- \frac{(1 - \kappa)^2}{m^2} g^2 k'_\mu \left\{ 2(\not{p} + m)(P_{11}^{1/2})^{\mu\nu} \right\} k_\nu$$

$$- \frac{\kappa^2}{m^2} g^2 k'_\mu (\not{p} + m) P_{3/2}^{\mu\nu} k_\nu$$

$$- \frac{2(1 - \kappa)\kappa}{m^2} g^2 \left( 2 \not{p} P_{11}^{1/2} + m P_{3/2} \right)^{\mu\nu}. \quad (18)$$

Observe that we get the same peak contribution (if we assume the same value of $g \equiv \frac{f_{\pi N\Delta}}{m_\pi}$ used when $g_2 = 0, g_1 = g$ but as $\kappa, (1 - \kappa) < 1$ the backgrounds are reduced by a smaller factor $\kappa^2, (1 - \kappa)^2$, and with an interference background (last term) coming from the last term in eq. (18) of the same order but enlarged by a factor 2. Nevertheless, as we will see, the $P_{11}^{1/2}, P_{3/2}$ contributions present in this term are small and also the $1/2$ backgrounds do not interfere with the peak $3/2$ amplitude while the $3/2$ one does.

3. Numeric calculations

Now we are going to calculate the resonance $s$ and $u$-channel contributions shown in fig. (2) to the $\pi N$ total elastic cross section. In addition, to reproduce experimental data we must include other backgrounds ($B_{\pi N}$) to the amplitude as shown in fig. (2) (see [11] for details). As the denominator of the pole term in eq. (18) is not defined at $p^2 = m^2$ for the $s$ amplitude we should include the unstable character of the $\Delta$. The simplest way to do it is the complex mass scheme (CMS) where we make the replacement $m \rightarrow m + i\Gamma$ in the full propagator, being $\Gamma$ the $\Delta$ width [11]. A more accurate procedure would be the use...
of the energy dependent $\Delta$ self-energy taking into account the $\Delta$ mixing with $\pi N$ states at one or higher loop bubbles to all order[12]. Nevertheless, since we are interested in the qualitative behavior of the different components of the $\pi N\Delta$ interaction, we use the simpler CMS scheme. Within this approach and with the $I_1$ interaction, $m$, $\Gamma$ and $g$ were fitted to describe the $\pi^+ p$ total elastic cross section satisfactorily [11] in the resonance region, giving $m = 1211.2\text{MeV}, \Gamma = 88.16\text{MeV}, g = 0.316$. As was shown previously [2], results with both interactions separately do not change appreciably the prediction of the total elastic cross section in the resonance region, the $I_1$ interaction fitting the experimental data slightly better. For this reason we will fix $g_1 + mg_2 = g = 0.316$ with the $m$ and $\Gamma$ values above reported. Nevertheless we want to analyze the cross section for energies above the resonance since in experiments the strong $\pi N$ interaction is typically probed for $\pi N$ invariant masses up to $2\text{GeV}$ in pion-photoproduction and weak-production, using models generalizing the CMS to energy-dependent width but the same $B_{\pi N}$ background [14]. In addition, there are channels for which the $\Delta$ s-channel contribution is suppressed by the isospin factors with respect to the $\Delta$ u-channel one, as in $\pi^- p$ scattering and then we want to analyze also the effect of using an interaction of the form $I_1 + I_2$ for this case.

Firstly we fix $\kappa$ in eq. (18) in order to get the closer approach to the $\pi^+ p$ elastic cross section experimental data, which were used previously for fixing the $\Delta(1232)$ parameters in the resonance region. The amplitude is built with the diagrams in figs. [2] and (2),
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i.e. $M_{\pi N} = R_{\pi N} + B_{\pi N}$, and reach pion kinetic energies ($T_{\text{lab}}$) of the order of 900 MeV ($W_{\pi N} \sim 2$ GeV). We find that for $\kappa = 1/2$ the model is closest to the data. In fig. (3), we show the results for the total elastic cross section for this value for $\kappa$ together with the results for $\kappa = 0$ (corresponding to $g_2 = 0$) and $\kappa = 1$ ($g_1 = 0$) to appreciate the improvement achieved. Also we show the results within the so called EXCMS approximation with $g_2 = 0$, obtained in a previous work by solving the Bethe-Salpeter equation for the $\Delta$ propagator in presence of a one-loop self energy and replacing at the end the renormalized mass as in the CMS approach [13], but still keeping the tree label approximation for the background $B_{\pi N}$. As can be observed, by no means within the CMS scheme the $I_2$ interaction gets a better description than the $I_1$ one, and the use of $I_1 + I_2$ improves the individual descriptions maintaining the right behavior in the pole peak.

In order to understand how both interactions cooperate to improve the high energy behavior of the amplitude we are going to separate the resonance peak and background amplitude contributions, which only makes sense for the $s$-channel $\Delta$ interchange, since for the $u$-channel the pole 3/2 term in eq. (18) alone is ill defined due to the possible vanishing of $p^2$. We begin with the $s$-channel: in fig. (4) we show the pole contribution from the resonant amplitude $R_{\pi N}(\text{pole})$ (first term in (18)) together with the background $R_{\pi N}(1/2)$ (second term) contribution when $\kappa = 0$ ($g_2 = 0$) which, as can be seen, is the largest term at high energies. We show also the $R_{\pi N}(3/2)$ (third term) background contribution when $\kappa = 1$ ($g_1 = 0$), and finally we separate 2 $\hat{p}P_{11}$ and $mP_{3/2}$ contributions in order to analyze the size of the last term in (18). Finally the full background for the $I_1 + I_2$ case $R_{\pi N}(1/2 + 3/2)$ when $\kappa = 0.5$ (second + third + fourth terms) is shown.

As can be seen, the second term in eq. (18) is strongly suppressed since its contribution is now weighted by a factor 1/16 in the cross section for $\kappa = 1/2$. Nevertheless, since it is larger than other contributions, it still contributes roughly a 25% of the $\kappa = 1$ $R_{\pi N}(3/2)$ background at $T_{\text{lab}} = 1100/$MeV, which we will use as a reference. Also the third contribution $R_{\pi N}(3/2)$ is reduced to 6.25% of our reference, making an even smaller contribution to the background. It might be surprising the similarity between the backgrounds $R_{\pi N}$ for $g_1 = 0$ and $\kappa = 0.5$, since they have clearly different origins in spin content. This similarity can be understood if we observe that we are plotting total (i.e. integrated) cross sections, since it can be shown that $\sum_{\text{spins}} \int d\theta |\tilde{u}(2\hat{p}P_{11}^\mu q_\nu q_\nu u)|^2 = 2 \sum_{\text{spins}} \int d\theta |u(2\hat{p}P_{3/2}^\mu q_\nu q_\nu u)|^2$. The effect of the interference between the $R_{\pi N}(\text{pole})$ and $R_{\pi N}(1/2 + 3/2)$ $s$-amplitudes to the total cross section is shown in fig. (5).

The improvement due to the weighting factors in eq. (18) is clear, but it gets still better when considering the interference between $R_{\pi N}$ and $B_{\pi N}$ as shown in fig. (3).

It is also important to analyze all the mentioned effects on the $u$-channel (second

A better description would be got by introducing the rescattering of the background, but this at the price of introducing form factors to regularize the integrals involved for intermediate pion-nucleon states.
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Figure 4. Contributions to the s-channel from the $R_{\pi N}(\text{pole})$ together with different contributions from background to $R_{\pi N}$ as indicated. We slightly extend the energy range to individualize better each contribution.

Figure 5. $R_{\pi N} = R_{\pi N}(\text{pole}) + R_{\pi N}(1/2 + 3/2)$ contribution to the cross section.
term in fig.2), which amplitude we call $R_{\pi N(\text{cross})}$ in fig. 6 and where now the different contributions are not separated in spin content since, as mentioned earlier, that separation leads to ill defined terms. In fig.6 we show the $u$-channel contribution to the total cross section in the $\pi^+p$ scattering together with the corresponding contributions if $g_2 = 0$ and if $g_1 = 0$. We see that for $I_1 + I_2$ the contribution lies between them. Observe that the cross section in the case $g_1 = 0$ is quite suppressed due to the fact that $p^2 < 0$ in the resonance region. This result represents an improvement, as can be seen when we calculate the elastic $\pi^-p$ cross section where the $u$-channel contribution is the most important while the $s$-channel one is suppressed by the isospin factors. In the fig.7 we compare our calculation with the different interactions to the elastic $\pi^-p$ cross section. Here can also be seen the improvement when using both interactions together, in spite that in this case for higher energies we have more excited resonances than the $\Delta$ that is the only included in our model. This is so because the $\pi^-p$ state has a stronger isospin 1/2 component, with many 1/2 isospin resonances in the region.

Finally, and for the sake of completeness, we show in fig.8 the same results for the charge exchange $\pi^-p \rightarrow \pi^0n$ channel. Here, the improvement using $I_1 + I_2$ is moderate since in this case the amplitude is built as $\sqrt{2}/3(s-u)$ contributions, thus the overvaluation at high energies in the $\pi^+p$ channel (mainly $s$-like) and the undervaluation in the $\pi^-p$ one (mainly $u$-like), leads to a worse behavior for this channel. In spite of this, using $I_1 + I_2$ is represents an improvement.
4. Conclusions

To date, publications devoted to the description of pion-production reactions describe the $\pi N\Delta$ vertex by either using the conventional first derivative pion field Lagrangian eq. (3) or the more recent second order derivative Lagrangian in eq. (4), as if they were mutually excluding possibilities. As mentioned earlier, both interaction vertexes have problems from the formal point of view, but can be used in a perturbative approach.
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We have shown that, within the spirit of Effective Field Theories, and especially what Weinberg called “folk theorem”, in order to reproduce low energy amplitudes these interaction terms are the first two of an infinite series respecting the point invariance $\Psi_\mu \to \Psi_\mu + \gamma_\mu \gamma^\nu \Psi_\nu$, which should provide increasing levels of accuracy. We show here that using both interaction terms together the fit to experimental data above the resonance region improves in all channels. Results for the total elastic cross section in the $\pi^+ p$ and $\pi^- p$ channels and also for the $\pi^- p \to \pi^0 n$ inelastic one, are improved when we work in a tree level approach within a complex mass scheme for the $\Delta$ resonance. This is physically sound, since we expect that, in order to reach higher energies, higher orders of a $p^2$ expansion of the interaction should be needed. Of course the description would be additionally improved by solving the Bethe-Salpeter equation for the $\Delta$ propagator using both interaction vertexes to account for the mixing of the $\Delta$ with $\pi$ and $N$ states, or by including rescattering of the non-resonance background contributions, but we will analyze this in a future publication. Without this it make no sense qualify our results trough a $\chi^2$ value.

5. Appendix

5.1. Free parameters in the $I_2$ interaction

To show the generality of the form of $\mathcal{L}_{I_2}$ observe that the general covariant form of the coefficients to contract with pion and a second derivative is, taking for simplicity $A = -1$ (see section 2), omitting isospin factors and integrating by parts

$$\mathcal{L}_{I_2}(A = -1) = -g_2 \bar{\Psi}_\mu R(\frac{-1}{2} - Z_2)_{\sigma}^\mu \mathcal{M}^{\sigma\alpha\nu} \partial_\alpha (\partial_\nu \Phi \psi) + c.c.,$$

(19)

where the most general tensor structure for $\mathcal{M}$ is $\mathcal{M}^{\mu\alpha\nu} = Z_1 \gamma^\mu \gamma^\alpha \gamma^\nu + Z_2 g^{\mu\alpha} \gamma^\nu + z_2 g^{\mu\nu} \gamma^\alpha + z_3 g^{\alpha\nu} \gamma^\mu$. Observe that in the free RS lagrangian in (1) and (2), there is no term containing $\dot{\Psi}^0$ for $A = -1$. So, the equation of motion for it is a true constraint, and $\Psi^0$ has no dynamics. It is necessary then that interactions do not change that (see ref.[10] appendix A). But since $\mathcal{L}_{I_2}$ contributes a $\dot{\Psi}^0$ in the equation of motion for $\psi$ via $R(\frac{-1}{2} - Z_2)_{\sigma}^0 \mathcal{M}^{\sigma\alpha\nu}$, the condition for $\dot{\Psi}^0$ not appearing in the equations of motion is that this contribution contains no time derivative of any of the other fields of the theory. This can be realized if $R$ is diagonal achieved with $Z_2 = -1/2$, and if $\mathcal{M}^{0\alpha0} = \mathcal{M}^{00\nu} = 0$. This leads to

$$\mathcal{M}^{\mu\alpha\nu} = Z_1 [\gamma^\mu \gamma^\alpha \gamma^\nu + g^{\mu\nu} \gamma^\alpha - g^{\mu\alpha} \gamma^\nu - g^{\alpha\nu} \gamma^\mu] = i\epsilon^{\mu\nu\alpha\rho} \gamma_\rho \gamma_5,$$

(20)

where we have used a property of gamma matrices. Finally if we replace eq.(20) in (19) we get eq.(4) for $A = -1$.

5.2. Spin projectors

We have introduced $P^k_{ij}$ which projects on the $k = 3/2, 1/2$ sector of the representation space, with $i, j = 1, 2$ indicating the subsectors of the 1/2 subspace, and are defined as
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\[
(P^{3/2})_{\mu\nu} = g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3p^2} [\not{p} \gamma_\mu p_\nu + p_\mu \gamma_\nu \not{p}], \\
(P^{1/2}_{22})_{\mu\nu} = \frac{p_\mu p_\nu}{p^2}, \\
(P^{1/2}_{11})_{\mu\nu} = g_{\mu\nu} - P^{3/2}_{\mu\nu} - (P^{1/2}_{22})_{\mu\nu}, \\
\quad = (g_{\alpha\alpha} - \frac{p_\mu p_\alpha}{p^2})(1/3\gamma_\alpha \gamma_\beta)(g_{\beta\nu} - \frac{p_\beta p_\nu}{p^2}), \\
(P^{1/2}_{12})_{\mu\nu} = \frac{1}{\sqrt{3p^2}} (p_\mu p_\nu - \not{p} \gamma_\mu p_\nu), \\
(P^{1/2}_{21})_{\mu\nu} = \frac{1}{\sqrt{3p^2}} (-p_\mu p_\nu + \not{p} p_\mu \gamma_\nu). \tag{21}
\]

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