Analysis on Static Performance of Flexible Bearing in Harmonic Reducer

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Abstract. The harmonic reducer has been widely used in military, industrial and agricultural fields because of its large transmission ratio, high precision, small backlash, stable transmission and small volume and weight. Flexible bearing is a key component of harmonic reducer, and its static mechanical properties are one of the main factors influencing the fatigue life and load carrying capacity of harmonic reducer. Mathematical model of contact force between rolling element and outer ring is established basing on three bending moment equations. This paper takes the CSF-40-80 flexible bearing with double eccentric arc cam as the research object, the finite element methods (FEM) analysis is used to study the deformation and stress of the flexible bearing under no-load and load. The effect of the double eccentric arc cam eccentricity on the stress is also presented, which provides theoretical support for the optimization of harmonic reducers and the manufacture of high-performance harmonic drive systems.

1. Introduction
The harmonic reducer was proposed by American scholar Musser [1] who based on the thin-shell elastic deformation theory in 1953. Harmonic drive system often works as a reducer; it consists of three components which are a wave generator, a non-rigid flexspline and a rigid circular spline. Since its inception, harmonic gear transmissions have been widely used in various fields of military, industrial and agricultural, because of their high precision, compactness, light weight and high reduction ratio [2, 3]. KIKUCHI et al calculated the stress on flexible bearing and flexspline, but did not consider the impact of the loads on the calculation results.[4] Bo Shuxin chose the better boundary conditions as a certain engagement range, but he did not analyses the stress state of flexspline with different loads [5]. However, few studies on the mechanical properties of flexible bearings have been analysed. There is a certain gap between the assumptions and the actual conditions.

In this paper, Mathematical model of contact force between rolling element and outer ring is established basing on three bending moment equations and the contact force of CSF-40-80 flexible bearing is obtained. The deformation and stress of the flexible bearing at non-load and load conditions are studied and analysed by the finite element methods analysis.
2. Theoretical model of flexible bearing

2.1. Load Analysis of outer ring
The load of the wave generator (including the flexible bearing) is related to the flexspline, which can be summarized from the working principle of the harmonic reducer [7]. The meshing force between flexspline and circular spline can be obtained from the experimental results [3.6]. Figure 1 shows the meshing force distribution on the flexspline.

\[ q_{th} = q_{th,\text{max}} \cos\left[\pi (\varphi - \varphi_1) / 2 \varphi_2\right] \]

\[ q_{rh} = q_{th} \tan \alpha \]

(1)

Where \( q_{th} \) is tangential component of meshing force; \( q_{rh} \) is radial component of the meshing force; \( \alpha \) is tooth engagement angle; \( \varphi_1 \) is angle between the symmetry axis CC' of the meshing range and the major axis AA' of wave generator; \( \varphi_2 \) is angles standing for the left meshing range; \( \varphi_3 \) is angles standing for the right meshing range.

Flexible bearing can only withstand the positive pressure from the flexspline. With the one-way contact condition, the load of the outer ring can be written as:

\[ q_r = q_{rh} = q_{th,\text{max}} \cos\left[\pi (\varphi - \varphi_1) / 2 \varphi_2\right] \tan \alpha \]

(2)

The effect of eccentric load is not considered due to the small load deflection angle. So \( \varphi_1 = 0 \) and \( \varphi_2 = \varphi_3 \). Therefore, the applied load of the outer ring can be written as:

\[ q_r = q_{rh,\text{max}} \cos\left(\frac{\pi \varphi}{\varphi_2}\right) \]

(3)

2.2. Load analysis of rolling element
The outer ring of the bearing is supported on a series of continuous rolling element, which can be regarded as a multi-span beam that continuously crosses over a series of supports. Three bending moment equations can be used to solve the problem. As shown in Figure 2. After the constraint
decomposition is completed, the equation [8] can be established for the bending moments $M_{n-1}$, $M_n$, $M_{n+1}$ of any three adjacent support points inside the bearing.

\[
M_{n-l_n} + 2M_n(l_n + l_{n+1}) + M_{n+1}l_{n+1} = -\frac{6\omega_n a_n}{l_n} - \frac{6\omega_{n+1} b_{n+1}}{l_{n+1}}
\]  

Where $l_n$, $l_{n+1}$ are Spans between adjacent support joints; $\omega_n$, $\omega_{n+1}$ are the area of the bending moment diagram under the load $q_n(\theta)$, $q_{n+1}(\theta)$; $a_n$, $b_{n+1}$ are the distances between the centroid of the bending moment diagram and the left and right ends.

From the above equation, the bending moment of each joint can be obtained. Each span can be regarded as a static structure [4]. Therefore, the contact force for each support joint can be obtained by static equilibrium conditions:

\[
M_{q_n} - M_n = M_{Q_n} \tag{5}
\]

\[
M_{q_{n+1}} - M_{n+1} = M_{Q_{n+1}} \tag{6}
\]

Where $M_{q_{n+1}}$ is the bending moment of the right end of the load $q_{(n+1)}(\theta)$ to the first $n+1$ internal loads; $M_{q_{n+1}}$ is the bending moment of the left end of the load $q_{(n+1)}(\theta)$ to the first $n+1$ internal loads; $M_{Q_n}$ is the bending moment of the left end of the support force $Q_n$ in the first $n$ span to the right end; $M_{Q_{n+1}}$ is the bending moment of the right end of the support force in the $Q_{n+1}$ first $n+1$ span to the left end.

Thus, the support forces at the left and right ends of each span can be written as:

\[
Q_{n} = M_{Q_n} / l_{n+1} \tag{7}
\]

\[
Q_{n+1} = M_{Q_{n+1}} / l_{n+1} \tag{8}
\]

The contact force of all the support joints can be obtained by the superposition of the two supporting forces of the adjacent span in the same joint.

\[
Q_n = Q_{n-1} + Q_{n+1} \tag{9}
\]
3. Exemplary calculation of flexible bearing

CSF-40-80 flexible bearing with double eccentric arc cam is studied and the contact force of the rolling element and the outer ring is calculated. Table 1 shows the main parameters of the research bearing.

| Parameter                                | Value   | Parameter                                | Value   |
|------------------------------------------|---------|------------------------------------------|---------|
| Inner ring width \( b_i / \text{mm} \)    | 12      | Outer ring width \( b_e / \text{mm} \) | 12      |
| Inner ring thickness \( t_i / \text{mm} \) | 2.5     | Inner ring thickness \( t_e / \text{mm} \) | 2.45    |
| Inner diameter \( d / \text{mm} \)        | 71.2    | Outer diameter \( D / \text{mm} \)       | 98.2    |
| Inner curvature coefficient \( f_i \)     | 0.511   | Outer curvature coefficient \( f_o \)    | 0.534   |
| Eccentricity of cam \( e / \text{mm} \)  | 72      | Radius of cam arc \( r_h \)            | 34      |
| Diameter of rolling element \( D_w / \text{mm} \) | 9.625 | Number of rolling element \( Z \)   | 23      |

The contact force of rolling element in CSF-40-80 flexible bearing is obtained from the above equations, and the results in shown in Figure.3.

Figure 3. Contact force distribution of rolling element

It can be seen from Figure.3 that only 11 rolling elements actually load among the 23 rolling elements. The maximum contact force of the rolling element occurs near the long axis of the wave generator. The load angle of the half circle above the short axis is approximately 62 degrees and the half circle below is 78 degrees.

4. Finite Element Model

An overall FEM model is established due to the number of rolling elements of the CSF-40-80 flexible bearing is odd. Wave generator cam in this paper is a double eccentric arc cam. To simplify the model, the chamfering of the cage and the inner and outer rings is neglected and initial geometric gap between the inner and outer raceway and the rolling elements is assumed to be zero.

The boundary condition is that the effect of load eccentricity is neglected, and the loading set two steps, each step is 1s. The external load imposed by the finite element methods model of the flexible bearing is

\[
q = \begin{cases} 
0 & t = 0 \sim 1s \\
\frac{q_{\text{max}} \cos(2\varphi)}{q_{\text{max}} \cos(2\varphi)} \cdot (t - 1) & t = 1 \sim 2s 
\end{cases}
\] (10)
Where $\varphi_2 = 45^\circ$; $q_{r_{max}}=1.8\text{MPa}$; $\varphi_2$ is position angel

Constraint of the cage on rolling element is simulated by constraining the circumferential freedom of rolling element. The axial freedom of the inner and outer rings is constrained and the centre line of the camshaft is fixed. Fully hexahedral meshing of flexible bearing is applied and the finite element model is shown in Figure 4.

![Figure 4. FEM model of flexible bearing](image)

Contact stiffness is the most important parameter influencing the results. For surface-surface contact element model, it is determined by the NSF (Normal Stiffness Factor), and NSF must not exceed 2.0. Assuming NSF equals to 1.6, the finite element results and theoretical results of the maximum contact force of outer ring and rolling element with different loads are shown in Figure. 5. The error between them is within 8%, so the contact stiffness is set to 1.6.

![Figure 5. Maximum contact force comparison](image)

5. Results and discussion
Total deformation and radial deformation nephogram of the flexible bearing without load are shown in Figure. 6.
It can be seen from Figure 6 that the maximum total deformation and radial deformation are both 0.4 mm. The minimum radial deformation is -0.38 mm, which indicates the flexible bearing has a large pre-deformation when the assembly is completed.

The deformation of inner ring raceway and outer ring raceway at load and no-load condition is shown in Figure 7.

Compared with non-load condition the deformation curve of the inner ring are almost coincident while the deformation curve of the outer ring has obvious deviations. The maximum radial and circumference deformation of the outer ring are both slightly reduced with load. Radial deformation can be fitted to $\omega \approx 0.4 \cos(2\psi)$, and circumferential deformation can be fitted to $\nu \approx 0.2 \sin(2\psi)$, the simulation results by Ansys is consistent with the thin-wall ring theory [4]. $\omega = d\nu / d\psi$

The equivalent stress nephogram of the flexible bearing at load and no-load condition is shown in Figure 8.
It can be seen from Figure 8 that the maximum equivalent stress at no load is 255.4 MPa, which indicates that the flexible bearing has a large equivalent stress already before loading. The maximum equivalent stress with load is 503.9 MPa. The increase of 97.3% indicates that the applied load has a great influence on the equivalent stress of the flexible bearing.

The stress curve of the inner and outer ring raceway before and after loading is shown in Figure 9. The stress characteristics of flexible bearing with or without load can be seen from Figure 9. The stress of the inner ring raceway hardly changes except at some crest positions while the equivalent stress of the outer ring raceway drops in most places especially near the short axis. Crest of the stress turn out to be larger when loaded while the positions and the number of the crests remain unchanged, which means the number and positions of the rolling elements actually loaded have not changed. The maximum equivalent stress of the flexible bearing occurs at the contact area of the inner ring raceway and the rolling element at the long axis of the cam.

6. Conclusion
In this paper, mathematical model of contact force between rolling element and outer ring is established basing on three bending moment equations and the contact force of CSF-40-80 flexible bearing is obtained. The static performance of the flexible bearing is analysed by FEM analysis and the following conclusions were drawn.
(1) The number of rolling elements actually loaded of the flexible bearing is 11. The maximum contact force occurs near the long axis of the cam while the contact force turns out to be zero near the short axis. The actual loaded range of the flexible bearing is smaller than the load range applied.

(2) The absolute values of the maximum radial deformation and the minimum radial deformation are not equal at no-load condition. The maximum equivalent stress of the flexible bearing increases remarkably after the load, and the maximum equivalent stress occur at the contact area between the inner ring raceway and the rolling element.

(3) The flexible bearing has a large equivalent stress already before loading the applied load has a great influence on the equivalent stress of the flexible bearing while the number and position of the actual rolling elements before and after loading are unchanged.

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