Performance of a Link in a Field of Vehicular Interferers with Hardcore Headway Distance

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Abstract—Even though many point processes have been scrutinized to describe the unique features of emerging wireless networks, the performance of vehicular networks have been largely assessed using mostly the Poisson Point Process (PPP) to model the locations of vehicles along a road. The PPP is not always a realistic model, because it does not account for the physical dimensions of vehicles, and it does not capture the fact that a driver maintains a safety distance from the vehicle ahead. In this paper, we model the inter-vehicle distance equal to the sum of two components: A constant hardcore headway distance, and a random distance following the exponential distribution. We would like to investigate whether a PPP for the locations of interfering vehicles can be used to describe adequately the performance of a link at the origin under the new deployment model. Unfortunately, the probability generating functional (PGFL) of the hardcore point process is unknown. In order to approximate the Laplace transform of interference, we devise simple approximations for the variance and the skewness of interference, and we select suitable probability functions to approximate the interference distribution. It turns out that the PPP (of equal intensity) gives a lower bound for the outage probability under the hardcore point process. When the coefficient of variation and the skewness of interference are high, the bound may become loose at the upper tail. Relevant scenarios are associated with urban street microcells and highway macrocells with low intensity of vehicles. We also show that the performance predictions using the PPP deteriorate with multi-antenna maximum ratio combining receiver and temporal performance indicators related to the performance of retransmission schemes. Our approximations generate good performance predictions in all considered cases.

Index Terms—Headway distance models, method of moments, probability generating functional, stochastic geometry.

I. INTRODUCTION

Inter-vehicle communication, e.g., dedicated short-range transmission IEEE 802.11p, and/or connected vehicles to roadside units, e.g., LTE-based Vehicular-to-Infrastructure (V2I) communication, will be critical for the coordination of roadside units, e.g., LTE-based Vehicular-to-Infrastructure (V2I) communication, and we select suitable probability functions to approximate the interference distribution. It turns out that the PPP (of equal intensity) gives a lower bound for the outage probability under the hardcore point process. When the coefficient of variation and the skewness of interference are high, the bound may become loose at the upper tail. Relevant scenarios are associated with urban street microcells and highway macrocells with low intensity of vehicles. We also show that the performance predictions using the PPP deteriorate with multi-antenna maximum ratio combining receiver and temporal performance indicators related to the performance of retransmission schemes. Our approximations generate good performance predictions in all considered cases.

Inter-vehicle communication, e.g., dedicated short-range transmission IEEE 802.11p, and/or connected vehicles to roadside units, e.g., LTE-based Vehicular-to-Infrastructure (V2I) communication, will be critical for the coordination of road traffic, automated driving and improved safety in emerging vehicular networks. In order to analyze the performance of vehicular networks, we need tractable but also realistic models for the locations of vehicles. The theory of point processes deals with random spatial patterns and can provide us with the general modeling framework. We have to construct carefully the deployment model for the vehicles along a roadway, balancing between accuracy and complexity, that describes the distribution of headway (or inter-vehicle) distance, i.e., the distance from the tip of a vehicle to the tip of its successor.

Despite its wide acceptance, the PPP has also received a lot of criticism, because it does not capture the repulsive nature of network elements, due to physical constraints and Medium Access Control (MAC) mechanisms. This criticism has sparked network modeling using many more point processes, which however, are not tailored to describe vehicular networks. Stationary determinantal point processes fit better than the PPP the Ripley’s K function (second-order spatial statistic) of real-world macro-base station datasets. The PGFL for some determinantal processes, e.g., Ginibre, Gauss, etc., can be computed and evaluated numerically. The repulsion induced by collision avoidance MAC protocols is better captured by Matérn rather than softcore processes. The locations of users in wireless networks may also exhibit clustering instead of repulsion, due to non-uniform population density and hotspots. The PGFL for some Poisson cluster processes is tractable, see example 6.3(a) and for the PGFL of the Neyman-Scott process.

Intuitively, a spatial model for vehicular networks should be broken down into two components; a model for the road infrastructure and another for the distribution of vehicles along the roads. The Manhattan Poisson line process can be used to model a vertical and horizontal layout of streets, while the Poisson line process is suitable to describe roads with random orientations. These line processes have been coupled with homogeneous one-dimensional (1D) PPPs for the distribution of vehicles to study coverage probability. The Laplace transform of interference from the line containing the typical receiver (defined as the origin) is calculated using the PGFL of PPP. The contribution of interference from the rest of the lines requires to work out their distance distribution to the origin and incorporate it into the PGFL. Alternatively,
we can map every road to the road containing the origin but with a non-uniform density of vehicles [17]. The PPP has also been used in 1D vehicular system set-ups for higher layer performance evaluation [18]. Non-homogeneous PPPs have been used to model the impact of random waypoint mobility on temporal statistics of interference over finite regions [19].

The distribution of vehicles along a road with few number of lanes, e.g., bidirectional traffic streams with restricted overtaking, will not resemble a PPP. The PPP allows unrealistically small headways with high probability, while in practice, the follower maintains a safety distance depending on its speed and reaction time plus the length of the vehicle ahead [20], [21]. The distribution of headways naturally depends on traffic status. Measurements have revealed that the log-normal distribution is suitable under free flow traffic, while log-logistic distribution is adequate in congestion status [22]. These distributions have been used to study the lifetime of inter-vehicle links [23], however, without considering interference.

We would like to identify whether the PPP for the locations of vehicular interferers can be used to describe adequately the performance of a link at the origin (not part of the point process generating the interference) and under which conditions. The simplest inter-vehicle distance model which contains the PPP as a special case, but can also be tuned to avoid small headways, consists of a constant hardcore distance plus a random component modeled by an exponential Random Variable (RV) [24]. The inter-vehicle distance follows the shifted-exponential distribution, and the PPP is obtained by setting the shift equal to zero. In [25] we have compared the variance and skewness of interference under two fields of equal intensity $\lambda$; one due to a PPP, and another due to a point process with shifted-exponential inter-arrivals and positive hardcore distance $c$. We have devised a simple formula scaling the variance due to a PPP with a factor, which depends on $\lambda$ and $c$. However, this expression cannot be directly translated into link performance, e.g., outage probability. In addition, the performance of the link over time, e.g., the distribution of local delay, and/or over space, e.g., the outage probability with multi-antenna receiver require the temporal and spatial correlation properties of interference which are different under the two deployment models.

The common methodology to assess the outage probability of a link over all possible network states needs the PGFL of the point process generating the interference. With a positive hardcore distance, the locations of vehicles become correlated, and we could not figure out how to use the PGFL of PPP as a building block for the calculation of the Laplace transform of interference originated from the hardcore process. Looking at the complications associated with the calculation of the second and third moments of interference [25], it does not seem promising to calculate higher-order terms in the series expansion of PGFL [26]. The expansion kernels in factorial moment representation of the PGFL are simple only for the PPP [27]. It has been recently shown that the outage probability in non-Poissonian wireless cellular networks can be well-approximated by shifting horizontally the outage probability due to a PPP [28]. This result is not applicable in our system set-up because the point process does not impact the distribution of useful signal power but only the distribution of interference level. We will also see that the distance distribution appears quite complicated to convert it into aggregate interference level distribution.

In order to assess the outage probability, we can also calculate few moments of interference, and select suitable distributions, with simple Laplace transform, to approximate it. The method of moments has been widely used for modeling wireless channels, e.g., composite fading [29], Signal-to-Noise-Ratio in composite fading [30], aggregate interference and spectrum sensing channels [31], [32].

Complementing our study in [25], we will generate a simple approximation for the skewness of interference, allowing us to capture the impact of different parameters on the skewness and the coefficient-of-variance (CoV) (the ratio of the standard deviation over the mean). The mean interference levels under the two models are equal. The CoV carries information about the behavior of the interference distribution around the mean, and the skewness about the symmetry between the tails. Note that the right/left tail of interference distribution is associated with the behavior of the outage probability at the low/high reliability regime. In addition, the sign of the skewness would be crucial in selecting appropriate distribution models. The simplicity of the approximations would enable us to deduce under which traffic conditions the PPP fails to approximate closely the CoV and the skewness. Under these conditions, the PPP does not describe accurately the interference distribution and subsequently the outage probability. In order to study the efficacy of PPP with temporal and spatial performance metrics, we will use the mean delay and the outage probability of dual-branch Maximum Ratio Combining (MRC) as relevant metrics. A positive hardcore distance reduces the temporal and spatial correlation of interference in comparison with the PPP. Because of that, the PPP performance predictions worsen. The contributions of this paper are listed below.

- We approximate the distance distribution between the nearest interferer and the origin for a point process with hardcore distance $c$ and intensity $\lambda$. Its complexity rules out the possibility to calculate the signal level distribution for the $k$-th nearest interferer, and convert it to aggregate interference level by summing over all $k \rightarrow \infty$.
- We show that for small hardcore distance as compared to mean inter-vehicle distance $\lambda^{-1}$, the skewness of interference is approximately equal to that due to a PPP of intensity $\lambda$ scaled by $(1 - \frac{k}{\lambda c})$. This complements [25], where it is shown that for small $\lambda c$, the variance of interference, and subsequently the CoV, are reduced in comparison with a PPP of equal intensity. Overall, a hardcore distance makes the distribution of interference more concentrated around the mean and less skewed.
- For fixed $\lambda c$, the skewness and the CoV of interference increase for smaller cell size and lower intensity of vehicles. The shifted-gamma distribution with parameters selected using the method of moments (matching the mean, the variance and the skewness) fits well the simulations in all scenarios, including those associated with a high CoV and skewness, e.g., urban microcells and sparse flows of vehicles along macrocells. In these scenarios, the outage
probability predicted using the PPP is a loose lower bound in the upper tail.

- Introducing hardcore distance reduces the spatial correlation of interference at the two branches of a MRC receiver. Under independent and identically distributed (i.i.d.) Rayleigh fading channels, the Pearson correlation coefficient scales down approximately by \((1 - \lambda c)\). The outage probability predicted by the PPP is not anymore a bound and worsens in the upper tail. A bivariate gamma approximation for the interference distribution with identical and correlated marginals gives a good fit.

- The PPP makes a pessimistic prediction for the mean delay with simple retransmissions and low mobility. The gamma approximation for the interference distribution can provide a good fit when the PPP fails.

The rest of this paper is organized as follows. In Section II, we present the system model and the correlation properties of the deployment. In Section III, we derive the distance distribution between the nearest interferer and the origin. In Section IV, we approximate the skewness of interference, and we select well-known Probability Distribution Functions (PDFs) for the distribution of interference. In Section V, we illustrate that the approximations fit well the simulations, while the bounds on the probability of outage based on the PPP and the Jensen’s inequality may not be tight. In Section VI, we test the validity of the approximations using the mean local delay and dual-antenna MRC receiver. In Section VII, we conclude this study and outline relevant topic for future work.

II. System Model

We consider 1D point process of vehicles \(\Phi\), where the inter-vehicle distance follows the shifted-exponential PDF. The shift is denoted by \(c > 0\), and the parameter of the exponential part by \(\mu > 0\). The average intensity \(\lambda\) of vehicles can be calculated from \(\lambda^{-1} = c + \mu^{-1}\), or equivalently \(\lambda = \frac{\mu}{1 + \mu c}\). This model has been proposed by Cowan [24], and due to the positive shift \(c\), it can avoid small inter-vehicle distances. The penalty paid, in terms of analytical model complexity, is the correlations introduced in the locations of vehicles. The correlation properties have been studied in statistical mechanics, see for instance [33], where the vehicles are the particles of a 1D hardcore fluid, and the shift is equal to the diameter of the rigid disk modeling the identical particles. Conditioning on the location of a particle at \(x\), the probability to find another particle at \(y\), is [33] equation (32)]

\[
\rho_k^{(2)}(y, x) = \left\{ \begin{array}{ll}
\lambda \sum_{j=1}^{k} \mu j \left( \frac{y-x-jc}{c} \right)^{\frac{j-1}{2}}, & y \in (x+kc, x+(k+1)c) \\
0, & \text{otherwise},
\end{array} \right.
\]  

(1)

where \(k \geq 1\) and \(\Gamma(j) = (j-1)!

One can derive equation (1) using basic probability theory. The \(k\)-th branch of the PCF, \(k \geq 1\), requires to sum over the probabilities of having \(\{0, 1, \ldots, (k-1)\}\) particles between \(x\) and \(y\). For \(k = 0\), we have \(\rho^{(2)}(y, x) = 0\), as no two particles can be found at distance separation less than \(c\). For \(y < x\), one has simply to inter-change \(y\) and \(x\) in (1). For more details, see [25] Section III. The derivation of (1) using thermodynamic equations is available in [33].
vehicles communicating in ad hoc mode. The vehicles are forced to stop their transmissions inside the guard zone.

The transmit power level is normalized to unity. The propagation pathloss exponent is denoted by $\eta > 2$. The distance-based pathloss for an interferer located at $r$ is $g(r) = |r|^{-\eta}$ for $|r| > r_0$, and zero otherwise, to filter out vehicles inside the cell. The fading power level over all interfering links, $h$, and over the transmitter-receiver link, $h_t$, is exponential (Rayleigh distribution for the fading amplitudes) with mean unity. The fading is i.i.d. over different links and time slots. The interferers and the transmitter are active in each time slot, and they are equipped with a single antenna. When multiple antennas are employed at the receiver, they are separated at least by half the wavelength and their fading samples are assumed i.i.d. The distance-based useful and interference signal levels at different antennas are assumed equal.

### III. Nearest Interferer Distance Distribution

The statistics of interference are closely related to the statistics of the distance between the interferers and the reference point. Let us denote by $X_1$ the RV describing the distance from the nearest interferer to the origin. For the PPP, due to the independence property, it suffices to calculate the distance distribution without the guard zone and shift the distribution by $r_0$. The contact distribution for a 1D PPP is exponential with parameter the intensity $\lambda$. Therefore the RV $X_1$ is distributed as $f_{X_1}(x) = 2\lambda e^{-\lambda \left( x - r_0 \right)}$, $x \geq r_0$. It is straightforward to verify that the CoV and the skewness of $X_1$ for the PPP are equal to $\frac{4}{1 + 2r_0}$ and $2$ respectively.

For the hardcore process, the distribution of $X_1$ follows easily, only if we ignore the guard zone. For $x \leq \frac{r_0}{2}$, the point process rules out any other interferer closer than $x$ to the origin. The nearest interferer is located uniformly in $[\frac{-r_0}{2}, \frac{r_0}{2}]$. The probability to find a vehicle within an infinitesimal $dx$ belonging to this interval is $\lambda dx$. As a result, the distance distribution is uniform in $[0, \frac{r_0}{2}]$, and the probability to observe any distance of this range is $2\lambda dx$. Therefore $P(X_1 \leq \frac{r_0}{2}) = \lambda r_0$.

For $x \geq \frac{r_0}{2}$, no interferer must be located within a distance $(2x - c)$ from the first interferer. Therefore $P(X_1 \geq x) = e^{-\lambda \left( 2x - c \right)}$. After deconditioning, $P(X_1 \geq x) = (1 - \lambda c) e^{-\mu (2x - c)}$, $x \geq \frac{r_0}{2}$. Finally, the Cumulative Distribution Function (CDF) for the RV $X_1$ takes the following form

$$P(X_1 \leq x) = \begin{cases} 2\lambda x, & x \in \left[ 0, \frac{r_0}{2} \right] \\ 1 - (1 - \lambda c) e^{-\mu (2x - c)}, & x \geq \frac{r_0}{2} \end{cases}$$

The guard zone raises the complexity of calculating the distribution of $X_1$, because the correlated locations of vehicles start to have an effect. In a practical system set-up, the locations of vehicles from the two sides of the guard zone are expected to be weakly correlated. In order to give a relevant approximation, we note that a high value for the dimensionless ratio $\frac{x}{\gamma}$ is $\frac{1}{\lambda r_0} = (1 + \mu c) > 1$ indicates that the PCF decorrelates slowly. The point process will decorrelate within $2r_0 c$ multiples of the hardcore distance if $2r_0 c \gg \frac{x}{\gamma} = (1 + \mu c) \approx \mu c$, or equivalently, $\mu \ll \frac{2r_0 c}{x}$. If this condition is true, we introduce minor error by treating as i.i.d. the distances of the nearest interferer from opposite sides of the guard zone. Then, the CDF of $X_1$ would be approximated by the CDF of the minimum of two i.i.d. RVs.

Let us denote by $X_1^\dagger$ the RV describing the distance between the nearest interferer from the positive half-axis and the origin. For $x \in (r_0, r_0 + c)$, $X_1^\dagger$ follows the uniform distribution. For $x \geq r_0 + c$, no other interferer must be located closer to the cell border, $P(X_1^\dagger \geq x | x \geq r_0 + c) = e^{-\mu (x - r_0 - c)}$, or $P(X_1^\dagger \geq x) = (1 - \lambda c) e^{-\mu (x - r_0 - c)}$, $x \geq r_0 + c$ after deconditioning. Finally,

$$P(X_1^\dagger \leq x) = \begin{cases} \frac{\lambda}{x - r_0}, & x \in [r_0, r_0 + c] \\ 1 - (1 - \lambda c) e^{-\mu (x - r_0 - c)}, & x \geq r_0 + c \end{cases}$$

The approximation for the CDF of $X_1$ follows from the minimum of two i.i.d. RVs $X_1^\dagger$.

$$P(X_1 \leq x) \approx \begin{cases} \frac{1}{x - r_0} - \frac{\lambda}{x - r_0}, & x \in [r_0, r_0 + c] \\ 1 - (1 - \lambda c) e^{-\mu (x - r_0 - c)}, & x \geq r_0 + c \end{cases}$$

(3)

In Fig. 3a we see that the above approximation essentially overlaps with the simulations even if the mean inter-vehicle distance, $\lambda^{-1} = 40$ m, becomes comparable to the guard zone size, $r_0 = 100$ m. This is because for $\lambda = 0.4$, the PCF converges to $\lambda^2$ after approximately $4c$, see Fig. 1 which is equal to 64 m, roughly one-third of the guard zone length. The condition $\mu \ll \frac{2r_0 c}{x}$ obviously holds. Differentiating (3), the approximation for the PDF becomes

$$f_{X_1}(x) \approx \begin{cases} 2\lambda (1 - \lambda (x - r_0)) e^{-\mu (x - r_0 - c)}, & x \in [r_0, r_0 + c] \\ 2\lambda (1 - \lambda c) e^{-\mu (x - r_0 - c)}, & x \geq r_0 + c \end{cases}$$

(4)

It is possible to verify that the CoV and the skewness of $X_1$ are less than $\frac{4}{1 + 2r_0}$ and $2$ respectively, the values associated with a PPP of equal intensity. In Fig. 3b we have simulated the distance distribution for the $k$-th nearest interferer, $k \leq 5$. The distance distributions for $k > 1$ follow the same trend as that proved for $k = 1$. The distributions of the hardcore process have lower CoV and skewness as compared to those of PPP. This complies with the intuition that a hardcore $c$ makes the point process less random. Based on this, we may conjecture that the distribution of interference due to the hardcore process will be more concentrated around the mean and less skewed as compared to that due to a PPP of equal intensity.

### IV. Interference Distribution

From the Campbell’s Theorem, we know that the mean interference for a stationary point processes of intensity $\lambda$ is $E[I] = 2\lambda \int_{r_0}^{\infty} x^{-\eta} dx = \frac{2\lambda r_0^{1-\eta}}{\eta - 1}$. The details for the approximation of the second moment of interference can be found in [23], Section V]. The main idea is to approximate the PCF with the PCF of PPP, $\rho^{(2)}(x, y) \approx \lambda^2$, for large distance separation $|y - x|$, and use the exact PCF only for small distances. According to Fig. 1 this approximation should be valid for $\lambda c \rightarrow 0$. Since the point process for $c > 0$ becomes less random than the PPP, the variance of interference should reduce [25, equation (14)].

$$\mathbb{V}[I] \approx \frac{4\lambda r_0^{1-\eta}}{2\eta - 1} \left( 1 - \lambda c + \frac{1}{2} \lambda^2 c^2 \right),$$

(5)

where the term in front of the parenthesis is the variance due to a PPP of intensity $\lambda$. 
Some preliminary calculations about the third moment of interference are available in Section IV. Here, we will derive a simple approximation relating it to that of PPP of intensity $\lambda$, similar to the approximation in [1] for the variance.

**Lemma 1.** The skewness of interference from a hardcore process of intensity $\lambda$ and hardcore distance $c$ can be approximated by the skewness due to a PPP of intensity $\lambda$, scaled by $(1 - \frac{\lambda c}{2})$. The approximation is valid for $\lambda c \to 0$ and $r_0 \to 0$.

$$S(I) \approx \frac{12\lambda^1 - 3\eta}{3\eta - 1} \left( \frac{4\lambda^1 - 2\eta}{2\eta - 1} \right) \left( 1 - \frac{\lambda c}{2} \right).$$

**Proof.** The proof can be found in the supplementary material (optional reading).

Few properties of the skewness can be drawn based on Lemma 1: (i) Introducing a small hardcore distance while keeping the intensity of interferers fixed, reduces the skewness but the distribution remains positively-skewed. (ii) The skewness of interference due to a PPP increases for increasing pathloss exponent $\eta$ and decreasing cell size $r_0$. Introducing hardcore distance for fixed $\lambda$ does not change this property. (iii) For a PPP, increasing the intensity $\lambda$ reduces the skewness of interference. This is also true for the hardcore process provided that the product $\lambda c$ is not decreasing.

The above properties can be observed in Fig. 4 where we have simulated the skewness for different cell size $r_0$, pathloss exponent $\eta$ and traffic parameters $\{\lambda, c\}$, with respect to the product $\lambda c$. We see that for the considered range of $\lambda c$, the approximations for the second- and the third-order correlation, $\rho^{(2)}(x, y), \rho^{(3)}(x, y, z)$ do not introduce practically any error as compared to the simulations. In addition, the approximation given in Lemma 1 is quite accurate for small $\lambda c$. While changing from the microcell to macrocell scenario, we have the interplay of two conflicting factors: On one hand, the intensity of vehicles decreases to account for the higher speed of vehicles, and this increases the skewness. On the other hand, the cell size increases which reduces the skewness. For the selected parameter values of Fig. 4 the skewness reduces because, according to Lemma 1 it is proportional to $\frac{1}{\sqrt{\lambda c}}$.

For a bounded pathloss model, the interference distribution strongly depends on the fading process [15]. In our system setup we note: (i) the positive skewness of interference, and (ii) the guard zone around the receiver which essentially bounds the pathloss model, along with the exponential PDF for the power fading. The gamma PDF has a positive skewness, and it includes the exponential PDF as a special case. The parameters $k, \beta$ of the gamma PDF, $f_\gamma(x) \approx \frac{x^{k-1}e^{-x/\beta}}{\Gamma(k)\beta^k}$, can be computed by matching two moments, the mean and the variance approximation in [5], resulting to $k = \frac{\mathbb{E}(I)}{\sqrt{\text{Var}(I)}}$ and $\beta = \frac{1}{k}$. The skewness of the gamma distribution is $\frac{1}{\sqrt{\text{Var}(I)}}$. For practical values of the pathloss exponent $\eta \in [2, 6]$ and $\lambda c < \frac{1}{4}$, one can verify that the skewness, $\frac{1}{\sqrt{\text{Var}(I)}}$, is less than the approximation given in Lemma 1. The shifted-gamma PDF, which matches also the skewness of interference, is expected to provide better fit than the gamma PDF.

$$f_\gamma(x) \approx \frac{(x-\epsilon)^{k-1}e^{-(x-\epsilon)/\beta}}{\Gamma(k)\beta^k}, x \geq \epsilon,$$

where $k = \frac{1}{\sqrt{\text{Var}(I)}}, \beta = \frac{1}{\sqrt{\text{Var}(I)}}$ and $\epsilon = \mathbb{E}(I) - k\beta.$
Fig. 5. Simulated PDF of the interference level for a hardcore point process along with gamma and shifted-gamma PDF approximations. Cell size \( r_0 = 100 \) m, \( \eta = 3 \) and \( \lambda_c = 0.4 \). A PPP with equal intensity is also simulated. 10^7 trials to generate each simulation curve.

| Table I | Standard Deviation (SD) and skewness of interference for a hardcore process with \( \lambda_c = 0.4 \) obtained by simulations, and estimated using [3] and Lemma I |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|         | simulations | gamma | shifted-gamma | PPP |
| sd., \( \lambda = 0.1 \) | 0.0024 | 0.0023 | 0.0024 | 0.0025 |
| skewn., \( \lambda = 0.1 \) | 0.60 | 0.46 | 0.58 | 0.66 |
| sd., \( \lambda = 0.025 \) | 0.0012 | 0.0012 | 0.0012 | 0.0014 |
| skewn., \( \lambda = 0.025 \) | 1.27 | 0.93 | 1.06 | 1.32 |

The approximation accuracy of the gamma and the shifted-gamma PDFs is illustrated in Fig. 5 for two intensities \( \lambda \), and \( \lambda_c = 0.4 \). For fixed \( \lambda_c \), a higher intensity of vehicles paired with a lower tracking distance can be associated with driving at lower speeds. The simulated standard deviation and skewness, along with their approximations, are included in Table I. We see in the table that: (i) the variance approximation in [3] is quite accurate, (ii) Lemma I estimates the skewness better than the gamma distribution, (iii) the PPP has higher variance and skewness than the hardcore process, justifying the approximations in [3] and Lemma I. In both cases, the PPP estimates the skewness (in an absolute sense) better than Lemma I because the value of \( \lambda_c = 0.4 \) is not close to zero. Nevertheless, the PPP gives much worse estimates for the standard deviation (see Table I) and the interference distribution (see Fig. 5) than the gamma approximations. For fixed \( \lambda_c \), the skewness and the CoV are both proportional to \( \sqrt{\lambda} \). We see in Fig. 5 that for the higher intensity of vehicles, \( \lambda = 0.1 \), the interference distribution becomes more concentrated and less skewed, and the gamma approximation provides a very good fit. For a lower intensity of vehicles, \( \lambda = 0.025 \), the skewness and the CoV of interference increase, and three moments clearly provide a better fit than two. Also, both figures indicate that the PPP will underestimate the PDF of the Signal-to-Interference Ratio (SIR) at the tails.

V. PROBABILITY OF OUTAGE

Under Rayleigh fading over the transmitter-receiver link, the probability of outage, \( P_{\text{out}}(\theta) = P(\text{SIR} \leq \theta) \), becomes equal to the complementary Laplace transform of the interference distribution. Even though the interference PDF is unknown, its Laplace transform could be computed provided that the PGFL of the hardcore process was available. Unfortunately, this is not the case. Thanks to [35, Theorem 2.1], we deduce that the outage probability due to a PPP of intensity \( \lambda \) is actually a lower bound to the outage probability due to the hardcore process of equal intensity \( \lambda_c \).

\[
P_{\text{out}}(\theta) = 1 - e^{-2\lambda_0 \int_0^\infty \frac{1}{1-sx^{-\eta}} \, dx} = P_{\text{out}}(\theta),
\]

where \( \theta \) is the SIR threshold, \( s = \frac{\theta}{\eta} \). \( a \) follows from exponentially i.i.d. interfering fading channels, \( b \) from the PGFL of PPP along with [35, Theorem 2.1], and the integral in the exponent can be expressed in terms of the \( _2F_1 \) Gaussian hypergeometric function [56, p. 566].

An upper bound to the probability of outage can be obtained using the Jensen’s inequality. This is roughly as tight as the lower bound using the PPP. The upper bound suggested in [35, equation (2.8)], which is essentially a first-order expansion of the PGFL around \( s \to 0 \), is tight only for small \( \theta \).

\[
P_{\text{out}}(\theta) = 1 - e^{-\mathbb{E}_x \left\{ \sum_k \log(1+sx_k^{-\eta}) \right\}}
\]

\[
\leq 1 - \exp \left\{ -\mathbb{E}_x \left\{ \sum_k \log(1+sx_k^{-\eta}) \right\} \right\}
\]

\[
\leq 1 - \exp \left\{ -2\lambda_0 \int_0^\infty \log(1+sx^{-\eta}) \, dx \right\} = P_{\text{out}}^{\text{est}}(\theta),
\]

where \( a \) is due to Jensen’s inequality, \( b \) follows from the Campbell’s theorem and the integral in the exponent can be expressed in terms of the \( _2F_1 \) function.

The gamma approximations for the PDF of interference studied in the previous section have simple Laplace transforms, and they can be used to generate simple approximations for the outage probability.

\[
P_{\text{out}}(\theta) \approx 1 - (1+\theta)^{-k} = P_{\text{out}}^{\text{g}}(\theta),
\]

\[
P_{\text{out}}(\theta) \approx 1 - e^{-\theta} (1+\theta)^{-k} = P_{\text{out}}^{\text{est}}(\theta).
\]

\(^{1}\)It follows by setting the local stability constraint \( c^* \) in [35, equation (2.8)] equal to the intensity \( \lambda \) of the Gibbs process.
the probability of successful reception. For the PPP, one may
interferers over time, it is challenging to calculate it. For
of transmissions required for successful reception. For a
A. Temporal performance
approximation for the distribution of interference.
of the link, and a dual-branch MRC receiver for the spatial
instance [9], investigates the properties of delay under (i ) i.i.d.
interference, with correlated marginals, would be needed. An
worsens, while the gamma approximations can be used to
take the complementary of the last line of (6) and invert it.
According to (6), the PPP sets a lower bound to the mean delay
with i.i.d. locations of interferers. For the hardcore process,
the mean delay would be approximated by \( (1 + s \beta)^k \), see (7).
Since \( k, \beta \) are positive, a loose upper bound can be set using
the Bernoulli inequality, \( (1 + s \beta)^k \leq e^{s \beta} = e^{s \mathbb{E}[\tau]} \).

In order to calculate the mean delay with static interferers,
one has to invert the probability of successful reception condi-
tioned on the realization of interferers, then average over their
locations [9]. The mean delay with Poisson interferers accepts
an elegant form for continuous transmissions, \( \mathbb{E}\{D\} = e^{s \mathbb{E}[\tau]} \),
which follows from substituting \( p = 1, q = 0 \) in [9] Lemma 2.
In order to overcome the lack of the PGFL for the hardcore
process, we use an alternative expression for the mean delay,
\( \mathbb{E}\{D\} = \sum_{T} \mathbb{P}_{\text{out}}(T) \) [7], where \( \mathbb{P}_{\text{out}}(T) \) is the
joint outage probability over \( T \) consecutive time slots.

\[
\mathbb{P}_{\text{out}}(T) = \mathbb{P}(\text{SIR}_1 \leq \theta, \text{SIR}_2 \leq \theta, \ldots, \text{SIR}_T \leq \theta) = \mathbb{E}\{(1 - e^{-s\mathbb{I}_1}) \ldots (1 - e^{-s\mathbb{I}_T})\}
= 1 + \sum_{t=1}^{T} (-1)^t \binom{T}{t} \mathbb{E}\{e^{-s \sum_{j=1}^{t} \mathbb{I}_j}\}
= 1 + \sum_{t=1}^{T} (-1)^t \binom{T}{t} \mathbb{E}\{e^{-s \sum_{k=1}^{t} h_{k,j}(g_{x_k})}\}
= 1 + \sum_{t=1}^{T} (-1)^t \binom{T}{t} \mathbb{E}\{e^{-s \sum_{k=1}^{t} h_k(t)g(x_k)}\},
\]
where \( \text{SIR}_j \) and \( \mathbb{I}_j \) describe the \( \text{SIR} \) and the instantaneous
interference respectively over the \( j \)-th time slot, and the RV
\( h_k(t) = \sum_{j=1}^{t} h_{k,j} \), as a sum of i.i.d. exponential RVs follows
the gamma distribution.

We deduce that the calculation of the joint Laplace func-
tional over \( t \) slots with static interferers is equivalent to the
calculation of the Laplace transform of interference for a single
time instance, but with a different fading distribution. The first
two moments of the RV \( h_k(t) = h(t) \ \forall k \) are \( \mathbb{E}\{h(t)\} = t \)
and \( \mathbb{E}\{h^2(t)\} = t(1+t) \). We will still utilize the gamma
approximation for the interference, but the fading is now
modeled by a gamma instead of an exponential RV. We
have the same simple expression for the Laplace transform

We see in Fig. 6 that the bounds, \( \mathbb{P}_{\text{out}}^{\text{ppp}}(\theta) \) and \( \mathbb{P}_{\text{out}}^{\gamma}(\theta) \), are
tight in the body of the distribution, but they start to fail in
the upper tail. Their error is more prominent in microcells and
macrocells with a low intensity of vehicles. Recall that smaller
cell sizes \( r_0 \) and lower intensities \( \lambda \) are associated with higher
CoV and skewness for the interference distribution. According
to [3] and Lemma 1 for a fixed \( \lambda c \), the absolute prediction
error of PPP increases for lower \( \{\lambda, r_0\} \), and subsequently, the
induced approximation errors for the interference distribution
and the outage probability would be higher. We claim that the
PPP cannot always describe accurately the outage probability
of a link in a field of interferers with hardcore headway
distance. We will illustrate next that for temporal performance
metrics and multiple antennas at the receiver the PPP accuracy

VI. APPLICATIONS

The two deployment models (hardcore vs. PPP) induce
different interference correlation over time and space. We will
use the mean local delay to describe the temporal performance
of the link, and a dual-branch MRC receiver for the spatial
performance. For notational brevity, we will use the gamma
approximation for the distribution of interference.

A. Temporal performance

The mean local delay is defined as the average number of
transmissions required for successful reception. For a
mobility model introducing correlations in the locations of
interferers over time, it is challenging to calculate it. For \( T \)
consecutive transmissions, the joint \( T \)-th dimensional PDF of
interference, with correlated marginals, would be needed. An
alternative study that can provide us with some insight, see for
instance [9], investigates the properties of delay under (i) i.i.d.
locations, and (ii) static interferers over time. The performance
can be associated with scenarios characterized by very high
and very low mobility of interferers respectively.

For i.i.d. locations, the mean delay is equal to the inverse of
the probability of successful reception. For the PPP, one may

![Fig. 6. Simulated probability of outage for a point process with \( \lambda c = 0.4 \), along with upper-bound, \( \mathbb{P}_{\text{out}}^{\text{ppp}}(\theta) \), lower-bound, \( \mathbb{P}_{\text{out}}^{\gamma}(\theta) \), and approximations, \( \mathbb{P}_{\text{out}}^{\gamma}(\theta) \) and \( \mathbb{P}_{\text{out}}^{\gamma}(\theta) \). 10^5 simulations. Pathloss exponents \( \eta = 4 \), mean useful received signal level \( P_x = 8 \times 10^{-6} \text{W} \) for \( r_0 = 100 \text{ m} \) and \( P_x = 5 \times 10^{-7} \text{W} \) for \( r_0 = 250 \text{ m} \). The approximations fit very well the simulations in the lower tail too, see also Fig 8.](image)
(1 + sβ(t))^{-k(t)}, where the parameters k, β now depend on t. Without showing the derivation details, the mean, and the variance of interference in the presence of Nakagami fading modeled by a gamma RV with shape t and scale unity are

$$E\{I(t)\} = \frac{2\lambda^{-1-t} t}{\eta-1}$$

$$\text{Var}(I(t)) \approx \frac{2\lambda^{1-2\eta t}}{2\eta - 1}(1 + t(1 - \lambda c)^2).$$

Finally, the mean delay can be read as

$$E\{D\} \approx \sum_{T=0}^{\infty} \sum_{t=0}^{T} (-1)^t \left( T \atop t \right) (1 + s\beta(t))^{-k(t)},$$

where k(t), β(t) are derived via moment matching using (8).

The above approximation can be turned into a single sum by changing the order of the summations and setting a sufficient maximum value T0 for the parameter T.

$$E\{D\} \approx \sum_{t=0}^{\infty} \sum_{T=t}^{T_0} (-1)^t \left( T \atop t \right) (1 + s\beta(t))^{-k(t)}$$

$$= \lim_{T_0 \to \infty} \sum_{t=0}^{\infty} \sum_{T=t}^{T_0} (-1)^t \left( T \atop t \right) (1 + s\beta(t))^{-k(t)}$$

$$= \lim_{T_0 \to \infty} \sum_{t=0}^{\infty} (-1)^t \left(T_0 + 1 \atop t + 1\right) (1 + s\beta(t))^{-k(t)}.$$  

Since it is not realistic to assume very low mobility across macrocells, we depict in Fig. 7 the mean delay for a microcell; the associated outage probabilities are shown in Fig. 6B. We observe that the interference field due to the hardcore process induces a much smaller increase in the mean delay in comparison with PPP, as we move from extreme mobile to static interferers. This is because the temporal correlation coefficient of interference due to a static PPP is equal to $\frac{1}{2}$ [49], while that due to the hardcore process is lower, and approximately $\frac{1}{2} (1 - \lambda c)$ [40]. Due to the lower correlation of interference, less retransmissions are needed to meet the SIR target, and the mean delay decreases in comparison with that due to static PPP. The approximation error of PPP in networks with low mobility blows up in the high reliability regime.

**B. Spatial performance**

The probability of successful reception for MRC with dual-branch receiver in the presence of spatially correlated interference requires the PGFL with respect to the reduced Palm measure [41] equation (25)]. In a Poisson field, due to the Slivnyak’s theorem, this is available, and the performance has been derived in [41] equation (26)]. Unfortunately, in our case, we will need again approximations about the distribution of interference in the two branches and their correlation. We will end up with a simple approximation for the outage probability, while the calculation in [41] equation (26)] requires the numerical computation of three integrals.

Let us denote by $I_1 = \sum_i h_{1,i} g(x_i)$ and $I_2 = \sum_i h_{2,i} g(x_i)$, the instantaneous interference, and by I the vector of $I_1, I_2$. Treating the interference as white noise, the post-combining SIR becomes equal to the sum of the SIRs at the two branches.

$$\mathbb{P}\{\text{SIR} \geq \theta\} = \mathbb{E}_I \left\{ \mathbb{P} \left( \frac{h_{1,1} P_r}{I_1} + \frac{h_{1,2} P_r}{I_2} \geq \theta \right| I \right\}.$$

Let us denote by $W = \frac{h_{1,1} P_r}{I_1}$ the RV describing the SIR at the second branch. Conditioning on the realization $w$, and using that the fading channel is Rayleigh, we have

$$\mathbb{P}\{\text{SIR} \geq \theta\} = \mathbb{E}_W \left\{ e^{-s_1 I_1} \right\} = \mathbb{E}_W \left\{ \int_0^{\infty} e^{-s_1 I_1} f_W(w) dw \right\},$$

where $s_1 = \frac{\max\{0, \theta - w\}}{P_r}$ and $f_W(w)$ is the conditional PDF of the SIR at the second branch.

Due to the fact that the fading channel is Rayleigh, $\mathbb{P}(W \geq w|I_2) = e^{-s_2 I_2}$, where $s_2 = \frac{w}{P_r}$. By differentiation, $f_W(w|I_2) = \frac{d}{dw} e^{-s_2 I_2}$. Therefore

$$\mathbb{P}\{\text{SIR} \geq \theta\} = \frac{1}{P_r} \int_0^{\infty} \mathbb{E}_I \left\{ I_2 e^{-s_1 I_1} e^{-s_2 I_2} \right\} dw$$

$$\approx \frac{1}{P_r} \int_0^{\infty} \mathbb{E}_I \left\{ I_2 e^{-s_1 I_1} e^{-s_2 I_2} \right\} dw +$$

$$\frac{1}{P_r} \int_0^{\infty} \mathbb{E}_I \left\{ I_2 e^{-s_2 I_2} \right\} dw,$$

where (a) follows from $s_1 = 0$ for $w > \theta$.

We will assume that the random vector I follows the bivariate gamma distribution with identical marginals following
the gamma distribution with parameters \( \{k, \beta\} \) calculated in Section VI. The correlation coefficient is denoted by \( \rho \). Using the differentiation property of the Laplace transform, the first expectation in \( (10) \),

\[
\mathbb{E}_1 \{ I_2 e^{-s_1 I_1 - s_2 I_2} \} \text{ becomes }
\]

\[
\mathcal{J} \approx -\frac{\partial}{\partial s_2} \left\{ \frac{(1 + s_1 \beta + s_2 \beta + s_1 s_2 \beta^2 (1 - \rho))^{-k}}{k \beta (1 + s_1 \beta (1 - \rho))} \right\} (1 + s_1 \beta + s_2 \beta + s_1 s_2 \beta^2 (1 - \rho))^{k+1}.
\]

The second expectation in \( (10) \) can be approximated as

\[
\mathbb{E}_1 \{ I_2 e^{-s_2 I_2} \} \approx k \beta (1 + s_2 \beta)^{-k-1}.
\]

After substituting \( (11) \) and \( (12) \) into \( (10) \), cancelling out some terms and carrying out the integration with respect to \( w \) for \( w > \theta \), we end up with

\[
P\{ \text{SIR} \geq \theta \} = P_r \gamma (P_r + \theta \beta)^{-k} + k \beta P_r^{2k} \times \\
\int_{\theta}^{\infty} (1 + \beta (\theta - w)^{-1} (1 - \rho)) dw.
\]

The above integral can be expressed in terms of \( \gamma F_1 \).

**Lemma 2.** For i.i.d. Rayleigh fading channels, the spatial correlation coefficient of interference \( \rho \) between the two antennas can be approximated as \( \rho \approx \frac{1}{2} (1 - \lambda c) \). The approximation is valid for \( \lambda c \rightarrow 0 \) and \( \frac{1}{r_0} \rightarrow 0 \).

**Proof.** Under the assumption of i.i.d. Rayleigh fading at the two antennas, the covariance of interference is

\[
\text{cov}\{ I \} = \mathbb{E}\{ h^2 \} \mathbb{E}\{ \sum_{x \in \Phi} g(x) g(y) \} + \mathbb{E}\{ h^2 \} \mathbb{E}\{ \sum_{x, y \in \Phi} g(x) g(y) \} - \mathbb{E}\{ I \}^2
\]

\[
= 2 \lambda r_0^{2 - 2\eta} + \int g(x) g(y) \rho (x, y) dxdy - \mathbb{E}\{ I \}^2.
\]

The variance of interference is \( \mathbb{E}\{ I \} = 2 \lambda r_0^{2 - 2\eta} + \int g(x) g(y) \rho (x, y) dxdy - \mathbb{E}\{ I \}^2 \).

The integral \( S = \int g(x) g(y) \rho (x, y) dxdy \) has been approximated in \( (25) \) Section VI for \( \lambda c \rightarrow 0 \) and \( \frac{1}{r_0} \rightarrow 0 \).

The first two dominant terms with respect to \( r_0 \) are

\[
S \approx \frac{4 \lambda^2 \lambda_0^{2 - 2\eta}}{(\eta - 1)^2} - \frac{4 \lambda^2 \lambda_0^{1 - 2\eta}}{2\eta - 1} + \frac{2 \lambda \lambda_0^{1 - 2\eta}}{2\eta - 1}.
\]

After substituting the above approximation for \( S \) in the expressions of the covariance and the variance, doing some factorization and cancelling out common terms, the correlation coefficient can be approximated as

\[
\rho = \frac{\text{cov}\{ I \}}{V\{ I \}} \approx \frac{(1 - \lambda c)^2}{2 - 2\lambda c + \lambda^2 c^2} \approx \frac{1}{2} (1 - \lambda c),
\]

and the Lemma is proved.

In Fig. 8 we depict the outage probability with dual-branch MRC. The performance prediction of PPP worsens in comparison with single-antenna receiver, and it is expected to deteriorate with more antennas and temporal performance metrics, e.g., mean local delay, in networks with low mobility. Even though the PPP still gives very accurate predictions for the outage probability with dual-antenna MRC in the lower tail, these predictions are not necessarily a bound to the outage probability due to the hardcore process. This is because the lower correlation of interference associated with the hardcore process, see Lemma 2, enhances the performance in comparison with the PPP. This is not visible in Fig. 8 because only two antennas are employed, but preliminary simulations with 8-antenna MRC indicated that the hardcore process achieves lower outage than PPP in the lower tail. Overall the use of PPP becomes limited with MRC. Despite the approximations involved in the derivation of \( (13) \), it fits quite well the simulations and can be used to get a quite good performance estimate with low computational complexity.

**VII. Conclusions**

The PPP model for vehicular networks allows small inter-vehicle distances with high probability. This is unrealistic in roads with few number of lanes. A more realistic point process, of equal intensity but with a hardcore distance
(shifted-exponential inter-arrivals), changes the properties of interference distribution and increases the outage probability for a link at the origin. The discrepancy in the outage probability predicted by the two models is clear when the coefficient of variation and the skewness of interference are high, e.g., in urban street microcells and motorway macrocells with sparse flows of vehicles. The discrepancy increases if we consider multiple antennas at the receiver because in that case, the spatial correlation of interference, which is different under the two deployment models, starts also to have an effect. Temporal performance indicators associated with the performance of retransmission schemes are affected by the correlation properties of interference too. The PPP may largely overestimate the mean delay for the link at the origin under low mobility of interferers. With high mobility, the temporal correlation of interference under both models vanishes, and the PPP underestimates the mean delay. Even if the PGFL of the hardcore point process is not available, a gamma approximation for the interference distribution gives better performance predictions than the PPP in all cases. In this paper, we assumed that the point process impacts the distribution of interferers while the transmitter-receiver link is fixed and known. It would be interesting to use a random link distance, and investigate whether the horizontal deployment gain for non-Poissonian point processes holds, see [23].

**APPENDIX**

The third moment of interference accepts contributions from a single user, from user pairs and also from triples of users.

\[
\mathbb{E}\{ I^3 \} = \mathbb{E}\{ h^3 \} \lambda \int g^3(x) \, dx + 3 \mathbb{E}\{ h^2 \} \int g^2(x) g(y) \rho^{(2)}(x, y) \, dx \, dy + \int g(x) g(y) g(z) \rho^{(3)}(x, y, z) \, dx \, dy \, dz \\
= 6 \lambda \int g^3(x) \, dx + 6 \int g^2(x) g(y) \rho^{(2)}(x, y) \, dx \, dy + \int g(x) g(y) g(z) \rho^{(3)}(x, y, z) \, dx \, dy \, dz \\
= \frac{12 \lambda r_0^{1-\eta}}{\eta - 1} + 6 \int g^2(x) g(y) \rho^{(2)}(x, y) \, dx \, dy + \int g(x) g(y) g(z) \rho^{(3)}(x, y, z) \, dx \, dy \, dz, \tag{A.1}
\]

where \( \mathbb{E}\{ h^3 \} = 6 \) and \( \mathbb{E}\{ h^2 \} = 2 \) for an exponential RV, and we have scaled the second term by three to count the ways to select a user pair out of a triple of users.

The contributions to the third moment from triples of users involve the third-order correlation in \([\ref{corr}]\). We will apply in both PCFs the approximation adopted in \([25]\). Similar to \([25]\), we will also assume that the guard zone is much larger than the tracking distance, \( r_0 \gg c \). In our approximations, we will keep up to the second order terms, and also the dominant \( r_0 \) terms with exponents larger or equal to \( 1 - 3\eta \).

Using the approximation for the PCF beyond 2\( c \), the term \( S' = \int x^{-2\eta} y^{-\eta} \rho^{(2)}(x, y) \, dx \, dy \) can be read as

\[
S' \approx 2\lambda \mu \int \int_{x+c}^\infty \int_{y+c}^\infty \frac{e^{\mu(y-x-c)}}{x^{2\eta} y^{\eta}} \, dx \, dy + 2\lambda \mu \int \int_{x+c}^\infty \int x^{-2\eta} y^{-\eta} \, dx \, dy + 2\lambda^2 \int \int_{x+c}^\infty \int_{y+c}^\infty \frac{e^{\mu(y-x-c)}}{x^{2\eta} y^{\eta}} \, dx \, dy + 2\lambda^2 \int \int_{x+c}^\infty \int x^{-2\eta} y^{-\eta} \, dx \, dy, \tag{A.2}
\]

where the factor 2 in front of the integrals accounts for \( x < -r_0 \).

The contribution to \( S' \) due to pairs at distances larger than 2\( c \) is the last two lines of \((A.2)\).

\[
S'_{>2c} = 2\lambda^2 \int \int_{x}^\infty \int_{y}^\infty x^{-2\eta} y^{-\eta} \, dx \, dy + 2\lambda^2 \int \int_{x+c}^\infty \int_{y+c}^\infty x^{-2\eta} y^{-\eta} \, dx \, dy.
\]

The first and the last term in the above expression are not equal due to asymmetry in the exponents of \( x \) and \( y \). After integrating and adding up the three terms we end up with

\[
S'_{>2c} = 2\lambda^2 \left( \frac{r_0^{2-3\eta} + r_0^{1-\eta}(2c + r_0)^{1-2\eta}}{(2\eta - 1)(\eta - 1)} \right) + 2\lambda^2 \frac{r_0^{2-3\eta}}{(3\eta - 2)(\eta - 1)} 2F_1 \left( \frac{3\eta - 2, \eta - 1, 3\eta - 1, \frac{2c}{r_0} }{2} \right) - 2F_1 \left( \frac{3\eta - 2, 2\eta, 3\eta - 1, -2c}{r_0} \right) - \frac{8\lambda^2 r_0^{2-3\eta}}{(2\eta - 1)(\eta - 1) - 3\eta - 1},
\]

where \((a)\) follows from expanding \( b = \frac{x}{r_0} \to 0 \).

The contribution to \( S' \) due to pairs of vehicles at distances \( |y - x| \) smaller than 2\( c \) is larger for \( y > x \) (there are no vehicles to filter out inside the cell in that case) than it is for \( y < x \). Nevertheless, for a small \( r_0 \), the two integrals in the first line of \((A.2)\) should be approximately equal. By making this assumption, we can avoid the approximation of the integral for \( y < x \), which is a little more tedious because for \( x \in (r_0, r_0 + 2c) \) we have to exclude the vehicles inside the cell. Finally, we can approximate the term \( S'_{<2c} \) as

\[
S'_{<2c} \approx 4\lambda \mu \int \int_{x+c}^\infty \int x^{-2\eta} y^{-\eta} \frac{e^{\mu(y-x-c)}}{x^{2\eta} y^{\eta}} \, dx \, dy. \tag{A.3}
\]

After integrating with respect to \( y \) we get

\[
S'_{<2c} \approx 4\lambda \mu^\eta \int_{r_0}^\infty x^{-2\eta} e^{\mu(x+c)} \left( \Gamma(1-\eta, \mu (c+x)) - \Gamma(1-\eta, \mu (2c+x)) \right) \, dx.
\]

In order to approximate the above integral, we first expand the integrand for \( \mu (c+x) \to \infty \). Due to the fact that \( \mu \geq \lambda \) and
where the expansion is valid for \( \lambda r_0 \gg 1 \), which is associated with a large number (on average) of vehicles inside the cell.

\[
S'_{2c} \approx \frac{4 \lambda e^{-\mu} r_0^{-3n} (\eta - c \nu + c \mu + e^\mu (\mu - \eta))}{3 \eta \mu} \times \frac{2 F_1 \left( 3 n, \eta + 1, 3 \eta + 1, -\frac{e}{r_0} \right)}{2 F_1 \left( 3 n - 1, \eta + 1, 3 \eta, -\frac{e}{r_0} \right)}.
\]

After substituting \( \mu = \frac{1}{\lambda \nu c} \), the above expression can be further approximated for \( \lambda c \to 0 \) and \( \frac{e}{r_0} \to 0 \). Keeping only the dominant term with respect to \( r_0 \), we end up with

\[
S'_{2c} \approx \frac{4 \lambda^2 c r_0^{-3n}}{3 \eta - 1} + \frac{2 \lambda^2 c r_0^{-3n}}{3 \eta - 1}.
\]

After summing up \( S'_{2c} \) with \( S'_{2c} \) and scaling the result by six, see equation (A.1), we have

\[
6 S' \approx \frac{24 \lambda^2 c r_0^{-3n}}{(2 \eta - 1) (\eta - 1)} + \frac{2 \lambda^2 c r_0^{-3n}}{3 \eta - 1} + \frac{12 \lambda^3 c^2 r_0^{-3n}}{3 \eta - 1}.
\]

The term \( S''_{11} \) can be approximated as follows

\[
S''_{11} = 12 \lambda^2 \mu \int_{r_0}^{x + 2c} \int_{y + 2c}^{\infty} \int_{y + 2c}^{\infty} x^{-\eta} y^{-\eta} e^w \mu^{-1} \left( \Gamma(1 - \eta, w) - \Gamma(1 - \eta, w + c \nu) \right) dy dz dx
\]

\[
\approx 12 \lambda^2 \mu \int_{r_0}^{x + 2c} \left( (n - c \nu + c \mu + e^\mu (\mu - \eta)) \right)^2 F_1 \left( 2 \eta, 1 + n, 2 \eta, 1, \frac{2 \mu}{2 x + z} \right)
\]

\[
+ \left( 1 - e^{-\mu} \right) (x + 2c)^{2n} 2 F_1 \left( \eta + 1, 2 \eta, 1, 2 \eta, \frac{-c}{x + 2c} \right)
\]

\[
\times \frac{2 \lambda^2 c r_0^{-3n}}{(2 \eta - 1) (3 \eta - 2)}
\]

where \( w = \mu (c + y) \), (a) follows from expanding at \( w \to \infty \) before integrating in terms of \( y \), and (b) from expanding around \( \frac{e}{r_0} \to 0 \) before integrating in terms of \( x \), then substituting \( \mu = \frac{1}{\lambda \nu c} \) and expanding at \( \lambda c \to 0 \).

The term \( S''_{12} \) does not involve any exponential but still, it cannot be expressed in semi-closed form, unless approximations are made in the integrand.

\[
S''_{12} = 12 \lambda^3 c^2 r_0^{-3n} \int_{r_0}^{x + 2c} \int_{y + 2c}^{\infty} x^{-\eta} y^{-\eta} z^{-\eta} (2 \eta + 2c) 2 F_1 \left( 2 \eta - 2 \eta, 2 \eta, 1, \frac{-2c}{2c + x} \right)
\]

\[
\times \frac{2 \lambda^2 c r_0^{-3n}}{(2 \eta - 1) (\eta - 1)}
\]

where the approximation is due to expansion \( \frac{e}{r} \to 0 \) before integrating.

The other two terms of \( S''_{1} \) can be approximated similarly,

\[
S''_{13} \approx \frac{12 \lambda^2 c^2 r_0^{-3n}}{3 \eta - 1}
\]

\[
S''_{14} \approx \frac{12 \lambda^2 c^2 r_0^{-3n}}{3 \eta - 1} - \frac{12 \lambda^2 c^2 r_0^{-3n}}{3 \eta - 1}.
\]

After adding up the approximations for the four terms consisting \( S''_{1} \), we end up with

\[
S''_{1} \approx \frac{2 \lambda^2 c r_0^{-3n} (9 \eta - 7)}{(\eta - 1)^3} - \frac{12 \lambda^3 c^2 r_0^{-3n}}{(\eta - 1) (2 \eta - 1)} + \frac{96 \lambda^3 c^2 r_0^{-3n}}{(\eta - 1)^3}.
\]

Assuming \( x < y < z \) and the user \( x \) at the opposite side of the cell as compared to the users \( y, z \), the term \( S''_{2} \) is

\[
S''_{2} \approx \lambda c \int_{r_0}^{x + 2c} \int_{x + 2c}^{\infty} y^{-\eta} z^{-\eta} dy dz dx
\]

\[
\times \frac{2 \lambda^2 c r_0^{-3n} (9 \eta - 1)}{(\eta - 1)^3} - \frac{12 \lambda^3 c^2 r_0^{-3n}}{(\eta - 1) (2 \eta - 1)} + \frac{96 \lambda^3 c^2 r_0^{-3n}}{(\eta - 1)^3}.
\]

where the factor 12 = 6 × 2 is due to six different orderings of the cell and the scaling by two is used to describe the case where the sides, with respect to the cell, of the user \( x \) and of the pair \( \{y, z\} \) are inter-changed.

The two integrals inside the parenthesis can be approximated similarly to the term \( S''_{1} \). After integrating the first, and
approximating the second for $\mu (x+c) \rightarrow \infty$ and $\lambda c \rightarrow 0$ we have

$$S'_2 \approx \frac{12\lambda r_0^{1-\eta}}{\eta-1} \left( \frac{\lambda^2 r_0^{1-2\eta}}{2(\eta-1)} \left( r_0^2 F_1(2(\eta-1), \eta, 2\eta-1, -2b) + \frac{4c_2 F_1(2\eta-1, \eta, 2\eta -2b)}{2(\eta-1)} + \frac{\lambda^2 c r_0^{1-2\eta} F_1(\eta-1, 2\eta-1, -2\eta + b)}{2(\eta-1)} \right) \right).$$

Expanding the above expression for small $b = \frac{c}{r_0}$ yields

$$S'_2' \approx \frac{6\lambda r_0^{3-3\eta}}{(\eta-1)^3} - \frac{12\lambda c r_0^{2-3\eta}}{(2\eta-1)(\eta-1)}.$$  \hfill (A.6)

Now, we can express the term $S''$ equal to the sum of $\left(A.3\right)$ and $\left(A.6\right)$

$$S'' \approx \frac{8\lambda r_0^{3-3\eta}}{(\eta-1)^3} - \frac{24\lambda^2 r_0^{2-3\eta}}{(2\eta-1)(\eta-1)} + \frac{96\lambda^3 c r_0^{1-3\eta}}{3(\eta-1)^3}.$$  \hfill (A.7)

Substituting $\left(A.4\right)$ and $\left(A.7\right)$ into $\left(A.1\right)$, and doing some rearrangement allows us to approximate the third moment of interference as

$$E\{I^3\} \approx \frac{12\lambda r_0^{1-3\eta}}{3\eta - 1} \left( 1 - 2\lambda c + 9\lambda^2 c^2 \right) - \frac{24\lambda^2 r_0^{2-3\eta}}{(2\eta - 1)(\eta - 1)}.$$  \hfill (A.8)

Using the approximation for the variance in $\left(A.5\right)$ and the third central moment, $E\{I^3\} = E\{(I - E\{I\})^3\}$, becomes

$$E\{I^3\} \approx \frac{12\lambda r_0^{1-3\eta}}{3\eta - 1} \left( 1 - 2\lambda c + 9\lambda^2 c^2 \right) - \frac{12\lambda^4 c^2 r_0^{2-3\eta}}{(2\eta - 1)(\eta - 1)}.$$  \hfill (A.9)

Using the approximations for the third central moment above and for the variance in $\left(A.5\right)$, the skewness is

$$S\{I\} \approx \left( \frac{12\lambda r_0^{1-3\eta}}{3\eta - 1} \left( 1 - 2\lambda c + 9\lambda^2 c^2 \right) - \frac{12\lambda^4 c^2 r_0^{2-3\eta}}{(2\eta - 1)(\eta - 1)} \right) \times \left( \frac{4\lambda r_0^{1-2\eta}}{2\eta - 1} \left( 1 - \lambda c + \frac{1}{2} \lambda^2 c^2 \right) \right)^{-\frac{3}{2}}.$$

Expanding the above expression for $\lambda c \rightarrow 0$ yields

$$S\{I\} \approx \frac{12\lambda r_0^{1-3\eta}}{3\eta - 1} \left( \frac{4\lambda r_0^{1-2\eta}}{2\eta - 1} \right)^{-\frac{3}{2}} \left( 1 - \frac{\lambda c}{2} \right).$$  \hfill (A.8)

where $\left( 1 - \frac{\lambda c}{2} \right)$ is the correction as compared to the skewness of interference due to a PPP of intensity $\lambda$.
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